Formulation of a Method for Assessment of Fatigue Life, Formation and Propagation of Cracks

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Abstract. An engineering method for assessing fatigue life, taking into account the accumulation of damage and allows determining the trajectory of the fatigue crack. The calculation is performed by successive approximations based on the ANSYS Mechanical APDL software package in both linear and nonlinear installations. Stages of calculation of fatigue, cracking and fatigue failure of the structure alternate.

1. Introduction
The aim of the work is to formulate a method for assessing the life of structures taking into account the accumulation of damage, the formation of fatigue cracks and the trajectory of their propagation. The object of study in this paper is fatigue damage.

The actuality of the work is due to the rationale for the use of structures under multi-cycle loads and the task of ensuring the operational characteristics of structures throughout their life. In figure 1 shows an example of the typical development of fatigue cracks of rail web near the holes.

![Fatigue crack in the rail web](image)

**Figure 1. Fatigue crack in the rail web**

The study tool is the numerical simulation of FEM using the ANSYS Mechanical APDL software package (as one of the most powerful, comprehensively verified and most common in practice), allowing...
you to optimally organize tests and “fill in” their results by determining the structural characteristics of interest in areas of complex stress-strain states.

When developing a method for assessing fatigue damage and crack propagation, the task was to simplify the implementation of the algorithm and the accuracy sufficient for engineering calculations with the most optimal time for preparing the model and performing calculations. This algorithm includes an assessment of the stresses acting near the holes, determining the number of loading cycles at which a defect occurs and determining the direction of its development. A method for estimating the time of crack development to an unacceptable length is discussed.

2. Description of the proposed calculation method

At present, when studying the processes of fatigue failure, a number of basic phenomena characterizing the peculiarities of the processes are distinguished. Destruction mechanisms are divided into the following main types [1-2]: dynamic (impulsive), for example, shock; long, for example, creep, relaxation; periodic (cyclic), for example, quasi-static, low-cycle fatigue, multi-cycle fatigue. Among the above processes of destruction, one of the most typical and frequently occurring causes of failure of elements of engineering structures is the process of multi-cycle fatigue.

When solving the problem of assessing multi-cycle fatigue, a number of main stages are traditionally distinguished [3]:

1. Determination of a dangerous point (cross section) in a structure in which the process of destruction during multi-cycle fatigue is located.
2. Determination of the loading history at the selected hazardous point.
3. Setting the properties of the material.
4. Specifying the characteristics of the structure or its element that affect the process under study.
5. Calculation of fatigue life.

Fatigue destruction involves the division into three periods: the initiation of a crack (initial stage), its development and the destruction of the structure.

At the initial stage, or the stage of origin, microcracks appear, because of the development of which cracks are formed, which grow with increasing velocity under the action of periodic external forces. At the stage of destruction, the structure loses its carrying capacity, because the remaining cross section is too small to resist external forces. The maximum length of the crack in the structure characterizes the final stage.

The previously presented initial stage takes part of the fatigue life of the structure, from 50 to 75%. However, more accurate studies at the microscopic level showed that microcracks appear after the loss of 1% of the resource [4]. In addition, small surface defects, which act as sources of crack initiation, may already be present in structures.

An important scientifically based reserve for extending the resource can be the calculation of the development of cracks formed, which can grow for a long period and not lead to negative consequences. To assess the rate of crack development and the time of implementation of an unacceptable length, a fracture mechanics apparatus can be used.

Methods for assessing fatigue life using an external load – loading cycle, characterized by average and amplitude values.

In the elements of real structures, a multiaxial stress state arises. Most of the experimental data on which the calculation of fatigue life is based were obtained in the case of uniaxial loading with a symmetric loading cycle. Therefore, when used to describe the external influence of the dependence of the stress tensor components at a dangerous point on time, one must be able to switch from a multiaxial stress state to a uniaxial and from an asymmetric loading cycle to a symmetric loading cycle.

The essence of the proposed method of calculation is as follows. At the first stage, the structure is calculated on a static action by the finite element method using the ANSYS Mechanical APDL software package. According to the results of the calculation, determine the greatest stress and the zone of their influence. The principal tensile stress is used as the calculated stress. The rationale for this choice can
serve as the assumption of quasi-brittle fracture of steel structures, that is, with the gradual accumulation of microcracks, brittle fracture of the structure occurs. The choice as a criterion for the destruction of principal stresses was because according to the results of experimental and theoretical work, the course of the process of multi-cycle fatigue is due to the accumulation of microcracks of the construction material [5–6]. The process of multi-cycle fatigue, by definition, refers to the processes of destruction that are not accompanied by developed plastic deformation. As a result, this process is localized near a certain point at which the most favourable conditions arise for the process under study. This point is determined not only by the type of stress state, but also by the characteristics of the structure itself.

At the second stage, the number of cycles $N$ corresponding to the intensity of this stress is found from the Makhutov equation [7-9], which has proved itself well in analysing the responsible nodes and elements. This Makhutov equation (1) below is one of the approximations of the fatigue curve.

$$\sigma^*_a = \frac{E}{4(N)^{m_p}} + \frac{\ln 100}{1 - r} + \frac{S_k}{4(N)^{m_e}} + \frac{1 + r^*}{1 - r^*}.$$  \hspace{1cm} (1)

In this equation: $\sigma^*_a$ – amplitude of conditional elastic stresses in the most loaded zones, equal to $\sigma^*_a = e_a E$ ($e_a$ is the amplitude of the elastic or elastic-plastic local deformation, $E$ is the elastic modulus); $\psi_k$ – relative narrowing of the sample (in %); $S_k$ – tear resistance in the neck; $m_e$ – characteristic (indicator) of low-cycle fatigue resistance; $m_e$ – characteristic (indicator) of resistance to high-cycle fatigue; $r$ – asymmetry coefficient of the cycle of local stresses in the zone under consideration; $r^*$ – asymmetry coefficient of the strain cycle. In this equation the inverse substitution was implemented. According to the known $\sigma^*_a$, the iteration parameter $N$ with step $n$ is determined.

