Gauge Boson Masses in the 3-d, SU(2) Gauge-Higgs Model

F. Karsch\textsuperscript{1}, T. Neuhaus\textsuperscript{2}, A. Patkós\textsuperscript{3} and J. Rank\textsuperscript{1}

\textsuperscript{1} Fakultät für Physik, Universität Bielefeld, P.O. Box 100131, D-33501 Bielefeld, Germany
\textsuperscript{2} Fachbereich Physik, Gesamthochschule Universität Wuppertal, D-42097 Wuppertal, Germany
\textsuperscript{3} Institute of Physics, Eötvös University, Budapest, Hungary

Abstract

We study gauge boson propagators in the symmetric and symmetry broken phases of the 3-d, SU(2) gauge-Higgs model. Correlation functions for the gauge fields are calculated in Landau gauge. They are found to decay exponentially at large distances leading to a non-vanishing mass for the gauge bosons. We find that the W-boson screening mass drops in the symmetry broken phase when approaching the critical temperature. In the symmetric phase the screening mass stays small and is independent of the scalar–gauge coupling (the hopping parameter). Numerical results coincide with corresponding calculations performed for the pure gauge theory. We find $m_w = 0.35(1)g^2T$ in this phase which is consistent with analytic calculations based on gap equations. This is, however, significantly smaller than masses extracted from gauge invariant vector boson correlation functions. As internal consistency check we also have calculated correlation functions for gauge invariant operators leading to scalar and vector boson masses. Finite lattice size effects have been systematically analyzed on lattices of size $L^2 \times L_z$ with $L = 4 \rightarrow 24$ and $L_z = 16 \rightarrow 128$. 
1 Introduction

The standard model of electroweak interactions predicts the existence of a phase transition to a high temperature symmetric phase \[1\]. The knowledge of fluctuation spectra in the high temperature phase is essential for understanding its physics.

In the high temperature phase the thermal contribution to the screening masses of $W$-gauge bosons dominates the contribution (if any) of the vacuum expectation value of the Higgs field. One has to distinguish here between the screening scales resulting from electric and magnetic gauge field fluctuations. The leading order electric screening mass is $O(gT)$. It is essentially a perturbative quantity, determined by the internal consistency of the resummed perturbative treatment of the thermodynamics of the system \[2, 3\]. Beyond leading order it requires a careful non-perturbative definition \[4\].

The non-vanishing magnetic screening mass does play an important role in controlling the infrared behaviour of the electroweak theory at high temperature and does influence the nature of the electroweak phase transition itself. For instance, it is expected that the existence or non-existence of a first order phase transition in the electroweak theory crucially depends on the magnitude of the thermal magnetic mass of the $W$-boson \[5, 6\]. The role of this mass in the symmetric high temperature phase of the electroweak theory is similar to that of the magnetic mass generated for the gluons in the high temperature, deconfined phase of QCD. Also, the thermal magnetic mass is crucial for the infrared behaviour of QCD at high temperature. In both cases these masses are expected to be of $O(g^2T)$ \[7\] up to possible logarithmic corrections \[8\].

The magnetic masses for the gauge bosons in the $SU(2)$ gauge-Higgs model as well as in QCD are not calculable within the context of high temperature perturbation theory. For instance, one might attempt to apply some sort of resummation to the magnetic sector, represented by a 3-dimensional effective theory in both cases \[9\]. A perturbative calculation of the free energy with a magnetic mass of $O(g^2T)$ introduced by adding and subtracting a corresponding term to the Lagrangian would lead to a series in $g^2T/m_w(T)$. All evidence gathered so far on this ratio suggest a large value for it. Any perturbative determination of it is therefore expected to fail.
Some form of a non-perturbative approach is needed. In the case of the $SU(2)$ gauge-Higgs model there have been various Monte Carlo calculations in which the thermal vector boson as well as the Higgs masses have been calculated with help of gauge invariant correlators of appropriate quantum numbers [10, 11, 12, 13].

