Numerical Estimation of the Fundamental Frequency of Improved Sites with Stone Columns

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Abstract

Background/Objectives: Installation of stone columns is one of the appropriate techniques for the improvement of soft soils. However, the seismic behavior of this reinforcing elements has been limitedly studied. Methods/Statistical Analysis: In this paper, effects of stone column construction on the fundamental frequency of the sites are studied numerically. Finite element analysis was performed using ABAQUS. The performed analysis was a modal analysis through the calculation of eigenvalues to determine of the fundamental frequency of improved sites with stone columns. The analyses was carried out in 3D and 2D in some cases. Findings: The results demonstrated that constructing the stone columns can increase the fundamental frequency of the site up to three times. Based on the performed analyses, the fundamental frequency amplification factor of the site (α) can be defined according to the dimensionless parameters including stone column to soil shear wave velocity, stone column height to its diameter, stone column distance to its diameter, and stone column arrangements. The results indicated that α decreased with a rise in the ratio of the stone column height to its diameter. A comparison of the stone column arrangements demonstrated that, in a triangle arrangement, the value of α was greater than the corresponding value in square arrangement. The results indicate that, in floating stone columns the effects of stone columns on the soil mass due to the absence of the bottom of the column with respect to the condition where the stone column was end bearing was considerably insignificant. Application/Improvements: Based on the results, α was presented for the square and triangle arrangements according to the dimensionless parameters and finally a 2D equivalent method was presented for the simplification of the 3D actual problem.

Keywords: ABAQUS Software, Fundamental Frequency, Modal Analysis, Soil Improvement, Stone Columns

1. Introduction

Construction of stone columns is a soil improvement method for increasing the bearing capacity or decreasing the settlement of soils¹. This method is based on the replacement of 15 to 30 percent of the poor soil through digging wells with certain diameter, depth, and distance, as well as filling these wells with sand, gravel, or cobblestone and compressing them to form vertical columns². Stone column techniques are employed to improve bearing capacity, slope stability, and drained rate as well as reducing the settlement and liquefaction potential of soils³. Stone column arrangements are either square or triangle (Figure 1). The number of stone columns in a constant area is greater in triangle compared to the square arrangements⁴.

In geotechnical earthquake engineering, stone columns are commonly employed to mitigate the liquefaction potential of loose granular soils⁵. However, the seismic performance of this reinforcing element has been very limitedly studied and requires more investigations⁶.

It is consequential to assessment a fundamental frequency of sites for the seismic design of structures⁷. For a uniform soil layer with height H on a rigid bedrock,
which has the constant shear wave velocity of $V_{\text{soil}}$, the fundamental frequency of site is as follows:

$$f_0 = \frac{V_{\text{soil}}}{4H}$$ (1)

If the same soil is improved by stone columns (Figure 2) and the fundamental frequency of this improved site is assumed equal to $f_s$, $\alpha$ can be defined as the fundamental frequency amplification factor of site as follows:

$$\alpha = \frac{f_s}{f_o}$$ (2)

This paper numerically studies the effective factors for the values of $\alpha$ in the improved sites with stone columns and evaluates the effects of various parameters including height of soil layers ($L$), soil’s shear wave velocity ($V_{\text{soil}}$), stone column’s shear wave velocity ($V_{\text{stone}}$), distance ($S$), diameter ($D$), height ($H$), and arrangement of stone columns (square or triangle) on the results.

In order to determine the fundamental frequency of the improved sites ($f_s$), finite element software ABAQUS was employed and analyses was carried out in 3D and 2D in some cases. The performed analysis was a modal analysis through the calculation of eigenvalues. According to the modal analysis of the problem, the behaviors of the soil and stone column were considered linear. Additionally, the shear wave velocity and density of the soil and stone columns were assumed constant in depth. Figure 3 demonstrates the meshing model of the 3×3 grid of the stone columns with ABAQUS. Subsequently, the effective parameters for $\alpha$ were introduced and, after the assessment of various parameters, $\alpha$ was defined on some dimensionless parameters.

2. Floating or End Bearing State of Stone Columns

Depending on the height of the column and depth of the bedrock, stone columns can be constructed as end bearing with their end on the bedrock or as floating with free end in the soil (Figure 4). For the comparison of floating and end bearing stone columns, the models with square arrangements ($D=1\text{m}$, $L=6\text{m}$, and $S=4\text{m}$) were taken into consideration and, for various values of $H/L$, the parameter $\alpha$ was estimated.

The results (Figure 5) indicate that, in floating stone columns ($H<L$), the effects of stone columns on the soil mass due to the absence of the bottom of the column with respect to the condition where the stone column was end bearing ($L=H$), was considerably insignificant. Considering the negligible effects of floating stone columns, compared to the end bearing stone columns on the fundamental frequency of the site, all of the

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**Figure 1.** Top Views of Stone Columns with Square and Triangle Arrangements.

