Multiple seesaw mechanisms of neutrino masses at the TeV scale

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Abstract

In pursuit of a balance between theoretical naturalness and experimental testability, we propose two classes of multiple seesaw mechanisms at the TeV scale to understand the origin of tiny neutrino masses. They are novel extensions of the canonical and double seesaw mechanisms, respectively, by introducing even and odd numbers of gauge-singlet fermions and scalars. It is thanks to a proper implementation of the global $U(1) \times Z_{2N}$ symmetry that the overall neutrino mass matrix in either class has a suggestive nearest-neighbor-interaction pattern. We briefly discuss possible consequences of these TeV-scale seesaw scenarios, which can hopefully be explored in the upcoming Large Hadron Collider and precision neutrino experiments, and present a simple but instructive example of model building.

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Solar, atmospheric, reactor and accelerator neutrino oscillation experiments have jointly provided us with very convincing evidence that three known neutrinos in the Universe must possess tiny and non-degenerate rest masses \([1]\). This great breakthrough is hopefully opening a low-energy window onto new physics beyond the Standard Model (SM) at very high energy scales. So far many theoretical and phenomenological attempts have been made towards understanding the observed neutrino mass hierarchy and lepton flavor mixing, and among them the seesaw ideas \([2, 3, 4]\) are most brilliant and might even lead us to a true theory of neutrino masses.

The canonical (type-I) seesaw mechanism \([2]\) can naturally work at a superhigh energy scale \(\Lambda_{SS} \sim 10^{14} \text{ GeV}\) to generate tiny neutrino masses of order \(\Lambda_{EW}^2 / \Lambda_{SS} \sim 0.1 \text{ eV}\) with \(\Lambda_{EW} \sim 10^2 \text{ GeV}\) being the electroweak scale. To be more specific, the effective Majorana mass matrix of three light neutrinos is given by

\[
M_\nu = -M_D M_R^{-1} M_D^T \tag{1}
\]

in the leading-order approximation, where \(M_D \sim \mathcal{O}(\Lambda_{EW})\) originates from the Yukawa interactions between the SM lepton doublet \(\ell_L\) and the right-handed neutrinos \(N_R^i\) (for \(i = 1, 2, 3\)), and \(M_R \sim \mathcal{O}(\Lambda_{SS})\) is a symmetric matrix coming from the lepton-number-violating Majorana mass term of \(N_R^i\). This seesaw picture is technically natural because it allows the relevant Yukawa couplings to be \(\mathcal{O}(1)\) and requires little fine-tuning of the textures of \(M_D\) and \(M_R\), but it loses the direct testability on the experimental side and causes a hierarchy problem on the theoretical side (as long as \(\Lambda_{SS} > 10^7 \text{ GeV}\) \([5]\)).

A possible way out of the impasse is to lower the seesaw scale down to \(\Lambda_{SS} \sim 1 \text{ TeV}\), an energy frontier to be soon explored by the Large Hadron Collider (LHC). However, to test such a TeV seesaw scenario necessitates an appreciable magnitude of \(M_D / M_R\) so as to make it possible to produce and detect heavy Majorana neutrinos at the LHC via their charged-current interactions. This prerequisite unavoidably requires a terrible fine-tuning of \(M_D\) and \(M_R\), because one has to impose \(M_R \sim 1 \text{ TeV}\), \(M_D / M_R \sim 10^{-3} \cdots 10^{-1}\) and \(M_\nu \sim 0.1 \text{ eV}\) simultaneously on the above seesaw relation \([6]\). It is therefore desirable to invoke new ideas to resolve this unnaturalness problem built in the TeV seesaw mechanism.

