Tunable Axicons Generated by Spatial Light Modulator with High-Level Phase Computer-Generated Holograms

Zhongsheng Zhai 1,2,*, Zhuang Cheng 1, Qinghua Lv 3,⋆ and Xuanze Wang 1,2

1 School of Mechanics and Engineering, Hubei University of Technology, Wuhan 430068, China; chengzhuang358@163.com (Z.C.); wangxz@hbut.edu.cn (X.W.)
2 Hubei Key Lab of Manufacture Quality Engineering, Wuhan 430068, China
3 Hubei Collaborative Innovation Center for High-efficient Utilization of Solar Energy, Hubei University of Technology, Wuhan 430068, China
* Correspondence: zs.zhai@hbut.edu.cn (Z.Z.); linsa080@hbut.edu.cn (Q.L.)

Received: 5 July 2020; Accepted: 24 July 2020; Published: 26 July 2020

Abstract: Axicon is an interesting optical element for its optical properties. This paper presents an approach to dynamically generated tunable axicons with a spatial light modulator (SLM). 256-level phase computer-generated holograms (CGHs) were loaded into the SLM to simulate the positive and negative axicons. The intensity distributions of beams passing through these axicons were analyzed with the principle of blazed grating and Fresnel diffraction; and the diffraction patterns were obtained theoretically in terms of zero-order Bessel beams and annular hollow beams, corresponding to the positive and negative axicons, respectively. Experimental results verified that the diffraction patterns have the same distribution as the real axicon. The types of the axicon and the axicon’s parameters can be easily altered through changing the CGHs.

Keywords: hologram; zero-order Bessel beams; annular hollow beams; axicon; blazed grating

1. Introduction

The ‘axicon’, first introduced by McLeod J. H. in 1954 [1], provides the most efficient method to produce zero-order Bessel beams. The zero-order Bessel beam, also called a nondiffracting beam, has many advantages in that the size and shape of the central spot remains constant [2]. Nondiffracting beams have been used in many fields: laser machining [3], extending the depth of a field [4], measuring multidegree-of-freedom error motions [5], and 3D shape measurement [6]. Ren O. et al. used the negative and positive axicons to change the diameter of the Bessel beam ring for ablating corneal material for corneal surgery [7].

The range of the nondiffracting beams, generated by a real axicon, is dependent on the parameters of the axicon (the diameter and refractive angle). Sometimes with the need of changing measurement ranges, it has to replace axicons with different refractive angles or tune the cone angle (refractive angle) of the axicon. It is not convenient to replace the axicon and adjust the optical path, and it is a difficult task to vary the cone angle [8]. The fabrication errors of the axicon will also influence the quality of the nondiffracting beams and its applications. Therefore, it is interesting to present a method to simulate axicons, which can dynamically and flexibly alter the parameters of the axicon without fabrication errors.

Many methods have been attempted to realize tunable axicons. Davidson N. et al. used computer-generated aspheric holographic optical elements to achieve an axilens [9], which combines the properties of an axicon and a spherical lens. In 2014, Algorri J. F. et al. proposed an approach based on nematic liquid crystal and phase-shifted electrical signals to obtain lensacons, logarithmic...
and linear axicons [8]. Milne G. et al. obtained tunable Bessel beams with a fluidic axicon, which is filled with refractive index variable liquid. The shortcoming of this method is that it needs a real glass axicon as a model [10]. Some scholars used the computer-generated holograms (CGHs) to achieve an approximately diffraction-free, arbitrary-order Bessel beam series [11–15]. The holograms were fabricated as phase-only structures by using the more-sophisticated techniques of photolithography. However, due to the limitation of the phase level (normally ≤4), the Bessel beams were not perfect, and this method cannot dynamically obtain tunable Bessel beams. Spatial light modulators, which can alert the amplitude and phase of the beams with programmable holograms, have been applied to generate tunable Bessel beams [16]. The authors used 64-level phase holograms to control the size and deflection angle of the beam. The phase level is not high, and the zero-order Bessel beam was not analyzed.

In this paper, the spatial light modulator (SLM) was applied to simulate the positive and negative axicons with 256-level phase CGHs. Based on the principle of blazed grating and Fresnel diffraction, the intensity distributions are derived when the axicons are illuminated by Gaussian beams. The diffraction patterns are obtained as zero-order Bessel beams and annular hollow beams, corresponding to the positive and negative axicons. Experimental results verified that the diffraction patterns have the same distribution as the real axicon. The types of the axicon and the axicon’s parameters can be easily altered through changing the CGHs. This method can dynamically generate tunable axicons without fabrication errors to suit different applications.

