Evidence for M-theory based on fractal nearly tri-bimaximal neutrino mixing

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Developing a theory that can describe everything in the universe is of great interest, and is closely relevant to M-theory, neutrino oscillation and charge-parity (CP) violation. Although M-theory is claimed as a grand unified theory, it has not been tested by any direct experiment. Here we show that existing neutrino oscillation experimental data supports one kind of high dimensional unified theory, such as M-theory. We propose a generalization of the tri-bimaximal neutrino mixing ansatz, and we find that the latest neutrino oscillation experimental data constraints dimension in a range between 10.46 and 12.93 containing 11, which is an important prediction of M-theory. This ansatz naturally incorporates the fractal feature of the universe and leptonic CP violation into the resultant scenario of fractal nearly tri-bimaximal flavor mixing. We also analyze the consequences of this new ansatz on the latest experimental data of neutrino oscillations, and this theory matches the experimental data. Furthermore, an approach to construct lepton mass matrices in fractal universe under permutation symmetry is discussed. The proposed theory opens an unexpected window on the physics beyond the Standard Model.

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Introduction.– M-theory, one of the most promising theories beyond the Standard Model, is suffering from pseudoscience questions [1] because of the lack of direct experimental evidence, and causes wide discussions [2]. Recently acquired Neutrino oscillation experimental data might provide promising chances to either support or decline the M-theory. However, the relationship between neutrino oscillation and M-theory has not been fully established yet. This is because the dimensions of these two theories are not identical. Neutrino theory is a low-dimensional theory while the M-theory is 11-dimensional [3, 4]. Usually, the high-energy M-theory has to be shrunk to 4 dimensions, forming a low-energy theory to match the experimental data such as those from Large Hadron Collider (LHC). However, none of these predictions have been supported by the LHC data yet because the low-dimension M-theory has not been completely developed.

Here, we expand the neutrino oscillation theory to 11-dimension using nonextensive statistics [5, 6]. This method has succeeded in many fields such as generalizations of relativistic and quantum equations [7], transverse momenta distributions at LHC experiments [8], dissipative optical lattices [9], plasmas [10], etc (see http://tsallis.cat.cbpf.br/biblio.htm, for a regularly updated bibliography). Nonextensive statistics is based on the fractal principle [11]. We bring it to modify the tri-bimaximal neutrino mixing pattern, which allows to incorporate CP violation and the fractal feature of the universe into the resultant scenario of fractal nearly tri-bimaximal flavor mixing. Results show that the dimension of a neutrino system using the nonextensive statistics is located between 10.46 and 12.93, which well covers the 11 predicted by the M-theory.

