Spectroscopic study of Be-shell stars: 4 Her and 88 Her

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Abstract We present an optical spectroscopic study based on 41 spectra of 4 Her and 32 spectra of 88 Her, obtained over a period of 6 months. We estimate the rotational velocity of these stars from He I lines in the blue spectral region (4000–4500 Å). We find that these stars are likely to be rotating at a fractional critical rotation of ∼0.80. We measure the average \( I_p/I_c \) ratio to quantify the strength of the H\( \alpha \) line and obtain 1.63 for 4 Her and 2.06 for 88 Her. The radius of the H\( \alpha \) emission region is estimated to be \( R_d/R_\ast \sim 5.0 \), assuming a Keplerian disk. These stars are thus found to be fast rotators with a relatively small H\( \alpha \) emission region. We detect \( V/R \) variation of the H\( \alpha \) spectral line during the observed period. We re-estimate the periods for both stars and obtain a period of ∼46 d and its harmonic of 23.095 d for 4 Her, and a period of ∼86 d for 88 Her. As these two cases are shell stars with binaries and have low H\( \alpha \) EW with the emission region closer to the central star, the \( V/R \) variation and a change in period may be an effect of the binary on the circumstellar disk.

Key words: stars: emission-line, Be — circumstellar matter — stars: individual (4 Her, 88 Her) — stars: rotation — techniques: spectroscopic

1 INTRODUCTION

A Classical Be-star is defined as a non-supergiant B-type star whose spectrum has, or had at sometime, one or more Balmer lines in emission (Collins 1987). The emission lines originate from the geometrically thin circumstellar disk rotating with near-Keplerian velocity surrounding the central star (Carciofi & Bjorkman 2006). This disk is said to be formed from the material ejected from the fast-spinning central star.

Be-shell stars are ordinary Be stars seen edge-on, so that the line of sight towards the star probes the circumstellar, equatorial disk (Porter & Rivinius 2003). They have sharp and deep absorption components in the centers of double-peaked emission lines. If the absorption in between the two peaks reaches below the continuum level, it is called a shell profile. Shell stars also have strongly rotationally broadened photospheric lines and additional narrow absorption lines (Rivinius et al. 2006). A significant fraction of Be stars, and thus also shell stars, undergoes long-term variability of \( V/R \) which is an indication of the Keplerian motion in the circumstellar disk (Hanuschik et al. 1995). One-third of all double-peaked profiles exhibit what is called violet-to-red emission peak height ratio (\( V/R \)) variations (Catanzaro 2013). It is one of the main characteristics describing the double-peaked emission lines of Be stars.

In this paper we mainly present the \( V/R \) variability of two such shell stars, i.e. 4 Her and 88 Her (see Table 1), based on spectroscopic data. We have obtained 41 spectra of 4 Her and 32 spectra of 88 Her over a period of about six months. We discuss the observed features and changes in the spectra of these stars. We have estimated the radius of the circumstellar disk using the H\( \alpha \) line. We also determined the rotational velocity of the central star from the He I lines.

The paper is arranged as follows. The following section gives a brief overview of these two stars, mainly focusing on their spectroscopic variability from previous studies. Section 3 addresses the details of spectral observations and data reduction techniques. In Section 4, we present the spectra and discuss the major results from the spectral line analysis of both stars. The conclusions drawn from this study are listed in Section 5.

2 PREVIOUS STUDIES

2.1 4 Her

4 Her is a well known and rather frequently studied Be-shell star. It was first recognized as a Be star by Heard
(1939) and Mohler (1940). Hubert (1971) reported remarkable spectral changes of the star which occurred between 1953 and 1970. Koubsky et al. (1994) identified two different long-term variation periods of 28 and 43 years when the Hα emission was absent. Harmanec et al. (1973) suggested the object is a single-line spectroscopic binary. Koubsky et al. (1997) showed that Hα V/R and radial velocities of the shell lines followed a 46.18 day period. This was also confirmed by Rivinius et al. (2006), who also described changes in the equivalent width (EW). Štefl et al. (2007) reported that the star showed orbital phase-locked variations and that the variations were coherent over more than 80 cycles. It also exhibits central quasi emission bumps (CQEB) which are the apparent doubling of some shell lines and were first described by Koubsky et al. (1997). The shell lines were reported to be typically broad for all stars that show CQEB by Rivinius et al. (1999). They proposed that further investigation of the period as well as the nature of its secondary component has to be carried out. Catanzaro (2013) estimated the radius of the circumstellar disk to be 4.3 $R_\odot$.

