GW170817-compatible Constant-roll Einstein-Gauss-Bonnet Inflation and Non-Gaussianities

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In this paper we investigate the inflationary phenomenology of an Einstein-Gauss-Bonnet theory compatible with the GW170817 event, by imposing the constant-roll evolution on the scalar field. We develop the constant-roll GW170817-compatible Einstein-Gauss-Bonnet formalism, and we calculate the slow-roll indices and the observational indices of inflation, for several models of interest. As we demonstrate, the phenomenological viability of the models we study is achieved for a wide range of the free parameters. In addition, for the same values of the free parameters that guarantee the inflationary phenomenological viability of the models, we also make predictions for the non-Gaussianities of the models, since the constant-roll evolution is known to enhance non-Gaussianities. As we show the non-Gaussianities are of the same order for the slow-roll and constant-roll case, and in fact in some cases, the amount of the non-Gaussianities is smaller in the constant-roll case.

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I. INTRODUCTION

After the striking neutron star merging GW170817 event [1], which was followed by a kilonova, the fact that the gravitational waves arrived almost simultaneously with the electromagnetic radiation emitted by the kilonova, it was obvious that the gravitational wave speed $c_T$ was nearly equal to that of light’s, that is $c_T^2 = 1$ in natural units. This fact has put several generalizations of Einstein’s theory of relativity into peril, since several extended theories of gravity predict a gravitational wave speed different from that of light’s, referring always to the speed of the tensor perturbations. One question with conceptual interest is, does it matter if some theories of extended gravity predict a primordial gravitational wave speed different than that of light’s? The answer could be easy to answer by simply thinking that the Universe during the inflationary era and in the post-inflationary era, is classical, and described by a four dimensional spacetime metric. Thus there is no particle physics reason for the Universe to change the mass of the primordial graviton. Thus indeed, the graviton, either it propagates in the form of primordial gravitational waves, or it propagates in the form of astrophysical originating gravitational waves, should be massless, or nearly massless. An extensive list of theories which were put into question after the GW170817 event, can be found in Ref. [2].

One of the theories that were put into question after the GW170817 event, were the Einstein-Gauss-Bonnet theories [3–44], see also the review [45] which form an appealing class of theories capable of describing the inflationary era and also several astrophysical objects. The reason for considering Einstein-Gauss-Bonnet theories as appealing candidate theories for the primordial era of our Universe is simply because these are string motivated theories, basically the whole theory is a string-corrected canonical scalar field theory minimally coupled to gravity. In several previous works [31–36] we demonstrated how Einstein-Gauss-Bonnet theories and their extensions, may actually be rectified in view of the GW170817 event, by simply demanding that the primordial gravitational wave speed is set equal to unity. In effect, this constraint results to a differential equation which constrains severely the functional form of the scalar potential $V(\phi)$ and of the scalar coupling function $\xi(\phi)$ of the scalar field with the Gauss-Bonnet invariant.

In this paper, we shall extend the formalism of our previous work [36], to take into account a constant-roll evolution for the scalar field. The constant-roll evolution is a widely used assumption for the evolution of the scalar field during the primordial era. The aim of this paper is two-fold: Firstly we shall investigate whether a viable phenomenology can be obtained by the constant-roll GW170817-compatible Einstein-Gauss-Bonnet theory. Secondly, we shall investigate what is the predicted amount of non-Gaussianities predicted by the GW170817-compatible Einstein-Gauss-Bonnet
theory, when the constant-roll assumption is used, since the constant-roll evolution is known to enhance the non-Gaussianities features. Our results are quite interesting, since we evince that the constant-roll evolution assumption for the scalar field can also yield the GW170817-compatible Einstein-Gauss-Bonnet theory viable and very good aligned with the latest Planck data \[46\], but more importantly, the non-Gaussianities in the case at hand are not enhanced, and in some cases are smaller in value, when compared to the slowly rolling scalar field scenario for the GW170817-compatible Einstein-Gauss-Bonnet theory. Our motivation to use modified gravity description for the inflationary era, comes from the fact that general relativity seems to fail to consistently describe several evolution eras of our Universe, such as the dark energy era, and in some cases the inflationary era, see for reviews \[45, 47–52\]. Also in some cases, it is possible that modified gravity can mimic dark matter, but also dark matter can also be a massive particle with no interaction or small interaction with other particles \[53–58\].

Before starting, an important discussion is in order. The Planck 2018 data on inflation are able to bring information relevant to the inflationary era, available and unaltered at late-times, due to the mechanism of inflation itself. Basically, the information measured in the CMB at present, is nothing else but the primordial modes which exited the Hubble horizon at the first time, at the time instance we assumed that inflation started. These modes were frozen after the horizon crossing, and re-entered the Hubble horizon during the radiation and matter domination eras, unaltered. For the latter reason the primordial modes carry information about the inflationary era, these are the frozen modes at early times. Now regarding the primordial tensor modes, the same principle applies, hence if primordial gravitational waves are ever found, these must be massless modes and which correspond to a gravitational wave speed equal to unity. Now the question is whether someone should expect these primordial modes to be massless, and why should an astrophysical gravitational wave speed equal to unity, impose constraints on the early-time primordial gravity waves. From a fundamental physics point of view, gravity is mediated by gravitons, so regardless the graviton mediates primordial gravity waves, or astrophysical gravity waves, the graviton is the same. From a particle physics point of view, there is no fundamental reason for the graviton to alter its mass during the inflationary and the post-inflationary era. This is why the constraint brought along by the kilonova related event GW170817 for a massless graviton, also affects the early-time tensor perturbation modes, thus the primordial gravity wave speed. For us, Einstein-Gauss-Bonnet theory is one of the most appealing extensions of minimally coupled scalar field theory, since it is string era. This is why the constraint brought along by the kilonova related event GW170817 for a massless graviton, also affects the early-time tensor perturbation modes, thus the primordial gravity wave speed. For us, Einstein-Gauss-Bonnet theory is one of the most appealing extensions of minimally coupled scalar field theory, since it is string

II. CONSTANT-ROLL INFLATIONARY EVOLUTION OF EINSTEIN-GAUSS-BONNET GRAVITY

In this section we shall investigate how the theoretical framework of Ref. \[36\] is modified if the constant-roll evolution is adopted for the scalar field. In order to render the article self-contained, we shall describe in brief the formalism of the GW170817-compatible Einstein-Gauss-Bonnet gravity developed in Ref. \[36\], and we shall consider a minimally coupled Einstein Gauss-Bonnet theory described by the gravitational action,

\[ S = \int d^4x \sqrt{-g} \left( \frac{R}{2\kappa^2} - \frac{1}{2} \omega g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) - \xi(\phi)\mathcal{G} \right) \]  

