Research on Position Servo System Based on Fractional-Order Extended State Observer

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ABSTRACT

The control quality of servo system requires not only stability, speed, and accuracy, but also strong anti-jamming ability. The permanent magnet synchronous motor (PMSM) is a multivariable and strongly coupled time-varying nonlinear system. It will be subject to different degrees of external interference. According to the advantages and limitations of fractional-order PID (FOPID) controller and active disturbance rejection control (ADRC), the fractional-order calculus concept is integrated into ADRC. In this paper, a control strategy of position servo system based on fractional-order extended state observer (FOESO) is proposed. The FOESO and fractional-order control law are designed to form a fractional-order active disturbance rejection control (FOADRC). The FOADRC controller is applied to the position loop control of the servo system. Through the experiment and performance analysis, it is proved that the FOESO can well observe the internal and external disturbances of the system, and the FOADRC has a good control effect in the position servo system.

INDEX TERMS

Permanent magnet synchronous motor (PMSM), fractional-order extended state observer (FOESO), fractional-order active disturbance rejection control (FOADRC), fractional-order calculus, position servo system.

I. INTRODUCTION

Permanent magnet synchronous motor (PMSM) uses permanent magnet to replace the excitation winding of wound synchronous motor. It is widely used in the actuator of modern AC servo system for its advantages of simple structure, high power density, high precision, and fast response speed. PMSM is a time-varying nonlinear system with multivariable and strongly coupled. And it will be subject to different degrees of external interference, including motor parameter changes, load disturbances, and other uncertain factors. Therefore, the control technology plays an important role in the comprehensive performance of the whole servo system. The efficient motion controller and effective control strategy are combined to improve the performance of the PMSM servo system.

In reference [1], a new control method based on disturbance observer is proposed to solve the problem of low-speed crawling of the AC servo system when the speed is lower than a certain critical value. In reference [2] a discrete-time of robust composite nonlinear controller to achieve fast and accurate set-point tracking for motor servo systems subject to actuator saturation and disturbances is designed. The idea is to use a combination of linear and nonlinear control, together with a disturbance rejection mechanism based on extended state observer. In reference [3] in order to improve the dynamic and static quality of the control system and reduce the chattering of the control signal, a fuzzy adaptive sliding mode controller is designed and applied to the rocket launcher position servo system by combining fuzzy control, adaptive control and sliding mode control.

The fractional-order PI$^\mu$D$^\beta$ controller has obvious advantages in response speed and application range compared with the integer-order PID controller [4]–[7]. The fractional integral means that the slope of the system integral amplitude-frequency characteristic response curve is not fixed at $-20 \text{db/dec}$. Improving the integer-order integral leads to the slow response speed of the system. The integral saturation causes the control quantity saturation and other negative effects. Fractional differential control has a
good inhibition to high-frequency noise. When the error change rate changes, the system response is not easy to have a sudden change. In reference [8], a fractional-order proportional derivative controller is designed in a systematic way based on the frequency, responses data model for the hard-disk-driver servo system. And the presented fractional-order proportional derivative controller is found to be efficient in improving the tracking-following control performance for the servo system. In reference [9], fractional-order PID torque control is applied to edge-driven coupled light rail vehicles. Through the comparison of models and experiments, it is found that the corresponding performance of the system under fractional PID control is better.

Active disturbance rejection control (ADRC) is proposed by Han Jingqing on the basis of PID controller. The extended state observer (ESO) is used to observe and compensate the internal and external disturbances of the system in real-time [10], [11]. Reference [12] summarizes the technology and engineering application of ADRC, and systematically expounds the research progress of ADRC theory. The ADRC proposed by researcher Han Jingqing is a nonlinear control, which has many problems, such as many parameters to be adjusted, complex adjustment and large calculation. According to this problem, based on the nonlinear active disturbance rejection control professor Gao Zhiquiang proposed the linear active disturbance rejection control (LADRC). In the LADRC the extended state observer is linearized and the parameters to be adjusted are related to the bandwidth of the observer, and replace the error feedback control law with PID control [13]–[15]. In reference [16], an active disturbance rejection adaptive control scheme via full state feedback is proposed for motion control of hydraulic servo systems. This controller is derived by effectively integrating adaptive control with ESO via backstepping method. And the experimental results verify that the control strategy has high tracking performance. In reference [17] an adaptive LADRC is proposed to achieve strong antidisturbance performance and reduce noise sensitivity for electromechanical actuators. And in order to improve the antidisturbance performance, a linear full-order ESO is integrated with the parallel controller.

