Sagnac interference in Carbon nanotube loops

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In this paper we study electron interference in nanotube loops. The conductance as a function of the applied voltage is shown to oscillate due to interference between electron beams traversing the loop in two opposite directions, with slightly different velocities. The period of these oscillations with respect to the gate voltage, as well as the temperatures required for the effect to appear, are shown to be much larger than those of the related Fabry-Perot interference. This effect is analogous to the Sagnac effect in light interferometers. We calculate the effect of interactions on the period of the oscillations, and show that even though interactions destroy much of the near-degeneracy of velocities in the symmetric spin channel, the slow interference effects survive.

Transport measurements on single-walled carbon nanotubes have provided many clear demonstrations of quantum strongly correlated phenomena in mesoscopic physics [1]. Particularly exciting examples include Luttinger-liquid behavior [2, 3], and the Fabry-Perot interference [4, 5] of electrons. These effects, in principle, allow the determination of the interaction parameters of the Luttinger liquid. Nevertheless, previous calculations showed only interference between spin and charge modes with energy scales that were of the same order of magnitude, making experimental observation challenging. In this paper, we propose, analyze, and show initial observations of a new mode of interference - Sagnac interference between two time-reversed paths [6]- in nanotubes forming a loop (i.e. a nanoloop), and with Fermi-surface away from the particle-hole symmetric points. This effect produces large-period fluctuations of the conductance as a function of gate-voltage, and source-drain voltage. Due to its large expected period, this fluctuation effect is expected to survive to very high temperatures.

The Sagnac interferometer measures the angular velocity of a ring, by measuring interference fringes between light propagating inside the ring in two opposite directions. Here we consider the Sagnac effect in an armchair nanotube with a loop (Fig. 1), i.e., two points along the nanotube between which electrons can tunnel (such tunneling effects were discussed in Ref. 7, 8). Electrons impinging on point \(X\) in Fig. 1, can either proceed and traverse the loop in a clock-wise direction, or tunnel to point \(X'\), and traverse the loop in a clock-wise direction, reproducing the Sagnac interference effect (Fig. 2b). Note that unlike Fabry-Perot interference, in which the interfering beams traverse a different physical distance (e.g. the loop a different number of times) the Sagnac interference is between two beams traversing exactly the same distance.

The role of the angular velocity for the light interferometer is replaced with a velocity difference for the two counter propagating electronic beams (Fig. 2b). Using a gate voltage \(V_g\), the Fermi-surface is tuned away from the band middle; ignoring interactions, right and left moving electrons in one node may have different velocities: \(v_R = v_F + u\), and \(v_L = v_F - u\) (Fig. 2a)[9]. A small velocity difference \(u\), like a small angular velocity in the light Sagnac effect [6], produces a slow fluctuation of the conductance as a function of \(V_g\) [10]. The phase difference between the two interfering beams is:

\[
\Delta \phi = Lk_L - Lk_R = \frac{L\epsilon_F}{\hbar v_L} - \frac{L\epsilon_F}{\hbar v_R} \approx \alpha eV_g L \frac{2u}{\hbar v_F}, \quad (1)
\]

where \(L\) is the length of the loop, and \(\epsilon_F = \hbar v_F / L k_F / L\). Also, \(\epsilon_F = \alpha eV_g\), where \(\alpha\) is the conversion factor between the gate voltage and change in chemical potential. Interference fringes repeat when \(\Delta \phi = 2\pi n\). Since roughly \(u \propto V_g\), the \(n\)'th fringe is at \(V_g \propto \sqrt{n}\); fringes are more dense as we move away from the middle of the nanotube’s band. For non-interacting electrons, the same fringes should appear as a function of a source-drain voltage, \(V_{sd}\). In the armchair-tube nanoloop, beams moving in the same direction around the loop, but in different nodes, also interfere (Fig. 2c). The two beams in this band-Sagnac effect differ by the same phase due to the time-reversal symmetry.

