The $g_2$ Structure Function

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Abstract. Polarized structure functions at low $Q^2$ have the physical interpretation of (generalized) spin polarizabilities. At high $Q^2$, the polarized parton distribution $g_2(x)$ provides access to quark-gluon correlations in the nucleon. We discuss the interpretation of the $x^2$ moment of $g_2(x)$ as an average transverse force on quarks in deep-inelastic scattering from a transversely polarized target. Qualitative connections with generalized parton distributions are emphasized. The $x^2$ moment of the chirally-odd twist-3 parton distribution $e(x)$ provides information on the dependence of the average transverse force on the transversity of the quark.

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INTRODUCTION

The electric polarizability $\alpha$ is the tendency of a charge distribution, such as the electron cloud of an atom or molecule, to be distorted from its normal shape by an external electric field $\vec{E}$ resulting in an electric polarization $\vec{P}$

$$\vec{P} = \alpha \vec{E}. \tag{1}$$

Likewise, the magnetic polarizability $\beta$ describes the response to an external magnetic field. Experimentally, these quantities can be accessed through the low energy Compton scattering amplitude

$$T(v) = \vec{e}'^* \cdot \vec{e} f(v) + i \vec{\sigma} \cdot \left( \vec{e}'^* \times \vec{e} \right) g(v), \tag{2}$$

where $f(v) = -\frac{e^2 \delta}{4\pi M} + (\alpha + \beta) v^2 + \mathcal{O}(v^4)$ and $e_N$ is the nucleon charge. Similarly, $g(v) = -\frac{e^2 \delta}{8\pi M} v + \gamma_0 v^3 + \mathcal{O}(v^5)$ can be related to the (forward) spin polarizability $\gamma_0$.

At nonzero $Q^2$ one can introduce the concept of ‘generalized polarizabilities’ $\alpha \rightarrow \alpha(Q^2)$, which, using dispersion relations [1], can be linked to the parton distributions $g_1 \equiv vG_1/M^2$ and $g_2 \equiv v^2G_2/M^4$ that appear in the polarized double-spin asymmetry in deep-inelastic scattering (DIS)

$$\frac{d^2 \sigma_{+-}}{d\Omega dE'} - \frac{d^2 \sigma_{++}}{d\Omega dE'} = \frac{4\alpha^2 E'}{\pi E Q^2} \left[ M (E + E' \cos \theta) G_1 - Q^2 G_2 \right]. \tag{3}$$

The leading twist PDF $g_1(x) = \sum_q e_q^2 \Delta q(x)$, has, after removing the quark charges the physical interpretation of the number density of quarks carrying momentum fraction $x$
with spin in the same direction as the nucleon spin minus that in the opposite direction
\[ \Delta q(x) = q^\uparrow(x) + q^\downarrow(x) - q^\downarrow(x) - \bar{q}^\downarrow(x). \]  
(4)

While the contribution from \( g_2 \) to the longitudinal double spin asymmetry (3) is suppressed compared to that of \( g_1 \) in the Bjorken limit, their different angular dependence still allows an unambiguous determination of \( g_2 \) from that asymmetry. An even better way to measure \( g_2 \) is to consider the longitudinal (beam) - transverse (target) double spin asymmetry

\[ \sigma_{LT} \propto g_T \equiv g_1 + g_2 \]  
(5)

to which \( g_2 \) contributes multiplied with the same factors as \( g_1 \), i.e. at the same power of \( \nu \). Having determined \( g_1 \) from \( \sigma_{LL} \), \( g_2 \) can then be obtained simply by subtracting the \( g_1 \) contribution from \( \sigma_{LT} \) in the Bjorken limit. This property of the polarized DIS cross section thus allows a clean extraction of higher twist matrix elements, and thus makes polarized deep-inelastic scattering (DIS) a rare opportunity for studying higher twist effects (for an overview, see Ref. [2]).

