On the long-term hydroelastic global structural response evaluation of a 100000 tdw oil-tanker

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Abstract. In this paper, the hydroelastic analysis for a 100000 tdw oil-tanker, with a length of 240 m, operation reference speed of 15 knots, on two loading cases full cargo and ballast, is developed. For a realistic approach of the dynamic response, the oil-tanker is considered under random waves’ loads, requiring several steps in the process of the short-term and long-term global structural analysis. The random waves have Longuet-Higgins formulation, with first-order harmonics by ITTC power density spectrum, with maximum wave significant height 12 m, and second-order interference waves’ harmonics. The short-term hydroelastic analysis includes the linear response by frequency-domain solutions, the non-linear response by time-domain solutions, the global strength significant response by FFT Fast Fourier Transformation spectral approach. The long-term analysis includes the hydroelastic global strength assessment for Word Wide Trade averaged wave’s height histogram, the fatigue hull structure evaluation, over a 25 years exploitation reference time, applying the cumulative damage ratio approach. The stress hot-spot factors, up to 1.5, influence is considered. The oil-tanker model by own program DYN is developed, modules for hydroelastic response and statistical processing, applicable in any ship’s design stage, stressing out to the 100000 tdw oil-tanker long-term operation limits on the two relevant loading cases.

1. Introduction
The ships with a length over 200 m have an elastic hull structure, with the fundamental natural global frequencies lower than 1 Hz [1, 2]. The global structural evaluation for large ships includes the vibrations response by advanced hydroelasticity approach [3-5]. The hydroelastic global response includes linear and non-linear oscillation and vibration components. Hydrodynamic phenomena slamming, whipping, springing by hydroelasticity are modeled, providing an extended evaluation of the long-term ships’ operation capabilities in extreme sea state conditions [6-8].

The hydroelastic numerical deterministic and statistic simulations focus on a 100000 tdw oil-tanker ship [9]. The tanker has significant geometric non-linearities at the fore part, V shape with bulb, and at the aft part, U shape. Amidships an extended prismatic shape is modeled (section 3).

The hydroelastic theoretical method (section 2) implemented in own program DYN [10], for a head Longuet-Higgins wave excitation with multi-harmonic interference components, includes several computational modules previously validated by experimental tests [11].

The 100000 tdw oil-tanker hydroelastic response evaluation includes three main criteria: bottom slamming and deck flooding occurrence prediction by short-term most probable vertical maximum displacements, long-term stresses check out by yielding stress limit, fatigue strength evaluation by cumulative damage approach, thus providing the ship’s operation limits assessment (sections 4, 5).
2. Fundamentals of long-term ship’s hydroelastic response in irregular waves

The ship’s hydroelastic response combines oscillations and vibrations hull global dynamic analyses, involving short-term linear frequency domain solutions and non-linear time-domain solutions, followed by long-term prediction response and structural fatigue analysis. The theoretical method implemented in own code DYN [10] has four main analysis steps with interlinked numerical modules.

First analysis step is the short-term linear hydroelastic ship’s response by module STABY (DYN) [10].

For the hydroelastic analysis as excitation head wave model Longuet-Higgins (L&H) [10] is considered. The wave elongation $\zeta_i(x,t)$ (1) includes first-order components $\zeta_i^1(x,t)$ (2) with the ITTC spectrum [12] and second-order positive and negative interference components $\zeta_i^{2(\pm)}(x,t)$ (3).

$$
\zeta_i^1(x,t) = \zeta_i^1(x,t) + \zeta_i^{2(+)}(x,t) + \zeta_i^{2(-)}(x,t) = \sum_{i,j=1}^{m} \left[ \zeta_i^{1+}(x,\alpha_j) \cos(\alpha_j t - \zeta_i^{1+}(x,\alpha_j) \sin(\alpha_j t) \right]; \ x \in [0,L]
$$

$$
\zeta_i^{2(\pm)}(x,t) = \sum_{i,j=1}^{m} \left[ \zeta_i^{2(\pm)}(x,\alpha_j,\beta_{ij}) \cos(\alpha_j t - \zeta_i^{2(\pm)}(x,\alpha_j,\beta_{ij}) \sin(\alpha_j t) \right]; \ \alpha_j = \omega_j + \frac{\omega_j^2}{g} \nu; \ m = 250
$$

where: $g$ gravity acceleration; $L$ ship speed and length; $\omega_j; j = 1, m$ wave circular-frequencies by ITTC spectrum [12]; $t$ time; $x$ longitudinal coordinate; $\omega_j \in \{ \omega_j, \omega_j^{(+)}; \omega_j^{(-)} \}; i, j = 1, m$.

The global vibration response analysis requires idealizing the hull by an elastic beam finite element model [10]. The ship’s stations idealization uses multi-parametric conformal transformation [10]. The strip theory hydrodynamic formulation includes the ship’s hull vertical deflection.

The linear hydroelastic response $w_{lin}(x,t)$ (4), according to the modal analysis approach, includes the heave ($r=0$) and pitch ($r=1$) oscillation modes and the first three vibration modes ($r=2, 3, 4$) [10].

$$
w_{lin}(x,t) = \sum_{r=0}^{4} w_r(x) \cdot p_r^{(0)}(t); \ \{ p_r^{(0)}(t) \}_{r=0}^{4}; \ x \in [0,L]
$$

where: $w_r(x), r = 0, 4$ displacement modal forms; $\{ p_r^{(0)}(t) \}_{r=0}^{4}$ first-order (linear) modal coordinates.

The linear hydroelastic equations system (5), under irregular Longuet-Higgins wave, has a frequency domain solution for each wave circular-frequency $\omega_j$ component [10]. The linear solution corresponds to the short-term linear oscillation and springing response components (8).

$$
[A(\omega_j)][p_r^{(0)}(t)] + [B(\omega_j)][p_r^{(1)}(t)] + [C(\omega_j)][p_r^{(2)}(t)] = \sum_{i,j=1}^{m} \left[ F^{(+)}(\omega_j,\omega_i) \sin(\alpha_j t - \zeta_i^{1+}(x,\alpha_j) \sin(\alpha_j t) \right]
$$

where: $[A(\omega_j)]$, $[B(\omega_j)]$, $[C(\omega_j)]$ the structural plus hydrodynamic inertial, damping, restoring matrices for wave $\omega_j$; $\{ F^{(+)}(\omega_j,\omega_i) \}_{i,j=1}^{m}$ the Longuet-Higgins generalized wave excitation force terms.

Second analysis step is the short-term non-linear hydroelastic response by module TRANZY (DYN) [10].

The non-linear hydroelastic response $w_{non}(x,t)$ (6) by the modal analysis approach formulation, corresponds to the short-time non-linear oscillation and springing, transitory whipping, due to bottom and side slamming, response components (8).

$$
w_{non}(x,t) = \sum_{r=0}^{4} w_r(x) \cdot \left[ p_r^{(0)}(t) + p_r^{(2)}(t) \right]; \ x \in [0,L]
$$

where: $\{ p_r^{(2)}(t) \}_{r=0}^{4}$ second-order modal coordinates non-linear hydroelastic response.
The non-linear hydroelastic system includes terms for the ship’s hull geometrical non-linearities and the bottom slamming impact pressure. Therefore, a non-linear hydrodynamic strip-theory formulation is required. Using the first-order modal coordinates (4) and the Longuet-Higgins wave elongation (1), the second-order modal coordinates (6) result from a non-linear iterative time-domain solution approach by the equations system (7). For each iteration, the simulation time is 80 s, with a time step of 0.01 s, using the β-Newmark method for the direct time-domain integration approach [10].

\[
\begin{bmatrix}
\ddot{\mathbf{u}}_w \\
\ddot{\mathbf{u}}_{\text{hel}} \\
\mathbf{f}_w \\
\mathbf{f}_{\text{hel}}
\end{bmatrix} = \begin{bmatrix}
A_{\text{w}} & B_{\text{w}} & C_{\text{w}} & D_{\text{w}} \\
A_{\text{hel}} & B_{\text{hel}} & C_{\text{hel}} & D_{\text{hel}}
\end{bmatrix}
\begin{bmatrix}
\mathbf{u}_w \\
\mathbf{u}_{\text{hel}} \\
\mathbf{f}_w \\
\mathbf{f}_{\text{hel}}
\end{bmatrix} + \begin{bmatrix}
\mathbf{f}_w (t) \\
\mathbf{f}_{\text{hel}} (t)
\end{bmatrix} \Rightarrow \begin{bmatrix}
\mathbf{u}_w (t) \\
\mathbf{u}_{\text{hel}} (t)
\end{bmatrix}
\]

(7)

where: \(A_{\text{w}}, A_{\text{hel}}, B_{\text{w}}, B_{\text{hel}}, C_{\text{w}}, C_{\text{hel}}, D_{\text{w}}, D_{\text{hel}}\) are the structural plus hydrodynamic inertial, damping, restoring matrices, for \(\omega_{\text{osc}} = (\omega_h + \omega_l) / 2\) the average oscillation natural circular-frequency and \(\omega_l\) the first-order vibration natural circular frequency; \(f_{\text{w}}, f_{\text{hel}}\) the non-linearities generalized force.