**Figure 2.** Uniform Soil Layer Improved by the Stone Columns.

**Figure 3.** Modeling Stone Columns in ABAQUS (3D).
subsequent analysis was carried out on the end bearing stone columns. Moreover, in modeling the end bearing columns, the end bearing columns were assumed to be completely located on the lower bedrock and its end did not have horizontal or vertical mobility.

3. Effects of the Shear Wave Velocity of Soil ($V_{\text{soil}}$) and Stone Columns ($V_{\text{stone}}$)

In unimproved soils, $f_0$ is affected by $V_{\text{soil}}$ and thus, in improved soil, it is assumed that $f_s$ is affected by shear $V_{\text{stone}}$. Thus, five various values of the shear wave velocity for the soil and stone columns were selected so that the ratio of $V_{\text{stone}}$ to $V_{\text{soil}}$ had a constant value (Table 1). The stone columns arrangement was square and other parameters were assumed equal in all five cases ($H=4m$, $D=1m$, and $S=2m$). Although the results demonstrated that $f_s$ was different in all five cases, the value of $\alpha$ was equal. Considering this fact, based on the ratio of $V_{\text{stone}}/V_{\text{soil}}$, the effects of $V_{\text{stone}}$ and $V_{\text{soil}}$ on the value of $\alpha$ were investigated.

4. Effects of Column Height ($H$) and Diameter ($D$)

To study the veracity of the results, five various values of height ($H$) and diameter ($D$) of the stone column were selected (Table 2). Values of $H$ and $D$ were selected so that the value of $H/D$ was equal to a constant value. The stone columns arrangement was square and other parameters were assumed identical in all five cases ($V_{\text{stone}}/V_{\text{soil}}=8$ and $S/D=2$). Although for different cases, the results demonstrated that the values of the fundamental frequency of the site were different, in all five cases, the values of $\alpha$ were almost identical. Taking this into account, in this paper, the effects of $H$ and $D$ of the stone columns on the value of $\alpha$ was investigated based on the ratio of $H/D$.

5. Determining $\alpha$ According To the Dimensionless Parameters of $H/D$ and $S/D$}

Based on the above results, $\alpha$ can be presented according to the dimensionless parameters $V_{\text{stone}}/V_{\text{soil}}$, $H/D$, $S/D$ as well as stone arrangements. Subsequently, based on the numerical analyses, $\alpha$ was presented for the square and triangle

Table 1. Effect of $V_{\text{stone}}/V_{\text{soil}}$ on $\alpha$

| Cases | $V_{\text{soil}}$ (m/s) | $V_{\text{stone}}$ (m/s) | $V_{\text{stone}}/V_{\text{soil}}$ | $f_s$ (ABAQUS) | $f_0 = V_{\text{soil}}/4H$ | $\alpha = f_s/f_0$ |
|-------|------------------------|------------------------|----------------------------------|----------------|------------------------|------------------|
| I     | 40                     | 320                    | 8                                | 4.27           | 2.50                   | 1.71             |
| II    | 60                     | 480                    | 8                                | 6.41           | 3.75                   | 1.71             |
| III   | 80                     | 640                    | 8                                | 8.54           | 5.00                   | 1.71             |
| IV    | 100                    | 800                    | 8                                | 10.68          | 6.52                   | 1.71             |
| V     | 120                    | 960                    | 8                                | 12.82          | 7.50                   | 1.71             |

Table 2. Effect of the $H/D$ on $\alpha$

| Cases | D (m) | H (m) | H/D | $f_s$ (ABAQUS) | $f_0 = V_{\text{soil}}/4H$ | $\alpha = f_s/f_0$ |
|-------|-------|-------|-----|---------------|------------------------|------------------|
| I     | 0.50  | 2.00  | 4   | 12.75         | 7.50                   | 1.70             |
| II    | 0.75  | 3.00  | 4   | 8.55          | 5.00                   | 1.71             |
| III   | 1.00  | 4.00  | 4   | 6.41          | 3.75                   | 1.71             |
| IV    | 1.25  | 5.00  | 4   | 5.14          | 3.00                   | 1.71             |
| V     | 1.50  | 6.00  | 4   | 4.28          | 2.50                   | 1.71             |
arrangements according to the dimensionless parameters (Figure 6). The results demonstrated that, while the value of S/D increased (effects of improvement were decreased), α was decreased, as in S/D=4, the fundamental frequency of the improved site was increased up to the maximum of 25% (α=1.25). Moreover, by increasing the value of H/D (slenderizing the stone column), the value of α was decreased. A comparison of the results of the square and triangle arrangements showed that, in triangle arrangements (for equal S/D), the parameter α was approximately 10% greater than the similar value in the square arrangement. The reason was that, in triangle arrangements, the zones of the influence of each column were greater than the similar value in the square arrangement.

**Figure 6.** α Values Versus H/D, S/D, and \( \frac{V_{\text{stone}}}{V_{\text{soil}}} \) for Square and Triangle Arrangements.
Range of the used values in the analyses, including stone columns and soil parameters \( V_{\text{stone}} \), \( V_{\text{soil}} \), \( S \), \( D \), and \( H \)) and dimensionless parameters \( V_{\text{stone}}/V_{\text{soil}} \), \( S/D \), and \( H/D \), are presented in Table 3.

