Design of Principal-agent Incentive Mechanism between Government and NPO
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Abstract: Based on principal-agent theory, this paper establishes an incentive contract mechanism between government and NPO under asymmetric information, and analyzes the impact of absolute risk aversion and output level on the expected utility of government, NPO and society. Research shows that risk aversion is negatively correlated with the expected utility of government, NPO and society. The output coefficient is positively correlated with the expected utility of government, NPO and society. Reducing absolute risk aversion, increasing output coefficient and increasing government incentives can effectively motivate NPO to actively participate in social rescue activities.

Key words: Principal-agent; Incentive intensity; Level of output; Expected utility

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1 Introduction
Non-profit organizations (NPO) are the "third sector" between the government and enterprises, aiming at promoting social welfare [2] and play an important role in making up for the infringement of public interests by the government or the market [3]. From "Guo Meimei Event", "Shangde False Donation Event" to "Wuhan Red Cross Event", a series of events have exposed the lack of supervision of China's NPO [4]. This not only affects the NPOs' own sustainable development, but also brings serious social impact.

Most scholars have studied the relationship between government and non-profit organizations based on principal-agent theory. Shu Hongjuan and Ning Xuanxi pointed out that there is a high degree of information asymmetry between public research and development institutions and the government, which makes the principal-agent cost very high, but the model results have not been verified by examples [5]. In order to further study the relationship between government and non-profit organizations, Wang Zhaozhong, Chen Xiqiang and others, represented by community organizations, made a quantitative analysis of the principal-agent relationship between government and community organizations from the perspective of input and output, but did not use examples to verify the analysis results [6]. Xue Yaoxuan used the game theory model to analyze the impact of institutional constraints on the performance of non-governmental environmental protection organizations under the condition of multiple principals. He did not use examples to analyze and verify the reliability of the model, nor did he propose solutions to the above problems from the perspective of the model, nor did he put forward clear and standardized research conclusions and recommendations [7].

Based on the above analysis, this paper studies the relationship between the government and NPO under the information asymmetry. Through the principal-agent model, it analyzes the impact of risk preference and output coefficient on the expected utility of the government and NPO, and puts forward targeted incentive strategies, which will contribute to improving the work efficiency of NPO and promoting NPO to actively participate in social emergency rescue activities.

2 Problem description and assumptions
Government departments entrust NPO to participate
and meet the needs of special groups and the allocation of resources. In order to maximize the interests of both parties, they form a principal-agent relationship and participate in the allocation of social resources. The principal-agent relationship is characterized by:

1. Information asymmetry exists between government and non-profit organizations. In the game between government and non-profit organizations, NPO holds specific information about emergency relief supplies, and the government can only judge the degree of efforts of non-profit organizations based on the completion of the tasks of non-profit organizations.

2. There is moral hazard between government and non-profit organizations. In the case of asymmetric information, the government can only observe the completion of the tasks of non-profit organizations. NPO may use its information advantages to hide its actual efforts, which may easily lead to moral hazard in the regulatory blind area.

In order to facilitate the research, the following assumptions are made without affecting the accuracy of the conclusions:

Assumption 1: A is a set of all possible behaviors of an agent, and \( a \in A \) represents a specific action. After the agent selects the action "a", the exogenous variable \( \mu \) is implemented, and thus the output function of NPO can be expressed as \( \pi = Ka + \mu \). \( K \) represents the output level. \( \mu \) is a random variable with an uncertain output, and \( \mu \sim N(0, \sigma^2) \).

Assumption 2: The government will allocate appropriate funds as compensation for NPO and record them as \( \alpha \) to motivate the work behavior of the agents. At the same time, some scholars pointed out that the income of the agent is the cost of the principal [8].

Hypothesis 3: The government and the NPO need to pay corresponding costs while making efforts. The cost function is in the form of \( C(a) = \frac{1}{2} \theta a^2 \), \( \theta(\theta > 0) \) represents the cost coefficient of effort, the higher the agent's effort level, the higher the cost.

3 Model building

The actual income of NPO:

\[
W = S(\pi) - C(a) = \alpha + Ka + \frac{1}{2} \theta a^2
\]  

The expected income of NPO:

\[
E(S(\pi) - C(a)) = \alpha + Ka + \frac{1}{2} \theta a^2
\]

According to the conclusion of document [9], \( Var(W) = \beta \sigma^2 \), the risk cost of the agent is:

\[
B = \frac{1}{2} \rho Var(W) = \frac{1}{2} \rho \beta^2 \sigma^2
\]

The definite income of NPO is:

\[
E_N = E(S(\pi) - C(a)) - B = \alpha + Ka + \frac{1}{2} \theta a^2 - \frac{1}{2} \rho \beta^2 \sigma^2
\]

The actual revenue of the government is:

\[
\pi - S(\pi) = (1 - \beta)(Ka + \mu) - \alpha
\]

The utility expected by the government is that the greater the social utility, the better, so its utility expression is:

\[
E_u = m(Ka) = \frac{\theta a^2}{2} - \frac{1}{2} \rho \beta^2 \sigma^2
\]

\[
E_s = m(Ka)^n
\] represents the social expected utility, and there is \( m>0, 0<n<1 \).