Constraints on dimension and mixing factors.– In order to obtain the dimension range of neutrino system, we analyze the latest neutrino oscillation experimental data with fractal nearly tri-bimaximal neutrino mixing theory (see Appendixes). In detail, adopting theoretical formula (see Eq. (15) in Appendixes) \( \sin^2 2\theta_{atm} = 1 - c^4 \), in which \( c \equiv \cos \theta \), combining with experimental data \( \sin^2 2\theta_{chz} = (8.5 \pm 0.5) \times 10^{-2} \), we obtain the allowed range of space-time dimension (there is an intimate relation \( q = d_f \) between q and fractal dimension \( d_f \) when the Euclidean dimension is one [13]): 10.46 \( \leq q \leq 12.93 \). This moment, theoretical formula and experimental data have no limit on \( \phi \) which is the source of leptonic CP violation in neutrino oscillations, so there is a set \( S_{chz,q} = \{ 10.46 \leq q \leq 12.93, -\infty < \phi < +\infty \} \), which can be expressed in Fig. 1 with the red strip area. For sake of seeing the limit of theoretical formula (see Eq. (15) in Appendixes) \( \sin^2 2\theta_{atm} = 1 - s^4 \), in which \( s \equiv \sin \theta \), and experimental data \( \sin^2 (2\theta_{23}) = 0.999^{+0.018}_{-0.010} \) for normal mass hierarchy and \( \sin^2 (2\theta_{23}) = 1.000^{+0.000}_{-0.017} \) for inverted mass hierarchy on the range of \( q \) and \( \phi \), we do the corresponding calculation and find that \( q \) can take any real number, namely, \( -\infty < q < +\infty \), and this moment, theoretical formula and experimental data also have no limit on \( \phi \). So, there is a set \( S_{atm,q} = \{ -\infty < q < +\infty, -\infty < \phi < +\infty \} \) which can be expressed in Fig. 1 with the blue strip area. There is relationship \( S_{atm,q} \supset S_{chz,q} \), seeing Fig. 1. Take the intersection of these two sets we conclude that the range of space-time dimension that our theory combined with the latest neutrino oscillation experimental data allowed is between 10.46 and 12.93 containing 11, which is an important prediction of M-theory. We can also see that the allowed range of space-time dimension will be further restricted with the improvement of the experimental accuracy. The neutrino oscillation experimental data becomes the first evidence of M-theory, which will effectively eliminate the people’s question to M-theory [1].
FIG. 1. The space of dimension $q$ and phase $\phi$. The figure expresses three sets and relationship among them: $S_{hz,q} = \{ 10.46 \leq q \leq 12.93, -\infty < \phi < +\infty \}$, $S_{atm,q} = \{ -\infty < q < +\infty, -\infty < \phi < +\infty \} \supseteq S_{hz,q}$, and $S_{sun,q}(\phi) \subset S_{atm,q}$, where the range of $q$ in set $S_{atm,q}$ is obtained based on experimental data $\sin^22\theta_{atm} = (8.5 \pm 0.5) \times 10^{-2}$, and $\phi$ is not limited now, so it can take any real number. $S_{stn,q}$ is decided by experimental data $\sin^2(2\theta_{23}) = 0.999900518$ for normal mass hierarchy and $\sin^2(2\theta_{13}) = 1.000000017$ for inverted mass hierarchy, and after checked there is $S_{atm,q} \supset S_{hz,q}$. In fact, $q$ in $S_{stn,q}$ can take all real number due to the fact that the experimental upper and lower limits of $\sin^2(2\theta_{23})$ are automatically satisfied. For upper limit, $\sin^22\theta_{atm} = 1 - s^2 \leq 1, \forall q \in \mathbb{R}$, and for lower limit one has $\sin^22\theta_{atm} \geq 0.99997675$, which satisfies the lower limit. At this moment $\phi$ is also not limited, so it can take any real number too. On the basis of meet the above conditions $S_{sun,q}(\phi)$ is decided by experimental data $\sin^2(2\theta_{12}) = 0.846 \pm 0.021$. $S_{sun,q}(\phi) \subset S_{sun,q}(\phi)$, and $S_{atm,q}(\phi)$ is obtained based on experimental data $\sin^2(2\theta_{atm}) = 0.99997675$, which satisfies the lower limit. The prediction of the strength of CP or T violation in neutrino oscillations under typical $q$ values are in Table 1, and especially, when $q = 11$, $-0.0011 \leq J_{q=11} \leq 0.0011$. Fig. 3 expresses the prediction intuitively. The predicted strength of CP violation can be determined by the T-violating asymmetry between $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\mu$ transitions or by the CP-violating asymmetry between $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions in a long-baseline neutrino oscillation experiment, when the terrestrial matter effects are under control or insignificant.

With the purpose of obtaining the range of $\phi$, we adopt the theoretical formula (see Eq. (15) in Appendixes) $\sin^22\theta_{sun} = \frac{3}{2} (1 - \frac{3}{2} s^2 - s c \cos \phi + \frac{3}{2} s c \cos \phi - 2 s^2 c^2 \cos^2 \phi)$ to analyze the experimental data [12] $\sin^2(2\theta_{12}) = 0.846 \pm 0.021$. We using numerical calculation find that the range of $\phi$ is depending on parameter $q$. The top and bottom limit of $\phi$ under the typical $q$ values are in Table 1. The $\phi$ set under the $q$ that allowed by all theoretical formula and experimental data, $S_{sun,q}(\phi)$, can be expresses with the yellow area in Fig. 1, and there is relationship $S_{sun,q}(\phi) \subset S_{hz,q}$.

