Jet Quenching in High Energy Heavy Ion Collisions
by QCD Synchrotron-like Radiation

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We consider synchrotron-like radiation in QCD by generalizing Schwinger’s treatment of quantum synchrotron radiation in QED to the case of a constant chromo magnetic field. We suggest a novel mechanism for jet quenching in heavy ion collisions, whereby high-
\( p_t \) partons get depleted through strong (classical) color fields. The latters are encountered in the color glass condensate or in the form of expanding shells of exploding sphalerons. Unlike bremsstrahlung radiation through multiple soft rescattering, synchrotron radiation converts a jet into a wide shower of soft gluons. We estimate the energy loss through this mechanism and suggest that it contributes significantly to the unexpectedly strong jet quenching observed at RHIC.

I. INTRODUCTION

A. Radiation in Various settings

By synchrotron-like radiation we mean radiation emitted by a charge moving in an external field that is strong enough not to allow for a perturbative treatment. The strong external field problem requires the exact trajectories classically, and the exact propagators quantum mechanically. Throughout, we will refer to synchrotron radiation as the part of the radiation stemming from within the strong field region, while the radiation from the outside field region (if present) will be still referred to as bremsstrahlung radiation. For ultrarelativistic particles of energy \( E = \gamma m \), bremsstrahlung radiation is usually collimated along the particle trajectory in two cones of opening angle \( 1/\gamma \ll 1 \), while synchrotron radiation diverges away from the particle trajectory.

In QED synchrotron radiation usually takes place within a magnet as illustrated in Fig. 1a. The classical trajectory includes the circular bending between the incoming and outgoing straight lines, and the radiation is emitted tangent to the arc length. Photons move on straight lines since in QED they are not affected by the external magnetic field.

Another interesting case of classical synchrotron-like radiation is that of an ultrarelativistic rotating charge in a strong gravitational field such as the one encountered near the horizon of a black-hole [1] (and subsequent literature). In this case, both the charge and the photon are gravitationally deflected as illustrated in Fig. 1b. The result is a significant reduction of the radiation loss: the total radiation yield is reduced by a factor of \( \gamma^2 \) in comparison to the yield from standard synchrotron radiation for the same curvature. Also, the radiation length for each particular direction is actually the entire circle, not just an arc length of order \( 1/\gamma \) as in the magnet.

In this paper we consider an ultrarelativistic charge in QCD (parton, quark, gluon) going through a constant chromomagnetic field as illustrated in Fig. 1c. The motion of the initial charge and the ensuing radiation are both strongly affected by the chromomagnetic field. If classical geometrical optics can be used [2] (and for hard jet quenching it is not the case) the classical motion of the external particle and the radiation is described by non-trivial trajectories. For soft radiation, the emitted gluons are bent along circular paths of smaller curvature radii. As a result, part of the radiation gets trapped in the near field region and never makes it to infinity. This makes standard radiation calculations obsolete. Moreover, the radiated gluons carry different charges and therefore move in different directions. The QCD chromomagnetic field resolves both momentum and color thereby acting as a double (squared) Newtonian prism.

Another instructive classical description to keep in mind is the instantaneous distribution of the radiation field in the ultrarelativistic case. In QED the nonzero field strength is mostly located around a spiral-like curve...
known as the evolute of the circle. Since both the charge and the photon moves light-like, the distance from the radiation point to the charge is the same as to the photon along a straight line. Details of the field distribution for QED synchrotron radiation can be found in [3]. Its analog for QCD in the simplest abelian-type external field when the radiated gluons undergo planar rotation has a cycloidal shape. The radiation field is the same at any instant of time but moves (together with the charge) at the speed of light.

B. Synchrotron Radiation versus Bremsstrahlung Radiation in QED

The number of soft prompt gluons per frequency $\omega$ expected from bremsstrahlung and synchrotron radiation can be estimated classically. The Lienard expression for the power radiated from a dipole

$$P = -\frac{2e^2}{3m^2} \left( \frac{dp_\Omega}{ds} \right)^2$$

was derived in 1898. It holds even ultrasoftly since the power (energy over time) is relativistically invariant. The acceleration in (1) relates to the field strength $F$ and the particle energy $E$ by the Lorentz formula. Classically, the energy loss depends quadratically on both $E$ and $F$, i.e. $P \sim e^2 F^2 E^2 / m^4$. This result does not hold at very large energies though. Indeed, it only holds in the range

$$e^3 (F/m^2) (E/m) \ll 1 ,$$

where we have separated 3 dimensionless physical factors: the coupling constant, the field strength in units of $m^2$ and the relativistic gamma factor. For classical QED synchrotron radiation in accelerators, this condition of course holds even with $\gamma = E/m \sim 10^5$ (LEP). The reason is the small electromagnetic coupling and the small magnetic fields $H/m^2 \ll 1$.

In the QCD problem we are interested in, the fields are strong, the external charge is massless and the induced radiation recoil is large. The condition (2) does not hold, and we have to reassess the power radiated from first principles. However, we expect that qualitatively the differential spectra to follow general lore at small frequencies. In particular, the number of bremsstrahlung photons per frequency should be of order

$$\frac{dN_B}{d\omega} \sim \frac{\alpha}{\pi} \frac{1}{\omega},$$

while the number of synchrotron photon per frequency (per orbit) should be of order

$$\frac{dN_S}{d\omega} \sim \frac{\alpha}{\pi} \left( \frac{\omega}{\omega_0} \right)^{1/3},$$

where $\omega_0 = eH/E$ is the synchrotron frequency with $E$ the energy of the external charge. At very low frequencies bremsstrahlung of course dominates, but not at large frequencies. Integrating (3) up to some maximal frequency yields

$$N_S \sim \frac{\alpha}{\pi} \left( \frac{\omega_{\text{max}}}{\omega_0} \right)^{1/3}$$

which amounts to an energy loss $\Delta E_S \sim \alpha \omega_{\text{max}}^{4/3} / \omega_0^{1/3}$. The maximum frequency is bounded by the classical characteristic frequency $\omega_c = 3\gamma^3 \omega_0$. As a result, the maximum number of synchrotron photons emitted per cycle is $N_S \sim \alpha \gamma$, which is seen to grow linearly with the energy. The maximum synchrotron energy loss $\Delta E_S \sim \alpha \gamma^4 \omega_0$ grows cubically with the energy. In comparison, the power following from the Lienard expression (1) grows quadratically with the energy $\Delta E \sim \alpha E^2$, and since one cycle takes time proportional to $E$, both expressions agree.

