T-T fatigue behaviors of composite T800/MTM46 cross-ply laminate and reliability analysis on fatigue life

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Abstract
Static and tension–tension fatigue experiments were conducted on T800/MTM46 cross-ply composite laminates. Fatigue limit and residual strength of the survival specimens were determined. Failure modes of static specimens are characterized by brittle fracture. However, failure modes of fatigue specimens are dominated by delamination, which tends to be more serious under low stress levels. Stiffness degradation can be presented by four stages, which is different from the previous researches. Matrix-dominated damage initiates first in the exterior plies especially in exterior 90° plies. Then, various types of damage appear in interior plies when damage in exterior plies has been very serious. Fatigue life data distribution was determined. Reliability fatigue life incorporating both reliability and confidence level was calculated using the single-side allowance factor. p-γ-S-N curve/surface was proposed to predict reliability fatigue life under various reliability and confidence levels.

Keywords
Carbon fiber–reinforced polymer, tension–tension fatigue, life distribution, reliability fatigue life, p-γ-S-N curve/surface

Introduction
Carbon fiber–reinforced polymer (CFRP) composites are widely used in aeronautics engineering because they exhibit excellent mechanical properties and good fatigue resistance, thus making the aircrafts lighter and improving performance.¹⁻⁵ So, they frequently replace the metallic counterparts for numerous primary and secondary load-bearing structures.⁶⁻⁸ Although exhibiting good fatigue resistance, fatigue performances of CFRP composites are still a big concern because of long service life (25 years or even more) for both military and civil aircrafts. Usually, with the progress of cyclic load, many damages such as intra-ply delamination (delamination within plies), inter-ply delamination (delamination between plies), matrix cracks, fiber breakage, and interface debonding may appear, which can lead degradation in the mechanical properties of composites such as tension/compression strength.⁹,¹⁰ However, some researchers also found the opposite results. For example, tensile strength might increase after a certain number of fatigue cycles, which was resulted from that much off-axis fibers had been reassembled to the loading direction.¹¹ Thus, the strength after fatigue is one of the major ways to study fatigue
issues in cost of material fracture. Later, many non-destructive evaluation (NDE) techniques were developed to evaluate the damage of CFRP composites under fatigue, such as infrared thermographic techniques, acoustic emission, X-ray radiography, and X-ray micro-computed tomography (µCT). Liang et al. studied the properties of flax/epoxy composites under fatigue loading. The stiffening phenomenon of flax reinforcements which were oriented parallel to the loading direction has been confirmed. Philippidis and Passipoularidis investigated the residual strength after fatigue in composite both theoretically and experimentally. They proposed several modified models to predict the residual strength, and the models could reach a good agreement with the experimental data. T Lotfi et al. studied the damage evolution in woven composite laminates using infrared techniques. They found that the average temperature in the gage section of specimen increased with the increase in fatigue cycles. JS Tate and AD Kelkar built a stiffness degradation model for biaxial braided composite of various braid angles. The proposed model could predict the stiffness degradation accurately compared to the experimental data. Kelkar et al. investigated the static and tension-compression behavior of unstitched, stitched, and Z-pinned plain-woven composites applied in aerospace industry. They found that the reinforcements in Z-direction (thickness direction) such as stitching and Z-pinning improved interlaminar shear strength, but at the expense of reduction in tensile and compression strength. However, there are still few investigations on how and where various types of damage initiate and develop.

For many traditional metallic materials, their fatigue life data usually obey to logarithm-normal or three-parameter (3P) Weibull distribution. However, the damage and failure caused by fatigue loading are more complex than metallic materials. Moreover, the complex damage and failure coupling with the inevitable defects generated in manufacturing may lead to a large dispersion in fatigue life data of composite materials, which can vary among two orders of magnitude or even more at a single-stress level. Thus, it is very necessary to explore distribution and reliability of fatigue life data of composite, which is important to guarantee the safety of composite applied in aircraft. Some researchers had pay attention to this problem. Raman studied the rotating bending fatigue using two-parameter (2P) Weibull distribution. The reliability fatigue life under different failure properties \( P_f \) (\( P_f = 1 - P_r \), \( P_r \) is the survival probability) was calculated, but the confidence level is not incorporated in the fatigue life. S Raif and AY Irfan studied the bending fatigue life data of glass-fiber-reinforced polyester composite using 2P Weibull distribution. They drew the S-N curves under various reliability. However, the confidence level is not involved. Lee et al. predicted the fatigue life of fiber composite materials using Weibull distribution. They drew the S-N curve under reliability 0.5 without confidence level. Other researchers also studied the reliability of composite materials using different distributions. However, as the authors know, the most of current studies on the reliability fatigue life just involve the reliability. Few investigations have include both reliability and confidence level to predict the reliability life.

In this article, the static and tension-tension (T-T) fatigue experiments were conducted on the T800/MTM46 cross-ply composite laminates. The fatigue performances were investigated. Fatigue life data were described by both logarithm-normal distribution and 3P Weibull distribution, and then, the better distribution was chosen. Based on the chosen distribution, the reliability fatigue life incorporating both reliability and confidence level was calculated.

