Dynamics of rotated spin states and magnetic ordering with two-component bosonic atoms in optical lattices

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(Dated: December 24, 2019)

The microscopic control available over cold atoms in optical lattices has opened new opportunities to study the properties of quantum spin models. While a lot of attention is focussed on experimentally realizing ground or thermal states via adiabatic loading, it would often be more straight-forward to prepare specific simple product states and to probe the properties of interacting spins by observing their dynamics. We explore this possibility for spin-1/2 and spin-1 models that can be realized with bosons in optical lattices, and which exhibit XY-ferromagnetic (or counterflow spin superfluid) phases. We consider the dynamics of initial spin-rotated states corresponding to a mean-field version of the phases of interest. Using matrix product state methods in 1D we compute both non-equilibrium dynamics and ground/thermal states for these systems. We compare and contrast their behaviour in terms of correlation functions and induced spin currents, which should be directly observable with current experimental techniques. We find that although spin correlations decay substantially at large distances and on long timescales, for induction of spin currents, the rotated states behave similarly to the ground states on experimentally observable timescales.

I. INTRODUCTION

Control over cold atoms in optical lattices has led to opportunities to realize a range of spin-model Hamiltonians, arising from superexchange of spinful fermions and bosons [1, 2]. Important recent progress has been made in the realization of magnetically ordered states in such systems, with the observation of antiferromagnetic ordering of fermions, corresponding to a Heisenberg spin model [3–7]. These states are generally produced by adiabatic loading of atoms into the lattice potential. In this context there have been a significant number of proposals for adiabatic manipulation of spin Hamiltonians in optical lattices, in order to obtain low-entropy states, even when the energy gap in the ground state is small [8–13]. This usually involves loading the lattice in a regime where the energy gap is large, and then manipulating the Hamiltonian parameters time-dependently.

At the same time, recent experiments can prepare well-defined initial product states, which are not eigenstates of the system, and then probe their subsequent non-equilibrium dynamics [14, 15]. Locally, these product states can appear as the mean-field state corresponding to the quantum phase associated with the ground state. For a particular phase, it is possible to probe directly in experiments to what extent the initial mean-field magnetic states, and the states they evolve into, are different from the true ground state for the same Hamiltonian parameters. For example, in the case of spin-1/2 models Barmettler et al. [16] considered the evolution of a perfect Néel state in 1D under an antiferromagnetic Heisenberg Hamiltonian. As a result of the dynamics, the magnetic ordering is found to decay exponentially in time, thus demonstrating important differences between mean-field and the true ground state in 1D.

In this article, we address such questions in the different context of spin-superfluid phases, which can be realized with multicomponent bosons in optical lattices [17]. Such spin-superfluids can also be identified with an XY-ferromagnet, and proposals for their adiabatic state preparation have been discussed, especially for the spin-1 case that occurs with two particles per site [13]. On the other hand, an ideal XY-ferromagnetic state can be also well approximated by a mean-field description, where all of the spins point in the XY-plane. For large spins, this corresponds to approximating the spin superfluid by a product of spin coherent states, analogously to a superfluid state of bosons on a lattice [18, 19]. Moreover, such states can be prepared in a relatively straight-forward experimental sequence. Beginning in a Mott Insulator (MI) state in which all spins are initially prepared aligned along the z-axis, we can apply an RF transition to rotate the state into the XY-plane, and we call this state the rotated state.

We compare and contrast exact quantum ground states of spin-1/2 and spin-1 models to their rotated-spin (mean-field) counterparts. Focusing on the 1D case, we compute ground and thermal states as well as the many-body dynamics of the system using tensor network methods based on Matrix Product State (MPS) and Matrix Product Operator (MPO) techniques [20–24]. We first quantify how far the initial spin coherent mean-field state is from the true XY-ferromagnetic ground state. We then study the evolution of this state, which is a consequence of inter-species interactions (anisotropies in the effective spin-models) leading to the initial state not being an ex-
act eigenstate of the Hamiltonian. We find that for short times “ideal” XY-correlations remain relatively robust. For longer times and small anisotropies the dynamics produces states with exponentially decaying spin correlations, resembling thermal states. We analyze the dependence of correlation lengths on anisotropies. For large anisotropies, the thermalization picture breaks down and a non-equilibrium state very different from the ferromagnet builds up quickly. Lastly, we show how the effective magnetic ordering, i.e. spin superfluidity can be probed by inducing spin currents. We propose a way to probe the magnetic ordering by measuring spin-currents generated by an effective magnetic field gradient. We compare spin-currents following from true ground-states of the system and for initial spin coherent states.

