Three-dimensional supergravity reloaded

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Abstract

The locally supersymmetric extension of the most general gravity theory in three dimensions leading to first-order field equations for the vielbein and the spin connection is constructed. Apart from the Einstein–Hilbert term with cosmological constant, the gravitational sector contains the Lorentz–Chern–Simons form and a term involving the torsion each with arbitrary couplings. The supersymmetric extension is carried out for vanishing and negative effective cosmological constant, and it is shown that the action can be written as a Chern–Simons theory for the supersymmetric extension of the Poincaré and AdS groups, respectively. Here we introduce a duality map between different gravity theories that greatly simplifies the construction. This map relies on the different ways to make geometry emerge from a single gauge potential. The extension for $\mathcal{N} = p + q$ gravitini is also performed.

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1. Introduction

Three-dimensional gravity is a useful toy model to give an insight into the problems of four-dimensional gravity, which is a notoriously difficult theory especially at the quantum level. Being a generally covariant theory of gravity in three dimensions, it captures many of the basic conceptual features of four-dimensional General Relativity but avoiding many of the computational difficulties.

The systematic study of three-dimensional gravity was put forward by Deser, Jackiw and ’t Hooft [1]. The essence of the simplicity is due to the fact that the gravitational field has no local degrees of freedom away from the sources. In fact, in vacuum the solutions are given by flat or constant curvature spacetimes in the case of a cosmological constant. In spite of this, the theory is not trivial as, for example, there are black hole solutions (known as BTZ black holes) in the case of negative cosmological constant [2].

One of the most remarkable facts about three-dimensional gravity is that it can be written as a gauge theory with Chern–Simons action [3, 4], where the gauge group is given by the
AdS, dS or Poincaré group for negative, positive and zero cosmological constant respectively. Chern–Simons theory in three dimensions is a theory of flat connections. This is related to the fact, mentioned above, that three-dimensional gravity has no local degrees of freedom. The locally supersymmetric extension of General Relativity in three dimensions has been done by Deser and Kay [5], and it has been shown that it can be written as a Chern–Simons theory for negative [3] or vanishing [6, 7] cosmological constant.

To be more precise, the Chern–Simons gauge theory is equivalent to General Relativity in the first-order formalism. That is, the spin connection and the vielbein are considered as independent variables\(^1\). However, there is a more general action for gravity in three dimensions which apart from the Einstein–Hilbert term with cosmological constant, contains the Lorentz–Chern–Simons form and a term involving the torsion each with arbitrary couplings [8, 9]. It has been shown that this model can also be written as a Chern–Simons theory [10, 11]. This is also known as the Mielke–Baekler model. This is the three-dimensional case of the gravitational theories with torsion in arbitrary dimension considered in [9]. This more general theory of gravity is also interesting because it admits a black hole solution with negative Riemannian curvature [12] which is essentially a BTZ black hole with torsion.

In this paper, we show how to construct the supersymmetric extension of this more general model for negative and vanishing cosmological constant, first for \( N = 1 \) and then for \( N \) gravitini. It is shown that the construction is greatly simplified by making use of the mapping between different theories of gravity presented here, which allows one to map between the standard theory without torsion and the more general theory including torsion. This mapping is shown to arise as a consequence of considering the different ways to make geometry emerge from a single gauge potential. That is to say, there are different ways of relating the gauge field of a Chern–Simons theory with the geometrical quantities, the vielbein and spin connection. Hence, although the action is the same in terms of the gauge field, the ambiguity in the identification of some of the gauge fields with the vielbein actually generates different possibilities that turn out to be physically inequivalent when one wants to make contact with gravity. This becomes more explicit when one couples the theory to some standard external source, which explicitly depends on the metric.

The structure of this paper is the following: in section 2, we give a brief review of the most general theory of gravity in first-order formalism in three dimensions. Then in section 3 we construct its \( N = 1 \) supersymmetric extension. Section 4 is devoted to exploring different ways of making geometry to emerge from a gauge field. This turns out to be useful in formulating a map between different theories of gravity, which is discussed in section 5. In section 6, we show how the supergravity theory constructed in section 3 can be obtained applying the map discussed above to the standard theory. Finally, in section 7 we show that the map is particularly useful in order to perform the extension to \( N = p + q \) supersymmetries.

2. The most general first-order action for three-dimensional gravity

The most general gravitational action in three dimensions that is invariant under local Lorentz transformations, up to a surface term, leading to first-order field equations for the dreibein \( e^a \),

\(^1\) Conformal gravity and supergravity can also be formulated along these lines, as done in [13] and [14] respectively. Another nontrivial development of 3D gravity is topologically massive gravity [15], where the Lorentz–Chern–Simons term for the spin connection is not independent, but a function of the vielbein. One obtains then higher derivative field equations with massive gravitons. Also in this theory black hole solutions are known and the thermodynamics has been studied (see e.g., [16] and references therein). The supersymmetric extension of topologically massive gravity has been studied in [5].
and the spin connection $\omega^{ab}$ is [8, 9]

$$I_G(e^a, \omega^a) = \int 2\alpha_1 e^a R^a_a - \frac{\alpha_2}{3} \epsilon_{abc} e^b e^c + \alpha_3 L_3(\omega^a) + \alpha_4 e_a T^a. \tag{1}$$

Here $\omega^a$ stands for the dual of the spin connection, $\omega^a := \frac{1}{2} e^{abc} \omega_{abc}$, and $L_3(\omega^a)$ is the Lorentz–Chern–Simons form given by

$$L_3(\omega^a) = \omega_ad\omega^a + \frac{1}{3} \epsilon_{abc} \omega^a \omega^b \omega^c. \tag{2}$$

The wedge product between forms is understood.

The field equations obtained from the variation of (1) with respect to the dreibein and the spin connection read

$$2\alpha_1 R^a_a - \alpha_2 \epsilon_{abc} e^b e^c + 2\alpha_4 T^a_a = 0, \tag{3}$$

$$2\alpha_1 T^a_a + 2\alpha_3 R^a_a + \alpha_4 \epsilon_{abc} e^b e^c = 0, \tag{4}$$

respectively, which implies that the three-dimensional geometry must be of constant curvature and torsion\(^2\) given by

$$R^a_a = \frac{1}{2} \left( \beta^2 + \frac{\sigma}{l^2} \right) \epsilon_{abc} e^b e^c, \tag{5}$$

$$T^a_a = -\beta \epsilon_{abc} e^b e^c, \tag{6}$$

where the constants $\beta$ and $l$ are fixed only by means of the couplings appearing in the action\(^3\)

$$\beta = -\frac{1}{2} \frac{\alpha_3 \alpha_2 + \alpha_4 \alpha_1}{\alpha_3 \alpha_4 - \alpha_1^2}, \quad \beta^2 + \frac{\sigma}{l^2} = -\frac{\alpha_1 \alpha_2 + \alpha_3^2}{\alpha_3 \alpha_4 - \alpha_1^2}. \tag{7}$$

We have introduced $\sigma = \pm 1$ or 0.