We stress that a multiple seesaw mechanism at the TeV scale may satisfy both naturalness and testability requirements. To illustrate, we assume that the small mass scale of three light neutrinos arises from a naive seesaw relation \(m \sim (\lambda \Lambda_{EW})^{n+1} / \Lambda_{SS}^n\), where \(\lambda\) is a dimensionless Yukawa coupling coefficient and \(n\) is an arbitrary integer larger than one. Without any terrible fine-tuning, the seesaw scale can be estimated from

\[
\Lambda_{SS} \sim \lambda^{\frac{n+1}{n}} \left[ \frac{\Lambda_{EW}}{100 \text{ GeV}} \right]^{\frac{n+1}{n}} \left[ \frac{0.1 \text{ eV}}{m} \right] \frac{1}{10^{\frac{2(n+6)}{n}}} \text{ GeV} .
\]

A numerical change of \(\Lambda_{SS}\) with \(n\) and \(\lambda\) is shown in Fig. 1, where \(\Lambda_{SS} \sim 1 \text{ TeV}\) may naturally result from \(n \geq 2\) and \(\lambda \geq 10^{-3}\). Hence the multiple seesaw idea is expected to work at the TeV scale and provide us with a novel approach to bridge the gap between theoretical naturalness and experimental testability of the canonical seesaw mechanism.

The simplest way to build a multiple seesaw model at the TeV scale is to extend the canonical seesaw mechanism by introducing a number of gauge-singlet fermions \(S_{nR}\) and
scalars $\Phi_n$ (for $i = 1, 2, 3$ and $n = 1, 2, \cdots$). We find that a proper implementation of the global $U(1) \times \mathbb{Z}_{2N}$ symmetry leads us to two classes of multiple seesaw mechanisms with the nearest-neighbor-interaction pattern — an interesting form of the overall $3(n + 2) \times 3(n + 2)$ neutrino mass matrix in which every $3 \times 3$ sub-matrix only interacts with its nearest neighbor. The first class contains an even number of $S^i_{nR}$ and $\Phi_n$ and corresponds to an appealing extension of the canonical seesaw mechanism, while the second class has an odd number of $S^i_{nR}$ and $\Phi_n$ and is actually a straightforward extension of the double seesaw mechanism [7]. Their possible collider signatures and low-energy consequences, together with a simple example of model building, will be briefly discussed.

The spirit of multiple seesaw mechanisms is to make a harmless extension of the SM by adding three right-handed neutrinos $N^c_R$ together with some gauge-singlet fermions $S^i_{nR}$ and scalars $\Phi_n$ (for $i = 1, 2, 3$ and $n = 1, 2, \cdots$). Allowing for lepton number violation to a certain extent, we can write the gauge-invariant Lagrangian for neutrino masses as

$$- \mathcal{L}_\nu = \bar{\ell}_L Y_\nu \tilde{H} N^c_R + \bar{N}^c_R Y_{S_i} S_{1R} \Phi_1 + \sum_{i=2}^n \overline{S^i_{(i-1)R}} Y_{S_i} S^i_{nR} \Phi_1 + \frac{1}{2} S^n_{nR} M_\mu S^n_{nR} + \text{h.c.},$$

where $\ell_L$ and $\tilde{H} \equiv i\sigma_2 H^*$ stand respectively for the SU(2)$_L$ lepton and Higgs doublets, $Y_\nu$ and $Y_{S_i}$ (for $i = 1, 2, \cdots, n$) are the $3 \times 3$ Yukawa coupling matrices, and $M_\mu$ is a symmetric Majorana mass matrix. After spontaneous gauge symmetry breaking, we arrive at the overall $3(n + 2) \times 3(n + 2)$ neutrino mass matrix $\mathcal{M}$ in the flavor basis defined by $(\nu_L, N^c_R, S^c_{1R}, \cdots, S^c_{nR})$ and their charge-conjugate states:

$$\mathcal{M} = \begin{pmatrix}
0 & M_D & 0 & 0 & 0 & \cdots & 0 \\
M_D^T & 0 & M_{S_1} & 0 & 0 & \cdots & 0 \\
0 & M_{S_1}^T & 0 & M_{S_2} & 0 & \cdots & 0 \\
0 & 0 & M_{S_2}^T & 0 & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & \cdots & \cdots & \cdots \\
\vdots & \vdots & \vdots & \cdots & M_{S_{n-1}}^T & 0 & M_{S_n} \\
0 & 0 & 0 & \cdots & \cdots & 0 & M_{S_n}^T \\
0 & 0 & 0 & \cdots & 0 & 0 & M_{\mu}
\end{pmatrix},$$

