Quantum Dot in the Kondo Regime coupled to p-wave superconductors

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This paper studies the physics of junctions containing superconducting (S) and normal (N) leads weakly coupled to an Anderson impurity in the Kondo regime (K). Special attention is devoted to the case where one of the leads is a $p - \text{wave}$ superconductor where mid-gap surface states play an important role in the tunneling processes and help the formation of Kondo resonance. The novel physics in these systems beyond that encountered in quantum dots coupled only to normal leads is that electron transport at finite bias $eV$ in $SKN$ and $SKS$ junctions is governed by Andreev reflections. These enable the occurrence of dissipative current even when the bias $eV$ is smaller than the superconducting gap $\Delta$. Using the slave boson mean field approximation the current, shot-noise power and Fano factor are calculated as functions of the applied bias voltage in the sub-gap region $eV < \Delta$ and found to be strongly dependent on the ratio $t_K$ between the Kondo temperature $T_K$ and the superconducting gap $\Delta$. In particular, for large values of $t_K$ the attenuation of current due to the existence of the superconducting gap is compensated by the Kondo effect. This scenario is manifested also in the behavior of the Josephson current as function of temperature.

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INTRODUCTION

The Kondo effect is one of the simplest manifestations of many-body physics exhibiting strong correlations. Originally, it has been introduced in order to explain the occurrence of a shallow minimum of the resistivity $\rho(T)$ of bulk metals doped with a small concentration of magnetic impurities. It has been confirmed that at low temperatures (below the Kondo temperature $T_K$) scattering from magnetic impurities is strong, leading to an enhanced resistivity. It is now well established that at this strong coupling regime the theory of perturbation fails due to logarithmic divergences of higher (than second) order diagrams.

The Kondo physics also plays an important role in electron transport through quantum dots, where instead of a magnetic impurity one now encounters localized electrons. Inspection of the dot-electron Green function reveals an enhancement of the local density of states at the Fermi energy (the Abrikosov-Suhl resonance). Thus, unlike the case of Kondo effect in bulk metals, its hallmark experimental manifestation in quantum dots is an enhancement of the conductance. An important milestone in this field has been recorded by recent observations of the Kondo effect in transport through quantum dots and in a finite length carbon nanotubes. These experiments have paved the way for a new class of experimental investigations into the physics of strongly correlated electrons in general and the Kondo effect in particular. Together with more recent experiments demonstrate the feasibility of exploiting tunable physical parameters of a quantum dot system in order to yield important information on Kondo systems and other many-body related phenomena. Such studies, for example, encompass the entire crossover region between the Kondo limit, the mixed valence regime and the non-Kondo (weak-coupling) domain.

While the Kondo physics in a quantum dot attached on both its sides to normal (N) metallic leads received much recent attention, it has been realized that novel physical effects emerge if (one or both) electrodes attached through a tunneling barrier to a quantum dot in the Kondo regime ($K$) is a superconductor ($S$). The central electron-transport mechanism in such $SKS$ and $SKN$ junctions is that of Andreev reflections when two particles tunnel together coherently to form a Cooper pair in the superconductor. In fact, the physics of $SKS$ and $SKN$ junctions is determined by the interplay of these Andreev reflections with the Kondo-formed resonance in the spectral density of states of the dot electron. This interplay between the Kondo effect and superconductivity in the sub-gap region is rather effective here. We then expect the the $I - V$ characteristics in the sub-gap region to be most important. Andreev reflections play a dominant role both in and out of equilibrium. In the former case they are responsible for occurrence of direct Josephson current while in the latter case they are the cause of dissipative currents at sub-gap voltages as well as for suppressing the zero bias anomaly in $NS$ junctions. The effect becomes even more significant when the effective Kondo temperature $T_K$ exceeds the superconducting energy gap $\Delta$. A natural candidate for such $SKN$ junction is that in which the role of the quantum dot or the Kondo impurity is played by a Carbon nano-tube (CNT), where relatively high values of $T_K$ can be achieved. Indeed, Kondo effect with rather high Kondo temperature ($T_K > 1K$) has been reported in $N - CNT - N$ junctions with a few conducting channels. Moreover, superconducting junctions with a weak link formed by CNT have already been fabricated.

