Isotope effect on filament dynamics in fusion edge plasmas

To cite this article: O H H Meyer and A Kendl 2017 Plasma Phys. Control. Fusion 59 065001

View the article online for updates and enhancements.
Isotope effect on filament dynamics in fusion edge plasmas

O H H Meyer\textsuperscript{1,2} and A Kendl\textsuperscript{1}

\textsuperscript{1}Institut für Ionenphysik und Angewandte Physik, Universität Innsbruck, A-6020 Innsbruck, Austria
\textsuperscript{2}Department of Physics and Technology, UiT—The Arctic University of Norway, N-9037 Tromsø, Norway

E-mail: ole.meyer@uibk.ac.at

Received 26 January 2017, revised 19 March 2017
Accepted for publication 27 March 2017
Published 20 April 2017

Abstract

The influence of the ion mass on filament propagation in the scrape-off layer of toroidal magnetised plasmas is analysed for various fusion relevant majority species, like hydrogen isotopes and helium, on the basis of a computational isothermal gyrofluid model for the plasma edge. Heavy hydrogen isotope plasmas show slower outward filament propagation and thus improved confinement properties compared to light isotope plasmas, regardless of collisionality regimes. Similarly, filaments in fully ionised helium move more slowly than in deuterium. Different mass effects on the filament inertia through polarisation, finite Larmor radius, and parallel dynamics are identified.

Keywords: isotope effect, plasma filament, blob, particle transport

(Some figures may appear in colour only in the online journal)

1. Introduction

In various tokamak experiments the confinement properties have been shown to scale favourably with increasing mass of the main (fusion relevant) ion plasma species, specifically hydrogen isotopes and helium [1–4]. Recently, similar features have been identified for a reversed field-pinch configuration [5]. The radial cross-field transport of coherent filamentary structures (commonly denoted ‘blobs’) in the scrape-off layer (SOL) of tokamaks accounts for a significant part of particle and heat losses [6,7] to the plasma facing components. Experimentally the ion mass effect on SOL filament dynamics has been studied in a simple magnetised torus [8]. Filamentary transport in tokamaks in general is an active subject of studies in experiments, analytical theory, and by two- and three-dimensional numerical computations.

The basic properties of filamentary transport are reviewed in [9]. Blob propagation results from magnetic drifts that polarise density perturbations, thus yielding a dipolar electric potential $\phi$ whose resulting $\mathbf{B} \times \nabla \phi$ drift in the magnetic field $\mathbf{B}$ drives the filaments down the magnetic field gradient and towards the wall. The basic physics is illustrated by accounting for the current paths involved upon charging of the blob by the diamagnetic current: the closure is via perpendicular polarisation currents in the drift plane and through parallel divergence of the parallel current [10].

Two-dimensional (2D) closure schemes are discussed in [11]. Depending on parallel resistivity, the dominant closure path features distinct dynamics: if closure is mainly through the polarisation current, the 2D cross-field properties are dominant, leading to a mushroom-cape shaped radial propagation. For reduced resistivity the closure parallel to the magnetic field direction in three-dimensional (3D) is dominant, and Boltzmann spinning leads to a more coherently propagating structure at significantly reduced radial velocity.

The isotope mass may have influence on the $\mathbf{E} \times \mathbf{B}$ shearing rate in the edge region [12], and flow shear in the edge region has been suggested to be a main agent which controls blob formation [13]. In addition, a finite ion temperature introduces poloidally asymmetric propagation of blobs [14,15]. The underlying finite Larmor radius (FLR) effects have been found to contribute to favourable isotopic transport scaling of tokamak edge turbulence [16].

In this work we study the isotopic mass effect on blob filament propagation by employing an isothermal gyrofluid model so that relevant FLR contributions to the blob evolution are effectively included, in addition to the mass dependencies in polarisation and in parallel ion velocities.
2. Gyrofluid model and computation

The present simulations on the isotopic dependence of 3D filament and 2D blob propagation in the edge and SOL of tokamaks are based on the gyrofluid electromagnetic model introduced by Scott [17]. In the local delta-f isothermal limit the model consists of evolution equations for the gyrocenter densities \( n_s \) and parallel velocities \( u_{||} \) of electrons and ions, where the index \( s \) denotes the species with \( s \in (e, i) \):

\[
\frac{d n_s}{dr} = -\nabla \cdot [u_{||} n_s] + K(\phi_s + \tau_s n_s),
\]

\[
\beta \frac{\partial A_s}{\partial t} + \epsilon_s \frac{d u_{||} n_s}{dr} = -\nabla (\phi_s + \tau_s n_s) + 2 \epsilon_s \tau_s K(u_{||}) - CJ_s
\]  

(1)

The plasma beta parameter

\[
\beta = \frac{4\pi n_e B^2}{\langle qR / L \rangle^2},
\]

controls the shear-Alfvén activity, and

\[
C = 0.51 \frac{\nu_L}{c_0} \frac{m_e}{m_i} \frac{qR}{L_i},
\]

mediates the collisional parallel electron response for \( Z = 1 \) charged hydrogen isotopes. The collisional response for other isotopes or ion species is discussed further below. The gyrofluid moments are coupled by the polarisation equation

\[
\sum_s a_s \left[ \Gamma n_s + \frac{\Gamma_0}{\tau_s} - 1 \phi_s \right] = 0,
\]

and Ampère’s law

\[
-\nabla_{||}^2 A_{||} = J_{||} = \sum_s a_s u_{||} n_s.
\]

