Gauged $D = 9$ Supergravities
and Scherk-Schwarz Reduction.

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ABSTRACT

Generalised Scherk-Schwarz reductions in which compactification on a circle is
accompanied by a twist with an element of a global symmetry $G$ typically lead to
gauged supergravities and are classified by the monodromy matrices, up to con-
jugation by the global symmetry. For compactifications of IIB supergravity on a
circle, $G = SL(2, \mathbb{R})$ and there are three distinct gauged supergravities that result,
corresponding to monodromies in the three conjugacy classes of $SL(2, \mathbb{R})$. There
is one gauging of the compact $SO(2)$ subgroup of the $SL(2, \mathbb{R})$ and two distinct
gaugings of non-compact $SO(1, 1)$ subgroups, embedded differently in $SL(2, \mathbb{R})$.
The non-compact gaugings can be obtained from the compact one via an analytic
continuation of the kind used in $D = 4$ gauged supergravities. For the super-
string, the monodromy must be in $SL(2, \mathbb{Z})$, and the distinct theories correspond
to $SL(2, \mathbb{Z})$ conjugacy classes. The theories consist of two infinite classes with
quantised mass parameter $m = 1, 2, 3, \ldots$, three exceptional theories correspond-
ing to elliptic conjugacy classes, and a set of sporadic theories corresponding to
hyperbolic conjugacy classes.
1. Scherk-Schwarz Reductions and Generalisations

The Scherk-Schwarz mechanism and its generalisations [1-11] introduces mass parameters into toroidal compactifications of supergravities and string theories. If the original theory in \(D'\) dimensions has a global symmetry \(G'\) acting on fields \(\phi\) by \(\phi \rightarrow g(\phi)\), then in a generalised Scherk-Schwarz reduction or twisted reduction the fields are not independent of the internal coordinates, but are chosen to depend on the torus coordinates \(y\) through an ansatz

\[
\phi(x^\mu, y) = g_y(\phi^i(x^\mu))
\]  

for some \(y\)-dependent symmetry transformation \(g_y = g(y)\) in \(G'\). Usually the twist will be contained in the subgroup \(K' \subseteq G'\) of symmetries of the action in \(D'\) dimensions, but in some cases it is possible to twist by symmetries that are symmetries of the field equations only (an example of this is given in [3]); here only twists in \(K'\) will be considered. In some cases such a generalised reduction leads to a spontaneous breaking of supersymmetry [1]. Typically, it results in a gauging of the reduced theory; see e.g. [5,6,9]. In such cases, if the standard reduction without twists from \(D'\) dimensions gives a theory in \(D\) dimensions with duality symmetry \(G\) and symmetry of the action \(K \subseteq G\), then twisting with an element of \(K' \subseteq K\) will result in a gauging of a subgroup \(L\) of \(K\). Consider compactifications from \(D' = D + 1\) dimensions to \(D\) dimensions on a circle, with periodic coordinate \(y \sim y + 1\). For example, for reducing a theory with a linearly realised \(G' = U(1)\) symmetry on a circle, a massless field \(\phi\) of charge \(q\) can be given a \(y\) dependence \(\phi(x, y) = e^{2\pi i q m y} \phi^i(x)\), so that the field \(\phi^i(x)\) is given a mass of \(qm\).

The map \(g(y)\) is not periodic, but has a monodromy

\[
\mathcal{M}(g) = g(1)g(0)^{-1}
\]  

for some \(\mathcal{M}\) in \(G'\). For maps of the form

\[
g(y) = \exp(My)
\]
for some Lie algebra element $M$, the monodromy is

$$\mathcal{M}(g) = \exp M$$

Then

$$M = g^{-1} \partial_\gamma g$$

is proportional to the mass matrix of the dimensionally reduced theory and is independent of $\gamma$.

