The parameters of the fermion-mixing and search restriction on a mass leptoquark.

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Abstract. It is shown that the inclusion of fermion mixing in the framework of four-colour quark-lepton symmetric model can be greatly reduces the restrictions on the mass vector leptoquark. The lower limit for the mass of the vector leptoquark obtained from processes $K_L^0 \rightarrow l_i^+ l_j^-$, $B^0 \rightarrow l_i^+ l_j^-$ and $B_s \rightarrow l_i^+ l_j^-$ can be of the order of 85 TeV.

1 Introduction

The search for a new physics beyond the Standard Model (SM) is now one of the aims of the high energy physics. One of the possible variants of such new physics can be the variant induced by the possible four color symmetry [1] between quarks and leptons. This symmetry predicts the existence of leptoquarks. Leptoquarks are color triplet bosons of spin 0 or 1, carrying lepton (L) and barion (B) number and fractional electric charge. They couple to lepton-quark pairs and appear in many extensions of the Standard Model (SM) which unify lepton and quark in one multiplet in the framework Grand United Theory (GUT) on grounds that $SU_5$ [2] and $SU_4$ [1], superstring-inspired $E_6$ model and other models. We study leptoquarks in the minimal extension SM with four color symmetry, that is Minimal Quark-Lepton Symmetry model (MQLS model)[3, 4]. The MQLS model predicted the existence of the vector leptoquarks with masses on the order of the mass scale, which $SU_c(4)$ symmetry is broken. The mass limits for the vector leptoquarks are well known and the most stringent of them are the indirect mass limits resulted from $K_L^0 \rightarrow e^\pm \mu^\mp$ decay and they are of order of $10^3$ TeV [5–7]. However, these restrictions can be lowered if consider additional fermion mixing, in this case masses can be order 40 TeV [8, 9] This paper describes constraints on the masses vector leptoquarks from the current experimental data on the leptonic decays $K_L^0 \rightarrow l_i^+ l_j^-$, $B^0 \rightarrow l_i^+ l_j^-$, $B_s \rightarrow l_i^+ l_j^-$ mesons.

2 Model

The MQLS model [3, 4] based on the group

$$G = SU_4(4) \times SU_2(2) \times U_R(1).$$

In the MQLS model the basic left- (L) and right- (R) handed quarks $Q^{L,R}_{i\alpha\beta}$ and leptons $l^{L,R}_{i\alpha}$ form the fundamental quartets of $SU(4)$ color group, and can be written, in general, as superpositions of the

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quark and lepton mass eigenstates \( Q^{LR}_{iaa} \) and \( l^{LR}_{iaa} \)

\[
\psi^{LR}_{iaa} = Q^{LR}_{iaa} = \sum_j (A^{LR}_{Q})_{ij} Q^{LR}_{jaa}, \quad \psi^{LR}_{iaa} = l^{LR}_{iaa} = \sum_j (A^{LR}_{l})_{ij} l^{LR}_{jaa},
\]

where \( i=1,2,3 \) are the generation indices, \( a=1,2 \) are the \( SU_L(2) \) indices and \( A = \alpha,4-SU_V(4) \) indices, \( \alpha=1,2,3 \) are the \( SU_U(3) \) color indices. The unitary matrices \( A^{LR}_{Q} \) and \( A^{LR}_{l} \) describe the fermion mixing and diagonalize the mass matrices of quarks and leptons. The combinations of these matrices give corresponding \( C_0 = (A^{LR}_{Q})^* A^{LR}_{Q} \) is the Cabibbo-Kobayashi-Maskawa matrix, \( C_1 = (A^{LR}_{l})^* A^{LR}_{l} \) is the Pontecorvo-Maki-Nakagawa-Sakata matrix that is analogous to the preceding one in the lepton sector and is not diagonal, this evident from neutrino oscillation, and

\[
K^{LR}_{a} = (A^{LR}_{Q})^* A^{LR}_{l}
\]

unitary mixing matrices that are due to the possible distinctions between the quarks and leptons mixing matrices \( A^{LR}_{Q} \) and \( A^{LR}_{l} \).

