TESTING METHODOLOGIES FOR THE CALIBRATION OF ADVANCED PLASTICITY MODELS FOR SHEET METALS: A REVIEW

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Abstract
Numerical simulations have become essential in engineering and manufacturing processes involving plasticity. The reliability and effectiveness of the simulations depend strongly on the accuracy of the adopted constitutive model. Accordingly, in recent years, an increasing interest is pointed towards experimental procedures and characterization methods that can be used to identify the constitutive parameters of advanced plasticity models, which allow to simulate properly the plastic behaviour of complex materials like, for instance, high-strength steel. This paper provides a thorough review of the current state-of-the-art, looking at both academia and industry. The available methodologies can be subdivided in two main areas: quasi-homogeneous material tests with analytical or numerical post-treatment of the experimental data and heterogeneous tests coupled with inverse methods for parameter identification. For each method, a brief description and references to norms and articles is provided, illustrating the advantages and the disadvantages.

KEYWORDS
constitutive modelling, homogeneous test, inverse methods, mechanical testing methods, plasticity

1 | INTRODUCTION

The constitutive behaviour of metals under plastic deformation is rather complex and can be described using a large number of theoretical models developed in the last century, since the first works of von Mises and Hill. Such models are continuously updated to meet the requirements of the industry in terms of new materials and the rapid advancements of the simulation capabilities through finite element (FE) analysis.

Every model relies on a set of constitutive parameters that must be identified by means of experiments. In order to correctly describe the plastic behaviour of materials, three aspects have to be considered: (i) the yielding point that triggers the phenomenon; (ii) the plastic flow rule that indicates how the material deforms under the prescribed stress field; and (iii) the hardening law that takes into account the changes in the material behaviour after plastic deformation occurs. Each of these three aspects can be very difficult to address properly. The main difficulties come from material
anisotropy (i.e., the properties depend on the material orientation), differential hardening (i.e., the shape of the yield surface changes as plastic strain increases), strain path change effects such as the Bauschinger effect (i.e., a different behaviour in compression after a prestrain in tension or more generally when reloading in the opposite direction), damage accumulation, plastic instabilities, and so on.

From the experimental point of view, the key feature is being able to investigate the major number of conditions in terms of different types of loading conditions and different levels of accumulated plastic deformation. To this purpose, the main available methodologies, which represents the current state-of-the-art, can be conveniently subdivided in two main areas: Quasi-homogeneous tests. In this case, the objective is producing a homogeneous mechanical state in a certain area of the specimen so that the stress and strain can be directly derived from the experimental raw data by making simple assumptions (e.g., volume constancy and plane stress). Each stress state is consequently studied using a dedicated test.

Heterogeneous tests coupled with inverse methods. The idea behind this approach is testing multiple stress and strain states within a single specimen through heterogeneous fields. As it is not possible to calculate directly the stress and strain components from the experimental raw data, and the identification problem must be solved iteratively using an inverse method.

Each approach has its pros and cons. For instance, quasi-homogeneous testings require a greater experimental effort to identify complex material models with a large number of parameters but, on the other hand, the experimental procedures are more simple and do not require a complex post-elaboration as often occurs using inverse methods. In the following sections, both approaches are thoroughly analysed, highlighting the recent developments.

Furthermore, in the last section, new experimental procedures and computational methods, for example, artificial neural network (ANN), that are starting to be used in the characterization of plasticity and are expected to mature in the near future, are briefly presented.

2 | QUASI-HOMOGENEOUS TESTS

Homogeneous material testing, historically, was the first approach used for the mechanical characterization of materials and consists in generating homogeneous stress and strain fields in a specimen designed to this purpose and cut from the material to be tested; by controlling simultaneously the stress (or strain) state and the strain rate during the test, the response of the material is measured and its mechanical properties are then derived. Starting from this premise, a wide range of tests have been devised over time in order to study the material behaviour under different mechanical states.

As an illustration of the relationship between the stress state and the mechanical test, Figure 1 summarizes some of the testing methods used for the characterization of sheet metals. Each point of the yield surface corresponds to a

**FIGURE 1** Overview of testing methods for sheet metal characterization
particular type of loading, for example, tension, compression, shear and equi-biaxial load. For each point, it is necessary to study and implement a particular type of specimen, boundary conditions, loading and testing procedure.

In this article, focused on metallic sheet materials, the types of homogeneous tests were further grouped into the following main categories:

1. **tensile tests**: where the state of stress is mainly uniaxial tension;
2. **multi-axial tests**: where the state of stress is obtained applying multiple loads with suitable actuators;
3. **stack compression tests**: where multiple layer of metallic sheets undergo a trough-thickness compression test;
4. **tension-torsion and shear tests**: where the state of stress in mainly shear;
5. **hydraulic bulge tests**: where the loading is obtained by means of the pressure of a fluid.

The main features and example of experimental set-ups for different categories will be detailed in the next sections.

### 2.1 Tensile tests

Uniaxial tensile tests are by far the most widespread tests used in both academic and industrial fields because of their ease to use and low cost, as they provide useful information about the strength and ductility of materials. ASTM E8/E8M-21,

[1] ISO 6892-1

[2] and JSA-JIS Z2241

[3] standards specify test methods that cover the quasi-static tension testing of metallic materials in any form at room temperature. An example of a tensile test set-up is shown in Figure 2.

The specimen geometry is defined by the standards in order to generate uniform stress and strain distributions on the cross-sectional area, and along a gauge length, used for measurement. Moreover, the external load is applied only in one direction, and only one stress tensor component is non-zero, corresponding to a uniaxial test. In this way, it is possible to obtain basic quantities such as yield strength, yield point elongation, tensile strength (or ultimate tensile strength), Young's modulus and elongation at break, and to determine the true (or Cauchy) stress $\sigma$ versus true (or logarithmic) strain $\varepsilon$ curve of the material as follows:

![Figure 2: Example of a tensile test on a flat specimen. The large length over width ratio, as well as the rather small sheet thickness, guarantees that only one stress component is non-zero](image-url)
\[ \varepsilon = \ln \left( \frac{L_0 + \Delta L}{L_0} \right) \]
\[ \sigma = \frac{F}{A_0} \exp(\varepsilon) \]

where \( F \) is the measured force, \( \Delta L \) is the axial displacement, \( A_0 \) is the initial area of the specimen and \( L_0 \) is the reference length.

However, necking is a very limiting phenomenon, which prevents an easy output of data beyond the localization point. At the onset of necking, indeed, the stress state departs gradually from uniaxiality and from uniformity across the neck section, as illustrated by a triaxiality ratio changing from 0.333 up to 0.45–0.5 (e.g., previous works\(^4\)). Consequently, during the post-necking phase the true stress-true strain curve obtained using Equation (1) is no longer valid and does not represent the real material behaviour, described in terms of the equivalent stress-strain curve. The problem is briefly sketched in Figure 3.

The problem of the post-necking hardening behaviour identification is still open. The first and most common approach is due to Bridgman (1952)\(^6\) who developed an analytical method for the determination of the equivalent stress-strain curve in the post-necking phase with the definition of a correction factor that correlates true stress with the equivalent stress. The method, valid for cylindrical specimens and extended later by Zhang et al. (1999)\(^7\) for specimens with a rectangular cross-section and a maximum aspect ratio of 8, assumes that equivalent stress and strain are uniform over the specimen minimum cross section and requires the measurement of the instantaneous curvature radius of the necking profile. Although the Bridgman correction is approximate and not very accurate, because of its simplicity, it is still often used in practical applications.

Alternative and more advanced approaches were developed to solve the post-necking problem. For example, Mirone suggested a material-independent solution that should achieve an error level less than half with respect to the Bridgman method.\(^8\) For sheet metals, Wang and Tong developed a correction method that exploits the full-field measurement over the diffuse necking surface extending the application to a general non-quadratic anisotropic flow function.\(^9\)

Recently, Tu et al. (2019) provided an overview on quasi-static standard tensile tests of metallic materials and on methods for characterization of the equivalent stress-strain curve in the post necking regime,\(^5\) where further details on this topic can be found.

