Design of a fast transfer alignment matching algorithm

Xingshou Geng, Kanghua Tang and Meiping Wu

College of Intelligence Science and Technology, National University of Defense Technology, Changsha, Hunan, 410073, China

*Corresponding author’s e-mail: 2099349490@qq.com

Abstract. In order to estimate the misalignment angle of align system effectively, this paper constructs a traditional 14 state rapid transfer alignment model, considers the problem that traditional velocity error definition is not completely accord with the consistency of coordinate system, the improved differential equation of velocity error is applied to transfer alignment, constructs an improved 12 state rapid transfer alignment model. Finally, the performance of the two alignment algorithms is tested by vehicle-mounted experimental data under the conditions of no maneuver, C shape maneuver and S shape maneuver, and the experimental result of the two algorithms is compared and analyzed. The experimental result shows that the improved 12 state rapid transfer alignment algorithm not only has better alignment accuracy than the traditional 14 state rapid transfer alignment algorithm, but also reduces the model complexity, reduces the system calculation, and avoids the influence of flexural deflection and dynamic misalignment angle. The yaw angle alignment accuracy is improved by 45%, 65% and 53% under the conditions of no maneuver, C shape maneuver and S shape maneuver, respectively.

1. Introduction

The transfer alignment algorithm is used to estimate and calibrate the errors of the misaligned slaver inertial navigation system through the high-precision master inertial navigation system. The key of the technique is how to estimate the attitude misalignment angle accurately and quickly[1]. Up to now, the design of transfer alignment algorithm mainly focuses on the design of filtering model (state and measurement equations), improvement of filtering method and selection of maneuver mode. Moreover, more and more transfer alignment algorithms have been widely used in space, airborne and shipborne weapon platforms which equipped with high-precision master inertial navigation system[1-2]. In reference[3], an adaptive incremental Kalman filtering algorithm is proposed to solve the problem of poor convergence of conventional Kalman filtering in airborne MEMS-IMU transfer alignment. In reference[4], an improved adaptive compensation H∞ filtering method is proposed to suppress the filtering divergence during transfer alignment. In reference[5], a transfer alignment method based on federated filter is proposed to achieve real-time and high-precision measurement of multi-point motion parameters during transfer alignment. Literature[6-7] prove the consistency of the attitude matching algorithm used in transfer alignment. Literature[8] believes that the z-direction flexural deflection between master and slaver inertial navigation system is the smallest, and proposes the adoption of speed plus z-direction attitude matching to transfer alignment. Literature[1] proposes a fast transfer alignment algorithm using velocity plus z-axis installation error angle. This method takes into account the advantages of traditional velocity plus attitude matching algorithm and avoids its disadvantages of being sensitive to time delay and deflection deformation during wing swing maneuver. In literature[9], 11-dimensional ψ angle and ψw angle transfer alignment model are established, and a dual-filter
parallel working idea of "installing angle filter plus gyroscope zero-bias filter" is proposed, which improves the flexibility and robustness of the transfer alignment filter. However, up to now, in all published papers about transfer alignment, the definition of velocity error only considers the magnitude of the velocity error vector, so it does not completely conform to the consistency of the coordinate system. In literature[10-12], the velocity error is derived in detail on the premise of taking into account the magnitude and direction of the velocity error vector, and it is applied to integrated navigation. Meanwhile, the single and two position alignment experiment is carried out in literature[10], but it has not been found that some scholars have applied it to transfer alignment.

Paper first constructs a traditional 14 state Kalman filtering model, and considers the problem that the definition of traditional velocity error does not consider the direction of velocity error vector, traditional velocity error definition does not completely accord with the consistency of coordinate system, so the improved differential equation of velocity error is applied to transfer alignment, constructs an improved 12 state rapid transfer alignment model, reduces the dimension of the filter model, reduces the system calculation, avoids the influence of flexural deflection and dynamic misalignment angle, improves the alignment accuracy. Finally, the algorithm is verified by vehicle experiment.

