DESIGN OF DIFFERENTIATED WARRANTY COVERAGE THAT CONSIDERS USAGE RATE AND SERVICE OPTION OF CONSUMERS UNDER 2D WARRANTY POLICY

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(Communicated by Bin Li)

Abstract. Warranty service providers usually provide homogeneous warranty service to improve consumer satisfaction and market share. Considering the difference of consumers, some scholars have carried out studies on maintenance strategies, service pricing, payment method, claim behaviour and warranty cost analysis in recent years. However, few scholars have focused on the differentiated coverage of warranty service that considers usage rate and service option of consumers. On the basis of previous classification criteria on usage rate, this paper divides consumers into heavy, medium and light usage rate groups with clear boundaries. To avoid discrimination in warranty service, this study divides 2D warranty coverage into disjoint sub-regions and adopts different maintenance modes in each sub-region. By formulating and calculating warranty cost model under warranty cost constraints, we can obtain the maximum warranty coverage under usage rate \( r \). Therefore, differentiated warranty scope for consumers in the three groups can be proposed, whilst consumers can choose the most suitable warranty service according to their usage rate. Evidently, the proposed warranty strategy can provide flexible warranty service for consumers, meet the requirements of the warranty cost constraints of warranty service providers and enable enterprises to occupy a favourable position in the market competition.

1. Introduction. For new products, manufacturers should offer consumers free base warranty service, which can be provided by the former or dealers (Reimann and Zhang 2013)[1]. If products break down under normal operating conditions, then consumers can apply warranty claims to warranty service providers. To protect consumers’ rights, governments have promulgated warranty regulations from the legal aspects. For example, the warranty policy for household vehicles in China clarifies the responsibilities of manufacturers within the stipulated warranty coverage. As an important component of marketing strategy, warranty service can enhance product

2010 Mathematics Subject Classification. Primary: 90B50; Secondary: 90B25.
Key words and phrases. Usage rate, warranty coverage, modelling, cost analysis, 2D warranty.
The first author is supported by NSF grant the National Natural Science Foundation of China (No. 71701200); the Postdoctoral Fund of China (No. 2016M590525); the Postdoctoral Fund of Jiangsu (No. 1601246C).
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market competitiveness and highlight product quality, whilst manufacturers tend to provide high-quality and wide coverage of warranty service to consumers (Huang et al. 2017; Xie 2017)[8, 23]. For example, the warranty policy for household vehicles in China stipulates that the warranty coverage shall not be less than 2 years or 50,000 km, whilst some manufacturers have extended base warranty coverage to 3 years or 100,000 km. Although warranty service satisfies the needs of consumers and facilitates the market expansion of manufacturers, such service also introduces additional warranty cost burden. Data from a previous studies have shown that automobile manufacturers bear over 25 billion dollars warranty cost in the United States (Tong et al. 2014)[29].

In view of the high warranty cost, previous studies have provided considerable attention to warranty cost analysis. At an earlier stage, these studies have mainly focused on one-dimensional (1D) warranty, in which warranty coverage variable is age (Akbarov and Wu 2012[1]; Jack and Schouten 2000[2]; Lin and Huang 2010[13]; Liu et al. 2015[14]; Nguyen and Murthy 1986[17]; Nguyen and Murthy 1989[18]; Park et al. 2014[19]; Shafiee et al. 2011[23]; Shang et al. 2016[25]; Vahdani et al. 2013[31]). For example, Nguyen and Murthy (1989)[18] proposed a replace–repair warranty strategy for 1D product and analysed the minimum expected warranty cost per product. Vahdani et al. (2013)[31] built a mathematical model with no-zero repair time for discretely degrading and repairable products under free replacement warranty. The model can enable warranty service providers to obtain the best rectification plan. However, warranty coverage is two-dimensional (2D) for many high-value durable products, whilst vehicle is the most commonly adopted product in previous 2D warranty studies. Given this research gap, many scholars have focused on 2D warranty service policy formulation by optimising maintenance models (Huang et al. 2015[7]; Park and Pham 2016[20]; Shahanaghi et al. 2013[24]; Varnosafaderani and Chukova 2012[32]; Wang et al. 2017[33]). Similar to the previous studies in 1D warranty, the 2D warranty service strategies were optimised with the pre-defined warranty period and often supposed that consumer usage rate remains unchanged.

