Fragment flow and the nuclear equation of state

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Abstract

We use the Boltzmann-Uehling-Uhlenbeck model with a momentum-dependent nuclear mean field to simulate the dynamical evolution of heavy ion collisions. We re-examine the azimuthal anisotropy observable, proposed as sensitive to the equation of state of nuclear matter. We obtain that this sensitivity is maximal when the azimuthal anisotropy is calculated for nuclear composite fragments, in agreement with some previous calculations. As a test case we concentrate on semi-central $^{197}\text{Au} + ^{197}\text{Au}$ collisions at 400 $A$ MeV.

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I. INTRODUCTION

It is fair to say that the field of heavy ion collisions is a flourishing area of contemporary research in physics. In its higher energy extension, it straddles high energy and nuclear physics. The main objectives there are to create higher energy densities than ever attained in terrestrial accelerators before. In this pursuit, one will surely learn a great deal about the behaviour of strongly interacting matter at high temperatures and densities. Ultimately, one will want to verify experimentally one of the most intriguing predictions of QCD: the formation of a plasma of quarks and gluons, deconfined over macroscopic portions of time and space. A vigorous experimental program is presently under way and the theoretical interest, both direct and indirect, being generated is considerable \[1\].

Heavy ion collisions in the so-called intermediate energy regime constitute a unique tool for the investigation of complex nuclear reaction dynamics. As one goes beyond the Fermi energy in kinetic energy per projectile nucleon, the phase space accessible to nucleons in microscopic two-body collisions opens up dramatically, owing to the disappearance of Pauli blocking effects. This area thus offers the intriguing possibility of studying the competition and the individual effects of the nuclear mean field and two–body collisions. In other words, the intermediate energy region stretches from a domain where mean field dynamics dominate to a regime where microscopic nucleon-nucleon collisions play a major role. One of the main goals of this line of research is an accurate determination of the bulk properties of nuclear matter as characterized by the nuclear equation of state (EOS). The EOS plays a crucial role in the dynamics of heavy ion collisions. It is also a key ingredient in the theory of stellar collapse leading to supernovæ formation. It also naturally has something to say on neutron star properties. One realizes the many facets of the nuclear EOS, thumbing through the proceedings of recent dedicated conferences \[2\]. Information on the EOS, as characterized generally by the coefficient of compressibility for nuclear matter in its ground state, K, can also be deduced from detailed Hartree–Fock plus RPA analyses of giant monopole resonances in finite nuclei \[3\]. These lower energy experiments probe regions of excitation energy adjacent to the nuclear ground state while intermediate energy heavy ion collisions will create zones of high density and temperature. Consistency requires that the value of the nuclear compressibility coefficient of equilibrium nuclear matter deduced from both sets of experiments be compatible with one another. Happily, after a period of apparent disagreement, this goal seems on the verge of being fulfilled. It now seems that the analysis of giant monopole resonances and of heavy ion flow data can both accommodate a value of $K \approx 210$ MeV \[4,5\]. We will further elaborate on flow measurements in intermediate energy heavy ion collisions.

It seems natural that the characteristics of the nuclear EOS would manifest themselves through some cooperative behaviour like nuclear collective flow. The experimental identification of this feature was made possible by the advent of a first generation of 4π detectors, capable of global event reconstruction. Successful schemes used to plot global variables and quantify this flow were the kinetic flow tensor distributions \[6\] and the average transverse momentum analysis \[7\]. Some other recent projections of the triple differential cross section include the azimuthal distributions \[8\] and correlation functions \[9\]. Calculations done with the very successful BUU model \[10\] have reported that the azimuthal anisotropy ratio \[8\] was an observable sensitive to the value of K used in the theory. We will address in this
paper the issue of the sensitivity of this particular observable to the nuclear equation of state.

As the sophistication in detection techniques increased, the separate measurement of the flow of nuclear clusters has revealed that “clusters go with the flow” \[1\] \textit{i.e.} the amount of directed flow, as characterized by the in-plane transverse momentum per nucleon, was found to \textit{increase} with fragment mass. This feature had in fact been predicted rather early \[12\]. Other calculations capable of producing nuclear fragments also contained the feature that flow effects should be stronger, the heavier the fragment \[13\] \[10\].

One purpose of this paper is to reconcile two apparently contradictory analyses of neutron azimuthal anisotropy \[14\] \[15\]. We also wish to provide a quantitative connection between composite flow and the coefficient of compressibility for equilibrium nuclear matter in the framework of the BUU model. For the purpose of briefness we will not elaborate on the well-documented transport model \[10\] here. We simply give our arguments in the following sections and we then conclude.

