Study on time-varying velocity measurement with self-mixing laser diode based on Discrete Chirp-Fourier Transform

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ABSTRACT

Laser’s optical output power and frequency are modulated when the optical beam is back-scattered into the active cavity of the laser. By signal processing, the Doppler frequency can be acquired, and the target’s velocity can be calculated. Based on these properties, an interferometry velocity sensor can be designed. When target move in time-varying velocity mode, it is difficult to extract the target’s velocity. Time-varying velocity measurement by self-mixing laser diode is explored. A mathematics model was proposed for the time-varying velocity (invariable acceleration) measurement by self-mixing laser diode. Based on this model, a Discrete Chirp-Fourier Transform (DCFT) method was applied. DCFT is analogous to DFT. We show that when the signal length N is prime, the magnitudes of all the side lobes are 1, whereas the magnitudes of the main lobe is \( N \). And the coordinates of the main lobe shows the target’s velocity and acceleration information. The simulation results prove the validity of the algorithm even in the situation of low SNR when N is prime.

Keywords: Velocity measurement, interferometry, laser diode, Self-mixing

1. INTRODUCTION

Laser Diode (LD) Self-Mixing (SM) interferometry is an emerging precision photoelectric measuring technology. When a small fraction of the light emitted by LD is backscattered or reflected by an external target and re-enter into the laser active cavity, the laser’s output power and frequency are modulated. Because LD output signal’s property is similar to the property of conventional two-beam interferometers, so it is called “Self-Mixing”. The self-mixing interference measurement system has the merits of simple architecture, naturally self-aligned optical characteristics, compact size and low cost. So, it can replace conventional interferometers in many situations for physical measurements, such as displacement [1-3], distance [4], velocity [5], and vibration [6].

In 1970s and 1980s, LDSM had been already used to measure velocity. For example, Shigenobu Shinohara reported a laser Doppler velocimeter using self-mixing in 1989 [7], which can discern the movement direction of the target, and the measurement range is 23 mm/s - 23 m/s. The measured results for a reciprocally moving target as well as a rotating target agrees well with the theoretical value. But all LDSM velocimeter documents we retrieved are aim at low-speed and invariable velocity motion target, variable velocity measurement is not covered. In many practical applications, the movement of objects is uniform, and therefore needs to solve the problems of variable velocity measurement by self-mixing interference, such as the signal processing method.

The method of FFT is usually adopted in laser self-mixing velocity measurement technology [8]. But FFT is used to analyze stationary signal. When the target moves with acceleration, the output signal is time-varying. If the method of FFT is still used to analyze the output signal, some problems will appear as follows: 1) lose the output SNR, leading to detection performance becoming bad, 2) the inherent resolution of Doppler frequency detection is down [9]. Therefore, this paper puts forward a method based on the Discrete Chirp-Fourier Transform (DCFT) to analyze laser self-mixing variable velocity measurement.

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2. MATHEMATICS MODEL OF SELF-MIXING VARIABLE VELOCITY MEASUREMENT

Fabry-Perot (F-P) cavity is used to analyze the self-mixing interference because of its simple and intuitionistic. Fig. 1 introduces a model of a solitary semiconductor laser diode in which laser facets and the external reflector form a three-mirror compound cavity. \( r_1 \) and \( r_2 \) form the laser’s inner cavity, \( r_3 \) and the external reflector form the laser’s external cavity. Length of the inner cavity and external cavity are \( L \) and \( L_{\text{ext}} \) respectively. Reflection coefficient of the three mirrors are \( r_1 \), \( r_2 \) and \( r_3 \).

