Getting Around the Nielsen-Ninomiya Theorem, towards the Rome Approach

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Abstract

The “no-go” theorem of Nielsen and Ninomiya has been the most tenacious obstacle against the construction of a chiral gauge theory with reasonable low energy spectrum, couplings and anomaly. In this paper we construct a model which supplements the usual (bilinear in the Fermi fields) lagrangian with quadrilinear fermionic terms. We show that in a certain region of the parameter space the difficulties of the “no-go” theorem may be overcome, and a “renormalized” perturbative strategy can be carried out, akin to the one followed in the Rome Approach (RA), whose counterterms are forced to be gauge invariant.

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1. The problem of defining a quantum chiral gauge field theory on a lattice lies at the roots of the Standard Model (SM). For even if space-time, the physical arena of the SM, is a continuum (as modeled in the generally accepted SM, which we shall call Continuum Standard Model (CSM)) well beyond the Planck length \( a_p \approx 10^{-33} \text{ cm} \), the very process of “renormalizing” the theory requires its definition on a space-time, like a discrete lattice, where the energy-momentum spectrum of the quantum fluctuations is bounded from above. When this can be done, i.e. when we can define our chiral gauge theory on a lattice with arbitrarily small lattice constant \( a \), the continuum limit is simply defined by the limit \( a \to 0 \).

It seems to us that this is the (essentially unique) way to give any sense to a Quantum Field Theory (QFT), and this for two reasons: one physical, the other mathematical. From the standpoint of physics the assumption of a mathematical continuum as the space-time basis of a generic QFT totally neglects the possible effects of the physics of Quantum Gravity in determining (according to the seminal ideas of Bernhard Riemann) the “fine-structure” of the (geometry of the) set of physical events. As for mathematics itself a continuum QFT contains actual infinities, a nightmare sternly but pointlessly denounced by Dirac, which can be possibly exorcized through a limiting procedure upon theories defined on lattices whose constants decrease to zero.

It is for the reasons just discussed that a most simple and general theorem \([1]\), found by H.B.Nielsen and M.Ninomiya at the beginning of the Eighties, appears as a most disappointing obstacle \([2]\) to giving a definite quantum field theoretical sense to the Standard Model itself, whose electroweak sector comprises precisely a quantum chiral gauge field theory. Prompted by the clear physical indication coming from the stunning phenomenological successes of the SM, L.Maiani and collaborators in Rome have elaborated a most reasonable philosophy to get around the unfavourable consequences of Nielsen and Ninomiya’s “no-go theorem”, without actually explicitly constructing a lattice QFT that evades the “no-go theorem”, and at the same time yields in the continuum limit \( a \to 0 \) the correct phenomenology. The analysis at one-loop level carried out by this group \([3]\) confirms the soundness of their point of view. However it is clear that a positive answer to the question whether the favourable signs now available will turn the Rome Approach (as it has been called, RA) into a consistent fool-proof strategy to calculate the chiral gauge theory in the continuum limit, can be given if we are able to explicitly construct a consistent lattice chiral gauge theory whose continuum limit
limit \((a \to 0)\) is just the “target theory” of the RA.

It is worthwhile recalling that in the last few years we have tried to develop a research program centered on the formulation of the SM on a Planck lattice, a lattice whose constant \(a\) is just the Planck length \(a_p\). The main physics motivation of our program is the expectation [4] that the violent quantum fluctuations of Quantum Gravity (QG) at the Planck scale do give rise to a discrete space-time structure, that can be usefully modeled by a 4-dimensional lattice, we have christened such attempt the Planck Lattice Standard Model (PLSM). A crucial aspect of the PLSM is the basic chiral nature of the electroweak sector, thus a basic prerequisite for its success is our ability to find an explicit formulation of a chiral gauge theory that

1. removes the “doubling” of the Weyl fermions and retains the correct anomaly;
2. maintains in the low-energy spectrum the basic chiral coupling to the gauge field;
3. realizes a “renormalized” perturbative strategy akin to the one known as the Rome Approach without gauge-variant counterterms and pushing the gauge bosons’ masses to the Planck mass, \(m_p\).

