Quantum information processing:
The case of vanishing interaction energy

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Abstract: We investigate the rate of operation of quantum "black boxes" ("oracles") and point out the possibility of performing an operation by a quantum "oracle" whose average energy equals zero. This counterintuitive result not only presents a generalization of the recent results of Margolus and Levitin, but might also sharpen the conceptual distinction between the "classical" and the "quantum" information.

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1. Introduction: information-processing bounds

In the realm of computation, one of the central questions is "what limits the laws of physics place on the power of computers?" [1,2]. The question is of great relevance to a wide range of scientific disciplines, from cosmology and nascent discipline of physical eschatology [3-5] to biophysics and cognitive sciences which study information processing in the conscious mind [6, 7]. One of the physical aspects of this question refers to the minimum time needed for execution of the logical operations, i.e. to the maximum rate of transformation of state of a physical system implementing the operation. From the fundamental point of view, this question tackles the yet-to-be-understood relation between the energy (of a system implementing the computation) on the one, and the concept of information, on the other side. Apart from rather obvious practical interest stemming from the explosive development of information technologies (expressed, for instance, in the celebrated Moore’s law), this trend of merging physics and information theory seems bound to offer new insights into the traditional puzzles of physics. Specifically, answering the question above might shed new light on the standard "paradoxes" of the quantum world [8-10].

Of special interest are the rates of the reversible operations (i.e. of the reversible quantum state transformations). To this end, the two bounds for the so-called "orthogonal transformations (OT)" are known; by OT we mean a transformation of a (initial) state $|\Psi_i\rangle$ to a (final) state $|\Psi_f\rangle$, while $\langle \Psi_i | \Psi_f \rangle = 0$. First, the minimum time needed for OT can be characterized in terms of the spread in energy, $\Delta \hat{H}$, of the system implementing the transformation [11-15]. However, recently, Margolus and Levitin [16, 17] have extended this result to show that a quantum system with average energy $\langle \hat{H} \rangle$ takes time at least $\tau = \hbar/4(\langle \hat{H} \rangle - E_0)$ to evolve to an orthogonal state, where $E_0$ is the ground state energy. In a sense, the second bound is more restrictive: a system with the zero energy (i.e. in the
ground state) cannot perform a computation ever. This however stems nothing about the nonorthogonal evolution which is still of interest in quantum computation.

Actually, most of the efficient quantum algorithms [18-21] employ the so-called quantum "oracles" (quantum "black boxes") not requiring orthogonality of the initial and the final states of the composite quantum system "input register + output register (I + O)" [22-24]. Rather, orthogonality of the final states of the subsystems' (e.g. O's) states is required, thus emphasizing a need for a new bound for the operation considered.

In this paper we show that the relative maximum of the rate of operation of the quantum "oracles" may point out the zero average energy of interaction in the composite system I + O. Actually, it appears that the rate of an operation cannot be characterized in terms of the average energy of the composite system as a whole. Rather, it can be characterized in terms of the average energy of interaction Hamiltonian. Interestingly enough, the ground state energy $E_0$ plays no role, here, and the absolute value of the average energy of interaction $\langle \hat{H}_{\text{int}} \rangle$ plays the role analogous to the role of the difference $\langle \hat{H}_{\text{int}} \rangle - E_0$ in the considerations of OT. Physically, we obtain: the lower the average energy, the higher the rate of operation. This result is in obvious contradistinction with the result of Margolus and Levitin [16, 17]—in terms of the Margolus-Levitin theorem, our result would read: the lower the difference $\langle \hat{H}_{\text{int}} \rangle - E_0$, the higher the rate of (nonorthogonal) transformation. On the other side, our result is not reducible to the previously obtained bound characterized in terms of the spread in energy [11-15], thus providing us with a new bound in the quantum information theory.

2. The quantum "oracle" operation

We concern ourselves with the bounds characterizing the rate of (or, equivalently, the minimum time needed for) the reversible transformations of a quantum system’s states. Therefore, the bounds known for the irreversible transformations are of no use here. Still, it is a plausible statement that the information processing should be faster for a system with higher (average) energy, even if—as it is the case in the classical reversible information processing—the system does not dissipate energy (e.g. [25]). This intuition of the classical information theory is justified by the Margolus-Levitin bound [16, 17]. However, this bound refers to OT, and does not necessarily applies to the nonorthogonal evolution.

