Gauge factor enhancement driven by heterogeneity in thick-film resistors

C. Grimaldi, P. Ryser, and S. Strässler
Département de Microtechnique, IPM, École Polytechnique Fédérale de Lausanne, CH-1015 Lausanne, Switzerland.

We present a simple picture of the gauge factor (GF) enhancement in highly heterogeneous materials such as thick-film resistors. We show that when the conducting phase is stiffer than the insulating one, the local strains within this latter are enhanced with respect to the averaged macroscopic strain. Within a simple model of electron tunneling processes, we show that the enhanced local strain leads to values of GF higher than those expected for a homogeneous system. Moreover we provide formulas relating the enhancement of GF to the elastic and microstructural characteristics of TFRs.

PACS number(s) 72.20.Fr, 72.80.Tm, 62.20.Dc

I. INTRODUCTION

Thick-film resistors (TFRs) are successfully used as piezoresistive sensors due to their high strain-sensitivity resistance. Their gauge factors (GFs), conventionally defined as \( GF = \frac{\delta R}{R \epsilon} \), where \( \delta R \) is the change of resistance \( R \) for an applied strain \( \epsilon \), range in fact from GF \( \sim 2 \) up to GF \( \sim 35 \) at room temperature. Of course, higher values of GF lead to a better piezoresistive response of a TFR, so that the knowledge of the factors which enhance GF is of primary importance in improving piezoresistive sensor performances.

Among these factors, the microstructure of TFRs plays certainly an important role in enhancing GF. A typical TFR is a composite material in which metallic grains are embedded in an insulating glassy matrix, and a positive correlation between the mean metallic grain size and GF has been established already since some time. Fabrication processes such as the peak firing temperature \( T_f \) modify the compositional and microstructure properties leading to important and complex modifications of GF. Typically, GF initially increases with \( T_f \) and after having reached a maximum, it shows a more or less pronounced decrease. Other important intrinsic factors affecting GF are the chemical compositions of both the metallic and insulating phases. Finally, in addition to these effects, there exists an empirical relationship between (sheet) resistance \( R \) and GF:

\[
GF \propto \ln R, \quad (1)
\]

which has been established for room temperatures. According to this relation therefore the most resistive TFRs show also the higher GFs.

Although these phenomenological properties are useful in improving the strain sensitivity in TFRs, the microscopic physical mechanisms enhancing GF are unclear, and the quest for high GF values rests only on empirical basis. An even more annoying aspect of this situation is that simple tunneling-like formulas for \( R \) are completely inadequate to explain the observed GF values. For example, at sufficiently high temperatures, transport is thought to be governed by tunneling between adjacent metallic grains separated by a mean distance \( d \).

\[
R \simeq R_0 \exp(2d/\xi), \quad (2)
\]

where \( \xi = h/\sqrt{2mV} \) is the localization length and \( m \) and \( V \) are the electron mass and the tunneling barrier potential, respectively. The prefactor in Eq.(2) is \( R_0 = \Phi R_H \) where \( R_H = 2\pi h/e^2 \simeq 26 \, \text{KΩ} \) is the Hall resistance and \( \Phi \) is a dimensionless geometrical factor. Assuming homogeneity, the main dependence on an applied strain \( \epsilon \) comes from the exponent of Eq.(2) through \( d \rightarrow d(1 + \epsilon) \), and the gauge factor reduces to:

\[
GF \equiv \frac{\delta R}{\epsilon R} \simeq 2d/\xi. \quad (3)
\]

Equations (2) and (3) are fully consistent with the empirical relation in Eq.(1), and \( R \) can be rewritten as \( R \simeq R_0 \exp(GF) \). However, typical values of the sheet resistance are of order \( 10 – 1000 \, \text{KΩ} \), while for example for GF = 20 we get \( R \simeq R_0 \exp 20 \sim 10^{10} \, \text{KΩ} \! \). This astronomically large value of \( R \) obtained by the simple tunneling argument gives an idea of the problem concerning the physical mechanism of GF in TFRs.

