On a parameter-dependent symbolic entropy in nonlinear dynamics complexity detection

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Abstract
Symbolization is prerequisite for symbolic time series analysis which is important in nonlinear dynamic complexity analysis. Shannon entropy for a parameter-dependent four-symbols transformation is analyzed and its complexity detection is validated in our works. We analyze the symbolic entropy in complexity detection of two chaotic models, logistic and Henon maps, and use ECG and HRV derived from ECG of CHF patients, healthy young and elderly subjects to test the impacts of parameter settings on complexity analysis. The complexity-loss theory of aging and diseased heartbeats is validated and reasons that may account for the paradox of symbolic entropy in ECG are discussed. Tests results prove that original parameter settings are not universally suitable for different time series, and it is necessary to adjust the controlling parameter accordingly.

Keywords: symbolization, nonlinear dynamic complexity, chaotic model, HRV

1. Introduction

Symbolic dynamic deals with robust properties of dynamics without digging into numbers and provides a rigorous way of looking at real dynamics with finite precision \cite{1}. Symbolic time series analysis, with the basic idea of simplify, improves sequence dynamic analysis effectively \cite{2,3}. Through symbolic transformation, it has excellent nonlinear dynamic complexity detections and better noise resistant features than traditional parameters such as Lyapunov exponent, fractal dimensions, therefore it has been commonly adopted in nonlinear dynamics analysis \cite{3,4,5,6,7}.

Symbolizations are classified into global static and local dynamic approaches, among which the global static method transform series into symbolic sequences by dividing series into finite partitions which are labeled with given symbols \cite{8,9}. A symbolic transformation belonging to global static ways in the works of Wessel N. et al. has effective applications in cardiac physiological signals \cite{10,11,12}, and it calculates the time series mean and sets a controlling parameter to transform series into symbol sequence on the basis of the alphabet $A = \{0, 1, 2, 3\}$. The critical controlling parameter affects the three parting lines and four intervals

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for the formulation of symbolic sequence. During some follow-up researches, scholars find it is effective in complexity detection of other biomedical signals such as ECG, EEG [13, 14, 15]. In these works, however, there is no in-depth research on the effects of parameter choices on different quantities and physiological signals.

In our contributions, we take advantage of classical chaotic model to test issues on complexity detection of the symbolic method and apply the symbolic approach to three groups of real-world physiological signals from PhysioBank, and make detailed analysis of controlling parameter’s effects on the complexity extraction of the parameter-dependent symbolic entropy.

2. Symbolization

Symbolic transformation of time series into sequence from a given alphabet is the first step of symbolic time series analysis. It is a coarse-graining procedure that some detailed information is lost while the coarse dynamic behavior can be analyzed [11]. The symbolic transformations make a compromise of extracting some dynamical information and of maintaining sufficient statistics for probability distribution estimation. A pragmatic symbolization is proposed for physiological time series, and it is context-dependent transformations having feature of close connection to physiological phenomena and easy interpretation [12]. Assuming $x_i$ belongs to the time series $X = \{x_1, x_2, \ldots, x_L\}$, symbolic transformation from $X$ to symbolic sequence $S$ in contributions of Wessel N. et al. is carried on as Eq. (1):

$$S_i(x_i) = \begin{cases} 
0 : \mu < x_i \leq (1 + \alpha)\mu \\
1 : (1 + \alpha)\mu < x_i < \infty \\
2 : (1 - \alpha)\mu < x_i \leq \mu \\
3 : 0 < x_i \leq (1 - \alpha)\mu 
\end{cases} \quad (1)$$

where $\mu$ is the series mean and $\alpha$ is special controlling parameter with reference from 0.03 to 0.07, and in the heart rate variability dynamic analysis, $\alpha$ is set to 0.05.

For the convenience of representation, in some physiological signals such as ECG there are negative values. In some symbolic works referring to this method, therefore, time series are divided into positive and negative parts which are symbolized separately, and the symbolization evolves into Eq. (2):

$$S_i(x_i) = \begin{cases} 
0 : \mu_1 < x_i \leq (1 + \alpha)\mu_1 \text{ or } (1 + \alpha)\mu_2 \leq x_i < \mu_2 \\
1 : (1 + \alpha)\mu_1 < x_i < \infty \text{ or } -\infty < x_i < (1 + \alpha)\mu_2 \\
2 : (1 - \alpha)\mu_1 < x_i \leq \mu_1 \text{ or } \mu_2 \leq x_i < (1 - \alpha)\mu_2 \\
3 : (1 - \alpha)\mu_2 \leq x_i \leq (1 - \alpha)\mu_1 
\end{cases} \quad (2)$$

where $\mu_1$ and $\mu_2$ are the means of positive and negative parts.