The typical nonorthogonal transformations in the quantum computing theory are operations of the quantum "oracles" employing "quantum entanglement" [18-21, 23, 24]. Actually, the operation considered is defined by the following state transformation:

$$|\Psi_i\rangle_I \otimes |0\rangle_O \rightarrow |\Psi_f\rangle_I \otimes |f(x)\rangle_O,$$

where $\{|x\rangle_I\}$ represents the "computational basis" of the input register, while $|0\rangle_O$ represents an initial state of the output register; by " $f$ " we denote the oracle transformation.

The key point is that the transformation (1) does not [18-21] require the orthogonality condition $I_O \langle \Psi_f | \Psi_f \rangle_I = 0$ to be fulfilled. Rather, orthogonality for the subsystem’s states is required [18-21]:

$$O\langle f(x) | f(x') \rangle_O = 0, \ x \neq x'$$

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for at least some pairs \((x, x')\), which, in turn, is neither necessary nor a sufficient condition for the orthogonality \(\langle \Psi_i | \Psi_f \rangle_{IO} = 0\) to be fulfilled.

Physical implementation of the quantum oracles of the type Eq. (1) is an open question of the quantum computation theory. However (and in analogy with the quantum measurement and the decoherence process [26-31]), it is well understood that the implementation should rely on (at least indirect, or externally controlled) interaction in the system \(I + O\) as presented by the following equality:

\[
|\Psi_f\rangle_{IO} \equiv \hat{U}(t) |\Psi_i\rangle_{IO} \equiv \hat{U}(t) \sum_x C_x |x\rangle_I \otimes |0\rangle_O = \sum_x C_x |x\rangle_I \otimes |f(x, t)\rangle_O,
\]  

(3)

where \(\hat{U}(t)\) represents the unitary operator of evolution in time (Schrodinger equation) for the combined system \(I + O\); index \(t\) represents an instant of time. Therefore, the operation (1) requires the orthogonality:

\[
\langle f(x, t) | f(x', t) \rangle_O = 0,
\]  

(4)

which substitutes the equality (2).

So, our task reads: by the use of Eq. (4), we investigate the minimum time needed for establishing of the entanglement present on the r.h.s. of Eq. (1), i.e. of Eq. (3).

3. The optimal bound for the quantum oracle operation

In this Section we derive the optimal bound for the minimum time needed for execution of the transformation (1), i.e. (3), as distinguished by the expression (4). We consider the composite system "input register + output register \((I + O)\)" defined by the effective Hamiltonian:

\[
\hat{H} = \hat{H}_I + \hat{H}_O + \hat{H}_{\text{int}}
\]  

(5)

where the last term on the r.h.s. of (5) represents the interaction Hamiltonian. For simplicity, we introduce the following assumptions: (i) \(\partial \hat{H} / \partial t = 0\), (ii) \([\hat{H}_I, \hat{H}_{\text{int}}] = 0\), \([\hat{H}_O, \hat{H}_{\text{int}}] = 0\), and (iii) \(\hat{H}_{\text{int}} = C \hat{A}_I \otimes \hat{B}_O\), where \(\hat{A}_I\) and \(\hat{B}_O\) represent unspecified observables of the input and of the output register, respectively, while the constant \(C\) represents the coupling constant. As elaborated in Appendix I, the simplifications (i)-(iii) are not very restrictive. For instance, concerning point (i)—widely used in the decoherence theory—one can naturally relax this condition to account for the wide class of the time dependent Hamiltonians; cf. Eq. (I.1) of Appendix I. Another way of relaxing the condition (i) is to assume the sudden switching the interaction in the system on and off.

