Electric-field driven long-lived spin excitations on a cylindrical surface with spin-orbit interaction

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Abstract

Based on quantum-kinetic equations, coupled spin-charge drift-diffusion equations are derived for a two-dimensional electron gas on a cylindrical surface. Besides the Rashba and Dresselhaus spin-orbit interaction, the elastic scattering on impurities, and a constant electric field are taken into account. From the solution of the drift-diffusion equations, a long-lived spin excitation is identified for spins coupled to the Rashba term on a cylinder with a given radius. The electric-field driven weakly damped spin waves are manifest in the components of the magnetization and have the potential for non-ballistic spin-device applications.

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I. INTRODUCTION

In the emerging field of spintronics, a main issue is the avoidance of spin randomization while stimulating controlled rotations of spins in a spin field effect transistor. Therefore, the recent proposal for a spintronic device, which operates in the non-ballistic regime, received considerable interest. According to this design, spin relaxation is suppressed, when the Rashba and Dresselhaus spin-orbit interaction (SOI) constants are tuned by an external gate voltage so that their couplings become equal. In fact, the suppression is a consequence of an exact spin-rotation symmetry, which leads to a strong anisotropy of the in-plane spin-dephasing time. At the presence of an in-plane electric field, the persistent spin helix is converted into a field-dependent internal eigenmode. In close analogy to space-charge waves in crystals, these field mediated spin excitations can be probed by optical grating techniques. Both spin and charge pattern, which are generated by polarized laser beams, provide the required wave vector for the excitation of internal eigenmodes. Suppressed spin-relaxation occurs not only in (001) GaAs/Al$_x$Ga$_{1-x}$As quantum wells with balanced Rashba and Dresselhaus SOI strengths but also in (110) quantum wells with Dresselhaus coupling.

Most activities in the field of spintronics that are based on the Rashba and Dresselhaus SOI refer to a plane two-dimensional electron gas (2DEG), which is confined by a semiconductor quantum well. However, the diversity of spin-related phenomena markedly increases for different geometries of the 2DEG. In dependence on the curvature of the surface, in which the 2DEG resides, additional contributions to the SOI appear that may lead to new spin effects. Examples of current interest provide microtubes fabricated by exploiting the self-rolling mechanism of strained bilayers. These rolled-up structures exhibit pronounced optical resonances arising from micron-sized cylindrical resonators or give rise to novel magnetoresistance oscillations, which were observed in the ballistic transport of electrons on cylindrical surfaces. For non-ballistic spintronic device applications, the prediction of a conserved spin component, which arises when the Rashba coupling constant $\gamma_1$ equals the quantity $h/2m^*R$ (with $R$ being the radius of the cylinder and $m^*$ the effective mass of the 2DEG) is most interesting. The identification of this novel long-lived spin mode by fabricated curved samples seems to be feasible with the present-day technology. It is the aim of this paper to study the field dependence of the predicted spin helix by a systematic consideration of electric-field mediated eigenmodes of a spin-charge coupled
2DEG that is confined to a circular cylinder. Both Rashba and Dresselhaus SOIs as well as spin-independent elastic impurity scattering are taken into account.

II. BASIC THEORY

While in classical mechanics the restricted motion of particles on curved surfaces is unambiguously described by equations of motion, the quantum-mechanical study of curved systems starts from two different perspectives. In the first, widely used method, the three-dimensional Schrödinger equation is converted to its two-dimensional counterpart by an appropriate confining procedure. This approach naturally accounts for the fact that the curved structures are embedded in a three-dimensional space, in which electric and magnetic fields could be present. The second alternative description of the carrier dynamics on curved samples completely rests on a two-dimensional model. In our study of SOI on the surface of a cylinder, we apply the widely accepted first approach, which was already used for studying spin effects on curved surfaces. The second-quantized version of the Hamiltonian has the form

