SINR and MI-based Robust Transmitted Waveform Design in the Situation of Multiple Targets

Xu Chen¹, Bin Wang¹*, Rihan Wu¹, Ruiyi Fan² and Lulu Wang³

¹School of Computer and Communication Engineering, Northeastern University at Qinhuangdao, China.
²State Grid Hebei Electric Power Supply Co. Ltd, China
³Artificial Intelligence Research Center, National Innovation Institute of Defense Technology, China.
*Email: wangbinneu@qq.com

Abstract. Most of the previous researches on transmitted waveform design assume that the radar can perceive the real target spectrum, as the environments of the radar target become more and more complex, the real target spectrum cannot be accurately captured by radar in many cases and the required performances of detection and estimation for radar cannot be met. Therefore, under the complex electromagnetic environment and the scenario of multiple targets, to ensure the detection and estimation performance of radar system, the SINR and MI-based robust transmitted waveform design methods are proposed in this paper. Simulation results indicate that the designed robust transmitted waveforms ensure the detection and parameter estimation performance effectively and provide useful guidance for energy allocation of transmitted waveform.

1. Introduction

Cognitive radar (CR) [1-2] is an intelligent radar which is recognized as a future system radar. Its main feature is the introduction of radar closed-loop system. Adaptive waveform design is an important part of the cognitive radar closed-loop system [3], which can improve the detection and estimation performance of radar system. In cognitive radar system, through using the prior information and the relationship between the transmitted waveform and the output signal-to-interference-plus-noise ratio (SINR), the transmitted waveform can be designed to maximize the SINR, thus the detection performance of radar can be improved [4]. To improve the estimation performance, the mutual information (MI) between the target and echo can be used as a criterion to design the transmitted waveform [5]. However, most of the recent SINR or MI-based transmitted waveform design works such as [6-8] do not consider more complex target model and also seldom pay attention to the complex model of multiple targets, while in practice the real target spectrum cannot be accurately captured by radar and there are multiple targets need to be detected in many cases. Therefore, under the complex
target model, as the prior information of the targets is not accurate, the traditional transmitted waveform design algorithm cannot meet the demands for detection and estimation performance of radar system.

In order to ensure the detection and estimation performance of radar system under the complex target model, the robust optimal transmitted waveform techniques based on SINR and MI are proposed in this paper, which can be used to maximize the efficiency of the transmitted waveform in complex environments. In this paper, the traditional transmitted waveform design methods based on SINR and MI are reviewed firstly, which assumes that the spectrum response of multiple targets is accurate. Then a blurred target model in a scenario of multiple targets is established, which assumes that each real target spectrum of multiple targets exists in a range of amplitudes known by the upper and lower bounds. Finally, the robust optimal transmitted waveforms based on SINR and MI are designed according to the established blurred target model, which guarantee the detection and estimation performance of radar system respectively in complex electromagnetic environments.

2. Signal Model and Problem Formulation

In CR system, we consider that the transmitted waveform is denoted by \( x(t) \). We can use this expression to represent the radar target echo signal [9]

\[
y(t) = x(t) * h(t) + x(t) * c(t) + n(t)
\]  

(1)

In the above-mentioned expression, \( h(t) \) means the target impulse response, \( X(f) \) means the spectrum response of \( x(t) \) and \( H(f) \) means the spectrum response of \( h(t) \). \( n(t) \) denotes a noise signal, the power spectral (PSD) of \( n(t) \) can be denoted by \( S_n(f) \). Within the entire frequency band, the amplitude of \( S_n(f) \) is greater than zero. \( c(t) \) represents the interference signal, the PSD of \( c(t) \) is \( S_c(f) \). The energy spectrum variance (ESV) of \( h(t) \) is represented by [9]. In the expression of (2), \( E[\cdot] \) represents the expectation of the expression in parentheses. \( \mu_{n}(f) \) in the expression of (2) is the average value of \( H(f) \) which is assumed to be 0.

\[
\sigma_n^2(f) = E\left[H(f) - \mu_{n}(f)\right]^2
\]  

(2)

In this paper, we suppose that the ESV of multiple targets is

\[
\sigma_{n,i}^2(f) = \sum_{i=1}^{M} p_i |H_i(f)|^2 - \sum_{i=1}^{M} p_i |H_i(f)|^2
\]

(3)

denotes the number of targets and \( p_i \) denotes the occurrence probability of \( i-th \) target.

