The strangeness content of the nucleon from effective field theory and phenomenology

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We revisit the classical relation between the strangeness content of the nucleon, the pion-nucleon sigma term and the SU(3)_F breaking of the baryon masses in the context of Lorentz covariant chiral perturbation theory with explicit decuplet-baryon resonance fields. We find that a value of the pion-nucleon sigma term of ~60 MeV is not necessarily at odds with a small strangeness content of the nucleon, in line with the fulfillment of the OZI rule. Moreover, this value is indeed favored by our next-to-leading order calculation. We compare our results with earlier ones and discuss the convergence of the chiral series as well as the uncertainties of chiral approaches to the determination of the sigma terms.

I. INTRODUCTION

We dedicate this study to the interplay between the nucleon sigma terms, \(\sigma_{\pi N}\) and \(\sigma_s\), which are defined as

\[
\sigma_{\pi N} = \frac{1}{2M_N} \langle N|\hat{m} (\bar{u}u + \bar{d}d)|N\rangle,
\]

\[
\sigma_s = \frac{1}{2M_N} \langle N|m_s\bar{s}s|N\rangle.
\]

Here, the up, down and strange quarks masses are indicated by \(m_u, m_d\) and \(m_s\), respectively, and \(\hat{m} = (m_u + m_d)/2\).

In the following, we restrict ourselves to the isospin limit, \(m_u = m_d = \hat{m}\), with the nucleon states having the Lorentz invariant normalization \(\langle N(p',s')|N(p,s)\rangle = 2E_N(2\pi)^3\delta(p' - p)\), where \(E_N = \sqrt{M_N^2 + |p|^2}\), \(M_N\) is the nucleon mass and \(s\) and \(s'\) are the spin indices.

Both \(\sigma_{\pi N}\) and \(\sigma_s\) are interesting observables and their non-vanishing values would clearly indicate that quark masses are not zero and give contribution to the nucleon mass. More precisely, the values of these two sigma terms embody the internal scalar structure of the proton and neutron. If they are small, most of the nucleon mass stems from the confinement of the lightest quarks in typical distances around 1 fm. Another property related to the nucleon scalar structure is the strangeness content of the nucleon, \(y\), which is defined as

\[
y = \frac{2\langle N|\bar{s}s|N\rangle}{\langle N|\bar{u}u + \bar{d}d|N\rangle} = \frac{2\hat{m} \sigma_s}{m_s \sigma_{\pi N}}.
\]

Notice that if the OZI rule (large \(N_C\) prediction) were exact then \(y = 0\). Besides their role in understanding the mass of the ordinary matter, \(\sigma_{\pi N}\) and \(\sigma_s\) are also necessary with respect to theoretical speculations on the origin of dark matter particles based on supersymmetry. An accurate determination of the sigma terms is needed to constrain the parameter space of the underlying supersymmetric models from the experimental bounds in direct searches of weakly interacting dark matter particles.

The determination of \(\sigma_{\pi N}\) is feasible from \(\pi N\) scattering data due to the low-energy theorem of current algebra that relates the value of the isospin even \(\pi N\) scattering amplitude at the Cheng-Dashen point with the nucleon scalar form factor. However, the situation is much more obscure for the strangeness scalar form factor of the nucleon, and then for the phenomenological determination of \(\sigma_s\) as well as of \(y\). Historically, the path to escape this end point is based on combining the definitions of Eqs. (1) and (2) as

\[
\sigma_{\pi N} = \frac{\sigma_0}{1 - y}
\]

where \(\sigma_0\) is the nucleon expectation value of the purely octet operator \(\bar{u}u + \bar{d}d - 2\bar{s}s\),

\[
\sigma_0 = \frac{\hat{m}}{2M_N} \langle N|\bar{u}u + \bar{d}d - 2\bar{s}s|N\rangle.
\]

The point to notice is that the latter operator is the only one in the QCD Lagrangian responsible for the hadronic mass splitting within an \(SU(3)\) multiplet. From the experimental values of the lightest baryon octet masses, \(M_{\Xi}, M_{\Sigma}\)
and $M_N$, we can then calculate approximately $\sigma_0$ by making use of $SU(3)$ flavor symmetry, with the result
\begin{equation}
\sigma_0 = \frac{\hat{m}}{m_s - \hat{m}} (M_\Xi + M_\Sigma - 2M_N) \simeq 27 \text{ MeV},
\end{equation}
where we have used $m_s/\hat{m} = 26(4)$.

