Abstract: The waveform optimisation problem in cognitive radar is non-convex and will have sub-optimal solutions when solved by the semi-definite relaxation (SDR) technique. Here, a novel nature-inspired waveform optimisation framework is proposed for range-spread target detection in cognitive radar. First, the waveform optimisation problem is formulated using maximum a posteriori probability and Kalman filtering to estimate the target scattering coefficients. To solve this problem more accurately and efficiently, three nature-inspired algorithms (modified particle swarm optimisation algorithm, Bat Algorithm, and Beetle Antennae Search algorithm), as a nature-inspired waveform optimisation (NIWO) approach is proposed. It is demonstrated through computer simulations that the proposed NIWO approach significantly outperforms the SDR approach, showing a promising tool for waveform optimisation in cognitive radar.

1 Introduction

Cognitive radar is a closed-loop system composed of transmitter, receiver, and the environment [1]. One of the key functionalities of the receiver is the ability to feedback the received environment information to the transmitter, the transmitter then optimises the transmitted waveform according to the feedback information, so as to continuously improve the detection and estimation performance of the system. One of the key problems in cognitive radar is waveform optimisation.

In the past, different approaches have been proposed for waveform optimisation, such as maximising mutual information (MI) [2], minimising Cramer–Rao bound (CRB) [3], maximising signal-to-interference-plus-noise ratio (SINR) [4], and signal-to-noise ratio (SNR) [5]. Among them, the method based on the normalised mean square error (NMSE) [6] of estimated target scattering coefficient (TSC), which is the Fourier transform of target impulse response (TIR), is a representative one with both convenience and simplicity. Moreover, maximum a posteriori probability (MAP) and Kalman filtering (KF) are also adopted to estimate the scattering coefficients of a target with improved performance.

However, the waveform optimisation problem is in general non-convex [7], and it is usually transformed into a convex form and then solved using the semi-definite relaxation (SDR) technique [8]. Although SDR is a powerful approximation technique for a range of very difficult optimisation problems, some constraints are often compromised in the approximation process, which unavoidably reduces the accuracy of a solution. Motivated by this, a nature-inspired waveform optimisation approach is proposed in this work.

Why Nature inspired: Nature-inspired optimisation algorithms have become increasingly popular due to their excellent performance in solving various complex optimisation problems [9], and some good examples include the particle swarm optimisation (PSO) algorithm [10], the bat algorithm (BA) [11], the genetic algorithm (GA) [12], and the firefly algorithm (FA) [13]. The beetle antennae search (BAS) algorithm is the newest meta-heuristic algorithm inspired by the searching behaviour of longhorn beetles [14]. These algorithms have been found to be efficient, especially when solving non-convex optimisation problems.

Contributions: There are two main contributions in this work.

i. A nature-inspired waveform optimisation (NIWO) framework is proposed for the first time. Given their ability in swarm intelligence and globe optimisation, we apply the nature-inspired algorithms to cognitive radar waveform optimisation and establish a mathematical link between target estimation and waveform optimisation. We propose three algorithms, the modified PSO, BA, and BAS, to solve the non-convex problem efficiently. The NIWO optimiser is iteratively updated and gradually converges to the optimum value for accurate target estimation.

ii. Our second contribution lies in developing a generalised waveform optimisation model for target detection and tracking. The corresponding theory on MAP and KF to minimise the NMSE is introduced, in order to accurately estimate the desired TSC.

The rest of the paper is arranged as follows. In Section 2, the cognitive radar system model is presented and the waveform optimisation problem is formulated. In Section 3, the NIWO approach is proposed for waveform optimisation. Simulation results are provided in Section 4, and conclusions are drawn in Section 5.

Notation: Symbols for vectors (lower case) and matrices (upper case) are in bold face. (·)T, diag(·), I, conj(·), CN(0,R), ∥·∥2, E, F, and Tr(·) denote the conjugate transpose (Hermitian), the diagonal matrix, the identity matrix, the conjugate operation, the complex Gaussian distribution with zero mean, and covariance R, the l2 norm, the expectation, the Fourier transform matrix, and the trace of a matrix, respectively.

2 Cognitive radar waveform optimisation model

2.1 Cognitive radar system model

The block diagram of a cognitive radar system is shown in Fig. 1. We consider a mono-static radar system and the range spread targets are described by the wide sense stationary-uncorrelated scattering (WSSUS) model [4, 15] Fig. 2.

We divide the range spread targets into P range cells and define the radial range of a target as the nearest range of all scatterings as illustrated in Fig. 3. The echo signal is represented as the
2.2 MAP and KF-based Target Estimation

The TSC can be estimated by the method of MAP as follows:

\[
\hat{g} = (C^{H} \times R_{c}^{-1} \times C + R_{n}^{-1})^{-1} \times C^{H} \times R_{n}^{-1} \times Y
\]

and (15) is transformed into

\[
\min \frac{\text{tr}(\text{MSE}_{\hat{g}})}{\text{tr}(\mathbf{R}_{s})} \\
\text{s.t.} \quad C = \text{diag}(\epsilon) \\
\| c \|_{2}^{2} \leq E_{c}
\]

This problem is non-convex and cannot be directly solved.