Since \( g_2(x, Q^2) \) involves higher twist, it does not have a parton interpretation as a single particle density. Indeed, the twist-3 part of \( g_2 \) is related to quark-gluon correlations whose intuitive interpretation may not be immediately clear. Since \( g_2(x, Q^2) \) is related to (electromagnetic) polarizabilities at low \( Q^2 \), these twist-3 matrix elements have been called color polarizabilities in the literature [3]. However, at high \( Q^2 \), the twist-3 piece of \( g_2(x, Q^2) \) is described by a local correlator and the physical interpretation as a polarizability no longer applies. Indeed, while nucleons need to be polarized in order to study \( g_2(x, Q^2) \), the nucleons are not distorted, but only ‘spin-alligned’. The quark-gluon correlations embodied in the twist-3 part of \( g_2(x, Q^2) \) are then obtained as a matrix elements of a certain operator in a spin-alligned, but undeformed, nucleon. This is very different from the usual use of the term ‘polarizability’ as the tendency of a charge or magnetization distribution to be distorted from its normal shape by an external field. Of course, one could broaden the notion of ‘polarizability’ to encompass matrix elements that are only non-zero when the nucleon is polarized, but within such a broadened definition, other spin-dependent observables, such as the polarized parton distribution \( \Delta q(x) \) or even the magnetic moment of the nucleon, would then also become ‘polarizabilities’ in the broader sense.

The main purpose of this paper is to explore an alternative physical interpretation of these particular twist-3 matrix elements as a force. First we summarize the connection between the \( x^2 \) moment of \( g_2(x, Q^2) \) and quark-gluon correlations. After discussing the connection between these correlations and the transverse force on the active quark in DIS, we then estimate sign and magnitude of that force based on DIS data, lattice calculations and heuristic pictures.
**$x^2$ MOMENTS AND QUARK-GLUON CORRELATIONS**

The chirally even spin-dependent twist-3 parton distribution $g_2(x) = g_T(x) - g_1(x)$ is defined in terms of light-cone correlations as

$$
\int \frac{d\lambda}{2\pi} e^{i\lambda x} \langle PS|\bar{\psi}(0)\gamma^\mu\gamma^5\psi(\lambda n)|Q^2|PS\rangle
\quad = \quad 2 \left[ g_1(x, Q^2) p^\mu (S \cdot n) + g_T(x, Q^2) S^\mu_+ + M^2 g_3(x, Q^2) n^\mu (S \cdot n) \right].
$$

where $p^\mu$ and $n^\mu$ are light-like vectors along the $-$ and $+$ light-cone direction with $p \cdot n = 1$. Using the equations of motion $g_2(x)$ can be expressed as a sum of a piece that is entirely determined in terms of $g_1(x)$ plus an interaction dependent twist-3 part that involves quark gluon correlations [4]

$$
g_2(x) = g_{WW}^2(x) + \bar{g}_2(x) \quad (6)
$$

$$
g_{WW}^2(x) = -g_1(x) + \int_x^1 \frac{dy}{y} g_1(y). \quad (7)
$$

Here we have neglected $m_q$ for simplicity. For example, the $x^2$ moment yields [5, 6]

$$
\int dxx^2 \bar{g}_2(x) = \frac{d_2}{3}
$$

with

$$
g \langle P, S | q(0) G^{+\gamma}(0) \gamma^+ q(0) | P, S \rangle = 2MP^+ P^S x d_2. \quad (8)
$$

In the limit where $Q^2$ is so low that the virtual photon wavelength is larger than the nucleon size, the electro-magnetic field associated with the two virtual photons appearing in the forward Compton amplitude corresponding to the structure function is nearly homogenous across the nucleon and the spin-dependent structure function $g_2(x, Q^2)$ can be related to spin-dependent polarizabilities. In contradistinction, in the Bjorken limit, the matrix elements describing the moments of $g_2(x, Q^2)$ are given by local correlation functions, such as (8). Nevertheless, because of the abovementioned low $Q^2$ interpretation of $g_2$, the local matrix elements appearing in (8)

$$
\chi_E 2M^2 \bar{S} = \langle P, S | q^+ \bar{\alpha} \times \epsilon E q | P, S \rangle \quad \quad \chi_B 2M^2 \bar{S} = \langle P, S | q^+ gBq | P, S \rangle,
$$

