THE KINEMATICAL CENTER AND MASS PROFILE OF THE LOCAL GROUP

ALAN B. WHITING
University of Birmingham, Edgbaston Road, Birmingham B15 2TT, UK
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ABSTRACT
Abandoning the assumption that light traces mass, I seek the location of the center of the Local Group of galaxies based solely on kinematic data and the plausible assumption of infall. The available set of positions and radial velocities is shown to be a misleading indicator of Local Group motions, giving a direction to the center offset from the true one; statistical techniques of moderate sophistication do not catch the offset. Corrected calculations show the center to lie in the direction of M31 within the uncertainty of the method, within a few degrees. The distance to the center is not well determined, lying about 0.5 Mpc from the Milky Way. The pattern of observed (galactocentric) radial velocities excludes both dynamically important “orphan halos” and any extended dark matter halo for the Group as a whole, and shows the Group to have formed from a much more extended volume than it presently occupies. Kinematics alone indicates that the mass of the Group is concentrated effectively in M31 and the Milky Way.

Key words: dark matter – galaxies: kinematics and dynamics – Local Group

1. THE LOCAL GROUP: MASS AND MOTION

The Local Group of galaxies, the concentration of perhaps four dozen galaxies around the Milky Way and M31, has proven useful for many years as a dynamical and cosmological laboratory. Not only does it contain those objects that may be studied in greatest detail (due to their proximity), it was for a long time the only such structure whose shape in three dimensions was well understood.

Perhaps the first important dynamical study based on the Local Group was the venerable timing argument of Kahn & Wolff (1959). Making the assumptions that the only significant mass in the Group resided in the Milky Way and M31 and that each of these might be treated as a point particle all the way back to the big bang, the resulting estimate of mass for the whole Local Group was an early indication of the presence of large amounts of dark matter. The argument was developed by Lynden-Bell (1981) to include other galaxies in the Group and was assumed to be massless and on radial trajectories directed toward or away from the barycenter. In spite of the simplistic picture of the Group that is assumed, the timing argument remains a useful point of reference, and updated treatments and versions of it may be found in, for example, Lynden-Bell (1999), Whiting (1999), and on pages 150 and 268 of Binney & Tremaine (2008).

A much more sophisticated look at Local Group dynamics is found in the Least Action calculations as developed, for instance, in Peebles et al. (2001) and found most recently in Peebles et al. (2011). Here, the trajectories of the various galaxies are subject to only reasonable restrictions. The assumptions, however, are retained of the Milky Way and M31 containing most of the Group’s mass (though their masses and those of others are allowed to vary somewhat between solutions) and of galaxies retaining their identities back to a very early time.

There is no question that galaxies have mass or that the Milky Way and M31 have rather a lot of it. However, the assumption of a close relation between mass and light remains an assumption on scales of, say, 100 kpc to several megaparsecs, and therefore should be examined if possible. This is especially so when there are indications pointing toward its modification. Dunn & Laflamme (1995) found significant discrepancies between the parameters of N-body simulations and the corresponding quantities calculated by treating the simulations as a least-action problem; discrepancies were attributed to the presence of “orphan” dark matter halos containing no galaxies. More recently, Whiting (2005) discovered that the kinematics of nearby galaxy groups does not match that expected qualitatively, a discrepancy that Martins-Vaquero et al. (2007) attribute again to “orphan” dark matter halos. Along similar lines to Dunn & Laflamme (1995), Li & White (2008) work out a correction to the timing argument based on more recent N-body work.

The purpose of the present study is to discard the assumption that light traces mass and to see how far we can go without it. In particular, what can the motions alone of galaxies within the Local Group tell us about its mass distribution? We will use galaxies as tracers of the velocity field and (assuming velocities to be due to gravity) use the velocity field to infer the mass distribution. In particular, we are looking to see whether the inferred mass matches the location of visible galaxies or perhaps indicates the existence of dynamically important “orphan halos” or an extended dark matter halo.