Additionally, with this value for $\sigma_0$ and by assuming the OZI rule to hold, so that $y = 0$, one obtains from Eq. (3) the naive estimation $\sigma_{\pi N} \simeq 30 \text{ MeV}$, that is much smaller than its phenomenological determinations from $\pi N$ scattering data. For instance, Gasser et al. [5] obtained the canonical result $\sigma_{\pi N} \simeq 45 \text{ MeV}$ in terms of a dispersive analysis of the pre-90s $\pi N$ elastic scattering data. A partial-wave analysis including the more modern $\pi N$ database carried out by the George-Washington University group [8], resulted in larger values of the pion-nucleon sigma term, $\sigma_{\pi N} = 64(8) \text{ MeV}$ [3]. Besides that, a study of $\pi N$ elastic scattering in Lorentz covariant baryon chiral perturbation theory (B$\chi$PT) agrees with the dispersive results, which depend on the data set employed [10]. Additionally, it also reveals that modern partial-wave analyses are, in general, more consistent with different scattering phenomenology than the older ones and lead to a relatively large value of the sigma-term, cf. $\sigma_{\pi N} = 59(7) \text{ MeV}$ [11].

The actual value of $\sigma_{\pi N}$ has important consequences on the strangeness content of the proton since, according to Eq. (3) and the result for $\sigma_0$ in Eq. (4), all these values for $\sigma_{\pi N}$ extracted from $\pi N$ scattering data would imply a very large result for $y$.

Now, at this point it is important to emphasize that Eq. (5) is an estimate obtained at leading order in a $SU(3)_F$-breaking expansion and the calculation of $\sigma_0$ from this equation could be affected by large higher order contributions. The next-to-leading order (NLO) chiral corrections were first calculated by Gasser in Ref. [17]. There he obtained $\sigma_0 = 35(5) \text{ MeV}$ by employing a chiral model for the meson cloud around the baryon which only considered contributions from the virtual octet baryons. Within the more evolved theoretical framework of B$\chi$PT Ref. [12] performed a calculation of the baryon masses and $\sigma_0$ in the heavy-baryon (HB) [13] expansion up to next-to-next-to-leading order (NNLO). In this work, the contributions of the decuplet-baryon resonances were not implemented explicitly but through resonance-saturation hypothesis they contributed to several of the many low-energy-constants (LECs) appearing at this order. All in all, they reported the value $\sigma_0 = 36(7) \text{ MeV}$, which was almost identical to the NLO result obtained by Gasser 15 years earlier. Later, Ref. [14] also included the decuplet-baryon resonances within HB$\chi$PT using a cut-off regularization scheme and still obtained basically the same result for $\sigma_0$. One should also notice that $\sigma_{\pi N} = 45 \text{ MeV}$ was taken as input in the analyses of Ref. [12, 14], which had a strong influence in the results of Ref. [14].

By employing $\sigma_0 \simeq 35 \text{ MeV}$ from the calculations of Refs. [12, 14, 17] in Eq. (3) one obtains that $y \simeq 0.2$ and $0.4$ for $\sigma_{\pi N} \simeq 45 \text{ MeV}$ and $\simeq 60 \text{ MeV}$, respectively. In particular, the latter value would imply a strangeness contribution to the mass of the nucleon of $\sim 300 \text{ MeV}$. Although not impossible, such a scenario with a strong breaking of the OZI rule is theoretically implausible, moreover after the experimental evidence pointing to a negligible strangeness contribution in other properties of the nucleon such as its electromagnetic structure [15] and spin [16]. Thus, if one gives credit to these results and translate them into a small value of $y$, then the present widely accepted value for $\sigma_0$ around $35 \text{ MeV}$ clearly discredits the relatively large values for $\sigma_{\pi N}$ favored by the most recent analysis of the $\pi N$ scattering data, cf. $\sigma_{\pi N} = 64(8) \text{ MeV}$ [3] and $\sigma_{\pi N} = 59(7) \text{ MeV}$ [11].

