Real-time optical micro-manipulation using optimized holograms generated on the GPU

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Abstract

Holographic optical tweezers allow the three dimensional, dynamic, multi-point manipulation of micron sized objects using laser light. Exploiting the massive parallel architecture of modern GPUs we can generate highly optimized holograms at video frame rate allowing the precise interactive micro-manipulation of 3D structures.

Keywords: optical trapping, digital holography, GPU computing, CUDA

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1. Introduction

Holographic optical tweezers (HOT) use light to manipulate matter at the micron scale \([1]\). Dielectric objects, whose refractive index is higher than the surrounding medium can be trapped in regions of high light intensity by electromagnetic forces arising from the scattering of light \([2]\). To achieve stable trapping in three dimensions, light has to be strongly focused using a microscope objective with high numerical aperture. Many objects can be trapped simultaneously if more than a single focal spot is generated around the objective’s focal plane. Digital holography provides a way to achieve this by applying a computer generated phase mask to a laser beam before it is sent through the microscope objective. The commercial availability of Spatial Light Modulators (SLM) has made this task easier by providing a reconfigurable support for computer generated holograms which is connected to a PC through the video output (usually DVI) on a standard video card \([3,4,5,6]\). The task of finding a phase mask that efficiently redistributes the available laser power among an array of target focal spots is not a straightforward one. Phase only modulation can easily give rise to unwanted focal spots (“ghost traps”) or large intensity variations. We recently proposed an iterative procedure that achieves optimal efficiency and uniformity in a few tens of steps \([7]\). However the resulting computational load is so high that the use of optimized algorithms for dynamic manipulation is limited to those circumstances when the sequence of moves is known in advance and holograms can be then pre-calculated. Such a slowness is often considered
as one of the major factors for preferring scanning beam techniques \[8\] over
digital holography for real-time applications.

In this paper we demonstrate that CUDA \[9\] enabled GPUs can generate
highly optimized holograms at a frame-rate that is fast enough to allow inter-
active micro-manipulation using strong and uniform trap arrays.

2. GPU device architecture

Graphic Processing Units (GPU) have brought the power of parallel calculus
to personal computers. The possibility of using a personal computer to easily
and cheaply achieve the performances of an expensive CPU cluster is revolu-
tionizing computational physics in a wide range of fields including molecular
dynamics \[10\], Monte Carlo simulations \[11\], finite element analysis \[12\], lattice
QCD \[13\]. The Compute Unified Device Architecture (CUDA) is a general pur-
pose parallel computing architecture introduced by NVIDIA. CUDA provides a
parallel programming model and software environment allowing to exploit the
massive parallel architecture of modern Graphic Processing Units (GPU) for
non-graphics applications. General purpose parallel algorithms can be imple-
mented on a CUDA enabled GPU using a small set of C extensions provided
by the CUDA SDK. The CUDA programming model closely reflects the GPU
hardware architecture. A CUDA enabled GPU is composed of a global memory
and a variable number of multiprocessors. Each multiprocessor includes eight
scalar processor cores, two special function units, 8192 registers, a multithreaded
instruction unit and one on-chip shared memory. As a result hundreds of cores
can collectively run thousands of computing threads that can share data without
sending it over the system memory bus. Threads are arranged in a grid of blocks
and each block is assigned to a multiprocessor. In this way threads that belong
to the same block can be synchronized and can cooperate using shared memory.
Within a block, threads are arranged in groups of 32 called warps, threads in a
warp are physically executed in parallel and are synchronized. Multiprocessors
can only execute one warp at time, however if threads in a warp are waiting
to access global memory the multiprocessor can stop executing that warp and
switch to another one eliminating memory latency time.

Such an execution model requires specific optimization strategies that, for
the purpose of the present work, can be summarized in three general rules:

Rule A) *Keep multiprocessors busy and hide memory latency.*

To this aim one should:

1. Group threads in a number of blocks that is multiple of the number
   of multiprocessors.
2. Choose the number of threads per block as a multiple of 32 to avoid
   wasting time with unfilled warps.
3. Maximize the number of active warps by using many threads per
   block.
4. When possible, avoid using conditional instructions that serialize the execution of a warp.

Rule B) *Minimize read/write operations on global memory.*

Writing and reading global memory is very slow and sometimes it can be even better to recalculate than to cache data. Shared memory must be used whenever it can reduce the access to global memory. Shared memory is hundreds of times faster than global memory but only 16k are currently available to any multiprocessor.

