Does gravity prefer the Poincaré dodecahedral space?

Boudewijn F. Roukema

Toruń Centre for Astronomy, Nicolaus Copernicus University, ul. Gagarina 11, 87-100 Toruń, Poland
boud@astro.uni.torun.pl
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Abstract

The missing fluctuations problem in cosmic microwave background observations is naturally explained by well-proportioned small universe models. Among the well-proportioned models, the Poincaré dodecahedral space is empirically favoured. Does gravity favour this space? The residual gravity effect is the residual acceleration induced by weak limit gravity from multiple topological images of a massive object on a nearby negligible mass test object. At the present epoch, the residual gravity effect is about a million times weaker in three of the well-proportioned spaces than in ill-proportioned spaces. However, in the Poincaré space, the effect is 10,000 times weaker still, i.e. the Poincaré space is about $10^{10}$ times “better balanced” than ill-proportioned spaces. Both observations and weak limit dynamics select the Poincaré space to be special.
1 The missing fluctuations problem and well-proportioned spaces

Through the Einstein field equations, differential geometry and astronomical observations have converged during the past decade on the concordance model of physical cosmology \cite{19}. The concordance model matches an impressive range of astronomical observational data sets, including both the cosmic microwave background (e.g., \cite{29}) and surveys of gravitationally collapsed astrophysical objects. Nevertheless, the concordance model is seriously incomplete: it does not say what 3-manifold describes the comoving space that we inhabit. At best, it only chooses between the three classes of constant curvature 3-manifolds, i.e. between those of negative, zero and positive curvature. The barely noticed “elephant in the room” is the topology of comoving space. This question was raised by Karl Schwarzschild \cite{26, 27}, but has been studied mostly during the last decade and a half (e.g. \cite{13, 17, 30, 15, 3, 20}). What are comoving space’s global symmetries – i.e. the holonomy transformations, which are isometries that map objects to themselves in the simply connected covering space, which is the apparent space from the “naïve” observer’s point of view (something like the apparent space seen in a mirror-lined room)?

A century ago, the missing ether problem revealed by the Michelson-Morley experiment was solved by dropping the assumption that space and time are independent. Now we have the missing fluctuations problem (e.g. \cite{5, 6}, and references therein) of the COsmic Background Explorer (COBE) and Wilkinson Microwave Anisotropy Probe (WMAP) all-sky cosmic microwave background (CMB) experiments. Cosmic topology solves this problem. It also leads to dropping the assumption that dynamics is independent of the global topology of comoving space.

The missing fluctuations problem is solved by the generic prediction that in a multiply connected 3-manifold, correlations between density perturbations should vanish above a length scale similar to that of the size of the 3-manifold, since (comoving) objects larger than space itself cannot exist. This argument is weakened, but remains approximately valid in the observer’s apparent space, in particular on the surface of last scattering (SLS), which is a thin 2-spherical shell of radius about $10h^{-1}$ Gpc (comoving) centred at the observer. CMB fluctuations are seen primarily on the (SLS).

A cut-off in correlations at large scales was suspected in the COBE data, and confirmed in the WMAP data (Fig. 16, \cite{29}). Estimates of the chance of this lack of large angular scale (“low l”) structure occurring in an infinite, flat model range from 0.3% (Sect. 7, \cite{29}) to 12.5% (Table 5, \cite{9}) for the first-year WMAP data, decreasing to 0.03% for the three-year and five-year WMAP data \cite{5, 6}.

Not all multiply connected spaces with a short length give a strong cut-off effect. Spaces whose fundamental lengths are approximately equal, called “well-proportioned”, are the most likely to provide a large-scale cut-off in

\footnote{See fig. 10 of \cite{15} for various definitions of “size”.
}
2 The Poincaré dodecahedral space $S^3/I^*$

Estimates of the curvature of the Universe on the scale of the SLS, via the total density parameter $\Omega_{\text{tot}}$, hint at positive curvature, e.g. $\Omega_{\text{tot}} = 1.014 \pm 0.017$ from the WMAP three-year data together with Hubble Space Telescope key project estimates of the Hubble constant $H_0$ (Table 12, [28]); $0.9915 < \Omega_{\text{tot}} < 1.0175$ from combining the WMAP five-year data, baryonic acoustic oscillations in galaxy surveys and supernovae data [12]. The missing fluctuations problem and the curvature estimates led to the proposal of one of the well-proportioned spaces, the Poincaré Dodecahedral Space (PDS), $S^3/I^*$ [16], as a candidate for comoving space.

