Observer-independent facts in the presence of a horizon

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In the famous thought experiment known as Wigner’s friend, Wigner assigns an entangled state to the composite quantum system consisting of his friend and her observed system. In the context of this thought experiment, Brukner recently derived a no-go theorem for observer-independent facts, i.e. those facts that would be common to both Wigner and his friend. In this article, we show that Brukner’s theorem fails in the presence of a horizon even if Wigner and his friend are on the same side of the horizon. However, we also show that the failure of the theorem comes with a price: the preservation of observer-independent facts requires that some of those facts must remain unknowable.

I. INTRODUCTION

In 1961 Eugene Wigner introduced a curious thought experiment that has recently garnered renewed interest in the physics literature \cite{1}. In the experiment, now known as “Wigner’s friend”, the aforementioned friend performs a measurement on a quantum system inside a sealed laboratory. A so-called “super-observer” (played by Wigner himself in the original paper) is placed outside the laboratory. The outcome of the friend’s measurement is reflected in some property of the device that is performing the measurement, e.g. a pointer reading or audible device click. Wigner describes the process unitarily based solely on the information to which he has access. At the conclusion of the process the friend’s description of the original quantum state consists of a projection of that state into one that corresponds to the outcome reported by the device. By contrast, Wigner assigns a specific entangled state to the system and the friend which he can verify through some further experiment (e.g. communicating with the friend). The main question that Wigner provoked with his thought experiment was what happens to the state from his (Wigner’s) point-of-view when his friend observes a definite outcome? Does the state collapse for Wigner at that exact moment or does it only collapse for Wigner when he receives information about the result of his friend’s measurement? If it is the latter, then how can we reconcile the two apparently different accounts of the original measurement process?

Wigner’s original intent with his thought experiment was to support his view that consciousness was necessary in order to collapse the wave function. It nevertheless serves as an interesting way to compare various interpretations of quantum mechanics. In particular, in objective collapse theories, Wigner’s state assignment can be statistically disproven by carrying out many verifications hence showing that the state is a definite, objective property of the universe \cite{2,3}. This is in contrast to other interpretations where the state is only a relative property, e.g. it is projected relative to the friend and in a superposition relative to Wigner \cite{5,6}. Both Wigner’s and his friend’s description of the state are equally valid in a relative sense since they accurately describe the world Wigner and his friend experience respectively. When they finally communicate, Wigner can update his description based on information he receives from his friend concerning the results of the friend’s measurement. Thus, strictly speaking, there is no inconsistency with quantum theory in this case. In other words, Wigner’s friend poses no apparent problems for epistemic interpretations of quantum theory.

In response to the recent work of Frauchiger and Renner \cite{7}, Brukner has recently derived a Bell-type no-go theorem for observer-independent facts that claims to show that there can be no theory in which Wigner’s and Wigner’s friend’s facts can jointly be considered to be locally objective properties of the universe \cite{8}. That is, Brukner’s theorem denies that either Wigner’s or his friend’s description of the state is a (locally) objective property of a given theory. As Brukner makes clear, the objective properties or “facts” described are understood to mean “immediate experiences of observers.” That is, it may refer to what some interpretations of quantum mechanics deems to be “real” (e.g. wave functions, Bohmian trajectories, etc.) only to the extent that these directly give rise to some sort of observable fact such as a pointer reading or detector click.

The relative state description of Wigner and his friend is analogous to the situation described by another famous thought experiment: Einstein’s twin paradox, the first explanation of which was due to Paul Langevin \cite{9}. In the twin paradox a pair of synchronized clocks becomes un-synchronized when one clock undergoes relativistic acceleration while the other remains inertial. In Langevin’s retelling, observers co-moving with the accelerated clock remain in regular contact with observers who remain in the frame of the inertial clock. Thus it is that the inertial observers will appear to see the accelerated clock slow down while observers co-moving with that same clock will see it behave normally. Each description is equally valid in a relative sense since both clocks (and their associated observers) maintain separate worldlines. When the worldlines re-intersect, both sets of observers can simply compare their respective results in order to

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arrive at a consistent description of the state of the clock at that instant. As such, special relativity contains no inconsistency in this case.

