Beyond The Colour-Singlet Model For Inelastic $J/\psi$ Photoproduction

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Abstract

Bound-state corrections to $J/\psi$ production from almost real photons are calculated in the colour-singlet model. A systematic, gauge-invariant, theory of hard quarkonium processes is used upto $O(v^2)$, where $v$ is the relative velocity of the quarks. The internal structure of the meson is characterised by two parameters, $\epsilon_B/M$ and $\nabla^2\phi(0)/M^2\phi(0)$, in addition to the usual wavefunction at the origin $\phi(0)$. These parameters are constrained to be consistent with measured leptonic decay of the $J/\psi$ and hadronic and radiative decays of $\eta_c$. The calculated corrections to the colour-singlet model, which include radiative effects, improve agreement with the experimental data.
The main production mechanism for $J/\psi$ particles in the inelastic process $\gamma + g \rightarrow J/\psi + X$ is believed to be the fusion, $\gamma + g \rightarrow J/\psi + g$. Because of this, $J/\psi$ production is an important tool for exploring the gluonic distribution inside nucleons. The first calculation of this in the so-called “colour-singlet” model was performed by Berger and Jones\cite{1} over 15 years ago. Subsequently, several authors applied the model to data as they became available. Recently radiative corrections to the basic model were calculated by Krämer et al.\cite{2}, who found these to be large at moderate photon energies $E_\gamma \approx 100\text{GeV}$. Earlier, relativistic corrections had been estimated by Jung et al.\cite{3} using a model proposed by Keung and Muzinich\cite{4}. These authors found the corrections to be fairly substantial, especially in the high-z ($z \geq 0.8$) region where the validity of the colour-singlet model is suspect. Inelastic photoproduction of the $J/\psi$ has been reviewed by Ali\cite{5}.

The purpose of this letter is to explore the effect of the binding of the quarks upon $J/\psi$ photoproduction in the colour-singlet model. In the original calculations\cite{1} this was totally neglected and the $c, \bar{c}$ were put on their mass shells. This is a sensible starting point because the binding energy $\epsilon_B = 2m - M$, and the quark relative velocity $v$, are small parameters: $\epsilon_B/M \ll 1$ and $v^2/c^2 \ll 1$. Since the $c$ quark mass is only $\sim 1.5 \text{GeV}$, substantial corrections could exist. This is indeed suggested by the calculations of refs.\cite{3,4}. However, we do not find the calculational method convincing for two important reasons. First, the model of ref\cite{4} does not treat gauge-invariance satisfactorily. Second, it incorpo-
rates binding energy corrections but not wavefunction corrections. Additionally, systematic improvement of the model seems difficult. Therefore a re-examination of bound-state corrections is important.

We have recently developed a systematically improvable gauge-invariant formalism for the one and two photon (gluon) decays of heavy quarkonia \[6\]. We extend and apply this here to $\gamma + g \rightarrow J/\psi + g$. This involves three bosons and is therefore considerably more complicated in computational terms. As noted in \[6\], the $J/\psi$ internal structure is described by more parameters than simply $\phi(0)$, the quark wavefunction at zero separation. These are $\epsilon_B/M$ and $\nabla^2\phi(0)/M^2\phi(0)$.

Our starting point is that the photoproduction amplitude $\gamma + p \rightarrow J/\psi + X$ is given by the sum of all distinct Feynman diagrams leading from the initial to the final state (Fig.1a). Each diagram can be written as an integral over the loop momenta which, for the lowest order diagram illustrated in Fig.1b, is

$$T_{\phi(1b)}^{\mu_1 \mu_2 \mu_3} = \int \frac{d^4k}{(2\pi)^4} Tr [ M(k) H^{\mu_1 \mu_2 \mu_3}(k)] . \quad (1)$$

The tensor $H^{\mu_1 \mu_2 \mu_3}(k)$ is the amplitude to produce a free gluon and two quarks, not necessarily on their mass-shells, from a photon and gluon. We call this the “hard” or perturbative part, and its expression can be read off from Fig.1b and permutations. The “soft” part is the zero-gluon, non gauge-invariant, Bethe-Salpeter amplitude,

$$M(k) = \int d^4x e^{ikx} \langle 0 | T[\bar{\psi}(-x/2)\psi(x/2)] | P, \epsilon \rangle. \quad (2)$$

In equations 1-2, $x^\mu$ is the relative distance between quarks and $k^\mu$ is the relative
momentum.

