Analysis of tubes filled with charged electron gas

Dr. Stefan Karrmann
Stuttgart
Germany

2011-02-23

Abstract
We show that tubes filled with electron gas, as presented by A. Bolonkin, are not possible with current materials. First, the pressure of the charges on the outer surface cancel almost all of the electrostatic pressure of the inner electrons. Second, due to the mutually repulsion most of the electrons are in the outmost shell of the tube and not individually free.

Contents
1 Introduction 1
2 Design of the tube 2
3 Gauss’s law 3
4 Charging of the tube 4
5 Capacitor model 4
6 Further arguments against free electron gas 6
7 Conclusion 7
A Material constants 7

1 Introduction
Tubes filled with electron gas, as described in [2], are very versatile devices. First, they are very light – almost as light as vacuum, i.e. much lighter than hydrogen filled tubes. This predestined them for e.g. ballons and high altitude platforms. Second, they are almost perfect conductors at huge range of temperatures, in particular at room temperature (circa 300 K). Thus, a detailed
analysis of its properties is necessary. The sketch of the construction is given in [2]. Here we present additional arguments which show that we cannot construct such tubes at a macroscopic scale with known materials.

We provide mainly two arguments to the analysis of the tubes. First, we pay attention to the forces which act on the positively charged outer surface. Second, we study the behaviour of electrons in the interior of the tube and compare this to the described behaviour of the electron gas in [2].

The applications of electron gas are based on one little-known fact – the electrons located within a cylindrical tube having a positively charged cover (envelope) are in neutral-charge conditions – the total attractive force of the positive envelope plus negative contents equals zero. That means the electrons do not adhere to positive charged tube cover. They will freely fly into an AB-Tube. ([2, page 2])

In section 2 we describe the construction of the tube as in [2] and name its parameters. In section 3 we use Gauss’s law to analyse the electrical force and particle acceleration of the electrons inside the tube. In section 4 we describe what happens if we charge the tube interior. Then we are ready to calculate all actual forces of the tube in section 5. Finally, we give additional arguments to support the behaviour of the electron gas in section 6.

2 Design of the tube

To describe the tube we use cylinder coordinates. The tubes centerline rests on the z-axis of the coordinate system. The interior of the tube is empty and is surrounded by a non-conducting envelope. This envelope has a conducting outer surface. The parameters and their units of this tube are:

- $d$ – the envelope thickness [m]
- $l$ – the length of the tube [m]
- $S_1$ – the inner surface of the envelope
- $S_2$ – the outer surface of the envelope
- $r_1$ – the radius of the inner surface $S_1$ [m]
- $r_2$ – the radius of the outer surface $S_2$ [m]
- $A_i$ – the area of the surface $S_i$ [m$^2$]
- $\tau_1$ – the one dimensional charge density of all charges on $S_1$ or inside the tube [C/m]
- $\tau_2$ – the one dimensional charge density of all charges on $S_2$ [C/m]
- $\epsilon$ – the permittivity $\epsilon_r \epsilon_0$ [F/m]
- $\epsilon_r$ – the relative permittivity of the envelope [1]
- $E_i$ – the electrical field generated by the charges $\tau_1$ on $S_1$ in the radial direction [N/C]

We have the relations $r_2 = r_1 + d$, $A_i = 2\pi r_i l$ by the geometry of the tube. We assume $\tau_2 = -\tau_1 > 0$ which is equivalent to electrically neutrality of the whole tube.
3 Gauss’s law

In this section we apply Gauss’s law, also known as Gauss’s flux theorem, to the tube. To simplify the calculation, we assume:

1. The system is in a dynamical equilibrium.

2. The length \( l \) of the tube is big enough such that we can neglect the bottom and the top of the tube.

3. The axial symmetry of the tube carries over to the charge distribution.

Assumption 1 is justified as electrons are quite mobile and have low inertia. We can approximate assumption 2 as good as necessary by construction of the tube. Finally, assumption 3 is common in physics and is justified here because of the homogeneity along the axis as long as we are far away from the ends of the tube which is given by assumption 2. We could drop it by introducing the position on the axis as a new parameter, but it complicates the calculation without leading to new insights.

