Electron spin qubits are a promising platform for quantum computation. Environmental noise impedes coherent operations by limiting the qubit relaxation ($T_1$) and dephasing ($T_2$) times. There are multiple sources of such noise, which makes it important to devise experimental techniques that can detect the spatial locations of these sources and determine the type of source. In this paper, we propose that anisotropy in $T_1$ and $T_2$ with respect to the direction of the applied magnetic field can reveal much about these aspects of the noise. We investigate the anisotropy patterns of charge noise, evanescent-wave Johnson noise, and hyperfine noise in hypothetical devices. It is necessary to have a rather well-characterized sample to get the maximum benefit from this technique. The general anisotropy patterns are elucidated. We calculate the expected anisotropy for a particular model of a Si/SiGe quantum dot device.

**INTRODUCTION**

Spin qubits are a promising platform for quantum information processing machines. Recent progress includes the development of high-fidelity single-qubit operations, coupling to resonators, a programmable quantum processor, universal quantum logic at 1.5 K, and capacitive coupling of spin qubits. The main obstacle, as in many quantum computing implementations, is the presence of noise that causes decoherence. The sources of decoherence are many: noise from the nuclear spin bath, evanescent-wave Johnson noise, random telegraph and $1/f$-type charge noise, and noise from phonons. The effective magnetic noise strength at lower frequencies is usually a measure of a weighted average of the noise power tensor by $\langle B_\text{eff}^\xi B_\text{eff}^\eta \rangle_{\omega}$. Since we have $1/T_2 = 1/2T_1 + 1/T_\phi$, all of the diagonal components of the noise tensor are accessible to experiment. For most spin qubits $T_1 \gg T_2$, so we have the simpler equation $T_2 = T_\phi$. Henceforth, we shall assume this to be the case and refer only to $T_2$.

This vector character of the coherence time equations means that one can investigate the nature of noise in spin qubit systems by measuring the anisotropy in $T_1$ and $T_2$ as a function of the direction of the applied field. Simply put, if the applied field is in the direction $R_2$, where $R$ is a rotation operator that takes $z$ into the direction with polar angles $(\Theta, \Phi)$, then $1/T_2(\Theta, \Phi) \propto \langle B_\text{eff}^\xi R^{\Theta\Phi}_x B_\text{eff}^\eta \rangle_{\omega} + \langle B_\text{eff}^\xi R^{\Theta\Phi}_y B_\text{eff}^\eta \rangle_{\omega}$, while $T_1(\Theta, \Phi)$ depends on $\langle B_\text{eff}^\xi B_\text{eff}^\eta \rangle_{\omega}$. The pattern in $(\Theta, \Phi)$ gives information about the nature of the noise sources and their positions relative to the qubit.

For charge qubits, the analog of the direction of the external field is the direction of the line in space that connects the two quantum dots. It is normally not possible to adjust this over a wide range and even in a narrow range, it is unlikely to be possible in a controllable way. As a result, the experiments we propose are only possible for spin qubits. In this paper, we limit the analysis to single-spin devices. The idea of using anisotropy to investigate noise should also be applicable to multi-dot qubit devices such as hybrid qubits or singlet-triplet qubits. The analysis is more complicated for these cases. For example, in the two-qubit experiment of ref.19, the change in the ratio of $g$-factors as the field is rotated introduces additional complications. We will not attempt any treatment of these more complex multi-qubit systems in this work, since we are mainly attempting to establish the basic principles involved, and for this purpose it is best to do the simpler cases first. Nevertheless, it seems likely that the anisotropy in decoherence times would be a useful tool even in these more involved situations.

We focus on the anisotropy of three different types of noise sources: charge noise, hyperfine noise, and evanescent-wave Johnson noise (EWJN) in silicon devices. Charge noise is the most important at low frequencies and generally determines $T_2$. EWJN is important at higher frequencies and low magnetic fields, and in...
many cases may determine $T_1$. Hyperfine noise is also important, particularly in GaAs systems. It is expected to be isotropic in $B_{0y}$, which from the viewpoint of this paper is a key experimental signature of this type of noise\textsuperscript{29}, as we shall discuss below. Phonon relaxation mediated by spin-orbit coupling, in contrast, is highly anisotropic, as has been shown previously\textsuperscript{15,21}. This mechanism is important at higher magnetic fields $B_0$. Since we do not include it, the results we present here hold only for $B_0 \leq 3-4T$\textsuperscript{22}. At these lower fields, $T_1$ saturates. It is important to note that the anisotropy due to phonon effects is determined by the orientation of $B_0$ relative to the crystal axes, while the anisotropies considered in this paper are relative to directions determined by the geometry of the device. Hot spots, where the valley and Zeeman levels cross, are also a strong source of decoherence\textsuperscript{23}. Fortunately, it is relatively easy to avoid this by tuning the strength of the applied field, and this would be necessary for the proposed experiment to work. In metal-oxide-semiconductor (MOS) structures the Dresselhaus interaction can be strong and is anisotropic. This gives rise to an anisotropic $T_2$ from charge noise that was measured in ref.\textsuperscript{24}. Here we focus on dots in heterostructures, where the angular variation of the spin-orbit coupling is expected to be much weaker.

