Scaling Law for a Magnetic Impurity Model with Two-Body Hybridization

Y. Yu*, Y. M. Li and N. d’Ambrumenil
Department of Physics, University of Warwick, Coventry, CV4 7AL, U. K.

ABSTRACT

We consider a magnetic impurity coupled to the hybridizing and screening channels of a conduction band. The model is solved in the framework of poor man’s scaling and Cardy’s generalized theories. We point out that it is important to include a two-body hybridization if the scaling theory is to be valid for the band width larger than $U$. We map out the boundary of the Fermi-non-Fermi liquid phase transition as a function of the model parameters.

* Address after September 1, 1994: Physics Department, University of Utah, Salt Lake City, UT84112, USA.
It has been known for some time that at energies much less than the Kondo temperature the ground states of simple models of magnetic impurities [1, 2] are Fermi liquids [3, 4]. Generalized versions of such models show a transition between Fermi liquid and non-Fermi liquid ground states [5,6,7,8]. It has recently been argued that analysis of this transition may lead to a better understanding of the apparently non-Fermi liquid-like behavior observed in the superconducting cuprates.

Non-Fermi liquid states appear in the phase diagrams of impurity models with finite range interactions. A spinless impurity model has been studied by multiplicative renormalization group [9] in [6] and using poor man’s scaling theory [10] in [7]. For a magnetic impurity model, non-Fermi liquid phases have also been found [5] using Wilson’s numerical renormalization group [3]. Something similar also occurs in a generalized Hubbard model in infinite dimensions [8]. In this paper, we adapt poor man’s scaling theory and its extension [11] to find the phase diagram for a magnetic impurity model at finite $U$ including the effect of screening channels in the particle-hole symmetric case. Until now scaling arguments have only been developed for the case of infinite $U$. We point out that for $U$ less than the bandwidth, a fully renormalizable model must include a ‘two-body hybridization process’ if the renormalization group equations are to remain consistent. This is the process in which two electrons with opposite spin hop from the conduction band directly onto the local orbital.

For finite $U$ and in the particle-hole symmetric case, we find that there are four regions in the phase diagram. These are characterized by three quantities, the usual hybridization $t_1$, a two-body hybridization $t_2$ and the spin exchange $V_y$. The first is the non-Fermi liquid fixed point at which all of the quantities are irrelevant. The second region corresponds to what we call the ‘$V_y$ relevant region’, where the fixed point Hamiltonian belongs to
the same universality class of Kondo’s strong fixed point model, and is therefore, the
Fermi liquid fixed point. The third one is the ‘$t_2$ relevant region’. We have calculated
the exponent of the leading term of the fixed point interaction Hamiltonian and found that
this fixed point is again the Fermi-liquid fixed point. We find that the region where all
of $g_1$, $g_y$ and $g_2$ are relevant can be divided into three sections. Two of them have the
same fixed point as that of the $g_y$-relevant and $t_2$-relevant regions. In the other section
the system behaves as ‘free orbital Fermi liquid’[2].

Our model Hamiltonian is

$$
H = \sum_{k>0,\sigma,l} \epsilon_k c_{k\sigma l}^{\dagger} c_{k\sigma l} + \epsilon_d n_d + Un_d^\dagger n_d^\dagger \\
+ t_1 \sum_{\sigma} (c_{\sigma 0}^{\dagger} d_{\sigma} + h.c.) + t_2 \sum_{\sigma} (c_{\sigma 0}^{\dagger} d_{-\sigma} + h.c.) \\
+ \sum_{\sigma,\sigma',l} V_{l} c_{\sigma l}^{\dagger} c_{\sigma l} d_{\sigma'}^{\dagger} d_{\sigma'} + \sum_{\sigma,l} V_{x,l} c_{\sigma l}^{\dagger} c_{\sigma l} d_{\sigma}^{\dagger} d_{\sigma} + \sum_{\sigma,l} V_{y,l} c_{\sigma l}^{\dagger} c_{-\sigma l} d_{\sigma}^{\dagger} d_{\sigma}.
$$

(1)

The first four terms in (1) are the usual Anderson model while $V_l$ term takes into account
the finite range interactions between the local ‘$d$ orbital’ and conduction electrons ($l = 0,\ldots,N_f$). The $V_{x,l}$ and $V_{y,l}$ terms describe the spin exchange interaction and are taken
to be zero for $l = 1,\ldots,N_f$ for convenience. The $t_2$ term [12] describes processes in which
two $l = 0$-channel conduction electrons with opposite spin hop onto the local orbital. To
renormalize the model in the framework of poor man’s scaling theory it is important that
this term as well as spin exchange terms are included.

