Phase transition and vacuum stability in the classically conformal B–L model

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Within classically conformal models, the spontaneous breaking of scale invariance is usually associated to a strong first order phase transition that results in a gravitational wave background within the reach of future space-based interferometers. In this paper we study the case of the classically conformal gauged B–L model, analysing the impact of this minimal extension of the Standard Model on the dynamics of the electroweak symmetry breaking and within cosmology. Particular attention is paid to the problem of vacuum stability and to the role of the QCD phase transition, which we prove responsible for concluding the symmetry breaking transition in part of the considered parameter space. Finally, we calculate the gravitational wave signal emitted in the process, finding that a large part of the parameter space of the model can be probed by LISA.

I. INTRODUCTION

The recent detection of the first gravitational wave signal by the LIGO collaboration [1] has opened a new observational window into the Universe. An important point of these investigations concerns the dynamics of phase transitions that occurred during the cosmological evolution, which may play a central role in an array of topics spanning from the problem of the baryon asymmetry of the Universe [2–11] to the quest for an ultraviolet completion of the Standard Model (SM) [12–31]. To provide a concrete example, gravitational wave astronomy has the potential to pinpoint the dynamics of the phase transition behind the generation of the electroweak scale, setting a new important benchmark for particle physics models. In fact, whereas the SM supports a second order electroweak phase transition, many of its extensions instead predict a first order phenomenon. In this case the electroweak phase transition proceeds through the nucleation and consequent expansion of bubbles that contain the true symmetry-breaking vacuum. Collisions between the bubbles and the motion in the plasma after bubble collisions then produce gravitational wave signals that can be detected in the present stochastic background by next-generation experiments such as the satellite-based interferometer LISA [32].

In regard of this, classically conformal – or scale-invariant – models [33–39] are an example of framework which typically induces a sizeable gravitational signature [19, 28, 40, 41], as thermal corrections here inevitably result in a potential barrier that separates the vacuum states of the theory. Presently the interest in conformal models has been revived for their possible connections with other problems in contemporary physics, involving for instance the origin of dark matter, the mechanism of cosmic inflation, vacuum stability or baryogenesis [34, 42–60].

In this work we continue these analyses by considering the classically conformal B–L model introduced originally in Refs. [61–63] and further studied in Refs. [64–68]. Differently from previous studies [19], we focus on the impact of the SM QCD phase transition, improving the analysis in Ref. [11] and demonstrating that in part of the considered parameter space it is responsible for concluding the dynamics of the electroweak symmetry breaking. In line with the general results of Ref. [74], we also find that thermal inflation [75, 76] is a feature of the model. In fact, the contribution of the potential energy difference between true and false vacuum states is large enough to dominate the Hubble parameter at temperatures below the critical one, and the inflationary regime may last until the onset of the QCD phase transition with non-trivial consequences on additional phenomenology [77].

The structure of the paper is as follows: after introducing the model in Sec. [1] we briefly discuss in Sec. [11] its phenomenology at collider experiments and within cosmology. The effective potential, including the contributions of thermal corrections and QCD phase transition, is presented in Sec. [V] whereas the relative analyses of perturbativity and vacuum stability are detailed in Sec. [V]. The electroweak phase transition is studied in Sec. [VI] and the resulting gravitational signature of the model is computed in Sec. [VII]. Finally, in Sec. [VIII] we gather our conclusions.

II. THE MODEL

The model we consider is based on the symmetry group $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, with quarks and leptons having a B–L charge of $+1/3$ and $-1$, respectively. The particle content of the SM is extended to include right handed neutrinos (RHN) $\nu_{Ri}$, required by the cancellation of the $U(1)_{B-L}$ anomaly, and a complex scalar $\phi = (\varphi + iG)/\sqrt{2}$ that only carries a $+2$
U(1)_{B-L} charge. Notice that the SM Higgs doublet $H = (G_+,(h+iG_0)/\sqrt{2})$ transforms as a singlet under U(1)$_{B-L}$.