The above qualitative reasoning is entirely classical. In quantum mechanics the photon back reaction cannot be ignored, especially for large energy losses with $\omega \sim E$ (in units $c = \hbar = 1$). Although in modern accelerators like LEP $\gamma \sim 10^5$ resulting into an enhancement in radiation per frequency that is about 15 orders of magnitude high, the photon energies are usually much smaller than the energy of the rotating particle and the back reaction can be ignored.

In the QCD case to be considered below, this is not true. Using quantum cutoff $\omega_{\text{max}} = E \ll \omega_c$, we then find that the number of bremsstrahlung photons grows logarithmically with the energy $N_B \sim \alpha \ln (E/\omega_{\text{min}})$ while the energy loss grows linearly with the energy $\Delta E_B \sim \alpha E$. These results are in total agreement with the ones obtained through standard quantum calculations such as Feynman graphs. For synchrotron radiation a cut at $\omega_{\text{max}} = E$ yields

$$N_S \sim \alpha \left( \frac{E^2}{eH} \right)^{1/3}$$

and the energy loss for a fixed width of the field region $\Delta z$ (rather than a circle) [4]

$$\Delta E_S \sim e^2 (eH)^{2/3} \Delta z E^{2/3},$$

which is seen to grow less than linearly. As a result, the relative energy loss by synchrotron radiation $\Delta E_S/E \sim 1/E^{1/3}$ is found to decrease with energy, although with a small power.

The present qualitative estimates are in agreement with the full quantum calculations to be described below. They show that bremsstrahlung radiation dominates over synchrotron radiation at large energies, although with a small relative power of 1/3. However, we recall that subsequent bremsstrahlung radiations at small frequencies...
angle interfere with each other and are further reduced by the Landau-Pomeranchuk-Migdal (LPM) effect, while the synchrotron emitted quanta are lost once and for all without further suppression.

C. Classical fields in heavy ion collisions

Synchrotron-like radiation may take place in QCD settings whenever classical and strong non-abelian fields can be formed. We think that such strong semiclassical gauge fields with amplitudes $A \sim 1/g$, naturally arise in the early stages of ultrarelativistic heavy-ion collisions. Currently, there are two first principle QCD reasonings in favor of prompt and strong non-abelian gauge fields in heavy ion collisions.

The first reasoning by McLerran and Venugopalan [7] suggests that the prompt phase of a heavy ion collision is a Color Glass Condensate (CGC). Schematically, consider a small element $\Delta x_\perp$ of the transverse plane defined by the disk-shaped boosted heavy nuclei. At high energy, the hadronic structure functions increase at small parton momentum fraction $x = p/E \to 0$. This increase results in an increase in the density $n_Q$ of color charges in the transverse 2-dimensional plane. The number of charges becomes ultimately large $N_Q = n_Q \Delta x_\perp \gg 1$, with a total charge of order $Q \sim \sqrt{N_Q}$. The large initial color charge $Q$ is at the origin of the classical color field. The smallest transverse size $\Delta x_\perp \sim 1/Q^2$ is fixed by the parton saturation scale $Q_s$ reached when the gauge field amplitude satisfies $\partial A \sim g A^2$. In this approach, the virtual classical fields are attached to the nuclear wavefunction and become real after being stripped off by the heavy ion collision.

The second reasoning suggests rather that the heavy ion collision converts the virtual classical vacuum fields to real fields. In other words the classical fields originate from the wavefunctional of the QCD vacuum. Yang-Mills fields have a rich topological structure in the vacuum, and insights from electroweak theory have argued recently [10–13] using arguments in favor of prompt and strong non-abelian gauge fields in heavy ion collisions. Schematically, color charge $Q$ is attached to the nuclear wavefunction whenever classical and strong non-abelian fields can tunnel into the virtual sector.

The QCD vacuum is filled with relatively small instantons (and also antiinstantons), with an average size $\rho \sim 1/3$ fm which is small in comparison to their relative separation which is of the order of 1 fm. The QCD vacuum is characterized by the small dimensionless instanton diluteness parameter $k = n\rho^4 \sim 0.01$. Therefore and immediately upon their release the QCD sphalerons are expected to be in a dilute phase as well. When produced in bunch like in a heavy ion collision, the QCD sphalerons evolve pretty much unshattered for a time of the order of 1 fm before they collide and get destroyed. More details for this process will be given below.

In a recent paper by one of us [14] the idea of jet quenching on coherent classical fields has been first discussed. The importance of the synchrotron-like radiation is due to both of us and was briefly advertised in [11]. The present paper elaborates further on this idea and presents detailed quantum calculations for jet quenching by synchrotron-like radiation.

II. QCD SYNCHROTRON RADIATION

In this section we proceed to estimate the QCD synchrotron radiation in a constant and Abelian-like chromomagnetic field. We will derive the exact classical and first quantum correction in the regime $\omega/E < 1$, and will provide an approximate expression for all frequencies $\omega$.

The problem of quantum synchrotron radiation in QED was addressed in a fundamental way by Schwinger [16] using the mass operator formalism. In this section, we extend this approach to the quantum synchrotron radiation in QCD. Two essential differences between the QED and QCD problem: i. the non-Abelian nature of the charge in QCD; ii. the emitted radiation also undergoes magnetic deflection. A quantum calculation is required in strong chromomagnetic fields owing to potentially large recoil corrections, essential for large jet quenching. The power radiated will be sought through the mass operator as

$$-\frac{1}{E} \Im M_{aa} = \int \frac{d\omega}{\omega} P_{aa}(\omega)$$

(8)

after pertinent kinematical identifications.

