USp(32) Special Grand Unification

Naoki Yamatsu *

Department of Physics, Kyushu University, Fukuoka 819-0395, Japan

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Abstract

We discuss a grand unified theory (GUT) based on a USp(32) GUT gauge group broken to its subgroups including a special subgroup. A GUT based on an SO(32) GUT gauge group has been discussed on six-dimensional (6D) orbifold space $M^4 \times T^2 / \mathbb{Z}_2$. It is inspired by the SO(32) string theory behind the SU(16) GUT whose SU(16) is broken to a special subgroup SO(10). Alternative direction is to embed an SU(16) gauge group into a USp(32) GUT gauge group, which is inspired by a non-supersymmetric symplectic-type USp(32) string theory. In a USp(32) GUT, one generation of the SM fermions is embedded into a 6D bulk Weyl fermion in a USp(32) defining representation. For a three generation model, all the 6D and 4D gauge anomalies in the bulk and on the fixed points are canceled out without exotic chiral fermions at low energies. The SM Higgs scalar is embedded into a 6D bulk scalar field in a USp(32) adjoint representation.

1 Introduction

The Standard Model (SM) in particle physics explains almost all experiments and observations at low energies, but the SM is usually regarded as a low-energy effective theory. There are many attempts to construct unified theories beyond the SM as below.

Grand unification [1] is one of the most attractive ideas to construct unified theories beyond the SM. As is well-known in e.g., Refs. [2,3], in four-dimensional (4D) space-time framework, the candidates for grand unified theory (GUT) gauge groups are only SU(n) ($n \geq 5$) [1,4], SO(4n + 2) ($n \geq 2$) [5–7], and E6 [8] because of ranks of groups and types of representations. In higher dimensional space-time frameworks [9], additional Lie groups such as SO(11) [9,15] SO(12) [16,17], and E7,8 are also candidates for GUT gauge groups. A lot of GUT models have been already proposed based on GUT gauge groups which are broken only to regular subgroups; e.g.,

$$E_8 \rightarrow E_7 \rightarrow E_6 \rightarrow SO(10) \rightarrow \begin{cases} SU(5) \\ G_{PS} \end{cases} \rightarrow G_{SM}, \tag{1.1}$$

where the subscript of arrows (R) stands for a regular subgroup breaking, $G_{SM} := SU(3)C \times SU(2)_L \times U(1)_Y$ stands for the SM gauge group, $G_{PS} := SU(2)_L \times SU(2)_R \times SU(4)C$ stands for the Pati-Salam group [15], and we omitted several U(1) subgroups. A few GUT models [19,21] have been already proposed based on GUT gauge groups which are broken not only to regular subgroups but also to special subgroups; e.g.,

$$SO(32) \rightarrow SU(16) \rightarrow SO(10) \rightarrow \begin{cases} SU(5) \\ G_{PS} \end{cases} \rightarrow G_{SM}, \tag{1.2}$$

*Electronic address: yamatsu.naoki@phys.kyushu-u.ac.jp
where the subscripts of arrows (R) and (S) stand for regular and special subgroup breakings, respectively, and we omitted several U(1) subgroups for regular subgroups. We note that a subgroup $H$ of a group $G$ is called a regular subgroup if all the Cartan subgroups of $H$ are also the Cartan subgroups of $G$; otherwise, the subgroup $H$ is called a special subgroup \[22\,23\,1\]

In Ref. \[19\], the author proposed a GUT model based on an SU(16) GUT gauge group broken to a non-maximal subgroup $G_{SM}$ shown in Eq. (1.2), where this type of GUTs are referred as special GUTs below. The results are summarized as follows. In a 4D SU(16) special GUT, one generation of quarks and leptons is embedded into a 4D SU(16) 16 Weyl fermion because one generations of the SM Weyl fermion fields are correctly unified into a Weyl fermion field in an SO(10) spinor representation 16. Only three 4D SU(16) 16 Weyl fermions suffer from a 4D SU(16) gauge anomaly \[20\]. It is possible to cancel out the anomaly by introducing three 4D SU(16) 16 Weyl fermions, where a representation $\bar{R}$ is the complex conjugate of $R$, but the matter content is vectorlike, and it is far from the SM whose matter content is chiral. So, to satisfy the 4D anomaly cancellation in a chiral gauge theory, we need to introduce a 4D Weyl fermion that belongs to a representation $R(\neq 16)$. One of the candidates is a 4D Weyl fermion in the SU(16) anti-symmetric tensor representation 120, where the value of the 4D anomaly coefficient of the SU(16) anti-symmetric tensor representation 120 is twelve times greater than one of the SU(16) defining representation 16. That is, the anomaly coefficient of the 4D SU(16) $(12 \times 16 \oplus 120)$ is zero, so 4D SU(16) $(12 \times 16 \oplus 120)$ Weyl fermions satisfy the 4D SU(16) anomaly cancellation condition, while the 4D SU(16) $(12 \times 16 \oplus 120)$ Weyl fermions are identified as twelve generations of quarks and leptons at a vacuum. We note that since a complex representation 120 of SU(16) is identified with a real representation 120 of SO(10), a 4D SU(16) 120 Weyl fermion is not a chiral fermion at a vacuum whose SU(16) is broken to SO(10). Regardless of a choice of a Weyl fermion in a representation $R$ of SU(16), a 4D SU(16) gauge anomaly cancellation condition restricts the minimal number of generations. Unfortunately, the minimal number is 12 in a 4D framework. In a 6D SU(16) special GUT on 6D orbifold space $M^4 \times T^2/\mathbb{Z}_2$, which is 4D Minkowski spacetime and two extra dimensional compactified space $T^2$ with a $\mathbb{Z}_2$ orbifold structure, one generation of quarks and leptons is identified as a set of the zero modes of a 6D SU(16) 16 Weyl fermion. Three generations of quarks and leptons are allowed without 4D exotic chiral fermions, which is consistent with the SU(16) gauge anomaly cancellation in the bulk and on the fixed points.

