A new approach for advanced plasticity and fracture modelling

N Park¹, T B Stoughton² and J W Yoon³,⁴*

¹ Metal Forming Technology R&D Group, Korea Institute of Industrial Technology, Incheon, South Korea
² General Motors Global Research and Development Center, Warren, USA
³ School of Mechanical, Aerospace and Systems Engineering, KAIST, Daejeon, Republic of Korea
⁴ School of Engineering, Deakin University, Geelong Waurn Ponds, Australia

E-mail: j.yoon@deakin.edu.au, j.yoon@kaist.ac.kr

Abstract. A solution to the challenge of modelling the general yielding behaviour including fracture initiation is proposed based on non-associated flow rule. A decoupled formulation with employment of a Lagrangian interpolation function is established to develop a general fracture criterion that considers the mutual effect of the material anisotropy, the strain rate, and the temperature on the onset of fracture. An advanced constitutive law is also developed to accurately describe the evolution of the anisotropy/ asymmetry-induced distortional yielding behaviour, using neither any interpolation nor optimization techniques for the calibration of the yield surface. The proposed models are successfully applied to various types of metallic materials to validate their noticeable flexibility and applicability in describing the general material behaviour during various forming processes.

1. Introduction
An accurate description of the material yielding and fracture behaviours is of great importance for the trustworthy numerical analysis that helps to develop an efficient and effective guideline for the real manufacturing process of the structural component. The metal sheet produced by cold rolling processes generally shows anisotropic characteristics in their deformation and fracture behaviours, and therefore, the stress and strain states at the onset of yielding and fracture are strongly dependent on both loading condition and direction. Normally, a yield surface evolves with variation of its shape and size during plastic deformation since the work hardening rate differs with respect to the loading direction. The non-uniform evolution of the yield surface becomes a critical issue especially when considering the materials that show the severe tension/compression asymmetry on the yielding behaviour. Meanwhile, structural components usually undergo a broad range of the deformation modes with variation of the strain rate, the temperature, etc. during complicated forming processes, which causes a primary difficulty in mathematically modelling both a constitutive law and a fracture criterion: for the constitutive modelling, an initial yield surface varies with respect to the strain rate and the temperature and it evolves in a different manner as the plastic deformation goes on [1]; for the fracture modelling, there exists no general tendency of the fracture limit over a wide range of the loading condition when considering the mutual influence of the material anisotropy, the strain rate, and the temperature on the fracture initiation
A major concern here is that the advanced yield criteria previously proposed normally features an isotropic evolution of the yield surface determined at an initial stage of the material yielding, and thereby neglect the evident changes in its shape as implied by the experimental tests. Therefore, it is challenging to develop the mathematical formulae that accurately describe the material yielding as well as the fracture limit.

In this paper, novel approaches proposed by Park et al. [7, 8] are reviewed in order to consider solutions to the challenges of modelling not only an advanced yield criterion that describes the evolution of the anisotropy/asymmetry-induced distortional yielding behaviour, but also a general fracture criterion that consider the effects of the material anisotropy, the strain rate, and the temperature on the fracture initiation, simultaneously. Comparisons of the experimental data including a virtual data set with predictions of the proposed criteria show that the proposed criteria provide sufficient predictability in describing the general material behaviour in consideration of the level of the plastic strain, the strain rate, and the temperature.

2. Advanced plasticity and fracture modelling

2.1. Constitutive modelling for describing symmetric and asymmetric yielding behaviours

An advanced yield criterion proposed by Park et al. [7] is defined by three multiplicative functions as follows:

\[
\phi(\sigma, \lambda) = \left[\frac{\sigma_{11}(\sigma_{11} - \sigma_{22}) - 2\sigma_{22}}{\sigma_0^2(\lambda)} + \frac{\sigma_{22}(\sigma_{22} - \sigma_{11})}{\sigma_0^2(\lambda)} + \frac{4\sigma_{12}^2}{\sigma_{45}^2(\lambda)} + \frac{\sigma_{11}\sigma_{22} - \sigma_{12}^2}{\sigma_{EB}^2(\lambda)}\right]^{1/2} \cdot X^i \cdot e^{K(\sigma, \lambda)}(\eta),
\]

with

\[
X = (C_0 - 1)\left[2\sin(\theta_L + \frac{\pi}{3})\right]^4 - 2(C_0 - 1)\left[2\sin(\theta_L + \frac{\pi}{3})\right]^2 + C_0,
\]

