Feedback of the electromagnetic environment on current and voltage fluctuations out of equilibrium

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A theory is presented for low-frequency current and voltage correlators of a mesoscopic conductor embedded in a macroscopic electromagnetic environment. This Keldysh field theory evaluated at its saddle-point provides the microscopic justification for our earlier phenomenological calculation (using the cascaded Langevin approach). The nonlinear feedback from the environment mixes correlators of different orders, which explains the unexpected temperature dependence of the third moment of tunneling noise observed in a recent experiment. At non-zero temperature, current and voltage correlators of order three and higher are no longer linearly related. We show that a Hall bar measures voltage correlators in the longitudinal voltage and current correlators in the Hall voltage. We go beyond the saddle-point approximation to consider the environmental Coulomb blockade. We derive that the leading order Coulomb blockade correction to the n-th cumulant of current fluctuations is proportional to the voltage derivative of the (n + 1)-th cumulant, generalizing to any n the earlier results for n = 1, 2.

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I. INTRODUCTION

A mesoscopic conductor is part of a macroscopic electrical circuit that influences its transport properties. This electromagnetic environment is a source of decoherence and plays a central role for single-electron effects [1, 2, 3, 4, 5]. Most studies address time-averaged properties. Time-dependent fluctuations of the electrical current are also affected by the environment, which reduces the low-frequency fluctuations by a feedback loop: A current fluctuation δI induces a counter-acting voltage fluctuation δV = −ZGδI over the conductor, which in turn reduces the current by an amount −GδV. (Here G and Z are, respectively, the conductance and the equivalent series impedance of the mesoscopic system and the equivalent series impedance of the macroscopic voltage-biased circuit.)

At zero temperature the macroscopic circuit does not generate any noise itself, and the feedback loop is the only way it affects the current fluctuations in the mesoscopic conductor, which persist at zero temperature because of the shot noise effect [6, 7, 8]. In the second cumulant C(2), or shot-noise power, the feedback loop may be accounted for by a rescaling of the current fluctuations: δI → (1 + ZG)−1δI. For example, the Poison noise C(2) = eI(1 + ZG)−2 of a tunnel junction is simply reduced by a factor (1 + ZG)−2 due to the negative feedback of the series impedance. We have recently discovered that this textbook result breaks down beyond the second cumulant [1]. Terms appear which depend in a nonlinear way on lower cumulants, and which can not be incorporated by any rescaling with powers of 1 + ZG. In the example of a tunnel junction the third cumulant at zero temperature takes the form

C(3) = eI(1 − 2ZG)(1 + ZG)−4.

Ref. [2] was restricted to zero temperature. In Ref. [10] we removed this restriction and showed that the nonlin-ear feedback of the electromagnetic environment drastically modifies the temperature dependence of C(3). Earlier theory [11, 12, 13] assumed an isolated mesoscopic conductor and predicted a temperature-independent C(3) for a tunnel junction. The coupling to an environment introduces a temperature dependence, which can even change the sign of C(3) as the temperature is raised. No such effect exists for the second cumulant. The predicted temperature dependence has been measured in a recent experiment [14]. The method we used in Ref. [10] to arrive at these results was phenomenological. The nonlinear feedback was inserted by hand into the Langevin equation, through a cascade assumption [15]. The purpose of the present paper is to provide a fully quantum mechanical derivation. Our results agree with Ref. [10], thereby justifying the Langevin approach.

The outline of this paper is as follows. In Secs. II and III we present the general framework within which we describe a broad class of electrical circuits that consist of mesoscopic conductors embedded in a macroscopic electromagnetic environment. The basis is a path integral formulation of the Keldysh approach to charge counting statistics [16, 17]. It allows us to compute correlators and cross-correlators of currents and voltages at arbitrary contacts of the circuit. The method is technically involved, but we give an intuitive interpretation of the results in terms of “pseudo-probabilities”. Within this framework we study in Secs. IV and V series circuits of two conductors. For concrete results we specialize to a low-frequency regime where the path integrals over fluctuating quantum fields can be taken in saddle-point approximation. The conditions of validity for this approximation are discussed. We obtain general relations between third order correlators in a series circuit and correlators of the individual isolated conductors. We specialize to the experimentally relevant case of a single meso-
We consider a circuit consisting of electrical conductors $G_i$, a macroscopic electromagnetic environment [with impedance matrix $Z(\omega)$], plus ideal current and voltage meters $M_i$. The current meter (zero internal impedance) is in series with a voltage source, while the voltage meter (infinite internal impedance) is in parallel to a current source. Any finite impedance of meters and sources is in series with a voltage source, while the voltage meter is in series with a current source at the contacts for a current measurement (left circuit) and a voltage source is present at the contacts for a voltage measurement (right circuit). The two circuits can also be combined into one larger circuit containing two conductors and both a current and a voltage meter. The electromagnetic environment is assumed to produce only thermal noise. To characterize this noise we consider the circuit without the mesoscopic conductors, see Fig. 1. Each pair of contacts to the environment is now attached to a current source and a voltage meter.

The impedance matrix is defined by partial derivatives of voltages with respect to currents,

$$Z = \left( \begin{array}{cc} Z_{GG} & Z_{GM} \\ Z_{MG} & Z_{MM} \end{array} \right) = \left( \begin{array}{cc} \frac{\partial V_{\alpha}}{\partial I_{\beta}} & \frac{\partial V_{\alpha}}{\partial I_{\gamma}} \\ \frac{\partial V_{\beta}}{\partial I_{\gamma}} & \frac{\partial V_{\beta}}{\partial I_{\delta}} \end{array} \right). \quad (2.1)$$

(All quantities are taken at the same frequency $\omega$.) If there is more than one pair of contacts of type $G$ or $M$, then the four blocks of $Z$ are themselves matrices. Positive and negative frequencies are related by $Z_{\alpha\beta}(-\omega) = Z_{\alpha\beta}^*(-\omega)$. We also note the Onsager-Casimir symmetry $Z_{\alpha\beta}(B, \omega) = Z_{\beta\alpha}(-B, \omega)$, in an external magnetic field $B$. The thermal noise at each pair of contacts is Gaussian. The covariance matrix of the voltage fluctuations $\delta V_\alpha$ is determined by the fluctuation-dissipation theorem,

$$\langle \delta V_\alpha(\omega) \delta V_\beta(\omega') \rangle = \pi \delta(\omega + \omega') \hbar \omega \coth(\frac{\hbar \omega}{2kT}) \times \left| Z_{\alpha\beta}(\omega) + Z_{\beta\alpha}^*(\omega) \right|, \quad (2.2)$$

with $T$ the temperature of the environment.