1.1. The Picture

We assume that there is a mass concentration associated with the Local Group, such as to produce an identifiable center. Next, in consideration of crossing times, the Group (at least beyond a certain distance from the center) must still be in the process of infall, and it is plausible that speeds of infall will be larger closer to the center. Outside a certain radius, galaxies will physically move away, though more slowly than in the general Hubble flow. The situation is illustrated in Figure 1.

This assumption of a mass concentration and radial infall is plausible, but must be checked when possible. Some possible signs indicating failure are mentioned below.

As observed from a position outside the center and participating in the infall, radial velocities will be positive toward the center and away from it, very negative on the far side, and smaller in magnitude looking off this axis, for purely geometric
Figure 1. An illustration of the assumed dynamical situation of the Local Group. Galaxies are falling in to the center (a few, not shown, may have passed it and are coming out again). Their speeds are larger as they are closer to the center. Outside the region shown, galaxies will begin to move away as they join the Hubble flow. As seen from one of the galaxies (marked with a circle), radial velocities will be of the smallest magnitude at roughly right angles to the direction of the center due to geometric effects. On the line to the center, galaxies on the same side as the observer will show positive radial velocities; beyond the barycenter, galaxies will show negative radial velocities. The greatest radial speeds will be found on this line.

reasons. Close to the center, the picture may be complicated by galaxies that have fallen in from the far side and are on their way out again, but in any case, the maximum observed radial speeds (positive or negative) will be in the direction of the center and directly away from it. Within this picture, then, the direction of the maximum magnitude of radial velocities will point toward the center of the Group. The distance to the center will be signaled by a sudden shift from positive to negative radial velocities or by a mixture of positive and negative velocities in some region.

The picture need not be exact to be useful. In particular, the center itself may be unoccupied by anything observable, and the velocity field may have holes in it at any point. Small-scale deviations and uncorrelated motions should average out, though in a sparsely sampled Group they might not do so as well as we would wish.

Conversely, a direction of maximum radial speeds can be calculated for any situation, even one in which there is no identifiable center, or in which the field of velocities does not have an overall pattern. We could detect the latter cases by taking various subsets of the observations and performing a calculation: the direction would fluctuate strongly depending on the particular galaxies included. In the case of the Local Group, if mass indeed follows light, we expect the center to lie in the direction of M31.

An off-center mass concentration within the Group with galaxies around it will show a large radial-velocity signal not associated with the Group as a whole; how this is dealt with in the case of M31 and the Milky Way is set out below.

For this calculation, we must correct heliocentric radial velocities for the motion of the Sun around the Galaxy, a correction that is not as accurately known as one might like, but we may use the resulting galactocentric radial velocities with no further correction.

2. FINDING THE VERTEX

2.1. The Sample

In attempting an analysis of the dynamics of the Local Group, one must first decide where its limits are. Including galaxies in the neighboring galaxy groups risks distorting conclusions by the actions of masses there, so no objects beyond 1.5 Mpc should be used, and those close to the border should be scrutinized for possible disturbance. For our purposes, a near limit also needs to be chosen. Including the many satellite galaxies of the Milky Way and M31 would show that those dominant galaxies have significant mass concentrations, which is not in question. However, we seek kinematic clues to mass distribution on a larger scale, so satellites must be excluded. How far out the influence of the bright galaxies dominates kinematics is not clear; McConnachie et al. (2009) discovered evidence that M33, over 200 kpc from M31, may have been influenced by the latter significantly. For the following calculations, I adopt 200 kpc as
a standard cutoff distance; however, I will occasionally explore others.

Data for galaxies used in the kinematic center calculations were taken from the NASA Extragalactic Database (NED),\(^1\) with the particular help of Ian Steers. All galaxies within 1.5 Mpc of either the Milky Way or M31 were initially selected, and those objects that are clearly part of the Galactic or Andromeda systems (such as the Sagittarius galaxy, now being cannibalized, and a few globular clusters) removed. Objects without a radial velocity could not be used and were dropped. Some of the averaged distances listed in NED were adjusted to give greater weight to newer and more accurate results. From the list of 44 galaxies thus obtained, an initial cut to remove satellite galaxies within 100 kpc of M31 or the Milky Way brought the total down to 33; most of the calculations were done on a final set of 25 using a 200 kpc cut. The galaxies and relevant data are listed in Table 1. The data here used for the 25 galaxy set are almost identical with the slightly larger sample of Peebles et al. (2011). The distribution of the final 25 galaxy sample on the sky is shown in Figure 2.

