Nucleosynthesis in a simmering universe

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Abstract

Primordial nucleosynthesis is considered a success story of the standard big bang (SBB) cosmology. The cosmological and elementary particle physics parameters are believed to be severely constrained by the requirement of correct abundances of light elements. We demonstrate nucleosynthesis in a class of models very different from SBB. In these models the cosmological scale factor increases linearly with time from the period during which nucleosynthesis occurs. It turns out that weak interactions remain in thermal equilibrium up to temperatures which are two orders of magnitude lower than the corresponding temperatures in SBB. Inverse beta decay of the proton can ensure adequate production of several light elements while producing primordial metallicity much higher than that produced in SBB. Other attractive features of these models
are the absence of the horizon, flatness and initial singularity problems, consistency with the age of globular clusters and consistent relationships between redshift and luminosity distance, angular diameter distance and the galaxy number count.
Early universe nucleosynthesis is regarded as a major “success story” of the standard big bang (SBB) model. The observed light element abundances are believed to severely constrain cosmological and particle physics parameters. Of late [Steigman, 1996] observations have suggested the need for a careful scrutiny and a possible revision of the status of SBB nucleosynthesis. While attempts to reconcile the cosmological abundance of deuterium and the number of neutrino generations within the framework of SBB are still on, we feel that alternative scenarios should be explored. Surprisingly, a class of models radically different from the standard one can produce the correct amount of helium as well as the metallicity observed in low metallicity objects. This paper is a status report on our ongoing efforts to study the cosmological implications of a class of models in which the cosmological scale factor $R(t)$ varies linearly with time.

A crucial assumption in the standard model is the existence of thermal equilibrium at temperatures around $10^{12}K$ or 100$MeV$. At these temperatures, the universe is assumed to consist of leptons, photons and a contamination of nucleons in thermal equilibrium. The ratio of weak reaction rates of leptons to the rate of expansion of the universe (the Hubble parameter) below $10^{11}K$ (age $\approx .01\text{secs}$) goes as [see eg., Weinberg, 1972]

$$\frac{\sigma n_l}{H} \approx \left(\frac{T}{10^{10}K}\right)^3$$

At these temperatures, the small nucleonic contamination begins to shift towards more protons and fewer neutrons because of the n-p mass difference. By $10^{10}K$ i.e. $T_9 = 10$, the neutrinos decouple. The distribution function of the $\nu$'s however maintains a Planckian profile as the universe expands. At $5 \times 10^9K$ (age of about 4 secs), $e^+,e^-$ pairs annihilate. The neutrinos having decoupled, all the entropy of the $e^+,e^-$ before annihilation, goes to heat up the photons - giving the photons some 40% higher temperature than the temperature corresponding to the neutrino Planckian profile. The decoupling of the neutrinos and the annihilation of the $e^+,e^-$ ensures the rapid fall of the neutron production rate $\lambda(p \rightarrow n)$ in comparison to the expansion rate of the universe. The n/p ratio freezes to about 1/5 at this epoch. This ratio now falls slowly.
on account of the decay of free neutrons. Meanwhile nuclear reactions and photo-disintegration of light nuclei ensure a dynamic buffer of light elements with abundances roughly determined by nuclear statistical equilibrium (NSE). Depending on the baryon-entropy ratio, at a critical temperature around $T_9 = 1$, the deuterium concentration is large enough for efficient evolution of a whole network of reactions leading up to the formation of the most stable light nucleus, viz. $^4\text{He}$. At slightly lower temperatures, the deuterium depletion rate becomes small compared to the expansion rate [see eg., Kolb & Turner, 1989] resulting in residual abundances of deuterium and $^3\text{He}$. Elaborate numerical codes have been developed by Wagoner and Kawano [WK] [1988] to describe the evolution of this phase. While the predicted abundances of deuterium, helium - 3, helium - 4 and lithium - 7 are believed to be consistent with observations, one does not see any astrophysical object with metallicity (abundance of lithium - 8 and heavier elements) as low as that predicted by primordial synthesis alone. The oldest objects are believed to be globular clusters. The metallicity reported in these systems is much higher than accounted for by SBB and much too low in comparison with that found in the atmosphere of population I stars and interstellar gas. Consistency of the light element abundances in SBB, moreover, is ensured only if the baryonic matter density is some two orders of magnitude less than the closure density. This is regarded as a respite in SBB. Using the rest of the (non-baryonic) matter in a suitable combination of hot and cold dark matter (with possibly a small cosmological constant also thrown in) to build up large scale structures in cosmology has developed into an industry. The current status is far from satisfactory [See eg., Ostriker et al 1995]. In particular, the age estimates of globular clusters are uncomfortably high in comparison with the age of the universe as set by conservative estimates.