2. System error model
In the North-East-Down(NED) local-level navigation coordinate system, the attitude error differential equation is:

\[ \dot{\phi}_n = -\omega_n^a \times \phi^a + \delta \omega_n^a - C^a_n \phi^b \]  

(1)

The traditional differential equation of velocity error is:

\[ \delta \dot{v}^s = f^s \times \phi^s - (2\omega_n^a + \omega_m^a) \times \delta \dot{v}^s + \dot{v}^s \times (2\delta \omega_n^a + \delta \omega_m^a) + C_n^a \delta \dot{v}^b \]  

(2)

The traditional velocity error is defined as: \( \delta v^s = \tilde{v}^s - v^s \). \( \tilde{v}^s \) represents the estimated value of the variable. Only the magnitude of the velocity error vector is considered, but the direction of the velocity error vector is not considered, which does not completely conform to the consistency of the coordinate system[10]. So the rigorous velocity error should be defined in the same frame of reference.

First, define: \( C_n^a \hat{C}_n^b = \exp(\phi^s \times) = I + \left[ \phi^s \times \right] + \left[ \phi^s \times \right] \left[ \phi^s \times \right] + \cdots \), \( \tilde{v}^s = \hat{C}_n^a \tilde{f}^s - (2\omega_n^a + \omega_m^a) \times \delta \tilde{v}^s + g^a \), \( \exp(\bullet) \) represents the exponential function of the matrix[13]. The rigorous velocity error defined in the same coordinate system[10-12] is: \( \delta v^s = C_n^a \hat{C}_n^b \tilde{v}^s - v^s = \tilde{v}^s - v^s = \hat{C}_n^a \tilde{v}^s - \tilde{v}^s \times g^a \). As can be seen from the equation, \( -\tilde{v}^s \times g^a \) is a term that represents the direction of velocity error vector. Compared with the traditional velocity error equation, the improved velocity error equation takes into account the magnitude and direction of the velocity error vector, makes the velocity error model more accurate. The improved differential equation of velocity error can be obtained by differentiating the above equation:

\[ \delta \dot{v}_1^s = \delta \dot{v}^s - \tilde{v}^s \times \phi^a - \tilde{v}^s \times \phi^a 
= f^s \times \phi^s - (2\omega_n^a + \omega_m^a) \times \delta \dot{v}^s + \dot{v}^s \times (2\delta \omega_n^a + \delta \omega_m^a) + C_n^a \delta \dot{v}^b + W_a 
- \left( \hat{C}_n^a \left( f^s + v^b + W_a \right) - (2\omega_n^a + \omega_m^a) \times \tilde{v}^s + \tilde{v}^s \times g^a - \tilde{v}^s \times \phi^a \right) - \omega_n^a \times \phi^a + \delta \omega_n^a - C_n^a \left( \phi^b + W_b \right) \]  

(3)

\( W_a, W_b \) represent the white noise vector of accelerometer and gyro error.
3. Algorithm design

3.1. Design of traditional 14 state fast transfer alignment algorithm
System state variables are selected as: horizontal velocity error, attitude error, installation angle error and zero deviation of gyro and accelerometer; process noise vector is selected as: accelerometer noise, gyroscope noise and installation angle noise, namely:

\[
\mathbf{X} = \left[ \delta v_n, \delta v_e, \varphi_s, \varphi_e, \varphi_z, \mu_s, \mu_e, \varepsilon_s, \varepsilon_e, \varepsilon_z, \nabla_x, \nabla_y, \nabla_z \right]^T
\]
\[
\mathbf{W} = \left[ w_{v_n}, w_{v_e}, w_{\varphi_s}, w_{\varphi_e}, w_{\varphi_z}, w_{\mu_s}, w_{\mu_e}, w_{\varepsilon_s}, w_{\varepsilon_e}, w_{\varepsilon_z}, w_{\nabla_x}, w_{\nabla_y}, w_{\nabla_z} \right]^T
\]

According to the system error model described above, the system state equation can be obtained:

\[
\dot{\mathbf{X}} = \mathbf{F}\mathbf{X} + \mathbf{G}\mathbf{W}
\]

\[
\mathbf{F} = \begin{bmatrix}
F_{11} & F_{12} & 0 & 0_{3 \times 3} & \mathbf{C}_b^x \\
F_{21} & F_{22} & 0 & -\mathbf{C}_b^y & 0_{3 \times 3} \\
0_{7 \times 3} & 0_{7 \times 3} & 0 & 0 & 0_{7 \times 3}
\end{bmatrix}
\]

\[
\mathbf{G} = \begin{bmatrix}
\mathbf{G}_{11} & 0_{2 \times 3} & 0_{2 \times 1} \\
0_{3 \times 3} & -\mathbf{C}_b^n & 0_{3 \times 1} \\
0_{6 \times 3} & 0_{6 \times 3} & 0_{6 \times 3}
\end{bmatrix}
\]

