Maximal Proper Acceleration and the Quantum-to-Classical Transition

Howard E. Brandt

Abstract

I first review the physical basis for the universal maximal proper acceleration. Next, I introduce a new formulation for a relativistic scalar quantum field which generalizes the canonical theory to include the limiting proper acceleration. This field is then used to construct a simple model of an uncorrelated many-body system. I next argue that for a macroscopic object consisting of more than Avogadro's number of atoms, any supposed quantum state of the object is negligibly small, so that for all practical purposes, the object is best described by classical mechanics. Thus, a new explanation is offered for the quantum-to-classical transition and the absence of quantum superposition of macroscopic objects in the everyday world.

Keywords: quantum field theory, quantum mechanics, quantum-classical transition, quantum measurement, maximal proper acceleration, Avogadro's number.

1 INTRODUCTION

There is no generally accepted theory of why the everyday world of macroscopic objects is not usefully described in terms of quantum states. For example, a planet is never observed to be in a quantum superposition state. It has however been speculated that for the description of a classical macroscopic many-body system of sufficient complexity, quantified by the number of atoms of which it is composed, quantum mechanics can be replaced by classical mechanics. [Of course for a highly correlated mesoscopic system such as a Bose condensate, which consists of a single quantum state, a quantum description is needed. Also, quantum mechanics is clearly needed to understand the atomic and molecular structure of macroscopic objects.] It is here to be argued that for ordinary macroscopic objects consisting of more than Avogadro's number of atoms, any supposed quantum state of the object as a whole is negligibly small, so that for all practical purposes the object is best described by classical mechanics. This follows from a natural extension of Lorentz-invariant quantum field theory to include the physics-based upper bound on physically possible proper accelerations [1]-[5].
2 MAXIMAL PROPER ACCELERATION

Heuristic arguments are first given for the existence of a universal upper bound on proper acceleration. In the presence of purely gravitational fields, macroscopic particles follow geodesic paths with vanishing proper acceleration. In the presence of non-gravitational forces, the proper acceleration is nonvanishing, however it follows from elementary physical reasoning that there is a maximum possible proper acceleration, the so-called maximal proper acceleration relative to the vacuum [1]. The physical basis for maximal proper acceleration is direct [5]. By the time-energy uncertainty principle, virtual particle-antiparticle pairs of mass \( m \) occur in the vacuum during a time \( \hbar/2mc^2 \) and over a distance \( \hbar/2mc \), the Compton wavelength of the particles. This is the ordinary vacuum polarization. Here \( \hbar \) is Planck’s constant divided by \( 2\pi \), and \( c \) is the velocity of light in vacuum. In an accelerated frame, the inertial force acts on such a virtual particle in the vacuum polarization, and if an amount of energy equal to its rest energy is imparted to it, the particle becomes real. The inertial force with magnitude \( ma \) on a virtual particle having proper acceleration \( a \) acts over a distance within which the particle can be created, namely, of the order of its Compton wavelength \( \hbar/2mc \), thereby doing work of order \( (ma)(\hbar/2mc) \). This follows from the fact that the proper acceleration is the magnitude of the ordinary acceleration in the instantaneous rest frame of the particle. If this work is equated to the rest energy \( mc^2 \) of the particle, it then follows that, for proper acceleration of order

\[
a_M \sim 2mc^3/\hbar,
\]

particles of mass \( m \) are copiously produced out of the vacuum. The larger the acceleration, the larger are the masses of the created particles. In the extreme, if the acceleration is sufficiently large, the created particles will be black holes. For this to occur, the size of a created particle (namely, of the order of its Compton wavelength \( \hbar/2mc \)) must be less than its Schwarzschild radius \( 2Gm/c^2 \), where \( G \) is the universal gravitational constant. Thus the minimum possible mass of a black hole is of the order of the Planck mass \( (\hbar c/G)^{1/2} \) [1]. Next, if the Planck mass \( (\hbar c/G)^{1/2} \) is substituted in Eq. (1), it follows that, for proper acceleration \( a_M \) given by

\[
a_M = 2\pi\alpha(c^7/\hbar G)^{1/2},
\]

where \( \alpha \) is a number of order unity, there will be copious production of Planck mass black holes out of the vacuum, resulting in the formation of a manifest spacetime foam and the topological breakdown of the classical spacetime structure, as well as the breakdown of the very concept of acceleration [1], [2], [5]. Thus Eq. (2) gives the maximum possible proper acceleration relative to the vacuum.