**Experiment**

**Materials**

Specimens are made of carbon/epoxy composite T800/MTM46 unidirectional (UD) prepreg of thickness 0.125 mm. The materials are cured in 130°C for 20 min and then in 180°C for 90 min in an autoclave facility. Material properties of the prepreg are listed in Table 1. The static and fatigue specimens are designed by ASTM D3039/D3039M, with the nominal dimensions of 250 mm (length) × 25 mm (width) × 2.5 mm (thickness). Ply sequences are cross-ply of [90/0]. Both edges of all specimens are wet sanded with ordinal 180, 320, and 600 grit to improve edge quality. All specimens are end tabbed with 2-mm-thick glass fiber-reinforced polymer to protect the specimen surface from the jaws of test machine, leaving a gauge length of 150 mm.

**Experimental method**

Static tension experiments were conducted according to ASTM D3039/D3039M on a MTS 810 servo-hydraulic test machine, with a constant loading rate of 1 mm/min. An average of 10 MPa was typically used for grip pressure applied at the end-tabs. During the experiment, an extensometer with 30 mm gage length was used to monitor the strain. A total of three specimens were involved in the static experiments.

**Table 1.** Material properties of T800/MTM46 (in MPa).

| \( E_{11} \) | \( E_{22} \) | \( E_{33} \) | \( G_{12} \) | \( G_{13} \) |
|---|---|---|---|---|
| 174,230 | 10,300 | 10,300 | 4960 | 4960 |
| \( G_{12} \) | \( v_{12} \) | \( v_{13} \) | \( v_{23} \) |
| 3860 | 0.31 | 0.40 | 0.31 |
T–T fatigue experiments were conducted under constant amplitude load according to ASTM D3479/D3479M. The fatigue experiment parameters were sinusoidal waveform, 10 Hz frequency, and stress ratio of 0.1. Fatigue loads were applied as percentages of ultimate tensile strength (UTS) determined by static tension experiments. Stress levels were starting from 70% UTS to 95% UTS in step of 5%. Fatigue tests were stopped at specimen failure or when cycles reached $10^6$ cycles. In this article, failure refers to total catastrophic fracture. The fatigue load, where specimens survived $10^6$ cycles, was referred as “fatigue limit” in this article. In each stress level, five specimens were involved. The experimental setups are shown in Figure 1.

**Experimental results and discussion**

**Static experiments**

The stress–strain curves of T800/MTM46 composite laminates are shown in Figure 2. It is obvious that all curves (three curves) show good linearity during the whole loading process. Results of the three specimens are very repeatable, which indicates that the experiments are convincible and reasonable (Table 2). According to the previous researches, for many fiber-reinforced polymer composite laminates whose 0° ply occupies a relatively large proportion, its stress–strain behavior can exhibit total linearity in the whole loading history. The average UTS is 1210.04 MPa with the $C_v$ (coefficient of variation, standard deviation/average value ratio, representing the dispersion of data) of 0.01362. The modulus $E$ in Table 2 is the tangent modulus in axial direction and determined based on the ratios of the UTS and ultimate strain. The average modulus $E$ and ultimate strain are 85.94 GPa and 0.01408, respectively, with $C_v$ of 0.01055 and 0.00325. Thus, the dispersion of static data is very small. The small dispersion of UTS may be caused by natural dispersion of fabrics and fiber mechanical properties.

Fracture modes of static specimens are shown in Figure 4(a). An obvious brittle fracture occurs around the middle section. The fracture part goes along the transverse direction (vertical to the loading direction) and exhibits linear type, which is located in the very limited region in the middle section compared to the whole scale gauge length of specimens. Lot of 0° fiber breakage can be found around the fracture part. However, no obvious delamination appears. Similar phenomenon is also found by F Ahamed et al. and S Marouani et al.

**Tension–tension fatigue experiments**

Fatigue life data are listed in Table 3, which show a relatively low scattering compared to the fatigue testing
results in Wang et al.\textsuperscript{39} As mentioned in previous researches about metallic materials, the fatigue life data under low stress level tend to show large scatter and that under high stress level has small scatter.\textsuperscript{40,41}

However, the \(Cv\) of fatigue life data under 95\%–75\% UTS is 0.6, 0.6, 0.72, 0.74, and 0.54. Thus, there is no clear trend in scatter of fatigue life data for T800/MTM46 composite laminates, which is far different from the fatigue life data of traditional metallic materials. Due to too much run-out fatigue life data under 70\% UTS, \(S-N\) curve was graphed without the 70\% UTS data in Figure 3. \(y\) axis is shown by the normalized stress level \(\left(\frac{s_{\text{max}}}{s_{\text{ult}}}\right)\) and \(x\) axis is the log-fatigue life. The data with arrows (run-out) stand for the interrupted experiments. Fitting function is \(y = 1.1677 - 0.0704x\) using the least square method. According to the survival cycle (10\(^6\) cycles) defined in this article, the fatigue limit is about 74.5\% UTS. When applied the stress level under 74.5\% UTS, the fatigue life can be considered as the infinite.

The fracture modes of fatigue specimens under various stress level (75\%–95\% UTS) are shown in Table 3.

