Variants of the Sliding Mode Control in Presence of External Disturbance for Quadrotor

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ABSTRACT In this paper, an Improved Integral Power Rate Exponential Reaching Law (IIPRERL) Sliding Mode Control strategy has been presented to cater to the chattering problem and stability issue with the focusing on to achieve minimum steady-state errors in the presence of matched disturbances for a quadrotor. Control strategies have been implemented on quadrotor which is 6 Degree of Freedom (DOF) underactuated model and it is derived via Newton-Euler(NE) equations. The main focus of this article is to design two flight control strategies for a quadrotor. Firstly, a novel IIPRERL-SMC is designed through Improved Integral Sliding Mode Control (II-SMC) via proper gain scheduling by system Eigenvalues. The control objective is to construct a controller such that would force the state trajectories to approach the sliding surface with an exponential policy. Meanwhile, a strong condition for reaching the law of sliding surface via Hurwitz stability has been introduced to the hovering of the quadrotor. Secondly, kinodynamic Rapidly Exploring Random Tree with Fixed Node (RRT*FN) which is an incremental sampling-based optimal algorithm has been implemented for online navigation and flight control of the quadrotor system. Online planning which is based on the offline one, is given on-board radar readings which gradually produces a smooth 3-D trajectory aiming at reaching a predetermined target in an unknown environment. The performance and stability of the quadrotor are completely examined by utilizing Lyapunov stability analysis. Simulations are presented to verify that the proposed scheme is effective with Hurwitz stability for both translational and rotational parts of the quadrotor.

INDEX TERMS Artificial intelligence, Improved Integral Sliding Mode Control, Lyapunov’s controller, Rapidly growing Random Tree, Vertical Take-Off Landing (VTOL).

I. INTRODUCTION
The importance of unmanned air vehicles (UAV)s, especially in control system engineering has increased manifold in recent years. Quadrotor optimization is an interesting area due to its significance in flight control to achieve multiple advantages [1]. The concept of quadrotor was first given by Breguet-Riched in 1907, since then it has been extensively considered in many areas i.e, in military and civil tasks it has remarkable significance due to its small size and ability to handle heavy payload [2]. Its applications in surveillance, search and rescue operations, navigation, aerial photography, monitoring and inspection along with accurate positioning with altitude control have more importance 3.

Control of quadrotor is a challenging task since it is an underactuated system and shows a complex degree of freedom. Research contributions including both linear and nonlinear control techniques in order to control of quadrotors are extensively available. Bouabdallah et.al have used the PID controller to control the quadrotor in the presence of minimal disturbance [3], which only deals with little disturbance. PID control via pole placement has been used in [4] for the low-speed propellers to control the inner loop (position) and outer loop (altitude). In another variant of the PID controller, Hongping Liu used the PID-PD controller (a combination of PD and PID controller) while using online gain regulation [5] for the closed-loop system design.

Noise removal and attitude control of quadrotors were achieved through bilinear modeling in [6], while PID and Model Reference Adaptive Control (MRAC) has been...
proposed in [7] to cater for adaptive noise. A non-linear PID controller was proposed by Aws Abdulsalam, to ensure Hurwitz stability in [8]. The technique presented in [9] is based on Linear Parameter Varying parameters with Linear Quadratic Gaussian (LPV-LQG) control. T.L. Wang also proposed an $H^\infty$ and a state feedback controller in the presence of noise and small algorithmic error for the design of a translational position and heading controller [10]. For S0(3) type quadrotor model a robust $H^\infty$ control scheme is suggested by Haibo Wang [11]. However, this method was required for experimental verifications. Combination of Linear Quadratic Estimation (LQE) and Karman Filter (KF) to form Linear Quadratic Gaussian (LQG) have also been proposed with Gaussian noise to cater for incomplete information of the system states [12]. All these above-given techniques work efficiently only under linear model constraints.

Several nonlinear control techniques have been also presented by researchers. Sliding mode control (SMC) has been a popular choice for researchers to control quadrotors. A Lyapunov based SMC was presented by Runcharonn.k in [13]. Trajectory tracking was achieved via robust SMC integrated with Human Machine Interface (HMI) by M.Reinoso in [14]. Payload variation has resulted in making quadrotors unstable and produces disturbance as well especially in the windy scenario. Mohsen et.al suggested an adaptive second fractional SMC technique based on backstepping in [15]. The proposed algorithm aims to attenuate wind disturbances and undesirable fluctuations caused by control angles and positions due to inertia. Recently, the Discrete-time Sliding Mode Control technique (DSMC) has been proposed for attitude and position control after continuous to discrete transformation [16].

To control the outer loop (position loop) the robust adaptive backstepping (AB) control to get the desired Euler-angles and the control laws. The inner loop (attitude loop) employs a new controller based on a combination of the Adaptive Backstepping technique and Fast Terminal Sliding Mode Control (ABFTSMC) to command the yaw angle and the tilting angles has been presented in [17]. In the same way, a new robust nonlinear adaptive control algorithm called Adaptive Nonsingular Fast Terminal Sliding-Mode Control (ANFT-SMC) has been proposed for orientation and tracking [18]. The ANFT-SMC ensures fast convergence, avoids singularities, and can cater chattering effect in the presence of disturbance and unknown effect. LMI-based sliding mode controller with an exponential policy for a class of under-actuated systems has been presented in [19] in which The asymptotic stability conditions on the error dynamical system are expressed in the form of linear matrix inequalities.

Integral backstepping sliding mode controller in the presence of external disturbances has been applied [20]. This algorithm proves to be robust to disturbances and preferable to solve the tracking problems. M.A-Vallejo, used feedback linearization (FL) and backstepping for robust design and achieving asymptotic stability [21]. To deal with disturbance parameters and optimal tracking problem, a robust method called Gaussian information fusion control (GIFC) has been proposed [22]. To control the underactuated systems, the study becoming a hot spots area. Numerous researchers are working with different methods such as closed-loop, introducing slack variable, and auxiliary coefficients to control state constraints [23]. They are designed in such a way that coupled between un-actuated and actuated states. By introducing such kind of auxiliary parameters, issues like output feedback and input saturation remain unchanged and should be addressed. Active disturbance rejection control based on flatness for robust tracking control has been implemented. Firstly, after linearization, an extended state observer is introduced to estimate the unknown state by neglecting nonlinear terms [24]. Recently a trend, that double-loop control technique has been developed for two subsystems of quadrotors. In this way, a robust backstepping SMC controller for the inner loop (attitude control) and integral sliding mode control for the outer loop of the quadrotor have been produced [25]. For multiple quadrotors, a distributed robust formation control technique has been developed with a backstepping approach. In this way, the position controller for translational control and attitude controller for the rotational controller. The study has been verified through simulations [26]. In the stability of the chaotic system, in the presence of external disturbance, a novel finite time compositive nonlinear feedback control technique developed. Ensuring that convergence of errors to zero base-on Lyapunov’s stability criteria [27]. Chattering is produced across switching surfaces when dynamics are forced to on it. For time-varying third-order systems and cater to fast terminal sliding mode behavior a disturbance observer control method has been developed [28].

