Holographic dark energy with time depend gravitational constant in the non-flat Hořava-Lifshitz cosmology

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Abstract We study the holographic dark energy on the subject of Hořava-Lifshitz gravity with a time dependent gravitational constant \((G(t))\), in the non-flat space-time. We obtain the differential equation that specify the evolution of the dark energy density parameter based on varying gravitational constant. we find out a relation for the state parameter of the dark energy equation of state to low redshifts which containing varying \(G\) corrections in the non-flat space-time.

Keywords Dark energy; future event horizen; Hořava-Lifshitz; gravitation constant;

1 Introductions

The observation that universe appears to be accelerating currently has caused one of the important probes to modern cosmology. Observational data such as, the type Ia (A.G. Riess et al.), supernovas cosmic microwave background (D. N. Spergel et al.), and large-scale structure (M. Tegmark et al.), indicates that the accelerated expansion of the universe is not proceeding as that predicted by general relativity. A first direction that could provide an explanation of this remarkable phenomenon is to introduce the concept of dark energy, with the most obvious theoretical candidate being the cosmological constant. However, at least in an effective level, the dynamical nature of dark energy can also originate from various fields, such is a canonical scalar field (quintessence) (B. Ratra and P. J. E. Peebles), a phantom field, that is a scalar field with a negative sign of the kinetic term (R. R. Caldwell), or the combination of quintessence and phantom in a unified model named quintom (B. Feng, X. L. Wang ang and X. M. Zhang). The second direction that could explain the acceleration is to modify the gravitational theory itself, such is the generalization to \(f(R)\)-gravity (S. Nojiri et al.), scalar tensor theories with non-minimal coupling (L. Perivolaropoulos) etc.

Although, there isn’t a quantum theory of gravity, one can proceed to investigate the nature of dark energy based on some principles of quantum gravity. Currently, an interesting such an attempt is the so-called “Holographic Dark Energy” (HDE) proposal (S. D. H. Hsu). The HDE is defined by

\begin{equation}
\rho_\Lambda = 3c^2 M_p^2 L^{-2},
\end{equation}

where \(c^2\) is a numerical constant of order unity and \(M_p\) is denoted the reduced planck mass \(M_p^{-2} = 8\pi G\). The holographic dark energy scenario has been tested and constrained by various astronomical observations (Q. G. Huang et al.) and it has been extended to various frameworks (Q. G. Huang et al., K. Enqvist et al., L. Amendola).

There are significant indications that Newton’s “constant” \(G\) can by varying, being a function of time or equivalently of the scale factor \(S. D’Innocenti et al\). In particular, observations of Hulse-Taylor binary pulsar (Damour T...et al.), G. S. Bisnovatyi-Kogan et al., helio-seismological data (D.B. Guenther et al.), and asteroseismological data from the pulsating white dwarf star G117-B15A (M. Biesiada et al.) lead to
\[
\left| \frac{\dot{G}}{G} \right| \lesssim 4.1 \times 10^{-11} \text{yr}^{-1}, \text{for } z \lesssim 3.5 \quad (\text{S. Ray and U. Mukhopadhyay})
\]

Thus, in previous paper (M. R. Setare et al.), we investigated the holographic dark energy scenario under a varying gravitational constant in the framework of Horava-Lifshitz gravity and we extracted the corresponding corrections to the dark energy equation-of-state parameter.

Recently, a power-counting renormalizable, ultra-violet (UV) complete theory of gravity was proposed by Horava in (P. Horava, P. Horava, P. Hořava). Although presenting an infrared (IR) fixed point, namely General Relativity, in the UV the theory possesses a fixed point with an anisotropic, Lifshitz scaling between time and space of the form \( x^i \rightarrow \ell^i x^i, t \rightarrow \ell^2 t \), where \( \ell, z, x^i \) and \( t \) are the scaling factor, dynamical critical exponent, spatial coordinates and temporal coordinate, respectively.

In the present work we are interested to study the Holographic dark energy in framework of Hořava-Lifshitz gravity. We extend previous study (M. R. Setare et al.) to the non-flat case and we want consider the effect of curvature constant of non flat space on the results which is obtained in the flat space, it is seen that the reasonable range of \( \lambda \) Eq. [3] in non flat space is larger than answer range of \( \lambda \) in flat space. The paper is organized as follows: in the next section we found the holographic dark energy with gravitation constant depend on time and derive the differential equation that specify the evolution of dark energy parameter. In Sec. 3, we obtain the parameter of the dark energy equation of state at the low redshift. Eventually, the latter section is devoted to conclusion.