### Table 3. Tension–tension fatigue life data.

| Stress level | Specimen no. | Cycles (N) | Average no. | Cv | Stress level | Specimen no. | Cycles (N) | Average no. | Cv |
|--------------|--------------|------------|-------------|----|--------------|--------------|------------|-------------|----|
| 95\% UTS     | 1            | 1694       | 1545        | 0.60| 90\% UTS     | 1            | 3048       | 4215        | 0.60|
|              | 2            | 1117       |             |     |              | 2            | 2016       |             |     |
|              | 3            | 1078       |             |     |              | 3            | 2940       |             |     |
|              | 4            | 739        |             |     |              | 4            | 8448       |             |     |
|              | 5            | 3097       |             |     |              | 5            | 4622       |             |     |
| 85\% UTS     | 1            | 76,466     | 36,169      | 0.72| 80\% UTS     | 1            | 413,439    | 273,769     | 0.74|
|              | 2            | 35,589     |             |     |              | 2            | 183,822    |             |     |
|              | 3            | 17,194     |             |     |              | 3            | 129,747    |             |     |
|              | 4            | 10,194     |             |     |              | 4            | 85,437     |             |     |
|              | 5            | 41,401     |             |     |              | 5            | 556,402    |             |     |
| 75\% UTS     | 1            | 720,211    | 554,610     | 0.54| 70\% UTS     | 1            | Run-out    | Run-out     |     |
|              | 2            | 468,521    |             |     |              | 2            | 1,000,000  |             |     |
|              | 3            | 375,210    |             |     |              | 3            | 1,000,000  |             |     |
|              | 4            | 221,915    |             |     |              | 4            | 955,526    |             |     |
|              | 5            | 987,194    |             |     |              | 5            | 1,000,000  |             |     |

UTS: ultimate tensile strength.

Figure 3. \(S-N\) curve.

Figure 4. Fracture modes of (a) static specimen, (b) run-out specimen, (c) 95\% UTS, (d) 90\% UTS, (e) 85\% UTS, (f) 80\% UTS, and (g) 75\% UTS in top view and (h) 85\% UTS and (i) 75\% UTS in edge view.
Figure 4(c)–(g). Ultimate failure of fatigue specimens is catastrophic and engender an explosive sound. The main fracture modes contain much 0° fibers breakage and extensive delamination. Especially, delamination is very dominating, which is far different from brittle fracture of static ones. Additionally, various-degree pull-out fibers and missing/fall-out matrix can be found during and after the experiments. This phenomenon indicates that much damage occur in fiber/matrix interface. So, the fatigue load applied on the composites can lead more matrix-dominated damage, such as delamination and fiber/matrix interface failure. However, the degree of delamination under different stress level tends to show large discrepancy. Fatigue specimens under low stress level tend to have more serious delamination. In Figure 4(c)–(g), the fatigue specimens under 95% and 90% UTS have relatively slight delamination. However, the delamination of fatigue specimens under 80% and 75% UTS is more extensive and catastrophic. Especially, the 75% UTS specimens tend to be the most serious ones. Obviously, the most extensive pull-out fibers and missing/fall-out matrix can be also observed in failure specimens of 75% UTS (Figure 4(i), in edge view), which is more obvious by comparing with specimens under 85% UTS (Figure 4(h), in edge view). Usually, more mechanical energy by test machine is imposed on the fatigue specimens under low stress level, which may contribute to this phenomenon.

Run-out specimens were also tested to evaluate residual tensile strength. The stress–strain curve of one run-out specimen is shown in Figure 2, which also shows obvious linearity through the whole loading process. (Because of the very repeating results, only one run-out specimen is shown in Figure 2. In total, three run-out specimens are tested.) The residual strength results of run-out specimens are listed in Table 4. UTS of the run-out specimens has no obvious change compared to non-cycle ones. However, the average modulus of run-out specimens descends about 14.03%. The fracture modes of run-out specimens are shown in Figure 4(b), which is different from the non-cycle specimen (Figure 4(a)). Many pull-out bundles of fibers and loose matrix sites can be observed. Additionally, obvious delamination also appears. Thus, although the run-out specimens survived 10⁶ cycles, much damage in interface has been accumulated during the fatigue, which leads to the obvious delamination in failure modes.

As mentioned before, the modulus or stiffness degradation often occurs during fatigue loading, which can show the progressive damage accumulation. So, the stiffness degradation under various stress level was monitored during fatigue loading in this article. The stiffness was calculated using equation (1)\(^\text{12}\)

\[
\frac{E_N}{E_0} = \left[ \frac{(\Delta h)^N}{(\Delta h)^{N-1}} \right]^{\frac{1}{N-1}}
\]

\(E_N\) is the stiffness at \(N\) cycle, \(E_0\) is the initial stiffness, \(\Delta h = h_{\text{max}} - h_{\text{min}}\), and \(\Delta h = h_{\text{max}} - h_{\text{min}}\). The stiffness degradation curves under various stress level are shown in Figure 5, which are drawn by normalized stiffness versus normalized fatigue life. In previous researches, the stiffness degradation is often characterized by three stages.\(^\text{1,19,43}\) Stage I is about 0%–10% fatigue life, in which the stiffness decreases very quickly. In stage II (about 10%–80% fatigue life), the stiffness tends to decrease at a slow constant rate. In stage III (after 80% fatigue life), stiffness tends to have a sharp decline. However, the stiffness degradation curves in this article show some discrepancy in the results of previous researches. In Figure 5, the stiffness decreases quickly at the beginning of fatigue life in about 0%–7% fatigue life (stage I), which is similar to stage I in Sneha and Scott.\(^\text{1}\) In this stage, many transverse matrix cracks and intra-ply delamination occurred. It should be noted that a transitory stable platform of curves occurs in about 7%–11% fatigue life after stage I, in which the stiffness tends to have no variations (stage II). The

| Specimen no. | UTS (MPa) | \(E\) (GPa) | Ultimate strain (mm/mm) |
|--------------|-----------|-------------|------------------------|
| 1            | 1200.03   | 73.88       | 0.01624                |
| 2            | 1224.11   | 73.89       | 0.16565                |
| 3            | 1188.00   | 73.89       | 0.01608                |

