Constraining Modified Theories of Gravity with Gravitational-Wave Stochastic Background

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The direct discovery of gravitational waves has finally opened a new observational window on our Universe, suggesting that the population of coalescing binary black holes is larger than previously expected. These sources produce an unresolved background of gravitational waves, potentially observable by ground-based interferometers. In this Letter we investigate how modified theories of gravity, modeled using the parametrized post-Einsteinian formalism, affect the expected signal, and analyze the detectability of the resulting stochastic background by current and future ground-based interferometers. We find the constraints that Advanced LIGO would be able to set on modified theories, showing that they may significantly improve the current bounds obtained from astrophysical observations of binary pulsars.

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I. Introduction.— The LIGO and Virgo Collaborations has recently announced the first direct detection of gravitational waves (GW) [1]. The loudness of the GW 150914 event, with an unexpected high signal-to-noise ratio (SNR), has allowed us to associate GW 150914 to the coalescence of a binary black hole (BBH) system with (source-frame) masses $36^{+5}_{-3} M_\odot$ and $29^{+4}_{-3} M_\odot$ at a luminosity distance of $\sim 400$ Mpc. This binary detection implies BBH masses and coalescence rates higher than previous theoretical predictions [2, 3], and in agreement with recent estimates, obtained with a population synthesis approach, which predicts the formation of a detectable BBH in the early Universe, and in low metallicity environments [4, 5].

As a consequence, also the stochastic gravitational wave background (GWB) produced by these coalescing cosmological BBH sources should be at the higher end of previous estimates [6–11], and could be potentially detectable by advanced detectors [12].

In this Letter we explore for the first time the ability of terrestrial interferometers to constrain the fundamental parameters of modified theories of gravity, through the detection of the GWB generated by the coalescing BBH. To this aim, we compare the fiducial GWB computed assuming general relativity (GR) [12] with the signal produced by modified theories. We model deviations from GR using the parametrized post-Einsteinian (PPE) formalism [13], which has been developed to capture GR modifications in the GW data. More specifically, similar to the post-Newtonian (PN) formalism, which is a low-velocity and weak-field expansion of the metric and matter variables [14], the PPE approach traces back model independent deviations from GR directly into GW templates. The relevance of such corrections in the gravitational emission of compact binaries has been deeply investigated in the literature [15–21], showing that GW parameter estimation can be strongly affected by GR deviations, if they are not properly taken into account [22].

Astrophysical constraints on the lowest order PPE coefficients have been set using observations of relativistic binary pulsars [23]. We refer the reader to the seminal manuscript [13], and to the review papers [24, 25] (and references therein), for an exhaustive description of this topic.

Following the PPE approach, in this work we do not focus on any specific theory of gravity. Conversely, we carry out a completely agnostic analysis. We investigate the regions of the PPE parameter space that are more likely to contribute to the GWB, producing significant deviations from GR. We consider both second and third generation detectors, analyzing their ability to detect the modified signal and to extract its physical properties. Finally, we show how terrestrial GW interferometers can improve the bounds on the PPE coefficients set by binary pulsar observations [23].

II. The stochastic background.— The normalized GWB spectral energy density is defined as

$$\Omega_{\text{GW}}(f) = \frac{1}{\rho_c} \frac{d\rho_{\text{GW}}}{d \ln f},$$

where $\rho_{\text{GW}}$ is the GW energy density and $\rho_c = \frac{3H_0^2}{8\pi G}$ is the critical energy density required to close the Universe. We assume $H_0 = 70$ km/s/Mpc, $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$. The function $\Omega_{\text{GW}}$ can also be written as

$$\Omega_{\text{GW}}(f) = \frac{f}{H_0 \rho_c} \int \frac{dE_{\text{GW}}}{df} [f(1+z)]R_{\text{coal}}(z) \left(\frac{1+z}{E(z)}\right) dz,$$

where $\frac{dE_{\text{GW}}}{df}$ is the rest-frame GW spectrum emitted by a
single source, $E(z) = \sqrt{\Omega_m (1+z)^3 + \Omega_\Lambda}$ for a flat Universe, and $\mathcal{R}_{\text{coal}}(z)$ is the BBH observed event rate per comoving volume. Following [12], we assume as the fiducial model $\mathcal{R}_{\text{coal}}(z)$ proportional to the cosmic star formation rate [26, 27], weighted by the fraction of stars with metallicity $Z < 0.5Z_\odot$ (see APPEnidx B in [28]).

