Systematic parametric design/calculation of the piston rod unit

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Abstract. In this article a modern and economic method for the strength calculation of the piston rod unit and its components under different operating conditions will be presented. Herefore the commercial FEA - Software will be linked with the company-owned calculation tools. The parametric user input will be followed by an automatic Pre- and Postprocessing. Afterwards the strength calculation is processed on all critical points of the piston rod connection, assisted by an extra module, based on general standards and special codes for reciprocating compressors. In this process most arrangements of the piston rod unit as well as the special geometries of the single-components (piston, piston rod and piston nut) can be considered easily. In this article the modeling of the notches, especially on the piston rod, piston as well as the piston nut will be covered in detail.

1. Introduction
The paper investigates the parametric design and calculation of all components of piston rod unit considering the contact properties of all clamped parts and different piston design. The piston rod unit generally includes piston, piston nut, ring and piston rod. On crankcase side the piston rod is connected to the crosshead.

These components are under extremely high loads during the operation of the reciprocating compressor. The forces acting on the components are the pre-load force, the gas force, the thermal related forces as well as the mass force. During one revolution of the crankshaft the gas and mass forces are changing their value and the direction. Additionally for each crankshaft angle the forces are different on different sections of the piston rod unit. That means the strength calculation must be performed for each angular step with the actual pressure and acceleration at the local section and for each notch on all components of the piston rod unit.

Furthermore the boundary conditions such as geometry, embedding of the contact surfaces, materials, strength grade, tight techniques as well as different operating cases of the reciprocating compressor are into consideration.

In this paper a systematic design and calculation of the piston rod unit will be presented. This consists of three main steps: structure modelling, (preprocessing), definition of boundary condition, meshing, loads apply, calculation for each crank angle and, post-processing (results). All steps are realized automatically using manufacturer calculation/design tools, commercial programming software VB6, C++, FEA-Software³, Scripting Language APDL³ (ANSYS Parametric Design Language) as well as guideline and standards.

The strength verification will be performed according the different guidelines¹,²,⁴,⁵.

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A calculation software was developed to perform an automatic strength analysis of the components of the piston rod unit. The special features of the software are:

- Reduction of the simulation time
- Increasing the quality of calculation
- Calculation of stress curves at each notch along the notch angle $\alpha$
- Systematic calculation/design and fault prevention through certified software.

2. Pre-processing
The software consists of several calculation modules and their interfaces between the components. The compressor data (like type, frame size, piston rod design, crosshead, connection rod etc) material-data of the main components (modulus of elasticity, ultimate tensile strength, yield strength, Poisson ratio, fatigue stress values, thermal expansion coefficient etc) can be generated automatically and internally from machine- and material databank. The basic module is the manufacturer program for the design/calculation of the reciprocating compressors. The interface enables every user after the selection and design of a specific compressor, to use the software for the piston unit calculation.

The main data/parameters are a result of the previous compressor selection:

- Stroke, type of piston (one piece, two piece, welded piston etc), compressor speed.
- Actual pressure variation over the crankshaft angle, in each cylinder compartments (HE, CE, idle or active compartment, SV-Unloader).
- Piston rod parameters: size, neck diameter, thread diameter, surface roughness, heat treatment.
- Piston nut data.

Data can be input/edited interactively by the user. Fig. 1 shows a screenshot of the user input form with the sketch of the two pieces piston and various input parameters.

- Notch geometry, radius, element type, element size or number of elements. Distance normal to the notch surface for the calculation of the stress gradient required for the fatigue analysis.

- Geometry of contact area between piston, piston rod and piston nut. Type and size of contact elements. Friction coefficient.
- Piston geometry (2D, 3D-model)
- Welded piston with ribs
- Temperature distribution.
- The assembly preload force of the unit

Hence the geometry and the loads of the piston rod unit are determined. The data are written to a temporary file. The next steps are the generating the FE-model, the definition of boundary condition, applying of the loads, the calculation, as well as the stress evaluation. All this steps are effected automatically. A special interface for the FEA-simulation in the script language (APDL) was generated.
Compressor - Software
- Compressor design/calculation
- compartment pressure calc.
- Main data

User
- Geometry, Material,
- Notch data, radius, element type, size...
- Contact parameters
- Preload force
- Control variable
- Input-file for FEA

Stress Evaluation
- Notch stress curves
- Stress verification
- Stress verif. on notches

Pre-Processing
- FEM - Modell
- Loads:
  - Prestress force
  - Pressure,
  - Temperature
  - Acceleration
  - Contact surface opt.