In this paper we present a theory of GF in TFRs which, on one hand, reconciles the observed values of GF with those of sheet resistances and, on the other hand, clarifies the key elements responsible for high gauge factor values. Basically, we claim that the high GF values are due to pronounced compositional heterogeneity of TFRs, so that the local strain \( \epsilon_{\text{loc}} \) can be substantially different from the applied macroscopic strain \( \epsilon \). In this situation, the mean inter-grain distance is modified by the local strain, \( d \rightarrow d(1 + \epsilon_{\text{loc}}) \), so that Eq.(1) acquires a prefactor:

\[
GF \simeq (2d/\xi) \frac{\epsilon_{\text{loc}}}{\epsilon}. \quad (4)
\]

The value of \( \epsilon_{\text{loc}}/\epsilon \) depends on the elastic properties of the metallic and glassy phases as well as on the mean
grain size and inter-grain distance. We show that when the insulating phase is less stiff than the conducting one, $\epsilon_{loc}$ can be substantially larger than $\epsilon$ leading therefore to an enhancement of GF. In the next section we provide analytical formulas relating $\epsilon_{loc}/\epsilon$ to the compositional and microstructural characteristics of TFRs by using a simplified model of electron hopping and strain distribution.

II. ORIGIN AND BASIC EFFECTS OF STRAIN HETEROGENEITY

Heterogeneity is an intrinsic feature of TFRs. As already stated in the introduction, TFRs are two-phases materials where metallic particles of sizes ranging between $\sim 200$ and $\sim 6000$ Å are embedded into an insulating glassy material. Depending on the fabrication processes, a certain amount of conducting material is dispersed in the glass and recent x-ray investigations have revealed that much smaller metallic particles (with sizes of order $\sim 20-70$ Å) are actually dispersed in the glass. At the present, it is not clear whether the electrons tunnel mainly between the large or the small particles, or if possible impurity states in the glass play a positive role in the observed conductivities. Several models of transport in TFRs have been proposed, having however the tunneling process as the common element. Hence, in the forthcoming discussion we assume Eq. (2) to capture the essential physics at least at high temperatures, and interpret $d$ as being a typical tunneling distance ignoring for the moment whether it refers to the large or the small grain mean separation.

An important cause of heterogeneity is the chemical composition of both the metallic and insulating phases. The empirical search for high GF values has selected some transition-metal oxides containing heavy elements (RuO$_2$, IrO$_2$, Bi$_2$Ru$_2$O$_7$ are typical examples) to be the metallic constituents which present the best TFR performances. An interesting aspect of this selection is that such metal-oxides are hard materials characterized by high values of bulk moduli $B$. For example RuO$_2$, which has $B \approx 270$ GPa is among the stiffest materials and IrO$_2$ should have comparable values of $B$. Conversely, the glassy phase has $B$ values typically of order 40-80 GPa depending on composition. Hence, the metallic phase is much stiffer than the insulating one so that there is a pronounced microscopical heterogeneity in the elastic properties of TFRs.

A straightforward consequence of this elastic heterogeneity is that TFRs under an applied macroscopic strain $\epsilon$ should develop highly variable local strain fields. In this case, Eq. (3) is inadequate because it has been obtained by assuming that the microscopic tunneling process, Eq. (6), is affected only by the macroscopic strain. Hence, a correct treatment of the GF problem in TFRs would require the knowledge of the local strains developed in a disordered two-phases material with different elastic properties. This is a quite difficult task which could be attacked only by extensive numerical calculations. However this is outside the scope of this paper, instead our aim here is to show that when a system is composed by interconnecting stiff conducting and soft insulating phases, the local strain within this latter is enhanced compared to the averaged one, and the tunneling process gets a higher strain sensitivity.

To illustrate in the simplest way the local strain enhancement effect, we consider a one-dimensional model defined as a periodic arrangement of $N$ conducting segments each of length $s$ separated each other by $N$ insulating segments of length $d$, so that the total length of the one-dimensional array is $L = N(s + d)$. The stress-strain relation is simply $\sigma(x) = 2\mu(x)\epsilon(x)$, where $\sigma(x)$ and $\epsilon(x)$ are the stress and strain functions which depend on the position $x$ along the array. In the two-phases model here considered, the elastic function $\mu(x)$ assumes the values $\mu_0$ or $\mu_1$ depending whether $x$ lies within the insulating or the conducting segments. The equilibrium strain distribution is governed by the equation $d\sigma(x)/dx = 0$ from which it is found that $\epsilon(x)$ is constant within each segment assuming the value $\epsilon(x) = \epsilon^0$ ($\epsilon(x) = \epsilon^1$) for $x$ within the insulating (conducting) segments, where $\mu_0\epsilon^0 = \mu_1\epsilon^1$.

