Flow rate estimation using acoustic field distortions caused by turbulent flows: time-reversal approach

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Abstract. A new acoustic technique for flow rate estimation is proposed here. This technique is based on the traditional ultrasonic cross-correlation flow meter, but instead of using a continuous wave or pulse trains in each transmitter-receiver pair, the acoustic time-reversal technique is applied. The system relies on the principle that a turbulent flow with multiple vortices will cause random distortions in a given acoustic field; hence, analyzing this noise caused in the ultrasound signal by the turbulence over time allows a “signature” or “tag” of the flow to be defined. In other words, the vortices modify the frequency response function of the flowing system uniquely, since the distortion is assumed to be random. The use of the time-reversal procedure in the cross-correlation flow meter provides improvements in several aspects: it simplifies the signal processing needed after the reception of the signals, avoiding the use of a demodulator to obtain the signature of the vortex; the signal is focused at the position of the reception transducer and; the sensitivity is also increased because the wave travels twice in the acoustic channel. The method is theoretically discussed showing its limitations and improvements. Experimental results in a laboratory water tank are also presented.

1. Introduction

Several methods of flow rate estimation using ultrasound have been proposed and tested over time. Since the early works, using transit time of an acoustic pulse [1], evolving to more complex models and techniques as Doppler shift detection [2], vortex shedding and the cross-correlation flow meters [3, 4], ultrasonic techniques are useful in flow measuring for different fluids and different regimens.

Ultrasonic techniques are no invasive, don’t introduce pressure drops, and are suitable for flows with two phases or solids in suspension. The guidelines to select the most adequate type of flow meter depend on the application and the flow rate of the setup [5].

Ultrasonic flow meters can be classified in three different groups:

- Transit time flow meter; averages the difference in measured transit time between ultrasound pulses propagating together with and against the flow.
- Doppler effect flow meter; detects the frequency shift introduced in the original signal by reflectors travelling along the flow, such as air bubbles, to estimate the average velocity of the fluid.
• Cross correlation flow meter; the average velocity of the fluid is estimated from the cross correlation maximum of the signals measured in two different sections in the flow, upstream and downstream respectively.

Cross correlation flow meters are a robust solution to measure the average (bulk) velocity in the case of a non uniform flow. Compared to transit time flow meters and Doppler effect flow meters, the cross correlation flow meter presents the advantage of not depending upon the velocity of sound in the fluid, therefore, variations of the sound velocity caused by changes in the fluid temperature and other factors don’t introduce measurement errors. Cross correlation flow meters can also acquire information from a larger portion of the velocity profile so that a more reliable estimate of the flow rate may be achieved [3].

Several devices are available in the market, including applications to measure ocean currents [6], blood flow [7] and mixture of gas-liquid flow [8]. Since this type of flow meter is dependent on the existence of perturbations within the flow, it fails in laminar flows in which there are no perturbations to correlate.

This work introduces an important modification in the cross correlation technique substituting the use of continuous wave and electronic demodulation of the signals for the time reversal of acoustic fields, thus using pulsed signals and avoiding problems associated with standing waves [9].

In section 2, the traditional cross correlation technique is reviewed. In section 3, the time reversal of the ultrasonic signals is introduced. Section 4 presents the experimental setup and the signal processing needed. Section 5 briefly explains the first experimental results obtained in laboratory and finally section 6 summarizes the conclusions.

2. Ultrasonic cross correlation flow meter
The basic arrangement of the cross correlation flow meter consist in two pairs of ultrasonic transducers axially distant, one upstream and the other downstream. Figure 1 shows a schematic diagram of the setup.

![Figure 1. Ultrasonic cross correlation flow meter schematic diagram.](image)

