Magnetotransport in 2D electron systems with a Rashba spin-orbit interaction

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The beating pattern of Shubnikov-de Haas oscillations in 2D electron system in the presence of a Rashba zero-field spin splitting is reproduced. It is shown, taking into account the Zeeman splitting, that the explicit formulae for the node position well describes the experimental data. The spin-orbit interaction strength obtained is found to be magnetic field independent in agreement with the basic assumptions of the Rashba model.

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There has been growing interest in the zero-magnetic-field spin splitting\(^1\) of the 2D electron gas (2DEG), associated with the spin-orbit interaction (SOI) caused by the structural inversion asymmetry in heterostructures\(^2\). Application of a gate voltage \(^3\) is known to be the most effective method to control the SOI strength. These 2D systems have been suggested for application in future spintronics devices, such as spin-based field-effect transistors\(^4\), spin-interference devices\(^5\), and nonmagnetic spin filters based on a resonant tunneling structure\(^6\). Usually, the beating-pattern analysis of Shubnikov-de Haas oscillations (SdHO)\(^3\) and the weak antilocalization method\(^7\) are used to determine the SOI strength in 2D systems. However, the former approach is known to lead to a certain controversy in determining the zero-field spin splitting \(\Delta\).

Namely, the spin splitting deduced from the SdHO beating node position at finite fields\(^3\) is different from that \(\Delta\) expected for \(B = 0\). In the present paper, this discrepancy is attributed to the contribution of the nonzero Zeeman spin splitting at finite fields. We support our idea by a rigorous analysis of the SdHO beating pattern caused by SOI spin splitting. The beating node positions reported in\(^3\) agree well with those predicted by the theory. Then, we demonstrate that the SOI strength is independent of the magnetic field.

Let us consider a 2DEG in the x-y plane, subjected to a magnetic field. In the Landau gauge, the one-electron Hamiltonian including the Rashba spin-orbit term\(^8\) is given by

\[
H = \frac{(\mathbf{p} + e\mathbf{A})^2}{2m} + \frac{\alpha}{\hbar}\sigma(\mathbf{p} + e\mathbf{A})|n + \frac{g\mu_B}{2}(\sigma\mathbf{B})
\]

where \(\mathbf{p}\) is the 2D momentum; \(m\), the effective mass; \(g\), the Zeeman factor; \(\mu_B\), the Bohr magneton; and, \(\mathbf{n}\), the unit vector in the z-direction. Then, \(\sigma\) is the Pauli spin matrix; \(\mathbf{B}\), the total magnetic field; and, \(\alpha\), the SOI strength.

It has been shown\(^3\) that the solution to Eq.\(^1\) has an explicit form in the case of a perpendicular magnetic field \(\mathbf{B} = B_z = B\). The spectrum for dimensionless energy \(\varepsilon = E/\mu\) (\(\mu\) is the Fermi energy) is given by\(^8\)

\[
\varepsilon_0 = \eta\beta, \quad \varepsilon_n = \eta(n \pm \sqrt{\gamma^2n + \beta^2}), n \geq 1
\]

where \(\eta = \hbar \omega_c/\mu\) is the dimensionless magnetic field; \(\omega_c = eB/mc\), the cyclotron frequency; \(\beta = \frac{1}{2}(1 - \chi)\), the term containing the Zeeman spin splitting; \(\chi = \frac{2\alpha n}{\hbar c}\), the spin susceptibility; and \(n\), an integer similar to that in the conventional description of the Landau levels (LL). Then, according to Ref.\(^3\)\(\gamma = \sqrt{\frac{\alpha}{\eta}} = \frac{\alpha k_F}{\mu \sqrt{\eta}}\), where \(\delta\) is the dimensionless SOI strength parameter; \(\Delta = 2\alpha k_F\), the zero-field spin-orbit splitting at the Fermi energy; and \(h k_F\), the Fermi momentum. Usually, the typical Fermi energy \(\mu \sim 80\text{meV}\) exceeds the SOI-induced splitting \(\Delta \sim 1\text{meV}\) (see\(^3\)), and, therefore \(\delta \ll 1\). It is noteworthy that the conventional spin-up(down) energy states associated with \(n\)-th LL number correspond to \(\varepsilon_n^+\) and \(\varepsilon_{n+1}^-\) states respectively. In the absence of SOI, Eq.\(^2\) reproduces well-known LL energy spectrum.