II. FRAGMENTS AND THE BUU MODEL

Exactly ten years of calculation with the BUU transport model \[19\] have left no doubt on the complexity of nucleus-nucleus reactions at intermediate energy and on the need for a complete transport approach. The model has been quite successful in reproducing single particle flow patterns and transverse momentum distributions \[20\] \[5\]. However, the BUU equation is the representation of a one-body theory. It yields the time evolution of the \textit{average} one-body density and consequently it is not well suited to describe aspects of nuclear reactions that deal with significant dynamical branching or fluctuations. Nuclear multifragmentation is a good example of this class of phenomena. A significant amount of theoretical activity has been devoted to incorporate the effects of fluctuations in the transport approaches. An attempt to extend the standard BUU in this direction was made by Bauer, Bertsch and Das Gupta \[21\]. Also, in an approach similar to the theory of hydrodynamic fluctuations, a Boltzmann-Langevin equation for the evolution of the one-body density was used \[22\]. A formalism for addressing stochastic one-body dynamics within the framework of transport theory was devised \[23\]. The above three techniques have been critically compared in a recent publication \[24\]. With the exception of the first approach, these scenarios are still not amenable to calculations that can directly be compared with experimental results. Some recent important developments involve the use of quantum many-body theory to derive transport equations with bound-state production and absorption \[25\] \[26\]. The work along these lines seems extremely promising. Note in passing that the BUU equation without the collision term is the Vlasov equation and the latter can be obtained from the quantal TDHF equation by taking a Wigner transform, then a semiclassical limit \[10\]. The presence of nuclear clusters in TDHF final states has also been indirectly observed \[27\].

Driven by the need to interpret the available experimental data in a plausible fashion, other more phenomenological avenues have been followed in the extraction of composite contributions from transport theory results. One simple and intuitively appealing approach relies on the idea of coalescence. This concept was introduced already long ago \[28\]. The
original formulation for heavy ion collisions was devised around the thermodynamic model. A discussion of the coalescence model and its comparison with other approaches has appeared in the literature \[29\]. Put simply, the picture stipulates that if two or more nucleons are close enough together in phase space when the momentum space configuration of the reacting system ceases to change, they will emerge as a self-bound cluster.

In performing theoretical analyses of intermediate energy heavy ion data and comparing the results of complete BUU calculations with measurements of single nucleon observables, the need to subtract the “spurious” (in this context) cluster contribution from the full simulation results has also arisen. Some early experimental measurements have concentrated on this independent cluster component. For example, the Plastic Ball group has observed relatively large triton yields in nuclear reactions at intermediate and high energies \[30\]. A coalescence prescription to study the transverse flow of intermediate mass fragments with a relativistic BUU model has been used previously and has been shown to provide a good description of the data \[31\]. At energies below and around 100 $A$ MeV, a six-dimensional coalescence model has also been used to filter the results of VUU simulations and to very successfully compare with experimental results \[32\]. At such energies, the composite to free nucleon ratio is larger than at the energies we will consider here.

It is important to remember that a cluster is really an entity correlated in six-dimensional phase space. However, in view of the fact that our BUU approach contains a binding mean field interaction, we shall adopt a somewhat simpler viewpoint. It is well known from transport theory calculations that the transverse momentum generation in heavy ion reactions begins quite early in the history of the reaction and then stops \[20\]. The amount of transverse momentum generated has then saturated and the momentum space distributions are approximately stable. Our idea is to apply a coalescence criterion in coordinate space only, at this point. Typical BUU calculations consist of several nucleus-nucleus collisions (“runs”) performed in parallel to enhance statistics and to provide a smooth initial state density profile in coordinate and momentum space \[10\]. The approach is then the following: within a given BUU run, a nucleon will be considered “free” only if no other nucleons are found within a certain critical three-dimensional distance, $d_c$. Otherwise, it will be considered a component of a bound cluster. We justify restricting our analysis to coordinate space by the fact that, owing to the dynamical nature of the problem and to single particle propagation in the transport model, particles nearby in coordinate space but far apart in momentum space will separate after a certain time. There are two parameters to our scenario: the time at which the coalescence model is applied, $t_c$, and the critical distance parameter, $d_c$. We choose $t_c$ as the time in the nucleus-nucleus centre of mass frame when the transverse momentum generation just starts to saturate. This is calculated for each reaction we study. The value of $d_c$ is left as a free parameter and adjusted to experimental data (see the next section). The coordinate space coalescence + BUU procedure is in fact not new \[33\] and has recently been applied to semi-central Au + Au reactions at 150, 250, 400 and 650 $A$ MeV \[34\]. At the two lower beam energies, the BUU calculations with the simple coalescence