In this paper we shall introduce an explicit chiral lattice QFT model and show that in a certain region of the relevant parameter space, the “physical wedge”, it satisfies the above three requirements. However, we should like to point out that we do not make any attempt\(^1\) to formulate a fully relativistic PLSM of quark and lepton families in terms of our chiral model, thus we can keep its structure as simple as possible.

The model we shall analyse below is the simplest \(SU(2)\) chiral gauge theory, which in order to avoid the sanctions of the “no-go theorem” will be supplemented with quadrilinear Fermi interactions terms[5][6], which we shall call the Nambu-Jona Lasinio (NJL) terms. Thus the fermionic part of the action on the lattice (of lattice constant \(a\)) is written as (repeated indices are summed over)

\[
S_F = \frac{1}{2a} \sum_n a^4 \bar{\psi}^i_L(n) \gamma_\mu D_\mu^i \psi_L^j(n) + \frac{1}{2a} \sum_n a^4 \bar{\psi}^i_R(n) \gamma_\mu \partial^\mu \psi^j_R(n)
\]

\(^1\)This will be reported in a future publication.
\[ +g_1 a^2 \sum_n a^4 \tilde{\psi}_i^I(n) \cdot \psi_R^I(n) \tilde{\psi}_R^I(n) \cdot \psi_L^I(n) \]
\[ +g_2 a^2 \sum_{n \mu \nu} a^4 \tilde{\psi}_L^I(n) \cdot (\partial^2 \mu \psi_R^I(n)) (\partial^2 \nu \psi_R^I(n)) \cdot \psi_L^I(n), \]
where \( \psi_L^I(n) \) \((i = 1, 2)\) is the \( SU(2)_L \) gauged doublet, \( \psi_R^I(n) \) \((l = 1, 2)\) is a doublet, invariant under the transformations of \( SU(2)_L \); both \( \psi_L^I \) and \( \psi_R^I \) are two-component Weyl fermions. In the second NJL-term
\[ \partial^2 \mu \psi_R^I(n) = \psi_R^I(n + \mu) + \psi_R^I(n - \mu) - 2 \psi_R^I(n), \]
in the continuum limit this term is a dimension-10 operator which is relevant only for “doublers”, i.e. for those field modes whose momenta
\[ p = \bar{p} + \Pi_A, \]
\( \Pi_A \) being one of the fifteen Brillouin momenta and \(|\bar{p}| \ll \pi a\). Note that in addition to the \( SU(2)_L \) chiral gauge invariance, and the global \( SU(2)_L \times SU(2)_R \) symmetry, the fermion action (1) possesses when \( g_1 = 0 \) an exact \( \psi_R \)-shift symmetry \( [7] \):
\[ \psi_R^I(n) \rightarrow \psi_R^I(n) + \text{const.} \]

2. Our goal now is to seek a possible region of the \((g_1, g_2, g)\)-space \((g\) is the \( SU(2) \) gauge coupling constant) where there “lives” a theory that obeys the requirements 1. – 3. above. To start with we shall assume that the gauge coupling is a good perturbative coupling, thus for the time being we shall take \( g \rightarrow 0 \).

In the \((g_1, g_2)\)-plane, in the weak coupling limit \( g_1, g_2 \ll 1 \), as indicated in fig.1, the action \([1]\) defines a \( SU(2)_L \times SU(2)_R \) chiral continuum theory with a “doubled” fermion spectrum, a theory that violates requirement 1.. On increasing \( g_1 \), while keeping \( g_2 \) in the perturbative region \( g_2 \ll 1 \), following the analysis and the discussion of Eichten and Preskill (EP) \([5]\), we shall hit a critical value \( g_1^c \), above which the theory undergoes a phase transition. Using a strong coupling expansion in \( g_1 \) \((g_1 > g_1^c)\) we can show that the fields \( \psi_L^I \) and \( \psi_R^I \) appearing in \([1]\) pair up with the composite Weyl fermions:
\[ \tilde{\psi}_L^I \cdot \psi_R^I \psi_i^I \quad \text{and} \quad \tilde{\psi}_R^I \cdot \psi_L^I \psi_i^I, \]

