Anomalous convective transport of the tokamak edge plasma, caused by the inhomogeneous ion cyclotron parametric turbulence

V. S. Mikhailenko,¹, b) V. V. Mikhailenko,², b) and Hae June Lee³, c)

¹) Plasma Research Center, Pusan National University, Busan 46241, South Korea
²) BK21 FOUR Information Technology, Pusan National University, Busan 46241, South Korea
³) Department of Electrical Engineering, Pusan National University, Busan 46241, South Korea

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In this paper, we develop the kinetic and hydrodynamic theories of the convective mesoscale flows driven by the spatially inhomogeneous electrostatic ion cyclotron parametric microturbulence in the pedestal plasma with a sheared poloidal flow. The developed kinetic theory predicts the generation of the sheared poloidal convective flow, and of the radial compressed flow with radial flow velocity gradient. The developed hydrodynamic theory of the convective flows reveals the radial compressed convective flow as the dominant factor in the formation of the steep pedestal density profile with density gradient exponentially growing with time. This gradient density growth is limited by the formation of the radial oscillating with time ion outflow of pedestal plasma to scrape-off layer.

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a) E-mail: vsmikhailenko@pusan.ac.kr
b) E-mail: vladimir@pusan.ac.kr
c) E-mail: haejune@pusan.ac.kr
I. INTRODUCTION

It was found in the fast wave (FW) heating experiments\textsuperscript{1-2} on the National Spherical Torus eXperiment (NSTX) that a significant part of the FW power loss occurs due to the anomalous convective flow of the collisionless dense hot plasma from the edge to the cold low density scrape-off layer (SOL) plasma. In the SOL, this lost hot plasma propagates mostly along SOL magnetic field lines to the divertor regions. It was estimated in Ref.\textsuperscript{3}, where the theory of the ion cyclotron (IC) electrostatic parametric turbulence, driven by the strong inhomogeneous FW electric field in the inhomogeneous plasmas, was developed, that the origin of this convective flow may be the interactions of ions and electrons with radially inhomogeneous microscale IC and drift turbulence. It was found that the radial and poloidal velocities of this flow are proportional to the spatial gradient of the spectral intensity of the turbulent electric field. This convective flow has a low flow velocity in the plasma core and in the SOL, where the levels of the IC parametric turbulence and of the drift turbulence are low, and the plasma parameters are weakly inhomogeneous. The edge transport barrier with steep density profile (commonly referred to as the pedestal) is, therefore, the most preferable region for the development of the intensive convective flows. The transport barrier establishes in the regimes of the enhanced confinement (or H-mode regime) which is the basic operational regime of tokamaks. This regime is characterized by the suppression of the drift type instabilities by the poloidal sheared flow, spontaneously developed in the tokamak edge, with the flow velocity shearing rate comparable with or above the maximum linear growth rate of the suppressed instabilities. Note, that the parametric IC instabilities have the growth rate much above the observed velocity shearing rate and are not suppressed by this flow. Beginning from the first observation of the H-mode of operation, it was found that this mode has been accompanied by periodic bursts of the ejected to SOL plasma structures, which are labelled as the edge localized modes (ELMs)\textsuperscript{4-5} and blobs\textsuperscript{6} with considerably larger density than the surrounding low density cold SOL plasma. It is well recognised now that the non-diffusive transport of the heat and matter outwards across the magnetic field, which occurs in addition to the anomalous diffusion, is the universal phenomenon\textsuperscript{6} experimentally observed in tokamaks.

It was found in Ref.\textsuperscript{3}, that the radial velocity of the convective flow is in the direction opposite to the gradient of the spectral intensity of the microscale turbulence. In the
tokamak plasmas, this convective flow moves outward of pedestal to the SOL region. It may be concluded therefore that the dynamical processes in the pedestal and near SOL are heavily determined by the convective flows formed in this region by the inhomogeneous microturbulence and by interplay of the convective flows with the sheared poloidal edge flow of a tokamak plasma, which is the inherent component of the pedestal development and sustainment.

The focus of this paper is the detailed kinetic and hydrodynamic theory of the non-diffusive convective flows, driven by the spatially inhomogeneous parametric microturbulence in the sheared poloidal edge (basic) flow of a tokamak plasma. In Sec. II, we present a detailed description of the two-scale approach to the kinetic theory of the IC parametric instabilities driven by the inhomogeneous FW in inhomogeneous plasmas, which is employed in this paper. In Sec. III, we present the theory of the mesoscale convective flows driven by the inhomogeneous parametric IC and drift microturbulence in the poloidal sheared flow. The derived Vlasov equations for ions and electrons, which govern the mesoscale evolution of the plasma, reveal the existence of the radial and poloidal convective flows with radially inhomogeneous flow velocities. Because of the presence of such flows, which make a dominant contribution to the evolution of inhomogeneous plasma to the equilibrium state, the conventional theory of the stability of the steady plasma cannot be applied to this problem. In this paper, the problem of the origin of the inhomogeneous convective flows and of their temporal evolution is analysed employing the nonmodal approach, which was developed in our theory of the temporal evolution and suppression of the instabilities of a plasma sheared flow across the magnetic fields\textsuperscript{8–10}. This approach employs the solution of the initial value problem instead of the application of the spectral transform in time variable, which is generally applied in the stability theory for the uniform plasmas with steady uniform flows. In Sec. III we develop the nonmodal theory of the convective flows with the radially inhomogeneous flow velocity, which develops in the poloidal sheared flow of the plasma edge. The convective flow involves the poloidal flow with a radially sheared velocity, and the radial compressed/stretched flow with a radially inhomogeneous flow velocity, the effect of which on the plasma stability was not considered yet.

In Sec. IV we derive, as the moments of the electron and ion mesoscale Vlasov equations, the fluid equations for the plasma with the sheared and compressed/stretched flows. These equations, which contain the continuity, the momentum, and the temperature equations for
both plasma species, are solved as the initial value problem which reveal the decisive effect of the compressed radial flow on the formation of the plasma outflows. Conclusions are presented in Sec. V.

II. TWO-SCALE APPROACH TO THE THEORY OF THE PARAMETRIC INSTABILITIES OF THE INHOMOGENEOUS PLASMA DRIVEN BY THE INHOMOGENEOUS FAST WAVE

Our theory of the microscale turbulence, driven by the inhomogeneous FW in the inhomogeneous plasmas, and the theory of the mesoscale convective flows, generated by the inhomogeneous microscale IC and drift turbulence, is based on the Vlasov-Poisson system of equation. In this theory we use the approximation of the slab geometry where $x, y, z$ directions are viewed as corresponding to the radial, poloidal and toroidal directions, respectively, of the toroidal coordinate system. Within this approximation the Vlasov equation employed in our theory for the velocity distribution function $F_\alpha$ of $\alpha$ plasma species ($\alpha = i$ for ions and $\alpha = e$ for electrons) in coordinates $\mathbf{r} = (x, y, z)$,

$$
\frac{\partial F_\alpha (\mathbf{v}, \mathbf{r}, t)}{\partial t} + \mathbf{v} \cdot \frac{\partial F_\alpha (\mathbf{v}, \mathbf{r}, t)}{\partial \mathbf{r}} + \frac{e_\alpha}{m_\alpha} \left( \mathbf{E}_{0x} (x) + \mathbf{E}_1 (x, t) + \tilde{\mathbf{E}} (r, t) \right) + \frac{1}{c} [\mathbf{v} \times (\mathbf{B}_0 + \mathbf{B}_1 (r, t))] \cdot \frac{\partial F_\alpha (\mathbf{v}, \mathbf{r}, t)}{\partial \mathbf{v}} = 0,
$$

contains the inhomogeneous radial electric field $\mathbf{E}_{0x} (x)$, which governs the basic poloidal sheared flow, the FW electric field $\mathbf{E}_1 (x, t)$, the electric field $\tilde{\mathbf{E}} (r, t)$ of the self-consistent plasma respond on FW, the uniform plasma-confining magnetic field $\mathbf{B}_0$ directed along the coordinate $z$, and FW magnetic field $\mathbf{B}_1 (r, t)$. In the edge layer of the tokamak plasma, the spatial inhomogeneity lengths of $\mathbf{E}_{0x} (x)$ and of FW fields are commensurable with a spatial inhomogeneity length of the pedestal plasma density. This mesoscale spatial inhomogeneity length on order of the pedestal width is much less than the wavelength of FW and the inhomogeneity lengths of the plasma parameters in the bulk of the tokamak plasma, but is much larger than the radial wavelength of the IC parametric microturbulence presented in Eq. (II) by electric field $\tilde{\mathbf{E}} (r, t)$. This microturbulence develops in the inhomogeneous pedestal plasma by the inhomogeneous FW. Therefore, electric field $\tilde{\mathbf{E}} (r, t)$ of IC microruurbation involves the microscale and mesoscale spatial inhomogenieties. Thus, the consistent theory which involves the theory of the IC parametric turbulence of the inhomogeneous plasma,
driven by the inhomogeneous FW, and the theory of the mesoscale plasma evolution caused
by the mesoscale inhomogeneities of the IC microturbulence, should include the treatment of
plasma evolution on the micro- as well as on the mesoscales. We employ for the development
of that theory the well known two - scale method (simple introduction to the variety versions
and applications of this method is presented in Ref. 11), which is used for problems in which
the solutions depend simultaneously on widely different scales. Applying this method to our
theory, we introduce, jointly with variables \( r = (x, y, z) \) for the microscale fast variations,
the spatial \( X = \varepsilon x \) variable, where \( \varepsilon \ll 1 \) is an artificial dimensionless small parameter,
which distinguishes the slow mesoscale variations along \( x \) direction, \( \partial/\partial x = \varepsilon \partial/\partial X \), in the
Vlasov equation.

