Probing anomalous $ZZH$ and $\gammaZH$ interactions at an $e^+e^-$ linear collider using polarized beams

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We examine anomalous $ZZH$ and $\gammaZH$ interactions in the process $e^+e^- \rightarrow HZ$ followed by $Z \rightarrow l^+l^-$ at a linear collider. We study the effects of beam polarization, both longitudinal and transverse, in probing these anomalous couplings. We study various correlations constructed out of the initial and final state lepton momenta and the spins of electron and positron. We evaluate the sensitivity of these observables to probe anomalous couplings at the linear collider.

1 Introduction

Although the standard model (SM) provides us an almost complete theory of electro-weak interactions, its mechanism of spontaneous breakdown of symmetry is still not well understood. Higgs, the center pillar of SM, has not been discovered yet. It is most likely that the Large Hadron Collider (LHC) will discover the SM Higgs. However, many alternative scenarios beyond SM offer solutions to electroweak symmetry breaking (EWSB) by incorporating more than one Higgs boson. Therefore, it is essential to carry out a thorough study of the properties of the Higgs, which is extremely difficult task at LHC. However, these precision tests can be meticulously performed at an $e^+e^-$ linear collider. The International Linear Collider (ILC), which is at the design stage and likely to become a reality [1], is definitely the well suited for this purpose.

At the lowest order in SM the $ZZH$ vertex is simply point-like whereas the $\gammaZH$ vertex vanishes. Interactions beyond SM can modify this vertex by means of a momentum-dependent form factor, as well as by adding more complicated momentum-dependent forms of anomalous interactions considered in [2, 3, 4]. Demanding Lorentz invariance, the most general coupling structure of $VZH$ vertex may be expressed as:

$$\Gamma_{\mu \nu} = g_{V} m_{Z} \left[ a_{V} g_{\mu \nu} + \frac{b_{V}}{m_{Z}^{2}} (k_{1\nu} k_{2\mu} - g_{\mu \nu} k_{1} \cdot k_{2}) + \frac{\tilde{b}_{V}}{m_{Z}^{2}} \epsilon_{\mu \nu \alpha \beta} k_{1}^{\alpha} k_{2}^{\beta} \right] \quad (1)$$

where $k_{1}$ and $k_{2}$ denote the momenta of the intermediate vector boson, $\gamma$ or $Z$ and outgoing vector boson $Z$ respectively and $a_{V}$, $b_{V}$ and $\tilde{b}_{V}$, in general complex, are form factors.

Here we consider in a general model-independent way the production of a Higgs mass eigenstate $H$ through the process $e^+e^- \rightarrow HZ$ mediated by $s$-channel virtual $\gamma$ and $Z$. We have studied how beam polarization (both transverse and longitudinal) can be used to constrain anomalous interactions of the Higgs with neutral electroweak bosons. There have been various recent works on the study of anomalous $VVH$ couplings at a linear collider [5, 6, 7, 8]. However, our approach is different from theirs in that we include $\gammaZH$ coupling, study the effects of both longitudinal as well as transverse $e^+$ and $e^-$ beams and construct simple asymmetries and correlations based on the $CP$ and $T$ transformation properties [9].
2 Use of beam polarization

Polarized beams are likely to be available at ILC, and several studies have shown the importance of longitudinal polarization in reducing backgrounds and improving the sensitivity of new effects. With longitudinal polarization, the cross section for $e^+e^-$ annihilation is given by

$$\sigma = (1 - P_L \overline{P}_L) \left[ \sigma_0 + P_{\text{eff}}^L \frac{\sigma_{RL} - \sigma_{LR}}{4} \right],$$

where $P_{\text{eff}}^L = \frac{P_L - \overline{P}_L}{1 - P_L \overline{P}_L}$ is the “effective polarization”, $\sigma_0$ is the unpolarized cross section, $\sigma_{LR}$ is the cross section with left helicity $e^-$ and right helicity $e^+$, and $P_L$ and $\overline{P}_L$ are degrees of longitudinal polarization of the $e^-$ and $e^+$ beams respectively. With opposite signs of beam polarization for $e^+$ and $e^-$, the factors $P_{\text{eff}}^L$ and $(1 - P_L \overline{P}_L)$ are large and hence the corresponding total cross section is enhanced. Since polarized beams may not be available for the full period of operation of the collider, we consider alternative options of integrated luminosities for individual combinations of polarization. We assume three runs of the collider with three different combinations: a) unpolarized beams, b) same signs of beam polarization, and c) opposite signs of beam polarization for $e^-$ and $e^+$. Half of the integrated luminosity is assumed for the run a) with unpolarized beams and other half to be equally divided between the other two runs b) and c).

