Correction to: Biframes and some of their properties

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Following the publication of the original paper [1], the authors would like to correct some errors.

Page 18, Example (6.6), part (i),

The sentences currently read:

We consider the Riesz basis \( \{ f_k \}_{k=1}^{\infty} = \{(-1, 2), (1, 0)\} \) for \( \mathbb{R}^2 \). Every orthonormal basis for \( \mathbb{R}^2 \) is in the following form. For \( a \in [0, 1] \),

\[ \{ e_k^a \}_{k=1}^{\infty} = \left\{ (a, \sqrt{1-a^2}), (-\sqrt{1-a^2}, a) \right\}. \]

The sentences should read:

We consider the Riesz basis \( \{ f_k \}_{k=1}^{\infty} = \{(-1, 2), (1, 0)\} \) for \( \mathbb{R}^2 \). Every orthonormal basis for \( \mathbb{R}^2 \) is of the following form: for \( a \in [-1, 1] \),

\[ \{ e_k^a \}_{k=1}^{\infty} = \left\{ (a, \sqrt{1-a^2}), (-\sqrt{1-a^2}, a) \right\}. \]

When we choose \( a \in [0, 1] \), we earn only a portion of the orthonormal bases for \( \mathbb{R}^2 \), not all of them. Since for \( a = -\sqrt{\frac{2}{5}} \), the pair \( \{ e_k^{-\sqrt{\frac{2}{5}}} \}_{k=1}^{\infty} \) also is an orthonormal basis for \( \mathbb{R}^2 \), and we can see by simple calculations that the pair \( \{ e_k^{-\sqrt{\frac{2}{5}}} \}_{k=1}^{\infty}, \{ f_k \}_{k=1}^{\infty} \) is a biframe with bounds \( \sqrt{\frac{2}{5}} \) and \( 4\sqrt{\frac{2}{5}} \). This means that there is an orthonormal basis \( \{ e_k^{-\sqrt{\frac{2}{5}}} \}_{k=1}^{\infty} \) for \( \mathbb{R}^2 \) such that \( \{ f_k \}_{k=1}^{\infty} \in \{ e_k^{-\sqrt{\frac{2}{5}}} \}_{k=1}^{\infty} \). This means that \( \mathcal{R} \subset \mathcal{E} \).

Example (6.6) affects the results after, which are changed as follows. After this example, it was found that there is a sequence \( \{ f_k \}_{k=1}^{\infty} \in \mathcal{R} \) such that \( \{ f_k \}_{k=1}^{\infty} \in \mathcal{E} \). Hence we change Proposition 6.8 in the following form. We add the proof of \( \mathcal{R} \subset \mathcal{E} \) in the continuation of the proof of the proposition.

Page 20, Proposition 6.8.
The sentences currently read:

**Proposition 6.8** The set $E'$ is the set of all Riesz bases.

The sentences should read:

**Proposition 6.8** The sets $E$ and $E'$ are equal to the set of all Riesz bases.

**Proof** Let $\{f_k\}_{k=1}^{\infty} \in \mathcal{R}$. There exist an invertible operator $V \in B(H)$ and an orthonormal basis $\{e_k\}_{k=1}^{\infty}$ for $H$ such that $f_k = Ve_k$ for all $k \in \mathbb{N}$. By polar decomposition [2, Chapter 5, Section 14] of $V^*$ we have $V^* = W(VV^*)^{\frac{1}{2}}$, where $W$ is a unitary operator. Therefore $V = (VV^*)^{\frac{1}{2}}W^* = S^\frac{1}{2}W^*$, where $S$ is the frame operator of $\{f_k\}_{k=1}^{\infty}$.

Now set $\delta_k := W^*e_k$. Then $\{\delta_k\}_{k=1}^{\infty}$ is an orthonormal basis for $H$, and we have

$$f_k = S^\frac{1}{2}\delta_k \ \forall k \in \mathbb{N}.$$  

This implies that $\{f_k\}_{k=1}^{\infty} \in \{\{\delta_k\}\}$, so $\{f_k\}_{k=1}^{\infty} \in E$. \qed

On page 21, after the proof of Proposition 6.8, the authors have given descriptions of the subsets $[[e_k]]'$ of $E'$. The above corrections change these results.

The sentences currently read:

The next example shows that unlike the subsets $[[e_k]]$ of $E$, the subsets $[[e_k]]'$ of $E'$ are not distinct. Hence, the collection of them cannot be a partition for the set $E' = \mathcal{R}$. The sentences should read:

The next example shows that the subsets $[[e_k]]'$ of $E'$ are not distinct in a real Hilbert space.

The errors have been updated in this correction article and the original article [1] has been corrected.

**Declarations**

**Competing interests**
The authors declare no competing interests.

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**References**

1. Firouzi Parizi, M., Alijani, A., Dehghan, M.A.: Biframes and some of their properties. J. Inequal. Appl. 2022, 104 (2022). https://doi.org/10.1186/s13660-022-02844-7

2. Kubrusly, C.S.: The Elements of Operator Theory. Birkhäuser, Brazil (2010)

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