Motivated by the above, we explore the possibility of obtaining a consistent scenario for nucleosynthesis in a class of models which are radically different from the standard one. In particular, we consider a cosmological model in which, at the epoch when $T \approx 10^{12}K$ and thereafter, the scale factor $R(t)$ increases as $t$. The linear evolution of the scale factor ensures a horizon-free cosmology. We shall describe later
how this can be ensured. We shall refer to $t$ as the age of the universe. The present value of the scale parameter and the present epoch $t_o$ are exactly determined by the present Hubble constant $H_o = 1/t_o$. The scale factor and the temperature of radiation are related by $RT = \text{constant}$. In such a model, the age of the universe when $T \approx 10^{10} K$ would be of the order of a few years. The universe takes some $10^2$ to $10^3$ years to cool from $10^{10} K$ to $10^8 K$. The rate of expansion of the universe is about $10^7$ times slower than the corresponding rates for the same temperature in standard cosmology. This makes a crucial difference and in fact ensures that the standard story does not go through.

The process of the neutrinos falling out of thermal equilibrium, for example, is determined by the rate of $\nu$ production per charged lepton:

$$\frac{\sigma_{\nu k}n_l}{c^6} \approx g_{\nu k}h^{-7}(kT)^5/c^6$$

and the expansion rate of the universe [$H = 1/t$]. Here $g_{\nu k} \approx 1.4 \times 10^{-45}$ erg- cm$^3$. For $kT > m_\mu$, $T > 10^{12} K$

$$\frac{\sigma_{\nu k}n_l}{H} \approx \left[\frac{T}{1.62 \times 10^8 K}\right]^4$$

Here we have normalized the value of $RT = tT = \text{constant}$ from the value $H_o = 55$ km/sec/Mpc for the Hubble constant. corresponding to $t_o \approx 12 \times 10^9$ years. Increasing $H_o$ by a factor of 2 would merely lead to a change in the denominator on the right side of eqn. 3 to $1.8 \times 10^8 K$. When $kT < m_\mu$, the number density of muons is reduced by a factor $[\exp(-m_\mu/kT)]$. Consequently, the rates of weak interactions involving muons get suppressed to

$$\frac{\sigma_{\nu k}n_l}{H} \approx \left[\frac{T}{1.62 \times 10^8 K}\right]^4 \exp[-\frac{10^{12}K}{T}]$$

The corresponding rates in the standard model are:

$$\frac{\sigma_{\nu k}n_l}{H} \approx \left[\frac{T}{10^{10} K}\right]^3$$
for \( kT > m_\mu \), and

\[
\sigma_{wk n_l}/H \approx \left[ \frac{T}{10^{10} K} \right]^3 \exp\left[ -\frac{10^{12}K}{T} \right]
\]

for \( kT < m_\mu \). This would lead to the weak interactions maintaining the \( \nu \)'s in thermal equilibrium to temperatures down to \( 1.62 \times 10^8 K \). This would then imply that the entropy released from the \( e^+e^- \) annihilation would heat up all the particles in equilibrium. Both the neutrinos and the photons would get heated up to the same temperature. The temperature then scales by \( RT = \text{constant} \) as the universe expands. The relic neutrinos and the photons (the CMBR) would therefore have the same Planckian profile \( (T \approx 2.7K) \) at present. This is in marked contrast to the standard result wherein the neutrino temperature is predicted to be lower than the photon temperature. The nuclear reaction rates are simply given by the expressions [Weinberg, 1972]:

\[
\lambda(n \rightarrow p) = A \int \left(1 - \frac{m_e^2}{(Q + q)^2}\right)^{1/2} (Q + q)^2 q^2 dq \times (1 + e^{q/kT})^{-1}(1 + e^{-(Q+q)/kT})^{-1}
\]

\[
\lambda(p \rightarrow n) = A \int \left(1 - \frac{m_e^2}{(Q + q)^2}\right)^{1/2} (Q + q)^2 q^2 dq
\]