Let us illuminate the somewhat subtle distinction between $s - \text{wave}$ and $p - \text{wave}$ $SKN$ junctions. In the case of an $SKN$ junction when one electrode is an $s - \text{wave}$ superconductor and the Kondo impurity is weakly coupled to the $S$ and $N$ electrodes, is determined by the
competition between two important effects. The first one is the formation of a Kondo singlet which screens the bare impurity spin and drives the system toward the unitary limit at very low temperatures. The second one is the existence of the superconducting gap which implies a vanishingly small density of low energy electron states \([2, 3, 4]\). These are precisely the electron states which are needed in order to screen the Kondo impurity. Note that if the Kondo temperature is larger than the superconducting gap \((T_K > \Delta)\) then electron states outside the gap can participate in the screening interaction. However, this condition cannot be easily realized. Consider, on the other hand, an SKN junction in which the \(S\) electrode consists of an unconventional superconductor which is oriented relative to the interface in such a way that the pair potential reverses its sign on the Fermi surface. In this case, mid-gap zero-energy states (ZES) are formed which are localized near the surface of the unconventional superconductor. These states can now participate in screening the impurity spin through the Kondo effect and emergence of sub-gap current is expected. In the experimentally feasible setup of \(S - CNT - N\) where just a few tunneling channels are present, charge is carried mainly by quasiparticles moving perpendicular to the interface. This restricts the possible values of the angle \(\theta\) between the superconducting surface and the direction of the injected quasiparticles \([13]\). Formation of ZES is possible when \(\Delta(\theta) = -\Delta(\pi - \theta)\). If the impurity is almost point-like, the relevant injection angle is of course \(\theta = 0\). Unconventional superconductors for which the symmetry of the pair potential is that of a triplet \((p - wave)\) superconductors satisfy the above condition. These have recently been discovered by Maeno, \textit{et al}\[10\] in \(Sr_2RuO_4\). For impurities of finite extent, one may also consider formation of ZES in \(d - wave\) superconductors. Thus, the physics of SKN junctions with an \(S\) electrode whose order parameter has a non-trivial symmetry is affected by the formation of ZES in the Kondo regime.

Beyond investigating the conductance dependence on the applied bias we propose and explore theoretically other novel experimental tools suitable for probing the Kondo regime. These include shot-noise measurements and the Josephson (direct) current. Like the conductance, we will calculate and analyze shot-noise power spectrum in SKN junctions at very low temperature where one lead is either an \(s - wave\) superconductor (mid-gap surface states are absent) or a \(p - wave\) superconductor (mid-gap surface states are present). We will also study the temperature dependence of the Josephson current in these junctions.

In order to study this strongly interacting non-equilibrium problem in the framework of workable approximation, we restrict ourselves to voltages that do not exceed the superconducting gap so that the Kondo temperature can satisfy the inequality \(T_K > \text{max}(\Delta, eV)\). Under this condition we can apply the well known approach familiar in the physics of the Anderson impurity Hamiltonian \([17]\), namely the mean field slave boson approximation (MFSBA) (see below). It consists of a workable scheme for calculating the conductance and the zero frequency shot-noise power. (Here we also apply the MFSBA to calculate Josephson current in SKS junctions.)

The combined effect of multiple Andreev reflections (MAR) and electron-electron interactions on the shot-noise in SKS junctions was studied in Ref. \([8, 16, 20]\). It was shown that in the strong coupling Kondo regime the effective transmission is enhanced, and a ballistic-like channel opens up. Hence, an interplay between MAR and the Kondo resonance yields an excess current at zero bias and the \(I - V\) curve behaves similarly to that of noninteracting ballistic junctions. At very high values of \(T_K\) and in the low voltage limit the current approaches the noninteracting value \(I_{AR} = 4e\Delta/h\). Analogously, the shot-noise power is shown to display a pronounced maximum at \(V = 0\) and a decay as \(1/V\) at small bias, familiar in the standard theory of noninteracting SNS junctions \([21]\).

