Resource Allocation and Power Control in Cooperative Small Cell Networks in Frequency Selective Channels with Backhaul Constraint

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Abstract

A joint resource allocation (RA), user association (UA), and power control (PC) problem is addressed for proportional fairness maximization in a cooperative multiuser downlink small cell network with limited backhaul capacity, based on orthogonal frequency division multiplexing. Previous studies have relaxed the per-resource-block (RB) RA and UA problem to a continuous optimisation problem based on long-term signal-to-noise-ratio, because the original problem is known as a combinatorial NP-hard problem. We tackle the original per-RB RA and UA problem to obtain a near-optimal solution with feasible complexity. We show that the conventional dual problem approach for RA cannot find the solution satisfying the conventional KKT conditions. Inspired by the dual problem approach, however, we derive the first order optimality conditions for the considered RA, UA, and PC problem, and propose a sequential optimization method for finding the solution. The overall proposed scheme can be implemented with feasible complexity even with a large number of system parameters. Numerical results show that the proposed scheme achieves the proportional fairness close to its outer bound with unlimited backhaul capacity in the low backhaul capacity regime and to that of a carefully-designed genetic algorithm with excessive generations but without backhaul constraint in the high backhaul capacity regime.

Index Terms

Orthogonal frequency division multiplexing (OFDM) downlink, proportional fairness maximization, user association, resource allocation, power control, cooperative small cells network, limited backhaul capacity

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I. INTRODUCTION

It has long been a challenge to suppress intercell interference in cellular mobile networks, and thereby improving the system throughput and spectral efficiency. Indeed, it is known that the cell densification may degrade the sum-rate unless the level of intercell interference is kept low enough compared to the desired signal [1]. The concept of “small cells” is one of the enablers of the next generation mobile network requiring extremely high data rate connections, where multiple small cell base stations (SBSs) in proximity are clustered to make a hotspot area providing high data rate connectivity. As the SBS cell size becomes smaller to further increase the sum-rate, user association (UA), resource allocation (RA), and power control (PC) should be carefully designed to mitigate intercell interference, particularly for cell-edge users. The optimisation for UA, RA, and PC can be considered to maximize the fairness of the users [2] or the sum-capacity of the total system [3]–[6]. The focus of this paper is on the proportional fairness maximization, where the aim is to maximize the geometric mean of users’ rates, compromising between the fairness and sum-capacity maximization.

A variety of literature have been around to tackle the joint optimisation problem of UA, RA, and PC for heterogenous multicell networks based on orthogonal frequency division multiplexing (OFDM) [5], [7]–[9]. In SBS networks, a reliable direct connection, i.e., X2 interface in LTE-A, is assumed between SBSs within a cluster, and hence the aforementioned problem can be solved cooperatively across the SBSs in a cluster, allowing the exchange of the channel gain values among the SBSs in the cluster.

A rich body of the literature assumes unique BS association [5], [7]–[18], in which a user can only be served by a unique BS, i.e., the user can receive data only from a single BS out of all the BSs. In frequency-selective fading channels, the RA and UA needs to be done on a per-resource-block (RB) or per-subcarrier basis, which generally results in a combinatorial NP-hard problem requiring exponential computational complexity with respect to the number of RBs and/or users to find the optimal solution. Several efforts have been made to relax this combinatorial problem to more tractable problems, i) reducing search space heuristically [5], [11], ii) simply relaxing integer variables to real-valued variables [15], [19], or iii) only using SNR averaged over all the RBs for each BS [10], [13]–[16], [20]. With the SNR averaged out for all the RBs, ignoring frequency selectivity, the RA problem is to find the proportion of the total bandwidth, allocated to each user, thus yielding a continuous optimisation problem with real-valued variables. In particular, the authors of [15] found a near-optimal solution for the UA and RA for the frequency non-selective fading case.

1Note that although the UA, RA, and PC can be optimized cooperatively across multiple SBSs, every user is associated with a unique BS for data reception.
On the other hand, a user can be jointly associated with multiple SBSs within a cluster to further improve the system throughput [6], [15], [21]-[25], in which a user can be simultaneously served via different RBs of different SBSs. Although the total throughput can be significantly enhanced using the multiple-BS association, a careful consideration is needed for the total amount of downlink data to be transmitted by each SBS not to exceed the backhaul capacity. In particular, non-ideal backhaul such as wireless backhaul with highly limited capacity is the highest priority of service operators of LTE-A [26].

In this paper, our focus is on multiple-BS association with limited backhaul capacity for frequency-selective fading channels. In particular, the per-RB UA, RA, and PC problem is tackled in pursuit of maximizing the proportional fairness.

A. Related Works

The authors of [15] formulate and solve a real-valued convex problem of joint UA and RA assuming frequency non-selective fading with fixed power. In [23], [25], a similar technique was used for UA and load balancing in the multicell frequency non-selective fading massive multi-input multi-output channel. Distributed UA schemes for given resource element (RE) with the SBSs harvesting energy are proposed in [21], where each user selects a SBS based on the statistical analysis of the amount of energy harvesting and energy consumption of the SBSs. These studies, however, assume frequency non-selective fading channels with ideal backhaul capacity, thus yielding a limited application in practical environment.

In [24], the joint problem of UA, RA, and PC is tackled with limited backhaul capacity constraint, where multiple-BS association is allowed. In particular, the authors of [24] merged the integer variables for UA and RA into the real-valued power variables, thereby yielding a continuous optimisation problem. However, this study also assumes frequency non-selective fading, where the problem is formulated on a long-term basis based only on long-term SNR. Therefore the backhaul constraint cannot be satisfied at all time, particularly if channel gain is temporarily high for all the REs at random.

To the best of the authors’ knowledge, in spite of its importance, the joint problem of UA, RA, and PC has never been solved with feasible computational complexity in frequency selective channels, where the RA, UA, and PC are carried out on a per-RB basis.

B. Contribution

We formulate the joint problem of UA, RA, and PC for frequency selective fading channels with limited backhaul capacity, and show that the well-known conventional dual problem approach cannot
be used for the considered problem. Inspired by the dual problem approach, however, we derive the first order optimality conditions, and then propose a two-step cascaded algorithm to find the solutions of the UA, RA, and PC problems sequentially with feasible computational complexity. In the proposed UA and RA algorithm, the gap between the 2-distance ring points, i.e., local optimal point, and the solution of the proposed algorithm is derived in terms of the lagrange variables, which asymptotically vanishes as the number of variables increases. For the PC problem, a zero-sum-game approach for the power allocated to the users of each SBS is proposed with sum-power constraint based on the first-order optimality condition.

Simulation results show that the proposed scheme exhibits the proportional fairness performance close to the outer bound with unlimited power assumption in the low backhaul capacity regime and to that of the genetic algorithm under unlimited backhaul capacity, which generally finds a near-global optimal solution with excessive generations, in the high backhaul capacity regime.

C. Organization of this paper
The remainder of this paper is organized as follow. Section II introduces the system model and formulates the problem. Section III analyzes the dual problem approach. Section IV presents the proposed UA, RA, and PC algorithms to find the solution. Section V provides numerical results, and Section VI concludes the paper.

II. SYSTEM MODEL AND PROBLEM FORMULATION
We consider a downlink SBS cluster composed of \( J \) SBSs and \( N \) users based on orthogonal frequency division multiplexing (OFDM). Assuming separate frequency carrier for the macro-cell BSs, e.g., Scenario 2a of the 3GPP small cell scenarios [26], there is no interference from the macro-cell BSs. Assuming frequency reuse 1 for the SBSs in the cluster, inter-cluster interference is neglected, which is dominated by intra-cluster interference. The UA and RA is carried out cooperatively across all the SBSs only within the cluster, assuming the exchange of per-RB channel gain information among SBSs via a direct interface, such as X2 interface in LTE-A. Each SBS is connected to the core network via backhaul link with limited capacity. The total bandwidth is divided into \( C \) frequency-division RBs, each of which is a group of multiple or single subcarriers. In frequency-selective fading, each RB has different channel gain for each user.

Let \( h_{ij}^{(c)} \) denote the channel gain from SBS \( j \) to user \( i \) on RB \( c \), where \( j \in B = \{1, 2, \cdots, J\} \), \( i \in N = \{1, 2, \cdots, N\} \), and \( c \in C = \{1, 2, \cdots, C\} \). Assuming quasi-static block fading, i.e., \( h_{ij}^{(c)} \) is constant for a frame and changes to the next value randomly, all the SBSs in the cluster share the
channel gain values for all the users within the cluster. This global CSI assumption within a cluster is feasible, because high data rate interface, such as optical fiber, between the SBSs in the same cluster is considered with high priority in the commercialized network [26]. The signal-to-interference-plus-noise ratio (SINR) when SBS \( j \) serves user \( i \) on RB \( c \) is denoted as

\[
\text{SINR}_{ij}^{(c)} = \frac{|h_{ij}^{(c)}|^2 p_j^{(c)}}{\sigma^2 + \sum_{k \neq j, k \in B} |h_{ik}^{(c)}|^2 p_k^{(c)}},
\]

where \( \sigma^2 \) represents the variance of additive white Gaussian noise (AWGN), and \( p_j^{(c)} \) is the transmission power of SBS \( j \) on RB \( c \), constrained by

\[
p_j^{(c)} \geq 0, \quad \forall j \in B, \quad c \in C,
\]

\[
\sum_{c \in C} p_j^{(c)} \leq P_{j,\text{max}}, \quad \forall j \in B,
\]

where \( P_{j,\text{max}} \) denotes the maximum total transmission power of SBS \( j \) across all the frequency blocks. The data rate of user \( i \) served by SBS \( j \) on RB \( c \) is represented by

\[
R_{ij}^{(c)} = W \log_2(1 + \text{SINR}_{ij}^{(c)}),
\]

where \( W \) denotes the bandwidth of each RB. We define a binary variable \( x_{ij}^{(c)} \) to represent UA and RA as follow:

\[
x_{ij}^{(c)} = \begin{cases} 1, & \text{if user } i \text{ is served by SBS } j \text{ on RB } c, \\ 0, & \text{otherwise.} \end{cases}
\]

We assume up to one user can be served on each RB. Note that each user can be served by different RBs of different SBSs, i.e., joint SBS association is allowed. These two conditions are denoted as

\[
\sum_{i \in N} x_{ij}^{(c)} = 1, \quad \forall j \in B, \quad c \in C.
\]

Each transmission of SBSs is constrained by backhaul capacity. The constraint for backhaul capacity is denoted as

\[
\sum_{c \in C} \sum_{i \in N} R_{ij}^{(c)} x_{ij}^{(c)} \leq Z_j, \quad \forall j \in B,
\]

where \( Z_j \) denotes the backhaul capacity of SBS \( j \).
The proportional fairness of the network with given UA and RA is denoted as

\[ U(X, P) = \sum_{i \in N} \log \left( \sum_{j \in B} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)} \right), \tag{8} \]

where \( X \in \{0, 1\}^{N \times B \times C} \) with \([X]_{ijc} = x_{ij}^{(c)}\) and \( P \in \mathbb{R}^{B \times C} \) with \([P]_{jc} = p_{j}^{(c)}\). The goal is to find the solution for \( X \) and \( P \) that maximize the proportional fairness under given backhaul constraint. The mixed integer optimisation problem is formulated as

\[
P1 : \max_{X, P} \sum_{i \in N} \log \left( \sum_{j \in B} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)} \right) \tag{9a} \]

subject to

\[
\sum_{i \in N} x_{ij}^{(c)} = 1, \forall j \in B, \forall c \in C \tag{9b}
\]

\[
\sum_{c \in C} \sum_{i \in N} R_{ij}^{(c)} x_{ij}^{(c)} \leq Z_{j}, \forall j \in B \tag{9c}
\]

\[
\sum_{c \in C} p_{j}^{(c)} \leq P_{j,\text{max}}, \forall j \in B \tag{9d}
\]

\[
R_{ij}^{(c)} = W \log_2 \left( 1 + \frac{|h_{ij}^{(c)}|^2 p_{j}^{(c)}}{\sigma^2 + \sum_{k \neq j} |h_{ik}^{(c)}|^2 p_{k}^{(c)}} \right), \forall i \in N, \forall j \in B, \forall c \in C \tag{9e}
\]

\[
x_{ij}^{(c)} \in \{0, 1\}, \forall i \in N, \forall j \in B, \forall c \in C \tag{9f}
\]

\[
p_{j}^{(c)} \geq 0, \forall j \in B, \forall c \in C \tag{9g}
\]

Because the joint optimisation problem (9) is not tractable for both \( X \) and \( P \), we decompose the problem to solve \( X \) and \( P \) sequentially in following Sections.