Tensile tests are also the main experimental procedure used to characterize the thermomechanical behaviour of metals at high temperatures. Two of the most important reference standards are ASTM E21-20\(^10\) and ISO 6892-2.\(^11\) In this field, many experiences come from the automotive industrial sector related to hot stamping process where, for instance, Merklein and Lechler studied an experimental set-up to determine thermomechanical properties of an Ultra High Strength Steel.\(^12\) The interest in the behaviour of steels at elevated temperatures comes also from the civil engineering field, where steel is used in the construction of buildings. In this context, Chen and Young\(^13\) performed an

**Figure 3** Difference between the true stress and the equivalent stress after diffuse necking\(^5\)
experimental study to investigate the behaviour of cold-formed steel at elevated temperatures using both steady and transient test methods; their set-up is illustrated in Figure 4. The interest of warm forming of aluminium alloys, to decrease the springback magnitude,\textsuperscript{14} has also arisen numerous investigations of the thermo-mechanical behaviour of these materials.\textsuperscript{15}

Finally, and as a transition to the next section, multi-axial stress states can be reached with only one loading direction but specific sample geometry and boundary conditions. As an example, tensile tests can be used to explore the plane strain condition using suitable specimen geometry where the width is much larger than the axial length. In this case, the deformation in the transverse direction is constrained and can be considered close to zero. The stress state is no longer uniaxial because the plane strain condition induces a positive tensile stress in the transverse direction. An example of plane strain test can be found in Aretz et al,\textsuperscript{16} where it is used to calibrate the Yld2003 model,\textsuperscript{17} and in Ha et al,\textsuperscript{18} where an inverse method is coupled with the plane strain test.

### 2.2 | Multi-axial tests

Mechanical components in actual applications are usually subject to complex stress states different from the simple uniaxial state; therefore it is very important to experimentally investigate what is the material behaviour under such conditions. Furthermore, foundry, rolling and forming processes can induce anisotropy related to the material crystallographic texture, leading to complex yielding phenomena that depend on the material orientation. In order to evaluate the yield surface, it is necessary to investigate several states that are not achievable with standard tensile tests.

To this purpose, multi-axial tests methods can be employed, where the specimens are loaded along two or more axes to investigate different loading paths. For sheet metals, the biaxial tensile test on cruciform specimens is certainly one of the most used, the ISO 16842\textsuperscript{19} standard specifies the method for measuring stress-strain curves at room temperature. One of the works that the standard takes as a reference is that of Merklein and Biasutti,\textsuperscript{20} who presented a standalone biaxial testing machine with out-of-plane loading system to study the anisotropic plastic behaviour of metal sheets on cruciform specimens. Nagayasu et al. were the first to propose a mechanism that enables the subjection of a specimen to biaxial tension using a standard uniaxial tensile machine.\textsuperscript{21} The biaxial test on cruciform specimen was also used under warm conditions and at several loading speeds.\textsuperscript{22,23}

A scheme of a biaxial testing machine, proposed by Kuwabara et al,\textsuperscript{24} is shown in Figure 5a: four hydraulic cylinders are used to load two perpendicular axes, a cruciform specimen is set in the centre of the machine and a pantograph-type link mechanism is used to equalize the displacements generated by the opposite cylinders. An example of an actual machine and set-up for the biaxial test is shown in Figure 5b.

**FIGURE 4** Experimental set-up used to characterize the thermomechanical behaviour of steel for building applications.
The design of the specimen as well as the position of the biaxial strain gauge is illustrated in Figure 6. In order to have a rather constant stress state in the centre of the cruciform specimen, it is necessary to introduce a series of slits, which avoid that the measurement gauge (i.e., the central area in which the stress state must be controlled) is geometrically constrained by the arms of the cruciform. Such specimen design was initially developed by Kuwabara et al.\cite{24} and used also in Andar et al.\cite{25} An important factor for the success of the biaxial tensile test is the minimization of the stress calculation error, and Hanabusa et al. suggested a method for determining the optimum strain measurement position.\cite{26} The design of the cruciform specimen is fundamental, as indicated also in Deng et al.\cite{27} because stresses are difficult to evaluate from the loads and the strains achievable in the test-section before failure are limited. It should be highlighted that such tests may also induce heterogeneous stress and strain fields, making them suitable for model calibration using inverse methods.\cite{28-30}

In the literature\cite{31,32} there are many other cases of multi-axial tests developed over time; for example, Calloch and Marquis presented a triaxial tension/compression test and a triaxial specimen for the study of cyclic plasticity under non-proportional loading.\cite{33} Seymen et al. studied a compact and portable apparatus for test of miniature cruciform specimens.\cite{34}
Multi-axial tests are usually combined with the standard uniaxial tests, but other homogeneous tests can be simultaneously considered to improve the quality of the characterization. For instance, Kim et al. studied the formability of steel sheet samples involving uniaxial tensile tests, biaxial (disk) compression tests, hydraulic bulge tests and biaxial tensile tests on cruciform specimens.\[35\]

2.3 | Stack compression test

The (uniaxial) Stack Compression Test (SCT) is a small-scale material test that essentially comes down to a compression test on a stack of sheet metal specimens cut from the base material by any suitable method, for example, spark erosion. The stack can consist of small circular discs or square specimens. During the test, the stack is compressed between parallel plates of a uniaxial testing machine. Figure 7 depicts a SCT in thickness direction of the sheets; in that case, the stress state is equivalent to balanced biaxial tension provided that the yield stress is independent of the hydrostatic pressure and friction can be safely ignored. In this regard, however, it must be noted that a recent study\[36\] reported a
discrepancy between the SCT and the hydraulic bulge test, indicating a positive correlation between the hydrostatic pressure and the flow stress. The SCT can also be used to subject sheet metal to in-plane compression, for example, to study the strength differential effect. To this end, stacks are glued together prior to testing as presented by Maeda et al.\(^{37}\)

A clear benefit is that the SCT requires only a small amount of test material which can be locally removed. The SCT enables measurement of the strain flow curve up to large strain because plastic instabilities, such as necking, are not present.\(^{38}\) Friction between the stack and the compression platen is inevitable and potentially leads to heterogeneous deformation of the stack.\(^{39}\) The average compressive true stress in the stack is calculated as follows:

\[
\sigma_{\text{avg}}^c = \frac{Fh}{\pi r_0^2 h_0}
\]  

(2)

where \(\sigma_{\text{avg}}^c\) is the average compressive true stress, \(F\) the measured force and \(h_0, r_0\) the initial stack height and radius, respectively. The logarithmic true compressive strain is simply:

\[
\varepsilon = \ln \left( \frac{h}{h_0} \right)
\]  

(3)

The friction-hill analysis of a homogenous compression of a single disc leads to\(^{40}\):

\[
\sigma = \frac{2\sigma_{\text{avg}}^c}{\left( \frac{h}{r} \right)^2 \left( \varepsilon^\mu - \frac{2\mu}{\pi} - 1 \right)}
\]  

(4)

with \(h, r\) are the instantaneous height and radius of the stack, respectively, and \(\mu\) is the friction coefficient. The height \(h\) of the stack is measured and \(r\) is derived assuming volume constancy.

However, when targeting the large strain flow curve, a small aspect ratio is favoured for the stability of the stack deformation and mitigating preliminary stack defects such as disc localization. The lower the aspect ratio of the stack, the more pronounced the frictional effect and the need for friction correction. Moreover, frictional effects lead to a triaxial stress state which further complicates the determination of the flow curve. Obviously, friction is a disadvantage of the SCT. The crux of the problem is that one must be able to guarantee a strain range for which a constant frictional condition prevails. In addition, correction of the flow curve requires a method to quantify the frictional condition (i.e., large plastic deformation, large contact pressure and small sliding lengths) and friction conditions might vary as lubrication deteriorates due to thinning of the film and extension of the surface. An and Vegter\(^{41}\) showed that oiled PFTE film yields a constant frictional behaviour when conducting the SCT. A correction can then be made based on the shear strength of the film. Coppieters\(^{42}\) adopted the modified two-specimen method\(^{43}\) to calibrate the coefficient of friction in the SCT on low carbon steel. Recently, Coppieters et al.\(^{44}\) adopted an inverse strategy to calibrate the coefficient of friction in the SCT on dual phase steel sheet.

