Quantitative Benchmarks and New Directions for Noise Power Estimation Methods in ISM Radio Environment

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Abstract

Noise power estimation is a key issue in modern wireless communication systems. It allows resource allocation by detecting white spectral spaces effectively, and gives control over the communication process by adjusting transmission power. Thus far, the proposed estimation methods in the literature are based on spectral averaging, eigenvalues of sample covariance matrix, information theory, and statistical signal analysis. Each method is characterized by certain stability, accuracy and complexity. However, the existing literature does not provide an appropriate comparison. In this paper, we evaluate the performance of the existing estimation techniques intensively in terms of stability and accuracy, followed by detailed complexity analysis. The basis for comparison is signal-to-noise ratio (SNR) estimation in simulated industrial, scientific and medical (ISM) band transmission. The source of used background distortions is complex noise measurement, recorded by USRP-2932 in an industrial production area. Based on the examined solutions, we also analyze the influence of noise samples separation techniques on the estimation process. As a response to the defects in the used methods, we propose a novel noise samples separation algorithm based on the adaptation of rank order filtering (ROF). In addition to simple implementation, the proposed method has a very good 0.5 dB root-mean-squared error (RMSE) and smaller than 0.1 dB resolution, thus achieving a performance that is comparable with the methods exploiting information theory concepts.

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Index Terms

Noise power estimation, SNR estimation, blind noise separation, rank order filtering.

I. INTRODUCTION

Noise power estimation is an important process associated with many different digital signal processing domains. The most common include fundamental problems of speech-enhancement and image denoising [1]–[4]. The dependence on the knowledge of noise variance also occurs in signal processing for segmentation, clustering, noise reduction, statistical inference etc. [4] [5]. Noise power estimation has also acquired a particular attention in wireless communication systems for its role in cognitive radio (CR) and link adaptation algorithms, which are the main motivation of this article.

In CR the detection of white spaces or spectral occupancy in a dynamic radio environment is imperative for opportunistic spectrum access. A simplest and commonly used method for evaluating spectral occupancy is energy detection (ED), which requires efficient noise power estimation in the band of interest for its reliable operation. Many studies show that an uncertainty in noise power level estimation severely limits the sensing capability of energy detector [6] [7]. As a result, the spectrum utilization is strongly associated with the ability to accurately estimate power in unoccupied channel [8].

With its direct impact on SNR estimation, noise power estimation is also an important parameter in SNR dependent link adaptation techniques such as adaptive coding and modulation, and power control. Link adaptation is a key denominator in radio systems for LTE, 5G and WLAN. For dynamic link adaptation, a real-time noise power estimator for continuous channel quality monitoring is required. Using power control as an example, the design parameters of a desired estimator can be well understood. Power control has a significant impact on communication range, achievable throughput and generated interference [9]. Rapid changes in power regulation, besides affecting energy efficiency, can be a source of serious impulse noise in adjacent channels. On the other hand, a fluent power regulation depends on the stability of the estimation results. Therefore, a proper estimation method must not only be accurate, but also characterized by possibly low standard deviation, which also approximates the resolution of the method.

Unfortunately, the existing literature is mainly focused on the design of new solutions for noise power estimation, and lacks a common quantitative benchmark for evaluating their performance. In this paper, we first develop a simulation model to overcome this gap that allows
for carrying out a comprehensive and realistic comparison of noise power estimation methods. The simulation model reliably substitutes a simple ISM band transmission with time-frequency resource allocation. Each estimation method, independently or by auxiliary algorithms, locates unoccupied parts of the spectrum and estimates the noise power. Our model allows to track the accuracy of the estimations over consecutive time intervals and collects the results to assess the long-term stability of each estimation technique. For realistic comparison of the studied estimators, we used real noise as a source of background distortion in the developed simulation model. To this end, we collected RF background noise traces in an industrial environment using National Instrument USRP-2932.

Our second contribution is a novel adaptation of rank order filtering (ROF) technique for samples separation. The ROF, in its original form [4], is used to search for local minima and replicate them among the filtering area i.e., it determines noise floor only. Our proposal adapts the filtering process to analyze energy changes and to identify groups of samples likely to be signal bands. This in turn leads us to an innovative combination of a simple ML estimation with ROF as samples separation technique. The proposed combination improves the accuracy of the estimation by 30% at the expense of only 14% higher variance. At the same time, the presented solution reduces the computational cost of samples separation by 50% with respect to the commonly used Fisher discriminant. Therefore, the proposed solution, easy to implement technique based on averaging, achieves the performance comparable with the estimation techniques originating from information theory.