6. Assumption of a Virtual Rigid Retaining Wall

In this section, the problem is done in plane strain with ABAQUS 2D. For this purpose, stone columns, which were in a row, were assumed as equivalent strips and these strips were supposed as a set of considerable rigid retaining walls in the soil profile according to Figure 7. Similar to the 3D case, \( \alpha \) can be presented by the values of \( H/D \), \( S/D \), and \( V_{\text{stone}}/V_{\text{soil}} \) (Figure 8). The process of determining the equivalent 2D width \( (D_{2D}) \) is investigated in Section 7.

7. An Approximate Method to Simplify an Actual 3D to An Equivalent 2D Problem

In this section, an equivalent method will be presented so that an actual 3D problem can be simplified as a 2D problem (plane strain). Conversion of 3D to 2D problem is a common technique in evaluation of improved sites. The 3D and 2D analyses were performed with ABAQUS 3D and 2D, respectively. According to Table 4, three various 2D equivalent cases were considered so that the most appropriate 2D equivalent answer could be estimated. In case I, diameter of the columns in the 2D and 3D modeling was assumed equal \( (D_{2D}=D_{3D}) \). In case II, diameter of the columns in the 2D was selected so that the area of the influences of stone columns was equal to 3D \( (A_{2D}=A_{3D}) \). In case III, diameter of the columns in the 2D was selected so that the inertial moment of stone columns would be equal to 3D \( (I_{2D}=I_{3D}) \). Moreover, column arrangements were assumed square and the height and distance of the columns in 2D and 3D modeling were assumed equal in all three cases \( (V_{\text{stone}}/V_{\text{soil}}=8.0 \) and \( S/D_{2D}=2) \). The results suggested that, in case III (equal inertial moment), a relatively good approximation existed between the actual 3D and the equivalent 2D results (Figure 9).

![Figure 7](image_url)  
**Figure 7.** (a) Top Views of Actual 3D and Equivalent 2D Geometries and (b) Soil and Stone Columns Profile in 2D Analysis.

![Figure 8](image_url)  
**Figure 8.** Effect of S/D and H/D on \( \alpha \) in 2D Problem (Plane Strain).

![Figure 9](image_url)  
**Figure 9.** Values of \( \alpha \) According to H/D\(_{3D}\) for Solving the Problem in 2D.

### Table 3. Range of parameters for stone columns and soil

| Parameter | \( V_{\text{stone}} \) (m/s) | \( V_{\text{soil}} \) (m/s) | \( H \) (m) | \( D \) (m) | \( S/D \) | \( H/D \) | \( V_{\text{stone}}/V_{\text{soil}} \) |
|-----------|----------------------------|----------------------------|----------|---------|---------|---------|----------------|
| Range     | 100–400                    | 40–100                     | 2–10     | 0.6–1.5 | 2–4     | 1.5–4   | 2–10           |

### Table 4. Different cases for 2D equivalent modeling

| Cases    | Stone distance (S) | Stone Height (H) | Stone diameter (D) |
|----------|--------------------|------------------|--------------------|
| Case I   | \( S_{2D} = S_{3D} \) | \( H_{2D} = H_{3D} \) | \( D_{2D} = D_{3D} \) |
| Case II  | \( S_{2D} = S_{3D} \) | \( H_{2D} = H_{3D} \) | \( D_{2D} = \frac{\pi \times D_{3D}^2}{4S} \) |
| Case III | \( S_{2D} = S_{3D} \) | \( H_{2D} = H_{3D} \) | \( D_{2D} = \sqrt{\frac{\pi \times D_{3D}^2}{4S}} \) |
8. Conclusion

This paper described the fundamental frequency amplification factor of the improved sites with stone columns ($\alpha$). This value was greater than or equal one and can be defined according to various parameters. The results indicated that, in floating stone columns, $\alpha$ was approximately equal to 1.0 and stone columns construction had no effect on $\alpha$. But, in end bearing stone columns, $\alpha$ can be equal to 4.0. The results showed that $\alpha$ can be defined according to the dimensionless parameters such as $V_{\text{stone}}/V_{\text{soil}}$, $H/D$, $S/D$, as well as square or triangle arrangements. The results also demonstrated that $\alpha$ was decreased with a rise in the ratio of the stone columns height to its diameter (slenderizing the stone column). A comparison of the stone columns arrangements showed that, in the triangle arrangement, the value of $\alpha$ was greater than the corresponding value in the square arrangement. Finally, a 2D equivalent method for the simplification of the 3D actual problem was presented by examining the various cases. The results suggested that, in the case the inertial moment of stone columns in 2D equaled 3D ($I_{2D}=I_{3D}$), relatively good approximation existed between the results of the actual 3D and the equivalent 2D.

9. References

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