Study on Slope Stability of Baoshi Expressway Based on the Analytic Hierarchy Process

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Abstract: Combining with the actual situation of section K13+165 in Baoshi Expressway, the hierarchical structure model of the slope stability is established based on the analytic hierarchy process (AHP) in operational research. Fourteen factors, which affected the slope stability, are analysed, quantitatively calculated and ranked. The consistency test was carried out. On this basis, the influence weights of each factor are sorted, and the main influencing factors of the slope stability are found out. The research can provide a basis for the maintenance and treatment of Baoshi expressway.

1. Introduction
The slope is a dynamic system and its stability is affected by many factors, whose analytical methods are also different. At present, the methods of slope stability analysis can be divided into qualitative and quantitative analysis. The qualitative analysis includes the engineering analogy and graphic method [1]. The quantitative analysis includes the limit equilibrium method [2-3], finite element method [4-6], finite difference method [7-8], and fuzzy mathematics method [9]. Based on previous research methods, this paper mainly introduces the analytic hierarchy process (AHP) in the stability analysis of the Expressway slope.

2. Introduction of the AHP and analysis procedure
The AHP is a multi-project and multi-objective decision-making method proposed by professor Saaty, an American operational researcher, in the 1970s. After years of development, the AHP has become the most popular and effective method in evaluation tools [10]. It divides complex problems into related orderly hierarchies and makes them into orderly multi-objective and multi-criteria decision-making methods. It is an effective analysis method combining the qualitative and quantitative analysis. The implementation process of the AHP can be divided into the following four steps.

2.1. Establishing the hierarchical structure model
After a full understanding of the system, the factors in the system are divided into several levels. The hierarchical structure and the subordinate relationship among various factors are illustrated in the form of block diagrams. When there are more factors at a certain level, the level can be further divided into several sub-levels.

2.2. Constructing the judgment matrix
The judgment matrix are the relative importance of the factors at each level, which are expressed numerically by introducing appropriate scales. The judgment matrix represents the comparison of
relative importance, which indicates the relationship between a factor of a higher level and its related factors of this level.

To quantify the evaluation, the numerical judgment matrix is formed. The AHP uses scaling methods of 1-9 and its reciprocal in Table 1. The practical ratio can explain the importance of comparative factors, and the corresponding elements of the judgment matrix can be taken as the ratio [11].

2.3. The weight calculation and consistency test

The weight is the relative importance of the corresponding factors at the same level to a certain factor at the next level. Concerning judgment matrix A, the eigenvalues and eigenvectors satisfying $A\omega = \lambda_{\text{max}} \omega$ are calculated, where $\omega$ is the normalized eigenvector corresponding to $\lambda_{\text{max}}$, i.e., the weight value. The consistency index $CI = (\lambda_{\text{max}} - n) / (n - 1)$ needs to be calculated for the consistency test. The average random consistency index $RI$ was listed in Table 2, which was calculated through 1000 repetitions in order 1-15. Gong et al. [12] of Tianjin University provided it in 1986. When the random consistency ratio satisfies $CR = CI / RI < 0.1$, it is considered that the results of hierarchical single ranking have the satisfactory consistency. Otherwise, it is necessary to adjust the element values of the judgment matrix.

| Scaling | Meaning |
|---------|---------|
| 1       | It means that the two factors are equally important. |
| 3       | It means that one factor is slightly more important than the other. |
| 5       | It means that one factor is obviously important than the other. |
| 7       | It means that one factor is strongly important than the other. |
| 9       | It means that one factor is extremely important than the other. |
| 2, 4, 6, 8 | It represents the median of the above adjacent judgments |
| Reciprocal | If the importance ratio of factor $i$ to $j$ is $b_{ij}$, then the importance ratio of factor $j$ to $i$ is $1/b_{ij}$. |

| Order $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | … |
|-----------|---|---|---|---|---|---|---|---|---|----|----|----|---|
| $RI$      | 0 | 0 | 0.52 | 0.89 | 1.12 | 1.26 | 1.36 | 1.41 | 1.46 | 1.49 | 1.52 | 1.54 | … |

2.4. Hierarchical population ordering

The hierarchical population ordering is to calculate the ranking value of the relative importance of all factors at the same level to the highest level (total objective). This process is carried out from the highest level to the lowest level one by one.