Further, an SO(32) special GUT was proposed in Ref. \[20\] whose gauge group SO(32) is broken to a non-maximal subgroup $G_{SM}$ shown in Eq. (1.2). Since an SO(32) group has only real representations \[2\,3\,22\,21\], any 4D SO(32) gauge theory is a vectorlike theory. To realize the SM, i.e., a 4D chiral gauge theory, an orbifold space construction \[27\,31\] is used. The results of the SO(32) special GUT on 6D orbifold space $M^4 \times T^2/\mathbb{Z}_2$ are almost the same as the above SU(16) special GUT.

The SO(32) special GUT was inspired by superstring theories \[82\,83\] that have been considered as a candidate of a unified theory to describe all the interaction including gravity. There are a lot of attempts to construct the SM as an effective theory derived from supersymmetric $E_8 \times E_8$ and SO(32) string theories \[34\,52\]. From the low-energy experiments, even if more fundamental theories beyond the SM have supersymmetry, the supersymmetry must be broken. So, it may be worth considering not only supersymmetric but also non-supersymmetric string theories.

Non-supersymmetric string theories has been investigated in e.g., Refs. \[43\,52\]. It was shown in Ref. \[45\] that a non-supersymmetric symplectic-type USp(32) string theory satisfies the gravitational and gauge anomaly cancellation conditions, which is further investigated in e.g., Refs. \[46\,48\]. Several attempts to construct the SM derived from non-supersymmetric

\[1\] For Lie groups and their regular and special subgroups, see e.g., Refs. \[2\,3\,22\,21\]. Branching rules of Lie groups and their subgroups such as SO(32)$\supset$SU(16) and SU(16)$\supset$SO(10) are explicitly written in Ref. \[3\]. Other branching rules used in the letter are calculated by a Mathematica program Susyno \[25\] and each projection matrix of Lie groups and their subgroups obtained by the wight diagram method discussed in e.g., Refs. \[3\,21\].

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\(SO(16) \times SO(16)\) string theories exist \[19, 52\], but those from a non-supersymmetric \(USp(32)\) string theory do not exist. In this letter, we propose a \(USp(32)\) model, but we will not discuss how to realize the model derived from string theories.

From another viewpoint of unified theories beyond the SM, gauge-Higgs unification (GHU) \[53–58\] is also an interesting idea, where the Higgs boson is identified as a zero mode of extra dimensional components of higher dimensional gauge fields. There are many GHU models discussed in e.g., Refs. \[27,28,59–67\]. Realistic GHU models are based on \(SU(3)_C \times SO(5) \times U(1)\) gauge theories in the Randall-Sundrum (RS) warped spacetime \[68–79\]. It gives nearly the same phenomenology at low energies as the SM. One of the \(SU(6)\) \[60,61,80,81\], \(SO(12)\) \[16,17\], \(E_6\) \[82,83\] are also discussed. As is known in e.g., Refs. \[2,3\], an \(SO(11)\) spinor representation \(32\) is a pseudo-real representation, so \(32\) of \(SO(11)\) cannot be identified as \(32\) of \(SO(32)\), while \(32\) of \(SO(11)\) can be identified as \(32\) of \(USp(32)\). We note that \(SO(11)\) is not a maximal subgroup of \(USp(32)\), and \(SO(12)(\supset SO(11))\) is a maximal subgroup of \(USp(32)\). In this letter, we propose a \(USp(32)\) model, but it is not a gauge-Higgs GUT model.

In the letter, we propose a \(USp(32)\) special GUT on 6D orbifold spacetime \(M^4 \times T^2/\mathbb{Z}_2\) whose \(USp(32)\) gauge group is broken to \(G_{\text{SM}}\) by orbifold symmetry breaking and the Higgs mechanism. We mainly focus on how to realize the SM matter content. It is an important task for constructing unified theories beyond the SM based on symplectic groups because almost people seems to believe that symplectic groups cannot be applied for GUT model buildings. The model building of a \(USp(32)\) special GUT is almost parallel to an \(SO(32)\) special GUT discussed in Ref. \[20\]. As we see below, the results of the \(USp(32)\) special GUT are almost the same as the ones of the \(SO(32)\) one.

This letter is organized as follows. In Sec. 2, we construct a 6D \(USp(32)\) special GUT on \(M^4 \times T^2/\mathbb{Z}_2\). Section 3 is devoted to a summary and discussion.