The spin connection can be decomposed as

$$\omega^{ab} = \tilde{\omega}^{ab} + k^{ab},$$

where $\tilde{\omega}^{ab}$ is the (torsion-free) Levi-Civita connection, and $k^{ab}$ stands for the contorsion, which by equation (6) is fixed as

$$k^{ab} = \beta \epsilon^{abc} e_c. \tag{8}$$

Thus, the curvature 2-form can be decomposed in terms of its Riemannian part $\tilde{R}^{ab}$ and the contorsion 1-form $k^{ab}$ as

$$R^{ab} = \tilde{R}^{ab} + \tilde{D} k^{ab} + \tilde{k}^a k^b,$$

where $\tilde{D}$ is the covariant derivative with the Levi-Civita connection $\tilde{\omega}^{ab}$. Hence, equations (5) and (8) imply that spacetime has constant curvature since the Riemannian part of the curvature acquires the form

$$\tilde{R}^{ab} = \frac{\sigma}{l^2} \epsilon^{ab} e^c. \tag{9}$$

where $\frac{\sigma}{l^2}$ is the effective cosmological constant.

The minimal local supersymmetric extension of the action (1) is carried out in the next section.

\(^2\) By constant torsion we mean that the torsion is constant and completely antisymmetric when its indices are lowered.

\(^3\) The case of $\alpha_3 \alpha_4 - \alpha_1^2 = 0$ is pathological, since the field equations (3) and (4) become linearly dependent.
3. Locally supersymmetric extension with $\mathcal{N} = 1$

Let us consider the following action principle

$$I(e^a, \omega^a, \psi) = I_G + I_\psi,$$

(10)

where the gravitational sector is given by $I_G$ in equation (1), and the fermionic term reads

$$I_\psi = -\left(\alpha_1 - \alpha_3 \left(\beta - \frac{1}{7}\right)\right) \bar{\psi} \left(D - \frac{1}{2} \left(\beta + \frac{1}{7}\right) e^a \Gamma_a\right) \psi,$$

(11)

where $\psi = \psi_\mu dx^\mu$ is the gravitino, whose Lorentz covariant derivative is given by $D\psi = d\psi - \frac{1}{2} \omega^a \Gamma_a$. Therefore, when the effective cosmological constant is negative or zero one can see that, the action (10) is invariant, up to a surface term, under the following local supersymmetry transformations spanned by $\epsilon = \epsilon(x)$,

$$\delta e^a = \frac{1}{2} \bar{\epsilon} \Gamma^a \psi,$$

$$\delta \omega^a = -\frac{1}{2} \left(\beta - \frac{1}{7}\right) \bar{\epsilon} \Gamma^a \psi,$$

$$\delta \psi = D\epsilon - \frac{1}{2} \left(\beta + \frac{1}{7}\right) e^a \Gamma_a \epsilon.$$

(12)

Note that apart from diffeomorphisms, local supersymmetry transformations (12), and the manifest local Lorentz symmetry spanned by, $\lambda^a = \lambda^a(x)$,

$$\delta e^a = \epsilon^{abc} e_b \lambda_c,$$

$$\delta \omega^a = D\lambda^a,$$

$$\delta \psi = -\frac{1}{2} \lambda^a \Gamma_a \psi,$$

(13)

the action (10) has an extra local symmetry which is spanned by a vector $\chi^a = \chi^a(x)$:

$$\delta e^a = D\chi^a + \beta \epsilon^{abc} e_b \chi_c,$$

$$\delta \omega^a = \frac{1}{2} \epsilon^{abc} e_b \chi_c - \beta \left(D\chi^a + \beta \epsilon^{abc} e^b \chi^a\right),$$

$$\delta \psi = \frac{1}{2} \chi^a \Gamma_a \psi.$$

(14)

One may check that the transformations (14) together with the local Lorentz symmetry (13) close, merging into the AdS algebra $SO(2, 2)$ which is isomorphic to $Sp(2) \times Sp(2)$. Once the supersymmetry transformations are included, one verifies that the whole set in equations (12), (13) and (14) closes off-shell and without the need of auxiliary fields into the $OSp(2|1) \times Sp(2)$ superalgebra, which is the minimal supersymmetric extension of the AdS algebra.

Analogously, in the limit $l \to \infty$, the transformations (12), (13) and (14) close off-shell in the super Poincaré algebra with $\mathcal{N} = 1$.

It is natural then to see whether the action principle (10) can be written in a manifestly covariant way under this gauge symmetry. It is worth pointing out that the local symmetries (12), (13) and (14) can be obtained from gauge transformations of a single super AdS connection given by

$$A = (\omega^a + \beta e^a) J_a + \frac{e^a}{l} P_a + \frac{1}{\sqrt{l}} \psi^a Q_a.$$

(15)
where $J_a$, $P_a$ and $Q_\alpha$ are the generators of Lorentz transformations, AdS boosts and supersymmetry, respectively, spanning the $OSp(2|1) \times Sp(2)$ algebra:

\[
\begin{align*}
[J_a, J_b] &= \epsilon_{abc} J^c, \\
[J_a, P_b] &= \epsilon_{abc} P^c, \\
[P_a, P_b] &= \epsilon_{abc} J^c, \\
[J_a, Q_\alpha] &= [P_a, Q_\alpha] = -\frac{1}{2} (\Gamma_{\alpha})_{\beta}^\gamma Q_\beta, \\
{\{Q_\alpha, Q_\beta\}} &= (C \Gamma^\gamma)_{\alpha\beta} (J^\gamma + P^\gamma).
\end{align*}
\] (16)

Indeed, for a gauge transformation

\[
\delta A = d\lambda + [A, \lambda],
\] (18)

choosing the Lie algebra valued parameter as $\lambda = \lambda^a J_a$ the Lorentz transformations (13) are recovered, and choosing $\lambda = \frac{1}{2} P_\alpha$ one obtains the AdS boosts in equation (14). Local supersymmetry transformations (12) are similarly obtained from (18) using $\lambda = \epsilon^\alpha Q_\alpha$.

Therefore, the dreibein, the spin connection and the gravitino can be seen as different components of a single connection $A$ for the gauge group $OSp(2|1) \times Sp(2)$ as in equation (15). This task is explicitly performed in section 6 and the extension for $N = p + q$ gravitini is done in section 7.