In contrast to the conventional formalism extensively used to find the low-B magnetoresistivity, we use the alternative approach\(^9\),\(^10\),\(^11\) which allows to resolve magnetotransport problem in both the SdHO and Integer Quantum Hall Effect (IQHE) modes. Moreover, this method was successfully used in a recent paper\(^12\) to reproduce the SdHO beating structure in the presence of the zero-field valley splitting (Si-MOSFET 2D system), and in both the crossed- and tilted-field configurations. Following the argumentation put forward in Ref.\(^10\), well above the classically strong magnetic field range \(\omega_c \tau \gg 1\), where \(\tau\) is the momentum relaxation time, 2DEG can be assumed dissipationless in strong quantum limit when the cyclotron energy \(\hbar \omega_c\) exceeds both the thermal energy \(kT\) and the energy related to LL-width \(h/\tau\). Here, \(\tau\) is the quantum relaxation time. Under the above assumptions \(\sigma_{xx}, \rho_{xx} \approx 0\). Nevertheless, routine dc measurements yield\(^10\) the finite magnetoresistivity associated with a combination of the Peltier and Seebeck thermoelectric effects. Within the scenario suggested\(^10\), we obtain the above magnetoresistivity in the form

\[
\rho = \rho_{yx} \frac{\alpha_{2D}^2}{L}
\]

where \(\alpha_{2D}\) is the 2DEG thermoelectric power; \(\rho_{yx}^{-1} = Nee/B\), the Hall resistivity; \(N = -(\partial \rho_{yx}/\partial T)\); the 2D den-
system, $\Omega = -kT \sum_n \ln \left( 1 + \exp \left( \frac{\mu_n - F_0}{kT} \right) \right)$, the thermodynamic potential; $\Gamma = \frac{eB}{hc}$, the zero-width LL density of states; $L = \frac{\pi^2 k_F^2}{3b^2}$, the Lorentz number; $k_B$, the Boltzmann constant. In fact, the 2D thermoelectric power in strong magnetic fields is a universal quantity \cite{s}, proportional to the entropy per electron: $\alpha_{2D} = -\frac{S}{N}$, where $S = -\ln(N)$ is the entropy. Both $S, N, \rho$ are universal functions of the dimensionless temperature $\xi = \frac{k}{\mu}$ and the magnetic field $\eta = 2/\nu$, where $\nu = N_0/\Gamma$ is the conventional filling factor, and $N_0 = \frac{m}{\hbar^2} \mu$ is the zero-field density of the strongly degenerate 2DEG in the absence of a SOI-induced splitting.

Using the Lifshitz-Kosevich formalism and, then, neglecting finite LL-width ($h/\tau_\eta \to 0$), we derive in Appendix asymptotic formulae for $\Omega$, and, hence, for $N, S, \rho_x, \rho_y$ which are valid at low temperatures and weak magnetic fields $\xi, \eta \ll 1$:

$$N = N_0 \xi F_0(1/\xi) + 2\pi N_0 \sum_{k=1}^{\infty} \frac{\sin(2\pi k/\eta)}{\sinh(r_k)} R(\eta), \quad (4)$$

$$S = S_0 - 2\pi^2 \xi k_B N_0 \sum_{k=1}^{\infty} \Phi(r_k) \cos(2\pi k/\eta) R(\eta),$$

where $S_0 = k_B N_0(2F_1(1/\xi) - F_0(1/\xi))$ is the entropy at $B = 0$; $F_0(z)$, the Fermi integral; and $\Phi(z) = 1 - e^{\pi i \coth(z)}$.

At $B = 0$ both the thermopower and 2D density are constants, i.e. $\alpha_{2D} = -\frac{\pi^2 k_F^2 k_B}{3b^2}$, hence the magnetoresisitivity is given by zero-field asymptote $\rho = \frac{1}{\eta} \frac{\pi^2 k_F^2}{3b^2}$. According to Eq. (4), for actual first-harmonic case ($k = 1$) the magnetoresistivity can be viewed as the zero-field background, on which the rapid SdHO modulated by long-period beatings (see Fig.2) are superimposed. It’s worthwhile to mention that at the beat nodes (i.e. when the form-factor $k = 1$ vanishes) the magnetoresistivity is given by zero-field asymptote. This is not, however, the case of low temperatures and/or high magnetic fields when the high-order terms ($k > 1$) in Eq. (4) may determine the amplitude of magnetoresistivity at the beat nodes. It turns out that the data reported in the above feature.

