A FORMAL REPRESENTATION OF THE
THEMATIC-RHEMATIC STRUCTURE OF SENTENCES
BASED ON A TYPED A-CALCULUS

Yoichi UE TAKE

Tokyo University of International Risue

ABSTRACT: In this paper, we give a formal representation of the thematic-rhematic (T-R) structure of a natural language discourse, based on a typed λ-calculus.

I. INTRODUCTION

In this paper, we give a formal representation of the thematic-rhematic (T-R) structure of a natural language discourse. Some pairs, triples, or in general n-tuples of sentences in a discourse may differ in the place of their information focus. The distribution of this information focus is called the thematic-rhematic (T-R) structure or dichotomy. In English, the use of particles the and a (an) is deeply related to the T-R structure. In general, a noun with the particle the constitutes a theme part of the sentence that appears at the beginning of the discourse or text, while that noun with the particle a appears in the second, third, etc., sentences as themes. In Japanese, the T-R dichotomy is well represented by postpositions wa and ga. The Korean language has a similar system. Meanwhile, in Slavic languages as Polish, Czech, and Russian, the word order is free and this degree of freedom is used for the representation of the T-R dichotomy. In Chinese, the word order is also used for the T-R dichotomy. Besides theme and rheme, similar terms as old-information and new-information, topic and comment, topic and focus etc. are used in the literature concerning functional linguistics (see, e.g., Valldavi). In our analysis, since we do not define these terms explicitly, it is not essential which terms are used. We give implicit definition of these concepts axiomatically. We consider the problem mainly for Japanese. We propose to use typed λ-calculus to analyze the problem. A logical notation is seen as a typed λ-term. Basic types are T and R. Roughly speaking, T and R stand for a theme part and a rheme part of a sentence, respectively. The difference of T-R dichotomy is given by different types. Thus the same sentence may have different types depending on the situation. For utterances, type inference will be performed. The correctness of a given discourse can be proved by checking the correctness of the types of each utterance. In this paper, we elaborate on this idea.

II. REPRESENTATION BASED ON A TYPED A-CALCULUS

The purpose of this paper is to propose a formal model for utterance interpretation of the thematic-rhematic structure of a Japanese sentence using a typed λ-calculus. In our analysis, a logical notation is seen as a typed λ-term. Basic types are T and R. Roughly speaking, T and R stand a theme part and a rheme part of a sentence, respectively. Although we analyze mainly Japanese sentences, the results can be applied to other languages. The T-R dichotomy of a Japanese sentence is represented by the postpositions wa and ga. For example, the following two sentences are different in T-R dichotomy, and used in different situations: (a) Taroo wa Gakusei desu. (Speaking of Taroo, he is a student.) (b) Taroo ga Gakusei desu. ((Of all the people we are talking about) Taroo (and only Taroo) is a student.) The meaning of both (a) and (b) is Taroo is a student, and thus may be written as student(Taroo). However this representation is obviously not sufficient for an account of the utterance interpretation of (a) and (b). The NP (noun phrase) of (a) marked with wa functions as a theme, i.e., it should have already appeared in the preceding discourse and thus can be considered as an old information. Therefore, in the discourse, sentence (a) should be preceded by a sentence that contains Taroo as a rheme (new information). For example, Taroo in the following sentence can be considered as a new information: (c) Taroo ga imasu. (Here is Taroo.) The pair (c), (a) in this order is a correct discourse utterance. On the other hand, the pair (c), (b) cannot be considered correct since student functions as a theme in (b) while it has not appeared in the preceding context. As is seen
from (b) and (c), an NP marked with postposition ga functions as a rheme (i.e., information focus). To explain the difference between (a) and (b) in the utterance level, we annotate $\lambda x.\text{student}(x)$ of (a) and (b) by different typed $\lambda$-terms. Roughly speaking we assign $T \to R$ and $R$ to each $\lambda x.\text{student}(x)$ of (a) and (b), respectively. Based on this, if we can show $\text{student}(\text{Taroo}) : R$ then we say sentence (a) (or (b)) of the discourse is correct. For example, if Taroo of (a) has a type $T$ then by the $\beta$-reduction of typed $\lambda$-calculus, we have $\text{student}(\text{Taroo}) : R$. For Taroo to have a type $T$, we impose a constraint that Taroo must have appeared in a preceding sentence. Other cases can be treated similarly. See the following descriptions for details. Thus the correctness of the discourse can be proved by checking the correctness of the types of each formula. In general, given a discourse $s_0, s_1, \ldots, s_n$ in logical forms, what we have to show is that $\Gamma \vdash s_0 : R$, $s_0 : R \vdash s_1 : R$, $\ldots$, $(s_0 : R, \ldots, s_n : R) \vdash s_n : R$, successively.

