Analysis of Drop Point Track of Ping Pong Ball After Hitting Based on Dynamic Analysis

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ABSTRACT
Table tennis is a popular sport in China. It has a low barrier to entry and also has difficult techniques. In order to improve the skills of table tennis match between players, this paper introduced the dynamic analysis of ping pong ball in flight after being hit. The main content was the stress analysis in the flight of ping pong ball. Moreover, the Magnus force caused by rotation was considered. The dynamic model of ping pong ball flight was built, the flight track of top spinning ball at 0, 30, 60, 90 and 120 r/s were simulated using Java software. The actual flight track of ping pong ball under the same conditions was obtained using the table tennis serving machine and high-speed camera to verify the effectiveness of the dynamic model. The results showed that the track obtained by the dynamic model basically coincided with the actual track, which verified the validity of the dynamic model; the maximum height and drop point distance of ping pong ball decreased with the increase of spinning speed of top spinning ball; the flight time of ping pong ball decreased with the increase of spinning speed of top spinning ball. To sum up, players can reduce the flight height, drop point distance and flight time of ping pong ball by increasing the top spinning speed, so as to improve the opponent's counterattack difficulty.

1. INTRODUCTION
Table tennis has a very large audience in China as the equipment required is very simple and the sport field needed is relatively small; hence China is also known as table tennis superpower [1]. Although table tennis has high popularity, is suitable for all ages, and has a low barrier to enter, the degree of appreciation and competitiveness of table tennis is not reduced at all, and the low difficulty of entry does not mean that table tennis is a sport with low difficult [2]. Table tennis is said to be a sport which is easy to get started but difficult to master. The high difficulty of table tennis lies in: ① fast speed: table tennis is one of the ball games with the fastest flight speed, and the small field of table tennis (table size: ) requires players to have extremely fast reaction [3]; ② fast rotation speed: hitting ping pong ball using the edge area of the racket not only can improve the hitting force, but also can make ping pong ball rotate because of the additional torque caused by the non-central collision between the hitting plane and the direction of velocity; ③ changeable skills: there are many skills that can be applied in the process of confrontation in table tennis, and the skills based on the rotation of the ball can be divided into cutting, chopping and driving [4]. Because of the above characteristics of table tennis, such as fast speed, fast rotation speed and changeable skills, there is not enough reaction time for the players to make countermeasures when the ball flies to the front of their

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eyes. Therefore, it is necessary to predict the track of the ball according to the angle and rotation of the ball at the beginning of the stroke. The analysis of the track of the drop point after hitting can provide an effective reference for players to defend. Bankosz et al. [5] conducted six series of hitting experiments on 12 female table tennis players in the certified biomechanics laboratory using the motion analysis system to present different kinds of topspin images and found that changing the type of topspin required changing the time, speed, main distance parameters and the direction of racket. Xiao et al. [6] studied the pen-hold forehand flip-loop combo technique using the experimental method of sports biomechanics and found that the movement and hitting speed of the technique were similar to the forehand flip shot technique, the ball rotation speed was between forehand flip shot and backhand flip shot, and the flight path was in a shape of hook, which was conducive to improving the forehand attack ability. This paper briefly introduced the dynamic analysis of ping pong ball in the flight process after being hit. The main content was the stress analysis in the flight process of ping pong ball. Moreover, Magnus force caused by rotation was considered. The dynamic model of the flight of ping pong ball was established, and the simulation of the flight path of the top spinning ball at 0, 30, 60, 90 and 120 r/s were carried out by Java software. The simulation of the flight path of the top spin ball is carried out by using java software. The actual flight track of ping pong ball under the same conditions was obtained by the table tennis serve machine and high-speed camera to verify the effectiveness of the dynamic model.

2. DYNAMICS PRINCIPLE OF FLIGHT PATH OF PING PONG BALL AFTER BEING HIT

In the competition of table tennis, players of both sides change the flight state of ping pong ball using the racket [7]. Although the flight track of ping pong ball before rebound is similar to parabola, the mass and volume of ping pong ball cannot ignore other forces except gravity. Therefore, the calculation and analysis of the track cannot be the same with the calculation of parabola.