Note that the identification of the components of the gauge potential in equation (15) with the vielbein and the spin connection is not performed in the standard way as in [3, 4]. For the purely gravitational sector, this general identification is shown to be equivalent to that performed in [10] for the AdS group, as well as that in [11] for the dS and Poincaré. This fact was used to show that the gravitational sector can be formulated as a Chern–Simons theory for any value of the effective cosmological constant $\sigma/l^2$, which determines the gauge group.

4. Different ways to make geometry emerge from a single gauge potential

Here we address the issue of different possibilities of making geometry emerge from a connection for the AdS, dS and Poincaré groups.

Any component of the gauge potential cannot be identified arbitrarily with the vielbein and the spin connection since in general they would not transform suitably under the corresponding Lorentz subgroup. Thus the possible identifications must satisfy the requirement that under a local Lorentz transformation with parameter $\lambda^a$ they transform as

\[
\delta e^a = \epsilon^{abc} e_b \lambda_c,
\] (19)

\[
\delta \omega^a = D\lambda^a.
\] (20)

Let us explicitly find different possible identifications between gauge fields for the AdS group and geometry in three dimensions. The AdS group is spanned by the Lorentz generators $J_a$ and the AdS boosts $P_a$ with the Lie algebra (16). A gauge connection for the AdS group then reads

\[
A = B^a J_a + C^a P_a,
\] (21)

where $B^a$ and $C^a$ are its components. Let us consider a Lie algebra valued parameter of the form

\[
\lambda = \lambda^a J_a + \chi^a P_a,
\] (22)

4 The relative sign on the rhs of the anticommutator corresponds to which of the copies of Sp(2) has been extended to OSp(2|1). Choosing the other copy amounts to making the replacement $l \to -l$ everywhere, and reversing the overall sign in the fermionic action. In the vanishing cosmological constant limit the super Poincaré algebra is recovered taking the Inönü–Wigner contraction. This is performed through rescaling the generators as $P_a \to l^{-1} P_a$, $Q_\alpha \to l^{-1/2} Q_\alpha$, and taking the limit $l \to \infty$. 


Thus, under an infinitesimal gauge transformation the connection transforms as
\[ \delta A = d\lambda + [A, \lambda], \]
which means that its components transform according to
\[ \delta B^a = d\lambda^a + B^a \lambda^b \epsilon_{abc} + C^a \chi^b \epsilon_{abc}, \]
\[ \delta C^a = d\chi^a + C^a \lambda^b \epsilon_{abc} + B^a \chi^b \epsilon_{abc}. \]
Now in order to relate the spin connection and dreibein to the gauge fields \( B^a \) and \( C^a \), it is natural to consider an arbitrary combination of them. We must verify that they transform as in equations (19) and (20) under a local Lorentz subgroup.

For simplicity let us consider a linear relation between the gauge fields \( B^a, C^a \) with the vielbein \( e^a \) and spin connection \( \omega^a \) of the form \(^5\)
\[ B^a = \alpha \omega^a + \beta e^a, \]
\[ C^a = \gamma e^a + \mu \omega^a. \]
Inserting these last equations in (23) and (24) we obtain the transformation law for the would-be vielbein \( e^a \) and spin connection \( \omega^a \). For the would-be vielbein one obtains that it transforms as
\[ (\mu \beta - \gamma \alpha) \delta e_c = d(\mu \lambda_c - \alpha \chi_c) + \mu(\alpha \omega^a \lambda^b \epsilon_{abc} + \beta e^a \lambda^b \epsilon_{abc} + \gamma e^a e^b \epsilon_{abc} + \mu \omega^a \chi^b \epsilon_{abc}) - \alpha(\gamma e^a \lambda^b \epsilon_{abc} + \mu \omega^a \lambda^b \epsilon_{abc} + \alpha \omega^a \chi^b \epsilon_{abc} + \beta e^a \chi^b \epsilon_{abc}). \]
Since the vielbein must transform as a vector under the Lorentz group as given in equation (19), the inhomogeneous term containing the exterior derivative in equation (27) has to do with AdS boosts, as it occurs in equation (14), with parameter \( \alpha \chi^a - \mu \lambda^a \). As we need to focus just on the local Lorentz transformations, without losing generality, we can impose the AdS boost parameter to vanish, i.e.,
\[ \alpha \chi^a = \mu \lambda^a. \]
One can then insert this back in (27), but in order to recover the correct transformation law for the vielbein, one must impose the cancellation of the terms containing \( \omega^a \), which leads to the condition
\[ \mu(\alpha^2 - \mu^2) = 0, \]
which is solved for \( \mu = 0 \) or \( \mu^2 = \alpha^2 \). The case \( \mu^2 = \alpha^2 \) is discarded since it would transform as a scalar, \( \delta e^a = 0 \), so that \( e^a \) cannot be identified with the vielbein. The remaining possibility, \( \mu = 0 \), exactly reproduces the vector transformation law (19) required for the vielbein.

The transformation law for the would-be spin connection \( \omega^a \) can then be found plugging the above results back into equation (23), and it reads
\[ a \delta \omega^a = d\lambda^a + a e^{abc} \omega_b \lambda_c. \]
Hence, \( a = 1 \) is required to reproduce the transformation (20), so that \( \omega^a \) can be identified with the spin connection.

\(^5\) Note this is not the most general linear combinations that can be considered. An example that is not contained in our ansatz is \( A = -e^0 P_0 + e^{0ij} P_{ij} + \alpha^0 J_0 + e^{0ij} \epsilon_{ij} \). For this choice the dreibein and the spin connection transform in the correct way under the Lorentz group. This possibility arises from the fact that the AdS group is \( SO(2, 2) \), so there is an ambiguity in choosing the time direction. Note that this is just an AdS transformation and so it does not change the form of the action.
In summary, according to our ansatz, the way to make geometry emerge from a connection for the AdS group is obtained extracting the spin connection and the vielbein from the gauge fields $B^a$ and $C^a$ from the following relation

$$B^a = \omega^a + \beta e^a, \quad C^a = \frac{1}{l} e^a,$$  \hspace{1cm} (28)

which depends on two arbitrary parameters $\beta$ and $l := \gamma^{-1}$, so that the gauge field is given by

$$A = (\omega^a + \beta e^a) J_a + \frac{1}{l} e^a P_a.$$  \hspace{1cm} (29)

Following the same reasoning, the same identification as in equation (28), or equivalently (29), can be readily found for the Poincaré and dS groups. For all the cases the algebra reads

$$[J_a, J_b] = \epsilon_{abc} J^c, \quad [J_a, P_b] = \epsilon_{abc} P^c, \quad [P_a, P_b] = -\sigma \epsilon_{abc} J^c,$$  \hspace{1cm} (30)

where

$$\sigma = \begin{cases} 
1 : & \text{dS} \\
0 : & \text{Poincaré} \\
-1 : & \text{AdS}.
\end{cases}$$