The scalar sector of the model is characterised by the following tree-level potential,

$$V = \lambda_H (H^\dagger H)^2 + \lambda_\phi (\phi^\dagger \phi)^2 - \lambda_p (H^\dagger H)(\phi^\dagger \phi),$$  

where we include the so-called ‘portal coupling’ $\lambda_p$ between the Higgs doublet and the new scalar field. Through the dimensional transmutation mechanism, radiative corrections induce non-trivial solutions for the minimization equation of the scalar potential and, consequently, both $h$ and $\phi$ develop non-vanishing vacuum expectation values (VEVs): $\langle \phi \rangle \equiv w > 0, \langle h \rangle \equiv v > 0$. According to the emerging phase transition pattern, the radiative symmetry breaking occurs along the $\varphi$ direction of the field space and is transmitted to the Higgs sector via the (positive) portal coupling.

We anticipate that our work focuses on the case where $w \gg v$, as collider bounds disfavour the complementary setup. In this limit, matching the observed Higgs boson mass $m_H = 126$ GeV and the electroweak VEV $v = 246$ GeV forces the Higgs boson quartic coupling approximately to its SM value, $\lambda_H \simeq m_H^2/(2v^2)$. The portal coupling is instead related to the VEV of $\varphi$ by $\lambda_p \simeq m_\varphi^2/v^2$.

The interactions of the RHOS are given by

$$-\mathcal{L}_\nu = Y^{ij}_{D} \overline{\nu_{Ri}} \bar{H} L_j + \frac{1}{2} Y^{ij}_{M} \phi \nu_{Ri} \nu_{Rj} + \text{h.c.},$$  

and after the symmetry breaking result in Dirac neutrino masses, as well as Majorana masses for the RHOS. This implements the seesaw mechanism in the model, which ascribes the smallness of the measured (active) neutrino masses to a suppression factor given by the ratio between the neutrino Dirac mass scale and the RHOS Majorana mass scale. In order not to clash with the bounds from Big Bang Nucleosynthesis we will consider RHOS with masses above 200 MeV.

The final ingredient of the model is a kinetic mixing term for the U(1)$_Y$ and U(1)$_{B-L}$ gauge fields, which is generally produced by quantum corrections even if set to zero at a scale. After diagonalising the kinetic term, the U(1)$_Y \times$ U(1)$_{B-L}$ part of the gauge covariant derivative is given by

$$D^\mu \simeq ig_Y q_Y B^\mu_Y + i(g_G + g_{B-L} q_{B-L}) B^\mu_{B-L},$$  

where $\bar{g}$ parametrizes the extent of the kinetic mixing and $q_Y$, $g_G$ and $B^\mu_Y$ are the charges, the gauge coupling and the gauge fields, respectively. Given the charge assignment of the extra scalar field $\phi$, the spontaneous symmetry breaking of the U(1)$_{B-L}$ symmetry will induce a mass for the corresponding gauge boson given after the diagonalization by $m_{Z'} = 2g_{B-L}w$.

III. PHENOMENOLOGICAL CONSEQUENCES

The interactions contained in Eq. (2) link the present framework to the problem of the origin of the baryon asymmetry detected in our Universe. In fact, as RHOS acquire a Majorana mass through the symmetry breaking of U(1)$_{B-L}$, it is possible to implement the leptogenesis mechanism for baryogenesis.

In standard scenarios of thermal leptogenesis RHOS with hierarchical Majorana masses $M_{ij} \gtrsim 10^9$ GeV are thermally produced in the plasma after inflation. As the Universe expands, an original lepton, or $B-L$, asymmetry is generated via the CP-violating out of equilibrium decays of the RHOS, and consequently partially converted into a baryon asymmetry by the SM sphi~leron processes. Remarkably, possible pre-existing $B-L$ asymmetries can be efficiently washed out owing to the interplay between flavour effects and the RHOS mass hierarchy. In terms of the present analysis, implementing a standard leptogenesis scenario would force the $Z'$ mass scale well above the reach of contemporary collider experiments, being this parameter sourced by the same VEV $w$ behind the RHOS mass scale. On general grounds, we also expect sizeable coupling in Eq. (2) that could drive the running of the scalar sector parameters.