After symbolization, the next step is to construct $m$-bit symbolic sequences which consist of $m$ bit symbols, leading to a maximum of $4^m$ different words. In our following complexity detection of classical chaotic models
and real-world physiological signals, 3-bit encoding is applied to symbolic sequences. There are several statistical approaches that characterize symbol sequences such as direct visual histograms or quantitative measures based on classical statistics and information theory [2]. In our symbolic entropy, we count probability of each code \( P(\pi) = \{ p(\pi_1), p(\pi_2), \ldots, p(\pi_N) \} \) and use classical Shannon entropy for nonlinear dynamic complexity extraction as Eq. (3), and its normalized form, W.N. symbolic entropy, is \( h(m) = \frac{H(m)}{\log_2 4} = \frac{H(m)}{2m} \).

\[
H(m) = -\sum_i p(\pi_i) \log_2 p(\pi_i), \quad \text{where} \quad p(\pi_i) \neq 0
\]  (3)

To measure especially low variability, the authors introduce another approach which transforms series into sequences consisting only of "0" and "1" by measuring difference between two successive beats. To obtain a consistent statistically appropriate estimate with the 4-symbol sequences, a 2m-bit encoding is applied to the symbolic sequences.

3. Chaotic models tests

3.1. Logistic map

The canonical form of logistic difference equation, \( x_{n+1} = r \cdot x_n (1 - x_n) \), is attractive by virtue of its extreme simplicity [16] and is widely applied in chaotic, nonlinear dynamical analysis. The two-degree polynomial mapping is often referenced as an archetypal example of nonlinear dynamical equations producing chaotic behaviors [17].

Referring to Lyapunov exponent of logistic map, W.N. symbolic entropy with controlling parameter 0.05 does not show satisfied results but it has desirable complexity detections in the whole regions with controlling parameter of 0.3 as shown in Fig. 1.

![Figure 1](image)

Figure 1: Logistic equations for varying control parameter from 3.5 to 4, where \( r^* = 3.567 \), exactly 3.569946, is the critical point to step into chaotic state. (a) Bifurcation diagram of logistic map. (b) Lyapunov exponent. (c) W.N. symbolic entropy of \( \alpha = 0.05 \). (d) W.N. symbolic entropy of \( \alpha = 0.3 \)
Showing in Fig.1c, W.N. symbolic entropy of $\alpha=0.05$ dose not achieve efficient nonlinear complexity extraction between $r=3.570$ and 3.611 when logistic map becomes chaotic. While when the controlling parameter is set to 0.3, W.N. symbolic entropy has accurate chaotic complexity captures at the cutting point $r^*$ and increases as chaotic behaviors of logistic map enhance.

To have more detailed knowledge about the parameters selection, we analyze and compare the logistic map complexity detections of the symbolic entropy with parameter from 0.01 to 0.99.

As revealed by Fig. 1 and Fig. 2, several values selected from the original interval cannot provide effectively complexity detections for W.N. symbolic entropy in the whole range of logistic map while the symbolic entropy with $\alpha$ chosen from [0.27, 0.30] have satisfied complexity detections, and that with parameter 0.30 has the best nonlinear dynamics analysis.
3.2. Henon equations

The Henon map is a classical two-dimensional dissipative quadratic map given by the coupled equations

\[ x_{i+1} = 1 - \alpha \cdot x_i^2 + y_i, \quad y_{i+1} = \beta \cdot x_i, \]

and the dynamical system is chaotic for the classical values \( \alpha = 1.4 \) and \( \beta = 0.3 \) [18].

We fix \( \beta = 0.3 \) and set \( \alpha \) to 1.35, 1.38 and 1.40 to observe the effects of controlling parameters on symbolic entropy in the chaotic system. Lyapunov exponents of the three Henon outputs are 0.3250, 0.3702 and 0.4077.

![Figure 3: Symbolic entropy of three Henon outputs with fixed \( \beta = 0.3 \) and \( \alpha = 1.35, 1.38 \) and 1.40. The symbolic method with \( \alpha \) smaller than 0.14 and bigger than 0.81 does not distinguish three Henon signals, and only in the second stage it has consistent results with Lyapunov exponent.](image)

From Fig. 3 the relationships of the three Henon signals’ symbolic entropy undergo a variety of changes as controlling parameter increases from 0.01 to 0.99. Referring to their Lyapunov exponents, only W.N. symbolic entropy with the second parameter interval, \([0.15, 0.45]\), has rational distinctions and if we want better differences among the three nonlinear outputs the interval should shrink to \([0.18, 0.32]\).

From complexity detections of W.N. symbolic entropy in the two chaotic models, we learn that for structural and dynamical differences between logistic and Henon systems, controlling parameter of the symbolic transformation should be adjusted accordingly in nonlinear complexity extraction.