In section II the Hamiltonian of SKN and SKS junctions is defined, and the bare action is introduced. After integrating out the fermion fields pertaining to the superconducting leads an effective action is obtained, depending solely on the dot variables. Calculations and presentations of conductance, shot-noise power and Josephson current are respectively detailed in sections III, IV and V.

**MODEL HAMILTONIAN AND EFFECTIVE ACTION**

The dynamics of systems like SKN or SKS junctions is governed by the Hamiltonian

\[
H = H_L + H_R + H_d + H_t + H_c,
\]

in which \(H_j\) (\(j = L, R\)) are the Hamiltonians of the electrodes which depend on the electron field operators \(\psi_{\sigma}(r, t)\) where \(r = (x,y)\) and \(\sigma = \pm \) is the spin index.

\[
j = \int dr [\Psi_{j\sigma}^\dagger(r)\xi(\nabla)\Psi_j - \gamma\Psi_{j\sigma}^\dagger(r)\Psi_{j\pi}^\dagger(r)\Psi_{j\pi}(r)\Psi_{j\sigma}(r)].
\]

Here \(\gamma\) is the BCS coupling constant and \(\xi(\nabla) = -\nabla^2/2m - \mu\) with \(\mu\) being the chemical potential at temperature \(T\). The Planck constant is set \(\hbar = 1\) and whenever appropriate, the spin, space and time dependence of all the field operators will not be explicitly displayed. As in Refs. \([8, 22]\) the dot is represented by a single level Anderson impurity with energy \(\epsilon_0 < 0\) and Hubbard repulsion parameter \(U\). In the Kondo regime of interest here we set \(U \rightarrow \infty\) and assume \(|\epsilon_0|\) to exceed any other energy scale except \(U\). In this case it is convenient to express the dot and the tunneling Hamiltonians \(H_d\) and \(H_t\) via slave boson (operators \(b, b^\dagger\)) and slave fermion
(operators \(c, c^\dagger\)) auxiliary fields\(^{23}\). Explicitly,

\[
H_d = \epsilon_0 \sum_{\sigma} c^\dagger_{\sigma} c_{\sigma},
\]
\[
H_t = \sum_{j} T_j c^\dagger_{\alpha} b_j \psi_{j\sigma}(0, t) + h.c.,
\]
where \(T_j\) is the tunneling amplitude. Finally, the Hamiltonian of the system must also include a term which prevents double occupancy in the limit \(U \to \infty\). This term reads,

\[
H_c = \lambda \sum_{\sigma} (c^\dagger_{\sigma} c_{\sigma} + b^\dagger b - 1),
\]

where \(\lambda\) is a Lagrange multiplier.

Following Ref. \(^{18}\) let us consider the dynamical “partition function”

\[
Z \sim \int \mathcal{D}[F] \exp(iS),
\]

where the path integral is carried out over all fields \([F]\) and the action \(S\) is obtained by integrating the Lagrangian pertaining to the Hamiltonian \([1]\) along the Keldysh contour. In performing the functional integrations the boson field operators are treated as \(c\)-numbers. As a result one arrives at an effective action expressed in terms of the Green functions of the leads,

\[
S_{\text{eff}} = -i \text{Tr} \ln \hat{G}^{-1} - \int dt [\hat{\sigma}_z(\hat{b} \hat{b} - 1)].
\]

Here \(\hat{\lambda} = (\lambda_1, \lambda_2), \hat{b} = (b_1, b_2)\) and \(\sigma_z\) are diagonal matrices acting in Keldysh space. The inverse propagator \(\hat{G}^{-1}\) depends on the Green functions of the electrodes\(^{22}\).

