A Charged Inflaton leaves Behind a Fractal Primeval Structure of Electromagnetic Fields

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The inflaton field is assumed to possess electric charge. The effect of the charge upon inflation is studied using a parameter $\omega_0$ that acts as a measure of the quantity of inflaton present. It is shown that at the end of inflation the charged inflaton should produce a fractal electromagnetic structure, which could act as a seed for the development of macroscopic galactic structures. Another consequence would be the presence nowadays of residual electromagnetic fields that would encompass galaxies, clusters, and larger structures.

PACS numbers: 98.65.Dx, 98.80.Cq, 11.10.Ef, 41.20.-q

1. Introduction

The Big-Bang Model of cosmology has such a strong experimental support that the odds are that it is fundamentally correct. But if the expansion factor of the universe has always been a power of the time, that is, if

$$ a(t) \propto t^n $$

always, then certain theoretical difficulties arise, some of which (perhaps the most important) are:

A. An unexplained large entropy density at the beginning of the universe, or, what is logically equivalent, the so-called flatness problem, which is the observation that there is no explanation for the fact the density parameter $\Omega \equiv \rho/\rho_c$ is so close to one at early times.

B. The homogeneity of our present universe, which seems to contradict the fact that it is made up of many patches that have always been causally disconnected.

C. The absence in our universe of monopoles and other relics that should have been created during early times.

While these conditions do not contradict the theoretical picture of the Big-Bang Model, they are not explained by it, and point to its incompleteness.

The inflation paradigm [1] postulates a scalar field $\varphi$, called the inflaton, which possesses a self-interacting potential $V(\varphi)$. Under suitable circumstances, the pressure $p$ produced by this field is negative, so that the work done by the expanding universe is negative and it exactly compensates the energy necessary to maintain a constant energy density while the volume continuously grows. According to the inflationary paradigm, at the beginning of this universe, or shortly after, the inflaton’s energy dominated over other types and as a result there was an exponential expansion

$$ a(t) = ct \times e^{\omega t} $$

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through which the universe becomes many times larger. This growth explains the previously mentioned theoretical peculiarities as follows: The inflationary expansion, which is caused by a cosmic potential energy density, drives the density parameter $\Omega$ to exactly the value one (as can be proven using the equations of general relativity). This same energy density has then to be converted into elementary particles in a dissipative process that is very generous in its entropy production. The expansion is so large that the entire observable universe comes from only one small patch in the early universe, and monopoles and other relics are spread over such a vast volume that they become unobservable from a practical point of view. So all the difficulties we listed are resolved.

At inflation’s end all that is left is the vacuum and the quantum fluctuations of the inflaton field. All the cosmic structure seen today would have to originate in these fluctuations. In the last few years redshift surveys of large numbers of galaxies [2] have shown the universe to possess hitherto unsuspected macrostructures. The galaxies are distributed along the walls of huge voids as large as 100 Megaparsecs forming a honeycomb structure. It could be possible that even larger structures can become apparent with a survey encompassing a larger volume. At present there is much interest in this question, and two major redshift surveys are underway. [3] Just how consistent the known macrostructures are with the usual inflation scenario, where the only possible source for structure are the inflaton’s quantum fluctuations, is not clear at present. But if even larger structures are found, it will be impossible to accommodate them within this scenario.

In this paper we are going to assume the inflaton to be electrically charged (positively, for simplicity) and study how its charge is going to affect the inflationary process. Besides being an alternative that should be worked out, there is a logical reason to undergo such a study. It seems that scalar bosons can have a dynamic character, that is, they can be gauge bosons. [4] In this case, in the context of a generalized grand unified theory, they would couple to the gauge vector bosons and thus to the photon. Since the photon is massless, the inflaton is going to produce EM fields far more than any other kind of field.