It is noteworthy here, that computational complexities increase with an increase in the DOF. Further, in previously the uses of backstepping are unrealistic for functional frameworks. This is one of the main reasons why such methodologies may not be utilized for an enormous class of under-actuated frameworks where linearization becomes troublesome. However, practical robustness is a must requirement in the control systems. In this respect, SMC based control approaches were engaged in recent years for controlling under actuated frameworks. Many researchers contributed to this field especially in SMC have vital importance [29]–[31].

In the context of path planning, multiple methodologies have been considered to take into account previous knowledge of the quadrotor flight data to generate maps for search area (offline), while for online or real-time coverage, sensor-based approaches have been used [32]. All of the presented control schemes can be considered as offline-control techniques but we need online flight planning techniques for quadrotor for on-spot calibrations also. Based on the offline and online planner may be proposed to give estimates about onboard radar readings that could be essential in generating a smooth 3D trajectory while aiming at reaching a predetermined target in an unknown environment. There is limited data that is presented to cater to both the control and planning of quadrotor at the same time. Our concern is to discover
progressively, productively and increasingly practical solutions for quadrotor UAVs path without making too many presumptions on the trajectories. For this, both the control and planning of quadrotors are presented here.

The term path is planning described as a collision-free path generation with optimal cost for the initial to the goal state. The optimum criteria could be based on one or more situations, including the smoothness, shortest physical distance, low risk, fuel, consumption needs and maximum coverage. As demand for artificial intelligence and robot technology is growing day by day, and it is difficult to use conventional methods to find an optimal path as a dimensional problem increase with the increase in the DOF of systems. Various sampling-based algorithms have been proposed and are classified into many domains based on their achievements [33].

Two basic approaches are vital which are classical and heuristic [34]. In classical based approaches we have cell decomposition, potential field, sampling-based algorithm, sub-goal network, vector field histogram, Voronoi digraph, accessibility graph, Partial Motion Planning, A*, D* and Probabilistic Road Map (PRM) [35]. The heuristic domain can be classified as an Artificial Intelligent (AI) network, Fuzzy based Nature-inspired algorithm, hybrid-based and cognitive approaches. In dealing with AI we have to face a lot of problems such as static, clattered and dynamic environments [36]–[38]. RRT*FN is an incremental sampling-based algorithm that could be utilized for optimal online motion planning of quadrotor [39].

This paper primarily deals with two scenarios. As the first objective, an SMC based control algorithm namely Improved Integral Power Rate Exponential Reaching Law (IIPRERL) is developed with the main focus on resolving the chattering issue without having much influence on the overshoot. IIPRERL ensures that robustness with minimal chattering. Furthermore, for a sliding mode controller, a well-defined and durable sliding surface is introduced which guarantees that force the state trajectories to approach the sliding surface with an exponential policy. An extensive relative investigation of the simulations results are then introduced which considers the tracking performance i.e., overshoots, settling time, sliding mode convergences, chattering reduction with control efforts.

The secondary theme of the paper is to apply the trajectory planning algorithm for the quadrotor system. For online planning, kinodynamic RRT*FN which is an incremental sampling-based algorithm has been presented for optimal motion planning of quadrotor. Main innovations and paper contributions can be summarized as follows:

1) Offline control and online flight planning of quadrotor have been proposed and implemented successfully.

2) Integral SMC (ISMC) and Improved Integral SMC (IISMC) techniques have been implemented to cater steady-state with strong condition for sliding surface in the presence of aerodynamic effect and external disturbances.

3) The proposed IIPRERL-SMC control algorithm effectiveness is validated successfully by simulations to achieve hovering stability of quadrotor via eigenvalues.

4) Kinodynamic RRT*FN which is an incremental sampling-based algorithm has been presented for optimal online motion planning of quadrotor.

The proposed novel algorithm IIPRERL-SMC has numerous advantages upon the well-known algorithms and we can utilize in every system especially UAVs like where we need.

- The small reduction error.
- To order reduction.
- To optimizing control.
- Optimal autonomous navigation planning like the quadrotor System.
- Nonlinear parameterized closed-loop systems and other UAVs.
- Better Performance design Robustness.

The remaining paper has been arranged in the following manner: Section II presents the quadrotor modeling through Newton’s Euler equations. Section III presents a comparison of IIPRERL with multiple control strategies. The simulation results of the modeling through the proposed algorithm has been presented in section IV. The basic description of RRT*FN algorithm and motion planning through this scheme has been implemented for a quadrotor in section V. In section VI, results with online trajectory planning have been verified through simulations. This is followed by a conclusion in the last section.

II. MATHEMATICAL MODELING

The mathematical model of quadrotor formulated through the Newton-Euler method is presented with basic schematic in Fig.1. [40]. It has four propellers with the control system placed at the center. The propellers are powered by
a DC motor. The propellers 1 and 3 are designed to move in an anti-clockwise direction whereas the other two move in the clockwise direction to balance the overall effect of thrust force. The quadrotor model under consideration is a 6 DOF \([\phi, \theta, \psi, x, y, z]\) model. For analysis sake, it is further sub-divided into rotational and translational parts. The rotational dynamics, \((\phi, \theta, \psi)\) represents roll, pitch and yaw, whereas \((x, y, z)\) represents translational dynamics around X, Y and Z axes respectively. Its four inputs and six states make it an under-actuated system that is difficult to control in flight. Angular torque and thrust force and applied for orientation, stabilization and tracking, stabilizing of the quadrotor. Following assumptions are considered for the quadrotor UAVs type vehicles:

- The origin of the body frame coincides with the center of gravity.
- Propellers are considered to be rigid with fixed Pitch.
- The body mass is evenly distributed.
- The angles Roll \((\phi)\), Pitch \((\theta)\), and Yaw \((\psi)\) are bounded in the interval \([-\pi/4,\pi/4]\).

The rotation \(R\) from the body frame to the inertial frame describes the orientation of the quadrotor which is given below:

\[
R = \begin{bmatrix}
 c\theta c\psi & s\phi s\theta c\psi & c\phi s\theta c\psi + s\phi s\psi \\
 c\theta s\psi & s\phi s\theta s\psi + c\phi c\psi & c\phi s\theta s\psi - s\phi c\psi \\
 -s\theta & -s\phi c\theta & c\phi c\theta
\end{bmatrix}
\] (1)

where:

\[
c = \cos, s = \sin
\] (2)

### A. ROTATIONAL DYNAMICS

Rotational equations for Roll, Pitch and Yaw \((\phi, \theta, \psi)\) have been derived from the body inertia frame \((x, y, z)\) with the help of Newton-Euler equations, so the total torque is given as:

\[
I\ddot{\Theta} + w \times I\dot{\Theta} + M_G = \tau
\] (3)

where \(I\) is the diagonal inertia matrix, \(w\) is the angular velocity, \(M_G\) is the gyroscopic moment and \(\tau\) is the total torque. The inertia matrix contains diagonal entries with their product is equal to zero having moments of inertia \(I_{xx}, I_{yy}\) and \(I_{zz}\) and their values is given in table 2.

\[
I = \begin{pmatrix}
 I_{xx} & 0 & 0 \\
 0 & I_{yy} & 0 \\
 0 & 0 & I_{zz}
\end{pmatrix}
\] (4)

From these moments rotation is produced from rotors and creates a force called aerodynamics forces or simply called lift forces. The forces and moments are shown in Fig.2. have been expressed by (5) and (6) as:

\[
F = \frac{1}{2}\rho AC_T r^2 \Omega_j^2 = K_f \Omega_j^2
\]