The proper operation of the flow meter depends on the presence of perturbations within the flow, such as, air bubbles, solids in suspension or differences in the density and local velocity produced by vortexes in the flow. These perturbations “modulate” the ultrasonic signal as it travels through the turbulent flow from the emitter transducers towards the receiver transducers. In the traditional version of the device, a continuous sinusoidal wave is used, and then the amplitude and phase of this wave are
modulated by the perturbations in the flow [3]. At the first observation, the signals recorded at receptor 1, \(Tr R1\), and at receptor 2, \(Tr R2\), look like noisy signals with low similarity. But when the distance between the transducers pairs (marked as \(d\) in fig 1) is lower than a certain value, defined here as the “correlation length” of the perturbations, the cross correlation between these two signals displays a well defined maximum. The time position of this maximum, \(\tau\), is displaced from zero and represents the average time taken by a perturbation to travel axially from the position of transducer pair 1 (\(Tr E1 – Tr R1\)) to the position of transducer pair 2 (\(Tr E2 – Tr R2\)). The analytical formulation of this correlation \(R_{y_1y_2}\) can be expressed as

\[
R_{y_1y_2}(\tau) = \frac{1}{T} \int_0^T y_1(t)y_2(t + \tau)dt
\]  

(1)

Here, \(y_1(t)\) is the signal recorded at the pair 1 and \(y_2(t)\) is recorded at pair 2. In practice the cross correlation is computed in the frequency domain as

\[
R_{y_1y_2}(\tau) = \mathcal{F}^{-1}(\mathcal{F}(y_1).\mathcal{F}^*(y_2))
\]

(2)

Were the operator \(\mathcal{F}\) is the Discrete Fourier transform and \(\mathcal{F}^*\) is its complex conjugate. The average velocity \(\langle v \rangle\) of the flow can be obtained from the correlation time \(\tau\) as

\[
\langle v \rangle = k \frac{d}{\tau}
\]

(3)

where \(k\) is the calibration factor, usually adjusted empirically to take into account the non uniformity in the velocity profile. There are several research works focused on finding the relation between the correlation time \(\tau\) and the average velocity \(\langle v \rangle\) [10, 11, 12].

The use of continuous sinusoidal waves introduces stationary waves in the pressure field generated by the transducers. To avoid this problem a pulsed version of the cross correlation flow meter was proposed [8], where the estimation of the average flow velocity is made taking into account only the amplitude of the emitted pulse.

The selection of the transducers depends on the distance between the emitter and the receiver (marked as \(D\) in fig 1), where distance of several meters are required, like rivers and intakes of hydroelectric turbines, low frequency transducers form 20 KHz to 100 KHz are suitable, for small distances such as pipe diameters, high frequency transducers are more sensitive to small perturbations.

### 3. Time reversal of ultrasonic signals

#### 3.1. Time reversal basis

The focusing of acoustic waves using time reversal in signal processing was developed by Mathias Fink and his collaborators during the late eighties [13]. In those first works, the main application scope was the improvement of acoustic images through inhomogeneous media. However, within the last years, several other practical applications using time reversal focusing of acoustic waves have been developed [14].

Time reversal of acoustic waves starts with the measure of the impulse response, \(h(r, t)\), of a given system. In practice a sinc pulse (flat frequency spectrum up to the cut-off frequency) is inputted in the emitter transducers (\(Tr E1\) and \(Tr E2\)). The received signal at the receivers (\(Tr R1\) and \(Tr R2\)) is recorded and time reversed, generating \(h(r, T – t)\) which is then retransmitted through the media, again from \(Tr E1\) and \(Tr E2\), to \(Tr R1\) and \(Tr R2\), respectively, where \(T\) is the temporal length of the signal. After this second emission, the signal is again received and recorded. This resultant signal can be obtained theoretically by the convolution between the time reversed signal and the impulse response of the system \(h(r, T – t) \otimes h(r, t)\), assuming that the impulsive response \(h\) does not change from the first to the second shot, this operation is given by

\[
\int_{-\infty}^{+\infty} h(r, \tau). h(r, t – T + \tau)d\tau
\]

(4)
Changing the variable $t - T = t'$ the product (4) is equivalent to the autocorrelation function of the impulse response $h$, given by

$$\int_{-\infty}^{+\infty} h(r, \tau) h(r, t' + \tau) d\tau$$  (5)

Expression (5) presents a maximum at $t' = 0$, which corresponds to the signal’s temporal length $T$. As these signals travel from $Tr E1$ and $Tr E2$ to $Tr R1$ and $Tr R2$ they are modulated by the perturbations and recorded in $Tr R1$ and $Tr R2$. It is important to notice that in the frequency domain, the time reversal process is equivalent to the product of the transfer function $H(r, \omega)$ by its complex conjugate $H^*(r, \omega)$. This product cancels the phase of each component in the frequency response.

$$H(r, \omega) H^*(r, \omega) = |H|^2$$  (6)

### 3.2. Application to detect changes in the media.

The time reversal maximum depends on the amplitude of each frequency component and also on the phase of each component. Every change in the frequency response affect this maximum, thus the time reversal maximum is sensitive to any change in $H$, both in amplitude and phase. As observed, the time reversal process is the autocorrelation function of the impulsive response; this fact was used to detect variations in the system’s response for example introduced by temperature [15].