## 2 Holographic Dark Energy With Gravitational Constant Depend on Time in a non-flat Universe

In the case where the space-time geometry is a non-flat Robertson-Walker:

\[
ds^2 = dt^2 - a^2(t) \left( \frac{1}{1 - kr^2} dr^2 + r^2 d\Omega^2 \right).
\]

with \( a(t) \) the scale factor, in comoving coordinates \( (t, r, \theta, \varphi) \), where \( k \) denotes the spacial curvature with \( k = -1, 0, 1 \) corresponding to open, flat and closed universe respectively. In this case, the first Friedmann equation in the framework of Hořava-Lifshitz gravity writes (M. R. Setare et al.)

\[
H^2 = \frac{k^2}{6(3\lambda - 1)} \rho - \frac{\beta k}{a^2},
\]

\( \rho \) is the energy density, \( \rho_m = \rho_m(a) \) and \( \rho_A \) is the dark matter density and dark energy density respectively, \( \lambda \) is a dimensionless constant and \( \beta = \frac{4a^2}{8(3\lambda - 1)} \).

Here \( k^2 = 8\pi G \), and \( \rho_m \) indicate the present value that quantity. Then, from the Eq. (3), we introduce the effective density parameter \( \Omega_{\lambda e} = \frac{\Omega_{\lambda e}}{2(3\lambda - 1)} = \frac{8\pi G \rho_m}{H^2} \)

Henceforth, we will use \( \ln a \), as an independent variable. Therefore, we define, \( \dot{X} = \frac{dX}{da} \), and \( \dot{X} = \frac{dX}{d\ln a} \), so that \( \dot{X} = X'/H \). A straightforward calculation using (1) and leads to (M. Jamil):

\[
\frac{\Omega_{\lambda}}{\Omega_{\lambda e}} = 2 \left( \frac{\sqrt{\Omega_{\lambda e}}}{c_e} \cos \sqrt{\frac{|k|}{y}} - 1 - \frac{\dot{H}}{H^2} \right).
\]

\[
\cos \sqrt{\frac{|k|}{y}} = \begin{cases} 
\cos y & k = +1, \\
1 & k = 0, \\
\cosh y & k = -1.
\end{cases}
\]

In order to clarify the effect of variety \( G \) on the \( \Omega_{\lambda e} \), we should to get rid of \( \dot{H} \) into Eq. (8). In this regard, differentiation of the Friedmann equation give rise to

\[
\dot{H} = \frac{2}{3(3\lambda - 1)} \left[ G' - 3G(1 + \omega) \right] \rho + \frac{\beta k}{a^2},
\]

where, we using from the fluid equation, \( \dot{\rho} = -3H(1 + \omega)\rho \). Since, \( \omega \) is

\[
\omega = \frac{\omega_A \rho_A}{\rho} = \frac{\omega_A \Omega_{\lambda e}}{\Omega_{\lambda e} + \Omega_{\lambda e}}.
\]
The Eos parameter for HDE is given by
\[ \omega_\Lambda = -\left( \frac{1}{3} + \frac{2\sqrt{\Omega_\Lambda}}{3c} \right) . \] 

(12)

Substituting the Eqs. (11), (12) into the Eq. (10) gives
\[ \frac{\dot{H}}{H^2} = \frac{G (\Omega_m + \Omega_\Lambda)}{2} - \frac{3 (\Omega_m + \Omega_\Lambda)}{2} + \frac{\Omega_\Lambda}{2} + \frac{\Omega_\Lambda}{c_e} - \Omega_{k_e}, \]

(13)

where \( \Omega_{k_e} = \beta \Omega_k = -\beta k/a^2 H^2 \), is the curvature parameter and \( G = G'/G \)
\[ \frac{\Omega'_\Lambda}{\Omega_\Lambda} = \frac{2\sqrt{\Omega_\Lambda}c_e^{-3/2}}{\omega_\Lambda} \cos(\sqrt{|k|}y) + (1 - \Omega_{k_e})(1 - G) - \Omega_\Lambda, \]