(5)

\[
M = \frac{1}{2}\rho AC_T r^2 \Omega_j^2 = K_m \Omega_j^2
\]

(6)

### B. TRANSLATIONAL DYNAMICS

Translational equations of motions can be derived by using Newton’s second law according to earth inertial frame as:

\[
\sum \vec{F} \rightarrow m \ddot{\vec{r}} \rightarrow m \ddot{\vec{r}} = \begin{pmatrix}
 0 \\
 0 \\
 mg
\end{pmatrix} + RF_b
\] (8)

where \(r = [x \ y \ z]^T\) is the quadratic distance, \(m\) is the quadrotor mass while \(F_b\) is the force without gravitational. Let the state vector \(X\) and input vector \(U\) can be defined as:

The state vector \(X\) is defined as:

\[
X = \begin{pmatrix}
x_1 \\
x_2 \\
x_3 \\
x_4 \\
x_5 \\
x_6 \\
x_7 \\
x_8 \\
x_9 \\
x_{10} \\
x_{11} \\
x_{12} \\
y \\
z \\
\dot{\phi} \\
\dot{\theta} \\
\dot{\psi} \\
\dot{x} \\
\dot{y} \\
\dot{z}
\end{pmatrix}
\] (9)

where the control input vector \(U = [U_1 \ U_2 \ U_3 \ U_4]^T\) are:

\[
U_1 = K_f(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2)
\]

(10)

\[
U_2 = K_f(-\Omega_1^2 + \Omega_4^2)
\]

\[
U_3 = K_f(\Omega_1^2 - \Omega_3^2)
\]

\[
U_4 = K_f(\Omega_2^2 - \Omega_3^2 - \Omega_4^2)
\]
Substituting equation (10) into (7) through mapping vector (9), we get the total moments acting of quadrotor.

\[ \tau = \begin{pmatrix} IU_2 \\ IU_3 \\ U4 \end{pmatrix} \] (11)

By substituting (5) into (7) and expanding all these to get the rotational sub-system model which is represented in (12) as:

\[ \dot{\phi} = \frac{l}{l_{XX}} \dot{U}_2 - \frac{J_c}{l_{XX}} \dot{\Omega}_e + \frac{l_{YY}}{l_{XX}} \dot{\psi} \dot{\theta} - \frac{l_{YY}}{l_{XX}} \dot{\phi} \dot{\psi} \]
\[ \dot{\theta} = \frac{l}{l_{YY}} \dot{U}_3 - \frac{J_c}{l_{YY}} \dot{\phi} \dot{\Omega}_e + \frac{l_{ZZ}}{l_{YY}} \dot{\phi} \dot{\psi} - \frac{l_{ZZ}}{l_{YY}} \dot{\theta} \dot{\psi} \]
\[ \dot{\psi} = \frac{l}{l_{ZZ}} \dot{U}_4 + \frac{l_{XX}}{l_{ZZ}} \dot{\phi} - \frac{l_{YY}}{l_{ZZ}} \dot{\theta} \dot{\phi} \] (12)

After simplifying
\[ \dot{\phi} = b_1 U_2 - a_2 x_4 \Omega_e + a_1 x_4 x_6 \]
\[ \dot{\theta} = b_2 U_3 - a_4 x_2 \Omega_e + a_3 x_2 x_6 \]
\[ \dot{\psi} = a_5 x_2 x_4 + b_5 U_4 \] (13)

Substituting equation (11) through mapping vector (9) we get the translational equation of motion as;

\[ \ddot{x} = \frac{-U_1}{m} (\sin x_1 \sin x_5 + \cos x_1 \cos x_5 \sin x_3) \]
\[ \ddot{y} = \frac{-U_1}{m} (\cos x_1 \sin x_5 \sin x_3 - \cos x_5 \sin x_3) \]
\[ \ddot{z} = g - \frac{U_1}{m} (\cos x_1 \cos x_3) \] (14)

The basic block diagram has been shown in Fig.3. for both translational and rotational system.

C. STATE SPACE REPRESENTATIONS
Let general state-space equation is

\[ X(t)^{(n)} = f(x, t) + g(x, t)u \]
\[ y = h(x) \] (15)

where \( X(t) \) is the state vector and \( u \) is the control input. The complete model of quadrotor has been represented in (16) by combining (13) and (14).

\[ \dot{x}_1 = \dot{x}_2 \]
\[ \dot{x}_2 = \dot{\phi} = b_1 U_2 - a_2 x_4 \Omega_e + a_1 x_4 x_6 \]
\[ \dot{x}_3 = \dot{\theta} = b_2 U_3 - a_4 x_2 \Omega_e + a_3 x_2 x_6 \]
\[ \dot{x}_4 = \dot{\psi} = a_5 x_2 x_4 + b_5 U_4 \]
\[ \dot{x}_5 = z = \dot{x}_8 \]
\[ \dot{x}_6 = \dot{z} = \dot{g} = \frac{U_1}{m} (\cos x_1 \cos x_3) \]
\[ \dot{x}_7 = y = x_{10} \]
\[ \dot{x}_10 = \dot{x} = \frac{-U_1}{m} (\sin x_1 \sin x_5 + \cos x_1 \cos x_5 \sin x_3) \]
\[ \dot{x}_{11} = y = x_{12} \]
\[ \dot{x}_12 = \dot{y} = \frac{-U_1}{m} (\cos x_1 \sin x_5 \sin x_3 - \cos x_5 \sin x_3) \] (16)

III. QUADROTOR CONTROL
Sliding mode control (SMC) based algorithms are presented in this section to achieve closed-loop tracking. The proposed scheme has been implemented in different phases for control of the quadrotor. First, a PID and SMC based control are discussed to remove the chattering. Then the other control strategies ISMC, IISMC and proposed IIPRERL-SMC have been implemented. A comparison of IIPRERL with ISMC and IISMC has also been implemented.

A. CONTROL WITH PID
The proportional integral and derivative control scheme with the feedback control-loop strategies have been widely considered in all domains especially in the industrial control system, it has vital importance. For hovering of the quadrotor, simple PID controller with a closed-loop system can be designed for Roll, Pitch and Yaw as:

\[ U_2 = K_p (\phi_d - \phi) + K_d (\dot{\phi}_d - \dot{\phi}) + K_i \int (\phi_d - \phi)dt \]
\[ U_3 = K_p (\dot{\theta}_d - \dot{\theta}) + K_d (\ddot{\theta}_d - \ddot{\theta}) + K_i \int (\dot{\theta}_d - \dot{\theta})dt \]
\[ U_4 = K_p (\dot{\psi}_d - \dot{\psi}) + K_d (\ddot{\psi}_d - \ddot{\psi}) + K_i \int (\dot{\psi}_d - \dot{\psi})dt \] (17)

And altitude control

\[ U_1 = K_p (z_d - z) + K_d (\dot{z}_d - \dot{z}) + K_i \int (z_d - z)dt \] (18)

where \( K_p, K_d \) and \( K_i \) are the gain of proportional, derivative and integral gains of PID respectively.