The Vlasov equation (1) in two-scales presentation has a form

\[
\frac{\partial F_\alpha (v, r, X, t)}{\partial t} + v \left( \frac{\partial F_\alpha (v, r, X, t)}{\partial r} + \varepsilon \frac{\partial F_\alpha (v, r, X, t)}{\partial X} \right) + \frac{e_\alpha}{m_\alpha} \left( E_{0x} (X) + E_1 (X, t) + \tilde{E} (r, X, t) \right) + e_\alpha \left[ v \times (B_0 + B_1 (X, t)) \right] \frac{\partial F_\alpha (v, r, X, t)}{\partial v} = 0,
\]

where FW fields

\[
E_1 (X, t) = E_{1x} (X) \cos \omega_0 t + E_{1y} (X) \sin \omega_0 t,
\]

\[
B_1 (X, t) = \frac{c}{\omega_0} \frac{dE_{1y} (X)}{dX} \cos \omega_0 t e_z.
\]

are inhomogeneous only on the mesoscale. The electric field \( \tilde{E} (r, X, t) \), which is inhomogeneous on the micro- and mesoscale, is governed by the Poisson equation,

\[
\nabla \cdot \tilde{E} (r, X, t) = 4\pi \sum_{\alpha=i,e} e_\alpha \int f_\alpha (v, r, X, t) dv,
\]

in which \( f_\alpha \) is the fluctuating part of the distribution function \( F_\alpha (v, r, X, t) \), \( f_\alpha (v, r, X, t) = F_\alpha (v, r, X, t) - F_{0\alpha} (v, X, t) \), \( F_{0\alpha} \) is the equilibrium distribution function of the inhomogeneous plasma; this function is governed by Eq. (2) with \( \tilde{E} (r, X, t) = 0 \).

The central point in the theory of the IC parametric instabilities driven by the spatially
inhomogeneous FW in the inhomogeneous plasma without sheared flow, developed Ref. 3,
is the transformation of the velocity \( v \), mesoscale coordinate \( X \) and microscale coordinates
\( r = (x, y, z) \) in Eq. (1) to the velocity \( v_i \), the mesoscale coordinate \( X_i \) and the microscale
coordinates \( \mathbf{r}_i \), determined in the frame of reverences which moves with velocity \( \mathbf{V}_i (X_i, t) \) equal to the velocity of an ion in the plasma-confining magnetic field \( \mathbf{B}_0 \), and FW fields relative to the laboratory frame. The transformation \((\mathbf{v}, X, \mathbf{r}, t) \rightarrow (\mathbf{v}_i, X_i, \mathbf{r}_i, t)\) is performed by employing the relation

\[
\mathbf{v} = \mathbf{v}_i + \mathbf{V}_i (X_i, t),
\]

and the inverse relation

\[
\mathbf{v}_i = \mathbf{v} - \mathbf{V}(X, t);
\]

the relation

\[
X = X_i + R_{ix} (X_i, t) = X_i + \int_{t_0}^{t} V_{ix} (X_i, t_1) \, dt_1,
\]

and the inverse relation,

\[
X_i = X - \int_{t_0}^{t} V_x (X, t_1) \, dt_1.
\]

The velocity \( \mathbf{V}_i (X_i, t) \) is determined by the equation

\[
\frac{d\mathbf{V}_i}{dt} = \frac{e_i}{m_\alpha} \left( \mathbf{E}_1 (X_i + R_{ix} (X_i, t), t) + \frac{1}{c} [\mathbf{V}_i \times \mathbf{B}_0] 
\right. \\
\left. + \frac{1}{c} [\mathbf{V}_i \times \mathbf{B}_1 (X_i + R_{ix} (X_i, t), t)] \right),
\]

with initial value \( \mathbf{V}_i (X_i, t = t_0) = 0 \). In this equation, the variable \( X_i \), which determines the spatial inhomogeneity of the plasma and FW field, enters to Eq. (10) as a parameter. It was derived in Ref. that when the amplitude of the ion displacement \( |R_{ix} (X_i, t)| \) in FW field is much less than the spatial scale length \( L_E \) of the radial inhomogeneity of the FW field, the solution to Eq. (10) may be derived in a form of a power series expansion in powers of \( R_{ix}/L_E \). It was found that when \( \omega_0 - \omega_{ci} \sim \omega_{ci} \), where \( \omega_{ci} \) is the ion cyclotron frequency, the terms on the order of \( O (R_{ix}/L_E) \) of this expansions with FW for pedestal parameters are negligibly small and may be neglected. It was derived in Ref. that with variables \( \mathbf{v}_i, \ X_i, \) and \( \mathbf{r}_i \), where

\[
\mathbf{r} = \mathbf{r}_i + \mathbf{R}_i (X_i, t) = \mathbf{r}_i + \int_{t_0}^{t} \mathbf{V}_i (X_i, t) \, dt_1,
\]
FW electric field is presented in the Vlasov equation (2) for the microscale perturbations only in terms of the order of \(|R_{i2}/L_E| \ll 1\) which may be neglected. Therefore the Vlasov equation for \(F_i (v_i, r_i, X_i, t)\) with great accuracy has a form as for a steady plasma in the uniform magnetic field \(B_0\) without FW fields, and contains the variable \(X_i\) as a parameter, i.e.

\[
\frac{\partial F_i (v_i, r_i, X_i, t)}{\partial t} + v_i \frac{\partial F_i}{\partial r_i} + \frac{e_i}{m_i c} [v_i \times B_0] \frac{\partial F_i}{\partial v_i} + \frac{e_i}{m_i} \tilde{E}_i (r_i, X_i, t) \frac{\partial F_i (v_i, r_i, X_i, t)}{\partial v_i} = 0. \tag{12}
\]

In fact, this equation is the result of the quantitative justification of the application of the local approximation to the theory of the parametric instabilities driven by the inhomogeneous FW field in the pedestal plasma. This result is completely applicable to the case of FW in the poloidal sheared flow of a tokamak plasma. In this case, velocity \(V_i\) is determined by Eq. (10) with the electric field \(E_1 (X_i + R_{i2} (X_i, t), t) + E_{0x} (X_i + R_{i2} (X_i, t))\).

The equation similar to Eq. (12) is derived for the Vlasov equation for the electron distribution function \(F_e (v_e, r_e, X_e, t)\), where \(v_e\) and \(r_e\) are the velocity and the position vector of electron in the frame of references which moves with velocity \(V_e (X_e, t)\) determined by Eq. (10) with ion species subscript \(i\) changed on the electron subscript \(e\). It follows from Eq. (12) that the Vlasov equation for the perturbation \(f_i (v_i, r_i, X_i, t)\) of the electron distribution function \(F_{i0} (v_i, X_i)\) and the similar equation for the perturbation \(f_e (v_e, r_e, X_e, t)\) of the electron distribution function \(F_{e0} (v_e, X_e)\) are the same as for a plasma without FW field,

\[
\frac{\partial f_i (v_i, r_i, X_i, t)}{\partial t} + v_i \frac{\partial f_i}{\partial r_i} + \frac{e_i}{m_i} [v_i \times B_0] \frac{\partial f_i}{\partial v_i} + \frac{e_i}{m_i} \tilde{E}_i (r_i, X_i, t) \frac{\partial f_i (v_i, r_i, X_i, t)}{\partial v_i} = - \frac{e_i}{m_i} \tilde{E}_i (r_i, X_i, t) \frac{\partial F_{i0} (v_i, X_i)}{\partial v_i}, \tag{13}
\]

\[
\frac{\partial f_e (v_e, r_e, X_e, t)}{\partial t} + v_e \frac{\partial f_e}{\partial r_e} + \frac{e_e}{m_e} [v_e \times B_0] \frac{\partial f_e}{\partial v_e} + \frac{e_e}{m_e} \tilde{E}_e (r_e, X_e, t) \frac{\partial f_e (v_e, r_e, X_e, t)}{\partial v_e} = - \frac{e_e}{m_e} \tilde{E}_e (r_e, X_e, t) \frac{\partial F_{e0} (v_e, X_e)}{\partial v_e}. \tag{14}
\]

The mesoscale spatial variables \(X_i\) and \(X_e\) are introduced to Eqs. (13), (14) by the spatial inhomogeneity of \(F_{i0} (v_i, X_i)\) and \(F_{e0} (v_e, X_e)\) functions and present in these equations as a parameters. Thus, the solutions to \(f_i\) and \(f_e\) also depend on \(X_i\) and \(X_e\). The solutions to Eqs. (13) and (14) were derived in Ref. 3, where the local approximation to the theory of the parametric IC microinstabilities driven by the inhomogeneous FW in the inhomogeneous
plasma was developed. These results for $f_i$ and $f_e$, as well as ones for the perturbed ion and electron densities, are applicable to the present investigations of the IC microinstabilities in the poloidal sheared flow.

The Vlasov equations (13) and (14) for $f_i(v_i, r_i, X_i, t)$ and $f_e(v_e, r_e, X_e, t)$, and the Poisson equation

$$\nabla \cdot \tilde{E}(r_i, X_i, t) = 4\pi \sum_{\alpha=i,e} c_{\alpha} \int f_{\alpha}(v_{\alpha}, r_{\alpha}, X_{\alpha}, t) \, dv_{\alpha}, \quad (15)$$

for the electric field $\tilde{E}_i(r_i, X_i, t) = -\frac{\partial \varphi_i(r_i, X_i, t)}{\partial r_i}$ compose the system of equation for the investigation of the microscale IC parametric instabilities of a plasma sheared flow in the inhomogeneous FW field, and the nonlinear evolution and saturation these instabilities. In the saturation state, the electric field $\tilde{E}$ of the electrostatic parametric microturbulence is determined in the ion frame in the form

$$\tilde{E}_i(r_i, X_i, t) = \int dk \tilde{E}_i(k, X_i) e^{-i\omega(k, X_i)t + ikr_i + i\theta(k)},$$

$$= -i \int dk k \varphi_i(k, X_i) e^{-i\omega(k, X_i)t + ikr_i + i\theta(k)}, \quad (16)$$

where the integration over $k$ is performed over wave numbers of the linearly unstable IC perturbations, and $\theta(k)$ is their initial phase. In the electron frame, oscillating relative to the ion frame, this electric field is determined by Eq. (16) with species subscript $i$ changed on $e$. The electric field $\tilde{E}_e(r_e, X_e, t)$ in variables $r_i, X_i$ has a form

$$\tilde{E}_e(r_e, X_e, t) = \int dk \tilde{E}_e(k, X_e) e^{-i\omega(k, X_e)t + ikr_e + i\theta(k)}$$

$$= \int dk \tilde{E}_i(k, X_i) e^{-i\omega(k, X_i)t + ikr_i + i\omega_p(k, X_i)t - ip\delta_{ie}(k, X_i) + i\theta(k)}, \quad (17)$$

where

$$\Omega_p(k, X_i) = \omega(k, X_i) + p\omega_0, \quad (18)$$

and $J_p(a_{ie})$ is the first kind Bessel function of order $p$ with argument $a_{ie}$. Parameters $a_{ie} \sim k\xi_{ie}$, where $\xi_{ie}$ is the amplitude of the relative displacement of electrons relative to ions in FW field, and $\delta_{ie}$ were determined in Ref. 3.

