Magnetostatics and optics of noncentrosymmetric metals

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The current in noncentrosymmetric metals in normal and superconducting state is found in frame of linear response theory. In line with usual terms corresponding diamagnetic response, the Landau diamagnetism and the Pauli paramagnetism the general expression contains also the terms corresponding to spacial dispersion specific for a medium without space parity. This, so called, gyrotropic current is calculated in zero frequency case as well in infrared frequency region. The static gyrotropic current yields negligibly small correction to the London magnetostatics in a superconductor without inversion center. Whereas the high frequency response produces the natural-optical activity revealed f.e. in the Kerr effect. The magnitude of the Kerr angle in infrared frequency region is proved to be in reasonable correspondence with recently reported observations of the Kerr effect in the pseudogap phase in several different high Tc materials.

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I. INTRODUCTION

The normal state of high Tc superconducting materials possesses many peculiar properties. The strange depletion of the density of states at the Fermi energy arising below some temperature T∗ in the underdoped region of the phase diagram is one of them. It is probably related to the onset of some nonsuperconducting electronic ordering. Indeed, quite recently several observations have been reported of the short-range charge density wave ordering detected by x-ray diffraction arising at about the same temperature. Another peculiar property is the Kerr rotation in the reflected light polarization revealed practically in all families of the hole-doped cuprates: underdoped YBCO, optimally doped Bi-2201 and 1/8 doped LBCO. The Kerr onset temperature TK is somewhere in the pseudogap regime. This implies a symmetry breaking phase transition at TK, despite the lack of thermodynamic evidence of such a transition (see, however, the recent paper by A. Shekhter et al).

There are two peculiar features of the observed Kerr effects in cuprate materials. First, if the sample is cooled down through TK in a field H = ± 60 Oe, and then measured in a zero field warm up, the sign and value of the Kerr signal is unchanged. Secondly, recent measurements have demonstrated that the sign of the Kerr angle is the same for reflection on the opposite crystal surfaces. Both of these observations are a strong indication that here we deal with the Kerr effect not due to ferromagnetism breaking the time inversion symmetry but with the Kerr rotation of gyrotropic origin revealing itself in media with broken space inversion symmetry. The latter property, called natural optical activity, is realized in a material with noncentrosymmetric crystal structure. Metals without inversion symmetry have recently become a subject of considerable interest following the discovery of superconductivity in CePt3Si, UL15, CeIrSi3, Y2C3, Li2(Pd1−x,Ptx)3B6, KOs2O16, and other compounds.

The Kerr effect for light reflection from media with chiral charge ordering of spinless electrons has been considered in the recent paper by P. Hosur and co-authors. The natural optical activity in this type of model originates from specific terms in periodic crystal field and the corresponding modification of the electron energy spectrum of the noncentrosymmetric crystal. In the case of slow enough noncentrosymmetric distortion, it can be mathematically described as a gauge field (Berry curvature) effectively acting as an magnetic field.

On the other hand, in noncentrosymmetric media there is another mechanism for the natural optical activity related with Bychkov-Rashba type spin-orbital interaction. Such a theory has been developed by the author and Y. Yoshioka. In that paper calculations were done at frequencies not exceeding the spin-orbital band splitting, whereas the Kerr effect measurements on high Tc materials have been performed in the infrared frequency region (more exactly at a wave length 1550 nm). So, it is appropriate to develop the corresponding theory for this frequency region. It is done in the present paper.

The modifications of electrodynamics introduced by space parity violation can be conveniently understood starting from the expression for the displacement current

\[ j_d = \frac{\varepsilon}{4\pi} \frac{\partial E}{\partial t}. \]

The current changes sign \( j_d \rightarrow -j_d \) both at time \( t \rightarrow -t \), and space \( \mathbf{r} \rightarrow -\mathbf{r} \) inversion. The latter property is lost in noncentrosymmetric media, hence the current expression has to be supplemented with the following term:

\[ j_d \rightarrow j_d + j_g, \quad j_g = \lambda \text{rot} \mathbf{E} + \nu \mathbf{B}, \]

where \( j_g \) is so called gyrocurrent current changing sign at time inversion \( t \rightarrow -t \), \( j_g \to -j_g \) but not at space inversion \( \mathbf{r} \rightarrow -\mathbf{r} \). To guarantee these properties \( \lambda \) must be an odd function of the time derivative \( \partial/\partial t \) (or frequency), \( \nu \) is an even function of frequency. Owing the relation between the Fourier components of magnetic induction and electric field \( B_i = c e_{ijk} \frac{\partial}{\partial x_j} E_k \), which is valid at finite frequency \( \omega \) and wave vector \( \mathbf{q} \), the \( \nu \) frequency
dependence can be included in the frequency dependence of \( \lambda \). Hence, \( \nu \) can be taken as frequency independent. Thus, the Fourier component of the frequency dependent part of the gyrotropy current can be written as

\[
j_{\gamma i}(\omega, \mathbf{q}) = i\epsilon_{ij} \lambda(\omega) q_j E_i(\omega, \mathbf{q}).
\]