Arrays of ultrasonic transducers (Time reversal mirrors) have been used to characterize vortex and to estimate velocity profiles of various flows, showing the sensitivity of the time reversal technique to vortex patterns [16, 17, 18].

The application of the technique starts with the characterization of the impulsive response; this is achieved emitting a sinc pulse both in $Tr E1$ and $Tr E2$. In this work the spectrum of the sinc is flat up to 5 MHz. In this application both emitters and receivers have central frequency 5 MHz.

![Figure 2](image)

**Figure 2.** A) Sinc signal used to obtain the impulsive response, B) Spectrum of the sinc, C) Impulsive response, D) Spectrum of the impulsive response. E) Response to the time reversed impulsive response or ASIR (Autocorrelation of the System’s Impulse Response). F) Spectrum of the signal E.

After recording the impulse response, figure 2C, the system is programmed to reverse it in the time domain, send it through the emitter transducer, acquire the new response and record the maximum of
the received signal. In this example, the acquisition window is 2.5 µs wide; this limits the maximum sampling frequency to 400 KHz, one point (the maximum of the acquisition window) for each window. However, the main limitation is the transmission time between the electronic device used to acquire the signals and the computer which lowers, in this case, the maximum sampling frequency to 50 Hz. Observe that the signal recorded after time reversal process, figure 2E, has a symmetrical shape. This is typical in an autocorrelation function, the presence of asymmetries shows that the impulsive response of the system has some differences in each shot. These differences are also reflected in the value of the correlation maximum.

Two noisy signals containing the information of the flow perturbations are then recorded; these signals are built as arrays whose elements are the maximums of each acquisition window at \( Tr R1 \) (downstream) and at \( Tr R2 \) (upstream). However the cross correlation of these signals shows a defined maximum, see figure 3C.

Results obtained for different flow rates and acquisition windows are presented in section 5.

4. Experimental setup

4.1. Hydrodynamic setup

The experiments were realized in a rectangular cross-section water channel, placed inside a water tank where the flow is generated by a submersible pump. The water channel is made of Plexiglas and its dimensions are: 1,000 mm length, 100 mm high and 100 mm width, being the axial distance between the two transmitter-receiver pair of 30 mm. The transducers stand 800 mm from the pump and the water depth is of 60 mm, measured from the base. The transducers are placed 30 mm high, measured from the base. Figure 4 shows the setup's schematic diagram.
The tank was filled with water up to 60 mm from the base, leaving the water channel’s cross-section not completely filled, since its high is of 100 mm. This partial filling combined with the pump placed close to the channel’s entrance create an adequate level of turbulence for the tests, generating non-stationary vortexes.

The flow is generated by a calibrated DC pump and its flow rate can be well controlled. The transducers are placed inside holes in the water channel walls, as shown in figure 5, in order to reduce signal decay and multiple reflections in the Plexiglas. All the measurements and acquisitions were made during system’s steady state operation.

4.2. Electronic setup and signal processing

The experimental prototype of the cross-correlation flow meter consists of 2 transmitter channels and 2 receiver channels, all connected to piezoelectric ultrasonic transducers with central frequency of 5.0 MHz.

The electronic apparatus is implemented with OPEN System, a Multi channel ultrasonic device made by Lecoeur Electronique. OPEN System provides accessible parameters and its open architecture makes it suitable for research, along with its analog transmitters that allow the generation of arbitrary waveforms used in this work for the emission of sinc and time-reversed signals. It has also transmitting and receiving functions, which concentrate all the signal emission and reception channels and functions in a single configurable electronic device. For this experiment, the system was used with 4 active channels and sampling rate of 80 MHz, 12 bits A/D and D/A converters, input impedance of
50 Ohms in the 2 transmitter channels and gain of 10dB at the receiver channels. The OPEN System’s interface is implemented through a PC linked to the device using USB 2.0.