The theory of the parametric microturbulence, developed in details in Ref. 3 for the case $V_0' = 0$, is completely applicable to the tokamak poloidal sheared flows, where always the
velocity shearing rate \( V_0' = -cE_0'/B_0 \) is much less than IC frequency \( \omega_{ci} \) and is less than the growth rate of the IC parametric instabilities. However, the poloidal sheared flow with the velocity shearing rate commensurable with velocity gradients of the inhomogeneous mesoscale convective flows strongly affects their structure and temporal evolution.

III. MESOSCALE CONVECTIVE FLOWS IN THE POLOIDALLY SHEARED PLASMA FLOW

On the nonlinear stage of the IC parametric microturbulence evolution at time above the inverse growth rate of the IC instabilities, \( t \gg \gamma^{-1}(k) > |\omega^{-1}(k)| \), electric field \([16]\) becomes the random function of the initial phase \( \theta (k) \). The motion of ions and electrons in this field has a form of the random scattering of particles by the turbulent electric field and mimics to the thermal motion of particles. Here we consider the average effect of the mesoscale inhomogeneity of the microscale IC turbulence on the mesoscale evolution of the ion and electron distribution functions. For this goal, we transform velocity \( \mathbf{v}_i \) and mesoscale \( X_i, Y_i \) and microscale \( r_i \) spatial variables to new velocity \( \tilde{\mathbf{v}}_i \) and coordinates \( \tilde{X}_i, \tilde{Y}_i, \tilde{r}_i \), in which the thermal ion motion and the ion motion in the turbulent electric field \( \tilde{\mathbf{E}}_i \) are separated. The transformation \( (\mathbf{v}_i, X_i, Y_i, r_i, t) \rightarrow (\tilde{\mathbf{v}}_i, \tilde{X}_i, \tilde{Y}_i, \tilde{r}_i, t) \) is determined by the relations

\[
\tilde{\mathbf{v}}_i = \mathbf{v}_i - \tilde{\mathbf{V}}_i (r_i, X_i, t),
\]

\[
\tilde{X}_i = X_i - \int_{t_0}^{t} \tilde{V}_{ix} (r_i, X_i, t_1) \, dt_1,
\]

\[
\tilde{Y}_i = Y_i - \int_{t_0}^{t} \tilde{V}_{iy} (r_i, X_i, t_1) \, dt_1,
\]

and

\[
\tilde{r}_i = r_i - \int_{t_0}^{t} \tilde{V}_{i} (r_i, X_i, t_1) \, dt_1,
\]

or by their inverse,

\[
\mathbf{v}_i = \tilde{\mathbf{v}}_i + \tilde{\mathbf{U}}_i (\tilde{r}_i, \tilde{X}_i, t),
\]
\[ X_i = \bar{X}_i + \int_{t_0}^{t} \bar{U}_{ix} \left( \bar{r}_i, \bar{X}_i, t_1 \right) dt_1, \]  

(24)

\[ Y_i = Y - V'_i X t = \bar{Y}_i + \int_{t_0}^{t} \bar{U}_{iy} \left( \bar{r}_i, \bar{X}_i, t_1 \right) dt_1, \]  

(25)

and

\[ \mathbf{r}_i = \bar{\mathbf{r}}_i + \mathbf{R}_i \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right) = \bar{\mathbf{r}}_i + \int_{t_0}^{t} \bar{\mathbf{U}}_i \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t_1 \right) dt_1. \]  

(26)

The velocity \( \bar{\mathbf{V}}_i (r_i, X_i, t) \) is determined by the Euler equation  

\[ \frac{\partial \bar{\mathbf{V}}_i}{\partial t} + \bar{V}_{ix} \frac{\partial \bar{\mathbf{V}}_i (r_i, X_i, t)}{\partial X_i} = \frac{e_i}{m_i} \left( \bar{\mathbf{E}}_i (r_i, X_i, t) + \frac{1}{c} \left[ \bar{\mathbf{V}}_i \times \mathbf{B}_0 \right] \right) \]  

(27)

as the velocity of an ion in the electric field \( \bar{\mathbf{E}}_i (r_i, X_i, t) \) of the IC parametric turbulence, where variables \( r_i, X_i \) are determined in the frame of references which moves with the velocity of an ion in the field of the fast wave in a poloidal sheared flow. In variables \( (\bar{X}_i, t) \), the convective nonlinear part \( \bar{V}_{ix} \frac{\partial}{\partial X_i} \) of the operator \( \frac{\partial}{\partial t} + \bar{V}_{ix} \frac{\partial}{\partial X_i} \) of Eq. (27) vanishes and this operator is transformed to the linear one, \( \frac{\partial}{\partial t} \). Then, Eq. (27) becomes the ordinary differential equation

\[ \frac{d}{dt} \bar{\mathbf{U}}_i \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right) = \frac{e_i}{m_i} \left( \bar{\mathbf{E}}_i \left( \bar{\mathbf{r}}_i + \bar{\mathbf{R}}_i \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right), \bar{\mathbf{X}}_i + \bar{R}_{ix} \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right), t \right), t \right) + \frac{1}{c} \left[ \bar{\mathbf{U}}_i \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right) \times \mathbf{B}_0 \right] \]  

(28)

for \( \bar{\mathbf{U}}_i \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right) = \bar{\mathbf{V}}_i \left( \mathbf{r}_i, \mathbf{X}_i, t \right) \), where \( \mathbf{r}_i, \mathbf{X}_i \) are determined as a functions of \( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \) by Eqs. (26) and (24). The spatial micro- and mesoscale variables \( \bar{\mathbf{r}}_i \) and \( \bar{\mathbf{X}}_i \) enter to Eq. (28) as parameters. Equation (28), as well as Eq. (10), reveals as a hybrid of Eulerian - Lagrangian description of the ion dynamics, in which the ion velocity and the ion displacement associated with the waves are defined as in the Eulerian dynamics by the functions of \( \bar{X}_i \) and \( t \) and not primarily as a function of the initial position of ion as in a purely Lagrangian description. At the same time, the nonlinear convective derivative is absent in Eqs. (28) and (10).

Using the expansion for the nonlinear electric field \( \bar{\mathbf{E}}_i \) in Eq. (28),

\[ \bar{\mathbf{E}}_i \left( \bar{\mathbf{r}}_i + \bar{\mathbf{R}}_i \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right), \bar{\mathbf{X}}_i + \bar{R}_{ix} \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right), t \right) \approx \bar{\mathbf{E}}_i \left( \bar{\mathbf{r}}_i + \bar{\mathbf{R}}_i \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right), \bar{\mathbf{X}}_i, t \right) \]  

\[ + \bar{R}_{ix} \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right) \frac{\partial}{\partial \bar{\mathbf{X}}_i} \bar{\mathbf{E}}_i \left( \bar{\mathbf{r}}_i + \bar{\mathbf{R}}_i \left( \bar{\mathbf{r}}_i, \bar{\mathbf{X}}_i, t \right), \bar{\mathbf{X}}_i, t \right), \]  

(29)
which is valid for the small displacement, $|\tilde{R}_{ix} (\tilde{r}_i + \tilde{R}_i, \tilde{X}_i, t)| \ll L_{E_i}$, of an ion in the inhomogeneous electric field $E_i$, the solution to Eq. (28) with the initial value $\tilde{U}_i (\tilde{r}_i, \tilde{X}_i, t_0) = 0$ is easily derived and may be presented in the form

$$
\tilde{U}_{ix} (\tilde{r}_i, \tilde{X}_i, t) \approx \tilde{U}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t) + \tilde{U}_{ix}^{(1)} (\tilde{r}_i, \tilde{X}_i, t),
$$

(30)

and

$$
\tilde{U}_{iy} (\tilde{r}_i, \tilde{X}_i, t) \approx \tilde{U}_{iy}^{(0)} (\tilde{r}_i, \tilde{X}_i, t) + \tilde{U}_{iy}^{(1)} (\tilde{r}_i, \tilde{X}_i, t).
$$

(31)

In Eqs. (30), (31)

$$
\tilde{U}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t) = \frac{e_i}{m_i} \int_{t_0}^{t} dt_1 \left[ \tilde{E}_x (\tilde{r}_i + \tilde{R}_i, \tilde{X}_i, t_1) \cos \omega_{ci} (t - t_1) + \tilde{E}_y (\tilde{r}_i + \tilde{R}_i, \tilde{X}_i, t_1) \sin \omega_{ci} (t - t_1) \right],
$$

(32)

$$
\tilde{U}_{ix}^{(1)} (\tilde{r}_i, \tilde{X}_i, t) = \frac{e_i}{m_i} \int_{t_0}^{t} dt_1 \left[ -\tilde{E}_x (\tilde{r}_i + \tilde{R}_i, \tilde{X}_i, t_1) \sin \omega_{ci} (t - t_1) + \tilde{E}_y (\tilde{r}_i + \tilde{R}_i, \tilde{X}_i, t_1) \cos \omega_{ci} (t - t_1) \right],
$$

(33)

$$
\tilde{U}_{iy}^{(0)} (\tilde{r}_i, \tilde{X}_i, t) = \frac{e_i}{m_i} \int_{t_0}^{t} dt_1 \left[ \frac{\partial}{\partial X_i} \left( \tilde{E}_x (\tilde{r}_i + \tilde{R}_i, \tilde{X}_i, t_1) \right) \cos \omega_{ci} (t - t_1) + \tilde{E}_y (\tilde{r}_i + \tilde{R}_i, \tilde{X}_i, t_1) \sin \omega_{ci} (t - t_1) \right],
$$

(34)

$$
\tilde{U}_{iy}^{(1)} (\tilde{r}_i, \tilde{X}_i, t) = \frac{e_i}{m_i} \int_{t_0}^{t} dt_1 \left[ -\frac{\partial}{\partial X_i} \left( \tilde{E}_x (\tilde{r}_i + \tilde{R}_i, \tilde{X}_i, t_1) \right) \sin \omega_{ci} (t - t_1) + \tilde{E}_y (\tilde{r}_i + \tilde{R}_i, \tilde{X}_i, t_1) \cos \omega_{ci} (t - t_1) \right].
$$

(35)