2. Theory

2.1. Real Axicon

An axicon can continuously converge the light to different positions along the axis, and it is widely used to produce nondiffracting beams. There are two types of axicons: a positive axicon and a negative axicon, as shown in Figure 1. From the view of geometrical optics, it can be seen that the positive axicon has a convergent effect on the beams, and the negative axicon has a divergent effect on the beams within a certain range of transmission distance.

![Figure 1. The principle of a real axicon: (a) positive axicon, (b) negative axicon.](image)

The transmittance function of the positive axicon is given by:

\[
l(r) = \begin{cases} 
\exp(-ik(n-1)r \tan \theta) & (r \leq D/2) \\
0 & (r > D/2)
\end{cases}
\]  

(1)

where \( r = (x^2 + y^2)^{1/2} \), \( k = 2\pi/\lambda \), \( \theta \) is the refracting angle formed by the conical surface with the flat surface, \( n \) is the refractive index, and \( D \) is the diameter of the axicon.

In general, the incident beam is a Gaussian beam, in which case the resulting beam after the axicon is called Gaussian–Bessel beam, and its intensity profile is given by [17]:

\[
I(r, z) = 2\pi k \left( \tan^2 \theta \right) (n-1)^2 z I_0 e^{-2(n-1)^2z\tan \theta/(\omega_0)} J_0^2(k(n-1)r \tan \theta)
\]  

(2)
where \(r\) and \(z\) are the radial and longitudinal coordinates respectively, \(I_0\) is the incident on-axis intensity, \(J_0\) is the zero-order of Bessel function, and \(\omega_0\) is the waist of the incident beam.

By finding the first zero of the Bessel function, the width of the central lobe \(r_0\) and the maximum transmission distance \(z_{p,\text{max}}\) of nondiffracting beams are given by the following equations:

\[
r_0 = \frac{2.4048}{k(n-1)\tan \theta}
\]

\[
z_{p,\text{max}} = \frac{r_p}{(n-1)\tan \theta}
\]

where \(r_p\) is the radius of the positive axicon, and the subscript \(p\) is represented of the positive axicon, as shown in Figure 1a.

When the parallel beam illuminates the negative axicon, the light is diverged and transformed into an annular beam, as show in Figure 1b, where the subscript \(n\) is represented of the negative axicon. From Figure 1b, the inner radius \(r_0\) and outer radius \(R_0\) can be obtained as:

\[
r_0 = z_n \tan \varphi
\]

\[
R_0 = (z_n - L_n) \tan \varphi + D/2
\]

where \(\varphi = \arcsin[(n-1)\theta_n]\), \(L_n\) is the thickness of the axicon and \(\theta_n\) is the conical angle. Then the width of the output beams from the negative axicon is:

\[
R_0 - r_0 = D/2 - L_n \tan \varphi = D/2 - L_n/\varphi = D/2 - L_n(n-1)\theta_n
\]

From Equation (7), we can see that the width remains constant within the transmission distance \(z_n\), and it is just related to the radius of the axicon \(D\), depth \(L_n\) and diverge angle \(\theta_n\).

2.2. Generation of Nondiffracting Beams with CGH

Although the ideal real axicon can effectively generate nondiffracting Bessel beams, the fabrication errors will influence the quality of nondiffracting beams. Therefore, an easy-to-process method is put forward. The method is loading the CGHs to the SLM to achieve the phase modulation of the incident beam. The hologram for simulating the positive axicon is present in Figure 2a, and Figure 2b is the central profile of the hologram. From Figure 2b, the profile can be looked at as an assembly of equally spaced grooves, and \(d\) and \(h\) are the period and the height of the grooves, respectively.

