Isospin Breaking in $B \to K^*\gamma$ Decays

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Abstract

A calculation of the leading isospin-breaking contributions to the $B \to K^*\gamma$ decay amplitudes based on the QCD factorization approach is presented. They arise at order $\Lambda/m_b$ in the heavy-quark expansion and are due to annihilation contributions from 4-quark operators, the chromo-magnetic dipole operator, and charm penguins. In the Standard Model the decay rate for $\bar{B}^0 \to \bar{K}^{*0}\gamma$ is predicted to be about 10–20% larger than that for $B^- \to K^{*-}\gamma$. Isospin-breaking effects are a sensitive probe of the penguin sector of the effective weak Hamiltonian. New Physics models in which the hierarchy of $B \to K^*\gamma$ decay rates is either flipped or greatly enhanced could be ruled out with more precise data.
1 Introduction

The study of radiative decays based on the flavor-changing neutral current transition $b \to s\gamma$ is of crucial importance for testing the flavor sector of the Standard Model and probing for New Physics. Whereas the inclusive mode $B \to X_s\gamma$ can be analyzed using the operator product expansion, it is usually argued that exclusive decays such as $B \to K^*\gamma$ and $B \to \rho\gamma$ do not admit a clean theoretical analysis because of their sensitivity to hadronic physics. However, it has recently been shown that in the heavy-quark limit the decay amplitudes for these processes can be calculated in a model-independent way using a QCD factorization approach [1, 2, 3], which is similar to the scheme developed earlier for non-leptonic two-body decays of $B$ mesons [4].

To leading order in $\Lambda/m_b$ (and neglecting isospin violation in the $B \to K^*$ form factors) one finds that the amplitudes for the decays $\bar{B}^0 \to \bar{K}^{*0}\gamma$ and $B^- \to K^{*-}\gamma$ coincide. Spectator-dependent effects enter at subleading order in the heavy-quark expansion. In this Letter the QCD factorization approach is used to estimate the leading isospin-breaking effects for the $B \to K^*\gamma$ decay amplitudes, the most important of which can be calculated in a model-independent way.

Experimental measurements of exclusive $B \to K^*\gamma$ branching ratios have been reported by the CLEO, Belle and BaBar Collaborations, with the results (averaged over CP-conjugate modes):

$$10^5 \text{Br}(\bar{B}^0 \to \bar{K}^{*0}\gamma) = \begin{cases} 4.55^{+0.72}_{-0.68} \pm 0.34 & [5] \\ 4.96 \pm 0.67 \pm 0.45 & [3] \\ 4.23 \pm 0.40 \pm 0.22 & [4] \end{cases}$$

$$10^5 \text{Br}(B^- \to K^{*-}\gamma) = \begin{cases} 3.76^{+0.89}_{-0.83} \pm 0.28 & [5] \\ 3.89 \pm 0.93 \pm 0.41 & [6] \\ 3.83 \pm 0.62 \pm 0.22 & [7] \end{cases}$$

The average branching ratios for the two modes are $\text{Br}(\bar{B}^0 \to \bar{K}^{*0}\gamma) = (4.44 \pm 0.35) \cdot 10^{-5}$ and $\text{Br}(B^- \to K^{*-}\gamma) = (3.82 \pm 0.47) \cdot 10^{-5}$. When corrected for the difference in the $B$-meson lifetimes, $\tau_{B^-}/\tau_{B^0} = 1.068 \pm 0.016$ [8], these results imply

$$\Delta_{0^-} \equiv \frac{\Gamma(\bar{B}^0 \to \bar{K}^{*0}\gamma) - \Gamma(B^- \to K^{*-}\gamma)}{\Gamma(\bar{B}^0 \to \bar{K}^{*0}\gamma) + \Gamma(B^- \to K^{*-}\gamma)} = 0.11 \pm 0.07.$$ 

