Modeling of control processes of spacecraft orbits with low-thrust engines

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Abstract. This paper describes development and modeling of the spacecraft transfer schemes with low-thrust engines from a high-elliptical (with eccentricity up to 0.7 and more) to a geostationary orbit. The problem of orbital elements optimal control is formulated according to Pontryagin maximum principle. We solved the problem of finding the optimal control law using local optimization method. We developed an algorithm and software to solve the problem of flight optimization between non-coplanar orbits. The comparison of calculation results with local optimal control law and precise solutions showed that for a wide range of boundary conditions the results of calculations differ within a margin of no more than 1.5 – 2 %. We given modeling examples of the developed control law for two cases of transfer from high-elliptical to a geostationary orbit. Results of calculations showed that the impact of the atmosphere drag on the transfer time is rather small, because the spacecraft spend a little time in atmosphere bounds and perigee of the orbit is rising above it in few revolutions.

1. Introduction

At present, a geostationary orbit (GSO) is preferred for communication satellite placement. In most cases spacecraft satellites are taken from support orbit to GSO by traditional impulse propulsion schemes, with high power cruising thrusters firing and shutting off at intervals. However, in recent times researchers have been increasingly interested in alternative geostationary orbit transfers, including a combined high and low thrust power plant. Successful missions of AEHF-1 [1], Express-AM5 and SES-9, that used electric propulsion at the insertion stage, fueled this interest.

A combination of high and low thrust propulsion increases the payload that can be delivered to orbit, as compared to traditional insertion schemes [2]. Among various types of space maneuvers, combined propulsion was demonstrated to significantly increase efficiency of geostationary orbit insertion missions, especially where there is considerable difference in inclinations between insertion and geostationary orbits.

An intermediate orbit can be formed with the help of the third stage of a heavy launch vehicle (e.g. Proton, Delta-IV, Arian). The insertion scheme with a circular intermediate orbit can be considered among the traditional methods, because rational insertion scheme and control laws for this type of
electric propulsion-powered spacecraft missions are well researched. Elliptical intermediate orbits, by now, have been much less studied. Additional margins of freedom (eccentricity and the perigee argument of the intermediate orbit) provide additional opportunities for optimization of the spacecraft’s insertion into operational orbit. Studies show that for many GSO insertion missions elliptical intermediate orbits are more efficient than circular. Moreover, optimal eccentricity value of an intermediate orbit can be reasonably high (up to 0.7 and more), so a scheme of insertion into high operational orbit with the help of a high elliptical intermediate orbit can be considered.

2. Factors to be considered in modeling controlled motion of a spacecraft

Spacecraft with electric propulsion can have long intervals of powered flight, measured sometimes in days and months, therefore analysis of the powered flight stages of the mission for EP spacecraft is based on a complete mathematical model in osculating elements [3], where thrust is considered as a perturbation (1).

\[
\begin{align*}
\frac{dA}{dt} &= \frac{2P}{(1-e^2)\mu} \left[ p \sin \theta \cdot S + (1 + e \cos \theta) \cdot T \right], \\
\frac{de}{dt} &= \frac{p}{\mu} \sin \theta \cdot S + \frac{e \cos \theta \cdot 2 \cos \theta + e}{1 + e \cos \theta} \cdot T, \\
\frac{di}{dt} &= \frac{p}{\mu} \cdot \frac{\cos u}{1 + e \cos \theta} \cdot W, \\
\frac{d\omega}{dt} &= \frac{p}{e \mu} \left[ -\cos \theta \cdot S + \frac{\sin \theta (2 + e \cos \theta)}{1 + e \cos \theta} \cdot T - \frac{e \sin u \cdot \cot \theta}{1 + e \cos \theta} \cdot W \right], \\
\frac{d\Omega}{dt} &= \frac{p}{\mu} \cdot \frac{\sin u}{\sin i(1 + e \cos \theta)} \cdot W, \\
\frac{du}{dt} &= \frac{\sqrt{\mu p}}{p^2} \left[ (1 + e \cos \theta)^2 - \frac{p^2}{(1 + e \cos \theta)^2} \cdot \cot \theta \cdot \sin u \cdot W \right],
\end{align*}
\]

where \( A \) is semi-major axis; \( p = A(1-e^2) \) is focal parameter; \( \theta = u - \omega \) is the true anomaly; \( e \) is eccentricity; \( \omega \) - argument of periapsis; \( \Omega \) is right ascension of ascending node; \( i \) orbit inclination; \( t \) is time; \( u \) is argument of latitude; \( S, T, W \) are projections of jet acceleration on direction of radial vector, on the perpendicular to it in the orbital plane, and on perpendicular to the orbital planet; \( \mu = fM \) is gravitational parameter of the central body.

