Dynamic analysis and continuous control of semiconductor lasers

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Stability control in laser is still an emerging field of research. In this paper the dynamics of External cavity semiconductor lasers (ECSLs) is widely studied applying the methods of chaos physics. The stability is analyzed through plotting the Lyapunov exponent spectra, bifurcation diagrams and time series. The results of the study show that the rich nonlinear dynamics of the electric field intensity ($|E|^2$) includes saddle node bifurcations, periodic, quasi periodic, chaotic and regular pulse packages. The issue of finding the conditions for creating stable domains has been studied. By considering the dynamical pumping current system coupled with laser, a method for the creation of the stable domain has been introduced.

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I. INTRODUCTION

The research on the laser dynamics has attracted a lot of interest for the past several years. Current investigations in this field include the study of different routes to chaos in lasers and the control of chaos in such systems. Most of these works are concentrated on semiconductor lasers with respect to their importance in optical communications. Chaotic behavior in semiconductor lasers has been investigated under various physical conditions such as external optical injection, and external optical feedback [1–3].

Our study is motivated from different points of view, in this study dimensionless ECSLs are investigated [4]. This class of dynamical systems can successfully be modeled by delay differential equations. Where the study has provided a realistic background for the investigation of time delay systems, which is very difficult as a problem in the infinite-dimensional space [5]. This allows us to present a consistent overall picture of the dynamics at the same time. In this study parameters such as feedback phase, feedback strength, and pump current are used as control parameters to obtain different dynamical regimes. Also the key idea underlying most chaos controlling schemes such as ECSLs is to take advantage of the unstable steady states and unstable periodic orbits of the system [6, 7]. They are embedded in the chaotic attractor characterizing the dynamics in phase space. Feedback schemes require real-time measurement of the state of the system and processing of a feedback signal [6, 7]. The location of the unstable fixed-point must be determined before control is initiated. The standard control method has the disadvantage of having a small region of convergence around the fix point [11, 13].

The aim of the present paper is studying the ECSLs dynamics by the Lyapunov exponent diagram as the analyzing tool and proposing a control method for the Laser. Regarding the pump current as one of the control parameter of the system, a discrete time dynamical system is built and coupled with Laser. The rest of the paper has been organized as follows: Section 2 describes Lang and Kobayashi model for ECSLs. The method of the control and analyzing the results are proposed in Section 3 and 4. In section 5, the stable domain of the introduced model before and after applying the control method is studied via Lyapunov exponents and bifurcation diagrams. Section 6 concludes the letter. The paper ends with an Appendix which contains algebraic calculations of the controller.

II. THE LASER MODEL

In the early 1980s, Lang-Kobayashi (LK) proposed model semiconductor lasers. The LK equations are model equations that have been used extensively in the past to describe a semiconductor laser subject to feedback from an external cavity [14, 15]. The LK equations are a sort of delay differential equations (DDEs) with an infinite-dimensional phase space [3]. For the (complex) electric field E and inversion N, we write the LK equations as the dimensionless and compact set of equations [4]:

$$\frac{dE}{dt} = (1 + i\alpha)NE + \eta E(t - \tau)e^{-ic_p},$$

(1)
Parameters describe the line width enhancement factor $\alpha$, the feedback strength $\eta$, the $2\pi$-periodic feedback phase $C_p$, the ratio between carrier and photon lifetime $T$ and the pump current $P$. In these equations, the time is normalized to the cavity photon lifetime (1ps) [10]. The external round trip time $\tau$ is also normalized to the photon lifetime. The remaining parameters are held fixed at $T = 1710$, $P = 0.8$, $\tau = 70$, $\alpha = 5.0$ [17]. In this paper, the stability of an electric field intensity $|E|^2$ is studied versus the feedback phase change $C_p$ and feedback strength change $\eta$.

\begin{equation}
T \frac{dN}{dt} = P - N - (1 - 2N) |E|^2.
\end{equation}

### III. THE CONTROL METHOD

Most recently, a different method of control has been shown to be successful in the experimental control of chaos. Based on:

- Determination of the stable and unstable directions in the Poincare section.
- Self-controlling feedback procedure.
- Introduction of small modulation of a control parameter.
- Knowledge of a prescribed goal dynamics.

