On the theory of strong pinning in high-temperature superconducting films

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Abstract. Several pinning mechanisms proposed to explain extremely high critical current densities \( j_c \) in \( c \)-oriented epitaxial YBaCuO films are analysed. It is concluded that only strong pin-breaking mechanisms, such as pinning by cores of edge dislocations threading the whole film and forming low-angle boundaries between single-crystal domains (at low and moderate fields) and the pinning by sparse large point defects, such as nanoscale inclusions of non-superconducting phases (at higher fields) are relevant. It is shown that available expressions for \( j_c \) for the last mechanism have certain limits of applicability, such as unphysical divergence of \( j_c \) with increasing anisotropy \( \Gamma \) leading to an essential overestimate even for YBaCuO with the moderate anisotropy \( \Gamma \approx 7 \). A nature of such inconsistency and the ways of its overcoming are elucidated. Some aspects of the strong pinning theory are discussed. In the case of thin films it is shown that for the accurate description both tilt and shear elastic moduli should be taken into account as competing factors. The obtained area of vortex trapping on finite-length pinning centre describes crossover from 0D to 1D case and allows a unification of strong pinning theory for different types of defects.

1. Introduction

Epitaxial films of cuprate layered high-\( T_c \) superconductors exhibit extremely high values of critical current densities \( J_c \) (up to \( 8 \times 10^{10} \) A/m\(^2\) at 77 K and self field for some SmBaCuO samples [1]).

Several pinning mechanisms have been proposed to explain such high \( J_c \) values. They can be distinguished by their nature (electromagnetic or core pinning) and dimensionality of defects themselves (1D or 2D correlated versus 0D random) as well as by dimensionality (2D or 3D) of the vortex line lattice (VLL) interaction with pin arrays. Suggested 0D (point-like) defects are considered to be: (1) microscopic (oxygen vacancies) and (2) macroscopic (normal phase nanoparticles). On the other hand, epitaxially grown YBCO films contain a multitude of intrinsic 1D pins, i.e., non-superconducting cores of out-of-plane screw or edge dislocations. The latter ones form quasi-periodical rows within low-angle boundaries between single-crystal domains.

A lot of theoretical expressions were deduced in the frameworks of different pinning models for \( J_c \) as a function of temperature \( T \), magnitude \( H \) and direction of applied magnetic field, material anisotropy \( \Gamma \), and film thickness \( d \). In order to recognize the governing pinning mechanism some simple characteristics (such as exponent values of various power dependences) are considered usually to compare with experimental curves. However, all the simple dependences have very limited ranges of applicability with some unphysical divergences of \( J_c \) out of these ranges. Many of such restrictions were already discussed, but some others still need elucidations being suggested in the present article.
The strong core-pinning model for thin films in a normal magnetic field is also generalized to unify the theory for 0D and 1D defects.

2. Pinning mechanisms for thin YBCO films and theoretical approaches to their treatment

Out-of-plane dislocations with non-superconducting cores are inherent for epitaxial film growth due to misfit with a substrate. For sufficiently thick films (~100 nm) the dislocations are threading films from the interface to the very surface [2]. Such 1D defects are ideal candidates for extremely strong pinning responsible for high $J_c$ values, as it was suggested for screw dislocations in the case of spiral growth mode [2, 3] and for edge dislocations in low-angle boundaries in the case of island growth mode [4, 5]. In the latter case the quantitative theory of collective VLL pinning was proposed, which attributed $J_c(H)$ dependence to a distribution of single-crystal domains by their size and mutual mean misalignment angle. Any thickness dependence was not supposed in this model with dislocations entirely threading the film.

