Research Article

Vibration-Based Damage Evaluation of a Reinforced Concrete Frame Subjected to Cyclic Pushover Testing

Burcu Gunes and Oguz Gunes

Civil Engineering Department, Istanbul Technical University, Istanbul, Turkey

Correspondence should be addressed to Burcu Gunes; bgunes@itu.edu.tr

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This research investigated the changes in vibration characteristics of a simple reinforced concrete (RC) frame subjected to incremental cyclic pushover testing as a basis for detection, quantification, and localization of damage in RC frames using vibration data obtained before and after a seismic event. A half-scale one-story one-bay plane frame was subjected to progressive damage through cyclic lateral loading to incrementally increasing drift ratios. Ambient and impact vibration tests were performed at each increment of drift ratio, and modal analyses of the acceleration responses obtained at seven locations on the frame were carried out with the acceleration responses measured at seven different locations on the frame to track changes in the dynamic characteristics. Linear degradation of the lowest two vibration frequencies was identified with increasing drift ratio, which was regarded as a promising result towards detection and quantification of damage. For localization, a flexibility-based damage localization procedure, the damage locating vector (DLV) approach, was explored. Localization results mostly agreed with the observed damage, and the approach was found to have potential for use in prioritizing the suspected damage locations in the structure for detailed inspections.

1. Introduction

Increased awareness of the economic and social effects of aging, deterioration, and natural hazards on civil infrastructure has brought the need to develop advanced health monitoring and damage detection tools for sustainability and mitigation purposes. Structural health monitoring (SHM) process includes implementation of a damage identification strategy encompassing detection, localization, and assessment of the extent of damage in a structure so that its remaining life can be predicted and possibly extended. To achieve this objective, SHM employs both local and global methods of damage identification. The local methods include visual inspections and nondestructive evaluation tools such as acoustic emission, ultrasonic, magnetic particle inspection, radiography, and eddy current [1]. All these techniques, however, require a priori localization of the damaged zone and easy access to the portion of the structure under inspection. Global methods are of particular interest due to their potential for rapid and cost-efficient—even remote—implementation of the health monitoring process [2].

SHM based on measured vibration data is a global method used to detect changes in system characteristics and to assess the integrity of the structure through on-site or remote sensing of its dynamic response [3–7]. The basic premise of the vibration-based techniques is that the vibration characteristics or the so-called modal parameters (frequencies, mode shapes, and modal damping) are functions of the physical properties of the structure (mass, energy dissipation mechanisms, and stiffness), and changes in these physical properties cause changes in the modal properties. The so-called before-and-after strategy, which is based on comparison of system characteristics obtained before and after a potentially damaging event or at the beginning and end of a monitoring period using the recorded sensor data, is one of the most widely used frameworks in this technique. This type of assessment, however, suffers from the fact that temperature changes, moisture, and other environmental factors also produce changes in dynamic characteristics. If
the causes of changes other than due to damage are considered to be noise in the measurement, then the changes due to damage must be significantly larger than the noise in order for the techniques to work. Hence, the modal parameters to be used as a basis for damage detection must be distinctively and distinguishably influenced by the damage in order to avoid failed or false detection.

2. Damage Identification Approaches and Vibration-Based SHM of Reinforced Concrete Frames

Most of the existing damage identification methods applied to structures can be classified into two groups: model-based and nonmodel or feature-based methods. The model-based methods are model updating procedures in which the mathematical model or the physical parameters of a structure are calibrated or updated using vibration measurements from the physical structure [8, 9]. Analytical sensitivities of response parameters to changes in physical properties are used to update modeling assumptions, physical sizing, elastic moduli, and so forth. The feature-based approaches detect changes in structural characteristics by detecting damage features in the measured data without the need for an analytical model of the structure. The main task here is the extraction of damage features sensitive to structural changes so that damage can be identified from the measured vibration response of the structure. The civil engineering community has been studying the vibration-based damage assessment of bridge and building structures since the early 1980s. Both model-based and non-model-based approaches utilizing the measured data in time domain, frequency domain, or modal domain have been investigated.