The lagrangian interaction vector leptoquarks with down fermions can be written in the following form

\[
L_{Vd} = \frac{g_4}{\sqrt{2}} \left( \bar{d}_{p i} (K^{L}_{\alpha})_{i \beta} \gamma^\mu P_L + (K^{R}_{\alpha})_{i \beta} \gamma^\mu P_R |\bar{l}|_i \right) V_{a \mu} + h.c.,
\]

where \( g_4 = g_{sU}(M_c) \) is the \( SU_V(4) \) gauge coupling constant, related to the strong coupling constant at the mass scale \( M_c \) of the \( SU_V(4) \) symmetry breaking, \( p,i=1,2,3,... \) are the quark and lepton generation indexes, \( \alpha=1,2,3 \) are \( SU_V(4) \) colour index, \( P_{LR} = (1\pm \gamma_5)/2 \).

3 The numerical results

3.1 Decays \( K_L^0 \to l^+_i l^-_j \)

The the branching ratio of the decays \( K_L^0 \to l^+_i l^-_j \) with \( i,j=1,2 \), \( l^{\pm}_i = e^\pm, \mu^\pm \) induced by the vector leptoquarks with neglect of electron and muon masses \( (m_e, m_\mu << R^{V}_{K} \bar{m}_{K^0}) \) in the case of the general fermion mixing can be written as [7]

\[
BR(K_{L}^{0} \to l_{i}^{+} l_{j}^{-}) = \frac{m_{K_{L}} \pi \alpha^{2} \bar{m}_{K_{L}}^{2} \bar{m}_{K^{0}}^{2} (R^{V}_{K_{L}})^{2}}{4m_{V}^{4} \Gamma_{K}^{total}} |k_{ij}^{(K_{L}^{0})}|^{2},
\]

where \( \bar{m}_{K^{0}} = m_{K^{0}}/(m_{s} + m_{d}) \), \( m_{s} \) and \( m_{d} \) - masses \( s \) and \( d \) quarks, \( m_{K^{0}} \) - mass \( K_{L}^{0} \)-meson, \( m_{V} \) - mass vector leptoquark, \( \Gamma_{K}^{total} \) - total width decay \( K_{L}^{0} \)-meson, \( f_{K^{0}} = 0.16 \) GeV - form factor \( K_{L}^{0} \)-meson, factor \( R^{V}_{K_{L}} = R_{K^{0}}(\mu_{K^{0}}, M_{L}) \) incorporate gluonic corrections, mixing factors incorporate fermionic mixing in general form

\[
|k_{ij}^{(K_{L}^{0})}| = \sqrt{|\nu_{ij}^{LR}|^{2} + |\nu_{ij}^{LR}|^{2}},
\]

\[
\nu_{ij}^{LR} = (K_{2}^{LR})_{2i} (K_{2}^{LR})_{1j}^{*} + (K_{2}^{LR})_{1i} (K_{2}^{LR})_{2j}^{*}.
\]

Matrices additional fermion mixing \( K_{2}^{LR} \) can be parametrize as Cabibbo-Kobayashi-Maskawa matrix using three angles \( \Theta_{12}, \Theta_{13}, \Theta_{23} \) and a phase \( \delta \) for each matrix

\[
K_{2}^{LR} = \begin{pmatrix}
K_{LR}^{LR} & K_{LR}^{LR} & K_{LR}^{LR} \\
K_{LR}^{LR} & K_{LR}^{LR} & K_{LR}^{LR} \\
K_{LR}^{LR} & K_{LR}^{LR} & K_{LR}^{LR}
\end{pmatrix}
\]

\[
\begin{pmatrix}
C_{13} & C_{13} & C_{13} \\
C_{13} & C_{13} & C_{13} \\
C_{13} & C_{13} & C_{13}
\end{pmatrix}
\]

\[
\begin{pmatrix}
K_{LR}^{LR} & K_{LR}^{LR} & K_{LR}^{LR} \\
K_{LR}^{LR} & K_{LR}^{LR} & K_{LR}^{LR} \\
K_{LR}^{LR} & K_{LR}^{LR} & K_{LR}^{LR}
\end{pmatrix}
\]
where \( s_{ij}^{LR} = \sin \Theta_{ij}^{LR} \), \( c_{ij}^{LR} = \cos \Theta_{ij}^{LR} \).

