Minimal Walking Technicolour, the Top Mass

and

Precision Electroweak Measurements

Nick Evans

School of Physics and Astronomy Southampton University,
Southampton, S017 1BJ, United Kingdom

Francesco Sannino

The Niels Bohr Institute, Blegdamsvej 17,
DK-2100 Copenhagen Ø, Denmark

Abstract

We consider a minimal technicolour theory with two techniflavours in the adjoint representation of an SU(2) technicolour gauge group which has been argued to feature walking dynamics. We show how to naturally embed this theory in an extended technicolour model capable of generating the top quark mass. We investigate the precision constraints and conclude that such models, in the light of the most recent precision data fit, are not ruled out.

*Electronic address: evans@phys.soton.ac.uk
†Electronic address: sannino@nbi.dk
I. INTRODUCTION

A dynamical mechanism for the breaking of electroweak symmetry, in the spirit of the chiral symmetry breaking driven by QCD’s dynamics, has long been an appealing idea. Technicolour is the prototype theory [1]. The light standard model fermion masses must be produced by the introduction of interactions linking those fermions to the techni-fermion sector. Here extended technicolour (ETC), where the technicolour group is combined with a gauged flavour symmetry of the standard model fermions, is the archetype [2]. The massive ETC gauge bosons generate light masses given by approximately $v^3/M^2_{ETC}$. Flavour physics has the potential to generate large flavour changing neutral currents (FCNCs) for the second family although this physics is at scales of at least 10s of TeV and hence hard to probe. Many and varied models exist of these high ETC scales including models with GIM mechanisms [3, 4, 5, 6] that survive flavour bounds.

In recent years precision electroweak constraints (neatly encapsulated by the measured values of the parameters $S$ and $T$ [7, 8, 9, 10, 11]) have exerted considerable pressure on such models, ruling out many simple cases. These constraints have tended to favour the standard model with a light higgs and zero or negative contributions to $S$ and $T$. Most technicolour models on the other hand give rise to a heavy higgs and large positive $S$ and $T$ contributions. Indeed generally mechanisms for generating negative $S$ and $T$ are thin on the ground. The full set of data from the LEP, SLC and Tevatron experiments is now available and a final set of limits on $S$ and $T$ have been produced [12]. Interestingly the central value has shifted, as a result of more precise measurements, and the data now shows some preference for positive $S$ and $T$ contributions. We reproduce a plot of the latest fit results in the $S,T$ plane in Figure 1. This data is more favourable to the generic idea of there being physics beyond the standard model.

Much has been made of the difficulties technicolour models have with $S$ although the most minimal models [13] (such as an SU(2) technicolour group and a single techni-fermion doublet) have always been reconcilable with the data. The most stringent hurdle to dynamical symmetry breaking models came instead from the large top mass. Generating such a large mass seems to require a very low ETC scale laying the mechanism open to test. This ETC sector must incorporate large isospin breaking to explain the top bottom mass splitting. The contributions to the $T$ parameter can be huge [14, 15]. The simplest models
of technicolour with QCD-like dynamics and a naive ETC mechanism are undoubtable ruled out.

QCD has traditionally been used as the testing ground of technicolour dynamics but it has always been recognized that non-QCD-like dynamics might play a part. The most discussed example is walking technicolour $^{16,17,18,19,20}$. Such theories are assumed to lie close to a strongly coupled infra-red fixed point so that the gauge coupling is strong over a significant energy regime. This behaviour enhances the size of the fermion condensate that is assumed to form and break electroweak symmetry. This in turn enhances the light fermion masses generated by ETC allowing the ETC scale to be raised. Originally the walking mechanism was used to address the FCNC problem requiring walking over many decades of energy regime. This seems to ask a bit much of the near conformality of the theory. Recently though the idea has been resurrected to push the ETC scale associated

FIG. 1: Latest electroweak precision measurements taken from $^{12}$. The ellipse is drawn for a reference higgs mass of 150 GeV.
with the top quark higher to reduce T contributions \cite{21}. Here conformality must only be approximated over a few TeV energy range. We will investigate this idea in a new setting below.