UTS: ultimate tensile strength.
possible reason for the existence of stage II may be a short-time stable situation in damage generation. In this transitory stable situation, the damage tends to have no obvious development, which leads to no obvious decrease in the stiffness. Then, the stiffness decreases at a quasi-constant slow rate during about 10%–82% fatigue life (stage III). Because the stage III occupied a major part of the fatigue life, many types of damages such as matrix cracks, inter/intra-ply delamination, and pull-out fibers have accumulated seriously. Stiffness has a sharp drop after about 82% fatigue life till to the ultimate failure (stage IV). In this stage, except for the damage mentioned above, fiber breakages have been observed, which might cause the sharp drop of stiffness. Thus, the stiffness degradation of T800/MTM46 composite laminates tends to be characterized by four stages. The stiffness degradation under 95% UTS–75% UTS is about 5.2%–9.6%. Moreover, the stiffness tends to decrease more seriously under lower stress level.

The failure of composite under fatigue load is very complex, which may result in many types of damage. Thus, the edge views of specimens were observed during fatigue loading. In this article, the edge view under 85% UTS (No. 2 specimen; \(N_f = 35,589\) cycles) is presented in Figure 6 to verify the damage evolution during the fatigue loading. Before the experiments, no obvious damage can be found (Figure 6(a)). However, various types of damage appear successively with the increase in fatigue cycles. In Figure 6(b) \((N_d = 3000\) cycles), intra-ply delamination and transverse cracks appear in the exterior 90° plies (especially surface 90° plies). On the contrary, the interior plies do not show obvious damage. When the cycles reach 10,000 (Figure 6(c)), clear inter-ply delamination can be observed between the exterior 90° and 0° plies. Additionally, transverse matrix cracks come into being in the exterior 90° plies. No obvious damage appears in the interior plies. In Figure 6(b) and (c), the matrix cracks form along the fiber–matrix interface, propagate in through-thickness direction, and terminate at the adjacent plies (another interface). In Figure 6(d) \((N_d = 15,000\) cycles), the interior plies begin to show damage such as intra-ply delamination in 0° plies, inter-ply delamination, and obvious pull-out fibers. At the same time, the damage in exterior plies develops more seriously. In Figure 6(e) \((N_c = 30,000\) cycles), a large number of fiber breakage and pull-out fibers can be found in interior 0° plies. Obvious matrix appears in interior 90° plies. It is interesting to note that \(N_c/N_f = 84.2\%\), around which the stiffness tends to decrease sharply in stage IV (Figure 5). Thus, the initiation of plenty of 0° fiber breakages and pull-out fibers in interior plies may contribute to the sharp decline in stiffness around 82% fatigue life and then lead to the ultimate failure.

The damage initiates in the exterior plies especially in the exterior 90° plies. Matrix cracks and intra-ply delamination appear first; 90° plies are matrix-dominated, which contribute to the easy initiation of damage such as matrix cracks and intra-ply delamination. With the increase in fatigue cycles, other types of damage appear successively such as inter-ply delamination, fiber breakages, and pull-out fibers both in exterior and interior plies. According to the observation, the damage in interior plies appears only when the damage in exterior plies reaches a very serious level or the exterior plies tend to hold no more damage. Once large amount of damage generate in interior plies, the stiffness may begin to decrease at high rate, which is the signal of ultimate failure of specimens.

**Distribution determination of fatigue life data**

In this article, logarithm-normal distribution and 3P Weibull distribution were both used to fit the fatigue life data. The better fitting distribution was determined according to the correlation coefficient \(r\).

**Distribution of fatigue life data**

**Logarithm-normal distribution.** Probability density function (PDF) and cumulative distribution function (CDF) of logarithm-normal distribution are shown in equations (2) and (3)

\[
f(lgx) = \frac{1}{\sqrt{2\pi\sigma}} \exp \left[ -\frac{(lgx - \mu)^2}{2\sigma^2} \right] \tag{2}
\]

\[
F(lgx) = \int_{-\infty}^{lgx} f(lgx) \, dlgx = \Phi \left( \frac{lgx - \mu}{\sigma} \right) \tag{3}
\]

where \(lgx\) is the random variable, \(\mu\) is the mathematical expectation, and \(\sigma\) is the standard deviation.

Equation (2) can be converted into equation (4)

\[
\Phi^{-1}[F(lgx)] = -\frac{\mu}{\sigma} + \frac{1}{\sigma}lgx \tag{4}
\]

By setting \(X = lgx, Y = \Phi^{-1}[F(lgx)], A = - (\mu/\sigma),\) and \(B = 1/\sigma,\) the standard linear regression equation can be obtained as equation (5)

\[
Y = A + BX \tag{5}
\]

The evaluating values of \(\mu, \sigma\) are calculated as shown in Table 5.

