To the solution problems of contact interaction in a two-roll module

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Abstract. The basic problems of the theory of contact interaction in a symmetric two-roll module are solved. Mathematical models of roll contact curves, friction stresses and distribution patterns of contact stresses in a symmetric two-roll module are obtained. Expressions that determine the indices of the roll contact curves in a symmetric two-roll module are derived. It was revealed that the neutral point in the drive roll is to the left of the centerline, and the maximum point of the diagrams of normal contact stresses does not coincide with the neutral point and is either in the no-slip zone or in the backward slip zone.

1. Introduction
The main tasks of the theory of contact interaction in two-roll modules are [1]: the determination of the angles of contact; analytical description of the curve shape of a contact roll; friction stress modeling; analytical description of the distribution patterns of normal and tangential contact forces.

Many publications [1 - 7] are devoted to solving these problems. In these studies, one or two of the above problems are solved. Currently, there is no comprehensive approach that considers all the problems of the theory of contact interaction in a two-roll module. This work is devoted to solving the basic contact problems in the symmetric two-roll module (figure 1).
2. Analytical solutions to the assigned tasks

Let us solve the first problem, that is, determine the angles of contact (the entering angle and the exit angle).

From figure 1 it follows that

\[ 2R \cos \varphi_1 + \delta_1 = 2R + \delta_0. \]

Hence

\[ \varphi_1 = \arccos \left( \frac{2R + \delta_0 - \delta_1}{2R} \right), \quad \text{(1)} \]

where \( \delta_1 \) – is the initial thickness of the material layer; \( \delta_0 \) – is the distance between the rolls; \( \varphi_1 \) – is the entering angle.

Similar to (1), we have

\[ \varphi_2 = \arccos \left( \frac{2R + \delta_0 - \delta_2}{2R} \right), \quad \text{(2)} \]

where \( \delta_2 \) – is the final thickness of the material layer; \( \varphi_2 \) – is the exit angle.

Let us move on to solving the second problem.

Despite the simplicity of a two-roll module, its work is complicated by various accompanying phenomena. First, a very significant role is played by the deformation properties of contacting bodies (roll coatings and a layer of processed material) in a two-roll module. The deformation properties of the contacting bodies are set by empirical or rheological dependencies [5]. In this article, we investigate contact problems for the case when the character of contacting bodies deformation is specified by the Kelvin-Voigt rheological models.

Studies show that the curve shape of the contact rolls in a two-roll module depends on the ratio of the deformation rates of the contacting bodies. In many publications, this ratio is considered constant [5]. We accept this hypothesis in solving the assigned tasks.

The contact zones of the rolls relative to the centerline are divided into compression zone I and recovery zone II.

In zone I, we select an elementary sector on the roll contact curve defined by the angle \( d\theta_1 \).
According to figure 1, the beginning and the end of the elementary sector are defined from

\[ 2(r_i + h_i) \cos \theta_i = H, \]  
\[ 2(r_i - dr_i + h_i - dh_i) \cos(\theta_i - d\theta_i) = H, \]

where \( H \) – is the center distance of the rolls.

We transform equation (4) taking into account equation (3). Ignoring the quantities of higher order of smallness, we obtain the following differential equation

\[ 2(1 + m_1)dr_i = H \frac{\sin \theta_i}{\cos^2 \theta_i} d\theta_i, \]

where \( m_1 = \frac{dh_i}{dr_i} \) – is the ratio of the compression rate of the material layer to the compression rate of the roll.

The solution to the differential equation is determined with account for the initial conditions: \( r_i = R \) at \( \theta_i = -\varphi_i \). Then we obtain the equation of the roll contact curves in the compression zone

\[ r_i = R + \frac{H}{2(1 + m_1)} \left( \frac{1}{\cos \theta_i} - \frac{1}{\cos \varphi_i} \right). \]  

Similarly, we derive the equation of the roll contact curves in the recovery zone

\[ r_2 = R + \frac{H}{2(1 + m_2)} \left( \frac{1}{\cos \theta_2} - \frac{1}{\cos \varphi_2} \right), \]

where \( m_2 \) – is the ratio of the material compression velocity to the roll compression velocity.