3 RELATIVISTIC QUANTUM FIELDS

There follows a new natural extension of Lorentz-invariant quantum field theory to include the physics-based upper bound on physically possible proper accel-
erations. In Minkowski spacetime with a negative-signature, the spacetime line element (interval), is given by

\[ ds^2 = dx_{\mu} dx^{\mu} = dx_0^2 - dx^2 - dy^2 - dz^2 = c^2 dt^2 - dx^2 - dy^2 - dz^2 \geq 0, \]  

which follows directly from the fact that the velocity of any object cannot exceed the velocity of light. Here \( x^0 = x_0 = ct \) where \( t \) is the time, and \( \{x^\mu\} = \{x^0, x^1, x^2, x^3\} = \{ct, x, y, z\} = \{x^0, x\} \) are the coordinates of a point in spacetime. In the present work, the \( x^\mu \) are also taken to be the coordinates of a macroscopic measuring device making measurements of a field or detecting a particle at the point \( x^\mu \) in spacetime. Associated with the Minkowski line element is the four-dimensional d’Alembertian operator

\[ \Box^{(4)} \equiv \frac{\partial^2}{\partial x_{\mu} \partial x^{\mu}}, \]  

which appears in the Klein-Gordon equation for a relativistic scalar quantum field \( \phi(x) \) describing particles of mass \( m \), namely

\[ \left( -\hbar^2 \frac{\partial^2}{\partial x_{\mu} \partial x^{\mu}} - m^2 c^2 \right) \phi = 0, \]  

or equivalently

\[ \left( \hbar^2 \Box^{(4)} + m^2 c^2 \right) \phi = 0. \]  

The four-velocity of a macroscopic measuring device, moving relative to a particle excitation of the quantum field at point \( x^\mu \) and measuring the particle, is given by

\[ v^\mu = dx^\mu / ds. \]  

It is to be emphasized here that \( v^\mu \) is not the four-velocity of the microscopic quantum particle [Since the measured particle is localized at a point in spacetime, its four-velocity is indeterminate due to the quantum uncertainty principle.] Also, the four-acceleration \( A^\mu \) of the measuring device is

\[ A^\mu = dv^\mu / ds. \]  

The corresponding proper acceleration \( A \) of the measuring device is defined by

\[ A^2 = -c^4 \frac{dv^\mu}{ds} \frac{dv^{\mu}}{ds}, \]  

which follows from the fact that proper acceleration is the magnitude of the ordinary acceleration in the instantaneous rest frame of an object. Under the constraint of the universal upper bound \( a_M \) on proper acceleration, one requires that

\[ A^2 \leq a_M^2. \]
It is useful to define
\[ \rho_0 \equiv \frac{c^2}{a_M}, \]  
which according to Eq. (2) is of the order of the Planck length. From Eqs. (9)-(11), it follows that
\[ -c^4 \frac{dv}{ds} \frac{d\mu}{ds} \leq \rho_0^2. \]  
Equivalently then
\[ d\sigma^2 \equiv ds^2 + \rho_0 \frac{d\sigma}{d\mu} d\mu \geq 0, \]  
or
\[ d\sigma^2 \equiv c^2 dt^2 - dx^2 - dy^2 - dz^2 + \rho_0^2 (dv^2_0 - dv^2_x - dv^2_y - dv^2_z) \geq 0, \]  
where \( v_0, v_x, v_y, \) and \( v_z \) are the time and spatial components of the four-velocity. Equation (14), expressing the fact that there is a maximum possible proper acceleration, is here taken to be the line element in the eight-dimensional spacetime-four-velocity space (tangent bundle of Minkowski spacetime [3], [4]). This is directly analogous to the fact that the ordinary spacetime line element Eq. (3) follows from the maximum possible velocity. Next, just as the four-dimensional d’Alembertian operator \( \Box^{(4)} \) is associated with the line element Eq. (3), one can take the eight-dimensional operator \( \Box^{(8)} \) defined by
\[ \Box^{(8)} \equiv \frac{\partial^2}{\partial x_\mu \partial x^\mu} + \frac{1}{\rho_0^2} \frac{\partial^2}{\partial v_\mu \partial v^\mu}, \]  
as the operator associated with the line element Eq. (14). [Equation (15) is the Laplace-Beltrami operator on the tangent bundle of Minkowski spacetime [6].] Next, by analogy with Eq. (5), one naturally defines a generalized scalar quantum field \( \phi(x, v) \) satisfying
\[ \Box^{(8)} \phi(x, v) \equiv 0, \]  
or equivalently,
\[ \left( \frac{\partial^2}{\partial x_\mu \partial x^\mu} + \frac{1}{\rho_0^2} \frac{\partial^2}{\partial v_\mu \partial v^\mu} \right) \phi(x, v) = 0. \]  
Equation (17) can also be written as
\[ (\Box_x + \rho_0^{-2} \Box_v) \phi(x, v) = 0, \]  
where the spacetime and four-velocity d’Alembertian operators are defined by
\[ \Box_x \equiv \frac{\partial^2}{\partial x_\mu \partial x^\mu}, \]  
and
\[ \Box_v \equiv \frac{\partial^2}{\partial v_\mu \partial v^\mu}. \]
respectively.

