Supersymmetric loop corrections induce potentially large CP-violating couplings of the Higgs bosons to nucleons and electrons that do not vanish in the limit of heavy superpartners. The Higgs-mediated CP-odd four-fermion operators are enhanced by \( \tan^2 \beta \) and induce electric dipole moments of heavy atoms which exceed the current experimental bounds for the electroweak scale Higgs masses and \( \tan \beta \gtrsim 10 \). If only the first two fermion generations are heavy, the Higgs-mediated contributions typically dominate over the Barr-Zee type two-loop diagrams at \( \tan \beta > 30 \).

Non-observation of the electric dipole moments (EDMs) of the neutrons\(^1\) and heavy atoms\(^2\) poses a serious problem for low-energy supersymmetry. This is because generically the EDMs are induced already at the one loop level and their predicted values exceed the experimental limits by orders of magnitude (see\(^3\) for recent analyses). The most straightforward way to circumvent this problem is to assume that the superpartner mass scale is rather high (over a few TeV) which leads to the suppression of all effective one-loop-generated CP-odd operators of dimension five and higher, and seemingly allows for arbitrary CP-violating phases. The arising Higgs mass fine-tuning problem can be alleviated if the third generation squarks are near the electroweak scale.

In this work, we consider sources of the EDMs which survive the decoupling of the superpartners while the Higgs masses are kept fixed. In this limit, the Minimal Supersymmetric Standard Model (MSSM) degenerates into a two-Higgs doublet model (2HDM) with an important one-loop-induced modification of the Yukawa sector compared to the usual type II models\(^4\). We show that the Higgs-mediated four-fermion operators induce potentially large electric dipole moments of heavy atoms that grow as \( \tan^2 \beta \), and that supersymmetric models with large \( \tan \beta \) face the SUSY CP problem even in the limit of heavy SUSY particles. We further compare the Higgs-mediated contributions with the two-loop induced contributions\(^5\) which are significant if the masses of the third generation squarks are near the electroweak scale.

At the tree level, the down type quarks and charged leptons obtain their masses from the interaction with the first Higgs doublet \( H_1 \). The finite one loop SUSY corrections induce considerable couplings of the second Higgs doublet \( H_2 \) to the \( D \)-quarks and charged leptons, that are absent in the limit of unbroken supersymmetry:

\[
- \mathcal{L}_Y = Y_D^{(0)} H_1 \bar{D}_L D_R + Y_D H_1^\dagger \bar{D}_L D_R \\
+ Y_E^{(0)} H_1 \bar{E}_L E_R + Y_E H_1^\dagger \bar{E}_L E_R + \text{h.c.}
\]

(1)

where \( Y_{D,E}^{(0)} \) are the tree level Yukawa couplings and \( Y_{D,E} \) are the loop induced couplings. The typical representatives of important threshold diagrams are given in Figs.1a and 1b. Ignoring flavor changing effects, we relate \( Y_{D,E}^{(0)} \) and \( Y_{D,E} \) by:

\[
Y_D = J_D Y_D^{(0)} , \quad Y_E = J_E Y_E^{(0)} .
\]

(2)

The loop functions \( J_D \) and \( J_E \) are complex and contain the dependence on the CP phases in the soft-breaking sector, which leads to the CP-odd interactions of the physical Higgses with the \( D \)-quarks and leptons. When one redefines the phase of the right-handed \( D \) and \( E \) fields, \( D_R \rightarrow e^{-i\delta_{D,E}} D_R \) and \( E_R \rightarrow e^{-i\delta_{D,E}} E_R \) such that the mass term in Eq.(1) becomes real, the induced CP-odd interactions in the Higgs interaction gets enhanced by \( \tan \beta = v_2/v_1 : \delta_{D,E} = \text{Arg}(1 + J_{D,E} \tan \beta) \). Thus, large \( \tan \beta \) can compensate the loop smallness of \( J_{D,E} \) so that the phases \( \delta_{D,E} \) can be order one. An exchange by physical Higgses will then produce CP-odd four-fermion interactions, Fig.2a. The relevant interactions are induced by the exchange of a CP-odd Higgs boson \( A \) and CP-even Higgs boson \( H \) between the \( D \)-quarks and the electron, and between the \( D \)-quarks:

\[
\mathcal{L}_f \simeq \frac{\tan^2 \beta}{2m_A} \sum_{i,j=e,d,s,b} Y_{SM}^{J_f} \sqrt{v} \left( \frac{\sin \delta_i - \sin \delta_j}{1 + J_i \tan \beta} \right) [1 + J_j \tan \beta] \times \bar{\psi}_i \psi_i \bar{\psi}_j \gamma_5 \psi_j .
\]