### 2.2 88 Her

88 Her was first discovered in 1959 to be a Be-shell star by Bidelman & Svolopoulos (1960). They described the Hα as showing a very strong double emission line, with an intense absorption core almost centrally dividing the broad emission into two very nearly equal components during 1959. Harmanec et al. (1974) improved their previous estimation of period and gave a much more precise value of 1959 d as well as the nature of its secondary component has to be carried out. Catanzaro (2013) estimated the radius of the circumstellar disk to be 4.3 $R_\odot$.

### 3 OBSERVATIONS AND DATA REDUCTION

The spectra of the two stars were obtained using the Universal Astronomical Grating Spectrograph (UAGS) at the Cassegrain focus of the 1.0 m Carl Zeiss reflector located at Vainu Bappu Observatory, Kavalur, India (12°34′) and operated by the Indian Institute of Astrophysics (IIA). The CCD used for imaging consists of 1024 × 1024 pixels with a size of 24 μm, where the central 1024 × 300 pixels were used for spectroscopy and the typical readout noise is about $4.8e^{-}$ and the gain is 1.22 $e^{-}$/ADU. All the spectra were acquired during several observing runs from February 2009 to July 2009 and the journal of observations is given in Table 2. The grating used for this particular observation is from Bausch and Lomb and has 1800 lines per millimeter, which in combination with the slit provided a resolution of 1 Å at Hα. The medium resolution data for the blue spectral region which included absorption lines like Hγ to Hθ and also He I lines were taken in the wavelength region 3800 – 4600 Å, and data for the red region having Hα in emission were taken in the range 6200 – 6800 Å.

The reduction of spectra, which included the subtraction of the bias frame, correction for the flat-field, the extraction of the aperture and wavelength calibration were performed using several routines in the NOAO/IRAF1 (Image Reduction and Analysis Facility) package. Dome flats were used to correct the pixel-to-pixel quantum efficiency variations. The wavelength calibration was performed using Fe-Ar arc lamp spectra. Typical signal-to-noise ratio (S/N) near Hα for 4 Her is $\sim$160 from 20 spectra and for 88 Her is $\sim$100 from 15 spectra.

All the spectra were initially normalized to the continuum. IRAF tasks were later used to measure parameters of the emission line profiles, such as EW, Full Width at Half Maximum (FWHM), V/R ratio, peak separation ($\Delta V$) and $I_p/I_e$. All the measurements are reported in the next section.

### 4 ANALYSIS AND DISCUSSION

4 Her was observed from February to July, 2009 and 88 Her was observed from April to July, 2009.

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1 IRAF is distributed by the National Optical Astronomy Observatory, which is operated by the Association of Universities for Research in Astronomy, Inc.
Representative sample spectra for the two stars are shown in Figures 1 and 2 respectively. In the following subsection, we discuss the rotational velocity study of both stars. Subsection 4.2 deals with the radius of the circumstellar disk and the method of its estimation. $V/R$ variation for the stars during the observation period is studied separately in Subsection 4.3.

### 4.1 Rotational Velocity Estimation

Rotational velocity ($v\sin i$) of Be stars is in general very high compared to B-type stars. Be stars are said to be rapid rotators and current statistics indicate that for a few Be stars, rotational velocity can reach their critical velocity (Rivinius et al. 2013). For rotational velocity estimation, we have used the He I absorption lines in the blue spectral region, He I $\lambda$4026, 4143, 4387 and 4471 (refer to Fig. 1). These lines are assumed not to be affected by emission from the disk. However, in the red spectral region (Fig. 2), the He I $\lambda$6678 absorption line is not used for the rotational velocity estimation, since there could be an emission component present in this line. For 88 Her, He I $\lambda$4387 was very weak and thus was not considered for $v\sin i$ estimation.

Steele et al. (1999) derived the spectral class and rotational velocities for a sample of 58 Be stars. They made a fit to the FWHM - $v\sin i$ correlation of Slettebak et al. (1975) and obtained relations shown in equations (1)–(4) in their paper for the four He I lines mentioned above. We estimated the $v\sin i$ for both 4 Her and 88 Her, using those relations between FWHM and $v\sin i$ for He I $\lambda$4026, 4143, 4387 and 4471. The FWHM of the He I lines and the estimated $v\sin i$ values are shown in Table 3 for 4 Her and in Table 4 for 88 Her. The average values of $v\sin i$ estimated for 4 Her and 88 Her are shown in Table 5. Out of the four He I lines, He I $\lambda$4026 was well resolved in most of the spectra and was consistent throughout the sample. The error tabulated for all the values corresponds to the standard deviation.

