Cooling nanorotors by elliptic coherent scattering

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Simultaneously cooling the rotational and translational motion of nanoscale dielectrics into the deep quantum regime is an open task of great importance for sensing applications and quantum superposition tests. Here, we show that the six-dimensional ground-state can be reached by coherent-scattering cooling with an elliptically polarized and shaped optical tweezer. We determine the cooling rates and steady-state occupations in a realistic setup, and discuss applications for mechanical sensing and fundamental experiments.

Introduction—Optically levitating nanoparticles in ultra-high vacuum yields an unprecedented degree of environmental isolation [1], rendering these systems ideally suited for precision sensing [2–5] and for mesoscopic quantum superposition tests [6–13]. For most applications it is crucial to cool the rotational and translational particle dynamics into the deep quantum regime. Recently, center-of-mass ground-state cooling has been achieved [14] using the method of coherent-scattering cooling [15, 16]. In this work, we show that elliptically polarized and shaped tweezers enable cooling of nanoparticles into their simultaneous rotational and translational groundstate.

The setup of coherent-scattering cooling consists of an optical tweezer levitating a nanoparticle inside a high-finesse cavity. If the tweezer is slightly red detuned from the cavity resonance, the particle motion loses energy by scattering tweezer photons into the cavity mode [17, 18]. In contrast to conventional optomechanical setups [19–26], the cavity mode is nearly empty in the system’s steady state, leading to a significant reduction of laser phase noise [15] and holding the prospect of reaching the strong coupling regime [27].

What distinguishes levitated nanoparticles from other optomechanical systems is their ability to rotate. Desired or not, rotations of any levitated object must therefore be controlled to fully exhaust its technological potential. The classical rotation dynamics of nanoscale objects can be manipulated with linearly and circularly polarised lasers [28–32], enabling high-precision pressure [33] and torque [34] sensing. In the quantum regime, the non-harmonicity of the rotational spectrum gives rise to pronounced interference effects [11, 34].

Recent experiments demonstrate rotational cooling [35, 36] by aligning particles in a space-fixed direction and damping their librations around this axis. While such schemes allow to reach the quantum regime with thin rod-shaped objects [25, 37], they cease to be efficient once the particle shape significantly deviates from this idealisation. Specifically, rotations around the axis of maximal susceptibility experience only weak cooling but strongly influence the librational dynamics [36].

In the present work, we demonstrate that this obstacle can be overcome by coherent-scattering cooling with an elliptically polarized and shaped tweezer, see Fig. 1. The elliptical polarization of the light field introduces two space-fixed axes to control the rotations [38], rather than a single polarization axis. When placed inside a cavity, the nanoparticle rotations couple to two orthogonally polarized cavity modes, which in turn cool different mechanical degrees of freedom. For suitably chosen optical parameters, the resulting coupling cools sub-wavelength aspherical objects into their joint 6D ro-translational quantum ground state, even if they are close to spherical. We calculate the cooling rates and steady-state occupations for experimentally realistic situations, show that the expected torque sensitivities surpass state-of-the-art estimates by several orders of magnitude, and discuss how this setup can be used to generate ultra-fast spinning, ultra-cold nanoparticles.

Nanoparticle-light interaction—We consider an ellipsoidal particle of mass \( m \) with three different principal-axis diameters \( \ell_a < \ell_b < \ell_c \) and associated moments of inertia \( I_a = m(\ell_b^2 + \ell_c^2)/20 \), \( I_b = m(\ell_a^2 + \ell_c^2)/20 \), and \( I_c = m\ell_a^2/20 \). The induced polarization field depends on the particle center-of-mass position \( \mathbf{r} \) and orientation \( \mathbf{\Omega} \), driving two cavity modes (blue lines) with orthogonal polarizations \( \mathbf{e}_1, \mathbf{e}_2 \).

The resulting coupled nanoparticle-cavity dynamics can cool the nanoparticle motion into the 6D ro-translational quantum groundstate.
\[ I_c = m(\ell^2_1 + \ell^2_2)/20. \] For sub-wavelength particles, the internal field is described by a linear susceptibility tensor \( \chi(\Omega) = R(\Omega)\chi_0 R^T(\Omega) \) (Rayleigh-Gans approximation), where \( R(\Omega) \) rotates from the space-fixed frame to the principal-axes frame and \( \Omega = (\alpha, \beta, \gamma) \) denotes the Euler angles in the \( z'y'z'' \)-convention. The susceptibility tensor \( \chi_0 = \text{diag}(\chi_a, \chi_b, \chi_c) \) contains the susceptibilities along the ellipsoid’s principal axes. They are given by \( \chi_i = (\varepsilon - 1)/[1 + (\varepsilon - 1)N_i] \), with the relative permittivity \( \varepsilon \) and the depolarization factors \( N_i \geq 0, \sum_i N_i = 1 \). The latter can be calculated in terms of elliptic integrals \([39]\), yielding \( \chi_a < \chi_b < \chi_c \).

For a given laser field \( E(r)e^{-i\omega t} \), the time-averaged force and torque acting on the particle are dominated by the conservative optical potential

\[ V_{\text{opt}}(r, \Omega) = -\frac{\varepsilon_0 V}{4} E^*(r) \cdot \chi(\Omega) E(r), \tag{1} \]

to first order of the particle volume \( V = \pi \ell_a \ell_b \ell_c/6 \) \([25]\). Thus, if the electric field is linearly polarized, the optical potential tends to align the particle axis of maximal susceptibility with the field polarization.