The characteristics of the ADRC algorithm that does not depend on the model information of the controlled object. It provides a unique solution for fractional-order object control. However, many problems such as high control bandwidth, short sampling interval, and large observation error need to be solved properly. In this paper, the idea of fractional calculus is introduced to ADRC, and the concept of the fractional-order active disturbance rejection control (FOADRC) is proposed. Compared with the traditional ADRC algorithm, the FOADRC controller introducing the fractional-order calculus operator according to the model characteristics of the controlled object, especially the highest order information. The FOADRC controller redesigns the ESO and replaces the error feedback control law with fractional-order control law. Although the traditional ESO can observe the internal and external disturbances of the system, it takes the fractional-order part of the control system and the position disturbance of the system as the total disturbance. This causes the observation bandwidth is too large. And when the total disturbance observation error is large, it is accompanied by the oscillation phenomenon. The improved fractional-order extended state observer (FOESO) can accurately observe the state and extended state of the fractional-order object with a lower observation bandwidth. And the robustness of the FOESO to the parameter change of the controlled object is also improved. In the position feedback error control rate, the fractional-order controller is used instead of the integer-order controller, which means that the adjustable range of the feedback control law is larger and the control signal is more efficient [18].

II. PMSM MATHEMATICAL MODEL

The mathematical model of the control object can accurately reflect the dynamic and static characteristics of the controlled system. The object of this paper is the surface-mounted permanent magnet synchronous motor. In order to achieve the maximum torque output, the \( i_d = 0 \) control method is adopted. According to the characteristics of PMSM, the mathematical model of PMSM in the d-q rotation coordinate system is [19], [20].

\[
\begin{bmatrix}
  i_d \\
  i_q \\
  \omega
\end{bmatrix} =
\begin{bmatrix}
  R_s & n_p \psi_f & 0 \\
  \frac{L_d}{J} & \frac{1.5n_p\psi_f}{J} & -B \\
  0 & \frac{L_d}{J} & -\frac{T_L}{J}
\end{bmatrix}
\begin{bmatrix}
  i_d \\
  i_q \\
  \omega
\end{bmatrix} +
\begin{bmatrix}
  u_d \\
  u_q \\
  0
\end{bmatrix}
\]

(1)

\[
\begin{bmatrix}
  u_d \\
  u_q \\
  0
\end{bmatrix} =
\begin{bmatrix}
  R_s + \frac{dL_d}{dt} - wL_q \\
  0 \\
  0
\end{bmatrix}
\begin{bmatrix}
  \frac{dL_d}{dt} \\
  \frac{dL_q}{dt}
\end{bmatrix} +
\begin{bmatrix}
  0 \\
  0 \\
  \psi_f
\end{bmatrix}
\]

(2)

In equation (1), \( u_d \) and \( u_q \) are the stator voltage component in the d-q coordinate system. The \( i_d \) and \( i_q \) are the stator current component in the d-q coordinate system. The \( L_d \) and \( L_q \) are the stator inductance component in the d-q coordinate system. The \( n_p \) is the number of motor pole pairs. The \( R_s \) is the stator resistance. The \( w \) is the angular frequency of the rotor. The \( J \) is the rotational inertia of the servo motor. The \( \psi_f \) is the rotor flux corresponding to the permanent magnet.