Next consider a possible separable single-mode solution $\phi(x, v) \equiv \phi(x^\mu, v^\mu)$ to Eq. (18) of the form

$$
\phi(x, v) = \phi_1(x)\phi_2(v),
$$

in which the dependence on the spacetime coordinates $x^\mu$ is separated from the dependence on the four-velocity coordinates $v^\mu$. If one substitutes Eq. (21) in Eq. (18), then for non-vanishing $\phi_1(x)$ and $\phi_2(v)$, one obtains

$$
\frac{\Box_x \phi_1(x)}{\phi_1(x)} + \rho_0^{-2} \frac{\Box_v \phi_2(v)}{\phi_2(v)} = 0.
$$

(22)

Since the first term of Eq. (22) depends only on $x$, and the second term depends only on $v$, both terms must be given by constants with the same absolute value, but with opposite signs. The constants can be defined in complete generality by $\pm(\mu c/\hbar)$, in which the constant $\mu$ is at this point an arbitrary constant to be determined. One therefore has

$$
\frac{\Box_x \phi_1(x)}{\phi_1(x)} = -\left(\frac{\mu c}{\hbar}\right)^2
$$

(23)

and

$$
\rho_0^{-2} \frac{\Box_v \phi_2(v)}{\phi_2(v)} = \left(\frac{\mu c}{\hbar}\right)^2.
$$

(24)

Possible solutions to Eqs. (23) and (24) are given by

$$
\phi_1^\pm(x) = \phi_{10} e^{\pm ik \cdot x},
$$

(25)

$$
\phi_2^\pm(v) = \phi_{20} e^{\mp q \cdot v} \theta(\pm q \cdot v),
$$

(26)

respectively, where $\phi_{10}$ and $\phi_{20}$ are constants, $k \cdot x \equiv k_\mu x^\mu$, $q \cdot v \equiv q_\mu v^\mu$, and $k^\mu$ and $q^\mu$ are Lorentz four-vectors, still to be determined. Also in Eq. (26), $\theta(x)$ is the Heaviside step function defined by

$$
\theta(x) = \begin{cases} 
1, & x \geq 0 \\
0, & x < 0
\end{cases}
$$

(27)

For $\phi_2^\pm(v)$ in Eq. (26) to be bounded for $|q \cdot v| \to \infty$, and the exponent to be decreasing, the negative sign must be chosen in the exponent for $q \cdot v > 0$, and the positive sign must be chosen for $q \cdot v < 0$. This is insured by the appearance of the Heaviside function $\theta(\pm q \cdot v)$ in Eq. (26). Next substituting Eqs. (25) and (26) in Eqs. (23) and (24), respectively, one obtains

$$
k^2 = \left(\frac{mc}{\hbar}\right)^2,
$$

(28)

and

$$
\rho_0^{-2} q^2 = \left(\frac{mc}{\hbar}\right)^2.
$$

(29)
respectively. Next, Eq. (23) can be rewritten as follows:

\[ \hbar^2 \Box \phi_1(x) + \mu^2 c^2 \phi_1(x) = 0, \]  

(30)

which is recognized to be the standard Klein-Gordon equation for a scalar quantum field describing particles of mass

\[ \mu = m. \]  

(31)