## 2 \(USp(32)\) special GUT

Before we introduce a \(USp(32)\) special GUT, we quickly check the \(USp(32)\) group and its subgroups. One of the maximal subgroups of a \(USp(32)\) group is a regular subgroup \(SU(16) \times U(1)\). The branching rules of \(USp(32) \supset SU(16) \times U(1)\) for \(USp(32)\) defining, rank-2 anti-symmetric tensor, and rank-2 symmetric tensor (adjoint) representations \(32, 495,\) and \(528\) are given by

\[
32 = (16)(1) \oplus (\overline{16})(-1), \quad (2.1)
\]

\[
495 = (255)(0) \oplus (120)(2) \oplus (\overline{120})(-2), \quad (2.2)
\]

\[
528 = (255)(0) \oplus (1)(0) \oplus (136)(2) \oplus (\overline{136})(-2), \quad (2.3)
\]

where the branching rules of \(SU(16) \supset SO(10)\) for \(SU(16)\) defining, rank-2 anti-symmetric tensor, rank-2 symmetric tensor, and adjoint representations \(16 (\overline{16}), 120 (\overline{120}), 126 (\overline{126}),\) and \(255\) are given in Ref. \[3\] as

\[
16 = (16), \quad (\overline{16}) = (\overline{16}), \quad (2.4)
\]

\[
120 = (120), \quad (\overline{120}) = (120), \quad (2.5)
\]

\[
136 = (\overline{126}) \oplus (10), \quad (\overline{136}) = (126) \oplus (10), \quad (2.6)
\]

\[
255 = (210) \oplus (45). \quad (2.7)
\]

From Eqs. (2.1) and (2.4), a \(USp(32)\) defining representation is decomposed into \(SO(10)\) spinor representations \(16\) and \(\overline{16}\). From Eqs. (2.3), (2.6), and (2.7), a \(USp(32)\) adjoint representation \(528\) is decomposed into \(SO(10)\) bi-spinor \(210\), adjoint \(45\), rank-2 symmetric tensor \(126\) and \(\overline{126}\), and two vector representations \(10\). In many \(SO(10)\) GUT models \[5,81,96\], not only
vector, spinor and adjoint representations $10, 16$ (T6) and $45$ but also rank-2 symmetric tensor and bi-spinor representations $126$ (T26) and $210$ are introduced. For example, an $SO(10)$ $126$ (T26) scalar field is introduced to generate neutrino masses via a 4D renormalizable cubic term $[55, 59]$, to reproduce realistic Yukawa coupling constants of quarks and leptons $[56, 57]$, to realize gauge coupling unification $[57, 59]$, and to introduce a dark matter candidate $[92, 94]$; an $SO(10)$ $210$ scalar field is also introduced to break $SO(10)$ to $G_{PS}$ and to realize gauge coupling unification $[57, 59]$, to introduce a dark matter candidate $[92, 94]$, and to prevent rapid proton decay $[95, 96]$.

We propose a $USp(32)$ special GUT in a 6-dimensional (6D) hybrid warped space $M_4 \times T^2 / \mathbb{Z}_2$. The metric is given by $[12, 97]$

$$ ds^2 = e^{-2\sigma(y)} (\eta_{\mu \nu} dx^\mu dx^\nu + dv^2) + dy^2, $$

where $\eta_{\mu \nu} = \text{diag}(-1, +1, +1, +1, +1, +1, +1)$, $\sigma(y) = \sigma(-y) = \sigma(y + 2\pi R_5)$, $\sigma(y) = k|y|$ for $|y| \leq \pi R_5$, and $R_5$ and $R_6$ represent 5th and 6th spatial extra dimensional radiiues, respectively. We identify spacetime points $(x^\mu, y, v)$, $(x^\mu, y + 2\pi R_5, v)$, $(x^\mu, y, v + 2\pi R_6)$, and $(x^\mu, -y, -v)$. There are four fixed points in the extra-dimensional space under $\mathbb{Z}_2$ parity: $(y_0, v_0) = (0, 0)$, $(y_1, v_1) = (0, 0)$, $(y_2, v_2) = (0, \pi R_5)$, and $(y_3, v_3) = (0, \pi R_6)$. Parity $P_j$ ($j = 1, 2, 3, 4$) around each fixed point is defined by $\psi_{(a)}(y, v) \rightarrow \psi_{(a)}(y, v)$, where $P_3 = P_1 P_0 P_2 = P_2 P_0 P_1$, 5th and 6th dimensional translations $U_5 : (x^\mu, y, v) \rightarrow (x^\mu, y + 2\pi R_5, v)$ and $U_6 : (x^\mu, y, v) \rightarrow (x^\mu, y, v + 2\pi R_6)$ satisfy $U_5 = P_1 P_0$ and $U_6 = P_2 P_0$, respectively. The metric given in Eq. (2.8) becomes a solution of the Einstein equations with the brane tension at $y = 0$ and $\pi R_5$ and a negative cosmological constant $\Lambda = -10k^2$. There are 4D branes at $y = 0$ and $\pi R_5$ referred to as the ultra-violet (UV) and infra-red (IR) branes, respectively.

