AN UPPER LIMIT ON THE TEMPORAL VARIATIONS OF THE SOLAR INTERIOR STRATIFICATION

A. EFF-DARWICH,1,2,3 S. G. KORZENNIK,1 S. J. JIMÉNEZ-REYES,2,4 AND F. PÉREZ HERNÁNDEZ2,5

Received 2002 April 18; accepted 2002 July 22

ABSTRACT

We have analyzed changes in the acoustic oscillation eigenfrequencies measured over the past 7 years by the GONG, MDI, and LOWL instruments. The observations span the period from 1994 to 2001 that corresponds to half a solar cycle, from minimum to maximum solar activity. These data were inverted to look for a signature of the activity cycle on the solar stratification. A one-dimensional structure inversion was carried out to map the temporal variation of the radial distribution of the sound speed at the boundary between the radiative and convective zones. Such variation could indicate the presence of a toroidal magnetic field anchored in this region. We found no systematic variation with time of the stratification at the base of the convection zone. However, we can set an upper limit to any fractional change of the sound speed at the level of $3 \times 10^{-5}$.

Subject headings: Sun: helioseismology — Sun: interior — Sun: magnetic fields

1. INTRODUCTION

Changes in the frequency of the solar $p$-mode oscillations have now been observed for more than a decade. Such changes affect both the central frequencies $\nu/C_2$ and the frequency splittings $\Delta\nu_{\text{fin}}$ of low-degree (Jiménez-Reyes et al. 2001), intermediate-degree (Howe, Komm, & Hill 1999; Dziembowski et al. 2000), and very high degree modes (Rajaguru, Basu, & Antia 2001). A number of mechanisms have been proposed to explain these variations in frequency. Libbrecht & Woodard (1990) have argued, on the basis of observations of intermediate-degree modes, that the source of the perturbations must lie near the solar surface. Gough & Thompson (1988) and Paternó (1990) concluded that magnetic fields located near the base of the convection zone, with strengths significantly lower than $10^6$ G, have no observable effect on $p$-mode frequencies. The stability analysis for magnetic fields by Moreno-Insertis, Schüssler, & Ferris-Mas (1992) has shown that fields with strengths significantly larger than $10^5$ G cannot be stored in this region.

The $p$-mode frequency variations track rather well the changes of the activity strength of the solar cycle with time. It is thus plausible that these frequency shifts are due to variations of the mean magnetic field near the photosphere (Gough & Thompson 1988; Goldreich et al. 1991). The influence of thin magnetic fibrils on the frequency shifts has been investigated by Bogdan & Zweibel (1985). Another possible cause for these frequency shifts is the presence of sunspots during solar activity (Braun et al. 1992). The dominant effect of sunspots on the propagation of acoustic waves is believed to be the dissipation of the acoustic energy, and therefore a decrease of their amplitudes and lifetimes would be expected (Hindman & Brown 1998; Rajaguru et al. 2001), while what has been observed is that frequencies increase with increased magnetic activity.

In the work presented here, we explore a mechanism first suggested by Kuhn (1988), in which the sound-speed perturbation associated with the observed changes of the photospheric latitudinal temperature distribution might be responsible for the frequency shifts seen during the solar cycle. Changes in the temperature distribution are themselves due to the heat transport through the convection zone induced by the solar dynamo.

Let us suppose that there is a magnetic field anchored below the base of the convection zone. To maintain pressure equilibrium, the gas pressure and thus the density inside the magnetized region must be lower than in their surroundings. The magnetized fluid will thus experience a larger radiative heating, and therefore the temperature at the top of the magnetized region will increase. This will also induce a change in the temperature gradient that could be large enough to make the region above the magnetic field convectively unstable. In such a scenario, the base of the convection zone would locally drop, allowing the magnetized fluid to ascend by convective upflows, transporting excess entropy to the photosphere (Kuhn & Stein 1996).