The local values $\epsilon^0$ and $\epsilon^1$ are related to the macroscopic strain $\epsilon$ by the following relation:

$$\epsilon = \frac{1}{L} \int_0^L dx \epsilon(x) = \frac{d\epsilon^0 + s\epsilon^1}{d + s},$$

from which we find that the strain within the insulating segments is

$$\epsilon^0 = \epsilon \frac{1 + s/d}{1 + (s/d)(\mu_0/\mu_1)},$$

while within the conducting segments is

$$\epsilon^1 = \epsilon \frac{1 + s/d}{1 + (s/d)(\mu_0/\mu_1)} \mu_0/\mu_1.$$

Equations (5) and (6) merely illustrate that, if $\mu_0 \neq \mu_1$, the strain is concentrated more within the softer phase. For example, in the limiting case where the conducting segments are perfectly rigid ($\mu_1 \rightarrow \infty$) we find $\epsilon^1 = 0$ and $\epsilon^0 = \epsilon(1 + s/d)$.

To investigate the effect of this heterogeneous strain distribution on transport, we model the electron tunneling between two adjacent conducting segments with a tunneling probability proportional to $\exp(2d/\xi)$, so that, if we neglect the contribution from the metallic segments, the total resistance reduces to:

$$R = NR_0 \exp(2d/\xi).$$

An applied macroscopic strain $\epsilon$ modifies the tunneling distance $d$ through the local strain, $d \rightarrow d(1 + \epsilon^0)$, so that the gauge factor $GF = \delta R/\epsilon R$ is
The above expression clearly illustrate the enhancement effect of heterogeneity driven by both geometrical, \( s/d \), and elastic, \( \mu_0/\mu_1 \), parameters. In fact, for metallic phases stiffer than the insulating one, \( \mu_1 > \mu_0 \), GF is always larger than 2\( d/\xi \), reaching the limiting value

\[
GF = (2d/\xi) \frac{\mu_1}{\mu_0},
\]  

(11)

in case of tunneling between large conducting phases \( (s/d \gg 1) \). Another limiting case of Eq. (10) is given for perfectly rigid conducting elements \( (\mu_1/\mu_0 \to \infty) \) which leads to

\[
GF = (2d/\xi)(1 + s/d),
\]  

(12)

so that the enhancement factor grows linearly with \( s/d \).

The simplicity of Eq. (11) stems from the assumption of one-dimensionality for which only one component of the stress and strain functions is needed. The analysis of a more realistic three dimensional case is of course much more complex. In fact, in addition to the complete solution of the strain field, one should also calculate the actual current path which is given by hopping processes along directions also different from the one where the field is applied. However, as we show in the following section, some approximated analytical results can also be obtained for the three-dimensional case by employing simplified models for the microstructure of TFRs.

III. THREE-DIMENSIONAL CASE

To analyze the piezoresistive response of a TFR let us consider a cantilever beam in the \( x-y \) plane with its main axis lying on the \( x \) direction. On top of the cantilever is deposited the TFR with its thickness measured in the \( z \) direction. A bending of the cantilever beam produces a strain \( \epsilon \) in the \( x \) direction and, if the thickness of the cantilever is sufficiently small, no strain in \( y \). If \( \nu \) is the Poisson ratio of the cantilever, the strain along the \( z \) direction is \( -\epsilon \nu/(1-\nu) \). The complete transfer of these strain values to the TFR leads to:

\[
\bar{\epsilon}_{xx} = \epsilon, \quad \bar{\epsilon}_{yy} = 0, \quad \bar{\epsilon}_{zz} = -\epsilon \nu/(1-\nu),
\]  

(13)

where \( \bar{\epsilon}_{xx}, \bar{\epsilon}_{yy}, \) and \( \bar{\epsilon}_{zz} \) are the macroscopic strain components within the TFR along the \( x, y, \) and \( z \) directions. Within this setup, the piezoresistive response is characterized by two distinct gauge factors: the longitudinal gauge factor, \( GF_L \), obtained when the potential difference is applied along the \( x \) direction, and the transversal gauge factor, \( GF_T \), when the field is applied along the \( y \) direction. It is important to distinguish between intrinsic and geometric contributions to the piezoresistive response. In fact, from quite general arguments, it can be shown that, under the strain field of Eq. (13), \( GF_L \) and \( GF_T \) satisfy the following relation:

\[
GF_L - GF_T = 2 + GF_L^{intr} - GF_T^{intr},
\]  

(14)

where the factor 2 stems from the geometrical deformation of the TFR, while \( GF_L^{intr} \) and \( GF_T^{intr} \) are intrinsic gauge factors which are governed by microscopic transport properties and are equal for isotropic systems. It is however experimentally observed that \( GF_L - GF_T > 2 \), suggesting that the piezoresistive response of TFRs is affected by a certain amount of anisotropy.