(14)

\[ - \frac{2\Omega_\Lambda^{3/2}}{c_e}. \]

3 Some Cosmology Application

Here, we should follow up an expression related to the state parameter of equation at the present time. Since we have extracted the expressions for \( \Omega'_\Lambda \), we can calculate \( w(z) \) for small redshifts \( z \), performing the standard expansions of the literature. In particular, since \( \rho_\Lambda \sim a^{-3(1+w)} \) we acquire Expanding \( \rho_\Lambda \) we have:
\[ \ln \rho_\Lambda = \ln \rho_\Lambda^0 + \frac{d\ln \rho_\Lambda}{d\ln a} \ln a + \frac{1}{2} \frac{d^2 \ln \rho_\Lambda}{2 d(\ln a)^2} (\ln a)^2 + \cdots, \]

(15)

here, the derivatives are taken at the present time \( a_0 = 1 \). Then, \( w(z) \) is given in the small red shifts \( \ln a = -\ln(1 + z) = -z \) up to second order, as:
\[ \omega(z) = -1 - \frac{1}{3} \left( \frac{d\ln \rho_\Lambda}{d\ln a} - \frac{d^2 \ln \rho_\Lambda}{2 d(\ln a)^2} (z)^2 \right). \]

(16)

We can rewrite (16) as:
\[ \omega = \omega_0 + \omega_1 z. \]

(17)

Since
\[ \rho_\Lambda = \frac{3 (3\Lambda - 1) H^2 \Omega_\Lambda}{4\pi G} = \frac{\Omega_\Lambda \rho_m}{\Omega_{m_e}} = \frac{\rho_\Lambda \Omega_\Lambda a^{-3}}{1 - \Omega_\Lambda - \Omega_{k_e}}, \]

(18)

after some calculation, and some simplification we achieve to \( \omega_0, \omega_1 \) as follows:
\[ \omega_0 = -\frac{1}{3} \left[ \frac{\Omega_{k_e}'}{\Omega_\Lambda} + (1 - \Omega_{k_e}) \Omega_\Lambda' \right] \]

(19)

\[ \omega_1 = \frac{1}{6} \left[ \left( \frac{\Omega_\Lambda'}{\Omega_\Lambda} \right)' + \frac{\Omega_\Lambda'' + \Omega_k' - \Omega_\Lambda' \Omega_k'}{(1 - \Omega_\Lambda - \Omega_{k_e})^2} \right] \]

(20)

\[ + \frac{1}{6} \frac{(\Omega_\Lambda' + \Omega_k')^2}{(1 - \Omega_\Lambda - \Omega_{k_e})^2} \]

Now, substituting Eq. (14) into Eqs. (19), (20) we obtain:
\[ \omega_0 = -\frac{1}{3} \left[ \frac{\Omega_{k_e}'}{1 - \Omega_{k_e} - \Omega_\Lambda} \right] \]

(21)

\[ \omega_1 = \frac{1}{6} \left[ (1 - \Omega_{k_e}) (\chi^2 + \Omega_\Lambda \zeta \chi - \xi) \right] \]

(22)

where, \( \chi, \zeta, \xi \) are define as follows:
\[ \chi = 2\frac{\Omega_{k_e}}{c_e} \cos(\sqrt{|k|}y) + (1 - \Omega_{k_e})(1 - G) \]

\[ - \frac{\Omega_\Lambda}{c_e} - \frac{2\Omega_\Lambda^{3/2}}{c_e}, \]

(23)

\[ \zeta = \frac{1}{c_e \sqrt{\Omega_\Lambda}} \cos(\sqrt{|k|}y) - 1 - \frac{3\Omega_\Lambda^{1/2}}{c_e}, \]

\[ \xi = +\Omega_{k_e} (1 - G) + G' (1 - \Omega_{k_e}) \]

\[ - \frac{2\sqrt{\Omega_\Lambda}}{aHc_e} \sinh(\sqrt{|k|}y). \]