Given the development of design and manufacturing technology, consumers increased their requirements for warranty service quality and scope. However, satisfying the consumer requirement of 2D warranty coverage by merely optimising maintenance strategy and expanding warranty service scope is difficult because of the huge warranty cost burden to manufacturers. These conditions have resulted in increasing attention to extended warranty service in recent years. Extended warranty can be considered an optional service for consumers with additional payment, whilst consumers can decide whether to purchase this service or not. In this area, Hartman and Laksana (2009)[6] proposed a 1D extended warranty strategy on the basis of the analysis of consumer service demand and behaviour. Similar to the studies on base warranty, maintenance strategy is also a research hotspot in the area of 2D extended warranty (Chang and Lin 2012[3]; Huang et al. 2017[8]; Shahanaghi et al. 2013[24]; Su and Shen 2012[26]; Su and Wang 2016[27]; Tong et al. 2014[29]). For example, Su and Shen (2012)[26] analysed warranty cost and profit with different options under extended warranty. The result shows that minimal repair should be adopted in the short warranty period; otherwise, complete and minimal repair should be adopted. Given that usage rate affects warranty cost, Tong et al. (2014)[29] divided extended warranty into two parts, namely, purchasing
extended warranty service at the point of product sale and base warranty expiration, and proposed a pricing model of 2D extended warranty contract for the three groups of consumers divided by usage rate. Different from previous studies which assumed that consumers claim for all failures, Gallego et al. (2015) [5] considered strategic claim behaviour and risk attitudes and proposed a residual value warranty to maximise the profit of warranty service providers. As preventive maintenance can prolong product life, reduce warranty cost and retain consumer loyalty, Huang et al. (2017) [8] proposed a customised 2D extended warranty strategy for consumers with different usage rate. Some studies have also focused on supply chain coordination issues related to extended warranty (Mai et al. 2017 [15]; Moura et al. 2017 [16]; Qin et al. 2017 [21]).

Through the aforementioned literature review, it can be found that warranty service providers usually provide homogeneous warranty service to consumers, including service coverage, maintenance strategy and service price. Considering the differences of consumers' usage habits, some literatures have studied the differentiated maintenance strategies (Ye and Murthy 2016) [36], service pricing (Tong et al. 2014) [29], claim behaviour (Gallego et al. 2015) [5] and warranty cost analysis (Huang et al. 2015) [7] in recent years. However, there are few studies have focused on differentiated warranty coverage that considers usage rate and service option of consumers. At present, it is essential to establish the scope of base warranty service for different usage rate consumer groups under the constraints of laws and warranty cost limits. Besides, previous studies have pre-defined homogeneous 2D warranty coverage and merely optimised maintenance strategies within the 2D warranty coverage (Huang et al. 2017 [8]; Jack et al. 2009 [11]; Tong et al. 2017 [30]). Given that warranty cost accounts for a large proportion of manufacturer profits, 2D warranty coverage under warranty cost constraints and maintenance strategy should be designed. However, research on this aspect is rather limited. Moreover, the distribution function of usage rate affects the expected warranty cost of consumers. Previous studies have generally assumed that the distribution function of consumer usage rate is uniform (Huang et al. 2017 [8]; Iskandar and Murthy 2003 [9]; Wang et al. 2015 [34]). Although using the distribution function form is easy, the accuracy of the formulated maintenance strategies will be affected. Furthermore, the consumer selection of warranty service set provided by warranty service providers will affect the expected warranty cost. Thus, the warranty coverage that considers consumer choice should be adjusted. The current study considers consumer usage rate difference and service choice based on preceding analysis. The set of warranty service coverage is determined by fitting the usage rate distribution function and constructing failure and cost analysis models, whilst the warranty coverage is further adjusted on the basis of service option under warranty cost constraints.

The remainder of this paper is organised as follows. Section 2 describes the notations and assumptions of this study. Section 3 explains the proposed model that considers consumer usage rate and service option under 2D warranty policy. Section 4 provides a case study to interpret the proposed model. Section 5 presents the conclusion and directions for future research.