To describe in a simple way the complex pattern of the current path and the local strain distribution, we introduce a model in which a cubic lattice of conducting spheres of diameter \( s \) with near-neighbour distance \( d \) is immersed in the insulating medium (Fig. 1). Moreover, the two phases have different isotropic elastic properties. As already assumed in the previous section, we disregard the contribution to transport coming from the conducting phase and assume for the moment direct tunneling between two neighbouring spheres. Within this arrangement of the conducting/insulating phases, we model the geometry of the current path and the anisotropy by assuming that, in average, charge transport can be described by \( N_{||} \) tunneling processes parallel and \( N_{\perp} \) tunneling processes orthogonal to the direction of the voltage drop \( \bar{V} \). Therefore, if the electric field is applied along the \( x \) direction, the resulting tunneling resistance is

\[
R_x = N_{||} R_x + N_{\perp} (R_y + R_z)/2,
\]  

where \( R_x, R_y, \) and \( R_z \) are resistances for tunneling along the \( x, y, \) and \( z \) directions, respectively. We make the further approximation that \( R_x, R_y, \) and \( R_z \) have the same average value. When the macroscopic strain field of Eq. (13) is applied, the different resistances correspond to the corresponding local strains:

\[
GF_x = (2d/\xi) \frac{\mu_0}{\mu_1},
\]  

(9)
where $\epsilon_{xx}^0 = \delta_x d/d$, $\epsilon_{yy}^0 = \delta_y d/d$ and $\epsilon_{zz}^0 = \delta_z d/d$. According to the previous definition and to Eq. (15), the longitudinal gauge factor reduces to:

$$\frac{\delta R_x}{R_x} = \frac{2d}{\xi} \epsilon_{xx}^0, \quad \frac{\delta R_y}{R_y} = \frac{2d}{\xi} \epsilon_{yy}^0, \quad \frac{\delta R_z}{R_z} = \frac{2d}{\xi} \epsilon_{zz}^0,$$  

where $\epsilon_{xx}^0 = \delta_x d/d$, $\epsilon_{yy}^0 = \delta_y d/d$ and $\epsilon_{zz}^0 = \delta_z d/d$. According to the previous definition and to Eq. (15), the longitudinal gauge factor reduces to:

$$\text{GF}_{\text{intr}}^L = \frac{\delta R_x}{\epsilon_{xx} R_x} = \frac{N \delta R_x + N_\perp (\delta R_y + \delta R_z)}{\epsilon_{xx}^0} \left( \frac{N}{R_x + N_\perp (R_x + R_y)/2} \right)$$

$$\simeq \frac{2d}{\xi} \left( 1 - \chi \right) \frac{\epsilon_{yy}^0}{\epsilon_{xx}^0} + \chi \left( \frac{\epsilon_{yy}^0}{\epsilon_{xx}^0} + \frac{\epsilon_{zz}^0}{\epsilon_{xx}^0} \right),$$

(16)

where $\chi = N_\perp/(N_\parallel + N_\perp)$. By following the same lines, the intrinsic transversal gauge factor $\text{GF}_{\text{intr}}^T$ is estimated by using the resistance for a field applied along the $y$ direction: $\delta R_y = N \delta R_y + N_\perp (R_x + R_z)/2$. Hence:

$$\text{GF}_{\text{intr}}^T = \frac{\delta R_y}{\epsilon_{xx} R_y} = \frac{N \delta R_y + N_\perp (\delta R_x + \delta R_z)}{\epsilon_{xx}^0} \left( \frac{N}{R_y + N_\perp (R_x + R_z)/2} \right)$$

$$\simeq \frac{2d}{\xi} \left( 1 - \chi \right) \frac{\epsilon_{yy}^0}{\epsilon_{xx}^0} + \chi \left( \frac{\epsilon_{yy}^0}{\epsilon_{xx}^0} + \frac{\epsilon_{zz}^0}{\epsilon_{xx}^0} \right).$$

(17)

Note that piezoresistive anisotropy, $\text{GF}_{\text{intr}}^L > \text{GF}_{\text{intr}}^T$, is obtained for $\chi < 2/3$.

