The Investigation of Photonic Band Gap of 2D Multi-fold Photonic Quasicrystals

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Abstract. Two dimensional photonic quasicrystals (PQCs) with different rotational symmetries are constructed based on holographic interference patterns. The photonic band gap (PBG) are obtained by calculating the transmission and reflection spectra based on the finite element method (FEM). The maximum PBG of different PQCs is obtained and investigated by considering the same dielectric constant and different fill factors. All the simulation settings are the same except for the structure itself in this process. The results show that 10-fold quasicrystal exhibit better band gap richness than the others. This may provide a reference to select appropriate quasicrystal structures for photonic devices.

1. Introduction

Light within the photonic band gap (PBG) are prohibited to propagate regardless of the propagation direction. Photonic crystals (PCs) has the ability to control light owing to PBG property so as to have attracted considerable attention. The small size, flexibility and efficiency of PCs also make them an excellent candidate for integrated optical circuits. Compared to PCs, photonic quasicrystals (PQCs) show better characteristics: complete band gaps with small dielectric constant[1-3], higher isotropy of PBG[3-5], the light of localization[6] and rich defect modes[7]. The advantage and richness of PQC also enable more micro-nano devices to be designed and fabricated, such as waveguides[4, 8], sensors[9], laser microcavities[10], and filters[2, 11] etc.

The PBG properties are widely studied in two dimensional 8-fold[1-5, 12], 10-fold[13, 14] and 12-fold[1, 6, 7] PQCs. J. Romero-Vivas et al.[2] found that the dielectric constant threshold value to open a complete PBG in 8-fold PQC could be as small as 1.6. And their study about the effect of dielectric constant on PBG is consistent with the results by Jianling Yin[5]. Besides, Jianling Yin et al. also find the band gap size for the optimum fill factor changes much more significantly than a given fill factor. Chongjun Jin et al.[4] and D. T. ROPER et al.[12] studied the effect of unit cell size on the PBG and showed PBG could be generated by small size PQC. Jianjun Liu et al.[13, 14] studied the influence of many structure parameters including medium layout on the PBG. Steven R. Sellers et al.[15] discussed the formation of PBG in PQCs. Weining Man et al.[16] compared the band gaps between PCs and PQCs. These studies help us to get a better understanding for PQCs.

Since the PBG property is the major feature of PQC, it is essential to study the structure possessing larger band gap. For the study of band gap properties of different quasicrystals, Mikael C. Rechtsman et al.[1] used a novel method to find the optimized pattern with the widest TM polarized gap for two-component materials and found 8-fold quasicrystal possessing lager band gaps with the relative dielectric constant greater than five among quasicrystals. Marian Florescu et al.[17] introduced an optimization method to design PQCs with substantial complete PBGs and showed the maximal
complete band gap in 5-fold symmetric structure were larger than that in 8-fold symmetric structure with the relative dielectric contrast equals 11.56. The studies about higher rotational symmetry quasicrystals exist, such as 14\cite{18, 19} and 18-fold\cite{19} PQCs, but are not extensive. It's necessary to extend study on more kinds of quasicrystals.

Since large-area PQC structures can be obtained by simple experimental setup\cite{20, 21}. The multiple rotational symmetry PQCs with dielectric rods in air are constructed based on the holographic interferogram. The band gap characteristics were studied by transmission spectra and reflection spectra. Then the band gaps of different quasicrystals are compared under the same settings. The results help to understand quasicrystals better and may be helpful to the design and application of photonic device.

2. Results and discussion
The two dimensional (2D) PQC structures are obtained by the multi-beam common-path interference method\cite{20, 21}. The interference intensity on the medium can be expressed as:

\[ I(r) = \sum_{j} E_j E_j^* \exp \left\{ -i \left[ (k_j - k_j^*) \cdot \vec{r} \right] \right\}, \]  

(1)

where \( E_j \) is the amplitude of beam \( j \), \( k_j \) is the wave vector of beam \( j \), \( \vec{r}=(x,y,z) \) is the spatial position vector. The wave vectors of \( N \) beams are:

\[ \vec{k}_m = \frac{2\pi m}{\lambda} \left( \cos \frac{2(m-1)\pi}{N} \sin \phi, \sin \frac{2(m-1)\pi}{N} \sin \phi, \cos \phi \right) \]  

(2)

where \( m=1-N \) is corresponding to the ordinal number of beam, \( n_w \) represents the refractive index of the medium in the writing laser wavelength, \( \phi \) is the crossing angle between the light beam and z axis. The holographic interferogram can be gotten by substituting \( \phi=44.3^\circ, n_w=1.5, \lambda=355\text{nm} \) into Eq.(1) and Eq.(2).

