Is the Cabibbo-Kobayashi-Maskawa Matrix Symmetric at the GUT Scale?

Zhi-zhong Xing

Department of Physics, Nagoya University, Chikusa-ku, Nagoya 464-01, Japan

Abstract

By use of the one-loop renormalization group equations and current experimental data, we study the off-diagonal asymmetries of the Cabibbo-Kobayashi-Maskawa (CKM) matrix at the GUT scale in the framework of the standard model as well as its two Higgs and supersymmetric extensions. It is concluded that the possibility of a symmetric CKM matrix at the GUT scale has almost been ruled out.

(Accepted for publication in J. Phys. G)

1Electronic address: xing@eken.phys.nagoya-u.ac.jp
In the standard SU(3) × SU(2) × U(1) gauge model, quark mixing and CP violation can be naturally described by the 3 × 3 Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$. The geometrical structure of $V$ is characterized by its two off-diagonal asymmetries, one about the $V_{ud} - V_{cs} - V_{tb}$ axis (denoted by $\Delta_1$) and the other about the $V_{ub} - V_{cs} - V_{td}$ axis (denoted by $\Delta_2$): 

$$
\Delta_1 \equiv |V_{us}|^2 - |V_{cd}|^2 = |V_{cb}|^2 - |V_{ts}|^2 = |V_{td}|^2 - |V_{ub}|^2,
$$
$$
\Delta_2 \equiv |V_{us}|^2 - |V_{cb}|^2 = |V_{cd}|^2 - |V_{ts}|^2 = |V_{tb}|^2 - |V_{ud}|^2.
$$

Some speculation has recently been made on the possibility of $\Delta_1 = 0$ and its consequences at low-energy scales [2, 3, 4]. This possibility is of particular interest on the point that it allows $V$ to be a symmetric matrix (about the $V_{ud} - V_{cs} - V_{tb}$ axis), which can be fully described in terms of only three independent parameters. A careful analysis shows that $\Delta_1 \sim 10^{-5} - 10^{-4}$ and $\Delta_2 \geq 400\Delta_1$ are favoured by current experimental data [5]. This implies that the possibility of a symmetric CKM matrix has almost been ruled out at low-energy scales.

A natural question to be asked is if there still exists the possibility for $\Delta_1 = 0$ at a superheavy scale, e.g., the grand unification theory (GUT) scale. In a specific GUT framework $\Delta_1 = 0$ is possible to hold as the consequence of a hidden symmetry, and the nonvanishing value of $\Delta_1$ at low-energy scales may come purely from the different evolution effects of $|V_{us}|^2$ and $|V_{cd}|^2$ (or other relevant CKM matrix elements). Although many renormalization-group analyses of the CKM matrix were made in the literature (see, e.g., [6, 7]), a careful look at the evolution behaviour of $\Delta_1$ and $\Delta_2$ has been lacking.

A symmetric CKM matrix at (or above) the GUT scale should be theoretically interesting. Since the CKM matrix is basically determined by quark mass matrices, its nine elements are expected to be calculable in terms of the quark mass ratios (as well as the possible CP-violating phases) in the framework of a theory more fundamental than the standard model. At low-energy scales we have known that it is impossible to derive the exact symmetric CKM matrix from any quark mass ansatz, due to the observed difference between quark mass ratios $m_u/m_c$ and $m_d/m_s$ as well as that between $m_c/m_t$ and $m_s/m_b$ [8]. Imposing specific but reasonable symmetries on quark mass eigenvalues at a superheavy scale (e.g., the left-right symmetry together with a $SU(2)_R$ symmetry of Yukawa couplings and a $U(1)$ horizontal family symmetry [9]), one would have some chance to obtain the symmetric quark flavor mixing matrix. Reversely, if a symmetric CKM matrix at the GUT scale were favored by
current experimental data, then it could provide us some hints towards the dynamical details or symmetries of quark mass generation.