Another consequence of conformality is that the technifermion self energy is enhanced at high energy. This reduces the contributions to the S parameter \cite{22,23,24,25}.

Finally it has been argued that walking theories are liable to generate a lighter higgs than traditional technicolour \cite{26}. To trigger chiral symmetry breaking the value of the coupling at the IR fixed point must lie close to a critical coupling value. If the fixed point was above, chiral symmetry breaking would be triggered before the fixed point was reached, if it was too low, there would be no chiral symmetry breaking. If we treat the number of techni-fermion flavours as a continuous variable then by adjusting $N_f$ one should be able to cross through this quantum phase transition. If one thinks in terms of an effective sigma model description of the symmetry breaking then, if the transition is continuous, it is reasonable that at the critical value of $N_f$ the higgs mass falls to zero \cite{42}. Thus if we lie close to the critical value in order to achieve walking the higgs mass might be expected to be lower than that in QCD.

Recently developments in the understanding of supersymmetric gauge theories \cite{27} have revealed that strongly coupled gauge theories can indeed have conformal IR behaviour for some range of the number of quark flavours. Arguments at large $N$ have been used to connect these theories to non-supersymmetric gauge theories with two index tensor matter \cite{28} (such a connection does not extend to confinement properties \cite{30}). In \cite{31} the implications of these theories to technicolour were raised. Using some of the results worked out in \cite{29} it was suggested, in \cite{32}, that the composite higgs can be light in these strongly coupled theories.

The electroweak data though forces one to propose small $N$ examples. The most minimal model proposed as having a conformal fixed point is SU(2) with two adjoint fermions \cite{31}. This is the model we will develop further in this letter. We note that it is always possible to propose that any technicolour model can be made walking by adding additional appropriate matter multiplets. If these are electroweak singlets then they will be invisible in precision data \cite{13,33}. It would be more elegant though if the electroweak content were sufficient and we pursue these examples since they are perhaps the most minimal matter content theories with possible walking dynamics.

The idea of technicolour models with matter fields in higher dimension representations of
technicolour have been discussed before [2, 34]. Such models are typically hard to combine with an extended technicolour sector since the standard model fermions are in the fundamental representation of their flavour symmetries [33]. The generation of the light fermion masses must be a crucial part of an electroweak symmetry breaking model. Here we will show that in fact the SU(2) model with adjoint matter can rather naturally be recast so that it is accessible to a standard ETC scenario. The crucial observation is that the model can also be written as an SO(3) technicolour model with fundamental matter. We will concentrate on the ETC mechanism for the generation of the top mass since only that sector will be experimentally accessible by the LHC. There is no block to using standard ETC technology to generate the lighter fermion masses too though.

We will finally address the crucial issue of the impact of the model on precision electroweak data. We estimate the contributions to S and T. We argue that the contributions, including the effects of the walking dynamics, are of order the allowed positive shifts in S and T with a somewhat heavy higgs. These estimates are as usual in technicolour only order of magnitude estimates but one must conclude that the dynamics of this model is not clearly ruled out. Thankfully the LHC will switch on soon and place such speculation on its correct footing.

II. THE MINIMAL WALKING MODEL

The technicolour sector we will consider is an SU(2) technicolour gauge group with two adjoint technifermions.

The two loop beta function (which is exact in ’tHooft gauge) of the more general theory with N colours and N_f adjoint fermions is given by

$$\beta = -\beta_0 \frac{g^2}{16\pi^2} - \beta_1 \frac{g^5}{(16\pi^2)^2},$$

(1)

with

$$\beta_0 = \frac{N}{3} (11 - 4 N_f), \quad \beta_1 = \frac{N^2}{3} (34 - 32 N_f).$$

(2)

The theory is therefore asymptotically free if N_f < 2.75. Below this number of flavours, at large N there is a perturbative Banks Zak fixed point. As N_f is lowered further this fixed point becomes non-perturbative.