where

$$
d_2 = \frac{1}{4} (\chi_E - 2\chi_M), \quad (10)
$$

(note that $\sqrt{2}G^{+\gamma} = B^c - E^\gamma$) are sometimes called color electric and magnetic polarizabilities [3]. In the following we will discuss why, at high $Q^2$, a better interpretation for these matrix elements is that of a color-Lorentz ‘force’. 
In electro-magnetism, the $\hat{y}$-component of the Lorentz force $F^y$ acting on a particle with charge $e$ moving, with (nearly) the speed of light along the $-\hat{z}$ direction, $\vec{v} \approx (0, 0, -1)$, reads

$$F^y = e \left[ \vec{E} + \vec{v} \times \vec{B} \right]^y = e \left( E^y - B^z \right) = -e \sqrt{2} F^{+y}, \quad (11)$$

which involves the same linear combination of Lorentz components that also appears in the gluon field strength tensor in (8). This simple observation already suggests a connection between $d_2$ and the color Lorentz force on a quark that moves (in a DIS experiment) with $\vec{v} \approx (0, 0, -1)$.

In order to explore this connection further we compare the matrix element defining $d_2$ with that describing the average transverse momentum of quarks in semi-inclusive DIS (SIDIS) [7]. The average intrinsic transverse momentum of quarks bound in a nucleon vanishes and therefore any net transverse momentum of quarks in a SIDIS experiment must come from the final state interactions (FSI) [8]. The average transverse momentum of the ejected quark (also averaged over the momentum fraction $x$ carried by the active quark in order to render the matrix element local in the position of the quark field operator) in a SIDIS experiment can thus be represented by the matrix element [9]

$$\langle k^y_\perp \rangle = -\frac{1}{2P^+} \left\langle P, S \left| \bar{q}(0) \int_0^\infty dx^- g G^{+y}(x^+ = 0, x^-) \gamma^+ q(0) \right| P, S \right\rangle, \quad (12)$$

where Wilson-line gauge links along $x^-$ are implicitly understood, but not written out explicitly. One way to derive this expression is to start from gauge invariantly defined quark momentum distributions with Wilson line gauge links extending to light-cone infinity. The integral over the gauge field is then obtained by acting with the transverse derivative (when averaging over $k^y_\perp$) on the gauge field appearing in the Wilson line [10, 11].

The matrix element appearing in (12) thus has a physical interpretation as the transverse impulse obtained by integrating the color Lorentz force along the trajectory of the active quark — which is an almost light-like trajectory along the $-\hat{z}$ direction, with $z = -t$. In order to make the correspondence more explicit, we now rewrite Eq. (12) as an integral over time

$$\langle k^y_\perp \rangle = -\frac{\sqrt{2}}{2P^+} \left\langle P, S \left| \bar{q}(0) \int_0^\infty dt G^{+y}(t, z = -t) \gamma^+ q(0) \right| P, S \right\rangle, \quad (13)$$

in which the physical interpretation of $-\frac{\sqrt{2}}{2P^+} \left\langle P, S \left| \bar{q}(0) G^{+y}(t, z = -t) \gamma^+ q(0) \right| P, S \right\rangle$ as the (ensemble averaged) transverse force acting on the struck quark at time $t$ after being struck by the virtual photon becomes more apparent. In particular,

$$F^y(0) \equiv -\frac{\sqrt{2}}{2P^+} \left\langle P, S \left| \bar{q}(0) G^{+y}(0) \gamma^+ q(0) \right| P, S \right\rangle = -\sqrt{2} M P^+ S^x d_2 = -M^2 d_2, \quad (14)$$

where the last equality holds only in the rest frame ($p^+ = \frac{1}{\sqrt{2}} M$) and for $S^x = 1$, can be interpreted as the averaged transverse force acting on the active quark in the instant right after it has been struck by the virtual photon.
Although the identification of \( \langle p | \bar{q} \gamma^+ G^{\gamma^0} q | p \rangle \) as a color Lorentz force may be intuitively evident after the above discussion, it is also instructive to provide a more formal justification. For this purpose, we consider the time dependence of the transverse momentum of the ‘good’ component of the quark fields (the component relevant for DIS in the Bjorken limit) \( q_+ \equiv \frac{1}{2} \gamma^- \gamma^+ q \)

\[
2p^+ \frac{d}{dt} \langle p^\gamma \rangle \equiv \frac{d}{dt} \langle PS | \bar{q} \gamma^+ (p^\gamma - gA^\gamma) q | PS \rangle = \frac{1}{\sqrt{2}} \frac{d}{dt} \langle PS | q_+^\dagger (p^\gamma - gA^\gamma) q_+ | PS \rangle = 2p^+ \langle PS | [\bar{q} \gamma^+ (p^\gamma - gA^\gamma) q + \bar{q} \gamma^+ (p^\gamma - gA^\gamma) \dot{q} - \bar{q} \gamma^+ gA^\gamma q] | PS \rangle.
\]