\[\text{[1]}\text{An alternate viewpoint is that each “physical” nucleon is represented by a number of “test nucleons” equal to the number of BUU runs.}\]
prescription overestimate somewhat the triple differential cross sections for free neutrons with laboratory polar angles above 15°. They however agree well with the neutron data from collisions at 400 and 650 A MeV over a wide range of laboratory polar angles. This approach has also successfully treated the case of Nb + Nb at 400 A MeV [18,35].

III. COLLECTIVE FLOW

There are several experimental observables that have been proposed as a quantitative measure of the collective flow in heavy ion collisions. Here we shall mainly concentrate on azimuthal distributions and on the so-called maximum azimuthal anisotropy ratio [8]. Those azimuthal distributions are measured event-by-event with respect to the reaction plane which can be estimated experimentally. There is always some error associated with the reaction plane determination [36]. The maximum global azimuthal anisotropy ratio can be defined as

\[ R = \frac{d\sigma}{d\phi}\bigg|_{\phi=0^\circ} / d\sigma\bigg|_{\phi=180^\circ}. \]  

Microscopic BUU calculations have shown that the maximum azimuthal anisotropy ratio for all test nucleons in a chosen rapidity range, \( R \), was sensitive to the value of the compressibility coefficient for equilibrium nuclear matter used in the theory [8]. A maximum azimuthal anisotropy ratio can also be defined at each polar angle, in a given rapidity range:

\[ r(\theta) = \frac{\sigma_3(\theta, \phi)|_{\phi=0^\circ}}{\sigma_3(\theta, \phi)|_{\phi=180^\circ}}, \]  

where

\[ \sigma_3(\theta, \phi) \equiv d^3\sigma / d(cos\theta) d(\phi - \phi_R) dy. \]  

The variables defined above have been determined in recent experiments measuring triple-differential cross sections of neutrons emitted in semi-central heavy ion collisions of Nb on Nb at 400 A MeV. The data were confronted with BUU calculations with the simple coalescence prescription to single-out the contribution of free neutrons [18,35]. This turned out to be a rather necessary and successful ingredient. With this prescription, the polar-angle dependence of the maximum azimuthal anisotropy ratio \( r(\theta) \) for the measured neutrons, emitted with rapidity \( 0.7 \leq (y/y_{beam})_{c.m.} \leq 1.2 \), could be reproduced by the calculations. We could also reproduce the triple-differential cross section for emitted neutrons both in magnitude and behaviour [18,35]. The coalescence parameter \( t_c \) was determined according to the criterion introduced in the previous section. The parameter \( d_c \) was adjusted such that the model would reproduce the double-differential cross section for emitted neutrons. This single parameter fit reproduced well the double differential cross section in magnitude and polar angle dependence. We have imposed a further check of our simple coalescence picture by scanning each identified cluster and keeping only those nucleons that could belong kinematically to a common Fermi sphere in momentum space. The change in our results was less than 1%. One conclusion of the experimental investigation and its comparison with
theory was that the maximal azimuthal anisotropy ratio \( r(\theta) \) of free neutrons turned out not to be sensitive to the nuclear EOS contrary to what was hoped previously. However, full one-body calculations of \( r(\theta) \) do exhibit considerable structure and sensitivity to \( K \) [17], as claimed originally [8]. They also overpredict the triple differential neutron cross sections.

For the purpose of clarifying the behaviour of the above observables in the theory, we concentrate here on the case of semi-central collisions of Au + Au at 400 \( A \) MeV. The impact parameter range we shall integrate over is \( 0 \leq b(\text{fm}) \leq 6.2 \), and \( d_c \) was set at 3.2 fm. These conditions were determined in a recent investigation of free neutrons emitted in heavy ion collisions [34]. The compressibility coefficient, \( K \), for equilibrium nuclear matter can be chosen by varying the set of constrained parameters \( A, B, C, \sigma \) and \( \Lambda \) in the MDYI momentum-dependent nuclear mean field [8]:

\[
U(\rho, \vec{p}) = A\left[\frac{\rho}{\rho_0}\right] + B\left[\frac{\rho}{\rho_0}\right]^\sigma + 2\frac{C}{\rho_0} \int d^3\vec{p}' \frac{f(r, \vec{p}')}{1 + \left[\frac{\vec{p} - \vec{p}'}{\Lambda}\right]^2}.
\]