Generalizing equations (5) and (6), we obtain the equations for the roll contact curves

\[ \begin{cases} 
  r_i = R + \frac{H}{2(1 + m_1)} \left( \frac{1}{\cos \theta_i} - \frac{1}{\cos \varphi_i} \right), & -\varphi_i \leq \theta_i \leq 0, \\
  r_2 = R + \frac{H}{2(1 + m_2)} \left( \frac{1}{\cos \theta_2} - \frac{1}{\cos \varphi_2} \right), & 0 \leq \theta_2 \leq \varphi_2. 
\end{cases} \]  

In [1], a model of friction stresses for a driven roll was obtained in the form:

\[ \begin{cases} 
  t_1 = tg(\theta_i - \psi_i + \xi)n_1, & -\varphi_i \leq \theta_i \leq 0, \\
  t_2 = tg(\theta_2 - \psi_i + \xi)n_2, & 0 \leq \theta_2 \leq \varphi_2, 
\end{cases} \]

where \( \psi_i = \arctg \frac{r_i'}{r_i}, \xi = \arctg \frac{F}{Q}, F, Q \) – are the forces acting on each roll of the two-roll module.

We transform system (8) taking into account the following considerations:

- a symmetrical two-roll module is considered in the study;
- the deviation of the curve shape of the contact from the circle practically does not affect the distribution pattern of tangential forces [8].

So, we have
Let us move on to solving the fourth problem. In the compression zone, we select an element of length $dl_1$. This element is acted upon from the roll by elementary normal $dN_1$ and tangential $dT_1$ forces and the response of the cut-off parts of the material layer. The components of forces $dN_1$ and $dT_1$ are balanced by force $\sigma dl_1$ (figure 1):

$$\sigma' dl_1 - dN_1 \cos \varphi - dT_1 \sin \varphi = 0,$$

$$\sigma' = n_1,$$

where $\sigma'$ is the compressive stress of the material layer, $n_1$ is the normal contact stress.

One of the conditions used in solving contact problems in a two-roll module is the equality of the compressive (recovery) stresses of the contacting bodies at each point of these curves, that is, $\sigma'_1 = \sigma^*_1$, where $\sigma'_1$ are the roll compression stresses [1]. Taking this condition into account, we have

$$\sigma^*_1 = n_1. \tag{10}$$

Let the character of the compression deformation of the roll elastic cover be given by the Kelvin-Voigt rheological model. Then

$$\sigma_1 = E_1 \varepsilon_1 + \mu_1 \frac{d\varepsilon_1}{dt}, \tag{11}$$

where $\sigma_1, \varepsilon_1, E_1, \mu_1$ are the stresses, deformation, deformation modulus, and viscosity coefficient of the elastic roll coating under compression, respectively.

From figure 1 it follows

$$\varepsilon_1 = \frac{R - r_1}{\lambda},$$

$\lambda$ – is the thickness of the roll elastic cover.

After substituting the expression $r_1$ from the system of equations (7), we obtain

$$\varepsilon_1 = \frac{H}{2\lambda(1 + m_1)} \left( \frac{1}{\cos \varphi_1} - \frac{1}{\cos \theta_1} \right). \tag{12}$$

Hence

$$\frac{d\varepsilon_1}{dt} = -\frac{H \omega}{2\lambda(1 + m_1)} \frac{\sin \theta_1}{\cos \theta_1} \tag{13},$$

where $\omega$ – is the angular velocity of the roll.

From expression (10), taking into account expressions (12) and (13), we obtain

$$\sigma_1 = \frac{H}{2\lambda(1 + m_1)} \left( \frac{E_1}{\cos \varphi_1} - \frac{E_1 + \omega \mu_1 \tan \theta_1}{\cos \theta_1} \right). \tag{14}$$
In expression (14), \( \sigma_1 \) reflects the compressive stress in experiments. Indeed, under contact interaction, \( \sigma_1^* \) reflects the compressive stress in the real process: at the beginning of the deformation zone it is zero, then it increases and reaches the value of \( \sigma_{1\text{max}} \) on the centerline. Therefore [1]

\[
\sigma_1^* = a_1 \sigma_1 + b_1, \quad -\varphi_1 \leq \theta_1 \leq 0.
\]