The remainder of this article is organized as follows: In Sec. II, we review the two effective spin models we consider in this work (spin-1 and spin-1/2), and how they arise from a two-species Bose-Hubbard model. In Sec. III, we explore the differences between ground, rotated, and thermal states in out-of-equilibrium dynamics. In Sec. IV we discuss methods to probe these states by observing spin currents of bosons in an optical lattice. Lastly, we provide a summary and an outlook in Sec. V.

II. SPIN MODELS FROM TWO-COMPONENT BOSONS

In this section, we introduce the two effective spin models that we will analyze in this work. Both can appear as effective models in Mott Insulating states of two-components (e.g. two internal spin states) of bosons trapped in the lowest band of an optical lattice [25, 26], and such systems exhibit a rich ground-state phase-diagram [17]. The system is described by a two-species Bose-Hubbard Hamiltonian

\[
\hat{H} = -\zeta \sum_{\langle i,j \rangle} (\hat{a}_i^\dagger \hat{a}_j + \hat{b}_i^\dagger \hat{b}_j) + U_{AB} \sum_j \hat{a}_j^\dagger \hat{a}_j \hat{b}_j^\dagger \hat{b}_j + \frac{U_A}{2} \sum_j \hat{a}_j^\dagger \hat{a}_j^\dagger \hat{a}_j \hat{a}_j + \frac{U_B}{2} \sum_j \hat{b}_j^\dagger \hat{b}_j^\dagger \hat{b}_j \hat{b}_j, \tag{1}
\]

Here, \(\hat{a}_i\) and \(\hat{b}_i\) are the bosonic annihilation operators for the two species denoted as A and B, respectively. The notation \(\langle i, j \rangle\) denotes a sum over all nearest-neighbour sites, \(\zeta\) is the tunneling rate, and \(U_A, U_B\) the intraspecies and \(U_{AB}\) the inter-species on-site interaction energy strength. We denote the average occupation of particles per site as \(n\). We will consider equal intra-species interactions \(U \equiv U_A = U_B\), which describes very well the situation for \(^{87}\text{Rb}\) atoms.

In the case of integer \(n\), when the intra-species interactions are large compared with the tunneling, \(U_A, U_B \gg \zeta\), the ground-state of the model is a MI state with particles exponentially localized at each lattice site and with small local number fluctuations. In second order perturbation theory, analogous to a Schrieffer-Wolf transformation producing the Heisenberg model from the Hubbard model [17], we obtain an effective spin-Hamiltonian acting in the low energy subspace.

A. Spin-1 Model

For \(n = 2\) the low-energy sub-space on a site \(l\) can be represented by three different states \(|+1\rangle_l, |0\rangle_l, |−1\rangle_l\), as depicted in Fig. 1(a), comprising effective eigenstates of a diagonal spin-1 operator, \(\hat{S}^z_l\), with eigenvalues \(S^z_l = +1, 0, −1\). The effective spin states correspond to the respective particle states \(\hat{a}_i^\dagger \hat{a}_j^\dagger |0\rangle, \hat{a}_i^\dagger \hat{b}_j^\dagger |0\rangle, \) and \(\hat{b}_i^\dagger \hat{b}_j^\dagger |0\rangle\), where \(|0\rangle\) denotes the empty lattice state.

Considering the case of equal number of \(A\) and \(B\) bosons \((n_A = n_B)\), the effective Hamiltonian is an anisotropic spin-1 Heisenberg model [17],

\[
\hat{H}_{\text{SP}1} = -J \sum_{\langle i,j \rangle} \hat{S}_i \hat{S}_j + u \sum_j (\hat{S}_j^z)^2. \tag{2}
\]

Here, \(u = U - U_{AB}\), \(J = 4\zeta^2/\Delta_{AB}\), and \(\hat{S}_i = (\hat{S}_i^x, \hat{S}_i^y, \hat{S}_i^z)\) is a vector of the three spin-1 operators.