The PDS has a positively curved solid dodecahedron as its fundamental domain. Several groups have investigated this model [24, 1, 2, 10, 11, 4, 18, 14]. When thinking of the PDS fundamental domain projected to $\mathbb{R}^3$, the identification of opposite faces must take place by a translation followed by a rotation of $\pm \pi/5 = \pm 36^\circ$. If the model is correct, then despite the observed average lack of correlations on large scales, the correlations in certain directions should be high, since certain regions of comoving space are multiply viewed.

The exact set of points seen twice is defined by the identified circles principle [4, 8], but a larger amount of information in the WMAP data can be used by cross-correlating temperature fluctuations between adjacent copies of the SLS [23, 22] in apparent space. This method gives an optimal astronomical orientation of the fundamental dodecahedron, and by allowing the search algorithm (Markov chain Monte Carlo) to investigate arbitrary twists (i.e. not constrained to $\pm 36^\circ$), it yields an optimal twist when matching opposite faces. The optimal orientation found in the WMAP data for the fundamental domain gives a strong cross-correlation, i.e. strong correlations exist between apparently distant points on the sky in a small number of directions, despite the missing fluctuations problem, and the optimal twist angle is $(+39 \pm 2.5)^\circ$, consistent with that required, despite the freedom allowed by the search algorithm [23, 22].

Is this just an empirically preferred space, or could the Poincaré space be favoured by gravity?

3 The residual gravity acceleration effect

It has been shown heuristically that global topology in a universe containing at least one density perturbation can feed back on local dynamics [21]. This can be seen most easily in a $T^1 \times \mathbb{R}^2$ model of length $L$, considering a massive object of mass $m$, its two adjacent images in the covering space $\mathbb{R}^3$, and a

$^1I^*$ is the binary icosahedral group.
massless test particle displaced $x$ from the massive object in the short direction (Fig. 3, [21]). In the (Newtonian) weak limit, in addition to being accelerated by the “local” copy of the massive object, the test particle is pulled in opposite directions towards the two distant copies of the massive particle. The latter two accelerations are nearly, but not quite, equal. The net effect is that the test particle has a small extra pull towards the closer of the two distant images of the massive object, i.e. it falls towards the “local” copy of the massive object more slowly than would be expected if multiple images were ignored. This is the “residual gravity effect”. In $T^1 \times \mathbb{R}^2$, for an object of fixed mass $m$, to first order in $x/L$, where $x/L \ll 1$, the residual acceleration $\ddot{x}$ is proportional to $x/L$.

What happens in other spaces, in particular the well-proportioned spaces? Perfectly regular well-proportioned spaces $T^3$, $S^3/T^*$, $S^3/O^*$, and $S^3/I^*$ are “better balanced” than ill-proportioned spaces such as $T^1 \times \mathbb{R}^2$. When considering all the adjacent topological images of a massive object and a negligible mass test particle displaced from it slightly in an arbitrary direction [21, 25], the first order term in $x/L$ (for $T^3$) or $x/r_C$ (for the spherical spaces, with curvature radius $r_C$) of the residual gravity effect vanishes. Small perturbations from perfect isotropy destroy this equilibrium. However, they induce a first order effect that tends to oppose the anisotropy and restore the equilibrium, favouring an equilibrium state in which the first order term cancels to zero.

However, what is especially surprising is that one of these four well-proportioned spaces is “more equal than the others”. The highest order term for the residual gravity effect in $T^3$, $S^3/T^*$, and $S^3/O^*$ is the third order term, but in the Poincaré space $S^3/I^*$, the third order term cancels, leaving the fifth order as the highest term [25]. Hence, at the present epoch, not only is the residual gravity effect about a million times weaker in three of the well-proportioned spaces than in ill-proportioned spaces, but in the Poincaré space, the effect is 10,000 times weaker still, i.e. the Poincaré space is at present about $10^{10}$ times “better balanced” than ill-proportioned spaces.

4 Conclusion

Through the residual gravity effect, global topology can affect dynamics. Moreover, the effect singles out a special role for the Poincaré space. It is likely that the effect was most relevant during early, pre-inflationary epochs. If weak limit gravity were physically relevant and if inhomogeneities existed at those epochs, then dynamics could have selected the Poincaré space as the best balanced 3-manifold, especially during the quantum epoch. The Poincaré space is also the space that seems to be favoured by observations. Is this just a coincidence, or are the missing fluctuations above scales of $60^\circ$ in the WMAP

\footnote{For a displacement relative to the cosmic web and observable cosmic topology, $x/L \sim x/r_C \sim 10^{-3}$.}
data a sign that gravity selected (the comoving spatial section of) the Universe to be a Poincaré dodecahedral space?

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