The Lie algebra element $M$ generates a one-dimensional subgroup $L'$ of $G'$, and this group becomes gauged in the dimensionally reduced theory. For a field $\phi$ transforming in some representation of $G'$ with $M$ acting through some matrix $\tilde{M}$, so that under $G'$, $\phi$ transforms as $\delta \phi = \lambda \tilde{M} \phi$ (with infinitesimal parameter $\lambda$), it is straightforward to show that on dimensional reduction to $D' - 1$ dimensions, the derivative of $\phi$ becomes the gauge covariant derivative $D\phi = d\phi + A\tilde{M} \phi$, where $A$ is the 1-form gauge potential arising from the reduction of the metric (the graviphoton), indicating that $L'$ has become a local symmetry for which the gauge field is the graviphoton. For Scherk-Schwarz reduction on $T^n$ with the twistings for the $n$ circles generated by $n$ commuting matrices $M_1, \ldots, M_n$, the resulting gauge group is the abelian group $L'_1 \times \ldots \times L'_n$ generated by the $M_i$.

The next question is whether two different choices of $g(\gamma)$ give inequivalent theories. The ansatz breaks the symmetry $G'$ down to the subgroup preserving $g(\gamma)$, consisting of those $h$ in $G'$ such that $h^{-1}g(\gamma)h = g(\gamma)$. Acting with a general constant element $k$ in $K'$ will change the massdependent terms, but will give a $D-1$ dimensional theory related to the original one via the field redefinition $\phi \to k(\phi)$. This same theory could have been obtained directly via a reduction using $k^{-1}g(\gamma)k$ instead of $g(\gamma)$, so two choices of $g(\gamma)$ in the same conjugacy class give equivalent reductions (related by field-redefinitions). A given monodromy can result from infinitely many different mass matrices [8], but these all give physically equivalent results (if all the Kaluza-Klein modes are kept) [12]. As a result, the reductions are classified by conjugacy classes of the monodromy-matrix $\mathcal{M}$ [8].
The map \( g(y) \) is a local section of a principal fiber bundle over the circle with fibre \( G' \) and monodromy \( \mathcal{M}(g) \) in \( G' \). Such a bundle is constructed from \( I \times G' \), where \( I = [0, 1] \) is the unit interval, by gluing the ends of the interval together with a twist of the fibres by the monodromy \( \mathcal{M} \). Two such bundles with monodromy in the same \( G' \)-conjugacy class are equivalent.

Of particular interest are the supergravity theories in \( D' = D + 1 \) dimensions with rigid duality symmetry \( G' \) and scalars taking values in \( G'/H' \) [13,14], which can be Scherk-Schwarz-reduced on a circle to \( D \) dimensions. The reduction requires the choice of a map \( g(y) \) of the form (1.3) from \( S^1 \) to \( G' \), which then determines the \( y \)-dependence of the fields through the ansatz (1.1), and any choice of Lie algebra element \( M \) is allowed.