We seek finite frequency cumulant correlators of the variables measured at the current and voltage meters,

$$\langle X_1(\omega_1) \cdots X_n(\omega_n) \rangle = 2\pi \delta \left( \sum_{k=1}^{n} \omega_k \right) C^{(n)}_{X}(\omega_1, \cdots, \omega_n). \quad (2.3)$$

Here $X_i$ stands for either $V_M$ or $I_M$. Fourier transforms are defined by $X_i(\omega) = \int dt \exp(i \omega t) X_i(t)$. Our aim is to relate the correlators at the measurement contacts to the correlators one would measure at the conductors if they were isolated from the environment.

### III. PATH INTEGRAL FORMULATION

Correlators of currents $I_M$ and voltages $V_M$ at the measurement contacts are obtained from the generating functional

$$Z_X[\vec{j}] = \left\langle T_{-} e^{-i \int dt [H + j^-(t)X]} T_{+} e^{i \int dt [H + j^+(t)X]} \right\rangle. \quad (3.1)$$

They contain moments of outcomes of measurements of the variable $X$ (equal to $I_M$ or $V_M$) at different instants of time. The symbols $T_{\pm} (T_{-})$ denote (inverse) time ordering, different on the forward and the backward part of the Keldysh contour. The exponents contain source terms $j^\pm$ and a Hamiltonian $H$, which we discuss separately.

The source term $j^\pm(t)$ is a charge $Q_M = \int^t dt' I_M(t')$ if $X = V_M$, whereas it is a phase $\Phi_M = \int^t dt' V_M(t')$ if $X = I_M$. (We have set $\hbar$ to unity.) The superscript $\pm$ determines on which part of the Keldysh contour the source is effective. The vector $\vec{j} = (j^cl, j^q)$ indicates the linear combinations

$$j^c = \frac{1}{2} \frac{\partial}{\partial t} (j^+ + j^-), \quad j^q = j^+ - j^- \quad (3.2)$$

\[ FIG. 1: Electrical circuits studied in this article. The black boxes represent conductors embedded in an electromagnetic environment (dashed rectangle). A voltage source is present at the contacts for a current measurement (right circuit) and a current source at the contacts for a voltage measurement (left circuit). The two circuits can also be combined into one larger circuit containing two conductors and both a current and a voltage meter. \]
The interaction term couples the phases $\Phi$ of the electromagnetic environment, which we model by a collection of harmonic oscillators at frequencies $\Omega$. We denote vectors in this two-dimensional “Keldysh space” by a vector arrow. The “classical” source fields $\mathbf{J} = (j_1^c, j_2^c, \ldots)$ account for current or voltage sources at the measurement contacts. Cumulant correlators of the measured variables are generated by differentiation of $\ln Z_{G}$ with respect to the “quantum” fields $j^q = (j_1^q, j_2^q, \ldots)$:

$$\left< \prod_{k=1}^{n} X_k(t_k) \right> = \prod_{k=1}^{n} \frac{\delta}{-i\delta j^q_k(t_k)} \ln Z_X \big|_{j^q=0}.$$  \hfill (3.3)

The Hamiltonian consists of three parts,

$$H = H_e + \sum_i H_G_i - \Phi_G I_G.$$  \hfill (3.4)

The term $H_e = \sum_i \Omega_i a_i^\dagger a_i$ represents the electromagnetic environment, which we model by a collection of harmonic oscillators at frequencies $\Omega_i$. The conductors connected to the environment have Hamiltonians $H_G_i$. The interaction term couples the phases $\Phi_G$ (defined by $i[H_e, \Phi_G] = V_G$) to the currents $I_G$ through the conductors. The phases $\Phi_G$, as well as the measured quantities $X$, are linear combinations of the bosonic operators $a_j$ of the electromagnetic environment,

$$\Phi_G = \sum_j \left( c_j^G a_j + c_j^{G*} a_j^\dagger \right),$$  \hfill (3.5a)

$$X = \sum_j \left( c_j^X a_j + c_j^{X*} a_j^\dagger \right).$$  \hfill (3.5b)

The coefficients $c_j^G$ and $c_j^X$ depend on the impedance matrix of the environment and also on which contacts are connected to a current source and which to a voltage source.

To calculate the generating functional we use a path integral formulation in Keldysh space [12, 21]. We first present the calculation for the case of a voltage measurement at all measurement contacts (so $X_k = V_{M_k}$ and $j_k = Q_{M_k}$ for all $k$). We will then show how the result for a current measurement can be obtained from this calculation. The path integral involves integrations over the environmental degrees of freedom $a_j$ weighted with an influence functional $Z_{I_G}$ due to the conductors. Because the conductors are assumed to be uncoupled in the absence of the environment, this influence functional factorizes:

$$Z_{I_G} = \prod_i Z_{I_G_i}.$$  \hfill (3.7)

An individual conductor has influence functional

$$Z_{I_G_i} = \left< T_- e^{i \int dt [H_{G_i} + \Phi_{G_i}(t) I_{G_i}]} \right> \times T_+ e^{-i \int dt [H_{G_i} + \Phi_{G_i}(t) I_{G_i}]}.$$  \hfill (3.8)

Comparing Eq. (3.8) with Eq. (3.1) for $X = I_M$, we note that the influence functional of a conductor $G_i$ is just the generating functional of current fluctuations in $G_i$ when connected to an ideal voltage source without electromagnetic environment. That is why we use the same symbol $Z$ for influence functional and generating functional.

The integrals over all environmental fields except $\Phi_G$ are Gaussian and can be done exactly. The resulting path integral expression for the generating functional $Z_{V_M}$ takes the form

$$Z_{V_M} = \int D[\Phi_G] \exp \left\{ -iS_e[\Phi_G, \Phi_G^\dagger] \right\} Z_{I_G} \Phi_G,$$  \hfill (3.9)

up to a normalization constant $Z$. We use for the integration fields $\Phi_G$ the same vector notation as for the source fields: $\Phi_G = (\Phi^c_G, \Phi^q_G)$ with $\Phi^c_G = \frac{1}{2}(\partial/\partial t)(\Phi^+_G + \Phi^-_G)$ and $\Phi^q_G = \Phi^+_G - \Phi^-_G$. The Gaussian environmental action $S_e$ is calculated in App. A. The result is given in terms of the impedance matrix $Z$ of the environment,
with the Bose-Einstein distribution $N(\omega) = [\exp(\omega/kT) - 1]^{-1}$. We have marked matrices in the Keldysh space by a check, for instance $\tilde{Y}$.

When one substitutes Eq. (3.10) into Eq. (3.14) and calculates correlators with the help of Eq. (3.9), one can identify two sources of noise. The first source of noise is current fluctuations in the conductors that induce fluctuations of the measured voltage. These contributions are generated by differentiating the terms of $\mathcal{S}$, that are linear in $\tilde{Q}_M$. The second source of noise is the environment itself, accounted for by the contributions quadratic in $\tilde{Q}_M$.

