Rainfall Threshold for Flash Flood Early Warning Based on Rational Equation: A Case Study of Zuojiao Watershed in Yunnan Province

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Abstract. Rainfall threshold plays an important role in flash flood warning. A simple and easy method, using Rational Equation to calculate rainfall threshold, was proposed in this study. The critical rainfall equation was deduced from the Rational Equation. On the basis of the Manning equation and the results of Chinese Flash Flood Survey and Evaluation (CFFSE) Project, the critical flow was obtained, and the net rainfall was calculated. Three aspects of the rainfall losses, i.e. depression storage, vegetation interception, and soil infiltration were considered. The critical rainfall was the sum of the net rainfall and the rainfall losses. Rainfall threshold was estimated after considering the watershed soil moisture using the critical rainfall. In order to demonstrate this method, Zuojiao watershed in Yunnan Province was chosen as study area. The results showed the rainfall thresholds calculated by the Rational Equation method were approximated to the rainfall thresholds obtained from CFFSE, and were in accordance with the observed rainfall during flash flood events. Thus the calculated results are reasonable and the method is effective. This study provided a quick and convenient way to calculated rainfall threshold of flash flood warning for the grass root staffs and offered technical support for estimating rainfall threshold.

1. Introduction
Flash flood early warning of small watershed is the main flash flood prevention measure. Rainfall threshold is widely used in China for flash flood early warning \cite{1}. When the forecast or measured rainfall reaches or exceeds the rainfall threshold, the early warnings are released, the masses in dangerous areas are transferred, and the casualties and property losses can be effectively reduced.

At present, the rainfall threshold was usually calculated by building hydrological model. For model calibration and verification, large amounts of meteorological and hydrological data are needed. Flash flood usually occurred in mountainous ungauged regions, the model calibrations are difficult. Thus hydrological model method is not suitable for the flood control personnel. A simple and easy method, using Rational Equation to calculate rainfall threshold, was proposed in this study. This method requires less data and is convenient for grass-roots staff to use. Based on local rainstorm hydrological data and Chinese Flash Flood Survey and Evaluation (CFFSE) results, considering the local rainfall, topography, rainfall threshold can be analyzed and estimated by the proposed method. This method can not only meet the demands of the flash flood prevention personnel at the grass-roots level, but also can be used for professional to calculate rainfall threshold rapidly to provide valuable time for early warning information releasing and people transferring.
2. Data sources and study area

2.1. Data sources
The basic geographic data of the study area are 1:50,000 and 1:250,000 digital lines graphic (DLG), which are produced by National Geomatics Center of China (NGCC), including administrative boundaries, water systems, roads, geographical annotations, etc. Small watersheds, land use/cover, soil type, and soil texture are generated by National Flash Flood Prevention Project Office. Hydrological data are obtained from *practical handbook for storm and flood calculation of Yunnan Province*. Rainstorm characteristics of small watersheds, the river control sections, critical water levels, and river roughness are collected from CFFSE results.

2.2. Study area
Zuojiao watershed is located in Qiaojia County of Zhaotong City, Yunnan Province. The local terrain is dominated by the plateau area (Figure 1-a). The watershed area is 12.71 km², the river length of the watershed is 8.6 km, and the average river slope is 20‰. According to the CFFSE results, average slope of Zuojiao watershed is 22 degrees, and the watershed is covered by a huge of vegetation. The land use/cover of this watershed are mainly woodland (56%), farmland (43.4%), and a little shrub land (0.5%) (Figure 1-b). The soil type of whole watershed is silty clay (Figure 1-c).

![Figure 1](image)

*Figure 1.* The image (a), land use/cover (b), and soil type (c) of Zuojiao watershed.

2.3. Rainstorm and channel characteristics
Based on *practical handbook for storm and flood calculation of Yunnan Province*, Zuojiao watershed is located in the 13th rainstorm section and runoff is in the first section. So the initial loss of Zuojiao is 15 mm, the soil stable infiltration is 2.2 mm/h. According to the CFFSE results, the design storms of Zuojiao watershed are listed in Table 1.