Following the terminology of [5,6], we call the 0-channel the hybridizing channel and
the others the screening channels. We deal with the screening channels by bosonization.
The spin of the screening channels is not important and we treat the screening channel as
spinless for simplicity. Let $b_{kl}$ be the bosonic operators corresponding to $c_{kl}$ for $l = 1,\ldots,N_f$
[13]. The effective Hamiltonian then is $H = H_0 + H_I$, where

3
\[
H_0 = \sum_{k>0,\sigma} \epsilon_k c^\dagger_{k\sigma} c_{k\sigma} + \epsilon_d n_d + U n_{d\uparrow} n_{d\downarrow} + V_0 \sum_{\sigma,\sigma'} c^\dagger_{\sigma} d^\dagger_{\sigma'} d_{\sigma'} + \sum_{k,l>0,\sigma} \frac{k}{\rho} b^\dagger_{kl} b_{kl},
\]

\[
H_I = t_1 \sum_{\sigma} (\Delta^\dagger c^\dagger_{\sigma} d_{\sigma} + \text{h.c.}) + t_2 \sum_{\sigma} (\Delta^2 c^\dagger_{\sigma} c^\dagger_{-\sigma} d_{-\sigma} + \text{h.c.}) + V_y \sum_{\sigma} c^\dagger_{\sigma} c^\dagger_{-\sigma} d^\dagger_{-\sigma} d_{\sigma},
\]

where the index 0 denoting the \(l = 0\) channel has been suppressed. The electron dispersion is taken to be \(\epsilon_k = (k - k_F)/\rho\) with \(\rho = (hv_F)^{-1}\) being the density of states at the Fermi surface. The operator \(\Delta\) is given by \(\Delta = \exp\{-\sum_k \rho V_l / \sqrt{kL(b_{kl} - b_{kl}^\dagger)}\}\). The Hilbert space of \(H_0\) can be projected onto four subspaces characterized by the four possible impurity electron states, \(|\alpha > = |0 >,|\sigma > \) and \(|3 > = |\uparrow\downarrow >\). Each term in the interaction \(H_I\) plays the role of a dipole operator causing transition among those subspaces.

The Hamiltonian in (2) can be treated using Haldane’s familiar procedure [14] by writing the partition function in terms of a sum over histories of the impurity. Expanding the partition function in \(H_I\) and labelling the (imaginary) times so that \(0 < \tau_1 < ... < \tau_n < \beta\), the partition function can eventually be written as a sum over all possible histories of the local degrees of freedom which fluctuate between the 4 local states. This gives

\[
Z = \sum_n \sum_{\alpha_1,...,\alpha_n} \int_0^{\beta - \tau} d\tau_n ... \int_0^{\tau_2 - \tau} d\tau_1 \exp(-S[\tau_1, ..., \tau_n; \alpha_1, ..., \alpha_n]),
\]

\[
S[\tau, \alpha] = \sum_{i<j} \sum_{a=\sigma,l} q_{\alpha_i,\alpha_{i+1}}^a q_{\alpha_j,\alpha_{j+1}}^a \ln \frac{\tau_j - \tau_i}{\tau} - \sum_i \ln(g_{\alpha_i,\alpha_{i+1}}) + \sum_i E_{\alpha_{i+1}} \frac{\tau_{i+1} - \tau_i}{\tau},
\]

where \(\alpha_1, ..., \alpha_n (\alpha_{n+1} = \alpha_1)\) and \(\tau_1, ..., \tau_n\) label a Feynman trajectory. \(\tau\) is the ultraviolet cut-off. With a simple shift of the ground state energy, \(E_\alpha\) can be chosen so that \(\sum_\alpha E_\alpha = 0\). We take \(E_0 = E_3 = -E_\sigma = -\epsilon_d \tau / 2\) for the particle-hole symmetric case (\(\epsilon_d = -U/2\)). \(g_{\alpha,\beta}\) are coupling constants, with \(g_{0\sigma} = g_{\sigma 3} = g_1 = t_1 \tau, g_{\sigma,-\sigma} = g_y = V_y \tau\) and \(g_{03} = g_2 = t_2 \tau\).
Equation (3) may be thought of as describing a one-dimensional four-component plasma of kinks carrying ‘charges’ $q^a_{\alpha\beta}$ and ‘fugacities’ $g_{\alpha\beta}$ [11,8]. $E_{\alpha}$ is a ‘magnetic field’. The ‘charges’ are given by