Therefore, the establishment of the model requires that the following two constraints be satisfied simultaneously:

1. **Participation constraint**: the expected utility obtained by the NPO after accepting the government entrustment is not less than the maximum expected utility obtained before accepting the entrustment;

2. **Incentive Compatible Constraint**: given that the government cannot observe the actions \( a \) and \( \mu \) of the NPO, no matter what incentive method is adopted, the NPO always tends to choose the action \( A \) that maximizes its expected utility. Therefore, action \( a \) expected by the government can only be achieved by maximizing the expected utility of NPO.

To sum up, the principal-agent model of government and NPO is:

\[
\begin{align*}
\text{Max} & \quad m(Ka)^n - \frac{1}{2} \theta a^2 - \frac{1}{2} \rho \beta^2 \sigma^2 \\
\text{s.t.} & \quad \alpha + Ka + \frac{1}{2} \theta a^2 - \frac{1}{2} \rho \beta^2 \sigma^2 \geq 0
\end{align*}
\]

\((\text{IR})\)

\((\text{IC})\) \( \text{argmax} \{ \alpha + Ka + \frac{1}{2} \theta a^2 - \frac{1}{2} \rho \beta^2 \sigma^2 \} \)

The \((\text{IC})\) condition can be replaced by the equivalent first derivative equal to zero [10].
Therefore, the principal-agent model is organized as follows:

\[
\text{Max } m(Ka)^n - \frac{1}{2} \theta a^2 - \frac{1}{2} \rho \beta^2 \sigma^2
\]

s.t.

\[
(\text{IR}) \quad \alpha + K \beta - \frac{1}{2} \theta a^2 - \frac{1}{2} \rho \beta^2 \sigma^2 \geq 0
\]

\[
(\text{IC}) \quad a = \frac{\beta K}{\theta}
\]

Substituting (7) into (6) to obtain the expected utility expression of the government is as follows:

\[
EU_g = m \left( \frac{K^2 \beta^2}{\theta} + \mu \right)^n - \frac{K^2 \beta^2}{20} - \frac{1}{2} \rho \beta^2 \sigma^2
\]

\[
= m \left( \frac{K^2 \beta^2}{\theta} \right)^n - \frac{K^2 \beta^2}{20} - \frac{1}{2} \rho \beta^2 \sigma^2
\]  

(8)

Based on the first-order partial derivative of (8) to \( \beta \), the optimal value of government incentive intensity can be obtained:

\[
\beta^* = \left[ \frac{K^2 + \rho \sigma^2}{\theta} \right]^{\frac{1}{n-2}}
\]

(9)

Simultaneously:

\[
a^* = \frac{K^2}{\theta} \left[ \frac{K^2 + \rho \sigma^2}{mn} \right]^{\frac{1}{n-2}}
\]

(10)

Based on \( \beta^* \) and \( A^* \), the most expected values of government and NPO are obtained as follows:

\[
E_g = \left( \frac{K^2}{\theta} \right) \frac{2n}{n-2} (mn)^{\frac{1}{n-2}} \left( \frac{K^2 + \rho \sigma^2}{\theta} \right)^{\frac{n}{n-2}} \left( \frac{1}{n} - \frac{1}{2} \right)
\]

(11)

\[
E_N = \alpha \frac{1}{2} \left( \frac{K^2}{\theta} \right)^{\frac{n}{n-2}} \left( \frac{\theta}{mn} \right)^{\frac{1}{n-2}} \left( \frac{K^2}{\theta} - \rho \sigma^2 \right)
\]

(12)

4 Case analysis

Assumed that the initial values of relevant parameters: \( m = 10000, \theta = 0.5, a = 10, \theta = 5000, k = 10000, \sigma^2 = 10000, \rho = 0 \). When the NPO tend to be risk-neutral, the relationship among the government's incentive degree (\( \beta \)), the NPO's own effort degree (\( a \)), the government's expected utility and the NPO's expected utility is shown in the following table.

| \( \rho \) | \( \beta \) | \( a \) | \( E_g \) | \( E_N \) |
|---|---|---|---|---|
| 0.11 | 0.61 | 20.78 | 3.42 \times 10^6 | 1.02 \times 10^6 |

4.1 The impact of different risk aversions

The changes of \( \beta, a, E_g \) and \( E_N \) are analyzed when the risk aversion value \( \rho \) changes and other conditions remain unchanged.