Eqs. (2), (3) are valid in an isotropic gyrotropy medium, otherwise the functions \( \lambda \) and \( \nu \) are tensors, such that in general the dielectric permeability is

\[
\varepsilon_{ij}(\omega, \mathbf{q}) = \varepsilon_{ij}(\omega, 0) + i\gamma_{ij}(\omega) q_i \varepsilon_{ij}(\omega) = -\frac{4\pi i}{\omega} \lambda_{ij}(\omega),
\]

where the off-diagonal tensor \( \gamma_{ij}(\omega) = -\gamma_{ji}(\omega) \) is an even real function of frequency \( \omega \). Like the Hall conductivity in media with broken time inversion symmetry, it describes optical activity in the case of broken space inversion symmetry.

In addition to the usual electromagnetic field action

\[
S = \frac{1}{2\varepsilon_0 c^2} \int dt d^3 r \left\{ \mathbf{B} \lambda \mathbf{E} + \mathbf{B} \nu \left[ A - \frac{\hbar c}{2} \nabla \varphi \right] \right\}.
\]

The gyrotropy current and magnetic moment are obtained from here as the variational derivatives

\[
j_\gamma = -\frac{\delta S_g}{\delta \mathbf{A}} = \lambda \text{rot} \mathbf{E} + \nu \mathbf{B},
\]

\[
\mathbf{M}_g = -\frac{\delta S_g}{\delta \mathbf{B}} = \frac{1}{2c} \mathbf{A} + \frac{1}{2} \nu \left[ A - \frac{\hbar}{2c} \nabla \varphi \right].
\]

Owing to gauge invariance, the last term \( \propto \nu \) in all these expressions exists only in the superconducting state.

We shall use the current response to the electromagnetic field with finite frequency and wave vector. This allows us to discuss magnetostatic phenomena in non-centrosymmetric superconductors like field penetration, magnetoelectric effects and also magnetic susceptibility anisotropy at finite wave vector. After that, we consider the optical properties of noncentrosymmetric metals. For simplicity, we shall discuss mostly the isotropic medium situation.

Finally, we will compare our results originating from the spin-orbital coupling in noncentrosymmetric materials with predictions of the model operating with chiral charge ordering of spinless electrons.

## II. CURRENT

The electron spectrum of a noncentrosymmetric metal has the following form\textsuperscript{20,21}

\[
\xi_{\alpha\beta}(\mathbf{k}) = (\varepsilon(\mathbf{k}) - \mu)\delta_{\alpha\beta} + \gamma(\mathbf{k})\sigma_{\alpha\beta} - \mu_B \mathbf{B} \sigma_{\alpha\beta},
\]

where \( \mu_B \) is the Bohr magneton, and the spin-orbital coupling is determined by the dot product of the Pauli matrix vector \( \mathbf{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \) and pseudovector \( \gamma(\mathbf{k}) \), which is odd with respect to momentum \( \gamma(-\mathbf{k}) = -\gamma(\mathbf{k}) \) and specific to each noncentrosymmetric crystal structure. For the cubic group \( G = O \), which describes the point symmetry of \( \text{Li}_2(\text{Pd}_{1-x}\text{Pt}_x)_3\text{B} \), as well as in isotropic media, the simplest form compatible with the symmetry requirements is

\[
\gamma(\mathbf{k}) = \gamma \mathbf{k},
\]

where \( \gamma \) is a constant. For the tetragonal group \( G = C_{4v} \), which is relevant for \( \text{CePt}_3\text{Si}, \text{CeRhSi}_3 \) and \( \text{CeIrSi}_3 \), the spin-orbit coupling is given by

\[
\gamma(\mathbf{k}) = \gamma(\mathbf{k}) \left[ k_y \hat{x} - k_x \hat{y} \right] + \gamma(\mathbf{k}) \left[ k_x k_y k_z (k_x^2 - k_y^2) \hat{z} \right].
\]

The current response of a clean metal to the electromagnetic field at finite \( \mathbf{q} \) and \( \omega \) is\textsuperscript{22}

\[
\hat{j}_i(\omega_n, \mathbf{q}) = -\frac{e^2}{c} T \left[ \hat{m}_{ij} \hat{n}_e + \int \frac{d^3 k}{(2\pi)^3} T \sum_{m=\pm\infty} \{ \hat{v}_i(\mathbf{k}) \hat{G}^{(0)}(K_+) \hat{v}_j(\mathbf{k}) \hat{G}^{(0)}(K_-) - \hat{v}_i(\mathbf{k}) \hat{F}^{(0)}(K_+) \hat{v}_j^\dagger(-\mathbf{k}) \hat{F}^{(0)}(K_-) \} \right] A_j(\omega_n, \mathbf{q}).
\]