On the other hand, advocates of pinning mechanisms by point defects usually appeal to thickness dependences of $J_c$ in YBCO films [6, 7]. Since these dependences for different film types as well as YBCO based coated conductors are essentially different, so do the claimed explanations. Let us consider them briefly. The crossover from $J_c \propto d^{-1/2}$ to nearly constant $J_c$ at $d > 0.6 \mu m$ observed for typical pulse laser deposited (PLD) films was interpreted in [6] as a crossover between 2D [8] and 3D [9] interaction regimes of VLL weak collective pinning by point defects. It should be emphasized that theories [8, 9] deal with weak random defects (microscopic, such as oxygen vacancies in CuO$_2$ layers or weak local fluctuations of the total oxygen content) and yield critical current density values $J_c \approx 10^{-4}J_0$ observed experimentally in bulk single crystal samples. Here $J_0 = 4 e_0/3 \sqrt{\phi_0 \xi}$ is the depairing critical current density, $e_0 = \phi_0^2/4 \pi \mu_0 \lambda^2$ is a typical vortex energy scale, where $\phi_0 = h/2e$ is the flux quantum, $\lambda$ is the London penetration depth for currents flowing in $ab$-plane (coinciding with CuO$_2$ layers). When strong point pinning centres, such as non-superconducting precipitates with a concentration $n_p$ and a characteristic radius $r_p$, were considered in the framework of weak collective pinning theories in [6], the estimation of critical $d$ value for the 2D - 3D crossover gave not only more than an order of magnitude discrepancy with the observed one, but the mean spacing between defects $L_p = n_p^{-1/3}$ estimated from the expression for $J_c$ in supposed 2D regime $J_c \approx J_0(\xi_c/L_p)^{3/2}d^{-1/2}$ and from experimental $J_c$ values yielded the absurd result $L \approx 17 \text{ nm}$ for $r_p = 20 \text{ nm}$, i.e., mean spacing between precipitates $L$ appeared to be less then the mean radius $r_p$. Thus, the weak pinning theories are obviously inapplicable for explanation of critical current densities in YBCO films.

Another kind of theories was used to explain nearly constant $J_c(d)$ for $d < 1 \mu m$ in successively ion milled YBCO coated conductors [7] and for a similar behaviour of $J_c$ in the range $0.2 \mu m < d < 0.5 \mu m$ in usual in situ grown PLD films [10]. In the first case the expression for $J_c$ determined by plastic vortex depinning from infinitely strong point pinning centres was used [7, 11]:

$$J_c \approx J_0(\xi_c/L_p) \log(L_p/\xi_c),$$

where $\xi_c$ is the coherence length in $c$-axis direction: $\xi_c = \xi/\Gamma$, $\Gamma = \lambda_c/\lambda$ is the anisotropy parameter, $\Gamma \approx 5-7$ for YBCO. Alternatively, more familiar pin breaking mechanism of vortex depinning from sparse large point pins was considered in [10]. Neglecting pinning reduction by thermal fluctuations in the low field single-vortex limit the maximum $J_c$ was estimated to be:

$$J_c \approx 0.14 J_0 \Gamma \left( \frac{r_p^c}{L_p} \right)^{3/2},$$

where $r_p^c$ is a characteristic defect size in the $c$-axis direction, for $ab$-plane only $r_p^{ab} > \xi$ is supposed.
At the first glance both mechanisms allow desirable maximum experimental values of $J_c \approx 0.05 J_0$ for mean spacing $L_p \approx 30$ nm. Actually, these theoretical results greatly overestimate $J_c$ due to essential model simplifications. The divergence in pin-breaking result (2.2), which allows $J_c$ values exceeding $J_0$ for high $\Gamma$ values, is obviously unphysical.

It should be noted that (2.1) and (2.2) yield from the different depinning mechanisms for the same type of pinning centres. In this case the physically observed $J_c$ value is determined by the minimal one for both the mechanisms (2.1) and (2.2), in a contrary to the case of coexisting different type of defects (such as 0D and 1D) with resulting $J_c$ governed by the maximal partial $J_c$ value due to additive pinning forces. For materials with high anisotropy $\Gamma$ the mechanism of plastic depinning (2.1) with $J_c \propto \Gamma^{-1}$ dominates, but for YBCO with the moderate anisotropy the value (2.1) tends to be rather high with respect to the pin breaking one (2.2) and that is why unimportant for actual $J_c$. Reasons of $J_c$ overestimation for the pin breaking mechanism are discussed below.

