NONPERTURBATIVE EFFECTS IN INCLUSIVE $\bar{B} \to X_s \gamma$

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Uncertainties in the theoretical prediction for the inclusive $\bar{B} \to X_s \gamma$ decay rate are examined. Certain nonperturbative effects involving a virtual $c\bar{c}$ loop, which are calculable using the operator product expansion, are discussed.

The inclusive $\bar{B} \to X_s \gamma$ decay is sensitive to physics beyond the standard model, and the photon spectrum can help us better understand nonperturbative effects in other $B$ decays. As the CLEO data excludes large deviations from the standard model, it is important to know the theoretical predictions as precisely as possible. With the completion of the next-to-leading order calculation, the theoretical uncertainty in perturbation theory is only about 10%.

The effective weak interaction Hamiltonian at a scale $\mu(\sim m_b)$ is

$$H_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_{i=1}^{8} C_i(\mu) O_i(\mu).$$

Here $O_2 = (\bar{s}_{L\alpha} \gamma_\mu b_{L\beta})(\bar{c}_{L\beta} \gamma_\nu c_{L\alpha})$, $O_1$ differs from $O_2$ in the contraction of color indices, $O_3 - O_6$ are other four-quark operators, and $O_7 = (i/16\pi^2) m_b \bar{s} L \sigma^{\mu\nu} F_{\mu\nu} b R$.

For large enough photon energies, the matrix element of $O_7$ dominates the $\bar{B} \to X_s \gamma$ rate. Since $m_b \gg \Lambda_{\text{QCD}}$, this contribution is calculable by performing an operator product expansion (OPE) for the time ordered product

$$T_{77} = \frac{i}{2m_B} \int d^4x e^{-iq \cdot x} \langle \bar{B}(v)| T\{O_7^+(x) O_7^-(0)\} |\bar{B}(v)\rangle g_{\mu\nu}. \quad (1)$$

Here $O_7^\pm = (i/8\pi^2) m_b \bar{s} L \sigma^{\mu\lambda} q_\lambda b R$. At fixed $q^2 = 0$, $T_{77}$ has cuts in the complex $v \cdot q$ plane along $v \cdot q < m_b/2$ and $v \cdot q > 3m_b/2$ corresponding to final hadronic states $X_s$ and $X_{bb\bar{s}}$, respectively. The $\bar{B} \to X_s \gamma$ decay rate is given by the discontinuity across the cut in the region $0 < v \cdot q < m_b/2$,

$$\frac{d\Gamma}{dE_\gamma} = \frac{4G_F^2 |V_{ts}^* V_{tb}|^2 C_7^2}{\pi^2} E_\gamma \text{Im} T_{77}. \quad (2)$$

Since the cuts are well-separated, this contribution can be computed assuming local duality at the scale $m_b$ ($m_{X_s} = m_B$ at $v \cdot q = 0$).

At leading order in the OPE, the dimension-three operator $\bar{b} \gamma_\mu b$ occurs. Its matrix element gives a calculable contribution proportional to $\delta(E_\gamma - m_b/2)$.

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which is equal to the free quark decay result. Higher dimension operators give terms proportional to derivatives of this delta function. The leading nonperturbative corrections suppressed by $\Lambda_{\text{QCD}}^2/m_b^2$ are quite small, about $-3\%$.

To justify retaining only the lowest dimension operators whose matrix elements are known, the photon energy must be averaged over a region $\Delta E_\gamma \gg \Lambda_{\text{QCD}}$, and cannot be restricted to be too close to its maximal (i.e., end-point) value. Currently this is a source of significant uncertainty, since the photon spectrum is only measured over a region about 500 MeV from the end-point.