III. Duality Analysis of UA and RA

In this section, \( X \), which denotes UA and RA, is obtained with given transmission power \( P \) and data rate \( R_{ij}^{(c)} \). The joint UA and RA problem is formulated with given \( P \) from problem (9). Then, the
problem is formulated by adding the variable $\lambda_i$ without losing any optimality:

\[
P_2 : \max_X \sum_{i \in \mathcal{N}} \log \lambda_i \tag{10a}
\]

s.t. $\lambda_i = \sum_{j \in \mathcal{B}} \sum_{c \in \mathcal{C}} R_{ij}^{(c)} x_{ij}^{(c)}$, $\forall i \in \mathcal{N}$ \hspace{1cm} (10b)

\[
\sum_{i \in \mathcal{N}} x_{ij}^{(c)} = 1, \forall j \in \mathcal{B}, \forall c \in \mathcal{C} \tag{10c}
\]

\[
\sum_{c \in \mathcal{C}} \sum_{i \in \mathcal{N}} R_{ij}^{(c)} x_{ij}^{(c)} \leq Z_j, \forall j \in \mathcal{B} \tag{10d}
\]

\[
x_{ij}^{(c)} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{B}, \forall c \in \mathcal{C} \tag{10e}
\]

The lagrangian expression of the problem (10) except constraints (10c) and (10e) is denoted as

\[
L(X, \lambda, \mu, \nu) = \sum_{i \in \mathcal{N}} \log \lambda_i + \sum_{i \in \mathcal{N}} \mu_i \left(\sum_{j \in \mathcal{B}} \sum_{c \in \mathcal{C}} R_{ij}^{(c)} x_{ij}^{(c)} - \lambda_i\right) + \sum_{j \in \mathcal{B}} \nu_j \left(Z_j - \sum_{i \in \mathcal{N}} \sum_{c \in \mathcal{C}} R_{ij}^{(c)} x_{ij}^{(c)}\right), \tag{11}
\]

where $\mu_i$ and $\nu_j$ are the lagrange multipliers corresponding to the constraints (10b) and (10d), $\mu \in \mathbb{R}^N$ with $[\mu]_i = \mu_i$, $\nu \in \mathbb{R}_+^B$ with $[\nu]_j = \nu_j$, and $\lambda \in \mathbb{R}^N$ with $[\lambda]_i = \lambda_i$. Then, the dual function of (10) is given by

\[
g(\mu, \nu) = \sup_{X \in \mathcal{X}_f, \lambda} L(X, \lambda, \mu, \nu), \tag{12}
\]

where $\mathcal{X}_f$ represents the domain of $X$ that satisfies the constraints (10c) and (10e), defined as

\[
\mathcal{X}_f = \left\{ X : \sum_{i \in \mathcal{N}} x_{ij}^{(c)} = 1, \forall (j, c), \quad x_{ij}^{(c)} \in \{0, 1\}, \forall (i, j, c) \right\}. \tag{13}
\]

The aim first is to solve $\sup_{X \in \mathcal{X}_f, \lambda} L(X, \lambda, \mu, \nu)$ in (12). To this end, a problem, which obtains $g(\mu, \nu)$, is defined as follow:

\[
\max_{X, \lambda} L(X, \lambda, \mu, \nu) \tag{14a}
\]

s.t. $\sum_{i \in \mathcal{N}} x_{ij}^{(c)} = 1, \forall j \in \mathcal{B}, \forall c \in \mathcal{C} \tag{14b}$

\[
x_{ij}^{(c)} \in \{0, 1\}, \forall i \in \mathcal{N}, \forall j \in \mathcal{B}, \forall c \in \mathcal{C} \tag{14c}
\]

An optimal value $\lambda^*$ maximizing the lagrangian (11) is obtained by partial derivative with respect to $\lambda_i$ as follow:

\[
\frac{\partial L(X, \lambda, \mu, \nu)}{\partial \lambda_i} = \frac{1}{\lambda_i} - \mu_i = 0 \rightarrow \lambda_i = \frac{1}{\mu_i}. \tag{15}
\]
Inserting (15) into the lagrangian (11) gives us

\[ \tilde{L}(X, \mu, \nu) = \sum_{i \in N} \log \frac{1}{\mu_i} + \sum_{j \in B} \sum_{c \in C} \sum_{i \in N} R_{ij}^{(c)} x_{ij}^{(c)} (\mu_i - \nu_j) + \sum_{j \in B} \nu_j Z_j - |N|. \] (16)

Now, the aim is to maximize \( \tilde{L}(X, \mu, \nu) \) over \( X \). Since \( X \) should satisfy the constraints (14b) and (14c), the optimal \( X \) is obtained by

\[ x_{ij}^{(c)} = F_{ij}^{(c)}(\mu, \nu), \quad \text{where} \quad F_{ij}^{(c)}(\mu, \nu) = \begin{cases} 1, & \text{if } i = \text{argmax}_{i \in N} R_{ij}^{(c)} (\mu_i - \nu_j), \\ 0, & \text{otherwise}. \end{cases} \] (17)

By substituting (17) into (16), the dual function \( g(\cdot) \) for given \( \mu \) and \( \nu \) is represented as

\[ g(\mu, \nu) = \sum_{i \in N} \log \frac{1}{\mu_i} + \sum_{j \in B} \sum_{c \in C} \max_{i \in N} R_{ij}^{(c)} (\mu_i - \nu_j) + \sum_{j \in B} \nu_j Z_j - |N|. \] (18)

The dual problem for \( \mu \) and \( \nu \) then is represented as follow:

\[
\begin{align*}
\min_{\mu, \nu} & \quad g(\mu, \nu) \\
\text{s.t.} & \quad \nu_j \geq 0, \quad \forall j \in B
\end{align*}
\] (19a)

(19b)

At this point, let us denote the optimal solution of \((\mu, \nu)\) of the problem \((19)\) as \( \mu^* \in \mathbb{R}^N \) with \( [\mu^*]_i = \mu^*_i \) and \( \nu^* \in \mathbb{R}^J_+ \) with \( [\nu^*]_j = \nu^*_j \). In addition, from (15) and (17), we define \( X^* \) and \( \lambda^* \) as

\[
\begin{align*}
\left\{\begin{array}{l}
x_{ij}^{(c)^*} = F_{ij}^{(c)}(\mu^*, \nu^*), \\
\lambda_i^* = \frac{1}{\mu_i^*}
\end{array}\right.
\] (20)

where \( X^* \in \{0, 1\}^{N \times B \times C} \) with \( [X^*]_{ij} = x_{ij}^{(c)^*} \) and \( \lambda^* \in \mathbb{R}^N \) with \( [\lambda^*]_j = \lambda_j^* \). That is, \((X^*, \lambda^*, \mu^*, \nu^*)\) is the dual solution of the problem \((10)\).

A closed-form solution of the problem \((19)\) is difficult to obtain, since \( g(\mu, \nu) \) is a convex function but may be non-differentiable for \( \mu^* \) and \( \nu^* \) as shown in the following lemmas.

**Proposition 1.** \( g(\mu, \nu) \) is a convex function with respect to \( \mu \) and \( \nu \).

**Proof:** The Hessian of the first term of \( g(\mu, \nu) \) in (18) is semi-positive definite, since it is a diagonal matrix with all positive diagonal elements. The second term of \( g(\mu, \nu) \) is also convex, since the maximum of the affine functions is convex. Therefore, the dual function \( g(\mu, \nu) \) is convex respect to \( \mu \) and \( \nu \). \[\blacksquare\]
Theorem 1. The solution \((\mu^*, \nu^*)\) of the dual problem (19) does not satisfy KKT condition if and only if there exist \(j \in B\) such that \(\nu_j^* > 0\), or \(\nu^* = 0\) and \(N < \sum_{j \in B} \sum_{c \in C} \max_{i \in \mathcal{N}} \frac{R_{ij}^{(c)}}{\sum_{k \in B} \sum_{l \in C} R_{ik}^{(l)} x_{ik}^{(l)}}\). In other words, KKT conditions are satisfied at \((\mu^*, \nu^*)\) only if \(\nu^* = 0\) and \(N \geq \sum_{j \in B} \sum_{c \in C} \max_{i \in \mathcal{N}} \frac{R_{ij}^{(c)}}{\sum_{k \in B} \sum_{l \in C} R_{ik}^{(l)} x_{ik}^{(l)}}\).

Proof: The KKT condition of the problem (19) is denoted as

\[
\begin{align*}
\frac{\partial g(\mu, \nu)}{\partial \nu_j} & \geq 0, \quad \forall j \in B \\
\nu_j \frac{\partial g(\mu, \nu)}{\partial \nu_j} & = 0, \quad \forall j \in B \\
\frac{\partial g(\mu, \nu)}{\partial \mu_i} & = 0, \quad \forall i \in \mathcal{N}
\end{align*}
\] (21)

i) \(\nu_j^* > 0\) for any \(j \in B\):
Let us assume that \(\nu_j^* > 0\) for some \(j \in B\). Because \(R_{ij}^{(c)}\), \(i = 1, \ldots, N\), \(j = 1, \ldots, B\), \(c = 1, \ldots, C\), are continuous random variables, for any given binary matrix \(X\) and real-valued constant \(Z\), we can have \(Z_j = \sum_{i \in \mathcal{N}} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)}\) with probability 0. Since inserting \((\mu^*, \nu^*)\) into (18) gives us

\[g(\mu^*, \nu^*) = \sum_{i \in \mathcal{N}} \log \frac{1}{\mu_i^*} + \sum_{j \in B} \sum_{c \in C} \sum_{i \in \mathcal{N}} R_{ij}^{(c)} x_{ij}^{(c)} \left(\mu_i^* - \nu_j^*\right) + \sum_{j \in B} \nu_j Z_j - |\mathcal{N}|,\] (22)

we have the following with probability 1:

\[
\left. \frac{\partial g(\mu, \nu)}{\partial \nu_j} \right|_{(\mu^*, \nu^*)} = Z_j - \sum_{i \in \mathcal{N}} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)} \neq 0, \forall j \in B.
\] (23)

From (23) and \(\nu_j^* > 0\), we have

\[
\left. \nu_j^* \frac{\partial g(\mu, \nu)}{\partial \nu_j} \right|_{(\mu^*, \nu^*)} \neq 0, \quad (24)
\]

which contradicts the second KKT optimality condition in (21).

ii) \(\nu^* = 0\):
We shall show that the solution \((\mu^*, \nu^*)\) of the dual problem (19) does not satisfies the KKT condition of (19) if \(\nu^* = 0\) and \(N < \sum_{j \in B} \sum_{c \in C} \max_{i \in \mathcal{N}} \frac{R_{ij}^{(c)}}{\sum_{k \in B} \sum_{l \in C} R_{ik}^{(l)} x_{ik}^{(l)}}\). Suppose that \(g(\mu^*, 0)\) satisfies the KKT conditions in (21), then we have

\[
\left. \frac{\partial g(\mu, \nu)}{\partial \mu_i} \right|_{(\mu^*, 0)} = -\frac{1}{\mu_i^*} + \sum_{j \in C} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)} = 0,
\] (25)

From (25), we have

\[
\mu_i^* = \frac{1}{\sum_{j \in C} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)}}, \quad \forall i \in \mathcal{N}.
\] (26)
Because $X^*$ is defined as the global maximization function $F_{ij}^{(c)}$ over $X_f$ as in (17), we have

$$g(\mu^*, 0) = \tilde{L}(X^*, \mu^*, 0) \geq \tilde{L}(X, \mu^*, 0), \quad \forall X \in X_f,$$  \hspace{1cm} (27)

which implies that

$$\sum_{i \in N} \log \frac{1}{\mu_i^*} + \sum_{j \in B} \sum_{c \in C} \sum_{i \in N} R_{ij}^{(c)} x_{ij}^{(c)} \mu_i^* \geq \sum_{i \in N} \log \frac{1}{\mu_i^*} + \sum_{j \in B} \sum_{c \in C} \sum_{i \in N} R_{ij}^{(c)} x_{ij}^{(c)} \mu_i^*, \quad \forall X \in X_f. \hspace{1cm} (28)$$

Inserting (26) into (28) gives us

$$N \geq \sum_{i \in N} \frac{V_i(X)}{V_i(X^*)}, \quad \forall X \in X_f,$$  \hspace{1cm} (29)

where $V_i(X) = \sum_{j \in B} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)}$ and $V_i(X^*) = \sum_{j \in B} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)}$.

