Collapse of the Fano Resonance Caused by the Nonlocality of the Majorana State

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One of the main features of the Majorana state, which attracts a considerable current interest to these excitations in solid-state systems, is related to its nonlocal character. It is demonstrated that the direct consequence of such nonlocality is the collapse of the Fano resonance manifesting itself in the conductance of an asymmetric interference device, the arms of which are connected by a one-dimensional topological superconductor. In the framework of the spinless model, it is shown that the predicted effect is associated with an increase in the multiplicity of the degeneracy of the zero-energy state of the structure arising at a specific case of the Kitaev model. Such an increase leads to the formation of a bound state in the continuum.

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The effective Rashba field, \( B_{SO} \), should be oriented along the perpendicular to the Zeeman field \( B \). Further on, in our calculations, all energy parameters are measured in the units of \( t = 1, \Delta = 0.25, \alpha = 0.2, \mu = 0 \).

The wires in the normal phase (NW), which are the arms of the ring (see Fig. 1), are assumed to be the same. Their Hamiltonians, \( H_{1-4} \), are obtained from \( H \) at \( \Delta = \alpha = 0 \). The coupling between the SW and NWs is described by the tunneling Hamiltonian,

\[
\hat{H}_T = -t_0 \sum_\sigma \left[ (b^+_{L,n\sigma} + b^+_{R,n\sigma}) a_{1\sigma} + (d^+_{1\sigma} + d^+_{R,1\sigma}) a_{N\sigma} \right] + h.c.,
\]

where \( t_0 \) is the hopping integral between the edge SW and NW sites; \( b^+_{L(R)ns} \) is the creation operator for an electron with spin projection \( \sigma \) at the last site in the left (right) upper NW; \( d^+_{L(R)1\sigma} \) is the creation operator for an electron with spin projection \( \sigma \) at the first site in the left (right) lower NW. In its turn, the coupling between the device (SW+NW) and contacts is also described by the tunneling Hamiltonian, which at the same time plays the role of interaction operator when we use the diagram technique for nonequilibrium Green’s functions

\[
\hat{V} = -\sum_{k\sigma} \left[ c^+_{L,k\sigma} (t_1 b_{L,1\sigma} + t_2 d_{L,\sigma}) + c^+_{R,k\sigma} (t_2 b_{R,1\sigma} + t_1 d_{R,\sigma}) \right] + h.c.,
\]

where \( c^+_{L(R)k\sigma} \) is the creation operator for an electron with wave vector \( k \) and spin projection \( \sigma \) at the left (right) contact; \( t_{1,2} \) are the hopping integrals between the contacts and device. Hamiltonian for the contact with wave vector \( k \) and spin projection \( \sigma \) at \( j \)th site of NW or SW \( b^+_{L(R)j\sigma} \) is the creation operator for an electron with spin projection \( \sigma \) at \( j \)th site of NW or SW \( b^+_{L(R)j\sigma} \). Then, we can specify the matrix Green’s function of the ring in the following form

\[
\hat{G}^{ab}(\tau, \tau') = -i \left( T_C \hat{\Psi} (\tau_a) \otimes \hat{\Psi}^\dagger (\tau'_b) \right),
\]

where \( T_C \) is the ordering operator at the Keldysh time contour consisting of the lower (superscript +) and upper (superscript −) parts [29]; \( a, b = +, \cdots, -; \hat{\Psi} \) has the dimension \( 4(N + 4n) \times 1 \), i.e., it includes the Nambu operators for both SW and all NWs,