We now analyze in detail the form-factor $R(\eta)$ (see Appendix) which determines the beating pattern of $S, N$ and, hence, $\rho$. For the actual first-harmonic case (i.e., $k = 1$), the beating nodes can be observed when $R(\eta) = 0$ or

$$\sqrt{\beta^2 + \frac{\delta}{\eta^2}} = \frac{j}{4}, \quad (5)$$

where we neglect the small quadratic term $\delta^2/4\eta^2 \ll \delta/\eta^2$ evaluating Eq. (4). Then, $j = 1, 3, \ldots$ is the beating node index. We emphasize that the first node cannot be observed in experiments, performed, for example, in Ref. [3]. Indeed, for real 2D In$_x$Ga$_{1-x}$As/In$_{0.52}$Al$_{0.48}$As system ($m = 0.049 m_0$, $g \simeq 4$) we find $\beta = 0.45$, and, therefore Eq. (5) cannot be satisfied for $j = 1$. With the help of Eq. (5), we analyze the nodes, reported in for three different samples, and then plot the dependence of the zero-field SOI splitting at the Fermi energy $\Delta$ against the node index (see Fig.1), starting from $j = 3$. For these samples $\Delta$ is nearly constant within the actual range of the magnetic fields, therefore we obtain the respective mean values $\Delta_0$ denoted in Table II. Note that the minor deviation of $\Delta$ with respect to its mean value in high-field limit (low-index nodes) can be associated with possible magnetic field dependence of the $g$-factor. In contrast, the non-parabolicity effects seem to be irrelevant for the actual low-field case $B < 1T$.

We emphasize that the node condition similar to Eq. (5) was previously discussed in literature. Following the analysis done in Ref. [3], the nodes occur when the spin-orbit-split subbands are shifted one with respect another by half a period at the Fermi energy. Namely, $1 \simeq \epsilon_n^+ = (\epsilon_{n+1} + \epsilon_{n+2})/2$, where $s = 0, 1, 2, \ldots$ corresponds to the node index as $j = 1 + 2s$. For actual high LL-number case $n \gg 1$ this condition reproduces Eq. (5).

Let us discuss the conventional method often used to extract the zero-field SOI splitting at the Fermi en-
energy. According to phenomenological arguments put forward by Das et al.\cite{3,4}, the nodes may occur when $\cos \left( \pi \frac{\Delta_{\text{tot}}}{\hbar \omega_c} \right) = 0$ or $\Delta_{\text{tot}} = \pm \frac{\Delta}{\hbar \omega_c}$, where the total spin splitting at the Fermi energy between spin-down $\varepsilon^-_{n+1}$ and spin-up $\varepsilon^+_n$ states yields $\Delta_{\text{tot}} = \hbar \omega_c - \sqrt{(2 \hbar \omega_c)^2 + \Delta^2}$. As expected, the total spin splitting $\Delta_{\text{tot}}$ coincides with the zero-field $+ \Delta$ and the Zeeman $\chi \hbar \omega_c$ spin splitting in low (high) magnetic field limit respectively. With the help of the dimensionless units the node condition suggested by Das et al. reads $\sqrt{\beta^2 + \frac{\delta}{\eta}} = \frac{1 + \beta/2}{2}$, hence, reproduces our result if one selects “+” set at $j \geq 1$. We argue that straightforward procedure (see Fig.1) used to extract $\Delta_0$ is, however, preferable compare to zero-field extrapolation method suggested in Ref.\cite{3,4}. Indeed, for low-density samples and (or) under the temperature enhanced conditions the SdHO amplitude is suppressed, hence, the low-field nodes become hidden. In this case the zero-field extrapolation method\cite{3} may lead to a subsequent errors.

Let us now reproduce (see Fig.2) the SdHO beating pattern with the nodes occurred in a typical sample (sample A($x = 0.65$)\cite{3}) at $B = 0.873; 0.46; 0.291; 0.227; 0.183; 0.153T$ using Eq.\cite{4}, and previously extracted value of zero-field SOI splitting $\Delta_0 = 2.34$meV. It’s worthwhile to mention that our results differ with respect to those, which can be obtained within the conventional formalism in the following: (i) the low-field quantum interference, classical magnetoresistivity and 3D substrate parallel resistivity\cite{3} background are excluded within our approach; (ii) in contrast to conventional SdHO analysis, our method determines the absolute value of magnetoresistivity, and moreover, can lead to a gradual transition\cite{16} from the SdHO to the IQHE mode.