There are two approaches to the strong pinning for the “pin-breaking” mechanism of current limitation. They can be called “dynamical” and “statistical” and differ by the method of calculation an area of vortex trapping by a defect, $u_0^2$. The dynamical approach comes from the early Labusch works [12] and takes into account the anisotropy of trapping lengths with respect to the Lorentz force direction (longitudinal $u_l$ and transversal $u_t$, $u_0^2 \approx 2u_l u_t$ and $u_l > u_t$). A simpler statistical approach ignores such anisotropy, because the trapping area is calculated from the energy criterion by equalizing the energy of VLL elastic deformation and the pinning energy in the absence of current. It is implicitly based on two assumptions: (a) a statistical accommodation function of VLL to an array of defects remains unchanged despite a rearrangement of vortices by applied current and (b) vortices effectively reach a thermal equilibrium for characteristic experimental time scales. The statistical approach gives somewhat underestimated values of the trapping area and $J_c$, taking very crudely thermalization processes into account, but it is incompatible with explicit consideration of flux creep. On the other hand, the dynamical approach somewhat overestimates $J_c$ and requires such an explicit flux creep account for a correct comparison with experiment. The advantage of dynamical approach (unattainable for the statistical one) is an ability to describe the weak to strong pinning crossover [13] based on the Labusch criterion for strong pinning: $\partial^2 f_p = \partial^2 e_p > C$, where $e_p(r)$ and $f_p(r)$ are the energy of vortex interaction with a defect and the respective pinning force (with derivatives taken at the maximal negative curvature of $e_p(r)$ above the inflection point), and $C$ is the effective elasticity constant [12]. This criterion is assumed far to be fulfilled and the statistical approach is used below for simplicity.

The detailed theory of strong pinning by sparse large point defects in dynamical approach was developed in [14] for bulk layered superconductors in inclined magnetic fields for the medium-field range. It was adapted to the case of moderately anisotropic superconductors and a few limits were also investigated in the statistical approach, such as the low field single-vortex limit (2.2) and the 2D case of strong pinning in thin film, for which the following result was obtained [10]:

$$J_c = \left( U_p / e_0 \right) \left( n_p \Gamma^2 f_p d / \phi_0 \right) \approx J_0 n_p d (r_p^f \Gamma)^2.$$ (2.3)
This expression was used to explain the experimentally observed $J_c$ reduction with film thickness below $d \approx 200$ nm [10]. However, it is obviously overestimated, because even for moderate YBCO anisotropy with $\Gamma \approx 5$–7 the unrealistic value of $J_c \approx J_0$ is obtained from (2.3) for easily achieved set of parameters: $L_p = r_p^{-1/3} \approx r_p^{1/3} \Gamma \approx 100$ nm. On the other hand, if the crossover of $J_c$ value with thickness from (2.3) to (2.2) is taken into account, the 2D result (2.3) is valid only for $d < d' = (L_p/\Gamma)(L_p/\pi r_p)^{1/2}$. It should be noted that $d' \approx 50$ nm for the proposed parameters, while the $J_c$ increase with $d$ was measured in the range 100–200 nm [10]. That is why the strong pinning $J_c$ dependences on anisotropy and film thickness (2.3) look rather doubtful.