Numerical experiments by Kuhn & Stein (1996) have shown that entropy perturbations in the deep convection zone can produce strongly peaked temperature changes in regions below $\tau = 1$ that have a substantial acoustic signature (where $\tau$ is the optical depth). They have also shown that the thermal perturbations that account for the solar acoustic variability are consistent with the observed solar irradiance and luminosity changes that occur during the 11 year solar cycle.

Luminosity changes, even if no larger than 0.1%, must come from the release of energy stored somewhere in the solar interior and must be accompanied by a change in the solar radius. The ratio between relative luminosity and radius changes, hereafter $W$, can help estimate the location of the region where this energy is stored (Gough 2000 and references therein). Theoretical calculations indicate that $W$ increases when increasing the depth of the source of the variations in luminosity. For instance, $W \approx 2 \times 10^{-4}$ if the
source is located in the outer layers of the convection zone, while $W \approx 0.5$ if the source is located in the solar core. Unfortunately, there is a large scatter in the observed values of $W$. Indeed, recent measurements of $W$ range from 0.021 as estimated by Dziembowski, Goode, & Schou (2001) and Antia et al. (2000a) to an upper limit of 0.08 derived by Emilio et al. (2000).

Kuhn (2000) estimated that a 0.1% luminosity perturbation integrated over a solar cycle corresponds to about $10^{39}$ ergs. If this energy originates in the tachocline and if the source is located in the outer layers of the convection zone, the near-surface layers (see Dziembowski et al. 1990 and references therein).

We have extended the previous analysis to the latest data available, including Low-Degree ($l$) Oscillation Experiment (LOWL) data. Only common modes to all data sets were used, in an attempt to obtain comparable and significant results to all instruments. This way, we can give a robust upper limit on the temporal variations of the solar internal stratification during the period 1994–2001.

2. INVERSION TECHNIQUE

The inversion for solar structure, in particular for sound speed $c$ and density $\rho$, is commonly based on the linearization of the equations of stellar oscillations around a reference model (Gough & Kosovichev 1988, 1990; Dziembowski, Pamyatnykh, & Sienkiewicz 1990). The differences of the structural profile between the actual Sun and a model are linearly related to differences between the observed frequencies and those calculated using that model. This relation is obtained using a variational formulation for the frequencies of adiabatic oscillations. A general relation for frequency differences is given by

$$\frac{\delta f_{nl}}{f_{nl}} = \int_0^{R_\odot} \left[ \frac{K^c_{nl}(r)}{c(r)} + \frac{K^\rho_{nl}(r)}{\rho(r)} \right] dr + E^{-1}_{nl} F(\nu) + \epsilon_{nl},$$

where $\delta f_{nl}$ are the frequency differences between the actual Sun and the model for the mode with radial order $n$ and degree $l$, and $\epsilon_{nl}$ is the corresponding relative error. The sensitivity functions, or kernels $K^c_{nl}(r)$ and $K^\rho_{nl}(r)$, are known functions that relate the changes in frequency to the changes in the model. The functions $\delta c/c$ and $\delta \rho/\rho$ are the unknown parameters to be inverted, i.e., the relative difference in the sound speed and the density, respectively, and $R_\odot$ is the solar radius. The term $E^{-1}_{nl} F(\nu)$ in equation (1) is introduced to take into account the so-called surface uncertainties; these include the dynamical effects of convection on the oscillation equations, as well as nonadiabatic processes in the near-surface layers (see Dziembowski et al. 1990 and references therein).

Following standard procedures, we represent $F(\nu_{nl})$ as a Legendre polynomial expansion. The quantity $E_{nl}$ is the inertia of the mode, normalized by the inertia that a radial mode of the same frequency would have (for more details, see Gough & Thompson 1991).

If we take $\delta f_{nl}$ as the differences in the observed frequencies at two different epochs, rather than the differences in frequency between the actual Sun and the model, $\delta c/c$ and $\delta \rho/\rho$ represent the variation with time of the Sun’s internal structure, as long as our underlying theoretical model is very close to the actual Sun.