For 4 Her, the average values obtained were compared with the values from Slettebak (1982) and Catanzaro (2013). Similarly for 88 Her, the average $v\sin i$ values obtained were compared with that of Slettebak (1976). All the individual averages of He I lines, as well as the collective average calculated from the individual averages for both the stars seem to be consistent with those from the literature except for that given by Catanzaro (2013). Also, all the averages are found to be within the error.

Shell stars, as defined by Rivinius et al. (2006), are stars with strongly rotationally broadened photospheric lines. Hence the $v\sin i$ values estimated in this study from the photospheric lines show a very high value as expected and also match well with the literature values. Catanzaro (2013) quotes 40 km s$^{-1}$ for 88 Her using Mg II 4481,
Fig. 1  Spectra of 4 Her (top) and 88 Her (bottom) in the wavelength region 3700 – 4600 Å along with prominent absorption lines marked on the spectra.

| Date of Observation | Het λ4026 | Het λ4143 | Het λ4387 | Het λ4471 |
|---------------------|-----------|-----------|-----------|-----------|
|                     | FWHM (Å)  | v sin i (km s⁻¹) | FWHM (Å)  | v sin i (km s⁻¹) | FWHM (Å)  | v sin i (km s⁻¹) | FWHM (Å)  | v sin i (km s⁻¹) |
| 06/04/09            | 7.4       | 338.2     | –         | –         | 7.8       | 326.2     | 6.2       | 254.2     |
| 14/04/09            | 5.6       | 275.5     | 7.7       | 342.8     | 11.7      | 493.4     | 5.8       | 238.8     |
| 02/05/09            | 6.4       | 291.3     | –         | –         | 7.2       | 302.1     | 6.9       | 285.4     |
| 02/05/09            | 6.2       | 285.5     | –         | –         | 8.4       | 354.1     | 5.5       | 227.1     |
| 03/05/09            | 5.2       | 238.4     | 4.6       | 204.8     | 6.3       | 265.5     | 6.0       | 245.8     |
| 03/05/09            | 6.6       | 300.4     | –         | –         | 7.8       | 329.0     | 5.9       | 245.1     |
| 05/05/09            | 6.1       | 279.4     | 5.8       | 257.5     | 7.1       | 298.7     | 7.0       | 290.2     |
| 05/05/09            | 6.5       | 296.9     | 8.4       | 374.2     | 7.0       | 292.5     | 7.8       | 302.1     |
| 06/05/09            | 6.4       | 293.6     | –         | –         | 9.7       | 408.5     | 6.3       | 259.8     |
| 06/05/09            | 8.3       | 379.1     | –         | –         | 7.8       | 327.8     | 6.6       | 272.1     |
| 10/05/09            | 8.7       | 397.0     | –         | –         | 8.3       | 348.4     | 7.1       | 291.2     |
| 10/05/09            | 8.5       | 390.7     | –         | –         | 7.7       | 325.6     | 7.2       | 296.5     |
| 11/05/09            | 6.3       | 286.8     | 9.3       | 413.0     | 7.7       | 322.4     | 5.4       | 222.0     |
| 11/05/09            | 6.1       | 277.9     | 6.9       | 305.0     | 7.3       | 307.9     | 7.2       | 299.0     |
| 13/05/09            | 6.2       | 285.3     | 7.1       | 316.6     | 6.9       | 291.4     | 7.0       | 287.7     |
| 13/05/09            | 7.8       | 356.6     | 7.8       | 345.6     | 7.7       | 323.8     | 6.4       | 264.9     |
| 13/05/09            | 7.8       | 357.7     | 7.9       | 349.8     | 3.4       | 141.7     | 6.7       | 274.6     |
| 02/06/09            | 6.2       | 285.0     | 8.8       | 392.2     | –         | –         | –         | –         |
| 16/06/09            | 6.7       | 307.8     | 5.4       | 241.8     | –         | –         | –         | –         |
| 23/06/09            | 6.9       | 317.2     | 7.5       | 333.2     | –         | –         | –         | –         |
| 23/06/09            | 6.3       | 287.9     | 6.5       | 288.7     | –         | –         | –         | –         |
Table 4 FWHM and $v \sin i$ Measurements for 88 Her from the Spectra using He I λ 4026, 4143 and 4471

