Transition to chaos in wide gap spherical Couette flow – experiment and DNS

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Abstract. The paper presents the results of the experimental and numerical investigations of transition to chaos in spherical layer $\delta = 1$ with counter rotating boundaries. In experiments the laminar-turbulent transition has been studied near the local maximum at the stability curve at $Re_o = -810$ ($-700 < Re_o < -950$). It was found that transition occurs through four or five bifurcations. Spectra of all supercritical flow states are formed from the frequencies, corresponding to the revealed flow structures. Transition to chaos occurs from one-frequency flow state. Its structure is symmetric with respect to the equator plane and includes three vortex tubes in each hemisphere. This symmetry-breaking bifurcation has a considerable hysteresis. Inner sphere acceleration may change the transition scenario – the last bifurcation leads to spatial-temporal intermittency. For the direct numerical simulation the case with $Re_o = -900$ was chosen. All flow states, including chaotic one, boundaries and hysteresis areas, observed in experiments, were obtained numerically. Furthermore, some details of flow structures were revealed only from numerical simulation.

1. Introduction
The isothermal flow in the spherical layer is determined by three similarity parameters, namely, the Reynolds numbers $Re_i = \omega_i r_i^2/\nu$ and $Re_o = \omega_o r_o^2/\nu$ and relative gap size $\delta = (r_o - r_i)/r_i$. Here $r_i$ and $r_o$ are the radius and angular velocity of the respective sphere: $i$ of the inner and $o$ of the outer, and $\nu$ is the kinematics viscosity of the fluid in the layer. Most previous experimental and numerical investigations were restricted to the case of only inner sphere rotation [1, 2]. A few papers were devoted to the case of both sphere rotation, but no more than two first bifurcations were examined [3-5]. Transition to chaos with counter rotating spheres in wide gap ($\delta = 1$) was investigated experimentally [6, 7] and afterward numerically [8].

Experimental setup represents two transparent spheres (the radius of outer sphere $r_o = 150$mm). Spheres rotate independently round the common axis. The fluctuations of the working fluid temperature and spheres angular velocities are no more than 0.05%. The experimental technique is based on the simultaneous visualization of the flow structure in the meridian plane and LDA measurements of the velocity pulsations. All flow states, presented here, were obtained in the following way: with the inner sphere at rest the outer sphere is slowly accelerated from rest to the chosen value of the rotation rate, after that the inner sphere is accelerated (quasi-static or not) from rest to the chosen value of the rotation rate.
The computational algorithm is finite-difference method for solving three-dimensional time-dependent Navier-Stockes equations for incompressible fluid in spherical coordinates. The algorithm consists of a second-order central difference approximation in space and a third-order semi-implicit Runge-Kutta scheme for time advancement. The no-slip and impermeability conditions are imposed on the spherical boundaries. The grids used for spatial resolution are non-uniform in the regions of the largest velocity gradients, namely, near the spherical boundaries and the equator plane. For the calculated results presented below, the grid point numbers ranged from 32 to 128 in the azimuth direction, from 50 to 100 in the radial direction, and from 64 to 180 in the meridian direction.

2. Transition to chaos, experiment

Transition to chaos was investigated in the vicinity of the local maximum on the stability curve \( a-b-h \) at \( \text{Re}_o = -810 \) (point \( b \)) in Fig.1a. Because of the different secondary flow states for \( \text{Re}_o > -810 \) and \( \text{Re}_o < -810 \) we treat this maximum as an intersection of two neutral curves, corresponding to different types of instability.

To the right of the maximum after the first bifurcation (curve \( b-h \)) the spiral waves state, with frequency \( f_s \) in the velocity spectrum, appears. The second bifurcation (curve \( I \), Fig.1a) leads to the quasi-periodical flow state, in the structure of this state the slow circumpolar motion is added to the spiral waves and in the spectrum frequency \( f_p \) is added to \( f_s \). After the third bifurcation (curve 2) the new flow state is formed, which was called the state of localized vortices. This state has one frequency \( f_s \) in the spectrum and equatorial-symmetric spatial structure: three equidistant complex vortices propagate in the direction of rotation of outer sphere, the vortices inclined with respect to equatorial plane. The forth bifurcation (curve 3) leads to chaotic flow state.

To the left of the maximum first bifurcation (curve \( a-b \)) leads to the slow circumpolar motion flow state with a single frequency \( f_p \) in the velocity spectrum. The second (curve 4) leads to the quasi-periodical state, when spiral waves are added to circumpolar motion. Up to \( \text{Re}_o = -900 \) the third bifurcation (curve 6) results in the state of localized vortices and the forth (curve 7) - in chaotic flow state. For \( \text{Re}_o < -900 \) the three frequency flow state appears between quasi-periodical and localized vortices states, namely between curves 5 and 6.