Based on linear (5) and non-linear (7) solutions, the power density spectra and the short-term statistical linear and non-linear hydroelastic response components (8, 9, 10, 11) by FFT Fast Fourier Transformation spectral method result [10].

\[
\text{RMS}_f^2 (x, h_{1/3}) = \int_{\omega_{\text{osc}}} S_f (\omega, x) |h_{1/3}| d\omega; \quad \text{RMS}_{\text{hel}}^2 (x, h_{1/3}) = \sqrt{\sum_{\omega} \text{RMS}_f^2 (x, h_{1/3})}; \quad x \in [0, L]
\]

(8)

where: \(\alpha \in \{\text{osc, vib1, vib2, vib3}\}\) oscillation and vibration response; \(\text{osc}\) includes both heave and pitch components that have close natural frequencies; \(\text{hel}\) includes all the hydroelastic components; \(\omega_{\text{osc}}, \omega_{\text{hel}}\) the circular-frequencies reference limits for hydroelastic components domain; \(S_f(\omega)\) power density spectrum for any hydroelastic response type; \(h_{1/3}\) wave significant height; \(\chi \in [\text{lin, ntl}]\) linear and non-linear hydroelastic response.

\[
Z_{\text{sw}} (x) = d_{\text{sw}} + (d_{\text{fore}} - d_{\text{aft}}) \frac{x}{L}; \quad w_{\text{RMS}}^2 (x, h_{1/3}) = \text{RMS}_{\text{hel}}^2 (x, h_{1/3}); \quad x \in [0, L]
\]

(9)

\[
Z_{\text{deck}} (x, h_{1/3}) = Z_{\text{sw}} (x) + w_{\text{RMS}}^2 |h_{1/3}| \leq Z_D (x); \quad Z_{\text{sw}}^2 (x, h_{1/3}) = Z_{\text{sw}} (x) - w_{\text{RMS}}^2 |h_{1/3}| \geq Z_K (x)
\]

(10)

where: \(w_{\text{RMS}}\) vertical displacement most probable amplitude (9); \(d_{\text{sw}}, d_{\text{fore}}, d_{\text{aft}}\) aft and fore draught in still water; \(Z_{\text{deck}}, Z_{\text{hel}}\) total short-term displacement at deck and keel into base plane reference; \(Z_D, Z_K\) vertical limit references at deck and keel line for the oil-tanker lines (figure 1) for bottom slamming and deck flooding occurrence evaluation (figures 10, 16).

\[
Q_{1/3} \left|_{\text{sw}} \right. (x, h_{1/3}) = 2 \cdot \text{RMS}_{f}^2 \left|_{\text{hel}} \right. (x, h_{1/3}) \quad ; \quad Q_{1/3} \left|_{\text{sw}} \right. (x, h_{1/3}) = Q_{\text{sw}} (x) + Q_{1/3} \left|_{\text{hel}} \right. (x, h_{1/3}) \quad ; \quad x \in [0, L]
\]

(11)

where: \(Q \in \{M, T\}\) bending moment or shear force in ship’s vertical plane; \(Q_{1/3}\) the bending moment or shear force significant amplitudes; \(Q_{\text{sw}}, Q_{1/3}\) still water and total short-term bending moment or shear force in ship’s vertical plane.

The short-term significant deck normal stress \(\sigma_{s\text{D}1/3}^x \left|_{\text{hel}} \right. \) and neutral axis tangential stress \(\tau_{s\text{NN}1/3}^x \left|_{\text{hel}} \right. \) result from expression (12).

\[
\sigma_{s\text{D}1/3}^x \left|_{\text{hel}} \right. (x, h_{1/3}) = K_x M_{1/3} \left|_{\text{hel}} \right. (x, h_{1/3}) \quad ; \quad \tau_{s\text{NN}1/3}^x \left|_{\text{hel}} \right. (x, h_{1/3}) = K_{1/3} T_{s\text{hel}}^x \left|_{\text{hel}} \right. (x, h_{1/3}) \cdot K_{\text{m}1/3} (x)
\]

(12)
where: \( W_{D,b}(x) = W_{D0,b0} \cdot f_{W_{D,b}}(x) \) deck and bottom bending modules with distribution functions \( f_{W_{D,b}}(x) \) and amidships \( W_{D0} W_{b0} \) values; \( K_{m0} = K_{m0} \cdot f_{K_{m0}}(x) \) the maximum shearing stress coefficient at neutral axis with distribution function \( f_{K_{m0}}(x) \) and amidships \( K_{m0} \) value (figure 2, table 2); \( K_y = 1.0 + 1.5 \) the average stress hot-spot coefficient.

- Third analysis step is the long-term hydroelastic response by module LTP (DYN) \[10\].

The long-term hydroelastic analysis uses the averaged Word Wide Trade (WWT) histogram of the wave’s significant height \( p(h_{1/3},i) = i, 12 \) (figure 5) \[12\]. The sea state long-term occurrence probability \( P_{LTP}(R) \) is function to the reference exploitation period \( R \) (year), with the values presented in table 1 for the WWT long-term wave’s significant height \( h_{1/3} \) histogram.

**Table 1.** Sea state long-term occurrence probability \( P_{LTP}(R) \) for WWT wave’s height histogram.

| R(years) | 1 hour | 1 day | 1 week | 1 month | 1 year | 5 years | 10 years | 15 years | 20 years | 25 years |
|----------|--------|-------|--------|---------|--------|---------|----------|----------|----------|----------|
| \( P_{LTP}(R) \) | 2.2 \times 10^{-3} | 9.1 \times 10^{-5} | 1.3 \times 10^{-5} | 3.0 \times 10^{-6} | 2.5 \times 10^{-7} | 5.0 \times 10^{-8} | 2.5 \times 10^{-9} | 1.7 \times 10^{-8} | 1.2 \times 10^{-9} | 1.0 \times 10^{-10} |

The long-term hydroelastic linear and non-linear stresses \( Y_{R}^L \) \( Y_{R}^W \) result by solving the non-linear equation (13) for each reference period \( R \) (table 1) and WWT wave’s significant height histogram, based on the short-term stresses (12). The total stress values \( (14) \) result by adding the still water stresses \( (\sigma_{SW0}, \sigma_{SW0}, \tau_{0
NNN}) \) to the long-term hydroelastic stress (13), with assessment by yielding stress limit criteria \( (\sigma_{adm}, \tau_{adm}, \text{table 2}) \).

\[
P_{LTP}(R)=\sum_{i=1}^{12} p(h_{1/3}) \cdot \exp \left[ -\frac{2}{\left( Y_{R}^L \right)_{h=0}^L} \right] \rightarrow Y_{R}^L \left( h_{1/3} \right) = \left\{ \sigma_{SW0}, \sigma_{SW0}, \tau_{0
NNN} \right\}_{h=0}^L \tag{13}\]

\[
\sigma_{SW0}^L(x) + \sigma_{SW0}^R(x) \leq \sigma_{adm}; \quad \tau_{0
NNN}^L(x) + \tau_{0
NNN}^R(x) \leq \tau_{adm}; \quad x \in [0, L] \tag{14}\]

- Fourth analysis step is the long-time hydroelastic fatigue analysis by module FAT (DYN) \[10\].

For the hydroelastic fatigue analysis, the damage cumulative ratio \( D \) approach (15, 16) is applied \[12\], and the predicted safety exploitation life \( R_E \) (17) results. For reference, we consider \( R = 25 \) years for long-term WWT wave’s height histogram and deck shell but-welded structural details \[12\].

\[
D^L(x) = \sum_{i=1}^{12} n_x(h_{1/3}) \cdot N_x(h_{1/3}) \cdot f_{SW} \cdot R; N_x(h_{1/3}) = F_{SW} \left[ \frac{f_{SW} \sigma_{SW0}^L(x)}{f_{SW} \sigma_{SW0}^L(h_{1/3})} \right] \tag{15}\]

\[
D^L(x) = D_{F}^{full,ball} + \frac{D_{F}^{full,ball}}{D_{F}^{ball,ball}} + \frac{D_{F}^{full,ball}}{D_{F}^{ball,ball}} + D_{F}^{ball,ball} \times \chi \in [lin, nl] \tag{16}\]

\[
D^L(x) = 0.5 \cdot D^L_{ball}(x) + 0.5 \cdot D^L_{ball}(x) \quad (R_E = L_D) \tag{17}\]

where: \( f_{sw} \) oscillations and vibrations natural frequencies (table 2); \( f_{sw} \) equivalent symmetric cycle factor \[12\]; \( f_{sw} \) welding quality factor \[12\]; \( F_{SW} \) stress-strain fatigue curves for a structural detail \[12\]; \( full, ball \) loading cases full-cargo and ballast considered with the same occurrence probability 50%.