Another attempt is based on an analytic treatment of coupled gap equations for the scalar and vector propagators on the mass shell [14, 15]. Such an analysis leads to the conjecture that also in the high temperature phase the magnetic $W$-boson mass is generated essentially by a Higgs-type phenomenon. The difference being that the expectation value of the order parameter is much smaller. It therefore is intuitively very appealing to continue the use of the same gauge invariant operators for the calculation of the magnetic mass (and also of the Higgs-mass), like in the low temperature phase.

The numerical calculations of gauge invariant correlators, however, do lead to a thermal mass for the vector boson which is substantially larger than the result of the analysis of gap equations in Landau gauge. This situation is somewhat similar to the case of $SU(N)$ gauge theories where the analysis of gauge invariant glueball operators with the quantum numbers of the glueon [16] does lead to much larger screening masses than the direct calculation of the gluon propagator in Landau gauge [17]. In this case the observed discrepancy was, however, expected. The gauge invariant correlation functions correspond to glueball states at low temperature and melt into several decoupled gluons with an effective thermal mass in the high temperature deconfined phase. The gauge invariant glueball correlation functions thus describe a multiple gluon state in the high temperature phase. The elementary thermal mass is only visible in a direct calculation of the gluon propagator.

In the $SU(2)$ gauge-Higgs model, however, on the basis of the above argument the discrepancy between analytic results and numerical calculations in the symmetric phase is somewhat unexpected. An explanation could be that analytic calculations are less stable in the symmetric phase. However, this discrepancy may also hint at a situation similar to the case of QCD, i.e. the gauge invariant operators may not project onto single $W$-boson states in the high temperature symmetric phase. Also the agreement of the bound state model of Dosch et al. with the screening masses obtained from the spectroscopy of gauge invariant operators [18] points to such an
interpretation.

In order to get closer to a clarification of this problem we need a more detailed quantitative understanding of both the behaviour of correlation functions for gauge fields in fixed gauges as well as that of gauge independent correlation functions with quantum numbers of the gauge bosons.

It is the purpose of this paper to study in detail the behaviour of correlation functions for the gauge potentials in Landau gauge. This will be done within the context of the 3-dimensional $SU(2)$ gauge-Higgs model which is obtained as an effective theory for the finite temperature electroweak model by integrating out heavy static modes corresponding to the zeroth component of the gauge fields \[19\]. We extract from the exponential fall-off of correlation functions of the gauge fields the $W$-boson magnetic mass in Landau gauge and compare with corresponding calculations of gauge invariant operators with quantum numbers of the $W$-boson as well as the Higgs boson. We have performed these calculations on a large number of different lattice sizes in order to control finite size effects. In addition we have performed calculations for the pure $SU(2)$ gauge theory in order to test the conjecture that the $W$-boson mass in the symmetric phase is closely related to the magnetic mass in the $SU(2)$ gauge theory. We do not address the problem of gauge invariance of the masses extracted from the gauge boson propagators and the related issue of influence of Gribov copies in this paper. These problems have been discussed in the context of calculations for photon and gluon propagators \[20\].

The paper is organized as follows. In section 2 we give the basic definitions for the 3-d gauge-Higgs model and its relation to the $(3+1)$-dimensional finite temperature $SU(2)$ Higgs model. In section 3 we discuss the calculation of the $W$-boson propagator in Landau gauge. The analysis of gauge invariant scalar and vector correlation functions is presented in section 4. Finally we give our conclusions in section 5.
2 Reduced EW-model

The physics of the longest range fluctuations of the finite temperature electroweak theory is described by an effective 3-d theory which is matched to the complete model at the distance scale $a \sim (gT)^{-1}$ [14, 21, 22, 23]. Though this effective theory coincides formally with the superrenormalisable 3-d gauge–Higgs model, the optimal quantitative description of the original physics is expected to be obtained by choosing the lattice spacing according to $\Theta \equiv aT \sim 1$.