![Figure 2](image_url)

**Figure 2.** (a) A hologram for realizing the positive axicon, (b) the central profile of the hologram.
Phase delay is determined by the phase difference of the light passing through different gray level in the SLM. Therefore, the transmission function of a single groove is:

\[ t(r) = \exp[-ik(n_1 - 1)r \tan \theta_B] = \exp\left[-ik(n_1 - 1)r \frac{h}{d}\right] \quad (8) \]

where \( r \) is the radial position, \( k = 2\pi/\lambda \), \( \theta_B \) is the base angle, and \( n_1 \) is the refractive index of the groove. In the single-groove condition, when the size of \( r \) is \( d \), the phase at this time is \( 2\pi \), then:

\[ k(n_1 - 1)h = 2\pi \quad (9) \]

Substituting \( k = 2\pi/\lambda \) into Equation (9), we can obtain the expression of groove height \( h \) that:

\[ h = \frac{\lambda}{n_1 - 1} \quad (10) \]

Because the hologram has circular symmetry, we take half of it to analyze. The transmittance function of the hologram can be looked at as the convolution between \( t(r) \) and Dirac comb function \( \text{comb}(r) \), as shown in Figure 3 [18].

\[ R(r) = t(r) \otimes \text{comb}(r) = \sum_{m=0}^{N-1} \exp\left[-ik(r-md)(n_1 - 1)\frac{h}{d}\right] \quad (11) \]

where \( \otimes \) is the convolution operator.

![Figure 3. The principle of calculating \( R(r) \) with convolution.](image)

If the SLM is illuminated with Gaussian beam, the electric field after \( N \) cycles behind SLM has the form:

\[ U(r) = A \exp\left(-\frac{r^2}{\omega^2}\right) R(r) = A \exp\left(-\frac{r^2}{\omega^2}\right) \sum_{m=0}^{N-1} \exp\left[-ik(r-md)(n_1 - 1)\frac{h}{d}\right] \quad (12) \]

where \( A \) is the amplitude of the Gaussian beam, \( \omega \) is the incident beam waist, and \( N \) is equal to \( D/2d \).

The hologram has circular symmetry, and, according to the theory of Fresnel diffraction, the diffraction field at distance \( z \) behind the SLM can be written as:

\[ E_1(r_1, z) = \frac{A}{\lambda z} \exp\left[ik \left(z + \frac{r_1^2}{2z}\right)\right] \times \int_0^{2\pi} \exp\left[-i k \frac{r_1^2}{2z} \cos(\theta - \xi)\right] d\xi \]

\[ \quad \times \int_0^{\pi/2} r \exp\left(i k \frac{r_1^2}{2z}\right) \sum_{m=0}^{N-1} \exp\left[-ikm(\omega^2)(n_1 - 1)\frac{h}{d}\right] \exp\left(-\frac{r_1^2}{\omega^2}\right) dr \]

where \( (r, \xi) \) are the polar coordinates in the plane \( z = 0 \), and \( (r_1, \theta) \) are the polar coordinates in the plane \( z \). Making use of:

\[ \int_0^{2\pi} \exp[ix(\varphi - \varphi_0)] d\varphi = 2\pi J_0(x) \quad (14) \]
Equation (13) can be rewritten as:

\[
E_1(r_1, z) = A \frac{2\pi}{\lambda} \exp \left[ i k \left( z + \frac{r_1^2}{2z} \right) \right] \times \int_0^{L/2} r_0(k_z) \exp \left( -i \frac{m}{\omega_d} N \right) \sum_{m=0}^{N-1} \exp \left[ i k \left( z + \frac{r_1^2}{2z} - (r - md)(n_1 - 1)\frac{h}{d} \right) \right] dr
\]  

(15)

The stationary-phase method provides a good approximation to the value of the integral in Equation (15). A function for the phase part of the integral in Equation (15) is defined as:

\[
f(r) = \frac{r^2}{2z} - (r - md)(n_1 - 1)\frac{h}{d}
\]  

(16)

\[
f'(r) = \frac{r}{z} - (n_1 - 1)\frac{h}{d} = 0
\]  

(17)

According to Equation (17), the stationary point \( r_q \) can be obtained as:

\[
r_q = z(n_1 - 1)\frac{h}{d}
\]  

(18)

Equation (15) can be expressed in the form:

\[
E_1(r_1, z) = Ak(n_1 - 1) \sqrt{\lambda z} f_0 \left( \frac{k(n_1 - 1)h}{d} \right) \times \exp \left[ -i \frac{(n_1 - 1)h}{d} \right] \exp \left[ i k \left( z + \frac{r_1^2}{2z} \right) \right] \times \sum_{m=0}^{N-1} \exp \left\{ \left[ k \left( \frac{(n_1 - 1)h}{d^2} + (n_1 - 1)mh \right) + \frac{m}{d} \right] \right\}
\]  