$$q^a_{0\sigma} = ((1 - \delta_x/\pi)\delta_{\lambda\sigma} - \delta_0/\pi, \delta_l/\pi),$$

$$q^a_{\sigma,\sigma'} = ((1 - \delta_x/\pi)\delta_{\lambda,\sigma'} - \delta_0/\pi, \delta_l/\pi)$$

and

$$q^a_{\sigma 3} = ((1 - \delta_x/\pi)\delta_{\lambda,\sigma} - \delta_0/\pi, \delta_l/\pi).$$

$q_{\alpha\beta} = -q_{\beta\alpha}$. The phase shifts are

$$\delta_0 = 2 \tan^{-1} \pi \rho V_0/2, \delta_x = 2 \tan^{-1} \pi \rho V_x/2 \text{ and } \delta_l = 2 \tan^{-1} \pi \rho V_l/2 [17].$$

The ‘charges’ obey the relations $q_{\alpha\beta} + q_{\beta\gamma} = q_{\alpha\gamma}$, which means that the model can be regarded as a special case of the general one-dimensional model with $1/r^2$ interaction considered by Cardy [11] and can be renormalized by poor man’s scaling theory. The Coulomb term in (3) can be rewritten as a one-dimensional spin chain model with interaction

$$\sum_{i<j} K(\alpha_i, \alpha_j) \tau_i^2/(\tau_i - \tau_j)^2,$$

where $K(\alpha, \beta) = -1/2 \sum_a (q^a_{\alpha\beta})^2$, i.e.

$K(0, \sigma) = K(\sigma, 3) = -\gamma_0, K(\sigma, \sigma') = -\gamma_x (1 - \delta_{\sigma,\sigma'})$ and $K(0, 3) = \gamma_x - 2\gamma_0$ with

$K(\alpha, \beta) = K(\beta, \alpha)$ and $K(\alpha, \alpha) = 0$. Here

$$\gamma_0 = (1 - \delta_x/\pi - \delta_0/\pi)^2 + (\delta_0/\pi)^2 + \sum_l (\delta_l/\pi)^2$$

and $\gamma_x = (1 - \delta_x/\pi)^2$. It is worth noting that if we consider the model with vanishing $t_2$, the relations $q_{\alpha\beta} + q_{\beta\gamma} = q_{\alpha\gamma}$ are not satisfied so that the model can not be mapped to the Cardy’s model.

Cardy has pointed out that the critical behavior of these kinds of model can be discussed using a generalized poor man’s scaling theory. Si and Kotliar have extended Cardy’s theory to the case in which small ‘magnetic fields’ $E_{\alpha}$ are included [15] as Haldane has also done [14]. For the particle-hole symmetric case of the present model, the
The renormalization group equations are given by

\[
\begin{align*}
\frac{dg_1}{d \ln \tau} &= (1 - \frac{1}{2} \gamma_0) g_1 - g_1 g_y e^{-E_\sigma} - g_1 g_2 e^{-E_0}, \\
\frac{dg_y}{d \ln \tau} &= (1 - \gamma_x) g_y - 2 g_1^2 e^{-E_0}, \\
\frac{dg_2}{d \ln \tau} &= (1 - 2 \gamma_0 + \gamma_x) g_2 - 2 g_1^2 e^{-E_\sigma}, \\
\frac{d\gamma_0}{d \ln \tau} &= -8 \gamma_0 (g_1^2 e^{-E_\sigma} + g_2^2 e^{-E_0}) + 4 \gamma_x [g_1^2 (e^{-E_\sigma} - e^{-E_0}) + g_2^2 e^{-E_0} - g_y^2 e^{-E_\sigma}], \\
\frac{d\gamma_x}{d \ln \tau} &= -4 \gamma_x (g_1^2 e^{-E_0} + g_y^2 e^{-E_\sigma}), \\
\frac{d\epsilon_d}{d \ln \tau} &= \epsilon_d \tau - 2 g_1^2 (e^{-E_0} - e^{-E_\sigma}) + 2 g_2^2 e^{-E_0} - 2 g_y^2 e^{-E_\sigma}.
\end{align*}
\]

(4)