To estimate the critical coupling for chiral symmetry breaking we require that the anomalous dimension of the quark mass operator must satisfy the relation $\gamma(2 - \gamma) = 1$ [20]. This
yields

\[ \alpha_c \simeq \frac{\pi}{3N}. \]  

(3)

The critical value of the number of flavors which gives this fixed point value is

\[ N_f^c \simeq 2.075. \]  

(4)

Since we are considering adjoint Dirac fermions the critical number of flavors, at the level of the approximations used here, is independent of the number of colors [43]. If we trust this result we can work at large \( N \) where the critical coupling vanishes to determine the critical value of \( N_f \) in a weakly coupled theory where we can trust the perturbative beta-function results. This makes the estimates of the conformal window here more solid than in the usual case with fundamental matter.

Finally we note that the critical coupling value for \( N = 2 \) is \( \alpha_c \simeq 0.52 \) [31]. We expect that the theory will enter a conformal regime unless the coupling rises above the critical coupling triggering the formation of a fermion condensate.

We conclude that a \( N_f = 2 \) theory is so close to \( N_f^c \) that it stands a good chance of being a walking theory - in other words the coupling might spend a large energy regime running just below the critical coupling before eventually just crossing it. When chiral symmetry breaking is triggered the fermions decouple from the theory and the pure glue dynamics then rapidly reach very strong coupling and generate a mass gap for the gluonic degrees of freedom.

To check the pattern of chiral symmetry breaking and for our analysis of ETC in the model it is helpful to recast the theory. We note that SU(2) with adjoint matter fields is equivalent to an SO(3) gauge theory with fundamental matter multiplets. Thus the two adjoint fermions may be written as

\[
\begin{pmatrix}
U^a \\
\bar{D}^a
\end{pmatrix}_L, \quad U^a_R, \quad D^a_R \quad a = 1, 2, 3
\]

(5)

with \( a \) the color index of SO(3). The left fields are arranged in three doublets of the SU(2)\(_L\) weak interactions in the standard fashion. The symmetry breaking condensate is expected to be the usual QCD-like vev for \( \langle \bar{U}U + \bar{D}D \rangle \) which breaks the electroweak symmetry in the standard technicolour pattern.
This model as described so far suffers from a topological anomaly - the Witten anomaly \[35\]. An SU(2) gauge theory must have an even number of fermion doublets to avoid the anomaly. Here there are three extra electroweak doublets added to the standard model and we are required to add a further doublet. We do not wish to disturb the walking nature of the technicolour dynamics so the doublet must be a technicolour singlet \[26\]. Our additional matter then is essentially a copy of a standard model fermion family with quarks (here transforming as a 3 of SO(3)) and a lepton doublet. It looks natural to make the same hypercharge assignments as we do across a normal fermion family (other possibilities exist as discussed in \[26\] but we will stick to this case here). Clearly this fourth family lepton will need to be made massive and the generation of such masses and the top quark mass will be the challenge for the next section.

III. EXTENDED TECHNICOLOUR

We have seen that adjoint multiplets of SU(2) can be written as fundamental representations of SO(3). This trick will now allow us to enact a standard ETC pattern from the literature - it is particularly interesting that for this model of higher dimensional representation techniquarks there is a simple ETC model. We have seen that the extra doublets of the model fill out a fourth family so it is tempting to try to enact a one family ETC type model. In fact though the techni-quarks do not transform under colour (the distinct SO(3) technicolour group replaces these interactions for the fourth family) and these schemes will not work. Instead we will follow the path proposed in \[4\] where we gauge the full flavour symmetry of the fermions.

If we were simply interested in the fourth family then the enlarged ETC symmetry is essentially a Pati-Salam type unification. We stack the doublets

\[
\begin{pmatrix}
U^a \\
D^a
\end{pmatrix}_L, \quad 
\begin{pmatrix}
N \\
E
\end{pmatrix}_L, \quad [U^a_R, N_R], \quad [D^a_R, E_R]
\]

(6)

into 4 dimensional multiplets of SU(4). One then invokes some symmetry breaking mechanism at an ETC scale (we will not speculate on the mechanism here)

\[
SU(4)_{ETC} \rightarrow SO(3)_{TC} \times U(1)_Y
\]

(7)
The technicolour dynamics then proceeds to generate a techniquark condensate $\langle \bar{U}U \rangle = \langle \bar{D}D \rangle \neq 0$. The massive gauge bosons associated with the broken ETC generators can then feed the symmetry breaking condensate down to generate fourth family lepton masses

$$m_N = m_E \simeq \frac{\langle \bar{U}U \rangle}{\Lambda^2_{ETC}}$$

We will estimate this in more detail below.