Performing the standard basis rotation in Keldysh space one finds,

\[
\hat{G}^{-1}(\epsilon, \epsilon') = \delta(\epsilon - \epsilon')(\epsilon - \tau_z \hat{\epsilon} - \frac{\Gamma^2}{2} \tau_{\pm} \hat{g}_{\pm}(\epsilon) \tau_{\pm}),
\]

where \(\hat{\epsilon} = \epsilon_0 + \lambda\) is the renormalized level position (in the Kondo limit one has \(\hat{\epsilon} \approx 0\)) and \(\Gamma = (\Gamma_L + \Gamma_R)/2 \propto T_{\text{el,R}}^2\) is the usual transparency parameter. The \(2 \times 2\) matrix representation (in Keldysh space) for \(g\) is composed of diagonal elements \(\hat{g}^{R/A}(\epsilon)\) and an upper off-diagonal element \(\hat{g}^{K}(\epsilon) = (\hat{g}^R - \hat{g}^A) \text{th}(\epsilon/2T)\). Here and below we define

\[
\hat{g}_{\pm} = \gamma_3 \hat{g}_L \pm \gamma_R \hat{g}_R,
\]

with asymmetry parameters \(\gamma_3 = \Gamma_j/\Gamma\). The matrix \(\hat{g}_R\) has the structure with retarded and advanced superconductor Green functions which in the \(s\)-wave case reads,

\[
\hat{g}^{R/A}(\epsilon) = i \left( \frac{\epsilon \pm i0}{} \right) + \frac{|\Delta| \tau_x}{\sqrt{(\epsilon \pm i0)^2 - |\Delta|^2}},
\]

while for the \(p\)-wave case with incidence angle \(\alpha\) it reads,

\[
\hat{g}^{R/A}(\epsilon) = \hat{g}_1^{R/A} + \hat{g}_2^{R/A} \tau_x,
\]

with

\[
\hat{g}_1^{R/A} = \frac{i \sqrt{|\epsilon |^2 - |\Delta|^2} \cos \alpha - \epsilon \sin \alpha}{\epsilon \cos \alpha + i \sqrt{|\epsilon |^2 - |\Delta|^2} \sin \alpha},
\]
\[
\hat{g}_2^{R/A} = \frac{i |\Delta|}{\epsilon \cos \alpha + i \sqrt{|\epsilon |^2 - |\Delta|^2} \sin \alpha}.
\]

Note that the Pauli matrices \(\tau_x, \tau_y, \tau_z\) act in Nambu space.

For a one channel system one can take \(\alpha = 0\) and simplify the expression for the Green functions,

\[
\hat{g}_1^{R/A} = \frac{i \sqrt{|\epsilon |^2 - |\Delta|^2}}{\epsilon \pm 0},
\]
\[
\hat{g}_2^{R/A} = \frac{i |\Delta|}{\epsilon \pm 0}.
\]

In the case of \(NK\) junction, the left lead is represented by the Keldysh Green’s function of a normal metal \(\hat{g}_L\): \(\hat{g}_L^{A/R} = \pm i\); the Keldysh Green function \(\hat{g}_R^K\) has only diagonal matrix elements equal to \(2i\text{th}(|\epsilon | \pm eV)/2T\). Performing the variation of the effective action with respect to the fields \(b\) and \(\lambda\) a couple of self-consistency equations are obtained that determine these fields. In order to explicitly write down these self-consistency equations let us introduce the bare Kondo temperature \(T_K = \text{Dexp}\left(-\pi|\epsilon_0|/(2|\Gamma|)\right)\) and define a parameter \(X\) by \(\text{Tr}b^2 = T_K^2 X\), where \(D\) is the energy bandwidth. Then the MFSBA equations take the form