In this article we investigate a scenario in which the experimental setup employed by Brukner in the proof of his theorem is embedded in a twin-paradox-like setting. The relative acceleration that is introduced occurs between two identical laboratories, each of which contains an observer in the role of Wigner’s friend who makes a measurement on a quantum system in their possession. Likewise, each laboratory includes a co-moving external super-observer in the role of Wigner who communicates with the observer inside the laboratory and whose description of the quantum system depends on the information received. We then extend the discussion to a single Wigner’s friend-type experiment in which the relative acceleration is then between Wigner and his friend.

Ultimately we find that Brukner’s no-go theorem fails in the presence of a horizon and that, as such, there is a point at which it again becomes possible to talk about the concept of an observer-independent (i.e. locally objective) fact. However, we also show that the failure of the theorem comes at the price of fully knowing those facts. We begin by briefly reviewing Brukner’s theorem and we reformulate his proof in terms of free bosonic states. We then show the dependence of this proof on the relative acceleration between the two halves of the experiment. Finally we show that, though the failure of Brukner’s theorem does mean that observer-independent facts can exist, some of those facts must remain unknowable. We begin with a review of Brukner’s theorem and its proof, but rather than using an entangled spin-1/2 system, our model uses an entangled two-mode free bosonic field which makes analysis in the presence of acceleration simpler since it reduces a portion of the problem to a discussion of the Unruh effect.

II. OBSERVER-INDEPENDENT FACTS FOR INERTIAL OBSERVERS

A typical Wigner’s friend thought experiment involves some two-level quantum system that can give rise to two outcomes upon measurement. The outcomes are recorded by a measurement apparatus that eventually is read by Wigner’s friend and committed to memory. The measurement apparatus as well as Wigner’s friend are taken to be inside an isolated laboratory. Wigner is placed outside this laboratory and can perform a quantum measurement on the overall system consisting of the two-level quantum system and the laboratory. It is usually assumed that all experiments are carried out a sufficient number of times in order to ensure reliable statistics.

Deutsch proposed a model of the Wigner’s friend thought experiment for which it was possible for Wigner to gain direct knowledge about whether or not the friend had observed a definite outcome without her (the friend) having to reveal which outcome she observed \[10\]. For example, let us assume that a detector that is sensitive to two modes \( j \) and \( k \) of a free bosonic field in Minkowski space \( M \) is used to make a measurement on such a field that is prepared in the state \( |\phi^+\rangle_B = \frac{1}{\sqrt{2}} (|0\rangle_M^j |1\rangle_M^k + |1\rangle_M^j |0\rangle_M^k) \) where \( |0\rangle_B^M \) and \( |1\rangle_B^M \) refer to the vacuum and single-particle states respectively. After the measurement has been completed, the measurement apparatus is found to be in one of many perceptively different macroscopic configurations corresponding to, for example, a particular pointer reading or an audible click. In Deutsch’s model, no assumptions need to be made concerning the friend’s formal description of the result. It is simply enough that she perceives a definite outcome.

Wigner then uses quantum theory to describe the friend’s measurement as a unitary transformation. The possible states of the field \( |0\rangle_B^j \) and \( |1\rangle_B^j \) are assumed to be entangled with the perceptively different macroscopic configurations of the apparatus, the laboratory, and the friend’s memory. We can represent these configurations using a pair of orthogonal states \( |F_0\rangle_F \) and \( |F_1\rangle_F \) respectively. The state of the composite system consisting of the field mode and the friend’s laboratory is thus

\[
|\Phi\rangle_{BF} = \frac{1}{\sqrt{2}} \left( |0\rangle_M^j |F_0\rangle_F + |1\rangle_M^j |F_1\rangle_F \right). \tag{1}
\]

The particular phase between the two amplitudes of equation \((1)\) is specified by Wigner via the measurement interaction. It is his specification of this phase that avoids the necessity of describing the state as an incoherent mixture of the two possibilities. Wigner can verify his state assignment by performing a Bell state measurement in the bases

\[
|\Phi^\pm\rangle_{BF} = \frac{1}{\sqrt{2}} \left( |0\rangle_M^j |F_0\rangle_F \pm |1\rangle_M^j |F_1\rangle_F \right),
\]

\[
|\Psi^\pm\rangle_{BF} = \frac{1}{\sqrt{2}} \left( |0\rangle_M^j |F_1\rangle_F \pm |1\rangle_M^j |F_0\rangle_F \right). \tag{2}
\]

