PROBING FOR MACHOS OF MASS $10^{-15} M_\odot$ TO $10^{-7} M_\odot$ WITH GAMMA-RAY BURST PARALLAX SPACECRAFT

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ABSTRACT

Two spacecraft separated by approximately 1 AU and equipped with gamma-ray burst (GRB) detectors could detect or rule out a cosmological density of massive compact halo objects (MACHOs) in the mass range $10^{-15} M_\odot \leq M \leq 10^{-7} M_\odot$, provided that GRBs prove to be cosmological. Previously devised methods for detecting MACHOs have spanned the mass range $10^{-15} M_\odot \leq M \leq 10^{-7} M_\odot$, but with a gap of several orders of magnitude near $10^{-9} M_\odot$. For MACHOs and sources both at a cosmological distance, the Einstein radius is approximately $1 \text{AU} (M/10^{-7} M_\odot)^{1/2}$. Hence, if a GRB lies within the Einstein ring of a MACHO of mass $M \leq 10^{-7} M_\odot$ as seen by one detector, it will not lie in the Einstein ring as seen by a second detector approximately 1 AU away. This implies that if GRBs are measured to have significantly different fluxes by the two detectors, this would signal the presence of a MACHO $\leq 10^{-7} M_\odot$. By the same token, if the two detectors measured similar fluxes for several hundred events, a cosmological abundance of such low-mass MACHOs would be ruled out. The lower limit is set by the time resolution $t_{\text{res}}$ of the detectors: $M \approx 10^{-15} M_\odot$ corresponds to $t_{\text{res}} \approx 10^{-2} \text{s}$. If low-mass MACHOs are detected, there are tests that can discriminate among events generated by MACHOs in the three mass ranges $M \approx 10^{-12} M_\odot$, $10^{-12} M_\odot \leq M \leq 10^{-7} M_\odot$, and $M \approx 10^{-7} M_\odot$. Further experiments would then be required to make more accurate mass measurements.

Subject headings: dark matter — gravitational lensing

1. INTRODUCTION

While there is no a priori reason to expect that massive compact halo objects (MACHOs) in the mass range $10^{-15} M_\odot \leq M \leq 10^{-7} M_\odot$ comprise a significant fraction of the density of the universe, neither is there any definitive argument ruling them out. There are three main candidates for MACHOs in this mass range: snow balls, black holes, and small molecular clouds. Snow balls are lumps of primordial baryons (H and He). Snow balls in this mass range are generally believed to evaporate on timescales short compared to the Hubble time (De Rújula, Jetzer & Massó 1992) and for this reason are often dismissed. However, the argument for evaporation might not be considered definitive. Primordial black holes in this mass range would not have had time to evaporate by Hawking radiation and generally would not radiate enough to be detected. On the other hand, there are no detailed scenarios in which significant numbers of primordial black holes in this mass range would form. Small molecular clouds have recently been advanced as a solution or partial solution to the dark matter problem, if for no other reason than they are extremely hard to detect (Pfenniger, Combes, & Martinet 1994). Small molecular clouds are not usually grouped as MACHOs since they are diffuse rather than "compact." However, for MACHOs the word "compact" in "massive compact halo object" does not refer to any specific density. It simply means that the object is compact enough to fit into its own Einstein ring. For Galactic MACHOs, this size is $r_e \approx 1 \text{AU} (M/0.05 M_\odot)^{1/2}$, thus excluding molecular clouds. For cosmological MACHOs, much larger radii are permitted:

$r_e \approx 1 \text{AU} (M/10^{-7} M_\odot)^{1/2}$, which would include many small molecular clouds. Small molecular clouds are expected to have a fractal structure, implying that they could be probed on small scales by gamma-ray burst (GRB) parallax measurements. (In this Letter we use the word "MACHO" to refer to any class of massive compact objects, whether or not they literally live in the halos of galaxies.)

The mass range probed by GRB parallaxes is quite a good scale for "hiding" dark matter, even if the candidate objects have not previously received a great deal of attention. Baryonic objects of this mass would be too cold to emit much light of their own—they would be truly "dark." In particular, no methods have previously been developed for probing the mass scale near $10^{-9} M_\odot$ because the objects are so dark. This mass scale has never previously been probed with gravitational lensing (Nemiroff 1993).

Here we show that GRB detectors aboard two spacecraft separated by approximately 1 AU could detect or rule out low-mass MACHOs.Refsdal (1966) was the first to point out that by observing a MACHO event from two platforms separated by solar system-scale distances, one could obtain significantly more information than from a single platform. Recently, this idea of a "parallax spacecraft" has been suggested as a method of better constraining the velocities and masses of Galactic microlensing events (Gould 1992b, 1994, 1995a; Druckier, Nemiroff, & Özerney 1994) and determining the transverse velocities of galaxies (Grieger, Kayser, & Refsdal 1986; Gould 1995b).