We argue that the noticeable increase in SdHO amplitude was observed\cite{3} at $B = 0.37T$. This value satisfies the criterion of the classically strong magnetic field since $\omega_c \tau = 4$ while the corresponding cyclotron energy $\hbar \omega_c = 8.2K$ correlates with that $\sim 9.8K$ expected from T-dependent SdHO-damping factor, i.e. when $2 \pi^2 \xi/\eta \sim 1$. We conclude that the energy associated with LL width $\sim \hbar/\tau_0$ is less or at least equal to the thermal energy. The above estimates point to validity of zero-width LL model in this particular case. Nevertheless, since both the temperature and finite LL width known to suppress the SdHO amplitude in a rather similar manner, we esteem reasonable to reproduce in Fig.4 the SdHO beating pattern using somewhat higher temperature $T = 1.6K$ than that $T = 0.5K$ reported in\cite{3}.

Note that our approach provides a correct number of oscillations between the adjacent nodes. For example, the number of oscillations confined between $j = 3, 5$ nodes\cite{37} correlates with that\cite{35} observed in\cite{3}. A minor point is that our approach predicts a somewhat lower amplitude of SdHO, compared with that in the experiment\cite{3}. For example, for $j = 3$ node ($B = 0.873T$ in Ref.\cite{3} ) we obtain $\rho = 0.0035$Ohm. Actually, one would expect the same order of magnitude for SdHO amplitude between the proximate nodes (see $j = 3, 5$ in Fig.2). Our estimation is, however, less than both the absolute magnetoresistivity 400Ohm at $B = 0.873T$ and SdHO amplitude $\sim 50$Ohm reported in Ref.\cite{3}.

In conclusion, we demonstrated the relevance of the approach\cite{10} regarding the beating pattern of SdHO caused by Rashba spin-orbit interactions. Taking into account the Zeeman splitting, the rigorous analysis of experimental data\cite{3} suggests a B-independent strength of the Rashba SOI. The above finding is consistent with the general theoretical assumptions\cite{3}. Our approach can be helpful for estimation of the SOI strength.

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### APPENDIX

Using the conventional Poisson formulae

$$\sum_{m_0}^{\infty} \varphi(n) = \int_{a}^{\infty} \varphi(n)dn + 2Re \sum_{k=-1}^{\infty} \int_{a}^{\infty} \varphi(n)e^{2\pi i k n}dn, \quad (6)$$

where $m_0 - 1 < a < m_0$, $m_0$ the initial value of the summation, the thermodynamic potential can be represented as the sum $\Omega = \Omega_0 + \Omega_3$ of the zero-field and oscillating parts as follows

$$\Omega_0 = -N_0 \mu \xi^2 F_1(1/\xi), \quad (7)$$
\[ \Omega_{\sim} = -N_0 \mu g \Re \sum_{k=1}^{\infty} \int_{0}^{\infty} e^{2\pi i k n} \ln \left( 1 + e^{-\frac{\varepsilon}{\Delta}} \right) \, dn, \]

where \( F_n(z) \) is the Fermi integral. For simplicity, we omit the SOI-induced splitting in the zero-field term \( \Omega_0 \) because \( \delta \ll 1 \). The special interest of the present paper is in the oscillating term \( \Omega_{\sim} \) of thermodynamic potential, which can be strongly affected by spin-orbit-split subbands(\( \pm \)). After a simple integration by parts, the oscillating term yields

\[ \Omega_{\sim} = N_0 \mu g \Re \sum_{k=1}^{\infty} \frac{i n k}{2\pi k} \int_{0}^{\infty} e^{2\pi i k n \varepsilon} \left( 1 + e^{-\frac{\varepsilon}{\Delta}} \right) \, d\varepsilon \quad (8) \]

Using Eq.\( (2) \), for a certain energy we calculate the actual high-order LL-like numbers, associated with both the spin-orbit-split subbands as

\[ n^\pm(\varepsilon) = \frac{\varepsilon}{\eta} + \frac{2}{\gamma^2} \pm \sqrt{\beta^2 + \frac{4}{\eta} \varepsilon + \frac{\gamma^4}{4}}. \quad (9) \]