The charge of the inflaton requires it to be a complex field. Incidentally, we shall assume a curved spacetime with a metric $g_{\mu\nu} = \text{diag}(1, -a^2, -a^2, -a^2)$ and a time coordinate $x^0$. We shall frequently use the 4-current $j_\mu = i\psi^* \vec{\partial}_\mu \psi = (j_0, j)$, where $\vec{\partial}_\mu \equiv \partial_\mu - \bar{\psi} \gamma_\mu \psi$. Complex inflatons have been treated before in the literature, but with different interests in mind. [5] In our treatment we include in the Lagrangian of the boson quantum theory a term $\omega_0 j_0$, that classifies homogenous states of complex bosons. Such states are exact solutions of $T = 0$ quantum field theories and are metastable under certain conditions. [6] The $\omega_0$ acts as a chemical potential in the sense that it is a measure of the amount of charge there is. This term is important in the understanding of the physical picture since, as we shall see, it forces the creation of electromagnetic (EM) fields as the universe expands. What this means physically is that the charged inflaton is forced to convert part of its energy into EM fields as the universe expands during inflation. Some of these EM fields have sizes comparable to the size of the patch that eventually results in our universe. Thus at inflation’s end there would exist in the universe a fractal array of EM fields that can act as seed for macrostructures.

2. Inflation with a complex inflaton
A typical Lagrangian for a complex field in flat spacetime is
\[ \mathcal{L} = |\partial_0 \varphi|^2 - |\nabla \varphi|^2 - m^2 |\varphi|^2 - \lambda |\varphi|^4. \tag{3} \]
The mass term has the usual sign, resulting in a potential with a minimum at \( \varphi = 0 \). An imaginary mass, on the contrary, would result in a minimum at a nonzero value for \( \varphi \). Recently the metastability of certain \( T = 0 \) solutions of the system given by (3) has been proven under fairly common conditions. [6] The metastability is brought into evidence by means of a field theory contact transformation, and the solutions are classified by a frequency \( \omega_0 \) that acts as a chemical potential (although there is no ensemble involved here) in the sense that it measures how much of the field there is. The last term in (3) is a self-interacting term that we will not include in our analysis.

We are interested in an electrically charged inflaton field \( \varphi \) that interacts with the EM potential \( A_\mu \), and so we use the quantum field theory Lagrangian:
\[ \mathcal{L} = |(i\partial_\mu - eA_\mu)\varphi|^2 - m^2 |\varphi|^2 - \frac{1}{4} F_{\mu \nu} F^{\mu \nu}. \tag{4} \]
This Lagrangian possess the local \( U(1) \) gauge invariance appropriate to electromagnetism. As an initial condition we assume that there is a uniform inflaton charge spread throughout the whole universe. We would like to measure the inflaton’s charge density in some convenient way. A reliable, if somewhat abstract way of doing this, is to perform the contact transformation (more specifically, the point transformation) generated by the functional
\[ \mathcal{F} = e^{i\omega_0 t} \Pi \varphi + e^{-i\omega_0 t} \varphi^* \Pi^* \tag{5} \]
between the fields \((\varphi, \pi)\) and \((\psi, \Pi)\), where \( \pi = \partial \mathcal{L} / \partial \dot{\varphi} \) and \( \Pi = \partial \mathcal{L}' / \partial \dot{\psi} \). [6] The resulting transformed Lagrangian, written with the metric in an expanded form, is:
\[ \mathcal{L}' = |\dot{\psi}|^2 - a^{-2} |\nabla \psi|^2 + [(\omega_0 - eA_0)^2 - m^2 - a^{-2} e^2 \mathbf{A} \cdot \mathbf{A}] |\psi|^2 + (\omega_0 - eA_0) j_0 + a^{-2} e \mathbf{A} \cdot \mathbf{j} - \frac{1}{4} F_{\mu \nu} F^{\mu \nu}. \tag{6} \]
The parameter \( \omega_0 \) serves then as a measure of the amount of charge present.

We need the Euler-Lagrange equations for the field \( \psi \), but one may wish first to add the term \( \Delta \mathcal{L}' = -\lambda (j_0)^2 \) to this Lagrangian. This term represents the repulsive electric self-interaction of the charge distribution \( j_0 \), and it takes such a simple form only when the distribution is fairly uniform. The electric energy density at the origin would be
\[ \int d^3 x' j_0(0) \frac{e^2}{|x'|} j_0(x') \approx \lambda [j_0(0)]^2, \tag{7} \]
where \( \lambda \) is an effective coupling constant that has a value depending on the size and type of universe. Technically speaking, this term arises from the quantum field theory Lagrangian as a second-order term in the perturbative expansion. The Euler-Lagrange equation of motion is:
\[ 0 = -2i \lambda (\partial_0 j_0) \psi - 4i \lambda j_0 \dot{\psi} + \ddot{\psi} + [3H - 2i(\omega_0 - eA_0)] \dot{\psi} - 2ia^{-2} e \mathbf{A} \cdot \nabla \psi - a^{-2} \nabla^2 \psi + [-3Hi(\omega_0 - eA_0) + e^2 a^{-2} \mathbf{A} \cdot \mathbf{A} + ie \dot{A}_0 - ia^{-2} e \nabla \cdot \mathbf{A} - (\omega_0 - eA_0)^2 + m^2] \psi, \tag{8} \]
where \( H = \dot{a}/a \) and the two first terms on the right are the ones due to \( \Delta \mathcal{L}' \). Our problem is to solve this equation simultaneously with the equations of electromagnetism and of general relativity.

