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EN©E LEVELS OF NUCLEI 40Sc AND 40K AS A FUNCTION OF SEMI-CLASSICAL COUPLING ANGLE θ₁₂ WITHIN THE MODIFIED SURFACE DELTA-INTERACTION

In this work, nuclear shell model was applied using modified surface delta-interaction to calculate, in particle-hole state, the energy levels of isobar nuclei 40Sc and 40K. Particles are in the model space (1f7/2) while the holes are found in the model space (1d5/2, 1s1/2, 1d3/2). The total angular momentum and parity are identified for possible particles and holes in nuclei above. Thus, we have used a theoretical study to find relationship between energy levels and the semi-classical coupling angle θ₁₂ at different orbitals within particle-hole configuration. We notice the energy levels seem to follow two universal functions which depend on the semi-classical coupling angles θ₁₂. We found the theoretical data agree to the experimental data.

Keywords: shell model, energy levels, modified surface delta-interaction, 40Sc, 40K, particle-hole.

1. Introduction

Having simple shell model in mind, Talmi [1] used the surface delta-interaction (SDI) to evaluate properties of nuclear states with few 'nucleons' on a magic core. Likewise, the shell model has been successful in describing configurations with a few nucleons outside closed shells or missing from them [2]. It employs assumption: at first there exists an inert core model of close shell [3], which acts with central forces upon valence nucleons; second, there exists a residual interaction caused by two-body forces acting between the valence nucleons. Schiffer [4] proposed very suggestive method. He consider only those nuclei where two particles (2p), two holes (2h) or particle and hole are present in addition to closed shell, and move in the orbits j₁ and j₂ of a self consistent field. He pointed out the universal behavior of the effective interaction in terms of the angle between the angular momenta of the interacting nucleons, property which was later shown to be related to the short-range character of the effective interaction [5, 6].The angle θ₁₂ between the proton (hole or particle) and neutron (hole or particle) angular momentum vectors j₁ and j₂ were reported in Refs. [7, 8]. There are a number of theoretical works discussing nuclear shell model, particle-hole (ph) configuration by means modified surface delta-interaction (MSDI). For example, shell model calculation of A = 41 - 43 nuclei [9]. The modified schematic shell model for ph configuration within SDI [1, 10] and MSDI has been applied to various nuclear structure states [11, 14] Lawson and Talmi [15, 16] studied the spectra 38Cl, 40K and 40Ca by employing the Pandya transformation between particle-particle (pp) spectra such as 38Cl and ph spectra as such 40K. Johnston [17] reported calculations for the one-hole states in potassium isotopes 40 ≤ A ≤ 46 using a model space based on the (f7/2p3/2)ν(d5/2s1/2)−1 configuration. The matrix elements of the residual interaction are treated as free parameters which are determined by a least-squares fit to the observed energy levels; good agreement is achieved in his results. In Ref. [18] residual interaction theory was applied to the study of the relation between pp and ph spectra using model space (1f7/2 1p3/2) for 40K. The result of this study was the exposition of the Pandya pp-ph relation. There are some noticeable differences among the 5 sets of proton and neutron matrix elements, although the 2− and 5− energies appear stable. The lowest ph multiplet, (1f7/2)(1d5/2)−1, is well known [19] in 40Sc, 40K and 40Ca. N. Schulz et al. [20] described an experimental analysis for the 40Ca(’He, ’He) reaction. Many experimental and theoretical studies on the structure of energy level in particle-hole states had been investigated [21, 22]. ph configuration is discussed in other book with the idea to use simple forces [23, 26]. Recently we studied the energy level of 208Po within the framework of the SDI [27]. Energy levels of isobaric nuclei 15N, 16F within the MSDI were studied [28] by D. N. Hameed and A. K. Hasan. Previous studies motivate the aim of the present work by application of MSDI to calculate the energy levels of isobaric nuclei (40Sc and 40K) in ph state. We have used a theoretical study to find relationship between energy levels and the semi-classical coupling angle θ₁₂ at different orbital within ph configuration.

