THE \( x \)-REGION OF SHADOWING CORRECTIONS IN NUCLEON STRUCTURE FUNCTIONS

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Abstract

We discuss the experimental indications on the behaviour of \( F_2(x, Q^2) \) at small \( x \) both in proton and nuclear targets. By comparing the parametrizations of the data we conclude that shadowing correction effects in a proton target can appear at a noticeable level for \( x = (2 \div 4) \times 10^{-4} \) and \( Q^2 \sim 10^4 \) GeV\(^2\), namely inside the HERA regime.

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1 Introduction

The experimental data of HERA on $F_2(x, Q^2)$ at small $x$, actually $10^{-4} < x < 10^{-2}$, show at moderate $Q^2$ (say $Q^2 \sim 10^1$ GeV$^2$) a singular $x$-behaviour. Both ZEUS [1] as well as H1 [2] Coll. data can be parametrized as $x^\delta$ with $\delta = -0.1 \div -0.25$. Evidently such a behaviour at $x \to 0$ is in contradiction with unitarity and it should be stopped by some shadowing mechanism [3, 4, 5]. Although the estimations of the $x$-region where this shadowing should appear are model dependent more or less realistic calculations [6, 7] predict significant (of the order of ten percent) shadowing corrections at $x > 10^{-4}$. Moreover, in accordance with [8] the discussed singular behaviour is in contradiction with unitarity at $x \sim (2 \div 3) \times 10^{-4}$.

However no shadowing effect is actually seen in the experimental data on $F_2(x, Q^2)$ obtained from proton targets at HERA. On the other hand the singularity of $F_2(x, Q^2)$ at $x < 10^{-2}$ observed in nuclear targets is significantly weaker, or, possibly, absent, when compared with the singularity of $F_2(x, Q^2)$ observed on protons. It seems therefore most likely that in the case of a proton target the parton density, in the $x$ region investigated till now, is not high enough to have shadowing, which is rather seen in nuclear targets as a consequence of the enhanced parton density in a nucleus (say, by factor $\sim A^{1/3}$). In this connection we present a new estimate of the $x$-region where shadowing corrections can be significant in a proton target. Our estimate is not based on specific model assumptions, it is rather obtained by comparing the behaviour $F_2(x, Q^2)$ on proton and nuclear targets.

2 Discussion of experimental data from proton and nuclear targets

Let us consider the experimental situation more detail. The H1 Coll. presents a global fit of their data of the singlet quark distribution, $q_{SI} = u + \bar{u} + d + \bar{d} + s + \bar{s}$, which determines practically the $F_2(x, Q^2)$ behaviour at small $x$ in the form [2]

$$xq_{SI}(x) = Ax^B (1 - x)^C (1 + Dx)$$

with $A = 1.15$, $B = -0.11$, $C = 3.10$ and $D = 3.12$, at $Q^2 = 4$ GeV$^2$ and $x > 2 \cdot 10^{-4}$. One can see that, at $x \leq 10^{-2}$, one can rewrite this expression as

$$xq_{SI}(x) = Ax^B$$

and one may neglect the non-singlet contribution within a few percent accuracy.
ZEUS Collaboration presented their data for singlet quark distribution in a similar form [1]

\[ xq_{SI}(x) = Ax^B(1 - x)^C(1 + D\sqrt{x} + Ex) \tag{3} \]

with \( A = 0.47, B = -0.26, C = 3.29, D = 2.45 \) and \( E = 6.38 \), at \( Q^2 = 7 \) GeV\(^2\) and \( x > (2 \div 3) \times 10^{-4} \). Also in this case one can rewrite the expression in the form in Eq.(2). One may point out a possible inconsistency in the two determinations of the value of \( B \), whose difference seems to be too large to be justified by the difference in the values of \( Q^2 \).

As already mentioned, the \( x \) dependences of \( F_2(x, Q^2) \) in nuclear targets is not so singular. Usually the ratio

\[ R_A(x) = F_2^{(A)}(x)/F_2^{(D)}(x) \tag{4} \]

is considered, where \( F_2^{(A)}(x) \) and \( F_2^{(D)}(x) \) are the structure functions per nucleon in a nucleus with mass number \( A \) and in the deuteron respectively. The distortion of parton distributions by nuclear medium leads to a deviation of \( R_A(x) \) from unity [1]. At small \( x \) the ratio can be expressed, independently of \( Q^2 \), as [3]

\[ R_A(x) = C \cdot x^\gamma, \tag{5} \]

where \( \gamma \) increases with \( A \) for light nuclei and possibly becomes a constant for \( A \simeq (40 - 60) \). By fitting the experimental data in [1] one obtains the values \( \gamma = 0.0874 \pm 0.0037, 0.0920 \pm 0.0125 \) and \( 0.119 \pm 0.020 \) for Ca, Cu and Pb, respectively. The values are in agreement with the older analysis of [10], where \( \gamma \sim 0.1 \) was obtained for gold at \( Q^2 = 4 \) GeV\(^2\) with weak \( Q^2 \)-dependence of \( \gamma \). One can therefore see that the values of \( F_2^{(A)}(x, Q^2) \) are practically independent on \( x \) in heavy nuclei, if one uses the H1 data for nucleon target, actually Eq.(1), and the singularity is about two times weaker if one uses ZEUS data, namely eq.(3). In both cases one concludes that shadowing of parton distributions in heavy nuclei is a significant effect.