Although there is no significant deviation of this quantity from zero, the fact that all three experiments see a tendency for a larger neutral decay rate raises the question whether the Standard Model could account for isospin-breaking effects of order 10% in the decay amplitudes.
2 Isospin-Breaking Contributions

In the Standard Model the effective weak Hamiltonian for $b \to s\gamma$ transitions is

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(s)} \left( C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\ldots,8} C_i Q_i \right),$$

(3)

where $\lambda_p^{(s)} = V_{ps}^* V_{pb}$ are products of CKM matrix elements, $Q_{1,2}^p$ are the current–current operators arising from $W$ exchange, $Q_{3,\ldots,6}$ are local 4-quark penguin operators, and $Q_7$ and $Q_8$ are the electro-magnetic and chromo-magnetic dipole operators. (We adopt the conventions of [4]; in particular, $C_1 \approx 1$ is the largest coefficient.) The Wilson coefficients $C_i$ and the matrix elements of the renormalized operators $Q_i$ depend on the renormalization scale $\mu$.

At leading power in $\Lambda/m_b$, and neglecting the tiny contribution proportional to $\lambda_u$, the $B \to K^*\gamma$ decay amplitude is given by

$$iA_{\text{lead}} = \frac{G_F}{\sqrt{2}} \lambda_c^{(s)} a_7 \langle \bar{K}^*(k,\eta)\gamma(q,\epsilon) | Q_7 | \bar{B} \rangle,$$

(4)

where the next-to-leading order (NLO) result for the coefficient $a_7^c = C_7 + \ldots$ is given in Eq. (42) of [1]. At this order contributions to the quantity $\Delta_0^-$ arise only from isospin violation in the $B \to K^*$ form factors. (The phase-space difference between the two decays is an effect of order $(\Lambda/m_B)^2$ and contributes the negligible amount $-4 \times 10^{-4}$ to $\Delta_0^-$. To first order we find

$$\Delta_0^\text{soft} = 1 - \frac{T_{B \to K^*}^0}{T_{B \to K^*}} \approx \frac{m_d - m_u}{m_s - m_d} \left( 1 - \frac{T_{B \to \rho^0}}{T_{B \to \rho^0}} \right) \approx 0.5\%,$$

(5)

where $T_{B \to K^*}$ is a form factor in the decomposition of the $B \to K^*$ matrix element of the tensor current $\bar{s}d_{\mu\nu}(1+\gamma_5)b$ evaluated at zero momentum transfer. We have used the form-factor predictions of [9] and assumed that the ratio of isospin to $U$-spin violation for the transition form factors scales approximately as the corresponding ratio of light quark masses. Although this estimate is rather uncertain, we believe it indicates that the “soft” isospin-breaking effect is negligible and could not account for a value of $\Delta_0^-$ as large as 10%.

At subleading order in $\Lambda/m_b$ “hard” isospin-violating effects appear in the form of spectator-dependent interactions. The leading contributions arise from the diagrams shown in Figure [1]. For a first estimate of these effects we adopt a simplification of the usual NLO counting scheme, in which we neglect terms of order $\alpha_s C_{3,\ldots,6}$ while retaining terms of order $\alpha_s C_{1,8}$. This is justified, because the penguin coefficients $C_{3,\ldots,6}$ are numerically very small. Also, it is a safe approximation to neglect terms of order $\alpha_s \lambda_u^{(s)}/\lambda_c^{(s)}$ given that $|$|$\lambda_u^{(s)}/\lambda_c^{(s)} |$ $\approx 0.01–0.02$ is very small. It then suffices to evaluate the contributions of the 4-quark operators shown in the first diagram at tree level. The terms neglected in this simplified NLO approximation will be estimated later.
Figure 1: Spectator-dependent contributions from local 4-quark operators (left), the chromo-magnetic dipole operator (center), and the charm penguin (right). Crosses denote alternative photon attachments.

