Wounded quarks and diquarks in heavy ion collisions

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A model in which the soft collisions of the nucleon are described in terms of interactions of its two constituents (a quark and a diquark) is proposed. When adjusted to describe precisely the elastic proton-proton scattering data and supplemented with the idea of wounded constituents, the model accounts rather well for the centrality dependence of particle production in the central rapidity region at RHIC energies.

I. INTRODUCTION

The data on production of particles in relativistic heavy ion collisions collected during the operation of the RHIC accelerator [1–3] indicate that the wounded nucleon model [4] does not describe correctly the observed particle multiplicities. Contrary to the model predictions, the particle density per one wounded nucleon in the central rapidity region (i) increases slowly with centrality of the collision and (ii) substantially exceeds that observed in nucleon-nucleon collisions.

This is not too surprising because the model was always considered only as a first, rough approximation. Indeed, soon after the original model was formulated, a possible improvement was proposed in the form of the wounded quark model [5]. Recently, it was suggested [6] that the wounded quark model can account for the main features of the data in the central rapidity region. To obtain this result, however, it was necessary to assume a rather large number of quark-quark collisions in the nucleon-nucleon scattering which is difficult to justify. If more realistic value of this number is used, the model predicts larger multiplicities than actually observed [7]. Nevertheless, these results show that the idea of a wounded constituent model may be not far from reality.

In the present paper we propose another generalization of the wounded nucleon model, based on the idea that the nucleon is composed of two constituents: one constituent quark forming a colour triplet and one constituent diquark forming a colour antitriplet. We assume, furthermore, (i) that particle production from these constituents is independent of the number of interactions they underwent and (ii) both constituents produce the same number of particles. It turns out that these two assumptions are sufficient to describe correctly the PHOBOS data [1–3] on particle production in the central (\(y = 0\)) rapidity region.

In the next section we formulate the model in more detail. In Section 3 the determination of the model parameters from proton-proton elastic scattering is described. Particle production in Au-Au collisions is discussed and compared with the PHOBOS data in Section 4. Our comments and conclusions are listed in the last section.

II. THE MODEL

As indicated in the introduction, we assume that in the soft collisions nucleon can be approximated by a system composed of the two constituents - a quark and a diquark - acting independently.

For the process of particle production we assume that each constituent (quark or diquark) which underwent at least one inelastic collision emits a certain amount of secondary partons. This number is independent of the number of collisions this constituent underwent afterwards. As
this is obviously a straightforward generalization of the wounded nucleon concept we call them wounded constituents. Furthermore, we assume that the wounded constituent quark and diquark emit secondary partons in approximately the same manner.

The arguments in favour of such an approach are collected in the last section. Here we would like to emphasize, however, that they may apply only to soft collisions, i.e. those in which the transverse mass of the emitted partons does not exceed \( \sim 200 \text{ MeV} \). The analogous limits for the observed hadrons are, naturally, somewhat higher.

It should be also pointed out that the proposed description can only be justified far from the fragmentation regions of the projectile and of the target. Therefore in this paper we restrict the comparison with the data to the central rapidity region.

The consequence of these assumptions, fundamental for our conclusions, is that the differential multiplicity of partons emitted in the nucleus-nucleus inelastic collisions can be represented as:

\[
\frac{dN_{AB}}{dy} = w_A^{(q+d)} F_+(y) + w_B^{(q+d)} F_-(y),
\]

where \( w_A^{(q+d)} \) and \( w_B^{(q+d)} \) are the numbers of wounded constituents (quarks and diquarks) in nucleus \( A \) and \( B \), whereas \( F_+, F_- \) are the differential multiplicities of partons emitted by one wounded constituent in \( A \) and \( B \), respectively.

When (1) is applied to nucleon-nucleon collisions we obtain:

\[
\frac{dN_{NN}}{dy} = w_N^{(q+d)} [F_+(y) + F_-(y)].
\]

These two equations summarize the relation between particle production in nucleon-nucleon and nucleus-nucleus collisions implied by the model.

