We derive a generalized master equation for multiphoton pulses interacting with multiple emitters in a waveguide-quantum electrodynamics system where the emitter frequency can be modulated. Based on this theory, we can calculate the real-time dynamics of the collective interacting emitters driven by an incident photon pulse which can be vacuum, coherent states, Fock states or their superpositions. We also derive generalized input-output relations to calculate the reflectivity and transmissivity of the incident field and the output photon pulse shapes can also be calculated. Our theory here can find important applications in the researches of waveguide-based quantum systems.

I. INTRODUCTION

In recent years, a major goal in quantum optics and quantum information is to build a large scale of quantum network from single quantum levels [1]. However, it is a big challenge to manufacture a single quantum system which contains a large number of qubits. In contrast, it is relatively feasible to build a small quantum system with high accurate controls and then connect these small systems into a large scale quantum network by certain quantum channels. Waveguide quantum electrodynamics (QED), which studies the interaction between emitters and waveguide photons, is a very good system for realizing large scale quantum network and has been attracted extensive studies in the past two decades [2–4]. The emitter-photon interaction can be significantly enhanced in the reduced dimension and the emission of an emitter to the waveguide modes can be nearly unit [5] which can find important applications in the high efficient single photon sources [6], single photon detection [7], and atom cavity [8–11]. The emitter-emitter interaction mediated by the one-dimension (1D) waveguide photons can be long-ranged which provides a unique system for studying many-body physics [12, 13] and long-ranged quantum information transfer [14, 15]. Due to the confinement of transverse field, the photon modes in a quasi-1D waveguide can have intrinsic direction dependent longitudinal angular momentum [16] which is extremely suitable for studying chiral quantum optics [17–23]. The waveguide-QED theory can be applied to a number of systems under extensive studies currently such as the photonic line defects coupling to quantum dots [24], cold atoms trapped along the alligator waveguide [25], superconducting qubits interacting with the microwave transmission lines [26, 27], and the plasmonic nanowire coupling to quantum emitters [28].

The theory of single photon transport in waveguide-QED is the basics for studying this system and has been extensively studied in the past two decades. Shen and Fan used the real-space Hamiltonian together with the Bethe-ansatz to study the stationary properties of single photon scattering by a single quantum emitter in a 1D waveguide [29] and this method is then extended to multi-emitter systems [30] and multi-level systems where many interesting effects can occur such as photon frequency conversion [31, 32], the electromagnetic induced transparency (EIT) effects [33], single photon transistor [34] and single photon switch [35]. In addition to the stationary spectrum, the real-time dynamics of the emitter system is also very interesting because the emitters are important units for quantum information processing and storage. Chen et al. applied the wavefunction approach to study the dynamics of a single photon pulse interacting with a single emitter [36]. We generalized the wavefunction approach to the multiple identical [37] and non-identical [38] emitters case from which we can study many interesting collective many-body physics and quantum information applications such as quantum state preparation [40] and waveguide-based quantum sensing [41]. Recently, Dinc et al. developed an analytical method based on Bethe-ansatz approach to study the time dynamics of a single photon transport problem [42].