We make some comments on these equations. The first three equations of (4) are exact for any value of \(\epsilon_d \tau\) while the last three are restricted to the case where the impurity level \(\epsilon_d\) is much less than the band width \(1/\tau\) [14]. (The assumption of a large band width is within the spirit of poor man’s scaling theory.) The last equation reflects the variation of \(\epsilon_d\) with \(1/\tau\):

\[\tau \frac{d\epsilon_d}{d \ln \tau} = -2 g_1^2 (e^{-E_0} - e^{-E_\sigma}) + 2 g_2^2 e^{-E_0} - 2 g_y^2 e^{-E_\sigma}.\]

(5)

The requirement \(\sum_\alpha E_\alpha = 0\) which we enforce throughout the scaling process, ensures that the separation of the field \(\epsilon_d \tau\) renormalization and the free energy renormalization is unambiguously determined.

We have assumed that \(g_{0\sigma} = g_{\sigma3},\ K(0, \sigma) = K(-\sigma, 3)\) and \(K(0, 3) = 4K(0, \sigma) - K(\sigma, -\sigma)\) in the present model. The scaling must preserve these equalities if the model is to remain consistent. This only happens in the particle-hole symmetric case. Away from the particle-hole symmetry, the scaling generates different flows for \(g_{0\sigma}\) and \(g_{\sigma3}, \ K(0, \sigma)\) and \(K(-\sigma, 3), \ etc.\)

As we have already said, if the model can be mapped to the special case of Cardy’s
model, \( t_2 \) can not vanish. Furthermore, from (4), we can see that even if we start with vanishing \( g_2 \) and \( g_y \), their absolute values all increase at a rate proportional to \( g_1 \). This means that the flows calculated assuming that \( g_y \) and \( g_2 \) vanish exactly are not correct renormalization flows. To describe a fully renormalizable model when the bandwidth is larger than \( U \), the Hamiltonian must include these two-body hybridization and spin exchange terms.

Since our renormalization group is perturbative in its treatment of \( g_{\alpha \beta} \), the renormalization of \( \gamma_0 \) and \( \gamma_x \) can be neglected in the first three equations of (4). This allows us to draw out the phase diagram in \( \gamma_0 - \gamma_x \) space (See Figure 1), which can be divided into four regions:

(i) For \( \gamma_0 > 2 \), \( \gamma_x > 1 \) and \( 2 \gamma_0 - \gamma_x > 1 \), all \( g_{\alpha \beta} \) are irrelevant and renormalize to zero. There exist weak coupling fixed points \( g^*_{\alpha \beta} = 0 \). The fixed point Hamiltonian is similar to the multi-channel X-ray edge problem [16]. The system exhibits a power-law decay of the correlation function with a non-universal exponent. This is the non-Fermi liquid phase. Since \( \gamma_0 = (1 - \delta_0/\pi - \delta_x/\pi)^2 + (\delta_0/\pi)^2 + \sum_l (\delta_l/\pi)^2 \), it is clear that it is the existence of the screening channels which allows \( \gamma_0 \) to exceed 2 and leads to the non-Fermi liquid phase [7].

(ii) For \( \gamma_0 > 2 \), \( 2 \gamma_0 - \gamma_x > 1 \) but \( \gamma_x < 1 \), \( g_1 \) and \( g_2 \) are irrelevant and renormalize to zero. \( g_y \) is relevant. We focus on the fixed point Hamiltonian. Choosing the parameters \( V_y = J_\perp \), \( V_0 = 2J_z \) and \( V_x = -J_z \) with \( J_z < 0 \) (this ensures \( \gamma_x < 1 \)), we see that the fixed point Hamiltonian \( H^* \) is just the single-channel Kondo Hamiltonian:

\[
H^* = \sum_{k>0,\sigma} \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + \epsilon_d n_d + U n_d^\dagger n_d + \sum_{k>0,l=1}^\infty k \frac{1}{l} b_{kl}^\dagger b_{kl} + \frac{J_\perp}{2} (S^+_c s^-_d + h.c.) + J_z S^z_c s^z_d.
\]

Here \( S_c \) and \( s_d \) are the spin operators of hybridizing and localized electrons respectively.
This $g_y$ ($J_\perp$)-relevant region is controlled by the strong-coupling fixed point of the Kondo problem. Furthermore, the values of the interaction parameters $V_0$ and $V_x$ are not important as long as they are within the parameter regime of the $g_y$ relevant region. Hence, the fixed point corresponding to this $g_y$ relevant phase is the Fermi liquid fixed point. Near the fixed point equation (5) reduces to, $\tau \frac{d \epsilon_d}{d \ln \tau} = -2g_y^2$, which implies that $U$ increases. This is consistent with the results for symmetric Anderson model: when the impurity level width $\Gamma \sim t_1^2$ is much less than $U$, the model is equivalent to the Kondo model [2,18]. We therefore call the $g_y$-region the Kondo region.

