Analysis and control for matrix rectifier by circuit DQ transformation

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Abstract: An input filter is indispensable for a matrix rectifier (MR), since it can improve the input current quality and reduce the ac supply voltage distortion. However, the characteristics of the input filter reduce the input power factor (IPF). Moreover, the unbalanced ac supply voltage could disturb the dc output voltage. Consequently, this paper provides a novel approach to achieve both tight dc output voltage and unity IPF at the main ac power supply by applying circuit DQ transformation to MR. Analyzing the DQ model of MR, sliding mode control (SMC) based on reaching law is used to achieve tight dc output voltage regulation of MR and PI control is applied to control the q-axis current to be zero. Finally, simulation and experiment results are shown to verify the effectiveness of the control scheme proposed in this paper.

Keywords: matrix rectifier, sliding mode control, DQ transformation, input power factor

Classification: Electronic instrumentation and control

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Introduction

The origins of the Matrix Converter (MC) can be traced to the late 1970s. The three-phase to two-phase ac-dc MC, i.e. matrix rectifier (MR), can be deduced from the conventional three-phase to three-phase MC [1]. MR has several advantages: (1) operation in all four quadrant operation; (2) no need of large value AC capacitor used on the output side; (3) easy safe current commutation; (4) intrinsic buck converter and (5) power factor regulation [2].

However, space vector modulation (SVM), the conventional modulation algorithm for MR, is open-loop control. Thus, applying convention SVM on MR, the dc output of MR is susceptible to voltage disturbance at the power supply [3]. Additionally, input filter is necessary for MR, since it can improve the main input current quality. Unfortunately, the existence of input filter definitely result in a displacement angle between input current and input voltage at the main power supply, which decrease the IPF. Moreover, in the condition of light dc load, IPF degrades tremendously [4]. In [5] the author analyzed power characteristic of MR including output voltage gain, input power factor, active power, reactive power and appearance power, yet its control strategy had not demonstrated. In [6] the author introduced a reduced general direct space vector modulation and this modulation can only achieve both dc output regulation and input power factor compensation in balanced ac supply condition. In order to solve such problems, in [7] the author present a convention sliding mode control to regulate the dc output voltage and input power factor. Although this control strategy can achieve high IPF, the chattering of dc output is inevitable. In [8], the author present a PI controller to improve IPF and dc output voltage in unbalanced ac supply condition. It is true that applying this strategy can achieve the purpose, but it can not obtain a quick response and its dynamic performance is poor.

This paper put forward a novel approach to solve the above-mentioned problems. Firstly, set up the DQ mode and analyze the static characteristics of MR. Secondly, sliding mode control with reaching law and PI control are introduced to achieve tight dc output voltage and unity IPF in the following sections. Finally, simulation and experiment are carried out to validate the effectiveness of the proposed compensation control algorithms.
2 Topology of matrix rectifier and space vector modulation

Fig. 1 shows the topology of MR and this topology consists by 6 part: a three-phase power source, input damping filters, main circuit with six bidirectional switches, an output filter and a resistive load.

In Fig. 1, $u_{sa}$, $u_{sb}$, $u_{sc}$ donate ac supply voltage and $i_{sa}$, $i_{sb}$, $i_{sc}$ donate the ac supply current. $L_i$ donates the inductance of the input filter and $R_i$ donates the its damping resistance in parallel. $C_i$ donates the capacitance of the input filter. $u_{ia}$, $u_{ib}$, $u_{ic}$ donate input voltage of MR and $i_{ia}$, $i_{ib}$, $i_{ic}$ donate the input current of MR. $S_{11}$–$S_{23}$ are the six bidirectional switches. $L_o$ and $C_o$ are the inductance and capacitance of the output filter. $V_{PN}$ is the output voltage of the main circuit and $V_o$ is the output voltage of MR. $R_L$ is the dc load. $I_{dc}$ and $I_o$ are the current of $L_o$ and $R_L$ respectively. SVM is used to provide sinusoidal input currents for the MR. Because SVM must satisfy the following two constrains simultaneously:

1. Existence theorem of matrix converter theories [9, 10];
2. Maximization of the dc output voltage to achieve full use of the input line–line voltages.

