THERMONUCLEAR EXPLOSION OF ROTATING MASSIVE STARS COULD EXPLAIN CORE-COLLAPSE SUPERNOVAE

Doron Kushnir\textsuperscript{1}

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ABSTRACT

It is widely thought that core-collapse supernovae (CCSNe), the explosions of massive stars following the collapse of the stars’ iron cores, is obtained due to energy deposition by neutrinos. So far, this scenario was not demonstrated from first principles. Kushnir and Katz (2014) have recently shown, by using one-dimensional simulations, that if the neutrinos failed to explode the star, a thermonuclear explosion of the outer shells is possible for some (tuned) initial profiles. However, the energy released was small and negligible amounts of ejected \( ^{56}\text{Ni} \) were obtained, implying that these one-dimensional collapse induced thermonuclear explosions (CITE) are unlikely to represent typical CCSNe. Here I provide evidence supporting a scenario in which the majority of CCSNe are the result of CITE. I use two-dimensional simulations to show that collapse of stars that include slowly (few percent of breakup) rotating \( \sim 0.1 - 10 \, M_\odot \) shells of mixed helium-oxygen, leads to an ignition of a thermonuclear detonation wave that unbinds the stars’ outer layers. Simulations of massive stars with different properties show that CITE is a robust process, and results in explosions with kinetic energies in the range of \( 10^{49} - 10^{52} \) erg, and \( ^{56}\text{Ni} \) yields of up to \( \sim M_\odot \), which are correlated, in agreement with observations for the majority of CCSNe. Stronger explosions are predicted from higher mass progenitors that leave more massive remnants, in contrast to the neutrino mechanism. Neutron stars are produced in weak (\( \lesssim 10^{51} \) erg) explosions, while strong (\( \gtrsim 10^{53} \) erg) explosions leave black hole remnants.

Subject headings: hydrodynamics — methods: numerical — supernovae: general

1. INTRODUCTION

There is strong evidence that CCSNe are explosions of massive stars, involving the collapse of the stars’ iron cores (Burbridge et al. 1957; Hirata et al. 1987; Smartt 2009) and ejection of the outer layers. It is widely thought that the \( \sim 10^{53} \) erg observed kinetic energy (KE) is obtained due to the deposition of a small fraction (\( \sim 1\% \)) of the gravitational energy (\( \sim 10^{53} \) erg) released in neutrinos (see Bethe 1990; Burrows 2013 for reviews). So far, this scenario was not demonstrated from first principles.

Burbridge et al. (1957) suggested a different mechanism for the explosion during core-collapse that does not involve the emitted neutrinos. In the proposed scenario, increased burning rates due to adiabatic heating of the outer shells as they collapse lead to a thermonuclear explosion (see also Hoyle & Fowler 1960; Fowler & Hoyle 1964). This collapse-induced thermonuclear explosion (CITE) naturally produces \( \sim 10^{51} \) erg from thermonuclear burning of \( \sim M_\odot \) (gain of \( \sim \text{MeV} / m_n \)). A few one-dimensional studies suggested that CITE fails (Colgate & White 1966; Woosley & Weaver 1982; Bodenheimer & Woosley 1983). It was recently shown, by using one-dimensional simulations, that CITE is actually possible for some (tuned) initial density and composition profiles (Kushnir & Katz 2014), which include shells of mixed helium and oxygen. An ignition of a thermonuclear detonation is obtained, \( \sim 10 \) seconds after the core collapsed, that unbinds the outer layers of the star. However, the energy released was small, \( \lesssim 10^{50} \) erg, and negligible amounts of ejected \( ^{56}\text{Ni} \) were obtained, implying that these one-dimensional CITE are not typical CCSNe.

Here I provide evidence supporting a scenario in which the majority of CCSNe are CITE of rotating massive stars. By using two-dimensional simulations with a fully resolved ignition process, I show that collapse of stars that include slowly (few percent of breakup) rotating \( \sim 0.1 - 10 \, M_\odot \) shells of He-O with densities of few \( \times 10^3 \, \text{g cm}^{-3} \), leads to an ignition of a thermonuclear detonation that unbinds the stars’ outer layers.

Previous studies of thermonuclear explosions of collapsing rotating stars (Bodenheimer & Woosley 1983; MacFadyen & Woosley 1999), without He-O mixtures, did not result in an ignition of a detonation. I show that the presence of He-O mixtures is a necessary condition for explosion, making the previous studies consistent with the present work.

Section 2 describes the simulations. Section 3 demonstrates the consistency of CITE with the majority of CCSNe, including new predictions.

2. NUMERICAL SIMULATIONS

Section 2.1 describes the initial profiles, Section 2.2 describes the numerical tool, Section 2.3 presents a simulation of a successful thermonuclear explosion that leads to a typical CCSNe, and Section 2.4 demonstrates that CITE is robust, by examining the sensitivity of the results to the assumed initial profile.