Rule C) *Access global memory with coalesced calls.*

When all threads in a half warp execute a read/write instruction, the hardware detects whether threads access consecutive global memory locations and coalesces all these accesses.

3. Optimized algorithms for holographic trapping

In back focal plane phase modulation we use an SLM to apply an array of phase shifts to a plane wave at the back focal plane of a focusing optical system (Fig. 1). Our task here is to calculate the best phase mask so that the modulated wavefront propagating through the optical system is focused onto an array of chosen target spots. Given the phase shift on each pixel $\phi_j$ the complex

$$z_{\text{SLM}} \quad \text{Fourier plane}$$

$$y(x_m, y_m, z_m)$$

$$x_j, y_j, 0$$

Figure 1: Schematic representation of Fourier optics propagation from SLM plane (back focal plane) to the front focal plane of the optical system.

field on a target point $m$, with coordinates $x_m, y_m, z_m$, is given by [7]:

$$V_m = \frac{1}{N} \sum_{j=1}^{N} e^{i(\phi_j - \Delta^m_j)} \quad (1)$$

Where $N$ is the number of pixels, $i$ is the imaginary unit and $\Delta^m_j$ is the phase acquired upon propagation:
\[ \Delta_j^m = \frac{z_m \pi}{\lambda f^2} (x_j^2 + y_j^2) + \frac{2\pi}{\lambda f} (x_j x_m + y_j y_m) \]  

where \( f \) is the effective focal length of the focusing optics (L3, L4, MO in Fig. 6), \( \lambda \) is the laser wavelength and \( x_j, y_j \) are the coordinates of the \( j \)th pixel. If we want to send all the light through a single point \( m = 1 \) then we should set \( \phi_j = \Delta_1^j \), so that \( V_1 = 1 \). When considering multiple traps, a phase only modulation might not be able to split all the available power uniformly among the target points. For each pixel we now have the multiple choices \( \Delta_j^m \) (the single trap holograms) and finding a compromise could seem a hopeless task. A first, reasonably fast recipe is that of taking the complex superposition of single trap holograms [14]:

\[ \phi_j = \arg \sum_{m=1,M} w_m e^{i(\Delta_j^m + \theta_m)} \]  

Where \( \phi_j \) is again the phase of the \( j \)th SLM’s pixel, \( m \)th is the trap index, \( M \) is the number of traps, \( \theta_m \) is a random phase relative to the \( m \)th trap. Such a procedure, usually referred as the random superposition algorithm (SR), is computationally rather fast but usually results in ghost traps and poor uniformities, especially when dealing with ordered structures. A quantitative measure of the hologram performance can be obtained by defining an efficiency (\( e \)) and a uniformity (\( u \)) parameters as a function of the fractions of total power flowing through the \( m \)th trap \( I_m = |V_m|^2 \):

\[ e = \sum_m I_m, \quad u = 1 - \frac{\max[I_m] - \min[I_m]}{\max[I_m] + \min[I_m]}, \]  

Figure 2: Flowchart representing \( K \) iterations of GSW algorithm.
A poor performance may result in particles getting trapped in unwanted ghost trap sites or bead escape from temporary low intensity traps. When such events are acceptable SR provides a good choice for real time manipulation, but if a higher degree of control is required a more performing algorithm is needed. A good candidate is the GSW algorithm (weighted Gerchberg-Saxton [7]) which gives excellent results in terms of efficiency and uniformity. The basic idea behind GSW is that, if aiming at uniform trap intensities with SR leads to nonuniformities, we may hope that there’s a choice of non uniform target traps’ intensities resulting in an evenly spread trapping light. GSW allows to calculate such non uniform weights \( w_m \) by the iterative procedure illustrated in the flowchart reported in Fig. 2. Angle brackets in the update weights box of Fig. 2 represent averaging over the trap index \( m \). After a few tens of iterations the procedure converges to almost perfectly uniform trap intensities so that \(|V_m| \approx \langle |V_m| \rangle\) and the weights \( w_m \) don’t get updated anymore.

