Radio Frequency Spectroscopy of Trapped Fermi Gases with Population Imbalance

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Motivated by recent experiments, we address, in a fully self consistent fashion, the behavior and evolution of radio frequency (RF) spectra as temperature and polarization are varied in population imbalanced Fermi gases. We discuss a series of scenarios for the experimentally observed zero temperature pseudogap phase and show how present and future RF experiments may help in its elucidation. We conclude that the MIT experiments at the lowest $T$ may well reflect ground state properties, but take issue with their claim that the pairing gap survives up to temperatures of the order of the degeneracy temperature $T_F$ at unitarity.

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The field of ultra-cold Fermi gases undergoing BCS-BEC crossover is particularly exciting because these superfluids exhibit a rather novel form of fermionic superfluidity: pairing begins at temperature $T^*$ while condensation take place at a significantly lower temperature $T_c$. Concomitantly, the normal state fermionic spectrum exhibits an excitation gap or "pseudogap" [1,2,3]. Experiments from MIT [4] on population imbalanced Fermi gases simultaneously observe vortices and condensate fractions to estimate $T_c$. Combined with RF spectroscopy [5] they have thereby established that a pairing gap is indeed visible above $T_c$. An even more striking claim from this group [5] is that in a highly polarized gas, one finds a ground state in which the pairing gap survives up to temperatures of the order of the degeneracy temperature $T_F$ at unitarity.

In this paper we use the standard one channel grand canonical Hamiltonian $H - \mu_1 N_1 - \mu_2 N_2$ which describes pairing between states $|1\rangle$ and $|2\rangle$ and for definiteness take state $|1\rangle$ as majority and state $|2\rangle$ as minority, unless indicated otherwise. We additionally ignore the interaction between state $|3\rangle$ and states $|1\rangle$ and $|2\rangle$, since mean-field energy shifts associated with the interaction between $|1\rangle$ and $|2\rangle$ and between $|1\rangle$ and $|3\rangle$ nearly cancel each other, as observed experimentally. Thus state $|3\rangle$ is associated with a noninteracting gas. In addition, there is a transfer matrix element $T_{k,p}$ from $|2\rangle$ to $|3\rangle$ given by $H_T = \sum_{k,p} \delta_{k,p} c_{k,p}^\dagger c_{2,k} + h.c.$. For plane wave states, $T_{k,p} = T \delta(q_L + k - p) \delta(\omega_{kp} - \omega_L)$. Here $q_L \approx 0$ and $\omega_L$ are the momentum and energy of the RF laser field, and $\omega_{kp}$ is the energy difference between the initial and final state. It should be stressed that unlike conventional quasi-particle tunneling, here one requires not only conservation of energy but also conservation of momentum. The RF current is defined as $I = \langle \hat{N}_3 \rangle = i \langle [H, \hat{N}_3] \rangle$. Using standard linear response theory one finds

$$ I = 2T^2 {\text{Im}}[X_{res}(-\omega_L + \mu_3 - \mu_2)]$$

where $\mu_3$ is the chemical potential of $|3\rangle$ and $\omega_{23}$ is the energy splitting between $|3\rangle$ and $|2\rangle$. After Matsubara summation and using $A_3(k, \nu) = 2\pi \delta(\nu - (\epsilon_k + \omega_{23} - \mu_3))$ as well as $A_2(k, \nu) = -2 \text{Im} A_3(k, \nu + i\nu^+) \equiv f(\epsilon_k - \omega - \mu) - f(\epsilon_k + \omega_{23} - \mu_3) \rangle$ rewrites the spectral functions for states $|3\rangle$ and $|2\rangle$, respectively, we have

$$ I(\omega) = \frac{T^2}{2\pi} \sum_k \left[ f(\epsilon_k - \omega - \mu) - f(\epsilon_k + \omega_{23} - \mu_3) \right],$$

where $\omega \equiv \omega_L - \omega_{23}$ is defined to be the RF detuning and $f(x)$ is the Fermi distribution function. In the above equations the
retarded response function $X_{sc}(\omega) = X(\omega_n \rightarrow \omega + i0^+) \), and we have expressed the linear response kernel $X$ in terms of single particle Green’s functions. We define $\omega_n$ and $\nu_n$ as even and odd Matsubara frequencies, respectively and $G_2$ is the fully dressed Greens function for the state 2 spins. (We use the convention $\hbar = k_B = 1$).