(19)

Then, the intensity distribution at plane \( z \) behind the SLM can be obtained as:

\[
I_{(r_1, z)} = 2A^2 \pi k z \left( \frac{h}{d} \right)^2 (n_1 - 1)^2 \exp \left[ -2 \left( \frac{(n_1 - 1)zh}{\omega d} \right)^2 \right] f_0^2 \left( k(n_1 - 1)r_1 \frac{h}{d} \right)
\]  

(20)

Equations (2) and (20) represent the intensity distribution of an incident beam passing through the real axicon and the SLM-loaded holograms, respectively. Comparing Equations (2) and (20), if \( \theta = h/d, \omega_0 = \omega \) and \( k_0 = A^2 \), the two equations are exactly the same. Therefore, we can use the hologram to simulate the positive axicon to generate zero-order Bessel beams (also called nondiffracting beams).

The width \( \rho_0 \) of the central lobe can be calculated by finding the first zero of the Bessel function and the maximum transmission distance \( z_{\text{max}} \) of nondiffracting beams are given by the following equations [10]:

\[
\rho_0 = \frac{2.4048\lambda}{2\pi \sin \beta} \approx \frac{2.4048}{k(n_1 - 1) \tan \theta_B} = \frac{2.4048d}{k(n_1 - 1)h}
\]  

(21)

\[
z_{\text{max}} = \frac{\omega}{\tan \beta} \approx \frac{\omega}{(n_1 - 1) \tan \theta_B} = \frac{\omega d}{(n_1 - 1)h}
\]  

(22)

By comparing Equations (3) and (21) and Equations (4) and (22), it can be found that the center-spot radius and the maximum transmission distance of the nondiffracting beams realized by the axicon and SLM are similar.

2.3. Generation of Annular Hollow Beams with CGH

The negative axicon has a divergent effect on the beams. When the parallel beams illuminate the negative axicon, the light is diverged and transformed into an annular beam, and the divergent beams remain parallel. The hologram for simulating the negative axicon is present in Figure 4a, and Figure 4b is the central profile of the hologram.
where, \( \beta_n = \arcsin((n_1 - 1)\theta_{Bn}) \), and \( \theta_{Bn} \) is the conical angle. Accordingly, the width of the output beams is expressed as:

\[
R_1 - r_1 = L/2 - h \tan \beta_n \approx L/2 - h(n_1 - 1) \tan \theta_{Bn} = Nd - \frac{h^2(n_1 - 1)}{d}
\]  

From Equation (25), it can be found that the width of annular hollow beams is only related to the cycles \( N \), the groove spacing \( d \), and the groove height \( h \).

3. Results and Discussion

To demonstrate the above theoretical analyses, we carried out experiments. The experimental setup is illustrated in Figure 5. A reflective SLM (Hamamatsu X10468-03, 256-level phase, made in Hamamatsu Photonics (China), Beijing, China) with 800 \( \times \) 600 pixels was used to load CGHs to simulate the positive and negative axicons.

Figure 4. (a) The hologram for realizing the negative axicon, and (b) its central profile.

From Figure 4b, we can get the inner radius \( r_1 \) and outer radius \( R_1 \) as:

\[
r_1 = z_n \tan \beta_n
\]

\[
R_1 = (z_n - h) \tan \beta_n + L/2
\]

where, \( \beta_n = \arcsin((n_1 - 1)\theta_{Bn}) \), and \( \theta_{Bn} \) is the conical angle. Accordingly, the width of the output beams is expressed as:

\[
R_1 - r_1 = L/2 - h \tan \beta_n \approx L/2 - h(n_1 - 1) \tan \theta_{Bn} = Nd - \frac{h^2(n_1 - 1)}{d}
\]  

Figure 5. Experimental setup for generating positive axicon.
3.1. Positive Axicon Simulated by SLM

The experimental setup is shown in Figure 5. The laser passed through Mirror 1 and a beam expander \((M \approx 3\times)\), and after reflection on Mirrors 2 and 3, illuminated on the SLM. A CCD (Charge Coupled Device) was used to receive Bessel beams generated by the SLM, and the images were sent to a computer. Meanwhile, the computer generated the holograms and loaded them into the SLM. Figure 6 presents the 256-level phase holograms for simulating the positive axicon on the conditions of \(d = 15\), \(20\) and \(25\) pixels (1 pixel = 12.5 \(\mu\)m).

