I. INTRODUCTION

The isotope effect in high-$T_c$ cuprate superconductors is unconventional in different respects. Optimally doped samples show a very small isotope exponent $\alpha$ of the order of 0.05 or even smaller, in contrast to the conventional Bardeen-Cooper-Schrieffer (BCS) value of 0.5, which one expects for a conventional phonon induced pairing interaction. This unusually small value in connection with the high value of $T_c$ lead to early suggestions that the pairing interaction in high-$T_c$ cuprates might be predominantly electronic in origin with a possible small phononic contribution. This scenario, however, is difficult to reconcile with the fact that the isotope coefficient also shows an unusually strong doping dependence, reaching values of 0.5, in some cases even higher, in the underdoped, $T_c$ reduced, compounds.

Many different models have been advanced in order to try to understand this unusual doping dependence in connection with the small isotope exponent at optimal doping, e.g. influence of van Hove singularities, anharmonic phonons, electron-phonon coupling in the presence of strong non-ferromagnetic correlations, pair breaking effects due to magnetic impurities or Jahn-Teller nonadiabaticity, but no consensus has been reached so far.

In recent years it became apparent that the physics of underdoped high-$T_c$ superconductors is governed by the pseudogap phenomenon. A behavior which is reminiscent of the presence of a pseudogap, growing upon successive underdoping, has been observed consistently in a large number of different experiments, e.g. nuclear magnetic resonance (NMR) Knight-shift and relaxation rate experiments, specific heat, angular-resolved photoemission spectroscopy (ARPES), tunneling, $c$-axis and $ab$-plane dynamical conductivity. Currently there is no consensus about the origin of this pseudogap and many different proposals exist. Williams et al. have shown that a phenomenological model for a pseudogap having $d$-wave symmetry can account well for thermodynamic quantities in the underdoped cuprates.

In the present manuscript we want to study whether the influence of the pseudogap might give a, perhaps more natural, explanation for the unusual isotope effect in underdoped high-$T_c$ cuprates. As has been shown by Carbotte et al. an energy dependence of the electronic density of states (DOS) varying on the pairing energy scale can modify the isotope effect, and therefore we ask whether there might be a link between the pseudogap phenomenon and the isotope effect. Since there exists no widely accepted theory for the pseudogap at present, here we will follow the idea of Williams et al and treat the pseudogap on a phenomenological basis, introducing it into the single particle excitation spectrum. Such a procedure is reasonable, if the pseudogap itself does not show an isotope effect, as suggested by recent NMR experiments.

In the following we will study the influence of such a pseudogap on the isotope effect within different models. We shall start with the weak-coupling (BCS) approximation where we consider $s$- and $d$-wave symmetry of the superconducting order parameter and of the pseudogap. In order to see whether these results are stable for more realistic cases, we will study two recently proposed models based on a spinfluctuation exchange pairing interaction, e.g. the nearly antiferromagnetic Fermi liquid (NAFL) model due to Monthoux and Pines and the self-consistent fluctuation-exchange (FLEX) approximation for the two dimensional Hubbard model.

II. WEAK COUPLING APPROXIMATION

The linearized gap equation in the weak-coupling limit for an anisotropic pairing interaction $V(\vec{k}, \vec{k}')$ reads:

$$\Delta(\vec{k}) = \frac{1}{N} \sum_{\vec{k}'} V(\vec{k}, \vec{k}') \tanh \left( \frac{\epsilon_k / 2T_c}{2\epsilon_{k'}} \right) \Delta(\vec{k}') \tag{1}$$

Here, $\epsilon_k$ is the band dispersion and $\Delta(\vec{k})$ the superconducting gap function. We want to assume that the pairing interaction consists of two parts: a phononic part $V_p(\vec{k}, \vec{k}')$ and an electronic part $V_e(\vec{k}, \vec{k}')$, such that...
The phononic part may consist of different contributions having different symmetries. However, since we assumed that the electronic part is dominating with a symmetry specified by \(\psi_\eta\), only the \(\psi_\eta\)-component of \(V_p\), having the same symmetry, will affect \(T_c\). Therefore we can assume without loss of generality

\[
V_p(\vec{k}, \vec{k}') = \begin{cases} V_{p0} \psi_\eta(\vec{k}) \psi_\eta(\vec{k}') & \text{if } |\epsilon_k|, |\epsilon_{k'}| \leq \omega_p \\ 0 & \text{else} \end{cases},
\]

where \(\omega_p\) is the characteristic phonon energy. In harmonic approximation, which we will adopt here, \(\omega_p\) varies with the isotopic mass \(M\) like \(1/\sqrt{M}\), while \(\omega_c\) is assumed to be independent of \(M\).

