Effective Range Corrections from Effective Field Theory with Dibaryon Fields and Perturbative Pions

Abstract We discuss a role of dibaryon fields in an effective field theory to study pion cloud effect around the unitary limit of two-nucleon systems at low energies.

1 Introduction

Nonperturbative renormalization for few-body systems is still an active issue in studies in effective field theories (EFTs). (See recent works, e.g., Refs. [1,2] and references therein.) A difficulty comes in when one solves the nonperturbative Lippmann–Schwinger (LS) equation with an \(NN\) potential constructed in an EFT: the potential can be singular, solving the LS equation with the potential generates another set of infinities, and thus those infinities need to be renormalized.

A nonperturbative renormalization procedure may be illustrated in a following way [3,4]: A calculated result \(a\) is renormalized by a physical quantity \(a_{\text{phy}}\), as

\[
a_{\text{phy}} = a(C_n(\Lambda), J_m(\Lambda)),
\]

where \(a_{\text{phy}}\) can be effective range parameters or phase shift data in the \(NN\) scattering, \(C_n\) are low energy constants (LECs) of the \(NN\) potential, and \(J_m\) are loop functions. The loop functions \(J_m\) become infinite in general, and thus one needs to choose a regularization scheme which introduces a renormalization scale \(\Lambda\) in both \(C_n(\Lambda)\) and \(J_m(\Lambda)\). Provided that a regularization method and a value of the scale \(\Lambda\) are specified, the LECs \(C_n(\Lambda)\) are fixed by using a set of relations in Eq. (1).

Because expressions of the renormalization relations in Eq. (1) are nonlinear in terms of \(C_n(\Lambda)\) and \(J_m(\Lambda)\) and quite complicated, it would be nontrivial that a perturbative series of the \(NN\) potential obeys naive counting rules under the constrains of Eq. (1); rather, it might be more likely that the counting rules or a validity of the theory significantly depend on detailed “specifications” which one employs in his or her calculation, such as which physical quantities and renormalization scheme are used for renormalization, the scale parameter \(\Lambda\) is kept at a finite value or sent to infinity, and so on.

Though the situation is complicated, to look at the problem from another point of view, we discuss an unconventional approach, introducing dibaryon fields [5,6]. Firstly, let us review some aspects of \(NN\) potentials with different values of pion mass.
2 Some Aspects of $NN$ Interactions with Various Values of Pion Mass

We briefly mention some aspects of $NN$ potentials with four values of the pion mass,

$$m_\pi = 0, \ 140, \ 198, \ 354 \text{ MeV}.$$  (2)

First two values above are well known; pion mass in chiral limit and physical mass, respectively (we do not discuss the nuclear forces with the physical pion mass here).

The $NN$ interactions in the chiral limit, $m_\pi^c = 0$, have been studied by many authors, but it may be worth mentioning one thing: a long-range part of the $NN$ interaction might be considered being a longer range in the chiral limit because of exchanging a massless particle, however, the interaction becomes rather a short range because of a (massless) pseudoscalar particle. One can easily see this, e.g., in one-pion-exchange potential at leading order in which momenta in two $\pi NN$ couplings are cancelled with momentum dependence of the potential pion propagator in the chiral limit, and thus the interaction becomes a point like, similar to that in pionless theory.

The third value of the pion mass, $m_\pi^{\text{crit.}} = 198$ MeV [7,8], in Eq. (2) is a critical pion mass at the unitary limit where all relevant scales vanish, i.e., scattering length of $S$-wave $NN$ scattering becomes infinite and deuteron binding energy vanishes. At this limit, three-body nucleon systems exhibit a universal feature, so-called Efimov effect, where infinitely many bound states with a series of geometric binding energies are accumulated at threshold.

The last value of the pion mass $m_\pi^{\text{latt.}} = 354$ MeV is the lightest pion mass in a lattice QCD simulation of $NN$ scattering [9]. With this lattice pion mass, very small values of the scattering lengths are reported; $a_0 = 0.63 \pm 0.50$ fm for $^1S_0$ channel and $a_1 = 0.63 \pm 0.74$ fm for $^3S_1$ channel. We note that physical values of them are $a_0 = -23.7$ fm and $a_1 = 5.42$ fm at $m_\pi^{\text{phys.}}$, significantly larger absolute values than those from the lattice simulation. Furthermore, they diverge in the unitary limit (at $m_\pi^{\text{crit.}}$).

3 Perturbative Pions Around the Unitary Limit

Now we discuss a perturbative expansion of pion cloud corrections around the unitary limit for few-body systems, instead of that around the chiral limit, and a role of the dibaryon fields.

The unitary limit is unique because an underlying theory of the few-body systems is totally masked by the universality, and thus one can never study QCD in few-nucleon systems in the limit. Dynamics of QCD in few-nucleon systems can be investigated in a deviation from it. In addition, Wigner's SU(4) symmetry can be realized in this limit [10]. Furthermore, the unitary limit for two-body systems is the critical point where the infinite scales (the infinite $S$-wave scattering lengths) appear. In the lattice studies of the $NN$ scattering, one should bring down his or her numerical results from the present large pion mass, $m_\pi^{\text{latt.}} = 354$ MeV, to the physical pion mass, $m_\pi^{\text{phys.}} = 140$ MeV, crossing over the critical point at the critical pion mass, $m_\pi^{\text{crit.}} = 198$ MeV, where the infinite scales might appear. Therefore, it may be more convenient in the few-body studies to choose the unitary limit than the chiral limit as a theoretical starting point.