3 Cognitive radar waveform optimisation based on the NIWO approach

3.1 The NIWO Framework

Nature-inspired optimisation algorithms have been found to be very efficient in solving non-convex problems. By modifying the PSO, BA, and BAS algorithms, the NIWO framework is proposed for solving the waveform optimisation problem in this section,
Step 1: Estimate TSC by the MAP method according to Eq. (6).
Step 2: Estimate TSC by the KF method according to Eq. (8-12).
Step 3: Calculate the objective function NMSE according to Eq. (18).
Step 4: Solve the non-convex problem using Nature-inspired algorithms (modified PSO, BA and BAS).
Step 5: Feedback the optimized transmit waveform for transmission.
Step 6: Use the optimized transmit waveform for TSC estimation.

Fig. 4 Algorithm 1: Nature-inspired waveform optimisation (NIWO)

Parameters: Dimension (number of signal samples): \( D \), number of iterations: \( \text{MaxDT} \).
Step 1: Initialization.
for each particle \( i \) do
    Randomly initialize particles’ velocity and position:
    \( V_{i}^{t}, X_{i}^{t} \)
    Initialize individual best position: \( \text{Pbest}_{i} = X_{i}^{t} \).
    Initialize fitness (object function): \( p(i) = F(\text{Pbest}_{i}) \)
end for
Initiate global best position: \( \text{Gbest} = \min \{ \text{Pbest} \} \).
Step 2: Waveform Optimization
for \( t = \text{MaxDT} \) do
    Dynamic inertia weight according to Eq. (25).
    for each particle \( i \) do
        Calculate \( V_{t+1} \) according to Eq. (19).
        Calculate \( X_{t+1} \) according to Eq. (20).
        if \( F(X_{t+1}) < p(i) \) do
            \( \text{Pbest}_{i} = X_{t+1} \).
            \( p(i) = F(X_{t+1}) \).
            for \( p(i) < F(\text{Gbest}) \) do
                \( \text{Gbest} = \text{Pbest}_{i} \).
                \( t = t + 1 \)
            end for
        end if
    end for
end for

Fig. 5 Algorithm 2: The modified PSO algorithm

which is described by Algorithm 1 (see Fig. 4). Next, the modified PSO algorithm is described in detail as an example of the NIWO approach.

3.2 The Modified PSO Algorithm
The PSO algorithm simulates bird predation by designing massless particles [16], which have two properties: velocity and position. Velocity represents speed and position represents moving direction. The position of each particle in the space has a fitness value corresponding to the cost function. Each particle is in search for its own best position (Pbest) and the global. After multiple iterations, the final global best position will be found. Each particle updates their velocity and position according to the following equations:

\[
V_{t+1} = wV_{t} + c_{1}r_{1}(P_{\text{best}} - X_{t}) + c_{2}r_{2}(G_{\text{best}} - X_{t}) \quad (19)
\]

\[
X_{t+1} = X_{t} + V_{t+1} \quad (20)
\]

Among them, \( w \) is the inertia weight that keeps the original velocity, \( c_{1} \) and \( c_{2} \) are the cognitive component and social component, respectively, and \( r_{1} \) and \( r_{2} \) are random numbers between 0 and 1. Each particle adjusts its own speed according to both their current local best solution and global best solution. The current position \( X \) and velocity \( V \) determine the next position of the particle. The PSO algorithm is applied to solve the waveform optimisation problem with the transmit power constraint. We modify the standard PSO algorithm by changing the fixed inertia weight into a dynamic inertia weight and the following is the steps involved:

3.2.1 Population definition and initialisation: First, we define and initialise a group of particles. Assume the total number of particles is \( M \). The position of a particle is a solution of the desired transmit waveform, and for particle \( i \), it is defined as \( X_{i} = \{x_{1}, x_{2}, \ldots, x_{D}\}, i \in [1, M] \). \( D \) is the length of the transmit signal, so that the solution is a \( D \)-dimensional vector. Initially, the position of each particle is selected randomly with the constraint of transmit power \( E_{t} \), i.e.

\[
||X_{i}||^{2} \leq E_{t} \quad (21)
\]

Next, we define and initialise the particles’ velocity. The velocity guides the particle about which position it should move to and how fast it could be. The velocity of particle \( i \) is defined as \( V_{i} = \{v_{1}, v_{2}, \ldots, v_{D}\} \). Then, we define and initialise the particles’ fitness value. The fitness function \( F \) is defined as the objective function value corresponding to the transmit signal, as follows,

\[
F = \frac{\text{tr}(MSE|i|^{-1} + \text{diag}(X_{i})^{-1} \times R_{t}^{-1} \times \text{diag}(X_{i}))^{-1}}{\text{tr}(R_{t})} \quad (22)
\]