Compared with single photon transport, the multi-photon scattering can have more interesting physics but its calculation is also much more complicated. The Bethe-ansatz approach can be extended to calculate the few-photon scattering problem where photon-photon bound states can occur [43–46]. However, when this method is generalized to more than two photons, the calculation becomes extremely cumbersome [47–50]. Alternative methods such as Lehmann-Symanzik-Zimmermann reduction method [51, 52], the Green function decomposition of multiple particle scattering matrix [53], the input-output formalism [54, 55], the Feynman diagrams [56], and the SLH formalism [57, 58] have also been proposed. In these stationary calculations, the photon is usually assumed to be a plane wave and the real-time dynamics of the emitter is usually ignored. Based on Heisenberg-Langevin approach, Domokos et al. studied
the coherent photon pulse scattering by a single emitter in a waveguide [61]. Kony and Gea-Banacloche generalized the wavefunction approach to study the one- and two-photon scattering by two emitters in 1D waveguide [62]. In 2015, Caneva et al. used the effective Hamiltonian approach to derive a master equation to calculate the emitter dynamics driven by a coherent photon pulse [63] and this method can be generalized to various systems [64, 65]. For the continuous-mode Fock states input, Gheri et al. derived a master equation to study the dynamics of a single emitter driven by a single and two photon wavepacket [66] and Baragiola generalized this method to the general N-photon case based on the Itô Langevin approach [67]. In their studies, they mainly focused on the single quantum system where the many-body interaction between the emitters is not considered. In 2018, we derived a master equation to study the dynamics of multiple emitters driven by continuous squeezed vacuum field in 1D waveguide [68] and found that steady-state population inversion of multiple Ξ-type emitters can occur in this system [69].

In this article, we consider a multiphoton pulse interacting with multiple emitters coupled with 1D waveguide. Since the input photon spectrum is not flat, we cannot use the usual way to derive a master equation using white noise limit. Instead, we generalize the method shown in Ref. [66] to a more general multi-photon-multi-emitter case where the emitter frequency can be modulated and the effects of non-waveguide modes can also be included. In addition, we also derived a general input-output theory to calculate the reflection and transmission properties of the system. The theory here can be applied to calculate transport of a large class of field like the vacuum, coherent states, Fock states and their superpositions which can have broad applications in the studying of waveguide-QED system.

This article is arranged as follows. In Sec. II, we derive a generalized master equation for the emitter dynamics and the generalized input-output theory to study the scattering field properties. In Sec. III, we apply this theory to the cases of coherent state input, the single and general N photon input. Finally, we summarize our results.

II. MULTIPHOTON SCATTERING THEORY

In this section, we first derive a generalized master equation for general multiphoton transport in 1D waveguide-QED system. Then we derive the generalized input-output relations of this system and use it to calculate the reflection and transmission properties of the field.

The model we studied is shown in Fig. 1 where a photon pulse containing multiple photons is injected into a 1D waveguide coupling to $N_a$ emitters with arbitrary spatial distributions. Here, we consider a general case where the emitters can have time modulating frequencies and they can couple to both the waveguide and non-waveguide photon modes. It is convenient to work in the rotating frame with the original emitter frequency $\omega_a$. The total Hamiltonian of the system and reservoir fields in the rotating frame is given by

$$H(t) = \frac{\hbar}{2} \sum_{j=1}^{N_a} \varepsilon_j(t) \sigma_j^+ \sigma_j^- + \hbar \sum_k \Delta \omega_k a_k^+ a_k + \hbar \sum_{q_\lambda} \Delta \omega_{q_\lambda} a_{q_\lambda}^+ a_{q_\lambda}$$

$$+ \hbar \sum_{j=1}^{N_a} (g_j^l e^{i k z_j} \sigma_j^+ a_k + H.c.)$$

$$+ \hbar \sum_{j=1}^{N_a} (g_j^r e^{i q_\lambda j} \sigma_j^+ a_{q_\lambda} + H.c.).$$

(1)

The physical meaning of each term in the Hamiltonian is as follows. The first term is the emitter Hamiltonian with time-dependent modulating frequency $\varepsilon_j(t)$ where $j = 1, 2, \ldots, N_a$. $\sigma_j^+$ and $\sigma_j^-$ are the $j$th component and the raising (lowering) Pauli operators of the $j$th emitter. The second term is the Hamiltonian of the waveguide photons with the detuning frequency $\Delta \omega_k = \omega_k - \omega_a$ and $a_k (a_k^\dagger)$ is the annihilation (creation) operator of the waveguide photon mode with frequency $\omega_k$. The third term is the non-guided reservoir field Hamiltonian where $a_{q_\lambda} (a_{q_\lambda}^\dagger)$ is the annihilation (creation) operator of the non-guided photon mode with frequency $\omega_{q_\lambda}$ and $\Delta \omega_{q_\lambda} = \omega_{q_\lambda} - \omega_a$. The fourth term is the emitter-waveguide photon interaction term with $g_j^l = \mu_j \cdot E_k (r_j)/\hbar$ being the coupling strength. The last term is the interaction between the emitters and the non-guided reservoir field with coupling strength $g_j^r = \mu_j \cdot E_{q_\lambda} (r_j)/\hbar$.