In this note we shall investigate the magnitudes of $\Delta_1$ and $\Delta_2$ at the GUT scale by use of the one-loop renormalization group equations and current data. For $\Delta_1 \sim 10^{-5} - 10^{-4}$ at low-energy scales, we find that the possibility of $\Delta_1 = 0$ has almost been ruled out at $\mu = 10^{16}$ GeV in the framework of the minimal supersymmetric standard model (MSSM) or at $\mu = 10^{14}$ GeV in the framework of the two Higgs doublet model (2HM) for all perturbatively allowed values of $\tan \beta$. In the context of the standard model (SM), $\Delta_1$ increases with energy while $\Delta_2$ decreases with energy. The evolution effect of $\Delta_2$ is only at the percent level for every model under discussion.

Without loss of generality, the one-loop renormalization group equations for the CKM matrix elements can be written as [7]:

$$16\pi^2 \frac{d}{dt} |V_{i\alpha}|^2 = 3c \left[ \sum_{j \neq i} \sum_{\beta = d,s,b} f_j^2 + 3a \sum_{\beta = d,s,b} f_\beta^2 + a \sum_{\beta' = e,\mu,\tau} f_{\beta'}^2 - G_U + \frac{3b}{2} f_i^2 + \frac{3c}{2} \sum_{\beta = d,s,b} f_\beta^2 |V_{i\beta}|^2 \right],$$

$$16\pi^2 \frac{df_i}{dt} = f_i \left[ 3 \sum_{j = u,c,t} f_j^2 + 3a \sum_{\beta = d,s,b} f_\beta^2 + a \sum_{\beta' = e,\mu,\tau} f_{\beta'}^2 - G_U + \frac{3b}{2} f_i^2 + \frac{3c}{2} \sum_{\beta = d,s,b} f_\beta^2 |V_{i\beta}|^2 \right],$$

$$16\pi^2 \frac{df_\alpha}{dt} = f_\alpha \left[ 3a \sum_{j = u,c,t} f_j^2 + 3 \sum_{\beta = d,s,b} f_\beta^2 + a \sum_{\beta' = e,\mu,\tau} f_{\beta'}^2 - G_D + \frac{3b}{2} f_\alpha^2 + \frac{3c}{2} \sum_{j = u,c,t} f_j^2 |V_{j\alpha}|^2 \right],$$

$$16\pi^2 \frac{df_{\alpha'}}{dt} = f_{\alpha'} \left[ 3a \sum_{j = u,c,t} f_j^2 + 3 \sum_{\beta = d,s,b} f_\beta^2 + a \sum_{\beta' = e,\mu,\tau} f_{\beta'}^2 - G_E + \frac{3b}{2} f_{\alpha'}^2 \right],$$

where $t \equiv \ln(\mu/M_Z)$ with $M_Z = 91.187$ GeV, the Latin (Greek) indices stand for the up (down) quarks, $c$ is a model-dependent coefficient, $f_i$ ($i = u, c, t$) and $f_\alpha$ ($\alpha = d, s, b$) are the eigenvalues of the Yukawa coupling matrices. The evolution equations of $f_i$, $f_\alpha$ and $f_{\alpha'}$ ($\alpha' = e, \mu, \tau$) read [7]:

$$G_F = \sum_{n=1}^{3} \left( C_n F_n g_n^2 \right), \quad 8\pi^2 \frac{dg_n^2}{dt} = b_n g_n^4.$$  \hspace{1cm} (4)

Here $C_n^F$ and $b_n$ are also model-dependent coefficients. The values of all such coefficients have been listed in Table 1 for the SM, 2HM and MSSM, respectively.
Table 1: The values of the model-dependent coefficients in the one-loop renormalization group equations for the CKM matrix elements, the eigenvalues of the Yukawa coupling matrices and the gauge couplings.