As an alternative, it is possible to consider a scenario where RHOS with masses comparable to, or below, the electroweak scale, can produce the required baryon asymmetry via CP-violating flavour oscillations. More in detail, the complex non-diagonal Majorana Yukawa matrices in Eq. (2) induce CP-violating $\nu_Ri \leftrightarrow \nu_Rj$ transitions, which conserve the overall lepton number but violate the lepton number of individual flavours. In this way, provided that at least one species of RHOS remains out of equilibrium while the SM sphi~lerons are active, the lepton asymmetry transmitted to the SM by the remaining RHOS species will be reprocessed by the same sphi~leron processes. Within the context of the conformal $B-L$ model, Ref. [63] adopted this mechanism to explain the observed baryon asymmetry of the Universe, and this is also the case that we implicitly assume in the present work. Given the allowed RHOS mass range, we can estimate the typical size of the neutrino Dirac Yukawas couplings via the seesaw formula,

$$y_D \simeq \sqrt{\frac{m_{\nu} M_{\nu}}{v^2}} \lesssim 10^{-5}$$  

where $M_{\nu}$ and $m_{\nu}$ are the mass scale of RH and active neutrinos, respectively. As a consequence, we can safely neglect the role of these parameters in determining the radiative corrections to the scalar sector. The same holds for the remaining couplings in Eq. (2), provided that RHOS do not exceed the electroweak scale:

$$y_M = 4g_{B-L} \frac{M_{\nu}}{m_{Z'}} \lesssim 0.1 g_{B-L}.$$  

For the above estimate we adopted a conservative bound,
scales as we find that the mass of the extra scalar approximately not occur above the QCD phase transition temperature, U fermions. with the negative sign for bosons and the positive for J. Here, the thermal integral
\[
\phi(\varphi) = \int_0^\infty \frac{d\varphi}{\sqrt{2\lambda\varphi}}
\]
indicates the intrinsic number of degrees of freedom \( k \). Depending on the values of these parameters, it is possible to stabilize the SM vacuum. The analytical minimization of the effective potential reduces to a single scale problem, \( \sum_j y_j \langle \psi_j \bar{\psi}_j \rangle h/\sqrt{2} \), which consequently acquires a non-zero VEV, \( v_{QCD} \equiv \langle h \rangle = \mathcal{O}(0.1 \text{GeV}) \). In turn, the portal coupling then induces a negative mass term for \( \varphi \), so that at \( T < T_{QCD} \) the effective potential along the \( \varphi \) direction becomes
\[
V_{\text{eff}}^{T<T_{QCD}} = -\frac{\lambda_\varphi(t)v_{QCD}^2}{4} \varphi^2 + V_{\text{eff}}^{T>T_{QCD}} ,
\]
where \( V_{\text{eff}}^{T>T_{QCD}} \) is given by Eq. [8]. In our analysis we take \( v_{QCD} = T_{QCD} = 0.1 \text{GeV} \).

V. VACUUM STABILITY AND PERTURBATIVITY

The high-energy behaviour of a model can be inferred by studying the renormalization flow of its parameters. The requirement of desirable properties, such as stability and perturbativity, then generally result in further constraints on the low-energy parameter space of the framework under examination. Within the SM, for example, renormalization group methods indicate the allowed top quark and Higgs boson mass windows through the requirements of i) limited interaction couplings (perturbativity), and ii) the absence of scalar background configurations with energies below the EW one (vacuum stability) \( \phi[89, 90] \), which crucially depends on the value of the top quark mass. In regard of this, according to present measurements of this parameter, the SM vacuum is only metastable \( [91-95] \).

Many extensions have been proposed in the attempt to overcome this puzzling feature of the SM. For instance, the simplest SM × \( U(1)_{B-L} \) framework with explicit symmetry breaking has been investigated up to next-to-leading precision in Refs. \( [72, 96-102] \). However, all these analyses confirm that the extra Yukawa terms introduced in this simple scenario generally worsen the overall high-energy behavior of the model, in spite of the stabilizing effect of scalar mixing and gauge couplings. An exception to this conclusion is provided by classical conformal models \( [63, 67] \), where the requirement of a radiative \( U(1)_{B-L} \) breaking bounds the magnitude of the parameters in the Majorana neutrino mass matrix.

