Modeling the cambering of the flapping wings of an insect using rectangular shell finite elements

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Received: January 9, 2020; Accepted: June 3, 2020; Published: June 16, 2020

Abstract. Cambering in the flapping wings of insects plays an important role in the aerodynamic performance of their flight. In a previous study, the authors proposed a wing model using shell finite elements to elucidate the mechanism of cambering. However, the analysis of a strongly coupled fluid–structure system using this model would be quite computationally expensive because of the necessity of robust mesh-moving techniques. Therefore, in this study, a new wing model using rectangular shell finite elements is proposed. In the proposed model, the veins and membranes are described as pseudo-elastic materials. The cambering of the proposed model is investigated by comparison with the previous model.

Keywords: Insect flight, Flapping wing, Cambering, Shell element, Finite element method

1. Introduction

Camber deformation is observed in the flapping wings of various insects [1]. The magnitude of the camber is defined as the ratio of the wing height \( C_h \) to the wing chord length \( C_l \), where \( C_h \) is defined as the distance between the points C and H, and \( C_l \) is defined as the distance between the points A and B in Fig.1(a), and the sign of the camber is positive if the shape of the camber is concave along the direction of the flapping translation. Camber deformation plays an important role in determining the aerodynamic performance of flapping wings [2,3,4]. Therefore, it is important to elucidate the mechanism by which cambering occurs. However, the details of this mechanism are unclear when the interaction between the wing and the surrounding air is considered.

A wing model composed of beam and shell elements has been proposed to elucidate the role of the vein in the cambering of an insect’s flapping wing, as shown in Fig.1(b) [5]. In this model, the complex network of veins is modeled using two veins and a leading edge, and the role of the vein is considered as a static large deformation problem. An unstructured mesh is used in this model, meaning it would be quite computationally expensive when applied to the finite element analysis of a system with strong coupling between the fluid and structure be-
cause of the necessity of robust mesh-moving techniques [6,7]. In this study, a wing model composed entirely of shell elements was developed to eliminate the need for robust mesh-moving techniques. The proposed model is composed of rectangular shell elements with pseudo-elastic material properties that reproduce the behavior of the veins and membranes of the wing. The fundamental validity of the proposed model is presented by comparison with the previously developed model. Furthermore, the deformation characteristics of the proposed model were investigated.

2. Proposed wing model

The rectangular wing model (wing length \( R_w \), wing chord length \( c_w \)) is composed of rectangular (structured) shell elements, as shown in Fig.2. The wing root is defined as the area at \( x = 0 \) along the \( y \)-axis, where the fixed support is imposed. The pseudo-elastic material properties of each element are defined nonhomogeneously over the space, as shown in Fig.3, to represent the different behavior of the wing membranes and veins. In this figure, the colors represent the different functional roles of the elements and correspond to different material properties. The Young’s modulus is given such that the flexural rigidity of each vein is equivalent to that of the corresponding vein in the wing of an actual insect.

The flapping of the wing model is schematically shown in Fig.4 to illustrate the analytical concept in this study. As shown in this figure, the leading edge of the wing model flaps and translates in the stroke plane with a stroke angle \( \Phi \). The magnitude of the twisting at any given time is defined as the slope angle of the current chord at that time with respect to the direction of the initial wing chord (feathering angle \( \theta \)).

Based on actual observations of the flapping wings of insects [1], the time history of the angular velocity can be expressed as a trapezoidal function, as shown in Fig.5 [8], where the velocity \( \omega_{\text{max}} \) at the middle of each half-stroke is constant. Therefore, the dynamic pressure is dominant at the middle of each half-stroke. Furthermore, the maximum camber occurs at the middle of each half-stroke. Therefore, the time instant of the middle of each half-stroke is considered throughout this study. The pressure at this time instant is given as

\[
P = \frac{1}{2} C_d \rho \frac{V_{\text{max}}^2}{2}
\]  

\[(1)\]

Figure 1: (a) Sectional view of wing model. (b) Wing model composed of beam and shell elements [5].
where $C_D$ is the drag coefficient for the flat plate, $\rho^f$ is the fluid mass density, $r$ is the distance from the wing root, $\Phi$ is the stroke angle, and $T_\phi$ is the flapping period. The acceleration and deceleration times $t_\phi$ were set to $T_\phi/8$, which is a typical value [8, 9]. This pressure (1) causes the large deformation in the wing models in the following section. Therefore, a finite element analysis considering the geometrical nonlinearity is used.

\[ V_{\text{max}} = \omega_{\text{max}} r = \frac{8\Phi r}{3T_\phi}, \quad (2) \]

Figure 2: Wing model composed of only shell elements.