The lattice formulation of the effective theory is given by

$$S_{3D}^{lat} = \frac{\beta}{2} \sum_P \text{Tr} U_P(x) + \frac{1}{2} \sum_{x,i} \text{Tr} \Phi_x^\dagger U_{x,i} \Phi_{x+\hat{i}} - \frac{1}{2\kappa} \sum_x \frac{1}{2} \text{Tr} \Phi_x^\dagger \Phi_x$$

$$- \frac{\lambda_3}{24} \sum_x (\frac{1}{2} \text{Tr} \Phi_x^\dagger \Phi_x)^2,$$

(2.1)

where $U_P$ is the standard plaquette variable of the $SU(2)$ lattice gauge theory and $\Phi_x$ is a complex $2 \times 2$ matrix which in terms of the real weak isosinglet-triplet decomposition of the complex Higgs doublet is given by $\Phi_x \equiv \Phi_0 + \tau_i \Phi_i$.

The relationship of the dimensionless lattice couplings $\beta$, $\lambda_3$, $\kappa$ to the couplings of the original $T = 0$, $SU(2)$ gauge–Higgs system is given by the following sequence of equations:

$$\beta = \frac{4}{g_3^2} \Theta, \quad g_3^2 = g^2(1 - \frac{g}{20\pi} \sqrt{\frac{5}{6}}),$$

(2.2)

$$\lambda_3 = \left( \frac{3 m_H^2}{4 m_W^2} g^2 - \frac{27}{160\pi} \sqrt{\frac{5}{6}} g^3 \right) \Theta, \quad m_H^2 = \frac{\lambda}{3} v^2, \quad m_W^2 = \frac{g^2}{4} v^2,$$

(2.3)

$$\frac{1}{\kappa} = m^2 a^2 + \left( \frac{3}{16} g^2 + \frac{1}{12} \lambda - \frac{3 g^3}{16\pi} \sqrt{\frac{5}{6}} \right) \Theta^2 - \Theta \Sigma(L) \left( \frac{3}{2} g^2 + \lambda - \frac{15 g^3}{32\pi} \sqrt{\frac{5}{6}} \right) + 6.$$ (2.4)

In these equations $g, \lambda, m^2$ represent the renormalised parameters of the original theory and $m_H, m_W$ are the $T = 0$ masses of the Higgs and $W$ bosons. $\Sigma(L)$ is a slightly size dependent geometrical factor which is known exactly for any lattice size $L$. In particular one obtains in the infinite volume limit $\Sigma(\infty) = 0.252731.$
The above relations are obtained when one integrates first over the non-static Matsubara modes with 1-loop accuracy and in a second reduction step also over the static $A_0$ mode \cite{22}. Then the coupling relations are accurate to $O(g^3, \lambda^{3/2})$. The $A_0$-integration has been realised with help of an iterated 1-loop calculation \cite{24}. Although it will not be relevant for the following discussion it is worth to note that the $O(g^3\Theta)$ term in Eq. \eqref{2.4} which has been estimated with this iterative technique deviates from the result given in \cite{22}.

Our numerical calculations have been performed with the parameters

$$\beta = 9.0 \quad , \quad \lambda_3 = 0.313646 \quad ,$$ \hspace{1cm} \eqref{2.5}

at the physical $W$-boson mass, $m_W = 80.6$ GeV. Choosing also $\Theta = 1$ this does correspond to a Higgs mass of about 80 GeV (Eq. \eqref{2.3}). Eq. \eqref{2.4} does then relate $\kappa$ to the ratio $m^2/T^2$. After finding $\kappa_c$, its value is easily translated into $T_c$ (with one more input of $v = 246$ GeV). This can be performed for each value of $\kappa$. Our correlation measurements covered the range $0.170 \leq \kappa \leq 0.180$. The $\kappa$—range can be translated with help of Eq. \eqref{2.4} into a temperature interval around the critical value, $0.6 \sim T_c \lesssim (4 - 5) T_c$. We expect that more accurate mappings between the couplings will not modify qualitatively the temperature interval covered.