Generating functionals $\mathcal{Z}_{IM}$ for circuits where currents rather than voltages are measured at some of the contacts can be obtained along the same lines with modified response functions. It is also possible to obtain them from $\mathcal{Z}_{VM}$ through the functional Fourier transform derived in App. B.

\begin{equation}
\mathcal{Z}_M(\vec{\Phi}_M) = \int \mathcal{D}[\tilde{Q}_M] e^{-i\tilde{Q}_M \times \vec{\Phi}_M} \mathcal{Z}_M[\tilde{Q}_M]. \quad (3.14)
\end{equation}

We have defined the cross product

\begin{equation}
\tilde{Q} \times \vec{\Phi} \equiv \int dt \, (Q^d \Phi^q - \Phi^d Q^q). \quad (3.15)
\end{equation}

This transformation may be applied to any pair of measurement contacts to obtain current correlators from voltage correlators.

Eq. (3.14) ensures that the two functionals

\begin{align*}
\mathcal{P}[V, I] &= \int \mathcal{D}[q] e^{i \int dt q V} \mathcal{Z}_V[\tilde{Q} = (I, q)], \quad (3.16) \\
\mathcal{P'}[V, I] &= \int \mathcal{D}[\varphi] e^{i \int dt \varphi I} \mathcal{Z}_I[\tilde{\Phi} = (V, \varphi)]. \quad (3.17)
\end{align*}

are identical: $\mathcal{P}[V, I] = \mathcal{P'}[V, I]$. This functional $\mathcal{P}$ has an intuitive probabilistic interpretation. With the help of Eq. (3.3) we obtain from $\mathcal{P}$ the correlators

\begin{align*}
\langle V(t_1) \cdots V(t_n) \rangle_I &= \int \mathcal{D}[V] \mathcal{D}[V(t_1) \cdots V(t_n)] \mathcal{P}[V, I] \\
&= \int \mathcal{D}[V] \mathcal{P}[V, I], \quad (3.18) \\
\langle I(t_1) \cdots I(t_n) \rangle_V &= \int \mathcal{D}[I] \mathcal{D}[I(t_1) \cdots I(t_n)] \mathcal{P}[V, I] \\
&= \int \mathcal{D}[I] \mathcal{P}[V, I]. \quad (3.19)
\end{align*}

This suggests the interpretation of $\mathcal{P}[V, I]$ as a joint probability distribution functional of current and voltage fluctuations. Yet, $\mathcal{P}$ can not properly be called a probability since it need not be positive. In the low frequency approximation introduced in the next section it is positive for normal metal conductors. However, for superconductors, it has been found to take negative values \cite{22}. It is therefore more properly called a “pseudo-probability”.

We conclude this section with some remarks on the actual measurement process. The time-averaged correlators (3.9) may be measured in two different ways. In the first way the variable $X$ is measured repeatedly and results at different times are correlated afterwards. In the second way (and this is how it is usually done \cite{22}) one uses a detector that measures directly time integrals of $X$ (for example, by means of a spectral filter). The correlators measured in the first way are obtained from the generating functional according to Eq. (3.10),

\begin{equation}
2\pi \delta \left( \sum_{k=1}^{n} \omega_k \right) C_X^{(n)}(\omega_1, \cdots, \omega_n) = \prod_{k} \left[ \int_{-\infty}^{\infty} dt \, e^{i\omega_k t} \right] \mathcal{X} e^{\delta(j(t))} \ln \mathcal{X} \bigg|_{j_q = 0}. \quad (3.20)
\end{equation}

The second way of measurement is modelled by choosing cross-impedances that ensure that an instantaneous measurement at one pair of contacts yields a time average at another pair, for example $Z_{MG}(\omega) \propto \delta(\omega - \omega_0)$. The resulting frequency dependent correlators do not depend on which way of measurement one uses.
IV. TWO CONDUCTORS IN SERIES

We specialize the general theory to the series circuit of two conductors $G_1$ and $G_2$ shown in Fig. 3 (lower panel). We derive the generating functional $Z_{V,I}$ for correlators of the voltage drop $V \equiv V_{M_1}$ over conductor $G_1$ and the current $I \equiv I_{M_1}$ through both conductors. (The voltage drop over conductor $G_2$ equals $V_{M_2} - V_{M_1} \equiv V_{\text{bias}} - V$, with $V_{\text{bias}}$ the non-fluctuating bias voltage of the voltage source.) To apply the general relations of the previous section we embed the two conductors in an electromagnetic environment, as shown in the top panel of Fig. 3. In the limit of infinite resistances $R_1$, $R_2$, and $R_3$ this 8-terminal circuit becomes equivalent to a simple series circuit of $G_1$ and $G_2$. We take the infinite resistance limit of Eq. (3.9) in App. C. The result

$$Z_{V,I}[\vec{Q}, \vec{\Phi}] = \int D[\vec{\Phi}'] e^{-i\vec{\Phi}' \times \vec{Q}} Z_1[\vec{\Phi}] Z_2[\vec{\Phi} - \vec{\Phi}']$$

shows that the generating functional of current and voltage correlators in the series circuit is a functional integral convolution of the generating functionals $Z_1 \equiv Z_{G_1}$ and $Z_2 \equiv Z_{G_2}$ of the two conductors $G_1$ and $G_2$ defined in Eq. (3.8).

Eq. (4.1) implies a simple relation between the pseudo-probabilities $P_{G_1+G_2}$ of the series circuit (obtained by means of Eq. (3.17) from $Z_{V,I}[\vec{Q}=0]$) and the pseudo-probabilities $P_{G_k}$ of the individual conductors (obtained by means of Eq. (3.17) from $Z_k$). We find

$$P_{G_1+G_2}[V,I] = \int D\Phi' P_{G_1}[V - \Phi', I] P_{G_2}[\Phi', I]. \quad (4.2)$$

This relation is obvious if one interprets it in terms of classical probabilities: The voltage drop over $G_1 + G_2$ is the sum of the independent voltage drops over $G_1$ and $G_2$, so the probability $P_{G_1+G_2}$ is the convolution of $P_{G_1}$ and $P_{G_2}$. Yet the relation (4.2) is for quantum mechanical pseudo-probabilities.

We evaluate the convolution (4.2) in the low-frequency regime, when the functionals $Z_1$ and $Z_2$ become local in time,

$$\ln Z_k[\vec{\Phi}] = -i S_k[\vec{\Phi}] = -i \int dt S_k(\vec{\Phi}(t)) \quad (4.3)$$

We then do the path integration in saddle-point approximation, with the result

$$\ln Z_{V,I}[\vec{Q}, \vec{\Phi}] = -i \text{extr}_\vec{\Phi} \left\{ \vec{\Phi}' \times \vec{Q} \right\} + \int dt \left[ S_1(\vec{\Phi}'(t)) + S_2(\vec{\Phi}(t) - \vec{\Phi}'(t)) \right] \quad (4.4)$$

The notation “extr” indicates the extremal value of the expression between curly brackets with respect to variations of $\vec{\Phi}'(t)$. The validity of the low-frequency and saddle-point approximations is addressed at the end of this section.