The parameters used in this work will correspond to \( K = 100, 150, 215, 250, \) and 380 MeV. Our calculations include Coulomb effects. Fig. 1a shows the maximum azimuthal anisotropy ratio \( r(\theta) \) of free nucleons plotted against laboratory polar angle \( \theta \) for near-central Au + Au collisions at 400 \( A \) MeV. For now, we restrict our analysis to the rapidity region \( 0.7 \leq (y/y_{\text{beam}})_{c.m.} \leq 1.2 \) [18,34,35]. Furthermore we implement a hard “spectator cut”, requiring that the momentum of the BUU test particles be larger than 0.25 GeV/c in the projectile rest frame and the laboratory frame. These cuts essentially remove particles emitted with \( \theta \lesssim 12^\circ \) and \( \theta \gtrsim 30^\circ \). The statistical uncertainties in the calculation will be slightly larger near the edges of the populated region. As claimed in Ref. [18,35], these results further confirm that the free nucleon azimuthal data are essentially insensitive to variations in the nuclear EOS. In Fig. 1b, we show \( r(\theta) \) vs. \( \theta \) for all BUU nucleon test particles. A much clearer sensitivity to the nuclear compressibility \( K \) can now be seen over most of the covered polar angle range. We may now subtract the free nucleons identified with our coalescence prescription to obtain a signal due to all the clusters averaged-over in the one-body BUU. This is show in Fig. 1c. Clearly, the highest values of the azimuthal anisotropy ratio are reached with the clusters only. A strong variation with \( K \) is also observed. In Fig. 2 we plot the polar angle integrated azimuthal distributions for free nucleons and clusters, using the procedure described above. Both distributions peak at \( \phi = 0^\circ \). The azimuthal differential cross sections for composites are considerably larger than those for free nucleons for small azimuthal angles \( \phi \) and become comparable at large \( \phi \). The width of the composite distribution is also significantly smaller. Thus our calculations are entirely consistent with the experimental observation that fragment flow is more correlated with the reaction plane than that of single particles [37].

We now calculate the maximum global anisotropy ratio \( R \), subject to the same kinematical rapidity and spectator cuts as before. We plot \( R \) against the coefficient of compressibility for equilibrium nuclear matter, \( K \), in Fig. 3. Each point in this figure represents a set of impact parameter integrated BUU calculations. The power and simplicity of this plot [17] is immediately apparent: an experiment measures one value of \( R \), given a well defined set of kinematical constraints. This would appear on this plot as a horizontal line. The intercept of this line with the appropriate theoretical curve would then directly yield a value of \( K \). The
free nucleons do not constitute a very sensitive observable, as we can see: the steeper the curve, the more accurate is the deduced value of $K$. All the test nucleons analyzed together are somewhat sensitive to the value of $K$, but the clusters alone are much more sensitive. As in the past one-body theories such as the BUU have compared with “pseudo-nucleons” obtained from folding all the measured particles (free nucleons and composites) together \[38,39\], an analogous procedure can be followed to produce a value of $R$ to be interpreted with the top curve in our Fig. 3. The values of $R$ obtained here with the full BUU test particle ensemble are comparable in magnitude with those of the original study \[8\], done with a different system at a slightly different bombarding energy.

The usefulness of the azimuthal anisotropy ratio depends largely on an accurate determination of the event reaction plane. A method which circumvents this difficulty is based on the azimuthal pair correlation function \[38\]:

$$C(\psi) = \frac{P_{\text{cor}}(\psi)}{P_{\text{uncor}}(\psi)},$$

(3.5)

where $\psi$ is the azimuthal angle between the transverse momenta of two particles. $P_{\text{cor}}(\psi)$ is the $\psi$ distribution for observed pairs from the same event, and $P_{\text{uncor}}(\psi)$ is the $\psi$ distribution for pairs from mixed events. Following Ref. \[8\], let us assume that the azimuthal cross section has the form

$$\frac{d\sigma}{d(\phi - \phi_R)dY} = a[1 + \lambda \cos(\phi - \phi_R)]$$

(3.6)

where $\phi_R$ is the azimuthal angle of the reaction plane. If this parametrization is exact, $C(\psi)$ can be written as

$$C(\psi) = 1 + \frac{1}{2} \lambda^2 \cos(\psi).$$

(3.7)