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2. *ph* formalism

The Hamiltonian can be written in the form [29]

$$ H = \sum_{\alpha j} \left( \alpha | E | \beta \right) \eta_{\alpha} \eta_{j} + $$

$$ + \frac{1}{4} \sum_{\alpha \beta \gamma \delta} \left( \alpha \beta \gamma | V | \delta \right) \eta_{\alpha} \eta_{\beta} \eta_{\gamma} \eta_{\delta} \, \eta_{\alpha} \eta_{\beta} \, \eta_{\gamma} \eta_{\delta} \, . \tag{1} $$

where $\eta_{\alpha}^{\dagger}$ and $\eta_{j}$ denote the (creation and annihilation) operators respectively, of a nucleon in state $[\alpha]$; $E$ is the single-particle (*s, p*) energy, and

$$ \left( \alpha \beta \gamma | V | \delta \right) $$

represents the normalized and antisymmetrized matrix element of nucleon -nucleon interaction. The wave function in one-particle (one-hole) (1p or 1h) configuration space are represented by [30]

$$ \left| 0; ph^{-1} \right> = a_{hole} a_{particle}^{\dagger} | 0 \right> \, \tag{2} $$

The accord to denote particle state by the label $p_{1}$, $p_{2}$, $p_{3}$, $p_{4}$ ... and hole state by label $h_{1}$, $h_{2}$, $h_{3}$, $h_{4}$ ...

$$ H_{s, p} \left| 0; ph^{-1} \right> = (e_{particle} - e_{hole}) a_{hole} a_{particle}^{\dagger} | 0 \right> = $$

$$ = (e_{particle} - e_{hole}) \left| 0; ph^{-1} \right> \, \tag{3} $$

Eq. (3) is the *s, p* Hamiltonian, $e_{particle}$ representing *s, p* energy for 'particle' and $e_{hole}$ symbolizing *s, p* energy for 'hole' the angular momentum coupling and, once a sufficiently large set of basis function is obtained, the coupled *ph* state are given by

$$ 0; ph^{-1}; JM = $$

$$ \sum_{mp_{particle}m_{hole}} \left( j_{p_{1}} m_{p_{1}} j_{p_{2}} m_{p_{2}} | JM \right> \hat{a}_{j_{p_{1}} m_{p_{1}}} \hat{a}_{j_{p_{2}} m_{p_{2}}} | 0 \right> \, \tag{4} $$

Thus one obtains for the matrix elements of the residual interaction (suppressing the symbol 0 for the close shell state)

$$ \left\langle j_{p_{1}} j_{h_{1}}^{-1} ; JM \right| V \left| j_{p_{2}} j_{h_{2}}^{1} ; JM \right> = \frac{1}{d} \sum_{\alpha \beta \gamma \delta} \left( \alpha \beta \gamma | V | \delta \right) \times \sum_{m_{p_{1}} m_{p_{2}} m_{p_{1}}^{m_{p_{2}} m_{h_{2}}}} (-1)^{j_{h_{1}}-m_{h_{1}}} \left( j_{p_{1}} m_{p_{1}} j_{p_{2}} m_{p_{2}} \right) \left| JM \right> \left\langle 0 \right| a_{i_{p_{1}} m_{p_{1}}} a_{i_{p_{2}} m_{p_{2}}} \left| 0 \right> (a_{i_{m_{p_{1}}} a_{i_{m_{p_{2}}}} a_{i_{m_{p_{1}}} a_{i_{m_{p_{2}}}}}}) a_{i_{m_{p_{1}}} m_{p_{1}} m_{p_{2}}} a_{i_{m_{p_{2}}} m_{p_{2}} j_{p_{2}} j_{p_{2}} m_{p_{2}}} \, \tag{5} $$

The ground state is taken to be a closed configuration and the excited state are mixed (1p, 1h) configuration this approach is usually referred to as the Tamm - Dancoff approximation.