2.1. Initial profiles

Pre-collapse profiles are uncertain (see, e.g. Smith & Arnett 2014), but nevertheless stellar evolution calculations of rotating massive stars generally predict the existence of a He-O shell (Hirschi et al. 2004, 2005; Hirschi 2007; Yusof et al. 2013). Instead of relying on some specific calculated profiles, I systematically examine the explosion’s properties as a function of the He-O shell properties. Profiles are composed as follows (this parametrization is somewhat different from Kushnir & Katz 2014):

- The profile is composed of shells with constant entropy
Pure oxygen (helium) is placed below (above) the He-O shell, the entropy of the He-O shell is calculated with cylindrical coordinates (Eulerian, adaptive mesh refinement; Dubey et al. 2009). The initial profiles are chosen to be in hydrostatic equilibrium without rotation. Rotation is added such that initially the pressure built from accumulating thermonuclear energy, at locations in the progenitor where $T > 2 \times 10^9 \, \text{K}$, oxygen is replaced with silicon to prevent fast initial burning, and the region between $r_{\text{inner}}$ and $r = 2 \times 10^8 \, \text{cm}$ is filled with $s = 1 \, k_B$ iron (satisfying hydrostatic equilibrium), which is prevented from burning. These regions collapse rapidly thorough the inner boundary and have negligible effect on the results. For simulations in which an explosion was obtained, the original resolution was reduced successively by factors of 2 for each doubling of the shock radius, starting with the first resolution increase made at a chosen threshold radius $r_{\text{res}}$ (5 $\times$ 10$^{10}$ cm for most simulations).

2.3. CITE is possible - example of a successful explosion which leads to a typical CCSN

A pre-collapse profile which leads to a typical CCSN is shown in Figure 1. The dynamical evolution of the collapse, calculated with a resolution of $\approx 30$ km, is shown in Figure 2. Collapsing material that includes angular momentum reaches a centrifugal barrier at a radius smaller by $\approx f_{\text{rot,0}}$ of its initial radius, leading to an increased pressure and to the formation of a rotation-induced accretion shock (RIAS), seen in panel (a). The in-fall velocity of the shock-heated oxygen is significantly reduced, allowing synthesis of $^{56}\text{Ni}$ (black contour) within the hot and dense downstream conditions. The energy released in this thermonuclear processing ($\approx 3 \times 10^{50}$ erg until $t = 28 \, \text{s}$, panel (d)) is sufficient for an expansion of the synthesized material. The base of the He-O shell, seen in panel (a), is deformed during collapse from its initial spherical shape. The RIAS hits the He-O shell at $t \approx 29 \, \text{s}$ which causes an ignition of a detonation (panel (b)). Because the shapes of the RIAS and the base of the He-O shell are different, the ignition position propagates from the symmetry axis towards the equator. The detonation propagates outward (panel (c)), producing thermonuclear energy at $\approx 5 \times 10^{50}$ erg s$^{-1}$ (panel (d)). The pressure built from accumulating thermonuclear energy, aided by the RIAS that acts as a piston, manages to halt the inward collapse and causes an expansion that leads to an outward motion. Once the detonation reaches outer layers with densities $\rho \lesssim 10^3$ g cm$^{-3}$ it decays and transitions to a hydrodynamic shock which continues to propagate outwards. The resulting ejecta has a mass of $\approx 4.5 \, M_\odot$ (leaving $\approx 6.5 \, M_\odot$ of material with negative spherical radial velocity, hereafter in-falling mass), KE $\approx 1.3 \times 10^{51}$ erg, and a $^{56}\text{Ni}$ mass, $M_{\text{Ni}}$, of $\approx 0.08 \, M_\odot$. The obtained KE and $M_{\text{Ni}}$ are typically observed in CCSNe (see Section 3). The KE of the ejecta may change slightly if a hydrogen envelope is added.

2.3.1. Numerical convergence

The KE and $M_{\text{Ni}}$ as a function of resolution are shown in panel (a) of Figure 3. The results are converged to $\approx 10\%$ for resolution of $\approx 30$ km, used throughout the paper. The $^{56}\text{Ni}$ yield is estimated as the total mass of $^{56}\text{Ni}$ with positive (spherical) radial velocity, $M_{\text{Ni}}(v_r > 0)$, at the latest time with the original resolution. At this time $M_{\text{Ni}}(v_r > 0)$ is roughly constant with time, and a decrease in $M_{\text{Ni}}(v_r > 0)$ was obtained as the resolution was reduced. In order to
verify that the decrease in $M_{\text{He}}(v_r > 0)$ is numerical, another calculation with $r_{\text{res}} = 10^{10}$ cm was performed (x-symbols in panel (a) of Figure 2), in which similar $M_{\text{Ni}}$ was obtained. The reduction of the resolution inhibits accurate calculation of the asymptotic $^{56}\text{Ni}$ distribution in the ejecta.

The KE and $M_{\text{Ni}}$ as a function of $r_{\text{inner}}$ are shown in panel (b) of Figure 2. The smaller $r_{\text{inner}}$, the earlier the RIAS is launched. As can be seen the KE and $M_{\text{Ni}}$ are not sensitive to the exact value of $r_{\text{inner}}$ with the $M_{\text{Ni}}$ slowly increasing for smaller $r_{\text{inner}}$. It would be desirable to decrease $r_{\text{inner}}$ below the minimal value tried here (100 km), however the in-fall velocities at this radius are already $\approx 0.4c$, leading to tens of percent error for a non-relativistic calculation. In what follows I employ $r_{\text{inner}} = 500$ km, keeping in mind that $M_{\text{Ni}}$ is probably under-predicted by a factor of a few.

