Abstract

Image segmentation is to separate an image into distinct homogeneous regions belonging to different objects. It is an essential step in image analysis and computer vision. This paper compares some segmentation technologies and attempts to find an automated way to better determine the parameters for image segmentation, especially the connectivity value of $\lambda$ in $\lambda$-connected segmentation. Based on the theories of the maximum entropy method and Otsu’s minimum variance method, we propose: (1) maximum entropy connectedness determination: a method that uses maximum entropy to determine the best $\lambda$ value in $\lambda$-connected segmentation, and (2) minimum variance connectedness determination: a method that uses the principle of minimum variance to determine $\lambda$ value. Applying these optimization techniques in real images, the experimental results have shown great promise in the development of the new methods. In the end, we extend the above method to more general case in order to compare it with the famous Mumford-Shah method that uses variational principle and geometric measure.

1 Introduction

Image segmentation is the basic approach in image processing and computer vision [22]. It is used to locate special regions and then extract information from them. Image segmentation is used to partition an image into different components or objects and is an essential procedure for image preprocessing, object detection and extraction, and object tracking. Image segmentation is also related to edge detection.

Even though there is no unified theory for image segmentation, some practical methods have been studied over the years such as thresholding, edge based segmentation, region growing, clustering (unsupervised classification), and split-and-merge segmentation, to name a few. $\lambda$-connected segmentation is a technique in the category of region growing segmentation. It was proposed to find an object having the property of gradual variation [3] [8] [4] [9] [10].

This paper attempts to find an automated way to better determine the parameters for image segmentation, especially the connectivity value of $\lambda$ in $\lambda$-connected segmentation.

This paper first reviews some major segmentation techniques to explain why segmentation is difficult, and how a special technique would be selected in specific applications. We then focus on our problem of determining segmentation parameters in $\lambda$-connected segmentation. Based on the philosophies of the maximum entropy method and Otsu’s minimum variance method, we propose: (1) maximum entropy connectedness determination: a method that uses maximum entropy to determine $\lambda$ value, and (2) minimum variance connectedness determination: a method that uses the principle of minimum variance to determine $\lambda$ value. Applying these optimization techniques in real images, the experimental results have shown the great promise of the new methods. In the end, we extend the above method to a more general case in order to compare it with the famous Mumford-Shah method that uses variational principle and geometric measure [20].

2 Image Segmentation Review

In this section, we first review current technology of image segmentation: five types of techniques, their characteristics, and uses. We then focus on the connectedness-based image segmentation technique.

2.1 Overview of Image Segmentation Approaches

As we know, there is no unified theory for image segmentation, some practical methods have been studied over
the years such as thresholding, edge based segmentation, region growing, clustering (unsupervised classification, e.g. k-mean or fuzzy c-mean), and split-and-merge segmentation. These segmentation algorithms have been developed for solving different problems. However, they are all based on one or more of the five philosophies listed below:

1. A segment is a class/cluster, so one can use a classification/clustering method to segment the image. Classification methods usually do not need to use the location/position information. Clustering for unsupervised classification technology can perform better to find an object for sampled points within the subset of data frames. Typical techniques include Isodata and k-mean or fuzzy c-mean.

K-mean or fuzzy c-mean is a standard classification method that is often used in image segmentation. This method classifies the pixels into different groups in order to minimize the total “errors,” where the “error” is the distance from the pixel value to the center of its own group.

2. A segment is a homogeneous region. If an object or region can be identified by absolute intensity (the pixel value), we usually use threshold segmentation. In other words, an object will be recognized as a geometrically connected region whose values/intensities are between a certain high-limit and a low-limit. We usually assume that the high limit is the highest value of the image. Therefore, in practice, one only needs to determine the low-limit. Maximum entropy and minimum variance (also called Otsu’s method) are two of the most popular methods for determining the best threshold for single image.

Multilevel thresholding is similar to threshold segmentation and uses the same philosophy, but multiple thresholds are produced at once. It needs an extremely high time cost for computation. We will discuss these two methods in detail in section 3.

3. A segment is a “smoothly-connected” region. In a region where intensity changes smoothly or gradually, the region is viewed as a segment. Smoothness can be measured by a limit.

A popular segmentation method is called mean-based region growing segmentation in this paper. A pixel will be included in a region if the updated region is homogeneous, meaning that the difference between pixel intensity and the mean of the region is limited by , a small real number.

The segmentation follows the same philosophy. This is to link all pixels that have the similar intensity.

