Neutrino-electron scatterings ($\nu - e$) are purely leptonic processes with robust Standard Model (SM) predictions. Their measurements can therefore provide constraints to physics beyond SM. Non-commutative (NC) field theories modify space-time commutation relations, and allow neutrino electromagnetic couplings at the tree level. Their contribution to neutrino-electron scattering cross-section was derived. Constraints were placed on the NC scale parameter $\Lambda_{NC}$ from $\nu - e$ experiments with reactor and accelerator neutrinos. The most stringent limit of $\Lambda_{NC} > 3.3$ TeV at 95% confidence level improves over the direct bounds from collider experiments.

The physical origin and experimental consequences of neutrino masses and mixings are not fully understood or explored [1]. Experimental studies on the neutrino properties and interactions can shed light to these fundamental questions and may provide hints or constraints to models on new physics. Reactor neutrino is an excellent neutrino source to address many of the issues, because of its high flux and availability. The reactor $\bar{\nu}_e$ spectra is understood and known, while reactor ON/OFF comparison provides model-independent means of background subtraction.

Neutrino-electron scatterings ($\nu - e$) are purely leptonic processes with robust Standard Model (SM) predictions [2]. It therefore provides an excellent probe to physics beyond SM [3, 4]. Experiments on $\nu - e$ scattering have played important roles in testing SM, and in the studies of neutrino intrinsic properties and oscillations. This article is a continuation of our previous work in which bounds were placed on Non-Standard Interaction (NSI) parameters and Unparticle physics [4]. The objective is to investigate the consequences and constraints of non-commutative physics (NC) using $\nu - e$ scattering data.

The differential cross-section in the rest frame of the initial electron for $\nu_\mu (\bar{\nu}_\mu) - e$ elastic scattering in SM, where only neutral current is involved, is given by [2, 3]:

$$
\frac{d\sigma (\bar{\nu}_\mu e)}{dT}_{SM} = \frac{G_F^2 m_e}{2\pi} \cdot \left[ (g_V + g_A)^2 \right. \\
+ \frac{(g_V - g_A)^2}{T^2} \left. \left( 1 - \frac{T}{E_{\nu}} \right)^2 \right) \\
- \left( g_V^2 - g_A^2 \right) \frac{m_e T}{E_{\nu}^2} \right] ,
$$

Figure 1: Schematic diagram of $\nu - e$ scattering via virtual photon exchange in non-commutative space-time, where $\alpha = e, \mu, \tau$ denotes the flavor state, while $k$ and $p$ ($k'$ and $p'$) are the initial (final) four-momenta of the neutrino probe and electron target, respectively. The NC coupling is given in Eq. 6 while $\theta^{\mu\nu\rho}$ is defined in Eq. 7.
charged-currents, neutral-currents and their interference

The weak mixing angle. In

where $G_F$ is the Fermi coupling constant, $T$ is the kinetic energy of the recoil electron, $E_{\nu}$ is the incident neutrino energy, $m_e$ is mass of the electron and $g_V, g_A$ are the vector and axial-vector couplings, respectively. The upper(lower) sign refers to the interactions with $\nu_e(\bar{\nu_e})$. The coupling constants in SM can be expressed by $g_V = -\frac{1}{2} + 2 \sin^2 \theta_W$ and $g_A = -\frac{1}{2}$, where $\sin^2 \theta_W$ is the weak mixing angle. In $\nu_e(\bar{\nu_e}) - e$ scattering, all of charged-currents, neutral-currents and their interference effects are involved [6], and the cross-section can be described through the replacement $g_{V,A} \rightarrow (g_{V,A} + 1)$ in Equation 1. Deviations of the measured electron recoil spectra with respect to SM predictions would indicate new physics.

Non-commutative (NC) field theories modify the space-time commutation relations. The idea dates back to the 1940’s when it was used to get rid of the divergences in quantum field theory before the renormalization concept was introduced [3]. Recent revival of interest on NC physics comes with the study of NC space-time in string theories, quantum gravity and Lorentz violation [5][7].

The space-time coordinates in NC are considered as operators, with the commutation relation:

$$[ \hat{x}_\mu, \hat{x}_\nu ] = i \theta_{\mu\nu} \ ,$$

where $\hat{x}_\mu$ denotes NC space-time coordinates. The real, antisymmetric matrix $\theta_{\mu\nu}$ is a constant with dimension (length)$^2$ (mass)$^{-2}$, and represents the smallest observable area in the $(\mu, \nu)$ plane, analogous to the Planck constant in space-momentum commutation relation. Ordinary space time relations are recovered at $\theta_{\mu\nu} = 0$.

