Dual Superconductor Mechanism of Confinement 
on the Lattice

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Abstract

We investigate the dual superconductor mechanism of confinement for pure SU(2) lattice gauge theory in the maximally abelian gauge. We focus on the dual Meissner effect. We find that the transverse distribution of the longitudinal chromoelectric field due to a static quark-antiquark pair satisfies the dual London equation. Moreover we show that the size of the flux tube scales according to asymptotic freedom.
Long time ago G. ’t Hooft and S. Mandelstam\cite{1} proposed that the confining vacuum is a coherent state of color magnetic monopoles. This proposal offers a picture of confinement whose physics can be clearly extracted. Indeed the dual Meissner effect causes the formation of chromoelectric flux tubes between chromoelectric charges leading to a linear confining potential. Following Ref. \cite{2}, one can study the monopole condensation by means of the so called Abelian projection. It turns out that \cite{3} it is possible to implement the Abelian projection on the lattice, where the Abelian projection amounts to fix the gauge by diagonalizing an operator which transforms according to the adjoint representation of the gauge group. In this paper we consider the SU(2) gauge group with standard Wilson action on a $12^4$ lattice in the maximally Abelian gauge \cite{4}. In this gauge one diagonalize the lattice operator

$$X(x) = \sum_{\mu} \left\{ U_\mu(x) \sigma_3 U_\mu^\dagger(x) + U_\mu^\dagger(x - \hat{\mu}) \sigma_3 U_\mu(x - \hat{\mu}) \right\} .$$

(1)

The gauge is fixed iteratively via overrelaxation like the Landau gauge \cite{3}. We adopted a convergence criterion which coincides with the one of Ref. \cite{6}.

The aim of this paper is to analyze the dual Meissner effect by studying the color fields distribution due to a static quark-antiquark pair. To measure the color fields we follow the method of Ref. \cite{7}. These authors measure the correlation of a plaquette $U_P$ with a Wilson loop $W$. The plaquette is connected to the Wilson loop by a Schwinger line $L$:

$$\rho_W = \frac{\langle \text{tr} (W U_P L^\dagger) \rangle}{\langle \text{tr} (W) \rangle} - \frac{1}{2} \frac{\langle \text{tr} (U_P) \text{tr} (W) \rangle}{\langle \text{tr} (W) \rangle} .$$

(2)

By moving the plaquette $U_P$ with respect to the Wilson loop one can scan the structure of the color fields. Note that in the naïve continuum limit, the operator $\rho_W$ is sensitive to the fields rather than to the square of the fields \cite{7}. 

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Following the Abelian dominance idea \cite{8} we investigate the color field distribution by measuring

\[
\rho_{W}^{ab} = \frac{\langle \text{tr} (W^A U_P^A) \rangle}{\langle \text{tr} (W^A) \rangle} - \frac{1}{2} \frac{\langle \text{tr} (U_P^A) \text{tr} (W^A) \rangle}{\langle \text{tr} (W^A) \rangle}.
\]  

(3)

where the superscript means that the Wilson loop and the plaquette are built from the abelian projected links.

We use Wilson loop of size 5. We average over 500 configurations (each one separated by 50 upgrades, after 3000 sweeps to equilibrate the lattice) in the range \(2.4 \leq \beta \leq 2.525\).

The authors of Ref. \cite{7} found a sizeable signal for the chromoelectric field parallel to the flux tube \((U_P \parallel W)\). So we focus on the longitudinal chromoelectric field. By moving the plaquette outside the plane of Wilson loop up to distance 5 in lattice units, we measure the transverse profile of the longitudinal chromoelectric field in the middle of the flux tube. In Fig. 1 we show the result for two different values of \(\beta\).

The authors of Ref. \cite{7} found that the transverse shape of the flux tube can be fitted in accordance with

\[
E_l(x_{\perp}) = A_G \exp \left[ -m_G x_{\perp} - \mu_G^2 x_{\perp}^2 \right], \quad x_{\perp} \geq 0.
\]  

(4)

Equation (4) describes the flux tube like a relativistic string with gaussian fluctuations \cite{12}.

We find that also our data can be fitted by Eq. (4). In Fig. 1 the dashed line is the result of the fit (4) \((A_G \text{ is fixed by } E_l(x_{\perp} = 0))\). Moreover in Fig. 2 we check the scaling of \(\mu_G\). We find that \(\mu_G\) scales and

\[
\frac{\mu_G}{\Lambda_L} = 83 \pm 2.
\]  

(5)
Such nice scaling property is not shared by $A_G$ and $m_G$. The value (3) is quite
close to the one $\frac{\mu_G}{\Lambda_L} = 75\pm2$ obtained in Ref. [7] by using non abelian quantities on
a $16^4$ cooled lattice. Thus, we feel that this result support the Abelian dominance
idea.