The fatigue analysis is done for each stress hot-spot coefficient \( K_y \), for the linear and non-linear hydroelastic response, for several relevant stations \( (x/L=0.25, 0.50, 0.75) \). A safe structural detail, by fatigue criteria, should ensure \( D \leq 1 \), \( R_E \geq 25 \) years, but at least an exploitation life of \( R_{E, min} \geq 5 \) years that covers one time between periodic ship’s hull class inspections \[12\].
3. The 100000 tdw oil-tanker numerical model

The hydroelastic analysis focuses on a 100000 tdw oil-tanker (table 2) with the 3D hull-lines from figure 1, modeling with accuracy the aft and fore hull geometric non-linearities. The prismatic shape with vertical sides extends over 50% of the ship hull, corresponding to the cargo-hold compartments. The hull-lines model is conformal for the linear and non-linear strip theory hydrodynamic formulation (section 2). Table 2 presents the 100000 tdw oil-tanker characteristics [9].

The hull structure preliminary design is according to the DNVGL rules [12]. Figure 2 presents the strength data simplified distribution functions and the amidships structure at the cargo-hold compartments. The 100000 tdw oil-tanker has a double hull into a longitudinal structural system, with a longitudinal central bulkhead. At the center structural part, the strength characteristics have a simplified uniform distribution, with the preliminary design values in table 2.

The hull structure is made of shipbuilding steel class A and AH32 (deck), isotropic material, according to the DNV-GL rules [12], with the yielding stress limit admissible values in table 2. For the vibration modes, the average structural damping is $\beta = 0.001$ [10].

**Table 2. Oil-tanker 100000 tdw characteristics.**

| Main data | Loading cases | Natural frequencies $f$ (Hz) | Steel material |
|-----------|---------------|-------------------------------|----------------|
| $L$ (m)  | 240.000       | $\Delta$ (t) 126457.16 81763.34 $\text{add.mass}$ | $E$ (N/m$^2$) 2.1x10$^7$ |
| $B$ (m)  | 42.000        | $\rho_w$ (t/m$^3$) 1.025 | $G$ (N/m$^2$) 8.1x10$^{10}$ |
| $H$ (m)  | 21.300        | $d_a$ (m) 15.000 | $R_{II}$ (MPa) 315, $\text{AH32}$ |
| Stations | 165(41)       | $d_b$ (m) 15.000 | $R_{II}$ (MPa) 235, $\text{AH32}$ |
| $g$ (m/s$^2$) | 9.81      | $d_{g0}$ (m) 15.000 | $R_{II}$ (MPa) 205, $\text{AH32}$ |
| $d_{g}$ (m) | 15.000       | $d_{g0}$ (m) 15.000 | $R_{II}$ (MPa) 120, $\text{AH32}$ |
| $v$ (kn) | 15            | $d_{g0}$ (m) 15.000 | $R_{II}$ (MPa) 263, $\text{AH32}$ |
| $I_{yy0}$ (m$^4$) | 417.481   | $d_{g0}$ (m) 15.000 | $R_{II}$ (MPa) 291, $\text{AH32}$ |
| $A_{0}$ (m$^2$) | 5.635    | $d_{g0}$ (m) 15.000 | $R_{II}$ (MPa) 120, $\text{AH32}$ |
| $W_{D0}$ (m$^3$) | 35.096   | $d_{g0}$ (m) 15.000 | $R_{II}$ (MPa) 263, $\text{AH32}$ |
| $W_{B0}$ (m$^3$) | 44.391   | $d_{g0}$ (m) 15.000 | $R_{II}$ (MPa) 291, $\text{AH32}$ |
| $f_{1/ktn}$ (m$^2$) | 1.522     | $d_{g0}$ (m) 15.000 | $R_{II}$ (MPa) 263, $\text{AH32}$ |

**Figure 1.** Oil-tanker 100000 tdw hull-lines model, (a) 3D view, (b) fore to aft view [9].

**Figure 2.** Oil-tanker 100000 tdw, (a) strength data distribution functions, (b) hull structure amidships.
For the 100000 tdw oil-tanker global hydroelastic analysis, we select two relevant loading cases, full cargo and standard ballast, with the mass distributions $m_x$ (figure 4), displacement, draughts in table 2. The oil-tanker hull structure is with a 1D equivalent beam numerical model [10] (section 2).

Figure 3 presents the hydrodynamic added mass $m_{33}$ and damping $N_{33}$ per unit length function to the motion frequency and instantaneous draught ($z \leq 21.3$ m) at station $x/L=0.5$. The natural heave and pitch oscillation frequencies, by a linear seakeeping method [10], have similar values with an average of 0.104 Hz (table 2). The natural vibration frequencies, by the Jacobi method [10], have first-order frequencies higher than 0.660 Hz (table 2). The hydrodynamic added masse and damping have changes in the oscillation domain. In the vibration domain, the added masses are no longer function to the frequency, the hydrodynamic damping becomes negligible, sustaining the formulation in (7).

Figure 5 presents the averaged WWT Word Wide Trade [12] wave significant height histogram, describing the long-term occurrence probability of the irregular waves. Figure 6 shows the short-term Longuet-Higgins wave time record (1) and FFT amplitude spectrum, reference $h_{1/3}=10.285$ m.
4. The short-term hydroelastic analysis of the 100000 tdw oil-tanker

4.1. The oil-tanker short-term hydroelastic response in full-cargo case

This section presents a selection of the relevant results for the short-term hydroelastic analysis of the oil-tanker, in full-cargo case (table 2), as following:

- Figure 7 presents the vertical displacement \( \omega \) (m) time record and FFT amplitude spectrum, at \( x/L = 0.25 \) section, non-linear solution (6), \( h_{L} = 10.285 \) m, full-cargo case.

- Figures 8 and 9 present the vertical bending moment \( M \) (kNm) time record and FFT amplitude spectrum, at \( x/L = 0.50 \) section, linear and non-linear solutions, \( h_{L} = 10.285 \) m, full-cargo case.

- Figure 10 presents the total short-term vertical displacement at deck \( \Delta Z_{\text{deck}} \) (m) and keel \( Z_{\text{keel}} \) (m), into base plane reference, non-linear solution, \( x=0-240 \text{m}, h_{L}=0-12 \text{ m}, \) full-cargo case.

- Figure 11 presents the total short-term vertical bending moment \( M_{\Delta Z_{\text{SW}}} \) (kNm) and vertical shear force \( T_{\Delta Z_{\text{SW}}} \) (kNm) (11), non-linear solution, \( x=0-240 \text{m}, h_{L}=0-12 \text{ m}, \) full-cargo case.

- Figure 12 presents the significant vertical bending moment \( M_{\omega} \) (kNm), oscillation, vibrations modes (11), linear and non-linear solution, \( x=0-240 \text{m}, h_{L}=0-12 \text{ m}, \) full-cargo case.

- Table 3 presents the most probable vertical displacement \( w_{\text{RMS}} \) (m) (9), at \( x/L = 0.25, 0.50, 0.75, \) the ratios between first, second, third vibrations and oscillation modes, \( h_{L}=10.285 \) m, full-cargo.

- Table 4 presents the significant vertical bending moments \( M_{\omega} \) (kNm) and the significant shear forces \( T_{\omega} \) (kNm) (11), at \( x/L = 0.25, 0.50, 0.75, \) including the ratios between first, second, third vibrations and oscillation modes, for reference \( h_{L}=10.285 \) m, full-cargo case.

For the hydroelastic vertical displacement response, the FFT amplitude spectrum (figure 7) points out as the main component oscillations \( f_{\omega} \approx 0.099 \text{ Hz} \) rigid body modes. The most probable vertical deflection on vibration modes is in the range of \( 21.4 \pm 81.1 \text{ mm} \) (\( x/L = 0.25-0.75 \)), which represent \( 2.98 \pm 3.03\% \) \( f_{\omega} = 0.664 \text{ Hz} \), \( 2.32 \pm 2.36\% \) \( f_{\omega} = 1.517 \text{ Hz} \), \( 1.30 \pm 1.33\% \) \( f_{\omega} = 2.482 \text{ Hz} \) from the most probable oscillation displacement (table 3). The differences between the non-linear and linear most probable vertical displacements are \( 3.68\% \pm 6.71\% \) (table 3) on average. The total most-probable vertical displacement points out that bottom slamming occurs at both end parts, aft \( h_{L} > 4 \text{ m}, \) fore \( h_{L} > 5 \text{ m} \) (figure 10). Side slamming occurs at both ends due to the ship’s flare forms (figure 1). The green sea (deck flooding) occurs only at the forepeak, for \( h_{L} > 10 \text{ m} \) (figure 10).