3 Angular distribution

We calculate the angular distribution arising from the square of the SM amplitude and from the interference between the SM amplitude and the amplitude arising from the anomalous $ZZH$ and $\gamma ZH$ couplings while ignoring terms bilinear in anomalous couplings assuming new physics contributions to be small. We have treated the two cases of longitudinal and transverse polarizations for the electron and positron beams separately. In all these cases, we have calculated traces using the symbolic manipulation program ‘FORM’ [10].

A. Angular distributions for longitudinally polarized beams

The angular distribution for the process $e^+e^- \rightarrow ZH$ with longitudinal beam polarization and including anomalous $ZZH$ and $\gamma ZH$ contributions may be written as

$$\frac{d\sigma_{Z^\gamma}}{d\Omega} \propto (1 - P_L \overline{P}_L) \left[ A_{Z^\gamma}^Z (1 + \sin^2 \theta) + B_{Z^\gamma}^Z + C_{Z^\gamma}^Z \cos \theta \right]$$

Apart from kinematic factors, the dependence of the coefficients $A$’s, $B$’s, etc on anomalous couplings and vector and axial couplings $g_V$ and $g_A$ is shown in Table 1.

| $A$ | $(g_V^2 + g_A^2) - 2g_V g_A P_{\text{eff}}^L$ | $g_V - g_A P_{\text{eff}}^L$ | $\Re \Delta a_Z, \Re a_\gamma$ |
| $B$ | $(g_V^2 + g_A^2) - 2g_V g_A P_{\text{eff}}^L$ | $g_V - g_A P_{\text{eff}}^L$ | $\Re b_Z, \Re b_\gamma$ |
| $C$ | $(g_V^2 + g_A^2) P_{\text{eff}}^L - 2g_V g_A$ | $g_A - g_V P_{\text{eff}}^L$ | $\Im b_Z, \Im b_\gamma$ |

Table 1: Dependence of coefficients $A$’s, $B$’s etc on vector and axial vector couplings and the anomalous couplings associated with them.

Immediate inferences from these expressions and Table 1 are:

(i) If coefficients $A$, $B$ and $C$ could be determined independently it would be possible to...
determine the anomalous couplings $\text{Re} a_\gamma$, $\text{Re} a Z$, $\text{Re} b_\gamma$, $\text{Re} b Z$, $\text{Im} b_\gamma$ and $\text{Im} b Z$.

(ii) Couplings $\text{Re} a Z$, $\text{Im} a_\gamma$, $\text{Im} b Z$, $\text{Im} b_\gamma$, $\text{Re} b Z$, $\text{Re} b_\gamma$ do not contribute to the angular distribution at this order, and hence remain undetermined.

(iii) Numerically $g_V$ is small, while $g_A = -1$. Hence, in unpolarized case, the terms $A Z$, $B Z$ and $C^\gamma$ dominate. On the other hand, there would be very low sensitivity to the remaining couplings, viz., $\text{Re} a_\gamma$, $\text{Re} b_\gamma$, and $\text{Im} b_\gamma$.

(iv) With longitudinal polarization turned on, with a reasonably large value of $P_L$, the coefficients $C^Z$, $A^\gamma$ and $B^\gamma$ would become significant. In that case, the sensitivity to $\text{Re} a_\gamma$, $\text{Re} b_\gamma$ and $\text{Im} b Z$ would be significant.

B. Angular distributions for transversely polarized beams

The angular distributions for the process $e^+ e^- \to Z H$ with transverse beam polarization is

$$\frac{d\sigma^{T \gamma}}{d\Omega} \propto \frac{d\sigma^{T \mu}}{d\Omega} + P_T P_T \sin^2 \theta [D^{Z \gamma}_T \cos 2\phi + E^{\gamma}_T \sin 2\phi],$$

where, $d\sigma^{T \mu}/d\Omega$ is the differential cross section with unpolarized beams.

From the above expression one can infer the following:

(i) To study any effects of transverse polarization, both electron and positron beams have to be polarized.