Throughout the article, we put \( \hbar = 1 \). All the quantities here such as single electron energy \( \xi_{\alpha\beta}(\mathbf{k}) \), velocity

\[
\mathbf{v}_{\alpha\beta}(\mathbf{k}) = -(c/e) \frac{\partial \xi_{\alpha\beta}(\mathbf{k} - e\mathbf{A}/c)}{\partial \mathbf{A}}(\mathbf{A} \rightarrow 0),
\]

the inverse effective mass \( m_{ij}^{-1} = \partial^2 \xi_{\alpha\beta}(\mathbf{k})/\partial k_i \partial k_j \), the Green functions \( G_{\alpha\beta}(\tau_1, \mathbf{k}; \tau_2, \mathbf{k}') \) \( = -\langle T_\tau a_{\alpha\beta}(\tau_1) a_{\alpha\beta}^\dagger(\tau_2) \rangle \) and \( F_{\alpha\beta}(\tau_1, \mathbf{k}; \tau_2, \mathbf{k}') \) \( = \langle T_\tau a_{\alpha\beta}(\tau_1) a_{-\alpha\beta}(\tau_2) \rangle \) are matrices in the spin space,
and $\sigma_0$ is the unit matrix.

To make calculations, it is more convenient to pass from the spin to the band representation, where the one-particle Hamiltonian has a diagonal form. In the absence of a magnetic field, it is

$$H_0 = \sum_k \xi_\lambda(k)c_{k\lambda}^\dagger a_{k\lambda} = \sum_{k,\lambda=\pm} \xi_\lambda(k)c_{k\lambda}^\dagger c_{k\lambda}. \quad (15)$$

Here, the band energies are

$$\xi_\lambda(k) = \varepsilon(k) - \mu + \lambda|\gamma(k)|, \quad (16)$$

such that the two Fermi surfaces are determined by the equations $\xi_\lambda(k) = 0$. For the simplest case of a quadratic energy spectrum in a cubic crystal, both Fermi surfaces keep the spherical form. The difference of the band energies $2|\gamma(k_F)|$ characterizes the intensity of the spin-orbital coupling.

The Fermi momentum with $\gamma = 0$ is determined by the equation $\varepsilon(k_F) = \varepsilon_F$. The corresponding density of states at the Fermi energy per one spin projection is $N_0 = m k_F/2\pi^2$. The split bands Fermi momenta are

$$k_\pm = k_F \mp m\gamma. \quad (17)$$

Here and in all the following calculations, we assume $\gamma k_F \ll \varepsilon_F$.

The diagonalization is made by the following transformation

$$a_{k\lambda} = \sum_{\lambda=\pm} u_{\alpha\lambda}(k)c_{k\lambda}, \quad (18)$$

with the coefficients

$$u_{\uparrow\lambda}(k) = \sqrt{\frac{|\gamma| + \lambda \gamma_z}{2|\gamma|}}, \quad u_{\downarrow\lambda}(k) = \lambda \frac{\gamma_x + i\gamma_y}{\sqrt{2|\gamma|(|\gamma| + \lambda \gamma_z)}}, \quad (19)$$

forming a unitary matrix $\hat{u}(k)$.

The zero field Green functions in the band representation are diagonal and have the following form:

$$G^{(0)}_{\lambda\lambda}(\omega_n, k) = \delta_{\lambda\lambda} G^0(\omega_n, k), \quad (20)$$

where

$$G^0(\omega_n, k) = -\frac{i\omega_n + \xi_\lambda}{\omega_n^2 + \xi_\lambda^2 + |\Delta_\lambda(k)|^2}, \quad (21)$$

and

$$t_\lambda(k) = -\lambda \frac{\gamma_x(k) - i\gamma_y(k)}{\sqrt{\gamma_x^2(k) + \gamma_y^2(k)}}, \quad (22)$$

The functions $\Delta_\lambda(k)$ are the gaps in the $\lambda$-band quasi-particle spectrum in the superconducting state. In the simplest model with BCS pairing interaction $v_g(k,k') = -V_g$, the gap functions are the same in both bands: $\Delta_+(k) = \Delta_-(k) = \Delta$ and we deal with pure singlet pairing.