At this point, the piezoresistive response can be calculated once the values of the local strains are known. However, also for the simplified model of spheres on a lattice, the general solution of the elastic problem is difficult. On the other hand, analytical results can be obtained by employing some approximations aimed to simplify as much as possible the elastic behavior of such a model. In the following we present two different approximation schemes.

A. Rigid spheres

In our model of transport described above, the current path is made of tunneling processes between the shortest distances ($d$) between two neighboring spheres. Hence, the piezoresistive response is governed by variations of $d$ along the $x$, $y$ and $z$ directions due to the applied macroscopic strains of Eq. (19). These modified inter-sphere distances are easily found in the limiting case of perfectly rigid spheres since in this case there is no deformation of the conducting phase ($\delta_z s = \delta_y s = \delta_z s = 0$). In fact, if $L_x$, $L_y$ and $L_z$ are the linear dimensions of the TFR, we have

$$\tilde{\epsilon}_{xx} = \frac{\delta L_x}{L_x} = \frac{\delta_d d}{s + d},$$

(18)

and

$$\tilde{\epsilon}_{yy} = \frac{\delta d y}{d y}, \quad \tilde{\epsilon}_{zz} = \frac{\delta d z}{d z},$$

and the corresponding expressions for $\tilde{\epsilon}_{yy}$ and $\tilde{\epsilon}_{zz}$. Upon substitution in Eqs. (19, 20), we find therefore:

$$\text{GF}_{\text{intr}}^L = \frac{2d}{\xi} \left( 1 - \frac{\chi - 2\nu}{2(1 - \nu)} \right) \left( 1 + s/d \right),$$

and

$$\text{GF}_{\text{intr}}^T = \frac{2d}{\xi} \left( 1 - \frac{\chi - 2\nu}{2(1 - \nu)} \right) \left( 1 + s/d \right).$$

(20)

where

$$\chi = N_\perp/(N_\parallel + N_\perp) = \frac{N}{R_y + N_\perp (R_x + R_z)/2}.$$  

(21)

Although the tortuous character of the current path enters in the three-dimensional case (through the parameter $\chi$), equations (20) and (21) have the same enhancement factor $1 + s/d$ as in the one-dimensional case of Eq. (10) in the limit $\mu_1/\mu_0 \to \infty$.

B. $s/d \gg 1$ limit

Another case which can be treated easily is given by assuming very short distances $d$ compared to the size of the conducting spheres ($s/d \gg 1$). In this limit and in the region where tunneling takes place, the boundaries separating two neighboring spheres can be treated in first approximation as planes parallel to each other. If we neglect the shear components of the stress and strain fields, however expected to be weak in this region, we find that the stress and strain fields are locally constant within each phase and equal to

$$\left( \begin{array}{ccc} \sigma_{xx}^i & \lambda i + 2\mu i & \lambda i \\ \lambda i & \lambda i + 2\mu i & \lambda i \\ \lambda i & \lambda i & \lambda i + 2\mu i \end{array} \right) \left( \begin{array}{c} \epsilon_{xx}^i \\ \epsilon_{yy}^i \\ \epsilon_{zz}^i \end{array} \right) = \left( \begin{array}{ccc} \alpha & \beta & \beta \\ \beta & \alpha & \beta \\ \beta & \beta & \alpha \end{array} \right) \left( \begin{array}{c} \epsilon_{xx}^i \\ \epsilon_{yy}^i \\ \epsilon_{zz}^i \end{array} \right),$$

(22)

where $\lambda_i$ and $\mu_i$ are the Lamé coefficients for the insulating ($i = 0$) and conducting ($i = 1$) phases. Since at the boundaries between the two phases the stresses must be equal, we find from Eq. (22)

$$\left( \begin{array}{c} \epsilon_{xx}^0 \\ \epsilon_{yy}^0 \\ \epsilon_{zz}^0 \end{array} \right) = \left( \begin{array}{ccc} \alpha & \beta & \beta \\ \beta & \alpha & \beta \\ \beta & \beta & \alpha \end{array} \right) \left( \begin{array}{c} \epsilon_{xx}^1 \\ \epsilon_{yy}^1 \\ \epsilon_{zz}^1 \end{array} \right),$$