For numerical calculations we used current experimental data [10]

\[
\begin{align*}
BR(K_L^0 \to e^+\mu^-) &< 4.7 \times 10^{-12}, \\
BR(K_L^0 \to \mu^+\mu^-) &= (6.8 \pm 0.11) \times 10^{-9}, \\
BR(K_L^0 \to e^+e^-) &= (9.6) \times 10^{-12}.
\end{align*}
\]

Use this data can represent the lower limit mass of the vector leptoquark as function of the angles \( \Theta_{ij}^{LR} \) and phases \( \delta_{ij}^{LR} \). Choosing these parameters we can obtain the minimal constraints on the masses of leptoquarks. So numerical calculations give the ranges of parameters where the contributions of the decays \( K_L^0 \to l^+_i l^-_j \) is zero. In particular, the choice of parameters in next values:

\[
\Theta_{12}^L = \Theta_{13}^L = \Theta_{23}^L = \Theta_{12}^R = \Theta_{13}^R = \Theta_{23}^R = \pi/2, \quad \delta^L = \delta^R = 0
\]

with correspondent matrices

\[
K^L_2 = K^R_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{pmatrix},
\]

obtain \( l_{21}^{(K)} = l_{11}^{(K)} = l_{22}^{(K)} = 0 \), and experimental limits performed for all values \( m_V \).

As shown in the case of fermion mixing into account only decays \( K_L^0 \to l^+_i l^-_j \) yields no constraints on the masses of vector leptoquarks.

### 3.2 Decays \( B^0 \to l^+_i l^-_j \) and \( B_s \to l^+_i l^-_j \)

The branching ratio decays \( B^0 \to l^+_i l^-_j \) can be represented [7] in the form

\[
BR(B^0 \to l^+_i l^-_j) = \frac{m_{B^0}^2 \pi \alpha_s^2 f_{B^0}^2 \sqrt{m_{B^0}^2 (R_{B^0})^2}}{4m_B^4 \Gamma_B} |k_i^{(B^0)}|^2,
\]

where \( \tilde{m}_{B^0} = m_{B^0}^2/(m_b + m_d) \), \( m_b, m_d \) – masses \( b \) and \( d \) quarks, \( f_{B^0} = 0.229 \) GeV – form factor \( B^0 \)-meson, factor \( R_{B^0} = R_{B^0}(\mu_{B^0}, M_c) \) incorporate gluonic corrections, \( \tilde{\mu}_{ij} = m_i^2/(\tilde{m}_{B^0} R_{B^0}^2) \).

\[
|k_i^{(B^0)}|^2 = \left( |\langle \phi_i^{L}\rangle|^2 + |\langle \phi_i^{R}\rangle|^2 \right)(1 - \mu_i^2 - \mu_j^2) + 2(\beta_{ij}^{LR} + \beta_{ij}^{LR} \beta_i^{LR} \beta_j^{LR}) |\mu_i \mu_j|
\]

\[
\beta_{ij}^{LR} = k_{ij}^{LR} - (\tilde{\mu}_{ij} k_{ij}^{LR} + \tilde{\mu}_{ji} k_{ji}^{LR})/2
\]

\[
k_{ij}^{LR} = (K_{12}^{LR})_{3i}(K_{23}^{LR})_{1j}, \quad k_{ij}^{LR} = (K_{23}^{LR})_{3i}(K_{12}^{LR})_{1j}
\]

For numerical calculations, we used current experimental data [10–12]

\[
\begin{align*}
BR(B^0 \to e^+\mu^-) &< 2.8 \times 10^{-9}, \\
BR(B^0 \to e^+\tau^-) &< 2.8 \times 10^{-5}, (*), \\
BR(B^0 \to \mu^+\tau^-) &< 2.2 \times 10^{-5}, (*), \\
BR(B^0 \to \mu^+\mu^-) &< 3.4 \times 10^{-10}, \\
BR(B^0 \to e^+e^-) &< 8.3 \times 10^{-8}, \\
BR(B^0 \to \tau^+\tau^-) &< 2.1 \times 10^{-3}.
\end{align*}
\]
The joint fitting parameters of the decays of $K^0_L$ and $B^0$ mesons gives some ranges of parameters where the lower limit of the mass vector leptoquark is minimal.