### 2.4 Tension-torsion and shear tests

In order to study the shear behaviour of metals, usually torsion or combined tension-torsion tests are conducted on cylindrical or tubular specimens. This type of test is particularly useful to investigate ductile fracture of metals under triaxial stress conditions. Examples of tension-torsion tests on aluminium alloys can be found in previous works.\(^{45,46}\) Cortese et al. (2016) used a tension-torsion machine to evaluate the damage accumulation of an isotropic steel using non-proportional tension-torsion loading conditions.\(^{47}\)

In order to study the shear behaviour of sheet metals, recently in-plane torsion tests are receiving an increasing interest. The test consists in clamping the internal and external area of a circular specimen and then rotating one with respect to the other to generate a shear deformation in the annular free region between the clamps, as illustrated in Figure 8, where both the specimen drawings and an experimental apparatus is shown.

Dealing with sheet metal anisotropy, the test described above presents the disadvantage of averaging the anisotropic behaviour for the whole circumference of the sample and sometimes having an incomplete optical access for...
measurements due to the structure of the testing machine, which applies an out-of-plane compression to avoid any slipping on the internal and external clamping areas. The first problem was solved by Brosius et al. (2011) who proposed a novel in-plane torsion test named Twin Bridge Shear Test, which uses a specimen based on a circular shape with partly tangential slots suitable to determine anisotropic yield behaviour. A solution to the second problem was recently proposed by Grolleau et al. (2019) who developed a novel in-plane torsion test using grooved specimens, already studied in Traphöner et al., with full optical access for 2D or stereo Digital Image Correlation measurements to characterize plasticity, under monotonic and reverse loadings, and fracture. However, a groove must be machined on the specimen, that prevents the use of very thin sheets.

FIGURE 8 In-plane torsion test: (a) drawing of the specimen, and (b) example of experimental apparatus.
In addition to torsion tests, shear tests for sheet metal characterization are also conducted using a translational displacement. An example of experimental set-up that uses this approach can be found in. Moreover, Yin et al. presented an overview of different shear test configurations for sheet metal characterization.

The main strengths of simple shear test on rectangular specimen are to reach high strains and perform load reversal using the same specimen. The specimens have a rectangular shape, a gauge area of length $L$ around 40–50 mm and width $h$ of 3–4 mm; the shear direction is along the length of the specimen, Figure 9. The samples are fixed using bolts that are tightened with a prescribed torque, which is calibrated for each tested material. The optimal value is obtained with the lowest torque that minimizes the sliding between the sample and the grips. Monotonic shear tests are performed on samples at the same orientations to the RD as the tensile test. Moreover, cyclic tests are easily performed in order to highlight the Bauschinger effect and to measure kinematic hardening parameters. These tests are composed of a loading up to several values of the shear strain followed by a reloading in the opposite direction. The shear strain $\gamma$, which corresponds to the non-diagonal component of the planar transformation gradient in the case of an ideal simple shear kinematics, is measured from a DIC system and is then defined as an average over a rectangular zone on the sample surface. Figure 9 shows a rather constant value of $\gamma$ except near the free edges of the specimen, over a distance of approximately 5 mm. Calibration of a mixed hardening model, using either simple shear or bending-unbending tests, has been carried out with either type of tests, showing a very good agreement.

Simple shear stress state involves the non-diagonal components of the stress tensor and brings therefore additional information. Experimental databases involving tensile, simple shear and hydraulic bulging tests have been presented for several sheet materials, including mild steel, dual phase steel DP 500, aluminium alloy 6022 and dual phase steel DP 600. The quality of the strain field was analysed by digital image correlation, showing that translational simple shear tests give a kinematics close to the theoretical one.

### 2.5 Hydraulic bulge tests

Another important class of mechanical tests on sheet metals exploits the pressure of a fluid, mainly water or oil, to deform the specimen. The hydraulic bulge test is probably the most used of such tests. The test consists in forming a flat specimen through the direct action of hydraulic pressure. The sample is fully clamped on its external edge with a circular die while on the internal area, during the test, the pressure of a fluid is continuously increased deforming the material until failure. The deformation of the sheet metal is monitored using an appropriate measurement system, which in

![Figure 9](image-url) Homogeneity of the strain distribution along three sections (S0, S1 and S2) parallel to the shear direction, for $\gamma$ equal to around 0.3 and 0.6. $X$ is along the sample length. Either the entire gauge surface is used (average 1) or a reduced area in the specimen centre (average 2)
the last years is mainly stereo-DIC. Figure 10a shows an example of a bulge test set-up with a circular die (clamping ring) and a stereo-DIC measurement system; Figure 10b shows an example of fractured specimen at the end of the test. The stress components during the test can be directly related to the curvature of the dome and used to retrieve the flow curve of the material in the equi-biaxial state. In particularly, the ISO 16808 standard specifies a method for the determination of the biaxial stress-strain curve by means of an optical measuring system. This method was compared by Mulder et al. with their alternative procedure, using the same measuring apparatus, showing that correction formulas can be used to improve the accuracy of the stress reconstruction. Before the publication of the ISO standard in 2014, KoÅ§ et al. presented an interesting work which compared different approaches, developed over the years, for the analysis of hydraulic bulge test data to determine the best combination in obtaining accurate flow curves models at room and elevated temperature conditions for different materials. Moreover, an empirical correction for the bending strain is suggested for small bulge diameter to sheet thickness ratios.

The principal advantage of the bulge test compared to the tensile test is the much larger achievable strain before necking due to the nearly-equibiaxial stress state at the apex of the deformed specimen; Gutsher et al., for example, performed viscous pressure bulge (VPB) tests and tensile tests for different steel and aluminium sheets highlighting the difference between the relative flow curves.

The hydraulic bulge test can also be used to characterize the anisotropic behaviour of sheet metal, for instance, in previous works, Chen et al. developed a methodology for incorporating anisotropy in the extraction of the material stress-strain response from a bulge test on aluminium alloys using the Yld2004 yield function. In the same way, data from elliptical hydraulic bulge testing were used by Williams and Boyle in the calibration of anisotropic yield function coefficients for titanium sheet, while a more general theory on the kinematic of bulge test is reported in Chen. Recently, Barnwal et al. studied the fracture characteristics of advanced high strength steels during hydraulic bulge test.

Another interesting experimental procedure that can be used to obtain multiaxial loading condition exploiting the pressure of a fluid is the multiaxial tube expansion testing for sheet metals developed by Kuwabara and Sugawara. The test consists in applying an axial load and an internal pressure to a tubular specimen, manufactured from a flat sheet, while a measurement system continuously monitor the axial and circumferential strain components from initial yield up to fracture. Figure 11a illustrates the servo-controlled machine that enables the execution of the test, two stereoscopic cameras for DIC measurement.
hydraulic cylinders are employed to apply the axial force and to pressurize the fluid in the tubular specimen, respectively. Results obtained in terms of deformations for different linear stress paths are shown in Figure 11b.

As a whole, several combinations of these quasi-homogeneous tests are nowadays used, for example, uniaxial and biaxial tensile tests and simple shear tests, to calibrate plasticity models under several mechanical states (e.g., Zang et al.\cite{57}). However, as it can lead to a huge number of tests, an alternative approach is becoming more and more attractive.

## 3 | HETEROGENEOUS TESTS COUPLED WITH INVERSE METHODS

Heterogeneous stress/strain states can be used in order to enrich the information about the material behaviour that can be achieved from a single test. This is particularly useful in case of anisotropic plasticity or complex yield surfaces, where several quasi-homogeneous tests are needed to explore the material behaviour. Dealing with heterogeneous tests, an inverse method is always required to identify the constitutive parameters; that is, the constitutive parameters are not directly identified from the test but become the unknowns of an optimization problem. In the next sections, first examples of unconventional heterogeneous tests developed for plasticity are described then the main inverse methods used to extract the constitutive parameters are presented.