In Deutsch’s proposal, Wigner obtains direct knowledge about whether or not the friend actually observed a definite outcome without requiring that the friend reveal the result. For instance, the friend could pass a note through a slot in the laboratory door on which is written either “I have observed a definite outcome” or “I have not observed a definite outcome” as show in Figure \[\square\]. As long as the message does not contain any information regarding the actual outcome of the friend’s measurement, the state of the encoded message can be factored out of the total state which then would read

\[
|\Phi\rangle_{BF_I} = \frac{1}{\sqrt{2}} \left( |0\rangle_M^j |F_0\rangle_F + |1\rangle_M^j |F_1\rangle_F \right) \otimes \left| \text{“I have observed a definite outcome.”} \right>_I. \tag{3}
\]

(Here we are assuming that the friend always observes a definite outcome when performing a measurement.) Because the state of the message can be factored out, we
see that Wigner can obtain direct evidence for the existence of his friend’s facts without knowing what those facts are. As such, both Wigner’s facts and his friend’s facts appear to coexist.

In a framework in which we can account for observer-independent facts, we should be able to jointly assign truth values to the observational statements $A_1$: “Wigner’s friend’s measurement apparatus indicates the bosonic field is in the vacuum state” and $A_2$: “Wigner’s measurement apparatus indicates the overall state is $\Phi$.” Wigner, of course, can learn the truth value of either of these two statements. If he performs a Bell measurement, he obtains the truth value for $A_2$ whereas if he, for example, simply opens the laboratory door and speaks to his friend, he can obtain the truth value for $A_1$. But in order for observer-independent facts to exist, we must be able to assign a truth value to both $A_1$ and $A_2$ independently of which measurement Wigner performs, i.e. independently of whether he makes a Bell measurement or opens the lab door. In other words, if the outcome $\Phi$ is observer-independent, then $A_1$ is true regardless of whether or not Wigner actually makes that measurement.

Brukner formalizes this by postulating that the truth values of the propositions $A_i$ for all observers form a Boolean algebra that is equipped with a countably additive positive measure $p(A) \geq 0$ for all propositions that correspond to the probability that a given proposition is true. For the scenario described here, we assume that we can jointly assign truth values ($+1 = \text{true}, -1 = \text{false}$) to the statements $A_1$ and $A_2$ and thus may also assign joint a joint probability $p(A_1 = \pm 1, A_2 = \pm 1)$. Brukner’s no-go theorem, which is a Bell-type theorem, then uses the fact that $A_1$ and $A_2$ do not commute and is stated as follows [3]:

**Theorem II.1** (Brukner’s no-go theorem). The following statements are incompatible, i.e. they lead to a contradiction:

1. Quantum predictions hold at any scale, even if the measured system contains objects as large as an “observer” (including her laboratory, memory, etc.). This is the assumption that quantum theory is universally valid.

2. The choice of the measurement settings of one observer has no influence on the outcomes of any other other distant observer(s). This is the assumption of locality.

3. The choice of measurement settings is statistically independent from the rest of the experiment. This is the freedom of choice assumption.

4. One can jointly assign truth values to the propositions about observed outcomes (“facts”) of different observers (as just described).

Proof. Consider a pair of super-observers (Alice and Bob) who play the role of Wigner in a Deutsch-like Wigner’s friend experiment as shown in Figure 2. That is, these super-observers may each carry out experiments on a system that includes a laboratory containing an observer (Charlie and Debbie respectively) who performs an experiment on a free bosonic field mode. We can thus perform a Bell-type test for which Alice chooses between measurements $A_1$ and $A_2$ which correspond to the statements that Charlie and Alice can respectively make about their measurement outcomes. Likewise, $B_1$ and $B_2$ are similar measurements that apply to Debbie and Bob respectively. The assumptions (2), (3), and (4) define hidden variables that pre-determine the values of $A_1$, $A_2$, $B_1$, and $B_2$ to be either $+1$ or $-1$. As such we can also define a joint probability $p(A_1, A_2, B_1, B_2)$ whose marginals satisfy the Clauser-Horne-Shimony-Holt (CHSH) inequality: $S = \langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle \leq 2\sqrt{2}$.