3. Application of the AHP in Baoshi Expressway

The geological conditions of K13+165 in Baoshi Expressway are special, and the tight folds and faults are well developed. What’s more, the structural deformation of strata is intense, and the distribution of joints and cleavages is extensive and complex. The tectonism is multistage and inherited, and there are many cataclastic zones. The above conditions have a great influence on the stability of K13+165 slope in Baoshi Expressway. The slope angle is determined by the analogy method in design, and the final slope angle is generally not more than 47 degrees. Therefore, it is necessary to analyse the slope stability in Baoshi Expressway and determine the influence of various factors on the slope stability. Consequently, it can ensure the slope stability and reduce the economic losses of personnel and equipment caused by landslides.
3.1. Grading of slope stability and selection of action factors

Up till now, the scholars at home and abroad have different criteria for slope stability classification. In this paper, the slope stability grade is divided into five grades, i.e., stability, comparatively stable, general stability, unstable and extremely unstable, which are expressed by symbols I, II, III, IV and V, respectively. According to the national standards, industry specifications, existing research results [13-14] and the analysis of slope stability, the qualitative or quantitative analysis relationship between each factor and slope stability grade is established, as shown in Table 3.

Table 3. The rating table of influencing factors.

| Grade of Slope Stability          | I         | II        | III       | IV        | V         |
|-----------------------------------|-----------|-----------|-----------|-----------|-----------|
| **The geological conditions of slope** |           |           |           |           |           |
| Rock properties                   | Hard      | Relatively hard | Relatively weak | weak      | Extremely weak |
| The relationship between structural plane and slope | Very advantageous | Advantageous | A little advantageous | Disadvantageous | Very disadvantageous |
| Internal friction angle / (°)     | >35       | 35-28     | 28-21     | 21-14     | <14       |
| Cohesive force /MPa               | >0.25     | 0.25-0.15 | 0.15-0.1  | 0.1-0.05  | <0.05     |
| **The geometric conditions of slope** |           |           |           |           |           |
| The angle of slope / (°)          | <20       | 20-30     | 30-40     | 40-50     | >50       |
| The total height of slope /m      | <30       | 30-60     | 60-100    | 100-200   | >200      |
| The morphology of slope           | Linear shapes | Between linear and convex shapes | Convex shapes | Between convex and concave shapes | Concave shapes |
| **The hydrologic al climate**     |           |           |           |           |           |
| The precipitation amount/mm       | <400      | 400-600   | 600-800   | 800-1000  | >1000     |
| Groundwater                       | Never     | Tiny      | A little obvious | Relatively abundant | Abundant |
| Temperature                       | Never     | Tiny      | Weak      | Relatively strong | Strong |
| **Other factors**                 |           |           |           |           |           |
| Weathering action                 | Never     | Tiny      | Weak      | Relatively strong | Strong |
| Blasting vibration                | Never     | Tiny      | Weak      | Relatively strong | Strong |
| Maximum earthquake intensity      | <3        | March 5th | May 7th   | July 8th  | >8        |
| Other anthropic factors           | Never     | Tiny      | Weak      | Relatively strong | Strong |

3.2. Establishment of the hierarchical structure model

According to the factors above-affected of the slope stability, the hierarchical structure model is established, as shown in Figure 1.
Figure 1. The tomographic structure model for slope stability evaluation.

The weights of each factor at each level are calculated.

3.3. The calculation of the hierarchical structure model

3.3.1. The calculation of approximate eigenvalues (weight vectors) of matrices. The calculation steps are as follows: the normalization of judgment matrix → the column addition of matrix → the row normalization of the judgment matrix. The equation for calculating the normalization of judgment matrix by column is as follows:

$$
\omega_{ij} = \frac{b_{ij}}{\sum_{k=1}^{n} b_{kj}}, \quad (i, j = 1, 2, \ldots, n)
$$