(1)

where \( g \) is the metric determinant, \( R \) denotes the Ricci scalar, \( \kappa = \frac{1}{M_P} \) is the gravitational constant where \( M_P \) denotes the reduced Planck mass, and \( V(\phi) \) is the scalar potential, while \( \mathcal{G} \) describes the Gauss-Bonnet invariant \( G = R^2 - 4R_{\alpha\beta}R^{\alpha\beta} + R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta} \), with \( R_{\alpha\beta} \) and \( R_{\alpha\beta\gamma\delta} \) being the Ricci and Riemann tensor respectively. Finally, \( \xi(\phi) \) denotes the Gauss-Bonnet coupling scalar function. Moreover, we shall assume that the geometric background is a flat Friedman-Robertson-Walker background, with the line element being,

\[ ds^2 = -dt^2 + a(t)^2 \delta_{ij} dx^i dx^j. \]  

(2)

According to this form, the metric tensor reads \( g_{\mu\nu} = diag(-1, a(t)^2, a(t)^2, a(t)^2) \). Furthermore, we shall also assume that the scalar field \( \phi \) is homogeneous, or in other words it is only time-dependent. Furthermore, since the metric is flat, the Ricci scalar and the Gauss-Bonnet invariant can be written in very simple forms, as \( R = 12H^2 + 6\dot{H} \) and \( \mathcal{G} = 24H^2(H + \dot{H}) \). Here, \( H \) signifies Hubble’s parameter and in addition, the “dot” denotes differentiation with respect to the cosmic time as usual. Finally, we note that the term \( \omega \) in the kinetic term will be set equal to unity in order to describe the canonical case, but for the time being we shall leave it as it is in order to show how the results depend on such term. However it shall be treated as a constant, independent of the scalar field.

By varying the gravitational action \( (1) \), one can extract the field equations easily. Consequently, the equations of motion are derived easily from the time and space components of the field equations for gravity and the continuity equation of the scalar field, which read,

\[ \frac{3H^2}{\kappa^2} = \frac{1}{2} \omega \dot{\phi}^2 + V + 24\xi H^3, \]  

(3)
\[-\frac{2\dot{H}}{\kappa^2} = \omega \dot{\phi}^2 - 16\dot{\xi}H\dot{H} + 8H^2(\ddot{\xi} - H\dot{\xi}), \] (4)

\[\ddot{\phi} + 3H\dot{\phi} + \frac{1}{\omega}(V' + \xi'G) = 0, \] (5)

where in contrast to the previous notation, the prime denotes differentiation with respect to the scalar field $\phi$. Describing the inflationary era properly implies an analytical solution of the system of equations of motion. Unfortunately, such a system is very difficult to study analytically. The solution can however be extracted by assuming certain approximations during inflation, after the first horizon crossing, or the initial moment of inflation. Here, we shall assume that the slow-roll approximations hold true and also impose the constant-roll condition on the scalar field. Mathematically speaking, we shall assume that the following conditions hold true,

\[\dot{H} \ll H^2, \quad \frac{1}{2}\omega \dot{\phi}^2 \ll V, \quad \ddot{\phi} = \beta H \dot{\phi}, \] (6)

where $\beta$ is the constant-roll parameter. These assumptions make the equations of motion simpler and we end up with the following expressions,

\[H^2 \simeq \frac{\kappa^2 V}{3}, \] (7)

\[\dot{H} \simeq -\frac{1}{2}\kappa^2 \omega \dot{\phi}^2, \] (8)

\[V' + (3 + \beta)\omega H\dot{\phi} + 24\xi' H^4 \simeq 0. \] (9)

These are the simplified equations of motion we shall use in order to produce results. However, before we proceed further, we shall impose certain additional constraints in order to achieve compatibility with recent striking observations.

The tensor perturbations of the flat FRW metric, or simply the primordial gravitational waves as they are called, propagate through spacetime with the velocity of light, as it was recently ascertained by the GW170817 event. This realization made it abundantly clear that theories which describe modified gravity and produce a different velocity must be discarded. A theory which belongs to that category is the Einstein-Gauss-Bonnet theory, since string corrections produce the following expression for their velocity in natural units,

\[c_T^2 = 1 - \frac{Q_f}{2Q_t}, \] (10)

where $Q_f = 16(\ddot{\xi} - H\dot{\xi})$ and $Q_t = \frac{1}{\kappa^2} - 8\dot{\xi} H$. The compatibility with the GW170817 event may be achieved only if we demand $c_T^2 = 1$. This in turn implies that the numerator of the second term becomes zero, or in other words $\ddot{\xi} = H\dot{\xi}$. This is an ordinary differential equation which can be solved easily. Although finding an expression for the term $\ddot{\xi}(\phi)$, which satisfies the aforementioned differential equation, is feasible [36], we shall choose a different approach taking advantage of the constant-roll condition. Let us expand the differential equation with respect to the scalar field. Since [30] holds true, and the differential operator $\frac{d}{dt}$ is equivalent to $\dot{\phi} \frac{d}{d\phi}$, then we have,

\[\ddot{\xi} + \beta H\dot{\xi} \dot{\phi} = H\dot{\xi} \dot{\phi}. \] (11)

Therefore, the expression for the derivative of the scalar field is,

\[\dot{\phi} = (1 - \beta)H \frac{\xi'}{\xi''}. \] (12)

Taking the limit $\beta = 0$, the above formula in equivalent to the analysis made in a previous work of ours [36], where we studied the slow-roll case, as expected. Thus, the equations of motion are rewritten in the case at hand as follows,

\[H^2 \simeq \frac{\kappa^2 V}{3}, \] (13)
\[ \dot{H} \simeq -H^2 \frac{\kappa^2 \omega (1 - \beta)^2 \left( \frac{\xi'}{\xi''} \right)^2}{2}, \]  
\[ V' + (1 - \beta)(1 + \frac{\beta}{3}\kappa^2 \omega \frac{\xi'}{\xi''} V + \frac{8}{3}\kappa^2 \xi' V^2 \simeq 0. \]

The above set of equations is much more easy to manipulate analytically, as we show in the next sections. Firstly, Eq. (14) is connected to the slow-roll index \( \epsilon_1 \) as we shall see in the subsequent calculations. It is a useful expression since it is interconnected to the constant-roll parameter \( \beta \) and the ratio of derivatives of the Gauss-Bonnet coupling function. Hence, designating an appropriate coupling function is of fundamental importance. Furthermore, the degrees of freedom have decreased by one, since constraints on the velocity of the gravitational waves were imposed. In consequence, specifying the coupling function leads to a differential equation which, once it is solved, it generates the scalar potential. Hence, these terms cannot be designated freely but they are in fact interconnected, although they have different origins. Let us now proceed with the evaluation of the slow-roll indices.