Suppose that Charlie and Debbie initially share an entangled two-mode free bosonic field that is in the state $|\psi\rangle_{B_1 B_2} = -\sin \theta \frac{1}{2} |\phi^-\rangle_{B_1 B_2} + \cos \theta \frac{1}{2} |\psi^+\rangle_{B_1 B_2}$ (4)

where

$$|\phi^-\rangle_{B_1 B_2} = \frac{1}{\sqrt{2}} \left( |0\rangle_B \langle 0|_B - |1\rangle_B \langle 1|_B \right)_{B_1 B_2}$$

(5)

$$|\psi^+\rangle_{B_1 B_2} = \frac{1}{\sqrt{2}} \left( |0\rangle_B \langle 0|_B + |1\rangle_B \langle 1|_B \right)_{B_1 B_2}$$

(6)
and, again, $j$ and $k$ refer to the respective modes. Here Charlie controls mode $j$ and Debbie controls mode $k$ where each is inside their own laboratory. The state given by equation (4) can be obtained by applying the appropriate pseudospin operators to the singlet state $|1\rangle$.

For Alice and Bob, the initial state together with the overall state of the laboratories is

$$|\Psi_0\rangle = |\psi\rangle_{B_1 B_2} |0\rangle_C |0\rangle_D$$

where the states $|0\rangle_C$ and $|0\rangle_D$ require no further characterization except to say that the observers are capable of completing a measurement.

Now we assume that Charlie and Debbie each have access to a detector that is sensitive to the single mode that is under their control (e.g. $j$ for Charlie and $k$ for Debbie). They each perform a measurement of their respective modes which amounts to determining if their mode is in the vacuum or single-particle state. From the point of view of Alice and Bob, these measurements are described by unitary transformations. Once the measurements are complete, we assume that the overall state of the entire system becomes

$$|^\Psi\rangle = -\sin\frac{\theta}{2} |\Phi^+\rangle + \cos\frac{\theta}{2} |\Psi^-\rangle$$

where

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}} (|A_g\rangle |B_e\rangle - |A_e\rangle |B_g\rangle)$$

$$|\Psi^-\rangle = \frac{1}{\sqrt{2}} (|A_g\rangle |B_e\rangle + |A_e\rangle |B_g\rangle)$$

and

$$|A_g\rangle = |0_g\rangle_{B_1} |C_0\rangle_C, \quad |A_e\rangle = |1_g\rangle_{B_1} |C_1\rangle_C,$$  \quad (10)

$$|B_g\rangle = |0_e\rangle_{B_2} |D_0\rangle_D, \quad |B_e\rangle = |1_e\rangle_{B_2} |D_1\rangle_D$$  \quad (11)

where the subscript $g$ refers to the vacuum state and the subscript $e$ refers to the single-particle state. Here the states $|C_0\rangle$ and $|C_1\rangle$ correspond to orthogonal states of Charlie’s lab associated with the two measurement outcomes (and similarly for Debbie’s lab). The details of these lab states is unimportant. All that matters is that they are orthogonal.

We next define two sets of binary observables in analogy to the Pauli spin operators along the $z$ and $x$ axes respectively:

$$A_z = |A_g\rangle \langle A_g| - |A_e\rangle \langle A_e|$$

$$A_x = |A_g\rangle \langle A_e| + |A_e\rangle \langle A_g|.$$  \quad (13)

Similar operators can be defined for $B_z$ and $B_x$. In the Bell experiment, Alice chooses between $A_1 = A_z$ and $A_2 = A_x$ and likewise for Bob. In this case, $A_1$ and $B_1$ represent a Wigner’s friend type of measurement while $A_2$ and $B_2$ represent a Wigner type of measurement. If we set $\theta = \pi/4$, then we find a violation of a CHSH-type inequality of

$$S = \langle A_1 B_1 \rangle + \langle A_1 B_2 \rangle + \langle A_2 B_1 \rangle - \langle A_2 B_2 \rangle \leq 2.$$  \quad (14)