The inverse problem defined by equation (1) is well known to be an ill-posed problem (Thompson 1995), the solution of which is not unique. It can be solved using inversion methodologies that can be classified into two different techniques: the regularized least-squares methods (RLS; see Craig & Brown 1986) and the optimal-localized-average methods (Backus & Gilbert 1968). Both methods compute an estimate of the solution at a target location from linear combinations of the observables, given a mesh of target locations. We have developed a variant of the RLS technique that we call the “optimal mesh distribution,” which optimizes the mesh of target locations to avoid undesired high-frequency oscillations of the solution. This optimization is achieved by computing a priori the spatial resolution of the solution from the set of available observables and their uncertainties (Eff-Darwich & Pérez-Hernández 1997). The smoothing function is itself also defined from the spatial resolution analysis, and it is weighted differently for each radial point. This method ensures that the smoothing constraint is properly applied over the optimal mesh.

3. OBSERVATIONAL DATA

The observational data consist of mode frequencies computed from time series spanning different epochs and observed with different instruments, namely, 57 sets based on 108 day long time series derived from GONG instruments (Leibacher et al. 1996) and spanning 1995 May to 2001 February, 27 sets based on 72 day long time series derived from the MDI instrument (Schou 1999) and spanning 1996 May to 2001 November, and 6 sets based on 1 yr long time series derived from the LOWL instrument (Jiménez-Reyes 2000; Tomczyk, Schou, & Thompson 1995) and spanning 1994 to 1999.

In order to use consistent data sets, only the modes common to all the sets for a given instrument were taken into account. As a consequence, the low-degree modes ($l < 13^\circ$) present in some GONG data sets had to be rejected. In addition, this selection reduced the number of MDI and LOWL modes by 30% and 4%, respectively.

The MDI and LOWL sets were further reduced to only include the modes common to both instruments. This was not done with the GONG data set because of the small amount of common modes present. Finally, and again for consistency, we deliberately restricted the range of degrees we included to correspond to the highest degree available in the LOWL data set (i.e., $l \leq 100^\circ$).

For each instrument and for each mode, we computed the temporal frequency average. We subsequently subtracted the respective averaged frequencies from each set, leaving us with frequency changes with respect to this temporal average as a function of epoch. For the GONG and MDI sets, we also computed averages corresponding to 1 yr long
epochs. Such averaging reduced the scatter of the data while producing data sets comparable to the LOWL sets.

4. RESULTS

Figure 1 shows the relative change of the sound speed as a function of radius inferred from 1 yr long MDI, GONG, and LOWL sets. These profiles show no significant changes at the level of a few \(10^{-5}\). The precision and resolution of the inversion is good enough to detect small variations of the stratification at the base of the convection zone. This is demonstrated in Figure 2, in which we show the sound-speed profiles inferred from the 1996 averaged MDI, GONG, and LOWL data sets, as well as sound-speed profiles obtained by inverting the same mode sets, but after injecting frequency changes that result from a perturbation in the sound speed (as small as \(3 \times 10^{-6}\) and \(3 \times 10^{-5}\)) between 0.68 and 0.70 \(R_\odot\). This figure indicates that a perturbation of the sound speed at the base of the convection zone on the order of, or slightly smaller than, \(5 \times 10^{-5}\) can be detected with the current precision resulting from a 1 yr long time series. Perturbations on the order of \(10^{-6}\) fall in the noise level of our inversions.

In an attempt to find temporal variations of the solar stratification at the base of the convection zone, we computed the mean value of \(\delta c/c\) in the radial interval \(0.69 < r/R_\odot < 0.72\). This interval contains not only the base of the convection zone, \(r \approx 0.7133 R_\odot\) (Basu & Antia 2001), but also the tachocline, \(r \approx 0.691 R_\odot\) (Corbard et al. 1999), both closely related to the toroidal magnetic field responsible for the solar cycle. The resulting values are shown as a function of time in Figure 3, for the inversions based on 1 yr long sets for all three instruments (GONG, MDI, and LOWL), as well as on the GONG 108 day long and MDI 72 day long sets.