\[
H_0 = \int_0^\infty \frac{d\varphi}{2\pi} \sum_{k_z} \left\{ \sum_s a^\dagger_{k_z,s}(\varphi) \left[ \frac{\hbar^2 k_z^2}{2m^*} + \frac{\hat{p}_\varphi^2}{2m^*} \right] a_{k_z,s}(\varphi) + \gamma_1 \sum_{s,s'} a^\dagger_{k_z,s}(\varphi) \left[ \sigma_z^{s,s'} \hat{p}_\varphi - i\hbar k_z \Sigma^{s,s'} \right] a_{k_z,s'}(\varphi) + \gamma_2 \sum_{s,s'} a^\dagger_{k_z,s}(\varphi) \left[ \frac{1}{2} (\Sigma^{s,s'} \hat{p}_\varphi + \hat{p}_\varphi \Sigma^{s,s'}) - i\hbar \hat{p}_\varphi \Sigma^{z,z'} \right] a_{k_z,s'}(\varphi) \right\},
\]

in which Rashba and Dresselhaus contributions appear with the coupling constants \(\gamma_1\) and \(\gamma_2\), respectively. The creation \([a^\dagger_{k_z,s}(\varphi)]\) and annihilation \([a_{k_z,s}(\varphi)]\) operators depend on the spin index \(s\), the wave vector component \(k_z\) along the axis of the cylinder, and the angle \(\varphi\). The SOI terms include the Pauli matrices \(\sigma\), the transverse momentum operator \(\hat{p}_\varphi\), and a matrix \(\hat{\Sigma}\) that introduces off-diagonal elements with respect to the spin index. These quantities are defined by

\[
\hat{p}_\varphi = -\frac{i\hbar}{R} \frac{\partial}{\partial \varphi}, \quad \hat{\Sigma} = \begin{pmatrix} 0 & -ie^{-i\varphi} \\ ie^{i\varphi} & 0 \end{pmatrix}.
\]
The periodic boundary conditions on the cylinder surface are accounted for by a discrete Fourier transformation, which is applied in the form

$$a_{kz \uparrow}(\varphi) = \sum_{m=-\infty}^{\infty} e^{im\varphi} a_{kz,m \uparrow}, \quad a_{kz \downarrow}(\varphi) = e^{i\varphi} \sum_{m=-\infty}^{\infty} e^{im\varphi} a_{kz,m \downarrow}. \quad (3)$$

By this transformation, the projection of the total angular momentum on the cylinder axis appears and the Hamiltonian simplifies considerably. In addition to the SOI, both elastic scattering on impurities with the short-range coupling strength $U$ and an external electric field $E$ (applied along the cylinder axis) are taken into account. As it is assumed throughout the paper that the radius $R$ of the cylinder is much larger than the lattice constant, we introduce the electron momentum vector

$$\mathbf{k} = (k_{\varphi}, k_z, 0), \quad k_{\varphi} = \left( m + \frac{1}{2} \right) / R, \quad (4)$$

in order to express the Hamiltonian in a form that is very similar to the case of planar geometry. We obtain

$$H = \sum_{k,s} \varepsilon(k) a_{ks}^\dagger a_{ks} + \sum_{k,k',s,s'} (\hbar \omega_1(k) \cdot \sigma_{ss'}) a_{ks}^\dagger a_{ks'}^\dagger$$

$$+ U \sum_{k,k'} \sum_{s} a_{ks}^\dagger a_{k's} - i \hbar E \cdot \sum_{k,s} \nabla_{\kappa} a_{k-z/2,s} a_{k+z/2,s} \bigg|_{\kappa=0},$$

with the parabolic dispersion relation

$$\varepsilon(k) = \frac{\hbar^2 k^2}{2m^*} - \frac{\hbar}{2R} \left( \gamma_1 - \frac{\hbar}{4m^* R} \right). \quad (6)$$

The main part of the SOI is included in the vector

$$\omega_1(k) = \begin{pmatrix} 0, -(&gamma_1 k_z - \gamma_2 k_{\varphi}), k_{\varphi}(\gamma_1 - \frac{\hbar}{2m^* R}) - \gamma_2 k_z \end{pmatrix}. \quad (7)$$

The model description of spin-independent scattering on impurities in Eq. (5) has the advantage of simplicity, permitting us an exact treatment of scattering in the Born approximation via the scattering time $\tau$ defined by

$$\frac{1}{\tau} = \frac{2\pi U^2}{\hbar} \sum_{k'} \delta(\varepsilon(k) - \varepsilon(k')).$$