In addition to the conservative potential \([1]\), the laser also exerts a radiation pressure force and torque. The latter follow from the electric-field integral equation as (see App. [A])

\[ \mathbf{F}_{\text{rad}} = \frac{\varepsilon_0 k^2 V^2}{12\pi} \text{Im} \left[ (\mathbf{\chi}^*) \cdot [\nabla \otimes (\mathbf{\chi})]^T \right], \tag{2} \]

and

\[ \mathbf{N}_{\text{rad}} = \frac{\varepsilon_0 k^2 V^2}{12\pi} \text{Im} \left[ (\mathbf{\chi}^2 \mathbf{E}^*) \times \mathbf{E} - (\mathbf{\chi} \mathbf{E}^*) \times (\mathbf{\chi} \mathbf{E}) \right], \tag{3} \]

where we omitted the dependence on \( r \) and \( \Omega \). Force and torque are proportional to the cubed wavenumber \( k \) and to the squared particle volume, and are consistent with Ref. \([23]\) for symmetric objects. The torque \([3]\) has been experimentally observed to accelerate nanorotors up to GHz frequencies \([29, 31, 33]\); it vanishes for linear field polarisation.

**Cavity dynamics** — If the particle is trapped inside a high-finesse optical cavity, the mechanical motion can couple strongly to two near-degenerate cavity modes, see Fig. 1 This interaction is determined by the total field,

\[ \mathbf{E}(r) = \sqrt{\frac{2\hbar}{\varepsilon_0 V_c}} \left[ \mathbf{e}_1 f_1(r) + \sum_{j=1,2} b_j \mathbf{e}_j f_c(r) \right], \tag{4} \]

where \( \omega \) denotes the tweezer frequency, \( V_c \) is the cavity mode volume, \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \) are the dimensionless tweezer and cavity amplitudes, \( \mathbf{e}_1 \) and \( \mathbf{e}_2 \) the corresponding polarization vectors, and \( f_1(r) \) and \( f_c(r) \) the mode functions.

The cavity resonance frequency \( \omega_c \) is detuned from the tweezer frequency by \( \Delta = \omega - \omega_c \). Near the cavity axis, the mode function can be approximated as a standing wave \( f_c(r) = \cos[k(\mathbf{e}_2 \times \mathbf{e}_1) \cdot \mathbf{r} + \phi] \), where \( \phi \) describes the relative positioning of cavity and tweezer. The Gaussian envelope of the cavity modes determines the mode volume \( V_c = \pi L w_c^2/4 \) with cavity waist \( w_c \) and length \( L \), but can be neglected for the dynamics.

The tweezer quadrature \( \epsilon \) is determined by the tweezer power and will be chosen real and positive. The tweezer mode function can be approximated by a traversing Gaussian beam with an elliptic intensity profile, \( f_1(r) = \exp[-(x^2/w_0^2 + y^2/w_0^2)/r^2(z)]e^{i[kz+\phi_1(z)/r(z)]} \) with the beam waists \( w_x, w_y \). The latter determine the Rayleigh range \( z_R = kw_x w_y/2 \), in turn sets the broadening factor \( r(z) = \sqrt{1+z^2/z_R^2} \). The Gouy phase is given by \( \phi_1(z) = \arctan(z/z_R) - kz(z^2 + y^2)/2(z^2 + z_R^2) \) \([13]\).

We assume the tweezer propagation direction \( \mathbf{e}_z \) to be orthogonal to the cavity axis, with \( \theta \) the angle between the latter and the \( y \)-axis, see Fig. 1. The cavity mode polarizations can be chosen as \( \mathbf{e}_1 = \cos \theta \mathbf{e}_x - \sin \theta \mathbf{e}_y \) and \( \mathbf{e}_2 = \mathbf{e}_z \). In order to achieve trapping in all three orientational degrees of freedom, we choose the tweezer to be elliptically polarized, \( \mathbf{e}_1 = \cos \psi \mathbf{e}_x + i \sin \psi \mathbf{e}_y \) with \( \psi \in (0, \pi/4) \).

The Hamiltonian of the combined nanoparticle-cavity dynamics takes the form

\[ H = H_0 + V_{\text{opt}} - \hbar \Delta (b_1^* b_1 + b_2^* b_2), \tag{5} \]

where \( H_0 \) denotes the free ro-translational particle Hamiltonian \([10]\). In principle, the resulting equations of motion must be augmented with the non-conservative radiation pressure force \([2] \) and torque \([3] \), and by the corresponding Rayleigh scattering contribution to the cavity field \([23]\). However, since the main effect of these terms is found to only shift slightly the steady-state particle configuration and cavity population we neglect them for now, see App. [A].

Thus, the equations for motion for the mechanical coordinates \( q \in \{r, \Omega\} \) follow from \([5]\) as \( \dot{q} = \partial_{p_q} H \), where \( p_q \in \{p, p_1\} \) are the canonical momenta. Taking the finite cavity line width \( k \) into account, the dynamics of the cavity modes \( b = (b_1, b_2) \) are given by \( \dot{b} = A(r, \Omega)b + \eta(r, \Omega) \), with the \( 2 \times 2 \)-matrix \( A(r, \Omega) = i \Delta_{\text{eff}}(r, \Omega) - \kappa \), the vector \( \eta(r, \Omega) \) is \( -iU_0 f_c(r) f_1(r) \mathbf{e}_j \cdot \chi \Omega \mathbf{e}_i \) and the coupling frequency \( U_0 = -\omega V/2V_c \). The matrix \( \Delta_{\text{eff}} \) is the effective detuning for a given particle position and orientation,

\[ \Delta_{\text{eff}}(r) = \Delta_{\text{eff}} - U_0 f_c^2 \mathbf{e}_j \cdot \chi \mathbf{e}_j^*. \tag{6} \]

It is real and symmetric, implying that it can be diagonalized with real eigenvectors and eigenvalues. The matrix \( A \) shares its eigenvectors with \( \Delta_{\text{eff}} \), so that its inverse \( A^{-1} \) exists for \( \kappa > 0 \).

