Vibration characteristics and optimization for panel elastically supported in mobile phone

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Abstract. In recent years, usage of smartphones and tablet terminals have spread around the world. These devices using touchscreen as a user interface are currently mainstream. Also, in order to let information of input or output surely know to users, there are some types of equipment having vibrational function in touchscreen. Here, the material of touchscreen consists of glass and the glass panel is fixed to a mobile phone's body by adhesive tapes along the edge of the panel. However, due to the difficulty of design of vibration, it needs investigation with a vast number of manufacturing prototypes. Moreover, the vibration characteristic of panels is not enough regarding intensity and a tactile impression. Therefore, in this study, the authors consider the vibration characteristic of glass panel elastically fixed by adhesive tapes along edges. First, they show modeling of adhesive tapes along edges of panel by using translational and rotational springs. Second, they show formulating vibration characteristic by using an energy method. Third, they optimize spring constants of translational and rotational springs by using Genetic Algorithm (GA) from the obtained expression. Finally, they consider natural frequencies and eigenmodes which were acquired from experiments and simulations.

1. Introduction
In the past two decades, mobile phones have spread extensively worldwide, and recently the majority of mobile phones are so-called smartphones which have more advanced usability than basic feature phones. Some of such advanced mobile phones have vibration function on display glass panels to help users well informed of arrived signals and reliable handling on touchscreen.

For improving the user-friendly operability, the analysis of panel vibration characteristics of mobile phones is required, and the mechanism design for effective excitation of panels should be considered with use of piezo-electric actuators. Such improvement will contribute not only to design of mobile phones but also to that of tablet computers. There have been a vast amount of technical papers on vibration of flat panels and plates [1,2]. Among them there are papers on the forced response of panels (plates) of metal (for example, [3-5]). None of these however focused on the features of panels used in the mobile phones. Also, Leissa [6], Khalili, Malekzadeh, Mittal [7] have resulted on the vibration problem of plates with idealized classical boundary conditions. Aydogdu and Timarci [8] studied about vibration analysis of cross-ply laminated square plates with general boundary conditions. Fares [9] dealt with the problem of optimal design and control of composite
laminated plates with classical boundary conditions using various plate theories. However, in practical situation, the edge restraints are not perfectly compatible with mathematically defined boundary conditions, and would rather be elastic to some extent. The past literature therefore includes some studies on vibration of laminated composite plates having elastic edge springs [10-13]. Mukhopadhya [14] presented the analysis of free vibration of rectangular plates with edges having different degrees of rotational restraint. Later, the same author also offered an analysis of the vibration and buckling of rectangular plates with varying degrees of rotational restraint along the edges [15]. Warburton and Edney [16] used Rayleigh-Ritz method for the analysis of vibrations of rectangular plates with elastically restrained edges.

On the other hand, the glass panel of mobile phones is fixed to the mobile phone’s body by an adhesive tape. There are some reports including elastic edge restraints in frequency calculation of the plate in the past. However, there have been no papers dealing with vibration of the glass panel fixed by an adhesive tape. Therefore, the present study analyzes the vibration characteristics of plates fixed by adhesive tapes by using the Rayleigh-Ritz method where the boundary conditions due to adhesive tape are assumed as elastic springs in the calculation. Measurement of the frequencies and mode shapes are also conducted based on the modal analysis technique. First, the authors measure frequencies under free edges condition, and verify the result of the calculation under classical boundary condition. Second, they measure frequencies of the glass panel fixed by adhesive tapes. Third, spring constants are identified by the GA. Finally, they confirm the validity of the present method from the obtained results.

2. Analysis Method
Figure 1 shows a structure around a smartphone plate fixed by adhesive tape. A glass panel is fixed to a case along the edges by adhesive tape. A cross-section of adhesive tape is homogeneous around the glass.

The strain energy due to bending deformation is given by

\[ U = U_1 + U_2 \]  

where \( U_1 \) and \( U_2 \) are the strain energy stored in a plate and the energy in the elastic restraints, respectively. We can get the strain energy of the plate as
where $A$ is the area of integration.

For bending problem, we can write

$$U_1 = \frac{1}{2} \int_A \left( M_x \kappa_x + M_y \kappa_y + M_{xy} \kappa_{xy} \right) dA$$

$$= \frac{1}{2} \int_A \left( \kappa^T \{M\} \right) dA$$

where

$$\{M\} = \begin{bmatrix} M_x \\ M_y \\ M_{xy} \end{bmatrix}, \quad \{\kappa\} = \begin{bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{bmatrix}$$

From stress-strain relationship below

$$\{M\} = [D]\{\kappa\}$$

We can rewrite the strain energy of the plate as follow

$$U_1 = \frac{1}{2} \int_A \{\kappa\}^T [D]\{\kappa\} dA$$

where

$$\{\kappa\} = \begin{bmatrix} -\frac{\partial^2 w}{\partial x^2} - \frac{\partial^2 w}{\partial y^2} - 2 \frac{\partial^2 w}{\partial x \partial y} \end{bmatrix}^T, \quad [D] = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix}$$

are the curvature vector and the stiffness matrix respectively. In the above equation, $w$ is the deflection of the plate.

