Experimental signature of the parity anomaly in a semi-magnetic topological insulator

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A three-dimensional (3D) topological insulator features a 2D surface state consisting of a single linearly dispersive Dirac cone. Under broken time-reversal symmetry, the single Dirac cone is predicted to cause half-integer quantization of Hall conductance, which is a manifestation of the parity anomaly in quantum field theory. However, despite various observations of quantization phenomena, the half-integer quantization has not been observed because most experiments simultaneously measure a pair of equivalent Dirac cones on two opposing surfaces. Here we demonstrate the half-integer quantization of Hall conductance in a synthetic heterostructure termed a semi-magnetic topological insulator, where only one surface state is gapped by magnetic doping and the other one is non-magnetic and gapless. We observe half-quantized Faraday and Kerr rotations with terahertz magneto-optical spectroscopy and half-quantized Hall conductance in transport at zero magnetic field. Our results suggest a condensed-matter realization of the parity anomaly and open a way for studying the physics enabled by a single Dirac fermion.

The realization of relativistic quantum phenomena is one of the central interests of contemporary condensed-matter physics. A prominent example is the emergence of Dirac fermions in two-dimensional (2D) systems, which are characterized by a linearly dispersive electronic band called a massless Dirac cone with spin-momentum locking and a crossing point (Dirac point) at zero energy. When time-reversal symmetry is broken, an energy (mass) gap opens at the Dirac point. Focusing on the spin degree of freedom in a single Dirac cone, it turns out that this gap opening is simultaneously accompanied by parity symmetry breaking. Specifically, the parity operation is defined as the reflection of one spatial coordinate (such as $x \rightarrow -x, y \rightarrow y$) in 2D space, and the mass gap of a Dirac fermion changes its sign under the parity operation. In high-energy physics, a similar phenomenon is known as the parity anomaly in (2+1)-D (space–time) quantum field theory, where a gauge-invariant regularization process (Pauli–Villars regularization) for a massless Dirac fermion coupled to the U(1) gauge (electromagnetic) field inevitably breaks the parity symmetry. A striking consequence of the parity anomaly is the generation of a parity-violating current with the half-integer-quantized Hall conductance, $e^2/(2h)$, under an electromagnetic field perturbation, where $e$ is the elementary charge and $h$ is Planck's constant. However, according to Nielsen–Ninomiya's fermion doubling theorem, the Dirac cones always appear in pairs and the paired Dirac cones cancel the parity anomaly, restoring the parity symmetry as a whole system. Therefore, the quantization phenomena reported so far, such as the quantum Hall effect and quantized magneto-optical effects, are characterized by integer topological indices. In the meantime, the possibility of a condensed-matter realization of the parity anomaly associated with two Dirac cones has been proposed by Haldane in a honeycomb lattice and realized in a cold atom system. However, the realization of the single Dirac cone and half-integer quantization of Hall conductance has remained elusive.

In this Letter we demonstrate that the half-integer quantization of Hall conductance associated with the parity anomaly can be realized in 3D topological insulators (TIs). The 3D TIs support a single gapless Dirac fermion cone at each surface as a consequence of the nontrivial Z$_2$ topological nature of wavefunctions in the insulating bulk. Unlike 2D lattice systems where the paired Dirac cones, degenerated by spin and orbital degrees of freedom, exist at different points of momentum space, a pair of spin-polarized single Dirac cones in a 3D TI appear separately on opposite surfaces, specifically the top and bottom surfaces in a crystal of thin-film form. The potential half-integer quantization at each surface as a result of breaking time-reversal symmetry was first recognized by Fu and Kane, then theoretically demonstrated in the layer-resolved Hall conductivity calculation by Essin et al. over a decade ago. More recently, various possible measurements of half-integer Hall conductivity have been proposed. In this experimental study, we utilize a layer-resolved doping technique with magnetic ions targeting only the vicinity of the top surface (Fig. 1a). It is possible to gap out only the top-surface Dirac cone with simultaneously broken 2D parity symmetry and time-reversal symmetry while keeping the bottom surface Dirac cone gapless (Fig. 1b); this breaks the inversion symmetry as a 3D structure (Supplementary Section I). Because only one of the paired Dirac cones is gapped, this semi-magnetic TI would be an ideal model for demonstration of the parity anomaly.

