Observational Constraints on Varying-α Domain Walls

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We consider the possibility that current hints of a spatial variation of the fine structure constant at high redshift could be due to a biased domain wall network described by a scalar field non-minimally coupled to the electromagnetic field. We show that in order to be cause of the reported spatial variation of the fine structure constant without being in conflict with the observed anisotropies of the cosmic microwave background, the characteristic scale of the network would have to be of the order of the Hubble radius and the fractional contribution of the domain wall network to the energy density of the Universe would need to be in the range $10^{-10} < \Omega_{\omega 0} < 10^{-5}$. We argue that for sufficiently large values of $\Omega_{\omega 0}$ in this range, domain walls could even be responsible for the large scale anomalies in the temperature distribution of the cosmic microwave background detected by Planck and WMAP and provide a significant contribution to the excess B-mode polarisation power detected by BICEP2. Since the domain wall contribution to the cosmic energy budget only becomes important at late times, domain wall networks cannot play a significant role as a seed for large scale structure formation and primary cosmic microwave background anisotropies.

I. INTRODUCTION

Recent analysis of a combined sample of quasar absorption line spectra obtained using UVES (the Ultraviolet and Visual Echelle Spectrograph) on the Very Large Telescope (VLT) and HIRES (the High Resolution Echelle Spectrometer) on the Keck Telescope have provided hints of a spatial variation of the fine structure constant ($\alpha$) that is well represented by an angular dipole model [1,2]. Subsequently, several authors have shown that these results could be induced by a domain wall network described by a scalar field non-minimally coupled to the electromagnetic field [3,6] (see also [7] for the first discussion of the cosmological implications of varying-\(\alpha\) walls).

Domain wall networks tend to dominate the energy density of the universe at late times, a property which is shared by all dark energy candidates. This means that, unlike cosmic strings, domain walls are not expected to lead to observable signatures on cosmic structure formation and to significantly contribute to the primary Cosmic Microwave Background (CMB) anisotropies. Although frozen domain wall networks have been proposed in the past as a dark energy source [8], detailed studies have since ruled out any significant contribution of domain walls to the dark energy budget [9,10] (see also [11,13]).

Notwithstanding, as happens with other topological defects, a small domain wall contribution to the energy budget (smaller than $10^{-5}$) is not ruled out by cosmological observations and could even be the source of the large scale anomalies in the temperature distribution of the CMB detected by Planck and WMAP (Wilkinson Microwave Anisotropy Probe) [14,17] if the characteristic length scale of the network is of the order of the Hubble radius (as is expected in the case of frictionless domain walls).

Recent results by the Background Imaging of Cosmic Extragalactic Polarization (BICEP2) experiment seem to be consistent with an excess of B-mode power on large angular scales (in the multipole range $30 < l < 150$), which has been interpreted as the first direct evidence for a primordial gravitational wave background produced at an early inflationary stage at the Grand Unified Theory scale [18] (see also [19,20] for alternative/complementary interpretations involving cosmic strings).

In this paper we will show that a domain wall network may be responsible for both the reported spatial variations of $\alpha$ based on VLT/UVES and Keck/HIRES observations and the Planck and WMAP large scale anomalies, and even contribute to the excess B-mode power observed by BICEP2.

II. DOMAIN WALL EVOLUTION: THE BASICS

In this section, we shall briefly review the essential aspects driving the cosmological evolution of a domain wall. In a flat homogeneous and isotropic Friedmann-Robertson-Walker background, the line element is given by

$$ds^2 = -dt^2 + a(t)^2 \left[ dr^2 + r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right) \right],$$

(1)
where $t$ is the physical time, $a$ is the scale factor and $(r, \theta, \phi)$ are spherical coordinates.

Let us start by considering the case of a spherically symmetric domain wall in a Minkowski spacetime (with $a = 1$). This will allow us to pinpoint the effect of curvatures on domain wall dynamics. In this case, the wall has an invariant area, $S = 4\pi \gamma r^2$ ($\gamma \equiv (1 - v^2)^{-1/2}$, where $v$ is the domain wall velocity), that is proportional to its energy (this is the domain wall analogue of the invariant perimeter of a cosmic string loop, $2\pi \gamma r$). Domain walls evolve in a way as to minimize their energy, and thus conserve their invariant area. It is straightforward to show that energy conservation then leads to the following equation of motion for a spherical domain wall in a Minkowski spacetime

$$\frac{dv}{dt} = n(1 - v^2)\frac{f_k}{r},$$

where $n = 2$ and $f_k$ can be equal to 1 or $-1$ depending, respectively, on whether the curvature is accelerating ($dv/dt > 0$) or decelerating ($dv/dt < 0$) the domain wall. Note that this expression also applies to point particles, if one sets $n = 0$, and to cosmic strings, for $n = 1$.