A considerable amount of this research has been dedicated to detect damage using the change in the natural frequencies. The use of natural frequency as a diagnostic tool for damage is based on the fact that it is a sensitive indicator of structural integrity. It was observed, however, that frequency changes due to damage may require either very precise measurements or high damage levels [4, 10, 11] and especially for large structures, using the frequency shifts as damage indicators may have practical limitations [6]. Mode shapes were used as an alternative to natural frequencies where the modal assurance and coordinate modal assurance criteria were used to correlate the mode shapes of the control and damaged beams [12–14]. Difficulties associated with choosing appropriate modes and pairing of the modes for comparison can be viewed as the drawbacks of this approach. Mode shape curvature was introduced as a means of improving the sensitivity of the damage indicator since higher derivatives of the mode shapes are more sensitive to damage [15, 16]. The limitations with the use of the mode shapes and the mode shape curvatures are that they require a very accurate and a large set of measurements to monitor the strain field over the entire structure. Another modal parameter, that is, damping, was explored as a means of crack detection. In cases where cracks lead to small or no frequency variations, damping changes may be useful to detect the dissipative effects produced by the cracks. The change in the strain energy stored in a particular vibration mode was also investigated as a potential indicator of damage. Implicit in the technique, however, is that if damage is located in an element that is not sensitive to changes in the modal parameter, it cannot be detected.

A promising approach for damage identification is the changes in flexibility or local stiffness [17–21]. The flexibility matrix is most sensitive to changes in the lower frequency modes because of the inverse relationship to the square of the modal frequencies, and usually, the lower vibration modes are the only ones that can be identified during an experimental survey due to practical difficulties in measuring the higher modes. These methods, yet, require a sufficient number of well-distributed sensors, and for small damage situations, the changes may be masked by numerical errors.

Pattern recognition methods, according to [22], are needed as supplements to the existing model-based techniques since damage detection can be viewed fundamentally as a pattern recognition problem. The main challenge associated with pattern recognition approaches is that they require significant amount of preprocessing. Artificial neural network approaches which do not require any knowledge on the physical relationships between the structural properties and damage necessitate large training samples for accurate damage detection [7, 23–27]. Recent advances in signal processing techniques based on Hilbert transforms, empirical mode decomposition, and wavelet analysis led to new avenues for structural health monitoring since they can be applied to nonlinear and nonstationary time series [28–34]. In addition to the requirement of fast computation algorithms, their limited physical meaning is a major drawback of these methods.

Early studies and applications of SHM to structures mostly focused on steel structures or structures made of homogenous materials. Unlike such structures, RC structures have—by design—concrete sections partly cracked in tension zones where the embedded steel reinforcement provides the load resistance. Hence, the vibration response of an RC structure differs significantly from that calculated based on gross sections disregarding concrete cracking. Opening and closing of cracks during dynamic response cause nonlinearities in behavior, and further complicated system damage mechanisms are growth of existing cracks and development of new ones which generally occur in a distributed fashion within the structure. Hence, vibration-based SHM requires use of customized identification approaches and metrics for detection and possible localization of damage in different type of structures.

Wang et al. [29] conducted an experimental study on small RC beams with various boundary conditions and noted an appreciable drop in natural frequencies and increase in damping ratios. Maeck et al. [35] implemented a method to determine the stiffness decrease of concrete beams to damage. Ndambi et al. [36] carried out an analysis of 6 m long RC beams to localize damage inflicted by static tests. Owen et al. [37] and Neild et al. [38] studied joint time-
frequency domain transforms for exploring the nonlinearity of the damaged reinforced concrete elements. Fang et al. [39] developed a sensitivity-based updating method to identify damage in a tested reinforced concrete frame by minimizing the discrepancies of modal frequencies and mode shapes. Reynders and De Roeck [40] developed a local flexibility method to localize damage and performed experiments with a reinforced concrete free-free beam and a three-span prestressed concrete bridge that are subjected to a progressive damage test. Frizzarin et al. [41] developed a time domain damage detection approach for concrete structures using the nonlinear damping identified from measured vibration responses. Demarie and Sabia [42] focused on identification of nonlinear damping and frequency of a progressively damaged beam-column node. Consuegra and Irfanoglu [43] performed shaking table tests of one-story two-way RC frames and employed signal processing and nonlinear identification approaches to evaluate damage. Consuegra and Irfanoglu [43] examined small scaled concrete beams and a full-scale three-story RC frame for displacement dependence of dynamic properties. Capazucca [44] related degradation of stiffness and development of cracking to frequency values measured in a frequency range through vibration tests on free-end beams.