The rest of the paper is organized as follows. Section II gives closely related works and motivation of our work. Section III introduces system model. Section IV describes commonly used noise power estimation methods. Section V gives a novel estimation algorithm. Section VI describes simulation model. Section VII compares the proposed estimation approach with the literature. Section VIII presents computational complexity analysis. Finally, conclusions are drawn in Section IX.

II. RELATED WORK AND MOTIVATION

Spectrum usage awareness and optimal exploitation of radio resources is particularly important for devices operating in ISM radio bands, where the spectrum utilization is not uniform. The devices operating in ISM bands use different communication standards, thus operate with
different bandwidths and transmit powers [10]. Therein, the accuracy of noise power estimation is central to the correct identification and efficient allocation of available radio channels.

Moreover, the ISM band is often used by wireless networks with nodes having limited-hardware resources. An important consideration to this end is the complexity of the estimation methods. With low-computing power, the nodes must remain aware of the radio environment and effectively schedule their transmissions. Therefore, the estimation techniques must not only be accurate and stable but also be easy to implement and execute.

Modern researchers have proposed a number of approaches for estimating noise power, based on various properties. Most often referred to in the literature are maximum likelihood (ML) and minimum variance unbiased (MVU) estimators [6], [11]–[14]. Both ML and MVU estimators are based on the principle of power spectrum averaging. However, the range of proposed solutions is much wider and includes, for instance, information theoretic methods as Akaike Information Criteria [15] [16] and correlation-based methods e.g., covariance based estimator [17] [18]. More specialized solutions, for example based on statistical relations, can also be found as in minimum mean squared error (MMSE) estimator [19]. All of these methods, although dedicated for similar applications, differ significantly in terms of accuracy, stability and complexity.

However, most of the published descriptions are largely focused on the parameters of the individual estimation methods and omitted the mutual relations between the existing solutions. As a response to the lack of relevant studies, we revisit the current estimation techniques and compare them in terms of the most important parameters. According to the authors, it is appropriate and necessary to conduct a comprehensive and realistic comparison of noise power estimation methods.

III. System Model

Energy analysis can be performed in either time domain or frequency domain. Performing time-domain energy analysis requires filter-banks to divide spectrum into frequency bands. In frequency domain, spectrum is already channelized into subbands by an FFT operation. In this work, estimation is performed on a resource block consisting of as a certain number of subcarriers in a given number of time frames. Therefore, a frame-by-frame processing of time-domain signals is considered, where consecutive frames are transformed to the spectral domain by applying FFT.

Let $S_m(n)$ and $W_m(n)$ be the complex signal and noise spectral coefficients, with $n$ the frequency-bin index and $m$ the time-frame index. The signal and the noise are assumed to be additive in
the short-time Fourier domain. The complex spectral noisy observation is thus given by

$$X_m(n) = S_m(n) + W_m(n).$$  \hspace{1cm} (1)

When information signal $S_m$ is absent, the received signal $X_m$ has power equal to the variance of the corresponding additive white Gaussian noise. Considering noise with zero mean and variance $\sigma_w^2$, and by invoking the central limit theorem, the distribution of the noise power can be assumed to be $N\left(\frac{\sigma^2_w \cdot \sigma_x^2}{MN}\right)$, where $M$ is the number of time frames $m \in [1, M]$ and $N$ is the number of frequency bins $n \in [1, N]$. Thus when $S_m(n)$ is present, the SNR can be expressed as

$$\text{SNR} = \frac{\sigma_x^2 - \sigma_w^2}{\sigma_w^2},$$ \hspace{1cm} (2)

where $\sigma_x^2$ and $\sigma_w^2$ are the received signal and the noise powers respectively.

IV. NOISE POWER ESTIMATION METHODS

A. Maximum Likelihood Estimator (ML)

The simplest and the most commonly used technique to estimate the noise power is ML estimator. The particular popularity of this method is due to its extremely simple implementation. In case of white Gaussian noise, ML is limited only to the averaging of the power spectrum over selected time interval [6] [11]. The energy contained in the frequency band of interest can be estimated as

$$\sigma_w^2 = \frac{1}{N} \sum_{n=1}^{N} |X_m(n)|^2,$$ \hspace{1cm} (3)

where $X_m(n)$ is the $n$-th frequency bin and $N$ is the number of FFT spectral components calculated for the noise samples separated from $m$-th time interval.