Lemma 1: Let \( \dot{x} = f(x) \rightarrow f(x, u) \) be a dynamic system with \( f(0) = 0. \) Let \( V \) be a Lyapunov function which is stickly positive definite, radially bounded, continuously differentiable with \( V(0) \) and \( \dot{V}(t) < 0. \) Let \( \Omega_e \subset R^n \) is a
compact set of initial conditions for the dynamic system. Also, it can be considered that every trajectory started on \( \Omega_c \) is also bounded by \( \Omega_c \).

**Definition 2:** Simple constant reaching law is consider as weak condition. \( s(t) = -K_i \text{sgn}(s_i) \) where \( K_i > 0 \), is positive constant, here robustness increases and in order increase fast reaching a speed \( K_i \) must be increased. However, chattering occurs. The signum function is defined as,

\[
\text{sgn}(s) = \begin{cases} 
+1 & \text{if } s > 0 \\
0 & \text{if } s = 0 \\
-1 & \text{if } s < 0 
\end{cases}
\]

**Definition 3:** Constant plus proportional reaching law is consider as strong condition. \( s(t) = q_i \text{sgn}(s_i) - K_i \) where \( q_i > 0, K_i > 0 \) are considered as positive constants. This is called strong condition. The addition of the extra term is responsible for the convergence rate. However, the proportional term reduces in the zone of the switching surface.

**Definition 4:** The expression \( \dot{S} = -K_i |S| \alpha \text{sgn}(S_i) - K_2(S_i) \) is power rate exponential reaching law, which is a very useful and vital role in sliding control law. Where (for robustness) and \( \alpha \) are positive constants with \( 0 < \alpha < 1 \), \(|S_i|\alpha \) is responsible for chattering with \( \alpha \) is nicely chosen to create a balance between robustness and chattering.

**Remark 2:** In IIPRERL, gain scheduling with eigenvalues make this case study powerful and efficient which improves the dynamic characteristics of the sliding mode.

**B. QUADROTOR CONTROL WITH SMC**

The sliding mode control [31] is consistently considered as a viable and productive methodology in control frameworks as a result of its invariance in sliding mode i.e., its outcomes in robustness to tackle uncertainties in sliding mode. Designing of the sliding mode controller, two stages are fundamental.

1) **Step 1:** The choice of the sliding surface.

2) **Step 2:** To produce strong control law which should be verified and in this way, state variables are enforced to remain in the sliding surface.

The finite-time convergence and the invariance property of the SMC are the two key advantages have been considered. The conventional SMC for the quadrotor system has been described in this section. The sliding phases of the SMC are shown in Fig.4. and the sliding surface is expressed in (19) while, \( e_1 = \phi - \dot{\phi}, e_3 = \theta - \dot{\theta}, e_5 = \psi - \dot{\psi} \) are the tracking errors, sliding surface is designed as,

\[
S(x_i) = ce_i + \dot{e}_i 
\]

where \( c \) is a positive constant value for control and verification. After this it obtained two control law called switching and equivalent terms:

\[
u = u_{eq} + u_s
\]

where:

\[
u_s = -K_1 \text{sgn}(s_i) - K_2(s_i)
\]

where \((K_2, K_1 > 0)\) are the positive values.
Similarly, the control inputs for Altitude, Pitch and Yaw have been given below.

\[ \dot{x}_1 = x_2 \dot{x}_2 = x_4 x_6 a_1 - x_4 \Omega_r a_2 + b_1 U_2 \] (33)

\[ e_1 = x_1 - x_{1ref} \]

\[ e_2 = \int e_1 dt \]

\[ e_3 = \int \left\{ \int e_1 dt \right\} dt \] (34)

After introducing double integral action (28) the sliding surface is defined as,

\[ s = c_1 e_1 + \dot{e}_1 + c_2 e_2 + \ddot{e}_2 + c_3 e_3 + \dddot{e}_3 \] (35)

\[ \dot{s} = c_1 \dot{e}_1 + \ddot{e}_1 + c_2 \dot{e}_2 + \dddot{e}_2 + c_3 \dddot{e}_3 + \dddddot{e}_3 \] (36)

\[ \therefore \dot{\dddot{e}}_3 = \dot{e}_1 \]

from (16),

\[ \dot{\dddot{e}}_3 = \dot{e}_1 \]

Also, to satisfy the sliding mode condition \( \dot{s} \dot{\dddot{e}} < 0 \), limits has to be set on \( K_1 \) and \( K_2 \) such as \( K_1 > 0 \) and \( K_2 > 0 \).

By equating the proposed reaching law (25) to the derivative of the sliding surface in (37) and substituting \( \dot{\phi} \) by its definition from (16), the control input \( U_2 \) is calculated to be,

\[ U_2 = \frac{1}{b_1} \left[ -(c_1 + 1) (x_2 - \dot{x}_{1ref}) - (c_2 + 1) e_1 - c_3 e_2 - x_4 x_6 a_1 + x_4 \Omega_r a_2 + b_1 U_2 - \dot{x}_{1ref} \right] \] (38)

Similarly, the control inputs for Altitude, Pitch and Yaw have been given in (39, 40 and 41) respectively as,

\[ U_1 = \frac{m}{b_4} \left[ -(c_1 + 1) (x_4 - \dot{x}_{3ref}) - c_3 e_2 - (c_2 + 1) e_1 + \dot{x}_{3ref} - c_3 e_2 - g - K_1 \text{sgn}(s_1) - K_2(s_2) \right] \] (39)

\[ U_3 = \frac{1}{b_2} \left[ -(c_1 + 1) (x_4 - \dot{x}_{3ref}) - (c_2 + 1) e_1 - c_3 e_2 - x_2 x_4 a_3 - x_2 \Omega_r a_4 + \dot{x}_{3ref} - K_1 \text{sgn}(s_1) - K_2(s_2) \right] \] (40)

Since the conventional SMC causes wear and tear of the system components, therefore, one of the main challenges which were solved via the integral sliding mode that steady-state errors in the presence of aerodynamic effects have been removed. Due to the hysteresis of the signum function in the actual control switch, both the sliding mode control and the integral sliding mode control faced chattering when the switching control law switches across the high-frequency domain. Although, the technique is preferable and makes easy tasks for practical implementation. However, one must be noticed that because of no reaching phase the system is no more robust from the initial state.

**C. INTEGRAL SLIDING MODE CONTROLLER**

The basic idea of ISMC is the addition of an integral term to the sliding manifold [41]. This integral term enables the system to start on the sliding manifold at the initial condition, hence eliminating the reaching phase the robustness throughout the entire system is guaranteed. Meanwhile, chattering effect can also be observed which degrades the system performance. Therefore, many researchers have been working on that issue. The ISMC controller has been used for the removal of the steady-state error. To drive control law mathematical modeling has been given below.

\[ \dot{x}_1 = x_2 \dot{x}_2 = x_4 x_6 a_1 - x_4 \Omega_r a_2 + b_1 U_2 \]

\[ e_1 = x_1 - x_{1ref} \]

\[ e_2 = \int e_1 dt \]

\[ e_3 = \int \left\{ \int e_1 dt \right\} dt \] (34)

After introducing double integral action (28) the sliding surface is defined as,

\[ s = c_1 e_1 + \dot{e}_1 + c_2 e_2 + \ddot{e}_2 + c_3 e_3 + \dddot{e}_3 \] (35)

\[ \dot{s} = c_1 \dot{e}_1 + \ddot{e}_1 + c_2 \dot{e}_2 + \dddot{e}_2 + c_3 \dddot{e}_3 + \dddddot{e}_3 \] (36)

\[ \therefore \dot{\dddot{e}}_3 = \dot{e}_1 \]

from (16),

\[ \dot{\dddot{e}}_3 = \dot{e}_1 \]

Also, to satisfy the sliding mode condition \( \dot{s} \dot{\dddot{e}} < 0 \), limits has to be set on \( K_1 \) and \( K_2 \) such as \( K_1 > 0 \) and \( K_2 > 0 \).