(ii) If the azimuthal angle $\phi$ of $Z$ is integrated over, there is no difference between the transversely polarized and unpolarized cross sections.

(iii) The $\cos 2\phi$ term of angular distribution (the $D_T$ term) determines a combination only of the couplings $\text{Re} a Z$ and $\text{Re} a_\gamma$.

(iv) A glaring advantage of using transverse polarization would be determination $\text{Im} a_\gamma$ from the $\sin 2\phi$ dependence of the angular distribution.

(v) Couplings $\text{Im} a Z$, $\text{Im} b Z$, $\text{Im} b_\gamma$, $\text{Re} b Z$, $\text{Re} b_\gamma$ remain undetermined with either longitudinal or transverse polarization.

4 Observables

We construct observables having definite $CP$ and $T$ transformation properties so as to probe couplings corresponding to interactions having those transformation properties under $CP$ and $T$. We have divided the observables into two categories : a) using the momenta of the $Z$ boson (listed in table 2), b) using the momenta of leptons coming from $Z$ decay (listed in table 3). From Table 2 we can see that with observables which are constructed from $Z$ boson momenta, couplings $\text{Im} a Z$, $\text{Im} b Z$, $\text{Im} b_\gamma$, $\text{Re} b Z$, $\text{Re} b_\gamma$ can not be probed. Since these couplings are $T$ odd, we need $T$-odd observables to probe them. $T$-odd observables include vector triple products, and so we need additional vectors which are only be available if we utilize the momenta of leptons coming from $Z$ decay and/or the spins of the initial $e^-$ and $e^+$. We have constructed such $T$ odd observables and have been listed in Table 3.

Observables $X_i$ are sensitive to longitudinal polarization and $Y_i$ are sensitive to transverse polarization.

5 Kinematical cuts

We need the following kinematical cuts for the identification of the decay leptons :

1. $E_f \geq 10$ GeV for each outgoing charged lepton,
2. $5^\circ \leq \theta_0 \leq 175^\circ$ for each outgoing charged lepton for it to remain away from the beam pipe,

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the couplings they probe. Here, \( P = p_1 + p_2 \) and \( q = p_3 + p_4 \).

### Table 2: Observables constructed using \( Z \) boson momenta \( q \), their CP and T properties and the couplings they probe. Here, \( P = p_1 + p_2 \) and \( q = p_3 + p_4 \).

| Symbol | Observable | CP | T | Couplings |
|--------|------------|----|---|-----------|
| \( A_{FB} \) | sign([p_1 - p_2], q) | - | + | \( \text{Im}\hat{b}_Z, \text{Im}\hat{b}_\gamma \) |
| \( A_T \) | sign(\(\sin 2\phi\)) | - | + | \( \text{Ima}_\gamma \) |
| \( X_1 \) | (p_1 - p_2), q | - | + | \( \text{Im}\hat{b}_Z, \text{Im}\hat{b}_\gamma \) |
| \( X_7 \) | (|p_1 - p_2|)^2 | + | + | \( \text{Re}b_Z, \text{Re}b_\gamma \) |
| \( Y_1 \) | (q_x q_y) | + | - | \( \text{Ima}_\gamma \) |
| \( Y_2 \) | (q_x^2 - q_y^2) | + | + | \( \text{Re}b_Z, \text{Re}a_\gamma, \text{Re}b_\gamma \) |

### Table 3: Observables constructed using lepton momenta from \( Z \) decay, their CP and T properties and the couplings they probe. Here, \( P = p_1 + p_2 \) and \( q = p_3 + p_4 \).