(3)

The gyroscreened electrostatic potential acting on the ions is given by

\[
\phi_s = \Gamma_1(\rho_s^2 k_i^2) \hat{\phi}_k,
\]

where \( \hat{\phi}_k \) are the Fourier coefficients of the electrostatic potential. The gyroaverage operators \( \Gamma_0(b) \) and \( \Gamma_1(b) = \Gamma_1^0(b)^2 \) correspond to multiplication of Fourier coefficients by \( I_0(b) e^{-b} \) and \( I_0(b/2) e^{-b/2} \), respectively, where \( I_0 \) is the modified Bessel function of zero’th order and we have introduced the shorthand notation \( b = \rho_s^2 k_i^2 \). We here use approximate Padé forms with \( \Gamma_0(b) \approx (1 + b)^{-1} \) and \( \Gamma_1(b) \approx (1 + b/2)^{-1} \) [18].

The perpendicular \( E \times B \) advecitive and the parallel derivative operators for species \( s \) are given by

\[
\frac{d_{||}}{dr} = \frac{\partial \phi_s}{\partial t} + \delta^{-1} \{ \phi_s \},
\]

\[
\nabla_{||} = \frac{\partial \phi_s}{\partial \xi} - \delta^{-1} \{ A_{||} \},
\]

where we have introduced the Poisson bracket as

\[
[f, g] = \frac{\partial f}{\partial \xi} \frac{\partial g}{\partial \eta} - \frac{\partial f}{\partial \eta} \frac{\partial g}{\partial \xi}
\]

(2)

In local 3D flux tube co-ordinates \( [x, y, z] \), \( x \) is a (radial) flux-surface label, \( y \) is a (perpendicular) field line label and \( z \) is the position along the magnetic field line. In circular toroidal geometry with major radius \( R \), the curvature operator is given by

\[
\mathcal{K} = \omega_y \left( \sin z \frac{\partial}{\partial x} + \cos z \frac{\partial}{\partial y} \right),
\]

where \( \omega_y = 2L_y / R \), and the perpendicular Laplacian is given by

\[
\nabla^2_{\perp} = \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right).
\]

Flux surface shaping effects [19, 20] in more general tokamak or stellarator geometry on SOL filaments [21] are neglected here for simplicity.

Spatial scales in each drift plane are normalised by the drift scale \( \rho_0 = \sqrt{T_e m_i / e V_B} \), where \( T_e \) is a reference electron temperature, \( B \) is the reference magnetic field strength and \( m_i \) is a reference ion mass, for which we use the mass of deuterium, \( m_{\text{D}} \). The parallel coordinate is normalised by the parallel connection length, \( L_{||} = 2 \pi q R \), where \( q \) is the safety factor at a reference location inside the separatrix. The influence of the connection length on turbulence properties across the separatrix is studied in [23]. The temporal scale is set by \( \tau_0 / L_i \), where \( \tau_0 = \sqrt{T_e / m_i} \), and \( L_i \) is a perpendicular normalisation length (e.g. a generalised profile gradient scale length), so that \( \delta = \rho_0 / L_i \) is the drift scale. The temporal scale may be expressed alternatively \( L_c / \tau_0 = L_c / (\rho_0 \Omega_0) = (\delta \Omega_0)^{-1} \), with the ion-cyclotron frequency \( \Omega_i = \omega_i / \rho_i \). In the following we employ \( \delta = 0.01 \) such that one normalised time unit corresponds to \( (\delta \Omega_0)^{-1} \approx 10^{-4} \) s. Fluctuation amplitudes are normalised according to \( n_s / n_0 \rightarrow n_s / e \phi / T_e \rightarrow \phi_s \), \( A_{||} / \rho_i B \rho_0 q R \rightarrow A_{||} / L_{||} / \rho_i B \rho_0 q R \rightarrow A_{||} / L_{||} / \rho_i B \rho_0 q R \rightarrow A_{||} / L_{||} \), with the electron beta

\[
\beta_e = 4\pi \rho_0 B^2 / \langle \rho R \rangle^2
\]

in terms of the background electron pressure \( p_0 = m_{\text{D}} T_e \). Note that this normalisation produces the factor \( \delta^{-1} \) with the Poisson brackets.

The main species dependent parameters are

\[
a_s = \frac{Z_i \rho_i}{n_i}, \quad \tau_s = \frac{T_e}{\xi_s T_e}, \quad \mu_s = \frac{m_i}{Z_i n_i},
\]

\[
\rho_i^2 = \mu_s \tau_i \rho_0^2, \quad \epsilon_s = \mu_s \frac{q R^2}{L_i},
\]

setting the relative concentrations, temperatures, mass ratios and FLR scales of the respective species. \( Z_i \) is the charge state of the species \( s \) with mass \( m_i \) and temperature \( T_e \). Note that the index ‘s’ denotes both electrons and ions, while the index ‘i’ represents ion species such as protium, deuterium, tritium or helium.

Unless stated otherwise we employ \( \beta = 0, \omega_y = 0.05, \delta = 0.01, \rho R / L_i = 18000 \), corresponding approximately to \( L_i = 4.25 \) cm, \( R = 165 \) cm, \( B = 2 \) T, \( T_e = 70 \) eV, \( q = 3 \) and \( L_i = 31 \) m, typical for ASDEX Upgrade conditions close to the separatrix. Different input parameters result in quantitative deviations with respect to the results presented in this work, however, qualitatively, the trends found are robust.
2.2. Parallel boundary conditions

We distinguish between two settings for parallel boundary conditions in 3D simulations. In the case of edge simulations a toroidal closed-flux-surface geometry is considered, and quasi-periodically global consistent flux-tube boundary conditions in the parallel direction [22] are applied on both state-variables \(n_e, \phi\) and flux variables \(v_L, u_L\).