(iii) For $\gamma_0 > 2$, $\gamma_x > 3$ but $2\gamma_0 - \gamma_x < 1$, $g_1$ and $g_y$ are irrelevant and $g_2$ is relevant. This parameter regime corresponds to the empty and doubly-occupied states being favored over singly-occupied states. We call this the $G_2$-region. The fixed point Hamiltonian $H^* = H_0^* + H_I^*$ where $H_0^*$ can be read off from (2) by replacing all parameters by their fixed point values. $H_I^*$ has its leading term $R = (g_2^*/\tau) \sum_{\sigma} (c^{\dagger}_{\sigma} c^{\dagger}_{-\sigma} d_{\sigma} d_{-\sigma} + h.c.)$. The matrix elements of $R$ in the fixed point basis can be evaluated by analogy with the X-ray edge problem [19]. As was done in [5], we denote by $|n_d = 0 \rangle$ the eigenstates of $H_0^*|n_d = 0 \rangle$ and by $|n_d = 2 \rangle$ the eigenstates of $H^*|n_d = 2 \rangle$. Then

$$< n_d = 0 | R | n_d = 2 > \sim \tau^\alpha. \quad (7)$$

The anomalous exponent $\alpha = (1 - 2\gamma_0 + \gamma_x)/2$. If $\alpha < 0$ the operator $R$ is irrelevant, and the fixed point corresponds to the first kind of fixed point we have discussed. In the present parameter regime, $\alpha > 0$ and the operator $R$ is relevant. The fixed point is regarded as a Fermi liquid one [5]. The line $\alpha = 0$ is marginal. (We note in passing that if $V_x = 0$, and hence $\gamma_x = 1$, our result appears to recover that of [5]. However, as the parameter region in which (7) holds requires $\gamma_x > 3$, this may be just a coincidence.)

(iv) For $\gamma_0 < 2$, all coupling constants are relevant. We divide this parameter region
into three sections (Figure 1). Although all coupling constants are relevant, (4) shows that $g^2_y \gg g^2_2$ in $I$, $g^2_y \gg g^2_2$ in $II$ and $g^2_y \sim g^2_2$ in $III$. According to (5), the Hubbard interaction $U$ increases and so singly-occupied states are favored in $I$ which is in the Kondo strongly coupling phase. In $II$, $U$ decreases and the empty and doubly-occupied states are favored. This suggests that in $II$ the system is the same FL phase as the $G_2$-region. In $III$, $U$ remains close to its initial small value and $|0\rangle$, $|\sigma\rangle$ and $|3\rangle$ are mixed. The impurity level width $\Gamma(\sim t_1^2) \gg U$ in $III$ implies that the system is in the ‘free orbital Fermi liquid phase’ [2].

We may summarise what we have learnt about the phase diagram as follows. There are two kinds of fixed point in the phase diagram: Fermi liquid and non-Fermi liquid. The part of the phase diagram controlled by Fermi liquid fixed point can be divided into three regions according to the behaviour expected at finite temperature. The three types of behaviour are Kondo strong-coupling, free-obital and what we have called ‘$G_2$’.

It is interesting to compare our results with those of the numerical renormalization group reported by Krishna-murthy, Wilson and Wilkins [2]. The case they studied corresponds to a point in our phase diagram: $\gamma_0 = \gamma_2 = 1$. For this case, we reproduce qualitatively their results from the renormalization group equations (4). They found that the system goes to a Fermi liquid fixed point with two regions: which they call free orbital region (this corresponds to our region $III$ in Fig. 1) and local moment region ($I$ in Fig. 1). Because they assume that $t_2$ is always zero, it is clear that they would not find the region corresponding to our region $II$. The ‘$G_2$-’ and ‘NFL-’ regions in Fig. 1 are physically relevant. If there is a direct Coulomb interaction between the conduction electrons and the impurity electron ($i.e.$ $V_0 \neq 0$ and $V_2 \neq 0$), the system could be driven into the $II$-region even if the two-body hybridization vanishes initially.
In conclusion, we have derived that the scaling laws for a magnetic impurity model including the ‘hybridization’ of an up and a down spin electron hopping onto the local orbital. The renormalization group analysis shows that there is a Fermi-non-Fermi liquid transition when the strength of the local interaction is varied. For the case of finite $U$ which we have considered, the model has to be particle-hole symmetric in order to preserve the consistency of the renormalization group equations derived from the generalized poor man’s scaling theory.

