Full penetration current variation of a superconducting tube.

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Abstract. It is well known that the critical current density \( J_c \) of a superconducting material depends on the magnetic field \( B \). If magnetic independent \( J_c \) is chosen for analytical calculation of current distribution, the critical current \( I_{c0} \) corresponds to full penetration current \( I_p \). \( I_{c0} \) is a measured current with 1\( \mu \)V/cm criterion and \( I_p \) is a calculated current. The aim of this paper is to present \( I_p \) variation of superconducting tube. To calculate \( I_p \), which is depending on the material itself, a linear function \( J_c(B) \) is sufficient to obtain realistic values by analytic way. We need to have a linear \( J_c(B) \) law \( (J_c(B)=J_{c0}(1-|B|/B_{J0})) \) that is close to the measured \( J_c(B) \) characteristics presented in this paper. The linear \( J_c(B) \) law chosen was used for the calculation of the distribution of both magnetic field \( B(r, t) \) and the current density \( J(r, t) \). These distributions allow the analytical calculation of \( I_p \). We present the variation of \( I_p \) with material characteristics and geometric parameters of the tube. The two main results are presented. \( I_p \) decreases when \( B_{J0} \) increases for same \( J_{c0} \). \( I_p \) increases when the tube internal radius \( R_{in} \) increases for same tube section. For AC losses versus current curves, \( I_p \) is the frontier between low and high losses. So the present results, allow designing future transport current superconducting cable.

1. Introduction
High critical temperature superconductors (HTS) used for transport of alternating electric current present lower electrical losses than resistive materials. AC losses have a deleterious effect on the cooling system and must be evaluated. The theoretical model used for these calculations was proposed by Bean [1]. This model assumed that the critical current density at any point of the sample can only take one of three constant values, the critical current density \( J_c \), \(-J_c\), and zero. Superconducting power cables have a geometry that resembles a tube comprised of superconducting filaments in a copper or silver matrix. Despite the inhomogeneous nature of these filament-matrix cables, the losses are calculated with the monoblock model using average current densities and thickness [2]. \( J_c(B) \) varies for low magnetic field in HTS [3], like in the case of transport current. The experimentalists showed how to obtain a linear law for \( J_c(B) \) starting from the measured \( J_{cM} \) according to a external magnetic field \( B_{ext} \) [3]. Expressions of the distributions of \( B(r, t) \), \( J(r, t) \), electric field \( E(r, t) \), and the variation of the full penetration current \( I_p \) are presented.

2. Magnetic field, current density and electric field distributions
The case of a superconducting tube with transport current \( i(t) \) is studied. The dimensions of the tube are internal radius \( R_{in} \), external radius \( R_{ex} \), section \( S \) and length \( h \), seen in (Fig. 1). The z-axis represents the axis of the tube.
The electromagnetic behaviour of superconductor is governed by the Maxwell equations:

\[ \text{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \]  

(1)

\[ \text{rot} \vec{B} = \mu_0 \vec{J}, \]  

(2)

For the superconducting material, we assume the approximation:

\[ \vec{B} = \mu_0 \vec{H}. \]  

(3)

\[ \text{E(J) is given by the critical state model and defines the relation between the electric field E and the current density J:} \]

\[ E \neq 0 : J = \pm J_c \]

(4)

\[ E = 0 : -J_c < J < +J_c \]

Due to symmetry and the current density being oriented along the z-axis, \( B(r, t) \), \( J(r,t) \), and \( E(r,t) \) have only one component:

\[ B(r, t) = B(r, t) \hat{u}_z, J(r, t) = J(r, t) \hat{u}_z, E(r, t) = E(r, t) \hat{u}_z \]

The shape of the current is unimportant for the calculations of the magnetic field distribution or the losses for the critical state model [4].

With the critical state model, \( i(t) \) must remain lower than \( I_p \) so that the material remains superconductive. \( I_p \) depends on the law \( J_c(B) \) of the material [6]. To allow an analytical calculation, \( J_c(B) \) is linearized:

\[ J_c(B) = J_{c0} \frac{B_{j0} - |B|}{B_{j0}} \]  

(5)

When the current rises, the current density penetrates from the external radius \( R_e \), toward the internal radius \( R_i \), and its direction is in the z-axis.