The maximum global azimuthal anisotropy ratio defined earlier can be expressed as

$$R = \frac{\left.\frac{d\sigma}{d\phi}\right|_{\phi=0^\circ}}{\left.\frac{d\sigma}{d\phi}\right|_{\phi=180^\circ}} = \frac{1 + \lambda}{1 - \lambda}.$$  

(3.8)

Thus, measuring the azimuthal pair correlation function can immediately provide us with the anisotropy ratio, $R$, without the ambiguities associated with event-by-event reaction plane determination. We can easily verify the validity of Eq. 3.6 as an accurate parametrization in the case at hand. Since the issue of statistics is quite important here, we shall slightly shift our rapidity window. We shall choose $0.4 \leq (y/y_{\text{proj}})_{cm} \leq 0.8$ as a sufficiently populated region with a reasonable sensitivity to the nuclear EOS, other kinematical constraints being the same. We study the same reaction as before, with the same reaction parameters. Fig. 4a is a plot of the azimuthal cross section for all test particles, calculated with three different equations of state: $K = 100$, $215$, and $380$ MeV. The curves represent a fit with the parametrization of Eq. 3.6. Fig. 4b represents the same exercise repeated for clusters as defined through our coalescence prescription. In both cases the accuracy of the cosine assumption is remarkable. We plot on Figs. 5a and 5b the azimuthal pair correlation function as calculated numerically together with the parametrization, Eq. 3.7. The value of
\( \lambda \) used are the same ones obtained by fitting the results in Fig. 4. Again, the agreement between the analytic formulæ and the numerical results is excellent. We can now easily plot the azimuthal anisotropy ratio as a function of compressibility as before. See Fig. 5. The results are very similar to those obtained with the rapidity window \( 0.7 \leq (y/y_{\text{beam}})_{c.m.} \leq 1.2 \), but with somewhat smaller numerical values. In cases where the cosine form is not appropriate or when the reaction plane is simply not measured, one can define a slightly modified definition of a global azimuthal anisotropy ratio:

\[
R' = \frac{C(\psi = 0^\circ)}{C(\psi = 180^\circ)}.
\]  

(3.9)

In the theory, this quantity has the same desirable behaviour with respect to variations of the nuclear matter compressibility coefficient as \( R \) as seen in Fig. 7. It can then also be useful in comparisons of experimental results with theoretical calculations.

IV. SUMMARY

In the framework of the BUU model, using a simple coalescence prescription we have provided an explanation for an apparent discrepancy between two calculations of the azimuthal anisotropy ratio in intermediate energy heavy ion collisions. We believe that it is quite important from the point of view of consistency to resolve this issue within the framework of the very model used to propose this experimental observable. This involved emphasizing the role played by the nuclear composite fragments and their participation in the collective nuclear flow. We have found behaviours in qualitative agreement with earlier calculations with different models and with existing experimental data. It is clear that our approach to clustering in heavy ion reactions is simple but we believe that more sophisticated scenarios should reach similar physical conclusions. What is also evident is that the aim of the theoretical efforts in understanding the dynamics of heavy ion collisions should not be restricted to the mere extraction of a single number, \( K \). High energy nuclear collisions clearly constitute a rich problem, with many possible projections of the experimental data to be dealt with.

Again, our aim was to extract some physics content in an admittedly phenomenological fashion, not to provide a rigorous and complete theory of cluster production. The theoretical problems associated with a complete time-dependent theory of composite formation are being addressed. We believe that therein lies the key to a more complete understanding of the nuclear dynamics involved in heavy ion collisions. In this respect, it is quite stimulating that very high quality data is becoming available. For example the EOS TPC collaboration has started to release some experimental results [40]. This data is virtually free of experimental biases. The EOS Time Projection Chamber, with its simple and seamless acceptance, good particle identification and high statistics, was designed to overcome some of the limitations of the previous generation of \( 4\pi \) detectors. Composites up to high \( Z \) have been measured in the TPC. This fact, combined with the active program under way at the GSI/SIS facility enables us to look towards the future with optimism.
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FIGURES

FIG. 1.  (a) We plot the azimuthal anisotropy ratio for free nucleons at a given polar angle, as a function of $\theta_{lab}$. The different symbols correspond to different values of the compressibility coefficient of equilibrium nuclear matter, in units of MeV. The reaction under scrutiny is $^{197}$Au + $^{197}$Au at 400 $A$ MeV. The multiplicity cut corresponds to semi-central collisions. Other kinematical constraints are specified in the main text. The statistical uncertainties are of the order of 10% in the middle of the populated area in polar angle and roughly 20% at the edges. (b) Same caption as (a), except that all BUU test particles satisfying the kinematical cuts are involved. (c) Same caption as (a), except that all free nucleons have been subtracted from the full set of BUU test particles satisfying the kinematical cuts.