\[
X = -\frac{i\Gamma}{2T_K} \text{Tr} \hat{G}^{K}\tau_z,
\]
\[
\lambda = \frac{i\Gamma}{8} \text{Tr} [\hat{G}^{K}\tau_z(\hat{g}_+^R + \hat{g}_+^A) + (\hat{G}^R + \hat{G}^A)\tau_z \hat{g}_+^K]\tau_\pm(4),
\]

where the trace includes energy integration as well. Eq. \(13\) effectively determines the Kondo temperature (through the parameter \(X\)), and reflects the constraint which prevents double occupancy in the limit \(U \to \infty\). The second self-consistency equation \(14\) defines the renormalized energy level position \(\hat{\epsilon}\). Let us briefly discuss the validity range of the present analysis. The MFSBA is known to encode the Kondo Fermi-liquid behavior at low temperatures. An important parameter here is the ratio between the Kondo temperature and the superconducting gap \(t_K \equiv T_K/|\Delta|\). For \(t_K \gtrsim 1\) a Fermi liquid behavior is expected. Accordingly, in this regime Eq. \(13\) should have a nonzero solution \(X \neq 0\) which corresponds to nonzero \(T_K\). On the other hand, in the limit of large \(\Delta\) the only possible solution is the trivial one \(b = 0\) (and, hence, \(T_K = 0\)). Quantitatively, the MFSBA is reliable only for sufficiently large values of \(t_K\). We believe, however, that it can provide useful qualitative information also for moderate values of \(t_K\) describing a crossover between the Kondo regime and the Coulomb blockade dominant domain\(^{23}\). It is worth noting here...
that, strictly speaking, the applied bias voltage $V$ attenuates the Kondo resonance and lowers $T_K$. Hence, for the reliability of the MFSBA in non-equilibrium situations, both $\Delta$ and $eV$ should not exceed the Kondo temperature. Attention below is mainly focused on the sub-gap voltage regime $eV \lesssim \Delta$ in which case $t_K$ appears to be the only relevant parameter.

**CONDUCTANCE**

The expression for the tunneling current obtained in Ref. [9] has a simple representation in terms of the dot Green function,

$$I = \frac{eX_{t_K}}{8\hbar} \text{Tr}[(\hat{G}^R\tau_z - \tau_z\hat{G}^A)\tilde{g}_K^L - \tilde{G}_K^L\tilde{g}],$$

(15)

where for SKN junctions we denote

$$\tilde{g} = -\gamma_R(\hat{g}^R\tau_z - \tau_z\hat{g}^A) - 2i\gamma_L\tau_z,$$

$$\tilde{g}_R^L = -\gamma_R(\tau_z\hat{g}_R^L + \hat{g}_R^L\tau_z).$$

The last quantity will be used later on as it enters the expression for the noise power spectrum. Being combined with eqs. (13) and (14) the result (15) can be conveniently used for computing the transport current and the differential conductance of an SKN junction in the Kondo regime for different values of $t_K$.

For sufficiently large $t_K$ we anticipate a strong Kondo resonance and the $G-V$ curves for both $s$-wave and $p$-wave superconductors is expected to resemble that of purely ballistic junctions (for $\gamma_R = \gamma_L = 1$) without interaction [4]. Indeed, in the limit of large $t_K \gg 1$ which corresponds to the unitary, pure ballistic case, our expression for the current (15) reduces to that derived in Ref. [9]. This agreement is further supported by our numerical calculations carried out for $t_K = 100$ (see Fig.1). Calculations of the current and conductance were also performed for $t_K = 5, 3, and 2$. The results are displayed in figure 1. For voltages in the sub-gap region, the conductance of an S(p-wave)KN junction is distinct from that of an S(s-wave)KN junction. Such difference shows that the ZES supports the formation of a Kondo singlet for the lower values $t_K = 5, 3$ and 2 and effectively turn the junction to be more transparent, approaching the BTK limit [3].