2.4. CITE is robust - the full set of simulations

In Sections 2.4.1 - 2.4.4 I examine the sensitivity of the results of Section 2.4.3 to $x_{\text{rot},0}$, $f_{\text{rot},0}$, $X_{\text{He}}, M_{\text{shell}}$, and $t_{b,0}/f_{j,0}$, while in Section 2.4.5 $M_{\text{base}}$ and $\rho_{\text{base}}$ are varied. The relevant figure for each section is indicated in the title. A list of the simulations is given in Table 1.

2.4.1. Sensitivity to the inner boundary of the rotation zone ($x_{\text{rot},0}$) – panel (c) of Figure 2

For $0 < x_{\text{rot},0} < 1$ ignition occurs immediately after the interaction with the RIAS, similarly to panel (b) of Figure 2. The smaller $x_{\text{rot},0}$, the earlier the RIAS is launched, leading to smaller KE (since the He-O shell is less compressed) but to higher $M_{\text{Ni}}$ (because of the higher density of the oxygen).

For $x_{\text{rot},0} = 1$ there is no RIAS inner to the He-O shell, and it ignites because of adiabatic compression during free-fall (self-ignition), similarly to one-dimensional simulations [Kushnir & Katz 2014]. In fact, in a one-dimensional simulation of this profile, while the explosion fails, $\approx 2.5 \cdot 10^{51}$ erg of thermonuclear energy is released, which is comparable to the $x_{\text{rot},0} = 1$ case ($\approx 2.9 \cdot 10^{53}$ erg). The one-dimensional case fails because only a small fraction of the thermonuclear energy is converted to outward motion and cannot overcome the binding energy of the star [Kushnir & Katz 2014], $E_{\text{bin}} \approx -9 \cdot 10^{50}$ erg (see the base of the He-O shell, corrected for thermal energy). For $x_{\text{rot},0} = 1$, the RIAS forms inside the He-O shell after ignition, and reaches the detonation front while acting as a piston that increases the fraction of thermonuclear energy that is converted to outward motion ($\approx 0.75$ in this case, $\lesssim 0.35$ in the equivalent one-dimensional case). Small amounts of $^{56}\text{Ni}$ are synthesized, since the RIAS forms late, where only a small amount of high density material is present. As $x_{\text{rot},0}$ is increased, a smaller fraction of the He-O shell is RIAS supported, and for $x_{\text{rot},0} = 1.2$ the explosion fails, similarly to the one-dimensional case.

For $x_{\text{rot},0} < 0.7$ more complicated dynamics are obtained, since the RIAS hits the He-O shell at early times, in which the density is too low for immediate ignition. However, later on ignition happens at the pole, where the induction time is shortest, and then the detonation slides towards the equator. The KE is smaller by a factor of a few compared to shock ignition, but the $M_{\text{Ni}}$ is somewhat higher because of the high density material that is being shocked.

2.4.2. Sensitivity to the rotation speed ($f_{\text{rot},0}$) – panel (d) of Figure 2

The KE depends weakly on $f_{\text{rot},0}$, as long as $f_{\text{rot},0} \geq 0.02$. For smaller values, KE drops sharply, and no explosion is obtained for $f_{\text{rot},0} = 0.01$, as no RIAS was launched at relevant times. The $M_{\text{Ni}}$ decreases with increasing $f_{\text{rot},0}$, since the RIAS is launched at larger radii, leading to collapsing material with smaller density. This demonstrates that rotation is required for CITE.

To test the effect of the artificial departure from hydrostatic equilibrium due to rotation, I performed a set of simulations where the initial profiles were first relaxed to equilibrium before the collapse. The relaxation was run for $\sim 1000$ s without inflow through $r_{\text{inner}}$ and without burning. The results of such simulations ($f_{\text{rot},0} = 0.05$ and $f_{\text{rot},0} = 0.1$) change by $\lesssim 50\%$ (x-symbols).

2.4.3. $X_{\text{He}}$ – panel (e) of Figure 2

For low (high) values of $X_{\text{He}}$ the KE decreases since the energy content of the He-O mixture is decreasing (since there are not enough target nuclei for alpha capture and the energy content of the He-O mixture cannot be extracted efficiently), until the explosions fails for $X_{\text{He}} = 0.1(0.9)$. This demonstrates the He-O mixture requirement for CITE. $^{56}\text{Ni}$ is produced below the He-O shell, and is not affected by its composition.

2.4.4. $M_{\text{shell}}$ and $t_{b,0}/f_{j,0}$ – panel (f) of Figure 2

Decreasing $M_{\text{shell}}$ decreases both the available thermonuclear energy and the binding energy $|E_{\text{bin}}|$. However, the obtained KE roughly follows $|E_{\text{bin}}|$ (see Section 2.4.5). $M_{\text{Ni}}$ is not sensitive to $M_{\text{shell}}$, unless weak explosion is obtained. For $t_{b,0}/f_{j,0} = 10^{13}$ the maximal He-O shell mass is only $M_{\text{shell,max}} \approx 1.02 M_{\odot}$, but at a given $M_{\text{shell}}$ the results depend weakly on $t_{b,0}/f_{j,0}$.