The simple analysis above, which ignores interactions, already provides a good picture of the Sagnac effect in nanotubes. Nevertheless, thin single-walled nanotubes are expected to have a Luttinger parameter \(g \sim 0.3\) [4]. Interactions change the hydrodynamic velocities in the nanotube dramatically, and may lift the near degeneracy
of the velocities between the interfering beams. In the following we analyze the Sagnac interference effect (and also the Fabry-Perot interference with \( u \neq 0 \)) of interacting electrons. We will show that interactions do not destroy the large-period Sagnac fringes. Yet strong modifications exist: the fringes in the conductance as a function of \( u \) are determined mostly by the bare, non-interacting, velocity spectrum, essentially reflecting Eq. (1), while the fringes in \( V_{sd} \) are modified dramatically, and reflect the four velocities of the tube's hydrodynamic modes.

The bosonized Lagrangian of the two Dirac nodes, with \( \lambda \) parametrizing the density-density interaction, is:

\[
L = \frac{\hbar v_F}{2\pi} \sum_{\sigma,a=1,2} \int dk \left[ \frac{1}{v_F} \partial_k^a \phi^a \nabla \phi^a - (\nabla \theta^a) \right]^2 - (\nabla \phi^a)^2 + (-1)^a \frac{2}{v_F} \partial_k^a \phi^a \nabla \theta^a + \int dk \lambda \left( \sum_{\sigma,a=1,2} \frac{1}{v_F} \nabla \theta^a \right)^2.
\]

The subscript \( a \) designates the Dirac node, \( \sigma \) is the spin. The charge mode, \( \frac{1}{\hbar v_F} \sum_{\sigma,a=1,2} \partial^a \phi^a \), has the interaction Luttinger parameter \( g = (1 + 8\lambda/\pi \hbar v_F)^{-1/2} \). The Fermionic operators are: \( \psi^a_R/L \sim e^{i(\phi^a + \theta^a)} \).

The Fourier-transform of (2) defines an 8 \times 8 quadratic form of the \( \theta \)'s and \( \phi \)'s. Its eight eigenvalues are square-roots of second degree polynomials of \( \omega \) and \( k \). The dispersion of the eight chiral modes is given by the values of

\[
\omega/k \quad \text{which make an eigenvalue vanish.}
\]

We find that the spin anti-symmetric channel consists of four untouched neutral chiral modes, with velocities:

\[
v_{1,2}^1 = \pm v_F + u \quad v_{1,2}^2 = \pm v_F - u
\]

(3)

where \( + \) and \( - \) indicate right and left movers respectively. More interestingly, the remaining four spin-symmetric chiral modes are given by (see also Ref. 13):

\[
v_{1,2}^{1,2} = \frac{1}{\sqrt{2}} \sqrt{1 + \frac{1}{g^2} + 2 \frac{u^2}{v_F^2} \pm \sqrt{\left(1 - \frac{1}{g^2}\right)^2 + 8 \frac{u^2}{v_F^2} \left(1 + \frac{1}{g^2}\right)}}
\]

(4)

\( v_1^1 \) and \( v_2^1 \) describe two right-left symmetric branches of the spectrum. \( v_1^2 \) is the charge mode velocity, when \( u = 0 \). The four velocities are depicted in Fig. 3. In the absence of spin scattering, only the spin-symmetric modes can interfere with each other, but the velocities related to these two modes are very different, as can be seen by Eq. (4). In the following, we determine which velocities appear, then, in the interference fringes of \( V_g \) and \( V_{sd} \).

\( V_g \) couples to the total electronic density, whereas \( V_{sd} \) couples to the density difference of right and left movers:

\[
\mathcal{L}_{g+sd} = \int dk \left( \alpha eV_g \frac{1}{\pi} \nabla \sum_{\sigma,a} \theta^a + eV_{sd} \frac{1}{\pi} \nabla \sum_{\sigma,a} \phi^a \right).
\]

(5)