For the symmetric collisions \((A = B)\) we obtain the particularly simple and elegant result:

\[
R_{AA} \equiv \frac{dN_{AA}/dy}{dN_{NN}/dy} = \frac{w_A^{(q+d)}}{w_N^{(q+d)}},
\]

Since the R.H.S. of this equation is independent of the phase-space region where \( R_{AA} \) is measured\(^2\) but depends on the centrality of the collision, this is indeed a very strong consequence of the model.

At the vanishing c.m. rapidity, (1) and (2) imply a simple relation even for asymmetric collisions:

\[
R_{AB}(y = 0) \equiv \frac{dN_{AB}(y = 0)/dy}{dN_{NN}(y = 0)/dy} = \frac{w_A^{(q+d)} + w_B^{(q+d)}}{2w_N^{(q+d)}}.
\]

To make a full use of (3) and (4), it is necessary to evaluate \( w_N^{(q+d)} \) and \( w_A^{(q+d)} \), \( w_B^{(q+d)} \) as function of the impact parameter of the collision. The corresponding formulae are obtained by a straightforward counting of probabilities \(^4\) \(^8\). For the average number of wounded quarks and diquarks in each of the colliding nuclei \( (w_A^{(q+d)} = W_q + W_d) \), at a fixed impact parameter \( b \), one obtains:

\[
W_a(b) = \frac{A}{\sigma_{AA}(b)} \int T(b - s) \left(1 - [1 - \sigma_{aq} T_{aq}(s)]^A [1 - \sigma_{ad} T_{ad}(s)]^A\right) d^2 s,
\]

\(^1\) This formula of course is valid for any distribution, not necessarily in rapidity.

\(^2\) Provided it is far enough from the fragmentation regions.
where \( a \) denotes \( q \) or \( d \), \( \sigma_{AA}(b) \equiv d^2\sigma_{AA}/db^2 \) is the inelastic differential \( AA \) cross-section\(^3\) \( T(b) \) is the nuclear thickness function (normalized to unity). \( T_{ab}(b) \) is given by:

\[
T_{ab}(b) = \frac{1}{\sigma_{ab}} \int \sigma_{ab}(s)T(b - s)d^2s, \tag{6}
\]

where \( \sigma_{ab}(s) \equiv d^2\sigma_{ab}/ds^2 \) are the differential cross-sections of the constituents (in impact parameter plane) and \( \sigma_{ab} = \int \sigma_{ab}(s)d^2s \) are the corresponding total inelastic cross-sections (\( ab \) denotes \( qq, qd \) or \( dd \)).

Note that the formulae (5) and (6) take into account the impact parameter dependence of the constituent cross-sections. If this dependence is neglected \( [\sigma_{ab}(s) = \sigma_{ab}d^2(s)], \ T_{ab}(b) \equiv T(b) \)\(^4\).

For the number of wounded constituents in nucleon-nucleon collisions we have, similarly, \( w^{(q+d)}_N = w_q + w_d \) with:

\[
w_q,d = \frac{1}{\sigma_{NN}} \int h_{q,d}(b)d^2b, \tag{7}
\]

where \( \sigma_{NN} \) is the total inelastic nucleon-nucleon cross-section and \( h_a \) is given by:

\[
h_a(b) = \int d^2s_q d^2s_d 'd^2s_qd'd^2Ds(q,s_d)D(s_q',s_d') \{1 - \left[1 - \sigma_{qq}(b + s_q' - s_d)\right]\left[1 - \sigma_{qd}(b + s_d' - s_a)\right]\}, \tag{8}
\]

with \( D(s_q,s_d) \) being the effective thickness of the nucleon.

It remains to determine the cross-sections of the constituents and their distribution inside the nucleon. This demands a detailed analysis of the nucleon-nucleon collisions, as discussed in the next section.

### III. NUCLEON-NUCLEON COLLISIONS

The distribution of the constituents inside the nucleon and their cross-sections are not known. We propose to determine them from the analysis of the data on proton-proton elastic scattering.

