Adaptive chaos synchronization of an attitude control of satellite: A backstepping based sliding mode approach

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1. Introduction

Chaos synchronization has gained more interest in the field of research, followed by the contribution of Pecora and Carroll in 1990 [1]. Several applications of chaotic synchronization are observed in different branches of engineering such as chemical, mechanical, information, communication and spacecraft. The phenomenon of chaotic synchronization is achieved between a dominant system which is as the drive or master system and a bounded set of response systems which are called slave systems. The slave systems are forced to match with the outputs of a master system through some physical interconnections or feedback loops [2]. Adaptive synchronization is to synchronize non-identical or identical systems in the existence of external disturbances, model uncertainties, unknown and time-varying parameters. The adaptive synchronization of a chaotic Chen system with unspecified parameters is obtained in [3]. In [4], a hybrid function projective synchronization is achieved for chaotic systems through adaptive control. An adaptive robust observer is put in to synchronize chaotic systems with time-delay [5]. Finite-time synchronization is obtained for a class of chaotic systems including external disturbances, fully unknown parameters and uncertainties [6]. Two chaotic gyros are synchronized via sliding mode control (SMC) in the presence of stochastic base excitation of white Gaussian noise [7].

With the advancement in space technology, the necessity of understanding satellite dynamics is of key importance for the improvement of satellite systems [8]. The nonlinear attitude dynamics of the satellite has higher dimensions [9] and chaotic behaviors [10]. Several control approaches has been used for the control problem of satellite attitude system in the presence of chaotic perturbations [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24]. Satellites show chaotic behavior in certain situations like solar radiation pressure, geomagnetic and gravitational fields [25]. A single satellite is sometimes insufficient to accomplish certain missions of space observations and earth observations. These tasks can be accomplished using satellites synchronization. Replacing a single satellite with a number of smaller satellites in clusters helps to reduce the launching cost as well as reduces the risk of failure of a difficult and complex mission. Therefore, reduction in the launching cost and reduction in the risk of the entire mission failure are the other benefits of satellites synchronization [26]. Therefore satellite attitude synchronization has become an attractive topic in the research field. Different strategies such as generalized projective synchronization [27], backstepping control [28, 29], predictive control based on T-S fuzzy modeling via linear matrix inequalities [30], active control [31], SMC [32, 33, 34, 35], neural and nonlinear control [36], time delay synchronization [37], attitude synchronization based on novel vector measurements [38], finite control technique [39, 40, 41, 42], auxiliary system technique [43], behavior-based control method [44, 45] has been adapted for chaotic satellite synchronization. Adaptive synchronization of a chaotic satellite attitude in the presence of external disturbances and uncertainties is difficult. Disturbances and uncertainties of the satellite attitude system are represented using auxiliary torques. The external disturbances are sunlight pressure torques, gravity gradient torques,
aerodynamics moment, etc., whereas internal disturbances are model uncertainties and parametric uncertainties \[46\].

In this paper, an adaptive backstepping sliding mode control (ABSMC) is proposed for synchronizing the chaotic satellites attitude with external disturbances. The SMC is incorporated with the backstepping method in order to achieve greater robustness against disturbances. Two different sets of external disturbances are taken to prove the robustness of the proposed control scheme. The external disturbances are the perturbing torques \[2\]. Therefore different adaptation strategies are integrated with the control laws in order to approximate the upper bounds of the disturbances and cancel out the undesirable effects of disturbances \[47\].

The paper is sequenced as follows: Section 2 describes the attitude dynamics of a chaotic satellite system. Section 3 presents the mathematical representation of the proposed ABSMC. In section 4, the simulation results are used to validate the theoretical derivations of the control law and the conclusion is drawn in section 5.

2. Representation of system dynamics and problem formulation

2.1. Satellite attitude dynamics

The equation of rotational motion for a rigid satellite is expressed under the following form \[2, 48, 49, 50, 51\].

\[
I\dot{\omega} + \omega \times (I\omega) = g + \eta
\]  

where, \(I\) denotes the inertia tensor of the symmetric body around the center of mass with \(I = \text{diag}(I_{xx}, I_{yy}, I_{zz})\), \(\omega\) the angular velocity with \(\omega = [\omega_x, \omega_y, \omega_z]^T\), \(g\) external disturbances torque with \(g = [g_x, g_y, g_z]^T\) and \(\eta\) the controlling torque with \(\eta = [\eta_1, \eta_2, \eta_3]^T\).

The cross product in Eq. (1) can be written as the product of the skew-symmetric cross product matrix \([\omega \times]\) and the vector term \((I\omega)\).