By considering equations (2), (4), and (5) one obtains the following differential equation:

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial B(r, t)}{\partial r} \right) = \mu_0 J_c(B) = \mu_0 J_{c0} \left( 1 - \frac{|B(r, t)|}{B_{j0}} \right) \]  

(6)

On the external radius there is the following boundary condition:
B(R, t) = b_c(t) = \frac{\mu_0 i(t)}{2\pi R_{ex}} \quad (7)

In this study i(t)>0, B(r,t) > 0 and (6) becomes :

\[ \frac{1}{r} \frac{\partial (r. B(r, t))}{\partial r} = \mu_0 J_c(B) = \mu_0 J_{c0} \left( 1 - \frac{B(r, t)}{B_{j0}} \right) \quad (8) \]

Considering (8) and (7), one deduces B(r,t):

\[ B_{j0} \left( \mu_0 J_{c0} \cdot r - B_{j0} \right) + \exp \left( \frac{\mu_0 J_{c0} \cdot (R_{ex} - r)}{B_{j0}} \right) \left( B_{j0}^2 + \mu_0 J_{c0} \cdot R_{ex} \left( b_c(t) - B_{j0} \right) \right) \]

\[ B(r,t) = \frac{\mu_0 J_{c0} \cdot r}{\mu_0 J_{c0} \cdot R_{ex}} \quad (9) \]

The full penetration current I_p can be calculated with the linear model:

B(r = R_{in}, i(t) = I_p) = 0 =

\[ B_{j0} \left( \mu_0 J_{c0} \cdot R_{ex} - B_{j0} \right) + \exp \left( \frac{\mu_0 J_{c0} \cdot (R_{ex} - R_{in})}{B_{j0}} \right) \left( B_{j0}^2 + \mu_0 J_{c0} \cdot R_{ex} \left( \frac{\mu_0 I_p}{2\pi R_{ex}} - B_{j0} \right) \right) \]

So :

\[ I_p = \frac{2\pi B_{j0}}{\mu_0 J_{c0}} \left( \mu_0 J_{c0} \cdot R_{ex} - B_{j0} + (B_{j0} - \mu_0 J_{c0} \cdot R_{in}) \exp \left( \frac{\mu_0 J_{c0} \cdot (R_{in} - R_{ex})}{B_{j0}} \right) \right) \quad (10) \]

3. I_p variation with B_{j0}, J_{c0} and tube dimensions

The ratio \( \frac{I_p}{I_{c0}} = J_{c0}, S \) versus B_{j0} for different internal radius for same section is represented on the figure 2.

![Fig. 2: Ratio I/I_{c0} versus B_{j0}](image)

We see on figure 2 that I_p is closer of I_{c0} for big internal radius because magnetic flux density is weaker and so the influence of J_{c}(B) is weaker.
Also we see on figure 2 that $I_p$ is close $I_{C0}$ for big value of $B_{J0}$. On the other hand, $I_C$ is much smaller than $I_{C0}$ for small values of $B_{J0}$. For this small values of $B_{J0}$, the self field creates by the tube (around 5mT) is close to $B_{J0}$. So we have to be careful to use this $I_p$ formula (10) for small values of $B_{J0}$. $I_p$ versus $I_{C0}$ for different $B_{J0}$ is represented on figure 3.

![Figure 3](image)

**Fig. 3 : $I_p$ versus $I_{C0}$ for different $B_{J0}$, Bean model for $I_p = I_{C0}$.**

We note that $I_p$ decreases when $B_{J0}$ decreases because we obtain Bean's model if $B_{J0}$ is infinite and $I_p = I_{C0}$. $I_p$ versus internal radius $R_{in}$ for same section $S$ on figure 4. We note that $I_p$ increases with $R_{in}$ for the same reason we explain for figure 2.

![Figure 4](image)

**Fig. 4 : $I_p$ versus $R_{in}$.**
4. Conclusion

We studied the influence of $J_c(B)$ on the current, electric field and magnetic field distributions in a superconductor tube fed by a current $i(t)$. We are able to calculation this distributions with the linear model. Kim model is the best model in relation on real $J_c(B)$ but Kim model and linear model are similar for small values of $B$. We showed that the influence of $J_c(B)$ is important for distributions of $B(r, t)$, $J(r, t)$ and $E(r, t)$. We saw that current penetrates deeper in the case of linear model in relation on Bean model. So we analytically calculated the influence of $J_c(B)$ on the full penetration current $I_p$. We noted that $I_p$ varies a lot with the parameter of $J_c(B)$, $J_{C0}$ and $B_{J0}$ and with the internal radius of the tube. To have higher $I_p$ that must take higher $J_{C0}$, $B_{J0}$ and $R_{in}$.

References

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