In most experiments on spin qubits, the relative orientation of the sample and the applied magnetic field is not allowed to vary. However, rotatable sample holders can give some variation in the angle between the growth direction and the applied field. See, for example, ref.\textsuperscript{25,26}. Full coverage of the whole solid angle can be obtained from vector magnet arrangements with appropriate parameters of the magnets. Indeed, experiments to optimize qubit operation by changing the direction of the applied field have been carried out\textsuperscript{26}. The present paper can be viewed as an aid to these kinds of efforts since the direction of maximum decoherence times can be inferred from the calculations. Other phenomena that have been investigated by rotating the field are the variations in the Rabi frequency of multi-hole qubits in Si\textsuperscript{27} and the profile of the spin–orbit interaction of a silicon double quantum dot in MOS structures\textsuperscript{28,29}.

This paper will focus on experiments that use a micromagnet to provide a field gradient at the position of the dot. The direction of the magnetization of the micromagnet can be affected by the rotation of the applied field in a way that is not well understood and that is difficult to measure. This means that to carry out the type of experiment that is proposed here, hard magnets must be used.

**RESULTS**

**Relaxation time**

The relaxation rate of a spin qubit in the noise magnetic field depends on the noise correlation function $\langle B_i^{(\text{eff})}(t) B_j^{(\text{eff})}(0) \rangle$, where $B_i^{(\text{eff})}$ is the effective noise magnetic field, $\omega_{\text{op}}$ is the operating frequency of the qubit. The effective noise magnetic field is any time-dependent field that couples to the spin in the usual way. Hence this could be a physical magnetic field, a field that comes from the motion of the qubit in an inhomogeneous field, a field that results from phonons mediated by spin-orbit coupling, etc.

$T_1$, the relaxation time, depends only on the transverse components of the correlation function. If we define $T_1^{(\text{y})}$ as the relaxation time when the applied field is in the i-direction, then:

$$
\frac{1}{T_1^{(\text{y})}} = \left( \frac{\hbar}{\pi} g \right)^2 \sum_j \left( \langle B_y^{(\text{eff})} B_y^{(\text{eff})} \rangle_{\omega_{\text{op}}} + \langle B_z^{(\text{eff})} B_z^{(\text{eff})} \rangle_{\omega_{\text{op}}} \right),
$$

$$
\frac{1}{T_1^{(\text{y})}} = \left( \frac{\hbar}{\pi} g \right)^2 \sum_j \left( \langle B_z^{(\text{eff})} B_z^{(\text{eff})} \rangle_{\omega_{\text{op}}} + \langle B_x^{(\text{eff})} B_x^{(\text{eff})} \rangle_{\omega_{\text{op}}} \right),
$$

$$
\frac{1}{T_1^{(\text{y})}} = \left( \frac{\hbar}{\pi} g \right)^2 \sum_j \left( \langle B_x^{(\text{eff})} B_x^{(\text{eff})} \rangle_{\omega_{\text{op}}} + \langle B_y^{(\text{eff})} B_y^{(\text{eff})} \rangle_{\omega_{\text{op}}} \right).
$$

where $\mu_0$ is the Bohr magneton. We take $g = 2$.

If the applied field is in an arbitrary direction $\hat{n} = \sin \theta \cos \phi \hat{x} + \sin \theta \sin \phi \hat{y} + \cos \theta \hat{z}$, where $\theta$ is the polar angle and $\phi$ is the azimuthal angle, then the relaxation rate becomes

$$
\frac{1}{T_1(\theta, \phi)} = \left( \frac{\hbar}{\pi} g \right)^2 \sum_j Q^{(1)}_{ij} \left\langle B_j^{(\text{eff})} B_j^{(\text{eff})} \right\rangle_{\omega_{\text{op}}},
$$

where

$$
Q^{(1)} = \begin{bmatrix}
\cos^2 \phi \cos^2 \theta + \sin^2 \phi & -\cos \phi \sin \phi \sin \phi \sin^2 \theta & -\cos \phi \cos \theta \sin \phi \\
-\cos \phi \sin \phi \sin \phi \sin^2 \theta & \sin^2 \phi \cos^2 \theta + \cos^2 \phi & -\cos \phi \cos \phi \sin \phi \\
-\cos \phi \cos \theta \sin \phi & -\cos \phi \cos \phi \sin \phi & \cos^2 \phi \cos^2 \theta \end{bmatrix}
$$

with $(x, y, z)$ as the basis for the matrix $Q^{(1)}$. Note that if there are any nonzero off-diagonal correlation functions $\left\langle B_i^{(\text{eff})} B_j^{(\text{eff})} \right\rangle_{\omega_{\text{op}}}$ ($i \neq j$), they are also needed in the expression for the relaxation time.