The isospin-breaking contributions to the decay amplitudes can be parameterized as $A_q = b_q A_{\text{lead}}$, where $q$ is the flavor of the spectator antiquark in the $\bar{B}$ meson. To leading order in small quantities $\Delta_0$ is then given by

$$\Delta_{0-} = \text{Re}(b_d - b_u).$$

The QCD factorization approach gives an expression for the coefficients $b_q$ in terms of convolutions of hard-scattering kernels with light-cone distribution amplitudes for the $K^*$ and $B$ mesons. When light quark masses are neglected, the leading and subleading projections for a transversely polarized vector meson with momentum $k$ and polarization $\eta$ are [10, 11]

$$\langle \bar{K}^*(k, \eta) | \bar{s}(-z) \cdots q(z) | 0 \rangle = \frac{f_{K^*}}{4} (\eta^* \bar{k} \beta) \int_0^1 dx e^{i z k \cdot x} \phi_\perp(x)$$

$$+ \epsilon_{\mu \nu \rho \sigma} \eta^* \nu k^\rho z^\sigma (\gamma^\mu \gamma_5) \beta \int_0^1 dx e^{i z k \cdot x} g_\perp^{(v)}(x)$$

$$+ \eta^* \frac{z}{k \cdot z} (\bar{k}) \beta \int_0^1 dx e^{i z k \cdot x} \left[ \phi_\parallel(x) - g_\perp^{(v)}(x) \right],$$

where $z^2 = 0$, and the ellipses on the left-hand side indicate a string operator required to make the non-local matrix element gauge invariant. The variable $x$ is the longitudinal momentum fraction of the strange quark, and $\zeta = (1 - x) - x$. All four distribution functions are normalized to 1. The asymptotic form of the leading-twist amplitudes $\phi_\parallel, \phi_\perp(x)$ is $6x\bar{x}$ with $\bar{x} = (1 - x)$. In the approximation where 3-particle distribution amplitudes of the kaon are neglected, the functions $\phi_\parallel$ and $g_\perp^{(v,a)}$ are related to each other by equations of motion [10, 11]. Using these relations we find that (the prime denotes a derivative with respect to $x$)

$$\frac{g_\perp^{(a)}(x)}{4\bar{x}} + \frac{1}{\bar{x}} \int_0^x dy \left[ \phi_\parallel(y) - g_\perp^{(v)}(y) \right] = g_\perp^{(v)}(x) - \frac{g_\perp^{(a)}(x)}{4},$$

(8)
which can be used to eliminate $\phi_\parallel$ from our results. This means that we neglect contributions to the quantity $K_2$ arising from $(g\bar{q}s)$ Fock states of the kaon, which are typically found to be suppressed with respect to two-particle contributions of the same twist. This approximation is justified, because numerically the effect of $K_2$ is about four times smaller than that of $K_1$. Including the 3-particle Fock states would, however, not invalidate factorization.

The leading-twist projection onto the $B$ meson involves two distribution amplitudes $\Phi_{B1}(\xi)$ and $\Phi_{B2}(\xi)$, where $\xi = O(\Lambda/m_b)$ is the light-cone momentum fraction of the spectator quark projected onto the direction of the kaon. The first inverse moment of the function $\Phi_{B1}(\xi)$ defines a hadronic parameter $\lambda_B$ via

$$\int_0^1 d\xi \frac{\Phi_{B1}(\xi)}{\xi} = \frac{m_B}{\lambda_B}.$$  \hspace{1cm} (9)

The function $\Phi_{B2}(\xi)$ does not enter our results. Note that 3-particle Fock states of the $B$ meson only contribute to the quantity $K_1$ at order $\alpha_s(m_b) C_{5,6}(m_b)$ and thus can be neglected in our approximation scheme.