Figure 1. Diagram of stress analysis of ping pong ball in the process of flight after hitting
Although ping pong ball has different hitting skills and flight track when it is hit in different ways, there are only two situations in the flight process of ping pong ball: rotation and non-rotation [8]. When ping pong ball does not rotate in the flight, its stress situation is as follows: it bears the vertical downward gravity, the air buoyancy opposite to the gravity direction, and the air resistance opposite to the movement direction of ping pong ball; when ping pong ball rotates in the flight process, in addition to the gravity, buoyancy and resistance, it will generate the Magnus force due to the disturbance of air fluid by the rotation of ball.

Ping pong ball is always affected by vertical downward gravity whether it rotates or not during flight. Its expression is as follows:

\[
G = \frac{\pi \rho t d^3 g}{6} = m_t g
\]  

(1)

where \(G\) is the gravity of ping pong ball, \(\rho_t\) is the density of ping pong ball, \(d\) is the diameter of ping pong ball, \(g\) is the acceleration of gravity, and \(m_t\) is the mass of ping pong ball.

In fact, according to the regulations of the International Table Tennis League, no matter in the regular competition or in the daily training, specifications such as size of table tennis ground, table tennis table and ping pong ball must be in accordance with the unified standards, among which the size of ping pong ball is specified as 40 mm diameter [9], and the mass is basically maintained at 2.7 g.

Both air and water are fluids, which will produce a buoyancy opposite to the direction of gravity for the objects in it, and its size is equal to the weight of the discharged fluid [10]. In general, daily activities, the air density is relatively small, even if a certain volume of air is discharged, the weight of the air can nearly be ignored compared with the gravity of the object. However, in the ping pong ball track analyzed in this study, the mass of the ping pong ball is relatively small, and the buoyancy that it bears cannot be ignored in the track calculation. Its expression is:

\[
f = \frac{\pi \rho_a d^3 g}{6}
\]  

(2)

where \(f\) is the buoyancy of the ping pong ball and \(\rho_a\) is the density of air.

When an object moves in the fluid, it will be subject to the resistance from the fluid. The size of the resistance is directly proportional to the speed of the object, and the direction is opposite to the speed direction. If the mass of the object itself is large enough and the moving speed is low, then the resistance of the fluid can be ignored, but in this study, the mass of the ping pong ball is only 2.7 g, and more importantly, the flying speed of the ping pong ball was generally about 5 m/s, so the air resistance produced [11] cannot be ignored, and its calculation formula is as follows:

\[
\begin{cases}
F = \frac{C_D \pi \rho_a d^2 v^2}{8} \\
24/R_e \quad R_e < 1 \\
24(1+R_e^{2/3}/6) \quad 1 \leq R_e < 1000 \\
0.44 \quad 1000 \leq R_e < 200000
\end{cases}
\]

where \(C_D\) is the drag coefficient.
where $F$ stands for air resistance, $C_D$ is the resistance coefficient of air, $v$ is the absolute speed of ping pong ball during flight, and $Re$ is Reynolds number [12].

If the ping pong ball does not rotate during the flight, the dynamic model of ping pong ball can be built according to the above-mentioned stress of ping pong ball according to Newton’s second law. However, ping pong ball often rotates during flight. Therefore, before the dynamic model is built, the Magnus force generated by rotation should be considered. The Magnus force is caused by the Magnus effect, and its calculation formula is:

$$F_m = \frac{\pi \rho a d v \omega}{8} \quad (4)$$

where $F_m$ stands for Magnus force and is the rotation speed of ball. Magnus force is calculated by equation (4), and its direction is determined by the right-hand method. Firstly, the direction of the rotation axis is determined by the right-hand spiral law [13]: the thumb indicates the rotation axis, and the other four fingers indicate the rotation direction, then the direction which the thumb points at is the direction of the rotation axis; then the thumb points at the direction of rotation axis and the index finger points at the direction of ball velocity, then the direction which is perpendicular to the hand side is the direction of Magnus force. Taking Figure 1 as an example, the rotation direction of the ball is clockwise. According to the right-hand spiral rule, the rotation axis direction of the ball is vertical to the paper surface, and after the index finger is pointed to the ball velocity direction, it can be seen that the direction of the Magnus force is parallel to the paper surface and perpendicular to the direction of ball velocity.