2. Notations and assumptions. \( W_n, U_n \) Age and usage parameters of the 2D warranty coverage, where \( n \in \{ l, m, h \} \); \( r_n \) Value of \( U_nW_n \), where \( n \in \{ l, m, h \} \); \( r_d \) Nominal design usage rate;
$W_r,U_r$: Age and usage parameters of the maximum warranty coverage under usage rate $r$;

$i$: $i$th failure type;

$C_{ir}$: Expected warranty cost of failure $i$ under usage rate $r$;

$C_{exp}$: Expected warranty cost constraint value per unit product;

$W^*,U^*$: Age and usage parameters of the 2D warranty service termination point;

$\gamma$: Proportion of the sub-regions to the 2D warranty coverage;

$g(r)$: Usage rate density function;

$C_{mi}$: Cost of minimal repair to rectify failure $i$;

$C_{ci}$: Cost of complete repair to rectify failure $i$;

$\rho_i$: Value of $C_{ci}/C_{mi}$;

Different from previous failure density functions which take product or component as object, this paper considers that there are many failure types in module and the failure density functions of different failure types are different. Hence, this paper set the failure density function of failure $i$ with usage rate $r$ to $f_i(w,r)$ and set the failure distribution function associated with $f_i(w,r)$ to $F_i(w,r)$. The function of survival rate relative to variable $w$ is called survivor function $F_i(w,r)$, which indicates that survival probability varies with variable $w$. The hazard function $h_i(w,r)$ refers to the probability that a component survives at $w$ and fails at the next moment, which is conditional probability. $H_i(w,r)$ is the cumulative hazard function associated with $h_i(w,r)$.

Similar to previous studies, this paper also assumes that product age is equivalent to its cumulative operation, whilst maintenance time is neglected (Huang et al. 2017[8]). By contrast, the differences in usage rate distribution functions of different modules are assumed. Additionally, warranty service providers are assumed to be willing to provide differentiated warranty coverage to consumers, who choose the most suitable warranty coverage option on the basis of their usage rate. Furthermore, consumer denial of service discrimination and the negative impact of service discrimination on manufacturers are assumed. Therefore, warranty service providers will not provide discriminatory differentiated warranty services to consumers. For example, $W_l > W_m$ and $U_l > U_m$.

3. Model construction.

3.1. Consumer classification and analysis of termination point under 2D warranty. For high-value durable products under 2D warranty, manufacturers should offer free base warranty service to consumers or entrust dealers. Given that warranty period directly affects the total expected warranty cost, warranty service providers must determine the coverage of warranty service. However, previous studies have often built maintenance models and performed cost analysis within the pre-set 2D warranty coverage (Chukova and Johnston 2006[4]; Iskandar et al. 2005[10]; Shahanaghi et al. 2013[24]; Tom 2002[28]). Given the difference in consumer usage rate, the termination points and warranty cost are also different. Hence, it is essential to focus on the difference of usage rate and classifying consumer groups on the basis of usage rate interval. Regarding this issue, Iskandar and Murthy (2003[9]) classified consumers into three categories, that is, light, medium and heavy usage rate, and considered the fuzziness of the classification criteria for the three different consumer groups with usage rate intensities of $[0.1,0.9]$, $[0.7,1.3]$ and $[1.1,2.9]$, respectively. Since then, many studies have adopted this classification standard for warranty strategy design and cost analysis (Huang et al. 2017[8]);
Table 1. The interval of usage rate intensity

| Usage intensity | Low limit of interval | Upper limit of interval |
|-----------------|-----------------------|-------------------------|
| Light           | $r_{l1}$              | $r_{l2}$                |
| Medium          | $r_{l2}$              | $r_{h1}$                |
| Heavy           | $r_{h1}$              | $r_{h2}$                |

Tong et al. 2014[29]; Wang et al. 2015[34]; Ye and Murthy 2016[36]). However, a partial overlap is observed in the boundaries of the usage rate intervals under this classification criterion. On this basis, Huang et al. (2017)[8] classified consumers into three types based on usage rate, in which the top 25%, last 25% and the rest of the ratios are classified as high, low and average usage rate, respectively. The boundaries of the usage rate classification method are clear. Unfortunately, only a few consumers have extremely high or low usage rate, whilst the estimation of warranty cost is inaccurate under the assumption of uniform distribution function. This study adopts the previous classification criteria as basis to assume that manufacturers set heavy, medium and light usage rate intervals on the bases of product characteristics and statistical data. Given that only a few consumer usage rate are approximately zero and tend to be infinite, the boundary parameters of usage rate intervals are set as $r_{l1}$, $r_{l2}$, $r_{h1}$ and $r_{h2}$, where $0 < r_{l1} < r_{l2} < r_{h1} < r_{h2} < \infty$.