**3P Weibull distribution.** PDF and CDF of 3P Weibull distribution are shown in equations (6) and (7)
Figure 6. Edge views of fatigued specimens under 85% UTS at (a) 0 cycles, (b) $N = 3000$ cycles, (c) 10,000 cycles, (d) 15,000 cycles, and (e) 30,000 cycles.

Table 5. The evaluating value of $\mu$, $\sigma$, $\alpha$, $\beta$, $b$, and $r$ for log-normal and 3P Weibull distribution.

| Stress level | Log-normal | 3P Weibull |
|--------------|------------|------------|
|              | $\mu$     | $\sigma$   | $r$   | $\alpha$ | $\beta$ | $b$ | $r$   |
| 95% UTS      | 3.1338    | 0.3061     | 0.9246 | 4.9783e2 | 1.6597e3 | 1.0002 | 0.9149 |
| 90% UTS      | 3.5697    | 0.3061     | 0.9237 | 1.3385e3 | 4.5284e4 | 1.0112 | 0.9150 |
| 85% UTS      | 4.4591    | 0.4508     | 0.9685 | 1.5230e3 | 4.2895e4 | 1.0811 | 0.9653 |
| 80% UTS      | 5.3342    | 0.4476     | 0.9568 | 1.7532e4 | 3.2092e5 | 1.0021 | 0.9159 |
| 75% UTS      | 5.6693    | 0.3340     | 0.9869 | 9.1098e4 | 6.4988e5 | 1.1908 | 0.9837 |

UTS: ultimate tensile strength; 3P: three parameter.
where $x$ is the random variable, $\alpha$ is the lower limit parameter of random variable, and $b$ is the shape parameter. $\beta$ is the characteristic parameter which is independent of other constant parameters because $F(\beta) = 1 - e^{-1} \approx 63.2$.

Equation (7) can be converted into equation (8)

$$\left( \frac{x - \alpha}{\beta - \alpha} \right)^b = \ln \left( \frac{1}{1 - F(x)} \right)$$

Taking logarithm of both sides of equation (7), it can be rewritten as equation (9)

$$\ln \left( \frac{1}{1 - F(x)} \right) = -b \ln(\beta - \alpha) + b \ln(x - \alpha)$$

Setting $X = \ln(x - \alpha)$, $Y = \ln[n/1 - F(x)]$, $A = -b \ln(\beta - \alpha)$, and $B = b$, standard linear regression equation can be obtained as equation (10)

$$Y = A + BX$$

The empirical frequency function is defined as equation (11)

$$F(x_i) = \frac{i}{n + 1}$$

where $n$ is the sample size and $i$ is the serial number of fatigue life data in the ascending order of the minimum to maximum. The values of $\alpha$, $\beta$, and $b$ are calculated as shown in Table 5.

Statistical test

To determine which distribution, logarithm-normal distribution or 3P Weibull distribution, is more precise to fit the fatigue life data, the correlation coefficient $r$ is applied. $r$ is defined as equation (12)

$$r = \frac{L_{XY}}{\sqrt{L_{XX}L_{YY}}}$$

where

$$L_{XX} = \sum_{i=1}^{n} (X_i - \bar{X})^2$$

$$L_{YY} = \sum_{i=1}^{n} (Y_i - \bar{Y})^2$$

Equation $|r| \leq 1$ is always a true statement. A good correlation can be affirmed between variable $X$ and $Y$ if $|r|$ is close to 1. So, the more precise distribution can be determined by comparing $r$ value. The bigger the value of $r$, the better the fitting. Usually, larger sample size numbers mean more precise fitting results. However, there are only five specimens in each stress level due to the high cost of this type of material. Thus, it is indispensable to find a threshold value for $r$ under different sample sizes to determine whether the distribution can describe the fatigue life data properly. The threshold value $r_a$ can be determined by equation (16)

$$t(n - 2) = r(\sqrt{n - 2}/\sqrt{1 - r^2})$$

where $t(n - 2)$ is the Student’s distribution with $n - 2$ degree of freedoms.

Based on equation (16), the threshold value $r_a$ can be obtained as equation (17) at a given confidence level $\alpha$. The statistical test will be successful if the calculated $r$ is bigger than the corresponding threshold value $r_a$, which means the fitting of fatigue life data is precise and acceptable. $r_a$ is 0.877 with sample size of 5 and confidence level of 0.95

$$r_a = \frac{t_a(n - 2)}{\sqrt{(n - 2) + t_a^2(n - 2)}}$$

The calculated correlation coefficient $r$ of logarithm-normal distribution and 3P Weibull distribution is listed in Table 5. It is obvious that $r$ of the two distribution at each stress level is larger than the threshold value 0.878 (at the sample size of $n = 5$). So, fatigue life data of T800/MTM46 materials obey to both logarithm-normal distribution and 3P Weibull distribution. However, the $r$ of logarithm-normal distribution is larger than that of 3P Weibull distribution at each stress level, which indicates that the logarithm-normal distribution is better to describe the fatigue life data of T800/MTM46 materials. Thus, the reliability fatigue life was analyzed later using logarithm-normal distribution.

Reliability fatigue life

Single-side allowance factor

In many fatigue experiments on composite materials, the numbers of specimens are limited because of high cost. Thus, it is necessary to study the reliability fatigue life. However, as mentioned above, almost all the current researches fail to incorporate both the reliability and confidence level to predict the reliability fatigue life. Thus, single-side allowance factor was used to solve this problem based on logarithm-normal distribution.

$$L_{XY} = \sum_{i=1}^{n} (X_i - \bar{X})(Y_i - \bar{Y})$$
distribution. Reliability fatigue life involving reliability and confidence level can be calculated using single-side allowance.