(23)

where

$$\alpha = \frac{(\lambda_1 + 2\mu_1)(\lambda_0 + \mu_0) - \lambda_1 \lambda_0}{(\lambda_0 + 2\mu_0)(\lambda_0 + \mu_0) - \lambda_0^2} = \frac{B_1}{B_0} \left[ 1 - 2 \frac{\nu_1 - \nu_0}{(1 + \nu_1)(1 + 2\nu_0)} \right],$$

(24)

and

$$\beta = \frac{\mu_0 \lambda_1 - \mu_1 \lambda_0}{(\lambda_0 + 2\mu_0)(\lambda_0 + \mu_0) - \lambda_0^2} = \frac{B_1}{B_0} \frac{\nu_1 - \nu_0}{(1 + \nu_1)(1 + 2\nu_0)}.$$

(25)
In the above expressions we have expressed the parameters \( \alpha \) and \( \beta \) in terms of the bulk modulus \( B_i \) and the Poisson ratio \( \nu_i \) of the corresponding phase, which are linked to the Lamé coefficients by the relations:

\[
\mu_i = \frac{3}{2} B_i \frac{1 - 2\nu_i}{1 + \nu_i}; \quad \lambda_i = 3 B_i \frac{\nu_i}{1 + \nu_i}.
\]  

(26)

Note that if the spheres are almost touching, the applied strain field \( \xi \) is transferred mostly to the conducting phase and, if we make the additional assumption that within the spheres the strains are almost constant, we obtain

\[
\begin{pmatrix}
\varepsilon_{xx}^1 \\
\varepsilon_{yy}^1 \\
\varepsilon_{zz}^1
\end{pmatrix}
\approx
\begin{pmatrix}
\varepsilon_{xx}^0 \\
\varepsilon_{yy}^0 \\
\varepsilon_{zz}^0
\end{pmatrix},
\]

(27)

which is an approximate but indicative relation. Finally, substituting Eqs.\((24,27)\) into Eqs.\((16,17)\), we find:

\[
GF_{\text{intr}} \approx \frac{2d}{\xi} \left[ 1 - \frac{\chi}{2} \frac{2 - \nu}{1 - \nu} - \left( 1 - \frac{3}{2} \chi \right) \frac{2 - \nu}{1 - \nu} \right. \\
\times \left. \frac{\rho_1 - \rho_0}{(1 + \rho_1)(1 - 2\rho_0)} \right] \frac{B_1}{B_0}
\]

(28)

for the longitudinal gauge factor and

\[
GF_T \approx \frac{2d}{\xi} \left[ \frac{\chi}{2} \frac{1 - 2\nu}{1 - \nu} + \left( 1 - \frac{3}{2} \chi \right) \frac{1 - 2\nu}{1 - \nu} \right. \\
\times \left. \frac{\rho_1 - \rho_0}{(1 + \rho_1)(1 - 2\rho_0)} \right] \frac{B_1}{B_0}
\]

(29)

for the transversal one.

The main result of Eqs.\((25,29)\) is that the gauge factors are proportional to the ratio of the bulk moduli \( B_1/B_0 \) which, as long as \( B_1 > B_0 \), determines the enhancement of the piezoresistive response with respect to the homogeneous limit \( B_1 = B_0 \). In RuO\(_2\)-based TFRs, this enhancement factor can in principle be as large as \( \chi \approx 4-5 \). Note however that hardening of the glassy phase due to diffusion of Ru particles (\( B_{\text{Ru}} \approx 220 \text{ GPa} \)) could reduce \( B_1/B_0 \) lowering therefore the piezoresistive response.