The first two methods are usually called feedback methods, while the 3rd and last are called non-feedback methods [18, 19]. Although control of chaos by small modulations has not been proved in general [20], this method involves the on chaotic behavior generation of an error signal from the difference between the output signal and its value at an earlier time. A great virtue of this method is that it does not require knowledge of more than one variable. The method should be very useful for applying in fast systems to control a chaotic nonlinear circuit. Until now, different parameters such as modulating current [21, 22], modulating voltage [23], and pump power variation response to a continuous error signal generator [24] have been used to modify the laser output. In this study, “pump current” has been selected as a tool to be implemented in the control method. By considering the Laser and the control system as a two dimensional dynamical system, the simple model for controlling the stability of Laser is introduced as follows [22]:

\begin{equation}
\begin{aligned}
\dot{E}(E, N, P_m) &= (1 + i\alpha)NE + \eta E(t - \tau)e^{-iC_p}, \\
T \frac{dN}{dt} &= P_m - N - (1 - 2N) |E|^2, \\
P_{m+1} &= \frac{4P_m}{(1 - P_m)}.
\end{aligned}
\end{equation}

The setup is shown schematically in Fig. 1. The most significant parts of the setup include the laser pump and the method whereby its output is modulated. The pump power is controlled by the use of an electric circuit. Figure 2 shows the desired controller. A photodiode is used to convert a sample of laser light to electric voltage “$V_s$”. This voltage is connected to non-inverting input of the comparators “$U_1$” and “$U_2$”. In the stable condition, $V_s$ is less than $V_1$ and more than $V_2$. Thus, the output voltage of $U_1$ and $U_2$ is in the low and high conditions, respectively. Consequently, $Q_1$ and $Q_2$ are off and $Q_3$ is on. As a result the relays 1 and 2 are off and $P_m$ is connected to the input of the $P_{m+1}$ function generator. When the unstable condition occurs and the laser intensity is increased, the voltage $V_s$ becomes more than $V_1$ and then the output voltage of $U_1$ changes to high. Due to high condition in the output of $U_1$, the transistor $Q_1$ and $Q_2$ conduct. At this stage the relay 2 disconnects $P_m$ from the input of the $P_{m+1}$ function generator and simultaneously relay 1 connects the $P_{m+1}$ function generator output to its input. The feedback from the output to the $P_{m+1}$ function generator input continuously decreases the $P_{m+1}$, and consequently decreases photodiode output voltage. Finally, when the output voltage of the photodiode becomes less than $V_1$, $Q_1$ and $Q_2$ become off. In this condition relay 1 disconnects but relay 2 is still on. And the capacitor “$C$” at the input of $P_{m+1}$ function generator holds the last $P_{m+1}$. After a while, if laser intensity decreases more, and “$V_s$” becomes less than $V_2$, then, the output voltage of $U_2$ changes to zero and then $Q_3$ changes to off. As a result, relay 2 becomes off and connects the $P_m$ to the input of the $P_{m+1}$ function generator. Finally, $P_{m+1}$ increases and the system changes to normal condition.

In this study, arbitrary chaotic regions of the system dynamical behavior, have been chosen to undergo the proposed control method [20–22].

