Porous medium convection in a chemically reacting ferrofluid with lower boundary subjected to constant heat flux

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Abstract. The effect of exothermic chemical reaction of zero-order on Bénard-Darcy ferroconvection is investigated using the technique of small perturbation. The eigenvalues associated with an adiabatic lower wall are determined by employing the Galerkin method. The Darcy-Rayleigh number is computed in terms of the parameters pertaining to chemical reaction and ferromagnetic fluid. It is established that, when chemical reaction escalates, there is a considerable shift from linearity and occurrence of asymmetry in the basic temperature profiles. It is ascertained that the threshold of Bénard-Darcy ferroconvection is augmented through the stresses of both mechanisms due to chemical reaction and magnetization, and the ferroconvective instability due to nonlinearity of magnetization is rather inconsequential when chemical reaction is present. It is also shown that the destabilizing feature of magnetic forces resulting from the fluid magnetization is less pronounced when chemical reaction is present.

1. Introduction

The engagement of magnetic fluids in engineering and other sophisticated applications is being facilitated owing to the fact that such fluids could successfully be controlled by external magnetic fields. The viability of many heat transfer applications concerning magnetic fluids is unquestionably associated with the enrichment of thermomechanics of ferromagnetic fluids (Bashtovoy et al [1]). The ferroconvective instability problem has been dealt with by numerous researchers since the critical values relating to ferroconvective instability could substantially be altered by means of magnetic forces resulting from the magnetization of such fluids. A comprehensive account of the ferroconvection problem involving magnetic fluids in horizontal layers was developed by Finlayson [2] who was competently able to envisage the critical values associated with the cases of magnetization induced ferroconvection and the occurrence of ferrocovction due to both magnetic and buoyancy mechanisms. When it comes to thin ferrofluid layers, it has been shown that magnetic mechanism is accountable for ferroconvection and the impact of buoyancy mechanism on the manifestation of ferroconvection is rather inconsequential.

Gupta and Gupta [3], Gotoh and Yamada [4] and Stiles and Kagan [5] extended the ferroconvective instability problem to include the effects of coriolis acceleration, ferromagnetic boundaries and a strong magnetic field respectively. The influence of a time-dependent sinusoidal magnetic field on ferroconvective instability, with the intention of addressing both harmonic and sub-harmonic modes of instability, has been studied by Aniss et al [6]. An analytical study of ferroconvection with micropolar
characteristics has been carried out by Abraham [7] who showed that magnetic fluids with micropolar features are always more stable than their Newtonian counterpart. Assuming that magnetic fluids absorb and emit thermal radiation and the boundaries as black bodies, Maruthamanikandan [8] studied the problem of ferroconvection influenced by radiative heat transfer. By adopting the Galerkin method, the combined effect of centrifugal acceleration and anisotropic porous medium on the threshold of ferroconvective instability has been examined by Saravanan [9] who found that anisotropies of magnetic fluid alter the stability of the system drastically.

Singh and Bajaj [10] ventured on the study of periodic flow of magnetic fluids subjected to temperature modulation by resorting to the Floquet theory. They showed that the flow patterns are in the form of oscillating time-periodic vertical magnetoconvective rolls subject to the parameters. Subcritical ferroconvective instability due to low frequency g-jitter has been predicted theoretically by Nisha Mary and Maruthamanikandan [11] who also explicated the fact that g-jitter mechanism works against the magnetic mechanism and reinforcement of g-jitter effect is feasible through the Vadasz number. Motivated by the implications of ferrofluids in the cooling of high-power electronics and in measuring thermal properties of magnetic fluids, Vatani et al [12] made an attempt to look at the investigation of thermomagnetic convection influenced by a vertical transient hot-wire cell by resorting to both analytical and experimental methods. They have established the dependence of thermomagnetic convection on the current experienced by the wire.

Inspired by the applications of Darcy-Brinkman ferroconvection model, Nisha Mary and Maruthamanikandan [13] and Soya Mathew and Maruthamanikandan [14] took up, respectively, the study of ferroconvective instability subjected to the influences of gravity modulation and temperature dependent viscosity. The enhancement of instability due to variable viscosity in the presence of magnetic mechanism is substantiated theoretically. Taking the effective viscosity of ferrofluids to be both thermally and magnetically responding, the combined effect of spatial heat source and variable viscosity on Marangoni ferroconvective instability has been dealt with by Maruthamanikandan et al [15] who expounded the fact that ferroconvection of Marangoni type is considerably affected by both variable viscosity and heat sources.