Consider first the inelastic nucleon-nucleon collisions. Following the idea that the interaction can be described as the interaction of two independent constituents in each of the nucleons, we have \(5, 9\):

\[
1 - \sigma(s_q, s_d; s_q', s_d'; b) = \left[1 - \sigma_{qq}(b + s_q' - s_q)\right]\left[1 - \sigma_{qd}(b + s_d' - s_d)\right], \tag{9}
\]

and

\[
\sigma(b) = \int d^2s_q d^2s_d' d^2s_qd'd^2Ds(q,s_d)D(s_q',s_d')\sigma(s_q,s_d; s_q', s_d'; b), \tag{10}
\]

where \( s_q(s_d') \), \( s_d(s_d') \) are transverse positions of the quarks and diquarks in the two colliding nucleons.

From the unitarity condition we deduce:

\[
t_{el}(b) = 1 - \sqrt{1 - \sigma(b)}. \tag{11}
\]

\(^3\) For heavy nuclei \( \sigma_{AA}(b) = 1 \), except at very large impact parameters which are of no interest even for most peripheral events measured at RHIC. In case of \( AuAu \) collisions, using the optical approximation, we have verified that \( \sigma_{AxAu}(b) = 1 \) for \( b \leq 14 \text{ fm} \), corresponding to \( W \geq 5 \).

\(^4\) We have verified that this is a poor approximation for peripheral collisions.
This allows one to evaluate the elastic and total cross-sections. By comparing them with data one can obtain information on the parameters of the model.

Since the nuclear cross-sections are not sensitive to the exact shape of the impact parameter dependence of the constituent cross-sections (as the nuclear radius is much larger than that of the nucleon), we parametrized \( \sigma_{ab}(s) \) using simple Gaussian forms:

\[
\sigma_{ab}(s) = A_{ab} e^{-s^2/R_{ab}^2}.
\]  

(12)

The radii \( R_{ab} \) were constrained by the condition \( R_{ab}^2 = R_a^2 + R_b^2 \) where \( R_a \) denotes the quark or diquark’s radius (a natural constraint for the Gaussians).

From (12) we deduce the total inelastic cross sections: \( \sigma_{ab} = \pi A_{ab} R_{ab}^2 \) and we also demand that the ratios of cross-sections satisfy the natural condition:

\[
\sigma_{qq} : \sigma_{qd} : \sigma_{dd} = 1 : 2 : 4,
\]  

(13)

expressing the idea that there are twice as many partons in the constituent diquark than those in the constituent quark. This allows to express \( A_{qd} \) and \( A_{dd} \) in terms of \( A_{qq} \).

For the distribution of the constituents we again take a Gaussian:

\[
D(s_q, s_d) = \frac{1 + \lambda^2}{\pi R^2} e^{-(s_q^2 + s_d^2)/R^2} \delta^2(s_d + \lambda s_q).
\]  

(14)

The parameter \( \lambda \) has the physical meaning of the ratio of the quark and diquark masses and satisfies, \( 1/2 \leq \lambda = m_q/m_d \leq 1 \) (the delta function guarantees that the center-of-mass of the system moves along the straight line).

Thus, finally, the model contains 5 free parameters.

Using this formulation and the formula (11) we have evaluated the elastic and total proton-proton cross-sections and adjusted the parameters by demanding that (i) total inelastic cross section (ii) slope of the elastic cross section (iii) position of the first diffractive minimum in elastic cross section and (iv) height of the second maximum in elastic scattering are in agreement with data.

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5 We have verified that the main results of this paper are not sensitive to the exact values of these ratios. A detailed analysis of elastic pp scattering (and, particularly of this assumption) will be given elsewhere [10].

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FIG. 1. Comparison of the experimental differential cross section with the results of the quark-diquark model. Data at \( \sqrt{s} = 30.5 \) GeV are taken from [11].
Data at all ISR energies [11, 12] were analyzed. It turns out that the model works very well indeed which is by no means a trivial conclusion. One example of such calculation is shown in Fig. 1 where the differential cross section $d\sigma/dt$ at the ISR energy $30.5$ GeV, evaluated from the model, is compared with experimental data [11]. One sees a rather impressive agreement. A detailed discussion goes beyond the scope of this paper and will be given elsewhere [10]. Here we are interested only in the resulting values for $\sigma_{qq}/\sigma_{NN}$ and $w_{N}^{(q+d)}$ which are necessary for evaluation of the R.H.S. of (3).