3. Methodology

3.1. Research Rationale and Approach. Previous research studies that explored detection of damage in structures based on vibration data have naturally involved numerous different definitions and forms of damage depending on the structure type, materials, geometry, and identification technique and parameters. Hence, a consistent definition of damage and a methodological framework that includes a systematic set of approaches and associated procedures as a basis for SHM of civil engineering structures are far from realization at the present time. A proper definition of damage that is "observable, identifiable, and measurable" is a controversial subject on its own at a fundamental level which makes it difficult to reach a consensus in many areas.

In the case of seismic performance evaluation of structures, the above-mentioned consensus has been established to a reasonable extent within the past two decades with the development of performance-based seismic engineering approach [45, 46]. Within this approach, the interstory drift—a measurable structural response parameter—is often used as an abstract damage measure, the limiting values of which are used to estimate the seismic performance level. Hence, integration of the vibration-based damage identification methods into the performance-based seismic engineering approach through the interstory drift measure is a promising SHM strategy with practical significance and high implementation potential.

Establishing a link between the changes in vibration characteristics of a concrete structure and its seismic performance through the use of interstory drift as a damage parameter requires a systematic series of forward problem implementations in experimental and numerical form. The primary objective of this research was to carry out the simplest experimental investigation to this effect, which involved measurement and identification of the changes in vibration characteristics of a one-story, one-bay RC frame as a result of increasing levels of drift—hence damage—to be inflicted by means of a cyclic pushover test that involves lateral loading only. This approach has significant advantages and limitations that require their due consideration to maximize the benefits. The obvious advantage is the simplicity of the experimental setup and the relative ease of performing the successive loading and vibration tests, especially in view of the requirement that the loading setup must be detached from the frame to perform the vibration tests. This simplicity, however, comes at the expense of a light and overly rigid frame with very high vibration frequencies. Hence, capturing the higher frequencies requires recording at very high sampling rates which is often limited by the capabilities of the data acquisition system.

In this preliminary study, vibration characteristics for only the two mode shapes shown in Figure 1 were targeted for identification. These two mode shapes are significant because they correspond to the lowest two vibration frequencies of a plane (2D) portal frame and are representative of the sway modes and floor vibration modes of a real structure. Although the vibration parameters obtained in this research for a simple portal frame were not expected to be representative of a real structure, investigation of possible correlations between the identified vibration parameters and the drift ratio constituted the main research objective. Once such correlations are identified, more realistic relationships representative of real structures can be obtained through more elaborate experimental investigations. Significance and impact of the experimental results and the obtained correlations towards accurate and reliable SHM of RC structures are discussed in the final sections of the manuscript.

3.2. Experimental Setup. The single-story one-bay reinforced concrete frame is comprised of a rectangular beam and a column with cross-sectional dimensions of 20 cm × 30 cm. The height of the frame structure is 1.5 m, and the length of the span is 2.0 m. The designed compressive strength of concrete is 27.5 MPa and the designed strength of the steel is 412 MPa. The dimensions and the design details of the constructed frame are presented in Figure 2.

The test frame, the accelerometers mounted on the structure, and the loading setup are displayed in Figures 3 and 4, respectively. The instrumentation used in the pushover experiments includes load cells to apply the lateral load on the structure and linear extensometers—one on the left beam-column connection and one on the footing—to monitor the displacements. A total of eight unidirectional accelerometers are mounted on the structure as shown in Figure 3. Seven of these accelerometers are placed on the structure and measure acceleration signals in the lateral direction on the columns and the transverse direction on the beam. The eighth accelerometer is mounted on the footing to measure the accelerations at the base of the structure. The vibration tests are carried out with an instrumented short-sledge impulse hammer with a 5k lbf (22.2 kN), and a 3 lbf...
Figure 1: (a) Lateral sway and (b) floor vibration mode shapes of a plane portal frame.

Figure 2: (a) Geometric and (b) reinforcement details of the RC test frame.

Figure 3: Schematic views of experimental setups for (a) lateral loading and (b) impact vibration tests.
3.3. Pushover Tests: Loading Protocol, Damage Levels, and Crack Propagation. The test frame is subjected to increasing levels of damage with the drift levels displayed in Table 1 through cyclic pushover tests. Each cyclic pushover scheme is performed in a displacement-controlled mode and followed by vibration testing of the frame. The loading protocol for the pushover tests and the associated load versus drift ratios are depicted in Figure 5. Figure 6 shows the displaced configuration of the frame loaded to 4% drift ratio, and the observed crack patterns for the inflicted damages are illustrated in Figure 7 for increasing drift ratios.