It can be seen that the parameters $l$ and $\beta$ acquire a precise geometrical interpretation when one deals with flat connections as follows. The identification (28) leads to the curvature $F = dA + A^2$, which in components reads

$$F = \left( R^a + \beta T^a + \frac{1}{2} \left( \beta^2 - \frac{\sigma}{l^2} \right) \epsilon^{abc} e_b e_c \right) J_a + \frac{1}{l} \left( T^a + \beta \epsilon^{abc} e_b e_c \right) P_a,$$

so that for flat connections, characterized by $F = 0$, the curvature and torsion 2-forms are given by

$$R_a = \frac{1}{2} \left( \beta^2 + \frac{\sigma}{l^2} \right) \epsilon_{abc} e^b e^c,$$  \hspace{1cm} (31)

$$T_a = -\beta \epsilon_{abc} e^b e^c,$$  \hspace{1cm} (32)

so that the geometry is described by spacetimes of constant curvature and torsion. Therefore, $\beta$ parametrizes the torsion and $l$ parametrizes the radius of curvature, since its Riemannian part turns out to be

$$\tilde{R}^{ab} = \frac{\sigma}{l^2} e^a e^b.$$

Note that since equations (31) and (32) were obtained from requiring the gauge field (29) to be locally flat, i.e., from $F = 0$, they correspond to the field equations for a Chern–Simons theory. On the other hand, since equations (31) and (32) are exactly the same as those coming from the most general action for gravity in three-dimensions described by (1) it goes without saying that, at least at the level of the field equations, both theories are equivalent.

The standard identification done for torsionless gravity, as in [3, 4], corresponds then to setting $\beta = 0$.

5. A map between different geometries and gravity theories

Here we show that the identification that allows one to extract the vielbein and the spin connection from a gauge potential given by equation (28) naturally provides a map between different geometries which are solutions of the Mielke–Baekler model with different coupling constants.
This can be seen as follows: suppose we have an AdS, dS or Poincaré gauge theory with connection $A$. Starting from the conventional identification $\beta = 0$ and $\gamma = \frac{1}{L^2}$, we can define the following mapping:

$$\omega^a \to \omega^a + \beta e^a,$$

$$e^a \to e^a,$$ \hspace{1cm} (33)

i.e. the spin connection is shifted by a term proportional to the vielbein. Because of the results of the previous section, this mapping does not spoil the identification between gauge fields and geometry. But it changes the form of the action when written in terms of the spin connection and the vielbein. Note that we could also have rescaled the vielbein. However this would just amount to a rescaling of the cosmological constant and so can be neglected without loss of generality.

For example, let us see the effect of this mapping on the AdS action. The most general action for gravity with negative cosmological constant giving field equations that imply vanishing torsion in three dimensions is given by the sum of the standard Einstein–Hilbert and the exotic action. This can be written as

$$I_{\text{AdS}} = \int \kappa \left( 2 R^a e_a + \frac{1}{3 L^2} \epsilon_{abc} e^b e^c \right) + k \left( L_3(\omega^a) + \frac{1}{L^2} T^a e_a \right).$$ \hspace{1cm} (34)

The existence of an exotic action is due to the fact that for the AdS algebra in three dimensions we have two nondegenerate invariant bilinear forms. This is not the most general gravity theory in three dimensions since it has only three free parameters. This action gives as equations of motion

$$2 R^a = - \frac{1}{L^2} \epsilon_{bc} e^b e^c, \hspace{1cm} T^a = 0.$$ 

Making now the shift in the spin connection as defined in equation (33) the action (34) is mapped into the following one:

$$I' = \int (2\kappa + 2k\beta) R^a e_a + \left( \kappa \frac{\beta^3}{3 L^2} + k \frac{\beta^2}{L^2} + \frac{k}{L^2} \beta \right) \epsilon_{abc} e^b e^c$$

$$+ k L_3(\omega) + \left( \frac{k}{L^2} + 2\kappa \beta + k \beta^2 \right) T^a e_a.$$ \hspace{1cm} (35)

The starting action (34) had three free parameters namely $\kappa$, $k$ and $1/L^2$ but now we have four free parameters, $\beta$ being arbitrary. The action $I'$ is therefore of the form given in (1) with coefficients

$$\alpha_1 = \kappa + k\beta,$$ \hspace{1cm} (36)

$$\alpha_2 = -3 \left( \frac{\kappa}{3 L^2} + \kappa \beta^2 + \frac{k}{L^2} \beta \right),$$ \hspace{1cm} (37)

$$\alpha_3 = k,$$ \hspace{1cm} (38)

$$\alpha_4 = \frac{k}{L^2} + 2\kappa \beta + k \beta^2.$$ \hspace{1cm} (39)

The shift in the spin connection therefore has the effect of mapping the action (34) into a Mielke–Baekler action (1) for every value of $\beta$. Furthermore, we can invert the relations (39)

$$k = \alpha_3,$$ \hspace{1cm} (40)

$$\kappa = \alpha_1 - \alpha_3 \beta.$$ \hspace{1cm} (41)
and $\beta$ and $l$ are given by (7). This means that, starting from the action (34), with a suitable choice of $\beta$ we can recover every point in the parameter space of the Mielke–Baekler model which has negative effective cosmological constant. Therefore, the solutions of the original action (34) are also mapped into solutions of the Mielke–Baekler model. We would like to stress that this fact can be used to greatly simplify the construction of the supersymmetric extension of the Mielke–Baekler model, since this task can be done by knowing the supersymmetry extension of the action (34), and applying the map.

The parameter $\beta$, which characterizes the field redefinition, is related to the contorsion of the resulting theory by (8). The effective cosmological constant $1/l^2$ is left unchanged by the field redefinition. Note that the map presented here can also be applied in the cases where the effective cosmological constant is non-negative.

As an application of the map, in the next section we show how the supergravity theory constructed in section 3 can be recovered from the standard supergravity theory with $\mathcal{N} = 1$.

6. Recovering the most general supergravity theory with $\mathcal{N} = 1$ from the map

It is useful to recall that in three dimensions, the AdS group is isomorphic to $Sp(2) \times Sp(2)$. This can be easily seen by changing the basis of the AdS algebra (16) according to

$$Ja = J^+_a + J^-_a$$

and

$$Pa = J^+_a - J^-_a,$$

where $J^+_a$ and $J^-_a$ are the generators of each individual copy of $Sp(2)$.

