Supporting Text S1. Equilibrium Analysis.

In the main text, we describe the development of a system of ordinary differential equations that we use to simulate population dynamics and population genetics of Aedes aegypti following the introduction of an R&R strain into a wild-type population. Analysis of this model is limited by the model complexity; however, we are able to obtain equilibrium population density of the wild-type population in the absence of transgenic releases.

In a completely wild-type population, the system

\[ \dot{J}_i = B_i(t) - \mu_J J_i - J_i \left( \alpha \sum_g J_g \right)^{\beta-1} - \nu J_i \]

\[ \dot{F}_i = \frac{1}{2} \nu \gamma_i J_i - \mu_F F_i + u_i^F \]

\[ \dot{M}_i = \frac{1}{2} \nu J_i - \mu_M M_i + u_i^M \]

\[ B_i(t) = w_i \lambda \sum_m F_m(t) \sum_n Pr(i|m,n) \frac{M_n(t)}{\sum_g M_g(t)} \]

for \( i = 1...9 \), where \( i = 9 \) represents the wild-type genotype, reduces to
\[
\begin{align*}
\dot{J}_9 &= \lambda F_9 - \mu J_9 - \alpha J_9^\beta - \nu J_9 \\
\dot{F}_9 &= \frac{1}{2} \nu J_9 - \mu F_9 \\
\dot{M}_9 &= \frac{1}{2} \nu J_9 - \mu M_9.
\end{align*}
\]

Here, \( \dot{M}_9 \) is decoupled from the system, so we can analyze the reduced system

\[
\begin{align*}
\dot{J}_9 &= \lambda F_9 - \mu J_9 - \alpha J_9^\beta - \nu J_9 \\
\dot{F}_9 &= \frac{1}{2} \nu J_9 - \mu F_9.
\end{align*}
\]

This system has a trivial equilibrium at \((J_9^{(1)}, F_9^{(1)}) = (0, 0)\), and one non-trivial equilibrium at

\[
\begin{align*}
J_9^{(2)} &= \frac{1}{\alpha} \left( \frac{\nu \lambda}{2 \mu_F} - \mu_j - \nu \right)^{1 - \frac{1}{\beta}} \\
F_9^{(2)} &= \frac{\nu}{2 \mu_F} J_9^{(2)}.
\end{align*}
\]

We rearrange the expression for \( J_9^{(2)} \) by noting that

\[
\begin{align*}
\frac{\nu \lambda}{2 \mu_F} - \mu J - \nu &= (\mu_j + \nu) \left( \frac{\nu \lambda}{2 \mu_F (\mu_j + \nu)} - 1 \right) \\
&= (\mu_j + \nu) (R_0 - 1),
\end{align*}
\]

where

\[
R_0 = \frac{1}{\mu_F} \cdot \frac{\lambda}{2} \cdot \frac{\nu}{\mu_j + \nu} = \frac{\nu \lambda}{2 \mu_F (\mu_j + \nu)}.
\]

Here, \( \frac{1}{\mu_j} \) is the average lifespan of adult females, \( \frac{\lambda}{2} \) is the rate of production of female offspring, and \( \frac{\nu}{\mu_j + \nu} \) is the fraction of juveniles that survive to emerge as adults. Thus, \( R_0 \) is the basic reproductive
number of the population. We rewrite (4) in terms of $R_0$.

$$J_9^{(2)} = \frac{1}{\alpha} ((\mu_J + \nu)(R_0 - 1))^{\frac{1}{\beta - 1}}$$  \hspace{1cm} (7)

$$F_9^{(2)} = \frac{\nu}{2\mu_F} J_9^{(2)}$$

In order for population to have a positive equilibrium (i.e., $J_9^{(2)} > 0$), $R_0 > 1$. Thus, we analyze the stability of the equilibrium only for the case when $R_0 > 1$.

To verify the stability of the equilibrium in (7), we first find the Jacobian of system (3).

$$\text{Jacobian}(J_9, F_9) = \begin{pmatrix} - (\mu_J + \nu + \beta(\alpha J_9)^{\beta - 1}) & \lambda \\ \frac{1}{2} \nu & -\mu_F \end{pmatrix}$$ \hspace{1cm} (8)

We then evaluate the Jacobian at the equilibrium in (7).

$$\mathcal{J} = \text{Jacobian}(J_9^{(2)}, F_9^{(2)}) = \begin{pmatrix} - (\mu_J + \nu + \beta(\mu_J + \nu)(R_0 - 1)) & \lambda \\ \frac{1}{2} \nu & -\mu_F \end{pmatrix}$$ \hspace{1cm} (9)

We now study the eigenvalues of $\mathcal{J}$ by studying the determinant and trace of $\mathcal{J}$. The equilibrium point $(J_9^{(2)}, F_9^{(2)})$ is stable when $\text{Tr}(\mathcal{J}) < 0$ and $\text{det}(\mathcal{J}) > 0$ (i.e., both eigenvalues of $\mathcal{J}$ must be negative). First, we calculate $\text{Tr}(\mathcal{J})$.

$$\text{Tr}(\mathcal{J}) = - (\mu_J + \nu + \beta(\mu_J + \nu)(R_0 - 1) + \mu_F)$$ \hspace{1cm} (10)

Since $R_0 > 1$, and because we require $\mu_J$, $\nu$, $\beta$, and $\mu_F$ to be positive, $\text{Tr}(\mathcal{J}) < 0$. Next, we calculate $\text{det}(\mathcal{J})$.

$$\text{det}(\mathcal{J}) = \mu_F (\mu_J + \nu + \beta(\mu_J + \nu)(R_0 - 1)) - \frac{\lambda \nu}{2}$$ \hspace{1cm} (11)

Rearranging the terms, we get
$$\det(J) = 1 + \beta(R_0 - 1) - \frac{\lambda \nu}{2\mu F(\mu J + \nu)}$$  

$$= 1 + \beta(R_0 - 1) - R_0 .$$  

(12)

In order for $\det(J) > 0$,

$$1 + \beta(R_0 - 1) - R_0 > 0$$

$$\beta(R_0 - 1) > R_0 - 1$$  

(13)

$$\beta > 1 .$$

So we have that the equilibrium $(J^*_g, F^*_g)$ is stable when $\beta > 1$.

**Equilibrium Values for Model Runs**

Here, we list the values of the equilibrium density of juveniles, adult males, and adult females that are used for model runs in the main text. Note that the release size of R&R individuals is always defined as a function of the equilibrium wild-type male population density so that release rates are always relative to the population density. This allows for a general study of R&R releases in an *Ae. aegypti* population. While changes in $\alpha$ will result in changes in the density of the population, the qualitative results for relative density are the same.

$$J^*_g = 10118.98$$

$$F^*_g = 7083.28$$  

(14)

$$M^*_g = 2529.74$$