According to the Heisenberg equation, the dynamics
of an arbitrary emitter operator $O_\sigma$ is given by

$$
\dot{O}_\sigma(t) = \frac{i}{2} \sum_{j=1}^{N_\sigma} \varepsilon_j(t)[\sigma_j^+(t), O_\sigma(t)]
$$

and using the Weisskopf-Wigner approximation we can obtain (see Appendix A)

$$
\dot{O}_\sigma(t) = \frac{i}{2} \sum_{j=1}^{N_\sigma} \varepsilon_j[\sigma_j^+(t), O_\sigma(t)]
$$

$$
+ i \sum_{j=1}^{N_\sigma} \sqrt{\frac{\Gamma_{jk}}{2}}[\sigma_j^+(t), O_\sigma(t)][a_j(t) + b_j(t)]
$$

$$
+ i \sum_{j=1}^{N_\sigma} \sqrt{\frac{\Gamma_{jk}}{2}}[a_j(t), O_\sigma(t)][\sigma_j^-(t), O_\sigma(t)]
$$

$$
+ \sum_{jl} \Lambda_{jl}[\sigma_j^+(t), O_\sigma(t)]\sigma_l^-(t)
$$

$$
- \sum_{jl} \Lambda_{jl}^*[\sigma_j^-(t)][\sigma_l^+(t), O_\sigma(t)],
$$

(11)

where $a_j(t) = \sqrt{\frac{\Gamma_j}{2}} \int_0^\infty e^{ikz_j a_k(0)} e^{-i\omega_k t} dk$ describes the absorption of the incident waveguide photons and $b_j(t) = \frac{\Gamma_j}{2} \int \int e^{i\phi} a_k(0) e^{-i\omega_k t} d^3q_k$ is the absorption of the incident nonguided photons. The collective interaction between the emitters can be calculated as [39]

$$
\Lambda_{jl} = \frac{\Gamma_j \Gamma_l}{2} e^{ik_{jl} |z_l|} + \frac{3}{4} \frac{\gamma_j \gamma_l}{(k_{jl})^2} \left[ \sin^2 \phi \frac{-i}{k_{jl}} \right]
$$

$$
+ (1 - 3 \cos^2 \phi) \left( \frac{1}{(k_{jl})^2} + \frac{i}{(k_{jl})^2} \right) e^{ik_{jl} |r_{jl}|},
$$

(12)

where the first term is the effective interaction mediated by the waveguide photons and the second term is the usual dipole-dipole interaction induced by the non-guided reservoir fields. $|r_{jl}| = |z_j - z_l|$ is the distance between the jth and lth emitters and $|z_{jl}| = |z_j - z_l|$ the distance in the zth direction. $\Gamma_j = 4\pi |q_{zj}|^2 / \nu_0$ is the decay rate due to the waveguide vacuum field and $\gamma_j$ is the spontaneous decay rate due to the nonguided photon modes. $\phi$ is the angle between the direction of the transition dipole moment and the waveguide direction.

