Calibration of the constitutive equations for materials with different levels of strength and plasticity characteristic based on the uniaxial tensile test data

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Abstract. Today finite elements method (FEM) modeling and calculation are used for analysis current state and forecasting reliability lifetime period of components and constructions. Essential issues for correctness of obtained results in FEM analysis are: proper interpretation of boundary conditions and right constitutive equation of material in terms of high stresses and strains. Standard procedure obtaining of material constitutive relation directly from experimental tensile test data does not proper result for every type of material. Method of constitutive equation creation for elastic-plastic material for a high level of plasticity based on data obtained during tensile test is presented. The proposed calibration method was verified on materials with different levels of strength and plasticity characteristics: S355JR and 42CrMo4 steels and ADI cast iron. Plastic deformation level, triaxiality stress state parameter and Lode factor were taken into account during created of the constitutive equation.

1 The idea of the constitutive equations calibration

When starting the analysis based on numerical modeling and calculation, especially if components are made from high-plasticity material, it is necessary to implement the calibration of the stress-strain constitutive equation, primary received from uniaxial loading test data. Some of the researchers propose fitted of the experimental and numerical test results by iteration selection of correlation parameters for the true stress-strain relationships obtained from uniaxial loading test data [1,2,3]. This method needs a few repetitions of the numerical calculations, until then numerical and experimental loading curves coming closer together (would be convergent).

Another method was proposed by Bai and Wierzbicki [4,5,6] and next developed by Neimitz and co-workers [7]. The yield stress were defined according to the next formula:

$$
\sigma_{yld} = \bar{\sigma}(\bar{\varepsilon}_p)\left[1 - \eta (\eta - \eta_0) \bar{\sigma}_0 \left( c_{ax}^{ax} - c_{ax}^{ax} \right) \left( \gamma - \frac{m+1}{m+1} \right) \right]
$$

In the presented formula, the function $\bar{\sigma}(\bar{\varepsilon}_p)$ describes relation between effective real stress and plastic strain. Symbol $\eta = \frac{\sigma_{ax}}{\sigma_{c}}$ is the triaxiality factor, where $\sigma_{m} = \frac{1}{3} (\sigma_{xx} + \sigma_{zz} + \sigma_{yy})$ and $\sigma_{c}$ is a effective stress; $\eta_0$ is a reference value of the triaxiality coefficient (for the uniaxial tensile test cylindrical specimen, $\eta_0=1/3$).

Bai and Wierzbicki postulated that the $\gamma$ function presents in the form:
\[ \gamma = 6.46[\sec(\theta - \pi/6) - 1] \]  

(2)

where \( \theta \) is the Lode angle, which must satisfy the inequality \( 0 \leq \theta \leq \pi/3 \). The Lode angle could be calculated using the below relationships (3)-(5):

\[ \bar{\theta} = 1 - \frac{6\theta}{\pi} = 1 - \frac{2}{\pi} \arccos \xi \]  

(3)

\[ \xi = L \frac{9 - L^2}{\sqrt{(L^2 + 3)^3}} \]  

(4)

\[ L = \frac{2\sigma_{II} - \sigma_I - \sigma_{III}}{\sigma_I - \sigma_{III}} \]  

(5)

The \( \sigma_I, \sigma_{II}, \sigma_{III} \) are principal stresses and \( \sigma_i \) and \( \sigma_{III} \) – maximum and minimum values of principal stresses, respectively. Quantity \( L \) called as the Lode parameter.

The parameters \( c, \xi, c_{\xi}, \xi_{\alpha} \) and \( m \) in Eq. 1 must be determined experimentally.

The equation describes the effect of material softening immediately before damage caused by growth and connection of voids was proposed by Neimitz [7].

\[ c_n' = c_n [1 + H(\varepsilon_{pl,0})(\varepsilon_{pl,i} - \varepsilon_{pl,0})]^\alpha \]  

(6)

In this equation, \( \varepsilon_{pl,0} \) is a value of plastic strain at which material softening beginning occurs (see Fig. 1) and \( \varepsilon_{pl,i} \) are further plastic strain values. \( H(\varepsilon_{pl,0}) \) is the H’eaviside function.