For SOL simulations, the state variables assume zero-gradient Neumann (sheath) boundary conditions at the limiter location and the flux variables are given as

\[
u_L = p_e L + \pm \Gamma _d n_e L, \quad v_T = \pm \Gamma _d [(\Lambda + 1) n_e L - \phi _L L],
\]

at the parallel boundaries \(z = \pm \pi\) respectively [23]. Note that in order to retain the Debye sheath mode in this isothermal model, the Debye current \(J_L = \pm \Gamma_d ([\Lambda + 1] n_e L - \phi _L L)\) is expressed as \(J_L = \pm \Gamma_d (\phi _L - \Lambda n_e)\) and the electron pressure \(p_e = n_e T_e\) is replaced by \(p_e = n_e [23]\). This edge/SOL set-up and its effects on drift wave turbulence has been presented in detail by Ribeiro et al in [23, 24].

The sheath coupling constant is \(\Gamma _d = \sqrt{(1 + \tau _i) / \mu _i}\). The floating potential is given by \(\Lambda = \Lambda _0 + \Lambda _i\), where \(\Lambda _0 = \log \sqrt{m_0 / (2 \pi e m_i)}\) and \(\Lambda _i = \log \sqrt{\mu _i / (1 + \tau _i)}\). Here terms with the index \(i\) apply only to the ion species. The expressions presented here are obtained by considering the finite ion temperature acoustic sound speed, \(c_i = \sqrt{(Z_i T_i + T_{ei}) / m_i}\), instead of \(c_0\) in [23]. This results in the additional \(\Lambda _i\), and the normalisation scheme yields the extra \(\sqrt{(1 + \tau _i) / \mu _i}\) in \(\Gamma _d\).

2.3. Numerical implementation

Our code TOEFL [25] is based on the delta-f isothermal electromagnetic gyrofluid model [17] and uses globally consistent flux-tube geometry [22] with a shifted metric treatment of the coordinates [26] to avoid artefacts by grid deformation.

In the SOL region a sheath boundary condition model is applied [23, 24]. The electrostatic potential is obtained from the polarisation equation by an FFT Poisson solver with zero-Dirichlet boundary conditions in the (radial) \(\phi\)-direction. Gyrofluid densities are adapted at the \(x\)-boundaries to ensure zero vorticity radial boundary conditions for finite ion temperature. An Arakawa–Karniadakis scheme is employed for advancing the moment equations [27–29].

3. Scaling laws from dimensional analysis

Blob velocity scalings are commonly deduced from the fluid vorticity equation. We follow this approach and construct the gyrofluid vorticity equation to deduce velocity scaling laws. The vorticity equation can be obtained upon expressing the gyrocenter ion density in terms of the electron density and polarisation contribution, inserting in the ion gyrocenter density evolution equation and subtracting the electron density evolution equation [15, 30]. Up to \(O(b)\) the ion gyrocenter density is

\[
n_i = n_e - \frac{1}{2} \mu _i \nabla _t ^2 p _L - \mu _i \nabla _t ^2 \phi _L,
\]

where the ion pressure is given in terms of the electron particle density \(p _L = \tau _i n_e\). The gyroaveraged potential for species \(s\) up to \(O(b)\) is

\[
\phi _s = \phi _L + \frac{1}{2} \mu _i \tau _i \nabla _t ^2 \phi _L.
\]

Following [30] we obtain

\[
\mu _i \nabla \cdot \frac{d}{dt} \nabla _t \phi _s = \nabla _t \cdot (L + \tau _i) \mathcal{K} (n_e).
\]

Here we have introduced the modified potential \(\phi _s = \phi _L + \mu _i\), the vorticity equation is equivalent to the quasi-neutrality statement of current continuity, \(\nabla \cdot J = 0\). We identify the divergence of the polarisation current,

\[
\nabla \cdot J _{pol} = - \mu _i \nabla \cdot \frac{d}{dt} \nabla _t \phi _s,
\]

and the divergence of the diamagnetic current,

\[
\nabla \cdot J _{dm} = -(L + \tau _i) \mathcal{K} (n_e).
\]

Blob propagation has in a linearisation of the present gyrofluid model been analytically analysed by Manz et al in [31]. Therein the dependence of blob velocity on the ion isotope mass is in principle present but not explicitly apparent. To clarify, we here restate the calculations of [31], but use the vorticity equation (9) with the explicit occurrence of \(\mu _i\). Neglecting parallel currents, employing the blob correspondence \(\partial _t, \partial _x \rightarrow 1 / \sigma\) and \(d / dt \rightarrow i \gamma = i v _b / \sigma\) [32], in terms of the blob width \(\sigma\), blob velocity \(v_b\) and linear growth rate of the instability \(\gamma\) and furthermore identifying \(\phi = i v _b\sigma\) (the radial component of the electric drift), we get (in normalised units):

\[
v_b = \frac{1}{\sqrt{2 \gamma}} \sqrt{1 + \frac{f^2 + g^2 - f}{g}} = \frac{1 + \tau _i \omega _b / \sigma}{\mu _i} A.
\]