With variables $\tilde{v}_i, \tilde{X}_i, \tilde{Y}_i, \tilde{r}_i$ the Vlasov equation for the distribution function $F_i (\tilde{v}_i, \tilde{X}_i, \tilde{Y}_i, \tilde{r}_i, t)$ of ions in the sheared poloidal flow for time $t \gg \tau_{corr} \sim \gamma^{-1}$ becomes

$$
\frac{\partial F_i}{\partial t} + \tilde{v}_{ix} \frac{\partial F_i}{\partial X_i} + (\tilde{v}_{iy} - V_0 t \tilde{v}_{ix}) \frac{\partial F_i}{\partial Y_i} - \tilde{v}_{ix} \int_{t_0}^{t} \frac{\partial}{\partial X_i} \tilde{V}_{ix} (\tilde{r}_i, X_i, t_1) dt_1 \frac{\partial F_i}{\partial X_i}
$$

(36)
for the ion distribution function $\bar{F}_i$ the hybrid of Eulerian - Lagrangian description the average at time to the positions $\tilde{r}_i, \tilde{X}_i, t$ of the electrostatic mesoscale perturbations. The potential $\varphi$ has nonlinear "extra term" originated from the convective part of the material time derivative. This term is absent in the averaged hybrid Eulerian - Lagrangian Eq. (28) for $U_i \left( \tilde{r}_i, \tilde{X}_i, t \right)$, averaged over the microscale initial phases $\theta (k)$, because $\langle \varphi_i \left( \tilde{r}, \tilde{X}_i, t \right) \rangle = 0$. It follows from Eq. (27), that the equation for the Eulerian mean velocity $\langle \tilde{V}_i \left( r_i, X_i, t \right) \rangle$ averaged over the initial phases $\theta (k)$, has nonlinear "extra term" originated from the convective part of the material time derivative. This term is absent in the averaged hybrid Eulerian - Lagrangian Eq. (28) for $U_i \left( \tilde{r}_i, \tilde{X}_i, t \right)$, averaged over the microscale initial phases

$$- \bar{v}_{ixz} \int_{t_0}^{t} \frac{\partial}{\partial X_i} \tilde{V}_{iy} \left( r_i, X_i, t_1 \right) dt_1 \frac{\partial F_i}{\partial Y_i} = \bar{U}_{ix} \left( \tilde{r}_i, \tilde{X}_i, t \right) \int_{t_0}^{t} \frac{\partial}{\partial X_i} \tilde{V}_{ix} \left( r_i, X_i, t_1 \right) dt_1 \frac{\partial F_i}{\partial X_i}$$

$$- \bar{U}_{ix} \left( \tilde{r}_i, \tilde{X}_i, t \right) \left( V_0^t + \int_{t_0}^{t} \frac{\partial}{\partial X_i} \tilde{V}_{iy} \left( r_i, X_i, t_1 \right) dt_1 \right) \frac{\partial F_i}{\partial Y_i}$$

$$+ \omega_c \bar{v}_{iy} \frac{\partial F_i}{\partial v_{iy}} - \left( \omega_c + V_0^t \right) \bar{v}_{ix} \frac{\partial F_i}{\partial v_{ix}}$$

$$- \frac{e_i}{m_i} \left( \frac{\partial}{\partial X_i} \varphi_i \left( \tilde{r}_i + \tilde{R}_i, \tilde{X}_i, \tilde{Y}_i, t \right) - V_0^t \frac{\partial}{\partial Y_i} \varphi_i \left( \tilde{r}_i + \tilde{R}_i, \tilde{X}_i, \tilde{Y}_i, t \right) \right) \frac{\partial F_i}{\partial v_{ix}}$$

$$- \frac{e_i}{m_i} \frac{\partial}{\partial Y_i} \varphi_i \left( \tilde{r}_i + \tilde{R}_i, \tilde{X}_i, \tilde{Y}_i, t \right) \frac{\partial F_i}{\partial v_{iy}} - \frac{e_i}{m_i} \frac{\partial}{\partial Y_i} \varphi_i \left( \tilde{r}_i + \tilde{R}_i, \tilde{X}_i, \tilde{Y}_i, t \right) \frac{\partial F_i}{\partial v_{ix}} = 0.$$ (36)

The electrostatic potential $\varphi_i$ in Eq. (36) depends on the micro- and mesoscales and can be expressed in the form

$$\varphi_i \left( \tilde{r}_i + \tilde{R}_i, \tilde{X}_i, \tilde{Y}_i, t \right) = \tilde{\varphi}_i \left( \tilde{r}_i + \tilde{R}_i, \tilde{X}_i, \tilde{Y}_i, t \right) + \Phi_i \left( \tilde{X}_i, \tilde{Y}_i, t \right),$$ (37)

where $\tilde{\varphi}_i$ is the potential of the microscale turbulence, and $\Phi_i \left( \tilde{X}_i, \tilde{Y}_i, t \right)$ is the potential of the electrostatic mesoscale perturbations. The potential $\varphi \left( \tilde{r}_i + \tilde{R}_i, \tilde{X}_i, \tilde{Y}_i, t \right)$, averaged over the initial phases of the microscale perturbations, is

$$\langle \varphi_i \left( \tilde{r}_i + \tilde{R}_i, \tilde{X}_i, \tilde{Y}_i, t \right) \rangle = \Phi_i \left( \tilde{X}_i, \tilde{Y}_i, t \right),$$ (38)

because $\langle \tilde{\varphi}_i \left( \tilde{r}, \tilde{X}_i, t \right) \rangle = 0$. The average of the microscale perturbations $\tilde{V}_i \left( r_i, X_i, t \right)$ has nonlinear "extra term" originated from the convective part of the material time derivative. This term is absent in the averaged hybrid Eulerian - Lagrangian Eq. (28) for $U_i \left( \tilde{r}_i, \tilde{X}_i, t \right)$, averaged over the microscale initial phases $\tilde{X}_i + \tilde{R}_{ix} \left( \tilde{r}_i, \tilde{X}_i, t \right)$ displaced by the microturbulence. The Vlasov equation for the ion distribution function $\bar{F}_i \left( \tilde{v}_i, \tilde{X}_i, \tilde{Y}_i, t \right)$, averaged over the microscale initial phases $\tilde{X}_i + \tilde{R}_{ix} \left( \tilde{r}_i, \tilde{X}_i, t \right)$, averaged over the microscale initial phases
for a time $t \gg \tau_{corr} \sim \gamma^{-1}$, is
\[
\frac{\partial F_i}{\partial t} + \bar{u}_{ix} \frac{\partial F_i}{\partial X_i} + (\bar{v}_{iy} - V_0 t \bar{v}_{ix}) \frac{\partial F_i}{\partial Y_i} - \bar{U}_{ix} \left( \bar{X}_i, t \right) \frac{\partial F_i}{\partial X_i} - \bar{U}_{iy} \left( \bar{X}_i, t \right) \frac{\partial F_i}{\partial Y_i} + \omega_i \bar{v}_{iy} \frac{\partial F_i}{\partial \bar{v}_{ix}} - (\omega_i + V_0) \frac{\partial F_i}{\partial \bar{v}_{iy}}
+ \omega_i \bar{v}_{iy} \frac{\partial F_i}{\partial \bar{v}_{ix}} - (\omega_i + V_0) \frac{\partial F_i}{\partial \bar{v}_{iy}}
- \frac{e_i}{m_i} \left( \frac{\partial \Phi_i}{\partial X_i} \left( \bar{X}_i, \bar{Y}_i, t \right) - V_0 t \frac{\partial \Phi_i}{\partial Y_i} \left( \bar{X}_i, \bar{Y}_i, t \right) \right) \frac{\partial F_i}{\partial \bar{V}_{ix}}
- \frac{e_i}{m_i} \frac{\partial \Phi_i}{\partial \bar{Y}_i} \left( \bar{X}_i, \bar{Y}_i, t \right) \frac{\partial F_i}{\partial \bar{v}_{iy}} - \frac{e_i}{m_i} \frac{\partial \Phi_i}{\partial \bar{v}_z} \left( \bar{X}_i, \bar{Y}_i, t \right) \frac{\partial F_i}{\partial \bar{v}_{iz}} = 0.
\]
(41)

In this equation, we use the relations
\[
\frac{\partial}{\partial X_i} \bar{V}_{ix} \left( \bar{r}_i, X_i, t_1 \right) = \frac{\partial}{\partial X_i} \bar{U}_{ix} \left( \bar{r}_i, \bar{X}_i, t_1 \right)
\approx \frac{\partial}{\partial X_i} \left( \bar{U}_{ix} \left( \bar{r}_i, \bar{X}_i, t_1 \right) \right) \left( 1 - \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{U}_{ix} \left( \bar{r}_i, \bar{X}_i, t_1 \right) dt_1 \right)
\]
(42)

and
\[
\frac{\partial}{\partial X_i} \bar{V}_{iy} \left( \bar{r}_i, X_i, t_1 \right) = \frac{\partial}{\partial X_i} \bar{U}_{iy} \left( \bar{r}_i, \bar{X}_i, t_1 \right)
\approx \frac{\partial}{\partial X_i} \left( \bar{U}_{iy} \left( \bar{r}_i, \bar{X}_i, t_1 \right) \right) \left( 1 - \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{U}_{ix} \left( \bar{r}_i, \bar{X}_i, t_1 \right) dt_1 \right).
\]
(43)

These relations stem from the identity $\bar{V}_i \left( \bar{r}_i, X_i, t \right) = \bar{U}_i \left( \bar{r}_i, \bar{X}_i, t \right)$, which follows from Eqs. (19) and (23), and from the identity
\[
\frac{\partial \bar{X}_i}{\partial X_i} = 1 - \int_{t_0}^{t} \frac{\partial \bar{V}_{ix} \left( \bar{r}_i, X_i, t_1 \right)}{\partial X_i} dt_1,
\]
which follows from Eqs. (20) and (24). In Eq. (41), $\bar{U}_{ix} \left( \bar{X}_i, t \right)$ and $\bar{U}_{iy} \left( \bar{X}_i, t \right)$ are the spatially inhomogeneous velocities of the convective flows along directions of $x_i$ and $y_i$, respectively. The velocity $\bar{U}_{ix} \left( \bar{X}_i, t \right)$ with accounting for the terms on the order of $O \left( \left| \bar{R}_{ix} / L_E \right|^3 \right)$ is determined by the equation
\[
\bar{U}_{ix} \left( \bar{X}_i, t \right) = \left\{ \bar{U}_{ix} \left( \bar{r}_i, \bar{X}_i, t \right) \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{V}_{ix} \left( \bar{r}_i, X_i, t_1 \right) dt_1 \right\}
\approx \bar{U}_{ix}^{(0)} \left( \bar{X}_i, t \right) + \bar{U}_{ix}^{(1)} \left( \bar{X}_i, t \right) + \bar{U}_{ix}^{(2)} \left( \bar{X}_i, t \right),
\]
(44)

where
\[
\bar{U}_{ix}^{(0)} \left( \bar{X}_i, t \right) = \left\{ \bar{U}_{ix}^{(0)} \left( \bar{r}_i, \bar{X}_i, t \right) \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{U}_{ix}^{(0)} \left( \bar{r}_i, \bar{X}_i, t_1 \right) dt_1 \right\},
\]
(45)
For the electric field \( \tilde{E} \) Velocity \( \bar{u} \) where determined by Eqs. (30) - (35) with the turbulent electric field \( \tilde{E} \) determined in the Appendix. A form similar to Eq. (41). The velocities \( \bar{u} \) are determined by Eqs. (19) - (25) with ion species subscript changed on electron, has by Eq. (17).