For the surface-mount PMSM, the electromagnetic torque equation in d-q coordinate system is:

\[ T_e = 1.5n_p\psi_f i_q \]

(3)

The equation of mechanical motion is [21]:

\[
\begin{align}
  w_m &= \frac{d\theta}{dt} \\
  \frac{d^2w_m}{dt^2} &= T_e - T_L - Bw_m
\end{align}
\]

(4)

where, \( T_L \) is the load torque. \( B \) is the viscous friction coefficient, and \( w_m \) is the mechanical angular velocity.
The vector control mode with \( i_d = 0 \) can be obtained from (4):

\[
\begin{}
\theta &= \frac{1}{J} (T_e - T_L - Bw_m) \\
\dot{\theta} &= \frac{1}{J} (1.5nP\Psi_i i_q - T_L - Bw_m) \\
\ddot{\theta} &= \frac{1}{J} (1.5nP\Psi_t (i_q^* - i_q) + T_L + Bw_m) \\
\dot{y} &= bu + a(t)
\end{align}
\]

where, \( b = \frac{1.5nP\Psi_t}{J} \); \( a(t) = -\frac{1}{J} [1.5nP\Psi_t (i_q^* - i_q) + T_L + Bw_m] \).

\( \theta \) is the rotor angular displacement, \( b \) is the compensation factor, \( u = i_q^* \) is the given value of the quadrature axis current, \( a(t) \) is the total disturbance, including all the disturbances inside and outside of the system.

Let the rotor angular displacement \( \theta \) of PMSM be the state variable \( x_1 \), the mechanical angular velocity \( w_m \) as the state variable \( x_2 \), and the total disturbance \( a(t) \) as a new state variable \( x_3 \). Then the expression is:

\[
\begin{align}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= bu + x_3 \\
\dot{x}_3 &= a(t) \\
y &= x_1
\end{align}
\]

III. FRACTIONAL-ORDER ADRC (FOADRC)

By integrating the ADRC technology with the fractional-order theory, the fractional-order control concept is integrated into the ESO to observe the internal and external disturbances of the system. The nonlinear state feedback error control law can be replaced by fractional-order PID controller [22]. Then, the controller and the control object have fast response speed, good robust performance, high control accuracy, and wide parameter adjustment range and so on [23], [24].

The traditional ADRC includes a tracking differentiator (TD), whose function is to arrange the ideal transition process and give the differential signal of the transition process. In order to prevent the step response from causing serious overshoot, the control amount in the initial stage is slowly increased. But it causes that the rise time is extended and the system response is slow.

According to the structural characteristics and fast control requirements of the controlled object, the TD is removed from the FOADRC controller. And the main purpose is to activate the object with the aid of the large error control signal at the beginning, so that it can realize the fast-tracking of the controlled system. Then, FOADRC designed in this paper includes fractional-order extended state observer and fractional-order control law. The structure diagram of fractional-order ADRC system is shown in Fig. 1.

The application of fractional calculus in ADRC is as follows:

\[
y(2\alpha) + a_2 y(\alpha) + a_1 y = w + bu
\]

where, \( u \) represents the input and \( y \) represents the output of the system respectively. \( \alpha \) \((0 < \alpha < 1)\) represents the fractional differential order of the system. The \( w \) is the external disturbance of the system. The \( b \) is the system gain parameter. The \( a_1, a_2 \) are the parameters of the system.

By simple linear transformation, the fractional-order system is regarded as the following integer order second-order system [25].

\[
y = \left[ \ddot{y} - (y(2\alpha) + a_2 y(\alpha) + a_1 y) + w \right] + bu
\]

where, \( f(\cdot) = \left[ \ddot{y} - (y(2\alpha) + a_2 y(\alpha) + a_1 y) + w \right] \) represents the total disturbance of the system. All fractional-order parts are regarded as the system disturbance, which is observed and compensated by the extended state observer.

Because the system is a second-order system, a third-order fractional-order ESO is designed. The system uses the third-order FOESO to accurately observe the system state and internal and external disturbances and compensates the error by fractional control rate. The advantages of the fractional-order controller, such as large parameter adjustment range and strong robustness, are combined with the characteristics that the ADRC can observe the internal and external disturbances of the system to realize the fractional-order ADRC.

A. FRACTIONAL-ORDER EXTENDED STATE OBSERVER (FOESO)

ESO is the core part of ADRC, which realizes the observation of internal and external disturbances. According to the idea of state observer, the disturbance that affects the output of the system is expanded into a new state variable. The process does not depend on the model of generating disturbance, and the disturbance of the system can be observed without direct measurement.