This method is related to a fuzzy method created by Rosenfeld who treated an image as a 2D fuzzy set. Then, he used -cut to segment the image into components. Another way is to measure two pixels to see if they are “fuzzy” connected. A pixel set is -connected if for any two points there is a path that is -connected where is a fuzzy value between 0 and 1. This is a generalization of threshold segmentation in some cases. This method can be used to divide (partition) different intensity levels without calculating different thresholds or clip-level values. However, for a complex image, how to calculate the value of remains unknown.

A fast algorithm can be designed to perform a segmentation. In fact, the simple form of both threshold segmentation and -connected segmentation can be done in linear time.

We can see that mean-based region growing segmentation is a statistical approach, but the -connected approach is a graph-theoretic method. We can combine them by requiring that the -connected segment also be within a limit of the mean. Or after the mean-based segmentation, we can do a -connected segmentation.

Split-and-merge segmentation uses quadtree to determine the order in which pixel(s) should be treated or computed. It is an algorithmic way to find an object or to force a merge order. This is because this method is based on the mean of the merged region. It does not guarantee a transitive relation. Again, the mean-based segmentation is not an equivalence relation. This method splits an image into four sections and checks if each part is homogeneous. The homogenous segments are then merged together. If the segments are not homogenous, the splitting process is repeated. This process is called quadtree segmentation. The method is more accurate for some complex images. However, it costs more time to segment an image. The time complexity of this method is . This was proved by Chen in 1991.

5. A segment is surrounded by one or several closed edges. If we can detect and track the edges, we can determine the location and outline of the segments.

The fifth philosophy is edge detection. To find low or high frequency pixels are very common in edge detection. However, not all edge-detection methods can be used in image segmentation since enhancing edge is not the primary purpose of image segmentation. The purpose of image segmentation is to find components. The number of edges should be relatively small. Otherwise, the extraction of the closed curves will be the major problem. Recent development indicated that the Mumford-Shah method is promising. The method uses the variational principal. This method has captured a considerable amount of attention.

The Mumford-Shah method considers three factors in segmentation: (1) the total length of all the segments, (2) the unevenness of the image without its edges, and (3) the total error between the original image and the proposed segmented images where each segment has unique or similar values in its pixels. When the three weighted factors are minimized, the resulting image is a solution of the Mumford-Shah method. Recently, Zhan and Vese proposed level-sets to simplify the Mumford-Shah method so that it produced better results. However,
level-sets use contour boundaries that may limit the flexibility of the original Mumford-Shah method. The Mumford-Shah method also needs an alternative process and its algorithm performance is still unknown.

2.2 Remarks on Selecting an Appropriate Segmentation Method

The $k$-mean or fuzzy $c$-mean, maximum entropy, and the Mumford-Shah method all require an iterated process that is good at detailed or fine segmentation. This is not a quick solution for fast segmentation. The fast segmentation methods are only used for region growing including the original threshold method, mean-based and lambda-connected search, and split-and-merge method.

In practice, $k$-mean or fuzzy $c$-mean and maximum entropy are still the most popular. However, for some types of sequential images, these two methods do not obtain stable results when we process a set of meteorological data to find areas with the most water vapor indicating (most likely) the location of a hurricane [19]. $\lambda$-connected segmentation, on the other hand, has worked very well. A problem the $\lambda$-connected method faced was that the $\lambda$ value needed to be assigned even though that value can be used throughout each data frame in the data set. To find a way to automatically determine the $\lambda$-value is a long-term goal for the author. Some methods have been proposed such as maximum connectedness spanning tree [5] [6] and the golden cut point [7].

In this paper, the author proposes optimization methods based on maximum entropy and minimum variance, respectively.

2.3 The Connectedness-based Segmentation

$\lambda$-Connected segmentation is based on the philosophy that an object must have a smooth inside and has a gap at its boundary (in terms of intensity). Trying to find a closed curve/boundary that indicates the intensity of the jump is the key to this method.

A measure of connectedness can be used to partition a set of data into connected components based on adjacency or neighborhood systems. Using connectedness to divide an image does not require transforming the image into binary form. After the data is partitioned, a fast algorithm such as the breadth-first-search algorithm can be applied to find a connected component [3] [8]. $\lambda$-connected search was introduced to segment such an image without transferring the image into a $\{0,1\}$-image. However, the value of $\lambda$, usually between 0 and 1, determines the fineness of the segmentation.