FEA

Post-Processing
- Stress results.
- Alternating stresses on piston rod
- Contact status
- Output-file.

Figure 1. User Input Form

Figure 2. Work process flowchart
3. Critical stress zones and contact areas

Figure 3 shows the critical high stress zones 1 to 9 and the contact areas I, II, III,…VII.

The critical notches 1, 2 and 5 are in the piston rod. Notch 1 on crosshead side of piston rod is a conical shoulder with radius. Notch 2 is a square shoulder with fillet (radius) in circular shaft. 5 represents a V-notch in a circular shaft (thread).

Notches 3, 4, 9 are located in bottom half of the piston 6, 7, 8 are in the top half of piston. These notches are representing changes on cross-section of piston with fillet of radius r. For welded piston the fillet radii shall be assumed to be 1mm (Radaj5).

4. Loads applied to the structure

The first load step is the preload force on the piston rod due to tightening of the piston nut. This load can be applied in ANSYS with pretension elements. This is the basic load. All other loads are superimposed on the basic load. When specifying the minimum axial assembly forces the loss of the preload force due to the plastic deformation of the contact surfaces must be taken into consideration. In API 618 the recommendation for the minimum preload is 1.5 times the maximum allowable continuous piston rod load. The preload force must be checked for each application under the actual operating loads: pressure, acceleration and temperature. Often the required preload force is much higher than the API recommendation. In The status of all contact areas, pressure and gap must be checked carefully for all operation condition. The prestress force has to prevent lifting or slide of all contact surfaces.

Figure 3, 4. Pressure versus crankshaft angle for double acting and stepped piston
Figure 5, 6. Acceleration and combined rod load at the piston rod neck

Figure 3 and 4 shows the typical pressure curves $p_o(\phi)$, $p_u(\phi)$ on CE-, HE- and idle-compartment for double acting and for stepped piston. These loads are applied on the corresponding areas of the structures (Fig. 7). Figure 5 represents the acceleration and Figure 6 shows the resulting combined rod load at the piston rod neck crosshead side.

Figure 7. Loads on the piston rod unit: Preload $F_v$, pressure $p_o(\phi)$, $p_u(\phi)$ and acceleration $a(\phi)$

Figure 8. Typical notch shape and required parameters

5. Notch geometry, parameters and stress evaluation.

The main parameters of notches are (Fig.8):
- Shape of notch, local coordinate systems
- Geometry parameters: $R$, $r$, $\alpha$, $x$, $y$
- Depth in vertical direction $\Delta s$
- Number of elements in circumferential direction 1
- Number of elements in radial direction 3

The principal stresses in the local coordinate system are calculated in three directions 1, 2, and 3 along the path (angle $\alpha$) and for each step of crankshaft ($\Delta \phi$ acc. $\Delta \phi = 5^\circ$). The direction 1 and 2 are always parallel to the free surface. The third direction is perpendicular to free surface of the component.

The stresses are calculated on three points: position a on the free surface $\sigma_d(\alpha, \phi) = (\sigma_{1a}, \sigma_{2a}, \sigma_{3a})$, position b in a depth of $\Delta s/2$ $\sigma_b(\alpha, \phi) = (\sigma_{1b}, \sigma_{2b}, \sigma_{3b})$, as well as position c in a depth of $\Delta s$ $\sigma_c(\alpha, \phi) = (\sigma_{1c}, \sigma_{2c}, \sigma_{3c})$, (Fig.8). The stress value for $\sigma_1$ in perpendicular direction ($\Delta s$) to the free surface can be approximated as a polynomial (second order): $\sigma_1 = A_0 + A_1s + A_2s^2$. The coefficients $A_0$, $A_1$, and $A_2$ can be calculated easy from the three stress values (position a,b,c) established by
the FEA-Analysis. Finally the stress gradient can be determined, as required in 2.5, for the strength verification. The formulas are listed in appendix. The equation for the gradient is:
\[
\frac{d\sigma}{ds} = A_1 + 2 \cdot A_2 \cdot s
\]