First of all let us elucidate a role of anisotropy. It controls all pin-breaking $J_c$ expressions consequently via the non-local VLL tilt modulus $C_{44} = \epsilon_0 \Gamma (\kappa a_0)^{-2}$ or the line tension $\epsilon_1 = \epsilon_0 \Gamma^{-2}$ in the single-vortex limit and then via the trapping area. In different limits and approaches $\Gamma$ is involved in $u_0^2$ with various but positive exponents up to 2 in the thin film 2D case (2.3). The same diminution of line tension with $\Gamma$ increases prohibits the plastic depinning process (vortex – pin decoupling starting from the annihilation of opposite-sign (anti-parallel) vortex sections outside the pin volume, see figure 1) and decreases the respective $J_c$ value (2.1). However, even without plastic limitation of $J_c$, the pin-breaking trapping area itself cannot diverge with $\Gamma$. The restriction of trapping length $u_0$ with increasing magnetic field by the VLL spacing $a_0$ was taken into account in [10]. It determines the high-field limit with $J_c \propto J_0 (r_p/\Gamma)^{-2}$ (without any dependence on anisotropy) valid for $B > \phi_0 (r_p^{1/3} \Gamma)^{-2}$. But in low and medium-field regimes the trapping length is also restricted by the mean pin spacing $L_p$. Other words, the general result for pin-breaking on sparse strong point pins $J_c = (f_p/\phi_0) n_0 u_0^2$ [10] is valid only for $u_0 < L_p$, which yields roughly $L_p^2 > r_p^{1/3} \Gamma a_0$ for medium-field and $L_p > r_p^{1/3} \Gamma^{-2}$ for low-field regimes. In the opposite case the pin-breaking critical current

$$J_c \approx (f_p/\phi_0) L_p^{-1} J_0 \left( r_p^2 / L_p \right)$$

(2.4)

is independent on anisotropy and magnetic field. Actually, low-field pin-breaking result (2.2) for rather high anisotropy and pin density is replaced by the lower value (2.4), while the plateau of $J_c(B)$ dependence is extended to the higher value $B' \approx \phi_0 (r_p^{1/3} \Gamma)^{-2}$.

If $r_p^{1/3}$ is formally increased in this consideration, result (2.4) is valid roughly for the parameter range $r_p^{1/3} < L_p < r_p^{1/3} \Gamma$, tending to $J_c \sim J_0$ for $L_p < r_p^{1/3} < d$. This means that with increasing length of non-superconducting cylindrical inclusions there is a continuous crossover from the pinning on sparse large point defects to the most effective pinning on 1D correlated pins threading the whole film. Such quasi-linear defects as finite-length (comparable with $d$) edge dislocations, which are terminated on nanoscale inclusions or some planar defects, may be rather important in real films. However, an accurate consideration of such pinning objects requires a generalization of thin-film pin-breaking limit presented in the following section.

3. Pin-breaking trapping area for film in normal magnetic field

It was mentioned above that in the statistical approach the pin-breaking trapping area is calculated by equalizing the energy of VLL elastic deformation to the pinning energy. The energy of elastic deformation of a vortex segment (length $L$) displaced by $u$ in the lattice cell is [10]:

$$U_{el} = C_{66} u^2 L + C_{44} \frac{u^2}{L} a_0^2$$

(3.1)

where $C_{66}$ is the shear elastic modulus. In the bulk medium-field case the optimal length of vortex fluctuation is obtained by minimizing (3.1) by $L$, that yields $L_0 \approx (C_{44}/C_{66})^{1/2} a_0$. This value represents a distance over which a vortex segment can fluctuate independently from its neighbours. The trapping
length $u_0$ is determined from the condition $U_{el}(u_0, L_0) = U_p$, yielding for the trapping area $u_0^2 = U_p/2a_0(C_{66}C_{44})^{1/2}$.