On the other hand, we find that the data are compatible with another functional form, namely

$$E_l(x_\perp) = A_M K_0 (\mu_M x_\perp) , \ x_\perp > 0 ,$$  \hspace{1cm} (6)

where $K_0$ is the modified Bessel function of order zero. Equation (3) is a straight-forward consequence of the dual superconductor hypothesis (see also the infrared effective theory of the monopole condensation proposed in Ref. [9]). Indeed, let us consider a second kind superconductor in an external static magnetic field. If we have an isolated vortex line, then in the London limit the magnetic field satisfies the London equation [10]

$$h - \lambda^2 \nabla^2 h = \varphi_0 \delta^{(2)} (x_\perp),$$  \hspace{1cm} (7)

where $h$ is the magnetic field parallel to the vortex line, $x_\perp$ the transverse distance
from the vortex line, and $\varphi_0$ the magnetic flux. The penetration depth $\lambda$ is related
to the photon mass by the well known relation

$$\lambda = \frac{1}{m_\gamma} .$$  \hspace{1cm} (8)

The solution of Eq. (7) is

$$h(x_\perp) = \frac{\varphi_0}{2\pi} \frac{1}{\lambda^2} K_0 \left( \frac{x_\perp}{\lambda} \right) \ x_\perp > 0 .$$  \hspace{1cm} (9)

Interchanging magnetic with electric, we are lead to consider the fit Eq. (3) with
$\mu_M = \frac{1}{\lambda}$ and $A_M \sim \mu_M^2$. 

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Recently, it has been shoved that \[11\] that the longitudinal electric field of the \(U(1)\) flux tube satisfies the dual London equation. We will compare our method with the one of Ref. \[11\] in a separate paper.

The results of the fit Eq. (6) are plotted as solid line in Fig. 1. Both fits Eqs. (4) and (6) give a comparable reduced \(\chi^2\). Moreover \(\mu_M\) scales (see Fig. 3):

\[
\frac{\mu_M}{\Lambda_L} = 132 \pm 2 .
\]

(10)

It turns out that (see Fig. 4):

\[
A_M = (0.23705 \pm 0.00841) \mu_M^2 .
\]

(11)

Following Ref. \[12\] we can define the width of the flux tube by

\[
D = \frac{\int d^2x_\perp x_\perp^2 E_l(x_\perp)}{\int d^2x_\perp E_l(x_\perp)} .
\]

(12)

From Equations (12) and (6) we obtain

\[
D = \frac{2}{\mu_M} .
\]

(13)

Using \[7\] \(\Lambda_L = 6.8 \pm 0.2\) MeV and Eq. (10) we get

\[
D = 0.44 \pm 0.02 \text{ fm}
\]

(14)

which is close to the value estimated in Ref. \[7\].

In a previous paper \[13\] we propose a method to measure the abelian photon mass by means of the connected correlation function of an operator with the quantum number of the photon. In order to check Eq. (8) we recorded also the abelian photon mass. In Figure (5) we display \(\mu_M/m_\gamma\) versus \(\beta\). Even though there are large statistical fluctuations, mainly due to the abelian photon mass, we see that the relation Eq. (8) is consistent with Monte Carlo data.
In conclusion we have showed that the transverse distribution of the longitudinal chromoelectric field due to a static quark-antiquark pair satisfies the dual London equation. However, care must be taken of self-energy effects from the Wilson line. So our results should be checked on larger lattices. We showed that the London penetration length scales according to asymptotic freedom. This suggests that the penetration length is a gauge invariant physical quantity. Some preliminary results on a larger lattice [14] support this conclusion. Needless to say, this matter will be deepen in future studies.

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FIGURE CAPTIONS

Fig. 1 Transverse distribution of the longitudinal chromoelectric field at a) $\beta = 2.4$ and b) $\beta = 2.5$. Dashed and solid lines refer to Eqs. (4) and (6) respectively.

Fig. 2 Asymptotic scaling of $\mu_G$. The dashed line is the fitted value Eq. (5).

Fig. 3 Asymptotic scaling of $\mu_M$. The dashed line is the fitted value Eq. (10).

Fig. 4 The ratio $A_M/m_M^2$ versus $\beta$.

Fig. 5 The ratio $\mu_M/m_\gamma$ versus $\beta$. The solid line corresponds to $\mu_M/m_\gamma = 1$. 
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