A stochastic periodic review inventory model for vendor–buyer system with setup cost reduction and service–level constraint

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ABSTRACT
This paper presents a joint economic lot-sizing problem under stochastic demand for a vendor–buyer system by synchronizing ordering and production cycles. The buyer manages its inventory periodically using a periodic review policy. The average demand and the buyer’s ordering cost are assumed to be fuzzy. The vendor’s production process is imperfect and the buyer’s inspection process is also imperfect. The vendor has opportunity to make an investment to reduce the setup cost. Since it is often difficult to calculate the shortage cost in inventory model, a service-level constraint corresponding to the buyer is considered in the mathematical model. The objective of the model is to simultaneously optimize the review period, production rate, setup cost, and the number of deliveries, such that minimize the joint total cost. We suggest an iterative procedure to find the optimal values of decision variables. A numerical example and a sensitivity analysis are given to show the application of the model and to investigate the effect of the changes in key parameters on the behavior of the proposed model.

1. Introduction
During the last decades, the research concerning with joint inventory problem has received a great deal of attention. The essence of the research was to coordinate and collaborate the parties in supply chain system by synchronizing production and inventory decisions in order to reduce the total cost. Many published works on joint inventory problem showed that making production and inventory decisions jointly will result a significant cost saving compared to an independent decision. In literature, the determination of production and ordering quantities to consider the benefits of all parties in the supply chain is called joint economic lot-sizing problem (JELP).

Recently, many researchers have developed a JELP under stochastic demand. Most of them have considered a continuous review policy to manage the buyer’s inventory level. However, only few researchers have addressed a stochastic demand environment under

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periodic review policy. Lin (2010) was the first who considered a stochastic periodic review vendor–buyer model under imperfect production process and backorder price discount. Lin and Lin (2016) studied imperfect items and recovery process in a single-supplier–single-retailer-integrated inventory model. Later, Tahami, Mirzazadeh, Arshadi-Khamsi, and Gholakami-Qadikolaei (2016) studied the effect of inflationary condition and ordering cost reduction in a vendor–buyer system. Jauhari, Sejati, and Rosyidi (2016a) extended the Lin (2010)’s model by investigating the impact of adjustable production rate and variable lead time on model’s behavior. Furthermore, Jauhari, Mayangsari, Kurdhi, and Wong (2017) gave an extension to their previous model by allowing the inclusion of adjustable production rate and inspection errors.

In most of aforesaid papers, the researchers have used a full cost model to develop the inventory model. In the full cost model, the objective of the inventory model is to find optimal policy which minimizes the joint total cost, including the shortage cost. In many practical situations, the shortage cost is very difficult to identify by an exact value for there will always be some intangible loss such as, the reputation and credibility losses and the delay of other parts, which consequently affects the customer loyalty and customer satisfaction (Li, Xu, & Ye, 2011). Therefore, a few researchers, including Jha and Shanker (2009, 2013) have used a service-level constraint to replace the shortage cost which implies that the shortage level per cycle is bounded. However, these models assumed that the production process is always perfect. In real situations, the defective items maybe produced due to unreliable process and/or damage in transit.

In most of the existing vendor–buyer models, usually the demand and the cost parameters are treated as crisp values. But in practical situations, it is difficult to set the parameters as crisp values due to e.g. new product introduction, single period setting and errors in data collections. Thus, here, the annual demand and the ordering cost are assumed to be fuzzy. In addition, the investment of setup cost reduction was often neglected in the previous inventory models. In reality, we often obtain a condition where a setup cost charged by the manufacturer is quite expensive due to high labor cost, long setup time, etc. Facing this condition, the manufacturer may invest an amount of money to reduce the setup cost. In real problem, an investment to reduce setup cost can be done by giving a comprehensive training to the operator, improving setup procedures and upgrading the tools. In this paper, we consider an investment in setup cost reduction which follows a logarithmic function and analyzed its impact in the proposed model.

In this paper, we develop a single-vendor–single-buyer model with stochastic demand, imperfect production process and periodic review policy. This paper contributes to the inventory literature in the following ways. First, we extend the model of Jauhari et al. (2017) by assuming that the annual demand and the ordering cost to be fuzzy in nature. Second, we give an opportunity to the vendor to reduce the setup cost by doing an investment. We intend to study how the investment made by the vendor can affect the inventory decisions, especially when the production rate can be adjusted flexibly. Third, we propose a mathematical model for vendor–buyer problem under the service-level constraint. We study the impact of the changes in service-level constraint on model’s behavior, especially when there are imperfect items and inspection errors. Furthermore, an iterative procedure is developed to find the optimal values of decision variables. A numerical example and a sensitivity analysis are presented to show the application of the model and to study the changes in key parameters on model’s behavior.
2. Literature survey

There have been plenty of published papers on vendor–buyer inventory model. The published models dealt with determining the optimal ordering and the production lots which can be achieved by integrating inventory decisions across the supply chain. Goyal (1976) was the first author who developed an integrated inventory model for a vendor and a buyer. He proposed an infinite production rate and used a lot for lot policy to deliver the product from the vendor to the buyer. Goyal’s model was then extended by Banerjee (1986) by proposing a finite production rate for a vendor system. He suggested a benefit sharing scheme to induce the buyer to joint with an integrated policy. Glock (2012a) developed a supply chain system consisting of a single buyer and multiple vendors. The author used an equal-sized batch shipment to deliver the single product from vendor to buyer. He investigated two different shipment policies, which are immediate deliveries and delayed deliveries, to coordinate the vendor’s production system and buyer’s ordering system. Later, Beck, Glock, and Kim (2017) extended the model of Glock (2012a) by considering a geometrically increasing batch shipment policy. Taleizadeh, Niaki, and Wee (2013) considered fuzzy-lead times and suggested two hybrid procedures to solve the integrated inventory problems. Fauza, Amer, Lee, and Prasetyo (2016) proposed a single-vendor-multiple buyers-integrated inventory model for food product involving different quality characteristics. Jia, Liu, Wang, and Lin (2016) studied postponed simultaneous delivery strategy in a single-vendor-multi buyer system under capacity utilization. Furthermore, the basic vendor–buyer model has been extended into various problems, such as imperfect production (Dey & Giri, 2014; Sana, 2011), unequal shipment policy (Giri & Roy, 2013), transportation cost (Wangsa, 2017; Wangsa & Wee, In press), environmental impacts (Li, Su, & Ma, 2017; Wahab, Mamun, & Ongkunaruk, 2011) and discount policy (Muniappan, Uthayakumar, & Ganesh, 2016; Zhang, Luo, & Duan, 2016).