Figure 3: Pseudo-elastic material properties in the proposed wing model.

Figure 4: Schematic of flapping insect wing model.

Figure 5: Time history of angular velocity of flapping motion.

3. **Nonlinear finite element analysis of wing models**

3.1. **Comparison of the camber deformation of the models**

The wing length $R_w$, the wing chord length $c_w$, the wing kinematic parameters, and the material properties of the wing membrane were defined based on data obtained from actual insects [1,10–12]. $R_w$ and $c_w$ are given as 0.0113 m and 0.00311 m, respectively. The stroke angle $\Phi$ is given as 108°. The flapping frequency $f_\phi$ is given as 161 Hz, and the flapping period $T_\phi$ is defined as $1/f_\phi$. The fluid mass density $\rho^f$ is given as 1.205 kg/m³. The drag coefficient $C_D$ is
The thickness of the wing is given as 2.0 μm. The material properties of the vein and the leading edge were also set based on the flexural rigidity of an actual insect wing [13]. The flexural rigidity along the wing length is given as 4.0 μNm². The flexural rigidity along the wing chord length is given as 0.35 μNm². The Young’s modulus and Poisson’s ratio for the wing membrane are given as 1.0 GPa and 0.49, respectively. These properties are equal to those of the previous wing model [5] in Fig.1(b). However, different from this model, the area of the veins and leading edge $S_v$ in the proposed model is arbitrary parameter. This is because $S_v$ cannot be determined from the morphological data of actual insects. Therefore, the effect of $S_v$ on the deformation of the wing model is investigated by changing $S_v$. Here, two values of $S_v = 1,489,000 \mu m^2$ (or smaller $S_v$) and $3,032,000 \mu m^2$ (or larger $S_v$) are tested. Note that $S_v$ is varied by changing the area, where the material properties of the veins and the leading edge are given.

Figure 6 shows the deformation of the wing model with the smaller $S_v$ (Fig. 3). Figure 7 shows the generated camber plotted against the distance from the wing root. As shown in these figures, the positive camber is observed in the wing model with the smaller $S_v$, and this camber is comparable with that of actual insect wings [1,14,15]. This indicates that the proposed model has the fundamental ability to simulate the cambering of insect wings. In contrast, the negative camber is observed in the wing model with the larger $S_v$.

The rotational displacements of the root vein $\theta_x$, $\theta_y$, and $\theta_z$ were then evaluated to elucidate the cause of the difference between the cambers of these wing models. As shown in Fig.8, $\theta_x$, $\theta_y$, and $\theta_z$ are the rotational displacements of the nodes located along the central axis of the root vein. Figure 9 shows the rotational displacements of the root veins for the two wing models with the different $S_v$ values. In this figure, the rotational displacements are plotted against the distance from the root of the vein. As can be seen, the rotational displacement $\theta_y$ about the $y$-axis in the model with the smaller $S_v$ is far larger than that in the model with the larger $S_v$. This is because the width of the root vein in the model with the smaller $S_v$ is less than that in the model with the larger $S_v$, meaning the rotational stiffness of the former is less than that of the latter.

Figure 10 shows the $z$-displacements of the wing chords for the two wing models at distances.
from the root of the wing of 10%, 20%, and 30% of the longitudinal length of the wing; these
displacements illustrate the shapes of the chords. As seen in Fig.10(a), in the model with the
smaller $S_v$, the dynamic pressure causes a concave deformation in the wing chord between the
leading edge and the root vein, which forms a positive camber, because the root vein rotates
significantly in the cross section along the wing chord. In contrast, in the model with the
larger $S_v$, such a deformation is not observed, as shown in Fig.10(b), because the rotational
displacement of the root vein is very small. It follows from these results that the rotation of
the root vein is important for camber generation.

