A Finite Element Model of Partial Discharge in Solid Medium

Wenqiang Wang\textsuperscript{1a*}, Zhen Shi\textsuperscript{1b*}

\textsuperscript{1}Guangxi Key Laboratory of Power System Optimization and Energy Technology, Guangxi University, Nanning 530004, China
\textsuperscript{a*}email: 1912391067@st.gxu.edu.cn, \textsuperscript{b*}email: eee0619@sina.com

Abstract—The mechanism of partial discharge (PD) is the theoretical basis of the PD detection, insulation monitoring and risk assessment. The finite element analysis of partial discharge is an effective measure to study the characteristics of PD. In this work, a physical model of PD of a gas gap in solid dielectric using finite element analysis (FEA) method has been developed to determine the relationship between the inception electric field with different applied stresses, the cavity diameter and air pressure has been considered as the physical parameters that affect PD characteristics. The change of potential distribution before and after the discharge and the field strength during the discharge are analyzed.

1. Introduction
With the development of the power industry, the voltage level and capacity of the power system are constantly improved, and the manufacturing cost of large electrical equipment is also getting higher. The failure of electrical equipment will consequently cause huge economic losses and adverse social impacts. The operation reliability of electrical equipment depends largely on the condition of insulation, which is significantly influenced by partial discharge. Some defects could remain inside the insulation during the manufacturing process such as tiny bubbles or cavities. As the dielectric constant of air is much smaller than that of solid dielectric materials, the electric field strength of the bubbles or cavities is therefore higher than that of the solid dielectric material, partial discharge could occur in these defects and accelerate the degradation of the insulation.

Abundant results have been obtained in the field of PD model research. There are four main PD models have been introduced to represent the cavity-dielectric system. According to the order of introduction, these models are the three-capacitance model (abc), the induced charge model (ICC), the finite element analysis model, and the multi-physicals field model. At present, most of the models adopted are three-capacitance models, which cannot present the charge accumulation process on the cavity surface, thus the physical mechanism of the partial discharge is not accurately revealed\cite{1}.

In this article the finite element analysis method is applied to build a description of partial discharge behavior in homogeneous dielectric material with spherical cavity model that is satisfied with the geometrical structure of the electric field, the distribution of the scalar potential numerical calculation, and the electric field distribution of the geometric shape with no limitation of the uniformity. The simulation results offered intuitive description of partial discharge process.

2. PD model

2.1. Physical field governing equations
In this work, the utility model describes the potential distribution in dielectric material. The basic
governing equation of the field model is presented as follows:

\[ \nabla \cdot \vec{D} = \rho_f \]  
(1)

\[ \nabla \cdot \vec{J}_f + \frac{\partial \rho_f}{\partial t} = 0 \]
(2)

Equation (1) and equation (2) is the potential field physics equation and the current continuity equation respectively. Where \( \vec{D} \) is the displacement field, \( \rho_f \) is the free or unpaired charge density, and \( \vec{J}_f \) is the free current density.

2.2. Streamer Inception Criterion
Under the external electric field, photoionization is generated due to the excitation, ionization and recombination of gas atoms or molecules, and a new electron collapse is caused by photoelectrons near the electron collapse. When the electron avalanche is caused, a discharge will occur. For the discharge to occur in the medium, two conditions must be met: first, the cavity electric field exceeds the critical electric field strength of the discharge, second, there is a free electron causing electron avalanche. For the ellipsoidal cavity, a critical electric field value that can guarantee the self-sustaining of the ionization process is proposed as the inception electric field of discharge, according to reference [2] which can be defined as:

\[ E_{\text{inc}} = \left( \frac{E_i}{p} \right)_{cr} p \left( 1 + \frac{B}{(2pa)^n} \right) \]
(3)

Where, \( \left( \frac{E_i}{p} \right)_{cr} \), \( B \) and \( n \) are parameters related to the ionization process in the gas and depend on each particular gas, for example, the values in air are 24.2 VPa^{-1}m^{-1}, 8.6 Pa^{1/2}m^{1/2} and 0.5, respectively \( p \) is the pressure in the cavity, and \( a \) is the radius of the cavity.