Space-momentum commutation relation gives rise to Heisenberg uncertainty principle. Similarly, the commu-
Both SM and NC contributions are displayed.

Bottom: CHARM-II experiment with accelerator νμ, LSND experiment with reactor νe from stopped-pion \[27\], and (c) TEXONO experiment with reactor \[28\].

![Graphs showing differential cross-sections averaged over the neutrino spectra as a function of the recoil energy](image)

Figure 2: Differential cross-sections averaged over the neutrino spectra as a function of the recoil energy for (a) Top: TEXONO experiment with reactor νe \[3\], (b) Middle: LSND experiment with νe from stopped-pion \[27\], and (c) Bottom: CHARM-II experiment with accelerator νμ(\bar{\nu}\mu) \[28\]. Both SM and NC contributions are displayed.

The commutation relation of Eq. 2 implies an uncertainty relation for space-time coordinates:

\[
\Delta x_\mu \Delta x_\nu \geq \frac{1}{2} |\theta_{\mu\nu}| .
\]

NC physics is characterized by an energy scale parameter \(\Lambda_{NC} = (1/\sqrt{|\theta_{\mu\nu}|})\), below which the space and time coordinates are incoherent. New physics induced by NC effects would be important above \(\Lambda_{NC}\). There is no theoretical bound on \(\Lambda_{NC}\). Observable NC effects may be studied in present and future experiments if \(\Lambda_{NC}\) would be of the order of TeV.

Non-commutative field theory based on the commutation relation of Eq. 2 has been constructed via Weyl-Moyal star product \[8\]. The ordinary functions in Minkowski space time is retained via the definition of the star product which characterizes the NC structure. That is,

\[
f(\hat{x})g(\hat{x}) \rightarrow \hat{f}(x) \ast \hat{g}(x) = \exp \left( \frac{i}{2} \theta_{\mu\nu} \partial_{x_\mu} \partial_{y_\nu} \right) f(x)g(y) \big|_{x=y} .
\]

The \(\hat{\ast}\) notation characterizes the NC-related variables which replace the SM analogs. The lepton spinor and gauge field in NC space can be expanded using Seiberg-Witten maps up to the first order of \(\theta\) as \[12, 13\], respectively:

\[
\hat{\psi} = \psi + \epsilon \theta^{\nu\rho} A_\rho \partial_\nu \psi
\]

and

\[
\hat{A}_\mu = A_\mu + \epsilon \theta^{\nu\rho} A_\rho \left[ \partial_\nu A_\mu - \frac{1}{2} \partial_\mu A_\nu \right] .
\]

Second order contributions in \(\theta\) have also been worked out \[13\].

Non-commutative QED has been constructed \[12, 16, 17\]. Weyl-Moyal correspondence allows neutral particles to couple with the \(U(1)\) gauge field, producing rich phenomenologies \[14, 18\]. The action describing a neutral fermion field in NC-QED can be written in terms of the usual field as \[12\]

\[
S = \int d^4x \bar{\psi} \left( (i\gamma^\mu \partial_\mu - m) - \frac{e}{2} \theta^{\nu\rho} (i\gamma^\mu (F_{\nu\rho} \partial_\mu + F_{\mu\rho} \partial_\nu + F_{\rho\mu} \partial_\nu) - mF_{\nu\rho}) \right) \psi
\]

where \(F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\). Non-commutative Standard Model (NC-SM) has been constructed using analogous approach \[19\].

Phenomenological studies of NC space-time and experimental constraints on \(\Lambda_{NC}\) were reviewed in Ref. \[20\]. They are summarized in Table II. High energy collider experiments probe possible NC-induced anomalous couplings between photons and leptons directly at the relevant scale \(\Lambda_{NC}\). The most stringent collider bounds are (i) \(\Lambda_{NC} > 141\) GeV from the LEP-OPAL experiment \[21\].
based on NC-QED induced $e^- + e^+ \rightarrow \gamma + \gamma$, and (ii) $\Lambda_{NC} > 1.5$ TeV from the Tevatron experiments via NC-SM induced the W-boson polarization in top quark decays \[22\].

Future experiments at LHC and linear collider may probe $\Lambda_{NC} < 10$ TeV. The other approaches study indirect manifestations of the NC-effects, which typically involve modeling of the atomic, QCD-hadronic and astrophysical systems. Another implicit assumption for the low energy experiments is that the formulation remains valid at an energy-momentum range significantly lower than $\Lambda_{NC}$.