In the quantum theory, the symmetry group \( G' \) is broken to a discrete subgroup \( G'({\mathbb{Z}}) \) [15]. A consistent twisted reduction of a string or M-theory, whose low-energy effective theory is the supergravity theory considered above, then requires that the monodromy be in the U-duality group \( G(\mathbb{Z}) \). (In the classical supergravity theory, any element of \( G \) can be used as the monodromy.) Then the choice of \( M \) is restricted by the constraint that \( e^M \) should be in \( G'({\mathbb{Z}}) \). As before, if two theories have \( M \)-matrices \( M, \tilde{M} \) related by \( M = k\tilde{M}k^{-1} \) where \( k \) is in \( G' \), the theories are related by field redefinitions. However, the data needed to specify the quantum theory includes the charge lattice \( \Gamma_p \) giving the allowed values of the quantised \( p \)-brane charge, and there is such a lattice for each of the values of \( p \) arising in the theory. For each \( p \), the \( p \)-brane charges will transform in some representation \( R_p \) of \( G' \) and so the \( G' \) transformation \( k \) taking \( M \) to \( \tilde{M} \) will take \( \Gamma_p \) to a new lattice \( \tilde{\Gamma}_p \). Thus the theories specified by \( (M, \Gamma_p) \) and \( (\tilde{M}, \tilde{\Gamma}_p) \) are related by field redefinitions and so are physically equivalent. One way to classify the distinct theories is to fix the lattices \( \Gamma_p \) and ask which monodromy matrices give distinct theories. The subgroup of \( G' \) which preserves the lattices \( \Gamma_p \) is the discrete U-duality subgroup, which will be denoted \( G'({\mathbb{Z}}) \), and \( G'({\mathbb{Z}}) \cap K' \) will be denoted \( K'({\mathbb{Z}}) \). Then acting with an element \( k \) of \( K'({\mathbb{Z}}) \) will preserve the lattices but change \( M \) to \( \tilde{M} = kMk^{-1} \). Then the two monodromies \( \mathcal{M} \) and \( \tilde{\mathcal{M}} \) will be
elements of $K'(\mathbb{Z})$ in the same $K'(\mathbb{Z})$ conjugacy class. Thus, for given charge lattices, two theories with $K'(\mathbb{Z})$ monodromies that are related by $\bar{M} = kMk^{-1}$ for some $k \in K'(\mathbb{Z})$ will be physically equivalent, as they are related by a field redefinition. Thus the distinct theories correspond to the distinct $K'(\mathbb{Z})$ conjugacy classes [8].

2. Scherk-Schwarz Reduction of IIB Supergavity on $S^1$.

The type IIB supergravity theory has $G = SL(2, \mathbb{R})$ global symmetry and any element $M$ of the $SL(2, \mathbb{R})$ Lie algebra can be used in the ansatz (1.1),(1.3) to give a Scherk-Schwarz reduction to 9-dimensions to obtain a class of massive 9-dimensional supergravity theories. Such reductions for particular elements of $SL(2, \mathbb{R})$ were given in [2,4,3], and the general class of $SL(2, \mathbb{R})$ reductions of IIB supergravity was obtained in [9,10,11]. This gives a 3-parameter family of theories, specified by the choice of matrix

$$M = \begin{pmatrix} m_1 & m_2 + m_3 \\ m_2 - m_3 & -m_1 \end{pmatrix}$$ (2.1)

The details of the reduction of the bosonic sector of the supergravity theory for general $M$ of this form were given in [9,10]. Note that this ansatz does not allow the monodromy to be an arbitrary $SL(2, \mathbb{R})$ group element, but requires it to be in the image of the exponential map. Acting with an $SL(2, \mathbb{R})$ transformation leaves the mass-independent part of the theory unchanged but changes the mass matrix by $SL(2, \mathbb{R})$ conjugation, and any two theories related by such a field redefinition are physically equivalent. There are then three distinct classes of inequivalent theories, corresponding to the hyperbolic, elliptic and parabolic $SL(2, \mathbb{R})$ conjugacy classes, represented by monodromy matrices of the form

$$\mathcal{M}_h = \begin{pmatrix} e^a & 0 \\ 0 & e^{-a} \end{pmatrix}, \quad \mathcal{M}_e = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}, \quad \mathcal{M}_p = \begin{pmatrix} 1 & a \\ 0 & 1 \end{pmatrix}$$ (2.2)
respectively, generated by the matrices

\[
M_h = \begin{pmatrix} a & 0 \\ 0 & -a \end{pmatrix}, \quad M_e = \begin{pmatrix} 0 & \theta \\ -\theta & 0 \end{pmatrix}, \quad M_p = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}
\] (2.3)

and each class is specified by a single coupling constant \((a\) or \(\theta\)). Thus the 3-parameter family of theories splits into three equivalence classes, with all the theories in a given class related by field redefinitions and rescalings of the single coupling constant.

