Surface flux concentrations and spherical $\alpha^2$ dynamo

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ABSTRACT

Context. In the presence of strong density stratification, turbulence can lead to a large-scale instability of a horizontal magnetic field if its strength is in a suitable range (within a few percent of the turbulent equipartition value). This instability is related to a suppression of the turbulent pressure so that the turbulence contribution to the mean magnetic pressure becomes negative. This results in the excitation of a negative effective magnetic pressure instability (NEMPI). This instability has so far only been studied for an imposed magnetic field.

Aims. We want to know how NEMPI works when the mean magnetic field is generated self-consistently by an $\alpha^2$ dynamo, whether it is affected by global spherical geometry, and whether it can influence the properties of the dynamo itself.

Methods. We adopt the mean-field approach which has previously been shown to provide a realistic description of NEMPI in direct numerical simulations. We assume axisymmetry and solve the mean-field equations with the PENCIL CODE for an adiabatic stratification at a total density contrast in the radial direction of $\approx 4$ orders of magnitude.

Results. NEMPI is found to work when the dynamo-generated field is about 4% of the equipartition value, which is achieved through strong $\alpha$ quenching. This instability is excited in the top 5% of the outer radius provided the density contrast across this top layer is at least 10. NEMPI is found to occur at lower latitudes when the mean magnetic field is stronger. For weaker fields, NEMPI can make the dynamo oscillatory with poleward migration.

Conclusions. NEMPI is a viable mechanism for producing magnetic flux concentrations in a strongly stratified spherical shell in which a magnetic field is generated by a strongly quenched $\alpha$ effect dynamo.

Key words. Sun: sunspots – Sun: dynamo – turbulence – magnetohydrodynamics (MHD) – hydrodynamics

1 Introduction

The magnetic field of stars with outer convection zones, including that of the Sun, is believed to be generated by differential rotation and cyclonic convection (see, e.g., Moffatt, 1978; Parker, 1979; Zeldovich et al., 1983; Brandenburg & Subramanian, 2005). The latter leads to an $\alpha$ effect, which refers to an important new term in the averaged (mean-field) induction equation, quantifying the component of the mean electromotive force that is aligned with the mean magnetic field (see, e.g., Steinbeck et al., 1996; Krause & Rädler, 1980; Brandenburg et al., 2013). However, what is actually observed are sunspots and active regions, and the description of such phenomena is not part of conventional mean-field dynamo theory (see, e.g., Priest, 1982; Six, 1989; Ossendrijver, 2003; Cally et al., 2003; Stenflo & Kosovichev, 2012).

Several models have been used to explain the formation of active regions and sunspots in an ad hoc manner. It is then simply assumed that a sunspot emerges when the magnetic field of the dynamo exceeds a certain threshold just above the bottom of the convection zone and for the duration of about a month (Chatterjee et al., 2004). Such models assume the existence of strong magnetic flux tubes at the base of the convection zone. They require magnetic fields with a strength of about 10$^6$ gauss (D’Silva & Choudhuri, 1993; Arlt et al., 2005). However, such strong magnetic fields are difficult to produce by dynamo action in turbulent convection (Guerrero & Käpylä, 2011).

Another possible mechanism for producing magnetic flux concentrations is the negative effective magnetic pressure instability (NEMPI), which can occur in the presence of strong density stratification, i.e., preferentially near the stellar surface, on scales encompassing those of many turbulent eddies. Direct numerical simulations (DNS; see Brandenburg et al., 2011; Kemel et al., 2012), mean-field simulations (MFS; see Brandenburg, Kleeorin, & Rogachevskii, 2010; Brandenburg et al., 2012; Kemel et al., 2012), and earlier analytic studies (Kleeorin et al., 1989, 1990, 1996; Käpylä et al., 2012), and earlier analytic studies (Kleeorin et al., 1989, 1990, 1996; Käpylä et al., 2012) now provide conclusive evidence about the physical reality of NEMPI. However, there remain open questions to be answered before it can be applied to detailed models of active regions and sunspots formation.