where $M_D \equiv Y_\nu \langle H \rangle$ and $M_{S_i} = Y_{S_i} \langle \Phi_i \rangle$ (for $i = 1, 2, \cdots, n$) are $3 \times 3$ mass matrices. Setting $N_R = S_{0R}$ for simplicity, one can observe that the Yukawa interactions between $S^i_{1R}$ and $S^i_{nR}$ exist if and only if their subscripts satisfy the selection rule $|i - j| = 1$ (for $i, j = 0, 1, 2, \cdots, n$). Note that $\mathcal{M}$ manifests a very suggestive nearest-neighbor-interaction pattern, which has attracted a lot of attention in the quark sector to understand the observed hierarchies of quark masses and flavor mixing angles [8]. Such a special structure of $\mathcal{M}$, or equivalently that of $\mathcal{L}_\nu$ in Eq. (2), may arise from a proper implementation of the global $U(1) \times \mathbb{Z}_{2N}$ symmetry. The unique generator of the cyclic group $\mathbb{Z}_{2N}$ is $\omega = e^{i\pi/N}$, which produces all the group elements $\mathbb{Z}_{2N} = \{1, \omega, \omega^2, \omega^3, \cdots, \omega^{2N-1}\}$. By definition, a field $\Psi$ with the charge $q$ transforms as $\Psi \rightarrow e^{i\pi q/N} \Psi$ under $\mathbb{Z}_{2N}$ (for $q = 0, 1, 2, \cdots, 2N - 1$). Hence we manage to assign the U(1) and $\mathbb{Z}_{2N}$ charges of the relevant fields in Eq. (2) in the following way:
1. The global U(1) symmetry can be identified with the lepton number \(L\), namely \(L(\ell_L) = L(E_R) = +1\), where \(E_R\) represents the charged-lepton singlets in the SM. We arrange the lepton numbers of gauge-singlet fermions and scalars to be \(L(N_R) = +1\), \(L(S_{iR}) = (-1)^k\) and \(L(\Phi_k) = 0\) (for \(k = 1, 2, \cdots, n\)). It turns out that only the Majorana mass term \(\overline{S}_{iR}M_\mu S_{iR}\) in \(\mathcal{L}_\nu\) explicitly violates the U(1) symmetry. After this assignment, other lepton-number-violating mass terms (e.g., \(\overline{N_R}M_R^T N_R\) in the canonical seesaw mechanism) may also appear in the Lagrangian, but they can be eliminated by invoking the discrete \(\mathbb{Z}_{2N}\) symmetry.

2. We assign the \(\mathbb{Z}_{2N}\) charge of \(S_{iR}\) as \(q(S_{iR}) = N\). Then it is easy to verify that the Majorana mass term \(\overline{S}_{iR}M_\mu S_{iR}\) is invariant under the \(\mathbb{Z}_{2N}\) transformation. If all the other gauge-singlet fermions \(S_{i\ell}\) (for \(k = 1, 2, \cdots, n - 1\)) take any charges in \(\{1, 2, \cdots, 2N - 1\}\) other than \(N\), their corresponding Majorana mass terms are accordingly forbidden. Given \(q(\ell_L) = q(E_R) = q(N_R) = 1\), both the charges of \(S_{i\ell}\) (for \(k = 1, 2, \cdots, n - 1\)) and those of \(\Phi_i\) (for \(i = 1, 2, \cdots, n\)) can be properly chosen so as to achieve the nearest-neighbor-interaction form of \(\mathcal{L}_\nu\) as shown in Eq. (2). But the solution to this kind of charge assignment may not be unique, because for a given value of \(n\) one can always take \(N \gg n\) to fulfill all the above-mentioned requirements [9]. Simple examples (with \(n = 1, 2, 3\)) will be presented below.