This simple technique accurately tracks the instantaneous value of noise power. Although it reacts quickly to the changes in noise power, each noise burst can seriously distort the measurement. As a consequence, this technique is highly susceptible to momentary inaccuracies.

B. Minimum Variance Unbiased Estimator (MVU)

The problem associated with ML estimator can be solved by extending the analysis over more time intervals. A solution called as minimum variance unbiased (MVU) estimator is based on averaging the noise power over the entire time-frequency block [12]. MVU can be described as

$$\sigma_w^2 = \frac{1}{MN} \sum_{m=1}^{M} \sum_{n=1}^{N} |X_m(n)|^2,$$ \hspace{1cm} (4)
where $X_m(n)$ is the $n$-th out of $N$ frequency bins in the $m$-th FFT realization. Also, $M$ is the number of consecutive time intervals processed into $M$ spectrum vectors.

The introduced extension results in stabilization of the estimation process in longer time period, which makes the MVU estimator more resistant to minor disturbances. However depending on the number of averaged intervals, the delay in the MVU response to noise changes increases. The delay in response time remains one of the major drawbacks of the method.

Note that while processing ML and MVU estimates, it is vital to separate noise samples precisely from the received signal. As a result, the accuracy of both methods is strongly dependent on the technique of samples separation. The process of samples selection can be realized in various ways. In [13], the authors proposed a hybrid technique by combining fine and fast sensing periods. The authors in [14] used the Fisher’s discriminant-based approach to divide the received signal into two groups. The other studies, e.g., [15] [16], refer to the Minimum Description Length (MDL) or Akaike Information Criterion (AIC) as an effective method of signal source separation.

C. Akaike Information Criterion (AIC)

Bearing in mind the constraints in ML and MVU estimators, a promising but complex estimation process is based on the AIC, which includes the samples separation without any knowledge of the received signal [15]. Therefore, it can be assumed as an independent estimation method. The estimation technique as described in its original form, i.e. in [15], is based on the eigenvalues of the covariance matrix. However, the estimation process has been significantly simplified in [16]. For white noise, it replaces the covariance matrix with the values of the sorted averaged periodogram. The main expression of AIC is given as

$$\text{AIC}(n) = (N - n) M \log (\alpha(n)) + n(2N - n),$$

where $N$ is the FFT size, $M$ is the number of intervals over which periodogram is averaged and $n$ is the number of frequency bins corresponding to the signal in $n$-th model. The function $\alpha(n)$ is defined as

$$\alpha(n) = \frac{1}{N - n} \left( \sum_{i=n+1}^{N} \lambda_i \right) \left( \prod_{i=n+1}^{N} \lambda_i \right)^{1/(N-n)},$$

where $\lambda_i$ is the power of the $i$-th frequency bin in averaged periodogram considered as a substitute of the eigenvalue.
The power bins $\lambda_i$ in the sorted periodogram, up to the index $n_{\text{min}}$ indicating the minimum value of $\text{AIC}(n)$, are assumed to represent the signal. The mean of the remaining periodogram values is considered to be an estimate of the noise power

$$
\sigma_w^2 = \frac{1}{N - n_{\text{min}}} \sum_{i=n_{\text{min}}}^{N} \lambda_i.
$$

(7)

This simplification makes the AIC-based estimation easier to implement and more useful as it remains effective and fully independent from the external signal separation techniques. However, it should be noted that the effectiveness of the method is based on the averaged periodogram created for the entire resource block. Thus it is problematic to maintain the reliability based on a single spectrum realization.

**D. Covariance Based Estimator (CBE)**

Another estimation approach, which departs from the averaging of the power samples, is based on the eigenvalues of the sample co-variance matrix. The estimation algorithm proposed in [17] uses two main assumptions. First, the ordered eigenvalue sequence can be separated into signal and noise groups according to the bandwidth occupancy ratio. It means that for the sample covariance matrix $C$

$$
C = \frac{1}{N}XX^T,
$$

(8)

in total number of $M$ eigenvalues $S$ of them are considered to represent the signal, where $X$ stands for $N \times M$ sized time-frequency block. This assumption is correct if the spectral occupancy ratio is equal to $M/S$. In such case $(M - S)$ eigenvalues can be assigned to the noise group.