2.4.5. $M_{\text{base}}$ and $\rho_{\text{base}}$ - Figure 2

In this section $t_{b,0}/f_{j,0} = 100$, $X_{\text{He}} = 0.5$, $M_{\text{shell}} = M_{\text{shell,max}}$, $f_{\text{rot},0} = 0.05$, $x_{\text{rot},0} = 0.8$, and the ranges $M_{\text{base}} \in [1.5, 16.5] M_{\odot}$, $\rho_{\text{base}} \in [0.5, 2.5] \cdot 10^{4} \text{ g cm}^{-3}$ are scanned.

The KE, $M_{\text{Ni}}$, and in-falling mass are shown in Figure 2 (only successful explosions are presented). The maximal possible KE ($M_{\text{Ni}}$) increases with $M_{\text{base}}$, and ranges between $10^{49}$ erg to $10^{52}$ erg (negligible amounts to $\approx 0.5 M_{\odot}$), covering the observed range for the vast majority of CCSNe (see Section 3). The in-falling mass is larger by $0.1 - 15 M_{\odot}$ than $M_{\text{base}}$.

As can be seen in panel (d) of Figure 2 the KE never exceeds the binding energy $|E_{\text{bin}}|$ by more than a factor of 2.5. This can be understood by comparing the available thermonuclear energy $\sim M_{\text{shell}} \times \text{MeV}/m_p$ with the binding energy $\sim -GM_{\text{base}}/r_{\text{base}}$ and noting that few $\times GM_{\text{base}}/r_{\text{base}} \approx \text{MeV}/m_p$ [Kushnir & Katz 2014]. In the absence of tuning between the released thermonuclear energy and $E_{\text{bin}}$, the minimal KE cannot be much smaller than $|E_{\text{bin}}|$ (KE $\gtrsim 0.25 |E_{\text{bin}}|$ for all calculations). Therefore, KE $\sim |E_{\text{bin}}|$ for CITE, in contrast to the neutrino mechanism, where larger KE are obtained for smaller $|E_{\text{bin}}|$ [Fryer 1999; Heger et al. 2003].

3. COMPARISON TO OBSERVATIONS AND PRELIMINARY PREDICTIONS

Estimates of the KE and $M_{\text{Ni}}$ for several observed supernovae are shown in Figure 3 where Type II (Type Ibc) supernovae, in which hydrogen is detected (not detected),
are compared with black crosses (x-symbols). The observations, listed in Table 3, were compiled from Hamuy (2003); Hendry et al. (2005); Pastorello et al. (2005); Inserra et al. (2011); Pastorello et al. (2012); Taddia et al. (2012); Tomasella et al. (2013); Dall’Ora et al. (2014); Lyman et al. (2014); Spiro et al. (2014); Utrobin & Chugai (2014), and include only CCSNe within comoving radial distance of 100 Mpc (to exclude rare events). The observations include modeling of the emitted light and are susceptible to systematic uncertainties. Moreover, the distribution of the sample in the KE–M Ni plane does not represent relative rates of events. Nevertheless, the range of observed KE (10^{50} – few × 10^{52} erg) and M Ni (10^{-3} – 1 M⊙), and the gross correlation between them (higher KE leads to higher M Ni), can be deduced. The lower limits of the KE and M Ni may be observationally biased.

The calculated KE and M Ni from Section 2 are compared to observations. Because of the sensitivity of the calculated results to the numerical treatment of the inner boundary (Section 2.3.1) and the partial parameter scan, this comparison, as well as the predictions made, are preliminary. As can be seen, the range of observed KE and M Ni, as well as the gross correlation between them, can be obtained from CITE. It is also possible to obtain KE ≈ few × 10^{50} erg without tuning (see Section 2.4.3) for small values of M base. One feature of CITE is that KE scales with M base, i.e. stronger explosions originated from higher mass progenitors, as observations suggest (Poznanski 2013), and in contrast to the neutrino mechanism Fryer (1999), Heger et al. (2003).

The calculated KE and remnant mass are shown in panel (b) of Figure 5. The remnant mass in the neutron star (NS) regime, M grav, includes a correction due to the negative binding energy by subtracting an assumed gravitational energy of 1.5 - 10^{50} (M grav/M⊙) erg (Lattimer & Yahil 1989). For gravitational mass above the maximal mass of a NS (taken here as 2.5 M⊙) the remnant mass represents the baryonic mass of the in-falling material. Stronger explosions are predicted to leave more massive remnants, in contrast to the neutrino mechanism (where strong explosions leave neutron stars while weak explosions leave black holes; Fryer 1999, Heger et al. 2003). CITE predicts that NSs can only be produced in weak (≤ 10^{51} erg) explosions (explaining, e.g., the low, < 10^{50} erg, KE of the Crab nebula; Smith 2013), while strong (≥ 10^{51} erg) explosions must leave black holes (BHs). This indicates that the vast majority of observed extragalactic CCSNe (including SN1987A) left behind BHs.