A. Abelian Chromomagnetic Field

For simplicity, we consider QCD synchrotron radiation in a constant and homogeneous chromomagnetic field

$$G^\mu_\nu(x) = \delta^{a8} G_\mu^a$$

(9)
where the abelian field strength corresponds to a constant magnetic field in the 3-direction, \( G_{12} = -G_{21} = H \). The background gauge field associated to (3) is

\[
A^a_{\mu}(x) = \delta^{a8} A_{\mu}(x) = \delta^{a8} \delta_{\mu 2} H x_1 .
\]

With our choice of the chromomagnetic background along the 8th color direction, the quarks and gluons can be diagonalized. The diagonal quarks in the fundamental representation carry color \((a = 1, 2, 3)\)

\[
e_a = g (T^a)_{aa} = \frac{g\sqrt{3}}{6} (1, 1, -2)
\]

and the diagonal gluons in the adjoint representation carry color

\[
g_A = (-1)^A \frac{g\sqrt{3}}{2},
\]

for \( A = 4, 5, 6, 7 \) and \( g_A = 0 \) for \( A = 1, 2, 3, 8 \). These two cases, as will be shown below, lead to qualitatively different radiation. The second case is basically QED-like.

Quantum synchrotron radiation will be sought for quarks and gluons interacting to all orders in \( H \) but to leading order in \( \alpha = g^2/4\pi \) between the quantized fields. We now present briefly the spin-1/2 case and discuss extensively the spin-0 case. In the semiclassical limit, both spins radiate at the same rate.

**B. Spin 0, 1/2 Jets**

Following Schwinger [16], to lowest order in perturbation theory the quark mass operator in the chromomagnetic field reads

\[
\mathbf{M}_{aa} = ig^2 (T^A)_{ab} (T^A)_{ba} \int \frac{d^4 k}{(2\pi)^4} \int_{-\infty - i0}^{\infty - i0} \frac{ds}{\cos(g_A H s)}
\]

\[
\times e^{-is(k^2 - k^2_{12} \tan(g_A H s)/(g_A H s) - 1)} (e^{2g_A sG})_{\mu\nu} \Phi(g_A)
\]

\[
\times \gamma^\mu (\gamma \cdot (\Pi_b - k) - m)^{-1} \gamma^\nu .
\]

We have defined the quark 4-momentum operator as

\[
\Pi_b(\mu) = i \partial_\mu - e_b A_\mu(x)
\]

and the Bohm-Aharanov line

\[
\Phi(g_A; x, y) = e^{i\frac{2A_x}{\sqrt{3}} (x_1 + y_1)(x_2 - y_2)} .
\]

The Bohm-Aharanov phase enforces gauge-invariance in the mass operator, but does not contribute to the radiation. Indeed, for an initial color-a quark emitting a color-b quark plus a color-A gluon,

\[
\Phi(e_a; x, y) = \Phi(e_b; x, y) \times \Phi(g_A; x, y)
\]

showing that the Bohm-Aharanov line in (13) on the gluon, can be redistributed to compensate the analogue ones on the quarks. This procedure will be assumed throughout, and thereby the gluon \( \Phi \) contribution reshuffled.

The occurrence of the synchrotron poles in the gluon propagator (13) implies that the \( s \)-integration is infinitesimally shifted below the real axis in the complex \( s \)-plane. The prescription follows the causal prescription for the free propagator,

\[
\frac{1}{k^2 + i0} = i \int_{0 - i0}^{\infty - i0} ds e^{-is k^2} .
\]

Also, since \( G \) is an antisymmetric matrix, its eigenvalues \( \pm iH \) are complex. The color precession factor is

\[
e^{2g_A sG} \rightarrow e^{\pm is(2g_A H)} .
\]

There is a subtlety due to the positive sign in (18) for \( k = 0 \), which is the analogue of the tachyonic mode of a spin-1 coupled to a constant chromomagnetic field in the first quantized approach. This mode is at the origin of the well-known Savvidy instability in QCD [17]. What it says, is that in QCD the chromomagnetic fields themselves are in general unstable against gluon emission. Although this phenomenon is interesting by itself, it clearly has nothing to do with jet energy losses.

The technique developed by Schwinger [16] can now be applied to (13) to derive the power radiated in a QCD synchrotron process whereby an energetic quark radiates through a chromomagnetic field. To avoid the unnecessary algebra triggered by the spin content of the quark, we present the results for the spin-0 case instead.

For a scalar quark in the fundamental representation, the analogue of (13) is

\[
\mathbf{M}_{aa} = ig^2 (T^A)_{ab} (T^A)_{ba} \int \frac{d^4 k}{(2\pi)^4} \int_{-\infty - i0}^{\infty - i0} \frac{ds}{\cos(g_A H s)}
\]

\[
\times e^{-is(k^2 - k^2_{12} \tan(g_A H s)/(g_A H s) - 1)} (e^{2g_A sG})_{\mu\nu} \Phi(g_A)
\]

\[
\times (\Pi_a - \Pi_b)^\mu ((\Pi_b - k)^2 - m^2)^{-1} (\Pi_a - \Pi_b)^\nu ,
\]

modulo counter-terms. The arrows on \( \Pi \)'s indicate the direction of the derivative. On mass-shell we expect \( \Pi_a \sim \Pi_b + k \), this will hold in the classical limit.