Second, by using descending ordered eigenvalues $\lambda_m$ of the matrix $C$, a range of $L$ linearly-spaced expected noise power values $\sigma_i^2 \in [\sigma_{\text{min}}^2, \sigma_{\text{max}}^2]$ can be generated, where $L$ is user defined spacing and

$$
\sigma_{\text{min}}^2 = \frac{\lambda_M}{\left(1 - \sqrt{M/N}\right)^2},
$$

(9)

$$
\sigma_{\text{max}}^2 = \frac{\lambda_{S+1}}{\left(1 - \sqrt{M/N}\right)^2}.
$$

(10)

Assuming that the empirical distribution of the noise eigenvalues denoted as $e.d.$ follows the Marcenko Pastuer distribution [18], goodness of fit testing can be used to find the best fitting power value

$$
D_l = \left\| e.d. - MP \left( (M - S)/N, \sigma_l^2 \right) \right\|_2,
$$

(11)
where \( l = 1 \ldots L \) and \( MP \) denotes the Marcenko Pastuer distribution. Following that, based on the conducted fit testing, the most accurate variance value is selected

\[
\sigma^2_w = \arg\min_{\sigma^2_i} (D_l).
\]  

(12)

Although a solid foundation in mathematics exhibits high reliability, the algorithm remains computationally expensive. In addition, it requires knowledge of the bandwidth occupancy which means an extension by an auxiliary algorithm such as MDL, as studied in [17], is needed.

E. Minimum Mean Squared Error Estimator (MMSE)

In MMSE estimation [19], the noise variance is estimated using an MMSE filter at each subcarrier. The filter coefficients are calculated using statistics of the noise over last \( M - 1 \) intervals. The estimator can be defined as

\[
\sigma^2_w = \sum_{n=1}^{N} w_n |X_M(n)|^2,
\]  

(13)

where \( X_M(n) \) are the frequency bins of the last FFT realization and \( w_n \) are the filter coefficients.

To determine \( w_n \) coefficients, it is necessary to calculate the variance \( \sigma^2_n \) for each \( n \)-th subcarrier

\[
\sigma^2_n = \frac{1}{M-1} \sum_{m=1}^{M-1} |X_m(n)|^2.
\]  

(14)

Following that, the correlation of variance in the frequency dimension needs to be calculated. The correlation expressed as

\[
r(\Delta) = E\{\sigma^2_n \sigma^2_{n+\Delta}\}
\]  

(15)

is used directly to generate the covariance matrix \( C \), where

\[
C(n, m) = r(n - m).
\]  

(16)

Based on matrix \( C \), vector \( r \) and a unit matrix \( I \), the coefficients vector \( w \) can be calculated as

\[
w = (C + r(0)I)^{-1} r.
\]  

(17)

The commonly used approach for noise power estimation in multichannel systems is however based on \( X_m(n) \) belonging to the distribution \( N(0, \sigma^2_n) \). This assumption requires subtraction of noisy received symbols by the best hypothesis of the noiseless ones. The subtraction operation requires the pool of transmitted symbols to be known, and therefore MMSE cannot be considered
fully blind. However, if these requirements are fulfilled, MMSE can be a convenient alternative to MVU, AIC and CBE. Unlike the other methods, it uses all the received samples to estimate noise power. Also, it does not require white noise distribution across the analyzed spectrum.

V. PROPOSED SEPARATION ALGORITHM

As mentioned earlier, in order to successfully perform ML estimation and strongly related with it MVU, it is necessary to use signal separation techniques such as the Fisher discriminant, MDL or AIC. However, it should be noted that the separation of signal is used in many other areas of signal processing. Thus the range of proposed solutions is very wide. However to perform ML or MVU in simple wireless network nodes effectively, it is important for the separation technique to remain simple. In this paper, we propose a low-complexity algorithm based on the adaptation of Rank Order Filtering (ROF, Fig. [1]).

![Fig. 1. A block diagram of rank order filter $R(k, l)$](image)

The ROF is originally dedicated to impulse noise reduction and automatic noise floor estimation [20]. We modify the ROF method to analyze power drop during the filtering process, which we utilize to separate spectral samples and perform ML or MVU on the selected noise group. A step-by-step operation of the proposed algorithm is shown in Fig. [2].

