The Need for a Photon-Photon Collider in addition to LHC & ILC for Unraveling the Scalar Sector of the Randall-Sundrum Model

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In the Randall-Sundrum model there can be a rich new phenomenology associated with Higgs-radion mixing. A photon-photon collider (\(\gamma C\)) would provide a crucial complement to the LHC and future ILC colliders for fully determining the parameters of the model and definitively testing it.

First, I review the essential features of the Randall-Sundrum (RS) model \cite{1}. There are two branes, separated in the 5th dimension, \(y\), and \(y \rightarrow -y\) symmetry is imposed. With appropriate boundary conditions, the 5D Einstein equations yield the metric

\[
ds^2 = e^{-2\sigma(y)}\eta_{\mu\nu}dx^\mu dx^\nu - b_0^2dy^2,
\]

where \(\sigma(y) \sim m_0b_0|y|\). Here, \(e^{-2\sigma(y)}\) is the warp factor which reduces scales of order \(M_{\text{Plank}}\) at \(y = 0\) on the hidden brane to scales of order a TeV at \(y = 1/2\) on the visible brane. Fluctuations of \(g_{\mu\nu}\) relative to \(\eta_{\mu\nu}\) are the KK excitations \(h_{\mu\nu}^n\). Fluctuations of \(b(x)\) relative to \(b_0\) define the radion field. In addition, we place a Higgs doublet \(\hat{H}\) on the visible brane. After various rescalings, the properly normalized radion and Higgs quantum fluctuation fields are denoted by \(\phi_0\) and \(h_0\). The action responsible for Higgs-radion mixing \cite{2} is

\[
S_\xi = \xi \int d^4x \sqrt{g_{\text{vis}}} R(g_{\text{vis}}) \hat{H}^\dagger \hat{H},
\]

where \(R(g_{\text{vis}})\) is the Ricci scalar for the metric induced on the visible brane.

A crucial parameter is the ratio \(\gamma \equiv v_0/\Lambda_\phi\) where \(v_0 = 246\) GeV is the SM Higgs vev and \(\Lambda_\phi\) is the vacuum expectation value of the radion field. The full quadratic structure of the Lagrangian, including \(\xi \neq 0\) mixing, takes a form in which the \(h_0\) and \(\phi_0\) fields for \(\xi = 0\) are mixed and have complicated kinetic energy normalization. We must diagonalize and rescale to get the canonically normalized mass eigenstate fields, \(h\) and \(\phi\) \cite{3}:

\[
h_0 \equiv dh + c\phi \quad \phi_0 \equiv a\phi + bh.
\]

In the above equations, \(a, b, c, d\) are functions of \(\xi, \gamma\) and the bare masses, \(m_{h_0}\) and \(m_{\phi_0}\). For given values of \(\xi, \gamma\) and the bare masses, \(m_{h_0}\) and \(m_{\phi_0}\), one must invert a set of equations to determine \(m_{h_0}\) and \(m_{\phi_0}\) and, thence, \(a, b, c, d\). Requiring consistency
leads to strong constraints on the allowed $\xi$ values for fixed $m_h$, $m_\phi$ and $\gamma$, leading to an hourglass shape for the theoretically allowed region in ($\xi$, $m_\phi$) parameter space at fixed $m_h$ and fixed $\gamma$ (equivalently, fixed $\Lambda_\phi$), as shown in Fig. 1 [4,5]. The precision EW studies of Ref. [6] suggest that some of the larger $|\xi|$ range is excluded, but we studied the whole range just in case.

![Contours of $g_{ZZh}^2 = (d + \gamma h)^2$](image1.png)

![Contours of $g_{ZZ\phi}^2 = (c + \gamma \phi)^3$](image2.png)

Figure 1: Contours of $g_{ZZh}^2 = g_{7Vh}^2$ and $g_{ZZ\phi}^2 = g_{7V\phi}^2$ [defined relative to the SM Higgs, Eq. (4)] for $\Lambda_\phi = 5$ TeV and $m_h = 120$ GeV. Regions outside the hour-glass shape are theoretically inconsistent. LEP direct discovery limits have been imposed and (at large $|\xi|$) cut out parts of the otherwise allowed $m_\phi < m_h$ parameter region. From [4].