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Figure 1. A pre-collapse profile (density, temperature, enclosed mass, burning to free-fall time ratio, and specific angular momentum on the equatorial plane ($j_{z=0}$)) that leads to a typical CCSN. For this profile $M_{\text{base}} = 6.0 M_\odot$, $\rho_{\text{base}} = 10^4$ g cm$^{-3}$, $t_{b,0}/t_{ff,0} = 100$, $X_{\text{He}} = X_{\text{O}} = 0.5$ and $M_{\text{shell}} = M_{\text{shell,max}} \approx 3.60 M_\odot$, leading to $r_{\text{base}} \approx 4.62 \times 10^9$ cm, burning time at $r_{\text{base}}$ of $t_{b,0} \approx 7.9 \times 10^2$ s and a total mass of $\approx 11.0 M_\odot$ (the profile below $r = 2 \times 10^8$ cm has negligible effect on the results and is not shown). The obtained density, temperature, and enclosed mass profiles are similar to pre-collapse profiles of a 30 $M_\odot$ star (dashed gray), calculated by Roni Waldman with the MESA stellar evolution code. The rotation parameters are $f_{\text{rot},0} = 0.05$ and $x_{\text{rot},0} = 0.8$ ($r_{\text{rot},0} = x_{\text{rot},0} \times r_{\text{base}} \approx 3.69 \times 10^9$ cm).
Figure 2. Dynamical evolution of the collapse for the initial conditions of Figure 1. Panels (a-c): logarithmic density maps at different times since collapse with isotope contours of He (red, $X_{\text{He}} = 0.1, 0.2, 0.45$) and $^{56}\text{Ni}$ (black, $X_{\text{Ni}} = 0.1$). Panel (d): rate of thermonuclear energy production, $\dot{E}_{\text{burn}}$ (red), accumulated thermonuclear energy produced, $E_{\text{burn}}$ (blue), and total KE of mass elements with positive radial (spherical) velocity, $E_{\text{kin}}(v_r > 0)$ (black).
Figure 3. KE (black) and $M_{\text{Ni}}$ (red) as a function of several numerical and physical parameters. panel (a): resolution, x-symbols are obtained from simulation with $r_{\text{res}} = 10^{10}$ cm; panel (b): $t_{\text{inner}}$, panel (c): $x_{\text{rot},0}$; panel (d): $f_{\text{tot},0}$, x-symbols are obtained from simulation with relaxed initial models; panel (e): composition; panel (f): $M_{\text{shell}}$, for $t_{\text{bf},0}/t_{\text{ff},0} = 10^2$ (solid) and $t_{\text{bf},0}/t_{\text{ff},0} = 10^3$ (dashed).
Figure 4. Results of calculations with $t_b/\tau_{ff,0} = 100$, $X_{\text{He}} = X_{\text{O}} = 0.5$, $M_{\text{shell}} = M_{\text{shell,max}}$, $f_{\text{rot},0} = 0.05$, $x_{\text{rot},0} = 0.8$, $M_{\text{base}} \in [1.5, 16.5] M_\odot$, and $\rho_{\text{base}} \in [0.5, 2.5] \cdot 10^4 g \text{ cm}^{-3}$ (different colors represent different $\rho_{\text{base}}$). Panel (a): KE; panel (b): $M_{\text{Ni}}$; panel (c): in-falling mass; panel (d): KE as a function of $-E_{\text{bin}}$.

Figure 5. The uniform sample of Figure 4 (circles, different colors represent different $M_{\text{base}}$) extended with the sensitivity calculations of Figure 3 (points). Panel (a): $M_{\text{Ni}}$–KE. The observed sample, black crosses (x-symbols) for type II (type Ibc), is shown without error bars (given in Table 3) for purpose of clarity. As can be seen, the range of observed KE and $M_{\text{Ni}}$, as well as the gross correlation between them, can be obtained from CITE. Panel (b): KE–remnant mass (see Section 3 for the definition of the remnant mass).
Table 1
The set of simulations examining the sensitivity of the $M_{\text{base}} = 6 M_{\odot}$, $\rho_{\text{base}} = 10^{14} \text{ g cm}^{-3}$, $t_{b,0}/t_{f,0} = 100$, $X_{\text{He}} = 0.5$, $f_{\text{tot},0} = 0.05$, $x_{\text{tot},0} = 0.8$ calculation with $\Delta R = 28.6 \text{ km}$, $r_{\text{res}} = 5 \cdot 10^{9} \text{ cm}$, $r_{\text{inner}} = 5 \cdot 10^{7} \text{ cm}$, to various numerical and physical parameters.

