Strategyproof Learning: Collecting Trustworthy User-Generated Datasets
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February 21, 2022

Abstract
We prove in this paper that, perhaps surprisingly, incentivizing data misreporting is not a fatality. By leveraging a careful design of the loss function, we propose Licchavi, a global and personalized learning framework with provable strategyproofness guarantees. Essentially, we prove that no user can gain much by replying to Licchavi’s queries with answers that deviate from their true preferences. Interestingly, Licchavi also promotes the desirable “one person, one unit-force vote” fairness principle. Furthermore, our empirical evaluation of its performance showcases Licchavi’s real-world applicability. We believe that our results are critical for the safety of any learning scheme that leverages user-generated data.

1 Introduction
Today’s large-scale algorithms, designed for autocompletion [LB21], conversational [SHL18] and recommendation [LJW19] applications, exploit the data generated from the activities of a large number of users [SSP13, WPN19, WSM19] to construct both global and personalized models [RRS11, FMO20, HHHR20].

However, the fact that strategic users may provide untrustworthy data challenges the classical theory of learning, which generally regards as desirable to fit available data, and to generalize them for future applications [Val84]. In applications such as content recommendation, activists, companies and politicians usually have strong incentives to promote certain views, products or ideologies [Hoa20, HFE21]. After all, two YouTube views out of three result from algorithmic recommendations [Sol18]. Quite naturally, this has led to vast amounts of fabricated activities to bias algorithms [BH19, NHK19], through “fake reviewing” [WNWW20], “astroturfing” [ZTK21] or automated harassment [KK22]. In fact, Facebook reportedly removed 15 billion fake accounts within two years [Do21]. This raises serious concerns, especially given today’s “stochastic parrots” [BGMS21]: today’s language models incentivize anti-vaccine groups to heavily pollute textual datasets with claims like “vaccines kill”, including through fake accounts, as autocompletion, conversational and recommendation algorithms trained on such data will more likely spread this view [MN20].

Arguably, in large-scale environments that naturally attract a large number of malicious entities, like social medias, any data that is not cryptographically signed by authentic trustworthy entities should not be trusted. In other words, a necessary condition for the safety of learning algorithms is to train them solely on signed data, that is, data that provably come from a known user. Yet, this is clearly not a sufficient condition for safety: even signed data cannot be wholeheartedly trusted. After all, even authentic users usually have preferences over what ought to be recommended to others, and thus have incentives to behave strategically.

Unfortunately, today’s state-of-the-art algorithms strongly incentivize and are extremely vulnerable to such strategic manipulations. In fact, it was shown by [FGHV22] that classical personalized federated learning algorithms like [DTN20, HHHR20] can be arbitrarily manipulable by a single strategic user, through the injection of a surprisingly small amount of poisonous data. In particular, such a user would be incentivized
to construct an attack model, and to provide data labeled with this attack model rather than with the user’s preferred model. Assuming most users behaving strategically, the data thereby collected would inevitably be hopelessly untrustworthy: any algorithm trained with such data could be dangerously manipulated and weaponized by malicious data providers.

In this paper, we ask whether an algorithm can achieve performant (personalized) learning, while incentivizing users’ honest data generation and reporting. In the parlance of social choice theory, such an algorithm is called strategyproof.

To pose and address the question in a precise manner, we propose a new and rigorous definition of strategyproofness in the context of learning from user-generated data. We then introduce Licchavi, a relatively simple, yet general, learning framework, based on a careful design of the underlying loss function. We assume that users generate data by labeling them. Equivalently, this can be regarded as users being given queries, and providing answers to these queries. Such answers can be given honestly, using the user’s implicit preferred model, or can be given strategically to bias the global model, or other users’ learned models. Licchavi then leverages users’ data to perform both global and personalized learning, by penalizing the discrepancy between the global model and users’ models. This essentially captures the trade-off, for each user, between fitting other users’ data and model personalization. Critically, we use a coordinate-wise pseudo-Huber penalization, which allows to derive strategyproofness guarantees.

Licchavi also has applications in high-dimensional voting, e.g., to determine the parameters of a content moderation algorithm on a social network. In the same vein as [NGA+18, LKK+19, FBS+20], Licchavi would query the voter, collect the voter’s answers and then use machine learning to model the voter’s preferences. In practice, however, especially in high dimensions, each voter often provides an insufficient amount of data. This prevents the model from reliably learning their preferences. Licchavi allows improving the sample complexity by leveraging other voters’ inputs to better learn a voter’s preferences. More importantly, the global model learned by Licchavi can then be regarded as the output of the high-dimensional voting.

Interestingly, in addition to providing strategyproofness guarantees, the use of such coordinate-wise pseudo-Huber penalizations also implies an intuitively appealing fairness principle that Licchavi satisfies. Basically, we show that Licchavi essentially fits the appealing fairness principle “one voter, (at most) one unit force” [EFGH21], assuming that this force is measured by the $\ell_\infty$-norm, while also accounting for the uncertainty Licchavi inevitably has on a user’s preferred model, when the user does not provide sufficiently many data. All in all, this makes Licchavi a very promising tool for scalable algorithmic governance, especially in controversial contexts where users’ preferences are expected to greatly diverge.

Contributions. Our main contribution is to introduce Licchavi and to analyze its strategyproofness, i.e., whether it is in each user’s interest to answer queries honestly. We first prove that, unfortunately, assuming that each user wants to minimize the Euclidean distance between a target user’s model and their preferred model, Licchavi cannot be guaranteed to always be $\alpha$-strategyproof. Fortunately, we also prove that, for gradient PAC* coordinate-wise separable local losses, Licchavi is guaranteed to be strategyproof. We also discuss how to leverage this result to tune Licchavi to obtain approximate strategyproofness in the general case, when local losses are not coordinate-wise separable.

Our second result is that, even without this tuning, in the asymptotic case of a large number of voters, Licchavi is $\alpha$-strategyproof, for a value of $\alpha$ that we explicitly compute based on the distribution of voters’ preferred models. In short, we argue that the study of the asymptotic strategyproofness of Licchavi can be reduced to the study of a related strategyproofness problem. We then go on proving that, for this related problem, Licchavi is $\alpha$-strategyproof. This result constitutes a fair argument for why strategic users will not have strong incentives to provide fabricated rather than honest data, in the general case.

Our paper also shows how easy Licchavi is to deploy for practical machine learning tasks. We do so by considering the case of the personalized federated fine-tuning of language models on a set of tweets published on Twitter. Our empirical evaluation[3] conveys the fact that Licchavi provides good performances, at least compared to classical variants [DTN20, HHHR20].

1The Licchavis were a clan in Ancient India, and are credited for their early form of proto-democracy.
2The code, the dataset, and the instructions for reproducibility can be found [here](https://example.com).
Related work. There is a large body of work on the strategyproofness of learning problems, including regression \cite{CPPS18, DFP10}, classification \cite{MPR12, CLP20, MAMR11, HMPW16}, statistical estimation \cite{CDP15}, and clustering \cite{PS03}. The goal has been mainly to train a single model that incentivizes the honesty of users who aim to bias the model in their favor (e.g., by pulling the regression model towards their own desired points or achieving a classifier correctly their own labels). But none of these papers studies the strategyproofness of a general global and personalized learning framework.

In the case of linear regression, \cite{CPPS18} and \cite{PP03} assume that each user can only provide a single data point. Unfortunately, this greatly restricts the users’ ability to contribute to the learning model. Whilst \cite{DFP10} allows users to provide multiple data points, they either require payments, which might not be possible (e.g., due to ethical reasons), or they restrict the model to one dimension or a constant function in \(\mathbb{R}^d\). Licchavi, in contrast, does not make use of any payment, nor does it restrict the dimension of the model, and yet enables users to contribute large datasets.

Note that other desirable properties of coordinate-wise regularizations in general (typically \(\ell_1\) regularization) have been previously observed, both in terms of generalization \cite{Tib96, Wan13, SKAZ22}, robustness \cite{XCM08, DJ17, PF20} and strategyproofness \cite{GH20, DFP10, FGFHV22} (in restricted settings). Here, we show how it can be used to provide strategyproofness guarantees for a very general global and personalized learning scheme.

Structure of the paper. The rest of the paper is organized as follows. Section 2 introduces Licchavi. Section 3 presents our first main contribution, the strategyproofness analysis for the non-asymptotic case. We also discuss the tuning of Licchavi for approximate strategyproofness. Section 4 introduces our second main contribution, the asymptotic strategyproofness analysis. Section 5 presents our empirical evaluation of Licchavi. Section 6 concludes. Proofs are provided in the Appendix.

2 Licchavi

We consider a set \(\{N\} = \{1, \ldots, N\}\) of users. Each user \(n \in [N]\) is repeatedly provided with queries \(Q\) (which they may select themselves), and is asked to provide answers \(A\). The set of user \(n\)'s query-answer pairs \((Q, A)\) forms the user’s reported dataset \(D_n\). We denote by \(\bar{D} \triangleq (D_1, \ldots, D_N)\) the tuple of users’ datasets.

Our goal is to perform both global and personalized learning (GPL). Namely, for each user \(n\), we want to recover a model \(\theta_n \in \mathbb{R}^d\) that fits and generalizes their reported data \(D_n\). We let \(\bar{\theta} \triangleq \{\theta_1, \ldots, \theta_N\}\) denote the the tuple of users’ local models. Additionally, we want to learn a common global model \(\rho \in \mathbb{R}^d\), which may typically be used for community-level decisions, e.g., in the context of content moderation. This amounts to constructing a GPL algorithm \(\text{Alg} : \bar{D} \mapsto (\rho^{\text{LIC}}(\bar{D}), \bar{\theta}^{\text{LIC}}(\bar{D}))\).

To do so, we consider that any user \(n\)'s dataset \(D_n\) defines a strongly convex and differentiable local loss function \(L(\theta_n | D_n)\). We then draw inspiration from personalized federated learning \cite{DTN20, HHR20} to improve sample efficiency, and learn appropriate models even for users whose datasets are very limited, by adding terms that penalize the discrepancies between users’ local models \(\theta_n\) and the global model \(\rho\).

Now, unfortunately, as shown by \cite{FGHV22}, some classical personalized federated learning algorithms like \cite{DTN20, HHR20} are extremely vulnerable to strategic attacks. To remedy this vulnerability, we introduce Licchavi. Essentially, Licchavi leverages coordinate-wise pseudo-Huber losses \cite{CBAB97, HZ06} to learn a global model. More precisely, given users’ datasets \(\bar{D}\), Licchavi outputs a minimum \((\rho^*, \bar{\theta}^*)\) of the following loss function:

\[
L_\text{CH}(\rho, \bar{\theta} | \bar{D}) \triangleq \sum_{n \in [N]} L(\theta_n | D_n) + w \sum_{n \in [N]} H_{B, \delta}((\theta_n - \rho), \sqrt{1 + \delta_n}),
\]

where \(H_{B, \delta}(z) \triangleq \sum_{i \in [d]} H_{B, \delta}(z_i) \triangleq \sum_{i \in [d]} \sqrt{\delta^2 + z_i^2}\), and where \(w, \delta > 0\) are hyperparameters of Licchavi. In spirit, \(H_B\) acts like an \(\ell_1\) penalty. In fact, when \(|D_n| \to \infty\), then the \(H_B\) term converges uniformly to the \(\ell_1\) loss.
More importantly, like with \( \ell_1 \) loss, the pull of each user on the global model in each direction is bounded by \( w \) (this will be formalized by Lemma 2). This enforces the fairness principle “one person, (at most) one unit force vote” [EFGH21]. This property turns out to be critical for strategyproofness (and also implies robustness!). But, interestingly, \( H_b \) has additional desirable properties. As opposed to \( \ell_1 \) loss, \( H_b \) is smooth, which makes it easier to optimize and more numerically stable. Also, the fact that it is closer to a quadratic loss for users with few data points means that such users will act on \( \rho \) with a weaker force. This is consistent with the idea that they ought to be more uncertain about how to pull on \( \rho \). In fact, we chose a typical uncertainty \( \frac{\delta}{\sqrt{1+|D_n|}} \) which decays with the square root of the number of user \( n \)'s data, to be consistent with the posterior’s standard deviation. Finally, unlike \( \ell_1 \) loss, \( H_b \) is strictly convex. Combining all these properties enables us to guarantee that Licchavi is well-defined.

**Proposition 1.** For any datasets \( \vec{D} \), LCH yields a unique minimum, which we denote by \( \rho^{LCH}(\vec{D}) \) and \( \vec{\theta}^{LCH}(\vec{D}) \).

**Sketch of proof.** The loss is clearly convex overall, and strictly convex in \( \theta_n \). But given \( \vec{\theta} \), it is then strictly convex with respect to \( \rho \). This proves uniqueness. Moreover, if \( \theta_n \) has a norm too large, then, by strong convexity, the global loss takes values larger than its value at 0. Thus the minimum must be reached for local models within a compact region. But then, for \( \theta_n \) in this region, when \( \rho \) has a norm too large, the global loss takes values larger than its value at 0. Hence the minimum must be reached within a bounded region for all models, which proves the existence of a minimum. The full proof is given in Appendix A.

3 Strategyproofness

In this section, we study the strategyproofness of Licchavi. We prove that, unfortunately, Licchavi provides no general guarantee of \( \alpha \)-strategyproofness. Remarkably, however, we identify a sufficient condition for Licchavi to guarantee strategyproofness. But before presenting our results, we first clearly define strategyproofness, and stress how challenging it is to make any participatory system strategyproof.

3.1 What is strategyproofness?

Essentially, a participatory system is strategyproof if it incentivizes honest participation. This means that, in a strategyproof system and in the context of machine learning, it should be in each user’s best interests to label data as they think the data should be labeled.

**Why strategyproofness matters.** We first stress that strategyproofness is critical for safely learning from user-generated data. After all, the theory of learning relies on the core principle that generalizing training data is desirable. However, if a learning algorithm strongly incentivizes data misreporting, perhaps because many users have strong desires or pressures to promote certain products, views or ideologies, and because dishonesty or misbehaviors strongly favor such outcomes, then we should expect the algorithm to generalize very misleading, and potentially dangerous, activities. More generally, learning algorithms are shaped by their training datasets. As a result, their safety strongly depends on the soundness of the data they are trained with. Strategyproofness is arguably one of the most needed properties to guarantee data soundness, especially in high-stake environments, e.g., involving information warfare [Lin19].

**How strategyproofness differs from Byzantine learning.** Over the last five years, a large body of research [BMGS17, MGR18, BBG19, EGG+20, KHJ21, MFG+21, KHJ22] has focused on Byzantine learning, which aims to guarantee the safety of learning despite the presence of participants with arbitrary (potentially maximally malicious) behaviors. This property is clearly important as well. After all, especially if the number \( N \) of users is large, then we should expect the presence of at least a few users with essentially nonsensical activities.
Having said this, we stress that strategyproofness is an orthogonal, complementary and equally important property in practical deployments. The main reason for this is that strategyproofness considers an arguably more common class of users. Namely, instead of assuming arbitrary or maximally malicious behaviors, strategyproofness considers strategic users. Such users are goal-directed. Typically, a strategic user will want the global model to promote their views, or they will want to make other users’ models recommend content aligned with the strategic user’s preferences.

Crucially, the Byzantine learning literature usually assumes that the vast majority of users behave honestly. This assumption often justifies them in erasing outliers. However, especially in a heterogeneous setting, such as a controversial political debate, erasing outliers can be argued to be unethical, as it amounts to silencing minorities’ views. Perhaps equally importantly, the honest majority assumption also dangerously fails, if most users behave strategically. If so, then the users’ reported datasets may be hopelessly dishonest; and generalizing any of it could be highly dangerous.

Strategyproofness is scarce. A reader unfamiliar with strategyproofness might feel underwhelmed by the positive results of our paper. Let us thus stress how rare this property is. In the 1970s, Gib73 and Sat75 independently proved that the only strategyproof, unanimous and deterministic voting algorithm is dictatorship. Later, Gib78 added that the only strategyproof, unanimous and neutral voting algorithm is random dictatorship. More positive results can be obtained by assuming additional structures on participants’ preferences; but even then, they are restrictive. For instance, KR83 proved that, in dimension 2 and assuming users want the output vector to be as close as possible (in Euclidean norm) to their preferred vector, preferences; but even then, they are restrictive. For instance, KR83 proved that, in dimension 2 and assuming users want the output vector to be as close as possible (in Euclidean norm) to their preferred vector, preferences; but even then, they are restrictive. For instance, KR83 proved that, in dimension 2 and assuming users want the output vector to be as close as possible (in Euclidean norm) to their preferred vector, preferences; but even then, they are restrictive.

Formal definition. We now formalize strategyproofness. The focus here will be on the incentives of any single, omniscient and strategic user \( s \in [N] \), with a preferred model \( \theta^*_s \). We consider that the user’s honest behavior consists of (randomly) drawing a large number of queries \( Q \), and to answer them using their preferred model \( \theta^*_s \). The precise way of answering the queries depends on the problem (see [FGHV22]). For instance, for linear regression, an answer could be of the form \( \theta^*_s = Q^T \theta^*_s + \xi \), where \( \xi \) may typically be a zero-mean noise. The honest dataset \( D^*_s \) would then be the set of pairs \((Q,Q^T \theta^*_s)\) thereby constructed.