**C. Power Radiated**

For spin-0, the power radiated follows from (8). Following Schwinger [16] we obtain the chromomagnetic synchrotron emission by a scalar quark in the classical limit in the following form

\[
\mathbf{P}_{aa}(\omega) = \frac{-\alpha}{\pi} (T^A)_{ab} (T^A)_{ba}
\]

\[
\times \omega \text{Im} \int_{0 - i0}^{\infty - i0} \frac{d\tau}{\tau} e^{-i(\omega \tau^3)/(24\omega)}
\]

\[
\times \left( \frac{m^2}{E^2} + \frac{1}{2} \omega^2 \tau^2 \right) e^{-i\omega^2 \tau^3/(24\omega)}
\]

\[
\times \left( \frac{m^2}{E^2} + \frac{1}{2} \omega^2 \tau^2 \right) e^{-i\omega^2 \tau^3/(24\omega)}
\]
where in (24) the $H = 0$ subtraction is not explicitly
shown but implied. The quark synchrotron and gluon
rescaled frequencies are $\omega_a = e_a H/E$ and $\omega_A = g_A H/E$
respectively.

In carrying out (21) the emitted gluon recoil effect on
the jet was ignored, and so the result is entirely classical.
We have checked that the gluon recoil effect amounts in
the first order to the shift
\[
\frac{1}{\omega} \rightarrow \left( \frac{1}{\omega} - \frac{1}{E} \right)
\]  
(21)
in the combination $P_{aa}/\omega$ (the gluon multiplicity),
thereby generalizing Schwinger’s first quantum correction
in QED to the QCD case. Of course, this substitution is
not the complete quantum answer, but will be discussed
below as an approximation.

Before we discuss the complete results, let us comment
the integrand of (21). The last exponent is due to charge
curving, and is the same as in QED. It provides a rapidly
oscillating phase at large $\omega$ and a corresponding cutoff.
The first exponent in the integrand of (21) is new. It
stems from the gluon rotation in the chromomagnetic
field. The phase follows from the transverse contribution
of the gluon propagator as is evident from (19). The
cosine in the denominator exhibits poles for
\[
\tau_n = \frac{\pi \omega}{E \omega_A} (2n + 1)
\]  
(22)
which are the gluon synchrotron orbits (classically the
 gluon spin and the tachyon problem drop). The second
contribution in (21) is the quark synchrotron contribution
as in QED.

Rewriting $1/\cos A$ as a geometrical sum of all powers
e$^{1A}$, we may bring (21) in the form of a sum
\[
\omega^{-1} P_{aa}(\omega) = \frac{\alpha}{\pi} (T^A)_{ab} (T^A)_{ba} \frac{2m^2}{\sqrt{3} E^2}
\times \sum_{n=0}^{\infty} e^{-i \pi n (1 - i 0)} \left( F(\xi_n) + 2\kappa_n K_{2/3}(\xi_n) \right)
\]  
(23)
with
\[
F(\xi) = \int_{\xi}^{\infty} dt K_{5/3}(t),
\]
\[
\left( \frac{\xi_n}{\xi} \right)^2 = \frac{(1 + (2n + 1)(2\beta)/(3\xi^{2/3}))^3}{(1 + \lambda^2)},
\]
\[
\kappa_n + 1 = \frac{1 + (2n + 1)(2\beta)/(3\xi^{2/3})}{(1 + \lambda^2)},
\]  
(24)
and
\[
\frac{\beta}{\lambda} = \left( \frac{3\omega_b}{2\omega} \right)^{1/3},
\]
\[
\lambda = \frac{E\omega_A}{\omega_b},
\]
\[
\xi = \frac{2\omega}{3\omega_b} \left( \frac{m}{E} \right)^3.
\]  
(25)
Again the quantum corrections follow from (23) through
the substitution (22) on the RHS. The K’s are modified
Bessel functions. The sum over $n$ in (23) sums over synchrotron
orbits of width $-i0$ except for the lowest orbit
which is zero. It is reminiscent of the sum over ‘Landau
levels’ in the Schroedinger formulation.

D. Small and Large $\omega$

The preceding results are easily analyzed for large and
small frequencies $\omega$. Since the abelian analysis with
$g_A = 0$ is known from QED, we focus on the non-abelian
part with $g_A \neq 0$. We will show that the non-abelian
contribution to the synchrotron radiation is strongly sup-
pressed at small $\omega$ due to strong ‘incoherence’ effects pro-
duced by the deflected radiation. At large $\omega$ the non-
abelian contribution is equal to the abelian contribution.
Specifically, the $g_A \neq 0$ contribution for small $\omega$ reads
\[
P_{aa}(\omega) = \frac{\alpha}{\pi} (T^A)_{ab} (T^A)_{ba} \frac{4\sqrt{\pi}}{9g\omega} \left( E\omega_A \right)^{3/4} \left( \frac{m}{E} \right)^2
\times \sum_{n=0}^{\infty} (-1)^n (2n + 1)^{3/4} e^{-\frac{3}{4}(2n+1)^{1/2}\sqrt{E\omega_A}/\omega},
\]  
(26)
which is characterized by an essential singularity at $\omega = 0$.
The $g_A \neq 0$ contribution for large $\omega$ reads
\[
P_{aa}(\omega) = \frac{\alpha}{\pi} (T^A)_{ab} (T^A)_{ba} \frac{2\sqrt{\pi}}{9} \left( \frac{m\omega}{E\omega_b} \right)^{5/2} \omega_b e^{-(m/E)^3} (2\omega/3\omega_b).
\]  
(27)

At small $\omega$ the non-abelian radiation is of order
$e^{-\#\omega/\sqrt{\omega}}$ which is much smaller than the abelian
radiation of order $\omega^{1/3}$. The reason is that the emitted non-
abelian gluon brings about its own phase which strongly
adds to the phase incoherence at small $\omega$ as is clearly
seen in (13) and (15). At large $\omega$ both the abelian and
non-abelian radiations are comparable and of order
$\omega^{5/2} e^{-2\omega/(k\omega\gamma^3)}$. Indeed, we note that (27) is inde-
dent of the gluon charge $g_A$. For ultrarelativistic jets, the
radiation frequencies are in the range $0 \leq \omega \leq E$. Thus
$\omega/\omega_b \leq \gamma$ which is way below the maximum of $\gamma^3$.
In light of the present observations we conclude that most of
the jet radiation is emitted through the ‘abelian’ part of
the gluon charge and in the small frequency range way
below the synchrotron maximum since $E/\omega_b \ll \gamma^3$. The
‘non-abelian’ part only enters as a correction.

