Strong Equivalence Relations for Iterated Models

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Abstract. The Iterated Immediate Snapshot model (IIS), due to its elegant geometrical representation, has become standard for applying topological reasoning to distributed computing. Its modular structure makes it easier to analyze than the more realistic (non-iterated) read-write Atomic-Snapshot memory model (AS). It is known that AS and IIS are equivalent with respect to wait-free task computability: a distributed task is solvable in AS if and only if it is solvable in IIS. We observe, however, that this equivalence is not sufficient in order to explore solvability of tasks in sub-AS models (i.e. proper subsets of AS runs) or computability of long-lived objects, and a stronger equivalence relation is needed.

In this paper, we consider adversarial sub-AS and sub-IIS models specified by the sets of processes that can be correct in a model run. We show that AS and IIS are equivalent in a strong way: a (possibly long-lived) object is implementable in AS under a given adversary if and only if it is implementable in IIS under the same adversary. Therefore, the computability of any object in shared memory under an adversarial AS scheduler can be equivalently investigated in IIS.

1 Introduction

Iterated memory models (see a survey in \cite{25}) proved to be a convenient tool to investigate and understand distributed computing. In an iterated model, every process passes, one by one, through a series of disjoint communication-closed memories $M_1$, $M_2$, \ldots Each memory $M_i$ is a distinct set of shared memory locations that can only be accessed a bounded number of times. The most popular iterated model is the Iterated Immediate Snapshot model (IIS) \cite{4}. Here a process accesses each memory with the immediate snapshot operation \cite{3} that writes to the memory and returns a snapshot of the memory contents. Once memory $M_k$ is accessed, a process never comes back to it. IIS has many advantages over the more realistic (non-iterated) read-write Atomic-Snapshot memory model (AS) \cite{1}. Moreover, nice geometrical representation of IIS \cite{21,24} makes it suitable for topological reasoning in analyzing algorithms and proving

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their correctness. It is therefore natural to look for a generic transformation that would map any problem in AS to an equivalent problem in IIS.

It has been shown by Borowski and Gafni [4] that the complete sets of runs of IIS and AS are, in a strict sense, equivalent: a distributed task is (wait-free) solvable in AS if and only if it is (wait-free) solvable in IIS. They established the result by presenting a forward simulation that, in every AS run, simulates an IIS run [3], and a backward simulation that, in every IIS run, simulates an AS run [4]. The equivalence turned out to be instrumental, e.g., in deriving the impossibility of wait-free set agreement [2, 19]. More generally, the equivalence enables the topological characterization of task solvability in AS [19,15].

However, tasks can be seen as one-shot abstraction: a process is invoking a task at most once. As a result, any nonblocking [20] task solution is also wait-free, so the equivalence established in [3, 4] allows for equating AS and IIS in terms of (wait-free) task solvability. However, for long-lived objects, this equivalence turns out to be insufficient. The goal of this paper is to establish a stronger one using elaborate model simulations.

Another motivation for a stronger equivalence between AS and IIS is the question of task solvability in sub-AS models (i.e., proper subsets of AS runs). We focus on adversarial sub-AS models [5, 22], specified by sets of processes that can be correct in a model run. Note that the original AS model is described by the adversary consisting of all non-empty sets of processes. Since the introduction of adversaries in [5], the models have become popular for investigating task computability [10, 11, 17]. But how to define an IIS “equivalent” for an adversarial sub-AS model?

In IIS, a correct yet “slow” process may be never noticed by other processes: a process may go through infinitely many memories $M_1, M_2, \ldots$ without appearing in the snapshots of any other process. Instead, we specify adversarial sub-IIS models using the sets of strongly correct processes [27] (originally referred to as fast processes [7]). Informally, a process is strongly correct in an IIS run if it belongs to the largest set of processes that “see” each other infinitely often in the run. More precisely, we match the run with a directed graph whose vertices are processes, and there is an edge from $i$ to $j$ if and only if $j$ sees $i$ infinitely often in the run. Now the processes that constitute the largest strongly connected component in the graph are called strongly correct (we show that there is exactly one such component in the graph).

A topological characterization of task computability in sub-IIS models has been recently derived [12]: given a task $T$ and an sub-IIS model $M$, solvability of $T$ in $M$ is equated with the existence of a specific continuous map between geometrical structures modelling inputs and outputs of the task. The characterization of [12] extends the celebrated asynchronous computability theorem [19] to sub-IIS models, and may provide deeper insights about task (in)solvability in sub-IIS than conventional operational reasoning can give. But is sub-IIS characterization of [12] relevant for (more realistic) adversarial sub-AS models?

In this paper, we show that the answer is “yes”. We prove that sub-models of IIS and AS that are governed by the same adversary are equivalent in a strong