These rates have the ratio:

\[
\frac{\lambda(p \rightarrow n)}{\lambda(n \rightarrow p)} = \exp\left(-\frac{Q}{kT}\right)
\]

The rate of expansion of the universe at a given temperature being much smaller than that in the standard scenario, the nucleons are expected to be in thermal equilibrium with the ratio \( X_n \) of neutron number to the total number of all nucleons given by:

\[
X_n = \frac{\lambda(p \rightarrow n)}{\lambda(p \rightarrow n) + \lambda(n \rightarrow p)} = [1 + e^{Q/kT}]^{-1}
\]
However the conditions of NSE would still hold. A buffer of light elements would emerge as before. The baryonic content of the universe at $T_9 \approx 1$ is constituted by protons (mainly), some neutrons (less than 1%) and a buffer of light elements in NSE. The strength of the buffer is enhanced by fresh neutron formation by the inverse beta decay of the proton and its capture into the buffer by the pn reaction. The buffer depletes by either: (i) the photodisintegration of any light element constituting the buffer followed by the decay of the resulting neutron before it can be recaptured into the buffer by the pn reaction; or (ii) the formation of $He_4$ which is the most stable nucleus at these temperatures. Once helium formation becomes more efficient than neutron decay, all subsequent neutrons formed would precipitate into $He_4$. This critical epoch is sensitive to the baryon-entropy ratio. If the ratio of number of protons that convert into neutrons after that epoch to the total baryon number of the universe is roughly 1/8, we would get the observed $\approx 25\%$ He. This simply translates into an appropriate requirement on the baryon-entropy ratio.

We have modified the (WK) numerical code outlined by Kawano to suit the taxing requirements of the much stiffer rate equations that we encounter in our slowly evolving universe. To get convergence of the rate equations for 26 nuclides and a network of 88 reactions, as given in Kawano, we executed some 500 iterations at each time step. We have incorporated an additional (89th) reaction: the pp reaction

$$p + p \rightarrow D + e^+ + \nu$$

As a consequence of this reaction, the lifetime of protons is around $10^{10}$ years in the core of a typical star at temperatures of the order of $T_9 \approx .01$ and densities some $100gmcm^{-3}$. The contribution of this reaction in the few minutes that the universe has temperature $\approx T_9 = 1$ and density $< 1gmcm^{-3}$ is negligible in SBB but in our model where the expansion rate is some $100yrs^{-1}$, it gives a substantial contribution. The results for different values of $\eta$ are described in table I. We find consistency with the $He_4$ abundances for $\eta \approx 10^{-8}$. The high metallicity produced is also a consequence of the slow expansion in this model.

To get the observed abundances of the light elements, one would have to rely on nucleosynthesis by secondary explosions of supermassive
objects [Wagoner 1969]. We feel we may be able to dynamically account for such explosions within the framework of models we shall outline later.

The expansion rate in this model does not depend on the background density and thus the abundances are independent of the number of neutrino species. The age of this universe (defined as the time elapsed from the hot epoch to the present) would be exactly 50% higher than the SBB age determination from the inverse of the present Hubble parameter. If one goes by the value of 80 km/sec/Mpc currently quoted for the Hubble parameter, our model would accommodate 15 giga-year old objects much more comfortably than SBB where one merely advocates disbelief in single-σ error bars [Longair 1995].