4. Implementing HOT algorithms on a CUDA device

![Figure 3: Schematic diagram of the kernel BackPropKer calculating the phase shifts on the SLM by a backward propagation of light emerging from the target spots.](image)

The parallel architecture of GPUs is particularly suited for digital holography, whose basic task is that of performing complex algebra over a large array of independent pixels. In the field of digital holography GPUs have been used for real-time holographic microscopy [15, 16] or holographic displays [17, 18]. In the field of optical trapping, the possibility of generating holograms with real-time frame rate is very attractive for interactive applications. Early attempts always suffered the slowness of CPU resulting in either slow or low efficiency holograms.
More recently, custom shading programs running on the GPU have been used to achieve a considerable speedup in hologram generation, although always being limited to quick and poorly performing algorithms [21, 22]. The CUDA architecture makes it a lot easier to implement more complex algorithms in a general purpose environment which is not limited to graphic applications. When using a CUDA enabled video card, results can be also computed directly on the frame buffer avoiding useless memory transfers.

| Rule A | Rule B | Rule C | t/trap (ms) | Speedup |
|--------|--------|--------|-------------|---------|
| Yes    | No     | No     | 1.22        | 100     |
| Yes    | Yes    | No     | 0.42        | 290     |
| No     | Yes    | Yes    | 0.47        | 260     |
| Yes    | Yes    | Yes    | 0.35        | 350     |

1 Yes: blocksize=32 × 12 = 384, No: blocksize=16
2 Yes: threads in a block access SLM pixels as a linear array
   No: threads in a block operate on square submatrices of SLM

Table 1: Comparison table summarizing time costs of SR algorithm and the relative importance of optimization rules. The GPU is a GeForce GTX 260 and speedup data are evaluated with respect to a Pentium D 3.2 GHz.

Both of the previously discussed algorithms require the common step of backward light propagation from the M target traps back to the N SLM pixels. In our case the SLM is placed in the back Fourier plane of the optical system so backward propagation is obtained by Eq. 3. As shown in Fig 3, the procedure can be translated into a kernel having as input arguments the full trap structure described by the M coordinates, weights and phases: \((x_m, y_m, z_m), w_m, \theta_m\). SR holograms are obtained by putting \(w_m = 1\) and choosing \(\theta_m\) as random phases. We implemented such a procedure in the single kernel BackPropKer having a number of threads equal to the number of SLM pixels. Each thread evaluates a single phase modulation \(\phi_j\) and stores it in a linear array residing in the global memory. According to rule C in section 2 it is important that contiguous threads write on contiguous pixels phase data so that coalesced memory access is guaranteed. As discussed in rule A in section 2 we use blocks containing a number of threads that is large and multiple of 32. Each thread needs to access the full trap structure so that a significant speedup can be achieved by preloading the trap data in the shared memory as prescribed by rule B. In each block only M threads cooperate to read the traps’ data. At this point we are ready to evaluate the time performance of BackPropKer in generating holograms using the SR algorithm. To this aim we first generate M random phases \(\theta_m\) on the CPU and then store the trap structure on the global memory. Using a GeForce GTX 260 we can generate \(768\times768\) SR holograms 350 times faster than using a Pentium D 3.2 GHz. The time spent by SR to compute a hologram grows linearly with the number of traps with a time per trap coefficient of 0.35 ms/trap. As an illustration of the relative importance of the considered optimization rules, we compare in Table 1 the most efficient kernel, where all
this rules are obeyed, to partially optimized kernels.

Turning now to the better performing GSW algorithm, in addition to a back propagation kernel we need a procedure to forward propagate the fields from SLM pixels to target traps (Eq. 1). Such a procedure can be decomposed into two main tasks: i) calculate the contribution of each pixel to the complex field $V_m$ on the $m^{th}$ trap’s location, ii) sum up all contribution to obtain $V_m$. The second task is a very common one and it’s widely discussed in the CUDA SDK examples. This procedure is based on the sum reduction kernel SumRedKer that performs partial sums, reducing the number of terms. A loop iterates SumRedKer until one single term is left containing the sum of all elements. A schematic representation of SumRedKer is reported in Fig. 4 where a single block is shown. Each block contains blocksize threads that perform the partial sum of $2 \times \text{blocksize}$ elements and writes the result back to the global memory. At the end of SumRedKer a number of terms equal to the number of used blocks still remains to be summed. Therefore a sequence of $\log(N)/\log(\text{blocksize})$ kernels is needed to perform the whole sum.