3. The SDI

Moszkowski and co-workers [31] have proposed a simple model able to describe the interaction between the valence nucleons. This model of interaction assumes the residual interaction $V_{1,2}$ between particles 1 and 2 takes place at the nuclear surface only [27, 32], interaction defined in this way possesses all the features of pairing interaction. It is short-ranging and allows only symmetric spatial states. It should be pointed out that the $SDI$ exists not only between particles coupled to $T = 1, J = 0$ but also between particles coupled to $J \neq 0$. Unlike pairing, $SDI$ interaction acts also in states with $T = 0$. Therefore the $SDI$ should be a better approximation then a pure pairing one. Following the work of Glaudemans [33] where the isospin dependence of the interaction was taken into account the 'interaction' may be written in the form [29, 32]

$$ V_{1,2} = -4 \pi A_{F} \delta \Omega_{1,2} \delta \hat{r}(1) - $$

$$ - R_{0} \delta \hat{r}(2) - R_{0} + B \tau_{1} \cdot \tau_{2} \, \tag{6} $$

where $\hat{r}(1)$, $\hat{r}(2)$ are the position vectors of interacting particles, $R_{0}$ is the nuclear radius [22, 34] the strength of interaction $A_{F}$. The correction term $B \tau_{1} \cdot \tau_{2}$ is introduced to account for the splitting between the groups of levels with different isospin. Such a form of interaction is called MSDI. The antisymmetrized matrix element of $V_{1,2}$ is given by [11, 12, 28, 29]

$$ \left\langle j_{j_{1}} j_{j_{2}} \right| V_{1,2} \left| j_{j_{1}} j_{j_{2}} \right> = \frac{A_{F}}{2(2J+1)} \frac{(2j_{j_{1}}+1)(2j_{j_{2}}+1)(2j_{j_{2}}+1)(2j_{j_{2}}+1)}{2(2J+1)(1+\delta_{1,2})} \frac{1}{(2J+1)} \times $$