**Adiabatic cavity cooling** — The above equations of motion describe how the cavity and the mechanical degrees of freedom exchange energy. To see how the detuning determines whether the mechanical degrees of freedom are heated up or cooled down \([19]\), we now calculate the local phase space contraction rate \( \Gamma_{\text{c}}(r, \Omega) \). For this sake,
we adiabatically eliminate the cavity degrees of freedom. In the bad cavity regime, the amplitude changes quickly as compared to the particle motion. We can thus write \( b = b_0 + \Delta b \), where \( b_0 = -A^{-1}\eta \) is the stationary cavity amplitude for a fixed particle position and orientation, and \( \Delta b \approx A^{-1} \sum q \partial q_b \) is a small stationary correction depending linearly on the momentum coordinates.

Inserting this expansion into the mechanical equations of motion yields to lowest order of the velocities the bad-cavity regime damping term. The associated phase-space contraction rate follows as (see App. [3])

\[
\Gamma_e \approx 2\hbar \sum_{qq'} \frac{\partial^2 H_0}{\partial p_q \partial p_{q'}} \text{Im} \left[ (\partial_q b^*_b)^\dagger (A^{-1})^\dagger A (\partial_p b^*_b) \right]. \tag{7}
\]

Since the Hessian of the free Hamiltonian is positive and symmetric, and since \( A \) is diagonalizable for all positions and orientations, the contraction rate (7) is positive if the effective detuning matrix (6) is negative definite. Thus the combined rotational and translational particle motion cools down if the tweezer is sufficiently far red-detuned, i.e. for \( \Delta < U_0 \chi_c \).

**Deep trapping regime** — In order to determine the cooling rates and limits in the resolved side-band regime, we harmonically expand the Hamiltonian (5) around the tweezer potential minimum \( q_{tw} = (r_{tw}, \Omega_{tw}) \), where the axis with the largest (intermediate) susceptibility aligns in parallel to the stronger (weaker) tweezer polarization axis \( \text{Re}(e_{i}) (\text{Im}(e_{i})) \) [38]. At the minimum \( r_{tw} = (0, 0, 0) \) and \( \Omega_{tw} = (0, \pi/2, 0) \) the cavity modes attain the amplitudes \( b_{tw} = b_s(q_{tw}) \), implying that the effective detuning matrix (6) is diagonal and that mode \( b_2 \) is empty.

The coupling to the cavity modes as well as the non-conservative tweezer torque displace the equilibrium configuration \( (q_{eq}, b_{eq}) \) slightly away from the tweezer minimum \( (q_{tw}, b_{tw}) \), see App. [3]. For small offsets, this does not affect the dynamics of the the small deviations \( \delta b = b - b_{eq} \) and the corresponding mechanical mode operators \( a_q = (q - q_{eq}) + \delta q/m\omega_q \). Here, the zero-point amplitude \( a_{eq} = \sqrt{\hbar/2m\omega_q} \) is determined by the effective masses \( m^{-1} = \delta b^*_b H_0(b_{tw}) \) and by the trapping frequencies \( \omega_q^2 = \delta b^*_b V_{\text{opt}}(q_{tw}, b_{tw})/m_q \). The mechanics-light coupling frequencies follow from the interaction potential (1) as \( g_{jq} = -q_{eq} \delta b_j \partial_q V_{\text{opt}}(q_{tw}, b_{tw})/h \). These couplings imply that the optical mode \( b_1 \) only interacts with coordinates that shift the effective detuning (6) along the \( e_1 \)-axis, i.e. \( q \in S_1 = \{x, y, z, \alpha\} \). In a similar fashion, mode \( b_2 \) only couples to \( q \in S_2 = \{\beta, \gamma\} \). The equilibrium orientations of \( S_2 \) coincide with their minimal values in the tweezer potential since \( \delta b_2 \) vanishes in the steady state.

In the harmonic approximation, the total Hamiltonian (5) therefore decomposes into two non-interacting contributions, \( H = H_1 + H_2 \), with

\[
\frac{H_j}{h} = \sum_{q \in S_j} \omega_q a_q^\dagger a_q - \sum_{qq' \in \epsilon_j} g_{jq}^a (a_q a_{q'} + a_{q'} a_q^\dagger) + \text{h.c.} - \sum_{q \in S_j} \left[ g_{jq} \delta b_j (a_q^\dagger + a_q) + \text{h.c.} \right] - \Delta_j \delta b_j^a \delta b_j. \tag{8}
\]

Here, the detunings \( \Delta_j = [\Delta_{\text{eff}}(q_{tw})]_{jj} \) follow from (6), the cavity-mediated mechanical coupling rates are \( g_{jq}^a = -q_{eq} \delta q_j \partial_q V_{\text{opt}}(q_{tw}, b_{tw})/h \), and we have switched to a quantum description.

We emphasize that the dynamics factorize into two non-interacting subsaces only due to our choice of the mode polarizations \( e_1 \) and \( e_2 \). Choosing rotated polarization vectors for \( b_1 \) and \( b_2 \), such as \( (e_1 \pm e_2)/\sqrt{2} \) would couple all mechanical degrees of freedom simultaneously to both optical modes.