(15)

where \( a_{11}, b_{11} \) - are the coefficients determined from the following conditions:

\[ \text{at } \theta_1 = -\varphi_1, \quad \sigma_1^* = 0; \quad \text{at } \theta_1 = 0, \quad \sigma_1^* = \sigma_{1\text{max}}. \]

After determining the coefficients \( a_{11} \) and \( b_{11} \), and substituting the resulting expression \( \sigma_1^* \) into equations (10), we find the distribution pattern of normal stresses in the roll compression zone

\[
n_1 = B_1 \left( E_1 - \mu_1 \omega \varphi_1 \frac{\cos \varphi_1}{\cos \theta_1} (E_1 + \mu_1 \omega \varphi_1 \theta_1) \right), \quad -\varphi_1 \leq \theta_1 \leq 0,
\]

(16)

where

\[
B_1 = \frac{\sigma_{1\text{max}}}{E_1 (1 - \cos \varphi_1) - \mu_1 \omega \tan \varphi_1}.
\]

The distribution pattern of normal stresses in the roll recovery zone is determined in a similar way:

\[
n_2 = B_2 \left( E_2 - \mu_2 \omega \varphi_2 \frac{\cos \varphi_2}{\cos \theta_2} (E_2 - \mu_2 \omega \tan \varphi_2) \right), \quad 0 \leq \theta_2 \leq \varphi_2,
\]

(17)

where

\[
B_2 = \frac{\sigma_{1\text{max}}}{E_2 (1 - \cos \varphi_2) - \mu_2 \omega \tan \varphi_2}.
\]

From the system of equations (9), taking into account expressions (16) and (17), we determine the distribution pattern of shear stresses.

\[
t_1 = B_1 \left( E_1 - \mu_1 \omega \varphi_1 \frac{\cos \varphi_1}{\cos \theta_1} (E_1 + \mu_1 \omega \varphi_1 \theta_1) \right) \tan (\theta_1 + \xi), \quad -\varphi_1 \leq \theta_1 \leq 0,
\]

(18)

\[
t_2 = B_2 \left( E_2 - \mu_2 \omega \varphi_2 \frac{\cos \varphi_2}{\cos \theta_2} (E_2 - \mu_2 \omega \tan \varphi_2) \right) \tan (\theta_2 + \xi), \quad 0 \leq \theta_2 \leq \varphi_2.
\]

(19)

Thus, the basic problems of the theory of contact interaction in a symmetric two-roll module are solved.

Contact stresses determined by formulas (16) - (19) are distributed over the contact curves of the rolls. The characteristic points of these distributions, such as the neutral point, are located on the roll contact curves.

There are seven characteristic points on the roll contact curves (figure 1): \( A_1 \) - the starting point, \( A_2 \) - the end point, \( A_3 \) - the point of the contact line, \( A_4 \) - the point separating the slide area of no-slip zone and the backward slip zone, \( A_5 \) - the point separating the no-slip zone and the slide area of forward slip zone, \( A_6 \) - the neutral point, \( A_7 \) - the point of the maximum of normal stresses. Let the points \( A_i \) \((i=1,7)\), be determined by angles \( \varphi_i \), respectively.

The angles defining the characteristic points of the roll contact curves are the main indices of the contact interaction in two-roll modules.
Two of the seven indices are determined by formulas (1) and (2). The third index, in this case, is zero, that is

$$\varphi_3 = 0. \quad (20)$$

To determine the indices, we use friction stress models. Refined models of friction stresses for the slip zone were proposed in [9]. These models for a symmetrical two-roll module are:

- for the backward slip zone

$$t_1 = -f_1 n_1 \frac{v_{ck} (\theta_1)}{v_{ck} (-\varphi_1)}, \quad -\varphi_1 \leq \theta_1 \leq -\varphi_4, \quad (21)$$

where $t_1$, $v_{ck} (\theta_1)$, $n_1$ – are the values of shear stress, sliding velocity and normal stresses at a point defined by angle $\theta_1$; $f_1$ – is the coefficient of friction in the backward slip zone.