We introduce three different dimensional fields in the $USp(32)$ special GUT. First, we introduce a 6D bulk gauge field $A_M$, which contains the SM gauge fields, where the gauge field belongs to the adjoint representation of $USp(32)$ $528$: a 6D bulk scalar field $\Phi_{528}$, which contains the SM Higgs field, where the subscript of fields, e.g., $528$ stands for the representation of $USp(32)$; three sets of 6D bulk positive and negative Weyl fermions $\Psi_{32}^{(a)}$ and $\Psi_{32}^{(a)}$ ($a = 1, 2, 3$), which contains the SM fermion fields. Second, to realize symmetry breaking at low energies, we introduce three 5D $USp(32)$ $86800$, $495$, and $32$ brane scalar fields $\Phi_{86800}$, $\Phi_{495}$, and $\Phi_{32}$ on the UV brane. Third, to realize 4D anomaly cancellation on all the fixed points, we introduce $\psi^{(b)}_{16}$, $\psi^{(b)}_{16}$ ($b = 1, 2, \cdots, 12$) at $(y, v) = (y_0, v_0)$, where the subscript of fields, e.g., $120$ stands for the representation of $SU(16)$. The matter content of the $USp(32)$ special GUT is summarized in Table 1.

Symmetry breaking consists of three stages in the $USp(32)$ special GUT whose $USp(32)$ is broken into $SU(3)_C \times U(1)_{EM}$:

1. Orbifold boundary conditions (BCs) break $USp(32)$ to $SU(16) \times U(1)$.

2. Nonvanishing vacuum expectation values (VEVs) of three 5D brane scalar fields $\Phi_{86800}$, $\Phi_{495}$, and $\Phi_{32}$ break $SU(16) \times U(1)$ to $G_{SM}$.

3. A VEV of a zero mode of a 6D bulk scalar field $\Phi_{528}$ breaks $G_{SM}$ to $SU(3)_C \times U(1)_{EM}$.

We summarize the above symmetry breaking chain as below:

$$ USp(32) \xrightarrow{\text{BCs}} SU(16) \times U(1) \xrightarrow{\langle \Phi_{86800} \rangle \neq 0} SO(10) \xrightarrow{\langle \Phi_{495} \rangle \neq 0} G_{SM} \xrightarrow{\langle \Phi_{32} \rangle \neq 0} SU(3)_C \times U(1)_{EM}, $$

where symmetry breaking chains between $SU(16)$ and $G_{SM}$ depend on the values of nonvanishing VEVs of the 5D brane fields $\Phi_{86800}$, $\Phi_{495}$, and $\Phi_{32}$ and their associated coupling constants.
The branching rule of $A_\sigma$ where $P$ stand for left- and right-handed Weyl fermions, respectively. (BCs stand for parity assignment and 5D brane fields, and the spacetime location of 5D and 4D brane fields are listed. Orbifold USp representations of 6D dimensional gauge fields $A$, SM gauge fields are introduced as a part of the 6D $USp(32)$ bulk field $A_M$, and 5th and 6th $USp(32)$ are the Pauli matrices. We take a convention for the relation between products $A \otimes B$ and explicit matrix forms given in e.g., Ref. [98]. The orbifold BCs $P_2$ and $P_3$ preserve $USp(32)$ symmetry, while the orbifold BCs $P_0$ and $P_1$ reduce $USp(32)$ to its regular subgroup $SU(16) \times U(1)$. The orbifold BC that breaks $USp(32)$ to $SU(16) \times U(1)$ is allowed under a

| 6D Bulk field | $A_M$ | $\Phi_{528}$ | $\psi^{(a)}_{32\pm}$ | $\psi^{(a)}_{32-}$ |
|---------------|-------|--------------|-----------------|-----------------|
| $USp(32)$     | 528   | 528          | 32              | 32              |
| $SO(5,1)$     | 6     | 1            | $4_+$           | $4_-$           |
| Orbifold BC   | $(+ +)$ | $(- -)$     | $(+ +)$         | $(+ +)$         |

| 5D Brane field | $\Phi_{86800}$ | $\Phi_{495}$ | $\Phi_{32}$ |
|----------------|-----------------|--------------|-------------|
| $USp(32)$      | 86800           | 495          | 32          |
| $SO(4,1)$      | 1               | 1            | 1           |
| Orbifold BC    | $(-)$           | $(+)$        | $(+)$       |
| Spacetime      | $y = 0$         | $y = 0$      | $y = 0$     |

| 4D Brane field | $\psi_{120}$ | $\psi_{16}$ | $\psi_{16}$ |
|----------------|--------------|-------------|-------------|
| $SU(16)$       | 120          | 16          | 16          |
| $U(1)$         | 0            | 0           | -1          |
| $SL(2, \mathbb{C})$ | $(1/2, 0)$   | $(1/2, 0)$  | $(1/2, 0)$  |
| Spacetime $(y, v)$ | $(0, 0)$     | $(0, 0)$    | $(0, 0)$    |

