Free-ion hyperfine fields and magnetic-moment measurements on radioactive beams: progress and outlook

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Abstract Hyperfine fields of highly-charged heavy ions carrying many electrons are discussed in view of their utility for magnetic-moment measurements on radioactive beams by the recoil in vacuum (RIV) technique. An overview of progress is given along with a review of the foundations for ab initio calculations of the relevant free-ion hyperfine interactions.

Keywords Hyperfine fields · Highly-charged ions · Magnetic moments · Radioactive beams

1 Introduction

This paper concerns free-ion hyperfine fields. It is motivated by the opportunity to use the recoil in vacuum (RIV) method to measure the magnetic moments of short-lived excited states of unstable nuclei produced as radioactive beams.

After briefly introducing radioactive beams and nuclear moments, in Section 2, and giving a description of the RIV method as applied to recent studies of neutron-rich nuclei in the region of $^{132}$Sn in Section 3, attention turns to the free-ion hyperfine fields. The formalism and some foundational features for modeling free-ion hyperfine interactions are introduced in Section 4. Section 5 concerns the experimental characterization of free-ion fields, particularly the case of Se and Ge ions leaving carbon foils with velocities of about 5 % of the speed of light. These notionally similar ions show unexpected differences which must stem from differences in their atomic structure. Section 6 discusses progress toward ab initio calculations of free-ion
hyperfine fields. An outlook for future experimental and theoretical work concludes the paper.

2 Motivation: radioactive beams and nuclear moments

New scientific opportunities have been opened up by the availability of beams of rare isotopes produced by radioactive beam accelerators. This frontier of nuclear physics research will continue for some decades as new facilities come on line.

The magnetic moment, which is measured by observing the response of the nuclear spin to an applied magnetic field, is an important observable in the study of the quantum mechanics of nuclear excitations. For example, it gives insights into how the nucleus carries its angular momentum. Magnetic moments are very sensitive to single-particle aspects of the wavefunction, and can strongly distinguish between proton versus neutron excitations.

The focus here is on measurements of the gyromagnetic ratios (i.e. the magnetic moment divided by angular momentum) of the first-excited $2^+$ states of neutron-rich nuclei near closed shells, produced as radioactive beams. These states typically have lifetimes of the order of picoseconds, thus requiring the use of intense hyperfine fields to perturb the orientation of the nuclear state by a measurable amount during its brief lifetime. For this purpose the recoil in vacuum method makes use of the free-ion hyperfine fields of highly charged ions recoiling out of a target into vacuum.

Experiments with radioactive beams are challenging in a number of ways. Firstly, the beam intensity is low, often orders of magnitude weaker than stable beams. To compensate requires the use of sophisticated high-efficiency detector arrays. Secondly, radioactive beams can be contaminated with unwanted isobaric ions. This complication must be evaluated case-by-case. Thirdly, because the beams are radioactive any experiment which aims to study the beam ion itself (rather than its decay products) must ensure that the beam does not stop in view of the $\gamma$-ray detectors. Finally, radioactive beam experiments almost all use ‘inverse kinematics’ wherein the roles of beam and target are interchanged. For example, a rare isotope radioactive beam of interest may be Coulomb excited on a stable target to discover excited states, and/or to measure reduced transition probabilities and magnetic moments.

Because it can often deal with these challenges better than alternative techniques, the recoil in vacuum method has a number of advantages for measurements of excited-state magnetic moments of radioactive beams.

3 The RIV method and its application to radioactive beams

When a free ion moves through vacuum the hyperfine interaction couples the atomic spin $J$ to the nuclear spin $I$ and together they precess about the total spin $F = I + J$, as illustrated in Fig. 1. The precession frequency $\omega_{FF'}$ is proportional to the nuclear $g$ factor and the magnitude of the hyperfine magnetic field at the nucleus. To measure the $g$ factor, the nuclear state of interest is excited by a suitable reaction and then allowed to recoil into vacuum. The effect of the hyperfine interaction is observed
Free-ion hyperfine fields

Fig. 1 Vector model of the free-ion hyperfine interaction

\[ F = I + J \]

via the reduced anisotropy of the angular correlation of the \( \gamma \)-rays de-exciting the state. The concept is illustrated in Fig. 2 using data from the Holifield Radioactive Ion Beam Facility (HRIBF) at Oak Ridge National Laboratory, USA [1].