If in passing to the band representation we neglect the difference between $\tilde{u}(k)$ and $\hat{u}(k \mp n/2)$, the expression for the current keeps the same form with diagonal Green function matrices $[20]$ and nondiagonal velocity matrices obtained from Eqs. [13], [14] by the substitutions $\hat{\sigma} \to \hat{\tau}(k) = \hat{u}^\dagger(k)\hat{\sigma}\hat{u}(k)$ and $\hat{\sigma}^\dagger \to \hat{\tau}^\dagger(-k)$:

$$v_j = k_j/m, \quad \hat{u}_j = \gamma \hat{\tau}_j, \quad \hat{u}_j = \frac{e}{\hbar} \mu_B e_{lmj} \hat{\tau}_m, \quad (23)$$

The expressions for the $\hat{\tau}(k)$ matrices are

$$\hat{\tau}_x = \left( \begin{array}{cc} \frac{\gamma_x}{\gamma_z} & -\frac{\gamma_z^*}{\gamma_z} \\ -\frac{\gamma_z^*}{\gamma_z} & \frac{\gamma_x}{\gamma_z} \end{array} \right), \quad \hat{\tau}_y = \left( \begin{array}{cc} \frac{\gamma_y}{\gamma_z} & -\frac{\gamma_z^*}{\gamma_z} \\ -\frac{\gamma_z^*}{\gamma_z} & \frac{\gamma_y}{\gamma_z} \end{array} \right), \quad \hat{\tau}_z = \left( \begin{array}{cc} \frac{\gamma_z}{\gamma_z} & -\frac{\gamma_z^*}{\gamma_z} \\ -\frac{\gamma_z^*}{\gamma_z} & \frac{\gamma_z}{\gamma_z} \end{array} \right), \quad (24)$$

The explicit form of the integrand in Eq. [11] in band representation is

$$\text{Tr}\{\hat{u}(k)\hat{G}^{(0)}(K_+)\hat{u}_j(k)\hat{G}^{(0)}(K_-) - \hat{u}(k)\hat{F}^{(0)}(K_+)\hat{u}_j(-k)\hat{F}^{(0)}(K_-)\} = \text{Tr}_{dia} + \text{Tr}_{para} + \text{Tr}_{gyro} \quad (26)$$
This formula contains rich information. The terms
\[ \text{Tr}_{\text{dia}} = v_i G_{+} v_j G_{+} + v_i F_{+} v_j F_{+}^\dagger + w_{++} G_{+} G_{+} + w_{++} F_{+} F_{+}^\dagger \]
\[ + v_i G_{+} w_{++} G_{+} + v_i F_{+} w_{++} F_{+}^\dagger + w_{++} G_{+} v_j G_{+} + w_{++} F_{+} v_j F_{+}^\dagger + (\leftrightarrow -) \] (27)
determine the diamagnetic current. In the zero frequency limit, the diamagnetic current is given by the sum of the London current and the current of Landau diamagnetic moment
\[ j_d = -\frac{c}{4\pi\delta_0^2} \left( \textbf{A} - \frac{c}{2e} \nabla \varphi \right) - c\chi_L [\textbf{q} \times [\textbf{q} \times \textbf{A}]]. \] (28)

The current corresponding to the Pauli paramagnetic moment
\[ j_p = -c\chi_p [\textbf{q} \times [\textbf{q} \times \textbf{A}]] \] (29)
originates from the terms
\[ \text{Tr}_{\text{para}} = u_{++} G_{+} u_{++} G_{+} + u_{++} F_{+} u_{++} F_{+}^\dagger + u_{++} G_{+} v_j G_{+} + u_{++} F_{+} v_j F_{+}^\dagger \]
\[ + u_{++} G_{+} w_{++} G_{+} + u_{++} F_{+} w_{++} F_{+}^\dagger + w_{++} G_{+} u_{++} G_{+} + w_{++} F_{+} u_{++} F_{+}^\dagger \]
\[ + u_{++} G_{+} w_{++} G_{+} + u_{++} F_{+} w_{++} F_{+}^\dagger + w_{++} G_{+} u_{++} G_{+} + w_{++} F_{+} u_{++} F_{+}^\dagger \]
\[ + w_{++} G_{+} w_{++} G_{+} + w_{++} F_{+} w_{++} F_{+}^\dagger + w_{++} G_{+} w_{++} G_{+} + w_{++} F_{+} w_{++} F_{+}^\dagger + (\leftrightarrow -) \] (30)