Table 1: The matter content in the $USp(32)$ special GUT on $M^4 \times T^2/\mathbb{Z}_2$ is shown. The representations of $USp(32)$ and 6D, 5D, 4D Lorentz group, the orbifold BCs of 6D bulk fields and 5D brane fields, and the spacetime location of 5D and 4D brane fields are listed. Orbifold BCs stand for parity assignment $\left( \begin{array}{cc} \eta_2 & \eta_3 \\ \eta_0 & \eta_1 \end{array} \right)$ for 6D fields and $\left( \begin{array}{c} \eta_2 \\ \eta_0 \end{array} \right)$ for 5D fields. The orbifold BCs of the 6D $USp(32)$ gauge field $A_M$ are given in Eq. (2.11). $(1/2, 0)$ and $(0, 1/2)$ of $SL(2, \mathbb{C})$ stand for left- and right-handed Weyl fermions, respectively. $(a = 1, 2, 3; b = 1, 2, \cdots, 12)$

The branching rule of $USp(32) \supset SU(16) \times U(1)$ for $86800$ is given by

$$86800 = (18240)(0) \oplus (14144)(0) \oplus (5440)(4) \oplus (5440)(-4) \oplus (255)(0) \oplus (1)(0) \oplus (21504)(2) \oplus (21504)(-2) \oplus (136)(2) \oplus (136)(-2).$$

(2.10)

In this letter, we assume the above symmetry breaking and we will not perform the potential analysis of the scalar fields and determine the values of their VEVs.

2.1 Bosonic sector

The SM gauge fields are introduced as a part of the 6D $USp(32)$ bulk gauge boson $A_M$. The 6D $USp(32)$ bulk gauge boson $A_M$ is decomposed into a 4D gauge field $A_\mu$ and 5th and 6th dimensional gauge fields $A_y$ and $A_v$, where $A_y$ and $A_v$ are 4D scalar fields. The orbifold BCs of the 6D $USp(32)$ gauge field are given by

$$\left( \begin{array}{c} A_\mu \\ A_y \\ A_v \end{array} \right)(x, y_j - y, v_j - v) = P_{32}^{-1} \left( \begin{array}{c} A_\mu \\ -A_y \\ -A_v \end{array} \right)(x, y_j + y, v_j + v),$$

$$P_{32} = \left\{ \begin{array}{ll} I_2 & \text{for } j = 2, 3 \\ I_6 \otimes \sigma_3 & \text{for } j = 0, 1 \end{array} \right.,$$

(2.11)

where $P_{32}$ is a projection matrix satisfying $(P_{32})^2 = I_2$, $I_n$ is an $n \times n$ identity matrix and $\sigma_a (a = 1, 2, 3)$ are the Pauli matrices. We take a convention for the relation between products $A \otimes B$ and explicit matrix forms given in e.g., Ref. [98]. The orbifold BCs $P_2$ and $P_3$ preserve $USp(32)$ symmetry, while the orbifold BCs $P_0$ and $P_1$ reduce $USp(32)$ to its regular subgroup $SU(16) \times U(1)$. The orbifold BC that breaks $USp(32)$ to $SU(16) \times U(1)$ is allowed under a
Table 2: Parity assignments \((\eta_2 P_{32} T P_{32}^{−1} \eta_3 P_{32} T P_{32}^{−1} \eta_3 P_{32} T P_{32}^{−1} \eta_3 P_{32} T P_{32}^{−1})\) of the 4D \(SU(16) \times U(1)\) gauge and scalar components of the 6D \(USp(32)\) gauge field \(A_M\) and scalar field \(\Phi_{528}\) are shown. \(\eta_j(j = 0, 1, 2, 3)\) stand for \(\eta_j = 1\) and \(\eta_j = −1\) for \(A_M = \mu\) and \(A_M = y, v\) regardless of \(j\); \(\eta_0 = \eta_1 = −1\) and \(\eta_2 = \eta_3 = 1\) for \(\Phi_{528}\). \(T\) represents \(T_A \otimes I_2 + T_{S_1} \otimes \sigma_3\) and \(T_{S_1} \otimes \sigma_1 + T_{S_2} \otimes \sigma_2\) for \((255)(0) \oplus (1)(0)\) and \((136)(+2) \oplus (136)(−2)\), respectively.

| \(SU(16) \times U(1)\) | \(M = \mu\) | \(M = y, v\) | \(\Phi_{528}\) |
|-----------------|----------|--------------|----------|
| \((255)(0) \oplus (1)(0)\) | \(+\) | \(−\) | \(+\) |
| \((136)(+2) \oplus (136)(−2)\) | \(+\) | \(−\) | \(+\) |

The 6D \(USp(32)\) gauge field \(\Phi_{528}\) is identified to a 4D scalar field itself. The orbifold BCs of the 6D \(USp(32)\) scalar field is given by

\[
\Phi_{528}(x, y_j − y, v_j − v) = \eta_j P_{j32} \Phi_{528}(x, y_j + y, v_j + v) P_{j32}^{-1},
\]

where \(P_{j32}\) is given in Eq. (2.11) and \(\eta_0 = \eta_1 = −1, \eta_2 = \eta_3 = 1\).