The connection in this basis is given by

$$A^a = A^+_a J^+_a + A^-_a J^-_a.$$

The standard way of making geometry emerge, in this basis reads

$$A^+_a = \omega^a + \frac{1}{l} e^a,$$

$$A^-_a = \omega^a - \frac{1}{l} e^a.$$

The most general AdS gravity theory, obtained by making the shift (33), in this basis is

$$A^+_a = \omega^a + \left(\beta + \frac{1}{l}\right) e^a,$$

$$A^-_a = \omega^a - \left(\beta - \frac{1}{l}\right) e^a.$$

This basis is particularly suitable for treating supergravity because the minimal supersymmetric extension of the AdS algebra is obtained just by supersymmetrizing one of the copies, that is $OSp(2|1) \times Sp(2)$. The $OSp(2|1)$ algebra is

$$[J^+_a, J^+_b] = \epsilon_{abc} J^+_c,$$

$$[J^+_a, Q_a] = -\frac{1}{2} (\Gamma_a)^{\beta \alpha} Q_\beta,$$

$$(Q_a, Q_b) = (C^{ab})_{\alpha \beta} J^+_\alpha.$$

The supersymmetric AdS action will be made from a $OSp(2|1)$ connection $A_+ = A^+_a J^+_a + \psi^a Q_a/\sqrt{l}$ and a $Sp(2)$ connection $A_- = A^-_a J^-_a$.

Let us first review the supersymmetric Einstein–Hilbert action with negative cosmological constant. This is constructed as a Chern–Simons theory whose Lagrangian, $L$, satisfies

$$\langle F^2 \rangle - \langle F^2 \rangle = dL,$$

making the standard identification

$$A^+_a = \omega^a + e^a/l$$ and $A^-_a = \omega^a - e^a/l$. We use the matrix representation of the generators given in appendix B. In this case the invariant bilinear form is the (super)trace. The action reads

$$I_1 = \hat{\kappa} \int_M \frac{1}{2} [L_{cs}(A_+ - L_{CS}(A_-))] - \frac{1}{l} \bar{\psi} \nabla \psi,$$

where $L_{CS}(A) = \langle A dA + \frac{3}{2} A^3 \rangle$. The action reduces to

$$I_1 = \frac{\kappa}{l} \int_M 2e^a R_a + \frac{1}{3l^2} \epsilon_{abc} e^a e^b e^c - \bar{\psi} \nabla \psi,$$
up to a surface term. Here $\nabla\psi$ is the covariant derivative with respect to the connection $A_+$, which, in terms of the vielbein and spin connection, is $\nabla\psi = (d\psi - \frac{1}{2}\omega^a \Gamma_a \psi - \frac{1}{2} \epsilon e^a \Gamma_a \psi) \equiv D\psi - \frac{1}{2} \epsilon e^a \Gamma_a \psi$.

In order to obtain the most general supersymmetric AdS action in three dimensions with $\mathcal{N} = 1$, we proceed as follows: we add to the standard supersymmetric Einstein–Hilbert action the supersymmetric ‘exotic action’. The exotic action is constructed from $\text{STr}(F_2^2) + \text{Tr}(F_2^2)$, and it reads

$$I_{\text{exotic}} = k \int_M \frac{1}{2} \epsilon e^a T_a + I_{\text{CS}}(\omega) - \frac{1}{l^2} \bar{\psi} \nabla \psi.$$  

The sum of the two actions $I_1$ and $I_{\text{exotic}}$ reads

$$I = \int_M \kappa \left( 2 e^a R_a + \frac{1}{3 l^2} \epsilon_{abc} e^a e^b e^c \right) + k \left( I_{\text{CS}}(\omega) + \frac{1}{l^2} \epsilon e^a T_a \right) - \left( \kappa + \frac{k}{l} \right) \bar{\psi} \nabla \psi$$

where we have redefined $\kappa = \hat{\kappa} / l$.

The last action is the starting point for recovering the $\mathcal{N} = 1$ supersymmetric extension of the most general gravity theory, described by the action (10). In fact applying the mapping $\omega^a \rightarrow \omega^a + \beta e^a$ the previous action is mapped

$$I \rightarrow I' \quad (44)$$

where $I'$ is given by

$$I' = (2\kappa + 2k\beta) R^a e_a + \left( \frac{\kappa}{3 l^2} + \kappa \beta^2 + \frac{k}{l^2} \beta + \frac{k}{3} \beta^3 \right) \epsilon_{abc} e^a e^b e^c$$

$$+ k I_{\text{CS}} \left( \frac{k}{l^2} + 2\kappa \beta + \kappa \beta^2 \right) T^a e_a - \left( \kappa + \frac{k}{l} \right) \bar{\psi} \left( \nabla - \frac{\beta}{2} e^a \Gamma_a \right) \psi. \quad (45)$$

We now make the same redefinitions as in (39) and so obtain the action $I_G$ of equation (1) plus the following fermionic term

$$- \left( \alpha_1 - \alpha_3 \left( \beta - \frac{1}{4} \right) \right) \bar{\psi} \left( D - \frac{1}{2} \left( \beta + \frac{1}{4} \right) e^a \Gamma_a \right) \psi,$$

and the supersymmetry transformations can be obtained by applying the shift (33) to the known transformations of the torsion-free case which are then as in (12). The Poincaré case is obtained for $\mathcal{N} = 1$ simply by taking the limit $l \rightarrow \infty$.

Note that for $\beta^2 - 1/l^2 = 0$ the vielbein decouples from the fermionic term and so the gravitino does not contribute to the stress–energy tensor. This is related to the fact that in this case one of the $\text{SL}(2, R)$ connections becomes $A_+ = \omega^a J_a$. This case is characterized by zero curvature but nonzero torsion, which in the literature is known as teleparallel gravity (for a review see e.g. [17]).

7. The $\mathcal{N} = p + q$ gravitini case

In this section, the locally supersymmetric extension of the most general theory of gravity in three dimensions for $\mathcal{N} = p + q$ gravitini is constructed. In order to do this we will start from the linear combination of the $\mathcal{N} = p + q$ action of standard three-dimensional supergravity of Achucarro and Townsend [3], and the $\mathcal{N} = p + q$ ‘exotic’ theory, and then apply the map in order to obtain the local supersymmetric extension of the more general theory. Let us therefore briefly recall the model of Achucarro and Townsend. As the AdS group is isomorphic to the $Sp(2) \times Sp(2)$ group we can obtain a supersymmetric extension with $\mathcal{N}$ gravitini of the AdS
theory by supersymmetrizing separately each copy of the $Sp(2)$ group. The result is then a Chern–Simons theory for the $OSp(2|p) \times OSp(2|q)$ group with $N = p + q$.