3.1 The bound derivation

Given the above simplifications (i)-(iii), the unitary operator \(\hat{U}(t)\) (cf. Eq. (3)) spectral form reads:

\[
\hat{U}(t) = \sum_{x, i} \exp \left\{ -it(\epsilon_x + E_i + C\gamma_{xi})/\hbar \right\} \hat{P}_{Ix} \otimes \hat{N}_{Oi}.
\]  

(6)
The quantities in Eq. (6) are defined by the following spectral forms: 
\[ \hat{H}_I = \sum_x \epsilon_x \hat{P}_{Ix}, \]
\[ \hat{H}_O = \sum_i E_i \hat{\Pi}_{Oi}, \]
and \[ \hat{H}_{\text{int}} = C \sum_{x,i} \gamma_{xi} \hat{P}_{Ix} \otimes \hat{\Pi}_{Oi}; \]
bearing in mind that \( A_I = \sum_x a_x \hat{P}_{Ix} \)
and \( B_O = \sum_i b_i \hat{\Pi}_{Oi}, \)
the eigenvalues \( \gamma_{xi} = a_x b_i. \)

From now on, we take the system’s zero of energy at the ground state by the exchange \( E_{xi} \rightarrow E_{xi} - E_0; \)
\( E_{xi} \equiv \epsilon_x + E_i + C \gamma_{xi}, \) \( E_0 \) is the minimum energy of the composite system—which Margolus and Levitin [16, 17], as well as Lloyd [1], have used. Then one easily obtains for the output-register’s states:
\[
|f(x,t)\rangle_O = \sum_i \exp\{-it(\epsilon_x + E_i + C \gamma_{xi} - E_0)/\hbar\}\hat{\Pi}_{Oi}|0\rangle_O.
\] (7)

Substitution of Eq. (7) into Eq. (4) directly gives:
\[
D_{xx'}(t) \equiv_O \langle f(x,t)|f(x',t)\rangle_O = \exp\{-it(\epsilon_x - \epsilon_{x'})/\hbar\} \times \sum_i p_i \exp\{-iCt(a_x - a_{x'})b_i/\hbar\} = 0, \quad \sum_i p_i = 1,
\] (8)
where \( p_i \equiv_O \langle 0|\Pi_{Oi}|0\rangle_O. \) The expression (8) represents the condition for ”orthogonal evolution” of subsystem’s (O’s) states bearing explicit time dependence; the ground energy \( E_0 \) does not appear in (8).

But this expression is already known from, e.g., the decoherence theory [26-31]. Actually, one may write:
\[
D_{xx'}(t) = \exp\{-it(\epsilon_x - \epsilon_{x'})/\hbar\} z_{xx'}(t),
\] (9)
where
\[
z_{xx'}(t) \equiv \sum_i p_i \exp\{-iCt(a_x - a_{x'})b_i/\hbar\} \quad \text{(10)}
\]
represents the so-called ”correlation amplitude”, which appears in the off-diagonal elements of the (sub)system’s (O’s) density matrix [26]:
\[
\rho_{Oxx'}(t) = C_x C_x^* z_{xx'}(t).
\]

So, we could make a direct application of the general results of the decoherence theory. However, our aim is to estimate the minimum time for which \( D_{xx'}(t) \) may approach zero, rather than calling for the qualitative limit of the decoherence theory [26]:
\[
\lim_{t \to \infty} |z_{xx'}(t)| = 0, \quad \text{(11)}
\]
or equivalently \( \lim_{t \to \infty} z_{xx'}(t) \to 0. \)

In order to obtain the more elaborate quantitative results, we shall use the inequality \( \cos x \geq 1 - (2/\pi)(x + \sin x), \) valid only for \( x \geq 0 \) [16, 17]. However, the use cannot be straightforward.
Namely, the exponent in the "correlation amplitude" is proportional to:

\[(a_x - a_{x'})b_i,\]  \hspace{1cm} (12)

which need not be strictly positive. That is, for a fixed term \(a_x - a_{x'} > 0\), the expression Eq. (12) can be both positive or negative, depending on the eigenvalues \(b_i\). For this reason, we will refer to the general case of eigenvalues of the observable \(\hat{B}_O\), \(\{b_i, -\beta_j\}\), where both \(b_i, \beta_j > 0, \forall i, j\).