Previous research on joint vendor–buyer problem focused on developing the model by considering stochastic demand. Ben-Daya and Hariga (2004) was the first author who introduced stochastic demand. He proposed an inventory model in which the demand follows normal distribution and the lead time is variable. Glock (2009) proposed an unequal shipment policy for delivering the product in a vendor–buyer system under stochastic demand. He showed that the proposed policy can achieve a significant saving compared to an equal shipment policy. Glock (2012b) extended Ben-Daya and Hariga (2004)’s model by allowing that the lead time can be reduced using a crashing cost. Kurdihi, Lestari, and Susanti (2015) developed a fuzzy continuous review vendor–buyer model by considering fuzzy demand and imperfect items. Braglia, Castellano, and Frosolini (2016) studied a safety stock management in an integrated inventory model consisting of a vendor and a buyer with continuous review policy and controllable lead time. They used a present value criterion to consider time value of money and inflation in the model. Jauhari, Fitriyani, and Aisyati (2016b) considered an inclusion of freight rate discount in a two-stage supply chain consisting of a vendor and a buyer. Two inventory models are developed under two types of discounts (all-weight discount and incremental discount) and two procedures are suggested to determine the decision variables. However, most of these models adopted a continuous review policy rather than periodic review policy to manage the buyer’s inventory level. In reality, we often see that the periodic review is adopted by practitioners for managing the inventories in some stores such as drugs store, grocery stores and retail stores. Silver et al.
(1998) showed some benefits of using periodic review policy, which are reducing reviewing cost and reducing reviewing errors.

In most of the above-mentioned papers, the production process and inspection process are assumed to be perfect. In reality, the production process is imperfect and produces a certain number of defective items. In addition, the inspection process carried out by the buyer is also imperfect due to human errors. Huang (2004) was the first who developed vendor–buyer model with defective items. He considered an uniform distribution for presenting the percentage of defective items. Ouyang, Wu, and Ho (2006) proposed two inventory models where the defective rate is taken to be constant and fuzzy. Khan and Jaber (2011) analyzed the effect of defective items obtained from suppliers in the inventory model consisting of a vendor and a buyer. Furthermore, Hsu and Hsu (2012) analyzed the inspection errors in a deterministic vendor–buyer model. Khan, Jaber, and Ahmad (2014) proposed a model with the inspection errors and learning. Jauhari (2016) investigated the effect of inspection errors and transportation cost in stochastic model. Castellano et al. (2017) studied a stochastic periodic review inventory model with imperfect production process and stochastic demand. The vendor has an opportunity to invest amount of money to improve the quality and to reduce the setup cost. Priyan and Uthayakumar (2017) proposed a vendor–buyer integrated system with defective items, setup cost reduction, and inspection errors. The defective rate is formulated using probabilistic function and the setup cost is reduced by means of investment. Pal and Mahapatra (2017) developed a three-layer supply chain involving of a supplier, a manufacturer and a retailer with imperfect production process, inspection errors and rework. Furthermore, Kim and Sarkar (2017) analyzed the impact of a quality improvement in a complex multi-stage production process under situation where the quality investment is made under a budget constraint.

The above literature review shows that a stochastic vendor–buyer inventory model has been extensively discussed in the past, but none has analyzed the impact of incorporating fuzzy annual demand, fuzzy ordering cost, imperfect production process, inspection errors, and adjustable production rate simultaneously. Therefore, here, an integrated inventory model for single-vendor and single-buyer system with imperfect items, inspection errors, periodic review, fuzzy annual demand, fuzzy ordering cost, adjustable production rate, setup cost reduction, and service-level constraint is developed. A comparison of the proposed model with some of the related papers in the inventory literature is shown in Table 1.

3. Notations and problem description

3.1. Notations

The following notations will be used to develop the model:

Decision variables:

\( T \)  review period (year)
\( P \)  production rate (units/year)
\( S \)  setup cost ($/setup)
\( n \)  number of deliveries from vendor to buyer
Table 1. A comparison of the proposed model and some of the published stochastic vendor–buyer models.

| Papers            | Fuzzy parameters | Production rate | Buyer’s policy | Setup cost reduction | Basic model | Production process | Inspection process |
|-------------------|------------------|-----------------|----------------|----------------------|-------------|-------------------|--------------------|
| Lin (2010)        | No               | Constant        | Periodic review| No                   | Full cost   | Imperfect          | Perfect            |
| Lin and Lin (2016)| No               | Constant        | Periodic review| No                   | Service-level constraint Full cost | Imperfect          | Perfect            |
| Glock (2009)      | No               | Constant        | Continuous review| No                   | Full cost   | Perfect            | No                 |
| Glock (2012b)     | No               | Adjustable      | Continuous review| No                   | Full cost   | Perfect            | No                 |
| Jauharia et al. (2016a) | No       | Adjustable      | Periodic review| No                   | Full cost   | Imperfect          | Perfect            |
| Jauharia et al. (2017) | Annual demand | Adjustable      | Periodic review| No                   | Full cost   | Imperfect          | Imperfect           |
| Kurdhi et al. (2015) | Annual demand | Constant        | Continuous review| Yes                  | Service-level constraint Full cost | Imperfect          | Perfect            |
| Castellano et al. (2017) | No            | Constant        | Periodic review| Yes                  | Full cost   | Imperfect          | Perfect            |
| Taleizadeh et al. (2013) | Lead time   | Constant        | Continuous review| No                   | Full cost   | Perfect            | No                 |
| Braglia et al. (2016) | No            | Constant        | Continuous review| No                   | Service-level constraint Full cost | Perfect            | No                 |
| Dey and Giri (2014) | No             | Constant        | Continuous review| No                   | Service-level constraint Full cost | Imperfect          | Perfect            |
| Proposed model    | Annual demand, Ordering cost | Adjustable | Periodic review| Yes                  | Service-level constraint | Imperfect          | Imperfect           |

Parameters:

\[ D \] demand rate (units/year)
\[ \sigma \] standard deviation of demand (units/year)
\[ V \] transportation cost ($/shipment)
\[ g \] transportation time (year)
\[ A \] ordering cost ($/order)
\[ S_0 \] original setup cost ($/setup)
\[ I(S) \] the capital investment required to reduce the setup cost ($)
\[ \gamma \] probability of defect
\[ c_p \] unit purchasing cost
\[ c_{dB} \] buyer's cost of a post-sale defective item ($/unit)
\[ c_i \] inspection cost ($/unit)
\[ c_{dV} \] vendor's cost of a post-sale defective item ($/unit)
\[ c_r \] cost of false rejection of non-defective item ($/unit)
\[ c_w \] vendor's warranty cost for producing defective item ($/unit)
\[ e_1 \] probability of type-I inspection error (classifying a non-defective item as defective)
\[ e_2 \] probability of type-II inspection error (classifying a defective item as non-defective)
\[ a_1 \] fixed production cost ($)
\[ a_2 \] variable production cost ($/unit)
\[ h_B \] holding cost for buyer ($/unit/year)
\[ h_V \] holding cost for vendor ($/unit/year)
3.2. **Problem description**

The following notations are used in our model:

1. The model considers single-vendor–single-buyer with single product.
2. The demand follows normal distribution with mean \( D \) and standard deviation \( \sigma \).
3. The buyer adopts periodic review policy to control the inventory level.
4. Inventory level at buyer will be reviewed for every \( T \) units of time. A sufficient quantity is ordered up to the target level \( R \) and the ordering quantity will arrive at the buyer after lead time.
5. The length of lead time must not exceed the review period \( T \). Thus, there is never more than a single order outstanding in any cycle.
6. The buyer orders quantity \( DT \) units and the vendor will manufacture \( nDT \) items per production setup with a finite production rate. It is assumed that \( P(1 - \gamma) > D \). Each lot will be delivered to the buyer in \( n \) equally sized shipments. The buyer will incur transportation cost \( V \) for each delivery of lot. The vendor will incur setup cost \( S \) for each production run.
7. The lot received by the buyer contains some defective items with probability \( \gamma \). Therefore, the buyer will conduct an inspection process to screen out the defective items.
8. The inspection process is imperfect due to human errors. The inspector may incorrectly classify the non-defective items as defective, which is called by type-I error, or incorrectly classify the defective items as non-defective, which is called by type-II error.
9. The production rate is adjustable. The vendor has an opportunity to control the production rate between \( P_{\text{min}} \) and \( P_{\text{max}} \). In practical situations, the production rate of a manufacturing facility can be influenced by setting the speed of machine and inserting idle time over production run (Buzacott & Ozkarahan, 1983; Schweitzer & Seidmann, 1991). Therefore, adjusting the production rate can reduce the vendor’s inventory level which leads to a significant saving on total cost.
10. There is an opportunity for the vendor to reduce the setup cost. The vendor can invest an amount of money to reduce the setup cost from original level \( S_0 \) to a target level \( S \). The setup cost investment done by the vendor is denoted by \( I(S) \), where \( I(S) \) is a convex and strictly decreasing function.

4. **Model development**

4.1. **Vendor’s cost model**

The vendor manufactures \( nDT \) units in each production cycle when the buyer orders a quantity of \( DT \) units. The length of production cycle and buyer’s ordering cycle are \( nT \) and \( T \), respectively. The setup cost charged to the vendor for each production cycle is \( S/nT \).
When the vendor produces the first lot, which is $DT$ units, he will deliver it quickly to the buyer. After that, the vendor will make the delivery on the average every $T$ unit of time until his inventory drops to zero. The average inventory level for the vendor can be calculated as follows:

$$\frac{DT}{2} \left[ \frac{D(2 - n)}{P} + (n - 1) \right]$$

Thus, the holding cost for the vendor is given by the following equation:

$$\text{IHV} = h_v \frac{DT}{2} \left[ \frac{D(2 - n)}{P} + (n - 1) \right]$$

As we assumed in the previous section, the vendor has an opportunity to adjust production rate between $P_{\text{min}}$ and $P_{\text{max}}$. Therefore, the production cost function will consequently be affected by the level of production rate. By adopting the formulation proposed by Khouja and Mehrez (1994), the production cost function is derived by:

$$\text{PCV} = \left( \frac{a_1}{P} + a_2 P \right) D$$

A capital investment will be made by the vendor to reduce the setup cost. The investment on setup cost reduction follows a logarithmic function. The formulation of investment is given by:

$$I(S) = d \ln \frac{S_0}{S}$$

where $0 < S \leq S_0$, $d = \frac{1}{\delta}$, $\delta$ is the percentage decrease in $S$ per dollar increase in $I(S)$. Therefore, the annual investment per year is $\tau I(S)$, where $\tau$ is a fractional opportunity cost per unit time.

As misclassifications occurs during inspection process, the vendor will endure type-I error cost, post-sales failure cost, and warranty cost. The type-I error cost, post-sales failure cost, and warranty cost are given by the Equations (5), (6) and (7), respectively.

$$\text{ETV} = c_{\gamma} (1 - \gamma) D e_1$$

$$\text{PSV} = c_{\gamma} D e_2$$

$$\text{WC} = c_{\gamma} D$$

Hence, the annual total cost for the vendor, which is composed of production cost, setup cost, investment cost, holding cost, post-sales failure cost, type-I error cost, and warranty cost, is expressed by:

$$\text{ATC}_V (T, P, S, n) = \left( \frac{a_1}{P} + a_2 P \right) D + \frac{S}{nT} + \tau d \ln \frac{S_0}{S} + h_v \frac{DT}{2} \left[ \frac{D(2 - n)}{P} + (n - 1) \right] + c_{\gamma} D e_2 + c_{\gamma} (1 - \gamma) D e_1 + c_{\gamma} D$$
4.2. Buyer’s cost model

Since $A$ is the ordering cost per order and $V$ is the transportation cost, the ordering cost and the transportation cost per year are given by $\frac{AT}{T}$ and $\frac{VT}{T}$, respectively. The purchasing cost can be calculated by multiplying the unit purchasing cost ($c_p$) and the demand ($D$). As we described in the previous section that the lot delivered from vendor to the buyer may contain a number of defective items with probability $\gamma$. The buyer then inspects the incoming lot and classifies the items into two categories, non-defective items and defective items. However, the inspection process is imperfect and produces two kinds of errors, which are type-I error and type-II error. The type-I error is occurred by categorizing non-defective products as defective products, while the type-II error is occurred by categorizing defective products as non-defective products. The defective items that pass the inspection process would be sold to end customers. End customers who detect the quality problems will return the products to the buyer. All the items classified by the inspector as defective items and all the items returned from end customers are sent back to the vendor in a single batch.

The formulation of holding cost is adopted from Lin (2010) and is presented in Equation (9). The inspection cost and the post-sales failure cost for the buyer are given by Equation (10) and Equation (11), respectively.

$$I_{HB} = h_B \left( \frac{DT}{2} + DT(1-\gamma)e_1 + DT\gamma(1-e_2) + \frac{DT\gamma e_2}{2} \right) \quad (9)$$

$$IC_B = c_pD \quad (10)$$

$$PS_B = c_{dB}\gamma De_2 \quad (11)$$

The annual total cost for the buyer is determined by the following equation:

$$ATC_B(T, n) = \frac{A}{T} + \frac{V}{T} + c_pD + c_{dB}\gamma De_2 + h_B \left( \frac{DT}{2} + DT(1-\gamma)e_1 + DT\gamma(1-e_2) + \frac{DT\gamma e_2}{2} \right) \quad (12)$$

We assume that the protection interval demand $X$ follows a normal distribution with mean $D(T+L)$ and standard deviation $\sigma \sqrt{T+L}$. By considering that the lead time consists of production time and transportation time, $\frac{DT}{P} + g$, so the target level is given by $R = D(T + \frac{DT}{P} + g) + k\sigma \sqrt{T + \frac{DT}{P} + g}$. Therefore, the expected shortage quantity $E = (X - R^+)$ at the end of the cycle can be calculated by:

$$E(X - R^+) = \int_R^\infty (x - R)f_X(x)dx$$

$$= \sigma \sqrt{T + \frac{DT}{P} + g} \psi(k) > 0 \quad (13)$$
where \( \psi(k) = \phi(k) - k[1 - \Phi(k)] \), \( \phi(k) \) and \( \Phi(k) \) denote the standard normal p.d.f and c.d.f, respectively.