III. The waveform model. — In the PPE approach the gravitational waveform in the frequency domain is modified both in amplitude and phase with respect to the PN waveforms,

$$h(f) = h_{\text{GR}}(f)(1 + \alpha u^3)e^{i6\beta f},$$

where $u = (\pi M f)^{1/3}$, $M$ is the chirp mass of the system, and $(\alpha, \beta, \delta, \zeta)$ are PPE parameters. $\beta$ and $\zeta$ define the type of modification introduced in the theory, while $\alpha$ and $\delta$ control the magnitude of the deviation, and have to be constrained by data. (In [15] Yunes and collaborators have shown the equivalence between the framework used by the LIGO Collaboration to test GR in [29], and the PPE formalism adopted in our paper. As pointed out in [15], the waveforms employed by the LIGO Collaboration represent a subset of the PPE models, since they allow only positive GR modifications in the phase.)

In this Letter, we focus on the information which can be extracted from the wave amplitude, therefore on $\alpha$ and $\beta$. $h_{\text{GR}}(f)$ is the phenomenological waveform described in [30], which combines the PN approximation with numerical relativity results to describe the whole binary coalescence. Moreover, we only consider the inspiral part of the signal, deferring to a forthcoming paper a detailed analysis of the impact of GR modifications on the merger and ringdown phases [31]. For $\alpha = \delta = 0$ we recover the standard PN waveform which, at lowest order, has amplitude

$$\mathcal{A}(f) = \sqrt{\frac{5}{24}} \frac{\mathcal{M}^{5/6}}{\pi^{2/3} d} f^{-7/6},$$

where $d$ is the source distance. In our analysis we truncate the template at the merger frequency in GR, which can be parametrized in terms of the mass components of the binary [30]. The waveform (3) enters quadratically into the GWB through the GW single source spectrum $dE_{\text{GW}}/df$, i.e., $\Omega_{\text{GW}}(f) \propto h_{\text{GR}}(f)^2(1 + \alpha u^3)^2$.

As noted in [13], Eq. (3) does not describe the most general modified waveform, and can be thought of as a single-parameter deformation of GR. Although multiple PPE coefficients may enter into the gravitational signals of a certain theory of gravity, the templates we consider parametrize the effects that are more relevant in the interferometer’s bandwidth. It should be mentioned that a feature of this approach is that the map between the PPE parameters and a specific theory is not unique; thus, there could be more than one model yielding the same result [15]. However, the detection of these coefficients would provide precious information on the theory of gravity. For example, a measurement of $\alpha \neq 0$ for $\beta = 1$ would identify a parity violation, while for $\beta = -8$ it would be a hint of anomalous acceleration, or violation of position invariance [15, 24].

In this Letter we do not choose any particular modified theory of gravity. Rather, being completely agnostic on the real nature of gravity, we explore the PPE parameter space to study how the modified waveform affects the GWB produced by the BBH. However, we assume the GR corrections in Eq. 3 as perturbative terms, and accordingly, in our analysis we consistently consider values of $\alpha$ and $\beta$ that satisfy the bound $|\alpha u^3| < 1$.

The PPE parameters have already been constrained by astrophysical observations. Using the data of double binary pulsars, strong bounds have been set on the amplitude $\alpha$ as a function of the specific considered theory, identified by the exponent $\beta$ [23]. It is shown that negative values of $\beta$ yield very tight constraints on $\alpha$. For gravity theories with $\beta < -2$, which gives $-1$ PN corrections in the amplitude, these observations imply $|\alpha| \lesssim 10^{-9}$. In general, theories with $\beta < 0$ predict corrections to GR that affect the low frequency regime and therefore, are well constrained by electromagnetic observations of binary systems far from coalescence, or by future GW space interferometers [32]. In this Letter we will focus our analysis on modified waveforms with $\beta > 0$. In these models the gravitational waveforms exhibit corrections at higher frequencies, and therefore are ideal candidates to be tested in the near future by ground-based GW interferometers.