The dynamics of inflation can be described by six parameters named the slow-roll indices, defined as follows \([3, 36]\),

\[ \epsilon_1 = -\frac{\dot{H}}{H^2}, \quad \epsilon_2 = \frac{\ddot{\phi}}{H\dot{\phi}}, \quad \epsilon_3 = \frac{\dot{F}}{2HF}, \quad \epsilon_4 = \frac{\dot{E}}{2HE}, \quad \epsilon_5 = \frac{\dot{F} + Q_a}{2HQ_t}, \quad \epsilon_6 = \frac{\dot{Q}_t}{2HQ_t}, \]  
\[ \left(16\right) \]

where \( F = \frac{1}{\kappa^2}, \) \( Q_a = -8\dot{\xi}H^2, \) \( Q_t = \frac{1}{\kappa} - 8\dot{\xi}H \) and \( E = \frac{1}{\kappa} + \frac{3Q_a^2}{2\kappa Q_t}. \) Hence, according to equations \([11], [12], [14] \) and \([16]\), the indices can be rewritten as follows,

\[ \epsilon_1 \simeq -\frac{\kappa^2 \omega}{2} (1 - \beta)^2 \left( \frac{\xi'}{\xi''} \right)^2, \]  
\[ \left(17\right) \]

\[ \epsilon_2 = \beta, \]  
\[ \left(18\right) \]

\[ \epsilon_3 = 0, \]  
\[ \left(19\right) \]

\[ \epsilon_4 = \frac{1 - \beta}{2} \frac{\xi'}{\xi''} \frac{E'}{E}, \]  
\[ \left(20\right) \]

\[ \epsilon_5 \simeq -\frac{4(1 - \beta)\xi'^2 \kappa^4 V}{3\xi'' - 8(1 - \beta)\xi'^2 \kappa^4 V}, \]  
\[ \left(21\right) \]

\[ \epsilon_6 \simeq -\frac{4(1 - \beta)\xi'^2 \kappa^4 V(1 - \epsilon_1)}{3\xi'' - 8(1 - \beta)\xi'^2 \kappa^4 V}. \]  
\[ \left(22\right) \]

Obviously, in the limit \( \beta = 0 \), all the slow-roll indices, apart from \( \epsilon_2 \), are restored as in the slow-roll case we studied in Ref. \([36]\). Moreover, it is clear that the value \( \beta = 1 \), which in turn implies that \( \dot{\phi} = H\dot{\phi} \), is not an accepted value for \( \beta \) due to the fact that it leads to zero slow-roll indices, apart from \( \epsilon_2 \), and this choice would lead to eternal inflation. This was obviously implied previously when we performed a division with \( \dot{\phi} \) in order to extract the form of \( \dot{\phi} \) depending on the coupling scalar function and Hubble’s parameter. On the other hand, there exists no physical constraint which prohibits \( \beta \) to obtain values greater than unity, so the inequality \( \beta > 1 \) could still yield interesting phenomenology. We also mention that the auxiliary functions are written as,

\[ E = \frac{1}{\kappa^2} + \frac{96}{\kappa^2 Q_t} \xi'^2 H^4, \]  
\[ \left(23\right) \]

\[ Q_a = -8(1 - \beta)\frac{\xi'^2}{\xi''} H^3, \]  
\[ \left(24\right) \]

\[ Q_t = \frac{1}{\kappa^2} - 8(1 - \beta)\frac{\xi'^2}{\xi''} H^2, \]  
\[ \left(25\right) \]
\[ Q_e = -32(1 - \beta) \frac{c^2}{\xi^2} \dot{H} \dot{H}. \]  

(26)

The term \( Q_e \) was introduced here, but will be used in the following relations. Lastly, we discuss the form of the observational indices in the case of the model at hand. The spectral index of primordial curvature perturbations \( n_s \), the spectral index of tensor perturbations \( n_T \) and the tensor-to-scalar ratio \( r \) in terms of the slow-roll indices, are defined as follows \[3, 36\],

\[ n_s = 1 - 2\frac{\epsilon_1 + \epsilon_2 + \epsilon_4}{1 - \epsilon_1}, \quad n_T = -2\frac{\epsilon_1 + \epsilon_6}{1 - \epsilon_1}, \quad r = 16 \left( \frac{\kappa^2 Q_e}{4H} - \epsilon_1 \right) \frac{c_A^2}{\kappa^2 Q_t}, \]

(27)

where \( c_A \) the sound wave velocity defined as,

\[ c_A^2 = 1 + \frac{Q_a Q_e}{3Q_a^2 + 2Q_e \omega \phi'^2}. \]

(28)

The aim in the rest of the paper is to evaluate the observational indices during the first horizon crossing. However, instead of using wavenumbers, we shall use the values of the scalar potential during the initial stage of inflation. Taking it as an input, we can obtain the actual values of the observational quantities. We can do so by firstly evaluating the final value of the scalar field. This value can be derived by equating slow-roll index \( \epsilon_1 \) in Eq. \( \text{(17)} \) to unity. Consequently, the initial value can be evaluated from the \( e \)-foldings number, defined as \( N = \int_{\phi_i}^{\phi_f} \frac{H d\phi}{\dot{\phi}} \), where the difference \( t_f - t_i \) signifies the duration of the inflationary era. Recalling the definition of \( \dot{\phi} \) in Eq. \( \text{(12)} \), one finds that the proper relation from which the initial value of the scalar field can be derived is,

\[ N = \frac{1}{1 - \beta} \int_{\phi_i}^{\phi_f} \frac{\epsilon''}{\xi'} d\phi. \]

(29)

From this equation, as well as equation \( \text{(17)} \), it is obvious that choosing an appropriate coupling function, is the key in order to simplify the results. In the following, we shall work with certain functional forms of this coupling function, derive the scalar potential from \( \text{(15)} \) and produce results for both the observational quantities introduced previously, but also we shall discuss the primordial non-Gaussianities, known to occur when the constant-roll condition is used, as we mentioned in the introduction. In the following section, we shall introduce the formalism of non-Gaussianities before continuing with examining the viability of certain models.