The leading spectator-dependent contributions summarized by the coefficients $b_q$ can be written as

$$b_q = \frac{12\pi^2 f_B Q_q}{m_b T^{B\to K^*} a_7^2} \left( \frac{f_{K^*}^2}{m_b} K_1 + \frac{f_{K^*} m_{K^*}}{6\lambda_B m_B} K_2 \right),$$  \hspace{1cm} (10)

where $m_b$ denotes the running $b$-quark mass. The product $m_b T^{B\to K^*} a_7^2$ is scale independent at NLO. The heavy-quark scaling laws $f_B \sim m_b^{-1/2}$ and $T_1^{B\to K^*} \sim m_b^{-3/2}$ imply that $b_q$ scales like $\Lambda/m_b$, and hence the spectator-dependent corrections contribute at subleading order in the heavy-quark expansion. However, because of the large numerical factor $12\pi^2$ in the numerator the values of $b_q$ will turn out to be larger than anticipated in \cite{1, 2}.

The dimensionless coefficients $K_i$ are given by ($N = 3$ and $C_F = 4/3$ are color factors)

$$K_1 = - \left( C_6 + \frac{C_5}{N} \right) F_\perp$$
$$+ \frac{C_F}{N} \frac{\alpha_s}{4\pi} \left\{ \left( \frac{m_b}{m_B} \right)^2 C_{8, X_\perp} - C_1 \left[ \left( \frac{4}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} \right) F_\perp - G_\perp(s_c) \right] + r_1 \right\},$$  \hspace{1cm} (11)

$$K_2 = \frac{\lambda_{(s)}^{(s)}}{\lambda_{(c)}^{(s)}} \left( C_1 + C_2 \right) \delta_{qu} + \left( C_4 + \frac{C_3}{N} \right) + \frac{C_F}{N} \frac{\alpha_s}{4\pi} \left\{ C_1 \left( \frac{4}{3} \ln \frac{m_b}{\mu} + \frac{2}{3} \right) - H_\perp(s_c) \right] + r_2 \right\},$$

where $r_1$ and $r_2$ are the residual NLO corrections neglected in our approximation scheme. The quantities

$$F_\perp = \int_0^1 dx \frac{\phi_\perp(x)}{3\bar{x}},$$

$$G_\perp(s_c) = \int_0^1 dx \frac{\phi_\perp(x)}{3\bar{x}} G(s_c, \bar{x}),$$
\[ H_\perp(s_c) = \int_0^1 dx \left( g_\perp^{(v)}(x) - g_\perp^{(a)}(x) \right) G(s_c, \bar{x}), \]

\[ X_\perp = \int_0^1 dx \, \phi_\perp(x) \frac{1 + \bar{x}}{3\bar{x}^2} \]

with \( s_c = (m_c/m_b)^2 \) are convolution integrals of hard-scattering kernels with the meson distribution amplitudes, and

\[ G(s, \bar{x}) = -4 \int_0^1 du \, u \bar{u} \ln(s - u\bar{u}\bar{x} - i\epsilon) \]

is the penguin function. The terms in \( K_2 \) arising from the current–current operators \( Q_{1,2}^u \) only contribute for \( q = u \), as indicated by the symbol \( \delta_{q_u} \). Since no distinction between CP-conjugate modes is made in the experimental determination of \( \Delta_0 \) we only need the real part of the ratio of CKM parameters, given in terms of Wolfenstein parameters as \( \text{Re}(\lambda_u^{(s)}/\lambda_c^{(s)}) = \lambda^2 \bar{\rho} = 0.011 \pm 0.005 \).