$\lambda$-connectedness can be defined on an undirected graph $G = (V,E)$ with an associated (potential) function $f : V \rightarrow R^m$, where $R^m$ is the $m$-dimensional real space [10]. Given a measure $\alpha_p(x,y)$ on each pair of adjacent points $x,y$ based on the values $\rho(x),\rho(y)$, we define

$$\alpha_p(x,y) = \begin{cases} \mu(\rho(x),\rho(y)) & \text{if } x \text{ and } y \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases}$$

where $\mu : R^m \times R^m \rightarrow [0,1]$ with $\mu(u,v) = \mu(v,u)$ and $\mu(u,u) = 1$. $\alpha_p$ is used to measure “neighbor-connectivity.” The next step is to develop path-connectivity so that $\lambda$-connectedness on $<G,\rho>$ can be defined in a general way.

In graph theory, a finite sequence $x_1,x_2,...,x_n$ is called a path, if $(x_i,x_{i+1}) \in E$. The path-connectivity $\beta$ of a path $\pi = \pi(x_1,x_n) = \{x_1,x_2,...,x_n\}$ is defined as

$$\beta_p(\pi(x_1,x_n)) = \min\{\alpha_p(x_i,x_{i+1})|i = 1,...,n-1\}$$

or

$$\beta_p(\pi(x_1,x_n)) = \prod\{\alpha_p(x_i,x_{i+1})|i = 1,...,n-1\}$$

Finally, the degree of connectedness or connectivity of two vertices $x,y$ with respect to $\rho$ is defined as:

$$C_\rho(x,y) = \max\{\beta(\pi(x,y))|\pi \text{ is a (simple) path.}\}$$

For a given $\lambda \in [0,1]$, point $p = (x,\rho(x))$ and $q = (y,\rho(y))$ are said to be $\lambda$-connected if $C_\rho(x,y) \geq \lambda$. In image processing, $\rho(x)$ is the intensity of a point $x$ and $p = (x,\rho(x))$ defines a pixel.

3 $\lambda$ Value Determination and Optimization

It is a natural and unavoidable question how we determine $\lambda$ value in connectedness-based segmentation? It is somehow similar to determine the clip level in threshold segmentation; however, lambda value is not as sensitive as the clip-level in thresholding. There are fewer $\lambda$ values to be selected than clip-levels. In Chapter 10 of [6], Chen provided the detail analysis on this issue.

Some techniques have been proposed and tested such as the binary search-based method and the maximum connectedness spanning tree method [6] [5]. We also proposed a golden-cut technique for bone density measurement in [7]. In this section, we propose two new methods to determine the $\lambda$ value for the segmentation. The new methods are based on maximum entropy and minimum variance, respectively.

3.1 Connectedness and Maximum Entropy

In this subsection, we propose a method that uses the maximum entropy method to determine the $\lambda$ value. It can be called maximum entropy connectedness optimization.
The maximum entropy method was first proposed by Kapur, Sahoo, and Wong [15]. It is based on the maximization of inner entropy in both the foreground and background. The purpose of finding the best threshold is to make both objects in the foreground and background, respectively, as smooth as possible. [15] [22] [1]

If \( F \) and \( B \) are in the foreground and background classes, respectively, the maximum entropy can be calculated as follows:

\[
H_F(t) = -\sum_{i=0}^{F} \frac{p_i}{p(F)} \ln \frac{p_i}{p(F)}
\]

\[
H_B(t) = -\sum_{i=t+1}^{255} \frac{p_i}{p(B)} \ln \frac{p_i}{p(B)}
\]

where \( p_i \) can be viewed as the number of pixels whose value is \( i \); \( p(B) \) is the number of pixels in background, and \( p(F) \) is the number of pixels in foreground. The maximum entropy is to find the threshold value \( t \) that maximizes \( H_F(t) + H_B(t) \).

Such an idea can be used for \( \lambda \)-connected segmentation. However, the total inner entropy for the image is to calculate the entropy for each segment (\( \lambda \)-connected component), not for the thresholding clipped foreground/background. This is because in \( \lambda \)-connected segmentation there is no specific background. Each \( \lambda \)-connected segment can be viewed as foreground, and the rest may be viewed as the background. It is different from the original maximum entropy where the range of pixel values determines the inclusion of pixels. Therefore, we need to summarize all inner entropies in all segments.

\[
H(\lambda) = \Sigma (\text{inner entropy of each } \lambda \text{-connected component})
\]

We will select the \( \lambda \) such that \( H(\lambda) \) will be maximized. We call this \( \lambda \)-value the maximum entropy connectedness. This unique value is a new measure for images.