The Vlasov equation for the average electron distribution \( \bar{F}_e \), expressed as

\[
\bar{U}_{ix}^{(1)} (\tilde{X}_i, t) = \left\langle \bar{U}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t) \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{U}_{ix}^{(1)} (\tilde{r}_i, \tilde{X}_i, t_1) \, dt_1 \right\rangle \\
+ \left\langle \bar{U}_{ix}^{(1)} (\tilde{r}_i, \tilde{X}_i, t) \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{U}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t_1) \, dt_1 \right\rangle ,
\]

(46)

\[
\bar{U}_{ix}^{(2)} (\tilde{X}_i, t) = - \left\langle \bar{U}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t) \right\rangle \\
\times \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{U}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t_1) \, dt_1 \int_{t_0}^{t_1} \frac{\partial}{\partial X_i} \bar{U}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t_2) \, dt_2 .
\]

(47)

Velocity \( \bar{v}_{iy} \) is expressed as

\[
\bar{v}_{iy} (\tilde{X}_i, t) = \left\langle \bar{v}_{ix} (\tilde{r}_i, \tilde{X}_i, t) \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{v}_{iy} (\tilde{r}_i, X_i, t_1) \, dt_1 \right\rangle \\
\approx \bar{v}_{iy}^{(0)} (\tilde{X}_i, t) + \bar{v}_{iy}^{(1)} (\tilde{X}_i, t) + \bar{v}_{iy}^{(2)} (\tilde{X}_i, t),
\]

(48)

where

\[
\bar{v}_{iy}^{(0)} (\tilde{X}_i, t) = \left\langle \bar{v}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t) \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{v}_{iy}^{(0)} (\tilde{r}_i, \tilde{X}_i, t_1) \, dt_1 \right\rangle ,
\]

(49)

\[
\bar{v}_{iy}^{(1)} (\tilde{X}_i, t) = \left\langle \bar{v}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t) \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{v}_{iy}^{(1)} (\tilde{r}_i, \tilde{X}_i, t_1) \, dt_1 \right\rangle \\
+ \left\langle \bar{v}_{ix}^{(1)} (\tilde{r}_i, \tilde{X}_i, t) \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{v}_{iy}^{(0)} (\tilde{r}_i, \tilde{X}_i, t_1) \, dt_1 \right\rangle ,
\]

(50)

\[
\bar{v}_{iy}^{(2)} (\tilde{X}_i, t) = - \left\langle \bar{v}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t) \right\rangle \\
\times \int_{t_0}^{t} \frac{\partial}{\partial X_i} \bar{v}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t_1) \, dt_1 \int_{t_0}^{t_1} \frac{\partial}{\partial X_i} \bar{v}_{ix}^{(0)} (\tilde{r}_i, \tilde{X}_i, t_2) \, dt_2 .
\]

(51)

For the electric field \( \tilde{E}_e \), given by Eq. (16), the velocities \( \bar{U}_{ix}^{(0)} (\tilde{X}_i, t) \) and \( \bar{U}_{iy}^{(0)} (\tilde{X}_i, t) \) are determined in the Appendix.

The Vlasov equation for the average electron distribution \( \bar{F}_e \), \( \bar{F}_e (\tilde{v}_e, \tilde{X}_e, \tilde{Y}_e, t) \), where \( \tilde{X}_e, \tilde{Y}_e \) are determined by Eqs. (19) - (25) with ion species subscript changed on electron, has a form similar to Eq. (41). The velocities \( \bar{U}_{ex} \) and \( \bar{U}_{ey} \) are determined in this equation by Eqs. (44) - (53) in which velocities \( \bar{U}_{ex} \) and \( \bar{U}_{ey} \) are determined by Eqs. (30) - (35) with the turbulent electric field \( \tilde{E}_e \), determined by Eq. (17).
The Vlasov equations \( \text{[1]} \) for \( \bar{F}_i \left( \bar{\mathbf{v}}_i, \bar{X}_i, \bar{Y}_i, t \right) \), and the similar equation for \( \bar{F}_e \left( \bar{\mathbf{v}}_e, \bar{X}_e, \bar{Y}_e, t \right) \), and the Poisson equation for the potential \( \Phi_i \left( \bar{X}_i, \bar{Y}_i, t \right) \),

\[
\frac{\partial^2 \Phi_i}{\partial X_i^2} + \frac{\partial^2 \Phi_i}{\partial Y_i^2} = -4\pi \left( e_i \int d\mathbf{v}_i \bar{F}_i \left( \bar{\mathbf{v}}_i, \bar{X}_i, \bar{Y}_i, t \right) \right) - |e| \int d\mathbf{v}_e \bar{F}_e \left( \bar{\mathbf{v}}_e, \bar{X}_e, \bar{Y}_e, t \right),
\]

(52)

compose the Vlasov-Poisson system, which governs the kinetic mesoscale evolution of a plasma under the average action of the spatially inhomogeneous microturbulence.

IV. HYDRODYNAMICS OF THE MESOSCALE CONVECTIVE FLOWS

The mesoscale spatial variations of a plasma are involved in the theory of the microturbulence through the dependences on \( X_i \) and on \( X_e \) of the ion and electron densities and temperatures. In this Section, we derive a closed set of equations which determine the mesoscale evolution of the ion and electron densities, velocities, temperatures and of the electrostatic potential in the poloidal sheared flow with radially inhomogeneous convective flows. These equations are the first three moments of the Vlasov equations \( \text{[1]} \) for \( \bar{F}_i \) and \( \bar{F}_e \). Starting with Eq. \( \text{[1]} \) we derive the moment equations as follows. By integrating over velocities \( \bar{\mathbf{v}}_i \) we derive the ion particles density conservation equation:

\[
\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial X_i} \left( n_i u_{ix} \right) + \frac{\partial}{\partial Y_i} \left( n_i \left( u_{iy} - V_0' t u_{ix} \right) \right) - U_{ix} \left( X_i, t \right) \frac{\partial n_i}{\partial X_i} - U_{iy} \left( X_i, t \right) \frac{\partial n_i}{\partial Y_i} = 0.
\]

(53)

By first multiplying by \( \bar{\mathbf{v}}_i \) of Eq. \( \text{[1]} \) and integrating over velocities \( \bar{\mathbf{v}}_i \), we obtain the momentum conservation equations:

\[
\frac{\partial u_{ix}}{\partial t} + u_{ix} \frac{\partial u_{ix}}{\partial X_i} + \left( u_{iy} - V_0' t u_{ix} \right) \frac{\partial u_{ix}}{\partial Y_i} - U_{ix} \left( X_i, t \right) \frac{\partial u_{ix}}{\partial X_i} - U_{iy} \left( X_i, t \right) \frac{\partial u_{ix}}{\partial Y_i} = -\frac{1}{m_i n_i} \left( \frac{\partial P_i}{\partial X_i} - V_0' \frac{\partial P_i}{\partial Y_i} \right) - \frac{e_i}{m_i} \left( \frac{\partial \Phi \left( X_i, Y_i, t \right)}{\partial X_i} - V_0' \frac{\partial \Phi \left( X_i, Y_i, t \right)}{\partial Y_i} \right) + \omega_c u_{iy},
\]

(54)

\[
\frac{\partial u_{iy}}{\partial t} + u_{ix} \frac{\partial u_{iy}}{\partial X_i} + \left( u_{iy} - V_0' t u_{ix} \right) \frac{\partial u_{iy}}{\partial Y_i} - U_{ix} \left( X_i, t \right) \frac{\partial u_{iy}}{\partial X_i} - U_{iy} \left( X_i, t \right) \frac{\partial u_{iy}}{\partial Y_i} = -\frac{1}{m_i n_i} \frac{\partial P_i}{\partial Y_i} - \frac{e_i}{m_i} \frac{\partial \Phi \left( X_i, Y_i, t \right)}{\partial Y_i} - \left( \omega_c + V_0' \right) u_{ix}.
\]

(55)
By first multiplying Eq. (11) by $\frac{1}{2} \left| \vec{v}_i - \vec{u}_i \left( \tilde{X}_i, \tilde{Y}_i, t \right) \right|^2$ and integrating over velocities $\vec{v}_i$ we obtain the equation

$$\frac{\partial T_i}{\partial t} + u_{ix} \frac{\partial T_i}{\partial X_i} + (u_{iy} - V_0' u_{ix}) \frac{\partial T_i}{\partial Y_i} - \tilde{U}_{ix} \left( \tilde{X}_i, t \right) \frac{\partial T_i}{\partial X_i} - \tilde{U}_{iy} \left( \tilde{X}_i, t \right) \frac{\partial T_i}{\partial Y_i} = - \frac{2}{3} T_i \left( \frac{\partial u_{ix}}{\partial X_i} - V_0' \frac{\partial u_{ix}}{\partial Y_i} + \frac{\partial u_{iy}}{\partial Y_i} \right),$$

(56)

which determines the evolution of the ion temperature in the poloidal sheared flow with convective flows. The definitions of the ion number density $n_i \left( \tilde{X}_i, \tilde{Y}_i, t \right)$, the velocity $\vec{u}_i \left( \tilde{X}_i, \tilde{Y}_i, t \right)$ and the ion temperature $T_i \left( \tilde{X}_i, \tilde{Y}_i, t \right)$ are the usual ones:

$$n_i \left( \tilde{X}_i, \tilde{Y}_i, t \right) = \int d\tilde{v}_i \bar{F}_i \left( \tilde{v}_i, \tilde{X}_i, \tilde{Y}_i, t \right),$$