| Coefficients | SM       | 2HM      | MSSM     |
|--------------|----------|----------|----------|
| \{a, b, c\}  | \{1, 1, -1\} | \{0, 1, \frac{1}{3}\} | \{0, 2, \frac{2}{3}\} |
| \{b_1, b_2, b_3\} | \{\frac{41}{6}, -\frac{19}{6}, -7\} | \{7, -3, -7\} | \{11, 1, -3\} |
| \{C_U^1, C_U^2, C_U^3\} | \{\frac{17}{12}, \frac{9}{4}, 8\} | \{\frac{17}{12}, \frac{9}{4}, 8\} | \{\frac{13}{9}, 3, \frac{16}{3}\} |
| \{C_D^1, C_D^2, C_D^3\} | \{\frac{5}{12}, \frac{9}{4}, 8\} | \{\frac{5}{12}, \frac{9}{4}, 8\} | \{\frac{7}{9}, 3, \frac{16}{3}\} |
| \{C_E^1, C_E^2, C_E^3\} | \{\frac{15}{4}, \frac{9}{4}, 0\} | \{\frac{15}{4}, \frac{9}{4}, 0\} | \{3, 3, 0\} |

Next we carry out a numerical analysis of the evolution effects for $\Delta_1$ and $\Delta_2$ by use of the above equations. The initial values of gauge couplings at $\mu = M_Z$ (i.e., $t = 0$) are taken as $g_1^2 = 0.127$, $g_2^2 = 0.42$ and $g_3^2 = 1.44$ \[7\]. We take the light quark masses at $\mu = 1$ GeV to be $m_u = 5.6$ MeV, $m_d = 9.9$ MeV, $m_s = 199$ MeV, $m_c = 1.35$ GeV and $m_b = 5.3$ GeV \[10, 11\]. These masses evolve up to $\mu = M_Z$ due only to gauge interactions, and this running effect can be approximately described by a common factor 0.58 for their values at $\mu = 1$ GeV \[12\]. The top-quark mass is typically taken as $m_t = 180$ GeV at $\mu = M_Z$. The charged lepton masses can be found from ref. \[11\]. The eigenvalues of the Yukawa coupling matrices are the ratios of the fermion masses to the Higgs vacuum expectation value $v$ (normalized to 175 GeV), and they may depend on the ratio of the two vacuum expectation values (defined by $\tan \beta$) in the 2HM and MSSM. We also choose $|V_{ud}| = 0.9744$, $|V_{us}| = 0.2205$ and $|V_{cb}| = 0.040$ as input parameters \[11\]. Since $\Delta_1 \sim 10^{-5} - 10^{-4}$ is allowed at low-energy scales \[2, 3\], we typically take $\Delta_1 = 1 \times 10^{-5}$, $5 \times 10^{-5}$ and $1 \times 10^{-4}$ at $\mu = M_Z$ in our calculations. The main results about the magnitudes of $\Delta_1$ and $\Delta_2$ at the GUT scales are illustrated in figs. 1 – 4. Some discussions are in order.

(1) One can see from fig. 1 that the value of $\Delta_1$ increases with energy in the context of the SM. Thus there is no possibility for $\Delta_1^{\text{sm}}$ to be vanishing at any superheavy scale.
(2) In the framework of the 2HM, the magnitude of $\Delta_1$ decreases with energy, and its behaviour as a function of $\tan \beta$ at the GUT scale $\mu = 10^{14}$ GeV is shown in fig. 2. We observe that $\Delta_{1\text{hm}}^2$ remains to be nonvanishing for the low-energy inputs $1 \times 10^{-5} \leq \Delta_1 \leq 10^{-4}$. If the value of $\Delta_1$ at $\mu = M_Z$ were much smaller than $10^{-5}$, $\Delta_{1\text{hm}}^2 = 0$ would be possible in the perturbative region of $\tan \beta$.