For such an interaction the gap function can be separated into two parts: \(\Delta(\vec{k}) = \Delta_{c,0}(\vec{k}) + \Delta_{c,p}(\vec{k})\), with

\[
\Delta_{c,p}(\vec{k}) = \begin{cases} \Delta_{c,0,0} \psi_\eta(\vec{k}) & \text{if } |\epsilon_k| \leq \omega_{c,p} \\ 0 & \text{else} \end{cases}.
\]

With this ansatz Eq. (3) becomes a \(2 \times 2\) matrix equation for the two order parameter components \(\Delta_{c,0}\) and \(\Delta_{c,p}\). Assuming a cylindrical Fermi surface with a constant density of states Eq. (4) can be written in the form

\[
\begin{pmatrix} \Delta_{c,0} \\ \Delta_{c,p} \end{pmatrix} = \begin{pmatrix} V_{c0}(\omega_c) & V_{c0}(\omega_c) \\ V_{p0}(\omega_c) & V_{p0}(\omega_p) \end{pmatrix} \begin{pmatrix} \Delta_{c,0} \\ \Delta_{c,p} \end{pmatrix},
\]

where we defined the function \(L(\omega)\)

\[
L(\omega) = N(0) \int_0^\omega \frac{\tanh(\epsilon/2T)}{\epsilon} \approx N(0) \ln \left( \frac{1.13\omega}{T} \right).
\]

The last expression holds in the weak coupling limit \(\omega \gg T\). \(N(0)\) denotes the density of states at the Fermi level. In deriving Eq. (5) we assumed \(\omega_c \ll \omega_p\), as is usually the case for spinfluctuation exchange models (see the following sections). However, the final result Eq. (12) does not depend on this choice. Letting \(L_p = L(\omega_p)\) and \(L_c = L(\omega_c)\) the leading eigenvalue of the matrix in Eq. (5) is

\[
\lambda(\omega_c, \omega_p, T) = \frac{V_{c0}L_c + V_{p0}L_p}{2} + \frac{1}{2} \sqrt{(V_{c0}L_c - V_{p0}L_p)^2 + 4V_{c0}V_{p0}L_c^2}\]

and \(T_c\) is determined from the implicit equation

\[
\lambda(\omega_c, \omega_p, T_c) = 1.
\]

From this the isotope exponent \(\alpha_0\) can be calculated:

\[
\alpha_0 = \frac{1}{2} \frac{d \ln T_c}{d \ln \omega_p} = \frac{1}{2} \frac{\omega_p}{T_c} \frac{\partial L_c}{\partial L_p} \frac{\partial L_p}{\partial L_c} + \frac{\partial L_p}{\partial L_c}.
\]

In the weak coupling limit \(\omega_p, \omega_c \gg T_c\) this gives

\[
\alpha_0 = \frac{1}{2} \frac{V_{p0}(1 - V_{c0}L_c)}{V_{p0}(1 + V_{c0}L_c) + V_{c0}(1 - V_{p0}L_p)}.
\]

Note, that for a purely electronic interaction \(V_{p0} = 0\) this expression yields \(\alpha_0 = 0\) and for a purely phononic interaction \(V_{c0} = 0\) it gives \(\alpha_0 = 0.5\), as one should expect. For a mixed interaction \(\alpha_0\) will generally lie between 0 and 0.5. In fact, one can easily show that for given values of \(\omega_p\) and \(\omega_c\) one can always choose \(V_{p0}\) and \(V_{c0}\) in such a way that a given value of \(T_c\) and \(\alpha_0 \in [0, 0.5]\) is reached.