In general, Eq. (10) reads:

\[
z_{xx'}(t) = z_{xx'}^{(1)}(t) + z_{xx'}^{(2)}(t), \hspace{1cm} (13a)
\]

where

\*
\[
z_{xx'}^{(1)} = \sum_i p_i \exp\{-iCt(a_x - a_{x'})b_i/\hbar\}, \hspace{1cm} (13b)
\]

\[
z_{xx'}^{(2)} = \sum_j p'_j \exp\{iCt(a_x - a_{x'})\beta_j/\hbar\}, \hspace{1cm} (13c)
\]

while \(\sum_i p_i + \sum_j p'_j = 1\). Since both \((a_x - a_{x'})b_i > 0, (a_x - a_{x'})\beta_j > 0 \ (\forall i, j)\), one may apply the inequality mentioned above.

Relaxed equality (4)—or relaxed equality (11)—is equivalent to \(\text{Re } z_{xx'} \cong 0\) and \(\text{Im } z_{xx'} \cong 0\). Now, from Eq. (13a-c) it directly follows:

\[
\text{Re } z_{xx'} = \sum_i p_i \cos[C(a_x - a_{x'})b_it/\hbar] + \sum_j p'_j \cos[C(a_x - a_{x'})\beta_j t/\hbar], \hspace{1cm} (14)
\]

which, after applying the above inequality gives:

\[
\text{Re } z_{xx'} > 1 - \frac{4}{\hbar}C(a_x - a_{x'})\left(B_1 + B_2\right)t - \frac{2}{\pi}\text{ Im } z_{xx'} - \frac{4}{\pi} \sum_i p_i \sin[C(a_x - a_{x'})b_it/\hbar], \hspace{1cm} (15)
\]

where \(B_1 \equiv \sum_i p_ib_i\), and \(B_2 \equiv \sum_j p'_j\beta_j\).

Since \(\sum_i p_i \sin[C(a_x - a_{x'})b_it/\hbar] \leq \sum_i p_i \equiv \alpha < 1, \forall t\), from Eqs. (11) and (15) it follows:

\[
0 \cong \text{Re } z_{xx'} + \frac{2}{\pi}\text{ Im } z_{xx'} > 1 - \frac{4\alpha}{\pi} - \frac{4}{\hbar}C(a_x - a_{x'})\left(B_1 + B_2\right)t. \hspace{1cm} (16)
\]

\[From (16) it is obvious that the condition imposed by Eq. (4) cannot be fulfilled in time intervals shorter than \(\tau_{xx'}\):

\[
\tau_{xx'} > \frac{(1 - 4\alpha/\pi)\hbar}{4C(a_x - a_{x'})\left(B_1 + B_2\right)}. \hspace{1cm} (17)
\]
The expression is strictly positive for $\alpha < \pi/4$, and which directly defines the optimal bound $\tau_{\text{ent}}$ as:

$$\tau_{\text{ent}} = \sup \{ \tau_{xx'} \}. \quad (18)$$

The assumption $\alpha < \pi/4$ is not very restrictive. Actually, above, we have supposed that neither $\sum_i p_i \approx 1$, nor $\sum_j p_j' \approx 1$, while the former is automatically satisfied with the condition $\alpha < \pi/4$.

### 3.2 Analysis of the results

The bound $\tau_{\text{ent}}$ is obviously determined by the minimum difference $a_x - a_{x'}$. This difference is virtually irrelevant (in the quantum computation models, it is typically of the order of $\hbar$). So, one may note, that the bound in Eq. (18) may be operationally decreased by the increase in the coupling constant $C$ and/or by the increase in the sum $B_1 + B_2$. As to the former, for certain quantum ”hardware” [32, 33], the coupling constant $C$ may be partially manipulated by experimenter. On the other side, similarly—as it directly follows from the above definitions of $B_1$ and $B_2$—by the choice of the initial state of the output register, one could eventually increase the rate of the operation by increasing the sum $B_1 + B_2$.