As shown in Fig.1 (a), 8-fold PQC can be gotten by introducing \( N=8 \). The structure is composed of dielectric rods with radius \( r \) in air background in the x-y plane. The central region of \( 8 \mu \text{m} \times 8 \mu \text{m} \) is selected for calculation, and the calculation settings are shown in Fig.1 (b). TE polarized wave (E-field perpendicular to x-y plane) is incident from the left side of the structure and then is detected on the right side. Angle of the incident light relative to x-axis is \( \theta \), which is depicted in Fig. 1 (b). Due to rotational symmetry in the range of 360\(^\circ\) and mirror symmetry in the range of 45\(^\circ\), \( \theta \) between 0\(^\circ\) and 22.5\(^\circ\) are sufficient to describe the propagation at all angles for 8-fold PQC. The band gap can be determined by combining transmission spectra and reflection spectra based on the finite element method (FEM).

![Figure 1. (a) The simulation model of 8-fold PQC, (b) the simulation setting.](image)

Since the band gap is the frequency at which light cannot pass, then light should not exist in the structure, where the band gap exists, the transmittance should be 0, and the reflectance should be 1. At the same time, the band gap for different incident angles is thought as a real band gap. Thus, five band gaps can be observed according to the transmission and reflection spectra shown in Fig. 2, among which the largest band gap is \( a/\lambda=0.521-0.559 \), where \( a/\lambda \) is normalized frequency, \( a=1\mu \text{m} \) is the lattice
constant and $\lambda$ is the wavelength of the source. In fact, PQCs are more likely to produce multiple band gaps than PCs\[16\].

Figure 2. Transmission spectrum (a) and reflection spectrum (b) for TE polarization at $\theta=0^\circ$, $5^\circ$, $10^\circ$, $15^\circ$, $20^\circ$ and $22.5^\circ$ with $r=0.12\mu m$ under $\varepsilon_1/\varepsilon_0=12$ of 8-fold PQC.

In the same way, according to Eq.(1) and Eq.(2), the two dimensional 10, 12, 14 and 18 fold PQC structures can be obtained by introducing $N=5$, 12, 7 and 9. The band gaps in these quasicrystals can also be obtained by the similar method as in the previous paper.

Figure 3. 2D holographic PQC structures (a)10-fold, (b) 12-fold, (c) 14-fold, (d)18-fold.

The maximum gap-midgap ratio $\Delta\omega/\omega_c$ versus fill factor $r/a$ were observed in these quasicrystals to compare which quasicrystal is more likely to generate band gaps, where, $\Delta\omega$ is the width of PBG, $\omega_c$ is the central frequency, respectively. The results are shown in Fig.4 and reveal the following information: (1) The optimum fill factor is different when the maximum band gap is generated in different quasicrystals. For example, 8-fold PQC have a larger band gap at $r/a=0.12$, while 10-fold PQC have a larger band gap at $r/a=0.14$. (2) The maximum band gap for different quasicrystals is different at the optimum fill factor, the order by size is 10-fold, 12-fold, 18-fold, 14-fold and 8-fold. (3) 10-fold PQC almost always generates the largest band gap with the change of fill factor, showing
better ability to generate band gaps than other structures. That is because the structure is hyperuniform and locally self-similar, which were known to be critical conditions for PBG formation\cite{15}. (4) On the whole, the band gap in 8-fold PQC is relatively small, the reason may be that its rotational symmetry and isotropy is lower. (5) The maximum band gap in 12-fold PQC keeps increasing with the increase of fill factor, which is different from that of other PQCs—the maximum band gap increases firstly and then decreases with the fill factor. This is because the fill factor in 12-fold PQC can be as high as 0.3, while the upper limit of fill factor in other PQCs is 0.18. In view of this, 12-fold PQC has great potential in generating wide band gaps.

Figure 4. The maximum gap-midgap ratios $\Delta \omega/\omega_c$ versus fill factor $r/a$ of different quasicrystals. The results show 8-fold PQC has no advantage in generating band gaps, which differ from the conclusion in the Ref. [1]. The ultimate cause is that the two 8-fold PQC’s patterns are different. Our conclusions are based on certain circumstances. The aim is to show which structure is more conducive to band gap generation in holographic interference-based structures. Besides, the calculation is done in the case of $\varepsilon_1/\varepsilon_0=12$, which is similar to the dielectric constant of silicon, so to a certain extent, it also reflects the band gaps in silicon-air structures.

3. Conclusion

The 2D multiple quasicrystal structures based on the holographic interferogram have been constructed and investigated. In the course of simulation, the parameter settings are same except for the structure itself. The maximum gap-midgap ratio $\Delta \omega/\omega_c$ at different fill factor is given for different quasicrystals. The results show that 10-fold and 12-fold PQCs are easier to produce band gaps than others. This possibly provides some guidance to the design and fabrication of quasicrystal based photonic device.

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