**Fig. 1** The conductance $G$ (in units of $e^2/h$) versus the bias $V$ (in units of $\Delta/\epsilon$) for an $s$-wave SKN (dash curves) and $Sp$-wave$KN$ (solid lines) junction at sub-gap voltages with $\Gamma/T_K^0 = 200$. The parameter $t_K = 2, 3, 5, 100$ (from down to top). The upper line corresponding to $t_K = 100$ coincides for $s$ and p wave superconducting leads.

Let us briefly summarize our results for the current and conductance of SKN junctions for $s$ and $p$ wave superconductors. In the limit of large $t_K$ the $G-V$ curve is practically independent of $t_K$ and resembles that of a ballistic junction, as indicated in the upper curve in Fig.1. For lower values of $t_K$, junctions with $s$-wave superconductor lead are driven away from the unitary limit and the underlying physics becomes much richer. It reflects the influence of both $\Delta$ and $V$ on the Kondo resonance and on the actual value of the Kondo temperature. For $t_K = 5$ the $G-V$ curve noticeably deviates from that obtained in the non-interacting limit. For $t_K = 2$ the competition between gap-related suppression of the Kondo effect and the effective transparency of the junction becomes essential, leading to further decrease of the conductance. However, it is interesting to note that for and S(s-wave)KN junction at $t_K = 2$ the conductance displays a small peak at the gap edge. Its interpretation is that the Kondo correlations strongly compete with superconductivity and influence the quasiparticle correlations when the energy exceeds the gap. Such an effect takes place even when $t_K < 1$ (see [4]). On the other hand, the S(p-wave)KN junction is less influenced to variations of $t_K$. This may be explained by the fact that due to the presence of ZES, superconductivity plays a minor role in the formation of the Kondo resonance and BTK behavior persists for smaller values of $t_K$. 


SHOT-NOISE

The shot-noise spectrum is usually defined as the symmetrized current-current correlation function [24, 25]

\[ K(t_1, t_2) = \hbar \langle [I(t_1)I(t_2)] - \langle I \rangle^2 \rangle, \]

(17)

where \( \langle ... \rangle \) denotes quantum averaging with the Hamiltonian [1]. The general expression for the zero frequency shot-noise power in junctions with one (or even two) superconducting leads (in MFSBA approximation) was obtained in our recent work [18]. Like in the case of SKS junctions, it is useful to write

\[ K = (K_1 + K_2)e^2\Delta/(8\hbar), \]

(18)

Expressions (18) and (19) (supplemented by the self-consistency eqs. (13) and (14)) are then solved numerically for the same set of parameters \( \Gamma/T_0 \) = 200, \( t_K = 100, 5, 3 \) and \( 2 \). The results for the shot-noise power spectrum \( K \) versus the applied voltage \( V \) are displayed in figure 2. These results are clearly correlated with those for the \( G-V \) curve and can be summarized as follows: In the limit \( t_K \gg 1 \) the characteristics of shot-noise power spectrum for both \( s \) and \( p \) wave superconductors are consistent with those obtained for purely ballistic junctions which exhibit strong suppressing of the shot-noise power in the sub-gap region. At lower \( t_K \) the physics is distinct. For \( t_K = 5 \) the noise spectrum for \( s \) wave superconducting lead still shows features typical for a junction with relatively high transparency, while the results for \( t_K = 2 \) are more similar to those for a low transparency junction. Such dependence is explicitly exposed in the plot of the Fano factor versus the applied voltage (see Fig. 3).

Though the Fano factor does not reach the maximum value of 2, it is strongly enhanced for the smaller value of \( t_K = 2 \). For \( p-wave \) superconductor the shot-noise power (as function of voltage) reflects the same physics as in the conductance: ZES makes the Kondo resonance less vulnerable to the impact of superconductivity and the junction remains close to the unitary limit nearly in the whole range of values of \( t_K \) considered here (see the solid curves on Fig. 2 and Fig. 3).