Since the maximum entropy means the minimum amount of information or minimum variation, we want the minimum change inside each segment. This matches the philosophy of the original maximum entropy method. In other words, the \( \lambda \)-connected maximum entropy has a better meaning in some applications. We use the \( \lambda_e \) such that

\[
H(\lambda_e) = \max \{H(x)|x \in [0, 1]\}.
\]

Assume there are \( m \) \( \lambda \)-components, define inner entropy of each \( \lambda \)-component \( S_i \):

\[
H_i(\lambda) = \sum_{k=0}^{255} -\frac{\text{Histogram}[k]}{n} \log \frac{\text{Histogram}[k]}{n}
\]

where \( n \) is the number of points in the component \( S_i \). \( \text{Histogram}[k] \) is the number of pixels whose values are \( k \) in the segment. Thus,

\[
H(\lambda) = \sum_{i=1}^{m} H_i(\lambda)
\]

The maximum entropy connectedness can be viewed as a measure of a special connectivity for the image. If \( \lambda \) value is calculated in the above formula for an image that makes \( H(\lambda) \) to be maximum, we call that the image have the maximum entropy connectedness \( \lambda \), denoted as \( \lambda_e \).

3.2 Experimental results with \( \lambda_e \)

In [7], we proposed a golden cut method for finding the \( \lambda \)-value for bone density connectedness calculation. We have obtained a \( \lambda = 0.96, 0.97 \) for a bone image (the size of the picture is different from the one used in this paper). For a similar image, using the maximum entropy connectedness presented in this section, we got \( \lambda_e = 0.95 \). The result is quite reasonable. The original image and both of the segmented images are shown in Fig. 1-4. No pre-cut (preprocessing) is performed in the segmentation.

![Figure 1. Bone Density Image Segmentation: the Original image](image1)

![Figure 2. Bone Density Image Segmentation: \( \lambda = 0.97 \)](image2)
0.99 (Fig. 6). An original maximum entropy arrived at the clip-level of 125 counts of the 8-bit gray level image (0-255 of the pixel value range. The reason is that the “Lena” image does not contain many “continuous” parts. In the $\lambda$-connected segmentation, we can see that $\lambda_e = 0.99$ connected segmentation has connected the continuous component especially at the face and shoulder. This matches the result of using the maximum entropy cut, Fig. 7 (We use NIH ImageJ to perform the cut.) Thus, we can say that our new method is still reasonable. When we use $\lambda = 0.98$ for the image, we get Fig. 8.

For the image having gradual variation property, $\lambda$-connected segmentation usually has an advantage. We have extensively tested a set of sequential images in order to find the outlier of meteorological data that indicates (most likely) the hurricane center [19]. The data frames we used are water vapor images.

Except the standard threshold method and the $\lambda$-connected segmentation method, all other methods we tested failed including famous $k$-mean and maximum entropy [19]. What we found was that the key for this sequential images is that the pre-cut is necessary. Interesting enough for us, 45% of the pick value of each image for the cut receives the best result. After that, we can use $\lambda=0.95$ for the segmentation parameter and extract the largest component for the outlier searching result.

The problem here is that 45% of the pick value as clip-level is not automatically calculated. If we want a totally au-
automatic process, we might to use maximum entropy to make the first cut. In this testing set, we have 12 image frames. For some beginning images, the method worked well as expected. For other images, the new method proposed in this paper consistently got wrong results since the \( \lambda_e \) calculated is always greater than or equal to 0.97. We cannot get \( \lambda \) to be 0.95 using "maximum entropy connectedness.” After looking into the detail images, we have found two problems: (1) the pixel values are not “continuous” around “the desired outlier,” and (2) The largest component criteria for the outlier is not quite represent the nature of hurricane centers (outliers), we need to change the criteria to “the largest and the brightest.”

For (1), we have done a smoothing process. For (2), we change the outlier criteria from the largest component to the total intensity of the component (not only testing its size). Smoothing preprocess is reasonable, and it is good for the \( \lambda \)-connected segmentation to find large component.

The following image shows such a treatment. We have applied an automatic pre-cut by using maximum entropy instead of the pre-cut using the 45% of pick value. Fig. 9 shows an original image.

Fig. 10 shows the result using our new method plus a maximum entropy threshold cut of preprocessing. A threshold value=23 is calculated by standard maximum entropy. Then, we perform the automated finding process to get \( \lambda_e \) described above. \( \lambda_e = 0.90 \) was obtained and used.

If we just use the standard maximum entropy at threshold = 23, we will have the following image Fig. 11.

3.3 Consider Outer Entropy in Maximum Entropy Connectedness

Using maximum entropy is a type of philosophical change. In fact, we can consider other formulas. For example we can select another way to calculate the entropy of one segment.

Here we propose a different formula. We can calculate the inner entropy of a component, then treat the rest of the image as the background for the component. The total entropy generated by this segment is the summation of both. We can apply this process to all components/segments while segmenting.