By equating the proposed reaching law (25) to the derivative of the sliding surface in (37) and substituting \( \dot{\phi} \) by its definition from (16), the control input \( U_2 \) is calculated to be,

\[ U_2 = \frac{1}{b_1} \left[ -(c_1 + 1) (x_2 - \dot{x}_{1ref}) - (c_2 + 1) e_1 - c_3 e_2 - x_4 x_6 a_1 + x_4 \Omega_r a_2 + b_1 U_2 - \dot{x}_{1ref} \right] \] (38)

Similarly, the control inputs for Altitude, Pitch and Yaw have been given in (39, 40 and 41) respectively as,

\[ U_1 = \frac{m}{b_4} \left[ -(c_1 + 1) (x_4 - \dot{x}_{3ref}) - c_3 e_2 - (c_2 + 1) e_1 + \dot{x}_{3ref} - c_3 e_2 - g - K_1 \text{sgn}(s_1) - K_2(s_2) \right] \] (39)

\[ U_3 = \frac{1}{b_2} \left[ -(c_1 + 1) (x_4 - \dot{x}_{3ref}) - (c_2 + 1) e_1 - c_3 e_2 - x_2 x_4 a_3 - x_2 \Omega_r a_4 + \dot{x}_{3ref} - K_1 \text{sgn}(s_1) - K_2(s_2) \right] \] (40)

D. DETERMINATION OF SWITCHING SURFACE COEFFICIENT II-SMC

Usually, sliding surface coefficients are designed through pole placement, hit-and-trial, and through simulations based techniques. In this article, a proper gain scheduling method has introduced by its stable eigenpoints to avoid overshoot which causes high voltage peaks. To determine the switching surface coefficient, the surface s, and its derivative \( \dot{s} \) should be taken as zero. The coefficients of the surface will be obtained by using \( (\lambda I - A) \) comparing with desirable stable points such as \( (\lambda + 1)(\lambda + 2) \) and assuming \( a_3 = 1 \) we get a sliding surface coefficient from definition II. The sliding surface and errors have given below.

Let by takings \( s \dot{s} = 0 \) sliding surface is as,

\[ S = a_1 e_1 + a_2 e_2 + a_3 e_3 \] (42)

when \( s \) taken as 0 then the errors are:

\[ e_1 = -\frac{a_2}{a_1} e_2 - \frac{a_3}{a_1} e_3 \]

\[ -\frac{a_2}{a_1} \int e_1 dt - \frac{a_3}{a_1} \int \left\{ \int x_1 - x_{1ref} \right\} dt \] (43)

Considering \( y_1 = e_3, y_2 = e_2 \) and \( y_3 = e_1 \) as,

\[ \dot{y}_1 = y_2 \Rightarrow \dot{y}_2 = -\frac{a_2}{a_1} y_2 - \frac{a_3}{a_1} y_1 \] (44)

After linearizing with Tayler series expansion around the equilibrium point, the set of the equation obtained as:

\[ [\dot{y}_1 \dot{y}_2]^T = \left( \begin{array}{cc} 0 & -\frac{a_2}{a_1} \\ -\frac{a_3}{a_1} & -\frac{a_2}{a_1} \end{array} \right) \Rightarrow \lambda^2 + \frac{a_2}{a_1} \lambda + \frac{a_3}{a_1} = 0 \] (45)

According to Hurwitz’s stability criteria, a matrix is stable if the real parts of Eigen-values lie on the left half-plane from definition 2.

\[ \frac{a_2}{a_1} = 3, \quad \frac{a_3}{a_1} = 2 \] (46)

and \( a_3 = 1 \)
Remark 4: The determination of the switching surface coefficient make this study is valuable in proposed techniques. In IIPRERL-SMC sliding mode control with designing proper switching surface constants by comparing its eigenvalues more powerful results have been observed.

E. IMPROVED INTEGRAL POWER RATE EXPONENTIAL REACHING LAW

In all the above-presented sliding mode control techniques, a trade-off such that by focusing on robustness, chattering can be seen. Also if the system is chattering-free than robustness is not granted. As every variant of SMC can counter matched uncertainty which is the most important form of robustness. So, there should be an exact method that will accomplish both chattering and robustness. Therefore IIPRERL-SMC has been introduced which satisfies these characteristics.

In SMC, different reaching laws have been considered. In conventional SMC, mostly simple constant reaching law has been taken. IIPRERL-SMC has been applied to cope with chattering and fast reaching of state. In this way reaching speed increase when states are far away from the sliding surface and also speed decreases when states near to the sliding surface to avoid this trade-off between chattering and robustness.

\[ \dot{S} = -K_1 |S_1|^\alpha \text{sgn}(S_1) - K_2(S_1) \]  

(47)

where (for robustness) and \( \alpha \) are positive constants with \( 0 < \alpha < 1 \). \( |S_1|^\alpha \) is responsible for chattering with \( \alpha \) is nicely chosen to create a balance between robustness and chattering. In our case, the value of \( \alpha \) is 0.7. After taking this sliding surface which is presented in (47) by equating to the derivative of the sliding surface in (37) and substituting \( \dot{\phi} \) by its definition from (16). Now after taking these assumption control input \( U_2 \) can be calculated.

\[ U_2 = \frac{1}{b_1} \left[ -(c_1 + 1)(x_2 - \dot{x}_{1ref}) - (c_2 + 1)e_1 - c_3e_2 - x_4\dot{\alpha} + x_4\Omega_r a_2 + \ddot{x}_{1ref} - K_1 |S_1|^\alpha \text{sgn}(S_1) - K_2(S_1) \right] \]  

(48)

Similarly, the control inputs for Attitude, Pitch and Yaw have been given in (72,73,and 74) respectively as,

\[ U_1 = \frac{m}{b_4} \left[ -(c_1 + 1)(x_8 - \dot{x}_{7ref}) - c_3e_2 - (c_2 + 1)e_1 + \dot{x}_{7ref} - c_3e_2 - g \right] - K_1 |S_1|^\alpha \text{sgn}(S_1) - K_2(S_1) \]  

(49)

\[ U_3 = \frac{1}{b_2} \left[ -(c_1 + 1)(x_4 - \dot{x}_{3ref}) - (c_2 + 1)e_1 - c_3e_2 - x_2\dot{\alpha} + x_2\Omega_r a_4 + \ddot{x}_{3ref} - K_1 |S_1|^\alpha \text{sgn}(S_1) - K_2(S_1) \right] \]  

(50)

\[ U_4 = \frac{1}{b_3} \left[ -(c_1 + 1)(x_6 - \dot{x}_{5ref}) - (c_2 + 1)e_1 - c_3e_2 - x_4\dot{\alpha} + \ddot{x}_{5ref} - K_1 |S_1|^\alpha \text{sgn}(S_1) - K_2(S_1) \right] \]  

(51)