For the hydroelastic vertical bending moment response, the FFT amplitude spectra (figures 8, 9) reveal that besides the oscillations, heave, pitch \( f_{\omega} = 0.097 \pm 0.101 \text{ Hz} \) rigid body modes, the first-order vibration mode \( f_{\omega} = 0.664 \text{ Hz} \) is also noticeable (figure 12). The total short-term vertical bending moment (figure 11) records maximum values around the amidships sections (\( x/L = 0.50 \)) at the cargo-holds compartments. Comparing the significant vertical bending moment values (figure 12), at selected sections \( x/L = 0.25, 0.50, 0.75, \) for oscillation mode as reference (table 4), results that the vibration modes represent \( 20.76\% \pm 38.00\% \) first-order \( (vib_{1}) \), \( 6.50\% \pm 7.72\% \) second-order \( (vib_{2}) \), \( 2.00\% \pm 2.67\% \) third-order \( (vib_{3}) \), supporting the linear and non-linear results in figures 8, 9, 12.

The total short-term vertical shear force (figure 11) records maximum values around the aft-quarter sections (\( x/L = 0.25 \)) at the ship’s structural transition region from cargo holds compartments to the engine compartment. For the significant vertical shear forces, at selected sections \( x/L = 0.25, 0.50, 0.75, \) the vibration modes represent \( 16.50\% \pm 30.34\% \) first-order \( (vib_{1}) \), \( 9.08\% \pm 11.23\% \) second-order \( (vib_{2}) \), \( 2.00\% \pm 3.07\% \) third-order \( (vib_{3}) \) from the significant oscillation modes shear force.

The differences between the non-linear and linear significant vertical bending moments (table 4) are \( 2.96\% \) \( (osc) \), \( 88.40\% \) \( (vib_{1}) \), \( 22.07\% \) \( (vib_{2}) \), \( 34.80\% \) \( (vib_{3}) \) on average. The differences between the non-linear and linear significant vertical shear forces (table 4) are \( 1.42\% \) \( (osc) \), \( 83.98\% \) \( (vib_{1}) \), \( 24.38\% \) \( (vib_{2}) \), \( 49.05\% \) \( (vib_{3}) \) on average. Thus, the non-linear solutions provide a noticeable increase of the bending moments (figure 12) and shear forces on vibration modes, especially on the first-order.

Comparing the ship’s responses in full-cargo case (tables 3, 4), for the hydroelastic phenomena result that the steady linear springing is small and the non-linear springing is medium. The whipping is high due to the occurrence of the bottom and side slamming at both ship’s extremities.
Non-linear Analysis - Time Record:

\[ w(m), \text{full-cargo Tanker 100000tdw, } v=15\text{kn}, x/L = 0.25, h_{1/3} = 10.285\text{m} \]

![Figure 7](image)

Figure 7. Oil-tanker 100000 tdw, non-linear analysis, \( w \) (m) vertical displacement, full-cargo, \( v=15 \text{ kn}, x/L = 0.25, \text{L&H wave } h_{1/3} = 10.285 \text{ m} \), (a) time record, (b) FFT amplitude spectrum.

Linear Analysis - Time Record:

\[ M(kNm), \text{full-cargo Tanker 100000tdw, } v=15\text{kn}, x/L = 0.50, h_{1/3} = 10.285\text{m} \]

![Figure 8](image)

Figure 8. Oil-tanker 100000 tdw, linear analysis, \( M \) (kNm) vertical bending moment, full-cargo, \( v=15 \text{ kn}, x/L = 0.50, \text{L&H wave } h_{1/3} = 10.285 \text{ m} \), (a) time record, (b) FFT amplitude spectrum.

Non-linear Analysis - Time Record:

\[ M(kNm), \text{full-cargo Tanker 100000tdw, } v=15\text{kn}, x/L = 0.50, h_{1/3} = 10.285\text{m} \]

![Figure 9](image)

Figure 9. Oil-tanker 100000 tdw, non-linear analysis, \( M \) (kNm) vertical bending moment, full-cargo, \( v=15 \text{ kn}, x/L = 0.50, \text{L&H wave } h_{1/3} = 10.285 \text{ m} \), (a) time record, (b) FFT amplitude spectrum.

Total short-term vertical displacement:

\[ Z_{\text{deck}} = Z_{\text{SW}} + w_{\text{RMS}}(m), \text{Tanker 100000tdw, full-cargo, } v=15 \text{ kn, non-linear} \]

![Figure 10](image)

Figure 10. Oil-tanker 100000 tdw, non-linear analysis, full-cargo, \( v=15 \text{ kn}, \text{L&H wave } h_{1/3} = 0-12 \text{ m} \), total short-term vertical displacement, (a) \( Z_{\text{deck}} \) (m) deck reference, (b) \( Z_{\text{keel}} \) (m) keel line reference.
Table 3. Oil-tanker 100000 tdw, full-cargo, short-term most-probable vertical displacements \( w_{\text{RMS}}(m) \), ratios between vibrations and oscillation modes, for reference wave \( h_{i/3} = 10.285 \) m.

| Section \( x(m) \) | Analysis | \( w_{\text{M1/3}}(m) \) | \( w_{\text{M1/3+SW}}(m) \) | \%vib1/osc | \%vib2/osc | \%vib3/osc |
|------------------|---------|----------------|-----------------|----------|----------|----------|
| 0.25L 60         | Linear  | 1.872          | 5.29 \( \times 10^{-2} \) | 2.83%    | 4.15 \( \times 10^{-2} \) | 2.22%    |
| 0.50L 120        | Linear  | 1.595          | 4.91 \( \times 10^{-2} \) | 3.07%    | 3.83 \( \times 10^{-2} \) | 2.39%    |
| 0.75L 180        | Linear  | 2.577          | 7.82 \( \times 10^{-2} \) | 3.03%    | 6.08 \( \times 10^{-2} \) | 2.36%    |

Average %: 2.98% 3.03% 2.32% 1.30%

| Phenomena       | Non-Linear | Bottom slamming | Side slaming | Green sea | ref. \( h_{i/3} \) |
|-----------------|------------|-----------------|--------------|-----------|-------------------|
|                 | 0 240      | \( h_{i/3} > 4 \) m | yes | no | \( h_{i/3} > 10 \) m |

M1/3+SW = TSW + M1/3 (Nm)

Figure 11. Oil-tanker 100000 tdw, non-linear analysis, full-cargo, \( v=15 \) kn, L&H wave \( h_{i/3} = 0-12 \) m, (a) \( M_{1/3+SW}(Nm) \) total short-term bending moment, (b) \( T_{1/3+SW}(N) \) total short-term shear force.

Figure 12. Oil-tanker 100000 tdw, linear and non-linear analyses, full-cargo, \( v=15 \) kn, L&H wave \( h_{i/3} = 0-12 \) m, significant bending moment components \( M_{1/3}(Nm) \), (a) oscillation (osc), (b) first vibration mode (vib1), (c) second vibration mode (vib2), (d) third vibration mode (vib3).
4.2. The oil-tanker short-term hydroelastic response in ballast case

Similar to the full-cargo case, this section presents a selection of the relevant results for the short-term hydroelastic analysis of the oil-tanker for the ballast case (table 2).