By contrast, when being strategic, user \( s \) can report any alternative strategic dataset \( D^\bullet_s \). Additionally, user \( s \) is assumed to know the datasets \( \bar{D}_{-s} \triangleq (D_{-s})_{n \neq s} \) provided by other users, and can adapt their choice of the strategic dataset \( D^\bullet_s \) accordingly. Importantly, user \( s \) is assumed to want to bias the learned global model (or a target user \( t \)’s local model) towards their preferred model \( \theta^*_s \). More precisely, we assume here that the strategic user’s goal is to minimize the Euclidean distance\(^{\text{3}}\) between \( \rho^\text{Alg}(\cdot, D^\bullet_s, \bar{D}_{-s}) \) and \( \theta^*_s \), or between \( \rho^\text{Alg}(\cdot, D^\bullet_s, \bar{D}_{-s}) \) and \( \theta^*_s \). Depending on where the strategic user’s focus is, we then have the two following definitions.

Definition 1. A global learning algorithm \( \text{ALG} \) is \( \alpha \)-strategyproof if, for any preferred model \( \theta^*_s \in \mathbb{R}^d \) and any other users’ datasets \( \bar{D}_{-s} \), given any \( \varepsilon, \delta > 0 \), there exists \( I \) such that, if \( D^\bullet_s \) is a dataset obtained by honestly answering at least \( \ell \) random queries with the preferred model \( \theta^*_s \), then with probability at least \( 1 - \delta \),

\[
\forall D^\bullet_s, \quad \| \rho^\text{Alg}(D^\bullet_s, \bar{D}_{-s}) - \theta^*_s \|_2 \leq (1 + \alpha) \| \rho^\text{Alg}(D^\bullet_s, \bar{D}_{-s}) - \theta^*_s \|_2 + \varepsilon. \tag{2}
\]

\(^{\text{3}}\)A vote is unanimous, if, when all users prefer the same alternative and vote honestly, then the vote outputs this unanimously preferred alternative.

\(^{\text{4}}\)A vote is neutral if the alternatives in contention in the vote play a symmetric role.

\(^{\text{5}}\)A vote is anonymous if the users play a symmetric role.

\(^{\text{6}}\)Appendix [B] generalizes our results to any norm invariant by coordinate-wise reflections, e.g., any \( \ell_p \) norm.
**Definition 2.** A personalized learning algorithm $\text{ALG}$ is user-targeted $\alpha$-strategyproof if, for any preferred model $\theta^*_t \in \mathbb{R}^d$, any other users’ datasets $\mathcal{D}_{-s}$, and any target user $t \in [N]$, given any $\varepsilon, \delta > 0$, there exists $I$ such that, if $\mathcal{D}_s^*$ is a dataset obtained by honestly answering at least $I$ random queries with the preferred model $\theta^*_s$, then with probability at least $1 - \delta$,

$$\forall \mathcal{D}_s^*, \quad \left\| \theta^\text{ALG}(\mathcal{D}_s^*, \mathcal{D}_{-s}) - \theta^*_t \right\|_2 \leq (1 + \alpha) \left\| \theta^\text{ALG}(\mathcal{D}_s^*, \mathcal{D}_{-s}) - \theta^*_s \right\|_2 + \varepsilon. \quad (3)$$

If the bound holds for $\alpha = 0$, then we simply say that $\text{ALG}$ is user-targeted strategyproof.

### 3.2 Main results

We can now state our main results of this section, which consist of both a negative and a positive theorem.

**Theorem 1.** For any $\alpha > 0$, LICCHAVI is neither global-targeted $\alpha$-strategyproof nor user-targeted $\alpha$-strategyproof.

Theorem 1 stresses the need for further assumptions to retrieve any strategyproofness. Here, we identify sufficient conditions to guarantee LICCHAVI’s strategyproofness. The first condition was first introduced by [FGHV22], who proved it to hold for linear and logistic regression under very mild conditions.

**Definition 3 (Gradient-PAC*), from [FGHV22].** Denote $\mathcal{E}(\mathcal{D}, \theta^1, I, A, B, \alpha)$ the event

$$\forall \theta \in \mathbb{R}^d, \quad (\theta - \theta^1)^T \nabla \mathcal{L}(\theta | \mathcal{D}) \geq A \mathbb{E} \min \left\{ \left\| \theta - \theta^1 \right\|_2, \left\| \theta - \theta^1 \right\|_2^2 \right\} - B \mathbb{E} \left\| \theta - \theta^1 \right\|_2.$$

The loss $\mathcal{L}$ is gradient-PAC* if, for any $K > 0$, there exist $A_K, B_K > 0$ and $\alpha_K < 1$ such that, for any preferred model $\theta^1 \in \mathbb{R}^d$ with $\left\| \theta^1 \right\|_2 \leq K$, assuming that the dataset $\mathcal{D}$ is obtained by answering random queries $Q$ with model $\theta^1$, $\mathbb{P} \left[ \mathcal{E}(\mathcal{D}, \theta^1, I, A_K, B_K, \alpha_K) \right] \rightarrow 1$ as $I \rightarrow \infty$.

Intuitively, gradient PAC guarantees that if a user $n$ answers sufficiently many queries by using a labeling model $\theta^*_n$, then the labeling model $\theta^*_n$ is robustly approximately reconstructed by minimizing the local loss. To guarantee strategyproofness, we also demand that the local loss $\mathcal{L}$ be coordinate-wise separable, which means that it can be written $\mathcal{L}(\theta | \mathcal{D}) = \sum_{i \in [d]} \mathcal{L}_i(\theta_i | \mathcal{D})$. Section 3.5 will discuss how this assumption can be removed, by tuning LICCHAVI to provide approximate strategyproofness.

**Theorem 2.** Assume that the local losses are gradient PAC* and coordinate-wise separable. Then LICCHAVI is both global and user-targeted strategyproof.

Let us now outline the nontrivial proofs of the two main theorems. Interestingly, we successfully decomposed them into lemmas, each of which uncovers insights about personalized federated learning in general, and about LICCHAVI in particular. The lemma proofs appear in Appendix [E].

### 3.3 Reduced losses

First, we note that the study of global-targeted strategyproofness can be reduced to the analysis of a loss which only depends on the global model. To do so, given a local dataset $\mathcal{D}$, we first define the reduced local loss

$$\mathcal{R}(\rho | \mathcal{D}) \triangleq \inf_{\theta \in \mathbb{R}^d} \mathcal{L}(\theta | \mathcal{D}) + w \text{HB} \frac{\frac{\varepsilon}{\sqrt{1 + |I|}}}{\sqrt{1 + |I|}} (\theta - \rho). \quad (4)$$

Below, we show that this reduced local loss is well-behaved.

**Lemma 1.** Equation (4) yields a unique minimum $\theta^*(\rho, \mathcal{D})$.

**Lemma 2.** $\mathcal{R}(\rho | \mathcal{D})$ is convex and differentiable. Moreover, $\nabla \mathcal{R} = w \text{HB} \frac{\frac{\varepsilon}{\sqrt{1 + |I|}}}{\sqrt{1 + |I|}} (\rho - \theta^*(\rho, \mathcal{D}))$, and $\| \nabla \mathcal{R} \|_\infty \leq w$.

Let $\mathcal{R}(\rho | \mathcal{D}_n) \triangleq \sum_{n \in [N]} \mathcal{R}(\rho | \mathcal{D}_n)$ and $\mathcal{R}(\rho | \mathcal{D}_{-s}) \triangleq \sum_{n \neq s} \mathcal{R}(\rho | \mathcal{D}_n)$ be the sum of (other) users’ reduced losses.

**Lemma 3.** $\rho^\text{LCH}(\mathcal{D})$ is the unique minimum of $\mathcal{R}(\rho | \mathcal{D})$, while $\theta^*_n \text{LCH}(\mathcal{D}) = \theta^*(\rho^\text{LCH}(\mathcal{D}), \mathcal{D}_n)$. 

6
3.4 Strong local PAC*

Another key step of our proofs is to reduce data reporting strategyproofness to model reporting strategyproofness, for gradient PAC* local losses. \cite{FGHV22} also proved that gradient PAC* implies local PAC* learning for a large class of personalized federated learning algorithm. In this paper, we prove a stronger result for the particular case of Licchavi. Namely, we prove that, under gradient PAC* local losses, LICCHAVI is strongly local PAC*.

**Definition 4.** A GPL algorithm ALG is strongly local PAC* if, for any user \( n \) and any preferred model \( \theta^\dagger_n \), any \( \varepsilon, \delta > 0 \), there exists \( I \) such that, if the user \( n \) provides a dataset \( D^\dagger_n \) with \( |D^\dagger_n| \geq I \) answers to random queries given using their preferred models \( \theta^\dagger_n \), then, with probability at least \( 1 - \delta \),

\[
\forall \bar{D} \in \mathcal{D}_n, \quad \left\| \theta^\text{ALG}_n \left( D^\dagger_n, \bar{D} \right) - \theta^\dagger_n \right\|_2 \leq \varepsilon \tag{5}
\]

Importantly, as opposed to local PAC* (introduced in \cite{FGHV22}), strong local PAC* guarantees the accuracy of the learning of \( \theta^\dagger_n \) independently from other users’ data \( \bar{D} \). This is a very desirable property in practice, as it guarantees that a user with sufficiently many data will never be hacked by a very active malicious user. Interestingly, this is a property that LICCHAVI guarantees.

**Lemma 4.** For gradient PAC* local losses, LICCHAVI is strongly local PAC*.

**Sketch of proof.** The key insight is that the pseudo-Huber regularization term of \( \Lch^{\dagger}_n \) has a bounded gradient. By contrast, by gradient PAC*, as a user \( n \) with preferred model \( \theta^\dagger_n \) provides more and more honest data \( D^\dagger_n \), for any \( \theta_n \) too far from the preferred model \( \theta^\dagger_n \), the negative gradient \(-\nabla_{\theta_n} \mathcal{L}(\theta_n|D^\dagger_n)\) of the local loss will point more and more towards \( \theta^\dagger_n \), so that it will eventually outweigh the gradient \( \nabla_{\theta_n} w \mathcal{H}_b(\theta_n - \rho) \) of the pseudo-Huber regularization term, no matter what value \( \rho \) takes. This guarantees that, for any value of \( \rho^{\text{LCH}}(\bar{D}) \), the optimum \( \delta^\text{LCH}_n(\bar{D}) = \theta^\ast(\rho^{\text{LCH}}(\bar{D}), D_n) \) will be close to \( \theta^\dagger_n \). \( \square \)

3.5 Reduction to model attack

By (strong) local PAC* and by providing enough data \( D_s^\bullet \) labeled with \( \theta^\bullet \), the strategic user \( s \) can essentially make LICCHAVI learn the model \( \theta^\text{LCH}_s \approx \theta^\bullet \). Moreover, by providing enough data, they can make the Huber loss essentially equal to an \( \ell_1 \) loss. This prompts us to consider the following modified Licchavi loss

\[
\mathcal{L}_s^\text{CH}(\theta^\bullet, \bar{D}^{-s}) \triangleq w \left\| \theta^\bullet - \rho \right\|_1 + \mathcal{R}(\rho|\bar{D}^{-s}). \tag{6}
\]

This loss can be easily shown to yield a unique minimum, which we denote by \( \theta^{\text{LCH}}_s(\theta^\bullet, \bar{D}^{-s}) \) and \( \delta^{\text{LCH}}_s(\theta^\bullet, \bar{D}^{-s}) \) for \( n \neq s \). Define also \( \theta^{\text{LCH}}_s(\theta^\bullet, \bar{D}) \triangleq \theta^\bullet \). The definition of \( \alpha \)-strategyproofness under model attack is then akin to the definitions of Section 3.1 but with models instead of data, and without any randomness and approximation, which removes the need of \( \varepsilon \) and \( \delta \). Typically, for the case of global-targeted \( \alpha \)-strategyproofness, the following must hold:

\[
\forall \theta^\bullet, \theta^\dagger, \forall \bar{D}^{-s}, \quad \left\| \rho^{\text{ALC}}(\theta^\dagger, \bar{D}^{-s}) - \theta^\bullet \right\|_2 \leq (1 + \alpha) \left\| \rho^{\text{ALC}}(\theta^\bullet, \bar{D}^{-s}) - \theta^\bullet \right\|_2. \tag{7}
\]

We can now adapt the equivalence proven by \cite{FGHV22} to the case of LICCHAVI’s strategyproofness.

**Lemma 5.** Assuming strong local PAC*, LICCHAVI is global-targeted \( \alpha \)-strategyproof under data attack if and only if it is global-targeted \( \alpha \)-strategyproof under model attack. The equivalence also holds for user-targeted \( \alpha \)-strategyproofness.

**Sketch of proof.** On one hand, any data attack \( D_s^\bullet \) yields the same outcome as the attack by model \( \theta^\bullet = \rho^{\text{LCH}}(D_s^\bullet, D^{-s}) \). On the other hand, by strong local PAC* (Lemma 4), an attack model \( \theta^\bullet \) yields essentially the same result as the dataset \( D_s^\bullet \) obtained by randomly a large number of queries and answering them with model \( \theta^\bullet \). The precise analysis, given in Appendix B.3, is however nontrivial. \( \square \)

In light of the lemma, to prove theorems 1 and 2 it is sufficient to (dis)prove strategyproofness under model attack.
3.6 Proof sketch of the negative result

Unfortunately, in general, no $\alpha$-strategyproofness guarantee holds for LICCHAVI.

**Sketch of proof.** Essentially, we construct a nasty instance for $d = 2$, by designing appropriately the other users’ reduced loss $\mathcal{R}(\rho)\mathcal{D}_{\rho}$, where $\rho$ is a (possibly unbounded) interval. But now, if $\theta^* < \inf(I)$, then $\rho^{\text{Licch}} = \inf(I)$. If $\theta^* > \sup(I)$, then $\rho^{\text{Licch}} = \sup(I)$. Finally, $\rho^{\text{Licch}} = \theta^*_s$. In any case, the learned value $\rho^{\text{Licch}}$ is closest to $\theta^*_t$ when $\theta^* = \theta^*_s$. Similar arguments apply to biasing a target user $t$’s model $\theta^*_t$. Appendix D details the proof.

3.7 Proof sketch of the positive result

**Sketch of proof.** Our assumptions allow to reduce strategyproofness to the one-dimension case. But then, in dimension 1, by behaving strategically, user $s$ can only achieve values for $\rho^{\text{Licch}}$ within a (possibly unbounded) interval $I$. But now, if $\theta^* < \inf(I)$, then $\rho^{\text{Licch}} = \inf(I)$. If $\theta^* > \sup(I)$, then $\rho^{\text{Licch}} = \sup(I)$. Finally, if $\theta^*_s \in I$, then $\rho^{\text{Licch}} = \theta^*_s$. In any case, the learned value $\rho^{\text{Licch}}$ is closest to $\theta^*_t$ when $\theta^* = \theta^*_s$. Similar arguments apply to biasing a target user $t$’s model $\theta^*_t$. Appendix D details the proof.

3.8 Approximate strategyproofness in the general case

In general, unfortunately, local loss functions are not coordinate-wise separable. Nevertheless, here, we discuss how our strategyproofness theorem can be leveraged to tune LICCHAVI and make it approximately strategyproof. The main trick is to tune each user $n$’s coordinate system depending on the sum of other users’ reduced loss $\mathcal{R}(\rho)\mathcal{D}_{\rho}$.

More precisely, denote $H_s \triangleq \nabla^2_{\rho=\rho^{-}} \mathcal{R}(\rho)\mathcal{D}_{\rho}$, where $\rho^{-}$ is the output of LICCHAVI executed on all users apart from user $s$. Since $\mathcal{R}$ is convex, we know that $H_s$ is semi-definite positive. Moreover, it is symmetric, thus there exists an orthogonal matrix $Q_s$ and eigenvalues $\lambda_1^s \geq \ldots \geq \lambda_n^s \geq 0$ such that $H_s = Q_s^T \text{Diag}(\lambda_1^s, \ldots, \lambda_n^s) Q_s$. Then, assuming there are many users, so that the effect of strategic user $s$ on the global model is small, and ignoring the additive constants, the reduced LICCHAVI loss becomes approximately

$$LCH_s(\rho, \rho, \mathcal{D}_{\rho}) \approx \sum_{i \in [d]} (Q_s \rho)_i^2$$

Now, in general, this loss has no guarantee of strategyproofness. However, we may now tune LICCHAVI for strategic user $s$ based on the orthogonal matrix $Q_s$ to fall back on the previous case. To do so, we introduce the following $\hat{Q}$-skewed LICCHAVI loss:

$$LCH(\rho, \hat{\rho}, \mathcal{D}, \hat{Q}) \triangleq \sum_{n \in [N]} \mathcal{L}(\theta_n) \mathcal{D}_{\theta} \mathcal{D}_{\rho} + \sum_{n \in [N]} \mathcal{H} \frac{1}{\sqrt{1+\beta_n}} (Q_n \theta_n - Q_n \hat{\rho})$$

Indeed, this loss corresponds to the following reduced loss for model attack:

$$LCH_s(\rho, \rho, \mathcal{D}_{\rho}) = \sum_{i \in [d]} (Q_s \rho)_i^2$$

assuming $\mathcal{R}(\rho)\mathcal{D}_{\rho} \approx \mathcal{R}(\rho)\mathcal{D}_{\rho}$. Importantly, this last approximation is coordinate-wise separable, which means that Theorem 3 would approximately apply here.
Unfortunately, the precise analysis of our approximations is highly nontrivial, and beyond the scope of this paper. In particular, we leave open the problem of determining how to (efficiently) compute matrices $\hat{Q}$ such that the vectors $\hat{Q}^T e^i$ are (approximately) eigenvectors of $\nabla^2_{\rho=\rho-n} R(\rho|\bar{D}_{-n}, \hat{Q}_{-n})$ for all users $n \in [N]$, where $e^i$ is the $i$-th vector of the canonical basis $\mathcal{E}$.