In previous analysis, the logarithm-normal distribution can describe the fatigue life data more accurately. Thus, the logarithm reliability life \( x_p \) with reliability \( p \) can be presented as equation (18)

\[
x_p = \mu + u_p \sigma
\]

(18)

\( \mu \) is the mathematical expectation, \( u_p \) is the upper-side factor of standard normal distribution, and \( \sigma \) is the standard deviation.

Due to the uncertainty of \( \mu \), the logarithm reliability life \( x_p \) can be expressed as equation (19)

\[
\hat{x}_p = \bar{x} + \mu_p \hat{\sigma} = \bar{x} + \mu_p \hat{k}s_x
\]

(19)

\( \bar{x} \) is the average value of logarithm fatigue life data according to the sample, \( \hat{\sigma} \) is the estimated value of \( \sigma \), \( s_x \) is the standard deviation of the sample, and \( \hat{k} \) is the correction factor of \( \hat{\sigma} \).

\( \hat{k} \) can be presented as equation (20)

\[
\hat{k} = \sqrt{\frac{n - 1}{n - 2}} \frac{\Gamma\left(\frac{n - 1}{2}\right)}{\Gamma\left(\frac{n}{2}\right)}
\]

(20)

\( n \) is the sample size and \( \Gamma \) is the gamma function.

\( \mu_p \) in equation (19) can be replaced by the single-side allowance factor \( k \). The probability that \( \bar{x} + \mu_p \hat{k}s_x \) is smaller than \( \mu + u_p \sigma \) is \( \gamma \) (shown in Figure 7).

Thus

\[
P\left(\bar{x} + k\hat{s}_x < \mu + u_p \sigma\right) = \gamma
\]

(21)

Random variable \( \xi \) can be expressed as

\[
\xi = \bar{x} + k\hat{s}_x
\]

(22)

Assuming \( \xi \) to obey the normal distribution, equation (18) can be expressed as equation (22) according to Figure 7

\[
\mu + u_p \sigma = E(\xi) + \mu_r \sqrt{Var(\xi)}
\]

(23)

According to the statistical theory, \( E(\xi) \) can be calculated by equation (24)

\[
E(\xi) = E(\bar{x}) + kE(\hat{s}_x)
\]

(24)

It is obvious that \( E(\bar{x}) = \mu, E(\hat{k}s_x) = E(\hat{\sigma}) = \sigma \), so equation (24) can be rewritten as equation (25)

\[
E(\xi) = \mu + k\sigma
\]

(25)

Based on equation (22), the variance \( Var(\xi) \) can be replaced by equation (26)

\[
Var(\xi) = Var(\xi) = Var(\bar{x}) + k^2 \hat{k}^2 Var(s_x)
\]

(26)

Here

\[
Var(\bar{x}) = \frac{\sigma^2}{n}
\]

(27)

\( \sigma^2 \) is defined by equation (28)

\[
Var(s_x) = \frac{\sigma^2}{n - 1 - 2 \left( \frac{\Gamma(\frac{\xi}{2})}{\Gamma(\frac{n-1}{2})} \right)^2}
\]

(28)

Substituting equations (20), (27), and (28) into equation (26), \( Var(\xi) \) can be expressed as equation (29)

\[
Var(\xi) = \frac{\sigma^2}{n} + k^2 \left( \frac{n - 1 - 2 \left( \frac{\Gamma(\frac{\xi}{2})}{\Gamma(\frac{n-1}{2})} \right)^2}{n - 1} \right)
\]

(29)

When \( n \geq 5 \), equation (30) can be obtained

\[
\frac{(n - 1)}{2} \left[ \frac{\Gamma(\frac{n-1}{2})}{\Gamma(\frac{n}{2})} \right]^2 \approx \frac{1}{2(n - 1)} + 1
\]

(30)

Thus, equation (29) can be simplified into equation (31)

\[
Var(\xi) = \frac{\sigma^2}{n} + \frac{k^2}{2(n - 1)}
\]

(31)
Substituting equations (25) and (31) into equation (23), equation (32) can be obtained

\[ \mu + u_p \sigma = \mu + k \sigma + u_r \sigma \sqrt{1 + \frac{k^2}{2(n-1)}} \]  

(32)

Then

\[ u_p = k + u_r \sqrt{\frac{1}{n} + \frac{k^2}{2(n-1)}} \]  

(33)

So, the single-side allowance factor \( k \) can be presented as equation (34)

\[ k = \frac{u_p - \mu \gamma \sqrt{\frac{1}{2} \left[ 1 - \frac{\nu^2}{2(n-1)} \right] + \frac{\nu^2}{2(n-1)}}}{1 - \frac{\nu^2}{2(n-1)}} \]  

(34)

When the reliability \( p \) and confidence level \( \gamma \) are given, the log-reliability fatigue life \( \hat{x}_p \) can be obtained by equation (35)

\[ \hat{x}_p = \bar{x} + k \hat{\sigma} \]  

(35)

Eventually, reliability fatigue life \( N_{p,\gamma} \) with reliability \( p \) and confidence level \( \gamma \) can be calculated by equation (36)

\[ N_{p,\gamma} = 10^{\hat{x}_p} \]  