The KK-graviton couplings to the $h$ and $\phi$ are determined by $\Lambda_\phi$. Fortunately, $\Lambda_\phi$ can be extracted using measurements of the KK-graviton spectrum at the LHC. In particular, the mass of the first KK-excitation is given by $m_1 = x_1 \frac{m_0 \Lambda_\phi}{\sqrt{6}}$, where $x_1$ is the first zero of the Bessel function $J_1$. 


while the excitation spectrum as a function of $m_{jj}$ in the vicinity of $m_1$ determines $m_0/M_{Planck}$ (see, for example, the plots in [7]). The ratio $m_0/M_{Planck}$ is related to the curvature of the brane and should be a relatively small number for consistency of the RS scenario. Sample parameters that are safe from precision EW data and Run I Tevatron constraints are $\Lambda_\phi = 5$ TeV and $m_0/M_{Planck} = 0.1$ (the latter is employed for all plots presented). These give $m_1 \sim 780$ GeV, well within the LHC reach. Once $\Lambda_\phi$ is determined, the goal will be to extract $\xi$, $m_h$ and $m_\phi$ from Higgs-radion measurements.

Crucial to determining these model parameters are the $f f$ and $V V$ couplings of the $h$ and $\phi$. For $V = W, Z$ and all $f$, the $h$ and $\phi$ couplings are rescaled relative to SM $h_{SM}$ couplings by the universal factors $g_{fV h}$ and $g_{fV \phi}$:

$$g_{fV h} = (d + \gamma b), \quad g_{fV \phi} = (c + \gamma a).$$  \hspace{1cm} (4)

In contrast, the $gg$ and $\gamma \gamma$ couplings of the $h$ and $\phi$ come from two sources: (1) the standard loop contributions computed using the above $f f / V V$ strength factors $g_{fV h}$ or $g_{fV \phi}$; and (2) "anomalous" contributions which are expressed in terms of the SU(3)$\times$SU(2)$\times$U(1) $\beta$ function coefficients. The complicated dependence of $g_{fV h}^2$ and $g_{fV \phi}^2$ on $\xi$ and $m_\phi$ is shown in Fig. 1 for $m_h = 120$ GeV and $\Lambda_\phi = 5$ TeV. Note that if $g_{fV h}^2 < 1$ is observed, then $m_\phi > m_h$, and vice versa, except for a small region near $\xi = 0$. Also note that the radion coupling $g_{fV \phi}^2$ is generally rather small and exhibits zeroes; however, if $m_\phi > m_h$ then at large $|\xi|$ the $ZZ\phi$ couplings can become sort of SM strength, implying SM type discovery modes could become relevant (see [4]).

A few notes on branching ratios (see [4]). The $h$ branching ratios are quite SM-like (even if partial widths are different) except that $h \to gg$ can be bigger than normal, especially when $g_{fV h}^2$ is suppressed. For $m_\phi < 2m_W$, $\phi \to gg$ is very possibly the dominant mode in the substantial regions near zeroes of $g_{fV \phi}^2$. However, for $m_\phi > 2m_W$ the $\phi$ branching ratios are sort of SM-like (except at $\xi \simeq 0$) but total and partial widths are rescaled.

We now turn to the LHC, ILC and $\gamma C$ capabilities. We will focus entirely on the case of $m_h = 120$ GeV. For the LHC and ILC, we summarize the work of Ref. [8]. For the LHC, we rescaled the statistical significances predicted for the SM Higgs boson at the LHC using $g_{fV h}^2$ or $g_{fV \phi}^2$ and the modified branching ratios. We found that the most important modes for Higgs-radion discovery are $gg \to h \to \gamma \gamma$ and $gg \to \phi \to ZZ^* \to 4\ell$. Also useful are $t\bar{t}h$ with $h \to bb$ and $gg \to h \to ZZ^* \to 4\ell$. Fig. 2 summarizes our results. It shows that the LHC can find either the $h$ or $\phi$ unless $m_\phi < m_h$ and $\xi > 0$ and large. $^c$ The region

\footnote{Note that $m_1$ is typically too large for KK graviton excitations to be present, or if present, important, in $h, \phi$ decays.}

\footnote{However, $|\xi| \lesssim 1.5$ is preferred by precision data in the $\Lambda_\phi = 5$ TeV case.}

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Figure 2: We consider $L = 30 \text{ fb}^{-1}$ at the LHC for $m_h = 120$ GeV. The hour-glass outer boundaries define the theoretically consistent parameter region. Within these, the blank (white) regions are where neither the $gg \rightarrow h \rightarrow \gamma\gamma$ mode nor the $gg \rightarrow h \rightarrow 4\ell$ mode yields a $> 5\sigma$ signal. The regions between dark blue curves are where $gg \rightarrow \phi \rightarrow 4\ell$ is $> 5\sigma$. The graphs are for $\Lambda_{\phi} = 2.5 \text{ TeV}$ (left) $\Lambda_{\phi} = 5 \text{ TeV}$ (center) and $\Lambda_{\phi} = 7.5 \text{ TeV}$ (right). From [8].