Therefore, if $g(\mu^*, 0)$ satisfies the KKT condition, $N \geq \sum_{i \in N} \frac{V_i(X)}{V_i(X^*)}, \forall X \in X_f$. At this point, the negation of the statement ‘$N \geq \sum_{i \in N} \frac{V_i(X)}{V_i(X^*)}, \forall X \in X_f$’ is given by

$$N < \max_{X \in X_f} \frac{\sum_{i \in N} V_i(X)}{\sum_{i \in N} V_i(X^*)} \hspace{1cm} (30)$$

$$= \max_{X \in X_f} \sum_{j \in B} \sum_{c \in C} \sum_{i \in N} \frac{R_{ij}^{(c)} x_{ij}^{(c)}}{\sum_{k \in B} \sum_{l \in C} R_{lk}^{(c)} x_{lk}^{(c)}} \hspace{1cm} (31)$$

$$= \max_{X \in X_f} \sum_{j \in B} \sum_{c \in C} \frac{R_{ij}^{(c)}}{\sum_{k \in B} \sum_{l \in C} R_{lk}^{(c)} x_{lk}^{(c)}} \hspace{1cm} (32)$$

$$= \sum_{j \in B} \sum_{c \in C} \max_{x_{ij} \in X_f} \frac{R_{ij}^{(c)}}{\sum_{k \in B} \sum_{l \in C} R_{lk}^{(c)} x_{lk}^{(c)}} \hspace{1cm} (33)$$

$$= \sum_{j \in B} \sum_{c \in C} \max_{i \in N} \frac{R_{ij}^{(c)}}{\sum_{k \in B} \sum_{l \in C} R_{lk}^{(c)} x_{lk}^{(c)}}. \hspace{1cm} (34)$$

Finally, the contrapositive of the aforementioned proposition gives us ‘if $N < \sum_{j \in B} \sum_{c \in C} \max_{i \in N} \frac{R_{ij}^{(c)}}{\sum_{k \in B} \sum_{l \in C} R_{lk}^{(c)} x_{lk}^{(c)}}$, then $(\mu^*, \nu^*)$ does not satisfy KKT condition (21).’

According to i) and ii), the solution $(\mu^*, \nu^*)$ does not satisfy the KKT condition (21) if and only if there exists $j \in B$ such that $\nu_j^* > 0$, or $\nu^* = 0$ and $N < \sum_{j \in B} \sum_{c \in C} \max_{i \in N} \frac{R_{ij}^{(c)}}{\sum_{k \in B} \sum_{l \in C} R_{lk}^{(c)} x_{lk}^{(c)}}$. \hspace{1cm} $\blacksquare$

Though happening with small probability, if the dual solution can satisfy the KKT conditions, then the solution becomes globally optimal, not just locally optimal, as shown by the following lemma.

**Lemma 1.** If the solution $(\mu^*, \nu^*)$ satisfies the KKT condition of the dual problem (19), then $X^*$ is the global optimal solution of the problem (10).

**Proof:** Suppose that there exists a global optimal solution of the problem (10) and $(\mu^*, \nu^*)$ satisfies
the KKT conditions (21) of the problem (19). Then, \( \nu^* = 0 \) because \( \frac{\partial g(\mu, \nu)}{\partial \nu_j} \bigg|_{\nu = \nu^*} \neq 0 \) for all \( j \in B \), as shown in (23). Suppose that the global maximum point \( X^g \) exists. Because \( \nu^* = 0 \), we need to have \( N \geq \sum_{i \in N} \frac{V_i(X^g)}{V_i(X^*)} \) as shown in (29). Because the objective function of the problem (10) is \( \log \prod_{i \in N} V_i(X) \), we have

\[
\prod_{i \in N} \frac{V_i(X^g)}{V_i(X^*)} \geq 1, \quad \forall X^* \in \mathcal{X}_f. \tag{35}
\]

We further have

\[
\frac{\sum_{i \in N} \frac{V_i(X^g)}{V_i(X^*)}}{N} \geq \left( \prod_{i \in N} \frac{V_i(X^g)}{V_i(X^*)} \right)^{\frac{1}{N}} \geq 1, \quad \forall X^* \in \mathcal{X}_f, \tag{36}
\]

where equality (a) holds if only if \( X^* = X^g \). This gives us

\[
\sum_{i \in N} \frac{V_i(X^g)}{V_i(X^*)} \geq N, \quad \forall X^* \in \mathcal{X}_f. \tag{37}
\]

From (29), we have \( \sum_{i \in N} \frac{V_i(X^g)}{V_i(X^*)} \leq N \), which, combined with (37), yields \( \sum_{i \in N} \frac{V_i(X^g)}{V_i(X^*)} = N \). Therefore, we get

\[
\frac{\sum_{i \in N} \frac{V_i(X^g)}{V_i(X^*)}}{N} = \left( \prod_{i \in N} \frac{V_i(X^g)}{V_i(X^*)} \right)^{\frac{1}{N}} = 1, \quad \forall X^* \in \mathcal{X}_f, \tag{38}
\]

and hence \( X^* = X^g \). Therefore, if \( X^* \) satisfies the KKT conditions (21), it is the global optimal solution \( X^g \) of the problem (10).

From Lemma 1 if the solution of the dual problem satisfies the KKT conditions, it gives us the global solution. However, this happens with small probability for varying channels, which can be shown by numerical simulations. Furthermore, not every global maximizer satisfies the condition (29), which can be shown by simple counter examples. As a result, an optimal solution in general is difficult to obtain from the dual problem approach with non-exponential computational complexity. In the next section, we propose an alternative approach to solve the original problem (10) with feasible-time computational complexity but achieving much higher proportional fairness.

IV. PROPOSED UA, RA, AND PC ALGORITHM
A. Proposed UA and RA Algorithm for Given PC

We first investigate the optimality condition of a mixed-integer maximization problem, starting with the following definition.

**Definition 1.** [27] [Sufficient conditions on local optimality] Suppose that \( \mathcal{D} \) is a domain of a combinatorial problem. Then, \( X \in \mathcal{D} \) satisfying the following conditions is defined as a \( p \)-distance (1 ≤ \( p \) ≤ \( n \))
ring solution:

\[ q(\mathbf{X}) \geq q(\mathbf{X}'), \quad \forall \mathbf{X}' \in \{ ||\mathbf{X} - \mathbf{X}'||_0 = p, \mathbf{X}' \in \mathcal{D} \} \tag{39} \]

where \( ||\mathbf{X} - \mathbf{X}'||_0 \) denotes the number of different elements of \( \mathbf{X} \) and \( \mathbf{X}' \).

The 2-distance ring solution of the problem (10) without the backhaul constraint (10d) is derived in Theorem 2.

**Theorem 2.** The indicator \( \mathbf{X} \), which satisfies the conditions (40) and (41), is a 2-distance ring solution of the problem (10) without the constraint (10d). In addition, for any given \( \mathbf{X}_0 \in \mathcal{X}_f, \mathbf{X}_i \) in (42), \( t = 1, 2, \ldots, \) converges to the 2-distance ring solution \( \mathbf{X}^* \).

\[
\zeta^{(c)}_{ij}(\mathbf{X}) = \frac{1}{\sum_{j'' \in \mathcal{B}} \sum_{c' \in \mathcal{C}} R^{(c')}_{ij} x^{(c')}_{ij} - x^{(c')}_{ij}} , \quad \forall i \in \mathcal{N}, \forall j \in \mathcal{B}, \forall c \in \mathcal{C}, \tag{40}
\]

\[
x^{(c)}_{ij} = \begin{cases} 1_{\{i = \text{argmax}_{i' \in \mathcal{N}} R^{(c)}_{i'j} \zeta^{(c)}_{i'j}(\mathbf{X})\}}, & \text{if } j = \left(\left\lfloor \frac{t}{c} \right\rfloor \mod J \right) + 1, c = (t \mod C) + 1, \\ x^{(c)}_{ij[t-1]}, & \text{otherwise}, \end{cases} \tag{41}
\]

where \( \mathbf{X}_i \in \mathcal{X}_f \) with \([\mathbf{X}_i]_{ij} = x^{(c)}_{ij[i]}\).

**Proof:** i) Note that the proportional fairness in (8) is different from all \( \mathbf{X} \) for given \( \mathbf{P} \), because \( R^{(c)}_{ij} \) is continuously distributed. A set of 2-distance points from \( \mathbf{X}^* \) is defined as

\[
A(\mathbf{X}^*) = \{ \mathbf{X} | ||\mathbf{X} - \mathbf{X}^*||_0 = 2, \mathbf{X} \in \mathcal{X}_f \}. \tag{43}
\]

For all \( \mathbf{X} \in A(\mathbf{X}^*) \), \( x^{(c)}_{ij} = x^{(c)*}_{ij} \) for all \((i, j, c) \in \mathcal{N} \times \mathcal{B} \times \mathcal{C}\) except for \( x^{(c')}_{ij'} \neq x^{(c')}_{ij''} \) and \( x^{(c')}_{i'j''} \neq x^{(c')}_{i''j'} \) for some \((j', c') \) and \( i'' \neq i' \). Let \( \mathbf{X}^* \) satisfies the conditions (41). Then, we have

\[
\sum_{i \in \mathcal{N}} R^{(c')}_{ij'} \zeta^{(c)}_{ij}(\mathbf{X}^*) x^{(c')}_{ij'} > \sum_{i \in \mathcal{N}} R^{(c')}_{ij'} \zeta^{(c)}_{ij'}(\mathbf{X}^*) x^{(c')}_{ij'}, \quad \forall \mathbf{X} \in A(\mathbf{X}^*), \tag{44}
\]

where \( \zeta^{(c)}_{ij} \) is represented in (40). A function \( \omega^{(c)}_{j}(\mathbf{X}) \) is defined as

\[
\omega^{(c)}_{j}(\mathbf{X}) = \prod_{i \in \mathcal{N}} \left( \sum_{m \in \mathcal{B}} \sum_{l \in \mathcal{C}} R^{(l)}_{im} x^{(l)}_{im} \right)_{x^{(c)}_{ij} = 0, \forall i \in \mathcal{N}}. \tag{45}
\]
Then, the proportional fairness function \( U(\mathbf{X}, \mathbf{P}) \) in (8) with given \( \mathbf{P} \) is represented as

\[
e^{U(\mathbf{X}, \mathbf{P})} = \omega_j^{(\nu)}(\mathbf{X}) \left( 1 + \sum_{i \in \mathcal{N}} R_{ij}^{(\nu)} (\mathbf{X}) x_{ij}^{(\nu)} \right), \quad \forall j \in \mathcal{B}, \forall \nu \in \mathcal{C}.
\] (46)