The measurement vector is the difference between master inertial navigation system and slaver inertial navigation system in the aspects of north and east velocity, attitude angle. The observation equation of the system can be obtained as follows:

\[
\mathbf{z} = \mathbf{H}\mathbf{x} + \mathbf{v}
\]

\[
\mathbf{H} = \begin{bmatrix}
I & 0_{3 \times 3} & 0_{3 \times 3} & 0_{2 \times 6} \\
0_{3 \times 2} & I & \mathbf{C}_b^n & 0_{3 \times 6}
\end{bmatrix}
\]

\[
\mathbf{v} \text{ is the measurement noise corresponding to each measurement, which can be regarded as white noise.}
\]

3.2. Design of improved 12 state fast transfer alignment algorithm
System state variables are selected as follows: horizontal velocity error, attitude error, yaw installation angle error and zero deviation of gyro and accelerometer; process noise vector is selected as: accelerometer noise, gyroscope noise and yaw installation angle noise, namely:

\[
\mathbf{X} = \left[ \delta v_n, \delta v_e, \varphi_s, \varphi_e, \varphi_z, \mu_s, \mu_e, \varepsilon_s, \varepsilon_e, \varepsilon_z, \nabla_x, \nabla_y, \nabla_z \right]^T
\]
\[
\mathbf{W} = \left[ w_{v_n}, w_{v_e}, w_{\varphi_s}, w_{\varphi_e}, w_{\varphi_z}, w_{\mu_s}, w_{\mu_e}, w_{\varepsilon_s}, w_{\varepsilon_e}, w_{\varepsilon_z}, w_{\nabla_x}, w_{\nabla_y}, w_{\nabla_z} \right]^T
\]

According to the system error model described above, the system state equation can be obtained:

\[
\dot{\mathbf{X}} = \mathbf{F}\mathbf{X} + \mathbf{G}\mathbf{W}
\]
\[ F = \begin{bmatrix} F_{11} & F_{12} & 0 & F_{14} & F_{15} \\ F_{21} & F_{22} & 0 & -C_b^a & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad G = \begin{bmatrix} G_{11} & G_{12} & 0_{2x4} \\ 0_{3x3} & -C_b^a & 0_{3x4} \\ 0_{3x1} & 0_{3x1} & 1 \\ 0_{6x3} & 0_{6x3} & 0_{6x3} \end{bmatrix} \]

\[ F_{11} = \begin{bmatrix} 0 & -2(\omega_n \sin L + \frac{v_e}{R_e + h} \tan L) \\ 2\omega_n \sin L + \frac{v_e}{R_e + h} \tan L & 0 \end{bmatrix}, \quad F_{12} = \begin{bmatrix} -v_e \omega_n \sin L - v_e \omega_n \cos L + g_e - v_e \omega_n \cos L \\ -g_e \end{bmatrix} \]

\[ F_{14} = \begin{bmatrix} v_D C_b^a(0) - v_N C_b^a(6) & v_D C_b^a(1) - v_N C_b^a(7) & v_D C_b^a(2) - v_N C_b^a(8) \\ v_D C_b^a(3) + v_e C_b^a(6) & -v_D C_b^a(4) + v_e C_b^a(7) & -v_D C_b^a(5) + v_e C_b^a(8) \end{bmatrix} \]

\[ F_{15} = \begin{bmatrix} C_b^a(0) & C_b^a(1) & C_b^a(2) \\ C_b^a(3) & C_b^a(4) & C_b^a(5) \end{bmatrix} \]

\[ G_{11} = \begin{bmatrix} C_b^a(0) & C_b^a(1) & C_b^a(2) \\ C_b^a(3) & C_b^a(4) & C_b^a(5) \end{bmatrix}, \quad G_{12} = \begin{bmatrix} -v_D C_b^a(3) + v_e C_b^a(6) & -v_D C_b^a(4) + v_e C_b^a(7) & -v_D C_b^a(5) + v_e C_b^a(8) \\ v_D C_b^a(0) - v_N C_b^a(6) & v_D C_b^a(1) - v_N C_b^a(7) & v_D C_b^a(2) - v_N C_b^a(8) \end{bmatrix} \]