We remark that our multiple seesaw picture should be the simplest extension of the canonical seesaw mechanism, since it does not invoke the help of either additional SU(2)\(_L\) fermion doublets [10] or a new isospin 3/2 Higgs multiplet [11]. We also stress that the double seesaw scenario [7] is only the simplest example in one class of our multiple seesaw mechanisms (with an odd number of \(S_{iR}\) or \(\Phi_n\)) and cannot reflect any salient features of the other class of multiple seesaw mechanisms (with an even number of \(S_{iR}\) or \(\Phi_n\)).

Now let us diagonalize \(\mathcal{M}\) in Eq. (3) to achieve the effective mass matrix of three light neutrinos \(M_\nu\) in the multiple seesaw mechanisms. Note that \(\mathcal{M}\) can be rewritten as

\[
\mathcal{M} = \begin{pmatrix}
0 & \tilde{M}_D^T \\
\tilde{M}_D & \tilde{M}_\mu
\end{pmatrix},
\]

where \(\tilde{M}_D = (M_D \ 0)\) denotes a \(3 \times 3(n + 1)\) matrix and \(\tilde{M}_\mu\) is a symmetric \(3(n + 1) \times 3(n + 1)\) matrix. Taking the mass scale of \(\tilde{M}_\mu\) to be much higher than that of \(\tilde{M}_D\), one can easily obtain \(M_\nu = -\tilde{M}_D\tilde{M}_\mu^{-1}\tilde{M}_D^T\) for three light Majorana neutrinos in the leading-order approximation. Because the elements in the fourth to \(3n\)-th columns of \(\tilde{M}_D\) are exactly zero, only the \(3 \times 3\) top left block of \(\tilde{M}_\mu^{-1}\) is relevant to the calculation of \(M_\nu\). Without loss of generality, the inverse of \(\tilde{M}_\mu\) can be figured out by assuming all the non-zero \(3 \times 3\) sub-matrices of \(\mathcal{M}\) to be of rank three. We find two types of solutions [9], depending on whether \(n\) is even or odd, and thus arrive at two classes of multiple seesaw mechanisms:

Class A of multiple seesaw mechanisms — they contain an even number of gauge-singlet fermions \(S_{iR}\) and scalars \(\Phi_n\) (i.e., \(n = 2k\) with \(k = 0, 1, 2, \cdots\)) and correspond to a novel extension of the canonical seesaw picture. The effective mass matrix of three light
Majorana neutrinos is given by

\[ M_\nu = -M_D \left[ \prod_{i=1}^{k} \left( M_{S_{2i-1}}^T \right)^{-1} M_{S_{2i}} \right] M_\mu^{-1} \left[ \prod_{i=1}^{k} \left( M_{S_{2i-1}}^T \right)^{-1} M_{S_{2i}} \right]^T M_D^T \]  

(5)

This effective multiple seesaw mass term is illustrated in Fig. 2(a). The nearest-neighbor-interaction pattern of \( \mathcal{M} \) with \( n = 2 \) can be obtained by imposing the global U(1) × \( Z_6 \) symmetry on \( \mathcal{L}_\nu \), in which the proper charge assignment is listed in Table 1.

\[ \Phi_1, \Phi_2 \]

Table 1: The charges of relevant fermion and scalar fields under the U(1) × \( Z_6 \) symmetry in the multiple seesaw mechanism with \( n = 2 \).

| \( \ell_\nu \) | \( H \) | \( E_R \) | \( N_R \) | \( S_{1R} \) | \( S_{2R} \) | \( \Phi_1 \) | \( \Phi_2 \) |
|------|------|------|------|--------|--------|--------|--------|
| \( L \) | +1   | 0    | +1   | −1     | +1     | 0      | 0      |
| \( q \) | +1   | 0    | +1   | +2     | +3     | +3     | 1      |

