Reliability assessment for systems suffering competing degradation and random shocks under fuzzy environment

Hongping Yu and Mao Tang
School of Mechanical Engineering, Chengdu University, Chengdu, China

Abstract
Reliability assessment of multi-component systems under competing degradation and random shocks has been intensively investigated in recent years. In most cases, the parameters associated with competing degradation and random shocks are represented by crisp values. However, due to insufficient data and vague judgments from experts, it may produce epistemic uncertainty with those parameters and they are befitting to be described as fuzzy numbers. In this article, the internal degradation is treated as a continuous monotonically increasing random process with respect to operating time, whereas the amount of cumulative damage produced by each external random shock is modeled by a geometric process. As components in a system suffer the same environmental condition, an external random shock will produce different amounts of cumulative damage to each component simultaneously. Each component fails when either the internal degradation or cumulative damage from the random shocks, whichever comes first, exceeds its corresponding random thresholds. Moreover, the parameters associated with the internal degradation and the random shocks are represented by triangular fuzzy numbers. The fuzzy reliability functions of components and the entire system are evaluated by a set of optimization models. A multi-component system, together with some comparative results, is presented to illustrate the implementation of the proposed method.

Keywords
Reliability evaluation, random shocks, degradation process, geometric process, fuzzy set, optimization model

Corresponding author:
Mao Tang, School of Mechanical Engineering, Chengdu University, No. 2025, Chengluo Avenue, Chengdu 610106, Sichuan, China.
Email: tangmao@cdu.edu.cn

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Introduction

Reliability modeling and assessment are critical issues for complex engineering systems. It is challenging to accurately assess the reliability for such systems because of many factors such as the failure competing or dependencies, impact from external environments, and epistemic uncertainty associated with parameters of component failure/degradation models. According to the failure mechanism, most of the industrial systems and their components, such as bearings, compressor blades of airplane engines, and primary coolant systems of nuclear power plants, fail not only due to the internal deterioration which is also called “degradation failure,” such as wear, crack growth, erosion, and corrosion, but also due to the cumulative damage from external random shocks, such as vibration and power surge, which could cause a sudden “catastrophic failure.”

In the literature, many models and approaches have been developed to analyze and evaluate the reliability of various degradation systems. For example, Guan et al.\textsuperscript{1} utilized the Wiener process to study the accelerated degradation test from a Bayesian perspective. Van Noortwijk\textsuperscript{2} reviewed the application of gamma processes in reliability modeling and maintenance. By employing an additive Wiener process model that consists of both a linear and a non-linear degradation part, Wang et al.\textsuperscript{3} proposed a general degradation modeling framework for hybrid deteriorating systems. Peng\textsuperscript{4} developed a degradation model based on an inverse normal–gamma mixture of inverse Gaussian process.

The aforementioned models are not suitable for some applications where the system can be damaged by external random shocks. There are many engineering systems that suffer from not only the internal degradation process but also the external random shocks. For instance, a DVD player suffers aging of a laser reader and random excess voltage and current to its circuits; the motor may fail under the degradation of bearing wearing and external random shocks such as sudden excess load, vibration, and switch. Research efforts on the reliability analysis in the case of internal degradation and external random shocks can be generally classified into two research directions. In the first direction, it is assumed that random shocks cause a jump in the degradation level of the system, leading either to failure if the magnitude of this jump is sufficiently large to cross the failure threshold or to incremental damage otherwise.\textsuperscript{5} For the first direction, Peng et al.\textsuperscript{6} developed reliability models and maintenance policies for systems subject to multiple dependent competing failure processes. In addition to continuous degradation, random shocks can result in incremental damage or direct failure depending on their magnitude. Song et al.\textsuperscript{7} proposed a reliability model for parallel systems with components experiencing dependent degradation processes and categorized shocks. A multi-objective optimization model for imperfect maintenance policy was proposed by Wang and Pham.\textsuperscript{8} The studied systems are subject to multiple competing and hidden failure processes. By taking account of the hard and soft failures with dependent shock effects, a reliability assessment method for multi-component systems was proposed by Song et al.\textsuperscript{9}

The second research direction assumes that random shocks influence the degradation rate of the system in contrast to causing a jump in the degradation level.
For example, Rafiee et al.\textsuperscript{10} developed a reliability model for systems subject to dependent competing failure processes of degradation and random shocks. The degradation rate of the system can change according to particular random shock patterns. Lin et al.\textsuperscript{11} extended a multi-state physics model framework for the reliability assessment of components subject to multi-state degradation and random shock processes, and a Monte Carlo simulation algorithm was developed to compute the reliability measures. Song et al.\textsuperscript{12} proposed a new system model where individual failure processes for each component and the component failure processes are statistically dependent. Huang and Askin\textsuperscript{13} investigated the reliability of an electronic device subject to multiple competing failure modes which can result in performance degradation, and this approach can be used to predict the dominant failure mode on the product. Moreover, by dividing the degradation trajectory into several discrete states, the multi-state systems have also received considerable attention in recent years.\textsuperscript{14–17} Some other research relevant to the deterioration system can be found in the literature.\textsuperscript{18–23,31}

Nevertheless, the aforementioned research efforts adhere to the assumption that the parameters associated with the internal degradation process and external random shocks were crisp values. Such an assumption may not always hold in reality. Due to the insufficient data and/or vague judgments from experts, it may inevitably produce epistemic uncertainty with these parameters. In contrast to the aleatory uncertainty which is irreducible uncertainty associated with natural stochastic variability and can be represented by existing probabilistic tools, the epistemic uncertainty arises due to insufficient data and/or vague judgments from experts. Many non-probabilistic methods and tools, such as the fuzzy set theory,\textsuperscript{24–26} the evidence theory,\textsuperscript{27,28} and the interval method,\textsuperscript{29,30} can be implemented to quantify the epistemic uncertainty associated with the parameters associated with the internal degradation process and external random shocks. The fuzzy set theory has been intensively implemented in reliability engineering as it can be constructed on the basis of expert vague attitudes/judgments rather than a large amount of objective information. For instance, the fuzzy set theory has been widely used in the reliability assessment of multi-state systems when the component state probability contains epistemic uncertainty. As shown by Ding and Lisnianski\textsuperscript{25} and Liu and Huang,\textsuperscript{26} the epistemic uncertainty of component state probability has propagated to the system reliability evaluation and the system reliability became a fuzzy number too.