Hence, by allowing the service-level constraint, the above buyer’s annual cost can be transformed to

\[
ATC_b(T, n) = \frac{A}{T} + V + c_p D + c_i D + c_{db}y D e_2 + h_b \left( \frac{DT}{2} + DT(1 - \gamma)e_1 + DT\gamma(1 - e_2) + \frac{DT\gamma e_2}{2} \right)
\]

subject to

\[
\frac{\sigma\psi(k) \sqrt{T + \frac{DT}{p} + g}}{D(T + \frac{DT}{p} + g)(1 - \gamma)(1 - e_1)} \leq \xi \quad (14)
\]

### 4.3. Joint total cost model

The annual joint total cost for a supply chain system consisting of a vendor and a buyer can be derived by adding the annual buyer’s total cost and annual vendor’s total cost, which is:

\[
AJTS(T, P, S, n) = \left( \frac{a_1}{P} + a_2 P \right) D + \frac{S}{nT} + \tau d \ln \frac{S_0}{S} + h_v \frac{DT}{2} \left[ \frac{D(2 - n)}{P} + (n - 1) \right]
\]

\[
+ c_d y D e_2 + c_v (1 - \gamma) D e_1 + c_w y D + \frac{A}{T} + \frac{V}{T} + c_p D + c_i D + c_{db}y D e_2
\]

\[
+ h_b \left( \frac{DT}{2} + DT(1 - \gamma)e_1 + DT\gamma(1 - e_2) + \frac{DT\gamma e_2}{2} \right) \quad (15)
\]

In reality, it is quite difficult to determine the annual demand and ordering cost because of various uncertainties. Therefore, in this paper, we develop a mathematical model for a supply chain system by allowing the inclusion of fuzzy annual demand and fuzzy ordering cost. Let us consider that the annual demand is located in an interval \( \bar{D} - q_1, \bar{D} + q_2 \) and the ordering cost is located in interval \( \bar{A} - p_1, \bar{A} + p_2 \), where \( q_1, q_2, p_1, \) and \( p_2 \) are determined by the decision-makers. Based on the intervals, we defuzzify \( \bar{D} \) and \( \bar{A} \) to be triangular fuzzy numbers, which are:

\[
\bar{D} = (\bar{D} - q_1, \bar{D}, \bar{D} + q_2) \quad (16)
\]

\[
\bar{A} = (\bar{A} - p_1, \bar{A}, \bar{A} + p_2) \quad (17)
\]

where \( 0 < q_1 \leq D, 0 < p_1 \leq A, 0 < q_2 \) and \( 0 < p_2 \). If \( \bar{D} \) and \( \bar{A} \) are fuzzified to be \( \tilde{D} \) and \( \tilde{A} \), respectively, the annual joint total cost is fuzzy value also. Then, it can be rewritten as follows:
subject to

\[
\begin{align*}
\sigma_w(k) \sqrt{T + \frac{(D + \frac{1}{4}(q_2 - q_1))T}{p} + g} \\
\frac{(D + \frac{1}{4}(q_2 - q_1)) - (D + \frac{1}{4}(q_2 - q_1))(T(1 - \gamma) + \frac{(D + \frac{1}{4}(q_2 - q_1))T \gamma e_2}{2})}{1 - \gamma(1 - e_1)} \leq \xi
\end{align*}
\]

\[P_{\text{min}} \leq P \leq P_{\text{max}} \tag{18}\]

5. Solution methodology

To find the solution of the above problem, we investigate the first partial derivatives of \(AJTS(T, P, S, n)\) with respect to \(T, P\), and \(S\), which are presented in Equation (19), (20) and (21), respectively.

\[
\frac{\partial AJTS(T, P, S, n)}{\partial T} = \frac{1}{2} \left( D + \frac{1}{4}(q_2 - q_1) \right) T \left( n - 1 + \frac{(D + \frac{1}{4}(q_2 - q_1))(2 - n)}{p} \right) - \frac{(A + \frac{1}{4}(p_2 - p_1))}{T^2}
\]

\[
-\frac{F}{T^2} - \frac{S}{nT^2} + h_s \left( \frac{(D + \frac{1}{4}(q_2 - q_1))}{2} + \left( D + \frac{1}{4}(q_2 - q_1) \right) T e_1(1 - \gamma) + \left( D + \frac{1}{4}(q_2 - q_1) \right) T \gamma e_2 \right)
\]

\[\tag{19}\]

\[
\frac{\partial AJTS(T, P, S, n)}{\partial P} = \left( D + \frac{1}{4}(q_2 - q_1) \right) \left( a_2 - \frac{a_1}{p} \right) - \frac{(D + \frac{1}{4}(q_2 - q_1))^2 h_v(2 - n)T}{2p^2}
\]

\[\tag{20}\]

\[
\frac{\partial AJTS(T, P, S, n)}{\partial S} = \frac{1}{nT} - \frac{\tau}{S}
\]

\[\tag{21}\]
By setting Equations (19–21) equal to zero, rearranging and simplifying we will obtain

\[ T^* = \sqrt{\frac{A + \frac{1}{4}(p_2 - p_1)n + Vn + S}{\frac{1}{2}(D + \frac{1}{4}(q_2 - q_1))nh_v \left( \frac{(D + \frac{1}{4}(q_2 - q_1))(n - 2) + (n - 1)}{p} \right) + nh_v \left( D + \frac{1}{4}(q_2 - q_1) \right)} + \frac{1}{2}(1 - \gamma)\lambda + 2\gamma\beta}} \]  

(22)

\[ P^* = \sqrt{\frac{2a_1(D + \frac{1}{4}(q_2 - q_1)) + \left( D + \frac{1}{4}(q_2 - q_1) \right)^2 h_v(2 - n)T}{2a_2(D + \frac{1}{4}(q_2 - q_1))}} = \sqrt{\theta} \]  

(23)

\[ S^* = \frac{nT\tau}{\delta} \]  

(24)

From Equation (23), it is clear that if the value of \( \theta < 0 \), the above equation may result in the infeasible solution. If we set \( P = P_{\text{min}} \) if \( \theta < 0 \), then Equation (23) can be rewritten as follows:

\[ P^* = \max\left[ P_{\text{min}}, \min\left( P_{\text{max}}, \sqrt{\theta} \right) \right] \]  

(25)

Theoretically, the optimal review period, production rate, and setup cost can be determined by Equations (22), (23) and (25), respectively. However, it is difficult to verify analytically that the annual joint total cost is a convex function. Although it can be easily shown (by taking the second derivative) that the annual joint total cost is convex in \( T^* \) and \( S^* \), the annual joint total cost may not be convex in \( P^* \). Hence, we note here, that the solutions obtained by the proposed procedure cannot be claimed as a global optimal solution and the proposed procedure is only guaranteed to converge to local optimal solutions. Nevertheless, many scholars, including Ben-Daya and Hariga (2004), Glock (2012b), Jauhari et al. (2016b, 2017), have used similar procedures to find the solutions in stochastic JELPs.