Let \( I \) be the image, \( C_i(\lambda) = I - S_i(\lambda) \) is the complement of component \( S_i(\lambda) \)

\[
H(C_i(\lambda)) = \{\text{Entropy for the set } C_i(\lambda)\}
\]

We can use the following formula for the basis of optimization.

\[
H(\lambda) = H(S_i(\lambda)) + H(C_i(\lambda))
\]

The outer (background) entropy is the total.

\[
H(\text{outer}) = \Sigma H(C_i)
\]

In \( H(\text{outer}) \), a pixel is calculated multiple times. We may need to use the average \( H(\text{outer})/m \) where \( m \) is the number of segments. The relationship between this formula and the formula we used in the previous subsection is also interesting.

Furthermore, we should consider the following general model.

\[
H_{optimal} = a \cdot H(\text{inner}) + b \cdot H(\text{outer})
\]

where \( a \) and \( b \) can be constant or function of segmentations.
3.4 Connectedness and Minimum (Inner) Variance

In this subsection, we develop a minimum variance-based method for finding the best $\lambda$ value in $\lambda$-connected segmentation. Minimum variance was first studied by Otsu in image segmentation [21] [11]. In other words, Otsu’s segmentation was the first global optimization solution for image segmentation. It is used to clip the image into two parts: the object and the background.

Assume that $\sigma^2(W), \sigma^2(B), \sigma^2(T)$ represent the within-class variance, between-class variance, and the total variance, respectively. The optimum threshold will be determined by maximizing one of the following criterion with respect to threshold $t$ [21] [11]:

$$H(\lambda) = \sum (\text{inner variance of each } \lambda - \text{component}) + c \cdot M$$

where $c$ is a constant. We could let $c = 1$. The calculation of the inner variance of a $\lambda$-component is to compute the variance (square of standard deviation) of the pixels in the component.

The following formula is to find minimum average variance (for each component).

$$H(\lambda) = \sum (\text{inner variance of each } \lambda - \text{component}) / M$$

We want to find $\lambda_0$ such that

$$H(\lambda_0) = \min \{H(\lambda) | \lambda \in [0, 1]\}$$

This strategy only works for the meteorological data. The experimental results show that the method is promising. For the other two kinds of images tested in maximum entropy connectedness, “Lena” and the Bone image, we still need to find an appropriate way under minimum variance philosophy.
The following images show the process on the same picture with a preprocessing threshold cut using the maximum entropy cut or 45% peak cut. Then, we perform the automated process of finding of \( \lambda \)-value. Fig. 12 shows that we arrived at \( \lambda_v=0.97 \) using the method of minimum variance connectedness determination described in this subsection. Without smoothing the original image, we pre cut the image using maximum entropy threshold. The result is not what we expected.

![Figure 12. Minimum variance connectedness determination without smoothing](image1)

When we smoothed the image, we got \( \lambda_v=0.90 \), and the result turned to be correct, see Fig. 13.

![Figure 13. Minimum variance connectedness determination with smoothing](image2)

### 3.5 \( \lambda \)-Connectedness and Mumford-Shah’s Method

How do we use Mumford-Shah’s idea to find the optimal segmentation? We can define \( L \) as the total length of the edge of all segments.

\[
H(\lambda) = \alpha \cdot \Sigma(\text{inner variance of each } \lambda\text{-component}) + \beta \cdot L
\]

(10)

where \( \alpha \) and \( \beta \) are constants. More generally, we can do the normal \( \lambda \)-connected fit \([4][9]\) on each \( \lambda \)-connected component. The total variance (or standard deviation) of the (normal \( \lambda \)-connected) fitted image is denoted as \( V \). \( L \) is still the total length of edges of segments (components), and \( D \) is the difference between the fitted image and the original image. Using Mumford-Shah’s Method, we can minimize the following equation to get the \( \lambda_p \).

\[
H(\lambda_p) = \min\{H(\lambda)|\lambda \in [0, 1]\}
\]

(12)

\[
H(\lambda_p) = \alpha \cdot (V) + \beta \cdot (L) + \gamma \cdot (D)
\]

(11)

### 4 Discussion

Even though we calculated the entropy or variance in each connected component that is different from the standard maximum entropy and the Otsu’s method in image segmentation, the philosophy remains the same as in these two popular methods. The results are very promising. These two new methods can be easily applied in other region-growing segmentations. A large amount of further research should be done to support and the new methods. We will implement the method proposed in subsection E in section III, and compare it with the results obtained in \([11]\).

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