In fact, as shown in Ref. \( [61] \), the presence of a radiatively generated minimum breaking the B–L symmetry can be inferred independently from the SM Higgs background. Along the \( \varphi \) direction, the minimization of the effective potential reduces to a single scale problem,
\[
\frac{dV}{d\varphi} = 0 ,
\]
with the negative sign for bosons and the positive for fermions. Depending on the values of these parameters, it is possible that the \( U(1)_{B-L} \) breaking phase transition does not occur above the QCD phase transition temperature, \( T_{QCD} = \mathcal{O}(0.1 \text{GeV}) \). In this case, the QCD phase transition induces an additional linear term for the Higgs field, \( \sum_j y_j \langle \psi_j \bar{\psi}_j \rangle h/\sqrt{2} \), which consequently acquires a non-zero VEV, \( v_{QCD} \equiv \langle h \rangle = \mathcal{O}(0.1 \text{GeV}) \). In turn, the portal coupling then induces a negative mass term for \( \varphi \), so that at \( T < T_{QCD} \) the effective potential along the \( \varphi \) direction becomes
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the $h-$independent part of the effective potential reaches its minimum at $\langle \varphi \rangle = w$ and increases elsewhere. Therefore, considering also the smallness of $\lambda_p$, in the conformal $B-L$ model instabilities can only be generated along the Higgs direction.

The sign of the second derivative of the effective potential in the $\varphi$ direction, as computed from the non-trivial solution of Eq. (13), receives a positive contribution from the gauge sector and a negative one from the Majorana Yukawa couplings. Hence, for $w$ to be in correspondence of a minimum of the potential (or, equivalently, to prevent a tachyonic scalar mass), the RHN Majorana mass scale must satisfy $M_{\nu} < m_{Z'}/2^{1/4}$ [66] (enforced in the present case by Eq. (5)), in line with similar bounds concerning the evolution of $\lambda_p$ and the following instability [72][100].

Figure 1. The evolution of $\lambda_H(t)$ as a function of the renormalization scale for $g_{B-L} = 0.1$, $m_{Z'} = 10^3$ GeV and three different values of $\tilde{g}$. All couplings have been set to the indicated values at the $w$ scale.

Within the classical conformal case, the high-energy behavior of the model is therefore shaped by the extended gauge sector, which enters the RGE of the scalar sector through the gauge coupling $g_{B-L}$ and the mixing parameter $\tilde{g}$. The kinetic mixing, in particular, affects the evolution of $\lambda_H$ already at the one-loop level, allowing to solve the issue of the SM instability when the mixing is sizeable. This is explicitly shown in Fig. 1 where $\lambda_H$ is plotted as a function of the renormalization scale for three different values of $\tilde{g}$. The indicated values of the couplings have been set at the $w$ scale.

The results of our analysis concerning the perturbativity and stability of the model are presented in Fig. 2. The shaded areas in both the panels indicate the region of the parameter space where the stability of the symmetry-breaking vacuum is ensured up to the scale indicated in the legend.

Figure 2. Top panel: $M_{Z'} = 10$ TeV. Bottom panel: $\tilde{g} = -0.5$. In both the panels, coloured areas indicate the region of the parameter space where the stability of the symmetry-breaking vacuum is ensured up to the scale indicated in the legend. Below the dotted, dashed and solid lines we have the values of the parameters which allow the model to retain perturbativity of all the couplings (all couplings $< \sqrt{4\pi}$) beyond the Planck scale, at most up to the Planck scale and at most up to the GUT scale, respectively. Beyond the continuous black line, the model has a maximum perturbativity scale not exceeding the pure SM instability scale.

2 Because of the hierarchy in the scales of the model, the RG equations have been solved by matching the SM evolution to the full model flow at the $Z'$ scale.
the following analysis, anticipating that the phase transition dynamics do not significantly depend on this choice: \( \tilde{g} \) does not directly affect the one-loop effective potential, entering only the running of \( g_{\text{B-L}} \). To conclude the section, we remark that the sudden change in the stability of the potential for values of \( \tilde{g} \simeq -0.4 \) is due to the RG flow of \( \lambda_h \) shown in Fig. 1.

**VI. PHASE TRANSITION**

At very high temperatures, thermal corrections dominate the potential and localize the fields at the origin, preventing the formation of new minima that would result in the spontaneous breaking of the symmetries of the model. This configuration is maintained until the temperature decreased enough to allow for the appearance of a second minimum in the potential, corresponding to a non-vanishing value of \( \varphi \). We can therefore define the critical temperature \( T_c \) as the temperature for which the new, symmetry-breaking, minimum becomes a global minimum of the potential, but thermal corrections still result in a potential barrier that prevents the fields from leaving the origin. At temperatures \( T < T_c \), the potential energy difference between the global minimum and the origin is then sizeable, whereas the height of the potential barrier progressively decreases. Quantum tunneling effects can then drive the fields to the global minimum of the potential, starting a first-order phase transition that proceeds via nucleation and consequent expansion of bubbles inside of which the symmetry is broken.