It is our aim in this work to emphasize that the situation concerning $\sigma_0$ is not settled yet, so that the previous conclusion does not necessarily hold. On one hand, the result of Gasser [17] is based on a model calculation of the meson cloud around the nucleon, whereas Refs. [12, 14] might be afflicted by the poor convergence of the chiral series typically shown by HB in the $SU(3)_F$ theory [18, 19].

A suitable approach that also includes explicitly the contributions from the decuplet-baryon resonances is a Lorentz covariant formulation of B$\chi$PT with a consistent power-counting via the extended-on-mass-shell renormalization (EOMS) scheme [20]. The relativistic corrections that results in this approach, in a way preserving the exact analytical properties of the Green functions, have been shown to tame the poorly convergent series of the HB expansion in baryonic observables as important as the magnetic moments [18] or masses [19, 22]. Moreover, once a prescription is taken to treat the problem of the interacting Rarita-Schwinger fields [23], this scheme is straightforwardly applicable to include the contributions of the decuplet-baryon resonances [21]. In this work we calculate $\sigma_0$ up to NLO using Lorentz covariant B$\chi$PT renormalized in the EOMS prescription and including explicitly the effects of the decuplet. We compare the results with those obtained in the HB expansion and estimate systematic higher-order effects through a partial calculation of NNLO pieces. All together, we find the remarkable result that the value of $\sigma_0$ becomes larger so that the modern experimental determinations of $\sigma_{\pi N} \sim 60 \text{ MeV}$ are then consistent with a small strangeness

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1 For a detailed exposition of the dispersive methods for obtaining $\sigma_{\pi N}$ from the analytic continuation of the $\pi N$ scattering amplitude to the Cheng-Dashen point see Refs. [3, 4].
FIG. 1: Feynman diagrams contributing to the nucleon mass up to $\mathcal{O}(p^3)$ in B$\chi$PT. The internal solid lines correspond, in general, to any octet baryon, double lines to decuplet-baryon resonances and dashed lines to mesons. The black dots indicate 1st-order couplings while crosses are insertions of $\mathcal{O}(p^2)$ operators given by the LECs $b_0$, $b_D$ and $b_F$ responsible for the leading $SU(3)_F$ breaking of the baryon-octet masses.

content of the nucleon, or with a small OZI rule violation. A first indication that the decuplet contributions could help to solve the strangeness puzzle concerning a relatively large $\sigma_{\pi N}$ was given by the HB calculation in Ref. [24]. Indirectly, this was also the case in Ref. [25] where very large and negative values of $\sigma_s$ were obtained when demanding $\sigma_{\pi N} = 45$ MeV, indicating a larger $\sigma_0$.

II. CALCULATION

The expressions for the sigma terms can be obtained either from the explicit calculation of the scalar form factor of the nucleon at $q^2 = 0$ or applying the Hellmann-Feynman theorem to the chiral expansion of its mass,

$$\sigma_{\pi N} = \hat{m} \frac{\partial M_N}{\partial \hat{m}} = \frac{m_N^2}{2} \left( \frac{1}{m_\pi} \frac{\partial}{\partial m_\pi} + \frac{1}{2m_K} \frac{\partial}{\partial m_K} + \frac{1}{3m_\eta} \frac{\partial}{\partial m_\eta} \right) M_N + \mathcal{O}(p^4),$$

$$\sigma_s = m_s \frac{\partial M_N}{\partial m_s} = (m_K^2 - m_\pi^2) \left( \frac{1}{2m_K} \frac{\partial}{\partial m_K} + \frac{2}{3m_\eta} \frac{\partial}{\partial m_\eta} \right) M_N + \mathcal{O}(p^4).$$

(6)