\( V_g \) changes the chemical potential and Fermi-surface of the electrons. This is seen by absorbing the new term in the \( \theta \) and \( \phi \) gradient terms (where the latter is involved only due to the difference in the original right and left moving velocities). Unlike \( V_g \), \( V_{sd} \) drives the system out of equilibrium: it induces a current. This entails a time-dependent transformation of the bosonic fields to absorb...
the term. The transformation:
\[ \tilde{\sigma}_{a}^{\prime} = \sigma_{a} - \frac{a g^{2} \psi_{a}}{\hbar v_{\psi}} x + \frac{\psi_{a}^{\dagger} h e}{v_{\psi}} t, \]
\[ \tilde{\sigma}_{a}^{\prime} = \sigma_{a} - (-1)^{a} \frac{a g^{2} \psi_{a}}{\hbar v_{\psi}} u / v_{\psi} x, \]
absorbs both \( V_g \) and \( V_{sd} \) in the bosonic fields, with \( \sigma = \uparrow, \downarrow \), and \( a = 1, 2 \) is the node. The slow, \( u \)-dependent, fluctuations can already be noticed in \( \sigma_{a}^{\prime} \)’s \( x \)-dependence.

The above procedure is drawn from Ref. 5.

Following are the possible scattering processes contributing to transport through the nanoloop. For simplicity, we define \( \tilde{\psi}_{a R/L} \sim e^{i(-1)^{a} \frac{a g^{2} \psi_{a}}{\hbar v_{\psi}} x - i(\sigma_{a}^{\prime} + \tilde{\sigma}_{a}^{\prime})} \). The simplest term is the same-node back-scattering:
\[ \tilde{B}(x, t) = b \sum_{\sigma, a} \left( \tilde{\psi}_{a R}^{\dagger}(x) \tilde{\psi}_{a L}^{\dagger}(x) e^{i k_{a} x - i \omega_{sd} u t} + \text{h.c.} \right), \]
with:
\[ k_{g} = \frac{\alpha g^{2} \psi_{a}}{\hbar v_{\psi}(1 - g^{2} u^{2} / v_{\psi})}, \quad \omega_{sd} = e V_{sd} / \hbar v_{F} \].

Second is cross-node back-scattering: \( \tilde{N}_{b}(x, t) = n_{b} \sum_{\sigma, a} \left( \tilde{\psi}_{a R}^{\dagger}(x) \tilde{\psi}_{b L}^{\dagger}(x) e^{i k_{a} x - i \omega_{sd} u t} + \text{h.c.} \right), \)
which is backscattering from node \( a \), to node, \( b \), \( \pi = 3 - a \).

Third is cross-node forward-scattering: \( \tilde{N}_{f}(x, t) = n_{f} \sum_{\sigma, a} \left( \tilde{\psi}_{a R}^{\dagger}(x) \tilde{\psi}_{a L}^{\dagger}(x) e^{i k_{a} x - i \omega_{sd} u t} + \text{h.c.} \right), \)
Most important is the backscattering term arising from tunneling between point \( X \) at \( x = 0 \), and \( X' \) at \( x = L \) (Fig. 2a), i.e., cross-loop back-scattering:
\[ \tilde{K}_{b}(t) = k_{b} \sum_{\sigma, a, b} \left( \tilde{\psi}_{a R}^{\dagger}(0) \tilde{\psi}_{b L}^{\dagger}(L) e^{i k_{a} x - i \omega_{sd} u t} + \text{h.c.} \right), \]

To calculate the conductance fluctuations, we must follow the Kubo/Keldysh formalism as it applies to the various scattering events, \( L_{m}(x, t) \), where \( L_{m} = B, \tilde{N}_{f/b}, \tilde{K}_{b} \). We defer an exact evaluation to a later publication, and concentrate here on the main features of the fluctuations:
\[ \Delta G_{m n} \sim \int_{0}^{\infty} dt \left[ \langle L_{m}(t), \tilde{L}_{n}(0) \rangle - \langle L_{m}, \tilde{L}_{n} \rangle |_{u = 2 e V_{sd} / \hbar} \right]. \]