### 4 Asymptotic Strategyproofness

In this section, we discuss the strategyproofness of Licchavi in the asymptotic setting of a large number of users. From a practical standpoint, this is arguably the most relevant setting for it allows us to approximate the loss restricted to other users by a quadratic function, as discussed below.

#### 4.1 Asymptotic setting

Let us first define the asymptotic setting, which is inspired from [EFGH21]. Intuitively, it corresponds to the limit where $N \to \infty$, when each user’s dataset $D_n$ is drawn independently from a distribution of datasets $\mathcal{D}$. This then naturally leads us to the following definition of strategyproofness which, for simplicity, we state in the case of model attack. By our equivalence lemma (Lemma 5), it is evidently equivalent to its (more wordy) data attack version.

**Definition 5.** A GPL algorithm $\text{Alg}$ is asymptotically global-targeted $\alpha$-strategyproof under distribution $\mathcal{D}$ if, for any $\varepsilon, \delta > 0$ and any preferred model $\theta^1_s$, there exists $N_0$ such that, if there are $N - 1 \geq N_0$ users (other than strategic user $s$) whose datasets $\bar{D}_{-s}$ are all drawn independently from $\mathcal{D}$, then with probability at least $1 - \delta$, we have

$$\forall \theta^\bullet_s, \left\| \rho^{\text{LCH}}(\theta^1_s, \bar{D}_{-s}) - \theta^1_s \right\|_2 \leq (1 + \alpha) \left\| \rho^{\text{LCH}}(\theta^\bullet_s, \bar{D}_{-s}) - \theta^1_s \right\|_2 + \varepsilon. \quad (10)$$

Now, when the number $N$ of users is large, the Licchavi loss under model attack can be approximated by

$$\text{LCH}(\rho|\theta^\bullet_s, \bar{D}_{-s}) \approx w \left\| \rho - \theta^\bullet_s \right\|_1 + (N - 1) \bar{R}(\rho), \quad (11)$$

where $\bar{R}(\rho) \triangleq \mathbb{E}_{\mathcal{D} \sim \mathcal{D}} \left[ R(\rho|\mathcal{D}) \right]$, with an expectation taken over the random dataset $\mathcal{D}$.

Now denote $\rho^\infty \triangleq \arg \min_\rho \bar{R}(\rho)$ the model obtained by ignoring the strategic user. We also define the achievable set $\text{AchSet}$ as the set of global models that could be obtained through model attack by the strategic user, i.e.

$$\text{AchSet}(\bar{D}_{-s}) \triangleq \left\{ \rho^{\text{LCH}}(\theta^\bullet_s, \bar{D}_{-s}) \left| \theta^\bullet_s \in \mathbb{R}^d \right. \right\}. \quad (12)$$

When $N$ is large, the strategic user’s attack model $\theta^\bullet_s$ will only have a small effect on the optimal global model. This means that, for large values of $N$, $\text{AchSet}(\bar{D}_{-s})$ gets arbitrarily small. As a result, over $\text{AchSet}(\bar{D}_{-s})$, and for a large enough number of users, the expected reduced loss $\bar{R}(\rho)$ in (11) can be approximated by a quadratic loss. More precisely, defining $\rho^\infty$ the minimum of $\bar{R}$ and $H_\infty \triangleq \nabla^2 \bar{R}(\rho^\infty)$, we then have

$$\text{LCH}_s(\rho|\theta^\bullet_s, \bar{D}_{-s}) \approx w \left\| \rho - \theta^\bullet_s \right\|_1 + (N - 1)(\rho - \rho^\infty)^T H_\infty (\rho - \rho^\infty). \quad (13)$$

Unfortunately, the precise formulation and derivation of this approximation is highly nontrivial, and left for future work. Importantly, however, it suggests that we can restrict our attention to this quadratic setting.

#### 4.2 The quadratic setting

In light of our discussion above, and without loss of generality in the asymptotic setting, we now focus on Licchavi against a quadratic function, with a unit voting right, i.e.

$$\text{LCH}(\rho|\theta^\bullet_s, S) \triangleq \left\| \rho - \theta^\bullet_s \right\|_1 + \rho^T S \rho. \quad (14)$$
To state our result, we define the crookedness of $S \succ 0$ by

$$\text{Crooked}(S) \triangleq \sup_{x \in \mathbb{R}^d} \inf_{y \in \mathbb{R}^d \text{s.t.} \ sgn(y) = sgn(x)} \frac{\|x\|_2 \|Sy\|_2}{x^T Sy} - 1, \quad (15)$$

where $sgn$ applies the sign function on each coordinate (and thus implies $y_i = 0$ whenever $x_i = 0$). We now have the following theorem.

**Theorem 3.** **Licchavi** against positive definite matrix $S$ is $\text{Crooked}(S)$-strategyproof.

**Sketch of proof.** The proof is nontrivial, as it involves understanding the function $\rho^{\text{Licch}}(\theta^\bullet, S)$, as well as its image for $\theta^\bullet \in \mathbb{R}^d$, which is the achievable set. Arguments based on orthogonal projection then allow us to lower bound the distance between $\theta^\dagger$ and the achievable set. The full proof is given in Appendix E.

Unfortunately, $\text{Crooked}$ does not seem to yield a closed form formula. Nevertheless, we point out that it takes lower values than another measure called $\text{Skew}$, introduced by [EFGH21].

**Proposition 2.** Let $\text{Skew}(S) \triangleq \sup_{x \in \mathbb{R}^d} \frac{\|x\|_2 \|Sx\|_2}{x^T Sx} - 1$. Then, for any $S \succ 0$, we have $\text{Crooked}(S) \leq \text{Skew}(S)$. Moreover, there are definite positive matrices $S \succ 0$ for which the inequality is strict.

**Sketch of proof.** The inequality is obtained by considering $y \equiv x$ in Equation (15). The strict inequality can be shown by considering a matrix $S$ whose eigenvectors are the canonical basis vectors, and whose eigenvalues differ.

Since [EFGH21] essentially showed that the geometric median is $\text{Skew}(S)$-strategyproof, and that this strategyproofness bound is tight, our theorem nicely shows that the coordinate-wise median (and variants like **Licchavi**) is essentially more strategyproof than the geometric median and its variants. Intuitively, by forcing agreements to be coordinate-wise, the coordinate-wise median (and variants like **Licchavi**) restricts the vulnerabilities to what happens only along the canonical basis vectors. In fact, in the specific case where each vector of the canonical basis $\mathcal{E}$ is an eigenvector of $S$, but with different eigenvalues, then Theorem 2 actually applies, and **Licchavi** is strategyproof ($\alpha = 0$). This is strictly better than what the geometric median guarantees in such a case.

## 5 Experimenting Licchavi

To test **Licchavi**, we consider a language fine-tuning task, on a language model with an embedding layer of dimension 256, two GRU with hidden size 200 and a fully connected layer with 10'000 output units (vocab size) using softmax, with cross-entropy loss on next token prediction. This yields $5 \times 10^6$ free parameters, half being in the embedding layer. A global model was pretrained on a pretraining dataset, and the model’s embedding layer was frozen.

We then considered a real Twitter dataset made of $2 \times 10^7$ hydrated tweets during the 2016 USA presidential election from $N = 100$ users. We performed federated fine-tuning of the last layer, with users’ tweets, using **Licchavi** (with $w \equiv 1$ and $\delta_c \equiv 10$) and the $\ell_2^2$ baseline [DNT20, HHHR20], which we implemented on top of Pytorch. We used a batch size of 32, 3 epochs per nodes per round, and a learning rate of $10^{-3}$. The performance was measured on another set of tweets by the $R_3$ measure, which is the average number of times our model contains the correct next word in its top 3 predictions. The results are displayed in Figure 1.

We observe that both **Licchavi** and $\ell_2^2$ fine tuning improve the $R_3$ measure of the global model in a similar way. This suggests that **Licchavi** can provide similar performances as classical personalized federated learning models, while additionally providing strategyproofness guarantees.
6 Conclusion

We introduced LICCHAVI, an algorithm for global and personalized learning, and we analyzed its strategyproofness. We proved both positive and negative theorems. Perhaps most importantly, we showed that LICCHAVI yields some asymptotic α-strategyproofness guarantees, and we sketched how to guarantee approximate strategyproofness in the general setting, by tuning LICCHAVI. We argue that such considerations are critical to guarantee the trustworthiness of training databases and, eventually, the security of deployed machine learning algorithms. We also implemented LICCHAVI for language fine tuning, and our experiments highlighted its practicality and performance.

Acknowledgment

The authors are thankful to Guillaume Le Mailloux for some useful preliminary work on strategyproof learning.

References

[BBG19] Gilad Baruch, Moran Baruch, and Yoav Goldberg. A little is enough: Circumventing defenses for distributed learning. In H. Wallach, H. Larochelle, A. Beygelzimer, F. d’Alché-Buc, E. Fox, and R. Garnett, editors, Advances in Neural Information Processing Systems, volume 32. Curran Associates, Inc., 2019.

[BGMS21] Emily M. Bender, Timnit Gebru, Angelina McMillan-Major, and Shmargaret Shmitchell. On the dangers of stochastic parrots: Can language models be too big? In Madeleine Clare Elish, William Isaac, and Richard S. Zemel, editors, FAccT ’21: 2021 ACM Conference on Fairness, Accountability, and Transparency, Virtual Event / Toronto, Canada, March 3-10, 2021, pages 610–623. ACM, 2021.

[BH19] Samantha Bradshaw and Philip N Howard. The global disinformation order: 2019 global inventory of organised social media manipulation. Project on Computational Propaganda, 2019.

[BMGS17] Peva Blanchard, El Mahdi El Mhamdi, Rachid Guerraoui, and Julien Stainer. Machine learning with adversaries: Byzantine tolerant gradient descent. In Isabelle Guyon, Ulrike von Luxburg, Samy Bengio, Hanna M. Wallach, Rob Fergus, S. V. N. Vishwanathan, and Roman Garnett, editors, Advances in Neural Information Processing Systems 30: Annual Conference on Neural Information Processing Systems 2017, 4-9 December 2017, Long Beach, CA, USA, pages 119–129, 2017.
[BPT17] Omer Ben-Porat and Moshe Tennenholtz. Best response regression. In I. Guyon, U. V. Luxburg, S. Bengio, H. Wallach, R. Fergus, S. Vishwanathan, and R. Garnett, editors, Advances in Neural Information Processing Systems, volume 30. Curran Associates, Inc., 2017.

[CBAB97] Pierre Charbonnier, Laure Blanc-Féraud, Gilles Aubert, and Michel Barlaud. Deterministic edge-preserving regularization in computed imaging. IEEE Trans. Image Process., 6(2):298–311, 1997.

[CDP15] Yang Cai, Constantinos Daskalakis, and Christos H. Papadimitriou. Optimum statistical estimation with strategic data sources. In Peter Grünwald, Elad Hazan, and Satyen Kale, editors, Proceedings of The 28th Conference on Learning Theory, COLT 2015, Paris, France, July 3-6, 2015, volume 40 of JMLR Workshop and Conference Proceedings, pages 280–296. JMLR.org, 2015.

[CLP20] Yiling Chen, Yang Liu, and Chara Podimata. Learning strategy-aware linear classifiers. In H. Larochelle, M. Ranzato, R. Hadsell, M. F. Balcan, and H. Lin, editors, Advances in Neural Information Processing Systems, volume 33, pages 15265–15276. Curran Associates, Inc., 2020.

[CPS18] Yiling Chen, Chara Podimata, Ariel D. Procaccia, and Nisarg Shah. Strategyproof linear regression in high dimensions. In Proceedings of the 2018 ACM Conference on Economics and Computation, EC’18, page 9–26, New York, NY, USA, 2018. Association for Computing Machinery.

[DFP10] Ofer Dekel, Felix Fischer, and Ariel D. Procaccia. Incentive compatible regression learning. Journal of Computer and System Sciences, 76(8):759–777, 2010.

[DJ17] Chris Ding and Bo Jiang. L1-norm error function robustness and outlier regularization. CoRR, abs/1705.09954, 2017.

[Dol21] Lara Dolden. Facebook removed over 15 billion fake accounts in 2 years. TechRound, 2021.

[DTN20] Canh T. Dinh, Nguyen H. Tran, and Tuan Dung Nguyen. Personalized federated learning with moreau envelopes. In Hugo Larochelle, Marc’Aurelio Ranzato, Raia Hadsell, Maria-Florina Balcan, and Hsuan-Tien Lin, editors, Advances in Neural Information Processing Systems 33: Annual Conference on Neural Information Processing Systems 2020, NeurIPS 2020, December 6-12, 2020, virtual, 2020.

[EGF21] El-Mahdi El-Mhamdi, Sadegh Farhadkhani, Rachid Guerraoui, and Lê Nguyên Hoang. Strategyproofness of the geometric median. CoRR, 2021.

[EGL+20] El-Mahdi El-Mhamdi, Rachid Guerraoui, Arsany Guirguis, Lê Nguyên Hoang, and Sébastien Rouault. Genuinely distributed byzantine machine learning. In Yuval Emek and Christian Cachin, editors, PODC ’20: ACM Symposium on Principles of Distributed Computing, Virtual Event, Italy, August 3-7, 2020, pages 355–364. ACM, 2020.

[FBS+20] Rachel Freedman, Jana Schaich Borg, Walter Simott-Armstrong, John P. Dickerson, and Vincent Conitzer. Adapting a kidney exchange algorithm to align with human values. Artif. Intell., 283:103261, 2020.

[FGH22] Sadegh Farhadkhani, Rachid Guerraoui, Lê Nguyên Hoang, and Oscar Villemot. An equivalence between data poisoning and byzantine gradient attacks, 2022.

[FMO20] Alireza Fallah, Aryan Mokhtari, and Asman E. Ozdaglar. Personalized federated learning with theoretical guarantees: A model-agnostic meta-learning approach. In Hugo Larochelle, Marc’Aurelio Ranzato, Raia Hadsell, Maria-Florina Balcan, and Hsuan-Tien Lin, editors, Advances in Neural Information Processing Systems 33: Annual Conference on Neural Information Processing Systems 2020, NeurIPS 2020, December 6-12, 2020, virtual, 2020.
[GH20] Sumit Goel and Wade Hann-Caruthers. Coordinate-wise median: Not bad, not bad, pretty good. *CoRR*, abs/2007.00903, 2020.

[Gib73] Allan Gibbard. Manipulation of voting schemes: a general result. *Econometrica: journal of the Econometric Society*, pages 587–601, 1973.

[Gib78] Allan Gibbard. Straightforwardness of game forms with lotteries as outcomes. *Econometrica: Journal of the Econometric Society*, pages 595–614, 1978.

[HFE21] Lê Nguyên Hoang, Louis Faucon, and El-Mahdi El-Mhamdi. Recommendation algorithms, a neglected opportunity for public health. *Revue Médecine et Philosophie*, 4(2):16–24, 2021.

[HHHR20] Filip Hanzely, Slavomír Hanzely, Samuel Horváth, and Peter Richtárik. Lower bounds and optimal algorithms for personalized federated learning. In Hugo Larochelle, Marc’Aurelio Ranzato, Raia Hadsell, Maria-Florina Balcan, and Hsuan-Tien Lin, editors, *Advances in Neural Information Processing Systems 33: Annual Conference on Neural Information Processing Systems 2020, NeurIPS 2020, December 6-12, 2020, virtual*, 2020.

[HMPW16] Moritz Hardt, Nimrod Megiddo, Christos Papadimitriou, and Mary Wootters. Strategic classification. In *Proceedings of the 2016 ACM Conference on Innovations in Theoretical Computer Science*, ITCS ’16, page 111–122, New York, NY, USA, 2016. Association for Computing Machinery.

[Hoa20] Lê Nguyên Hoang. Science communication desperately needs more aligned recommendation algorithms. *Frontiers in Communication*, 5:115, 2020.

[HZ06] Andrew Harltey and Andrew Zisserman. *Multiple view geometry in computer vision (2. ed.)*. Cambridge University Press, 2006.

[IJW+19] Eugene Ie, Vihan Jain, Jing Wang, Sanmit Narvekar, Ritesh Agarwal, Rui Wu, Heng-Tze Cheng, Tushar Chandra, and Craig Boutilier. Slateq: A tractable decomposition for reinforcement learning with recommendation sets. In Sarit Kraus, editor, *Proceedings of the Twenty-Eighth International Joint Conference on Artificial Intelligence, IJCAI 2019, Macao, China, August 10-16, 2019*, pages 2592–2599. ijcai.org, 2019.

[KHJ21] Sai Praneeth Karimireddy, Lie He, and Martin Jaggi. Learning from history for byzantine robust optimization. In Marina Meila and Tong Zhang, editors, *Proceedings of the 38th International Conference on Machine Learning, ICML 2021, 18-24 July 2021, Virtual Event*, volume 139 of *Proceedings of Machine Learning Research*, pages 5311–5319. PMLR, 2021.

[KHJ22] Sai Praneeth Karimireddy, Lie He, and Martin Jaggi. Byzantine-robust learning on heterogeneous datasets via bucketing. In *International Conference on Learning Representations*, 2022.

[KK22] Ayushman Kaul and Devesh Kumar. Tek fog: An app with bjp footprints for cyber troops to automate hate, manipulate trends. *The Wire*, 2022.

[KR84] K.H. Kim and F.W. Roush. Nonmanipulability in two dimensions. *Mathematical Social Sciences*, 8(1):29–43, 1984.

[LB21] Florian Lehmann and Daniel Buschek. Examining autocompletion as a basic concept for interaction with generative AI. *i-com*, 19(3):251–264, 2021.

