TESTING FOR NEW COUPLINGS IN TOP QUARK DECAY

Charles A. Nelson and L. J. Adler, Jr.

Department of Physics, State University of New York at Binghamton

Binghamton, N.Y. 13902-6016

Abstract

Tests of the Lorentz structure of $t \rightarrow W^+ b$ decay will be carried out at the Tevatron, and later at the LHC and at a NLC. To quantitatively assay future measurements of competing observables, we consider the $g_{V-A}$ coupling values of the helicity decay parameters versus “$(V-A) + $ Single Additional Lorentz Structures”. Three phase-type ambiguities exist, but measurement of the sign of the large interference between the $W$ longitudinal/transverse amplitudes could exclude the two due dynamically to additional $(S + P)$ and $(f_M + f_E)$ couplings. Sizable $T$-violation signatures can occur for low-effective mass scales ($< 320 \text{ GeV}$), but in most cases can be more simply excluded by 10% precision measurement of the probabilities $P(W_L)$ and $P(b_L)$. Signatures for the presence of $T$-violation associated with the dynamical phase-type ambiguities, $CP$-violation signatures, and $\Lambda_b$ polarimetry are also discussed.

1Contributed Paper for ICHEP2000; More detailed paper is hep-ph/0007086

2Electronic address: cnelson @ binghamton.edu
1 Motivations, and Content Versus Ref.[3]

In physics at the highest available energies, it is always important to exploit simple reactions and decays so as to search for new forces, for new dynamics, and for discrete symmetry violations. Because the t-quark weakly decays before hadronization effects are significant, and because of the large t-quark mass, t-quark decay can be an extremely useful tool for such fundamental searches. Initial tests of the Lorentz structure and of symmetry properties of $t \rightarrow W^+ b$ decay will be carried out at the Tevatron[1], but the more precise measurements will be possible at the CERN LHC [2] and at a NLC [2].

It is important to be able to quantitatively assay future measurements of competing observables consistent with the standard model (SM) prediction of only a $g_{V-A}$ coupling and only its associated discrete symmetry violations. For this purpose, without consideration of possible explicit $T$ violation, in Ref.[3] plots were given of the values of the helicity parameters in terms of a “$(V-A) + Additional Lorentz Structure$” versus effective-mass scales for new physics, $\Lambda_i$, associated with each additional Lorentz structure.

In this contributed paper, to assay future measurements of helicity parameters in regard to $T$ violation, the effects of possible explicit $T$ violation are briefly reported. A more detailed paper on this latter subject will soon be available. In effective field theory, $\Lambda_i$, is the scale [4] at which new particle thresholds or new dynamics are expected to occur; $\Lambda_i$ can also be interpreted as a measure of a top quark compositeness/condensate scale. In measurement of some of the helicity parameters, the LHC should be sensitive to $\sim 3 \%$ and the Tevatron in a Run 3 to perhaps the $\sim 10 \%$ level (“ideal statistical error levels”) [5].
2 Consequences of Single Additional Lorentz Structures in Absence of Explicit $T$ Violation

In this section, we briefly review the work reported in Ref. [3]. This published paper contains a more detailed discussion, useful simple formulas relating the “$\alpha, \beta, \gamma$” relative phases of Fig.1a and the helicity parameters of Fig.1b, and plots of the values of the associated helicity parameters in the case of single additional Lorentz structures.

The attached Figs. 1a, 1b provide a good orientation to this topic: a complete measurement of on-shell properties of the $t \rightarrow W^+b$ decay mode will have been accomplished when the 4 moduli are determined and any 3 of the relative phases of the helicity amplitudes $A(\lambda_{W^+}, \lambda_b)$. The helicity parameters appear directly in various polarization and spin-correlation functions such as those obtained in Ref.[5].

The top lines of the first two tables list the standard model (SM)’s numerical values for the quantities shown in Figs. 1a, 1b. In the SM, all the relative phases are either zero or $\pm \pi$ so the primed helicity parameters are zero. In Table 1 in the top line are the standard model expectations for the numerical values of the helicity amplitudes $A(\lambda_{W^+}, \lambda_b)$ for $t \rightarrow W^+b$ decay in $g_L = 1$ units. The input values are $m_t = 175GeV$, $m_W = 80.35GeV$, $m_b = 4.5GeV$. The $\lambda_b = 1/2$ b-quark helicity amplitudes would vanish if $m_b$ were zero. For this reason, if one is guided by the SM expectations, the most accessible quantities experimentally should be the two moduli and the relative phase shown on the right of Fig. 1a. If the SM is correct, one expects that the $A(0, -1/2)$ and $A(-1, -1/2)$ moduli and relative phase $\beta_L$ will be the first quantities to be determined. The $\lambda_b = 1/2$ moduli are factors of 30 and 100 smaller in the SM. Interference measurements

2
between the two columns are of order $O(LR)$. $L$ and $R$ denote the $b$ quark’s helicity $\lambda_b = \mp 1/2$.

Throughout this moduli-phase analysis of top decays, intrinsic and relative signs of the helicity amplitudes are specified in accordance with the standard Jacob-Wick phase convention.

In Table 2 in the top line are the SM’s numerical values of the associated helicity parameters. Explicit formulas for the standard model helicity amplitudes and for experimental distributions in terms of these helicity parameters are given in Ref.[5].

The layout of the corners in Fig. 1 has been chosen to reflect the layout in the probability plots for $P(W_L)$ versus $P(b_L)$, see Ref.[3] and Figs.5-6 below. The quantities

$$P(W_L) = \text{Probability } W^+ \text{ is longitudinally polarized, } \lambda_{W^+} = 0$$

$$P(b_L) = \text{Probability } b \text{ is left-handed, } \lambda_b = -1/2$$

In terms of the first two helicity parameters of Table 2, $P(W_L) = \frac{1+\sigma}{2} = 0.705(SM)$ and $P(b_L) = \frac{1+\xi}{2} = 1.00(SM)$. So in the standard model, the emitted $W$ boson should be 70% longitudinally polarized and the emitted $b$-quark should be almost completely left-handed polarized.