$$ \times \left[ (-1)^{j_{j_{1}}+j_{j_{2}}} \langle j_{j_{2}}-1/2 j_{j_{2}}+1/2 | J0 \rangle \cdot \langle j_{j_{1}}-1/2 j_{j_{1}}+1/2 | J0 \rangle \left[ 1 - (-1)^{j_{j_{1}}+j_{j_{2}}+T} \right] \right] - $$
where \( \langle j_2 - 1/2 j_1 1/2 | J_0 \rangle \) and \( \langle j_4 - 1/2 j_3 1/2 | J_0 \rangle \) are the Clebsh - Gordon coefficients [23, 29, 32] and \( j_1, j_2, j_3 \) and \( j_4 \) are the spin states of particles. Correspondingly, \( J \) and \( T \) indicate the spin and isospin of a 2p state. The diagonal matrix elements with \( j_1 = j_2 \) and \( j_3 = j_4 \) correspond to pure states. The \[ p_\hbar^{-1} | H | p_\hbar^{-1} \rangle_\Gamma = (\epsilon_{p_1} - \epsilon_{p_2})\delta_{p_1 p_2}\delta_{\hbar \hbar_2} + \langle p_\hbar^{-1} | V | p_\hbar^{-1} \rangle_\Gamma, \] \[ \langle p_\hbar^{-1} | V | p_\hbar^{-1} \rangle_\Gamma = \frac{1}{4} \left\{ (2j_{p_1} + 1)(2j_{p_2} + 1)(2j_{e_1} + 1)(2j_{e_2} + 1) \right\} ^{0.5} (-1)^{Q} \left[ A_{0} \left[ 1 + 2(-1)^{T} \right] \right] \times \left\{ \langle j_{p_1} - 1/2 j_{h_1} 1/2 | J_0 \rangle \langle j_{p_2} - 1/2 j_{h_2} 1/2 | J_0 \rangle + (-1)^{T} \left[ A_{0} - A_{1} \left[ 1 + 2(-1)^{T} \right] \right] \langle j_{h_1} - 1/2 j_{p_1} 1/2 | J_0 \rangle \times \langle j_{p_2} - 1/2 j_{h_2} 1/2 | J_0 \rangle \right\} \delta_{\hbar p_1}^{even} \delta_{\hbar \hbar_2} ^{even}, \] where \( \Gamma = TJ, Q = j_{h_1} + j_{h_2} + l_{p_1} + l_{h_1} + j + T, \) and \( Z = j_{h_1} + j_{p_2} + l_{p_2} + l_{h_2}, J \) is the total angular momentum, and values of \( A_{0}, A_{1}, B \) and \( C \) as functions of the mass number \( \Lambda [11, 29] \) are obtained from fits to experimental data in the varied mass region, where \( A_{0} = 2A_{0} = B = C = 25/A. \) The \( J \)-dependence of the diagonal two-body matrix element as a function of the spin \( J \) of 2p state is very characteristic when their value are plotted in a particular way. Consider two particles in orbits \( j_1 \) and \( j_2 \) with \( J = j_1 + j_2. \) One can write then \[ J^2 = (j_1 + j_2)^2 = j_1^2 + j_2^2 + 2(j_1 j_2) \cos \theta_{12}, \] \[ \cos \theta_{12} = \frac{J + 1}{2j_1 j_2 (j_1 + 1)(j_2 + 1)}. \] The \( J \)-dependence of the matrix element \( \langle j_{1} j_{2} | V_{12} | j_{1} j_{2} \rangle \) can thus be plotted as a function of the angle \( \theta_{12}. \) The radial overlaps of the particle orbits for light nuclei differ from those for heavy nuclei. The ensuing mass dependence can be removed [8] by dividing each matrix element by the absolute value of the average two-body energy. For 2-particles in the same orbit \( j \) this quantity \( E \) is defined by \[ E = \frac{\sum_{j} (2J + 1) \langle j^{2} | V | j^{2} \rangle_{JT} }{ \sum_{j} (2J + 1) }. \] The behavior of 2\( h \) nuclei is very much the same that 2\( p \) nuclei except that the s. p. energy \( \epsilon_{\text{two-body}} = -\epsilon_{\text{particle}}. \) The residual interaction is given by the MSDI defined in Eq. (6), the matrix of the Hamiltonian is \[ \sum_{j} (2J + 1) E_{j} \] behavior of 2\( h \) nuclei is very much the same that 2\( p \) nuclei except that the s. p. energy \( \epsilon_{\text{two-body}} = -\epsilon_{\text{particle}}. \) The residual interaction is given by the MSDI defined in Eq. (6), the matrix of the Hamiltonian is \[ \sum_{j} (2J + 1) E_{j} \] behavior of 2\( h \) nuclei is very much the same that 2\( p \) nuclei except that the s. p. energy \( \epsilon_{\text{two-body}} = -\epsilon_{\text{particle}}. \) The residual interaction is given by the MSDI defined in Eq. (6), the matrix of the Hamiltonian is \[ \sum_{j} (2J + 1) E_{j} \] behavior of 2\( h \) nuclei is very much the same that 2\( p \) nuclei except that the s. p. energy \( \epsilon_{\text{two-body}} = -\epsilon_{\text{particle}}. \) The residual interaction is given by the MSDI defined in Eq. (6), the matrix of the Hamiltonian is \[ \sum_{j} (2J + 1) E_{j} \]
1,2 = 90, the particle orbits have a small overlap, which result in a weak interaction. This interaction makes clearly why the curve in Fig. 1, b, c and Fig. 2, b, c have appositive slope for 1,2 varying from 180 to 90. For smaller angle 1,2 the Pauli exclusion principle become important. For 1,2 = 0 and j1 = j2, one must distinguish the two possibilities of isospin coupling. In the former case the particle occupy a spatially symmetric ph state which due to the strong short range attraction lead to a large negative matrix element. In the case T = 1 case the ph form a spatially antisymmetric state and hence their relative distance increases for decreasing angle 1,2 to 0.

4. Results and discussion

Nuclear properties of many nuclei are compared with the experimental data [36]. The energy levels of 40Sc and 40K nucleus are calculated according to the following details.

4.1. 40Sc nucleus

In the frame of the shell model, we have considered 40Sc nucleus. In this case, there are one proton (particle) and one neutron (hole) outside of the inert core 40Ca. One proton occupies the model space 1f1/2, and one neutron occupies the model space 1d5/2 2s1/2 1d3/2. Energy levels can be obtained by using the s.p. energy [36, 37], where:

\[
e_p(\text{particle}) = e_1f_{1/2} = -1.085 \text{ MeV}, \quad e_h(\text{hole}) = e_1d_{5/2} = -15.64 \text{ MeV}, \quad e_2s_{1/2} = -13.17 \text{ MeV} \quad \text{and} \quad e_2d_{3/2} = -10.995 \text{ MeV}.
\]

The parameters A0 = 0.7, A1 = 0.8, B = 0.65 and C = -0.3. Table 1 and Fig. 1, a show comparison between theoretical and experiment values of 40Sc nucleus [38]. The MSDI calculations of the energies and parity are in good agreement with the experimental values.