3 Possible connection between shadowing effects in a nucleon and in a nucleus

Our main assumption to establish a possible connection between the parton distributions in nuclei and in free nucleons is that the parton distributions at \( Q^2 >> 1 \) GeV\(^2\) are determined by hard processes which, in principle, can be described by QCD (possibly with some non-perturbative contribution).

\(^1\)It is assumed usually that the nucleons inside a deuteron are practically free.
In such a case all nuclear medium effects should also be determined by hard parton interactions and not by some soft physics (say, interactions with pion fields, etc.). As a consequence all shadowing effects in nuclei should be connected with the possibility of interacting with partons from different nucleons, which effectively increases the parton density. The highest enhancement of the parton density in a nucleus is therefore $\sim A^{1/3}$, when all parton at some given impact parameter can be involved in the interaction.

By parametrizing the data on $R_A(x)$ in the range $0.5 \text{ GeV}^2 < Q^2 < 200 \text{ GeV}^2$ as in Eq.(5), one finds that for comparatively heavy nuclei (say, for $Cu$) the values of the parameters in Eq.(5) are $C \approx 1.3$ and $\gamma \approx 0.1$. One obtains therefore $R_A(x) = 1$ at $x \approx 0.06$ and the ratio decreases to the value $R_A(x_1) = 0.9$ (10\% shadowing effect) at $x_1 \approx 0.025$. Following the modern sets of phenomenological parton distributions [11] one can see that the value of $F_2^{(N)}(x_1)$, for a proton target at $x_1 \approx 0.025$ and $Q^2 \sim 10^{1} \text{ GeV}^2$, is about 0.45. In the case of copper, where shadowing is observed, the parton density can be enhanced by a factor as big as $A^{1/3} = 4$. It seems therefore quite reasonable to expect the same shadowing effect for a proton target in the $x$-region where $F_2^{(N)}(x_2) = 4 \times F_2^{(N)}(x_1) \sim 1.8$. From the review of parton distributions [11] one can see that the correspondent $x_2$ value is $x_2 \sim 3 \times 10^{-4}$, which is inside the kinematical domain accessible at HERA.

A similar estimate can be obtained from the parametrization of $R_A(x)$ in Eqs.(36) and (37) of Ref. [12]:

$$R_A(x) = 1 + 1.19[x^3 - 1.5(x_0 + X_L)x^2 + 3x_0x_Lx] \ln^{1/6}(A)$$

$$- \left[ \alpha_A - \frac{1.08(A^{1/3} - 1)}{\ln(A + 1)} \sqrt{x} \right] \exp^{-x^2/x_0^2} \quad (6)$$

with $x_0 = 0.1$, $x_L = 0.7$ and $\alpha_A = 0.1(A^{1/3} - 1)$. The main difference between this parametrization and Eq.(5) is the behaviour at $x \to 0$. Eq.(5) presents the power behaviour, whereas Eq.(6) gives a constant value at $x \leq 10^{-4}$. If one looks for the value of $x$ where $R_A(x_1) = 0.9$ one obtains $x_1 \sim 0.04$. The value of $F_2^{(N)}(x_1)$ is about 0.4 which gives $F_2^{(N)}(x_2) = 4 \times F_2^{(N)}(x_1) \sim 1.6$, corresponding to the value $x_2 \sim 4 \times 10^{-4}$.

\footnote{If we do the same estimate from the parametrization of heavier nuclei we obtain smaller values of $x_2$ because the values of parameters $C$ and $\gamma$ are practically the same but the value of $A^{1/3}$ is larger. Possibly, in this kinematical regime, only a fraction of all partons at a given impact parameter are allowed to interact in heavier nuclei.}
4 Conclusion

It seems quite probable that shadowing corrections can be seen at the smallest values of $x$ accessible at HERA as a deviation of $F_2(x, Q^2)$ from the behaviour in Eq.(2).

Two are the main sources of uncertainty in our estimate. The first is the quality of DIS data on proton and nuclear targets, which are used to obtain the parameters in Eq.(1) and in Eq.(3). The second is the enhancement factor. Actually $A^{1/3}$ has to be understood only as an order of magnitude estimate. The real enhancement factor is determined by the number of target nucleons which can interact with the wee partons emitted by the virtual photon. It seems nevertheless rather plausible that, by increasing the precision of the experimental measurements of $F_2(x, Q^2)$ at the smallest values of $x$ available at HERA, the first evidence of shadowing in proton structure functions might be actually observed.

Acknowledgements

We are grateful to M.G.Ryskin and G.I.Smirnov for useful discussions. This work is supported in part by INTAS grant 93-0079.
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