In the presence of vacuum deorientation, the particle-\( \gamma \) angular correlation after Coulomb excitation takes the form (see e.g. [2] and references therein)

\[
W(\theta_p, \theta_\gamma, \Delta \phi) = \sum_{kq} B_{kq}(\theta_p) Q_k F_k D_{q0}^{k^*}(\Delta \phi, \theta_\gamma, 0),
\]

where the angles are defined schematically in Fig. 2 and \( \Delta \phi = \phi_p - \phi_\gamma \). The attenuation coefficients, \( G_k \), specify the vacuum deorientation effect; \( B_{kq}(\theta_p) \) is the statistical tensor, which defines the spin alignment of the initial state. \( F_k \) represents the usual \( F \)-coefficient for the \( \gamma \)-ray transition, \( Q_k \) is the attenuation factor for the finite size of the \( \gamma \)-ray detector, and \( D_{q0}^{k^*}(\Delta \phi, \theta_\gamma, 0) \) is the rotation matrix. In the applications of interest \( k = 0, 2, 4 \).

Recoil-in-vacuum can refer to two quite distinct experimental techniques, depending on whether the ion has a very simple, few-electron configuration, or whether it has a complex many-electron configuration [3]. A version of the RIV technique to measure the first-excited state \( g \) factors of H-like light ions \( (Z < 20) \) produced by fast radioactive beams has been proposed [4]. It is planned to test this technique in the near future [5]. The discussion here is on the application of the RIV technique to slower-moving many-electron radioactive ions as produced by the Isotope Separation On Line (ISOL) type facilities like HRIBF [1]. In these \( g \)-factor measurements the experimental procedures are identical to those in a measurement of the reduced transition probability, or \( B(E2) \), by Coulomb excitation. The focus of the analysis, however, is on the angular correlation pattern of the \( \gamma \) radiation de-exciting the state of interest rather than its total intensity.

Figure 3 illustrates the procedure used to determine the \( g(2^+ \rightarrow 0^+) \) value in neutron-rich \(^{132}\)Te based on a calibration using stable \(^{126}\)Te and \(^{130}\)Te. The measured attenuation factors are plotted versus the product \( g \tau \). The \( g \) factors for the stable isotopes were measured precisely at the Australian National University [9]. The curves represent a model-based fit to the calibration data [10], which will be described in
**UNPERTURBED**

130Te beam

\( ^{12}\text{C} \) recoil

\( \gamma \) ray emitted at angle \( (\theta, \phi) \)

scattered 130Te stopped in copper

**PERTURBED**

130Te beam

\( ^{12}\text{C} \) recoil

\( \gamma \) ray emitted at angle \( (\theta, \phi) \)

scattered 130Te 'recoils' in vacuum

\[ W(\Delta \phi) \]

\[ \Delta \phi \text{ [degrees]} \]

\[ \theta, \phi \]

\[ \theta_\gamma, \phi_\gamma \]

\[ g_\tau \text{ [ps]} \]

\[ G_k \text{ versus } g_\tau \]

\[ G_2, G_4 \]

\[ G_{134}\text{Te} \]

\[ G_{132}\text{Te} \]

\[ G_{130}\text{Te} \]

\[ G_{281}\text{Te} \]

**Fig. 2** Unperturbed and perturbed angular correlations for the \( 2^+_1 \rightarrow 0^+_1 \) transition in \(^{130}\text{Te} \) [1]. *Left* Unperturbed angular correlations following implantation into copper. *Right* Perturbed angular correlations with reduced anisotropy resulting from vacuum deorientation of the nuclear spin.

**Fig. 3** Attenuation factors versus \( g_\tau \) for several isotopes of Te. For semimagic \(^{134}\text{Te} \) the three points correspond to predicted \( g \) factors: Monte Carlo Shell Model (MCSM) [6], Quasiparticle Random Phase Approximation (QRPA) [7], Shell Model (SM) [8]

greater detail below. Figure 3 also shows the locations of points on the \( G_k \) versus \( g_\tau \) curve as predicted by various nuclear models for \( g(2^+_1) \) in \(^{134}\text{Te} \) [6–8]. It is evident that the range of predicted \( G_k \) values for semimagic \(^{134}\text{Te} \) is near the maximum slope of the \( G_k \) vs \( g_\tau \) curve, and hence the RIV measurement has the sensitivity
required to distinguish between the theoretical models. An RIV measurement on $^{134}$Te, performed at HRIBF in February 2012 will be published elsewhere [11].