Semi-magnetic TI heterostructure films consisting of (Bi,Sb)$_2$Te$_3$ and Cr-doped (Bi,Sb)$_2$Te$_3$ were grown by molecular-beam epitaxy (MBE; Fig. 1c, Methods and Supplementary Section II). The magnetic element Cr was modulation-doped only near the top surface (2 nm). The exchange interaction between the surface electrons and the magnetic ions (Cr) opens an energy gap on the top-surface Dirac cone. The Fermi energy $E_F$ was carefully tuned so that $E_F$ lay within the magnetic gap of the top surface Dirac cone, as confirmed by gate-voltage-dependent transport measurements.
Fig. 1 | Semi-magnetic topological insulator and the parity anomaly. a, b, Schematic of a semi-magnetic TI (a), where the top-surface Dirac state is gapped and the bottom surface Dirac state is gapless (b). Orange arrows and red (blue) arrows indicate the magnetization and the spins for the top (bottom) surface Dirac cone, respectively. The 2D parity symmetry of the gapped surface state is broken in that the spin direction is not conserved by the parity operation (that is, reflection of one spatial coordinate such as \( x \rightarrow -x, y \rightarrow y \)) (Supplementary Fig. 1). c, Schematic layout of an MBE-grown semi-magnetic TI film used for the experiments. The total thickness of 10 nm makes the hybridization between the top and bottom surface states negligible. In the upper part (5 nm from the top surface), the Bi:Sb ratio is fixed so that \( E_F \) is located in the magnetically induced gap, whereas, in the lower part, the energy level of the Dirac point (\( E_{\text{DP}} \)) can be tuned by varying the Sb fraction, \( x \).

( Supplementary Section V). The energy level of the Dirac point for the bottom surface (\( E_{\text{DP}} \)) was tunable by changing the Bi:Sb ratio, \( x \).

One experimental method used to observe half-integer quantization makes use of terahertz magneto-optical Faraday and Kerr rotation measurements\(^1\). Previous studies reported the integer quantization of Faraday and Kerr rotations in uniform TI films with broken time-reversal symmetry\(^{13-15}\). In these experiments, the top and bottom surface states simultaneously contributed to the quantized rotations because the magneto-optical polarization integrations integrate all the rotational contributions along the light path\(^{13,29,30}\). By contrast, in a semi-magnetic TI, half-quantized magneto-optical rotations are expected because only the gapped top surface can be a source of the magneto-optical rotations.

We first present the Faraday and Kerr rotation spectra for the magnetic TI film, which exhibits the quantum anomalous Hall (QAH) effect. The spectra for this are shown in Fig. 2c,d as a function of the incident photon energy \( \hbar \omega \) (where \( \hbar = h/2\pi \)), taken at zero magnetic field \( \mu_B H = 0 \) T and \( T = 1 \) K after aligning the magnetization up to 2 T, and obtained by analysing the time-domain waveforms of the transmitted terahertz pulses (Fig. 2a,b)\(^{13,14}\). We find that the real parts of the Faraday (\( \theta_F \)) and Kerr (\( \theta_K \)) rotation angles (with negligible imaginary parts (ellipticities), \( \eta_F \) and \( \eta_K \)) are consistent with the theoretical values for the quantized Faraday and Kerr rotations—\( \tan^{-1}\left( \frac{\alpha_{\text{DP}}}{n_i + 1} \right) = 3.27 \) mrad and \( \tan^{-1}\left( \frac{2 \alpha_{\text{DP}}}{n_i + 1} \right) = 9.20 \) mrad, respectively—where \( n_i \approx 3.46 \) is the refractive index of the InP substrates, \( \alpha = \frac{\varepsilon_{\text{vac}}}{\varepsilon_{\text{vac}}} \approx \frac{1}{1.33} \) is the fine-structure constant, \( \varepsilon_{\text{vac}} \) is the vacuum permittivity and \( c \) is the speed of light. This result is in agreement with previously reported results\(^5\), and the signal-to-noise ratio is much improved (Extended Data Fig. 1).