Table 1. Matrix element values Nsc and Nk (MeV) for nuclei 40Sc and 40K, respectively

| Jp | Jh | Nsc | Nk |
|----|----|-----|-----|
| 7/2 | 3/2 | 1.843 | 5.536 |
| 7/2 | 3/2 | 2.351 | 5.357 |
| 7/2 | 3/2 | 3.073 | 4.799 |
| 7/2 | 3/2 | 1.993 | 5.018 |

\[E_{\text{J}, \text{MeV}} = J|\langle j_1^p f_{j_4-1}^p |V| j_2^p f_{j_4-1}^h \rangle| \]
Continuation of the Table 1

| J | N_e | N_h |
|---|---|---|
| 7/2 | 1/2 | 7/2 | 1/2 | 3 | 4.463 | 2.694 | 7/2 | 5/2 | 7/2 | 5/2 | 5 | 9.093 | 2.132 |

4.2. $^{40}$K nucleus

The model space includes $1f_{7/2}$ particle orbits and $1d_{5/2}$, $2s_{1/2}$ hole orbits in $^{40}$K nucleus. It is one-neutron (particle) outside the inert core $^{40}$Ca and one proton (hole). The spectrum of this nucleus was calculated by using Eqs. (8) and (9) and from these matrix element plus ($s.p.$ energy for particle – $s.p.$ energy for hole) to obtain the energy we have the $s.p.$ energy which are [29] the $s.p.$ energy value [36, 37] where: $e_p(particle) = e_1f_{7/2} = 8.329$ MeV, $e_h(hole) = e_1d_{5/2} = -8.363$ MeV, $e_2s_{1/2} = -5.843$ MeV and $e_1d_{5/2} = -3.973$ MeV. The parameters $A_0 = 0.8$, $A_1 = 0.9$, $B = 0.6$ and $C = -0.2$. Table 2 and Fig. 2, a show comparison between theoretical and experimental values of $^{40}$K nucleus.

Plotting energy levels of these states as a function of the corresponding angle $\theta_{1,2}$ determined according to Eq. (12) one can draw the smooth curve shown in Figs. 1 and 2 case (b) show the behaviour for even values $j_1 + j_2 + J = \text{even}$ and case (c) that for odd values of $j_1 + j_2 + J = \text{odd}$ effective interaction deduced from the data. The energy levels have been normalized to the centroid of each multiplet and are plotted against the angle $\theta_{1,2}$ defined in the text. Note that maximum $J$ values correspond to minimum angle and vice versa. Curvature is a measure of a short-range attractive force. The lines are drawn to connect these points. The energy level seem to follow two universal functions which depend on the semi-classical coupling angles $\theta_{1,2}$ but are otherwise independent on $j$. For $j_1 \neq j_2$ several “typical” functions $\theta_{1,2}$ can be constructed which fit subsets of the data and differ in a predictable way.

Fig. 2.
**Table 2. Comparison between theoretical and experimental values**

| $J^*$ | Energy, MeV | Energy, MeV | $J^*$ | Energy, MeV | Energy, MeV |
|-------|-------------|-------------|-------|-------------|-------------|
| $^{40}$Sc | Theor. | Exp. [39] | $^{40}$K | Theor. | Exp. [38] |
| $^{40}$Sc | $J^*$ | $J^*$ |
| $4_1$ | 0 | 0 | $4_1$ | 0 | 0 |
| $3_1$ | 0.03 | 0.034 | $3_1$ | 0.03 | 0.029 |
| $2_1$ | 0.77 | 0.772 | $2_1$ | 0.801 | 0.800 |
| $1_1$ | 0.89 | 0.893 | $1_1$ | 0.881 | 0.891 |
| $2_1$ | 2.27 | 2.37 | $2_1$ | 1.999 | 2.047 |
| $4_1$ | 2.911 | 2.933 | $4_2$ | 3.011 | 2.069 |