We will consider separately the case that both conductors $G_1$ and $G_2$ are mesoscopic conductors and the case that $G_1$ is mesoscopic while $G_2$ is a macroscopic conductor. The action of a macroscopic conductor with impedance $Z$ is quadratic,

$$S_{\text{macro}}[\vec{\Phi}] = \frac{1}{2} \int \frac{d\omega}{2\pi} \vec{\Phi}' \Phi' Y \vec{\Phi}'$$

(4.5)

corresponding to Gaussian current fluctuations. The matrix $Y$ is given by Eq. (3.14), with a scalar $Z_{GG} = Z$. The corresponding pseudo-probability $P_{\text{macro}}$ is positive,

$$P_{\text{macro}}[V,I] = \exp \left\{ - \int \frac{d\omega}{4\pi\omega} \frac{|V - ZI|^2}{\Re Z} \tanh \left( \frac{\omega}{2kT} \right) \right\} \quad (4.6)$$

Substitution of $P_{\text{macro}}$ for $P_{G_2}$ in Eq. (4.2) gives a simple result for $P_{G_1+G_2}$ at zero temperature,

$$P_{G_1+G_2}[V,I] = P_{G_1}[V - ZI,I], \quad \text{if } T = 0. \quad (4.7)$$

The feedback of the macroscopic conductor on the mesoscopic conductor amounts to a negative voltage $-ZI$ produced in response to a current $I$.

The action of a mesoscopic conductor in the low-frequency limit is given by the Levitov-Lesovik formula.

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FIG. 3: Top panel: Circuit of two conductors $G_1$, $G_2$ in an electromagnetic environment modelled by three resistances $R_1$, $R_2$, $R_3$. In the limit $R_1$, $R_2$, $R_3 \to \infty$ the circuit becomes equivalent to the series circuit in the lower panel.
\[ S_{\text{meso}}(\Phi) = \frac{1}{2\pi} \sum_{n=1}^{N} \int d\varepsilon \, \ln[1 + \Gamma_n(e^{i\varepsilon \varphi} - 1)n_R(1 - n_L) + \Gamma_n(e^{-i\varepsilon \varphi} - 1)n_L(1 - n_R)], \] (4.8)

with \( \Phi = (V, \varphi) \). The \( \Gamma_n \)'s \((n = 1, 2, \ldots, N)\) are the transmission eigenvalues of the conductor. The two functions \( n_L(\varepsilon, T) = [\exp(\varepsilon/kT) + 1]^{-1} \) and \( n_R(\varepsilon, T) = n_L(\varepsilon + eV, T) \) are the filling factors of electron states at the left and right contacts, with \( V \) the voltage drop across the conductor and \( T \) its temperature.

The criterion for the applicability of the low-frequency and saddle-point approximations to the action of a mesoscopic conductor depends on two time scales. The first scale \( \tau_1 = \min(1/eV, 1/kT) \) is the mean width of current pulses due to individual transferred electrons. The second scale \( \tau_2 = e/I \approx (e^2/G)\tau_1 \) is the mean time between current pulses. At frequencies below \( 1/\tau_1 \) the action of the conductor becomes local in time. Below the second scale \( 1/\tau_2 \) the action of the conductor becomes local in space. For frequencies \( \omega \gg \tau_2 \) we indeed have \( S_{\text{meso}} \approx \tau I \varphi \approx \tau I/e \approx \tau_2 \gg 1 \).

These two approximations together are therefore justified if fluctuations with frequencies \( \omega \) above \( \Lambda \approx \min(1/\tau_1, 1/\tau_2) \) are suppressed by a small effective impedance: \( Z(\omega) \ll h/e^2 \) for \( \omega \gtrsim \Lambda \). A small impedance acts as a heavy mass term in Eq. \text{(4.1)}, suppressing fluctuations. This is seen from Eq. \text{(5.3)} for a macroscopic conductor and it carries over to other conductors. In Sec. \text{VII} we will examine the Coulomb blockade effects that appear if \( Z(\omega) \) is not small at high frequencies.

\section{V. THIRD CUMULANTS}

\subsection{A. Two arbitrary conductors in series}

We use the general formula \text{(4.4)} to calculate the third order cumulant correlator of current and voltage fluctuations in a series circuit of two conductors \( G_1 \) and \( G_2 \) at finite temperature. We focus on correlators at zero frequency \( (\text{finite frequency generalizations are given later}). \)

The zero-frequency correlators \( C^{(n)}(V) \) depend on the average voltage \( \overline{V} \) over \( G_1 \), which is related to the voltage \( V_{\text{bias}} \) of the voltage source by \( \overline{V} = V_{\text{bias}}(1 + G_1/G_2)^{-1}. \) The average voltage over \( G_2 \) is \( V_{\text{bias}} - \overline{V} = V_{\text{bias}}(1 + G_2/G_1)^{-1}. \) Our goal is to express \( C^{(n)}(V) \) in terms of the current correlators \( C^{(n)}(V) \) and \( C^{(n)}(V) \) that the conductors \( G_1 \) and \( G_2 \) would have if they were isolated and biased with a non-fluctuating voltage \( V \). These are defined by

\[ \langle I_i(\omega_1) \cdots I_i(\omega_n) \rangle_V = 2\pi\delta \left( \sum_{k=1}^{n} \omega_k \right) C^{(n)}(V), \] (5.1)

where \( I_i \) is the current through conductor \( i \) at fixed voltage \( V \).