To perform the correction for solar motion, I follow Peebles et al. (2011). Following the indications in Reid et al. (2009) that the accepted value of 220 km s\(^{-1}\) for the circular velocity of the Sun may be too small, I perform calculations for 230 km s\(^{-1}\) and separately for 260 km s\(^{-1}\), covering the indicated range. Results for the 230 km s\(^{-1}\) correction are designated “w1” hereafter, and those for 260 km s\(^{-1}\) “w2.” To correct for solar motion relative to the local standard of rest, I use the figures of Schönrich et al. (2010).

A plot of the (w1) corrected radial velocities on the sky is given in Figure 3. Although there are deviations, in overall appearance, it agrees with the picture of Figure 1: the largest magnitudes of radial velocity occur in two roughly opposite directions, with one of them containing the most negative figures. A plot of the w2 velocities shows the same pattern.

The effect of observational uncertainties on our calculations is expected to be negligible. Radial velocities are known to a few km s\(^{-1}\), two orders of magnitude more precise than the solar motion. Positions are known to a fraction of a degree; as will be seen, noise in the velocity field (that is, the fact that the Local Group does not strictly follow a radial infall pattern) dominates any error from this source. Distances are not used in the following calculation, which seeks only the direction to the center. They will be used to infer the mass profile, and distance errors will be considered in that section.

2.2. Calculations

Our goal is to find the direction in which the observed radial velocities have a maximum in magnitude. The most straightforward formulation is to calculate the quantity

\[ U(\mathbf{r}) = \sum_{j} |\mathbf{r} \cdot \mathbf{v}_j|, \]

where \( \mathbf{r} \) is the direction we are varying and \( \mathbf{v}_j \), the observed radial velocities, and find the direction that gives a maximum. This was done with the w1 and w2 corrections for solar motion, and as a check, also with 170 and 200 km s\(^{-1}\) corrections. For the w1 correction, a bootstrap calculation yielded uncertainties in longitude and latitude. Finally, a jackknife was run to estimate bias. The results are given in Table 2 and Figure 4.

The immediately obvious feature of the results is their offset from M31, over 30 degrees away. It is not due to an inaccurate correction for solar motion in the Milky Way, changing that by some 90 km s\(^{-1}\) has very little effect. The jackknife calculation shows some bias, but nowhere near enough to explain the offset, and it does not lead toward M31.

