High-precision measurement of the polarized hadron beam energy in circular accelerator

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Abstract. A method for the high-precision measurement of the hadron polarized beam energy in circular accelerators by measuring the spin precession frequencies is offered. The problem of the particles energy measurement with accuracy better, than the beam energy spread, first of all, is connected with influence of synchrotron oscillations on spin motion. The given method develops the well-known method of high-precision measurement of electron-positron beams mean energy in case of a wide spin frequency spread.

The high-precision measurement of polarized beam energy in electron-positron storage rings had been offered in papers [1–4] and known as the resonance depolarization technique. In principal this technique allows to measure the mean value of the beam energy with accuracy better than the spread of the beam energy. The masses \(\Phi, K^0, K^\pm, J/\Psi, \Psi', \Upsilon\)-mesons had been measured with very high precision based on this technique [3]. The comparison of anomalous parts of gyromagnetic ratio of electron and positron had been made with relative precision \(10^{-5}\) [4]. It was on two orders of the magnitude better than had been measured that time.

Let’s overview the main foundations of the resonance depolarization technique using traditional flat accelerator model with vertical leading field \(B_y\). In this case the spin precession frequency is proportional to energy \(\gamma mc^2\):

\[ \Omega = \omega_s(1 + \gamma G), \quad \omega_s(\gamma) = \frac{\langle B_y \rangle}{\gamma mc}. \]

Here \(\omega_s\) is the synchronous particle revolution frequency, \(G\) is an anomalous part of the gyromagnetic ratio. Measurements are carried out by application of the RF- solenoid with RF-field frequency equal to \(\omega_d\). The resonance condition induced by RF- solenoid can be written in the form

\[ \Omega = \omega_d + k\omega_s, \quad k = 0, \pm 1, \pm 2 \ldots \]

Measuring the beam depolarization, one can find out the resonance spin frequency and then calculate the mean value of the beam energy.

The precision of beam energy measuring is determined by average beam spin frequency spread

\[ \langle \delta \Omega \rangle = G\gamma \omega_s \left( \frac{\gamma - \gamma_s}{\gamma_s} + \frac{\gamma_s \theta \omega \langle (\Delta \gamma)^2 \rangle}{\omega_s \partial \gamma \gamma^2} \right). \]
Here $\Delta \gamma mc^2$ is a deviation of particle energy from the synchronous particle energy value $\gamma_s mc^2$. Using the phase stability principle, the deviation of average beam energy $\bar{\gamma}_m mc^2$ from the synchronous particle energy can be calculated through mean square of beam betatron oscillations amplitudes $\langle a^2 \rangle$:

$$\bar{\gamma} - \gamma_s = -\frac{1}{2\gamma_s} \left[ \frac{\partial^2 \omega}{\partial \gamma^2} \langle (\Delta \gamma)^2 \rangle + \frac{2 \partial \omega}{\partial a^2} \langle a^2 \rangle \right]$$

(1)

Taking into account (1), the spin frequency spread can be written as

$$\frac{\langle \delta \Omega \rangle}{G\gamma \omega_s} = \frac{1}{K \omega_s \partial a^2} \langle a^2 \rangle + \left[ \frac{1}{2K} \frac{\gamma_s^2}{\omega_s} \left( \frac{\partial^2 \omega}{\partial \gamma^2} \right) - K \right] \frac{\langle (\Delta \gamma)^2 \rangle}{\gamma^2}, \quad K = \frac{\gamma_s}{\omega_s} \left( \frac{\partial \omega}{\partial \gamma} \right)$$

(2)

The contribution to the first term in equation (2) is basically connected with nonlinearity of the leading field in the accelerator and can be compensated by application of additional magnetic elements. In this case the measurement precision is proportional to $\langle (\Delta \gamma)^2 \rangle / \gamma^2$ factor. In principle, by special choosing of the accelerator lattice, it is possible to compensate also the contribution of the second term in equation (2). Then the measurement precision will be determined by next members of spin frequency series expansion.