**III. PRIMORDIAL NON GAUSSIANITIES UNDER THE CONSTANT-ROLL CONDITION**

Until now, the perturbations in the Cosmic Microwave Background (CMB) are described perfectly as Gaussian distributions, since no practical evidence is found pointing out a non-Gaussian pattern in the CMB. It is possible though, not probable for the moment, that in the following years the observations may reveal a non-Gaussian pattern in the CMB primordial power spectrum. In this section we shall discuss how to evaluate the non-Gaussianities quantitatively in the context of the GW170817-compatible Einstein-Gauss-Bonnet gravity, using the formalism and notation of \[59\]. We first define the following quantities,

\[ \delta_\xi = \kappa^2 H \xi, \quad \delta_X = \frac{\kappa^2 \omega \phi'^2}{H^2}, \quad \epsilon_s = \epsilon_1 - 4\delta_\xi, \quad n = \frac{\dot{\epsilon}_s}{H \epsilon_s}, \quad s = \frac{\dot{c}_A}{H c_A}. \]

(30)

Here, we shall implement a different formula for the sound wave speed, however equivalent to the previous, which is based on these newly defined quantities for convenience and reads,

\[ c_A^2 \simeq 1 - \frac{64\delta_\xi^2(2\delta_\xi + \delta_X)}{\delta_X}. \]

(31)

Recalling equations \[\text{(12)}, \text{(14)} \text{ and (15)}\], one finds that the previous auxiliary terms have the following forms,

\[ \delta_\xi \simeq \frac{1 - \beta}{3} \kappa^4 V \frac{\xi'^2}{\xi''}, \]

(32)

\[ \delta_X \simeq \kappa^2 \omega (1 - \beta)^2 \left( \frac{\xi'}{\xi''} \right)^2 = 2\epsilon_1, \]

(33)
\[ \epsilon_s \simeq (1 - \beta) \left( \frac{\kappa \xi'}{\xi''} \right)^2 \left( \frac{\omega(1 - \beta)}{2} - \frac{4}{3} \kappa^2 \xi'' V \right), \quad (34) \]

\[ n \simeq 2(1 - \beta) \left( 1 - \frac{\xi' \xi''}{\xi'' \xi'''} - 4 \kappa^2 \frac{\xi'}{\xi''} \frac{V' \xi'' + V \xi'''}{3 \omega(1 - \beta) - 8 \kappa^2 \xi'' V} \right), \quad (35) \]

\[ s = (1 - \beta) \frac{\xi' c_A}{\xi'' c_A}. \quad (36) \]

In certain examples, we shall see that by choosing appropriately the coupling function simplifies greatly the quantity \( \epsilon_s \), as it shall also coincide with \( \epsilon_1 \), along with \( \delta_X \). No matter the form of the sound wave velocity, the derivative \( c_A \) is very complex, so we omit its analytic expression. These forms are very useful due to the fact that the power spectra \( P_S \) of the primordial curvature perturbations and the equilateral momentum approximation term \( f_{NL}^{eq} \) can be derived from such terms. These quantities are defined as,

\[ P_S = \frac{\kappa^4 V}{24\pi^2 \epsilon_s c_A}, \quad (37) \]

\[ f_{NL}^{eq} \simeq \frac{55}{36} \epsilon_s + \frac{5}{12} n + \frac{10}{3} \delta \xi. \quad (38) \]

In the following we shall appropriately specify the value of the term \( f_{NL}^{eq} \) during the first horizon crossing, to see what the constant-roll condition brings along. The evaluation shall be performed by using the values of the free parameters \( m \) in such a way so that the viability of the observational indices of inflation are compatible with the 2018 Planck data [46].

It is useful to note that, the spectral indices of scalar and tensor perturbations and the tensor-to-scalar ratio which will be numerically evaluated in the subsequent sections for appropriately chosen models, can be derived using the auxiliary parameters of this section, as follows,

\[ n_S = 1 - 2\epsilon_s - n - s - 8\delta \xi, \quad n_T = -2\epsilon_s - 8\delta \xi, \quad r = 16 \frac{\epsilon_s c_A}{1 - 8\delta \xi}. \quad (39) \]

These are obviously equivalent to the definitions presented in the previous section, but we shall proceed with the slow-roll expression.

**IV. SPECIFIC MODELS AND THEIR COMPATIBILITY WITH RECENT OBSERVATIONS**

As it was mentioned before, our main aim is to extract the value of the scalar field during the first horizon crossing and insert it as an input in Eq. (27). Firstly, we shall define the Gauss-Bonnet coupling scalar function. Afterwards, we shall derive the scalar potential from Eq. (15) corresponding to the selected coupling function. Accordingly, we shall equate the slow-roll index \( \epsilon_1 \) with unity in order to find the final value of the scalar field and finally, from Eq. (29) the initial value of the scalar field will be extracted.

Let us now discuss several models which can produce viable results.

**A. Model I: Power-Law Coupling Function**

Suppose that the Gauss-Bonnet coupling scalar function is defined as follows,

\[ \xi (\phi) = \lambda_1 (\kappa \phi)^{m_1}, \quad (40) \]

where \( \lambda_1 \) is an unspecified for the time being dimensionless constant. This is a very appealing function since the ratio \( \xi' \xi'' \) which appears in our calculations is greatly simplified, since,

\[ \xi'' = \frac{m_1 - 1}{\phi} \xi'. \quad (41) \]
In this case, we shall use the positive values of the scalar field. Assuming that in reduced Planck Units, where the auxiliary parameters. These are, incomplete from below gamma function. Let us now proceed with the evaluation of the slow-roll indices and certain observational quantities and the predicted non-Gaussianities of the model, can be extracted directly from equation (29). The resulting value is, As a result, the initial value of the scalar field, which is also the one that we need in order to evaluate both the parameters of the theory have the values $(\omega, \lambda_1, N, c, \beta, m_1)=(1, -1, 60, 0, 0.017, 10)$ then the observed quantities in Eq. (47) obtain values compatible with the current observational data. In fact, the spectral indices of the scalar and tensor perturbations, along with the tensor to scalar ratio, obtain the values $n_S = 0.965992$, $n_T = -0.06329 \cdot 10^{-6}$ and $r = 3.2506 \cdot 10^{-5}$ which are accepted values according to the recent Planck 2018 collaboration [46]. Furthermore, we mention that the initial and final value of the scalar field are $\phi_i = 0.0184556$ and $\phi_f = 12.948$ which indicates an increase in the scalar field. Lastly, we note that the slow-roll indices obtain the values $\epsilon_1 = 2.03164 \cdot 10^{-6}$, $\epsilon_4 = 6 \cdot 10^{-29}$, $\epsilon_5 = 6 \cdot 10^{-18}$ and $\epsilon_6 = 7 \cdot 10^{-18}$ which are extremely small.