Table 1

| Name          | \(l\) | \(b\) | \(\mu\) | \(rv\) |
|---------------|------|------|-------|------|
| Milky Way     | 0    | 0    | 0.05  | 2    |
| LMC*          | 280.5| −32.9| 18.48 | 0.1  | 278 ± 2 |
| SMC*          | 302.8| −44.3| 18.85 | ±0.1 | 158 ± 4 |
| UMi*          | 105.0| 44.8 | 19.45 | ±0.1 | 247 ± 1 |
| Draco*        | 86.4 | 34.7 | 19.65 | ±0.2 | 292 ± 1 |
| Sextans*      | 243.5| 42.3 | 19.85 | ±0.1 | 224 ± 2 |
| Sculptor*     | 287.5| −83.2| 19.64 | ±0.05| 110 ± 1 |
| Carina*       | 260.1| −22.2| 20.05 | ±0.05| 229 ± 60 |
| Fornax*       | 237.1| −65.7| 20.65 | ±0.1 | 53 ± 1 |
| Leo I         | 226.0| 49.1 | 22.0 ± | 0.5  | 285 ± 2 |
| Leo II        | 220.2| 67.2 | 21.65 | ±0.05| 79 ± 1 |
| Phoenix       | 272.2| −68.9| 23.04 | ±0.05| 56 ± 29 |
| NGC 6822      | 25.3 | −18.4| 23.45 | ±0.05| −57 ± 2 |
| M31           | 121.2| −21.6| 24.45 | ±0.05| −300 ± 4 |
| M32*          | 121.2| −22.0| 24.4 ± | 0.1  | −200 ± 6 |
| NGC 205*      | 120.7| −21.1| 24.5 ± | 0.1  | −241 ± 3 |
| And I*        | 121.7| −24.8| 24.7 ± | ±0.05| −368 ± 11 |
| And III*      | 119.4| −26.3| 24.38 | ±0.05| −351 ± 9 |
| NGC 147***    | 119.8| −14.3| 24.2 ± | ±0.05| −193 ± 3 |
| And V***      | 126.2| −15.1| 24.52 | ±0.08| −403 ± 4 |
| And II*       | 128.9| −29.2| 24.03 | ±0.1  | −188 ± 3 |
| NGC 185***    | 120.8| −14.5| 24.0 ± | ±0.1  | −202 ± 3 |
| Cassiopeia    | 109.5| −10.0| 24.4 ± | ±0.1  | −307 |
| IC 10         | 119.0| −3.3 | 24.57 | ±0.05| −348 ± 1 |
| Pegasus dSph  | 106.0| −36.3| 24.53 | ±0.06| −354 ± 3 |
| LGS 3         | 126.8| −40.9| 24.9 ± | ±0.2  | −183 |
| DDO 216       | 94.8 | −43.6| 24.9 ± | ±0.2  | −287 |
| Leo T         | 214.9| 43.7 | 23.09 | ±0.05| 35 |
| Leo V**       | 261.9| 58.5 | 21.23 | ±0.02| 173 ± 3 |
| And XIV**     | 123.0| −33.2| 24.33 | ±0.33| −481 ± 1 |
| And IX*       | 123.2| −19.7| 24.45 | ±0.04| −216 |
| UGC 4879      | 164.7| 42.9 | 25.22 | ±0.2  | −70 ± 15 |
| IC 1613       | 129.7| −66.0| 24.4 ± | ±0.1  | −234 ± 1 |
| Cetus         | 101.5| −72.9| 24.44 | ±0.03| −87 |
| Leo A         | 196.9| 52.4 | 24.5 ± | ±0.1  | 24 |
| WLM           | 75.9 | −73.6| 24.9 ± | ±0.1  | −122 ± 2 |
| Tucana        | 322.9| −47.4| 24.72 | ±0.05| 194 ± 4 |
| DDO 210       | 34.0  | −31.3| 24.97 | ±0.1  | −141 ± 2 |
| SagDIG        | 21.1 | −16.3| 25.15 | ±0.5  | −79 ± 1 |
| NGC 3109      | 262.1| 23.1 | 25.54 | ±0.05| 403 ± 1 |
| Antlia        | 263.1| 22.3 | 25.55 | ±0.05| 362 |
| Sextans A     | 246.1| 39.9 | 25.75 | ±0.1  | 324 ± 1 |
| Sextans B     | 233.2| 43.8 | 25.75 | ±0.1  | 300 |
| M33           | 133.6| −31.3| 24.5 ± | ±0.1  | −179 ± 3 |

Notes. The galaxy sample used for kinematic center calculations: abbreviated name, galactic longitude in degrees, galactic latitude in degrees, distance modulus with uncertainty, heliocentric radial velocity in km s\(^{-1}\) with uncertainty. All data are taken from the NED compilation, in some cases with adjustments to favor more recent and more accurate distance determinations. Some radial velocity uncertainties were not included in NED and are not shown. Galaxies with an asterisk after the name are closer than 100 kpc to either M31 or the Milky Way; those with two asterisks are within 200 kpc.