| Date of Observation | He I $\lambda$ 4026 FWHM (Å) | He I $\lambda$ 4143 FWHM (Å) | He I $\lambda$ 4471 FWHM (Å) |
|---------------------|-------------------------------|-------------------------------|-------------------------------|
| 14/04/09            | 6.6                           | 7.5                           | 8.2                           |
| 02/05/09            | 7.2                           | 4.2                           | 5.9                           |
| 03/05/09            | 5.4                           | 6.3                           | 7.5                           |
| 03/05/09            | 7.6                           | –                             | 7.4                           |
| 05/05/09            | 7.0                           | 4.9                           | 7.8                           |
| 05/05/09            | 5.1                           | 7.2                           | 6.6                           |
| 06/05/09            | 7.5                           | –                             | 6.6                           |
| 11/05/09            | 6.5                           | 6.2                           | 6.8                           |
| 11/05/09            | 6.6                           | 6.3                           | 6.2                           |
| 13/05/09            | 6.9                           | 5.5                           | 7.7                           |
| 16/06/09            | 7.1                           | 6.2                           | 24.7                          |
| 23/06/09            | 5.7                           | 11.7                          | 519.4                         |
| 23/06/09            | 9.2                           | 7.3                           | 323.1                         |
| 25/06/09            | 6.6                           | 8.2                           | 365.6                         |
| 25/06/09            | 5.7                           | –                             | 278.0                         |

Table 5 Rotational velocity parameters. $v \sin i$ measured from the spectra for both stars is compared with two other estimations; $\omega$ is calculated using $v_c$ given in table 2 of Yudin (2001), who interpolated values by Moujtahid et al. (1999).

| Parameter | 4 Her | 88 Her |
|-----------|-------|--------|
| $v \sin i$ (km s$^{-1}$) | 4026 Å | 310 ± 10 (21) | 316 ± 13 (17) |
|           | 4143 Å | 320 ± 17 (13) | 303 ± 25 (12) |
|           | 4387 Å | 321 ± 17 (17) | –                |
|           | 4471 Å | 269 ± 7 (17)  | 288 ± 9 (11)    |
| Average   | 305 ± 13 | 302 ± 16    |                   |
| Slettebak | 300*  | 300**     |                   |
| Catanzaro | 275   | 40        |                   |
| $v_c$ (km s$^{-1}$) |       |        | Yudin 362 | 374 |
| $\omega$  | 4026 Å | 0.86   | 0.85   |
|           | 4143 Å | 0.88   | 0.81   |
|           | 4387 Å | 0.89   | –      |
|           | 4471 Å | 0.74   | 0.77   |
| Average   | 0.84  | 0.81    |        |

Notes: † Total number of spectra from which the value was averaged; * Slettebak (1982), ** Slettebak (1976).

which is underestimated compared to the values we have obtained.

We have estimated the critical fractional rotation velocity of these stars, as these are expected to rotate close to the critical or break-up velocity. The value for the critical velocity $v_c$ was taken from Yudin (2001) which was estimated for a particular spectral class and luminosity class. Thus, for the spectral type indicated in Table 1, $v_c$ was obtained and critical fractional rotation, $\omega$ as given by $v \sin i/v_c$, was estimated. The uncertainty in $\omega$ is not only from $v \sin i$ but also from the spectral and luminosity classes (Rivinius et al. 2006). The critical fractional rotation obtained in our study using the He I lines was compared with the values obtained by Rivinius et al. (2006).

They estimated an average critical fractional rotation $\bar{\omega}$ to be $81 \pm 12\%$, for shell stars, by considering 27 shell stars. We find that our estimation is similar to the literature value.

4.2 Disk Radius Estimation

The material ejected from the central star forms an equatorially flattened disk also called the circumstellar decretion disk. We estimated the size of the disk by using the Hα line.

Figure 2 shows the Hα line which is seen as double-peaked emission with the central absorption core between the two peaks going below the continuum for both stars, as normally seen for any Be-shell star. The time series of the
Hα profile for both stars is also shown in Figure 3. 4 Her was observed 20 times in 11 d and 88 Her was observed 15 times in 8 d in the Hα region.