In \(D = 10\), the duality group is \(K' = SL(2, \mathbb{R})\) and there are two 2-form fields \(\hat{B}_2^i\) \((i = 1, 2)\) transforming as an \(SL(2, \mathbb{R})\) doublet. On reduction to \(D = 9\), the duality group \(K\) is \(SL(2, \mathbb{R}) \times \mathbb{R}\) and the \(\hat{B}_2^i\) reduce to a doublet of 2-forms \(B_2^i\) and a doublet of 1-forms \(B_1^i\). In addition, there is a third vector field \(A_1\) from the reduction of the metric, and this is an \(SL(2, \mathbb{R})\) singlet. The 4-form potential in \(D = 10\) gives a 4-form \(C_4\) and a 3-form \(C_3\), but the self-duality constraint in \(D = 10\) implies that \(C_4\) is the dual of \(C_3\) in \(D = 9\), and the theory can be written in terms of \(C_3\) alone \([9,10,]\). The field strengths for these gauge fields include

\[
\begin{align*}
H_2^i &= dB_1^i - M^{ij} B_2^j \\
H_3^i &= dB_2^i - H_2^j \wedge A_1 \\
G_4 &= dC_3 + \frac{1}{2} \epsilon_{ij} (-B_1^i \wedge H_3^j + [B_2^i + A_1 \wedge B_1^i] \wedge H_2^j)
\end{align*}
\] (2.4)

In particular, these are invariant under the following gauge symmetry with 1-form parameter \(\lambda_1^i\)

\[
\begin{align*}
\delta B_1^i &= M^{ij} \lambda_1^j \\
\delta B_2^i &= d\lambda_1^i \\
\delta C_3 &= -\frac{1}{2} \epsilon_{ij} \lambda_1^i \wedge H_2^j
\end{align*}
\] (2.5)

This Stuckelberg symmetry is a shift symmetry for \(B_1\) and, when \(M^{ij}\) is invertible, can be used to set \(B_1^i = 0\), so that the two 1-forms \(B_1^i\) are eaten by the 2-forms \(B_2^i\), which become massive. For the parabolic case in which \(M\) is not invertible,
one of the $B_1^i$ is eaten by one of the $B_2^j$, which becomes massive, so the physical spectrum has one massive 2-form, a massless 2-form and a massless 1-form gauge field. The action includes the terms

$$\int G_{ij}H_2^i \wedge *H_2^j + g_{ij}H_3^i \wedge *H_3^j$$

with scalar-dependent matrices $g_{ij}(\phi), G_{ij}(\phi)$ given explicitly in [9,10]. For the case in which $M$ is invertible, this becomes

$$\int \tilde{G}_{ij}B_2^i \wedge *B_2^j + g_{ij}DB_2^i \wedge *DB_2^j$$

in the gauge $B_1^i = 0$, where

$$\tilde{G}_{ij} = G_{kl}M_{i}^{k}M_{j}^{l}$$

and

$$DB_2^i = dB_2^i + M^i_\ jB_2^j \wedge A_1$$

is a covariant derivative invariant under the gauge transformation

$$\delta A_1 = d\alpha, \quad \delta B_2^i = -\alpha M^i_\ jB_2^j$$

The first term in (2.7) is a mass term for the $B_2$ field, and (2.10) indicates that the symmetry with parameter $\alpha(x)$ is the 1-dimensional subgroup of $SL(2, \mathbb{R})$ generated by $M$, which has been gauged, with gauge field $A_1$; this is confirmed by checking the other sectors of the theory. The gauged subgroup in the elliptic case is the compact rotation group $SO(2)$ of matrices generated by $M_e$, while in the hyperbolic case it is the non-compact group $SO(1,1)$ generated by $M_h$. The parabolic case is similar, and is the gauging of the of the non-compact group $SO(1,1)$ generated by $M_p$. The parameter $\alpha$ or $\theta$ is then the gauge coupling constant. In [16], it was conjectured that the hyperbolic case corresponds to an $SO(1,1)$ gauging.
The group manifold $SL(2, \mathbb{R})$ with the Cartan-Killing metric is a Lorentzian space with signature $(+, +, -)$. The three distinct theories arise from gauging the 1-dimensional subgroup of $SL(2, \mathbb{R})$ generated by a Lie algebra element that is timelike (the elliptic case), spacelike (the hyperbolic case) or null (the parabolic case).

