Low primordial information content in the Milky Way with warm dark matter

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ABSTRACT

We speculatively examine some issues related to the information content of primordial patches that collapse to form galaxies like the Milky Way. If the dark matter is warm, or if some other process dramatically suppressed small-scale density fluctuations, then the patch that formed the Milky Way would have had low primordial information content. Depending on assumptions about the accuracy with which the initial conditions are specified, the patch would have contained only several billion independent information-carrying ‘pixels’ if the warm-dark-matter (WDM) particle had a mass of 1 keV. This number of ‘pixels’ is much less than even the number of stars in the Milky Way. Like other recent observational tests, this would provide an argument disfavoring such a low mass, under two strong assumptions: (1) a high degree of structure in the Milky Way cannot arise from very smooth initial conditions, and (2) non-primordial information/randomness sources are negligible. An example of a non-primordial information source is a central black hole with an accretion disk and jets, which in principle could broadcast small-scale quantum fluctuations throughout the galaxy. This brings up a question, and even, in principle, a test (if the dark matter is warm) about the scale at which structure in the Galaxy is entirely deterministic from the initial conditions.

Key words: large-scale structure of Universe – cosmology: theory

1 INTRODUCTION

Intuitively, any sensible measure of the complexity, or information content, of the Milky Way, would be staggeringly vast, compared to that usually encountered by humans. A convenient definition to use is the Kolmogorov complexity (Kolmogorov 1963), which is the length (e.g. in digital bits) of the shortest-possible algorithm that describes the object. This description can be considered to consist of two parts: a program, and data fed to the program. We denote the Kolmogorov complexity as $I_K$.

How to find a minimal description of the Milky Way? An entirely full description would include all positions and momenta of all particles, to the accuracy allowed by the uncertainty principle. This description would carry a huge amount of raw data, with no processing necessary. But this description can surely be compressed. Particles form organized structures, like molecules, crystals, lifeforms, planets, stars, star clusters, and spiral arms. But we aim at an exact description; explicitly encoding descriptions of such structures with all their possible variations rapidly becomes unsatisfying and overwhelming.

Assuming that the formation of structure in the Galaxy is entirely deterministic, a plausibly minimal complete description would consist of accurate knowledge of its initial conditions, together with an algorithm, i.e. the set of physical laws that formed the Galaxy from these initial conditions. In the current cosmological paradigm, the primordial patch that formed the Milky Way was imprinted with a random pattern of scalar density fluctuations during inflation, occurring in the first instants after the Big Bang. It is this patch, and its surroundings that govern the tidal field in which it collapses, that we refer to as ‘initial conditions.’ This pattern is thought to have been completely random, arising from quantum processes. Further degrees of freedom were present initially in the form of tensor fluctuations (present on large scales) and even decaying modes, but these are likely to be negligible for galaxy formation.

How deterministic is the formation of structure in the Galaxy? That is, to what degree is there a one-to-one correspondence between initial and final conditions? Some simulators assume that if sufficient resolution and computational resources existed, there would be a one-to-one correspondence between initial and final conditions. But since we are still very far from the regime where all relevant processes are entirely resolved, the issue of the scale at which structure in the Galaxy is deterministic is not yet a crucial one for modeling.

But the following questions are still interesting: On large, linear scales, the dynamics are deterministic, but at what scale does a one-to-one correspondence between initial and final conditions fail? When it fails, does a volume of initial-conditions phase space...
generally shrink or expand in the final-conditions phase space? That is, do information sinks or sources dominate?

### 1.1 Information sinks

Information ‘sinks’ occur when many sets of initial conditions (microstates) correspond to indistinguishable final conditions (macrostates). This gives an entropy, the logarithm of the number of microstates per macrostate. In this paper, we only use the word ‘entropy’ to mean information that has gone down a sink, but not primordial information. The Kolmogorov information $I_K$ behaves oppositely; neglecting information sources, $I_K$ decreases and is converted into entropy.