(57)

$$\vec{u}_i \left( \tilde{X}_i, \tilde{Y}_i, t \right) = \frac{1}{n_i \left( \tilde{X}_i, \tilde{Y}_i, t \right)} \int d\tilde{v}_i \tilde{v}_i \bar{F}_i,$$

(58)

$$T_i \left( \tilde{X}_i, \tilde{Y}_i, t \right) = \frac{m_i}{3 n_i \left( \tilde{X}_i, \tilde{Y}_i, t \right)} \int d\tilde{v}_i \left| \tilde{v}_i - \vec{u}_i \right|^2 \bar{F}_i,$$

(59)

and $P_i = n_i T_i$ is the ion thermal pressure. Equations (50) - (53) for ions and the similar equations for electrons, and the Poisson equation for the potential $\Phi_i \left( \tilde{X}_i, \tilde{Y}_i, t \right)$,

$$\frac{\partial^2 \Phi_i}{\partial X_i^2} + \frac{\partial^2 \Phi_i}{\partial Y_i^2} = -4 \pi \left( e_i n_i \left( \tilde{X}_i, \tilde{Y}_i, t \right) - |e| n_e^{(i)} \left( \tilde{X}_i, \tilde{Y}_i, t \right) \right),$$

(60)

where $n_e^{(i)} \left( \tilde{X}_i, \tilde{Y}_i, t \right)$ is the electron density perturbation, determined in the ion frame variables $\tilde{X}_i, \tilde{Y}_i$, in which potential $\Phi_i \left( \tilde{X}_i, \tilde{Y}_i, t \right)$ in Eq. (60) is determined, compose the basic system of the fluid equations, which governs the mesoscale evolution of the poloidal sheared plasma flow with radially inhomogeneous convective flows across the magnetic field.

As it follows from Eqs. (A1), (A2) and (A5), (A6), the velocities $\bar{U}_i \left( \tilde{X}_i, t \right)$, $\bar{U}_e \left( \tilde{X}_i, t \right)$ of the ion and electron convective flows are proportional to the mesoscale gradient of the spectral intensity $\left| \varphi_i \left( \vec{k}, \tilde{X}_i \right) \right|^2$ of the turbulent electric field. Therefore the maximum of the ion convective flow velocities could be in the pedestal, where the maximum gradients of the plasma parameters are observed, i. e. for the $\tilde{X}_i$ values in the interval $\tilde{X}_{iT} > \tilde{X}_i > \tilde{X}_{iB}$, where $\tilde{X}_{iT}$ and $\tilde{X}_{iB}$ are the coordinates of the pedestal top and of the pedestal bottom, respectively. Outside this interval, i. e. for the plasma core, $\tilde{X}_i > \tilde{X}_{iT}$, and in the far SOL, $\tilde{X}_i < \tilde{X}_{iB}$, the plasma parameters and of FW field are much more uniform and the
convective flow velocities are more slower. Thus, the problem of the mesoscale evolution of the pedestal plasma involves the problem of the stability of a plasma with convective flows with spatially inhomogeneous velocities. As a first step to the solution of the system of Eqs. (53) - (56), we find the characteristics of the operator

\[
\frac{D}{Dt} = \frac{\partial}{\partial t} - \bar{U}_{ix} \left( \bar{X}_i, t \right) \frac{\partial}{\partial \bar{X}_i} - \bar{U}_{iy} \left( \bar{X}_i, t \right) \frac{\partial}{\partial \bar{Y}_i},
\]  

(61)

which are determined by the system

\[
dt = - \frac{d\bar{X}_i}{\bar{U}_{ix} \left( \bar{X}_i, t \right)} = - \frac{d\bar{Y}_i}{\bar{U}_{iy} \left( \bar{X}_i, t \right)}. \tag{62}
\]

For deriving the simplest solution to system (62), which reveals the effects of the spatial inhomogeneity of the convective flow velocities, we use in Eq. (62) the expansions

\[
\bar{U}_{ix} \left( \bar{X}_i, t \right) = \bar{U}_{ix}^{(0)} + \bar{U}_{ix}' \left( \bar{X}_i^{(0)}, t \right) \left( \bar{X}_i - \bar{X}_i^{(0)} \right),
\]  

(63)

and

\[
\bar{U}_{iy} \left( \bar{X}_i, t \right) = \bar{U}_{iy}^{(0)} + \bar{U}_{iy}' \left( \bar{X}_i^{(0)}, t \right) \left( \bar{X}_i - \bar{X}_i^{(0)} \right)
\]  

(64)

at the vicinity of an arbitrary coordinate \( \bar{X}_i^{(0)} \), and consider the case of the time-independent velocity compressing rate, \( \bar{U}_{ix}' = \text{const} \), and of the velocity shearing rate, \( \bar{U}_{iy}' = \text{const} \). The solution to system (62) for the case of the flows with stationary uniform compressing and shearing rates is simple and has a form

\[
\bar{X}_i = \frac{1}{\bar{U}_{ix}'} \left[ \left( \bar{U}_{ix}^{(0)} + \bar{U}_{ix}' \left( \bar{X}_i - \bar{X}_i^{(0)} \right) \right) e^{\bar{U}_{ix}' t} - \bar{U}_{ix}^{(0)} \right], \tag{65}
\]

and

\[
\bar{Y}_i = \bar{Y}_i + \left( \bar{U}_{iy}^{(0)} - \bar{U}_{ix}^{(0)} \frac{\bar{U}_{iy}}{\bar{U}_{ix}} \right) t - \frac{\bar{U}_{iy}'}{\left( \bar{U}_{ix}' \right)^2} \left( \bar{U}_{ix}^{(0)} + \bar{U}_{ix}' \left( \bar{X}_i - \bar{X}_i^{(0)} \right) \right), \tag{66}
\]

\( \bar{X}_i \) and \( \bar{Y}_i \) are the integrals of system (62) with expansions (63), (64). Note, that at \( t = 0 \)

\( \bar{X}_i = \bar{X}_i - \bar{X}_i^{(0)} \).

Now we perform the transformations of variables \( \bar{X}_i, \bar{Y}_i \) in Eqs. (53) - (56) to variables \( \bar{X}_i, \bar{Y}_i \) and derive the following equations:

\[
\begin{align*}
\frac{\partial n_i \left( \bar{X}_i, \bar{Y}_i, t \right)}{\partial t} + e^{\bar{U}_{ix}' t} \frac{\partial}{\partial \bar{X}_i} \left( n_i u_{ix} \left( \bar{X}_i, \bar{Y}_i, t \right) \right) \\
- \frac{\bar{U}_{iy}'}{\bar{U}_{ix}'} \frac{\partial}{\partial \bar{Y}_i} \left( n_i u_{ix} \left( \bar{X}_i, \bar{Y}_i, t \right) \right) + \frac{\partial}{\partial \bar{Y}_i} \left( n_i \left( u_{iy} \left( \bar{X}_i, \bar{Y}_i, t \right) - V_0'^\prime t u_{ix} \right) \right) &= 0,
\end{align*}
\]  

(67)
\[
\frac{\partial u_{ix}}{\partial t} + u_{ix} \left( e^{U_{ix}^t} \frac{\partial u_{ix}}{\partial X_i} - \frac{U_{iy}^t}{U_{ix}^t} \frac{\partial u_{ix}}{\partial Y_i} \right) + (u_{iy} - V_0^t u_{ix}) \frac{\partial u_{ix}}{\partial Y_i}
= - \frac{1}{m_i n_i} \left( \frac{X_i - \bar{X}_i, Y_i, t}{(X_i, Y_i, t)} \right) \left( e^{U_{ix}^t} \frac{\partial P_i}{\partial X_i} - \frac{U_{iy}^t}{U_{ix}^t} \frac{\partial P_i}{\partial Y_i} - V_0^t \frac{\partial P_i}{\partial Y_i} \right)
- \frac{e_i}{m_i} \left( e^{U_{ix}^t} \frac{\partial \Phi}{\partial X_i} (\bar{X}_i, \bar{Y}_i, t) - \frac{U_{iy}^t}{U_{ix}^t} \frac{\partial \Phi}{\partial Y_i} (\bar{X}_i, \bar{Y}_i, t) \right) + \omega_{ci} u_{iy}, \quad (68)
\]

\[
\frac{\partial u_{iy}}{\partial t} + u_{ix} \left( e^{U_{ix}^t} \frac{\partial u_{iy}}{\partial X_i} - \frac{U_{iy}^t}{U_{ix}^t} \frac{\partial u_{iy}}{\partial Y_i} \right) + (u_{iy} - V_0^t u_{ix}) \frac{\partial u_{iy}}{\partial Y_i}
= - \frac{1}{m_i n_i} \frac{\partial P_i}{\partial Y_i} - \frac{e_i}{m_i} \frac{\partial \Phi}{\partial Y_i} (\bar{X}_i, \bar{Y}_i, t) - (\omega_{ci} + V_0^t) u_{ix}, \quad (69)
\]

\[
\frac{\partial T_i}{\partial t} + u_{ix} \left( e^{U_{ix}^t} \frac{\partial T_i}{\partial X_i} - \frac{U_{iy}^t}{U_{ix}^t} \frac{\partial T_i}{\partial Y_i} \right) + (u_{iy} - V_0^t u_{ix}) \frac{\partial T_i}{\partial Y_i}
= - \frac{2}{3} T_i \left( e^{U_{ix}^t} \frac{\partial u_{ix}}{\partial X_i} - \frac{U_{iy}^t}{U_{ix}^t} \frac{\partial u_{ix}}{\partial Y_i} + \frac{\partial u_{iy}}{\partial Y_i} \right), \quad (70)
\]

\[
\left( e^{2U_{ix}^t} \frac{\partial^2}{\partial X_i^2} - 2e^{U_{ix}^t} \frac{U_{iy}^t}{U_{ix}^t} \frac{\partial^2}{\partial X_i \partial Y_i} + \left( \frac{U_{iy}^t}{U_{ix}^t} \right)^2 \frac{\partial^2}{\partial Y_i^2} \right) \Phi (\bar{X}_i, \bar{Y}_i, t)
= -4\pi (e_i n_i (\bar{X}_i, \bar{Y}_i, t) - |e| n_i^{(e)} (\bar{X}_i, \bar{Y}_i, t)). \quad (71)
\]