We now present the results for the magneto-optical rotations for a semi-magnetic TI film using the same experimental set-up. As shown in Fig. 2e, \( \theta_F \) is \( \sim 1.6 \) mrad and almost independent of the incident light frequency. This value is almost half that measured for the QAH (Fig. 2c) and integer (\( \nu = 1 \)) quantum Hall (QH) systems\(^{13-15,18,19,29,30}\). This half Faraday rotation angle is accounted for by the half-integer quantization in the magneto-optical rotation. The Faraday rotation, for small rotation angles, is described by the summation of the dynamical 2D Hall conductivity \( \sigma_{\text{xy}}^{\text{2D}}(\omega, z) \) along the light path (\( z \) direction), \( \tan \theta_F = \frac{1}{(n_i + 1)\mu B} \sum \sigma_{\text{xy}}^{\text{2D}}(\omega, z_i) \) \( (z_i \) is the \( z \) coordinate of the \( i \)th layer)\(^5\). Given that the top and bottom surface states are well decoupled and that the gapped top surface state has half-quantized Hall conductivity and the gapless bottom surface bulk states make no contribution, the rotation angle should be \( \theta_F = \tan^{-1}\left( \frac{\alpha_{\text{DP}}}{n_i + 1} \right) \approx 1.63 \) mrad, consistent with the experimentally measured value of \( \sim 1.6 \) mrad.

Similarly, the Kerr rotation angle \( \theta_K \) was measured by utilizing multiple reflections of the terahertz pulses inside the substrate (Fig. 2b)\(^{13,14}\). As shown in Fig. 2f, the value of \( \theta_K \) is \( \sim 4.5 \) mrad, which is again almost half that for the QAH state (Fig. 2d). This also agrees with the predicted half-integer quantization for a thin-film limit (\( \theta_K = \tan^{-1}\left( \frac{2 \alpha_{\text{DP}}}{n_i + 1} \right) \approx 4.60 \) mrad; Supplementary Section III). The ellipticities \( \eta_F \) and \( \eta_K \) are quite similar compared with \( \theta_F \) and \( \theta_K \), respectively, suggesting no resonance feature due to a sufficient magnetic gap opening of the top surface state compared to the incident terahertz photon energy \( \hbar \omega < 5 \) meV; Supplementary Section IV). Thus, the observation of half-quantized rotations under \( \mu_B H = 0 \) T is consistent with the picture that only the gapped top surface works as a source of magneto-optical rotations, and the contribution of the gapless bottom surface to the rotations is negligibly small (Supplementary Section III).

The half-quantized magneto-optical rotations in the low-frequency region motivated us to study the Hall conductance at the zero-frequency limit (that is, d.c. electrical transport measurements). In the magneto-optical rotation measurements, the observed Faraday and Kerr rotations can be regarded as being a consequence of the topological nature in the interior of the gapped top surface. In d.c. electrical transport measurements, on the other hand, only the gapless bottom surface state where the electric current is running through is to be probed. Thus, confirming the correspondence between the magneto-optical rotation and d.c. electrical transport measurements is important for understanding the surface–bulk correspondence of the low-energy half-integer-quantized electrodynamics in TIs. Experimental efforts to probe this have been made in similar magnetic/non-magnetic TI thin flakes under high magnetic fields\(^{13,14}\); however, this should result in integer quantization or arbitrary values due to the contributions from both the top and bottom surface states (Supplementary Section IX). The constraints introduced by flake shapes with invasive metal contacts will inevitably lead to the occurrence of artefacts in the Hall conductance (converted from the resistance) due to the non-uniform current densities and equipotential lines in the samples\(^5\).

In Fig. 3a,b we show the temperature (\( T \)) dependence of the Hall resistance (\( \rho_H \)) and sheet resistance (\( \rho_s \)) at zero magnetic field, respectively, for five Hall-bar devices (Fig. 3d inset and Extended Data Fig. 2), with well-defined sample dimensions (via photolithography) so as to achieve accurate Hall measurements (Methods).
Different Bi:Sb ratios (x) correspond to different values of \( E_{\nu} \) (the energy level of the Dirac point for the bottom surface state; Fig. 3a inset and Fig. 1b). Apparently, there are no characteristic features common to the samples. By contrast, when the resistance is converted to conductance, the values of \( \sigma_{xy} \) for all the samples converge to the half quantum conductance 0.5\( e^2/h \) at low temperatures, as clearly seen in Fig. 3c. The quantization occurs below 2 K, which is comparable to the typical quantization temperature of the QAH effect (Fig. 3a,b). This result indicates that, regardless of the different \( E_{\nu} \) values, \( \sigma_{xy} \) is quantized to the half-integer value as long as \( E_{\nu} \) resides in the magnetic gap of the top surface state. Figure 3d presents the data for the sheet conductance \( \sigma_{xy} \) showing a relatively large variation with \( x \). \( \sigma_{xy} \) can be attributed to the dissipative electron transport in the bottom surface where the carrier densities are changed by \( x \) (Supplementary Figs. 6 and 7).