![Figure 6](image)

**Figure 6.** Bessel beams generated by the holograms with different periods: (a) \(d = 15\), (b) \(d = 20\), (c) \(d = 25\) pixels.

We can see that the interval of Bessel rings increased with the extension of the holographic period \(d\). From Equations (3) and (21), it was found that the relationship between the real axicon and the hologram is \(h/d = \tan \theta \approx \theta\). When \(h\) remains constant \((h = 0.0012\) mm\), increasing the holographic period, \(d\) is the equivalent of decreasing the refracting angle \(\theta\) of the real axicon. It means that changing the period \(d\) can obtain axicons with different refracting angles. In the same optical path, it can change the holograms dynamically to simulate a different axicon. This will be convenient in some applications.

3.2. Comparing with Real Axicon

The real axicon was manufactured with mechanical methods, which will generate inevitable fabrication errors. In order to compare the quality of Bessel beams produced by these two methods, we chose a real axicon \((r = 12.7\) mm\, and \(\theta = 0.008\) rad\) and generated the equivalent holograms \((d = 13\) pixels\, h = 0.0012 mm\). Figure 7 presents the images of the Bessel beams on different \(z\) planes. We can see that the Bessel beams generated by the SLM + CGHs remained in perfect form with the whole range. However, the Bessel beams generated by the real axicon could not stay in the same form, and deformation appeared when the distance was far away from the axicon, as shown in Figure 7, when \(z \geq 930\) mm. Therefore, using CGHs is not only a flexible way to generate the Bessel beams but can also avoid the fabrication errors.

At the same time, we compared the axial intensity distribution of the Bessel beams generated by these two methods. Some other parameters of the holograms and the incident beams are given by \(n_1 = 1.51637\, \omega = 6\) mm, \(\lambda = 650\) nm, and \(l_0 = 1\). Table 1 presents axial intensity distributions of Bessel beams of theory (calculated by Equation (20)), generated by the SLM and the real axicon within the \(z_{\text{max}}\), and their varying trends and fitting curves are shown in Figure 8. From Table 1 and Figure 8, we can see that the axial intensity distributions of the SLM are well matched with the theoretical analysis. The distributions of the real axicon are consistent with the theoretical analysis in the near distance,
but it began to deviate from the theoretical value after the peak. The reason for the deviation is the
deformation of the Bessel beams due to the manufacture errors. This also proved that the axial intensity
distributions are affected by the manufacture precision of the axicon.

Figure 7. Bessel beams generated by the spatial light modulator (SLM) and the real axicon at different
z planes.

Table 1. Axial-intensity distributions of Bessel beams of theory generated by the SLM and the real axicon.

| z Plane (mm) | Axial Intensity (a.u.) |
|--------------|------------------------|
|              | Theory | SLM | Axicon |
| 240          | 92.52  | 88.47 | 93.20  |
| 370          | 132.53 | 120.47 | 137.73 |
| 470          | 155.58 | 147.43 | 147.59 |
| 580          | 172.19 | 166.32 | 158.47 |
| 700          | 179.74 | 173.31 | 170.70 |
| 810          | 177.76 | 179.48 | 172.27 |
| 930          | 167.50 | 176.57 | 178.30 |
| 1040         | 152.58 | 167.73 | 163.02 |
| 1160         | 132.54 | 112.73 | 63.78  |

Figure 8. The axial-intensity distributions of Bessel beams generated by theoretical analysis, the SLM,
and the axicon.

The results show that both the axicon and SLM satisfy the principles that the intensity first increases,
and, after reaching a peak, it begins to decrease with the extension of the transmission distance.
3.3. Negative Axicon Generated by SLM

Negative axicons can generate annular beams, which can be applied in laser drilling to improve the efficiency [19,20]. Since demonstrating the SLM can also produce the annular beams with the CGHs, experiments were carried out with the same experimental setup in Figure 5 by adding a lens (focal length \( f = 110 \) mm). From Figure 4b, it can be seen that the beams from the SLM have a divergence angle, and its distribution area will exceed the receiving size of the CCD. Therefore, the lens was used to converge the beams and guarantee that the CCD will capture them, as shown in Figure 9. The hologram used to simulate the negative axicon, as shown in Figure 4a, was loaded into the SLM, and the period was set as \( d = 15 \) pixels, the distance from the SLM to the lens \( l_1 = 230 \) mm, and the height of the grooves \( h = 0.0012 \) mm.