We check zero modes of the 6D \(USp(32)\) adjoint bulk scalar field \(\Phi_{528}\). The \(USp(32)\) scalar field \(\Phi_{528}\) has Neumann BCs at the fixed points \((y_2, v_2)\) and \((y_3, v_3)\). Since the \(USp(32)\) symmetry is broken to \(SU(16) \times U(1)\) at the fixed points \((y_0, v_0)\) and \((y_1, v_1)\), the \(SU(16) \times U(1)\) components of the scalar field \(\Phi_{528}\) have Neumann and Dirichlet BCs at the fixed points \((y_0, v_0)\) and \((y_1, v_1)\); they have zero modes corresponding to 4D \(SU(16)\) and \(U(1)\) gauge fields; since the other components of \(A_M\) and any component of \(A_y\) and \(A_v\) have four Dirichlet BCs or two Neumann and two Dirichlet BCs at the four fixed points, they do not have zero modes. The orbifold BCs reduce \(USp(32)\) to \(SU(16) \times U(1)\). Parity assignments of \(A_M\) are shown in Table 2.

The SM Higgs field is introduced as a part of a 6D \(USp(32)\) adjoint bulk scalar field \(\Phi_{528}\). The 6D \(USp(32)\) bulk scalar \(\Phi_{528}\) is identified to a 4D scalar field itself. The orbifold BCs of the 6D \(USp(32)\) scalar field is given by
In this case, for $\Phi$ of $G$, the (16) component. For the $\kappa$ of $G$ contains 1 of $SO(10)$; for $\Phi_{32}$, the $SU(16) \times U(1)$ (16) components have zero modes, where $16$ and $16$ of $SO(10)(\subset SU(16))$ contains 1 of $SU(5)$; and for $\Phi_{495}$, the $SU(16) \times U(1)$ (255) components have zero modes, where $255$ of $SU(16)$ contains $(1,1,0)$ of $G_{SM}(\subset SU(5) \subset SO(10) \subset SU(16))$. The corresponding parity assignments are realized by choosing appropriate signs of $\eta_{\Phi}$. The scalar field $\Phi_{86800}$ is responsible for breaking $(USp(32) \supset)SU(16) \times U(1)$ to $SO(10)$; the nonvanishing VEV of the scalar field $\Phi_{32}$ breaks $(USp(32) \supset)SO(10)$ to $SU(5)$; the nonvanishing VEV of $\Phi_{495}$ breaks $(USp(32) \supset)SU(5)$ to $G_{SM}$. The above symmetry breaking pattern from $SO(10)$ to $G_{SM}$ is $SO(10) \rightarrow SU(5) \rightarrow G_{SM}$. There is another symmetry breaking pattern $SO(10) \rightarrow G_{PS} \rightarrow G_{SM}$. In this case, for $\Phi_{495}$, the $SU(16) \times U(1)$ (255)(0) components have zero modes, where 255 of $SU(16)$ contains $(1,1,1)$ of $G_{PS} = SU(2)_L \times SU(2)_R \times SU(4)_C(\subset SO(10))$; for $\Phi_{32}$, the $SU(16) \times U(1)$ (16)(1) components have zero modes, where $16$ and $16$ of contains $(1,1,0)$ of $G_{SM}(\subset G_{PS})$. Thus, the nonvanishing VEV of $\Phi_{495}$ breaks $(USp(32) \supset)SO(10)$ to $G_{PS}$, the nonvanishing VEV of the scalar field $\Phi_{32}$ breaks $(USp(32) \supset)G_{PS}$ to $G_{SM}$, the symmetry breaking patterns of $USp(32)$ broken to $G_{SM}$ are almost the same as that of $SO(32)$ broken to $G_{SM}$ discussed in Ref. [20].