Transform (7) as follow:

\[
y(2\alpha) = -a_2 y(\alpha) - a_1 y + w + bu = g(y^{(\alpha)}, y, w) + bu
\]

Taking the second-order position loop as the control object, it will be expanded into the following form of state space.

\[
\begin{align}
x^{(\alpha)} &= Ax + Bu + Eh(t) \\
y &= Cx
\end{align}
\]
The total disturbance is \( g(\cdot) = -a_2 y^{(\alpha)} - a_1 y + w \). It includes the internal and external disturbances of the system and expands them to the state variables of the system: \( x_3^{(\alpha)} = g(\cdot) \).

Where,

\[
\begin{align*}
    x &= \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} : A &= \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} ; B &= \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \\
    E &= \begin{bmatrix} 0 \\ 0 \end{bmatrix} ; C &= [1 \\ 0 \\ 0].
\end{align*}
\]

According to the state-space structure of the object, the fractional-order extended state observer is designed as follows:

\[
\begin{align*}
    e_1 &= z_1 - y \\
    z_1^{(\alpha)} &= z_2 - \beta_1 e_1 \\
    z_2^{(\alpha)} &= z_3 - \beta_2 e_1 + bu \\
    z_3^{(\alpha)} &= -\beta_3 e_1
\end{align*}
\]  

(11)

The outputs \( z_1, z_2, z_3 \) respectively observe the system state \( x_1, x_2, x_3 \). The parameters \( \beta_1, \beta_2, \beta_3 \) are the gain coefficients of the extended state observer. The parameter \( b \) is the system gain parameter. The Fractional-order ESO structure is shown in Fig. 2.

In Fig.2, \( z_1 \) is the observed value of the motor position. The \( z_2 \) is the fractional-order differential of \( z_1 \). The \( z_3 \) is the fractional-order differential of \( z_2 \). And the \( z_3 \) is also the observed system disturbance.

The stability of fractional-order system is analyzed by the root locus method. According to (10), the FOESO is a linear equal proportion fractional-order system. Through the mapping relation: \( w = s^\alpha \), the equal proportion fractional-order of \( s \) plane is mapped to the integral-order system of \( w \) plane, where \( \alpha \) is the equal proportion order of the original system.

\[
G(s) = \frac{b_m w^m + b_{m-1} w^{m-1} + \ldots + b_i w^i + b_0}{a_n w^n + a_{n-1} w^{n-1} + \ldots + a_1 w^1 + a_0}
\]

(12)

According to (12), draw the root locus of the integer-order system with \( w \) as the variable. The commensurate fractional-order system’s stability region in \( w \)-plane is shown in Fig.3. The stability of the original fractional-order system can be judged by the relationship between the trajectory and the stable region of the \( w \) plane. For the fractional-order system of equal proportional order \( \alpha \), the slope of the two lines in the stable region is \( \pm \alpha \pi / 2 \) [26, 27]. If the poles of the mapped integer order system about \( w \) are all in the stable region, the system is stable, otherwise, the system is unstable.

According to (10) and (11), the FOESO error \( e_i = z_i - x_i \) (\( i = 1, 2, 3 \)) is:

\[
\begin{bmatrix}
    e_1 \\
    e_2^{(\alpha)} \\
    e_3^{(\alpha)}
\end{bmatrix}
= \begin{bmatrix}
    -\beta_1 & 1 & 0 \\
    -\beta_2 & 0 & 1 \\
    -\beta_3 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    e_1 \\
    e_2 \\
    e_3
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    -1
\end{bmatrix} h(t)
\]

(13)

Through the mapping condition \( w = s^\alpha \), the fractional-order system is mapped to the integer-order system of \( w \) plane. The stability of the system is analyzed by using the root locus method.

\[
\begin{bmatrix}
    e_1 \\
    e_2 \\
    e_3
\end{bmatrix}
= \begin{bmatrix}
    -\beta_1 & 1 & 0 \\
    -\beta_2 & 0 & 1 \\
    -\beta_3 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    e_1 \\
    e_2 \\
    e_3
\end{bmatrix} + \begin{bmatrix}
    0 \\
    0 \\
    -1
\end{bmatrix} h(t)
\]

(14)