In this article, the system reliability assessment with the internal degradation process and external random shocks is conducted under the fuzzy environment. The system consists of multiple components, and every component has its own degradation path but suffers the same external random shocks simultaneously. Thus, the system degrades with its components. The internal degradation is assumed following a continuous random process and the cumulative damage resulting from random shocks is presented by a geometric process with a Poisson arrival pattern. Moreover, the parameters associated with the internal continuous degradation process and the geometric process are represented by triangular fuzzy numbers (TFNs). The approximate reliability functions of components and the
The entire system at any $\alpha$-cut level are evaluated by a set of optimization models. A multi-component system, together with some comparative results, is presented to illustrate the implementation of the proposed method.

The rest of this article is organized as follows: Section “System description” presents the basic assumptions and definitions of the studied systems. Some preliminaries of the fuzzy set theory are provided in section “Preliminaries.” The mathematical models to derive the analytical reliability of each component and the entire system are formulated in section “Reliability evaluation under the fuzzy environment.” Two numerical cases are given to demonstrate the implementation of the proposed method, and an approximating algorithm is used to reduce the computational complexity in section “Case studies.” Section “Conclusion and future work” presents a brief conclusion.

System description

In many engineering cases, components in a system are subjected to several distinct degradation processes. Each process has its respective growing path and pattern, for example, gears in a transmission system suffer crack growth and wear. As a system consists of multiple components, each component in the system is functionally and physically different from one another, and hence they suffer various internal degradation processes and failure mechanisms. In this study, the internal continuous degradation of component $l$ is modeled by a random process function, denoted as $Y_l(t)$. In addition, external random shocks imposed on component $l$ will also lead to failure due to the cumulative damage, denoted as $D_l(t)$, produced by random shocks. The arrival of external random shocks follows a Poisson process and the cumulative damage caused by each random shock. The cumulative damage is also a random process as the damage caused by shocks will be randomly cumulated with respect to operating time. Therefore, components will fail due to either internal degradation process or cumulative damage caused by random shocks whenever one of the two reaches its threshold. Moreover, due to insufficient data and vague judgments from experts, the parameters associated with the internal continuous degradation of component $l$, that is, $Y_l(t)$, and the cumulative damage, that is, $D_l(t)$, contain epistemic uncertainty, and the TFNs under the fuzzy set theory are utilized to quantify the epistemic uncertainty. Before deriving the fuzzy reliability measures of the studied systems, some basic assumptions are summarized as follows:

1. Under the fuzzy set theory, the state of component $l$ at any time instant $t$ is characterized by the continuous internal degradation process $\tilde{Y}_l(t)$ and the cumulative damage $\tilde{D}_l(t)$ caused by external random shocks. The system state is completely determined by the states of components and the system configuration.
2. The fuzzy cumulative damage $\tilde{D}_l(t)$ of component $l$ is statistically independent of $\tilde{Y}_l(t)$.
3. It is assumed that the arrival of random shocks is memoryless. Based on this memoryless assumption, the homogeneous Poisson process is used to characterize the arrival of the external random shocks with a constant intensity $\lambda$. It should be noted that the cumulative damage produced by each individual shock becomes greater with the increasing number of shocks. Moreover, as components in a system are exposed to the same environmental condition, random shocks for all the components have the same arrival rate $\lambda$, but different magnitudes for each cumulative damage.

4. The thresholds of internal degradation and external random shocks, denoted by $L_l$ and $S_l$, respectively, are not fixed values for all the components. Rather, they are treated as fuzzy distributions $\tilde{L}_l$ and $\tilde{S}_l$, respectively, with all the parameters being TFNs.

As the damage caused by the external random shocks increases with the number of shocks, the geometric process is used in this work to model this phenomenon by increasing the cumulative damage produced by each individual shock. The geometric process is defined as follows.

**Definition 1.** Suppose that $X$ and $Y$ are two $s$-independent random variables. $X$ is said to be stochastically greater than $Y$ or $Y$ is stochastically less than $X$ if $P(X > \alpha) \geq P(Y > \alpha)$ for any real value of $\alpha$. The above stochastic ordering of two random quantities can be denoted as $X \succeq_{st} Y$ or $Y \preceq_{st} X$. Moreover, a stochastic process $\{X_n, n = 1, 2, \ldots\}$ is said to be stochastically decreasing if $X_n \geq_{st} X_{n+1}$ for $n = 1, 2, \ldots$ or stochastically increasing if $X_n \leq_{st} X_{n+1}$ for $n = 1, 2, \ldots$.

**Definition 2.** Assume that $\{\xi_n, n = 1, 2, \ldots\}$ is a sequence of non-negative and independent random quantities. If the cumulative distribution function (CDF) of $\xi_n$ is defined as $F_n(t) = F(b^{n-1}t)$ for $n = 1, 2, \ldots$, where $b$ is a positive parameter, then $\{\xi_n, n = 1, 2, \ldots\}$ form a geometric process. The parameter $b$ is also called the ratio of the geometric process. For example, if $b > 1$, $\{\xi_n, n = 1, 2, \ldots\}$ are stochastically decreasing, while if $0 < b < 1$, $\{\xi_n, n = 1, 2, \ldots\}$ are stochastically increasing. Particularly, if $b = 1$, the geometric process degenerates to a renewal process. Nevertheless, in this work, due to insufficient data and vague judgments from experts, the parameter associated with the geometric process, that is, $b$, also contains epistemic uncertainty and it is represented by a TFN.