Now we wish to consider the influence of a pseudogap. In the presence of a pseudogap we have to modify the single particle excitation spectrum. Following Williams et al, we replace in Eq. (11)

\[
\epsilon_k \quad \Rightarrow \quad \sqrt{\epsilon_k^2 + E_g^2(\vec{k})},
\]

where \(E_g(\vec{k})\) is the pseudogap and will be chosen to be either \(E_{g,s}(\vec{k}) = E_{g0} = \text{const}\) for an s-wave pseudogap or \(E_{g,d}(\vec{k}) = E_{g0} \cos 2\Theta_k\) for a d-wave type pseudogap. Note, that this symmetry of the pseudogap not necessarily has to be identical with the pairing symmetry and we will allow them to be independent in this section. However, the study in Ref. [7] suggests that both symmetries are of d-wave type in underdoped high-\(T_c\) compounds and we will focus on this case in the following sections. With the replacement Eq. (11) the function \(L(\omega)\) becomes

\[
L(\omega) = N(0) \int_0^{2\pi} d\Theta \psi_\eta^2(\Theta) \int_0^{\omega} \frac{d\epsilon}{\epsilon} \frac{\tanh \left( \frac{\sqrt{\epsilon^2 + E_g^2(\Theta)}}{2T} \right)}{\sqrt{\epsilon^2 + E_g^2(\Theta)}}.
\]

Eqs. (10) and (11) still remain valid, if one uses this expression for \(L(\omega)\). In the weak-coupling limit \(\omega_p, \omega_c \gg T_c, E_g\) we then find for the isotope exponent

\[
\alpha = \alpha_0 \left( \frac{1}{4\pi T_c} \int_0^{2\pi} d\Theta \int_0^\infty \frac{\psi_\eta^2(\Theta)}{\cosh^2 \left( \frac{\sqrt{\epsilon^2 + E_g^2(\Theta)}}{2T} \right)} \right)^{-1}.
\]
where $\alpha_0$ is the isotope exponent Eq. (10) in the absence of a pseudogap. Eq. (13) shows that $\alpha/\alpha_0$ only depends on $E_g(0)/T_c$, the pairing symmetry $\psi_0(\Theta)$ and the symmetry of the pseudogap. Since $T_c$ is a function of $E_g(0)$, determined from Eq. (9), for a given symmetry of both the pseudogap and the pairing state $\alpha/\alpha_0$ is a universal function of $T_c/T_c(0)$. Here, $T_c(0) = T_c(E_g = 0)$. In Fig. 1 we show $\alpha/\alpha_0$ as a function of $T_c/T_c(0)$ for different symmetries. The solid line shows the isotope exponent for an $s$-wave pseudogap. This result is independent of the pairing symmetry, as can be seen by performing the angular integration in Eq. (13). For an anisotropic pseudogap having $d_{x^2-y^2}$-wave symmetry, however, the pairing symmetry does affect the result. The dotted line shows the result for an $s$-wave superconductor with a $d_{x^2-y^2}$-wave pseudogap, while the dashed-dotted line shows the result for a $d_{x^2-y^2}$-wave superconductor with a $d_{x^2-y^2}$-wave pseudogap. The weakest $T_c/T_c(0)$ dependence is found for a $d_{xy}$ superconductor with a $d_{x^2-y^2}$-wave pseudogap (dashed line). In all cases one can see from Eq. (13) that $\alpha/\alpha_0$ diverges for $T_c \to 0$. Thus, in principle arbitrarily high values of $\alpha$ can be reached. As an illustration experimental results on Pr-doped YBCO are shown in this figure as solid squares. Here, it should be noted that experimental results on different compounds can differ somewhat and also vary with the dopant used (see Ref. 1). Certainly these differences need further explanation (e.g. see the review in Ref. 4) and cannot be understood solely due to the influence of the pseudogap. Here, we only want to focus on the influence of a pseudogap alone and investigate the general tendency and order of magnitude of the effect, which is similar in many compounds.