[Lin19] Herbert Lin. The existential threat from cyber-enabled information warfare. *Bulletin of the Atomic Scientists*, 75(4):187–196, 2019.

[LKK+19] Min Kyung Lee, Daniel Kusbit, Anson Kahng, Ji Tae Kim, Xinran Yuan, Alissa Chan, Daniel See, Ritesh Nookthagattu, Siheon Lee, Alexandros Psomas, and Ariel D. Procaccia. Webuildai: Participatory framework for algorithmic governance. *PACMHCI*, 3(CSCW):181:1–181:35, 2019.
[MAMR11] Reshef Meir, Shaull Almagor, Assaf Michaely, and Jeffrey S. Rosenschein. Tight bounds for strategyproof classification. In The 10th International Conference on Autonomous Agents and Multiagent Systems - Volume 1, AAMAS '11, page 319–326, Richland, SC, 2011. International Foundation for Autonomous Agents and Multiagent Systems.

[MFG+21] El Mahdi El Mhamdi, Sadegh Farhadkhani, Rachid Guerraoui, Arsany Guirguis, Lé-Nguyen Hoang, and Sébastien Rouault. Collaborative learning in the jungle (decentralized, byzantine, heterogeneous, asynchronous and nonconvex learning). In Thirty-Fifth Conference on Neural Information Processing Systems, 2021.

[MGR18] El Mahdi El Mhamdi, Rachid Guerraoui, and Sébastien Rouault. The hidden vulnerability of distributed learning in byzantium. In Jennifer G. Dy and Andreas Krause, editors, Proceedings of the 35th International Conference on Machine Learning, ICML 2018, Stockholmsmässan, Stockholm, Sweden, July 10-15, 2018, volume 80 of Proceedings of Machine Learning Research, pages 3518–3527. PMLR, 2018.

[MN20] Kris McGuffie and Alex Newhouse. The radicalization risks of GPT-3 and advanced neural language models. CoRR, abs/2009.06807, 2020.

[MPR12] Reshef Meir, Ariel D. Procaccia, and Jeffrey S. Rosenschein. Algorithms for strategyproof classification. Artificial Intelligence, 186:123–156, 2012.

[NGA+18] Ritesh Noothigattu, Snehalkumar (Neil) S. Gaikwad, Edmond Awad, Sohan Dsouza, Iyad Rahwan, Pradeep Ravikumar, and Ariel D. Procaccia. A voting-based system for ethical decision making. In Sheila A. McIlraith and Kilian Q. Weinberger, editors, Proceedings of the Thirty-Second AAAI Conference on Artificial Intelligence, (AAAI-18), the 30th innovative Applications of Artificial Intelligence (IAAI-18), and the 8th AAAI Symposium on Educational Advances in Artificial Intelligence (EAAI-18), New Orleans, Louisiana, USA, February 2-7, 2018, pages 1587–1594. AAAI Press, 2018.

[NHK19] Lisa-Maria Neudert, Philip Howard, and Bence Kollanyi. Sourcing and automation of political news and information during three european elections. Social Media+ Society, 5(3):2056305119863147, 2019.

[PF20] Scott Pesme and Nicolas Flammarion. Online robust regression via SGD on the l1 loss. In Hugo Larochelle, Marc’Aurelio Ranzato, Raia Hadsell, Maria-Florina Balcan, and Hsuan-Tien Lin, editors, Advances in Neural Information Processing Systems 33: Annual Conference on Neural Information Processing Systems 2020, NeurIPS 2020, December 6-12, 2020, virtual, 2020.

[PPP04] Javier Perote and Juan Perote-Peña. Strategy-proof estimators for simple regression. Mathematical Social Sciences, 47(2):153–176, 2004.

[PS03] Javier Perote and Olavide Sevilla. The impossibility of strategy-proof clustering. Economics Bulletin, 2003.

[RRS11] Francesco Ricci, Lior Rokach, and Bracha Shapira. Introduction to recommender systems handbook. In Francesco Ricci, Lior Rokach, Bracha Shapira, and Paul B. Kantor, editors, Recommender Systems Handbook, pages 1–35. Springer, 2011.

[Sat75] Mark Allen Satterthwaite. Strategy-proofness and arrow’s conditions: Existence and correspondence theorems for voting procedures and social welfare functions. Journal of economic theory, 10(2):187–217, 1975.

[SHL18] Heung-Yeung Shum, Xiaodong He, and Di Li. From eliza to xiaoice: challenges and opportunities with social chatbots. Frontiers Inf. Technol. Electron. Eng., 19(1):10–26, 2018.
[SKAZ22] Xudong Shi, Qi Kang, Jing An, and MengChu Zhou. Novel L1 regularized extreme learning machine for soft-sensing of an industrial process. *IEEE Trans. Ind. Informatics*, 18(2):1009–1017, 2022.

[Sol18] Joan E. Solsman. Youtube’s ai is the puppet master over most of what you watch. *CNET*, 2018.

[SSP+13] Jason R. Smith, Herve Saint-Amand, Magdalena Plamada, Philipp Koehn, Chris Callison-Burch, and Adam Lopez. Dirt cheap web-scale parallel text from the common crawl. In *Proceedings of the 51st Annual Meeting of the Association for Computational Linguistics, ACL 2013, 4-9 August 2013, Sofia, Bulgaria, Volume 1: Long Papers*, pages 1374–1383. The Association for Computer Linguistics, 2013.

[Tib96] Robert Tibshirani. Regression shrinkage and selection via the lasso. *Journal of the Royal Statistical Society: Series B (Methodological)*, 58(1):267–288, 1996.

[Val84] Leslie G. Valiant. A theory of the learnable. *Commun. ACM*, 27(11):1134–1142, 1984.

[Wan13] Lie Wang. The l1 penalized lad estimator for high dimensional linear regression. *Journal of Multivariate Analysis*, 120:135–151, 2013.

[WNWW20] Yuanyuan Wu, Eric W. T. Ngai, Pengkun Wu, and Chong Wu. Fake online reviews: Literature review, synthesis, and directions for future research. *Decis. Support Syst.*, 132:113280, 2020.

[WPN+19] Alex Wang, Yada Pruksachatkun, Nikita Nangia, Amanpreet Singh, Julian Michael, Felix Hill, Omer Levy, and Samuel R. Bowman. Superglue: A stickier benchmark for general-purpose language understanding systems. In Hanna M. Wallach, Hugo Larochelle, Alina Beygelzimer, Florence d’Alché-Buc, Emily B. Fox, and Roman Garnett, editors, *Advances in Neural Information Processing Systems 32: Annual Conference on Neural Information Processing Systems 2019, NeurIPS 2019, December 8-14, 2019, Vancouver, BC, Canada*, pages 3261–3275, 2019.

[WSM+19] Alex Wang, Amanpreet Singh, Julian Michael, Felix Hill, Omer Levy, and Samuel R. Bowman. GLUE: A multi-task benchmark and analysis platform for natural language understanding. In *7th International Conference on Learning Representations, ICLR 2019, New Orleans, LA, USA, May 6-9, 2019*. OpenReview.net, 2019.

[XCM08] Huan Xu, Constantine Caramanis, and Shie Mannor. Robust regression and lasso. In Daphne Koller, Dale Schuurmans, Yoshua Bengio, and Léon Bottou, editors, *Advances in Neural Information Processing Systems 21, Proceedings of the Twenty-Second Annual Conference on Neural Information Processing Systems, Vancouver, British Columbia, Canada, December 8-11, 2008*, pages 1801–1808. Curran Associates, Inc., 2008.

[ZTK21] Thomas Zerback, Florian Töpfl, and Maria Knöpfle. The disconcerting potential of online disinformation: Persuasive effects of astroturfing comments and three strategies for inoculation against them. *New Media Soc.*, 23(5), 2021.
Appendix

A  Existence and uniqueness of the optimum

Proposition. For any family of datasets $\mathcal{D}$, LCH yields a unique minimum.

Proof. Let us first prove the existence of the minimum. Define $L_0 \triangleq \text{LCH}(\emptyset,0|\mathcal{D})$, the value of the the LICCHAVI loss at 0. Since the local loss functions are strongly convex, there exists a constant $c$ such that if for any $n \in [N]$ we have $\|\theta_n\|_2 \geq c$, then $\mathcal{L}(\theta_n|\mathcal{D}_n) \geq L_0$. This implies that at the infimum, we must have $\|\theta_n\|_2 \leq c$, for all $n \in [N]$. On the other hand, if $\|\rho\|_2 \to \infty$ then $\|\theta_n\|_2 \leq c$ implies that $\text{HB} \frac{4\rho}{\sqrt{1+|D|}} (\theta_n - \rho)$ goes to infinity and in particular becomes larger than $L_0$ for $\|\rho\|_2$ large enough. Therefore, the infimum of LCH must be reached in a bounded and close region around the origin which is a compact set. The infimum is thus a minimum, which proves the existence of a minimum.

We now move on to proving the uniqueness. Consider two minima $(\rho^{(1)}, \theta^{(1)})$ and $(\rho^{(2)}, \theta^{(2)})$. By the strict convexity of $\mathcal{L}$ we have

$$\forall n \in [N], \mathcal{L} \left( \frac{\theta^{(1)}_n + \theta^{(2)}_n}{2} \right) \leq \frac{1}{2} \left( \mathcal{L}(\theta^{(1)}_n|\mathcal{D}_n) + \mathcal{L}(\theta^{(2)}_n|\mathcal{D}_n) \right), \quad (16)$$

with strict inequality if $\theta^{(1)}_n \neq \theta^{(2)}_n$. Similarly, by the strict convexity of HB, for all $n \in [N]$, we obtain

$$\text{HB} \frac{4\rho}{\sqrt{1+|D|}} \left( \frac{\theta^{(1)}_n - \rho^{(1)}}{2} + (\theta^{(2)}_n - \rho^{(2)}) \right) \leq \frac{1}{2} \left( \text{HB} \frac{4\rho}{\sqrt{1+|D|}} (\theta^{(1)}_n - \rho^{(1)}) + \text{HB} \frac{4\rho}{\sqrt{1+|D|}} (\theta^{(2)}_n - \rho^{(2)}) \right), \quad (17)$$

with a strict inequality $\theta^{(1)}_n - \rho^{(1)} \neq \theta^{(2)}_n - \rho^{(2)}$. Now combining all of the above inequalities yields

$$\text{LCH} \left( \frac{\rho^{(1)} + \rho^{(2)}}{2}, \frac{\theta^{(1)} + \theta^{(2)}}{2} \right) \leq \frac{1}{2} \left( \text{LCH} \left( \rho^{(1)}, \theta^{(1)} \right) \mathcal{D} + \text{LCH} \left( \rho^{(2)}, \theta^{(2)} \mathcal{D} \right) \mathcal{D} \right), \quad (18)$$

and the above inequality becomes strict if at least one of the inequalities in (16) or (17) are strict. But since, by optimality of the solutions, the right-hand side takes the minimum value of LCH, we must have equality. This implies that $\theta^{(1)}_n = \theta^{(2)}_n$ and $\theta^{(1)}_n - \rho^{(1)} = \theta^{(2)}_n - \rho^{(2)}$ for all users $n \in [N]$. Considering any user, say $n = 1$, in the second equality then implies $\rho^{(1)} = \rho^{(2)}$. All in all, we thus have uniqueness. \hfill \qed

B  Reductions to model attacks

B.1 Reduced losses

Lemma 1. For any data $\mathcal{D}$ and $\rho$, the infimum problem defining $\mathcal{R}$ yields a unique minimum $\theta^*(\rho, \mathcal{D})$.

Proof. Given that the local loss $\mathcal{L}$ is strongly convex and that the pseudo-Huber loss is convex, we know that their sum is strongly convex, which guarantees the existence and uniqueness of $\theta^*(\rho, \mathcal{D})$. \hfill \qed

Lemma 2. For any data $\mathcal{D}$, the reduced loss $\mathcal{R}(\rho|\mathcal{D})$ is convex and differentiable. Moreover, $\nabla \mathcal{R} = w \nabla \text{HB} \frac{4\rho}{\sqrt{1+|D|}} (\rho - \theta^*(\rho, \mathcal{D}))$, and thus $\|\nabla \mathcal{R}\|_\infty \leq w$.

Proof. The convexity and differentiability of the reduced loss follows straightforwardly from Lemma 9 of [FGHV22]. By the same lemma, we have $\nabla \mathcal{R} = w \nabla \text{HB} \frac{4\rho}{\sqrt{1+|D|}} (\rho - \theta^*(\rho, \mathcal{D}))$. In particular, $\partial_{\rho_i} \mathcal{R} = w \frac{\rho_i - \theta^*_i}{\sqrt{1+|D|}(\rho_i - \theta^*_i)^2}$, whose absolute value is at most $w$. \hfill \qed
Lemma 6. \( L^{\text{LCH}}(\mathcal{D}) \) is the unique minimum of \( R(\rho|\mathcal{D}) \), while \( \theta_n^{LCH}(\mathcal{D}) = \theta^*(L^{\text{LCH}}(\mathcal{D}), \mathcal{D}_n) \).

Proof. Clearly, we have

\[
\inf_{\rho, \theta} LCH(\rho, \theta|\mathcal{D}) = \inf_{\rho} \left\{ \inf_{\theta} LCH(\rho, \theta|\mathcal{D}) \right\} = \inf_{\rho} R(\rho|\mathcal{D}). \tag{19}
\]

This shows that \( \rho \) minimizes \( LCH \) (with some value of \( \theta^* \)) if and only if it minimizes the reduced loss \( R \). Since the former has a unique minimum, so does the latter, which is \( L^{\text{LCH}}(\mathcal{D}) \). Moreover, similar computations clearly show that

\[
LCH \left( L^{\text{LCH}}(\mathcal{D}), \theta_{n}^{LCH}(\mathcal{D}) \middle| \mathcal{D} \right) = LCH \left( L^{\text{LCH}}(\mathcal{D}), \theta^*(L^{\text{LCH}}(\mathcal{D}), \mathcal{D}_n) \middle| \mathcal{D} \right). \tag{20}
\]

By the uniqueness of the minimum, we then conclude that \( \theta_{n}^{LCH}(\mathcal{D}) = \theta^*(L^{\text{LCH}}(\mathcal{D}), \mathcal{D}_n) \). Or, put differently, for each user \( n \), we have \( \theta_{n}^{LCH}(\mathcal{D}) = \theta^*(L^{\text{LCH}}(\mathcal{D}), \mathcal{D}_n) \). \( \square \)

B.2 Strong local \( \text{PAC*} \)

In this section, to prove Lemma 6, we prove an even stronger result, which asserts that, assuming user \( n \) provides enough data, then, given any global model, \( \theta_{n}^* \) is successfully probably approximately correct. This result will be useful in the proof of Lemma 6.

Lemma 6. Assume gradient \( \text{PAC*} \) local losses. Then, for any model \( \theta_{n}^* \) and any \( \varepsilon, \delta > 0 \), there exists \( \mathcal{I} \) such that, if user \( n \) provides a dataset \( \mathcal{D}_n^\dagger \) with at least \( \mathcal{I} \) answers to random queries with model \( \theta_{n}^\dagger \), then, with probability at least \( 1 - \delta \), we have

\[
\forall \rho \in \mathbb{R}^d, \quad \| \theta_n^* (\rho, \mathcal{D}_n^\dagger) - \theta_n^\dagger \|_2 \leq \varepsilon. \tag{21}
\]

Proof. Consider a user \( n \in [N] \) and their preferred model \( \theta_{n}^\dagger \). Fix \( \varepsilon, \delta > 0 \). Define \( K \triangleq \| \theta_{n}^\dagger \|_2 \). Denote \( \mathcal{I} \) the number of data points provided by user \( n \). By the optimality of \( \theta_{n}^* \triangleq \theta_{n}^* (\rho, \mathcal{D}_n^\dagger) \), we have

\[
0 \in (\theta_{n}^* - \theta_{n}^\dagger)^T \nabla L(\theta_{n}^* | \mathcal{D}_n^\dagger) + (\theta_{n}^* - \theta_{n}^\dagger)^T \nabla \theta_{n} \left( \frac{w \text{HB} |D_n| \varepsilon_{\text{d}}} {\sqrt{1 + \| \rho \|}} (\theta_{n}^* - \rho^*) \right) \tag{22}
\]

\[
\geq (\theta_{n}^* - \theta_{n}^\dagger)^T \nabla L(\theta_{n}^* | \mathcal{D}_n^\dagger) - \| \theta_{n}^* - \theta_{n}^\dagger \|_2 \| \nabla \theta_{n} \left( \frac{w \text{HB} |D_n| \varepsilon_{\text{d}}} {\sqrt{1 + \| \rho \|}} (\theta_{n}^* - \rho^*) \right) \|_2 \tag{23}
\]

\[
\geq (\theta_{n}^* - \theta_{n}^\dagger)^T \nabla L(\theta_{n}^* | \mathcal{D}_n^\dagger) - w \sqrt{d} \| \theta_{n}^* - \theta_{n}^\dagger \|_2, \tag{24}
\]

where, in the last line, we used the fact the infinite norm of the gradient \( \text{HB} \) is bounded by \( 1 \) and \( \| \theta \|_2 \leq \sqrt{d} \| \theta \|_\infty \).