3.4. Vibration Tests: Impact and Ambient Vibration. Impact excitation is one of the most common methods used for experimental modal testing [47]. Hammer impacts produce a broadband excitation signal ideal for modal testing with a minimal amount of equipment and setup. Although it has limitations with respect to its precise positioning and force level control, its advantages such as its versatility, mobility, and reliability of results greatly outweigh its disadvantages and make it extremely effective for modal testing situations. The auto spectrum calculated for a sample impact force with a sampling frequency of 500 Hz is displayed in Figure 8. Examination of this plot reveals that a broadband force is produced provided that the frequency range of interest for the test structure is less than 100 Hz.

Measured acceleration response signals corresponding to the undamaged state of the frame when it is subjected to a lateral impact at the top of the column (at A3 location) and a vertical impact at the midpoint of the beam (at A5 location) are displayed in Figures 9(a) and 10(a). This dataset, obtained with a sampling interval of 0.002 sec, is presented with a constant shift to enable comparison of the measured accelerations at different locations. Figures 9(b) and 10(b) present the associated frequency response functions for accelerometers A3, A4, and A5.

To distinguish the structural modes from those that may arise as rigid body modes due to the experimental setup, the in-plane horizontal accelerations measured at the base of column (A1) are processed and the associated FRFs are provided in Figures 11(a) and 11(b). The peaks observed at 17 Hz and 30 Hz in these plots are interpreted as the “rigid body modes” that appear due to boundary conditions. The variations in the FRFs are displayed in Figure 12 for different drift ratios.
Ambient vibration measurements, a sample of which is displayed in Figure 13, are also collected at each drift ratio for all the sensor locations. The power spectral density (PSD) functions are obtained using Welch’s technique as described above for initial screening of the data. Figure 14 displays the comparison of the PSDs obtained from ambient vibration of the test frame at sensor locations on the column (A3) and the beam (A5) at the selected damage levels.

**4. Modal Identification**

The literature on modal identification methodologies is vast but it is not the intent of this paper to provide a review on these techniques. For the current study, Eigensystem Realization Algorithm (ERA) is adopted to process the impact test data in the time domain. The details of the technique can be found in [48]. The technique can be briefly summarized as leading to realization with the following underlying model structure:

\[ x_{k+1} = Ax_k + Bu_k, \]  
\[ y_k = Cx_k + Du_k, \]  

where \( x_k \) is the state vector that holds the current state of the system, \( u_k \) is the vector of system inputs, \( y_k \) is the vector of system outputs at time instant \( k \), and \( A, B, C, \) and \( D \) are the discrete-time state-space matrices. ERA uses the measured input and output response of the structure with the principles of minimum realization to estimate the system matrices as follows.
\[ A \Sigma^{-1/2} R^T H(1) \Sigma^{1/2}, \quad (2a) \]
\[ B = \Sigma^{1/2} S^T E, \quad (2b) \]
\[ C = E^T R \Sigma^{1/2}, \quad (2c) \]

where \( H(1) \) is a time-shifted Hankel matrix obtained from the output measurements, \( S \) and \( R \) are the matrices of nonzero singular values and the associated vectors of the Hankel matrix, and \( E = [I 0] \) with \( I \) being the identity matrix 0 a matrix of zeros with appropriate dimensions. Modal parameters can then be identified using the system matrices \( A \) and \( C \).

Table 2 lists the identified modal parameters corresponding mainly to the sway of the columns \([f_1, \xi_1]\) and the bending up-and-down of the beam \([f_2, \xi_2]\) using ERA based on accelerometer data at each damage state. In each ERA realization, a Hankel matrix of size \((1050 \times 7)\) was constructed based on the impulse response data sampled at 500 Hz. Then after performing a singular value decomposition, a relatively high system order \((n = 20)\) is assumed in order not to miss any physical modes. Since the system order of real-life structures is not known a priori, the number of assumed modes is usually set to be two to ten times the number of true modes in the frequency range of interest. The computational spurious modes are eliminated from the
Figure 9: (a) Acceleration measurements after impact at A3 location on the column. (b) FRF of the measured accelerations (undamaged state).

Figure 10: (a) Acceleration measurements after impact at A5 location on the beam. (b) FRF of the measured accelerations (undamaged state).