We follow the latter strategy since the explicit expressions for the baryon masses in the different schemes treated in this paper can be directly obtained using the Appendix of Ref. [19]. Thus, the chiral expansion of the sigma terms up to NLO from Eq. (6) is written as,

$$\sigma_{\pi N} = -4(2b_0 + b_D + b_F) \frac{m_N^2}{2} +$$

$$\frac{1}{(4\pi F_\phi)^2} \sum_{\phi=\pi,K,\eta} \left( \xi_{N,\phi}^{(B)} \Sigma_{\pi}^{(B)}(m_\phi) + \xi_{N,\phi}^{(T)} \Sigma_{\pi}^{(T)}(m_\phi) \right) + \mathcal{O}(p^4),$$

$$\sigma_s = -4(b_0 + b_D - b_F) \left( m_K^2 - m_\pi^2 \right) +$$

$$\frac{1}{(4\pi F_\phi)^2} \sum_{\phi=\pi,K,\eta} \left( \xi_{N,\phi}^{(B)} \Sigma_{s}^{(B)}(m_\phi) + \xi_{N,\phi}^{(T)} \Sigma_{s}^{(T)}(m_\phi) \right) + \mathcal{O}(p^4).$$

(7)

The first line in these formulas corresponds to the LO contribution given at tree-level by the same $\mathcal{O}(p^3)$ LECs that appear in the chiral expansion of the baryon masses. While $b_0$ provides a $SU(3)_F$-singlet contribution that cannot be disentangled from the bulk mass of the octet baryons, the LECs $b_D$ and $b_F$ induce a splitting of octet-baryon masses (tree-level in diagram (a) in Fig. 1) which gives rise to the GMO relation [19]. The second lines enclose the NLO or $\mathcal{O}(p^3)$ corrections that stem from the loop topologies shown in Fig. 1 (b) and (c). Thus, the effect of virtual octet ($B$) and decuplet ($T$) baryons is explicitly accounted for. Their contributions are weighted by the coefficients $\xi_{N,\phi}^{(X)}$, which are combinations of $SU(3)$ Clebsch-Gordan coefficients and the meson-baryon couplings $D$, $F$ (octet contributions) and $C$ (decuplet contributions). The loop functions $\Sigma_{\pi,s}^{(X)}$ depend, exclusively, on the mass of the virtual pseudoscalar meson and on the ones of the octet and decuplet baryons in the chiral limit, $M_B$ and $M_T$ respectively. Strictly speaking, the $SU(3)_F$ breaking of the baryon masses in these loops, which are represented by the crosses in Fig. 1 (b) and (c), are contributions that start at NNLO or $\mathcal{O}(p^4)$.

For the baryon masses we use the results obtained in Ref. [19] in Lorentz covariant B$\chi$PT up to $\mathcal{O}(p^3)$ in EOMS. The chiral loops contain divergences and analytic pieces breaking the power-counting formula [11] that are removed in dimensional regularization by the proper redefinition of the bare LECs (EOMS scheme [20]). The contributions of
the decuplet baryons are included taking the octet and decuplet masses in the chiral limit of approximately the same order. Namely, the octet and decuplet contributions are considered on the same footing for power-counting purposes and no specific expansion in \( \delta = (M_B - M_O) \) is performed. The HB formulas \( \Delta \) can be always recovered from the renormalized covariant results by taking the non-relativistic expansion \( M_B \sim M_O \sim \Lambda_{\chi PT} \). In particular, the HB results within the small-scale-expansion (SSE) \( 26 \), that it is used to include explicitly the decuplet resonances,\(^2\) are retrieved once the HB expansion is performed in our results \( 25 \).

| Tree level \( O(p^2) \) | Octet \( O(p^2) \) | Octet+Decuplet \( O(p^2) \) |
|--------------------------|----------------|-------------------------------|
|                         | \( b_O \) [GeV\(^{-1}\)] | \( b_F \) [GeV\(^{-1}\)] | \( b_O \) [GeV\(^{-1}\)] | \( b_F \) [GeV\(^{-1}\)] | \( b_O \) [GeV\(^{-1}\)] | \( b_F \) [GeV\(^{-1}\)] |
|                         | 0.060(4)    | -0.213(2)   | 0.061(4)    | -0.502(2)   | 0.315(4)    | -0.420(2)   |
|                         | 0.061(4)    | -0.213(2)   | 0.061(4)    | -0.502(2)   | 0.161(4)    | -0.420(2)   |

TABLE I: Values of the \( O(p^2) \) LECs \( b_O \) and \( b_F \) determined from the baryon octet mass splittings in the different B\( \chi \)PT approaches considered in this paper.