On the other hand, the study of chemical reaction-driven convection in porous media, keeping in mind its implications into reactors and synthesis and oxidation of materials, has led to numerous meaningful and worthwhile applications (Kordylewski and Krajewski [16] and Farr et al [17]). By resorting to a zero order chemical reaction of exothermic type, Malashetty et al [18] laid the foundations of the work concerning buoyancy and reaction-driven convection in porous media. They elucidated that a nonlinear temperature profile, associated with the quiescent state and resulting from chemical reaction, is liable, to a great extent, for the destabilizing impact of chemical reaction. Motivated by this groundwork, Mubeen Taj et al [19] and Khudeja et al [20, 21] extended the work to include the features of couple stresses and viscoelasticity respectively. Zhang et al [22] investigated convection of nanofluids due to radiation and chemical reaction. Reaction-driven boundary layer flow of ferrofluids of Maxwell type subjected to Soret and suction effects has been dealt with by Majeed et al [23]. Very recently, Nisha Mary and Maruthamanikandan [24] addressed the problem of chemical reaction induced ferroconvection taking into account isothermal and non-magnetic boundaries.

It should be stressed that, for chemical reaction induced convection of Rayleigh-Bénard type, substantial modification of critical values is possible when the lower wall is treated as adiabatic rather than isothermal (Malashetty et al [18]). It is found that the reported research works on ferroconvective instability have not included the effect of chemical reaction with an adiabatic lower boundary. Moreover, the resulting eigenvalue problem turns out to be more formidable because finding an analytical solution of the problem is impossible. Taking advantage of the gap in the aforementioned research domain, we intend to address the problem of ferroconvection of Bénard-Darcy type when both chemical reaction and an adiabatic lower wall are present. The higher order Galerkin method is adopted to tackle the numerical solution of the boundary value problem involved. The special cases of the study at hand are pointed out when the magnetic mechanism and chemical reaction are absent.
2. Mathematical formulation
A chemically reactive ferrofluid filled horizontal porous layer located between two surfaces of infinite length with finite thickness $h$ is considered. The fluid layer is cooled at a temperature of $T_c$ from the top. An exothermic reaction of zero order could be reckoned when the temperature varies slightly in the whole domain from $T_c$. The governing equations pertinent to the problem at hand are

$$\nabla \cdot \nabla = 0$$  \hspace{1cm} (2.1)

$$\rho_R \left[ \frac{1}{\varepsilon} \frac{\partial \tilde{V}}{\partial t} + \frac{1}{\varepsilon^2} (\tilde{V} \cdot \nabla) \tilde{V} \right] = -\nabla p + \rho g - \frac{\mu_f}{k} \nabla + \nabla \cdot (\overline{H} B)$$ \hspace{1cm} (2.2)

$$\varepsilon C_f \left[ \frac{\partial T}{\partial t} + (\tilde{V} \cdot \nabla) T \right] + (1-\varepsilon)(\rho_R C_o) \frac{\partial T}{\partial t} + \mu_o T \left( \frac{\partial \tilde{M}}{\partial t} \right) \cdot \left( \frac{\partial \overline{H}}{\partial t} + (\tilde{V} \cdot \nabla) \overline{H} \right)$$

$$= K \nabla^2 T + \varepsilon C_f Q_r \exp \left( -\frac{E}{R_{m} T} \right)$$ \hspace{1cm} (2.3)

$$\rho = \rho_R \left[ 1 - \alpha (T - T_c) \right]$$ \hspace{1cm} (2.4)

$$\dot{M} = \frac{\dot{H}}{H} M (H, T)$$ \hspace{1cm} (2.5)

$$M = M_o + \chi (H - H_o) - K_m (T - T_c).$$ \hspace{1cm} (2.6)

Various physical quantities used in equations (2.1) through (2.6) have their conventional interpretations (Finlayson [2], Malashetty et al [18] and Nisha Mary and Maruthamanikandan [24]).

Maxwell’s equations for the problem at hand turn out to be

$$\nabla \times \overline{H} = 0, \quad \nabla \cdot \tilde{B} = 0, \quad \tilde{B} = \mu_o \left( \overline{H} + \dot{M} \right).$$ \hspace{1cm} (2.7)
The temperature boundary conditions in line with the objective are

\[ T = T_c \text{ at } z = h \quad \text{and} \quad \frac{\partial T}{\partial z} = 0 \text{ at } z = 0 \]  

(2.8)

with the temperature \( T_h \) of the lower wall being higher than the temperature \( T_c \) of the upper wall.