3. SIMULATION ANALYSIS

3.1 Experimental Environment
In this study, the drop point track of ping pong ball was simulated and analyzed after hitting. The simulation experiment was carried out in a laboratory server with configurations of Windows 7 system, i7 processor and 16 G memory.

3.2 Experiment Setup

Firstly, a coordinate system was established, as shown in Figure 2. The short edge of the table tennis table was taken as the origin, the direction of the long edge was taken as the X axis, and the direction which was perpendicular to the table and passed the origin was taken as the Y axis. Then, as described above, the stress analysis of ping pong ball after hitting was analyzed, and a dynamic model was established:
\[
\begin{align*}
\frac{d^2x}{dt^2} = & -\frac{C_D \rho a v^2}{2} + \pi \rho r^3 \omega \frac{dy}{dx} \\
\frac{d^2y}{dt^2} = & -G - f - \frac{C_D \rho a v^2}{2} - \pi \rho r^3 \omega \frac{dx}{dt}
\end{align*}
\]

(5)

where \( r \) is the radius of table tennis, \( x \) represents the horizontal displacement of ping pong ball, and \( y \) stands for the displacement of ping pong ball in the vertical direction. \( d \) in equation (5) is not the diameter of ping pong ball mentioned above but the differential sign.

Figure 3. The calculation flow of ping pong ball track

The flow of calculation of ping pong ball track by Java software is shown in Figure 3. Firstly, the initial conditions were input, and the stress condition of the ping pong ball was calculated. Then the track of ping pong ball was calculated according to the dynamic model. After the calculation, the track was compared with the specification of the table to determine whether the track is out of bounds or touching the net; if not, the effective track of the ping pong ball was output, and if out of bounds or touching the net, only the effective track before touching or out of bounds was output. The initial parameters to be input are shown in Table 1. Firstly, the initial position was set as (0.0 m, 0.1 m); the ambient temperature was 25 ℃, the atmospheric pressure was 1 atm; the initial speed of ping pong ball after being hit was 10 m/s, and the direction was 30° with the x axis; the specification of ping pong ball was 2.7 g and 40 mm diameter; moreover, five kinds of rotation conditions after being hit were set, which were 0, 30, 60, 90 and 120 r/s respectively. In general motion, 120 r/s is a high-speed rotation. The rotation speed above 120 r/s is hard to hit for the players, and the higher rotation speed is the demand of theoretical analysis, with relatively little value for practical application. Therefore, the rotation parameter below 120 r/s was set in this study, and moreover the rotation direction of ping pong ball was topspin, as shown in Figure 1.

In order to ensure the validity of the track calculated by the dynamic model, the actual track was obtained by actual experiment. The operation is as follows. Five ping pong balls with different rotation speeds were launched by a table tennis serving machine under the conditions shown in Table 1, and the flight track of the ping pong balls was captured by a high-speed camera [14] and compared with the track calculated by the dynamic model.
Table 1. Initial conditions for trajectory calculation using dynamic model

| Track number | Initial position coordinates | Environment specifications | Initial velocity | Specification of ping pong ball | Initial rotation speed | Rotation direction |
|--------------|------------------------------|----------------------------|-----------------|---------------------------------|-----------------------|-------------------|
| 1            | X=0.0 m; y=0.1 m            | Temperature: 25 ℃; atmospheric pressure: 1 atm | The speed is 10 m/s; the direction is 20° with the x axis | Mass: 2.7 g; diameter: 40 mm | 0 r/s               | Top spin          |
| 2            |                              |                            |                 |                                  | 30 r/s               |                   |
| 3            |                              |                            |                 |                                  | 60 r/s               |                   |
| 4            |                              |                            |                 |                                  | 90 r/s               |                   |
| 5            |                              |                            |                 |                                  | 120 r/s              |                   |

3.3. Experimental Results

The drop point tracks of ping pong balls calculated by the dynamic model are shown in Figure 4. The actual flight track was obtained by using the table tennis server under the same setting conditions. The actual track experiment was conducted three times. It was found from the comparison between the actual track and calculated track that the fitting degree of the two tracks under the same initial conditions was quite high. Therefore, only the track calculated by the dynamic model was displayed in Figure 4, and the error between the experimental track and calculated track is shown in Table 2. It was found that the average error between No. 1 track obtained from three experiments and the calculated track was 0.79%, the average error between No. 2 track obtained from three experiments and the calculated track was 0.80%, the average error between No. 3 track obtained from three experiments and the calculated track was 0.81%, the average error between No. 4 track obtained from three experiments and the calculated track was 0.83%, and the average error between No. 5 track obtained from three experiments and the calculated track was 0.84%. It was seen from the errors that the track obtained by the dynamic model had enough validity.