5. Conclusion

In this work, the agreement between theoretical and experimental levels is satisfactory for excitation energies. There are many unconfirmed experimental energy levels confirmed by our calculations and new values for energy levels which were not specified in the experimental data. The choice of model space MSDf effective interactions are suitable in this mass region. Note that maximum $J$ values correspond to

References

1. A. Heusler, R.V. Jolos, P. von Brentano. Excitation
energies of particle-hole states in $^{208}$Pb and the surface delta interaction. Phys. At. Nucl. 76 (2013) 807.
2. I. Talmi. The shell model-successes and limitation. Nucl. Phys. A 507 (1990) 295.
3. R. Arvien, S.A. Moszkowski. Generalized seniority and the surface delta interaction. Phys. Rev. A 45 (1996) 831.
4. A. Molinari et al. Effective two-body interaction in simple nuclear spectra. Nucl. Phys. A 239 (1975) 45.
5. P. Van Isacker. Geometry of shell-model matrix elements. EPJ Web Conferences 78 (2014) 03004.
6. P. Van Isacker. A geometry for the shell model. EPJ Web Conferences 178 (2018) 05002.
7. E. Courrier et al. The shell model as a unified view of nuclear structure. Rev. Mod. Phys. 77 (2005) 427.
8. J.P. Schiffer. The spectra of near-magic odd-odd nuclei and the effective interaction. Ann. Phys. 66 (1971) 798.
9. D.S. Chuu et al. Shell-model calculations of one-hole states in the nuclei of $A = 41 - 43$. Phys. Rev. C 27 (1983) 380.
10. A. Heusler et al. Complete identification of states in $^{208}$Pb below $E_x = 6.2$ MeV. Phys. Rev. C 93 (2016) 054321.
11. A. Heusler, R.V. Jolos, P. von Brentano. Description of one-particle-one-hole configurations coupled to the 3-yrast state in the doubly magic nucleus $^{208}$Pb. Phys. Rev C 99 (2019) 034323.
12. A.H. Taqi. Particle-particle and hole-hole random phase approximation calculations for $^{42}$Ca and $^{50}$Ca. Acta Phys. Pol. B 41 (2010) 1327.
13. N.A.F.M. Poppelier, P.W.M. Glaudemans. Particle-hole excitations in the $^{208}$Pb mass region. Z. Phys. A 329 (1988) 275.
14. F.A. Majeed, R.A. Radhi. Dipole and quadrupole electron excitations of the isovector $T = 1$ particle-hole states in $^{12}$C. Chin. Phys. Lett. 23 (2006) 2699.
15. M. Moinester et al. Multipole analysis of particle-particle or particle-hole multiplets. Phys. Rev. 179 (1969) 984.
16. I. Talmi. Simple Models of Complex Nuclei (Switzerland: Harwood Academic Publishers, 1993).
17. P. Johnstone. Effective interaction for one-hole states in K isotopes. Phys. Rev. C 22 (1980) 2561.
18. W.W. Daehnick. The residual interaction of bound nucleons-two-nucleon matrix elements deduced from transfer experiments. Phys. Rep. 96 (1983) 317.
19. P.M. Endt. Energy levels of $A = 21 - 44$ nuclei. Nucl. Phys. A 521 (1990) 1.
20. N. Schulz et al. Identification of particle-hole multiplets in $^{40}$Sc. Nucl. Phys. A 162 (1971) 349.
21. V.Y. Hansper et al. Measurement of the $^{40}$Ca$(^1He,t)^{40}$Sc reaction. Phys. Rev. C 61 (2000) 028801.
22. H.T. Fortune, R. Sherr. $2p_{3/2}$ strength in $^{40,41}$Sc and the $^{40}$Ca$(p, p')$ reaction. Phys. Rev. C 65 (2002) 067301.
23. K. Heyde. The Nuclear Shell Model. Study Edition (Berlin: Springer-Verlag, 1994).
24. S.M. Austin, G.M. Crawley. The Two-Body Force in Nuclei (London-New York: Plenum Press, 1972).
25. R.F. Casten. Nuclear Structure from a Simple Perspective (Oxford: University Press, 1990).
26. J. Suohon. From Nucleons to Nucleus. Concepts of Microscopic Nuclear Theory (Berlin: Springer, 2007).
27. A.K. Hasan, A.R.H. Subber. Level structure of $^{210}$Po by means of surface delta interaction. Turk. J. Phys. 37 (2013) 348.
28. D.N. Hameed, A.K. Hasan. Energy levels of isobaric nuclei ($^{16}$N, $^{16}$F) within the modified surface delta-
interaction model. Ukr. J. Phys. 63 (2018) 579.