![Figure 1. Scheme for determining the \( \varepsilon_{pl,0} \) value.](image)

2 Materials used in experimental tests and numerical analysis

Experimental tests and following numerical calculations were carried out on specimens made of steel types – S355JR, 42CrMo4 and cast iron ADI EN-GJS-1059-6. The steel S355JR after normalization has ferrite-pearlite microstructure type. Microstructure of 42CrMo4 steel after quenching (850 °C, oil) and tempering (500 °C) was tempered martensite type. Selection of those materials was justified of
different levels of strength and plasticity characteristics. Uniaxial tension tests were carried out used cylindrical specimens with a 5 mm in diameter and base length of 25 mm (Fig. 2) at temperature 20°C. The characteristics of strength and plasticity are presented in Tab. 1. Relationship between true stress and true strain was determined on the basis of the nominal stress and strain from uniaxial tensile tests results [8]. The diagrams of stress-strain relationships for nominal and true values are shown in figure 3. The nominal and true stress and strain calculated from recorded signals of load F and extensometer elongation ΔL as (Eq. 7):

\[ \varepsilon_{\text{nom}} = \frac{L - L_0}{L_0}; \ \sigma_{\text{nom}} = \frac{F}{S_0}; \ \varepsilon_{\text{true}} = \ln(1 + \varepsilon_{\text{nom}}); \ \sigma_{\text{true}} = \sigma_{\text{nom}}(1 + \varepsilon_{\text{nom}}) \]  

Table 1. The strength and plasticity properties of analyzed materials.

| Materials | \( \sigma_{\text{LYS}} \)/\( \sigma_{\text{HYS}} \) [MPa] | \( \sigma_{\text{UTS}} \) [MPa] | \( E \) [GPa] | \( n \) | \( A_s \) [%] |
|-----------|------------------|-----------------|--------------|------|--------|
| S355JR    | 366/375          | 377/380         | 495          | 613  | 201    | 7.90   | 4.78   | 35.43 |
| 42CrMo4   | 1126             | 1135            | 1211         | 1287 | 202    | 25.64  | 15.24  | 14.94 |
| ADI       | 775              | 781             | 1125         | 1232 | 159    | 7.89   | 6.11   | 9.55  |

\( \sigma_{\text{LYS}} \) - lower yield strength, \( \sigma_{\text{HYS}} \) - higher yield strength, \( \sigma_{\text{UTS}} \) - ultimate strength, \( E \) - Young's modulus

Figure 2. The specimen for uniaxial loading test.  
Figure 3. The nominal and true stress-strain diagrams of tested materials.

The lowest strength characteristic and the highest plasticity were obtained for S355JR steel. The highest strength characteristics showed 42CrMo4 steel. The pronounced necking was presented in both steel types specimens before damage during uniaxial test, while the necking was not observed before cracking for specimens of cast iron ADI.

3 Calibration process of material constitutive equations

The true stress-strain curves obtained on the basis data of uniaxial tensile test. Taking into account the data set which are limited of maximum stress value, that is associated with necking process formation during tension-loaded specimen process. The stress-strain relationship in range from start yielding to necking initiation is written by power function and known as Ramberg-Osgood or Ludvik laws [9].
Further strain process in necking area takes place in 3D stress state with participation of voids nucleation, growth and coalescence, and required more complex mathematical expression to written these physical processes.

Based on the true stress-strain relationship was conducted the procedure of material constitutive equations creation in domain of large strain and stress values.

In the first stage the numerical simulation of specimens same as uniaxial loading were carried out and true stress-strain relationships obtained from tensile tests were used with data approximation according to liner dependence. The program ABAQUS [10] was used for numerical modeling and calculation. It was modeled of ¼ of the tensile specimens because its axially symmetrical [Fig. 4].

![Figure 4. Numerical model in Abaqus program for uniaxial tensile specimen.](image)

The load and elongation data obtained of numerical calculation were compared to the experimental results. Only for specimens made of cast iron ADI numerical and experimental data are completely overlaps. For specimens made of S355JR and 42CrMo4 steels were observed significant differences between calculated and experimental data, namely for data which corresponding with necking process.

For the specimens in which did not occur necking before damage the constitutive equation of material can be used in form of true stress-strain relationship without any corrections. The data obtained of numerical calculation and experimental testing are similar (Fig. 5).

Other situation has place for specimens with necking (S355JR and 42CrMo4 steels). Takes into account a fact, that strain level during necking process reaches more 2.0, then true strain obtained from uniaxial tensile test are limited to 0.2, it is necessary extrapolated the true stress-strain relationship data to higher values. Initially the extrapolation was carried out in form of power function of true stress – plastic strain dependence to 2.5 and more of plastic strain level. As we can see after this type extrapolation are significant differences in numerical calculated and experimental data in ranges graphs corresponding to necking process.