Let’s emphasize the importance of synchrotron oscillations in this technique. This oscillations average deviated particles motion and the beam behaves as though it has zero energy spread near to the single isolated spin resonance. Such representation is correct when the spin frequency spread is much less than synchrotron oscillations frequency $\sigma_\gamma \ll \omega_\gamma$ that takes place in electron-positron rings. The spin motion significantly is changed for hadron beams in case of wide spin frequency spread ($\sigma_\gamma \gg \omega_\gamma$). It is well-known, that the synchrotron oscillations influence leads to the basic resonance splitting into the series of modulation resonances which are equidistant from the basic one [5]

$$\Omega_{km} = \Omega_k \pm m \omega_\gamma, \quad m = 0, \pm 1, \pm 2 \ldots$$

The strength of modulation resonance is $w_{km} = w_k F_m$, where $w_k$ is basic resonance strength, $F_m$ is a modulation resonance weight which for Gaussian distribution on synchrotron oscillations amplitudes is determined by the modulation resonance index $k_\gamma = \sigma_\gamma / \omega_\gamma$ and equal to

$$F_m = e^{-k_\gamma^2/2} \sqrt{I_m(k_\gamma^2)}$$

Here $I_m(x)$ is the modified Bessel functions of the first kind. From equation $e^{-x} \sum_{m=-\infty}^{\infty} I_m(x) = 1$ one can derive the next property of modulation resonances

$$\sum_{m=-\infty}^{\infty} |w_{km}|^2 = w_k^2$$

(3)

The modulation resonance index also determines quantity of the strongest modulation resonances. The modulation resonance strengths are very small if $k_\gamma \ll 1$ and situation practically does not differ from the single isolated basic resonance. The basic resonance is split into the series of modulation resonances if $k_\gamma > 1$. It is possible to consider for estimations, that the strongest resonances are with numbers $m < k_\gamma$. The strengths of the strongest resonances giving the main contribution to the sum (3) are approximately equal to $w_k / \sqrt{F_\gamma}$.

Figure 1 shows the change of single isolated resonance spectrum under synchrotron oscillations for case of $k_\gamma = 4$. Here $\nu_\gamma = \omega_\gamma / \omega_s$ is dimensionless synchrotron oscillations tune. One can see that resonance strengths are decreased in almost two times at $|m| > 4$.

The modulation resonance index influence on spin motion has been demonstrated on COSY accelerator in papers [6, 7]. In these works the induced resonance strength was measured for
A single isolated resonance without synchrotron oscillations

Series of modulation resonances ($k_{\gamma}=4$)

**Figure 1.** The change of single isolated resonance spectrum under synchrotron oscillations.

Proton and deuteron beams. Experiments have shown that for the unbunched deuteron beam the resonance width was proportional to spin frequency spread ($\sim 25$ Hz). At presence of synchrotron oscillations ($\sim 215$ Hz) it is possible to locate resonance frequency with a narrow width (5 Hz). So the beam behaves as if it has not energy spread. Really, in this case the modulation resonance index $k_{\gamma} = 0.1$ and it influence can be neglected (see Figure 2). For the protons beams the situation was contrary. If for the unbunched proton beam, as well as in the first case, the resonance width was proportional to spin frequency spread ($\sim 1.3$ kHz) for the bunch proton beam with synchrotron oscillations frequency ($\sim 25$ Hz) the resonance width had not decreased, and even had increased twice. In this case the modulation resonance index was $k_{\gamma} \sim 60$. Therefore single isolated resonance was split into a series of modulation resonance on all width of spin frequency spread with the strengths reduced approximately in 8 times (see Figure 2).

**Figure 2.** The modulation resonance spectrum on COSY: a) case of the deuteron beam, b) case of the proton beam.

As it has been noted, the modulation resonance weights $F_m$ are approximately equal in the area $m \leq k_{\gamma}$, and are decreased outside one. Therefore the main difficulty of application of a resonance depolarization technique for a hadron beam is a recognizing of the basic resonance frequency among a series of identical modulation resonances.
In a real situation the synchrotron oscillations nonlinearity leads to the synchrotron oscillations frequency spread which is of the order of $\delta \nu_\gamma \sim 0.1 \div 0.5 \nu_\gamma$. The synchrotron oscillations frequency spread causes the side modulation resonances spectrum broadening that sharply selects the central resonance. Basically the spectrum broadening mechanism is connected with change of $m$-th resonance position by value $m \delta \nu_\gamma$ (see Figure 3).