4. Real-world physiological data

Three groups of heart signals [19] from different subjects, CHF patients [20], healthy elderly and young people [21], are applied to test the symbolization. The 15 patients, among which there are 11 men aged 22 to 71 and 4 women aged 54 to 63, with severe congestive heart failure underwent about 20 hours data collection. The two healthy groups are 20 young (21 to 34 years old) and 20 elderly (68 to 85 years old) subjects which all have 2 hours of continuous supine resting while continuous ECG. Each subgroup of the healthy subjects
includes equal numbers of men and women. ECG and heart rate derived from it are both applied in our contributions.

4.1. Effects of parameter $\alpha$ on real-world physiological data

Heart rates derived from ECG is typical nonlinear signal with high degree of complexity, non-linearity and non-stationarity. In this part, effects of parameter in complexity detection of real-world heartbeats are analyzed. The controlling parameter increases from 0.01 to 0.99 with step of 0.01, and relationships of the three kinds heartbeats’ symbolic entropy are depicted in Fig. 4.

![Figure 4: W.N. symbolic entropy of three kinds of heartbeats with increasing controlling parameter.](image)

The choice of parameter is important to three kinds of HRV nonlinear dynamic analysis indicated by Fig.4. Healthy young volunteers’ heart rates maintain higher symbolic entropy than that of the elderly ones, and CHF patients’ heart signals have overall lowest nonlinear complexities. The three symbolic entropy curves share similar changes, first-rise and then-fall tendency, and come to their convergences eventually.

The symbolic entropy relationships of three groups of subjects in Fig.4 are divided into three parts, less than 0.05 to the first interval, between 0.06 and 0.50 to the second part, and more than 0.50 to the third one. The three groups experience rapid increases to their maximum in the first stage. In the second section, symbolic entropy of three kinds of heartbeats undergo varying degrees of decline and come to their respective convergences. Healthy young volunteers’ heart signals complexity converges to around 0.4 when $\alpha$ is between 0.30 to 0.40, and that of elderly people drops to about 0.36 when controlling parameter increases from 0.15 to 0.20. Symbolic entropy values of CHF heart rates, however, have slower declining rate than those of healthy subjects, which leads that complexities of the CHF heartbeats are even higher than that of the elderly between $\alpha$ of 0.10 to 0.20. When the controlling parameter is bigger than 0.50, three entropy curves are convergent and their relationships, the healthy young $>$ the healthy elderly $>$ CHF patients, are unchanging.
The ‘complexity-loss’ theories of aging and disease about heart rates of related literatures \[22, 23, 24, 25\] are obvious in our nonlinear dynamics analysis. ECG-derived heartbeat variability contains valuable cardiac regulation information and has typical nonlinear dynamical behaviors which attribute to nonlinear complexity analysis for underlying mechanism of heart activities. Heartbeats of the healthy young generally yield more dynamical complexity than the elderly ones whose declined integrative cardiac control system leads to dynamical complexity reduction. Generally, it is accepted that fluctuations patterns of heartbeat intervals in CHF patients become quite regular and nonlinear dynamics of their heart rates are severely damaged.

Symbolic entropy of three kinds of heartbeats with six typical \(\alpha\) from different stages are chosen for statistically comparative analysis, where 0.03, 0.04 and 0.05 are in the referenced range of the original literatures, and 0.40, 0.50 and 0.60 are chosen from convergent stage. Independent samples t tests for each two heart rates’ entropy values are carried out as Table 1 listed.

| \(\alpha\)  | 0.03 | 0.04 | 0.05 | 0.40 | 0.50 | 0.60 |
|-------------|------|------|------|------|------|------|
| CHF-Elderly | 0.000| 0.000| 0.006| 0.036| 0.016| 0.011|
| CHF-Young   | 0.000| 0.000| 0.000| 0.000| 0.000| 0.000|
| Elderly-Young| 0.023| 0.000| 0.000| 0.013| 0.006| 0.001|

W.N. symbolic entropy with the above six controlling parameters all achieve significant differences among the three kinds of heartbeats statistically (\(p < 0.001\)), and it seems results of parameter 0.04 and 0.05 have better distinctions. In the first stage, however, symbolic entropy values of three kinds of heart signals under dramatic changes, so complexity detections are sensitive to the change of controlling parameters. When parameters are bigger than 0.50, nonlinearity characterized by symbolic entropy tend to convergent, so parameters in the third stage should be better choices.

4.2. Effects of data length on W.N. symbolic entropy

In some other symbolic entropy analysis, such as permutation entropy [5, 26], base-scale entropy [6], the methods have fast calculation and have satisfied results even with very short data sets. In the two chaotic models analysis, we find that data length does not affect results of complexity detection significantly, so in this section we conduct research on data length effects on heart rate nonlinear dynamic analysis.