To introduce the unitary limit into a theory treating pions perturbatively, we make use of the dibaryon fields (some details can be found in Ref. [11]). A naive picture of this choice for the $NN$ scattering is that in the conventional approach (its theoretical starting point is in the chiral limit) we have two nucleon cores, which has a baryon number, $B = 1$, and infinitely heavy mass, surrounded by the pion cloud and they interact with each other, whereas in the unconventional approach with the dibaryon fields (that is in the unitary limit), we have one core, which has the baryon number, $B = 2$, and nil-binding energy or infinite scattering length, surrounded by the pion cloud. It may be interesting to employ this picture in studying the few-body systems.

4 EFT with Dibaryons and Perturbative Pions

The EFT with dibaryon fields and perturbative pions, briefly discusses above, is applied to studying effective range corrections in the $S$-wave $NN$ scattering at low energies [11].

Though we employ the dibaryon fields to incorporate the picture discussed above, as discussed in the introduction, our result would depend on details of the calculation. Specifications of our calculation are in the following: we calculate $NN$ scattering amplitudes for $^1S_0$ and $^3S_1$ channels, treat the pions perturbatively and consider only leading one-pion-exchange diagrams around the leading order amplitude generated by the
Effective range parameters of $S$ wave $NN$ scattering are given as

$$ p \cot \delta_0 = -\frac{1}{a} + \frac{1}{2} r p^2 + v_2 p^4 + v_3 p^6 + v_4 p^8 + \cdots , $$

where $p$ is the relative momentum of two nucleons, $\delta_0$ are $S$-wave phase shifts, and $v_2$, $v_3$, and $v_4$ are effective range parameters in higher orders. After the renormalization of $a$ and $r$, we obtain expressions of the effective range parameters $v_2$, $v_3$, and $v_4$ as

$$ v_2 = \frac{g^2 m_N}{16 \pi f^2} \left\{ -\frac{16}{3} a_d^2(\mu) m_\pi^4 + \frac{32}{5} a_d(\mu) m_\pi^3 - \frac{2}{m_\pi^2} \left[ 1 + \frac{r_d(\mu)}{a_d(\mu)} \right] + \frac{4 r_d(\mu)}{3 m_\pi} \right\}, $$

$$ v_3 = \frac{g^2 m_N}{16 \pi f^2} \left\{ -\frac{16}{5} a_d^2(\mu) m_\pi^6 - 128 a_d(\mu) m_\pi^5 + \frac{16}{3} \frac{1}{m_\pi^2} \left[ 1 + \frac{r_d(\mu)}{a_d(\mu)} \right] \right\}, $$

$$ v_4 = \frac{g^2 m_N}{16 \pi f^2} \left\{ -\frac{256}{5} a_d^2(\mu) m_\pi^8 + \frac{512}{9} a_d(\mu) m_\pi^7 - \frac{16}{m_\pi^2} \left[ 1 + \frac{r_d(\mu)}{a_d(\mu)} \right] \right\}, $$

where $a_d(\mu)$ and $r_d(\mu)$ are bare scattering length and effective range generated from the dibaryon fields, and $\mu$ is a scale parameter from the DR and PDS scheme. We obtain new corrections to $v_2$, $v_3$, $v_4$ from the effective range $r$ ($r_d(\mu)$ in the above expressions) compared to those previously obtained by Cohen and Hansen [12,13].

We note that though the bare parameters are usually replaced by the physical ones, we retain the $\mu$ dependence in them because we renormalize the effective range parameters $a$ and $r$ and regard the higher order ones $v_2$, $v_3$, and $v_4$ as contributions from higher energies depending on the scale parameter $\mu$. We find a large sensitivity of $v_2$, $v_3$, and $v_4$ to the $\mu$ value. Subsequently, we fit $v_2$ to a $v_2$ value obtained from partial wave analysis (PWA) by adjusting the value of $\mu$ where we regard the scale $\mu$ as a fine-tuning parameter of the high energy contributions and have $\mu = 178$ and $330$ MeV for the $^1S_0$ channel and $\mu = 167$ and $246$ MeV for the $^3S_1$ channel. We find a fairly good agreement of the effective range parameters $v_3$ and $v_4$ to the values of PWA with those $\mu$ values.

In our results of the phase shifts $\delta_0$, if we require a $\mu$-independence to the results of $\delta_0$, they agree to those of the accurate potential model calculation up to $p \sim 50$ MeV mainly because of our choice of renormalization method using two effective range parameters $a$ and $r$. If we use the adjusted $\mu$-values mentioned above, the situation is improved and the $S$-wave phase shifts show a better agreement with those from the accurate potential model up to $p \sim m_\pi$. More details can be found in Ref. [11].

Acknowledgments The author would like to thank Chang Ho Hyun for collaboration. This work is supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science, and Technology (2012R1A1A2009430).

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