| description | $r_{\text{base}}$ [10$^{9}$ cm] | $\rho_{\text{He-O}}$ [k$g$] | $M_{\text{shell}}$ [$M_{\odot}$] | total mass [$10^{51}$ erg] | $-E_{\text{kin}}$ | kinetic energy [$10^{51}$ erg] | $^{56}\text{Ni}$ mass [$10^{-2} M_{\odot}$] | in-falling mass$^a$ [$M_{\odot}$] |
|-------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| reference   |                  |                  |                  |                  |                  |                  |                  |                  |
| $\Delta R$ = 4.3 km | 1.40             | 8.06             | 6.45             |                  |                  |                  |                  |                  |
| $\Delta R$ = 57.2 km | 1.37             | 5.18             | 6.61             |                  |                  |                  |                  |                  |
| $r_{\text{res}} = 10^{10}$ cm | 1.33             | 8.33             | 6.46             |                  |                  |                  |                  |                  |
| $r_{\text{inner}} = 100$ km | 1.54             | 16.2             | 6.31             |                  |                  |                  |                  |                  |
| $r_{\text{inner}} = 250$ km | 1.40             | 11.1             | 6.38             |                  |                  |                  |                  |                  |
| $r_{\text{inner}} = 1000$ km | 1.19             | 0.0147           | 6.87             |                  |                  |                  |                  |                  |
| $x_{\text{tot},0} = 0.4$ | 0.5533           | 14.3             | 7.42             |                  |                  |                  |                  |                  |
| $x_{\text{tot},0} = 0.5$ | 0.545            | 15.6             | 7.22             |                  |                  |                  |                  |                  |
| $x_{\text{tot},0} = 0.6$ | 0.515            | 13.6             | 7.31             |                  |                  |                  |                  |                  |
| $x_{\text{tot},0} = 0.7$ | 0.912            | 10.6             | 6.57             |                  |                  |                  |                  |                  |
| $x_{\text{tot},0} = 0.9$ | 1.56             | 3.96             | 6.61             |                  |                  |                  |                  |                  |
| $x_{\text{tot},0} = 1$ | 1.24             | 0.329            | 6.95             |                  |                  |                  |                  |                  |
| $x_{\text{tot},0} = 1.1$ | 0.25             | 0.0907           | 9.31             |                  |                  |                  |                  |                  |
| $f_{\text{tot},0} = 0.015$ | 0.362            | 0.211            | 9.30             |                  |                  |                  |                  |                  |
| $f_{\text{tot},0} = 0.02$ | 1.20             | 6.92             | 7.27             |                  |                  |                  |                  |                  |
| $f_{\text{tot},0} = 0.025$ | 1.46             | 21.3             | 6.95             |                  |                  |                  |                  |                  |
| $f_{\text{tot},0} = 0.03$ | 1.59             | 17.8             | 6.72             |                  |                  |                  |                  |                  |
| $f_{\text{tot},0} = 0.04$ | 1.47             | 14.7             | 6.55             |                  |                  |                  |                  |                  |
| $f_{\text{tot},0} = 0.06$ | 1.26             | 5.23             | 6.52             |                  |                  |                  |                  |                  |
| $f_{\text{tot},0} = 0.07$ | 1.16             | 2.98             | 6.55             |                  |                  |                  |                  |                  |
| $f_{\text{tot},0} = 0.08$ | 0.972            | 2.07             | 6.63             |                  |                  |                  |                  |                  |
| $f_{\text{tot},0} = 0.09$ | 0.970            | 1.46             | 6.65             |                  |                  |                  |                  |                  |
| $f_{\text{tot},0} = 0.1$ | 0.742            | 1.37             | 6.86             |                  |                  |                  |                  |                  |
| relaxed $f_{\text{tot},0} = 0.1$ | 1.80             | 9.77             | 6.73             |                  |                  |                  |                  |                  |
| $X_{\text{He}} = 0.2$ | 4.66             | 7.00             | 3.84             | 11.4             | 0.886            | 0.144            | 9.23             | 9.03             |
| $X_{\text{He}} = 0.3$ | 4.61             | 7.23             | 3.56             | 11.0             | 0.839            | 0.467            | 8.12             | 7.56             |
| $X_{\text{He}} = 0.4$ | 4.60             | 7.51             | 3.52             | 10.9             | 0.835            | 1.13             | 8.98             | 6.61             |
| $X_{\text{He}} = 0.6$ | 4.05             | 8.13             | 3.78             | 11.3             | 0.895            | 1.11             | 9.57             | 6.65             |
| $X_{\text{He}} = 0.7$ | 4.69             | 8.46             | 4.06             | 11.6             | 0.953            | 0.631            | 8.74             | 7.33             |
| $X_{\text{He}} = 0.8$ | 4.75             | 8.83             | 4.50             | 12.2             | 1.04             | 0.288            | 6.59             | 6.02             |
| $X_{\text{He}} = 0.9$ | 2.71             | 8.19             | 0.444            | 6.72             | 0.173            | 0.135            | 3.68             | 6.27             |
| $X_{\text{He}} = 1.0$ | 3.23             | 8.06             | 0.929            | 7.47             | 0.295            | 0.326            | 5.32             | 6.34             |
| $X_{\text{He}} = 1.1$ | 3.58             | 7.99             | 1.42             | 8.19             | 0.404            | 0.495            | 8.03             | 6.39             |
| $X_{\text{He}} = 1.2$ | 3.86             | 7.93             | 1.89             | 8.96             | 0.506            | 0.657            | 10.7             | 6.41             |
| $X_{\text{He}} = 1.3$ | 4.09             | 7.89             | 2.36             | 9.48             | 0.602            | 0.824            | 9.20             | 6.42             |
| $X_{\text{He}} = 1.4$ | 4.30             | 7.86             | 2.81             | 10.1             | 0.694            | 1.03             | 9.18             | 6.44             |
| $X_{\text{He}} = 1.5$ | 4.48             | 7.83             | 3.25             | 10.6             | 0.783            | 1.11             | 8.28             | 6.51             |

| $t_{b,0}/t_{f,0} = 10^{4}$ | 3.01             | 7.18             | 0.389            | 6.65             | 0.154            | 0.179            | 2.99             | 6.18             |
| $t_{b,0}/t_{f,0} = 10^{3}$ | 3.59             | 7.08             | 0.799            | 7.11             | 0.267            | 0.555            | 6.59             | 6.24             |
| $t_{b,0}/t_{f,0} = 10^{3}$ | 3.82             | 7.05             | 1.02             | 7.40             | 0.320            | 0.466            | 8.66             | 6.21             |