If we substitute equation (7) into equation (6) we can get

$$U_1 = \frac{1}{2} \int_A \left( D_{11} \kappa_x^2 + 2D_{12} \kappa_x \kappa_y + D_{22} \kappa_y^2 + 2D_{16} \kappa_x \kappa_{xy} + 2D_{26} \kappa_y \kappa_{xy} + D_{66} \kappa_{xy}^2 \right) dA$$

This is the strain energy equation of a plate.
Figure 2 shows a rectangular plate of dimensions $a \times b$ with thickness $h$, having elastic line restraints along the edges. Each edge is elastically restrained by translational and/or rotational spring. A Cartesian coordinate system $O$-$xy$ is taken with its origin at the center of the plate.

Next, we will consider about the energy stored in the elastic restraints (elastic springs). The energy equation is written by

$$U_2 = \frac{1}{2} \int_{-b/2}^{b/2} k_1 w^2 \left( -\frac{a}{2}, y \right) dy + \int_{-a/2}^{a/2} k_2 w^2 \left( x, -\frac{b}{2} \right) dx$$

$$+ \int_{-b/2}^{b/2} k_3 w^2 \left( \frac{a}{2}, y \right) dy + \int_{-a/2}^{a/2} k_4 w^2 \left( x, \frac{b}{2} \right) dx$$

$$+ \frac{1}{2} \left\{ \int_{-b/2}^{b/2} k_{r1} \left[ \frac{dw}{dx} \left( -\frac{a}{2}, y \right) \right]^2 dy \right\}$$

$$+ \int_{-a/2}^{a/2} k_{r2} \left[ \frac{dw}{dy} \left( x, -\frac{b}{2} \right) \right]^2 dx$$

$$+ \int_{-b/2}^{b/2} k_{r3} \left[ \frac{dw}{dx} \left( \frac{a}{2}, y \right) \right]^2 dy + \int_{-a/2}^{a/2} k_{r4} \left[ \frac{dw}{dy} \left( x, \frac{b}{2} \right) \right]^2 dx \right\}$$

(9)

where $k_i$ ($i=1,2,3,4$) are stiffness of translational springs [N/m] and $k_{ri}$ ($i=1,2,3,4$) are of rotation springs [N] per unit length.

The kinetic energy $T$ is given by
where $\rho$ is the mass per unit volume [kg/m$^3$]. The deflection is assumed in sinusoidal time variation as

$$w(x, y, t) = \tilde{w}(x, y)\sin\omega t = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} C_{mn} X_m(x) Y_n(y)\sin\omega t$$

(11)

In equations (11), the $X_m$ and $Y_n$ are the functions that satisfy the kinematical boundary conditions at the edges, and are expressed in previous studies [17] for classical boundary conditions by

$$X_m(\xi) = \xi^m (1 + \xi)^{BC_1} (1 - \xi)^{BC_3}, Y_n(\eta) = \eta^n (1 + \eta)^{BC_2} (1 - \eta)^{BC_4}$$

(12)

where $BC_1, BC_2, BC_3$ and $BC_4$ are the boundary index [17]. The classical boundary conditions may be one of clamped (denoted by C), simply supported (S) and free (F) edges along the boundary, and the total edge condition of a rectangular plate was given by a combination of C, S and F. In addition to such classical boundary conditions, translational and rotational elastic restraints can be added to each of the four edges, allowing the model to be capable of calculating frequency of laminated plate with non-ideal boundary condition.

In the following analysis, non-dimensional quantities are used for simplicity of analysis.

$$\xi = 2x/a, \quad \eta = 2y/b$$

(non-dimensional coordinates)

$$\Omega = \omega a^2 \left( \frac{\rho h}{D_0} \right)^{1/2}, \quad D_0 = \frac{E_0 h^3}{12(1-\nu_{12}^2 \nu_{21}^2)}$$

(frequency parameter, reference stiffness)

$$\bar{k}_i = \frac{k_i a^4}{d_0}, \quad \bar{k}_{ri} = \frac{k_{ri} a}{d_0}$$

(translational and rotational stiffness)

$$\alpha = a/b$$

(aspect ratio)

After rewriting energy equation by using equations (13), the functional is minimized with respect to unknown coefficients $C_{mn}$

$$\frac{\partial}{\partial C_{\bar{m}\bar{n}}} (U_{\text{max},1} + U_{\text{max},2} - T_{\text{max}}) = 0$$