4 Modeling recoil in vacuum: foundations

This section reviews the formalism and develops the foundational concepts needed to model the free-ion hyperfine interactions associated with the RIV method. Further discussion may be found in Stuchbery and Stone [10] and the review by Goldring [3].

4.1 Formalism and features

As indicated by the vector model in Fig. 1, the free-ion hyperfine interaction is determined by angular momentum algebra. The starting point for calculating the $\gamma$-ray angular correlation is the statistical tensor of the nuclear state, which specifies the alignment of the nuclear angular momentum. To evaluate the effect of the hyperfine interaction, we begin with the statistical tensor of coupled (nuclear + electronic) system. At all times the atomic, nuclear and total statistical tensors are related by

$$\rho_{kq}(FF') = \sum_{k,\ell,\ell'} \rho_{\ell,\ell'}(I) \rho_{k,\ell'}(J) \hat{F} \hat{F'} \hat{k}_i \hat{k}_j |k_i q_i k_j q_j |kq$$

where $\rho_{kq}(FF')$ is the tensor of the coupled system, $\rho_{k,q}(I)$ is the tensor of the nucleus, and $\rho_{k,q}(J)$ is the tensor of the atomic electrons. $\hat{F} = \sqrt{2F + 1}$, etc. The Clebsch–Gordan coefficient and 9J symbol have their usual designations. In the notation of (1), the nuclear statistical tensor $B_{kq} = \sqrt{2k + 1} \rho_{kq}(I)$.

The coupled tensor evolves in time as

$$\rho_{kq}(FF'; t) = \rho_{kq}(FF'; 0) e^{-i\omega_{FF} t},$$

where $\omega_{FF} = (E_F - E_{F'})/\hbar$. Since we are concerned with magnetic dipole interactions

$$\omega_{FF'} = \frac{A_J}{2\hbar} \left[ F(F + 1) - F'(F' + 1) \right].$$

The hyperfine interaction constant $A_J$ is

$$A_J = \frac{g \mu_N B}{J},$$

where $g$ is the nuclear $g$ factor, $\mu_N$ is the nuclear magneton, $B$ is the hyperfine magnetic field at the nucleus and $J$ is the angular momentum of the electronic
configuration. The hyperfine interaction constant \( A_J \) is usually given as a frequency in MHz (i.e. as \( A_J/\hbar \)). The hyperfine field in tesla is related to \( A_J \) in MHz by

\[
B(\text{tesla}) = 0.1312 \frac{J}{g} A_J(\text{MHz}). \tag{7}
\]

The initial statistical tensor of the nucleus is determined by the nuclear excitation mechanism, and in the case of Coulomb excitation, it can be calculated very accurately. If the atomic system is initially randomly oriented, it follows that the time dependence of the nuclear tensor is given by

\[
\rho_{kj}(I; t) = \rho_{kj}(I; 0) G_k(t), \tag{8}
\]

where the time-differential attenuation coefficient is given by

\[
G_k(t) = \sum_{F, F'} \frac{(2F + 1)(2F' + 1)}{2J + 1} \left\{ \frac{F F' k}{I I J} \right\}^2 \cos(\omega_{FF'} t). \tag{9}
\]

The 9J symbol in (2) has reduced to a 6J.

The experiments considered here determine the time-integral attenuation factors

\[
G_k^\infty(\tau) = \int_0^\infty G_k(t)e^{-t/\tau} dt/\tau, \tag{10}
\]

where \( \tau \) is the mean life of the nuclear state.

Examples of \( G_k(t) \) and \( G_k^\infty(\tau) \) for \( I = 2 \) and the same hyperfine field strength are shown in Fig. 4. A single frequency is present for \( J = 1/2 \). As \( J \) increases the number of different frequencies present increases and the \( G_k(t) \) function becomes more complex.

For a given hyperfine field strength \( B \), the time-integral coefficients for the different \( J \) values all decrease from unity at a similar rate, but they approach different ‘hard core’ values. The hard core, usually denoted \( \alpha_k \), is given when \( F = F' \), or \( \omega_{FF'} = 0 \), in (9). Specifically,

\[
\alpha_k = \sum_F \frac{(2F + 1)^2}{2J + 1} \left\{ \frac{F F k}{I I J} \right\}^2. \tag{11}
\]

The right side of Fig. 4 shows how the hard-core parameters depend on the atomic angular momentum \( J \) (for \( I = 2 \)). The magnitude of the hard-core is a very strong function of \( J \) for \( J < 2 \), but it becomes almost independent of the atomic angular momentum for \( J \geq 2 \). This behavior has implications for the application of RIV to nuclear moment measurements.