A is the initial blob amplitude. In the limit of large blobs, \(g \gg f\), so that

\[
v_b \approx \frac{1 + \tau _i \omega _b / \sigma}{\mu _i} A.
\]

and for smaller blobs satisfying \(g \ll f\) we get

\[
v_b \approx \frac{1 + \tau _i \omega _b / \sigma}{\mu _i} A.
\]

The correspondence with the result of [31] is made explicit upon renormalising, i.e. letting \(v_b \rightarrow v_b / \sigma c_0\), \(A \rightarrow A / \delta, \sigma \rightarrow \sigma / \rho _0, \omega _b \rightarrow 2 \omega _c / R\). The limits then are

\[
v_b \approx \frac{1 + \tau _i \omega _b / \sigma}{\mu _i} c_0 \sqrt{\frac{2 \sigma}{R}} A, \quad \text{for} \quad g^3 \gg \frac{\mu _i \tau _i^2 A}{4(1 + \tau _i) \omega _b},
\]
and

\[ v_B \approx 2e_0 \frac{1 + \tau_i}{\mu_i \tau_i} \left( \frac{\sigma}{\rho_i} \right)^2, \quad \text{for } \sigma^3 \ll \frac{\mu_i m_i A}{4(1 + \tau_i)\omega_B}. \]  

(16)

For 2D computations of sufficiently large blobs we consequently expect \( v_B \sim 1/\sqrt{\mu_i} \), whereas for the 3D model the expected scaling is not \textit{a priori} that clear. In [31] 3D (linear) scaling laws were presented, where the parallel dynamics was approximated by the Hasegawa–Wakatani closure, \( \nabla |J_i| = \frac{1}{c} \nabla (n_e - \phi) \).

In the following we are going to compare reduced 2D and full 3D dynamical blob simulations for various isotope species with the analytical \( 1/\sqrt{\mu_i} \)-scaling.

### 4. 2D blob computations

In this section we numerically analyse the dependence of filament dynamics on the normalised ion mass \( \mu_i \) by reduced 2D blob simulations of the isothermal gyrofluid equations (1) and (2). For the computations in this section we use as parameters: curvature \( \omega_B = 0.05 \), drift scale \( \delta = 0.01 \), grid size \( L_x = L_y = 128\rho_0 \), grid points \( N_x = N_y = 256 \), initial blob amplitude \( A = 1 \) and Gaussian blob width \( \sigma = 10\rho_0 \).

Figure 1 shows contours plots of the electron particle density at different times of evolution of a seeded blob for several species of cold ions (\( \tau_i = 0 \)). The initial Gaussian density perturbation \( n_e(x, y, t = 0) = A \exp[-(x^2 + y^2)/\sigma^2] \) undergoes the familiar transition towards a mushroom-shaped structure before the blob eventually breaks up due to secondary instabilities. This figure illustrates the main point for the following discussion: lighter isotopes propagate faster than heavier isotopes. In terms of the (normalising) deuterium mass we consider \( \mu_H = 1/2 \), \( \mu_D = 1 \), \( \mu_T = 3/2 \) and \( \mu_{\text{He}^+} = 2 \) with \( \mu_i = m_i/(Z_i m_D) \). The species index \( \text{He}^+ \) here denotes \textit{singly} charged helium-4 with \( Z_{\text{He}^+} \equiv 1 \). The case of (fully ionised) doubly charged helium-4 isotopes will be discussed further below in context of 3D simulations in section 5.2, since the charge-state dependent collision parameter \( C \) is absent in the 2D subset. Note that for the 2D setting the doubly charged helium-4 can be interpreted as deuterium at half the ion temperature. Observe that the lighter the ion species are, the further the blob is developed in its radial propagation and evolution at a given snapshot in time.

For warm ions (\( \tau_i > 0 \)) the blob propagation depends on the relative initialisation of the electron and ion gyrocenter densities. Commonly, a zero \( E \times B \) vorticity blob initialisation is assumed where \( n_i(x, y, t = 0) = A \Gamma_i^{-1} n_e(x, y, t = 0) \); inserting these into the polarisation equation (3) results in vorticity \( \Omega \equiv \nabla \times \phi = 0 \). This initialisation for most parameters leads to an FLR induced rapid development of a perpendicular propagation component in addition to the radial
propagation of the blob, and thus a pronounced up–down asymmetry in y direction. Alternatively, the electron and ion gyrocenter densities can be chosen as equal with \( n_e(x, y, t = 0) = n_i(x, y, t = 0) \), so that \( \phi \sim n_i \). In this case the initial vorticity mostly cancels the FLR asymmetry effect, and the blob remains more coherent and steady in its radial propagation \[33\]. The truth may be somewhere in between: as in the experiment blobs are not ‘seeded’ (in contrast to common simulations), but appear near the separatrix from \( E \times B \) drift wave vortices or are sheared off from poloidal flows, in general some phase-shifted combination of electric potential and density perturbations will appear. For comparison we perform simulations with both of these seeded blob density initialisations.

We note that the \( x \) coordinate is effectively pointing radially outwards (in negative magnetic field gradient direction) at a low-field midplane location in a tokamak, and the magnetic field here points into the \( (x, y) \) plane \((e_y = e_x \times e_z)\), so that the effective electron diamagnetic drift direction of poloidal propagation is in the present plots downwards (in negative \( y \) direction).