### 3.1 | Unconventional mechanical tests

The label “unconventional” defines all the mechanical tests where the application of complex loading conditions, the use of complex specimen geometries, or the combination of both, generate a heterogeneous response of the material. Generally speaking, unconventional tests require full-field techniques to measure the heterogeneous strain field that
can be combined with an inverse method to calibrate the parameters of a given constitutive model. In this section, an extensive overview on these unconventional experiments for the characterization of the plastic response of metals is presented, making a first classification based on the loading modes used to deform the specimen.

Grooves, notches and holes are widely used for generating heterogeneous mechanical fields on specimens subjected to uniaxial loading. For instance, in Meuwissen, a mixed numerical-experimental method for the mechanical characterization of sheet metals is presented, employing the heterogeneous data from uniaxial tensile tests on an irregular plate with two perforations (Figure 12a).

In Kajberg and Lindkvist, the deformation fields from tensile tests on specimens with lateral notches (Figure 12c), retrieved with Digital Speckle Photography (DSP), are used to feed an inverse procedure based on FE simulations to identify the hardening behaviour of hot-rolled steels.

Geometries with notches are also investigated in Belhabib et al. where the authors compared, numerically and experimentally, the strain fields produced by a classical dog-bone specimen, a plane tensile specimen and a deep-notched tensile specimen. The analysis reveals that the latter geometry offers a better sensitivity of the strain field to the parameters of the constitutive model, enhancing the quality of the identification via Finite Element Model Updating (FEMU) approach.

Cooreman made a similar study with a uniaxial specimen test on a perforated specimen as well as on a more complex geometry and a biaxial tensile test with a perforated cruciform (Figures 12b and 12d). The material coefficients of the Hill48 anisotropic yield criterion were successfully identified, reducing the number of the required experimental tests from at least three standard tensile tests to only one complex experiment.

The work in Pottier et al. also compared three specimen geometries using uniaxial tensile tests: a classic uniaxial tensile specimen, a perforated specimen, and a shear-like tensile test (depicted in Figure 12f). The authors applied a finite-element-based inverse method coupled with DIC to identify the six constitutive parameters of an anisotropic elasto-plastic model. In particular, the study shows an improvement of the parameter identification when heterogeneous strain fields are used, where the shear-like tensile specimen provides the best performance for the shape prediction in a deep-drawing process.

A novel specimen for combined tension-compression (Figure 12e) is proposed in Ishiki et al. where it is used to evaluate the anisotropic plastic behaviour of pure titanium sheets under linear stress paths. The authors combined the results from the proposed test with biaxial tensile tests, tension-internal pressure test on tubular specimens and uniaxial in-plane compression test, analysing the differential work hardening behaviour by means of spline-based approach.

The study in Güner et al. explores the effects of notch’s radius and material orientation for the identification of an advanced anisotropic plasticity model, namely, the Yld2000-2d, employing a FEMU identification framework on full-field data. After a first numerical analysis for the evaluation of the optimal specimen configuration, the identification procedure was used to characterize an AA6016-T4 sheet metal. The work motivates the use of heterogeneous mechanical field for the inverse characterization; however, the material parameters obtained were not accurate due to the assumption of neglecting the kinematic hardening.

Kim et al. developed a specimen for a uniaxial tensile test based on trial and error which can provide heterogeneous stress states (Figure 12g–j). The authors analysed four sample shapes using authors analysed four Virtual Fields Method (VFM) to find the constitutive parameters of Swift hardening law with the Hill48 yield function. It was concluded that the simultaneous identification of all anisotropic plastic parameters was reasonable only when the tests could offer enough heterogeneous information relevant to each anisotropic parameter. Geometry (j) offered a heterogeneous stress field but did not provide enough information for each parameter. Therefore, the mentioned geometry by itself was unsuitable for the simultaneous identification of all anisotropic parameters. Even though geometry (g) yielded very satisfactory identification results from the numerical simulation data, the sample tended to buckle in the experiments. Geometry (h) provided several heterogeneous stress states and the identification of the anisotropic parameters was carried out successfully. Geometry (i) is capable of generating biaxial stress states in the central area between the two holes and necking occurred at very early stages in the hole areas during the experiments, resulting in unsuccessful identification.

Also the study in Jones et al. is focused on the identification of the flow stress by means of the VFM. With the aim of maximizing the stress heterogeneity and the range of strain rates, preventing buckling and ensuring a uniaxial loading direction, the authors developed the specimen geometry illustrated in Figure 12 via an iterative process. The numerical analysis showed a good heterogeneity of the stress field and the most probable stress states were tension along the vertical axis, with some amounts of biaxial and shear.
FIGURE 12  Heterogeneous specimens for uniaxial tensile tests: (a) Meuwissen,\textsuperscript{74} (b) Cooreman,\textsuperscript{75} (c) Kajberg and Lindkvist,\textsuperscript{76} (d) Cooreman,\textsuperscript{75} (e) Ishiki et al.,\textsuperscript{77} (f) Pottier et al.,\textsuperscript{78} (g–j) Kim et al.,\textsuperscript{79} (l) Jones et al,\textsuperscript{80} and (m) Küsters and Brosius\textsuperscript{81}
A complex geometry of a heterogeneous specimen for a uniaxial tensile test was proposed by Küsters and Brosius\cite{Kusters1981} and here reported in Figure 12m. The sample was studied for damage characterization of sheet metals, and provide a wide range of different stress conditions with only one experiment.

One of the first works on unconventional mechanical tests was motivated by the lack of standard cruciform specimens to investigate the biaxial loading condition.\cite{86} The paper is aimed to develop a new geometry using statistical tools of factorial and response surface designs. The result is the cruciform specimen with a circular reduced central region depicted in Figure 13a.

A further study on the shape optimization of cruciform specimen is reported in Zidane et al.\cite{87} where the performance of several specimen configurations is investigated numerically and a new specimen geometry (Figure 13b), inspired by Johnston et al.\cite{90} is proposed. Another comparison between three biaxial specimen shapes is proposed in Schmaltz and Willner\cite{91}: the heterogeneous fields produced by these geometries were used as input of an inverse identification protocol based on the minimization between the displacement fields measured experimentally and the outcome fields of a numerical simulation of the test.

An inverse strategy exploiting FE analyses is used for retrieving the parameters of a plastic constitutive model, employing biaxial tensile tests on cruciform specimens.\cite{88} The corresponding geometry is illustrated in Figure 13c, as result of an optimization procedure used to cover many strain paths as possible from uniaxial to biaxial tension in a balanced way, maximizing the amount of strain achieved in the centre of the gauge area.

A novel cruciform specimen for the identification of the plastic behaviour of metals is also presented in Liu et al.\cite{89} The authors proposed a cruciform specimen shape as in Figure 13d, that is used for the characterization of an
aluminium alloy AA5086 sheet metal through FEMU technique. Differently from the previous case, the specimen geometry presents a circular zone in the centre with reduced thickness that was optimized by means of numerical simulations.

A numerical approach, followed by an experimental validation, was also exploited in Zhu et al.\textsuperscript{92} for studying miniaturized cruciform specimens which can be used in testing machines with maximum load below 5000 N. The work compares three main geometries and evaluates the effects associated with important specimen features such as chamfers, centre thinning and open slits.

Other examples of unconventional mechanical tests on sheet metals can be found, for instance, in Mohr and Henn,\textsuperscript{93} where an experimental approach to investigate the onset of fracture in metals at low and intermediate stress triaxialities on flat specimens is presented. The test is performed using a custom-made testing device, a schematic is given in Figure 14a. The flat specimen (Figure 14b) exhibits different stress states within its gauge section depending on the direction and orientation of the displacement loading. For 90° loading, the predominant stress state is uniaxial tension at moderate strains; at 0°, it is pure shear and for in-between angle values, the stress distribution is non-uniform. The sample was designed in order to develop cracks in the centre area, due to the amplitude of the strains. Thereby, the test offers different strain states depending on the loading conditions.

\textsuperscript{94} proposed a heterogeneous mechanical test using a new sample geometry, based on out-of-plane deformations, for the inverse identification of an elasto-plastic constitutive model. The sample was designed to exhibit tensile, shear and expansion behaviours.