We have chosen to work at a rather large value of the Higgs mass. For this choice of parameters the nature of the phase transition has not yet been clarified. It could be either continuous or only very weakly first order. The study of the gap equations in \cite{14} indicates a smooth crossover. We shall give an argument below in favor of a second order transition. One therefore should be able to find a critical value of the hopping parameter at which the symmetry restoring takes place. This has been determined by us as

$$\kappa_c = 0.17467(2) \quad .$$ \hspace{1cm} \eqref{2.6}

We will report in more detail about the determination of $\kappa_c$ and an analysis of the order of the phase transition elsewhere.
3 W-boson propagator in Landau gauge

3.1 Landau gauge fixing

Our analysis of the W-boson propagator in Landau gauge closely follows the approach used in the calculation of the gluon propagator in finite temperature QCD [17, 25]. We define the gauge fields, $A_\mu(x)$, from the $SU(2)$ link variables, $U_{x,\mu}$

$$A_{\mu}(x) = \frac{i}{2g} \left( U_{x,\mu}^\dagger - U_{x,\mu} \right) .$$

(3.1)

The Landau gauge condition $|\partial_\mu A_\mu(x)|^2 = 0$ is then realized on each lattice configuration by maximizing the trace of the link fields, $U_{x,\mu}$,

$$\Sigma = \sum_{x,\mu} \text{Tr} \left[ U_{x,\mu} + U_{x,\mu}^\dagger \right] .$$

(3.2)

The maximization has been performed using an overrelaxation algorithm combined with a FFT-algorithm [26, 27] until the Landau gauge condition has been satisfied within an accuracy of $10^{-9}$. Typically this required about 500 iterative maximization steps.

On the gauge fixed configurations we analyze correlation functions of gauge fields averaged over $(x, y)$-planes,

$$\tilde{A}_\mu(z) = \sum_{x,y} A_\mu(x, y, z) ,$$

(3.3)

in order to improve the projection onto the zero momentum excitations. The correlation functions of these averaged fields are then calculated in the transverse $z$-direction,

$$G_w(z) = \langle \text{Tr} \tilde{A}_\mu(0) \tilde{A}_\mu(z) \rangle .$$

(3.4)

For large separations $z$ these correlation functions do project onto the W-boson propagator mass.


3.2 W-boson propagator

We have analyzed the $W$-boson propagator in Landau gauge in the 3-d $SU(2)$ gauge-Higgs model. Calculations have been performed for a large number of hopping parameter values. We have used an overrelaxed heat-bath algorithm and performed typically 50,000 iterations\footnote{We call an iteration a combination of 4 overrelaxation steps followed by one heat-bath update. The heat-bath update of the scalar fields was optimized by shifting a suitably chosen quadratic term of the Higgs field from the accept reject decision into the generation of the cartesian gaussian components.} per $\kappa$-value. After every tenth iteration we have then fixed the Landau gauge and calculated the gauge fixed correlators as well as a set of gauge invariant operators. Most calculations have been performed on a lattice of size $16^2 \times 32$. In order to get control over finite lattice size effects we have performed additional calculations on lattices of size $L^2 \times 32$ with $L$ ranging from 4 to 24 as well as $16^2 \times L_z$ with $L_z$, ranging from 16 to 128. These calculations have been performed at two values of the hopping parameter below and above $\kappa_c$. The statistics accumulated for our detailed finite size analysis at these $\kappa$-values is summarized in Table 1.

The analysis of the volume dependence allowed us to select a suitable ansatz for the fits of correlation functions on the $16^2 \times 32$ lattice which minimize finite size effects in the determination of the $W$-boson propagator mass. In Fig. 1 we show the correlation function $G_w(z)$ defined in Eq.(3.4) for various lattice sizes at $\kappa = 0.1745$ and $\kappa = 0.17484$. The correlation functions show a slower decay at short distances which also has been observed in the analysis of the gluon propagator [17, 28]. This behaviour is also evident from the analysis of local masses which approach a plateau at large distances from below. We define local masses in two different ways either as solution of the equation

\[
\frac{G_w(z - 1) - G_w(z)}{G_w(z) - G_w(z + 1)} = \frac{G_w^{fit}(z - 1) - G_w^{fit}(z)}{G_w^{fit}(z) - G_w^{fit}(z + 1)},
\]  

or

\[
\frac{G_w(z - 1)}{G_w(z)} = \frac{G_w^{fit}(z - 1)}{G_w^{fit}(z)}.
\]
Here $G_w(z)$ denotes the calculated values for the correlation functions and

$$G_w^{\text{fit}}(z) = A \left( \exp(-m_w z) + \exp(-m_w (L_z - z)) \right) + B. \quad (3.7)$$