(14)

where $(\bar{m} = 0, 1, ..., (M - 1); \bar{n} = 0, 1, ..., (N - 1))$ and the frequency equation is obtained as
\begin{equation}
\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \left\{ \left( \frac{D_{11}}{D_0} \right)^{(200)} I^{(2000)} + \left( \frac{D_{12}}{D_0} \right) \alpha^2 \left( I^{(2002)} + I^{(0220)} \right) + \left( \frac{D_{22}}{D_0} \right) \alpha^4 I^{(0022)} \right. \\
+ 2 \left( \frac{D_{16}}{D_0} \right) \alpha \left( I^{(2101)} + I^{(1210)} \right) + 2 \left( \frac{D_{26}}{D_0} \right) \alpha^3 \left( I^{(0121)} + I^{(1012)} \right) \\
+ 4 \left( \frac{D_{66}}{D_0} \right) \alpha^2 I^{(1111)} \right\} \\
\left[ \left( \tilde{k}_1 \psi_{m\bar{m}}(00)(-1) + \tilde{k}_3 \psi_{m\bar{m}}(00)(1) \right) \Phi_{m\bar{n}}(00) \right. \\
\left. + \alpha \left( \tilde{k}_2 \psi_{n\bar{n}}(00)(-1) + \tilde{k}_4 \psi_{n\bar{n}}(00)(1) \right) \Phi_{m\bar{n}}(00) \right] \\
+ \frac{1}{2} \left[ \left( \tilde{k}_{r1} \psi_{m\bar{m}}(11)(-1) + \tilde{k}_{r3} \psi_{m\bar{m}}(11)(1) \right) \right. \\
\left. + \alpha^2 \left( \tilde{k}_{r2} \psi_{n\bar{n}}(11)(-1) + \tilde{k}_{r4} \psi_{n\bar{n}}(11)(1) \right) \Phi_{m\bar{n}}(00) \right] \\
- \frac{1}{16} \Omega^2 I^{(0000)} \int_{m\bar{m}n\bar{n}} C_{mn} = 0 \tag{15}
\end{equation}

where $I$'s are the definite integrals that can be evaluated exactly by

\begin{equation}
I_{m\bar{m}n\bar{n}}(p\bar{p}q\bar{q}) = \Phi_{m\bar{m}}(p\bar{p}) \times \Phi_{n\bar{n}}(q\bar{q}) \tag{16}
\end{equation}

where $p, \bar{p}, q$ and $\bar{q}$ are times of differential. In equation (16), the $I_{m\bar{m}n\bar{n}}(p\bar{p}q\bar{q})$ is a product of two integrals in $x$ and $y$ directions, such as

\begin{equation}
\Phi_{m\bar{m}}(p\bar{p}) = \int_{-1}^{1} \left[ \frac{d^{(p)} X_m(\xi)}{d\xi^{(p)}} \right] \left[ \frac{d^{(p)} X_{\bar{m}}(\xi)}{d\xi^{(p)}} \right] d\xi, \quad \Phi_{n\bar{n}}(q\bar{q}) = \int_{-1}^{1} \left[ \frac{d^{(q)} Y_n(\eta)}{d\eta^{(q)}} \right] \left[ \frac{d^{(q)} Y_{\bar{n}}(\eta)}{d\eta^{(q)}} \right] d\eta \tag{17}
\end{equation}

The integrals along the elastic edges are evaluated by

\begin{equation}
\psi_{m\bar{m}}(p\bar{p}) = \frac{d^{(p)} X_m(\xi)}{d\xi^{(p)}} \frac{d^{(p)} X_{\bar{m}}(\xi)}{d\xi^{(p)}}, \quad \psi_{n\bar{n}}(q\bar{q}) = \frac{d^{(q)} Y_n(\eta)}{d\eta^{(q)}} \frac{d^{(q)} Y_{\bar{n}}(\eta)}{d\eta^{(q)}} \tag{18}
\end{equation}
3. Identification method

The present study employs the GA [18] method to identify spring constants of elastically supported edges due to its high searching ability, high versatility, and simple algorithm. The GA is a metahuristic searching method and mimics the mechanics of natural genetics. Design variables are represented by bit strings as chromosome strings, and new generation is reproduced based on each fitness by genetic operations such as crossover and mutation. In this study, as seen in equation (9), design variables are four translational spring constants and four rotational spring constants. The GA parameters employed in this study is listed in Table 1.

| Parameter           | Value |
|---------------------|-------|
| Generation number $N_g$ | 100   |
| Population size $N_p$   | 50    |
| Crossover probability $P_c$ | 0.9   |
| Mutation probability $P_m$ | 0.05  |

