$N^*(1535) \to N$ transition form-factors due to the axial current

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Abstract

The form-factors for the transition $N^*(1535) \to N$ induced by isovector and isoscalar axial currents within the framework of light-cone QCD sum rules by using the most general form of the interpolating current are calculated. In numerical calculations, we use two sets of values of input parameters. It is observed that the $Q^2$ dependence of the form-factor $G_A$ can be described by the dipole form. Moreover, the form-factors $G_P^{(S)}$ are found to be highly sensitive to the variations in the auxiliary parameter $\beta$. 

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1 Introduction

Nucleon form-factors are fundamental quantities for understanding the inner structure of hadrons at low energies. The electromagnetic form-factors of nucleon are studied in a wide range of momentum transfer square (see [1]). The electromagnetic form-factors within the light-cone sum rules (LCSR) were comprehensively studied in many works (see for example [2, 3, 4, 5, 6]). Unlike the electromagnetic form-factors, those induced by isovector and isoscalar axial currents are not measured. Only the nucleon axial charge is experimentally well determined from the neutron $\beta$ decay, and the latest value is $g_A = 1.2724$ [7]. The determination of the axial form-factors can give very useful information on the flavor structure and spin content of nucleon resonances. Therefore, the study of these form-factors receives special attention for understanding the structure of nucleon resonances.

The LCSR results for the nucleon axial form-factors and for tensor factors are studied in [8, 9] and [10, 11], respectively. The axial form-factors of the nucleon within the holography approach are investigated in [12]. In the lattice QCD, the momentum-transfer dependence of the axial form-factor is studied in [13, 14, 15, 16, 17]. The study of the momentum transfer dependence of the axial form-factor is possible only in pion electro-production and neutrino-induced charged-current reactions.

The experiments planned at Jefferson Laboratory (JLab), MAMI, are aimed to study properties of baryon resonances in photo- and electro-production reactions [18].

Motivated by the prospective experiments at JLab, MAMI, we aim to study the form-factors for the transition $N^*(1535) \rightarrow N$ induced by isovector and isoscalar axial currents. To this end, we introduce the following vacuum to $N^*(1535)$ correlation function:

$$\Pi_\mu(p,q) = i \int d^4x \ e^{iqx} \langle 0 | T\{\eta(0)A_\mu(S)(x)\} | N^*(p) \rangle.$$ \hspace{1cm} (1)

In Eq. (1), $\eta$ is the interpolating current for the nucleon and $A_\mu(S)$ is the isovector (isoscalar) axial vector current,

$$A_\mu^{(S)} = \bar{u}\gamma_\mu\gamma_5u \mp \bar{d}\gamma_\mu\gamma_5d.$$ \hspace{1cm} (2)

2 Sum rules for the transition $N^*(1535) \rightarrow N$ form-factors induced by axial current

In this section, we derive the LCSR for the form-factors for the transition $N^*(1535) \rightarrow N$ induced by isovector and isoscalar axial currents. To this end, we introduce the following vacuum to $N^*(1535)$ correlation function:

$$\Pi_\mu(p,q) = i \int d^4x \ e^{iqx} \langle 0 | T\{\eta(0)A_\mu^{(S)}(x)\} | N^*(p) \rangle.$$
The nucleon interpolating current, in general, can be written as

$$\eta = 2\varepsilon^{abc} \sum_{\ell=1}^{2} (u^{aT} C A^{\ell}_{1} d^{b}) A^{\ell}_{2} u^{c}$$  \hspace{1cm} (3)$$

where \(a, b,\) and \(c\) are color indices, \(C\) is the charge-conjugation operator, \(A^{1}_{1} = I, A^{1}_{2} = \gamma_{5},\) \(A^{2}_{1} = \gamma_{5},\) and \(A^{2}_{2} = \beta I.\) The case \(\beta = -1\) corresponds to the Ioffe current.

The LCSR for the relevant quantities is obtained by calculating the correlation function from the hadronic and QCD sides and then matching the result of two representations. From the hadronic side, the result can be obtained by inserting the total set of the nucleon states between the nucleon interpolating current and the axial current \(A^{(S)}_{\mu}\), and separating the contribution of the ground-state nucleon.

The standard procedure for obtaining QCD sum rules is the calculation of the correlation function from hadronic and QCD sides. We begin with the calculation of the hadronic side of the correlation function \(\Pi_{\mu}\).

Now we will give a few details of this procedure. We explicitly keep the contribution of the state \(N(940)\) in the hadronic dispersion relation and, according to the quark-hadron duality ansatz, represent higher states by a dispersion integral, starting from some threshold, \(s_{0}\).

After separating the contribution from the ground-state \(N\) in the hadronic part, we get

$$\Pi_{\mu} = \frac{\langle 0 | \eta | N \rangle \langle N | A^{(S)}_{\mu} | N^{*} \rangle}{m_{N}^{2} - p^{2}} + \cdots$$  \hspace{1cm} (4)$$

where \(\cdots\) denotes the contributions from higher states and the continuum. The coupling of \(N\) to the interpolating current \(\eta\), that is, the decay constant or the residue, is defined as

$$\langle 0 | \eta | N(p) \rangle = \lambda_{N} u_{N}(p).$$  \hspace{1cm} (5)$$

The matrix element of the axial current between \(N\) and \(N^{*}\) in terms of three form-factors is given by

$$\langle N(p') | A_{\mu} | N^{*}(p) \rangle = \bar{u}_{N}(p') \left[ \gamma_{\mu} \gamma_{5} G_{A}(q^{2}) + \frac{q_{\mu}}{m_{N} + m_{N^{*}}} \gamma_{5} G_{P}(q^{2}) \right.$$

$$\left. + i\sigma_{\mu\nu} \frac{q_{\nu}}{m_{N} + m_{N^{*}}} \gamma_{5} G_{T}(q^{2}) \right] \gamma_{5} u_{N^{*}}(p) + \cdots$$  \hspace{1cm} (6)$$