In Fig. 4a we directly compare the results of the magneto-optical rotation and the d.c. electric transport measurements using an identical semi-magnetic TI sample (\( x = 0.93 \)) by using the relation \( \tan \theta_0 = \frac{2 m_0}{\hbar^2} \sigma_{xy} \) (refs. 18,29,30). The results of magneto-optics and transport show good agreement with each other, meaning that the half-integer quantization is verified regardless of measurement methods. When we apply a perpendicular magnetic field, both \( \sigma_{xy} \) and \( \theta_0 \) are almost doubled and exhibit integer quantization:\( ^{12} \)

\[
\sigma_{xy} = e^2/h \quad \text{and} \quad \theta_0 = \tan^{-1} \left( \frac{\sigma_{xy}}{\sigma_{xx}} \right) \approx 9.20 \text{ mrad} \text{ (Extended Data Fig. 3)}.
\]

The doubled rotations under high magnetic fields can be understood by Landau level formation on the bottom surface, which works as an additional source of half-quantized rotations. The \( \nu = 1 \) quantization under a magnetic field supports that the Fermi level resides within the gap of the half-quantized top surface considering the original surface bands at zero magnetic field (Fig. 4a inset).

The half-integer quantization in transport can be explained by the summation of the half-quantized 2D Hall conductivity \( (\sigma_{xy})^{\text{2D}} \) with zero 2D longitudinal conductivity \( (\sigma_{xx})^{\text{2D}} = 0 \) for the top surface and finite 2D conductivity \( (\sigma_{xx})^{\text{2D}} \neq 0 \) with zero 2D Hall conductivity \( (\sigma_{xy})^{\text{2D}} = 0 \) for the bottom surface (Supplementary Fig. 11) by assuming surface parallel conduction channels, where superscripts \( t \) and \( b \) denote the top and bottom surfaces, respectively. To provide more insight into the transport, we sketch the current distribution for the top and bottom surfaces in Fig. 4b,c.

Here the electric current \( I = (i_t + i_b)W \) (\( W \), sample width) is driven along the \( x \) direction (Fig. 4c). The dissipative bottom surface current density \( i_b \) generates the longitudinal electric field as described by \( E_x = \frac{\sigma_{xx}}{\rho_{xx}}i_x \). Because the top and bottom surfaces are electrically connected at the side surface (note that the gapless side surface contribution to conductivity is so small as to be ignored, compared with the gapless bottom surface in the thin-film case), this \( E_x \) also generates the non-dissipative top surface Hall current \( j_t = \sigma_{xy}^{\text{2D}t}E_x = -\frac{\sigma_{xy}}{\sigma_{xx}}E_x \) because the top surface state is gapped. Importantly, because the \( y \)-direction current densities on the top and bottom surfaces must compensate for each other, \( j_x = j_t + j_b = 0 \) (Fig. 4c).