While the ansatz given in Eq. (3.5) is independent of the constant $B$ the second version given in Eq. (3.6) is not. In the latter case we use $B \equiv 0$ to define the local masses. Results for these are shown in Fig. 4. We note that there is no apparent dependence on $L_z$ visible in the analysis based on Eq. (3.5) while there is a significant volume dependence if we use the second ansatz. We do, however, find that both forms yield consistent results for $L_z \geq 64$. This suggests that we can minimize finite lattice size effects by allowing for a constant in our ansatz for a global fit to the correlation functions. We also see from Fig. 2 that the local masses do develop a plateau for $z \geq 8$. We therefore have fitted the correlation functions only for distances $z \geq 8$ using the ansatz given in Eq. (3.7). The fit results for the correlation functions shown in Fig. 1 are summarized in Table 1. We note that these fits yield values for the propagator masses $m_w$ which are within errors independent of $L_z$. Moreover, the constant $B$ rapidly drops to zero with increasing $L_z$. We find that it is well described by an exponential decrease, $B \sim \exp(-0.1L_z)$. We also note that the constant $B$ is consistent with being zero in the symmetry broken phase.
| $L_z$ | $m_w$  | $A$   | $B$          | # iterations |
|------|--------|-------|--------------|--------------|
|      | $\kappa = 0.1745$ |        |              |              |
| 32   | 0.166(7) | 10.3 (2) | -0.80(15)   | 190.000      |
| 40   | 0.194(14) | 10.9 (6) | -0.18(13)   | 40.000       |
| 48   | 0.179(11) | 10.1 (9) | -0.14(8)    | 40.000       |
| 64   | 0.174(9)  | 9.3 (8)  | -0.045(22)  | 90.000       |
| 128  | 0.163(12) | 8.6 (9)  | -0.012(13)  | 60.000       |
|      | $\kappa = 0.17484$ |        |              |              |
| 32   | 0.308(6)  | 3.4 (1)  | -0.012(19)  | 80.000       |
| 64   | 0.291(11) | 3.2 (2)  | -0.009(4)   | 40.000       |

Table 1: Results of fits to the correlation functions shown in Fig. 1.

already for $L_z = 32$. A similar analysis has been performed for the dependence of the correlation functions on the transverse lattice size. In that case simulations have been performed on lattices of size $L^2 \times 32$ with $L$ ranging from 4 to 24.

From the above analysis of finite size effects on propagator masses below and above $\kappa_c$ we conclude that masses can reliably be extracted from correlation functions already on lattices of size $16^2 \times 32$ using a fit of the form given in Eq. (3.7). The results obtained this way for a large number of $\kappa$ values are shown in Fig. 3. We note that within our numerical accuracy the propagator mass $m_w$ is independent of $\kappa$ in the symmetric phase while it rises rapidly above $\kappa_c$. We also have performed a calculation at $\kappa = 0$, $\lambda_3 = 0$, i.e. in the pure $SU(2)$ gauge theory. This yields a value for the propagator mass which is consistent with those obtained in the symmetric phase of the $SU(2)$ gauge-Higgs model close to $\kappa_c$. This pure gauge value also is shown in Fig. 3 as a filled circle. A fit to the data for $m_w$ below $\kappa_c$ yields

$$m_w = 0.158 \pm 0.002 \quad , \quad \kappa \leq \kappa_c \ . \quad (3.8)$$

Above $\kappa_c$ the mass increases rapidly. A good fit to the data in the entire range $\kappa \geq \kappa_c$ is obtained with the ansatz

