DYNAMICAL SCREENING OF GRAVITATIONAL INTERACTION AND PLANETARY MOTIONS IN MODIFIED SOLAR POTENTIAL

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Abstract

A density disturbance in a system of gravitating mass, induced by a moving selected body gives rise to a dynamical screening of Newtonian potential of this body. When applied to the solar planetary system it means that as a result of the motion of the Sun in the Galaxy its effective force potential appears more weak than the Newtonian potential. The relevant modifications of main relations of the solar dynamics are considered here and it is found in particular that the reestimated period of the Earth revolution around the Sun rises in 1 second per year and semimajor axis of the Earth orbit increases on 4 kilometers. Similar relations are obtained for other planets too. It may be supposed that the inclusion of these effects can help to explain the observable anomalous acceleration of spacecrafts Pioneer 10 and 11.

1 Introduction

It is a matter of general experience that a resting test charge in a plasma produces the Debye screened potential. When this charge is moving the static screening decreases [1] and the potential becomes anisotropic [2]. These properties are due to collective effects in systems of charge particles. Here the similar effects for systems of gravitating particles are discussed. For simplicity we shall restrict our consideration to the model steady state system of a homogeneous Maxwellian gas of gravitating bodies.

The problem of motion of the test star in a fluctuating force field of another stars was first studied by Chandrasekhar [3] and discussed later by Marochnik [4] using methods of kinetic gas theory as an initial problem for a distribution function describing the test star. In contrast to his approach, here we consider a perturbation of the equilibrium state of a gas of gravitating bodies as a response to a force field of the test moving body following to our general approach developed in refs. [5-7].

The main advantage of our approach is a derivation in Sec.2 of the expression for a renormalized (effective) potential of the Sun and calculation of its difference from the Newtonian solar potential for planets of the solar system.

Next we calculate in Sec.3 corresponding corrections to the periods and semimajor axis of revolutions of the Earth and other planets.

2 Gravitational screening

In contrast to plasma or electrolyte where the screening of the Coulomb potential of a charge particle is presumably due to the presence of particles of an opposite charge and takes a place both for the moving test particle (dynamical screening) and for the fixed one

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(static Debye–Hückel screening) the screening of the Newtonian potential of a selected test body in a system of gravitating bodies is possible for the moving test body only.

In the model discussed below the Galaxy is considered (neglecting the dark matter) as an equilibrium gravitating system of stars with a chaotic distribution of velocities described by the Maxwellian \( f_0(v) = \rho_0(2\pi \bar{v}^2)^{-3/2}\exp\{-v^2/2\bar{v}^2\} \), where \( \rho_0 \) is the density of stellar matter and \( \bar{v} \) velocities dispersion. The Sun is excluded from the whole system and considered as the additional gravitating body of mass \( M_\odot \) inserted at the instant \( t_0 \) at the point \( r_0 = 0 \) with the determined steady velocity \( \mathbf{u} \). Introducing the test particle gives rise to a perturbation \( f_1(r, v, t) \) of the system, so its distribution function takes the form \( f(r, v, t) = f_0(v) + f_1(r, v, t) \).

Then, in the linear approximation in the perturbation the system is described by the Vlasov equation

\[
\frac{\partial f_1(r, v, t)}{\partial t} + \mathbf{v} \cdot \frac{\partial f_1(r, v, t)}{\partial r} + \nabla \Phi \cdot \frac{\partial f_0(r, v, t)}{\partial v} = 0. \tag{1}
\]

This is supplemented by the Poisson equation for the effective potential

\[ \nabla^2 \Phi = -4\pi Gm \int d^3v f_1(r, v, t) - 4\pi GM_\odot \delta(r - \mathbf{u})(t - t_0). \tag{2} \]

We are interested in a steady–state behavior of the gravitating system. It means physically that processes are considered at times \( t > \tau \), where \( \tau = 1/\sqrt{4\pi G\rho_0} \) is the characteristic relaxation time. For our Galaxy (where \( \rho_0 = 1.9 \cdot 10^{-23} \text{g cm}^{-3} \)) it is of order \( 10^7 \) years what falls far short of the time of existence of the solar system (in the opposite case transition processes would be to take into consideration). To exclude transition processes, we put \( t_0 \to -\infty \). Then the formal solution to equations (1) and (2) is

\[ \nabla^2 \Phi = 4\pi Gm \int d^3v \int_{-\infty}^{t} dt' \nabla \Phi(r - v(t - t'), t') \cdot \frac{\partial f_0}{\partial v} - 4\pi GM_\odot \delta(r - \mathbf{u}t) \tag{3} \]

or in the Fourier transform:

\[ [k^2 - 4\pi Gm \int d^3v \frac{k \cdot \partial f_0/\partial v}{\omega - kv + i\nu}] \Phi(k, \omega) = 8\pi^2 GM_\odot \delta(\omega - ku), \tag{4} \]

whence

\[ \Phi(k, \omega) = \frac{8\pi^2 GM_\odot}{k^2 \varepsilon_g(k, \omega)} \delta(\omega - ku). \tag{5} \]