![Figure 9. The principle of the annular beams focused by a lens.](image)

Figure 10 presents the images of annular beams generated by the hologram at the planes \( z = 75, 80, 85, \) and \( 90 \) mm behind the lens, and the corresponding radii of the inner and outer are shown in Table 2. From Table 2, it can be found that the width of annular beams decreased with the increase of the \( z \) place positions, and the experimental results were consistent with the theory. The images prove that the hologram can work as a negative axicon.

![Figure 10. The experimental results for the simulating negative axicon with the SLM. (a) \( z = 75 \) mm, (b) \( z = 80 \) mm, (c) \( z = 85 \) mm, (d) \( z = 90 \) mm.](image)

| \( z \) Plane (mm) | Theory (mm) | Generated by the SLM (mm) |
|-----------------|-------------|---------------------------|
|                 | \( r_1 \)   | \( r_o \)                  | \( r_o - r_1 \) | \( r_1 \) | \( r_o \) | \( r_o - r_1 \) |
| 75              | 0.474       | 1.275                     | 0.801           | 0.464   | 1.271   | 0.806          |
| 80              | 0.445       | 1.132                     | 0.687           | 0.439   | 1.122   | 0.684          |
| 85              | 0.352       | 0.988                     | 0.637           | 0.355   | 0.980   | 0.626          |
| 90              | 0.387       | 0.845                     | 0.458           | 0.381   | 0.839   | 0.464          |
4. Conclusions

This paper presents a method in which 256-level phase CGHs were loaded into the SLM to dynamically generate tunable axicons. Based on the principle of convolution and Fresnel diffraction, the intensity distributions of beams passing through these axicons were analyzed. The diffraction patterns were obtained as zero-order Bessel beams and annular hollow beams corresponding to the positive and negative axicons, respectively. The experimental results proved the theoretical analysis. Using SLM to simulate axicons has many advantages. It can dynamically generate tunable axicons for the demand of different transmission distances, and it can obtain better Bessel beams than those generated by the real axicon, which has inevitable manufacture errors. With the aid of high-level phase CGHs, the simulated positive and negative axicons will be widely used in the fields of precision measurement and laser processing. The limitation of the proposed method using SLM to generate axicon is that the SLM cannot cope with high laser power r, which may damage the liquid crystal inside.

Tunable axicons are easily generated by the SLM, and one of significant advantages of the axicon is that it can generated tunable Bessel beams, which is expected to apply in laser dicing, laser grooving, and laser drilling. However, the application is constrained by the worst uniformity of the Bessel beams along their axial propagation length, as shown in Figure 8. Further work is needed to improve axial intensity distribution and make sure that the beam has a constant intensity along its propagation length.

Author Contributions: Z.Z. and Z.C. deduced the formulas, conducted the simulations, and wrote the manuscript. Q.L. and X.W. contributed the main conceptual ideas. All authors read and approved the final manuscript.

Funding: This work was supported by the National Natural Science Foundation of China (Nos. 51575164, 51405143), the Science and Technology Research Project of Department of Education of Hubei Province (No. D20161406), the Open Fund of State Key Laboratory of Precision Measurement and Instrument (No PIL1602), the Fund of Hubei University of Technology (Nos. HSKFZD2014007, BSQD13048), the China Postdoctoral Science Foundation (Grant No. 2016M602269), and the Natural Science Foundation Youth Project of Hubei Provincial (Grant No. 2018CFB290).