To satisfy these constrains above, only nine feasible switching states of the six bidirectional switches exist, and these nine states determine nine vectors which can be divided into two categories: active vectors ($I_1$–$I_6$) and zero vectors ($I_7$–$I_9$). Moreover, the space vector hexagon is separated into six sectors by the active vectors, as shown in Fig. 2. The reference current vector $I_{ref}$ synthesized from two adjacent vectors $I_{1a}$ and $I_{1b}$, and a zero vector, as shown in Fig. 2. $\theta$ is the angle between $I_{ref}$ and its right adjacent active vector $I_{1a}$, and $\theta \in [0, \frac{\pi}{3}]$.

The reference current vector $I_{ref}$ can be calculated as following expression:

$$I_{ref} = \frac{T_a(\theta)}{T_S} I_a + \frac{T_b(\theta)}{T_S} I_b + \frac{T_0(\theta)}{T_S} I_0$$

(1)
where $T_a(\theta)T_b(\theta)$ and $T_0(\theta)$ are the durative times of $I_a$, $I_b$ and $I_0$ in one switching period $T_S$. The duty cycle $d_a(\theta)$, $d_b(\theta)$ and $d_0(\theta)$ can be calculated as following expressions:

$$
\left\{
\begin{array}{l}
    d_a(\theta) = T_a(\theta)/T_S = m \sin\left(\frac{\pi}{3} - \theta\right) \\
    d_b(\theta) = T_b(\theta)/T_S = m \sin(\theta) \\
    d_0(\theta) = T_0(\theta)/T_S = 1 - d_a(\theta) - d_b(\theta)
\end{array}
\right.
$$

(2)

where $m$ is the modulation index, $m \in [0, 1]$. According to the above-mentioned SVM, the modulation matrix can be given as:

$$
\Psi_{rec}^T = m \begin{bmatrix}
    \cos(\omega t + \phi_i) \\
    \cos\left(\omega t + \phi_i - \frac{2\pi}{3}\right) \\
    \cos\left(\omega t + \phi_i + \frac{2\pi}{3}\right)
\end{bmatrix}^T
$$

(3)

The averaged output voltage in a switching period can be calculated as:

$$
V_{PN} = \Psi_{rec}^T \begin{bmatrix}
    u_{ia} \\
    u_{ib} \\
    u_{ic}
\end{bmatrix} = 1.5mV_{im} \cos \phi_i
$$

(4)

where $V_{im}$ is the amplitude of the input phase voltage, $\phi_i$ is the desired input current displacement angle.

### 3 DQ model of MR and its static analysis

Matrix rectifier is a strongly nonlinear, time-varying and coupling system, so its characteristic analysis is difficult to carry out. However, by means of DQ transformation can solve this problem effectively. Assuming the $\phi$ is the phase between d-axis and a-axis. According to the mathematical model of MR in Fig. 1, using circuit DQ transformation, DQ model of MR can be given as following:

![Space vector diagram in SVM and the synthesis of reference current vector.](image)
where \(i_{sd}\) and \(i_{sq}\) are the d-axis and q-axis input current of ac supply. \(u_{id}\) and \(u_{iq}\) are the d-axis and q-axis input voltage of MR.

Fig. 3 and Fig. 4 are the DQ transformed equivalent circuit of MR and its mathematical model respectively.
In steady-state, according to the above-mentioned two figures, the current and voltage of d-axis and q-axis at the main power supply can be expressed as:

\[
\begin{align*}
    u_{td} &= \frac{1}{1 - \omega^2 C_i L_i} \sqrt{\frac{3}{2} V_{im} \cos \varphi} \\
    u_{tq} &= \frac{-1}{1 - \omega^2 C_i L_i} \sqrt{\frac{3}{2} V_{im} \sin \varphi} \\
    i_{sd} &= \frac{3 \omega C_i V_{im} \sin \varphi}{2 (1 - \omega^2 C_i L_i)} + \sqrt{\frac{3}{2} \frac{3 m^2 V_{im} \cos \varphi_1 \cos (\varphi_1 - \varphi)}{2 R_L (1 - \omega^2 C_i L_i)^2}} \\
    i_{sq} &= \frac{3 \omega C_i V_{im} \cos \varphi}{2 (1 - \omega^2 C_i L_i)} + \sqrt{\frac{3}{2} \frac{3 m^2 V_{im} \cos \varphi_1 \sin (\varphi_1 - \varphi)}{2 R_L (1 - \omega^2 C_i L_i)^2}}
\end{align*}
\]

Also the dc output voltage of main circuit \( V_{PN} \) is as:

\[
V_{PN} = \sqrt{\frac{3}{2} m [u_{td} \cos (\varphi_1 - \varphi) - u_{tq} \sin (\varphi_1 - \varphi)]}
\]