The magnetic ordering in the ground state depends on the interactions. When \(U \gg U_{AB}\), the ground state will exhibit a spin insulator or spin-Mott state configuration, with \(S^z_i \to 0\) for all sites \(i\). Interactions of similar size, \(U_{AB} \lesssim U\), lead to a XY-ferromagnetic ground state, induced by the superexchange term. The rotated product-state, which would represent a mean-field XY ferromagnetic state is a superposition of all three spin
states on each site (as sketched in Fig. 1(d)). Note that, while in the Mott phase of two species of atoms the net overall transport of atoms is suppressed, the XY phase corresponds to a state with a counterflow, (i.e. the currents of the two species are equal in absolute values but opposite directions), and can be nondissipative (supercounterflow) [26, 27]. Finally, for $U_{AB} > U$, the ground state is a $z$-ferromagnet.

### B. Spin-1/2 Model

In the case of $n = 1$, the resulting effective Hamiltonian is a spin-1/2 XXZ Heisenberg model, where a single $A$ boson is mapped to spin-up $|\uparrow\rangle_j$ and $B$ boson to spin-down $|\downarrow\rangle_j$ on site $j$ (cf. Fig. 1(b)) [17]

$$\hat{H}_{\text{spin-1/2}} = -J \sum_{\langle i,j \rangle} \hat{s}_i \cdot \hat{s}_j + \Delta \sum_{\langle i,j \rangle} \hat{s}_i^x \hat{s}_j^x,$$  \hspace{1cm} (3)

where $J = 4\zeta^2/U_{AB}$ and $\Delta = 8\zeta^2/U_{AB} - 8\zeta^2/U$ is the anisotropy. $\vec{s}_i = (\hat{s}_i^x, \hat{s}_i^y, \hat{s}_i^z)$ is a vector of the three spin-1/2 operators.

We note that in an experiment, variations in $\Delta$ correspond to variations of $U - U_{AB}$, and that if we set $u = U - U_{AB}$ as we did in the spin-1 case, that we can rewrite $\Delta/J = 2u/U$. The realisable range of $\Delta/J$ values is thus dependent on our ability to tune $U_{AB}$ in an experiment.

In this case, the only phase transition is at $U_{AB} = U$. When $U_{AB} < U$, the ground state of the system is the XY-ferromagnet (or spin superfluid), in contrast to the Z-ferromagnet for $U_{AB} > U$, as shown in Fig. 1(e).

### III. ROTATED STATES AND OUT-OF-EQUILIBRIUM DYNAMICS

In this section, we will first discuss the preparation of the spin-rotated states, and then study their differences to true XY-ferromagnetic ground states. Secondly, we look at the dynamics of the system and analyze the dynamically prepared states, e.g. as function of the anisotropies.

#### A. Preparation of spin rotated states

The ideal mean-field XY-ferromagnetic state can be prepared by beginning with all spins aligned along the $z$-axis ($|\psi_0\rangle = |\uparrow\uparrow\uparrow\uparrow\uparrow\uparrow\rangle$), and then rotating that state locally into the XY-plane, of every atom simultaneously (see Fig. 1(c)). In an experiment, this could be achieved by beginning in a single-component MI with the correct filling factor, and then applying an RF/microwave drive that corresponds to a $\pi/2$ rotation around the $x$-axis, as generated by the operator

$$\hat{R}_x = \prod_{j} e^{-i\hat{s}_j^x}. \hspace{1cm} (4)$$

The rotated initial state is then

$$|\psi_r\rangle = \hat{R}_x |\psi_0\rangle. \hspace{1cm} (5)$$

The operation is calculated analogously for the spin-1/2 with operators $\hat{s}^x$. We will now analyze how close the state $|\psi_r\rangle$ is to the true XY-ferromagnetic ground-state and how the dynamics will modify it.