6. Meshing strategy of the structure

The meshing of the structure must be practice-oriented and the model should have a reasonable size. On the one hand the meshing of the components should be as coarse as possible to reduce the simulation time and on the other hand as fine as necessary with reference to the engineering tasks. For notches (Fig. 10) and the contact areas a fine controlled meshing is used to get the correct information for the required stresses, contact status as well as the contact pressure. For other areas of the structure a free coarse mesh can be used. The element type and size can be adjusted by user. The following figure shows the different piston designs (Fig. 9): stepped piston 2D-model, two pieces double acting piston 2D and 3D-model, double acting welded piston with ribs 3D-model. All these models can be calculated automatically with the computer program.

7. Finite Element Analysis and results

The first simulation is the calculation of the prestressed structure under the preload force \( F_v \). The next calculations are successively under operating condition: pressure, temperature as well as acceleration for each crankshaft angle.

The main results of the Finite Element Analysis are the forces / stresses at the piston rod neck, thread (piston side and crosshead side) and the stresses on all notches.

The strength verification of pre-stressed connection piston rod, ring, piston and piston nut is performed according guideline. The reduction of preload force of bolted joint due to the deformation of the contact surfaces is taken into consideration. From the FEA the actual preload force, the amplitude of the alternating force/stresses, maximum forces/stresses in the piston rod neck /thread as well as the resilience of the superimposed clamped parts and piston rod can be determined. From these values the safety factors are calculated. Furthermore the status and the pressure on all contact areas is checked.

The next step is the stress evaluation on the notches. From the FEA results the distribution (stress curves) of stresses on the free surface, on the depth \( \Delta s / 2 \) as well as on the depth \( \Delta s \) along the angle \( \alpha \) for each notch and crankshaft angle \( \phi \) are calculated. Further the required alternating stress amplitude \( \sigma_a \), average stress \( \sigma_m \) as well as the stress gradient \( d\sigma/ds \) for the fatigue strength verification according to are calculated.

Figure 11 to 14 shows the stress tangential to the free surface (direction 1) along the angle \( \alpha \) in Notch 1, 2, 3 and 4 for two piston positions: Bottom-Dead-Center BDC and Top-Dead-Center TDC.
Figure 11. Notch 1, Tangential stress curves at BDC, TDC, amplitude and average stress on free surface and in depth $\Delta s$.

Figure 12. Notch 2, Tangential stress curves at BDC, TDC, amplitude and average stress on free surface and in depth $\Delta s$.

Figure 13. Notch 3, Tangential stress curves at BDC, TDC, amplitude and average stress on free surface and in depth $\Delta s$.

Figure 14. Notch 4, Tangential stress curves at BDC, TDC, amplitude and average stress on free surface and in depth $\Delta s$. 
8. Summary

The components of the piston rod unit are under very high dynamic loads due to: pressure, temperature and acceleration. Depending on the operation mode of the reciprocating compressor, normal operation, SV-Unloaders, VSD, variable operation conditions the load profile of piston rod unit changes. The individual single calculation for each load step requires an enormous effort. The traceability and documentation of analysis would also be a big effort. The method presents a so-called expert system connects different tested and proved modules like company tools, standards, company owned software as well as commercial software together. The calculation is carried out automatically. The main steps of the calculation are:
- Compressor calculation
- User input data
- Modelling of the structure
- Finite Element Analysis
- Verification of the preload force
- Status of the contact surfaces
- Stress verification on the relevant critical notches.

Instead of the suggested factor 1.5 in API 618 5th ed., para.6.10.1, a minimum factor of 1.75 is recommended.

References
1 VDI 2230 Richtlinien. Systematic calculation of high duty bolted joints. Joints with one cylindrical bolt.
2 FKM-Richtlinie. Rechnerischer. Festigkeitsnachweis für Maschinenbauteile.
3 ANSYS , Simulation Software.
4 API Standard 618. Reciprocating Compressors for Petroleum, Chemical, and Gas Industry Services.
5 Hobbacher. Empfehlungen zur Schwingfestigkeit geschweißter Verbindungen und Bauteile.