This system does not contain any explicit dependences on the radial coordinate \( \bar{X}_i \). Instead, these equations involve the explicit time dependences, which reveal the effects of the basic poloidal sheared flow and of the convective compressed and sheared flows on the mesoscale temporal evolution of the inhomogeneous turbulent plasmas. It follows from this system, that the basic poloidal sheared flow with the velocity shearing rate \( V_0' \) leads to the appearance of the linearly growing with time coefficients in Eqs. (67) - (71). This time dependence reveals the effect of the continuous distortion with time of the perturbations by the basic poloidal sheared flow. This distortion grows with time and forms a time-dependent nonmodal processes, the linear and the renormalized nonlinear stages of which were investigated in Refs. 8-10. It was found, that this nonmodal process is at the foundation of the experimentally observed effect of the suppression of the drift-type instabilities by the poloidal sheared flow, which have the growth rates less than the poloidal flow velocity shearing rate \( V_0' \). Equations (67) - (71) display also principally different effect of the compressed convective flow along \( \bar{X}_i \). It reveals in the exponentially growing with time coefficients \( e^{U_{ix}^t} \) in Eqs. (67) - (71), which at time \( t > (U_{ix}^t)^{-1} \) determine the compressed flow as the dominant factor.
in the evolution of the plasma with a radially inhomogeneous turbulence. For that time, system (67) - (71) for \( n_i, n_e, T_i, T_e, u_{ix}, u_{ex}, u_{iy}, u_{ey}, \) and \( \Phi \) can be substantially reduced by neglecting the derivatives over \( \dot{Y}_i \), which are exponentially small with respect to the terms containing the derivatives over \( \dot{X}_i \). The reduced system,

\[
\frac{\partial n_i (\dot{X}_i, t)}{\partial t} + e^{\dot{U}_{ix}^i t} \frac{\partial}{\partial \dot{X}_i} (n_i (\dot{X}_i) u_{ix} (\dot{X}_i, t)) = 0, \tag{72}
\]

\[
\frac{\partial u_{ix} (\dot{X}_i, t)}{\partial t} + e^{\dot{U}_{ix}^i t} u_{ix} \frac{\partial u_{ix}}{\partial \dot{X}_i} = -\frac{e}{m_i} \left( \frac{1}{n_i (\dot{X}_i)} \frac{\partial P_i}{\partial \dot{X}_i} - e_i \frac{\partial \Phi (\dot{X}_i, t)}{\partial \dot{X}_i} \right) + \omega_{ci} u_{iy}, \tag{73}
\]

\[
\frac{\partial u_{iy} (\dot{X}_i, t)}{\partial t} + e^{\dot{U}_{ix}^i t} u_{ix} \frac{\partial u_{iy}}{\partial \dot{X}_i} = -\omega_{ci} u_{ix}, \tag{74}
\]

\[
\frac{\partial T_i (\dot{X}_i, t)}{\partial t} + e^{\dot{U}_{ix}^i t} u_{ix} \frac{\partial T_i}{\partial \dot{X}_i} = -e^{\dot{U}_{ix}^i t} \frac{2}{3} T_i \frac{\partial u_{ix}}{\partial \dot{X}_i}, \tag{75}
\]

and the equations for \( n_e, T_e, u_{ex}, u_{ey} \) derived from Eqs. (72) - (75) by changing species index \( i \) on \( e \), supplemented by the equation for the electrostatic potential \( \Phi (\dot{X}_i, t) \),

\[
e^{2\dot{U}_{ix}^i t} \frac{\partial^2 \Phi (\dot{X}_i, t)}{\partial \dot{X}_i^2} = -4\pi \left( e_i n_i (\dot{X}_i, t) - |e| n_e (\dot{X}_i, t) \right), \tag{76}
\]

compose the nonlinear system of equations, which governs the mesoscale dynamics of the pedestal plasma. The more simple presentation of all these equations may be derived using the ion ”compressed” time variable \( \tau_i \), determined by the relation

\[
\dot{U}_{ix}^i \tau_i = e^{\dot{U}_{ix}^i t}, \tag{77}
\]

with which Eqs. (72) - (76) are presented in the following much more simple form:

\[
\frac{\partial n_i (\dot{X}_i, \tau_i)}{\partial \tau_i} + \frac{\partial}{\partial \dot{X}_i} (n_i u_{ix} (\dot{X}_i, \tau_i)) = 0, \tag{78}
\]

\[
\frac{\partial u_{ix} (\dot{X}_i, \tau_i)}{\partial \tau_i} + u_{ix} \frac{\partial u_{ix}}{\partial \dot{X}_i} = -\frac{1}{m_i} \left( \frac{1}{n_i (\dot{X}_i, \tau_i)} \frac{\partial P_i}{\partial \dot{X}_i} - e_i \frac{\partial \Phi (\dot{X}_i, \tau_i)}{\partial \dot{X}_i} \right) + \frac{\omega_{ci}}{\dot{U}_{ix}^i} u_{iy} (\dot{X}_i, \tau_i), \tag{79}
\]
\[
\frac{\partial u_{iy}(\dot{X}_i, \tau_i)}{\partial \tau_i} + u_{ix} \frac{\partial u_{iy}}{\partial \dot{X}_i} = -\frac{\omega_{ci}}{U_{ix}'' \tau_i} u_{ix}(\dot{X}_i, \tau_i),
\]

(80)

\[
\frac{\partial T_i(\dot{X}_i, \tau_i)}{\partial \tau_i} + u_{ix} \frac{\partial T_i}{\partial \dot{X}_i} + T_i \frac{\partial u_{ix}(\dot{X}_i, \tau_i)}{\partial \dot{X}_i} = 0.
\]

(81)

Using Eqs. (77) - (81) with species index \(i\) changed on \(e\), we derive the equations for \(n_e(\dot{X}_e, \tau_e)\), \(T_e(\dot{X}_e, \tau_e)\), \(u_{ex}(\dot{X}_e, \tau_e)\), \(u_{ey}(\dot{X}_e, \tau_e)\). Also, Eq. (76) for the electrostatic potential \(\Phi(\dot{X}_i, \tau_i)\), will get a form

\[
\frac{\partial^2 \Phi_i(\dot{X}_i, \tau_i)}{\partial \dot{X}_i^2} = -\frac{4\pi}{(U_{ix}' \tau_i)^2} \left( e_i n_i(\dot{X}_i, \tau_i) - |e| n_e(i)(\dot{X}_i, \tau_i) \right).
\]

(82)

For understanding the temporal evolution of the plasma compressing flow it is instructive to derive the linearised solutions to system (78) - (82). We present the ion density and the ion temperature in Eqs. (78) and (81) in the forms

\[
n_i(\dot{X}_i, \tau_i) = n_{i0}(\dot{X}_i, \tau_i) + n_{i1}(\dot{X}_i, \tau_i),
\]

(83)

and

\[
T_i(\dot{X}_i, \tau_i) = T_{i0}(\dot{X}_i, \tau_i) + T_{i1}(\dot{X}_i, \tau_i),
\]

(84)

where \(n_{i0}(\dot{X}_i, \tau_i)\) and \(T_{i0}(\dot{X}_i, \tau_i)\) are the equilibrium values of the ion density and of the ion temperature, and \(n_{i1}, T_{i1}\) are their perturbations caused by the self-consistent electrostatic respond of a plasma on the relative motion of the ion and electron compressed flows. As it follows from Eqs. (78) and (81), the evolution with time of the ion density \(n_{i0}(\dot{X}_i, \tau_i)\) and of the ion temperature \(T_{i0}(\dot{X}_i, \tau_i)\) in compressed flow is determined by the equations

\[
\frac{\partial n_{i0}(\dot{X}_i, \tau_i)}{\partial \tau_i} = 0,
\]

(85)

\[
\frac{\partial T_{i0}(\dot{X}_i, \tau_i)}{\partial \tau_i} = 0,
\]

(86)

or, for \(n_{i0}(\dot{X}_i, \tau_i)\) and \(T_{i0}(\dot{X}_i, \tau_i)\), by the equations

\[
\frac{\partial n_{i0}}{\partial t} - \left( \bar{U}_{ix}' + \bar{U}_{ix} \left( \dot{X}_i - \dot{X}_i^{(0)} \right) \right) \frac{\partial n_{i0}}{\partial X_i} = 0,
\]

(87)
\[
\frac{\partial T_{i0}}{\partial t} - \left( \bar{U}_{ix}^{(0)} + \bar{U}_{ix}' \left( \bar{X}_i - \bar{X}_i^{(0)} \right) \right) \frac{\partial T_{i0}}{\partial X_i} = 0,
\]
where expansion (63) was used. The solution to Eq. (85),
\[
n_{i0} = n_{i0} \left( \bar{X}_i \right),
\]
is independent on time, whereas the solution \(n_{i0} \left( \bar{X}_i, t \right)\) to Eq. (87) for the simplest initial condition
\[
n_{i0} \left( \bar{X}_i, t = 0 \right) = n_{i0} \left( \bar{X}_i \right) = n_{i0} \left( \bar{X}_{i0} \right) + \frac{\partial n_{i0} \left( \bar{X}_i, t = 0 \right)}{\partial \bar{X}_i} |_{\bar{X}_i = \bar{X}_{i0}} \bar{X}_i
\]
is equal to
\[
n_{i0} \left( \bar{X}_i, t \right) = n_{i0} \left( \bar{X}_{i0} \right) + \frac{\partial n_{i0} \left( \bar{X}_i, t = 0 \right)}{\partial \bar{X}_i} |_{\bar{X}_i = \bar{X}_{i0}} \bar{X}_i
\]
\[
\times \frac{1}{\bar{U}_{ix}'} \left[ \left( \bar{U}_{ix}^{(0)} + \bar{U}_{ix}' \left( \bar{X}_i - \bar{X}_i^{(0)} \right) \right) e^{\bar{U}_{ix}' t} - \bar{U}_{ix}^{(0)} \right].
\]
Equation (91) displays that the value of \(n_{i0}\) at \(\bar{X}_i = \bar{X}_{i1}\), detected at \(t = t_1\), is shifted by the compressed flow and is observed in \(\bar{X}_i = \bar{X}_{i2} < \bar{X}_{i1}\) at time \(t = t_2 > t_1\). The solution similar to Eq. (91) is derived also to Eq. (88) for the temporal evolution of the equilibrium ion temperature \(T_{i0}\) in compressed flow.