Table 1 shows the usage rate interval of usage intensity.

Given that the expected warranty cost varies with usage rate, predicting the usage rate of each consumer is difficult for warranty service providers. Moreover, there is no need to provide customised warranty service scope for each usage rate. This study supposes that warranty service providers can design warranty service strategies for different usage rate groups, whilst consumers can choose the warranty service coverage which is most conducive to their usage rate. As $(0, W_n)$ is the interval of age axis and $(0, U_n)$ is the interval of usage axis, the coverage of 2D warranty service can be expressed as $(0, W_n) \times (0, U_n)$ which is similar to previous studies (Tong et al. 2014[29]; Tong et al. 2017[30]) (see details in Figure 1).

As can be seen in Figure 1, if $r \geq r_n = U_n/W_n$, then the age and usage parameters of termination point are $U_n/r$ and $U_n$. Otherwise, the parameters are $W_n$ and $W_n r$, where $n \in \{l, m, h\}$

For consumers, the optimal termination of a 2D warranty service is that the termination point simultaneously determines the critical value of time and usage. Therefore, 2D warranty cost model with usage rate $r$ should be built and the maximum scope of a 2D warranty service under the constraint of warranty cost limit should be determined. The usage rate distribution function should be determined to calculate the expected warranty cost of consumer groups. Given that the density function of usage rate varies and affects the formulation of warranty coverage, this study supposes that usage rate density function is $g(r)$. In the application process, the function expression must be obtained by function type determination and parameter fitting.

3.2 Failure model. In the design of warranty scope, the failure model of a product or component should be formulated. The failure density functions of consumers with different usage rate are different because consumer usage rate varies for products under 2D warranty. However, the empirical analysis of warranty claims has enabled this study to realise the difficulty of determining the type of failure function and
Figure 1. Termination point of 2D warranty service

expression on the 2D warranty plane for consumers. The scale parameters of the actual usage rate and nominal design usage rate are assumed to satisfy the following relationships on the basis of previous studies (Jack et al. 2009)[11]:

$$\lambda_r = \left( \frac{\lambda_0}{r} \right)^{\varepsilon} \lambda_0.$$  \hfill (1)

where \(\varepsilon\) is the acceleration factor and \(\lambda_0\) and \(\lambda_r\) are the scale parameters of \(r_d\) and \(r_r\), respectively.

3.3. Maintenance strategy. Maintenance and cost analysis models have consistently been the focus of previous warranty studies. Many scholars have conducted comprehensive studies on maintenance strategies under 2D warranty. And, the studies have often divided disjoint sub-regions within the presented warranty scope and determined the optimal maintenance strategies by assigning different regional maintenance modes and optimising regional boundaries (Jack et al. 2009)[11]. However, these sub-regions can hardly provide differentiated warranty coverage for consumer groups with different usage rate. Moreover, the optimisation process and results of maintenance strategies under 2D warranty are complex and difficult to operate in practice. Besides, these maintenance strategies provide different maintenance services to consumers, thereby leading to discrimination in maintenance service. For example, when consumers with low usage rate lodge warranty claims, warranty service providers adopt minimal repair to rectify failures. However, complete repair is adopted only for heavy usage rate consumers. This study uses the aforementioned considerations as bases to assume that warranty service providers set the same proportion of sub-regions within the warranty coverage and specify the maintenance modes in the sub-regions. On the basis of previous studies, the first warranty claim in \((0, \gamma W_n) \times (0, \gamma U_n)\) is assumed to adopt complete maintenance to fix the failure. Otherwise, minimal repair is used to rectify the failure. Figure 2 shows the
schematic of the maintenance strategy under 2D warranty. The shadows in Figure 2 represent the areas where different maintenance modes are adopted in the sub-regions.