Our proposal requires that GRBs occur at cosmological distances. For a discussion on the arguments for and against this hypothesis see Paczyński (1995) and Lamb (1995).
Paczyński (1986, 1987) was the first to point out that if they are at such great distances, GRBs could undergo a detectable microlensing effect. Mao & Paczyński (1992) and Grossman & Nowak (1994) estimated the chance that GRBs might undergo a galactic (macro-) lensing effect, and Blaes & Webster (1992) suggested that a cosmological abundance of MACHOs $M \sim 10^{-10} \, M_\odot$ could be found by searching for autocorrelations in the time series of measured GRB fluxes. Nemiroff et al. (1993) have excluded a density of MACHOs between 10^{-13} and 10^{-14} $M_\odot$ for a conservative estimate of GRB redshifts, with dim bursts lying near a redshift of unity. Gould (1992a) suggested the possibility of observing femtolensing of GRBs by MACHOs with $D = 10^{10} \, M_\odot$ near the upper end of the range, the distribution of flux ratios measured in different events will give a clue to the mass. We also discuss possible follow-up experiments that could provide additional information about $M$.

Our basic idea is extremely simple. For MACHOs and sources at a cosmological distance, and for MACHOs of sufficiently low mass ($M \lesssim 10^{-7} \, M_\odot$), the size of the Einstein ring projected onto the solar system will be $\tilde{r}_e < 1$ AU. In this case, were the source to lie within the Einstein ring as seen by one spacecraft, it would not lie within the Einstein ring as seen by a second spacecraft approximately 1 AU away. The signature of a lensing event is simply that a single GRB is observed to have significantly different fluxes as seen from the two spacecraft.

In §2 we discuss this idea quantitatively and show that it is sensitive to a range of mass $10^{-15} \, M_\odot \lesssim M \lesssim 10^{-7} \, M_\odot$. If no events were detected, this would place limits on the cosmological density of such objects. If some events were detected, one would know only the total optical depth of MACHOs within the mass range to which the experiment is sensitive, but one would not generally have any additional information constraining the mass. In §3, we discuss how such additional information might be obtained. For MACHOs near the lower end of the range, the ratio of the fluxes at the two spacecraft will vary with time, permitting a rough estimate of the mass. For MACHOs near the upper end, the distribution of flux ratios measured in different events will give a clue to the mass. We also discuss possible follow-up experiments that could provide additional information about $M$.

2. ANALYSIS

A lens magnifies a point source by different amounts as seen by different observers. An observer directly in line with the lens and the source would detect a ring of formally infinite magnification: the Einstein ring (Einstein 1936). The observer sees the Einstein ring as having some angular radius $\theta_e$ and, given the distance to the lens, can compute the radius $r_e$ of the Einstein ring in the lens plane. An observer for whom the source and lens were separated by an angle $\theta$ would see two images of the source with combined magnification,

$$A(x) = \frac{x^2 + 2}{x^2 + 4} \theta_e^2, \quad x = \frac{\theta}{\theta_e}.$$ (1)

The physical Einstein radius $r_e$ can be projected onto either the observer or the source plane, yielding two new length scales $\tilde{r}_e$ and $\tilde{r}_e$. If two observers are separated by $\tilde{r}_e$, then the angle between the unlensed source and the lens changes by $\theta_e$. Two sources separated by $\tilde{r}_e$ have unlensed angular positions relative to the lens that differ by $\theta$. The four quantities so defined are related by

$$\theta_e = \frac{2 R_6 D_{\text{LS}}}{\sqrt{D_{\text{OL}} D_{\text{OS}}}} = \frac{r_e}{D_{\text{OL}}} = \frac{\tilde{r}_e}{D_{\text{OL}}} = \frac{\tilde{r}_e D_{\text{LS}}}{D_{\text{OL}} D_{\text{OS}}},$$ (2)

where $D_{\text{OL}}, D_{\text{OS}}$, and $D_{\text{LS}}$ are the distances between the observer, lens, and source, respectively, and $R_6$ is the Schwarzschild radius of the lens: $R_6 = 2GM/c^2 = 3 \, \text{km} (M/M_\odot)$. In a cosmological setting, the distances discussed are all angular diameter distances (see, e.g., Turner, Ostriker, & Gott 1984).

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The generic term “Einstein ring” when used in the literature usually refers to $r_e$. Note that this quantity is related to $\tilde{r}_e$ as absolute to relative parallax.