\[
K(\sigma, \lambda) = \frac{k_u(\lambda)\sigma_{11}(\sigma_{11} - \sigma_{22}) + k_{90}(\lambda)\sigma_{22}(\sigma_{22} - \sigma_{11}) + k_{45}(\lambda)\sigma_{12}^2 + k_{EB}(\lambda)(\sigma_{11}\sigma_{22} - \sigma_{12}^2)}{\sigma_{0M}^2(\sigma)},
\]

where \(\theta_L\), \(\sigma_{0M}\), and \(\lambda\) denote the Lode angle parameter, von Mises effective stress, and the plastic compliance factor associated with the equivalent plastic strain, respectively. It is noted that \(\varepsilon^p = (\partial \sigma_{0M}/\partial \sigma)\lambda\) and \(\lambda = \int \lambda\). Here, \(\sigma_{0M}\) denotes the plastic potential. In equation (3), the evolutionary tension/compression asymmetric coefficients of \(k_u\), \(k_{45}\), \(k_{90}\), and \(k_{EB}\) are defined as \(k_i(\lambda) = \frac{\ln|\sigma_{0M}^i(\lambda)/\sigma_{0M}^f(\lambda)|}{\sigma_{0M}^i(\lambda)}\) where the subscript of \(i\) substitutes for 0, 45, 90, and EB, denoting the loading angle to the rolling direction and the Equi-Biaxial stress state. It is noted that a form of a condition function proposed by Lee et al. [9] was employed in the formulation of the asymmetry function. The superscripts of \(T\) and \(C\) indicate uniaxial tension and compression loading states, respectively. In Park et al. [7], the advanced yield criterion was constructed based on the Stoughton and Yoon (2009) criterion [10] (hereinafter, S–Y 2009 criterion), which is given in the first part of the right side of equation (1), describing the symmetric evolution of the yield surface. Since the S–Y 2009 criterion features the non-associated flow rule, the advanced yield criterion accordingly needs a plastic potential to properly describe the direction of plastic flow. The second and third parts of the right side of equation (1) take part in controlling the flatness and the asymmetry of the yield surface, respectively, so that the advanced yield criterion is capable of describing the general yielding behaviours of various metals and alloys with not only Face-Centred Cubic (FCC) and Body-Centred Cubic (BCC) structures, but also hexagonal Closed Pack (HCP) crystal structure. The additional term of \(I(\eta)\) in equation (1) was devised with the stress triaxiality to distinguish typical stress states in dealing with the tension/compression asymmetry on the material yielding. In reliable consideration of the material hardening behavior under the uniaxial tension and compression, a couple of constraint sets are enforced with the formulation of
In (\( \eta \)): i.e., \( I(\eta) = 0 \) for \( \eta = 1/3, 2/3 \) and \( I(\eta) = -1 \) for \( \eta = -1/3, -2/3 \), respectively. In this paper, a 3rd-degree polynomial that satisfies all the constraints aforementioned was used for the sake of simplicity in the mathematical formulation. As in the case of the S–Y 2009 criterion, the material deformation modes are distinguished by the value of \( \phi(\sigma, \tilde{\lambda}) \), i.e., \( \phi(\sigma, \tilde{\lambda})<1 \) for elastic deformation; \( \phi(\sigma, \tilde{\lambda})=1 \) for elasto–plastic deformation.

A key advantage of the proposed criterion is that it directly employs the eight independent flow stress curves of \( \sigma_0^1(\tilde{\lambda}), \sigma_5^1(\tilde{\lambda}), \sigma_{90}^1(\tilde{\lambda}), \sigma_{90}^1(\tilde{\lambda}), \sigma_0^1(\tilde{\lambda}), \sigma_5^1(\tilde{\lambda}), \sigma_{90}^1(\tilde{\lambda}), \) and \( \sigma_{90}^1(\tilde{\lambda}) \), enabling no need of either interpolation or optimization procedure for the calibration of the model coefficients to deal with the yield surface evolution with the plastic deformation. In addition, the proposed criterion can be simply extended to the general yield criterion that considers the mutual effect of the plastic strain, the strain rate, and the temperature by imposing the strain rate and temperature effect into the basic strain hardening model, i.e., \( \tilde{\sigma} = \tilde{\sigma}(\tilde{\varepsilon}^p, \dot{\varepsilon}^p, T) \).