Even in idealized collisionless dark-matter dynamics, coarse-graining gives an information sink. A related effect is the reduction in Fisher information in the power spectrum on small scales (e.g. Rimes & Hamilton 2006). Much of this can be recovered by a change of variables to the log-density (Neyrinck et al. 2009), but the restoration is not complete: fluctuations that were once imprinted on a structure that has collapsed are still completely lost, when resolved on a cosmological scale. See Neyrinck & Yang (2013) for a discussion of how information imprinted at different scales behaves globally, in different density variables.

Baryonic physics give further sinks, spoiling the deterministic classical Vlasov-Poisson system of collisionless dark matter. In the linear regime, baryonic fluctuations are the same as the dark matter, except further damped by photon diffusion and gas pressure forces (e.g. Silk 1968; Greenin et al. 2003). So there is no additional information present in the baryons that is not in the cold dark matter. Even for slightly nonlinear objects like filaments, the gas is a smoothed version of the dark matter (Harford & Hamilton 2011). Small-scale dark-matter structures likely persist within them, but are affected gravitationally by the baryons.

A black hole is a perfect information sink; collapse to form one produces physical (Bekenstein-Hawking) entropy. Another example of a cosmologically, physically relevant entropy is the dust heated by stars (Bousso et al. 2007). In both cases, especially the first, information from the initial conditions is lost. We do not claim that the information difference $\Delta I_K = I_{K,\text{prim}} - I_{K,\text{init}}$ is exactly a physical entropy, especially not one that takes into account all gravitational effects. But it does behave in a similar, increasing way as physical entropy.

### 1.2 Information sources

Information ‘sources’ are processes that inject new, non-primordial randomness into the system. Each time an atom radiates a photon from an excited energy state, it is a probabilistic process, and the randomness in the time of photon emission adds information to the system. Because these quantum processes occur far into the large-number regime for most astrophysical processes, it is likely that information sources are negligible after coarse-graining on an astronomical scale.

However, in astrophysical processes, there are many instabilities that can amplify small fluctuations chaotically. There are also processes that propagate structure on very small scales to larger scales within a galaxy, e.g. supernovae and jets. The fine-scale structure of a supernova explosion, perhaps partially arising from quantum randomness, can grow to scales comparable to a galaxy, especially for small galaxies. Active galactic nuclei (AGN’s), and other jets, are another possible non-primordial information source, potentially broadcasting small fluctuations within a few Schwarzschild radii of a black hole throughout a galaxy. In could be that non-primordial randomness accumulates over time, eventually dominating the information budget on all galactic scales. The non-astronomical-scale structure on Earth, for example, may be dominated by non-primordial information, although that is not clear.

### 1.3 Information conservation and warm dark matter

In this paper, we explore the consequences of conservation (or, if anything, net destruction) of $I_K$ from initial to final conditions. This could plausibly hold on scales larger than a critical scale $F_{\text{deterministic}}$, small compared to a characteristic size of a galaxy, such that structure, after coarse-graining on that scale, is determined by the initial conditions. We assume that, at this scale, $F_{K,\text{init}} \leq I_{K,\text{prim}} + I_{K,\text{physics}}$, where $F_{K,\text{init}}$ is the Kolmogorov complexity of the Galaxy at $z = 0$ after coarse-graining at $F_{\text{deterministic}}$. $I_{K,\text{prim}}$ is the Kolmogorov complexity of the initial conditions on which the present structure depends, and $I_{K,\text{physics}}$ is the information necessary to encode the algorithm used to advance the initial data to the present structure. We neglect $I_{K,\text{physics}}$ as $\ll I_{K,\text{prim}}$. ‘Structure after coarse-graining’ here means, for instance, a 6D (position and momentum) phase-space map of all matter within the Galaxy with position-space pixels of size $F_{\text{deterministic}}$, and some commensurate pixel size in velocity coordinates. We assume that $F_{\text{deterministic}}$ is large enough to smooth over thermal motions, but do not explicitly use a particular scale below, since it is so uncertain.

In §2 we show how the primordial information depends on small-scale power cutoffs. In particular, if the dark matter was warm, i.e. slightly relativistic at decoupling, then primordial fluctuations were damped. In §3 we discuss how this primordial information tally might be used, and conclude.