The bubble nucleation rate per unit of time and volume can be estimated as [103]

\[
\Gamma(T) \simeq T^4 \left( \frac{S_3}{2\pi T} \right)^\frac{3}{2} \exp \left( -\frac{S_3}{T} \right),
\]

where

\[
S_3 = 4\pi \int r^2 dr \left[ \frac{1}{2} \left( \frac{d\varphi}{dr} \right)^2 + V_{\text{eff}}(\varphi, T) \right]
\]

is the action for an \( O(3) \)-symmetric bubble. The largest contribution into the above quantity arises from the classical path which minimizes \( S_3 \), corresponding to the solution of

\[
\frac{d^2\varphi}{dr^2} + 2 \frac{d\varphi}{r \ dr} = \frac{dV_{\text{eff}}}{d\varphi}
\]

with boundary conditions \( d\varphi/dr = 0 \) at \( r = 0 \), and \( \varphi \to 0 \) at \( r \to \infty \).

Figure 3. The evolution of \( S_3/T \) for a benchmark point with \( m_{Z'} = 10 \text{ TeV}, \ g_{\text{B-L}} = 0.26 \), and \( \tilde{g}(w) = -0.5 \). The Majorana Yukawa couplings are assumed negligible.

The evolution of \( S_3/T \) as a function of temperature is shown for a benchmark case in Fig. 3. As temperature decreases, \( S_3/T \) also decreases and eventually results in a sizeable bubble nucleation rate. However, below \( T = T_{\text{QCD}} \), the QCD phase transition changes the behaviour of \( S_3/T \) inducing a negative mass term which cancels the thermal potential barrier, in a way that \( S_3/T \) eventually vanishes.

We can then define the bubble nucleation temperature \( T_n \) as the temperature at which the probability of producing at least one bubble per horizon volume in a unit of Hubble time approaches unity [104].

\[
\frac{\Gamma(T_n)}{H(T_n)^4} \simeq 1.
\]

Notice that the Hubble rate \( H \) includes the contribution due to the energy difference between the symmetry-conserving and symmetry-breaking vacua, \( \Delta V(T = 0) \). This vacuum energy dominates over the radiation contribution at

\[
T < T_v \simeq 0.3T_c
\]

as long as the phase transition is ongoing, and results in an epoch of thermal inflation.

Following Ref. [74], we proceed by computing the volume fraction converted to the symmetry-broken phase at temperature \( T \),

\[
I(T) = \frac{4\pi}{3} \int_T^{T_p} \frac{dT'}{T'^4 H(T')} \left( \int_T^{T'} \frac{d\tilde{T}}{H(\tilde{T})} \right)^3.
\]

The percolation temperature \( T_p \) is defined as \( I(T_p) = 0.34 \), and here always satisfies the condition [74]

\[
3 + T dI/dT < 0
\]

enforcing that the physical volume of the patches still in the symmetric phase of the theory decrease.

The lines in the bottom panel of Fig. 4 show the behaviour of the different temperatures defined above as a
log10 \left( R^* \right) \sim -10 \quad -10 \quad -2 \quad -1 \quad 0 \quad 0 \quad 2 \quad 4 \quad 6 \quad 8 \quad 10 \quad N \quad T_c \quad T_v \quad T_{co} \quad T_{reh} \quad T_p \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4 \quad 0.5 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad \log10 \left( T/\text{GeV} \right) \quad \log10 \left( \left( R^* H^* \right) \right) \quad \log10 \left( \left( R^* H^* \right) \right)

function of the B–L gauge coupling, assuming \( m_{Z'} = 10 \text{ TeV} \) and \( \tilde{g} = -0.5 \) and neglecting the Majorana Yukawa couplings. We see that percolation always takes place right after nucleation. The relative difference between the two corresponding temperatures is at its maximum immediately before the QCD phase transition starts to affect the dynamics. The temperature \( T_{co} \), represented by the lowest dashed line, signals the vanishing of the thermal potential barrier. For small values of \( g_{B-L} \) this is due to the negative mass term induced by the QCD phase transition, whereas for larger \( g_{B-L} \) this is caused by the running of \( \lambda_5 \).