In conclusion, the set of $q$ and $\phi$ allowed by theoretical formula and experimental data is the intersection of sets $S_{hz,q}$, $S_{atm,q}$ and $S_{sun,q}(\phi)$, namely, $S_{sun,q}(\phi)$, i.e. the yellow area in Fig. 1.

**Change on range of mixing factors.** Next, we investigate the change on range of $\phi$ after the dimension increased with the theoretical formula (see Eq. (15) in Appendixes) $\sin^22\theta_{sun} = \frac{3}{2} (1 - \frac{3}{2} s^2 - s c \cos \phi + \frac{3}{2} s c \cos \phi - 2 s^2 c^2 \cos^2 \phi)$ under the cases of $q = 1$ and $10.46 \leq q \leq 12.93$, respectively. From Table 1 and Fig. 2 we find that when $q = 1$, $0.49 \leq \phi_{q=1} \leq 1.27$ the order of magnitude is 1; but when $10.46 \leq q \leq 12.93$, the range of $\phi$ increases with the increase of $q$ namely, from $-1537.79 \leq \phi_{q=10.46} \leq 1537.79$ to $-6372.47 \leq \phi_{q=12.93} \leq 6372.47$ with the order of magnitude $10^4$. The order of magnitude of $\phi$ range increases 3 order after the dimension increased, which eliminates the question of small range of $\phi$ values. Specifically, when $q = 11, -2162.81 \leq \phi_{q=11} \leq 2162.81$.

**Prediction on CP violation.** To examine the theory proposed in this paper, we give a prediction of the strength of CP or T violation in neutrino oscillations. No matter whether neutrinos are Dirac or Majorana particles, the strength of CP or T violation in neutrino oscillations is measured by a universal parameter $J$ which is defined as [14]: $Im (V_{\alpha \beta} V_{\gamma \lambda}^* V_{\gamma \lambda}^* V_{\beta \lambda}^* J_\gamma J_\lambda) = J \sum_\gamma \sum_\lambda (\varepsilon_{\alpha \beta \gamma} \varepsilon_{\gamma \lambda})$, in which the Greek subscripts run over $(e, \mu, \tau)$, and the Latin subscripts run over $(1, 2, 3)$. Considering the lepton mixing scenario proposed above, one has (see Eq. (18) in Appendixes): $J = \frac{1}{15} s c \sin^2 \theta (c^2 + s^2 \rho^2_2 (\phi))$. The prediction of the strength of CP or T violation in neutrino oscillations under typical $q$ values are in Table 1, and especially, when $q = 11$, $-0.0011 \leq J_{q=11} \leq 0.0011$. Fig. 3 expresses the prediction intuitively. The predicted strength of CP violation can be determined by the T-violating asymmetry between $\nu_\mu \rightarrow \nu_\tau$ and $\nu_e \rightarrow \nu_\mu$ transitions or by the CP-violating asymmetry between $\nu_\mu \rightarrow \nu_e$ and $\bar{\nu}_\mu \rightarrow \bar{\nu}_e$ transitions in a long-baseline neutrino oscillation experiment, when the terrestrial matter effects are under control or insignificant.