The peak-to-continuum intensity $I_p/I_c$ was obtained by considering the highest peak among the two peaks of Hα. The de-blending technique in IRAF was used to estimate the EW of the Hα line for all the days and is shown in separate tables for the two stars (Tables 6 and 7). These were measured along with the $V/R$ ratio and $\Delta V$ for 4 Her and 88 Her and all the quantities are shown in Tables 6 and 7. The average values for $I_p/I_c$, EW, $\Delta V$ and $V/R$ for 4 Her from 20 observations and 88 Her from 15 observations are shown in Table 8. The errors in EW and $\Delta V$ are the standard deviations of the available observations.

Another quantity measured from the spectra for the two stars is the velocity separation, i.e. $\Delta V$, between the red and violet peaks in the double-peaked profile. It will give an estimation of the region of emission for the Hα profile. These were measured using the $V/R$ ratio for 4 Her and 88 Her and all the quantities are shown in Tables 6 and 7. The average values for $\Delta V$ and $V/R$ for 4 Her from 20 observations and 88 Her from 15 observations are shown in Table 8. The errors in EW and $\Delta V$ are the standard deviations of the available observations.

Rotational velocity of the disk is approximated by the power law as shown in Equation (1).

$$\frac{R_d}{R_*} = \left(\frac{2 \sin i}{\Delta V}\right)^{\frac{1}{2}}$$ (1)

The extent of the Hα emission region $R_d$ is estimated in terms of the stellar radius $R_*$ by assuming the region to be in a Keplerian orbit around the star (Huang 1972). $R_d$ is also estimated for a non-Keplerian orbit by changing the rotational parameter $j$ from 1/2 to 1. Values of $R_d/R_*$ estimated for both cases using the average $\Delta V$ are shown in Table 8. The $\sin i$ is taken from Table 5.

We have not estimated the radius of the disk using Catanzaro (2013), since the $\sin i$ itself is too low. In the case of 4 Her, the radius of the disk is found to be in the range 3.9 – 5.6 $R_*$ and for 88 Her it is found to be in the range 3.4 – 4.2 $R_*$. Slettebak et al. (1992) assumed Keplerian geometry for the circumstellar disk of 41 stars and concluded that the Hα emission region in general is seen in the range 7 – 19 $R_*$ from the central star. Comparing our estimation of $\sim 5 R_*$ to this range, we can conclude that the Hα emission region is closer to the star for both 4 Her and 88 Her and also that the Hα emission region is smaller than the previously estimated range. We have not estimated errors in the values of the radius for each star, but present a range of values for them. We expect the errors to be about 10%. The parameter which is susceptible to large errors is the EW where the error can be large when the continuum is not too well defined due to an inadequate S/N.

4.3 $V/R$ Variation

The material ejected from the central star moves around in the disk and when this system is viewed in a highly inclined or edge-on system, it results in the splitting of the Hα line into two peaks. The Hα emission line may be either single or double-peaked, but the most common profile seen is the double-peaked emission lines (Hanuschik 1996). The variation of the $V/R$ ratio is called the $V/R$ variation. The variation in the peaks can be seen clearly in the spectra for both the stars and is illustrated in Figure 3. Both 4 Her and 88 Her were previously known to show variations in the $V/R$ and the period of variations is well known to be 46.18 d (Koubsky et al. 1997) for 4 Her and 86.72 d (Doazan et al. 1982) for 88 Her.

In this study, we looked for the short-term $V/R$ variations in these two stars using different period search methods. Koubsky et al. (1997) used spectroscopic and photometric observations of 4 Her from 1969–1997 which also included data from Harmance et al. (1976). They determined the period using the radial velocity measurements of the Hα line. They reported a phase-locked $V/R$ variation with period 46.18 d by considering two distinct shell episodes twenty years apart. Rivinius et al. (2006) confirmed this period from their data set and reported a decrease of EW from –6 to –1 Å from early 1997 to mid 1999 and again an increase to –5.5 Å after which it remained unchanged till early 2003.

In our observation, 4 Her shows Hα emission with $V > R$ in all the spectra taken before July but showed a reversal, i.e. $V < R$, for the three spectra obtained in the month of July. There is also a slight variation of EW for 4 Her from –6.4 to –8.6 Å in a span of six months in 2009 which indicates that the EW has increased from the value quoted by Rivinius et al. (2006).