(3)

Here \( Y_{SM}^{J_f} \) denote the Standard Model values for the Yukawa couplings, \( Y_{SM}^{J_f} \equiv \sqrt{2} m_f / v \) (\( v = 246 \text{ GeV} \)), and the CP-phases are understood modulo \( \pi \). In the derivation of (3), we have used the relations \( m_A \simeq m_H \), \( \cos^2 \alpha \simeq 1 \), where \( \alpha \) is the neutral Higgs mixing angle, and dropped all \( \tan \beta \)-suppressed terms. This approximation works well even for moderately large \( \tan \beta \).

![FIG. 1. SUSY threshold corrections in the down quark sector, (a) and (b), and in the Higgs sector, (c).](image)

CP-odd contact interactions can also be induced via \( A - H \) mixing, Fig.2b, which appears due to CP-violating Higgs couplings to the third generation squarks\(^6\) (Fig.1c):
\[ \mathcal{L}_{AH} \simeq \frac{(AH) \tan^2 \beta}{2m_A^4} \sum_{i,j=e,d,s,b} Y_{i}^{SM} Y_{j}^{SM} \bar{\psi}_i \psi_j \frac{1}{|1 + J_i \tan \beta||1 + J_j \tan \beta|}. \]  

Here we have used \( m_H^2 \simeq m_A^2 \gg (AH) \). Such effects were studied previously in the context of 2HDMs with spontaneous breaking of CP by Barr \[10\].

Inspection of Eq. (3) reveals that the CP-odd coupling grows as \( \tan^3 \beta \) because \( \sin \delta_i \simeq \text{Im} J_i \tan \beta / |1 + J_i \tan \beta| \), until the radiative corrections become comparable to the tree-level values. Here we treat \( m_A \) as an independent variable (it is proportional to the SUSY \( B_H \) parameter). The cubic growth is different from a \( \tan^2 \beta \)-behavior in 2HDMs with spontaneous CP violation \[11\]. Thus, generally Eq. (4) represents a subleading effect, as \( (AH) \) contains a loop smallness not compensated by large \( \tan \beta \).

We note that the QCD renormalization group flow for the electron–quark interactions from \( m_A \) to 1 GeV is trivial at one loop and one can simply take \( Y_i^{SM} \) normalized at 1 GeV.

![Diagram](image)

FIG. 2. Higgs-mediated four-fermion interactions with CP violation in the Higgs-fermion vertex (a) and on the Higgs line (b).

Using Eq. (3), we calculate the EDMs of paramagnetic atoms and estimate the EDMs of diamagnetic atoms and neutrons. The semi-leptonic operators in (3) induce two types of \( T \)-odd nucleon-electron interaction

\[ \mathcal{L}_{CP} = C_S \bar{N}N \bar{\psi}\gamma_5 e + C_P \bar{N}i\gamma_5 N \bar{e} \]  

with possible isospin dependence. \( C_S \) and \( C_P \) are severely constrained by the recently improved experimental bounds on the EDM of the thallium and mercury atoms \[12\]. With the use of the standard technique for the QCD matrix elements \[13\] of a heavy quark over a nucleon state, the isospin-singlet coupling \( C_S \) can be expressed as

\[ C_S \simeq \frac{5.5 \times 10^{-10} \tan^2 \beta}{m_A^4} \left( \frac{1 - 0.25\kappa}{|1 + J_a \tan \beta|} \left[ \frac{(1 - 0.25\kappa)\sin \delta_i - \sin \delta_e}{|1 + J_a \tan \beta|} + \frac{3.2\kappa\sin \delta_i - \sin \delta_e}{|1 + J_a \tan \beta|} + \frac{0.5\sin \delta_i - \sin \delta_e}{|1 + J_a \tan \beta|} \right] \right). \]  