The time-differential factors \( G_k(t) \) can be measured for stable beams by using a plunger-type apparatus. The ions recoil through vacuum between the target and a foil set at a distance \( D \) behind the target where they are stopped and hyperfine interaction is quenched. For ions that reach the stopper before the nucleus decays the attenuation factor is \( G_k(D/v) \), where \( v \) is the ion velocity. \( G_k(t) \) can thus be measured by varying the distance \( D \). The Yale group has recently reported such measurements on \(^{98}\)Ru [12]. Unfortunately this method has limited applicability to radioactive beams because they cannot be stopped in view of the \( \gamma \)-ray detectors.
4.2 Decoherence, irreversibility and classical limits

The manner in which quantum systems become classical is of interest for the foundation of quantum mechanics as well as for practical applications such as quantum computing. Free-ion hyperfine interactions provide a flexible laboratory to explore aspects of quantum decoherence and the interaction of an almost isolated quantum system with its environment. In fact, the measurement of the nuclear $g$ factor via the attenuation coefficients $G_k^\infty(\tau)$ is, in effect, a measurement of the rate at which the free-ion system decoheres and irreversibly loses information to its environment.

Two aspects are particularly relevant: First, there is the irreversibility that emerges from the incoherent superposition of hyperfine fields, and second there is irreversibility as a consequence of atomic and nuclear decays.

For an ensemble of many-electron ions with a distribution of electron configurations the attenuation coefficient is given by

$$G_k = \sum_i w_i G^i_k,$$

where $G^i_k$ is the deorientation coefficient for an ion in the state $i$ and $w_i$ is the fraction of ions in that state. The weights are normalized so that $\sum w_i = 1$. Figure 5 shows an example of the attenuation factors $G_2(t)$ for $I = J = 2$ that result from the superposition of 5, 10 and 100 individual fields of randomly chosen strength up to a set cut-off. This example approximates the situation that applies to the RIV $g$-factor measurements. It is evident that the periodicity, and hence reversibility, is lost, even with the superposition of a moderate number of fields. The interpretation of this behavior as a classical limit is supported by the observation that a quasi-exponential drop of the attenuation factor to the hard core also emerges in the limit of $\hbar \to 0$, which may be evaluated by letting $I$ and $J$ both become very large.

Atomic and nuclear decays cause a ‘loss of information’ to the environment. Because the nuclear state of interest ceases to exist, the nuclear decay clearly has dramatic irreversible consequences. Indeed, the integral coefficients $G_k^\infty(\tau)$, which average over the nuclear decay, show no periodicity at all.
The effect of atomic decays is more profound and requires more detailed analysis. Usually the atomic decay will be accompanied by the emission of an $E1$ photon that carries away $1\hbar$ of angular momentum. A rigorous treatment of atomic decays should propagate the statistical tensor of the combined system using (4) to the point in time at which the atomic decay takes place. At that moment, the atomic tensor should be projected out and modified according to the consequences of the atomic decay. The modified atomic tensor should then be inserted into (2) to obtain the tensor for the combined system after the decay, whereupon the propagation in time can resume according to (4). In practice, this rigorous procedure has rarely been used. The loss of angular momentum from the atomic system is generally ignored, and the effect of the atomic decays has been evaluated in terms of products of attenuation factors. This simplified approach will be illustrated here for a single atomic decay. A more detailed description of attenuation factors for a cascade of transitions has been given in [13]. There the interest was in a sequence of nuclear transitions, however the formalism is identical for a sequence of atomic transitions.

Consider an atomic decay from a state $a$ to state $b$, where the upper state has mean life $\tau_a$, and the lower level is assumed stable. Let the time-dependent attenuation factors for the two states be denoted $G_k^a(t)$ and $G_k^b(t)$, respectively. Ignoring changes in the statistical tensor due to the atomic decay, the time dependent attenuation factor that results is given by

$$G_k(t) = G_k^a(t)e^{-t/\tau_a} + \int_0^t G_k^a(t_a) G_k^b(t-t_a)e^{-t_a/\tau_a} dt_a/\tau_a.$$  \hfill (13)

The first term in this equation is the attenuation factor of the state $a$ weighted by the probability that it survives until time $t$. The second term accounts for decays from state $a$ to state $b$ at a time $t_a \leq t$.