The separation process starts with the iterative erosive filtering $R(k, 1)$ performed on the power spectrum vector $P_m(n)$. The initial value of $k$ is set to 2 and $k$ is increased in each iteration until it reaches the size of spectrum vector $N$. After filtering the entire input vector with filter size $k$, energy $E_{m,k}$ of the output signal $F_{m,k}(n)$ is calculated. The consistent increase in the size of the filter $k$ allows to find the size $K$, after which the energy decrease $D_{m,k}$ in relation to that of the previous filtering process is highest.

The found $K$ indicates the bandwidth of the signal with the highest power. To sensitize the algorithm also for possible signals with wider bands carrying less power, index $K$ is iteratively increased until analyzed energy decrease $D_{m,k}$ achieves a value lower than the assumed threshold $\lambda_1$. The resulting filter size $K$ defines the broadest signal band in the analyzed spectrum.
Fig. 2. Noise power estimation algorithm using ROF-based sample separation.

1. Calculate power spectrum
   \[ P_x(n) = |X_x(n)|^2 \]

2. Proceed Rank Order Filtering
   \[ F_{n,k}(n) = P_x(n) \ast R(k,1) \]

3. Calculate energy
   \[ E_{n,k} = \sum_{n=1}^{N} F_{n,k}(n) \]

4. Calculate energy drop
   \[ D_{n,k} = \frac{(E_{n,k-1} - E_{n,k})}{E_{n,k-1}} \]

5. Check if \( k \geq N \)

6. Search for index \( K \)
   \[ K = \text{arg} \{ \max(D_{n,k}) \} \]

7. If \( D_{n,k} \geq \lambda_1 \), then \( K+1 \)

8. Proceed moving average
   \[ P_x(n) = P_x(n) \ast \text{MAV}(k) \]

9. Define sign of derivative
   \[ S_x(n) = \text{sgn} \{ \Delta P_x(n) \} \]

10. Mark rejected indexes \( n \)
    \( (n_i, n_j) \in \text{Rejected Group} \)
    If \( \forall n \in (n_i, n_j), S_x(n) > 0 \)
    and \((n_i - n_j) > \lambda_2 \)

11. Calculate power estimation
    \( n' \in \text{Rejected Group} \)
    \[ \sigma^2_n = \frac{1}{N} \sum_{n'} P_x(n') \]
    \[ \sigma^2_x \]
Fig. 3. Process of noise samples separation using modified ROF

Using a $K$-sized moving average on the spectrum $P_m(n)$ results in a smoothed spectrum $\bar{P}_m(n)$ in which the long-rising edges of the signals are feasible indicators of the currently occupied subbands.

The identification of signal bands is based on the sign of a derivative function $\Delta \bar{P}_m(n)$. The areas $(n_a, n_b)$ of the averaged spectrum for which derivative has positive values and which are wider than the assumed threshold $\lambda_2$ indicate signal subbands. These areas of the spectrum can
be rejected. The remaining samples $P_m(n)$ indicated by the indexes $n'$, not assigned to the signal group, can be used to estimate the noise power $\sigma_w^2$.

Fig. 3 shows the operation of separation algorithm on an exemplary signal frame. Fig. 3(a) depicts the input power vector, which represents a noisy channel with two signal subbands. Fig. 3(b) presents percentage energy drop after erosive filtering in terms of the increase of the filter size $k$. The desired size $K$ is marked in red, while assumed threshold $\lambda_1$ (equal to 5%) is marked as a blue line. Fig. 3(c) shows the power spectrum trace after $K$-size moving average. Results of the derivative calculation from the averaged power spectrum are given in Fig. 3(d). The blue lines indicate positive areas $(n_1, n_2)$ and $(n_3, n_4)$ wider than the assumed threshold $\lambda_2$ (equal to 5% of the total bandwidth). Positive areas between the lines are marked as a signal group while the rest of the spectrum is assumed to represent noise. The separation into signal and noise group is shown in Fig. 3(e). The samples from Fig. 3(a) with indexes marked on Fig. 3(e) as signal should be rejected. Rest of the samples should be used to estimate the noise power.