Bearing in mind the obvious equality:

$$\langle \hat{H}_{\text{int}} \rangle = C\langle \hat{A}_I \rangle \langle \hat{B}_O \rangle = C\langle \hat{A}_I \rangle (B_1 - B_2), \quad (19)$$

one directly concludes that adding energy to the composite system as a whole, does not necessarily increase the rate of the operation considered. Rather, the rate of the operation is determined by the absolute value of the average energy of interaction, $|\langle \hat{H}_{\text{int}} \rangle|$. For instance, if $B_1 \neq 0$ while $B_2 = 0$ (or $B_2 \neq 0, B_1 = 0$), from Eq. (19) it follows that the increase in $B_1$ (or in $B_2$, and/or in the coupling constant $C$) coincides (for $\langle \hat{A}_I \rangle \neq 0$) with the increase in $|\langle \hat{H}_{\text{int}} \rangle|$, as well as with the decrease in the bound Eq. (18). This observation is in accordance with the Margolus-Levitin bound [16, 17]: the increase in the average energy (of interaction) gives rise to the increase in the rate of the operation (still, without any restrictions posed by the minimum energy of either the total, or the interaction Hamiltonian). Therefore, the absolute value $|\langle \hat{H}_{\text{int}} \rangle|$ plays, in our considerations, the role analogous to the role of the difference $\langle \hat{H}_{\text{int}} \rangle - E_0$ in the considerations of the ”orthogonal transformations”.

However, for the general initial state of the output register, both $B_1 \neq 0$ and $B_2 \neq 0$. Then, e.g., for $B_1 > B_2$:

$$B_1 + B_2 = B_1(1 + \kappa) \leq 2B_1, \quad \kappa \leq 1, \quad (20)$$

which obviously determines the relative maximum of the rate of the operation by the following equality:

$$B_1 = B_2, \quad \kappa = 1, \quad (21a)$$

which, in turn (for $\langle \hat{A}_I \rangle \neq 0$), is equivalent with:

$$\langle \hat{H}_{\text{int}} \rangle = 0. \quad (21b)$$
But this result is in obvious contradistinction with the result of Margolus and Levitin [16, 17]. Actually, the expressions (21a,b) stem that, apart from the concrete values of \( B_1 \) and \( B_2 \), the relative maximum of the rate of the operation requires (mathematically: implies) the zero average energy of interaction, \( \langle \hat{H}_{\text{int}} \rangle = 0 \)—which (as distinguished above) is analogous to the equality \( \langle \hat{H}_{\text{int}} \rangle - E_0 = 0 \) for "orthogonal transformations."

4. Discussion

Intuitively, the speed of change of a system’s state should be directly proportional to the average energy of the system. This intuition is directly justified for the quantum OT by the Margolus-Levitin theorem [16, 17]. Naively, one would expect this statement to be of relevance also for the nonorthogonal evolution. Actually, in the course of the orthogonal evolution, the system’s state "passes" through a "set" of nonorthogonal states, thus making nonorthogonal evolution faster than the orthogonal evolution itself. (At least this physical picture is justified for "realistic" interpretations of quantum mechanics, like the dynamical-reduction or many-worlds theories.)

This intuition is obviously incorrect for the cases studied. In a sense, the expressions (21) state the opposite: the lower the difference \( B_1 - B_2 \) (i.e. the lower the average energy of interaction, \( |\langle \hat{H}_{\text{int}} \rangle| \)), the faster the operation considered. Therefore, our main result, Eq. (21), is in obvious contradistinction with the conclusion drawn from the Margolus-Levitin bound [16, 17]: the zero energy quantum information processing is possible and, in the sense of Eq. (21), is even preferable. From the operational point of view, the bound \( \tau_{\text{ent}} \) can be decreased by manipulating the interaction in the combined system \( I + O \), as well as by the proper local operations (e.g., the proper state preparations increasing the sum \( B_1 + B_2 \)) performed on the output register.

As it can be easily shown, the increase in the sum \( B_1 + B_2 \) coincides with the increase in the spread in \( \hat{B}_O \), \( \Delta \hat{B}_O \), i.e. with the increase in the spread \( \Delta \hat{H}_{\text{int}} \). This observation however cannot be interpreted as to suggest reducibility of the bound in Eq. (17) onto the bound characterized in terms of the spread in energy [11-15]—in the case studied, \( \Delta \hat{H}_{\text{int}} \). Actually, as it is rather obvious, the increase in the spread \( \Delta \hat{H}_{\text{int}} \) does not pose any restrictions on the average value \( \langle \hat{H}_{\text{int}} \rangle \). Therefore, albeit having a common element with the previously obtained bound [11-15], the bound in Eqs. (17) and (18), represents a new bound in the quantum information theory.*