![Figure 1. Block diagram representing design procedure of proposed technique.](image)

![Figure 2. Chaotic state trajectories of the drive system.](image)
From Eq. (2), the angular velocity can be then expressed by differential equations given in Eq. (3):

\[
\dot{\omega}_x = \frac{\omega_y \omega_z}{C_0} I_{yy} - \frac{\omega_x \omega_z}{C_0} I_{zz} + g_x + \eta_1
\]

\[
\dot{\omega}_y = \frac{\omega_z \omega_x}{C_0} I_{zz} - \frac{\omega_y \omega_x}{C_0} I_{xx} + g_y + \eta_2
\]

\[
\dot{\omega}_z = \frac{\omega_x \omega_y}{C_0} I_{xx} - \frac{\omega_z \omega_y}{C_0} I_{yy} + g_z + \eta_3
\]

where the principal moments of inertia values are considered as \( I_{xx} = 3 \) kgm\(^2\), \( I_{yy} = 2 \) kgm\(^2\) and \( I_{zz} = 1 \) kgm\(^2\).

2.2. Formulation of synchronization problem

Consider two identical satellites attitude systems as in Eq. (3), the master system is given by Eq. (4)

\[
\dot{\omega}_1 = \frac{1}{3} (\omega_2 \omega_3 - h_2 h_3) + \frac{1}{3} (d_1 - f_1) + \eta_1
\]

\[
\dot{\omega}_2 = -\omega_1 \omega_3 + \frac{1}{2} (d_2 - f_2) + \eta_2
\]

\[
\dot{\omega}_3 = \omega_1 \omega_2 + d_3 + \eta_3
\]

where \( \omega_i \) (\( i = 1,2,3 \)) are the state variables, \( d_i \) (\( i = 1,2,3 \)) the external perturbing torques of the slave system, and \( \eta_i \) (\( i = 1,2,3 \)) is the control inputs to synchronize the master system with the slave system.

We define the errors \( e_i(t) = \omega_i(t) - \hat{h}_i(t) \) (\( i = 1,2,3 \)), which yield the error dynamics as

\[
\dot{e}_1 = \frac{1}{3} (\omega_2 \omega_3 - h_2 h_3) + \frac{1}{3} (d_1 - f_1) + \eta_1
\]

\[
\dot{e}_2 = -\omega_1 \omega_3 + \frac{1}{2} (d_2 - f_2) + \eta_2
\]

\[
\dot{e}_3 = \omega_1 \omega_2 + d_3 + \eta_3
\]

Assumption 1. External disturbances are generally unknown and cannot be measured. The only information available is that the upper bounds of the disturbances are some positive (known or unknown) constants as given in Eq. (7):

\[
|d_1| \leq D_1, |d_2| \leq D_2, |d_3| \leq D_3, |f_1| \leq F_1, |f_2| \leq F_2, |f_3| \leq F_3
\]
3. Control law formulation for chaos synchronization

Backstepping is a control strategy which follows a recursive technique to stabilize the origin of a strict-feedback nonlinear system \cite{47}. This nonlinear robust control strategy is based on the stability theory of Lyapunov \cite{52}. SMC is a nonlinear controller which provides robustness against uncertainties and external disturbances. Some of the advantages of the SMC include quick response and superior transient performance \cite{53}. SMC is designed based on the concept of the Lyapunov stability theorem. The designing consists of two steps. In the first step, a fictitious surface is designed in a manner that the expected performance is achieved by the trajectory along the surface of the system. This fictitious surface is named as the sliding mode surface. In the second step, a switching control (discontinuous) signal is designed, which drives the trajectories to the sliding surface \cite{54}. A careful selection of the switching gain parameters is essential in order to avoid the problem of chattering. This technique employed to adjust the gain parameters in order to adaptively estimate the upper bounds of disturbances and ensure the stability of the nonlinear system is called adaptive control \cite{55}.

The objective of designing the proposed controller is to asymptotically synchronize the slave system with the master system, which means that whatever external disturbances may be present or whatever the initial values are chosen, the error $\|e(t)\| \rightarrow 0$ as $t \rightarrow \infty$.

3.1. Derivation of control input $\eta_1$

Let’s define the first Lyapunov function candidate for $e_1$ is given by Eq. (8)

$$V_1 = \frac{1}{2} e_1^2$$

whose derivative of $V_1$ with respect to time $t$ is given by Eq. (9)

$$V_1' = e_1 \dot{e}_1 = e_1 (\sigma_1 - b_1 e_1) = \sigma_1 e_1 - b_1 e_1^2$$

where $\sigma_1$ is the first sliding surface chosen as

$$\sigma_1 = \alpha_1 e_1$$

We choose the virtual control input as expressed in Eq. (11)

$$\dot{e}_1 = \sigma_1 - b_1 e_1 (b_1 \geq 1).$$

If $\sigma_1 \rightarrow 0$, $V_1' = -b_1 e_1^2 \leq 0$, which is quadratic and negative definite.

The first control input $\eta_1$ is obtained using another Lyapunov function $V_1'$ as given in Eq. (12)

$$V_1' = V_1 + \frac{1}{2} e_1^2$$

The time derivative of $V_1'$ is given as

$$V_1'' = V_1' + \sigma_1 \dot{e}_1 = \sigma_1 e_1 - b_1 e_1^2 + \sigma_1 (\alpha_1 e_1)$$

The term $\dot{e}_1$ in Eq. (13) consists of the first control input, therefore $\dot{e}_1$ is substituted from Eq. (6a) and $V_1'$ can be rewritten as
and the control input is obtained as

\[ V_\text{eq} = \frac{1}{C_0} (w_2 w_3 - h_2 h_3) \]  

Since disturbances cannot be known in advance, therefore it is not incorporated into the equation of control input. Satisfying the reaching condition \((\sigma_1 \dot{\sigma}_1 < 0)\) in Eq. (14), the first corresponding control input is acquired as

\[ \eta_{\text{eq}} = -\frac{\alpha_1}{\alpha_2} - \frac{1}{3} (w_2 w_3 - h_2 h_3) \]  

A switching control law \(\eta_{\text{eq}} = \xi_1 \text{sign} (\sigma_1)\) is augmented with Eq. (15) and the control input is obtained as

\[ \eta_1 = -\frac{\alpha_1}{\alpha_2} - \frac{1}{3} (w_2 w_3 - h_2 h_3) + \xi_1 \text{sign} (\sigma_1) \]  

In practical applications, obtaining the upper bounds of disturbances and uncertainties is difficult. In order to suppress any disturbances and model uncertainties with unknown upper bounds \(\xi_1\) is replaced with \(\tilde{\xi}_1\).

Eq. (16) can be rewritten as

\[ \eta_1 = -\frac{\alpha_1}{\alpha_2} - \frac{1}{3} (w_2 w_3 - h_2 h_3) + \tilde{\xi}_1 \text{sign} (\sigma_1) \]  

\(\tilde{\xi}_1\) is obtained using another Lyapunov function \(V_\text{eq}\) is given by Eq. (18)

\[ V_\text{eq} = V_1 + \frac{1}{2r_1} \tilde{\xi}_1^2 \]  

where \(\tilde{\xi}_1\) is the error in estimation with \(\dot{\xi}_1 = \xi_1 - \tilde{\xi}_1\).

The time derivative of \(V_1\) is given by

\[ V_1' = V_1' + \frac{1}{r_1} \tilde{\xi}_1 \]  

Substituting Eq. (14) into Eq. (19) gives

\[ V_1'' = V_1' + \frac{1}{r_1} \tilde{\xi}_1 = -b_1 \xi_1^2 + \alpha_1 \left( \frac{1}{3} (w_2 w_3 - h_2 h_3) + \frac{1}{3} (d_1 - f_1) + \eta_1 \right) \]  

Eqs. (21) and (22) are obtained by substituting Eq. (17) into Eq. (20).

\[ V_1'' = V_1' + \frac{1}{r_1} \tilde{\xi}_1 = -b_1 \xi_1^2 + \alpha_1 \left( \frac{1}{3} (w_2 w_3 - h_2 h_3) + \frac{1}{3} (d_1 - f_1) + \eta_1 \right) \]  

\[ \leq -b_1 \xi_1^2 + \alpha_1 \left( \frac{1}{3} (D_1 - F_1) + \alpha_1 \tilde{\xi}_1 \right) |\sigma_1| + \tilde{\xi}_1 \left( \frac{1}{r_1} - \alpha_1 |\sigma_1| \right) \]  

The time derivative of \(\tilde{\xi}_1\) can be written as \(\dot{\tilde{\xi}}_1 = 0 - \tilde{\xi}_1\).

From (23), \(V_1\) will be semi negative definite if the adaptation law is formulated as
\[ \frac{d}{dt} \xi_1 = -\eta_1 |\xi_1| \]  
(25)

for adaptively estimating the unknown upper bounds.

Substituting Eq. (24) and Eq. (25) into Eq. (23) gives

\[ \dot{V}_1 = -b_1 e_1^2 - \phi_1 |\xi_1| \leq 0 \]  
(26)

which implies that \( \dot{V}_1 \) is a negative semi-definite function and therefore the error trajectories will converge and stay on the sliding surface.