Based on the group of samples classified as noise, effective ML estimation for a single time interval can be performed. Therefore, performing the separation process also for the following intervals allows for MVU estimation.

Note that the thresholds $\lambda_1$ and $\lambda_2$ were chosen empirically. A change in these thresholds may effect the efficiency of the algorithm. However, the optimization of $\lambda_1$ and $\lambda_2$ remains open for further research.

VI. SIMULATION MODEL

A fair performance comparison of the estimation methods requires a common signal platform. The adopted model should reliably approximate transmission in ISM band, where various heterogeneous devices occupy resources using different bandwidths and powers. In addition, the used model should also simulate variability of the tracked noise power and allow analysis of the responses to SNR changes in the band of interest.

We developed a simulation model that handles the requirements of ISM radio environment. In our simulations, the subject of analysis is a time-frequency block (Fig. 4). It consists of $N$-point FFTs calculated for $M$ non-overlapping time intervals. The resource block is divided into four equal frequency subbands, to which the signal with regulated band fulfillment and power can be added.
The signal collected by a radio receiver may significantly differ from the pseudorandom sequences adjusted to the theoretical assumptions of the white Gaussian noise. The latter is the most common approach followed in performance studies. However, the real noise characteristics include uneven distribution of the noise among frequencies and distortions caused by the receiver’s limitations. Therefore, the most important advantage of the presented simulation model is the use of real noise traces as a source of distortion (see Fig. 5).

The real noise traces are the collection of I/Q data of the RF background noise in channel 26 of IEEE 802.15.4. We collected the traces in an industrial production area using National Instrument USRP-2932. Fig. 6 shows a layout of the industrial plant. The channel bandwidth is 5 MHz with the center frequency at 2480 MHz. The sampling frequency is set to 10 Msamples/s and the I/Q samples are stored in 32-bit single precision format.

The simulation model determines the reference noise power based on the variance $\sigma_w^2$ of the noise time samples collected in $M$ consecutive time intervals. The adjustment of the reference noise power is obtained by rescaling the specified variance $\sigma_w^2$.

The examined estimation scenario also required to append a deterministic signal with given bandwidth and power. Due to the simplicity of the parameters manipulation, we used a rectangular
Fig. 5. Comparison of a real industrial environment noise and a pseudorandom white Gaussian noise.

Fig. 6. Industrial environment where the traces are collected.

function in the frequency domain, defined as

$$S_m(n) = A_m \text{rect} \left( \frac{n - n_c}{n_a - n_b} \right).$$

(18)

where $A_m$ is the amplitude of a rectangular function, $n_a$ and $n_b$ are the boundary indexes of the
spectral bins of the rectangular function and, $n_c = (n_a + n_b) / 2$ is the central bin of the function $\text{rect}(\cdot)$. Therefore the required spectral energy of a signal can easily be defined as

$$E_m = A_m^2 (n_a - n_b)$$

which can be regulated by either changing the bandwidth or amplitude.

VII. COMPARISON OF THE ESTIMATION METHODS

Comparison is carried out for the real noise realization rescaled to 1mW average power. The simulated scenario assumes spectrum division into four subbands and the presence of a deterministic signal in a single subband occupied in 100% of width. Thus, $n_a$ and $n_b$ are set to 2.5 MHz and 3.75 MHz respectively. The regulated SNR is set to 0 dB and -3 dB modified by signal amplitude equal to 63.2 mV and 44.7 mV in particular cases.

In our study, we analyze the ability of estimation methods to track noise power in a variable environment (Fig. 7). However, the main focus is on the utility of each method in SNR mapping (Fig. 8). Each technique is evaluated in terms of accuracy by the root-mean-squared error analysis and stability expressed by standard deviation (see Table I).

For comparison, the MMSE method is adapted for the purpose of the blind analysis. The knowledge about the shapes of the transmitted symbols is replaced by the reduction in the average value of the frequency bins over time in appropriate subbands. Thus, the processed signal is reduced to the theoretical distribution $\mathcal{N}(0, \sigma^2)$, as postulated in [18].

In case of real noise, which does not completely fulfill the theoretical assumptions of the normal distribution, MMSE results in the underestimation of the noise power. This leads to overestimation of the SNR and as a result may cause too small power increase in the channel to keep transmission effective.