FIG. 3. An accelerated observer in Minkowski space is a static observer in a Rindler space. Worldlines of accelerated observers in Minkowski space form a parabola $x^2 - t^2 = a^{-2} e^{2a\xi}$ for constant $a$ and $\xi$. Observers on such a trajectory in region II are causally disconnected from region I and vice-versa.

with a value of $S = 2\sqrt{2}$ which is a maximal violation in agreement with Brukner’s results for spin-1/2 systems.

III. OBSERVER-INDEPENDENT FACTS IN THE PRESENCE OF A HORIZON

Let us now consider the situation if there is relative acceleration between the two halves of the experiment. That is, let us uniformly co-accelerate Bob, Debbie, and her laboratory with some proper acceleration $a$. Proper acceleration is interpreted as a physical acceleration, i.e. an acceleration that is measured by some device such as an accelerometer. It is therefore measured relative to an inertial observer who is momentarily at rest with respect to the object under acceleration. In this case, the states of mode $k$ must be specified in Rindler coordinates in order to properly describe what Debbie and Bob observe. If we consider just a single spatial dimension $z$, the worldlines of uniformly accelerated observers in Minkowski space correspond to hyperbolae to the left (region I) and right (region II) of the origin on a spacetime diagram as in Figure 3. These regions are bounded by light-like asymptotes that form the Rindler horizon. Rindler coordinates are defined as

$$t = a^{-1} e^{a\xi} \sinh a\tau, \quad z = a^{-1} e^{a\xi} \cosh a\tau, \quad |z| < t$$

$$t = -a^{-1} e^{a\xi} \sinh a\tau, \quad z = a^{-1} e^{a\xi} \cosh a\tau, \quad |z| > t$$  \quad (15)
where $a$ is the proper acceleration as defined above, $\xi$ is a space-like coordinate, and $\tau$ is the proper time. The Minkowski vacuum state,

$$|0\rangle^M = \Pi_j |0_j\rangle^M,$$

which is defined as the absence of any particle excitation in any of the $j$ modes can be expressed in terms of a product of two-mode squeezed states of the Rindler vacuum [12] as

$$|0_k\rangle^M \sim \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r \, |n_k\rangle_I \, |n_k\rangle_{II},$$

(17)

where $\cosh r = (1-e^{-2\pi \Omega})^{-1/2}$, $\Omega := |k|c/a$, and $a$ is the acceleration. The states $|n_k\rangle_I$ and $|n_k\rangle_{II}$ are Fock states and refer to the mode decomposition in regions I and II of Rindler space respectively. Likewise, the single-particle states can be written as [13]

$$|1_k\rangle^M = \frac{1}{\cosh^2 r} \sum_{n=0}^{\infty} \tanh^n r \sqrt{n+1} \, |(n+1)_k\rangle_I \, |n_k\rangle_{II}.$$  

(18)

For the sake of argument, let us assume that Debbie, her laboratory, and Bob are all uniformly accelerated in a direction such that they are causally disconnected from region I, i.e. they follow a trajectory given by the hyperbola $x^2 - t^2 = a^{-2}e^{2a\xi}$ as in Figure 3. As such, equation (11) becomes

$$|B_y\rangle = \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r \, |n_k\rangle_I \, |n_k\rangle_{II} \, |C_0\rangle_C$$

$$|B_x\rangle = \left( \frac{1}{\cosh r} \sum_{n=0}^{\infty} \tanh^n r \sqrt{n+1} \right) \times |(n+1)_k\rangle_I \, |n_k\rangle_{II} \, |C_1\rangle_C.$$  

(19)