For the first step of sending a spike excitation through the two emitter channels, a sinc function of 1.25 µs was used, as shown in the figure 2A, being its formula given by

\[ \text{sinc}(2\pi f_0 t) \] (7)

Where \( f_0 = 2.0 \text{ MHz} \). The signal has a cut-off frequency of 5.0 MHz in order to match the 5.0 MHz transducers used, as shown in the figure 2B. The sinc signal was built digitally with 100 samples for all tests, based in the sampling rate of 80 MHz. The signal is then treated with a Blackman window for smoothing its edges and is then normalized before being sent to the OPEN System.

The reception of the impulse response is implemented through A-scans with transducers \( Tr R1 \) and \( Tr R2 \), using acquisition windows of 2.5 µs as shown in the figure 2C.

The cycle of emission of the sinc signal and reception of the impulse response is repeated 128 times for each transducer pair in order to estimate a statistical mean of the impulse response for both pairs. The calculated mean signals are then time reversed and loaded into the OPEN System through USB 2.0 to be emitted through transducers \( Tr E1 \) and \( Tr E2 \).

The emission of these signals generates responses as the one shown in the figure 2E, which will be called ASIR (Autocorrelation of the System’s Impulse Response) from now on. The ASIR signals are obtained at both receiver transducers, \( Tr R1 \) and \( Tr R2 \), also with sampling frequency of 80 MHz and acquisition window of 2.5 µs. After the acquisition, the ASIR signals are then sent to the PC where a peak detection algorithm stores the value of the signal’s maximum. This cycle is repeated continuously in order to build arrays with 90,000 samples of ASIR’s maximums for pairs 1 and 2, forming the digital signals \( A1(t) \) and \( A2(t) \) respectively.

Although the ASIR signals are sampled with 80 MHz, the signals \( A1(t) \) and \( A2(t) \) are built with a sampling frequency of 50Hz, therefore, with a Nyquist frequency of 25Hz [19], due to the time spent in the USB 2.0 communications between PC and the OPEN System.

Offsets of \( A1(t) \) and \( A2(t) \) are removed and the signals are normalized. \( A1(t) \) and \( A2(t) \) are then interpolated using the algorithm 8.1 [20] to emulate a sampling rate 20 times higher, forming the signals \( AI1(t) \) and \( AI2(t) \), as the ones shown in the figures 3A and 3B. These interpolated signals are then cross-correlated using a PC-implemented digital algorithm based on the equation (2), then generating a signal as the one shown in the figure 3C.

As explained in section 3.2, the position of the maximum of the cross-correlation signal provides an estimation of the transit time (\( \tau \)) of perturbations travelling from pair 1 to pair 2. As the distance between the two pairs of transducers is \( d = 30 \text{ mm} \), the bulk velocity of the flow can be calculated by equation (3).

5. Experimental results
The system was tested for flow rates ranging from 202 ml/s to 391 ml/s, with Reynolds numbers from 4023 to 7793 respectively, being the Reynolds number given by [21]

\[ Re = \frac{\rho \cdot V \cdot D_H}{\mu} \] (8)

Where \( \rho \) is the water’s density; \( V \) is the flow’s average velocity; \( \mu \) is the dynamic viscosity and \( D_H \) is the hydraulic diameter defined as

\[ D_H = 4 \cdot A/P \] (9)

Where \( A \) is the cross sectional area and \( P \) is the wetted perimeter [21]. Reynolds numbers were calculated assuming the following properties for the water: \( \mu = 1.002 \times 10^{-3} \text{ N s/m}^2 \), \( \rho = 1.000 \text{ kg/m3} \).
The experiment was realized acquiring signals during steady state operation of five flow rates: 202 ml/l, 254 ml/s, 307 ml/s, 351 ml/s and 391 ml/s. For each of the four flow rates, the samples of $\tau$ were acquired and saved continuously for 30 minutes, forming arrays with 90,000 samples.

The tests showed a high dependency of the cross-correlation results on the flow rate applied to the water channel, where the peak of the cross-correlation becomes narrower and closer from $t=0$ as the flow rate is increased. This is primarily expected since the reduction of flow rate brings the Reynolds number close to the boundary between turbulent and transition flows. Thus as the Reynolds number becomes smaller, the level of turbulence decreases and so the distortions in the acoustic field. Once the level of distortions decreases, the modulation on the signal’s amplitude becomes smoother, making the cross-correlation peak broader and smaller until the point where it vanishes at the beginning of the laminar regimen, increasing the errors associated with the estimation.