This additional dissipative current density \( j_t \) generates the transverse electric field \( E_y = \frac{\sigma_{xy}}{\rho_{xy}}E_x \), which induces the longitudinal electric field \( E_x = \frac{\sigma_{xx}}{\rho_{xx}}E_y \). Using the longitudinal Hall current \( j_t = \sigma_{xy}^{\text{2D}t}E_y = \frac{\sigma_{xy}}{\sigma_{xx}}E_y \) on the top surface. Eventually, because the experimentally measured resistance components are expressed as \( \rho_{\text{meas}}^{\text{2D}x} = E_x/(W/\rho_{xx}) \) and \( \rho_{\text{meas}}^{\text{2D}y} = E_y/(W/\rho_{xy}) \), the Hall conductance becomes \( \sigma_{xy}^{\text{meas}} = \frac{\rho_{\text{meas}}^{\text{2D}x}}{\rho_{\text{meas}}^{\text{2D}y}} = \frac{\sigma_{xy}^{\text{2D}x}}{\sigma_{xy}^{\text{2D}y}} = \frac{e^2}{\pi h} \). Thus, \( \sigma_{xy}^{\text{meas}} \) exhibits the half-integer quantization in the transport measurement, irrespective of \( \sigma_{xx}^{\text{meas}} \), which well describes the observations in Fig. 3c.d. Although the current distribution can be comprehensively understood by the above parallel surface conduction picture, the Hall current from the parity anomaly flows through the delocalized gapless states in the side and bottom surfaces in the real-space picture (Supplementary Section VII). This situation is analogous to the plateau-to-plateau transition in ordinary QH systems, where the delocalized states in the bulk support the Hall current. Here the gapless Dirac fermion in the bottom surface plays the role of such delocalized states (Supplementary Section IX).

Finally, we mention the precision of the half-quantized \( \sigma_{xy} \). Experimentally, the measured deviation of \( \sigma_{xy} \) from \( e^2/(2h) \) was \( \sim 2.6\% \) (Fig. 3c inset). Because \( \sigma_{xx} \) remains finite, the precision of the half-integer quantization cannot be as high as the QAH effect, in principle. Furthermore, in the ground state, namely \( T = 0 \text{ K} \), all the electronic states would be localized, as seen in the decrease of \( \sigma_{xx} \).
The inset presents a magnified view of $\sigma_{\text{a}}$ as a function of a linear scale of T.

Fig. 3 | Half-integer quantization in electrical transport. a–d, Zero-field Hall resistance $\rho_H$ (a), sheet resistance $\rho_s$ (b), Hall conductance $\sigma_H \equiv \rho_H/(\rho_{xx} + \rho_{xy})$ (c) and sheet conductance $\sigma_s \equiv \rho_s/(\rho_{xx} + \rho_{xy})$ (d) as a function of the logarithmic scale of temperature T. Data for semi-magnetic TI films with various values of Bi:Sb ratio ($x = 0.73, 0.80, 0.85, 0.89$ and 0.93) and a typical QAH film sample (purple) are shown. The resistance measurement was conducted using Hall-bar-shaped samples as shown in the inset of d, which shows an optical microscope image of a Hall bar with a top-gate electrode. White scale bar, 100 µm. The inset of c presents a schematic of the surface band structures for the semi-magnetic TI film, in which the carrier type is electrons (holes) when $x \leq 0.85$ ($\geq 0.89$) (Supplementary Figs. 6 and 7). The inset of c presents a magnified view of $\sigma_{\text{a}}$ as a function of a linear scale of T.

Fig. 4 | Correspondence between transport and magneto-optics. a, Magnetic field $\mu_0H$ dependence of $\theta_\text{c}$, averaged over $\hbar\omega = 1$ to 3 meV (red circles) and $\theta_\text{c}$ (black line) measured by d.c. transport at $T = 1$ K. The green square point represents $\theta_\text{c}$ for the QAH film averaged over $\hbar\omega = 1$ to 3 meV. The ticks of the left and right ordinates are adjusted by using the relation $\tan \theta_\text{c} = \frac{1}{\sqrt{2}} \frac{\rho_{xx}}{\rho_{xy}} \sigma_{\text{a}}$. The error bars represent standard deviation. The inset presents a band diagram of the top (red line) and bottom (blue line) surface states, showing Landau level formation to produce the QH effect when applying magnetic fields. b, c, Schematics of the top view (b) and side view (c) of current flow in the parallel conduction picture. The magnetic (top) and non-magnetic (bottom) layers are drawn in green and blue colours, respectively.