Figure 6 shows an example of $G_2(t)$ for an atomic decay from a state with $J = 3/2$ to a state with $J = 1/2$. The $G_2(t)$ for the two contributing states, which have rather simple periodicity, are also shown. It is evident that their regularity is destroyed by the atomic decay. The hard-core value of the lower $J = 1/2$ state is also reduced. In general, persistent atomic transitions are required to reduce the hard-core attenuation coefficients to zero.
4.3 Static model

We have seen that a superposition of many hyperfine frequencies gives a quasi-exponential time dependence to the vacuum attenuation factors, \( G_k(t) \). The alignment of the nuclear state, and hence the anisotropy of the \( \gamma \)-ray angular correlation, decreases approximately exponentially with time, at a rate that depends on the magnitude of the nuclear \( g \) factor. (See Figs. 3 and 5.) Although atomic transitions can potentially have a large influence on the attenuation factors for an ensemble of multielectron ions recoiling through vacuum, it will become evident from the following discussion that their impact is in fact small. It is useful therefore to formulate a static model of the free-ion hyperfine interaction based on (12). In the case where the distribution of fields, \( w_i \), is a Lorentzian distribution centered at \( B = 0 \), the time-dependent attenuation factors show an exponential decay and the time-integrated attenuation coefficients are given by

\[
G_k^\infty(\tau) = \alpha_k + (1 - \alpha_k) \frac{1}{1 + |\Gamma_k| \tau},
\]

where \( \Gamma_k \) is proportional to \( |g| \). The ‘hard core’ parameter \( \alpha_k \) gives the asymptotic value of \( G_k \) at long times, whereas \( \Gamma_k \) is the time constant for the quasi-exponential decay of the attenuation coefficient. Physically, \( \Gamma_k/g \) is related to the average strength of the hyperfine fields acting on the nucleus, while \( \alpha_k \) is determined by the average angular momentum of the atomic electron configurations. Although a Lorentzian distribution of hyperfine fields is not realistic, (14) has proven useful for fitting data.
Stuchbery and Stone [10] introduced a static model of the form

$$G_k = \sum_{i,j} w_J(J_i)w_B(B_j)G_k(J_i, B_j),$$  

(15)

where the contributing attenuation coefficients, $G_k(J_i, B_j)$, are evaluated from (9). In this model the weights associated with the atomic spin and the hyperfine field, $w_J$ and $w_B$, respectively, are assumed to be independent normalized Gaussian distributions. The mean and standard deviations are denoted $\bar{J}$ and $\sigma_J$ for $w_J$, and $\bar{B}$ and $\sigma_B$ for $w_B$. These distributions, which have $\bar{B} \geq 0$ and $\bar{J} \geq 0$, are cut off at $J = 0$ and $B = 0$. Clearly $J \geq 0$ and $J$ takes only integer or half-integer values. The magnetic field distribution is assumed to be continuous. Because the attenuation coefficients are independent of the sign of $B$, the calculations can be restricted to $B \geq 0$.

4.4 Atomic transitions

Despite the strong effect of the atomic transition shown in Fig. 6, the RIV interaction is primarily static, with atomic transitions playing a relatively minor role. This conclusion is supported by the consistent observation of non-zero hard-core terms in experimental attenuation coefficients.

Stuchbery and Stone [10] investigated the impact of atomic transitions in fits to the integral attenuation coefficients for $^{122}$Te, $^{126}$Te, and $^{130}$Te recoiling in vacuum at velocity $v/c \sim 0.06$. By means of an empirical Monte Carlo model and fits to the data they concluded that very few atomic transitions occur in these ions in the time interval up to 11 ps (the mean life of the $2^+_1$ state in $^{122}$Te). Their results are redrawn in Fig. 7 where the static model fit is superimposed on the fit of the model with atomic transitions. In this case the static model is indistinguishable from the model with transitions up to $g\tau \sim 2.5$, which corresponds to times up to about 7 ps.

Calculated atomic state lifetimes for heavy ions carrying many electrons are significantly longer than the nuclear lifetime for a large number of the low-excitation atomic states that contribute to free-ion hyperfine fields [14]. The conclusion drawn from the comparison in Fig. 7 is therefore expected to be generally applicable for such ions.

4.5 Evaluating hyperfine fields and core polarization

The discussion so far has used arbitrary hyperfine field strengths, or values from empirical fits to data. We now consider the evaluation of hyperfine field strengths from atomic calculations such as the Hartree-Fock method.