Also, on the first stages of the flight, with low value of the perigee of the intermediate orbit it’s worthwhile to allow for perturbations caused by atmosphere and non-centrality of Earth’s gravity field.

3. Setting the optimization problem and methods of its solution

In this paragraph we’ll formulate a problem of optimal semi-major axis, eccentricity, and inclination control, minimizing discrepancies on these elements. For the mathematical model of motion of an EP powered spacecraft let us consider a system of differential equations in osculating elements.

It is known that the system has its peculiarities at \( e = 0 \) and \( i = 0 \). In practice, the most common solution for this is transfer to equinoctial elements. In this case, however, a common system of differential equations in osculating elements is preferable, therefore before integrating we shall set fixed final values of eccentricity and inclination, different from zero and meeting the error margin requirements for the solution of the problem.

Let us introduce two right-hand systems of coordinates: orbital (Onrb) and spacecraft-centered (OXYZ). The thrust vector \( \vec{P} \) is directed along the \( OX \) axis.
The components of reactive acceleration in orbital system of coordinates look as follows (2):

$$ T = \ddot{a} \cos \lambda \cos \phi, \quad S = \ddot{a} \sin \lambda \cos \phi, \quad W = \ddot{a} \sin \psi, $$

where $a$ is the module of complete reactive acceleration ($a = a \cdot (1 - a t / c)^{-1}$), $\delta$ is the thruster switch on/off function ($\delta = \{0, 1\}$); $\lambda$ is the thrust vector orientation angle in the local orbital plane ($\lambda \in [0^\circ; 180^\circ]$); $\psi$ is the thrust vector orientation angle in the local horizon plane ($\psi \in [-90^\circ; 90^\circ]$) (figure 1). Obviously, the minimum transfer time is achieved at continuous thruster burning ($\delta = 1$) with shifts in components of reactive acceleration.

In flat maneuvers (changes in semi-major axis and eccentricity of the orbit) the biggest input comes from transversal component of the reactive acceleration $T$, while only the binormal component $W$, with changes in its signs twice for revolution, is used to control the inclination of the orbit.

The borderline conditions are stated below (3):

$$ t = t_0 : A(t_0) = A_0, \quad e(t_0) = e_0, \quad i(t_0) = i_0; \quad t = t_K : A(t_K) = A_K, \quad e(t_K) = e_K, \quad i(t_K) = i_K $$

The variables $\omega$, $\Omega$, $u$, are unconditioned, so equations $\frac{d\omega}{dt}, \frac{d\Omega}{dt}, \frac{du}{dt}$ can be excluded from the mathematical model of the variation problem, but considered in the course of further modeling. In this way we can obtain the limits of the optimality criterion for a dynamic problem - thrust time of the transfer (that is equal to total time of the transfer at $\delta = 1$).

The terminal criterion stated in the form of square functional as representing as multiplying as a squared discrepancies sum for semi-major axis, eccentricity and inclination of the orbit and corresponding weighted (undefined) coefficients (4) [4]:

$$ I = \Delta x_K^T \alpha \Delta x_K \rightarrow \min, $$

where $\alpha = \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{22} & 0 \\ 0 & 0 & \alpha_{33} \end{bmatrix}$, $\sum \alpha_{ij} = 1$. $\Delta x = [\Delta A, \Delta e, \Delta i]^T$, $\Delta A = A(t) - A_1$, $\Delta e = e(t) - e_1$, $\Delta i = i(t) - i_1$. and $\alpha_{ii} = \alpha_{11}$, $\alpha_{ii} = \alpha_{22}$, $\alpha_{ii} = \alpha_{33}$ are weighed coefficients (elements of diagonal matrix) for semi-major axis, eccentricity and inclination respectively [5].

3.1 Method based on Pontryagin maximum principle

The variation problem will be solved according to Pontryagin maximum principle [5] conditional to constant firing of the thruster ($\delta = 1$), because in this case minimum transfer time is achieved.

The solution of the problem according to Pontryagin maximum principle provided analytical equations for thrust vector orientation angles.
Application of the maximum principle allows to reduce an optimization problem to a boundary value problem for a system of common differential equations. The solution of a problem of optimal control over orbit parameters in strict setting, resulting from Lagrange-Pontryagin formalism, requires very complicated computing. In addition, the issue of convergence and stability of the solution algorithm for the boundary value problem becomes paramount. Therefore the initial problem was broadened to a local optimization problem.