Here we have used \( (m_u + m_d)(\langle N|\bar{\tau}u + \bar{d}d|N\rangle/2 = 45 \text{ MeV} \) and \( (m_u - m_d)(\langle N|\bar{\tau}d - \bar{u}d|N\rangle/90 \text{ MeV} \ll 1 \). The coefficient \( \kappa \equiv \langle N|m_3s\bar{s}|N\rangle/220 \text{ MeV} \) parametrizes the uncertainty in the value of \( \langle N|m_3s\bar{s}|N\rangle \) matrix element. Its “best” value \( \kappa = 1 \) is inferred from the leading order flavour \( SU(3) \) analysis of the baryon octet mass splittings. An assumption of the strange quark behaves as a heavy quark would lead to a smaller value, \( \kappa = 0.3 \). It is important to note that a significant source of uncertainty – the poorly known masses of the light quarks – does not affect Eq. (6).

Using (6), and the results of the atomic calculation that relates \( d_{Tl} \) and \( C_S \)[2],

\[ d_{Tl} \simeq -8.5 \times 10^{-17} \text{ cm} \times C_S(100 \text{ GeV})^2, \]  

one can express the thallium EDM in terms of the SUSY parameters. Comparison of (7) with the experimental data provides the bound \( C_S < 1.1 \times 10^{-8} (100 \text{ GeV})^{-2} \).

The dimensionless loop functions \( J_i \)'s depend on the pattern of the soft masses. To get an idea of the size of the induced EDMs, let us first consider a toy model with \( m_{\text{fermion}} = m_{\text{gaugino}} = |\mu| = |A_i| = M \gg M_Z \). The dominant contribution comes from the squark-gluino and stop-Higgsino exchange:

\[ J_e = 0; \quad J_d = J_s = \frac{\alpha_s}{3\pi} \exp\{i\phi_{\mu} + i\phi_3\}; \]  

\[ J_b = \frac{\alpha_s}{3\pi} \exp\{i\phi_{\mu} + i\phi_3\} + \frac{(Y_i^{SM})^2}{32\pi^2} \exp\{i\phi_{\mu} + i\phi_{A_i}\}, \]  

where \( \phi_{\mu}, \phi_3, \phi_{A_i} \) are the phases of the \( \mu \)-parameter, the gluino mass, and the \( A_i \) parameter, respectively. In the case of general soft terms, the gluino contribution to \( J_i \) should be multiplied by \( |\mu|M_3^2/(m_{\text{th}}^2, m_{\text{th}}^2, |\mu|^2) \) \[14\] , where \( m_{\text{th}}, 1, 2 \) are the squark mass eigenvalues, \( M_3 \) is the gluino mass, and the loop function is defined by

\[ I(a, b, c) = \frac{2ab \ln(a/b) + bc \ln(b/c) + ac \ln(c/a)}{(a - b)(b - c)(c - a)} \]  

such that \( I = 1/M^2 \) for \( a = b = c = M^2 \). In the same limit, the CP-odd Higgs mixing is given by

\[ \langle AH \rangle = \frac{\alpha_s^2}{64\pi^2} \left\{ (Y_i^{SM})^4 \sin(2\phi_{\mu} + 2\phi_{A_i}) + (Y_b^{SM})^4 \tan^4 \beta \sin(2\phi_{\mu} + 2\phi_{A_b}) \right\}. \]  

Obviously, both \( J_i \)'s and \( \langle AH \rangle \) are independent of the superpartner mass scale \( M \). An expression for \( \langle AH \rangle \) in a more general case can be found in \[15\]. As we will see, the effect of the \( A - H \) mixing does not impose significant constraints for \( m_A \geq 150 \text{ GeV} \), so henceforth we will mainly concentrate on the effect of the vertex corrections.

