Spin-orbit induced anisotropy in the magnetoconductance of two-dimensional metals

R. Raimondi, M. Leadbeater

INFM e Dipartimento di Fisica "E. Amaldi", Università di Roma Tre, Via della Vasca Navale 84, 00146 Roma, Italy

P. Schwab

Institut für Physik, Universität Augsburg, 86135 Augsburg Germany

E. Caroti, C. Castellani

INFM e Dipartimento di Fisica, Università "La Sapienza", piazzale A. Moro 2, 00185 Roma Italy

(Received January 16, 2022)

Over the last five years there has been a significant amount of experimental evidence of a metallic-like temperature dependent resistivity for a certain electron density range in Si-MOSFETs and semiconductor heterostructures. This has called for a better understanding of the possible ground states of two-dimensional (2D) systems. For such an endeavour it is important from the outset to identify the relevant physical mechanisms at play. A recent summary of the main experimental facts and suggested theoretical models can be found in Ref.1. In the 2D systems which are experimentally investigated, the spin-orbit interaction, as due to the lack of structural inversion symmetry, has been proposed as a potentially relevant mechanism for the observed resistivity behaviour. However, there is no consensus at present about the importance of this spin-orbit interaction.

Magneto-transport experiments are very powerful tools in selecting processes which are affected by the presence of a magnetic field. In the case of 2D systems one of the most puzzling features is the magnetoresistance in the metallic regime. Here we calculate the conductivity in the framework of the Drude-Boltzmann theory. We do not worry about the microscopic origin of the scattering which is, for example, responsible for the strong magnetic field dependence of the conductivity. Our concern is the anisotropy of the conductance and its behaviour as a function of the various physical parameters. The most important findings are: a) the magnetoresistance is anisotropic; b) the sign of the anisotropy depends on the type of scattering potential seen by the charge carriers; c) the anisotropy factor has a maximum as function of the magnetic field, with the maximum position scaling with the density. In the following we present details of our calculations and discuss its possible experimental relevance.

We start from the model Hamiltonian

$$H = \frac{p^2}{2m} + \alpha \sigma \cdot \mathbf{p} \wedge \mathbf{e}_z - \frac{1}{2} g \mu_B \sigma \cdot \mathbf{B}$$

with $\alpha$ a parameter describing the spin-orbit interaction due to the confinement field and $\sigma$ being a vector whose components are the Pauli matrices. The unit vector $\mathbf{e}_z$ is perpendicular to the 2D plane and defines the $z$-axis. The magnetic field is chosen to lie in $x$-direction. One finds then two bands with the dispersion

$$E_{\pm}(p) = p^2/2m \pm \Omega(p),$$

with $\Omega(p) = |\alpha \mathbf{p} \wedge \mathbf{e}_z - \omega_c \mathbf{e}_z|$. In the problem we have several energy scales: the Fermi energy $E_F$ (in the absence of magnetic field with no spin-orbit coupling), the Zeeman energy $\omega_c = \frac{1}{2} g \mu_B B$ (note that this differs by a factor two from the standard definition), and the spin-orbit energy $\alpha p F$. It is useful to make a few estimates. In Si inversion layers one has a density-of-states $N_0 = 1.59 \times 10^{11} \text{ cm}^{-2} \text{meV}^{-1}$, which for the range of densities considered in Ref.1.
We obtain then

\[ \omega_\alpha = 5.1 - 9.5K. \]

By assuming \( g = 2 \) a magnetic field of \( 1 \) T gives \( \omega_\alpha = 0.6K. \) For the spin-orbit interaction, we take from Ref. 1 \( \alpha = 6 \times 10^{-6}K \) cm, which gives a spin-orbit energy \( \alpha p_F = 3.36 \) K. Considering transport introduces another scale, which we characterize by the scattering rate \( 1/\tau. \) We assume throughout this paper that \( \alpha p_F > \omega_s \) the polarization vector \( e_\alpha \) makes a full rotation when averaging over the Fermi surface. In the limit \( \omega_s > \alpha p_F \) however the polarization vector becomes locked with \( e_\alpha \approx -e_y. \) It is clear that one then finds a negative anisotropy in the conductance. Going explicitly through the algebra we arrive at