The evaluation of Eq. 1 also requires the task of calculating the contribution of the field radiating from each pixel to the total trap field $V_m$. Such a contribution is obtained calculating the phase shifts $\Delta_j^m$ in Eq. 2 and building the complex exponentials $e^{i(\phi_j - \Delta_j^m)}$. In order to reduce read/write operations on global memory we use a slightly different version of the sum reduction kernel

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Figure 4: Schematic diagram of the kernel SumRedKer. For the sake of simplicity we only show a single block of four threads. In the first step each thread sums up two elements of the global memory and stores the result on shared memory. In the following steps the number of active threads and the terms to sum are halved until a block is left with one single term. The $i^{th}$ block will write its partial sum on the $i^{th}$ address the global memory array.
as the first partial sum step. The first \textbf{SumRedKer} will begin having the phase modulations $\phi_j$ on the global memory locations $g_j$ in Fig. 4 so that we need to calculate complex exponential before the first write on shared memory (i.e. $s_1 = e^{i(\phi_1 - \Delta m_1)} + e^{i(\phi_5 - \Delta m_5)}$). The phases $\Delta m_j$ (M*N in total) are needed both for forward and backward propagation routines. Observing that such phases are fixed for a chosen trap geometry, one could think that precaching them in global memory could save computational time. However we checked that direct calculation is always faster (see rule B). Once $V_m$s are known, the calculation of the weights $w_m$ is quick and straightforward. The time required by GSW grows almost linearly with the number of traps or iterations. In Fig. 5 we report the computational time per trap per iteration as a function of traps number. Deviations from linearity are observed for small traps number evidencing the presence of a time cost which is essentially independent from the number of traps and is probably due to memory read/write operations. As we can see from the figure, we can neglect the small deviations from linearity and define a time per trap per iteration. Using a GeForce GTX 260 we obtain 0.44 ms/trap/iteration obtaining a 45x speedup respect to a Pentium D 3.2 GHz.

5. Real-time manipulation

Our optical tweezers are based upon a Nikon TE2000U inverted microscope with a 100x objective lens, NA 1.4. To form the trap we use a Nd:YAG laser, frequency-doubled to give a maximum power of 3 W at 532 nm (LaserQuantum Opus). After expansion and collimation, the beam from this laser is reflected off a computer-controlled SLM (HoloEye LCR 2500). Our SLM is based on a liquid crystal reflective micro-display. A laser beam reflecting off the SLM will...
emerge with a phase retardation that can be modulated on a pixel by pixel basis. Phase modulation is achieved by electrically addressing the pixels and therefore locally reorienting the nematic axis of liquid crystal molecules. When a grayscale, 8bit depth image is displayed on the SLM, a proper pattern of voltages is relayed to the pixels so that each grayscale value is linearly mapped to a phase shift ranging from 0 to $2\pi$. Light reflected off the SLM is then coupled to the microscope by projecting a demagnified image of the SLM plane on the back focal plane of the microscope objective. An array of optical traps is then produced around the front focal plane of the objective located in a colloidal water suspension above the coverslip. The SLM was controlled by a host PC equipped with a NVidia GeForce GTX 260 video card. User input is managed by a GUI mainloop thread (Tkinter) running in a Python shell while a Python module wraps the CUDA library functions providing a high level interface to the GPU hologram generation.

As a demonstration of real-time manipulation using optimized GPU generated holograms, we show the simultaneous trapping and manipulation of eight silica beads (2 $\mu$m diameter) in water. Optimized holograms are obtained with 5 GSW iterations at a rate of 48 Hz following user input. Fig. 8 shows three frames from the corresponding SLM and CCD timelines. While a hologram movie is displayed on the SLM (lower timeline) based on user input, a dynamic 3D micro-hologram, consisting of an array of moving bright light spots, is projected in the sample volume providing dynamical, and real-time reconfigurable optical traps. Trapped beads are imaged with bright light illumination on a CCD camera (upper timeline). The actual frame-rate is slightly lowered due
Figure 7: GSW performance. We report the efficiency ($e$) and uniformity ($u$) for GSW generated holograms as a function of the number of traps. The number of GSW iterations is always such to work at a fixed framerate of 20 Hz. Holograms with a performance above 90% can be generated at 20Hz for trap arrays as large as 16.

In conclusion, we have used a CUDA enabled video card to generate optimized holograms for optical trapping with a speedup of 350x (SR) and 45x (GSW) over the host CPU. The obtained speedup allowed us to trap and manipulate multiparticle 3D structures with efficient and uniform trap arrays in real time. Our results demonstrate that the high computational load of hologram generation cannot be considered any longer as a limiting factor of holographic trapping for real time applications. We acknowledge support from INFM through the Seed-project.

[1] D.G. Grier, A revolution in optical manipulation, Nature 424 (2003) 810-
Figure 8: Frames from a movie showing the interactive micro-manipulation of 8 silica beads with 2 \( \mu m \) diameter (watch the full movie in supplementary online material). The beads are arranged on the vertices of a 5 \( \mu m \) side cube which is then rigidly rotated. Bottom row shows the corresponding frames (holograms) displayed on the SLM.