Moreover, we make also predictions for the amount of non-Gaussianities in the primordial power spectrum of the curvature perturbations. From equations [33], the expected value of $f_{NL}$, for the exact same set of parameters we
FIG. 1: Contour plots of the spectral index of primordial curvature perturbations (right) and the tensor-to-scalar ratio (left) depending on parameters $\beta$ and $m$, ranging from $[0.01, 0.09]$ and $[4, 10]$ respectively. Concerning the spectral index, it is clear that the dominant parameter which defines its value is the constant-roll parameter and in fact there exists a very narrow area of acceptance which ranges approximately from 0.015 to 0.02, exactly where the value in our example resides.

FIG. 2: Parametric plot of the tensor-to-scalar ratio (x axis) and the spectral index of scalar perturbations (y axis) depending on parameters $\beta$ and $m$, ranging from $[-0.01, 0.09]$ and $[4, 26]$ respectively. Even in this case, it is clear that there exists a narrow area of acceptance for this set parameters due to the rage of compatible with the observations values of the spectral index $n_S$.

used to obtain the viability of the model with the Planck data, is $f_{NL}^{\tau\tau} = 0.0910216$ which is also an accepted value and may explain why non Gaussianities have yet to be observed. Finally, the parameters used to derive such values are equal to $\delta \xi = -10^{-18}$, $\epsilon_s = 2.0316 \cdot 10^{-6}$ and $\eta = 0.21844$ which means that one one of them is in fact dominant. These results imply that $\epsilon_s = \epsilon_1$.

At this point, it is also worth mentioning that the observed quantities $n_S$ and $r$ experience different changes when the values of the free parameters alter. For instance, the constant-roll parameter $\beta$ is the only one which affects the spectral index of scalar perturbations while the exponent $m$ of the coupling scalar function along with the constant-roll
parameter affect the tensor-to-scalar ratio, with the first being more decisive factor. This can easily be observed in Fig. 1 where one sees that the spectral index of scalar perturbations is depicted by a simple plot resembling vertical lines. In addition, while the term $f^{NL}_{\eta}$ is independent of parameter $\lambda$, it can be enhanced by decreasing the exponent $m$ but such a change leads to a subsequent decrease in the tensor-to-scalar ratio. For instance, choosing $m = 1.5$ leads to $f^{NL}_{\eta} = 1.22875$, $n_S = 0.966$ and the effective value of the tensor-to-scalar ratio is 0, since numerically speaking, $r \sim \mathcal{O}(10^{-102})$. Further information for the behavior of the spectral index and of the tensor-to-scalar ratio can be found in Fig. 2 where we present the parametric plot of the tensor-to-scalar ratio (x axis) and of the spectral index of scalar perturbations (y axis) depending on parameters $\beta$ and $m$, ranging from [-0.01, 0.09] and [4,26] respectively.

Another comment that should be briefly discussed here is the form of the scalar potential. From the continuity equation, it becomes apparent that (42) is quite complex, however this is not true. Since $\mathcal{G} \sim \mathcal{O}(10^{-15})$ whereas $\xi' \mathcal{G} \sim \mathcal{O}(10^{-17})$ while $16 \xi H \mathcal{H} \sim \mathcal{O}(10^{-23})$, which explains why these terms, compared to the scalar potential and the kinetic term, can be neglected. Lastly, $V' \sim \mathcal{O}(10^{-3})$ whereas $\xi' \mathcal{G} \sim \mathcal{O}(10^{-15})$ which explains why the second form of the scalar potential presented in Eq. (52) is equivalent to that of (42).

B. Model II: Advanced Exponential Model

Let us now assume that the coupling scalar function has the following form,

$$\xi(\phi) = \kappa \lambda_2 \int e^{\gamma_2 \phi} dx,$$  \hspace{1cm} (53)

where $\gamma$ is an auxiliary integration variable. This may seem like a strange choice, but it can be justified due to the simple form of the ratio $\xi'/\xi''$ which appears in our calculations, as

$$\xi'' = m_2 \gamma_2 \phi \xi'/\xi'.$$  \hspace{1cm} (54)

In order to find the expression of the scalar potential, we must make use of Eq. (13). However, the differential equation is not so easy to solve. To do so, we must make an additional approximation which is reasonable and it realized in the following equation,

$$V' + \kappa^2 \omega (1 - \beta) \left( 1 + \frac{\beta}{3} \right) \frac{\xi'}{\xi} V \simeq 0.$$  \hspace{1cm} (55)

Using this differential equation, the resulting scalar potential is,

$$V(\phi) = \psi \exp (\alpha_2 (\kappa \phi)^{2-m_2}),$$  \hspace{1cm} (56)

where here, $\alpha_2 = \kappa (\gamma_2^2 + 2 \gamma_2 - 3) \left[ \gamma_2 \phi \right]^{2-m_2}$ and $V_2$ the integration constant with mass dimensions $[m]^4$. Continuing, the resulting expressions for several terms of interest are shown below,

$$\delta_\epsilon \simeq \frac{(1 - \beta) \kappa \phi \lambda_2}{3 \gamma_2 m_2} \psi \exp \left( \phi (\kappa \phi)^{-m_2} e^{\gamma_2 (\kappa \phi)^{m_2}} \right),$$  \hspace{1cm} (57)

$$\epsilon_\delta \simeq \frac{(1 - \beta) (\kappa \phi)^{1-2m_2} \left( 3(1 - \beta) \kappa \phi \omega - 8 \gamma_2 \lambda_2 m_2 \kappa^4 \psi \exp \left( \phi (\kappa \phi)^{m_2} e^{\gamma_2 (\kappa \phi)^{m_2}} \right) \right)}{6 \gamma_2 m_2^2}.$$  \hspace{1cm} (58)
\[
\epsilon_1 \simeq \frac{\omega}{2} \left( \frac{1 - \beta}{m_2 \gamma_2} \right)^2 (\kappa \phi)^2 (1 - m_2),
\]
(59)

\[
\epsilon_2 = \beta,
\]
(60)

\[
\epsilon_3 = 0,
\]
(61)

\[
\epsilon_5 \simeq \frac{4(\beta - 1)\kappa \phi \lambda_2 \kappa^4 V(\phi) e^{\gamma_2 (\kappa \phi)^{m_2}}}{3 \gamma_2 m_2 (\kappa \phi)^{m_2}} + 8(\beta - 1)\kappa \phi \lambda_2 \kappa^4 V(\phi) e^{\gamma_2 (\kappa \phi)^{m_2}},
\]
(62)

\[
\epsilon_6 \simeq -\frac{4(\beta - 1)^2 \lambda_2 (\kappa \phi)^{1 - m_2} e^{\gamma_2 (\kappa \phi)^{m_2}} (m_2 \kappa^4 V(\phi) (\gamma_2 (\kappa \phi)^{m_2} - 1) + \kappa \phi \kappa^3 V'(\phi) + \kappa^4 V(\phi))}{\gamma_2 m_2 (3 \gamma_2 m_2 (\kappa \phi)^{m_2} + 8(\beta - 1)\kappa \phi \lambda_2 \kappa^4 V(\phi) e^{\gamma_2 (\kappa \phi)^{m_2}})}.
\]
(63)