- Figure 13 shows the vertical displacement \( w \) (m) time record and FFT amplitude spectrum, at \( x/L = 0.25 \) section, non-linear results (6), reference L&H wave \( h_{1/3} = 10.285 \text{ m} \), in ballast case.
- Figures 14 and 15 present the vertical bending moment \( M \) (kNm) time record and FFT amplitude spectrum, at \( x/L = 0.50 \) section, linear and non-linear results, L&H wave \( h_{1/3} = 10.285 \text{ m} \), ballast case.
- Figure 16 presents the total short-term vertical displacement at deck \( Z_{\text{deck}} \) (m) and keel line \( Z_{\text{keel}} \) (m) (10), non-linear results, \( x=0-240\text{m} \), L&H wave \( h_{1/3} = 0-12 \text{ m} \), ballast case.
- Figure 17 presents the vertical total short-term bending moment \( M_{1/3} \) (kNm) and shear force \( T_{1/3} \) (kNm) (11), non-linear results, \( x=0-240\text{m} \), L&H wave \( h_{1/3} = 0-12 \text{ m} \), ballast case.
- Figure 18 presents the significant vertical bending moment \( M_{1/3} \) (kNm), oscillation, vibrations modes (11), linear and non-linear results, \( x=0-240\text{m} \), L&H wave \( h_{1/3} = 0-12 \text{ m} \), ballast case.
- Table 5 presents the most-probable vertical displacement \( w_{\text{RMS}} \) (m) (9), at \( x/L = 0.25, 0.50, 0.75 \) reference sections, the ratios between first, second, third vibrations and oscillation modes, for reference Longuet-Higgins (1) wave \( h_{1/3} = 10.285 \text{ m} \), ballast case.
- Table 6 presents the significant vertical bending moments \( M_{1/3} \) (kNm) and the shear forces \( T_{1/3} \) (kNm) (11), at \( x/L = 0.25, 0.50, 0.75 \) reference sections, the ratios between first, second vibrations and oscillation modes, for reference L&H wave \( h_{1/3} = 10.285 \text{ m} \), ballast case.

Analog to the full-load case, the hydroelastic vertical displacement response (figure 13) has the oscillation modes \( f_{\text{r}1} \approx 0.109 \text{ Hz} \) dominant. Taking as reference the most-probable oscillation displacement, for vibration modes the most-probable vertical deflections are in the range of 16.8-69.8 mm \( (x/L = 0.25-0.75) \), representing 2.91+2.94% \( (f_{\text{r}1} \approx 0.81 \text{ Hz}) \), 2.33+2.37% \( (f_{\text{r}2} \approx 1.828 \text{ Hz}) \), 1.28+1.32% \( (f_{\text{r}3} \approx 2.961 \text{ Hz}) \) (table 5). The non-linear and linear most probable vertical displacements have 1.47% + 4.28% (table 5) on average differences. The bottom slamming occurs at both end parts for any wave significant height, \( h_{1/3} > 0 \text{m} \) (figure 16), due to smaller draught as for full-cargo case (table 2). Due to the ship’s flare forms at extremities (figure 1), the side slamming occurs at both ends. In the ballast case, the deck flooding (green sea) does not occur (figure 16).

| Section (m) | Analysis | \( M_{1/3} \) (kNm) | \( M_{1/3} \) (kNm) | % vib/osc | % vib/osc | % vib/osc | % vib/osc |
|-------------|----------|-----------------|-----------------|------------|------------|------------|------------|
| 0.25L 60    | Linear   | 7.41 \times 10^{-3} | 1.63 \times 10^{-3} | 22.00% | 6.75 \times 10^{-4} | 9.11% | 1.78 \times 10^{-3} | 2.40% |
| 0.5L 120    | Linear   | 1.75 \times 10^{-3} | 3.74 \times 10^{-3} | 21.37% | 5.33 \times 10^{-4} | 3.05% | 2.87 \times 10^{-3} | 1.64% |
| 0.75L 180   | Linear   | 9.99 \times 10^{-3} | 1.89 \times 10^{-3} | 18.92% | 7.34 \times 10^{-4} | 7.35% | 1.95 \times 10^{-3} | 1.95% |
| Average%:   |          | -                | 20.76%          | 6.50% | - | 2.00% |

| Section (m) | Analysis | \( T_{1/3} \) (kN) | \( T_{1/3} \) (kN) | % vib/osc | % vib/osc | % vib/osc | % vib/osc |
|-------------|----------|-----------------|-----------------|------------|------------|------------|------------|
| 0.25L 60    | Linear   | 2.14 \times 10^{-3} | 4.67 \times 10^{-3} | 21.82% | 1.0 \times 10^{-3} | 5.14% | 3.56 \times 10^{-2} | 1.66% |
| 0.5L 120    | Linear   | 1.27 \times 10^{-3} | 9.26 \times 10^{-3} | 7.29% | 2.29 \times 10^{-3} | 18.03% | 3.14 \times 10^{-2} | 2.47% |
| 0.75L 180   | Linear   | 2.50 \times 10^{-3} | 5.10 \times 10^{-3} | 20.40% | 1.02 \times 10^{-3} | 4.08% | 4.64 \times 10^{-2} | 1.86% |
| Average%:   |          | -                | 16.50%          | 9.08% | - | 2.00% |

| Phenomena | Linear springing small | Non-linear springing medium | Whipping high |
|-----------|------------------------|-----------------------------|-------------|
| Analysis  | ref. \( h_{1/3} \) | 10.285 m |

### Table 4. Oil-tanker 100000 tdw, full-cargo, short-term significant bending moments \( M_{1/3} \) (kNm) and shear forces \( T_{1/3} \) (kNm), ratios between vibrations and oscillation modes, wave \( h_{1/3} = 10.285 \text{ m} \).
Figure 13. Oil-tanker 100000 tdw, non-linear analysis, \( w \) (m) vertical displacement, ballast, \( v=15 \text{ kn} \), \( x/L=0.25 \), L&H wave \( h_{1/3}=10.285 \text{ m} \), (a) time record, (b) FFT amplitude spectrum.

Figure 14. Oil-tanker 100000 tdw, linear analysis, \( M \) (kNm) vertical bending moment, ballast, \( v=15 \text{ kn} \), \( x/L=0.50 \), L&H wave \( h_{1/3}=10.285 \text{ m} \), (a) time record, (b) FFT amplitude spectrum.

Figure 15. Oil-tanker 100000 tdw, non-linear analysis, \( M \) (kNm) vertical bending moment, ballast, \( v=15 \text{ kn} \), \( x/L=0.50 \), L&H wave \( h_{1/3}=10.285 \text{ m} \), (a) time record, (b) FFT amplitude spectrum.

Figure 16. Oil-tanker 100000 tdw, non-linear analysis, ballast, \( v=15 \text{ kn} \), L&H wave \( h_{1/3}=0-12 \text{ m} \), total short-term vertical displacement (a) \( Z_{\text{deck}} \) (m) deck reference, (b) \( Z_{\text{keel}} \) (m) keel line reference.
Table 5. Oil-tanker 100000 tdw, ballast, short-term most-probable vertical displacements \( w_{RMS}(m) \), ratios between vibrations and oscillation modes, for reference wave \( h_{1/3}=10.285 \ m \).

| Section \( x/(m) \) | Analysis | \( w_{RMS}(m) \) | \( w_{RMS}(m) \) | \%vib1/osc | \( w_{RMS}(m) \) | \%vib2/osc | \( w_{RMS}(m) \) | \%vib3/osc |
|---------------------|----------|----------------|----------------|-------------|----------------|-------------|----------------|-------------|
| 0.25L 60            | Linear   | 1.989          | 6.87 \times 10^{-2} | 3.45%       | 5.55 \times 10^{-2} | 2.79%       | 3.05 \times 10^{-2} | 1.53%       |
| 0.50L 120           | Linear   | 1.373          | 3.81 \times 10^{-2} | 2.78%       | 3.06 \times 10^{-2} | 2.23%       | 1.68 \times 10^{-2} | 1.23%       |
| 0.75L 180           | Linear   | 2.360          | 5.87 \times 10^{-2} | 2.49%       | 4.68 \times 10^{-2} | 1.98%       | 2.57 \times 10^{-2} | 1.09%       |
|                     |          |                |                | Average % : | -              | 2.91%       | -              | -           | 1.28%       |
| 0.25L 60            | Non-linear | 1.998         | 6.98 \times 10^{-2} | 3.49%       | 5.66 \times 10^{-2} | 2.83%       | 3.16 \times 10^{-2} | 1.58%       |
| 0.50L 120           | Non-linear | 1.387         | 3.96 \times 10^{-2} | 2.85%       | 3.19 \times 10^{-2} | 2.29%       | 1.77 \times 10^{-2} | 1.27%       |
| 0.75L 180           | Non-linear | 2.429         | 6.04 \times 10^{-2} | 2.49%       | 4.80 \times 10^{-2} | 1.98%       | 2.67 \times 10^{-2} | 1.10%       |
|                     |          |                |                | Average % : | (1.47%)         | (2.81%)     | (2.94%)        | (2.93%)     | 2.37%       | (4.28%)     | 1.32%       |

Figure 17. Oil-tanker 100000 tdw, non-linear analysis, ballast, \( v=15 \) kn, L&H wave \( h_{1/3}=0-12 \) m, (a) \( M_{1/3+SW}(kNm) \) total short-term bending moment, (b) \( T_{1/3+SW}(kN) \) total short-term shear force.