Inverse methods can also be used as driving criteria for the shape optimization of heterogeneous specimens, as described in Souto et al\textsuperscript{[95,96]} for uniaxial tensile test and biaxial tensile test, respectively. Here, optimized shapes were generated by describing the boundaries of specimens by means of spline functions, resulting, in the butterfly-shaped geometry and the cruciform specimen displayed in Figure 15. In particular, the uniaxial butterfly specimen exhibited a strain state range between simple shear to plane strain tension, while the biaxial tensile test provided more mechanical information. Moreover, an experimental validation of the butterfly shape was performed in Aquino et al.\textsuperscript{97}

In order to minimize the cost function that provides the optimum heterogeneous specimen shape for a uniaxial tensile test, based on the indicator developed by Souto et al.,\textsuperscript{98} Andrade-Campos et al. investigated the best curve parameterization and optimization algorithm.\textsuperscript{99} Seven optimization strategies were used, but only four of them were able to converge to a solution: the Nelder–Mead, pattern search, SDBOX (globally-convergent derivative-free algorithm for the minimization of a continuously differentiable function), and CMA-ES (covariance Matrix Adaptation Evolution Strategy). The number of control points that minimizes the cost function regarding the different curve parameterization was also determined. The result was seven control points for B-splines and NURBS, and six for Splines. The specimen that was able to provide more mechanical information was obtained by using NURBS with CMA-ES. The resulting sample shows a butterfly shape, providing mainly strain states in the neighbourhood of uniaxial tension, due to the tensile loading conditions of the test and the best solutions offered shear and uniaxial compression states for small values of

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure14.png}
\caption{Universal biaxial testing device presented in Mohr and Henn\textsuperscript{[93]}: (a) schematic of the test and (b) details of the flat specimen. All dimensions are in millimeteres}
\end{figure}
strain. Using a unique test, however, it was not possible to obtain at the same time large levels of plastic strain and a wide range of stress heterogeneity. Moreover, none of the tests were able to produce the biaxial stress state.

It was also proposed to use topology optimization for the specimen design. The aim was to find the geometry of a uniaxial tensile test sample that maximizes the sensitivity of the measured displacement field to the material parameters to be identified, under volume fraction constraints and without any a priori information on the specimen shape. Numerical results and experimental validation have confirmed the method. The design obtained from the optimization procedure (Figure 16a) revealed a part of the specimen not linked to the remainder of the structure after filtering. One problem of this approach is the practical feasibility of specimens with such a complex shape. Further research is required on this topic, for instance the manufacturing constraints could be introduced in the cost function of the optimization algorithm. Topology optimization was also used to design a new specimen for a tensile test (Figure 16b). The obtained specimen presented pure shear, compression and tensile stress states in the plastic region. However, no

**FIGURE 15** Specimen shape optimization through spline functions under uniaxial and biaxial loading conditions

**FIGURE 16** Heterogeneous specimens geometries generated by means of topology optimization in Chamoin et al. for identification of the elastic properties (a), and Barroqueiro et al. for the plastic response (b)
experimental validation took place. Finally, it is important to underline that a key point in this field is the definition of suitable metrics to evaluate and compare different specimen geometries.\textsuperscript{102}

### 3.2 Inverse methods in plasticity testing

Inverse methods are applied in many different fields of mechanical engineering, a review of different applications can be found in previous works.\textsuperscript{73,103,104} A comparison in terms of performance of different inverse methods can be found in Martins et al.\textsuperscript{105} With regard to plasticity, the most used methods are the FEMU and the VFM, which will be detailed in the following sections. Further methodologies will be also mentioned afterwards.

#### 3.2.1 Finite Element Model Updating

The FEMU is a widely accepted inverse method. Since the pioneering work of Kavanagh and Clough,\textsuperscript{106} the FEMU has been developed and applied to different types of models. In the case of plasticity models, the work of Meuwissen et al. is recognized as the first attempt to address this type of models.\textsuperscript{107} They applied the FEMU to the calibration of two yield criteria, based on an experimental database generated using a uniaxial test with a non-standard specimen geometry. Following this study, more complex models and specimen geometries were tested (e.g., previous works\textsuperscript{78,94,108}). Some studies were also dedicated to identify the best specimen shape for identification with FEMU using a topological optimization approach.\textsuperscript{95,97} Guery et al. proposed the use of a weighted version of the FEMU method to calibrate a crystal plasticity model.\textsuperscript{109} This study combined the displacement fields measured at the microstructural scale and the load level at the macroscale. More recently, Rose and Menzel proposed a staggered identification methodology, which considered three calibration phases dedicated to elastic, plastic and thermal material parameters.\textsuperscript{110} In this case, a thermo-mechanical heterogeneous test was used.

The FEMU relies on the simple idea of updating the material parameters on a chosen constitutive model until the difference between numerical predictions and experimental measurements reach a minimum. The numerical predictions are generated by a FE model that represents the mechanical test from which the experimental measurements were collected. The updating process is generally driven by an optimization method, which relies on an objective function that measures the difference between numerical and experimental data.

One of the strengths of this method is its flexibility. The experimental and numerical data used to build the objective function can be of different types (e.g., displacements, strains, forces and temperatures) and it does not require a complete field, partial measurements of it can be used. Therefore, the selection of data can be tailored to the problem at hand.

The data selected to build the objective function is an open topic. The objective function can assume different forms, which depend on the type of data and weights. Generally, the least square structure is adopted. An example of a simple objective function based on the in-plane components of the strain tensor can be defined as

\[
J_{FEMU}(\xi) = \sum_{i=1}^{n_p} \sum_{j=1}^{n_s} \left[ (\varepsilon_{xx} - \bar{\varepsilon}_{xx}(\xi))^2 + (\varepsilon_{yy} - \bar{\varepsilon}_{yy}(\xi))^2 + (\varepsilon_{xy} - \bar{\varepsilon}_{xy}(\xi))^2 \right],
\]

\(\varepsilon_{xx}, \varepsilon_{yy}, \varepsilon_{xy}\) and \(\bar{\varepsilon}_{xx}, \bar{\varepsilon}_{yy}, \bar{\varepsilon}_{xy}\) are the in-plane components of the experimental and numerical strain tensor, respectively.

In case of full-field measurements, the experimental data envelope is composed of a grid of measurement points at the surface of the sample for different time instants \((\bar{\varepsilon}(x,t))\). In this sense, \(n_p\) and \(n_s\) represent the number of measurement points on the surface of the sample and the number of time steps, respectively.

It is common practice to use weighting coefficients in the objective function formulation (e.g., Mathieu et al.\textsuperscript{111}), but their selection is not intuitive and usually depends on the data and experience of the user.\textsuperscript{112}

An important remark regarding the evaluation of the objective function is that the numerical and experimental data must be available in the same positions. Otherwise, an interpolation procedure is required. However, to avoid the computational effort of interpolation at each numerical iteration, the majority of works interpolate the experimental values at the numerical integration points.
A general flowchart for FEMU method is presented in Figure 17. The FE model is built according to the geometry and boundary conditions (B.C.) of the experimental test. The process starts with an initial set of material parameters ($\xi^i$) that is used to run the FE analysis. Then, the evaluation of the objective function is performed. If the value of the objective function is above a threshold value, an iterative process begins and an optimization method leads the updating of the initial set of material parameters ($\xi^u$). This process is repeated until the objective function reaches a value below the threshold or until the updating process stagnates. The threshold value is defined by the user and represents the admissible global difference between numerical and experimental data. The type of optimization methods used in this kind of problem lies within two main families: (i) gradient-based methods (e.g., Gauss–Newton method or Levenberg–Marquardt method) and (ii) direct methods (e.g., evolutionary or simplex).[104,112,113] The first family of methods requires the value of the objective function and its gradient to compute a new set of parameters, while the direct methods only require the value of the objective function. In terms of computational cost, gradient-based methods are usually more efficient than the direct methods, since these tend to require a lower number of evaluations of the objective function. Nevertheless, gradient-based methods do not guarantee the finding of global minimum, which is much dependent on the selected initial set of material parameters.[108,115] The selection of the optimization method is another topic on which there is no consensus and is a common point across all inverse methods.