It should be noted that the integrand equation in Eq.\( (8) \) is a rapidly oscillating function, which is, in addition, strongly damped when \( \varepsilon > 1 \). The major part of the magnitude of the integral results from the energy range close to the Fermi energy, when \( \varepsilon \sim 1 \). Therefore, \( n^\pm(\varepsilon) \) can be regarded as smooth functions of energy, and, hence, can be re-written as \( n^\pm = n^\pm(1)(\varepsilon - 1) \), where we use the designation \( n^\pm = n^\pm(1) \). Under the above assumption, we can change the lower limit of integration to \(-\infty\) and then use the textbook expression

\[ \int_{-\infty}^{\infty} \frac{j^k y}{\sin(\pi k y)} \, dy = \frac{i^k}{\sin(\pi k)} \]

for the integral of the above type. Finally, the thermodynamic potential yields

\[ \Omega = \Omega_0 + N_0 \mu g 2\pi^2 \xi^2 \sum_{k=1}^{\infty} \frac{\cos(\pi k(n^+_1 + n^-_1)) R(\eta)}{r_k \sinh(r_k)} \]

where we assume that \( (\frac{2n^\pm}{\pi k} \frac{1}{\Delta}) \sim \frac{1}{\Delta} \) is valid for the actual case of high-order Landau levels \( n^\pm \gg 1 \), and \( r_k = 2\pi^2 \xi \eta / \eta \) is a dimensionless parameter related to T-damping of SdH amplitude. Then, \( R(\eta) = \cos(\pi k(n^+_1 + n^-_1)) \) is the form-factor. The oscillatory part of the thermodynamic potential consists of rapid oscillations

\[ \cos(\pi k(n^+_1 + n^-_1)) \approx \cos(2\pi k/\eta), \]

on which long-period beatings governed by the form-factor are superimposed. As expected, the form-factor is reduced in absence of SOI to a field-independent constant \( R(\eta) = \cos(2\pi k/\eta) \), and, therefore, the beating structure is absent. Using the conventional thermodynamic definition, we can easily obtain both the entropy and the density of 2D electrons, specified by Eq.\( (1) \).

[1] G. Lommer, F. Makler, and U. Ressler, Phys. Rev. B, 32, 6965 (1985).
[2] J. Luo, H. Munekata, F.F. Fang, and P.J. Stiles, Phys. Rev. B 38, 10142 (1988).
[3] B. Das, D.C. Miller, S. Datta, R. Reifenberger, W.P. Hong, P.K. Bhattacharaya, J. Singh, and M. Jaffe, Phys. Rev. B 39, 1411 (1989).
[4] B. Das, S. Datta, R. Reifenberger, Phys. Rev. B 41, 8278 (1990).
[5] E.I. Rashba, Fiz. Tverd. Tela (Leningrad) 2, 1224, 1960 [Sov. Phys. Solid State 2, 1109 (1960)]; Y.A. Bychkov and E.I. Rashba, J. Phys. C 17, 6039 (1984).
[6] V.A. Bychkov, V.I. Mel’nikov, and E.I. Rashba, Zh. Eksp. Teor. Fiz. 98, 717 (1990), [Sov. Phys. JETP 71, 401 (1990)].
[7] J. Nitta, T. Akazaki, H. Takayanagi, and T. Enoki, Phys. Rev. Lett. 78, 1335 (1997).
[8] G. Engels, J. Lange, Th. Schpers and H. Lth, Phys. Rev. B 55, R1958 (1997).
[9] S. Datta and B. Das, Appl. Phys. Lett. 56, 665 (1990).
[10] Tie-Zheng Qian and Zhao-Bin Su, Phys. Rev. Lett. 72, 2311 (1994).
[11] J. Nitta, F.E. Meijer, and H. Takayanagi, Appl. Phys. Lett. 75, 695 (1999).
[12] T. Koga, J. Nitta, H. Takayanagi, and S. Datta, Phys. Rev. Lett. 88, 126601 (2002).
[13] T. Koga, J. Nitta, T. Akazaki, and H. Takayanagi, Phys. Rev. Lett. 89, 046801 (2002).
[14] C.G.M. Kirby and M.J. Laubitz, Metrologia 9, 103 (1973).
[15] M.V. Cheremisin, Zh. Eksp. Teor. Fiz. 119, 409 (2001), [Sov. Phys. JETP, 92, 357, 2001].
[16] M.V. Cheremisin, Physica E, 28, 393 (2005).
[17] M.V. Cheremisin, Physica E 27, 151 (2005).
[18] S.M. Girvin and M. Jonson, J.Phys.C 15, L1147 (1982).
[19] Can-Ming Hu, J.Nitta, T.Akazaki et al, Phys. Rev. B 60, 7736 (1999).
[20] M. Dobers, K. von Klitzing, G. Weimann, Phys. Rev. B 38, 5453 (1988).
[21] M. Dobers, J.P.Viereit, Y. Guldner et al, Phys. Rev. B 40, 8075 (1989).