![Figure 1](image1.png)

**Figure 1.** The relationship between \( \rho \) and \( \beta \) and \( a \)

![Figure 2](image2.png)

**Figure 2.** The relationship between \( \rho \) and \( E_g \) and \( E_N \)

From fig. 1, when \(-1<\rho<1\), there exists a negative correlation between \( \rho \) and \( \beta \) and \( a \). When \( \rho \) tends to 1, the non-profit organization is risk-biased, indicating that it will bear more risks in its activities. This will encourage the government to increase the incentive intensity to third-party organizations. With
the effective support of the government, non-profit organizations will also continuously improve their efforts. On the contrary, when \( \rho \) tends to -1, non-profit organizations become risk evaders. In order to reduce the impact of external risks on the organization's operation, non-profit organizations undertake low-risk tasks based on actual conditions in emergency rescue activities. Due to this limitation, the government's incentive intensity will gradually decrease, and therefore, the efforts of non-profit organizations will gradually decrease in a stable operating environment.

As can be seen from fig. 2, there is a negative correlation between \( \rho \) and \( E_G \) and \( E_N \). When NPO tend to have complete risk preference, the expected utility of government is the same as that of NPO. When NPO tend to avoid risks, the expected utility of government is higher than that of NPO. In contrast, NPO are more sensitive to risks than the government, so the decline rate of expected utility is faster than the decline rate of government expected utility.

### 4.2 Impact of different output levels

The changes of \( \beta \), \( a \), \( E_G \) and \( E_N \) are analyzed when the output level \( K \) changes and other conditions remain unchanged.

| \( K \) | \( \beta \) | \( a \) | \( E_G \) | \( E_N \) |
|---|---|---|---|---|
| 1000 | 1 | 10 | \( 7.5 \times 10^5 \) | \( 2.5 \times 10^5 \) |
| 5500 | 0.32 | 17.65 | \( 2.34 \times 10^6 \) | \( 7.79 \times 10^5 \) |
| 10000 | 0.22 | 21.54 | \( 3.48 \times 10^6 \) | \( 1.16 \times 10^6 \) |

The specific relationship between each parameter is shown in figs. 3 to 4.

As can be seen from Figure 3, on the one hand, the output level of NPO has a positive correlation with their own efforts and a negative correlation with the government incentives. On the other hand, the image shows that the government incentives are more significantly affected than the efforts of NPO.

As Figure 4, when the output level is in the range of 0 to 10000, the expected utility of the government is higher than that of NPO. The output of non-profit organizations has a positive impact on both principals and agents. For non-profit organizations, an appropriate output means that the cost of the organization's operation is within a reasonable range. The government's incentives make the organization optimistic about the expected performance. Therefore, it is beneficial to reduce the occurrence of moral hazard.

### 4.3 Factors affecting social utility

From (6) and (7), the expression of social utility is:

\[
E_s = \phi \left( \frac{K^2 + \beta}{0} \right) + \frac{K^2}{\sigma^2} + \frac{\rho \sigma^2}{\sigma^2} \frac{\sqrt{2 \pi}}{\rho^2}
\]

The changes of \( \beta \), \( a \), \( E_G \) and \( E_N \) are analyzed when the output level \( K \) changes and other conditions remain unchanged. The relationship among output level \( k \), absolute risk aversion \( \rho \) and \( (E_s) \) is analyzed with other conditions unchanged, as shown in fig. 5.
Figure 5. The relationship between \( E_s \) and \( K \) and \( \rho \)

As can be seen from Figure 5, the higher the output level of NPO, the higher the service level, and therefore the higher the social expected utility; The greater the absolute risk aversion of NPO, the smaller the risk tolerance, and therefore the smaller the social expected utility, which is consistent with the results of simple mathematical model analysis. At the same time, according to the different degree of curvature of the curved surface in the figure, we can see that the output level of NPO has a greater impact on the social expected utility.

5 Conclusion

This paper establishes a model to analyze the risk aversion (\( \rho \)) of NPO and the output level (K) of NPO, which are the common influencing factors of the government's expected utility, the NPO's expected utility and the society's expected utility, puts forward some measures and suggestions to motivate the principal-agent relationship between government and NPO.

(1) Reduce the absolute risk aversion of non-profit organizations. Due to the characteristics of non-profit, non-profit organizations have poor ability to resist risks. When the government gives a certain amount of funds and policy support, the non-profit organizations' capital strength is strengthened and their ability to resist risks is significantly improved. Therefore, reducing the degree of risk aversion will not only increase the efforts of non-profit organizations, but also increase the expected revenue of the government and non-profit organizations.

(2) Improve the output level of NPO. The output level of NPO is directly related to their service capabilities. Therefore, the government can choose NPO with strong capabilities in all aspects to cooperate and give support, which will help to improve the expected social utility and the expected government utility.

(3) Appropriately increase the government's incentives. Within a reasonable range, through tax, talent and other incentives, the government can increase the intensity of incentives for NPO, which can not only motivate NPO to work hard and obtain more support and higher social influence, but also strengthen cooperation with the government, so as to better carry out social emergency rescue activities and achieve a win-win situation.

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