The widely adoption of FEMU is mainly due to its flexibility, which is a great advantage compared to other methods. As mentioned, it is not constrained to full-field measurements, specimen shapes or types of loads.[94,114] This advantage combined with the ease of implementation, make FEMU an attractive method. However, it suffers from a well-known weakness. Each evaluation of the objective functions requires a FE model run, which leads to a high computational cost.[105] Moreover, it also requires an accurate FE model that represents the experimental test as close as possible to reality. Depending on the boundary conditions, this can be difficult to attain. For instance, the force distribution on a boundary is usually unknown. The results can also be mesh sensitive, which is an aspect inherent to every method that makes use of FE analysis.

### 3.2.2 Virtual fields method

The VFM was first introduced by Grédiac.[115] Contrary to FEMU, the VFM requires full-field measurements, which may have delayed the interest on this method. Nevertheless, due to the recent advances and maturity of full-field measurements, this interest has grown in the last years.

![Detailed flowchart for the Finite Element Model Updating (FEMU) (Figure 17)](image-url)
measurements, it has received significant attention from the scientific community. The first application in plasticity models was presented by Grédiac and Pierron in 2006.\cite{116} The VFM was used to calibrate an elasto-plastic model using a non-standard specimen that generated heterogeneous strain fields. Since then, several papers dedicated to VFM in plasticity were published.\cite{117,118} Pierron et al. extended the VFM to elasto-plastic material identification with cyclic loads and combined kinematic/isotropic hardening.\cite{119} Rossi and Pierron applied VFM to extract the constitutive parameters of a plasticity model from three-dimensional (within the volume) displacement fields.\cite{120} Kim et al. used the VFM for the characterization of the post-necking strain hardening behaviour when the stress and strain distributions over the necking area become heterogeneous.\cite{121} The VFM was recently exploited to develop a calibration process to identify the flow stress curve from circular bulge test data in.\cite{122}

VFM was also extensively used to identify the constitutive parameters of anisotropic plasticity models. For instance, Kim et al. used steel sheet specimens of a complex shape.\cite{79} Rossi et al. presented the application of the VFM to large strain anisotropic plasticity.\cite{123} Martins et al. explored the combination of a biaxial test with a cruciform specimen and the VFM to develop an efficient strategy for simultaneous identification of material parameters related with hardening and anisotropy in plasticity.\cite{50} Fu et al. used the VFM to identify homogeneous anisotropic hardening models in previous work.\cite{124} Application of VFM to viscoplasticity can be found in Notta-Cuvier et al.\cite{125} while in previous works\cite{126,127} temperature dependent viscoplasticity was investigated using isothermal tests and a heterogeneous thermo-mechanical, respectively.

There are two crucial components behind VFM, the Principle of Virtual Work and an appropriate set of virtual fields. In the absence of body-forces, the Principle of Virtual Work expresses that the internal virtual work must equal the external virtual work performed by the external forces and can be written as follows

\[
\int_{\Omega} \sigma(\xi, \varepsilon) : \varepsilon^* \, dV = \int_{\Gamma_f} F \cdot u^* \, dS, \tag{6}
\]

where \(\varepsilon^*\) is the virtual strain tensor and \(u^*\) is a virtual displacement vector. \(dV\) and \(dS\) are the infinitesimal volume and area of a given solid body, respectively. The principle of virtual work is independent of any constitutive model, theoretically it can be applied to all types of constitutive models. Provided an appropriate set of virtual fields, the force distribution \(\{F\}\) along the boundary \(\Gamma_f\) is not required. Instead, the resultant of the applied force can be used, which is usually measured during an experimental test. The Cauchy stress tensor \(\sigma(\xi)\) is computed by means of a constitutive model and a set of material parameters \(\varepsilon\).

In case of linear elasticity, the material parameters can be evaluated directly from a linear system of equations, which will have the same number of equations as number of unknown material parameters of the model. For the case of a non-linear model, such as plasticity models, it is no longer possible to derive the linear system of equations and the identification process becomes an optimization process, which relies on the minimization of an objective function.\cite{119,128} This objective function is based on the Principal of Virtual Work and expresses the gap between the internal and the external virtual works as

\[
\mathcal{F}_{\text{VFM}}(\xi) = \sum_{i=1}^{n_t} \left( \int_{\Omega} \sigma(\xi, \varepsilon) : \varepsilon^* \, dV - \int_{\Gamma_f} F \cdot u^* \, dS \right)_i^2, \tag{7}
\]

where the calculation of the stress field from the experimental full-field strain field is performed through a stress update algorithm.\cite{129,130}

It is worth mentioning that the general formulation of the VFM in Equation (6) is defined in the 3D space, requiring full-strain components to compute the internal virtual work. However, gathering deformation data in the bulk of a wide class of engineering materials is extremely challenging: for materials such as steels, even advanced 3D full-field techniques like the Digital Volume Correlation (DVC)\cite{131} may not resolve the volume deformation due to the absence of a pattern in the material bulk. Therefore, the VFM is often applied assuming that the mechanical fields are uniform along one direction,\cite{132} solving VFM problem by using only the full-field strain data measured on the specimen surface. This assumption is particularly convenient for characterization of thin sheet metals, where 2D plane stress material models are widely used to describe their material behaviour. On the other hand, with thick metal plates the plane stress
A detailed flowchart for VFM is presented in Figure 18. The flowchart contains two different paths. The path indicated by the black line represents the calibration process of linear models. In this type of models, the material parameters are retrieved after solving a system of linear equations. The path indicated by the red line corresponds to non-linear constitutive models. In this case, the material parameters are retrieved after an iterative process that relies on the minimization of the objective function presented in Equation (7). In this case, an optimization method is required.

The selection of a set of virtual fields is part of the VFM identification process. A proper choice is crucial to set the method and has an impact on the quality of the final set of material parameters. The selection of the virtual fields has been pointed out as the major weakness of VFM, especially in non-linear cases.[108,113] The virtual displacement and strain fields are mathematical test functions, which can be seen as weights[128,134] and are independent of the measured displacement and strain fields.[128] The virtual fields are usually selected to satisfy two conditions. In the first condition, the displacement boundary conditions has to be satisfied, hence at the boundary $\Gamma_u$, the virtual displacement field must be zero ($\mathbf{u}^v = \mathbf{0}$).[104,128] The second condition relates to the computation of the second integral in Equation (7) and the use of the resultant force instead of its distribution. The virtual displacement fields must be chosen in order to be constant along the boundary $\Gamma_f$ and collinear with $\mathbf{f}$ to eliminate the components of the resultant force that are unknown.[128,135] Apart from that, the virtual fields must have to assure a $C^0$ continuity.

Regarding the use of the applied force in VFM, in special cases such as dynamic testing, obtaining accurate measurements of the applied force can be difficult. A different formulation of Equation (6), which includes the virtual work of the inertial forces, can then be used and the external virtual work term can be cancelled out using suitable virtual fields.[128,136,137] This approach has been exclusively applied to the identification of dynamic mechanical characteristics; thus, it will not be addressed in this work.

As mentioned above, the selection of virtual fields is a critical part of the method, which has undergone significant advancements to become less user-dependent. Currently, there are three strategies to select the virtual fields:

![Detailed flowchart for VFM](image)
i. Manually defined virtual fields: In this strategy, the selection of the virtual fields depends entirely on the user. Polynomials or sine/cosine functions are selected to generate the virtual fields. The search for a function that meets the conditions above mentioned depends on the expertise of the user and there is no guarantee that the chosen function produces the best results. Nevertheless, this procedure is still the most used in non-linear cases due to ease of implementation.

ii. Stiffness-based virtual fields: This second procedure has been a great step to overcome the previous drawbacks. It was first proposed by Avril et al. for the case of anisotropic elasticity and then extended to elasto-plasticity by Pierron et al. This strategy relies on a statistical approach to minimize the effect of noise. The computation of the stiffness-based virtual fields requires the derivation of the tangent stiffness matrix. Its major disadvantage is the implementation due to the derivation of the stiffness matrix, which is a cumbersome process in non-linear models.

iii. Sensitivity-based virtual fields: Proposed by Marek et al., the sensitivity-based virtual fields are determined based on the sensitivity of the stress maps to the material parameters. During the reconstruction of the stress field, the sensitivity of the stress map to a respective parameter is evaluated and transformed into a set of virtual fields. These sets capture the regions of the specimen where the information on a particular parameter is more relevant. The sensitivity of the stress maps for each parameter should be evaluated and the number of virtual fields is at least equal to the number of unknowns. This strategy offers an easier implementation procedure than the stiffness-based virtual fields. Its computational cost is dependent on the number of material parameters to identify and the computational effort to reconstruct the stress fields.

