A efficient urban surface flood model based on cellular automate technique and its application in Wuhan

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Abstract. To achieve fast flood modelling for urban area, a efficient two-dimensional model based on cellular automata was developed and further coupled with a hydrological model. The model is tested using an analytical and a real event case. The analytical case results show that the model is capable of simulating water-depth process with good agreement with the analytical solution under a reasonable slope threshold and grid resolution. Slope threshold and grid resolution are the two key factors affecting the computational efficiency and model accuracy. In the case of the real event case, the results are also reasonably. In general, this model can be applied to rapid flood analysis in large urban areas.

1. Introduction
As a powerful tool for urban flood control and disaster mitigation, the urban flood modelling has become an important research topic in recent years [1]. Recently, 2D urban flood model has become popular in urban flood modelling since the traditional 1D model cannot simulate the evolution of surface water accumulation. However, the typical 2D models requiring high computational resources to solve the Shallow Water Equations (SWEs). In order to improve the efficiency of the 2D model as much as possible without significantly reducing the calculation accuracy, some scholars have made improvements by simplifying the SWEs. Moreover, some researchers used parallel computing technique to improve the efficiency [2]. However, these simplified physical based models are still time-consuming because of the complex formulas and the sequence calculation flow.

CA technique has the capacity of modelling the spatiotemporal evolution process of complex physical systems by only using simple operations and it is well suited for parallel computation. Recently, the CA technique has used in developing simple and efficient 2D urban flood models. Dottori and Todini [3] developed a 2D CA model using Manning's equation to calculate discharge between cells while Ghimire et al [4] using a ranking system to calculate the volume of water flowed between cells.

The main idea of this work is to present a high-precision and efficient 2D urban flood by coupling a 2D CA model followed the Guidolin [5] and precipitation-runoff model. The new model was applied to an analytical case proposed by Hunter et al [6] and a real flood event in the area of Wuhan, China.
2. Methodology

2.1. 2D CA model

The proposed 2D CA model was developed following the work of Guidolin et al [5]. The calculation space is designed as rectangular grid with von-Neumann (VN) neighbourhood which allows water transfer in four directions. The major feature of our new model is that it adopted a simplified ranking system which has four steps:

- Calculate the available storage volume of each neighbour cell by multiplying cell area and the positive water level differences between the central cell and the neighbour cells (equations (1) and (2));
- Calculate the weights of each neighbour cell according to the available storage volume of each neighbour cell (equations (2)-(7));
- Calculate the amount of water flowing out of the central cell (equations (8) and (9));
- Compute the eventual amount of water leave the central cell for each directions (equation (10)).

\[
\Delta Z_i = Z_0 - Z_i \quad \forall i \in \{1, 2, 3, 4\} \tag{1}
\]

\[
A v_i = A_{i} \max \{\Delta Z_i, 0\} \quad \forall i \in \{1, 2, 3, 4\} \tag{2}
\]

\[
A v_{\min} = \min \{A v_i | i = 1, 2, 3, 4\} \tag{3}
\]

\[
A v_{\max} = \max \{A v_i | i = 1, 2, 3, 4\} \tag{4}
\]

\[
A v_T = \sum_{i=1}^{4} A v_i \tag{5}
\]

\[
w_i = \frac{A v_i}{A v_T + A v_{\min}} \tag{6}
\]

\[
w_0 = \frac{A v_{\min}}{A v_T + A v_{\min}} \tag{7}
\]

\[
v_M = \min \{\sqrt{h g}, \frac{1}{n} h^{2/3} \sqrt{\frac{\Delta Z_{0,M}}{\Delta x_{0,M}}} \} \tag{8}
\]

\[
I_M = \min \{v_M h_i \Delta e_i, h_i \Delta A_i\} \tag{9}
\]

\[
I_i = w_i I_T \tag{10}
\]

Where \(i\) is index of the central cell; \(j\) is the index of the neighbour cells; \(Z(m)\) is the water level; \(A(m^2)\) is the cell area; \(A v_i (m^3)\) is the available storage volume between the central cell and the \(i\)th neighbour cell; \(A v_{\min} (m^3)\), \(A v_{\max} (m^3)\) and \(A v_T (m^3)\) are minimum, maximum and sum of the available storage volumes, respectively; \(w_i\) is the weight of the \(i\)th neighbour cell; \(h(m)\) is the water depth; \(g (m/s^2)\) is gravitational acceleration; \(n (m^{1/3} /s)\) is the Manning's roughness; \(M\) is the index of the neighbour cell with the largest weight; \(v_M (m/s)\) is the maximum permissible intercellular velocity; \(\Delta x_{0,M} (m)\) is the
distance between the central cell and the cell with the largest weight; \( I_M (m^3) \) is the maximum volume of water can be moved into the cell with the largest weight; \( \Delta t \) (s) is the calculation time step; \( e_M \) (m) is the length of cell edge.