Integrating Eq. (26) from \( t = 0 \) to \( \infty \) gives

\[ \int_0^\infty (b_1 e_1(t)^2 + \phi_1 |\xi_1(t)|) dt \leq \int_0^\infty \dot{V}_1(t) dt \]

\[ = V_1(0) - V_1(\infty) < \infty \]  
(27)

For satisfying the condition in Eq. (27), \( \lim_{t \to \infty} \int_0^t (b_1 e_1(s)^2 + \phi_1 |\xi_1|) ds \) is bounded. By Barbalat’s lemma [55], we deduce that \( \lim_{t \to \infty} (b_1 e_1(s)^2 + \phi_1 |\xi_1(s)|) = 0 \). In other words, \( b_1 e_1(t)^2 + \phi_1 |\xi_1(t)| \) asymptotically converges to zero.

### 3.2. Derivation of control input \( \eta_2 \)

We choose another Lyapunov function candidate as given in Eq. (28)

\[ V_2 = V_1 + \frac{1}{2} \sigma_2^2 \]  
(28)

whose time derivative is given by Eq. (29)

\[ \dot{V}_2 = V_1 + e_2 \dot{e}_2 = \sigma_1 e_1 - b_1 e_1^2 + \sigma_1 e_2 - b_2 e_2^2 \]  
(29)

We choose the virtual control input as given in Eq. (30)

\[ \dot{e}_2 = \sigma_1 - b_2 e_2 (b_2 > 1) \]  
(30)

If \( \sigma_1 \to 0 \), \( V_2 = -b_1 e_1^2 - b_2 e_2^2 \leq 0 \) which implies that \( V_2 \) is negative definite.

In order to obtain the second control input \( \eta_2 \), another Lyapunov function \( V_2' \) is selected and presented in Eq. (31)

\[ V_2' = V_2 + \frac{1}{2} \sigma_2^2 \]  
(31)

where the second sliding surface \( \sigma_2 \) is selected as

\[ \sigma_2 = \alpha_1 e_1 + \alpha_2 e_2 \ (\alpha_2 \geq 1) \]  
(32)

The time derivative of \( V_2' \) is given as

\[ \dot{V}_2' = V_2' + \sigma_2 \dot{\sigma}_2 = \sigma_1 (e_1 + e_2) - b_1 e_1^2 - b_2 e_2^2 + \sigma_2 (\alpha_1 e_1 + \alpha_2 e_2) \]  
(33)

Substituting \( \sigma_1 \) in Eq. (33) from Eq. (32)

\[ V_2' = -b_1 e_1^2 - b_2 e_2^2 - \alpha_2 e_2 e_2 - \alpha_2 e_2^2 + \sigma_2 (e_1 + e_2 + \alpha_1 e_1 + \alpha_2 e_2) \]  
(34)

where another virtual control input is chosen and expressed by Eq. (35).
\[ e_2 = a_1 + k_1 e_1 \ (k_1 \geq 1) \]  
(35)

which makes \( a_2 e_2 e_2 \geq 0 \).

The term \( e_2 \) in Eq. (34) consists of the second control input, therefore, \( e_2 \) is substituted from Eq. (6b) in Eq. (34). Satisfying the reaching condition \( (\sigma_2 \sigma_2 < 0) \) in Eq. (34), the second equivalent control input is obtained as

\[ \eta_{eq} = a_2^{-1} \left( -e_1 - e_2 - a_1 (\sigma_1 - h_1 h_3) \right) \]  
(36)

A switching control law \( \eta_2 = \tilde{\xi}_2 \text{sign}(\sigma_2) \) is augmented with Eq. (36) and the control input is obtained as given in Eq. (37)

\[ \eta_2 = a_2^{-1} \left( -e_1 - e_2 - a_1 (\sigma_1 - h_1 h_3) \right) \]  
(37)

In order to suppress any disturbances and model uncertainties with unknown upper bounds \( \tilde{\xi}_2 \) is replaced with \( \hat{\tilde{\xi}}_2 \)

\[ \eta_2 = a_2^{-1} \left( -e_1 - e_2 - a_1 (\sigma_1 - h_1 e_1) \right) - \left( w_1 w_3 - h_1 h_3 \right) + \hat{\tilde{\xi}}_2 \text{sign}(\sigma_2) \]  
(38)

\( \hat{\tilde{\xi}}_2 \) is obtained using another Lyapunov function and expressed by Eq. (39)

\[ V_2" = V_2' + \frac{1}{2r_2} \hat{\tilde{\xi}}_2^2 \]  
(39)

where \( \hat{\tilde{\xi}}_2 \) is the error in estimation and is given by \( \hat{\tilde{\xi}}_2 = \tilde{\xi}_2 - \hat{\tilde{\xi}}_2 \).