(36)

The value of \( k \) with various reliability (50%–95%, increasing in step of 5%) and confidence level (50%–95%, increasing in step of 5%) was calculated based on equation (34) shown in Figure 8 (n = 5). It is obvious that single-side allowance factor, value of \( k \), will decrease when the reliability \( p \) and confidence level \( \gamma \) increase. However, the value of \( k \) decreases more quickly when reliability increases to the same degree than confidence level. For instance, \( k \) is 0 (reliability \( p = 50\% \) and confidence level \( \gamma = 50 \)), \( k \) is \(-0.114 \) with reliability \( p = 50\% \) and confidence level \( \gamma = 60 \), and \( k \) is \(-0.253 \) with reliability \( p = 60\% \) and confidence level \( \gamma = 50 \). This result indicates that the reliability imposes more serious effect on the reliability fatigue life than confidence level. In other words, a safer fatigue life will be obtained by increasing the reliability than confidence level to a same degree from a same point.

The value of \( k \) is graphed against the sample size \( n \) under various confidence level (Figure 9) and various reliability (Figure 10). \( k \) increases with the sample increasing at a given confidence level or reliability.
One of the most popular ways to describe the fatigue life is the S-N curve/surface. The changing trend of Npγ with reliability γ = 95% under various confidence level (p = 95%) is calculated in Table 6. Npγ with confidence level γ = 95% and reliability p = 50%, 65%, 80%, and 95% is calculated in Table 6. Npγ with confidence level γ = 95% and reliability p = 50%, 65%, 80%, and 95% is calculated in Table 7. It is obvious that the reliability fatigue life Npγ decreases with reliability or confidence level increases. Especially, Npγ with reliability p = 90% and γ = 50% is smaller than Npγ with reliability p = 90% and γ = 50% if t < s. The changing trend of Npγ is consistent to single-side allowance factor k.

One of the most popular ways to describe the fatigue life data is power function. Thus, the S-N curve is presented as equation (37)

\[ S^m N = C \]  

(except when p = 90% and γ = 50, k remains stable when n changes). This result indicates that a larger reliability fatigue life can be obtained with a larger sample size number when a certain confidence level and reliability are given. Obviously, the value of k increases fast when the sample size increases from 5 to 10. On the contrary, the increasing rate slows down when sample size exceeds 10. So, 10 can be defined as economical sample size number when a certain confidence level and reliability fatigue life can be obtained with a larger sample size in fatigue tests, and it is most economical to set the sample size around 10.

### Table 6. Reliability fatigue life Npγ under various confidence level (p = 95%).

| Stress level | Confidence level 0.5 | Confidence level 0.65 | Confidence level 0.80 | Confidence level 0.95 |
|--------------|-----------------------|------------------------|------------------------|------------------------|
|              | LgNpγ                 | Npγ                    | LgNpγ                 | Npγ                    | LgNpγ                 | Npγ                    | LgNpγ                 | Npγ                    |
| 95% UTS      | 2.7441                | 555                    | 2.6687                | 466                    | 2.5484                | 354                    | 2.1411                | 138                    |
| 90% UTS      | 3.1800                | 1514                   | 3.1047                | 1272                   | 2.9844                | 965                    | 2.5770                | 377                    |
| 85% UTS      | 3.8972                | 7893                   | 3.7885                | 6145                   | 3.6151                | 4122                   | 3.0277                | 1066                   |
| 80% UTS      | 4.7736                | 59,383                 | 4.6652                | 46,264                 | 4.4922                | 31,060                 | 3.9062                | 8058                   |
| 75% UTS      | 5.2752                | 188,457                | 5.1953                | 156,767                | 5.0676                | 116,849                | 4.6355                | 43,198                 |

**UTS**: ultimate tensile strength.

### Table 7. Reliability fatigue life Npγ under various reliability (γ = 95%).

| Stress level | Confidence level 0.95 | Reliability 0.95 | Reliability 0.65 | Reliability 0.80 | Reliability 0.95 |
|--------------|-----------------------|------------------|------------------|------------------|------------------|
|              | LgNpγ                 | Npγ              | LgNpγ            | Npγ              | LgNpγ            | Npγ              |
| 95% UTS      | 2.9196                | 831              | 2.8225           | 665              | 2.5558           | 360              | 2.1411              | 138                    |
| 90% UTS      | 3.3555                | 2267             | 3.2583           | 1813             | 2.9917           | 981              | 2.5770              | 377                    |
| 85% UTS      | 4.1502                | 14,132           | 4.0102           | 10,238           | 3.6257           | 4223             | 3.0277              | 1066                   |
| 80% UTS      | 5.0260                | 106,180          | 4.8864           | 76,982           | 4.5027           | 31,824           | 3.9062              | 8058                   |
| 75% UTS      | 5.4614                | 289,302          | 5.3583           | 228,220          | 5.0754           | 116,849          | 4.6355              | 43,198                 |

**UTS**: ultimate tensile strength.

S is the stress level, N is the fatigue life, and m and C are the constants.

Replace N by Npγ, thus the p-γ-S-N can be obtained as equation (38)

\[ S^m N_p\gamma = C_{p\gamma} \]  

Npγ is the reliability fatigue life with reliability p and confidence level γ, and mγ and Cγ are the constants.