It is obvious that the parameters \( T, P, \) and \( S \) are dependent of each other. For example, to calculate \( T \) we need to obtain \( P \), which in turn is a prerequisite for determining \( T \). Thus, to find the solution of the above problem, we propose an iterative procedure which is developed based on Ben-Daya and Hariga (2004). A proposed algorithm is provided below:

**Step 1** Fuzzify \( D \) and \( A \) to be a triangular fuzzy numbers, \( \bar{D} = (\bar{D} - q_1, \bar{D}, \bar{D} + q_2) \) and \( \bar{A} = (\bar{A} - p_1, \bar{A}, \bar{A} + p_2) \) where \( q_i \) and \( p_i \) for \( i = 1, 2, \ldots \) are decided by the decision-makers.

**Step 2** Defuzzify the fuzzy numbers \( D \) and \( A \) using the signed distance method.

**Step 3** Set \( n = 1 \) and JAITS \( (T^*_{n-1}, P^*_{n-1}, S^*_{n-1}, n - 1) = \infty \).

**Step 4** Set \( i = 1 \) and compute \( P_i = \sqrt{\frac{2}{\sigma^2}} \).

**Step 6** Set \( i = 1 \), and compute \( T_i = \sqrt{\frac{A + \frac{1}{4}(p_2 - p_1)(n + 4b) + \frac{1}{2}(e^{\frac{1}{2}(q_2 - q_1)})_n h_v \left( \frac{(e^{\frac{1}{2}(q_2 - q_1)})_n (n - 2) + (n - 1)}{p} \right) + nh_v \left( e^{\frac{1}{2}(q_2 - q_1)}(n - 2) + (n - 1) \right) + nh_v \left( e^{\frac{1}{2}(q_2 - q_1)} \right) \right)} + \frac{1}{2}(1 - \gamma)\lambda + 2\gamma\beta} \) (26)
Table 2. Optimization results for the proposed model.

| n' | T (years) | P* (units) | S* (S) | SLC  | ATC0 (S) | ATC1 (S) | AJTS (S) |
|----|-----------|------------|--------|------|----------|----------|----------|
| 1  | .4704     | 2000       | 670.6446 | .0136| 67,215   | 57,465   | 124,680  |
| 2  | .3644     | 2000       | 897.7843 | .0152| 66,990   | 57,467   | 124,457  |
| 3  | .2898     | 1771.9     | 871.2200 | .0165| 66,867   | 57,331   | 124,198  |
| 4  | .2733     | 1541.5     | 1096.2  | .0165| 66,847   | 57,461   | 124,308  |

Note: SLC = \[ \frac{\sigma \gamma k}{\sqrt{\left(\frac{1}{\gamma} + (\frac{\gamma - \sigma}{\tau})\right)^2 + \left(\frac{\gamma}{1 - (\gamma - \sigma)^2}\right)}} \]

Step 7 Set i = 1 and compute \( S_i \) using Equation (24).

Step 8 For a given previous value of \( T_i \), compute \( P_{i+1} \), from Equation (25).

Step 9 For a given previous value of \( P_{i+1} \) and \( S_i \), compute \( T_{i+1} \) from Equation (22).

Step 10 For a given previous value of \( T_{i+1} \), compute \( S_{i+1} \) from Equation (24).

Step 11 If \( P_i = P_{i+1} \), \( T_i = T_{i+1} \) and \( S_i = S_{i+1} \), go to step 12, otherwise go to step 4 with \( i = i + 2 \).

Step 12 Compare \( S_i \) with \( S_{i+1} \),

a. If \( S_i < S_{i+1} \), then the solution obtained in step 11 is optimal for given \( n \). We denote the solution by \((T^*, P^*, S^*)\).

b. If \( S_i \geq S_{i+1} \), then for given \( n \) set \( S_{i+1} = S_i \) and compute the corresponding values of \( T_{i+1} \) and \( P_{i+1} \) from Equation (22) and Equation (25), respectively.

Step 13 Set \( AJTS(T^*, P^*, S_{i+1}^*, n) = AJTS(T^*, P^*, S^*, n) \).

Step 14 If \( AJTS(T^*, P^*, S_{i+1}^*, n) \leq AJTS(T_{i+1}^*, P_{i+1}^*, S_{i+1}^*, n - 1) \) and the service-level constraint in Equation (18) is satisfied, then go to step 4 with \( n = n + 1 \), otherwise go to step 15.

Step 15 Set \( AJTS(T^*, P^*, S^*, n^*) = AJTS(T_{i+1}^*, P_{i+1}^*, S_{i+1}^*, n - 1) \) then \((T^*, P^*, S^*, n^*)\) is the optimal solution for the proposed problem.

6. Numerical example

To illustrate the above proposed algorithm, let us consider an integrated inventory system with the data adopted from Jauhari, Sejati et al. (2016a) and Jauhari et al. (2017), \( D = 1000 \) units/year, \( A = $100/order, V = $25/delivery, q_1 = 100, q_2 = 50, p_1 = 20, p_2 = 50, k = 1.645, \sigma = 10 \) units/week, \( h_B = 5/unit/year, h_V = 3/unit/year, y = .05, e_1 = .04, e_2 = .04, c_w = 65/unit, c_i = 1/unit, c_{db} = 200/unit, c_{dv} = 300/unit, c_r = 100/unit, c_w = 50/unit, \)

Table 3. The impact of the changes in \( q_1 \) on the proposed model (for \( q_2 = 50 \)).