Since Bob, Debbie, and her laboratory are not causally connected to region II, we must trace over the states in this region which gives a mixed state, $\rho_{AB}$. We can then form a pair of binary observables for Bob in analogy to equations [12] and [13] and, once again setting $\theta = \pi/4$, we can test a CHSH-type inequality of the form shown in equation (14). The expectation values in this inequality are of the form $\langle A_1 B_1 \rangle = \text{Tr}(\rho_{AB} A_1 B_1)$. The problem with evaluating this inequality, however, is that equation (19) includes an infinite (vector) sum over all values of $n$ which represents the fact that Fock space is infinite in size. However, the sum is convergent and the size of the Fock space can be truncated by setting $n_{\text{max}} = N$ when $\tanh^n r < c$ for some value of $\epsilon$. That is, we choose to ignore any vectors $\tanh^n r \, |n\rangle$ in the sum for which $\tanh^n r$ is sufficiently small. Nevertheless, the size of the density matrix is still quite large even for small values of $N$ (e.g. it is $81 \times 81$ for $N = 3$). As such it is necessary to evaluate this numerically. Of particular interest to us here is the dependence of $S$ on the proper acceleration $a$:

- does Brukner’s theorem hold for all values of $a$, some values of $a$, or none?

We chose to implement our model using Python’s QuTiP package which is especially well-suited for working with Fock states [14][15]. It turns out that the value of $\epsilon$ must be chosen carefully to ensure numerical stability while also not sacrificing any relevant physics. It is reasonable to expect that the dependence of $S$ on $a$ ought to be consistent with the dependence of the logarithmic negativity on $r$ as given in the work of Fuentes and Mann [13]. Values of $\epsilon$ below roughly 0.1 demonstrate strong instabilities for values of $r$ ranging to approximately 6.0. With $\epsilon = 0.1$, however, we obtain results that are consistent with those of Fuentes and Mann for the logarithmic negativity. Our results are shown in Figure 4. Classicality is reached when $a/|k|c \sim 5.3$. Thus we can see that Brukner’s theorem does not hold for all values of $a$. The the fact that the vacuum and single-particle states can be written as equations (17) and (18) was first shown by Davies and Unruh [17][18]. As such, the failure of Brukner’s theorem for free bosonic fields is seen to be a consequence of the Unruh effect.

Due to the equivalence principle our results should hold for gravitationally induced horizons as well and thus we expect that this is a general statement. As such, it turns out to have interesting implications for any spacetime that includes a horizon.

### IV. IMPLICATIONS

Brukner has suggested that one possible way around this problem is to assume that since Bob, Debbie, and her laboratory are beyond a horizon, in some sense they can be thought to not exist from the standpoint of Alice and Charlie [19]. We now describe a scenario in which it is possible for Alice and Charlie to maintain knowledge...
of Bob’s and Debbie’s existence even if they pass beyond a horizon.

If Bob (and Debbie) exist then Alice should be able to assign a truth value to the proposition $A_3$: “Bob, Debbie, and her laboratory exist.” In order to address Brukner’s point, it is merely necessary for Bob, Debbie, and her laboratory to possess some objective property that ensures that they are somehow part of the same “universe” as Alice in the sense that they contribute some lasting effect on that universe. It is not necessary that they be distinguishable in any way. It is simply necessary that Alice be able to say that, in some way, they are still “in” the same universe. Since we assume that Bob, Debbie, and her laboratory collectively possess a (relativistically invariant) mass $m$ \[1\]. As long as the mass $m$ leads to some measurable effect somewhere in the universe, then Alice can assign a truth value to proposition $A_3$.

Let us now suppose that Bob’s half of the experiment is near a black hole of mass $M$. Further suppose that $m$ is a non-trivial fraction of $M$ (Debbie’s laboratory, for example, could be a planet and Bob could be orbiting that planet in a spacecraft). We will motivate this requirement in a moment. We assume that Alice has the ability to monitor the total mass of the black hole plus Bob’s half of the experiment (perhaps through some standard astrophysical method), i.e. she has the ability to monitor that value $m + M$. As long as the value of $m + M$ does not change, then the truth value of proposition $A_3$ should be knowable since $m$ contributes to the overall effect of $m + M$ on their surroundings.