Taking logarithm on both side of equation (38), equation (39) can be obtained

\[ \log(S) = \frac{\log(C_{p\gamma})}{m_{p\gamma}} - \left(\frac{1}{m_{p\gamma}}\right) \log(N_{p\gamma}) \]

Through setting \( y = \log(S), x = \log(N_{p\gamma}), Q = \log(C_{p\gamma})/m_{p\gamma}, \) and \( W = -1/m_{p\gamma} \), equation (39) can be simplified to equation (40)

\[ y = Q + Wx \]

Thus, the p-γ-S-N curves under various confidence level (reliability p = 95%) are graphed in Figure 11 using the fatigue life data in Table 6. The correlation coefficient r for γ = 50%, 65%, 80%, and 95% is 0.9861, 0.9877, 0.9891, and 0.9812, respectively, which indicates that the fitting is very precise. The reliability fatigue life Npγ increases rapidly from γ = 95%–80%.
tends to slow down gradually. Thus, with a certain reliability, \( N_{p,\gamma} \) will decrease rapidly when a high confidence level is satisfied. The \( p-\gamma \)-S-N curves under various reliability (confidence level \( \gamma = 95\% \)) are graphed in Figure 12 using the fatigue life data in Table 7. The correlation coefficient \( r \) for \( \gamma = 50\%, 65\%, 80\%, \) and \( 95\% \) is 0.9800, 0.9838, 0.9891, and 0.9812, respectively, which also shows a good agreement between the fatigue life data and fitting lines. The reliability fatigue life \( N_{p,\gamma} \) increases relatively rapidly from \( p = 95\% \) to 65\%, especially from \( p = 95\% \) to 80\%. However, the increasing rate is relatively slow between 65\% and 50\%. Thus, with a certain confidence level, \( N_{p,\gamma} \) will decrease rapidly when a high reliability is satisfied.

Actually, the S-N curves in Figure 3 are obtained under reliability \( p = 50\% \) and \( \gamma = 50\% \), which is a special case in \( p-\gamma \)-S-N curves (the dashed line in Figures 11 and 12). Obviously, the \( p-\gamma \)-S-N curves predicted in this article are all located on the left side of S-N curve, which also indicate that the reliability fatigue life decreases when the reliability and confidence level increase. So, we can get a safer prediction of fatigue life using \( p-\gamma \)-S-N curve (\( p \geq 50\% \) and \( \gamma \geq 50\% \)) than conventional S-N curve.

To predict the whole scale \( N_{p,\gamma} \), the \( p-\gamma \)-S-N surface under various stress level (sample size \( n = 5 \)) is drawn from reliability 50\% to 99\% and confidence level 50\% to 99\% in Figure 13. Under high reliability and confidence level, \( N_{p,\gamma} \) descends sharply in each stress level. It is obvious that the low stress level leads to the high reliability fatigue life. According to the \( p-\gamma \)-S-N surface, \( N_{p,\gamma} \) under arbitrary reliability and confidence level can be obtained.

**Conclusion**

T–T fatigue behaviors of T800/MTM46 composite laminates and its reliability fatigue life involving both reliability and confidence level were studied in this article. The major results and conclusions are summarized as follows:

1. Various degree of delamination is the most notable failure modes of fatigued specimens, which is far different from the brittle fracture of static specimens. Additionally, the fatigued failure specimens under low stress level (75\% and 80\%UTS) have more serious delamination (through the whole gauge length) than those under high stress level (90\% and 95\% UTS).

2. Average UTS of the run-out specimens do not show obvious change compared to that of non-cycle specimens. However, the modulus has a 14.03\% decline. The stress–strain curves of run-out specimens still show clear linearity. Scatter of fatigue life data under various stress level does not show clear trend, which differs with
the traditional metallic materials. The fatigue limit is about 74.5% UTS.

3. Stiffness degradation is presented by four stages. In stage I (about 0%–7% fatigue life), the stiffness decreases quickly. Then, a notable stable platform follows behind, which is stage II (about 7%–11% fatigue life). After that, the stiffness decreases at a quasi-constant low rate in a time-consuming manner, which is stage III (about 11%–82% fatigue life). In stage IV (about 82%–100% fatigue life), the stiffness tends to have a sharp drop, which is the signal of ultimate failure.

4. Matrix-dominated damage including the matrix cracks and intra-ply delamination initiate first in exterior plies, especially in exterior 90° plies. Then, the interior plies generate the damage only when the exterior plies have very serious damage and tend to have no ability to carry more damage. Lot of 0° fiber breakages and pull-out fibers in interior piles in the later period of fatigue life may contribute to the sharp decline of stiffness in stage IV.

5. Log-normal and 3P Weibull distribution both can describe the fatigue data well but the log-normal distribution is the better one. Reliability fatigue life $N_{p,γ}$ incorporating both reliability and confidence level was calculated using single-side allowance factor based on log-normal distribution. The most economical sample size number is around 10. With a certain confidence level/reliability, $N_{p,γ}$ will decrease rapidly when a high reliability/confidence level is satisfied.

6. $p$-$γ$-S-N curve is proposed using power function to predict $N_{p,γ}$. To predict the whole scale $N_{p,γ}$, $p$-$γ$-S-N surface is graphed. The results show that $N_{p,γ}$ decreases with the increase in reliability/confidence level. Reliability imposes more notable influence on $N_{p,γ}$ than confidence level, comparing with a same point.

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