The second relation, connecting the integral to the correlation’s Fourier transform, is due to the time dependence of the integrand being \( e^{2 i \omega_{sd} u t} = e^{2 i e V_{sd} u t / \hbar} \). In the case of \( L_{m} \) occurring at \( x = 0 \) and \( L_{m} \) at \( x = L \), the dependence of the oscillating part of \( \Delta G_{m n} \) on \( V_{sd} \) is easily seen to be of the form:
\[ \Delta G_{m n} = f \left( e^{2 i \omega_{sd} L / v_{F}^{\dagger}}, e^{2 i \omega_{sd} L / v_{F}^{\dagger}}, e^{2 i \omega_{sd} L / v_{F}^{\dagger}}, e^{2 i \omega_{sd} L / v_{F}^{\dagger}} \right), \]

where \( \psi_{F}^{\dagger} / v_{F} \) are the four propagation velocities of the various hydrodynamic modes found in Eqs. (4) and (3).

Thus we find that the interference fringes as a function of \( V_{sd} \) are determined by the velocities of the interaction-induced four hydrodynamic modes.

Our main result is the interference pattern in \( G \) vs. \( V_{sd} \). From the various scattering events we infer that only two expressions give rise to interference effects in \( V_{sd} \): \( \exp(2 i k_{g} L) \) and \( \exp(2 i k_{g} L / v_{F}^{\dagger}) \), with \( k_{g} \) defined in Eq. (8).

Table I enumerates the conductance fluctuations due to second-order scattering. Our focus is the loop-Sagnac interference between counter-propagating beams in an interacting nanotube, which is the first line in Table I. Indeed, it coincides with the band-Sagnac interference (B-SAG); it is possible, however, to distinguish the two Sagnac modes by applying a magnetic flux to the loop. The band-Sagnac fringes will be unaffected, whereas the loop-Sagnac phase-difference will be shifted by \( 2 e \Phi / \hbar \), where \( \Phi \) is the flux through the loop.

A remarkable difference between the Sagnac and Fabry-Perot interference is the sensitivity to temperature. Using a simple argument we estimate the maximum temperature for each interference. The kinetic energy of a single electron has uncertainty of order \( T \). This could be thought of as an uncertainty in the gate voltage: \( \Delta V_{g} \sim T_{c} / \alpha g^{2} e \). When \( \Delta V_{g} \approx V_{g} \), an interference fringe disappears, which is how we obtain \( T_{c} \) in Table I.

A subtle point is the absence of \( g^{2} \) in the \( T_{c} \) column in Table I; since temperature only smears the kinetic energy of electrons, the effects of interactions should be omitted to first approximation, hence the cancellation of \( g^{2} \).

We find that \( T_{c} \) for the Sagnac modes is \( v_{F} / u \) larger than that of the Fabry-Perot interference.

\[ T_{c}^{SAG} \sim \frac{\pi \hbar v_{F}}{L} \left( 1 - \frac{u^{2} g^{2}}{v_{F}^{2}} \right) v_{F} u \sim \frac{v_{F} T_{FP}}{u} \]

Table I: Interfering contributions to the conductance as a function of \( V_{sd} \). The first row indicates the type of interference. The loop-Sagnac (L-SAG) and band-Sagnac (B-SAG) correspond to Fig. 2b,c respectively. The three Fabry-Perot modes originate from optical-path difference: of two-loops in node 1 or 2 (B-FP±), of one loop in node 1 and one in 2 (FP), or, due to loop-tunneling, one loop in node 1 or 2 (L- FP±). The coherence temperature of each interference mode is determined heuristically by assuming \( T_{c} \sim \alpha g^{2} e V_{sd} / k_{B} \).
Hence the limiting factor for the observation of the Sagnac effect is most likely phonon scattering.