Of course, a lack of small-scale power from warm dark matter (WDM) would show up in other ways, more directly observable than information content, but all related to the smoothing of initial power that underlies our information-content investigation. WDM reduces the population of Milky Way satellites compared to CDM; it was the ‘missing satellites’ problem with CDM that first prompted the recent burst of interest in WDM (e.g. Bode et al. 2001 B01). However, currently allowed WDM masses do not seem to solve these issues by themselves (Schneider et al. 2014; Weinberg et al. 2013) give a recent review of the status of these issues. Hogan & Dalcanton (2000) also investigate how WDM and dark-matter self-interaction influence the structure and stability of dark-matter haloes. There have also been several proposals to look at structure within the Milky Way, its stellar streams, and its arrangement of satellites for similar cosmological information (e.g. Lynden-Bell & Lynden-Bell 1995; Metz et al. 2009; Starkenburg et al. 2009; Li & Helmi 2008; Law & Majewski 2010; Cooper et al. 2011; Bozek et al. 2013; Ngan & Carlberg 2014).

## 2 QUANTIFYING THE PRIMORDIAL INFORMATION OF THE MILKY WAY

Observations of the cosmic microwave background (CMB) constrain the primordial Universe to have very nearly Gaussian, small density fluctuations (e.g. Planck Collaboration et al. 2013). The essential implication for present purposes is that the initial pattern of fluctuations, if pixelized, can be considered to consist of independent random Gaussian densities in each pixel, with the resulting field multiplied in Fourier space by $P_{\text{init}}(k)$, the square root...
of the initial power spectrum. The statistical independence of each pixel implies that the primordial information is incompressible, at least if power is substantial at all scales. Then the initial information at resolution \( r \) of a primordial patch \( I^\text{prim}_{\text{pix}}(r) = bN_{\text{pix}}(r) \), where \( N_{\text{pix}}(r) \) is the number of pixels of size \( r \) necessary to cover the patch, and \( b \) is the number of bits necessary to encode each pixel. The ambiguity in specifying \( b \) is unfortunate, but necessary given that the concept of Kolmogorov complexity is defined for discrete, not continuous, systems.

Note that enumerating information in pixels is at odds with the holographic principle, which posits that the amount of information in a volume is proportional to its surface area, not volume (for a review, see Bousso 2002). This distinction is crucial for black holes. If Planck volumes are considered to be the fundamental information-carrying units, counting the number of them in a small patch gives an information overestimate, because many available high-energy arrangements of states would give black-hole collapse (Bousso 2002). Here, however, we consider lower energies; a fiducial epoch to measure \( I^\text{prim}_{K} \) is when the CMB was emitted. The \( 10^{-5} \) fluctuations in the modest-density plasma present there are non-relativistic. We also consider galaxy-forming patches that are much smaller than the cosmological horizon. So, we adopt a count by volume instead of surface area.

Power should not exist at arbitrarily scales primordially. By ‘primordial’ here we mean at an epoch after inflation, and after free-streaming thermal motions had finished smoothing out dark-matter fluctuations (e.g. Bond & Szalay 1983), but while fluctuating in the modest-density plasma present there are non-relativistic. We also consider galaxy-forming patches that are much smaller than the cosmological horizon. So, we adopt a count by volume instead of surface area.

\[
I^\text{prim}_{K} = \frac{M_{\text{b}}}{\rho_{\text{cut}}M_{\Omega}} = \frac{M}{1.1 \times 10^{11} M_{\odot}} \frac{\Omega_{\text{DM}}}{0.27} \left( \frac{1 h^{-1} \text{Mpc}}{r_{\text{cut}}} \right)^{3} b, \tag{1}
\]

where \( \rho_{\text{b}} \) is the mean matter density (i.e. the conversion factor between mass and Lagrangian volume occupied in the initial conditions), \( \Omega_{\text{DM}} \) is the fraction of the critical density comprised of dark matter, and \( b \) is the number of bits required to encode each pixel. By Birkhoff’s theorem, distant fluctuations can only affect local motions through the tidal field (e.g. Peebles 1993, p. 94), so it is a reasonable assumption that the information determining its final structure was entirely contained in this Lagrangian patch (plus a negligible set of extra quantities to specify the tidal field). Note that \( I^\text{prim}_{K} \) decreases steeply with \( r_{\text{cut}} \), implying high sensitivity to this parameter if it could be measured.