The channel control section of Zuojiao village is roughly triangular and the shape is ‘V’. The critical water level of this village along river is 2430.9 m (Figure 2). The riverbed consists of sand gravel, cobble stone soil, etc. The river landscape is shown in Figure 3.

| Name          | Duration | Return Period of Rainfall($H_p$) |
|---------------|----------|----------------------------------|
|               | 100-year | 50-year | 20-year | 10-year | 5-year |
|               | Return Period | Return Period | Return Period | Return Period | Return Period |
| Zuojiao watershed | 10 min    | 25       | 23       | 20       | 18   | 15   |
|                | 1 h       | 71       | 64       | 54       | 46   | 38   |

*Table 1.* Design storms of Zuojiao.
3 Calculation of rainfall threshold for flash flood early warning

3.1 Critical flow calculation

According to control section, critical water level and roughness data of Zuojiao village along river, the Manning equation (Equation 1) \[^{[3]}\] was used to calculate the flow velocity, and then the critical flow was calculated using Equation 2.

$$v = \frac{1}{n} \cdot R^{\frac{2}{3}} \cdot J^{\frac{1}{2}}$$

(1)

$$Q_m = A \cdot v$$

(2)

Where $v$ is flow velocity (m/s), $n$ is roughness, $R$ is hydraulic radius (m), $J$ is gradient ratio, $Q_m$ is flow (m$^3$/s), and $A$ is the area of flow section (m$^2$). $A$ and $R$ are calculated using the control section data, $J$ is obtained from the vertical section, and $n$ is determined by the underlying surface condition of the watershed.

According to the measure data and generalization section (Figure 2 and Table 2), flood calculation can be carried out. The channel gradient $J$ is 20‰, the area of flow section $A$ is about 17.82 m$^2$, the wetted perimeter is about 14.63 m, and calculated hydraulic radius is 1.22 m. Based on the composition of river bed, the roughness value is 0.04, and the critical flow is calculated as 71.84 m$^3$/s. Specific calculation parameters and results are shown in Table 3.

### Table 2. Generalized parameters of control section.

| Flow Section Area $A$ (m$^2$) | Wetted Perimeter $L$ (m) |
|-------------------------------|-------------------------|
| $A$                           | $L1$                    | $L2$        | $L$       |
| 17.82                         | 10.74                   | 3.89        | 14.63     |
Table 3. The parameters and results of critical flow calculation.

| Critical Level (m) | Flow Section Area (m²) | Wetted Perimeter (m) | Hydraulic Radius (m) | Roughness Value | Gradient Ratio (%) | Flow Velocity (m/s) | Flow (m³/s) |
|--------------------|------------------------|----------------------|----------------------|-----------------|--------------------|--------------------|-------------|
| 2430.9             | 17.82                  | 14.63                | 1.22                 | 0.04            | 20                 | 4.03               | 71.84       |

3.2. Confluence time analysis

Confluence time of the watershed is estimated by using Equation (3) \(^{[3-5]}\) and Equation (4) \(^{[2]}\).

\[
\tau = 0.278 \frac{L}{m^{1/3} Q^{1/4}} \tag{3}
\]

Where \(\tau\) is confluence time (h), \(L\) is river length (m), \(J\) is river gradient, \(Q\) is critical flow (m³/s), \(m\) value can be calculated Equation (4) \((\theta=L/J^{1/3})\).

\[
m = \begin{cases} 
0.895 \theta^{0.064} & (\theta < 100) \\
0.380 \theta^{0.25} & (\theta \geq 100) 
\end{cases} \tag{4}
\]

The river length of the small watershed \(L\) is 8.6 km, river gradient \(J\) is 20‰, critical flow is 71.84 m³/s, and \(m\) is 1.06. The confluence time of Zuojiao watershed is about 2 h (Table 4).