First consider the following discourse consisting of a single sentence.

$\text{Taroo ga imasu. (Here is Taroo.)} \quad (1)$

The meaning of this sentence is:

$s_0 = \text{here.is}(\text{Taroo}) \quad (2)$

We define this discourse to be correct if $s_0 : R$. This is done in the following way: Translate $\text{Taroo ga}$ into $\lambda f.\text{(Taroo)}$. We let this formula have either type of $T \to R$ or $R \to R$ when the proper noun Taroo is marked with the postposition ga. Thus we have the following translation rules:

$\text{Taroo ga} \Rightarrow \lambda f.\text{(Taroo)} \subseteq s_0 : T \to R \quad (3.1)$

$\text{Taroo ga} \Rightarrow \lambda f.\text{(Taroo)} \subseteq s_0 : R \to R \quad (3.2)$

This can be written for short as

$\text{Taroo ga} \Rightarrow \lambda f.\text{(Taroo)} \subseteq s_0 : (T \lor R) \to R \quad (4)$

In the above, $t \subseteq s_0$ means that $t$ is a typed $\lambda$-term component of the logical formula $s_0$. That is

$t \subseteq s_0 \iff (t_1, t_2) t_1 t_2 = s_0 \quad (5)$

Here $t_1$ and/or $t_2$ may be empty. Thus $s_0 \subseteq s_0$. From (3), we have

$\Gamma \vdash \lambda f.\text{(Taroo)} \subseteq s_0 : (T \lor R) \to R \quad (6)$

The verb imasu allows a neutral description. A neutral description has the following T-R dichotomy:

$\text{Taroo ga imasu.}$

\begin{tabular}{c c c}
Rheme & Rheme & \end{tabular} \quad (7)

A sentence of neutral description in the Japanese language was first found and named by Kuroda (1965). This kind of sentence has no theme part. For this kind of verb, we assign a type $R$ and write as follows:

$\Gamma \vdash \lambda x.\text{here.is}(x) \subseteq s_0 : R \quad (8)$

Now by (6) and (8) we can deduce the following judgement.

$\epsilon_0 : A_0, \epsilon_1 : A_1 \vdash$

$\lambda f.\text{(Taroo)}(\lambda x.\text{here.is}(x))$

$= (\lambda x.\text{here.is}(x))(\text{Taroo})$

$= \text{here.is}(\text{Taroo}) = s_0 : R \quad (9)$

where $\epsilon_0 : A_0$ and $\epsilon_1 : A_1$ stand for (6) and (8), respectively. Thus $s_0 : R$ has been proved and the correctness of the discourse (1) has been established. To deduce (9), we have of course used the inference rule of the typed $\lambda$-calculus given by

$\epsilon_0 : \alpha \to \beta, \epsilon_1 : \alpha \vdash \epsilon_0 \epsilon_1 : \beta \quad (10)$

Note that the type used for $(\lambda f.\text{(Taroo)})$ in deduction (9) is $R \to R$. In general, for a neutral description, $\beta$-reduction for $R \to R$ and $R$ occur. Next we consider the discourse consisting of the following two sentences.