The experimental observation motivating this work is shown in Fig. 4. At $T = 32K$ fast oscillations with period $\delta V_g \sim 0.3V$ appear; we identify them with the loop-FP mode of Table I (Coulomb blockade is determined by the total wire length, and is expected at a much lower $\delta V_g \sim 10^{-3} - 10^{-2}V$). Another fast mode appears at $T = 12K$, with a doubled frequency, $\delta V_g \sim 0.15V$, and therefore fits the regular FP mode. But in addition, a slowly oscillating envelope of the conductance is already evident at $T = 64K$, with the first period being roughly $\delta V_g \sim 20V[14]$. If we identify this with the Sagnac effects, then $\delta V_g^{SAG}/\delta V_g^{FP} \sim 130$. To see if this is indeed feasible, we approximate the nanotube’s dispersion as parabolic, $\pm \varepsilon_k = -\gamma(1 - (2\pi/3m)^2 k^2)$; and $\gamma \approx e/2\pi$. The first fringe due to the Sagnac interference appears when the accumulated phase difference between the two interfering beams is $\pi$:

$$\Delta \phi = \int_0^L \frac{d\varepsilon}{\hbar v_F} \approx \frac{L}{\hbar v_F} \left(\frac{\gamma_s^{SAG}}{\gamma_s^{FP}}\right)^2 = \pi,$$

where the integral is necessary due to the dependence of $u/v_F$ on $\varepsilon$. The first fringe due to Fabry-Perot interference is when $\mu^{FP}L/\hbar v_F = \pi$. Using $L = 7 \mu m$ and $v_F = 8 \cdot 10^5 m/s$, and $\gamma \approx 2.5eV[9]$, we obtain $V_g^{SAG}/V_g^{FP} = \gamma_s^{SAG}/\gamma_s^{FP} \sim 300$, which agrees with the experiment up to a factor of 2. This extra factor might be due to the Fabry-Perot interference arising not from the loop, but from the shorter sections of the nanotube. The experimental results are also consistent with the fact that the Fabry-Perot interference are expected roughly at $T^{FP} \sim \frac{2\hbar v_F}{\gamma} \sim 10K$. Since currently only two samples of the loop geometry are available, we limit ourselves to the order-of-magnitude analysis above, and defer a detailed analysis of the experiment to a future publication.

In addition to the Sagnac interference, we also obtained the modification of the Fabry-Perot interference in the presence of right-left asymmetry and interactions. Most interesting in this respect are the loop- and band-FP effects. In a non-interacting system, these arise from one of the interfering beams going through the loop once, or twice, more than the other, but in a single band. These effects, with a minus sign in Table I, $\delta V_g = \frac{e\hbar}{\gamma s L} \frac{1 - \gamma^2/\gamma_s^2}{v_F - u}$, present the only interference that may increase their period as $V_g$ tunes away from the Dirac nodes. Slow nanotube conductance oscillations were also seen in Ref. 15, and at first interpreted as Fabry-Perot interference between two closely-spaced localized impurities on the nanotube. Ref. 11, tried to explain them as an impurity interference effect, similar to what we call the band-Sagnac effect (Fig. 2c). But the period of these oscillations increases with distance from the middle of the band, contrary to Ref. 15 observations. Thus we conclude that the slow interference of Ref.[15] may indeed be due to band-Fabry-Perot interference between closely-spaced impurities.

In this paper we discussed the Sagnac and Fabry-Perot interference effects in interacting nanotube loops, with $v_R - v_L = 2u \neq 0$. We found that $V_g$ changes the ‘carrier wave’ and induces fluctuations that depend mostly on the bare dispersion of the nanotube, while $V_{sd}$ produces fluctuations whose periods depend on the velocities of the non-equilibrium hydrodynamic modes. By studying these conductance fluctuations experimentally, one could in principle extract all hydrodynamic velocities, the interaction parameter, and the bare electron dispersion. We also provided rough estimates of the coherence temperatures, $T_c$, required to see the Sagnac interference, and showed that it is much higher than that of the Fabry-Perot interference. The estimates of $T_c$ are expected to be modified by a precise inclusion of interactions; this we will pursue in a future publication, in addition to the explicit dependence of the conductivity on $V_{sd}$. The Sagnac interference is closely related to the interference giving rise to weak localization, therefore its analysis could directly determine the temperature and interaction dependence of the electronic dephasing time $\tau_\phi$ (see further Ref. [16, 17]). Here we showed that the Sagnac effect clearly survive interactions at $T = 0$, therefore our results can be interpreted as evidence for the divergence of $\tau_\phi$ in an interacting electronic system.

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