Core polarization effects are very important in determining the hyperfine fields of atoms and near-neutral ions. Small admixtures of configurations in which an inner electron is left unpaired can have large contributions. An example is provided by the $3s^22S_{1/2}$ state of Na for which the experimental hyperfine interaction constant is $A_{1/2} = 885.81$ MHz. The Hartree-Fock value for the configuration $1s^22s^22p^63s^1$ is $A_{1/2}(\text{HF}) = 626.6$ MHz, only 70 % of the experimental value. As shown by Froese Fischer et al. [15] (Table 8.9, p. 176), a simple configuration interaction (CI) calculation of the polarization effects gives $A_{1/2}(\text{CI}) = 761.0$ MHz (86 % of
Fig. 7 Comparison of the static model and a model with atomic transitions for the Te isotopes. The data points in order from right to left are for $^{132}\text{Te}$, $^{130}\text{Te}$, $^{126}\text{Te}$, and $^{122}\text{Te}$ [10].

Fig. 8 Calculated hyperfine fields (lower panel) and the effect of core polarization (upper panel) for the $^2S_{1/2}$ ground state of Na-like ions.

experiment), whereas a very large-scale CI calculation is needed to bring theory within 1% of experiment [16].

Before embarking on a quest to calculate the hyperfine fields associated with the highly charged free-ions present in RIV measurements, it is important to ask: How important are polarization effects?

To address this question, the ratio $A_{1/2}(\text{HF})/A_{1/2}(\text{CI})$ was evaluated for the $3s^2^2S_{1/2}$ state along the Na-like sequence from $Z = 11$ to $Z = 35$, using the configuration-state function expansion of Froese Fischer et al. [15]. The results are shown in the upper panel of Fig. 8. The lower panel shows the hyperfine field at the nucleus corresponding to $A_{1/2}$ from the CI calculation.
It is immediately evident from the upper panel of Fig. 8 that the Hartree Fock approximation approaches the CI calculation as $Z$—and hence the charge on the ion—increases. This behavior can be understood as stemming from two effects. Firstly, as $Z$ increases, the diagonal energies in the Hamiltonian of the CI problem increase, and hence the diagonal energy differences increase as well. At the same time, the off diagonal matrix elements from electron-electron interactions, which cause the configuration interactions, remain unchanged. The effect is that the core-polarization contributions to the wavefunction decrease markedly as $Z$ increases. Secondly, as $Z$ increases, the strength of the hyperfine field due to the $3s$ electron increases relative to the fields due to the inner electrons. For example, assuming the hyperfine fields scale approximately with $(Z - \sigma_{nl})^3$, where $\sigma_{nl}$ is the screening charge, and taking $\sigma_{1s} \sim 0.5$ and $\sigma_{3s} \sim 6$ from Hartree Fock calculations, gives $B_{1s}/B_{3s} \sim 10$ for $Z = 11$ and $B_{1s}/B_{3s} \sim 2$ for $Z = 30$.

In view of the general origin for the convergence of the HF and CI calculations for highly-charge ions seen in Fig. 8, similar behavior can be expected for any valence state. Provided the ion of interest is highly charged, as it is the cases of present interest, a Hartree Fock calculation for the valence configuration will give the hyperfine field with an accuracy of about 5%.

5 Characterizing free-ion fields: experimental

The $G_k$ versus $gr$ curve, such as that in Fig. 3, must be accurately characterized to extract $g$ factors from RIV measurements. To this end, at the Australian National University we have been studying the free-ion hyperfine fields of stable nuclei having known moments. Four HPGe $\gamma$-ray detectors operate in coincidence with an array of photodiode particle detectors, as shown in Fig. 9, which allow measurements around the beam axis ($\phi$ angle) similar to the radioactive beam measurements. Also like the radioactive beam measurements, the stable-beam studies have been performed in inverse kinematics with the beam ions Coulomb excited on either $^{12}$C or $^{27}$Al targets; the different targets serve to vary the exit velocity and hence charge-states of the ions entering vacuum.