![Figure 3](image-url)  
**Figure 3.** The modulation resonances spectrum broadening due to synchrotron frequency spread.

There are different procedures of the central resonance position recognizing. For example, it is possible to use fast resonance crossing with a constant crossing speed $\varepsilon'$. The vertical polarization changes by value $\pi w_k^2 / \varepsilon'$ during time of the order of $1 / \sqrt{\varepsilon'}$ after crossing next modulation resonance. Using equation (3) one see, that the result value of vertical polarization after all modulation resonance crossings is equal to $P_z = 1 - \pi w_k^2 / \varepsilon'$ that is equivalent to the basic isolated resonance crossing. To distinguish each resonance influence it is necessary that the resonance area was noticeably less then distances between them: $\varepsilon' \ll \nu_\gamma$. An optimum resonance area value is determined by the spin frequency spread. The maximal polarization jump on the central resonance will be if one choose the induced resonance strength equal to $w_k = \sqrt{\varepsilon' / \pi}$.

![Figure 4](image-url)  
**Figure 4.** Fast crossing of a single isolated resonance under synchrotron oscillations.

Figure 4 shows the dependence of the vertical beam polarization versus the resonance detune $\varepsilon$ during resonance crossing at $k_\gamma = 4$ in the cases of synchrotron frequency spread (black line) and without last one (gray line). One can see that at the central resonance crossing there is a
sharp polarization jump (jump time $\sim 1/\sqrt{\varepsilon'}$). It can be used to recognize the basic resonance frequency position and further to calculate the beam energy.

The fast crossing of an induced spin resonance with controllable strength and crossing speed can be realized by application of the RF-solenoid as it was done, for example, in papers [6,7].

In summary it should be noted that for optimization of resonance depolarization technique parameters in the real accelerator it is necessary to consider influence of the nearest spin resonances, vertical deviations of particles trajectories, second-order terms of spin perturbation, the accelerator lattice accuracy of manufacturing, etc. Also one ought to estimate influence of oscillations of the particle phase in the bunch and the associated electric field induced by the RF-solenoid.

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References
[1] Bukin A, Derbenev Ya, Kondratenko A, Kurdadze L, Serednyakov S, Sidorov V, Skrinsky A, Tumaikin G and Shatunov Yu 1975 Absolute calibration of beam energy in the storage ring. $\Phi$-meson mass measurement (Preprint INP 75-64)
[2] Derbenev Ya, Kondratenko A, Serednyakov S, Skrinsky A, Tumaikin G and Shatunov Yu 1980 Accurate calibration of the beam energy in a storage ring based on measurement of the spin precession frequency of polarized particles Part. Accel. 10 177-180
[3] Skrinsky A 1982 Accelerator and detector prospects of elementary particle physics Sov. Phys. Usp. 25 639-661
[4] Serednyakov S, Skrinsky A, Tumaikin G and Shatunov Yu 1977 High accuracy comparison of the electron and positron magnetic moments Phys. Lett. 66 B (1) 102-104
[5] Golubeva N, Issinsky I, Kondratenko A, Kondratenko M, Mikhailov V and Strokovsky E 2002 The study of deuteron and proton beam polarization in the Nuclotron ring (Preprint JINR P9-2002-289) Dubna
[6] Morozov V, Chao A, Krisch A, Leonova M, Raymond R, Sivers D, Wong V and Kondratenko A 2008 Narrow spin resonance width and spin flip with an rf-bunched proton beam Phys. Rev. Lett. 103, 144801
[7] Morozov V, Chao A, Krisch A, Leonova M, Liu J, Raymond R, Sivers D, Wong V and Kondratenko A 2009 Wide spin resonance with an rf-bunched proton beam arXiv:1001.1456v1 [physics.acc-ph]