One advantage of VFM compared to FEMU is the lower computational cost. Since VFM does not require FE simulations, the identification process becomes less computationally demanding. This difference in computational cost is very significant as highlighted by Zhang et al. and Martins et al. Another crucial advantage of VFM is related to the boundary conditions, as it does not require the exact distribution of the applied force on the boundary $\Gamma^f$. With a proper choice of virtual fields, it only requires the force resultant in one direction, which is usually measured during experiments. The main disadvantage of VFM is the flexibility in terms of data, it requires full-field experimental data over the entire domain. A method to improve the robustness of the measured full-field data from DIC was recently proposed in, exploiting a FE scheme and the Gauss quadrature rule for integration.

Moreover, in non-linear models, it is required a stress reconstruction algorithm. These algorithms are often based on the so-called return-mapping algorithm, but new developments in this field have brought more efficient algorithms to combine with VFM.

3.2.3 Other inverse methods for plasticity

Although FEMU and VFM are the most used inverse methods in plasticity, other approaches can be found in the literature. Several of these approaches rely on an energy balance. Strano and Altan, for example, presented an inverse energy approach to determine the flow stress of tubular materials. Coppieters et al. and Coppieters and Kuwabara proposed an inverse approach which minimizes a cost function based on the comparison of the internal and external work within the diffuse neck of a tensile test, where, instead of using virtual fields, the actual measured displacement/strain field is used. The latter approach can be considered indeed as a special case of the nonlinear VFM.

Other approaches based on VFM were proposed, for instance, Rossi et al. proposed the linear stress-strain curve identification (LSSCI). In this method, the hardening curve identification is reduced to a linear problem and identified without any iteration. Lattanzi et al. proposed an integrated strategy that combines different approaches to efficiently characterize the material behaviour of anisotropic metals by decoupling the identification of the hardening curve and the anisotropic yield surface.

Another inverse method widely used in several engineering applications is the Integrated Digital Image Correlation (IDIC). With this approach, the image correlation procedure and mechanical identification procedure are integrated into a one-step, where the constitutive parameters become the parameters of the shape function used in the image correlation algorithm. An application of IDIC to anisotropic plasticity can be found in Bertin et al. Mathieu et al. and Ruybalid et al. compared the performance of FEMU and IDIC.
4 | FURTHER METHODOLOGIES

Other practices for material characterization are emerging in the last years. An observable trend is the integration of multiple experimental techniques in order to obtain more information about the material behaviour from a single test. For example, different temperature ranges or strain rates can be investigated in a single test. Moreover, machine learning and deep learning techniques have started to be used in material characterization too, although their use is very limited at the current state-of-the-art; however, the interest in this topic is rapidly growing.

4.1 | Integrated thermomechanical characterization

Examples of integrated approaches is the use of bulge test at warm temperatures to combine multiaxial loading and temperature effect. A possible set-up is shown in Figure 19 where the mechanical characteristics of AA5052 and AA6061 sheet blanks are investigated using both tensile and bulge testing methods. Similarly, Lee et al. measured the stress-strain behaviour under warm conditions (about 100°C) of advanced high strength steel (AHSS) sheets through hydraulic bulge tests.

Van Rooyen and Becker presented a numerical-experimental methodology that employs digital image correlation to measure temperature-specific tensile properties of a steel. The methodology presents the ability to obtain temperature-specific properties from a single sample: the sample is subjected to a longitudinal thermal gradient through resistive heating in a thermomechanical loading system, while the full-field capabilities of digital image correlation and thermal imaging are used to measure strain and temperature surface maps, respectively. A similar approach was used by Lattanzi et al. to identify the mechanical properties of laser-tailor-heat-treated blanks. A heterogeneous tensile test was conducted up to high temperature for a DP980 steel, to calibrate a viscoplastic model. Both the thermal and mechanical fields exhibit a gradient along the specimen length.

4.2 | Machine learning

In the recent years, the use of machine learning and deep learning is remarkably increasing in many engineering applications. For instance, in computer vision, machine learning is nowadays a standard approach widely used both in research and industrial applications. Machine learning requires that a learning algorithm, mainly NNs, is trained with
experimental or synthetic data in order to learn some complex functions. A NN can be viewed as an extremely powerful fitting algorithm able to reproduce very complex functions, provided that a sufficient number of training examples is available. A typical scheme of a NN, shown in Figure 20, is composed by an input layer with the input data, an output layer with the target output and a series of hidden units that need to be trained. The forward propagation algorithm allows to go from the input layer to the output layer and is used to make predictions. The backward propagation algorithm, instead, is used to train the NN starting from known data. Many NN architectures are available, usually, if the number of hidden units is particularly high, the term deep learning is used.

The use of machine learning in metal forming can be seen back to the 90's.[157–159] The use of NNs (defined by the authors as a subfield of artificial intelligence) to model material behaviour was preliminary made by Ghaboussi in 1991[157] and their promising results have been reproduced and extended over the last decades. The advantages that all behaviour can be represented within a unified environment and that the NN is built directly from experimental data were very attractive. Although only plane stress behaviour of concrete was reproduced, monotonic biaxial loading and compressive uniaxial cycle loading were modelled with success using a backward propagation NN. Later, several works applying artificial NNs to sheet metal works have been presented,[159] however the majority have the purpose to minimize the dependency on human expertise and time taken in design of parts and dies. To identify material behaviour, the works of Furukawa (e.g., Furukawa and Yagawa[160]), defining an implicit viscoplastic constitutive model using NNs, should be highlighted. To validate the approach, Furukawa and Yagawa[160] used synthetic data retrieved from the Chaboche constitutive model as a virtual material. Nevertheless, real material behaviour was also reproduced, though with less success. Several works followed the same approach.[161,162]

In material characterization and plasticity, another example of machine learning was presented by Lin et al. that developed an artificial NN model to predict the constitutive flow behaviours of 42CrMo steel during hot deformation.[156] Mandal et al.[163] investigated the deformation behaviour of type 304L stainless steel during hot torsion using ANN and showed that the machine learning-based model is an effective tool to evaluate and predict the deformation behaviour of stainless steels. Later, based on the experimental data obtained from Gleeble-1500 Thermal Simulator,[161] developed a constitutive relationship model for Ti40 alloy using a back propagation (BP) NN. Absolute relative errors of less than 8% were found for the predicted flow stress values in comparison with the experimental values.

Jenab et al.[164,165] applied NNs to predict, respectively, the hot deformation behaviour of AA7075 and the tensile flow behaviour of AA5182-O sheets. Recently, Li et al. (2019) used a machine-learning approach to identify a modified Johnson-Cook model for describing the strain rate and temperature dependent plasticity of dual-phase steels.[166]

A recent review of artificial NNs in the constitutive modeling of materials, with emphasis to composite materials, is presented in Liu et al.[167]

Today, different machine learning architectures are applied to material modelling. However, the major problems of using machine learning models to reproduce the material behaviour mentioned by researchers are (i) the lack of large

![Schematic structure of backward propagation neural network](image)
amount of data, not only for training but also for testing and cross validation and (ii) the black-box and full data-driven nature of ANNs. This last disadvantage is a serious drawback for the understanding of the phenomena during the deformation of the materials and, as data-driven models, these do not satisfy conservation laws, symmetry and invariance requirements seen in conventional models.

5 | CONCLUSIONS

This article addresses the topic of the mechanical characterization of metallic sheets, using either quasi-homogeneous tests or heterogeneous tests coupled with an inverse identification procedure. An extensive review of classical quasi-homogeneous tests is presented, highlighting the strength of this approach, that is, the direct calculation of stress and strain components from raw data. However, as the number of tests increases with the complexity of the mechanical models, and hence the number of material parameters to be identified, an alternative approach consists of using heterogeneous tests. The number of such tests is rather large, each of them having advantages and drawbacks, and are usually validated for a limited number of materials. There is a need for common practices and guidelines to ensure a widespread use of these tests. The virtual mechanical design of such tests, based on the increase of the richness of the mechanical information, is still a topic under investigation. Finally, inverse identification procedures, that need to be linked to the heterogeneous tests, are reviewed as well as very up-to-date methods based on artificial intelligence.