In the limit of the vanished compressing rate \(\bar{U}_{ix}'\), Eq. (91) displays the transport of the ion density inhomogeneity with velocity \(\bar{U}_{ix} \left( \bar{X}_{i0} \right)\),
\[
n_{i0} \left( \bar{X}_i, t \right) = n_{i0} \left( \bar{X}_{i0} \right) + \frac{\partial n_{i0} \left( \bar{X}_i, t = 0 \right)}{\partial \bar{X}_i} |_{\bar{X}_i = \bar{X}_{i0}} \bar{X}_i \cdot \left( \bar{X}_i - \bar{X}_{i0} + \bar{U}_{ix}^{(0)} t \right).
\]
When the level of IC parametric turbulence is much lower than the level of the low frequency drift turbulence, the electric field (16) is determined by the drift turbulence. The suppression of the edge drift turbulence by the poloidal sheared flow entails the formation of the stagnation point for the compressed flow velocity at the pedestal bottom, where \(\bar{U}_{ix} \left( \bar{X}_{iB} \right) \approx 0\) and \(n_{i0} \left( \bar{X}_i < \bar{X}_{iB} \right) \approx 0\). The solution (91) for the ion density in the region \(\bar{X}_i > \bar{X}_{iB}\) of the pedestal bottom becomes equal to
\[
n_{i0} \left( \bar{X}_i, t \right) = \frac{\partial n_{i0} \left( \bar{X}_i \right)}{\partial \bar{X}_i} |_{\bar{X}_i = \bar{X}_{iB}} e^{\bar{U}_{ix}' (\bar{X}_{iB}) t} \left( \bar{X}_i - \bar{X}_{iB} \right).
\]
It follows from Eqs. (93), that the gradient of the ion density at \(\bar{X}_i > \bar{X}_{iB}\) grows exponentially with time as \(e^{\bar{U}_{ix}' t}\). This effect of the fast stepping up with time of the density profile.
in the pedestal region by the compressed flow looks like the instability development with the growth rate equal to $U'_{ix}$ for ions and $U'_{ex}$ for electrons.

It follows from Eq. (73) that due to the fast growing coefficient $e^{U'_{ix}t}$ the radial ion pressure force at some time $t \gtrsim t_*$ can be larger than the radial component of the ion Lorentz force. The crude estimate for the transition time $t_*$ may be derived from the balance relation, which follows from Eq. (73),

$$e^{U'_{ix}t_*} \frac{v_{T_i}}{L_n} \sim \omega_{ci},$$

where $L_n$ is the spatial scale of the ion density gradient of the pedestal plasma. For the ion temperature $T_i \sim 200$ eV, $L_n = 2$ cm, and $B_0 = 2$ T, we have $e^{U'_{ix}t_*} \sim 10$. For the numerical sample, considered in Ref. 3, $U_{ix} \sim 0.1v_{T_i} \sim 10^6$ cm/s and $U'_{ix} \sim 5 \cdot 10^5$ s$^{-1}$ we derive $t_* \sim 6 \cdot 10^{-6}$ s. At time $t > t_*$, the radial outflow of the temporally unconfined ions forms.

V. CONCLUSIONS

In this paper, we develop the kinetic and hydrodynamic theories of the convective mesoscale flows, driven by the spatially inhomogeneous IC parametric microturbulence of the pedestal plasma with a sheared poloidal flow. The amplitude $\tilde{E}(k, X_i)$ of the electric field (16) of the microscale IC parametric turbulence of the inhomogeneous plasma, driven by the inhomogeneous FW field, is spatially inhomogeneous and depends on the mesoscale coordinate $X_i$. In this paper, the IC microturbulence is assumed to be weakly nonlinear with known dependence of $\tilde{E}$ on $X_i$, which established by the linear theory of the IC instability, and by the weak nonlinear theory which determines the local saturation this instability at position $X_i$. The basic result of the developed theory is the Vlasov equation (41) which determines the mesoscale evolution of the ion/electron distribution functions resulted from the interaction of ions/electrons with the inhomogeneous IC microturbulence. This theory predicts the generation of the sheared poloidal convective flow, and of the radial compressed flow with radial flow velocity gradient.

The developed hydrodynamic mesoscale theory reveals the radial compressed convective flow as the dominant factor in the formation of the steep pedestal density profile with density gradient exponentially growing with time. This gradient density growth is limited
by the radial oscillating with time outflow of the pedestal ions to SOL. The process of the
temporal steeping and smoothing of the plasma density profile in the pedestal reveals as the
manifestation of the ultimate stage of the relaxation of the unstable radially inhomogeneous
density of the pedestal plasma. It was initiated as the microturbulence and finalised as
the mesoscale nonlinear relaxation oscillations. The analytical treatment of the temporal
evolution of the plasma compressed flow near the pedestal bottom at time \( t > t^* \) may be
performed as the solution of the initial-boundary problem for the system of equations which
includes Eqs. (81) - (84) for \( n_{i1} (\tilde{X}_i, \tau_i) \), \( T_{i1} (\tilde{X}_i, \tau_i) \), \( u_{ix} (\tilde{X}_i, \tau_i) \), \( u_{iy} (\tilde{X}_i, \tau_i) \), Eqs. (81) -
(84) for the electron component for \( n_{e1} (\tilde{X}_e, \tau_e) \), \( T_{e1} (\tilde{X}_e, \tau_e) \), \( u_{ex} (\tilde{X}_e, \tau_e) \), \( u_{ey} (\tilde{X}_e, \tau_e) \), and
Eq. (85) for potential \( \Phi_i (\tilde{X}_i, \tau_i) \). The solution of this problem will be done in the separate
paper.

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DATA AVAILABILITY

The data that support the findings of this study are available from the corresponding
author upon reasonable request.

Appendix A: Velocities \( \bar{U}_{ix}^{(0)} (\tilde{X}_i, t) \) and \( \bar{U}_{iy}^{(0)} (\tilde{X}_i, t) \)

For the electric field \( \tilde{E}_i \), determined by Eq. (16), velocities \( \bar{U}_{ix}^{(0)} (\tilde{X}_i, t) \) and \( \bar{U}_{iy}^{(0)} (\tilde{X}_i, t) \)
were determined in Ref.3. For completeness, we present these results here, i. e.

\[
\bar{U}_{ix}^{(0)} (\tilde{X}_i) = \frac{1}{2 \omega_{ci} m_i} \int d \mathbf{k} \left[ a_{i1} (\mathbf{k}) \tilde{E}_{ix} (\mathbf{k}, \tilde{X}_i) \frac{\partial}{\partial X_i} \left( \tilde{E}_{iy}^* (\mathbf{k}, \tilde{X}_i) \right) 
+ a_{i2} (\mathbf{k}) \tilde{E}_{iy} (\mathbf{k}, \tilde{X}_i) \frac{\partial}{\partial X_i} \left( \tilde{E}_{ix}^* (\mathbf{k}, \tilde{X}_i) \right) \right]
\]

\[
= \frac{1}{4 \omega_{ci} m_i} \int d \mathbf{k} k_x k_y \left( a_{i1} (\mathbf{k}) + a_{i2} (\mathbf{k}) \right) \frac{\partial}{\partial X_i} \left| \varphi (\mathbf{k}, \tilde{X}_i) \right|^2, \quad (A1)
\]
and \[
\tilde{U}_{iy}^{(0)} (\tilde{X}_i) = -\frac{1}{4\omega_{ci} m_i} \int dk \left[ a_{i1} (k) \frac{\partial}{\partial \tilde{X}_i} \left| \tilde{E}_{ix} (k, \tilde{X}_i) \right|^2 - a_{i2} (k) \frac{\partial}{\partial X_i} \left| \tilde{E}_{iy} (k, \tilde{X}_i) \right|^2 \right]
\]
\[
= -\frac{1}{4\omega_{ci} m_i} \int dk \left( k^2 a_{i1} (k) - k^2 a_{i2} (k) \right) \frac{\partial}{\partial \tilde{X}_i} \left| \varphi (k, \tilde{X}_i) \right|^2,
\] (A2)
where the asterisk in Eq. (A1) implies the operation of complex conjugate. The coefficients \(a_{i1} (k)\) and \(a_{i2} (k)\) are determined as
\[
a_{i1} (k) = \left[ \frac{\omega_{ci}}{\omega (k) (\omega_{ci} + \omega (k))^2} + \frac{\omega_{ci}}{\omega (k) (\omega_{ci} - \omega (k))^2} + \frac{1}{(\omega_{ci}^2 - \omega^2 (k))} \right],
\] (A3)
and
\[
a_{i2} (k) = \left[ \frac{1}{(\omega_{ci} + \omega (k))^2} + \frac{1}{(\omega_{ci} - \omega (k))^2} + \frac{1}{(\omega_{ci}^2 - \omega^2 (k))} \right].
\] (A4)
The velocities of electrons \(\tilde{U}_{ex}^{(0)} (\tilde{X}_i)\) and \(\tilde{U}_{ey}^{(0)} (\tilde{X}_i)\) in the ion frame, with electric field \(E_e (\hat{r}_i, \tilde{X}_i, t)\) determined by Eq. (17), are
\[
\tilde{U}_{ex}^{(0)} (\tilde{X}_i) \approx \frac{c^2}{2 B_0^2} \int dk \tilde{E}_{ix} (k, \tilde{X}_i) \frac{\partial}{\partial \tilde{X}_i} \left( \tilde{E}_{ix}^* (k, \tilde{X}_i) \right)
\times \sum_{p=-\infty}^{\infty} J_p^2 (a_{ie} (k, \tilde{X}_i)) \frac{1}{\Omega_p (k, \tilde{X}_i)},
\] (A5)
and
\[
\tilde{U}_{ey}^{(0)} (\tilde{X}_i) \approx -\frac{1}{2 B_0^2} \int dk \frac{\partial}{\partial X_i} \left| \tilde{E}_{ix} (k, \tilde{X}_i) \right|^2
\times \sum_{p=-\infty}^{\infty} J_p^2 (a_{ie} (k, \tilde{X}_i)) \frac{1}{\Omega_p (k, \tilde{X}_i)},
\] (A6)
where limit \(|\omega_{ce}| \gg \Omega_p \sim \omega_{ci}\) was used.

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