As of the symmetry we have a one dimensional charge density \( \tau(r) \leq 0 \) which depends only on the distance to the central axis. It describes the charge per length in a sub-tube of radius \( r \) whichs axis is identical with the axis of the tube. As we neglect the end sides of the tube we have that the direction of the electrical field \( E \) is radial. Now we can apply Gauss’s law the the Volume \( V(r) \) of a small finite cylinder whichs axis is on the axis of the tube, radius \( r \), and length \( L \).

\[
\oint_{\partial V(r)} \mathbf{E} \cdot d\mathbf{A} = \frac{Q(V(r))}{\varepsilon_0} \quad (1)
\]

\[
\iff \quad E(r)2\pi rL = \frac{\tau(r)L}{\varepsilon_0} \quad (2)
\]

\[
\iff \quad E(r) = \frac{\tau(r)}{2\pi r\varepsilon_0} \quad (3)
\]

This means that an electron at distance \( r \) from the axis reacts to the field \( E(r) \) such that it accelerates radially by

\[
a(r) = E(r)q_e/m_e \geq 0 \quad (4)
\]

where \( q_e = -1.602176487(40) \cdot 10^{-19} \) C (charge of an electron) \( (5) \)

\( m_e = 9.10938215(45) \cdot 10^{-31} \) kg \( \) (mass of an resting electron) \( (6) \)

Obviously the electrons in the tube are not free as long as there are electrons nearer to the axis. They repel each other such that they accelerate radially by

\footnote{Figures are taken from [1].}
Thus, there is no such thing as an electron gas which consists of individually free electrons.

4 Charging of the tube

If we charge the tube interior, e.g. by a hot cathode or by field electron emission, the electrons will repel each other following equation (4). Sooner or later they hit the non-conducting inner surface $S_1$ of the tube. There, some may be elastically or inelastically reflected other will adhere to the insulator as static electricity. The reflected electrons are accelerated according to equation (4) and will hit $S_1$ again and again until they adhere there as static electricity.

The charge distribution of the static electricity on $S_1$ will be uniform mainly because of two reasons. First, the electrons have higher probability to hit regions with (temporarily) lower charge, i.e. electron density. Second, every real insulator has a finite electric resistance such that any non-uniform charge distribution adjust over time to the uniform distribution by small electric currents.

Finally, all electrons which have been injected into the tube ends as static electricity of the insulator at the inner surface $S_1$ and that they are uniformly distributed.

Nota bene, an uniformly distributed charge on $S_1$ does not generate an electrical field inside the tube because of Gauss’s law and the symmetry resulting in equation 3.

5 Capacitor model

Given that the charges inside the tube adhere uniformly distributed to the inner surface $S_1$ we can calculate the forces which acts on the tube. Since the outer surface $S_2$ is conducting and positively charged such that the total charge is 0 C the tube is a cylinder capacitor.

In [2] the analysis focuses on the force which acts at the inner surface $S_1$ and that the force is generated by the electrical field as electrical pressure, c.f. [2] equation (1) on page 6.

In the discussion in [2] all forces are ignored which act on the positive charges at the outer surface $S_2$ of the envelope. To understand the situation and the forces at the envelope, we first analyse the effects of the electrical field next to the electrically insulating envelope.

We use the result [3] to calculate $E_i$. Since we are interested in the electrical field next to the surfaces $S_i$ but in the material of the envelope we use the limit
to get the result.

\[ E_1 = \lim_{h \to 0^+} E(r_1 + h) = \frac{\tau_1}{2\pi r_1 \varepsilon} \]  
\[ E_2 = \lim_{h \to 0^+} E(r_2 - h) = \frac{\tau_1}{2\pi r_2 \varepsilon} \]  

The pressure of the electric field is given by

\[ p_i = \frac{\varepsilon}{2} E_i^2 \]  

The direction of the force is in each case into the envelope, i.e. \( p_1 \) is outward radial pressure and \( p_2 \) is inward radial pressure. This is the case since if we would move the charges each in its direction we would decrease the total energy of the system. Equivalently, we would reduce the volume which contains the electrical field without increasing it due to Gauss’s law. The energy of the electric field, whichs energy density is \( 2E^2/\varepsilon \), in the volume difference would be set free.