Figure 10 shows measured attenuation coefficients for 175 MeV beams of the stable $^{32}$Ge and $^{34}$Se isotopes excited on a carbon target. The $g$ factors of these isotopes are taken from Mertzimekis et al. [17], and the lifetimes from Raman et al. [18]. The observed difference in the attenuation factors for the longer-lived isotopes, i.e. $^{74,76}$Ge compared with $^{76,82}$Se, was surprising. Some important conclusions can be drawn based on empirical and semi-empirical fits to the data. The left panel in Fig. 10 shows an empirical fit based on (14); the right panel shows a semi-empirical model-based fit using (15). To limit the parameters it has proved effective to (1) fix the standard deviation of the atomic spin distribution to $\sigma_J = \hbar$, and (2) for the distribution of hyperfine fields, to set $\sigma_B = \bar{B}$, where $\bar{B}$ is the average hyperfine field strength at the nucleus. Two parameters, $\bar{J}$ and $\bar{B}$, then determine $G_2$ and $G_4$.

Both the empirical fit (with 4 parameters) and the semi-empirical model-based fit (with 2 parameters) give comparable descriptions of the data. Moreover, both fits suggest that effectively the same average hyperfine field strength is experienced by the Ge and Se ions. What differs for the Ge versus Se ions is the magnitude of the hard core term in the empirical fit or, equivalently, the value of $\bar{J}$ in the model-based fit: $\bar{J} = 1.2$ for Ge and $\bar{J} = 1.7$ for Se.
Charge-state measurements (performed at the Australian National University) show that the Ge ions here carry on average 12 to 13 electrons, whereas the Se ions on average carry 15. The Ge ions are therefore mainly Mg-like and Al-like, with ground state configurations of $3s^2$ and $3s^23p^1$, respectively. These configurations produce the terms $^1S$ and $^2D$, respectively, with corresponding ground-state atomic spins of $J = 0$ and $J = 1/2$. For P-like ions the lowest configuration is $3s^23p^3$, which produces the terms $^4S$, $^2D$, and $^2P$, and the ground-state spin is $J = 3/2$. Also, the low-excitation spectra of excited Mg-, Al- and P-like ions are consistent with an average atomic spin in the range $1 \leq \bar{J} \leq 2$. Looking back to Fig. 4, it can be concluded that the strong sensitivity of the hard-core attenuation observed for Se and Ge ions here is a consequence of their average atomic spin being less than $2\hbar$.

It is important to observe that the RIV measurements on radioactive beams in the $^{132}$Sn region are not affected in this way. In those cases the ions carry around 22 electrons and the low-excitation configurations have several electrons in the 3$d$ orbit. The average atomic angular momentum is expected to be considerably greater than $2\hbar$; indeed fits to data imply that $\bar{J} \sim 4.5$ [10].
6 Ab initio modeling of free-ion fields

With the computer power available today, along with comprehensive atomic structure codes such as the multiconfiguration Hartree-Fock (MCHF) Atomic Structure Package [15, 19] and GRASP2K [20], it has become reasonable to attempt ab initio calculations of the free-ion hyperfine fields of relevance to RIV and magnetic moment measurements.

The simplest microscopic approach to model the RIV attenuation for many-electron ions is to superimpose the deorientation coefficients for the calculated hyperfine fields up to a cut-off in excitation energy, assuming a weighting factor of \((2J + 1)\) for each atomic state. Stone et al. [21] have reported such calculations for Mo, Ru and Pd ions with \(v/c \sim 0.05\).

This static model can then be improved by including the effect of atomic transitions, based on the calculated atomic level lifetimes. Chen et al. [22] have developed a Monte Carlo method to evaluate the effect of atomic transitions and applied it to the tellurium isotopes.

In this section we describe the steps being taken to model the RIV data for Ge and Se ions shown in Fig. 10, aiming eventually to obtain a first-principles explanation of the difference observed for the isotopes having longer-lived states. The approach is to build up the ingredients for an ab initio calculation step-by-step, and confront theory and experiment at each stage.