** Recoil heating** — So far, we discussed the conservative interaction between the mechanical degrees of freedom and the cavity modes. Recoil heating by incoherently scattered tweezer photons and collisional heating by residual gas atoms limits the cooling process. For recoil heating, the resulting diffusion rates can be calculated either in a semiclassical picture [37] or from the Lindbladians of the exact quantum master equation [11]. For the center-of-mass masses \( q \in \{x, y, z\} \), this yields the phonon heating rates

\[
\xi_{qq} = \frac{\gamma_s c}{5} k^2 q_{eq}^2 \left[ \chi_c^2 \cos^2 \psi + \chi_b^2 \sin^2 \psi \right] (2 + u \delta_{qq})
\]

\[
-\chi_c^2 \cos^2 \psi \delta_{xz} - \chi_b^2 \sin^2 \psi \delta_{qq},
\]

with the Rayleigh scattering rate \( \gamma_s = \omega k^3 V^2 c^2/6\pi V_c \), and \( u = 5(1 + 1/k_{2\pi}^2) \). A similar calculation for the

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**Table I.** Stationary phonon occupations for three silicon ellipsoids with principal axes \( (e_a, e_b, e_c) \) in the weak-coupling approximation. Room temperature (r.t.) indicates that the respective degree of freedom is not cooled in the deep trapping regime. The occupations are obtained for a 1550 nm cavity, at ellipticity angle \( \ell = 25, 40, 100 \) second row: same as for \( 70, 70, 70 \). \( \ell = 2 \mu m, w_x = 40 \mu m, \) linewidth \( \kappa = 300 \text{kHz}, \) tweezer power \( P = 0.5 \text{W}, \) waist \( w_x = 1.6 \text{um}, \) detuning \( \Delta = -500 \text{kHz}, \) power \( P = 0.1 \text{W}, \) frequency \( \theta = \pi/4, \) and \( \phi = 3\pi/8. \) (25, 40, 100) first row: same as for \( 70, 70, 70 \), except \( P = 0.1 \text{W}, \) frequency \( \Delta = -11 \text{MHz}, \) power \( P = 0.1 \text{W}, \) waist \( w_x = 30 \text{um}, \) frequency \( \theta = 600 \text{kHz}, \) and \( \phi = 3\pi/8. \) (25, 40, 100) second row: same as for \( 70, 70, 70 \). \( 69, 70, 71 \): \( \ell = 1.5 \text{mm}, w_x = 30 \text{um}, \) frequency \( \kappa = 600 \text{kHz}, \) waist \( \Delta = -18 \text{kHz}, \) and \( \phi = 3\pi/8. \) The steady-state occupations for the \( x'- \) and \( y'-\)modes are approximate because they are not in the strict weak-coupling regime for the given parameters.

| particle shape \((e_a, e_b, e_c) [\text{mm}]\) | cooled by \( b_1 \) | cooled by \( b_2 \) |
|-----------------|-----------------|-----------------|
| \( (70, 70, 70) \) | \( 0.1 \) | \( 0.2 \) | \( 0.4 \) | \( \text{r.t.} \) | \( \text{r.t.} \) |
| \( (25, 40, 100) \) | \( \text{r.t.} \) | \( \text{r.t.} \) | \( \geq 10^3 \) | \( 0.1 \) | \( 0.2 \) | \( \geq 10^3 \) | \( \geq 10^3 \) |
| \( (69, 70, 71) \) | \( 0.1 \) | \( 0.1 \) | \( 0.7 \) | \( 0.3 \) | \( 0.9 \) | \( 0.2 \) |
librational degrees of freedom $q \in \{\alpha, \beta, \gamma\}$ yields the diffusion rates

$$\xi_q = \gamma_q g_{qz}^2 \Delta \chi_q^2 \left[ \delta_{\alpha q} + \cos^2 \psi \delta_{\beta q} + \sin^2 \psi \delta_{\gamma q} \right], \quad (10)$$

where $\Delta \chi_q = |\chi_q - \chi_c|$ and cyclic permutations.

The corresponding diffusion rates due to collisions with residual gas atoms are $\xi_g = k_B \gamma_g^* T / \hbar \omega_q$, where $T$ is the gas temperature and $\gamma_g^*$ denotes the gas damping constant. The total heating rates are thus given by $\xi_q = \gamma_q^* + \xi_g^*$.

Ground-state cooling—The cavity-mediated coupling between the mechanical degrees of freedom leads to the appearance of hybrid mechanical modes $S'_j = \{x', y', z', \alpha'\}$ and $S'_2 = \{\beta', \gamma'\}$, which can be determined by diagonalising the first line of Eq. (8). However, since the cavity population is much smaller than the tweezer amplitude, $|b_j| \ll \epsilon$, the couplings $g_{qy}$ are small so that the hybridized modes $S'_j$ are well approximated by the original modes $S_j$ [43].