For the numerical values of the couplings, we use \( D = 0.80 \) and \( F = 0.46 \) \( 27 \). The decuplet coupling \( C \) can be fixed from the \( \Delta(1232) \to \pi N \) decay rate, giving \( C = 1.0 \) \( 21 \). However, there is some evidence from LQCD that this coupling is somewhat smaller \( 22 \). Indeed, an \( SU(3)_F \)-average among the different decuplet-to-octet pionic decay channels gives \( C = 0.85 \pm 0.15 \), that is the value we use.\(^3\) As mentioned above, the \( O(p^2) \) LECs \( b_O \) and \( b_F \) are determined using the experimental baryon-octet mass splittings. Their values for the different B\( \chi \)PT schemes analyzed in this paper can be found in Table I \( 19 \). For the meson decay constant we also take the \( SU(3)_F \)-average \( F_\pi \equiv 1.17 f_\pi \) with \( f_\pi = 92.4 \) MeV. Variations in these values of \( D \), \( F \), \( C \) and \( F_\pi \) were discussed in Ref. \( 19 \) and do not influence the final results once their correlations are taken into account. For the masses of the pseudoscalar mesons we use \( m_\pi = m_{\pi \pm} = 139 \) MeV, \( m_K = m_{K \pm} = 494 \) MeV, while for the baryon masses in the loops we use the chiral-limit baryon masses obtained at LO, \( M_B^{(1)} = 1.151 \) GeV and \( M_T^{(1)} = 1.382 \) GeV. The mass of the \( \eta \) meson is fixed with the Gell-Mann-Okubo mass relation, \( 3 m_\pi^2 = 4 m_K^2 - m_\eta^2 \) which is accurate enough up to the order we work.

Finally, we restrain our analysis to \( O(p^3) \) despite of the fact that the extension of formulas to \( O(p^4) \) accuracy is straightforward, albeit affected by a dramatic loss of predictability, and have been reported in the literature \( 12, 28-31 \). At the latter order, 15 new LECs contribute to the baryon masses and sigma-terms. Eight of them correspond to \( O(p^3) \) operators which appear through diagrams with the topology of a tadpole (see Ref. \( 28 \) for details). These also contribute to the chiral expansion of the meson-baryon scattering amplitudes, although their LECs have not been determined yet from the associated experimental data or LQCD results. The other loop diagrams appearing at this order are the ones at \( O(p^3) \) but with the \( SU(3)_F \) breaking of the baryon masses in the loop taken into account by insertions of the \( O(p^2) \) LECs \( b_O \), \( b_F \) and \( b_D \) (crosses in the diagrams \( b \) and \( e \) of Fig. \( 1 \)). The remaining 7 LECs correspond to \( O(p^2) \) operators and they renormalize the loop divergences appearing at this order. Therefore, a quantitative analysis of the sigma terms at NNLO without any further assumption on the values of the LECs (such as Large \( N_c \) constraints \( 29 \) or resonance saturation hypothesis estimates \( 12 \)) is affected, at present, by a large uncertainty. On the other hand, a promising source of theoretical information on the values of the LECs is becoming available through LQCD calculations. An application in this direction within EOMS B\( \chi \)PT at \( O(p^3) \) and \( O(p^4) \) can be found in \( 19 \) and \( 30, 31 \), respectively.