The “arrows” in the upper part of Fig. 1 define the measurable $\alpha, \beta, \gamma$ relative phases between the four amplitudes. For instance,

$$\alpha_0 = \phi_0^R - \phi_0^L, \quad \beta_L = \phi_{-1}^L - \phi_0^L, \quad \gamma_+ = \phi_1^R - \phi_0^L \quad (1)$$

where $A(\lambda_{W^+}, \lambda_b) = |A| \exp(i\phi_{\lambda_{W^+}}^{LR})$. So for a pure $V - A$ coupling, the $\beta$’s vanish and all the $\alpha$’s and $\gamma$’s equal $+\pi$ (or $-\pi$) to give the intrinsic minus sign of the standard model’s $b_R$ amplitudes, see top row of Table 1.

The lower part of Fig. 1 displays the real part and imaginary part (primed) helicity parameters corresponding to interference measurements of the respective relative phases. For instance, c.f.
Appendix B of Ref.[3],

\[ \eta_L = \frac{1}{\Gamma} |A(-1, -\frac{1}{2})||A(0, -\frac{1}{2})| \cos \beta_L \] (2)

\[ \eta'_L = \frac{1}{\Gamma} |A(-1, -\frac{1}{2})||A(0, -\frac{1}{2})| \sin \beta_L \]

and

\[ \eta_{L,R} = \frac{1}{2} (\eta \pm \omega) \] (3)

Because of the relative magnitudes of the moduli predicted by the SM, in our consideration of information from b-quark polarization measurements, we concentrate on the two b-quark interference parameters \( \kappa_0 \) and \( \epsilon_+ \) and on their primed analogues. If surprises are discovered in top quark decay, other phases and/or helicity parameters might be more useful and certainly would be useful as checks and/or constraints. By \( \Lambda_b \) polarimetry[5], or some other \( b \)-polarimetry technique, it would be important to measure the \( \alpha \) and \( \gamma \) relative phase. In the standard model, the two helicity parameters between the amplitudes with the largest moduli are

\[ \kappa_0 = \frac{1}{\Gamma} |A(0, \frac{1}{2})||A(0, -\frac{1}{2})| \cos \alpha_0 \]

\[ \epsilon_+ = \frac{1}{\Gamma} |A(1, \frac{1}{2})||A(0, -\frac{1}{2})| \cos \gamma_+ \] (4)

We refer to \( \kappa_0, \epsilon_+ \) as the "b-polarimetry interference parameters". For \( \kappa'_0, \epsilon'_+ \), the sine function replaces the cosine function in Eqs.(4). Unfortunately from the perspective of a complete measurement of the four helicity amplitudes, the tree-level values of \( \kappa_0, \epsilon_+ \) in the SM are only about 1%. See the top line in both parts of Table 2. Two dimensional plots of the type \( (\epsilon_+, \eta_L) \) and \( (\kappa_0, \eta_L) \), and of their primed counterparts, have the useful property that the unitarity limit is a circle of radius \( \frac{1}{2} \) centered on the origin[3].

In the plots in Ref.[3] and below, the values of the helicity parameters are given in terms of a "\((V-A) + \text{Single Additional Lorentz Structure}\)". Generically, in the case of no explicit \( T \) violation,
we denote these additional couplings by

\[ g_{\text{Total}} \equiv g_L + g_X \]  \hspace{1cm} (5)

\[ X = \begin{cases} 
X_c = \text{chiral} = \{ V + A, S \pm P, f_M \pm f_E \} \\
X_{nc} = \text{non-chiral} = \{ V, A, S, P, f_M, f_E \}.
\end{cases} \]

For \( t \to W^+b \), the most general Lorentz coupling is

\[ W^* \gamma_\mu \bar{u}_b (p) \Gamma_\mu u_t (k) \]  \hspace{1cm} (6)

where \( k_t = q_W + p_b \), and

\[ \Gamma^\mu_V = g_V \gamma^\mu + \frac{f_M}{2\Lambda} \sigma^{\mu\nu} (k - p)_\nu + \frac{g_S}{2\Lambda} (k - p)^\mu \\
+ \frac{g_T}{2\Lambda} (k + p)^\mu + \frac{g_P}{2\Lambda} \sigma^{\mu\nu} (k + p)_\nu \]  \hspace{1cm} (6)

\[ \Gamma^\mu_A = g_A \gamma^\mu \gamma_5 + \frac{f_E}{2\Lambda} \sigma^{\mu\nu} (k - p)_\nu \gamma_5 + \frac{g_P}{2\Lambda} (k - p)^\mu \gamma_5 \\
+ \frac{g_T}{2\Lambda} (k + p)^\mu \gamma_5 + \frac{g_P}{2\Lambda} \sigma^{\mu\nu} (k + p)_\nu \gamma_5 \]  \hspace{1cm} (7)

For \( g_L = 1 \) units with \( g_i = 1 \), the nominal size of \( \Lambda_i \) is \( \frac{m_t}{2} = 88 \text{GeV} \), see below.

Lorentz equivalence theorems for these couplings are treated in Appendix A of Ref.[3]. Explicit expressions for the \( A(\lambda_{W^+}, \lambda_b) \) in the case of these additional Lorentz structures are given in Ref. [5]. Other recent general analyses of effects in \( t \to W^+b \) decay associated with new physics arising from large effective- mass scales \( \Lambda_i \) are in Refs. [6-12]. Some work on higher order QCD and EW corrections has been done in [13].

The partial width \( \Gamma \) for \( t \to W^+b \) is the remaining and very important moduli parameter for testing for additional Lorentz structures. Since \( \Gamma \) sets the overall scale, it cannot be well measured by spin-correlation techniques, which better measure the ratios of moduli and relative phases, so we consider \( \Gamma \) separately; see also [14,15]. From the perspective of possible additional Lorentz
structures, measurement of the partial width \( \Gamma \) is an important constraint. In particular, this provides a strong constraint on possible \( V + A \) couplings in contrast to measurement of \( P(W_L) \) which does not[3]. \( \Gamma \) provides a useful constraint for the possibility of additional \( V \) and \( A \) couplings which are appealing from the perspective of additional gauge-theoretic structures.