Thus, substituting Eq. (31) in Eq. (30), one has

\[ \hbar^2 \Box \phi_1(x) + m^2 c^2 \phi_1(x) = 0. \]  

(32)

Lorentz invariance requires that the four-momentum of the particle must satisfy the standard relativistic relation between energy, momentum, and mass, namely,

\[ p^2 = p_\mu p^\mu = p_0^2 - |\mathbf{p}|^2 = m^2 c^2, \]  

(33)

where \( p^\mu \) is the four-momentum of the particle, with spatial component \( \mathbf{p} \) and time component \( p^0 \). Also, substituting Eq. (25) in Eq. (32), then

\[ k^\mu = \frac{p^\mu}{\hbar}. \]  

(34)

In earlier work [6], [8], [9], the four-vector \( q^\mu \) in Eq. (26) was taken to be given by \( q^\mu = p_0 p^\mu / \hbar \), which clearly satisfies Eq. (29). It was also pointed out that there are other possible choices for \( q^\mu \) which one might consider [9]. In the present work, the following new choice is made for the four-vector \( q^\mu \):

\[ q^\mu = \frac{p_0}{\lambda_0} a^\mu a, \]  

(35)

where

\[ \lambda_0 = \frac{\hbar}{mc} \]  

(36)

is the wavelength for a particle at rest with rest mass \( m \), \( a^\mu \) is the four-acceleration of the particle, and \( a \) is the corresponding proper acceleration of the particle, namely,

\[ a^2 = a_0^2 - |\mathbf{a}|^2, \]  

(37)

where \( \mathbf{a} \) is the spatial component of four-acceleration, and \( a_0 = a_0 \) is the time component. [Note that in standard quantum field theory, the momentum, velocity, and acceleration of a particle at a specific spatial location lacks meaning because of the uncertainty principle; however, in quantum field theory the particle momentum is merely a parameter, and in the present work the particle acceleration is also a parameter. Also, it is well to recall that in the Bohm interpretation of quantum mechanics, the particle position, velocity, and acceleration of a particle at a point are simultaneously meaningful [10], [11], [12].] Next one notes that the four-vector

\[ n^\mu \equiv \frac{a^\mu}{a} \]  

(38)
is a unit vector, namely,
\[ n^2 = n_\mu n^\mu = \frac{a^2}{a^2} = 1, \quad (39) \]
and using Eqs. (35), (38), and (39), one has
\[ q^2 = q_\mu q^\mu = \frac{\rho_0^2}{\lambda_0^2} = \left( \frac{\rho_0 mc}{\hbar} \right)^2, \quad (40) \]
which agrees with Eq. (29). Next substituting Eqs. (34) and (35), respectively, in Eqs. (25) and (26), respectively, one obtains
\[ \phi_1^\pm (x) = \phi_{10} e^{\pm ipx/\hbar}, \quad (41) \]
and
\[ \phi_2^\pm (v) = \phi_{20} e^{\pm \frac{\rho_0 a^2}{\lambda_0^2} \theta \left( \pm \frac{a \cdot v}{a} \right)}, \quad (42) \]
respectively, where
\[ p \cdot x = p_\mu x^\mu, \quad (43) \]
and
\[ a \cdot v = a_\mu v^\mu. \quad (44) \]

Next, substituting Eq. (42) in Eq. (24), one obtains
\[ \Box_v \phi_2^\pm (v) = \phi_{20} \left( \frac{\rho_0}{\lambda_0} \right)^2 \left[ \delta^\prime \left( \pm \frac{a \cdot v}{a} \right) - 2\delta \left( \pm \frac{a \cdot v}{a} \right) + \theta \left( \pm \frac{a \cdot v}{a} \right) \right] e^{\mp \rho_0 a \cdot v \over \lambda_0}, \quad (45) \]
where \( \delta(x) \) is the Dirac delta function. From Eq. (37), one has
\[ \{ a^\mu \} = \{ a_0, a \} = \left\{ \left( a^2 + |a|^2 \right)^{1/2}, a \right\}. \quad (46) \]
Also, using Eqs. (7) and (3), one has
\[ \{ v^\mu \} = \left\{ \frac{dx^0}{ds}, \gamma \frac{dx}{dt} / c \right\} = \left\{ \gamma, \gamma \frac{dx}{dt} / c \right\}, \quad (47) \]
where
\[ \gamma = \left( 1 - \left( \frac{dx}{dt} / c \right)^2 \right)^{-1/2}. \quad (48) \]
Using Eqs. (46) and (47), it can be shown that [See Appendix]:
\[ \frac{a \cdot v}{a} \equiv a_\mu v^\mu a = \gamma \left[ \left( a^2 + |a|^2 \right)^{1/2} - a \cdot \frac{dx}{dt} / c \right] \geq 1, \quad (49) \]
and therefore
\[ \delta \left( \pm \frac{a \cdot v}{a} \right) = 0, \quad (50) \]
and
\[ \delta' \left( \pm \frac{a \cdot v}{a} \right) = 0. \]  
(51)