### IV. STABILITY ANALYSIS

#### A. Lyapunov exponent spectrum

Lyapunov exponents and entropy measures, can be considered as “dynamic” measures of attractors complexity and are called “time average” [20]. The Lyapunov exponent $\lambda$ is useful for distinguishing various orbits. Lyapunov Exponents quantify sensitivity of the system to initial conditions and give a measure of predictability. The Lyapunov exponents are a measure of the rate at which the trajectories separate one from another. A negative exponent implies that the orbits approach to a common fixed point. A zero exponent means that the orbits maintain their relative positions; they are on a stable attractor. Finally, a positive exponent implies that the orbits are on a chaotic attractor, so the presence of a positive Lyapunov exponent indicates chaos. The Lyapunov exponents are defined as follows:

Consider two nearest neighboring points in phase space at time 0 and $t$, with the distances of the points in the $ith$
direction $\|\delta x_i(0)\|$ and $\|\delta x_i(t)\|$, respectively. The Lyapunov exponent is then defined by the average growth rate $\lambda_i$ of the initial distance,

$$\lambda_i = \lim_{t \to \infty} \frac{1}{t} \ln \frac{\|\delta x_i(t)\|}{\|\delta x_i(0)\|} \tag{4}$$

The existence of a positive Lyapunov exponent is the indicator of chaos showing neighboring points with infinitesimal differences at the initial state abruptly separate from each other in the $ith$ direction $\|\delta x_i\|$. Using the algorithm of Wolf [28], the Lyapunov exponent is calculated versus a given control parameter. Then, the value of the control parameter increases a little and the Lyapunov exponent is calculated for the new control parameter. By continuing this procedure Lyapunov exponent spectrum of the system is plotted versus the control parameter.

### B. Bifurcation diagrams

Bifurcation means a qualitative change in the dynamical behavior of a system when a parameter of the system is varied [26]. A bifurcation diagram provides a useful insight into the transition between different types of motion that can occur as one parameter of the system alters. It enables one to study the behavior of the system on a wide range of an interested control parameter. In this paper the dynamical behavior of the system is studied through plotting the bifurcation diagrams of the of intensity $|E|^2$ versus feedback phase $C_p$ and feedback strength change $\eta$, as control parameters. This procedure continued by increasing the control parameter and the new resulting points were plotted in the bifurcation diagram versus the new control parameter.

### V. RESULTS AND DISCUSSIONS

#### A. Introducing the dynamics of the master laser

The dynamical behavior of the master laser as a function of the control parameter, $C_p$, can be divided into two areas:

- low feedback strengths
- high feedback strengths

In the low feedback strengths, such as; $\eta_m = 0.0455$ [16], the output of laser contains chaotic (CH) [29], periodic ($P1, P2, ..$) and quasi periodic (QP) [30], behaviors, respectively. Fig. 3(a) shows that the dynamical behavior of the output intensity for master laser is CH, P7, P1 and QP. In the high feedback strengths, such as; $\eta_m = 0.135$, the laser output contains regular pulse package (RPP), P1, and QP behaviors [16, 17], which is confirmed in Fig. 4. As it is shown, the dynamical behavior of the output intensity for master laser is RPP, QP and P1. For understanding the general faces of the ECSLS one could refer to Fig. 5, where the bifurcation curve of the laser output in terms of feedback strength $\eta$, in a constant feedback phase $C_p$ is presented. As it is shown, the bifurcation curve contains periodic, QP, CH, and RPP behaviors. The maximum Lyapunov exponents are presented in Fig. 3(b) to verify the corresponding characteristics. As can be observed, the positive and negative values in Lyapunov exponent spectrum verify the aperiodic and periodic behaviors in bifurcation diagram. In addition, extended numerical analysis on the ECSLSs dynamics has been carried out on several control parameters. The system exhibits extremely nonlinear behaviors when the control parameters are varied.

#### B. Applying the chaos control method

In order to provide a fair justification on employing a periodic perturbation as the control technique, it is first necessary to consider two facts. First, the similar chaos control methods should be evaluated, and their practicability and associated advantages should be clarified. Second, we address this question that: can such a method be easily implemented in the real world applications? In fact, a method should be chosen so as to be best suited for an experimental implementation. In this way, as it will be discussed in the following, the pump current sources will be able to satisfy both highlighted remarks.