Figure 2 shows blob propagation for warm ions with \( \tau_i = 1 \) initialised with the zero vorticity condition. For comparison, we present in figure 3 the propagation for the same parameters but initialised with equal electron and ion gyrocenter densities. Clearly, the latter cases with initial non-zero vorticity \( \Omega = \nabla^2 \phi \) results in faster and more coherent radial propagation, whereas the zero vorticity cases exhibit significant poloidal translation through the FLR induced spin-up. Regardless of initialisation, blobs of light ion species with small \( \mu_i \) travel faster and are further developed at a given time compared to heavier species.

Relevant quantities which determine the intermittent blob related transport properties of the tokamak SOL are the maximum blob velocity and acceleration. In figure 4 we present maximum radial centre-of-mass velocities \( V_{x,\text{max}} \) and the average radial acceleration \( A \) as a function of the ion mass parameter \( \mu_i \). The different symbols/colours represent cases with cold \((\tau_i = 0, \text{ blue lower curves})\) and warm \((\tau_i = 1)\) ions, with both types of initial conditions used on the latter: the zero vorticity condition is depicted in red (middle curves) and the \( n_e = n_i \) condition in green (upper curves).

It can be seen that the maximum radial blob velocity is slightly larger for \( n_e = n_i \) initialisation due to the mainly radial propagation (left figure), but the average acceleration is for both cases nearly equal (right figure).

The radial centre-of-mass position is given by \( X_r = \int \frac{dx dy \, n_i}{dX dy \, n_e} \). Taking the temporal derivative gives the radial centre-of-mass velocity, \( V_r = dX_r / dt \).
The maximum of $V(t), V_{\text{max}} = \max \{ V(t) \}$ and the corresponding time for the occurrence of the maximum, $t_{\text{max}}$, then give a measure of the average radial acceleration, $A = V_{\text{max}}/t_{\text{max}}$.

Clearly, an inverse dependence of velocities and acceleration on effective ion mass $\mu_i$ can be inferred for all cases.

For cold ions, the only mass dependence in the present 2D isothermal gyrofluid model, lies in the gyrofluid polarisation equation, carrying over the mass dependence of the polarisation drift in a fluid model. As deduced from the basic linear considerations in section 3, the maximum blob velocity scales inversely with the square root of the ion species or...
isotope mass: the plotted fits are close to the expected lines $V_{e,max} \sim \mu^{-0.5}$.

From dimensional analysis it follows that the acceleration should scale according to $A \sim \gamma^2 \sigma$, where $\gamma$ is the growth-rate of the linear instability. For $\gamma \sim 1/\sqrt{\mu}$, we expect $A \sim 1/\mu$ for cold ions: this is confirmed in figure 4 (right) where the fitted exponents are close to $-1$. Warm ion simulations also feature $\mu$ species mass dependence through the FLR operators $\Gamma_1(b)$ and $\Gamma_2(b)$, where $b = \mu^2 k_2^2 = \mu \tau_i \rho_i k_e^2$.

We find that for the parameters at hand, the maximum radial velocity for warm ions with zero vorticity initialisation is higher compared to cold ions, with a slightly increased isotopic dependence (seen in an exponent $-0.548$ compared to $-0.500$).

Initialising with non-zero vorticity yields approximately 50% increased velocities compared to cold ions, and slightly weakens the isotopic dependence (expressed by an exponent $-0.488$). This can be attributed to the mass dependence in the FLR operators, which we further discuss below in section 5.1.

5. Three-dimensional filament computations

In three dimensions, when the blob extends into an elongated filament along the magnetic field lines, additional physics enters into the model. The basic picture of interchange driving of filaments by charging through $\nabla B$ and curvature drifts to produce a net outward $E \times B$ propagation still remains valid. However, the total current continuity balance now also involves parallel currents: $-\nabla \cdot J_{pol} = \nabla \phi_{nl} + \nabla \cdot J_{diss}$.

The detailed balance among the current terms determines the overall motion of the filament. Furthermore, blob filaments in the edge of toroidal magnetised plasmas generally tend to exhibit ballooning in the unfavourable curvature region along the magnetic field. The parallel gradients in a ballooned blob structure also lead to a parallel Boltzmann response, mediated mainly through the resistive coupling of $\nabla \phi - n_e \nabla \phi$ to $C_{diss}$ in equation (2). This tends towards (more or less phase shifted) alignment between the electric potential and the perturbed density, which strongly depends on the collisionality parameter $C$.

For low collisionality, the electric potential in the blob evolves towards establishment of a Boltzmann relation in phase with the electron density along $B$, so that $n_e \sim \exp(-\phi) \sim \phi$. This leads to reduced radial particle transport, and the resulting spatial alignment of the potential with the blob density perturbation produces a rotating vortex along contours of constant density, the so-called Boltzmann spinning [34, 35]. Large collisionality leads to a delay in the build-up of the potential within the blob, so that the radial interchange driving can compete with the parallel evolution, and the perpendicular propagation is similar to the 2D scenario.

In the following we investigate how 3D filament dynamics is depending on the ion mass. Clearly, we expect an impact in addition to the 2D effects found in the previous section, since (i) the parallel ion velocity is inversely dependent on ion mass (but is for any ion species slow compared to the electron velocity), (ii) the shear boundary coupling constants are mass dependent, and (iii) the basic dependence on the ion mass in the polarisation current will play a more role complicated role compared to the 2D model.