29. P.J. Brussaard, P.W.M. Glaudemans. Shell-Model Applications in Nuclear Spectroscopy (North-Holland, Amsterdam, 1977).

30. A.E.L. Dieperink et al. An investigation of the odd-parity states of $^{40}$Ca with the Tabakin interaction and the MSDI. Nucl. Phys. A 116 (1968) 556.

31. R. Arvien, S.A. Moszkowski. Generalized seniority and the surface delta interaction. Phys. Rev. 145 (1966) 830.

32. R.D. Lawson. Theory of the Nuclear Shell Model (Oxford: Clarendon Press, 1980).

33. P.W.M. Glaudemans et al. Two-body matrix elements from a modified surface delta interaction.

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ЕНЕРГЕТИЧНІ РІВНІ ЯДЕР $^{40}$Sc та $^{40}$K ЗАЛЕЖНОСТІ ВІД НАПІВКЛАСИЧНОГО КУТА ЗВ’ЯЗКУ $\theta_{1,2}$ В МОДИФІЦУВАНИЙ ПЕРВОНАЧАЛЬНІЙ ДЕЛЬТА-ВЗАЄМОСІДІЙ

Оболонкову модель ядра з використанням модифікованої поверхневої дельта-взаємодії було застосовано в цій роботі для обчислення енергетичних рівнів ізобарних ядер $^{40}$Sc та $^{40}$K в стані частинка-дірка. Частинки знаходяться в стані $(1f_{5/2})$, тоді як дірки знаходяться в стані $(1d_{3/2}, 1s_{1/2}, 1d_{5/2})$. Загальний кутовий момент і парність визначено для можливих частинок і дір в усіх назначених ядрах. Автори знайшли таким чином залежність між рівнями енергії та напівкласичним кутом зв’язку $\theta_{1,2}$ для різних орбіталей у межах конфігурації частинка-дірка. Можно відзначити, що рівні енергії наближено описуються двома універсальними функціями, які залежать від напівкласичного кута зв’язку $\theta_{1,2}$. Показано, що теоретичні розрахунки узгоджуються з експериментальними даними.

Ключові слова: оболонкова модель, енергетичні рівні, модифікована поверхнева дельта-взаємодія, $^{40}$Sc, $^{40}$K, частинка-дірка.

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ЕНЕРГЕТИЧЕСКИЕ УРОВНИ ЯДЕР $^{40}$Sc И $^{40}$K ЗАВИСИМОСТИ ОТ ПОЛУКЛАССИЧЕСКОГО УГЛА СВЯЗИ $\theta_{1,2}$ В МОДИФИЦИРОВАННОМ ПЕРВОНАЧАЛЬНОМ ОДНОСТУПЕНЧАТОМ ДЕЛЬТА-ВЗАИМОДЕЙСТВИИ

Оболочечная модель ядра с использованием модифицированного поверхностного дельта-взаимодействия была применена в этой работе для вычисления энергетических уровней изобарных ядер $^{40}$Sc и $^{40}$K в состоянии частица-дырка. Частички находятся в состоянии $(1f_{5/2})$, тогда как дырки находятся в состоянии $(1d_{3/2}, 1s_{1/2}, 1d_{5/2})$. Общий угловой момент и четность определены для возможных частиц и дырок у вышеуказанных ядер. Авторы нашли таким образом зависимость между уровнями энергии и полукласическим углом связи $\theta_{1,2}$ для различных орбиталей в пределах конфигурации частица-дырка. Можно отметить, что уровни энергии приближенно описываются двумя универсальными функциями, которые зависят от полукласического угла связи $\theta_{1,2}$. Показано, что теоретические расчеты согласуются с экспериментальными данными.

Ключевые слова: оболочечная модель, энергетические уровни, модифицированное поверхностное дельта-взаимодействие, $^{40}$Sc, $^{40}$K, частица-дырка.

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