In the following, we want compare the diagram of the state parameter equation at the present time versus \( \lambda \) on both flat space and non-flat space for different redshifts. In the Fig.1 we have plotted the equation of state parameter \( \omega \), versus \( \lambda \) on both flat and non-flat space time for \( z = 0.01 \) and \( G = 0 \) and small \( \Omega_{k_e} \). We have obtained interval \( \Delta \lambda_1 = (0.32, 0.93) \) and \( \Delta \lambda_2 = (0.24, 1.32) \) for flat space time (a) and non-flat space time (b) respectively, in which \( \omega \) accept the allows values between (−1, 1), as well as this action in the Fig.2 and Fig.3, also have plotted for \( z = 0.5 \), and \( z = 0.9 \) on both flat (a) and non-flat (b) space time respectively. It is clearly seen that is \( \Delta \lambda_2 > \Delta \lambda_1 \). This result show that the suitable interval for \( \lambda \) which \( \omega \) accept the values between (−1, 1) in non flat space time is larger than the relevant interval with flat space time.
Astrophysical observations imply that the state parameter of dark energy must be changeable on the cosmic time. In this regard, one obvious contender is the holographic dark energy. In the HDE, the parameter $c$ can take the various values, but we set $c = 1$. In this paper we have discussed the holographic dark energy whit the time dependent gravitation constant in the framework of Hořava gravity in the non flat universe. By evaluating Eqs. (21), (22), and their diagram, it is considerable that anyone from values of limeted $\lambda$ gives a suitable estimate from the state parameter, which is agreeable with experience data. Choosing $\Omega_\Lambda = 0.73$, for the average vlue $0.32 < \lambda < 0.84$, one can attain to $-0.4 < \omega < 1$ in flat and nonflat space time and the only effect of the curvature constant is that the interval of the $\lambda$ variations outgrow lightly from that one. However, by applying the term curvature and the cos term due to curvature parameter, although variation of $\omega$ isn’t notable, nevertheless these could be significant in the overall of cosmological evaluation. The last statement is that, however, the effect of variation $k$ is very small, but it has an important role in the evolution of cosmological and it is shown that the equation of state of dark energy for lower red shift is generalized due to contribution of applying $k$.

4 Conclusion

Astrophysical observations imply that the state parameter of dark energy must be changeable on the cosmic time. In this regard, one obvious contender is the holographic dark energy. In the HDE, the parameter $c$ can take the various values, but we set $c = 1$. In this paper we have discussed the holographic dark energy whit the time dependent gravitation constant in the framework of Hořava gravity in the non flat universe. By evaluating Eqs. (21), (22), and their diagram, it is considerable that anyone from values of limeted $\lambda$ gives a suitable estimate from the state parameter, which is agreeable with experience data. Choosing $\Omega_\Lambda = 0.73$, for the average vlue $0.32 < \lambda < 0.84$, one can attain to $-0.4 < \omega < 1$ in flat and nonflat space time and the only effect of the curvature constant is that the interval of the $\lambda$ variations outgrow lightly from that one. However, by applying the term curvature and the cos term due to curvature parameter, although variation of $\omega$ isn’t notable, nevertheless these could be significant in the overall of cosmological evaluation. The last statement is that, however, the effect of variation $k$ is very small, but it has an important role in the evolution of cosmological and it is shown that the equation of state of dark energy for lower red shift is generalized due to contribution of applying $k$. 

![Fig. 1](image1.png)

Fig. 1 the diagram Variations the state parameter, $\omega$ with respect to $\lambda$:(a) for flat space tim. (b)non flat space time.

![Fig. 2](image2.png)

Fig. 2 the diagram Variations the state parameter, $\omega$ with respect to $\lambda$ for $z = 0.5$ : (a) for flat space time.It is seen that the interval $(0.32, 0.857)$ is satisfyed for $\lambda$ in which $\omega$ accept the allows values between $-1, 1$. (b) for non flat space time. It is seen that interval $(0.32, 0.89)$ is satisfyed for $\lambda$ to the same interval of $\omega$.

![Fig. 3](image3.png)

Fig. 3 the diagram Variations the state parameter, $\omega$ with respect to $\lambda$ for $z = 0.9$: (a) for flat space time. It is seen that the interval $(0.32, 0.827)$is satisfyed for $\lambda$ in which $\omega$ accept the allows values between $-1, 1$. (b)on non flat space. It is seen that the interval $(0.32, 0.827)$ is satisfyed for $\lambda$ in which $\omega$ accept the allows values between $(-1,1)$,
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