Figure 11: (a) FRF of the measured acceleration at the base (A1) for the undamaged case with (a) impact at A3 and (b) impact at A5.
Figure 12: Continued.
physically true modes through mode accuracy indicators [48].

The decreasing tendency in the identified values of the natural frequencies as damage progresses can be clearly observed in Figure 15. The changes in the damping ratios of the first mode, however, show an increasing trend with no clear tendency. For the second mode, this tendency is not consistent with the progression of damage. Although an extensive experimental campaign is carried out on the test frame with three repetitions of impacting each sensor location with a hammer and recording the accelerations, the identification results reported here use only two impact locations: one at the midpoint of the beam and the other one at the right corner of the frame.

Vibration measurements from the frame under operating conditions are also recorded and processed for modal identification. The enhanced frequency domain decomposition (EFDD) method [49] working in frequency domain and subspace identification (SSI) method [50] operating in time domain are explored for extracting the modal parameters. The EFDD method is a nonparametric method based on singular value decomposition of the spectral density matrix of measured accelerations \( S_{YY}(w) \):

\[
S_{YY}(w) = U(w)\Sigma(w)U^H(w),
\]

where the matrix \( U \) is a unitary matrix holding the singular vectors, \( S \) is a diagonal matrix holding the singular values, and \( H \) denotes the complex conjugate transpose.

This operation leads to a set of auto spectral density functions, each corresponding to a single-degree-of-freedom (SDOF) system. An inverse Fourier transform of these functions yields the free-decay function of the SDOF system from which the modal frequencies and damping ratios can be estimated. It has been shown that the result is exact where the loading is white noise, the structure is lightly damped, and the mode shapes of the close modes are orthogonal [49].

The subspace identification technique provides a parametric method for estimating the system by fitting a discrete-time stochastic space realization with the following underlying model structure:

\[
x_{k+1} = Ax_k + \nu x_{k+1},
\]

\[
y_k = Cx_k,
\]
where $A$ is the state matrix including all the system dynamics, $x_k$ is the state vector that holds the current state of the system at time instant $k$, and $v_k + 1$ is the Gaussian white noise process that is driving the system. $y_k$ is the output of the system and $C$ is the observation matrix. The procedure identifies the matrices $A$ and $C$ using the covariance matrices of the measured accelerations. A singular value decomposition of the matrix is performed, and a least square solution is sought. Once the state-space model of the structure is obtained, modal parameters are extracted from the identified matrices.

As can be seen from Table 3, it was only possible to identify the first mode with the ambient data and both methods yield similar values for the fundamental frequency. The decreasing tendency in the estimated values is also evident with the ambient data. The damping ratios, however, are much more scattered and do not display a clear trend. It can be noted that although the frequency estimations are not exactly identical for the impact and ambient tests, they can be considered to be close enough whereas the same conclusion cannot be drawn for the damping ratios. Although the operational modal analysis techniques provide accurate estimates of natural frequencies and mode shapes, they are known to produce poor structural damping estimates due to inherent random and/or bias. Since the measured data includes noise, the fit of the correlation function also includes noise, resulting in larger mean errors in the damping estimation [51]. It should also be noted that the damping ratios are identified by the logarithmic decrement method which uses the temporal information from a vibration response. This gives rise to the occurrence of measurement error since temporally measured data is easily contaminated by noise [52].

5. Localization of Damage Using the Damage Locating Vector (DLV) Approach

The DLV method is a flexibility-based damage localization procedure that hinges on extraction of a set of damage locating vectors, DLVs using the changes in the flexibility matrix. These vectors, theoretically, when treated as static loads on the structure at sensor locations produce zero stress fields in the damaged elements. A brief summary of the DLV method is provided in this section for completeness; however, for further details, the reader may refer to [53]. Assuming that a set of linearly independent load vectors, $L$, that produce identical deformations at the reference ($U$) and

Figure 14: Normalized PSDs measured at A3 and A5 locations with the progression of damage: (a) undamaged, (b) 0.5%, (c) 2%, and (d) 4% drift ratios.
### Table 2: Identified modal frequencies and damping ratios after impact tests.