Figure 18. Oil-tanker 100000 tdw, linear and non-linear analyses, ballast, \( v=15 \) kn, L&H wave \( h_{1/3}=0-12 \) m, significant bending moment components \( M_{1/3}(kNm) \) (a) oscillation (osc), (b) first vibration mode (vib1), (c) second vibration mode (vib2), (d) third vibration mode (vib3).
Table 6. Oil-tanker 100000 tdw, ballast, short-term significant bending moments $M_{ib}$ (kNm) and shear forces $T_{ib}$ (kN), ratios between vibrations and oscillation modes, reference wave $h_{ref}=10.285$ m.

| Section (m)   | Analysis $M_{ib}$ (kNm) | $M_{osc}$ (kNm) | $\%vib/osc$ | $M_{ib}$ (kNm) | $M_{osc}$ (kNm) | $\%vib/osc$ |
|---------------|------------------------|----------------|-------------|----------------|----------------|-------------|
| 0.25L 60      | 6.95 $\times 10^3$     | 4.98 $\times 10^4$ | 7.17%       | 1.42 $\times 10^4$ | 20.43% | 1.52 $\times 10^4$ | 2.19% |
| 0.50L 120     | 1.81 $\times 10^6$     | 1.25 $\times 10^5$ | 6.91%       | 5.04 $\times 10^4$ | 2.78% | 2.45 $\times 10^4$ | 1.35% |
| 0.75L 180     | 1.02 $\times 10^6$     | 6.35 $\times 10^4$ | 6.23%       | 1.52 $\times 10^5$ | 14.90% | 2.00 $\times 10^4$ | 1.96% |
| Average %     | -                      | -               | 6.77%       | -               | 12.70% | -               | 1.83% |

Table 6 (continued)

| Section (m)   | Analysis $T_{ib}$ (kN) | $T_{osc}$ (kN) | $\%vib/osc$ | $T_{ib}$ (kN) | $T_{osc}$ (kN) | $\%vib/osc$ |
|---------------|------------------------|----------------|-------------|----------------|----------------|-------------|
| 0.25L 60      | 2.13 $\times 10^4$     | 1.50 $\times 10^5$ | 7.04%       | 2.48 $\times 10^4$ | 11.64% | 2.80 $\times 10^4$ | 1.31% |
| 0.50L 120     | 1.37 $\times 10^4$     | 5.50 $\times 10^3$ | 4.01%       | 5.25 $\times 10^3$ | 38.32% | 3.39 $\times 10^2$ | 2.47% |
| 0.75L 180     | 2.66 $\times 10^4$     | 1.75 $\times 10^4$ | 6.58%       | 1.55 $\times 10^3$ | 5.83% | 4.21 $\times 10^2$ | 1.58% |
| Average %     | -                      | -               | 5.88%       | -               | 18.60% | -               | 1.79% |

The FFT vertical bending moment spectra (figures 14, 15) make visible the first-order ($f_{osc}=0.810$ Hz) and second-order ($f_{osc}=1.828$ Hz) modes by linear and non-linear hydroelastic solutions (figure 18). The natural frequencies in the ballast case are higher in comparison to the full-cargo case (table 2). Thus, the hydrodynamic damping becomes effective zero (figure 3) for second-order vibration mode, the bending response becoming visible for that mode (figures 14, 15).

Analog to the full-cargo case, the oscillation vertical bending moment is the dominant ($f_{osc}=0.10%\pm0.11$ Hz) modal component (figures 14, 15). The maximum total short-term vertical bending moments, for any wave condition, are obtained at the amidships sections $x/L=0.50$ (figure 17). For the selected sections ($x/L=0.25$, 0.50, 0.75), the vibration significant vertical moment modes represent $6.77%\pm20.21$ first-order ($vib_1$), $12.70%\pm18.61$ second-order ($vib_2$), $1.83%\pm5.18%$ third-order ($vib_3$), in comparison to the reference oscillation component (tables 6, figure 18).

For this loading case, the total short-term vertical shear force (figure 17) has maximum values at aft-quarter, amidships, fore-quarter sections ($x/L=0.25$, 0.50, 0.75). The vibration significant vertical shear forces at the selected sections with maximum values represent $5.88%\pm16.94%$ first-order ($vib_1$), $18.60%\pm26.19%$ second-order ($vib_2$), $1.79%\pm6.25%$ third-order ($vib_3$), from the oscillation mode references value (table 6).

The non-linear and linear significant vertical bending moments have $0.74%$ ($osc$), $113.06%$ ($vib_1$), $67.00%$ ($vib_2$), $184.30%$ ($vib_3$) on average differences (table 6). The non-linear and linear significant vertical shear forces have $2.23%$ ($osc$), $187.64%$ ($vib_1$), $56.13%$ ($vib_2$), $255.96%$ ($vib_3$) on average differences (table 6). Thus, the non-linear hydrodynamic solutions lead to a significant increase for the vibration moments on the bending moments (figure 18) and shear forces on vibration modes. Analog to the full-cargo case, the linear springing is small, the non-linear springing is medium (tables 5, 6). Because the impact bottom and side slamming can occur in any wave condition (figure 16), with hydrodynamic damping considered zero for frequencies over 1 Hz (figure 3), the transitory whipping is higher (figure 18).

The total short-term bending moments and shear forces in the full-cargo case are higher than in the ballast case (figures 11, 17) so that the first loading case (table 2) is the extreme case in this study for the assessment of the 100000 oil-tanker tdw strength capabilities.
5. The long-term hydroelastic and fatigue analysis of the 100000 tdw oil-tanker

5.1. The oil-tanker long-term hydroelastic and fatigue analysis in full-cargo case

This section presents a selection of the relevant results for the long-term hydroelastic (13) and fatigue (15) analysis of the oil-tanker, WWT waves significant height histogram, in the full-cargo case.

- Figure 19 presents the total long-term hydroelastic deck normal-stress $\sigma_{D,R}$ (MPa) and neutral axis tangential-stress $\tau_{NN,R}$ (MPa) (14), non-linear solution, $x=0-240\text{m}$, $R\leq 25\text{years}$, $K_s=1.2$, full-cargo case.

- Figure 20 presents the trend-line for the maximum total long-term hydroelastic deck normal-stress $\sigma_{D,R}$ (MPa) and neutral axis tangential-stress $\tau_{NN,R}$ (MPa) (14), linear and non-linear solutions, $R=0+25\text{years}$, reference hot-spot stress $K_s=1.2$, full-cargo case.

- Table 7 presents the evaluation of the yielding stress limit criteria (14) with admissible values in table 2. The assessment includes the total long-term deck, bottom, neutral axis stresses (14), linear and non-linear solutions, hot-spot stress factor $K_s=1.0-1.5$, for $R=25\text{years}$ reference, full-cargo case.

- Table 8 presents the evaluation of the fatigue criteria (15, 16) and safety exploitation life $R_e$ (17) prediction. The assessment has as reference the butt-welded deck shell structural detail at section $x/L=0.50$, linear and non-linear solutions, hydro elastic and only oscillation responses, hot-spot stress factor $K_s=1.0-1.5$, welding quality standard and high quality (h.q.), full-cargo case.

The total long-term stresses (13, 14) have significant increase over the whole reference period $R=25\text{years}$, being maximum amidships at deck shell and aft-quarter at the neutral axis of side, double side, longitudinal bulkhead shells (figures 19, 20).

The yielding stress criteria evaluation (table 7) points out that critical strength level results. Thus, $K_s\geq 1.3$ for the non-linear hydroelastic solutions, $K_s\geq 1.4$ for the linear hydroelastic solutions are the hot-spot stress factor limits. For the $K_s=1.5$ factor, the yielding stress criteria (14) imposes to reduce the long-term reference period significantly under 25 years for the full-cargo case.

Figure 19. Oil-tanker 100000 tdw, non-linear analysis, full cargo, $v=15\text{ kn}$, WWT wave histogram, $K_s=1.2$, long-term (a) $\sigma_{D,R}$ (MPa) deck normal-stress, (b) $\tau_{NN,R}$ (MPa) neutral axis tangential-stress.

Figure 20. Oil-tanker 100000 tdw, linear and non-linear analyses, full cargo, $v=15\text{ kn}$, WWT wave histogram, $K_s=1.2$, long-term maximum stress (a) $\sigma_{D,R}$ (MPa) deck, (b) $\tau_{NN,R}$ (MPa) neutral axis.
The fatigue analysis takes as reference a butt-welded structural detail from the deck shell, amidships at x/L=0.5, where the long-term stresses (figure 19) are maximum.

For the non-linear hydroelastic response, by the fatigue strength criteria (15, 16, 17), the exploitation life has restrictions for \( K_s \geq 1.1, R_e = 3.9 \) years (\( K_r = 1.5 \)), standard welding quality, and improving for high-quality welding \( K_s \geq 1.3, R_e = 10.2 \) years (\( K_r = 1.5 \)) (table 8). Also, for the linear hydroelastic response for \( K_s \geq 1.2, R_e = 6.7 \) years (\( K_r = 1.5 \)) restrictions result.