where \(G_{A}, G_{P},\) and \(G_{T}\) are the axial, induced pseudoscalar, and induced tensor form-factors, respectively. The matrix element of the isoscalar axial-vector current between \(N\) and \(N^{*}\) is obtained from (6) by replacing \(A_{\mu} \rightarrow A_{\mu}^{S}, G_{A} \rightarrow G_{A}^{S}, G_{P} \rightarrow G_{P}^{S},\) and \(G_{T} \rightarrow G_{T}^{S}.\) The \(G\)-parity invariance and taking the exact isospin symmetry limit lead to the result that \(G_{T}^{S}\) and \(G_{T}^{S}\) form-factors must vanish. Therefore, in all the following discussions these form-factors are taken to be zero. Taking into account this fact and putting Eqs. (6) and (5) into (4) for the hadronic contributions to the correlation function, we obtain

$$\Pi_{\mu} = \frac{\lambda_{N}}{m_{N}^{2} - p^{2}} \left[ \gamma_{\mu} \gamma_{5} G_{A} + \frac{q_{\mu}}{m_{N} + m_{N^{*}}} \gamma_{5} G_{P}(q^{2}) \right] \gamma_{5} u_{N^{*}}(p) + \cdots$$  \hspace{1cm} (7)$$
Using the equation of motion, namely \(\phi u_{N^*}(p) = m_{N^*} u_{N^*}(p)\), in the correlation function from hadronic part, we get

\[
\Pi_\mu = \frac{\lambda_N}{m_{N^*}^2 - p'^2} \left\{ (m_{N^*} - m_N) \gamma_\mu G_A + 2p_\mu G_A - \not{q} \gamma_\mu G_A \right. \\
+ \left. \frac{G P q_\mu}{m_{N^*} + m_N} (m_{N^*} + m_N - \not{q}) \right\} + \cdots
\]

(8)

where \(p' = p - q\). From this expression, we see that the correlator function can be decomposed into the following Lorentz structures:

\[
\Pi_\mu(p, q) = \Pi_1 p_\mu + \Pi_2 \gamma_\mu + \Pi_3 \not{q} \gamma_\mu + \Pi_4 q_\mu + \Pi_5 q_\mu \not{q}.
\]

(9)

Now, let us turn our attention to the calculation of the correlation function from the QCD side. For \(p'^2, q^2 \ll 0\), the product of the two currents in Eq. (1) can be expanded around the light cone, \(x^2 \sim 0\). The result of the operator-product expansion (OPE) is obtained by a sum over the distribution amplitudes (DAs) of \(N^*\), with an increasing twist multiplied by their corresponding coefficient functions.

At this point, we would like to present some details of the calculations. Using the forms of interpolating current and axial current given above, we get

\[
(\Pi_\mu)_\rho = \frac{1}{2} \int d^4x \ e^{iq \cdot x} \sum_{\ell=1}^2 \left\{ (CA_1^\ell)_{\alpha\gamma} [A_2^\ell S_u(-x) \gamma_\mu \gamma_5]_{\rho\beta} \right. \\
+ (A_2^\ell)_{\rho\beta} [(CA_1^\ell)^T S_u(-x) \gamma_\mu \gamma_5]_{\gamma\beta} \\
\left. + (A_2^\ell)_{\rho\alpha} (CA_1^\ell S_u(-x) \gamma_\mu \gamma_5)_{\gamma\gamma} \right\} 4\varepsilon^{a\beta}(0)|u^a(0)u^b(x)d^c(0)|N^*(p)) .
\]

(10)

In Eq. (10), the upper (lower) sign in the last line corresponds to the isovector (isoscalar) axial current, and \(S_q(x)\) is the light-quark propagator. The light-quark propagator in the presence of external electromagnetic and gluonic background fields were calculated in [20] and are given by

\[
S_q(x) = \frac{ig}{2\pi^2 x^4} \int_0^1 du[u, ux] \{\bar{u} \not{\sigma} \gamma_5 [u] \} G^\alpha\beta(ux)[ux, 0] \\
- \frac{ig}{16\pi^2 x^2} \int_0^1 du[u, ux] \{\bar{u} \not{\sigma} \gamma_5 [u] \} F^\alpha\beta(ux)[ux, 0] .
\]

(11)

In Eq. (11),

\[ [x, y] := P \exp \left\{ i \int_0^1 dt(x - y) \not{e} A^\mu(t x - \not{t} y) + g B^\mu(t x - \not{t} y) \right\} \]

is the path-ordering exponent. In the calculations, we use the Fock-Schwinger gauge, i.e. \(x_\mu A^\mu(B^\mu) = 0\), hence these factors can be safely omitted.

The matrix element of the three quarks between the vacuum and the states \(N^*\), defined in terms of the DAs of \(N^*\) with increasing twist, is given in [21]. For the sake of completeness, we present these DAs in Appendix A.
Using the explicit expressions of the DAs of $N^*$, performing the integration over $x$, and selecting the coefficients of the structures $\slashed{q} \gamma_{\mu}$ and $\slashed{q} q_{\mu}$ in both representations, we get

\[
\frac{\lambda_N}{m_N^2 - p^2} G_A^{(S)} = \Pi_3^{(S)},
\]
\[
- \frac{\lambda_N}{m_N^2 - p^2} G_P^{(S)} = \Pi_5^{(S)}.
\]

In Eq. (12), $\Pi_5^{(S)}$ and $\Pi_3^{(S)}$ are given by

\[
\Pi_5^{(S)} = m_N^2 \int_0^1 dx_2 \left\{ \frac{(1 - \beta) N_1(x_2) + (1 + \beta) N_2(x_2)}{\Delta^2(x_2)} + \frac{(1 + \beta) N_3(x_2)}{\Delta^3(x_2)} \right\}
\]
\[
\mp m_N^2 \int_0^1 dx_3 \left\{ \frac{(1 - \beta) N_4(x_3) + (1 + \beta) N_5(x_3)}{\Delta^2(x_3)} + \frac{(1 + \beta) N_6(x_3)}{\Delta^3(x_3)} \right\}
\]

and

\[
\Pi_3^{(S)} = m_N^2 \int_0^1 dx_2 \left\{ \frac{(1 - \beta) N_7(x_2) + (1 + \beta) N_8(x_2)}{\Delta(x_2)} + \frac{(1 - \beta) N_9(x_2) + (1 + \beta) N_{10}(x_2)}{\Delta^2(x_2)} \right\}
\]
\[
\mp m_N^2 \int_0^1 dx_3 \left\{ \frac{(1 - \beta) N_{11}(x_3) + (1 + \beta) N_{12}(x_3)}{\Delta(x_3)} + \frac{(1 - \beta) N_{13}(x_3) + (1 + \beta) N_{14}(x_3)}{\Delta^2(x_3)} \right\}
\]