E. Energy Loss

The ‘abelian’ part of the jet radiation follows from the
summation over $A = 1, 2, 3, 8$ for which $g_A = 0$, thus
e$a = e_b$. In this case (23) simplifies to the QED-like answer
\( \omega^{-1} P_{aa}(\omega) = \frac{\alpha}{\pi} (T^A)_{ab} (T^A)_{ba} \frac{m^2}{\sqrt{3} E^2} F(\xi) \) (28)

since \( \xi_u = \xi \) and \( \kappa_u = 0 \). In this case, the total power emitted follows by integrating (28) over the gluon frequency, explicitly including the recoil effects (23).

\[
P_{aa} = \frac{\alpha}{\pi} (T^A)_{aa} (T^A)_{aa} \frac{m^2}{\sqrt{3}}
\times \int_0^E d\omega \frac{\omega}{E} F(\xi_{corr})
\] (29)

with a different (quantum corrected) \( \xi_{corr} \)

\[
\xi_{corr} = \frac{2}{3} \left( \frac{m}{E} \right)^3 \frac{\omega / \omega_a}{1 - \omega / E}.
\] (30)

In Fig. 2 we compare two spectra for some particular selection of parameters. The general result for (29) can be obtained by expanding around the classical result with the first quantum correction included

\[
P_{aa} \approx \frac{\alpha}{\pi} (T^A)_{aa} (T^A)_{aa} \ C (\omega_a E^2)^{2/3}
\] (31)

with the constant

\[
C = -\frac{\pi (3/2)^{6/3}}{10 \sin(\pi/3) \Gamma(-2/3)} \frac{2^{5/3}}{21} (1 - \frac{8}{21}) \approx 0.52.
\] (32)

The contributions in \((1 - 8/21)\) are the classical contribution and the first quantum recoil correction respectively. So the recoil of the emitted gluon decreases the net radiation by a factor 0.62. The ratio of the contributions of the classical to recoil corrected spectra is 0.48. Within few percents these results agree with direct numerical estimates of the integrals including the approximate all orders quantum recoil corrections. This justifies a posteriori the use of the first order correction in our calculations.

In order to further compare losses related with emission of QED-like and QCD-like gluons, we show in Fig. 2 the leading \((n=0)\) integral

\[
I = \int_0^\infty \frac{d\tau}{\tau} \left( \frac{m^2}{E^2} + \frac{1}{2} \omega_b \tau^2 \right)
\times \sin \left[ \frac{(E \omega) \tau^3}{24 \omega} + \frac{E \omega \tau}{2 \omega} + \frac{\omega m^2 \tau}{2 E^2} + \frac{\omega \omega_b \tau^3}{24} \right]
\] (33)

with the \( H = 0 \) subtraction implied, for \( E = m = H = 1 \) and 4 different sets of charges (see the figure caption).

III. APPLICATIONS TO HEAVY ION COLLISIONS

In this section we will first recall for completeness some qualitative arguments regarding jet quenching in heavy ion collisions, most of which have already been discussed.
in [1]. We will then use the above synchrotron radiation results to assess jet energy loss in the color glass condensate approach and the exploding sphaleron approach.

A. Introduction to Jet Quenching

Jet quenching is a sort of “tomography” of the prompt excited system, created in high energy heavy ion collisions. Even very hard jets radiate and lose some energy during their passage through the system, thereby providing information about the early stages of the collision.

The so called quenching factor $Q(p_t)$ is defined as the observed number of jets normalized to the expected number of jets calculated in the parton model without account for final state interactions [11] ⁵. It is usually assumed that hard QCD probes are under good theoretical control, and that one can assess the initial jet production reliably.

Experimentally, jet reconstruction in a heavy ion environment is very difficult to achieve. Therefore, all currently reported results for jet quenching refer to the observed/expected ratio of the yields of single hadrons. Furthermore and in so far, large $p_t$ means $p_t = 2$-6 GeV. Which part of this transverse momentum comes from genuine high energy jets is anybody guess. The first direct evidences for jets have been recently reported by the STAR collaboration [18], whereby a second particle conversion to produced number of jets calculated in the parton model without account for final state interactions [11] ⁵. It is usually assumed that hard QCD probes are under good theoretical control, and that one can assess the initial jet production reliably.

In the early theoretical studies on the subject [11] ⁵, a rather modest jet re-scattering in the Quark-Gluon Plasma (QGP) has been considered. Accounting for the radiation effect [24] has significantly increased expectations for the magnitude of the result, while accounting for the Landau-Pomeranchuck-Migdal (LPM) effect [21] has somewhat decreased the magnitude of the result. We will not discuss this involved subject, but only recall that the expected quenching factor from such studies is $Q(p_t) = 0.5$-0.7 for jets with $p_t = 10$-20 GeV.

Experimentally, a relatively modest jet quenching has been first observed in deep inelastic scattering for a forward jet going through cold nuclear matter (for recent discussion and references see [22]). The heavy ion data at the CERN SPS has also shown modest quenching effects, but already the very first RHIC data [23] (especially for $\pi^0$ from the PHENIX collaboration) have shown that quenching of jets is very strong, with $Q(p_t) < 1/3$. Subsequent discoveries that (i) at $p_t > 2$ GeV protons and anti-protons dominate the charge particle spectra; (ii) that the azimuthal asymmetry remains very strong even at large $p_t$; (iii) that development of very successful hydro and/or cascade description of spectra without jets even at $p_t = 2$-3 GeV, have all led to suspect that the real jet quenching factor may even stronger. Furthermore, one of us even found [24] that the very high degree of azimuthal asymmetry observed by the STAR experiment at $p_t = 2$-6 GeV [18] cannot be reproduced by any amount of jet quenching, no matter how strong.