$$m_w = 0.158 + a(\kappa - \kappa_c)\beta \quad \kappa \geq \kappa_c \ . \quad (3.9)$$
In order to be more sensitive to the critical behaviour close to \( \kappa_c \) we have restricted the fit to the interval \( \kappa_c \leq \kappa \leq 0.176 \). In this case we find for the two free parameters \( a = 4.0(4) \) and \( \beta = 0.384(15) \). We note that the exponent \( \beta \) turns out to be consistent with that of the \( O(4) \) spin model in 3-dimensions. In a recent Monte Carlo analysis [29] this exponent has been found to be \( \beta = 0.3836(46) \) which is in agreement with results obtained from the \( (4 - \epsilon) \)-expansion. Through the Higgs-mechanism the \( W \)-boson mass in the \( SU(2) \) gauge-Higgs model is linked to the scalar field expectation value. It thus seems plausible that also the temperature dependence of the \( W \)-boson mass close to \( \kappa_c \) is controlled by the exponent \( \beta \). Although the agreement is quite striking we stress that without a more detailed finite size analysis we can, at present, not rule out a smooth crossover as found in Ref. [14] or a critical behaviour controlled by a one-component scalar field as suggested in Ref. [12].

The results for the propagator mass discussed so far are in good quantitative agreement with analytic calculations of Ref. [14]. Expressing our result in terms of \( g^2 \) we find in the symmetric phase \( m_w = 0.35(1)g^2T \). This should be compared with the approximate value \( m_w \simeq 0.28g_0^2 \) quoted in Ref. [14] for large values of \( \lambda/g^2 \). Also the functional form of \( m_w(\kappa) \) in the symmetry broken phase is in good agreement with results obtained from the analysis of gap equations when the \( \kappa \)-dependence
is transformed into temperature dependence with help of Eq. (2.4). However, the emerging physical picture is rather different from what is proposed in [14].

Based on an analysis of coupled gap equations for the scalar and vector propagators in Landau gauge it has been concluded in [14] that also above $T_c$ the propagator mass is determined by the vacuum expectation value of the Higgs field. This expectation value is actually much smaller than at $T = 0$ (mini-Higgs mechanism) and is proportional to $g^2$. For scalar and gauge couplings corresponding to equal Higgs and $W$-boson masses at zero temperature they do find a smooth crossover between the low and high temperature regimes which are distinguished by different magnitudes of the scalar field expectation values. However, it also is found that the high temperature magnetic mass is rather insensitive to the actual value of $\lambda/g^2$.

Our calculation suggests a different physical mechanism. Since the pure gauge propagator mass is compatible with the result of the Higgs-model in the symmetric phase, we believe that the magnetic mass is of fully thermal origin, without any high $T$ (low $\kappa$) Higgs effect. Above $\kappa_c$ a non-analytic contribution adds to the thermal value which can be attributed to the onset of the Higgs-effect in the low temperature
phase. Although we can, at present, not rule out a smooth crossover behaviour we note that the temperature dependence in the vicinity of $\kappa_c$ is well described by a non-analyticity characteristic for continuous phase transitions in Higgs models.

4 Gauge invariant vector and scalar correlators

In the symmetry broken phase the $W$-boson propagator mass increases rapidly with $\kappa$. This tendency agrees well with both the low temperature vector mass extracted from the gap equation (when $g \simeq 2/3$ and $a = 1/T$ of our simulation are taken into account) and with the masses obtained from gauge invariant correlators in lattice units by [11, 12]. These latter simulations do, however, use a smaller value for $\lambda$ than we do. It therefore is important to compare for our choice of couplings the agreement of the masses extracted from gauge invariant vector operators with the propagator mass.