All information about the spin-orbit coupled electron ensemble is provided by the spin-density matrix

$$f_{ss'}(\mathbf{k}, \mathbf{k}'|t) = \langle a_{ks}^\dagger a_{k's'} \rangle_t, \quad (9)$$
which is calculated from quantum-kinetic equations. A transparent physical interpretation of final results is facilitated by considering the projected spin vector on a local trihedron. This transformation is achieved by

$$f = \sum_{s,s'} f_{s}^{s'} S_{ss'},$$

with the following transformation matrices

$$S^{\varphi} = \frac{1}{2} \begin{pmatrix} 0 & i e^{2i\varphi} \\ -ie^{-2i\varphi} & 0 \end{pmatrix}, \quad S^{z} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad S^{r} = \frac{1}{2} \begin{pmatrix} 0 & e^{2i\varphi} \\ e^{-2i\varphi} & 0 \end{pmatrix},$$

which project to the cylinder axis ($S^{z}$), as well as to the tangential ($S^{\varphi}$) and normal ($S^{r}$) directions. To proceed, the wave vectors are shifted according to $k \rightarrow k + \kappa/2$ and $k' \rightarrow k - \kappa/2$ with $k_{\varphi} = (m + m')/2R$ and $\kappa_{\varphi} = (m - m')/R$. The derivation of spin-charge coupled kinetic equations is carried out by applying the same steps as in our previous study of the planar geometry. The final result is expressed by the kinetic equations

$$\frac{\partial}{\partial t} f(k, \kappa|t) - \frac{i\hbar}{m*} k \cdot \kappa f + i\omega(\kappa) \cdot f + \frac{e}{\hbar} E \cdot \nabla_{k} f = \frac{1}{\tau}(\overline{f} - f),$$

$$\frac{\partial}{\partial t} f(k, \kappa|t) - \frac{i\hbar}{m*} (k \cdot \kappa) f - 2\omega(k) \times f + i\omega(\kappa) f + \frac{e}{\hbar} (E \cdot \nabla_{\kappa}) f = \frac{1}{\tau}(\overline{f} - f) - \frac{\hbar \omega(k)}{\tau} \frac{\partial}{\partial \epsilon(k)} \overline{f} + \frac{1}{\tau} \frac{\partial}{\partial \epsilon(k)} \frac{\hbar \omega(k)}{\tau} f,$$

in which a new SOI vector appears given by

$$\omega(\kappa) = (\omega_{1y}(\kappa) \sin(2\varphi), \omega_{1y}(\kappa) \cos(2\varphi), -\omega_{1z}(\kappa)).$$

On the right-hand side of Eq. (13), there remain scattering contributions that are proportional to the SOI. These terms are necessary for a consistent treatment of a homogeneous 2DEG. The bar over quantities in Eqs. (12) and (13) indicates an integration over the polar angle $\alpha$ of the vector $k = k(\cos \alpha, \sin \alpha, 0)$. The kinetic Eqs. (12) and (13) serve as a starting point for various studies of spin effects on cylindrical surfaces. We mention only the field-induced spin accumulation and the influence of the spin degree of freedom on the charge current (as well as the appearance of a "spin current" on the cylinder). In this paper, we do not follow this interesting line of reasoning but look for a solution of Eqs. (12) and (13) in the long-wavelength and low-frequency drift-diffusion regime.
The envisaged macroscopic behavior of the coupled spin-charge system is established during an evolution period, in which a nonequilibrium spin polarization and charge density still exist, whereas the energy of particles already thermalized.\textsuperscript{20} In this transport regime, the following separation ansatz for the mean components $\overline{f}$ and $\overline{\mathbf{f}}$ is justified

$$
\overline{f}(\mathbf{k}, \boldsymbol{\kappa}|t) = -F(\boldsymbol{\kappa}|t) \frac{n'(\varepsilon(\mathbf{k}))}{dn/d\varepsilon_F}, \quad \overline{\mathbf{f}}(\mathbf{k}, \boldsymbol{\kappa}|t) = -\mathbf{F}(\boldsymbol{\kappa}|t) \frac{n'(\varepsilon(\mathbf{k}))}{dn/d\varepsilon_F},
$$