For the non-linear oscillation response, by the fatigue strength criteria (15, 16, 17), the exploitation life has restrictions for \( K_s \geq 1.2, R_e = 6.4 \) years (\( K_r = 1.5 \)), standard welding quality, and improving for high-quality welding \( K_s \geq 1.4, R_e = 15.4 \) years (\( K_r = 1.5 \)) (table 8). Also, for the linear oscillation response for \( K_s \geq 1.2, R_e = 7.7 \) years (\( K_r = 1.5 \)) restrictions result. Thus, the fatigue analysis based on hydroelastic response leads to higher restraints in comparison to the fatigue analysis based on oscillation response.

For standard welding quality, taking as reference the non-linear hydroelastic response, the hot-spot stress must be lower than \( K_s \leq 1.4 \) so that the exploitation life \( R_e = 5.6 \geq 55 \) years is over the regular inspection period [12]. A selection of high-quality welding for the full-cargo case is increasing the exploitation life up to twice of the inspection period \( R_e = 10.2 \geq 10 \) years (\( K_r = 1.5 \)).
5.2. The oil-tanker long-term hydroelastic and fatigue analysis in ballast case
This section presents a selection of the relevant results for the long-term hydroelastic (13) and fatigue (15) analysis of the oil-tanker, WWT waves significant height histogram, in the ballast case.

- Figure 21 presents the total long-term hydroelastic deck normal-stress $\sigma_{D,R}$ (MPa) and neutral axis tangential-stress $\tau_{zNN,R}$ (MPa) (14), non-linear solution, stress hot-spot factor $K_s=1.2$, ballast case.

- Figure 22 presents the trend-line for the maximum total long-term hydroelastic deck normal-stress $\sigma_{D,R}$ (MPa) and neutral axis tangential-stress $\tau_{zNN,R}$ (MPa) (14), $K_s=1.2$, ballast case.

- Table 9 presents the evaluation of the yielding stress limit criteria (14) with admissible values in table 2 and total long-term deck, bottom, neutral axis stresses (14), ballast case.

- Table 10 presents the evaluation of the fatigue criteria (15, 16) and safety exploitation life $R_e$ (17) prediction for butt-welded deck shell structural detail at section $x/L=0.50$ reference, ballast case.

Analog to the full-cargo case, the total long-term stress is maximum at deck amidships (figure 21).

The total long-term transversal stress is maximum at the aft and fore-quarter neutral axis (figure 21).

The influence of the reference period for $R=10$ years is significant on the total long-term stresses (13, 14), continuing to increase the stress values up to $R=25$ years (figure 22).

In the ballast case, the yielding stress criteria (table 9) leads to no restrictions of $R=25$ years for any stress hot-spot factor $K_s=1.0-1.5$, due to lower total long-term stresses (figures 21, 22) in comparison to the full-cargo case (figures 19, 20).

Analog to the full-cargo case, for the ballast case a deck shell butt-welded detail, at $x/L=0.5$ with maximum long-term stress (figure 21), is the reference structure for the fatigue analysis.

In the ballast case, for the non-linear hydroelastic response, by the fatigue strength criteria (15, 16, 17), the exploitation life has restrictions for $K_s\geq1.3$, $R_e=12.5$ years ($K_s=1.5$), standard welding quality. For high-quality (h.q.) welding no restrictions $R_e>25$ years ($K_s=1.5$) (table 10) result. Also, it results in restrictions for the linear hydroelastic response for $K_s\geq1.3$, $R_e=13.5$ years ($K_s=1.5$).

Figure 21. Oil-tanker 100000 tdw, non-linear analysis, ballast, $v=15$ kn, WWT wave histogram, $K_s=1.2$, long-term (a) $\sigma_{D,R}$ (MPa) deck normal-stress, (b) $\tau_{zNN,R}$ (MPa) neutral axis tangential-stress.

Figure 22. Oil-tanker 100000 tdw, linear and non-linear analyses, ballast, $v=15$ kn, WWT wave histogram, $K_s=1.2$, long-term maximum stress (a) $\sigma_{D,R}$ (MPa) deck, (b) $\tau_{zNN,R}$ (MPa) neutral axis.
In ballast cases, for the non-linear oscillation response, by the fatigue strength criteria (15, 16, 17), the exploitation life has restrictions for $K_{1.1}=13.6$ years ($K_{1.5}=1.5$), standard welding quality, for high-quality (h.q.) welding no restrictions $R_{E}=25$ years ($K_{1.5}=1.5$) (table 10) occur. For the linear oscillation response restrictions for $K_{1.1}=13.6$, $R_{E}=25$ years ($K_{1.5}=1.5$) occur.

The exploitation life is more than twice of the inspection period $R_{E}=12.5>10$ years ($K_{1.5}=1.5$) for standard welding quality and without restrictions $R_{E}=25$ years for high-quality welding condition, taking as reference the non-linear hydroelastic response.

**Table 9.** Oil-tanker 100000 tdw, ballast, total long-term maximum deck, bottom, neutral axis stresses (MPa), WWT wave histogram, linear and non-linear solutions, for $R_{E}=25$ years reference.

| Panel selection Stress (MPa) (14) | DYN | adm | $K_{1.0}$ | $S/adm$ | $R(years)$ | $K_{1.1}$ | $S/adm$ | $R(years)$ | $K_{1.2}$ | $S/adm$ | $R(years)$ |
|----------------------------------|-----|-----|-----------|---------|------------|-----------|---------|------------|-----------|---------|------------|
| $\sigma_{D,L}$ deck              | Linear | 263 | 158.88    | 0.605   | $>25$      | 174.77    | 0.665   | $>25$      | 190.66    | 0.725   | $>25$      |
| $\sigma_{D,B}$ bottom            | 205  | 125.62 | 0.613  | $>25$      | 138.18    | 0.674   | $>25$      | 150.74    | 0.735   | $>25$      |
| $\tau_{N,N,L}$ n-n axis         | 120  | 74.22 | 0.619   | $>25$      | 81.65     | 0.680   | $>25$      | 89.07     | 0.742   | $>25$      |
| $\sigma_{D,L}$ deck              | Non-linear | 263 | 160.99 | 0.613 | $>25$ | 177.09 | 0.674 | $>25$ | 193.19 | 0.735 | $>25$ |
| $\sigma_{D,B}$ bottom            | 205  | 127.28 | 0.621 | $>25$ | 140.01 | 0.683 | $>25$ | 152.74 | 0.745 | $>25$ |
| $\tau_{N,N,L}$ n-n axis         | 120  | 77.12 | 0.643 | $>25$ | 84.83 | 0.707 | $>25$ | 92.55 | 0.771 | $>25$ |

**Table 10.** Oil-tanker 100000 tdw, ballast, long-term fatigue analysis, WWT wave histogram, linear and non-linear analysis, oscillations and vibrations, $K_{1}=1.0-1.5$, reference section $x/L=0.50$, deck shell.

| Model type | Welding quality | $D_{oc}$ | $D_{ocb}$ | $D_{ocb2}$ | $D_{ocb3}$ | $D_{hel}$ | $R_{Eosc}$/R_{Enc} (year/year) | $D_{oc}$ | $D_{ocb}$ | $D_{ocb2}$ | $D_{ocb3}$ | $D_{hel}$ | $R_{Eosc}$/R_{Enc} (year/year) |
|------------|----------------|---------|-----------|-----------|-----------|---------|-------------------------------|---------|---------|-----------|-----------|---------|-------------------------------|
| Linear     | standard       | 3.0.10^{-3} | 3.0.10^{-3} | 1.2.10^{-5} | 5.4.10^{-6} | 0.301 | $>25$ | 4.8.10^{-9} | 5.1.10^{-9} | 2.0.10^{-9} | 9.1.10^{-10} | 0.476 | $>25$ |
| Non-linear | standard       | 3.2.10^{-4} | 3.8.10^{-4} | 3.8.10^{-4} | 6.10^{-5} | 5.9.10^{-6} | $>25$ | 5.10^{-9} | 9.0.10^{-9} | 6.4.10^{-9} | 9.8.10^{-10} | 0.509 | $>25$ |
| Linear     | h.q.           | 1.1.10^{-1} | 1.8.10^{-1} | 1.3.10^{-1} | 1.9.10^{-1} | 0.108 | $>25$ | 1.8.10^{-3} | 3.0.10^{-3} | 2.1.10^{-3} | 3.2.10^{-3} | 0.179 | $>25$ |
| Linear     | standard       | 7.1.10^{-8} | 8.1.10^{-8} | 3.2.10^{-8} | 1.4.10^{-8} | 0.715 | $>25$ | 1.0.10^{-3} | 4.9.10^{-4} | 2.2.10^{-4} | 1.011 | 24.720.0 |
| Non-linear | standard       | 7.5.10^{-9} | 1.4.10^{-8} | 1.0.10^{-7} | 1.6.10^{-8} | 0.766 | $>25$ | 1.1.10^{-2} | 1.6.10^{-4} | 2.4.10^{-4} | 1.086 | 23.0 23.5 |
| Linear     | h.q.           | 2.8.10^{-4} | 4.7.10^{-4} | 3.3.10^{-4} | 5.1.10^{-4} | 0.281 | $>25$ | 4.1.10^{-1} | 7.3.10^{-5} | 5.1.10^{-9} | 7.9.10^{-7} | 0.419 | $>25$ |

5.3. The oil-tanker long-term fatigue analysis in combined full-cargo and ballast cases

For a realistic evaluation of the 100000 tdw oil-tanker safety exploitation life (17), the results from fatigue analysis in full-cargo (table 8) and ballast (table 10) must be combined, considering equal occurrence probability of both loading cases.