Using the QCD equations of motion

\[
\ddot{q} = (igA^0 + \gamma^i \vec{\gamma} \cdot \vec{D}) q,
\]

where \( -iD^\mu = p^\mu - gA^\mu \), yields

\[
2p^+ \frac{d}{dt} \langle p^\gamma \rangle = \langle PS | \bar{q} \gamma^+ g (G^{00} + G^{\gamma^0}) q | PS \rangle + \langle PS | \bar{q} \gamma^+ \gamma^\nu D^\nu \gamma D^\nu \gamma \gamma q | PS \rangle = \sqrt{2} \langle PS | \bar{q} \gamma^+ gG^{\gamma^0} q | PS \rangle + \langle PS | \bar{q} \gamma^+ \gamma^\nu D^\nu \gamma \gamma q | PS \rangle,
\]

where \( \langle PS | \bar{q} \gamma^+ \gamma^- D^\nu D^\nu q | PS \rangle \) symbolically stands for all those terms that involve a product of \( \gamma^+ \gamma^- \) as well as a \( \gamma^\perp \) and that also involve only transverse derivatives \( D^\nu \).

Now it is important to keep in mind that we are not interested in the average force on the ‘original’ quark fields (before the quark is struck by the virtual photon), but after absorbing the virtual photon and moving with (nearly) the speed of light in the \( -\hat{z} \) direction. In this limit, the first term on the r.h.s. of (18) dominates, as it contains the largest number of \( \gamma^+ \gamma^- \) Lorentz indices. Dropping the other terms yields (14).

### OTHER HIGHER TWIST MATRIX ELEMENTS

A measurement of the \( x^2 \)-moment \( f_2 \) of the twist-4 distribution \( g_3(x) \) allows determination of the expectation value of a different linear combination of Lorentz/Dirac components of the quark-gluon correlator appearing in (8) \( \chi_E \)

\[
f_2 M^2 S^\mu = \frac{1}{2} \langle p, S | \bar{q} g G^{\mu\nu} \gamma_5 q | p, S \rangle.
\]

Using rotational invariance, to relate various Lorentz components one thus finds a linear combination of the matrix elements of electric and magnetic quark-gluon correlators \( \chi_E \)

\[
f_2 = \chi_E - \chi_M, \quad (20)
\]

that differs from that in (10). In combination with (8) this allows a decomposition of the force into electric and magnetic components \( F^\gamma = F^\gamma_E + F^\gamma_M \), using

\[
F^\gamma_E(0) = -\frac{M^2}{4} \chi_E \quad \quad F^\gamma_M(0) = -\frac{M^2}{2} \chi_B \quad (21)
\]
for a target nucleon polarized in the \(+\hat{x}\) direction, where \([3, 13]\):

\[
\chi_E = \frac{2}{3} (2d_2 + f_2) \quad \chi_M = \frac{1}{3} (4d_2 - f_2).
\]

(22)

A relation similar to (14) can be derived for the \(x^2\) moment of the twist-3 scalar PDF \(e(x)\). For its interaction dependent twist-3 part \(\bar{e}(x)\) one finds for an unpolarized target

\[
4M P^+ P^+ e_2 = \sum_{i=1}^{2} g \langle p | \bar{q} \sigma^{+i} G_i q | p \rangle,
\]

(23)

where \(e_2 \equiv \int_0^1 dx x^2 \bar{e}(x)\). The matrix element on the r.h.s. of Eq. (23) can be related to the average transverse force acting on a transversely polarized quark in an unpolarized target right after being struck by the virtual photon. Indeed, for the average transverse momentum in the \(+\hat{y}\) direction, for a quark polarized in the \(+\hat{x}\) direction \((k^2 \perp\) moment of the Boer-Mulders function \(h_{1+}^{\perp}\) integrated also over \(x\)), one finds

\[
\langle k^y \rangle = \frac{1}{4P} \int_0^{\infty} dx^- g \langle p | \bar{q} \sigma^{+y} G^{+y} (x^-) q(0) | p \rangle.
\]

