Temperature square dependence of the low frequency $1/f$ charge noise in the Josephson junction qubits

O. Astafiev,$^{1,2}$ Y. A. Pashkin,$^{1,2}$ Y. Nakamura,$^{1,2}$ T. Yamamoto,$^{1,2}$ and J. S. Tsai$^{1,2}$

$^1$NEC Fundamental and Environmental Research Laboratories, Tsukuba, Ibaraki 305-8501, Japan
$^2$The Institute of Physical and Chemical Research (RIKEN), Wako, Saitama 351-0198, Japan

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To verify the hypothesis about the common origin of the low frequency $1/f$ noise and the quantum $f$ noise recently measured in the Josephson charge qubits, we study temperature dependence of the $1/f$ noise and decay of coherent oscillations. $T^2$ dependence of the $1/f$ noise is experimentally demonstrated, which supports the hypothesis. We also show that dephasing in the Josephson charge qubits off the electrostatic energy degeneracy point is consistently explained by the same low frequency $1/f$ noise that is observed in the transport measurements.

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Due to potential scalability, Josephson quantum bits are good candidates for building quantum computers$^1$. To have a long decoherence time, the qubits should be well decoupled from all noise sources, in particular, charge noise from uncontrollable charge fluctuations. Therefore, the noise and decoherence in the qubits are now the key issue of the qubit research. Although the noise has been studied in a number of works$^2,3,4,5,6,7$, its nature is still not yet understood.

The low frequency noise in metallic single electron transistors (SETs) has been studied intensively a while ago$^2,3,4,10$. It has been found that the noise is produced by charge fluctuators and its spectral density is close to $1/f$. Recently, it has been shown that the low frequency charge noise gives the main contribution to dephasing of coherent oscillations in the Josephson charge qubits$^3,4$. The high frequency quantum noise of the environment has been investigated in Ref.$^2$ by using a qubit as a quantum spectrometer. The qubit relaxation is caused by the asymmetric quantum noise with a non-monotonic spectrum, which tends to have a linear frequency dependence ($f$-noise) in a wide energy range (from 2 to 100 GHz $\times h$) like in the case of a simple ohmic environment. The quantum noise originates from absorption of the energy of excited qubits by the cold environment and, therefore, should be nearly temperature independent in the range of qubit energies higher than $k_B T$. Surprisingly, it turned out that the amplitude of the $f$ quantum noise crosses the low frequency $1/f$ noise extrapolated to the gigahertz range at a frequency $\omega_c \sim k_B T / h$, which implies that both noises may have a common origin. Furthermore, if one assumes that the crossover frequency $\omega_c$ scales linearly with temperature then the strength of the $1/f$ charge noise should be proportional to temperature square ($T^2$ dependence)$^2$. Although $T^2$ dependence has been observed in the critical current noise of the Josephson junction$^11$, and maybe related to charge fluctuations in the junction, the dependence appears to be unexpected in the charge noise measurements as it contradicts to the linear temperature dependence of the $1/f$ noise in glasses$^12,13$. Therefore, the $T^2$ dependence in the charge $1/f$ noise has to be experimentally confirmed. In addition, temperature dependence gives important information on the density of states of the fluctuators and will help to verify theoretical models of the $1/f$ noise intensively studied recently in Refs.$^14,15,16,17,18$. Some of these works are based on the prediction of the $T^2$ dependence reported in Ref.$^2$. Unfortunately, temperature dependence of the $1/f$ noise in SETs has not been studied in details in earlier works. It was found that the noise increases with temperature and saturates in the low temperature range$^15,20$. In Ref.$^20$, the quadratic temperature dependence was expected but has not been actually demonstrated.

In this work, we study temperature dependence of the $1/f$ noise in the Josephson charge qubits by measuring dc transport in the SET regime. We have found that the noise exhibits $T^2$-dependence at temperatures from 200 mK up to $\sim 1$ K. We also study the effect of temperature on dephasing of the qubit during coherent oscillations. Decay of the coherent oscillations away from the degeneracy point of electrostatic energy is consistently explained by dephasing due to the low frequency $1/f$ charge noise. We also briefly discuss phenomenological models of $T^2$ dependences from the experimental point of view.