To quantify the differences between the GR and the modified background, we introduce the optimized SNR for a given integration time $T$ [6]:

$$\text{SNR} = \frac{3H_0^2}{\sqrt{50\pi^2}} \sqrt{T} \int_0^\infty df \left[ \frac{\gamma^2(f) \Omega_{\text{GW}}(f)}{f^6 S_1(f) S_2(f)} \right]^{1/2},$$

where $S_1(f), S_2(f)$ are the power spectral noise densities of two detectors, and $\gamma(f)$ is the normalized overlap reduction function. We have computed the SNR for Advanced LIGO (AdLIGO) and the Einstein Telescope (ET), assuming the ZERO DET high P anticipated sensitivity for both Livingston and Hanford sites [33], and the ET-B configuration for the [34]. For the latter, $\gamma(f)$ is assumed to be constant, i.e., $\gamma = -3/8$, while for AdLIGO the overlap function is given by the numerical results described in [35].

IV. Results. — In Fig. 1 we show the GWB spectra for modified theories with exponent $\beta = (2, 1)$ and different values of the parameter $\alpha$, compared to the fiducial GR case. (We note that the fiducial model considered in this work differs from the GWB of [12], which is computed also taking into account the merger and the ringdown phases.) and assuming a mean chirp mass of $M = 28M_\odot$.

The power-law integrated sensitivity curve for AdLIGO with an integration time of 1 year is also shown.

The net effect of positive (negative) values of $\alpha$ is to increase (decrease) the spectral energy density of the back-
ground, which, for certain values, is significantly different from that predicted by GR. As an example, a gravity theory with \( \beta = 2 \), which yields a 1 PN correction to the amplitude of the waveform, and \( \alpha \sim 6 \), would produce a background three times larger than the fiducial. A similar behaviour is shown for GWBs with \( \beta = 1 \). As expected by the PN character of the PPE approach, for a fixed \( \alpha \), smaller values of the exponent \( \beta \) yields larger deviations.

When \( \alpha < 0 \) the amplitude of the GWB decreases, limiting the possibility to detect these backgrounds with advanced detectors. However, they are potentially observable by a third generation of ground based interferometers. The left panel of Fig. 2 shows the GWB for some values of \( \alpha < 0 \) and \( \beta = (2, 1.5, 1) \), compared to the power-law integrated sensitivity curve of ET, assuming 1 year of observation.

In the right panel of Fig. 2 we show how the SNR changes as a function of the integration time, for AdLIGO and different PPE models. For some of the considered configurations the SNR increases to a factor \( \gtrsim 10 \) after 24 months. The fiducial GR background would require \( \sim 30 \) years to reach the same value.

In Fig. 3 we extend our analysis, showing the contour lines for detection thresholds SNR= \((3, 5, 8)\), computed for AdLIGO with 1 and 3 years of integration, for theories with \( \alpha > 0 \) and \( \beta \in [0.2, 2] \). The long-dashed curve identifies the region where the parameters \( \alpha \) and \( \beta \) are constrained by binary pulsar observations, as computed in [23]: the allowed region is on the right side of this curve. The shaded region defines the range where \( \alpha \) and \( \beta \) satisfy the condition \(|\alpha \beta| < 1\). After 1 year of integration, AdLIGO would be able to identify GWBs with SNR = 5, produced by modified theories with \( \beta \gtrsim 0.9 \) and values of \( \alpha \) lying on the red dashed curve. Three years of observation would be needed to detect the same signals with \( 8 \lesssim \text{SNR} \lesssim 10 \).