Now, gradient \( \text{PAC*} \) implies the existence of an event \( \mathcal{E} \) that occurs with probability at least \( P(K, \mathcal{I}) \), under which we have

\[
0 \geq A_K \mathcal{I} \min \left\{ \| \theta_{n}^* - \theta_{n}^\dagger \|_2, \| \theta_{n}^* - \theta_{n}^\dagger \|_2^2 \right\} - B_K \mathcal{I} \alpha \| \theta_{n}^* - \theta_{n}^\dagger \|_2 - w \sqrt{d} \| \theta_{n}^* - \theta_{n}^\dagger \|_2. \tag{25}
\]

Note that the event \( \mathcal{E} \) is independent from \( \rho. \) If \( \| \theta_{n}^* - \theta_{n}^\dagger \|_2 \geq 1 \), this implies

\[
0 \geq (A_K \mathcal{I} - B_K \mathcal{I} \alpha - w \sqrt{d}) \| \theta_{n}^* - \theta_{n}^\dagger \|_2, \tag{26}
\]

which cannot hold for \( \mathcal{I} > \mathcal{I}_1 \triangleq \max \left\{ 2w \sqrt{d}/A_K, (2B_K/A_K)^{\frac{1}{\alpha}} \right\} \). Thus, for \( \mathcal{I} > \mathcal{I}_1 \), we have

17
\[ 0 \geq A_K \|\theta_n^* - \theta_n^1\|_2^2 - (B_K T^\alpha + w \sqrt{d}) \|\theta_n^* - \theta_n^1\|_2. \]  

(27)

As a result,
\[ \|\theta_n^* - \theta_n^1\|_2 \leq \frac{B_K T^\alpha + w \sqrt{d}}{A_K I}. \]  

(28)

Considering \( I \) large enough such that \( I > I_1 \) and \( P(K, I) \geq 1 - \delta \) and \( \frac{B_K T^\alpha + w \sqrt{d}}{A_K I} \leq \varepsilon \), we obtain the result. \( \square \)

Lemma 4 then follows straightforwardly.

**Lemma 4.** For gradient PAC* local losses, LICCHAVI is strongly local PAC*.

**Proof.** This follows from Lemma 6 and the fact that \( \theta_n^{LCH}(\bar{D}) = \theta_n^*(\rho^{LCH}(\bar{D}), D_n) \) (Lemma 3). \( \square \)

### B.3 Equivalence between data attack and model attack

Our equivalence proof will leverage the following lemma, largely drawn from [FGHV22].

**Lemma 7 (Lemma 1 from [FGHV22]).** Consider any data \( \bar{D} \) and any user \( s \in [N] \). Then having user \( s \) reporting \( D_s \) is equivalent to having them reporting the model \( \theta_s^{LCH}(\bar{D}) \), i.e.
\[ \rho^{LCH}(D_s, \bar{D}_s) = \rho^{LCH}(\theta_s^{LCH}(\bar{D}), \bar{D}_s) \quad \text{and} \quad \forall n, \theta_n^{LCH}(D_s, \bar{D}_s) = \theta_n^{LCH}(\theta_s^{LCH}(\bar{D}), \bar{D}_s). \]  

(29)

**Sketch of proof.** This is derived from the fact that the loss as a function of \( \rho \) and \( \bar{D}_s \) is unchanged. \( \square \)

We will also need the following lemma, adapted from Lemma 2 of [FGHV22] (or, rather, by its generalization, which is Lemma 14 in [FGHV22]). However, a bit more work is needed to adapt their proof, as, here, we need to transform a pseudo-Huber loss into an \( \ell_1 \) loss. We bound this transformation by the following uniform bound.

**Lemma 8.** For any \( \delta > 0 \) and \( t \in \mathbb{R} \), we have \( 0 \leq \sqrt{\delta^2 + t^2} - |t| \leq \delta. \)

**Proof.** Clearly, \( \delta^2 + t^2 \geq t^2 \), which implies \( \sqrt{\delta^2 + t^2} \geq |t| \), and thus \( \sqrt{\delta^2 + t^2} - |t| \geq 0 \). Moreover, we have \( (\sqrt{\delta^2 + t^2} - \sqrt{t^2})^2 = \delta^2 + t^2 - 2\sqrt{\delta^2 t^2} + t^2 \leq \delta^2 + 2t^2 - 2\sqrt{t^4} = \delta^2 \), using the inequality \( \sqrt{\delta^2 + t^2} \geq \sqrt{t^2} \). Taking the square root yields the lemma. \( \square \)

**Lemma 9.** We have \( 0 \leq \text{Hub}_\delta(x) - \|x\|_1 \leq \delta d. \)

**Proof.** By the previous lemma, on each coordinate \( i \), we have \( 0 \leq \text{Hub}_\delta(x_i) - |x_i| \leq \delta d. \) Adding up all the coordinates yields the lemma. \( \square \)

**Lemma 10.** Assume strong local PAC* learning. Consider a user \( s \in [N] \), any model \( \theta_s^1 \) and fix other users’ datasets \( \bar{D}_s \). For any \( \varepsilon, \delta > 0 \), there exists \( I \) such that, if user \( s \) provides a dataset \( D_s^1 \) by answering at least \( I \) random queries with model \( \theta_s^1 \), then with probability at least \( 1 - \delta \), we have
\[ \left\| \rho^{LCH}(\theta_s^1, \bar{D}_s) - \rho^{LCH}(D_s^1, \bar{D}_s) \right\|_2 \leq \varepsilon \quad \text{and} \quad \forall n, \left\| \theta_n^{LCH}(\theta_s^1, \bar{D}_s) - \theta_n^{LCH}(D_s^1, \bar{D}_s) \right\|_2 \leq \varepsilon. \]  

(30)

**Proof.** Define the compact set \( C \) of models that are \( \varepsilon \)-close to \( \rho^{LCH} \triangleq \rho^{LCH}(\theta_s^1, \bar{D}_s) \) and \( \theta_s^{LCH} \triangleq \theta_s^{LCH}(\theta_s^1, \bar{D}_s) \), i.e.
\[ C \triangleq \left\{ (\rho, \theta) \left\| \rho - \rho^{LCH}(\theta_s^1, \bar{D}_s) \right\|_2 \leq \varepsilon \quad \text{and} \quad \forall n \neq s, \left\| \theta_n - \theta_n^{LCH}(\theta_s^1, \bar{D}_s) \right\|_2 \leq \varepsilon \right\}. \]  

(31)

Denote \( D \triangleq \mathbb{R}^{d \times (1 + N)} \setminus C \) the closure of the complement of \( C \). By the same arguments as Proposition 1, we know that \( \text{LCH}(\rho, \bar{D}_s)_{\theta_s^1, \bar{D}_s} \) yields a minimum over \( D \). But by the uniqueness of the minimum, we know that
\[ \eta \triangleq \inf_{(\rho, \bar{D}_s) \in D} \text{LCH}(\rho, \bar{D}_s)_{\theta_s^1, \bar{D}_s} - \text{LCH}(\rho^{LCH}, \bar{D}_s)_{\theta_s^1, \bar{D}_s} > 0. \]  

(32)
Thus, for any \((\rho, \vec{\theta}) \in D\), we have \(\text{LCH}(\rho, \vec{\theta}; \bar{D}_s, \bar{D}_\neg s) \geq \eta + \text{LCH}(\rho^{LCH}, \vec{\theta}^{LCH}; \bar{D}_s, \bar{D}_\neg s)\). We now invoke strong local PAC* learning. More precisely, consider the event

\[
\mathcal{E} \triangleq \{ \forall \rho \in \mathbb{R}^d, \| \theta^*_s(\rho, D^\dagger_s) - \theta^s \|_2 \leq \min \{ \epsilon, \eta/6w \} \} .
\]

By Lemma 6, we know that there exists \(I_1\) such that, if user \(s\) provides a dataset \(D^\dagger_s\) at least \(I_1\) answers to random queries, then the event \(\mathcal{E}\) occurs with probability at least \(1 - \delta\). Now consider \(I_2 \triangleq \max \{ I_1, 9w^2d^2 \delta^2/\eta^2 \}\). We now assume that the dataset \(D^\dagger_s\) contains at least \(I_2\) answers to random queries. Then \(\mathcal{E}\) still occurs with probability at least \(1 - \delta\). By optimality of \(\theta^s_{LCH}(\rho, D^\dagger_s)\), under \(\mathcal{E}\), we then have

\[
\forall \theta, \rho \in \mathbb{R}^d, \mathcal{L}(\theta^s(\rho, D^\dagger_s)|D^\dagger_s) + wHB \frac{s_\epsilon}{\sqrt{1 + |D^\dagger_s|}} (\rho - \theta^s(\rho, D^\dagger_s)) \leq \mathcal{L}(\theta_s|D^\dagger_s) + wHB \frac{s_\epsilon}{\sqrt{1 + |D^\dagger_s|}} (\rho - \theta_s) \tag{34}
\]

Given Lemma 3, applying this inequality to \(\rho \triangleq \rho^{LCH}\) and \(\theta_s \triangleq \theta^s(\rho, D^\dagger_s)\) then yields

\[
\mathcal{L}(\theta^s(\rho, D^\dagger_s)|D^\dagger_s) \geq \mathcal{L}(\theta^s_{LCH}|D^\dagger_s) + wHB \frac{s_\epsilon}{\sqrt{1 + |D^\dagger_s|}} (\rho^{LCH} - \theta^s_{LCH}) - wHB \frac{s_\epsilon}{\sqrt{1 + |D^\dagger_s|}} (\rho^{LCH} - \theta^s(\rho, D^\dagger_s)) \tag{35}
\]

Then, for any models \((\rho, \vec{\theta}_s) \in D\) and \(\theta_s\), under \(\mathcal{E}\), we then have

\[
\text{LCH}(\rho, \vec{\theta}; D^\dagger_s, \bar{D}_\neg s) \geq \text{LCH}(\rho, \vec{\theta}_s; \theta^s(\rho, D^\dagger_s)|D^\dagger_s, \bar{D}_\neg s) \tag{36}
\]

\[
= \left( \sum_{n \neq s} \mathcal{L}(\theta_n|D_n) + wHB \frac{s_\epsilon}{\sqrt{1 + |D_n|}} (\rho - \theta_n) \right) + \mathcal{L}(\theta^s(\rho, D^\dagger_s)|D^\dagger_s) + wHB \frac{s_\epsilon}{\sqrt{1 + |D^\dagger_s|}} (\rho - \theta^s(\rho, D^\dagger_s)) \tag{37}
\]

\[
\geq \left( \text{LCH}(\rho, \vec{\theta}_s; \theta^s, \bar{D}_\neg s) - w \| \rho - \theta^s \|_1 \right) + \left( \mathcal{L}(\theta^s_{LCH}|D^\dagger_s) + wHB \frac{s_\epsilon}{\sqrt{1 + |D^\dagger_s|}} (\rho^{LCH} - \theta^s_{LCH}) - wHB \frac{s_\epsilon}{\sqrt{1 + |D^\dagger_s|}} (\rho^{LCH} - \theta^s(\rho, D^\dagger_s)) \right) \tag{38}
\]

\[
\geq \eta + \text{LCH}(\rho^{LCH}, \vec{\theta}^{LCH}; \theta^s, \bar{D}_\neg s) + \mathcal{L}(\theta^s_{LCH}|D^\dagger_s) + wHB \frac{s_\epsilon}{\sqrt{1 + |D^\dagger_s|}} (\rho^{LCH} - \theta^s_{LCH}) - w \| \rho - \theta^s \|_1 \tag{39}
\]

\[
\geq \eta + \text{LCH}(\rho^{LCH}, \vec{\theta}^{LCH}; D^\dagger_s, \bar{D}_\neg s) + w \| \rho^{LCH} - \theta^s \|_1 - w \| \rho - \theta^s \|_1 \tag{40}
\]

\[
\geq \eta + \frac{w \delta_d}{\sqrt{1 + |D^\dagger_s|}} \sqrt{1 + |D^\dagger_s|} \tag{41}
\]

\[
\geq \text{LCH}(\rho^{LCH}, \vec{\theta}^{LCH}; D^\dagger_s, \bar{D}_\neg s) + w \left( \frac{\delta_d}{\sqrt{1 + |D^\dagger_s|}} - 2w \| \theta^s - \theta^s(\rho, D^\dagger_s) \|_1 \right) \tag{42}
\]

\[
\geq \text{LCH}(\rho^{LCH}, \vec{\theta}^{LCH}; D^\dagger_s, \bar{D}_\neg s) + \eta - \frac{2\eta}{3} = \text{LCH}(\rho^{LCH}, \vec{\theta}^{LCH}; D^\dagger_s, \bar{D}_\neg s) + \frac{\eta}{3} \tag{43}
\]

\[
> \text{LCH}(\rho^{LCH}, \vec{\theta}^{LCH}; D^\dagger_s, \bar{D}_\neg s). \tag{44}
\]
This proves that any \((\rho, \bar{D}_s) \in D\) cannot be the unique minimum of LICCHAVI given datasets \((D_s^1, \bar{D}_s)\). Thus \((\rho^{\text{LCH}}(D_s^1, \bar{D}_s), \bar{\theta}^{\text{LCH}}(D_s^1, \bar{D}_s)) \in C\). Adding to this the guarantee of event \(E\) yields the lemma.

**Lemma 5** Assuming strong local PAC*, LICCHAVI is global-targeted \(\alpha\)-strategyproof under data attack if and only if it is global-targeted \(\alpha\)-strategyproof under model attack. The equivalence also holds for user-targeted \(\alpha\)-strategyproofness.

**Proof.** Let us first assume that LICCHAVI is global-targeted \(\alpha\)-strategyproof under model attack. We then fix \(\varepsilon, \delta > 0\), and we consider the event \(E\) defined by

\[
E \triangleq \left\{ \forall D_s, \left\| \rho^{\text{LCH}}(D_s^1, \bar{D}_s) - \rho^{\text{LCH}}(\theta_s^1, \bar{D}_s) \right\|_2 \leq \frac{\varepsilon}{1 + \alpha} \right\}. \tag{45}
\]

Note that \(E\) is random because it depends on the random honest dataset \(D_s^1\), whose random queries are answered with model \(\theta_s^1\). Given strong local PAC*, we know that there is \(I\) large enough such that \(\mathbb{P}[E] \geq 1 - \delta\). Assume \(E\). Now fix other users’ datasets \(\bar{D}_s\), and consider any strategic dataset \(D_s^\dagger\) that \(s\) could inject. By Lemma 3 we know that there exists \(\theta_s^\dagger \triangleq \theta_s^{\text{LCH}}(D_s^\dagger, \bar{D}_s)\) such that \(\rho^{\text{LCH}}(\theta_s^\dagger, \bar{D}_s) = \rho^{\text{LCH}}(D_s^\dagger, \bar{D}_s)\). Then

\[
\left\| \rho^{\text{LCH}}(D_s^\dagger, \bar{D}_s) - \theta_s^\dagger \right\|_2 \leq \left\| \rho^{\text{LCH}}(\theta_s^\dagger, \bar{D}_s) - \theta_s^1 \right\|_2 \tag{46}
\]

\[
\leq (1 + \alpha) \left\| \rho^{\text{LCH}}(\theta_s^\dagger, \bar{D}_s) - \theta_s^1 \right\|_2 \tag{47}
\]

\[
\leq (1 + \alpha) \left( \left\| \rho^{\text{LCH}}(\theta_s^\dagger, \bar{D}_s) - \rho^{\text{LCH}}(D_s^1, \bar{D}_s) \right\|_2 + \left\| \rho^{\text{LCH}}(D_s^1, \bar{D}_s) - \theta_s^1 \right\|_2 \right) \tag{48}
\]

\[
\leq \varepsilon + (1 + \alpha) \left\| \rho^{\text{LCH}}(D_s^1, \bar{D}_s) - \theta_s^1 \right\|_2, \tag{49}
\]

which proves \(\alpha\)-strategyproofness under data attack.

Reciprocally, assume that LICCHAVI is global-targeted \(\alpha\)-strategyproof under data attack. Fix any target model \(\theta_s^1\), attack model \(\theta_s^\dagger\), and any \(\varepsilon > 0\). We then define the following events, which depend on the datasets \(D_s^1\) and \(D_s^\dagger\), whose random queries are answered respectively with models \(\theta_s^1\) and \(\theta_s^\dagger\):

\[
E_1 \triangleq \left\{ \forall D_s, \bar{D}_s, \left\| \rho^{\text{LCH}}(D_s^1, \bar{D}_s) - \theta_s^1 \right\|_2 \leq (1 + \alpha) \left\| \rho^{\text{LCH}}(D_s, \bar{D}_s) - \theta_s^1 \right\|_2 + \varepsilon \right\}, \tag{50}
\]

\[
E_2 \triangleq \left\{ \forall \bar{D}_s, \left\| \rho^{\text{LCH}}(D_s^1, \bar{D}_s) - \rho^{\text{LCH}}(\theta_s^1, \bar{D}_s) \right\|_2 \leq \varepsilon \right\}, \tag{51}
\]

\[
E_3 \triangleq \left\{ \forall \bar{D}_s, \left\| \rho^{\text{LCH}}(D_s^\dagger, \bar{D}_s) - \rho^{\text{LCH}}(\theta_s^\dagger, \bar{D}_s) \right\|_2 \leq \varepsilon \right\}. \tag{52}
\]

By \(\alpha\)-strategyproofness under data attack, we know that, when the datasets answer sufficiently many queries, \(E_1\) occurs with probability at least 3/4. By Lemma 3 we also know that, when the datasets answer sufficiently many queries, each of events \(E_2\) and \(E_3\) also occurs with probability at least 3/4. As a result, we know that, when the datasets answer sufficiently many queries, the intersection \(E_1 \cap E_2 \cap E_3\) occurs with probability at least 1/4. Under \(E_1 \cap E_2 \cap E_3\), we then have

\[
\left\| \rho^{\text{LCH}}(\theta_s^1, \bar{D}_s) - \theta_s^1 \right\|_2 \leq \left\| \rho^{\text{LCH}}(\theta_s^1, \bar{D}_s) - \rho^{\text{LCH}}(D_s^1, \bar{D}_s) \right\|_2 + \left\| \rho^{\text{LCH}}(D_s^1, \bar{D}_s) - \theta_s^1 \right\|_2 \tag{53}
\]

\[
\leq \varepsilon + (1 + \alpha) \left\| \rho^{\text{LCH}}(D_s^\dagger, \bar{D}_s) - \theta_s^1 \right\|_2 \tag{54}
\]

\[
\leq \varepsilon + (1 + \alpha) \left( \left\| \rho^{\text{LCH}}(D_s^\dagger, \bar{D}_s) - \rho^{\text{LCH}}(\theta_s^\dagger, \bar{D}_s) \right\|_2 + \left\| \rho^{\text{LCH}}(\theta_s^\dagger, \bar{D}_s) - \theta_s^1 \right\|_2 \right) \tag{55}
\]

\[
\leq \varepsilon + (1 + \alpha) \varepsilon + (1 + \alpha) \left\| \rho^{\text{LCH}}(\theta_s^\dagger, \bar{D}_s) - \theta_s^1 \right\|_2. \tag{56}
\]

But this event is deterministic. Since it occurs with a positive probability, it must thus hold with probability 1. We conclude by noting that it holds for any \(\varepsilon > 0\). Taking the limit \(\varepsilon \to 0\) proves global-targeted \(\alpha\)-strategyproofness under model attack.