In summary, we have experimentally demonstrated the parity anomaly with half-integer quantization of Hall conductance in a semi-magnetic TI, using two different methods—terahertz magneto-optical spectroscopy and transport measurements. Although the precision of terahertz spectroscopy is not so high as that of the d.c. transport measurements, the magneto-optical rotation directly reflects the Hall conductivity, whereas the d.c. transport measurements require a resistance-to-conductance conversion using a well-defined sample shape. Our transport measurements using carefully fabricated Hall-bar samples enable highly accurate measurements, providing data consistent with the terahertz measurements. Thus, our dual measurements are complementary to each other, confirming the parity anomaly. The semi-magnetic TI structure with the parity anomaly can be used as a platform for exotic single Dirac fermion physics. For example, dyon particles arise from the fractional topological magneto-electric effect on a magnetically gapped (top) surface\textsuperscript{5,6}. Furthermore, additional superconducting proximity coupling on the gapless (bottom) surface can open a superconducting gap, which potentially produces more accessible non-Abelian Majorana edge states under relaxed conditions of magnetic and superconducting gaps\textsuperscript{31–42}. 

On lowering the temperature (Fig. 3d) due to broken time-reversal symmetry as a whole system\textsuperscript{1}, such as from a tiny amount of magnetic (Cr) impurities possibly contained in the bottom surface (Supplementary Section VIII) or due to a tiny energy gap formed on the bottom surface by top–bottom surface hybridization (Supplementary Section X). This would cause a shift of $\sigma_{\text{a}}$ to either $\epsilon^2/h$ or 0, respectively. Nevertheless, the experimentally observed half-integer quantization has surprisingly high robustness in the parameter ranges of the present experiments in which we vary $E_g$ (Fig. 3 and Supplementary Fig. 10), $T$ (down to 50 mK) and sample size (10 µm to 1.5 mm) (Supplementary Section XI) and in another semi-magnetic TI heterostructure doped with a different magnetic dopant (V) (Supplementary Section XII).
Online content
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**Methods**

Thin-film growth. The Cr-doped (Bi, Sb)\textsubscript{2}Te\textsubscript{3} heterostructure films were grown on epi-ready semi-insulating (>10\textsuperscript{4}\Omega \textpercm) InP (111)A substrates maintained at 200°C (growth temperature), employing an MBE chamber equipped with standard Knudsen cells under a back pressure of about 1 \times 10\textsuperscript{-10} Pa. The composition of the films was nominally determined with the beam equivalent pressures of Cr, Bi and Sb fluxes, and the beam pressure of Te relative to Bi and Sb was kept at a ratio of \(\sim 40:1\) to suppress Te vacancies. The Cr-modulation doping was controlled by opening and closing the shutter of the Cr cell. By cross-sectional scanning energy-dispersive X-ray spectrometry, as presented in a previous study\textsuperscript{9} in which the growth conditions, including growth temperature (200°C), were the same as in the present study, interlayer diffusion of Cr atoms from the Cr-modulation-doped layer was found to be negligibly small. The growth rate of the films was \(\sim 0.2\) nm min\textsuperscript{-1}, as characterized by X-ray reflectivity in representative samples. To prevent deterioration of the films, these were capped with Al\textsubscript{2}O\textsubscript{3} \(\sim 5\) nm deposited by atomic layer deposition (ALD) at room temperature immediately after taking them out of the MBE chamber.