$^a$ Without the binding energy correction used in panel (b) of Figure 4.
### Table 2

The set of simulations with $t_{90}/t_{ff,0} = 100$, $X_{He} = 0.5$, $M_{\text{shell}} = M_{\text{shell, max}}$, $f_{\text{rot,0}} = 0.05$, and $x_{\text{rot,0}} = 0.8$.

| $M_{\text{base}}$ [$M_\odot$] | $\rho_{\text{base}}$ [$10^4$ g cm$^{-3}$] | $r_{\text{base}}$ [$10^5$ cm] | $s_{\text{He-4}}$ | $M_{\text{shell}}$ [$M_\odot$] | total mass | $-E_{\text{bin}}$ | kinetic energy | $^{56}\text{Ni}$ mass | in-falling mass* |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|-----------------|-----------------|-----------------|
| 1.5 | 1.5 | 1.50 | 7.06 | 0.540 | 2.97 | 0.0714 | 0.0621 | - | 2.01 |
| 1.5 | 1.75 | 1.36 | 6.75 | 0.297 | 2.18 | 0.0443 | 0.0481 | - | 1.69 |
| 2.0 | 2.0 | 1.28 | 6.51 | 0.187 | 1.85 | 0.0319 | 0.0365 | - | 1.63 |
| 2.25 | 2.25 | 1.16 | 6.31 | 0.130 | 1.71 | 0.0252 | 0.0305 | - | 1.63 |
| 2.5 | 2.5 | 1.09 | 6.15 | 0.0964 | 1.64 | 0.0210 | 0.0207 | - | 1.63 |
| 2.75 | 2.75 | 1.03 | 6.01 | 0.0755 | 1.60 | 0.0182 | 0.0126 | - | 1.63 |
| 2.25 | 1.25 | 2.00 | 7.88 | 4.91 | 9.06 | 0.4599 | 0.3134 | 0.009044 | 6.48 |
| 2.25 | 1.25 | 2.63 | 7.31 | 2.21 | 6.07 | 0.2821 | 0.2741 | 0.000804 | 2.46 |
| 2.25 | 1.5 | 2.35 | 6.91 | 1.33 | 4.43 | 0.1883 | 0.3053 | 0.00130 | 2.46 |
| 2.25 | 1.75 | 2.12 | 6.62 | 0.842 | 3.54 | 0.1351 | 0.3111 | 0.00378 | 2.42 |
| 2.25 | 1.94 | 6.39 | 0.357 | 2.80 | 0.1033 | 0.2553 | 0.01533 | 2.41 |
| 2.25 | 2.25 | 1.79 | 6.20 | 0.403 | 2.78 | 0.0829 | 0.2111 | 0.01982 | 2.40 |
| 2.25 | 2.5 | 1.67 | 6.05 | 0.300 | 2.62 | 0.0689 | 0.1671 | 0.00950 | 2.40 |
| 2.25 | 2.75 | 1.56 | 5.92 | 0.232 | 2.53 | 0.0588 | 0.1181 | 0.000793 | 2.40 |