To evaluate Eq. \text{(5.1)} it is convenient to discretize frequencies \( \omega_n = 2\pi n/\tau \). The Fourier coefficients are \( f_n = \tau^{-1} \int_0^\tau dt e^{in\omega t} f(t) \). The detection time \( \tau \) is sent to infinity at the end of the calculation. For zero-frequency correlators the sources at non-zero frequencies vanish and there is a saddle point configuration such that all fields at non-zero frequencies vanish as well. We may then write Eq. \text{(5.1)} in terms of only the zero-frequency fields \( \bar{\Phi}_0 = (V_0, \varphi_0), \bar{\Phi}'_0 = (V'_0, \varphi'_0), \) and \( \bar{Q}_0 = (I_0, q_0) \), with actions

\[ \tau^{-1} S_k(\bar{\Phi}'_0) = G_k \varphi'_0 V'_0 + i \sum_{n=2}^{\infty} \frac{(-i \varphi'_0)^n}{n!} C^{(n)}(V'_0). \] (5.2)

For \( \bar{\Phi}_0 = (V_{\text{bias}}, 0) \) and \( \bar{Q}_0 = (0, 0) \) the saddle point is at \( \bar{\Phi}'_0 = (\overline{V}, 0) \). For the third order correlators we need the extremum in Eq. \text{(1.3)} to third order in \( \varphi_0 \) and \( q_0 \). We have to expand \( S_k \) to third order in the deviation \( \delta \Phi'_0 = \bar{\Phi}'_0 - (\overline{V}, 0) \) from the saddle point at vanishing sources. We have to this order

\[ \tau^{-1} S_1(\bar{\Phi}'_0) = G_1 \varphi'_0 (V + \delta V'_0) - \frac{i}{2} C^{(2)}(V) \varphi_0^2 - \frac{1}{6} C^{(3)}(V) \varphi_0^3 - \frac{i}{2} \frac{d}{dV} C^{(2)}(V) \varphi'_0 \delta V'_0 + \mathcal{O}(\delta \Phi'_0^4), \] (5.3)

\[ \tau^{-1} S_2(\bar{\Phi}'_0 - \bar{\Phi}_0') + G_2 \varphi'_0 (V_{\text{bias}} - \overline{V} + \delta V'_0) - \frac{i}{2} C^{(2)}(V_{\text{bias}} - \overline{V}) \varphi_0^2 - \frac{1}{6} C^{(3)}(V_{\text{bias}} - \overline{V}) \varphi_0^3 + \frac{i}{2} \frac{d}{dV} C^{(2)}(V_{\text{bias}} - \overline{V}) \varphi'_0 \delta V'_0 + \mathcal{O}(\delta \Phi'_0^4). \] (5.4)

Minimizing the sum \( S_1(\bar{\Phi}'_0) + S_2(\bar{\Phi}'_0 - \bar{\Phi}_0') \) to third order in \( q_0 \) and \( \varphi_0 \) we then find the required relation between the correlators of the series circuit and the correlators of the isolated conductors. For the second order correlators we
find
\begin{align}
C_{1I}^{(2)}(\mathbf{V}) &= (R_1 + R_2)^{-2}[R_1^2 C_{1I}^{(2)}(\mathbf{V}) + R_2^2 C_{2I}^{(2)}(V_{\text{bias}} - \mathbf{V})], \\
C_{V}^{(2)}(\mathbf{V}) &= (R_1 + R_2)^{-2}(R_1 R_2)^2[C_{1I}^{(2)}(\mathbf{V}) + C_{2I}^{(2)}(V_{\text{bias}} - \mathbf{V})], \\
C_{1I}^{(2)}(\mathbf{V}) &= (R_1 + R_2)^{-2}R_1 R_2[2 R_2 C_{2I}^{(2)}(V_{\text{bias}} - \mathbf{V}) - R_1 C_{1I}^{(2)}(\mathbf{V})],
\end{align}

with $R_k = 1/G_k$.

The third cumulant correlators contain extra terms that depend on the second-order correlators,
\begin{align}
C_{III}^{(3)}(\mathbf{V}) &= (R_1 + R_2)^{-3}[R_1 C_{I}^{(3)}(\mathbf{V}) + R_2 C_{2}^{(3)}(V_{\text{bias}} - \mathbf{V})] + 3 C_{IV}^{(2)} \frac{d}{d\mathbf{V}} C_{II}^{(2)}, \\
C_{VII}^{(3)}(\mathbf{V}) &= (R_1 + R_2)^{-3}(R_1 R_2)^3 C_{2}^{(3)}(V_{\text{bias}} - \mathbf{V}) - C_{1}^{(3)}(\mathbf{V})] + 3 C_{V}^{(2)} \frac{d}{d\mathbf{V}} C_{VI}^{(2)}, \\
C_{VII}^{(3)}(\mathbf{V}) &= (R_1 + R_2)^{-3}(R_1 R_2)^2[R_1 C_{I}^{(3)}(\mathbf{V}) + R_2 C_{2}^{(3)}(V_{\text{bias}} - \mathbf{V})] + 2 C_{V}^{(2)} \frac{d}{d\mathbf{V}} C_{V}^{(2)} + C_{IV}^{(2)} \frac{d}{d\mathbf{V}} C_{VI}^{(2)}, \\
C_{III}^{(3)}(\mathbf{V}) &= (R_1 + R_2)^{-3}R_1 R_2 [2 R_2 C_{2}^{(3)}(V_{\text{bias}} - \mathbf{V}) - R_1^2 C_{1}^{(3)}(\mathbf{V})] + 3 C_{IV}^{(2)} \frac{d}{d\mathbf{V}} C_{II}^{(2)} + C_{IV}^{(2)} \frac{d}{d\mathbf{V}} C_{VI}^{(2)}. \\
\end{align}

These results agree with those obtained by the cascaded Langevin approach \([10]\).

B. Mesoscopic and macroscopic conductor in series

An important application is a single mesoscopic conductor $G_1$ embedded in an electromagnetic environment, represented by a macroscopic conductor $G_2$. A macroscopic conductor has no shot noise but only thermal noise. The third cumulant $C_2^{(3)}$ is therefore equal to zero. The second cumulant $C_2^{(2)}$ is voltage independent, given by [7]
\begin{equation}
C_2^{(2)}(\omega) = \omega \coth\left(\frac{\omega}{2kT}\right) \text{Re} G_2(\omega),
\end{equation}
at temperature $T_2$. We still assume low frequencies $\omega \ll \text{max}(eV/kT_1)$, so the frequency dependence of $S_1$ can be neglected. We have retained the frequency dependence of $S_2$, because the characteristic frequency of a macroscopic conductor is typically much smaller than of a mesoscopic conductor.