Figure 2. Schematic of the maintenance strategy under 2D warranty

3.4. Warranty cost model. For usage rate $r$, warranty service providers should calculate the parameters of $W_r$ and $U_r$ under cost constraint and maintenance strategy. Thereafter, the maximum 2D warranty coverage of usage rate $r$ is determined as $(0, W_r) \times (0, U_r)$.

Complete repair is adopted for the first failure occurrence in the former $\gamma$ proportion of the 2D warranty coverage. As hazard function is a conditional probability function, the expected warranty cost during the remaining warranty period can be calculated using Equation (2).

$$C_{ci} + C_{mi} \int_{W_r}^{W} h_i (x - w, r) \, dx.$$  \hfill (2)

Otherwise, the expression of the expected warranty cost of the remaining warranty coverage can be obtain:

$$C_{mi} \int_{\gamma W_r}^{W_r} h_i (w, r) \, dw.$$ \hfill (3)

Using Equations (2) and (3), we can obtain the expected warranty cost of failure $i$ under usage rate $r$ as follows:
As can be seen in Equation (4), the first half of the first equality is a double
integral, which integrates \( x \) and \( w \) respectively. Using Equation (4), the parameters
of \( W_r \) and \( U_r \) can be obtained.

Using Equation (5), we can obtain the expected warranty cost constraint value
per unit product.

\[
C_{exp} = \sum_{i=1}^{I} C_{ir}.
\] (5)

Generally, a change in usage rate \( r \) results in a change in the value of \( W_r \) and \( U_r \),
and the change trend of \( W_r \) and \( U_r \) can be divided into positive correlation and
negative correlation. Figure 3 shows the trend diagram of \( W_r - U_r \).

As can been seen in Figure 3, Figure 3-a shows the change trend of \( W_r \) and \( U_r \)
is positive, and Figure 3-b shows a negative correlation between \( W_r \) and \( U_r \).

This study assumes that warranty service providers provide differentiated war-
nanty coverage to various consumer groups on the basis of the classification of usage
rate in Table 1. Given that the warranty service coverage set is non-continuous,
some consumers whose usage rate is not \( r_n \) would choose the warranty service with
coverage of \( (0,W_n) \times (0,U_n) \). Therefore, the scope of 2D warranty service should
be determined on the basis of Equations (1)–(5) (see details in Figure 4).

As can been seen in Figure 4, warranty service providers present differentiated
warranty coverage to consumers with heavy, medium and light usage rate, whilst
consumers choose the most suitable warranty service period on the basis of their
own usage rate. To ensure the warranty scope \( (0,W_n) \times (0,U_n) \) can represent the
usage rate of the three groups of consumers, the values of \( r_l \), \( r_m \) and \( r_h \) can be
obtained as follows:

\[
\frac{\int_{r_{lh}}^{r_{lh}} r g(r) dr}{\int_{r_{ll}}^{r_{lh}} g(r) dr} = \frac{\int_{r_{lh}}^{r_{lh}} h_{1}(x - w,r) dx}{\int_{r_{ll}}^{r_{lh}} h_{1}(x - w,r) dx} + \int_{W_r}^{W_r} h_{1}(w,r) dw f_{i}(\gamma W_r,r) \] (4)

\[
c_{ir} = \int_{0}^{W_r} \left[ C_{cr} + C_{mi} \int_{0}^{W_r} h_{i}(x - w,r) dx \right] f_{i}(w,r) dw + C_{mi} \int_{0}^{W_r} h_{i}(w,r) dw \bar{F}_{i}(\gamma W_r) \]

\[
= C_{mi} \int_{0}^{W_r} \left[ \rho_{i} + H_{i}(W_r - w,r) \right] f_{i}(w,r) dw + \int_{0}^{W_r} \left[ H_{i}(W_r,r) - H_{i}(\gamma W_r,r) \right] \bar{F}_{i}(\gamma W_r,r)
\]
For light usage rate consumers, the warranty coverage is \((0, W) \times (0, U)\), where \(r_l = U_l/W_l\). Using the Equations formulated above, we can obtain the warranty coverage for light usage rate consumers as follows:

\[
C_{exp} = \sum_{i=1}^{I} \int_{r_{l1}}^{r_{l2}} C_{mi} \int_0^{W^*_l} \left[ \rho_i + H_i \left( W^* - w, r \right) \right] f_i(w, r) \cdot dw \\
+ \left[ H_i \left( W^*, r \right) - H_i \left( \gamma W^*, \gamma r \right) \right] F_i \left( \gamma W^*, r \right) g(r) dr \\
\frac{\int_{r_{l1}}^{r_{l2}} g(r) dr}{\int_{r_{l1}}^{r_{l2}} g(r) dr}
\]

If \(r_{l1} < r < r_{l2}\), then \(W^* = W_l\) and \(U^* = W_l/r\). And if \(r_l < r < r_{l2}\), then \(W^* = U_l/r\) and \(U^* = U_l\).

Similarly, the warranty coverages for medium and heavy usage rate consumers are \((0, W_m) \times (0, U_m)\) and \((0, W_h) \times (0, U_h)\), and they can be obtained as follows:

\[
C_{exp} = \sum_{i=1}^{I} \int_{r_{m1}}^{r_{m2}} C_{mi} \int_0^{W^*_m} \left[ \rho_i + H_i \left( W^* - w, r \right) \right] f_i(w, r) \cdot dw \\
+ \left[ H_i \left( W^*, r \right) - H_i \left( \gamma W^*, \gamma r \right) \right] F_i \left( \gamma W^*, r \right) g(r) dr \\
\frac{\int_{r_{m1}}^{r_{m2}} g(r) dr}{\int_{r_{m1}}^{r_{m2}} g(r) dr}
\]

where \(U_m = W_m r_m\). If \(r_{m2} < r < r_m\), then \(W^* = W_m\) and \(U^* = W_m/r\). And if \(r_{m} < r < r_{h1}\), then \(W^* = U_m/r\) and \(U^* = U_m\).

\[
C_{exp} = \sum_{i=1}^{I} \int_{r_{h1}}^{r_{h2}} C_{mi} \int_0^{W^*_h} \left[ \rho_i + H_i \left( W^* - w, r \right) \right] f_i(w, r) \cdot dw \\
+ \left[ H_i \left( W^*, r \right) - H_i \left( \gamma W^*, \gamma r \right) \right] F_i \left( \gamma W^*, r \right) g(r) dr \\
\frac{\int_{r_{h1}}^{r_{h2}} g(r) dr}{\int_{r_{h1}}^{r_{h2}} g(r) dr}
\]

where \(U_h = W_h r_h\). If \(r_{h2} < r < r_h\), then \(W^* = W_h\) and \(U^* = W_h/r\). And if \(r_h < r < r_{h2}\), then \(W^* = U_h/r\) and \(U^* = U_h\).

Given that \(r_n\) is the value of \(U_n/W_n\), where \(n \in \{l, m, h\}\), and it may result in discrimination in 2D warranty service, warranty service providers must adjust the value on the basis of the actual data in the application process.
4. **Case study.** Prior studies have often set the function types and parameters of the usage rate and failure density functions. Simultaneously, these studies assumed that only one type of failure occurred and rarely considered the occurrence of multiple failures. However, various types of failures occur during warranty period. The failure distribution functions and expected warranty cost of these failures are also different. Therefore, this study considers that multiple types of failures occur during warranty period, whilst the failure functions and expected repair cost vary. The empirical data of warranty claim in a previous study is adopted in this section. For data confidentiality, current research uses A to represent the module, whilst the manufacturer encrypts the failure types.