In Table I we show the SNR computed for the advanced and third generation interferometers, for different integration times, for the GWB computed using the PPE waveforms with \( \beta = 2 \). Large SNRs are expected for ET. (We note that such SNRs may be biased since Eq. (5) is defined in the small signal approximation, whereas ET should be able to detect these backgrounds directly.) but for some values of \( \alpha \) and \( \beta \) the GWB could be potentially detectable also by AdLIGO. The analysis presented above shows that a region of the PPE parameter space does exist, where the spectral energy density \( \Omega_{GW}(f) \) of the GWB produced by binary black hole coalescence could be detected by AdLIGO. To further clarify this point, we assess the ability of current interferometers to distinguish these GWB from the GR counterpart, and to extract physical information. We follow the strategy adopted in [28], where it has recently been pointed out that second generation detectors may not be able to distinguish between a BBH GWB and a generic power-law background. This would strongly affect our ability to...
extract information on the background shape. Here, we apply a model selection procedure to determine whether the modified GWB can be discerned by one computed in GR, or assuming a power-law behavior. To this aim we compare the likelihood functions between two models $\Omega_{1,2}(f)$: $L(\Omega_1, \Omega_2) \propto \exp \left[-\frac{1}{2}(\Omega_1 - \Omega_2 |\Omega_1 - \Omega_2|)\right]$, where $(A|B) = 2T \left(\frac{3H_0^2}{10^9}\right)^2 \int_{\text{min}}^{\text{max}} df \gamma^2(f) \frac{A(f)B(f)}{P(f)S(f)}$.

Then, we compute their likelihood ratios, 

$$R_{\text{PPE}} = \frac{L(\Omega_{\text{GR}}|\Omega_{\text{GR}})}{L(\Omega_{\text{GR}}|\Omega_{\text{PPE}})} , \quad R_{\text{PL}} = \frac{L(\Omega_{\text{PPE}}|\Omega_{\text{PPE}})}{L(\Omega_{\text{PPE}}|\Omega_{\text{PL}})} ,$$

(6)

where $\Omega_{\text{PPE}} = \Omega_{\text{PPE}}(\alpha, \beta)$ and as usual $\Omega_{\text{GR}} = \Omega_{\text{PPE}}(\alpha = 0)$, while the power-law density is given by $\Omega_{\text{PL}} = \Omega_0 (f/f_0)^{2/3}$, with $f_0$ being the arbitrary reference frequency and $\Omega_0$ the amplitude that can be computed analytically [28]. If the likelihood ratio approaches 1, the two GWBs cannot be distinguished, while large values of $R$ identify a preferred model. In particular $R_{\text{PPE}} > 1$ suggests that $\Omega_{\text{PPE}} \neq \Omega_{\text{GR}}$, and $R_{\text{PL}} > 1$ reveals that the detected GWB differs significantly from a power-law energy spectrum. To assess the full detectability of the features of the GW signal, both ratios in Eq. (6) must be greater than 1. The top panels of Fig. 4 show the values of $\alpha$ and $\beta$ for which $R_{\text{PPE}} = 1$ and $R_{\text{PL}} = 1$, with 1 and 3 years of integration with AdLIGO, compared against the binary pulsar constraints (long-dashed curve).

We note that for $\beta \gtrsim 1$ all configurations in the allowed region (shaded zone on the right of the long-dashed curve) lead to $R_{\text{PPE}} > 1$, and then can potentially be distinguished from the fiducial model, with a cumulative SNR $\lesssim 4$. Three years of observation would improve this picture, allowing us to discern among gravity theories with $R_{\text{PPE}} >> 1$ and SNR $\gtrsim 5$. However, larger values of the PPE amplitude, outside the permitted parameter space, are needed to satisfy the condition $R_{\text{PL}} \gg 1$. For example, a gravity theory with $\beta = 1.5$ requires $\alpha \gtrsim 12$.