3 Moduli Parameters and Phase-Type Ambiguities

Versus predictions based on the SM, two dynamical phase-type ambiguities were found by investigation of the effects of a single additional “chiral” coupling \( g_i \) on the three moduli parameters

\[
\sigma = P(W_L) - P(W_T), \quad \xi = P(b_L) - P(b_R), \quad \text{and} \quad \zeta = \frac{1}{\Gamma}(\Gamma_{b_L}^{b_R} - \Gamma_{b_L}^{b_T}).
\]

For an additional \( S + P \) coupling with \( \Lambda_{S+P} \sim -34.5 \text{GeV} \) the values of \( (\sigma, \xi, \zeta) \) and also of the partial width \( \Gamma \) are about the same as the SM prediction, see Table 2. This is the first dynamical ambiguity. Table 1 shows that this ambiguity will also occur if the sign of the \( A_X(0, -\frac{1}{2}) \) amplitude for \( g_L + g_X \) is taken to be opposite to that of the SM’s amplitude. An additional \( S \pm P \) only effects the longitudinal \( W^\pm \) amplitudes and not the transverse \( \lambda_W = \mp 1 \) ones. By requiring that

\[
\frac{A_X(0, -\frac{1}{2})}{A_X(-1, -\frac{1}{2})} = -\frac{A_L(0, -\frac{1}{2})}{A_L(-1, -\frac{1}{2})}
\]

for \( X = S + P \), we obtain a simple formula

\[
\Lambda_{S+P} = -\left(\frac{g_{S+P}}{g_L}\right)\frac{m_t}{2(E_W + q_W)} \sim -\left(\frac{g_{S+P}}{g_L}\right)\frac{m_t}{4} (1 - \frac{m_W^2}{m_t^2}).
\]

It is important to regard these ambiguities from (i) the signs in their \( b_L \) amplitudes versus those for the SM and from (ii) the tensorial character and \( \Lambda \) value of the associated Lorentz structure.

For an additional \( f_M + f_E \) coupling with \( \Lambda_{f_M+f_E} \sim 53 \text{GeV} \) the values of \( (\sigma, \xi, \zeta) \) are also about the same as the SM prediction, see Table 2. This is the second dynamical ambiguity. In this case,
the partial width $\Gamma$ is about half that of the SM due to destructive interference. Table 1 shows that this ambiguity will also occur if the sign of the $A_X(-1, -\frac{1}{2})$ amplitude for $g_L + g_X$ is taken to be opposite to that of the SM’s amplitude. Again, from (8) for $X = f_M + f_E$, we obtain

$$\Lambda_{f_M+f_E} = \left(\frac{g_{f_M+f_E}}{g_L}\right) \frac{m_t E_W}{2(E_W + q_W)} \sim \left(\frac{g_{f_M+f_E}}{g_L}\right) \frac{m_t}{4} \left(1 + \left(\frac{m_W}{m_t}\right)^2\right)$$

(10)
since $\frac{m_b \sqrt{E_b - q_W}}{m_t \sqrt{E_b + q_W}} \sim 10^{-3}$.

Besides the $f_M + f_E$ construction of this second phase-type ambiguity, it should be kept in mind that some other mechanism might produce the relative sign change shown in Table 1, but without also changing the absolute value of the $b_L$ amplitudes. In this case the measurement of the partial width $\Gamma$ would not resolve this phase ambiguity.

From consideration of Table 1, a third (phase) ambiguity can be constructed by making an arbitrary sign-flip in the $b_L$ amplitudes, so $A_X(\lambda_W, \lambda_b = -\frac{1}{2}) = -A_{V-A}(\lambda_W, \lambda_b = -\frac{1}{2})$, with no corresponding sign changes in the $b_R$ amplitudes.

Resolution of this third ambiguity, as well as determination of two remaining independent relative phases (e.g. $\alpha_0$ and $\gamma_+$) necessary for a complete amplitude measurement of $t \to W^+b$ decay, will require direct empirical information about the $b_R$-amplitudes. One way would be from a $\Lambda_b$ polarimetry measurement [5] of the $b$-polarimetry interference parameters $\epsilon_+$ and $\kappa_0$. Even at an NLC, such measurements will be difficult unless certain non-SM couplings occur. In particular, here additional $S + P$ and $f_M + f_E$ couplings have negligible effects, but non-chiral couplings like $V$ or $A$, $f_M$ or $f_E$ (for $\epsilon_+$), $S$ or $P$ (for $\kappa_0$) can produce large effects[3].

Since the helicity parameters appear directly in the various polarization and spin-correlation functions, it is clearly more model independent to simply measure them rather than to set limits on an “ad hoc” set of additional coupling constants. The large $m_b$ effects displayed in some of
the plots in Ref.[3] explicitly demonstrate this point. In many cases, finite $m_b$ effects in both $b_L$ and $b_R$ amplitudes lead to sizable “oval shapes” as the effective mass scale $\Lambda_i$ varies. There do not exist “Lorentz equivalence theorems” with-respect-to both $m_b$ dependence and a minimal set of couplings when $m_b$ is allowed to vary.

In summary, in the absence of explicit $T$ violation, three phase-type ambiguities versus the SM prediction exist: two dynamical ones with low effective mass scales, $g_{V-A} + g_{S+P}$ with $\Lambda_{S+P} \sim -35 GeV$ and $g_{V-A} + g_{S+P}$ with $\Lambda_{S+P} \sim 53 GeV$, and a third due to an arbitrary sign-flip in the $b_L$-amplitudes $A_X(\lambda_b = -1/2) = -A_{V-A}(\lambda_b = -1/2)$. The two dynamical ambiguities can be resolved by measurement of the sign of the large interference between the $W$ longitudinal/transverse amplitudes. Measurement of the sign of the $\eta_L$ helicity parameter will determine the sign of $\cos \beta_L$ where $\beta_L$ is the relative phase of the two $b_L$-amplitudes ( $\eta_L = \pm 0.46$ where the upper sign is for the SM ). Both from the perspective of carefully testing the SM and that of searching for new physics, we believe that it is very important that experiments measure both this $W$ longitudinal/transverse interference parameter and its associated $T$ violation parameter $\eta_L'$. The latter parameter is very important in the following analysis in this paper.