2.3. Electron generation rate
Surface emission and volume ionization are the main mechanisms for generating initial electrons. Compared with surface emission, the electron generation rate of volume ionization is relatively small, and the related parameters of volume ionization are difficult to determine, so volume ionization is generally ignored when calculating the total electron generation rate. According to references [3,4], total electron generation rate can be expressed as:

\[ N_{\text{tot}}(t) = N_{PD} \exp \left( \left[ E_{\text{cav}}(t) / E_{\text{inc}} \right] \exp \left( - \left( t - t_{PD} \right) / \tau_{\text{decay}} \right) \right) \]
(4)

Where \( N_{PD} \) is the number of electrons available after the last discharge event, \( E_{\text{cav}} \) is the electric field value at the symmetry axis of the cavity, \( E_{\text{inc}} \) is the inception electric field of the discharge, \( t_{PD} \) is the time of the last discharge, \( \tau_{\text{decay}} \) is the charge decay constant. The number of residual electrons available in the last event (\( N_{PD} \)) can be expressed as:

\[ N_{PD} = N_{e0} \left| E_{\text{cav}} \left( t_{PD} \right) / E_{\text{inc}} \right| \]
(5)

Where \( N_{e0} \) is the initial number of electrons when the cavity electric field reaches the inception electric field value.

2.4. Modeling progress of partial discharge
When the electric field of the cavity exceeds \( E_{\text{inc}} \), it is vital to evaluate the availability of free electrons for the occurrence of discharge. Therefore, the probability of the existence of free electrons at each time step at the beginning of discharge can be calculated as follows:

\[ P(t) = N_{\text{tot}}(t) \Delta t \]
(6)

Where \( \Delta t \) is time interval of simulation, in this work, equations (4) and (5) are used to determine \( N_{\text{tot}}(t) \). Once the value of \( P(t) \) is determined, a random number \( R \) between zero and unity is generated.
If $P > R$, then PD happens; otherwise, another time step is added to the delay time for the next PD ignition.

When partial discharge conditions are met, the discharge event can be represented by changing the non-conduction state of the cavity. This can be modeled by increasing the cavity conductivity from the initial conductivity $\sigma_{\text{cav0}}$ to the maximum cavity conductivity $\sigma_{\text{cavmax}}$ during discharge events. $\sigma_{\text{cavmax}}$ is the conductivity of the cavity during discharge that causes the electric field $E_{\text{cav}}(t)$ across the center of the cavity to decrease to less than the extinction electric field $E_{\text{ext}}$. During the discharge, the conductivity of the streamer channel is determined by the conductivity of the electrons in the plasma, because the conductivity of the ions is negligible. Therefore, according to reference [5] $\sigma_{\text{cavmax}}$ can calculated as:

$$\sigma_{\text{cavmax}} = \left( \frac{\alpha e^2 N_e \lambda_e}{m_e c_e} \right)$$

(7)

Where $\alpha$ is the coefficient (~0.85) related to the energy distribution and the mean free path of the electron, $e$ is the charge of the electron, $m_e$ is the mass of the electron, $\lambda_e$ is the mean free path of the electron (~4μm), $c_e$ is the heat velocity of the electron (~3x10^8m/s), $N_e$ is the electron density.

3. Result

The geometry of the model is realized using two-dimensional (2D) axisymmetric finite element analysis software, which is based on the field model solving the potential and interfaces with the code. The model, as shown in Fig.1, consists of a homogeneous dielectric material, a hemispherical cavity (due to the central axis of symmetry), and a cavity surface in order to simulate surface charge attenuation through conduction along the cavity wall. The specific geometric parameters are shown in Table 1.