The first three convolution integrals above exist for any reasonable choice of the distribution amplitudes. This shows that, to the order we are working, the QCD factorization approach holds at subleading power for the matrix elements of the 4-quark and current–current operators (including penguin contractions). Factorization of these matrix elements is indeed expected to hold to all orders in perturbation theory as a consequence of color transparency \([1]\). This implies, in particular, that long-distance contributions to the first diagram in Figure 1, which have been analysed using QCD sum rules in \([14, 15]\), must be suppressed by at least two powers of \( \Lambda/m_b \) and so do not contribute to the order we are working. On the other hand, if the function \( \phi_\perp(x) \) vanishes only linearly at the endpoints (as indicated by its asymptotic behavior), then the convolution integral \( X_\perp \) suffers from a logarithmic endpoint singularity as \( x \to 1 \), corresponding to the region where the light spectator in the \( K^* \) meson is a soft quark. This indicates that at subleading power factorization breaks down for the matrix element of the chromo-magnetic dipole operator \( Q_8 \) (second diagram in Figure 1). In the phenomenological analysis below we regulate this singularity by introducing a cutoff such that \( x < 1 - \Lambda_h/m_B \), where \( \Lambda_h = 0.5 \text{ GeV} \) is a typical hadronic scale. Since \( X_\perp \) is dominated by soft physics, we assign a large uncertainty to this estimate.

The Wilson coefficients \( C_3,...,6 \) entering the expressions for \( K_1 \) and \( K_2 \) must be evaluated at NLO, while the remaining coefficients can be taken at leading order. Throughout, the two-loop expression for the running coupling \( \alpha_s(\mu) \) is used. The explicitly scale-dependent terms in the expressions for the coefficients \( K_i \) arise from the charm-penguin diagrams. They are necessary to cancel the scale and scheme dependence of the Wilson coefficients \( C_3,...,6 \). In a complete NLO calculation there would be additional logarithmic terms proportional to \( \alpha_s C_3,...,6 \), which can be deduced using the renormalization group (RG). This gives for the remainders

\[ r_1 = \left[ \frac{8}{3} C_3 + \frac{4}{3} n_f (C_4 + C_6) - 8(N C_6 + C_5) \right] F_\perp \ln \frac{\mu}{\mu_0} + \ldots, \]

\[ r_2 = \left[ -\frac{44}{3} C_3 - \frac{4}{3} n_f (C_4 + C_6) \right] \ln \frac{\mu}{\mu_0} + \ldots, \]

(14)
where $n_f = 5$, and $\mu_0 = O(m_b)$ is an arbitrary normalization point. The sensitivity to $\mu_0$ provides an estimate of the residual non-logarithmic NLO terms denoted by the ellipses, whose calculation is left for future work. After the addition of the $r_i$ pieces the quantity $K_2$ is RG invariant at NLO. In the case of $K_1$ a scale dependence remains, which cancels against the scale dependence of the tensor decay constant, $f_{K^*}^\perp(\mu) \sim [\alpha_s(\mu)]^{C_F/\beta_0}$, and of the running $b$-quark mass.

An important element of the phenomenological analysis are the convolution integrals. We adopt the shapes of the light-cone distribution amplitudes obtained (at second order in the Gegenbauer expansion) using QCD sum rules [10] and vary the amplitude parameters within their respective error ranges. This leads to $F_\perp = 1.21 \pm 0.06$, $G_\perp(s_c) = (2.82 \pm 0.20) + (0.81 \pm 0.23)i$ and $H_\perp(s_c) = (2.32 \pm 0.16) + (0.50 \pm 0.18)i$, where $m_c/m_b = 0.26 \pm 0.03$ has been used for the ratio of quark masses. Next, we find $X_\perp = (3.44 \pm 0.47)X - (3.91 \pm 1.08)$, where $X = \ln(m_B/\Lambda_b) (1 + \varrho e^{i\varphi})$ parameterizes the logarithmically divergent integral $\int dx/\bar{x}$. Following [4] we allow $\varrho \leq 1$ and an arbitrary strong-interaction phase $\varphi$ to account for the theoretical uncertainty due to soft rescattering in higher orders. The above results for the convolution integrals refer to a renormalization point of $\sqrt{5}\text{GeV}$. (The expressions for $r_i$ given earlier are valid if the convolution integrals are normalized at a fixed scale.)