After transforming to the hybrid modes, each mechanical degree of freedom $Q \in S'_j$ with trapping frequencies $\omega_Q$ linearly interacts with the respective light mode $b_j$, as quantified by the coupling constant $g_Q$. In the weak coupling approximation, the resulting cooling/heating rates are $\gamma_Q^\pm = 2|g_Q|^2 \kappa / [\kappa^2 + (\Delta_q \pm \omega_Q)^2]$. These expressions are valid for $\gamma_Q^\pm \ll \kappa$ and $\gamma_Q^\pm \ll (\omega_Q - \omega_Q')^2$, where $Q, Q' \in S'_j$. From this, one obtains the stationary mechanical mode occupations $n_Q = (\gamma_Q^+ + \xi_Q) / (\gamma_Q^- + \gamma_Q^+)$. The resulting weak coupling steady-state occupations for three ellipsoidally shaped particles are listed in Tab. I. Based on this we conclude that the 6D quantum groundstate is realistically achievable by elliptic coherent-scattering cooling. Even though the rotation of an exactly spherical particle (first row) cannot be cooled, the orientation can be driven into the quantum ground state in the other cases considered. For increasingly anisotropic particles (second and third row) the librational and translational frequencies diverge, rendering simultaneous cooling inefficient. Nonetheless, appropriately choosing the detuning allows one to efficiently cool either the rotations or the center-of-mass motion to the ground state. For slightly aspherical particles (fourth row), all six degrees of freedom can be simultaneously cooled into their quantum groundstate. In Tab. I gas scattering is only relevant for the $\beta'$ and $\gamma'$ degrees of freedom in the fourth row. The corresponding damping constants are $\gamma_g^* \approx 5p_B \ell_B^2 \sqrt{2\pi \mu / 6m} \sqrt{kbT}$ [2] with $T = 300$ K and the mass of Helium $\mu$.

The cooling timescale $\log(k_B T / \hbar \omega_Q n_Q) / (\gamma_Q^- - \gamma_Q^+)$ are on the order of a few hundred microseconds for the translation and in the two-digit millisecond regime for the librations, assuming a starting temperature of 40 K (librational trapping). Non-harmonicities in $H_0$ give rise to higher harmonics in the cavity output fields as long as the phonon number exceeds a few hundred. This is illustrated in Fig. 2 which shows the power spectral densities of the two cavity output modes for three different gas pressures, App. C.

Discussion—Coherent-scattering cooling with an elliptically polarized tweezer offers an attractive setup for efficiently cooling nanoparticles into the deep quantum regime in all their ro-translational degrees of freedom. This has great potential for sensing applications and for fundamental quantum experiments, even with spherical objects since exact sphericity can never be guaranteed.

Specifically, torque sensing can be best performed by monitoring the mode $b_2$ which is unaffected by the center-of-mass motion. The minimally detectable torque from
measuring the two librational degrees of freedom $\beta, \gamma$ is $N^\text{min}_2/\sqrt{B} \approx 4N/\omega^2 \chi_0 k_0^2 \approx 3.9 \times 10^{-30}$ Nm/Hz, where $B$ is the measurement bandwidth [15]. This would improve current experiments by orders of magnitude [9] and enable the observation of rotational quantum friction [10] and the Casimir torque [47]. Simultaneously monitoring both cavity output modes also opens the door to combined force and torque measurements using different degrees of freedom of a single levitated object.

Even for particle shapes where 6D cooling is inefficient, elliptic coherent scattering can prepare their orientational degrees of freedom in the deep quantum regime. This setup is therefore ideally suited for preparing rotational quantum superposition tests with nanoscale rotors [11, 34]. For instance, cooling an asymmetric rotor [34], generates a cold but rapidly rotating state as required to observe quantum persistent tennis racket flips [34] and acousto-rotational coupling [18]. For the particle considered in the second row of Tab. 1 GHz rotation frequencies can be achieved in a matter of milli-seconds.

Finally, the isolated librational $S_2^m$ modes are ideally suited for trapped quantum experiments [12, 13, 49] because they are weakly affected by Rayleigh scattering of tweezer photons [10], leading to coherence times larger than that of the other mechanical modes by about one order of magnitude. For instance, the expected $\gamma'$ coherence time is on the order of $0.6 - 1.1$ ms for the 6D groundstate setup in Tab. 1.

In summary, coherent-scattering cooling with an elliptically polarized tweezer enables simultaneous rotational ground-state cooling of nanoscale dielectrics. This setup may well serve as a building block for future quantum experiments and sensors with levitated nanoparticles.

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Appendix A: Radiation pressure force and torque

To derive the expression for the radiation pressure force and torque Eqs. (2) and (3) exerted by light on small dielectrics of arbitrary shape, we consider a particle of homogeneous relative permittivity $\varepsilon$, illuminated by electromagnetic radiation characterized by the complex field $E(r)$ and the frequency $\omega$. For particle extensions much smaller than the laser wavelength $2\pi/k$, the time-averaged dipole force and torque can be written as [50]

$$ F_{\text{dip}} = \frac{1}{2} \text{Re} \left[ p^* \cdot (\nabla \otimes E)^T \right] \quad \text{and} \quad N_{\text{dip}} = \frac{1}{2} \text{Re} \left( p^* \times E \right). \quad (A1) $$

Here, $r$ denotes the particle position, and we introduced the complex dipole vector $p$, which is determined by the internal field $E_{\text{tot}}$. In the quasi-static approximation $p = \varepsilon_0 \chi^V V E(r)$, where $\chi$ is the shape-dependent susceptibility tensor, Eqs. (A1) correspond to the force and torque described by the optical potential [4].

The non-conservative radiation pressure force and torque appear as corrections to Eq. (1) in lowest order of $\chi^V V k^3$. The correction to the electric dipole moment $p$ can be calculated from the electric-field integral equation for the total electric field $E_{\text{tot}}$ inside the dielectric [71].

$$ E_{\text{tot}}(r) = E(r) + \frac{\varepsilon - 1}{4\pi} (\nabla \otimes \nabla + k^2) \int_{\nu} d^3 r' e^{-ik|r-r'|} \frac{E_{\text{tot}}(r')}{|r-r'|}. \quad (A2) $$

Expanding the exponential function to third order in $k$, noting that the first order term vanishes and treating second and third order expressions as small corrections to the electrostatic solution $E_{\text{stat}} = \chi E/\varepsilon$ yields

$$ p = \varepsilon_0 V \chi \left( 1 + \frac{k^2 V}{6\pi} \right) E, \quad (A3) $$

where we neglected the $k^2$ contribution because it only minimally modifies the real part of $p$ and can therefore be included in the susceptibility tensor. The small correction due to the finite volume of the particle gives rise to a non-conservative force and torque when inserted into Eq. (A1).