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1 This research has made use of the NASA/IPAC Extragalactic Database (NED), which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.
some strange results because the data are sparse and not at all evenly distributed. In an effort to correct for this, we divide by the “shape-function” and instead maximize

\[ V(\mathbf{\hat{r}}) = \frac{\sum |\mathbf{\hat{r}} \cdot \mathbf{v}_i|}{\sum |\mathbf{\hat{r}} \cdot \mathbf{r}_i|^2} \]

For various reasons, it is actually easier to handle the related quantity

\[ W(\mathbf{\hat{r}}) = \frac{\sum (\mathbf{\hat{r}} \cdot \mathbf{v}_i)^2}{\sum (\mathbf{\hat{r}} \cdot \mathbf{r}_i)^2} ,\]

and most calculations will be done with this.

Several variations on the \( W \)-function calculation were performed. The \( W \) function was run with four values for solar motion, as before, and a jackknife. The absolute-value \( V \) function was calculated for the \( w1 \) and \( w2 \) corrections. In an attempt to discover the influence of any unusual galaxies or small groups of them, a bootstrap calculation using only 20 galaxy samples was included. Finally, calculations including galaxies within 200 kpc, but outside 100 kpc of the Milky Way and M31, and then imposing no distance cutoff at all, were performed. The results are tabulated in Table 3 and displayed in Figure 5. (Some of the results do not appear in Figure 5 for reasons of clarity.)
The various $V$- and $W$-function calculations show directions much different from that of the $U$ function, indicating that the shape of the Local Group has a significant effect on the latter, as we might expect. Also as expected, we find that including the satellite galaxies within 200 kpc and then 100 kpc of Andromeda pulls the maximum in that direction. The bootstrap calculations again tell us that the offset is real and that the direction of maximum radial speed is well separated from the line between the two bright galaxies in the Group. In addition, the 20 galaxy bootstraps indicate that this maximum is a feature of the kinematics of the Group as a whole, not changing greatly when various subsets are excluded. Corrections for the solar motion make little difference except for the absolute-value $V$ function. The jackknife estimate of bias, however, seems rather wild; it does not even fit on the plot of Figure 5.

### 2.3. Asking the Right Question

At this point, we ask a different question: if the kinematical center were located exactly in the Andromeda direction, what would the calculations come up with? Of course, the various algorithms were tested on toy velocity models before being employed on actual data, but these were symmetrical. Now, we take the positions of the Local Group galaxies as we find them on the sky, but assign each a radial velocity equal to 100 km s$^{-1}$ times the cosine of the apparent angle from M31. The $U$ calculation now gives a center in the direction of $l = 82, b = -43$, plotted in Figure 6. It clearly matches that calculated from observed radial velocities.

### Table 2

| Calculation          | Longitude | Latitude |
|----------------------|-----------|----------|
| w1 correction        | 86 ± 9    | -44 ± 5.5|
| w2 correction        | 82        | -46      |
| 200 km s$^{-1}$ correction | 88       | -44      |
| 170 km s$^{-1}$ correction | 89       | -43      |
| Jacknife             | 99        | -46      |

Notes. Directions in Galactic longitude and latitude, for the $U$-function calculations of the Local Group center. Uncertainties for the w1 solution are taken from a bootstrap calculation. M31 itself lies at 121, -21.

### Table 3

| Calculation          | Longitude | Latitude |
|----------------------|-----------|----------|
| 25-galaxy W, w1      | 92 ± 8    | -0.5 ± 8 |
| 25-galaxy W, w2      | 91 ± 10   | -2.6 ± 11|
| 25-galaxy W, 200 km s$^{-1}$ | 93       | 0.5      |
| 25-galaxy W, 170 km s$^{-1}$ | 93       | 2        |
| 25-galaxy V, w1      | 110       | -2       |
| 25-galaxy V, w2      | 90        | 13       |
| 25-galaxy jacknife W, w1 | 108      | 36       |
| 20-galaxy average W, w1 | 90 ± 17  | -1 ± 19  |
| 20-galaxy average W, w2 | 90 ± 17  | -6 ± 22  |
| 100-kpc cutoff, w1   | 107       | -12      |
| 100-kpc cutoff, w2   | 119       | -26      |
| No cutoff, w1        | 111       | -19      |
| No cutoff, w2        | 119       | -26      |

Notes. Directions in Galactic longitude and latitude, for various calculations of the Local Group barycenter. The 25 galaxy sample includes only objects more than 200 kpc from both M31 and the Milky Way; calculations using a 100 kpc cutoff and none at all are shown for comparison. M31 itself lies at 121, -21.