Figure 6 shows the effect of the flow rate on the cross-correlation peak, with its maximums normalized.

![Figure 6. Normalized cross-correlation for the first four flow rates tested.](image)

Another important factor concerning the cross-correlation peaks is the length of the signals $A11(t)$ and $A12(t)$ used in the calculations. Estimations of $\tau$ were made using signals with 90,000, 67,500, 45,000 and 22,500 samples. The tests showed that the length of signals (the length of the acquisition window) used for the calculation of the cross-correlation has a significant influence over results, where the longer is the acquisition window; the bigger is the peak in the cross-correlation which provides more reliable estimations. Figure 7 shows the influence of the acquisition window’s length on the peak of the cross-correlation.
Figure 7. Dependency of the cross-correlation peak on the length of the acquisition window.

The dependency of the cross-correlation peak on the length of the acquisition window shows strong linearity, as expected by the theoretical model, for the reason that with longer signals, the integral represented by equation (5) will in fact be a discrete sum with more non-zero elements. Figure 8 shows the linear relation between the cross-correlation peak and the time length of the acquisition window.

Figure 8. Value of the cross-correlation peak (Y axis) plotted against the time length of the acquisition window (X axis) and a linear regression curve, showing strong linearity.

The mean values of $\tau$ estimated from the acquisitions allow the estimation of the calibration factor for this experiment. The real mean velocity can be calculated as follows
\[ \langle \text{Vr} \rangle = \frac{Q}{A_{CS}} \tag{10} \]

Where \( Q \) is the flow rate given by the pump and \( A_{CS} \) is the cross-sectional area of the flow. Substituting (11) in the equation (3) the calibration factor is given by

\[ k = \frac{Q \cdot \tau}{A_{CS} \cdot d} \tag{11} \]

Table 1 shows the values of \( k \) estimated from equation (11) using the five mean values of \( \tau \) estimated with the acquisitions.

**Table 1.** Estimation of the calibration factor.

| Flow Rate [l/s] | \( \tau \) | \( k \)  |
|-----------------|------------|---------|
| 0.202           | 0.0972     | 81.6    |
| 0.254           | 0.0735     | 77.7    |
| 0.307           | 0.0654     | 83.6    |
| 0.351           | 0.0598     | 87.4    |
| 0.391           | 0.0505     | 82.2    |

Average \( k \): 82.5

Figure 9 shows the average estimated transit time of the perturbations (\( \tau \)) between ultrasonic beams 1 and 2 as a function of the actual flow rate.

![Figure 9](image)

**Figure 9.** Average estimated transit time of perturbations between ultrasonic beams 1 and 2 (Y axis) plotted against the actual flow rate (X axis) and a linear regression.

After the calculus of the calibration factor, the flow rate estimation was then executed using the ultrasonic apparatus configured with the calculated \( k \). Table 2 shows a comparison between the actual flow rate, estimated by measuring the time taken by the flow to fill a known volume, and the flow rate estimated by the ultrasonic system developed here.
Table 2. Comparison between the actual flow rate and the flow rate estimated with the ultrasonic system after calibration.

| Actual Flow Rate [l/s] | Measured Flow Rate [l/s] | Error (%) |
|------------------------|--------------------------|-----------|
| 0.202                  | 0.204                    | 1.1       |
| 0.254                  | 0.269                    | 6.2       |
| 0.307                  | 0.303                    | 1.3       |
| 0.351                  | 0.331                    | 5.6       |
| 0.391                  | 0.392                    | 0.4       |

6. Conclusions
As table 2 shows, the initial results for the flow rate estimation in a pipe with rectangular cross section were considerably accurate, presenting errors smaller than 7%. An important point here is the characterization of the vortex, bubbles and other kinds of disturb generators, since it is important to know the frequency band width of these perturbations in order to develop suitable hardware for the detection of all the information carried across the flow. The implementation of hardware which does not require a high level of communication between the PC and de OPEN System is the next step in this research, intending to improve the system’s sampling rate through the use of DSP directly in the OPEN System.

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