According to the bandwidth parameter method of the integer-order extended state observer, if the error equation converges, the poles need to be set in the stable region of the graph. Since \( 0 < \alpha < 1 \), it can be seen that the root locus of its transfer function is located in the left half-plane of \( s \). That is to say, the stable region includes the whole left half-plane. The integer-order ESO bandwidth tuning method is suitable for the parameter tuning of linear FOESO. Then, the idea of the bandwidth parameter method of the integer-order extended state observer can be used. The poles of above are all assigned to the same multiple roots [28–30]. And the concept of observer bandwidth \( w_o \) is proposed by professor Gao Zhiqiang can be used to describe it [13], [31]. The characteristic polynomial of the extended observer satisfies:

\[
s^{3\alpha} + \beta_1 s^{2\alpha} + \beta_2 s^{\alpha} + \beta_3 = (s^\alpha + w_o^3)
\]

(15)

Thus, the observer gain is parameterized as \( \beta_1 = 3w_o^3 \); \( \beta_2 = 3w_o^2 \); \( \beta_3 = w_o^3 \). It can be seen that the parameters to be adjusted are uniquely related to the bandwidth \( w_o \), which makes the parameter adjustment process of the fractional-order extended state observer simple.
B. FRACTIONAL-ORDER CONTROL LAW

According to the independence of each part of ADRC [32], [33], a fractional-order controller is proposed to replace the state feedback error control law of ADRC. Compared with the traditional PID controller, PI\(^\mu\)D\(^\lambda\) controller has two more adjustable parameters: integral order \(\mu\) and differential order \(\lambda\). Therefore, when the fractional-order controller is applied to the control system, it shows better control performance. The block diagram of fractional-order PI\(^\mu\)D\(^\lambda\) controller is shown in Fig. 4.

Since the disturbance in ADRC has been used to compensate and observe, the fractional-order PD\(^\lambda\) controller is shown in Fig. 4. The transfer function of the fractional-order PI\(^\mu\)D\(^\lambda\) controller is:

\[
G_c(s) = \frac{U(s)}{E(s)} = K_P + K_I s^\mu + K_D s^\lambda
\]  

(16)

The output of the fractional-order controller in the time domain is:

\[
u(t) = K_P e(t) + K_I s^\mu e(t) + K_D s^\lambda e(t)
\]

(17)

where, \(\mu < 0, \lambda > 0\). error \(e(t)\) is the input of fractional-order PI\(^\mu\)D\(^\lambda\) controller. The \(u(t)\) is the output of fractional-order controller.

The fractional-order control law designed in this paper is PD\(^\lambda\) control, and the fractional-order differential approximation is carried out by Oustaloup method [34]–[36]. First, determine the frequency band to be approximated \([w_h, w_l]\), then the rational approximation function can be obtained by cascading rational functions as follows:

\[
G_N(s) = K \prod_{k=-N}^{N} \frac{s + w_k}{s + w_k}
\]

(18)

where, \(w_k = w_h \left(\frac{w_h}{w_k}\right)^{\frac{k-N+1}{2N+1}}\), \(w_k = w_h \left(\frac{w_h}{w_k}\right)^{\frac{k-N+1}{2N+1}}\),

\[
K = \left(\frac{w_h}{w_h}\right)^{-\frac{N}{2}} \prod_{k=-N}^{N} \frac{w_k}{w_k}
\]

The \(\lambda\) is the fractional-order, and the order in (18) is \(n = 2N + 1\). Generally, when \(n = 5\), it is the best approximation order.

The FOADRC designed in this paper will be applied to the position loop of the servo system. According to the performance requirements of the servo system without overshoot, the designed fractional order control law is only fractional order PD control. The structure diagram of fractional-order ADRC is shown in the Fig. 5. Then the final output of the fractional ADRC is \(u = \frac{w_h - w_l}{b}\).

C. PROOF OF CONVERGENCE OF THE FOESO

From the state error (13), the system state error equation can be expressed as:

\[
E^{(\alpha)} = AE + Bh(t)
\]

(19)

where, \(E^{(\alpha)} = \begin{bmatrix} e^{(\alpha)}_1(t) \\ e^{(\alpha)}_2(t) \\ \vdots \\ e^{(\alpha)}_{n+1}(t) \end{bmatrix}, A = \begin{bmatrix} -b_1 & 1 & 0 & \cdots & 0 \\ -b_2 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ -b_{n+1} & 0 & 0 & \cdots & 1 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}, h(t) = g^{(\alpha)}(\cdot)\).