For the thin film case $d < L_0$ just $L$ was substituted in (3.1) by $d$. Neglecting the first term $u_0^2 = U_p d/(C_{44}a_0^2)$ for trapping area and (2.3) for $J_c$ is obtained [10]. But such a crude procedure neglects a possibility of defect trapping by vortex in two different ways: by vortex bending with distortion $u_1$ (described by $C_{44}$) and through a shift of vortex centre from VLL site by $u_2$ (described by $C_{66}$), with $u = u_1 + u_2$ (figure 2). Implicitly, (2.3) corresponds to the condition $u_1 = u_2$, which is the case only at $d = L_0$ as it will be shown below. However, at $d < L_0$, the relation between partial distortions $u_1$ and $u_2$ should be evaluated by minimizing the energy of elastic deformation in the form:

$$ U_{el} = C_{66}u^2 r_p^c + C_{66}(u - u_1)^2(d - r_p^c) + C_{44} \frac{u_1^2}{d - r_p^c} a_0^2, \quad (3.2) $$

that yields for optimal distortions

$$ u_{01} = \frac{C_{66}(d - r_p^c)^2}{C_{66}(d - r_p^c)^2 + C_{44}a_0^2} u, \quad u_{02} = \frac{C_{44}a_0^2}{C_{66}(d - r_p^c)^2 + C_{44}a_0^2} u. \quad (3.3) $$

It follows easily from (3.3) that $u_{01} = u_{02}$ only at $d = L_0$. Substituting (3.3) into (3.2) and equalizing the last to pinning energy the following expression for the trapping area in thin film is obtained:

$$ u_0^2 = U_p \frac{C_{66}(d - r_p^c)^2 + C_{44}a_0^2}{C_{44}C_{66}a_0^2 + C_{44}C_{66}(d - r_p^c)^2 r_p^c}. \quad (3.4) $$

It should be noted that even for the point-pin case $r_p^c << d$ linear by the film thickness $d$ result for $J_c$ (2.3) is obtained only if the second term in numerator of (3.4) is omitted. On the other hand, in the cases $r_p^c \rightarrow d$, as well as $C_{66}d^2 << C_{44}a_0^2$, the trapping area (3.4) is reduced to a simple 2D form $u_0^2 = U_p/C_{66}d$, which equivalent to that used in [4, 5] (with $u_0$ denoted as $\delta$) for pinning by threading dislocations. Taking into account that $U_p \approx e_0 \sigma_{p}^c F(\tau)$, where $F(\tau)$ is a temperature dependent smearing factor, $\tau = 1 - T/T_c$, $C_{66} \approx e_0/4a_0^2$ and $C_{44} \approx e_0/\Gamma a_0^2$, (3.4) yields

![Figure 2](https://example.com/fig2.png)

**Figure 2.** Two types of VLL distortions by a strong point-like pin in a thin film with $d < L_0$. 

\[ u_0^2 = 4a_0^2r_p^2F(\tau) \frac{\Gamma^2(d - r_p^c)}{4a_0^2d + r_p^c(d - r_p^c)}. \] (3.5)

The value of trapping distance \( \delta_e \equiv u_0 = r_0a_0/\xi \propto (\tau B)^{1/2} \) [4, 5] is recovered from (3.5) for threading edge dislocations with \( r_p^c = d \) and \( F(\tau) \approx (r_0/2\xi(\tau))^2 \) for \( r_0 \ll \xi(\tau) \) [15], where \( r_0 \) is the radius of non-superconducting dislocation core and \( \xi(\tau) = \xi_0\tau^{-1/2} \) is the coherence length.

4. Conclusion
The presented generalization of strong pinning theory in thin film limit \( d < L_0 \) allows unification of 0D and 1D pin cases. The pin-breaking trapping area is calculated as a function of the film thickness \( d \), material anisotropy \( \Gamma \), induction of the applied normal magnetic field (VLL spacing \( a_0 \)), and the pin extension in c-axis direction, \( r_p^c \). It is shown that the experimental thickness dependences of \( J_c \) cannot be explained in frameworks of simple fundamental crossovers, such as 2D – 3D in the weak pinning limit. These dependences differ drastically for various kinds of substrates and film deposition technologies. So, they are governed by specific structure peculiarities like a threading edge dislocation structure and a spatial distribution of precipitates and finite-length dislocations.

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