![Figure 5. Experimental and numerical force – elongation curves.](image)
The second type of the extrapolation was done in liner function form based on fitting of ~200 true stress-strain data points directly before initial necking. As a result of implementing to numerical calculation the extrapolation in liner form, the graphs obtained of numerical data are located slightly higher than one of experimental data in necking process range (Fig. 5).

So for specimens in which necking are observed before damage, during creation the constitutive equations, should be introduced correction, which takes into account processes leading to material softening. For this purpose the equation 6 was used [7]. The appropriate coefficients values for equation 1, which calculated by iteration method, are presented in table 2. As a result introduced this correction the experimental and numerical simulation data are similar (Fig. 6).

**Table 2.** The use coefficients values for equation 1.

| Material:        | $c_\eta$ | $c_\theta^x$ | $c_\theta^x$ | m   | $c_{PLO}$ | $\alpha$ |
|------------------|---------|--------------|--------------|-----|-----------|---------|
| S355JR steel     | 0.20    | 1.03         | 1.00         | 1.00| 0.06      | 2.35    |
| 42CrMo4 steel    | 0.20    | 1.01         | 1.00         | 1.00| 0.06      | 1.40    |

![Figure 6. Experimental and numerical corrected curves of force – elongation: a) S355JR steel, and b) 42CrMo4 steel.](image)

**4 Summary**

The tests carried out in presents paper clearly indicate on the necessity to perform the right calibration procedure of stress-strain constitutive relation for obtained properly result during numerical calculation. For materials, which do not shown necking and softening effect, calibration procedure can be limited to interpolation and extrapolation of the true stress – logarithmic strain relationship by linear function. While for materials, which shown necking during tensile loading [11,12], calibration procedure should additionally take into account members writing softening effect. The softening process of metallic materials by damage increase are consideration also in GTN-model [13,14]. Taking into account of softening effect in steels allows obtained correct stress and strain distributions directly in front of the crack or defects and right described fracture process construction members [7,13].
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References

[1] Zhao K and Wang L and Chang Y and Yan J 2016 Identification of post-necking stress–strain curve for sheet metals by inverse method *Mechanics of Materials* 92 pp 107-118
[2] Joung MS and Eom JG and Lee MC 2008 A new method for acquiring true stress–strain curves over a large range of strains using a tensile test and finite element method *Mechanics of Materials* 40 pp 586-593
[3] Depreński Ł and Seweryn A 2011 Experimental Research into Fracture of EN-AW 2024 and EW-AW 2007 Aluminum Alloy Specimens with Notches Subjected to Tension *Experimental Mechanics* 51 pp 1075-1094
[4] Bao Y and Wierzbicki T 2004 On fracture locus in the equivalent strain and stress triaxiality space *International Journal of Mechanical Science* 46 pp 81-98
[5] Bai Y and Wierzbicki T 2010 Application of extended Mohr–Coulomb criterion to ductile fracture *International Journal of Fracture* 161 pp 1-20
[6] Bai Y and Wierzbicki T 2008 A new model plasticity and fracture with pressure and Lode dependence *International Journal of Plasticity* 24 pp 1071-1096
[7] Neimitz A and Galkiewicz J and Dzioba I 2018 Calibration of constitutive equations under conditions of large strains and stress triaxiality *Archives of Civil and Mechanical Engineering* 18 pp 1123-1135
[8] ASTM E8 2003 Standard test method for tension testing of metallic materials International West Conshohocken
[9] Anderson TL 2008 Fracture Mechanics New York
[10] Abaqus 6.12 Getting Started with Abaqus Interactive Edition.
[11] Dzioba I and Lipiec S and Furmańczyk P and Pała R 2018 Investigation of fracture process of S355JR steel in transition region using metallographic, fractographic tests and numerical analysis *Acta Mechanica et Automatica* 12 pp 145-150
[12] Dzioba I 2010 Properties of the 13KHM steel after operation and degradation under the laboratory conditions *Materials Science* 12 46 (3) pp 357-364
[13] Defaisse C and Maziere M and Marcin L and Besson J 2018 Ductile fracture of an ultra-high strength steel under low to moderate stress triaxiality *Engineering Fracture Mechanics* 194 pp 301-318
[14] Zhang Y and Lorentz E and Besson J 2017 Ductile damage modelling with locking-free regularized GTN model *International Journal for Numerical Methods in Engineering* 113 (3) pp 1871-1903