The next category of diagrams contain a single gluon exchange between the blob and one of the two hard propagators (Fig.2). These all have the general form,

$$T_{1}^{\mu_{1} \mu_{2} \mu_{3}} = \int \frac{d^{4}k}{(2\pi)^{4}} \frac{d^{4}k'}{(2\pi)^{4}} \text{Tr} [M^\rho(k, k') H_{\rho}^{\mu_{1} \mu_{2} \mu_{3}}(k, k')].$$

(3)

Again $H_{\rho}^{\mu_{1} \mu_{2} \mu_{3}}$ may be directly read off from the diagrams, and the soft part is,

$$M^\rho(k) = \int d^{4}x d^{4}z \ e^{ik \cdot x} e^{ik' \cdot z} \langle 0 | T [\bar{\psi}(-x/2) A^\rho(z) \psi(x/2)] | P, \epsilon \rangle.$$

(4)

This is a matrix in colour space since, $A^\rho \equiv \frac{1}{2} \lambda^a A^{a\rho}$. The soft gluon which originates from the blob has its momentum $k'$ bounded by $R^{-1} \sim k' \ll M$, where $R$ is the meson’s spatial size. The two gluon diagram can be included in the same way,

$$T_{2}^{\mu_{1} \mu_{2} \mu_{3}} = \int \frac{d^{4}k}{(2\pi)^{4}} \frac{d^{4}k'}{(2\pi)^{4}} \frac{d^{4}k''}{(2\pi)^{4}} \text{Tr} M^\rho\rho''(k, k', k'') H_{\rho\rho''}^{\mu_{1} \mu_{2} \mu_{3}}(k, k', k''),$$

(5)

with,

$$M_{\rho' \rho''}(k, k', k'') = \int d^{4}x d^{4}x' d^{4}x'' e^{i(k \cdot x + k' \cdot x' + k'' \cdot x'')} \langle 0 | T [\bar{\psi}(-x/2) A_{\rho'}(x') A_{\rho''}(x'') \psi(x/2)] | P, \epsilon \rangle.$$

(6)

The gluon self-interaction diagram in Fig.3b is similarly included.

As the next step, the hard parts are expanded in the quark and gluon relative momenta. The various amplitudes are combined and the Ward identity $\partial^\alpha S_F = -S_F \gamma^\alpha S_F$ is freely used. The upshot of the calculations is that the ordinary derivatives combine with gauge fields to yield covariant derivatives, i.e.,
a gauge-invariant result for the amplitude for $\gamma + g \rightarrow J/\psi + X$ is obtained,

$$(T_0 + T_1 + T_2)^{\mu_1 \mu_2 \mu_3} = Tr[\langle 0 | \bar{\psi} \psi | P, \epsilon \rangle h^{\mu_1 \mu_2 \mu_3} + \langle 0 | \bar{\psi} i \not{D}_\alpha \psi | P, \epsilon \rangle \partial^\alpha h^{\mu_1 \mu_2 \mu_3} + \langle 0 | \bar{\psi} i \not{D}_\alpha \not{i} \not{D}_\beta \psi | P, \epsilon \rangle \frac{1}{2} \partial^\alpha \partial^\beta h^{\mu_1 \mu_2 \mu_3} + \langle 0 | \bar{\psi} F^{\alpha \beta} \psi | P, \epsilon \rangle \frac{i}{2} \partial_\alpha H^{\mu_1 \mu_2 \mu_3} + \ldots].$$

In the above, $h^{\mu_1 \mu_2 \mu_3} = H^{\mu_1 \mu_2 \mu_3}(k = 0)$. The last term shall not concern us here since it is of higher order than $v^2$.