In the present paper we take a first step toward combining NEMPI, which is well described using mean-field theory, with the $\alpha$ effect in mean-field dynamos. To study the dependence of NEMPI on the magnetic field strength, we assume that $\alpha$ is quenched. This allows us to change the magnetic field strength by changing the quenching parameter. We employ spherical geometry using axisymmetry, i.e., azimuthal derivatives are neglected. Furthermore, $\alpha$ is a pseudo-scalar that changes sign about the equator, so we assume that $\alpha$ is proportional to $\cos \theta$, where $\theta$ is colatitude (Roberts, 1972). We arrange the quenching of $\alpha$ such that the resulting mean magnetic field is of ap-
propriate strength to allow NEMPI to work. This means that the effective (mean-field) magnetic pressure has locally a negative derivative with respect to increasing normalized field strength \( (\text{Kemel et al., 2012b}) \), so the mean toroidal magnetic field must be less than about 20% of the equipartition field strength.

The choice of using spherical geometry is taken because the dynamo-generated magnetic field depends critically on the geometry. Therefore, to have a more realistic field structure, we felt it profitable to carry out our investigations in spherical geometry. Guided by the insights obtained from such studies, it will in future be easier to design simpler Cartesian models to address specific questions regarding the interaction between NEMPI and MFS of NEMPI. Unlike most of the earlier calculations, we adopt an adiabatic equation of state. This results in a stratification to assess the dependence on numerical resolution, and to look for new effects that are due to spherical geometry. We begin by describing first the basic model.

### 2. The model

The evolution equations for mean velocity \( \mathbf{U} \), mean density \( \rho \), and mean vector potential \( \mathbf{A} \), are

\[
\frac{\partial \mathbf{A}}{\partial t} = \mathbf{U} \times \mathbf{B} + \alpha \mathbf{B} - \eta \mathbf{J},
\]

\[
\frac{\partial \rho}{\partial t} = \nabla \cdot \mathbf{J} + \nabla \left( Q_p \frac{\mathbf{B}^2}{2 \mu_0} \right) - \nu_T \mathbf{Q} - \nabla \mathbf{P},
\]

\[
\frac{\partial \mathbf{J}}{\partial t} = -\mathbf{P} \cdot \nabla \mathbf{U},
\]

where \( D/ Dt = \partial/ \partial t + \mathbf{U} \cdot \nabla \) is the advective derivative, \( \mathbf{P} = \mathbf{T} + \Phi \) is the mean reduced enthalpy with \( \mathbf{T} = c_p T \) being the mean enthalpy, \( T \propto \rho^{4/5} \) the mean temperature, \( \gamma = c_p / c_v \) is the ratio of specific heats at constant pressure and constant density, respectively, \( \Phi \) is the gravitational potential, \( \rho \) is the mean density, \( \eta_T = \eta_t + \eta \) and \( \nu_T = \nu_t + \nu \) are the sums of turbulent and microphysical values of magnetic diffusivity and kinematic viscosities, respectively,

\[
-\mathbf{Q} = \frac{\nabla^2 \mathbf{U}}{2} + 2 \nabla \nabla \ln \rho
\]

is a term appearing in the viscous force and \( \mathbf{S}_{ij} = \frac{1}{2} ( \mathbf{U}_{ij} + \mathbf{U}_{ji} ) - \delta_{ij} \nabla \cdot \mathbf{U} \) is the traceless rate of strain tensor of the mean flow, \( \mathbf{J} = \nabla \times \mathbf{B} / \mu_0 \) is the mean current density, \( \mu_0 \) is the vacuum permeability, and the second term, \( \nabla \left( q_p \mathbf{B}^2 / 2 \mu_0 \right) \), on the right-hand side of Equation (2) determines the turbulent contribution to the mean Lorentz force. Here, \( q_p \) depends on the local field strength (see below). Note that this term enters with a plus sign, so a positive \( q_p \) corresponds to a suppression of the total turbulent pressure. The net effect of the mean field leads to an effective mean magnetic pressure \( p_{\text{eff}} = (1 - q_p) \mathbf{B}^2 / 2 \mu_0 \), which becomes negative for \( q_p > 1 \), which can indeed be the case for magnetic Reynolds numbers well above unity \( (\text{Brandenburg et al., 2012}) \).