The fuzzy cumulative damage $\tilde{D}_l(t)$ of component $l$ caused by external random shocks can be computed by

$$\tilde{D}_l(t) = \sum_{i=1}^{N(t)} \tilde{X}_{li}$$

(1)

where $\tilde{X}_{li}$ is the fuzzy cumulative damage of component $l$ caused by the $i$th external random shock. Suppose that $\tilde{F}_{li}(t)$ is the fuzzy CDF of the fuzzy cumulative damage $\tilde{X}_{li}$ produced by the $i$th shock; based on the definition of the geometric process, one has
Fli(t) = F(\tilde{b}_i^{i-1}) t) for i = 1, 2, \ldots, where 0 \leq \tilde{b}_i \leq 1 \tag{2}

Hence, \{\tilde{X}_i, i = 1, 2, \ldots, N(t)\} forms an increasing fuzzy geometric process, representing that the fuzzy cumulative damage produced by each shock becomes greater with the increase in the number of shocks.

**Preliminaries**

In this section, we will briefly review the fundamentals of the fuzzy set theory, including the TFN, the extension principle, and the parametric programming, which are used in the subsequent sections.

**TFN**

A TFN \( \tilde{X} \) can be represented by a triplet \( \tilde{X} = (a, b, c) \), and its membership function, as shown in Figure 1, can be defined as follows \(^{24}\)

\[
\mu_{\tilde{X}}(x) = \begin{cases} 
\frac{x - a}{b - a}, & a \leq x < b \\
1, & x = b \\
\frac{x - c}{b - c}, & b < x \leq c \\
0, & \text{otherwise}
\end{cases}
\tag{3}
\]

where \( a \leq b \leq c \) and \( a, b, \) and \( c \) are crisp numbers. In this work, all the fuzzy variables are treated as TFNs because of its wide implementations in reliability engineering.\(^{25,26}\)

Let \( \tilde{X}_1 = (a_1, b_1, c_1) \) and \( \tilde{X}_2 = (a_2, b_2, c_2) \) be two TFNs. Then the arithmetical operations for the two TFNs are stipulated as follows

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**Figure 1.** The membership function of a TFN.
\[ \tilde{X}_1 + \tilde{X}_2 = (a_1 + a_2, b_1 + b_2, c_1 + c_2), \quad \tilde{X}_1 - \tilde{X}_2 = (a_1 - c_2, b_1 - b_2, c_1 - a_2) \]

\[ \tilde{X}_1 \times \tilde{X}_2 = (a_1 \times a_2, b_1 \times b_2, c_1 \times c_2), \quad \tilde{X}_1 \div \tilde{X}_2 = \left( \frac{a_1}{c_2}, \frac{b_1}{b_2}, \frac{c_1}{a_2} \right) \]

**The extension principle and parametric programming technique**

The extension principle, introduced by Zadeh,\textsuperscript{24} allows one to obtain the membership function of a function with respect to \( n \) input fuzzy numbers

\[
\mu_{\tilde{P}(\tilde{x})}(z) = \sup_{x \in \mathbb{R}^n} \min \{ \mu_{\tilde{X}}(x) \} \\
= \sup_{x_1 \in R_1, x_2 \in R_2, \ldots, x_n \in R_n} \min \{ \mu_{\tilde{X}_1}(x_1), \mu_{\tilde{X}_2}(x_2), \ldots, \mu_{\tilde{X}_n}(x_n) \} \tag{4}
\]

where \( \tilde{X} \) represents the set of \( n \) input fuzzy numbers \( \{ \tilde{X}_1, \tilde{X}_2, \ldots, \tilde{X}_n \} \), \( x \) is the set of \( n \) input fuzzy variables \( \{ x_1, x_2, \ldots, x_n \} \), \( \mathbb{R}^n = \{ R_1, R_2, \ldots, R_n \} \) is the universal set of real numbers, and \( m(\bullet) \) is a mapping function from input \( x \) to output \( z \). Based on the extension principle, the interval of \( \alpha \)-cut level set of fuzzy number \( \tilde{P}(\tilde{x}) \) can be represented by

\[
\tilde{P}_\alpha(\tilde{x}) = [\min \tilde{P}(x; \mu_{\tilde{X}}(x) \geq \alpha), \max \tilde{P}(x; \mu_{\tilde{X}}(x) \geq \alpha)] = [\tilde{P}_\alpha^L, \tilde{P}_\alpha^U] \tag{5}
\]

while the lower and upper bounds of \( \tilde{P}(\tilde{x}) \) at any \( \alpha \)-cut level can be calculated, respectively, by a pair of parametric programming as

\[
\tilde{P}_\alpha^L = \min P(x_1, x_2, \ldots, x_n) \\
\text{s.t.} \quad \tilde{x}_{1\alpha}^L \leq x_1 \leq \tilde{x}_{1\alpha}^U \]
\[
\tilde{x}_{2\alpha}^L \leq x_2 \leq \tilde{x}_{2\alpha}^U \\
\vdots \]
\[
\tilde{x}_{n\alpha}^L \leq x_n \leq \tilde{x}_{n\alpha}^U 
\]

and

\[
\tilde{P}_\alpha^U = \max P(x_1, x_2, \ldots, x_n) \\
\text{s.t.} \quad \tilde{x}_{1\alpha}^L \leq x_1 \leq \tilde{x}_{1\alpha}^U \]
\[
\tilde{x}_{2\alpha}^L \leq x_2 \leq \tilde{x}_{2\alpha}^U \\
\vdots \]
\[
\tilde{x}_{n\alpha}^L \leq x_n \leq \tilde{x}_{n\alpha}^U 
\]
Reliability evaluation under the fuzzy environment

The reliability of a single component and a multi-component system under the fuzzy environment is discussed in this section under the internal degradation and external random shocks.