![Fig. 1 Schematic diagram of a three-mirror cavity self-mixing interference](image)

From three-mirror cavity model, the change of laser’s output gain and frequency are \(^{[8]}\)

\[
g_{\text{th}} = g_o - \frac{\xi}{L} \cos \left( 2\pi f \tau_{\text{ext}} \right) \\
f = f_o - \frac{\xi}{2\pi} \sqrt{1+\alpha^2} \sin \left( 2\pi f \tau_{\text{ext}} + \arctan \alpha \right)
\]

Where, \( g_o \) is initial gain, \( g_{\text{th}} \) is threshold gain, \( f = f_o + \Delta f \), \( f_o \) is laser’s initial frequency, \( \Delta f \) is the change of frequency, \( \tau = 2L/c \) is optical round-trip time in inner cavity, \( \tau_{\text{ext}} = 2L_{\text{ext}}/c \) is optical round-trip time in external cavity, \( \xi = r_3 (1-|r_1|^2) \) is the coupling coefficient of the laser coupled from external cavity to inner cavity, \( \alpha \) is the laser’s linewidth enhancement factor.

From Eq. (1), the laser’s threshold gain \( g_{\text{th}} \) is modulated by feedback light. Because the laser’s output power is proportion to the laser’s threshold gain, so the laser’s output power is also modulated by feedback light. The output power of LD can be expressed as

\[
p = p_o \left[ 1 + m \cos \left( 2\pi f \tau_{\text{ext}} \right) \right]
\]

Where, \( m \) is the undulation coefficient.

From Eq. (2), the laser’s output power changes periodically when the length \( L_{\text{ext}} \) of external cavity changes. When the target moves away from laser in velocity \( v \), the angle between target moving direction and light axial is \( \theta \). the external cavity length can be expressed as \( L_{\text{ext}} = L_x + vt \cos \theta \) (when move converse \(-v\)), then \( \tau_{\text{ext}} = 2L_{\text{ext}}/c \), the Eq.(2) can be expressed as

\[
p = p_o \left[ 1 + m \cos \left( 4\pi f \frac{L_{\text{ext}} + vt \cos \theta}{c} \right) \right]
\]

From Eq. (3), the laser output power’s frequency fluctuations can be expressed as
\[ \Delta f = \frac{2v}{\lambda} \cos \theta = f_0 \] (4)

Namely, self-mixing frequency is equal to the target’s Doppler frequency. So, based on the change of laser output power, the change of frequency \( \Delta f \) can be attained, then the target’s velocity can be acquired. This is the principle of LD SM velocity measurement.

From Eq. (3) and Eq. (4), when the target moves in invariable velocity, the target’s Doppler can be extracted by FFT, and then obtain the target’s velocity through Eq. (4). But, when the object moves in variable velocity, this method is not feasible.

If the target moves in variable velocity and the acceleration is \( a \) and invariable, \( V = V_o + at \), Eq. (3) can be expressed as

\[
p = p_o \left[ 1 + m \exp(i2\pi) \left( \frac{2f}{c} \frac{L}{c} + \frac{2fV_o \cos \theta}{c} t \right) \right]
\] (5)

If the sampling frequency is \( f_s \), sampling number is \( N \), then \( t = n/f_s \), Eq. (5) can be expressed as

\[
p = p_o + E \exp \left( \frac{i2\pi}{N} (l_n^2 + k_n) \right) \quad n = 0, \ldots, N - 1
\] (6)

Where

\[
E = p_o m \exp \left( \frac{i4\pi L}{c} \right)
\] (7)

\[
l_o = \frac{2fa \cos \theta N}{cf_r^2}
\] (8)

\[
k_o = \frac{2fV_o \cos \theta N}{cf_r^2}
\] (9)

In order to facilitate subsequent signal processing, Eq. (6) can be expressed as

\[
s(n) = p - p_o = E \exp \left( \frac{i2\pi}{N} (l_n^2 + k_n) \right)
\] (10)

Consider the output signal noise, the output signal is

\[
y(n) = s(n) + \omega(n)
\] (11)

Where \( \omega(n) \) is white noise with Zero mean and variance \( \sigma^2 \). So, the mathematical model of self-mixing variable velocity measurement is established. And then, we will use the signal processing method of Chirp-Fourier transform to get the object velocity and acceleration information.