To proceed, one can perform a Lorentz and CPT invariant decomposition of each of the hadronic matrix elements in Eq. (7). This is somewhat complicated and involves a large number of constants which characterize the hadron. Considerable simplification results from choosing the Coulomb gauge, together with the counting rules of Lepage et al. The result of using this analysis is that, in this particular gauge, the gluons contribute at $O(v^3)$ to the reaction $^3S_1 \rightarrow \gamma + X$ and hence can be ignored. Even this leaves us with too many parameters, and forces us to search for a dynamical theory describing the essential dynamics of a $Q\bar{Q}$ system going beyond the usual non-relativistic potential models. A possible, but by no means unique, description is provided by the Bethe-Salpeter equation with an instantaneous kernel. This has been conveniently reviewed by Keung and Muzinich and we shall use their expression for the B-S amplitude in terms of the non-relativistic wavefunction $\phi(p)$. By projecting appropriately from their
wavefunction it is readily established that for $1^{-}$ states,

$$\langle 0 | \bar{\psi} \sigma_{\alpha} \bar{\psi} | P, \epsilon \rangle = \frac{1}{2} M^{1/2} \left( 1 + \frac{\nabla^2}{M^2} \right) \phi \left( 1 + \frac{P}{M} \right) \gamma^\nu \frac{1}{3 M^2} \left( 1 + \frac{\nabla^2}{M^2} \right) \phi, \quad \langle 0 | \bar{\psi} i \sigma_{\alpha} \bar{\psi} | P, \epsilon \rangle = \frac{1}{3} M^{3/2} \frac{\nabla^2 \phi}{M^2} \epsilon^\beta \left( -g_{\alpha \beta} + i \epsilon_{\mu \nu \alpha \beta} \frac{P^\nu}{M} \gamma^\mu \gamma_5 \right), \quad \langle 0 | \bar{\psi} i \sigma_{\alpha} i \sigma_{\beta} \bar{\psi} | P, \epsilon \rangle = \frac{1}{6} M^{5/2} \frac{\nabla^2 \phi}{M^2} \left( g_{\alpha \beta} - \frac{P_\alpha P_\beta}{M^2} \right) \left( 1 + \frac{P}{M} \right) \frac{\nabla^2 \phi}{M^2} \phi. \quad (8)$$

Using Eqs. 7 and 8, the differential crosssection for the subprocess $\gamma + g \rightarrow J/\psi + g$ comes out to be $^{1}$,

$$\frac{d\sigma}{dt} = \frac{256}{3 s^2} \frac{\alpha_s \alpha_s^2 \pi^2 M^2 \epsilon_q^2 |\phi(0)|^2 [\eta_0 f_0(s, t, u) + \eta_B f_B(s, t, u) + \eta_W f_W(s, t, u)]}{(s - M^2)^2 (t - M^2)^2 (u - M^2)^2}. \quad (9)$$

In the above $s$, $t$, and $u$ are the partonic level Mandelstam variables (we omit the usual carets) which obey the relation $s + t + u = M^2$. The appropriate average over initial gluon colours and sum over final gluon colours has been made. If radiative corrections are ignored,

$$\eta_0 = 1, \quad \eta_B = \frac{\epsilon_B}{M}, \quad \eta_W = \frac{\nabla^2 \phi}{M^2 \phi}. \quad (10)$$

The function $f_0$ is the standard, leading order, result:

$$f_0(s, t, u) = \frac{s^2 t^2 + t^2 u^2 + u^2 s^2 + M^2 s t u}{(s - M^2)^2 (t - M^2)^2 (u - M^2)^2}. \quad (11)$$

The binding energy and wavefunction corrections, $f_B$ and $f_W$ respectively, are slightly more complicated:

$$f_B(s, t, u) = \frac{1}{4D} \left[ -7 s t u(s^4 + t^4 + u^4) + 7 M^2 (s^3 t^3 + t^3 u^3 + u^3 s^3) \right]$$

$^{1}$We used Mathematica [9], supplemented by the HIP package [10], for computation of traces and simplification of algebra.
\[ + (s^2t^2 + t^2u^2 + u^2s^2)(s^3 + t^3 + u^3 + 15stu) \]

\[ + M^2stu(s^3 + t^3 + u^3 + 29M^2s^2t^2u^2), \quad (12) \]

and,

\[
f_W(s, t, u) = \frac{1}{6D} \left[ 141stu(s^4 + t^4 + u^4) - 85M^2(s^3t^3 + t^3u^3 + u^3s^3) \right. \\
- 27(s^2t^2 + t^2u^2 + u^2s^2)(s^3 + t^3 + u^3 + \frac{205}{27}stu) \\
- 139M^2stu(s^3 + t^3 + u^3) - 463M^2s^2t^2u^2 \right]. \quad (13) \]