Finally, the gyrotrropic current originates from the terms
\[ \text{Tr}_{\text{gyro}} = v_i G_{+} u_{++} G_{+} + v_i F_{+} u_{++} F_{+}^\dagger + u_{++} G_{+} v_j G_{+} + u_{++} F_{+} v_j F_{+}^\dagger \]
\[ + u_{++} G_{+} w_{++} G_{+} + u_{++} F_{+} w_{++} F_{+}^\dagger + w_{++} G_{+} u_{++} G_{+} + w_{++} F_{+} u_{++} F_{+}^\dagger \]
\[ + u_{++} G_{+} w_{++} G_{+} + u_{++} F_{+} w_{++} F_{+}^\dagger + w_{++} G_{+} u_{++} G_{+} + w_{++} F_{+} u_{++} F_{+}^\dagger \]
\[ + w_{++} G_{+} w_{++} G_{+} + w_{++} F_{+} w_{++} F_{+}^\dagger + w_{++} G_{+} w_{++} G_{+} + w_{++} F_{+} w_{++} F_{+}^\dagger + (\leftrightarrow -) \] (31)

The notation \((\leftrightarrow -)\) means that all these expressions are supplemented by the corresponding terms with sign ” + “ substituted by sign ” − “ and vice versa.

Below we consider several properties derived from the current expression.

III. PAULI SUSCEPTIBILITY

A simple example of an unusual property of non-centrosymmetric superconductors is shown by the Pauli susceptibility, which can be found from the paramagnetic current, as determined by the Eq.(30) type terms at \(\omega_n = 0, \textbf{q} = 0\). When the band splitting strongly exceeds the superconducting gap \(\gamma k_F \gg \Delta\), the susceptibility has a finite value at \(T = 0\), even for pure s-wave pairing interaction. For isotropic superconductors it is\[ \chi_p = \frac{2}{3} \mu_B^2 N_0 (2 + Y(T)) \] (32)
where
\[ Y(T) = -T \int_{-\infty}^{\infty} d\xi \sum_{m=-\infty}^{\infty} (G^2(\Omega,m,k) + |F(\Omega,m,k)|^2) \]
\[ = -\int_{-\infty}^{\infty} d\xi \frac{\partial f}{\partial E} \]
is Yosida function (concentration of normal electrons), \(f(E_k) = (e^{E_k/T} + 1)^{-1}\) is the Fermi distribution function, \(E_k = \sqrt{\xi^2 + \Delta^2}\). The corresponding paramagnetic limiting field
\[ H_p = \sqrt{\frac{3}{2}} \frac{\Delta_f=0}{\mu_B} \] (33)
is \(\sqrt{3}\) times larger than in centrosymmetric superconductors.

Another peculiar property is the static susceptibility anisotropy arising at finite wave vectors \(\textbf{q}\). This has been theoretically predicted by Takimoto\(^{22}\), and was recently measured by B.Fáš at a\(^{23}\). The analytical expression for susceptibility anisotropy in the case of a tetragonal crystal with broken space parity, point group \(C_{4v}\) was found in Ref.24. As the susceptibility at \(\textbf{q} = 0\) the susceptibility anisotropy at finite wave vectors can be also re-derived from the paramagnetic current determined by the Eq.(30) type terms at \(\omega_n = 0, \textbf{q} \neq 0\). The result is
\[ \chi_{xx} - \chi_{yy} = \mu_B^2 N_0 \gamma_1^2 (E_F - q_z^2) \] (34)
Here, the function \(f(\textbf{q}) \sim O \left( \frac{2q}{2\pi} \right)^2\) is fully symmetric with reference to the tetragonal symmetry. Thus, the \(\textbf{q}\)-dependent basal plane anisotropy proved to be quadratic on the band splitting.

The spin susceptibility in a crystal with cubic symmetry and broken space parity also loses its diagonal form:
\[ \chi_{xy} \approx \mu_B^2 N_0 (i\gamma q_z/E_F + \gamma^2 q_x q_y/E_F^2 + \ldots). \] (35)
IV. GYROTROPIC CURRENT

The four lines in Eq. (31) correspond to four integrals with different structure determining gyrotropy current

\[ j_{gI}(\omega, \mathbf{q}) = -2e\mu_B [I_{1ij} + I_{2ij} + I_{3ij}] B_j + i\frac{e^2}{c} \epsilon_{ijl} A_j I_{4lj}, \]  

(36)

where for cubic symmetry crystal with \( \hat{\gamma} = \hat{k} \)

\[
I_{1ij}(\omega_n, \mathbf{q}) = \int \frac{d^3k}{(2\pi)^3} T \sum_{m=-\infty}^{\infty} \nu \delta_{ij} [G_+(K_+)G_+(K_-) + F_+(K_+)F_+(K_-) - G_-(K_+)G_-(K_-) - F_-(K_+)F_+(K_-)], \]  