To study temperature dependence we fabricate qubits with the same geometry and junction properties as the qubits used in Ref.$^2$. The Al structure is fabricated on top of 400 nm thick Si$_3$N$_4$ insulation layer deposited on a gold ground plane. The total capacitance of the qubit island is about 600 aF (the charging energy is $E_C = e^2 / 2C \approx 30$ GHz$\times h$) and is mainly formed by its Josephson junction. Instead of the trap island used in Ref.$^2$, we fabricate an electrical lead connected to the qubit through a small highly resistive tunnel junction with a resistance of $10 - 50$ M$\Omega$ (as it was in our earlier works$^3,4$) to measure current through the qubit in the SET regime.
We use the qubit as an SET and measure the low frequency charge noise, which causes the SET peak position fluctuations. Temperature dependence of the noise is measured from the base temperature of 50 mK up to 900 mK. The SET is normally biased to $V_g = 4\Delta/e$ ($\sim 1$ mV), where Coulomb oscillations of the quasiparticle current are observed. Figure 1(a) exemplifies the position of the SET Coulomb peak as a function of the gate voltage at temperatures from 50 mK up to 900 mK with an increment of 50 mK. The current noise spectral density is measured at the gate voltage corresponding to the slope of the SET peak (shown by the arrow), at the maximum (on the top of the peak) and at the minimum (in the Coulomb blockade). Normally, the noise spectra in the two latter cases are frequency independent in the measured frequency range (and usually do not exceed the noise of the measurement setup). However, the noise spectra taken on the slope of the peak show nearly $1/f$ frequency dependence (see examples of the current noise $S_\eta$ at different temperatures in Fig. 1(b)) saturating at a higher frequencies (usually above $10 - 100$ Hz depending on the device properties) at the level of the noise of the measurement circuit. The fact that the measured $1/f$ noise on the slope is substantially higher than the noises on the top of the peak and in the blockade regime indicates that the noise comes from fluctuations of the peak position, which can be translated into charge fluctuations in the SET.

To obtain the $1/f$ charge noise spectral density

$$S_\eta(\omega) = \frac{\alpha}{\omega}$$

(defined for frequencies $\omega > 0$) we first take the low frequency part of the current noise spectral density $S_I(f)$ and find the parameter $A$ of the fitting curve $A/f$ as $A = \langle \sigma_f(f) \rangle / \langle 1/f \rangle$. Next, we transform the current noise into the charge noise $\alpha = A/(dI/dV_g)^2/(\Delta V_g/e)^2$ using the transfer function $dI/dV_g$ on the slope of the peak at the measurement point, where $\Delta V_g$ is the spacing in gate voltage between two adjacent peaks (corresponding to the change of the SET charge by $e$). Dimensionality of $\alpha$ is $e^2$ and a typical value of $\alpha$ is of the order of $(10^{-3} e)^2$ at $T \leq 200$ mK.

Solid dots in Fig. 1(c) represent $\alpha^{1/2}$ as a function of temperature. $\alpha^{1/2}$ saturates at temperatures below 200 mK at the level of $2 \times 10^{-6}e$ and exhibits nearly linear rise at temperatures above 200 mK with $\alpha^{1/2} \approx \eta^{1/2} T$, where $\eta \approx (1.0 \times 10^{-2}e/K)^2$ (the solid line in Fig. 1(c)). $T^2$ dependence of $\alpha$ is observed in many samples, though sometimes the noise is not exactly $1/f$, having a bump from the Lorentzian spectrum of a strongly coupled low frequency fluctuator. In such cases, switches from the single two-level fluctuator are seen in time traces of the current $S_I$.

Note that at a fixed bias voltage the average current through the SET increases with temperature (see Fig. 1(a)). However, it has almost no effect on the noise as we confirmed from the measurement of the current noise dependence. Nevertheless, to avoid possible contribution from the current dependent noise we adjust the bias voltage in the next measurements so that the average current is kept nearly constant at the measurement point, where $\Delta$ is the spacings in gate voltage between two adjacent peaks (corresponding to the change of the SET charge by $e$).

The Hamiltonian of our qubit written in the charge representation is

$$H_{\text{qubit}} = \frac{1}{2} \frac{\hbar e}{\Delta} \theta \frac{\Delta^2}{U} \sin \theta \sin \phi,$$

where $\theta$ is the angle between the currents of the Cooper pair and the Josephson current. The measured temperature dependence of $\alpha$ with $\eta \approx (1.3 \times 10^{-2}e/K)^2$.