As noted above, the Ge ions in Fig. 10 are mainly Mg-like and Al-like, whereas the Se ions are predominantly P-like. As a first step towards a microscopic model of the RIV hyperfine interactions for these ions, the hyperfine fields at the nucleus were evaluated for Mg-like and Al-like Ge ions using the MCHF codes [15, 19]. Multiconfiguration calculations were performed to evaluate (at least approximately) the effects of core polarization. The lowest 35 levels of Ge XXI from the configurations \(3s^2, 3s3p, 3p^2, 3s3d,\) and \(3d^2\) were considered. These atomic levels are estimated to have lifetimes of the order of the nuclear lifetimes (up to \(\sim 20\) ps) or longer [14]. Higher atomic states tend to be shorter lived than the nuclear lifetimes and therefore will have less influence. Similar calculations were made for the lowest 46 levels of Ge XX from the configurations \(3s^23p, 3s3p^2, 3s^23d, 3p^3, 3s3p3d,\) and \(3p3d^2\). The calculated hyperfine fields are shown in Fig. 11. For the Mg-like ions, the \(^3P_J\) \((J = 0, 1, 2)\) levels from the \(3s3p\) configuration are noteworthy. These are the lowest excited states of the ion, produce the strongest hyperfine fields \((J = 1, 2)\), and have lifetimes that are orders of magnitude longer than the nuclear lifetimes. It is likely that these atomic states are important in determining the hard-core values of the attenuation coefficients. The related states in the Al-like ions are the \(^2D\) and \(^4P\) levels from the \(3s3p^2\) configuration. For these states the highest fields are for the \(J = 5/2\) states (see the right panel of Fig. 11). The conjecture that the low-excitation atomic levels largely determine the hard-core attenuation coefficients can therefore be tested by varying the balance of Mg-like and Al-like ions in the charge-state distribution (i.e. by varying the velocity at which the ion leaves the target foil).

As a first attempt to model the RIV attenuation for Ge ions, the deorientation coefficients for the calculated hyperfine fields in Fig. 11 were superimposed, assuming a weighting factor of \((2J + 1)\) for each atomic state. Al-like and Mg-like ions were assumed equal in abundance and other charge-states were ignored. The result of this calculation, shown by the broken curves in Fig. 12, is that the average hyperfine
field is too weak. Although there are a number of approximations in this simple calculation, it firmly indicates that the higher excited atomic states must have less weight than the lower excited states. In effect a cut-off in the atomic excitation energy is needed, well below 250 eV. Since a sharp cut-off is unrealistic, we instead modify the weighting of the atomic states using a Boltzmann distribution:

$$w_i \propto (2J + 1)e^{-E_i/T},$$

(16)

where $E_i$ is the excitation energy of the atomic state and $T$ is a parameter resembling temperature. Unfortunately at present there is no clear method to define this parameter from first principles. However a rough fit to the Ge data in Fig. 12, assuming an equal population of Mg-like and Al-like ions, gives $T \sim 25$ eV (the solid curves). The implication is that the lowest atomic states are preferentially populated as the ion emerges from the target foil into vacuum. This aspect in modeling the free-ion hyperfine fields associated with recoil in vacuum will require further investigation and scrutiny.

The next steps for more realistic modeling are: first, to include the full range of ionic charge states contributing to the observed attenuation coefficients, and second,
to make quantitative estimates of the atomic decays taking place. In principle, particular long-lived atomic states could ‘trap’ the atomic population and have an enhanced impact on the observed attenuation factors. Once these well defined processes have been treated rigorously, we can better evaluate the initial population distribution of the atomic states of the ions as they enter vacuum.

7 Summary and outlook

To sum up, the recoil in vacuum method has proved to be very powerful for excited-state magnetic-moment measurements on radioactive beams, particularly near the neutron-rich double magic nucleus \(^{132}\text{Sn}\). Results of such measurements on neutron-rich Te \([11]\) and Sn \([23]\) isotopes, performed recently at HRIBF, Oak Ridge, USA, will be published soon.

Several groups have commenced atomic structure calculations with the goal to develop a quantitative microscopic model of the free-ion hyperfine fields in RIV measurements. As a challenge to these calculations, and an opportunity to refine them, atomic structure effects are seen in otherwise similar measurements of free-ion hyperfine fields for Ge and Se ions carrying \(\sim 12 – 15\) electrons. The difference apparently stems from a difference in the average atomic angular momentum in the range between \(\bar{J} = 1\) and \(\bar{J} = 2\).

The main obstacle to ab initio calculations is that the initial population of atomic states, when the ion enters vacuum, is not well known. In the case of the Ge ions discussed here it appears that there is a strong preference to populate lower excited states. This inference requires further investigation. Along with more refined theoretical calculations that include atomic decays, more comprehensive experimental data are needed. For example, measurements on the variation of the attenuation factors with the ion velocity can test sensitivity to the charge state. Precise measurements of the time-dependent attenuation factors for selected cases would also be of immense value.

Overall, great progress has been made recently in understanding the free-ion hyperfine fields relevant to nuclear spectroscopy measurements. Despite the remaining unknowns, magnetic-moment measurements by the RIV method have been placed on a firm footing by the combination of experimental and theoretical work on free-ion hyperfine interactions reviewed here.

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