Complicating the picture is the exact form of a dark-matter streaming cutoff in power, which would not be completely sharp in either Fourier or configuration space. For a neutrino-like WDM particle, B01 obtain a polynomial fit over scales near \( \alpha \).

\[
P_{\text{DM}(k)} = P_{\text{CDM}(k)}[1 + (\alpha k)^{-10}], \tag{2}
\]

where \( \alpha \), a characteristic cutoff scale, is given by

\[
\alpha \approx 0.05 \frac{\Omega_{\text{DM}}}{0.4}^{0.15} \left( \frac{h}{0.65} \right)^{1.3} \left( \frac{\text{keV}}{m_{\text{DM}}} \right)^{1.15} h^{-1} \text{Mpc}. \tag{3}
\]

For CDM, the dark-matter power cutoff is much smaller than Galactic scales, \( \sim 0.6 \) pc for a 100 GeV neutralino, or even \( \sim 0.003 \) pc for an axion (Diemand et al. 2005). For WDM, however, \( r_{\text{cut}} \) may be comparable to Galactic scales. In a fiducial WDM case suggested to ameliorate small-scale problems in CDM (B01), \( \alpha = 0.05 h^{-1} \text{Mpc} \), corresponding to a WDM particle mass of 1 keV. However, recent analyses of Lyman-\( \alpha \) fluctuations and ultra-faint dwarf galaxies to simulations favor a larger dark-matter mass, \( m_{\text{DM}} \gtrsim 4 \text{ keV} \) (Polisensky & Ricotti 2011; Viel et al. 2013). A 4-keV WDM particle gives \( \alpha = 0.01 h^{-1} \text{Mpc} \), using \( \Omega_{\text{DM}} = 0.27, h = 0.7 \), in Eq. (3).

To make progress in quantifying \( I_{K} \), we assume that the cutoff is steep enough to erase structure to zero at the effective sharp cutoff \( r_{\text{cut}} \), which is only an order of magnitude or two smaller than \( \alpha \). Specifying a fiducial precision of \( 10^{-6} \) that corresponds to single precision gives \( r_{\text{cut}} \approx \alpha/50 \) for cutoffs of the form in Eq. (3). See the Appendix for details.

Including dark matter, the Milky Way halo has mass \( M_{\text{MW}} \approx 10^{12} M_{\odot} \) (e.g. McMillan 2011), giving a Lagrangian volume of \( \sim 8 \) \( (h^{-1} \text{Mpc})^{3} \). For a 1-keV particle, setting \( r_{\text{cut}} = 10^{-8} h^{-1} \text{Mpc} \), \( I_{K}^\text{prim} = 8 \) gigapixels, fitting into a cube 2,000 pixels on a side. Using single-precision, 32-bit floating-point numbers, this could be encoded in \( \sim 30 \) gigabytes. \( N_{\text{pixels}} \sim \alpha^{-3} \approx \times \Omega_{\text{DM}} \). For a 4-keV particle, \( r_{\text{cut}} = 2 \times 10^{-4} h^{-1} \text{Mpc} \), and \( I_{K}^\text{prim} = 1,000 \) gigapixels, fitting into a cube 10,000 pixels on a side. This could be encoded at single precision with 4 terabytes. In contrast, in a CDM scenario, with a characteristic cut at \( r_{\text{cut}} \approx 0.012 \) (assuming the same factor of 50 as above, although the cutoff may have a different shape), \( I_{K}^\text{prim} \) enlarges to \( \sim 10^{24} \) pixels, encodable at single-precision in \( \sim 10^{12} \) bytes, or 10 yottabytes.

Fig. 3 shows patches of Gaussian random fields, each with the same Fourier phases, that might collapse to form the Milky Way, in different dark-matter scenarios. According to the above estimate, the smoothest, 1 keV blob is specified with 2000\(^2\) pixels. Visually, there is so little structure that the field seems specifiable with many fewer pixels than that, suggesting that the may be an overestimate of the true information content. Quantifying \( I_{K}^\text{prim} \) unambiguously is a clear necessity if these ideas are used to make precise constraints.