As an important conclusion we can draw from these weak-coupling results that a pseudogap in general leads to an increase of the isotope exponent $\alpha$ over its value $\alpha_0$ in the absence of the pseudogap. The quantitative size of this effect depends on the symmetries of the pseudogap and the pairing state. However, the qualitative behavior is very similar in all cases. The size of $\alpha_0$ can become small, if a strong electronic coupling constant $V_{e0}$ and a small phononic coupling $V_{p0}$ is considered.

III. STRONG COUPLING EFFECTS: NAFL MODEL

Having seen that a pseudogap can lead to an increase of the isotope exponent in a weak-coupling superconductor, one might wonder whether this effect will survive in more realistic models for superconductivity. In order to see, how strong-coupling effects affect the results, we want to consider a recently proposed spinfluctuation exchange model, the nearly antiferromagnetic Fermi liquid (NAFL) model due to Monthoux and Pines. Within this model the pairing interaction is provided by exchange of antiferromagnetic spinfluctuations and the pairing symmetry is $d_{x^2-y^2}$. The (frequency dependent) pairing interaction is given by

$$V(q, \nu_m) = g^2 \chi(q, \nu_m),$$

where $g$ is a coupling constant, $\nu_m$ the Bose-Matsubara frequencies, and the spinususceptibility $\chi$ is given by

$$\chi(q, \nu_m) = \frac{\chi_0}{1 + \xi^2(q - Q)^2 + \nu_m/\omega_s}$$

Here, $Q = (\pi, \pi)$ is the antiferromagnetic wavevector, $\xi$ is the magnetic correlation length and $\omega_s$ the characteristic spinfluctuation frequency. Using this interaction the Migdal-Eliashberg equations for strong-coupling superconductors are solved selfconsistently. Here one has to solve for the self-energy $\Sigma$

$$\Sigma(k, \omega_n) = \frac{1}{N} \sum_{k', \nu_n'} V(k' - k, \omega_n - \omega_n') G(k', \omega_n)$$

along with Dyson’s equation for the Green’s function $G$

$$G(k, \omega_n) = \frac{1}{\omega_n - \epsilon_k - \Sigma(k, \omega_n)}$$

selfconsistently. Using this solution, $T_c$ is determined from the linearized gap-equation

$$\phi(k, \omega_n) = -\frac{1}{N} \sum_{k', \nu_n'} V(k' - k, \omega_n - \omega_n') \times |G(k, \omega_n)|^2 \phi(k, \omega_n)$$

where $\phi$ is the gap-function. For the bandstructure $\epsilon_k$ a tight-binding band with next nearest neighbor hopping has been used in Ref. 30 and we will adopt that here.

FIG. 1. Weak-coupling result Eq. (13) for the isotope exponent $\alpha/\alpha_0$ as a function of $T_c/T_c(0)$ in the presence of a pseudogap. $\alpha_0$ and $T_c(0)$ denote the values in the absence of the pseudogap. The solid line shows the result for an $s$-wave pseudogap. For a $d_{x^2-y^2}$-wave pseudogap the results for an $s$-wave pairing symmetry (dotted line), a $d_{x^2-y^2}$-wave pairing symmetry (dashed-dotted line), and a $d_{xy}$-wave pairing symmetry (dashed line) are shown. The solid squares are experimental results on Pr-doped YBCO from Ref. 1.
In order to have a small nonzero isotope exponent at optimal doping we consider coupling to an additional phonon mode, the 'buckling' mode studied in Refs. 10 and 17. This mode provides an attraction in the \( d_{x^2-y^2} \)-wave channel and its pairing interaction reads

\[
V_p(\hat{q}, \omega_m) = V_{p0} (\cos^2 \frac{q_x}{2} + \cos^2 \frac{q_y}{2}) \frac{\omega_m^2}{\omega^2_p} \frac{\omega^2}{\Omega^2_m + \omega^2_p}
\]  \quad (19)

We do not expect the main results to depend strongly on the details of the electron-phonon spectrum, as long as its coupling strength is small compared with the spin-fluctuation interaction. It is important, however, that the electron-phonon interaction has an attractive component in the \( d_{x^2-y^2} \)-wave channel and its pairing interaction is fixed and does not change with the electron-phonon coupling strength. For higher values of the spin-fluctuation frequency \( \omega_s \), the renormalization of the pseudogap \( E_g \) due to the self-energy \( \Sigma \) is taken into account in this approximation. In contrast to the weak-coupling approximation in the previous section, the pseudogap as seen in the density of states is washed out now and thus is a real 'pseudo'-gap.