We study decay of coherent oscillations away from the electrostatic energy degeneracy point at different temperatures by measuring pulse induced current $I(t)$.

The Hamiltonian of our qubit written in the charge basis $|0\rangle$ and $|1\rangle$ (with and without the Cooper pair in the island) is $H = \frac{1}{\hbar} \frac{\Delta}{\cos \theta + \sigma_z \sin \theta}$, where $\theta$ is the angle between the currents of the Cooper pair and the Josephson current $E_J$. The electrostatic energy $\Delta U$ is controlled by the qubit gate voltage. Adjusting a dc gate voltage to the point far away from the degeneracy ($\Delta U \gg E_J$, $\theta \approx 0$), where the ground state is nearly $|0\rangle$ we apply a rectangular voltage pulse of length $t$ bringing the qubit in the vicinity of the degeneracy point ($\Delta U$ is of the order or smaller than $E_J$ and $\theta \approx \pi/2$), that is the Hamiltonian changes non-adiabatically to $\hat{H}_1$ for time $t$. The coherent evolution can be presented as $\exp(-\frac{i}{\hbar} \int_0^t \hat{H}_1 dt) |0\rangle$. Applying a sequence of identical pulses we detect a pulse induced current, which is proportional to the probability to find out the qubit in the state $|1\rangle$ after the pulse manipulation.

The typical current oscillation as a function of $t$ away...
from the degeneracy point ($\theta \neq \pi/2$) is exemplified in the inset of Fig. 2(a). If dephasing is induced by the Gaussian noise, the oscillations decay as $\exp(-t^2/2T_2^*)$ with

$$\frac{1}{T_2^*} \approx \frac{\cos^2 \theta}{\hbar^2} \left(\frac{4EC}{e}\right)^2 \int_0^\infty S_q(\omega) \left(\frac{2\sin(\omega t/2)}{\omega t}\right)^2 d\omega,$$

where $\omega_0 \approx 1/\tau$ is the low frequency integration limit defined by the measurement time constant $\tau$. In the case of the $1/f$ Gaussian noise of Eq. 1

$$T_2^* \approx \frac{\epsilon h}{4EC \sqrt{\alpha \ln(\omega_1 \tau)} \cos \theta},$$

where $\omega_1 \leq \pi/T_2$ is the effective high frequency limit.

Qubit dephasing due to the non-Gaussian noise is treated in Refs. 22, 23. For instance, in the case of a strongly coupled fluctuator, the decay is slower than Gaussian. However, importantly, Eq. 23 used for fitting the initial part of the oscillations still gives a reasonably good agreement for estimation of the amplitude of the $1/f$ noise. We have analyzed time traces of the noise and found that the noise is often close to the Gaussian, but sometimes it is clearly not, for example, in the presence of a strongly coupled fluctuator.

The solid line in the inset of Fig. 2(a) shows decay of coherent oscillations measured at $T = 50$ mK and the dashed envelope exemplifies a Gaussian with $T_2^* = 180$ ps. We derive $\alpha^{1/2}$ from Eq. 24 and plot it in Fig. 2(a) by open dots as a function of temperature. The low frequency integration limit and the high frequency cutoff are taken to be $\omega_0 \approx 2\pi \times 25$ Hz and $\omega_1 \approx 2\pi \times 5$ GHz for our measurement time constant $\tau = 0.02$ s and typical dephasing time $T_2^* \approx 100$ ps 24.

The saturation of the $1/f$ noise at low temperatures has also been observed in earlier works 14, 20. Although its origin is not clear, we can suggest the following possible mechanisms: (1) heating of an electron system, (2) freezing out fluctuators, so that the effective number of active fluctuators decreases down to a few per decade (in this case the $1/f$ noise saturates to the level of a single fluctuator amplitude).

To collect more information about the $1/f$ noise we perform an additional experiment studying the noise in the SET fabricated on a different substrate. Although the results are not conclusive and require more systematic study we think that it is instructive to present these data here. Fig. 2(b) demonstrates the $1/f$ noise temperature dependence for an SET fabricated on single-crystal GaAs. The GaAs substrate has been chosen to reduce the number of defects (as a possible origin of fluctuators) typical for amorphous materials like CVD grown Si$_3$N$_4$ or thermal oxide on top of bare silicon. Again, clear $T^2$-dependence is observed above 200 mK, and $\eta \approx (0.75 \times 10^{-2} e/K)^2$ is lower but close to what was measured in the case of Si$_3$N$_4$.