From the point of view of the present investigation, the most important observation is that both the ratio $\sigma_{qq}/\sigma_{NN}$ and the average number of wounded constituents in nucleon-nucleon collisions $w_{N}^{(q+d)}$ seem almost entirely independent of the details of the model (provided that, as explained above, the parameters are adjusted to describe correctly the proton-proton elastic data). The obtained values are:

$$\frac{\sigma_{qq}}{\sigma_{NN}} = 0.147 - 0.148; \quad w_{N}^{(q+d)} = w_{q} + w_{d} = 1.182 - 1.186.$$  

These values, supplemented by the relation [13], are used for evaluation of the R.H.S. of (3).

IV. AU+AU COLLISIONS

Having determined the parameters of the model from the elastic pp data, we could evaluate its predictions for the particle production in Au-Au collisions which is the main goal of this investigation.

![FIG. 2. Predictions of the wounded quark-diquark model (for $W \geq 5$) compared with those from the wounded nucleon model.](image)

Since the PHOBOS data are presented versus the number of the wounded nucleons in both colliding nuclei ($2w_{A}^{(N)} = W$), we have to calculate also $w_{A}^{(N)}$ as a function of the impact parameter of the collision. This can be obtained from the well-known formula [4]:

$$w_{A}^{(N)}(b) = \frac{A}{\sigma_{AA}(b)} \int T(b - s) \{1 - [1 - \sigma_{NN}T_{NN}(s)]A\} d^{2}s,$$  

(16)

6 For instance, an analogous calculation performed in the model with the assumption that the proton consists of three uncorrelated constituent quarks led to negative conclusion [13].

7 Note that $\sigma_{qq}/\sigma_{NN} > 1/9$, indicating presence of shadowing.
with $\sigma_{NN}(s)$ in a Gaussian form with $\sigma_{NN}(0) = 0.92$ taken from our estimates (it agrees very well with the data [12]).

For the nuclear density we have been using the standard Woods-Saxon formula with the nuclear radius $R_{Au} = 6.37$ fm, and $d = 0.54$ fm.

In Fig. 2 the predicted ratio $R_{AuAu}/w_{Au}^{(N)}$ (which shows explicitly the deviation of our model from the traditional wounded nucleon model) is plotted versus $2w_{Au}^{(N)} = W$ for $W \geq 5$. One sees that the model explains naturally the increase of the production multiplicity from one wounded nucleon with increasing centrality of the collision.

The comparison of the model with the PHOBOS data on particle production per one wounded nucleon is shown in Fig. 3.

![Figure 3](image) FIG. 3. The predictions of the wounded quark-diquark model (for $W \geq 5$) compared with the data from PHOBOS coll. [1][3]. The shaded areas reflect the inaccuracy in the pp data.

The data on particle production in pp collisions, necessary to obtain the model predictions, were taken from UA5 collaboration, as quoted in [2][3][8]. They are also shown in the Fig. 3 as points at $W = 2$. This introduces some uncertainty, as indicated by the shadowed areas [9].

The inelastic proton-proton cross sections needed for this calculation were taken as $\sigma_{NN} = 32$ mb, 36 mb, 41 mb and 42 mb at energies $\sqrt{s} = 19.6$, 62.4, 130, and 200 GeV, respectively. Our model then gives the following values of the ratio of the integrated inelastic quark-quark to proton-proton cross sections: $\sigma_{qq}/\sigma_{NN} = 0.147$, 0.148, 0.148, 0.149. Finally, the number of wounded quarks and diquarks in a single proton-proton collision $w_q + w_d = 1.183, 1.185, 1.186, 1.187$.

One sees that, within the experimental accuracies a very good agreement both in shape and in absolute value is obtained.

V. COMMENTS AND DISCUSSION

Several comments are in order.