**Dephasing time**

The calculation of the dephasing time is more complicated than that for the relaxation time, since it depends more sensitively on the full frequency spectrum of the noise and higher-level correlation functions. For the purposes of this paper, only ratios of $T_2$ for different applied field angles are important. Hence the specific approximation used to compute $T_2$ is not so crucial. It will be sufficient to assume that the field fluctuation obeys Gaussian statistics. Then if the applied field is in the $z$-direction, the off-diagonal components of the density matrix of the qubit decay according to the expression $\exp(-T_2 t)$ with

$$
\Gamma(t) = \frac{t^2}{2} \left( \frac{2 \hbar \omega_{\text{op}}}{\pi} \right)^2 \int_{-\infty}^{\infty} \sin^2(\omega t/2) \cdot Q^{(2)}_{ij}(\theta, \phi)
$$

where

$$
Q^{(2)} = \begin{bmatrix}
\cos^2 \phi \sin^2 \theta & \cos \phi \sin \phi \sin^2 \theta & \cos \phi \cos \phi \sin \phi \\
\cos \phi \sin \phi \sin^2 \theta & \sin^2 \phi \cos^2 \theta + \cos^2 \phi & \cos \phi \cos \phi \sin \phi \\
\cos \phi \cos \phi \sin \phi & \cos \phi \cos \phi \sin \phi & \cos^2 \phi \cos^2 \theta + \cos^2 \phi
\end{bmatrix}
$$

in the same basis as used for $Q^{(1)}$.

The results of this subsection and the previous one make it clear that the entire tensor structure of the noise correlation function is in principle accessible simply by measuring $T_1$ and $T_2$.

**Effective magnetic field noise from a micromagnet**

Single-qubit logic gates in spin systems are often implemented using a micromagnet to set up a magnetic field gradient. This has the unwanted complication that electric field noise moves the spin up and down the gradient, causing a time-dependent magnetic field $B^{(0)}$ that can decohere the qubit. Here we outline how this plays into the anisotropy effect.

We will take a simple model of a quantum dot in a harmonic potential. The Hamiltonian is

$$
H = -\sum_i \frac{\hbar^2}{2m_i} \frac{\partial^2}{\partial x_i^2} + \sum_i \frac{1}{2} k_i x_i^2 - q \sum_i x_i E_i(t)
$$

where $i$ is a Cartesian index, $m_i$ and $k_i$ are the effective mass and spring constant of the electron in the i-direction, $q$ is the electric charge of the electron and $E_i(t)$ is the noise electric field component in the i-direction. The frequency of the electric noise
is much smaller than the natural frequencies of the harmonic motion, so the Born-Oppenheimer approximation applies and the effect of $E_i(t)$ is to shift the position of the minimum of the potential by $\Delta x_i(t) = -q E_i(t)/2k$. The confinement in the $z$-direction is much stronger, though the effects of excited states in this potential are measurable \cite{10,31}. To the level of approximation needed in this paper, we can drop the $z$ term in Eq. (7) in the potential and treat the dot as two-dimensional.

The micromagnet sets up a static field $B^\text{m}$ that varies strongly in space. The associated effective noise field that acts on the spin is

$$B_i^{(E)}(t) = \frac{\partial B^\text{m}}{\partial x_i} \Delta x_i(t) = q \sum_j \frac{1}{2k} \frac{\partial B^\text{m}}{\partial x_j} E_j(t). \quad (7)$$

The field gradient and the spring constant are device parameters. This equation already shows that any interesting information about the noise field $E_i(t)$.

### Magnet hardness

In qubit experiments carried out to date, the micromagnet is made of pure cobalt \cite{32,33}. Co is hexagonal with the easy axis for magnetization along the $z$-direction. The anisotropy parameter (the difference in energy between the $z$-axis and the $x$-$y$ plane) is about $10^{6}$ J/m$^3$, which corresponds to an anisotropy field $H_A$ of about 0.5 T when the magnetization is saturated, as it surely is at the low temperatures of the experiments. This and shape anisotropy will mainly determine the coercive field, though sample-dependent domain wall pinning will also contribute. The anisotropy parameter quoted above is consistent with the results of ref. \cite{34}, though it is sometimes assumed that the magnet will rotate freely \cite{35,36}. In any case, in the fields of present-day experiments (a fraction of 1 T), the magnet cannot be considered to be hard and some rotation of the direction of the magnetization of the Co micromagnet is certainly to be expected. This will change the field gradient tensor that is used to calculate $T_1$ and $T_2$.

There are three possible solutions to this problem, the first two of which involve changing the magnet.

The first is to use a softer ferromagnet with a cubic crystal structure such as Fe, where one could expect the magnetization follows the applied field. This would erase much too not all of the anisotropy in the decoherence times. It is certainly not ideal but might still give useful information about the noise sources.