From Eq. (11), we can derive a corresponding master equation for the emitters. Since $T_{RS}[O_S(t)\rho_S] = T_{RS}[O_S\rho_S(t)]$ where $\rho_S(t) = T_{RS}[\rho(t)]$ is the emitter system density operator, by time derivation on both sides we have $T_{RS}[O_S\rho_S(t)] = T_{RS}[O_S\rho_S(t)]$ and from Eq. (11) we can obtain (see Appendix A)

$$
\dot{\rho}_S(t) = -\frac{i}{2} \sum_{j=1}^{N_\sigma} \varepsilon_j(t)[\sigma_j^+, \rho_S(t)] - \frac{i}{2} \sum_{j=1}^{N_\sigma} \sqrt{\frac{\Gamma_{jk}}{2}}[\sigma_j^+, \rho_j'(t)]
$$

$$
- \frac{i}{2} \sum_{j=1}^{N_\sigma} \sqrt{\frac{\Gamma_{jk}}{2}}[\sigma_j^-, \rho_j'(t)] - i \sum_{jl} \text{Im}(\Lambda_{jl})[\sigma_j^+\sigma_l^-, \rho_S(t)]
$$

$$
- \sum_{jl} \text{Re}(\Lambda_{jl})[\sigma_j^+\sigma_l^-, \rho_S(t) + \rho_S(t)[\sigma_j^+\sigma_l^-, \rho_S(t)]]
$$

(13)

Here, we consider the case that the incident photons are from the waveguide photons and the nonguided reser-
voir field is initially in the vacuum. Since $a_{j1}(0) = 0$, we have $b_{j1}(0) = 0$. Therefore $\rho_j(t) = Tr_{\mathcal{R}}[U(t)a_j(t)\rho(0)U^\dagger(t)]$ is due to the contribution of the incident waveguide photons. This is the main equation of this section. The first term in Eq. (13) is the frequency modulation term. The second and third terms describe the excitation and deexcitation due to the incident photon field. The forth term describes the dipole-dipole interactions between the emitters induced by the guided and nonguided vacuum field. The last term is the collective dissipation due to the guided and nonguided vacuum fluctuation. However, we should note that Eq. (13) itself is in general not closed because we have the new operators like $\rho_j'(t)$ and $\rho_j''(t)$. In some special cases, Eq. (13) is closed. For example, if there is no external driving field, the second and third terms disappear and the equation is closed from which the emitter excitation transport can be studied. Another example is that if the incident field is a coherent field or superposition of coherent fields, the $\rho_j'(t)$ and $\rho_j''(t)$ terms can then be reduced to a complex number multiplying $\rho_j(t)$ and Eq. (13) becomes closed again from which the full dynamics of the emitters driven by a coherent field can be calculated. In general cases such as the Fock state input, we have to repeat the above procedures to derive equations for $\rho_j'(t)$ until all the equations are closed.

B. The generalized input-output theory

In the previous subsections, we derive the master equations for the emitter system which allows to calculate the real dynamics of the emitters for an arbitrary photon wavepacket input. In this subsection, we derive the generalized input-output relations of this system by expressing the output field operators as the function of input operators and the system operators. Together with the master equations derived in the previous subsection, we can then study the reflection and transmission properties of this system.

If we integrate Eq. (3) from $t$ to $t_f$ where $t_f > t$, we can obtain

$$ a_k(t) = a_k(t_f) e^{i\Delta \omega_k(t_f - t)} + i \sum_{j=1}^{N_a} g_{k,j}^* e^{-ikz_j} \int_t^{t_f} \sigma_j^+(t') e^{i\Delta \omega_k(t' - t)} dt'. $$

Comparing Eq. (7) with Eq. (14) it is readily to obtain that

$$ a_k(t_f) e^{i\Delta \omega_k(t_f - t)} = a_k(0) e^{-i\Delta \omega_k t} - i \sum_{j=1}^{N_a} g_{k,j}^* e^{-ikz_j} \int_0^{t_f} \sigma_j^+(t') e^{i\Delta \omega_k(t' - t)} dt'. $$