| \( q_1 \) | T (years) | P* (units) | S* (S) | n' | SLC  | ATC0 (S) | ATC1 (S) | AJTS (S) |
|---------|-----------|------------|--------|----|------|----------|----------|----------|
| 10      | .2857     | 1769.9     | 859.0267 | 3  | .0162| 68,375   | 58,600   | 126,975  |
| 20      | .2862     | 1772.6     | 860.3547 | 3  | .0162| 68,207   | 58,459   | 126,666  |
| 30      | .2866     | 1770.3     | 861.6892 | 3  | .0162| 68,040   | 58,318   | 126,358  |
| 40      | .2871     | 1770.5     | 863.0304 | 3  | .0163| 67,872   | 58,177   | 126,059  |
| 50      | .2875     | 1770.8     | 864.3783 | 3  | .0163| 67,705   | 58,036   | 125,741  |
| 60      | .2880     | 1771.0     | 865.7330 | 3  | .0163| 67,537   | 57,895   | 125,432  |
| 70      | .2884     | 1771.2     | 867.0944 | 3  | .0164| 67,370   | 57,754   | 125,124  |
| 80      | .2889     | 1771.5     | 868.4627 | 3  | .0164| 67,202   | 57,613   | 124,815  |
| 90      | .2893     | 1771.7     | 869.8379 | 3  | .0165| 67,035   | 57,472   | 124,507  |
| 100     | .2898     | 1771.9     | 871.2200 | 3  | .0165| 66,867   | 57,331   | 124,198  |
The optimization results for the above numerical example are shown in Table 2. The iteration will stop at $n^* = 3$ with service level $1 - \xi = 98.35\%$. Thus, the optimal values of decision variables, are $T^* = .2898$ years, $P^* = 1777.9$ units/year, $S^* = $871.22 and the buyer cost, vendor cost, and joint total cost are $66,867, $57,331, and $124,198, respectively.

We further analyze the impact of the changes in key parameters, including $q_1, q_2, p_1, p_2, \delta, e_2, e_2$, and $\xi$, on the model’s behavior. The sensitivity analysis is performed by observing the optimal values of $T^*, P^*, S^*, n^*$ and their impacts on the buyer cost, vendor cost, and the joint total cost. As we described in the previous section that the demand and the ordering cost are considered to be fuzzy rather than constant. Fuzzy numbers denotes numbers that influence the crisp values to determine the fuzzy annual demand and fuzzy ordering cost.

In this section, we first investigate how the fuzzy annual demand influence the model’s behavior. Tables 3 and 4 presents the impacts of the changes in $q_1$ and $q_2$ on the model, respectively. We observe that the increased of $q_1$ and $q_2$ lead to the constant value of $n$. The impact of the increase in $q_1$ and $q_2$ on the buyer cost, vendor cost, and the joint total cost is quite different. When $q_1$ is increased gradually, the buyer cost, vendor cost, and the joint total cost are decreased significantly. But when $q_2$ is increased continually, the buyer cost, vendor cost and the joint total cost are increased as well. In addition, when the value of annual demand is considered to be a constant ($q_1 = q_2 = 50$) the model will result in $67,705, 58,036$, and $125,741$ for the buyer cost, vendor cost, and joint total cost, respectively. It is clear that when $q_2 - q_1$ is getting higher, the annual demand will also increases, which

| $q_1$ | $T^*$ (years) | $P^*$ (units) | $S^*$ ($) | $n^*$ | $SLC$ | $ATC_B$ ($) | $ATC_V$ ($) | $AJTS$ ($) |
|------|--------------|---------------|-----------|------|-------|-----------|-----------|----------|
| 10   | .2917        | 1772.8        | 876.8188  | 3    | .0166 | 66,197    | 56,767    | 122,964  |
| 20   | .2912        | 1772.6        | 875.4084  | 3    | .0166 | 66,365    | 56,908    | 123,273  |
| 30   | .2907        | 1772.4        | 874.0052  | 3    | .0166 | 66,532    | 57,049    | 123,581  |
| 40   | .2903        | 1772.1        | 872.6091  | 3    | .0166 | 66,700    | 57,190    | 123,890  |
| 50   | .2898        | 1771.9        | 871.2200  | 3    | .0165 | 66,867    | 57,331    | 124,198  |
| 60   | .2893        | 1771.7        | 869.8379  | 3    | .0165 | 67,035    | 57,472    | 124,507  |
| 70   | .2889        | 1771.5        | 868.4627  | 3    | .0164 | 67,202    | 57,613    | 124,815  |
| 80   | .2884        | 1771.2        | 867.0944  | 3    | .0164 | 67,370    | 57,754    | 125,124  |
| 90   | .2880        | 1771.0        | 865.7330  | 3    | .0164 | 67,537    | 57,895    | 125,432  |
| 100  | .2875        | 1770.8        | 864.3783  | 3    | .0163 | 67,705    | 58,036    | 125,741  |

| $p_2$ | $T^*$ (years) | $P^*$ (units) | $S^*$ ($) | $n^*$ | $SLC$ | $ATC_B$ ($) | $ATC_V$ ($) | $AJTS$ ($) |
|-------|--------------|---------------|-----------|------|-------|-----------|-----------|----------|
| 10    | .2845        | 1776.3        | 855.3728  | 3    | .0166 | 66,825    | 57,320    | 124,145  |
| 20    | .2858        | 1775.2        | 859.3730  | 3    | .0166 | 66,836    | 57,323    | 124,159  |
| 30    | .2872        | 1774.1        | 863.3472  | 3    | .0166 | 66,846    | 57,325    | 124,171  |
| 40    | .2885        | 1773.0        | 867.2961  | 3    | .0166 | 66,857    | 57,328    | 124,185  |
| 50    | .2898        | 1771.9        | 871.2200  | 3    | .0165 | 66,867    | 57,331    | 124,198  |
| 60    | .2911        | 1770.8        | 875.1193  | 3    | .0165 | 66,877    | 57,334    | 124,211  |
| 70    | .2924        | 1769.7        | 878.9947  | 3    | .0164 | 66,888    | 57,336    | 124,224  |
| 80    | .2937        | 1768.7        | 882.8464  | 3    | .0164 | 66,898    | 57,339    | 124,237  |
| 90    | .2950        | 1767.6        | 886.6749  | 3    | .0164 | 66,908    | 57,342    | 124,250  |
| 100   | .2963        | 1766.5        | 890.4806  | 3    | .0163 | 66,918    | 57,344    | 124,262  |

$\alpha_1 = $2000/batch, $\delta = .0001, \tau = .1/year, $a_2 = $1/2000/unit, $P_{min} = 1400$ units/year, $P_{max} = 2000$ units/year, $g = .0833/year, \xi = .025$.

The optimization results for the above numerical example are shown in Table 2. The iteration will stop at $n^* = 3$ with service level $1 - \xi = 98.35\%$. Thus, the optimal values of decision variables, are $T^* = .2898$ years, $P^* = 1777.9$ units/year, $S^* = $871.22 and the buyer cost, vendor cost, and joint total cost are $66,867, $57,331, and $124,198, respectively.