\[ \frac{\sigma_{xx} - \sigma_{yy}}{\sigma_0} = \frac{1}{2} \left( \frac{\alpha p_F}{\epsilon_F} \right)^2 \left( \frac{\omega_s}{\alpha p_F} \right)^2 \Theta \left( \frac{\omega_s}{\alpha p_F} - 1 \right), \]

which has an edge at \( \omega_s = \alpha p_F \) and becomes constant when \( \omega_s > \alpha p_F. \) In the strong field limit the arguments are different. For large field only the lower band is occupied. The Fermi surface again is almost rotational symmetric, but there is a weak elliptic distortion which leads to an anisotropy:

\[ \frac{\sigma_{xx} - \sigma_{yy}}{\sigma_0} = \frac{1}{2} \frac{(\alpha p_F)^2}{\omega_s \epsilon_F}. \]

From these considerations we find that the anisotropy for \( \omega_s \sim \epsilon_F \) scales as \( (\alpha p_F/\epsilon_F)^2. \)

In order to have a more microscopic calculation of the conductivity we now go beyond the relaxation time approximation. We introduce a disorder potential \( U, \) which will be treated in the self-consistent Born approximation and the conductivity will be calculated using the Green function formalism. For simplicity we assume a impurity potential with short range Gaussian correlations

\[ \langle U(x)U(x') \rangle = \frac{1}{2\pi N_0} \delta(x - x'). \]

The self-energy is determined self-consistently by the equation

\[ \Sigma = \frac{1}{2\pi N_0 \tau} \sum_p G(p), \]

where \( G(p) \) is the Green function. Notice that \( \Sigma \) and \( G(p) \) are \( 2 \times 2 \) matrices. Expanding the self-energy in Pauli matrix components, it turns out that the self-energy has no \( \sigma_2 \) and \( \sigma_3 \) components. As a result the self-energy will have the form \( \Sigma = \Sigma_0 \sigma_0 + \Sigma_1 \sigma_1. \) The real part of the self energy \( \Sigma_0 \) shifts the energy spectrum by a constant. Since we have to adjust the chemical potential in order to keep the particle number at a given value \( \text{Re} \Sigma_0 \) can be safely neglected. The real part of \( \Sigma_1 \) gives rise to a renormalization of the Zeeman energy. For the imaginary part, we find in the limit of weak disorder

\[ \text{Im} \Sigma_0 = -1/2 \tau_0 \approx -(N_+ + N_-)/4N_0 \tau \]

\[ \text{Im} \Sigma_1 = -1/2 \tau_1 \approx -(N_+ - N_-)/4N_0 \tau \]

The sum or difference \( 1/\tau = 1/\tau_0 \pm 1/\tau_1 \) are roughly the scattering rates for the two subbands. For weak magnetic field \( (\omega_s \sim \epsilon_F), \) the density of states in the two subbands

\[ n = 0.7 - 1.3 \times 10^{11} \text{cm}^{-2}, \]

\[ \frac{\omega_\alpha}{\epsilon_F} \]

\[ \epsilon_F \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]

\[ \sigma \]

\[ \langle \rangle \]

\[ \alpha \]

\[ \sum \]

\[ \pi \]

\[ \delta \]

\[ \delta \]

\[ \sum \]

\[ \tau \]
are identical and therefore $1/\tau_{\pm} = 1/\tau_0 = 1/\tau$. In the strong field limit ($\omega_s > \epsilon_F$) the upper band is depopulated, so that $1/\tau_1 = 1/\tau_0 = 1/2\tau$.

The current operator, which is necessary for the evaluation of the conductivity, is modified in the presence of the spin-orbit interaction:

$$j = \frac{P}{m} - \alpha \sigma \wedge e_z.$$  \hspace{1cm} (11)

The conductivity is obtained by the formula

$$\sigma_{ij} = \frac{e^2}{4\pi} \sum_{p} \text{Tr} \left[ j_i G^R j_{RA} G^A - j_i G^{RR} j_{RA} G^R + j_i^* G^R j_{RA}^* G^A - j_i^* G^{RR} j_{RA}^* G^R \right].$$  \hspace{1cm} (12)

The dressed vertex $J^i$ depends upon whether it is connected to a pair of retarded and advanced Green functions or a pair of Green functions with equal analytic properties.