In a FRW universe, the motion of the domain walls will be damped as a result of the expansion of the background. For planar domain walls, the wall momentum per unit comoving area should be conserved. One thus has that

$$\frac{d(v^rn^{n+1})}{dt} = 0,$$

or, alternatively,

$$\frac{dv}{dt} = -(1 - v^2)(n + 1)Hv,$$

with $n = 0, 1$ and $2$, for point particles, cosmic strings and domain walls, respectively. Here, $H = (da/dt)/a$ is the Hubble parameter.

Eqs. (2) and (4) may be combined in order to obtain the equation of motion of a spherically symmetric domain wall in a FRW background. Note that, despite the fact that this equation describes the microscopic evolution of a single spherical wall, it captures the essential features of the large-scale dynamics of domain wall networks. As a matter of fact, a close analogue has been shown to accurately describe the root-mean-square velocity of domain wall networks. We refer the reader to Refs. [9, 10, 12, 13, 21–23], for a detailed description of the semi-analytical velocity-dependent one-scale (VOS) model for domain wall networks.

### III. THE BIASED EVOLUTION OF VARYING-$\alpha$ WALLS

For the remainder of this paper, we shall assume that the domain walls are associated with the variation of a scalar field $\phi$ that interpolates from $\phi_-$ to $\phi_+$ (or vice-versa) at the domain wall. Note that this is the simplest realisation of a domain wall network without junctions and that more complex networks with junctions may be constructed in models with more than two minima of the scalar field potential [9, 10]. Let us also assume that the value of the fine structure constant $\alpha$ is a function of the scalar field $\phi$ ($\alpha = \alpha(\phi)$) — so that it takes different values on each of the wall’s domains — and let $\alpha_- = \alpha(\phi_-)$ and $\alpha_+ = \alpha(\phi_+)$ be the values of $\alpha$ on each side of the domain wall. This promptly implies that the baryons on opposite sides of a domain wall have slightly different masses. The fractional mass difference is given approximately by

$$\zeta \equiv \frac{\Delta m}{m} = \frac{\Delta \alpha}{\alpha}$$

where $\zeta$ corresponds to the fractional electromagnetic contribution to the baryon mass. Note that $\zeta$ takes different values depending on the nature of the particles being considered: $\xi = -0.0007$ and $\xi = 0.00015$, for the proton and neutron respectively [24]. Protons are significantly more abundant than neutrons and their electromagnetic contribution is considerably larger than that of neutrons. For this reason, and for the remainder of this paper, we shall assume that $\xi = -0.0007$.

The domains on opposite sides of a wall have the same baryon number density, however, as a consequence of the difference in baryon mass, that must have different baryon energy densities. This energy difference between the domains introduces a bias on the dynamics of the domain wall, favouring the domains with a smaller value of the baryon energy density. Biased domain walls were originally envisioned as means to evade the Zel’dovich bound [25], however they have also served as the basis of the devaluation scenario that attempted (and failed) to solve the cosmological constant problem [26]. The dynamics of biased domain walls has been studied in detail in these contexts (see e.g [27–31]). Here, we follow closely the approach in Ref. [30].

Let us denote the values of the baryon densities by $\rho_B-$ and $\rho_B+$ and introduce a bias parameter defined by

$$\epsilon = \rho_{B+} - \rho_{B-} = \zeta \rho_B.$$  

In order to model the effect of the bias on the dynamics of the domain wall, we start by considering a planar domain wall in a Minkowski spacetime. Energy conservation implies that

$$d(\sigma \gamma) = v\epsilon dt,$$

or equivalently

$$\frac{dv}{dt} = \epsilon f_v \frac{f_v}{R \gamma^3},$$

where $\sigma$ is the domain wall energy per unit area and $f_v$ can be equal to 1 or $-1$ depending on whether the bias
is accelerating \((dv/dt > 0)\) or decelerating \((dv/dt < 0)\) the domain wall. When a domain wall is moving towards a region with higher baryon energy density, the wall gains momentum to balance the resulting energy loss. Therefore, the domain walls feel a pressure which favours the suppression of the regions with a larger value of the baryon energy density. One may infer from Eq. \(\xi\) that the effect of the energy difference between the domains is to accelerate the domain wall and, in that sense, \(\epsilon\) acts as an effective curvature characterised by a constant length scale \(R_c = \sigma/\epsilon\).