(37)

\[
I_{2ij}(\omega_n, \mathbf{q}) = \gamma \int \frac{d^3k}{(2\pi)^3} T \sum_{m=-\infty}^{\infty} \hat{\gamma}_i \hat{\gamma}_j [G_+(K_+)G_+(K_-) + F_+(K_+)F_+(K_-) + G_-(K_+)G_-(K_-) + F_-(K_+)F_+(K_-)], \]  

(38)

\[
I_{3ij}(\omega_n, \mathbf{q}) = \gamma \int \frac{d^3k}{(2\pi)^3} T \sum_{m=-\infty}^{\infty} (\delta_{ij} - \hat{\gamma}_i \hat{\gamma}_j) [G_+(K_+)G_-(K_-) + F_+(K_+)F_-(K_-) + G_-(K_+)G_+(K_-) + F_-(K_+)F_+(K_-)], \]  

(39)

and

\[
I_{4lj}(\omega_n, \mathbf{q}) = \gamma^2 \int \frac{d^3k}{(2\pi)^3} T \sum_{m=-\infty}^{\infty} \hat{\gamma}_l [G_+(K_+)G_-(K_-) + F_+(K_+)F_-(K_-) - G_-(K_+)G_+(K_-) - F_-(K_+)F_+(K_-)]. \]  

(40)

In all the integrals, one has to perform summation over the Matsubara frequencies \( \Omega_m \) and to pass from the discrete set of frequencies \( \omega_n \) into entire half-plane \( \omega > 0 \) by substitution \( \omega_n \to \omega + i0 \).

V. STATIC GYROTROPY PROPERTIES

A. Static gyrotropic current

The gyrotropic current in the static limit \( \omega = 0 \) was first calculated for an uniaxial crystal with the Rashba spin-orbital coupling by V. Edelstein\(^{22} \). He found that

\[ j_g(\omega = 0) = \nu (\hat{c} \times \mathbf{B}), \]  

(41)

\[ \nu \propto \varepsilon \mu_B \gamma n_s(T) / \varepsilon_F, \]

where \( n_s(T) \) is the density of superconducting electrons, \( \hat{c} \) is direction of anisotropy axis.

For an isotropic medium in the static limit \( \omega = 0, q \to 0 \), the integral \( I_{4lj} \) calculated in Ref. 19 above \( T_c \) vanishes. In the superconducting state it takes negligibly small value order of \( \propto \Delta^2 / \varepsilon_F^2 \). The other integrals are

\[ I_{1ij} = 2N_0 \gamma Y(T) \delta_{ij}, \]  

(42)

\[ I_{2ij} = -\frac{2}{3} N_0 \gamma Y(T) \delta_{ij}, \]  

(43)

\[ I_{3ij} = -\frac{4}{3} N_0 \gamma \delta_{ij}. \]  

(44)

Thus, for a metal with cubic symmetry and broken space parity, the gyrotropy current in the static limit is

\[ j_g = \nu \mathbf{B}, \quad \nu = \frac{8}{3} \varepsilon \mu_B \gamma N_0 (1 - Y(T)). \]  

(45)

Like the expression for the spin susceptibility \(^{32} \), this formula is valid when the band splitting strongly exceeds the superconducting gap \( \gamma k_F \gg \Delta \). The expression for the static gyrotropy current free of this limitation has been found in the paper Ref. 26.

B. London magnetostatics

The generalization of the London magnetostatics to the noncentrosymmetric case has been made by Levitov et al\(^{27} \). In this case the London current is

\[ j = -\frac{e}{4\pi \delta^2} \left( \mathbf{A} - \frac{\hbar c}{2e} \nabla \varphi \right) + \nu \mathbf{B}, \]  

(46)
where $\delta$ is the London penetration depth and the corresponding London equation acquires the form

$$\Delta B = \frac{1}{\delta^2} B - \frac{4\pi}{c} \nu B. \quad (47)$$

For a superconductor occupying the half the plane $z > 0$ and the external magnetic field $H$ directed along $x$ direction, the boundary conditions according to equation (7) are

$$B^{\text{int}} - B^{\text{ext}} = 4\pi M = \frac{2\pi\nu}{c} A \quad (48)$$

that is

$$B^{\text{int}}_x = H, \quad B^{\text{int}}_y = \frac{2\pi\nu}{c} A_y. \quad (49)$$

The solution of Eq. (47) with boundary conditions (49) yields

$$B_x + iB_y = H(1 + i\tan \beta) \exp\left(-\frac{z e^{i\beta}}{\delta}\right) \approx H(1 + i\beta) \exp\left(-\frac{z}{\delta}\right) \quad (50)$$

showing that the magnetic field inside the superconductor acquires an $y$ component, and rotates in $x, y$ plane. The dimensionless parameter determining these magnetostatic gyrotropy properties is an angle $\beta$:

$$\sin \beta = \frac{2\pi\nu\delta}{c}. \quad (51)$$

A simple estimation of this angle value at $T = 0$ is

$$\beta(T = 0) = \frac{4}{3\pi} \frac{e^2 \gamma}{\hbar c} k_F \delta \approx 10^{-3}. \quad (52)$$

Here we used the band splitting $\gamma k_F \approx 10^3$ Kelvin (see $^{28}$). This means that London magnetostatics in noncentrosymmetric superconductors undergoes a negligibly small deviations from that in the centrosymmetric case.