Nevertheless, the analysis of part of the \( O(p^3) \) corrections can be useful to assess the convergence of the chiral series and to give a credible estimate on the systematic error to the \( O(p^3) \) results on the sigma terms due to the truncation of their chiral expansions. Indeed, we have calculated explicitly the respective corrections arising from the \( SU(3)_F \) breaking of the baryon masses in the loops \( b \) and \( e \) of Fig. \( 1 \). The divergences have been renormalized in the EOMS scheme and the uncertainty on the unknown values of the \( O(p^4) \) LECs has been explored by varying the renormalization scale in the interval \( 0.7 \) GeV \( \leq \mu \leq 1.3 \) GeV. The maximal contribution obtained for these corrections in both, the octet and decuplet diagrams, is quoted as our theoretical uncertainty. That is,

\[
\Delta \sigma_{\pi N}^{\chi PT} \simeq 20 \text{ MeV}, \quad \Delta \sigma_{\pi N}^{\chi EOMS} \simeq 6 \text{ MeV}, \quad \Delta \sigma_{\pi N}^{\chi EOMS} \simeq 6 \text{ MeV}.
\]
This explicit calculation of higher-order pieces already confirms the expectation that the convergence in the covariant approach is substantially better than the one obtained in the HB case [18, 19].

| \( b_0^{\text{Expt}} \) [GeV\(^{-1}\)] | \( y \) | \( \sigma_s \) [MeV] |
|----------|----------|----------------|
| \( \sigma_{\pi N} = 45(7) \) MeV | -0.79(9) | -0.28(13)(10) -150(80)(60) |
| \( \sigma_{\pi N} = 59(7) \) MeV | -0.97(9) | 0.02(13)(10) 16(80)(60) |

TABLE II: Value of the LEC \( b_0 \) and of the observables related to the strangeness of the nucleon, \( y \) and \( \sigma_s \), obtained in Lorentz covariant B\(\chi\)PT including decuplet contributions and using the phenomenological determinations of \( \sigma_{\pi N} \) as input.

\[ \Delta \sigma_{\pi N} = 10 \text{ MeV} \]

This indicates a stabilization of the final outcome at the O\(\chi\)PT level. The difference \( \Delta \sigma_{\pi N} \) between the calculations excluding/including the explicit decuplet of baryon resonances is only of around 10 MeV.

| \( \sigma_0 \) [MeV] | Tree level \( O(p^2) \) | Octet \( O(p^2) \) | Octet-Decuplet \( O(p^2) \) |
|--------------------|-----------------|-----------------|--------------------|
| \( b_0^{\text{OZI}} \) [GeV\(^{-1}\)] | HB Covariant | HB-SSE Covariant | HB-SSE Covariant |
| \( \sigma_0 \) [MeV] | 27 | 58(23) | 46(8) | 89(23) | 58(8) |
| \( b_0^{\text{OZI}} \) [GeV\(^{-1}\)] | -0.274 | -0.90(15) | -0.70(5) | -1.52(15) | -0.95(5) |

TABLE III: Values of \( \sigma_0 \) and the \( O(p^2) \) LEC \( b_0 \) given by the exact fulfillment of the OZI rule for the different B\(\chi\)PT approaches considered in this paper.

In order to appreciate the improvement in the chiral expansion that results by employing Lorentz covariant B\(\chi\)PT in the EOMS we compare our results for \( y = 0 \), quite close to the last line value in Table III with the HB\(\chi\)PT calculations with/without the decuplet-baryon resonances in Table III. As we can see, the corrections to the LO result on \( \sigma_0 \) studied are large. This occurs despite that the discrepancy of the Gell-Mann-Okubo equation is correctly predicted in any of these schemes and, in fact, the description of the experimental octet mass splittings improves at \( O(p^4) \) \[19\]. As already anticipated by the calculation of the \( O(p^4) \) pieces in Eqs. [3], the \( SU(3)_F \) HB expansion has severe problems of convergence in the description of the sigma terms at \( O(p^3) \). The huge central value and errors of \( \sigma_0 \) for the HB-SSE expansion has to be regarded as a clear manifestation of these problems. Another one is the large variation in the value of \( \sigma_0 \) between HB and HB-SSE. On the contrary, for the covariant calculation the difference between the calculations excluding/including the explicit decuplet of baryon resonances is only of around 10 MeV, much smaller than the difference between the LO and NLO results in the purely octet formulation of the theory. This indicates a stabilization of the final outcome at the \( O(p^3) \) value for the covariant case. These conclusions are consistent with those derived from the analyses of other observables [18, 19, 21] that also indicated similar problems of convergence for the HB studies in the \( SU(3)_F \) sector. Similar comments can be done concerning the value of \( b_0 \) and its variations when comparing with the different levels of sophistication in the calculation. E.g. one observes a change in \( b_0 \) between HB and HB-SSE in Table III that is a factor 3 times larger than for the covariant calculations.