#### B. Comparison of the rotated state with the ground state

To obtain a first idea about the similarities of $|\psi_r\rangle$ and the ground-state of the system, we compute the energy difference $\Delta E = E_r - E_{GS}$ per spin of the two states. Fig. 2 shows $\Delta E$ for various values of the anisotropy $(u/J$ and $\Delta/J$) for spin-1 and spin-1/2, respectively. The results are for a 1D chain with a varying number of sites $M$, and computed using MPS techniques. There is no energy difference without anisotropy, and the result is very close to the ground state for small values of $u/J, \Delta/J$ in each case. The energy difference increases with the anisotropy, and for large system sizes the value of energy difference per spin is independent of the system size.

It is difficult to compare these differences directly between the two models, because the Hamiltonians and their corresponding energy scales are significantly different. We note also that the variation in $U - U_{AB}$ that is required for a change in $\Delta/J$ for the spin-1/2 model is much larger than the variation for a given value of $u/J$ in the spin-1 model, as for the Mott Insulator regime in which we are working, $U/J$ is substantial.

Naturally, the energy difference only gives us a first indication of similarities or differences between the rotated state and the ground states. In the next sections we will look at the time evolution of correlation functions, and then the behaviour of spin currents induced in the system.

#### C. Dynamics of correlation functions

We now look at the out-of-equilibrium dynamics after a preparation of $|\psi_r\rangle$, in particular we will focus on the dynamics of spin-spin correlations in the system. To compute the time evolution under each Hamiltonian, we use the Time Evolving Block Decimation (TEBD) algorithm [23, 24, 28, 29] for MPS. Corresponding bond dimensions required for convergence are indicated in the figure captions.

The correlation functions are calculated as

$$\Theta_j = |\langle \hat{S}_i^+ \hat{S}_{i+j}^- \rangle | = \frac{1}{M - 2b - j} \sum_{i=1+b}^{M-b-j} |\langle \hat{S}_i^+ \hat{S}_{i+j}^- \rangle |. \hspace{1cm} (6)$$
imaginary-time evolution of the density matrix in MPO large system sizes) at finite temperatures, we use an effect for simple observables if it thermalizes, e.g. for rotated state, energy of the thermal state matches the energy of the \( \beta \) obtained states to a thermal state \( \hat{\rho} \). We use a purification formalism [23]. At the initial point of the evolution the system is considered at the infinite temperature, i.e. its density matrix is proportional to the identity, \( \rho_0 \propto I \), where all states have equal probability of occupation. Then, the next step is to evolve the density matrix to finite temperatures \( \hat{\rho}(\beta) \propto e^{-\beta \hat{H}} \). We use a purification technique [23, 37] to preserve positive semi-definiteness of the density matrix, and hence rewrite this expression as \( \hat{\rho}(\beta) \propto e^{-\beta H/2} \rho_0 e^{-\beta H/2} \). Since \( \rho_0 = \bar{\rho}_0 = \rho_0 \rho_0^\dagger \) only one side of the above expression needs to be evolved, \( \bar{\rho}(\beta) \equiv e^{-\beta H/2} \rho_0 \), and the thermal expectation value of an arbitrary operator \( \hat{O} \) can be obtained as

\[
\langle \hat{O} \rangle_\beta = \frac{\text{tr}(\hat{O} \rho(\beta) \rho(\beta)^\dagger)}{\text{tr}(\rho(\beta) \rho(\beta)^\dagger)}.
\]

We find that the Time-Dependent Variational Principle (TDVP) algorithm [38–41] is a very efficient integrator for time-propagation at finite temperatures in terms of the balance between the speed and accuracy, however other methods maybe useful in terms of accuracy, for instance Runge-Kutta [42].

The accuracy of the method is verified by comparing the numerical calculations with the exact solution (using exact diagonalization) in the case of smaller systems. For bigger system sizes the convergence of numerical results to the exact solution is checked by increasing the bond dimension for the MPS, \( D \). We verified the validity of all
10 partially decrease with increasing distance $r$ thermal effects: at high temperatures, spin orientations lines in Fig. 3). However, this effect will be destroyed by our results by comparing the convergence of this method with respect to the observables we are interested in, and we confirm the convergence in the bond dimension by running multiple calculations with increasingly large $D$.