3. Stability analysis

Resorting to the stability analysis of small perturbations involving normal modes (Finlayson [2], Malashetty et al [18], Nisha Mary and Maruthamanikandan [24] and Chandrasekhar [25]), the dimensionless equations turn out to be

\[
\frac{\xi}{V_d} \left( D^2 - a^2 \right) U = -R_d a^2 \gamma - \left( D^2 - a^2 \right) U - N a^2 \left( \frac{d \theta_b}{dz} \right) \nabla \psi + N a^2 \left( \frac{d \theta_b}{dz} \right) \gamma
\]

(3.1)

\[
\xi \gamma + \frac{d \theta_b}{dz} U = \left( D^2 - a^2 \right) \psi + FK \text{Exp}(\theta_b) \gamma
\]

(3.2)

\[
\left( D^2 - M_3 a^2 \right) \psi - D \gamma = 0
\]

(3.3)

where \( \xi \) is the growth rate and \( \psi \) the magnetic scalar potential. The parameters of the study at hand are \( V_d = \frac{\epsilon \mu_f h^2}{\rho_kk_c} \), the Vadasz number, \( N = \frac{\mu_0 K_m^2 T_c^2 k}{\mu_f k (1 + \chi)} \), the Darcy-magnetic number, \( R_d = \frac{\alpha \rho_R g R_{un} T_c^2 h k}{\mu_f \kappa E} \), the Darcy-Rayleigh number and \( M_3 = \frac{H_o + M_o}{H_o (1 + \chi)} \), the non-buoyancy magnetization parameter.

The expression for the temperature of the quiescent state reads

\[
\theta_b = \text{Log} \left( \frac{V_1}{2FK} \right) + \text{Log} \left[ 1 - \left( 1 - V_2 \text{Exp} \left( -\sqrt{V_1} \right) \right)^2 \right]
\]

(3.4)

where \( FK = \frac{C h^2}{\kappa} \) is the Frank-Kamenetskii number, \( \kappa = \frac{K}{\epsilon C_f} \), \( C = \frac{Q_r \text{Exp} \left( -\frac{E}{R_{un} T_c} \right)}{T_r} \) and \( T_r = \frac{R_{un} T_c^2}{E} \). The constant \( V_1 \) is determined implicitly from the equation

\[
\text{Exp} \left( \sqrt{V_1} \right) \left[ 1 - \frac{1 - 2FK}{V_1} \right] = \left[ 1 + \frac{1 - 2FK}{V_1} \right]
\]

(3.5)

and

\[ V_2 = 1. \]  

(3.6)
3.1. Stationary instability
Since oscillatory instability could be established to be absent, the simultaneous differential equations associated with the stationary mode are

\[
\left( D^2 - a^2 \right) U + R_d a^2 Y - N a^2 \left( \frac{d\theta_b}{dz} \right) Y + N a^2 \left( \frac{d\theta_b}{dz} \right) D\psi = 0
\] (3.7)

\[
\left( D^2 - a^2 \right) Y + FK \exp(\theta_b) Y - \left( \frac{d\theta_b}{dz} \right) U = 0
\] (3.8)

\[
\left( D^2 - M_1 a^2 \right) \psi - D\psi = 0.
\] (3.9)

The boundary conditions pertinent to the problem at hand are

\[
U = D\gamma = D\psi = 0 \text{ at the lower wall } \}
\]

\[
U = \gamma = D\psi = 0 \text{ at the upper wall } \}
\] (3.10)

4. Method of solution
Keeping in mind the variable coefficients arising in equations (3.7) and (3.8), we resort to the Galerkin method (Finlayson [26]) to determine the numerical solution of the associated boundary value problem. Accordingly, we assume that

\[
U = \sum A_j U_j, \quad \gamma = \sum B_j \gamma_j, \quad \psi = \sum C_j \psi_j
\]

where \( A_j, B_j \) and \( C_j \) are constants. On applying the Galerkin method, we obtain the following simultaneous algebraic equations of homogeneous type

\[
\begin{bmatrix}
D_{ij} A_j + E_{ij} B_j + F_{ij} C_j = 0 \\
G_{ij} A_j + H_{ij} B_j = 0 \\
K_{ij} B_j + L_{ij} C_j = 0
\end{bmatrix}
\] (4.1)