For our present study we chose the free computational parameters basically identical to the 2D case above: drift scale $\delta = 0.01$, curvature $\omega_b = 0.05$, blob amplitude $A = 1$ and perpendicular blob width $\sigma = 10 \rho_0$. The Gaussian width of initial parallel density perturbation is given by $\Delta_x = \sqrt{32} = \sqrt{4 L_i}$, which represents a slight ballooning with some initial sheath connection:

$$n_e^{3D}(t = 0, x, y, z) = n_e^{diss} \cdot \exp \left[ \frac{(z - z_0)^2}{\Delta^2} \right].$$

where $z_0$ is the parallel reference coordinate at the outboard mid-plane and $n_e^{diss}(x, y)$ is the perpendicular Gaussian initial perturbation introduced in section 4. In this section we first focus on zero vorticity initial conditions, non-zero conditions will be discussed further below.

The perpendicular domain size is $L_x = L_y = 128 \rho_0$ with a grid resolution of $N_x = N_y = 256$. The number of parallel grid points is varied between $N_z = 8$ and 16. The filament simulations have been tested for convergence with respect to the number of drift planes up to $N_z = 32$: $N_z = 8$ yields qualitatively and quantitatively similar results to $N_z = 16$. The plots showing colour cross sections throughout this article are taken from simulations with $N_z = 8$, and the presented quantitative results have been obtained for $N_z = 16$.

We here set $\beta = 0$ as electromagnetic effects in the SOL are thought to be of minor importance for the present discussion [31]. The collisionality parameter is chosen in $C = 0.5$–100 to cover a likely range of tokamak SOL values. Typical values for the collisionality parameter for the SOL in ASDEX Upgrade L-mode plasmas have in the literature [31] been reported as $C \sim 1$–100, and a reference characteristic collisionality in [36] for MAST has been given as $C \sim 2$.

Figure 5 illustrates the dependence of filament evolution with respect to collisionality dependent Boltzmann spinning for warm deuterium ions. The case with $C = 10$ represents the strong Boltzmann spinning (drift wave) regime, where density and potential perturbations are closely aligned and the radial filament motion is strongly impeded. Increasing the collisionality to $C = 100$ reduces parallel electron dynamics and so effectively increases the lag of potential build-up within the density blob perturbations, so that the Boltzmann spinning is reduced. The substantial perpendicular motion component is in this case partly caused by FLR effects like in the corresponding 2D case for $\tau_i = 1$. The $\tau_i$ contribution to the ion diamagnetic curvature term results in enhanced radial driving of the blob compared to cold ion cases.

A measure for blob compactness can be introduced [14] by

$$I_\xi(t) = \frac{\int dx \int dy n_e(x, y, t) h(x, y, t)}{\int dx \int dy n_e(x, y, t = 0) h(x, y, t = 0)},$$

where $h$ is a (in general peaked) function that characterises the density distribution. Figure 5 shows the time evolution of $I_\xi$ for the 3D simulation at $C = 100$ and $C = 1$, where $I_\xi$ is first reduced and then saturated for $C = 100$ but increases in time for $C = 1$. The difference in $I_\xi$ between $C = 100$ and $C = 1$ can be attributed to the collisional damping of the interchange wave: the higher collisionality $C = 100$ attenuates the interchange activity in time, and the lower collisionality $C = 1$ allows for the interchange wave to propagate and overlap with the subsequent sheath wave.
where the Heaviside function \( h(x, y, t) \) is defined as
\[
h(x, y, t) = 1 \quad \text{if} \quad (x - x_{\text{max}}(t))^2 + (y - y_{\text{max}}(t))^2 < \sigma^2,
\]
and zero elsewhere. That is, the integral takes non-zero values for density contributions located inside a circle of radius \( \sigma \) around its maximum. Figure 6 quantifies the above observations. In the strong Boltzmann spinning regime \((C = 10, \text{left})\) the blob retains much of its initial shape, so that the compactness is higher compared to the weak Boltzmann spinning regime \((C = 100, \text{right})\), where filaments feature a more bean-shaped structure which reduces the compactness measure. At the time of measurement \((t = 5)\), the heavier isotopic blobs show slightly more compactness, which is an indirect result of decreased velocity; at a given time, the lighter isotopic blobs are further developed and thus less circular. The observed trends are similar for cold \((\tau_i = 0)\) and warm \((\tau_i = 1)\) ions.

For increased collisionality, the deviation from circularity is more pronounced, as the mushroom-cape shape is realised. Blobs in light isotopic plasmas are then again further developed, i.e. finer scales have emerged at the time of recording, resulting in a sharper mass dependence of blob compactness compared to \(C = 10\), where smaller scales are less prominent.

Figure 7 shows filament propagation for cold ions \((\tau_i = 0)\) and weak Boltzmann spinning. This can be compared to figure 8 which shows propagation for warm ions \((\tau_i = 1)\) and also weak Boltzmann spinning. It is observed that there is poloidal propagation also for the cold ion case, which is a consequence of the non-vanishing Boltzmann spinning that is also present, although greatly reduced, for these high collisionality \((C = 100)\) cases. The resulting maximum radial centre-of-mass velocities at the outboard midplane \((z = z_0)\) are shown in figure 9 for weak (right) and strong (left) Boltzmann spinning.