Consequently, the voltage gain of MR can be shown as:

\[
G_v = \frac{V_{PN}}{V_{im}} = \frac{1.5 m \cos \varphi_1}{(1 - \omega^2 C_i L_i)}
\]

Moreover, the expressions of input active power and reactive power are as:

\[
P_s = u_{sd} i_{sd} + u_{sq} i_{sq} = \frac{9 t^2 m^2 \cos^2 \varphi_1}{4 R_L (1 - \omega^2 C_i L_i)^2}
\]

\[
Q_s = u_{sq} i_{sd} - u_{sd} i_{sq} = \frac{-3 V_{im}^2}{2 (1 - \omega^2 C_i L_i)} \left[ \omega C_i + \frac{3 m^2 \sin (2 \varphi_1)}{4 R_L (1 - \omega^2 C_i L_i)} \right]
\]

Obviously Eq. (10) explains the relationship between the input reactive power and the parameters of whole circuit including the modulation index \( m \).

4 Compensation algorithms

Eq. (4) explains that \( V_{PN} \) is ripple if ac voltage supply is under unbalanced conditions. Thus, adjusting the value \( m \) can offset the effect of the unbalanced ac supply when \( \varphi_1 \) changes very slightly.

The integral sliding surface can definitely eliminate static system errors and enhance control precision [11]. Thus, it is necessary to apply SMC to compensate the dc output voltage. Fig. 5 shows a compensation diagram of dc output voltage. The sliding surface of \( V_{PN} \) is utilized as:

\[
S = e_v + c \dot{e}_v
\]

Where \( e_v \) is voltage error, and \( e_v = V_{ref} - V_o \). \( V_o \) is the voltage of \( C_o \) and \( c \) is a positive constant. Besides, derivative of sliding surface \( S \) is as:

\[
\dot{S} = \left( -1 + \frac{c}{R_L C_o} \right) \dot{e}_v - \frac{c}{R_L C_o} (V_{PN} - V_o)
\]

When \( c < R_L C_o \) and \( - \frac{c}{R_L C_o} (V_{PN} - V_o) \gg \left| - \frac{c}{R_L C_o} (V_{PN} - V_o) \right| \), Eq. (10) can be simplified as

\[
\dot{S} = - \frac{c}{R_L C_o} (V_{PN} - V_o)
\]
In order to gain excellent dynamic performance and weaken the ripple of dc output, exponent reaching law, a novel approach, is applied on SMC [12, 13]:

\[ \dot{S} = -\varepsilon \text{sgn} S - kS \]  

(14)

Where \( \varepsilon \) and \( k \) are both positive constant. Substituting Eq. (14) into Eq. (13),

\[ -\frac{c}{R_1C_0} (V_{PN} - V_0) = -\varepsilon \text{sgn} S - kS \]  

(15)

Substituting Eq. (4) into Eq. (15), the expression \( m \) can be shown as:

\[
m = \begin{cases} 
\frac{V_o}{1.5V_{im} \cos \varphi_i} - \frac{L_o C_o}{1.5cV_{im} \cos \varphi_i} (\varepsilon + kS) & S > 0 \\
\frac{V_o}{1.5V_{im} \cos \varphi_i} + \frac{L_o C_o}{1.5cV_{im} \cos \varphi_i} (\varepsilon + k|S|) & S < 0 
\end{cases} \]  

(16)

For easy calculation, let \( \varphi = 0 \), which means \( u_{sq} = 0 \). Eq. (10) indicates that input reactive power \( Q_s \) can be controlled to be zero only when both \( u_{sq} = 0 \) and \( i_{sq} = 0 \). Consequently, when \( \varphi = 0, \varphi_i < \pi/12 \) and \( (1 - \omega^2 C_i L_i) \approx 1 \), the expression of \( i_{sq} \) can be simplified as:

\[ i_{sq} = \sqrt[3]{\frac{2}{3}} \left( \omega C_i V_{im} + \frac{3V_{im}}{2R_L} \frac{m^2}{3} \phi_1 \right) \]  

(17)

Let \( i_{sq} = 0 \), the expression of \( \phi_i \) is as:

\[ \phi_i = -\frac{2\omega R_L C_i}{3m^2} \]  

(18)

Thus this paper present a new approach to control input power factor angle. In detail, \( \phi_i = -\frac{2\omega R_L C_i}{3m^2} \) can achieve \( i_{sq} = 0 \), which means that the input reactive power can be controlled to be zero. However, \( R_L \) and \( C_i \) may change to a certain extent, which means setting \( \phi_i = -\frac{2\omega R_L C_i}{3m^2} \) cannot ensure \( i_{sq} = 0 \). Consequently a PI controller as shown in Fig. 6 is necessary.