The improved velocity error measurement equation can be obtained as follows:

\[ \delta \vec{z} = \vec{v}^* - (\vec{v}^* + [\vec{v}^* \times \vec{g}^*]) \quad (7) \]

The measurement vector is the difference between master inertial navigation system and slaver inertial navigation system in the aspects of north and east velocity, yaw angle. The observation equation of the system can be obtained as follows:

\[ z = Hx + v = \begin{bmatrix} H_{\vec{v}_x} \v_n \\ H_{\vec{v}_y} \v_n \end{bmatrix} \]

\[ H = \begin{bmatrix} 1 & 0 & 0 & -v_D & v_e \quad 0_{1x6} \\ 0 & 1 & v_D & 0 & -v_N \quad 0_{1x6} \end{bmatrix} \]

After Kalman filtering, the velocity status is updated as follows:

\[ \vec{v}_x^* = \vec{v}_x - \delta \vec{v} - (\vec{v} + [\vec{v} \times \vec{g}]) \quad (8) \]

\[ \delta \vec{v}^* = \vec{v} - (\vec{v}^* + [\vec{v}^* \times \vec{g}^*]) \quad (9) \]

4. Experimental results and analysis

The traditional algorithm and the improved algorithm designed in this paper are verified by vehicle-mounted experimental data. The master inertial navigation system used in the experiment is an optical fiber integrated navigation system, the accuracy of pure inertial navigation is maintained: velocity \( \leq 0.5m/s \), position \( \leq \text{inmile}(\text{C}50\%) \), course angle \( \leq 0.01'(\text{RMS}) \), attitude angle \( \leq 0.005'(\text{RMS}) \). The slaver inertial navigation gyroscope zero deviation is \( 20'/h (1\sigma) \), zero bias stability is \( 4.5'/h \), random walk is \( 0.1'/\sqrt{h} \), accelerometer bias, the zero deviation stability and zero deviation repeatability are both 1mg. The experimental result is as follows. Note: When calculating the mean value of multiple experimental result, just consider the absolute value of the data.

Figure 1, Figure 3 and Figure 5 show respectively the experimental trajectory of no maneuver, C shape maneuver and S shape maneuver. Figure 2, Figure 4 and Figure 6 show respectively the attitude angle alignment error of no maneuver, C shape maneuver and S shape maneuver. Table 1, Table 2 and
Table 3 show respectively the attitude angle alignment error of the traditional algorithm and the improved algorithm under the conditions of no maneuver, C shape maneuver and S shape maneuver. According to the comparative analysis of Figure 2 and Table 1, Figure 4 and Table 2, Figure 6 and Table 3, the alignment effect of the improved algorithm on roll angle and pitch angle is similar to that of the traditional algorithm, but the alignment accuracy of yaw angle is better than that of the traditional algorithm, the yaw angle alignment accuracy is improved by 45%, 65% and 53% under the conditions of no maneuver, C shape maneuver and S shape maneuver, respectively.

| Experiment | Roll (degree) | Pitch (degree) | Yaw (degree) |
|------------|--------------|---------------|--------------|
| 1          | 0.0319       | -0.4970       | 0.4809       |
| 2          | 0.0317       | -0.4972       | -0.4972      |
| 3          | 0.0170       | -0.4878       | 0.4212       |
| Mean       | 0.0266       | 0.4933        | 0.4935       |

Figure 1. The trajectory of no maneuver.
Figure 2. Attitude angle alignment error.
Figure 3. C shape maneuver trajectory.
Figure 4. Attitude angle alignment error.
Table 2. Attitude angle alignment error of C shape maneuver.

|                  | Experiment 1 | Experiment 2 | Experiment 3 | Mean  |
|------------------|--------------|--------------|--------------|-------|
| Roll Angle(degree) | -0.0906      | -0.0911      | 0.0114       | 0.0644|
| Improved algorithm | -0.0899      | -0.0918      | 0.0116       | 0.0644|
| Pitch Angle(degree) | -0.2355      | -0.2211      | -0.5059      | 0.3208|
| Improved algorithm | -0.2357      | -0.2216      | -0.5061      | 0.3211|
| Yaw Angle(degree)  | -1.1694      | -1.1052      | 0.3911       | 0.8886|
| Improved algorithm | -0.2630      | -0.4581      | 0.2113       | 0.3108|

Figure 5. S shape maneuver trajectory.