Table 4. The parameters and results of confluence time.

| Watershed | \(Q_{m}\) (m³/s) | \(L\) (km) | \(\theta\) | \(m\) | \(J\) (%) | \(\tau\) (h) |
|-----------|-------------------|-----------|---------|-------|----------|-----------|
| Zuojiao   | 71.84             | 8.6       | 14.8    | 1.06  | 20       | 1.32      |

3.3. Net rainfall estimation

Because of the uncertainty in the calculation of critical flow, although the calculating result of confluence time is 1.32 h, the confluence time is considered to be 2 h in practice, slightly longer than the calculated time. According to the equation of net rainfall (Equation 5) deduced from the Rational Equation \(^{[3-4]}\), the net rainfalls for each duration were estimated (Table 5).

\[
h_{t} = 3.6 \frac{Q_{m}}{F} t, \quad t \leq \tau \tag{5}
\]

Where \(h_t\) is the net rainfall corresponding to durations (mm), \(F\) is watershed area (km²) and \(Q_m\) is flow value (m³/s).

Table 5. The results of net rainfall estimation.

| Duration | \(Q_{m}\) (m³/s) | \(F\) (km²) | Net Rainfall (mm) |
|----------|------------------|-------------|-------------------|
| 1h       | 71.84            | 12.71       | 20.3              |
| 2h       | 71.84            | 12.71       | 40.6              |

3.4. Rainfall losses estimation

The rainfall losses mainly include depression storage, vegetation interception and soil infiltration. Because of average slope of watershed is 22 degrees, which is out-of-flatness ground, depression storage was determined as 6 mm \(^{[6]}\). Due to Zuojiao watershed covered by plenty of vegetation, the loss of vegetation interception was determined as 15 mm \(^{[7-8]}\). Considering the characteristic of soil infiltration, the steady infiltration carried out after 3 h of rainfall \(^{[9]}\). The confluence time of Zuojiao is about 2 h. Therefore, the condition of steady infiltration is not analyzed in terms of soil infiltration. The estimated rainfall losses are shown in Table 6.

Table 6. Results of rainfall losses.

| Duration | Depression Storage (mm) | Vegetation Interception (mm) | Steady Infiltration (mm) | Rainfall Losses (mm) | Notes                  |
|----------|-------------------------|-----------------------------|-------------------------|----------------------|------------------------|
| 1h       | 6                       | 15                          | 7.5                     | 28.5                 | Initial soil infiltration 7.5 mm/h |
| 2h       | 6                       | 15                          | 12.5                    | 33.5                 | Medium soil infiltration 5 mm/h  |
3.5. Critical rainfall estimation

According to the Equation 6, the critical rainfall is the sum of net rainfall and rainfall losses (Table 7).

\[ H_c = h_t + L_t = 3.6 \frac{Q_m}{F} t + L_t, \ t \leq \tau \]  

(6)

Where \( H_c \) is critical rainfall (mm), \( L_t \) is value of rainfall losses.

Table 7. The calculation results of critical rainfall.

| Duration | Net rainfall (mm) | Rainfall losses (mm) | Critical rainfall (mm) |
|----------|------------------|----------------------|------------------------|
| 1h       | 20.3             | 28.5                 | 48.8                   |
| 2h       | 40.6             | 33.5                 | 74.1                   |

3.6. Rainfall threshold determination

On the basis of obtaining critical rainfall, watershed soil moisture should be taken into account in order to analyze rainfall threshold [10-11]. The critical rainfall is calculated under the condition of less soil and no rain (i.e., dry soil). So the depression storage, vegetation interception, and soil infiltration are basically estimated with large values. On the calculation of rainfall threshold, the depression storage and vegetation interception were also estimated under the conditions of moderate and larger soil moisture capacity (Table 8).