From Eq. 5.9 (and a straightforward generalization to frequency dependent correlators) we can obtain the third cumulant correlators by setting $C_2^{(3)} = 0$ and substituting Eq. 5.10. We only give the two correlators $C_{III}^{(3)}$ and $C_{VII}^{(3)}$, since these are the most significant for experiments. To abbreviate the formula we denote $G = G_1$ and $Z(\omega) = 1/G_2(\omega)$. We find
\begin{align}
C_{III}^{(3)}(\omega_1, \omega_2, \omega_3) &= \frac{C_{III}^{(2)}(\mathbf{V}) - (dC_{III}^{(2)}/d\mathbf{V}) \sum_{j=1}^{3} Z(-\omega_j) [C_{I}^{(2)}(\mathbf{V}) - G Z(\omega_j) C_{2}^{(2)}(\omega_j)] [1 + Z(-\omega_j) G]^{-1}}{[1 + Z(\omega_1) G][1 + Z(\omega_2) G][1 + Z(\omega_3) G]}, \\
C_{VII}^{(3)}(\omega_1, \omega_2, \omega_3) &= \frac{C_{VII}^{(2)}(\mathbf{V}) - (dC_{VII}^{(2)}/d\mathbf{V}) \sum_{j=1}^{3} Z(-\omega_j) [C_{I}^{(2)}(\mathbf{V}) + C_{2}^{(2)}(\omega_j)] [1 + Z(-\omega_j) G]^{-1}}{[1 + Z(\omega_1) G][1 + Z(\omega_2) G][1 + Z(\omega_3) G]}. \\
\end{align}

We show plots for two types of mesoscopic conductors: a tunnel junction and a diffusive metal. In both cases it is assumed that there is no inelastic scattering, which is what makes the conductor mesoscopic. The plots correspond to global thermal equilibrium ($T_1 = T_2 = T$) and to a real and frequency-independent impedance $Z(\omega) \equiv Z$. We compare $C_{I}^{(3)} \equiv C_{III}^{(3)}$ with $C_{V}^{(3)} \equiv -C_{VII}^{(3)}/Z^3$. (The minus sign is chosen so that $C_{I}^{(3)} = C_{V}^{(3)}$ at $T = 0$.)

For a tunnel junction one has
\begin{equation}
C_{I}^{(2)}(V) = Ge^{2V}, \quad C_{I}^{(3)}(V) = Ge^{2V}. 
\end{equation}

The third cumulant of current fluctuations in an isolated tunnel junction is temperature independent [11], but this is changed drastically by the electromagnetic environment [11]. Substitution of Eq. 5.10 into Eqs. 5.5 and 5.7 gives the curves plotted in Fig. 4 for $ZG = 0$ and $ZG = 1$. The slope $dC_{I}^{(3)}(V)/dV$ becomes strongly tem-
FIG. 4: Third cumulant of voltage and current fluctuations of a tunnel junction (conductance $G$) in an electromagnetic environment (impedance $Z$, assumed frequency independent). Both $C^{(3)}_I$ and $C^{(3)}_V$ are multiplied by the scaling factor $A = (1 + ZG)^3/eGkT$. The two curves correspond to different values of $ZG$ (solid curve: $ZG = 1$; dashed curve: $ZG = 0$). The temperatures of the tunnel junction and its environment are chosen the same, $T_1 = T_2 = T$.

FIG. 5: Same as Fig. 4 but now for a diffusive metal.

cumulant is already temperature dependent even in the absence of the electromagnetic environment. In the limit $ZG \to \infty$ we recover the result for $C^{(3)}_V$ obtained by Nagaev from the cascaded Langevin approach [27].

VI. HOW TO MEASURE CURRENT FLUCTUATIONS

In Fig. 4 we have plotted both current and voltage correlators, but only the voltage correlator has been measured [14]. At zero temperature of the macroscopic conductor there is no difference between the two, as follows from Eqs. (5.8) and (5.9): $C^{(3)}_{III} = -C^{(3)}_{VVV}/Z^3$ if $C^{(2)}_I = 0$, which is the case for a macroscopic conductor $G_2$ at $T_2 = 0$. For $T_2 \neq 0$ a difference appears that persists in the limit of a non-invasive measurement $Z \to 0$ [10]. Since $V$ and $I$ in the series circuit with a macroscopic $G_2$ are linearly related and linear systems are known to be completely determined by their response functions and their temperature, one could ask what it is that distinguishes the two measurements, or more practically: How would one measure $C^{(3)}_{III}$ instead of $C^{(3)}_{VVV}$?

To answer this question we slightly generalize the

\[ C^{(2)}_I(V) = \frac{1}{3} GeV \left( \coth p + 2/p \right), \]

\[ C^{(3)}_I(V) = e^2 GV \frac{p(1 - 26e^{2p} + e^{4p}) - 6(e^{4p} - 1)}{15p(e^{2p} - 1)^2}. \]
macroscopic conductor to a four-terminal, rather than two-terminal configuration, see Fig. 6. The voltage $V_M$ over the extra pair of contacts is related to the current $I_G$ through the series circuit by a cross impedance, $\partial V_M/\partial I_G = Z_{MG}$. The full impedance matrix $Z$ is defined as in Eq. (2.1). For simplicity we take the zero-frequency limit. For this configuration the third cumulant $C_{V_M V_M V_M}^{(3)}$ of $V_M$ is given by

$$
\frac{C_{V_M V_M V_M}^{(3)}}{Z_{MG}^3} = C_{I_G I_G I_G}^{(3)} + \frac{Z_{GM} + Z_{MG}}{2Z_{GM}} \left( \frac{C_{V_G V_G V_G}^{(3)}}{Z_{GG}^3} - C_{I_G I_G I_G}^{(3)} \right). \quad (6.1)
$$

It contains the correlator $\langle \delta V_M(\omega) \delta V_M(\omega') \rangle = 2\pi \delta(\omega + \omega')C_{GM}$ of the voltage fluctuations over the two pairs of terminals of the macroscopic conductor, which according to the fluctuation-dissipation theorem [22] is given in the zero-frequency limit by

$$
C_{GM} = kT_2(Z_{GM} + Z_{MG}). \quad (6.2)
$$

The correlator $C_{GM}$ enters since $C_{V_M V_M V_M}^{(3)}$ depends on how thermal fluctuations in the measured variable $V_M$ correlate with the thermal fluctuations of $V_G$ which induce extra current noise in $G_1$.

We conclude from Eq. (6.1) that the voltage correlator $C_{V_M V_M V_M}^{(3)}$ becomes proportional to the current correlator $C_{I_G I_G I_G}^{(3)}$ if $Z_{GM} + Z_{MG} = 0$. This can be realized if $V_M$ is the Hall voltage $V_H$ in a weak magnetic field $B$. Then $Z_{MG} = -Z_{GM} = R_H$, with $R_H \propto |B|$ the Hall resistance. The magnetic field need only be present in the macroscopic conductor $G_2$, so it need not disturb the transport properties of the mesoscopic conductor $G_1$. If, on the other hand, $V_M$ is the longitudinal voltage $V_L$, then $Z_{MG} = Z_{GM} = R_L$, with $R_L$ the longitudinal resistance. The two-terminal impedance $Z_{GG}$ is the sum of Hall and longitudinal resistances, $Z_{GG} = R_L + R_H$. So one has

$$
C_{V_L V_L V_L}^{(3)} = \left( \frac{R_L}{R_L + R_H} \right)^3 C_{V_G V_G V_G}^{(3)}, \quad (6.3)
$$

$$
C_{V_H V_H V_H}^{(3)} = R_H^3 C_{I_G I_G I_G}^{(3)} . \quad (6.4)
$$

VII. ENVIRONMENTAL COULOMB BLOCKADE

The saddle-point approximation to the path integral [44] for a mesoscopic conductor $G_1$ in series with a macroscopic conductor $G_2$ (impedance $Z$) breaks down when the impedance at the characteristic frequency scale $\lambda = 1/\max(\tau_1, \tau_2)$ discussed in section [24] is not small compared to the resistance quantum $\hbar/e^2$. It can then react fast enough to affect the dynamics of the transfer of a single electron. These single-electron effects amount to a Coulomb blockade induced by the electromagnetic environment [4]. In our formalism they are accounted for by fluctuations around the saddle point of Eq. (4.1).