The second option is to use a harder ferromagnet, so that the field gradient tensor is fixed once and for all when the magnet is cooled in a field. There are many possibilities, but one that suggests itself is SmCo$_5$, very close chemically to the Co magnets in current use. The anisotropy field can be as high as $H_A = 55$ T, an order of magnitude higher than pure Co \cite{37}. The magnetization would not be significantly affected by the rotation of the external field. Then the anisotropy in $T_1$ and $T_2$ is more pronounced and more information can be extracted from it.

The third possibility is to stick with the Co magnet but to use an external field that is considerably smaller (<0.05 T, say) so that the field from the micromagnet is fixed. This has the problem that the energy level splitting of the qubit becomes smaller than $k_B T$, and initializing the spin for measurements is problematic. It is possible only at temperatures of the order of 1 mK, considerably lower than the temperatures in use today for these experiments. One might still be able to do spin blockade-based measurements, however.

In this paper, we shall assume that the second alternative is chosen, since this gives the richest phenomenology, and seems feasible with fairly modest changes in fabrication techniques. So we take the field gradients to be fixed. Of course, this means that the experiment cannot be carried out with the sample of ref. \cite{33} which uses a Co magnet and electron temperatures of order 150 mK.

### Evanescent-wave Johnson noise

Evanescent-wave Johnson noise (EWNJ) is due to the random motion of charges in the metallic elements of the device. This motion produces random electric and magnetic fields on the qubits in the vicinity of the metal. For the discussion of this effect, let us take the growth direction for the device to be the $z$-direction, the distance of the qubit from the gate layer as $d$, the gate thickness as $w$, and the dielectric constant of the intervening insulating material as $\epsilon_f$.

For the case of noise from a conducting half-space, rather simple formulas are available \cite{39,40}. In most Si/SiGe heterostructure and Si MOS devices, the gates form sheets of metal that can be approximated as a uniform layer from the standpoint of noise production. Thus the theory of EWNJ from a film with a finite $w$ is more appropriate. It has been worked out in detail \cite{39}, though the results are somewhat complicated, and depend on whether we consider electric field noise or magnetic field noise. For the values of $d$ of interest to us, electric field noise is slightly enhanced for the film case as compared to the half-space case, while the opposite is true for the magnetic field noise, and the effect of finite $w$ is larger. A very good approximation is to use the half-space formula for the electric noise and a modified formula for the magnetic case.

Given these considerations, the noise correlation functions for the electric field are

$$\langle E_i E_j \rangle_{\text{op}} = \frac{h \omega_p \epsilon_{fi} \epsilon_{fo}}{2d} \coth \frac{h \omega_p}{2k_B T}. \quad (8)$$

Here $\sigma$ is the conductivity. The other elements of the noise tensor are $\langle E_i E_z \rangle_{\text{op}} = \langle E_z E_j \rangle_{\text{op}} = (1/2) \langle E_i E_j \rangle_{\text{op}}$, while the off-diagonal elements of the tensor vanish. This electric noise is converted into effective magnetic field noise using the techniques of the previous section.

The magnetic EWNJ correlation function is given by

$$\langle B_i B_j \rangle_{\text{op}} = \frac{h \omega_p \mu_0 \sigma w}{8d} \coth \frac{h \omega_C}{2k_B T}. \quad (9)$$

This is reduced from the half-space result by a factor of $w/d$. The other elements of this noise tensor are $\langle B_i B_j \rangle_{\text{op}} = (1/2) \langle B_i B_j \rangle_{\text{op}}$, while the off-diagonal elements of the tensor vanish. Unlike the electric noise, magnetic EWNJ acts directly on the qubit spin to produce decoherence.

### Charge noise sources

The exact nature of the low-frequency charge noise remains controversial. There are two leading models for the source of the two-level systems (TLS) that give rise to this noise.

The first model is the TLS proposal of Anderson, Halperin, and Varma, and independently Phillips, of 1972 \cite{40,41}. The picture, shown in Fig. 1, is that the noise source is the motion of some atom or group of atoms in a potential that supports bistability in some range of parameters. The motion also produces a fluctuating dipole moment, which we take as $a \mathbf{p}$ in the two stable positions. For the most part, this idea has formed the conceptual background of the field in physics experiments for the last half century. We will assume that the orientation of these dipoles is uniformly random on the unit sphere, and that they are distributed uniformly in space in the oxide layer above the qubits. We call this the random dipole model.