(i) The concept of wounded nucleons and of wounded constituents is based on two ideas: (a) during the interaction, the "wounded" object acts as one unit and (b) particle emission process takes much longer time than it is needed for the projectile to pass the internuclear distance. These

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8 For $\sqrt{s} = 62.4$ and 200 GeV they are taken directly from UA5 data, for $\sqrt{s} = 19.6$ and 130 GeV they are interpolated.

9 Unfortunately, the pp data from PHOBOS coll. are still not available.
assumptions can be qualitatively justified only for soft collisions where the momentum transfer and transverse mass of the emitted partons are small enough.

The first condition requires that the momentum transferred to the projectile is smaller than its inverse size. For the size of order of 1 fm, this limits the momentum transfer to about 200 MeV.

The second condition demands that the emission time:

\[ t \sim \frac{\gamma}{m_\perp} \approx \frac{e^y}{2m_\perp}, \]

where \( \gamma \) is the Lorentz factor of the emitted particle (parton) in the rest frame of the target nucleus and \( y \) is its rapidity in the same frame, should be significantly greater than the intranuclear distance. For \( y > 2 \) this limits the transverse mass of the emitted partons below 200 MeV. Of course for the observed final hadrons this limit may be significantly higher.

These estimates are, surely, rather crude. A more detailed verification of the model for particles with various masses and transverse momenta will be of great interest, as it may help to understand better the very concept of a wounded constituent.

(ii) As seen from these arguments, the model is not expected to apply in the fragmentation region of the projectile and target where, moreover, important energy-momentum conservation effects, as well as secondary interactions inside the nucleus must be present. Therefore we focus our attention on the central rapidity region which, at RHIC energies, is well separated from the fragmentation regions.

(iii) The model assumes that a wounded quark produces the same number of secondary partons as a wounded diquark. This is not unreasonable since the colour content of both constituents is the same (3 and \( \bar{3} \)) and that, probably, the colour charge of the projectile is the main factor determining the emission intensity. We admit, however, that since the actual dynamics of the soft production process is not yet understood, this argument can be questioned. The good agreement of the model with data, as presented in this paper, may thus serve as an (indirect) confirmation of the important role of colour dynamics in the process of particle production.

(iv) The predictions of the model described in this paper refer to the emission of "primary" partons and do not take into account further development of the system during its expansion and final formation of hadrons. The observed agreement with data indicates that the space-time development of the system, despite presence of the well-known collective effects, does not introduce drastic changes in its global characteristics. This may be considered as an argument in favour of the laminar hydrodynamic expansion suggested already by MC simulations of this process which seems to indicate very small viscosity of the created medium [14].

(v) We have verified that the main properties of the quark-diquark picture of the nucleon obtained from analysis of the pp elastic data are not sensitive to the details of the calculations (several forms of the distribution of constituents and of their cross-sections were considered). The typical values of the parameters are \( R_q \approx 0.3 \text{ fm}, R_d \approx 0.75 \text{ fm}, R \approx 0.3 \text{ fm}, A_{dd} \approx 0.55, A_{qd} \approx 0.5, A_{qq} \approx 1.1 \). Thus in our model the diquark appears to be rather large, comparable to the size of the proton. It is remarkable that this feature agrees well with other estimates [15], based on rather different arguments.

(vi) In this paper we have only discussed the symmetric Au-Au collisions. It would be also interesting to check the model for asymmetric collisions. We have verified that in case of d-Au collisions at \( y = 0 \) the formula [11] gives the result which does not differ very much (less than 10 \%) from that of the wounded nucleon model. Thus the observed good agreement of the wounded nucleon model [16] with the PHOBOS data on d-Au collisions [17] is not destroyed in our approach.

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10 This argument is due to W. Czyz (private communication).

11 \( A_{qq} \) depends somewhat on the value of \( \lambda \) which is not very well determined. For \( \lambda \) not far from \( 1/2 \), \( A_{qq} = 1 \).
In conclusion, we have formulated a model in which the soft collisions of the nucleon are described in terms of interactions of its two constituents: a quark and a diquark. The model can be adjusted to describe very precisely the elastic proton-proton scattering data. Supplemented with the idea of wounded constituents, the model accounts rather well for the centrality dependence of particle production in the central rapidity region at RHIC energies.

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