Doazan et al. (1982) used 1963 – 1979 spectroscopic data and 1968 – 1981 photometric observations and determined the period for 88 Her as 86.7221 d and also reported that the period seems to have been stable since 1912. They also reported that the $V/R$ variation was in phase with the radial velocity of the Hα line. Dzemmler et al. (1988) observed this star from 1977 – 1987 and studied different lines in the shell spectrum. Menickent & Vogt (1991) published that the $V/R$ period of 88 Her is about 0.24 yr and also gave the EW as 3.9 Å during their observation before 1989. During our observation, 88 Her showed only slight fluctuations in the $V/R$ ratio but had $V < R$ for all the spectra. Rivinius et al. (2006) also found $V < R$ in their observation. We see a significant variation of EW for 88 Her from –9.0 to –13.4 Å in a span of four months in 2009.

To search for the period of $V/R$ variation in both the stars, we collected all the $V/R$ values that were available in the literature and combined them with our data to obtain robust results. For 4 Her, we collected $V/R$ values from three previous data sets, i.e. from Harmance et al. (1976) with 27 data points during 1969 – 1973, from Koubsky et al. (1997) with 89 data points during 1975 – 1997 and Rivinius et al. (2006) with 56 data points during 1998 – 2003. We added 20 $V/R$ values from our data set in 2009 which provided us altogether with 192 $V/R$ data points for 4 Her. For 88 Her, we collected $V/R$ values from two previous data sets, i.e. from Doazan et al. (1982) with 51 data points during 1971 – 1979 and from Dzemmler et al. (1988) with 15 data points during 1981 – 1987. We collec-
Fig. 2 Spectra of 4 Her (top) and 88 Her (bottom) in the wavelength region 6200 – 6800 Å showing Hα double-peaked emission along with the HeI absorption line.

Fig. 3 Left: Time Series of the 4 Her Hα line from February to July 2009; Right: Time Series of the 88 Her Hα line from April to July 2009. (Spectra are offset and labeled with the observation date, with the oldest appearing at the bottom and most recent at the top. Note that although the spectra are displayed evenly spaced, they are not evenly distributed in time.)
Table 6  Measurements of Hα Emission Line Parameters for 4 Her

| Date of Observation | \((I_p/I_c)\) | EW (Å) | \(\Delta V\) (km s\(^{-1}\)) | \(V/R\) |
|---------------------|--------------|--------|------------------|--------|
| 03/02/09            | 1.47         | −7.0   | 280.0            | 1.16   |
| 04/02/09            | 1.44         | −6.4   | 281.0            | 1.16   |
| 26/04/09            | 1.64         | −8.6   | 257.7            | 1.03   |
| 26/04/09            | 1.64         | −7.5   | 274.3            | 1.01   |
| 27/04/09            | 1.62         | −7.4   | 260.7            | 1.03   |
| 27/04/09            | 1.61         | −7.2   | 281.6            | 1.06   |
| 28/04/09            | 1.69         | −7.2   | 262.8            | 1.11   |
| 28/04/09            | 1.58         | −6.8   | 262.0            | 1.07   |
| 29/04/09            | 1.73         | −8.4   | 274.4            | 1.13   |
| 29/04/09            | 1.66         | −7.2   | 253.0            | 1.15   |
| 30/04/09            | 1.64         | −7.2   | 253.9            | 1.10   |
| 30/04/09            | 1.63         | −6.9   | 256.7            | 1.13   |
| 01/05/09            | 1.64         | −7.6   | 274.0            | 1.17   |
| 01/05/09            | 1.64         | −7.4   | 280.6            | 1.18   |
| 11/07/09            | 1.62         | −8.0   | 275.3            | 0.94   |
| 11/07/09            | 1.62         | −7.4   | 300.1            | 0.94   |
| 22/07/09            | 1.69         | −8.2   | 292.0            | 0.91   |
| 22/07/09            | 1.68         | −7.8   | 281.3            | 0.94   |
| 23/07/09            | 1.60         | −7.3   | 273.0            | 0.86   |
| 23/07/09            | 1.69         | −8.3   | 284.7            | 0.96   |