In our $T$-matrix formalism [11, 12], $G_2(\mathbf{k}, \nu)$ contains two self-energy contributions deriving from condensed Cooper pairs ($\Sigma_{sc}$) as well as from finite momentum pairs ($\Sigma_{pg}$). The latter represent pseudogap effects which first appeared in the spectral function, in Ref. [18]. We have $\Sigma = \Sigma_{pg} + \Sigma_{sc}$, where $\Sigma_{pg}(\mathbf{k}, \nu) = \frac{\Delta_{pg}^2}{\nu + \xi_{k,1} + \gamma_k}$ and $\Sigma_{sc}(\mathbf{k}, \nu) = \frac{\Delta_{sc}^2}{\nu + \xi_k}$, here $\Delta_{sc}$ is the superfluid order parameter, and $\gamma \neq 0$ is associated with the life time effects of noncondensed pairs. The resulting spectral function can readily be computed as

$$A_2(\mathbf{k}, \nu) = \frac{2\Delta_{pg}^2\gamma(\nu + \xi_k)^2}{(\nu + \xi_k)^2(\nu - E_k^2) + \gamma^2(\nu^2 - \xi_k^2 - \Delta_{sc}^2)^2}.$$ 

(3)

Here $\xi_{k,1} = \epsilon_k - \mu_1$, $\xi_k = \epsilon_k - \mu$, $\mu = (\mu_1 + \mu_2)/2$, $\hbar = (\mu_1 - \mu_2)/2$, and $\nu = \nu - \hbar$. In the quasiparticle dispersion, $E_k$, $\Delta^2(T) = \Delta_{sc}^2(T) + \Delta_{pg}^2(T)$. The precise value of $\gamma$, and even its $T$-dependence is not particularly important, as long as it is non-zero at finite $T$. In practice, we choose its value based on the experimental atomic peak width. As is consistent with the standard ground state constraints, $\Delta_{pg}$ vanishes at $T = 0$, where all pairs are condensed. Above $T_c$, we have Eq. (3) with $\Delta_{sc} = 0$. Because the energy level difference $\omega_{23}$ ($\approx 80$ MHz) is so large compared to other energy scales in the problem, the state $|3\rangle$ is initially empty and thus $f(\epsilon_k + \omega_{23} - \mu_3) = 0$ in Eq. (2). Once the trap is incorporated, Eqs. (2) and (3) can then be used to compute the local current density $I(r, \omega)$ and then to obtain the total net current $I_F(\omega) = \int d^3r I(r, \omega)n_{\sigma}$ with $\sigma = 1, 2$. Unless stated otherwise the energy unit $T_F$ represents the Fermi temperature for the noninteracting unpolarized Fermi gas with the same total particle number.

To treat the trap, we assume a spherically symmetrical harmonic oscillator potential $V(r) = m\omega_r^2r^2/2$. The density, excitation gap and chemical potential will vary along the radius. These quantities can be self-consistently determined using the local density approximation (LDA). The phase diagram, representing the stable regimes for phase separation, the Sarma phase as well as the normal Fermi gas phases as a function of temperature and polarization has been mapped out [2, 10]. Since it is at the heart of the current experiments, one must also determine [3] where pairing occurs without superfluidity. These non-condensed pair effects (which are generally ignored in the literature) are also essential for arriving at physical values for $T_c$. Important for the present purposes, the phase separated state is not associated with pseudogap effects, unlike the Sarma state. The same behavior is mirrored in the density profiles. The Sarma phase consists of a superfluid core followed by a correlated “mixed normal” or pseudogap regime, followed by a Fermi gas in the outer regions of the trap. The phase separated state, by contrast has an essentially unpolarized superfluid core separated from a non-correlated normal Fermi gas by a sharp interface. The phase boundary is determined [9] by the balance of pressure.