The Higgs–quark vertex corrections lead to the following thallium EDM normalized to the current 90\% C.L. experimental bound \( d_{Tl}^{\text{exp}} \equiv 9.4 \times 10^{-25} \text{ cm} \)[2]:

\[ \frac{d_{Tl}}{d_{Tl}^{\text{exp}}} \simeq \frac{\tan^3 \beta}{350 \times m_{100}^2} \sin \phi_{\mu} + 0.04 \sin(\phi_{\mu} + \phi_{A_i}) \]  

where we have set \( \phi_3 = 0 \), \( |1 + J_i \tan \beta| \simeq 1 \), and \( \kappa = 1 \). \( m_{100} \) is \( m_A \) measured in the units of 100 GeV. Already at \( \tan \beta \simeq 7 \) the r.h.s. of (11) may reach 1 (while the standard EDM contributions are suppressed for the SUSY masses of 10 TeV).
We conclude that even for arbitrarily heavy superpartners, the SUSY CP problem reappears if \( \tan \beta (m_A/100 \text{ GeV})^{-2/3} \gtrsim 10 \). For instance, with \( \phi_\mu \sim 1 \), \( m_A \sim 100 \text{ GeV} \), and \( \tan \beta \sim 60 \) the induced EDM exceeds the experimental bound by almost three orders of magnitude! It is important to note that these calculations are free of large nuclear uncertainties [13].

The CP-odd constant \( C_\gamma \) also induces EDMs of diamagnetic atoms through the mixing with the hyperfine interaction [13,14]. Our prediction for the mercury EDM is

\[
\frac{d_{\text{Hg}}(C_\gamma)}{d_{\text{Hg,exp}}} \simeq \frac{\tan^3 \beta}{900 \times m^2_{100}} \left[ \sin \phi_\mu + 0.04 \sin(\phi_\mu + \phi_{A_t}) \right],
\]

where the current experimental bound is \( |d_{\text{Hg,exp}}| \equiv 2 \times 10^{-28} \text{ e cm} \). This imposes a slightly weaker bound than the thallium EDM. However, in the case of diamagnetic atoms there are two additional classes of contributions, induced by \( C_\gamma \) and the nuclear Schiff moment. To evaluate \( C_\gamma \), we follow the strategy of Ref. [14]. Unlike the previous case, there is a strong dependence of the result on \( m_{u}/m_{d} \), and within the error bars for this ratio the matrix elements of \( \bar{s}t\gamma_5 s \) and \( \bar{b}r\gamma_5 \) b over the neutron are compatible with zero. The d-quark contribution gives

\[
C_\gamma(n) \simeq 6.3 \times 10^{-9} \frac{\tan^2 \beta (\sin \delta_d - \sin \delta_s)}{m^2_A} \left| \frac{1 + J_e \tan \beta}{1 + J_d \tan \beta} \right|,
\]

where we have used \( m_u/m_d = 0.55 \). Using the results of the atomic calculation [12], we convert this into the bound

\[
\frac{d_{\text{Hg}}(C_\gamma)}{d_{\text{Hg,exp}}} \simeq \frac{\tan^3 \beta}{3500 \times m^2_{100}} \sin \phi_\mu.
\]

This is clearly a subleading contribution, compared to [13]. Finally, the \( \bar{t}r\gamma_5 d \bar{t}d \) interactions in [3] induce \( d_{\text{Hg}} \) via the nuclear Schiff moment. It is known that a T-odd one pion exchange between nucleons is the dominant source of the Schiff moment. We estimate the T-odd pion-nucleon coupling \( \tilde{g}_{\pi N N} \) following the approach of [13]. Since \( \sin \delta_s = \sin \delta_d \) in our case, there are cancellations in the sum [3] and the dominant contribution comes from the operator \( \bar{d}r\gamma_5 d \bar{t}d \). The resulting isospin-triplet coupling \( \tilde{g}_{\pi N N} \bar{\pi}^0 N N \) is

\[
\tilde{g}_{\pi N N} \sim 2.7 \times 10^{-13} \frac{\tan^2 \beta (\sin \delta_d - \sin \delta_b)(1 - 0.25\kappa)}{m^2_{100}} \left| \frac{1 + J_b \tan \beta}{1 + J_d \tan \beta} \right|.
\]