Table 8. Calculation results of rainfall threshold.

| Duration | Scenario | Depression Storage (mm) | Vegetation Interception (mm) | Soil Infiltration (mm) | Loss Rainfall (mm) | Net Rainfall (mm) | Critical Rainfall (mm) | Rainfall Threshold (mm) | Notes |
|----------|----------|--------------------------|-----------------------------|------------------------|-------------------|-------------------|----------------------|------------------------|-------|
| 1h       | Dry      | 6                        | 15                          | 7.5                    | 28.5              | 20.3              | 48.8                 | 45                     | Initial soil infiltration 7.5mm/h |
|          | Normal   | 5                        | 14                          | 5.0                    | 24                | 20.3              | 44.5                 | 40                     | Medium soil infiltration 5mm/h    |
|          | Wet      | 4                        | 12                          | 2.5                    | 18.5              | 20.3              | 38.8                 | 35                     | Initial soil infiltration 7.5mm/h |
| 2h       | Dry      | 6                        | 15                          | 12.5                   | 33.5              | 40.6              | 74.1                 | 70                     | Initial soil infiltration 7.5mm/h |
|          | Normal   | 5                        | 14                          | 10.0                   | 29                | 40.6              | 69.6                 | 65                     | Medium soil infiltration 5mm/h    |
|          | Wet      | 4                        | 12                          | 7.5                    | 23.5              | 40.6              | 64.1                 | 60                     |                                 |

3.7. Rationality analysis

The rainfall threshold was calculated by Rational Equation, and the rationality analysis was necessary, which included the critical flow calculation and the rainfall losses rationality analysis.

Table 9 is shown the CFFSE rainfall threshold results of Zuojia village. Compared Table 9 and Table 8, the rainfall thresholds are very close under the condition of wet moisture soil, and the main difference lies on the analysis of depression storage and vegetation interception. Therefore, the rainfall thresholds calculated by Rational Equation are reasonable, and the Rational Equation method can be used for the people at the grass roots level to calculate rainfall threshold rapidly and conveniently.

Table 9. Rainfall thresholds of CFFSE.

| Duration | \( Q_m \)(m³/s) | F(km²) | h(mm) | L(mm) | Rainfall Threshold(mm) |
|----------|----------------|--------|-------|-------|------------------------|
| 1h       | 64.5           | 12.71  | 18.3  | 15    | 33                     |
| 2h       | 64.5           | 12.71  | 36.6  | 17.2  | 54                     |

4. Conclusions and prospects

Driving by CFFSE results, a rapidly and conveniently method for calculating rainfall threshold was proposed based on the Rational Equation. Firstly, net rainfall was calculated by critical flow. Then rainfall losses were analyzed in order to estimate the critical rainfall of the village along river. Finally, rainfall thresholds were estimated in different soil moisture conditions.
In this study, Zuojiao watershed in Qiaojia County of Yunnan Province was selected as the study area. The calculation results of net rainfall were 20.3 mm in 1 h and 40.6 mm in 2 h, and the net rainfall of different durations showed linear correlation. In the rainfall losses calculation, depression storage, vegetation interception, and soil infiltration were considered as the main factors and were analyzed. Depression storage and vegetation interception is the same under different soil moisture conditions (dry, normal, and wet) and the main difference comes from soil infiltration quantity in different durations. The initial soil infiltration is mainly considered for 1 h and the initial and medium-term soil infiltration considered for 2 h. The results of rainfall losses for 1 h and 2 h are 28.5 mm, 33.5 mm. As a result, the critical rainfall for 1 h and 2 h is 48.8 mm, 74.1 mm. On the basis of critical rainfall, taking into account the soil moisture content of the watershed, the rainfall threshold are analyzed under different soil moisture conditions. The calculation results of rainfall threshold are close to the results of CFFSE and are agreement with observed rainfalls. Therefore, the calculation method is reasonable. The Rational Equation method provides a quick and convenient method for calculating the rainfall threshold of flash flood and offered an effectively way to conduct the flash flood early warning tasks.

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