From (46), we have

\[
e^{U(\mathbf{X}^*, \mathbf{P})} = \frac{\omega_j^{(\nu)}(\mathbf{X}^*) \left( 1 + \sum_{i \in \mathcal{N}} R_{ij}^{(\nu)} (\mathbf{X}^*) x_{ij}^{(\nu)*} \right)}{\omega_j^{(\nu)}(\mathbf{X}) \left( 1 + \sum_{i \in \mathcal{N}} R_{ij}^{(\nu)} (\mathbf{X}) x_{ij}^{(\nu)} \right)} = \frac{\omega_j^{(\nu)}(\mathbf{X}^*) \left( 1 + \sum_{i \in \mathcal{N}} R_{ij}^{(\nu)} (\mathbf{X}^*) x_{ij}^{(\nu)*} \right)}{\omega_j^{(\nu)}(\mathbf{X}) \left( 1 + \sum_{i \in \mathcal{N}} R_{ij}^{(\nu)} (\mathbf{X}) x_{ij}^{(\nu)} \right)} > 1, \quad \forall \mathbf{X} \in A(\mathbf{X}^*),
\] (47)

which follows from the fact that \( \mathbf{X} = \mathbf{X}^* \) except for \( x_{ij}^{(\nu)*} \neq x_{ij}^{(\nu)} \) and \( x_{ij}^{(\nu)*} \neq x_{ij}^{(\nu)} \) for some \( (j', \nu') \) and \( i'' \neq i' \), and by the definition of \( U(\mathbf{X}, \mathbf{P}) \) in (46). From (44) and (48),

\[
U(\mathbf{X}^*, \mathbf{P}) > U(\mathbf{X}, \mathbf{P}), \quad \forall \mathbf{X} \in A(\mathbf{X}^*).
\] (49)

Therefore, \( \mathbf{X}^* \) is a 2-distance ring solution if \( \mathbf{X}^* \) satisfies (40)-(41).

ii) Note that \( \mathbf{X}_{[t]} \in A(\mathbf{X}_{[t-1]}) \cup \{ \mathbf{X}_{[t-1]} \} \). The definition of \( \mathbf{X}_{[t]} \) in (42) gives us

\[
\sum_{i \in \mathcal{N}} R_{ij}^{(\nu)} (\mathbf{X}_{[t-1]}) x_{ij}^{(\nu)} \geq \sum_{i \in \mathcal{N}} R_{ij}^{(\nu)} (\mathbf{X}_{[t-1]}) x_{ij}^{(\nu)} \forall t \geq 1,
\] (50)

where \( j = \left\lfloor \frac{t}{C} \right \rfloor \mod J + 1 \) and \( \bar{c} = (t \mod C) + 1 \). Then, from (46) and (50), we have

\[
e^{U(\mathbf{X}_{[t]}, \mathbf{P})} = \frac{\omega_j^{(\nu)}(\mathbf{X}_{[t]}) \left( 1 + \sum_{i \in \mathcal{N}} R_{ij}^{(\nu)} (\mathbf{X}_{[t-1]}) x_{ij}^{(\nu)} \right)}{\omega_j^{(\nu)}(\mathbf{X}_{[t-1]}) \left( 1 + \sum_{i \in \mathcal{N}} R_{ij}^{(\nu)} (\mathbf{X}_{[t-1]}) x_{ij}^{(\nu)} \right)} \geq 1, \quad \forall t \geq 1.
\] (51)

Let \( \mathbf{Y}_n \) be defined as \( \mathbf{Y}_n = \mathbf{X}_{[n \bar{c}]} \) for all \( n = 0, 1, \ldots \). Then, from (51), \( U(\mathbf{Y}_n, \mathbf{P}) \) is monotonically increasing with respect to \( n \), i.e., \( U(\mathbf{Y}_n, \mathbf{P}) \leq U(\mathbf{Y}_{n+1}, \mathbf{P}) \) for all \( n = 0, 1, \ldots \). Because the resultant proportional fairness is different for all \( \mathbf{X} \in \mathcal{X}_J \), \( U(\mathbf{Y}_n, \mathbf{P}) \) is strictly increasing for all \( n \leq k \), where \( k = \arg \min \{ \mathbb{M} \mid \mathbf{Y}_m = \mathbf{Y}_{m+1} \} \). From the result in the part i), \( \mathbf{Y}_n \) is a 2-distance ring solution of the problem if \( U(\mathbf{Y}_n, \mathbf{P}) = U(\mathbf{Y}_{n+1}, \mathbf{P}) \). Therefore, \( \mathbf{Y}_n \) converges to a local optimal 2-distance ring solution, which shows that \( \mathbf{X}_{[t]} \) converges to a 2-distance ring solution \( \mathbf{X}^* \).

From Theorem 2, the local optimal 2-distance ring solution \( \mathbf{X}^* \) is obtained from (42) for the problem (10) without the consideration of the backhaul constraint. Now, the aim is to take into account the backhaul constraint. In the dual problem approach, the lagrange variable \( \nu_j \) for the \( j \)-th backhaul constraint plays a role of pricing in (18) on the amount of data transmission of SBS \( j \). Specifically, if
the amount of backhaul left at SBS \( j \) becomes small, \( \nu_j \) increases, resulting in negative \( R_{ij}^{(c)}(\mu_i - \nu_j) \) in the cost function of (18). Thus, the user \( i \) with large \( R_{ij}^{(c)}, c \in C \), shall have larger magnitude of \( R_{ij}^{(c)}(\mu_i - \nu_j) \) with a negative sign, which results in less chance to be allocated for RB \( c \) of SBS \( j \) in the max operation of (18). Therefore, users with smaller rates \( R_{ij}^{(c)} \) are allocated for SBS \( j \) so that the backhaul constraint (10d) of SBS \( j \) is satisfied.

Inspired by the pricing approach, we modify the conditions (40) and (41) as follows:

\[
\zeta_{ij}^{(c)}(X^*) = \frac{1}{\sum_{j' \in B} \sum_{c' \in C} R_{ij'}^{(c')} x_{ij'}^{(c')} x_{ij'}^{(c')^*}} = 0, \quad \forall i \in N, \forall j \in B, \forall c \in C, \quad (52)
\]

\[
x_{ij}^{(c)^*} = \mathbf{1}_{\{i = \arg\max_{i' \in N} R_{ij'}^{(c)}(\zeta_{ij'}^{(c)}(X^*) - \nu_{j'})\}}, \quad \forall i \in N, \forall j \in B, \forall c \in C. \quad (53)
\]

The lagrange multiplier \( \nu_j \) is imposed for pricing the data rate of SBS \( j \) and thereby satisfying the backhaul constraint. Specifically, \( \nu_j \) is obtained from the sub-gradient method as

\[
\nu_j := \left[ \nu_j - \alpha \frac{\partial g(\mu, \nu)}{\partial \nu_j} \right]^+ = \left[ \nu_j - \alpha \left( Z_j - \sum_{i \in N} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)^*} \right) \right]^+, \quad (54)
\]

where \( \alpha \) denotes the step size.

An alternative of sub-gradient method (54) is also proposed based on the cyclic coordinate descent method for faster convergence. In the cyclic coordinate descent method, only one variable from \( \nu_j \) is sequentially updated with the other variables fixed. That is, \( \nu_j \) at the \((t+1)\)-th iteration, denoted by \( \nu_j^{(t+1)} \), is updated by

\[
\nu_j^{(t+1)} = \arg\min_{\gamma \geq 0} g(\nu_1^{(t)}, \ldots, \nu_{j-1}^{(t)}, \gamma, \nu_{j+1}^{(t)}, \ldots, \nu_J^{(t)}), \quad (55)
\]

where \( \gamma \) is obtained by sub-gradient method for \( \nu_j \) in (54). The details of the cyclic coordinate descent algorithm is presented in Algorithm 1.

Note that Algorithm 1 finds the 2-distance ring solution, because the conditions (52) and (53) are identical to (40) to (41) if \( \nu_j = 0 \), i.e., the backhaul constraint is strictly satisfied. If the backhaul constraint of SBS \( j \) is not satisfied for previously found \( X \), \( \nu_j \) is updated by a positive value of \(-\alpha \left( Z_j - \sum_{i \in N} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)^*} \right) \). At the next iteration then, \( \zeta_{ij}^{(c)}(X^*) - \nu_j \) in (53) may become negative. As a result, users with smaller rates \( R_{ij}^{(c)} \) are selected for SBS \( j \) so that the backhaul constraint is satisfied.

In fact, unlike in Theorem 2, the sub-gradient or cyclic coordinate descent method does not always guarantee a 2-distance ring solution due to the additional backhaul constraint. Hence, a gap between the solution of Algorithm 1 and 2-distance points of the solution is derived in Lemma 2.
Lemma 2. The gap of the proportional fairness between the solution of Algorithm \[X^*\] and 2-distance points from the solution is bounded by \(\max_{j \in B} \epsilon_j \nu_j\), where \(\epsilon_j = Z_j - \sum_{i \in N} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)}\), which is 0 if \(C \to \infty\).

Proof: The proof is shown in Appendix A. 

From Lemma 2 as the number of RBs per SBS increases, the local optimal solution is asymptotically guaranteed even with the modified optimality conditions (52) and (53), i.e., Algorithm 1 where \(x_{ij}^{(c)}\) and \(c_{ij}^{(c)}\) are sequentially updated. In this sequential update, because \(X\) does not have any changes if \(\nu_j\) changed with very small amount, the closed-form update for \(\nu_j\) is adjusted. Let \(X_{old} \in X_f\) with \([X_{old}]_{ijc} = x_{ij,old}^{(c)}\) satisfy the conditions (52) and (53). Then, resource allocation of SBS \(j\) on RB \(c\) is changed into user \(i'\), i.e., \(x_{ij}^{(c)} : 1 \to 0\) and \(x_{ij}^{(c)} : 0 \to 1\), if \(R_{ij}^{(c)}(c_{ij}^{(c)}(X_{old}) - \nu_j) \geq \sum_{i \in N} R_{ij}^{(c)}(c_{ij}^{(c)}(X_{old}) - \nu_j)^{x_{ij,old}^{(c)}}\). Then, the \(\nu_j\) value which changes allocation of SBS \(j\) on RB \(c\) into user \(i'\) is denoted

\[
\beta_{i',j,c} = \frac{R_{ij}^{(c)} s_{ij}^{(c)}(X_{old}) - \sum_{i \in N} R_{ij}^{(c)} s_{ij}^{(c)}(X_{old}) x_{ij,old}^{(c)}}{R_{ij}^{(c)} - \sum_{i \in N} R_{ij}^{(c)} x_{ij,old}^{(c)}}, \quad \forall i \in N, \forall j \in B, \forall c \in C. \tag{56}
\]

For any \(j \in B\), because the sign of gradient step for \(\nu_j\) is same with the sign of \(\sum_{i \in N} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)} - Z_j\), the nearest \(\beta_{ij}^{(c)}\) from \(\nu_j\) is denoted as

\[
\nu_j^{(new)} = \begin{cases} 
\min_{i,c} \beta_{i,j,c} | \beta_{i,j,c} > \nu_j \, , & \text{if } Z_j \leq \sum_{i \in N} \sum_{c \in C} R_{ij}^{(c)} x_{ij}^{(c)} \\
\max_{i,c} \beta_{i,j,c} | \beta_{i,j,c} < \nu_j \, , & \text{otherwise}.
\end{cases} \tag{57}
\]

Then, because the aim is to find a nearest value of \(\nu_j\) that changes \(X\), In other words, the step size \(\alpha\) in (54) is replaced by dynamic step size \(\alpha_{\text{dynamic}}\) as follow:

\[
\alpha_{\text{dynamic}} = \max \left( \alpha, \left| \frac{\nu_j - \nu_j^{(new)}}{Z_j - \sum_{i \in N} \sum_{j \in B} R_{ij}^{(c)} x_{ij}^{(c)}} \right| \right). \tag{58}
\]

B. Proposed Power Control Algorithm for Given UA and RA

In Section IV-A the RA and UA for maximizing the proportional fairness is considered for given \(P\). Here, a per-RB PC is proposed to maximize the proportional fairness by allocating power on each RB with given \(X\). In Section III and IV-A the constraint (6) assumes that each RB is always allocated to one of the users. However, allocating an RB to no user should also be considered. Fortunately, this can be taken into account by allocating zero power on the RB. Then, the PC problem for given \(X\) is
Algorithm 1: Cyclic coordinate descent method for the proposed UA and RA algorithm

Initialization: set $\nu_j = 0$, $\forall j$. set $c_{ij}^{(e)} = 0$, $\forall i$.

repeat
   for $\forall j \in B$
      repeat
         for $\forall c \in C$
            1) Update $x_{ij}^{(c)}$ according to (53).
            2) Update $\zeta_{ij}^{(c)}$ according to (52).
      end
      until $\zeta_{ij}^{(c)}$ converges
   3) Update $\nu_j$ according to (55).
end
until $\nu_j$ converges and $Z_j \geq \sum_{i \in \mathcal{N}} \sum_{c \in \mathcal{C}} R_{ij}^{(c)} x_{ij}^{(c)}$, $\forall j \in B$.