The \(h(t)\) is taken as input and state error as output [31], [37]. Then the error can be expressed as:

\[
E = (s^\alpha I - A)^{-1} Bh
\]

(20)

Then,

\[
(s^\alpha I - A)^{-1} B = \frac{1}{F(s)} \begin{bmatrix} 1 \\ s^\alpha + \beta_1 \\ \vdots \\ s^{(n+1)\alpha} + \beta_n s^{n\alpha} + L + \beta_n \end{bmatrix}
\]

(21)

where, \(F(s)\) is the characteristic polynomial of the state error equation [31]. According to the design principle of bandwidth parameterization:

\[
F(s) = s^{(n+1)\alpha} + \beta_1 s^{n\alpha} + \ldots + \beta_n s^\alpha + \beta_{n+1} = \left(s^\alpha + w_0\right)^{n+1}
\]

(22)
According to the final value theorem:

$$
\begin{bmatrix}
e_1(+) \\
e_2(+) \\
\vdots \\
e_{n+1}(+)
\end{bmatrix}
= \lim_{s \to 0} s \left( s^\alpha I - A \right)^{-1} B h(s) = \begin{bmatrix}
-1 \\
-\beta_1 \\
\vdots \\
-\beta_{n+1}
\end{bmatrix} h(+\infty)$$

When $|h(+\infty)| \leq M$, the error is:

$$|e_1(+\infty)|_{\text{max}} = \frac{C_{n+1}}{w_0^{n+2-1}} |h(+\infty)| \leq \frac{M C_{n+1}}{w_0^{n+2-1}}$$

Therefore, the bandwidth of the fractional-order expanded state observer has a great influence on the tracking error, and the increase of the bandwidth will reduce the observation error. However, the bandwidth of the observer is not unlimited, and the upper limit of the bandwidth is limited by the sampling frequency.

### IV. EXPERIMENTAL VERIFICATION

The fractional-order ADRC proposed in this paper is verified by experiments, and the TMS320F28335 of TI company is used as the control center to design the experimental hardware platform. The hardware structure diagram of the system is shown in Fig. 6.

The permanent magnet synchronous motor selected in this system is NaZhi 130st-M06050. The parameters are shown in Table 1:

| Parameters          | Value   |
|---------------------|---------|
| Power (kw)          | 1.5     |
| Rated Speed (r/min)| 2500    |
| Rated Torque (N·m) | 6       |
| Pole Pairs          | 4       |
| Rotor Inertia (kg·m²)| 1.26×10⁻³|
| Winding Resistance (Ω)| 1.21    |
| Winding Inductance (mH)| 3.87    |

According to the final value theorem:

$$
\begin{bmatrix}
e_1(+) \\
e_2(+) \\
\vdots \\
e_{n+1}(+)
\end{bmatrix}
= \lim_{s \to 0} s \left( s^\alpha I - A \right)^{-1} B h(s) = \begin{bmatrix}
-1 \\
-\beta_1 \\
\vdots \\
-\beta_{n+1}
\end{bmatrix} h(+\infty)$$

When $|h(+\infty)| \leq M$, the error is:

$$|e_1(+\infty)|_{\text{max}} = \frac{C_{n+1}}{w_0^{n+2-1}} |h(+\infty)| \leq \frac{M C_{n+1}}{w_0^{n+2-1}}$$

The FOADRC designed in this paper is applied to the position loop control of the servo system. Both the speed closed-loop control and the current closed-loop control adopt the traditional PI controller. The current vector control adopts the $i_d = 0$ control method to achieve maximum torque output. The motor position and the internal and external disturbances of the system are observed by FOESO, which are feedbacked to form the position closed-loop control of the servo system. The error is compensated by the fractional-order control law (FOPD). The structure diagram of the position servo system is shown in Fig. 8.

Where, APR is automatic position regulator; ASR is automatic speed regulator; ACR is automatic current regulator.

The differential order is determined according to the empirical method and the actual situation of the position servo system. According to (18), the fractional-order differential approximation is carried out by the method of oustaloup. The parameters of FOESO are adjusted according to the bandwidth debugging method proposed by Professor Gao Zhiqiang.