![Fig. 5. Diagram of dc output voltage compensated by SMC based on reaching law.](image1)

![Fig. 6. Diagram of PI controller for \( i_{sq} \).](image2)
5 Simulation and experimental results

5.1 Simulation and its analysis

A detailed simulation is built in this section to verify the proposed control strategy on MATLAB/SIMULINK. Parameters of simulation are as shown in Table I.

| Parameter                          | Value       |
|------------------------------------|-------------|
| Source voltage and frequency       | 50 V/50 Hz  |
| Inductance of input filter $L_i$   | 2 mH        |
| Capacitor of input filter $C_i$    | 20 µF       |
| Damping resistance of input filter $R_i$ | 15 Ω      |
| Inductance of output filter $L_o$  | 5 mH        |
| Capacitor of output filter $C_o$   | 20 µF       |
| Load resistance $R_L$              | 30 Ω        |

Table I. Parameters of simulation

Three comparative simulation were carried out in this section: 1) three-phase balanced input voltage, using SVM without any compensation algorithm; 2) three-phase balanced input voltage, using SVM with proposed compensation algorithm; 3) three-phase unbalanced input voltage, using SVM with proposed compensation algorithm. The simulation waveform are shown from Fig. 7 to Fig. 9.

Fig. 7 indicates that setting different value of modulation index $m$ can get different tight dc output voltage, when ac supply voltage is balanced. However the IPF phase is not zero. Fig. 8 and Fig. 9 shows that adapt the SVM with proposed

![Fig. 7. Simulation wave of using conventional SVM when the ac supply is balanced.](image1)

![Fig. 8. Simulation wave of SVM compensated by SMC based on reaching law when the ac supply is balanced.](image2)
compensation algorithm can successfully achieve both tight dc output voltage and high IPF no matter when the ac supply voltage is balanced or not.

5.2 Experimental results and its analysis
An experimental prototype is built to further illustrate and validate the proposed control approach. Experiment parameters are the same as in simulations. Three comparative experiment were carried out: 1) using SVM without any compensation algorithm when ac supply is balanced and unbalanced; 2) using SVM with proposed compensation algorithm when ac supply is balanced; 3) using SVM with proposed compensation algorithm when ac supply is unbalanced. Experimental waveform are listed as following:

Fig. 9. Simulation wave of SVM compensated by SMC based on reaching law when the ac supply is unbalanced.

Fig. 10a. Experimental waveform of using conventional SVM when the ac supply is balanced.

Fig. 10b. Experimental waveform of using conventional SVM when the ac supply is unbalanced.
Fig. 11a. Experimental waveform of using SVM compensated by SMC based on reaching law when ac supply is balanced.

Fig. 11b. Experimental waveform of using SVM compensated by conventional SMC when ac supply is balanced.

Fig. 11c. Experimental waveform of using SVM compensated by SMC based on reaching law when ac supply is balanced and \( \varphi_t = -2\omega R_i C_i / 3m^2 \).

Fig. 12. Experimental waveform of using SVM compensated by SMC based on reaching law when ac supply is unbalanced.
Fig. 10a and 10b show that conventional SVM can only achieve tight dc output voltage when the ac supply is balanced. If the ac supply voltage is unbalanced, dc output is ripple tremendously. Meanwhile, the IPF is not high in two above-mentioned waveforms.

Fig. 11a and 11b indicate that using SMC based on reaching law, the dc output voltage is more tight than using conventional SMC. Moreover, Fig. 11a and 11c point that if \( \varphi = -2\omega R_L C_i/3m^2 \) is fixed, settling time is longer.

Fig. 12 point out that even if the ac supply is unbalanced, using SVM compensated by SMC based on reaching law can successfully achieve two goals: tight dc output voltage and high IPF.

6 Conclusion

The novel approach of achieving both tight dc voltage regulation and high IPF by using SVM compensated by the SMC based on reaching law have been theoretically justified and experimentally verified. Its effectiveness has been verified through simulated and experimental results.

The experimental results also demonstrate that the by applying SMC based on reaching law, dynamic performance of controlling ac-dc matrix rectifier is excellent. Comparing to other compensation algorithms, the compensation strategy this paper present is simple and with less calculation. Furthermore, the proposed converter is also capable of wide leading or lagging power factor operation which may be of much relevance in certain application.

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