$\text{Taroo ga imasu. (Here is Taroo.)} \quad (11.1)$

$\text{Taroo wa gakusei desu. (Taroo is a student.)} \quad (11.2)$

The T-R dichotomies of the above sentences are as follows:

$\text{Taroo ga imasu.}$

\begin{tabular}{c c c}
Rheme & Rheme & \end{tabular} \quad (12.1)

$\text{Taroo wa gakusei desu.}$

\begin{tabular}{c c c}
Theme & Rheme & \end{tabular} \quad (12.2)

The NP (noun phrase) of (12.2) marked with wa functions as a theme. It should have already appeared in the preceding discourse as a rheme. The discourse (12) satisfies this constraint since Taroo appears as a theme in (12.1) since it is marked with the postposition ga. The discourse (12) is actually correct. We now formally state the correctness of (12). The logical forms of (12.1) and (12.2) are given as

$s_0 = \text{here.is}(\text{Taroo}) \quad (13.1)$

$s_1 = \text{student}(\text{Taroo}) \quad (13.2)$

First we must show $s_0 : R$, however we have already seen this. Thus we show $s_1 : R$. Note that $s_0 = (\lambda x.\text{student}(x))(\text{Taroo})$. It is natural to assign $\lambda x.\text{student}(x)$ a type $T \to R$ since (12.2) contains

$s_0 = \text{here.is}(\text{Taroo}) \quad (13.1)$

$s_1 = \text{student}(\text{Taroo}) \quad (13.2)$
the postposition was. This postposition is called the thematic wa. We write this as follows.

\[
\text{wa flakusci desu} \Rightarrow \\
\lambda x. \text{student}(x) \subseteq s_1 : T \rightarrow R
\] (14)

Thus we have

\[
\vdash \lambda x. \text{student}(x) \subseteq s_1 : T \rightarrow R
\] (15)

Therefore if Taroo has a type \( T \), we have \( s_1 : R \) by \( \beta \)-reduction. The NP can be a theme if it has already appeared in the preceding discourse as a theme. This rule can be written as follows:

\[
\lambda f. f(\text{Taroo}) \subseteq s_0 : (T \text{ or } R) \rightarrow R \vdash \text{Taroo} \subseteq s_1 : T
\] (16)

Now \( s_1 : R \) can be shown as follows. By (6) and (16),

\[
\vdash \text{Taroo} \subseteq s_1 : T
\] (17)

Applying the \( \beta \)-reduction rule to (15) and (17), we have \( s_1 : R \). Thus the discourse (11) is correct.

In Japanese, the following sentence at the beginning of the discourse is not natural.

\[
\text{Taroo wa gakusei desu. (Taroo is a student.)}
\] (18)

This is because Taroo appears as a theme but it is not preceded by a sentence in which Taroo appears as a theme. In our formal description, the incorrectness of the discourse (18) is described as a failure of type checking. We define the discourse to be incorrect if either \( s_0 : R \) or \( s_1 : R \) is not proved. Indeed, \( s_0 : R \) where \( s_0 = \text{student}(\text{Taroo}) \) is not proved since we do not have \( \text{Taroo} \subseteq s_0 : T \).

We now consider the following discourse consisting of two sentences.

\[
\text{Gakusei ga imasu.}
\] (19.1)

\[
\text{Taroo ga gakusei desu.}
\] (19.2)

The logical forms for (19.1) and (19.2) are given as follows.

\[
s_0 = (\exists x)\text{student}(x) \land \text{here_is}(x)
\] (20.1)

\[
s_1 = \text{student}(\text{Taroo})
\] (20.2)

Since gakusei (student) is marked with the postposition ga, and the verb imasu allows a neutral description, we have

\[
(\exists x)\text{student}(x) \land \text{here_is}(x) \subseteq s_0 : R
\] (21)

From this we have

\[
(\exists x)\text{student}(x) \subseteq s_0 : R
\] (22)

In general we impose the following postulate.