In Ref. [17] it has been found that the Coulomb blockade correction to the mean current calculated to leading order in $Z$ is proportional to the second cumulant of current fluctuations in the isolated mesoscopic conductor $(Z = 0)$. More recently, the Coulomb blockade correction to the second cumulant of current fluctuations has been found to be proportional to the third cumulant [18]. It was conjectured in Ref. [18] that this relation holds also for higher cumulants. Here we give a proof of this conjecture.

We show that at zero temperature and zero frequency the leading order Coulomb blockade correction to the $n$-th cumulant of current fluctuations is proportional to the voltage derivative of the $(n+1)$-th cumulant. To extract the environmental Coulomb blockade from the other effects of the environment we assume that $Z$ vanishes at zero frequency, $Z(0) = 0$. The derivation is easiest in terms of the pseudo-probabilities discussed in Sec. [31].

According to Eq. (5.19), cumulant correlators of cur-
rent have the generating functional
\[ F_{G_1+G_2} [\Phi] = \langle (V, \varphi) \rangle = \ln \int D I e^{-i \int dt I \varphi} P_{G_1+G_2} [V, I]. \] (7.1)
Zero frequency current correlators are obtained from
\[ \langle \langle 0 \rangle \rangle_{G_1+G_2} = i^n \delta^n \left[ \frac{\partial^n}{\partial \varphi (0)} \right]_0 F_{G_1+G_2} [\Phi] \bigg|_{\varphi = 0}. \] (7.2)
We employ now Eq. (4.7) and expand \( F_{G_1+G_2} [\Phi] \) to first order in \( Z \),
\[ F_{G_1+G_2} [\Phi] = F_{G_1} [\Phi] \]
\[ - \int D I e^{-i \int dt I \varphi} \int \frac{d \omega}{2 \pi} Z (\omega) I (\omega) \frac{\delta^2}{\delta V (\omega) \delta \varphi (\omega)} F_{G_1} [\Phi]. \]
(7.3)
The last equality holds since single derivatives of \( F_{G_1} [\Phi] \) with respect to a variable at finite frequency vanish because of symmetry reasons. Substitution into Eq. (7.2) gives
\[ \langle \langle 0 \rangle \rangle_{G_1+G_2} = \langle \langle 0 \rangle \rangle_{G_1} \]
\[ - \int \frac{d \omega}{2 \pi} Z (\omega) \frac{\delta}{\delta V (\omega)} \langle \langle I (\omega) 0 \rangle \rangle_{G_1}, \]
(7.4)
which is what we had set out to prove.

VIII. CONCLUSION

In conclusion, we have presented a fully quantum mechanical derivation of the effect of an electromagnetic environment on current and voltage fluctuations in a mesoscopic conductor, going beyond an earlier study at zero temperature [2]. The results agree with those obtained from the cascaded Langevin approach [10], thereby providing the required microscopic justification.

From an experimental point of view, the nonlinear feedback from the environment is an obstacle that stands in the way of a measurement of the transport properties of the mesoscopic system. To remove the feedback it is not sufficient to reduce the impedance of the environment. One also needs to eliminate the mixing in of environmental thermal fluctuations. This can be done by ensuring that the environment is at a lower temperature than the conductor, but this might not be a viable approach for low-temperature measurements. We have proposed here an alternative method, which is to ensure that the measured variable changes sign under time reversal. In practice this could be realized by measuring the Hall voltage over a macroscopic conductor in series with the mesoscopic system. The field theory developed here also provides for a systematic way to incorporate the effects of the Coulomb blockade which arise if the high-frequency impedance of the environment is not small compared to the resistance quantum. We have demonstrated this by generalizing to moments of arbitrary order a relation in the literature [17, 18] for the leading-order Coulomb blockade correction to the first and second moment of the current. We refer to Ref. [12] for a renormalization group analysis of the Coulomb blockade corrections of higher order.

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APPENDIX A: DERIVATION OF THE ENVIRONMENTAL ACTION

To derive Eq. (3.10) we define a generating functional for the voltages \( V = (V_M, V_G) \) in the environmental circuit of Fig. 2
\[ Z_e(\tilde{Q}) = \begin{pmatrix} T_+ e^{i \int dt [H + Q^-(\omega) V]} T_- e^{-i \int dt [H + Q^- (\omega) V]} \end{pmatrix}. \]
(A1)

We have introduced sources \( Q = (Q_M, Q_G) \). Since the environmental Hamiltonian is quadratic, the generating functional is the exponential of a quadratic form in \( \tilde{Q} \),
\[ Z_e(\tilde{Q}) = \exp \left( -\frac{i}{2} \int \frac{d \omega}{2 \pi} \tilde{Q}^\dagger (\omega) \tilde{G}(\omega) \tilde{Q}(\omega) \right). \]
(A2)

The off-diagonal elements of the matrix \( \tilde{G} \) are determined by the impedance of the circuit,
\[ \frac{i}{\delta Q^\dagger (\omega) \delta Q^\dagger (\omega)} \ln Z_e \bigg|_{\tilde{Q}=0} = \frac{\delta}{\delta V_b (\omega)} \langle V_a (\omega) \rangle \]
\[ = 2 \pi \delta (\omega - \omega') Z_{ab} (\omega). \] (A3)
The upper-diagonal \( (c, c) \) elements in the Keldysh space vanish for symmetry reasons \( (Z_e | \tilde{Q} = 0 = 0, \text{cf. Ref. } [22]) \). The lower-diagonal \( (q, q) \) elements are determined by the fluctuation-dissipation theorem [22],
\[ \frac{-i}{\delta Q^\dagger (\omega) \delta Q^\dagger (\omega)} \ln Z_e \bigg|_{\tilde{Q}=0} = \langle \delta V_a (\omega) \delta V_b (\omega) \rangle \]
\[ = \pi \delta (\omega + \omega') \cosh \left( \frac{\omega}{2 k T} \right) [Z_{ab} (\omega) + Z^\dagger_{ab} (\omega)]. \]
(A4)
Consequently we have
\[ \tilde{G}(\omega) = \begin{pmatrix} 0 & Z(\omega) - \frac{i}{2 \pi} \coth \left( \frac{\omega}{2 k T} \right) [Z(\omega) + Z^\dagger(\omega)] \end{pmatrix}. \]
(A5)
FIG. 8: Circuit to relate voltage to current measurements.