4 Consequences of Explicit $T$ Violation

To assay future measurements of helicity parameters in regard to $T$ violation, the next five sets of figures, Figs. 2-6, are for the case of a single additional pure-imaginary coupling, $ig_i/2\Lambda_i$ or $ig_i$, associated with a specific additional Lorentz structure, $i = S, P, S+P, \ldots$. 
In the $t$ rest frame, the matrix element for $t \to W^+b$ is

$$
\langle \theta_1^t, \phi_1^t, \lambda_{W^+}, \lambda_b | \frac{1}{2}, \lambda_1 \rangle = D_{\lambda_1, \mu}^{(1/2)*}(\phi_1^t, \theta_1^t, 0) A(\lambda_{W^+}, \lambda_b)
$$

(11)

where $\mu = \lambda_{W^+} - \lambda_b$ in terms of the $W^+$ and $b$ helicities. $\lambda_1$ gives the $t$ quark’s spin component quantized along a $z_1^t$ axis, see Fig.1 in 2nd paper in Ref.[5]. So, upon a boost back to the ($tt$) center-of-mass frame, or to the $\bar{t}$ rest frame, $\lambda_1$ also specifies the helicity of the $t$ quark. By rotational invariance there are only two amplitudes $A(0, -1/2), A(-1, -1/2)$ for $\lambda_b = 1/2$, and two with $\lambda_{W^+} = 0, 1$ for $\lambda_b = -1/2$. For the $CP$-conjugate process, $\bar{t} \to W^-\bar{b}$, in the $\bar{t}$ rest frame

$$
\langle \theta_2^t, \phi_2^t, \lambda_{W^-}, \lambda_{\bar{b}} | \frac{1}{2}, \lambda_2 \rangle = D_{\lambda_2, \bar{\mu}}^{(1/2)*}(\phi_2^t, \theta_2^t, 0) B(\lambda_{W^-}, \lambda_{\bar{b}})
$$

(12)

with $\bar{\mu} = \lambda_{W^-} - \lambda_{\bar{b}}$.

As shown in Table 3 a specific discrete symmetry implies a specific relation among the associated helicity amplitudes. In the case of $T$ invariance, the helicity amplitudes must be purely real. The $T$ invariance of Table 3 will be violated if either (i) there is a fundamental violation of canonical $T$ invariance, or (ii) there are absorptive final-state interactions. In the SM, there are no such final-state interactions at the level of sensitivities considered in the present analysis. In our earlier papers[5], we have kept this assumption of “the absence of final-state interactions” manifest by referring to the $T$ invariance of Table 3 as “$\tilde{T}_{FS}$ violation”.

Barred parameters $\bar{\xi}, \bar{\zeta}, \ldots$ have the analogous definitions for the $CP$ conjugate process, $\bar{t} \to W^-\bar{b}$. Therefore, any $\bar{\xi} \neq \xi, \bar{\zeta} \neq \zeta, \ldots \implies CP$ is violated. That is, “slashed parameters” $\zeta \equiv \xi - \bar{\xi}, \ldots$, could be introduced to characterize and quantify the degree of CP violation. This should be regarded as a test for the presence of a non-CKM-type CP violation because, normally, a CKM-phase will contribute equally at tree level to both the $t \to W^+b_L$ decay amplitudes and so a
CKM-phase will cancel out in the ratio of their moduli and in their relative phase. There are four tests for non-CKM-type CP violation[5].

A recent review of CP-violation in t-quark physics is in [16].

4.1 Additional $S \pm P, f_M \pm f_E, S, P, f_M, \text{ or } f_E$ couplings

The two plots displayed in Fig.2 are for dimensional couplings with chiral $S \pm P, f_M \pm f_E$ and non-chiral $S, P, f_M, f_E$ Lorentz structures. The upper plot displays the $\eta_L'$ helicity parameter versus the effective-mass scale $\Lambda_i$ with $g_i = 1$ in $gL = 1$ units. The lower plot displays the induced effect of the additional coupling on the partial width for $t \to W^+b$. The standard model limit is at the “wings” where $|\Lambda_i| \to \infty$ for each additional dimensional coupling.

Fig.3 displays plots of the b-polarimetry interference parameters $\epsilon_+'$ and $\kappa_0'$ versus $\Lambda_i$ for the case of a single additional $S, P, f_M, f_E$ and $S \pm P, f_M - f_E$ coupling: Curves are omitted in the plots in this paper when the couplings produce approximately zero deviations in the helicity parameter of interest, e.g. this occurs for $f_M + f_E$ in both the $\epsilon_+'$ and $\kappa_0'$ helicity parameters. The unitarity limit for $\epsilon_+'$ and $\kappa_0'$ is also 0.5.

4.2 Additional $V + A, V, \text{ or } A$ couplings

An additional $V - A$ type coupling with a complex phase versus the SM’s $g_L$ is equivalent to an additional overall complex factor in the SM’s helicity amplitudes. This will effect the overall partial width $\Gamma$, but it can’t otherwise be observed by spin-correlation measurements.

For a single additional gauge-type coupling $V, A, \text{ or } V + A$, in Fig.4 are plots of the b-polarimetry interference parameters $\epsilon_+'$ and $\kappa_0'$, and of the partial width for $t \to W^+b$ versus pure-imaginary
coupling constant $i g_i$. The $g_i$ value is in $g_L = 1$ units. In the cases of the additional dimensionless, gauge-type couplings, the standard model limit is at the origin, $g_i \to 0$.