\[
A \land B \subseteq s_i : R \vdash A \subseteq s_i : R
\] (23)

Furthermore we add the following postulate.

\[
Qx.f(x) \subseteq s_0 : R \vdash \lambda x.f(x) \subseteq s_1 : T
\] (24)

where \( Q \) stands for a quantifier \( V \) or \( \exists \). This postulate means that a predicate that appeared as a theme can be treated as a theme in the succeeding sentences. From this and (22) we can deduce

\[
\lambda x.\text{student}(x) \subseteq s_1 : T
\] (25)

We now show \( s_1 : R \). First by (4) we have (6). Applying the \( \beta \)-reduction rule (10) to (6) and (25) we have \( s_1 = \text{student}(\text{Taroo}) : R \). Therefore, the discourse (19) is correct. Note that the type used for \( \lambda f. f(\text{Taroo}) \) is \( T \rightarrow R \). Compare this with (9).

We now consider the following discourse consisting of a single sentence.

\[
\text{Taroo ga gakusei desu.}
\] (26)

In the above sentence type checking fails as follows. Since the postposition ga is attached to Taroo, we have (6). Therefore, \( \lambda x.\text{student}(x) \subseteq s_0 \) must have a type of either \( T \) or \( R \). However this is impossible. Since gakusei desu cannot be used in a sentence of neutral description, \( \lambda x.\text{student}(x) \subseteq s_1 \) never has a type \( R \). The sentence \( x \text{ ga gakusei desu} \) always means that it is \( x \) who is a student and is used only in the situation where gakusei is a theme. According to Kuno (1973), this use of predicate is called the exhaustive-listing. On the other hand, \( \lambda x.\text{student}(x) \) can have a type \( T \) only when student has appeared as in (21) in the preceding context and the postulate (24) can be used. Since (26) does not have a preceding text, it never happens. Thus it fails to prove \( s_0 : R \) and it has been established that (26) is not a correct discourse.

So far we have considered discourses consisting of two sentences. However the above method can be easily extended to a discourse that is consisting of more than three sentences. In this case, the inference rules used over several sentences are modified. For example, (15) can be modified as follows:

\[
\vdash \text{Taroo} \subseteq s_j : T
\] (16')

where \( s_j \) denotes the logical form corresponding to the \( j \)-th sentence of a discourse. Furthermore, \( \text{Taroo} \) can of course be arbitrary term, and thus we can establish the following more general rule:

\[
\lambda f.t \subseteq s_i, i < j : (T \text{ or } R) \rightarrow R \vdash t \subseteq s_j : T
\] (16'')
III. CONCLUSIONS

In this paper, we have given a formal representation of the T-R structure of a natural language discourse. We have proposed using a notion of typed $\lambda$-calculus. A logical notation has been seen as a typed $\lambda$-term. The correctness of a given discourse can be proved by checking the correctness of the types of each utterance. Although we have analysed mainly Japanese sentences, the results can be applied to other languages by considering adequate translation rules to encode a given sentence to formal representations.

In Uetake (1993, 1994), the author has proposed another tool for the analysis of the T-R structure. The tool used there is a logical notation called ontological promiscuity of Hobbs (1985), which is first-order and nonintensional. Using this description, a proof process of utterance interpretation of a discourse is obtained. It is interesting that two concepts similar to these (i.e., typed $\lambda$-calculus and ontological promiscuity) used in the analysis of the T-R structure of a discourse are used in the theory of constructive mathematics ($r$-realizability and constructive type theory). The concept of ontological promiscuity in Uetake (1993, 1994) corresponds to the $r$-realizability and the typed $\lambda$-calculus of this paper to the constructive type theory. See Uetake (1994) for more detailed discussion.

One of the reviewers noted that Barbara Partee is recently working on logically reconstructing the Prague school's notion of topic-focus articulation. The author would like to thank him/her for this information.

ACKNOWLEDGEMENTS

The author would like to thank Professor Akira Ishikawa for valuable discussions and comments.

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