To ensure the calculation stability, the time step \( \Delta t \) is updated by computing on each cell and the formula is provided by Hunter et al [6] as follow:

\[
\Delta t = \frac{\Delta x^2}{4} \min \left( \frac{2n}{h^{5/3}} S_{i_{\text{min}}}^{1/2}, S_i > \sigma \right)
\]  

(11)

where \( i \) is the index of the calculation cell; \( S_{i_{\text{min}}} \) is minimum water surface gradient of \( i \)th cell for all directions; \( \sigma \) is the slope tolerance which used to prevent the simulation time become too small.

The state update interval \( (\Delta t_u) \) is further used to reduce the calculation time. The time step is recalculated every \( \Delta t_u \) instead of each time step.

2.2. Precipitation–runoff model

In this paper, the runoff simulation is carried out over each cell which is further divided into different hydrological response units according to the land use type. The each permeable unit includes a vegetation canopy, a surface reservoir layer, a soil layer, and a groundwater layer from top to bottom. A physical equation describing the movement of the water flow is established on each hydrological response unit. The Richards equation is used to describe the water movement, and the soil flow along the slope direction is considered. The mass conservation equation and Darcy’s law are used to describe the water exchange between groundwater and river channel. The detailed calculation principle can be found in the reference [7]. Table 1 gives the important parameters of four main urban hydrological response units.

| Parameter                  | Pavement | Road | Building | Greenspace |
|----------------------------|----------|------|----------|------------|
| Manning roughness (m\(^{-1/3}\)s) | 0.04     | 0.028| 0.05     | 0.08       |
| Saturation conductivity (mm/hr) | -        | -    | -        | 1.2        |
| Saturated moisture content | -        | -    | -        | 0.4        |
| Residual moisture content   | -        | -    | -        | 0.15       |
| Depression storage (mm)     | 2        | 1    | 5        | 5          |

3. Case and study areas

3.1. Analytical solution

The analytic solution of water flow over a horizontal plane with a constant flow velocity is derived by Hunter et al [6] as follow:

\[
h(x,t) = \left[ \frac{7}{3} (C - n^2 u^2 (x - ut)) \right]^{7/3}
\]  

(12)

where \( h \) (m) is the water depth; \( u \) (m/s) is the depth-averaged velocity in the \( x \) direction; \( C \) is a constant which can be fixed by initial conditions.

Equation (12) can be used to verify the CA model by providing the benchmark data with specific boundary conditions and initial conditions. In this study, the model was implemented with open downstream boundary and a dry initial condition. The value of \( u \) and \( n \) are 1 m/s and 0.01 m\(^{1/3}\)s, respectively. The simulation duration is 3600 s.

3.2. Real-word case

The Fruit Lake Gate area (figure 1), in the middle of the Wuhan city, China is a challenging case due
to high degree of urbanization. The catchment area is the seat of the provincial government of Hubei Province, and is also an intensive residential gathering place. The research area is 2.48 km$^2$ with 27.38% of building, 8.29% of green space, 18.96% of road and 45.36% of paved area. During June 30 to July 6, 2016, Wuhan suffered a heavy rainfall storm. The rainfall in several rainfall stations are exceeded the historical extreme value, reaching 565.7~719.1 mm. In this study, the 10 minutes interval precipitation with a total number of 249 mm during July 5 and July 6 is used for the simulation.

![Figure 1. (a) DEM and (b) Landuse Study area.](image)

### 4. Results and discussion

#### 4.1. Analytical solution

The computation grid has a length of 5 km in the flow direction and a width of 1 km. Figure 2 and table 2 shows the comparison between analytical solution described by equation (1) and the model simulated at 3600 s of different grid resolutions (10 m, 25 m and 50 m). As shown in figures 2(a) and 2(b), the simulated water depth is good agreement with the analytical solution, with the RMSE are 0.0042, 0.0072, and 0.0104 m for three dx. As expected, the model is sensitive to the grid resolution. Therefore, in practical applications, it is necessary to select an appropriate spatial step size to ensure the accuracy of the simulation.