Next, using Eqs. (20), (45), (50), (51) and (42), one obtains
\[ \Box_v \phi_2^\pm (v) = \left( \frac{\rho_0}{\lambda_0} \right)^2 \phi_2^\pm (v). \]  
(52)

Also, using Eqs. (31) and (36), Eq. (24) can be written as
\[ \Box_v \phi_2 (v) = \left( \frac{\rho_0}{\lambda_0} \right)^2 \phi_2 (v). \]  
(53)

Comparing Eq. (52) with Eq. (53), one concludes that \( \phi_2^\pm (v) \) given by Eq. (42) solves Eq. (53). Next substituting Eqs. (41) and (42) in Eq. (21), it follows that possible solutions to Eq. (17), representing the positive and negative frequency modes of the quantum field are given by
\[ \phi^\pm (x, v) = \phi_0 e^{\pm ip \cdot x/\hbar} e^{\pm \frac{\rho_0}{\lambda_0} (a \cdot v)} \theta \left( \pm \frac{a \cdot v}{a} \right), \]  
(54)

where
\[ \phi_0 = \phi_{10} \phi_{20}. \]  
(55)

and, in accord with Eq. (33), the components of the particle four-momentum are given by
\[ \{p^\mu\} = \{p^0, p\} = \left\{ m c^2 + |p|^2, p \right\}. \]  
(56)

The general solution \( \phi(x, v) \) for a free relativistic scalar quantum field is obtained by integrating over the spatial components of four-momentum and four-acceleration of the invariant positive and negative frequency modes \( \phi^\pm (x, v) \) of Eq. (54), including appropriate particle creation and annihilation operators. It follows from Eqs. (54), (49), and (27) that for the positive-frequency particles and the negative-frequency antiparticles, respectively, nonvanishing support is provided by positive and negative values of \( \frac{a \cdot v}{a} \), respectively. Thus the relativistic Lorentz-invariant scalar quantum field for particles of mass \( m \) is given by
\[ \phi(x, v) = 2 \int \frac{d^3p}{N^{1/2}(2\pi\hbar)^{3/2}(2\rho_0)^{3/2}} \int \frac{d^3a}{(2\pi\hbar)^{3/2}(2\lambda_0)^{3/2}} \left[ e^{-ip \cdot x/\hbar} e^{-\frac{\rho_0}{\lambda_0} \frac{a \cdot v}{a}} \theta \left( \frac{a \cdot v}{a} \right) b(p, a) \right. \]  
\[ + \left. e^{ip \cdot x/\hbar} e^{\frac{\rho_0}{\lambda_0} \frac{a \cdot v}{a}} \theta \left( -\frac{a \cdot v}{a} \right) b^\dagger(p, a) \right]. \]  
(57)

Here, to recapitulate, \( \theta(x) \) is the Heaviside step function, \( x^\mu \) is both the particle spacetime coordinate and the spacetime coordinate of the measuring device that measures the quantum field, \( p^\mu \) is the particle four-momentum, \( a^\mu \) is the particle four-acceleration, \( v^\mu \) is the four-velocity of the measuring device, \( \hbar \) is Planck’s constant divided by \( 2\pi \), \( \rho_0 = c^2/a_M \), \( a_M \) is the maximal proper acceleration given by Eq. (2), and \( \lambda_0 = \hbar/mc \). Also, \( N \) is a normalization constant, \( b^\dagger(p, a) \) and \( b(p, a) \) are particle creation and annihilation operators.
for the spatial component of four-momentum $p$ and spatial component of four-acceleration $a$, and the following natural extensions of the standard bosonic commutation relation are adopted:

$$ [b(p,a), b^\dagger(p', a')] = \delta^3(p - p')\delta^3(a - a'), \quad (58) $$

$$ [b(p,a), b(p', a')] = 0, \quad (59) $$

$$ [b^\dagger(p,a), b^\dagger(p', a')] = 0. \quad (60) $$

Note that since the field is localized in spacetime, then even though the proper acceleration $a$ is bounded by $a_M$, consistent with Eq. (37), the magnitude of the spatial component of acceleration $|a|$ of the particle may range from zero to infinity in the integral. Also, one notes that according to Eqs. (2) and (11), $\rho_0$ is of the order of the Planck length, and in the mathematical limit of infinite limiting proper acceleration $a_M$, one has vanishing $\rho_0$, and Eq. (57) then reduces to the same form as a canonical Lorentz-invariant scalar quantum field (as it must). In this sense, the modification of the relativistic quantum field introduced here is negligible, except at energies beyond the Planck energy $(\hbar c^5/G)^{1/2}$, as may be seen from the following.

Using Eqs. (2), (11), (36), and (49), one sees that both the positive and negative frequency terms in Eq. (57) are proportional to

$$ \exp\left(-\frac{\rho_0}{\lambda_0} \frac{a \cdot v}{a}\right) = \exp\left[-\frac{1}{2\pi\alpha} \frac{\gamma m}{m_{Pl}} \left(1 + \left(\frac{|a|}{a}\right)^2\right)^{1/2} - \frac{a}{c} \frac{dx/dt}{c}\right] $$

$$ \exp\left(-\frac{\rho_0}{\lambda_0} \frac{a \cdot v}{a}\right) = \exp\left[-\frac{1}{2\pi\alpha} \frac{\gamma m}{m_{Pl}} \left(1 + \left(\frac{|a|}{a}\right)^2\right)^{1/2} - \frac{a}{c} \frac{dx/dt}{c}\right], \quad (61) $$

where $m$ is the rest mass of the particle, $m_{Pl}$ is the Planck mass,

$$ m_{Pl} = \left(\frac{\hbar c}{G}\right)^{1/2}, \quad (62) $$

and

$$ \gamma = \left(1 - \frac{dx/dt}{c}\right)^{-1/2}, \quad (63) $$

where $dx/dt$ is the velocity of the measuring device relative to the particle. For velocities of the measuring device much less than the velocity of light, and for particles masses much less than the Planck mass, Eq. (61) is for all practical purposes unity because $\rho_0$ is so small (of the order of the Planck length), and Eq. (57) effectively reduces to the canonical scalar quantum field.

### 4 QUANTUM-TO-CLASSICAL TRANSITION

Next, a possible implication of the limiting proper acceleration $a_M$ is that for a particle with four-momentum $p^\mu$ and four-acceleration $a^\mu$, and for a measuring
device with spacetime coordinates \( x^\mu \) and four-velocity coordinates \( v^\mu \) with respect to a particle in Minkowski spacetime, the quantum state is given by

\[
\psi(x, v) = \langle 0 | \phi(x, v) | p, a \rangle.
\]

(64)

Here \( |0\rangle \) is the vacuum state, \( |p, a\rangle \) is the state of the particle with four-momentum \( p^\mu \) and four-acceleration \( a^\mu \), and \( \phi(x, v) \), given by Eq. (57), is the scalar quantum field associated with the particle. From Eqs. (57), (61), and (63), in the case in which the measuring device is at rest with respect to the particle, namely \( dx/dt = 0 \), it follows that the quantum state is given by

\[
\psi(x, d\vec{x}/dt = 0) = A \exp \left( -\frac{1}{2\pi \alpha} \frac{m}{m_{Pl}} \left( 1 + \left( \frac{|a|}{a} \right)^2 \right)^{1/2} \right) \exp \left( -ip \cdot \frac{x}{\hbar} \right),
\]

(65)

where \( A \) is a normalization constant. Equation (65) can be expected to hold for any bosonic or fermionic state, ignoring spin. For standard elementary particle masses \( (m/m_{Pl} \ll 1) \), the wave function Eq. (65) reduces to the standard plane wave. But suppose that Eq. (65) is taken to describe a macroscopic object containing many more than Avogadro’s number of atoms, in which case its mass \( m \) is such that

\[
m \gg N_A m_n,
\]