Finally, we define and initialise Pbest and Gbest. For the initial positions of the population, their fitness values are calculated according to (22). The current fitness value of particle \( i \) is its own best position. Therefore, the individual best position is

\[
\text{Pbest}_{i} = \arg \min_{X_{i}} F(X_{i}), i = 1, 2, \ldots, N \quad (23)
\]

The global best position is the position corresponding to the minimum fitness value of all particles, i.e.

\[
\text{Gbest} = \min \{ \text{Pbest} \} \quad (24)
\]

3.2.2 Population Update: The movement of the particles is defined as in (19) and (20).

When \( w \) is larger, it has a better global searching ability. For particle population, it needs to have powerful global searching ability at the beginning due to random initialisation, while not as randomness will decrease in the process. Therefore, we define a dynamic weight as

\[
w = w_{\text{max}} - (w_{\text{max}} - w_{\text{min}}) \times \left( \frac{t}{\text{MaxDT}} \right) \quad (25)
\]

\( w_{\text{max}} \) and \( w_{\text{min}} \) are maximum inertia weight and minimum inertia weight, respectively. \( t, \text{MaxDT} \) are the current iteration time and the maximum iteration number, respectively.

Then, we can update Pbest and Gbest according to (19) and (20) as before. The pseudo code of the modified PSO algorithm is described in Algorithm 2. (see Fig. 5)

3.3 Bat Algorithm
BA is a meta-heuristic optimisation algorithm. It simulates the echo-location behaviour of bats in nature and realises the global search and local search by changing the pulse amplitude and pulse frequency timely. Each virtual bat has a random flight speed and
position (problem solution), and bat has different frequencies or wavelengths, loudness, and pulse emissivity. When bat discovers and hunts prey, it changes the frequency, loudness, and pulse emissivity, and selects the best solution until the target is stopped or the condition is satisfied. The BA applies this principle to find the global optimal solution.

3.4 BAS Algorithm

Beetles look for food according to the strength of the food smell. They have two long tentacles for sensing the odour intensity, and if the intensity detected at left tentacle is higher than the right side, the beetle will fly to the left, otherwise it will fly to the right. The BAS algorithm applies this principle to find the optimal solution. The smell of food is equivalent to the objective function and by imitating the behaviour of beetles, we can optimise the function efficiently.

4 Simulation

In this section, we compare the SDR approach with the NIWO approach. The length of the target TIR is assumed to be \( p = 10 \), and the target-scattering functions are illustrated as the blue line in Fig. 6. The simulation parameters are given in Table 1.

The target estimation results are shown in Fig. 6, where the waveform optimised by NIWO approach is consistent with the actual TSC and is better than SDR method. By comparing the waveforms optimised by modified PSO, BA, and BAS, we can find that the estimated result by BAS algorithm is closest to the actual value and has the best performance.

The initial value of NMSE estimated by MAP after optimisation using the SDR and NIWO approaches is shown in the Fig. 7, where it can be observed that the NMSE of the NIWO approach outperforms the SDR approach. By comparing the waveform optimised by the modified PSO, BA, and BAS, we can find that the initial value of NMSE estimated by MAP after optimisation using the BAS algorithm is the smallest.

Fig. 8 shows the value of NMSE estimated by KF versus the number of iterations using different optimisation methods. As can be seen, large errors appear at low iteration numbers due to an inaccurate initialisation. Then, the errors decrease quickly when the number of iterations increases. When the number reaches a certain value, the errors reach a steady value. Among the three nature-inspired algorithms, BAS has provided the best performance.

5 Conclusion

In this paper, a nature-inspired waveform optimisation framework has been proposed for range spread targets in cognitive radar. Under this framework, three nature-inspired algorithms including the revised PSO algorithm, the revised BA and the revised BAS algorithm have been studied, where the transmit, waveform is adaptively optimised under the criterion of minimising NMSE of the TSC. As shown by simulation results, compared with the SDR approach, the proposed NIWO approach can provide a better estimation result, and the estimated NMSE is \(<0.1\). Overall, the NIWO approach can be considered as an effective and promising tool for detection and tracking of range spread targets in cognitive radar.

Table 1 Simulation parameters

| Parameter                  | Value |
|----------------------------|-------|
| TIR length \( p \)         | 10    |
| PRI T                      | 1 ms  |
| time interval \( r \)      | 1 s   |
| signal length \( D \)      | 10    |
| transmit power ES          | 1     |
| number of KF iterations MaxDT | 20 |
| SNR of the echo signal     | 10 Db |
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