A similar smallness is specific for other gyrotropy magnetostatic properties such as the magnetoelectric influence on the upper critical field or the creation of helical superconducting phases. This subject was discussed in detail in Ref.29. One must make, however, an important comment. The treatment of the paper $^{28}$ written properly from the symmetry point of view, has been supported by a microscopic theory calculation. The latter was done for the single band case using the limitation $\mu_0 H < \gamma k_F$. The corresponding calculations for two bands split by the spin-orbit coupling reproduces the same results, but all the magnetoelectric terms linear in gradients $\eta^a K_{ij} H_i D_j q_i$ in the free energy density acquire an additional reduction of order $\gamma/v_F$.

VI. DYNAMIC GYROTROPY PROPERTIES

At finite frequencies, making use the relations

$$E_i = \frac{i\omega}{c} A_i, \quad B_i = ce_{ijk} \frac{q_j}{\omega} E_k$$

one can rewrite Eq. (50) as

$$j_{gi}(\omega, \mathbf{q}) = -\frac{e^2}{m\omega} (I_{1ij} + I_{2ij} + I_{3ij}) e_{ijk} q_i + m e_{ikl} I_{4l} E_k, \quad (53)$$

Here both the frequency $\omega$ and the wave vector $\mathbf{q}$ dependence of the integrals are essential. An important simplification takes place in the high frequency region $\omega > q v_F$, where one can neglect the $\mathbf{q}$-vector dependence of the integrals $I_{1ij}, I_{2ij}, I_{3ij}$ and calculate $I_{4i}$ in first order in $\mathbf{q}$. Then we have

$$I_{1ij} = 0, \quad I_{2ij} = 0, \quad I_{3ij} = \delta_{ij} I_3, \quad I_{4i} = \frac{q_i}{m} I_4, \quad (54)$$

$$I_3 = \frac{1}{3\pi^2} \left\{ k_x^2 - k_y^2 + a^2 \ln \frac{k_y^2}{k_x^2 - a^2} \right\}, \quad (55)$$

$$I_4 = -\frac{a^2}{12\pi^2} \left\{ m \gamma \left( \frac{k_x + m \gamma}{k_x^2 - a^2} + \frac{k_x - m \gamma}{k_x^2 - a^2} \right) \right. \quad (56)$$

$$+ \frac{3}{2} \ln \frac{k_x^2 - a^2}{k_x^2 - a^2} - a^2 \left( \frac{1}{k_x^2 - a^2} - \frac{1}{k_x^2 - a^2} \right) \right\},$$

where $a = \frac{\omega}{2\gamma}$. The integration over momenta has been performed for the case of spherical Fermi surfaces.

So, for $\omega > q v_F$ we arrive at the following expression for the current

$$j_{gi} = i e_{ijl} \lambda(\omega) q_j E_l, \quad \lambda(\omega) = \frac{ie^2}{m\omega} [I_3(\omega) + I_4(\omega)]. \quad (57)$$

As it should be, $\lambda(\omega)$ is thus proved to be a pure imaginary odd function of frequency.

Physically, the wave vector magnitude is determined by the inverse skin penetration depth $q \approx \delta^{-1}$. The latter in the infrared frequency region is of order $10^{-5}$ cm (see the book $^{29}$), such that the integral values written above are correct for $\omega > q v_F \approx 10^{13}$ rad/sec. The band splitting in noncentrosymmetric metals $\sim \gamma k_F$ can be smaller or larger than this value. In any case at frequencies higher than the band splitting $\omega > \gamma k_F$, and making use Eqs. (55), (56), we obtain a more simpler formula for gyrotropy current

$$j_{gi} = i e_{ijl} \lambda(\omega) q_j E_l, \quad \lambda(\omega) = \frac{4i}{\pi^2} \frac{e^2}{\hbar} \left( \frac{\gamma k_F}{\hbar \omega} \right)^3. \quad (58)$$

Here we return to dimensional units.