The simplest solution, and the one that has evident physical interest, is the one where imaginary terms like \(-3Hi(\omega_0 - eA_0)\psi\) are basically zero for the short while inflation lasts because \( \omega_0 \approx eA_0 \). The inflaton’s charged current produces EM fields that contain a relatively small part of its energy, but are of theoretical interest. The resulting EM structure does not affect very much the dynamics of inflation for two reasons: first, is coupled to the inflaton through the small fine structure constant, and, second, the induced fields are weakened very fast due to the rapid volume growth. The field \( \psi \) has a solution that is similar to the one Linde employed in his chaotic inflation model. \[7\] Linde’s solution requires a large initial value for the inflaton field (of the order of Planck’s energy) and a steeply descending potential \( V(\psi) \), which, in our case, is given by the mass term \( m^2|\psi|^2 \).

In it the expansion factor \( a \) grows exponentially at a fixed rate throughout inflation, as in (2). The inflaton begins the inflationary period decreasing exponentially at a rate \( \beta \); that is, initially,

\[
\psi = ct. \times e^{-\beta t}.
\] (9)

However, the rate of decrease \( \beta \) of the inflaton increases slowly throughout the inflationary period until it stops the inflationary process, as we shall presently see.

The first two terms on the right in equation (8), that are due to the second order contribution \( \Delta \mathcal{L}' \), vanish because both \( j_0 = i\psi^* \partial_0 \psi = 0 \) and \( \partial_0 j_0 = i\psi^* \dddot{\psi} - i\dddot{\psi}^* \psi = 0 \) are zero for an inflaton obeying (9).

The equations of general relativity for a Robertson-Walker universe (with zero space curvature and cosmological constant) are

\[
H^2 = M_P^{-2}\rho \quad \text{and} \quad \ddot{a}/a = -\frac{1}{2}M_P^{-2}(\rho + 3p),
\] (10)

where \( \rho \) is the density, \( p \) is the pressure, and \( M_P^{-2} = \frac{8}{3}\pi G \). For the inflationary expansion described by (2), we have that \( H^2 = \ddot{a}/a = \alpha^2 \), so we conclude that

\[
\rho = -p = ct.,
\] (11)

which is a necessary and sufficient condition for inflation. Neglecting again terms with the factor \( (\omega_0 - eA_0) \) and the induced EM fields, the density is given by \( \rho = T_{00} = |\dot{\psi}|^2 + m^2|\psi|^2 \) and the pressure by \( p = \frac{1}{3}T_{kk} = |\dot{\psi}|^2 - m^2|\psi|^2 \). It is evident from these expressions that condition (11) is satisfied only if \( |\dot{\psi}|^2 \ll m^2|\psi|^2 \). This is the same as requiring \( \beta^2 \ll m^2 \) initially. Something interesting happens here. Neglecting in (8) terms with the factor \( (\omega_0 - eA_0) \) and also the smaller inhomogeneous, anisotropic terms due to induced EM fields, equation (8) can be simplified into the form

\[
0 = \dddot{\psi} + 3H\dot{\psi} + m^2\psi.
\] (12)

Notice that the same requirement \( \beta^2 \ll m^2 \) that we made to insure that (11) hold also assures us that the second time derivative in (12) is initially negligible! This, plus the fact
that initially $\psi \approx M_P$, are the technical details that allow in this case inflation to last long enough to solve many of the theoretical problems of the Big-Bang. Now, substituting $\psi$, as given by (9), in (12) leads us to the relation $3\alpha \beta = m^2$, from which we conclude that $\beta \ll \alpha$. This last inequality tells us that the inflaton field is not going to be zero at inflation’s end, because its decay rate is far smaller than the expansion factor’s growth rate.