The fits of the exponent in \( \mu^\alpha \) to the simulation data in figure 9 carry evidence that the additional mass dependences introduced by the 3D model via parallel sheath-boundary conditions and parallel ion velocity dynamics causes the clear deviation from a \(1/\sqrt{\mu_i} \) scaling.
For high collisionality (and thus reduced Boltzmann spinning), the parallel current is impeded and the dynamics is more 2D than for lower collisionalities. The competing nature of the parallel divergence versus current continuity via the divergence of the polarisation current with collisionality is shown in figure 10: for each value of collisionality $C$ we compute the isotopic dependence of the outboard-midplane maximum centre-of-mass radial velocity, $V_{\text{20max}}$, contained in the scaling exponent, $\alpha_{\mu_i}$. For large values of $C$ the resulting dynamics strongly features 2D propagation characteristics, since the diamagnetic current is almost exclusively closed via the polarisation current, which gives the $1/\sqrt{\Gamma_i}$ scaling introduced in section 4. Decreasing the collisionality parameter results in weaker isotopic dependence, reflecting the competing influence of parallel- and perpendicular dynamics on the filament motion. Note that the scaling with respect to $C$ cannot be inferred from linear models.

5.1. Non-zero gyrofluid vorticity initialisation

So far we have in this section applied the initial condition $n_x \equiv \Gamma_1 n_1$ (recall $\Gamma_1 = (1 + \mu_1 \tau_1 k_i^2)^{-1}$) associated with zero initial vorticity. Now the case for non-zero initial vorticity by the condition $n_x = n_1$ is considered.

Figure 11 shows results for warm ion ($\tau_i = 1$) computations in the strong ($C = 10$, blue) and weak ($C = 100$, green) Boltzmann spinning regimes. (Recall that for $\tau_i = 0$ this discussion is redundant since $\Gamma_1(\tau_i = 0) = 1$.) The left figure depicts the maximum radial centre-of-mass velocity, and the right figure shows the corresponding average acceleration.

Comparing with figure 9 we notice that the resulting filament velocities are similar to those obtained from zero initial vorticity. We also find that the isotopic dependence $\sim \mu_i$ is not significantly altered.

Recalling the results from 2D computation in section 4, we may conclude that the initialisation does not affect substantially the maximum radial filament velocity in 3D numerical simulations. The slight impact of the initial condition on the resulting scaling exponent for the 2D case may then be connected to the more prominent mass dependence in the polarisation current, which is weakened when parallel currents are taken into account.

5.2. Comparison of filaments in deuterium and helium plasmas

When comparing blob filament propagation in deuterium and in fully ionised helium-4 plasmas in the present model, the dynamical evolution is identical in the cold ion limit: in the
The model parameter \( \mu_i = m_i/(Z_i m_D) \) the doubled mass of the helium nucleus exactly cancels with the doubled positive charge, \( Z_{He} = 2 \).

Differences are only appearing in warm ion cases. The normalised mass ratio is now identical for both species, \( \mu_D = \mu_{He} = 1 \). The only model parameter that is different, is the helium temperature ratio, \( \tau_{He} = T_{He}/Z_{He} T_e \). The species mass effects thus appears in the combined \( b \sim \mu_i \tau_i \). In the following we consider plasmas at equal temperature, \( T_D = T_{He} = 2 T_e \) such that \( \tau_D = 2 \) and \( \tau_{He} = 1 \).
The higher charge state of the helium nucleus is also indirectly evident in the reduced electron–ion collision frequency contained in the C-parameter. For electron–ion collisions where the ions are in charge state \(Z_i\) we have \(C \sim \alpha_c \nu_c^\ast\), with [37]

\[
\alpha_c \approx 1 + 1.198Z_i^1 + 0.222Z_i^2 + 1 + 2.966Z_i^1 + 0.753Z_i^2
\]

For \(Z_i = 1\) we have \(\alpha_c \approx 0.51\) and \(Z_i = 2\) gives \(\alpha_c \approx 0.43\).

To account for this dependence, we in the following consider two cases: (1) equal non-normalised collision frequencies, i.e. \(C \to 0.51C\) for deuterium and \(C \to 0.43C\) for helium; (2) using the same \(C\) for both deuterium and helium computations.

In case (2), setting first the normalised collisionality parameter \(C = 10\) identical for both D and He computations results in (recall the drift-scale \(\delta = \rho_i/L_i = 0.01\)) \(V_D = 1.41\delta c_0\) and \(V_{\text{He}} = 1.14\delta c_0\). Setting \(C = 100\) identical for both D and He gives \(V_D = 3.2\delta c_0\) and \(V_{\text{He}} = 2.6\delta c_0\), respectively.

For case (1) we set the electron–ion collision frequency equal for both species, so that different \(C\) parameters are used according to equation (21): \(C_D = 10\) corresponds to \(C_{\text{He}} = 8.43\), and \(C_D = 100\) to \(C_{\text{He}} = 84.3\). In these cases, maximal radial He velocities are \(V_{\text{He}}(C = 10) = 1.09\delta c_0\) and \(V_{\text{He}}(C = 100) = 2.46\delta c_0\).

We find that regardless of how the charge state dependency for the relative value of the collision parameter is treated, the filaments in deuterium plasmas move faster than in helium plasmas at identical temperature. This is visualised in figure 12 showing filament propagation at equal electron–ion collision frequency and electron temperature. Note that the resulting increased ion-temperature ratio for deuterium, \(\tau_i = T_i/Z_i T_e\), produces increased radial velocities, agreeing approximately with the factor \(\sqrt{3}/2\) suggested by equation (15).