The middle panel of Fig. 4 shows instead that the number of \( e \)-folds of thermal inflation, \( N \equiv \log(T_v/T_p) \), significantly increases for lower values of \( g_{B-L} \). Consequently, only for \( g_{B-L} \simeq 0.42 \) the phase transition concludes during the radiation dominated epoch. We find that \( N \) behaves as \( N \sim \log(m_{Z'}/\text{GeV}) + C \) for \( T_p < T_{QCD} \), where \( C \) is a constant that is mildly dependent on \( g_{B-L} \), and \( C(g_{B-L} = 0.1) \simeq 1.0 \). CMB studies then bound \( m_{Z'} \) from above through the requirement that \( N \lesssim 60 \), as required by the observed perturbation spectra. The bound however is not very efficient: for \( g_{B-L} = 0.1 \) we have \( m_{Z'} \lesssim 10^{25} \text{ GeV} \).

After the phase transition concludes, the vacuum energy decays into radiation, consequently producing a plasma thermalised at a temperature \( T_v < T_{reh} < T_{QCD} \) shown by the red line in the bottom panel of Fig. 4. We find that \( T_{reh} \) scales as \( m_{Z'} \), and if \( T_p \ll T_v \), the reheating temperature is \( T_{reh} \approx T_v \approx 0.09 m_{Z'} \). Given the parameter values indicated by collider experiments, the resulting temperature is high enough to restore the electroweak symmetry. This will be then broken in the same way as in the pure SM, as the temperature decreases below \( T \approx 140 \text{ GeV} \) with the expansion of the Universe.

Our results concerning the phase transition dynamics are also presented in the first panel of Fig. 4 as a function of \( m_{Z'} \) and \( g_{B-L} \). The color code indicates the percolation temperature, the dashed lines show the number of \( e \)-folds of thermal inflation, and the dot-dashed lines represent the reheating temperature. The thick black line highlights the contour \( T_p = T_{QCD} \), below which the transition happens only after the QCD one.

VII. GRAVITATIONAL WAVE SIGNAL

We now discuss the gravitational wave signal emitted at the phase transition, focusing on strongly supercooled dynamics, \( T_p \ll T_v \), that occur during the vacuum energy dominance. We assume here that the bubble walls do not reach terminal velocity before they collide, as the energy density of the plasma is strongly depleted during the thermal inflation period. Bubble collisions then source the GW spectrum, which in the source frame is given in by [105],

\[
\Omega_{GW}(k) = (R_s H_s \Omega_v)^2 \frac{0.035(k/\tilde{k})^3}{(1 + 1.99(k/\tilde{k})^{2.07})^{2.18}},
\]

as a function of the wave-number \( k = 2\pi \nu \). Here \( \tilde{k} = 3.2/R_s \) corresponds to the peak frequency \( \nu = \nu_{env} \) of the spectrum, \( H_s = H(T_p) \) is the Hubble rate,

\[
\Omega_v = \frac{8\pi \Delta V(T = 0)}{3M_p^2 H_s^2}
\]
is the vacuum energy density parameter, and

\[
R_s^{-3} = T_p \int_{T_p}^{T_v} \frac{dT'}{T'^2} \frac{\Gamma(T')}{H(T')} e^{-I(T')}
\]
is the average bubble separation \( [74] \) at the percolation temperature. Our computations indicate that \( R_s \) scales

\[ \text{Notice that there is no observational lower bound on the percolation temperature. Only after the phase transition, the reheating dynamics must bring the plasma to a temperature above the Big Bang nucleosynthesis one.} \]
roughly as $m_{Z'}^2$, so the product $H_\ast R_\ast$ is basically independent of $m_{Z'}$. The dependence of $H_\ast R_\ast$ on the B–L gauge coupling is shown in the top panel of Fig. 4.

In order to predict the corresponding signal detectable at gravitational wave observatories, we let the gravitational waves emitted at the phase transition propagate until today. This amounts to a scaling of amplitude and frequency given by [100]

$$\frac{\Omega_{GW}(T_0)}{\Omega_{GW}(T_{\text{reh}})} = 2.46 \times 10^{-5} \left(\frac{100}{g_\ast}\right)^{1/3},$$

$$\frac{\nu(T_0)}{\nu(T_{\text{reh}})} = 1.65 \times 10^{-7} \nu_{\text{reh}} \left(\frac{g_\ast}{100}\right)^{1/6},$$

where $g_\ast = g(T_{\text{reh}})$ is the effective number of relativistic degrees of freedom at the reheating temperature.