(66)

where \( N_A \) is Avogadro’s Number, and \( m_n \) is the mass of a nucleon. Then according to Eqs. (65) and (66), in this simplified model, one obtains for the quantum state \( \psi_{mac} \) of this macroscopic many-body object:

\[
\psi_{mac} \ll A' \exp \left( -\frac{1}{2\pi \alpha} \frac{N_A m_n}{m_{Pl}} \left( 1 + \left( \frac{|a|}{a} \right)^2 \right)^{1/2} \right) \exp (-iN_A p \cdot x/\hbar),
\]

(67)

in which \( A' \) is the normalization constant for \( N_A \) particles. Equivalently, one can write the state \( \psi_{mac} \) as a product of \( N_A \) copies of Eq. (65), since the exponents add. Substituting \( N_A = 6 \times 10^{23} \), \( m_n = 1.7 \times 10^{-27} \text{kg} \), and \( m_{Pl} = 2.2 \times 10^{-8} \text{kg} \) in Eq. (67), the macroscopic many-body wave function \( \psi_{mac} \) is seen to be negligible. This suggests that, for all practical purposes, a macroscopic object, such as a macroscopic measuring device, should not be described by quantum mechanics, and instead is best described by classical mechanics. This is consistent with Bohr’s requirement that the macroscopic measuring device be classical, and that it be clearly distinguished from the microscopic quantum object being measured \[13, 14, 10\]. It is also significant to note that in the present theory, the quantum-to-classical transition is not sharp, but is gradual beyond Avogadro’s number of atoms. According to Eq. (67), the transition depends on the total mass of the object being measured. Also noteworthy is that the transition also depends on the universal gravitational constant \( G \), which, according to Eq. (62), appears in Eq. (67). In this connection, one recalls that Diosi introduced a theory of statistical mass localization with a strength proportional to the gravitational constant \( G \) \[15, 10\]. It is also noteworthy that
in other theories of the quantum-to-classical transition, new universal constants were introduced, whereas in the present theory there are no new independent universal constants. For example, in the popular theory of G. C. Ghirardi, A. Rimini, and T. Weber (GRW theory), a time scale was introduced for the rate of spontaneous localization in spacetime, and also a length scale characterizing the narrowness of the localization \[10\]. In the present theory, all of the universal constants are standard, namely, \( G, \hbar, c, N_A \), and known particle masses. Also of note is that with current values of the parameters in the GRW theory, significant tests of the theory could be obtained with objects having more than \( 10^8 \) nucleons, which is far less than Avogadro’s number \[10\]. It is also well to note that a bound on the number of particles which can exist in a quantum entangled state, as in a quantum computer, has earlier been conjectured on the basis of a cosmological information bound in which more bits of information would be needed to specify the state than can be accommodated in the observable universe \[17\]. In that conjecture, the required number of particles for the transition to occur would be far less than Avogadro’s number. It is also well to note that the present theory in itself places no practical upper bound on the size of a quantum computer register because Avogadro’s number is so large. Also, in the present theory, any quantum parallelism involving the macroscopic every-day world, as in the Everett interpretation of quantum mechanics, would be negligible \[18\].

5 CONCLUSION

A new formulation of a relativistic scalar quantum field is here introduced which generalizes the canonical theory to include the limiting proper acceleration. This field is used to construct a simple model of a many-body system, and it is argued that for a macroscopic object consisting of more than Avogadro’s number of atoms, any supposed quantum state of the object is negligibly small, so that for all practical purposes, the object is best described by classical mechanics. Thus, a new explanation is offered for the quantum-to-classical transition and the absence of quantum superpositions of macroscopic objects in the everyday world.