| Table 1 | Geometric structure parameters of simulation model |
|---------|--------------------------------------------------|
| Thickness of material $h_{\text{mat}}$ (mm) | 3 |
| Radius of material $r_{\text{mat}}$ (mm) | 5 |
| Radius of cavity $r_{\text{cav}}$ (mm) | 0.5 |
| Thickness of cavity surface $h_{\text{surf}}$ (mm) | 0.01 |

| Table 2 | Parameters applied to the finite element simulation model |
|---------|----------------------------------------------------------|
| Applied voltage $U_{\text{pp}}$ (kV) | 18 |
| Applied frequency $f$ (Hz) | 50 |
| Inception electric field $E_{\text{inc}}$ (kV/m) | 4.32 |
| Minimum of cavity conductivity $\sigma_{\text{cav0}}$ (S/m) | 0 |
| Maximum of cavity conductivity $\sigma_{\text{cavmax}}$ (S/m) | 0.0004 |
| Surface charge decay constant $\tau_{\text{decay}}$ (s) | 0.002 |

Boundary conditions were set in the finite element model. An AC voltage of frequency $f$ is applied to the upper electrode was applied to the upper electrode, while the lower electrode was always grounded. Current continuity conditions were applied to the boundary conditions of the medium and cavity, and finite element software parameters were initialized as Table 2.
Fig. 2 shows the potential distribution of cavity in the before and after discharge, before discharge, due to the applied voltage and surface charge accumulation, lead to voltage over discharge inception voltage on both ends of the cavity, the conductivity of cavity increasing as discharge progressing, generally speaking, discharge maintain dozens of nanoseconds, electric field will decline sharply, dropped to extinction electric field, the discharge end, the conductivity of cavity returns to its original value $\sigma_{cav0}$.

Fig. 2 Potential distribution of medium during simulation (a) before discharge, (b) after discharge

Fig. 3 shows the relationship between discharge inception electric field $E_{inc}$ and pressure. The inception field strength is an increasing function of pressure. With the increase of pressure, $E_{inc}$ increases, which is because the ideal gas law is taken into account. Temperature can indirectly affect PD properties by changing the exponential term of surface emission. This is consistent with the conclusion of reference [6].

Fig. 3 Relationship between inception electric field strength and pressure

Fig. 4 shows the relationship between the inception electric field and the diameter of cavity. The inception electric field is a decreasing function of the cavity diameter, which decreases with the cavity diameter increase. Compared with large cavity, the surface charge conduction along the cavity wall in smaller cavity decay more significantly, resulting in a lower rate of electron generation, the result is consistent with the conclusion in reference [7].
Fig. 4 Relationship between inception field strength and cavity diameter

As shown in Fig. 5, the electric field strength of the first discharge and the next long interval discharge is higher than $E_{inc}$, this is due to the discharge for the first time, the surface electron generation rate is low, initial electronic mainly provided by the volume ionization, so when the electric field strength of cavity exceeds $E_{inc}$, discharge will not happen immediately. Only when the electric field is higher, the discharge will occur, so there will be a period of delay. When the discharge occurs, the electrons accumulated on the surface will decay with time, during the decay process, the electrons may jump from the shallow trap to the deep trap, and the electrons in the deep trap are difficult to capture. It is also possible that during the discharge, electrons are diffused into the deep part of the insulating material and become inactive, resulting in the electric field strength of the next discharge being higher than that of $E_{inc}$. The simulation results are consistent with reference [8,9].

Fig. 5 Electric field strength during discharge in one cycle

4. Conclusion
This article established a description homogeneous solid medium single cavity two-dimensional model of partial discharge. The model is controlled by finite element analysis software and related codes. The model is used to simulate the partial discharge process dynamically, and to study the influence of external conditions on the discharge. The simulation results are compared with relevant literatures and experiments, and the accuracy of the model is verified.

From the simulation model of partial discharge, it can be seen that when the diameter of cavity increases, the corresponding inception electric field decreases, which is consistent with the results in
literature. When the cavity pressure is considered, the inception electric field of discharge increases with pressure. The simulation results also show that the electric field at the first discharge is higher than the inception electric field, because the initial electron generation rate is lower and a stronger electric field is needed to ignite the discharge.

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