The elliptic $SO(2)$ gauging was considered in detail in [10]. The other two cases can be obtained from this using the analytic continuation techniques of [17]. Consider starting from the $SO(2)$ gauging with

$$M = g \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$ \hfill (2.11)

where $g = m_3$. Consider acting on this theory with the $SL(2, \mathbb{R})$ transformation

$$k(t) = \begin{pmatrix} e^t & 0 \\ 0 & e^{-t} \end{pmatrix}$$ \hfill (2.12)

for some parameter $t$. This will take the theory to a similar theory, but with $M$ replaced by $M' = k(-t) Mk(t)$, so that a conjugate $SO(2)$ subgroup has been gauged. This is of course equivalent to the original $SO(2)$ gauging, via a field redefinition. Next rescale the coupling constant $g \to ge^{-2t}$, so that $M$ is now replaced with

$$M(t) = e^{-2t} k(-t) Mk(t) = g \begin{pmatrix} 0 & 1 \\ -\xi & 0 \end{pmatrix}$$ \hfill (2.13)

where

$$\xi = e^{-2t}$$ \hfill (2.14)

throughout the action and supersymmetry transformations. For all finite real $t$, this gives an $SO(2)$ gauging equivalent to the original one. However, taking the limit $t \to \infty$ gives a well-defined theory, but with $\xi = 0$ in (2.13), so that the
generator is now the parabolic generator $M_p$ in (2.3) with $a = g$. Similarly, continuing to $\xi = -1, t = i\pi/2$, $M(t)$ becomes the generator of a hyperbolic subgroup, congruent to $M_h$ in (2.3) with $a = g$. By the arguments of [17], these limits of the original $t$-dependent theory are guaranteed to give supersymmetric gauged supergravity theories, giving an independent check that the generalised Scherk-Schwarz reduction gives a locally supersymmetric theory.

The mass matrix $M$ (2.1) corresponds to a vector $v = (m_1, m_2, m_3)$ that transforms as a vector under $SO(2,1) \sim SL(2, \mathbb{R})$, and has norm $m_1^2 + m_2^2 - m_3^2$ with respect to the Cartan-Killing metric. The $SO(2)$ gauging has $v = (0,0,m_3)$ and acting with $k(t)$ corresponds to a boost with rapidity $t$ taking $v$ to another time-like vector of the same norm. The limit $t \to \infty$ is an infinite boost taking $v$ to a null vector proportional to $(0,1,1)$ and requires a rescaling of the components of the vector. Continuing to $t = i\pi/2$, the boost becomes a ‘rotation’ taking $v$ to a spacelike vector $(0, m, 0)$.

3. Scherk-Schwarz Reduction of IIB Superstring on $S^1$.

In the quantum IIB theory, the quantization of string and 5-brane charges breaks the classical $SL(2, \mathbb{R})$ invariance to the discrete $SL(2, \mathbb{Z})$ U-duality symmetry [15]. The quantum-consistent Scherk-Schwarz reductions of this theory to 9 dimensions are those for which the monodromy is in $SL(2, \mathbb{Z})$. For given string and 5-brane charge lattices, acting with an $SL(2, \mathbb{Z})$ transformation $k$ will preserve the lattices but change the monodromy $\mathcal{M} \to k\mathcal{M}k^{-1}$. This will take the theory to a physically equivalent one, so that the distinct theories are represented by monodromies in the distinct $SL(2, \mathbb{Z})$ conjugacy classes [8]. The $SL(2, \mathbb{Z})$ conjugacy classes have been discussed in [18,19]. There is the trivial class $\mathcal{M} = 1$, together with $\mathcal{M} = -1$. For any conjugacy class $\mathcal{M}$, $-\mathcal{M}$ and $\pm \mathcal{M}^{-1}$ also represent conjugacy classes, so for each $\mathcal{M}$ in the following list, there are also conjugacy classes $-\mathcal{M}$ and $\pm \mathcal{M}^{-1}$. There are an infinite number of parabolic $SL(2, \mathbb{Z})$ conjugacy
classes with \( Tr(\mathcal{M}) = 2 \), represented by \( T^n \):