The main result of our study is having shown that \( \sigma_{\pi N} \sim 60 \) MeV is perfectly compatible with a rather accurate fulfillment of the OZI rule and, hence, with a small strangeness content of the nucleon. In other words, there is no argument against relatively large values of \( \sigma_{\pi N} \) based on the OZI rule, as \( \sigma_0 \) can be afflicted by important...
systems as those driven by properly accounting of the relativistic corrections and the explicit inclusion of the decuplet resonances. Other outcome of our work is that the value of $\sigma_0$ obtained using the experimental baryon-octet mass splittings, Lorentz covariant B$\chi$PT in EOMS up to $O(p^3)$ with explicit decuplet-baryon resonances as degrees of freedom, favors $\sigma_{\pi N} \sim 60$ MeV. However, for this result to hold higher order corrections should be under control. Indeed, this seems to be the case as indicated by our calculation of a sub-set of known $O(p^3)$ diagrams. Unfortunately, first complete studies of LQCD results at $O(p^4)$ in SU(3) B$\chi$PT are not conclusive on this respect yet, as large variations in the sigma terms are found between different strategies. Ref. \[29\] concludes $\sigma_{\pi N} = 32 \pm 2$ MeV, $\sigma_g = 22 \pm 20$ MeV and $y \approx 0.05 \pm 0.04$, while Ref. \[31\] results with $\sigma_{\pi N} = 46(2)(12)$ and $\sigma_s = 157(25)(68)$. Further work in this direction, ideally including more observables, experimental data and LQCD results to tackle the large number of unknown LECs appearing at $O(p^4)$, will be necessary to settle this question.

We also show two results from LQCD without employing B$\chi$PT, Ref. \[33\] obtained $\sigma_{\pi N} = 39(4)(+18)\pm 7$ MeV, $\sigma_s = 33(14)(+22)\pm 17$ MeV and $y = 0.20(7)(\pm 13)$, while Ref. \[34\] determines $y$ with the value $y = 0.135(46)$. These direct LQCD calculations clearly suggest a small value for $y$ and a subsequent contribution to the nucleon mass due to strangeness of around the same size as from the lightest quark masses. Other calculations supplying LQCD results with different formulations of B$\chi$PT are \[32, 35, 36\]. In these studies a small value for $y$ results, compatible with our own determination in the last line of Table \[11\]. There is also a tendency in the LQCD results favoring the phenomenological determination $\sigma_{\pi N} \simeq 45(7)$ MeV \[3\], although within present uncertainties the result of Ref. \[33\] is compatible at the level of one sigma with the larger value $\sigma_{\pi N} = 59(7)$ MeV \[10\].

IV. CONCLUSIONS

In summary, we have revisited an old empirical relation between the strangeness content of the nucleon and the pion-nucleon sigma term in the context of covariant B$\chi$PT and employing only phenomenological information. Earlier estimates of $\sigma_0$ made at different levels of accuracy in $\chi$PT agreed on that a small violation of the OZI rule in the nucleon requires a value of $\sigma_{\pi N}$ close to $\sim 35$ MeV. A long-standing puzzle \[37\] has arisen from sustained experimental evidence pointing to a value of this quantity close to 60 MeV, reinforced by the values obtained from modern $\pi N$ databases. We have shown that the previous calculations of $\sigma_0$ are afflicted by important systematic effects, in particular those given by relativistic corrections and by the omission of the decuplet resonances. Once these are incorporated, we obtain a larger $\sigma_0$ so that a relatively large value of $\sigma_{\pi N}$ is not necessarily inconsistent with a negligible strangeness content of the nucleon as currently indicated by experiment and LQCD. In fact, our calculation at NLO in Lorentz covariant B$\chi$PT in the EOMS with explicit decuplet-baryon resonances favors this scenario.

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