(24)

A comparison with Eq. (23) shows that the average transverse force at \(t = 0\) (right after being struck) on a quark polarized in the \(+\hat{x}\) direction reads

\[
F^y(0) = \frac{1}{2\sqrt{2}p^+} g \langle p | \bar{q} \sigma^{+y} G^{+y} q | p \rangle = \frac{1}{\sqrt{2}} M P^+ S^x e_2 = \frac{M^2}{2} e_2,
\]

(25)

where the last identify holds only in the rest frame of the target nucleon and for \(S^x = 1\). In the physical interpretation of (25) it is important to keep in mind that, for a given flavor, the number of quarks on which the force in (25) is only half that in (14) as only half the quarks in an unpolarized nucleon will be polarized in the \(+\hat{x}\) direction.

**HEURISTIC PICTURES AND NUMERICAL STUDIES**

When the target nucleon is transversely polarized, e.g. in the \(+\hat{x}\) direction the axial symmetry in the transverse plane is broken. In particular, the quark distribution (more precisely the distribution of the \(\gamma^+\)-density that dominates in DIS in the Bjorken limit) in the transverse plane is deformed [16]. The average deformations can be related to the contribution from each quark flavor to the anomalous magnetic moment of the nucleon and was predicted to be quite substantial [16] and has also been observed in lattice QCD [17]. Given the fact that, for a nucleon polarized in the \(+\hat{x}\) direction the \(\gamma^+\)-distribution for \(u\) \((d)\) is shifted towards the \(\pm\hat{y}\) direction suggests that these quarks also ‘feel’ a nonzero color-electric force pointing on average in the \(\mp\hat{y}\) direction, i.e. one would expect that \(d_2\) is positive (negative) for \(u\) \((d)\) quarks. This is also consistent with a negative (positive) sign for the Sivers on the proton function as observed by the HERMES collaboration [18] and the vanishing Sivers function for deuterium in the COMPASS experiment [19]. In fact, in the large \(N_C\) limit, one would expect that \(d_2\) for \(u\)
FIGURE 1. Distribution of the $j^+$ density for $u$ and $d$ quarks in the $\perp$ plane ($x_B j = 0.3$ is fixed) for a nucleon that is polarized in the $x$ direction in the model from Ref. [16]. For other values of $x$ the distortion looks similar.

$(d)$ quarks are equal and opposite. Note that while $d_2$ for $u$ $(d)$ quarks being exactly equal and opposite would imply the same for protons (neutrons), any deviation from being exactly equal and opposite is enhanced for proton (neutron) since there is a significant cancellation between the two quark flavors in the nucleon.

If all spectators in the FSI were to ‘pull’ in the same direction, the force on the active quark would be of the order of the QCD string tension $\sigma \approx (450 \text{MeV})^2$, which would translate into a value $d_2 \sim 0.2$. However, it is more natural to expect a significant cancellation between forces from spectators pulling the active quark in different directions, the actual value of $d_2$ is probably about one order of magnitude smaller, i.e. $d_2 \sim 0.02$ appears to be more natural. Instanton based models have suggested an even smaller value [20].

Heuristic arguments/lattice calculations [21, 17] also suggest that the deformation of (the $\gamma^+$-distribution for) transversely polarized quarks in an unpolarized nucleon is more significant than that of unpolarized quarks in a transversely polarized nucleon. When applied to the final state interactions, this observation suggests $|e_2| > |d_2|$ (the fact that in an unpolarized nucleon only half the quarks are polarized in the $+\hat{x}$-direction is compensated by the factor $\frac{1}{2}$ in (25).

Lattice calculations of the twist-3 matrix element yield [22]

$$d_2^{(u)} = 0.020 \pm 0.024 \quad d_2^{(d)} = -0.011 \pm 0.010$$

renormalized at a scale of $Q^2 = 5 \text{GeV}^2$ for the smallest lattice spacing in Ref. [22]. Note that we have multiplied the numerical results from [22] by a factor of 2 to account for the different convention for $d_2$ being used.