Let $v$ be the transverse speed of the lens relative to the observer-source line of sight, and let $t_b$ be the duration of the GRB. Then, the magnification will be essentially constant during the GRB, provided that

$$\frac{vt_b}{r_e} \ll 1.$$ (3)

We initially assume that equation (3) holds. (As we show in §3, it is possible to detect the event even when eq. [3] fails, but the analysis differs somewhat.) In addition, we assume that the event is detected by two observers separated by a distance $a$ that is large compared to the projected Einstein ring, $\tilde{r}_e/a \ll 1$. It follows immediately that if the GRB is within the Einstein ring as seen by first observer ($x_1 < 1$), then it will be very far from the Einstein ring as seen by the other ($x_2 > 1$). Hence, the ratio of the fluxes as seen by the two observers (which is equal to the ratio of magnifications $\tilde{r}/a$) is just equal to $A$. For a given event, the probability that an observer lies inside the Einstein ring of some MACHO is by definition the optical depth, $\tau$. Let $f(\tilde{r})d\tilde{r}$ be the probability that the observed ratio of fluxes lies between $\tilde{r}$ and $\tilde{r} + d\tilde{r}$. And let $F(\tilde{r}) = \int_{\tilde{r}}^{\infty} d\tilde{r}'f(\tilde{r}')$. From equation (1) one then finds that

$$f(\tilde{r}) = 2(\tilde{r}^2 - 1)^{-3/2}, \quad F(\tilde{r}) = 2\tilde{r}(1 - \tilde{r}^{-2})^{-1/2} - 1.$$ (4)

The ratio $\tilde{r}/A$ has the same distribution.

Consider then an experiment that is sensitive to flux ratios that differ from unity by at least $\tilde{r}/A$. For definiteness, we take $\tilde{r}/A = 1.34$, corresponding to $x_1 < 1$. And assume that $N$ GRBs are observed. Then, one would expect $2NF(\tilde{r}/A) = 2N\tilde{r}/a$ events, where one or the other observer saw significantly more flux than the other. Hence, if no such events were observed, Poisson statistics would rule out optical depths $\tau > 1.5N$ at the 95% confidence level.

The expected optical depth is given by

$$\tau = g\Omega_L,$$ (5)

where $\Omega_L$ is the density of lenses in units of the closure density of the universe. While the parameter $g$ depends on distribution of the sources and to a lesser extent on the geometry of the universe, one finds that for a mean source redshift of $z \sim 0.5$ appropriate under one set of estimates for the brighter GRBs (Wickramasinghe et al. 1993; Norris et al. 1994; Cohen, Kolatt, & Piran 1995), $g \sim 0.05$ (Gould 1992a; Nemiroff et al. 1993). This means that with only a few hundred events, one could probe to densities $\Omega_L \sim 0.1$, the value characteristic of dark matter on galactic scales.

Let us assume for the moment that indeed no events were
detected with flux ratios significantly different from unity. What range of masses would be ruled out? The upper mass limit is set by the assumption that the two observers are separated by more than an Einstein ring. The most probable scenario for a detectable lensing event occurs when the lens distance is a reasonable fraction of the source distance (Turner et al. 1984; Nemiroff 1989). From equation (2), \( a > (R_{5} \cdot D_{OS})^{1/2} \). For definiteness, we adopt \( D_{OS} \sim 1 \text{ Gpc} \) and find

\[
M \lesssim 10^{-12} M_{\odot} \left( \frac{a}{\text{AU}} \right)^{2} \left( \frac{D_{OS}}{\text{Gpc}} \right)^{-1}.
\]

The lower mass limit is set by our initial assumption (codified in eq. [3]) that the event is instantaneous. The vast majority of GRB models (see, e.g., Nemiroff 1994) are explosive: gamma-ray emitting material expands at a highly relativistic speed. We will assume they expand at the speed of light and rewrite equation (3) as \( \tau_{e} \sim c t_{r} \). Typical GRBs last a few seconds. We therefore find

\[
M \gtrsim 10^{-12} M_{\odot} \left( \frac{t_{e}}{1 \text{ s}} \right)^{2} \left( \frac{D_{OS}}{\text{Gpc}} \right)^{-1}.
\]

As we show in § 3, it is possible to detect events with smaller Einstein rings, provided that \( \omega \tau_{e} \ll 1 \), where \( \omega = v/r_{e} \) and \( \tau_{e} \) is the time resolution of the detector. Assuming infinite time resolution, the fundamental lower mass limit is set by the requirement that the source radius \( r_{s} > r_{e} \). Even at the outset of the burst, the source must be at least the size of a neutron star, implying a fundamental limit \( M \gtrsim 10^{-21} M_{\odot} \). However, at \( M \sim 10^{-15} M_{\odot} \), it is already straightforward to detect the event by means of interference effects, and this requires only a single Earth-orbiting spacecraft (Gould 1992a). We adopt this as the lower limit, which, according to equation (7), can be achieved with time resolution \( \tau_{e} \approx 10^{-2} \text{ s} \).