![Figure 2. Comparison between analytical solution and the model simulated at 3600 s of different grid resolutions for (a) whole length of flow direction (b) length between 3 km and 5 km.](image)
Table 2. RMSE, minimum time step and total runtime for different $dx$ and $\sigma$.

| $Dx$ (m) | $\Sigma$ (‰) | RMSE (m) | $\text{Min}\Delta t$ (s) | Runtime (s) |
|----------|---------------|----------|--------------------------|-------------|
| 10       | 1             | 0.2671   | 0.0206                   | 522.59      |
| 10       | 0.5           | 0.1994   | 0.0143                   | 835.31      |
| 10       | 0.1           | 0.0042   | 0.0061                   | 2916.14     |
| 10       | 0.01          | 0.0042   | 0.0061                   | 2886.78     |
| 25       | 1             | 0.2981   | 0.1000                   | 70.09       |
| 25       | 0.5           | 0.1823   | 0.0890                   | 135.62      |
| 25       | 0.1           | 0.0072   | 0.0380                   | 460.24      |
| 25       | 0.01          | 0.0072   | 0.0380                   | 462.96      |
| 50       | 1             | 0.2396   | 0.1000                   | 23.63       |
| 50       | 0.5           | 0.1389   | 0.1000                   | 33.59       |
| 50       | 0.1           | 0.0104   | 0.1000                   | 117.16      |
| 50       | 0.01          | 0.0104   | 0.1000                   | 113.28      |

The grid resolution and slope threshold are expected as two important parameters that affect the computational accuracy and computational efficiency. The model is applied with a combination of different slope thresholds (1‰, 0.5‰, 0.1‰, 0.01‰) and different grid resolutions (10 m, 25 m and 50 m) with a 60s update time. The RMSE, minimum time step ($\text{Min}\Delta t$) and total runtime are shown in table 1 and figure 3. The results show that the calculation accuracy of the model decreases with the increase of the grid resolution, and decreases with the increase of the slope threshold. When the slope threshold is less than 0.1‰, the RMSE is no longer reduced with the slope threshold decrease (figure 3(a)). The relationship between RMSE and spatial grid resolution various for different slope thresholds.

![Figure 3](image-url)(a) ![Figure 3](image-url)(b)

Figure 3. Relationship between RMSE and (a) slope threshold (b) total runtime.

Table 3. Correlation coefficient matrix of grid resolution, slope threshold, RMSE, minimum time step and total running time.

| $Dx$ (m) | $\Sigma$ (‰) | RMSE (m) | $\text{Min}\Delta t$ (s) | Runtime (s) |
|----------|---------------|----------|--------------------------|-------------|
| 1        | 0.0000        | -0.0758  | 0.8726                   | -0.6464     |
| 1        | 0.9710        | 0.2738   | -0.4127                  |             |
| 1        | 0.2512        | -0.4261  |                         |             |
| 1        | -0.7609       |          |                         |             |

As expected, the computational efficiency decreases significantly as the grid resolution increases, and decreases as the slope threshold increases. However, the total runtime increase slightly when slope threshold is less than 0.1‰ or greater than 0.5‰ (figure 3(b)). Table 3 gives the correlation coefficient
matrix of grid resolution, slope threshold, RMSE, minimum time step and total running time. The correlation coefficient between RMSE and slope threshold is 0.9710 which indicating that simulation accuracy is primarily controlled by the slope threshold. The minimum time step is mainly related to the grid resolution with a correlation coefficient of 0.8726. There is a significant negative correlation between computational efficiency (run time) and grid resolution or slope threshold. In general, an appropriate slope threshold can be selected to ensure the calculation accuracy, and a larger space step can be selected to improve the calculation efficiency of the model in simulation.

4.2. Real event case
In the real case, the model were performed using DEMs with spatial resolutions 5 m with main parameters showed in table 1. There is no official measured data on the flooding during the event. Therefore, this study did a field survey for one point specifically. The water depth change progress simulated at the point is shown in figure 4(a). The maximum water depth is 0.51 m which is close to the result of field survey (0.6 m) and the progress is also in line with the description of the residents. Figure 4(b) shows the submerged area change progress during the rain storm. The area of flooding increases as the rain increase and reach their peaks after the rain peak 50 minutes. The peaks of flooding area over 0.4 m and 0.15 m are 23.16 ha and 47.91 ha. Since the model does not consider the drainage pipe, the lower area is still flooded after the rain stop. Figure 5 shows the water depth distribution at the peak rainfall time (figure 5(a)) and the maximum water depth during simulation (figure 5(b)). The lower part of the study area suffer more serious of water lag which is realistic.

Figure 4. Water depth change at monitoring point (a) and submerged area (b).

Figure 5. Water depth at peak rainfall time (a) and (b) maximum water depth during simulation.
5. Conclusions
In this paper we have presented an efficient two-dimensional flood model for urban area based on CA technology and a hydrology model. It is applied to the analytical solution and a real case simulation. The main conclusions are as follows:

- The model simulation results are in good agreement with the analytical solution, with a minimum RMSE of 0.0042m.
- The calculation accuracy of the model is mainly affected by the slope threshold. The calculation efficiency of the model is mainly affected by the grid resolution and the slope threshold. An appropriate slope threshold can be selected to ensure the calculation accuracy, and a larger grid resolution can be selected to improve the calculation efficiency of the model.
- The results of the Wuhan case show that the model can simulate the flood process in the urban area well which indicates that the model has a good application prospect.

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