Particles, monopoles and EM fields are weakened into oblivion due to the tremendous inflationary expansion. In traditional inflation only quantum fluctuations are left to generate structure; however, if the inflaton has an electric charge, it generates EM fields throughout inflation. Of those, the ones induced just before the end of the inflationary process remain. In other words, the EM fields induced at the end of inflation are not weakened by further expansion and can be the seed of a complex structure. As we said before, the small decay rate of the inflaton (compared with the large expansion rate of the expansion factor) assures us that there is going to be inflaton left at inflation’s end. From a mathematical point of view, the origin of these induced fields that survive inflation can be traced to the imaginary terms of equation (8), that have to cancel among themselves, and that have $\omega_0$ to give them a permanent scale. From a physical point of view, they are the last fields induced by the charged inflaton just before inflation’s end.

The inflaton’s electric charge density left at the end of inflation is furthered weakened by the usual Big-Bang expansion, but there should still be an asymmetry in the electric charge present nowadays in the cosmos. Gradients in space or time of the inflaton produce EM fields that in turn can produce other EM or inflatons. (Notice, regarding this point, that inflatons can couple directly with two photons, so that it is possible for a photon to decay directly into two inflatons and another photon, as long as it has energy $E > 2m$.) Therefore, as a result of their interaction with the inflaton field, a complicated fractal picture of electric currents and EM fields is formed, that ranges from macroscopic EM structures down to incoherent photons and particles. It is perhaps possible that this thermalization is enough to account for present-day matter, in which case no other dissipative mechanism would be necessary. The fractal macrostructures resulting from the charged inflaton’s expansion call to mind previous statistical correlation analysis that did not agree with a scale-invariant fluctuation spectrum. [8]

3. Concluding remarks

The Euler-Lagrange equation of Lagrangian (6) for the field $A_0$ is

$$a^{-2} \nabla \cdot (\nabla A_0 + \dot{A}) = -e\dot{j}_0 + 2e(eA_0 - \omega_0)|\psi|^2.$$  \hspace{1cm} (13)

Notice the strong weakening effect of the $a^{-2}$ on the induced EM fields. Thus throughout inflation $\omega \approx eA_0$, but at inflation’s end the expansion factor will not affect so decisively those fields and $A_0$ will begin to strongly depend on time. The question arises as to what direction the field $\mathbf{A}$ is going to take every time it appears at a different place of what should be a fairly isotropic universe. Every such occurrence represents a spontaneous breaking of the rotational symmetry. We suspect that quantum fluctuations $\delta \mathbf{A}$ of the vector field must be behind these symmetry breakings.
The scenario we are contemplating is the following. Near the end of inflation the vacuum is populated with quantum fluctuations and EM fields. Thermalization occurs and the universe begins an expansion that goes as a power of time. After 300,000 years go by, the universe becomes transparent to electromagnetic radiation and the radiation bath can suddenly propagate throughout the whole universe. About $10^{10}$ years later here on earth we study the radiation (now redshifted to the microwave) that was emitted precisely at this moment. So we are studying the structure impressed on a sphere with precisely a $10^{10}$ light-years radius. [9] We see a certain anisotropy of one part in 100,000 in the cosmic microwave background radiation, that could be due to the inflaton’s quantum fluctuations, or perhaps to the currents and density gradients induced in the plasma by the EM fields.

The information we get from the redshift surveys of galaxies has a free parameter, the redshift itself. By focusing on a specific redshift, we can study the structure of a sphere of a specific radius. By varying the redshift we can obtain three-dimensional mappings of the galaxies. The other sphere, the microwave one, is much farther away than the ones we study through redshifts, and the structure we see, of a two-dimensional nature, has yet to be enhanced by the effect of gravity working through billions of years. The microwave sphere is a cross section of the possible three-dimensional structure that could exist at the time. There seems to be no contradiction, at least in principle, between the small anisotropic microwave signal and the macroscopic structures seen at optical wavelengths. The data that will become available in the next few years will very likely clarify these topics.

One final comment. If our hypothesis is true and the field that drove inflation had an electric charge, the EM fields it induced should exist even today, although much weakened by the usual Big Bang expansion. They must encompass all galaxies, clusters and macrostructures.

ACKNOWLEDGEMENT

We wish to thank Dr. Walter Fernández, head of the Laboratorio de Investigaciones Atmosféricas y Planetarias, University of Costa Rica, for kindly allowing us to become a rather frequent users of the computers there during this past year.

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