(1)
\[ A \rightarrow \begin{bmatrix} 0.545 & 0.5 & 0.6 & 0.5 \\ 0.1364 & 0.125 & 0.1 & 0.125 \\ 0.1818 & 0.25 & 0.2 & 0.25 \\ 0.1364 & 0.125 & 0.1 & 0.125 \end{bmatrix} \rightarrow \begin{bmatrix} 2.1455 \\ 0.4864 \\ 0.8818 \\ 0.4864 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5451 \\ 0.1236 \\ 0.2240 \\ 0.1236 \end{bmatrix} = \omega^{(B)} \] (2)

\[ B_1 \rightarrow \begin{bmatrix} 0.462 & 0.5 & 0.429 & 0.429 \\ 0.231 & 0.25 & 0.286 & 0.286 \\ 0.154 & 0.125 & 0.143 & 0.143 \\ 0.154 & 0.125 & 0.143 & 0.143 \end{bmatrix} \rightarrow \begin{bmatrix} 1.82 \\ 1.053 \\ 0.565 \\ 0.565 \end{bmatrix} \rightarrow \begin{bmatrix} 0.455 \\ 0.263 \\ 0.141 \\ 0.141 \end{bmatrix} = \omega^{(C)}_1 \] (3)

\[ B_2 = \begin{bmatrix} 0.2 & 0.2 & 0.2 \\ 1.2 & 1.2 & 1.2 \\ 0.3 & 0.3 & 0.3 \end{bmatrix} \rightarrow \begin{bmatrix} 1.5 \\ 1.5 \\ 0.6 \end{bmatrix} = \omega^{(C)}_2 \] (4)

\[ B_3 \rightarrow \begin{bmatrix} 0.588 & 0.5 & 0.615 \\ 0.384 & 0.118 & 0.077 \\ 0.294 & 0.4 & 0.308 \end{bmatrix} \rightarrow \begin{bmatrix} 1.703 \\ 0.295 \\ 1.002 \end{bmatrix} \rightarrow \begin{bmatrix} 0.5677 \\ 0.0983 \\ 0.334 \end{bmatrix} = \omega^{(C)}_3 \] (5)

\[ B_4 = \begin{bmatrix} 0.0625 & 0.03 & 0.0566 & 0.087 \\ 0.25 & 0.1212 & 0.0943 & 0.1304 \\ 0.3125 & 0.3636 & 0.283 & 0.2609 \\ 0.375 & 0.4848 & 0.566 & 0.5217 \end{bmatrix} \rightarrow \begin{bmatrix} 0.2364 \\ 0.5959 \\ 1.9475 \\ 1.9475 \end{bmatrix} \rightarrow \begin{bmatrix} 0.0591 \\ 0.149 \\ 0.3052 \\ 0.4871 \end{bmatrix} = \omega^{(C)}_4 \] (6)

3.3.2. Calculating the maximum eigenvalue. According to the equation \( \lambda_{\text{max}} = \frac{1}{n} \sum_{i=1}^{n} (A \omega)_i / \omega_i \), the following results can be obtained, \( \lambda^{(B)}_{\text{max}} = 4.01, \lambda^{(C)}_{\text{max}} = 4.01, \lambda^{(C)}_{\text{max}} = 3, \lambda^{(C)}_{\text{max}} = 2.89, \) and \( \lambda^{(C)}_{\text{max}} = 4.139. \)

3.3.3. The consistency test. According to formula \( CI = (\lambda_{\text{max}} - n) / (n - 1) \) and consistency test standard \( CR = CI / RI < 0.1 \) in Table 2, the matrix consistency can be tested. According to the calculation, the \( CR \) of the five judgment matrices is less than 0.1. Accordingly, the consistency test of the five judgment matrices has passed the test.

3.4. Hierarchical population ordering

To obtain the weights of the lowest influencing factors on the highest level, i.e., overall goal, it is necessary to rank in the hierarchy. The ranking results are shown in Table 4.

| Level \( C \) | Level \( B \) | \( \text{The total} \) | \( \text{Total} \) |
|-------------|-------------|----------------|-------------|
|             | \( B_1 \)    | \( B_2 \)    | \( B_3 \)    | \( B_4 \)    | \text{sorting} | \text{ranking} |
| \( C_1 \)   | 0.455       | 0            | 0            | 0            | 0.2480        | 1            |
| \( C_2 \)   | 0.263       | 0            | 0            | 0            | 0.1434        | 2            |
| \( C_3 \)   | 0.141       | 0            | 0            | 0            | 0.0769        | 4            |
| \( C_4 \)   | 0.141       | 0            | 0            | 0            | 0.0452        | 5            |
| \( C_5 \)   | 0           | 0            | 0            | 0            | 0.0368        | 8            |
| \( C_6 \)   | 0           | 0            | 0            | 0            | 0.0342        | 9            |
| \( C_7 \)   | 0           | 0            | 0            | 0            | 0.0325        | 11           |
| \( C_8 \)   | 0           | 0            | 0.5677       | 0            | 0.0316        | 3            |
| \( C_9 \)   | 0           | 0            | 0.0983       | 0            | 0.0308        | 12           |
| \( C_{10} \)| 0           | 0            | 0.334        | 0            | 0.0289        | 6            |