where $n'(\varepsilon(\mathbf{k}))$ denotes the Fermi distribution function and $n = \int d\varepsilon \rho(\varepsilon)n(\varepsilon)$ is the equilibrium carrier density (with $\rho(\varepsilon)$ being the density of states). $n'(\varepsilon(\mathbf{k}))$ is a short-hand notation for $dn/d\varepsilon_F(\mathbf{k})$. Adopting this approximation also for the field contributions on the left-hand side of Eqs. (12) and (13), we obtain a set of linear equations for the components of the spin-density matrix. The solution is expanded with respect to $\kappa$ and integrated over the angle $\alpha$. This procedure leads to coupled equations for the charge $F$ and spin $\mathbf{F}$ distribution functions, the solution of which is easily integrated over the energy $\varepsilon(\mathbf{k})$. The resulting coupled spin-charge drift-diffusion equations take the form

$$
\left[ \frac{\partial}{\partial t} - i\mu \mathbf{E} \cdot \boldsymbol{\kappa} + D\kappa^2 \right] \mathbf{F} + \frac{i}{\mu_B} \mathbf{\omega}(\boldsymbol{\kappa}) \cdot \mathbf{M} + \frac{2im^*\tau}{\hbar\mu_B} ([\Lambda \times \mathbf{\omega}(\boldsymbol{\kappa})] \cdot \mathbf{M}) = 0, \quad (16)
$$

$$
\left[ \frac{\partial}{\partial t} - i\mu \mathbf{E} \cdot \boldsymbol{\kappa} + D\kappa^2 + \hat{\Gamma} \right] \mathbf{M} + \frac{e}{m^*c} \mathbf{M} \times \mathbf{H}_{\text{eff}}
$$

$$
-\chi(\hat{\Gamma} \mathbf{H}_{\text{eff}}) \frac{\mathbf{F}}{n} - \frac{2i}{c} \frac{m^*\mu}{e} [\Lambda \times \mathbf{\omega}(\boldsymbol{\kappa})] \mathbf{F} = \mathbf{G}, \quad (17)
$$

for the charge density $\mathbf{F}$ and the magnetization $\mathbf{M} = \mu_B \mathbf{F}$ with $\mu_B = e\hbar/2mc$ being the Bohr magneton. The vector $\mathbf{G}$ on the right-hand side of Eq. (17) accounts for the source of an external spin generation. Furthermore, $D$ and $\mu$ denote the diffusion coefficient and the mobility that are related to each other via the Einstein relation $\mu = eDn'/n$. The Pauli susceptibility is given by $\chi = \mu_B n'$. Scattering times that refer to various spin components are collected by the symmetric matrix $\hat{\Gamma}$, which is given by

$$
\hat{\Gamma} = \frac{4Dm^*^2}{\hbar^2} \begin{pmatrix}
a_{11}^2 + a_{12}^2 + a_{31}^2 + a_{32}^2 & -(a_{22}a_{32} + a_{21}a_{31}) & -(a_{11}a_{21} + a_{22}a_{12}) \\
-(a_{22}a_{32} + a_{21}a_{31}) & a_{11}^2 + a_{12}^2 + a_{21}^2 + a_{22}^2 & -(a_{12}a_{32} + a_{11}a_{31}) \\
-(a_{11}a_{21} + a_{22}a_{12}) & -(a_{12}a_{32} + a_{11}a_{31}) & a_{21}^2 + a_{22}^2 + a_{31}^2 + a_{32}^2
\end{pmatrix}, \quad (18)
$$

where the quantities $a_{ij}$ are expressed by the spin-orbit coupling constants

$$
a_{11} = \gamma_2 \sin(2\varphi), \quad a_{21} = \gamma_2 \cos(2\varphi), \quad a_{12} = -\gamma_1 \sin(2\varphi), \quad a_{22} = -\gamma_1 \cos(2\varphi), \quad (19)
$$
\[ a_{31} = -\left( \gamma_1 - \frac{\hbar}{2m^*R} \right), \quad a_{32} = \gamma_2. \]  

(20)

The electric field is accounted for by the vector

\[ \Lambda = (a_{21}\mu E_\phi + a_{22}\mu E_z, a_{31}\mu E_\phi + a_{32}\mu E_z, a_{11}\mu E_\phi + a_{12}\mu E_z), \]  

(21)

from which an effective magnetic field

\[ H_{\text{eff}} = \frac{2m^*c}{e\hbar}(\Lambda + 2iD\omega(\kappa)), \]  

(22)

is derived that enters Eq. (17) for the field-induced magnetization. The appearance of a magnetic field \( H_{\text{eff}} \), which is solely due to the electric field, illustrates why there is a perfect electric-field analog of the Hanle effect.\(^{21}\)

The drift-diffusion Eqs. (16) and (17) for the charge density \( F(\kappa, t) \) and magnetization \( F(\kappa, t) \) provide the basis for the study of many spin-related phenomena of a curved 2DEG in the drift-diffusion regime. Here, we shall focus on spin-related eigenmodes of the system.