Table 7  Measurements of Hα Emission Line Parameters for 88 Her

| Date of Observation | \((I_p/I_c)\) | \(W\) (Å) | \(\Delta V\) (km s\(^{-1}\)) | \(V/R\) |
|---------------------|--------------|----------|------------------|--------|
| 26/04/09            | 2.16         | −11.8    | 316.9            | 0.85   |
| 26/04/09            | 2.11         | −10.2    | 313.4            | 0.82   |
| 27/04/09            | 2.21         | −12.0    | 317.6            | 0.83   |
| 28/04/09            | 2.03         | −9.8     | 298.8            | 0.91   |
| 28/04/09            | 2.05         | −9.2     | 318.6            | 0.85   |
| 29/04/09            | 2.11         | −10.8    | 319.4            | 0.88   |
| 29/04/09            | 2.02         | −9.4     | 318.7            | 0.87   |
| 30/04/09            | 2.06         | −9.0     | 326.0            | 0.86   |
| 30/04/09            | 2.04         | −9.5     | 318.1            | 0.85   |
| 01/05/09            | 2.07         | −10.2    | 318.4            | 0.90   |
| 01/05/09            | 2.01         | −10.5    | 301.0            | 0.93   |
| 01/05/09            | 2.05         | −9.8     | 296.1            | 0.88   |
| 11/07/09            | 1.93         | −11.2    | 297.8            | 0.96   |
| 11/07/09            | 1.95         | −13.4    | 297.3            | 0.96   |
| 27/07/09            | 2.12         | −11.4    | 312.4            | 0.93   |

tively had 81 \(V/R\) values along with our 15 data points for 88 Her.

Two different approaches were adopted for the \(V/R\) period analysis, namely string length and a Fourier-based method. A time-series analysis has been performed with the rigorous analysis of variance (AoV) method in multi-harmonic (MAOVMH) mode (Schwarzenberg-Czerny 1996). In this method, no implicit assumption is made on the shape of the variations. The NASA Exoplanet Archive Periodogram Service with the Lomb-Scargle algorithm (Scargle 1982) was also used to estimate the Lomb-Scargle power spectra. In addition to this, we also used the Schuster algorithm (Schuster 1898) to derive the Schuster periodogram. In the latter case, we have estimated the Scargle False Alarm Probability (FAP) and S/N of individual frequencies. The periods obtained from all the three methods are summarized in Table 9 for both stars. Periodograms for both stars using all three methods are shown in Figure 4.

All the three methods yielded almost the same frequency spectrum (Fig. 4) for both stars. In addition to the longer ∼46 d period of 4 Her, we detected a smaller period of 23.095 d which is the harmonic of the ∼46 d period. Agreement between the results of using different methods was an indication of reliability of a detection. In order to check whether the shorter period was already there in the older data set, we checked in the data sets from Harmanec et al. (1976), Koubsky et al. (1997) and Rivinius et al. (2006). We obtained the above mentioned shorter period in the data set from Koubsky et al. (1997) which had 89
Table 8  Hα Emission Line Parameters and Estimation of the Radius of the Disk

| Star Name | $I_p/I_c$ | $\Delta V$ | $v_{\sin i}$ | $R_d/R_*$ |
|-----------|----------|------------|---------------|------------|
| 4 Her     | 1.63     | $-7.48 \pm 0.13$ | 272.95 ± 2.9 |            |
| 88 Her    | 2.06     | $-10.55 \pm 0.32$ | 311.36 ± 2.61|            |

$^\dagger$Refer to Table 5

Table 9  Periods obtained from different periodograms for 4 Her and 88 Her. FAP and S/N of the peak in the case of the Schuster periodogram are given. A $p$-value in the case of the Lomb-Scargle algorithm is also given.

| Star Name | AoV | Schuster | Lomb-Scargle |
|-----------|-----|----------|--------------|
|           | Period (d) | Period (d) | FAP | S/N | Period (d) | $p$-value |
| 4 Her     | 46.296 | 46.296 | 7.57811E–015 | 8.333 | 46.173 | 0 |
|           | 23.095 | 23.095 | 4.11814E–013 | 8.678 | 23.095 | 2.68922E–008 |
| 88 Her    | 86.797 | 86.957 | 1.11085E–002 | 6.474 | 86.884 | 3.61152E–003 |

Fig. 4  Periodograms for 4 Her (left panels) and 88 Her (right panels) using different period search algorithms.
Fig. 5 Phase plot of $V/R$ for 4 Her with period 46.296 d (top) and 23.095 d (bottom) along with a fit of a spline function.

$V/R$ values, but it was not as significant as what we obtained after combining all the data with our data. This was also checked by combining the other data sets with these 89 $V/R$ values, where we observed an increasing trend in significance of the period with the addition of more data. Thus, the new period we estimated might have been missed earlier due to a sparser data set. We conclude that the $V/R$ variation in 4 Her is more than singly periodic. But without having a larger dataset available, further discussions on multiple periodicity would seem meaningless. This suggests that continuous monitoring of this star is necessary to obtain the $V/R$ variability period. A phase plot of $V/R$ has been shown in Figure 5 for 4 Her with two periods detected. No smaller periods were detected in the case of 88 Her except for the already known $\sim$86 d period. The phase plot for 88 Her was not included since the data set is very sparse. However, more continuous observations are required to understand the nature of its variability. We discuss the implication of this result in the next subsection.