![Figure 1](image_url)

Fig.1b demonstrates the dependences of the frequencies (normalized to the rotation frequency of the outer sphere \( f_o \)) on the ratio \( \text{Re}/\text{Re}_{o_i} \). Here \( \text{Re}_{o_i} \) corresponds to the formation of spiral waves (that is \( \text{Re}_i \) at curves 4 and \( b-h \) in Fig.1a). The relative frequencies of the spiral waves are described by curve \( s \), which originate at the point \( \text{Re}/\text{Re}_{o_i} = 1 \). For \( \text{Re}_o > -810 \) this state appears at the stability threshold.
and for $Re_o < -810$ this state appears after second bifurcation. Curve $p$ describes circumpolar motion for all values of $Re_o$. For $Re_o < -810$ the onset of the circumpolar motion occurs at the descending branch of the curve $p$ ($Re/Re_o < 1$), while for $Re_o > -810$ at the ascending part ($Re/Re_o > 1$). Curve $g$ represents the third frequency in the three frequency state, existing at $Re_o < -900$. And curve $t$ represents the frequency of the localized-vortex state. So we may say that at any $Re_o$, the relative frequencies of each supercritical flow state depend only on the ratio $Re/Re_o$.

Not quasi-static acceleration of the inner sphere from the localized vortices state leads to the formation of the spatial-temporal intermittency flow state \cite{7}. In this state the samples of chaotic flow state interchange by the samples of localized vortices state. The intervals of the chaotic flow state are increased with the increase of $Re_i$.

3. Numerical simulation

For numerical simulation the transition to chaos at $Re_o = -900$ was chosen. The calculations were carried out for the parameters corresponding to the above presented experiments: $\nu = 5 \times 10^{-5} \text{ m}^2/\text{s}$, $r_i = 0.075 \text{ m}$, $r_o = 0.15 \text{ m}$, $\Omega_o = -2 \text{ s}^{-1}$, and $\Omega_i = 2.9 - 4.2 \text{ s}^{-1}$ ($Re_i = 300 - 460$).

The structure of corresponding axisymmetric flow and its linear stability analysis were preliminary investigated for azimuth numbers $m$ from 1 to 6, separately for equatorial-symmetric and equatorial-asymmetric modes. The first mode growing in accordance with the linear theory is the equatorial-symmetric mode with $m = 3$ at $Re_i = 315$. Two modes, symmetric and antisymmetric with $m = 2$, begin grow at $Re_i = 359$ and $Re_i = 358$ respectively. At $Re_i = 414$ all the modes do not decay.

The three-dimensional solution at $Re_i = 340$ is time periodic, equatorial-symmetric and consists of three vortices in each hemisphere propagating in the direction of the outer sphere rotation. The oscillation frequency is in good agreement with the experimental value of the frequency $f_p$. The solution is consistent with the first growing linear mode. The next state is quasi-periodical one ($Re_i = 360$). The nonlinear solution becomes asymmetric about the equator plane. Three equatorial-symmetric vortices, which have existed between the pole and the equator, are now supplemented by two asymmetric vortices. The whole flow structure propagates in the direction of the inner sphere rotation. As in the experiment, two frequencies are observed in the velocity signal spectrum: one frequency ($f_p$) corresponds to slow circumpolar motion, and the other ($f_s$), to a rapid motion with two vortices asymmetric about the equator plane – spiral waves.

At $Re_i = 378$ the flow remains asymmetric about the equator plane; three frequencies observed in the velocity spectrum are in satisfactory agreement with the experimental data. The whole flow structure propagates in the direction of the inner sphere rotation.

After the fourth bifurcation the flow becomes periodic and symmetric about the equator plane. In each hemisphere, three complex vortices, equidistant in the azimuth angle, propagate in the direction of the outer sphere rotation. Fig. 2 represents transition from localized vortices state to chaos in calculations. Time dependence of one velocity component is shown in Fig. 2a. At the beginning the
behavior of the velocity corresponds to the localized vortices state, one frequency with numerous harmonics is observed in its spectrum (Fig2b). During the transition the behavior of the velocity looses its regularity, velocity spectrum becomes continuous (Fig2c).

All boundaries between flow states and the dependence of the average torque $M$ exerted by the fluid on the inner sphere on the $Re$ are presented in Fig.3. At the top of Fig.3 the experimentally obtained boundaries are shown. $0$ denotes the area of existence of basic flow, $1$ – circumpolar periodical motion, $2$ – quasi-periodical flow state, $3$ – three-frequency flow state, $4$ – localized vortices flow state, $5$ – chaotic flow state. Two hysteresis areas were obtained both in calculations and experiments. Enlarged hysteresis areas are shown in Fig.3a, b. Formation of the chaotic flow state is followed by jump decrease of $M$ (Fig.3b), on the contrary to formation of localized vortices flow state (Fig.3a).

4. Concluding remarks
Thus, the laminar-turbulent transition in gap $\delta = 1$ with counter rotating boundaries was investigated both experimentally (-700 < $Re_o$ < -950) and numerically ($Re_o$ = -900). According to the calculations all flow states, except for chaotic one, represent a combination of symmetric and asymmetric traveling waves with the azimuth numbers $m = 2$ and/or 3. Before chaos all symmetric about the equator plane flow states propagate in the direction of the outer sphere rotation, while the asymmetric ones in the direction of the inner sphere rotation. Calculated and experimental results are in good agreement.

For the first time the chaotic flow state in isothermal Spherical Couette Flow was obtained by direct numerical simulation. Transition to stochastic behavior always occurs from the periodical localized vortices flow state. All bifurcation points preceding this state arise according to the Ruelle-Takens scenario.

5. Acknowledgments
This work was supported by the Russian Foundation for Basic Research, projects 05-01-00607 and 07-08-00247.

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