Following \( \text{Kemel et al., 2012d} \), the function \( q_p(\beta) \) is approximated by:

\[
q_p(\beta) = \frac{q_{p0}}{1 + \beta^2/\beta_p^2} = \frac{\beta_x^2}{\beta_p^2 + \beta^2},
\]

where \( q_{p0}, \beta_p, \) and \( \beta_x = \beta_p q_{p0}^{1/2} \) are constants, \( \beta = |\mathbf{B}| / B_{\text{eq}} \) is the modulus of the normalized mean magnetic field, and \( B_{\text{eq}} = \sqrt{\mu_0 p_{\text{eq}}} \) is the equipartition field strength.

For NEMPI to work, there has to be a mean horizontal field strength in a suitable range such that the derivative, \( dp_{\text{eff}}/d\beta^2 \), is negative somewhere within the computational domain. Unlike the Cartesian cases investigated in earlier work \( (\text{Brandenburg, Kleerbin, & Rogachevskii, 2010}; \text{Brandenburg et al., 2012}; \text{Kemel et al., 2012d}) \), where it is straightforward to impose a magnetic field in a sphere, it is easier to generate a magnetic field by a mean-field dynamo. In that case, we have to include a term of the form \( \alpha \mathbf{B} \) in the expression for the mean electromotive force [see the second term on the right-hand side of Equation (1)]. When the mean magnetic field is generated by a dynamo, the resulting magnetic field strength depends on the nonlinear suppression of the dynamo. We assume here a simple quenching function for the \( \alpha \) effect, i.e.,

\[
\alpha(\theta, \beta) = \frac{\alpha_0}{1 + Q_\alpha \beta^2},
\]

where \( Q_\alpha \) is a quenching parameter which determines the typical field strength to be of the order of \( Q_\alpha^{1/2} B_{\text{eq}} \). The value of \( Q_\alpha \) must be chosen large enough so that the nonlinear equilibration of the dynamo process results in that \( dp_{\text{eff}}/d\beta \) is indeed negative within the computational domain. In analogy with the \( \beta_p \) parameter in Equation (5) we can define a parameter \( \beta_\alpha = Q_\alpha^{1/2} \), which will be quoted occasionally.

The strength of the dynamo is also determined by the dynamo number,

\[
C_\alpha = \alpha_0 R / \eta_T.
\]

For our geometry with \( 0.7 \leq r/R \leq 1 \), the critical value of \( C_\alpha \) for the onset of dynamo action is around 18. The excitation conditions for dipolar and quadrupolar parities are rather close together. This is because the magnetic field is strongest at high latitudes, so the hemispheric coupling is weak. In the following we restrict ourselves to solutions with dipolar parity. We adopt the value \( C_\alpha = 30 \), so the dynamo is nearly twice supercritical.

As mentioned before, our gravitational potential \( \Phi \) is that of a point mass. We define \( \Phi \) such that it vanishes at a radius \( r_* \), i.e.,

\[
\Phi(r) = -GM \left( \frac{1}{r} - \frac{1}{r_*} \right).
\]

The radial component of the gravitational acceleration is then \( g = GM / r^2 \). We adopt an initially adiabatic stratification with \( c_p T = -\Phi(r) \), so \( T \) vanishes at \( r = r_* \), whose value has to be chosen some distance above \( r = R \). For our reference model

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1. http://pencil-code.googlecode.com
adiabatic stratification with

\[ \frac{\rho}{\rho_0} \]

where \( \rho/\rho_0 \approx 0.0068 \) and \( H_p(r) = H_p(\rho) \) by definition. The dotted line marks the value of \( \eta_t/\beta_u \text{rms} H_p \).

we use \( r_s/R = 1.001 \). The radius \( r_s \) determines the density contrast. The pressure scale height is given by

\[ H_p(r) = \frac{r(1-r/r_s)}{n+1}, \tag{9} \]

where \( n = 1/(\gamma - 1) = 3/2 \) is the polytropic index for an adiabatic stratification with \( \gamma = 5/3 \). The density scale height is \( H_p = r(1-r/r_s)/n \). Table 1 gives the density contrast for different values of \( r_s \). The initial density profile is given by

\[ \frac{\rho}{\rho_0} = (-\Phi/mc^2)^n. \tag{10} \]