Now, that we know the direction of the pressure we can calculate the total force \( F_{\text{tot}} \) which acts on the envelope in the radial direction by:

\[ F_{\text{tot}} = F_2 + F_1 = p_2 A_2 + p_1 A_1 = \pi d \left( r_1 E_1^2 - r_2 E_2^2 \right) \]
\[ = \frac{lr_2^2}{4\pi \varepsilon} \left( \frac{1}{r_1} - \frac{1}{r_2} \right) = \frac{lr_2^2}{4\pi \varepsilon} \frac{r_2 - r_1}{r_2 r_1} \]
\[ = \frac{lr_2^2}{4\pi \varepsilon} \frac{d}{r_1^2 + r_1 d} \]  

We see, that for infinite thin envelopes, i.e. \( d \to 0 \), the total force is zero \( F_{\text{tot}} = 0 \). For \( d \ll r_1 \) we can approximate

\[ F_{\text{tot}} \approx \frac{lr_2^2}{4\pi \varepsilon} \frac{d}{r_1^2} \]  

To overcome the atmospheric pressure \( p_{\text{air}} \) we need a total force \( F_{\text{tot}} \) such that:

\[ F_{\text{tot}} \geq p_{\text{air}} A_2 = 2\pi r_2 l p_{\text{air}} \]  

On the other hand we must limit the electrical field \( E_i \) by the dielectric strength \( E_{\text{max}} \) of real materials, c.f. appendix A. As the field \( E \) is stronger at the inner surface \( S_1 \) we must obey:

\[ E_1 \leq E_{\text{max}} \]  

We simplify the two conditions (12), (13). The result turns out to be:

\[ \frac{ed}{2p_{\text{air}}} E_{\text{max}}^2 \geq \frac{r_1^2 + 2r_1 d + d^2}{r_1} \rightarrow r_1 \]  

5
For $E_{\text{max}} = 670 \text{ MV/m}$, $\epsilon_r = 2.3$ (for Polyethylen) and $d = 1 \text{ \mu m}$ we obtain $r_1 \approx 45.11 \text{ \mu m}$. The exact solution turns out to be

$$r_1 = \frac{d}{4\rho_{\text{air}}} \left( \epsilon E_{\text{max}}^2 - 4\rho_{\text{air}} + \sqrt{\epsilon^2 E_{\text{max}}^4 - 8\rho_{\text{air}}\epsilon E_{\text{max}}^2} \right)$$  \hspace{1cm} (15)$$

which results in $r_1 = 43.08749 \text{ \mu m}$. If we increase the thickness of the envelope $d$ the resulting radius $r_1$ increases exactly linearly.

If the pressure of the electric field must stabilize a macroscopic tube with radius of 1 m against standard atmospheric pressure $p_{\text{air}}$, we need new materials which have a dielectric strength of about $102 \text{ GV/m}$, i.e. they must be more than 150 times as strong as current existing materials, c.f. [5]. Therefore, it is today impossible to create macroscopic tubes with thin envelopes which are stabilized by electrostatic pressure.

Moreover, for a given envelope material the relative buoyancy is proportional to the relative density if we neglect the end caps of the tube. This implies that we do not obtain a balloon if we increase the radius or length of the tube. We give the proof in the remainder of this section.

Let $m_{\text{air}}$ the mass of the displaced air [kg], and $m_{\text{env}}$ the mass of the envelope of the tube [kg]. Using (15) we derive

$$m_{\text{air}} = \pi r_1^2 * l * \rho_{\text{air}} = \pi (r_1 + d)^2 * l * \rho_{\text{air}}$$  \hspace{1cm} (16)$$

$$m_{\text{env}} = \pi (r_2^2 - r_1^2) * l * \rho_{\text{env}} = \pi (2r_1d + d^2) * l * \rho_{\text{env}}$$  \hspace{1cm} (17)$$

$$\frac{m_{\text{env}}}{m_{\text{air}}} = \frac{2r_1d + d^2}{r_1^2 + 2r_1d + 2d^2} \frac{\rho_{\text{env}}}{\rho_{\text{air}}} = \frac{2x + 1}{x^2 + 2x + 1} \frac{\rho_{\text{env}}}{\rho_{\text{air}}}$$  \hspace{1cm} (18)$$

where $x = \frac{1}{4\rho_{\text{air}}} \left( \epsilon E_{\text{max}}^2 - 4\rho_{\text{air}} + \sqrt{\epsilon^2 E_{\text{max}}^4 - 8\rho_{\text{air}}\epsilon E_{\text{max}}^2} \right)$ by (15).