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DATA AVAILABILITY STATEMENT

Data sharing is not applicable to this article as no new data were created or analysed in this study.

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REFERENCES

[1] ASTM E8/E8M-21, Standard test methods for tension testing of metallic materials, 2021.
[2] ISO 6892-1, Metallic materials – tensile testing – part 1: method of test at room temperature, 2019.
[3] JIS Z2241, Metallic materials – tensile testing at room temperature, 2012.
[4] A. Pradeau, S. Thuillier, J. W. Yoon, Int. J. Mech. Sci. 2016, 119, 23.
[5] S. Tu, X. Ren, J. He, Z. Zhang, Fatigue Fract. Eng. M. 2020, 1, 3.
[6] P. W. Bridgman, Studies in Large Plastic Flow and Fracture, McGraw Hill 1952.
[7] Z. L. Zhang, M. Hauge, J. Ødegård, C. Thaulow, Int. J. Solids Struct. 1999, 36, 3497.
[8] G. Mirone, Int. J. Solids Struct. 2004, 41, 3545.
[9] L. Wang, W. Tong, Int. J. Solids Struct. 2015, 75, 12.
[10] ASTM E21-20, Standard test methods for elevated temperature tension tests of metallic materials, 2021.
[11] ISO 6892-2, Metallic materials – tensile testing – part 2: method of test at elevated temperature, 2018.
[12] M. Merklein, J. Lechler, J. Mater. Process. Technol. 2006, 177, 452.
[13] J. Chen, B. Young, Thin Wall Struct. 2007, 45(1), 96.
[14] R. Grèze, P. Y. Manach, H. Laurent, S. Thuillier, L. F. Menezes, Int. J. Mech. Sci. 2010, 52, 1094.
[15] J. Coër, C. Bernard, H. Laurent, A. Andrade-Campos, S. Thuillier, Exp. Mech. 2011, 51, 1185.
[16] H. Aretz, O. Hopperstad, O.-G. Lademo, J. Mater. Process. Technol. 2007, 186, 221.
[17] H. Aretz, J. Mater. Process. Technol. 2005, 168, 1.
[18] J. Ha, S. Coppétiere, Y. P. Korkolis, Int. J. Mech. Sci. 2020, 182, 105706.
[19] ISO 16842, Metallic materials – sheet and strip – biaxial tensile testing method using a cruciform test piece, 2014.
[20] M. Merklein, M.Biasutti, J. Mater. Process. Technol. 2013, 213, 939.
[21] T. Nagayasu, S. Takahashi, T. Kuwabara, In International Deep-Drawing Research Group (IDDRG), 2010.
[22] W. Liu, D. Guiness, L. Leotoing, E. Ragneau, Mater. Science Eng. A 2016, 676, 366.
[134] A. Marek, F. M. Davis, F. Pierron, Comp. Mech. 2017, 60, 409.
[135] D. Notta-Cuvier, B. Langrand, F. Lauro, E. Markiewicz, Int. J. Solids Struct. 2015, 69, 415.
[136] G. Le Louëdec, F. Pierron, M. A. Sutton, C. Siviour, A. P. Reynolds, J. Dyn. Behav. Mater. 2015, 1, 176.
[137] D. Leem, J.-H. Kim, F. Barlat, J. H. Song, M.-G. Lee, Metals Mater. Int. 2018, 24, 351.
[138] J. Fu, F. Barlat, J.-H. Kim, F. Pierron, Int. J. Solids Struct. 2016, 102, 30.
[139] J. Fu, W. Xie, J. Zhou, L. Qi, Int. J. Mech. Sci. 2020, 181, 105756.
[140] S. Avril, M. Grédiac, F. Pierron, Comp. Mech. 2004, 34, 439.
[141] A. Marek, F. M. Davis, M. Rossi, F. Pierron, Int. J. Mater. Form. 2019, 12, 457.
[142] L. Zhang, S. G. Thakku, M. R. Beotra, M. Baskaran, T. Aung, J. C. H. Goh, N. G. Strouthidis, M. J. A. Girard, Biomech. Model. Mechan. 2017, 16, 871.
[143] C. Kim, M. G. Lee, Int. J. Solids Struct. 2021, 233, 111204.
[144] M. Strano, T. Altan, J. Mater. Process. Technol. 2004, 146, 92.
[145] S. Coppieters, S. Cooreman, H. Sol, P. Van Houtte, D. Debruyne, J. Mater. Process. Technol. 2011, 211, 545.
[146] S. Coppieters, T. Kuwabara, Exp. Mech. 2014, 54, 1355.
[147] M. Rossi, L. Cortese, K. Genovese, A. Lattanzi, F. Nalli, F. Pierron, Exp. Mech. 2018, 58, 1181.
[148] A. Lattanzi, F. Barlat, F. Pierron, A. Marek, M. Rossi, Int. J. Mech. Sci. 2020, 173, 105422.
[149] H. Leclerc, J.-N. Périé, S. Roux, F. Hild, International Conference on Computer Vision/Computer Graphics Collaboration Techniques and Applications, Springer 2009.
[150] M. Bertin, F. Hild, S. Roux, Strain 2017, 53, e12233.
[151] A. P. Ruybalid, J. P. M. Hoefnagels, O. van der Sluis, M. G. D. Geers, Int. J. Numer. Meth. Eng. 2016, 106, 298.
[152] S. Mahabunphachai, M. Koç, Mater. Des. 2020, 242, 2010, 31.
[153] J.-Y. Lee, L. Xu, F. Barlat, R. H. Wagoner, M.-G. Lee, Exp. Mech. 2013, 53, 1681.
[154] M. van Rooyen, T. H. Becker, Strain Anal. J. Eng. Des. 2018, 53, 117.
[155] A. Lattanzi, A. Piccininni, P. Guglielmi, M. Rossi, G. Palumbo, Int. J. Mech. Sci. 2021, 192, 106134.
[156] Y. C. Lin, J. Zhang, J. Zhong, Comp. Mater. Sci. 2008, 43, 752.
[157] J. Ghaboussi, J. H. Garrett, X. Wu, J. Eng. Mech. 1991, 117, 132.
[158] Z.-C. Lin, D.-Y. Chang, Artif. Intell Eng. 1996, 10, 21.
[159] S. Kashid, S. Kumar, Am. J. Intell. Syst. 2012, 2, 168.
[160] T. Furukawa, G. Yagawa, Int. J. Numer. Meth. Eng. 1998, 43, 195.
[161] Y. Sun, W. D. Zeng, Y. Q. Zhao, X. M. Zhang, Y. Shu, Y. G. Zhou, Mater. Des. 2011, 32, 1537.
[162] M. Gaspar, A Andrade-Campos AIP Conference Proceedings, 2019.
[163] S. Mandal, P. V. Sivaprasad, S. Venugopal, K. P. N. Murthy, Appl. Soft Comp. 2009, 9, 237.
[164] A. Jenab, A. K. Taheri, K. Jenab, J. Mater. Eng Perform. 2013, 22, 903.
[165] A. Jenab, I. S. Sarraf, D. E. Green, T. Rahmaan, M. J. Worswick, Mater. Des. 2016, 94, 262.
[166] X. Li, C. C. Roth, D. Mohr, Int. J. Plasticity 2019, 118, 320.
[167] X. Liu, S. Tian, F. Tao, W. Yu, Compos. Part B-Eng. 2012, 224, 109152.
[168] M. Raissi, P. Perdikaris, G. E. Karniadakis, J. Comp. Phys. 2019, 378, 686.
[169] Y. Heider, K. Wang, W. Sun, Comp. Meth. App. Mech. Eng. 2875, 363(11), 2020.
[170] F. Masi, I. Stefanou, P. Vanucci, V. Maffi-Berthier, J. Mech. Phys. Solids 2012, 147, 104277.

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