A similar overestimation tendency is exhibited by CBE method. However, the detection based on the eigenvalues is characterized by significantly higher stability. In comparison with other, simpler techniques, CBE does not provide significant improvement in estimation accuracy.

Among the analyzed estimation techniques ML, MVU and AIC can be considered a group of related solutions, i.e. estimators based on the averaging of selected power samples. In this group, AIC undoubtedly deserves particular attention. AIC, despite replacing the eigenvalues with averaged periodogram, reliably approximates the noise power. Thus allows steady and accurate SNR estimation, and remains unsusceptible to sudden power fluctuations.
Fig. 7. Comparison of the noise power estimation for different estimation methods.

Fig. 8. Comparison of SNR estimation for different estimation methods.

TABLE I
SNR ESTIMATION ACCURACY AND STABILITY FOR DIFFERENT ESTIMATION METHODS

| Parameter  | CBE  | MMSE | AIC  | ML   | MVU  |
|------------|------|------|------|------|------|
| RMSE [dB]  | 0.391| 1.337| 0.406| 0.547| 0.572|
| Std. dev. [dB] | 0.173| 0.523| 0.084| 0.206| 0.066|
Fig. 9. Comparison of the noise power estimation for different signal separation methods.

Fig. 10. Comparison of SNR estimation for different signal separation methods.

### TABLE II

| Parameter | AIC  | MVU(ID) | MVU(FD) | MVU(ROF) |
|-----------|------|---------|---------|-----------|
| RMSE [dB] | 0.406| 0.572   | 0.706   | 0.497     |
| Std. dev. [dB] | 0.084| 0.066   | 0.064   | 0.073     |
On the other hand, ML effectively tracks the current noise power. In terms of power regulation, the use of ML compared to CBE, will result in small but sudden power changes. However, due to increased inaccuracy, it exhibits higher SNR overestimation than eigenvalue based method.

MVU significantly decreases susceptibility for short-term changes. Although in terms of stability it exhibits the results comparable to those achieved by AIC, in accuracy it causes 50% higher SNR overestimation.

It should be noted that despite the overestimation of SNR, both ML and MVU remain very flexible solutions. Although in the fast processing cases it can be more convenient to use only ML estimator, in more complex cases method can be simply extend to more reliable MVU.

As we mentioned in Section [1], it is necessary to use signal separation techniques for noise power estimation using ML and MVU. In such a case, it is reasonable to assume that the problem of limited accuracy can be mitigated not only by more advanced estimation methods, but also by precise detection of spectrum parts free from deterministic signal.

We analyze ML and MVU approximations with different separation methods in terms of noise power estimation (Fig. [9]) and correctness of SNR tracking (Fig. [10]). ML and MVU methods are combined with ideal separation (IS), Fisher discriminant (FD) and proposed rank order filtering (ROF). MDL implementation is omitted, due to its similarity with the AIC method [15]. The AIC is observed to be the most competitive estimation method in terms of accuracy and complexity of estimation. Therefore it is used as the main reference technique for the proposed solution.

The ideal splitting case exhibits an underestimation effect because of the reduced size of the noise group compared to the reference and underestimation resulting from inaccurate transition to the frequency domain. The effect increases with the use of Fisher’s discriminant. Due to the recognition of high amplitude noise samples as part of the signal group, the use of discriminant results in the highest SNR overestimation in the analyzed group of methods.

In the presented case, ROF results in the opposite effect i.e., a lower underestimation. As the signal samples with near-noise values at signal boundaries are not rejected, it results in the extension of the noise group by a small number of low signal samples and improves the noise power underestimation. The proposed solution provides better sample separation and as a consequence in conjunction with ML and MVU improves SNR estimation (see Table [II]). With the small cost of decrease in stability, the use of ROF significantly improves the accuracy of long-term response to SNR changes. The proposed solution clearly outperforms use of Fisher discriminant in terms of accuracy and remains a convenient alternative to AIC.
VIII. Computational Complexity

To complete the comparison it is necessary to determine the computational complexity of each estimation method. Assuming the basic operations performed in IEEE arithmetic machine at \( O(1) \) cost each, complexity is defined as the number of operations necessary to perform detection on \( N \) received new samples. To simplify the calculation, an \( N \times N \) block size is adopted. Each estimation method precedes FFT at cost \( N \log N \).