Third generation detectors, would be able to fully extract the physical information of the GWB in the allowed parameter space, and constrain the change in slope in Figs. 1-2 due to the PPE correction, which is frequency dependent. In the bottom panel of Fig. 4 we show $R_{\text{PPE}}$ and $R_{\text{PL}}$ for two PPE theories with $\alpha = 0.1$ and $\beta = (1.5, 2)$, as a function of the integration time, for ET. For both theories we find high values of both likelihood ratios, even after 6 months of observation.

It is interesting to note that even a null detection of the GW signal would provide information on the allowed space for the PPE parameters. We propose here a simple strategy to exploit this feature. As a rule of thumb we can assume that the GWB is potentially observable if the SNR is greater than a defined threshold SNR_T. Looking at Fig. 3, after 1 year of integration and assuming SNR_T = 3, if no GWB is detected, we could exclude the parameter space outside the two green dot-dashed curves. A SNR threshold of 5 rules out the values of $\alpha$ and $\beta$ outside the red short-dashed curves. Two or more years of observation would provide additional restrictions on the PPE coefficients. This simple strategy would constrain the PPE amplitude $\alpha$ to values $O(10)$, with a large impact on models with $\beta \gtrsim 1$, where current bounds are quite loose. In fact, for a gravity theory with $\beta = 2$, binary pulsar observations can only constrain $\alpha$ to be $|\alpha| \lesssim 2000$. Our approach would improve this bound by 2 orders of magnitude.

**IV. Conclusions.**— In this paper we have analyzed how GR modifications affect the GWB produced by the coalescence of BBH systems, showing that the parameter space available for modified theories may yield an enhancement of the background energy density.

As pointed out in [12] the fiducial GWB has an uncertainty band which depends on different assumptions on

| $\beta = 2$ AdLIGO | AdLIGO | ET |
|-----------------|-------|----|
| $\alpha$       | (1 yr) | (3 yrs) | (1 yr) |
| GR             | 1.39  | 1.96  | 223   |
| 1              | 2.31  | 4.00  | 656   |
| 4              | 3.54  | 6.13  | 1060  |
| 6.5            | 4.78  | 8.28  | 1470  |
| -1             | 1.64  | 2.85  | 446   |
| -4             | 0.887 | 1.54  | 216   |
| -6.5           | 0.47  | 0.814 | 102   |

**TABLE I:** SNR of AdLIGO and ET computed for different integration times, $\beta = 2$ and different values of $\alpha$. 

FIG. 4: (Top) Values of $\alpha$ and $\beta$ yielding $R_{\text{PL}} = 1$ and $R_{\text{PPE}} = 1$, computed for AdLIGO with 1 and 3 years of integration. The long-dashed line and the shaded region correspond to the pulsar constraints and the parameter space where $|\alpha u^\beta| < 1$, respectively. The green curves represent the contour line for the GWB detectable with SNR =3 and 5. (Bottom) Likelihood ratios $R_{\text{PPE}}$ and $R_{\text{PL}}$, computed for ET as a function of the integration time, for PPE models with $\alpha = 0.1$ and $\beta = (2, 1.5)$. 
the formation and evolution of the binary progenitors. Alternative theories introduce another source of degeneracy. However, for a given astrophysical scenario, every GWB computed in GR has a modified PPE counterpart that, for $\alpha > 0$, is larger in amplitude and different in slope. These features imply that a GWB detection will still be able to constrain the PPE parameters in large regions of the parameter space. In addition, it should be mentioned that in the future more detectors, Virgo [36], KAGRA [37] and LIGO-India [38], will become operational; the sensitivity of the network of these detectors will significantly increase with respect to that of AdLIGO alone, making accessible further regions of the PPE parameter space.

A detailed investigation of the regions of $\alpha$ and $\beta$ where the GWB is distinguishable from the GR background requires a more sophisticated statistical analysis, like the one presented in [39]. We plan to include this analysis in a forthcoming extended publication [31], in which we will also consider the effect of a network of detectors. Finally, we remark that a comprehensive study of how the merger and the ringdown may affect the GWB in GR was carried out in [7, 10], pointing out that only ET would be able to identify the contribution of these two phases. It would be interesting to reexamine these results for AdLIGO, as far as modified theories of gravity are considered.

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