| Symbol | Observable | CP | T | Couplings |
|--------|------------|----|---|-----------|
| \( X_2 \) | \( P, (p_3 - p_4) \) | - | + | \( \text{Im}\hat{b}_Z, \text{Im}\hat{b}_\gamma \) |
| \( X_3 \) | (\(p_3 \times p_4\)) \( L \) | - | - | \( \text{Re}b_Z, \text{Re}b_\gamma \) |
| \( X_4 \) | (p_1 - p_2), (p_3 - p_4), (p_1, p_2) \( L \times \), \( p_3, p_4 \) \( L \) | - | - | \( \text{Re}b_Z, \text{Re}b_\gamma \) |
| \( X_5 \) | (p_1 - p_2), q, (p_3 \times p_4) \( L \) | - | - | \( \text{Im}\hat{b}_Z, \text{Im}\hat{b}_\gamma \) |
| \( X_6 \) | \( P, (p_3 - p_4), (p_3 \times p_4) \) \( L \) | - | - | \( \text{Im}\hat{b}_Z, \text{Im}\hat{b}_\gamma \) |
| \( X_8 \) | (p_1 - p_2), (p_3 - p_4), z \( L \) | - | - | \( \text{Re}b_Z, \text{Re}b_\gamma \) |
| \( Y_3 \) | (p_3 - p_4), (p_3 - p_4) y | - | - | \( \text{Ima}_\gamma, \text{Imb}_\gamma \) |
| \( Y_4 \) | q_x q_y (p_3 - p_4) z | - | - | \( \text{Re}b_Z, \text{Re}b_\gamma \) |
| \( Y_5 \) | (p_3 - p_4), z (p_3 - p_4) y | - | - | \( \text{Im}\hat{b}_Z, \text{Im}\hat{b}_\gamma \) |
| \( Y_6 \) | [(p_3)^2 - (p_4)^2] - [(p_3)_y^2 - (p_4)_y^2] | - | - | \( \text{Im}\hat{b}_Z, \text{Im}\hat{b}_\gamma \) |

3. \( \Delta R_\eta \geq 0.2 \) for the pair of charged lepton, where \((\Delta R)^2 \equiv (\Delta \phi)^2 + (\Delta \eta)^2\), \(\Delta \phi\) and \(\Delta \eta\) being the separation in azimuthal angle and rapidity, respectively.

4. Cut on the invariant mass of the \( f \) to confirm onshellness of the \( Z \) boson, which is \( \Delta_{fL} \equiv |m_{fL} - M_Z| \leq 5 \Gamma_Z \). This cut also reduce the contamination from \( \gamma \gamma H \) couplings.

### 6 Sensitivities

We now obtain numerical results for the asymmetries, the correlations and the sensitivities of these observables for a definite configuration of the linear collider. For our numerical calculations, we have made use of the following values of parameters: \( M_Z = 91.19 \) GeV, \( \alpha = 1/128 \), \( \sin^2 \theta_W = 0.22 \). For the parameters of the linear collider, we have assumed \( \sqrt{s} = 500 \) GeV, \( P_L = 0.8, \text{Re}P_L = \pm 0.6, P_T = 0.8, \text{Re}P_T = \pm 0.6 \), and an integrated luminosity \( \int L dt = 500 \) fb\(^{-1}\). We have chosen to work with Higgs mass of 120 GeV.

Each observable depends on a combination of a limited number of couplings, dependent
on CP and T properties. We can determine, from a single observable, limits on individual couplings by combining the results from more than one observable, or from more than one combination of polarization. We will refer to limits on a coupling as an individual limit if all other couplings are zero. If no such assumption is made, and more than one observable is used simultaneously to put limits on all couplings contributing to these observables, we will refer to the limits as simultaneous limits.

We demand that the contributions to the observable coming from the anomalous parts are less than the statistical fluctuation in these quantities at a chosen level of significance and study the sensitivity of a LC to probe them.

Statistical fluctuations in the cross-section or in an asymmetry for a given luminosity \( \mathcal{L} \) can be written as: 
\[
\Delta \sigma = \sqrt{\frac{\sigma_{SM}}{\mathcal{L}}} \quad \text{and} \quad (\Delta A)^2 = \frac{(1 - A_{SM}^2)}{(\sigma_{SM} \mathcal{L})}
\]
respectively.

Transverse polarization, using \( A_T \), enables the determination of \( \text{Im} \tilde{b}_\gamma \) independent of all other couplings, with a possible 95% CL limit of about 0.04.

For the purpose of illustrating how we determine simultaneous limits, let us take, for example, the observable \( A_{FB} \) which determines a combination of \( \text{Im} \tilde{b}_Z \) and \( \text{Im} \tilde{b}_\gamma \). For three different polarization combinations chosen by us, we have three different combinations of these couplings determined by \( A_{FB} \) which we have plotted in fig 1. The best simultaneous limits is obtained by looking at the smallest region enclosed by the lines. In fig 1, the smallest region is enclosed by lines corresponding to unpolarized beams and beams having opposite sign of polarization of \( e^+ \) and \( e^- \). The best limits are obtained by looking at the extremeties of the region. These are \( |\text{Im} \tilde{b}_\gamma| \leq 4.69 \times 10^{-3}; |\text{Im} \tilde{b}_Z| \leq 5.61 \times 10^{-3} \). In a similar fashion, we can put simultaneous limits on all other couplings by using different observables or using various combination of polarizations. In Table 4 we show 95% individual limits obtained on couplings \( \text{Im} \tilde{b}_Z \) and \( \text{Im} \tilde{b}_\gamma \) using the forward-backward asymmetry \( A_{FB} \) for various combinations of beam polarizations.