Table 4. Total ranking and weights of factors affecting slope stability.
3.5. Result analysis
It can be seen that among all the factors, the rock properties, structural plane, slope relationship, process precipitation, internal friction angle and cohesion have a greater impact on slope stability from Table 4. Therefore, in the process of slope stability evaluation and slope maintenance, these factors should be taken into consideration.

4. Conclusion
Through the stability analysis of K13+165 slope in Baoshi Expressway, it is proved that the AHP is scientific and practical in multi-level and multi-scheme analysis. What’s more, the conclusion is relatively accurate and reliable. However, the accuracy of the AHP is easily affected by subjective and objective factors, such as the knowledge level and work experience of the evaluators. Therefore, it is necessary to adjust the evaluation indexes according to the actual situation of the slope. Thus, to ensure the safety of the slope and better economic benefits, the analysis results, which are according to the actual situation of the slope, can be obtained.

References
[1] Gao J Y, Xing Y C, Chen Y X. (2011) Prediction model for stability of high loess slopes. J. Chinese Journal of Geotechnical Engineering, 33(1): 163-169.
[2] Zhou J, Li E M, Yang, S, Wang M Z, Shi X Z, Yao S, Mitri H S. (2019) Slope stability prediction for circular mode failure using gradient boosting machine approach based on an updated database of case histories. J. Safety Science, 118: 505-518.
[3] Liu Y D, Yang X Q, Xu L, Lin Y K, Gong X. (2018) A stability analysis of slope considering strength anisotropy in soils. J. Journal of Guangdong University of Technology, 35(6): 61-66.
[4] Chen X Y, Zhang L L, Chen L H, Li X, Liu D S. (2018) Slope stability analysis based on the coupled Eulerian-Lagrangian finite element method. J. Bulletin of Engineering Geology and the Environment, 78(06): 4451-4463.
[5] Belytschko T, Lu Y Y, Gu L. (1994) Element-free Galerkin methods. J. International Journal of Numerical Methods Engineering, 37(2): 229-256.
[6] Cárdenas I C. (2018) On the use of Bayesian networks as a meta-modelling approach to analyse uncertainties in slope stability analysis. J. Georisk: Assessment and Management of Risk for Engineered Systems and Geohazards, 13(1): 53-65.
[7] Zhou S L, Wang Y S, Wei J F, Chen X. (2010) Influence factors of slope stability based on the finite difference method. J. Journal of Chongqing Jiaotong University (Natural Science), 29(5): 737-741.
[8] Liu X N, Shi C, Li D J, Wang F. (2014) Application of limit equilibrium finite difference method in slope engineering based on slip-line cluster analysis. J. Science Technology and Engineering, 14(35): 97-103.
[9] Giasi C I, Masi P, Cherubini C. (2003) Probabilistic and fuzzy reliability analysis of a sample slope near Aliano. J. Engineering Geology, 67: 391-402.
[10] Kang F, Han S X, Salgado R, Li J J. (2015) System probabilistic stability analysis of soil slopes using Gaussian process regression with Latin hypercube sampling. J. Computers and Geotechnics, 63(9): 13-25.
[11] Sahoo S, Dhar A, Kar A. (2016) Environmental vulnerability assessment using grey analytic hierarchy process based model. J. Environmental Impact Assessment Review, 56(1): 145-154.

[12] Zhao H C, He J S, Xu S B. (1985) Application of analytic hierarchy process in science and technology management. J. Science of Science and Management of S. & T, (6): 23-25.

[13] Liu D L, Tham G H, Lee P Q G, Tsui Y, Wang Y H. (1999) The stability of rock slope and fuzzy comprehensive evaluation method. J. Chinese Journal of Rock Mechanics and Engineering, 18(2): 170-175.

[14] Sun S H, Cao L Z, Zhang L X. (2007) Fuzzy comprehensive judgment method used in slope stability of strip mine. J. Journal of Liaoning Technical University, 26(2):177-179.