| Drift ratio (%) | $f_1$ (Hz) | $\zeta_1$ (%) | $f_2$ (Hz) | $\zeta_2$ (%) |
|-----------------|------------|---------------|------------|---------------|
|                 | Test1      | Test2         | Test3      | Test1         | Test2         | Test3         | Test1         | Test2         | Test3         |
| 0               | 41.27      | 40.87         | 40.58      | 2.43          | 2.10          | 2.22          | 92.26         | 94.12         | 92.12         | 0.53          | 1.04          | 0.96          |
| 0.25            | 40.99      | 39.67         | 40.01      | 3.53          | 3.2           | 2.85          | 92.10         | 93.19         | 92.01         | 0.82          | 1.11          | 1.02          |
| 0.35            | 36.66      | 35.44         | 34.28      | 3.99          | 2.62          | 2.38          | 88.74         | 87.99         | 88.48         | 0.37          | 0.62          | 1.28          |
| 0.5             | 33.49      | 33.67         | 32.10      | 4.29          | 3.16          | 3.18          | 87.60         | 86.65         | 88.38         | 0.75          | 1.47          | 1.08          |
| 0.75            | 32.12      | 33.26         | 32.76      | 4.44          | 4.20          | 4.48          | 86.30         | 87.21         | 86.82         | 0.90          | 1.61          | 2.18          |
| 1               | 31.20      | 29.76         | 32.00      | 4.92          | 9.86          | 9.75          | 85.18         | 84.86         | 85.51         | 1.64          | 1.95          | 1.17          |
| 1.5             | 27.32      | 25.99         | 24.80      | 5.30          | 11.10         | 9.14          | 77.62         | 79.12         | 80.42         | 1.30          | 2.86          | 2.33          |
| 2               | 23.28      | 20.42         | 20.93      | 7.09          | 3.68          | 3.46          | 62.90         | 63.52         | 64.16         | 0.74          | 1.27          | 1.37          |
| 2.5             | 20.86      | 19.30         | 19.71      | 7.32          | 9.67          | 8.71          | 60.71         | 60.45         | 60.59         | 1.11          | 2.25          | 2.26          |
| 3.2             | 17.54      | 17.81         | 17.47      | 8.05          | 9.89          | 8.69          | 56.41         | 55.80         | 56.99         | 1.39          | 3.06          | 1.36          |
| 4               | 15.56      | 15.60         | 12.07      | 7.45          | 9.14          | 8.39          | 51.32         | 51.12         | 51.6          | 3.48          | 2.12          | 6.35          |
| 0.25            | 40.99      | 39.67         | 40.01      | 3.53          | 3.2           | 2.85          | 92.10         | 93.19         | 92.01         | 0.82          | 1.11          | 1.02          |
damaged ($D$) states of the structure exist and taking the difference between the two states, one can write

$$ (F_D - F_U)L = 0, \text{ or } DF.L = 0. \quad (5) $$

Except the trivial case of no damage ($DF = 0$), this equation can only be satisfied if the change in the flexibility matrix, $DF$, is rank deficient. Following a singular value decomposition of the change in the flexibility matrix, these vectors in $L$ can be computed. Since the DLVs induce no stress in the damaged elements, the damages in those elements do not affect the displacements at the sensor locations. Therefore, the DLVs are indeed the vectors in $L$.

Applying each DLV to the undamaged analytical model of the structure, the internal stresses in all members can be calculated and characterizing stress, $\sigma_i$, for each element based on the class type of the member can be obtained.

**Table 3:** Identified parameters for the first mode: SSI versus EFDD methods.

| Drift (%) | SSI $f_1$ (Hz) | SSI $\zeta_1$ (%) | SSI $f_2$ (Hz) | EFDD $\zeta_2$ (%) |
|-----------|----------------|-------------------|----------------|-------------------|
| 0         | 40.91          | 13.55             | 39.06          | 1.22              |
| 0.35      | 34.82          | 12.15             | 35.72          | 4.09              |
| 0.5       | 31.97          | 9.07              | 30.78          | 3.67              |
| 0.75      | 29.48          | 1.22              | 29.55          | 1.38              |
| 1         | 28.91          | 2.57              | 29.59          | 1.64              |
| 1.5       | 24.16          | 4.60              | 24.41          | 6.30              |
| 2         | 19.92          | 7.71              | 21.48          | 2.46              |
| 2.5       | 19.03          | 5.10              | 20.66          | 4.73              |
| 3.2       | 15.18          | 6.41              | 15.63          | 3.30              |
| 4         | 13.84          | 4.88              | 12.28          | 3.48              |
Elements having zero stresses can then be identified as potentially damaged. However, in practice when measured data is utilized, the stress may not be exactly zero due to noise and uncertainties. Furthermore in the case of multiple DLVs, the information gathered from each DLV may not be identical. A weighted stress index (WSI) that discriminates between “large” and “small” stresses and combines all the vectors is proposed in which the potentially damaged elements can be identified as those having WSI < 1:

$$\text{WSI} = \sum_{\text{dlv}} \frac{n_{\text{si}}}{n_{\text{dlv}}},$$

where \( n_{\text{dlv}} \) is the number of DLVs and \( n_{\text{si}} \) and \( n_{\text{svn}} \) can be computed as

$$n_{\text{si}} = \frac{\sigma_i}{[\sigma_i]_{\text{max}}},$$

$$n_{\text{svn}} = \frac{\sqrt{\sigma_i^2 - q_{i,j}^2}}{q_{i,j}},$$

in which \( \sigma_i \) are the singular values obtained from the singular value decomposition of \( DF \) and \( q_{i,j} \) is the maximum value of \( (r_i)^2 \) calculated for all the vectors in \( L \).

Using the identified modes with impact tests, flexibility matrices are synthesized at each stage of the frame and DLV approach is applied with the difference in the estimated flexibility matrices to localize damage. Bending moment values are selected as the characterizing stress \( \sigma \) that led to the WSI values shown in Table 4. Using the threshold of WSI less than 1.0 for potential damage location, the set of damaged members can be identified. Examination of the values in Table 4 suggests that after the first two pushover tests corresponding to drift ratios of 0.35% and 0.5%, no potential damage is identified. The localization approach appears to be unable to identify the damage in the beam-column connection which was observed during visual inspections at 0.5% drift level. Although at 1.0% drift level, none of the WSI values appear to be smaller than 1, joint 3 has the smallest WSI value (1.13) and misses the list of potentially damaged set by a small margin. The data corresponding to the frame after being pushed-over to 1.5% drift level suggests potential damage around joint 3. With further progression of damage to a 2% level, DLV approach suggests damage around the base joint of the frame. This classification for the base joint complies with the visual inspections; however, joint 3, for which potential damage was identified in the previous stage, does not make the WSI cut. However, it should be noted that it is still at the top of the list of members sorted based on the WSI values. Pushing the frame to a drift ratio of 2.5% changes the set of potentially damaged regions to those around joints 3, 7, 1, and 9. The same list of potentially damaged regions is obtained from the pushover test corresponding to the drift ratio of 3.2%. The damage localization approach employed in this study focuses on finding the differences between the undamaged and damaged structure, more specifically interrogating changes in synthesized flexibility matrices (DF). The basic idea is that the intersection of the null stress regions corresponding to the load distributions defined by the null space of DF can be used to localize damage. Depending on the number and location of the sensors, the intersection of the null stress regions may contain elements that are not damaged in addition to the damaged ones. Elements that are undamaged but which cannot be discriminated from the damaged ones by changes in flexibility appear in the list of potentially damaged members since they are inseparable for the given sensor set. In practice when measured data is utilized, the stress may not be exactly zero due to noise and uncertainties, and the approach recommends utilizing a threshold value of WSI < 1 for identifying potentially damaged elements. With the measured data, joint 2 with WSI = 0.94 and joint 8 with WSI = 1.14 can be considered in the list of potentially damaged locations with WSI values close to the recommended threshold. This may be due to the fact that they become inseparable for this specific arrangement of sensor measurements. It should be noted that these thresholds are recommended such that the DLV approach should not miss any damage location at the expense of including additional ones. For the final pushover test, no DLV is found since none of the \( svn \) values are smaller than 0.2.

## 6. Concluding Remarks

Comparison of the modal properties revealed variations with the progression of damage. With the progressive pushover test data, while sharp decreases in modal frequencies are identified, mode shape changes are observed to
remain small. Although damage leads to variations in the identified damping ratios, the changes do not display a general trend. This may be due to the fact that in concrete structures the cracks open and close, hence causing varying trends in the changes of the damping ratios. The DLV approach resulted in damage localizations, which complies with the visual inspections, and appears to be a viable candidate for prioritizing the physical domain of the structure for further damage investigation.

Data Availability

The data used to support the findings of this study are available from the corresponding author.

Conflicts of Interest

The authors declare no conflicts of interest.

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