where

\[
N_1(x) = -\tilde{A}_{123}(x) + \tilde{A}_{1345}(x) - 2 \tilde{A}_{34}(x) - \tilde{V}_{123}(x) + \tilde{V}_{34}(x),
\]
\[
N_2(x) = -\tilde{P}_{12}(x) + \tilde{S}_{12}(x) - 2 \tilde{T}_{123}(x) - 3 \tilde{T}_{127}(x) - 5 \tilde{T}_{158}(x) - 10 \tilde{T}_{78}(x) + \frac{2}{x} \tilde{T}_{234578}(x),
\]
\[
N_3(x) = \frac{2}{x} \left( Q^2 + m_N^2 x^2 \right) \tilde{T}_{234578}(x),
\]
\[
N_4(x) = \tilde{A}_{123}(x) + \tilde{A}_{34}(x) + \tilde{V}_{123}(x) - \tilde{V}_{34}(x),
\]
\[
N_5(x) = -\tilde{P}_{12}(x) + \tilde{S}_{12}(x) - \tilde{T}_{127}(x) - \tilde{T}_{158}(x) - 2 \tilde{T}_{78}(x) + \frac{2}{x} \tilde{T}_{234578}(x),
\]
\[
N_6(x) = \frac{2}{x} \left( Q^2 + m_N^2 x^2 \right) \tilde{T}_{234578}(x),
\]
\[
N_7(x) = -A_{3}(x) + V_{1}(x) - V_{3}(x) - \frac{1}{2x} \left[ \tilde{A}_{123}(x) + \tilde{V}_{123}(x) \right],
\]
\[
N_8(x) = P_{1}(x) + S_{1}(x) + 2T_{1}(x) - 4T_{7}(x) - \frac{1}{2x} \left[ \tilde{T}_{123}(x) + 3 \tilde{T}_{127}(x) \right],
\]
\[
N_9(x) = -\frac{1}{2x} \left( Q^2 + m_N^2 x^2 \right) \left[ \tilde{A}_{123}(x) + \tilde{V}_{123}(x) \right] - m_N^2 \left[ \tilde{A}_{123456}(x) - \tilde{V}_{123456}(x) \right],
\]
\[
N_{10}(x) = -\frac{1}{2x} \left( Q^2 + m_N^2 x^2 \right) \left[ \tilde{T}_{123}(x) + 3 \tilde{T}_{127}(x) \right] + m_N^2 \left[ \tilde{T}_{234578}(x) \right],
\]
\[
N_{11}(x) = \frac{1}{2} \left[ A_{1}(x) - V_{1}(x) \right],
\]
\[
N_{12}(x) = \frac{1}{2} \left[ P_{1}(x) + S_{1}(x) - T_{1}(x) + 2T_{7}(x) \right] + \frac{1}{4x} \left[ \tilde{T}_{123}(x) + \tilde{T}_{127}(x) \right],
\]
\[ N_{13}(x) = -m_N^2 \left[ \tilde{A}_{123456}(x) + \tilde{V}_{123456}(x) \right], \]
\[ N_{14}(x) = \frac{1}{4x} \left( Q^2 + m_N^2 x^2 \right) \left[ \tilde{T}_{123}(x) + \tilde{T}_{127}(x) \right] + \frac{m_N^2}{4} \tilde{T}_{234578}(x), \]
and
\[ \tilde{S}_{12} = \tilde{S}_1 - \tilde{S}_2, \]
\[ \tilde{P}_{12} = \tilde{P}_2 - \tilde{P}_1, \]
\[ \tilde{V}_{123} = \tilde{V}_1 - \tilde{V}_2 - \tilde{V}_3, \]
\[ \tilde{V}_{1345} = -2\tilde{V}_1 + \tilde{V}_3 + \tilde{V}_4 + 2\tilde{V}_5, \]
\[ \tilde{V}_{34} = \tilde{V}_4 - \tilde{V}_3, \]
\[ \tilde{V}_{123456} = -\tilde{V}_1 + \tilde{V}_2 + \tilde{V}_3 + \tilde{V}_4 + \tilde{V}_5 - \tilde{V}_6, \]
\[ \tilde{A}_{123} = -\tilde{A}_1 + \tilde{A}_2 - \tilde{A}_3, \]
\[ \tilde{A}_{1345} = -2\tilde{A}_1 - \tilde{A}_3 - \tilde{A}_4 + 2\tilde{A}_5, \]
\[ \tilde{A}_{34} = \tilde{A}_3 - \tilde{A}_4, \]
\[ \tilde{A}_{123456} = \tilde{A}_1 - \tilde{A}_2 + \tilde{A}_3 + \tilde{A}_4 - \tilde{A}_5 + \tilde{A}_6, \]
\[ \tilde{T}_{123} = \tilde{T}_1 + \tilde{T}_2 - 2\tilde{T}_3, \]
\[ \tilde{T}_{127} = \tilde{T}_1 - \tilde{T}_2 - 2\tilde{T}_7, \]
\[ \tilde{T}_{158} = -\tilde{T}_1 + \tilde{T}_5 + 2\tilde{T}_8, \]
\[ \tilde{T}_{234578} = 2\tilde{T}_2 - 2\tilde{T}_3 - 2\tilde{T}_4 + 2\tilde{T}_5 + 2\tilde{T}_7 + 2\tilde{T}_8, \]
\[ \tilde{T}_{78} = \tilde{T}_7 - \tilde{T}_8, \]
\[ \tilde{T}_{125678} = -\tilde{T}_1 + \tilde{T}_2 + \tilde{T}_5 - \tilde{T}_6 + 2\tilde{T}_7 + 2\tilde{T}_8. \]

The generic expressions for the DAs with and without tildes are given as
\[ F(x_2) = \int_0^{1-x_2} dx_1 \ F(x_1, x_2, 1-x_1 - x_2), \]
\[ \tilde{F}(x_2) = \int_1^{x_2} dx_2' \int_0^{1-x_2'} dx_1 \ F(x_1, x_2', 1-x_1 - x_2'), \]
\[ \tilde{\tilde{F}}(x_2) = \int_1^{x_2} dx_2' \int_1^{x_2'} dx_2'' \int_0^{1-x_2''} dx_1 \ F(x_1, x_2'', 1-x_1 - x_2''). \]

A similar set of expressions holds true when the argument is \( x_3 \).