Skipping a long chain of nuclear and atomic matrix elements that relate \( \tilde{g}_{\pi N N} \) and \( d_{\text{Hg}} \) (see Ref. [12] for details), we get:

\[
\frac{d_{\text{Hg}}(\text{Schiff})}{d_{\text{Hg,exp}}} \simeq \frac{\tan^3 \beta}{1.1 \times 10^2 m^2_{100}} \sin(\phi_\mu + \phi_{A_t}).
\]

Remarkably, Eq. (15) has the same sensitivity to \( \phi_\mu + \phi_{A_t} \) as Eq. (11), yet this calculation involves considerable uncertainties. A combination of \( d_{Tl} \) and \( d_{Hg} \) constrains both phases, \( \phi_\mu \) and \( \phi_{A_t} \), once again exemplifying the complementarity of the two measurements [13].

Let us now consider a popular scenario where only the first two sfermion generations are assumed to be heavy (> 10 TeV). We further assume the most conservative case when the gluinos are also heavy such that the leading term in Eq. (11) disappears. Then we have

\[
J_d = J_s = J_0 = 0; \quad J_b \simeq \frac{(Y_{SM})^2}{32\pi^2} A_\mu I (m_t^2, m_b^2, |\mu|^2).
\]

We note, however, that in order to suppress the contribution of the gluino exchange diagram in \( J_b \) one would have to require \( M_3 \gtrsim 60 \mu \).

Fig. 3 shows a \( \tan \beta \) dependence of \( C_S / |C_S|_{\text{exp}} = d_{Tl} / |d_{Tl}|_{\text{exp}} \) in this model. We use the parameters of Ref. 8, namely \( |A_{t,b}| = |\mu| = 1 \text{ TeV}, \, \text{Arg}(A_{t,b,\mu}) = \pi/2 \), and \( m_{sq} = 0.6 \text{ TeV} \) (assuming that the left and right stop and sbottom mass parameters are given by \( m_{sq} \), while other squarks are decoupled), and treat the Higgs masses as independent parameters. The three curves v1,v2,v3 correspond to \( d_{Tl} \) induced by the Higgs vertex corrections with \( m_A = 100, 200, 300 \text{ GeV} \), respectively, while the curve m1 corresponds to \( d_{Tl} \) induced by the \( A-H \) mixing with \( m_A = 100 \text{ GeV} \) (for \( m_A \geq 150 \text{ GeV} \) this effect does not impose any considerable constraints). At \( \tan \beta \gtrsim 15 \) the generated EDM is comparable to the experimental limit and at \( \tan \beta \sim 60 \) it exceeds the experimental limit by up to two orders of magnitude. We observe that for this choice of the parameters, the sensitivity to \( \phi_\mu + \phi_{A_t} \) is better than that in Eq. (11) by a factor of a few. The sensitivity to \( \phi_\mu + \phi_{A_t} \) will be as good or better if \( M_3 \lesssim 60 \mu \). We remark that the considered parameter space is constrained by the observed \( B \rightarrow s \gamma \) branching ratio. When the effect of the SUSY threshold corrections is taken into account, the constraints become rather weak [17].

We further compare the Higgs-mediated EDMs with the two-loop effects considered in Ref. 8. The strongest constraint was obtained from the electron or, equivalently, the thallium atom EDM (the neutron EDM imposes weaker constraints, at least in the naive quark model). At low \( \tan \beta \) the two-loop diagrams are more important than the ones considered here, while at \( \tan \beta \gtrsim 35 \) the Higgs induced four–fermion operators provide stronger constraints (see Fig. 2 of Ref. 8 subject to the factor of two correction). This statement, of course, depends on \( m_A \) and the soft masses. Concerning the \( m_A \) dependence, the EDM due to the Higgs vertex scales as \( 1/m^2_A \) and the EDM due to the \( A-H \) mixing falls off faster than \( 1/m^4_A \). The Barr–Zee contribution scales down roughly linearly with \( m_A \) and therefore is more significant at heavier \( m_A \). On the other hand, the Higgs-mediated EDMs become dominant in the case of heavier superpartners.