To improve the detection performance of the radar transmitter, SINR is adopted as the criterion to design the transmitted waveform. \( E_x \) indicates the energy constraint of the transmitted waveform. \( BW \) is the bandwidth and the transmitted waveform PSD is essentially limited to it. Based on the above information, the optimization criterion can be indicated as

\[
SINR = \int_{-\frac{BW}{2}}^{\frac{BW}{2}} \frac{\sigma_{n,i}^2(f)|X(f)|^2}{S_n(f)|X(f)|^2 + S_{n,i}(f)} df
\]  

(4)

The designed optimal transmitted waveform for SINR should conform [9]

\[
\max_{|X(f)|^2} SINR\left(|X(f)|^2\right)
\]  

(5)
Under the energy constraint (5), the optimal transmitted waveform can be obtained by the Lagrange multiplier method that maximizes the SINR (3), which can be denoted as

$$\|X(f)\|^2 = \max \left[ 0, \sqrt{\frac{\sigma^2_{H}(f)S_{w}(f)}{S_{w}(f)}} \left( A \cdot \sqrt{\frac{S_{w}(f)}{\sigma^2_{H}(f)}} \right) \right]$$

(6)

respectively. $A$ is a constant that can be obtained by the energy constraint of the transmitted waveform

$$\int_{0}^{\infty} \max \left[ 0, \sqrt{\frac{\sigma^2_{H}(f)S_{w}(f)}{S_{w}(f)}} \left( A \cdot \sqrt{\frac{S_{w}(f)}{\sigma^2_{H}(f)}} \right) \right] df = E_x$$

(7)

To improve the estimation performance of the radar transmitter, MI is adopted as the criterion to design the transmitted waveform. The energy constraint of the transmitted waveform is also set to be $E_x$, $T_f$ denotes the duration of the target echo $y(t)$. Therefore, the optimization criterion is shown as

$$MI = T_f \int_{0}^{\infty} \ln \left[ 1 + \frac{\sigma^2_{H}(f)\|X(f)\|^2}{T_f \left( S_{w}(f)\|X(f)\|^2 + S_{w}(f) \right)} \right]$$

(8)

The designed optimal transmitted waveform for MI should conform [9]

$$\max_{\|X(f)\|^2} MI \left(\|X(f)\|^2\right)$$

s.t. $\int_{0}^{\infty} \|X(f)\|^2 df \leq E_x$

(9)

(10)

Under the energy constraint (10), the optimal transmitted waveform can be obtained by the Lagrange multiplier method that maximizes the MI (8), which can be denoted as

$$\|\tilde{X}(f)\|^2 = \max \left[ 0, \frac{\sigma^2_{H}(f)}{2T_f \cdot S_{w}(f) + \sigma^2_{H}(f)} \left( \tilde{A} \cdot \frac{T_f S_{w}(f)}{\sigma^2_{H}(f)} \right) \right]$$

(11)

$\tilde{A}$ is also a constant that can be obtained by the energy constraints of the transmitted waveform.

$$\int_{0}^{\infty} \max \left[ 0, \frac{\sigma^2_{H}(f)}{2T_f \cdot S_{w}(f) + \sigma^2_{H}(f)} \left( \tilde{A} \cdot \frac{T_f S_{w}(f)}{\sigma^2_{H}(f)} \right) \right] df \leq E_x$$

(12)

In the above-mentioned SINR and MI-based radar transmitted waveform design process, we assume that the real target spectrum is known, while in many practical cases the real target spectrum cannot be accurately captured. When the target model is blurred, the transmitted waveform design methods described above will not meet the need for detection and estimation performance of radar system. Therefore, in order to reduce the loss of radar system performance, the SINR and MI-based robust optimal transmitted waveform methods will be considered next.