The $OSp(2|p)$ algebra is spanned by the generators $J_\alpha^a$ of $Sp(2)$, $M_{ij}$ of $SO(p)$ and $Q_a^i$ satisfying

\[
\left[ J_\alpha^a, M_{ij} \right] = 0,
\left[ J_\alpha^a, J_\beta^b \right] = \epsilon_{abc} J_\gamma^c,
\left[ J_\alpha^a, Q_\beta^i \right] = -\frac{1}{2} (\Gamma_a)^i_\alpha Q_\beta^i,
\left[ M_{ij}, M_{kl} \right] = \eta_{ij} M_{kl} - \eta_{ki} M_{lj} - \eta_{kj} M_{il},
\left[ M_{ij}, Q_\alpha^k \right] = (m_{ij})^k_l Q_l^\alpha,
\left[ Q_\alpha^k, Q_\beta^l \right] = (C/\Gamma^a_1)^\alpha_\beta J_a^k + \frac{1}{2} c_{\alpha\beta} M_{kl},
\]

where $(m_{ij})^k_l$ are given by

\[
(m_{ij})^k_l = \left( \eta^{ik} \delta^j_l - \eta^{il} \delta^j_k \right)
\]

and provide a representation for $SO(p)$. We define a superconnection $A_{\pm}$:

\[
A_{\pm} = A_a^+ J_a + \frac{\psi^a}{\sqrt{l}} Q_a^i - \frac{1}{2} b_{ij}^i M_{ij}
\]

with the index $i$ running from 1 to $p$. Similarly we define a superconnection for the $OSp(2|q)$ algebra:

\[
A_- = A_a^- J_a + \frac{\psi^a}{\sqrt{l}} Q_a^I - \frac{1}{2} b_{IJ}^I M_{IJ}
\]

with the index $I$ running from 1 to $q$.

The matrix representation for the connection $A$ is given in appendix C where the Chern–Simons term was derived. The Chern–Simons Lagrangian for $OSp(2|p)$ is

\[
L_+ = \frac{1}{2} L_{CS}(A_+) - L_{CS}(a_{ij}) - \bar{\psi}^i \nabla_+ \psi_k
\]

where the covariant derivative is

\[
\nabla_+ \psi_j = d\psi_j - \frac{1}{2} A_a^+ \Gamma_a \psi_j - a_{ij}^k \psi_k.
\]

For $OSp(2|q)$ the Lagrangian is

\[
L_- = \frac{1}{2} L_{CS}(A_-) - L_{CS}(b_{IJ}) - \bar{\psi}^I \nabla_- \psi_J.
\]

The covariant derivative is

\[
\nabla_- \psi_I = d\psi_I - \frac{1}{2} A_a^- \Gamma_a \psi_I - b_{IJ}^I \psi_J.
\]

Our starting point in constructing the most general supergravity action in three dimensions is to take a combination of the $L_+$ and $L_-$ actions of the form

\[
I = \kappa l (L_+ - L_-) + k (L_+ + L_-).
\]

With the standard identification $A^a_{\pm} = \omega^a \pm \epsilon^a / l$, the action $I$ can be rewritten as

\[
I = I_{AdS} - l \left[ \kappa + \frac{k}{l} \right] \text{Tr} \left( ada + \frac{2}{3} a^3 \right) + l \left[ \kappa - \frac{k}{l} \right] \text{Tr} \left( bdb + \frac{2}{3} b^3 \right)
- \left[ \kappa + \frac{k}{l} \right] \psi^i \left( D - \frac{1}{2l} \epsilon^a \Gamma_a \right) \psi_i - a_{ij} \psi^i
+ \left[ \kappa - \frac{k}{l} \right] \psi^I \left( D + \frac{1}{2l} \epsilon^a \Gamma_a \right) \psi_I - b_{IJ} \psi^I,
\]

where

\[
\left[ J_\alpha^a, J_\beta^b \right] = \epsilon_{abc} J_\gamma^c,
\left[ J_\alpha^a, Q_\beta^i \right] = -\frac{1}{2} (\Gamma_a)^i_\alpha Q_\beta^i,
\left[ M_{ij}, M_{kl} \right] = \eta_{ij} M_{kl} - \eta_{ki} M_{lj} - \eta_{kj} M_{il},
\left[ M_{ij}, Q_\alpha^k \right] = (m_{ij})^k_l Q_l^\alpha,
\left[ Q_\alpha^k, Q_\beta^l \right] = (C/\Gamma^a_1)^\alpha_\beta J_a^k + \frac{1}{2} c_{\alpha\beta} M_{kl},
\]

and

\[
(m_{ij})^k_l = \left( \eta^{ik} \delta^j_l - \eta^{il} \delta^j_k \right)
\]
where the gravitational part of the action, $I_{\text{AdS}}$, is given by equation (34). This is the known $(p, q)$ AdS supergravity action [3].

We now construct the more general AdS supergravity action. This is obtained by applying the map $\omega^a \rightarrow \omega^a + \beta e^a$. The action is then mapped

$$I \rightarrow I'$$

with

$$I' = I_{G} + I_a + I_b + I_\psi' + I_{\bar{\psi}}',$$

where $I_{G}$ is the most general gravity action in three dimensions given by equation (1) and the pieces involving the gauge fields are

$$I_a := l\alpha_+ \text{Tr}(aa + \frac{2}{3}a^3), \quad I_b := l\alpha_- \text{Tr}(bb + \frac{2}{3}b^3),$$

and the fermionic pieces are given by

$$I_\psi' := \alpha_+ \bar{\psi}^i \left( \left[ D - \frac{1}{2} \left( \beta + \frac{1}{l} \right) e^a \Gamma_a \right] \psi_j - a_{ij} \psi^j \right),$$

$$I_{\bar{\psi}}' := \alpha_- \bar{\psi}^i \left( \left[ D - \frac{1}{2} \left( \beta - \frac{1}{l} \right) e^a \Gamma_a \right] \psi_j - b_{ij} \psi^j \right).$$

The coefficients $\alpha_{\pm}$ are defined by

$$\alpha_{\pm} := \mp \left[ \alpha_1 - \alpha_3 \left( \beta \mp \frac{1}{l} \right) \right].$$

The action is invariant under Lorentz transformations, AdS boosts and supersymmetry as well as the $SO(p) \times SO(q)$ gauge symmetry.