The denominator \( D \) is:

\[
D = (s - M^2)^3(t - M^2)^3(u - M^2)^3. \quad (14) \]

The total \( \gamma p \) crossection is obtained by convolution of \( d\sigma/dt \) with the gluon distribution \( G(x) \) in the proton.

\[
d^2\sigma \over dxdt = G(x) {d\sigma \over dt}. \quad (15) \]

Integration of Eq.9 over \( t \) in the interval \( M^2 - s \) to 0 yields,

\[
\sigma(s) = \frac{256}{3}\pi^2e^2\alpha_s^2e_q^2\phi(0)^2 \frac{1}{M^5}[\eta_0F_0 + \eta_BF_B + \eta_WF_W + \text{rad.cor.}], \quad (16) \]

We note that Eq.9 reduces to Eq.23 of Jung et al.\[3\] if the condition \( \eta_W = \frac{1}{2}\eta_B \) is imposed. This latter condition is equivalent to \( \frac{1}{M} \nabla^2\phi(0) = \frac{1}{2}\epsilon_B\phi(0) \), which is the Schrödinger equation for quark relative motion in a potential which vanishes at zero separation. It is also worthy of note that the same condition emerges as a renormalization condition in the treatment of positronium by Labelle et al.\[11\] (see their Eqs.11 and 12). However, in our treatment there is no principle which a priori constrains \( \eta_B \) to bear a fixed relation to \( \eta_W \) and therefore both will be considered adjustable parameters.
where,

\[ F_0 = \left[ -1 - 4\xi + 2\xi^3 + \xi^4 + 2\xi^5 - 2(1 + 2\xi + 5\xi^2) \log \xi \right] / (1 - \xi)^2 \xi^2 (1 + \xi)^3, \]

\[ F_B = \frac{1}{2} \left[ -2 + 16\xi - 10\xi^2 + 48\xi^3 + 10\xi^4 - 64\xi^5 + 2\xi^6 \right. \]
\[ \left. - (1 - 3\xi + 14\xi^2 - 106\xi^3 + 17\xi^4 - 51\xi^5) \log \xi \right] / (1 - \xi)^3 \xi^2 (1 + \xi)^4, \]

and

\[ F_W = \frac{1}{3} \left[ 26 - 14\xi + 210\xi^2 - 134\xi^3 - 274\xi^4 + 150\xi^5 + 38\xi^6 - 2\xi^7 \right. \]
\[ \left. + (27 + 50\xi + 257\xi^2 - 292\xi^3 + 205\xi^4 - 78\xi^5 - 41\xi^6) \log \xi \right] / (1 - \xi)^3 \xi^2 (1 + \xi)^5. \tag{17} \]

In the above, \( \xi = s/M^2 \). The curves for \( F_0, F_B \) and \( F_W \) as a function of \( \xi \) are shown in Fig[4]. Note that \( F_W \) is a large negative number for very small values of \( s \) which rises and becomes positive for \( \sqrt{s} > 4.75 \, GeV \) while \( F_B \) is a negative quantity for all values of the incoming photon energies. The radiative correction to \( \sigma(s) \) has already been calculated by Krämer et al.[2]. They have taken into account the modification of the initial gluon densities as well as the box diagrams and the splitting of the final gluon into gluon and light quark-antiquark pairs. Like the behaviour of the scaling functions plotted in [2], the corrections presented in Fig.[4] are also large at moderate photon energies but decrease with incoming energies.

The differential crosssection \( d\sigma/dz \) calculated for \( \sqrt{s} = 14.7 \, GeV \) is shown in Fig.[5] where \( z = E_{J/\psi}/E_\gamma \). We use a simple gluon distribution function \( xG(x) = 3(1 - x)^5 \). For the numerical evaluations we take \( \alpha_s = 0.19 \) and \( m = 1.43 \, GeV \). Additionally, we take \( \eta_W = -0.073 \) and \( |R_{J/\psi}|^2 = 0.978 \, GeV^3 \). This set
of parameters is the same as given in Ref. [6]. These values, when inputted into the theoretical formulae, with radiative corrections evaluated at $\mu = m$ yield the following values for the decay widths [6],