One technical but important point made in the last paper in [24], is that when the quenching is strong it cannot be evaluated using the mean energy loss. Specifically, the quenching factor can be seen as the ratio of produced-and-quenched to produced spectra

$$Q(p_t) = \frac{\int d\epsilon \frac{dN}{dp_t^2}(p_t + \epsilon)}{\int d\epsilon \frac{dN}{dp_t^2}(p_t)}$$  \hspace{1cm} (34)$$

where $\epsilon$ is the energy lost in the medium and $D(\epsilon)$ its normalized distribution. For small $\epsilon$ one can expand it to first order, obtaining a correction proportional to the mean energy loss $\langle \epsilon \rangle = \int \epsilon d\epsilon D(\epsilon)$. However, because the spectrum is so steep, this is only valid when $\epsilon/p_t$ is not larger than few percent.

Therefore and for a qualitative assessment of the magnitude of the effect needed, we suggest a different simple approximation. Using a power parameterization of the spectrum

$$\frac{dN}{dp_t^2} \sim \frac{1}{p_t^n}$$  \hspace{1cm} (35)$$

for both the observed and “hard” distributions, we obtain

$$Q(p_t) = \int d\epsilon D(\epsilon) \left( \frac{1}{1 + \epsilon/p_t} \right)^n \sim \int d\epsilon D(\epsilon) e^{-\epsilon/p_t}$$  \hspace{1cm} (36)$$

instead of [24]. Using a simple delta-like distribution peaked at some fractional loss,

$$D(\epsilon) = \delta(\epsilon - \kappa p_t)$$  \hspace{1cm} (37)$$

we have $Q(p_t) = 1/(1 + \kappa)^n$. With $n \approx 12$ in the few-GeV domain at RHIC energies, a jet quenching by one order of magnitude would correspond to $\kappa \sim 1/4$. This means that a mean loss of about 15-20% of the produced jet momentum is sufficient. However, this conclusion is oversimplified. As one can see from [30] the quenching factor is dominated by small losses $\epsilon/p_t < 1/n \sim 1/12$. What this means is that what matters is the probability to escape with as small losses as possible.

This conclusion changes the relative role of early (synchrotron-like) versus late (multiple bremsstrahlung with LPM) effects. While the latter can be very large for specific geometry (LPM energy loss [21] is $\Delta E \sim L^2$ where $L$ is the path in matter), the integral [30] would be dominated by surface emission with small $L \sim 1$ fm or so. Early effects emphasized in this paper, even if producing

⁵ Effects due to initial state interaction should be also included. What this means is that the parton distribution functions should be nuclear rather than hadronic, following from lepton-nuclei experiments. Parton rescattering in nuclei at the origin of the so called Cronin effect, should also be included in the expected yield.
less average losses, are expected to affect the probability \( D(\epsilon) \) at small \( \epsilon \), preventing easy escape of some jets. Detailed numerical simulations (which are well beyond the limits of this work) are needed to understand their relative role.

Summarizing this subsection, we say that the traditional approach in which excited matter such as a QGP consists of a collection of uncorrelated quarks and gluons acting as scattering centers for the high energy partons, has difficulties accounting for the large jet losses reported at RHIC. This is the primary motivation for considering synchrotron-like radiation in this work.

![Diagrams](image)

**FIG. 3.** The l.h.s. shows three diagrams illustrating QCD bremsstrahlung radiation: the small circle represents the source of the perturbative field. The r.h.s. shows three similar diagrams illustrating QCD bremsstrahlung radiation (a,b) and synchrotron-like radiation (c). The strong chromomagnetic field is assumed to be in between two thin vertical lines, where the full (dressed) propagators are indicated by thicker lines.

### B. QCD bremsstrahlung

For completeness we start with a brief reminder of ordinary QCD bremsstrahlung. This effect was first considered by Bertsch and Gunion [25], who calculated the three diagrams shown in the l.h.s of Fig. 3 for soft gluon emission. Adding their squares at high collision energies relative to both the momentum transfer \( \vec{t}_i \) from the target parton and also the transverse momentum of the radiated gluon \( \vec{q}_t \), they have shown that the number of gluons emitted is

\[
\frac{dN_g}{dyd^2q_t} = \frac{C_s^2 \alpha_s}{\pi^2} \frac{P_t^2}{Q_t^2 (\vec{q}_t - \vec{t}_i)^2}
\]

(38)

where \( C_s^2 \) is a color factor (3 for \( qq \) scattering with gluon emission). Note that at zero momentum transfer \( \vec{t}_i = 0 \) the radiation vanishes. Also, when the denominators become small, they generate two cones of radiation, along the initial and final direction of the jet. Integrating this result over the rapidity \( y \) and the transverse momentum \( q_t \) of the gluon, yields the standard logarithmic factor.