4.1. **Warranty claim data description.** The number of warranty claim data used in this study is 410725 with product sales volume of 236361. The usage rate density function is as follows:

\[ g(r) = 0.043r^{0.79}e^{(-r/g)^{1.79}}. \]  

(12)

As mentioned above, we can hardly determine the failure type and expression on the 2D warranty plane for consumers with different usage rate. Through investigation, the nominal design usage rate \( r_d \) of this product can be obtained as \( 1.0 \times 10^4 \) km per year. Table 2 shows the information on major failure types.

| Failure | \( C_{m1}(\text{Yuan}) \) | \( C_{cl}(\text{Yuan}) \) | \( f_i(w, r_d) \) | \( \lambda_0 \) | \( k \) |
|---------|-----------------|-----------------|-----------------|--------|--------|
| A31     | 1000            | 5000            | 6.32E-04        | 0.0106| 4.03   | 2.06  |
| A18     | 3200            | 6400            | 2.20E-02        | 0.0592| 2.32   | 3.09  |
| A88     | 800             | 4800            | 2.53E-03        | 0.0452| 2.48   | 4.55  |
| A20     | 2600            | 7800            | 5.67E-02        | 0.0497| 2.90   | 4.99  |
| A30     | 4500            | 9000            | 4.78E-03        | 0.0622| 4.30   | 4.62  |
| A10     | 3000            | 12000           | 5.82E-02        | 0.0514| 2.94   | 4.51  |
| A16     | 2900            | 8700            | 9.24E-03        | 0.0426| 2.76   | 4.42  |
| A50     | 3500            | 10500           | 6.98E-02        | 0.0857| 3.28   | 1.85  |
| A15     | 1700            | 3400            | 1.36E-02        | 0.0514| 3.14   | 4.51  |
| A40     | 2800            | 5600            | 1.33E-02        | 0.0497| 2.69   | 4.19  |
| A17     | 4600            | 9200            | 1.89E-02        | 0.0555| 3.22   | 1.55  |

The accelerated failure coefficients should be determined by the manufacturer in combination with the design and experimental data. This study considers two cases, namely, \( \varepsilon = 0.9 \) and \( \varepsilon = 1.1 \). Using Equation (1) and Table 2, we can obtain the failure density functions of failure i with usage rate \( r \) as follows:

\[ f_i(w, r) = \frac{k \varepsilon^w}{\lambda_0 r d^\varepsilon} \left( \frac{w r^\varepsilon}{\lambda_0 r d^\varepsilon} \right)^{k-1} e^{-(w r^\varepsilon / \lambda_0 r d^\varepsilon)} \]  

(13)

where \( r_d = 1 \times 10^4 \) km per year and \( C_{exp} = 2400 \) Yuan.

The analysis of the warranty claim data indicates that the usage rate is within the range of \([0.31, 2]\), whilst 95.87% of the usage rate is within the range of \([0.1, 6]\). Therefore, this study defines the first third of the interval of \([0, 2]\) as light usage rate, the last third as heavy usage rate and the rest as medium usage rate. The
boundary parameters are also approximated as \( r_{l1} = 0.1, r_{l2} = 1.2, r_{h1} = 2.4 \) and \( r_{h2} = 6 \), whilst the sub-intervals of the usage rate are \([0.1,1.2], (1.2,2.4] \) and \((2.4,6]\), thereby accounting for 32.6%, 31.2% and 32.0%, respectively.

Using Equations (6)-(8), (12) and the feedback from the warranty service provider, this paper approximates \( r_{l} = 0.65 \), \( r_{m} = 1.8 \) and \( r_{h} = 3.0 \). Assumed that manufacturer set the boundary parameters of sub-warranty region, and \( \gamma = 0.5 \) is assumed in this paper. That is, the first failure occurrence in the first-half warranty coverage adopts complete repair, whereas minimal repair is used for the other failures.

4.2. **Case A:** \( \varepsilon = 0.9 \). The curve of \( W_r - U_r \) under \( \varepsilon = 0.9 \) can be obtained by using the preceding equations, see details in Figure 5.

![Figure 5. Curve of \( W_r - U_r \) (\( \varepsilon = 0.9 \))](image)

Table 3. Age and usage parameters of the 2D warranty coverage
(\( \varepsilon = 0.9 \))

| \( w_n \) | Value | \( u_n \) | Value |
|-----------|-------|-----------|-------|
| \( W_l \) | 3.35  | \( U_l \) | 2.18  |
| \( W_m \) | 2.44  | \( U_m \) | 4.39  |
| \( W_h \) | 1.83  | \( U_h \) | 5.49  |

Hence, the warranty coverages for the three groups of consumers are \((0, W_l) \times (0, U_l) = (0, 3.35) \times (0, 2.18), (0, W_m) \times (0, U_m) = (0, 2.44) \times (0, 4.39) \) and \((0, W_h) \times (0, U_h) = (0, 1.83) \times (0, 5.49)\).