In Fig. 2 we show \( \alpha \) as a function of \( T_c \) for three different values of the spin-fluctuation frequency \( \omega_s \) along with the weak-coupling result for a \( d_{x^2-y^2} \)-wave superconductor with a \( d_{x^2-y^2} \)-wave pseudogap from Fig. 1. For higher values of the spin-fluctuation frequency \( \omega_s \), the weak-coupling result as one should expect. For small \( \omega_s = 0.03t \) there are some deviations from the weak-coupling limit. However, the results are not affected very much.

Here it is necessary to take into account the real part of the self-energy \( \Sigma \), since the pseudogap opens at the Fermi surface, which is renormalized due to the self-energy. Table I also shows the amplitude of the pseudogap, denoted by \( E_{g,\text{supp}} \), which completely suppresses \( T_c \). Experimentally, the ratio of \( E_{g,\text{supp}} / T_{c0} \) is about 6-15, depending on the material. The values found here indeed turn out to be of this order of magnitude. Note, that the renormalization of the pseudogap \( E_g(k) \) due to the self-energy \( \Sigma \) is taken into account in this approximation.

In order to study the influence of a pseudogap we introduce a \( d_{x^2-y^2} \) pseudogap \( E_g(k) = E_{g0} \cos 2\Theta_k \), as suggested by the analysis of Williams et al. 16 into the single particle excitation spectrum by replacing in the single particle Green’s function Eq. (17):

\[
\epsilon_k + \text{Re} \Sigma \quad \Rightarrow \quad \pm \sqrt{(\epsilon_k + \text{Re} \Sigma)^2 + E_g^2(k)}. \quad (21)
\]

---

**TABLE I.** Coupling constants for the NAFL model with an additional coupling to the buckling phonon mode Eq. (17) for different values of the spin-fluctuation frequency \( \omega_s \). \( E_{g,\text{supp}} \) denotes the value of the pseudogap \( E_{g0} \), which completely suppresses \( T_c \).

| \( \omega_s / t \) | \( g / t \) | \( \lambda_{ph} \) | \( E_{g,\text{supp}} / T_{c0} \) |
|------------------|------------|-----------------|-----------------|
| 0.03             | 5.1        | 0.31            | 12.3            |
| 0.06             | 3.2        | 0.15            | 6.5             |
| 0.2              | 2.4        | 0.10            | 4.0             |

---

**FIG. 2.** Isotope coefficient \( \alpha \) as a function of \( T_c \) for the NAFL model with an additional coupling to the buckling phonon mode. The opening of the pseudogap leads to a suppression of \( T_c \) and an increase of \( \alpha \). The solid line shows the corresponding weak-coupling result from Fig. 1. Results are shown for different values of the characteristic spin-fluctuation frequency: \( \omega_s = 0.03t \) (dashed line), \( \omega_s = 0.06t \) (dotted line), and \( \omega_s = 0.2t \) (dashed-dotted line). In each case the coupling constants have been adjusted such that \( T_{c0} = 90K \) and \( \alpha_0 = 0.05 \).

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**IV. SELF-CONSISTENT FLEX APPROXIMATION**

Within the NAFL model the spinfluctuation pairing interaction is fixed and does not change with the electronic properties. However, it is clear both experimen-
tally and theoretically that the pseudogap does affect the spinsusceptibility and thus should affect the spinfluctuation pairing interaction itself. To study such kind of effects it is necessary to treat the electronic properties and the electronic pairing interaction in a self-consistent way. Such a self-consistent treatment is provided by the so-called fluctuation-exchange (FLEX) approximation for the two dimensional Hubbard model and also yields a \(d_{x^2-y^2}\)-wave superconducting state. The main difference with the NAFL model is that the spinsusceptibility is calculated from the interacting Green’s functions within an RPA-type approximation. Then the pairing interaction reads:

\[
V(\vec{q}, i\nu_m) = \frac{3}{2} U^2 \frac{\chi_0(\vec{q}, i\nu_m)}{1 - U\chi_0(\vec{q}, i\nu_m)},
\]

where the bubble susceptibility \(\chi_0\) is calculated from the fully dressed single-particle Green’s function \(G\) (Eq. (2)) self-consistently:

\[
\chi_0(\vec{q}, i\nu_m) = - \frac{1}{N} \sum_{k,n} G(\vec{k} + \vec{q}, i\omega_n + i\nu_m)G(\vec{k}, i\omega_n)
\]

where \(\alpha = 0\). For \(U \leq 3.6t\), \(\alpha(T_c/T_{\alpha})\) very much follows the weak-coupling limit. Only if \(U\) reaches values of the order of \(4t\) or higher, deviations from the weak-coupling limit become apparent. For higher values of \(U\), \(\alpha(T_c/T_{\alpha})\) becomes flatter and starts to rise only at smaller values of \(T_c\). This is a consequence of the influence of the pseudogap on the spinfluctuation pairing interaction. In contrast to the NAFL model the opening of the pseudogap not only reduces \(T_c\) via the reduction of single particle spectral weight at the Fermi level, but it also suppresses the lowest frequency spinfluctuations. Since these are predominantly pair-breaking, this increases the effective coupling strength of the spinfluctuations as compared with the phonons and thus leads to a reduction of \(\alpha\).

**V. CONCLUSIONS**

We studied the influence of a pseudogap on the isotope exponent for different models having an electronic pairing interaction with a subdominant electron-phonon interaction. In the weak-coupling limit we found that the introduction of a pseudogap leads to a strong increase of the isotope exponent above its value in the absence of a pseudogap. For \(T_c \rightarrow 0\) the isotope exponent diverges, allowing arbitrarily high values. The symmetries of the order parameter and the pseudogap only lead to quantitative, but not qualitative changes of these results. Strong-coupling effects within the NAFL model do not affect the results very much. The size of the pseudogap compared with \(T_c\) turns out to be of the right order of magnitude. Self-consistent treatment of the spinfluctuation pairing interaction in the presence of the pseudogap can lead to stronger deviations from the weak-coupling limit. The general tendency that the isotope exponent rises upon opening of the pseudogap still remains, however. From these results it does not seem unreasonable that the pseudogap indeed can have an important influence on the isotope effect and might be, at least partially, responsible for the increasing isotope exponent in the underdoped cuprates.