6 APPENDIX

Equation (49) is to be proved. From Eq. (49, one obtains

\[
\frac{a \cdot v}{a} = \gamma \left[ \left( 1 + \left( \frac{|a|}{a} \right)^2 \right)^{1/2} - \frac{|a|}{a} \frac{dx/dt}{c} \right],
\]

(68)
or, equivalently,

\[
\frac{a \cdot v}{a} = \gamma F (|a|/a),
\]

(69)
where the function \( F(x) \) is defined by
\[
F(x) \equiv (1 + x^2)^{1/2} - \beta x \cos \theta,
\] (70)
and
\[
\beta = \frac{|dx\,/\,dt|}{c} < 1,
\] (71)
\[
\theta = \cos^{-1} \left( \frac{\mathbf{a} \cdot dx\,/\,dt}{|a| |dx\,/\,dt|} \right),
\] (72)
and using Eq. (71) in Eq. (63), one has
\[
\gamma = (1 - \beta^2)^{-1/2} \geq 1.
\] (73)
Next, the function \( F(x) \) has a minimum for
\[
0 = \frac{\partial F}{\partial x} = x \left(1 + x^2\right)^{-1/2} - \beta \cos \theta,
\] (74)
and it follows that the minimum is at
\[
x = x_{\text{min}} = \frac{\beta \cos \theta}{(1 - \beta^2 \cos^2 \theta)^{1/2}}.
\] (75)
Then substituting Eq. (75) in Eq. (70), one obtains the minimum value of the function \( F(x) \), namely,
\[
F(x_{\text{min}}) = (1 - \beta^2 \cos^2 \theta)^{1/2}.
\] (76)
Finally from Eqs. (69), (73), and (76), it follows that
\[
\frac{a \cdot v}{a} > \frac{(1 - \beta^2 \cos^2 \theta)^{1/2}}{(1 - \beta^2)^{1/2}} > 1,
\] (77)

References

[1] Howard E. Brandt, "Maximal Proper Acceleration Relative to the Vacuum," Lett. Nuovo Cimento 38 522-524 (1983); 39, 192 (1984).

[2] Howard E. Brandt, "Maximal Proper Acceleration and the Structure of Spacetime," Foundations of Physics Letters, 2, 39-58 (1989).

[3] Howard E. Brandt, "Structure of Spacetime Tangent Bundle," Found. Phys. Lett. 4, 523-536 (1991).

[4] Howard E. Brandt, "Finslerian Spacetime," Contemporary Mathematics 196, 273-287 (1996).
[5] Howard E. Brandt, "Quantum Vacuum Heuristics," *J. Mod. Optics* **50**, 2455-2463 (2003).

[6] Howard E. Brandt, "Quantum Fields in the Spacetime Tangent Bundle," *Found. Phys. Lett.*, **11**, 265-275 (1998).

[7] Howard E. Brandt, “Particle Geodesics and Spectra in the Spacetime Tangent Bundle,” Reports on Mathematical Physics, Vol. 45, 389-405 (2000).

[8] Howard E. Brandt, "The Quantum-Classical Boundary," in Quantum Information and Computation XI, edited by Eric Donker, Andrew R. Pirich, and Howard E. Brandt, Proceedings of SPIE, Vol. 8749, 87490E1-3, Bellingham WA (2013).

[9] Howard E. Brandt, "Lorentz-Invariant Quantum Fields in the Spacetime Tangent Bundle," *International Journal of Mathematics and Mathematical Sciences* **24**, 1529-1546 (2003).

[10] Franck Laloe, Do We Really Understand Quantum Mechanics, Cambridge University Press (2012).

[11] Detlef Durr and Stefan Teufel, Bohmian Mechanics, Springer-Verlag, Berlin (2009).

[12] P. Holland, The Quantum theory of Motion, Cambridge University Press, Cambridge (1995).

[13] Niels Bohr, "Can Quantum-Mechanical Description of Physical Systems be Considered Complete?" *Phys. Rev.* **48**, 696-702 (1935).

[14] John von Neumann, Mathematical Foundations of Quantum Mechanics, Princeton University Press, Princeton NJ (1955).

[15] L. Diosi, "Models for Universal Reduction of Macroscopic Quantum Fluctuations," *Phys. Rev. A* **40**, 1165-1174 (1989).

[16] G. C. Ghirardi, A. Rimini, and T. Weber, "Unified Dynamics for Microscopic and Macroscopic Systems," *Phys. Rev. D* **34**, 470-401 (1986).

[17] P. C. W. Davies, "The Implication of a Cosmological Information Bound for Complexity, Quantum Information, and the Nature of Physical Law," arXiv:quant-ph/0703041 (2007).

[18] Hugh Everett III, "The Theory of the Universal Wave Function," in Many-Worlds Interpretation of Quantum Mechanics, Editors Bryce S. DeWitt and Neill Graham, Princeton University Press Princeton NJ (1973).