\[
\mathcal{M}_{I_n} = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}
\]  \hspace{1cm} (3.1)

for integer \( n \). There are three elliptic \( SL(2, \mathbb{Z}) \) conjugacy classes with \( Tr(\mathcal{M}) < 2 \), represented by

\[
\mathcal{M}_{II} = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathcal{M}_{III} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathcal{M}_{IV} = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}
\]  \hspace{1cm} (3.2)

There are an infinite number of hyperbolic \( SL(2, \mathbb{Z}) \) conjugacy classes with \( Tr(\mathcal{M}) > 2 \), represented by

\[
\mathcal{M}_{H_n} = \begin{pmatrix} n & 1 \\ -1 & 0 \end{pmatrix},
\]  \hspace{1cm} (3.3)

for integers \( n \) with \( |n| \geq 3 \), together with sporadic monodromies \( \mathcal{M}_t \) of trace \( t \).

\[
\mathcal{M}_8 = \begin{pmatrix} 1 & 2 \\ 3 & 7 \end{pmatrix}, \quad \mathcal{M}_{10} = \begin{pmatrix} 1 & 4 \\ 2 & 9 \end{pmatrix}, \quad \mathcal{M}_{12} = \begin{pmatrix} 1 & 2 \\ 5 & 11 \end{pmatrix}, \quad \mathcal{M}_{13} = \begin{pmatrix} 2 & 3 \\ 7 & 11 \end{pmatrix}, \quad \mathcal{M}_{14} = \begin{pmatrix} 1 & 2 \\ 6 & 13 \end{pmatrix}, \ldots
\]  \hspace{1cm} (3.4)

where this is the complete list of sporadic classes for \( 3 \leq t \leq 15 \) [19].

For any \( \mathcal{M} \), the monodromies \( \mathcal{M} \) and \( \mathcal{M}^{-1} \) define physically equivalent theories, related by changing the mass parameter \( m \to -m \). The \( SL(2, \mathbb{Z}) \) element \(-I\) acts trivially on the scalars, and reverses the sign of the 2-form potentials. The relation between the theory obtained by a Scherk-Schwarz reduction with monodromy \( \mathcal{M} \) and that with monodromy \(-\mathcal{M}\) will be discussed in [12].

Then the physically distinct theories obtained by Scherk-Schwarz reduction of the IIB string theory are those with monodromies (3.1),(3.3), consisting of two
infinite series, each labelled by an integer $n$, the three exceptional cases with monodromies (3.2), and the hyperbolic monodromies (3.4). (The ones corresponding to (3.1),(3.2),(3.3) are also discussed in [16].) These will now be compared to the three classes of classical supergravity theories obtained with monodromies (2.2). The theories given by reduction with monodromy $\mathcal{M}_I$, correspond to gauging the parabolic subgroup of $SL(2,\mathbb{R})$ generated by $M_p(a)$ in (2.3). This reduction was first performed in [2], and was shown to give the same 9-dimensional supergravity as the conventional (untwisted) reduction of the massive IIA supergravity theory of Romans to 9 dimensions, with mass parameter $m = a$. In the quantum theory, this mass is quantized [2], $a = n$ for some integer $n$, as was seen from a different point of view in [20]. The monodromies $\mathcal{M}_{H_n}$, $|n| \geq 3$ and $\mathcal{M}_t$ given in (3.4) are conjugate to the hyperbolic gauging with monodromy $\mathcal{M}_h(a)$ and will again give a quantum-consistent theory.