We have calculated correlators for the standard gauge invariant vector and scalar operators

$$O_{v,i}(x) = \text{Tr} \sigma_3 \Phi_x^\dagger U_{x,i} \Phi_{x+i}, \quad i = 1, 2,$$

as well as two different scalar operators,

$$O_s^a = \det \Phi_x, \quad O_s^b = \sum_{i=1}^{2} \text{Tr} \Phi_x^\dagger U_{x,i} \Phi_{x+i},$$

In analogy to Eq. (3.4) we define operators $\hat{O}$ which project onto zero momentum states. The long distance behaviour of the scalar correlator $G_s^\alpha(z) = \langle \hat{O}_s^\alpha(0) \hat{O}_s^\alpha(z) \rangle$, $\alpha = a, b$, does then yield the scalar (Higgs) mass while the correlator $G_v(z) = \sum_{i=1,2} \langle \hat{O}_{v,i}(0) \hat{O}_{v,i}(z) \rangle$ defines the mass of a vector particle with the quantum numbers of the $W$-boson.

In both cases we also have performed an analysis of the finite size dependence of masses extracted from these correlation functions. Some results are given in Fig. 4. This shows that also in this case the masses may reliably be analyzed on lattices of size $16^2 \times 32$.  

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The scalar (Higgs) mass has been calculated on the same set of configurations used for the analysis of the $W$-boson propagator in Landau gauge. Fits have been performed for distances $z \gtrsim (2 - 4)$. We do find quite a different behaviour for the scalar mass in the symmetric phase while it is similar in magnitude and functional dependence to the propagator mass in the symmetry broken phase. In all cases we obtained consistent results from the two scalar operators defined in Eq. (4.2). The masses extracted from $G_s^a$ are shown in Fig. 5.

The scalar mass becomes very small at $\kappa_c$. The behaviour is consistent with a second order or very weak first order phase transition. The functional dependence of the scalar mass below and above $\kappa_c$ is clearly different. We therefore have fitted the masses to the ansatz

$$m_s = c_\pm + a_\pm |(\kappa - \kappa_c)|^{\nu_\pm},$$

(4.3)

where the subscript +/- refers to the broken/symmetric phases. We find consistent fits for $c_\pm \equiv 0$ as well as $c_\pm \neq 0$. The fit parameters for both cases are summarized in Table 2. We note that also this behaviour is quite similar to the results obtained from gap equations [14].
Figure 5: Scalar masses obtained from fits to the gauge invariant correlation function $G_s^a$ calculated on lattices of size $16^2 \times 32$.

From Table 2 we conclude that the determination of critical exponents is quite sensitive to the inclusion or exclusion of a constant term in the fits. This shows that also here we will need a rather detailed finite size analysis close to $\kappa_c$ to draw definite conclusions. Still a few observations may be appropriate already at this point: When approaching $\kappa_c$ from below (in the symmetric phase) we find that $\nu$ is consistent with the mean field value $1/2$. This has also been observed in another simulation of the 3-dimensional model [12]. However, when approaching $\kappa_c$ in the broken phase we find a smaller value for the exponent $\nu$. This is particularly true

| $\kappa \leq \kappa_c$ | $\kappa \geq \kappa_c$ |
|-------------------------|-------------------------|
| **fit with $c_\pm \neq 0$** | **fit with $c_\pm \equiv 0$** |
| $c_-$ | $a_-$ | $\nu_-$ | $c_+$ | $a_+$ | $\nu_+$ |
| $0.04$ (2) | $26$ (9) | $0.58$ (6) | $0.073$ (20) | $6$ (2) | $0.45$ (6) |
| $-$ | $14$ (2) | $0.48$ (4) | $-$ | $3.0$ (5) | $0.31$ (3) |

Table 2: Results of the fits to the masses extracted from the scalar correlation function $G_s^a$. In Fig. 6. we have shown the fits with $c_\pm \equiv 0$. 
when we exclude the constant term in our fits. In any case, it does seem that a large exponent, i.e. $\nu > 1/2$, like in the 3-d Ising model ($\nu \sim 2/3$) or the $O(4)$-model ($\nu \simeq 0.75$ [29]) is ruled out by our data. It seems that the temperature dependence of the scalar mass is very similar to that of the $W$-boson propagator mass, although in the case of a second order phase transition both should depend on different critical exponents. As we have argued in the previous section the temperature dependence of the $W$-boson propagator mass is expected to be controlled by the exponent $\beta$ while the scalar mass is controlled by the correlation length exponent $\nu$. This may hint at are more complex dynamics close to $\kappa_c$ than described by the universality class of 3-d, scalar spin models and may even indicate the possible existence of a tricritical point.