JOSEPHSON CURRENT

In this section we study an equilibrium property (Josephson effect) of an SKS junction in which both electrodes are \( p-wave \) superconductors (for comparison we also represent the results for \( s-wave \) superconducting leads). In the Kondo regime, the self-consistent equations
consistent equations for the case of phase difference between two superconductors. Self-consistent equations with respect to the phase of the order parameter, leads were derived in Ref. [20] as a function of the phase difference \( \theta \) close to 0. Dash and solid curves correspond to s and p-wave superconductors, correspondingly. The parameters are the same as in Fig.1. The value of \( t_K \) decreases from top downward.

In the s-wave case (dashed lines in figure 4) the current at \( t_K = 100 \) qualitatively corresponds to the unitary limit \([26]\), while its amplitude is decreased at smaller values of \( t_K \). Nevertheless, in this region of parameters, the Kondo effect strongly competes with superconductivity and therefore there are no traces of a \( \pi \)-junction. The mid-gap states for \( p-wave \) superconductors act as if they increase the effective normal region of the junction and therefore, the weight of the contribution to the Josephson current which is proportional to \( \sin \delta \) is reduced. When \( t_K \) becomes smaller, the Josephson current for \( p-wave \) junction tends to be similar to that in an SIS junction.

The Josephson current is obtained by variation of free energy with respect to the phase of the order parameter,

\[
I = \frac{2e}{h} \frac{\partial F}{\partial \delta}.
\]  

Explicitly we then find,

\[
I = \frac{e}{h} \sum_\omega \omega^2 (1 + \alpha(\omega))^2 \epsilon^2 + (\beta(\omega) \Delta^2 \cos^2 \frac{\epsilon}{2})
\]  

The self-consistency equation and the expression for the Josephson current can easily be extended to the case of an anisotropic coupling : \( \Gamma_{K/R} = \Gamma(1 \pm p) \). For this, one should replace \( \cos^2 \frac{\epsilon}{2} \rightarrow (\cos^2 \frac{\epsilon}{2} + p^2 \sin^2 \frac{\epsilon}{2}) \) and \( I \propto \sin \delta \rightarrow I \propto (1 - p^2) \sin \delta \).

For a small anisotropy parameter \( p = 0.1 \) and for values of \( t_K = 100, 5, 3, 2 \), we calculate the Josephson current as a function of the phase difference \( \delta \) at temperatures close to \( T = 0 \). The plot is depicted in Fig.4.

\[
\tilde{\epsilon} + \frac{2T}{\pi} \log \frac{\tilde{\epsilon}}{T_K} =
\sum_\omega \left[ \frac{2\Gamma(1 + \alpha(\omega)) \sqrt{\omega^2 + \Delta^2} + \beta(\omega) \Delta^2 \cos^2 \frac{\epsilon}{2}}{(1 + \alpha(\omega))^2 \omega^2 + \epsilon^2 + \beta(\omega) \Delta^2 \cos^2 \frac{\epsilon}{2}} - \frac{2\Gamma|\omega|}{\omega^2 + \epsilon^2} \right]
\]  

(20)

(21)

Here \( \alpha(\omega) = \tilde{\Gamma} \sqrt{\omega^2 + \Delta^2} / \omega, \beta(\omega) = \tilde{\Gamma} / \omega \) and \( \delta \) is the phase difference between two superconductors.

Fig. 4 Josephson current versus phase difference \( \delta \) at \( T \rightarrow 0 \). Dash and solid curves correspond to s and p-wave superconductors, correspondingly. The parameters are the same as in Fig.1. The value of \( t_K \) decreases from top downward.