\[
\hat{\Psi} = \left( b_{L,1} \cdots b_{L,n} a_{L,1} \cdots a_{L,n} \cdots b_{R,1} \cdots b_{R,n} a_{R,1} \cdots a_{R,n} \right)^T.
\]
The electron current in the left contact is written as
\[ I = e \int d\tau \text{Tr} \left[ \hat{\sigma} \text{Re} \left\{ i \tilde{t}^\tau_1 (\tau) \tilde{G}^{+-}_{k,L,1} (\tau, \tau) + i \tilde{t}^-_n (\tau) \tilde{G}^{+-}_{k,L,n} (\tau, \tau) \right\} \right], \]
(7)
where \( \tilde{t}_1,n = \frac{t_{1,2}}{2} \text{diag} \left( e^{-i \frac{\omega \tau}{\hbar}}, e^{i \frac{\omega \tau}{\hbar}}, e^{-i \frac{\omega \tau}{\hbar}}, e^{i \frac{\omega \tau}{\hbar}} \right) \cdot \tilde{\sigma} \)
(8)
In [7], the mixed Green’s functions have the form
\[ G^{+-}_{k,L,1} = i \left( \hat{b}^\dagger_{k,1} \otimes \hat{c}_{L} \right) \] and \( G^{+-}_{k,L,n} = i \left( \hat{d}^\dagger_{L,n} \otimes \hat{c}_{L} \right). \]
In the Nambu operator space, \( \hat{H}_D \) has the form of Hamiltonian for free particles, therefore, in specifying averages in \( G^{+-}_{k,L,1} \) and \( G^{+-}_{k,L,n} \) we should use the same guidelines as those for the averages for \( T_c \)-ordered product of the second quantization operators [32, 33]. As a result, at \( t \to 0 \) expression (7) transforms to
\[ I = 2e \int \frac{d\omega}{\pi} \text{Tr} \left[ \hat{\sigma} \text{Re} \left\{ \tilde{t}^\tau_1 (\omega) \tilde{G}^{+-}_{L,1,1} (\omega) + \tilde{t}^-_n (\omega) \tilde{G}^{+-}_{L,n,n} (\omega) \right\} \right]. \]
(9)
where \( \tilde{\Sigma}^\tau_1,1,1 (\omega) \) and \( \tilde{\Sigma}^-_n,n,n (\omega) \) are self-energies of the left contact \((i,j, n); \) \( \tilde{G}^{+-}_{L,1,1} (\omega) \) is the bare Green’s function of the left contact. Integrating over time \( \tau_1 \) and using the Fourier transform, we get
\[ I = e \sum_{i,j=1,4} \int_{-\infty}^{+\infty} \frac{d\omega}{\pi} \text{Tr} \left[ \hat{\sigma} \text{Re} \left\{ \tilde{\Sigma}^\tau_{i,j,i,j} (\omega) \tilde{G}^{+-}_{i,j,i,j} (\omega) + \tilde{\Sigma}^-_{i,j,i,j} (\omega) \tilde{G}^{+-}_{i,j,i,j} (\omega) \right\} \right]. \]
(10)
The further transformation of (10) makes it possible to obtain an explicit form of the components associated with the local Andreev reflection and the nonlocal transfer of charge carriers. However, these expressions are quite cumbersome, and we do not present them here.

Note that many-particle interactions are absent in the system, therefore the Green’s functions in the integrand [10] are determined taking into account all the tunneling processes between the device and contacts [33]. In particular, \( \tilde{G}^{a}_{i,j,i,j} \) block-matrices of the advanced Green’s function of the whole device, \( \tilde{G}^{a} \), are determined by the Dyson equation,
\[ \tilde{G}^{a} = \left[ (\omega - \hbar D - \tilde{\Sigma}^\tau (\omega))^{-1} \right]^+, \]
(11)
where \( \tilde{\Sigma}^\tau (\omega) \) is the matrix of retarded self-energy revealing the effect of both contacts on the ring. In the course of further numerical calculations, we will use the popular approximation of wide-band contacts, for which the real parts of the self-energy functions can be neglected and the imaginary parts can be considered as constants (see, for example, [33]). Then, we have the following nonzero blocks \( \tilde{\Sigma}^\tau \):
\[ \tilde{\Sigma}^{\tau}_{L,1,1} = \tilde{\Sigma}^{\tau}_{R,n,n} = -\frac{i}{2} \Gamma_{11}, \tilde{\Sigma}^{\tau}_{1,R,1} = \tilde{\Sigma}^{\tau}_{L,n,n} = -\frac{i}{2} \Gamma_{22}, \]
\[ \tilde{\Sigma}^{\tau}_{L,1,n} = \tilde{\Sigma}^{\tau}_{R,n,L} = \tilde{\Sigma}^{\tau}_{L,n,L} = -\frac{i}{2} \Gamma_{12}. \]
(12)
where \( \Gamma_{ii} = \Gamma_{ii} \hat{I}_i, \Gamma_{ij} = 2 \pi \ell_{ij}^2 \rho \) is the function characterizing the broadening of energy levels of the device due to its interaction with the contact \((i = 1, 2); \) \( \rho \) is the density of states in the contact; \( \Gamma_{12} = \sqrt{\Gamma_{11} \Gamma_{22}}; \) \( \hat{I}_i \) is the unit \( 4 \times 4 \) matrix. Considering directly the asymmetric (symmetric) ring, we assume that \( \Gamma_{22} = \Gamma_{11}/2 = 0.01 \) \( \Gamma_{22} = \Gamma_{11} = 0.01. \)