For example if set parameters as
\[
\Theta^L_{12} = \Theta^R_{12} = \pi/4, \quad \Theta^L_{13} = \Theta^R_{13} = \pi/2, \quad \Theta^L_{23} = \Theta^R_{23} = 0, \quad \delta^L = \delta^R = \pi/2,
\]
then
\[
K^L_2 = K^R_2 = \begin{pmatrix} 0 & 0 & -i \\ \frac{-i}{\sqrt{2}} & \frac{i}{\sqrt{2}} & 0 \\ \frac{i}{\sqrt{2}} & \frac{-i}{\sqrt{2}} & 0 \end{pmatrix}
\]
in this case $k^{(K_0)}_{ij} = 0$ and contribution $K^0_L$–meson to lower limit mass vector leptoquark negligible, but contribution $B^0$ meson are considerable (contribution give $B^0 \rightarrow e^\pm \tau^\mp$ and $B^0 \rightarrow \mu^\pm \tau^\mp$ decays)
\[
k^{(B^0)}_{ij} = \begin{pmatrix} 0 & 0 & 0.61 \\ 0 & 0 & 0.62 \\ 0 & 0 & 0 \end{pmatrix}.
\]

The restriction on the lower limit mass vector leptoquarks obtained from decays $K^0_L$ and $B^0$ mesons together are 10 TeV.

Include consideration of experimental data on the decays of $B_s$ mesons [10–12]
\[
\begin{align*}
BR(B_s \rightarrow e^+ \mu^-) &< 1.1 \times 10^{-8} (*) \\
BR(B_s \rightarrow \mu^+ \mu^-) &= (3.0^{+0.67}_{-0.63}) \times 10^{-10} (*) \\
BR(B_s \rightarrow e^+ e^-) &< 8.3 \times 10^{-8} \\
BR(B_s \rightarrow \tau^+ \tau^-) &< 6.8 \times 10^{-3}
\end{align*}
\]
increases constraints on the masses vector leptoquarks.

For example if set parameters as
\[
\Theta^L_{12} = \Theta^R_{12} = 9\pi/50, \quad \Theta^L_{13} = \Theta^R_{13} = \pi/2, \quad \Theta^L_{23} = \Theta^R_{23} = 0, \quad \delta^L = \delta^R = 0,
\]
then
\[
K^L_2 = K^R_2 = \begin{pmatrix} 0 & 0 & 1 \\ -0.54 & 0.84 & 0 \\ -0.84 & -0.54 & 0 \end{pmatrix}.
\]

In this case
\[
k^{(K^0)}_{ij} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}
\]
and contribution $K^0_L$ negligible,
\[
k^{(B^0)}_{ij} = \begin{pmatrix} 0 & 0 & 0.87 \\ 0 & 0 & 0.35 \\ 0 & 0 & 0 \end{pmatrix}
\]
and contribution $B^0$ small,
\[
k^{(B_s)}_{ij} = \begin{pmatrix} 0.41 & 0.99 & 0 \\ 0.16 & 0.4 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]
contribution $B_s$ from $B_s \to e^\pm \mu^\mp$ and $B_s \to \mu^+ \mu^-$ decays significant.

The restriction on the mass vector leptoquark obtained from decays $K_L^0$, $B^0$ and $B_s$ mesons together are 85 TeV.

4 Conclusion

Accounting fermion mixing in leptonic decays of $K_L^0$, $B^0$ and $B_s$ mesons leads to a significant weakening of restrictions on the lower limit mass vector leptoquark. The current limit is about 85 TeV, it is essentially weaker, than restriction without taking into account fermion mixing. This result is in good agreement with the result of prof. A.D. Smirnov presented in this seminar. The main result is to avoid strong constraints from decay $K_L^0 \to e^\pm \mu^\mp$ on the masses of vector leptoquarks. Also the result does not contradict the experimental constraints on the vector leptoquark type of Pati-Salam from $B_s \to e^\pm \mu^\mp$ and $B^0 \to e^\pm \mu^\mp$ decays [13].

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