FIG. 2. (a) The azimuthal distributions of free nucleons with respect to the reaction plane are plotted. The calculations shown correspond to three different values of K. The units of K are MeV. The curves were drawn to guide the eye. (b) Same caption as (a) but for nuclear clusters.

FIG. 3. We plot the maximum global azimuthal anisotropy ratio as defined in the main text, as a function of the compressibility coefficient for equilibrium nuclear matter. The kinematical cuts are such that particles with rapidity $y$ such that $0.7 \leq (y/y_{beam})_{c.m.} \leq 1.2$ were accepted. The spectator cut as defined in the main text was also implemented. The curves were drawn to guide the eye.

FIG. 4. (a) The azimuthal distributions of free nucleons with respect to the reaction plane are plotted. The calculations were repeated from three different values of K. The units of K are MeV. The rapidity window has been shifted to $0.4 \leq (y/y_{beam})_{c.m.} \leq 0.8$. The curves are not drawn through the data but represent a fit with Eq. (3.6). (b) Same caption as (a) but for nuclear clusters.

FIG. 5. (a) The azimuthal correlation function, calculated with all the BUU test nucleons is plotted. The curves represent Eq. (3.7) with values of $\lambda$ obtained from Fig. 4. (b) Same caption as (a) but for nuclear clusters.

FIG. 6. We plot the maximum global azimuthal anisotropy ratio as defined in the main text, as a function of the compressibility coefficient for equilibrium nuclear matter. The kinematical cuts are such that particles with rapidity $y$ such that $0.4 \leq (y/y_{beam})_{c.m.} \leq 0.8$ were accepted. The spectator cut as defined in the main text was also implemented. The curves were drawn to guide the eye.

FIG. 7. We plot the modified global azimuthal anisotropy ratio as defined in the main text, as a function of the compressibility coefficient for equilibrium nuclear matter. The kinematical cuts are such that particles with rapidity $y$ such that $0.4 \leq (y/y_{beam})_{c.m.} \leq 0.8$ were accepted. The spectator cut as defined in the main text was also implemented. The curves were drawn to guide the eye.
Free Nucleons

- $K=100$
- $K=150$
- $K=215$
- $K=380$

$J.~Zhang$ and $C.~Gale$, Fragment flow..., P.R.C Fig.1 (a)
All Test Particles

J. Zhang and C. Gale, Fragment flow..., P.R.C Fig.1 (b)
Free Nucleons

\( \frac{d\sigma}{d\phi} (\text{b}) \)

\( \circ \) \( K=100 \)
\( \blacktriangle \) \( K=215 \)
\( \square \) \( K=380 \)

J. Zhang and C. Gale, Fragment flow..., P.R.C Fig.2 (a)
Clusters

\( d\sigma / d\phi \) (b)

\( \phi \) (deg)

- K=100
- K=215
- K=380

J. Zhang and C. Gale, Fragment flow..., P.R.C Fig.2 (b)
All Test Particles

\( \frac{d\sigma}{d\phi} \) (b)

\( \phi \) (deg)

- ○ K=100
- ▲ K=215
- □ K=380

J. Zhang and C. Gale, Fragment flow..., P.R.C Fig.4 (a)
Clusters

○ $K=100$

△ $K=215$

□ $K=380$

$\frac{d\sigma}{d\phi}$ (b) vs $\phi$ (deg)

J. Zhang and C. Gale, Fragment flow..., P.R.C Fig.4 (b)
All Test Particles

○ $K=100$

△ $K=215$

■ $K=380$

$C(\psi)$ vs. $\psi$ (deg)

J. Zhang and C. Cole, Fragment flow..., P.R.C Fig.5 (a)
Clusters

- ○ $K=100$
- ▲ $K=215$
- ■ $K=380$

$C(\psi)$ vs $\psi$ (deg)

J. Zhang and C. Gale, Fragment flow..., P.R.C Fig. 5 (b)
J. Zhang and C. Gale, Fragment flow..., P.R.C Fig.6
J. Zhang and C. Gale, Fragment flow..., P.R.C Fig.7