3.2 Local optimization method. Applicability conditions. Loss estimate.

The local optimal control \( u(t, x) \) allows to minimize subintegral expression, derivative \( \frac{dI}{dt} \), at any moment in time, rather than the functional of the dynamic problem \( I \). If the subintegral expression does not change its sign and is a monotonous function, this setting of the problem is equivalent to the initial one [4].

In a general case, synthesis of local optimal controls does not guarantee an absolute optimum in the initial setting of the problem, however, there is a certain class of problems with a minor parameter (in particular, reactive acceleration created by EP engine), wherein the local optimal controls are the closer to optimal the lesser is the minor parameter, i.e., the smaller is the correcting control.

The problem of finding the optimal control law is solved in the assumption that changes in semi-major axis, eccentricity and inclination of the orbit is monotonous. Then the problem can be reduced to a problem of selecting a local optimal control law, followed by verification of the condition of monotonous change in the functional. Let us find a local optimal law of combined control over semi-major axis, eccentricity and inclination of the orbit, that ensures the minimum of functional \( I \), defined by equation (4), at set initial conditions.

Replace this functional with a local criterion ensuring maximum change in \( I \) (5):

\[
\frac{dI}{dt} = 2\alpha \left( \frac{A - A_v}{A_v} \right) \frac{dA_v}{dr} + 2\alpha_i \frac{de}{dr} + 2\alpha_e \frac{di}{dr} \rightarrow \max
\]

The result of finding the maximum of the expression (5) for two variables \( \lambda(t), \psi(t) \) are analytical expressions for the spatial angles of the thrust vector \( \lambda \) and \( \psi \), where \( \psi \) is the angle between thrust vector and local orbital plane, and \( \lambda \) is the angle between projection of the thrust vector on local orbital plane and the transversal \( T \).

The control law obtained thereby \( \dot{\psi}(t), \dot{\lambda}(t) \) features a simple structure and makes it possible to compute a dynamic maneuver without solving a boundary value problem.

As follows from (5), the rate of change in semi-major axis, eccentricity and inclination of the orbit depends on the values of weighed coefficients \( \alpha_i, \alpha_e, \alpha \). Due to selecting the values of the weighed coefficients it’s possible to provide orbital elements simultaneously achievement of their final values. In first approximation it is possible to accept the following approach: \( \alpha_i = \alpha_e = \alpha \). The subsequent selection of weighting coefficients may be carried out manually or with use of special algorithm.

The results of calculations with local optimal control laws were compared with precise solutions of problems for transfers between non-coplanar orbits (V. N. Lebedev). The comparison showed that for a wide range of boundary conditions the results of calculations differ within a margin of no more than 1.5 - 2 %. Therefore, local optimal control is good first approximation for solving variational problems of low thrust space transfers.

4. Algorithm and software for a problem of optimization of a transfer between elliptical and geostationary orbits

Calculations for orbit raising of a heavy geostationary communication satellite (type Express AM-5, AMOS-6) with electric propulsion are given below. The initial ellipsis is formed by a powerful
impulse of the third stage of the heavy class launch vehicle. Initial data for the transfer is represented
in Table 1. The time required for reaching the needed value of every orbital parameter is given in Table 2.

Table 1. Initial data, where \( H_p \) is the perigee altitude; \( H_a \) is the apogee altitude; \( i \) is inclination; \( m_0 \) is launch mass of the spacecraft; \( m_{dry} \) is the dry mass of the spacecraft; \( P \) is thrust; \( I_{sp} \) is specific impulse.

| \( H_p \), km | \( H_a \), km | \( i \), deg. | \( m_0 \), kg | \( m_{dry} \), kg | \( P \), mN | \( I_{sp} \), s |
|--------------|--------------|-------------|-------------|-------------|--------|--------|
| 200          | 80000        | 28          | 3500        | 2600        | 360    | 1600   |
| 200          | 80000        | 51.6        | 3500        | 2600        | 360    | 1600   |

Table 2. Target values of orbital parameters, where \( e \) is eccentricity; \( R_p \) is perigee radius, \( R_a \) is apogee radius; \( A \) is semi-major axis; \( p \) is focal parameter.