F. LYAPUNOV STABILITY ANALYSIS

Let \( x = 0, \dot{x} = f(x) \) and \( D \in \mathbb{R}^n \) is a domain that contains \( x = 0 \), for which \( V: D \rightarrow \mathbb{R} \) is continuously differentiable. Then \( x = 0 \), it is stable in a given domain \( D \) and stickly speaking if we have \( \dot{V}(x) < 0 \) given in \( D \) then this is called asymptotically stable. In case of the quadrotor system, the Lyapunov candidate is considered in (52) as;

\[ V(e, s) = \frac{1}{2}(e^2 + s^2) \]  

(52)

According to Lyapunov condition as: \( \dot{V} = SS' \leq 0 \), with \( \dot{S} = -K_1 \text{sgn}(S_1) - K_2 \text{sgn}(S_2) \) is to make positive invariant set \( \{|s| \leq c| \} \) such that trajectories remain on it \( \dot{V} = -sK_1 \text{sgn}(S_1) - K_2 s^2 \). The Lyapunov theorem is satisfied with the dominant term \( K_2 S^2 > 0 \). In this section, Lyapunov’s control has been driven to show the effectiveness of the proposed technique. In the case of perfect cross, VTOL (\( I_{XX} = I_{YY} \)) is a drastically reduced where \( \Omega_e \) is completely supposed to be zero and the best Lyapunov candidate \( V(X) \) can be taken as;

\[ V(X) = \frac{1}{2}(x_1 - x_1^d)^2 + x_2^2 + (x_3 - x_3^d)^2 + x_4^2 + (x_5 - x_5^d)^2 + x_6^2 \]  

(53)

whereas the \( \dot{V}(X) \) is the negative semi-definite is

\[ \dot{V}(X) = \left( x_1 - x_1^d \right)^2 x_2 + \frac{1}{I_{XX}} U_2 + (x_3 - x_3^d)^2 x_4 + x_4 \frac{1}{I_{YY}} U_3 + x_2^2 + (x_5 - x_5^d)^2 x_6 + \frac{1}{I_{ZZ}} U_4 \]  

(54)

By simplifying control law for the Roll (\( \phi \)), Pitch (\( \theta \)) and Yaw (\( \psi \)) can be chosen and given in (55-57) as:

\[ U_2 = -\frac{1}{I_{XX}} (x_1 - x_1^d)^2 - K_1 x_2 \]  

(55)

\[ U_3 = -\frac{1}{I_{YY}} (x_3 - x_3^d)^2 - K_2 x_4 \]  

(56)

\[ U_4 = -\frac{1}{I_{ZZ}} (x_5 - x_5^d)^2 - K_3 x_6 \]  

(57)

Theorem 1: Consider the system (16), suppose there exist \( V(x) \) and \( K_1, K_2 \) which are satisfied by given lemma 1, (26) and (27). Let \( u_0 = -K_1 \text{sgn}(s_1) - K_2(s_2) \) is given by (21) than for all \( (x_1(0), x_2(0), \ldots, x_{12}(0)) \subset \Omega \) where \( \Omega_e \) is the positive invariant compact set defined by lemma 1 than the trajectories \( (x_1(t), x_2(t), \ldots, x_{12}(t)) \) are bounded for all \( t \geq 0 \). Moreover, if all assumption holds globally and \( V(x) \) is radially unbounded then the foregoing conclusion holds that trajectories can globally asymptotically track according to their desired position.

Proof:

\[ V(t) = \frac{1}{2} S_1^T S_1 + \frac{1}{2} S_2^T S_2 \]  

(58)

Invoking time derivative we have

\[ \dot{V}(t) = S_1^T \dot{S}_1 + S_2^T \dot{S}_2 \]  

(59)

\[ = S_1^T (-K_1 |S_1|^\alpha \text{sgn}(S_1) - K_2(S_1)) \]  

(60)
The trajectories are globally asymptotically track their desired positions.

\begin{align}
+ s_2^2 (-K_2 |S_2|^{\alpha} sgn(S_2) - K_2(S_2)) \\ 
\sum_{i=1}^{2} s_i^2 (-K_i |S_i|^{\alpha} sgn(S_i)) < 0
\end{align}

\(\forall S_1, S_2 \neq 0\). So, if above theorem satisfied which sure the trajectories are globally asymptotically track their desired positions.

**G. IMPROVED INTEGRAL POWER RATE EXPONENTIAL REACHING LAW IN THE PRESENCE OF Matched DISTURBANCE**

To control the quadrotor here we introduced matched disturbance \(w(t)\) to show the robustness. Let general state-space equation with matched disturbance is

\[
X^*(t)^{(\alpha)} = f(x, t) + g(x, t)u + w
\]

\[
y^* = h(x)
\]

Further in (16) we augment \(w(t)\) which satisfies (62).

**Theorem 2:** There exists some known function \(\rho(t)\) such that the matched perturbation, \(w(t)\) which satisfies (15),

\[
|w| \leq \rho(t) \quad \forall t \geq 0
\]

**Proof:**

Consider the Lyapunov function

\[
V^*(x) = \frac{1}{2} \sigma_0^2
\]

Taking the derivative of the Lyapunov function

\[
\dot{V}^* = \sigma_0 \delta_0 = \sigma_0 (a_1 \hat{x}_1 + \hat{x}_2)
\]

Using (61)

\[
\dot{V}^* = \sigma_0 (a_1 x_2 + f(x_1, x_2) + g(x_1, x_2) u + w)
\]

Given in (71)

\[
\dot{V}^* = \sigma_0 (-[\kappa + \rho(t)] sgn(\sigma_0) + w)
\]

\[
\dot{V}^* = -\kappa \sigma_0 sgn(\sigma_0) + \sigma_0 [w - \rho(t) sgn(\sigma_0)]
\]

by using the theorem (63)

\[
\sigma_0 [w - \rho(t) sgn(\sigma_0)] \leq 0
\]

Hence,

\[
\dot{V}^* \leq -|\sigma_0| (\kappa) \leq 0
\]

\[
|w| \leq \rho(t) \quad \forall t \geq 0
\]

The trajectories are globally asymptotically track their desired positions.

Taking sliding surface which is presented in (47) by equating to the derivative of the sliding surface in (37) and introduced matched disturbance from(70). After this substituting \(\dot{\phi}\) by its definition from (63) and taking these assumptions control input \(U_2\) can be calculated which varifies the theorem 2.

\[
U_2^* = \frac{1}{b_1} [-(c_1 + 1)(x_2 - \dot{x}_{1_ref})]
\]

Similarly, the control inputs for Altitude, Pitch and Yaw have been given in (50, 51 and 52) respectively as,

\[
U_1^* = \frac{m}{b_4} [-(c_1 + 1)(x_8 - \dot{x}_{7_ref})]
\]

\[
-(c_2 + 1)e_1 - c_3 e_2 - x_4 a_1 + x_4 \Omega_r a_2
\]

\[
+ \dot{x}_{1_ref} - K_i |S_i|^{\alpha} sgn(S_i) \{k + \rho(t) - K_1(s_i)\}
\]

\[
U_3^* = \frac{1}{b_2} [-(c_1 + 1)(x_4 - \dot{x}_{3_ref})]
\]

\[
-(c_2 + 1)e_1 - c_3 e_2 - x_2 a_3 - x_2 \Omega_r a_4
\]

\[
+ \dot{x}_{3_ref} - K_i |S_i|^{\alpha} sgn(S_i) \{k + \rho(t) - K_2(s_i)\}
\]

\[
U_4^* = \frac{1}{b_3} [-(c_1 + 1)(x_6 - \dot{x}_{5_ref})]
\]

\[
-(c_2 + 1)e_1 - c_3 e_2 - x_4 a_5 + \dot{x}_{5_ref}
\]

\[
-K_i |S_i|^{\alpha} sgn(S_i) \{k + \rho(t) - K_2(s_i)\}
\]