### III. LONG-LIVED SPIN WAVES

A solution of Eq. (17) is searched for under the condition that the retroaction of spin on the charge density can be neglected so that the carrier density is given by its equilibrium value \( (F = n) \). Performing a Laplace transformation with respect to the time variable \( t \) and introducing the abbreviations

\[ M' = M - \chi H_{\text{eff}}, \quad \Sigma = s - i\mu E \cdot \kappa + D\kappa^2, \]  

(23)

with \( s \) being the Laplace variable, Eq. (17) is converted into the linear equations

\[ \Sigma M' + \tilde{\Gamma} M' + \frac{e}{m^*c} M' \times H_{\text{eff}} \]  

\[ = M(0) + G/s - 2i\frac{m^*\mu}{c} \Lambda \times \omega(\kappa)n - \chi \Sigma H_{\text{eff}}, \]  

which are symbolically written as \( \tilde{T} M' = Q \). Eigenmodes of the spin subsystem are calculated from the zeros of the determinant of the matrix \( \tilde{T} \). A simple but cumbersome algebra leads to the result

\[ \det \tilde{T} = \Sigma(\sigma^2 + \omega_H^2) + g_2 \left( \sigma + \frac{(\mu E)^2}{D} \right), \]  

(25)
in which the short-hand notations $\omega_H = (e/m^*c)H_{\text{eff}}$ and $\sigma = \Sigma + g_1$ are used. The coupling constants $g_1$ and $g_2$ are given by

\begin{align*}
g_1 &= 2\frac{4Dm^{*2}}{\hbar^2}\left[\gamma_1^2 + \gamma_2^2 - \frac{\hbar}{2m^*R}\left(\gamma_1 - \frac{\hbar}{4m^*R}\right)\right], \\
g_2 &= \left(\frac{4Dm^{*2}}{\hbar^2}\right)^2\left[\gamma_2^2 - \gamma_1\left(\gamma_1 - \frac{\hbar}{2m^*R}\right)\right]^2.
\end{align*}

The cubic equation $\det \hat{T} = 0$ with respect to the Laplace variable $s$ has three solutions, which give the dispersion relations of spin excitations. Most eigenmodes have a finite lifetime. However, there is one long-lived spin excitation, whose damping completely disappears for a given wave number $\kappa_z$. This mode appears for a model without any Dresselhaus SOI ($\gamma_2 = 0$), when the coupling constant $\gamma_1$ matches the quantity $\hbar/2m^*R$. In this case, we obtain ($s \to i\omega$)

$$\omega_{1,2} = -\mu E_z (\kappa_z \pm K) - iD (\kappa_z \pm K)^2,$$

with $K = 2m^*\gamma_1/\hbar$ being a wave number that is built from the Rashba spin-orbit coupling constant $\gamma_1$. This soft mode becomes increasingly undamped in the limit $\kappa_z \to K$. The persistent spin mode of this kind, which is a consequence of a new spin-rotation symmetry, has no counterpart in the planar Rashba model and is a distinct feature that solely appears on a cylinder surface.

In order to excite the persistent spin wave, a regular lattice of spin polarization $Q_r$ perpendicular to the cylinder surface is provided by laser pulses. For simplicity, the spin generation is assumed to have the form

$$Q_r = \frac{Q_{r0}}{2}[\delta(\kappa_z - \kappa_0) + \delta(\kappa_z + \kappa_0)].$$

Under the condition $Q_\varphi = Q_z = 0$, the solution $M = \chi H_{\text{eff}} + \hat{T}^{-1}Q$ of Eq. (17) is expressed by

$$M_z = \frac{\mu E_zK}{\sigma^2 + \omega_H^2}Q_r, \quad M_r = \frac{\sigma}{\sigma^2 + \omega_H^2}Q_r.$$