Another interesting feature is the effect of the Poisson’s ratios of the conducting (\( \nu_1 \)) and insulating (\( \nu_0 \)) phases. In fact, since we consider \( \chi < 2/3 \), when \( \nu_1 > \nu_0 \) the longitudinal (transversal) gauge factor is diminished (enhanced) with respect to the limit \( \nu_1 = \nu_0 \). For \( \nu_1 < \nu_0 \) this situation is reversed. This is explained by noting that Poisson’s ratio measures the lateral contraction relative to the longitudinal extension for a homogeneous material in a tensile stress. Therefore, in the heterogeneous system we consider, the phase with higher Poisson’s ratio will have a more pronounced transversal contraction than the phase with lower value of \( \nu \). This affects the piezoresistive response as schematically illustrated in Fig. 2, where a cut along the \( x-y \) plane of our model of TFRs is shown. For simplicity, we consider that the only nonzero macroscopic strain component is \( \varepsilon_{xx} = \epsilon \) [i.e., \( \nu = 0 \) in Eq.\((13)\)]. The dashed and solid lines represent the contour of the elastically deformed conducting spheres by \( \varepsilon_{xx} \) when \( \nu_1 = \nu_0 \) and \( \nu_1 > \nu_0 \), respectively. Compared to the \( \nu_1 = \nu_0 \) case, when \( \nu_1 > \nu_0 \) the inter-sphere distances are reduced in the \( x \) direction and enhanced in the \( y \) direction. Therefore, in the limiting case in which all the tunneling processes are aligned with the voltage drop (\( \chi = 0 \)) we expect that the longitudinal GF is reduced while the transversal one is enhanced, in agreement with the trend given by Eqs.\((28,29)\). Of course, finite values of \( \chi \) tend to lower this effect which vanishes in the isotropic limit \( \chi = 2/3 \).

**IV. DISCUSSION AND CONCLUSIONS**

The results presented above clearly indicate two essential elements which favour high piezoresistive responses in TFRs:

(i) the conducting phase must be stiffer than the glassy one (\( B_1 > B_0 \)),

(ii) the mean linear size \( s \) of the metallic particles must be much larger than the typical tunneling distance \( d \).

Both points (i) and (ii) seem to justify some of the trends and properties of TFRs. For example, the use of hard oxide-metals as constituent of the metallic phase is in agreement with (i), while the observed enhancement of the gauge factor with the mean metallic grain size is in conformity with (ii). Hence, the problem of how the fabrication processes affect the piezoresistive response could be approached by investigating the effects on (i) and (ii). For example, the reduction of GF at high peak firing temperature \( T_l \) could be due to an enhanced dispersion

![FIG. 2. Pictorial explanation of the effect of different Poisson’s ratios of the conducting (\( \nu_1 \)) and insulating (\( \nu_0 \)) phases. It is assumed that the macroscopic strain is only along the \( x \) direction. The dashed and solid lines represent the contours of the elastically deformed conducting spheres when \( \nu_1 = \nu_0 \) and \( \nu_1 > \nu_0 \), respectively. For an electric field applied along the \( x \) direction, the gauge factor is diminished while for a field along the \( y \) direction the transversal response is enhanced.](image-url)
of the metallic phase into the glass. This would enhance the effective bulk modulus of the glass leading to a lower ratio $B_1/B_0$.

In conclusion, we have proposed a theory of piezoresistivity in TFRs based on elastic heterogeneity. Within a simplified model of TFRs, we have shown that when the conducting phase is stiffer than the insulating one, the local strains within this latter are enhanced with respect to the averaged macroscopic strains. This enhancement leads to higher piezoresistive responses leading to higher GF values compared to those expected for a homogeneous system.

**ACKNOWLEDGMENTS**

The authors would like to thank M. Prudenziati for interesting discussions.

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14. We are not aware of reported measurements of $B$ in IrO$_2$. However, we can estimate the bulk modulus of IrO$_2$ by using the relation $\theta_D \simeq C \sqrt{aB/M}$, where $\theta_D$ is the Debye temperature, $a$ the lattice constant and $M$ the molecular mass ($C$ is a constant). Among the oxides, IrO$_2$ is the compound most similar to RuO$_2$ concerning both the electronic and structural properties. In normal conditions, IrO$_2$ has the same tetragonal (rutile) structure of RuO$_2$ with very close lattice parameters (their unit-cell parameters are equal to within 0.04 the above relation we find $\theta_D1/\theta_D2 \simeq \sqrt{B_1M_2/B_2M_1}$, where the indexes 1 and 2 refer to RuO$_2$ and IrO$_2$, respectively. Since $\theta_D1/\theta_D2 \simeq \sqrt{M_2/M_1}$ we find therefore $B_1 \simeq B_2$. This estimate holds true also when more refined relations between $\theta_D$ and elastic constants are used [H. Siethoff and K. Ahlborn, J. Appl. Phys. 79, 2968 (1996)].