Appendix, Stress verification according FKM [2]

Static stress verification with local stresses
-Material Properties

Ultimate and yield strength of material from standard

\[ R_{m,N}, R_{p,N}, K_{d,m}, K_{d,p} \]

Technology Factor

\[ a_{d,m}, a_{d,p} \]

Constants

\[ K_A \]

Anisotropy factor

\[ d_{eff}, d_{eff,N}, d_{eff,P} \]

Effective diameter

Ultimate and yield strength of prefab. piece from the draw.

\[ R_{m,p} = K_{d,m} \cdot K_A \cdot R_{m,N}, R_p = K_{d,p} \cdot K_A \cdot R_{m,p} \]

Tension and shear stress factor

\[ f_\sigma, f_\tau \]

Compressive strength, shear strength

\[ R_{c,m} = f_\sigma \cdot R_m, R_{c,p} = f_\sigma \cdot R_p \]

\[ R_{s,m} = f_\tau \cdot R_m, R_{s,p} = f_\tau \cdot R_p \]
**Characteristic construction values**

Plastic stress concentration factor

\[ K_{p, \sigma_i}, K_{p, \tau}, K_{p, \tau} \]

Plastic notch sensitivity factor

\[ n_{pl, \sigma_i}, n_{pl, \sigma_i}, n_{pl, \tau} \]

Stress concentration factor squer shoulder

\[ K_{SK, \sigma_i} = \frac{1}{n_{pl, \sigma_i} \cdot K_{NL}}, K_{SK, \sigma_i} = \frac{1}{n_{pl, \tau} \cdot K_{NL}}, K_{SK, \sigma_i} = 1 \]

**Component strength**

Static local strength

\[ \sigma_{SK, 1} = \frac{f_{\sigma} \cdot R_m}{K_{SK, \sigma_1}}, \sigma_{SK, 2} = \frac{f_{\sigma} \cdot R_m}{K_{SK, \sigma_2}}, \sigma_{SK, 3} = \frac{f_{\sigma} \cdot R_m}{K_{SK, \sigma_3}} \]

**Safety factors**

Overall safety factor

\[ f_{ges} \]

**Stress characteristics**

The stress characteristics are result of FEA. The maximum extreme stresses will be evaluated from the stress curves in each notch Fig.11, 12, 13, 14

\[ \sigma_{max, ex, 1}, \sigma_{max, ex, 2}, \sigma_{max, ex, 3} \]

**Stress verification**

Static degree of utilization in three directions:

\[ a_{sk, \sigma_i} = \frac{\sigma_{max, ex, 1}}{f_{ges}} \leq 1 \]

\[ a_{sk, \sigma_i} = \frac{\sigma_{max, ex, 2}}{f_{ges}} \leq 1 \]

\[ a_{sk, \sigma_i} = \frac{\sigma_{max, ex, 3}}{f_{ges}} \leq 1 \]

**Fatigue strength calculation with local stresses**

**Stress characteristics**

The stress characteristics are result of FEA. The stress values: principal, maximum, minimum, mean as well as amplitude will be evaluated from the stress curves in each notch Fig.11, 12, 13, 14

Principal stresses in three directions \[ \sigma_{1, zd} = \sigma_1, \sigma_{2, zd} = \sigma_2, \sigma_{3, zd} = \sigma_3 \]

Maximum and minimum stresses \[ \sigma_{max, 1}, \sigma_{max, 2}, \sigma_{max, 3}, \sigma_{min, 1}, \sigma_{min, 2}, \sigma_{min, 3} \]

Stress amplitude, mean stresses in three directions, \( i=1,2,3 \)

\[ \hat{\sigma}_{i,j} = \frac{\sigma_{max, i} - \sigma_{min, i}}{2}, \sigma_{m, i} = \frac{\sigma_{max, i} + \sigma_{min, i}}{2} \]

Stress ratio

\[ R_{\sigma, i} = \frac{\sigma_{max, i}}{\sigma_{min, i}} \]