In order to evaluate the conductivity, we have to determine the renormalized vertices and perform the momentum integrals. Before showing the complete results it is useful to consider the conductivity in the absence of vertex corrections and compare with the above Boltzmann equation analysis.

The result is shown in Fig. 2 (dashed lines). The conductivity may be expressed as a sum of intra- and interband contributions. Because of the gap between the two spin subbands, the interband terms do not contribute much unless the disorder is strong enough to produce a broadening of the Fermi surface of the two bands. The condition for weak disorder is then $\Omega \tau \gg 1$. In the weak disorder limit one reproduces – when $\omega_s < \epsilon_F$ – the results obtained from the Boltzmann equation (the solid line). For large magnetic field the Boltzmann result is not reproduced, even for very weak disorder. The reason is that even an isotropic scattering potential generates an anisotropic quasi-particle lifetime as demonstrated here below. Let us denote the scattering probability from state $p$ to $p'$ by $W_{pp'}$. When going from the spin- to the eigenstate basis the scattering probability in the lower band becomes

$$W_{pp'} \rightarrow W_{pp'} \frac{1}{2} \left( 1 + e_p \cdot e_{p'} \right) \approx W_{pp'} \left( 1 - \alpha^2 (p_x - p'_x)^2 / 4 \omega_s^2 \right).$$  \hspace{1cm} (13)

From this anisotropic scattering probability the inverse lifetime of an electron in state $p$ is determined as $1/\tau_p \approx 1 - (\alpha p_x / \omega_s)^2 / 4\omega_s^2 / \tau$ which definitely depends on the position on the Fermi surface. The anisotropic scattering rate then also contributes to the anisotropy in the magnetoconductance.

We are now ready to consider the vertex corrections. For the choice of disorder potential we have made, these corrections usually vanish expressing the fact that the relaxation of momentum is governed by the quasiparticle lifetime. However, in the present problem there are specific current vertices due to the spin-orbit interaction which acquire a vertex correction. The vertex corrections are obtained in the standard way by solving the equation

$$\Gamma_{i} = \frac{1}{2\pi N_0 \tau} \sum_{p} G^R j_{RA} G^A.$$  \hspace{1cm} (14)

By separating the $p$-dependent (i.e., proportional to the $p$ vector) and $p$-independent parts $\gamma^i = \gamma^i_{ss'} + \gamma^i_{aa'}$ and $\gamma^i_{ss'} = \gamma^i_{ss'} + \gamma^i_{aa'}$, one notices that as a consequence of isotropic scattering, the $p$-dependent part is not dressed, i.e., $\gamma^i_{ss'} = \gamma^i_{ss'} + \gamma^i_{aa'}$. The equation for the momentum independent part of the current vertex reads

$$\Gamma_{i} = \frac{1}{2\pi N_0 \tau} \sum_{p} G^R j_{RA} G^A.$$  \hspace{1cm} (15)

We included here the matrix (=spin) indices $s$, $s'$, $a$, and $b$. The quantities $\tilde{\gamma}^i_{ss'}$ are a sum of the bare vertices $\gamma^i_{ss'}$ and a term which is generated by $p/m$,

$$\tilde{\gamma}^i_{ss'} = \gamma^i_{ss'} + \frac{1}{2\pi N_0 \tau} \sum_{p} G^R_{ss'} (p^i/m) G^A_{aa'}.$$  \hspace{1cm} (16)

The above equation may be solved by expanding all matrices in terms of the Pauli matrix basis, e.g., $\Gamma = \sum_{\eta = 0,1,2} \sigma_{\eta} \Gamma_{i}^{\eta}$. By skipping the details of the derivation we present the numerical results for the renormalized vertices and the conductivity in Figs. 3 and 4, respectively. One observes that for weak magnetic field the quantities $\Gamma$ become very small so that $\Gamma_{i} \approx p / m$ i.e. the anomalous velocity is cancelled by the vertex corrections! Only for fields which are at least of the order of the Fermi energy does an anomalous velocity contribution survive.