The gravitational wave spectra generated for three benchmark points are shown in Fig. 5. The signal is the strongest for $g_{B-L} = 0.26$, as for smaller values of this parameter the phase transition takes place only after the QCD one. In fact, as shown in Fig. 3 the slope of $S_1/T$ changes after the QCD phase transition has induced a negative mass term for $\varphi$, which effectively speeds up the process. This is manifest in the top panel of Fig. 4 showing the average bubble separation at $T_b$, as a function of $g_{B-L}$.

Our final results are summarised in Fig. 6. The left panel shows the percolation temperature, the number of $e$-folds of thermal inflation and the reheating temperature, as discussed in the previous section. In all panels the thick solid black line indicates where $T_p = T_{\text{QCD}}$. Below this line, the phase transition happens after QCD has already induced a negative mass term for the $\varphi$ field.

The middle and right panels of Fig. 6 characterise the GW emission consequent to the phase transition. The middle panel shows the peak frequency $\nu_{\text{env}}$, and the amplitude of the spectrum at the corresponding frequency, $\Omega_{GW}(\nu = \nu_{\text{env}})$. We see that the strongest GW signal is obtained when the transition takes place immediately before the QCD one. The shape of the peak frequency contours follows from the $g_{B-L}$ dependence of $R_\ast$, shown in the top panel of Fig. 4 and the peak frequency increases as a function of $m_{Z'}$ as the spectrum is redshifted. The blue dashed line highlights the frequency $\nu = 3\text{mHz}$ to which LISA is sensitive the most. The mild dependence of $H_\ast R_\ast$ on $m_{Z'}$ is also evident from the behaviour of the $\Omega_{GW}(\nu = \nu_{\text{enc}})$ contours (dot-dashed lines).

Finally, our prospect for the detection at LISA of the GW spectrum emitted in the considered model is shown in the right panel of Fig. 6. The color code indicates here the amplitude of the GW signal relative to the best sensitivity of the experiment. In the A5M5 setup, (see Fig. 5), LISA would be able to probe the whole region between the blue dashed contours.

We remark that our prediction of the GW spectrum is not to be trusted in the region above the black dashed line, where the phase transition concludes in the radiation dominated era. In fact, in this case Eq. (22) is not applicable as GWs originate from sound waves and turbulence in the plasma rather than bubble collisions. Notice however that entering such a region requires a substantial $g_{B-L}$ coupling, which according to the bottom panel of Fig. 3 sets the perturbativity scale of the model below the Planck scale.

VIII. CONCLUSIONS

In this work we furthered the study of the conformal B–L extension of the Standard Model. After introducing the framework and briefly reviewing its general phenomenology, we focused on the phase transition dynamics that the scenario supports and on the high-energy properties of the theory.

With the RG-improved potential for the scalar sector of the theory at hand, we have identified a region in the parameter space of the model that ensures the stability of the potential and the perturbativity of its parameters up to scales well beyond the Planck one. In particular, we have found that the electroweak vacuum instability is here rescued by the effect of the gauge mixing, once the mixing parameter is set to $\hat{g} \approx -0.5$.

Assuming this value in the following analysis (which is rather insensitive to this parameter), we then studied the symmetry breaking pattern supported by the model, originated by the extra scalar field responsible for the radiative breaking of the B–L symmetry. We find that thermal corrections prevent the transition to the emerging symmetry breaking minimum of the effective potential in a large part of the considered parameter space. As a consequence, we see the rise of an epoch of thermal inflation sourced by the potential energy difference between the false and true vacua of the theory.

At the latest, the inflationary regime concludes soon after the onset of the QCD phase transition, as the additional term induced in the scalar potential weakens the thermal contribution and a first order electroweak phase transition takes place. The potential energy density of the false vacuum is then transferred to radiation in a sec-
The spectrum is sizeable enough to fall within the reach of next-generation interferometers. In particular, LISA will probe most of the parameter space considered in the present analysis.

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