GENERATION OF GRAVITATIONAL WAVES AND SCALAR PERTURBATIONS
IN INFLATION WITH EFFECTIVE $\Lambda$-TERM
AND T/S STORY

V.N.LUKASH and E.V.MIKHEEVA
Astro Space Center of P.N.Lebedev Physical Institute,
Profsoyuznaya Street 84/32, Moscow, 117810, Russia

We argue that gravitational wave contribution to the cosmic microwave background anisotropy at angular scale $\sim 10^0$ may exceed 50% for some models of hybrid inflation producing standard cosmology with the density perturbation slope $n \simeq 1$.

1 Introduction

Two types of metric perturbations $h_{\alpha\beta}$ generating at inflation driven by scalar field, can contribute to large-scale cosmic microwave background anisotropy through the SW effect\textsuperscript{1}. They are the Scalar (density) and Tensor (gravitational waves) perturbations which are presented as follows: $h_{\alpha\beta}/2 = A\delta_{\alpha\beta} + B_{\alpha\beta} + G_{\alpha\beta}$. The first and second terms correspond to the scalar mode whereas the third one is for the tensor one ($G_{\beta}^\alpha = G_{\alpha,\beta}^\beta = 0$). Their contribution into $\Delta T/T$ may be standardly separated over $S$ and $T$ parts:

$$\left\langle \left( \frac{\Delta T}{T} \right)^2 \right\rangle_1 = \sum_\ell \langle a_\ell^2 \rangle = \sum_\ell \left( \langle a_\ell^2 \rangle_S + \langle a_\ell^2 \rangle_T \right) = S + T. \quad (1)$$

The available experimental data are not sufficient yet to recognize and find gravitational wave fraction in $\Delta T/T$ (that could be possible, in principle, by means of a joint analysis of the dependence of the temperature fluctuation spectrum on scale, the amplitude of Dopper peak, etc., which is the matter of future observations). Nevertheless, today we can evaluate $T/S$ theoretically for various inflationary models to test the possible effect and see how large it may deviate from model to model and which particular properties of the model correlate with large $T/S$. Actually, we construct a counterexample to the common prejudice that “$T/S$ is negligible for the Harrison-Zel’dovich spectrum”, introducing here a general class of models which face the opposite conclusion: “$T/S$ may be about or even larger than 1 for $n \simeq 1$”. 

2 Generation of cosmological perturbations in inflationary model with effective Λ-term

A lot of inflation models has been discussed in the literature for T/S. The result was that large T/S could be achieved at the expense of the rejection from the Harrison-Zeldovich spectrum: T/S ≥ 1 for nS ≤ 0.8, the “red” spectra. Thus, it would be interesting to study possible T/S in models producing “blue” (nS > 1) spectra of density perturbations. Here we present a general class of models based on the only scalar field with the potential:

\[ V = V_0 + \frac{m^2 \varphi^2}{2}, \]

where \( V_0 \) and \( m \) are constants, \( V_0 \) describe the effective (metastable) Λ-term. The mechanism of its decay is not fixed here and may be arbitrary, for example, with help of another scalar field like in the hybrid inflation.

It is clear that the inflaton dynamics in the potential (2) has two regimes and, consequently, the spectra of cosmological perturbations has two asymptotics. The regimes are separated by the critical value of field \( \varphi_{cr} \) at which the first term is equal to the second \( \varphi_{cr}^2 = 2V_0/m^2 \).

At the first stage (we call it “red” asymptotic — by the form of the density perturbation spectrum) \( \varphi \) is large and the massive term dominates the evolution. This is similar to the case of well-known chaotic inflation, so it is easy to write down the spectra of the perturbations as follows (\( c = \hbar = 8\pi G = 1 \)):

\[ q^r_k = \frac{m \varphi^2}{4\sqrt{6}\pi}, \quad h^r_k = \frac{m \varphi}{2\sqrt{3}\pi}, \]

For \( \varphi < \varphi_{cr} \) the constant term predominates. Here we obtain the “blue” asymptotic spectrum of density perturbations:

\[ q^b_k = H_0 c^2 n_{cr} \varphi_{cr}^2, \quad h^b_k = \frac{H_0}{\sqrt{2}\pi}, \]

where \( H_0 = (V_0/3)^{1/2}, \quad c_n^2 = 2^{1/2-n_s/2}(7-n_S)\Gamma(2-n_S/2)(3\sqrt{\pi})^{-1}, \quad \Gamma(x) \) is gamma function, \( q \) coincides with \( A \) in comoving frame and \( h_k^b \) is the spectrum of \( G_{\alpha\beta}G^{\alpha\beta}. \) \( \varphi \) in (3), (4) is taken at horizon crossing \( k = a(\varphi)H(\varphi), \) \( a \) and \( H = (V/3)^{1/2} \) are scale and Hubble factors, respectively. Note that we have the following relation between the spectrum slope at the “blue” asymptotic \( n_S \) and \( \varphi_{cr}: (n_S-1)(7-n_S) = 24\varphi_{cr}^{-2}, \varphi_{cr}^2 \geq 8/3, 1 < n_S < 4. \)

As for gravitational waves their spectrum remains universal for any \( k, \) but the density perturbation spectrum requires approximation (fitting both
asymptotics and becoming exact in the slow-roll limit):

\[ q_k = \frac{H_0 \varphi_{cr}}{4\pi y} (1 + y^2)^{1/2} (c_n^2 + y^2), \quad h_k = \frac{H_0}{\sqrt{2\pi}} (1 + y^2)^{1/2}, \quad (5) \]

where \( y = \varphi/\varphi_{cr} \). It is easy to see that the ratio of tensor to scalar spectra achieves its maximum at \( \varphi \simeq \varphi_{cr} \), where the minimum of density perturbation power occurs:

\[ \frac{h_k}{q_k} = \frac{2\sqrt{2}}{\varphi_{cr} c_n^2 + y^2} \leq \frac{\sqrt{2}}{c_n \varphi_{cr}} = D_n, \quad (6) \]

where \( D_2 = 0.8, D_3 = 1 \). We may suppose that here \( T/S \) gets its maximum as well.

\( T/S \) was calculated for potential (2) with DMR COBE window function. We obtained that \( T/S \) function is a two-parametric one and depends on \( n_S \) and \( k_{cr} \), the latter corresponds to \( \varphi_{cr} \). However, the earlier introduced approximation formula, \( T/S \simeq -6n_T \), holds in our case as well if both \( T/S \) and local \( |n_T| \) are taken at their maxima. We found a phenomenological relation between \( n_T \) and \( n_S \): \( |n_T| \simeq (n_S - 1)/4 \), thus, another formula for \( T/S \) can be proposed for this model: \( T/S|_{max} \simeq 1.5(n_S - 1) \).

### 3 Conclusions

Finally, we conclude: The inflation model predicting \( T/S > 1 \) has been constructed. A property of this model is “blue” spectrum of density perturbations for scales \( k \gg k_{cr} \). However in the region, where \( T/S \geq 1, k \sim k_{cr} \) and therefore the locally observed spectrum of cosmological perturbations is close to scale-invariant \( (n \simeq 1) \).

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