We further analyze the impact of the changes in key parameters, including $q_1, q_2, p_1, p_2, \delta, e_2, e_2$, and $\xi$, on the model’s behavior. The sensitivity analysis is performed by observing the optimal values of $T^*, P^*, S^*, n^*$ and their impacts on the buyer cost, vendor cost, and the joint total cost. As we described in the previous section that the demand and the ordering cost are considered to be fuzzy rather than constant. Fuzzy numbers denotes numbers that influence the crisp values to determine the fuzzy annual demand and fuzzy ordering cost. In this section, we first investigate how the fuzzy annual demand influence the model’s behavior. Tables 3 and 4 presents the impacts of the changes in $q_1$ and $q_2$ on the model, respectively. We observe that the increased of $q_1$ and $q_2$ lead to the constant value of $n$. The impact of the increase in $q_1$ and $q_2$ on the buyer cost, vendor cost, and the joint total cost is quite different. When $q_1$ is increased gradually, the buyer cost, vendor cost, and the joint total cost are decreased significantly. But when $q_2$ is increased continually, the buyer cost, vendor cost and the joint total cost are increased as well. In addition, when the value of annual demand is considered to be a constant ($q_1 = q_2 = 50$) the model will result in $67,705, 58,036$, and $125,741$ for the buyer cost, vendor cost, and joint total cost, respectively. It is clear that when $q_2 - q_1$ is getting higher, the annual demand will also increases, which
consequently leads to the increase in all costs. On the contrary, when \( q_2 - q_1 \) is getting lower, the annual demand reduces and leads to the decrease in all costs.

It is also interesting to investigate how the changes in \( q_1 \) and \( q_2 \) will affect \( S, P, \) and \( T \). We observe from Tables 3 and 4 that \( S \) is significantly affected by the changes of \( q_2 \) and \( q_1 \). When \( q_2 - q_1 \) is getting higher, we observe that \( S \) is getting lower. But when \( q_2 - q_1 \) is getting lower, \( S \) is getting higher. Facing a greater annual demand the vendor will have a higher setup frequency resulting in an expensive setup cost. Thus, the vendor tends to decrease the setup cost to avoid from the increasing of annual setup cost. However, the vendor still have an opportunity to reduce the setup cost using an investment. Moreover, the value of \( P \) is slightly changing, when there is an increases in \( q_1 \) and \( q_2 \). Facing the changes in annual demand, the vendor tends to balance the inventory level by adjusting the production rate. Thus, the vendor will receive benefits from applying adjustable production rate policy.

### Table 6. The impact of the changes in \( p_1 \) on the proposed model (for \( p_2 = 50 \)).

| \( p_1 \) | \( T \) (years) | \( P^* \) (units) | \( S^* \) ($) | \( n^* \) | \( SLC \) | \( ATC_0 \) ($) | \( ATC_y \) ($) | \( AJTS \) ($) |
|---|---|---|---|---|---|---|---|---|
| 10 | .2911 | 1770.8 | 875.1193 | 3 | .0165 | 66,877 | 57,334 | 124,211 |
| 20 | .2898 | 1771.9 | 871.2200 | 3 | .0165 | 66,867 | 57,331 | 124,198 |
| 30 | .2885 | 1773.0 | 867.2961 | 3 | .0165 | 66,857 | 57,328 | 124,185 |
| 40 | .2872 | 1774.1 | 863.3472 | 3 | .0166 | 66,846 | 57,325 | 124,171 |
| 50 | .2858 | 1775.2 | 859.3730 | 3 | .0166 | 66,836 | 57,323 | 124,156 |
| 60 | .2845 | 1776.3 | 855.3728 | 3 | .0166 | 66,825 | 57,320 | 124,145 |
| 70 | .2831 | 1777.4 | 851.3461 | 3 | .0167 | 66,814 | 57,317 | 124,131 |
| 80 | .2817 | 1778.6 | 847.2926 | 3 | .0167 | 66,804 | 57,314 | 124,118 |
| 90 | .2804 | 1779.7 | 843.2115 | 3 | .0167 | 66,793 | 57,311 | 124,104 |
| 100 | .2790 | 1780.8 | 839.1024 | 3 | .0169 | 66,782 | 57,308 | 124,090 |

### Table 7. The impact of the changes in \( \delta \) on the proposed model.

| \( \delta \) | \( T^* \) (years) | \( P^* \) (units) | \( S^* \) ($) | \( n^* \) | \( SLC \) | \( ATC_0 \) ($) | \( ATC_y \) ($) | \( AJTS \) ($) |
|---|---|---|---|---|---|---|---|---|
| .0001 | .2898 | 1771.9 | 871.2200 | 3 | .0165 | 66,867 | 57,331 | 124,198 |
| .0002 | .2186 | 1830.7 | 328.3748 | 3 | .0186 | 66,810 | 56,676 | 123,486 |
| .0003 | .1980 | 1847.1 | 198.6110 | 3 | .0194 | 66,813 | 56,465 | 123,278 |
| .0004 | .1882 | 1855.1 | 141.4016 | 3 | .0198 | 66,820 | 56,360 | 123,180 |
| .0005 | .2710 | 2000 | 134.1289 | 1 | .0173 | 66,844 | 56,388 | 123,232 |
| .0006 | .2594 | 2000 | 111.7741 | 1 | .0176 | 66,833 | 56,316 | 123,149 |
| .0007 | .2508 | 2000 | 95.8064 | 1 | .0178 | 66,825 | 56,260 | 123,085 |
| .0008 | .2441 | 2000 | 83.8306 | 1 | .0180 | 66,820 | 56,217 | 123,037 |
| .0009 | .2388 | 2000 | 74.5161 | 1 | .0183 | 66,815 | 56,182 | 123,019 |
| .0010 | .2345 | 2000 | 67.0645 | 1 | .0183 | 66,815 | 56,153 | 122,968 |

### Table 8. The impact of the changes in \( e_1 \) on the proposed model (for \( e_2 = .04 \)).

| \( e_1 \) | \( T^* \) (years) | \( P^* \) (units) | \( S^* \) ($) | \( n^* \) | \( SLC \) | \( ATC_0 \) ($) | \( ATC_y \) ($) | \( AJTS \) ($) |
|---|---|---|---|---|---|---|---|---|
| .01 | .2963 | 1766.5 | 890.5692 | 3 | .0158 | 66,834 | 54,530 | 121,364 |
| .02 | .2941 | 1768.3 | 884.0001 | 3 | .0161 | 66,845 | 55,464 | 122,309 |
| .03 | .2919 | 1770.1 | 877.5515 | 3 | .0163 | 66,856 | 56,397 | 123,253 |
| .04 | .2898 | 1771.9 | 871.2200 | 3 | .0165 | 66,867 | 57,331 | 124,198 |
| .05 | .2877 | 1773.6 | 865.0021 | 3 | .0167 | 66,878 | 58,265 | 125,143 |
| .06 | .2857 | 1775.3 | 858.8948 | 3 | .0169 | 66,889 | 59,198 | 126,087 |
| .07 | .2837 | 1777.0 | 852.8948 | 3 | .0172 | 66,899 | 60,132 | 127,031 |
| .08 | .2817 | 1778.6 | 846.9994 | 3 | .0174 | 66,910 | 61,066 | 127,976 |
| .09 | .2798 | 1780.3 | 841.2055 | 3 | .0177 | 66,920 | 62,000 | 128,920 |
| .10 | .2779 | 1781.8 | 835.5105 | 3 | .0179 | 66,930 | 62,934 | 129,864 |
Furthermore, as \( q_1 \) and \( q_2 \) increases from 10 to 100, which is 10 times greater, the review period does not vary substantially.