### 3. SIGNAL PROCESSING OF CHIRP-FOURIER TRANSFORM

For a signal \( y(n) \) with length \( N \), its N-point DCFT is defined as

\[
Y(l,k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} y(n) W_N^{l_n^2 + k_n} \quad 0 \leq l, k \leq N - 1
\] (12)
Where \( W_N = \exp\left(-j \frac{2\pi}{N}\right) \). When \( l = 0 \), the DCFT is transformed to DFT, this indicates that DFT belongs to DCFT.

Let’s consider following lemma.

**Theorem 1:** When \( N \) is a prime, \( 0 \leq l, k \leq N - 1 \), we have the following equation:

\[
\sum_{k=0}^{N-1} W_N^{-l o k} = \begin{cases} 
N & l = 0, k = 0 \\
\sqrt{N} & l \neq 0 \\
0 & l = 0, k \neq 0
\end{cases} \tag{13}
\]

**Theorem 2:** Let \( x(n) = \sum_{k=0}^{N-1} W_N^{l o k} \), for some integers \( l_o \) and \( k_o \) with \( 0 \leq l_o, k_o \leq N - 1 \). If the \( N \) is a prime, its DCFT magnitude has the following form:

\[
|X(k,l)| = \begin{cases} 
\sqrt{N} & l = l_o, k = k_o \\
1 & l \neq l_o \\
0 & l = l_o, k \neq k_o
\end{cases} \tag{14}
\]

Theorem 2 tells us that for the signal \( x(n) \), the peak or the mainlobe of its DCFT appears at \((l_o, k_o)\), and its value is \( \sqrt{N} \).

Based on the theorem 2, for the signal \( y(n) \) in the equation (11), the peak of its DCFT appears at \((l_o, k_o)\), and its value is \( |E|\sqrt{N} \). So for the output signal \( y(n) \), has its DCFT, and the value \( l_o \) and \( k_o \) where its mainlobe appears can be obtained, and then through the equations (8) and (9), the target’s velocity and acceleration can be acquired.

**4. NUMERICAL SIMULATION**

When the object moves in variable speed, \( \nu_o = 1103 m/s \) and \( a = 4.27 \times 10^3 m/s^2 \), if use conventional FFT to analyze the output signal, the result is as figure 2.

![Fig. 2 Simulation result by FFT method](image-url)

From the simulation result, the FFT can not be used to extract target’s movement information when the target moves in variable speed. Let’s use DCFT to analyze the output signal in the following.
Set the simulation parameters: \( N = 127 \), \( v_o = 1103 m/s \), \( a = 4.27 \times 10^{13} m/s^2 \), \( E = 1 \), \( f_c = 3 \times 10^6 \), then \( l_o = 3.9987 \), \( k_o = 30.9877 \). When \( SNR = 0 \) dB, and \( SNR = 7 \) dB, the simulation result is \( l_o = 4 \), and \( k_o = 31 \), just as figure 3. The simulation result tells us that under low SNR conditions, through the DCFT method can get object’s velocity and acceleration information.

![Simulation result by DCFT method](image)

Fig. 3 Simulation result by DCFT method

This method has a big advantage when used to analyze high-speed and high acceleration movement. But this method has some limitations: 1) require the sampling points \( N \) is prime, otherwise the measurement accuracy becomes low, 2) measurement results are only integer solutions, so the measured parameters \( l_o \) and \( k_o \) should be approximate integers, otherwise the measurement error become much bigger. So, in order to make this method is more effective and adapt much broader field, it needs further improvement.

5. CONCLUSION

This paper discusses the problem of laser self-mixing variable velocity measurement firstly. On the basis of self-mixing variable velocity measurement model, the method of Discrete Chirp-Fourier Transform is proposed to analyze self-mixing variable velocity measurement signal. Through the simulation analysis, this method can effectively obtain object’s velocity and acceleration information, and has good noise performance. This paper is a useful exploration for self-mixing variable velocity measurement, promoting its practical process. And the corresponding experimental study will be carried out.

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