4. Experimental setup

An experiment is conducted to compare with analytical results, and the applicability of the classical plate theory to non-metal, glass plates is verified. Here, the modal analysis technique is applied. Vibration characteristics of glass panel is measured for two types of boundary conditions. As shown in figure 3, one is totally free and it is simulated by supporting soft elastic cord. Since the mass of the glass panel is light, the measurement by the non-contact is desirable. However, in that case, as a measurement point greatly moves under the free boundary condition, the accelerometer is fixed to the glass. The accelerometer is fixed to the position of 1/4 from the lower edge on the centerline of the longer direction of the glass. The sensitivity, frequency range and mass of accelerometer are 1.0 mV/(m/s$^2$), 20 kHz and 0.5g, respectively. Also, as shown in figure 4, a standard impulse hammer is used to excite the panel and its response is measured with a FFT analyzer. The roving hammer technique is applied and sampling frequency is 10 kHz. Material constants and dimensions of the glass panel are as follows.

A glass test model:

\[
E = 69.1 \text{ GPa}, \quad v = 0.21, \quad \rho = 2510 \text{ kg/m}^3
\]
\[
a \times b \times h = 60.4 \times 105.6 \times 0.55 \text{ mm}
\]
\[
m = 8.81 \text{ g}
\]
Thickness ratio: $a / h = 60.4 / 0.55 = 109.8$
(a) Glass plate supported by soft elastic cord
(b) Accelerometer
(PCB Piezotronics, 352C22)

**Figure 3.** System of measurement under totally free condition

(a) Hammer with steel tip covered with vinyl impact cap (Ono-sokki, GK-2110)
(b) FFT analyzer (Ono-sokki, DS-2000) with Modal analysis software (Vibrant Technology, ME’scope VES)

**Figure 4.** System of measurement under both totally free and elastically restrained condition

As shown in figure 5, the other was the panel attached to a large steel block by the adhesive tape along the edge. In this case, a miniature hammer and laser Doppler vibrometer (LDV) were employed.
5. Results and Discussions

Table 2 presents the lowest three natural frequencies [Hz] of glass plates obtained by the present Ritz and the experiment under free edge condition. The calculated results gave slightly lower values compared with the experiment. The maximum difference was 3.0% and it was known from that the both methods resulted in good agreement.

|          | 1st   | 2nd   | 3rd   |
|----------|-------|-------|-------|
| Exp.     | 269.8 | 300.1 | 687.2 |
| Ritz     | 265.7 | 300.3 | 666.3 |
| diff     | -1.5% | 0.1%  | -3.0% |

The mode shapes were also in good agreement, as seen in figure 6.

![Comparison of mode shapes of glass plates under free edge condition](image)

**Figure 6.** Comparison of mode shapes of glass plates under free edge condition
Table 3 lists the results of the experiment and the calculation under elastic edge restraints. The spring constants were identified to have minimum differences between the experiment and calculation by the GA. And the stiffnesses of the translational springs are set to $k_1 = k_3 = 178$ MN/m, $k_2 = k_4 = 21$ MN/m and the rotational springs to $k_{r1} = k_{r3} = 10$ N, $k_{r2} = k_{r4} = 6$ MN. The difference of first frequency and that of second one are as small as that under the free edge condition. The maximum difference is $10.1\%$ for the third mode and larger than the other ones. It is possible that dependency of velocity of elastic modulus of adhesive tape causes large difference.

| Table 3. Lowest three natural frequencies [Hz] obtained by modal analysis experiment and present Ritz method. |
|----------------------------------|-----------------|-----------------|
| Exp.                            | 481             | 971             | 1197            |
| Ritz                            | 486             | 1000            | 1074            |
| diff                            | +0.8\%          | +2.8\%          | -10.1\%         |

On the other hand, both methods give similar mode shapes as figure 7 shown.

![Mode shapes comparison](image)

**Figure 7.** Comparison of mode shapes of glass plates fixed by adhesive tape

6. Conclusions
In this paper, the authors discussed vibration of the glass panel fixed by the adhesive tape along the edges. In the formulation using the Rayleigh-Ritz method the adhesive tape was modeled by both translational and rotational springs. First, the authors conducted the experiment under free edges condition and confirmed the results calculated from the Rayleigh-Ritz method under classical boundary condition compared with the experimental results. Second, they evaluated natural frequencies of the glass panel fixed to the large block by adhesive tapes and identified the translational and rotational spring constants by the GA. As a result, although the difference of frequencies was slightly large in the third mode, the mode shapes were in good agreement. Also, the differences in the
other two modes are as small as with the free edge condition. Therefore, it was found that the present method is effective to predict the vibration behavior of the panel fixed by the adhesive tape.

7. References
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