Finally, for the derivation of the sum rules for the relevant form-factors, we perform a Borel transformation in the variable \(-(q-p)^2\) in order to suppress the contributions from higher states and the continuum and to enhance the contribution of the ground state. This
can be achieved with the help of the following subtraction rules:

\[
\int dx \frac{\rho(x)}{\Delta(x)} \to - \int_{x_0}^{1} dx \frac{\rho(x)}{x} e^{-s(x)/M^2},
\]

\[
\int dx \frac{\rho(x)}{\Delta^2(x)} \to \frac{1}{M^2} \int_{x_0}^{1} dx \frac{\rho(x)}{x^2} e^{-s(x)/M^2} + \frac{\rho(x_0)}{x_0^2 m_{N^*}^2 + Q^2},
\]

\[
\int dx \frac{\rho(x)}{\Delta^3(x)} \to - \frac{1}{2M^4} \int_{x_0}^{1} dx \frac{\rho(x)}{x^3} e^{-s(x)/M^2} - \frac{1}{2x_0} \frac{\rho(x_0)}{(Q^2 + x_0^2 m_{N^*}^2) M^2} + \frac{1}{2} \frac{x_0^2 e^{-s_0/M^2}}{Q^2 + x_0^2 m_{N^*}^2} \frac{d}{dx_0} \left[ \frac{1}{x_0 Q^2 + x_0^2 m_{N^*}^2} \right].
\]

(18)

In Eq. (18), the denominator on the left-hand side is defined as

\[\Delta(x) := (xp - q)^2 = x(p - q)^2 - \bar{x} Q^2 - x \bar{x} m_{N^*}^2,\]

where \(\bar{x} = 1 - x\). \(x_0\) is the solution of \(s(x) = s_0\) where

\[s(x) = \frac{\bar{x} Q^2 + x \bar{x} m_{N^*}^2}{x}.
\]

3 Numerical analysis

Having the explicit expressions of the form-factors \(G^{(S)}_A\) and \(G^{(S)}_P\), now we perform the numerical analysis of the sum rules for them.

The main non-perturbative input ingredients of the LCSR for the form-factors \(G^{(S)}_A\) and \(G^{(S)}_P\) are the DAs of \(N^*\). They are presented in [21]. The values of the parameters appearing in the DAs can also be found in [21]. In numerical calculations, we use two set of values of parameters, i.e. LCSR-1 and LCSR-2.

The sum rules for the form-factors \(G^{(S)}_A\) and \(G^{(S)}_P\) contain three auxiliary parameters, i.e. the Borel mass squared, \(M^2\), the continuum threshold, \(s_0\), and the parameter \(\beta\). The form-factors should be independent of these parameters. Therefore, the primary aim of any sum rules analysis is to find the appropriate domains of these auxiliary parameters for which the form-factors exhibit good stability to their variations.

The working regions of \(M^2\) and \(s_0\) are determined from the standard criterion that the contributions from higher states and the continuum and as well as higher-twist terms should be suppressed. Our analysis shows that the working regions of \(M^2\) and \(s_0\), which satisfy the aforementioned criterion are \(1 \text{ GeV}^2 \leq M^2 \leq 3 \text{ GeV}^2\) and \(s_0 = (2.25 \pm 0.25) \text{ GeV}^2\). In [22], the mass and residues of octet baryons are analyzed, and in the present work, we use the result for the residue of the nucleon given in the same reference. The uncertainties of the input parameters and intrinsic uncertainties carried by the LCSR method (namely the factorization scale, \(M^2\), higher-twist corrections) can introduce an uncertainty about \((15 \pm 5)\%\). Therefore, we expect that the sum rules for the form-factors work effectively in the domain

\[2 \text{ GeV}^2 \leq Q^2 \leq 6 \text{ GeV}^2.\]

(19)
Note that the LCSR approach is not dependable for $Q^2 \leq 1 \text{ GeV}^2$. It is because the mass corrections are proportional to $m^2_N/Q^2$, which becomes very large for $Q^2 < 1 \text{ GeV}^2$, and hence the LCSR becomes unreliable.

4 Results and discussion

In Fig.s (1)–(8), we depict the dependence of the form-factors $G_A^{(s)}$ and $G_P^{(s)}$ on $Q^2$ for various values of $\beta$ in its working region and at the fixed values of $M^2 = 2 \text{ GeV}^2$ and $s_0 = 2.25 \text{ GeV}^2$ for two sets of values of input parameters entering the DAs.

From the figures, we make the following observations:

- The values of $G_A$ and $G_A^S$ are positive (negative) for negative (positive) values of $\beta$, whereas $G_P$ and $G_P^S$ have the same sign as $\beta$ for both sets of parameters, LCSR-1 and LCSR-2.

- The moduli of the form-factors $G_A$, $G_A^S$, $G_P$, and $G_P^S$ for the second set of values are larger than those for the first set of parameters.

- The form-factors $G_P$ and $G_P^S$ are sensitive to the variations in $\beta$ when $\beta > 0$ for both sets of input parameters. For example, the values of the form-factors $G_P$ and $G_P^S$ at $\beta = 1$ are nearly twice as large as the corresponding values of these form-factors for other values of $\beta > 0$.

- All the form-factors considered display a similar dependence on $Q^2$ for both sets (LCSR-1 and LCSR-2) and for all values of $\beta$.

- The set LCSR-1 gives the form-factor $G_P^S$ approximately twice that predicted by the set LCSR-2 for $\beta < 0$.

From our results, it follows that the form-factor $G_A(Q^2)$ can be parametrized by the dipole form

$$G_A(Q^2) = \frac{g_A}{(1 + Q^2/m_A^2)^2}$$  \hspace{1cm} (20)

where we tabulated the average of the values of $g_A$ and $m_A$ for both parameter sets and for all the values of $\beta$ that we considered in Table 1.

| $\beta$ | $g_A$  | $m_A$ (GeV) |
|---------|--------|-------------|
| $< 0$   | 3.65 ± 0.70 | 4.55 ± 0.66 | 1.05 ± 0.02 | 1.20 ± 0.02 |
| $> 0$   | −5.82 ± 0.40 | −6.61 ± 0.53 | 1.00 ± 0.02 | 1.16 ± 0.01 |

From Table 1 it follows that the values of $g_A$ and $m_A$ for the LCSR-2 case is slightly larger than the ones for the LCSR-1 one. The magnitude of $g_A$ for the case $\beta > 0$ is larger
than that for $\beta < 0$. Note that for $\beta = -1$ (the case of the Ioffe current), for the values of $g_A$ and $m_A$ we find
\[
g_A = \begin{cases} 2.66, & \text{LCSR-1}, \\ 3.61, & \text{LCSR-2}, \end{cases} \quad \text{and} \quad m_A = \begin{cases} 1.08 \text{ GeV}, & \text{LCSR-1}, \\ 1.23 \text{ GeV}, & \text{LCSR-2}. \end{cases}
\] (21)

We observe that the value of $m$ is practically insensitive to the variations of both positive and negative values of $\beta$.