Here the identity $M^2 \approx 5 \text{GeV}/\text{fm}$ is useful to better visualize the magnitude of the force.

$$F_{(u)} = -25 \pm 30 \text{MeV/fm} \quad F_{(d)} = 14 \pm 13 \text{MeV/fm}.$$  (27)

In the chromodynamic lensing picture, one would have expected that $F_{(u)}$ and $F_{(d)}$ are of about the same magnitude and with opposite sign. The same holds in the large $N_C$ limit.
A vanishing Sivers effect for an isoscalar target would be more consistent with equal and opposite average forces. However, since the error bars for $d_2$ include only statistical errors, the lattice result may not be inconsistent with $d_2^{(d)} \sim -d_2^{(u)}$.

The average transverse momentum from the Sivers effect is obtained by integrating the transverse force to infinity (along a light-like trajectory)

$$\langle k_y \rangle = \int_0^\infty dt F_y(t) \quad (13).$$

This motivates us to define an 'effective range'

$$R_{\text{eff}} \equiv \frac{\langle k_y \rangle}{F_y(0)}. \quad (28)$$

Note that $R_{\text{eff}}$ depends on how rapidly the correlations fall off along a light-like direction and it may thus be larger than the (spacelike) radius of a hadron. Of course, unless the functional form of the integrand is known, $R_{\text{eff}}$ cannot really tell us about the range of the FSI, but if the integrand does not oscillate.

Fits of the Sivers function to SIDIS data yield [23] one finds about $|\langle k_y \rangle| \sim 100$ MeV [23]. Together with the (average) value for $|d_2|$ from the lattice this translates into an effective range $R_{\text{eff}}$ of several fm. It would be interesting to compare $R_{\text{eff}}$ for different quark flavors and as a function of $Q^2$, but this requires more precise values for $d_2$ as well as the Sivers function.

Note that a complementary approach to the effective range was chosen in Ref. [24], where the twist-3 matrix element appearing in Eq. (14) was, due to the lack of lattice QCD results, estimated using QCD sum rule techniques. Moreover, the ‘range’ was taken as a model input parameter to estimate the magnitude of the Sivers function.

The impact parameter distribution for quarks polarized in the $+\hat{x}$ direction was found to be shifted in the $+\hat{y}$ direction [25, 17, 21]. Applying the chromodynamic lensing model implies a force in the negative $-\hat{y}$ direction for these quarks and one thus expects $e_2 < 0$ for both $u$ and $d$ quarks. Magnitude: since $\kappa_\perp > \kappa$, expect odd force larger than even force and thus $|e_2| > |d_2|$.

It would be interesting to study not only whether the effective range is flavor dependent, but also whether there is a difference between the chirally even and odd cases. It would also be very interesting to learn more about the time dependence of the FSI by calculating matrix elements of $\bar{q}Y^+ (D^+ G^{+\perp}) q$, or even higher derivatives, in lattice QCD. Knowledge of not only the value of the integrand at the origin, but also its slope and curvature at that point, would be very useful for estimating the integral in Eq. (12).

**DISCUSSION**

The quark-gluon correlations in the $x^2$-moment $d_2$ of the twist-3 polarized PDF $g_2$ can be identified with the transverse component of the color-Lorentz force acting on the struck quark in the instant after absorbing the virtual photon. The direction of the the force for $u$ and $d$ quarks can be understood in terms of the transverse deformation of parton distributions for a transversely polarized target. In combination with a measurement of the $x^2$ moment of the twist-4 polarized PDF $g_3$ one can even decompose this force into color-electric and magnetic components. Although still quite uncertain, first experimental/lattice results suggest values around $25 - 50$ MeV/fm for the net force. This should
be compared with the net transverse momentum due to the Sivers effect which is on the order of 100MeV.

The $x^2$ moment $e_2$ of the chirally odd twist-3 (scalar) PDF $e(x)$ can be related to the transverse force acting on transversely polarized quarks in an unpolarized target. Therefore, $e_2$ is to the Boer-Mulders function $h_{1\perp}$, what is $d_2$ to the Sivers function $f_{1\perp T}$.

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