4.4 Discussion

In this study, we have estimated various parameters of the two shell stars, 4 Her and 88 Her. We estimated the rotational velocity $v \sin i$ of the stars to be 305 ±13 km s$^{-1}$ for 4 Her and 302 ±16 km s$^{-1}$ for 88 Her. These stars are well known to be edge-on systems, hence these velocities are considered to be very close to the true velocity. By assuming the critical velocity based on the spectral class, we estimated that the fractional critical rotation is about 0.8, suggesting that these stars are rotating very close to their breakup velocity. We also estimated the radius of H$\alpha$ emission, which is found to be in the range 3.4–5.6 $R_\odot$. 
In summary, we find that these stars are fast rotating stars with the Hα emission region located very close to the star.

We also notice that there is a $V/R$ variation in the Hα profile. $V/R$ variation only occurs when the disk is assumed to be an eccentric Keplerian disk. This is understood as a global density wave pattern in the circumstellar disk as shown in figure 2 of McDavid et al. (2000) of a one-arm density wave for ζ Tau. It is the precession of the density wave about the central star. This is based on the model of global one-armed oscillations of equatorial disks in Be stars as given by Okazaki (1991).

4 Her is reported to have $V/R$ variations phase-locked to the orbit of the binary component (Štefl et al. 2007). Rivinius et al. (2006) compared the $V/R$ variations of 4 Her and ε Cap with ϕ Per and mentioned that the variations are periodic for the phase-locked $V/R$ variation rather than being cyclic and also observed that they have shorter time-scales. Mennickent & Vogt (1991) found that 6 among their sample of 33 stars showed short term $V/R$ variations and these are stars with small envelopes. They suggest that the short term $V/R$ variations could be caused by the rotation of inhomogeneities in the circumstellar envelopes. As the two shell stars studied are binaries, it is quite possible that the inhomogeneities in the circumstellar material are caused by the gravitational effect of the binary. This is also supported by the fact that one of the observed periods of 4 Her is a harmonic of the previously estimated period. Thus, the change in the value of the period estimated suggests that there might be an inhomogeneity present in the disk and probably the binary star may be responsible for this perturbation.

Further observations and continuous monitoring of these stars will increase the number of data points and help us in identifying the period more accurately. Our study also reveals that short-term monitoring of these systems is important for understanding the effect of the binary star and its perturbation on the circumstellar disk. This study finds that 4 Her and 88 Her are ideal targets for continuous monitoring of $V/R$ variation in the Hα profile, especially since these observations can be performed with a moderate-sized telescope equipped with a spectrograph.

5 CONCLUSIONS

(1) We have presented the spectroscopic analysis of the two Be-shell stars, 4 Her and 88 Her, which were observed for about six months in 2009.

(2) The rotational velocity $v \sin i$ was calculated using Hβ lines and is found to be $\sim 300 \text{ km s}^{-1}$ for both stars. The fraction of critical rotation for the two stars is found to be $\sim 0.8$, suggesting them to be rapid rotators.

(3) The radius of the circumstellar disk $R_d/R_e$, using the Hα double-peaked emission profile is found to be $\sim 5.0$, assuming a Keplerian orbit for both 4 Her and 8 Her. This implies that the Hα emission disk is very small for both stars.

(4) The EW of the Hα emission line profile varied from $-6.4$ to $-8.6$ Å for 4 Her in a span of six months and from $-9.0$ to $-13.4$ Å for 88 Her in a span of four months.

(5) $V/R$ variation was observed for both stars and the period was re-estimated by different period search techniques. For 4 Her, periods of $\sim 46$ d and a harmonic of $23.095$ d were detected. For 88 Her, a longer period of $\sim 86$ d, which is very close to the period given in literature, was obtained; but further confirmation is required after continuous monitoring.

(6) We conclude and confirm that these two stars are rapid rotators with a smaller Hα emitting region. As they have binaries (with 4 Her being a phase locked binary), the observed $V/R$ variation may be due to the gravitational effect of the binary on the circumstellar disk.

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