From Eq. (17), one directly determines the absolute maximum of the rate of the operation, i.e. the absolute minimum of the r.h.s. of Eq. (17). Actually, for \( \hat{B}_O \) bounded (which is generally the case for quantum computation models), the inequality \( B_{\text{min}} \leq B_1 + B_2 \leq B_{\text{max}} \) determines the absolute minimum of the r.h.s. of Eq. (17):

\[
\frac{(1 - 4\alpha/\pi) \hbar}{4C(a_x - a_{x'})B_{\text{max}}},
\]

* This bound is of interest also for the decoherence theory, but it does not provide us with magnitude of the "decoherence time", \( \tau_D \). Actually, one may write—in our notation—that \( \tau_D \propto (a_x - a_{x'})^{-2} \), while—cf. Eq. (17)—\( \tau_{\text{ent}} \propto (a_x - a_{x'})^{-1} \), which therefore indicates \( \tau_D \gg \tau_{\text{ent}} \). This relation is in accordance with the general results of the decoherence theory: the entanglement formation should precede the decoherence effect.
where $B_{\text{max}}$ ($B_{\text{min}}$) is the maximum (minimum) in the set $\{b_i, \beta_j\}$. Interestingly enough, the minimum Eq. (22) is achievable also in the following special case: if $B_1 + B_2 = p_1 b_{\text{max}} + p'_1 \beta_{\text{max}}$, while $p_1 = p'_1 = 1/2$ and $b_{\text{max}} = \beta_{\text{max}} = B_{\text{max}}$, one obtains, again, that $\langle \hat{H}_{\text{int}} \rangle = 0$; by $b_{\text{max}}$ ($\beta_{\text{max}}$) we denote the maximum in the set $\{b_i\}$ ($\{\beta_j\}$).

It cannot be overemphasized: the zero (average) energy quantum information processing is in principle possible. Moreover, the condition $\langle \hat{H}_{\text{int}} \rangle = 0$ determines the relative maximum of the operation considered. But this result challenges our classical intuition, because it is commonly believed that the efficient information processing presumes an "energy cost". In the classical domain, this was established in 1960s by Brillouin [25], following the ground-breaking studies of Szilard and others on the problem of Maxwell’s demon. So, one may wonder if ”saving energy” might allow the efficient information processing ever. Without ambition to give a definite answer to this question, we want to stress: as long as the ”energy cost” in the classical information processing (including the quantum-mechanical ”orthogonal evolution”) is surely necessary, this need not be the case with the quantum information processing, such as the entanglement establishing. Actually, the ”classical information” refers to the orthogonal (mutually distinguishable) states, while dealing exclusively with the orthogonal states is a basis of the classical information processing [23]. Nonorthogonal states (i.e. nonorthogonal transformations) we are concerned with necessarily refer to the nonclassical information processing. So, without further ado, we stress that Eq. (21) exhibits a peculiar aspect of the quantum information (here: of the entanglement formation), so pointing to the necessity of its closer further study.

The roles of the two registers (I and O) are by definition asymmetric, as obvious from Eqs. (1) and (3). This asymmetry is apparent also in the bound given in Eq. (17), which is the reason we do not discuss in detail the role of the average value $\langle \hat{A}_I \rangle$. Having in mind the considerations of Section 3, this discussion is really an easy task not significantly changing the conclusions above.

Finally, applicability of the bound (17) for the general purposes of the quantum computing theory is limited by the defining expression Eq. (1). The bound in Eq. (17) is of no use for the algorithms not employing quantum entanglement. As such an example, we may consider Grover’s algorithm [34], which does not employ quantum entanglement in its oracle operation. As another example, we mention the so-called ”adiabatic quantum computation” (AQC) model [35, 36]. This new computation model does not employ any ”oracles” whatsoever. Moreover, the AQC algorithms typically involve non-persistent entanglement (as distinct from those in Eq. (1)) of states of neighbor qubits (cf. Eq. (II.1) in Appendix II). Therefore, the bound in Eq. (17) is of no direct use in AQC, and cannot be used for analyzing this non-circuit model of quantum computation (cf. Appendix II).