In the ground state of our spin models, we see that the correlations decay algebraically (as shown by the black lines in Fig. 3). However, this effect will be destroyed by thermal effects: at high temperatures, spin orientations become randomized, and the correlations will exponentially decrease with increasing distance $r$,

$$
\langle \hat{S}_i^+ \hat{S}_{i+r}^- \rangle \propto e^{-\frac{\xi}{r}},
$$

with $\xi$ being the correlation length (analogously for the spin-1/2 with operators $\hat{s}_i^+, \hat{s}_{i+r}^-$). The properties of the thermal states corresponding to the energies of the rotated states are summarized in Fig. 4. There we compare results for the correlation lengths (obtained from an exponential fit), and for the entropy per lattice site.

In both models the correlation length decreases with the anisotropy. For the spin-1 model, we find that a large correlation length is attained for smaller anisotropy, demonstrating that for a thermal state (in the long-time limit) a state with significant correlations may be stabilized for small $u/J$. In contrast, for the spin-1/2 case we find that except for very small $\Delta/J$ the correlation lengths obtained are shorter.

Note that in performing an effective imaginary time evolution as described above, the exponential factor will always provide an instability towards the ground state, where numerical noise biases the final state towards the ground state, especially for large $\beta$. Thus, in our calculation for thermal states, the calculations become inaccurate in the low-temperature limit. For the spin-1 model, this limited our comparisons to the regime $u \gtrsim 0.2J$. However, we see in Fig. 4 that the spin-1 model shows a long correlation length already at $u = 0.2J$, and it is reasonable that for smaller $u$ the correlation length will further drastically increase, leading ultimately to an algebraic decay in the limit where the rotated state becomes the true ground state.

### E. Thermalization dynamics

In general, closed quantum systems thermalize in the long-time limit. This is meant in the sense that local observables in a small subsystem appear to be described by a thermal density matrix $\rho_{th} \propto \exp(-\beta \hat{H})$, with the (inverse) temperature set by the energy matching condition with the initial state, $\langle \psi_0 | \hat{H} | \psi_0 \rangle = \text{tr}(\rho_{th} \hat{H})$. In general, this thermalization behavior is expected for Hamiltonians without simple/local conserved degrees of freedom (integrable models) and in situations without disorder. The mechanism behind such thermalization can be explained, e.g., via the well-studied eigenstate thermalization hypothesis [30–36].

In order to analyze in which regimes such thermalization might or might not occur in our system, we consider the time dependent expectation value of the correlations, here $\langle \hat{O}_j \rangle = \langle \hat{S}_i^+ \hat{S}_{i+j}^- \rangle$ (analogously for the spin-1/2 with operators $\hat{s}_i^+, \hat{s}_{i+j}^-$). If the system relaxes to a steady state, this state should be identical to the infinite-time average,

$$
\overline{\langle \hat{O}_j \rangle} = \lim_{T \to \infty} \frac{1}{T} \int_0^T dt \langle \psi(t) | \hat{O}_j | \psi(t) \rangle = \sum_\alpha |c_\alpha|^2 \langle \alpha | \hat{O}_j | \alpha \rangle,
$$

where for the last line we have expanded the time-dependent states into the Hamiltonian eigenbasis with eigenvalues $\{E_\alpha\}$ and eigenstates $\{|\alpha\rangle\}$, $|\psi(t)\rangle = \sum_\alpha c_\alpha \langle \alpha | \exp(-iE_\alpha t) | \alpha \rangle$ [33] (here, $\hbar = 1$ and we assumed non-degenerate eigenstates). The last line denotes the expectation value in a “diagonal ensemble”. We want to test if time-averaged correlations - either over our simulation window, or the limiting case of the diagonal ensemble - can be described by a thermal density matrix.

Numerically it is difficult to simulate systems with large $M$ to very long times, but we can investigate the behaviour for small system sizes, where the whole spectrum can be numerically computed, and then extrapolate to larger systems. We compute $\langle \hat{O}_j \rangle$ averaged over a time-scale $t \in [0, t_0]$, $\overline{\langle \hat{O}_j \rangle}_{t_0}$, and compare the result to a thermal state with the energy of the initial state, $\rho_{th}$, and we then evaluate

$$
\Delta \langle \hat{O}_j \rangle_{th} = \frac{|\overline{\langle \hat{O}_j \rangle}_{t_0} - \text{tr}(\hat{O}_j \rho_{th})|}{|\text{tr}(\hat{O}_j \rho_{th})|}.
$$