Table 3. Attitude angle alignment error of S shape maneuver.

|                  | Experiment 1 | Experiment 2 | Experiment 3 | Mean  |
|------------------|--------------|--------------|--------------|-------|
| Roll Angle(degree) | 0.0233       | 0.0193       | 0.0321       | 0.0249|
| Improved algorithm | 0.0228       | 0.0194       | 0.0323       | 0.0248|
| Pitch Angle(degree) | -0.4770      | -0.4922      | -0.4987      | -0.4893|
| Improved algorithm | -0.4760      | -0.4922      | -0.4988      | -0.4890|
| Yaw Angle(degree)  | 0.4732       | 0.2562       | 0.2569       | 0.3288|
| Improved algorithm | 0.2910       | 0.0892       | 0.0748       | 0.1517|

5. Conclusion
In this paper, a traditional 14 state fast transfer alignment model is constructed to estimate the attitude misalignment angle. By analyzing the definition of the traditional velocity error equation, the improved differential equation of velocity error is applied to the transfer alignment, and an improved 12 state rapid transfer alignment model is built to estimate the attitude misalignment angle. Finally the algorithm is verified through the vehicle-mounted experimental data under the conditions of no maneuver, C shape maneuver and S shape maneuver, the experimental result shows that the improved algorithm on roll angle and pitch angle is similar to that of the traditional algorithm, but the alignment accuracy of yaw angle is better than that of the traditional algorithm. And the improved algorithm reduces the dimension of the filtering model and the amount of calculation, avoids the influence of flexural deflection and dynamic misalignment angle. The yaw angle alignment accuracy is improved.
by 45%, 65% and 53% under the conditions of no maneuver, C shape maneuver and S shape maneuver, respectively.

References

[1] Xia J.H., Zhang J.L., Lei H.J. (2017) Improved velocity plus attitude matching fast transfer alignment algorithm. Chinese journal of inertial technology, 25(01): 17-21.

[2] Havinga M.C.(2013) Flight Test Results of a MEMS IMU Based Transfer Alignment Algorithm for Short Range Air-to-Air Missiles. In: AIAA Guidance, Navigation, & Control. pp.5244-5346.

[3] Chu, H.R., Sun, T.T., Zhang, B.Q., et al. (2017) Rapid Transfer Alignment of MEMS SINS Based on Adaptive Incremental Kalman Filter. Sensors. 17(12): 152-165.

[4] Lyu W., Cheng X., Wang J.(2019)An Improved Adaptive Compensation H∞Filtering Method for the SINS' Transfer Alignment Under a Complex Dynamic Environment. Sensors. 19(2).

[5] Gong X., Zhang J. (2016) An innovative transfer alignment method based on federated filter for airborne distributed POS. Measurement. 86: 165-181.

[6] Chen K., Lu H., Zhao G., et al. (2008) Uniformity of transfer alignment attitude matching algorithm. Chinese journal of inertial technology. 16(2): 127-131.

[7] Chen K., Lu H., Yan G. (2008) Research on the consistency between rapid transfer alignment equation and traditional transfer alignment equation. Journal of northwest university of technology. 26(3): 326-329.

[8] Reiner J. (1996) In-flight transfer-alignment using aircraft-to-wing stiff-angle estimation. In: AIAA Guidance, Navigation and Control Conference. 237-245.

[9] Liu B., Mu R.J., Zhang X., Mi C.W., Cui N.G. (2016) Application of two rapid transfer alignment methods in airborne guidance weapons. Chinese journal of inertial technology. 24(02):141-147.

[10] Wang M.S., Wu W.Q., He X.F., Pan X.F. (2019) Further explanation and application of state transformation extended Kalman filter. Chinese journal of inertial technology. 27(04):499-504+509.

[11] Wang M.S., Wu W.Q., Zhou P.Y., et al. (2018) State transformation extended Kalman filter for GPS/SINS tightly coupled integration[J]. GPS Solutions. 22(4):112-.

[12] Wang M.S., Wu W.Q., He X.F., et al. (2019) State Transformation Extended Kalman Filter for SINS based Integrated Navigation System. In: 2019 DGON Inertial Sensors and Systems (ISS).

[13] Barfoot, Timothy D. (2017) State Estimation for Robotics [M]. Cambridge University Press, Cambridge.