One could now naturally proceed to include the third (second, first) family by raising the ETC symmetry group to SU(8) (SU(12), SU(16)) and a series of appropriate symmetry breakings. This would generate masses for all the standard model fermions but no isospin breaking mass contributions within fermion doublets. The simplest route to generate such splitting is to make the ETC group chiral so that different ETC couplings determine the isospin +1/2 and -1/2 masses. Let us only enforce such a pattern for the top quark and fourth family here since the higher ETC scales are far beyond experimental probing.

We might for example have the SU(7) multiplets

$$\left[ \begin{array}{c} U^a \\ D^a \end{array} \right]_L, \quad \left[ \begin{array}{c} N \\ E \end{array} \right]_L, \quad \left[ \begin{array}{c} t^c \\ b^c \end{array} \right]_L, \quad [U^a_R, N_R, t^c_R]$$

(9)

here $a$ will become the technicolour index and $c$ the QCD index. We also have a right handed SU(4) ETC group that only acts on

$$[D^a_R, E_R]$$

(10)

The right handed bottom quark is left out of the ETC dynamics and only has proto-QCD SU(3) dynamics. The bottom quark will thus be left massless. The symmetry breaking scheme at, for example, a single ETC scale would then be

$$SU(7) \times SU(4) \times SU(3) \rightarrow SO(3)_{TC} \times SU(3)_{QCD}$$

(11)

The top quark now also acquires a mass from the broken gauge generators naively equal to the fourth family lepton multiplet. Traditional estimates of the electroweak vev and ETC generated masses based on one loop diagrams are given by

$$v^2 = \frac{N}{4\pi^2} \Sigma(0)^2, \quad m = \frac{N}{4\pi^2} \frac{\Sigma(0)^2}{\Lambda^2_{ETC}} \Sigma(0)$$

(12)
here $\Sigma(0)$ is the techniquark self energy at zero momentum which must lie around 1TeV. Such estimates suggest that a top quark mass beyond a few 10s of GeV would be hard to achieve even with an ETC scale of 1 TeV.

Walking dynamics has many features though that one would expect to overcome the traditional small size of the top mass in ETC models. Firstly the enhancement of the techniquark self energy at high momentum enhances the ETC generated masses by a factor potentially as large as $\Lambda_{ETC}/\Sigma(0)$. In [21] it is argued that this effect alone may be sufficient to push the ETC scale to 4 TeV and still maintain the physical top mass.

In fact the dynamics of walking models is most likely yet more complicated! The technicolour coupling is near conformal and strong so the ETC dynamics will itself be quite strong at its breaking scale which will tend to enhance light fermion masses [39]. In our ETC model the top quark will also feel the effects of the extra massive octet of axial gluon-like gauge fields that may induce a degree of top condensation a là top colour models [40, 41]. A precise estimate of this strongly coupled system is beyond current theoretical understanding. Gap equation analyses in NJL type models do support the enhancement. Note that because there is a standard ETC mechanism present that generates a basic contribution to the top mass the NJL type interactions do not have to be tuned to criticality to provide some enhancement of the top mass. In any case such a tuning is already implicit in the walking mechanism.

We conclude that a 4-8 TeV ETC scale for generating the top mass does seem possible and is certainly an interesting possibility. In this model the fourth family lepton would then have a mass of the same order and well in excess of the current search limit $M_Z/2$.

IV. PRECISION DATA

We next come to estimating the contributions of the technifermion sector of the minimal walking model to the S and T electroweak precision data parameters.