Furthermore we compare the infinite time-average, i.e. the diagonal expectation value, to the thermal one by computing

$$
\Delta \langle \hat{O}_{d.e.} \rangle = \frac{|\overline{\langle \hat{O}_j \rangle}_{d.e.} - \text{tr}(\hat{O}_j \rho_{th})|}{|\text{tr}(\hat{O}_j \rho_{th})|}.
$$
We expect that the lack of thermalization for larger anisotropies and for various system sizes, and we note that the results are qualitative similar between the diagonal ensemble (13). The difference is shown as function of the anisotropies and for various system sizes, and we note that the results are qualitative similar between the diagonal ensemble and the explicit time average, despite the relatively short simulation time. All calculations are done with exact diagonalization and for periodic boundaries.

The results are summarized in Fig. 5 for spin-1 and spin-1/2.

Strikingly, for anisotropies larger than 1, both the diagonal ensemble expectation value and the finite time average differ clearly from thermal states in both models. For smaller anisotropies, within finite size effects the states obtained have expectation values of the chosen observables that are consistent with the thermal state. We expect that the lack of thermalization for larger anisotropies is related to increasing interactions between magnons, which will lead to integrability in 1D.

IV. PROBING SPIN CURRENTS

Lastly, we consider how the rotated initial spin states and ground states respond to imposed spin-currents. In an experiment we can realize this by applying a magnetic field gradient in an experiment, which induces a spin current.

We define the spin current \( \hat{C} \) as

\[
\hat{C} = \frac{1}{M} \sum_{l} c_l,
\]

with operators

\[
c_l = -\frac{1}{2i} \left( \hat{S}_l^+ \hat{S}_{l+1}^- - \hat{S}_l^- \hat{S}_{l+1}^+ \right),
\]

arising from the continuity equation [43, 44]:

\[
\frac{d}{dt} \hat{S}_l^z = \left[ iH, \hat{S}_l^z \right] = c_{l+1} - c_l.
\]

Note that in contrast to single particle current measurements [45], the spin-currents correspond to relative momentum distributions of the two atomic species, and correlations between them could be probed via noise correlation measurements [46, 47].

We compute the currents for spin-1/2 and spin-1, in each case considering the behaviour of the current in the ground state and the rotated state. We perform calculations by applying the “kick” operator as a Matrix Product Operator to the MPS representation of our state, and computing the corresponding time evolution. For an ideal (non-interacting) superfluid, we would expect no decay of the current, but interactions will always lead to decay of the current once a critical strength of the kick is exceeded.

The resulting currents are compared in Fig. 6, for both spin-1 and spin-1/2, and for situations where the kick is applied to the ground states (on the left hand side of the figure) or the rotated states (on the right hand side of the figure), for different values of the anisotropies and momenta \( \Omega/\pi \). We can clearly distinguish regimes where the currents are stable and regimes where they are unstable.

Comparing the behaviour of the rotated and ground states, we see qualitatively that up to \( tJ = 2 \) the decay of the current is very similar for both states in the case of each model. We further observe that the rotated state exhibits a smaller initial current, this is related to a broader initial relative momentum distribution for the two spins (as when the momentum distribution is broader, typically the same translation in quasimomentum will cause less of a change in the average group velocity [48]). Furthermore, we find that where in some cases (especially for spin-1/2 as in Fig. 6(f)), the current is non-decaying for the ground state, we notice a decay for the rotated state with time, as the decay of the long-range correlations (shown in Fig. 3(d)) becomes important. The reason why this is particularly visible for spin-1/2 is because the most robust currents occur for larger anisotropy, where there is a bigger difference between the rotated state and the ground state, and hence a faster decay of the correlations.

To emphasize the dependence of the current stability on anisotropy and \( \Omega \), in Fig. 7 we plot the relative difference between the current after short time evolutions and
at the beginning of the evolution,

$$\Delta \langle \hat{C} \rangle = \frac{|\langle \hat{C} \rangle_{tJ=1} - \langle \hat{C} \rangle_{tJ=0^+}|}{|\langle \hat{C} \rangle_{tJ=0^+}|}. \quad (18)$$

We find that also when the kick is applied to the rotated states, we can clearly quantify a crossover between two regimes of persistent and decaying currents, in both models.