The situation when the frequency is smaller than the band splitting $\omega < \gamma k_F$ but at the same time larger than $q v_F$ has been considered in the paper $^{19}$, where the integrals $I_{1ij}, I_{2ij}, I_{3ij}$ were not taken into account and the gyrotropy current was calculated in terms of the integral $I_4$ only.
VII. KERR ROTATION

Now we can apply the standard procedure to calculate the Kerr rotation for linearly polarized light that is normally incident from the vacuum to the flat boundary of a medium. The reflected light is elliptically polarized with the major axis rotated relative to the incident polarization by an amount

\[
\theta = \frac{2n\kappa \Delta n}{(1 - n^2 + \kappa^2) + (2n\kappa)^2}.
\]

(59)

Here \( n \) and \( \kappa \) are the real and imaginary part of medium refraction index neglecting gyrotropy. The difference in the refraction indices of gyrotropy medium for circularly polarized light with the opposite polarization is

\[
\Delta \hat{n} = \hat{n}_+ - \hat{n}_- = -\frac{4\pi}{c} \Im(\lambda).
\]

(60)

Thus, at frequencies exceeding the band splitting,

\[
\Delta \hat{n} = -\frac{16}{\pi} \frac{\varepsilon^2}{\hbar c} \left( \frac{\gamma k_F}{\hbar \omega} \right)^3.
\]

(61)

Although the band splitting \( \gamma k_F \) is not known for many noncentrosymmetric materials, one can expect it to be about thousand Kelvin or in frequency units \( \sim 10^{14} \text{rad/sec} \). As an example, we consider the region where the frequency of light is of order \( \omega \approx \frac{\sqrt{4\pi n^2 e^2/m^*}}{m^*} \), the plasma frequency. In this frequency region, the real and imaginary part of the conductivity are \( \sigma' \approx \omega_p^2/4\pi\omega^2 \tau \) and \( \sigma'' \approx \omega_p^2/4\pi\omega \). Then, one can find \( 2n\kappa \approx \omega_p^2/\omega^3 \tau \) and \( \kappa^2 - n^2 \approx \omega_p^2/\omega^2 \). Thus, for the Kerr angle we obtain

\[
\theta \approx -\frac{32}{\pi} \frac{\varepsilon^2}{\hbar c \omega_p^2} \left( \frac{\gamma k_F}{\hbar \omega} \right)^3.
\]

(62)

To obtain a numerical estimate of the Kerr angle, let us take \( \omega_p^2 \tau \approx 10^3 \), \( \omega/\omega_p \approx 10^{-1} \), \( \gamma k_F/\hbar \omega \approx 1/3 \). Then we find

\[
\theta_{Kerr} \approx 1 \text{ \mu rad},
\]

(63)

which is in reasonable agreement with measured Kerr angles in the cuprate compounds reported in Refs. 4-7.

VIII. DISCUSSION

It is interesting to compare the results obtained here with the results of calculations performed within the model of chiral charge ordering spinless electrons. Namely, in the paper the authors considered a system with Hamiltonian

\[
H = \sum_k E(k; z)\psi_k^\dagger \psi_k z - t_\perp \sum_{k,z} (\psi_k^\dagger \psi_{k,z+1} + H.c.),
\]

(64)

\[
E(k; z) = \frac{1}{2m}(k^2 + |k \cdot n(z)|^2) - E_F,
\]

\[
n(r) = n_0[\cos(\pi Q z), \sin(\pi Q z), 0].
\]

Making use the following assumptions

\[
n_0 \ll 1, \quad t_\perp^2 \ll n_0^2 E_F, \quad \hbar \omega \gg |t_\perp|
\]

they have found

\[
\lambda_{Kerr} (\omega) \approx \frac{i}{4\pi} \frac{\varepsilon^2 n_0^3 t_\perp^2 E_F}{\hbar (\hbar \omega)^3},
\]

(65)

which is related to \( \gamma(\omega) \) used in Ref.17 by

\[
\gamma(\omega) = -\frac{4\pi i}{\omega} \lambda(\omega).
\]

It is worth noting that the results given by Eqs. 65 and 58, which originate from completely different models, have the same frequency dependence of the gyrotropy coefficient \( \lambda(\omega) \).

For completeness it should be mentioned that two researches have recently been put forward to explain the observed Kerr effect in cuprates. Both of them are based on the loop-current model by Varma possessing charge chirality violating space parity.

IX. CONCLUSION

The Kerr onset seen in the pseudogap phase of a large number of cuprate high-temperature superconductors, arising at about the same temperature as the short range charge density wave order, can serve as evidence of a gyrotropic ordering that breaks space inversion symmetry but preserves time-reversal invariance. Here we proposed a simple microscopic model of an isotropic metal where inversion symmetry breaking reveals itself as a spin-orbital coupling, lifting the band degeneracy and creating the electron band splitting. The magnitude of the Kerr angle in the infrared frequency region given by Eq. 63 is proved to be in reasonable agreement with recently reported observations of the Kerr effect in high Tc materials.

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