Neutrino-photon interactions are forbidden at the tree level in SM and can proceed only as loop corrections. However, NC-QED allows neutrino-photon couplings at tree level due to new couplings to the U(1) gauge field \[12\]. Consequently, $\nu - e$ interactions can take place via exchange of virtual photons as depicted in Figure 1. This new channel will contribute to the measurable $\nu - e$ cross-section in addition to the SM charged- and neutral-currents.

The NC-QED coupling at the photon-neutrino vertex is given by \[12\] \[13\],

$$\Gamma^\mu(\nu\nu\gamma) = -e\theta^{\mu\nu\rho}k_\nu q_\rho \left(1 - \frac{\gamma^5}{2}\right),$$ \(6\)

where

$$\theta^{\mu\nu\rho} = \theta^{\mu\rho\nu} + \theta^{\rho\nu\mu} + \theta^{\nu\mu\rho}$$ \(7\)

while $k$ and $p$ ($k'$ and $p'$) are the initial (final) four-momenta of the neutrino and electron, respectively, and $q = p' - p$. In contrast, the NC-QED photon-charged lepton vertex coupling \[23\] \[24\] is given by

$$\Gamma^\mu(l^\pm f^\pm \gamma) = ie\gamma^\mu \exp[i(p^\nu \theta^{\nu\rho} p^\rho)]$$ \(8\)

in which the NC-effects manifest as anomalous phases. This characteristic feature also applies to the case for NC-QED three- and four-photon coupling vertices \[23\].

Using the NC-QED $\nu\nu\gamma$ vertex factor of Eq. 8, the matrix element for $\nu - e$ scattering in leading order of $\theta_{\mu e}$ can be written as:

$$-iM_{NC} = \frac{e^2}{2q^2} \left[\bar{u}(p')\gamma^\mu u(p)\right] \left[\bar{u}(k')\theta_{\mu\nu\rho}k^\nu q^\rho\left(1 - \frac{\gamma^5}{2}\right)u(k)\right].$$ \(9\)

Ignoring small neutrino mass, averaging over initial spin states and summing over final states, the squared amplitude of NC contribution is

$$|M|^2 = \frac{32e^4}{q^4} \left(\frac{\vec{\theta} \cdot (\vec{k} \times \vec{k}')}{2}\right)^2 \left[(p \cdot k)(p' \cdot k') + (p' \cdot k')(p' \cdot k) - m^2(k \cdot k')\right],$$ \(10\)

where $\theta_{\mu e} k^\mu k'^\nu = \vec{\theta} \cdot (\vec{k} \times \vec{k}')$ under the definition $\vec{\theta} = (\Theta \sin \xi, 0, \Theta \cos \xi)$, where $\xi$ is a phase angle and $\Theta = |\theta_{\mu e}| = (1/\Lambda_{NC})$. Taking average over $\xi$, the NC-QED induced differential cross-section of $\nu - e$ scattering is:

$$\frac{d\sigma(\nu e)}{dT}(E_\nu) = \frac{\pi\alpha^2\Theta^2 E_\nu^2}{T} \left[\frac{1}{T} \left(\frac{2}{E_\nu}\right) + \frac{3T - 2m_e}{2E_\nu^2} - \frac{T^2 - 2m_e T}{2E_\nu^2} - \frac{m_e T^2}{4E_\nu^4} \left(1 - \frac{m_e}{T}\right)\right].$$ \(11\)

The expression is valid for all types of $\nu(\bar{\nu}) - e$ scatterings at $E_\nu \ll \Lambda_{NC}$. The $\alpha^2\Theta^2 E_\nu^2$ dependence resembles that in NC-induced $e^+ + e^- \rightarrow \nu\bar{\nu} + \nu\bar{\nu}$ process. There is no interference between the SM and NC channels in first order of $\Theta$ \[17\]. Interaction of $\nu - e$ via the exchange of $Z$ and $W$-bosons with NC-SM is in principle possible. However, in neutrino beam fixed-target experiments the four-momentum transfer is much smaller than $Z$-boson mass: $|q^2| = 2m_e T << m_Z^2$, the NC-SM contribution is suppressed by $\sim [(m_e/E_\nu)/(\alpha \cdot m_Z^2)]^2$ relative to that of NC-QED in Eq. 11 and hence can be neglected.