**TABLE I.** The range of $\phi$ and strength of CP or T violation. $q = 1$ is the ideal one-dimensional case; $q = 10.46$ and 12.93 are dimension lower and upper limits allowed by existing neutrino oscillation experimental data, respectively; $q = 11$ is the prediction of M-theory. After the dimension increased, the range of phase spanned from $0.49 \leq \phi_{q=1} \leq 1.27$ to $-2162.81 \leq \phi_{q=11} \leq 2162.81$, increasing 3 orders; The predicted strength of CP violation is $-0.0011 \leq J_{q=11} \leq 0.0011$, which can be determined by the T- or CP-violating asymmetry in a long-baseline neutrino oscillation experiment.

| $q$       | $\phi_{\min}$ | $\phi_{\max}$ | $J_{\min}$ | $J_{\max}$ |
|-----------|---------------|---------------|------------|------------|
| 1.00      | 0.39          | 1.27          | 0.0054     | 0.0110     |
| 10.46     | -1537.79      | 1537.79       | -0.0012    | 0.0012     |
| 11.00     | -2162.81      | 2162.81       | -0.0011    | 0.0011     |
| 12.93     | -6372.47      | 6372.47       | -0.0009    | 0.0009     |
Further discussions and remarks.— Our findings reveal a strong association between neutrino oscillation and M-theory at the point of 11 dimensions of space-time. This would mean that the neutrino oscillation experiment is the initial robust evidence of M-theory, broking the spell that the M-theory has no experimental evidence, eliminating pseudoscience questions [1], and opening an unexpected window on the physics beyond the Standard Model. However, we should realize that in spite of the M-theory have part truth, but not completely developed yet, and there may be other way. Fractal theory and practice [15] have illuminated that the world is of fractal. The definition of fractal dimension is more universal than the one of Euclidean dimension. Euclid dimension is just a special case of fractal dimension, and there is intimate relation \( q = d_f \) between \( q \) and fractal dimension \( d_f \) when the Euclidean dimension is one [13]. As the \( q \to 1 \) limit case of the fractal nearly tri-bimaximal neutrino mixing pattern under discussion, the nearly tri-bimaximal neutrino mixing pattern, as Xing [16] expected, serves as the leading-order approximation of a more complicated flavor mixing matrix (see Eq. (12) in Appendixes), though its prediction on \( \sin^2(2\theta_{13}) \) is not consistent with the experimental data. Although existing neutrino oscillation experiment data limits the range of space-time dimension
between 10.46 and 12.93 (see Fig. 1), the range of space-time dimension will be narrowed down with the increasing of experimental accuracy, and we expect an exclusion of 12 dimension. In addition, we find the order of magnitude of \( \phi \) range increases 3 orders after the dimension increased (see Fig. 2). Moreover, this theory yields a prediction (see Fig. 3) which can be determined by the T-violating asymmetry between \( \nu_\mu \rightarrow \nu_e \) and \( \nu_e \rightarrow \nu_\mu \) transitions or by the CP-violating asymmetry between \( \nu_\mu \rightarrow \bar{\nu}_e \) and \( \bar{\nu}_e \rightarrow \nu_\mu \) transitions in a long-baseline neutrino oscillation experiment, when the terrestrial matter effects are under control or insignificant. Note that our scenario predicts that \(-0.0011 \leq J_{q=11} \leq 0.0011\), and when \( \phi = 0 \), \( J_{q=11} = 0 \), namely, there is no CP violation. Therefore, our theory can be applied whether CP is violated or not.

Finally, let us remark that the fractal nearly tri-bimaximal mixing pattern and its possible extensions require some peculiar flavor symmetries to be imposed on the charged lepton and neutrino mass matrices. It is likely that the fractal nearly tri-bimaximal neutrino mixing pattern under discussion serves as the more complicated flavor mixing matrix that scientists are looking for [16], and one of the nearly tri-bimaximal neutrino mixing patterns is its leading-order approximation. We expect that more delicate neutrino oscillation experiments in the near future will be able to verify the fractal nearly tri-bimaximal mixing pattern, from which one may get some insight into the underlying flavor symmetry and its breaking mechanism responsible for the origin of both lepton masses and leptonic CP violation.