To begin, it is useful to present the prototypical behavior for the RF spectra. Quite generally we find that in the phase separated state (low $T$) there is a single pairing peak, whereas in the pseudogap phase (higher $T$) there are two peaks. And the Sarma phase (intermediate $T$) may have either one or two, depending on $T$ and $\delta$. Finally, at high $T$, we have only one atomic peak, located precisely at the atomic level separation $\omega_{23} = 0$. For a range of lower $T$, the atomic peak persists deriving from the effectively noninteracting Fermi gas contribution at the trap edge; the pairing peak arises from the superfluid or pseudogap region in the trap center.

Figure 1 presents self consistent numerical results for the minority RF spectra at unitarity and polarization $\delta = 0.5$, for (a) $T/T_F = 0.4$, (b) 0.25, and (c) 0.15, respectively. The insets in (b) and (c) are, respectively, the pairing peak position and the energy gap $\Delta(T)$ at the trap center as a function of $T/T_F$, (in units of the majority Fermi energy $E_{F}^{(1)}$), for $\delta = 0.1$ (black), 0.5 (red), and 0.8 (green lines), as labeled. The corresponding $T_c/T_F = 0.28$ (black), 0.25 (red), and 0.19 (green lines), respectively, and the estimated $T^*$ can also be read off from the insets where the gap vanishes. Here we choose $\gamma = 0.05$. 

Figure 1: RF spectra for polarized gases in a harmonic trap at unitarity and polarization $\delta = 0.5$, for (a) $T/T_F = 0.4$, (b) 0.25, and (c) 0.15, respectively. The insets show the pairing peak position and the energy gap $\Delta(T)$ at the trap center as a function of $T/T_F$, (in units of the majority Fermi energy $E_{F}^{(1)}$), for $\delta = 0.1$ (black), 0.5 (red), and 0.8 (green lines), as labeled. The corresponding $T_c/T_F = 0.28$ (black), 0.25 (red), and 0.19 (green lines), respectively, and the estimated $T^*$ can also be read off from the insets where the gap vanishes.

The black, red and green curves
correspond to three polarizations: $\delta = 0, 1, 0.5$ and 0.8 respectively. One can see that the higher polarization is associated with a smaller peak position and energy gap. We see that the magnitudes of the pairing gap are rather comparable to their experimental counterparts. As in experiment the pairing gap increases with decreasing temperature. The energy scale at which it smoothly vanishes can be read off in the insets which yield $T^*$. There is no sharp feature at $T^*$, so experimentally it cannot be precisely defined. Nevertheless, we see that there is a clear separation between the peak location curves for the three polarizations. By contrast the experimental data for all measured polarizations lie on the same (approximately) universal curve, with substantially higher $T^*$ (by a factor of 2 or so).

We now turn to a first scenario for elucidating the exotic non-superfluid phase at high polarizations [5] by considering the possibility [19] that this state is a Fermi gas or liquid. The loss of superfluidity would be due to a destabilization (arising from more benign Hartree-like corrections) in the competing normal Fermi gas phase. This scenario is not compatible with a zero temperature pseudogap phase (since the presence of an excitation gap for fermions means that it is not in a Fermi gas or Fermi liquid state). Nevertheless, this scenario would give rise to a single, nearly symmetric RF peak at low temperatures and high polarizations, similar to that observed experimentally, albeit associated with an atomic rather than pairing peak.

Figure 2 plots the self consistently determined RF spectra at very low temperatures $T = 0.01T_F$ in the unitary (left) and the noninteracting limit (right panels), assuming state $|2\rangle$ is the majority (red dashed) and minority (black solid lines), respectively. The top two panels correspond to high polarizations, $\delta = 0.7$ and 0.8. The bottom panel presents a comparison with an unpolarized gas. This low temperature phase corresponds to superfluidity in all cases in the left column, since that is what is found in our self consistent calculations. The two high polarizations correspond to phase separation. In the noninteracting gas case (right column), the results are very simple. We find, as expected, only atomic peaks in the majority and minority curves. They are located at precisely the same position – at the zero of our frequency scale. Comparing the two curves in (a) and (b) with (d) and (e) one sees that with future majority spectra there is a simple way to rule out this particular Fermi gas scenario. At low $T$ the majority curves in (a) and (b) (unlike the minority) have atomic peaks as well as pairing peaks. The larger atomic peaks of the majority plots are associated with the fact that the majority has a much larger noninteracting gas tail in its particle density profile. By contrast for the minority curves on the left, all fermions are paired at these low $T$ and we see only a single pairing peak.