The second model is the related but physically quite distinct idea of McWhorter \cite{42}, in which a conducting layer serves as a reservoir for electron traps near the surface of the layer whose energies are close to the Fermi level of the layer. Electrons from the reservoir can hop on and off, again changing the distribution of charge in the system, the change being well approximated by a
fluctuating dipole perpendicular to the layer. In this case, the fluctuation is between a zero and a fixed nonzero value of the dipole moment, while in the dipole model the fluctuation is between two different nonzero values. The trap-type TLS is in fact widely thought to be the most important for the noise in field-effect transistors in the engineering community. However, as in the case of the dipole model, real proof of the details of the model is hard to come by. We call this the trap model. An illustration of the model is given in Fig. 1b.

It is evident that the two models are not easy to distinguish experimentally, since they will both give random telegraph noise with a distribution of switching rates, and reasonable assumptions about the distribution will lead to $1/T$ type noise. They differ in the orientation of the effective dipoles, however, suggesting that an experiment that can detect anisotropy in the noise correlation tensor will distinguish the two models. This forms a chief motivation for the current work.

**Source positions**

Anisotropy in the decoherence rates can come from several sources. We have seen that the noise tensor from EJWJN can itself be anisotropic, and the dipole and trap models also have characteristic anisotropy signatures. In addition to this, it is possible for charge noise sources to be clumped, either from a tendency to adhere to different device elements, or, particularly in the case of only a few sources, to cluster by random chance. Of course, if the noise is coming from a certain direction this is also a source of anisotropy.

This leads to distinguishing four models all together, which we call the uniformly distributed dipole model (UD), uniformly distributed trap model (UT), localized cluster dipole model (CD) and localized cluster trap model (CT). The U-type models assume that the sources are many in number and uniformly distributed, while the C-type models assume that the sources are relatively few in number. The total number of sources is of course also very important to determine.

The U- and C-type models represent limiting cases of very many and just a few closely spaced noise sources, respectively. Of course, it is not possible to rule out in advance a lumpy set of say 10 to 100 noise sources. The current computational method would need to be developed considerably further to become a useful characterization tool in this difficult intermediate case. In particular, multiple qubits and cross-correlation functions among them would most likely be needed.

**Noise correlation functions**

The noise correlation functions for all models are calculated as follows.

First, we note that because of the metallic elements in the device, it is important to include screening of the electric noise. Electric dipoles near a metal surface at $z = 0$ are screened in an anisotropic fashion. A dipole oriented perpendicularly to the surface is anti-screened since the image dipole is in the same direction as the original one. This is in sharp contrast to a dipole oriented parallel to a metal surface, which is strongly screened, with the image dipole opposite in direction to the original one.

These image charge effects are taken into account in our calculations by adding an image dipole $\pm p$. If the bare dipole $p = (p_x, p_y, p_z)$ is located at $r = (x, y, z)$ relative to a metallic layer at $z = 0$ the image dipole is located at $(x, y, z)$ and its moment is given by $p_{im} = (-p_y, -p_x, p_z)$. Examples of a bare dipole and corresponding image dipole for the random dipole model and trap model are shown in Figs. 1(c) and (d).
The isotropic hyperfine noise is taken into account by estimating it from other experiments. It should not differ too much from one device to another. The material in the case study device is natural silicon. If the decoherence rate in isotopically purified silicon\(^2\) is subtracted from that in natural silicon\(^3\), we find a hyperfine contribution to the rate of \(1/T^2_{\text{hyper}} = (1.83 \times 10^{-1}) - (20.4 \times 10^{-1}) = (20.01 \times 10^{-1})\). \(1/T^2_{\text{hyper}}\) is then simply added to the dephasing rates from charge noise. Because of its isotropy, its effect is to smooth the resulting plots.

For charge noise, \(\Gamma(t)\) for the applied field along the \((\theta, \phi)\) direction can be written as

\[
\Gamma(t; \theta, \phi) = \sum_i Q_i^{(2)} \left[ \frac{\partial \gamma_i}{\partial \theta} \delta \theta + \frac{\partial \gamma_i}{\partial \phi} \delta \phi \right],
\]

where \(\gamma_x(t)\) and \(\gamma_y(t)\) are the prefactors related to the gradients in \(x\)- and \(y\)-direction respectively.

Now we assume that the TLS noise is a Poisson process with an exponential time correlation functions with characteristic relaxation time \(\tau\) and carry out the necessary integrations. For the UD model we have

\[
\gamma_x(t) = \gamma_y(t) = \left(\frac{2\pi}{\omega_0} \right)^2 \left(\frac{a_0^2}{\omega_0^2} \right)^2 \frac{\partial \rho_a}{\partial \omega} \left(\frac{1}{\tau} - \frac{1}{\omega^2}\right) \times 2\pi \tau (t + \omega^{-1} - 1)\tau).
\]

For the UT model we find

\[
\gamma_x(t) = \left(\frac{2\pi}{\omega_0} \right)^2 \left(\frac{a_0^2}{\omega_0^2} \right)^2 \frac{\partial \rho_a}{\partial \omega} \left(\frac{1}{\tau} - \frac{1}{\omega^2}\right) \times 2\pi \tau (t + \omega^{-1} - 1)\tau).
\]

The temporal part of \(\gamma_x\) and \(\gamma_y\) results from the integration of the product of a Lorentzian \(g(\omega) = 2\sqrt{t}/(1 + \omega^2)\) and \(\sin^2(\omega t/2)\). Those equations are obtained by converting electric field noise correlations (Eq. (12) or Eq. (13)) into effective magnetic field correlation using Eq. (7) (See details in Supplementary Notes). It is important to note first that the details of the noise spectrum and thus the choice of an exponential correlation are not crucial for the anisotropy patterns, since they depend only on ratios of noise strengths. On the negative side, if some parameter of the noise such as \(\tau\) itself depends on position in the sample, then the extraction of useful information from the analysis of the data would become far more complicated.