The \( \tilde{\Sigma}^{\tau}_{i,j,i,j} \) blocks in (10) are obtained by the solution of Keldysh equation, \( \tilde{G}^{+-} = \tilde{G}^{a} \tilde{\Sigma}^{+-} \tilde{G}^{a} \). Note that we consider the regime, when all the transient processes have ended and the bare Green’s functions of the device are not involved to this equation [33]. Here, nonzero blocks \( \tilde{\Sigma}^{+-} \) are given by the expressions
\[ \tilde{\Sigma}^{\tau-}_{1,1,1,1} = -2 \tilde{\Sigma}^{\tau-}_{1,1,1,1} \hat{F}^{a}, \quad \alpha = L, R, \quad i,j = 1, n, \]
\[ \hat{F}^{a}_{L,R} = \text{diag} \left( n (\omega \pm eV/2), n (\omega \mp eV/2) \right), \]
\[ n (\omega \pm eV/2), n (\omega \mp eV/2) \]
Zeeman splitting (see the dashed lines in Fig. 2a). As a result, the number of conductance peaks increases; this is illustrated by the dashed curve in Fig. 2b. However, not all zeros in the excitation energies manifest themselves as resonances in the conductance, which is a signature of arising BSC \([36, 37]\). There are several ways to achieve a finite value of their lifetime. For example, it is possible to break the spatial symmetry of the eigenstates of a closed system with zero energy. Hence, the vanishing of the Fano resonance could suggest an increase in the multiplicity of the degeneracy of this state if the overlap of the Majorana wave functions becomes negligible. To test this hypothesis, we consider the spinless model of the ring with \(n = 1\). In this situation, we use the Kitaev chain with an even number of sites in the bridge \([24]\). Then, at \(\epsilon_d = \mu = 0\) the Hamiltonian of the ring has the form

\[
H_D = \sum_{j=1}^{N-1} \left( -t a_j^+ a_{j+1} + \Delta a_j^+ a_{j+1}^+ \right) - t_0 \left( b_{L0} + b_{R0} \right) - t_0 a_N^+ \left( d_{L1} + d_{R1} \right) + \text{h.c.}
\]

The diagonalization of Hamiltonian (14) leads to the following equation for the excitation spectrum

\[
E_i \left( E \cdot P_1 - 4t_0^2 \delta_1^{N/2-1} \right) \left( E \cdot P_2 + 2t_0^2 \delta_1^{N/2-1} \right) \cdot \left( E \cdot P_3 - 2t_0^2 \delta_2^{N/2-1} \right) \left( E \cdot P_4 + 2t_0^2 \delta_2^{N/2-1} \right) = 0,
\]

where \(\delta_{1,2} = t \mp \Delta; P_i\) is the \(i\)th polynomial of power \(N/2\) for which \(P_{2,4} = P_{1,3} (E \rightarrow -E)\) due to the electron?hole symmetry. It follows from \([15]\) that at the specific case of the Kitaev model, \(\Delta = \pm \epsilon\), where the wave functions of the Majorana fermions do not overlap, the multiplicity of the degeneracy for the zero-energy state increases at
$N > 2$. This is just the cause of the suppression of the narrow Fano resonance illustrated in Fig. 3b.

To make the situation clearer, let us turn to the study of our system using the Majorana representation, $a_i = (\gamma_{1j} + i\gamma_{2j})/2$, where $\gamma_{ij} = \gamma_{ij}^+$ ($i = 1, 2$). In Figs. 4a and 4b, we schematically present the device in the framework of such description at the specific case of Kitaev model, $\Delta = t$, for $N = 2$ and $N > 2$, respectively (straight lines denote the interaction between Majorana fermions of different kinds). We can see that in the first case, the upper and lower arms remain connected due to the absence of superconducting pairing in horizontal directions. In the second case, the device is divided into upper and lower identical subsystems. Each of them includes two chains of interacting quasiparticles. The self-energies of a chain with only two bonds in the horizontal direction are $E_{1,2} = 0$ and $E_{2,3} = \pm t_0/\sqrt{2}$. If the vertical bond is included (similarly to the Fano-Anderson model), we have $E_{1,2} = 0$ and $E_{3,4} = \pm \sqrt{t^2 + t_0^2}/2$. Thus, it is the formation of the T-shaped structures of Majorana fermions that leads to the suppression of the Fano resonance in the asymmetric ring. Note that the nonlocality of the MS does not depend on the ratio of the tunneling parameters between the subsystems (contacts, NW, and SW), therefore, the effect under discussion has the universal nature and arises in the most general situation typical of experiment, namely, when all these parameters are different. In addition, from Fig. 4b, it becomes clear that simply at $t = 0$, i.e. in the case of two noninteracting arms, the Fano resonance is not suppressed. We should emphasize that in the symmetric case, the described Fano resonance does not arise in principle.

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