The proof for user-targeted \(\alpha\)-strategyproofness is essentially the same. \(\square\)
C Proof of non-strategyproofness

To prove Theorem 1, we propose a counter example, which will be parametrized by $A > 1$ (and we will consider the limit $A \to \infty$).

C.1 The counter example

Namely, consider $d = 2$, $N = 3$ and $w = 1$. Now assume that users 1 and 2 are honest, and provide the same dataset $D = D_1 = D_2$ of at least $A^4/\delta^2$ inputs, and for which

$$L(\theta|D) = \frac{A^2}{2} (A\theta_1 - \theta_2)^2 + \frac{1}{2} \theta_2 + \frac{1}{2A^2} (\theta_1^2 + \theta_2^2). \quad (57)$$

It is clear that this loss is strongly convex and differentiable, and thus satisfies the assumptions of the paper. Moreover, intuitively, it locks $\theta^*(\rho)$ essentially along the line $A\theta_1 = \theta_2$, while favoring lower values of $\theta_2$ along this line, at least while $\theta_2 \geq -A^2$.

Moreover, since the loss looks the same from user 1 and user 2’s perspectives, and by uniqueness of the minimum, we know that, for any model attack $\theta^{\bullet}$ by strategic user $s \neq 3$, we will have $\theta^{LCH}_{\bullet}(\theta^{\bullet}, \bar{D}_s) = \theta^{LCH}_s(\theta^{\bullet}, \bar{D}_s)$. Thus, without loss of generality, we assume that both users are always assigned the same model $\theta$. In particular, denoting $\nu \triangleq \delta c/\sqrt{1 + |D|} \leq 1/A^2$, and assuming strategic user $s$ reports model $\theta^{\bullet}$ (with $\bullet \in \{\dag, \blacklozenge\}$), we can consider the following modified LICCHAVI loss (we leave the dependence on $D$ implicit):

$$LCH(\rho, \theta|\theta^{\bullet}) \triangleq 2L(\theta|D) + 2H_B(\rho - \theta) + \|\rho - \theta^{\bullet}\|_1. \quad (58)$$

Indeed, it is immediate to verify that the minimum $\rho^{LCH}_{\bullet}, \theta^{LCH}$ of this loss will coincide with the LICCHAVI computation, i.e., $\rho^{LCH} = \rho^{LCH}(\theta^{\bullet}, \bar{D}_s)$ and $\theta^{LCH} = \theta^{LCH}_s(\theta^{\bullet}, \bar{D}_s)$ for $n \in \{1, 2\}$.

We consider the target model $\theta^\dag \triangleq (0, 1)$, and the attack model $\theta^\blacklozenge \triangleq (1/A, 1)$. We will show that the strategic user can get both $\rho^{LCH}$ and $\theta^{LCH}$ much closer to $\theta^\dag$, by reporting $\theta^{\bullet}$ rather than $\theta^\dag$. More precisely, we will prove that $\|\rho^{LCH}(\theta^\dag) - \theta^\dag\|_2 = O(1)$ as $A \to \infty$, while $\|\rho^{LCH}(\theta^{\blacklozenge}) - \theta^\blacklozenge\|_2 = O(1/A)$. This will prove Theorem 1.

C.2 Bounding the optimal global model

In this section, we prove that $\rho^{LCH} \approx \theta^{LCH}$. In fact, we will prove that for any fixed value of $\theta$, if we optimize $\rho$, then the distance between $\theta$ and the optimized value $\rho^*$ will be at most $1/A^2$. Intuitively, this should not be surprising; indeed since the honest users 1 and 2 form a majority, they should be deciding where $\rho^{LCH}$ is. To prove this, denote $u \triangleq \nabla \rho H_B(\rho - \theta) \in (-1, 1)^2$. The partial derivatives with respect to the global model, given strategic user $s$’s reported model $\theta^{\bullet}$, are then given by

$$\frac{\partial \rho^LCH}{\partial \rho_1} = 2u_1 + \text{sgn}(\rho_1 - \theta^{\bullet}_1), \quad (59)$$
$$\frac{\partial \rho^LCH}{\partial \rho_2} = 2u_2 + \text{sgn}(\rho_2 - \theta^{\bullet}_2). \quad (60)$$

**Lemma 11.** For $i \in [d] = \{1, 2\}$, either $\rho^{LCH}_i = \theta^{\bullet}_i$ or $\rho^{LCH}_i - \theta^{LCH} = \text{sgn}(\rho_i - \theta^{\bullet}_i)\nu$.

**Proof.** If $\rho^{LCH}_i \neq \theta^{\bullet}_i$, then $\text{sgn}(\rho_i - \theta^{\bullet}_i) \in \{-1, 1\}$. By the optimality condition, we know that equations $(59)$ and $(60)$ must equal zero. This implies that $u^{LCH} = -\frac{1}{2} \text{sgn}(\rho_i - \theta^{\bullet}_i)$. Solving this yields the lemma. □

Denote $\rho^*(\theta, \theta^{\bullet})$ the optimal value of $\rho$ when $\theta$ is fixed, and given the strategic user’s reported model $\theta^{\bullet}$.

**Lemma 12.** $\|\rho^*(\theta, \theta^{\bullet}) - \theta\|_\infty \leq 1/A^2$. 

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Proof. By the optimality condition on $\rho^*(\theta)$, we know that for each coordinate $i$, we must have $0 \in \partial_{\rho_i} \text{LCH} = 2u_i + \text{sgn}(\rho_i - \theta_i)$. Since $\text{sgn}(\rho_i - \theta_i) \in [-1, 1]$, there must thus exist $\kappa_i \in [-1, 1]$ such that $2u_i + \kappa_i = 0$, which implies that $u_i = -\frac{1}{2}\kappa_i \in [-1/2, 1/2]$. Thus in particular $|u_i| = \frac{|\rho_i - \theta_i|}{\sqrt{\rho_i^2 + |\rho_i - \theta_i|^2}} \leq 1/2$. This implies that, at the optimum, $\left(1 + \frac{\nu^2}{|\rho_i^*(\theta, \theta) - \theta_i|}\right) \leq 1/2$, which can only occur if $|\rho_i^*(\theta, \theta) - \theta_i| \leq \nu \leq 1/A^2$. This is the lemma.

Lemma 13. $\text{HB}_\nu(\rho^*(\theta) - \theta) \leq 2\sqrt{2}/A^2$.

Proof. This follows straightforwardly from the previous lemma.

C.3 Model reduced loss

The previous lemmas prompt us to consider the following model-reduced loss

$$S(\theta | \theta^\bullet) \triangleq \inf_{\rho} \text{LCH}(\rho, \theta|\theta^\bullet) = \text{LCH}(\rho^*(\theta), \theta|\theta^\bullet).$$

(61)

Note that we can write $S(\theta | \theta^\bullet) = S_{\text{simple}}(\theta | \theta^\bullet) + \text{ERR}(\theta | \theta^\bullet)$, where $S_{\text{simple}}(\theta | \theta^\bullet) \triangleq 2\mathcal{L}(\theta|\mathcal{D}) + \|\theta - \theta^\bullet\|_1$ is what we will call the simplified model reduced loss, and where $\text{ERR}$ is the error function due to model reduced loss simplification, given by

$$\text{ERR}(\theta | \theta^\bullet) \triangleq 2\text{HB}_\nu(\rho^*(\theta, \theta^\bullet) - \theta) + \|\rho^*(\theta, \theta^\bullet) - \theta^\bullet\|_1 - \|\theta - \theta^\bullet\|_1.$$ (62)

Interestingly, the error function is uniformly small, so that we can essentially know $S$ by only studying $S_{\text{simple}}$.

Lemma 14. For any $\theta, \theta^\bullet$, we have $|\text{ERR}(\theta | \theta^\bullet)| \leq 7/A^2$.

Proof. By triangle inequality, we have

$$|\text{ERR}(\theta | \theta^\bullet)| \leq 2\text{HB}_\nu(\rho^*(\theta, \theta^\bullet) - \theta) + \|\rho^*(\theta, \theta^\bullet) - \theta^\bullet\|_1 - \|\theta - \theta^\bullet\|_1$$

(63)

$$\leq 2\frac{2\sqrt{2}}{A^2} + \|\rho^*(\theta, \theta^\bullet) - \theta\|_1 \leq \frac{4\sqrt{2}}{A^2} + 2\|\rho^*(\theta, \theta^\bullet) - \theta\|_\infty$$

(64)

$$\leq \frac{4\sqrt{2}}{A^2} + 2 \leq 7/A^2,$$ (65)

where we used the two previous lemmas.

Given the lemma, we can provide the following bounds on interesting values of the reduced loss $S$:

$$S(0 | \theta^\bullet) = S_{\text{simple}}(0 | \theta^\bullet) + \text{ERR}(0 | \theta^\bullet) \leq 1 + 7A^{-2},$$

(66)

$$S(\theta^\bullet | \theta^\bullet) = S_{\text{simple}}(\theta^\bullet | \theta^\bullet) + \text{ERR}(\theta^\bullet | \theta^\bullet) \leq 1 + \frac{A^{-2} + 1}{A^2} + 7A^{-2} \leq 1 + 9A^{-2},$$ (67)

using $A > 1$. In particular, if we can guarantee that $S(\theta | \theta^\bullet) > 1 + 9A^{-2}$ for $\theta$ in some regions of space, then we can exclude the possibility that $\theta_{\text{LCH}}(\theta^\bullet)$ belongs there.

C.4 The optimal model is bounded along the second coordinate

Lemma 15. Consider any $\theta^\bullet$ and suppose $A \geq \|\theta^\bullet\|_1 + 7$. Then $|\theta_{\text{LCH}}(\theta^\bullet)| \leq 2A^2$.

Proof. First note that

$$S(0 | \theta^\bullet) = \|\theta^\bullet\|_1 + \text{ERR}(0 | \theta^\bullet) \leq \|\theta^\bullet\|_1 + 7A^{-2} \leq \|\theta^\bullet\|_1 + 7,$$ (68)
using $A > 1$. Now assume that $|\theta_2| \geq 2A^2$, and consider any $\theta_1 \in \mathbb{R}$. Then

$$S(\theta|\theta^\bullet) \geq \theta_2 + \frac{1}{A^2}\theta_2^2 \geq \frac{1}{A^2}|\theta_2|^2 - |\theta_2| \geq 4A^2 - 2A^2 = 2A^2 > A \geq \|\theta^\bullet\|_1 + 7,$$

using $A > 1$. Thus $S(\theta|\theta^\bullet) > S(0|\theta^\bullet)$, which implies that $\theta$ cannot be optimal. Thus we must have $|\theta_2|^{\text{err}}(\theta^\bullet)| \leq 2A^2$. □

### C.5 Further model reduced loss

Now, interestingly, the simplified reduced loss $S_{\text{simple}}$ has a simple closed form, which allows us to study it directly. In particular, given a a fixed value of $\theta_2$, the parameter $\theta_1$ is easily optimized with respect to $S_{\text{simple}}$. Indeed, note that

$$\partial_{\theta_1}S_{\text{simple}} = 2A^3(A\theta_1 - \theta_2) + \frac{2}{A^4}\theta_1 + \text{sgn}(\theta_1 - \theta_1^\bullet).$$

Thus, defining $\theta_1^\star(\theta_2, \theta_1^\bullet) \triangleq \arg \min_{\theta_1} S_{\text{simple}}(\theta|\theta^\bullet)$, we must have $(2A^4 + \frac{2}{A^4})\theta_1^\star(\theta_2, \theta_1^\bullet) \in 2A^3\theta_2 - \text{sgn}(\theta_1 - \theta_1^\bullet)$, which then implies

$$\theta_1^\star(\theta_2, \theta_1^\bullet) \in \frac{\theta_2}{A + A^{-5}} - \frac{\text{sgn}(\theta_1 - \theta_1^\bullet)}{2A^4 + 2A^{-2}}.$$  

Define $\text{ERR}_2(\theta_2|\theta_1^\bullet) \triangleq \theta_1^\star(\theta_2, \theta_1^\bullet) - A^{-1}\theta_2$ the error when estimating $\theta_1^\star$ with $A^{-1}\theta_2$, we then have the following bound on this error function.

**Lemma 16.** For all $\theta_2, \theta_1^\bullet$, we have $|\text{ERR}_2(\theta_2|\theta_1^\bullet)| \leq A^{-6}|\theta_2| + A^{-4}$.

**Proof.** Indeed, we have

$$|\text{ERR}_2(\theta_2|\theta_1^\bullet)| \leq \left|\frac{\theta_2}{A} - \frac{\theta_2}{A + A^{-5}}\right| + \frac{1}{2A^4 + 2A^{-5}} \leq \frac{A^{-5}|\theta_2|}{2A} + \frac{1}{2A^4} \leq A^{-6}|\theta_2| + A^{-4},$$

which is the lemma. □

**Lemma 17.** Assume $A \geq 9$. Then $|\text{ERR}_2(\theta_2|\theta_1^\bullet)| \leq 3A^{-4}$ and $|\text{ERR}_2(\theta_2^{\text{err}}|\theta_1^\bullet)| \leq 3A^{-4}$.

**Proof.** Note that $\|\theta^\bullet\|_1 \leq 2$ and $\|\theta^\bullet\|_1 \leq 2$ (using $A > 1$). Thus for $A \geq 9$, Lemma 15 applies to $\theta^\bullet = \theta^\dagger$ and $\theta^\bullet = \theta^\bullet$. Combining this with the previous lemma yields the new lemma. □

Put differently, any point $(\theta_1^\star(\theta_2, \theta_1^\bullet), \theta_2)$ can hardly deviate from the line $\theta_2 = A\theta_1$ along the first coordinate, especially as $A \to \infty$. Now define the further model reduced loss $S_2(\theta_2|\theta^\bullet) \triangleq S(\theta_1^\star(\theta_2, \theta_1^\bullet), \theta_2|\theta^\bullet)$, which now only depends on the scalar $\theta_2$.

**Lemma 18.** For $|\theta_2| \leq 2A^2$ and $\bullet \in \{\dagger, \bullet\}$, we have $S_2(\theta_2|\theta^\bullet) \geq 1 + \left|A^{-1}\theta_2 - \theta_1^\bullet\right| - 10A^{-2}$.

**Proof.** Indeed, we then have

$$S_2(\theta_2|\theta^\bullet) = S(\theta_1^\star, \theta_2|\theta^\bullet) \geq S_{\text{simple}}(\theta_1^\star, \theta_2|\theta^\bullet) - 7A^{-2} \geq \theta_2 + \left|A^{-1}\theta_2 - \theta_1^\bullet\right| - |\text{ERR}_2(\theta_2|\theta_1^\bullet)| - 7A^{-2} \geq 1 + \left|A^{-1}\theta_2 - \theta_1^\bullet\right| - 10A^{-2},$$

using the inequality $\theta_2 + \left|\theta_2^\bullet - \theta_2\right| \geq \theta_2 + \theta_2^\bullet - \theta_2 = \theta_2^\bullet = 1$, for $\bullet \in \{\dagger, \bullet\}$. □

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C.6 Weakness of honest model report

We now consider the case of an honest model report \( \theta^0 = \theta^1 = (0, 1) \), and show that \( \theta^\text{LCH}(\theta^1) \) must then be at a distance \( \Omega(1) \) from \( \theta^1 \), as \( A \to \infty \).

**Lemma 19.** For \( A \geq 40 \), \( \| \theta^\text{LCH}(\theta^1) \|_2 \leq 1/2 \).

**Proof.** By contradiction, consider \( \theta_2 \geq 1/2 \) and \( |\theta_2| \leq 2A^2 \). By Lemma 18, we then have \( S_2(\theta_2|\theta^1) \geq 1 + A^{−1} − 10A^{−2} \geq 1 + A^{−1}/2 - 10A^{−1}/40 \geq 1 + A^{−1}/4 \). We now use the fact that \( A \geq 40 > 36 \), thus \( A/4 > 9 \). Multiplying both sides by \( A/2 \) then yields \( A^{−1} > 9A^{−2} \). Therefore \( S_2(\theta_2|\theta^1) > 1 + 9A^{−2} \geq S_2(\theta^\text{LCH}(\theta^1)) \). Thus \( \theta^\text{LCH}(\theta^1) \) cannot be optimal if \( \theta_2 \geq 1/2 \) and \( |\theta_2| \leq 2A^2 \). Since, by Lemma 15, we already know that it cannot be optimal with \( |\theta_2| \geq 2A^2 \), we conclude that we must have \( \| \theta^\text{LCH}(\theta^1) \|_2 \leq 1/2 \).