(3) It is more interesting to look at the evolution effect of $\Delta_1$ in the framework of the MSSM. At the GUT scale $\mu = 10^{16}$ GeV, the changes of $\Delta_{1\text{mssm}}^1$ with $\tan \beta$ are illustrated in fig. 3. Clearly $\Delta_{1\text{mssm}}^1 < \Delta_{1\text{hm}}^2$ for the same input of $\tan \beta$. Taking $\Delta_1 = 0$ at low-energy scales, we find that $\Delta_{1\text{mssm}}^1 < 0$ in the perturbative region of $\tan \beta$. This implies that there would exist the possibility for $\Delta_{1\text{mssm}}^1 = 0$ if the low-energy value of $\Delta_1$ were well below $10^{-5}$ (e.g., $\Delta_1 \sim 10^{-7}$ or smaller).

(4) As indicated by current experimental data [2, 5], the chance for $\Delta_1$ to be larger than $10^{-4}$ or smaller than $10^{-5}$ at low-energy scales is very tiny. Although the possibility of $|V_\text{us}| = |V_\text{cd}|$ has not been absolutely ruled out by direct measurements, it is expected to be impossible from the combined data on quark mixing and CP violation [11]. In this sense, we may conclude that the possibility of a symmetric CKM matrix at the GUT scales is almost ruled out by current low-energy data.

(5) The evolution behaviour of $\Delta_2$ with energy is opposite to that of $\Delta_1$ in the context of the SM, 2HM or MSSM. Since $\Delta_2 \approx |V_\text{us}|^2$ at low-energy scales, it has no possibility to be vanishing at any GUT scale. For illustration, we plot the ratio $\Delta_2(\mu)/\Delta_2(M_Z)$ as a function of $\tan \beta$ in fig. 4, where $\Delta_2(M_Z) = 0.047$ has been used as the input. We see that the running factor of $\Delta_2$ in each model is only at the percent level.

In summary, we have studied the off-diagonal asymmetries of the CKM matrix ($\Delta_1$ and $\Delta_2$) at the GUT scale by use of the one-loop renormalization group equations. For $\Delta_1 \sim 10^{-5} - 10^{-4}$ at low-energy scales, the possibility of $\Delta_{1\text{mssm}}^1 = 0$ (or $\Delta_{1\text{hm}}^2 = 0$) has almost been ruled out at $\mu = 10^{16}$ GeV (or $\mu = 10^{14}$ GeV) for all perturbatively allowed values of $\tan \beta$. In contrast, $\Delta_1$ increases with energy in the framework of the SM. The magnitude of $\Delta_2$ decreases with energy in the context of the SM, but it increases with energy in the context of the 2HM or the MSSM.
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Figure 1: The off-diagonal asymmetry $\Delta_1^{\text{em}}$ as a function of the scale $\mu$ (up to $\mu = 10^{14}$ GeV) in the framework of the SM. Here the low-energy inputs are $\Delta_1 = 1 \times 10^{-4}, 5 \times 10^{-5}$ and $1 \times 10^{-5}$, respectively.

Figure 2: The off-diagonal asymmetry $\Delta_1^{\text{2hm}}$ (at the scale $\mu = 10^{14}$ GeV) as a function of $\tan \beta$ in the framework of the 2HM. Here the low-energy inputs are $\Delta_1 = 1 \times 10^{-4}, 5 \times 10^{-5}$ and $1 \times 10^{-5}$, respectively.
Figure 3: The off-diagonal asymmetry $\Delta_1^{\text{msm}}$ (at the scale $\mu = 10^{16}$ GeV) as a function of $\tan \beta$ in the framework of the MSSM. Here the low-energy inputs are $\Delta_1 = 1 \times 10^{-4}$, $5 \times 10^{-5}$ and $1 \times 10^{-5}$, respectively.

Figure 4: The ratio of the off-diagonal asymmetry $\Delta_2$ at the GUT scale to that at the low-energy scale as a function of $\tan \beta$ in the framework of the SM ($\mu = 10^{14}$ GeV), 2HM ($\mu = 10^{14}$ GeV) and MSSM ($\mu = 10^{16}$ GeV), respectively. Here $\Delta_2(M_Z) = 0.047$ has been used.