The total force and torque contain the dipole expressions (A1) and a correction accounting for the interaction of the scattered field with itself originating from different volume elements within the particle. This additional contribution yields no net force, but can yield a net torque. This scattering torque $N_s$ is determined by Maxwell’s stress tensor $T$ of the scattered electromagnetic field, integrated over a spherical surface of infinite radius centred at the particle center of mass [50]

$$ N_s = \lim_{r' \to \infty} \frac{1}{r'^3} \int d^3 n \times [T(r'n)n]. \quad (A4) $$

measuring the two librational degrees of freedom $\beta, \gamma$ is $N^\text{min}_2/\sqrt{B} \approx 4N/\omega^2 \chi_0 k_0^2 \approx 3.9 \times 10^{-30}$ Nm/Hz, where $B$ is the measurement bandwidth [15]. This would improve current experiments by orders of magnitude [9] and enable the observation of rotational quantum friction [10] and the Casimir torque [47]. Simultaneously monitoring both cavity output modes also opens the door to combined force and torque measurements using different degrees of freedom of a single levitated object.

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In summary, coherent-scattering cooling with an elliptically polarized tweezer enables simultaneous rotational ground-state cooling of nanoscale dielectrics. This setup may well serve as a building block for future quantum experiments and sensors with levitated nanoparticles.

Acknowledgments— JS and HR contributed equally.

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Appendix A: Radiation pressure force and torque

To derive the expression for the radiation pressure force and torque Eqs. (2) and (3) exerted by light on small dielectrics of arbitrary shape, we consider a particle of homogeneous relative permittivity $\varepsilon$, illuminated by electromagnetic radiation characterized by the complex field $E(r)$ and the frequency $\omega$. For particle extensions much smaller than the laser wavelength $2\pi/k$, the time-averaged dipole force and torque can be written as [50]

$$ F_{\text{dip}} = \frac{1}{2} \text{Re} \left[ p^* \cdot (\nabla \otimes E)^T \right] \quad \text{and} \quad N_{\text{dip}} = \frac{1}{2} \text{Re} \left( p^* \times E \right). \quad (A1) $$

Here, $r$ denotes the particle position, and we introduced the complex dipole vector $p$, which is determined by the internal field $E_{\text{tot}}$. In the quasi-static approximation $p = \varepsilon_0 \chi^V V E(r)$, where $\chi$ is the shape-dependent susceptibility tensor, Eqs. (A1) correspond to the force and torque described by the optical potential [4].

The non-conservative radiation pressure force and torque appear as corrections to Eq. (1) in lowest order of $\chi^V V k^3$. The correction to the electric dipole moment $p$ can be calculated from the electric-field integral equation for the total electric field $E_{\text{tot}}$ inside the dielectric [71].

$$ E_{\text{tot}}(r) = E(r) + \frac{\varepsilon - 1}{4\pi} (\nabla \otimes \nabla + k^2) \int_{\nu} d^3 r' e^{-ik|r-r'|} \frac{E_{\text{tot}}(r')}{|r-r'|}. \quad (A2) $$

Expanding the exponential function to third order in $k$, noting that the first order term vanishes and treating second and third order expressions as small corrections to the electrostatic solution $E_{\text{stat}} = \chi E/\varepsilon$ yields

$$ p = \varepsilon_0 V \chi \left( 1 + \frac{k^2 V}{6\pi} \right) E, \quad (A3) $$

where we neglected the $k^2$ contribution because it only minimally modifies the real part of $p$ and can therefore be included in the susceptibility tensor. The small correction due to the finite volume of the particle gives rise to a non-conservative force and torque when inserted into Eq. (A1).

The total force and torque contain the dipole expressions (A1) and a correction accounting for the interaction of the scattered field with itself originating from different volume elements within the particle. This additional contribution yields no net force, but can yield a net torque. This scattering torque $N_s$ is determined by Maxwell’s stress tensor $T$ of the scattered electromagnetic field, integrated over a spherical surface of infinite radius centred at the particle center of mass [50]

$$ N_s = \lim_{r' \to \infty} \frac{1}{r'^3} \int d^3 n \times [T(r'n)n]. \quad (A4) $$
Asymptotically expanding the scattered fields \( \mathbf{E}_s \) and \( \mathbf{B}_s \) in \( 1/r' \) as \( \mathbf{E}_s = \mathbf{E}_{s1/r'} + \mathbf{E}_{s1/r'^2} + O(1/r'^3) \) and likewise for \( \mathbf{B}_s \), and using the transversality of electromagnetic radiation, \( \mathbf{n} \cdot \mathbf{E}_{s1/r'} = 0 \) and \( \mathbf{n} \cdot \mathbf{B}_{s1/r'} = 0 \), yields after time-averaging

\[
\mathbf{N}_u = \lim_{r' \to \infty} \frac{r'^3}{2} \text{Re} \left[ \int d^2 \mathbf{n} \varepsilon_0 (\mathbf{n} \cdot \mathbf{E}_{1/r'}) \nabla \times \mathbf{E}_{1/r'} + \frac{1}{\mu_0} (\mathbf{n} \cdot \mathbf{B}^{*}_{1/r'}) \mathbf{n} \times \mathbf{B}_{1/r'} \right]. \tag{A5}
\]