We can define the following input-output operators [70]

$$ a_{in}^R(t) = \sqrt{\frac{\nu_0}{2\pi}} \int_0^{\infty} a_k(0) e^{-i\Delta \omega_k t} dk, $$

$$ a_{in}^L(t) = \sqrt{\frac{\nu_0}{2\pi}} \int_{-\infty}^0 a_k(0) e^{-i\Delta \omega_k t} dk, $$

$$ a_{out}^R(t) = \sqrt{\frac{\nu_0}{2\pi}} \int_0^{\infty} \int_0^t a_k(t_f) e^{i\Delta \omega_k(t-t_f)} dk, $$

$$ a_{out}^L(t) = \sqrt{\frac{\nu_0}{2\pi}} \int_{-\infty}^t a_k(t_f) e^{-i\Delta \omega_k(t-t_f)} dk, $$

where $z_1$ is the position of the left most emitter and $z_N$ is the position of the right most emitter. Since the right output field propagates freely after scattering by the right most emitter and the left output field propagates freely after scattering by the first emitter, phase factors $e^{ikz_N}$ and $e^{-ikz_1}$ are added in the definitions of the right and left output operators, respectively [63]. From Eq. (15) we can obtain the generalized input-output relations (see Appendix B)

$$ a_{out}^R(t) = a_{in}^R(t - z_N/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\nu_0}{2\pi}} e^{-ik_0 z_j} \sigma_j^-(t), $$

$$ a_{out}^L(t) = a_{in}^L(t + z_1/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\nu_0}{2\pi}} e^{ik_0 z_j} \sigma_j^-(t), $$

where $z_{Nj} = z_N - z_j$. From these two generalized input-output relations we can calculate the properties of the scattering field of this system. We can define the instant field intensity propagating to the right and to the left at time $t$ by $r(t) = \langle a_{out}^R(t)a_{out}^R(t) \rangle$ and $l(t) = \langle a_{out}^L(t)a_{out}^L(t) \rangle$, respectively, which given by

$$ r(t) = \langle a_{in}^R(t - z_N/v_g)a_{in}^R(t - z_N/v_g) \rangle - 2 \sum_{j=1}^{N_a} \sqrt{\frac{\nu_0}{2\pi}} \text{Im}[e^{ik_0 z_j} \langle \sigma_j^+(t)\sigma_j^- \rangle] $$

$$ + \sum_{j=1}^{N_a} \sqrt{\frac{\nu_0}{2\pi}} e^{ik_0 (z_j - z_1)} \langle \sigma_j^+(t)\sigma_j^- \rangle, $$

$$ l(t) = \langle a_{in}^L(t + z_1/v_g)a_{in}^L(t + z_1/v_g) \rangle - 2 \sum_{j=1}^{N_a} \sqrt{\frac{\nu_0}{2\pi}} \text{Im}[e^{-ik_0 z_j} \langle \sigma_j^+(t)\sigma_j^- \rangle] $$

$$ + \sum_{j=1}^{N_a} \sqrt{\frac{\nu_0}{2\pi}} e^{ik_0 (z_j - z_1)} \langle \sigma_j^+(t)\sigma_j^- \rangle. $$

On the right hand side of Eqs. (22) and (23), the first terms are the incident field intensities, the second terms are the absorption and stimulated emission of the system, and the last terms are the spontaneous emission of the system. From $r(t)$ and $l(t)$, we can obtain the pulse shape propagating to the right and to the left after the scattering process. The field intensity reflected to the left and
the right in the whole scattering process are then given by
\[ R = \frac{I_R}{I_R + I_L}, \]  
(24) 
and the transmissivity \( T = 1 - R \).