Tables 5 and 6 show the sensitivity analysis to investigate the behavior of the model when there is only one change in parameters \( p_2 \) and \( p_1 \), while the other parameters remain unchanged. The higher \( p_2 \), the higher the setup cost and the lower production rate, which lead to an increase in the vendor cost and a decrease in the buyer cost. We observe that as \( p_2 - p_1 \) increases, the ordering cost and the buyer cost will eventually be higher, which causes the total cost to be higher as well. It can also be seen that the setup cost continually increases as \( p_2 \) adopts higher value. The production rate is reduced as there is an increase in \( p_2 \). The vendor tends to have a greater inventory level and a higher setup cost which ends up in an increasing of the vendor cost. Moreover, \( p_1 \) has different impacts as \( p_2 \) does on model’s behavior. Facing the increase in \( p_1 \), the setup cost decreases, while the production rate increases which leads to the decrease in all parties costs. We note here, that the review period is mostly affected by the changes in \( p_1 \) and \( p_2 \) rather than the changes in \( q_1 \) and \( q_2 \). It is obvious that when \( p_1 \) and \( p_2 \) are changed up to ten times higher, the review period varies substantially.

The impact of the variation of \( \delta \) on the proposed model is shown in Table 7. The value of \( \delta \) reflects how efficient the investment made by the vendor can reduce the setup cost. We observe interesting results which indicate that the value of \( \delta \) has a significant impact on the decision variables and costs. When the value of \( \delta \) gradually increases, the optimal setup cost decreases. For example, when \( \delta \) is increased from .0002 to .0003, the setup cost is decreased from $328.37 to $198.61. It implies that if an investment becomes more efficient, the vendor will obtain more benefits from a lower setup cost. The number of deliveries also changes as there is an increase in \( \delta \). For a fixed \( n \), the review period decreases due to the increase in \( \delta \). The supply chain system seems to shorten the review period which ends up in reducing inventory level. For a case when \( \delta \) is increased from .0001 to .0004 \((n = 3)\), the production rate is decreased but for a case when \( \delta \) is increased from .000 to .001 \((n = 1)\), the production rate is relatively constant. Furthermore, the cost associated with parties in the supply chain system is reduced, as there is an increase in \( \delta \). It can be seen that the savings earned by the vendor is much higher than the buyer.

Table 8 shows the impact of the variations of type-I error \((e_1)\) on the model. It shows that as \( e_1 \) increases from .01 to .1, the cost of rejecting non-defective items and the cost of warranty increase which ends up in an increasing of all parties costs and supply chain cost. The higher value of \( e_1 \) indicates that the probability of the inspector make a mistake is getting higher which leads to the greater of amount of defective items returned from the buyer to
the vendor. Consequently, the warranty cost paid by the vendor will raise significantly. We may also see that the values of \( T \), \( P \), and \( S \) also vary when the value of \( e_1 \) is set to be higher value. However, the optimal number of deliveries remains unchanged. When \( e_1 \) is changed from \( .01 \) to \( .1 \), the setup cost and the review period are decreased from \$890.57 \) to \$835.51 \) and from \(.2963 \) years to \(.2779 \) years, respectively, while the production rate is increased from \(1766.5 \) units to \(1781.8 \) units.

Table 9 summarizes the effects of the changes in type-II errors (\( e_2 \)) on the proposed model. In the previous section, we defined that the type-II errors is any error made by the inspector in classifying the defective items as non-defective items. This condition will lead to a higher amount of defective items returned from the customers to the buyer. Consequently, the post-sales failure paid by the buyer to the customers increases as well. From Table 9, we may observe that the buyer cost, vendor cost and the joint total cost increase due to the increase in \( e_2 \). The increases are majorly caused by the post-sales failure costs incurred by the vendor and the buyer, which substantially cause joint total cost to be higher. It obvious that the percentage increase in the joint total cost in Table 9 is lower than those in Table 8. It implies that the type-I error has much more impact on the joint total cost than that of the type-II error. Therefore, the managers and practitioners need to pay more attention to controlling the type-I errors to reduce the joint total cost.

Finally, we also investigate how the service-level constraint gives impact on the decision variables and costs. Table 10 shows the impact of the changes in \( \xi \) on the proposed model. The higher the service level will lead to positive influences on customer satisfaction and loyalty which are important to strengthening competitive advantage in the market. As the value of service level (\( 1 - \xi \)) becomes higher, the review period is increased, while the number of deliveries is decreased. However, the values of \( P \) and \( S \) are insensitive to the changes in \( \xi \). By adopting a higher service level, the inventories held in the system seems to increase to prevent from stockout. However, the all parties costs are increased as there is an increase in service level.

### 7. Conclusions and future research directions

This paper developed an integrated inventory model for vendor–buyer system by addressing some considerations, including fuzzy annual demand, fuzzy ordering cost, imperfect production, inspection errors, setup cost reduction, and service-level constraint. Previous research on this problem mostly neglected the investment in reducing setup cost and used a shortage cost to formulate a mathematical model for vendor–buyer system. Whereas in real system, the vendor may have an option to invest amount of money to reduce the setup cost. In this paper, the annual demand and the ordering cost are taken to be fuzzy rather than constant. We also considered imperfect production and imperfect inspection.
Misclassifications occur during inspection, where some errors may influence the inspector’s fault in categorizing items to be defective or non-defective. We proposed a mathematical model which minimizes the joint total cost and suggested an iterative procedure to obtain the optimal review period, production rate, setup cost and the number of deliveries. A numerical example and a sensitivity analysis are given to show the application of the model and to study the behavior of the model in dealing with uncertainty of the values of some parameters.

The results from a sensitivity analysis show that the parameters of fuzzy annual demand and fuzzy ordering cost affect the key parameters and costs. Thus, the determination of fuzzy parameters by the decision parameters must be accordance with the actual annual demand and ordering cost trends in the real system. In addition the managers or practitioners need to control type-I error carefully since this kind of error gives more impact to the costs than that of type-II error. We also found that the optimal number of deliveries is only influenced by the changes of service-level constraint. Furthermore, the result shows that the higher service level, the higher joint total cost and the longer review period.

The model can be extended by allowing the inspection to be taken in vendor side. Thus, the defective items found by the inspector can be reworked. Further study may investigate the effect of both imperfect items and inspection errors on carbon emissions in the supply chain system. Finally, interesting managerial insights may be resulted by studying how the changes in fuel price affect the costs in an integrated vendor–buyer system.

Disclosure statement

No potential conflict of interest was reported by the authors.

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