3. Robust Transmitted Waveform Design Methods

Consider that the spectrum model of multiple targets is blurred, which is an assumption that each real target spectrum of the multiple targets is within an uncertainty range, in which the upper and the lower bound are known. The expression of this model can be denoted as
\[ H_i(f) \in \varepsilon_i = \{ \xi_{ik} \leq H_i(f_k) \leq u_{ik}, k = 1, 2, \ldots, K \} \]  \hspace{1cm} (13)

where \( f_k \) represents the sampling frequency, \( K \) represents the number of sampling points and \( i = 1, 2, 3, \ldots \) represents different targets. The uncertainty range \( \varepsilon_i \) corresponding to each target is different in this model.

For each specific target ESV, there will be an optimal transmitted waveform for SINR and MI respectively. However, as the real target spectrum for each target in the multiple targets varying in its corresponding uncertainty range, the maximin robust techniques for SINR and MI are good ways which can guarantee the worst-case performance. In this part, the maximin robust transmitted waveform techniques of SINR and MI are proposed as follows.

3.1. SINR-based robust transmitted waveform technique

The SINR-based maximin robust transmitted waveform technique for multiple targets should satisfy

\[
\max_{\{\mathcal{F}(f)\}} \left\{ \min_{\{\mathcal{P}(f)\} \in \varepsilon_i} \text{SINR} \left( X(f)^2, \sigma_H^2(f) \right) \right\} \int_{\mathcal{F}(f) \notin E_i} \| V(f) \| \, df \leq \varepsilon_i
\]  \hspace{1cm} (14)

The solution of the optimal problem described in (14) is

\[
X^{\text{max min}}(f)^2 = \max \left\{ 0, \frac{\varepsilon_z^2(f) - \lambda_z}{\sigma_z^2(f)} \left( -\frac{S_m(f)}{\sqrt{\sigma_z^2(f)}} \right) \right\} \int_{\mathcal{F}(f) \notin E_i} \| V(f) \| \, df \leq \varepsilon_i
\]  \hspace{1cm} (15)

where \( \eta_i(f) = \{ \xi_{ik}, k = 1, 2, \ldots, K \} \) denotes the lower bound of \( i \)-th target spectrum uncertainty range, and \( \varepsilon_z^2(f) = \sum_{m} R_m(f) \left( \sum_{m} P_m(f) \right)^2 \) in the expression of (15). \( \hat{\lambda} \) is a constant which can be solved by

\[
\int_{\mathcal{F}(f) \notin E_i} \| V(f) \| \, df \leq \varepsilon_i
\]  \hspace{1cm} (16)

3.2. MI-based robust transmitted waveform technique

The MI-based maximin robust transmitted waveform technique for multiple targets should satisfy

\[
\max_{\{\mathcal{F}(f)\}} \left\{ \min_{\{\mathcal{P}(f)\} \in \varepsilon_i} \text{MI} \left( X(f)^2, \sigma_H^2(f) \right) \right\} \int_{\mathcal{F}(f) \notin E_i} \| V(f) \| \, df \leq \varepsilon_i
\]  \hspace{1cm} (17)

The solution of the optimal problem described in (17) is

\[
X^{\text{max min}}(f)^2 = \max \left\{ 0, \frac{\varepsilon_z^2(f) - \lambda_z}{2T_z \cdot \sigma_z^2(f) + \sigma_z^2(f)} \left( -\frac{T_z S_m(f)}{\sigma_z^2(f)} \right) \right\} \int_{\mathcal{F}(f) \notin E_i} \| V(f) \| \, df \leq \varepsilon_i
\]  \hspace{1cm} (18)

where \( \varepsilon_z^2(f) = \sum_{m} R_m(f) \left( \sum_{m} P_m(f) \right)^2 \) in the expression of (18), which is the same as \( \sigma_z^2(f) \) in subsection 3.1 and \( \hat{\lambda} \) is a constant which can be solved by

\[
\int_{\mathcal{F}(f) \notin E_i} \| V(f) \| \, df \leq \varepsilon_i
\]  \hspace{1cm} (19)
Therefore, under the condition of the blurred target model, taking the above-mentioned maximin robust techniques into account can optimize the detection and estimation performances of the radar system effectively.