These blobs are slices through a 512\(^3\) Gaussian random field with a power spectrum generated with CAMB (Lewis et al. 2000) using the parameters given above. The blob shown exceeds an average spherical-collapse overdensity \( \delta = 1.09 \). The contours outlining the blob isodensity contours in the field after applying a 1 h\(^{-1}\) Mpc-dispersion Gaussian filter to the CDM field, with the level of the contour set to give a Milky-Way-mass object.

3 DISCUSSION

Above, we argued that if the dark matter is a 1 keV WDM particle, then the primordial patch that formed the Milky Way contained on the order of 8 gigapixels of information; if the dark matter is of mass 4 keV, this increases to \( \sim 1000 \) gigapixels. If the Milky Way can be shown to have more information \( I_{K}^\text{prim} \) in its structure than these, in a way that excludes random information introduced after the primordial fluctuations were imprinted, this could constrain a power cutoff \( r_{\text{cut}} \), sensitive for example to dark-matter warmth.

But what are the prospects for estimating \( I_{K}^\text{prim} \) from the Galaxy’s current structure? A naive estimate would assign some information to each structure that might have some dependence on the initial conditions, such as a star. There are 10\(^{11}-12\) stars in the Galaxy (e.g. Franck et al. 2001), depending on what one counts as a ‘star’. This number far exceeds the \( \sim 10^{5} \) primordial pixel...
count for a 1-keV WDM particle, and is of the order of the $\sim 10^{12}$ pixel count at 4 keV. Under the strong assumption that each star contains one independent unit of information of comparable information content to a primordial pixel of size $r_{\text{pix}}$, i.e. that $b$, the ‘number of bits per pixel’ above, is the same in both cases, and that Galactic structure formation is deterministic from the initial conditions on scales of typical stellar separation, this suggests that the high degree of apparent complexity in the Milky Way makes low-mass WDM less plausible. Here, we assume $\Gamma_{\text{deterministic}} \sim 1$ pc, some typical interstellar scale.

Of course, the assumption that each star has a unit of independent information is very optimistic; in reality, many stars originate in multiple-star systems or even clusters, causing some of their information to be degenerate. On the other hand, an independent star in phase space already carries six numbers, which we have already condensed to one. Also, much information about the assembly of the Galaxy lies in the character of its stellar populations, metallicities, etc. It may be a stretch to claim that primordial information determines these, but it is not inconceivable. And, there is independent information in the arrangement of gas and dark matter, that we have excluded.

What could make the comparison of initial and final information more meaningful and rigorous? Comparing $z = 0$ observables of stellar distributions and populations in WDM versus CDM, using hydrodynamic simulations, would help to identify to what degree each star’s information is degenerate, and to identify where the underlying primordial simplicity in WDM might show up. WDM presents special difficulties in simulations (Wang & White 2007; Angulo et al. 2013). Still, there is an intriguing finding that the morphology of star formation is much different in a WDM scenario (Gao & Theuns 2007): it tends to form in cosmic-web filaments rather than in conventional galaxies. A recent hydrodynamic simulation extended this conclusion to the case of a Milky-Way-type galaxy (Gao et al. 2014). Unsurprisingly, they find that the Milky Way protogalaxy shows very little structure in WDM, but plentiful structure in CDM.

Another line of further investigation would be to clarify the relationship between the sinks that destroy information about the primordial fluctuations (e.g. thermal processes in the gas, and black holes) and physical, thermodynamic entropy.

Along the lines of this complexity argument, WDM could impart some simplicity in the way the stars and gas occupy position–velocity six-dimensional phase space. With WDM, some residual structure from the phase-space ‘catastrophes’ (Arnold et al. 1982; Hidding et al. 2014) which initially formed the Galaxy, or proto-galaxies, may still be present on sufficiently large scales; looking for evidence of these structures may also help to guide the search for underlying simplicity. If such ordered motions are present in the vast dataset of stellar positions and velocities from the recently launched Gaia satellite, this would suggest a simple assembly for the Milky Way, favoring a WDM scenario.