To perform covariance based estimation (CBE) on \( N \) incoming samples, it is necessary to perform matrix multiplication with \( N^2 \) operations. Then, CBE requires determination of eigenvalues with cost of \( N^2 \) calculations assuming power iteration algorithm. The goodness of fit testing with MP distribution has a complexity of \( N^2 \). The algorithm requires further \( 8N \) additions and multiplications. Thus, the total cost of the estimation will be \( N \log N + 8N + 2N^2 + N^2 \).

MMSE estimation requires an update of the variance vector at cost \( 3N^2 + 2N \) and update of the autocorrelation at cost of \( N \log N \). Based on the autocorrelation, a sample covariance matrix is written at cost \( 2N \). Calculation of the power spectrum requires another \( 5N \) operations, while determining the coefficients for the weighted summation costs \( 3N^2 \) operations. The total computational complexity of the estimation is therefore \( 2N \log N + 8N + 6N^2 \).

In case of AIC, calculation proceeds with \( 7N \) operations to obtain the average periodogram. Sorting requires further \( N \log N \) operations. The \( 2N^2 + 4N \) calculations are necessary to determine the function \( \alpha \). Calculating AIC costs another \( 6N \). Finding a minimum of the function and calculating the power from the remaining samples of the periodogram will cost maximum \( 2N \) operations. The total cost of AIC estimation will therefore be \( 2N \log N + 17N + 2N^2 \).

Determining the ML or MVU estimates for a single \( N \) samples vector with respect to the Fisher discriminant will require firstly \( 4N \) operations to determine the amplitude spectrum. Finding the maximal Fisher’s discriminant involves the cost of \( 4N \) each time it needs to be designated, whereas the discriminant can be found in \( N \) steps. This gives the cost of \( 4N^2 \) operations. Determination of power on the basis of separated samples in the maximal case will require \( 2N \) operations. Thus, the total cost of the estimate is \( N \log N + 6N + 4N^2 \).

In the proposed algorithm, that is merging ML or MVU with ROF, it is necessary to determine the power spectrum at the expense of \( 5N \) operations. Filtering process and power calculations require further \( 2N^2 \) operations. Later, the algorithm is loaded with the sum of \( 5N \) operations resulting from calculating differences in powers, comparisons to find desired power drop point,
TABLE III

COMPUTATIONAL COMPLEXITY OF STUDIED ESTIMATION METHODS FOR N-SAMPLES INPUT

| Estimation method | Computational complexity |
|-------------------|-------------------------|
| CBE               | $8N + N \log N + 2N^2 + N^{2.38}$ |
| MMSE             | $8N + 2N \log N + 6N^2$ |
| AIC              | $17N + 2N \log N + 2N^2$ |
| ML (FD)          | $6N + N \log N + 4N^2$ |
| ML (ROF)         | $11N + N \log N + 2N^2$ |

moving average, derivative and samples rejection. The cost of conducting ML on the remaining power spectrum samples will require maximally $N$ operations. The total cost of the proposed estimation technique will be $N \log N + 11N + 2N^2$.

The comparison of computational complexity in Big-O notation shows for most methods the same $O(n^2)$ square complexity. However a deeper analysis of all necessary operations (see Table III), shows that the proposed method exhibits lower complexity with respect to competing solutions. These small computational differences are of particular importance in cases of frequently performed fast-sensing based on a small number of input samples.

IX. CONCLUSION

In this paper, we performed a comprehensive analysis of noise power estimation methods in terms of their stability, accuracy and complexity. In our analysis, a special attention is given to the background noise process to obtain a real-world performance of the existing estimation techniques. For this purpose, we utilized complex noise measurement data collected in an industrial plant. We observed that the measured noise process has completely different behavior on the performance of the studied estimators when compared with pseudorandom noise.

Our analysis reveals that exploring the new solutions for effective noise power estimation techniques is one side of the coin. The other side is to find the efficient methods of samples separation for prior estimation techniques. In this respect, we developed a novel algorithm for samples separation that finds its roots in rank order filtering. The proposed algorithm, while being simple to implement thanks to its low-complexity, is effective in improving the accuracy of noise power and SNR estimation when combined with the commonly used ML and MVU estimation.
algorithms. Our proposal, results in less than 0.5 dB RMSE and 0.075 dB standard deviation, respectively. The achieved performance makes our method comparable to more complex solutions based on covariance matrix analysis, and also the ones exploiting information theory concepts.

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