The tube is buoyant if and only if $\frac{m_{\text{env}}}{m_{\text{air}}} < 1$. This is only fulfilled for a medium of rather hight density and low pressure. E.g. for $E_{\text{max}} = 670 \text{ MV/m}$, $\epsilon_r = 2.3$, $\rho_{\text{env}} = 0.94 \text{ g/cm}^3$ [6] (for Polyethylen) and $\rho_{\text{air}} = 1.204 \text{ kg/m}^3$ [7] we get $\frac{m_{\text{env}}}{m_{\text{air}}} > 35$.

6 Further arguments against free electron gas

Although we have proven already in section 3 that there is no free electron gas in the tube, we present here additional arguments.

First, if an electron in the tube moves the charges on the envelope reacts on this. For a perfect conductor they react such that the total electrical field outside the tube vanishes. For a real conductor this is only partially achieved or rather with a time delay. Since the time of Sir Isaac Newton we know actio equals reactio [3, Lex. III., page 13]. Thus, the moving of the charges on the envelope causes forces on all the electrons in the tube. Therefore, the electrons are not free.

Second, we consider the situation of conductors. Vacuum is a conductor as soon as there are charge carriers in it, e.g. electrons. By definition of vacuum...
there are no particles which hinder the motion of the charge carriers but the
carriers themselves. Because of the electrostatic induction the mobile electrical
charges resides on the surface of the conductor. This is a well known fact of
electrostatic theory. The electrons are not in the interior of the vacuum but at
its boundary, i.e. the surface \( S_1 \).

Third, from particle accelerator experiments we know for sure that we must
contain an ensemble of electrons by some means, e.g. magnetic fields or the self
magnetic field if there is an electric current in the vacuum. In the latter case
the self magnetic field reduces the radial pressure of the electrons. Without
containment the cloud of electrons expands until it reaches the boundaries of
the container.

7 Conclusion

The properties of the tube as described in [2] are quite promising. Unfortunately
our analysis shows that such tubes are not possible with today, i.e. 2011, known
materials. Thus we cannot build ballons with high lift capacity, high altitude
platforms, or quasi-superconductors as proposed in [2] with this technique.

This analysis uses that the electrons inside the tube adhere as static elec-
tricity to the envelope. The resulting pressure is that of the electric field.

A. Bolonkin’s opinion is that most of the electrons are reflected by the enve-
lope and as such exchange momentum with it which creates outward pressure.
Other contributions of the pressure are “charge pressure”, and “magnetic pres-
sure”. This effects are additional to the electric pressure. [4]

We recommend an experiment to observe the real behaviour of the electrons
in the tube.

A Material constants

| Substance           | Dielectric Strength [MV/m] |
|---------------------|-----------------------------|
| Helium              | 0.15                        |
| Air                 | 0.4 - 3.0                   |
| Alumina             | 13.4                        |
| Window glass        | 9.8 - 13.8                  |
| Silicone oil        | 10 - 15                     |
| Polystyrene         | 19.7                        |
| Neoprene rubber     | 15.7 - 27.6                 |
| Water               | 65 - 70                     |
| Salt                | 150                         |
| Benzene             | 163                         |
| Teflon              | 87 - 173                    |
| L.D. Polyethylene film | 300                      |
| Fused silica        | 470 - 670                   |

Source: [5]
References

[1] K. Nakamura et al. (Particle Data Group), J. Phys. G 37, 075021 (2010) Cut-off date for this update was January 15, 2010. http://pdg.lbl.gov/ retrieved 2010-02-19.

[2] Alexander Bolonkin, AB Electronic Tubes and Quasi-Superconductivity at Room Temperature, C&R, 1310 Avenue R, #F-6, Brooklyn, NY 11229, USA, T/F 718-339-4563, aBolonkin@juno.com aBolonkin@gmail.com http://Bolonkin.narod.ru Presented to http://arxiv.org on 8 April, 2008, http://arxiv.org/abs/0805.0230

[3] Sir Isaac Newton, Philosophiae Naturalis Principia Mathematica, July 5 1687, http://www.ntnu.no/ub/spesialsamlingene/ebok/02a019654.html retrieved 2011-02-18T17:35:42 GMT

[4] Private communication with Alexander Bolonkin

[5] Dielectric strength, http://en.wikipedia.org/wiki/Dielectric_strength retrieved 2009-03-02T02:37:19 GMT

[6] Polyethylen, http://en.wikipedia.org/wiki/Polyethylene retrieved 2010-06-01

[7] Air, http://en.wikipedia.org/wiki/Air retrieved 2010-06-01