The time derivative of \( V_2" \) is given by

\[ V_2." = V_2.' + \frac{1}{r_2} \hat{\tilde{\xi}}_2 \]  
(40)

Eq. (41) is obtained by substituting Eq. (34) into Eq. (40)

\[ V_2." = -b_1 e_1^2 - b_2 e_2^2 - a_2 e_1 e_2 - a_2 e_2^2 + \sigma_1 (a_2 (d_2 - f_2) + a_2 \hat{\tilde{\xi}}_2 \text{sign}(\sigma_2)) \]  
(41)

Substituting \( \hat{\tilde{\xi}}_2 \) from Eq. (6b) and \( \eta_2 \) from Eq. (38) into Eq. (40) gives Eqs. (42) and (43)

\[ V_2." = -b_1 e_1^2 - b_2 e_2^2 - a_2 e_1 e_2 - a_2 e_2^2 + \frac{a_2^2}{2} (d_2 - f_2) + a_2 \hat{\tilde{\xi}}_2 \text{sign}(\sigma_2) \]  
(42)

\[ + \frac{1}{r_2} \hat{\tilde{\xi}}_2 \]  
(43)

\[ \leq -b_1 e_1^2 - b_2 e_2^2 - a_2 e_1 e_2 - a_2 e_2^2 + \left( \frac{a_2^2}{2} (D_2 - F_2) + a_2 \hat{\tilde{\xi}}_2 \right) \sigma_2 \]  
(44)
The time derivative of $\xi_2$ can be written as

$$\dot{\xi}_2 = 0 - \frac{1}{C_0} \xi_2.$$ 

From Eq. (44), $V_2$ will be semi negative definite if the adaptation law is formulated as

$$\alpha_2 \xi_2 (D_2 - F_2) + \alpha_2 \xi_2 = -\varphi_2$$

if upper bounds are known, where $\varphi_2$ is a positive constant.

$$\dot{\xi}_2 = -\sigma_2 \alpha_2 |\xi_2|$$

for adaptively estimating the unknown upper bounds. Substituting Eq. (45) and Eq. (46) into Eq. (44) gives

$$\dot{V}_2 = -b_1 e_1^2 - b_2 e_2^2 - \alpha_2 e_1 e_2 - \alpha_2 e_2^2 - \varphi_2 |\xi_2| \leq 0$$

$V_2$ is a positive definite function candidate and according to Eq. (47) $\dot{V}_2$ is a negative semi-definite function which implies that the error trajectories will converge and stay on the sliding surface.

Integrating Eq. (47) from $t = 0$ to $\infty$ gives

$$\int_0^\infty \left( b_1 e_1(t)^2 + b_2 e_2(t)^2 + \alpha_2 e_1(t) e_2(t) + \alpha_2 e_2(t)^2 + \varphi_2 |\xi_2(t)| \right) dt \leq V_2(0) - V_2(\infty) < \infty$$

Similar to the previous case, by the boundedness of Eq. (48) and applying Barbalat’s lemma, $b_1 e_1(t) + b_2 e_2(t)^2 + \alpha_2 e_1(t) e_2(t) + \alpha_2 e_2(t)^2 + \varphi_2 |\xi_2(t)|$ asymptotically converges to zero.

### 3.3. Derivation of control input $\eta_3$

We choose another Lyapunov function candidate as given by Eq. (49)

$$V_3 = V_2 + \frac{1}{2\sigma_3^2}$$

and its time derivative is expressed by Eq. (50)

$$\dot{V}_3 = \dot{V}_2 + \sigma_3 e_3 = \sigma_3 e_1 - b_1 e_1^2 + \sigma_3 e_2 - b_2 e_2^2 + \sigma_3 e_3 - b_3 e_3^2$$

where virtual control input is given by Eq. (51)

$$\sigma_3 = \sigma_3 - b_3 e_3 (b_3 \geq 1)$$

If $\sigma_3 = 0$, $V_3 = -b_1 e_1^2 - b_2 e_2^2 - b_3 e_3^2 \leq 0$ which implies that $V_3$ is negative definite.