The effect of the bias on domain walls with curvature in an expanding background is determined by the interplay of the surface pressure — caused by the curvature —, the volume pressure — resulting from \(\epsilon\) — and the damping caused by Hubble expansion. Eqs. \(6\), \(7\) and \(8\), may be combined into the following equation

\[
\frac{dv}{dt} = (1 - v^2) \left( \frac{2f_k}{R} + \frac{f_r}{Rc\gamma} - 3Hv \right),
\]

where \(R = ar\) is the physical radius of the domain wall. Biased domain walls may be long-lived or disappear almost immediately, depending on the relative importance of the different terms in Eq. \(9\). As the domain walls evolve and the physical size of the domains increases, the importance of the bias term, when compared to the curvature and damping terms, grows with cosmic time. Once the characteristic length scale of the domain wall network \(L\) becomes larger than \(R_c\) — or, equivalently, when

\[
\frac{\sigma}{\epsilon L} = \frac{\rho_w}{\epsilon} = \frac{\Omega_w}{\Omega B \zeta} < 1,
\]

— domains with larger baryon energy density disappear exponentially fast \(26\). Here \(\rho_c\) is the critical density of the universe, \(\rho_w = \sigma/L\) is the average energy density of the domain wall network, \(\Omega_B = \rho_B/\rho_c\) and \(\Omega_w = \rho_w/\rho_c\). Varying-\(\alpha\) domain walls have, thus, an additional source of instability — the bias introduced by the energy difference between domains — and, as we shall see in the next section, this will allow us derive a lower bound on the admissible domain wall energy density.

**IV. OBSERVATIONAL CONSTRAINTS**

Observations of the quasar absorption spectra at high redshifts obtained using HIRES on the Keck Telescope were shown to be consistent with a variation of the fine structure constant of \((0.57 \pm 0.11) \times 10^{-5}\) \(32\) (see also \(33\) and \(35\)). However, ensuing studies based on VLT/UVES data have indicated a much smaller variation of \(\Delta \alpha\) \(36\) \(37\). These observational constraints can be reconciled by allowing for a spatial variation of the fine structure constant. In fact, it has been shown in \(2\) that the data from a large sample of 295 absorbers from the VLT and Keck telescopes is compatible with an angular dipole model with amplitude \(\Delta \alpha/\alpha = 0.97^{+0.22}_{-0.26} \times 10^{-5}\) \(2\) (see also \(1\)).

It has been suggested that spatial variations of \(\alpha\) consistent with current observations could also be originated by a domain wall network described by a scalar field non-minimally coupled to the electromagnetic field \(3\) \(4\) \(6\) (see also \(1\)). This scenario was shown to be compatible with the current data, although a detailed verification will require a new generation of high-resolution ultra-stable spectrographs such ESPRESSO (the Echelle SPectrograph for Rocky Exoplanet and Stable Spectroscopic Observations). Note that in a model with spatial variations in \(\alpha\) caused by the existence of a domain wall network, the stringent laboratory constraints (see, for example, \(38\) and references there in) are naturally evaded.

The domain wall network would have to have a characteristic scale of the order of the Hubble radius in order to be able to induce spatial variations of order \(\beta\) on a cosmological scale. The redshift of the closest domain wall may be estimated by solving the equation

\[
\Delta \eta \equiv \eta_0 - \eta = \beta L/a,
\]

with \(\beta \lesssim 1\), using the VOS model for domain walls. Here \(\eta\) is the conformal time (or comoving particle horizon) defined by \(\eta = \int dt/a\), and the subscript ‘0’ refers to the present time. Using the calibration of the VOS model given in Ref. \(39\) and assuming fractional matter and cosmological constant energy densities compatible with the Planck results \(14\) (\(\Omega_m = 0.315\) and \(\Omega_A = 1 - \Omega_m\), respectively), we have found that if the wall closest to us is at a redshift \(z = 0.5 - 1\) — as indicated by the results obtained in \(6\) — its existence would be consistent with a plausible range for the corresponding value of \(\beta\) \((0.3 - 0.7)\). Results from high resolution field theory simulations have shown no significant dependence of the characteristic velocity on the properties of the network. However, the characteristic scale could vary significantly and be smaller by a factor of up to 3 in the case of complex domain wall networks with junctions \(10\) (or even by a larger factor if friction is important \(12\) or in the presence of massive junctions \(11\)). The above estimate of the value of \(\beta\) appears to favour the simplest frictionless domain wall models without junctions over more complex scenarios with junctions.