**Lemma 20.** For \( A \geq 40 \), \( \| \theta^\text{LCH}(\theta^1) \|_2 \geq 1/4 \) and \( \| \rho^\text{LCH}(\theta^1) \|_2 \geq 1/4 \).

**Proof.** By the previous lemma, we know that \( \| \theta^\text{LCH}(\theta^1) \|_2 \geq 1/4 \). By triangle inequality, and using Lemma 12, we then have \( \| \rho^\text{LCH}(\theta^1) \|_2 \geq \| \rho^\text{LCH}(\theta^1) - \theta^1 \|_2 \geq \| \theta^\text{LCH}(\theta^1) - \theta^1 \|_2 \geq 1/2 - \sqrt{2} \| \theta^\text{LCH}(\theta^1) - \theta^1 \|_\infty \geq 1/2 - \sqrt{2}/A^2 \geq 1/4 \), for \( A \geq 40 \).

C.7 Effectiveness of strategic model report

We now consider the case where strategic user \( s \) reports \( \theta^0 = (1/A, 1) \), and prove that in this case, \( \theta^\text{LCH}(\theta^0) \) is at a distance \( O(1/A) \) from \( \theta^1 = (0, 1) \), when \( A \to \infty \).

**Lemma 21.** For \( A \geq 9 \), \( \| \theta^\text{LCH}(\theta^0) \|_2 \leq 20A^{-1} \).

**Proof.** By Lemma 15, we know that \( \theta^\text{LCH}(\theta^0) \) must have an absolute value at most \( 2A^2 \). Now consider \( \theta_2 \) such that \( |\theta_2| \leq 2A^2 \) and for which \( |\theta_2| \geq 20A^{-1} \). By Lemma 18, we then have

\[
S_2(\theta_2|\theta^0) \geq 1 + 20A^{-2} - 10A^{-2} \geq 1 + 10A^{-2} > 1 + 9A^{-2},
\]

and thus \( \theta_2 \) cannot be optimal. Hence the lemma.

**Lemma 22.** For \( A \geq 40 \), \( \| \theta^\text{LCH}(\theta^0) \|_2 \geq 35A^{-1} \) and \( \| \rho^\text{LCH}(\theta^0) \|_2 \leq 35A^{-1} \).

**Proof.** By the previous lemma, we know that \( \| \theta^\text{LCH}(\theta^0) \|_2 \leq 20A^{-1} \leq 1 \), using \( A \geq 40 \). Thus \( \| \theta^\text{LCH}(\theta^0) \|_2 \leq 2 \). As a result,

\[
\| \theta^\text{LCH}(\theta^0) - \theta^1 \|_2 = \| A^{-1}\theta^\text{LCH}(\theta^0) - A^{-1} + \rho(\theta^\text{LCH}(\theta^0)|\theta^0) \|_2 \leq A^{-1}\| \theta^\text{LCH}(\theta^0) - 1 \|_2 + \rho(\theta^\text{LCH}(\theta^0)|\theta^0) \leq A^{-2} + 2A^{-6} + A^{-4} \leq 23A^{-2},
\]

using \( A > 1 \). We then have

\[
\| \theta^\text{LCH}(\theta^0) - \theta^1 \|_2 = \| \theta^\text{LCH}(\theta^0) - \theta^1 \|^2 \leq (A^{-1} + \| \theta^\text{LCH}(\theta^0) - \theta^1 \|^2) + 200A^{-2} \leq (24A^{-1})^2 + 200A^{-2} = 976A^{-2} \leq (32A^{-1})^2 \leq (35A^{-1})^2.\]

Finally, we invoke Lemma 12, which yields

\[
\| \rho^\text{LCH}(\theta^0) - \theta^1 \|_2 \leq \| \rho^\text{LCH}(\theta^0) - \theta^\text{LCH}(\theta^0) \|_2 + \| \theta^\text{LCH}(\theta^0) - \theta^1 \|_2 \leq 33A^{-1} + \sqrt{2}A^{-2} \leq 35A^{-1}.
\]

C.8 Combining it all

**Theorem** For any \( \alpha > 0 \), LICCHAVI is neither global-targeted \( \alpha \)-strategyproof nor user-targeted \( \alpha \)-strategyproof.

**Proof.** Our previous lemmas show that, when \( A \geq 40 \), by reporting \( \theta^0 \) rather than \( \theta^1 \), strategic user \( s \) gains a factor \( A/140 \), both in biasing other users’ models \( \theta^\text{LCH} \) and in biasing the global model \( \rho^\text{LCH} \), as

\[
\| \rho^\text{LCH}(\theta^1) - \theta^1 \|_2 \geq 1/4 > \left(1 + \frac{A}{140}\right)35A^{-1} \geq \left(1 + \frac{A}{140}\right)\| \rho^\text{LCH}(\theta^0) - \theta^1 \|_2 \sin\text{,}
\]

and similarly \( \| \theta^\text{LCH}(\theta^1) - \theta^1 \|_2 \geq (1 + \frac{A}{140})\| \theta^\text{LCH}(\theta^0) - \theta^1 \|_2 \). Therefore, for any \( A \geq 40 \), LICCHAVI fails to be global-targeted \( (A/140) \)-strategyproof; and it also fails to be user-targeted \( (A/140) \)-strategyproof. Given any \( \alpha > 0 \), taking \( A \triangleq \max \{40, 140\alpha\} \) proves the theorem. 

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D Proof of strategyproofness

In this section, we now prove Theorem 2, namely, the strategyproofness of LICCHAVI for gradient PAC* and coordinate-wise separable local losses.

D.1 Disentangling the coordinates

In this section, we show how the assumption of coordinate-wise separable local loss functions allows to reduce the study of strategyproofness to one-dimensional functions. Namely, recall that the local losses are coordinate-wise separable if $L(\theta | D) = \sum_{i \in [d]} L_i(\theta_i | D_i)$. We can then define the coordinate-wise LICCHAVI loss function along dimension $i$ by

$$L_{CH_i}(\rho_i | \theta_{si}, \tilde{D}_{-i}) \triangleq \sum_{n \neq s} L_i(\theta_{ni} | D_n) + w | \rho_i - \theta_{si} |.$$  \hspace{1cm} (79)

The global loss function is then the sum of the coordinate-wise loss functions, i.e.,

$$L_{CH}(\rho | \theta, \tilde{D}_{-s}) = \sum_{i \in [d]} L_{CH_i}(\rho_i | \theta_{si}, \tilde{D}_{-i}).$$  \hspace{1cm} (80)

From this, we derive trivially the following lemma.

**Lemma 23.** $\rho^*_i^{\text{LCH}}(\theta^*_s, \tilde{D}_{-s})$ minimizes $L_{CH_i}(\rho_i | \theta_{si}, \tilde{D}_{-i})$.

**Proof.** This is straightforward. \hfill \square

D.1.1 Strategyproofness in dimension 1

In particular, this means that the strategic user $s$ can focus on coordinate-wise attacks.

**Lemma 24.** Consider a strictly convex function $f : \mathbb{R} \rightarrow \mathbb{R}_+$, and denote $\rho^*(\theta^*) = \arg \min_{\rho \in \mathbb{R}} \{f(\rho) + | \rho - \theta^* | \}$. Then there exists $\rho^*_\min, \rho^*_\max \in \mathbb{R} \cup \{-\infty, +\infty \}$, with $\rho^*_\min \leq \rho^*_\max$, such that, $\rho^*(-\infty, \rho^*_\min) = \{\rho^*_\min\}$, $\rho^*(\rho^*_\max, +\infty) = \{\rho^*_\max\}$, and $\rho^*(\theta^*) = \theta^*$ for all $\theta^* \in [\rho^*_\min, \rho^*_\max]$.

**Proof.** Denote $F(\rho, \theta^*) \triangleq f(\rho) + | \rho - \theta^* |$. First, let us verify that $\rho^*(\cdot)$ is well-defined, by showing that, for all $\theta^*, F(\cdot, \theta^*)$ has a unique minimum. For $| \rho - \theta^* | \geq f(\theta^*)$, we then have $F(\rho, \theta^*) \geq f(\rho) + | \rho - \theta^* | \geq f(\theta^*)$. Thus, the infimum of $F$ on $\mathbb{R}$ is its infimum on $[\theta^* - f(\theta^*), \theta^* + f(\theta^*)]$, which is a compact set. Thus the infimum is reached by a minimum $\rho^*(\theta^*) \in [\theta^* - f(\theta^*), \theta^* + f(\theta^*)]$. The uniqueness of $\rho^*(\theta^*)$ is then guaranteed by the strict convexity of $f$, which implies that of $F$.

Let us now show that $\rho^*$ must be nondecreasing. Since $f$ is strictly convex, its subgradients $\partial f$ are nondecreasing. The same holds for $\text{SGN}(\cdot) = \partial | \cdot |$. Now assume $\theta^*_1 \leq \theta^*_2$. Then, $0 \in \partial_1 F(\rho^*(\theta^*_1), \theta^*_1) = \partial f(\rho^*(\theta^*_1)) + \text{SGN}(\rho^*(\theta^*_1) - \theta^*_1) \geq \partial f(\rho^*(\theta^*_1)) + \text{SGN}(\rho^*(\theta^*_1) - \theta^*_2) = \partial_1 F(\rho^*(\theta^*_1), \theta^*_2)$. Thus the subderivatives $F(\cdot, \theta^*)$ at $\rho = \rho^*(\theta^*)$ are negative or nil. This implies that the optimum of $F(\cdot, \theta^*)$ is on the right of $\rho^*(\theta^*)$. In other words, we must have $\rho^*(\theta^*_1) \leq \rho^*(\theta^*_2)$.

Let us now define $\rho^*_\min \triangleq \inf_{\theta^* \in \mathbb{R}^d} \rho^*(\theta^*) \in \mathbb{R} \cup \{-\infty \}$ and $\rho^*_\max \triangleq \sup_{\theta^* \in \mathbb{R}^d} \rho^*(\theta^*) \in \mathbb{R} \cup \{+\infty \}$. Now consider $\theta^* \in (\rho^*_\min, \rho^*_\max)$. We thus know that there exists $\theta^*_{s1}, \theta^*_{s2} \in \mathbb{R}$ such that $\rho^*(\theta^*_{s1}) \leq \theta^*_s \leq \rho^*(\theta^*_{s2})$. By the monotonicity of $\rho^*$, we know that $\theta^*_s \leq \theta^*_{s1} \leq \theta^*_{s2}$. Moreover, the optimality of $\rho^*(\theta^*)$ implies that $0 \in \partial f(\rho^*(\theta^*_1)) + \text{SGN}(\rho^*(\theta^*_1) - \theta^*_1) \geq \partial f(\rho^*(\theta^*_1)) - 1$, since the minimal value of the sign function is $-1$. Similarly, by the optimality of $\rho^*(\theta^*_2)$, we have $\partial f(\rho^*(\theta^*_2)) \leq 1$. Since $\partial f$ is nondecreasing, we must then have $\partial f(\theta^*_s) \in [-1, 1]$. But then, denoting $g \in \partial f(\theta^*_s)$, since $\text{SGN}(\theta^*_s - \theta^*_s) = \text{SGN}(0) = [-1, 1]$, we have $\partial_1 F(\theta^*_s, \theta^*_s) \geq g + [-1, 1] = [g - 1, g + 1]$. Since $g \in [-1, 1]$, we know that $\partial_1 F(\theta^*_s, \theta^*_s)$ intersects 0, which proves that $\rho^*(\theta^*_s) = \theta^*_s$.

Now consider $\theta^*_s < \rho^*_\min$. By the definition of $\rho^*_\min$, we know that $\rho^*_\min \leq \rho^*(\theta^*_s)$. As a result, $\text{SGN}(\rho^*(\theta^*_s) - \theta^*_s) = -1$ We then know that $0 \in \partial_1 F(\rho^*(\theta^*_s), \theta^*_s) = \partial f(\rho^*(\theta^*_s)) - 1$. But note that this
equality property holds for all $\theta^{\bullet} < \rho_{min}^\ast$. Therefore, $\rho^\ast(\rho_{min}^\ast - 1) = \rho^\ast(\theta^{\bullet})$ for all $\theta^{\bullet} < \rho_{min}^\ast$. But since $\rho^\ast$ is nondecreasing, we also know that $\rho_{min}^\ast = \lim_{\rho \to \infty} \rho^\ast(\theta^{\bullet}) = \rho^\ast(\rho_{min}^\ast - 1)$. Thus, in fact, for any $\theta^{\bullet} < \rho_{min}^\ast$, we have $\rho^\ast(\theta^{\bullet}) = \rho_{min}^\ast$. From this, it also follows that $\partial f(\rho_{min}^\ast)$ contains $-1$, which implies that $\rho^\ast(\rho_{min}^\ast) = \rho_{min}^\ast$.

Finally, we deal similarly with the case of $\rho_{max}^\ast$. Namely, similarly, we show that for all $\theta^{\bullet} \geq \rho_{max}^\ast$, we have $\rho^\ast(\theta^{\bullet}) = \rho_{max}^\ast$.

Lemma 25 (Strategyproofness in dimension 1). Consider a strictly convex function $f : \mathbb{R} \to \mathbb{R}_+$, and denote $\rho^\ast(\theta^{\bullet}) = \arg \min_{\rho \in \mathbb{R}} \{ f(\rho) + |\rho - \theta^{\bullet} | \}$. Then reporting $\theta^{\bullet}$ honestly minimizes the distance to the honest preferences, i.e.,

$$\forall \theta^{\bullet} \in \mathbb{R}, \ |\rho^\ast(\theta^{\bullet}) - \theta^{\bullet}| \leq |\rho^\ast(\theta^{\bullet}) - \theta^{\bullet}|.$$  

Proof. As in Lemma 24 we simply distinguish the three cases $\theta^{\bullet} \leq \rho_{min}^\ast$, $\rho_{min}^\ast \leq \theta^{\bullet} \leq \rho_{max}^\ast$, and $\theta^{\bullet} \leq \rho_{max}^\ast$. In the second case, the left-hand side of the lemma is zero, which makes the inequality clear. In the first and third case, the left-hand side is equal to $|\rho_{min}^\ast - \theta^{\bullet}|$ and $|\theta^{\bullet} - \rho_{max}^\ast|$ respectively. The inequality then follows from the definition of $\rho_{min}^\ast$ and $\rho_{max}^\ast$.

Lemma 26. Consider two strictly convex functions $f$ and $g$ (we also allow $g = 0$). We define $\rho^\ast(\theta^{\bullet}) = \arg \min_{\rho} f(\rho) + |\rho - \theta^{\bullet}|$ and $\theta^{\bullet} = \arg \min_{\rho} g(\rho) + |\rho - \theta^{\bullet}|$. Then,

$$\forall \theta^{\bullet} \in \mathbb{R}, \ |\rho^\ast(\theta^{\bullet}) - \theta^{\bullet}| \leq |\rho^\ast(\theta^{\bullet}) - \theta^{\bullet}|.$$  

Proof. Denote $\rho_{min}^\ast = \min \rho^\ast$ and $\rho_{max}^\ast = \max \rho^\ast$ the minimal and maximal values of $\rho^\ast$. By Lemma 24 for $\theta^{\bullet} = [\rho_{min}^\ast, \rho_{max}^\ast]$ (or if $g = 0$), we know that $|\rho^\ast(\rho) - \theta^{\bullet}|$ is minimized for $\rho = \theta^{\bullet}$, which is achieved by reporting $\theta^{\bullet} = \theta^{\bullet}$.

Now assume $\theta^{\bullet} \leq \rho_{min}^\ast$. By Lemma 24 for any $\theta^{\bullet}$, we know that $\rho^\ast(\theta^{\bullet}) \geq \rho_{min}^\ast = \rho^\ast(\theta^{\bullet})$. Then, by monotonicity of $\rho^\ast$ (Lemma 24), then, for any $\theta^{\bullet}$, we have $\rho^\ast(\theta^{\bullet}) = \rho^\ast(\theta^{\bullet})$.

Now, if $\rho^\ast(\rho^{\ast}(\theta^{\bullet})) = \max \rho^\ast(\rho^{\ast}(\theta^{\bullet}))$, then we must have $\rho^\ast(\rho^{\ast}(\theta^{\bullet})) = \rho^\ast(\rho^{\ast}(\theta^{\bullet}))$, and thus Equation (82) is actually an equality (and thus the inequality holds). Otherwise, if $\rho^\ast(\rho^{\ast}(\theta^{\bullet})) < \max \rho^\ast(\rho^{\ast}(\theta^{\bullet}))$, then by Lemma 24 we must have $\rho^\ast(\rho^{\ast}(\theta^{\bullet})) \geq \rho^\ast(\theta^{\bullet})$. We then have $\rho^\ast(\rho^{\ast}(\theta^{\bullet})) \geq \rho^\ast(\rho^{\ast}(\theta^{\bullet})) \geq \rho^\ast(\rho^{\ast}(\theta^{\bullet})) \geq \rho_{min}^\ast = \theta^{\bullet}$. In particular, we thus have $\rho^\ast(\rho^{\ast}(\theta^{\bullet})) \geq \rho_{min}^\ast = \theta^{\bullet}$, from which the lemma follows.