Figure 4. The drop point tracks of ping pong balls after hitting calculated by the dynamic model

It was seen from Figure 4 that the five tracks ascended first and then descended; in the early stage of the flight track after the ping pong was hit, the top spinning tracks under the five rotation speeds basically coincided, but in the middle and later stages, the five tracks were obviously different, and the biggest difference that can be seen was the track radian.
The greater the rotation speed of the up-spin ball, the greater the track radian in the later stage of the flight and the closer the first drop point of the ping pong ball to the origin of coordinates. In the competition of table tennis, on the premise of ensuring that the ball is over the net and not out of the boundary, the closer the first drop point of the ping pong ball is to the net, the more difficult it is for the catcher to hit back to the ball. Therefore, improving the spinning speed of ping pong ball after hitting will help to shorten the distance between the drop point of ping pong ball and net, create the difficulty for the opponent to hit back, and improve the aggressiveness.

Table 2. Error between the track obtained from the actual experiment and the track calculated by the dynamic model

| Track number | 1   | 2   | 3   | 4   | 5   |
|--------------|-----|-----|-----|-----|-----|
| Error between the first experiment and simulation/% | 0.79 | 0.80 | 0.81 | 0.83 | 0.84 |
| Error between the second experiment and simulation/% | 0.78 | 0.81 | 0.80 | 0.82 | 0.82 |
| Error between the third experiment and simulation/% | 0.81 | 0.79 | 0.82 | 0.84 | 0.85 |
| Average error% | 0.79 | 0.80 | 0.81 | 0.83 | 0.84 |

Figure 5. Time required for the top spinning ball with different rotation speeds flying to the first drop point
As shown in Figure 5, the time taken for a ping pong ball with a rotating speed of 0 to the first drop point was 0.59 s, the time taken for a top spinning ball with a rotating speed of 30 r/s to the first drop point was 0.44 s, the time taken for a top spinning ball with a rotating speed of 60 r/s to the first drop point was 0.35 s, the time taken for a top spinning ball with a rotating speed of 90 r/s to the first drop point was 0.27 s, and the time taken for a top spinning ball with a rotating speed of 120 r/s to the first drop point was 0.21 s. It was seen intuitively from Figure 5 that the time for the ping pong ball to fly to the first drop point after being hit decreased with the increase of top spinning speed.

The flight time of ping pong ball in the air depends on the movement state in the vertical direction. In addition to the buoyancy and gravity, the rotating ping pong ball has the air resistance and the component of the Magnus force in the vertical direction. The vertical component of the Magnus force is downward; the greater the rotation speed, the greater the Magnus force, the greater the vertical component, and the larger the downward resultant force of the ping pong ball in the vertical direction; as a result the motion time of the ping pong ball in the vertical direction becomes shorter, i.e., the flight time becomes shorter.

According to the above law, in the actual combat of table tennis, the flight time of ping pong ball can be reduced by increasing the top spinning speed of ping pong ball, so as to reduce the reaction time of the opponent and increase the difficulty of return stroke.

4. CONCLUSION

This paper briefly introduced the dynamic analysis of the flight process of ping pong ball after being hit. The main content is the stress analysis in the flight process of ping pong ball. Moreover, the Magnus force caused by rotation was considered. The dynamic model of ping pong ball flight was established, and simulation was carried out. The model was verified by the actual track under the same conditions. The results are as follows: (1) the track of ping pong ball calculated by the dynamic model basically coincided with the actual track under the same conditions, which verified the validity of the dynamic model; (2) the track calculated by the dynamic model showed that the maximum height that the ping pong ball could reach gradually reduced with the increase of the rotation speed of the top spinning ball, and the distance from the final drop point to the origin also gradually reduced; (3) the time needed for the ping pong ball to fly to the first drop point calculated by the dynamic model decreased with the increase of the top spinning speed.

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