In our case the field crossed by a jet parton is classical and non-perturbative with \( A \sim 1/g \), and the radiation is described by the modified diagrams shown in the r.h.s. of Fig. 3. The motion inside the field is described by fully dressed propagators, and the power radiated from the slab has been described above. Assuming the chromomagnetic field to be confined to a finite slab, requires that we also add the bremsstrahlung diagrams from the in and out motion as illustrated in the first two diagrams of Fig. 3. The standard QCD variant of the QED Weizsacker-Williams (WW) approximation can be used, with the so called “parton-in-parton” DGLAP splitting functions [11]

\[
\frac{\partial x_1}{\partial \ln \mu} (\epsilon, \mu) = \frac{\alpha_s(\mu)}{2\pi} P_{ij} (x)
\]

(39)

where \( x \) is the standard parton momentum fraction and i,j=q,g. The splitting functions are known for all values of \( x \), and the total energy loss for quark/gluon due to splitting is [11]

\[
\frac{\epsilon_q}{p} \approx 0.28 \alpha_s (Q_{\text{high}}^2/Q_{\text{low}}^2)
\]

(40)

\[
\frac{\epsilon_g}{p} \approx 1.0 \alpha_s (Q_{\text{high}}^2/Q_{\text{low}}^2)
\]

(41)

where the second integral was regulated by setting \( x_{\text{max}} = m^*/P \), taken to be 0.95 in the estimate. The parton-to-parton splitting happens twice, i.e. in and out. On the way in, the 2 scales that define the DGLAP evolution of the jet are the kick in the scattering process and the parton virtuality while hitting the magnet. On the way out, the 2 scales are the kick from the slab and the final scale of the parton in matter or its hadronization scale (whichever is larger).

We close this summary section by the following qualitative comments: i. The strong bending of the partons happens rarely, so it is not subject to the LPM effect; ii. The in and out bremsstrahlung effect depends weakly (logarithmically) on the field strength provided it is large enough to allow for the separation of the 2 cones of radiation; iii. The magnitude of the relative energy loss by bremsstrahlung alone is of order \( \Delta E/E \sim 1/4 \), and
maybe alone sufficient to explain the expected energy loss for gluon jets at RHIC; iv. The contribution from diagram (a) is in general small if the jet has been just produced in a hard collision, since the virtual field does not have sufficient time to form.

C. Jet quenching in the Color Glass Condensate

If the initial excited glue is not a set of incoherent gluons with occupation numbers \( n \sim O(1) \), as is the case in an equilibrated QGP, but a coherent classical field with large occupation numbers \( n \sim O(1/g^2) \), the radiation losses are synchrotron-like. Coherence helps, because all coherent quanta work in the same direction in space and color space, providing larger acceleration.

To assess the amount of synchrotron radiation in the CGC phase, we recall some useful numbers from the numerical analysis of the SU(3) version of the CGC carried in [24]. At RHIC energies the saturation scale \( Q_s \) was found to be 1.3 GeV. The initial classical CGC field was found to abelianize in a time \( \tau_{\text{CGC}} \) of order \( Q_s \tau_{\text{CGC}} \sim 3 \). In this regime, the gluon energy density was found to be \( \epsilon/Q_s^4 \approx 0.17/g^2 \) with an approximately thermal momentum distribution. The transverse energy per quantum was found to be 1.66\( Q_s \), resulting in an effective temperature of 1 GeV. This is of course only apparent as the underlying evolution is classical and originates from a coherent state. The field strength \( F \) (the r.m.s. combination of electric and magnetic fields) is about

\[
g F \sim 0.58 Q_s^2 \sim 1 \text{ GeV}^2
\]

giving a quark with \( e_a = g \sqrt{3}/6 \) a kick of order \( e_a F \tau \sim 0.66 \) GeV, and about twice that for a gluon.

Before substituting these numbers into our expressions for the synchrotron radiation loss, we need to do some relevant color sums

\[
C_A = \sum_a (T^A)^2_{aa} |e_a|^{2/3}
\]

The values for “penetrating gluons” of kind 3 and 8 are 0.30 and 0.22, respectively. For all gluons, we have \( \sum_{A=1,8} C_A = 2.06 \), with about 1/2 originating from the undeflected gluons and 1/2 originating from the deflected ones (of course with the extra penalty factor at low \( \omega \) as explained above). For the estimate to follow we will use a color factor of 1.5.

Using the CGC numbers just quoted, we find that the relative energy loss of a quark by synchrotron radiation in a time \( \tau_{\text{CGC}} \) is

\[
\frac{\Delta E_{\text{CGC}}}{E} \approx 0.3 \left( \frac{H}{1 \text{ GeV}^2} \right)^{2/3} \left( \frac{\Delta \tau_{\text{CGC}}}{0.5 \text{ fm}} \right) \left( \frac{1 \text{ GeV}}{E} \right)^{1/3}.
\]

The gluon loss is about twice the quark loss.

D. Jet Quenching on the Exploding Sphalerons

The energy and the Chern-Simons number of the released (turning) coherent state is

\[
E = \frac{3\pi}{4\alpha\rho} \mathcal{E}
\]

\[
N_{\text{CS}} = \frac{1}{2} \mathcal{N}
\]

In so far, the dimensionless parameters have been determined in two ways. First, by minimizing the QCD potential for fixed Chern-Simons number with the result \([10]\)

\[
\mathcal{E} = (1 - \kappa^2)^2
\]

\[
\mathcal{N} = \text{sign}(\kappa)(1 - |\kappa|)^2(2 + |\kappa|)/2
\]

Eliminating \( \kappa \) yields the potential profile \( E(N_{\text{CS}}) \). Second, by maximizing the partial parton-parton cross section with the result \([12]\)

\[
\mathcal{E} = (E/M_S)
\]

\[
\mathcal{N} = (E/M_S)^{2/5}
\]

At the sphaleron point the partial cross section is maximum, with \( \mathcal{E} = N \). Only this case will be considered here. Using instanton vacuum physics we obtain a sphaleron mass \( M_S = 3 \) GeV and a sphaleron size \( \rho = 1/3 \) fm.