![Figure 1: The region in the \( \text{Im} \tilde{b}_Z - \text{Im} \tilde{b}_\gamma \) plane accessible at the 95% CL with observable \( X_1 \) with different longitudinal beam polarization configurations.](image)

| \( P_L \) | \( \mathcal{P}_L \) | \( |\text{Im} \tilde{b}_\gamma| \) | \( |\text{Im} \tilde{b}_Z| \) |
|---|---|---|---|
| Unpolarized | | 0.00392 | 0.0108 |
| \( P_L = 0.8, \mathcal{P}_L = +0.6 \) | | 0.00543 | 0.0229 |
| \( P_L = 0.8, \mathcal{P}_L = -0.6 \) | | 0.00320 | 0.00262 |

Table 4: 95% CL individual limits on couplings \( \text{Im} \tilde{b}_Z \) and \( \text{Im} \tilde{b}_\gamma \) using forward-backward asymmetry, \( A_{FB} \).

In Tables 5 and 6 we show 95% CL individual limits on anomalous couplings using observables for different combinations of beam polarizations. From Table 6, we see that longitudinal polarization helps in enhancing the sensitivities of the couplings which are not much sensitive in unpolarized case.

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Table 5: The 95 % C.L. individual limits on the anomalous $ZZH$ and $\gamma ZH$ couplings, with transversely polarized beams for $\sqrt{s} = 500$ GeV and $\int L \, dt = 500$ fb$^{-1}$.

| Observable | Coupling | $(P_T, P_T)$ |
|------------|----------|-------------|
| $Y_1$ | $(q_y q_y)$ | $\text{Im} a_y$ | $1.98 \times 10^{-1}$ |
| $Y_2$ | $(q_x^2 - q_y^2)$ | $\text{Re} b_y$ | $2.65 \times 10^{-2}$ |
| $Y_3$ | $(p_3 - p_4)_x (p_3 - p_4)_y$ | $\text{Im} b_y$ | $4.72 \times 10^{-2}$ |
| $Y_4$ | $q_y (p_3 - p_4)_z$ | $\text{Im} b_z$ | $1.58 \times 10^{-1}$ |
| $Y_5$ | $(p_3 - p_4)_x (p_3 - p_4)_y q_z$ | $\text{Re} b_z$ | $5.56 \times 10^{-2}$ |
| $Y_6$ | $[(p_3)_x^2 - (p_4)_x^2] - [(p_3)_y^2 - (p_4)_y^2]$ | $\text{Im} b_z$ | $1.10 \times 10^{-1}$ |

Table 6: The 95 % C.L. individual limits on the anomalous $ZZH$ and $\gamma ZH$ couplings, for $\sqrt{s} = 500$ GeV and integrated luminosity $\int L \, dt = 500$ fb$^{-1}$.