Return: indicator $x_{ij}^{(c)}$

formulated from (9) as:

$$P3 : \max_{\mathcal{P}} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{B}} \sum_{c \in \mathcal{C}} R_{ij}^{(c)} x_{ij}^{(c)}$$

s.t
$$\sum_{c \in \mathcal{C}} \sum_{i \in \mathcal{N}} R_{ij}^{(c)} x_{ij}^{(c)} \leq Z_j, \forall j \in \mathcal{B}$$

$$\sum_{c \in \mathcal{C}} p_j^{(c)} \leq P_{j,\text{max}}, \forall j \in \mathcal{B}$$

$$R_{ij}^{(c)} = W \log_2 \left( 1 + \frac{|h_{ij}^{(c)}|^2 p_j^{(c)}}{\sigma^2 + \sum_{k \neq j} |h_{ik}^{(c)}|^2 p_k^{(c)}} \right), \forall i \in \mathcal{N}, \forall j \in \mathcal{B}, \forall c \in \mathcal{C}$$

$$p_j^{(c)} \geq 0, \forall j \in \mathcal{B}, \forall c \in \mathcal{C}$$

To solve the problem (59), we first relax the problem without the constraint (59b), which shall be considered later, as follow:

$$P4 : \max_{\mathcal{P}} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{B}} \sum_{c \in \mathcal{C}} W \log_2 \left( 1 + \frac{|h_{ij}^{(c)}|^2 p_j^{(c)} x_{ij}^{(c)}}{\sigma^2 + \sum_{k \neq j} |h_{ik}^{(c)}|^2 p_k^{(c)}} \right)$$

s.t
$$\sum_{c \in \mathcal{C}} p_j^{(c)} \leq P_{j,\text{max}}, \forall j \in \mathcal{B}$$

$$p_j^{(c)} \geq 0, \forall j \in \mathcal{B}, \forall c \in \mathcal{C}.$$
The aim is to obtain the KKT conditions of the problem (60). The Lagrangian of the problem (60) is represented as

\[ L(P, \xi, \varphi) = \sum_{i \in N} \log \left( \sum_{j \in B} \sum_{c \in C} W \log_2 \left( 1 + \frac{\left| h_{ij}^{(c)} \right|^2 p_j^{(c)} x_{ij}^{(c)}}{\sigma^2 + \sum_{k \neq j} \left| h_{ik}^{(c)} \right|^2 p_k^{(c)}} \right) \right) \]

\[ + \sum_{j \in B} \xi_j \left( P_{j,\text{max}} - \sum_{c \in C} p_j^{(c)} \right) + \sum_{j \in B} \sum_{c \in C} \varphi_j^{(c)} p_j^{(c)}, \]

where \( \xi \in \mathbb{R}^B_{\geq 0} \) with \( \lfloor \xi \rfloor = \xi_j \), and \( \varphi \in \mathbb{R}^{B \times C} \) with \( \lfloor \varphi \rfloor_{jc} = \varphi_j^{(c)} \) are Lagrange multipliers corresponding to the constraints (60b) and (60c). The KKT conditions of the problem (60) are established in Lemma 3.

**Lemma 3.** The KKT conditions of the problem (60) are given by

\[
\begin{align*}
\frac{\partial U(X, P)}{\partial p_j^{(c)}} & = \xi_j, \quad \forall (j, c) \in B \times C \setminus \{(k, l) | p_k^{(l)} = 0\} \quad (63a) \\
\frac{\partial U(X, P)}{\partial p_j^{(c)}} & \leq \xi_j, \quad \forall j \in B, \forall c \in C \quad (63b) \\
p_j^{(c)} & \geq 0, \quad \forall j \in B, c \in C \quad (63c) \\
P_{j,\text{max}} - \sum_{c \in C} p_j^{(c)} & \geq 0, \quad \forall j \in B \quad (63d) \\
\xi_j & \geq 0, \quad \forall j \in B \quad (63e) \\
\xi_j (P_{j,\text{max}} - \sum_{c \in C} p_j^{(c)}) & = 0, \quad \forall j \in B \quad (63f)
\end{align*}
\]

**Proof:** Proof in Appendix B

Now, we propose an algorithm to find a solution that satisfies the KKT conditions in Lemma 3 with the consideration of the backhaul constraint (59b). We first start with the following theorem.

**Lemma 4.** For a local optimal point of the problem (60), there exists at least one \( j \in B \) which satisfies \( \sum_{c \in C} p_j^{(c)} = P_{j,\text{max}} \).

**Proof:** We only consider \( P \neq 0 \), because \( P = 0 \) cannot be a local optimal solution. For convenience, let us vectorize \( P \in \mathbb{R}^{B \times C} \) as \( p = \text{vect}(P) \in \mathbb{R}^{B \times C \times 1} \). For any \( w \in \mathbb{C}^{B \times C \times 1} \) with \( \|w\|_2 = 1 \), the directional derivative of the function \( U(X, p) \) is represented as

\[ D_w(U) = \lim_{\Delta \to 0^+} \frac{U(X, p + \Delta \cdot w) - U(p)}{\Delta} = w^T \nabla_p U(X, p), \quad (64) \]
From (70), we have
\[ \frac{\partial U(X, p)}{\partial p_j} = \left[ \frac{\partial U(X, p)}{\partial p_1}, \frac{\partial U(X, p)}{\partial p_2}, \ldots, \frac{\partial U(X, p)}{\partial p_k} \right]^T. \]

Let \( P_j \) denote the set of \( p \)’s satisfying the constraints (60b) and (60c). Then, for all \( \tilde{p} \in P_j \) except for the trivial case \( \tilde{p} = 0 \), choosing the direction as \( w = \frac{\tilde{p}}{||\tilde{p}||_2} \) gives us
\[ D_w(U) = \frac{1}{||\tilde{p}||_2} \tilde{p}^T \nabla_p U(X, \tilde{p}) \]
\[ = \lim_{\Delta \to 0^+} \frac{U(X, \tilde{p} + \Delta \cdot \frac{\tilde{p}}{||\tilde{p}||_2}) - U(\tilde{p})}{\Delta} \]
\[ = \frac{1}{||\tilde{p}||_2} \frac{\partial U(X, \tilde{p}(1 + \Delta))}{\partial \Delta} \bigg|_{\Delta=0}, \]
where (67) follows from the l’Hôpital’s law. Then, we further have
\[ \frac{\partial U(X, \tilde{p}(1 + \Delta))}{\partial \Delta} \bigg|_{\Delta=0} = \sum_{i \in \mathcal{N}} \sum_{k \in \mathcal{B}} \frac{1}{\sum_{l \in \mathcal{C}} R_{ik}^{(l)}} \sum_{j \in \mathcal{B}} \sum_{c \in \mathcal{C}} \frac{\partial R_{ij}^{(c)}}{\partial p_j^{(c)}(1 + \Delta)} x_{ij}^{(c)} \bigg|_{\Delta=0}. \] (68)

Here, we get
\[ \frac{\partial R_{ij}^{(c)}}{\partial \Delta} \bigg|_{\Delta=0} = \frac{|h_{ij}^{(c)}|^2 \tilde{p}_j^{(c)}}{(1 + \sum_{b \neq j} |h_{ib}^{(c)}|^2 \tilde{p}_b^{(c)}) (1 + \sum_{b \in \mathcal{B}} |h_{ib}^{(c)}|^2 \tilde{p}_b^{(c)})} \geq 0. \] (69)

From (65) to (69) and the fact that \( \tilde{p}_j^{(c)} \geq 0 \) and \( p \neq 0 \), we have
\[ \lim_{\Delta \to 0^+} \frac{U(X, \tilde{p} + \Delta \cdot \frac{\tilde{p}}{||\tilde{p}||_2}) - U(\tilde{p})}{\Delta} = \frac{1}{||\tilde{p}||_2} \tilde{p}^T \nabla_p U(X, \tilde{p}) > 0. \] (70)

From (70), we have
\[ \frac{1}{||\tilde{p}||_2} \tilde{p}^T \nabla_p U(X, \tilde{p}) = \frac{1}{||\tilde{p}||_2} \sum_{j \in \mathcal{B}} \sum_{c \in \mathcal{C}} p_j^{(c)} \frac{\partial U(X, \tilde{p})}{\partial p_j^{(c)}} > 0, \ \forall p \in P_j \setminus \{0\}. \] (71)

From (71), since \( p_j^{(c)} \geq 0 \) for all \( j \in \mathcal{B} \) and \( c \in \mathcal{C} \), there must exist at least one index \((j^*, c^*)\) for some \( j^* \in \mathcal{B} \) and \( c^* \in \mathcal{C} \) such that
\[ \frac{\partial U(X, P)}{\partial p_j^{(c^*)}} > 0. \] (72)

Therefore, for any \( p \in P_j \), there exists at least one positive element of \( \nabla_p U(X, p) \).

Now, we prove the converse. Let us denote the local optimal solution of the problem (60) by \( P^* \). Now, suppose that for all \( j \in \mathcal{B} \), \( P_{j,\text{max}} - \sum_{c \in \mathcal{C}} p_j^{(c^*)} > 0 \). Then, since the local optimal solution satisfies
the KKT conditions in (\ref{eq:KKT}), $\xi_j = 0$ from (\ref{eq:KKT}). In addition, from (\ref{eq:KKT}), we have
\[
\frac{\partial U(X, P)}{\partial p_j^{(c)}} \leq 0, \quad \forall j \in \mathcal{B}, \forall c \in \mathcal{C}.
\] (\ref{eq:73})

Because (\ref{eq:73}) contradicts to (\ref{eq:72}), for local optimal solution $P^*$, it is not possible for all $j \in \mathcal{B}$, $P_{j,\max} - \sum_{c \in \mathcal{C}} P_j^{(c)} > 0$.