Upon release in Minkowski space, the sphaleron state evolves classically in real time through the classical Yang-Mills equations, as was originally done in the electroweak theory \([27]\). For SU(2) Yang-Mills, this evolution was recently carried out in [10] both numerically and analytically. The turning states were found to explode into thin shells of coherent gluonic fields. For our purposes, we just recall that the shell for \( t, r \gg \rho \) has a very simple radial energy density

\[
4\pi r^2 e(r, t) = \frac{8\pi}{g^2 \rho^2} \left( \frac{\rho^2}{\rho^2 + (r - t)^2} \right)^3
\]

At large time \( t \gg \rho \) the corresponding gauge field is purely transverse, with equal chromoelectric and chromomagnetic fields. The prompt sphaleron configuration released in parton-parton scattering carries initially a very strong chromomagnetic field,

\[
\sqrt{H} \sim \left( \frac{2M_S}{\rho^2} \right)^{1/4} \sim 1 \text{ GeV}.
\]

In the early phase of the prompt process in heavy ion collisions, the escaping sphalerons form a dilute gas. So unlike the CGC they cannot affect most of the jets initially for times \( t \sim \rho \). They do affect them as they expand into exploding shells. The net synchrotron radiation loss involves also the transverse density of sphalerons per unit rapidity \( n_S \) and their typical collision volume \( \sigma(t) \, dt \). Specifically,
\[ \Delta E = \int P(t) n_S \sigma(t) dt \sim \int dt t^{-2/3 + 2} \quad (50) \]

where we used the cross section \( \sigma \sim t^2 \) and the radiation loss \( P \sim H^{2/3} t^{-2/3} \). The result formally diverges for large times. However, the above reasoning is only valid till the single shell expansion remains coherent. As we now show, the originally dilute gas of shells quickly evolves into a foam-like structure for times \( t \sim 2-3 \rho \) providing a natural cutoff in the time integral.

Recent estimates of the number of clusters produced in pp and AA collisions as a function of the collision energy and centrality, are still rather uncertain. The theoretical calculations of the cross section such as [12] are carried to only exponential accuracy, while phenomenological studies such as [28] have only resulted into an upper estimate. Assuming the whole growth with \( H \) valid till the single shell expansion remains coherent. As we now show, the originally dilute gas of shells quickly evolves into a foam-like structure for times \( t \sim 2-3 \rho \) providing a natural cutoff in the time integral.

The chief idea of this work is that if the initial stage of a heavy ion collision produces a coherent classical field rather than an a gas of incoherent quanta, one should reconsider the theory of all prompt processes (Drell-Yann, photons, dileptons, heavy quarks, ...), including the current theory of jet energy losses. Instead of multiple small angle scattering subject to the Landau-Pomeronchuk-Migdal suppression, we have synchrotron-like QCD radiation. The radiation is enhanced by the coherent classical fields, providing larger acceleration and radiation compared to independent quanta.

There are currently two mechanisms for the formation of strong and prompt classical color fields in heavy ion collisions. First, the color glass condensate (CGC) [8] is a classical Weizacker-Williams field of virtual gluons initially part of the wavefunction of the colliding nuclei, that is made real by the collision. Second, the exploding sphaleron-like clusters which are the remnants of (singular) instantons in the QCD vacuum. The clusters are not only coherent, but evolve into thin shells of strongly localized fields and become foam-like. Any jet has a probability of about 1 to interact with walls of exploding clusters.

We have shown that synchrotron radiation loss from a 6 GeV quark jet is about 0.17 in the CGC and about 0.12 in the exploding sphalerons. Gluon jet losses are about twice larger. The corresponding bremsstrahlung radiation loss on the in and out motion results also in a loss of a similar magnitude. At larger jet energies, bremsstrahlung loss dominates over synchrotron loss, although the latter is found to decrease slowly with increasing energy (as \( 1/E^{1/3} \)). All in all, the mechanisms of jet quenching considered here provide in total about 20-30\% loss, which is about consistent with the empirically reported jet quenching at RHIC.

The present considerations of jet quenching in the context of the color glass condensate or the exploding sphalerons is rather schematic, with quantum effects carried only to leading order. As emphasized above, however, what really matters is not the average radiation loss for a jet, but rather the probability for a jet to escape without losses. To assess this, detailed simulations with realistic nuclei geometry are needed. Also, as suggested in [13, 14], it is possible to experimentally measure whether the radiated gluons are produced in a narrow cone around the jet (bremsstrahlung) or not (synchrotron). One can also do tagged jets by measuring photon-jet correlations, and search for acomplementarity and \( p_t \) disbalance. Clearly much more theoretical work is needed to make the present estimates more quantitative.

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**APPENDIX A: CLASSICAL REDUCTION**

In this Appendix we give some helpful steps leading to the classical formulae [20]. We will show how to reduce the gluon part, since the quark part follows from
Schwinger’s paper [16] to which we refer to. We first exponentiate the scalar quark propagator in [19], combine it with the already exponentiated gluon propagator and change the proper-time variables (Feynman parametrization) to obtain

\[ \int_0^\infty ds \int_0^1 \frac{du}{\cos(g_A H s(1-u))} e^{-ism^2 u} e^{-is H_0} \times e^{-i \alpha(1-u) k_1^2 ((\tan(g_A H s(1-u))/g_A H s(1-u))-1)}, \]  
(A1)

for the propagators only. The proper-time Hamiltonian is

\[ H_0 = (k - u \Pi_0)^2 + u(1 - u) \Pi_b^2, \]  
(A2)

with a ‘mass-shell’ condition \( \partial \Pi_0 / \partial k \sim 0 \) leading to \( k \sim u \Pi_0 \) or \( k^2 \sim u^2 m^2 \) in the classical limit. This saddle point relation receives corrections at the quantum level.

Following [16] we introduce the key change of variables that facilitates the identification with the classical radiation problem [4]

\[ s = \frac{\tau}{2 \omega}, \]  
\[ u = \frac{\omega}{E} \ll 1, \]  
(A3)

where the last inequality will be relaxed through the first quantum correction. Using (A3) and the substitution \( k_1^2 \rightarrow \omega^2 \) valid to leading order in \( \omega/E \) (classical), we find the classical limit to the gluonic contribution to be

\[ e^{-i(E \omega_A \tau)/(24\omega)} \cos(E \omega_A \tau)/(2\omega) \]  
(A4)

which is the part quoted in [20]. The remaining quark part follows exactly Schwinger’s argument and will not be repeated here.

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