Table 11 presents the evaluation of the fatigue criteria (15, 16) and safety exploitation life $R_{E}$ (17) prediction for buttwelded deck shell structural detail, at section $x/L=0.50$ reference, for the combined full-cargo and ballast cases (17).
The long-term fatigue analysis for combined full-cargo and ballast cases (table 11) leads to:
- for linear hydroelastic response, standard welding quality, restrictions for $K_s=1.3$, $R_E=9.0$ years = 1.8·$T_R$ ($K_x=1.5$), over 80% to the technical inspection period $T_w=5$ years [12];
- for linear oscillation response, standard welding quality, restrictions for $K_s=1.3$, $R_E=9.8$ years = 1.96·$T_R$ ($K_x=1.5$), almost twice the technical inspection period;
- for non-linear hydroelastic response, standard welding quality, restrictions for $K_s=1.2$, $R_E=5.9$ years = 1.18·$T_R$ ($K_x=1.5$), close to the technical inspection period;
- for non-linear oscillation response, standard welding quality, restrictions for $K_s=1.2$, $R_E=8.6$ years = 1.72·$T_R$ ($K_x=1.5$), over 72% to the technical inspection period;
- for non-linear hydroelastic response, high-quality welding, restrictions for $K_s=1.4$, $R_E=15.2$ years = 3.04·$T_R$ ($K_x=1.5$), three times the technical inspection period;
- for non-linear oscillation response, high-quality welding, restrictions for $K_s=1.5$, $R_E=20.5$ years = 4.10·$T_R$ ($K_x=1.5$), four times the technical inspection period.

The fatigue results are pointing out that the hydroelastic response is delivering more restrictions in comparison to the oscillations response. For the long-term ship’s behavior improvement, the best solution is the consideration of the high-quality welding process solution, so that the oil-tanker safety exploitation life is around $R_E=15.2$ years $\approx 3$·$T_R$ in the extreme case of stress hot-spot factor $K_x=1.5$.

### 6. Conclusions

For ships with elastic hull girder, the long-term safe exploitation life evaluation requires an advanced hydroelastic approach with coupled oscillations and vibrations formulation. This study involves the own hydroelastic code, validated by towering tank experimental tests [11]. For best accuracy, the hydroelastic approach includes the Longuet-Higgins model wave excitation, short-term and long-term hydroelastic response, with linear and non-linear solutions, enhanced by fatigue analysis (section 2).

The hydroelastic analysis includes hydrodynamic phenomena as bottom slamming, side slamming, steady springing and transitory whipping [10]. Thus, it can generate non-linear vibration components up to 5 Hz that requires a significant amount of computational time.

This study focuses on a 100000 tdw oil-tanker with 240 m length and first-order natural frequency lower than 1 Hz (section 3). Thus, the hull girder has much sensitivity on global vibration modes, which qualifies this ship for a long-term hydroelastic assessment.

The short-term hydroelastic analysis (section 4) points out that the oscillation component represents the main response component, with small changes from the linear to the non-linear solutions. The global vibration components record a significant increase in the non-linear solution in comparison to the linear solutions. For the sectional efforts in the full-cargo case, the non-linear first-order vibration component becomes significantly higher in comparison to the other two vibration modes. In the ballast case, the non-linear sectional efforts of second and first-order vibration components become significant due to higher natural vibration frequencies in comparison to the full-cargo loading case that practically leads to zero hydrodynamic damping in the vibration domain. Including the still water

### Table 11. Oil-tanker 100000 tdw, combined full-cargo and ballast cases, long-term fatigue analysis, WWT wave histogram, linear and non-linear analysis, $K_s=1.0-1.5$, reference section $x/L=0.50$, deck shell.

| Model type | Welding quality | $D_{osc}$ | $D_{hel}$ | $R_{hel}$ (year) | $K_{osc}$ (year) | $R_{osc}$ (year) | $D_{osc}$ | $D_{hel}$ | $R_{hel}$ (year) | $K_{osc}$ (year) | $R_{osc}$ (year) |
|------------|-----------------|-----------|-----------|-----------------|-----------------|-----------------|-----------|-----------|-----------------|-----------------|-----------------|
| Hot-spot stress factor | $K_x=1.0$ | $K_x=1.1$ | $K_x=1.2$ |                 |                 |                 |           |           |                 |                 |                 |
| Linear     | standard        | 0.378     | 0.397     | $>25$           | $>25$           | 0.610           | $>25$     | 0.925     | $>25$           | 0.983           | $>25$           |
| Non-linear | standard        | 0.414     | 0.527     | $>25$           | $>25$           | 0.672           | $>25$     | 1.030     | $>25$           | 1.360           | $>25$           |
| linear     | h.q.            | 0.143     | 0.180     | $>25$           | $>25$           | 0.241           | $>25$     | 0.385     | $>25$           | 0.493           | $>25$           |
| Hot-spot stress factor | $K_x=1.3$ | $K_x=1.4$ | $K_x=1.5$ |                 |                 |                 |           |           |                 |                 |                 |
| Linear     | standard        | 1.341     | 1.434     | **17.4**        | **18.6**        | 1.232           | **20.2**  | 2.555     | **28.7**       | 2.781           | **33.4**       |
| Non-linear | standard        | 1.507     | 2.041     | **12.3**        | **16.6**        | 2.139           | **17.7**  | 2.979     | **20.5**       | 5.943           | **31.0**       |
| linear     | h.q.            | 0.587     | 0.761     | $>25$           | $>25$           | 0.856           | $>25$     | 1.218     | $>25$           | 1.641           | **20.5**       |

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sectional efforts, the total short-term hydroelastic bending moments and shear forces result higher for the full-cargo case in comparison to the ballast case. The bottom and side slamming occur for mid-sea state conditions in the full-cargo case and any sea state in the ballast case. The deck flooding (green sea) occurs only in the full-cargo case, at the forepeak, for extreme sea state conditions. For both cases, the linear springing is small; the non-linear springing becomes medium due to the geometric non-linearities. The transitory whipping is high for both loading cases due to much reduced hydrodynamic and structural damping on vibration modes.

The long-term hydroelastic stresses (section 5) have a significant increase function to the reference period. In the full-cargo case, the stresses hot-spot larger than 1.3 lead to restrictions by the yielding stress criteria, without restrictions in the ballast case. For both cases, the total long-term deck stress is maximum at amidships over the cargo-hold compartments. Thus, the selected reference structure for fatigue analysis is a butt-welded deck shell structural detail.

The long-term fatigue analysis (section 5) points out that the vibration components have a negative effect on the fatigue strength, reducing the expected exploitation life. More restrictions occur in the full-cargo case in comparison to the ballast case, due to higher stress values. More differences result between the linear and non-linear solutions, the stresses hot-spot values having a negative influence. For combined loading cases, stress hot-spot 1.5, non-linear solutions, the safe exploitation life is 5.9 years (standard welding) and 15.2 years (high-quality welding) (table 11), covering at least one inspection time reference (5 years).

In conclusion, the long-term hydroelastic analysis of the 100000 tdw oil-tanker points out that the best accuracy results by the non-linear hydroelastic solutions. For the increase of the predicted long-term safe exploitation life, the high-quality welding process is the best solution. Also, by design, the structural details stress hot-spot should reduce as much as possible.

Further studies shall continue to study the influence of the oil-tanker speed and other loading cases on the long-term hydroelastic response. An extended analysis of the stress hot-spot factors for the oil-tanker shall continue by 3D structural models. The sensitivity assessment of the DYN code for hydroelastic analysis shall continue by other experimental tests on new ship types.

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