In Fig. 4 we present the behavior of different EDM contributions in this “decoupling” limit, which leads us basically to the pattern of SUSY breaking considered before.
In order to observe a cross-over from the two-loop contributions to the Higgs mediation in $\delta T_1$, we fix $\tan \beta = 40$ and scale the SUSY mass parameters by the common factor $X$: $|A_{t,b}| = |\mu| = 1X\text{ TeV}$, $\text{Arg}(A_{t,b}\mu) = \pi/2$, and $m_{sq} = 0.6X\text{ TeV}$.

While the Barr–Zee contributions decouple rather quickly, the effects we consider stay constant in the limit of heavy superpartners. Finally, we note that there are the top and bottom quarks in the loop. In SUSY models, these are 3-loop effects and do not lead to significant constraints (see [18] for the analysis in general 2HDMs).

To summarize, we have considered the EDMs of heavy atoms induced by the Higgs exchange in SUSY models with CP violation in the supersymmetric sector. The mechanism of Higgs-mediation is insensitive to an overall scale of the superpartner masses and grows as $\tan^3 \beta$ for fixed values of $m_{A,H}$. It provides significant and free of large hadronic and nuclear uncertainties constraints on the SUSY CP-phases for $\tan \beta \gtrsim 10$ and electroweak scale $m_{A,H}$.

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[1] P.G. Harris et al., Phys. Rev. Lett. 82 (1999) 904.
[2] B. C. Regan et al., Phys. Rev. Lett. 88 (2002) 071805.
[3] M. V. Romalis, W. C. Griffith and E. N. Fortson, Phys. Rev. Lett. 86 (2001) 2505.
[4] M. A. Rosenberry and T. E. Chupp, Phys. Rev. Lett. 86 (2001) 22; D. Cho, K. Sangster, E.A. Hinds, Phys. Rev. Lett. 63 (1989) 2559.
[5] S. Abel, S. Khalil and O. Lebedev, Nucl. Phys. B 606 (2001) 151; V. D. Barger et al., Phys. Rev. D 64 (2001) 056007.
[6] R. Hempfling, Phys. Rev. D 49, 6168 (1994); L. J. Hall, R. Rattazzi and U. Sarid, Phys. Rev. D 50, 7048 (1994); C. Hamzao˘glu, M. Pospelov and M. Toharia, Phys. Rev. D 59 (1999) 095005; K. S. Babu and C. F. Kolda, Phys. Rev. Lett. 84, 228 (2000).
[7] S. Weinberg, Phys. Rev. Lett. 63, 2333 (1989); S. M. Barr and A. Zee, Phys. Rev. Lett. 65, 21 (1990) [E-ibid. 65, 2920 (1990)].
[8] D. Chang, W. Y. Keung and A. Pilaftsis, Phys. Rev. Lett. 82, 900 (1999) [E-ibid. 83, 3972 (1999)].
[9] A. Pilaftsis, Phys. Lett. B 435, 88 (1998); A. Pilaftsis and C. E. Wagner, Nucl. Phys. B 553, 3 (1999); D. A. Demir, Phys. Rev. D 60, 055006 (1999).
[10] S. M. Barr, Phys. Rev. Lett. 68, 1822 (1992); Phys. Rev. D 47, 2025 (1993).
[11] M. A. Shifman, A. I. Vainshtein and V. I. Zakharov, Phys. Lett. B 78 (1978) 443.
[12] I.B. Khriplovich and S.K. Lamoreaux, “CP Violation Without Strangeness”, Springer, 1997.
[13] M.G. Kozlov, Phys. Lett. A 130 (1988) 426.
[14] A. A. Anselm et al., Phys. Lett. B 152 (1985) 116.
[15] V.M. Khatsimovsky, I.B. Khriplovich, A.S. Yelkhovsky, Annals Phys. 186 (1988) 1; M. Pospelov, Phys. Lett. B 530 (2002) 123.
[16] T. Falk, K.A. Olive, M. Pospelov and R. Roiban, Nucl. Phys. B 560 (1999) 3.
[17] G. Degrassi, P. Gambino and G. F. Giudice, JHEP 0012, 009 (2000).
[18] T. Hayashi et al., Phys. Lett. B 348, 489 (1995).