When comparing with existing experiments, it should be noted that if the single peak in the zero temperature pseudogap phase were an atomic peak such as in the calculations of Ref. [19], there would be a shift in its position (relative to that computed here) though probably not large enough to match the experimental presumed pairing peak. In summary, this figure shows that the combined measurement of both majority and minority curves can serve to establish whether a single peak is coming from paired atoms or noninteracting atoms. In this way it can address the scenario which associates the non-superfluid state at high polarizations with a Fermi gas phase.

In Fig. 3 we turn to another possible scenario for the mysterious zero temperature pseudogap phase and calculate the self consistent RF spectra within a finite temperature (normal) pseudogap phase which arises in the Sarma portion of the phase diagram at very high polarizations [9]. Here we consider $\delta = 0.95$, in order to have polarization and temperatures consistent with the computed phase diagram [9]. The temperature gradually increases from $T = 0.15T_F$ to $0.17T_F$ as we go from bottom to top panels. This figure is based on the implicit possibility that the purported $T = 0$ pseudogap phase [5] is a finite temperature observation. We thus use RF experiments as a type of thermometry and probe whether the experimental temperatures are sufficiently low to be in the true ground state. Importantly, we see from the figure that a two peaked structure is clearly visible at the lowest $T$ of this intermediate temperature scale, $0.15T_F$. It will be even better resolved at somewhat lower polarizations, as studied experimentally. The two peaks start to merge at $0.17T_F$, where we are left with an atomic peak only. The observation of two peaks in this figure, in contrast with experiment, suggests that the MIT experiments were conducted at sufficiently low $T$.

A third possible scenario for the observed zero temperature pseudogap phase follows from BdG-based calculations [16] which suggest that the ground state is not phase separated as in LDA theories at unitarity, but instead a superfluid with a complex order parameter – in a Fulde-Ferrell-Larkin-Ovchinnikov (FFLO) state [20]. We have conducted a finite temperature study (importantly including noncondensed pairs) of the simplest such state [21] which suggests that this oscillatory order parameter phase rapidly becomes unstable with increasing...
Figure 3: RF spectra at unitarity when $|2\rangle$ is the minority with polarization $\delta = 0.95$, for different temperatures (a) $T/T_F = 0.17$, (b) $0.16$, and (c) $0.15$, respectively. Here we take $\gamma = 0.05$.

temperature. Because it is not sufficiently robust, we argue that the FFLO phase is not likely to be a candidate for the exotic ground state. Indeed, experiments from both groups seem to support phase separation [6,8]. We stress that the phase separation that is consistent with the current theory seems to be more akin to that in Ref. [6] than that in Ref. [8] where there is very little, if any, pseudogap regime, in either the density profiles or the phase diagram.

As a fourth scenario, we note that the most natural way to obtain $T_c = 0$ with $T^{*} \neq 0$ is associated with phases in which there is a frustration of pair mobility which leads to localization of pairs. This state appears in recent theoretical work on unitary gases [22] in the presence of optical lattices. Nevertheless, it appears difficult to understand how it can arise from Zeeman-like effects, which primarily break pairs apart. Most likely (but for very different reasons) this phase has been observed in high $T_c$ superconductors under various perturbations where $T_c$, but not $T^{*}$, is driven to zero [2].

In summary, this paper has shown that future RF experiments are needed to arrive at a more conclusive understanding of the observed pairing peaks, hopefully, both by reducing the unexpectedly high estimates of $T^{*} \gtrsim T_F$ (in order to be consistent with essentially all estimates), and via providing majoring spectra. The latter can confirm the presence of a pairing, as contrasted with, an atomic peak. We cannot rule out the possibility that the purported $T = 0$ pseudogap phase has some form of superfluid order. However, if instead a non-superfluid but paired ground state is confirmed, it will very likely contain some degree of “bosonic” order.

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