The elliptic monodromy $\mathcal{M}_e(\theta)$ gives the gauging of the compact $SO(2)$ subgroup of $SL(2,\mathbb{R})$, giving the $SO(2)$-gauged supergravity discussed in [10] with mass parameter $m = \theta$. In the quantum theory, the monodromies $\mathcal{M}_{II}, \mathcal{M}_{III}, \mathcal{M}_{IV}$ arise from $SO(2)$ gaugings at special values of the angle $\theta$. The monodromy $\mathcal{M}_{III}$ is clearly a rotation through $\pi/2$,

$$\mathcal{M}_{III} = \mathcal{M}_e(\pi/2)$$

while the other two are $SL(2,\mathbb{R})$ conjugate to rotations through $\pi/3, 2\pi/3$, i.e. there are matrices $U, V$ in $SL(2,\mathbb{R})$ such that

$$\mathcal{M}_{II} = U\mathcal{M}_e(\pi/3)U^{-1}, \quad \mathcal{M}_{IV} = V\mathcal{M}_e(2\pi/3)V^{-1}$$

Thus in the Scherk-Schwarz reduction with elliptic monodromy giving an $SO(2)$ gauging, quantum consistency forces the ‘mass parameter’ $\theta$ to take the discrete values $n\pi/2$ or $n\pi/3$ for integer $n$, giving just three non-trivial physically distinct cases, $\theta = \pi/2, \pi/3, 2\pi/3$. 

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Thus Scherk-Schwarz dimensional reduction leads to a quantization of the mass parameters in the 9-dimensional gauged supergravity theories. The Scherk-Schwarz reduction of IIB string theory on a circle with monodromy $\mathcal{M} \in SL(2,\mathbb{Z})$ can be viewed as F-theory reduced on a $T^2$ bundle over a circle with monodromy $\mathcal{M}$ [8], and monodromies in the same $SL(2,\mathbb{Z})$ conjugacy class give equivalent bundles, so the quantization condition has a topological origin. The M-theory dual of these reductions was given in [8].

BPS solutions of these 9-dimensional theories have been considered in [2,10]. The parabolic gauged theory has an exponential potential, so that the only critical point is for infinite values of the scalars. It has no maximally supersymmetric vacuum, but has half-supersymmetric domain wall solutions in which the wall separates regions with different values of the quantized mass $m$ [2]. The theories on either side of the wall, with masses $m, m'$, are related by an $SL(2,\mathbb{Z})$ transformation $\mathcal{M}_n$ with parameter $n = m - m'$. The $SO(2)$ gauged theory has a potential with a minimum at which the potential vanishes. It thus has a Minkowski vacuum [10], which breaks all supersymmetries [11]. It also has BPS domain wall solutions that separate regions with mass parameters $\theta = \pm \theta_0$ [10]. For $\theta_0 = \pi/2$, the theory with mass parameter $-\pi/2$ is obtained from that with mass parameter $\pi/2$ by acting with the $SL(2,\mathbb{Z})$ transformation $-\mathbf{1}$, which acts by changing the sign of the 2-form gauge fields. The theory with $\theta = -\pi/3$ is obtained from the one with $\theta = 2\pi/3$ by acting with the $SL(2,\mathbb{Z})$ transformation $-\mathbf{1}$. The theories with mass parameters $\theta = \pm \pi/3$ are related by an $SL(2,\mathbb{Z})$ transformation $\mathcal{M}_{IV}$, conjugate to a rotation through $2\pi/3$. General BPS domain walls of all three classes of supergravity theories with monodromies (2.2) will be considered in [16], together with further properties of these theories, and in particular the structure of the scalar potentials.
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