The work on application of Eq. (17) in optimizing entangling circuits is in progress, and will be published elsewhere.

5. Conclusion

We show that the zero average energy quantum information processing is theoretically possible. Specifically, we show that the entanglement establishing in the course of operation of some typical quantum oracles employed in the quantum computation algorithms distinguishes the zero average energy of interaction in the composite system "input register
output register”. This result challenges our classical intuition, which plausibly stems a need for the “energy cost” in the information processing. To this end, our result, which sets a new bound for the nonorthogonal evolution in the quantum information processing, sets a new quantitative relation between the concept of information on the one, and of the physical concept of energy, on the other side—the relation yet to be properly understood.

**Literature:**

[1] S. Lloyd, *Nature* **406**, 1047 (2000).
[2] A. Galindo and M. A. Martin-Delgado, *Rev. Mod. Phys.* **74**, 347 (2002).
[3] S. Lloyd, *Phys. Rev. Lett.* **88**, 237901-1 (2002).
[4] F. J. Tipler, *Int. J. Theor. Phys.* **25**, 617-661 (1986).
[5] F. C. Adams and G. Laughlin, *Rev. Mod. Phys.* **69**, 337-372 (1997).
[6] C. H. Woo, *Found. Phys.* **11**, 933 (1981).
[7] S. Hagan, S. R. Hameroff, and J. A. Tuszynski, *Phys. Rev. E* **65**, 061901 (2002).
[8] B. d’Espagnat, ”Conceptual Foundations of Quantum Mechanics” (Benjamin, Reading, MA, 1971).
[9] J. A. Wheeler and W. H. Zurek, (eds.), ”Quantum Theory and Measurement” (Princeton University Press, Princeton, 1982).
[10] Cvitanović et al, (eds.), ”Quantum Chaos–Quantum Measurement” (Kluwer Academic Publishers, Dordrecht, 1992).
[11] S. Braunstein, C. Caves and G. Milburn, *Ann. Phys.* **247**, 135 (1996).
[12] L. Mandelstam and I. Tamm, *J. Phys. (USSR)* **9**, 249 (1945).
[13] A. Peres, ”Quantum Theory: Concepts and Methods” (Kluwer Academic Publishers, Hingham, MA, 1995).
[14] P. Pfeifer, *Phys. Rev. Lett.* **70**, 3365 (1993).
[15] L. Vaidman, *Am. J. Phys.* **60**, 182 (1992).
[16] N. Margolus and L. B. Levitin, in *Phys. Comp96*, (eds.) T. Toffoli, M. Biafore and J. Leao (NECSI, Boston 1996).
[17] N. Margolus, L. B. Levitin, *Physica D* **120**, 188 (1998).
[18] D.R. Simon, *SIAM J. Comp.* **26**, 1474 (1997).
[19] P.W. Shor, *SIAM J. Comp.* **26**, 1484 (1997).
[20] Peter W. Shor, ”Introduction to Quantum Algorithms”, e-print arXiv quant-ph/0005003.
[21] M. Ohya and N. Masuda, *Open Sys. & Information Dyn.* **7**, 33 (2000).
[22] A.M. Steane, *Rep. Prog. Phys.* **61**, 117 (1998).
[23] M. Nielsen and I. Chuang, ”Quantum Information and Quantum Computation” (Cambridge University Press, Cambridge, 2000).
[24] J.Preskill, in ”Introduction to Quantum Computation and Information”, (eds.) H.K. Lo, S. Popescu, J. Spiller, World Scientific, Singapore, 1998.
[25] L. Brillouin, “Science and Information Theory” (New York: Academic Press, 1962).
[26] W.H. Zurek, *Phys. Rev. D* **26**, 1862 (1982).
[27] W.H. Zurek, *Prog. Theor. Phys.* **89**, 281 (1993).
[28] D. Giulini, E. Joos, C. Kiefer, J. Kupsch, I.-O. Stamatescu and H.D. Zeh, ”Decoherence and the Appearance of a Classical World in Quantum Theory” (Springer, Berlin, 1996).
Appendix I

Relaxing the simplifications (i)-(iii) of Section 3 does not lead to significant changes of our results. This can be seen by employing the arguments of Dugić [29, 30], but for completeness, we briefly outline the main points in this regard.