If the dark matter is proven to be warm, e.g. through direct detection, the rich structure in the Galaxy could have only two explanations, both surprising philosophically (to the author).

First, mechanisms in star formation and hydrodynamics could allow fine-scale structure to develop even from smooth initial conditions, in a deterministic way. This would be like an apparently highly complex fractal, generated from a simple algorithm. This possibility can in principle be tested in simulations, by comparing statistics of Galactic structure in a ‘simple’ WDM scenario to those in CDM. The degree of determinism could also be studied with simulations, varying initial conditions slightly and studying the effect in the final state. The number of possible Milky Ways, $\sim 2^L$, would still be incomprehensibly large in the case of several billion pixels, but curiously still finite.

The second possibility is that Galactic structure is dominated by non-primordial randomness, supplied, for instance, in microscopic processes operating during star-formation, or in AGN, which can broadcast small-scale fluctuations to large scales. If the first possibility can be ruled out with simulations, this opens the

Figure 1. Initial-conditions (Lagrangian) patches that might collapse to form the Milky Way, assuming three different dark matter possibilities. The patch, color-coded by overdensity (linearly extrapolated to $z = 0$), is a 2D slice of a high-density 3D blob with about the mass of the Milky Way (see text for details). Pixels have side length 0.01 comoving $h^{-1}$ Mpc, the value of $\alpha$ in a 4 keV model (middle panel). Note that even at 4 keV, there is noticeable smoothing on larger scales than $\alpha$. There is very little structure visible in the 1 keV blob. In the CDM case, the structure in the blob continues to much smaller scales than the pixels.
window to a test of how deterministic galaxy formation is from the initial conditions on that scale.

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APPENDIX: SMOOTH POWER CUTOFFS

One way to specify $r_{\text{cut}}$, from a parameter like $\alpha$ is with a precision to which the initial density field should be known. $r_{\text{cut}}(b)$ can be defined as the critical scale where additional pixel fluctuations from scales smaller than $r_{\text{cut}}$ are unresolved at the precision specified by $b$, the number of bits used to represent each pixel density. The size of these fluctuations can be quantified by $\sigma^2(r)$, the variance in cells of radius $r$; this may be computed from an integral over $P(k)$ multiplied by the square of the spherical top-hat (using spherical cells for simplicity) pixel window function.

The fiducial fractional precision we use for a pixel density, to determine the effective sharp $r_{\text{cut}}$, is $\text{Err}(r_{\text{cut}}) \equiv \sigma(0) - \sigma(r_{\text{cut}})/\sigma(0) \approx 10^{-6}$. Attenuating a CAMB linear power spectrum (generated using the above concordance cosmological parameters) with the cutoff in Eq. (2), we found that $\text{Err}(r_{\text{cut}}) \approx 10^{-6}$ at $r_{\text{cut}} \approx \alpha/50$, for $\alpha \lesssim 1 h^{-1}$ Mpc.

This choice of $10^{-6}$ is admittedly quite arbitrary, but there are some reasons for the choice. It is the precision at which representational discreteness becomes visually obvious in $\text{Err}(r)$, log-log plotting it as a function of $r$, if $\sigma(r)$ is encoded with 32-bit floating-point single precision. $10^{-6} \approx 2$--$2^{-16}$ is 16 times the fundamental precision of the mantissa at single precision, which is encoded with 24 bits. Going all the way down to $2^{-24}$ could seem a more obvious choice at single precision, but we wish to allow resolved modes to be able to contribute more than in the last couple of bits of precision. Another reason we do not assume higher precision is that we do not want the results to depend on precise knowledge of the cutoff shape for many orders of magnitude smaller than $\alpha$.

An undesirable aspect of the definition in Eq. (1) is the high sensitivity of $I_k$ to the sharp $r_{\text{cut}}$, which is strange for a smooth cutoff. One strategy to reduce this effect might be to encode a field of pixels in Fourier space, encoding each Fourier mode with a number of bits that depends on the mode’s contribution to the total variance of the field ($\propto P(k)k^3$). Modes with higher $P(k)$ could be encoded with more bits. This would give a gradual plateau in information as resolution is increased. However, leave this investigation to future work.