Brukner’s argument is that if at any point Bob’s half of the experiment were to cross a horizon (in this case, the event horizon of the black hole) then, since Alice would no longer have knowledge of Bob’s half, Bob, Debbie, and her laboratory could be thought of as no longer existing to Alice in some sense. However, if Bob’s half of the experiment has crossed the event horizon, the black hole’s mass is now $m + M$. In other words, while the black hole and Bob’s half of the experiment are no longer distinguishable from one another, the overall mass of the system remains the same and thus Alice is still able to assign a truth value to proposition $A_3$. The requirement that $m$ is a non-trivial fraction of $M$ ensures that any change in the mass of the black hole after it absorbs Bob’s half of the experiment, is measurable by Alice and, within some range of error, is consistent with an absorption of mass $m$. In other words, this requirement ensures that Alice can assign a definite truth value to proposition $A_3$.

We thus see that it is possible for Alice to conclude that Bob, Debbie, and her laboratory still exist even after passing through a horizon. As such, it is possible for Brukner’s theorem to fail in the presence of a horizon while simultaneously ensuring that some proposition involving an “existence criteria” can always be assigned a definite truth value. This would seem to suggest that the presence of a horizon anywhere within a given spacetime is enough to restore the notion of observer-independent facts.

This has another consequence, however. Imagine, instead, the usual Wigner’s friend experiment in the form proposed by Deutsch. Recall that In a framework in which we can account for observer-independent facts, we should be able to jointly assign truth values to the propositions $A_1$ and $A_2$ as defined in Section 1. Let us add to this the proposition $A_4$: “Wigner’s friend exists.” Crucially this implies that truth values can exist for $A_1$ and $A_2$. Note that this is different from proposition $A_3$ which referred to a collective Wigner’s friend experiment in which Bob played the role of Wigner and Debbie played the role of his friend. The existence of observer-independent facts is assumed to require that the truth values for $A_1$ and $A_2$ are independent of which measurement Wigner performs. As we have just shown, it appears that observer-independent facts are preserved in the presence of a horizon.

But now consider a situation in which Wigner’s friend and her laboratory are in the presence of a black hole and their collective mass is a non-trivial fraction of the black hole’s mass. Wigner can continually monitor the total mass of the black hole plus his friend and her laboratory and thus can, at any point, assign a truth value to proposition $A_4$. Now suppose that at some point prior to Wigner carrying out a measurement of type $A_1$ or type $A_2$, his friend and her laboratory pass the black hole’s event horizon. Due to the results outlined above, Wigner can still assign a truth value to proposition $A_4$ yet cannot assign a truth value to either proposition $A_1$ or $A_2$. As such it appears that it is possible for Wigner to know that truth values for $A_1$ and $A_2$ exist and yet be unable to determine them.

This appears to offer an interesting compromise on the part of physics in regard to observer-independent facts. While it is clear that Brukner’s theorem fails in the presence of a horizon and thus observer-independent facts can exist, it also appears to be the case that this failure comes at the cost of fully knowing those facts. In other words, it may be that in order for observer-independent facts to exist, some of those facts must be unknowable.

V. CONCLUSION

In this article we have examined the recent no-go theorem for observer-independent facts proposed by Brukner and have shown that the theorem fails in the presence of a horizon. Our model employed entangled modes of a free bosonic field our results for the dependence of the Bell parameter $S$ as a function of relative acceleration $a$ are consistent with known results for the logarithmic negativity. However, we have also shown that the failure of Brukner’s theorem comes with a price. In order to preserve the notion of observer-independent facts, we

1 In order to ensure relativistic invariance, we define the mass here as the magnitude of the four-momentum.
found that it must be that some of those facts remain unknowable.

It is worth noting here that the use of definite particle states for testing a situation such as this can prove problematic \cite{20, 21}. Specifically, the bosonic field modes, though confined to the respective laboratories of Charlie and Debbie, are still highly non-local, which can lead to superluminal signaling between two observers within the laboratory, one to the past of the spacelike hypersurface on which the observable in question is measured, and one to the future. While it is not clear that this applies in the scenario discussed in this paper, it is nevertheless worth mentioning.

Regardless, however, the fact remains that the degradation of entanglement by relative acceleration and, equivalently, gravitation, is a well-established fact. The specific form of entangled state that is shared by Charlie and Debbie is largely immaterial as long as this is true. Therefore, the results of Section \[IV\] remain valid.

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