$$
\begin{align*}
\Gamma(J/\psi \rightarrow e^+e^-) &= 5.61 \text{ KeV} \\
\Gamma(\eta_c \rightarrow \text{hadrons}) &= 9.99 \text{ MeV} \\
\Gamma(\eta_c \rightarrow 2\gamma) &= 6.48 \text{ KeV}
\end{align*}
$$

which are quite close to the values of the experimentally measured decay widths [15], 5.36 ± .28 KeV, 10.3 ± 3.6 KeV and 8.1 ± 2 KeV respectively. As remarked in Ref. [6], the value of $\alpha_s$ chosen for the numerical calculations differs from its value deduced from deep inelastic scattering, $\alpha_s(m_c) \approx 0.3$. Larger values inserted into the expressions for the decay rates result in large differences between the wavefunctions at the origin of $J/\psi$ and $\eta_c$, violating the assumption that these are of $O(v^2)$.

In Fig. [5], we show various theoretical results for $d\sigma/dz$, together with the experimental data taken from the EMC and NMC [12, 13, 14]. The dash-dot line is the zeroth order term calculated by setting $\eta_B = \eta_W = 0$ in Eq. [9]. $d\sigma/dz$ calculated from Eq. [9] with the identification $\eta_W = \frac{1}{2}\eta_B$, but without radiative corrections, is shown with a dotted line (see footnote 2). The dashed line is the differential crosssection for $\eta_W = -0.073$ without radiative corrections. Finally the solid line is the full result (including radiative corrections) calculated up to $O(v^2)$. A $K$ factor of 3.5 has been used, obtained by fitting the crossection including
all the corrections upto $O(v^2)$ to the experimental data. The overall effect of including these terms is that they describe the experimental data quite well in the region $0.5 < z < 0.9$.

In Fig.[6a]-[6f], the double differential crosssection $d\sigma/dzdp_t^2$ is shown for different $z$-bins and a comparison is made with the experimental data [12, 13]. We see that the relativistic corrections move the theoretical predictions in the right direction. However the theoretical curve does not describe the data for the highest $z$ region ($0.95 < z < 1$) where the elastic $J/\psi$ production is expected to be important. Our formalism works best for the moderate $z$ region i.e, $0.7 \leq z \leq 0.95$. Note that the validity of the colour singlet model hinges essentially upon all propagators being hard, a condition which only holds away from the end points. Gluonic vacuum fluctuation, a non-perturbative effect, may otherwise be important [16].

In conclusion, we have estimated the $O(v^2)$ corrections to the inelastic $J/\psi$ production from $\gamma p$ collisions and shown that the agreement of theory with experiment is improved if these corrections are included.

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3The value of the $K$ factor depends upon the value of $\alpha_s$. For $\alpha_s = 0.3$, it is lowered to 1.25.
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Figure Captions

Figure:1 a) The amplitude $\gamma + p \rightarrow J/\psi + X$, b) One of the 6 leading order diagrams.

Figure:2 a) One gluon exchange diagram.

Figure:3 a) Two gluon exchange diagram. b) Three gluon vertex diagram.

Figure:4 The functions $F_0$, $F_B$ and $F_W$ as a function of $\xi = s/M^2$. Note that their contribution is large at moderate incoming photon energies and decreases as $s$ increases.

Figure:5 The differential crosssection $d\sigma/dz$ plotted as a function of $z$ at $\sqrt{s} = 14.7 GeV$. The solid line is the full crosssection (including the radiative correction calculated by Krämer et al.\cite{3}). $d\sigma/dz$ calculated from Eq.\cite{9} with $\eta_B = -0.076$ and $\eta_W = -0.073$ is shown by the dashed line while the dotted line corresponds to the curve with the choice $\eta_W = \frac{1}{2} \eta_B$ (see footnote 2). The curve represented by dash-dot line is the crosssection with $\eta_B = 0$ and $\eta_W = 0$. A $K$ factor of 3.5 has been used to account for the overall normalization. The data points are taken from EMC and NMC\cite{12,13,14}.

Figure:6 The double differential crosssection $d\sigma/dz/dp_t^2$ for different $z$-bins. The solid curve is predicted from the model including $\eta_B$ and $\eta_W$ corrections ($K = 3.5$). The dotted curve is for the choice $\eta_W = \frac{1}{2} \eta_B$. The dashed curve is for $\eta_B = 0$ and $\eta_W = 0$. 

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