Here, all fields are evaluated at \( r' \mathbf{n} \). The scattered fields in (A5) follow from the electric field integral equation (A2) and Maxwell’s equations as \( \mathbf{n} \cdot \mathbf{B}_{1/r'^2} = 0 \) together with

\[
\mathbf{n} \cdot \mathbf{E}_{1/r'^2} = -\frac{ik e^{ikr'}}{2\pi \varepsilon_0 r'^2} \mathbf{n} \cdot \mathbf{p} e^{-i\mathbf{k} \cdot \mathbf{n} \mathbf{r}} \quad \text{and} \quad \mathbf{n} \times \mathbf{E}_{1/r'} = -\frac{k^2 e^{ikr'}}{4\pi \varepsilon_0 r'} \mathbf{p} \times \mathbf{n} e^{-i\mathbf{k} \cdot \mathbf{r}}. \tag{A6}
\]

Inserting these expressions into (A5) and adding them to the dipole force and torque (A1) yields the total force and torque acting on the particle

\[
\mathbf{F} = \nabla \left( \frac{\varepsilon_0 V}{4} \mathbf{E}^* \cdot \chi \mathbf{E} \right) + \frac{\varepsilon_0 k^3 V^2}{12\pi} \text{Im} \left[ (\chi \mathbf{E}^*) \cdot (\nabla \otimes (\chi \mathbf{E}))^T \right], \tag{A7a}
\]

and

\[
\mathbf{N} = \frac{\varepsilon_0 V}{2} \text{Re} \left[ (\chi \mathbf{E}^*) \times \mathbf{E} \right] + \frac{\varepsilon_0 k^3 V^2}{12\pi} \text{Im} \left[ (\chi^2 \mathbf{E}^*) \times \mathbf{E} - (\chi \mathbf{E}^*) \times (\chi \mathbf{E}) \right], \tag{A7b}
\]

where the field is always evaluated at the particle position \( \mathbf{r} \). The first terms are described by the conservative optical potential \[1\], and the second terms are the radiation pressure force \[2\] and torque \[3\].

When writing the equations of motion in terms of canonical phase-space coordinates with Hamiltonian \[5\], the non-conservative force and torque appear as a generalized force \( \mathbf{F}_q \), i.e., \( \dot{\mathbf{p}}_q = -\partial_q H + \mathbf{F}^\text{rad}_q \). This generalized force can be written as

\[
\mathbf{F}^\text{rad}_q = -\frac{\varepsilon_0 k^3 V^2}{12\pi} \text{Im} \left[ (\chi \mathbf{E}^*) \cdot \partial_q (\chi \mathbf{E}) \right], \tag{A8}
\]

and is equivalent to the expression derived in Ref. \[25\] for rod-shaped particles.

For the setup considered in the main text the non-conservative force and torque are suppressed by a factor of \( k^3 V \chi_c / 6\pi \approx 10^{-3} \) as compared to the conservative optical potential. For elliptical polarization, the main effect of the generalized forces (A8) is thus to shift the equilibrium configuration of the nanoparticle by a few nanometers and arc seconds. However, for circularly polarized light, where \( \alpha \) is not suppressed, the torque may efficiently spin up the particle \[29\][31].

### Appendix B: Phase-space contraction rate

To quantify how non-conservative generalized forces lead to a local phase-space contraction or expansion, we consider the dynamics of the \( 2N \)-dimensional phase-space point \( \mathbf{z}_t = (q_1, \ldots, q_N, p_1, \ldots, p_N) \), where \( N \) denotes the number of degrees of freedom. Including non-conservative generalized contributions, we can write the equations of motion in the form \( \mathbf{z}_t = J \dot{\mathbf{q}}_t H + \mathbf{K} \),

where the \( J \) is the \( (2N \times 2N) \)-symplectic matrix, \( J \delta_{\mathbf{k}} = (\partial_{p_1}, \ldots, \partial_{p_N}, -\partial_{q_1}, \ldots, -\partial_{q_N}) \), \( H \equiv H(\mathbf{z}_t) \) is the Hamiltonian and the vector field \( \mathbf{K} \equiv \mathbf{K}(\mathbf{z}_t) \) describes the non-conservative contributions.

For non-vanishing \( \mathbf{K} \), the infinitesimal volume element \( dV(\mathbf{z}_t) \) is in general a function of time \( t \). To quantify the resulting contraction or expansion rate, we introduce the dynamical mapping \( \Phi_t \) from an arbitrary phase space point \( \mathbf{z}_0 \) to its time-evolved coordinates \( \mathbf{z}_t = \Phi_t(\mathbf{z}_0) \). Thus, we can write \( dV(\mathbf{z}_t) = \Phi_t(\mathbf{z}_0) dV(\mathbf{z}_0) \), where \( \Phi_t(\mathbf{z}_0) = \det[\partial_\mathbf{z}_0 \Phi_t(\mathbf{z}_0)] \) is the Jacobian of the dynamical mapping \( \Phi_t \), and \( \partial_\mathbf{z} \) denotes the derivative with respect to the initial conditions \( \mathbf{z}_0 \).