Thus, a single pair of GRB spacecraft separated by 1 AU could detect or rule out a cosmological abundance of lenses over eight decades of mass. Most of this range has proven inaccessible by other search techniques. A larger baseline could reach larger masses.

3. MASS MEASUREMENTS AND CONSTRAINTS

If GRB lensing events are detected, it will in general require additional experiments in order to estimate the mass of the lenses. However, even without additional experiments, it will be possible to obtain some constraints. Understanding these constraints also allows one to extend somewhat the range of sensitivity of the experiments relative to the limits set in the previous section.

At the upper mass limit the constraint arises from the distribution of flux ratios. Consider the opposite limit from the one examined in § 2: \( a \ll \tau_{e} \). For this case, if one observer sees a lensed GRB, so will the other. In general, the magnification difference will only be significant if the observers are fairly close to the center of the Einstein ring, \( x \ll 1 \). In this limit, the flux ratio is given by \( R = (1 + \alpha^{2} - 2 \alpha \cos \phi)^{1/2} \), where \( \alpha = a/(x r_{e}) \), and \( \phi \) is the angle between the spacecraft separation vector and the source-lens axis as seen from one of the spacecraft. Hence, for fixed \( \phi \), \( R > \tau_{e} \), provided that \( x < g(\hat{R}, \phi) \), where \( g(\hat{R}, \phi) = (a r_{e}) (\hat{R}^{2} - \sin^{2} \phi)^{1/2} - \cos \phi (\hat{R}^{2} - 1) \). The probability that an event will have dimensionless lens separation \( x \) as seen from one satellite and relative orientation \( \phi \) is \((\tau/2\pi) dxd\phi\). The differential and cumulative probabilities are then found to be

\[
f(\hat{R}) = \frac{a^{2}}{(\hat{R})^{2}} \hat{R}^{3} + \hat{R}, \quad F(\hat{R}) = 2\pi \left( \frac{a}{\hat{R}} \right)^{2} (\hat{R} - \hat{R}^{-1})^{-2},
\]

respectively, and the same distributions apply for \( \hat{R} \). For large ratios, this distribution is almost identical to equation (4). Thus, if the experiment were sensitive only to high-ratio events, one could not distinguish between an optical depth \( \tau = \tau_{e} \) for lenses \( M \approx 10^{-7} M_{\odot} \) and a larger optical depth \( \tau = (\hat{R} \alpha)^{2} \tau_{e} \) for much larger masses. However, if, for example, the experiment were sensitive to ratios as small as \( \hat{R}_{\min} = 1.2 \), then for the low-mass lenses, one-half the events would have ratios \( \hat{R} > 1.42 \), whereas for the high-mass lenses the fraction would be 26%. Hence, the two distributions could be distinguished with modest statistics.

The lower mass limit was set by demanding that the event be shorter than the Einstein ring crossing time. However, if the GRBs are longer than this, lensing events will still be observable, provided that the time resolution is shorter than the crossing time. In this case, the ratio of magnifications (and thus fluxes) will be time dependent, with a profile exactly like that of a classical microlensing event: \( R(t) = A[x(t)] \), where \( A(x) \) is given by equation (1), \( x(t) = [\omega^{2}(t - t_{0})^{2} + \beta^{2}]^{1/2} \), \( \omega = v/r_{e}, t_{0} \) is the time of maximum magnification, and \( \beta \) is the dimensionless impact parameter. If \( R \) has no significant time dependence, this would constrain the mass \( M \gtrsim 10^{-12} M_{\odot} \). If time dependence were detected, the measured timescale would serve to characterize the size of the Einstein ring and thus approximately characterize the mass. The principal uncertainty in these determinations would be the assumption that the transverse speed is approximately \( c \).

If MACHOs were detected by a pair of such spacecraft, then the above constraints could be used to design new experiments that could determine the mass more precisely. If the mass were constrained to be \( 10^{-12} M_{\odot} \lesssim M \lesssim 10^{-7} M_{\odot} \), then a set of, say, five spacecraft could be launched with separations of, say, 0.3, 0.1, 0.003, 0.001, and 0.0003 AU. If the mass lay near the lower limit, then the nearest spacecraft would see similar magnifications, while the most distant would see no magnification at all. If the mass were somewhat higher, several spacecraft would see similar magnifications, while the most distant would see no magnification. In this way the Einstein rings could be estimated to a factor of about 3, corresponding to mass estimates of a factor of about 10. If the masses were determined to be \( \approx 10^{-7} M_{\odot} \), then the mass of the nearest spacecraft would see similar magnifications, while the most distant would see no magnification.

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