THE DIXMIER-DOUADY CLASS IN THE
SIMPLICIAL DE RHAM COMPLEX

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Abstract

On the basis of A. L. Carey, D. Crowley, M. K. Murray’s work, we exhibit a cocycle in the simplicial de Rham complex which represents the Dixmier-Douady class.

1. Introduction

In [5, Carey, Crowley, Murray], they proved that when a Lie group $G$ admits a central extension $1 \to U(1) \to \hat{G} \to G \to 1$, there exists a characteristic class of principal $G$-bundle $p: Y \to M$ which belongs to a cohomology group $H^2(M, U(1)) \cong H^3(M, \mathbb{Z})$. Here $U(1)$ stands for a sheaf of continuous $U(1)$-valued functions on $M$. This class is called a Dixmier-Douady class associated to the central extension $\hat{G} \to G$.

On the other hand, we have a simplicial manifold $\{NG(p)\}$ for any Lie group $G$. It is a sequence of manifolds $\{NG(p) = G^p\}_{p=0, 1, \ldots}$ together with face maps $e_i: NG(p) \to NG(p - 1)$ for $i = 0, \ldots, p$ satisfying relations the $e_ie_j = e_{j-1}e_i$ for $i < j$. (The standard definition also involves degeneracy maps but we do not need them here.) Then the $n$-th cohomology group of classifying space $BG$ is isomorphic to the total cohomology of a double complex $\{\Omega^q(NG(p))\}_{p+q=n}$. See [3] [6] [9] for details.

In this paper we will exhibit a cocycle on $\Omega^*(NG(\ast))$ which represents the Dixmier-Douady class due to Carey, Crowley, Murray. Such a cocycle is also studied in a general setting by K. Behrend, J.-L. Tu, P. Xu and C. Laurent-Gengoux [1] [2] [13] [14], and G. Ginot, M. Stiènon [7] but our construction of the cocycle is different from theirs, and the proof is more simple. Stevenson [12] also exhibited a cocycle which represents the Dixmier-Douady class in singular cohomology group instead of the de Rham cohomology. As a consequence of our result, we can show that if $G$ is given a discrete topology, the Dixmier-Douady class in $H^3(BG^d, \mathbb{R})$ is 0. Furthermore, we can exhibit the “Chern-Simons form” of Dixmier-Douady class on $\Omega^*(\overline{NG}(\ast))$. Here $\overline{NG}$ is a simplicial manifold which plays the role of universal bundle.
The outline is as follows. In section 2, we briefly recall the notion of simplicial manifold $NG$ and construct a cocycle in $\Omega^\ast(NG(\ast))$. In section 3, we recall the definition of a Dixmier-Douady class and prove the main theorem. In section 4, we give the Chern-Simons form of the Dixmier-Douady class.

2. Cocycle on the double complex

In this section first we recall the relation between the simplicial manifold $NG$ and the classifying space $BG$, then we construct the cocycle on $\Omega^{\ast\ast}(NG)$.

2.1. The double complex on simplicial manifold

For any Lie group $G$, we define simplicial manifolds $NG$, $N\overline{G}$ and a simplicial $G$-bundle $g : NG \to NG$ as follows:

\[ NG(p) = \underbrace{G \times \cdots \times G}_{p\text{-times}} \ni (g_1, \ldots, g_p) : \]

face operators $e_i : NG(p) \to NG(p - 1)$

\[ e_i(g_1, \ldots, g_p) = \begin{cases} (g_2, \ldots, g_p) & i = 0 \\ (g_1, \ldots, g_1, g_{i+1}, \ldots, g_p) & i = 1, \ldots, p - 1 \\ (g_1, \ldots, g_{p-1}) & i = p \end{cases} \]

\[ N\overline{G}(p) = \underbrace{G \times \cdots \times G}_{p+1\text{-times}} \ni (h_1, \ldots, h_{p+1}) : \]

face operators $\bar{e}_i : N\overline{G}(p) \to N\overline{G}(p - 1)$

\[ \bar{e}_i(h_1, \ldots, h_{p+1}) = (h_1, \ldots, h_i, h_{i+2}, \ldots, h_{p+1}) \quad i = 0, 1, \ldots, p \]

And we define $\gamma : N\overline{G} \to NG$ as $\gamma(h_1, \ldots, h_{p+1}) = (h_1 h_2^{-1}, \ldots, h_p h_{p+1}^{-1})$.

To any simplicial manifold $X = \{X_s\}$, we can associate a topological space $\|X\|$ called the fat realization. Since any $G$-bundle $\pi : E \to M$ can be realized as the pull-back of the fat realization of $\gamma$, $\|\gamma\|$ is an universal bundle $EG \to BG$ [11].

Now we construct a double complex associated to a simplicial manifold.

**Definition 2.1.** For any simplicial manifold $\{X_s\}$ with face operators $\{e_s\}$, we define double complex as follows:

\[ \Omega^{p,q}(X) \overset{\text{def}}{=} \Omega^q(X_p) \]

Derivatives are:

\[ d' := \sum_{i=0}^{p+1} (-1)^i e_i^* \quad d'' := \text{derivatives on } X_p \times (-1)^p \]

For $NG$ and $N\overline{G}$ the following holds ([3] [6] [9]).
THEOREM 2.1. There exists a ring isomorphism

\[ H(\Omega^*(NG)) \cong H^*(BG), \quad H(\Omega^*(N\overline{G})) \cong H^*(EG) \]

Here \( \Omega^*(NG) \) and \( \Omega^*(N\overline{G}) \) means the total complexes. \( \square \)

For a principal G-bundle \( Y \to M \) and an open covering \( \{U_i\} \) of \( M \), the transition functions \( g_{ij}: U_{ij} \to NG(p) \) induce the cocycle map \( H^*(NG) \to H^*_\text{Cech-deRham}(M) \). The elements in the image are the characteristic class of \( Y \) [9].

2.2. Construction of the cocycle

Let \( \hat{G} \to G \) be a central extension of a Lie group \( G \) and we recognize it as a \( U(1) \)-bundle. Using the face operators \( \{e_i\}: NG(2) \to NG(1) = G \), we can construct the \( U(1) \)-bundle over \( NG(2) = G \times G \) as \( \delta \hat{G} := e_0^*\hat{G} \otimes (e_1^*\hat{G})^{\otimes -1} \otimes e_2^*\hat{G} \).

Here we define the tensor product \( S \otimes T \) of \( U(1) \)-bundles \( S \) and \( T \) over \( M \) as

\[ S \otimes T := \bigcup_{x \in M} (S_x \times T_x) / (s, t) \sim (su, tu^{-1}), \quad (u \in U(1)) \]

LEMMA 2.1. \( \delta\hat{G} \to G \times G \) is a trivial bundle.

Proof. We can construct a bundle isomorphism \( f: e_0^*\hat{G} \otimes e_2^*\hat{G} \to e_1^*\hat{G} \) as follows. First we define \( f \) to be the map sending \( \{((g_1, g_2), \hat{g}_2), ((g_1, g_2), \hat{g}_1)\} \) s.t. \( \rho(\hat{g}_2) = g_2 \), \( \rho(\hat{g}_1) = g_1 \) to \( \{((g_1, g_2), \hat{g}_2), ((g_1, g_2), \hat{g}_1)\} \) s.t. \( \rho(\hat{g}_2) = g_2 \).

For any connection \( \theta \) on \( \hat{G} \), there is the induced connection \( \delta \theta \) on \( \delta\hat{G} \) [4, Brylinski].

PROPOSITION 2.1. Let \( c_1(\theta) \) denote the 2-form on \( G \) which hits \( \left(\frac{1}{2\pi i}\right) d\theta \) in \( \Omega^2(\hat{G}) \) by \( \rho^* \), and \( \hat{s} \) any global section of \( \delta\hat{G} \). Then the following equation holds.

\[ (e_0^* - e_2^* - e_2^*) c_1(\theta) = \left(\frac{1}{2\pi i}\right) d(\hat{s}^*(\delta\theta)) \in \Omega^2(NG(2)). \]

Proof. Choose an open cover \( p' = \{V_{\lambda}\}_{\lambda \in \Lambda} \) of \( G \) such that there exist local sections \( \eta_{\lambda}: V_{\lambda} \to \hat{G} \) of \( \rho \). Then \( \{e_0^{-1}(V_{\lambda'}) \cap e_1^{-1}(V_{\lambda'}) \cap e_2^{-1}(V_{\lambda'})\}_{\lambda', \lambda'' \in \Lambda} \) is an open cover of \( G \times G \) and there are the induced local sections \( e_0^* \eta_{\lambda} \otimes (e_1^* \eta_{\lambda'})^{\otimes -1} \otimes e_2^* \eta_{\lambda''} \) on that covering.

If we pull back \( \delta\theta \) by these sections, the induced form on \( e_0^{-1}(V_{\lambda'}) \cap e_1^{-1}(V_{\lambda'}) \cap e_2^{-1}(V_{\lambda''}) \) is \( e_0^* (\eta_{\lambda'}^* \theta) - e_1^* (\eta_{\lambda'}^* \theta) + e_2^* (\eta_{\lambda''}^* \theta) \). We restrict
From direct computations we can check that the pull-back of $\delta \theta$ is equal to 0. This means $c_1(\theta) = \sum (-1)^{2ni} d(\eta^\ast \theta)$. Also $d(e_0^\ast(\eta^\ast \theta) - e_1^\ast(\eta^\ast \theta) + e_2^\ast(\eta^\ast \theta)) = d(\delta^\ast(\delta \theta))|_{e_0^\ast(V_\ast) \cap e_1^\ast(V_\ast) \cap e_2^\ast(V_\ast)}$ since $\delta \theta$ is a connection form. This completes the proof.

**Proposition 2.2.** For the face operators $\{e_i\}_{i=0,1,2,3} : NG(3) \to NG(2)$,

$$(e_0^* - e_1^* + e_2^*- e_3^*)(\delta^*(\delta \theta)) = 0.$$  

**Proof.** We consider the $U(1)$-bundle $\delta(\delta \mathcal{G})$ over $NG(3) = G \times G \times G$ and the induced connection $\delta(\delta \theta)$ on it. Composing $\{e_i\} : NG(3) \to NG(2)$ and $\{e_i\} : NG(2) \to G$, we define the maps $\{r_i\}_{i=0,1,2,3} : NG(3) \to G$ as follows.

$$r_0 = e_0 \circ e_1 = e_0 \circ e_0, \quad r_1 = e_0 \circ e_2 = e_1 \circ e_0, \quad r_2 = e_0 \circ e_3 = e_2 \circ e_0$$

Then $\{\bigcap r_i^{-1}(V_{j(i)})\}$ is a covering of $NG(3)$. Since each $\bigcap r_i^{-1}(V_{j(i)})$ is equal to

$$e_0^\ast(e_0^\ast(V_\ast) \cap e_1^\ast(V_\ast) \cap e_2^\ast(V_\ast)) \cap e_1^\ast(e_0^\ast(V_\ast) \cap e_2^\ast(V_\ast)) \cap e_2^\ast(e_0^\ast(V_\ast) \cap e_2^\ast(V_\ast))$$

there are the following induced local sections on that.

$$e_0^\ast(e_0^\ast(V_\ast) \cap e_1^\ast(V_\ast) \cap e_2^\ast(V_\ast)) \cap e_1^\ast(e_0^\ast(V_\ast) \cap e_2^\ast(V_\ast)) \cap e_2^\ast(e_0^\ast(V_\ast) \cap e_2^\ast(V_\ast)) \cap e_3^\ast(e_0^\ast(V_\ast) \cap e_2^\ast(V_\ast))$$

From direct computations we can check that the pull-back of $\delta(\delta \theta)$ by this section is equal to 0. This means $\delta(\delta \theta)$ is the Maurer-Cartan connection. Hence if we pull back $\delta(\delta \theta)$ by the induced section $e_0^\ast \delta \times (e_1^\ast \delta)^{\otimes -1} \otimes e_2^\ast \delta \times (e_3^\ast \delta)^{\otimes -1}$, it is also equal to 0 and this pull-back is nothing but $(e_0^* - e_1^* + e_2^* - e_3^*)(\delta^*(\delta \theta))$. 

The propositions above give the cocycle $c_1(\theta) = \sum (-1)^{2ni} \delta^*(\delta \theta) \in \Omega^3(NG)$ below.

$$
\begin{array}{ccc}
0 & \xrightarrow{d} & \Omega^2(G) \\
& & \xrightarrow{e_0^* - e_1^* + e_2^*} \\
c_1(\theta) & \xrightarrow{\delta^*} & \Omega^2(G \times G) \\
\end{array}
$$

$$
\begin{array}{ccc}
\Omega^2(G \times G) & \xrightarrow{-d} & \Omega^1(G \times G) \\
& & \xrightarrow{\delta^*} \\
(\delta^*(\delta \theta)) & \xrightarrow{\sum (-1)^{2ni} \delta^*(\delta \theta)} & 0
\end{array}
$$
Proposition 2.3. The cohomology class \[ c_1(\theta) - \left( -\frac{1}{2\pi i} \right) \hat{s}^*(\delta \theta) \] does not depend on \( \theta \).

Proof. Suppose \( \theta_0 \) and \( \theta_1 \) are two connections on \( \hat{G} \). Consider the \( U(1) \)-bundle \( \hat{G} \times [0, 1] \to G \times [0, 1] \) and the connection form \( t\theta_0 + (1 - t)\theta_1 \) on it. Then we obtain the cocycle \( c_1(t\theta_0 + (1 - t)\theta_1) - \left( -\frac{1}{2\pi i} \right) \hat{s}^*(\delta(t\theta_0 + (1 - t)\theta_1)) \) on \( \Omega^3(NG \times [0, 1]) \). Let \( i_0 : NG \times \{0\} \to NG \times [0, 1] \) and \( i_1 : NG \times \{1\} \to NG \times [0, 1] \) be the natural inclusion map. When we identify \( NG \times \{0\} \) with \( NG \times \{1\} \), \( (i_0)^{-1}i_1^*: H^3(\Omega^*(NG \times \{0\})) \to H^3(\Omega^*(NG \times \{1\})) \) is the identity map. Hence \[ c_1(\theta_0) - \left( -\frac{1}{2\pi i} \right) \hat{s}^*(\delta \theta_0) = c_1(\theta_1) - \left( -\frac{1}{2\pi i} \right) \hat{s}^*(\delta \theta_1) \].

3. Dixmier-Douady class on the double complex

First, we recall the definition of Dixmier-Douady classes, following [5]. Let \( \pi : Y \to M \) be a principal \( G \)-bundle and \( \{U_x\} \) a Leray covering of \( M \). When \( G \) has a central extension \( \rho : \hat{G} \to G \), the transition functions \( g_{ab} : U_{ab} \to G \) lift to \( \hat{G} \), i.e. there exist continuous maps \( \hat{g}_{ab} : U_{ab} \to \hat{G} \) such that \( \rho \circ \hat{g}_{ab} = g_{ab} \). This is because each \( U_{ab} \) is contractible so the pull-back of \( \rho \) by \( g_{ab} \) has a global section. Now the \( U(1) \)-valued functions \( c_{ab} \) on \( U_{ab} \) are defined as \( c_{ab} := \hat{g}_{ab} \hat{g}_{ab}^{-1} \). Note that here they identify \( \hat{g}_{ab} \hat{g}_{ab}^{-1} \) with \( U_{ab} \times U(1) \). Then it is easily seen that \( \{c_{ab}\} \) is a \( U(1) \)-valued \( \hat{G} \)-cocycle on \( M \) and hence define a cohomology class in \( H^2(M, U(1)) \approx H^3(M, \mathbb{Z}) \). This class is called the Dixmier-Douady class of \( Y \).

Here \( G \) can be infinite dimensional, but we require \( G \) to have a partition of unity so that we can consider a connection form on the \( U(1) \)-bundle over \( G \). A good example which satisfies such a condition is the loop group of a finite dimensional Lie group [4] [10].

Secondly, we fix any trivialization \( \delta \hat{G} \cong \hat{G} \times U(1) \). Then since \( g_{ab} G \otimes (g_{ab} G)^{-1} \otimes g_{ab} \hat{G} \) is the pull-back of \( \delta \hat{G} \) by \( (g_{ab}, g_{ab}) : U_{ab} \to G \times G \), there is the induced trivialization \( g_{ab}^* \hat{G} \otimes (g_{ab}^* \hat{G})^{-1} \otimes g_{ab} \hat{G} \cong U_{ab} \times U(1) \). So we have the Dixmier-Douady cocycle by using this identification.

Now we are ready to state the main theorem.

Definition 3.1. For the global section \( \hat{s} : G \times G \to 1 \), we call the sum of \( c_1(\theta) \in \Omega^2(NG(1)) \) and \( -\left(\frac{1}{2\pi i}\right) \hat{s}^*(\delta \theta) \in \Omega^1(NG(2)) \) the simplicial Dixmier-Douady cocycle associated to \( \theta \) and the trivialization \( \delta \hat{G} \cong \hat{G} \times U(1) \).

Theorem 3.1. The simplicial Dixmier-Douady cocycle represents the universal Dixmier-Douady class associated to \( \rho \).
Proof. We show that the \([C_{2,1} + C_{1,2}]\) below is equal to \(\left\{ \left( \frac{-1}{2\pi i} \right) d \log c_{\beta_f} \right\} \) as a Čech-de Rham cohomology class of \(M = \bigcup U_x\).

\[
C_{2,1} \in \prod \Omega^2(U_{\beta_f})
\]

\[
\prod \Omega^1(U_{\beta_f}) \xrightarrow{\delta} C_{1,2} \in \prod \Omega^1(U_{\beta_f})
\]

\[
C_{2,1} = \{(g^*_{\beta_f}c_1(\theta))\}, \quad C_{1,2} = \left\{ -\left( \frac{-1}{2\pi i} \right) (g_{\beta_f}, g_{\beta_f})^*s^*(\delta \theta) \right\}
\]

Since \(g^*_{\beta_f}c_1(\theta) = \hat{g}^*_{\beta_f}p^*(c_1(\theta)) = d\left( \frac{-1}{2\pi i} \right) \hat{g}^*_{\beta_f} \delta^\theta\), we can see \( [C_{2,1} + C_{1,2}] = \left[ \delta \left( \left( \frac{-1}{2\pi i} \right) \hat{g}^*_{\beta_f} \delta^\theta \right) \right] + C_{1,2} \). By definition \( (s \circ (g_{\beta_f}, g_{\beta_f}))(p) \cdot c_{\beta_f}(p) = (\hat{g}_{\beta_f} \otimes \hat{g}^*_{\beta_f})^{-1}(p) \) for any \( p \in U_{\beta_f} \). Hence \( (g_{\beta_f}, g_{\beta_f})^*s^*(\delta \theta) + d \log c_{\beta_f} = \delta(\hat{g}^*_{\beta_f} \delta^\theta) \).

\[
\text{Corollary 3.1. If the principal } G \text{-bundle over } M \text{ is flat, then its Dixmier-Douady class is 0 in } H^3(M, \mathbb{R}).
\]

Proof. This is because the cocycle in Theorem 3.1 vanishes when \( G \) is given a discrete topology.

\[
\text{Corollary 3.2. If the first Chern class of } \rho : \hat{G} \to G \text{ is not 0 in } H^2(G, \mathbb{R}), \text{ then the corresponding Dixmier-Douady class of the universal } G \text{-bundle is not 0.}
\]

Proof. In that situation, any differential form \( x \in \Omega^1(NG(1)) \) does not hit \( c_1(\theta) \in \Omega^2(NG(1)) \) by \( d : \Omega^1(NG(1)) \to \Omega^2(NG(1)) \).

4. Chern-Simons form

As mentioned in section 2.1, \( NG \) plays the role of the universal \( G \)-bundle and \( NG \), the classifying space \( BG \). Then, the pull-back of the cocycle in Definition 3.1 to \( \Omega^*(NG) \) by \( \gamma : NG \to NG \) should be a coboundary of a cochain on \( NG \). In this section we shall exhibit an explicit form of the cocochain, which can be called Chern-Simons form for the Dixmier-Douady class.

Recall \( NG(1) = G \times G \) and \( \gamma : NG(1) \to NG \) is defined as \( \gamma(h_1, h_2) = h_1h_2^{-1} \). Then we consider the \( U(1) \)-bundle \( \delta, \hat{G} := \delta^1 \hat{G} \otimes \gamma^1 \hat{G} \otimes (\delta^1 \hat{G})^{-1} \) over \( G \times G \) and the induced connection \( \delta, \theta \) on it. We can check \( \delta, \hat{G} \) is trivial using the same argument as that in Lemma 2.1, so there is a global section \( s_\gamma : G \times G \to \delta, \hat{G} \).
Theorem 4.1. If we take \( s_g = 1 \), the cochain \( c_1(\theta) - \frac{-1}{2\pi i} s_g^*(\delta, \theta) \in \Omega^2(N\mathcal{G}) \) is a Chern-Simons form of \( c_1(\theta) - \frac{-1}{2\pi i} s_g^*(\delta, \theta) \in \Omega^3(N\mathcal{G}) \).

Proof. Repeating the same argument as that in Proposition 2.1, we can see \((e_0 + \gamma^* - \overline{e_1})(c_1(\theta)) = \frac{-1}{2\pi i} d(s_g^*(\delta, \theta)) \in \Omega^2(N\mathcal{G}(1)).\) Because \((e_0, e_1, e_2) \circ \gamma = (\gamma \circ e_0, \gamma \circ e_1, \gamma \circ e_2), (\overline{e_0} G) \otimes (\overline{e_1} G) \otimes (\overline{e_2} G) \) is \( \gamma^*(\delta \mathcal{G}). \) Hence \((\overline{e_0} - \overline{e_1} + \overline{e_2}) s_g^*(\delta, \theta) = \gamma^*(\delta \mathcal{G}). \)

By restricting the Chern-Simons form on \( \Omega^*(N\mathcal{G}) \) to the edge \( \Omega^*(G) \), we obtain the cocycle on \( \Omega^*(G) \). So there is the induced map of the cohomology class \( H^*(BG) \cong H^*(\Omega^*(NG)) \rightarrow H^{*-1}(G) \). This map coincides with the transgression map for the universal bundle \( E\mathcal{G} \rightarrow BG \) in the sense of J. L. Heitsch and H. B. Lawson in [8]. Hence as a corollary of theorem 4.1, we obtain an alternative proof of the following theorem from [5] [12].

Theorem 4.2. The transgression map of the universal bundle \( E\mathcal{G} \rightarrow BG \) maps the Dixmier-Douady class to the first Chern class of \( \mathcal{G} \rightarrow G \).

Remark 4.1. Here the meaning of the terminology “transgression map” is different from those in [5] [12], but the statement is essentially same.

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