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

SLM Spatial light modulator
CGH Computer generated hologram

References

1. McLeod, J.H. The axicon: A new type of optical element. *J. Opt. Soc. Am.* 1954, **44**, 592–597. [CrossRef]
2. Zhai, Z.; Zhao, B. Diffraction intensity distribution of an axicon illuminated by polychromatic light. *J. Opt. A Pure Appl. Opt.* 2007, **9**, 862–867. [CrossRef]
3. Rioux, M.; Tremblay, R.; Belanger, P.A. Linear, annular, and radial focusing with axicons and applications to laser machining. *Appl. Opt.* 1978, **17**, 1532–1536. [CrossRef] [PubMed]
4. Zhai, Z.; Ding, S.; Lv, Q.; Wang, X.; Zhong, Y. Extended depth of field through an axicon. *J. Mod. Opt.* 2009, **56**, 1304–1308. [CrossRef]
5. Zhai, Z.; Lv, Q.; Wang, X.; Shang, Y.; Yang, L.; Kuang, Z.; Bennett, P. Measurement of four-degree-of-freedom error motions based on non-diffracting beam. *Opt. Commun.* 2016, **366**, 168–173. [CrossRef]
6. Zhou, L.; Zhou, F.; Qu, D.; Liu, X.; Lu, W. Error analysis of the non-diffraction grating structured light generated by triangular prism. *Opt. Commun.* 2013, **306**, 174–178. [CrossRef]
7. Ren, O.; Birngruber, R. Axicon: A new laser beam delivery system for corneal surgery. *IEEE J. Quantum Electron.* 1990, **26**, 2305–2308. [CrossRef]
8. Algorri, J.; Urruchi, V.; García-Cámara, B.; Sánchez-Pena, J. Liquid crystal lenascons, logarithmic and linear axicons. *Materials* 2014, **7**, 2593–2604. [CrossRef] [PubMed]
9. Davidson, N.; Friesem, A.A.; Hasman, E. Holographic axilens: High resolution and long focal depth. *Opt. Lett.* 1991, **16**, 523–525. [CrossRef] [PubMed]
10. Milne, G.; Jeffries, G.D.; Chiu, D.T. Tunable generation of Bessel beams with a fluidic axicon. *Appl. Phys. Lett.* 2008, **92**, 261101. [CrossRef] [PubMed]
11. Turunen, J.; Vasara, A.; Friberg, A.T. Holographic generation of diffraction-free beams. *Appl. Opt.* **1988**, *27*, 3959–3962. [CrossRef] [PubMed]

12. Vasara, A.; Turunen, J.; Friberg, A.T. Realization of general nondiffracting beams with computer-generated holograms. *J. Opt. Soc. Am. A* **1989**, *6*, 1748–1754. [CrossRef] [PubMed]

13. Vieira, T.A.; Yepes, I.S.; Suarez, R.A.; Gesualdi, M.R.; Zamboni-Rached, M. Optical reconstruction of non-diffracting beams via photorefractive holography. *Appl. Phys. B* **2017**, *123*, 134. [CrossRef]

14. El Halba, E.M.; Khouilid, M.; Boustimi, M.; Belafhal, A. Generation of generalized spiraling Bessel beams by a curved fork-shaped hologram with Bessel-Gaussian laser beams modulated by a Bessel grating. *Optik* **2018**, *154*, 331–343. [CrossRef]

15. Dudley, A.; Vasilyeu, R.; Belyi, V.; Khilo, N.; Ropot, P.; Forbes, A. Controlling the evolution of nondiffracting speckle by complex amplitude modulation on a phase-only spatial light modulator. *Opt. Commun.* **2012**, *285*, 5–12. [CrossRef]

16. Chattrapiban, N.; Rogers, E.A.; Cofield, D.; Hill III, W.T.; Roy, R. Generation of nondiffracting Bessel beams by use of a spatial light modulator. *Opt. Lett.* **2003**, *28*, 2183–2185. [CrossRef] [PubMed]

17. Alexeev, I.; Leitz, K.H.; Otto, A.; Schmidt, M. Application of Bessel beams for ultrafast laser volume structuring of non-transparent media. *Phys. Procedia* **2010**, *5*, 533–540. [CrossRef]

18. Joseph, W.G. *Introduction to Fourier Optics*, 2nd ed.; Academic Press: New York, NY, USA, 1996.

19. Kuang, Z.; Perrie, W.; Edwardson, S.P.; Fearon, E.; Dearden, G. Ultrafast laser parallel microdrilling using multiple annular beams generated by a spatial light modulator. *J. Phys. D Appl. Phys.* **2014**, *47*, 115501. [CrossRef]

20. Zeng, D.; Latham, W.P.; Kar, A. Characteristic analysis of a refractive axicon system for optical trepanning. *Opt. Eng.* **2006**, *45*, 094302.

© 2020 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (http://creativecommons.org/licenses/by/4.0/).