### 4.3 Indirect effects of $T$ violation on other helicity parameters

The plots in Fig.5 show the indirect effects of a single additional pure-imaginary chiral coupling, $i g_i/2 \Lambda_i$ or $i g_i$, on other helicity parameters. For the coupling strength ranges listed in the “middle table”, the upper plot shows the effects on the probability, $P(W_L)$, that the emitted $W^+$ is “Longitudinally” polarized and the effects on the probability, $P(b_L)$, that the emitted b-quark has “Left-handed” helicity. Each curve is parametrized by the magnitude of the associated $g_i$ or $\Lambda_i$. On each curve, the central open circle corresponds to the region with a maximum direct $T$ violation signature, e.g. for $f_M + f_E$ from Fig. 2 this is at $|\Lambda_{f_M+f_E}| \sim 50 GeV$. The large/small solid circles correspond respectively to the ends of the ranges listed in the middle table where the direct signatures fall to about 50% of their maximum values. Similarly the lower plot is for the W-polarimetry interference parameters $\eta, \omega$. Curves are omitted for the remaining moduli parameter $\zeta$ since a single additional pure-imaginary coupling in these ranges produces approximately zero deviations from the pure $V - A$ value of $\zeta = 0.41$.

The plots in Fig.6 show the indirect effects of a single additional pure-imaginary non-chiral coupling on other helicity parameters. Versus the middle table given here, the curves are labeled as in Fig. 5. The upper plot is for the two probabilities $P(W_L)$ and $P(b_L)$. The lower plot is for the W-polarimetry interference parameters $\eta, \omega$.

In summary, sizable $T$-violation signatures can occur for low-effective mass scales ($< 320 GeV$) as a consequence of pure-imaginary couplings associated with a specific additional Lorentz struc-
ture. However, in most cases, such additional couplings can be more simply excluded by 10% precision measurement of the probabilities $P(W_L)$ and $P(b_L)$. The W-polarimetry interference parameters $\eta$ and $\omega$ can also be used as indirect tests, or to exclude such additional couplings.

5 Tests for $T$ Violation Associated with the Dynamical Phase-Type Ambiguities

In Fig. 7 are plots of the signatures for a partially-hidden $T$ violation associated with a $S+P$ phase-type ambiguity: We require Eq.(8) to hold when the additional $S+P$ coupling, $g_{S+P}/2\Lambda_{S+P}$ has a complex effective mass scale parameter $\Lambda_{S+P} = |\Lambda_{S+P}| \exp(-i\theta)$ where $\theta$ varies with the mass scale $|\Lambda_{S+P}|$. For $m_b = 0$, the resulting function $\theta(|\Lambda_{S+P}|)$ is very simple. This construction maintains the standard model values in the massless b-quark limit for the four moduli parameters, $P(W_L), P(b_L), \zeta,$ and $\Gamma$. The function $\theta(|\Lambda_{S+P}|)$ is then used for the $S+P$ coupling when $m_b = 4.5\text{GeV}$. The SM values for the moduli parameters are essentially unchanged. There are two cases, $\sin \theta \geq 0$ and $\sin \theta \leq 0$. The phase choice of $\phi^R_1 = \pm \pi$, cf. top line in Table 1, has no consequence since it is a $2\pi$ phase difference.

For $\sin \theta \geq 0$ in Fig.7 is the solid curve for the $\eta_L'$, the $T$ violation W-polarimetry interference parameter, plotted versus $1/|\Lambda_{S+P}|$. The dashed curve is for the W-polarimetry interference parameters $\eta_L, \eta, \omega$ which are degenerate. The dark rectangles show the standard model values at the $|\Lambda_{S+P}| \to \infty$ endpoint where $\theta = \pi/2$. At the other endpoint $|\Lambda_{S+P}| \sim 34.5\text{GeV}$, or $1/|\Lambda_{S+P}| = 0.029\text{GeV}^{-1}$, the coupling is purely real with $\theta = \pi$. The unitarity limit for each of these helicity parameters is 0.5.
From the perspectives of (i) measuring the $W$ interference parameters and of (ii) excluding this type of $T$ violation, it is noteworthy that where $\eta'_{L}$ has the maximum deviation, there is a zero in $\eta_{L}, \eta, \omega$. So if the latter parameters were found to be smaller than expected or with the opposite sign than expected, this would be consistent with this type of $T$ violation.

At the maximum of $\eta'_{L}$, $|\Lambda_{S+P}| \sim 49 GeV$ and the other $T$ violation parameters are also maximum. The curves for these parameters have the same overall shape as $\eta'_{L}$ but their maxima are small, $\epsilon_{+}' \sim 0.015$ and $\kappa_{0}' \sim 0.028$.

For the other case where $\sin \theta \leq 0$, all these $T$ violation primed parameters have the opposite overall sign. The signs of other helicity parameters are not changed.

In Fig. 8 are plots of the signatures for a partially-hidden $T$ violation associated with a $f_{M} + f_{E}$ phase-type ambiguity: As above for the analogous $S + P$ construction, the additional $f_{M} + f_{E}$ coupling $g_{f_{M}+f_{E}}/2\lambda_{f_{M}+f_{E}}$ now has an effective mass scale parameter $\Lambda_{f_{M}+f_{E}} = |\Lambda_{f_{M}+f_{E}}| \exp -i\theta$ in which $\theta$ varies with the mass scale $|\Lambda_{f_{M}+f_{E}}|$ to maintain standard model values in the massless b-quark limit for the moduli parameters $P(W_{L}), P(b_{L})$, and $\zeta$. For the case $\sin \theta \geq 0$, in Fig. 8 the upper plot shows by the solid curve the $T$ violation W-polarimetry interference parameter $\eta'_{L}$ versus $1/|\Lambda_{f_{M}+f_{E}}|$. By the dashed curve, it shows the W-polarimetry interference parameters $\eta_{L}, \eta, \omega$ which are degenerate. At the endpoint $|\Lambda_{f_{M}+f_{E}}| \sim 52.9 GeV$, or $1/|\Lambda_{f_{M}+f_{E}}| = 0.0189 GeV^{-1}$, the coupling is purely real with $\theta = 0$.