Fuzzy reliability evaluation for a single component

The fuzzy reliability $\tilde{R}_l(t)$ of component $l$ in terms of the operating time instant $t$ can be written as

$$
\tilde{R}_l(t) = \tilde{P}(\tilde{Y}_l(t) \leq \tilde{L}_l, \tilde{D}_l(t) \leq \tilde{S}_l)
$$

where $\tilde{Y}_l(t)$ and $\tilde{D}_l(t)$ are $s$-independent events, and thus one has

$$
\tilde{R}_l(t) = \tilde{P}(\tilde{Y}_l(t) \leq \tilde{L}_l) \times \tilde{P}(\tilde{D}_l(t) \leq \tilde{S}_l)
$$

The $\alpha$-cut level of the reliability function of component $l$ is given by

$$
\tilde{R}_{la}(t) = \left[ \tilde{R}_{la}^L(t), \tilde{R}_{la}^U(t) \right]
$$

with the lower bound

$$
\tilde{R}_{la}^L(t) = \tilde{P}_\alpha^L(\tilde{Y}_l(t) \leq \tilde{L}_l) \tilde{P}_\alpha^L(\tilde{D}_l(t) \leq \tilde{S}_l)
$$

and the upper bound

$$
\tilde{R}_{la}^U(t) = \tilde{P}_\alpha^U(\tilde{Y}_l(t) \leq \tilde{L}_l) \tilde{P}_\alpha^U(\tilde{D}_l(t) \leq \tilde{S}_l)
$$

where the detailed procedures of computing $[\tilde{P}_\alpha^L(\tilde{D}_l(t) \leq \tilde{S}_l), \tilde{P}_\alpha^U(\tilde{D}_l(t) \leq \tilde{S}_l)]$ and $[\tilde{P}_\alpha^L(\tilde{Y}_l(t) \leq \tilde{L}_l), \tilde{P}_\alpha^U(\tilde{Y}_l(t) \leq \tilde{L}_l)]$ are as follows.

Suppose that the arrival of the external random shocks follows a Poisson process with a fixed arriving rate $\lambda$. The fuzzy probability of the cumulative damage caused by shocks is given by

$$
\tilde{P}(\tilde{D}_l(t) \leq \tilde{S}_l) = \tilde{P}\left( \sum_{i=1}^{N(t)} \tilde{X}_{li} \leq \tilde{S}_l \right) = \tilde{P}(\tilde{X}_{l1} + \tilde{X}_{l2} + \ldots + \tilde{X}_{ln(t)} \leq \tilde{S}_l)
$$

$$
= \sum_{n=0}^{\infty} \Pr\{N(t) = n\} \tilde{P}(\tilde{X}_{l1} + \tilde{X}_{l2} + \ldots + \tilde{X}_{ln(t)} \leq \tilde{S}_l | N(t) = n)
$$

(12)

As the sequence of the cumulative damage $\{\tilde{X}_{li}, i = 1, 2, \ldots, N(t)\}$ produced by each shock forms a fuzzy geometric process, $\tilde{F}_{li}(t)$ is the fuzzy CDF of random variable $\tilde{X}_{li}$. The fuzzy CDF of $\tilde{Z}_{ln} = \tilde{X}_{l1} + \tilde{X}_{l2} + \cdots + \tilde{X}_{ln}$ of component $l$ can, therefore, be obtained by the following recursive formulas

$$
\tilde{Z}_{ln}(s) = \tilde{F}_{ln}(s) = \tilde{P}(\tilde{X}_{l1} \leq s)
$$

(13)
\[\tilde{Z}_{12}(s) = \tilde{P}(\tilde{X}_{12} \leq s) = \tilde{P}(\tilde{X}_{11} \leq s - u, \tilde{X}_{12} = u)\]
\[= \tilde{F}_1(\tilde{b}_i) * \tilde{F}_1(s)\]  
(14)

\[\tilde{Z}_{ln}(s) = \tilde{P}(\tilde{X}_{11} + \tilde{X}_{12} + \cdots + \tilde{X}_{ln} \leq s)\]
\[= \tilde{F}_1(\tilde{b}_i^{n-1}) * \tilde{F}_1(\tilde{b}_i^{n-2}) * \cdots * \tilde{F}_1(s)\]  
(15)

where * is the convolution operator. The \( \alpha \)-cut level of \( \tilde{Z}_{ln}(s) \) is given by \( \tilde{Z}_{lna}(s) = [\tilde{Z}_{lna}^l(s), \tilde{Z}_{lna}^u(s)] \) with the lower bound
\[\tilde{Z}_{lna}^l(s) : \min \tilde{Z}_{ln}(s) = \tilde{F}_1(\tilde{b}_i^{n-1}) * \tilde{F}_1(\tilde{b}_i^{n-2}) * \cdots * \tilde{F}_1(s)\]
\[\text{s.t.} \]
\[\tilde{F}_{la}^l(\tilde{b}_i^{n-1}) \leq \tilde{F}_1(\tilde{b}_i^{n-1}) \leq \tilde{F}_{la}^u(\tilde{b}_i^{n-1})\]
\[\tilde{F}_{la}^l(\tilde{b}_i^{n-2}) \leq \tilde{F}_1(\tilde{b}_i^{n-2}) \leq \tilde{F}_{la}^u(\tilde{b}_i^{n-2})\]
\[\vdots\]
\[\tilde{F}_{la}^l(s) \leq \tilde{F}_1(s) \leq \tilde{F}_{la}^u(s)\]  
(16)

and the upper bound
\[\tilde{Z}_{lna}^u(s) : \max \tilde{Z}_{ln}(s) = \tilde{F}_1(\tilde{b}_i^{n-1}) * \tilde{F}_1(\tilde{b}_i^{n-2}) * \cdots * \tilde{F}_1(s)\]
\[\text{s.t.} \]
\[\tilde{F}_{la}^l(\tilde{b}_i^{n-1}) \leq \tilde{F}_1(\tilde{b}_i^{n-1}) \leq \tilde{F}_{la}^u(\tilde{b}_i^{n-1})\]
\[\tilde{F}_{la}^l(\tilde{b}_i^{n-2}) \leq \tilde{F}_1(\tilde{b}_i^{n-2}) \leq \tilde{F}_{la}^u(\tilde{b}_i^{n-2})\]
\[\vdots\]
\[\tilde{F}_{la}^l(s) \leq \tilde{F}_1(s) \leq \tilde{F}_{la}^u(s)\]  
(17)

where the \( \alpha \)-cut level of fuzzy CDF \( \tilde{F}_{la}(\tilde{b}_i) = [\tilde{F}_{la}^l(\tilde{b}_i), \tilde{F}_{la}^u(\tilde{b}_i)] \) \((i = 1, 2, \ldots, n - 1)\) in equations (16) and (17) can be calculated, respectively, by
\[\tilde{F}_{la}^l(\tilde{b}_i) : \min \tilde{F}_1(\tilde{b}_i)\]
\[\text{s.t.} \]
\[\tilde{b}_i^l \leq \tilde{b}_i \leq \tilde{b}_i^u\]  
(18)

and
\[\tilde{F}_{la}^u(\tilde{b}_i) : \max \tilde{F}_1(\tilde{b}_i)\]
\[\text{s.t.} \]
\[\tilde{b}_i^l \leq \tilde{b}_i \leq \tilde{b}_i^u\]  
(19)