When the experimental results will be obtained, there may appear the possibility to compare our predictions with experimental data.

Finally, we would like to note that our results can be improved by taking into account radiative corrections and the contributions of four and five particles as well as more precise determination of the input parameters.

## Conclusion

In the present work, we calculated the $N^*(1535) \rightarrow N$ transition form-factors induced by isovector and isoscalar axial currents within the light-cone QCD sum rules using the general form of the interpolating current. In performing numerical analysis, we used two sets of values of input parameters. We obtained that the form-factor $G_A(Q^2)$ exhibits the dipole form dependence on $Q^2$ and we find the values of relevant parameters, namely $g_A$ and $m_A$.

It was also obtained that $G_P^{(s)}$ shows a strong dependence on the variations of the auxiliary parameter $\beta$.

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Appendix A: \( N^\ast \) distribution amplitudes

In this Appendix, we present the \( N^\ast \) DAs, which are necessary to calculate the \( \Lambda \rightarrow N^\ast \) transition form-factors. The DAs of the \( N^\ast \) baryon are defined from the matrix element \( \langle 0 \left| e^{abc}u^a_\alpha(a_1x)d^b_\beta(a_2x)d^c_\gamma(a_3x) \right| N^\ast(p) \rangle \). The general decomposition of this matrix in terms of the DAs of the \( N^\ast \) baryon is given below. (see [21]),

\[
4\langle 0 \left| e^{abc}u^a_\alpha(a_1x)d^b_\beta(a_2x)d^c_\gamma(a_3x) \right| N^\ast(p) \rangle = S_1m_{N^\ast}C_{\alpha\beta}N^\ast_\gamma - S_2m^2_{N^\ast}C_{\alpha\beta}(\not p N^\ast)_\gamma \\
+ \mathcal{P}_1m_{N^\ast}(\gamma_5C)_{\alpha\beta}(\gamma_5N^\ast)_\gamma + \mathcal{P}_2m^2_{N^\ast}(\gamma_5C)_{\alpha\beta}(\gamma_5\not p N^\ast)_\gamma - \left( \mathcal{V}_1 + \frac{x^2m^2_{N^\ast}}{4}\not{p} M \right) (\not p C)_{\alpha\beta}N^\ast_\gamma \\
+ \mathcal{V}_2m_{N^\ast}(\not p C)_{\alpha\beta}(\not p N^\ast)_\gamma + \mathcal{V}_3m_{N^\ast}(\not p C)_{\alpha\beta}(\not p \gamma_5N^\ast)_\gamma - \mathcal{V}_4m^2_{N^\ast}(\not p C)_{\alpha\beta}N^\ast_\gamma \\
- \mathcal{V}_5m^2_{N^\ast}(\gamma_5C)_{\alpha\beta}(\gamma_5\not p N^\ast)_\gamma + \mathcal{V}_6m^3_{N^\ast}(\gamma_5C)_{\alpha\beta}(\not p N^\ast)_\gamma \\
- \left( \mathcal{T}_1 + \frac{x^2m^2_{N^\ast}}{4}\mathcal{T}_1^M \right) (i\sigma_{\mu\nu}p_\mu C)_{\alpha\beta}(\gamma^\mu N^\ast)_\gamma + \mathcal{T}_2m_{N^\ast}(i\sigma_{\mu\nu}x^\mu p^\nu C)_{\alpha\beta}N^\ast_\gamma \\
+ \mathcal{T}_3m_{N^\ast}(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^\mu N^\ast)_\gamma + \mathcal{T}_4m_{N^\ast}(\sigma_{\mu\nu}p^\nu C)_{\alpha\beta}(\sigma^{\mu\nu}x_\rho N^\ast)_\gamma \\
- \mathcal{T}_5m^2_{N^\ast}(i\sigma_{\mu\nu}x^\nu C)_{\alpha\beta}(\gamma^\mu N^\ast)_\gamma - \mathcal{T}_6m^2_{N^\ast}(i\sigma_{\mu\nu}x^\mu p^\nu C)_{\alpha\beta}(\not p N^\ast)_\gamma \\
- \mathcal{T}_7m^2_{N^\ast}(\sigma_{\mu\nu}C)_{\alpha\beta}(\sigma^\mu N^\ast)_\gamma + \mathcal{T}_8m^3_{N^\ast}(\sigma_{\mu\nu}x^\nu C)_{\alpha\beta}(\sigma^{\mu\nu}x_\rho N^\ast)_\gamma .
\]

The functions labeled with calligraphic letters in the above expression do not possess definite twists but they can be written in terms of the \( N^\ast \) distribution amplitudes (DAs) with definite and increasing twists via the scalar product \( p \cdot x \) and the parameters \( a_i, i = 1, 2, 3 \). The relations between the two sets of DAs for the \( N^\ast \), and for the scalar, pseudo-scalar, vector, axial-vector, and tensor DAs for nucleons are:

\[
S_1 = S_1 \]
\[
2(p \cdot x)S_2 = S_1 - S_2 \]
\[
\mathcal{P}_1 = P_1 \]
\[
2(p \cdot x)P_2 = P_2 - P_1 \]
\[
\mathcal{V}_1 = V_1 \]
\[
2(p \cdot x)V_2 = V_1 - V_2 - V_3 \]
\[
2V_3 = V_3 \]
\[
4(p \cdot x)V_4 = -2V_1 + V_3 + V_4 + 2V_5 \]
\[
4(p \cdot x)V_5 = V_4 - V_3 \]
\[
4(p \cdot x)^2V_6 = -V_1 + V_2 + V_3 + V_4 + V_5 - V_6 \]
\[
\mathcal{A}_1 = A_1 \]
\[
2(p \cdot x)A_2 = -A_1 + A_2 - A_3 \]
\[
2A_3 = A_3 \]
\[ 4(p \cdot x) A_4 = -2A_1 - A_3 - A_4 + 2A_5 \]
\[ 4(p \cdot x) A_5 = A_3 - A_4 \]
\[ 4(p \cdot x)^2 A_6 = A_1 - A_2 + A_3 + A_4 - A_5 + A_6 \]
\[ T_1 = T_1 \]
\[ 2(p \cdot x) T_2 = T_1 + T_2 - 2T_3 \]
\[ 2T_3 = T_7 \]
\[ 2(p \cdot x) T_4 = -T_1 + T_3 + 2T_8 \]
\[ 4(p \cdot x)^2 T_5 = 2T_3 - 2T_4 + 2T_5 + 2T_7 + 2T_8 \]
\[ 4(p \cdot x) T_7 = T_7 - T_8 \]
\[ 4(p \cdot x)^2 T_8 = -T_1 + T_2 + T_3 - T_6 + 2T_7 + 2T_8 \]

where the terms in \( x^2, V_i^M, A_i^M \) and \( T_i^M \) are left aside.

The distribution amplitudes \( F[a_i(p \cdot x)] = S_i, P_i, V_i, A_i, T_i \) can be represented as:
\[
F[a_i(p \cdot x)] = \int dx_1 dx_2 dx_3 \delta(x_1 + x_2 + x_3 - 1) e^{i p \cdot x_i} F(x_i) .
\]

where, \( x_i \) with \( i = 1, 2 \) and \( 3 \) are longitudinal momentum fractions carried by the participating quarks.

The explicit expressions for the \( \Lambda \) DAs up to twist 6 are given as:

**Twist–3 DAs:**
\[
V_1(x_1, \mu) = 120 x_1 x_2 x_3 \left[ \phi_3^0(\mu) + \phi_3^+(\mu)(1 - 3x_3) \right] ,
\]
\[
A_1(x_1, \mu) = 120 x_1 x_2 x_3 (x_2 - x_1) \phi_3^-(\mu) ,
\]
\[
T_1(x_1, \mu) = 120 x_1 x_2 x_3 \left[ \phi_3^0(\mu) - \frac{1}{2} (\phi_3^+ - \phi_3^-)(\mu)(1 - 3x_3) \right] .
\]

**Twist–4 DAs:**
\[
V_2(x_1, \mu) = 24 x_1 x_2 \left[ \phi_4^0(\mu) + \phi_4^+(\mu)(1 - 5x_3) \right] ,
\]
\[
A_2(x_1, \mu) = 24 x_1 x_2 (x_2 - x_1) \phi_4^-(\mu) ,
\]
\[
T_2(x_1, \mu) = 24 x_1 x_2 \left[ \xi_4^0(\mu) + \xi_4^+(\mu)(1 - 5x_3) \right] ,
\]
\[
V_3(x_1, \mu) = 12 x_1 \left[ \psi_3^0(\mu)(1 - x_3) + \psi_3^+(\mu)(1 - x_3 - 10x_1 x_2) + \psi_3^-(\mu) (x_1^2 + x_2^2 - x_3(1 - x_3)) \right] ,
\]
\[
A_3(x_1, \mu) = 12 x_1 (x_2 - x_1) \left[ (\psi_3^0 + \psi_3^+(\mu) (\mu)(1 - 2x_3) \right] ,
\]
\[
T_3(x_1, \mu) = 6 x_1 \left[ (\phi_4^0 + \psi_4^0 - \xi_4^0)(\mu)(1 - x_3) + (\phi_4^+ + \psi_4^+ + \xi_4^+)(\mu)(1 - x_3 - 10x_1 x_2) \right] ,
\]
\[
S_1(x_1, \mu) = 6 x_3 (x_2 - x_1) \left[ (\phi_4^0 + \psi_4^0 - \xi_4^0)(\mu)(1 - x_3) + (\phi_4^+ + \psi_4^+ - \xi_4^+)(\mu)(1 - x_3 - 10x_1 x_2) \right] ,
\]
\[
P_1(x_1, \mu) = 6 x_3 (x_1 - x_2) \left[ (\phi_4^0 + \psi_4^0 - \xi_4^0 + \phi_4^+ + \psi_4^+ - \xi_4^+)(\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) + \psi_3^- (\mu)(2x_1 x_2 - x_3(1 - x_3)) \right] ,
\]

**Twist–5 DAs:**
\[
V_4(x_1, \mu) = 3 \left[ \psi_5^0(\mu)(1 - x_3) + \psi_5^+ (\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) + \psi_5^- (\mu)(2x_1 x_2 - x_3(1 - x_3)) \right] ,
\]
\[ A_4(x_i, \mu) = 3(x_2 - x_1) \left[ -\psi_5^0(\mu) + \psi_5^+ (\mu)(1 - 2x_3) + \psi_5^- (\mu)x_3 \right], \]
\[ T_4(x_i, \mu) = \frac{3}{2} \left[ (\phi_5^0 + \psi_5^0 + \xi_5^0)(\mu)(1 - x_3) + (\phi_5^+ + \psi_5^+ + \xi_5^+)(\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) + (\phi_5^- - \psi_5^- + \xi_5^-)(\mu)(2x_1x_2 - x_3(1 - x_3)) \right], \]
\[ T_8(x_i, \mu) = \frac{3}{2} \left[ (\phi_5^0 + \psi_5^0 - \xi_5^0)(\mu)(1 - x_3) + (\phi_5^+ + \psi_5^+ - \xi_5^+)(\mu)(1 - x_3 - 2(x_1^2 + x_2^2)) + (\phi_5^- - \psi_5^- - \xi_5^-)(\mu)(2x_1x_2 - x_3(1 - x_3)) \right], \]
\[ V_5(x_i, \mu) = 6x_3 \left[ \phi_5^0(\mu) + \phi_5^+ (\mu)(1 - 2x_3) \right], \]
\[ A_5(x_i, \mu) = 6x_3(2x_2 - x_1)\phi_5^- (\mu), \]
\[ T_5(x_i, \mu) = 6x_3 \left[ \xi_5^0(\mu) + \xi_5^+ (\mu)(1 - 2x_3) \right], \]
\[ S_2(x_i, \mu) = \frac{3}{2}(x_2 - x_1) \left[ - (\phi_5^0 + \psi_5^0 + \xi_5^0)(\mu) + (\phi_5^+ + \psi_5^+ + \xi_5^+)(\mu)(1 - 2x_3) + (\phi_5^- - \psi_5^- + \xi_5^-)(\mu)x_3 \right], \]
\[ P_2(x_i, \mu) = \frac{3}{2}(x_2 - x_1) \left[ - (\phi_5^0 - \psi_5^0 + \xi_5^0)(\mu) + (\phi_5^0 - \psi_5^0 + \xi_5^0)(\mu)(1 - 2x_3) + (\phi_5^- - \psi_5^- + \xi_5^-)(\mu)x_3 \right]. \]