The $E_\nu^2$ and $1/T$ dependence in the NC-QED cross-section of Eq. 11 is significantly different from that of SM and suggests that experimental investigations would favor the studies of high energy neutrinos and low energy electron recoils. Three experiments with different ranges of neutrino energies were selected for the analysis: (I) TEXONO experiment with reactor $\nu_e - e$ at MeV range with $T \sim 10 - 100$ keV using high-purity germanium detector \[26\] and $\sim 3 - 8$ MeV using CsI(Tl) crystal scintillators \[3\]; (II) LSND experiment with $\nu_e - e$ in a stopped pion beam, at $T \sim 18 - 50$ MeV using liquid scintillator \[27\]; and (III) CHARM-II experiment with high

| Experiment | $\nu$ | $< E_\nu >$ (MeV) | $T$ | Measured $\sin^2 \theta_W$ | Best-Fit on $\Theta^2$ (MeV$^{-4}$) | $\Lambda_{NC}$ (95% CL) |
|------------|------|-------------------|----|-------------------|-------------------------------|-------------------|
| TEXONO-HPGe \[26\] | $\bar{\nu}_e$ | 1–2 MeV | 12–60 MeV | — | $(9.27 \pm 6.65) \times 10^{-22}$ | > 145 GeV |
| TEXONO-CsI(Tl) \[5\] | $\bar{\nu}_e$ | 1–2 MeV | 3–8 MeV | 0.251 ± 0.039 | $(0.81 \pm 5.74) \times 10^{-21}$ | > 95 GeV |
| LSND \[27\] | $\nu_e$ | 36 MeV | 18–50 MeV | 0.248 ± 0.051 | $(0.38 \pm 2.06) \times 10^{-21}$ | > 123 GeV |
| CHARM-II \[28\] | $\nu_\mu$ | 23.7 GeV | 3–24 GeV | 0.2324 ± 0.0083 | $(0.20 \pm 1.03) \times 10^{-26}$ | > 2.6 TeV |

| $\nu_\tau$ | 19.1 GeV | 3–24 GeV | } | 0.2324 ± 0.0083 | $(0.38 \pm 2.06) \times 10^{-21}$ | > 123 GeV |
energy $\nu_e(\bar{\nu}_e) - e$ in a proton-on-target neutrino beam at $T \sim 3 - 24$ GeV [28]. The key experimental parameters are summarized in Table II. The differential cross-section in electron recoil energy is given by:

$$\frac{d\sigma}{dT} = \int \frac{d\sigma}{dE_\nu}(E_\nu) \phi_\nu(E_\nu) \, dE_\nu,$$  \hspace{1cm} (12)

where the neutrino spectrum $\phi_\nu(E_\nu)$ is normalized with $\int \phi_\nu(E_\nu) \, dE_\nu = 1$. The measurable electron recoil spectra for the cases of SM and NC in these experiment are displayed in Figures [a,b,c]. Both SM and NC contributions at typical values of $\Lambda$ are overlaid. The sawtooth structures for $T \lesssim 10$ keV in Figure [2] are from correction due to atomic binding [29].

The combined contributions of SM and NC

$$\left( \frac{d\sigma}{dT} \right)_{SM} + \left( \frac{d\sigma}{dT} \right)_{NC}$$  \hspace{1cm} (13)

are placed in the integrand of Eq. (12) and compared with experimental data, using the current values of $\sin^2 \theta_W$ at the respective $Q^2$. Constraints on $\sin^2 \theta_W$ are derived with a minimum-$\chi^2$ analysis, from which lower bounds of $\Lambda_{NC}$ at 95% confidence level (CL) were derived. The results are summarized in Table II. With the sensitivities enhanced by a $E_\nu^2$ factor, the most stringent lower limit comes from the CHARM-II experiment with high energy accelerator neutrinos, where

$$\Lambda_{NC} > 3.3 \text{ TeV}$$  \hspace{1cm} (14)

at 95% CL. This improves over the best direct bounds from collider experiments. We note also that a similar analysis was attempted with reactor neutrinos data [26] in Ref. [14]. However, an error in the differential cross-section formula equivalent to Eq. (11) together with a missing factor in $\alpha$ in the numerical evaluation make the results invalid.

It is possible that NC physics can also give rise to flavor-changing transitions in $\nu - e$ scattering. The NC analysis of this work can be extended to include two parameters ($\Lambda_{NC}, \lambda_{\alpha \beta}$), where $\lambda_{\alpha \beta}$ denotes the branching ratio of the flavor-changing process $\nu_\alpha + e \rightarrow \nu_\beta + e$. The bounds would be relevant to the analysis of the precision neutrino oscillation measurements, in which sub-leading NC-induced effects may appear at the sources, during propagation through matter and at the detectors. Such studies would be analogous to the combined analysis of non-standard neutrino interactions and oscillation parameters [10].

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