Radial profiles of \( \rho/\rho_0 \) are shown in Figure 1 for \( r_s/R \) varying between 1.1 and 1.001. The analytic estimate of the growth rate of NEMPI, \( \lambda \), based on an isothermal layer with \( H_p = H_p = \text{const} \) is given by (Kemel et al., 2012b)

\[ \lambda \approx \beta_u \text{rms}/H_p - \eta_t k^2. \tag{11} \]

Table 1. Dependence of the density contrast on the value of \( r_s \).

Note that \( R - r = 35 \text{ Mm} \) corresponds to \( r/R = 0.95 \).

\begin{tabular}{cccc}
\hline
\( r_s/R \) & \( H_p(\text{top})/R \) & \( H_{p0}/R \) & \( \rho_{\text{max}}/\rho_{\text{min}} \) \\
\hline
1.100 & \( 3.6 \times 10^{-2} \) & 0.052 & \( 1.4 \times 10^4 \) \\
1.010 & \( 4.0 \times 10^{-3} \) & 0.023 & \( 2.9 \times 10^2 \) \\
1.001 & \( 4.0 \times 10^{-4} \) & 0.019 & \( 8.9 \times 10^3 \) \\
\hline
\end{tabular}

Assume that this equation also applies to the current case where \( H_p \) depends on \( r \) and setting \( k = H_p^{1/3} \), the normalized growth rate is

\[ \frac{\lambda H_p^{0}}{\beta_u H_p^{0}} - \frac{\eta_t}{H_p^{0}} \tag{12} \]

In Figure 1, we compare therefore \( H_p^{0}/H_p \) with \( \eta_t/\beta_u \text{rms} H_p^{0} \) and see that the former exceeds the latter in our reference model with \( r_s/R = 1.001 \). This suggests that NEMPI should be excited in the outer layers.

For the magnetic field, we adopt perfect conductor boundary conditions, on the inner and outer radii, \( r_0 = 0.7 R \) and \( R \), respectively, i.e.,

\[ \frac{\partial \mathbf{B}}{\partial r} = \mathbf{A} = 0, \quad \text{on } r = r_0, R. \tag{13} \]

On the pole and the equator, we assume

\[ \frac{\partial \mathbf{A}}{\partial \theta} = \mathbf{B} = 0, \quad \text{on } \theta = 0^\circ \text{ and } 90^\circ. \tag{14} \]

Since our simulations are axisymmetric, the magnetic field is conveniently represented via \( \mathbf{B}_{\text{pol}} = \mathbf{A} \). In particular, contours of \( r \sin \theta \mathbf{A} \) give the magnetic field lines of the poloidal magnetic field, \( \mathbf{B}_{\text{pol}} = \mathbf{A} \times (\mathbf{A} \times \hat{r}) \). The magnetic field strength is given either in units of the local equipartition value, \( B_{\text{eq}}(r) \), or in units of the equipartition value \( B_{\text{eq}}^{0} \equiv B_{\text{eq}}(r_{\text{ref}}) \) at the reference radius \( r_{\text{ref}} = 0.95 R \).

In all cases presented in this paper, we adopt a numerical resolution of \( 256 \times 1024 \) mesh points in the \( r \) and \( \theta \) directions. This is significantly higher than what has been used previously, even in mean field calculations with stratification and hydrodynamical feedback included; see Brandenburg et al. (1992), where a resolution of just \( 41 \times 81 \) meshpoints was used routinely. In principle, lower resolutions are possible, but in some cases we found certain properties of the solutions to be sensitive to the resolution.

3. Results

In our model, the dynamo growth rate is about \( 170 \eta_t R^2 \). Although both dynamo and NEMPI are linear instabilities, this
optimistic set of parameters describing NEMPI, namely
see the effect of NEMPI more clearly, we consider a somewhat
begin by discussing the effects of varying the stratification. To
associate with that of NEMPI.