In the derivation of these equations, it was considered that the inverse Fourier transformation with respect to $\kappa_\varphi$ leads to $\varphi = 0$. The inverse Laplace transformation and the integration over $k_z$ give for the non-vanishing components of the field-mediated magnetization the final
results

\[ M_r(z, t) = \frac{Qr_0}{2} \left\{ e^{-D(\kappa_0+K)^2t} \cos[\kappa_0 z + \mu E_z(\kappa_0 + K)t] + e^{-D(\kappa_0-K)^2t} \cos[\kappa_0 z + \mu E_z(\kappa_0 - K)t] \right\}, \tag{31} \]

\[ M_z(z, t) = M_z^(-)(z, t) - M_z^+(z, t), \tag{32} \]

with

\[ M_z^\pm(z, t) = \frac{\mu E_z}{(\mu E_z)^2 + (2D\kappa_0)^2} \frac{Qr_0}{2} \left\{ 2D\kappa_0 \cos[\kappa_0 z + \mu E_z(\kappa_0 \pm K)t] - \mu E_z \sin[\kappa_0 z + \mu E_z(\kappa_0 \pm K)t] \right\} e^{-D(\kappa_0 \pm K)^2t}. \tag{33} \]

Both components \( M_z \) and \( M_r \) consist of a strongly and weakly damped oscillating term. Under the resonance condition \( \kappa_0 = K \), the first mode quickly disappears, whereas the second mode becomes completely undamped. A smooth dependence on the electric field \( E_z \) persists in the magnetization \( M_z \) along the cylinder axis. A slight detuning of the resonance, however, leads to the appearance of an electric-field driven spin wave, the damping of which is extremely weak. The frequency of this long-lived spin excitation is directly controlled by the applied electric field. The situation is similar to the persistent spin helix of a planar 2DEG. Therefore, it is supposed that the robust spin wave on a cylinder and its direct manipulation by an electric field has the potential to be utilized in future spintronic device applications.

IV. SUMMARY

Nanostructures with a great variety of novel geometries like curved graphene systems and rolled-up 2DEG are now experimentally available. Hence, the rigorous theoretical study of the dynamics on such curved surfaces became a subject of recent interest. Especially spin effects have been treated because the curvature of the surface gives rise to additional contributions to the SOI. Consequently, the number of possible spin effects considerably increases in nanostructures with curved geometries. This observation further stimulates activities in the field of spintronics. A key point regarding spin-field-effect transistors is the exclusive manipulation of spin by means of an electric field. Particularly attractive is the proposal for a device working in the non-ballistic regime, where spin scattering in a
planar 2DEG is suppressed due to a spin-rotation symmetry. A similar effect that occurs on the surface of a cylinder was studied in the present paper. Based on quantum-kinetic equations for the spin-density matrix, rigorous coupled spin-charge drift-diffusion equations were systematically derived for a cylinder, whose radius is much larger than the lattice constant. From the solution of these equations, the dispersion relation of field-dependent spin eigenmodes are identified. In general, there are three damped spin excitations, the character of which is determined by the coupling constants $\gamma_1$ and $\gamma_2$ of the Rashba and Dresselhaus SOI. For the pure Rashba model ($\gamma_2 = 0$), a long-lived spin wave exists, when the radius $R$ of the cylinder matches the condition $R = \hbar/2m^*\gamma_1$. This finding is of particular interest as an applied electric field stimulates a nearly undamped spin wave. The excitation mechanism of spin waves has the same character as the excitation of space-charge waves, which are normally strongly damped. Nevertheless, space-charge waves in crystals were clearly demonstrated in experiment. To my knowledge, completely undamped space-charge waves do not exist. Their damping is reduced, however, in the regime of negative differential conductivity due to a negative Maxwellian relaxation time. The complete disappearance of the damping of an excitation is a novelty that occurs in special spin subsystems with a $k$-linear SOI. The above mentioned peculiarity of the Rashba model on a cylindrical surface has no counterpart in a planar 2DEG. Unfortunately, the experimental demonstration of this effect is rendered more difficult because the huge internal strain within the tube breaks the bulk inversion symmetry so that an appreciable Dresselhaus contribution to the SOI is expected, which detunes the strong spin resonance. If this problem can be circumvented, the long-lived field-mediated spin excitations on a cylinder have the potential to be utilized in spintronic devices that work even in the non-ballistic regime.

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