Similar to the previous model, index \( \epsilon_4 \) was omitted due to its intricate form. Finally, as was the case with the previous model, we present the initial and final value of the scalar field,

\[
\phi_f = \frac{1}{\kappa} \left( \sqrt{\frac{\omega}{2}} \frac{1 - \beta}{m_2 \gamma_2} \right)^{\frac{1}{m_2 - 1}},
\]
(64)

\[
\phi_i = \frac{1}{\kappa} \left( (\kappa \phi_f)^{m_2} - \frac{N(1 - \beta)}{\gamma_2} \right)^{\frac{1}{m_2}}.
\]
(65)

Assuming that in Planck Units, \((\omega, \lambda_2, N, V_2, \beta, m_2, \gamma) = (1, 100, 60, 1, 0.017, 3, -1)\) then the resulting spectral index of primordial curvature perturbations and the tensor-to-scalar ratio are compatible with the observations, as \( n_S = 0.965059 \) and \( r = 0.003732 \) are acceptable values. Furthermore, we mention that the unobserved spectral index of tensor perturbations obtains the value \( n_T = -0.000466 \) and for the scalar field, \( \phi_i = 3.89501 \) and \( \phi_f = 0.481347 \) which shows that the scalar field decreases with time. Finally, when it comes to the slow-roll indices, the majority of them have extremely small values as \( \epsilon_1 = 0.00023, \epsilon_4 = 4.17 \cdot 10^{-47} \) and \( \epsilon_5 = 5.7 \cdot 10^{-26} = \epsilon_6 \). Concerning the non-Gaussianities issue, it turns out that the term \( f^{eq}_{NL} \) obtains the value \( f^{eq}_{NL} = 0.009598 \) which is obviously small. Moreover, \( \delta_\xi = -1.43 \cdot 10^{-26}, \epsilon_s = 0.00025 \) and \( \eta = 0.02218 \) In this model, not only does \( \epsilon_s \) coincide with index \( \epsilon_1 \), but two of the three parameters for evaluating the non-Gaussianities have non negligible values.
In this case, the exponent $m$ affects both the spectral index of scalar perturbations and the tensor-to-scalar ratio. The same applies to the constant-roll parameter $\beta$ but in this case, only the spectral index experiences a significant change. Lastly, $\gamma_2$ alters both values and as a matter of fact in not so significant rate. Decreasing $\gamma_2$ to the value $-10$ alters the fourth decimal in each magnitude. In contrast, if $\beta$ was to obtain the value 0.015, which is a really small change, the spectral index takes a non-compatible value with the observations, which indicates the great impact such a change in the parameters has. The dependence on $\beta$ and $\gamma_2$ can be viewed in Fig. 3. Finally, the exponent may vary in the range $[3,14]$ and the only change which the observed quantities shall experience is in the fourth and fifth decimal, with the tensor-to-scalar ratio experiencing a decrease.

Let us proceed with the dynamics of the system in terms of altering certain free parameters. It turns out that many parameters leave the results unaltered, while others play a significant role. For instance, changing the constant-roll parameter $\beta$ to 0.015, while it affects greatly the spectral index of primordial curvature perturbations, it does not alter the order of magnitude of $f_{NL}$, just changes the numerical value in the same order. Even if it changes to, lets say $\beta = 0.6$, the results are the same. In contrast, the exponent $m$ affects significantly the parameters, since an increase in the exponent, leads to a decrease in the tensor-to-scalar ratio but also enhances the non-Gaussianities and even by a lot. As it was shown in the slow-roll case [36], the exponent $m$ alters the order of magnitude of $f_{NL}$, just changes the numerical value in the same order. Even if it changes to, lets say $m = 100$ leads to viable results and also increases the non linear term only by one order, meaning that $f_{NL} = 0.0137485$.

Finally, we discuss the approximations made throughout the equations of motion. When it comes to the slow-roll approximations, we note that $H \sim \mathcal{O}(10^{-5})$ while $H^2 \sim \mathcal{O}(10^{-1})$ and similarly, $\frac{1}{3} \omega \dot{\phi}^2 \sim \mathcal{O}(10^{-5})$ and $V \sim \mathcal{O}(10^{-1})$ which shows that the approximations in fact do apply. In addition, the following terms are $24 \xi H^3 \sim \mathcal{O}(10^{-25})$, $16 \xi H \dot{H} \sim \mathcal{O}(10^{-29})$ which justifies why they were neglected in equations (3) and (4). Furthermore, for equation (5), $V' \sim \mathcal{O}(10^{-2})$ in contrast to $24 \xi' H^4 \sim \mathcal{O}(10^{-23})$ so it is reasonable why the latter was neglected.

As a last comment, we mention that the dominant parameters which affect the results are mainly the constant-roll parameter $\beta$ and parameters $\gamma_2$ and $m_2$ while $\lambda_2$ seems to not cause any change to the results along with $V_2$.

C. Comparison Between Slow-roll And Constant-Roll

In this case, we shall work with the previous model but implement a different formalism. Here, we shall set $\beta$ equal to zero, so visually it will disappear from all the previous equations but in reality, for the scalar field we assume that in addition to the slow-roll approximations (6), the approximation $\dot{\phi} \ll \dot{\phi}H$ holds. This will lead to a change with the new set of equations being,

$$\xi(\phi) = \kappa \lambda_3 \int_{e^{\gamma_2 \lambda m}}^{e^{\phi}} e^{\gamma_2 z^m} dx,$$

$$\dot{\phi} \simeq H \frac{\xi'}{\xi''},$$

$$H^2 \simeq \frac{\kappa^2 V}{3},$$

$$\dot{H} \simeq -\frac{H^2}{2} \kappa^2 \omega \left( \frac{\xi'}{\xi''} \right)^2,$$

$$V' + \omega \kappa^2 \frac{\xi'}{\xi''} V \simeq 0.$$  

This model was studied thoroughly in [36] and it is capable of producing viable results. Before we proceed with the results however, it is worth mentioning the changes to which the scalar potential and the slow-roll indices will be subjected to. From the previous set of equations, the resulting scalar potential is,

$$V(\phi) = V_3 e^{-\frac{(\lambda^2 - m_3)}{\gamma_3 \lambda^2 - m_3}},$$
which as expected is the same as before, \cite{55} with $\beta = 0$. Similarly,

$$
\delta_\xi \simeq \frac{\kappa \phi \lambda_3 \kappa^4 V(\phi)(\kappa \phi)^{-m_3} e^{\gamma_3 (\kappa \phi)^{m_3}}}{3 \gamma_3 m_3},
$$