There are four extra massive electroweak doublets in this model - the three SO(3) colours of techni-quarks and the extra lepton doublet. Assuming these doublets are degenerate then one would perturbatively expect that there is an S parameter contribution

$$S \simeq 4 \times \frac{1}{6\pi} \simeq 0.2 \quad (13)$$
If the technicolour dynamics were QCD-like then the contribution from the techniquarks would be expected to double due to the exchange of techni-glue \cite{4}. However, with walking dynamics the techniquark self energies are expected to fall off less quickly and the naive perturbative computation with a hard mass is expected to be a conservative estimate. Indeed, one can show that the S value in a walking theory is reduced with respect to the naive perturbative estimate near the conformal window \cite{22,24}.

The ETC gauge bosons in the model break custodial isospin to generate the top bottom mass splitting and one must worry that this will feed into the techniquark sector generating large T contributions \cite{14,15}. There are a number of such contributions. Firstly one can consider the exchange of an ETC gauge boson (associated with a diagonal generator) across a techni-quark loop - in our model there are such bosons that couple to $Q_L$ and $U_R$ but not $D_R$. One can naively estimate these contributions from perturbative diagrams but the result is essentially just a dimensional estimate

$$T \sim \frac{\nu^2}{\alpha \Lambda^2_{ETC}} \sim 0.3$$

Here $\alpha = e^2/4\pi$, we have used an ETC scale of 5 TeV and there will also be order one multiplicative factors.

A second contribution comes from the exchange of ETC gauge bosons enhancing the techni-quark masses. Again since there is isospin breaking we might expect techni-up techni-down mass splitting. If we use the perturbative estimate for the resulting contribution to T we find

$$T \sim \frac{N_{TC}}{12\pi^2\alpha} \frac{\Delta \Sigma^2}{\nu^2} \sim 0.6N_{TC} \frac{\Delta \Sigma^2}{m_t^2}$$

Whether the techniquark mass splitting ($\Delta \Sigma^2$) is actually as large as the top bottom mass splitting depends on the precise origin of the top mass. For example if the top colour interactions play a large role, the splitting in the techni-sector may be smaller.

Finally there is the fourth family lepton multiplet. In the ETC model above if we keep the SU(7) and SU(4) ETC groups couplings equal then this sector will only see isospin splitting as a result of the top bottom splitting. The lepton doublet might reasonably be expected to have the same mass splitting as the techni-quark doublet shifting $N_{TC} \rightarrow N_{TC} + 1$ in the above estimate.

These estimates are rather naive and the most we can claim is that the T contribution
will be of order 0.5 multiplied by a number of order one. If we now refer back to Figure 1 we see that if the walking dynamics produces a higgs with mass between 150 GeV to 1 TeV and we allow an S contribution of 0.2 and a T contribution of 0.5 we are consistent with the data (note that only the shift in the latest precision data allows such a conclusion). We conclude that the model is not ruled out.

V. SUMMARY

It has become clear from the study of supersymmetric and large N gauge dynamics that theories with strongly coupled IR fixed points do exist. A minimal version of such a theory might provide a viable walking technicolour model. Here we considered a minimal model SU(2) with two adjoint fermions that has been suggested as a walking theory. Our main goal was to show that it can be naturally extended to an ETC model. Only when the top mass generation mechanism is explicit can one even begin to test whether the model has too much isospin violation and hence too large a T parameter - the biggest constraint on technicolour models. Our naive estimates suggest that the T parameter contributions are of the order of those now allowed by the precision data. We would caution that such models can not yet be ruled out and that the latest precision measurements have alleviated some of the stress these models are under.

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[42] The mean field result for the higgs mass as function of number of flavors found in [26] may acquire corrections [36, 37]. A possible criticism would be that near the conformal phase transition other states may become light (although no formal proof exists). Although this may affect the argument supporting universal behavior near the phase transition [38], it need not influence the result [26]. We also stress that the fact that the chiral symmetry breaking scale vanishes as well does not imply that the chiral partner of the pions does not become parametrically lighter and narrow near the phase transition. This is so since the self coupling of the scalar is also a parameter which is expected to vanish near a continuous phase transition.
[43] We thank D. Dietrich and K. Tuominen for pointing this feature to us.