Inversion profiles inferred from any linear inversion technique always correspond to the convolution of the underlying solution by the resolution kernel (Thompson 1995). Therefore, even if the mode set used in a sequence of inversions remains identical, the resolution kernels will, at some level, change with time, since the uncertainties change with time. Such variation could produce an apparent temporal behavior of the inferred profiles that does not correspond to a real variation of the underlying true solution.

To quantify this effect, we computed the averaging kernels at \(r = 0.69 R_\odot\) for all the inversions based on 1 yr long data sets. These were convolved with an artificial sound-speed perturbation of the form

\[
\frac{\delta c}{c} = \begin{cases} 
3 \times 10^{-5}, & 0.67 \leq \frac{r}{R_\odot} \leq 0.71, \\
5 \times 10^{-5}, & 0.91 \leq \frac{r}{R_\odot} \leq 0.93, \\
0, & \text{otherwise}
\end{cases}
\]

where the sound-speed perturbation centered at 0.92 \(R_\odot\) is intended to reproduce the results found by Dziembowski et al. (2000) and Antia et al. (2001), while a second perturbation at the base of the convection zone is also introduced. The convolutions \(q(t)\) are then calculated in the following way:

\[
q(t) = \int K(r, r_0, t) \frac{\delta c(r)}{c(r)} dr.
\]

The resulting values of \(q(t)\), at \(r_0 = 0.69 R_\odot\) relative to the average \(\bar{d}_{av}\) of all the convolutions calculated for every instrument, are shown in Figure 4. This figure demonstrates that for the data from all three instruments, the effect of the changes in the observed uncertainties on the inverted profiles is negligible, corresponding to levels well below \(10^{-3}\).

The averaging kernel corresponding to the solution at \(r = 0.69 R_\odot\) obtained from the 1997 MDI data set is shown in the right panel of Figure 5. This is well located and indicates that our radial resolution corresponds to 0.04 \(R_\odot\). We should also point out that the averaging kernels have an important negative nonlocal contribution near the surface.
a feature that affects at some level the solution at the base of the convection zone. The effect of this nonlocal component of the averaging kernels was quantified, in the case of MDI and GONG data, by performing inversions with data sets expanded to higher degrees, i.e., up to $l = 150$. The averaging kernel at $r = 0.69 \, R_\odot$ obtained from the MDI 1997 data shows a substantial reduction of the negative component located at $r \approx 0.90 \, R_\odot$, when including higher degree modes. However, the temporal behavior of the stratification at the base of the convection zone does not significantly differ when the MDI data sets are expanded from $l \leq 100^\circ$ to $l \leq 150^\circ$, as illustrated in the left panel of Figure 5.

Therefore, by including the data from all three instruments, and after assessing the effects of temporal changes of the resolution of the solutions, we can safely conclude that there are no significant systematic variations of the stratification at the base of the convection zone at the level of $3 \times 10^{-5}$ and that this upper limit is constrained by the scatter present in the data.

Vorontsov (2000) analyzed MDI data spanning from 1996 to 2000 and found systematic variations of the radial solar stratification with time, expressed as relative changes of radius of $2 \times 10^{-5}$. Our results obtained with MDI data are in good agreement with those found by Vorontsov (2000), in the sense that there is a systematic variation in the relative sound-speed difference that is well correlated to the magnetic activity in the Sun. The maximum in solar activity corresponds to the maximum variation in sound speed. Since these results are not seen when analyzing LOWL and GONG data, they should still be taken with some degree of skepticism.

The Solar Oscillations Investigation-MDI project on the Solar and Heliospheric Observatory (SOHO) is supported by NASA grant NAS 5-3077 at Stanford University. SOHO is a project of international cooperation between ESA and NASA. The GONG project is funded by the National Science Foundation (NSF) through the National Solar Observatory, a division of the National Optical Astronomy Observatories, which is operated under a cooperative agreement between the Association of Universities for Research in Astronomy and the NSF. The LOWL instrument has been operated by the High Altitude Observatory of the National Center for Atmospheric Research, which is supported by the NSF. This work was partially supported by NASA-Stanford contract PR-6333 and by NSF grant AST 95-2177.
REFERENCES