Under a SUSY transformation the fields transform as

$$\delta e^a = \frac{1}{2} \bar{\epsilon}^i \Gamma^a \psi_i + \frac{1}{2} \bar{\epsilon}^i \Gamma^a \psi^i,$$

$$\delta \psi^i = D\bar{\epsilon}^i - \frac{1}{2} \left( \beta + \frac{1}{l} \right) e^a \Gamma_a \bar{\epsilon}^i,$$

$$\delta \psi^i = D\bar{\epsilon}^i - \frac{1}{2} \left( \beta - \frac{1}{l} \right) e^a \Gamma_a \bar{\epsilon}^i,$$

$$\delta \sigma^a = -\frac{1}{2} \left( \beta - \frac{1}{l} \right) \bar{\epsilon}^i \Gamma^a \psi_i - \frac{1}{2} \left( \beta + \frac{1}{l} \right) \bar{\epsilon}^i \Gamma^a \psi^i,$$

$$\delta a_{ij} = -\frac{1}{2\sqrt{l}} (\bar{\epsilon}^i \psi_j - \psi^j \epsilon_i),$$

$$\delta b_{ij} = -\frac{1}{2\sqrt{l}} (\bar{\epsilon}^i \psi_j - \bar{\psi}^j \epsilon_i).$$

(54)

In the case of more than one supersymmetry, one cannot naively take the limit $l \rightarrow \infty$ to obtain the Poincaré case as it does not give a supersymmetric theory. The correct approach is shown in [7] where a direct sum of the standard $(p, q)$ super AdS algebra and a $so(p) \oplus so(q)$ algebra is considered. With a suitable change of basis one obtains the correct Poincaré limit.
Therefore in order to obtain the most general Poincaré supergravity theory one can apply the map to the construction described in [7].

8. Conclusions and discussion

We have constructed the locally supersymmetric extension of the most general gravity theory in three dimensions leading to first-order field equations for the vielbein and the spin connection. We have seen that the construction of this theory can be simplified by making use of a mapping between different theories of gravity described in section 5. This mapping is obtained by a shift in the spin connection of the form $\omega^a \rightarrow \omega^a + \beta^e e^e$. Note that it is a special feature of three dimensions that the spin connection can be dualized to have one Lorentz index.

It is worth pointing out that the mapping is also useful for finding solutions. For example, it is known that the black hole solution with torsion [12] has the same metric as the BTZ black hole but the spin connection is given by $\omega^a = \tilde{\omega}^a - \beta e^a$, where $\tilde{\omega}^a$ is the Riemannian connection. From our discussion, this result can be seen immediately. We know that the theory that gives torsion $-\beta$ can be obtained from the torsion-free theory by replacing $\tilde{\omega}^a$ with $\omega^a + \beta e^a$ everywhere in the action. This means that this mapping is also a mapping between solutions of the two theories. Given that the BTZ is a solution of the torsionless theory, the corresponding solution of the new theory is simply $\omega = \tilde{\omega} - \beta e$.

It is also interesting to study the black hole solution with torsion for its thermodynamical properties. In fact it was shown [18, 19] that the black hole entropy acquires a correction to the usual Bekenstein–Hawking formula proportional to the inner horizon. Also the conserved charges of gravity with torsion have been studied [20]. An interesting application is given by the computation of the mass and angular momentum of the BTZ black hole with torsion. It is found that there is a mixing between the integration constants $M$ and $J$ which in the torsion-free case are the mass and angular momentum. It should be possible to obtain the same results by use of the mapping.

It would also be interesting to find the BPS states and the asymptotic dynamics of our proposed supergravity model with torsion. By virtue of the map presented here, one naturally expects that the boundary conditions should generally be different for two different theories related by the map, so that the physics of both theories is clearly inequivalent.

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Appendix A. Notation and conventions

We use the metric with the signature $(-, +, +)$. The Levi-Civita symbol $\epsilon_{abc}$ is totally antisymmetric with $\epsilon_{012} = 1$, $\epsilon_{012} = -1$ and it satisfies the following identities:

$$\epsilon_{abc}e^{ade} = -(\delta^d_b \delta^e_c - \delta^d_c \delta^e_b), \quad \epsilon_{abc}e^{abd} = -2 \delta^d_c .$$

The Dirac matrices satisfy the Clifford algebra $[\Gamma_a, \Gamma_b] = 2 \eta_{ab}$. We choose $\Gamma_0 = i \sigma_z$, $\Gamma_1 = \sigma_y$, $\Gamma_2 = \sigma_x$. 

where the $\sigma$ are the usual Pauli matrices. Then we have the identity
\[
\Gamma_{ab} = \epsilon_{abc} \Gamma^c.
\]
For a spinor $\epsilon^a$ we define the Majorana conjugate $\bar{\epsilon}_\beta := C_{\beta\alpha} \epsilon^\alpha$, where $C$ is the charge conjugation matrix satisfying $C^T = -C$ and $(\Gamma_\mu)^T = C \Gamma_\mu$.

Appendix B. Matrix representation of $OSp(2|1)$ and construction of the $\mathcal{N} = 1$ Lagrangian

We choose a set of matrices that form a representation of the $OSp(2|1)$ Lie algebra (42) given by
\[
J_\alpha = \begin{pmatrix} -\frac{1}{2} (\Gamma_\alpha)^\mu_\beta & 0 \\ 0 & 0 \end{pmatrix}, \quad Q_\alpha = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \delta^\alpha_\beta \\ -\frac{1}{\sqrt{2}} C_{\alpha\beta} & 0 \end{pmatrix}.
\]
(\text{B.1})

The $OSp(2|1)$ connection defined as $A = A^a J_a + \frac{i}{\sqrt{2}} Q_\alpha \bar{\psi}$, in this representation takes the form
\[
A = \begin{pmatrix} -\frac{1}{2} A^a \Gamma_a & \frac{1}{\sqrt{2}} \bar{\psi} \\ \frac{1}{\sqrt{2}} \bar{\psi} & 0 \end{pmatrix}.
\]
(\text{B.2})

Hence we can compute the curvature defined as $\mathcal{F} = dA + A^2$ reads
\[
\mathcal{F} = \begin{pmatrix} -\frac{1}{2} F^a \Gamma_a & \frac{1}{\sqrt{2}} \bar{\psi} \nabla \bar{\psi} \\ \frac{1}{\sqrt{2}} \bar{\psi} \nabla \psi & 0 \end{pmatrix}.
\]
(\text{B.3})

Here $\nabla$ is the covariant derivative given by $\nabla \psi = d\psi - \frac{1}{2} A^a \Gamma_a \psi$. We can use the fact that for Chern–Simons theories we have $d L = \langle F^2 \rangle$, where the invariant 4-form is the supertrace of $\mathcal{F}^2$ to write the Lagrangian in terms of the components of $A$.
\[
\text{STr}(\mathcal{F}^2) = \frac{1}{2} F^a F_a - \frac{1}{2} F^a \bar{\psi} \Gamma_a \psi - \frac{1}{2} \bar{\psi} \nabla \bar{\psi} \nabla \psi
= d \left( \frac{1}{2} L_{cs}(A) - \frac{1}{\sqrt{2}} \bar{\psi} \nabla \psi \right) = d L.
\]
(\text{B.4})

Appendix C. Matrix representation for $OSp(2|p)$ and the construction of the $\mathcal{N} = p + q$ supergravity Lagrangian