3.
respectively to build two massive Dirac fermions $\psi^I_n$, neutral with respect to the gauge transformations of $SU(2)_L$, and $\psi^I_c$, carrying a $SU(2)_L$ charge. Their propagators in the strong coupling limit are given by

\[ S_{nk}^{II}(p) = \frac{i}{a} \sum_{\mu} \gamma^\mu \sin(p_\mu) + M \frac{1}{a^2} \sum_{\mu} \sin^2(p_\mu) + M^2 \delta_{lk}; \]

\[ S_{ij}^{Ij}(p) = \frac{i}{a} \sum_{\mu} \gamma^\mu \sin(p_\mu) + M \frac{1}{a^2} \sum_{\mu} \sin^2(p_\mu) + M^2 \delta_{ij}, \]

where the chiral invariant mass is given,

\[ M = 16 \frac{g_1}{a} (1 + \frac{g_2}{g_1} w(p)^2); \quad w(p) = \sum_{\mu} (1 - \cos(p_\mu)). \]

These propagator show that in this region not only the “doublers” have acquired a $O(\frac{1}{a})$ mass but also the “normal” modes ($\Pi_a = 0$). And this is clearly not acceptable.

Let us now discuss the phase structure of the theory in the neighbourhood of the critical value $g_{c1}$ (while keeping $g_2$ very small). Such critical value is determined by the propagators of the 8 composite scalars

\[ A_{1i}^I = \frac{1}{\sqrt{2}} (\bar{\psi}^I_L \cdot \psi^I_R + \bar{\psi}^I_R \cdot \psi^I_L), \quad A_{2i}^I = \frac{i}{\sqrt{2}} (\bar{\psi}^I_L \cdot \psi^I_R - \bar{\psi}^I_R \cdot \psi^I_L), \]

corresponding to the real and imaginary parts of the complex fields $\bar{\psi}^I_R \cdot \psi^I_L$. In the strong coupling limit one easily obtains

\[ G_{1,2}^{ii,jk}(q) = \delta_{ij} \delta_{lk} \frac{1}{a^2 \sum_{\mu} \sin^2 q_\mu + \mu^2}, \]

where

\[ \mu^2 = 16 \frac{g_1}{a^2} g_1 \left( 1 + \frac{4g_2}{g_1} w(p) w(p') \right) - \frac{8}{a^2}, \]

and $p, p'$ are external momenta. It is only when the composites become bound states, i.e. when $\mu^2 > 0$, that one obtains what we might call the EP-phenomenon, leading to the appearance of the chiral symmetric phase (that
we call EP-phase) where Dirac fermions propagate according to (6) and (7). The critical value \( g_1^c \) can be determined by setting \( \mu^2 = 0 \) Thus, due to the momentum dependence (through the \( g_2 \)-term) of \( \mu^2 \), for \( g_2 \neq 0 \) we shall have different critical coupling constants \( g_1^c \) for different “doublers”, in particular in the strong coupling limit for the “normal” mode one has \( g_1^{c_{\text{normal}}} = 0.5 \). Thus in the \((g_1, g_2)\)-plane from the point \((g_1^{c_{\text{normal}}}, 0)\) there start (See Fig.1) different critical lines, one for each \( \Pi_A \), separating the symmetric EP-phase from another phase, that Eichten and Preskill expected to be a weak coupling symmetric phase. Thus, one can define continuum chiral gauge theories on one of these critical lines where all doublers are decoupled, chiral fermions remain and gauge symmetry is not broken. We call this expectation as the EP scenario. However, as pointed out by ref.[8], this EP scenario fails owing to the fact that critical lines \( g_1^c(g_2^c) \) do not separate two symmetric phases and there is a spontaneous symmetry breaking phase, which we shall call the NJL-phase, in between the two symmetric phases. This can be quickly figured out by looking at what one gets \( \mu^2 > 0 \) (symmetric phase), \( \mu^2 = 0 \) (critical lines) and \( \mu^2 < 0 \) (broken phase).

In order to check this latter statement, let us return to the weak coupling \( g_1, g_2 \) limit. Based on the analysis of the large-\( N_f \) (\( N_f \) is the multiplicity associated with an extra fermionic index, e.g. \( N_{\text{color}} \)) weak coupling expansion, we argue that the NJL-terms in the action (1) induce a spontaneous chiral symmetry breaking [9]. Indeed in the new vacuum the Weyl fields \( \psi^i_L \) and \( \psi^i_R \) pair up to become massive Dirac fields that violate \( SU(2) \)-chiral symmetry, a fact that can be ascertained in the following way. The inverse propagator of this massive Dirac fermion can be written as

\[
S^{-1}(p) = \begin{pmatrix}
P_L \frac{1}{\alpha} \sum_{\mu} \gamma_{\mu} f^\mu_L(p) P_L \delta_{ij} & P_L \Sigma^d(p) P_R \\
P_R \Sigma^k(p) P_L & P_R \frac{1}{\alpha} \sum_{\mu} \gamma_{\mu} f^\mu_R(p) P_R \delta_{lk}
\end{pmatrix},
\]

where the fermion self-energy function, in the \( N_f \to \infty \) limit, obeys the “gap-equation” \( (\Sigma^d(p) = \delta^d \Sigma(p)) \):

\[
\Sigma(p) = 2 \int_q \frac{\Sigma(q)}{\text{den}(q)} (\tilde{g}_1 + \tilde{g}_2 w(p) w(q)),
\]

where

\[
\int_q = \int_{-\pi}^{\pi} \frac{d^4 q}{(2\pi)^4}, \quad \text{den}(q) \equiv \sum_{\rho} \sin^2(q\rho) + (a \Sigma(q))^2,
\]
and $\tilde{g}_{1,2} \equiv g_{1,2}N_f$. Using the parametrization\footnote{The details of the calculation will be presented elsewhere.}

$$\Sigma(p) = \Sigma(0) + \tilde{g}_2 v w(p); \quad \Sigma(0) = \rho v,$$

where $\rho$ depends on $\tilde{g}_{1,2}$ only, we can solve the gap-equation (13) in a straightforward manner. For $v = O(\frac{1}{a})$, one gets

$$\rho = \frac{\tilde{g}_1 \tilde{g}_2 I_1}{1 - \tilde{g}_1 I_0}, \quad \rho = 1 - \frac{4\tilde{g}_2 I_2}{4I_1},$$

where

$$I_n = 4\int_{q} \frac{w(q)^n}{\text{den}(q)}.$$

For $g_1 = 0$ Eq.(16) implies:

$$\rho = 0, \quad \text{i.e.} \quad \Sigma(0) = 0.$$

This means that on the line $g_1 = 0$ the “normal” modes ($\Pi_A = 0$) remain massless, while all the “doublers” acquire a mass $O(\frac{1}{a})$, thus violating chiral invariance through the $g_2$ coupling alone.

As for the function $f_L^\mu(p)$ ($f_R^\mu(p)$) in Eq.(12), the $\psi_R$-shift symmetry for $g_1 \to 0$ allows us to derive for $f_R^\mu(p)$ the form

$$f_R^\mu(p) = \sin(p_\mu),$$

while for $f_L^\mu(p)$ in the large $N_f$-limit one obtains:

$$f_L^\mu(p) = Z_2(p) \sin(p_\mu),$$

with the wave-function renormalization $Z_2(p = \bar{p} + \Pi) = \text{const.}$ Thus the massive Dirac inverse propagator can be written for both members of the $SU(2)$ doublet

$$S^{-1}(p) = i \frac{1}{a} \sum_{\mu} \gamma_\mu \sin(p^\mu) Z_2(p) P_L + i \frac{1}{a} \sum_{\mu} \gamma_\mu \sin(p^\mu) P_R + \Sigma(p),$$

showing that the $SU(2)_L \times SU(2)_R$ symmetry is spontaneously broken to $SU(2)$, giving rise to three Goldstone bosons and one massive $O(\frac{1}{a})$ Higgs mode, as discussed in Ref.[6].
For \( v \to 0 \) Eq (16) yields a critical line \((\tilde{g}_1, \tilde{g}_2)\) separating the NJL-phase from the weak coupling symmetric phase, which goes from \((0.4, 0)\) to \((0, 0.006)\) as indicated in Fig.1. Thus the NJL-phase stands between the chiral symmetric doubled weak coupling phase and the chiral symmetric EP-phase. This fact, that according to our analysis holds even for \( N_f = 1 \), shows that the EP-scenario for getting around the no-go theorem, unfortunately cannot be realized. Indeed, such scenario contemplates the contiguity of the EP and the symmetric weak phase, so that there would exist a chiral line (See Fig.1) where the theory is chirally symmetric and all the unwanted doublers get a mass \( O(\frac{1}{a}) \). Our analysis shows instead that the EP-phase is contiguous to the NJL-phase, that we have just described and agrees with analysis of ref.[8].

Unfortunately also the NJL-phase is physically not entirely acceptable, for even when \( g_1 = 0 \) the \( SU(2)_L \times SU(2)_R \) chiral symmetry is violated by a hard breaking Wilson term [10] (See eq. (14)), corresponding to a dimension 5-operator. As a consequence, in the well known phenomenon that mixes the gauge bosons and the Goldstone modes one expects the gauge bosons to acquire a mass \( O(\frac{1}{a}) \), contrary to physics expectations.

### 3. In our investigation we are finally left with another strong coupling region \((g_1 \ll 1, g_2 \gg 1)\) (See Fig.1) where the \( \psi_R \)-shift symmetry is exact. This region can be treated exactly as the one dealt with above. We find that the vacuum of the theory retains the \( SU(2)_L \times SU(2)_R \) symmetry, and its excitation spectrum contains both chiral invariant \((\psi^i_n)\) and chiral \((\psi^i_c)\) massive Dirac fermions, whose propagators in the strong coupling limit \( g_2 \gg 1 \) can be determined to be of the same form as eqs.(6,7). The chiral invariant mass, instead of eq.(8), this time is given by:

\[
M^2 = \frac{16}{a^2 g_2^2} \left( w(p)^2 + \frac{g_1}{g_2} \right)^2.
\]  

As for the charged fermion field \( \psi^i_c \), even when \( g_1 \ll 0 \) and \( g_2 \gg 0 \), the low energy spectrum still comprises the massive Dirac mode built by the “normal” modes of \( \psi^i_L \) and the composite \((\bar{\psi}_R^k \cdot \psi^i_L)\psi^k_R \), a situation that is phenomenologically undesirable. One can solve this problem if in the \((g_1, g_2)\)-plane there existed a wedge where only the unwanted doublers are in
the EP-phase, while the “normal” modes would still be massless (does not undergo the NJL spontaneous symmetry breaking) and live upon a chirally symmetric vacuum. In order to see whether such circumstance really occurs, let us proceed as above. We consider again the scalars $A_{i1}^1, A_{i2}^2$ (See Eq.(9)) and their propagators, whose mass $\mu^2$ this time is not given (approximately) by (11), but by:

$$\mu^2 = \frac{16}{a^2} g_2 \left( 4w(p)w(p') + \frac{g_1}{g_2} \right) - \frac{8}{a^2}. \tag{23}$$

The critical lines for different doublers acquiring chiral invariant masses are determined, as above, by the condition $\mu^2 = 0$; thus the first “threshold” (See Fig.1) is encountered when $g_1 = 0$ at $g_2 = 0.002$, for the doubler:

$$p = p' = (\pi, \pi, \pi, \pi), \tag{24}$$

while the last threshold appears for $g_2 = 0.03$, corresponding to the four doublers with

$$p = p' = (\pi, 0, 0, 0), \quad (0, \pi, 0, 0), .... \tag{25}$$

Thus in the strong coupling approximation for $g_2 > 0.03$ all excitations but the “normal” modes $(p, p' = \tilde{p})$, i.e. all doublers, acquire chiral-invariant $O(\frac{1}{a})$ masses (6,7,22), while the latter modes remain massless, on a chirally invariant vacuum, so long as $g_1 = 0$.

In order to check latter statement that normal mode remains massless, we go back to eq.(18) that shows normal mode is massless when the coupling $g_1 = 0$, even for very large value of the coupling $g_2$. This is actually resulted by the exact shift-symmetry (4) when the coupling $g_1 = 0$. Thus, the failure of the EP-scenario, due to very existence of the NJL broken phase in between two symmetric phases discussed in our previous section and ref.[8] can be cured when $g_1 = 0$, where normal modes always remain in the chirally symmetric phase and we can send $g_2 \to \infty$ to guarantee that all the doublers are chirally invariantly decoupled. Note that such an important feature is absent, when $g_1 = 0$, in the original EP’s model[3].