Below we discuss a phenomenological model explaining the properties of the noise. Our qubit is coupled to charge dipoles $ed$ in the insulator, which, in turn, induce a charge $\delta q$ in the qubit island (the fluctuators at distances smaller than the characteristic size of the island $R$ produce $\delta q \sim ed/R$, and the matrix element of the Cooper pair transition is at most $4EC\delta q$). Based on the phenomenology from Ref. 2, it has been pointed out in Refs. 14, 16 that the $T^2$ dependence may originate from two-level fluctuators (characterized by bias energy $\epsilon$ and tunneling energy $\Delta$) and linearly distributed in $\epsilon$ with an energy independent amplitude $\langle \delta q \rangle$. Summation of the Lorentzian spectra over many fluctuators with energies $\epsilon$ below $k_B T$ gives $(k_B T)^2$ term in the noise spectrum. Such linear energy distribution, for example, appears in the models treating charge fluctuations between the superconducting island and an insulator 15, 16. The switching rate of the thermally activated fluctuators ($\epsilon < k_B T$) can be presented as $\gamma \sim \gamma_0(\Delta/\epsilon)^2$, where $\gamma_0$ depends on the coupling of the fluctuators to the external thermal bath, and $\Delta$ is the tunneling energy of electrons in the fluctuators 23. The slow fluctuators, contributing to the low-frequency $1/f$ noise should have a strong suppression factor $(\Delta/\epsilon)^2 \ll 1$. On the other hand, to efficiently absorb qubit energy, the two-level systems should have $\Delta$ of order or larger $\epsilon$. Note that the two-level systems producing the $1/f$ noise and absorbing the qubit energy are characterized by very different values of $\Delta$. A commonly used assumption that two-level
systems are distributed according to $P(\Delta) \propto 1/\Delta$ gives rise to the crossover frequency of $\omega_c \approx k_BT/h$.

To show relationship of the model with experiments we provide some numbers from experiments. The $1/f$ noise in this work is studied in the temperature range from 0.05 to 1 K, which means that only fluctuators with the activation energies lower than $k_BT$ (from 1 to 20 GHz $\times$ $h$) produce the noise. Note that this thermal energy range overlaps with the qubit energy $2 - 100$ GHz $\times$ $h$ for which the $f$ quantum noise has been studied in Ref. [3]. On the other hand, the $1/f$ noise is measured in the frequency range 0.1 – 100 Hz, which gives typical values of $\gamma$ for the fluctuators contributing to the measured noise. The high frequency cutoff of the $1/f$ noise (which may give rough estimate of $\gamma_0$) is not known for our qubits. For rough estimations, we take $\gamma_0 = 1$ MHz from Ref. [4] and find that only fluctuators with $(\Delta/e)^2 \approx \gamma/\gamma_0 \sim 10^{-7} - 10^{-4}$ contribute to the measured noise. Note that to have necessary relationship between the fluctuators and the two-level system absorbing the qubit energy, distribution $P(\Delta) \propto 1/\Delta$ should hold in a very wide range of $\Delta$ from $10^6$ Hz $\times$ $h$ or less up to $10^{11}$ Hz $\times$ $h$.

Apart from the phenomenology, the most important question now is what is the detailed mechanism of the $1/f$ and $f$ noises? A few microscopic models have been proposed in Refs. [15, 16, 18]. Test experiments have to be done to verify the models. For instance, behavior of the noise as a function of magnetic field and superconducting-normal state transition would be important for the theories involving superconductivity.

In conclusion, we have observed $T^2$ temperature dependence of the low frequency $1/f$ noise, which supports the idea that the $1/f$ and $f$ noises have a common origin. Typically, the noise spectral density is $(10^{-2} e)^2 (T/K)^2 / \omega$ at $T > 200$ mK and saturates to the level of the order of $(10^{-2} e)^2 / \omega$ at $T < 200$ mK. We demonstrated that free induction decay is consistently explained by dephasing on the low frequency $1/f$ noise. $T^2$ dependence of similar amplitude is observed for two different substrate materials amorphous Si$_3$N$_4$ and single-crystal GaAs.

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