Concerning the conductivity the most striking result is that the vertex corrections, for not too strong magnetic field, change the sign of the anisotropy. (See Fig. 3).
The dressed vertices $\Gamma_0 \ldots \Gamma_3$ in units of $\alpha$ as a function of the magnetic field. Here we chose $\alpha \rho F = 0.4 e_F$ and $1/(e_F^F \tau) = 0.4$. Remember that the bare vertices are $\gamma_0 = \gamma_3 = 0$ and $\gamma_1 = -\gamma_2 = \alpha$; the asymptotic values in the strong field limit are $\Gamma_1 = -\Gamma_3 = \alpha/2$, $\Gamma_2 = -\alpha$, and $\Gamma_3 = 0$. Notice that $\Gamma_2$ reaches the asymptotic value for high magnetic field only very slowly.

FIG. 4. Anisotropy in the magneto-conductance for $1/(e_F^F \tau) = 0.2$ and various spin-orbit energies. For comparison we included the results of the Boltzmann theory ($\alpha \rho F = 0.4 e_F$; full line). For not too strong fields, the vertex corrections change the sign of the effect.

To understand the effect of the vertex correction, it is again useful to consider the scattering rate in the eigenstate basis, see eq. (13). One then observes that along the $x$ direction, forward scattering is enhanced compared to backward scattering, while along the $y$ direction there is no difference between forward and backward. Because vertex corrections increase conductivity if forward scattering is favoured, one expects that vertex corrections enhance $\sigma_{xx}$ most contributing to $\Delta \sigma$ with a positive term, which for low and moderate magnetic fields overcomes the negative contribution discussed previously.

We now comment on our results in the light of the available experiments. Both in the relaxation time approximation and in the Green function calculation the maximum anisotropy in the conductivity occurs at $\omega_x \sim e_F$ irrespective of the value of the spin-orbit energy $\alpha$. This means that the energy of the peak scales with the electron density. This is in agreement with the experiments. A second feature is that the peak strength is controlled by the spin-orbit energy in units of $e_F$. Hence the peak strength must scale with the inverse density, so that the effect is expected to decrease going more deeply into the metallic regime. This again is in agreement with the experimental findings [4]. Concerning the sign of the effect, inclusion of the vertex corrections in the Green function calculation spoils the agreement with the experiments. A possible reason for this discrepancy may be due to our simplifying assumption of a short range $p$-independent disorder potential. Our analysis has shown in fact that the sign of the anisotropy depends crucially on the effective $p$-dependence of the scattering rate in the eigenstate basis. It is then reasonable to expect that the specific choice of the scattering potential may indeed play a role in determining the sign of the effect a low fields. This analysis, while it is worth pursuing, is however beyond the scope of the present paper.

In conclusion, our theory explains how the spin-orbit coupling may give rise to the anisotropy in the conductivity, although the interpretation of the experiments may require more realistic models of disorder, as for example that due to scattering by impurities located outside the 2D plane, which favours small-angle scattering. It is also our hope that this work will stimulate more experimental effort toward the investigation of anisotropic conductivity as a function of the scattering potential or in samples with different mobilities.

This work was partially supported by MURST under contract no. 9702265437 (R.R. and C.C.), by INFM under the PRA-project QTMD (R.R.) and the European Union TMR program (M.L.) and the DFG through SFB 484 (P.S. and R.R.).
kler, Phys. Rev. Lett. 84, 5592 (2000) (cond-mat/9911239).

9 V.M. Pudalov, G. Brunthaler, A. Prinz, and G. Bauer, cond-mat/0004206.

10 V.S. Khrapai, E.V. Deviatov, A.A. Shashkin, V.T. Dolgopolov, cond-mat/0005377.

11 For hole carrier GaAs heterostructures one has a density-of-states of $N_0 = 1.8 \times 10^{11} \text{ cm}^{-2}\text{meV}^{-1}$ with $m = 0.38m_0$ and for the density range $n = 1 - 6 \times 10^{10} \text{ cm}^{-2}$ considered in the experiments one has a Fermi energy $\epsilon_F = 0.6 - 3.8$ K.