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1 J. P. Franck, in *Physical Properties of High Temperature Superconductors IV*, ed. D. M. Ginsberg (World Scientific, Singapore, 1994), p. 189.
2 F. Marsiglio, R. Akis, and J. P. Carbotte, Solid State Comm. **64**, 905 (1987).
3 J. P. Franck, J. Jung, M. A.-K. Mohamed, S. Gyrax, and G. I. Sproule, Phys. Rev. B **44**, 5318 (1991).
4 Unless one wants to assume that the electron-phonon coupling constant changes strongly with doping. See T. Dahm, D. Manske, and L. Tewordt, Phys. Rev. B **54**, 12006 (1996). Within this scenario one quickly reaches global structural instability, however.
5 J. Labbé and J. Bok, Europhys. Lett. **3**, 1225 (1987).
6 C. C. Tsuei, D. M. Newns, C. C. Chi, and P. C. Pattnaik, Phys. Rev. Lett. **65**, 2724 (1990).
7 R. J. Radtke and M. R. Norman, Phys. Rev. B **50**, 9554 (1994).
8 Unless one wants to assume that the electron-phonon coupling constant changes strongly with doping. See T. Dahm, D. Manske, and L. Tewordt, Phys. Rev. B **54**, 12006 (1996). Within this scenario one quickly reaches global structural instability, however.
9 L. Pietronero and S. Strassler, Europhys. Lett. **18**, 627 (1992).
10 A. Nazarenko and E. Dagotto, Phys. Rev. B **53**, R2987 (1996).
11 A. Greco and R. Zeyher, Phys. Rev. B **60**, 1296 (1999).
12 J. P. Carbotte, M. Greeson, and A. Perez-González, Phys. Rev. Lett. **66**, 1789 (1991).
13 V. Z. Kresin, A. Bill, S. A. Wolf, Yu. N. Ovchinnikov, Phys. Rev. B **56**, 107 (1997).
14 A. Bill, V. Z. Kresin, and S. A. Wolf, in *Pair Correlations in Many-Fermion Systems*, ed. V. Z. Kresin (Plenum, New York, 1998), p. 25.
15 For a more complete list of references see the reviews in Refs. [3] and [4].
16 For a review see T. Timusk and B. Statt, Rep. Prog. Phys. **62**, 61 (1999).
17 G. V. M. Williams, J. L. Tallon, and J. W. Loram, Phys. Rev. B **58**, 15053 (1998); G. V. M. Williams, J. L. Tallon, E. M. Haines, R. Michalak, and R. Dupree, Phys. Rev. Lett. **78**, 721 (1997).
18 V. J. Emery and S. A. Kivelson, Nature (London) **374**, 434 (1995).
19 S.-C. Zhang, Science **275**, 1089 (1997).
20 D. Pines, Physica C **282-287**, 273 (1997).
21 X.-G. Wen and P. A. Lee, Phys. Rev. Lett. **76**, 503 (1996).
22 A. S. Alexandrov, V. V. Kabanov, and N. F. Mott, Phys. Rev. Lett. **77**, 4796 (1996).
23 K. Maki and H. Won, Physica C **282-287**, 1839 (1997).
24 V. J. Emery, S. A. Kivelson, and O. Zachar, Phys. Rev. B **56**, 6120 (1997).
25 T. Dahm, D. Manske, and L. Tewordt, Phys. Rev. B **56**, 11419 (1997).
26 S. G. Lie and J. P. Carbotte, Solid State Comm. **34**, 599 (1980).
27 E. Schachinger, M. G. Greeson, and J. P. Carbotte Phys. Rev. B **42**, 406 (1990).
28 G. V. M. Williams, J. L. Tallon, J. W. Quilty, H. J. Trodahl, and N. E. Flower, Phys. Rev. Lett. **80**, 377 (1998).
29 F. Raffa, T. Ohno, M. Mali, J. Roos, D. Brinckmann, K. Conder, and M. Eremin, Phys. Rev. Lett. **81**, 5912 (1998). In this work a small isotope exponent of the pseudogap $\alpha_{E_g} \sim 0.06$ was found from $^{63}$Cu NQR spin-lattice relaxation measurements, which are sensitive to the spin-susceptibility at the antiferromagnetic wavevector. In the present work the null result in the Knight-shift study from Ref. [28], which is sensitive at $q = 0$, is the more appropriate measure, however.
30 P. Monthoux and D. Pines, Phys. Rev. B **49**, 4261 (1994).
31 N. E. Bickers, D. J. Scalapino, and S. R. White, Phys. Rev. Lett. **62**, 961 (1989); N. E. Bickers and D. J. Scalapino, Ann. Phys. (N.Y.) **193**, 206 (1989).
32 T. Dahm (unpublished).
33 J. Song and J. F. Annett, Phys. Rev. B **51**, 3840 (1995).
34 N. Bulut and D. J. Scalapino, Phys. Rev. B **54**, 14971 (1996).
35 C.-H. Pao and N. E. Bickers, Phys. Rev. Lett. **72**, 1870 (1994).
36 P. Monthoux and D. J. Scalapino, Phys. Rev. Lett. **72**, 1874 (1994).
37 T. Dahm and L. Tewordt, Phys. Rev. Lett. **74**, 793 (1995).
38 P. Monthoux and D. J. Scalapino, Phys. Rev. B **50**, 10339 (1994).
39 A. J. Millis, S. Sachdev, and C. M. Varma, Phys. Rev. B **37**, 4975 (1988).
40 For a study of the influence of the superconducting gap on the spin-fluctuation pairing interaction see T. Dahm, Solid State Comm. **101**, 487 (1997).