The time derivative of the Jacobii determinant can be calculated by differentiating the determinant function,

\[
\partial_t \Phi_t(\mathbf{z}_0) = \text{tr} \left[ \left[ \partial_\mathbf{z}_0 \Phi_t(\mathbf{z}_0) \right]^{-1} \partial_\mathbf{z}_0 \Phi_t(\mathbf{z}_0) \right] \Phi_t(\mathbf{z}_0). \tag{B1}
\]

Thus each infinitesimal volume element \( dV(\mathbf{z}_t) \) locally contracts or expands with a rate quantified by the right-hand side of Eq. (B1). Using the equations of motion and applying the chain rule, e.g., \( \partial_\mathbf{z}_0 \mathbf{K}(\mathbf{z}_t) = [\partial_\mathbf{z}_0 \Phi_t(\mathbf{z}_0)][\partial \otimes \mathbf{K}(\mathbf{z}_t)] \) yields the local phase space contraction rate

\[
\Gamma_c(\mathbf{z}_t) = -\partial_\mathbf{z}_0 \Phi_t(\mathbf{z}_0). \tag{B2}
\]

If the non-conservative contribution only acts as a force \( F_q \), the contraction rate takes the form of a momentum divergence, \( \Gamma_c = -\sum_q \partial_q F_q \). Inserting the corresponding forces in the bad cavity regime yields Eq. \[7\].
Appendix C: Power Spectral Density

We calculate the power spectral densities of the cavity mode deviations \(\delta b_j\) with Hamiltonian (8) by formulating the quantum Langevin equations with noises (9). The power spectral densities are defined by

\[
S_{b_jb_j}[\omega] = \frac{1}{2\pi} \int_{-\infty}^{\infty} dt \ e^{-i\omega t} \langle \delta b_j(t) \delta b_j(0) \rangle = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\omega' \langle b_j[\omega] b_j[\omega'] \rangle,
\]

where we defined the Fourier representation of the cavity mode deviations as

\[
b_j[\omega] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt \ e^{-i\omega t} \delta b_j(t).
\]

In Fourier space, the quantum Langevin equations for the cavity modes \(b_j[\omega]\) and the mechanical normal modes \(a_Q[\omega]\) read as

\[
-i\omega b_j[\omega] = (i\Delta_j - \kappa) b_j[\omega] + \sqrt{2\kappa} \eta_j[\omega] + i \sum_{s_j} g_{s_j}^* \left( a_Q[\omega] + a_Q^*[\omega]\right),
\]

and

\[
-i\omega a_Q[\omega] = -i\omega a_Q[\omega]* + i\sqrt{\xi Q} \eta Q[\omega] + i \left( g_Q b_j[\omega] + g_Q^* b_j^*[\omega]\right),
\]

with the independent cavity noise operators \(\eta_j[\omega]\) and the (classical) white noise \(\eta Q[\omega]\), accounting for Rayleigh scattering and gas collisions. In the time domain, they fulfill

\[
\left[ \eta_j(t), \eta_j^*(t') \right] = \delta_{jj'} \delta(t-t'), \quad \langle \eta_j(t) \eta_j^*(t') \rangle = 0, \quad \langle \eta_j^*(t) \eta_j(t') \rangle = 0, \quad \langle \eta_Q(t) \eta_Q^*(t') \rangle = \delta_{QQ'} \delta(t-t'),
\]

since the thermal photon occupation of the cavity is negligible. These relations fully determine all correlators in the frequency domain. The mechanical noise terms \(\eta_Q(t)\) describe real white noise, resulting in \(\eta_Q[\omega] = \eta Q[\omega]\). Also, in writing Eqs. (C3), we used that gas damping of the mechanical modes is negligibly small for the pressures considered in the manuscript.

Solving the equation (C3) for \(b_j[\omega]\) yields

\[
b_j[\omega] = \chi_j[\omega] \left[ \sqrt{2\kappa \eta_j[\omega]} + i \sum_{s_j} g_{s_j}^* \left( a_Q[\omega] + a_Q^*[\omega]\right) \right],
\]

with the cavity susceptibility \(\chi_j[\omega] = 1/[\kappa - i(\Delta_j + \omega)]\). Inserting this solution into Eq. (C3b), neglecting all off-resonant contributions and evaluating \(\chi_j[\omega]\) at the mechanical resonance \(\omega Q = \omega Q + |g_Q|^2 \text{Im}(\chi_j[\omega]) + \chi_j[-\omega Q]\) and damping constants \(\gamma Q = 2|g_Q|^2 \text{Re}(\chi_j[\omega])\), yields that \(a_Q\) evolve independently with frequencies \(\omega Q = \omega Q + |g_Q|^2 \text{Im}(\chi_j[\omega]) + \chi_j[-\omega Q]\) and damping constants \(\gamma_Q = 2|g_Q|^2 \text{Re}(\chi_j[\omega])\).

Solving the resulting equation shows that

\[
a_Q[\omega] = \chi Q[\omega] \left[ i \sqrt{\xi Q} \eta Q[\omega] + i \sqrt{2\kappa} \left( g_Q \chi_j[\omega] \eta Q[\omega] + g_Q^* \chi_j^*[\omega] \eta Q^*[\omega]\right) \right],
\]

with the mechanical susceptibilities \(\chi Q[\omega] = 1/[\gamma Q/2 + i(\omega Q - \omega)]\). Finally, combining Eqs. (C1), (C4) and (C5) gives the power spectral densities as

\[
S_{b_jb_j}[\omega] = \frac{1}{2\pi} |\chi_j[\omega]|^2 \left[ 2\kappa |\chi_j[-\omega]|^2 \sum_{s_j'} (g_{s_j'}^*)^2 \left( \chi_j[\omega] - \chi_j^*[\omega]\right)^2 + \sum_{s_j'} |g_{s_j'}|^2 \xi Q |\chi_j[\omega] - \chi_j^*[\omega]|^2 \right].
\]

The first term of the PSD originates from cavity vacuum shot noise, which limits cavity cooling even in the absence of mechanical noise. The second term describes heating due to scattering of tweezer photons and collisions with residual gas atoms.

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