In the s-wave case (dashed lines in figure 4) the current at \( t_K = 100 \) qualitatively corresponds to the unitary limit \([26]\), while its amplitude is decreased at smaller values of \( t_K \). Nevertheless, in this region of parameters, the Kondo effect strongly competes with superconductivity and therefore there are no traces of a \( \pi \)-junction. The mid-gap states for \( p-wave \) superconductors act as if they increase the effective normal region of the junction and therefore, the weight of the contribution to the Josephson current which is proportional to \( \sin \delta \) is reduced. When \( t_K \) becomes smaller, the Josephson current for \( p-wave \) junction tends to be similar to that in an SIS junction.

\[
\tilde{\epsilon} + \frac{2T}{\pi} \log \frac{\tilde{\epsilon}}{T_K} =
\sum_\omega \left[ \frac{2\Gamma(1 + \alpha(\omega)) \sqrt{\omega^2 + \Delta^2} + \beta(\omega) \Delta^2 \cos^2 \frac{\epsilon}{2}}{(1 + \alpha(\omega))^2 \omega^2 + \epsilon^2 + \beta(\omega) \Delta^2 \cos^2 \frac{\epsilon}{2}} - \frac{2\Gamma|\omega|}{\omega^2 + \epsilon^2} \right]
\]  

(20)

(21)

Here \( \alpha(\omega) = \tilde{\Gamma} \sqrt{\omega^2 + \Delta^2} / \omega, \beta(\omega) = \tilde{\Gamma} / \omega \) and \( \delta \) is the phase difference between two superconductors. Self-consistent equations for the case of s-wave superconducting leads were derived in Ref. [20].

The Josephson current is obtained by variation of free energy with respect to the phase of the order parameter,

\[
I = \frac{2e}{h} \frac{\partial F}{\partial \delta}.
\]  

(22)

Explicitly we then find,

\[
I = \frac{e}{h} \sum_\omega \omega^2 (1 + \alpha(\omega))^2 \epsilon^2 + (\beta(\omega) \Delta^2 \cos^2 \frac{\epsilon}{2})
\]  

(23)

The temperature dependence of the maximum Josephson current for s-wave (dashed curves) and p-wave(solid curves) cases when \( t_K = 100, 5, 3, 2 \) (from the top to the bottom). The phase \( \delta \) of each curve is chosen so as to give the maximum value of Josephson current at \( T = 0 \) in Fig. 4. The rest parameters are the same as in Fig. 1.

By the same reasoning the temperature dependence (see Fig. 5) of the Josephson current is more similar to the usual SIS junction for the p-wave case, while for s-wave superconductors the current behavior is closer to the temperature dependence of the Josephson current in SNS junctions. To be more precise, Fig. 5 displays the temperature dependence of the maximum Josephson current \( I(T) \) for s-wave (dashed lines) and p-wave (solid lines) cases. For p-wave junctions, \( I(T) \) assumes relatively large value near \( T = T_C \) while for s-wave junctions, it rapidly decreases as \( T \) increases. This difference stems from the question of whether the mid-gap state appears or not. The situation is similar to the one encountered in Josephson current through in an SIS system \([27]\). The difference becomes more prominent when the Kondo effect is suppressed; \( I(T) \) drops more rapidly for s-wave junctions than for p-wave ones as \( t_K \) decreases.

In conclusion, we have analyzed an important physical problem involving strong correlations, the Kondo effect and superconductivity. These aspects can be combined in an \( S_K \) junction consisting of an Anderson impurity (in the Kondo regime). We have developed a theoretical framework by which it is possible to investigate an
interplay between Andreev reflections and the formation of Kondo resonance in the Kondo regime $T < \Delta < T_K$. In this limit we calculated Josephson Current. We have also investigated non-equilibrium aspects and elucidated the nonlinear $G-V$ characteristics. Moreover, we calculated the shot-noise power spectrum of SKN junctions at voltages $eV \leq \Delta$. It is found that at sufficiently large $t_K$ the Kondo resonance effectively turns the junction behavior to be similar to that of highly transparent non-interacting weak links for both $s$ and $p-\text{wave}$ superconductors. However, when the ratio of the Kondo temperature to the superconducting gap becomes smaller the behavior of these two types of junctions is quite different: the Kondo resonance persists much more effectively for junctions with $p-\text{wave}$ leads than it does for $s-\text{wave}$ leads.

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