To summarize the above discussion, let us make a journey in the $(g_1, g_2)$-plane taking off from the origin, and identify the different phases of the theory described by the lattice action (1). We walk first in the weak coupling symmetric phase, populated by massless, doubled fermions: a strange, alien
world. Diffusing away from the origin we hit a line, the NJL-line, that pushes the doublers to the cut-off, i.e. gives them a chirally variant $O(\frac{1}{a})$ mass, but unfortunately does so also to the chiral gauge bosons due to the hard (dimension-5) Wilson term, that possesses now a non-trivial expectation value in the new NJL-vacuum. Thus, again, we are in a world completely different from our own. Continuing to move out, always keeping $g_1 < 0.5$, we hit several other critical lines where different doublers acquire a chirally invariant $O(\frac{1}{a})$ mass, up to the last line where the four doublers physical with $|p_\mu| = \pi$ get their chiral invariant $O(\frac{1}{a})$ masses \(^{1}\). Past that line and go down onto $g_1 = 0$, we are in the sought “physical wedge” (See Fig. 1): here the low energy spectrum is undoubled, it only contains “normal” modes and, most importantly, the original chiral $SU(2)_L$-gauge symmetry for $|p_\mu| \ll 1$ is exact when $g_1 = 0$. More precisely, in this wedge the “normal” mode of $\psi^i_L$, due to the vanishing of $\Sigma(0)$ \(^{18}\), interacts in a completely chiral invariant way. From Eq.\((21)\) we can write the propagators of such massless modes as:

$$S_L(p)_{ij}^{-1} = i\gamma_\mu \tilde{p}^\mu \tilde{Z}_2 P_L \delta_{ij},$$

(26)

and

$$S_R(p)_{ik}^{-1} = i\gamma_\mu \tilde{p}^\mu P_R \delta_{ik},$$

(27)

where $\tilde{Z}_2$ is the wave-function renormalization for $p = \tilde{p}$ and $g_1 = 0$. Thus we see that the scenario envisaged by Eichten and Preskill can be finally realized, but only in the physical wedge $g_1 \to 0$, $g_2 > 0.03$. We should also stress at this point that, even though our arguments are based on various kinds of approximations, strong coupling and large $N_f$, there are absolutely no reasons to expect that more accurate calculations will change qualitatively the whole picture.

This shows that there exists a well defined, concrete way to get around the “no-go theorem” of Nielsen and Ninomiya that has so far barred the road to viable chiral gauge theories. The way around the “no-go theorem” requires the extension of the simple Wilson gauge lagrangian through the addition of two quadrilinear fermionic interactions terms (See Eq.\((1)\)), which in a particular physical wedge of the $(g_1, g_2)$-plane give rise to a ground state where the unwanted doublers acquire a large chiral invariant $O(\frac{1}{a})$ mass by the EP mechanism, while the “normal” (low momenta) fermionic modes do not get their masses through a Nambu-Jona Lasinio spontaneous symmetry
breaking mechanism, thus keeping the low-momentum interaction chirally invariant. This provides a physical arena to realize the scenario of the RA. The theory defined by the action Eq.(1) upon the vacuum of the “physical wedge” has all the features of the target theory of the RA. As a consequence it can be looked at as a concrete justification of the feasibility of the perturbative strategy envisaged in the RA. However, instead of utilizing in the “renormalized” perturbation theory the gauge non-invariant Wilson propagator to remove doublers and the gauge variant counterterms to enforce the chiral gauge symmetries, one has now in the “physical wedge” an undoubled low-energy effective theory with the gauge invariant propagators (6), (7) for doublers and (20), (27) for normal modes respectively, thus allowing gauge-invariant counterterms only, as expected in gauge invariant Quantum Field Theories like QCD and QED.

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Figure Captions

Figure 1: The phase diagram for the theory (1) in the $g_1 - g_2$ plane (at the gauge coupling $g \simeq 0$).