(72)

$$
\epsilon_s \simeq \frac{(\kappa \phi)^{1-2m_3} (3 \kappa \phi \omega - 8 \gamma_3 \lambda_3 m_3 \kappa^4 V(\phi)(\kappa \phi)^{m_3} e^{\gamma_3 (\kappa \phi)^{m_3}})}{6 \gamma_3^2 m_3^2},
$$

(73)

$$
\epsilon_1 \simeq \frac{\omega (\kappa \phi)^2 (1-m_3)}{2 (\gamma_3 m_3)^2},
$$

(74)

$$
\epsilon_2 \simeq \frac{(\kappa \phi)^{-2m_3} (\omega (\kappa \phi)^2 + 2m_3 (m_3 - 1) \gamma_3 (\kappa \phi)^{m_3})}{2 \gamma_3^2 m_3^2},
$$

(75)

$$
\epsilon_3 = 0,
$$

(76)

$$
\epsilon_5 \simeq \frac{4 \kappa \phi \lambda_3 \kappa^4 V(\phi) e^{\gamma_3 (\kappa \phi)^{m_3}}}{8 \kappa \phi \lambda_3 \kappa^4 V(\phi) e^{\gamma_3 (\kappa \phi)^{m_3}} - 3 \gamma_3 m_3 (\kappa \phi)^{m_3}},
$$

(77)

$$
\epsilon_6 \simeq \frac{-4 \kappa \phi \lambda_3 (\kappa \phi)^{-m_3} e^{\gamma_3 (\kappa \phi)^{m_3}}}{\gamma_3 m_3 (\kappa \phi)^{m_3}} (m_3 \kappa^4 V(\phi) (\gamma_3 (\kappa \phi)^{m_3} - 1) + \kappa \phi \kappa^3 V''(\phi) + \kappa^4 V(\phi)).
$$

(78)

Similarly, apart from $\epsilon_2$, all the indices coincide with those previously for $\beta = 0$, as expected. Following, the exact same steps, the initial and final value of the scalar field read,

$$
\phi_f = \frac{1}{\kappa} \left( \sqrt{\frac{\omega}{2}} \left| \frac{1}{\gamma_3 m_3} \right| \right)^{\frac{1}{m_3-1}},
$$

(79)

$$
\phi_i = \frac{1}{\kappa} \left( (\kappa \phi)^{m_3} \frac{N}{\gamma_3} \right)^{\frac{1}{m_3}}.
$$

(80)

Assuming that in Planck Units, $(\omega, \lambda_3, N, V_3, m_3, \gamma_3) = (1, 1, 60, 1, 20, -0.001)$ then from Eq \cite{27}, we obtain the values $n_S = 0.968331$, $n_T = -2.08667 \cdot 10^{-5}$ and $r = 1.6693 \cdot 10^{-5}$ which are compatible results with the observations results. Moreover, $\phi_i = 1.7335$ and $\phi_f = 1.20642$ in Planck Units, which shows a decrease with time. And finally, $\epsilon_1 = 1.04 \cdot 10^{-6}$, $\epsilon_4 = 7 \cdot 10^{-52}$ and $\epsilon_5 = 10^{-29} = \epsilon_6$ which indicates that the slow-roll conditions indeed apply.

The main aim of the analysis performed in this subsection however, was to evaluate and predict the amount of non-Gaussianities in the power spectrum. The above set of parameters leads to the value $f_{NL}^{eq} = 0.013196$. Similarly, $\delta_\xi = -4 \cdot 10^{-30}$, $\epsilon_s = \epsilon_1$ and $\eta = 0.03164$. A quick comparison between the models corresponding to the slow-roll and constant-roll case, indicate that in the constant-roll case the value of $f_{NL}^{eq}$ decreases, since viability can be achieved for smaller values of the exponent $m$. Thus the main difference between the two phenomenologies is the set of values for the free parameters that can achieve both viability for the observed the spectral index of primordial curvature perturbations and the tensor-to-scalar ratio. In conclusion, both the slow-roll and the constant-roll condition of this particular model are more than capable of describing a viable phenomenology and in fact are able to predict the same amount of non-Gaussianities, and remarkably in the constant-roll case, slightly smaller amount of non-Gaussianities.

V. PHENOMENOLOGY BY IMPOSING THE CONDITION $\kappa \xi' / \xi'' \ll 1$

In this section we shall assume that the following condition holds true $\kappa \xi' / \xi'' \ll 1$, and we shall examine the phenomenological implications for an appropriately chosen model. Thus the differential equation that connects the scalar potential and the scalar coupling function takes the form,

$$
V' + \frac{8}{3} \kappa^4 \xi' V^2 \simeq 0.
$$

(81)
This is a simple ordinary differential equation which has the following solution,

\[
V(\phi) = \frac{1}{\frac{2}{3} \kappa^4 \xi(\phi) - \Lambda},
\]

(82)

where \(\Lambda\) is an integration constant with mass dimensions \([m]^{-4}\). By appropriately choosing the Gauss-Bonnet coupling scalar function, specifies immediately the scalar potential. In this model, let us assume that the coupling function is defined as,

\[
\xi(\phi) = \lambda_4 E r f(\gamma_4 \kappa \phi).
\]

(83)

This is a model which was also studied in our previous work \([36]\). It is an appropriate function since,

\[
\xi'' = -2(\gamma_4 \kappa)^2 \phi \xi',
\]

(84)

thus the ratio \(\xi'/\xi''\) is greatly simplified. In addition, the corresponding slow-roll indices are written as,

\[
\delta_\xi \simeq -\frac{(1 - \beta) \lambda_4 \kappa^4 V(\phi) e^{-(\gamma_4 \kappa \phi)^2}}{3 \sqrt{\pi \gamma_4 \kappa \phi}},
\]

(85)

\[
\epsilon_s \simeq \frac{(1 - \beta) \left(32 \gamma_4^3 \kappa \phi \lambda_4 \kappa^4 V(\phi) e^{-(\gamma_4 \kappa \phi)^2} + 3 \sqrt{\pi \omega}(1 - \beta)\right)}{6 \sqrt{\pi}(2 \gamma_4^2 \kappa \phi)^2},
\]

(86)

\[
\epsilon_1 \simeq \frac{\omega}{2} \left(\frac{1 - \beta}{2 \gamma_4^2 \kappa \phi}\right)^2,
\]

(87)

\[
\epsilon_2 = \beta,
\]

(88)

\[
\epsilon_3 = 0,
\]