In order to obtain the third control input $\eta_3$, another Lyapunov function $V_3$ is selected and given in Eq. (52)

$$V_3 = V_3 + \frac{1}{2\sigma_3^2}$$

where $\sigma_3$ is the third sliding surface and is selected as
\[ \sigma_3 = \alpha_1 e_1 + \alpha_2 e_2 + \alpha_3 e_3 \quad (\alpha_3 \geq 1) \]

The time derivative of \( V_3' \) is given as

\[ V_3' = \dot{V}_3 + \sigma_1 \dot{\sigma}_3 = -b_1 e_1^2 - b_2 e_2^2 - b_3 e_3^2 \]

Substituting \( \sigma_1 \) in Eq. (54) with Eq. (53)

\[ V_3' = (\alpha_2 - \alpha_2 e_2)(e_1 + e_2) + \sigma_3(e_1 + \alpha_2 e_2 - b_1 e_1) - b_3 e_3^2 \]

Using another virtual control input as given in Eq. (56)

\[ e_3 = \sigma_3 + k_1 e_1 \]

in Eq. (55) makes \((\alpha_2 - \alpha_3 e_3)(e_1 + e_2 + e_3) \leq 0\).

\[ \eta_{m(3)} = \alpha_3^{-1}(-e_1 - e_2 - e_3 - \alpha_1(\sigma_1 - b_1 e_1) - \alpha_2(\sigma_1 - b_2 e_2)) \]

A switching control law \( \eta_{m3} = \xi_3 \text{sign}(\sigma_3) \) is augmented with Eq. (57) and the control input is obtained as given in Eq. (58)

\[ \xi_3 = \alpha_3^{-1}(-e_1 - e_2 - e_3 - \alpha_1(\sigma_1 - b_1 e_1) - \alpha_2(\sigma_1 - b_2 e_2)) - (w_1 w_2 - h_1 h_2) + \xi_3 \text{sign}(\sigma_3) \]

In order to suppress any disturbances and model uncertainties with unknown upper bounds \( \xi_3 \) is replaced with \( \hat{\xi}_3 \)

\[ \eta_{m3} = \alpha_3^{-1}(-e_1 - e_2 - e_3 - \alpha_1(\sigma_1 - b_1 e_1) - \alpha_2(\sigma_1 - b_2 e_2)) - (w_1 w_2 - h_1 h_2) + \hat{\xi}_3 \text{sign}(\sigma_3) \]

\( \hat{\xi}_3 \) is obtained using another Lyapunov function and given in Eq. (60)

\[ V_3'' = V_3' + \frac{1}{2 \hat{\xi}_3^2} \]

where \( \hat{\xi}_3 \) is the error in estimation and is given by \( \hat{\xi}_3 = \xi_3 - \hat{\xi}_3 \).

Differentiating \( V_3'' \) with respect to \( t \), we get

\[ \dot{V}_3'' = \dot{V}_3' + \frac{1}{\hat{\xi}_3^2} \hat{\xi}_3 \]

Substituting Eq. (55) into Eq. (61) gives

\[ \dot{V}_3'' = (\alpha_2 e_2 - \alpha_2 e_3)(\sigma_1 + e_1) - b_1 e_1^2 - b_2 e_2^2 - b_3 e_3^2 \]

\[ + \sigma_3(e_1 + e_2 + \sigma_1 e_1 + \sigma_2 e_2 + \sigma_3 e_3) - \frac{1}{r_5 \hat{\xi}_3^2} \]
Substituting $e_2$ from Eq. (6c) and $\eta_3$ from Eq. (59) into Eq. (62) yields Eqs. (63) and (64)

$$
\dot{V}_3^\alpha = (-\alpha_2 e_2 - \alpha_3 e_3)|e_1 + e_2 + e_3| - b_1 e_1 ^2 - b_2 e_2 ^2 - b_3 e_3 ^2 + \sigma_3 (\alpha_3 (D_3 - F_3) + \frac{1}{t_3} \gamma _3)
$$

(63)

$$
+ \alpha_3 (\gamma _3 + \sigma_3 |e_1 + e_2 + e_3| - \frac{1}{t_3} \gamma _3)
$$

(64)

$$
\leq (-\alpha_2 e_2 - \alpha_3 e_3)|e_1 + e_2 + e_3| - b_1 e_1 ^2 - b_2 e_2 ^2 - b_3 e_3 ^2
$$

$$
+ (\alpha_3 (D_3 - F_3) + \alpha_3 \gamma _3) + \gamma _3 (\frac{1}{t_3} \gamma _3 - \alpha_3 |e_1|)
$$

(65)

The time derivative of $\gamma _3$ can be written as $\gamma _3 = 0 - \dot{\gamma}_3$.

From (65), $V_3^\alpha$ will be semi negative definite if the adaptation law is chosen as

$$\alpha_3 (D_3 - F_3) + \alpha_3 \gamma _3 = - \dot{\eta}_3$$

(66)

in case upper bounds are known, where $\eta_3$ is a positive constant.

$$\dot{\gamma}_3 = - \dot{\eta}_3 |\sigma_3|$$

(67)

for adaptively estimating the unknown upper bounds.