where neither the $h$ nor the $\phi$ can be detected grows (decreases) as $m_h$ decreases (increases). It diminishes as $m_h$ increases since the $gg \rightarrow h \rightarrow 4\ell$ rate increases at higher $m_h$. The regions where the $h$ is not observable are reduced by considering either a larger data set or $qqh$ Higgs production, in association with forward jets. Figure 2 also exhibits regions at large $|\xi|$ with $m_\phi > m_h$ in which both the $h$ and $\phi$ mass eigenstates will be detectable. In these regions, the LHC will observe two scalar bosons somewhat separated in mass, with the lighter (heavier) having a non-SM-like rate for the $gg \rightarrow h \rightarrow \gamma\gamma$ ($gg \rightarrow \phi \rightarrow ZZ$) final state. Additional information will be required to ascertain whether these two Higgs bosons derive from a multi-doublet or other type of extended Higgs sector or from the present type of model with Higgs-radion mixing. For this, we must turn to the ILC and $\gamma C$.

At an $e^+ e^-$ ILC, any light scalar, $s$, will be detected in the $Z^* \rightarrow Z s$ mode if $g_{ZZs}^2 / g_{ZZh_{SM}}^2 \geq 0.01$. Since $g_{ZZh}^2 / g_{ZZh_{SM}}^2 = g_{V_{h}}^2 \geq 0.2$ throughout all of the allowed parameter region, see Fig. 1, observation of the $h$ at the ILC is guaranteed. In contrast, Fig. 1 shows that $g_{V_{\phi}}^2 \leq 0.01$ for a significant part of parameter space (smaller $|\xi|$, especially when $m_\phi > m_h$). Unfortunately, as shown in Ref. [8], this is also the region where precision measurements of the $h$ properties at the ILC will deviate by $< 2.5\sigma$ from SM expectations and we could mistakenly conclude that the Higgs sector was that of the SM.

Can a $\gamma\gamma$ collider at the ILC or $\gamma\gamma$ collider based on a few CLIC modules help? To assess, we recall the results for the SM Higgs boson obtained in the CLIC study of [9]. There, a SM Higgs boson with $m_{h_{SM}} = 115$ GeV was examined. After cuts, one obtains signal and background rates of $S = 3280$ and $B = 1660$ in the $\gamma\gamma \rightarrow h_{SM} \rightarrow b\bar{b}$ channel, corresponding to $S/\sqrt{B} \sim 80!$

First, consider the $h$. By rescaling to obtain $S_h$ from $S_{h_{SM}}$, one finds that
the $\gamma\gamma \rightarrow h \rightarrow b\bar{b}$ rate is either changed very little or somewhat enhanced for $m_\phi < m_h$ and only modestly suppressed for $m_\phi > m_h$ (e.g., a factor of 2 at $m_\phi = 200$ GeV). Thus, at worst, we would have $S_h/\sqrt{B} \sim 480 \sim 40$, which is still a very strong signal. In fact, we can afford a reduction by a factor of 16 before we hit the 5$\sigma$ level! Thus, the $\gamma\gamma$ collider will allow $h$ discovery (for $m_h = 120$) throughout the entire hourglass, which is something the LHC cannot absolutely do. In contrast, using the factor of 16 mentioned above, the $\phi$ with $m_\phi < 120$ GeV is very likely to elude discovery in the $\gamma\gamma \rightarrow \phi \rightarrow b\bar{b}$ mode. For the $m_\phi > m_h$ region, $\gamma\gamma \rightarrow \phi \rightarrow WW, ZZ$ would be the best mode, but our current results are not encouraging.

It is important to emphasize that the $\gamma C$ can play a very special role even if we only observe the $h$ there. Indeed, let us suppose that the $\phi$ is not seen at any of the three colliders. The $h$ is very likely to be seen at the LHC for $L > 100 \text{ fb}^{-1}$ and, as discussed, will be seen at the $\gamma C$ and the ILC. Since $m_h$ will be well-measured, only $m_\phi$ and $\xi$ need to be determined ($\Lambda_\phi$ having been determined as outlined earlier). This requires two measurements, with three or more measurements needed to test the model. If we could trust LHC and $\gamma C$ and ILC absolute rates (systematics being the question), their different dependencies on the parameters imply that we could then determine $m_\phi$ and $\xi$ and test the model even if we don’t see the $\phi$. An interesting way to phrase the LHC and $\gamma C$ rate measurements is in terms of the ratio of the rates:

$$\frac{\text{Rate}(gg \rightarrow h \rightarrow \gamma\gamma)}{\text{Rate}(\gamma\gamma \rightarrow h \rightarrow b\bar{b})} = \frac{\Gamma(h \rightarrow gg)}{\Gamma(h \rightarrow b\bar{b})},$$

which is the most direct probe for the presence of the anomalous $ggh$ coupling [10]. In particular, $R_{hgg} = 1$ if the only contributions to $\Gamma(h \rightarrow gg)$ come from quark loops and all quark couplings scale in the same way. Since the RS model predicts anomalous $gg$ coupling contributions in addition to rescaled standard loop contributions, substantial deviations from $R_{hgg} = 1$ are predicted, as shown in Fig. 3.