From Lemma \ref{lemma:4}, at least one $j \in \mathcal{B}$ exists that satisfies $P_{j,\max} = \sum_{c \in \mathcal{C}} P_j^{(c)}$. Let $\mathcal{B}_{eq} = \{j | P_{j,\max} = \sum_{c \in \mathcal{C}} P_j^{(c)} \}$ and $\mathcal{B}_{neq} = \{j | P_{j,\max} > \sum_{c \in \mathcal{C}} P_j^{(c)} \}$. Then, from (\ref{eq:KKTa}) and (\ref{eq:KKTb}), we have
\[
\frac{\partial U(X, P)}{\partial p_j^{(c)}} = \xi_j, \quad \text{for all } j \in \mathcal{B}_{eq}, c \in \mathcal{C} \setminus \{(k, l) | p_k^{(l)} = 0\} \text{ at the optimal point, where } \xi_j > 0.
\]
On the other hand, we have
\[
\frac{\partial U(X, P)}{\partial p_j^{(c)}} = 0, \quad \text{for all } j \in \mathcal{B}_{neq}, c \in \mathcal{C} \setminus \{(k, l) | p_k^{(l)} = 0\} \text{ at the optimal point, where } \xi_j = 0.
\]
Therefore, we aim to design $P$ such that $\frac{\partial U(X, P)}{\partial p_j^{(c)}}$ for the RBs with non-zero transmission power become all identical within the same SBS. Since it is difficult to obtain the solution $P$ in a closed-form, we propose a sequential update of $P$ from a SBS to another SBS.

Fig. \ref{fig:1} depicts the example of the proposed algorithm in case of SBS $j \in \mathcal{B}_{eq}$, i.e., $\frac{\partial U(X, P)}{\partial p_j^{(c)}}$ for all RBs of SBS $j$ with non-zero transmission power are identical to a positive constant. The left figure of Fig. \ref{fig:1} depicts $\frac{\partial U(X, P)}{\partial p_j^{(c)}}$ with the initial $P$. We select two RBs, RB 2 and 4, and take $\Delta p$ from the transmission power for RB 4 to give it to RB 2. For small $\Delta p$, this only makes the cost function $U(X, P)$ increase, because $\frac{\partial U(X, P)}{\partial p_j^{(c)}} > 0$ and $\frac{\partial U(X, P)}{\partial p_j^{(c)}} < 0$. In addition, the exchange of $\Delta p$ between the two RBs does not break the sum-power constraint. We do this until convergence as in the right figure of Fig. \ref{fig:1}.

Now, the concern is two-fold: 1) consideration of the RBs with zero power and 2) SBSs with $P_{j,\max} > \sum_{c \in \mathcal{C}} P_j^{(c)}$.

The first concern can be completely resolved if we take $\Delta p$ from the transmission power for the RB with the smallest $\frac{\partial U(X, P)}{\partial p_j^{(c)}}$ at each iteration. Suppose that all the transmission power of RB $c$ is taken at the \( t \)-th iteration. According to our design choice, this means $\frac{\partial U(X, P)}{\partial p_j^{(c)}} \leq \frac{\partial U(X, P)}{\partial p_j^{(c)}}$ for all $c' \in \mathcal{C} \setminus c$. At the next iteration, the power exchange shall be continued for the RBs with non-zero transmission power.
At convergence, we will have \( \frac{\partial U(X, P)}{\partial p_j^{(e)}} \leq \frac{\partial U(X, P)}{\partial p_j^{(e)}} \) and \( \left| \frac{\partial U(X, P)}{\partial p_j^{(e)}} - \xi_j \right| \leq \varepsilon, \varepsilon > 0 \), for all \( \tilde{c} \in C_{\text{active}} \) and \( \tilde{c} \in C_{\text{inactive}} \), where \( C_{\text{active}} = \{ c | p_j^{(e)} > 0, c \in C \} \) and \( C_{\text{inactive}} = \{ c | p_j^{(e)} = 0, c \in C \} \). Therefore, the KKT condition for the RB with zero-transmission power, (635), can be also satisfied.

The second concern can be resolved as follow. After convergence, if all the \( \frac{\partial U(X, P)}{\partial p_j^{(e)}} \) values for SBS \( j \) become identical to a positive constant or zero, our assumption \( P_{j, \text{max}} = \sum_{c \in C} p_j^{(e)} \) holds true. On the other hand, if \( \frac{\partial U(X, P)}{\partial p_j^{(e)}} \) values for SBS \( j' \) become identical to a negative constant, the full transmission power for SBS \( j' \) cannot be assumed at the local optimal point. Instead, we should have \( P_{j', \text{max}} > \sum_{c \in C} p_j^{(e)} \). Then, we can repeat the algorithm modifying the total transmission power as \( \sum_{c \in C} p_j^{(e)} = P_{j', \text{max}} - \Delta p_{\text{total}} \) for a step size \( \Delta p_{\text{total}} > 0 \).

In what follows, detailed parameters optimization are presented, followed by the overall proposed algorithm.

1) Selection of the Two RBs for the Transmission Power Exchange: Let us update \( p_j^{(c_1)} \) and \( p_j^{(c_2)} \) at the \( t \)-th iteration in design of the transmission power for SBS \( j \). The updated power is denoted as \( P_{[t]} \in \mathbb{R}^{B \times C} \), where \( [P_{[t]}]_{jc} = p_j^{(e)} \) is defined by

\[
p_j^{(e)} = \begin{cases} p_j^{(e)}_{[t-1]} + \Delta p_j^{(e)}_{[t]}, & \text{if } c = c_1, \\ p_j^{(e)}_{[t-1]} - \Delta p_j^{(e)}_{[t]}, & \text{if } c = c_2, \\ p_j^{(e)}_{[t-1]}, & \text{otherwise.} \end{cases} \tag{74}
\]

Here, \( \Delta p_j^{(e)}_{[t]} \) denotes the amount of transmission power exchange between the selected RBs at the \( t \)-th design iteration in SBS \( j \). Then, the proportional fairness with \( P_{[t]} \) is approximated for small \( \Delta p_j^{(e)}_{[t]} \) by Taylor series with respect to \( \Delta p_j^{(e)}_{[t]} \) as follow:

\[
U(X, P_{[t]}) = U(X, P_{[t-1]}) + \left( \frac{\partial U(X, P)}{\partial p_j^{(c_1)}} - \frac{\partial U(X, P)}{\partial p_j^{(c_2)}} \right)_{P=P_{[t-1]}} \Delta p_j^{(e)}_{[t]} + O(\Delta p_j^{(e)}_{[t]}^2). \tag{75}
\]

In order to maximize (75) for given \( \Delta p_j^{(e)}_{[t]} \), the two RBs \( c_1 \) and \( c_2 \) are chosen by

\[
c_1 = \arg \max_{c \in C} \frac{\partial U}{\partial p_j^{(c_1)}} \bigg|_{P=P_{[t-1]}} \quad \text{and} \quad c_2 = \arg \min_{c \in C, p_j^{(e)}_{[t-1]} \neq 0} \frac{\partial U}{\partial p_j^{(c_2)}} \bigg|_{P=P_{[t-1]}}. \tag{76}
\]

2) Design of the Power Exchange: For given RB indices \( (c_1) \) and \( (c_2) \) for SBS \( j \), \( \Delta p_j^{(e)}_{[t]} \) is designed to satisfy the optimality condition \( \frac{\partial U(X, P)}{\partial p_j^{(c_1)}} = \frac{\partial U(X, P)}{\partial p_j^{(c_2)}} \). Let us denote the difference in the partial derivatives at the \( t \)-th iteration as

\[
f(P_{[t]}) = f_1(P_{[t]}) - f_2(P_{[t]}), \tag{77}
\]
where \( f_i(P_{[t]} = \frac{\partial U(X, P)}{\partial p_j^{(c)}} |_{P=P_{[t]}} \) is defined over \( i \in \{1, 2\} \). The following proposition establishes the approximate of the optimal \( \Delta p_{j,[t]} \) in a closed-form.

**Proposition 2.** For any given RB \( c_1 \) and \( c_2 \) of SBS \( j \), \( \Delta p_{j,[t]} \) leading to \( f(P_{[t]} = O(\Delta p_{j,[t]}^3) \) is obtained by

\[
\Delta p_{j,[t]}^* = -f'(P_{[t-1]} + \text{sgn}[f'(P_{[t-1]}]) \frac{\sqrt{f'(P_{[t-1]}^2)} - 2f''(P_{[t-1]}f'(P_{[t-1]})}{f''(P_{[t-1]})},
\]

(78)

where \( \text{sgn}[\cdot] \) denotes the sign function, \( f''(P_{[t-1]} = f_1^{(2)}(P_{[t-1]} - f_2^{(2)}(P_{[t-1]}), \) and \( f'(P_{[t-1]} = f_1^{(1)}(P_{[t-1]} - f_2^{(1)})(P_{[t-1]}). \) Here, with the assumption \( \lambda_i \gg W \), the functions are given by

\[
f_i(P_{[t-1]} = \frac{\partial U(X, P)}{\partial p_j^{(c)}} |_{P=P_{[t-1]}} = \sum_{k \in B} \lambda_{i(k,c_i)} \ln 2 - \sum_{k \neq j} \lambda_{i(k,c_i)} \ln 2,
\]

(79)

\[
f_i^{(1)}(P_{[t-1]} = \frac{\partial f_i(P_{[t-1]} = 0}{\partial \Delta p_j^{(c)}} |_{\Delta p_j^{[t]}=0} = \sum_{k \in B} (-1)^j S^2 \lambda_{i(k,c_i)} \ln 2 - \sum_{k \neq j} (-1)^j Q^2 \lambda_{i(k,c_i)} \ln 2,
\]

(80)

\[
f_i^{(2)}(P_{[t-1]} = \frac{\partial^2 f_i(P_{[t-1]} = 0}{\partial \Delta p_j^{(c)}} |_{\Delta p_j^{[t]}=0} = \sum_{k \in B} \frac{2S^3}{\lambda_{i(k,c_i)} \ln 2} - \sum_{k \neq j} \frac{2Q^3}{\lambda_{i(k,c_i)} \ln 2},
\]

(81)

where \( S = W \left[ \frac{\text{SNR}_{k,c_i}^{(c)}}{1+\sum_{b \in B} \text{SNR}_{k,c_i}^{(c)} p_b^{(c)} \text{SNR}_{k,c_i}^{(c)} p_{b[k-1]}^{(c)}} \right], Q = W \left[ \frac{\text{SNR}_{k,c_i}^{(c)}}{1+\sum_{b \neq k} \text{SNR}_{k,c_i}^{(c)} p_b^{(c)} \text{SNR}_{k,c_i}^{(c)} p_{b[k-1]}^{(c)}} \right], i(k,c) \) denotes the user served by SBS \( k \) on RB \( c \), \( \text{SNR}_{ij}^{(c)} = \frac{|h_{ij}^{(c)}|^2}{\sigma^2}, i(k,c) = \arg\max_{i \in N} x_{i,k}^{(c)} \) and \( \lambda_{i(k,c)} = \sum_{j \in B} \sum_{c \in C} R_{ij}^{(c)} x_{i,j}^{(c)}, \)

\[\text{Proof: Proof is in Appendix C.}\]

According to Proposition 2, \( \Delta p_{j,[t]}^* \) chosen as (78) gives us the KKT optimality \( f(P_{[t]} \rightarrow 0 \) as iteration grows, where \( \Delta p_{j,[t]} < 1 \). On the other hand, however, the backhaul constraint (59b) should be taken into consideration by limiting the maximum possible value of \( \Delta p_{j,[t]} \), as shown in Proposition 3.