Substituting Eq. (66) and (67) into Eq. (65) gives

$$
\dot{V}_3^\alpha = (-\alpha_2 e_2 - \alpha_3 e_3)|e_1 + e_2 + e_3| - b_1 e_1 ^2 - b_2 e_2 ^2 - b_3 e_3 ^2 - \eta_3 |\sigma_3| \leq 0
$$

(68)

$V_3^\alpha$ is a positive definite function candidate and Eq. (68) implies that $\dot{V}_3^\alpha$ is a negative semi-definite function and therefore the error trajectories will converge and stay on the sliding surface.

Integrating Eq. (68) from $t = 0 \to \infty$ gives

$$
\int_0 ^\infty (\alpha_2 e_2(t) + \alpha_3 e_3(t)) (e_1(t) + e_2(t) + e_3(t)) + b_1 e_1(t) ^2 + b_2 e_2(t) ^2 + b_3 e_3(t) ^2 + \sigma_3 |\sigma_3(t)| dt \leq - \int_0 ^\infty V_3^\alpha(t) dt
$$

(69)

$$
= V_0^\alpha (0) - V_0^\alpha (\infty) < \infty
$$

Similar to the previous case, by the boundedness of Eq. (69) and applying Barbiala's lemma, $\alpha_2 e_2(t) + \alpha_3 e_3(t) (e_1(t) + e_2(t) + e_3(t)) + b_1 e_1(t) ^2 + b_2 e_2(t) ^2 + b_3 e_3(t) ^2 + \sigma_3 |\sigma_3(t)|$ asymptotically converges to zero.

4. Simulation results

In this section, two simulation works are performed and analyzed to implement the master-slave synchronization. The block diagram representing the design procedure of the proposed technique is shown in Figure 1. The external disturbances are represented using external perturbing torques. Two different perturbing torques are stated in (Example 1 and Example 2) to validate the efficiency of the proposed control methodology in terms of robustness. The simulations (in Example 1 and Example 2) are carried out with the same controller parameters and the same initial states of the system. The initial values of the master system are selected as $[w_0 (0) w_0 (0) w_0 (0)] ^T = [3 4 1.2]^T$, whereas those of the slave system are selected as $[h_1 (0) h_2 (0) h_3 (0)] ^T = [3.5 5 2.5]^T$.

**Example 1**

In this example, we apply our results to the synchronization problem with the following perturbing torques as in [56].

$$g_x = -1.2 \omega_x + \frac{\sqrt{6}}{2} \omega_y$$

(70)

$$g_y = 0.35 \omega_y$$

$$g_z = -\sqrt{6} \omega_x - 0.4 \omega_y$$

With Eq. (70), the master system can be rewritten as given in Eq. (71)

$$\dot{w}_1 = -\frac{1}{3} \omega_2 w_3 - 0.4 w_1 + \frac{\sqrt{6}}{2} w_2$$

(71)

$$\dot{w}_2 = -w_1 w_3 + \frac{0.35}{2} w_2$$

$$\dot{w}_3 = w_1 w_2 - \sqrt{6} w_1 - 0.4 w_3$$

and the slave system can be rewritten as given in Eq. (72)

$$\dot{h}_1 = -h_1 h_3 - 0.4 h_1 + \frac{\sqrt{6}}{2} h_3 + \eta_1$$

(72)

$$\dot{h}_2 = -h_1 h_3 + \frac{0.35}{2} h_2 + \eta_2$$

$$\dot{h}_3 = h_1 h_2 - \sqrt{6} h_1 - 0.4 h_3 + \eta_3$$

The system displays chaotic motion as depicted in Figure 2. It is shown in Figure 3 that without control, the respective states of the system cannot be synchronized. For implementing the synchronization of the master-slave system with the different initial values, the controller parameters are chosen as $b_1 = 2, b_2 = 2, \xi_1 (0) = 3, \xi_2 (0) = 3, \xi_3 (0) = 3, r_1 = 5, r_2 = 4, r_3 = 2, a_1 = 1, a_2 = 1, a_3 = 1$. Figure 4 depicts the state responses of the master and slave systems for Example 1 using the proposed scheme.

It can be seen in Figure 4 that the proposed control input helps to synchronize the two systems as in Example 1 within 2 s, even if the initial conditions are significantly different.

The adaptation of the switching gains parameters $\xi_1$, $\xi_2$ and $\xi_3$ for Example 1 are shown in Figure 5(a), (b) and (c), respectively. These gain parameters reach its final value within 2 s and successfully cancel out the disturbance torques. The sliding surfaces take 2 s to converge to zero as presented in Figure 6(a), (b) and (c).