Here

\[ \varepsilon_g(k, \omega) = 1 - \frac{4\pi G}{k^2} \int d^3v \frac{k \cdot \partial f_0/\partial v}{\omega - kv + i\nu} \tag{6} \]

is the gravitational permittivity of the system. This function determines a reply of the gravitating system on a gravitational disturbance. The concept of the gravitational permittivity is well–known in the theory of gravitating system (see, e.g. [4, 8, 9]). Its zeroes define a dispersion equation of a gravitating system of which complex roots determine a spectrum of elementary excitations.

An infinitesimal imaginary value \( i\nu \) is added to the frequency \( \omega \) in equations (4) and (6), which may be considered as a result of introducing a ”shadow” of the Boltzmann collision integral \(-\nu f_1\) into the r.h.s. of the Vlasov equation. Such addition assures a
selection of retarded solutions and accounts for Landau damping in collisionless plasma. Then the permittivity \( \varepsilon_g(k, \omega) \) becomes a complex function and can be expressed as

\[
\varepsilon_g(k, \omega) = 1 - \frac{\kappa_J^2}{k^2} W(z), \quad z = \frac{\omega}{\sqrt{2k\nu}},
\]

where \( \kappa_J = (4\pi G \rho_0 / \bar{v}^2)^{1/2} \) and \( W(z) = 1 + i(\pi)^{1/2} \exp \{-z^2\} \text{erfc}(-iz) \).

Calculating integrals of reverse Fourier transform

\[
\Phi(r, t) = \frac{1}{(2\pi)^4} \int_{-\infty}^{\infty} d\omega \int d^3 k e^{-i(\omega t - kr)} \Phi(k, \omega)
\]

we choose the axis \( z \) along \( u \) and introduce the dimensionless variables \( Z = (z - ut)\kappa_J, \quad X = x\kappa_J, \quad K = k/\kappa_J, \quad V = u/(\sqrt{2}\nu) \). Then, taking into account cylindrical symmetry we get

\[
\Phi(r, t) = \frac{GM_\odot \kappa_J}{2\pi^2} \int_{-\pi}^{\pi} d\phi \int_{-\pi}^{\pi} d\theta \int dKK^2 \sin \theta \frac{\exp\{iK(X \sin \theta \cos \phi + Z \cos \theta)\}}{K^2 - W(V \cos \theta)} = \\
\frac{GM_\odot \kappa_J}{(X^2 + Z^2)^{3/2}} \left[ 1 + (X^2 + Z^2)^{3/2} I(X, Z, V) \right],
\]

where

\[
I(X, Z, V) = \frac{1}{2\pi^2} \int_{-\pi}^{\pi} d\phi \int_{0}^{\pi} d\theta \int dK W(V \cos \theta) \sin \theta \frac{\exp\{iK(X \sin \theta \cos \phi + Z \cos \theta)\}}{K^2 - W(V \cos \theta)}.
\]

This integral determines a deviation of the effective potential from the Newtonian one described by the first term in the square bracket of the equation (9). After taking the Cauchy integral over \( K \) we find

\[
I(X, Z, V) = \frac{1}{4\pi^2} \int_{-\pi}^{\pi} d\phi \int_{0}^{\pi} d\theta \sin \theta \sqrt{-W} \left[ -2i \sinh(\Delta \sqrt{-W}) \text{Ci}(-i \Delta \sqrt{-W}) + \right. \\
\left. + \cosh(\Delta \sqrt{-W}) \left( \pi + 2i \text{Shi}(\Delta \sqrt{-W}) \right) \right],
\]

where \( \Delta = X \sin \theta \cos \phi + Z \cos \theta \). To calculate numerically values of this integral over an arbitrary range of variations of the parameters \( V \) and \( Z \) should present no problems but for the discussed subject of planetary motion in the solar system we can confine our consideration to concrete values of these parameters.