Let \( \tilde{S}_l \) be the fuzzy threshold of the cumulative damage of component \( l \) due to shocks; one has
\[
\bar{H}_{ln} = \bar{P}(X_{l1} + X_{l2} + \cdots + X_{ln} \leq \bar{S}_l) = \bar{P}(\bar{Z}_{ln} \leq \bar{S}_l) = \int_0^\infty \bar{Z}_{ln}(u) d\bar{S}_l(u) \quad (20)
\]

Then, the \(\alpha\)-cut level of \(\bar{H}_{lna}\) can be represented by \([\bar{H}^L_{lna}, \bar{H}^U_{lna}]\) with the lower and upper bounds of \(\bar{H}_{ln}\) at any \(\alpha\)-cut level calculated, respectively, by a pair of parametric programming as

\[
\bar{H}^L_{lna} = \min_{0}^\infty \int_0^\infty \bar{Z}_{ln}(u) d\bar{S}_l(u)
\]

s.t.
\[
\bar{Z}^L_{lna}(u) \leq \bar{Z}_{ln}(u) \leq \bar{Z}^U_{lna}(u)
\]
\[
\bar{S}^L_{lna}(u) \leq \bar{S}_l(u) \leq \bar{S}^U_{lna}(u)
\]

and

\[
\bar{H}^U_{lna} = \max_{0}^\infty \int_0^\infty \bar{Z}_{ln}(u) d\bar{S}_l(u)
\]

s.t.
\[
\bar{Z}^L_{lna}(u) \leq \bar{Z}_{ln}(u) \leq \bar{Z}^U_{lna}(u)
\]
\[
\bar{S}^L_{lna}(u) \leq \bar{S}_l(u) \leq \bar{S}^U_{lna}(u)
\]

Hence, the fuzzy probability \(\bar{P}(\bar{Y}_l(t) \leq \bar{L}_l)\) at any \(\alpha\)-cut level can be calculated by equations (21) and (22).

Without loss of generality, the internal continuous degradation process of component \(l\) is assumed to follow a Weibull distribution with the time-varying fuzzy shape parameter \(\tilde{\beta}_l(t)\) and the time-varying fuzzy scale parameter \(\tilde{\eta}_l(t)\). The fuzzy probability that the fuzzy cumulative degradation is less than a constant \(u\) at any time instant \(t\) can be written as

\[
\bar{P}(\bar{Y}_l(t) \leq u) = 1 - \exp \left( - \left( \frac{u}{\tilde{\eta}_l(t)} \right)^{\tilde{\beta}_l(t)} \right) \quad (23)
\]

By plugging equation (23) into \(\text{Pr}\{Y_l(t) \leq L_l\}\), one has

\[
\bar{G}_l(t) = \bar{P}(\bar{Y}_l(t) \leq \bar{L}_l)
\]
\[
= \int_0^\infty \bar{P}(\bar{Y}_l(t) \leq u) d\bar{L}_l
\]
\[
= \int_0^\infty \left( 1 - \exp \left( - \left( \frac{u}{\tilde{\eta}_l(t)} \right)^{\tilde{\beta}_l(t)} \right) \right) d\bar{L}_l \quad (24)
\]
Therefore the $\alpha$-cut level of the fuzzy survival probability of component $l$ under the internal continuous degradation process, denoted as $\tilde{G}_{la}(t) = [\tilde{G}^L_{la}(t), \tilde{G}^U_{la}(t)]$, can be calculated, respectively, by a pair of parametric programming as

$$\tilde{G}^L_{la}(t) = \min_{0}^{\infty} \left(1 - \exp \left(-\left(\frac{u}{\tilde{\eta}(t)}\right)\tilde{\beta}(t)\right)\right) d\tilde{L}_l$$

s.t.

$$\tilde{\eta}^L_{la}(t) \leq \tilde{\eta}_l(t) \leq \tilde{\eta}^U_{la}(t)$$

$$\tilde{\beta}^L_{la}(t) \leq \tilde{\beta}_l(t) \leq \tilde{\beta}^U_{la}(t)$$

$$\tilde{L}^L_{la} \leq \tilde{L}_l \leq \tilde{L}^U_{la}$$

and

$$\tilde{G}^U_{la}(t) = \max_{0}^{\infty} \left(1 - \exp \left(-\left(\frac{u}{\tilde{\eta}(t)}\right)\tilde{\beta}(t)\right)\right) d\tilde{L}_l$$

s.t.

$$\tilde{\eta}^L_{la}(t) \leq \tilde{\eta}_l(t) \leq \tilde{\eta}^U_{la}(t)$$

$$\tilde{\beta}^L_{la}(t) \leq \tilde{\beta}_l(t) \leq \tilde{\beta}^U_{la}(t)$$

$$\tilde{L}^L_{la} \leq \tilde{L}_l \leq \tilde{L}^U_{la}$$

Then, by substituting equations (21), (22), (25), and (26) into equation (9), the $\alpha$-cut level of the reliability function of component $l$ under both the fuzzy internal continuous degradation process and the fuzzy external random shocks, denoted by $\tilde{R}_{la}(t) = [\tilde{R}^L_{la}(t), \tilde{R}^U_{la}(t)]$, can also be calculated by a pair of parametric programming with the lower bound

$$\tilde{R}^L_{la}(t) : \min \tilde{R}_l(t) = \tilde{G}_l(t) \sum_{n=0}^{\infty} \frac{\left(\lambda t\right)^n e^{-\lambda t}}{n!} \tilde{H}_{ln}$$

s.t.