\(\rho_a\) and \(\rho_x\) are poorly known, so we use them as fitting parameters. \(T^2_1 = 840\) ns was measured for only a single direction of the field, indicated by the red dots in Fig. 2. This yields \(\rho_a = 2.93 \times 10^{20}\) m\(^{-3}\) and \(\rho_x = 2.66 \times 10^{11}\) m\(^{-2}\). For \(T^2_1\) in Fig. 3, we have chosen to show the full angular ranges since in some instances the topology of the function is clearer this way. The number of maxima \(N_{\text{max}}\), minima \(N_{\text{min}}\), and saddle points \(N_S\) in each map are shown in Table 1. When the dephasing and relaxation times in the maps are continuous functions without higher-order critical points, Morse theory can be applied. In particular for such functions on a 2D sphere (genus zero), they follow the relation: \(N_{\text{max}} + N_{\text{min}} = N_S + 2\). Table 1 shows how the topology of the function can change when parameters affecting the noise are varied. For the maps of Fig. 2 and 3, Morse theory cannot be applied because the ridges in the maps correspond to a line of higher-order critical points.

Anisotropy of \(T_2\)

The goal of this section is to determine signatures of the different types of noise in \(T_2\) data. We first show that EWJN is not important for \(T_2\). Then we point out that hyperfine noise is isotropic and estimate its magnitude.

In the experiment of ref. \(^{33}\), \(T_2^*\) of the device is measured to be \(840 \pm 70\) ns and corresponds to \(T_2^*(\theta)\) of our simulation because the applied magnetic field \(\mathbf{B}_0\) is in the \(x\)-direction. For EWJN, the dephasing rate in the \(i\)-direction is

\[
\frac{1}{T_2^{(i)}} = 2nk_BT_1 \left(\frac{2\mu_0}{h} \right)^2 \lim_{\omega \to 0} \left[ \frac{1}{\omega} \left( b_i^{(\text{eff})} b_j^{(\text{eff})} \right) \right].
\]

This is the dephasing time for the exponential regime when the time \(t \gg h/k_BT\). The calculated \(T_2^{(i)} = 1.19\) s from EWJN is six orders of magnitude larger than the experimental one. Thus the dominant mechanism for the decoherence of the qubit should be charge noise, not EWJN.
The background \( T_{\text{type}} = 2.01 \mu s \) sets an upper bound on the plotted values in Fig. 2. The white regions in Fig. 2a and d represent angular regions where the charge noise contribution is negligible and this upper bound is reached. Figure 2a shows the results for the UD model and Fig. 2b for the UT model. The horizontal (vertical) axis denotes polar (azimuthal) angle with respect to the device’s z-direction. The key feature of these two models is that anisotropy of Fig. 2 results only from the magnetic field gradients. The patterns are not too dissimilar, with the ratio between maximum and minimum values being around 3 for the UD model and 2 for the UT model. The main difference between the UD and UT models is that the peaks and valleys are broader in the UT model. In the UD model, the dipoles are oriented randomly, while in the UT model they are in the z-direction. The differences in the anisotropy maps between UD and UT can be traced back to the different behavior of electric field lines from these two different types of sources. However, this does not manifest itself in a simple way because of the complexity of the gradient tensors that mediate the electric noise. Because of that, it is difficult to develop much physical intuition about the distinction between UD and UT charge noise sources from inspection of the anisotropy maps, and it appears that a full calculation is necessary to test the differences between the two noise models.

The anisotropy maps for the CD model, a localized dipole cluster, are shown in Fig. 2c and d. The cluster is located at \((x, y, z) = (37, 0, 37)\) nm and \((x, y, z) = (0, 37, 37)\) nm respectively. The maps for the CT model, a localized trap cluster, are shown in Fig. 2e and f. The trap is located at \((x, y, z) = (37, 0, 137)\) nm and \((x, y, z) = (0, 37, 137)\) nm respectively. In both CD and CT model, the qubit is located at the origin. Thus, Fig. 2c is directly comparable to Fig. 2e and d is directly comparable to Fig. 2f. The overall dipole strength \( \rho_0 \) is used for the fitting parameter of these single cluster models, once more by using the experimental value measured at the red point. Figure 2c-f exhibits more anisotropy relative to the uniform distribution models. This is expected since the localization of the source itself introduces anisotropy. On the other hand, one might expect that C- and U-type models would be easy to distinguish because of a simpler azimuthal dependence for the latter. But once more because of the mediation of the noise by the complicated field gradient tensors, such simple expectations are not borne out.