We now address the issue of realising the linear evolution within the framework of a Friedman cosmology. Such an evolution can be accounted for in a universe dominated by ‘K - matter’ for which the density scales as \( R^{-2} \). The Hubble diagram (luminosity distance-redshift relation), the angular diameter distance - redshift relation and the galaxy number count-redshift relations do not rule out such a “coasting” cosmology [Kolb, 1989; Sethi et al 1996]. However, if one requires this matter to dominate even during the nucleosynthesis era, the K - matter would almost close the universe. There would be hardly any baryons. An alternative way of achieving a linear evolution of the scale factor is an effective Einstein theory with a repulsive gravitational constant at long distances. Such possibilities follow from effective gravitational actions that have been considered in the past. Ellis and Xu [1995], for example, considered a fourth order theory with action:

\[
S = \int d^4x \sqrt{-g} [\alpha R^2 - \beta R] \tag{11}
\]

In the weak field approximation, the effective Newtonian potential in such a theory is:

\[
\phi = -\frac{a}{r} + b \exp\left(\frac{-\mu r}{r}\right) \tag{12}
\]

For \( \mu r \ll 1 \) we can have a canonical effective attractive theory. At large distances, the effective potential is dominated by the first term alone - corresponding to repulsive gravity. A similar possibility occurs
in the conformally invariant proposal of Manheim and Kazanas [1990]. Choosing the gravitational action to be the square of the Weyl tensor gives rise to an effective induced action:

\[ S = \int d^4x \sqrt{-g} [\alpha C^2 - \beta R] \] (13)

The dynamics of a conformally flat FRW metric is driven by the anomalous repulsive term alone. Canonical attractive domains occur in the model as non-conformally flat perturbations in the FRW spacetime. Repulsive long range gravitation with spherical attractive domains also occur in a variant of a model by Zee [1982]:

\[ S = \int d^4x \sqrt{-g} [ -\epsilon \phi^2 R + \frac{1}{2} (\partial_{\mu} \phi)(\partial^{\mu} \phi) + V(\phi) + \beta_{\text{ind}} R + \Lambda_{\text{ind}} + L_m ] \] (14)

Here \( \beta_{\text{ind}} \) and \( \Lambda_{\text{ind}} \) are induced gravitational and cosmological constants, \( L_m \) the action for the rest of the matter fields and \( V(\phi) \) the effective potential for the scalar field. In these models, static non-topological soliton solutions exist. For such solutions, \( \phi \) is constant inside a sphere and rapidly goes to zero near its surface. These solutions would have an effective canonical attractive gravitation in their interior and have repulsive gravitation outside.

We have been exploring the possibility that such non-topological domains - gravity balls (g-balls) - of the size of a typical galactic halo (or larger) play an essential role in cosmology. The formation of large scale structure by the splitting of gravity balls, the growth of density perturbations and gravitational lensing are some of the areas which are the subject of our continuing investigation. In this context, we find the results of this paper quite encouraging.

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TABLE I

Abundances of Some Light Elements and Metals.

| $\eta$ | $^2\text{H}$ ($10^{-9}$) | $^3\text{H}$ ($10^{-18}$) | $^3\text{He}$ ($10^{-25}$) | $^4\text{He}$ ($10^{-14}$) | $^7\text{Be}$ ($10^{-11}$) | $^8\text{Li}$ & above ($10^{-9}$) |
|--------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 9.0    | 2.007           | 1.25            | 8.65            | 2.03            | 1.39            | 8.06            |
| 9.1    | 2.008           | 1.26            | 8.63            | 2.06            | 1.32            | 8.63            |
| 9.2    | 2.009           | 1.26            | 8.60            | 2.10            | 1.23            | 9.35            |
| 9.3    | 2.010           | 1.27            | 8.59            | 2.11            | 1.19            | 9.75            |
| 9.4    | 2.014           | 1.26            | 8.56            | 2.15            | 1.11            | 10.66           |
| 9.5    | 2.015           | 1.27            | 8.50            | 2.18            | 1.05            | 11.41           |
| 9.6    | 2.016           | 1.28            | 8.52            | 2.19            | 1.01            | 11.88           |
| 9.7    | 2.017           | 1.28            | 8.49            | 2.22            | 0.96            | 12.69           |
| 9.8    | 2.020           | 1.29            | 8.47            | 2.25            | 0.91            | 13.51           |
| 9.9    | 2.020           | 1.29            | 8.45            | 2.28            | 0.86            | 14.47           |
| 10.0   | 2.020           | 1.30            | 8.43            | 2.30            | 0.83            | 15.19           |

Initial Temperature $10^{11}K$
Final Temperature $10^7K$
No. of iterations at each step 550