In the multiple seesaw mechanism (i.e., \( M_\nu = -M_D M_R^{-1} M_D^T \) by setting \( S_{0R} = N_R \) and \( M_\mu = M_R \)). If \( M_{S_{2i}} \sim M_D \sim \mathcal{O}(\Lambda_{EW}) \) and \( M_{S_{2i-1}} \sim M_\mu \sim \mathcal{O}(\Lambda_{SS}) \) hold (for \( i = 1, 2, \ldots, k \)), Eq. (5) leads to \( M_\nu \sim \Delta_{EW}^{2(k+1)} / \Lambda_{SS}^{2k+1} \), which can effectively lower the conventional seesaw scale \( \Lambda_{SS} \sim 10^{14} \text{ GeV} \) down to the TeV scale as illustrated in Fig. 1.

Taking \( n = 2 \) (i.e., \( k = 1 \)) for example [12], we arrive at the minimal extension of the canonical seesaw mechanism:

\[ M_\nu = -M_D \left( M_{S_1}^T \right)^{-1} M_{S_2} M_\mu^{-1} M_{S_2} \left( M_{S_1} \right)^{-1} M_D^T . \]  

(6)

This effective multiple seesaw mass term is illustrated in Fig. 2(a). The nearest-neighbor-interaction pattern of \( \mathcal{M} \) with \( n = 2 \) can be obtained by imposing the global U(1) × \( Z_6 \) symmetry on \( \mathcal{L}_\nu \), in which the proper charge assignment is listed in Table 1.

Class B of multiple seesaw mechanisms — they contain an odd number of gauge-singlet fermions \( S^i_{1R} \) and scalars \( \Phi_n \) (i.e., \( n = 2k + 1 \) with \( k = 0, 1, 2, \ldots \) and correspond to an interesting extension of the double seesaw picture. The effective mass matrix of three light Majorana neutrinos reads

\[ M_\nu = M_D \left[ \prod_{i=1}^{k} \left( M_{S_{2i-1}}^T \right)^{-1} M_{S_{2i}} \right] \left( M_{S_{2k+1}}^T \right)^{-1} M_\mu \left( M_{S_{2k+1}} \right)^{-1} \left[ \prod_{i=1}^{k} \left( M_{S_{2i-1}}^T \right)^{-1} M_{S_{2i}} \right]^T M_D^T \]  

(7)

in the leading-order approximation. The \( k = 0 \) case just corresponds to the double seesaw scenario with a very low mass scale of \( M_\mu \) [7]: \( M_\nu = M_D \left( M_{S_1}^T \right)^{-1} M_\mu \left( M_{S_1} \right)^{-1} M_D^T \). Note that the nearest-neighbor-interaction pattern of \( \mathcal{M} \) in the double seesaw mechanism is guaranteed by an implementation of the global U(1) × \( Z_4 \) symmetry with the following charge assignment: \( L(\ell_\nu) = L(E_R) = L(N_R) = +1 \), \( L(S_{1R}) = -1 \), \( L(H) = L(\Phi_1) = 0 \), \( q(\ell_L) = q(E_R) = q(N_R) = q(\Phi_1) = +1 \), \( q(H) = 0 \) and \( q(S_{1R}) = +2 \).

If \( M_{S_{2i}} \sim M_D \sim \mathcal{O}(\Lambda_{EW}) \) and \( M_{S_{2i-1}} \sim \mathcal{O}(\Lambda_{SS}) \) hold (for \( i = 1, 2, \ldots, k \)), the mass scale of \( M_\mu \) is in general unnecessary to be as small as that given by the double seesaw mechanism. To be more specific, let us consider the minimal extension of the double
seesaw picture by taking \( n = 3 \). In this case, we impose the \( U(1) \times Z_{10} \) symmetry on \( \mathcal{L}_\nu \) with a proper charge assignment listed in Table 2 to assure the nearest-neighbor-interaction form of \( \mathcal{M} \). The corresponding formula of \( M_\nu \) is

\[
M_\nu = M_D \left( M_{S_1}^T \right)^{-1} M_{S_2} \left( M_{S_3}^T \right)^{-1} M_\mu \left( M_{S_3}^T \right)^{-1} M_D^T.
\]