Further input parameters are $m_b(m_b) = 4.2\text{GeV}$, the decay constants $f_B = (200 \pm 20)\text{MeV}$ [14, 17], $f_{K^*} = (226 \pm 28)\text{MeV}$ and $f_{K^*}^\perp = (175 \pm 9)\text{MeV}$ (at $\mu = \sqrt{5}\text{GeV}$) [10], and the parameter $\lambda_B = (350 \pm 150)\text{MeV}$ defined in terms of the first inverse moment of the $B$-meson distribution amplitude $\Phi_{B1}$ [4]. A dominant uncertainty in the prediction for $\Delta_0^-$ comes from the tensor form factor $T_1^{B\to K^*}$, recent estimates of which range from $0.32^{+0.04}_{-0.02}$ [13] to $0.38 \pm 0.06$ [4]. On the other hand, a fit to the $B \to K^*\gamma$ branching fractions yields the lower value $0.27 \pm 0.04$ [4]. To good approximation the result for $\Delta_0^-$ is inversely proportional to the value of the form factor. Below we take $T_1^{B\to K^*} = 0.3$ (at $\mu = m_b$) as a reference value.

From the diagrams in Figure [1] it is seen that in all cases the operators are probed at momentum scales of order $\mu \sim m_b$. Hence, following common practice we vary the renormalization scale between $m_b/2$ and $2m_b$. The result for $\Delta_0^-$ is shown in Figure [2]. The width of the band reflects the sensitivity to input parameter variations. The three curves correspond to different choices of the scale $\mu_0$ in the expressions (14) for the remainders $r_i$. The excellent stability under variation of both $\mu$ and $\mu_0$ shows that our approximation scheme captures the dominant terms at NLO.

Combining all sources of uncertainty we obtain

$$\Delta_0^- = (8.0^{+2.1}_{-3.2})\% \times \frac{0.3}{T_1^{B\to K^*}}. \quad (15)$$

The three largest contributions to the error from input parameter variations are due to $\lambda_B$ ($^{+1.0}_{-2.5}\%$), the divergent integral $X_\perp$ ($\pm 1.2\%$), and the decay constant $f_B$ ($\pm 0.8\%$). The perturbative uncertainty is about $\pm 1\%$. Our result is in good agreement with the current central experimental value of $\Delta_0^-$ including its sign, which is predicted unambiguously. By far the most important source of isospin breaking is due to the 4-quark
Figure 2: Prediction for the quantity $\Delta_0^-$ as a function of the renormalization scale, assuming $T_1^{B\to K^*} = 0.3$. The dark lines refer to $\mu_0 = m_b$ (solid), $m_b/2$ (upper dashed) and $2m_b$ (lower dashed). The band shows the theoretical uncertainty.

penguin operator $Q_6$, whose contribution to $\Delta_0^-$ is about 9% (at $\mu = \mu_0 = m_b$). The other terms are much smaller. In particular, the contribution of the chromo-magnetic dipole operator, for which factorization does not hold, is less than 1% in magnitude and therefore numerically insignificant. Hence, the most important isospin-breaking contributions can be calculated using QCD factorization. It follows from our result that these effects mainly test the magnitude and sign of the ratio $C_6/C_7$ of penguin coefficients.

3 New Physics

Because of their relation to matrix elements of penguin operators, isospin-breaking effects in $B \to K^*\gamma$ decays are sensitive probes of physics beyond the Standard Model. For example, scenarios in which the sign of $\Delta_0^-$ is flipped could be ruled out with more precise data. In addition, in certain extensions of the Standard Model there exist local 4-quark operators yielding an isospin-breaking contribution to the decay amplitudes at leading power in the heavy-quark expansion. Precise measurements of radiative decay rates would tightly constrain the corresponding Wilson coefficients. Here we confine ourselves to a brief illustration of the most interesting potential New Physics effects in $B \to K^*\gamma$ decays. A more detailed study will be presented elsewhere.