APPENDIXES

The mixing factors of solar, atmospheric and CHOOZ neutrino oscillations read:

\[
\begin{align*}
\sin^2 2\theta_{\text{sun}} &= 4 |V_{e1}|^2 |V_{e2}|^2, \\
\sin^2 2\theta_{\text{atm}} &= 4 |V_{\mu3}|^2 \left( 1 - |V_{\mu3}|^2 \right), \\
\sin^2 2\theta_{\text{chz}} &= 4 |V_{e3}|^2 \left( 1 - |V_{e3}|^2 \right).
\end{align*}
\]

A. Fractal nearly tri-bimaximal neutrino mixing

The tri-bimaximal neutrino mixing pattern \( U_v = V_0 \) can be constructed from the product of two modified Euler rotation matrices:

\[
R_{12} (\theta_x) = \begin{pmatrix} c_x & s_x & 0 \\ -s_x & c_x & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

\[
R_{23} (\theta_y) = \begin{pmatrix} 0 & c_y & s_y \\ 0 & -s_y & c_y \\ 1 & 0 & 0 \end{pmatrix},
\]

where \( s_x \equiv \sin \theta_x \), \( c_x \equiv \cos \theta_x \), and so on. Functions \( \exp_q (iu) \) and \( \cos_q (u) \) can be defined with \( \exp_q (u) \) which is the one-dimensional q-exponential function that naturally emerges in nonextensive statistics [5] spawned by fractal thought [11]. For a pure imaginary \( iu \), one defines \( \exp_q (iu) \) as the principal value of

\[
\exp_q (iu) = \left[ 1 + (1 - q) iu \right]^{1/(1-q)}.
\]

The above function satisfies [6]:

\[
\exp_q (\pm iu) = \cos_q (u) \pm i \sin_q (u),
\]

\[
\cos_q (u) = \rho_q (u) \cos \left\{ \frac{1}{q-1} \arctan [(q-1)u] \right\},
\]

\[
\sin_q (u) = \rho_q (u) \sin \left\{ \frac{1}{q-1} \arctan [(q-1)u] \right\},
\]

\[
\rho_q (u) = \left[ 1 + (1-q)^2 u^2 \right]^{1/[2(1-q)]},
\]

\[
\exp_q (iu) \exp_q (-iu) = \cos^2_q (u) + \sin^2_q (u) = \rho^2_q (u).
\]

Note that \( \exp_q [i (u_1 + u_2)] \neq \exp_q (iu_1) \exp_q (iu_2) \) for \( q \neq 1 \). Then we obtain:

\[
V_0 = R_{23} (\theta_y) \otimes R_{12} (\theta_x)
\]

\[
= \begin{pmatrix} c_x & s_x & 0 \\ -s_x c_y & c_x c_y & s_y \\ s_x s_y & -s_y c_x & c_y \end{pmatrix}.
\]

The general form of the corresponding neutrino mass matrix \( M_\nu \) is
\[
M_{\nu} = V_0 \begin{pmatrix}
    m_1 & 0 & 0 \\
    0 & m_2 & 0 \\
    0 & 0 & m_3
\end{pmatrix} V_0^T
= \begin{pmatrix}
    c_{\nu}c_{\mu} (m_1 - m_2) & c_{\nu}s_{\mu} (m_1 - m_2) & c_{\nu}c_{\gamma} s_{\mu} (m_1 - m_2) \\
    -c_{\nu}c_{\mu} s_{\mu} (m_1 - m_2) & s_{\mu} s_{\mu} (m_1 + c_{\mu}^2 m_2 + s_{\mu}^2 m_3) & -c_{\nu} s_{\mu} (m_1 + c_{\mu}^2 m_2 - m_3) \\
    c_{\nu} s_{\mu} (m_1 - m_2) & -c_{\nu} s_{\mu} (m_1 + c_{\mu}^2 m_2 - m_3) & s_{\mu}^2 (m_1 + c_{\mu}^2 m_2 + c_{\mu}^2 m_3)
\end{pmatrix}.
\]

(10)

Taking \( q = 1 \), \( \theta_x = \arctan(1/\sqrt{2}) \approx 35.3^\circ \) and \( \theta_y = 45^\circ \), the results in usual space-time are reproduced [16].