When operators in $H_{\text{eff}}$ other than $O_7$ are included, there are contributions from diagrams in which the photon couples to light quarks. Typically, the leading logarithms are calculable for such processes, but there are uncalculable contributions suppressed by a logarithm (or equivalently by $\alpha_s$, but not by a power of the scale of the process), which can only be estimated using the fragmentation functions $D_{q\rightarrow \gamma X}$ and $D_{g\rightarrow \gamma X}$ deduced from other experiments or from models. Perturbative computations indicate that the contribution of light quark loops and the effects related to decay functions of light partons into a photon are both very small for decays into hard photons. Assuming that these calculations provide correct order of magnitude estimates, such not power suppressed effects constitute less than five percent uncertainty in the theoretical prediction for the $\bar{B}\rightarrow X_s\gamma$ decay rate.

Nonperturbative effects involving the photon coupling to the charm quark contain matrix elements of local operators only suppressed by $\Lambda_{\text{QCD}}^2/m_c^2$ rather than $\Lambda_{\text{QCD}}^2/m_b^2$. Such effects could be sizable, since $m_b^2/m_c^2 \sim 10$ and the $\Lambda_{\text{QCD}}^2/m_c^2$ corrections are 3%. For a sufficiently heavy charm quark, nonperturbative corrections to the interference of $O_2$ and $O_7$ can be computed from the discontinuity of the diagram in Fig. 1. Analogous diagrams with more gluons are suppressed by additional powers of $\Lambda_{\text{QCD}}/m_c$. Denote the photon and gluon momenta by $q$ and $k$, respectively. Working to all orders in $k\cdot q/m_c^2$ since $|q| \sim m_b$, but neglecting $k\cdot q/m_b^2$, $k^2/m_b^2$, and $m_s/m_b$, gives

\[
T_{27} = -\frac{1}{2m_B} \langle \bar{B}(v) | \bar{b} m_b \sigma^{\mu\rho} q_\rho \frac{m_b \hat{g} - \hat{q}}{(m_b v - q)^2 + i\epsilon} \gamma^\mu (1 - \gamma_5) I_{\mu\nu} | \bar{B}(v) \rangle. \tag{3}
\]

Here $I_{\mu\nu}$ is a complicated operator involving all powers of $q \cdot ID/m_c^2$,

\[
I_{\mu\nu} = \left( \frac{e}{16\pi^2} \right)^2 \frac{2}{9m_c^2} \sum_{n=0}^\infty \frac{3[(n+1)!]^2 2^{n+3}}{(2n+4)!} \left( -\frac{q \cdot ID}{m_c^2} \right)^n \epsilon_{\mu\nu\lambda\delta} q^\delta q_\eta g_s G^{\lambda\eta}. \tag{4}
\]

\footnotetext{b}{For soft photons these are important. Interference effects where the photon couples to a light quark and either to the charm quark or through $O_7$ are also small for hard photons.}
Figure 1: Feynman diagram that gives rise to $T_{27}$ in Eq. (3). Interchange of the photon and gluon couplings to the charm loop is understood.

The covariant derivatives, $D$, act on the gluon field $G^\lambda$, so the matrix elements of these operators are determined by the spacetime dependence of the chromomagnetic field in the $B$ meson. The contribution of $T_{27}$ to the $\bar{B} \to X_s \gamma$ decay rate is given by Eq. (2) with $C_7^2 T_{27}$ replaced by $2C_2 C_7 T_{27}$.

For the leading $n = 0$ term in Eq. (4), the matrix element in Eq. (3) is known from the $B^* - B$ mass splitting. This gives

$$\frac{\delta \Gamma(\bar{B} \to X_s \gamma)}{\Gamma(B \to X_s \gamma)} = -\frac{C_2}{9C_7} \frac{\lambda_2}{m_c^2} \simeq 2.5\%.$$ (5)

Here we used $C_2 = 1.11$, $C_7 = -0.32$, $\lambda_2 = 0.12\text{GeV}^2$, and $m_c = 1.4\text{GeV}$.