**Example 2**

In this example, we consider the perturbing torques as

$$g_x = -1.2 \omega_x + \frac{\sqrt{6}}{2} \omega_y + 10 \cos \gamma \sin \theta$$

(73)

$$g_y = 0.35 \omega_y + 10 \cos \psi \sin \gamma$$

$$g_z = -\sqrt{6} \omega_x - 0.4 \omega_y + 10 \cos \theta \sin \psi$$

With Eq. (73), the master system is given by Eq. (74)

$$w_1 = -\frac{1}{3} \omega_2 w_2 - 0.4 w_1 + \frac{\sqrt{6}}{2} w_2 + 10 \cos \gamma \sin \theta$$

(74)

$$w_2 = -w_1 w_3 + \frac{0.35}{2} w_2 + 10 \cos \psi \sin \gamma$$

$$w_3 = w_1 w_2 - \sqrt{6} w_1 - 0.4 w_3 + 10 \cos \theta \sin \psi$$

and the slave system is given by Eq. (75)

$$h_1 = -h_1 h_3 - 0.4 h_1 + \frac{\sqrt{6}}{2} h_3 + \frac{10}{3} \cos \gamma \sin \theta + \eta_1$$

(75)

$$h_2 = -h_1 h_3 + \frac{0.35}{2} h_2 + \frac{10}{2} \cos \psi \sin \gamma + \eta_2$$

$$h_3 = h_1 h_2 - \sqrt{6} h_1 - 0.4 h_3 + 10 \cos \theta \sin \psi + \eta_3$$

where $\psi, \gamma$ and $\theta$ are taken as -0.0077, -0.0077 and -0.7509, respectively [10, 11].
For this synchronization problem, we also use the same initial values of the states and similar controller parameters as those used in Example 1. Figure 7 shows the synchronization error responses between the master and slave system.

Observing Figure 7, the synchronization errors for Example 2 exactly converge to zero within 3 s, as concluded in Remark 3. The evolution of the switching gains parameters $\xi_1, \xi_2$ and $\xi_3$ for Example 2 are shown in Figure 8(a), (b), and (c), respectively. The gain parameters converge to its final value within 2 s and successfully nullify the disturbance torques. The sliding surfaces for Example 2 converge to zero within 2 s, as presented in Figure 9.

Remark 2. Disturbances are present in three error subsystems represented by Eqs. (6a), (6b) and (6c). Three different sliding mode surfaces are introduced for each of the three subsystems. Remark 3. Eqs. (25), (46), and (67) represents the updating laws which are obtained as functions of the sliding surfaces given by Eqs. (10), (32), and (53), respectively. The control laws proposed in Eqs. (17), (38), and (59) along with the updating laws given by Eqs. (25), (46), and (67), not only cancel out the undesirable effects of the unknown perturbing torques but also minimizes the synchronization error to the smallest neighborhood of the equilibrium point. Therefore, the control scheme proposed is considered adaptive.

Remark 4. The asymptotic stability of the closed-loop system is guaranteed using two types of virtual control inputs (with different structures) through the Lyapunov stability theorem. The first structure of virtual control input is taken as $e_i = c_1 - b_i e_i, i = 1, 2, 3$ which ensures the closed-loop system stability through the time derivative Lyapunov function candidate $V_i, i = 1, 2, 3$. The second structure of virtual control input is taken as $e_i = c_{i,i} + k_i e_i, i = 1, 2, 3$ which guarantees the closed-loop system stability through the time derivative Lyapunov function candidate $V_{i,i}, i = 2, 3$.

5. Conclusion

A novel adaptive sliding mode control based on the backstepping approach is proposed for synchronizing chaos attitude in a satellite system. The adaptation laws are derived into the proposed control laws to suppress any unwanted effects of the unknown perturbing torques and also reduce the synchronization error to the smallest neighborhood of the equilibrium point. These adaptive laws are obtained based on the Lyapunov stability theorem. The adaptation parameters are updated to effectively estimate the upper bounds of disturbances in situations when it is not easy to perceive the bounds of disturbances. Virtual inputs of two different structures are employed to prove the stability of the closed-loop system through the Lyapunov stability theorem. Simulation results confirm the robustness of the control methodology in chaos synchronization in the existence of different perturbing torques. Besides, internal disturbances (model uncertainties and parametric uncertainties) will be conducted in addition to external disturbances in the future work.

Declarations

Author contribution statement

Pikaso Pal: Conceived and designed the experiments; Performed the experiments; Analyzed and interpreted the data; Wrote the paper.

Gang Gyoo Jin, S. Bhakta, V. Mukherjee: Analyzed and interpreted the data; Wrote the paper.

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