So although the $U$ algorithm works well on symmetrical data, the actual distribution of Local Group galaxies on the sky forces it to a wrong answer. Importantly, it is one not identified by the jacknife/bootstrap technique.

Applying the same prescription to the $W$ function, we obtain the result plotted in Figure 7. This time we do not find such a nice agreement. The new approach cuts down the offset greatly but stubbornly remains outside the calculated error bars. We should not make too much of these; remember that the jackknife bias correction does not even fit on this plot. At any rate, if we take this together with the $U$ result, we can conclude with reasonable certainty that there is no offset.

Why are the bootstrap/jacknife techniques fooled? Clearly the problem lies in the uneven distribution of Local Group galaxies on the sky. Beyond that it is difficult to answer. A detailed look at the statistical properties of this distribution with
Figure 6. Direction of maximum observed radial velocity according to the \( U \) function. As in Figure 4, the squares show the calculation using observed radial velocities and four different corrections for solar motion relative to the Milky Way; the error bars are derived from bootstrap realizations. The position of M31 is shown with a diamond. If all galaxies are assigned radial velocities that vary as the cosine of angular distance from Andromeda, the idealized case, the calculated direction is marked by the asterisk. The latter is clearly well within the uncertainty of the data.

Figure 7. Direction of maximum radial velocity according to the \( W \) function. As in Figure 4, the boxes show positions based on observed data, with two different corrections for solar motion and error bars derived from bootstrap calculations, and a diamond marks the position of Andromeda. The triangle shows the kinematical center under the cosine prescription; it remains outside the calculated uncertainties.

3. DISTANCE TO THE CENTER

Having satisfied ourselves that the kinematic center of the Local Group lies in the direction toward Andromeda, can we constrain its distance? This would indeed be useful, possibly showing the relative masses of the two big spirals. Keeping in mind Figure 1, we now plot galactocentric radial velocities in the direction of M31 and perpendicular to that line in Figure 8; the galaxies are identified in Figure 9.

Unfortunately, there are no galaxies between the Milky Way and M31, in the region where we expect the center to lie, so there is no direct indication by a change of sign of radial velocity of the distance to the center. We can only be reasonably certain that it lies between the generally positive radial velocities on the left and the negative values on the right.\(^2\) To make further headway, we have to consider just how the infall velocities vary with distance.

4. THE VELOCITY–DISTANCE LAW

Let us designate the position of a galaxy as seen from the center of the Group by a vector \( \mathbf{r} \), with ourselves as observers at \( r_0 \). The angle from \( r_0 \) to \( \mathbf{r} \) at the center is \( \theta \). The motion of a galaxy is radial, \(-a\hat{r}\). The observed radial velocity is then

\[
V_{\text{obs}} = \frac{-ar - a_0 r_0 + (a_0 + a) r \cos \theta}{\sqrt{r^2 + r_0^2 - 2rr_0 \cos \theta}}. \tag{1}
\]

\(^2\) Working only from the kinematics, in principle, M31 (at \(-91 \text{ km s}^{-1} \) in this figure) could have fallen through the center and now be emerging. This would require an enormous, completely dark mass, however, somehow located just along the Milky Way–M31 line and beyond the latter, such an implausible situation is not further considered.
It would be straightforward to fit the observed $V_{\text{obs}}$, with a function $a$ and a distance to the center $r_0$ by a standard least-squares or $\chi^2$ technique. However, we have just seen how misleading straightforward procedures can be when applied to the Local Group data. Instead, we seek simply to match the clearest overall feature of Figure 8, the division between positive and negative observed radial velocities. Note that uncertainties in distance, of 10% or less, may blur the picture slightly but leave this main feature unchanged. Since the positive–negative division is roughly radial, distances have little effect on it.