| Coupling | $P_L = 0$ | $P_L = 0.8$ | $P_L = 0.8$ |
|----------|-----------|-------------|-------------|
| $X_1$ | $\text{Im} b_Z$ | $4.11 \times 10^{-2}$ | $8.69 \times 10^{-2}$ | $9.94 \times 10^{-2}$ |
| | $\text{Im} b_{y}$ | $1.49 \times 10^{-2}$ | $2.06 \times 10^{-2}$ | $1.22 \times 10^{-2}$ |
| $X_2$ | $\text{Im} b_Z$ | $4.12 \times 10^{-2}$ | $5.99 \times 10^{-2}$ | $3.84 \times 10^{-2}$ |
| | $\text{Im} b_{y}$ | $5.23 \times 10^{-1}$ | $3.12 \times 10^{-1}$ | $5.52 \times 10^{-2}$ |
| $X_3$ | $\text{Re} b_Z$ | $1.41 \times 10^{-1}$ | $2.97 \times 10^{-1}$ | $3.40 \times 10^{-2}$ |
| | $\text{Re} b_{y}$ | $5.09 \times 10^{-2}$ | $7.05 \times 10^{-2}$ | $4.15 \times 10^{-2}$ |
| $X_4$ | $\text{Re} b_Z$ | $2.95 \times 10^{-2}$ | $4.29 \times 10^{-2}$ | $2.75 \times 10^{-2}$ |
| | $\text{Re} b_{y}$ | $3.81 \times 10^{-1}$ | $2.24 \times 10^{-1}$ | $3.95 \times 10^{-2}$ |
| $X_5$ | $\text{Im} b_Z$ | $7.12 \times 10^{-2}$ | $1.04 \times 10^{-1}$ | $6.64 \times 10^{-2}$ |
| | $\text{Im} b_{y}$ | $9.10 \times 10^{-1}$ | $5.42 \times 10^{-1}$ | $9.53 \times 10^{-2}$ |
| $X_6$ | $\text{Im} b_Z$ | $7.12 \times 10^{-2}$ | $1.50 \times 10^{-1}$ | $1.72 \times 10^{-2}$ |
| | $\text{Im} b_{y}$ | $2.58 \times 10^{-2}$ | $3.57 \times 10^{-2}$ | $2.10 \times 10^{-2}$ |
| $X_7$ | $\text{Re} b_Z$ | $1.75 \times 10^{-2}$ | $2.54 \times 10^{-2}$ | $1.63 \times 10^{-2}$ |
| | $\text{Re} b_{y}$ | $2.23 \times 10^{-1}$ | $1.34 \times 10^{-1}$ | $2.35 \times 10^{-2}$ |
| $X_8$ | $\text{Re} b_Z$ | $1.53 \times 10^{-2}$ | $2.22 \times 10^{-2}$ | $1.42 \times 10^{-2}$ |
| | $\text{Re} b_{y}$ | $1.94 \times 10^{-1}$ | $1.16 \times 10^{-1}$ | $2.04 \times 10^{-2}$ |

7 Summary and conclusions

We have obtained angular distributions for the process $e^+ e^- \rightarrow ZH$ in the presence of anomalous $\gamma ZH$ and $ZZH$ couplings to linear order in these couplings in the presence of longitudinal and transverse beam polarizations. We have then looked at observables and asymmetries which can be used in combinations to disentangle the various couplings to the extent possible. We have also obtained the sensitivities of these observables and asymmetries to the various couplings for a definite configuration of the linear collider.

In some cases, the contribution of some couplings is suppressed due to the vector coupling of $Z$ to $e^+ e^-$ being very small. In those cases longitudinal polarization helps to enhance the contribution of these couplings and thereby improving the sensitivity.

We find that with a linear collider operating at a c.m. energy of 500 GeV with the
capability of 80% electron polarization and 60% positron polarization with an integrated luminosity of 500 fb$^{-1}$, with the observables described above, it would be possible to place 95% CL individual limits of the order of a few times $10^{-2}$ on all couplings taken nonzero one at a time with use of an appropriate combination ($P_L$ and $P_L'$ of opposite signs) of longitudinal beam polarizations. This is an improvement by a factor of 5 to 10 as compared to the unpolarized case. The simultaneous limits possible are, as expected, less stringent. While they continue to be better than $5 \times 10^{-2}$ for most couplings. Transverse polarization enables the determination of $\text{Im}a_\gamma$ independent of all other couplings. Also, in the angular distribution of $Z$, all $T$ odd anomalous couplings $\text{Im}a_Z$, $\text{Im}b_Z$, $\text{Re}b_Z$ and $\text{Re}b_\gamma$ are absent, and therefore, inorder to probe these couplings, we need to construct $T$-odd observables which involve vector triple products. For this, we study the decay of the $Z$ boson to a charged lepton pair and construct various $T$-odd observables using the momenta of the leptons coming from $Z$ decay. With $T$-odd observables, we could probe all the anomalous couplings which are earlier absent.

We have assumed that only one leptonic decay mode of $Z$ is observed. Including both $\mu^+\mu^-$ and $\tau^+\tau^-$ modes would trivially improve the sensitivity. In case of observables like $X_1$, $Y_1$, $Y_2$, which do not need charge identification, even hadronic decay modes of $Z$ can be included, which would considerably enhance the sensitivity.

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