**Proposition 3.** For given \( X \), the maximum possible \( \Delta p_{j,[t]} \) that satisfies the backhaul capacity is obtained in terms of the remaining backhaul capacity of SBSs as

\[
\Delta p_{j,[t]} \leq \min \left\{ \min_{k \in B \setminus \{j\}} \left\{ \frac{\frac{-L_j(X, P_{[t-1]})}{w}}{p_{j,[t-1]} - 1} \right\}, \left\{ \frac{\frac{-L_j(X, P_{[t-1]})}{w}}{p_{j,[t-1]} - 1} \right\}, \Delta p_{j,[t]}^* \right\},
\]

(82)

where \( L_j(X, P_{[t]} = Z_j - \sum_{i \in N} \sum_{c \in C} R_{ij}^{(c)} x_{i,j}^{(c)} \left|_{P=P_{[t]}} \right. \left| \right. \)

\[\text{Proof: shown in Appendix D}\]

From the Proposition 3 the power exchange with backhaul consideration \( \Delta p_{j,[t]} \) is denoted as

\[
\Delta p_{j,[t]} = \min \left\{ \min_{k \in B \setminus \{j\}} \left\{ \frac{\frac{-L_j(X, P_{[t-1]})}{w}}{p_{j,[t-1]} - 1} \right\}, \left\{ \frac{\frac{-L_j(X, P_{[t-1]})}{w}}{p_{j,[t-1]} - 1} \right\}, \Delta p_{j,[t]}^* \right\},
\]

(83)
Then, we may have $P_{[t]}$ that satisfies the KKT condition (63) except (63e) with the proposed algorithm; that is, $\frac{\partial U(X,P)}{\partial p_j^{(e)}} \leq \xi_j$ and $\frac{\partial U(X,P)}{\partial \hat{p}_j^{(e)}} = \xi_j$ for all $j \in B$, $\hat{c} \in C_{\text{active}}$, $\tilde{c} \in C_{\text{inactive}}$, if the backhaul constraint is irrelevant, i.e., $Z_j - \sum_{i \in N} \sum_{c \in C} R_{ij} x_{ij}^{(c)}$ is relatively large.

3) Design of $\Delta p_{\text{total}}$: If the backhaul constraint is very tight, the KKT condition (63) cannot be satisfied with the current transmission power assumption. In such case, although the proposed algorithm ends up with the same conditions, $\xi_j$ is negative, which contradicts to (63e). At convergence after repeating Section IV-B1 and IV-B2 at $t = T_n$, if $\xi_j < 0$, then the sum-power should be reduced to satisfy the KKT conditions (63). Since the gradient of the Lagrangian $L(P, \xi, \varphi)$ in (61) is $\xi_j$, the gradient method yields the updated sum-power constraint for SBS $j$ with $\xi_j < 0$ being $\sum_{c \in C} p_{j,[T_n]}^{(c)} = \sum_{c \in C} p_{j,[T_n]}^{(c)} + \Delta p_{\text{total}}^{(n)}$, which denoted by

$$
\Delta p_{\text{total}}^{(n)} = \min\{\gamma \xi_j, P_{j,\text{max}} - \sum_{c \in C} p_{j,[T_n]}^{(c)}\},
$$

(84)

where $\gamma$ denotes step size.

C. Overall Algorithm

The overall proposed PC algorithm is summarized in Fig. 2.
TABLE I
COMPLEXITY COMPARISON IN FLOPS

| Algorithm                        | UA and RA | PC            |
|----------------------------------|-----------|---------------|
| energy-constrained FFRA [14]     | $O \left( \frac{JN}{\epsilon^2} \right)$ | -             |
| modified unconstrained FFRA [13] | $O((I_{SBS})JN)$ | $O((I_{SBS})J^2)$ |
| proposed algorithm               | $O(I_{\xi}I_{\nu}(NC))$ | $O(I_P(J^2 + J^2C))$ |
| genetic algorithm                | $O(G_{\text{max}}M_pJ^2NC^2)$ | -             |
| global optimal solution          | $O \left( N^{BC} \right)$ | -             |

D. Complexity Analysis

In this subsection, the computational complexity in flops for the proposed RA algorithm based on Algorithm 2 and PC algorithm based on Fig. 2 is analyzed in comparison to the existing approaches in Table I. For comparison, the energy-constrained fractional frequency RA (FFRA) algorithm [14] and unconstrained FFRA algorithm [13] are considered, where the RA problem is relaxed to a continuous optimization problem with real-valued variables. For fair comparison, we have modified the unconstraint FFRA algorithm to consider the backhaul constraint and PC.

The overall proposed algorithm has computational complexity of $O \left( I_{\xi}I_{\nu}(NC) + I_P(J^2 + J^2C) \right)$, where $I_{\xi}$, $I_P$, and $I_{\nu}$ denote the numbers of iterations needed for $\xi$, $P$, and $\nu$ to converge, respectively. The global optimal solution of the problem (10) requires exponential computational complexity of $O \left( N^{BC} \right)$. The energy-constrained FFRA has the computational complexity of $O \left( \frac{JN}{\epsilon^2} \right)$ overall, where $\epsilon$ denotes the convergence threshold. The computational complexity of the modified unconstrained FFRA is $O((I_{SBS})BN) + O((I_{SBS})B^2)$, where $I_{SBS}$ denotes the number of iterations. The overall computational complexity of the genetic algorithm is $O(G_{\text{max}}M_pJ^2NC^2)$, where $G_{\text{max}}$ and $M_p$ denote the maximum generation and the population size, respectively. Numerical results for the computational complexity shall be presented in Section V.

V. NUMERICAL RESULTS

The proportional fairness of the proposed scheme is numerically evaluated in the 3GPP small cell scenario 2a. That is, a small cell is interfered only by other small cells, not macro cells [26]. The system parameters in table II are based on [26], [28], which are used for the simulations.

The proportional fairness $U(X, P)$ of the proposed scheme, i.e. UA and RA in Algorithm I and PC in Fig. 2 versus the number of iterations is depicted in Fig. 3 in order to show the convergence of $\xi$, $\nu$ and $P$. As shown in the figure, $\xi$ converges in Algorithm I within a reasonable number of iterations,
TABLE II
SIMULATION PARAMETERS

| Parameter                              | Value                        |
|----------------------------------------|------------------------------|
| Network model                          | 3GPP scenario 2a [26]        |
| Number of small cells                  | 4                            |
| Number of users in the cluster         | 40                           |
| Number of resource blocks              | 100                          |
| $W$ (kHz)                              | 180                          |
| Backhaul capacity (Mbps)               | 20-100                       |
| Transmission power (dBm)               | 35                           |
| Noise power (dBm/Hz)                   | -174                         |
| Cluster diameter (m)                   | 1000                         |
| Pathloss (dB)                          | $38 + 30 \log_{10}(d)$      |

which is usually less than 10. Though, the iterations needed for convergence of $\nu$ and $P$ are around 400 and 2000, respectively, only a few iterations also give us relatively high performance.

In Fig. 4, the proportional fairness is shown with respect to the backhaul capacity. The proposed scheme is implemented with the high complexity ($I_{\zeta} = 10$, $I_{\nu} = 400$, $I_{P} = 2000$) and low complexity ($I_{\zeta} = 1$, $I_{\nu} = 40$, $I_{P} = 10$) settings, requiring orders of 176160 and $2 \cdot 10^7$ flops, respectively. On the other hand, the energy-constrained FFRA with $\epsilon = 0.03$ [14] and modified unconstrained FFRA with $I_{SBS} = 56$ [13] require the overall complexity of orders of 177777 and 98564 flops, respectively. It is shown that the proposed scheme with both of the settings significantly outperform the previous approaches, obtaining higher frequency diversity gain due to finer per-RB UA, RA, and PC. Particularly, the proposed scheme with the low complexity setting achieves higher proportional fairness than the energy-constrained FFRA even with lower computational complexity for the backhaul capacity higher than 20Mbps.

For comparison, the outer bound without power constraint is considered, which can be derived using the inequality/equality for arithmetic and geometric averages, as shown in Proposition 1 of [15]. The proposed scheme asymptotically achieves this outer bound as the backhaul capacity becomes small, where the power constraints become satisfied in the derivation of the outer bound, yielding a tight upper bound. On the other hand, in pursuit of finding the global optimal solution, we also consider the genetic algorithm but without backhaul capacity. For the genetic algorithm, we considered 100 population, 0.8 crossover fraction, $10^5$ maximum generations, and 50 elite counts. For convergence of one simulation environment, the genetic algorithm required about 5 hours whereas the proposed scheme with the high complexity converged within 1 min on average. As the backhaul capacity increases, the proposed scheme asymptotically achieves the performance of the genetic algorithm without power.
constraint with much lower computational complexity.

VI. CONCLUSIONS

We have considered a UA, RA, and PC problem with limited backhaul capacity to maximize the proportional fairness of cooperative multicell networks. We have proposed a cascaded iterative algorithm to solve the problem and show the achievability of the optimal solution. The simulation results have shown that the proposed scheme closely achieves the globally optimal proportional fairness, which can be obtained with exponential computational complexity, at the cost of reasonably increased computational complexity compared with the existing schemes.

APPENDIX A
PROOF OF LEMMA 2

Let \( X^* \) be a solution from Algorithm 1 and \( \epsilon_j = Z_j - \sum_{i \in \mathcal{N}} \sum_{c \in \mathcal{C}} R_{ij}^{(c)} x_{ij}^{(c)*} \). Then, for any \( X' \in A(X^*) \) in (43) that satisfies the constraint of backhaul capacity (100), we have \( \epsilon_j \geq 0 \) for all \( j \in \mathcal{B} \) and

\[
\sum_{i \in \mathcal{N}} R_{ij}^{(c)} \left( \zeta_{ij}^{(c)}(X^*) - \nu_j \right) x_{ij}^{(c)*} \geq \sum_{i \in \mathcal{N}} R_{ij}^{(c)} \left( \zeta_{ij}^{(c)}(X^*) - \nu_j \right) x_{ij}^{(c)*}, \forall (j, c) \in \mathcal{B} \times \mathcal{C}, \forall X' \in A(X^*), \quad (85)
\]

From (85), we get

\[
\sum_{i \in \mathcal{N}} R_{ij}^{(c)} \zeta_{ij}^{(c)}(X^*) x_{ij}^{(c)*} - R_{ij}^{(c)} \zeta_{ij}^{(c)}(X^*) x_{ij}^{(c)*} \leq \sum_{i \in \mathcal{N}} \nu_j R_{ij}^{(c)} (x_{ij}^{(c)*} - x_{ij}^{(c)*}) \leq \epsilon_j \nu_j, \forall (j, c) \in \mathcal{B} \times \mathcal{C}, \forall X' \in A(X^*), \quad (86)
\]
Let X and X' be different for only \((j, c)\). Then, first term of (86) is bounded by

\[
\sum_{i \in \mathcal{N}} \left( R_{ij}^{(e)} \zeta_{ij}^{(e)}(X^*) x_{ij}^{(e)} - R_{ij}^{(e)} \zeta_{ij}^{(e)}(X^*) x_{ij}^{(e)} \right) = \frac{R_{ij}^{(e)}(X^*)}{V_T(X^*, j, c)} - \frac{R_{ij}^{(e)}(X^*)}{V_T(X^*, j, c)} = \sum_{i \in \mathcal{N}} \left( \frac{V_T(X^*, i)}{V_T(X^*, j, c)} - \frac{V_T(X^*, i)}{V_T(X^*, j, c)} \right) = \frac{\prod_{i \in \mathcal{N}} V_i(X') - \prod_{i \in \mathcal{N}} V_i(X^*)}{\prod_{i \in \mathcal{N}} V_i(X^*)} \geq e^{U(X', P)} - \frac{e^{U(X', P)}}{e^{U(X^*, P)}} - 1 \tag{87}
\]

where \(T(X, j, c) = \arg\max_{i \in \mathcal{N}} x_{ij}^{(e)}\) and \(V_i(X) = \sum_{j \in B} \sum_{c \in C} R_{ij}^{(e)} x_{ij}^{(e)}\). From (86) and (87), because \(\hat{j}\) is determined by \(X'\), we have

\[
U(X', P) - U(X^*, P) \leq \log(1 + \epsilon_j \nu_j) \leq \max_{j \in B} \log(1 + \epsilon_j \nu_j) \leq \max_{j \in B} \epsilon_j \nu_j, \ \forall X' \in A(X^*). \tag{88}
\]

Then, the proportional fairness gap between the solution \(X^*\) and \(X' \in A(X^*)\) is bounded by \(\max_{j \in B} \epsilon_j \nu_j\).