Next, the most stressed element is removed, the parameter $N$ is written, which indicates the number of loading cycles corresponding to this stress, and a comparison is made with the maximum allowable crack. The iterative process is repeated until the crack reaches the maximum allowable length.

Plotting the function of the crack size to the number of cycles allows determining the zones of stable and accelerated growth of cracks. The analysis of these graphs makes it possible to identify threshold values of loading cycles, at which a sharp increase in the rate of growth of fatigue cracks occurs and to take measures to reduce the negative impact of fatigue phenomena in the operation of structures.

The advantages of the method include the possibility of its use in automatic mode. In the programming language APDL, a program was written that implements the entire considered cycle. The implementation of this algorithm is possible in both linear and non-linear formulation.

The block diagram of the algorithm in the linear formulation is shown in figure 2.

Most often, the work of the real material is complex and not always, it can be taken into account by a linear relationship of stresses and strains or between displacement and load. To solve the main problem of structural analysis in a broad sense, it is necessary to abandon the simple assumptions of the linear theory and move to a broader and more complex justification of the nonlinear theory. First, when implementing this algorithm in a non-linear setting, it is necessary to abandon the premise of calculation on the undeformed state, which involves a small amount of movement. These results in a nonlinearity called geometric nonlinearity. In this case, the theory introduces nonlinear relations between deformations and displacements, which allow taking into account the influence of changes in the shape and size of the structure on its stress-strain state. Thus, the shape and dimensions of the structure in different states are not identified.
The replacement of Hooke's law with nonlinear dependencies between stresses and strains is the essence of the so-called physical nonlinearity. 

Due to design, features in the deformation process may change the design scheme of the structure. For example, new connections may be formed or, conversely, old ones may be destroyed. In such cases, the system can be called structurally nonlinear. This approach relates to the question of complex interaction between structures (contact problems).

Implementation of physical nonlinearity in ANSYS Mechanical APDL is executed in the statement describing the diagram of working steel tensile or piecewise linear MULTILINEAR ISOTROPIC HARDENING, bilinear method or the BILINEAR ISOTROPIC HARDENING (figure 3).

**Figure 2.** A block diagram of the linear algorithm
Figure 3. Stress-strain diagrams for ANSYS PC plasticity options [10]

Block diagram of the algorithm in a nonlinear formulation is shown on figure 4.
3. Results and discussions
The proposed method was tested on the example of calculating a 500 mm long rail with two holes as a symmetrical beam on two supports loaded with a force \( P = 800 \text{ kN} \) in the middle of the span (figure 5). The result of the calculation with the development of an inclined crack is shown in figure 5. The number of cycles to failure was \( N = 232000 \). The calculation confirmed the quantitative and qualitative picture of fatigue cracks development.

![Flowchart](image)

**Figure 4.** A block diagram of the nonlinear algorithm
The scientific novelty of the proposed method of assessing fatigue damage and crack propagation is as follows:
1. Using a combined approach to identify the location of the crack formation and to estimate the number of loading cycles sufficient for its propagation.
2. Extension of Mahutov's algorithm for solving problems with continuation of fatigue calculation of crack development. This algorithm is used not only to assess the appearance of the first crack, but also for its further growth.
3. Reliable determination of the place of occurrence of cracks and the number of cycles of cracks to unacceptable sizes.
4. Combining empirical methods of resource estimation with FEM based on APDL algorithm.

In our opinion, the advantages of the proposed method are:
1. Determining the location of the fatigue crack.
2. Combination of crack formation and crack propagation approach (solution of two problems). 3. The ability to determine the qualitative nature of the development of cracks, and quantitative.
3. There is no need to rebuild or thicken the grid to simulate crack growth during the counting process.

As a further development of the method is proposed:
1. Verification of the proposed algorithm with a refined solution of fracture mechanics and experimental data.
2. Expanding the range of tasks.
3. Creating a universal resource estimation algorithm for an arbitrary metalwork.

4. Conclusions
1. An engineering calculation method has been developed for assessing the fatigue life and the propagation of cracks in perforated structures, taking into account damage accumulation and the formation of fatigue cracks.
2. Tasks of the resource of the construction and propagation of cracks are possible both in linear and non-linear settings, moreover, taking into account non-linearity allows extending the possibilities of performing calculations by taking into account geometric, physical and constructive non-linearity.
3. The method of fatigue strength is based on the strength criteria for the greatest tensile stresses, which is a reasonable choice when solving problems of determining the life of structures.

4. The main stages of fatigue strength and fatigue failure were considered.

5. Using the built-in programming language APDL based on ANSYS allows you to expand the capabilities of the program with built-in calculation modules written by the user. This approach allows you to adapt to the needs of the user.

6. The use of the combined approach allows determining the threshold values of loading cycles, at which there is a sharp increase in the growth rate of fatigue cracks and taking measures to reduce the negative impact of fatigue phenomena in the operation of structures.

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