### 2.2. Modelling of a general fracture criterion based on a decoupled formulation

In Park et al. [8], a phenomenological fracture criterion was proposed in order to accurately model the onset of fracture in consideration of the mutual effect of the material anisotropy, the strain rate, and the temperature. The proposed criterion was formulated based on a decoupled formation with the use of a Lagrangian interpolation function as follows:

\[
H(\theta, \dot{\varepsilon}^p, T, \sigma) = \sum_i N_i(\theta, \dot{\varepsilon}^p, T) h_i(\sigma),
\]

where

\[
N_i(\theta, \dot{\varepsilon}^p, T) = N_j^p(\theta) N_j^p(\dot{\varepsilon}^p) N_j^K(T).
\]

Here, the superscripts of \( p, q, \) and \( r \) denote the number of sub-divisions in each coordinate system defined by the loading direction, the strain rate, and the temperature. The functions of \( N_i \) and \( h_i \) respectively denote the Lagrangian interpolation function and the fracture limit at a certain loading condition of \( \theta, \dot{\varepsilon}^p, \) and \( T \) where the subscript of \( i \) represents a typical node denoted by the three numbers of \( I, J, \) and \( K \) corresponding to the positions of the local coordinates \( \theta, \dot{\varepsilon}^p, \) and \( T \). Note that the Lagrangian interpolation function has the delta function property by its mathematical definition, and therefore, the following relationships can be held:

\[
N_i(\theta, \dot{\varepsilon}^p, T) = \delta_{ij}, \sum_i N_i(\theta, \dot{\varepsilon}^p, T) = 1,
\]

where the subscript of \( j \) denotes the nodal point regarding a selected set of the loading conditions for which fracture quantities are considered to be evaluated. From the merit of the delta function property, the fracture envelope over a broad range of loading conditions can be constructed while maintaining the fracture predictability at a certain loading condition. In modeling the function \( h \) associated with the physical quantity that defines the occurrence of fracture, any kinds of fracture criteria including an uncoupled damage approach can be used once the model deals with the fracture quantities affected by various physical factors aforementioned. In this paper, the Magnitude of Stress Vector (MSV), which is an \( L^2 \)-norm of the principal stress vector defined as \( ||\sigma|| = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2} \), was simply employed for the construction of the function \( h \) to validate the flexibility of the proposed modeling approach. It is noted that Khan and Liu [11] found there exists an evident relationship between the MSV and the magnitude of the first stress invariant \( I_1 \) at the onset of fracture. The polynomial function was employed to define the fracture quantities, which can provide more flexibility for the prediction of physical quantities at the onset of fracture. The form of the general fracture criterion can be, then, written in terms of \( I_1 \) as below:

\[
H(\theta, \dot{\varepsilon}^p, T, \sigma) = \sum_{i=1}^m \sum_{k=0}^n N_i(\theta, \dot{\varepsilon}^p, T) c_{ik} I_1^k.
\]
where $c_{ik}$ denotes the material constant of the fracture criterion. It is worth to mention that the entire shape of the fracture envelope is strongly dependent on the order of the Lagrangian interpolation function.

3. Verification of the model performance

3.1. Flexibility of the proposed fracture criterion

In this paper, we simply focused on the validation of the anisotropic fracture criterion constructed based on the decoupled formulation as below:

$$H(\theta, \sigma) = \sum_{i=1}^{m} N_i(\theta) \cdot \|\sigma\| = \sum_{k=0}^{n} N_k(\theta) \cdot c_{ik} I_k^2. \quad (8)$$

![Figure 1. Prediction of the anisotropic fracture limit over a wide range of stress states.](image)
Due to the lack of experimental data, the virtual data set was used in order to confirm the model performance of the proposed fracture criterion. From the comparison of the virtual data with the predictions from the proposed fracture criterion, as represented in figure 1, it can be concluded that the proposed one has a noticeable flexibility in modelling the anisotropic fracture limit across a broad range of loading states while keeping non-directionality of the equi-biaxial stress state on the onset of fracture, and this mainly results from the decoupled formulation between the stress state and the loading direction dependencies on the fracture initiation.