(8)

This effective multiple seesaw mass term is illustrated in Fig. 2(b). It becomes obvious that the proportionality of \( M_\nu \) to \( M_\mu \) in Eq. (8) is doubly suppressed not only by the ratio \( M_D/M_{S_1} \sim \Lambda_{\text{EW}}/\Lambda_{\text{SS}} \) but also by the ratio \( M_{S_2}/M_{S_3} \sim \Lambda_{\text{EW}}/\Lambda_{\text{SS}} \), and thus \( M_\nu \sim 0.1 \text{ eV} \) can naturally result from \( Y_\nu \sim Y_{S_1} \sim Y_{S_2} \sim Y_{S_3} \sim \mathcal{O}(1) \) and \( M_\mu \sim 1 \text{ keV} \) at \( \Lambda_{\text{SS}} \sim 1 \text{ TeV} \).

Charged-current interactions of neutrinos — they are important for both production and detection of light and heavy Majorana neutrinos in a realistic experiment. To define the neutrino mass eigenstates, we diagonalize the overall mass matrix \( \mathcal{M} \) in Eq. (4) by means of the following unitary transformation:

\[
\begin{pmatrix}
V & \tilde{R} \\
\tilde{S} & \tilde{U}
\end{pmatrix}
\begin{pmatrix}
0 & \tilde{M}_D \\
\tilde{M}_D^T & \tilde{M}_\mu
\end{pmatrix}
\begin{pmatrix}
V & \tilde{R} \\
\tilde{S} & \tilde{U}
\end{pmatrix}^* =
\begin{pmatrix}
\tilde{M}_\nu & 0 \\
0 & \tilde{M}_{N+S}
\end{pmatrix},
\]

(9)

where \( \tilde{M}_\nu \equiv \text{Diag}\{m_1, m_2, m_3\} \) contains the masses of three light Majorana neutrinos (\( \tilde{\nu}_1, \tilde{\nu}_2, \tilde{\nu}_3 \)), and \( \tilde{M}_{N+S} \) denotes a diagonal matrix whose eigenvalues are the masses of \( 3(n+1) \) heavy Majorana neutrinos (\( \tilde{N}, \tilde{S}_1, \cdots, \tilde{S}_n; \) and each of them consists of three components). The SM charged-current interactions of \( \nu_e, \nu_\mu \) and \( \nu_\tau \) can therefore be expressed, in terms of the mass eigenstates of light and heavy Majorana neutrinos, as

\[
-L_{cc} = \frac{g}{\sqrt{2}} (e^{-\mu - \tau})_L \gamma^\mu \left[ V \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \\ \tilde{\nu}_3 \end{pmatrix}_L + \tilde{R} \begin{pmatrix} \tilde{N} \\ \tilde{S}_1 \\ \vdots \\ \tilde{S}_n \end{pmatrix}_L \right] W^- + \text{h.c.}
\]

(10)

in the basis where the mass eigenstates of three charged leptons are identified with their flavor eigenstates. Note that \( V \) is the \( 3 \times 3 \) neutrino mixing matrix responsible for neutrino oscillations, while the \( 3 \times 3(n+1) \) matrix \( \tilde{R} \) governs the strength of charged-current interactions of heavy Majorana neutrinos. Note also that both \( VV^\dagger + \tilde{R}\tilde{R}^\dagger = 1 \) and \( V\tilde{M}_\nu V^T + \tilde{R}\tilde{M}_{N+S}\tilde{R}^T = 0 \) hold, and thus \( V \) must be non-unitary. It is \( \tilde{R} \) that measures the deviation of \( V \) from unitarity in neutrino oscillations and determines the collider signatures of heavy Majorana neutrinos at the LHC.
We expect that our multiple seesaw idea can lead to rich phenomenology at both the TeV scale and lower energies. For simplicity, here we only mention a few aspects of the phenomenological consequences of multiple seesaw mechanisms.