Symmetry breaking via the non-vanishing VEVs of the 5D brane scalar fields $\Phi_{86800}$, $\Phi_{495}$, and $\Phi_{32}$ on the UV brane affects BCs of some components of 6D bulk fields through 4D brane localized interaction terms [12][13]. For the 6D bulk gauge field $A_M$, the $USp(32)/G_{SM}$ components of the 4D components $A_\mu$ have Neumann BCs on the UV brane without the symmetry breaking effects through the UV brane. When we take into account the symmetry breaking effects, the BCs ($y = 0$) become effectively Dirichlet BCs and originally zero modes acquire masses of $O(m_{KK})$, which depends on coupling constants of corresponding 4D brane interactions and the values of the VEVs ($\Phi_\pm$). In addition, for the 6D bulk scalar field $\Phi_{528}$, a 4D brane localized mass term $\mu \Phi_{528} \Phi_{528}$, where $\mu$ is a mass parameter, is allowed. So, zero modes of $\Phi_{528}$ have masses, which are expected to $O(m_{KK})$. That is, all the $SU(16) \times U(1)$ (136)(2) ($\pm$ (336)(2)) component fields have a common mass. 4D brane localized interaction terms between the 6D bulk scalar field $\Phi_{528}$ and 5D brane scalar fields $\Phi_{86800}$ and $\Phi_{495}$, $\kappa \Phi_{86800} \Phi_{528} \Phi_{528}$ and $\kappa' \Phi_{495} \Phi_{528} \Phi_{528}$, where $\kappa$ and $\kappa'$ are parameters, are allowed because a symmetric tensor product of $USp(32)$ is given by $(528 \otimes 528) = (52360) \otimes (3080) \otimes (495) \otimes (1)$. A term $\kappa \Phi_{86800} \Phi_{528} \Phi_{528}$ leads to a mass term of zero modes of the $SU(16) \times U(1)$ (136)(2) ($\pm$ (336)(2)) component. The mass of the $(SU(16) \times U(1) \supset)SO(10) \times U(1)$ (126)(2) ($\pm$ (216)(-2)) component is different from that of the $(10)(2) \otimes (10)(-2)$ component because those Clebsch-Gordan coefficients (CGCs) are different. Further, a term $\kappa' \Phi_{495} \Phi_{528} \Phi_{528}$ leads to a mass term of zero modes of the $SU(16) \times U(1)$ (136)(2) ($\pm$ (336)(2)) component. For the $(SU(16) \times U(1) \supset)SO(10) \times U(1)$ (10)(2) ($\pm$ (10)(-2)) component, the mass of the $(SU(16) \supset SO(10) \supset)G_{PS} (1,1,6)$ component is different from that of the $(SU(16) \supset SO(10) \supset)G_{PS} (2,2,1)$ component because those CGCs are different. Therefore, we can realize an almost massless $G_{PS} (2,2,1)$ scalar field, which can be identified as the SM Higgs field when we take into account the VEV of $\Phi_{32}$ that breaks $G_{PS}$ to $G_{SM}$, by choosing parameters carefully.

### 2.2 Fermionic sector

The SM Weyl fermions are identified with zero modes of 6D $USp(32)$ 32 Weyl bulk fermions, whose orbifold BCs are given by

$$\Psi_{32}(x, \ell - y, v_j - v) = \eta_{\ell} \gamma P_{32} \Psi_{32}(x, \ell + y, v_j + v),$$

(2.14)
where the subscript of $\Psi \pm$ stands for 6D chirality, $\eta_{j\pm}$ is a positive or negative sign satisfying $\prod_{j=0}^{3} \eta_{j\pm} = 1$. 6D gamma matrices $\gamma^a$ ($a = 1, 2, \cdots, 7$) satisfy $\{ \gamma^a, \gamma^b \} = 2\eta^{ab} (\eta^{ab} = \text{diag}(-I_1, I_3))$. $\tau := -i\gamma^5 \gamma^6 = \gamma_6^{16} \gamma_{4D}^5$, where $\gamma_{4D}^5$ are 4D left- and right-handed Weyl fermions $\psi^a_{L/R}$. $\gamma^a_{L/R} := (P_{L/R} \gamma_{4D}^a)$, where subscripts $L/R$ and $\pm$ stand for 4D and 6D chiralities, respectively:

$$
\Psi_{6D} := \left( \begin{array}{c}
\psi_{4D}^L \\
\psi_{L}^{-}
\end{array} \right), \quad \Psi_{6D}^+ = P_{6D} \Psi_{6D} = \left( \begin{array}{c}
\psi_{4D}^R \\
\psi_{L}^{+} \\
0
\end{array} \right), \quad \Psi_{6D}^- = P_{6D} \Psi_{6D} = \left( \begin{array}{c}
0 \\
0 \\
\psi_{R}^{-}
\end{array} \right). \quad (2.15)
$$

We omit the superscripts such as 6D and 4D below.

Here we check zero modes of a 6D $\text{USp}(32)$ 32 positive Weyl fermion with orbifold BCs $\eta_j = -1$. In this case, only a 4D $SU(16) \times U(1)$ (16) left-handed Weyl fermion has a zero mode. This is because only the left-handed Weyl fermion component has all the Neumann BCs at all the fixed points $(y_j, v_j)$.

To realize three generations of the SM chiral fermions, we need to introduce three 6D $\text{USp}(32)$ 32 positive Weyl fermions. However, only 6D $\text{USp}(32)$ 32 positive Weyl fermions suffer from 6D gauge anomalies. It is known that the 6D gauge anomalies can be canceled out by introducing additional 6D $\text{USp}(32)$ 32 negative Weyl fermions with different BCs as discussed in Refs. [19-21]. Parity assignments of $\Psi_{32}^{(a)}$ are shown in Table B.

We check contributions to 4D brane anomalies from the above 6D Weyl fermion sets. At two fixed points $(y_j, v_j)$ ($j = 2, 3$), there is no 4D pure $\text{USp}(32)$ gauge anomaly because any 4D anomaly coefficient of $\text{USp}(32)$ is zero. At the other two fixed points $(y_j, v_j)$ ($j = 0, 1$), there can be 4D pure $SU(16)$, pure $U(1)$, mixed $SU(16) - SU(16) - U(1)$, and mixed grav. - grav. $- U(1)$ anomalies. At a fixed point $(y_1, v_1)$, the anomalies generated from the 6D $\text{USp}(32)$ 32 positive and negative Weyl fermions are canceled each other. Finally, at the other fixed point $(y_0, v_0)$, the 6D $\text{USp}(32)$ 32 positive and negative Weyl fermions generate 4D pure $SU(16)$, pure $U(1)$, mixed $SU(16) - SU(16) - U(1)$, and mixed grav. - grav. - $U(1)$ anomalies. As the same discussion in Ref. [20], the 4D gauge anomalies can be canceled out by introducing 4D brane Weyl fermions in appropriate representations of $SU(16) \times U(1)$ shown in Table I.