The difference between the CD and CT models lies in the dipole orientation. In the CD model, it is assumed that the cluster contains dipoles of all orientations and the noise electric field is averaged over the solid angle. This washes out the anisotropy to some extent, but the pattern still depends on the direction of the line connecting the dipole and the qubit. The distance between the dipole and qubit just changes the overall magnitude of \( T_2 \). In the CT model, however, the trap generates a noise electric field with more directionality, so the overall anisotropy patterns are sharper and both the direction and the distance are important.

Comparing Figs. 2c to 2d and 2e to 2f indicates that the source position has a large effect. To understand this in more detail, let us focus on the CT model in Fig. 2e and f. Note that \( \partial B_x/\partial x, \partial B_y/\partial y, \) and \( \partial B_z/\partial y \) are an order of magnitude greater than the other gradient terms. In Fig. 2e, the electric field at the qubit has only \( x \) and \( z \) components. The \( x \) component contributes to \( 1/T_2 \) after multiplication by \( \partial B_x/\partial x \) and \( \partial B_z/\partial z \). Thus small \( T_2 \) is expected when the applied field is in the \( x \) and \( z \) directions, which can be identified on the map with \((\theta, \phi) = (n/2, 0)\) and \((\theta, \phi) = (0, 0)\), respectively. On the other hand, in Fig. 2f, the electric field at the qubit has only \( y \) and \( z \) components. The leading contribution to \( 1/T_2 \) is the result of the product of the \( y \) component and \( \partial B_y/\partial y \). A small \( T_2 \) is expected with the applied field is in the \( y \)-direction, which is seen at the point \((\theta, \phi) = (n/2, \pi/2)\) on the map. Thus for the distinction between CD and CT models, some relatively simple physical considerations can help to decipher the anisotropy map.

It is important to point out that if relatively few two-level systems contribute to the dephasing of the qubit, as is often hypothesized based on deviations for power-law spectra, the anisotropy can be used to determine the source position and to distinguish between random dipole and trap models for the

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**Fig. 2** The anisotropy maps of decoherence time \( T_2 \). x and y axes are the polar angle \( \theta \) and the azimuthal angle \( \phi \) with respect to the device’s z-direction, respectively. The models used for simulations are: a uniformly distributed random dipoles (UD), b uniformly distributed traps (UT), c and d single dipole cluster (CD) located at \((x, y, z) = (37, 0, 37)\) nm and \((x, y, z) = (0, 37, 37)\) nm respectively, e and f single trap cluster (CT) located at \((x, y, z) = (37, 0, 137)\) nm and \((x, y, z) = (0, 37, 137)\) nm respectively. The qubit is located at the origin. The applied field direction \((\theta, \phi) = (n/2, 0)\) used in the experiment is indicated by the red dot. For the uniform distribution models in a and b, the volume density \( \rho_v \) and areal density \( \rho_a \) are respectively used as fitting parameters to match \( T_2(n/2, 0) \) to the experimental value, 840 ns, and for single cluster models in c-f the dipole strength \( \rho_0 \) is used as a fitting parameter.

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charge noise. The present method can be extended to models
with very few sources by eliminating the averaging we have
performed, but the analysis quickly becomes complicated.

**Anisotropy of T₁**

In the experiment, T₁ of the device is in the order of 1 s. To
estimate the contribution of charge noise, we use the results from
II A together with the determination of densities from T₂. This
leads immediately to an estimate in the range of 10⁹ S/m so we
conclude that charge noise is not important for spin relaxation in
the single-qubit system considered here. To exclude phonon
relaxation we need to stipulate for the moment that the external
field strength is less than about 1 T. This leaves EWJN as the
dominant mechanism.

The relaxation rate with applied field direction in (θ, φ) for EWJN
can be written as

\[
\frac{1}{T₁(θ, φ)} = a + β \sum_i Q^{(1)}_i \left( \frac{∂T}{∂x} \frac{∂Q_i}{∂x} + \frac{∂T}{∂y} \frac{∂Q_i}{∂y} \right)
\]

where

\[
a = \frac{hω_{μω} ω}{4d_ω} \coth \frac{hω}{k_BT},
\]

\[
β = \left( \frac{q}{2mω_{μω}} \right)^2 \frac{hω_{μω} ω}{4d_ω} \coth \frac{hω}{k_BT}.
\]

The anisotropy pattern for the dephasing time of a spin qubit
is shown in Fig. 3 with (a) σ = 2 × 10⁸ S/m, (b) σ = 2 × 10⁷ S/m, and (c) σ = 2 × 10⁹ S/m. The anisotropy pattern
in Fig. 3a is fairly simple because the magnetic noise is dominant
and the direct magnetic EWJN itself is not very anisotropic, as can be seen from Eq. (9) and the text following it.