Fig. 3. Meridional cross-sections of \( \overline{B}_\phi \)/\( B_{eq0} \) (color coded) together with magnetic field lines of \( \overline{B}_{pol} \) for different stratification parameters \( r_\ast \) and \( Q_\alpha = 10^3 \). The dashed lines indicate the latitudes 70.3°, 73.4°, 75.6°, and 76.4°.

is no longer the case in our coupled system, because NEMPI de-
stands on the magnetic field strength, and only in the nonlinear
regime of the dynamo does the field reach values large enough
for NEMPI to overcome turbulent magnetic diffusion. This is
shown in Figure 2 where we plot the growth of the magnetic
field and compare with runs with different values of \( q_{p0} \). For
\( q_{p0} = 100 \) we find a growth rate of about 270 \( \eta_T / R^2 \), which
we associate with that of NEMPI.

Let us now discuss the resulting magnetic field structure. We
begin by discussing the effects of varying the stratification. To
see the effect of NEMPI more clearly, we consider a somewhat
optimistic set of parameters describing NEMPI, namely \( q_{p0} =
100 \) and \( \beta_0 = 0.05 \), which yields \( \beta_\ast = 0.5 \); see Equation (5).
This is larger than the values 0.23 and 0.33 found from numeri-
cal simulations with and without small-scale dynamo action, re-
spectively (Brandenburg et al., 2012). The effect of lowering the
value of \( q_{p0} \) can be seen in Figure 4 and will also be discussed
below. We choose \( Q_\alpha = 1000 \) for the \( \alpha \) quenching parameter
so that the local value of \( \overline{B}_\phi / B_{eq0} \) near the surface is between 10
and 20 percent, which is suitable for the excitation of NEMPI
(Kemel et al., 2012a). Meridional cross sections \( \overline{B}_\phi / B_{eq0} \) to-
tgether with magnetic field lines of \( \overline{B}_{pol} \) are shown in Figure 3.
Note that a magnetic flux concentration develops near the sur-
face at latitudes between 70° and 76° for weak and strong strat-
ification, respectively. Structure formation from NEMPI occurs
in the top 5% by radius, and the flux concentration is most pro-
nounced when \( r_\ast \leq 1.01 \).

Next, if we increase the magnetic field strength by making
\( Q_\alpha \), smaller, we see that the magnetic flux concentrations move
toward lower latitudes down to about 49° for \( Q_\alpha = 100 \); see Figure 4. However, while this is potentially interesting for the
Sun, where sunspots are known to occur preferentially at low
latitudes, the magnetic flux concentrations become also weaker
at the same time, making this feature less interesting from an
astrophysical point of view. For comparison with the parameter
\( \beta_{p0} = 0.05 \) in Equation (5) we note that \( \beta_\ast = Q_\alpha^{-1/2} \) takes
the values 0.1, 0.07, 0.04, and 0.03 for \( Q_\alpha = 100, 200, 500, \) and
1000, respectively. Thus, for these models the quenchings of the
non-diffusive turbulence effects in the momentum and induction
equations are similar.

Also, if we decrease \( q_{p0} \) to more realistic values, we expect
the magnetic flux concentrations to become weaker. This is
indeed borne out by the simulations; see Figure 5 where we show
meridional cross-sections for \( q_{p0} \) in the range 40 \( \leq q_{p0} \leq 100 \)
for \( Q_\alpha = 10^3 \). This corresponds to the range 0.32 \( \leq \beta_\ast \leq 0.5 \).

For magnetic weaker fields, i.e., for larger values of the
quenching parameter \( Q_\alpha \), we find that NEMPI has a modifying
effect on the dynamo in that it can now become oscillatory. A
butterfly diagram of \( \overline{B}_\phi \) and \( \overline{B}_{\phi} \) is shown in Figure 6. Meridional
cross-sections of the magnetic field at different times covering
half a magnetic cycle are shown in Figure 7. It turns out that,
at sufficiently weak magnetic field strengths, NEMPI causes the
oscillatory solutions with poleward migrating flux belts. The rea-
son for this is not well understood, but it is reminiscent of the
poleward migration observed in the presence of weak rotation
(Losada et al., 2012). Had this migration been equatorward, it
might have been tempting to associate this with the equatorward
migration of the sunspot belts in the Sun.