4. Simulation and Results
This paper put forward many simulation analyses to verify if the SINR and MI-based robust transmitted waveform techniques for multiple targets proposed above are valid. The blurred model of the multiple targets is shown in Fig. 1. For each target of multiple targets, the primary energy is distributed around the normalized frequency -0.3, -0.1, 0.1, and 0.3, with the probability of occurrence 0.1, 0.2, 0.3, and 0.4 respectively. The amplitude of the upper bound is the real amplitude added a random value, and the amplitude of the lower bound is the real amplitude subtracted a random value. Fig. 2 shows the SINR and MI-based waveform spectrum results. The real target ESV is displayed in the top panel of Fig. 2. In addition, the most unfavorable target ESV and the spectrum response of signal dependent interference are also shown in the top panel. The total energy of the transmitted waveform spectrum is assumed to be 1 W. The SINR and MI-based optimal transmitted waveform spectrum and robust transmitted waveform spectrum are presented in the middle panel and bottom panel of Fig. 2 respectively. The waveform spectrum result is the same as expected, the SINR-based optimal waveform places its main energy in few frequency bands, while the MI-based optimal waveform based on MI places its energy in multiple frequency bands. The designed transmitted waveforms assign the limited energy in frequency bands with the strong target spectrum response and the weak interference waveform spectrum response. Assume that energy constraint of the transmitted waveform varies from 1W to 10W, the SINRs corresponding to the SINR-based optimal transmitted waveform and the SINR-based robust transmitted waveform for the most unfavorable target ESV are compared in Fig. 3, and also the MIs corresponding to the MI-based optimal transmitted waveform and the MI-based robust transmitted waveform for the most unfavorable target ESV are compared in Fig. 4. In Fig. 3 and Fig. 4, the most unfavorable performance can be enhanced through using maximin robust techniques, which finally optimize the detection and estimation performances of the radar system effectively.
5. Conclusion
In this paper, the SINR and MI-based optimal transmitted waveform techniques are proposed firstly, which assumes that the real information of the multiple targets is fully known. The designed SINR and MI-based optimal transmitted waveforms are suitable for limited energy condition. Then, the blurred model of multiple targets is considered. This model assumes that each real target spectrum of the multiple targets is within an uncertainty range, which has its known upper and lower bounds. According to the established target model above, the SINR and MI-based maximin robust transmitted waveform has been designed respectively. Results show that the designed SINR and MI-based robust transmitted waveforms improve the detection and parameter estimation performances effectively and provide useful guidance for energy allocation of transmitted waveform.

6. Acknowledgement
This work was supported by the Natural Science Foundation of Hebei Province (No. F2018501051) and the National Natural Science Foundation of China (No. 61701502).

7. References
[1] Simon H 2006 Cognitive radar: a way of the future IEEE Signal Processing Magazine 23 30-40
[2] Joseph R G 2010 Cognitive radar: a knowledge-aided fully adaptive approach IEEE Radar Conference 1365-70
[3] Haykin S, Yanbo X and Davidson T N 2008 Optimal waveform design for cognitive radar Signals, Systems and Computers 52 3-7
[4] Kay S 2007 Optimal signal design for detection of gaussian point targets in stationary gaussian clutter/reverberation IEEE Journal of Selected Topics in Signal Processing 1 31-41
[5] Yang Y and Blum R S 2006 Radar waveform design using minimum mean-square error and mutual information IEEE Workshop on Sensor Array and Multichannel Processing 4 234-8
[6] Kim H S, Goodman N A, et al 2017 Improved waveform design for radar target classification Electronics Letters 53 879-81
[7] Kay S and Raghavan R S 2017 Information-theoretic optimal radar waveform design IEEE Signal Processing Letters 24 274-8
[8] Kay S and Raghavan R S 2018 Locally optimal radar waveform design for detecting doubly spread targets in colored noise *IEEE Signal Processing Letters* **25** 833-7

[9] Romero R A, Junhyeong B and Goodman N A 2011 Theory and application of SNR and mutual information matched illumination waveforms *IEEE Transactions on Aerospace and Electronic Systems* **47** 912–27