- **Non-unitary neutrino mixing and CP violation.** Since $V$ is non-unitary, it generally involves a number of new flavor mixing parameters and new CP-violating phases [13]. Novel CP-violating effects in the medium-baseline $\nu_\mu \rightarrow \nu_\tau$ and $\nu_\mu \rightarrow \nu_\tau$ oscillations may therefore show up and provide a promising signature of the unitarity violation of $V$, which could be measured at a neutrino factory [14].

- **Signatures of heavy Majorana neutrinos at the LHC.** Given $\tilde{M}_D \sim \mathcal{O}(\Lambda_{EW})$ and $\tilde{M}_\mu \sim \mathcal{O}(\Lambda_{SS}) \sim \mathcal{O}(1)$ TeV, it is straightforward to obtain $\tilde{R} \approx \tilde{M}_D \tilde{M}_\mu^{-1} \tilde{U} \sim \mathcal{O}(0.1)$, which actually saturates the present experimental upper bound on $|\tilde{R}|$ [15]. For Class A of multiple seesaw mechanisms, their clear LHC signatures are expected to be the like-sign dilepton events arising from the lepton-number-violating processes $pp \rightarrow l_\alpha^{\pm}l_\beta^{\pm}X$ (for $\alpha, \beta = e, \mu, \tau$) mediated by heavy Majorana neutrinos [16]. For Class B of multiple seesaw mechanisms with $M_\mu \ll \Lambda_{EW}$, the mass spectrum of heavy Majorana neutrinos generally exhibits a pairing phenomenon in which the nearest-neighbor Majorana neutrinos have nearly degenerate masses and can be combined to form pseudo-Dirac particles. This feature has already been observed in the double seesaw model [7]. Therefore, the discriminating collider signatures at the LHC are expected to be the $pp \rightarrow l_\alpha^{\pm}l_\beta^{\pm}l_\gamma^{\pm}X$ processes (for $\alpha, \beta, \gamma = e, \mu, \tau$) [17].

- **Possible candidates for dark matter.** One or more of the heavy Majorana neutrinos and gauge-singlet scalars in our multiple seesaw mechanisms could be arranged to have a sufficiently long lifetime. Such weakly-interacting and massive particles might therefore be a plausible candidate for cold dark matter [18].

One may explore more low-energy effects of multiple seesaw mechanisms, such as their contributions to the lepton-flavor-violating processes $\mu \rightarrow e\gamma$ and so on. It should also be interesting to explore possible baryogenesis via leptogenesis [19], based on a multiple seesaw picture, to interpret the cosmological matter-antimatter asymmetry.

As a flexible and testable TeV seesaw scheme, the multiple seesaw mechanisms can also provide us with plenty of room for model building. But the latter requires further inputs or assumptions. Here we present a simple but instructive example, in which all the textures of $3 \times 3$ sub-matrices in the overall neutrino mass matrix $M$ are symmetric and have the well-known Fritzsch pattern [3].

\[
M_a = \begin{pmatrix}
0 & x_a & 0 \\
x_a & 0 & y_a \\
0 & y_a & z_a
\end{pmatrix}
\]

with $a = D, S_1, \cdots, S_n$ or $\mu$, for illustration. Choosing the Fritzsch texture makes sense because it coincides with the nearest-neighbor-interaction form of $M$ itself. We make an
additional assumption that the ratio \( x_a/y_a \) is a constant independent of the subscript \( a \). Then it is easy to show that the effective mass matrix of three light Majorana neutrinos \( M_\nu \) has the same Fritzsch texture in the leading-order approximation:

\[
M_\nu = - \left( \begin{array}{ccc}
0 & \frac{x_D^2}{x_\mu} \left[ \prod_{i=1}^{k} \frac{x_{S_{2i}}^2}{x_{S_{2i-1}}^2} \right] & \frac{y_D^2}{y_\mu} \left[ \prod_{i=1}^{k} \frac{y_{S_{2i}}^2}{y_{S_{2i-1}}^2} \right] \\
\frac{x_D^2}{x_{S_{2k+1}}} \left[ \prod_{i=1}^{k} \frac{x_{S_{2i}}^2}{x_{S_{2i-1}}^2} \right] & 0 & \frac{y_D^2}{y_{S_{2k+1}}} \left[ \prod_{i=1}^{k} \frac{y_{S_{2i}}^2}{y_{S_{2i-1}}^2} \right] \\
0 & \frac{y_D^2}{y_{S_{2k+1}}} \left[ \prod_{i=1}^{k} \frac{y_{S_{2i}}^2}{y_{S_{2i-1}}^2} \right] & 0 
\end{array} \right) \tag{12}
\]

derived from Eq. (5) for Class A of multiple seesaw mechanisms (with \( n = 2k \) for \( k = 0,1,2,\cdots \)); and

\[
M_\nu = \left( \begin{array}{ccc}
0 & \frac{x_D^2}{x_{S_{2k+1}}} \left[ \prod_{i=1}^{k} \frac{x_{S_{2i}}^2}{x_{S_{2i-1}}^2} \right] & \frac{y_D^2}{y_\mu} \left[ \prod_{i=1}^{k} \frac{y_{S_{2i}}^2}{y_{S_{2i-1}}^2} \right] \\
\frac{x_D^2}{x_{S_{2k+1}}} \left[ \prod_{i=1}^{k} \frac{x_{S_{2i}}^2}{x_{S_{2i-1}}^2} \right] & 0 & \frac{y_D^2}{y_{S_{2k+1}}} \left[ \prod_{i=1}^{k} \frac{y_{S_{2i}}^2}{y_{S_{2i-1}}^2} \right] \\
0 & \frac{y_D^2}{y_{S_{2k+1}}} \left[ \prod_{i=1}^{k} \frac{y_{S_{2i}}^2}{y_{S_{2i-1}}^2} \right] & 0 
\end{array} \right) \tag{13}
\]

generated from Eq. (7) for Class B of multiple seesaw mechanisms (with \( n = 2k + 1 \) for \( k = 0,1,2,\cdots \)). This seesaw-invariant property of \( M_\nu \) is interesting since it exactly reflects how two classes of multiple seesaw mechanisms work for every element of \( M_\nu \). Note that it is possible to interpret current experimental data on small neutrino masses and large flavor mixing angles by taking both the texture of the light neutrino mass matrix \( M_\nu \) and that of the charged-lepton mass matrix \( M_l \) to be of the Fritzsch form [20]. Hence the above examples are phenomenologically viable. Once the texture of \( M_\nu \) is fully reconstructed from more accurate neutrino oscillation data, one may then consider to quantitatively explore the textures of those \( 3 \times 3 \) sub-matrices of \( M \) in such a multiple seesaw model.

To conclude, new ideas are eagerly wanted in the upcoming LHC era to achieve a proper balance between theoretical naturalness and experimental testability of the elegant seesaw pictures, which ascribe the small masses of three known neutrinos to the existence of some heavy degrees of freedom. In the present work we have extended the canonical and double seesaw scenarios and proposed two classes of multiple seesaw mechanisms at the TeV scale by introducing an arbitrary number of gauge-singlet fermions and scalars into the SM and by implementing the global U(1) \( \times Z_{2N} \) symmetry in the neutrino sector. These new TeV-scale seesaw mechanisms are expected to lead to rich phenomenology at low energies and open some new prospects for understanding the origin of tiny neutrino masses and lepton number violation.

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Figure 1: A numerical illustration of the seesaw scale $\Lambda_{SS}$ changing with $n$ and $\lambda$ as specified in Eq. (1). Here the horizontal line stands for the TeV scale.

Figure 2: The origin of light Majorana neutrino masses in multiple seesaw mechanisms: (a) the minimal extension of the canonical seesaw mechanism (with $n = 2$); and (b) the minimal extension of the double seesaw mechanism (with $n = 3$).