$$\tilde{G}^L_{la}(t) \leq \tilde{G}_l(t) \leq \tilde{G}^U_{la}(t)$$

$$\tilde{H}^L_{ln} \leq \tilde{H}_l \leq \tilde{H}^U_{ln}$$

and the upper bound

$$\tilde{R}^U_{la}(t) : \max \tilde{R}_l(t) = \tilde{G}_l(t) \sum_{n=0}^{\infty} \frac{\left(\lambda t\right)^n e^{-\lambda t}}{n!} \tilde{H}_{ln}$$

s.t.

$$\tilde{G}^L_{la}(t) \leq \tilde{G}_l(t) \leq \tilde{G}^U_{la}(t)$$

$$\tilde{H}^L_{ln} \leq \tilde{H}_l \leq \tilde{H}^U_{ln}$$
It should be noted that the fixed arriving rate $\lambda$ is used for all the components in a system. Because all the components suffer the same external random shocks simultaneously, but with different damage magnitudes.

**Fuzzy reliability evaluation for a multi-component system**

In the previous subsection, the fuzzy reliability of a single component is derived. In this section, we focus on the scenario that a system consists of multiple components. Actually, a complex system can be simplified to some basic structures such as series and parallel, and the system reliability indices can be calculated iteratively. Thus, the reliability functions of series and parallel systems can be computed, respectively, as follows.

The $\alpha$-cut level of fuzzy reliability for a series-connected system with $N$ components is given by $\tilde{R}_\alpha(t) = [\tilde{R}_L^L(t), \tilde{R}_U^U(t)]$. The lower and upper bounds can be calculated by

$$R_{\alpha}^L(t) = \prod_{l=1}^{N} \tilde{R}_{la}^L(t)$$

and

$$R_{\alpha}^U(t) = \prod_{l=1}^{N} \tilde{R}_{la}^U(t)$$

respectively, where $\tilde{R}_{la}^L(t)$ and $\tilde{R}_{la}^U(t)$ can be calculated by equations (27) and (28), respectively.

In the same fashion, the fuzzy reliability function of a parallel system at any $\alpha$-cut level with $N$ components is given by $\tilde{R}_\alpha(t) = [\tilde{R}_L^L(t), \tilde{R}_U^U(t)]$ with the lower bound

$$R_{\alpha}^L(t) = 1 - \prod_{l=1}^{N} (1 - R_{la}^L(t))$$

and the upper bound

$$R_{\alpha}^U(t) = 1 - \prod_{l=1}^{N} (1 - R_{la}^U(t))$$

where $\tilde{R}_{la}^L(t)$ and $\tilde{R}_{la}^U(t)$ can also be calculated by equations (27) and (28), respectively.

**Case studies**

**The approximating numerical algorithm**

In fact, it is very difficult to obtain the analytical solution for the system reliability from the convolution reliability function given in equations (21) and (22). In this
section, an approximating numerical algorithm of calculating equations (21) and (22) is derived. First, the approximating numerical algorithm is conducted under the fixed parameters associated with the external random shocks. Then, the approximating numerical algorithm is extended under the fuzzy set theory. The Poisson process has an arrival rate \( \lambda = 0.1 \), and thus the average arrival time between two random shocks is \( 1/\lambda = 10 \).

Under the fixed parameters associated with the external random shocks, the survival probability under the cumulative damage by shocks, denoted as \( \Pr\{X_1 + X_2 + \cdots + X_n \leq S\} \), can be expressed as a normal random variable as follows:

\[
\Pr\{X_1 + X_2 + \cdots + X_n \leq S\} = \Phi \left( \frac{S - (\mu_1 + \mu_2 + \cdots + \mu_n)}{\sqrt{\sigma_1^2 + \sigma_2^2 + \cdots + \sigma_n^2}} \right)
\]

where \( \Phi \) is the CDF of the standard normal distribution.

Based on the CDF of the Poisson distribution, a threshold value of \( N_L \) is assumed in our study; the number of the arrivals of shocks being more than \( t\lambda + N_L \) and less than \( t\lambda - N_L \) is ignorable. Hence, \( \Pr\{D(t) \leq S\} \) can be approximately calculated by \( \sum_{n=0}^{\infty} \frac{(\lambda t)^n e^{-\lambda t}}{n!} \Pr\{X_1 + X_2 + \cdots + X_n \leq S\} \)

\[
= \sum_{n=0}^{\infty} \frac{(\lambda t)^n e^{-\lambda t}}{n!} \Phi \left( \frac{S - (\mu(1-b^{-n}))}{\sqrt{\sigma^2(1-b^{-2n})}/(1-b^{-2})} \right)
\]

where \( \sum_{n=0}^{\infty} \Pr\{N(t) = n\} \) is approximated by \( \sum_{n=0}^{\infty} \Pr\{N(t) = n\} \).

If \( \Pr\{t\lambda - N_L \leq N(t) \leq t\lambda + N_L \} \) is close to 1, then the approximate result becomes accurate, meaning that the mostly possible situations are all included. Figure 2 demonstrates the sum of the probabilities of \( \Pr\{N(t) = k\} \) corresponding to \( t \), where \( t\lambda - N_L \leq k \leq t\lambda + N_L \) and \( N_L = 20 \). As shown in Figure 2, the minimum sum of the probabilities, where \( t \in [0, 500] \), is 0.99611, and it means that the probability of the number of shocks being more than 70 and less than 30 at \( t = 500 \) is only 0.00389. The minimum sum of the probabilities for different settings of \( N_L \) is tabulated in Table 1. Considering the precision and computational efficiency, \( N_L = 20 \) is used in our study.