(89)

\[
\epsilon_5 \simeq \frac{4(1 - \beta) \lambda_4 \kappa^4 V(\phi)}{8(1 - \beta) \lambda_4 \kappa^4 V(\phi) + 3 \sqrt{\pi \gamma_4 \kappa \phi} e^{(\gamma_4 \kappa \phi)^2}},
\]

(90)

\[
\epsilon_6 \simeq \frac{2(1 - \beta)^2 \kappa \lambda_4 \left(-\kappa \phi e^{3V'(\phi)} + (2(\gamma_4 \kappa \phi)^2 + 1) \kappa^4 V(\phi)\right)}{(\gamma_4 \kappa \phi)^2 \left(8(1 - \beta) \lambda_4 \kappa^4 V(\phi) + 3 \sqrt{\pi \gamma_4 \kappa \phi} e^{(\gamma_4 \kappa \phi)^2}\right)}.
\]

(91)

Finally, we mention that the values of the scalar field during the initial and final moment of inflation are,

\[
\phi_i = -\sqrt{\frac{\omega |1 - \beta|}{2 \gamma_4^2 \kappa}},
\]

(92)

\[
\phi_f = \frac{1}{2 \gamma_4^2 \kappa} \sqrt{8 N \gamma_4^2 + \omega(1 + \beta^2 - 2\beta)}.
\]

(93)

Assuming that in Planck Units, \((\omega, \lambda_4, N, \Lambda, \beta, \gamma_4) = (1, 10^4, 60, 0, 0.013, 1)\) then the resulting values for the spectral index of primordial curvature perturbations and the tensor-to-scalar ratio are \(n_S = 0.965829\) and \(r = 0.0324065\), which are are both compatible results with the Planck 2018 data \([46]\). Furthermore, the spectral index of tensor perturbations is \(n_T = -0.00405904\) and the values of the scalar field are \(\phi_i = 7.75382\) and \(\phi_f = -0.348957\) which indicates a decrease in the scalar potential. And finally, when it comes to the slow-roll indices, \(\epsilon_1 = 0.00202, \epsilon_4 = 1.13 \cdot 10^{-52}, \epsilon_5 = 2.7 \cdot 10^{-28} = \epsilon_6\). The effective value of the last three is obviously zero.

In addition, the predicted values for the non-Gaussianities are also compatible results. We mention that the equilateral non linear term obtains the value \(f_{NL}^0 = 0.009934\) which is quite a small value. Also, \(\delta_\xi = -6.95 \cdot 10^{-28}, \epsilon_s = \epsilon_1, \eta = 0.0164167\). The \(f_{NL}^0\) term can obtain a greater value by decreasing \(\gamma_4\), but such decrease leads
to a subsequent increase in the tensor-to-scalar ratio so it must be made with care. Choosing $\gamma_4 = 0.8$ leads to $f_{NL}^{eq} = 0.011662$ while producing also viable spectral indices and tensor-to-scalar ratio. Here, $\gamma_4$ affects the following quantities, the tensor-to-scalar ratio, the term $f_{NL}^{eq}$ and the spectral index of scalar perturbations, but mainly the first two, while the latter are affected greatly by the constant-roll parameter $\beta$.

Lastly, we examine the validity of our approximations. Concerning the slow-roll approximations, we note that $\dot{H} \sim O(10^{-8})$ compared to $H^2 \sim O(10^{-5})$ similarly $\frac{1}{2} \omega \dot{\phi}^2 \sim O(10^{-8})$ in contrast to $V \sim O(10^{-5})$. Indeed, the approximations in (3) are valid. Moreover, the string terms in the equations of motion (3) and (4) are $24 \xi H^3 \sim O(10^{-32})$ and $16 \dot{H} H \sim O(10^{-35})$ which justifies the reason they were neglected. Also, the ratio $\xi / \xi''$ is of order $O(10^{-3})$. The term $V'$ is of same order as $\xi' V^2$.

This set of values for the free parameters of the theory is interesting due to the fact that selecting $\Lambda = 0$ implies that,

$$V(\phi) = \frac{3}{8 \kappa^4 \xi(\phi)}, \quad (94)$$

which is an interesting relation between the scalar functions of the model.

VI. CONCLUSIONS

In this work we investigated the quantitative effects of imposing a constant-roll evolution on the scalar field for a GW170817-compatible Einstein-Gauss-Bonnet theory. Our focus was on the inflationary era, and we calculated the slow-roll indices and the observational quantities of inflation, and we confronted several models with the observational data coming from the Planck 2018 collaboration. As we demonstrated, the resulting inflationary phenomenology can be compatible with the latest Planck data, for a wide range of the free parameters of the theory, and with the constant-roll condition holding true. In our calculations we demonstrated that all the assumptions we made were satisfied for all the models we examined, and for the values of the free parameters that yield inflationary viability with respect to the latest Planck data. For all the models we studied, we also investigated the amount of non-Gaussianities that are predicted from the models, by calculating the quantity $f_{NL}^{eq}$ in the equilateral momentum approximation. Interestingly enough, we demonstrated that the amount of non-Gaussianities is quite small in the constant-roll case, and also in some cases, where we compared the slow-roll and constant-roll cases explicitly, we showed that the quantity $f_{NL}^{eq}$ is even smaller in the constant-roll case, compared to the slow-roll case. Finally, we performed an analytic approximation in the differential equation that connects the scalar field potential and the scalar coupling function, and we examined the phenomenology of inflation in this case too. As we evinced, the model can also be compatible with the Planck 2018 too.

A future study should address the important feature of having the constraint $c^2_f = 1$ holding true after the slow-roll or constant-roll era, during the reheating era and beyond. In that case, the differential equation that connects the scalar field potential $V(\phi)$ and the scalar coupling function $\xi(\phi)$ is not simplified, as it was during the inflationary era, thus one may use the exact form of the differential equation, and impose the constraint that it holds true for all the post-inflationary eras, and that it exactly defines the interconnection of the scalar potential and of the scalar-Gauss-Bonnet coupling. This differential equation could be taken as an additional constraint in the theory, and may affect the reheating era if the constant-roll assumption is used, like for example in Ref. [30], or even if the slow-roll assumption is used. The point is that in our previous work [30], and in the present work, we found only an approximate relation for the scalar potential and the scalar-Gauss-Bonnet coupling function, however, in the post-inflationary era, the full differential equation should be taken into account, and thus, one may have the exact relation between the two scalar functions, without the need of any approximation. Thus, one may use this differential equation as an additional constraint, and solve thus numerically problems of astrophysical or cosmological interest. This task is in our future plans and we aim to materialize this in the next years, motivated by the current astrophysical and cosmological interest on the gravitational wave speed [61, 62].

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