Setting $V_{\text{obs}} = 0$ and dividing through by $a_0 r_0$, we arrive at the condition

$$\alpha \rho + 1 = (\alpha + \rho) \cos \theta,$$

where $\alpha = a/a_0$ and $\rho = r/r_0$. In all that follows, we take the distance to the center as 0.5 Mpc; any reasonable changes make no perceptible difference to our conclusions. Taking $a$ as a power law, $a \propto r^{-n}$, with $n = 0.1, 0.2, 0.3, 0.4, 0.5$ we arrive at the curves in Figure 10.

A steeper variation of infall velocity with distance allows a larger region of positive observed radial velocity, as one would expect. For the steeper law, galaxies closer to the center are being pulled away from us more strongly (though no galaxies actually appear here), and we are being pulled inward more strongly than more distant galaxies. However, note that the steepest law shown is not strong enough: there are still observed positive velocities in the negative region, with no negative velocities in the positive region. We need to look more closely at our picture of infall.

**4.1. Falling into a Potential Well**

We suppose that small galaxies are falling into the general gravitational potential of the Local Group, trading potential energy for kinetic. We have

$$\frac{1}{2} v^2(r) - \frac{1}{2} v_0^2 = \Phi_0 - \Phi(r),$$

where $v_0$ is the velocity at some chosen distance where the gravitational potential is $\Phi_0$. If the components of the Local Group started their infall with zero velocity at an infinite distance, or at any rate from so far away that $\Phi_0$ may be neglected, the infall velocity varies with distance as

$$v(r) = \sqrt{-2\Phi(r)},$$

which, with the point-mass potential of $\Phi = -GM/r$, gives the $r^{-1/2}$ curve that is not quite good enough.

Of course, galaxies are not point masses and are indeed generally taken to be embedded in dark matter halos. We will look at two representations of a large class of halo profiles (Binney & Tremaine (2008), pp. 71–72). The NFW profile (Navarro et al. (1996)) is based on N-body simulations and has a potential

$$\Phi = -\frac{\ln(1 + r/a)}{r/a},$$

with $a$ a parameter we will call the core radius, while that suggested by Lynden-Bell & Lynden-Bell (1995) was intended to reproduce a flat rotation curve and has the potential

$$\Phi \propto \ln \left| \frac{\sqrt{s^2 + 1} + 1}{s} \right|$$

with $s = r/a$, $a$ again being a parameter we will call the core radius.

NFW profiles, with core radii of 0.2, 0.4, 1.0, and 2.0 Mpc give the curves of Figure 11 (the $1/\sqrt{r}$ is the solid curve, shown for reference). For the Lynden–Bell profile, we have the corresponding curves of Figure 12.

It was not to be expected that spreading out the mass of the Local Group would give a steeper velocity law, and indeed none of the curves does as well as our $r^{-1.2}$. However, we have shown that there is no general dark matter halo around the Local Group, as assumed by Cox & Leob (2008), nor indeed any significant amount of mass apart from the M31–Milky Way pair. Note that any halo with $a$ smaller than about 100 kpc is indistinguishable from a point mass by this analysis, so we cannot say anything about the mass distribution this close to the center.

Trying some exotic dynamics, one can generate a promising curve by pairing a point mass with a linear repulsion term.
Unfortunately, to show a good fit to the galactocentric radial velocity data, the repulsion term must be orders of magnitude larger than the observed cosmological constant or dark energy. Modified Newtonian dynamics (Milgrom 1983) does not fit into the picture at all since it has a logarithmic potential (and thus one cannot have galaxies falling in from infinity).

Several obvious ways to relax our simplifying assumptions do not help. We could allow galaxies to have different total energies, that is, different $v=0$ radii. However, that would only flatten the $v(r)$ function, as closer galaxies would fall in from smaller radii and thus have lower velocities than before. It might be possible to arrange things so that the closer galaxies have systematically fallen in from farther away, overtaking the slower ones, but this is contrived and seems unlikely.

We could allow some angular momentum so that galaxies do not fall along strictly radial orbits. Indeed, Benson (2005) found that in his $N$-body simulations that galaxies crossing the virial radius of a dark matter halo had tangential speeds similar to radial speeds. However, this also would flatten the $v(r)$ function, as potential energy is transformed into tangential motion that (unless carefully arranged to be always away from the Milky Way) would be unobservable or show up only as noise. In any case, the virial radius of any dark matter halo is, as we have seen, likely to be smaller than this kind of analysis can distinguish.