We can estimate the accuracy with which $R_{hgg}$ can be measured as follows. Assuming the maximal reduction of $S_h/S_{hSM} = 1/2$, we find that $\Gamma(h \rightarrow \gamma\gamma)/\Gamma_{tot}$ can be measured with an accuracy of about $\sqrt{S_h + B}/S_h \sim \sqrt{3200}/1600 \sim 0.035$. The dominant error will then be from the LHC which will typically measure $\Gamma(h \rightarrow gg)/\Gamma_{tot}$ with an accuracy of between 0.1 and 0.2 (depending on parameter choices and available $L$). From Fig. 3, we see that 0.2 fractional accuracy will reveal deviations of $R_{hgg}$ from 1 for all but...
Figure 3: We plot the ratios $R_{hgg}$ and $R_{\phi gg}$ obtained after including the anomalous $ggh$ and $g\phi g$, respectively, coupling contributions. Results are shown for $m_h = 120$ GeV and $\Lambda_{\phi} = 5$ TeV as functions of $\xi$ for $m_\phi = 20$, $55$ and $200$ GeV. (The same type of line is used for a given $m_\phi$ in the right-hand figure as is used in the left-hand figure.) From [4].

the smallest $\xi$ values. Given the measured $m_h$, the direction and magnitude of those deviations will give a strong constraint on $m_\phi$ relative to $\xi$ (although, for instance, you can’t tell if $m_\phi < m_h$ and $\xi < 0$ or $m_\phi > m_h$ and $\xi > 0$).

Now suppose we also observe the $\phi$. If $|\xi|$ is large, this is possible at the ILC for any $m_\phi$ (see the $g_{T\phi}^V = 0.01$ contour of Fig. 1) and at the LHC if $m_\phi > m_h$ (see Fig. 2). The value of $R_{hgg}$ combined with knowing $m_\phi$ will then determine $\xi$ without relying on any absolute rates. In addition, the $e^+e^\to Z^* \to Z\phi$ rate will have reliable absolute normalization and it directly determines $g_{Z\phi}^2/g_{ZH}^2_{SM} = g_{T\phi}^V$. Since $g_{T\phi}^V$ is wildly varying as a function of the model parameters (see Fig. 1), its measured value will over constrain and test the model. If the LHC also sees the $\phi$ we get the model-testing $gg \to \phi \to ZZ$ rate, leading to a further cross check on the model.

We summarize assuming that $\Lambda_\phi \lesssim 20$ TeV. First, $\Lambda_\phi$ will be measured from the KK $m_{jj}$ spectrum at the LHC. Further, for such $\Lambda_\phi$, the $\gamma C$, like the ILC, can see a light $h$ for all of the $(\xi, m_\phi)$ RS parameter space. Both colliders can see the $h$ where the LHC can’t, although the “bad” LHC regions are not very big for full $L$. The ability to measure $R_{hgg}$ may be the strongest reason for having the $\gamma C$ as well as the LHC and ILC, not only in the RS context but also since most non-SM Higgs theories predict $R_{hgg} \neq 1$ for one reason or another,
unless one is in the decoupling limit. Further, if the $\phi$, as well as the $h$, is detected at the ILC, the motivation for building the $\gamma C$ becomes even stronger since the measured values of $m_h$, $m_\phi$, $R_{hgg}$ and $g_{VV\phi}^2$ provide a very definitive over constrained test of the RS model. If $m_\phi > m_h$ and $|\xi|$ is large enough for detection of $gg \to \phi \to ZZ$ at the LHC to be possible, the ILC would not be critical (but the $\gamma C$ would be) since we could get a definitive determination of $\xi$ using the measured $m_h$, $m_\phi$ and $R_{hgg}$ values and then the $gg \to \phi \to ZZ$ rate would test the model. Further model tests would be possible if we could accurately measure the rate for $h$ production in other LHC and/or $\gamma C$ channels — something that is certainly possible, but not guaranteed (especially with high accuracy). Overall, there is a nice complementarity among the machines — each brings new abilities to probe and definitively test the scalar sector of the RS model. Very generally, the case for a (low-energy) $\gamma C$ is compelling if a Higgs boson is seen at the LHC that has non-SM-like rates and properties.

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