The asymmetric model can be discussed only in the infinite $U$ limit, when the doubly-occupied states are completely suppressed leaving only local states $|0>$ and $|\sigma>$. This is then just a special case of the spin-$N+1$ model with added screening channels and has been discussed by Si and Kotliar [15]. Results of a numerical renormalization group calculation were also reported in [5]. At finite $U$, the normal assumption that the usual hybridization remains particle-hole symmetric after renormalization is valid only for the particle-hole symmetric case. Results of more general treatment will published separately [20].

The authors thank Professor Z. B. Su who brings us to pay our attention to this type of problems and presented a prior. We are also very grateful for useful discussions with him and Professor L. Yu. This work was supported in part by SERC of the United Kingdom under grant No.GR/E/79798 and also by MURST/British Council under grant No.Rom/889/92/47.
[1] J. Friedel, Can. J. Phys. 34, 1190 (1956); A. Blandin and J. Friedel, J. Phys. Radium 20, 160 (1959).

[2] P. W. Anderson, Phys. Rev. 124, 41 (1961); H. R. Krishna-murthy, K. G. Wilson and J. W. Wilkins, Phys. Rev. Lett. 16, 1101 (1975).

[3] K.G. Wilson, Rev. Mod. Phys. 47, 773 (1975).

[4] P. Nozieres, J. Low Temp. Phys. 17, 31 (1974).

[5] I. E. Perakis, C. M. Varma and A. E. Ruckenstein, Phys. Rev. Lett. 70, 3467 (1993).

[6] T. Giamarchi, C. M. Varma, A. E. Ruckenstein and P. Nozieres, Phys. Rev. Lett. 70, 3967 (1993).

[7] G. M. Zhang, L. Yu and Z. B. Su, Phys. Rev. B49, 7759 (1994).

[8] Q. M. Si and G. Kotliar, Phys. Rev. Lett. 70, 3143 (1993).

[9] J. Solyom, Adv. Phys. 28, 209 (1979); J. Phys. F 4, 2269 (1975); P. Nozieres, and A. Blandin, J. Phys. (Paris) 41, 193 (1980).

[10] P. W. Anderson, G. Yuval and D. R. Hamann, Phys. Rev. B1, 4464 (1970); P. W. Anderson, J. Phys. C3, 2436.

[11] J. L. Cardy, J. Phys. A14, 1407 (1981); See also S. Chakravarty and J. Hirsch, Phys. Rev. B25, 3273 (1982) and Q. Si and G. Kotliar, Phys. Rev. B48, 13881 (1993).

[12] In [5], this term was discussed as the leading term of the fixed point Hamiltonian.

[13] D. C. Mattis and E. Lieb, J. Math. Phys. 6, 304 (1965).

[14] F. M. D. Haldane, Phys. Rev. Lett. 40, 416 (1978); J. Phys. C11, 5015 (1978).
[15] See Q. M. Si and G. Kotliar in refs.[11] and [8].

[16] G. Mahan, Phys. Rev. 153, 882 (1967); P. Nozieres and C. T. de Domincis, Phys. Rev. 178, 1097 (1969); P. W. Anderson, Phys. Rev. Lett. 18, 1049 (1967).

[17] P. B. Wiegmann and A. M. Feinkel’stein, Sov. Phys. JETP 48, 102 (1978).

[18] P. W. Anderson and A. M. Clogston, Bull. Am. Phys. Soc. 6, 124(1961); J. Kondo, Prog. Theor. Phys. 28, 846(1962); J. R. Schrieffer and P. A. Wolff, Phys. Rev. 149, 491(1966).

[19] See, for example, P. Nozieres and C. T. de Dominicis in [16]; Also see K. D. Schotte and U. Schotte, Phys. Rev. 182, 479 (1969); Recent application, see [5].

[20] Y. Yu, Y. M. Li and N. d’Ambrumenil, in preparation.
Fig. 1 The phase diagram in $\gamma_0 - \gamma_x$ space. The thick lines are phase boundaries. The thin lines divide the Fermi liquid phase into the different regions characterised by different behaviour at finite temperature.
FIGURE 1