For the spin-1 there is a phase transition (crossover) between spin superfluid (XY) and spin-Mott at around $u \approx 0.6J$, where the currents rapidly decay as the system becomes strongly anisotropic, in analogy to spin currents for bosons in a 1D Bose-Hubbard model [48–51]. For infinitesimal kicks $\Omega \to 0$, the currents remain constant in the XY-ferromagnetic phase regime, and start to decay once entering into the spin-Mott phase. This can be seen along the vertical axis in Fig. 7(a). For a larger $\Omega$, as we go towards the isotropic point, the current will still decay after a certain critical $\Omega$ value is reached. This value decreases as we go towards the critical value of $u$ to enter the spin-Mott phase. Again, as in the 1D case for currents in a Bose-Hubbard model [48], this is not a sharp transition, but rather a gradual crossover, as shown in Fig. 7(a).

In contrast, for spin-1/2, we are always in the XY-ferromagnetic phase, where the currents will remain constant for any infinitesimal kick strength $\Omega$, except exactly at the isotropic point. From [50] we know at the same time that the critical value of $\Omega$ increases from zero with increasing anisotropy $\Delta/J$, and we see that the value of $\Omega$ above which we observe substantial decay of the current increases with increasing $\Delta/J$. For $\Delta = J$, we see essentially non-decaying currents at any time and kick strength from the ground state, which we expect as the XX model can be mapped to non-interacting fermions.

V. SUMMARY AND OUTLOOK

For XY-ferromagnet states of 1D spin models for two bosonic species in an optical lattice, we have compared and contrasted the dynamics of the ground state and a product state of spins rotated into the XY-plane, as a function of anisotropy in both spin-1 and spin-1/2 models. By computing the out-of-equilibrium dynamics, we have shown that in both cases, if we begin in a rotated product spin states, the correlations decrease rapidly in time, faster for a higher anisotropy. We also compared the rotated state to thermal correlation lengths and entropies. For the time evolution of spin-currents we observed different behavior between the spin-1/2 and the spin-1 models. For the spin-1/2 model currents are more...
stable for higher anisotropies, in contrast to the spin-1 case. This is due to the critical velocity in the system increasing with system size. At longer times, we begin to see decay of currents for the rotated states that occur earlier than for the ground states, which is where the influence of the correlation decay becomes significant in the dynamics. For the spin-1 model, we observed a cross-over between regions where the currents were essentially stable (counterflow superfluid regime, or XY-ferromagnet) and unstable (moving towards a spin-Mott state), in analogy to to similar results in superfluid states of 1D Bose-Hubbard models.

By using RF techniques to rotate an initial single-species Mott Insulator state, these states can be directly realized in ongoing experiments with optical lattices. It is an interesting prospect to probe the difference between mean-field spin states and the true ground states experimentally, for the effective spin models not only in 1D. For larger dimensions we expect the rotated state to be closer to the true ground-state as the mean-field assumption generally becomes better with the dimensionality. In particular in this regime, which is hard for fully exact numerical approaches, an experimental investigation would be interesting. Such efforts provide an interesting basis for further investigation of spin superfluidity in multi-component bosonic lattice models.

Acknowledgments

This work was supported by AFOSR MURI FA9550-14-1-0035. Work at the University of Strathclyde was supported by the EPSRC Programme Grant DesOEQ (EP/P009565/1), and by the EOARD via AFOSR grant number FA9550-18-1-0064. Numerical calculations here utilized the ARCHIE-WeSt High Performance Computer. J.S. is supported by the French National Research Agency (ANR) through the Programme d’Investissement d’Avenir under contract ANR-11-LABX-0058 NIE within the Investissement d’Avenir program ANR-10-IDEX-0002. W.K. receives support from the NSF through the Center for Ultracold Atoms and Award No. 1506369, from ARO-MURI NonEquilibrium Many-Body Dynamics (Grant No. W911NF14-1-0003), from AFOSR-MURI Quantum Phases of Matter (Grant No. FA955014-10035), from ONR (Grant No. N00014-17-1-2253), and from a Vannevar-Bush Faculty Fellowship.

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