| \( i \), deg. | Orbital parameter | Target value | Time to reach the target value, days |
|--------------|------------------|--------------|-------------------------------------|
| 28           | \( e \)          | 0            | 287.2                               |
|               | \( A \), km      | 42164        | 289.5                               |
|               | \( i \), deg.    | 0            | 289.0                               |
|               | Discrepancy      | 0            | \(<10^{-4} \) to 282.2             |
| 51.6         | \( e \)          | 0            | 323.7                               |
|               | \( A \), km      | 42164        | 327.4                               |
|               | \( i \), deg.    | 0            | 326.0                               |
|               | Discrepancy      | 0            | \(<10^{-4} \) to 318.0             |

Propellant mass required for the transfer with \( i = 28^\circ \) is 552.9 kg. With inclination of 51.6\(^\circ\) the propellant mass required for the transfer is 618.1 kg.

As the results of calculations for the two transfers, weighed coefficients were selected, to meet the rule of orbital parameters meeting their target values of the same time. They are given in Table 3.

Table 3. Values of weighed coefficients.

| Weighed coefficient | \( i \), degrees | \( 28 \) | \( 51.6 \) |
|---------------------|------------------|---------|---------|
| \( e \)             | 0.1              | 0.15    |
| \( A \)             | 0.5              | 0.4     |
| \( i \)             | 0.4              | 0.45    |

Differences in time between achievement of the orbital parameters the target values is due to lack of precision in selecting weighed coefficients. If smaller margin between achieving target values of orbital parameters in time is required, iterative selection of weighed coefficients can be carried out.

Modeling the perturbation effect of the Earth atmosphere on the transfer showed that the impact of the atmosphere is rather small, because the spacecraft leaves the atmosphere in just a few loops and spends only a short period of time travelling through it.

Anyway, perturbation of the atmosphere increases flight time by less than one hour. The insignificance of impact is explained by the fact that the force of aerodynamic drag \( F_a \) at the part of the trajectory with lowest altitude (maximum drag on first revolution) is approximately equal to the thrust \( F_T \) (\( F_A = 320 \) mN, \( F_T = 360 \) mN) while the length and duration of the interval where drag is efficient is very small (see Figure 2). Because of that, the perigee of the orbit is rising above atmosphere in 3-4 revolutions.

Modeling yielded the following dependencies on time (for \( i = 28^\circ \)) (Figure 3-8).
Figure 2. Spacecraft trajectory with impact from the atmosphere.

Figure 3. The spacecraft spatial motion trajectory.

Figure 4. Inclination vs flight time.

Figure 5. Semi-major axis, apogee and perigee radius vs flight time.

Between day 0 and day 200 of the transfer on figure 8 we can observe gradual monotonous decrease in positive amplitude values of the inclination of $\lambda$ from 70 to 7°, related to increase of semi-major axis. From day 200 to day 280 the value of semi-major axis is decreasing, first slowly, and since day 260 abruptly, and the increase of $\lambda$ to 85° is related to that. Asymmetry relative to zero is explained by high eccentricity of the initial orbit.

Figure 8 shows that for duration of the transfer $\psi$ is changing across the range of -90 to +90 degrees, and change in the direction of vector's orientation happens twice per loop.
Mode ling demonstrated that the derivative of the functional does not change its sign, and the functional monotonously decreases to zero, minimizing discrepancies along semi-major axis, eccentricity and inclination of the orbit.

Figure 9 shows the main window of the software for calculation and modeling transorbital transfers.

5. Conclusion

A control law for small thrust spacecraft orbit, that is characterized by rather simple structure and allows to compute dynamic maneuvers without solving a boundary value problem, is obtained.

Transorbital flights between arbitrary non-coplanar orbits in a wide range of boundary conditions were modeled. A qualitative analysis of the changes in orbital parameters, thrust vector orientation vectors for EP-powered spacecraft for a local optimal control scheme for orbit parameters was carried
out, on example of calculating orbit raising for a heavy geostationary communication satellite of Express AM-5 / AMOS-6 class, powered by an electric propulsion unit.

Figure 9. The main window of the software showing sample calculation of a transfer from a high elliptic to a geostationary orbit.

A regular algorithm for solving a dynamic optimization problem for arbitrary boundary conditions was suggested. The algorithm allows to build a module for calculating dynamic characteristics of small-thrust transfers within a general framework of ballistic and design parameter optimization.

An interactive research software complex for calculating ballistic and design parameters and graphic visualization of the motion scheme for small thrust spacecraft was developed.

6. References

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