4.3. **Case B:** \( \varepsilon = 1.1 \). Similarly, the curve of \( W_r - U_r \) under \( \varepsilon = 1.1 \) can be obtained as shown in Figure 6.

Equations (9)–(11) are used to obtain the age and usage parameters of the 2D warranty coverage. See details in Table 4.
Figure 6. Curve of $W_r - U_r$ ($\varepsilon = 1.1$)

Table 4. Age and usage parameters of the 2D warranty coverage ($\varepsilon = 1.1$)

| $w_n$ | Value | $u_n$ | Value |
|-------|-------|-------|-------|
| $W_l$ | 3.53  | $U_l$ | 2.29  |
| $W_m$ | 2.28  | $U_m$ | 4.10  |
| $W_h$ | 1.50  | $U_h$ | 4.50  |

The warranty coverages for the three groups of consumers can be obtained as $(0, W_l) \times (0, U_l) = (0, 3.53) \times (0, 2.29)$, $(0, W_m) \times (0, U_m) = (0, 2.28) \times (0, 4.10)$ and $(0, W_h) \times (0, U_h) = (0, 1.50) \times (0, 4.50)$.

From above case study we see that, the model proposed above can provide different usage rate consumers with flexible warranty service. In order to facilitate the implementation of this model, warranty service providers need to approximate the calculation results. For example, the flexible warranty coverages can be approximated to $(0, 3.50) \times (0, 2.30)$, $(0, 2.30) \times (0, 4.00)$ and $(0, 1.50) \times (0, 4.50)$ when $\varepsilon = 1.1$. In practice, warranty service providers must determine the calculation parameters of the model on the basis of the design parameters, experimental data and market conditions.

To simply the calculation, this case study set the proportion of the sub-regions to the 2D warranty coverage as 0.5. In practice, warranty service providers set the value of $\gamma$ in maintenance strategy. From previous literature review, we can find that many scholars have carried out the study on the optimization of maintenance strategies in detail (Baik et al. 2004[2]; Chukova and Johnston 2006[4]; Iskandar et al. 2005[10]; Jack et al. 2009[11]; Tom 2002[28]; Varnosafaderani and Chukova 2012[32]). When determine the value of $\gamma$, warranty service providers can refer to the maintenance models of these references.

5. Conclusion. For high-value durable products under 2D warranty, manufacturers offer homogeneous warranty services to consumers or entrust dealers. Given the influence of usage rate on failure density function, the expected warranty cost is
different during 2D warranty period. Meanwhile, a variety of failures occur in warranty coverage, whereas previous models have considered only one type of failure occurrence. This study considers usage rate and service option under 2D warranty and conducts a research on differentiated warranty coverage. The proposed model sets the same proportion of sub-regions to 2D warranty coverage to avoid discrimination in warranty service content and scope. Meanwhile, warranty service providers should calculate and adjust the value of \( r_n \), where \( W_l > W_m > W_h \), \( U_h > U_m > U_l \), \( r_n = U_n/W_n \) and \( n \in l, m, h \). In the process of model construction, the accelerated failure model is used to describe the failure model, whilst the boundary parameters are determined under the expected warranty cost constraint value per unit module. The case study adopts a set of warranty claim data, which has been used in a previous study to formulate differentiated warranty coverage for consumers with light, medium and heavy usage rate. The proposed model can assist warranty service providers formulate differentiated warranty service strategies and meet the usage habits of consumers with diverse usage rate. Differentiated warranty service strategies can also be formulated for products with multi-components on the basis of the proposed model in the actual operation process, which is applicable to warranty service demand under product diversification.

Acknowledgments. This paper was supported by the National Natural Science Foundation of China (No. 71701200, 71532008, 71971210), the Postdoctoral Fund of China (No. 2016M590525), the Postdoctoral Fund of Jiangsu (No. 1601246C) and the Humanities and Social Science Foundation in Hubei Provincial Education Department (17Q088).

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Received June 2019; revised September 2019.
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