In the Theorem \(B\) \(Z_j \neq \sum_{i \in \mathcal{N}} \sum_{c \in C} R_{ij}^{(e)} x_{ij}^{(e)}\) with probability 1 since \(R_{ij}^{(e)}\) is a random variable and the dimension of \(R_{ij}^{(e)}\) is finite. As \(C \rightarrow \infty\), the dimension of \(R_{ij}^{(e)}\) is infinite. Then, there exists \(X\) such that \(Z_j = \sum_{i \in \mathcal{N}} \sum_{c \in C} R_{ij}^{(e)} x_{ij}^{(e)}\) with probability 1.

**APPENDIX B**

**PROOF OF LEMMA 3**

The KKT condition obtained from the lagrangian (61) is denoted as

\[
\begin{align*}
\frac{\partial U(X, P)}{\partial p_j^{(c)}} - \xi_j + \varphi_j^{(c)} &= 0, \ \forall j \in B, c \in \mathcal{C} \tag{89a} \\
\xi_j \left( P_{j, \text{max}} - \sum_{c \in C} p_j^{(c)} \right) &= 0, \ \forall j \in B \tag{89b} \\
\varphi_j^{(c)} p_j^{(c)} &= 0, \ \forall j \in B, c \in \mathcal{C} \tag{89c} \\
P_{j, \text{max}} - \sum_{c \in C} p_j^{(c)} &\geq 0, \ \forall j \in B \tag{89d} \\
\varphi_j^{(c)} &\geq 0, \ \forall j \in B, c \in \mathcal{C}. \tag{89e}
\end{align*}
\]

In addition, \(P\) should satisfy the constraints (60b) and (60c) of the problem (60). From (89a), (89b) and (89c), we have \(\frac{\partial U(X, P)}{\partial p_j^{(c)}} = \xi_j - \varphi_j^{(c)}\), where \(\xi_j = 0\) if \(\sum_{c \in C} p_j^{(c)} < P_{j, \text{max}}\) and \(\varphi_j^{(c)} = 0\) if \(p_j^{(c)} > 0\).
Then, the partial derivatives \( \frac{\partial U(X,P)}{\partial p_j^{(c)}} \) is represented in terms of \( \xi_j \) and \( \varphi_j^{(c)} \) as

\[
\frac{\partial U(X,P)}{\partial p_j^{(c)}} = \begin{cases} 
\xi_j, & \text{if } p_j^{(c)} > 0, \sum_{c \in C} p_j^{(c)} = P_{j,\text{max}}, \\
\xi_j - \varphi_j^{(c)}, & \text{if } p_j^{(c)} = 0, \sum_{c \in C} p_j^{(c)} = P_{j,\text{max}}, \\
0, & \text{if } p_j^{(c)} > 0, \sum_{c \in C} p_j^{(c)} < P_{j,\text{max}}, \\
-\varphi_j^{(c)}, & \text{if } p_j^{(c)} = 0, \sum_{c \in C} p_j^{(c)} < P_{j,\text{max}}.
\end{cases}
\] (90)

From (90), the KKT condition (89a) is represented by (63).

**APPENDIX C**

**PROOF OF PROPOSITION 2**

The second-order approximated \( f_i(P_{[t]}) \) from Taylor series with small \( \Delta p_{j,[t]} \) is written as

\[
f_i(P_{[t]}) = f_i(P_{[t-1]}) + f'_i(P_{[t-1]})\Delta p_{j,[t]} + f''_i(P_{[t-1]})\Delta p_{j,[t]}^2 + O(\Delta p_{j,[t]}^3),
\] (91)

where \( f^{(n)}_i(P_{[t-1]}) = \left. \frac{\partial^n f_i(P_{[t]})}{\partial P_{j,[t]}^n} \right|_{P_{j,[t]}=0} \). From (91), we have

\[
f(P_{[t]}) = f_1(P_{[t]}) - f_2(P_{[t]}) = f''_1(P_{[t-1]})\Delta p_{j,[t]}^2 + f'_1(P_{[t-1]})\Delta p_{j,[t]} + f(P_{[t-1]}) + O(\Delta p_{j,[t]}^3),
\] (92)

where \( f''_1(P_{[t-1]}) = f'_1(P_{[t-1]}) - f''_2(P_{[t-1]}) \) and \( f'(P_{[t-1]}) = f'_1(P_{[t-1]}) - f''_2(P_{[t-1]}) \). Then, the solution of \( f(P_{[t]}) = O(\Delta p_{j,[t]}^3) \) is denoted as

\[
\Delta p_{j,[t]} = \frac{-f'(P_{[t-1]}) \pm \sqrt{\{f''(P_{[t-1]})\}^2 - 2f''(P_{[t-1]})f'(P_{[t-1]})}}{f''(P_{[t-1]})}.
\] (93)

Because the aim is to obtain \( \Delta p_{j,[t]} \) such that \( f(P_{[t]}) = O(\Delta p_{j,[t]}^3), \Delta p_{j,[t]} = 0 \) in (93) if \( f(P_{[t-1]}) = O(\Delta p_{j,[t]}^3) \). To this end, the sign of the square-root term should be \( \text{sgn} \left[ f'(P_{[t-1]}) \right] \). Then, \( \Delta p_{j,[t]} \) is denoted as

\[
\Delta p_{j,[t]} = \frac{-f'(P_{[t-1]}) + \text{sgn} \left[ f'(P_{[t-1]}) \right] \sqrt{\{f''(P_{[t-1]})\}^2 - 2f''(P_{[t-1]})f'(P_{[t-1]})}}{f''(P_{[t-1]})}.
\] (94)

The partial derivatives \( f_i(P_{[t-1]}), f_i^{(1)}(P_{[t-1]}) \) and \( f_i^{(2)}(P_{[t-1]}) \) can be immediately derived by definition as in (79), (80), and (81), respectively.


\textbf{APPENDIX D}

\textbf{PROOF OF PROPOSITION 3}

From the constraint (60c) and (74), we have

\[ \Delta p_{j,\lfloor t \rfloor} \leq p_{j,\lfloor t-1 \rfloor}^{(c_1)}. \]  

(95)

In transferring transmission power from RB \( c_1 \) to RB \( c_2 \), increased power on RB \( c_1 \) of SBS \( j \) increases the SINR of SBS \( j \) on RB \( c_1 \), and reduced power on RB \( c_2 \) of SBS \( j \) increases the SINR of other SBSs on RB \( c_2 \). Here, the maximum possible \( \Delta p_{j,\lfloor t \rfloor} \) that satisfies the backhaul constraint is obtained in terms of remaining backhaul capacity. The remaining backhaul capacity of SBS \( j \) for given \( X, P_{\lfloor t \rfloor} \) is denoted as

\[ L_j(X, P_{\lfloor t \rfloor}) = Z_j - \sum_{i \in \mathcal{N}} \sum_{c \in \mathcal{C}} \left( R_{ij}^{(c)} x_{ij}^{(c)} \right) \bigg|_{P = P_{\lfloor t \rfloor}}. \]  

(96)

i) SINR of SBS \( j \) on RB \( c_1 \)

Because the power of SBS \( j \) on RB \( c_1 \) increases, the SINR \( \text{SINR}_{i,\lfloor j, c_1 \rfloor}^{(c_1)} \) increases. In order to satisfy backhaul constraint (59b),

\[ W \log_2(1 + \text{SINR}_{i,\lfloor j, c_1 \rfloor}^{(c_1)}) \bigg|_{P = P_{\lfloor t \rfloor}} - W \log_2(1 + \text{SINR}_{i,\lfloor j, c_1 \rfloor}^{(c_1)}) \bigg|_{P = P_{\lfloor t-1 \rfloor}} \leq L_j(X, P_{\lfloor t-1 \rfloor}), \]  

(97)

where \( i = \arg \max_{i \in \mathcal{N}} x_{i,\lfloor j, c_1 \rfloor}^{(c_1)} \). The left-hand side of (97) is bounded as follow:

\[ W \log_2(1 + \text{SINR}_{i,\lfloor j, c_1 \rfloor}^{(c_1)}) \bigg|_{P = P_{\lfloor t \rfloor}} - W \log_2(1 + \text{SINR}_{i,\lfloor j, c_1 \rfloor}^{(c_1)}) \bigg|_{P = P_{\lfloor t-1 \rfloor}} \leq W \log_2 \left( 1 + X \left( 1 + \frac{\Delta p_{j,\lfloor t \rfloor}}{p_{j,\lfloor t-1 \rfloor}^{(c_1)}} \right) \right) - \log_2(1 + X) \]  

(98)

\[ \leq W \log_2 \left( 1 + \frac{\Delta p}{p_{j,\lfloor t-1 \rfloor}^{(c_1)}} \right), \]  

(99)

where \( X = \text{SINR}_{i,\lfloor j, c_1 \rfloor}^{(c_1)} \bigg|_{P = P_{\lfloor t-1 \rfloor}} \). Then, from (97) and (100), \( \Delta p_{j,\lfloor t \rfloor} \) satisfies the backhaul constraint if

\[ \Delta p_{j,\lfloor t \rfloor} \leq \left( 2 \frac{L_j(X, P_{\lfloor t-1 \rfloor})}{W} - 1 \right) p_{j,\lfloor t-1 \rfloor}^{(c_1)}. \]  

(101)

ii) SINR of other SBSs on RB \( c_2 \)

The SINR of SBSs except SBS \( j \) on RB \( c_2 \) increases because the interference on RB \( c_2 \) is decreased.
Then, the backhaul constraint (59b) on SBS $k$ is satisfied if

$$W \log_2 \left( 1 + \text{SINR}_{i(k,c_2)k}^{(c_2)} \right) \bigg|_{P=P_{[t]}} - W \log_2 \left( 1 + \text{SINR}_{i(k,c_2)k}^{(c_2)} \right) \bigg|_{P=P_{[t-1]}} \leq L_k(X, P_{[t-1]}),$$  \hspace{1cm} (102)$$

where $i^{(j,c_2)} = \arg\max_{i \in N} x_{ij}^{(c_2)}$. The left-hand side of (102) is bounded by

$$W \log_2 \left( 1 + \text{SINR}_{i(k,c_2)k}^{(c_2)} \right) \bigg|_{P=P_{[t]}} - W \log_2 \left( 1 + \text{SINR}_{i(k,c_2)k}^{(c_2)} \right) \bigg|_{P=P_{[t-1]}}$$

$$= W \log_2 \left( 1 + \frac{\text{SNR}_{i(k,c_2)k}^{(c_2)} p_{k,[t-1]}^{(c_2)}}{Y - \text{SNR}_{i(k,c_2)j}^{(c_2)} \Delta p_j,[t]} \right) - W \log_2 \left( 1 + \frac{\text{SNR}_{i(k,c_2)k}^{(c_2)} p_{k,[t-1]}^{(c_2)}}{Y} \right)$$

$$= W \log_2 \left( M - \text{SNR}_{i(k,c_2)j}^{(c_2)} \Delta p_j,[t] \right) - W \log_2 \left( M \frac{\text{SNR}_{i(k,c_2)j}^{(c_2)} \Delta p_j,[t]}{Y} \right)$$

$$\leq -W \log_2 \left( 1 - \frac{\Delta p_j,[t]}{p_{j,[t-1]}^{(c_2)}} \right),$$  \hspace{1cm} (106)$$

where $Y = \sigma^2 + \sum_{l \in B} p_{l,[t-1]}^{(c_2)} \text{SNR}_{i(k,c_2)l}^{(c_2)}$, $M = Y + \text{SNR}_{i(k,c_2)k}^{(c_2)} p_{k,[t-1]}^{(c_2)}$. Then, from (102) and (106), we have

$$\Delta p_j,[t] \leq \left( 1 - 2 \frac{L_k(X, p_{[t-1]}^{(c_2)})}{W} \right) p_{j,[t-1]}^{(c_2)}.$$  \hspace{1cm} (107)$$

From (95), (100) and (106), The condition of $\Delta p_j,[t]$ that satisfied the backhaul constraint (59b) is denoted in (82).

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