The case $\theta^{\bullet} \geq \rho_{max}^\ast$ is derived similarly.

D.2 Combining it all

Lemma 27. If $|x_i| \geq |y_i|$ for all coordinates $i \in [D]$, then $\|x\|_2 \geq \|y\|_2$.

Proof. This is clear, given that $\|x\|_2^2 = \sum |x_i|^2$ is an increasing function of all terms $|x_i|$.

Theorem 2. Assume that the local losses are gradient PAC* and coordinate-wise separable. Then LICCHAVI is both global and user-targeted strategyproof.

Proof of Theorem 2. We apply lemmas 25 and 26 with functions

$$f_i(\rho_i) = \frac{1}{\lambda_i \Delta_{si}} \min_{\rho_i} \text{LCH}_{-si}(\rho, \tilde{\theta}_{-si}, \tilde{D}_{-si}),$$

$$g_{ti}(\theta_{ti}) = \frac{1}{\lambda_i \Delta_{ti}} \text{LCH}_{ti}(\theta_{ti}).$$

From this it follows that, for any $i \in [d]$, any target user $t \in [N]$, any honest parameter $\theta^\ast$ and any strategic vector $\theta^{\bullet}$, we have

$$|\rho^\ast(\theta^\ast) - \theta^\ast| \leq |\rho^\ast(\theta^{\bullet} ; \tilde{D}_{-si}) - \theta^\ast|,$$

$$|\rho^\ast(\theta^\ast) - \theta^\ast| \leq |\rho^\ast(\theta^{\bullet} ; \tilde{D}_{-si}) - \theta^\ast|.$$  

Combining Lemma 5 and Lemma 27 then allows to conclude.
E  The quadratic setting

In this section, we detail the proof of Theorem 3, which states the $\alpha$-strategy proofness of LICCHAVI against a quadratic loss.

E.1 Characterizing the effect of model attacks

Lemma 28. AchSet($S$) = \{ $S^{-1}z \mid z \in [-1,1]^d$ \}.

Proof. For each coordinate $i \in [d]$, we have $\partial_i \text{LCH} = \text{sgn}(\rho_i - \theta_{si}) + (S\rho)_i$. The optimality of of $\rho^{LCH}$ implies $(S\rho^{LCH})_i \in \text{sgn}(\rho_i - \theta_{si}) \subseteq [-1,1]$. Thus $\rho^{LCH} \in \{ S^{-1}z \mid z \in [-1,1]^d \}$, which proves that AchSet is included in the deformed hypercube.

Conversely, let $z \in [-1,1]^d$. We consider $\theta^{\bullet} \triangleq S^{-1}z$. Then, for $\rho = \theta^{\bullet}$, we have $\partial_i \text{LCH} = \text{sgn}(\rho_i - \theta_{si}) + (S\rho)_i = [-1,1] + (S^{-1}z)_i = [-1,1] + z_i$. Because $z \in [-1,1]^d$, this set contains 0. Thus the partial derivatives of LCH at $\rho = \theta^{\bullet}$ are all nil, which implies $\rho^{LCH}(\theta^{\bullet}) = \theta^{\bullet} = S^{-1}z$. Thus, in particular, we have $S^{-1}z \in \text{AchSet}(S)$, which is the needed opposite inclusion. \hfill $\square$

To state our result, we now define the crookedness of $S \succ 0$ by

$$\text{Crooked}(S) \triangleq \sup_{x \in \mathbb{R}^d} \inf_{y \in \mathbb{R}^d} \frac{\|x\|_2 \|Sy\|_2}{x^TSy} - 1,$$  \hfill (87)

where $\text{sgn}$ applies the $\text{sgn}$ function on each coordinate (and thus implies $y_i = 0$ whenever $x_i = 0$). Denote $\mathcal{E}$ the canonical basis. For any $\kappa \in \{-1,1\} \times \{-1,1\} \times \{0\}$, we consider the corresponding hypercube face defined by $\text{FACE}(\kappa) \triangleq \{ \kappa \}$, and we denote $S^{-1} \cdot \text{FACE}(\kappa) \triangleq \{ S^{-1}z \mid z \in \text{FACE}(\kappa) \}$. Let us also define $\text{EDGE}(\kappa) \triangleq \{ i \in [d] \mid \kappa_i = -1 \}$ or $\kappa_i = +1 \}$. Now denote

$$\text{ker}(\kappa) \triangleq S^{-1} \cdot \text{FACE}(\kappa) + \sum_{i \in \text{EDGE}(\kappa)} \kappa_i \mathbb{R} e_i.$$  \hfill (88)

Lemma 29. $\rho^{LCH}(\text{ker}(\kappa)) = S^{-1} \cdot \text{FACE}(\kappa)$.

Proof. We show that for any $\theta \in \text{ker}(\kappa)$, we must have $\rho \in S^{-1} \cdot \text{FACE}(\kappa)$. Consider $\theta \in \text{ker}(\kappa)$. Then there exists $\rho \in S^{-1} \cdot \text{FACE}(\kappa)$ and nonnegative scalars $x_i \geq 0$ for $i \in \text{EDGE}(\kappa)$ such that $\theta = \rho + \sum_{i \in \text{EDGE}(\kappa)} x_i \kappa_i e_i$. Now note that $\partial_i \text{LCH} = \text{sgn}(\rho_i - \theta_i) + (S\rho)_i$, which means

$$\partial_i \text{LCH} = \begin{cases} [-1,1] + (S\rho)_i, & \text{if } i \notin \text{EDGE}(\kappa) \\ -\kappa_i + (S\rho)_i, & \text{if } i \in \text{EDGE}(\kappa) \end{cases}. $$  \hfill (89)

But now for any $i \notin \text{EDGE}(\kappa)$, we have $(S\rho)_i \in \kappa_i = (-1,1)$, and thus 0 $\in \partial_i \text{LCH}$. Also, for $i \in \text{EDGE}(\kappa)$, $(S\rho)_i = \kappa_i$, and thus $\partial_i \text{LCH} = 0$. Therefore, all of the partial derivatives of LCH at $\rho$ are 0 which means $\rho^{LCH}(\theta) = \rho$. This concludes the proof. \hfill $\square$

Lemma 30. For any $\theta \in \text{ker}(\kappa)$, there exist unique nonnegative numbers $x_i \geq 0$ for $i \in \text{EDGE}(\kappa)$ such that $\theta = \rho^{LCH}(\theta) + \sum_{i \in \text{EDGE}(\kappa)} \kappa_i x_i e_i$ and $\rho^{LCH}(\theta) \in S^{-1} \cdot \text{FACE}(\kappa)$.

Proof. By definition, since $\theta \in \text{ker}(\kappa)$, there must exist $\rho \in S^{-1} \cdot \text{FACE}(\kappa)$ and $x_i \geq 0$ for $i \in \text{EDGE}(\kappa)$ such that $\theta = \rho + \sum_{i \in \text{EDGE}(\kappa)} \kappa_i x_i e_i$. Now, in a similar manner to (89) in Lemma 29, we obtain that $0 \in \nabla \text{LCH}(\rho)$, and thus $\rho^{LCH}(\theta) = \rho$. Now note that by the strict convexity of LCH, we know that $\rho^{LCH}(\theta)$ is unique. We now show that scalars $x_i$ are also unique. Assume we have two sets of non-negative scalars $\{x_i\}$ and $\{y_i\}$ such that $\theta = \rho^{LCH}(\theta) + \sum_{i \in \text{EDGE}(\kappa)} \kappa_i x_i e_i = \rho^{LCH}(\theta) + \sum_{i \in \text{EDGE}(\kappa)} \kappa_i y_i e_i$. This implies that $\sum_{i \in \text{EDGE}(\kappa)} \kappa_i (x_i - y_i) e_i = 0$. Now since $e_i$'s are linearly independent, we must have $x_i = y_i$ for all $i \in \text{EDGE}(\kappa)$. This proves that the set of scalars $\{x_i\}$ is unique. \hfill $\square$
Lemma 31. \( \theta \in \ker(\kappa) \) if and only if \( \rho^{LCH}(\theta) \in S^{-1} \cdot \text{FACE}(\kappa) \).

Proof. The first direction is proved by Lemma 30. Here we prove the opposite direction, i.e., if \( \rho^{LCH}(\theta) \in S^{-1} \cdot \text{FACE}(\kappa) \) then \( \theta \in \ker(\kappa) \). By the optimality of \( \rho^{LCH}(\theta) \), we must have \( 0 \in \partial_{\kappa} \text{LCH}(\rho^{LCH}(\theta), S) \), for all \( i \in [d] \), which means

\[
\forall i \in [d], 0 \in \text{sgn}\left((\rho^{\text{LCH}}(\theta))_i - \theta_i\right) + (S \rho^{\text{LCH}}(\theta))_i.
\]

(90)

Now, if \( i \in \text{EDGE}(\kappa) \), then \((S \rho^{LCH}(\theta))_i = \kappa_i \), and thus \( \text{sgn}(\rho^{LCH}(\theta))_i - \theta_i) = -\kappa_i \). Therefore, we must have \( \theta_i = (\rho^{LCH}(\theta))_i + x_i \kappa_i \) for \( x_i \geq 0 \). On the other hand, if \( i \notin \text{EDGE}(\kappa) \), then \( 1 < (S \rho^{LCH}(\theta))_i < 1 \), which implies \(-1 < \text{sgn}(\rho^{LCH}(\theta))_i - \theta_i) < 1 \). For this inequality to hold, we must have \( \theta_i = (\rho^{LCH}(\theta))_i \). This proves the other direction and hence the lemma.

Lemma 32. The faces \( S^{-1} \cdot \text{FACE}(\kappa) \) partition \([-1, 1]^d \).

Proof. It is clear that the faces \( \text{FACE}(\kappa) \) partition \([-1, 1]^d \). Since \( S^{-1} \) is invertible, the lemma follows.

Lemma 33. The spaces \( \ker(\kappa) \) partition \( \mathbb{R}^d \).

Proof. We show that any \( \theta \in \mathbb{R}^d \) belongs to \( \ker(\kappa) \) for exactly one choice of \( \kappa \). Consider \( \theta \in \mathbb{R}^d \). By the strong convexity of \( \text{LCH}(\rho, S) \), we know that \( \rho^{\text{LCH}}(\theta) \) is unique. Using Lemma 32, and the fact that \( S^{-1} \cdot \text{FACE}(\kappa) \) partitions \([−1, 1]^d \) (Lemma 32), we obtain that there exists a unique \( \kappa \) such that \( \rho^{LCH}(\theta) \in S^{-1} \cdot \text{FACE}(\kappa) \). Lemma 31 then concludes.

E.2 Proof of \( \alpha \)-strategyproofness

Theorem 3. Licchavi against positive definite matrix \( S \) is Crooked(\( S \))-strategyproof.

Proof. By Lemma 28, we know that the achievable set \( \text{ACHSET}(S) \) of all possible local models for the strategic user is the deformed unit hypercube (parallelepiped) \( S^{-1} \cdot [-1, 1]^d \). Now we consider two different cases separately:

Case i) \( \theta_s \in \text{ACHSET}(S) \). In this case we have \( 0 \in \nabla \text{LCH}(\rho, S) \) for \( \rho = \theta_s \), and thus \( \rho(\theta_s) = \theta_s \). Therefore, it is not possible for the strategic user to gain by misreporting their local model.

Case ii) \( \theta_s \notin \text{ACHSET}(S) \). Note that the achievable set \( \text{ACHSET}(S) \) can be characterized using 2d inequalities as

\[
\{ z : \forall i \in [d], \forall j \in \{-1, 1\}, (je^i)^T S z \leq 1 \}.
\]

(91)

Now by Lemma 33, there must exist \( \kappa \in \{-1\}, \{-1, +1\}, \{+1\}\}^d \) such that \( \theta_s \in \ker(\kappa) \). Plus since \( \theta_s \) does not belong to the achievable set, \( \text{EDGE}(\kappa) \) is not empty. Now by Lemma 30, we have \( \theta = \rho^{\text{LCH}}(\theta) + x \) for \( \rho^{\text{LCH}}(\theta) \in S^{-1} \cdot \text{FACE}(\kappa) \) and \( x = \sum_{i \in \text{EDGE}(\kappa)} \kappa_i x_i e_i \), which implies \( (\kappa e^i)^T S \rho^{LCH}(\theta) = 1 \) for all \( i \in \text{EDGE}(\kappa) \). We now lower bound the distance between \( \theta_s \) and any point \( z \) in the achievable set. For this, consider the inequalities associated to \( i \in \text{EDGE}(\kappa) \), i.e., for any \( i \in \text{EDGE}(\kappa) \), we have \( (\kappa_i e^i)^T S z \leq 1 \). Now consider any convex combination of these inequalities, yielding \( y^T S z \leq 1 \), for \( y = \sum_{i \in \text{EDGE}(\kappa)} y_i \kappa_i e^i \) with non-negative scalars \( y_i \geq 0 \) such that \( \sum y_i = 1 \). Each of these inequalities for any set \( \{y_i\} \) defines a closed half space containing the achievable set and with \( \rho^{\text{LCH}}(\theta_s) \) on its boundary. Therefore, for any point \( z \in \text{ACHSET}(S) \), the distance between \( z \) and \( \theta_s \) is at least the distance between \( \theta_s \) and its orthogonal projection on the half space \( y^T S z \leq 1 \). In equations, this implies

\[
\|\theta_s - z\|_2 \geq \frac{(\theta_s - \rho^{LCH}(\theta_s))^T (S y)}{\|S y\|_2} = \frac{x^T S y}{\|S y\|_2}.
\]

(92)

Now note that this inequality holds for any \( y \). Thus, we obtain

\[
\|\theta_s - z\|_2 \geq \sup_y \frac{x^T S y}{\|S y\|_2},
\]

(93)

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Note that as the magnitude of $y$ cancels out in the nominator and the denominator, the above inequality holds for any $y$ such that $y_i \geq 0$ for all $i \in \text{Edge}(\kappa)$, i.e.,

$$\|\theta_s - z\|_2 \geq \sup_{y_i \geq 0} \|T_{Sy}\|_2 \geq \sup_{y \in \mathbb{R}^d, \text{sgn}(y) = \text{sgn}(x)} \frac{x^T Sy}{\|Sy\|_2},$$

where the second inequality comes from the fact that $\text{sgn}(y) = \text{sgn}(x)$ implies $y_i \geq 0$ for all $i \in \text{Edge}(\kappa)$.

We then obtain

$$\inf_{z \in \text{AcSet}(S)} \|\theta_s - z\|_2 \leq \sup_{y \in \mathbb{R}^d, \text{sgn}(y) = \text{sgn}(x)} \frac{\|x\|_2 \|Sy\|_2}{x^T Sy} \leq \text{Crooked}(S) + 1. \quad (95)$$

Hence, the theorem.

### E.3 Crookedness is smaller than skewness

To prove Proposition 2, which says that crookedness is smaller than skewness, with strict inequality for some matrices, we first recall a lemma from [EFGH21] about skewness.

**Lemma 34** (Proposition 12 in [EFGH21]). Denote $\Lambda \triangleq \max \text{Sp}(S)/\min \text{Sp}(S)$ the ratio of extreme eigenvalues. Then,

$$\text{Skew}(S) \geq 1 + \frac{\Lambda}{2\sqrt{\Lambda}} - 1. \quad (96)$$

We now prove the proposition.

**Proposition 2.** Let $\text{Skew}(S) \triangleq \sup_{x \in \mathbb{R}^d} \frac{\|x\|_2 \|Sy\|_2}{x^T Sy} - 1$. Then, for any $S \succ 0$, we have $\text{Crooked}(S) \leq \text{Skew}(S)$. Moreover, there are definite positive matrices $S \succ 0$ for which the inequality is strict.

**Proof.** The inequality $\text{Crooked} \leq \text{Skew}$ is evident by setting $y = x$ in the definition of $\text{Crooked}$ (Equation (15)).

We now prove for some matrices the inequality is strict. Consider a diagonal matrix $S = \text{Diag}(\lambda_1, \ldots, \lambda_d)$ with eigenvalues $\lambda_1 \geq \ldots \geq \lambda_d > 0$. Now for any vector $x \in \mathbb{R}^d$, define $y(x) \triangleq S^{-1}x$. This implies that $\text{sgn}(y(x)) = \text{sgn}(x)$. We then obtain that

$$\frac{\|x\|_2 \|Sy\|_2}{x^T Sy} = \frac{\|x\|_2 \|x\|_2}{x^T x} = 1. \quad (97)$$

As this is true for any arbitrary vector $x \in \mathbb{R}^d$, we obtain that

$$\text{Crooked}(S) \triangleq \sup_{x \in \mathbb{R}^d} \inf_{y \in \mathbb{R}^d, \text{sgn}(y) = \text{sgn}(x)} \frac{\|x\|_2 \|Sy\|_2}{x^T Sy} - 1 = \sup_{x \in \mathbb{R}^d} \inf_{y \in \mathbb{R}^d, \text{sgn}(y) = \text{sgn}(x)} 1 - 1 = 0. \quad (98)$$

But now by Lemma 34, we have $\text{Skew}(S) \geq \frac{1 + \Lambda}{2\sqrt{\Lambda}} - 1$ for $\Lambda = \lambda_1/\lambda_d$. Therefore, $\text{Skew}(S)$ may take arbitrarily large values for $\Lambda$ large enough. In particular, for $\Lambda > 0$, we have $\text{Crooked}(S) < \text{Skew}(S)$.

\[29\]