### Table 3: Parity assignments of $\Psi_{32}^{(a)}$

| $SU(16) \times U(1)$ | $\Psi_{32}^{(a)+}$ | $\Psi_{32}^{(a)-}$ |
|-----------------------|---------------------|---------------------|
| $(16)(+1)$           | $++$                | $+$                 |
| $(\overline{16})(-1)$| $++$                | $+$                 |

3. Summary and discussion

In this letter, we proposed a $\text{USp}(32)$ special GUT by using a special breaking $SU(16)$ to $SO(10)$. Zero modes of a 6D $\text{USp}(32)$ 32 Weyl fermion are identified with one generation of quarks and leptons; the 6D $\text{USp}(32)$ and the 4D $SU(16) \times U(1)$ gauge anomalies on the fixed points allow
a three generation model of quarks and leptons in a 6D framework; as in the $SU(16)$ special GUT \[19\], exotic chiral fermions do not exist. The SM Higgs scalar field is introduced as a part of a 6D $USp(32)$ adjoint bulk scalar field. Unfortunately, to realize an almost massless SM Higgs scalar field and the other massive scalar fields, the fine-tuning is inevitable.

In the $USp(32)$ special GUT, the SM fermions and the SM Higgs scalar are embedded into zero modes of the 6D bulk fields, so the masses and mixing matrices are given by the overlap integral of the wave functions of zero modes of the 6D bulk fermion fields $\Psi^{(a)}_{32}$ and the 6D bulk scalar field $\Phi_{328}$. For an $SO(11)$ gauge-Higgs GUT model in the 6D hybrid warped space \[12\text{[13]},\] hierarchical masses of fermions are realized by taking the values of bulk vector masses in the 6D hybrid warped space, which corresponds to bulk scalar masses in the 5D RS space. To explain tiny neutrino masses by a see-saw mechanism \[101\text{[107]},\] additional $SO(11)$ singlet brane fermions are introduced on the UV brane, which satisfy a symplectic Majorana condition \[108\text{[109]}\]. The additional brane fermions, bulk fermions, and the $SO(11)$ breaking brane scalar fields lead to additional brane mass terms on the UV brane. The mass terms generate tiny neutrino masses. A similar discussion is given in Ref. \[110\]. Also, a mixing matrix in the quark sector, the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix \[111\text{[112]},\] is introduced by brane interaction terms \[77\text{[113][116]}\]. We will leave further discussions for future studies.

We comment on the possibility of gauge-Higgs unification in a $USp(32)$ special GUT. As is discussed in the letter, a 6D bulk scalar field in the adjoint representation of $USp(32)$ contains the SM Higgs scalar component, so the 4D scalar component of a 6D $USp(32)$ bulk gauge field contains also the SM Higgs scalar component, but the BCs in Eq. \[2.11\] do not allow zero modes of the 4D scalar component. Instead of the BCs in Eq. \[2.11\], if we take the BCs $P_{32} = I_{16} \otimes \sigma_3$ ($j = 0, 1, 2, 3$), the 4D scalar fields of the $SU(16) \times U(1)$ \((\begin{pmatrix} 136 \end{pmatrix}(2) \oplus \begin{pmatrix} 136 \end{pmatrix}(-2))\) component have zero modes, where a part of the 4D scalar fields can be identified the SM Higgs scalar field. However, the BCs do not allow three chiral generations of the SM fermions because each 6D bulk Weyl fermion in a $USp(32)$ defining representation 32 has two or no zero modes of the 4D left-handed or right-handed Weyl fermions in a $SO(10)$ spinor representation 16 depending on each parity assignment. We need some additional ideas to construct unified models satisfying both $USp(32)$ grand unification and gauge-Higgs unification. We will leave further discussions for future studies.

Finally, we comment on symmetry breaking. Many people vaguely believe that symmetry groups are broken to only regular subgroups, not to special subgroups. However, the symmetry breaking of $SU(n)$ to its special subgroups such as $SO(n)$ and $USp(2[n/2])$ are known to be realized by a nonvanishing VEV of a fundamental scalar field in rank-2 symmetric and anti-symmetric tensor representations of $SU(n)$ \[117\text{[119]}\], a nonvanishing VEV of a composite scalar field made by fermion pair condensation in fundamental and rank-2 anti-symmetric tensor representations of $SU(n)$ \[120\], orbifold boundary conditions (BCs) by $Z_2$ outer automorphisms on $S^1/Z_2$ orbifold space \[99\]. Also, other symmetry groups such as $SO(n)$ and $E_6$ broken to their special subgroups are discussed in Ref. \[117\] for fundamental scalar fields; in Refs. \[121\text{[122]}\] for composite scalar fields; in Ref. \[99\] for $Z_2$ orbifold space.

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