A necessary requirement for the domain walls to be able to seed the spatial variations of \(\alpha\) is that the bias introduced by the interaction with the baryons does not lead to the suppression of the domain wall network prior to the present epoch. This requirement (see Eq. \(10\)) yields the following stringent lower limit on the contribution of domain walls to the cosmic energy budget

\[
\Omega_w > \Omega_B \left| \frac{\Delta \alpha}{\alpha} \right| \sim 3 \times 10^{-5} \left| \frac{\Delta \alpha}{\alpha} \right| \sim 3 \times 10^{-10}. \quad (12)
\]

Here, we have taken \(\Omega_B h^2 = 0.02205\), with \(h = 0.673\), as indicated by Planck data \(14\). On the other hand, a conservative constraint on the contribution of a domain
wall network with a characteristic length of the order of the Hubble radius to the cosmic energy budget is

$$\Omega_{w0} < 10^{-5},$$

(13)
in order to prevent domain walls from providing the dominant contribution to the large scale temperature anisotropies of the CMB. The scenario of domain wall induced spatial variations of the fine structure constant is thus tightly constrained, both on the nature of the walls — which should be frictionless — and on their fractional energy density.

Note that, the presence of a domain wall network with a characteristic scale comparable to the Hubble radius may leave a variety of other observational signatures (in particular if its fractional energy density is close to the upper limit set in Eq. (13)). As happens in the case of cosmic strings [10–16], as domain wall networks evolve and interact, a fraction of their energy is released in the form of vector and tensor modes that may contribute to the B-mode polarisation of the CMB. In fact, it has been shown in Ref. [17] using field theory simulations that the spectrum of gravitational waves emitted by domain walls peaks at the scale corresponding to the size of the Hubble radius at the time of their decay. This makes domain walls a candidate for the source of the B-mode signatures recently detected by BICEP2 [18]. Note also that the fact that domain walls — which only become cosmologically relevant at recent times — cannot give a significant contribution to the matter power spectrum makes domain walls a stronger contender to explain the excess of B-mode power on large angular scales detected by BICEP2 than other defects (such as cosmic strings).

In [20] it was found that, in order for cosmic strings to be the dominant contributor to excess of B-mode power on large angular scales detected by BICEP2, the inter-string distance would need to be extremely large. Although this analysis does not directly apply to domain wall networks, the fact that the characteristic length of a domain wall network is significantly larger than that of local strings is reassuring. Also, domain walls only become cosmologically relevant at recent times, thus implying that the small scale B-mode power induced by domain walls will be strongly suppressed on small angular scales when compared to a corresponding contribution from cosmic strings. Another interesting observation is the apparent tension between the BICEP2 results and the Planck upper limit on the tensor-to-scalar ratio $r$. A recent suggestion to solve this discrepancy involves a spatial variation of $r$ which could also account for the large scale anomalies in the temperature distribution of the cosmic microwave background detected by Planck and WMAP [48]. A similar signature would also be expected in the case of the domain wall scenario studied in the present paper. The very large characteristic length scale of the domain wall at the present time could be naturally associated with large scale temperature and polarisation CMB power asymmetries and with a very large cosmic variance that more detailed studies will need to tackle.

The detection of B-modes by BICEP2 is still a matter of controversy, with several authors questioning the validity of the methods used (see e.g. [49, 50]). However, even if this detection is not confirmed, stronger observational constraints on the fractional energy density of domain walls may be inferred from the absence of signal. In either case, valuable information will be gained with more detailed studies of the large scale anisotropies and CMB polarisation signatures generated by varying-$\alpha$ domain walls.

V. CONCLUSIONS

In this paper we have considered the possibility that a domain wall network may be responsible for both the reported spatial variations of $\alpha$ based on VLT/UVES and Keck/HIRES observations, the excess B-mode power observed by BICEP2 and the Planck and WMAP large scale anomalies. We have shown that in order to explain the former the dynamics of the domain wall network needs to be essentially frictionless — so that its characteristic length scale at the present time is of the order of the Hubble radius — and that its energy density is constrained within five orders of magnitude ($10^{-10} < \Omega_{w0} < 10^{-5}$). For sufficiently large values of $\Omega_{w0}$ ($\sim 10^{-6} - 10^{-5}$) the domain walls are expected to provide a significant contribution to the B-mode polarisation power spectrum on large angular scales, associated to vector and tensor perturbations, and to the large scale temperature anisotropies, thus being a plausible explanation of the large scale anomalies in the temperature distribution of the CMB detected by Planck and WMAP and of the B-mode polarisation signature detected by BICEP2. A more detailed study taking into account the large cosmic variance will be needed in order to further test this scenario. This study will require an accurate computation of the scale dependence of both vector and tensor perturbations generated by domain wall networks around the present time for a wide range of representative domain wall network models (with or without junctions). Unlike other defect models, where a significant contribution to the CMB temperature and polarisation late time anisotropies would be naturally associated with a significant contribution to the matter power spectrum and to the primary CMB anisotropies, domain walls may only become cosmologically relevant at late times thus providing a negligible contribution to the later.

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