**Material characteristics**

Tension, compressive and shear fatigue strength factor

\[ f_{W, \sigma}, f_{W, \tau}, \tau_{W, zd} = f_{W, \tau} \cdot \sigma_{W, zd} \]

\[ \sigma_{W, zd} = f_{W, \sigma} \cdot R_m \]

Polynomial function for the stress in direction \( i=1,2,3 \)

\[ \sigma_i(s) = A_{i, j} + A_{i, j} \cdot s + A_{2, j} \cdot s^2 \]
-Construction characteristics
Stress gradient on free surface of the notch, direction $i=1,2,3$
\[ \overline{G}_{\sigma,i} = \frac{d\sigma_i}{ds} = A_{i,i} \]

Constants, mean arithmetic roughness
\[ a_G, b_G, A_{R,\sigma}, R_{m,N,min}, R_{z}, \overline{K}_f, K_{NL,E} \]

Notch sensitivity factor
\[ n_{\sigma,i} = \begin{cases} 
1 + \overline{G}_{\sigma,i} \cdot 10 \left( \frac{e_{\sigma,i}}{R_{m,N,min}} \right) & \text{für } \overline{G}_{\sigma,i} \leq 0.1 \\
1 + \sqrt{\overline{G}_{\sigma,i} \cdot 10} \left( \frac{e_{\sigma,i}}{R_{m,N,min}} \right) & \text{für } 0.1 \leq \overline{G}_{\sigma,i} \leq 1 \\
1 + \sqrt{\overline{G}_{\sigma,i} \cdot 10} \left( \frac{e_{\sigma,i}}{R_{m,N,min}} \right) & \text{für } 1 \leq \overline{G}_{\sigma,i} \leq 100 
\end{cases} \]

Roughness factor for normal and shear stresses
\[ K_{R,\sigma} = 1 - A_{R,\sigma} \cdot \log(R_z) \cdot \log \left( \frac{2 \cdot R_z}{R_{m,N,min}} \right) \]
\[ K_{R,x} = 1 - f_{w,x} \cdot A_{R,\sigma} \cdot \log(R_z) \cdot \log \left( \frac{2 \cdot R_z}{R_{m,N,min}} \right) \]

Surface layer factor, coat factor construction factor
\[ K_{f}, K_S \]

-Component strength
Alternate fatigue strength direction $i = 1, 2, 3$
\[ \sigma_{WK,i} = \frac{\sigma_{W,\sigma,i}}{K_{WK,i}} \]

Residual stress factor, constants, mean stress sensitive factor
\[ K_{E,\sigma} = a_m, b_m \]
\[ M_\sigma = a_m \cdot 10^{-3} \cdot R_m + b_m, M_\sigma = f_{w,x} \cdot M_\sigma \]

Mean stress factor
\[ K_{M,\sigma} = \begin{cases} 
\frac{1}{1 - M_\sigma} & \text{Zone } I \ R_\sigma > 1 \\
\frac{1}{1 + M_\sigma} & \text{Zone } II \ -\infty \leq R_\sigma \leq 0 \\
\frac{1 + M_\sigma \cdot \frac{\sigma_m}{\sigma_\sigma}}{1 + M_\sigma} & \text{Zone } III \ 0 < R_\sigma < 0.5 \\
\frac{1 + M_\sigma \cdot \frac{\sigma_m}{\sigma_\sigma}}{3 \cdot (1 + M_\sigma)} & \text{Zone } IV \ R_\sigma > 0.5 
\end{cases} \]

Fatigue strength amplitude
\[ \sigma_{AK,i} = K_{AK,i} \cdot K_{E,\sigma} \cdot \sigma_{WK,i} \]

Endurance strength factor
\[ K_{AK,i} = \begin{cases} 
1 & \text{für } N = \infty \n \end{cases} \]

Endurance strength amplitude
\[ \sigma_{EAK,i} = K_{AK,i} \cdot \sigma_{AK,i} \]

-Safety factors
Safety factor
\[ j_D \]

- Fatigue stress verification
Degree of utilization in three directions $i = 1, 2, 3$
\[ a_{BK,i} = \frac{\hat{\sigma}_i}{\sigma_{BK,i}} \leq 1 \]