The anisotropy is increased as shown in Fig. 3b where the
magnetic noise is somewhat more comparable to the electric
noise. The anisotropy becomes even larger in Fig. 3c where the
magnetic noise is one order of magnitude smaller than the electric
noise. This pattern looks like the reversal of the anisotropy map
of T₂ in Fig. 2a and b. This is natural since T₂ of a spin qubit is due to longitudinal noise while T₁ is due to transverse noise. From
a practical point of view, the qubit performance would be improved
when the applied field direction is set to the angles that give
maximal T₂ (in the case of T₂ ≪ T₁).

**DISCUSSION**

The anisotropy pattern for the dephasing time of a spin qubit
comes from the combination of the magnetic field gradient and
the noise electric field, the latter being determined by the
configuration of noise dipoles. By introducing a vector magnet in a
quantum dot device, noise characteristics such as noise dipole
type and/or spatial distribution of noise dipoles can be
experimentally investigated. Another way to obtain similar
information is to exploit a controllable magnetic field gradient
for a spin qubit on a nitrogen-vacancy center in a diamond⁴⁵,⁴⁶. In
this case, the gradient can be varied instead of the direction of
applied magnetic field to study noise characteristics.

The anisotropy maps of relaxation times can be explained by a
combination of direct magnetic noise and indirect electric noise.
The magnetic noise part resulting from EWJN is isotropic in the x
plane in typical device structures. The electric noise is
mediated by magnetic field gradients, which is the only source
of anisotropy. As a result, the anisotropy gets bigger as the
influence of the electric noise part increases.

To summarize, we have shown anisotropy in relaxation times
and dephasing times using device parameters taken from
quantum dot of Kawakami et al. Making the anisotropy map can
help to understand the noise mechanisms. Specifically, this will
benefit the understanding of solid state quantum processors.
where the causes of noise are still being investigated. Our work contributes to science in the noisy intermediate-scale quantum era by suggesting a new experimental method for noise characterization of spin qubit devices.

METHODS

Device parameters

The parameters for the device of ref. are as follows. The field gradients at the qubit in units of mT/100 nm are $\frac{\partial B}{\partial x} = -0.20$, $\frac{\partial B}{\partial y} = -0.05$, $\frac{\partial B}{\partial z} = -0.27$, $\frac{\partial B}{\partial y} = 0.03$, and $\frac{\partial B}{\partial y} = 0.18$, and $\frac{\partial B}{\partial y} = 0.02$. The z-direction of the device is taken to be the growth direction. The variation in the z-direction is not needed in the two-dimension approximation we are using. Other important parameters are the thickness of the aluminum oxide layer $l = 100$ nm, the dielectric constant $\varepsilon = 13.05$ for SiGe, and the transverse effective mass $m = 0.19 m_e = 1.73 \times 10^{-5}$ kg. The lowest orbital excitation frequency is taken as $\omega_{orb} = 6.84 \times 10^{11}$ s$^{-1}$ and it is related to the spring constants by the equations $k_x = k_y = m \omega_{orb}^2$. As mentioned above, we take $k_z \rightarrow \infty$ since confinement is strong along the growth direction. The base temperature is 25 mK, while the electron temperature is about 150 mK, the value we use for the calculations.

Another parameter needed as input to the theory is the conductivity $\sigma$ of the Au gate. This was not measured in this device, but under similar growth conditions for Au films a value of $\sigma = 2 \times 10^5$ S m$^{-1}$ was obtained at the temperatures of the experiment. We should regard this as a probably somewhat high order-of-magnitude estimate of $\sigma$ in the actual device, and we used a range of values for $\sigma$. We take the gate thickness as $w = 25$ nm, and the distance from the qubit to the gates $d = 137$ nm. The qubit operating frequency is $\omega_{op} = 2\pi \times 12.9$ GHz $= 8.11 \times 10^{14}$ Hz$^{-1}$. Two micromagnets made of cobalt are defined on top of the gates, approximately 162 nm above the qubit. In the experiments reported in ref. the applied field $B_z$ was in the z-direction. This paper concerns what happens if this direction is varied (but with the caveats discussed in “Magnet Hardness”).

DATA AVAILABILITY

Authors can confirm that all relevant data are included in the article and/or its supporting files.

CODE AVAILABILITY

The code that produced the figures in the text is available from the corresponding author upon reasonable request.

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AUTHOR CONTRIBUTIONS

R.J.J. conceived and supervised the study. Y.C. modeled the problem and obtained the simulated results. All authors discussed the results and prepared the manuscript.

COMPETING INTERESTS

The authors declare no competing interests.

ADDITIONAL INFORMATION

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