3.2. General applicability of the proposed yield criterion
With employment of a shape adjustment term of $X^k$ in equation (1) in modelling the advanced yield criterion based on the S–Y 2009 criterion, various kinds of material yielding behaviours having quadratic (Hill-like), non-quadratic (Tresca-like), and intermediate type of yield surface can be reasonably described. Note that the flatness of the yield surface is controlled by the exponent of $k$, and the form of the shape adjustment term allows to have its first derivatives and the values being unity at $\theta = 0, \pi / 3$ so that it makes possible to control the entire shape of the yield surface while guaranteeing the prediction for the work hardening behaviour under the uniaxial and equi-biaxial stress state. In the formulation, the S–Y 2009 criterion plays a role of the upper boundary on the shape of the yield surface, as represented in figure 2, since the shape adjustment term becomes unity when the exponent of $k$ is set to zero. It is noted Lee et al. [12] also proposed a yield criterion through coupling of quadratic and non-quadratic functions to describe the anisotropic hardening based on the S–Y 2009 criterion. For the purpose of comparison, the yld2000-2d, which is well-known as one of the most reliable models that describes an initial yielding state, is also reviewed. Figure 3 reveals that the proposed and the Lee–Stoughton–Yoon 2017 criteria well match with the experimental data as plastic deformation goes on; however, the yld2000-2d slightly overpredicts the work hardening, especially under the equi-biaxial stress state, since it basically assumes the isotropic expansion of the initial yield surface with a fixed shape. Therefore, it is not able to provide a sufficient prediction of the material yielding behaviour especially in the case that a severe anisotropic work hardening takes place with plastic deformation.

![Figure 2](image2.png)
**Figure 2.** Variation of the yield locus according to the value of the exponent $k$.

![Figure 3](image3.png)
**Figure 3.** Prediction of the yield surface evolution from the Yld2000-2d, S–Y 2009, Lee–Stoughton–Yoon 2017, and the proposed criteria (after Park et al. [7]).
By including the asymmetry function $e^{K\varepsilon f(\theta)}$ in the formulation of the advanced yield criterion, the subsequent evolution of the anisotropy/asymmetry-induced distorted yield surface can be properly described as represented in figure 4. In light of the mathematical characteristic of the shape adjustment term, the flatness of the distorted yield surface can be also controlled by the exponent of $k$ as shown in figure 5.

**Figure 4.** Evolution of the distorted yield locus of Ti–3Al–2.5V alloy (after Park et al. [7]).

**Figure 5.** Influence of the shape adjustment term on the shape of the distorted yield surface (after Park et al. [7]).

**Figure 6.** Prediction of the initial yield loci of Ti–6Al–4V alloy at various loading conditions (after Park et al. [7]).

**Figure 7.** Normalized yield loci of Ti–6Al–4V alloy at various loading conditions (after Park et al. [7]).
Table 1. Material parameters of the KHL model for Ti–6Al–4V alloy (data after Khan et al. [1]).

| Loading Condition | Loading Direction | \(A\) [MPa] | \(B\) [MPa] | \(n_0\) | \(n_1\) | \(c_k\) | \(m\) |
|-------------------|-------------------|-------------|-------------|--------|--------|--------|-------|
| Compression       | RD                | 1273.12     | 868.44      | 0.5447 | 1.5343 | 0.0284 | 1.9368|
|                   | TD                | 1104.52     | 509.23      | 0.3688 | 0.8693 | 0.0213 | 2.6770|
|                   | ND                | 1108.45     | 759.00      | 0.5181 | 0.8693 | 0.0176 | 2.6450|
| Tension           | RD                | 1124.07     | 604.98      | 0.7626 | 1.4628 | 0.0215 | 2.5183|
|                   | TD                | 1054.02     | 668.07      | 0.8343 | 1.2887 | 0.0182 | 2.5642|
|                   | ND                | 1052.37     | 439.84      | 0.6201 | 0.9509 | 0.0177 | 2.7110|

where \(\dot{\varepsilon}^p = 10^{-1} \text{s}^{-1}\), \(D_0^p = 10^6 \text{s}^{-1}\), \(T_m = 1933\text{K}\), and \(T_r = 296\text{K}\), respectively.