Antia, H. M., Basu, S., Hill, F., Howe, R., Komm, R. W., & Schou, J. 2001, MNRAS, 327, 1029
Antia, H. M., Basu, S., Pintar, J., & Pohl, B. 2000a, Sol. Phys., 192, 459
Antia, H. M., Chitre, S. M., & Thompson, M. J. 2000b, A&A, 360, 335
Backus, G., & Gilbert, F. 1968, Geophys. J. RAS, 16, 169
Basu, S., & Antia, H. M. 2001, MNRAS, 324, 498
Bogdan, T. J., & Zweibel, E. G. 1985, ApJ, 298, 867
Braun, D. C., et al. 1992, ApJ, 391, L113
Corbard, T., Blanc-Féraud, L., Berthomieu, G., & Provost, J. 1999, A&A, 344, 696
Craig, I. J. D., & Brown, J. C. 1986, Inverse Problems in Astronomy: A Guide to Inversion Strategies for Remotely Sensed Data (Bristol: Hilger)
Dziembowski, W., Pamyatnykh, A. A., & Sienkiewicz, R. 1990, MNRAS, 244, 542
Dziembowski, W. A., Goode, P. R., Kosovichev, A. G., & Schou, J. 2000, ApJ, 537, 1026
Dziembowski, W. A., Goode, P. R., & Schou, J. 2001, ApJ, 553, 897
Eff-Darwich, A., & Pérez-Hernández, F. 1997, A&AS, 125, 391
Emilio, M., Kuhn, J. R., Bush, R. I., & Scherrer, P. 2000, ApJ, 543, 1007
Goldreich, P., Murray, N., Willette, G., & Kumar, P. 1991, ApJ, 370, 752
Gough, D. O. 2000, Nature, 272, 1282
Gough, D. O., & Kosovichev, A. G. 1988, in Seismology of the Sun and Sun-like Stars (ESA SP-286; Noordwijk: ESA), 195
Hindman, B. W., & Brown, T. M. 1998, ApJ, 504, 1029
Howe, R., Komm, R., & Hill, F. 1999, ApJ, 524, 1084
Jimenez-Reyes, S. J. 2000, Ph.D. thesis, Univ. La Laguna, Tenerife
Jimenez-Reyes, S. J., Corbard, T., Palle, P. L., Roca Cortes, T., & Tomczyk, S. 2001, A&A, 379, 622
Kuhn, J. 1988, ApJ, 331, L131
Kuhn, J. R., & Stein, R. F. 1996, ApJ, 463, L117
Leibacher, J., et al. 1996, Science, 272, 1282
Llibrech, K., & Woodward, M. F. 1990, Nature, 345, 779
Moreno-Insertis, F., Schüssler, M., & Ferrés-Mai, A. 1992, A&A, 264, 686
Paternó, L. 1990, in Progress of Seismology of the Sun and Stars, ed. Y. Osaki & H. Shibahashi (New York: Springer), 41
Rajaguru, S. P., Basu, S., & Antia, H. M. 2001, ApJ, 563, 410
Schou, J. 1999, ApJ, 523, L181
Thompson, M. J. 1995, Inverse Problems, 11, 709
Tomczyk, P., Schou, J., & Thompson M. J. 1995, ApJ, 448, L57
Vorontsov, S. 2000, in Helio- and Asteroseismology at the Dawn of the Millennium (ESA SP-464; Noordwijk: ESA), 363

Fig. 5.—Left: Temporal variations of the average of $b/c$ over the radial points that lie within the interval $0.69 \leq r/R_\odot \leq 0.72$. Results correspond to 1 yr long MDI data with degrees expanding up to $l = 100$ (diamonds) and 150 (circles). Right: Averaging kernels corresponding to the inversion of the 1996 data at $r = 0.69 R_\odot$. The solid line corresponds to the averaging kernel obtained from the inversion of data with degrees up to $l = 100$, while the dotted line represents the averaging kernel obtained from the inversion of data with degrees up to $l = 150$. 

Eff-Darwich et al.