**Magneto-optical terahertz spectroscopy.** In the time-domain terahertz measurements, a mode-locked Ti:sapphire laser generated laser pulses with wavelength of 800 nm and duration of 100 fs, which were split into two paths to emit and detect terahertz pulses using a photoconductive antenna and a dipole antenna, respectively. The terahertz photon energy (\(h\omega\)) in the range of 1–6 meV (0.2–1.5 THz in frequency), centred at \(\sim 2\) meV. The film samples \((1 \times 1 \times 1\) cm\) were mounted on a copper stage with a 5-mm-diameter hole to achieve sufficient transmission of terahertz light and were cooled in a cryostat equipped with a superconducting magnet (7 T) and a 3He probe insert down to \(\sim 1\) K. We measured the transmitted terahertz electric fields \(E_z(t)\) and \(E_y(t)\) using two wire grid polarizers (parallel for the \(E_x\) measurements or orthogonal for the \(E_y\) measurements) across the samples. To eliminate the background signal except for the magneto-optical rotations, \(E_x(t)\) was deduced by anti-symmetrizing the waveforms of \(E_z(t+\Delta t)\) and \(E_z(t-\Delta t)\), where \(H\) is the magnetic field applied perpendicular to the film plane. When the measurements were performed at \(\Delta t=0\), the anti-symmetrizing procedure was performed for \(E_z(t+\Delta t)\) and \(E_z(t-\Delta t)\), by reversing the spontaneous magnetization \(M\) along the \(z\) (film normal) direction. The energy spectra of the complex rotation angles are obtained by the Fourier analysis of \(E_z(t)\) and \(E_y(t)\) using the tensor relations \(\sigma_{x\text{y}}=\rho_{x\text{y}}+\rho_{y\text{x}}\) and \(\rho_{x\text{y}}=\rho_{x\text{y}}/2\), where \(\rho_{x\text{y}}\) is the ellipticity, for the small rotation angles. The magneto-optical Kerr measurements were performed with multiple reflections inside the substrate (InP). With respect to the waveform of the terahertz pulse transmitted through the sample (Fig. 2b), the second pulse comes from the multiple reflections inside the substrate after the first main pulse with a time delay \(\Delta t=2d/c\), where \(d\) is the thickness of the InP substrate \((d=360 \mu\text{m})\), \(c\) is the speed of light and \(n_i\) is the refractive index of the InP substrate. By using \(\alpha\) M. Ogino, Y. Hayashi, H. Shishikura, D. Murata and Y. D. Kato for support of the terahertz measurements. This research project was partly supported by the JSPS/MEXT Grant-in-Aid for Scientific Research (nos. 15H05853, 15H05687, 17K0179, 18H04229 and 18H01155) and JST CREST (nos. JPMJCR16F1 and JPMJCR1874).

**Author contributions**

Y. Tokura conceives and supervised the project. M.M., R.Y. and K.Y. fabricated the samples with help from A.T., K.S.T. and M. Kawasaki. M.M., Y.O. and Y. Takahashi performed the terahertz spectroscopy measurements and analysed the data. M.M. and M. Kawamura performed the transport measurements and analysed the data. T.M. and N.N. contributed to the theoretical discussions. M.M., M. Kawamura, T.M., N.N. and Y. Tokura wrote the manuscript, with input from all the other authors.

**Competing interests**

The authors declare no competing interests.

**Additional information**

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Extended Data Fig. 1 | Quantized Faraday and Kerr rotations in a QAH state. a, $\theta_F$ and $\eta_F$ (a) and $\theta_K$ and $\eta_K$ (b) spectra at $T = 1$ K for the QAH insulator film, which is partly the same as Fig. 2c and d in the main text, with a slight variation of external magnetic fields ($\mu_0H = 0, 0.01$, and $1$ T). c, Measured fine-structure constant $\alpha_{\text{meas}}$, which is calculated from (a) and (b) by using a relation of $\alpha_{\text{meas}} = (\tan\theta_F\tan\theta_K - \tan^2\theta_F)/(\tan\theta_K - 2\tan\theta_F)$ (refs. 29,30). d, $\theta_F$, $\eta_F$ (d) and $\theta_K$, $\eta_K$ (e) spectra at $\mu_0H = 0$ T and at various temperatures ($T = 1, 1.6, 4.2, 15.6, 32.6$, and $56.7$ K). f, $T$ dependence of $\theta_F$ and $\theta_K$ taken at $\hbar\omega = 2$ meV, suggesting that the Curie temperature is about $50$ K and that the integer quantization subsists possibly up to $4.2$ K. The inset is the magnified view of f. The error bars in a-f represent the standard error of the mean.
**Extended Data Fig. 2** Optical microscope image of a typical Hall bar device used in the transport measurements. The black broken lines indicate the shape of the TI film below the gate electrode, formed into the Hall bar structure.
Extended Data Fig. 3 | Kerr rotation in the semi-magnetic Ti under magnetic fields. a, Representative complex Kerr rotation spectra for the semi-magnetic Ti film used for Fig. 4a in the main text. The open circles at $\hbar \omega = 0$ meV indicate the values anticipated from the measured dc conductivity values. b, Background Kerr spectra of the InP substrate without any Ti films at 7 T. The inset shows the Faraday rotation spectra at 7 T, where an observable polarization rotation occurs at $\hbar \omega > 2$ meV, possibly due to the magnetic resonance of magnetic impurities involved in InP substrates. The error bars in a and b represent the standard error of the mean.