Here, as in Fig. 7, where $\eta'_{L}$ has the maximum deviation, there is a zero in $\eta_{L}, \eta, \omega$. The lower plot shows the indirect effect of such a coupling on the partial width $\Gamma$ for $t \rightarrow W^+b$.

At the maximum of $\eta'_{L}$, $|\Lambda_{f_{M}+f_{E}}| \sim 63 GeV$. The curve for the $T$ violation parameter $\kappa_{0}'$ has the same shape and is also maximum at the same position with a value $\kappa_{0}' \sim 0.005$. $\epsilon_{+}'$ remains
very small. For the other case where $\sin \theta \leq 0$, each of these $T$ violation primed parameters has the opposite overall sign.

In summary, sufficiently precise measurement of the $W$-interference parameter $\eta_L$ and of the $\eta_L'$ parameter can exclude partially-hidden $T$ violation associated with either of the two dynamical phase-type ambiguities. However, if $\eta_L = (\eta + \omega)/2$ were found to be smaller than expected or with a negative sign, this would be consistent with this type of $T$ violation.

Acknowledgments

For computer services, one of us (CAN) thanks John Hagan and Ted Brewster. This work was partially supported by U.S. Dept. of Energy Contract No. DE-FG 02-86ER40291.

References

[1] F. Abe, et. al. (CDF collaboration), Phys. Rev. Lett. 74, 2626(1995); S. Abachi, et. al. (D0 collaboration), Phys. Rev. Lett. 74, 2632(1995).

[2] ATLAS Technical Proposal, CERN/LHCC/94-43, LHCC/P2 (1994); CMS Technical Design Report, CERN-LHCC- 97-32; CMS-TDR-3 (1997). Reports on work for Next Linear Colliders by B. Wiik (DESY), H. Sugawara (KEK), and B. Richter (SLAC) at ICHEP98, Vancouver, Canada, and at Lepton-Photon Sym. 1999, Stanford, CA.. A recent working-group review of top quark physics is M.Beneke, et. al., hep-ph/0003033.

[3] C.A. Nelson and A.M. Cohen, Eur. Phys. J. C8, 393(1999). The sign in Table 2 for $\kappa_0$ in the case of the $S + P$ ambiguity has been corrected in the present paper.
[4] G. ’t Hooft, THU-94/15; S. Weinberg, in “Unification of Elementary Forces and Gauge Theories”, eds. D.B. Cline and F.E. Mills (Harwood, London, 1978).

[5] C.A. Nelson, B.T. Kress, M. Lopes, and T.P. McCauley, Phys. Rev. D56, 5928(1997); D57, 5923(1998). The relations between the chiral and non-chiral couplings are $g_{chiral} = g_1 \pm g_2$; e.g. $g_L = g_V - g_A, g_R = g_V + g_A, g_{S\pm P} = g_S \pm g_P, g_\pm = f_M \pm f_E$. Since in $g_L = 1$ units the numerical value of a specific $\Lambda_i$ is determined by fixing its $g_i = 1$, the sign of $\Lambda_i$ gives the relative sign of the $g_i/(2\Lambda_i)$ coupling versus $g_L$.

[6] G.J. Gounaris, F.M. Renard and C. Verzegnassi, Phys. Rev. D52, 451(1995); G.J. Gounaris, D.T. Papadamou and F.M. Renard, Z. Phys. C76, 333(1997).

[7] A. Bartl, E. Christova, and W. Majerotto, Nucl. Phys. B460, 235(1996); A. Bartl, E. Christova, T. Gajdosik, and W. Majerotto, Phys. Rev. D58, 074007 (1998); hep-ph/9803426.

[8] B. Lampe, Phys. Lett. B415, 63(1997); B. Grzadkowski, Z. Hioki, Phys.Rev. D61, 014013(2000).

[9] K. Whisnant, J.M. Yang, B.-L. Young and Z. Zhang, Phys. Rev. D56, 467(1997); J.M. Yang and B.-L. Young, ibid., D56, 5907(1997); J.-J. Cao, et.al., Phys.Rev. D58, 094004(1998).

[10] F. Larios, E. Malkawi, C.-P. Yuan, Acta Phys. Polon. B27, 3741(1996); hep-ph/9704288; H.-J. He, Y.-P. Kuang, C.-P. Yuan, hep-ph/9704276.

[11] G. Mahlon and S. Parke, Phys. Rev. D55, 7249(1997); Phys.Lett. B476, 323-330 (2000); B411, 173(1997); G. Mahlon hep-ph/9811219, hep-ph/9811281.

[12] Y.Zeng-Hui, H. Pietschmann, M. Wen-Gan, H. Liang, J. Yi, Eur.Phys.J. C9, 463 (1999).
[13] M. Jezabek and J.H. Kuhn, Phys. Lett. B207, 91(1988); B329, 317(1994); Nuc. Phys. B314, 1(1989); B320, 20(1989); C.S. Li, R. Oakes and T.C. Yuan, Phys. Rev. D43, 3759(1991); T. Mehen, Phys. Lett. B417, 353(1998); A. Czarnecki and K. Melnikov, Nucl.Phys. B544, 520(1999); B. Lampe, Eur.Phys.J. C8, 447(1999).

[14] A.S. Belyaev, E.E. Boos and L.V. Dudkov, Phys.Rev. D59, 075001(1999); A.P. Heinson, A.S. Belyaev and E.E. Boos, ibid. D56, 3114(1997).

[15] S.S. Willenbrock, hep-ph/9905498; M.C. Smith and S.S. Willenbrock, Phys. Rev. D54, 6696(1996); T.Stelzer, Z. Sullivan and S. Willenbrock, Phys.Rev. D58, 094021(1998); R. Pittau, Phys. Lett. B386, 397(1996).

[16] D. Atwood, S. Bar-Shalom, G. Eilam, and A. Soni. hep-ph/0006032.
Table Captions

Table 1: For the ambiguous moduli points, numerical values of the associated helicity amplitudes $A(\lambda_{W^+}, \lambda_b)$. The values for the amplitudes are listed first in $g_L = 1$ units, and second as $A_{new} = A_{g_L=1}/\sqrt{\Gamma}$ which removes the effect of the differing partial width, $\Gamma$ for $t \to W^+b$. [$m_t = 175 GeV, m_W = 80.35 GeV, m_b = 4.5 GeV$].

Table 2: For the ambiguous moduli points, numerical values of the associated helicity parameters. Listed first are the four moduli parameters. Listed second are the values of the $W$-polarimetry interference parameters which could be used to resolve these dynamical ambiguities.

Table 3: The helicity formalism is based on the assumption of Lorentz invariance but not on any specific discrete symmetry property of the fundamental amplitudes, or couplings. For instance, for $t \to W^+b$ and $\bar{t} \to W^-\bar{b}$ a specific discrete symmetry implies a definite symmetry relation among the associated helicity amplitudes.

Figure Captions

FIG. 1: For $t \to W^+b$ decay, display of the four helicity amplitudes $A(\lambda_{W^+}, \lambda_b)$ relative to the $W^+$ boson and b-quark helicities. The upper sketch defines the measurable “$\alpha, \beta, \gamma$” relative phases, c.f. Eqs(1). The lower sketch defines the real part and imaginary part (primed) helicity parameters corresponding to these relative phases. Measurement of a non-zero primed helicity parameter would be a direct signature for $T$ violation.

FIG. 2: To assay future measurements of helicity parameters in regard to $T$ violation, the next five sets of figures are for the case of a single additional pure-imaginary coupling, $ig_i/2\Lambda_i$ or $i\lambda_i$, associated with a specific additional Lorentz structure, $i = S, P, S + P, \ldots$. The two plots
displayed here are for dimensional couplings with chiral $S \pm P, f_M \pm f_E$ and non-chiral $S, P, f_M, f_E$ Lorentz structures. The upper plot displays the $\eta_L'$ helicity parameter versus the effective-mass scale $\Lambda_i$ with $g_i = 1$ in $g_L = 1$ units. The lower plot displays the induced effect of the additional coupling on the partial width for $t \to W^+b$. The standard model limit is at the “wings” where $|\Lambda_i| \to \infty$ for each additional dimensional coupling. The unitary limit for $\eta_L'$ is 0.5.

FIG. 3: Plots of the b-polarimetry interference parameters $\epsilon_+'$ and $\kappa_0'$ versus $\Lambda_i$ for the case of a single additional $S, P, f_M, f_E$ and $S \pm P, f_M - f_E$ coupling: Curves are omitted in the plots in this paper when the couplings produce approximately zero deviations in the helicity parameter of interest, e.g. this occurs for $f_M + f_E$ in both the $\epsilon_+'$ and $\kappa_0'$ helicity parameters. The unitarity limit for $\epsilon_+'$ and $\kappa_0'$ is also 0.5.

FIG. 4: For a single additional gauge-type coupling $V, A$, or $V + A$, plots of the b-polarimetry interference parameters $\epsilon_+'$ and $\kappa_0'$, and of the partial width for $t \to W^+b$ versus pure-imaginary coupling constant $ig_i$. The $g_i$ value is in $g_L = 1$ units. In the cases of the additional dimensionless, gauge-type couplings, the standard model limit is at the origin, $g_i \to 0$.

FIG. 5: These plots show the indirect effects of a single additional pure-imaginary chiral coupling, $ig_i/2\Lambda_i$ or $ig_i$, on other helicity parameters. For the coupling strength ranges listed in the “middle table”, the upper plot shows the effects on the probability, $P(W_L)$, that the emitted $W^+$ is “Longitudinally” polarized and the effects on the probability, $P(b_L)$, that the emitted b-quark has “Left-handed” helicity. Each curve is parametrized by the magnitude of the associated $g_i$ or $\Lambda_i$. On each curve, the central open circle corresponds to the region with a maximum direct $T$ violation signature, e.g. for $f_M + f_E$ from Fig. 2 this is at $|\Lambda_{f_M+f_E}| \sim 50 GeV$. The large/small solid circles correspond respectively to the ends of the ranges listed in the middle.
table where the direct signatures fall to about 50% of their maximum values. Similarly the lower plot is for the W-polarimetry interference parameters $\eta, \omega$. Curves are omitted for the remaining moduli parameter $\zeta$ since a single additional pure-imaginary coupling in these ranges produces approximately zero deviations from the pure $V - A$ value of $\zeta = 0.41$. A dark rectangle denotes the value for the pure $V - A$ coupling of the standard model.

FIG. 6: These plots show the indirect effects of a single additional pure-imaginary non-chiral coupling on other helicity parameters. Versus the middle table given here, the curves are labeled as in Fig. 5. The upper plot is for the two probabilities $P(W_L)$ and $P(b_L)$. The lower plot is for the W-polarimetry interference parameters $\eta, \omega$.

FIG. 7: Plots of the signatures for a partially-hidden $T$ violation (see text) associated with a $S + P$ phase-type ambiguity: In this case, the additional $S + P$ coupling, $g_{S+P}/2\Lambda_{S+P}$, has an effective mass scale parameter $\Lambda_{S+P} = |\Lambda_{S+P}| \exp -i\theta$ where $\theta$ varies with the mass scale $|\Lambda_{S+P}|$ to maintain standard model values in the massless b-quark limit for the four moduli parameters, $P(W_L), P(b_L), \zeta$, and $\Gamma$. Plotted versus $1/|\Lambda_{S+P}|$ for the case $\sin \theta \geq 0$ is the solid curve for the $\eta_L'$, the $T$ violation W-polarimetry interference parameter and the dashed curve for the W-polarimetry interference parameters $\eta_L, \eta, \omega$ which are degenerate. For $\sin \theta \leq 0$, the $\eta_L'$ sign is opposite. The dark rectangles show the standard model values at the $|\Lambda_{S+P}| \rightarrow \infty$ endpoint where $\theta = \pi/2$. At the other endpoint $|\Lambda_{S+P}| \sim 34.5 GeV$, or $1/|\Lambda_{S+P}| = 0.029 GeV^{-1}$, the coupling is purely real with $\theta = \pi$. Where $\eta_L'$ has the maximum deviation, there is a zero in $\eta_L, \eta, \omega$.

FIG. 8: Plots of the signatures for a partially-hidden $T$ violation (see text) associated with a $f_M + f_E$ phase-type ambiguity: The additional $f_M + f_E$ coupling, $g_{f_M+f_E}/2\Lambda_{f_M+f_E}$, has an effective mass scale parameter $\Lambda_{f_M+f_E} = |\Lambda_{f_M+f_E}| \exp -i\theta$ where $\theta$ varies with the mass scale...
$|\Lambda_{f_M+f_E}|$ to maintain standard model values in the massless b-quark limit for the moduli parameters $P(W_L), P(b_L),$ and $\zeta$. Versus $1/|\Lambda_{f_M+f_E}|$ for $\sin \theta \geq 0$, the upper plot shows by the solid curve $\eta_L'$. By the dashed curve, it shows $\eta_L, \eta, \omega$ which are degenerate. At the endpoint $|\Lambda_{f_M+f_E}| \sim 52.9 GeV$, or $1/|\Lambda_{f_M+f_E}| = 0.0189 GeV^{-1}$, the coupling is purely real with $\theta = 0$. For $\sin \theta \leq 0$, the $\eta_L'$ sign is opposite. The lower plot shows the indirect effect of such a coupling on the partial width $\Gamma$ for $t \to W^+b$. 
Table 1: Amplitudes in Standard Model and at Ambiguous Moduli Points

|                      | $A(0, -\frac{1}{2})$ | $A(-1, -\frac{1}{2})$ | $A(0, \frac{1}{2})$ | $A(1, \frac{1}{2})$ |
|----------------------|----------------------|----------------------|----------------------|----------------------|
| $A_{g_{L=1}}$ in $g_{L=1}$ units |                      |                      |                      |                      |
| $V - A$              | 338                  | 220                  | -2.33                | -7.16                |
| $S + P$              | -338                 | 220                  | -24.4                | -7.16                |
| $f_M + f_E$          | 220                  | -143                 | 1.52                 | -4.67                |
| $A_{\text{new}} = A_{g_{L=1}}/\sqrt{T}$ |                      |                      |                      |                      |
| $V - A$              | 0.84                 | 0.54                 | -0.0058              | -0.018               |
| $S + P$              | -0.84                | 0.54                 | -0.060               | -0.018               |
| $f_M + f_E$          | 0.84                 | -0.54                | 0.0058               | -0.018               |

Table 2: Helicity Parameters in Standard Model and at Ambiguous Moduli Points

|                      | $\sigma$ | $\xi$ | $\zeta$ | $\bar{T}[GeV]$ |
|----------------------|-----------|-------|----------|-----------------|
| $V - A$              | 0.41      | 1.00  | 0.41     | 1.55GeV         |
| $S + P$              | 0.41      | 0.99  | 0.40     | 1.55GeV         |
| $f_M + f_E$          | 0.41      | 1.00  | 0.41     | 0.66GeV         |
| $\eta$               |           |       | $\eta_b$ | $\kappa_0$ | $\epsilon_+$ |
| $V - A$              | 0.46      | 0.46  | 0.46     | -0.005         | -0.015        |
| $S + P$              | -0.45     | -0.46 | -0.46    | 0.05           | 0.015         |
| $f_M + f_E$          | -0.46     | -0.46 | -0.46    | 0.005          | -0.015        |

Table 3: Discrete Symmetry Relations and the Helicity Amplitudes

| Invariance | Symmetry Relation |
|------------|-------------------|
| P          | $A(-\lambda_{W^+}, -\lambda_b) = A(\lambda_{W^+}, \lambda_b)$ |
|            | $B(-\lambda_{W^-}, -\lambda_{\tilde{b}}) = B(\lambda_{W^-}, \lambda_{\tilde{b}})$ |
| C          | $B(\lambda_{W^-}, \lambda_{\tilde{b}}) = A(\lambda_{W^-}, \lambda_{\tilde{b}})$ |
| CP         | $B(\lambda_{W^-}, \lambda_{\tilde{b}}) = A(-\lambda_{W^-}, -\lambda_{\tilde{b}})$ |
| T          | $A^*(\lambda_{W^+}, \lambda_b) = A(\lambda_{W^+}, \lambda_b)$ |
|            | $B^*(\lambda_{W^-}, \lambda_{\tilde{b}}) = B(\lambda_{W^-}, \lambda_{\tilde{b}})$ |
| CPT        | $B^*(\lambda_{W^-}, \lambda_{\tilde{b}}) = A(-\lambda_{W^-}, -\lambda_{\tilde{b}})$ |
This figure "n11.gif" is available in "gif" format from:

http://arxiv.org/ps/hep-ph/0006342v2
This figure "n22.gif" is available in "gif" format from:

http://arxiv.org/ps/hep-ph/0006342v2
This figure "n32.gif" is available in "gif" format from:

http://arxiv.org/ps/hep-ph/0006342v2
This figure "n42.gif" is available in "gif" format from:

http://arxiv.org/ps/hep-ph/0006342v2
This figure "n52.gif" is available in "gif" format from:

http://arxiv.org/ps/hep-ph/0006342v2
This figure "n62.gif" is available in "gif" format from:

http://arxiv.org/ps/hep-ph/0006342v2
This figure "n72.gif" is available in "gif" format from:

http://arxiv.org/ps/hep-ph/0006342v2
This figure "n82.gif" is available in "gif" format from:

http://arxiv.org/ps/hep-ph/0006342v2