This result is an order of magnitude larger than the perturbative estimate of the $O_2 O_7$ contribution (which contains a gluon in the final state). The $n = 1$ matrix element vanishes due to the equations of motion. The $n > 1$ terms in Eq. (4) depend on an infinite series of unknown matrix elements. Estimating $\langle \bar{B}(v) \mid \Gamma^{\alpha\beta}(q, v) \langle iD_{\mu_1} \cdots iD_{\mu_n} g_s G_{\alpha\beta} \rangle b \mid B(v) \rangle/(2m_B) \sim (\Lambda_{QCD})^{n+2}$, the $n > 1$ terms are “suppressed” compared to the $n = 0$ term only by powers of $m_b \Lambda_{QCD}/m_c^2$. (A different estimate was given by Grant et al.\cite{Grant}.

In the limit where $m_c$ is fixed and $m_b \to \infty$, the higher order terms in Eq. (4) become successively more important and the expansion we have made is inappropriate. (The whole contribution in Eq. (3) is still suppressed by $\Lambda_{QCD}/m_b$.) In the limit where $m_b/m_c$ is fixed and $m_c, b \to \infty$, the $n = 0$ result dominates the sum in Eq. (4) as the $n > 1$ terms are suppressed by powers of $\Lambda_{QCD}/m_c$. In the physical world, $m_b \Lambda_{QCD}/m_c^2 \sim 0.6$ is of order unity. As the coefficients of the operators in Eq. (3) are already small for small $n$, and decrease asymptotically as $3\sqrt{n}/(2^{n+1} n^{3/2})$, the $n > 1$ terms probably do not introduce an uncertainty larger than the size of the leading $n = 0$ term. Nonperturbative effects from the $O_1 O_7$ interference are expected to be smaller.

Consider next the contribution of $(C_1 O_1 + C_2 O_2)^2$. Diagrams like that in Fig. 1 should give a smaller result than for the interference of $O_2$ with $O_7$ (of order $\Lambda_{QCD}^4/m_c^4$ instead of $\Lambda_{QCD}^2/m_c^2$). But in this case there is a contribution to the $\bar{B} \to X_s \gamma$ decay rate from $\bar{B} \to X_s J/\psi$ followed by $J/\psi \to \gamma X$,
which is much larger than the perturbative calculation of the effect of $(C_1O_1 + C_2O_2)^2$. The combined branching ratio for this process is about as large as the total $\bar{B} \to X_s \gamma$ decay rate: of order $10^{-4}$. This might not present a serious difficulty for the comparison of experiment with theory, since $\bar{B} \to X_s J/\psi$ followed by $J/\psi \to \gamma X$ does not favor hard photons. Moreover, it can be treated as a background and subtracted away. However, if such a subtraction is made, is it not double counting to include the perturbative result for the $(C_1O_1 + C_2O_2)^2$ contribution into the theoretical prediction? In any event, $B \to X_s J/\psi$ followed by $J/\psi \to \gamma X$ is a long distance contribution, while the charm quarks are far off-shell when nonperturbative effects suppressed by powers of $\Lambda_{\text{QCD}}/m_c$ are calculable. Further work on these issues is warranted.

In summary, the nonperturbative contribution to the matrix element of $C_7 O_7$, of order $\Lambda_{\text{QCD}}^2/m_c^2$, is about $-3\%$ with small uncertainty. Although there is no OPE for the contribution of photon coupling to light quarks, such effects give less than five percent uncertainty for hard photons. For the contribution of photon coupling to charm quarks, there are nonperturbative effects of order $\Lambda_{\text{QCD}}^2/m_c^2$, whose magnitude is $2.5\%$, with a similar uncertainty. However, it is possible that larger nonperturbative effects may exist. For comparison with the present data, which focuses on the region $E_\gamma \geq 2.2$ GeV, the largest theoretical uncertainty is due to the contribution of higher dimension operators to $T_{77}$ which become important in the end-point region. This uncertainty would be substantially smaller if the photon energy cut were reduced.

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