Figure 8: Definition of the rotational dis-
placements $\theta_x$, $\theta_y$, and $\theta_z$ of the root vein.
This wing model uses the smaller $S_v$.

Figure 9: Rotational displacements of the root
veins for the wing models with the smaller and
larger $S_v$.

Figure 10: Displacements of the wing chords in the $z$-direction. As the positions of these
chords, 10%, 20%, and 30% of the longitudinal length of the wing from the wing’s root are
considered. The models with the smaller and larger $S_v$ are used in (a) and (b). The circles
show the positions of the root vein, and the squares show the positions of the leading edge.

$S_v = 1,489,000 \mu m^2$  $S_v = 3,032,000 \mu m^2$

$\theta_x$  $\theta_y$  $\theta_z$  $\theta_x$  $\theta_y$  $\theta_z$

$\theta_x$  $\theta_y$  $\theta_z$
3.2. Modification of the constraint condition

The area in which the constraints are applied at the base was then varied to reduce the rotational displacement of the root vein, as shown in Fig.11. The purpose of this was to verify the importance of the rotation of the root vein in cambering, as was discussed in the previous section. In the analysis in the previous section, the length of the constrained area at the base $l_c$ was approximately 6% of the wing chord length $c_w$, whereas in this section, it was increased to range from approximately 8% to 17% of the wing chord length $c_w$.

Figure 12 shows an example of the deformation of the wing model in the case where $r_c$, which is defined as the ratio of $l_c$ to the wing chord $c_w$, was set to 15%. As can be seen, the cambering and twisting are adequately represented in the wing model at various values of $r_c$. The feathering angle $\theta$ at the tip of the wing is plotted against $r_c$ in Fig.13 to show its relationship to $l_c$. As shown in this figure, the increase of the constrained area at the wing’s base results in a reduction in $\theta$. Figure 14 shows the camber distributions for $r_c$ ranging from 6% to 17% along with that for the model wing using the beam and shell elements for comparison, and Fig.15 shows the rotational displacement $\theta_y$ of the root vein for the same range of $r_c$ values. As with Fig.13, Fig.14 demonstrates that the increase in the constrained area at the wing’s base results in reduced camber. This is because the increase in the constrained area at the base reduces $\theta_y$. It follows from these results that the rotational displacement of the root vein has a strong effect on camber generation. In addition, the magnitudes of the camber and the twisting can be controlled by modifying the constrained area at the base. The camber distribution in the case of $r_c = 15\%$ is very close to that of the previous model [5] in Fig. 1 (b). It shows the fundamental validity of the proposed model.

The model with $S_v = 1,489,000 \mu m^2$

(Fig. 3)

Figure 11: Increase of the constrained area at the wing’s root.

Figure 12: Deformation of the wing model for $r_c = 15\%$.

Figure 13: Feathering angle $\theta$ at the tip of the wing plotted against the ratio $r_c$ of $l_c$ to the wing chord length $c_w$. 
Figure 14: Camber distributions for the various \( r_c \), which ranges from 6\% to 17\%, and that for the previous model wing [5] in Fig. 1 (b).

4. Concluding remarks

In this study, the wing model composed of the regular shell elements was developed, and its fundamental validity was demonstrated. Then, the deformation characteristics of the proposed model were investigated. We found that the rotational displacement of the root vein has the strong effect on the camber generation. Furthermore, we found that the magnitude of the camber can be controlled by varying the constrained area at the base. This is because the increase of the constrained area at the base decreased the rotational displacement of the root vein. In our future work, the proposed model will be used for the finite element analysis considering the strong fluid–structure interaction in order to elucidate the cambering mechanism of insect wings.

Acknowledgement

This work was supported by JSPS KAKENHI Grant Numbers 17H02830 and 20H04199.

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