**IV. CONTROL RESULTS AND SIMULATIONS**

In this section, simulations of the proposed scheme with three different control techniques have been simulated in the MATLAB /simulink environment to verified the results. In Table 2 and Table 3 all of the parameters are given which are used in simulations. As our main concern is to avoid chattering and peak voltages, although slow responses are obtained. In this way Fig.(5-7), explain three different controllers with graphical representations for SMC, ISMC and IISMC. In first, the PID controller has been tested, after that, a robust SMC controller is implemented. From SMC chattering phenomena is happened, to avoid this ISMC has better results in a sense of chattering. Fig.(5-7), show the responses of states Roll, Pitch, Yaw, and Altitude respectively. The green line represents the conventional SMC, blue line for ISMC and the red line shows improved ISMC with gain scheduling. In these figures, it can easily be observed that these results of SMC with green lines and ISMC with blue lines have many fluctuations in the sense of chattering, which is undesirable which makes a high-cost control input. This can reduce by using Integral-SMC, Improved Integral SMC and IIPRERL-SMC. Meanwhile, it is also observed that we get low overshoot to avoid high peaks voltages.

**FIGURE 5.** SMC, I-SMC and II-SMC response for the Roll angle.
TABLE 1. Comparative analysis SMC, ISMC, IISMC and IIPRERL-SMC.

| Characteristics     | SMC       | ISMC      | IISMC     | IIPRERL-SMC |
|---------------------|-----------|-----------|-----------|-------------|
| Tracking Control    | Slow (Not Precise) | Slow (Not Precise) | Very fast | Slow (Highly Precise) |
| Settling Time       | Low       | Low       | Low       | High        |
| Overshoot           | low       | Very      | Low       | Minimal     |
| Chattering Analysis | Severe    | Low       | Moderate  | Minimal     |
| Control Effort      | Low       | High      | Low       | Lowest      |
| Computational       |           |           |           |             |
| Complexity          |           |           |           |             |

TABLE 2. Quadrotor parameters.

| Parameter | Value   | Unit |
|-----------|---------|------|
| $l_{XX}$  | 7.5e-3  | $kg\cdot m^2$ |
| $l_{YY}$  | 7.5e-3  | $kg\cdot m^2$ |
| $l_{ZZ}$  | 1.4e-2  | $kg\cdot m^2$ |
| $l$       | 0.24    | $m$   |
| $J_r$     | 6e-5    | $kg\cdot m^2$ |
| $m$       | 0.65    | $kg$  |
| $K_f$     | 3.13e-5 | $Ns$  |
| $K_t$     | diag(0.1, 0.1, 0.15) | $Nm\cdot s$ |
| $K_m$     | 7.5e-7  | $Nm\cdot s$ |

TABLE 3. Simulation parameters of quadrotor control.

| Parameter | Value   | Parameter | Value   |
|-----------|---------|-----------|---------|
| $a_1$     | $(l_{YY} - l_{ZZ})/l_{XX}$ | $c_1$ | 6.98    |
| $a_2$     | $J_r/l_{XX}$   | $c_2$ | 4.68    |
| $a_3$     | $(l_{ZZ} - l_{XX})/l_{YY}$ | $c_3$ | 4.68    |
| $a_4$     | $J_r/l_{YY}$   | $c_4$ | 5.24    |
| $a_5$     | $(l_{XX} - l_{YY})/l_{ZZ}$ | $K_1$ | 2.05    |
| $b_1$     | $l_{XX}$      | $K_2$ | 2.64    |
| $b_2$     | $l_{YY}$      |       |         |
| $b_3$     | $l_{ZZ}$      |       |         |

The initial positions for inner loop rotational angles are $(\phi, \theta, \psi) = (0, 0, 0)$ with their desired states $(5, 5, 5)$ rad respectively. In all these cases, it has been observed that the SMC controller is good but due to switching surface, it produces chattering around steady-state which degrades the system performance. Therefore, several researchers have been working on that issue. To accomplish this problem IIPRERL-SMC has been introduced. For more precisely to avoid high voltages and chattering, the IIPRERL-SMC based results have been shown in Fig. (8-10). which shows that the proposed technique outperforms the rest of the techniques with the elimination of high voltages peaks completely. In Fig.(8-10). the comparison of SMC, IIPRERL-SMC and Layapnov’s controller has been presented for Roll, Pitch and Yaw angles respectively. In IIPRERL-SMC the speed increase when states are far away from the sliding surface and speed decreases when states near to the sliding surface to avoid this trade-off between chattering and robustness. IIPRERL-SMC has been applied to cope with chattering and fast reaching of state. It has been observed that under IIPRERL-SMC system perfectly work on it. A comparison of different control approaches have been shown in table 1. The control efforts have been shown in Fig.(11-13) for the Roll, Pitch and Yaw angles respectively in the presence of matched disturbance. As in the conventional SMC, a lot of control effort need for stability due to chattering. In table 1 it can be seen that conventional SMC need more control effort than IIPRERL-SMC. The green line in the control effort represents SMC and the red line represents the IIPRERL-SMC control effort. We used simulations constants which have been shown in table 2 and table 3.
I. Ahmad et al.: Variants of the SMC in Presence of External Disturbance for Quadrotor

V. ONLINE FLIGHT CONTROL OF QUADROTOR

Online planning, based on the offline one, is given on-board radar readings which gradually produces a smooth 3-D trajectory aiming at reaching a predetermined target in an unknown environment. It is commonly prefer for low level altitude. For this, RRT*FN is illustrated which gained noteworthiness in sampling-based path planning in the dynamic and cluttered environment, has been implemented for the underactuated nonlinear system model. Robot Motion Planning (RMP) has turned out to be an interesting area in encouraging research domains due to its randomness and wide applications over the last decades. Kinodynamic planning deals with motion planning problems in which robot constraints like velocity and acceleration bounds must be satisfied to address the issues of motion planning. Encountered difficulties like environmental structure, computational requirements and assumptions are considered. To deal with such kind of complexities, this section focus on kinodynamic path planning especially, in control system engineering, to find the best optimal trajectory to attain global minima. Simulations results are verified through the MATLAB/Simulink environment.

Autonomous navigation is among the most crucial requirements of an intelligent vehicle. Robots are designed to navigate and find an optimal path from the initial point to its final point while avoiding obstacles within the minimum cost. The best way to understand the whole scenario is by dividing it into four main parts which have been shown in Fig.14.

The first one is perception, in which sensors are work with the robot to find information from the environment. The second one is localization and map building, here we get information from the environment which helps the robot to find its position. The third one is cognition and path planning, in which steering of the robot is involved to find its goal with optimal cost. The last step of this structure is motion control in which robot find its desired trajectory. This complete process is optimally done within the obstacle-free environment shown in Fig.14.