**A. PERFORMANCE VERIFICATION OF FOESO**

According to the FOESO designed in this paper, it is applied to the position loop of the servo system and design the
FIGURE 9. Motor position and observed position waveform.

FIGURE 10. The bandwidth test of PID controller system.

FIGURE 11. The bandwidth test of ADRC controller system.

verification experiment. In the experiment, the position is set to 100 rad. From the actual position and observation position waveforms in Fig. 9, it can be seen that the FOESO can observe the system output, indicating that the FOESO has a high observation accuracy.

B. EXPERIMENTAL TEST OF SYSTEM BANDWIDTH

From Fig. 10 to Fig. 12 are the test results of the bandwidth of PID controller, ADRC, and fractional ADRC respectively. In the experiment, the position is set as a sinusoidal signal with different frequencies.

According to the definition of closed-loop bandwidth in control theory, when the amplitude of the position feedback signal is reduced to 0.707 times the amplitude of the given sine signal or the phase of the position feedback signal lags behind 90 degrees of the given signal, it is considered that the given signal frequency is considered as the cut-off frequency.

The bandwidth test results of the three controllers are shown in Table 2. It can be seen that the position loop bandwidth of FOADRC is the largest, followed by ADRC, and the PID controller is the smallest.

TABLE 2. Experimental results of different controller bandwidth.

| Controller | Bandwidth (HZ) |
|------------|----------------|
| PID        | 6              |
| ADRC       | 7              |
| FOADRC     | 8.6            |

TABLE 3. Adjustment time of different controllers after being disturbed.

| Controller | Adjustment Time(ms) |
|------------|---------------------|
| PID        | 550                 |
| ADRC       | 400                 |
| FOADRC     | 280                 |

C. IN THE EXPERIMENT OF ANTI-JAMMING PERFORMANCE OF DIFFERENT CONTROLLERS

In order to verify the performance of FOADRC controller designed in this paper, the FOADRC controller, ADRC controller, and PID controller are tested for anti-jamming performance. The fractional-order control law of FOADRC adopts PD$^\lambda$ control. The parameters of FOESO are adjusted by according to the bandwidth debugging method put forward by professor Gao Zhiqiang.

In the experiment, the position is set 20 rad for each controller. And in order to ensure the stable operation of the system, the experiment limits the amplitude of each controller at speed of 1200 r/min in the speed loop. After the no-load positioning of the motor, the load of 2 N·m is suddenly added, and then the load is suddenly discharged to the no-load after a period of time. The loading process is realized by the load controller independently developed by our laboratory. The load controller can realize any torque within the allowable range of the load motor. The load torque given by all controllers is the same as the load time applied. From Table 3 and Fig. 13, according to the system response state, from the degree of interference and the adjustment time of the system, it can be seen that the fractional-order ADRC has the strongest anti-jamming ability, followed by the ADRC, and PID has the worst anti-jamming performance in the comparative test.
According to the characteristics of fractional-order controller, such as fast response, wide range of parameter adjustment, and attenuation of fractional order differential to high-frequency noise, combined with the ADRC algorithm, which can eliminate disturbance before the final output of the system is affected by internal and external disturbances. Based on the performance requirements of servo system, this paper analyzes the existing problems in current servo control system, and the method of position servo based on fractional-order ESO is proposed. The stability of the fractional-order ESO is proved in this paper. The experimental verification is carried out by using the self-designed servo driver platform and the load controller platform. The experimental results show that the FOESO has high observation accuracy and can observe the internal and external disturbances of the system. By testing the bandwidth and anti-jamming performance of the integer-order PID controller, the ADRC and the FOADRC, the experimental results show that the FOADRC has higher bandwidth, faster response speed, and better anti-jamming performance. Therefore, under the same experimental conditions, FOADRC has better control effect than integer-order PID and LADRC.

In this paper, the FOADRC is only applied to the position loop control of servo system. The next step is to study the application of FOADRC in the current loop of the servo system. At the same time, the FOADRC is more complex than the traditional PID controller. In order to reduce the computing pressure of the CPU, the dual cpu DSP and FPGA architecture will be used. The high-speed parallel processing ability of FPGA will be used to reduce the computing pressure of DSP and improve the response bandwidth of the system.

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