Symbolic entropy of the three different groups of subjects with data length increasing from 100 to 2900 on step of 200 is shown in Fig. 5.
As can be seen from Fig. 5a and 5b, when controlling parameters are 0.04 and 0.05, symbolic entropy relationships of three groups of heart signals do not change as data length increases although distinctions between entropy of CHF heartbeats and that of healthy elderly heart rates deteriorate when data length is 500, so length of data sets should be bigger than 700 to get satisfied complexity detections. In Fig. 5c and 5d when data length are less than 1300, complexities of CHF patients’ heartbeats are bigger than those of the healthy elderly heart signals which is contradictory to conventional researches, and when data length are 1700 or bigger the three kinds of heart rates have rational complexity extraction and can be distinguished effectively. Symbolic entropy of the healthy young subjects’ heart signals maintain highest in the four subplots and are not affected by data length.

From complexity extractions of the three groups of heart rates, we reach that controlling parameters in interval of \([0.03, 0.05]\) or bigger than 0.50 have efficient symbolic entropy analysis and distinguish different heartbeats of different states. For stability nonlinear analysis of and insensitivity to parameter selection, it is more appropriate to choose bigger than 0.50. Parameter between 0.03 to 0.05 is preferred for symbolic transformation if data length is not that large.

5. Discussions

The symbolic transformation in works of Wessel N. et al. are analyzed in this paper while there are still some other issues that need further discussions.

ECG is one of the most direct manifestations of cardiac activity, and it has clear regularity due to periodic cardiac behaviors. Some physiological abnormalities are reflected by differences particular waveforms, and cardiac dynamical complexity may highly hide in ECG. The symbolic entropy for the three kinds of ECG analysis, however, yields some confusing results illustrating in Fig.
According to Fig. 6 in the whole range of increasing $\alpha$ healthy young subjects' ECG have lower entropy values than those of the healthy elderly, and CHF patients' ECG symbolic entropy has significantly different changes to the two heartbeats groups which leads to entropy changes among the three kinds of ECG. The results are rather confusing that symbolic entropy values of three kinds of heart signals are opposite to the aforementioned complexity-loss results in related researches. We suppose the perplexed results may be caused by the following reasons. 1. Multiscale factor. Our symbolic entropy analysis is based on single scale, however, physiologic time series like ECG may generate complexity over multiple time scale associated with a hierarchy of interacting regulatory mechanisms \cite{22}. The paradox maybe due to the fact that the symbolic entropy fails account for multiple time scales inherent in healthy physiologic dynamics \cite{22,24,25,27}. Multiscale analysis is to construct coarse-grained series $\{y_j\}$ for the original series $\{x_i\}$, $y_j^\tau = 1/\tau \Sigma_{i=0}^{\tau-1} x_{i+1}$, $1 \leq j \leq N/\tau$ where $\tau$ is scale factor. 2. Multi-dimensional vector construction. One-dimensional time series, like ECG, in fact contain all variables of the system, but much information is hidden higher dimensions and an approach for this situation is to construct multiple dimensional vectors. Generally, the phase space dimension of a nonlinear system may be high and vector reconstruction can expose the hidden structural behaviors. The vector reconstruction goes as $Y_m(i) = \{x(i), x(i + \tau), \ldots, x(i + (m - 1)\tau)\}$, where $m$ is embedding dimension and $\tau$ is delay time \cite{28,29}. 3. Other reasons attributing to the inconsistency may be that some nonlinear information of aging or disease is related to particular waveforms or their changes, for instance HRV reflects subtle changes between RR interval or so called NN interval. The relevant complexities may be reflected only by special features extractions or some preprocessings.

Symbolization is just the first step in symbolic time series analysis, and it can be followed by a variety of different analytical methods, such as classical statistics, entropy methods, irreversibility measures and so on \cite{2}. Through application of Shannon entropy, we find that the original parameter selections are not
suitable in complexity detection of two chaotic models and real-world heart signals. Each quantities may target different aspects of information and different signals have different structural or dynamical properties, therefore we guess that one cannot find an optimum parameters interval for different complexity measures and signals. In cases of other statistical measures, however, it need to verify whether the parameters setting should be adjusted accordingly.

Whether our hypothesis about the confusing results of complexity extraction in ECG are real reasons or which one is the effective solution to the problem, and if our parameter adjusting thoughts are proper in other situations, we would like to emphasize that our arguments have to be validated by more in-depth and comprehensive researches.

6. Conclusions

Through W.N. symbolic entropy analysis to chaotic models and real-world heart signals, we reach conclusions that the parameter-dependent symbolic method should adjust its controlling parameter according to different time series to get efficient nonlinear dynamical complexity detections. And we further prove the complexity-loss theory about aging and diseased cardiac control system.

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