New Physics effects arising at some high energy scale manifest themselves at low energy through new contributions to the effective weak Hamiltonian. As a first, popular example in which the operator basis is not enlarged, consider the minimal supersymmetric Standard Model (MSSM) with minimal flavor violation, and with contributions to the $B \to X_s\gamma$ decay rates that are enhanced in the large-$\tan \beta$ limit taken into account.
where $O$ taken as (a factor of $\text{Model}$, and a plethora of local 4-quark operators, the most general set of which can be any of the ten combinations $\left( V_1, V_2, V_3, V_4, V_5, V_6, V_7, V_8, V_9, V_{10} \right)$, additional contributions to the coefficients of the dipole operators $Q$ graphs.

Beyond leading order [19, 20, 21]. In this scenario new contributions to $Q_{3,\ldots,6}$ and $Q_8$ are too small to have a significant effect. For low $\tan \beta$, $\text{Re}(C_7)$ is negative as in the Standard Model. However, for large $\tan \beta$ the coefficient $\text{Re}(C_7)$ can take on both positive or negative values. Positive values, which would flip the sign of $\Delta_{\text{pole}}$, become more probable as $\tan \beta$ increases (see, e.g., [22]). With more precise data, isospin breaking in $B \to K^*\gamma$ decays could rule out significant regions of the MSSM parameter space at large $\tan \beta$. We note that the sign of $\text{Re}(C_7)$ can also be flipped in supersymmetric models with non-minimal flavor violation, independently of $\tan \beta$, via gluino–down-squark loop graphs.

In a more general scenario, New Physics effects can be parameterized in terms of additional contributions to the coefficients of the dipole operators $Q_7$ and $Q_8$, contributions involving dipole operators $\hat{Q}_7$ and $\hat{Q}_8$ with opposite chirality than in the Standard Model, and a plethora of local 4-quark operators, the most general set of which can be taken as (a factor of $\frac{G_F}{\sqrt{2}} \lambda^{(s)}$ is included here for convenience only)

$$\mathcal{H}^{4\text{-quark}}_{\text{eff,NP}} = \frac{G_F}{\sqrt{2}} \lambda^{(s)} \sum_{q=u,d,\ldots} \sum_{\Gamma=\Gamma_1 \otimes \Gamma_2} \left( c^{q}_{\Gamma} O^q_{\Gamma} + \tilde{c}^{q}_{\Gamma} \tilde{O}^q_{\Gamma} \right),$$

where $O^q_{\Gamma} = s \Gamma_1 b \bar{q} \Gamma_2 q$ and $\tilde{O}^q_{\Gamma} = \bar{s} \Gamma_1 \bar{b} \bar{q} \Gamma_2 q_i$. The Dirac structure $\Gamma = \Gamma_1 \otimes \Gamma_2$ can be any of the ten combinations $(V \otimes A) \otimes (V \otimes A)$, $(V \otimes A) \otimes (V \otimes \pm A)$, $(S \otimes \pm P) \otimes (S \otimes \pm P)$, $(S \otimes P) \otimes (S \otimes \pm P)$, and $T_{L,R} \otimes T_{L,R}$. Here, as usual, $S = 1$, $P = \gamma_5$, $V = \gamma_{\mu}$, $A = \gamma_\mu \gamma_5$, and $T_{L,R} = \sigma_{\mu\nu}(1 \pm \gamma_5)$. As an example, note that such an extensive list of 4-quark operators can arise in general supersymmetry models [23, 24].

In a generic New Physics model the Wilson coefficients $c^q_{\Gamma}$ and $\tilde{c}^q_{\Gamma}$ need not be flavor independent [23]. Therefore, isospin-breaking effects can arise even if the photon in the first diagram in Figure 1 is emitted from the $b$ or $s$-quark lines. Taking this possibility into account, we find that at leading order the contributions to the coefficients $K_i$ are