6. Conclusions

We have investigated filament propagation in SOL conditions characteristic for tokamak fusion devices. Quasi-2D dynamics is restored in high resistivity regimes, where the maximum radial blob velocity scales inversely proportional with the square root of the ion mass. In 2D simulations the diamagnetic current drive is closed solely via the polarisation current, yielding this simple characteristic scaling.

The larger inertia through polarisation of more massive ion species effectively slows the evolution of filaments, and the maximum radial velocity occurs later compared to blobs in plasmas with lighter ions.

For non-zero initial vorticity condition, the 2D warm ion blobs show compact radial propagation, where the isotopic effect through the mass dependent FLR terms is slightly less pronounced.

Boltzmann spinning appears in 3D situations particularly for low collisionality regimes, and leads to a reduced dependence on the ion isotope mass. The exponent in the scaling \(V \sim \mu_i^\ast\) has been found to be typically within the range \(\alpha \in [-0.1, -0.3]\) for \(C < 10\), which is a regime relevant for the edge of most present tokamaks.

Considering current continuity, the closure via the parallel current divergence dynamically competes with current loops being closed through the polarisation current. For high collisionalities the parallel current is effectively impeded and the polarisation current characteristics dominate the blob evolution, resulting in a dependence upon the ion mass similar to what has been found in the 2D case. The initial condition has been found to have little influence on the maximum radial velocity when in 3D the parallel closure of the current is taken into account.

For similar ion temperatures and electron–ion collision frequencies, it has been found that helium filaments travel more slowly compared to deuterium filaments in both high and low collisionality regimes.

This work was devoted to the identification of isotopic mass effects on seeded (low amplitude) blob filaments in the tokamak SOL by means of a delta-f gyrofluid model. Naturally, blobs emerge near the separatrix within coupled edge/SOL turbulence. The dependence of fully turbulent SOL transport on the ion mass therefore will have to be further studied within a framework that consistently couples edge and SOL turbulence, preferably through a full-f 3D gyrofluid (or gyrokinetic) computational model that does not make any smallness assumption on the relative amplitude or perturbations compared to the background [15, 33, 38, 39].
Acknowledgments

We acknowledge main support by the Austrian Science Fund (FWF) project Y398. This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014–2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

References

[1] Bessenrodt-Weberpals M et al 1993 Nucl. Fusion 33 1205
[2] Hawryluk R J 1998 Rev. Mod. Phys. 70 537
[3] Liu B et al 2016 Nucl. Fusion 56 056012
[4] Xu Y et al 2015 Phys. Rev. Lett. 110 265005
[5] Lorenzini F et al 2015 Nucl. Fusion 55 043012
[6] Boedo J A et al 2003 Phys. Plasmas 10 1670
[7] LaBombard B et al 2001 Phys. Plasmas 8 2107
[8] Theiler C et al 2009 Phys. Rev. Lett. 103 065001
[9] D’Ipollito D A, Myra J R and Zweben S J 2011 Phys. Plasmas 18 060501
[10] Krasheninnikov S I 2001 Phys. Lett. A 283 368
[11] Krasheninnikov S I, D’Ipollito D A and Myra J R 2008 J. Plasma Phys. 74 679
[12] Hahm T S et al 2013 Nucl. Fusion 53 072002
[13] Manz P et al 2015 Phys. Plasmas 22 022308
[14] Madsen J et al 2011 Phys. Plasmas 18 112504
[15] Wiesenberger M, Madsen J and Kendl A 2014 Phys. Plasmas 21 092301
[16] Meyer O H H and Kendl A 2016 Plasma Phys. Control. Fusion 58 115008
[17] Scott B D 2005 Phys. Plasmas 12 102307
[18] Dorland W et al 1993 Phys. Fluids B 5 815–35
[19] Kendl A and Scott B D 2006 Phys. Plasmas 13 012504
[20] Kendl A, Scott B D, Ball R and Dewar R L 2003 Phys. Plasmas 10 3684
[21] Riva F, Lanti E, Jolliet S and Ricci R 2017 Plasma Phys. Control. Fusion 59 055001
[22] Scott B D 1998 Phys. Plasmas 5 2334
[23] Ribeiro T T and Scott B D 2005 Phys. Plasmas Control. Fusion 47 1657
[24] Ribeiro T T and Scott B D 2008 Plasma Phys. Control. Fusion 50 055007
[25] Kendl A 2014 Int. J. Mass Spectrom. 365/366 106–13
[26] Scott B D 2001 Phys. Plasmas 8 447
[27] Arakawa A 1966 J. Comput. Phys. 1 119
[28] Karniadakis G E, Israeli M and Orszag S A 1991 J. Comput. Phys. 97 414
[29] Naulin V and Nielsen A 2003 J. Sci. Comput. 25 104
[30] Scott B D 2007 Phys. Plasmas 14 102318
[31] Manz P et al 2013 Phys. Plasmas 20 102307
[32] Krasheninnikov S I 2001 Phys. Lett. A 283 368
[33] Held et al 2016 Nucl. Fusion 56 126005
[34] Angus J R et al 2012 Contrib. Plasma Phys. 52 348
[35] Angus J R and Umansky V M 2014 Phys. Plasmas 21 012514
[36] Easy L et al 2014 Phys. Plasmas 21 122515
[37] Hirshman S P 1977 Phys. Fluids 20 589
[38] Kendl A 2015 Plasma Phys. Control. Fusion 57 045012
[39] Nespoli F et al 2017 Plasma Phys. Control. Fusion 59 055009