As stated in Section 2.1, the proposed yield criterion can be easily extended to the general one that describes the continuous evolution of the distorted yield surface whose shape is strongly dependent on the level of the plastic strain, the strain rate, and the temperature. With employment of the Khan–Huang–Liang (KHL) model [1] in equation (9), the strain rate- and temperature-dependent distorted yield surface can be described, using neither any interpolation nor optimization techniques for the calibration of the yield surface.

\[
\bar{\sigma}(\dot{\varepsilon}^p, \dot{\varepsilon}^p, T) = A + B \left( 1 - \frac{\ln \dot{\varepsilon}^p}{\ln D_0^p} \right)^{n_1} \left(\frac{\dot{\varepsilon}^p}{\dot{\varepsilon}^p} \right)^k \left[ \frac{T_m - T}{T_m - T_r} \right]^m,
\]

where \(A, B, n_0, n_1, c_k\), and \(m\) are the material parameters. \(T_m\) and \(T_r\) are the melting and reference temperatures. \(\dot{\varepsilon}^p\) and \(D_0^p\) are the reference strain rate and the constant used to non-dimensionalize the strain rate term. Figures 6 and 7 show the prediction of the variation of the yield locus for Ti–6Al–4V alloy with the use of the KHL model parameters given in Table 1. The initial yield stress surface under the uniaxial tension and compression loading states are represented in figure 8. It is worth to mention that, in Khan et al. [1], the yield stresses under equi-biaxial tension and compression conditions were obtained equivalently by those under equi-biaxial tension and compression along the normal direction (ND) of the metal sheet based on the experimental observation that the material yielding behavior of Ti
alloy is not sensitive to the hydrostatic pressure. In comparison of the experimental results with the predictions from the proposed yield criterion, it can be concluded that the proposed yield criterion has remarkable applicability in describing the subsequent evolution of the anisotropy/asymmetry-induced distorted yield surface for various types of metallic materials. It is worth to note that the convexity of the yield surface can be guaranteed by confirming a general graphical description of 3D yield surface in consideration of its practical engineering application.

4. Conclusions

Novel approaches for advanced plasticity and fracture modelling have been discussed. Comparisons of the experimental data including the virtual data set with predictions from the proposed criteria indicate that the proposed modelling approaches have not only remarkable flexibility but also general applicability in describing the material behaviour in conjunction with the mutual effect of the strain rate and the temperature while producing reasonable predictions on both yielding and fracture initiation. Therefore, it is concluded that the proposed modelling approaches can contribute to the establishment of the simple but effective way to reliably describe complicated material behaviours.

5. Acknowledgements

The authors acknowledge the support from the General Motors Research and Development Center. This work is also partially supported by the BK21 (Brain Korea 21) Plus Project.

References

[1] Khan A S, Yu S and Liu H 2012 Deformation induced anisotropic responses of Ti–6Al–4V alloy Part II: A strain rate and temperature dependent anisotropic yield criterion. Int. J. Plast. 38, pp 14–26.
[2] Lou Y and Yoon J W 2017 Anisotropic ductile fracture criterion based on linear transformation. Int. J. Plast. 93, pp 3–25.
[3] Yoon J W, Zhang S and Stoughton T B 2017 Orthotropic ductile fracture criterion based on linear transformation. In J. Phys. Conf. 896, p. 012110.
[4] Rahmaan T, Zhou P, Butcher C and Worswick M J 2018 Strain rate and thermal softening effects in shear testing of AA7075-T6 sheet. In EPJ Web of Conf. 183, p 02037.
[5] Park N, Huh H, Lim S J, Lou Y, Kang Y S and Seo M H 2017 Fracture-based forming limit criteria for anisotropic materials in sheet metal forming. Int. J. Plast. 96, pp 1–35.
[6] Park N, Huh H, Yoon J W 2018 Anisotropic fracture forming limit diagram considering non-directionality of the equi-biaxial fracture strain. Int. J. Solids and Struct. 151, pp 181–194.
[7] Park N, Stoughton T B and Yoon J W 2019 A criterion for general description of anisotropic hardening considering strength differential effect with non-associated flow rule. Int. J. Plast. (Submitted)
[8] Park N, Stoughton T B and Yoon J W 2019 A new approach for fracture prediction considering general anisotropy of metal sheets. Int. J. Plast. (Submitted)
[9] Lee E H, Stoughton T B and Yoon J W 2018 Kinematic hardening model considering directional hardening response. Int. J. Plast. 110, pp 145–165.
[10] Stoughton T B, Yoon J W 2009 Anisotropic hardening and non-associated flow in proportional loading of sheet metals. Int. J. Plast. 25, pp 1777–1817.
[11] Khan A S and Liu H 2012 A new approach for ductile fracture prediction on Al 2024-T351 alloy. Int. J. Plast. 35, pp 1–12.
[12] Lee E H, Stoughton T B and Yoon J W 2017 A yield criterion through coupling of quadratic and non-quadratic functions for anisotropic hardening with non-associated flow rule. Int. J. Plast. 99, pp 120–143.