For stellar (barionic) matter in the Galaxy we have \( \kappa_J \simeq 2.6 \cdot 10^{-21} \text{cm}^{-1} \), thus the values \( Z \) and \( X \) can be estimated as \( 10^{-9} < |Z| < 10^{-6} \), \( 10^{-9} < |X| < 10^{-6} \) for planets of the solar system of which orbits remote from the Sun at distances \( 10^{12} < r < 10^{15} \text{cm} \). Since a peculiar velocity of the Sun in the Galaxy is \( 1.95 \cdot 10^6 \text{cm/c} \), and the velocity dispersion in the Galaxy is \( \bar{v} \simeq 1.55 \cdot 10^6 \text{cm/c} \), we put \( V = 1 \).

As a result of calculation of the integral (11) at \( V = 1 \) we get \( I = -0.3677 \). When varying \( V \) within the range from 0.3 to 3.0, the numerical value of the integral \( I \) varies from -0.3 to -0.47.

# 3 Correction to revolution periods

According to the equation (9) and obtained numerical estimation of the integral \( I \) the energy of the renormalized gravitational interaction between the Earth and the Sun can be
represented as
\[ U(R) = U_0 + \delta U, \quad U_0 = -A/r, \quad \delta U = A\gamma, \quad A = GM_\odot M_\oplus, \quad \gamma = -I\kappa J. \] (12)

Thus, in such approximation the correction \( \delta U \) to the Newtonian energy of interaction is constant value independent on \( r \).

Before proceeding to a calculation of its influence on the Kepler period of the Earth revolution it is necessary to check whether it disturb the closure of the finite trajectory of the Earth around the Sun.

In the classical two–body problem (after its reduction to the problem of the one body motion in a central force field) an angle of precession of the perihelion \( \Delta \varphi \) (see, e.g. [10]) is defined as
\[ \Delta \varphi = 2 \int_{r_{\text{min}}}^{r_{\text{max}}} \frac{M}{r} dr \sqrt{2m(E - U_0 - \delta U)} - \frac{M^2}{r}, \] (13)
where \( r_{\text{max}} \) and \( r_{\text{min}} \) are the maximum and minimum of the radius–vector \( r \), \( m = \frac{M_\odot M_\oplus}{M_\odot + M_\oplus} \) reduced mass,
\[ M = mr^2 \dot{\varphi}, \quad E = \frac{m\dot{r}^2}{2} + \frac{M^2}{2mr^2} + U. \]
The last two values, the moment \( M \) and energy \( E \), are the motion integrals.

The condition of closure of a trajectory is that the angle of precision to be of the form \( \Delta \varphi = 2\pi k/n \) where \( k, n \) are integers. For Newtonian potential this condition is fulfilled ( \( \Delta \varphi = 2\pi \) ).

As a result of expanding of the integrand of the equation (13) in \( \delta U \) the zeroth member gives \( 2\pi \), and the first member determines an additional precession of the perihelion as
\[ \delta \varphi = \frac{\partial}{\partial M} \left( \frac{2m}{M} \int_0^\pi r^2 dU d\varphi \right) \bigg|_0, \] (14)
where the expression within the brackets \( (...)_0 \) is estimated for the unperturbed motion. With the use of the parameter \( p \) and eccentricity \( e \) of an elliptic closed orbit we have
\[ r = \frac{p}{(1 + e \cos \varphi)}; \quad p = \frac{M^2}{ma}; \quad e = \sqrt{1 + \frac{2EM^2}{mA^2}}. \] (15)
Then
\[ \delta \varphi = \frac{2\gamma}{mA} \frac{\partial}{\partial M} \left( \frac{M^3 \pi}{(1 + e \cos \varphi)^3/2} \right) = \frac{2\gamma}{mA} \frac{\partial}{\partial M} \left( \frac{M^3}{2|m|A^3/2} \right) = 0. \]
Therefore, the perturbed trajectory remains closed.

We estimate now a change of the revolution period \( T \) under influence of the perturbation to the energy \( \delta U \). It is not difficult to check that all the mathematical treatment leading [10] to the third Kepler law for undisturbed Newtonian potential
\[ T_0 = \pi A \sqrt{\frac{m}{2|E_0|^{3/2}}}, \quad E_0 = \frac{m\dot{r}^2}{2} + \frac{M^2}{2mr^2} + U_0(r) \] (16)
remains valid when we change \( E_0 \) for \( E = E_0 + \delta U \) and leads to the modified form
\[
T = \pi A \sqrt{\frac{m}{2|E_0 + \delta U|}} = \pi A \sqrt{\frac{m}{2|E_0|}} \left(1 + \frac{\delta U}{E_0}\right)^{-3/2} \approx T_0 \left(1 + \frac{3A\gamma}{2|E_0|}\right),
\]
whence
\[
\frac{T - T_0}{T_0} = \frac{3A\gamma}{2|E_0|}.
\]