Finally, we discuss the change of kinetic, magnetic, and cur-
cent helicities due to NEMPI. We do this by using a model
that is close to our reference model with \( r_\ast / R = 1.001 \) and
\( Q_\alpha = 1000 \), except that \( q_{p0} \) = 0 in the beginning, and then at
\( t_0 \) we change it to \( q_{p0} = 100 \). The two inverse length scales
based on magnetic and current helicities,

\[
k_M = \left( \frac{\int V^2 B_{\phi}}{\int V B^2} \right)^{-1} \quad \text{and} \quad k_C = \mu_0 \frac{\int V J \cdot B}{\int V B^2} \quad (15)
\]

increase by 25%, while the inverse length scale based on the
kinetic helicity,

\[
k_K = \frac{\int V \cdot W \cdot U}{\int V^2} \text{d}V,
\]

drops to very small values after introducing NEMPI, see e.g.
Figure 8. Here, \( \nabla \times U \) is the mean vorticity. This be-
behavior of $k_k$ is surprising, but it seems to be associated with an increase in kinetic energy. The reason for the increase of the two magnetic length scales, on the other hand, might be understandable as a consequence of increasing gradients associated with the resulting flux concentrations.

4. Conclusions

The present investigations have shown that NEMPI can occur together with the dynamo, i.e., both instabilities can work at the same time and can even modify each other. It was already clear from earlier papers that NEMPI can only work in a limited range of magnetic field strengths. Hence we have adopted a simple $\alpha$ quenching prescription to arrange the field strength in the desired range. Furthermore, unlike much of the earlier work on NEMPI, we have used an adiabatic stratification instead of an isothermal one. This implies that the pressure scale height is no longer constant and now much shorter in the upper layers than in the bulk of the domain. This favors the appearance of NEMPI in the upper layers, because the growth rate is inversely proportional to the pressure scale height.

There are two lines of future extensions of the present model. On the one hand, it is important to study in more detail the interplay between NEMPI and the dynamo instability. This is best done in the framework of a local Cartesian model which is more easily amenable to analytic treatment. Another important exten-
mission would be the inclusion of differential rotation. At the level of a dynamically self-consistent model, where the flow speed is a solution of the momentum equation, differential rotation is best implemented by including the $\Lambda$ effect (Rüdiger, 1980, 1989). This is a parameterization of the Reynolds stress that is in some ways analogous to the parameterization of the electromotive force via the $\alpha$ effect. Mean-field models with both $\alpha$ and $\Lambda$ effects have been considered before (Brandenburg et al., 1992), so the main difference would be the additional parameterization of magnetic effects in the Reynolds stress giving rise to NEMPI. In both cases, our models would be amenable to verification using DNS by driving turbulence through a forcing function. In the case of a spherical shell, this can easily be done in wedge geometry where the polar regions are excluded. However, in that case the mean-field dynamo solutions are oscillatory with equatorward migration (Mitra et al., 2010). At an earlier phase of the present investigations we have studied NEMPI in the corresponding mean-field models and found that NEMPI can revert the propagation of the dynamo wave from equatorward to poleward. However, owing to time dependence, the effects of NEMPI are then harder to study, which is why we have refrained from studying such models in further detail.

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Fig. 6. Butterfly diagram of $\overline{B}_r$ (upper panel) and $\overline{B}_\phi$ (lower panel) for $Q_\alpha = 10^{14}$, $r_\star = 1.001$, $\omega = 11.3 \eta_t/R^2$.

Fig. 7. Meridional cross-sections of $\overline{B}/B_{eq0}$ at different times, for $Q_\alpha = 10^{14}$, $r_\star = 1.001$. The cycle frequency here is $\omega = 11.3 \eta_t/R^2$. Furthermore, the toroidal field is normalized by the local equipartition value, i.e., the colors indicate $\overline{B}_\phi/B_{eq}(r)$.
Fig. 8. The three inverse length scales $k_C$, $k_M$ and $k_K$ as a function of time. At time $t_0$, the value of $q_{\nu_0}$ has been changed from 0 to 100.

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