The approximating numerical algorithm can be extended under the fuzzy set theory when the parameters associated with external random shocks are represented by TFNs. The \( \alpha \)-cut level of \( \tilde{P}_\alpha(D(t) \leq \tilde{S}) \) can be represented by \( [\tilde{P}_\alpha^L(D(t) \leq \tilde{S}), \tilde{P}_\alpha^U(D(t) \leq \tilde{S})] \) with the lower and upper bounds of \( \tilde{P}(D(t) \leq \tilde{S}) \) at any \( \alpha \)-cut level being calculated, respectively, by a pair of parametric programming as...
Subsequently, equations (35) and (36) will be used to approximately calculate the fuzzy survival probability of components caused by external random shocks.

\[
\tilde{P}_\alpha^L(D(t) \leq \tilde{S}) = \tilde{H}_{\text{ina}}^L = \min \int_{0}^{\infty} \sum_{n=1-N_l}^{\infty} \frac{(\lambda t)^ne^{-\lambda t}}{n!} \Phi\left(u - \frac{\left(\frac{1-b^{-n}}{1-b}\right)}{\sqrt{\sigma^2(1-b^{-2n})}}\right) d\tilde{S}(u)
\]

s.t. \( \tilde{S}_\alpha^L(u) \leq \tilde{S}_f(u) \leq \tilde{S}_\alpha^U(u), \mu_\alpha^L \leq \mu \leq \mu_\alpha^U, \sigma_\alpha^L \leq \sigma \leq \sigma_\alpha^U, b_\alpha^L \leq b \leq b_\alpha^U \)

and

\[
\tilde{P}_\alpha^U(D(t) \leq \tilde{S}) = \tilde{H}_{\text{ina}}^U = \max \int_{0}^{\infty} \sum_{n=1-N_l}^{\infty} \frac{(\lambda t)^ne^{-\lambda t}}{n!} \Phi\left(u - \frac{\left(\frac{1-b^{-n}}{1-b}\right)}{\sqrt{\sigma^2(1-b^{-2n})}}\right) d\tilde{S}(u)
\]

s.t. \( \tilde{S}_\alpha^L(u) \leq \tilde{S}_f(u) \leq \tilde{S}_\alpha^U(u), \mu_\alpha^L \leq \mu \leq \mu_\alpha^U, \sigma_\alpha^L \leq \sigma \leq \sigma_\alpha^U, b_\alpha^L \leq b \leq b_\alpha^U \)

Subsequently, equations (35) and (36) will be used to approximately calculate the fuzzy survival probability of components caused by external random shocks.
A multi-component system

The studied system is composed of three components, where components #2 and #3 are connected in parallel and then in series with component #1 (see Figure 3). As mentioned in section “Reliability evaluation under the fuzzy environment,” the internal degradation processes of components are governed by Weibull distributions. The time-dependent shape and scale parameters of component $l$ are given, respectively, by

$$\eta_l(t) = a_{l1} t^{b_{l1}} \exp(c_{l1} t)$$ (37)

and

$$\beta_l(t) = a_{l2} \left( \frac{1}{t} + 1 \right)^{b_{l2}} \exp\left(\frac{c_{l2}}{t}\right)$$ (38)

However, due to insufficient data and/or vague judgments from experts, the parameters associated with the time-dependent shape and scale parameters of component $l$ contain epistemic uncertainty and are represented by the TFNs under the fuzzy set theory. Then, the $\alpha$-cut level of the shape parameter of component $l$ can be represented as $\eta_{l\alpha}(t) = [\eta_{l\alpha}^L(t), \eta_{l\alpha}^U(t)]$ with the lower bound

$$\eta_{l\alpha}^L(t) = \min a_{l1} t^{b_{l1}} \exp(c_{l1} t)$$

s.t.

$$a_{l1\alpha}^L \leq a_{l1} \leq a_{l1\alpha}^U$$

$$b_{l1\alpha}^L \leq b_{l1} \leq b_{l1\alpha}^U$$

$$c_{l1\alpha}^L \leq c_{l1} \leq c_{l1\alpha}^U$$ (39)

and the upper bound

$$\eta_{l\alpha}^U(t) = \max a_{l1} t^{b_{l1}} \exp(c_{l1} t)$$

s.t.

$$a_{l1\alpha}^L \leq a_{l1} \leq a_{l1\alpha}^U$$

$$b_{l1\alpha}^L \leq b_{l1} \leq b_{l1\alpha}^U$$

$$c_{l1\alpha}^L \leq c_{l1} \leq c_{l1\alpha}^U$$ (40)
In the same fashion, the $\alpha$-cut level of the scale parameter of component $l$ can be represented as $\beta_{la}(t) = [\beta_{la}^L(t), \beta_{la}^U(t)]$ with the lower bound

$$\beta_{la}^L(t) = \min a_{l2} \left( \frac{1}{t} + 1 \right)^{b_{l2}} \exp \left( \frac{c_{l2}}{t} \right)$$

s.t.

$$a_{l2}^L \leq a_{l2} \leq a_{l2}^U$$
$$b_{l2}^L \leq b_{l2} \leq b_{l2}^U$$
$$c_{l2}^L \leq c_{l2} \leq c_{l2}^U$$

and the upper bound

$$\beta_{la}^U(t) = \max a_{l2} \left( \frac{1}{t} + 1 \right)^{b_{l2}} \exp \left( \frac{c_{l2}}{t} \right)$$

s.t.

$$a_{l2}^L \leq a_{l2} \leq a_{l2}^U$$
$$b_{l2}^L \leq b_{l2} \leq b_{l2}^U$$
$$c_{l2}^L \leq c_{l2} \leq c_{l2}^U$$

In this example, all the fuzzy parameters are represented by TFNs and are tabulated in Table 2. Based on equations (27) and (28), the fuzzy survival probability of each component can be calculated under the internal degradation processes. As an illustration, the fuzzy survival probability of each component at time $t = 10$ months under the internal degradation processes is depicted in Figure 4. As shown in

**Figure 4.** The fuzzy survival probability for each component under internal degradation processes.
Table 2. Fuzzy parameter settings for each component.