There are several kinds of sampling-based approaches, but Probabilistic Road-Maps (PRM) [25] and Rapidly Exploring Random Tree (RRT) [26], [27] algorithms have gained significant importance among path planning algorithms. Both PRM and RRT are converged to the optimal solution when...
I. Ahmad et al.: Variants of the SMC in Presence of External Disturbance for Quadrotor

FIGURE 14. Robot navigation structure.

the number of iterations approaches to infinity [25]. The basic idea of PRM and RRT is to generate nodes in the sample space through a random process, then a tree of linkages is built-in sample space to determine a possible path towards goal position. After this, the shortest path is found between initial point to the goal point which tends to avoid obstacles in that given space. The kinodynamic RRT* algorithm can control the high dimensional configuration space UA Vs to fly in a complex environment. The deferential system can be described as the evolution of quadrotor, which is represented by (16) with the \((x, y)\) position and \(\theta\) are orientation w.r.t a frame attached to a reference space. The control inputs \(\sum_{n=1}^{4} U_n\) is represented by angular velocities with \([-2\pi, 2\pi]\) boundaries.

Algorithm 1

\[
\begin{align*}
T &= (V,E) \leftarrow \text{RRT}(Z_{\text{init}}) \\
R &\leftarrow \text{InitializeTree}(); \\
T &\leftarrow \text{InsertNode}(\theta, Z_{\text{init}}, T); \\
\text{For } i = 1 \text{ to } i = N \text{ do,} \\
Z_{\text{rand}} &\leftarrow \text{Sample} \\
Z_{\text{nearest}} &\leftarrow \text{Nearest}(Z_{\text{rand}}); \\
(Z_{\text{new}}, u_{\text{new}}) &\leftarrow \text{Steer}(Z_{\text{nearest}}, Z_{\text{rand}}); \\
\text{if } \text{ObstacleFree}(Z_{\text{new}}, Z_{\text{nearest}}) \text{then} \\
T &\leftarrow \text{InsertNode}(Z_{\text{new}}, Z_{\text{new}}); \\
\text{End if} \\
\text{End For} \\
\text{return } T \\
= 0
\end{align*}
\]

A. RRT* ALGORITHM

In Sampling-Based Problem (SBP), some basic terminologies are needed to be understood, such as configuration space has called as \(C - \text{space}\), free space as \(C_{\text{free}}\), obstacle space as \(O_{\text{obs}}\) and \(g\) is used to represent position. Other terminologies such as path \(P\), starting position \(q_{\text{start}}\) and goal position is denoted by \(q_{\text{goal}}\). RRT* based algorithms work in the configuration which consists of a set of all possible transformations necessarily required for the path planning of robot. Configuration space can also be denoted by \(Z \subset R_n, n \in N\) where \(n\) represent dimension in the given space and \(N\) is a set of positive integers. \(Z_{\text{new}}\) represents a new node and \(Z_{\text{near}}\) is the nearest node from \(Z_{\text{new}}\). The basic pseudocode of RRT* is given in algorithm 1. The basic steps and expansion of trees in RRT* are shown below to understand the whole scenarios.

1) Sampling: Core planner.
2) Metric: Measures the cost or return value.
3) Nearest Neighbour (NN): Return procedure from the new node.
4) Select Parent: Connect the existing node to the newly sampled node.
5) Local planning: Connect nodes to straight.
6) Collision checking (CC): Probability of success or failure is represented by a Boolean function.

The basic structure of the RRT planning algorithm is implemented on MATLAB in Fig.15, and the results show that a physical path can be found by using RRT in which the red line shows the goal to final point using Euclidean distance. The axes are taken of 1000 by 1000. In the case of the trajectory planning of quadrotor this basic algorithm is applied which has been shown in section VI.

In this presented algorithm it has been observed that the trajectories can be computed effectively making asymptotically planning which is computationally feasible for the kinodynamic system even in high dimensional space. Since there are no earlier restrictions on the number of nodes in the tree created by the RRT* calculation, the multidirectional RRT* expends numerous computational limits. where advantageous memory is required to store the additional these nodes. The number of nodes can be fixed to ease this issue. In this manner, an augmentation of RRT*, called RRT* Fixed Nodes (RRT*FN), that utilizes a nodes removal procedure from the skeleton of the RRT* which is utilized [27]. The reasoning of the system comes out from the way that: If a node doesn’t have a child node, it likewise implies that it isn’t on a way arriving at the objective state. By removing this node the issue can be solved.
VI. PATH PLANNING OF QUADROTOR

In this section, the basic RRT*FN algorithm has been implemented for the quadrotor. The sampling-based RRT algorithm can solve the high dimensional problem of UAVs with non-holonomic constraints. To generate the trajectory for the 6-DOF kinematic model of a quadrotor concerning its global coordinates can be seen in (16). RRT algorithm creates a goal trajectories that system leads to its goal points. The goal of the planner is the find goal states of quadrotor given a series of torque with the assist of the Euclidean metric shown in equation as,

$$\rho = \sqrt{(\Delta \phi + \Delta \dot{\phi})^2}$$  (75)

In general, it’s difficult for motors to lift quadrotors to their desired states in a smooth motion. Therefore, the quadrotor swings back and forth until the required sufficient velocity achieved to get the goal configurations. This is due to the kinematic constraints of the quadrotor model. Therefore, the shape has been shown in Fig.16. and Fig.17. due to gradual changes in the pose of the quadrotor model. As it can be seen in Fig.16, Fig.17 and Fig.18 the trajectory planning of the Roll $\phi$, Pitch $\theta$ and Yaw $\psi$ respectively. The pseudocode of basic RRT*FN which has been implemented on quadrotor trajectory planning has been shown in algorithm 2.

![Algorithm 2](image)

The experimental results show that the quadrotor can reach its goal position with an optimal cost created by the target position despite the number of nodes by removal methodology. As indicated by the paper [27], it will converge an optimal arrangement. An economical system for quadrotor has presented for its navigation. In this way, RRT* can control high dimensional C-space of UAVs to fly clattered environment. We have made the plan to make our source code online which we have implemented so that available for download. In the future, we believe that the running time further can be improved with a visual tracking system based on the online library.

VII. CONCLUSION

This paper presented an efficient robust control technique IIPRERL-SMC for a quadrotor that is robust to uncertainties and to tackle undesired chattering phenomena. In this way, reaching speed increase when states are far away from the sliding surface, and also speed decreases when states near to the sliding surface to avoid this trade-off between chattering and robustness. This work has been done in phases with variants of SMC. Firstly, start with the development of a dynamic model, SMC for a quadrotor has been designed. Steady-state errors of the control system have been eliminated by augmented improved integral action, taking the
strong condition of the sliding surface. Furthermore, rather than making the assumption, and by introducing the parameterization technique through eigenvalues which made this scheme is valuable. Finally, IIPRERL with these strong controller techniques has been developed. The simulations results proved satisfactory results addressed by IIPRERL-SMC and proved the effectiveness of the proposed scheme. For path planning kinodynamics RRT*FN algorithm has been implemented for local path planning of the quadrotor model. The sampling-based RRT algorithm can solve the high dimensional problem of UAVs with Non-Holonomic constraints. Author has made a plan that these algorithms should be implemented in a 3D environment this would be a great hike in the control system.

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