* Without the binding energy correction used in panel (b) of Figure 5.
| Name   | Kinetic energy [10^41 erg] | 56Ni mass [M⊙] | Type | Reference | Name   | Kinetic energy [10^41 erg] | 56Ni mass [M⊙] | Type | Reference |
|--------|---------------------------|----------------|------|-----------|--------|---------------------------|----------------|------|-----------|
| 69L    | 2.4±0.7                   | 0.082±0.034    | II 3 | 73R       | 2.7±1.2 | 0.084±0.044    | II 3           |    |           |
| 83I    | 1                         | 0.15           | Ibc 3| 83N       | 1      | 0.15           | Ibc 3          |    |           |
| 84L    | 1                         | 0.15           | Ibc 3| 86L       | 1.3±0.5 | 0.034±0.013    | II 3           |    |           |
| 87A    | 1.7                       | 0.075          | II 3 | 88A       | 2.2±1.2 | 0.062±0.031    | II 3           |    |           |
| 89L    | 1.2±0.6                   | 0.015±0.008    | II 3 | 90E       | 3.4±1.0 | 0.062±0.022    | II 3           |    |           |
| 91G    | 1.2±0.6                   | 0.022±0.006    | II 3 | 92H       | 3.1±1.3 | 0.129±0.037    | II 3           |    |           |
| 92ba   | 1.3±0.5                   | 0.019±0.007    | II 3 | 93J       | 2.4±1.1 | 0.13±0.04     | II 11          |    |           |
| 94I    | 1.2±0.5                   | 0.08±0.01      | Ibc 11| 96cb     | 2.1±0.9 | 0.12±0.03      | II 11          |    |           |
| 97D    | 0.9                       | 0.006          | II 3 | 97ef      | 8      | 0.15           | Ibc 3          |    |           |
| 98A    | 5.6                       | 0.11           | II 7 | 98bw      | 38.3±13 | 0.76±0.11      | Ibc 11         |    |           |
| 99br   | 0.6                       | 0.0016±0.00011 | II 3 | 99cr     | 1.9±0.6 | 0.09±0.027     | II 3           |    |           |
| 99dn   | 7.3±2.6                   | 0.12±0.01      | Ibc 11| 99em     | 1.3±0.1 | 0.036±0.009    | I 1            |    |           |
| 99em   | 1.2±0.3                   | 0.042±0.019    | II 3 | 99ex     | 3.6±1.3 | 0.18±0.04      | Ibc 11         |    |           |
| 99gi   | 1.2±0.7                   | 0.018±0.009    | II 3 | 99bb     | 4.4±0.3 | 0.083±0.039    | I 1            |    |           |
| 02ap   | 6.3±2.9                   | 0.09±0.01      | Ibc 11| 03Z      | 0.24±0.018 | 0.0063±0.0066 | I 4            |    |           |
| 03bg   | 3.9±1.8                   | 0.19±0.02      | II 11| 03gd     | 1.4±0.3 | 0.016±0.006   | II 5           |    |           |
| 03jd   | 7.2±2.4                   | 0.51±0.09     | Ibc 11| 04aw     | 6.0±3.3 | 0.26±0.04     | Ibc 11         |    |           |
| 04dk   | 5.3±2.2                   | 0.27±0.04     | Ibc 11| 04dn     | 7.1±3.5 | 0.22±0.03     | Ibc 11         |    |           |
| 04et   | 2.1±0.3                   | 0.069±0.009   | II 1  | 04fe     | 3.6±1.7 | 0.3±0.05      | Ibc 11         |    |           |
| 04ff   | 2.9±1.6                   | 0.22±0.04     | II 11| 04gg     | 5.2±2.1 | 0.14±0.05     | Ibc 11         |    |           |
| 05az   | 3.9±1.7                   | 0.38±0.07     | Ibc 11| 05bf     | 0.5±0.3 | 0.09±0.02     | Ibc 11         |    |           |
| 05cs   | 0.42±0.03                 | 0.0082±0.0016 | II 1  | 05cs     | 0.16±0.03 | 0.0066±0.0024 | I 2            |    |           |
| 05hg   | 2.1±1.2                   | 0.76±0.1      | Ibc 11| 06T      | 1.7±0.5 | 0.1±0.02      | II 11          |    |           |
| 06au   | 3.2                       | 0.073         | II 8 | 06el     | 6.4±4.1 | 0.16±0.03     | II 11          |    |           |
| 06ep   | 4.1±2.2                   | 0.08±0.02     | Ibc 11| 06ov     | 2.4     | 0.127         | II 8           |    |           |
| 07C    | 3.5±2.3                   | 0.02±0.04     | Ibc 11| 07Y      | 1.9±1.8 | 0.05±0.03     | Ibc 11         |    |           |
| 07gr   | 2.9±1.1                   | 0.1±0.02      | Ibc 11| 07od     | 0.5     | 0.02         | II 6           |    |           |
| 07ru   | 13.5±3.4                  | 0.52±0.05     | Ibc 11| 07uy     | 10.8±5.9 | 0.34±0.05     | Ibc 11         |    |           |
| 08B    | 4.0±1.7                   | 0.1±0.01      | Ibc 11| 08ax     | 2.6±1.1 | 0.16±0.04     | Ii 11          |    |           |
| 08In   | 0.505±0.034               | 0.015±0.005   | II 1  | 08in     | 0.48±0.098 | 0.012±0.005 | I 2            |    |           |
| 09E    | 0.6                       | 0.04          | II 7 | 09bb     | 9.2±3.2 | 0.31±0.04     | Ibc 11         |    |           |
| 09bw   | 0.3                       | 0.022         | II 10| 09jfl    | 8.9±3.3 | 0.24±0.02    | Ibc 11         |    |           |
| 11bm   | 14.3±5.6                  | 0.71±0.09     | Ibc 11| 11dh     | 1.5±0.7 | 0.09±0.03     | II 11          |    |           |
| 11hs   | 1.1±0.5                   | 0.04±0.01     | II 11| 12A      | 0.48     | 0.011         | II 9           |    |           |
| 12A    | 0.525±0.056               | 0.016±0.006   | II 1  | 12aw     | 1.5      | 0.06         | II 10          |    |           |
| iPTF13bnv | 1.8±0.8                  | 0.07±0.02     | Ibc 11|          |        |              |                |    |           |

**Note.** — REFERENCES: (1) Urobin & Chugai (2014);(2) Spiero et al. (2013);(3) Hamuy (2003);(4) Hendry et al. (2005);(5) Inserra et al. (2011);(6) Pastorello et al. (2012);(7) Pastorello et al. (2005);(8) Lattia et al. (2013);(9) Tomasella et al. (2013);(10) Dal Ora et al. (2014);(11) Lyman et al. (2014)