Twist-6:
\[ V_6(x_i, \mu) = 2 \left[ \phi_6^0(\mu) + \phi_6^+(\mu)(1 - 3x_3) \right], \]
\[ A_6(x_i, \mu) = 2(x_2 - x_1)\phi_6^-, \]
\[ T_6(x_i, \mu) = 2 \left[ \phi_6^0(\mu) - \frac{1}{2} (\phi_6^+ - \phi_6^-)(1 - 3x_3) \right]. \]

Finally the \(x^2\) corrections to the corresponding expressions \(V_1^M, A_1^M, T_1^M\) for the leading twist DAs \(V_1, A_1\) and \(T_1\) in the momentum fraction space are given as:
\[ V_1^M(x_2) = \int_{0}^{1-x_2} dx_1 V_1^M(x_1, x_2, 1 - x_1 - x_2) \]
\[ = \frac{x_2^2}{24} \left[ f_N C_f^u(x_2) + \lambda_1^{N^*} C_\lambda^u(x_2) \right], \]

where
\[ C_f^u(x_2) = (1 - x_2)^3 \left[ 113 + 495x_2 - 552x_2^2 - 10A_1^u(1 - 3x_2) \right. \]
\[ + 2V_1^d(113 - 951x_2 + 828x_2^2) \left], \]
\[ C_\lambda^u(x_2) = -(1 - x_2)^3 \left[ 13 - 20f_1^d + 3x_2 + 10f_1^u(1 - 3x_2) \right]. \]

The expression for the axial–vector function \(A_1^{M(u)}(x_2)\) is given as:
\[ A_1^{M(u)}(x_2) = \int_{0}^{1-x_2} dx_1 A_1^M(x_1, x_2, 1 - x_1 - x_2), \]

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\[ = \frac{x_2^2}{24}(1 - x_2)^3 \left[ f_{N^*}D_f^u(x_2) + \lambda_1^{N^*}D_\lambda^u(x_2) \right], \]

with

\[ D_f^u(x_2) = 11 + 45x_2 - 2A_1^u(113 - 951x_2 + 828x_2^2) + 10V_1^d(1 - 30x_2), \]
\[ D_\lambda^u(x_2) = 29 - 45x_2 - 10f_1^u(7 - 9x_2) - 20f_1^d(5 - 6x_2). \]

Similarly, we get for the function \( T_1^{M(u)}(x_2): \)

\[ T_1^{M(u)}(x_2) = \int_0^{1-x_2} dx_1 T_1^M(x_1, x_2, 1 - x_1 - x_2), \]
\[ = \frac{x_2^2}{48} \left[ f_{N^*}E_f^u(x_2) + \lambda_1^{N^*}E_\lambda^u(x_2) \right], \]

where

\[ E_f^u(x_2) = -\left\{ (1 - x_2) \left[ 3(439 + 71x_2 - 621x_2^2 + 587x_2^3 - 184x_2^4) \right. \right. \]
\[ + 4A_1^u(1 - x_2)^2(59 - 483x_2 + 414x_2^2) \]
\[ - 4V_1^d(1301 - 619x_2 - 769x_2^2 + 1161x_2^3 - 414x_2^4) \right\} - 12(73 - 220V_1^d)\ln|x_2|, \]
\[ E_\lambda^u(x_2) = -\left\{ (1 - x_2) \left[ 5 - 211x_2 + 281x_2^2 - 111x_2^3 + 10(1 + 61x_2 - 83x_2^2 + 33x_2^3)f_1^d \right. \right. \]
\[ - 40(1 - x_2)^2(2 - 3x_2)f_1^u \right\} - 12(3 - 10f_1^d)\ln|x_2|. \]

The following functions are encountered to the above amplitudes and they can be defined in terms of the 8 independent parameters, namely \( f_{N^*}, \lambda_1, \lambda_2 \) and \( f_1^u, f_1^d, f_2^d, A_1^u, V_1^d). \)

\[ \phi_3^0 = \phi_6^0 = f_{N^*}, \]
\[ \phi_4^0 = \phi_5^0 = \frac{1}{2}(f_{N^*} + \lambda_1^{N^*}) \]
\[ \xi_4^0 = \xi_5^0 = \frac{1}{6}\lambda_2^{N^*} \]
\[ \psi_4^0 = \psi_5^0 = \frac{1}{2}(f_{N^*} - \lambda_1^{N^*}), \]
\[ \phi_3^- = \frac{21}{2}f_{N^*}A_1^u, \quad \phi_3^+ = \frac{7}{2}\lambda_1^{N^*}f_{N^*} \ln|1 - V_1^d|, \]
\[ \phi_4^+ = \frac{1}{4}[f_{N^*}(3 - 10V_1^d) + \lambda_1^{N^*}(3 - 10f_1^d)], \]
\[ \phi_4^- = -\frac{5}{4}[f_{N^*}(1 - 2A_1^u) - \lambda_1^{N^*}(1 - 2f_1^d - 4f_1^u)], \]
\[ \psi_4^- = -\frac{1}{4}[f_{N^*}(2 + 5A_1^u - 5V_1^d) - \lambda_1^{N^*}(2 - 5f_1^d - 5f_1^u)], \]
\[ \psi_4^+ = \frac{5}{4}[f_{N^*}(2 - A_1^u - 3V_1^d) - \lambda_1^{N^*}(2 - 7f_1^d + f_1^u)]. \]
\[ \xi_4^+ = \frac{1}{16} \lambda_2^{N^*} (4 - 15 f_2^d) , \]
\[ \xi_4^- = \frac{5}{16} \lambda_2^{N^*} (4 - 15 f_2^d) , \]
\[ \phi_5^+ = -\frac{5}{6} \left[ f_{N^*} (3 + 4 V_1^d) - \lambda_1^{N^*} (1 - 4 f_1^d) \right] , \]
\[ \phi_5^- = -\frac{5}{3} \left[ f_{N^*} (1 - 2 A_1^u) - \lambda_1^{N^*} (f_1^d - f_1^u) \right] , \]
\[ \psi_5^+ = -\frac{5}{6} \left[ f_{N^*} (5 + 2 A_1^u - 2 V_1^d) - \lambda_1^{N^*} (1 - 2 f_1^d - 2 f_1^u) \right] , \]
\[ \psi_5^- = \frac{5}{3} \left[ f_{N^*} (2 - A_1^u - 3 V_1^d) + \lambda_1^{N^*} (f_1^d - f_1^u) \right] , \]
\[ \xi_5^+ = \frac{5}{36} \lambda_2^{N^*} (2 - 9 f_2^d) , \]
\[ \xi_5^- = -\frac{5}{4} \lambda_2^{N^*} f_2^d , \]
\[ \phi_6^+ = \frac{1}{2} \left[ f_{N^*} (1 - 4 V_1^d) - \lambda_1^{N^*} (1 - 2 f_1^d) \right] , \]
\[ \phi_6^- = \frac{1}{2} \left[ f_{N^*} (1 + 4 A_1^u) + \lambda_1^{N^*} (1 - 4 f_1^d - 2 f_1^u) \right] , \]