It can be noted that \( a_0 = \frac{A}{2|E_0|} \) is the semimajor axis. Substituting here the value of \( a_0 \) we get
\[
\frac{T - T_0}{T_0} = 3\gamma a_0 = 4.25 \times 10^{-8} \left[\frac{I}{-0.3677}\right] \left[\frac{G}{6.67 \times 10^{-8}}\right]^{1/2} \left[\frac{\rho_0}{1.9 \times 10^{-23}}\right]^{1/2} \times 
\]
\[
\times \left[\frac{v}{1.55 \times 10^6}\right]^{-1} \left[\frac{a_0}{1.496 \times 10^{13}}\right]
\]
which corresponds to \( \Delta T_{\oplus} \approx 1.3 \) sec/year (1 year \( \approx 3.15 \times 10^7 \) sec)

Thus, accounts for screening of the Newtonian potential gives rise to lengthen of the ET year on 1 second approximately what compensate (in the order of the value) the systematic difference \( \Delta T = UT - ET = UT - TAI \approx 0.8 \) sec.

Another consequence of the renormalization of the Newtonian potential is modification of parameters of the Earth orbit. In particular, the Earth semimajor axis \( a_{\oplus} \propto |E|^{-1} \) increase on the value
\[
\frac{\Delta a_{\oplus}}{a_{\oplus}} \approx \frac{A\gamma}{|E_0|} \approx 2.83 \times 10^{-8},
\]
that is, \( \Delta a_{\oplus} \approx 4.25 \times 10^5 \) cm.

Similar calculations may be performed for other planets of the solar system. Their results are represented in the Table.

| Planet | \( \Delta T/T \) | \( \Delta T \), sec | \( \Delta a/a \) | \( \Delta a \), km |
|--------|----------------|----------------|---------------|---------------|
| Venus  | 3.1 \times 10^{-8} | 0.6 | 2.1 \times 10^{-8} | 2.2 |
| Earth  | 4.3 \times 10^{-6} | 1.3 | 2.8 \times 10^{-8} | 4.2 |
| Mars   | 6.5 \times 10^{-8} | 3.8 | 4.3 \times 10^{-8} | 9.9 |
| Jupiter| 2.2 \times 10^{-7} | 0.83 \times 10^{2} | 1.5 \times 10^{-7} | 1.1 \times 10^{2} |
| Saturn | 4.1 \times 10^{-7} | 3.8 \times 10^{2} | 2.7 \times 10^{-7} | 3.9 \times 10^{2} |
| Uranus | 8.2 \times 10^{-7} | 2.2 \times 10^{3} | 5.4 \times 10^{-7} | 1.6 \times 10^{4} |
| Neptune| 1.3 \times 10^{-6} | 6.7 \times 10^{3} | 8.5 \times 10^{-7} | 3.8 \times 10^{4} |

Thus, we see that relative time and space corrections to the far giant planets motion resulted from the renormalization of the solar potential are of order \( 10^{-6} \). It may be supposed that they can be detected with the use of spacecrafts on far solar orbits.

The above results are based on some approximations. The most important of them are our choice of galactic disk volume density \( \rho_0 = 1.9 \times 10^{-23} \) g cm\(^{-3} \) having an influence on the value of the Jeans wave number \( \kappa_J \) and total ignoring of a dark matter. As it was shown in [6], inclusion of the dark matter gives rise to a sufficient decrease of an effective Jeans wave number and enhancing of the dynamical screening. Therefore, an experimental checking of the above results could give an additional information related to the dark matter in the Galaxy.
It would be interesting to apply our results to discussion of the observed [11, 12] anomalous Pioneer 10 and 11 acceleration with a magnitude $\sim 8 \times 10^{-8} \text{ cm s}^{-2}$. Its relative value is of order of $10^{-4}$, that is much greater our corrections to orbits of the giant planets being at the same distance (20 ÷ 30 AU) from the Sun. An expected value of the spacecraft acceleration was calculated with the use of two independent programs, that are JPL’s Orbit Determination Program (ODP) and Aerospace Corporation’s Compact High Accuracy Satellite Motion Program (CHASMP). Both programs use the planetary ephemeris, and timing inputs. Without knowledge of the programs we can only suppose that taking into account of our corrections to these data can produce a change of the expected acceleration and, probably, provide a reasonable explanation for the observed anomalous acceleration.

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