We cannot get a steeper potential well than that of a point mass, clearly. However, by relaxing one of our assumptions, we can add higher-order terms to the potential. If we have two point masses $a$ distance $b$ apart, the first two terms in the multipole expansion are

$$
\Phi = -\frac{GM}{r} - \mu \frac{GM}{r^3} \frac{b^2(3\cos^2\theta - 1)}{2},
$$

where $M$ is the total mass, $r$ the distance from the center of mass, and $\mu$ depends on the mass ratio, being one-fourth for equal masses and two-ninths for a 2:1 ratio. Using this potential as we did for the NFW and Lynden–Bell profiles,3 we get the zero-velocity curves of Figure 13.

Two separated masses clearly fit the observations better than any sort of centered mass profile. A distance of 1 Mpc between the Milky Way and Andromeda is already a significant improvement over a point mass, and 2 Mpc may be a reasonable estimate of the time-averaged distance between the two galaxies; 3 Mpc fits just a bit better—but at this point, the assumption that $r > b$, upon which the multipole expansion depends, has completely broken down. We are at the limit of what our simple picture can deliver.

To check the effect of another of our simplifying assumptions, that of cylindrical symmetry about the Milky Way–Andromeda axis, consider the Local Group projected perpendicular to that axis. It is rather flattened, stretching almost 2.5 Mpc from Tucana on one side to Sextans A on the other, but less than 1.4 Mpc along a perpendicular line from NGC 3109 to SagDIG. If we separate the two sides of this very rough plane, like opening a book in the center, and plot the information in Figure 13 again, we arrive at Figure 14.

Comparing the two sides, we see that the zero-radial-velocity curve is somewhat farther to the left on the top than on the bottom. The difference, however, is not great and depends upon one or two galaxies. Overall, the symmetry holds up; we are not being misled by a couple of rogue objects. However, it is also clear that the Local Group population is sparse in the

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3 The plotted curves assume a 2:1 mass ratio; equal masses give curves that are indistinguishable.
interesting regions, and any further analysis must be done in a more sophisticated way.

5. CONCLUSIONS

The primary conclusions of this study are not surprising, but neither are they trivial. It has long been assumed that the center of mass of the Local Group lies in the direction of M31 and that the Milky Way and M31 have most of the mass. Confirmation of these assumptions have been arrived at purely kinematically, using the plausible picture of infall but without introducing any relations between light and mass at all. They rule out any extended dark matter halo in the Group as a whole, as well as any dynamically significant population of orphan halos. (They do not rule out orphan halos entirely—those are alive and well among N-body researchers; see, for example, Sawala et al. (2014)—but the Local Group’s orphans do not seem to have the dynamic effect that those in other nearby groups have.) They agree with a very recent dynamical study (Diaz et al. 2014), approaching the problem in quite a different way. In addition, it has been shown that the region from which the Local Group has been assembled is much larger than its present volume, large enough that the gravitational potential of most galaxies at the beginning of infall (strictly speaking, the differences between them) were negligible. This could be a very useful result.

An important secondary conclusion is that the kinematics of the Local Group is not well sampled by the visible galaxies. In fact, their sparseness and asymmetry managed to fool statistical techniques of moderate sophistication. The Local Group can be a misleading place! In turn, we reach the conclusion that, while things like extended Group halos and dynamically important orphan halos are clearly worse fits to the data, it is very difficult to arrive at a convincing numerical estimate of how much worse they are.

A minor point is that a plausible algorithm (here, a shape correction term) may not have the desired effect. In this case, moving from the $U$ to the $V$ and $W$ quantities made the results even more deceiving.

Many years later, the basic simplifying assumptions of Kahn & Woltjer (1959) have been justified. Although the sophistication of the dynamical analyses applied to the Local Group has increased tremendously, the results have been, overall, a process of refinement rather than revolution.

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