To make CP violation and the fractal feature of the universe be naturally incorporated into \( V \), we adopt the following complex rotation matrices:

\[
R_{12} (\theta, \phi) = \begin{pmatrix}
    c & s e^{-i\phi} & 0 \\
    -se^{-i\phi} & c & 0 \\
    0 & 0 & 1
\end{pmatrix},
\]

(11)

where \( c \equiv \cos \theta \), \( s \equiv \sin \theta \), and \( e^{i\phi} = \exp(i\phi) \). In this case, we obtain the lepton flavor mixing of the following pattern:

\[
V = R_{12}^T (\theta, \phi) \otimes V_0
= \begin{pmatrix}
    \frac{1}{\sqrt{2}} (2c + s e^{-i\phi}) & \frac{1}{\sqrt{2}} (c - s e^{-i\phi}) & -\frac{1}{\sqrt{2}} s e^{-i\phi} \\
    -\frac{1}{\sqrt{2}} (c - s e^{-i\phi}) & \frac{1}{\sqrt{2}} (c + s e^{-i\phi}) & \frac{1}{\sqrt{2}} s e^{-i\phi} \\
    \frac{1}{\sqrt{2}} s e^{-i\phi} & -\frac{1}{\sqrt{2}} s e^{-i\phi} & \frac{1}{2} c
\end{pmatrix}.
\]

(12)

This matrix \( V \) represents a fractal nearly tri-bimaximal flavor mixing scenario, if the rotation angle \( \theta \) is small. The parameter \( \phi \) in \( V \) are the source of leptonic CP violation in neutrino oscillations.

B. Constraints on dimension, mixing factors and CP violation

A proper texture of \( M_l \) which may lead to the flavor mixing pattern \( V \) is

\[
M_l = \begin{pmatrix}
    0 & C_l & 0 \\
    C_l^T & B_l & 0 \\
    0 & 0 & A_l
\end{pmatrix},
\]

(13)

where \( A_l = m_\tau \), \( B_l = m_\mu - m_\tau \), and \( C_l = \sqrt{m_\tau m_\mu} e^{i\phi} \). Then the mixing angle \( \theta \) in \( V \) reads

\[
\tan_q (\theta) = \frac{\sin_q \theta}{\cos_q \theta} = \sqrt{\frac{m_\tau}{m_\mu}}.
\]

(14)

It is easy to prove that when \( q \to 1 \), the results in usual space-time are recovered, namely [16], \( C_l = \sqrt{m_\tau m_\mu} e^{i\phi} \), \( \tan (2\theta) = 2 \frac{\tan \theta}{1 - \tan^2 \theta} \).

In the next step we calculate the mixing factors of solar, atmospheric and reactor neutrino oscillations. According to this theory, one obtains

\[
\begin{align*}
\sin^2 2\theta_{sun} &= \frac{s}{2} \left( 1 - \frac{3}{2} s^2 - s c \cos_q \phi + \frac{3}{2} s^3 c \cos_q \phi - 2s^2 c^2 \cos_q \phi \right), \\
\sin^2 2\theta_{atm} &= 1 - s^4, \quad \sin^2 2\theta_{chz} = 1 - c^4.
\end{align*}
\]

(15)

Note when \( q \to 1 \), the results in usual space-time are recovered [16]:

\[
\begin{align*}
\sin^2 2\theta_{sun} &= \frac{s}{2} \left( 1 - \frac{3}{4} \sin^2 \theta - \sin \theta \cos \theta \cos \phi + \frac{3}{2} \sin^3 \theta \cos \theta \cos \phi - 2\sin^2 \theta \cos^2 \theta \cos^2 \phi \right), \\
\sin^2 2\theta_{atm} &= 1 - \sin^4 \theta, \quad \sin^2 2\theta_{chz} = 1 - \cos^4 \theta.
\end{align*}
\]