where

\[
D_{ij} = \left\{ U_i D^2 U_j \right\} - a^2 \left\{ U_i U_j \right\}, \quad E_{ij} = R_d a^2 \left\{ U_i \gamma_j \right\} - N a^2 \left\{ U_i \frac{d\theta_b}{dz} \gamma_j \right\},
\]

\[
F_{ij} = N a^2 \left\{ U_i \frac{d\theta_b}{dz} D\psi_j \right\}, \quad G_{ij} = \left\{ \psi_i \frac{d\theta_b}{dz} U_j \right\},
\]

\[
H_{ij} = a^2 \left\{ \gamma_i \gamma_j \right\} - \left\{ \gamma_i D^2 \gamma_j \right\} - F K \left\{ \gamma_i \exp(\theta_b) \gamma_j \right\}, \quad K_{ij} = \left\{ \psi_i D\psi_j \right\},
\]

\[
L_{ij} = M_1 a^2 \left\{ \psi_i \psi_j \right\} - \left\{ \psi_i D^2 \psi_j \right\}.
\]

The trial functions \( U_m = \sin[m\pi z], \gamma_m = \cos \left[ \frac{m\pi z}{2} \right] \) and \( \Psi_m = \cos[m\pi z] \) are taken into account in order to warrant the satisfaction of orthogonality and boundary conditions.
5. Results and discussion
The problem of chemical reaction-driven porous medium ferroconvective instability influenced by an adiabatic lower wall is dealt with in the present work. Since oscillatory instability is non-existent and determination of analytic solution is impossible, the Galerkin technique is employed to work out the associated numerical solution. The quiescent state temperature profiles concerning $\theta_b$ and $FK$ are presented in Figure 2. As can be seen, an increase in $FK$ results in greater asymmetry and nonlinearity of the profiles on quiescent state temperature. This particular feature of the temperature profiles pertaining to the quiescent state should be estimated in conjunction with equation (3.4). Physically speaking, the observed increase in asymmetry and nonlinearity of the quiescent state temperature profiles could be attributed to the rise in the heat generation rate caused by chemical reaction.

![Figure 2. Basic state temperature profiles.](image)

Figure 3 depicts the impact of both magnetic state and chemical reaction. The physical parameter $FK$ is the ratio of the characteristic flow time to reaction time and $N$ is the proportion of the release of energy due to the stress associated with magnetic forces and dissipation of energy resulting from viscous and thermal fluctuations. It is seen from Figure 3 that the Rayleigh number $R_{dc}$ drops off when both $FK$ and $N$ are on the increase. As a result, the threshold of Bénard-Darcy ferroconvection is augmented through the stresses of both mechanisms of chemical reaction and magnetization. Noticeably, the instability caused by the magnetic mechanism becomes more marked only when the parameter $FK$ is large enough. On the other hand, Figure 4 delineates the influence of $M_3$ on chemical reaction-driven porous medium ferroconvection. The parameter $M_3$ signifies the shift towards nonlinearity in magnetization. It is clear from Figure 4 that the destabilising effect of $M_3$ is rather inconsequential provided chemical reaction is present and that curtailment of ferroconvection due to chemical reaction is plausible when there is significant intensification in the nonlinearity of magnetization. Further, when $N = FK = 0$, the celebrated values $a_c = \pi$ and $R_{dc} = 4\pi^2$ (Nield and Bejan [27]) can be obtained.
**Figure 3.** Plot of $R_{dc}$ versus $FK$ with variations in $N$.

**Figure 4.** Plot of $R_{dc}$ versus $FK$ with variations in $M_3$. 
6. Conclusions
Ferroconvective instability of Bénard-Darcy type influenced by a zero order chemical reaction of exothermic type is investigated by resorting to the stability analysis of infinitesimal perturbations involving normal modes followed by the Galerkin method of higher order. The following conclusions could be drawn from the analysis of the present work:

- The manifestation of ferroconvective instability resulting from chemical reaction is largely due to the asymmetry and nonlinearity of the quiescent state profiles of temperature.
- The threshold of ferroconvective instability could be augmented by virtue of magnetic and chemical reaction mechanisms and the destabilizing feature of magnetic forces is less pronounced when chemical reaction is present.
- The instability due to nonlinearity of magnetization is rather insignificant when chemical reaction is present.

The implications of the present work may have major sway on heat transfer applications wherein microfluidic devices are adopted. Nonlinear effects, other types of chemical reactions, anisotropic porous medium, non-Newtonian ferrofluids and other related constraints leading to internal heat generation could be part of the future work of the present study.

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