The environmental action $S_e$ is defined by

$$Z_e[\vec{Q}] = \int D[\Phi_G] \exp \left(-iS_e[\vec{Q}_M, \vec{Q}_G] - i\vec{\Phi}_G \times \vec{Q}_G\right).$$  \hspace{1cm} (A6)

One can check that substitution of Eq. (3.10) into Eq. (A6) yields the same $Z_e$ as given by Eqs. (A2) and (A5).

**APPENDIX B: DERIVATION OF EQ. (3.14)**

In the limit $R \to \infty$ a voltage measurement in the circuit of Fig. 8 corresponds to a voltage measurement at contacts $M$ and $M'$ of the circuit $C$. We obtain the generating functional $Z_V$ of this voltage measurement from Eq. (3.9). The influence functional is now due to $C$ and it equals the generating functional $Z_I$ of a current measurement at contacts $M$ and $M'$ of $C$. From Eq. (3.10) with $Z_{MM} = Z_{GG} = -Z_{MG} = -Z_{GM} = R$ we find in the limit $R \to \infty$ that the environmental action takes the simple form $S_e[\vec{Q}_M, \vec{\Phi}_G] = \vec{\Phi} \times \vec{Q}$, with the cross-product defined in Eq. (3.15). Consequently, we have

$$Z_V[\vec{Q}] = \int D[\Phi] e^{-i\vec{\Phi} \times \vec{Q}} Z_I[\vec{\Phi}].$$  \hspace{1cm} (B1)

This equation relates the generating functionals of current and voltage measurements at any pair of contacts of a circuit.

**APPENDIX C: DERIVATION OF EQ. (4.1)**

To derive Eq. (4.1) from Eq. (3.9) we need the environmental action $S_e$ of the circuit shown in Fig. 3. The impedance matrix is

$$Z = \frac{1}{R_1 + R_2 + R_3} \begin{pmatrix} R_1(R_2 + R_3) & -R_1R_2 & -R_1R_2 & -R_1R_2 \\ -R_1R_2 & R_2(R_1 + R_3) & -R_1R_2 & -R_1R_2 \\ -R_1(R_2 + R_3) & -R_1R_2 & R_1(R_2 + R_3) & -R_1R_3 \\ -R_1R_3 & -R_1R_3 & -R_1R_3 & R_3(R_1 + R_2) \end{pmatrix}. \hspace{1cm} (C1)$$

We seek the limit $R_1, R_2, R_3 \to \infty$. The environmental action (3.10) takes the form

$$S_e[\vec{Q}_M, \vec{\Phi}_G] = \vec{\Phi}_G \times \vec{Q}_M + \vec{\Phi}_G \times \vec{Q}_M + \vec{\Phi}_G \times \vec{Q}_M.$$

Substitution into Eq. (3.29) gives $Z_{VV}$. Employing Eq. (3.14) to obtain $Z_{VI}$ from $Z_{VV}$ we arrive at Eq. (4.1).

[1] E. Ben-Jacob, E. Mottola, and G. Schön, Phys. Rev. Lett. 51, 2064 (1983).
[2] G. Schön, Phys. Rev. B 32, 4469 (1985).
[3] M. H. Devoret, D. Esteve, H. Grabert, G.-L. Ingold, H. Pothier, and C. Urbina, Phys. Rev. Lett. 64, 1824 (1990).
[4] G.-L. Ingold and Yu. V. Nazarov, in Single Charge Tunneling, edited by H. Grabert and M. H. Devoret, NATO ASI Series B294 (Plenum, New York, 1992).
[5] H. Lee and L. S. Levitov, Phys. Rev. B 53, 7383 (1996).
[6] Sh. Kogan, Electronic Noise and Fluctuations in Solids (Cambridge University, Cambridge, 1996).
[7] Ya. M. Blanter and M. Böttiker, Phys. Rep. 336, 1 (2000).
[8] C. W. J. Beenakker and C. Schönberger, Physics Today
56 (5), 37 (2003).

[9] M. Kindermann, Yu. V. Nazarov, and C. W. J. Beenakker, Phys. Rev. Lett. (in press).
[10] C. W. J. Beenakker, M. Kindermann, and Yu. V. Nazarov, Phys. Rev. Lett. 90, 176802 (2003).
[11] L. S. Levitov and M. Reznikov, cond-mat/0111057.
[12] D. B. Gutman and Y. Gefen, cond-mat/0201007.
[13] K. E. Nagaev, Phys. Rev. B 66, 075334 (2002).
[14] B. Reulet, J. Senzier, and D. E. Prober, cond-mat/0302084.
[15] Yu. V. Nazarov, Ann. Phys. (Leipzig) 8, 507 (1999).
[16] Yu. V. Nazarov and M. Kindermann, cond-mat/0107133.
[17] A. Levy Yeyati, A. Martin-Rodero, D. Esteve, and C. Urbina, Phys. Rev. Lett. 87, 046802 (2001).
[18] A. V. Galaktionov, D. S. Golubev, and A. D. Zaikin, cond-mat/0212494.
[19] M. Kindermann and Yu. V. Nazarov, cond-mat/0304078.
[20] H. B. G. Casimir, Rev. Mod. Phys. 17, 343 (1945).
[21] A. Kamenev, in Strongly Correlated Fermions and Bosons in Low-Dimensional Disordered Systems, edited by I. V. Lerner, B. L. Altshuler, V. I. Fal’ko, and T. Giamarchi, NATO Science Series II Vol. 72 (Kluwer, Dordrecht, 2002); cond-mat/0109316.
[22] M. Kindermann and Yu. V. Nazarov, in Quantum Noise, edited by Yu. V. Nazarov and Ya. M. Blanter, NATO Science Series II Vol. 97 (Kluwer, Dordrecht, 2003); cond-mat/0303590.
[23] W. Belzig and Yu. V. Nazarov, Phys. Rev. Lett. 87, 197006 (2001).
[24] U. Gavish, Y. Imry, L. Levinson, and B. Yurke, in Quantum Noise, edited by Yu. V. Nazarov and Ya. M. Blanter, NATO Science Series II Vol. 97 (Kluwer, Dordrecht, 2003); cond-mat/0211646.
[25] L. S. Levitov and G. B. Lesovik, JETP Lett. 58, 230 (1993).
[26] L. S. Levitov, H. Lee, and G. B. Lesovik, J. Math. Phys. 37, 4845 (1996).
[27] K. E. Nagaev, cond-mat/0302008.