$$K^{{\text{NP}}}_1 = \frac{m_B}{6\lambda_B} \left( \tilde{c}^{q}_{\Gamma R \otimes TR} - \frac{1}{2} \tilde{c}^{q}_{(S+P)\otimes (S+P)} \right) - \left( F_{\perp} + \frac{Q_4}{Q_4} F_{\perp} \right) \tilde{c}^{q}_{(V-A)\otimes (V+A)}$$

$$+ \frac{C_F \alpha_s}{N} \frac{X_{\perp}}{4\pi} c^8_{\text{NP}} ,$$

$$K^{{\text{NP}}}_2 = \tilde{c}^{q}_{(V-A)\otimes (V-A)} - \frac{1}{2} \tilde{c}^{q}_{(S-P)\otimes (S+P)} ,$$

where $c^q_{\Gamma} \equiv \tilde{c}^q_{\Gamma} + c^q_{\Gamma}/N$, and we have introduced the new convolution integral

$$\widetilde{F}_{\perp} = \int_0^1 dx \frac{\phi_\perp(x)}{3x} = 0.84 \pm 0.06 .$$

The New Physics contribution from the chromo-magnetic dipole operator is included in (17) despite its $O(\alpha_s)$ suppression, because it could potentially be large in models with strongly enhanced $C_8$. In analogy with (10), we define a quantity $\tilde{b}^{{\text{NP}}}_q$ with corresponding coefficients $\tilde{K}^{{\text{NP}}}_1$ given by equivalent expressions with all Wilson coefficients replaced by
their opposite-chirality counterparts. Its contribution to the decay amplitude is \( \hat{A}_q = \hat{b}_q^{NP} \hat{A}_{\text{lead}} \), where \( \hat{A}_{\text{lead}} \) is defined in analogy with (4) in terms of the matrix element of the opposite-chirality operator \( \hat{Q}_7 \). Because this amplitude does not interfere with the leading Standard Model amplitude for \( B \to K^* \gamma \) its effect is likely to be suppressed.

A remarkable fact is that there can exist local 4-quark operators yielding a leading (i.e., not power-suppressed) contribution to the decay amplitudes, as indicated by the factor \( \sim m_B/\lambda_B \) in the first term in \( K_1^{NP} \) and \( \hat{K}_1^{NP} \), which compensates the power suppression from the prefactor \( f_{K^*}/m_b \) in (10). The origin of these terms can readily be understood by noting that the tensor operators \( \bar{s}\sigma_{\mu\nu}(1 \pm \gamma_5)q \bar{q}\sigma^{\mu\nu}(1 \pm \gamma_5)b \) have a leading-twist projection onto the \( K^* \) meson. When Fierz-transformed into the basis used above, such operators turn into tensor operators and operators with structure \( (S \pm P) \otimes (S \pm P) \). The fact that such operators enter the amplitudes for exclusive radiative \( B \) decays with large coefficients means that future, precise measurements of radiative decay rates will provide tight constraints on the corresponding Wilson coefficients.

4 Summary

We have presented a model-independent analysis of the leading isospin-breaking contributions to the \( B \to K^* \gamma \) decay amplitudes. In the Standard Model these effects appear first at order \( \Lambda/m_b \) in the heavy-quark expansion. They can be expressed in terms of convolutions of hard-scattering kernels and meson light-cone distribution amplitudes of leading and subleading twist. We have evaluated these contributions at NLO in perturbation theory, neglecting however some numerically suppressed \( O(\alpha_s) \) terms. With the exception of the matrix element of the chromo-magnetic dipole operator, whose contribution is numerically small, factorization of the leading isospin-breaking contributions is expected to hold to all orders in perturbation theory.

To our knowledge, the analysis presented here provides the first example of a quantitative test of QCD factorization at the level of power corrections. As such, it lends credibility to the idea of factorization as a leading term in a well-behaved expansion in inverse powers of the \( b \)-quark mass. Our prediction for the magnitude and sign of isospin-breaking is in good agreement with the present central experimental value of this effect. If this agreement persists as the data become more precise, it will be possible to place novel constraints on flavor physics beyond the Standard Model.

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