where the parameters \( A_1^u, V_1^d, f_1^d, f_1^u, \) and \( f_2^d \) are defined as [21],

\[
A_1^u = \varphi_{10} + \varphi_{11} ,
\]
\[
V_1^d = \frac{1}{3} - \varphi_{10} + \frac{1}{3} \varphi_{11} ,
\]
\[
f_1^u = \frac{1}{10} - \frac{1}{6} \lambda_1^{N^*} - \frac{3}{5} \eta_{10} - \frac{1}{3} \eta_{11} ,
\]
\[
f_1^d = \frac{3}{10} - \frac{1}{6} \lambda_1^{N^*} + \frac{1}{5} \eta_{10} - \frac{1}{3} \eta_{11} ,
\]
\[
f_2^d = \frac{4}{15} + \frac{2}{5} \xi_{10} .
\]

The numerical values of the parameters \( \varphi_{10}, \varphi_{11}, \varphi_{20}, \varphi_{21}, \varphi_{22}, \eta_{10}, \eta_{11}, \) and \( f_{N^*}/\lambda_1^{N^*}, \)
and \( \lambda_1^{N^*}/\lambda_1^N \) are presented in Table 2 (this Table is taken from [21]).

| Model   | \( \lambda_1^{N^*}/\lambda_1^N \) | \( f_{N^*}/\lambda_1^{N^*} \) | \( \varphi_{10} \) | \( \varphi_{11} \) | \( \varphi_{20} \) | \( \varphi_{21} \) | \( \varphi_{22} \) | \( \eta_{10} \) | \( \eta_{11} \) |
|---------|----------------------------------|-------------------------------|-----------------|-----------------|----------------|----------------|----------------|----------------|----------------|
| LCSR–1  | 0.633                            | 0.027                         | 0.36            | -0.95           | 0              | 0              | 0              | 0              | 0.94           |
| LCSR–2  | 0.633                            | 0.027                         | 0.37            | -0.96           | 0              | 0              | 0              | -0.29          | 0.23           |
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Figure captions

Fig. (1) The dependence of the form-factor $G_A(Q^2)$ on $Q^2$ for various values of $\beta$ with the data set LCSR-1.

Fig. (2) The same as Fig. (1), but with the LCSR-2.

Fig. (3) The same as Fig. (1), but for $G_A^S(Q^2)$.

Fig. (4) The same as Fig. (3), but with LCSR-2.

Fig. (5) The same as Fig. (1), but for $G_P(Q^2)$.

Fig. (6) The same as Fig. (5), but with LCSR-2.

Fig. (7) The same as Fig. (1), but for $G_P^S(Q^2)$.

Fig. (8) The same as Fig. (7), but with LCSR-2.
$s_0 = 2.25 \text{ GeV}^2, \ M^2 = 2 \text{ GeV}^2, \text{ LCSR-1}$

Figure 1:
$s_0 = 2.25 \text{ GeV}^2$, $M^2 = 2 \text{ GeV}^2$, LCSR-2

Figure 2:
\(s_0 = 2.25 \text{ GeV}^2, \ M^2 = 2 \text{ GeV}^2, \ \text{LCSR-1}\)

\[ s_0 = 2.25 \text{ GeV}^2, \ M^2 = 2 \text{ GeV}^2, \ \text{LCSR-1} \]

\[ G_A(Q^2) \]

\[ Q^2 \ (\text{GeV}^2) \]

\[ \beta = -5 \quad \beta = 1 \]
\[ \beta = -3 \quad \beta = 1.4 \]
\[ \beta = -2 \quad \beta = 2 \]
\[ \beta = -1.4 \quad \beta = 3 \]
\[ \beta = -1 \quad \beta = 5 \]

Figure 3:
\( s_0 = 2.25 \text{ GeV}^2, \ M^2 = 2 \text{ GeV}^2, \ \text{LCSR-2} \)

\[ G_{SA}(Q^2) \]

\( Q^2 (\text{GeV}^2) \)

\( \beta \leq 0 \)

\( \beta = -5 \)
\( \beta = -3 \)
\( \beta = -2 \)
\( \beta = -1.4 \)
\( \beta = -1 \)

\( \beta \geq 0 \)

\( \beta = 1 \)
\( \beta = 1.4 \)
\( \beta = 2 \)
\( \beta = 3 \)
\( \beta = 5 \)

Figure 4:
$s_0 = 2.25 \text{ GeV}^2$, $M^2 = 2 \text{ GeV}^2$, LCSR-1

Figure 5:
$s_0 = 2.25 \text{ GeV}^2, \ M^2 = 2 \text{ GeV}^2$, LCSR-2

Figure 6:
$s_0 = 2.25 \text{ GeV}^2$, $M^2 = 2 \text{ GeV}^2$, LCSR-1

Figure 7:
$s_0 = 2.25 \text{ GeV}^2$, $M^2 = 2 \text{ GeV}^2$, LCSR-2

Figure 8: