Diffusion with Local Resetting and Exclusion

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Stochastic resetting models diverse phenomena across numerous scientific disciplines. Current understanding stems from the renewal framework, which relates systems subject to \textit{global} resetting to their non-resetting counterparts. Yet, in interacting many-body systems, even the simplest scenarios involving resetting give rise to the notion of \textit{local} resetting, whose analysis falls outside the scope of the renewal approach. A prime example is that of diffusing particles with excluded volume interactions that \textit{independently} attempt to reset their position to the origin of a 1D lattice. With renewal rendered ineffective, we instead employ a mean-field approach whose validity is corroborated via extensive numerical simulations. The emerging picture sheds first light on the non-trivial interplay between interactions and resetting in many-body systems.

As allegorized in the popular nursery rhyme on a small spider’s Sisyphean attempt to ascend a waterspout during questionable weather, surmounting a long and volatile endeavor is often quite challenging and full of setbacks. Similar setbacks also arise in many natural phenomena that are prone to “resetting”: the abrupt cessation of a dynamical process, which consecutively starts anew. Examples for resetting phenomena are abundant and come from a diverse range of research fields including search, first-passage, and animal foraging \cite{1-10}, algorithmics \cite{11-13}, optimization \cite{14-15}, and reaction kinetics \cite{19-24}.

The renewal framework constitutes the current state-of-the-art in the analysis of resetting phenomena \cite{6, 7, 25}. This powerful approach utilizes the understanding that a resetting event forces the system back to its initial state to formulate stochastic renewal equations, relating the properties of resetting systems to those of their non-resetting counterparts. Such relations have proven salient in uncovering steady-state, transport, relaxation, and first-passage properties in a long list of model systems \cite{26-45}. Moreover, they have been instrumental in the discovery of universal phenomena that emerge as a result of resetting \cite{6, 7, 15, 46-50}.

In spite of its centrality to the study of resetting phenomena, the renewal framework’s applicability is limited. Specifically, in many-body systems, it only applies when resetting acts to bring the \textit{entire} system back to its initial configuration, a process which we hereby term “global resetting” \cite{51, 53}. Yet, generically, resetting will have a more local nature that can, in turn, be affected by interactions. As a concrete example, consider diffusive particles, subject to repulsive short-ranged interactions, that move along a 1D track immersed in a fluid. In this case, resetting can be used to model the unbinding of a particle from the track and its subsequent rebinding at a uniquely favorable location. However, due to interactions, rebinding cannot occur if this location is already occupied by a different particle. This, in fact, is precisely the picture that arises in bio-polymerization, where RNA polymerases and ribosomes play the role of resetting particles. Since a resetting event leaves the system’s configuration mostly intact, aside from the new position of the resetting particle, the renewal framework does not apply and alternative approaches must be sought. The absence of a unifying theoretical framework renders the study of “local” resetting in interacting many-body systems extremely challenging.

In this Letter, we take a first step towards establishing an understanding of local resetting within an analytically tractable model. In particular, we study a 1D ring lattice of \(L\) sites, occupied by diffusive particles which interact via volume exclusion and \textit{independently} attempt to reset their position to the origin site, if it is vacant (see Fig. 1). These dynamics give rise to a non-

Figure 1. An illustration of diffusion with exclusion and local resetting on a ring lattice. Each particle attempts to hop to its left and right neighboring sites with rate 1, and to reset its position to the origin with rate \(r\). In both cases, an attempt is successful only if the target site is vacant.
trivial steady-state density profile, whose analysis lies well beyond the scope of renewal theory. Instead, taking a mean-field (MF) approach, we are able to derive a closed-form solution for the stationary density profile and analyze its scaling properties in the limit of large \( L \). The MF description is corroborated against extensive numerical simulations to remarkable accuracy. The results established herein pave the way to the extension of current experimental studies of single-particle resetting to interacting many-body systems.

**The Model** - Consider a 1D periodic lattice of \( L \) sites labeled \( \ell = 0, \ldots, L - 1 \), occupied by \( N \) particles of average density \( \bar{\rho} \equiv N / L \). The particles are subject to hard-core, exclusion interactions by which each site may hold one particle at most [54-59]. The system then evolves in continuous time via the dynamical rules illustrated in Fig. 1. Each particle attempts to hop to its left and right neighboring sites with rate 1 and to reset its position to site \( \ell = 0 \) with rate \( r \). In both cases, an attempt is successful only if the target site is vacant.

These dynamics can be formulated in terms of a Markov chain. Let \( \tau_\ell (t) \) denote the occupation of site \( \ell \) at time \( t \), taking the value 0 if the site is vacant and 1 otherwise. The model’s dynamics imply the following evolution of \( \tau_\ell (t) \)

\[
\tau_\ell (t + dt) - \tau_\ell (t) = \Gamma_\ell (t),
\]

where \( \Gamma_\ell (t) \) is given by

\[
\Gamma_\ell (t) = \begin{cases} 
\tau_{\ell \pm 1} & \text{w.p. } (1 - \tau_\ell) dt \\
-\tau_\ell & \text{w.p. } (1 - \tau_{\ell \pm 1}) dt \\
\sigma_\ell & \text{w.p. } (1 - \tau_0) R_\ell dt
\end{cases}
\]

and w.p. abbreviates “with probability”. Clearly, a distinction must be made between site \( \ell = 0 \), which experiences an influx of resetting particles, and the remaining sites \( \ell \neq 0 \). Equation (2) for \( \Gamma_\ell (t) \) indeed serves as short-hand notation for

\[
\sigma_\ell = -1 \text{ and } R_\ell = r \tau_\ell \\
\sigma_\ell = +1 \text{ and } R_\ell = r \sum_{m=1}^{L-1} \tau_m
\]

for \( \ell \neq 0 \) and \( \ell = 0 \), respectively. Note that the expression for \( R_0 \) is not arbitrary, originating from the model’s particle conservation, i.e.

\[
\sum_{\ell=0}^{L-1} [\tau_\ell (t + dt) - \tau_\ell (t)] = 0.
\]

**Main Results** - Using an analytical mean-field approach and extensive numerical simulations, we show that the stationary density profile of resetting and interacting random walkers behaves very differently from that found in the absence of interactions. As in the non-interacting case, resetting acts to concentrate particles at the origin site \( \ell = 0 \). However, here, exclusion prevents the origin from being occupied by more than a single particle. Moreover, resetting cannot occur when the origin is occupied, giving rise to a non-trivial interplay between exclusion, diffusion, and resetting.

For a fixed mean density \( \bar{\rho} \), the model’s exclusion interactions and diffusive dynamics yield a density profile that is a scaling function of \( L / \bar{\rho} \), asserting that the density substantially deviates from \( \bar{\rho} \) throughout the entire system. This stands in stark contrast to the non-interacting picture, where the width of the profile is \( \sim \sqrt{D / r} \), with \( D \) denoting the diffusion coefficient [2].

Here we find that, for large systems, the density profile is entirely independent of the resetting rate \( r \). In fact, it only depends on the mean density \( \bar{\rho} \), as seen in Eqs. (3). A different picture emerges when the number of particles \( N = \bar{\rho} L \) is instead kept fixed, in which case the density profile is shown to be a function of \( L \) alone, as obtained in Eq. (4). These MF predictions are verified, to remarkable precision, in data collapses obtained from extensive numerical simulations. Figure 2 demonstrates the density profile’s scaling form for a fixed mean density while Fig. 3 shows its behavior for fixed particle number \( N \). Additional data is provided in the supplemental material (SM) for different parameters.

**MF analysis** - To evoke the MF approximation, we first average over the Markov chain in Eqs. (1) and (2), replacing the mean occupation \( \langle \tau_\ell \rangle \) by the corresponding density field \( \rho_\ell \in [0, 1] \). The MF approximation is then manifested in the factorization of products of the form \( \langle \tau_\ell \rangle = \langle \tau_\ell \rangle \langle \tau_k \rangle \rightarrow \rho_\ell \rho_k \). The rates \( R_{\ell=0} \) and \( R_{\ell=0} \) in Eq. (3) respectively become \( R_{\ell=0} = r \sum_{m=1}^{L-1} \rho_m \) and

![Figure 2](image)

**Figure 2.** Data collapse of the density profile for \( r = 1 \) and \( \bar{\rho} = 0.2 \) versus the scaling variable \( x = \ell / L \). The different markers denote various values of the system size \( L \) while the solid black curve denotes the theoretical MF prediction in Eq. (12) for \( L \rightarrow \infty \). Note that the site indices \( \ell \) have been shifted to \( \ell = -L/2 + 1, \ldots, 0, \ldots, L/2 \) for convenience of presentation. Correspondingly, \( x \in [-0.5, 0.5] \).
Using the dynamic’s symmetry around site \( \ell \), To this end, we set our interest lies in the stationary behavior of the den-

tation, and at site \( \ell \) the following equation for \( \rho_L \) has proven to be remarkably successful for analyzing

t rigorously justify the MF approximation, this approach

\[ R_{\ell \neq 0} = r \rho_{\ell}. \]

While it is generally quite difficult to rigorously justify the MF approximation, this approach has proven to be remarkably successful for analyzing numerously many lattice models with exclusion interactions. As we show below, MF also provides a remarkably accurate description of the present model.

In the limit \( dt \to 0 \), the MF approximation yields the following equation for \( \rho_{\ell} (t) \),

\[ \partial_t \rho_{\ell} = \rho_{\ell+1} - 2 \rho_{\ell} + \rho_{\ell-1} + \sigma_{\ell} (1 - \rho_{\ell}) \langle R_{\ell} \rangle. \] (4)

Our interest lies in the stationary behavior of the density profile. To this end, we set \( \partial_t \rho_{\ell} = 0 \) and separately analyze Eq. (4) at sites \( \ell \neq 0 \), termed the “bulk” equation, and at site \( \ell = 0 \), which we call the “boundary” equation. Solving the stationary bulk equation gives

\[ \rho_{\ell} = c_1 A^\ell + c_2 A^\ell, \] (5)

where \( c_{1,2} \) are constants and we have defined

\[ A_{\pm} = 1 + \frac{a}{2} \left( 1 \pm \sqrt{1 + \frac{4}{a}} \right), \] (6)

and

\[ a = r (1 - \rho_0). \] (7)

Using the dynamic’s symmetry around site \( \ell = 0 \), i.e. \( \rho_{\ell} = \rho_{L-\ell} \), gives \( c_2 = c_1 A_+ L \). The particle conservation condition

\[ N = \bar{\rho} L = \rho_0 + \sum_{\ell=1}^{L-1} \rho_{\ell}, \] (8)

is then used to determine \( c_1 \), such that \( \rho_{\ell} \) becomes

\[ \rho_{\ell} = \frac{(1 - A_-) (L \bar{\rho} + a r - 1)}{2 (A_- - A_+)} (A_+^{\ell} + A_-^{\ell}). \] (9)

Although we have obtained a formal solution for the MF density profile \( \rho_{\ell} \), our work is not yet done since \( \rho_{\ell} \) still depends on the density at site \( \ell = 0 \) through \( a \) in Eq. (7). To determine \( \rho_0 \), we revisit Eq. (4) for the density profile and consider its behavior at \( \ell = 0 \). In the stationary limit, this equation can be written as

\[ \rho_1 = \rho_0 - \frac{r}{2} (1 - \rho_0) (N - \rho_0), \] (10)

where \( N - \rho_0 = \sum_{m=1}^{L-1} \rho_m \) and we have again used the \( \ell \to L - \ell \) symmetry to replace \( \rho_1 + \rho_{L-1} \to 2 \rho_1 \). To make progress, we separately consider two distinctly different physical scenarios: the case of a constant particle density \( \bar{\rho} \) and the case of a constant particle number \( N \).

**Fixed density \( \bar{\rho} \)** - In this case the number of particles in the system \( N \) grows linearly with system-size \( L \). For large \( L \), and a correspondingly large number of particles, there is a small time gap between the instance site \( \ell = 0 \) is vacated by a particle hopping to a vacant neighboring site, and the time it is reoccupied due to a resetting event. Reoccupation due to hopping from neighboring sites is negligible, since it is attempted with rate 1 while resetting events are attempted with rate \( L \bar{\rho} = N r \gg 1 \). In the limit of \( L \to \infty \), where the system contains infinitely many particles, the total rate of resetting attempts becomes infinite and reoccupation is immediate. We thus expect that the density near the origin be unity, up to small finite-\( L \) corrections. We thus consider the ansatz

\[ \begin{align*}
\rho_0 &\cong 1 - (\alpha/L)^\mu, \\
\rho_1 &\cong 1 - (\beta/L)^\nu,
\end{align*} \] (11)

generally allowing for a different scaling with \( L \) at sites \( \ell = 0 \) and \( \ell \neq 0 \). Substituting this ansatz into Eq. (10) yields the relation \( \mu = 1 + \nu \). We can then determine the value of \( \nu \) by substituting Eq. (11) into Eq. (9) for \( \rho_{\ell} \), evaluated at site \( \ell = 1 \). This self-consistency requirement sets \( \nu = 1 \) and, correspondingly, \( \mu = 2 \) (see the SM for details). With this, we obtain the density profile at sites \( \ell \neq 0 \) as

\[ \rho_{\ell} \cong 2^{-1} a \bar{\rho} (e^{\alpha} - 1)^{-1} \left( e^{\alpha (L-\ell)/L} + e^{\alpha \ell/L} \right). \] (12)

The parameter \( \alpha \) is determined by demanding that the ansatz for \( \rho_0 \) in Eq. (11) be consistent with the density profile \( \rho_{\ell} \) in Eq. (12) at site \( \ell = 0 \). This gives

\[ \sqrt{2} a \coth (\alpha/2) = 2, \] (13)
With this insight, we return to Eq. (9) in order to re-
setting \( \ell \) dependent of \( \rho = 0 \) \( \ell \) is vacated and the instance it is reoccupied now
remains \( \text{finite} \), even as \( L \) is increased. Correspondingly, both \( \rho_0 \) and \( \rho_1 \) are asymptotically independent of \( L \).
With this insight, we return to Eq. (9) in order to re-
late \( \rho_1 \) at \( \ell = 0 \) to \( \rho_0 \) for finite \( N \), in the limit of large \( L \). Substituting \( \bar{\rho} = N/L \) and \( a = r (1 - \rho_0) \) into Eq. (9), setting \( \ell = 0 \) and equating to \( \rho_0 \) asymptotically yields

the polynomial equation

\[
\frac{r (1 - \rho_0) (N - \rho_0)}{\sqrt{r (1 - \rho_0) (r (1 - \rho_0) + 4) - r (1 - \rho_0)}} \approx \rho_0, \quad (16)
\]

where we have used the fact that \( A_{\ell L}^{\perp} \) and \( A_{\ell L}^{L} \) both
vanish as \( L \to \infty \), for any \( L \)-independent \( a \) (see Eq. (9)). This equation for \( \rho_0 \) has three roots, of which only one satisfies \( \rho_0 \in [0, 1] \). While its precise analytical form is rather involved and is thus deferred to the SM,
in the limit \( 1 \ll N \ll L \), where the system contains many particles but the average density is low, it reduces to
\( \rho_0 = 1 - 4r^{-1}N^{-2} + O(N^{-4}) \). By Eq. (7), this translates into \( a \approx 4/N^2 \) which, when substituted into
Eq. (9), reveals that the density profile has a width of \( \sim N/2 \) around site \( \ell = 0 \). Since \( N \) is fixed, the
density profile remains a function of \( \ell \) alone, implying that it only extends over a finite \( \sim O(N) \) region near
the origin, as demonstrated in Fig. 3.

Conclusions - In this Letter, we have gone beyond the
popular renewal framework to study the effect of \( \text{local} \)
resetting on a many-body interacting system. Employ-
ing a MF approach, we derived the stationary density
profile of \( N \) particles diffusing on a 1D ring lattice of
\( L \) sites while being subject to exclusion interactions
and local resetting with rate \( r \). For large systems occupied
by many particles, we find that the profile’s width is
independent of \( r \), being solely a function of the position
and average density \( \bar{\rho} \) (or the number of particles \( N \),
depending on which was kept fixed while taking the large
system limit). This intriguing behavior directly follows
from the delicate interplay between local resetting and
exclusion interactions, which prohibit resetting if the
origin is already occupied. Indeed, for non-interacting
particles the density profile is known to adopt a width
\( \sim r^{-1/2} \) [2], irrespective of \( \bar{\rho} \) or \( N \), which stands in stark
contrast to our findings here.

Two recent experimental studies explored resetting in
the context of diffusive single-particle systems, clearly
marking interacting many-body systems as the next
frontier [54, 55]. Exclusion is one complication that is
sure to arise in such systems, and the results established
herein are thus especially well-posed to serve as a bench-
mark for comparison. Moreover, the approach we
presented can be extended further, as required, to capture
more realistic scenarios featuring excluded volume in-
teractions and local resetting. These will be considered
elsewhere.

Acknowledgments - A. M. wishes to thank David
Mukamel for his ongoing encouragement and support.
A. M. also thanks the support of the Center of Scien-
tific Excellence at the Weizmann Institute of Science.
S. R. acknowledges support from the Azrieli Founda-
tion, from the Raymond and Beverly Sackler Center for
Computational Molecular and Materials Science at Tel
Aviv University, and from the Israel Science Foundation
(grant No. 394/19).
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Supplemental Material

I. ADDITIONAL COMPARISON BETWEEN SIMULATIONS AND MEAN-FIELD PREDICTIONS

In this section we further establish the effectiveness of the mean-field (MF) approach in the study of the model presented in the main text. We do so by extending the comparison provided in the main text, between the MF result for the density profile $\rho_{\ell}$ of the main text Eq. (14) and direct numerical simulations of the model, for additional parameters.

The main text Fig. 2 compares a data collapse (for different values of the system size $L$) of the density profile versus $x = \ell/L$ to our MF results, for a resetting rate $r = 1$ and a mean density $\bar{\rho} = 0.2$. Figure S1 complements this by providing the same comparison for the mean density $\bar{\rho} = 0.6$ and the resetting rate $r = 1$. In Fig. S2 we provide an additional comparison for a resetting rate which grows linearly with $L$ and a mean density $\rho = 0.1$. The remarkable agreement between MF theory and numerical simulations of the density profile versus $x = \ell/L$ reassures that MF provides a very good description for a broad range of model parameters and implies that the profile indeed remains a scaling function of $\ell/L$, even when $r \propto L$.

II. SYSTEM-SIZE SCALING OF $\rho_{\ell}$

Here we set the values of $\nu$ and $\mu$ in the ansatz $\rho_0 = 1 - \left(\frac{a}{L}\right)^{\mu}$ and $\rho_1 = 1 - \left(\frac{a}{L}\right)^{\nu}$ that appear in the main text Eq. (11). To this end, we first substitute them into the stationary boundary equation $\rho_1 = \rho_0 - \frac{r}{2} \left(1 - \rho_0\right) \left(L\bar{\rho} - \rho_0\right)$ of the main text Eq. (10). We obtain

$$\left(\frac{\beta}{L}\right)^{\nu} - \frac{r\bar{\rho}^{\mu}}{2L^{\nu-1}} - \frac{r}{2} \left(\frac{a}{L}\right)^{2\mu} - \left(1 - \frac{r}{2}\right) \left(\frac{a}{L}\right)^{\mu} = 0.$$  (17)

As $L \to \infty$, this can only be satisfied if $\mu > 1$, in which case Eq. (17) becomes

$$\frac{r\bar{\rho}^{\mu}}{2L^{\nu-1}} \approx \left(\frac{\beta}{L}\right)^{\nu}.$$  (18)

Correspondingly, we conclude

$$\begin{cases} \mu = 1 + \nu \\ \beta^{\nu} \approx \left(\frac{a}{L}\right)^{1+\nu} \end{cases}.$$  (19)

Having related $\mu$ to $\nu$, we next set out to derive the value of $\nu$. To do this, we substitute the ansatz at site $\ell = 0$, i.e. $\rho_0 = 1 - \left(\frac{a}{L}\right)^{1+\nu}$, into the main text Eq. (9) for the density profile $\rho_{\ell}$ at site $\ell = 1$

$$\rho_1 = \frac{(1 - A_{-})}{2(A_{-} - A_{+}^{L})} \left(\frac{a}{L} - A_{-} + A_{+}^{L-1}\right),$$  (20)

and demand that it identifies with our ansatz, $\rho_1 = 1 - \left(\frac{a}{L}\right)^{\nu}$. Recall that $A_{\pm}$ is provided in the main text.
Eq. (6) as $A_{\pm} = 1 + \frac{3}{2} \left( 1 \pm \sqrt{1 + \frac{4}{a}} \right)$ and that $a = r (1 - \rho_0)$. Using the ansatz for $\rho_0$, we can rewrite $A^L_{\pm}$ in the limit of large $L$ as

$$A^L_{\pm} \simeq \left[ 1 + \frac{r}{2} \left( \frac{\alpha}{L} \right)^{\nu+1} \left( 1 \pm \sqrt{\frac{4}{r} \left( \frac{L}{\alpha} \right)^{\nu+1}} \right) \right]^L \equiv 1 \pm \sqrt{r} \left( \frac{\alpha}{L} \right)^{\frac{\nu+1}{2}} L^{\frac{1}{2} - \nu}.$$ (21)

Depending on the value of $\nu$, $A^L_{\pm}$ exhibits one of three distinct behaviors as $L \to \infty$: For $0 < \nu < 1$, it exponentially decays/blows-up, while for $\nu > 1$ we get $A^L_{\pm} \simeq 1 \pm O \left( L^{\frac{1}{2} - \nu} \right)$. Yet, for $\nu = 1$, we obtain the $L$-independent expression $A^L_{\pm} \simeq e^{\pm \sqrt{r} \alpha^{\frac{1}{2}} L^{\frac{1}{2} - \nu}}$. We finally use this understanding to derive the value of $\nu$. It is straightforward to show that the leading, large-$L$ behavior of $\rho_1$ in Eq. (20) is

$$\rho_1 \simeq L^{\frac{1}{2} - \nu}. \quad (22)$$

This can only self-consistently agree with the ansatz $\rho_1 = 1 - \left( \frac{3}{2} \right)^{\nu}$ as $L \to \infty$ if $\nu = 1$, which immediately also allows us to deduce $\mu = 2$. The large-$L$ scaling of the deviation of the density $\rho_0$ from unity is numerically verified in Fig. S3.

### III. FIXED NUMBER OF PARTICLES $N$

In the case of a fixed number of particles $N$, the polynomial main text Eq. (16) is solved by

$$\rho_0 = \frac{4 + r - (-1)^{1/3} \chi^{1/3} \left( 1 + \frac{1+3N^2)^2+8r+16}{(-1)^{3/3}\chi^{2/3}} \right)}{3r}, \quad (23)$$

where

$$\chi(r, N) = (1 - 9N^2) r^3 + 6 (2 + 3N^2) r^2 + 48 r + 64 + 3iN \sqrt{3r^3} \times \sqrt{(1 - N^2)^2 r^3 + 4 (3 + 5N^2) r^2 + 4 (12 + N^2) r + 64}. \quad (24)$$

Although it is far from obvious at first sight, $\rho_0$ in Eqs. (23) and (24) satisfies the requirement $\rho_0 \in [0,1]$ for $r, N > 0$.

Figure S3. Log-log plot of $(1 - \rho_0)$ versus $L$ for $r = 1$ and $\rho = 0.2$. The blue dots denote the model’s numerical simulation results while the solid orange line a least-squares fit to $c_0/L^{\mu}$. 

Figure S3