Before evaluating the potential chaos control methods for an ECSLSs system, we should consider the parameters that can be implemented in the control technique. It is clear that, to control the chaotic oscillations of a dynamical system, one of the control parameters of the system should be perturbed or modified in order to reach the regular desirable behavior. However, in the application phase, we are restricted to apply changes only to the external forcing elements such as feedback phase, feedback strength that are determined by the type of the application, and the pump current which has been chosen in the present study.

The results are depicted in Figs. 6-9. Under the collective system parameters, every category corresponds to original system and controlled system; see Figs. 6(a), 6(b) and Figs. 7(a), 7(b). Also the control method has been tested through Lyapunov exponent diagrams; see Figs. 6(c) and 7(c). The dashed line represents the original system, while the solid line represents the controlled system. This figure indicates a significant abatement of the Lyapunov exponents from positive values to negative ones indicating that stable dynamics can be achieved after the proposed technique is engaged. The results have also been confirmed by plotting the intensity $|E|^2$ versus time in a certain value of the feedback phase $C_p$ before and after the control process. Respectively Figs. 8(a) and 9(a) show before control process and Figs. 8(b) and 9(b) depict after control process.
VI. CONCLUSION

Our main contribution in this paper is to develop the simple method for study and control the laser dynamics. In this paper, the concept of feedback chaos control has been re-defined by taking into account the control parameter as a variable in the time and is changed by using another chaotic map, for which a new and effective control scheme has been presented. The result of the present study broadens our understanding of the complex dynamics of laser, and also helps us to control its nonlinear dynamics.

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Appendix A: Detail of control circuit

The circuits in the Figs. 10, 11 and 12 are used to generate $P_{m+1}$ Function generator. Figure 11 shows the multiplier circuit. This circuit multiplies $(1 - P_m)$ by $(1 - P_m)$ and makes the denominator of the function $(1 - P_m)$ and makes the denominator of the

$$V_c = V_T \left( \frac{V_a}{R_{IS}} + 1 \right) \left( \frac{V_b}{R_{IS}} + 1 \right)$$

The op amp $U_5$ adds the two equal voltages $V_b$ to its inverting input. Then, the voltage $V_c$ at the output $U_5$ is:

$$V_c = V_T \left( \frac{V_a}{R_{IS}} + 1 \right) \left( \frac{V_b}{R_{IS}} + 1 \right)$$

and the voltage at the output of $U_6$ is:

$$V_d = -RI_{D1} = -RI_s \left( e^{\left( \frac{V_a}{V_T} \right)} - 1 \right)$$

$$= -\frac{V_a^2}{RI_s} - 2V_a, \quad (A3)$$

The op amp $U_7$ adds the voltages $V_d$ and $V_a$, and therefore, cancels “$-2V_a$” at its output. Then, the voltage at $V_g$ will be as follows:

$$V_g = \frac{V_a^2}{RI_s}, \quad (A4)$$

By considering $R = 1M\Omega$ and $I_a = 1\mu\text{sec}$, the equation (A4) can be written as:

$$V_g = V_a^2 = (1 - P_m)^2, \quad (A5)$$

In Fig. 12, a part of multiplier circuit including op amp, $U_2$, $U_4$, $U_5$ and $U_6$ is used as a divider. In this figure, the voltages at points a and b are respectively $4P_m$ and $-(1 - P_m)^2$, therefore, the voltage at point c is as below:

$$V_c = V_T \left( \frac{4P_m}{R_{IS}} + 1 \right) - Ln\frac{(1 - P_m)^2}{R_{IS}} + 1 \right)$$

Assuming $4P_m \gg RI_s$ and $(1 - P_m)^2 \gg RI_s$, the equation (A6) can be written as below:

$$V_c = V_T Ln \left( \frac{4P_m + RI_s}{(1 - P_m)^2 + RI_s} \right)$$

and then $V_d$ is:

$$V_d = -RI_s e^{\frac{V_a}{V_T}} - 1 \equiv -RI_s \frac{4P_m}{(1 - P_m)^2}, \quad (A8)$$

Finally $V_f$ is:

$$V_f = \frac{4P_m}{(1 - P_m)^2}, \quad (A9)$$

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