First, for a time dependent Hamiltonian, which is still a "nondemolition observable", \([\hat{H}(t), \hat{H}(t')] = 0\), the spectral form [30]:

\[
\hat{H} = \sum_{x,i} \gamma_{xi}(t) \hat{P}_{ix} \otimes \hat{\Pi}_{Oi}.
\]  

(I.1)

This is a straightforward generalization of the cases studied and also a wide class of the time dependent Hamiltonians. E.g., from (I.1) it easily follows that the term \(\alpha_{xi}(t) = \int_{0}^{t} \gamma_{xi}(t') dt'\) substitutes the term \(\gamma_{xi}t\) in the exponent of the expression (6). Needless to say, this relaxes the constraint (i) of Section 3 and makes the link to the realistic models of the quantum "hardware".

To this end, it is worth emphasizing: in the realistic models one assumes the actions performed on the qubits in order to design the system dynamics. Interestingly enough, such actions usually result in the (effective) time independent model Hamiltonians [23, 33]. Moreover, some time dependent models allow direct applicability of the above notion; e.g., for the controlled Heisenberg interaction, \(\hat{H} = J(t)\vec{S}_1 \cdot \vec{S}_2\), the action reads: \(\hat{U}(t) = \exp(-iK(t)\vec{S}_1 \cdot \vec{S}_2)\), where \(K(t) \equiv \int_{0}^{t} J(t') dt'\). As an illustration of the models not fitting (i), one can consider NMR models which, in turn, are known to be of only limited use in the large-scale quantum computation [23, 32, 33]. We conclude that the realistic models of the large-scale quantum computation fit with the relaxed point (i) of our considerations.

Similarly, relaxing the exact compatibilities (cf. the point (ii) in Section 3) leads to the approximate separability—i.e., in Eq. (I.1), there appear terms of small norm—which does not change the results concerning the "correlation amplitude" \(z_{xx'}(t)\) [26], and consequently concerning \(D_{xx'}(t)\).

Finally, generalization of the form of the interaction Hamiltonian (cf. point (iii) of Section 3) does not produce any particular problems, as long as the Hamiltonian is of (at least approximately) separable kind, and also a "nondemolition observable". E.g., from
the general form for $\hat{H}_{\text{int}}$ [30], $\hat{H}_{\text{int}} = \sum_k C_k \hat{A}_k \hat{B} \otimes \hat{B}_k$, one obtains the term $\sum_k C_k (a_{k,x} - a_{k,x'}) b_{ki}$, instead of the term of Eq. (12).

The changes of the results may occur [30] if the Hamiltonian of the composite system is not of the separable kind and/or not a "nondemolition observable"; for an example see Appendix II.

For completeness, we notice: a composite-system observable is of the separable kind if it can be proved diagonalizable in a noncorrelated (the tensor-product) orthonormal basis of the Hilbert space of the composite system [30].

Appendix II

By "nonpersistent entanglement" we mean the states of a composite system which can be written as:

$$|\Psi\rangle = \sum_i C_{it} |i_t\rangle |i_t\rangle,$$

i.e. states whose Schmidt (canonical) form is labeled by an instant of time, $t$ (continuously varying with time). The occurrence of such forms for AQC can be easily proved by the use of the method developed in Ref. [30] applied to, e.g., Eq. (3.5) of Ref. [35]. Needless to say, states of the Eq.(II.1)-form are exactly what should be avoided in the situations described by Eq. (1).

To this end, the problem addressed in Ref. [30] reads: "what characteristics of the system Hamiltonian are required in order to attain the persistent entanglement (cf. Eq. (1))?". The answer is given by the points (i)-(iii) (but see Appendix I). In other words, as long as the conditions (i)-(iii) are fulfilled, nonpersistent entanglements do not occur in the system. As a corollary, having (i)-(iii) in mind, the nonpersistent entanglement of AQC cannot be (at least not directly) addressed within the present considerations.