A similar detailed analysis of the gauge invariant vector correlation function on our data sets failed because the signal disappeared already at rather short distances ($z \simeq 4$) in the statistical noise. The construction of improved operators may help in this channel [13]. We could calculate the vector mass at the two $\kappa$ values close to $\kappa_c$, where we did perform the finite size analysis (see Fig. 4). Here we extracted the masses in the vector channel from a fit to the correlation functions for $z \geq 2$. This yields

$$m_v = \begin{cases} 0.557 \pm 0.087 & , \quad \kappa = 0.1745 \\ 0.356 \pm 0.028 & , \quad \kappa = 0.17484 \end{cases} \quad (4.4)$$

We note that in the symmetric phase at $\kappa = 0.1745$ the mass in the vector channel is more than twice as large as the mass extracted from the $W$-boson propagator ($m_w \simeq 0.158$), whereas these masses are similar in the symmetry broken phase. For instance, we find from Table 4 at $\kappa = 0.17484$ for the $W$-boson propagator mass $m_w = 0.308$ (6), which is compatible with the value given in Eq. 4.4 for $m_v$.

5 Conclusions

In this paper we have established the existence of the exponential decay of gauge field (link-link) correlations of the 3-d gauge-Higgs system in Landau gauge which leads to a non-vanishing magnetic screening mass in the $W$-boson propagator. In
the high temperature (small $\kappa$) regime this characteristic length scale agrees with the same quantity of the pure gauge system.

This equality cannot be a coincidence, and gets further support from the study of the gauge invariant excitation spectrum by Philipsen et al. [13]. They report the non-mixing of a $0^{++}$ state composed of gauge plaquettes with those operators having the same quantum numbers and also involving Higgs fields. This decoupling phenomenon is actually expected at high temperature; it corresponds to the separation of the heavy scalar modes from the dynamics of the weakly screened magnetic fluctuations which are described by an effective theory both in the case of QCD and the gauge-Higgs system [9].

On the basis of this apparent decoupling we have argued that the magnetic vector fluctuations do not receive any contribution to their screening mass from a Higgs-type mechanism in the high temperature phase. The onset of the additional mass generation through the Higgs-mechanism can be observed as a well-localized increase of the effective mass above $\kappa_c$. Our present calculations, which have been performed with a set of couplings corresponding to $m_H \approx 80\text{GeV}$, suggest the existence of a second order phase transition. While the temperature dependence of the $W$-boson propagator mass close to $\kappa_c$ is consistent with the critical behaviour expected from an $O(4)$ symmetric effective theory, we seem to find deviations from this picture for the scalar mass. However, only a very careful finite size analysis can substantiate this observation and should allow to distinguish from a smooth crossover suggested by [14] or the critical behaviour of an effective theory possibly controlled by a one-component scalar field as suggested in [12].

The propagator masses and the gauge invariant spectrum agree well in the broken symmetry phase.

An important issue is to clarify why the two kinds of operators which yield the same mass in the symmetry broken phase cease to couple to the same state in the symmetric phase. Further investigations of gauge invariant and gauge dependent correlation functions should lead to progress on this question. One possibility would be, for instance, to construct also simple non-gauge invariant two-particle operators whose correlators in the Landau gauge could reproduce the results of the gauge invariant spectroscopy.
Another important next step towards the clarification of the nature of the symmetric phase and its fundamental degrees of freedom is the thorough investigation of the contribution of the static sector to the equation of state of the finite temperature gauge-Higgs system. Our analysis suggests that the thermodynamics in the symmetric phase may be described in terms of almost free massive degrees of freedom having the mass explored in the present paper.

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