(16)

In this scenario, adopting experimental data [12], \( \sin^2 2\theta_{chz} = (8.5 \pm 0.5) \times 10^{-2} \), one obtains \( 10.46 \leq q \leq 12.93 \); thus there is \( 0.999987 \leq \sin^2 2\theta_{atm} \leq 0.999999 \), which is highly consistent with the experimental data [12]: \( \sin^2 (2\theta_{23}) = 0.999^{+0.001}_{-0.018} \) for normal mass hierarchy and \( \sin^2 (2\theta_{23}) = 1.000^{+0.000}_{-0.017} \) for inverted mass hierarchy; in addition, to make \( \sin^2 2\theta_{sun} \leq 0.867 \) to accord with the experimental data \( \sin^2 (2\theta_{12}) = 0.846 \pm 0.021 \), one needs only \( 1357.79 \leq \phi_{q=10.46} \leq 1537.79 \) or \(-6372.47 \leq \phi_{q=12.93} \leq \phi_{q=1} \leq 1.27 \).

Additionally, given that \( q \) is close to 11 and the intime relation \( q = d_f \) between \( q \) and fractal dimension \( d_f \) when the Euclidean dimension is one [13], we assume \( q = 11 \), then this scenario gives the predicted values of \( \sin^2 2\theta_{chz} = 0.082456 \) and \( \sin^2 2\theta_{atm} = 0.999987 \) which amazingly fit in with the current data [12] \( \sin^2 (2\theta_{13}) = (8.5 \pm 0.5) \times 10^{-2} \) and \( \sin^2 (2\theta_{23}) = 0.999^{+0.001}_{-0.018} \) for normal mass hierarchy \( \sin^2 (2\theta_{23}) = 1.000^{+0.000}_{-0.017} \) for inverted
mass hierarchy), respectively; the range of parameter $-2162.81 \leq \phi_{q=11} \leq 2162.81$ limited by current data $\sin^2(2\theta_{12}) = 0.846 \pm 0.021$ is also much better than that in usual space-time ($0.49 \leq \phi_{q=1} \leq 1.27$). According to the calculations above, we come to the following conclusions: i) the universe is fractal with high dimension; ii) some high dimensional space-time theories, such as M-theory, can be in line with expectations. A numerical illustration of $\sin^2 2\theta_{\odot}$ as the function of $q$ and $\phi$ is shown in Fig. 2, where the two horizontal lines are the top and bottom limits of experimental data. As can be seen from the figure, in $\phi = 0$ case, $\sin^2 2\theta_{\odot}$ very sensitively dependent on $\phi$.

The strength of CP or T violation in neutrino oscillations is measured by a universal parameter $J$ which is defined as [14]:

$$ Im \left( V_{\alpha i} V_{\beta j}^* V_{\gamma k}^* \right) = J \sum_{\gamma,k} (\varepsilon_{\alpha \beta \gamma} \varepsilon_{ijk}) . $$

(17)

Considering the lepton mixing scenario proposed above, one has

$$ J = \frac{1}{6} s c \sin_q \phi \left( c^2 + s^2 \rho_q^2 (\phi) \right) . $$

(18)

Obviously, when $q \to 1$, the result in usual space-time is recovered [16]:

$$ J = \frac{1}{6} s c \sin \phi . $$

(19)

Based on Figs. 2 and 3 as well as the numerical calculations, one obtains the table I.

The strength of CP or T violation $J$ in fractal nearly tri-bimaximal neutrino mixing patterns is predicted as: $-0.0011 \leq J_{q=11} \leq 0.0011$. The experimental data of strength of CP or T violation may limit the range of parameter $\phi$, but unfortunately at present, there is no experimental information on the Dirac and Majorana CP violation phases in the neutrino mixing matrix is available [12].