| Parameter | Component #1 | Component #2 | Component #3 |
|-----------|--------------|--------------|--------------|
| \(a_1\)   | \((0.6552, 0.6652, 0.6752)\) | \((1.2366, 1.2466, 1.2566)\) | \((0.8227, 0.8327, 0.8427)\) |
| \(b_1\)   | \((0.3853, 0.3953, 0.4053)\) | \((0.3113, 0.3213, 0.3313)\) | \((0.2450, 0.2550, 0.2650)\) |
| \(c_1\)   | \((0.0019, 0.0029, 0.0039)\) | \((0.0031, 0.0041, 0.0051)\) | \((0.0041, 0.0051, 0.0061)\) |
| \(a_2\)   | \((6.446, 6.546, 6.646)\) | \((-529.96, -528.96, -527.96)\) | \((541.128, 542.128, 543.128)\) |
| \(b_2\)   | \((-529.96, -528.96, -527.96)\) | \((-353.12, -352.12, -351.12)\) | \((-353.12, -352.12, -351.12)\) |
| \(c_2\)   | \((489.25, 490.25, 491.25)\) | \((611.51, 612.51, 613.51)\) | \((611.51, 612.51, 613.51)\) |

\(F(t)\) for each component.

\(N_0^{(9, 10, 11)}(0.09, 0.1, 0.111)\)
Figure 4, all the survival probabilities under the internal degradation processes are extended to fuzzy numbers. Component #3 has the largest fuzzy survival probability, whereas component #2 has the smallest fuzzy survival probability under internal degradation processes.

Furthermore, based on equations (35) and (36) and the approximating numerical algorithm, the fuzzy survival probability of each component can be calculated under the external random shocks. The results are delineated in Figure 5. As shown in Figure 5, all the survival probabilities under the external random shocks are extended to fuzzy numbers and component #2 has the smallest fuzzy survival probability under the external random shocks.

The fuzzy reliability functions of components #1, #2, and #3 can be calculated by equations (27) and (28), and the results are shown in Figure 6, respectively. Based on the series–parallel configuration of the system, the fuzzy system reliability function is given by

\[
\tilde{R}(t) = \tilde{R}_1(t) \cdot \left[1 - \left(1 - \tilde{R}_2(t)\right) \cdot \left(1 - \tilde{R}_3(t)\right)\right] \\
= \sum_{n=\lambda t - 20}^{\lambda t + 20} \frac{(\lambda t)^n e^{-\lambda t}}{n!} \left(\tilde{G}_1(t)\tilde{H}_{1n} \cdot \left(1 - \left(1 - \tilde{G}_2(t)\tilde{H}_{2n}\right) \cdot \left(1 - \tilde{G}_3(t)\tilde{H}_{3n}\right)\right)\right)
\]  

(43)

Therefore, the \(\alpha\)-cut level of the fuzzy system reliability can be represented as \(\tilde{R}_\alpha^L(t), \tilde{R}_\alpha^U(t)\] with the lower bound
By setting time instant $t$ in equations (43) and (44) to be any time of interest, the fuzzy system reliability over time can be evaluated. The results, as shown in
Figure 7, indicate that the system reliability extended to fuzzy values due to the epistemic uncertainty of the insufficient data and/or vague judgments from experts.

**Conclusion and future work**

In this article, the fuzzy reliability evaluation for systems suffering two competing failure modes, that is, internal continuous degradation process and external random shocks, was conducted. The internal continuous degradation process was treated as a continuous monotonically increasing random process with respect to operating time, whereas the geometric process was utilized to characterize the increasing cumulative damage caused by each shock. All the parameters associated with the internal continuous degradation process and external random shocks were represented by TFNs. The fuzzy reliability function of the series–parallel systems was formulated by a set of optimization models. Two numerical examples were presented to illustrate the implementation of the proposed method and the approximating numerical algorithm. The result from the approximating numerical algorithm was very close to that of the analytical method.

It is worth noting that there are still some works needed to be explored in the future. First, it is worth considering to relax the independent property of continuous and discrete (external random shocks) degradation processes. One may employ the discretization method to represent the continuous process through a discrete process with finite state space, and then the system state is determined based on the combinational matrix of continuous and discrete degradation processes. Another promising direction is to examine the availability of the system under various
inspection policies, and how to minimize the inspection cost and maximize the availability under different criteria is worth discussing.

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**ORCID iD**

Mao Tang https://orcid.org/0000-0003-4737-3579

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**Author biographies**

**Hongping Yu** is currently an Associate Professor in the School of Mechanical Engineering, Chengdu University. His research interests include computer-aided design, computer-aided manufacturing, mechanical reliability, and mechanical optimization design.

**Mao Tang** is currently an Associate Professor in the School of Mechanical Engineering, Chengdu University. He has authored or coauthored many papers in international conferences and published some books in mechanical design. His research interests include advanced manufacturing, electromechanical system integration, and reliability engineering.

**Appendix 1**

**Notation**

| Symbol | Description |
|--------|-------------|
| $\tilde{b}_l$ | fuzzy coefficient of the geometric process of component $l$ |
| $\tilde{D}_l(t)$ | fuzzy cumulative damage of component $l$ at time $t$ resulting from the external random shocks |
| $\tilde{F}_l(t)$ | fuzzy distribution of the cumulative damage resulting from the random shocks to component $l$ |
| $\tilde{L}_l$ | fuzzy random threshold of component $l$ for the internal degradation process |
| $N$ | total number of components in a system |
| $\tilde{R}(t)$ | fuzzy system reliability function |
| $\tilde{R}_l(t)$ | fuzzy reliability function of component $l$ at time instant $t$ |
| $\tilde{S}_l$ | fuzzy random threshold of component $l$ for the cumulative damage from shocks |
| $\tilde{X}_{li}$ | fuzzy cumulative random damage caused from the $i$th random shocks to component $l$ |
| $\tilde{Y}_l(t)$ | fuzzy internal continuous degradation process for component $l$ |
| $\leq_{st}$ | strictly satisfies the inequality. |
| $\tilde{\beta}_l(t)$ | fuzzy shape parameter of the Weibull distribution of component $l$ at time $t$ |
| $\tilde{\eta}_l(t)$ | fuzzy scale parameter of the Weibull distribution of component $l$ at time $t$ |