- for the forward slip zone

$$t_2 = -f_2 n_2 \frac{v_{ck} (\theta_2)}{v_{ck} (\varphi_2)}, \quad \varphi_{12} \leq \theta_{12} \leq \varphi_{12}, \quad (22)$$

where $t_2$, $v_{ck} (\theta_2)$, $n_2$ – are the values of shear stress, sliding velocity and normal stresses at the point defined by angle $\theta_2$; $f_{12}$ – is the coefficient of friction in the forward slip zone.

According to [1], the sliding velocity in the backward slip zone can be expressed by the following formula

$$v_{ck} (\theta_1) = \omega R - v_u \cos \theta_1, \quad -\varphi_1 \leq \theta_1 \leq -\varphi_4. \quad (23)$$

Hence, we have $v_{ck} (-\varphi_1) = \omega R - v_u \cos \varphi_1.$

As follows from equation (22)

$$\frac{t_1 (-\varphi_1)}{n_1 (-\varphi_1)} = -\frac{f_1 v_{ck} (-\varphi_1)}{v_{ck} (-\varphi_1)}. \quad (25)$$

or, taking into account equations (23) and (24)

$$\frac{t_1 (-\varphi_1)}{n_1 (-\varphi_1)} = -\frac{f_1 (\omega R - v_u \cos \varphi_1)}{\omega R - v_u \cos \varphi_{11}}. \quad (26)$$

On the other hand, equation (9) implies

$$\frac{t_1 (-\varphi_1)}{n_1 (-\varphi_1)} = tg (-\varphi_4 + \xi). \quad (27)$$

Considering equations (26) and (27), and assuming that $\sin \varphi_{44} \approx \varphi_{44}, \cos \varphi_{44} \approx 1$, we obtain the expressions for the exponent $\varphi_4$

$$\varphi_4 = \frac{f_1 (\omega R - v_u)}{\omega R - v_u \cos \varphi_{11}} + \xi. \quad (28)$$

We define the index $\varphi_5$ in a similar way:

$$\varphi_5 = \frac{f_2 (\omega R - v_u)}{\omega R - v_u \cos \varphi_{11}} - \xi. \quad (29)$$
The remaining two indices $\varphi_6$ and $\varphi_7$ from the seven indices of the roll contact curves, are determined by the force conditions.

At the neutral point $A_6$, and accordingly, at the neutral angle ($-\varphi_6$), the shear stress is zero. From the first formula of system (9), it follows that

$$\tan(-\varphi_6 + \xi) = 0$$

Hence, we have

$$\varphi_6 = \xi.$$  \hfill (30)

At the last characteristic point $A_7$, and accordingly, at the angle ($-\varphi_7$), the normal stress reaches its maximum value. The problem of determining $\varphi_7$ is solved by the well-known methods of defining the maximum point: first, we find the derivative $n'_i(\theta_i)$, then from the condition of equality to zero of the resulting expression at the point ($-\varphi_7$), we obtain

$$\omega \mu_1 (1 + \sin^2 \varphi_7) - E_1 \sin \varphi_7 \cos \varphi_7 = 0$$

Then, assuming that $\sin \varphi_7 \approx \varphi_7$, $\cos \varphi_7 \approx 1$, we determine

$$\varphi_7 = \frac{\omega \mu_1}{E_1}.$$  \hfill (31)

Thus, expressions (1), (2), (20), (28), (29), (30), and (31) are obtained, which determine the curve indices of the rolls contact in a symmetric two-roll module.

3. Results
The basic problems of the theory of contact interaction in a symmetric two-roll module are solved.

Mathematical models of roll contact curves, friction stresses and distribution patterns of contact stresses in a symmetric two-roll module are obtained.

Expressions are derived that determine the indices of the roll contact curves in a symmetric two-roll module.

4. Conclusions
Based on the analysis of the distribution patterns of contact stresses obtained, the following aspects were revealed:

- the maximum point of the diagrams of normal contact stresses does not coincide with the neutral point; it is either in the no-slip zone or in the backward slip zone;
- tangential contact stresses change their signs at the neutral point, which is located to the left of the centerline in the drive roll.

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