[2] A. Ashkin, J. M. Dziedzic, J. E. Bjorkholm, Observation of a single-beam gradient force optical trap for dielectric particles, Opt. Lett. 11 (1986) 288-290.

[3] M. Reicherter, T. Haist, E.U. Wagemann, H.J. Tiziani, Optical particle trapping with computer-generated holograms written on a liquid-crystal display, Opt. Lett. 24 (1999) 608-610.

[4] J. Liesener, M. Reicherter, T. Haist, H.J. Tiziani, Multi-functional optical tweezers using computer-generated holograms, Opt. Commun. 185 (2000) 77-82.

[5] E.R. Dufresne, G.C. Spalding, M.T. Dearing, S.A. Sheets, D.G. Grier, Computer-generated holographic optical tweezers arrays, Rev. Sci. Instrum. 72 (2001) 1810-1816.

[6] J. Curtis, B.A. Koss, D.G. Grier, Dynamic holographic optical tweezers, Opt. Commun. 207 (2002) 169-175.

[7] R. Di Leonardo, F. Ianni, G. Ruocco, Computer generation of optimal holograms for optical trap arrays, Opt. Express 15 (2007) 1913-1922.
[8] K. Visscher, G. J. Brakenhoff, and J. J. Kroll, Micromanipulation by multiple optical traps created by a single fast scanning trap integrated with the bilateral confocal scanning laser microscope, Cytometry 14 (1993) 105-114.

[9] http://www.nvidia.com/object/cuda_home.html

[10] W. Liu, B. Schmidt, G. Voss, and W. Müller-Wittig, Accelerating molecular dynamics simulations using Graphics Processing Units with CUDA, Comp. Phys. Comm. 179 (2008) 634-641.

[11] T. Preis, P. Virnau, W. Paul, J.J. Schneider, GPU accelerated Monte Carlo simulation of the 2D and 3D Ising model, J. Comp. Phys. 228 (2009) 4468-4477.

[12] E. Elsen, P. LeGresley, E. Darve, Large calculation of the flow over a hypersonic vehicle using a GPU, J. Comp. Phys. 227 (2008) 10148-10161.

[13] G. I. Egri, Z. Fodor, C. Hoelbling, S.D. Katz, D. Ngrdi, K.K. Szab, Lattice QCD as a video game, Comp. Phys. Comm. 177 (2007) 631-639.

[14] L.B. Lesem, P.M. Hirsch, J.A. Jordan, The kinoform: a new wavefront reconstruction device, IBM J. Res. Dev. 13 (1969) 150-155.

[15] T. Shimobaba, Y. Sato, J. Miura, M. Takenouchi and T. Ito, Real-time digital holographic microscopy using the graphic processing unit, Opt. Express 16 (2008) 11776-11780.

[16] F.C. Cheong, B. Sun, R. Dreyfus, J. Amato-Grill, K. Xiao, L. Dixon, D.G. Grier, Flow visualization and ow cytometry with holographic video microscopy, Opt. Express 17 (2009) 13071-13079.

[17] N. Masuda, T. Ito, T. Tanaka, A. Shiraki, and T. Sugie, Computer generated holography using a graphics processing unit, Opt. Express 14 (2006) 603-608.

[18] L. Ahrenberg, P. Benzie, M. Magnor, and J. Watson, Computer generated holography using parallel commodity graphics hardware, Opt. Express 14 (2006) 7636-7641.

[19] J. Leach et al., Interactive approach to optical tweezers control, Appl. Optics 45 (2006) 897-903.

[20] E. Pleguezuelos, A. Carnicer, J. Andilla, E. Martin-Badosa and M. Montes-Usategui, Fast generation of holographic optical tweezers by random mask encoding of Fourier components Comp. Phys. Comm. 176 (2007) 701-709.

[21] M. Reicherter, S. Zwick, T. Haist, C. Kohler, H. Tiziani, and W. Osten, Fast digital hologram generation and adaptive force measurement in liquid-crystal-display-based holographic tweezers, Appl. Opt. 45 (2006) 888896.
[22] D. Preece, R. Bowman, A. Linnenberger, G. Gibson, S. Serati and M. Padgett, Increasing trap stiffness with position clamping in holographic optical tweezers, Opt. Express 17 (2009) 22718-22725.