We have the following matrix representation of the $OSp(2|p)$ Lie algebra (46):
\[
J_a = \begin{pmatrix} -\frac{1}{2} (\Gamma_a)^\mu_\beta & 0 \\ 0 & 0 \end{pmatrix},
(\text{C.1})

M_{ki} = \begin{pmatrix} 0 & 0 \\ 0 & -(m_{ki})^j_\gamma \end{pmatrix},
(\text{C.2})

Q^k_\gamma = \begin{pmatrix} 0 & \frac{1}{\sqrt{2}} \delta^k_\gamma \\ -\frac{1}{\sqrt{2}} C_{\gamma\delta} \eta^{k\delta} & 0 \end{pmatrix},
(\text{C.3})

where $(m_{i})^j_\gamma$ is defined as in equation (47).
The matrix representation for the connection $A = A^a J_a + \frac{\psi^a}{\sqrt{2}} Q_a - \frac{1}{2} a^{ijk} M_{ij}$ is

$$A = \left( -\frac{i}{2} A^a \Gamma_a, \frac{1}{\sqrt{2}} \psi_j \frac{1}{a^j} \right).$$ (C.4)

So the field strength $\mathcal{F} = dA + A^2$ becomes

$$\mathcal{F} = \left( \frac{F^a}{2} \psi^k \psi^j + \frac{i}{\sqrt{2}} \nabla \psi_j \frac{1}{2} \psi^k \nabla \psi^i \right).$$ (C.5)

where we have defined the field strength of the $SO(P)$ gauge field, $f^{ij} = da^i_a + a^i_a d^a_j$. We then obtain

$$\text{Str}[\mathcal{F}^2] = \frac{i}{2} F^a F_a - f_{ij} f^{ij} - d(\bar{\psi}^k \nabla \psi^k) = dL.$$ (C.6)

So we obtain the Lagrangian

$$L = \frac{1}{2} L_{\text{CS}}(A) - L_{\text{CS}}(a^i_a) - \bar{\psi}^k \nabla \psi_k.$$ (C.7)

**Note added.** After the completion of this paper, the structure of asymptotic symmetries for the supergravity theory presented in section 7 for the case of $N = 1 + 1$ was studied in [21], where it was found that the asymptotic Poisson bracket algebra of the canonical generators has the form of two independent super Virasoro algebras with different central charges.

**References**

[1] Deser S, Jackiw R and ‘t Hooft G 1984 Three-dimensional Einstein gravity: dynamics of flat space Ann. Phys. 152 220

Deser S and Jackiw R 1984 Three-dimensional cosmological gravity: dynamics of constant curvature Ann. Phys. 153 405

[2] Barados M, Teitelboim C and Zanelli J 1992 The Black hole in three-dimensional space-time Phys. Rev. Lett. 69 1849 (Preprint hep-th/9204099)

Barados M, Henneaux M, Teitelboim C and Zanelli J 1993 Geometry of the $(2+1)$ black hole Phys. Rev. D 48 1506 (Preprint gr-qc/9302012)

[3] Achucarro A and Townsend P K 1986 A Chern–Simons action for three-dimensional anti-de Sitter supergravity theories Phys. Lett. B 180 89

[4] Witten E 1988 $(2+1)$-Dimensional gravity as an exactly soluble system Nucl. Phys. B 311 46

[5] Deser S and Kay J H 1983 Topologically massive supergravity Phys. Lett. B 120 97

Deser S 1984 Cosmological topological supergravity Quantum Theory of Gravity: Essays in Honor of the 60th birthday of Bryce S DeWitt ed B S DeWitt (Bristol: Adam Hilger) p 374

[6] Achucarro A and Townsend P K 1989 Extended supergravities in $d = (2 + 1)$ as Chern–Simons theories Phys. Lett. B 229 383

[7] Howe P S, Izquierdo J M, Papadopoulos G and Townsend P K 1996 New supergravities with central charges and Killing spinors in $(2 + 1)$-dimensions Nucl. Phys. B 467 183 (Preprint hep-th/9505032)

[8] Mielke E W and Baekler P 1991 Topological gauge model of gravity with torsion Phys. Lett. A 156 399

[9] Mardones A and Zanelli J 1991 Lovelock–Cartan theory of gravity Class. Quantum Grav. 8 1545

[10] Blagojevic M and Vasilic M 2003 3D gravity with torsion as a Chern–Simons gauge theory Phys. Rev. D 68 104023 (Preprint gr-qc/0307078)

[11] Cacciatori S L, Caldarelli M M, Giaconnet A, Klemm D and Mansi D S 2005 3D Chern–Simons formulation of three-dimensional gravity with torsion and nonmetricity J. Geom. Phys. 56 2523 (Preprint hep-th/0507200)

[12] Garcia A A, Hehl F W, Heinicke C and Macias A 2003 Exact vacuum solution of a $(1 + 2)$-dimensional Poincaré gauge theory: BTZ solution with torsion Phys. Rev. D 67 124016 (Preprint gr-qc/0302097)

[13] Horne J H and Witten E 1989 Phys. Rev. Lett. 62 501

[14] Lindström U and Rocek M 1989 Phys. Rev. Lett. 62 2905

For more recent developments, see e.g. Nishino H and Gates S J J 1993 Int. J. Mod. Phys. A 8 3371

[15] Deser S, Jackiw R and Templeton S 1982 Topologically massive gauge theories Ann. Phys. 140 372

Deser S, Jackiw R and Templeton S 1988 Topologically massive gauge theories Ann. Phys. 185 406 (erratum)
Deser S, Jackiw R and Templeton S 1988 Topologically massive gauge theories Ann. Phys., NY A 281 409

Solodukhin S N 2006 Holography with gravitational Chern–Simons Phys. Rev. D 74 024015 (Preprint hep-th/0509145)

Park M I 2006 BTZ black hole with gravitational Chern–Simons: thermodynamics and statistical entropy Preprint hep-th/0608165

De Andrade V C, Guillen L C T and Pereira J G 2000 Teleparallel gravity: an overview Preprint gr-qc/0011087

Blagojevic M and Cvetkovic B 2006 Class. Quantum Grav. 23 4781 (Preprint gr-qc/0601006)

Blagojevic M and Cvetkovic B 2007 Class. Quantum Grav. 24 129 (Preprint gr-qc/0607026)

Blagojevic M and Cvetkovic B 2006 J. High Energy Phys. JHEP10(2006)005 (Preprint gr-qc/0606086)

Blagojevic M and Cvetkovic B 2005 8th Workshop on What Comes Beyond the Standard Models (Bled, Slovenia, 19–29 July 2005)

Cvetkovic B and Blagojevic M 2007 Preprint gr-qc/0702121