The scattering power spectrum can be usually obtained from the two-time correlation function of the output field operator
\[ S(\omega) = \int_0^\infty \int_0^\infty \langle a_{\text{out}}^+(t_1) a_{\text{out}}(t_2) \rangle e^{i\omega(t_1-t_2)} dt_1 dt_2, \]  
(25) 
where the average is over the initial state of the whole system. According to the generalized input-output relation shown in Eqs. (20) and (21), \( a_{\text{out}}(t) \) can be expressed as the summation of the input field operator \( a_{\text{in}}(t) \) and the emitter operators \( \sigma_j^+(t) \). The results when \( a_{\text{in}}(t) \) operator acts on the initial state can be readily worked out. Usually, the two-time average of the emitter operators \( \langle \sigma_j^+(t)\sigma_j^-(t+\tau) \rangle \) can be calculated from the master equation according to the quantum regression theorem [71]. However, to apply the quantum regression theorem to calculate the two-time correlation function, it usually requires that the reservoir field does not change significantly. This condition may not be very well satisfied in the waveguide-QED system because the waveguide photon can be significantly absorbed by the emitters especially near the resonance frequency. Therefore, direct use of the quantum regression theorem to numerically calculate the spectrum here may cause some errors and need to be treated carefully. However, at the plane wave limit, an alternative strategy can be used to calculate the scattering spectrum. If the incident photon pulse has a very narrow bandwidth, we can calculate its reflectivity and transmissivity from the above discussions. After calculating the reflectivity and transmissivity for each frequency, we can then obtain the whole scattering power spectrum of the system at the plane wave limit.

### III. Application to Different Photon Wavepackets

In this section, we take the coherent states and the Fock states as example to show how to apply the theory we developed in the previous section to study the emitter dynamics and the field scattering property.

#### A. Coherent state wavepacket

We first consider the case when the incident field is a coherent state. Suppose that the incident field is a continuous-mode coherent state with wavefunction \( |\Psi_{cs}\rangle = \Pi_k|\alpha_k\rangle \) where
\[ |\alpha_k\rangle = e^{-|\alpha_k|^2/2} \sum_{n_k=0}^{\infty} \frac{(\alpha_k)^{n_k}}{\sqrt{n_k!}} |n_k\rangle. \]  
(26) 
The average photon for the \( k \)th mode \( \bar{n}_k = |\alpha_k|^2 \).

Since \( a_k|\Psi_{cs}\rangle = \alpha_k|\Psi_{cs}\rangle \), the operator \( \rho_j(t) = Tr_R[U(t)a_j(t)\rho(0)U^\dagger(t)] = \alpha_j(t)\rho_S(t) \) where
\[ \alpha_j(t) = \sqrt{v_g^{\dagger}} \int_{-\infty}^{\infty} e^{ikz_1} e^{-i\bar{\delta} k z_1} |\alpha_k\rangle dk \]  
(27) 
is the real-time incident coherent photon pulse. Therefore, the operator \( \rho_j(t) \) is reduced to a number multiplying the system density operator \( \rho_S(t) \). The master equation shown in Eq. (13) then becomes
\[ \dot{\rho}_S(t) = -\frac{i}{2} \sum_{j=1}^{N_a} \varepsilon_j(t)[\sigma_j^+, \rho_S(t)] + i \sum_{j,l} \text{Im}(\Lambda_{jl})[\rho_S(t), \sigma_j^+ \sigma_l^-] + \mathcal{L}[\rho_S(t)], \]  
(28) 
where \( \mathcal{L}[\rho_S(t)] = -\sum_{j,l} \text{Re}(\Lambda_{jl})[\sigma_j^+ \sigma_l^- \rho_S(t) + \rho_S(t)\sigma_j^+ \sigma_l^- - 2\sigma_l^- \rho_S(t)\sigma_j^+] \) describes the collective dissipation process. Equation (28) is the master equation of the waveguide-QED system when the incident photon pulse is in a coherent state.

The master equation shown in Eq. (28) is itself a closed equation from which we can calculate the real-time dynamics of the emitters for arbitrary coherent pulse input. Here, without loss of generality we assume that the photon pulse has Gaussian shape throughout this paper. Supposing that the incident coherent field has a Gaussian pulse shape with average photon number \( \bar{n} \), its spectrum can be written as
\[ \alpha_k = \frac{\sqrt{\bar{n}}}{\pi^{1/4}\Delta} \frac{\sqrt{2\Delta}}{e^{-(k-k_0)^2/2\Delta}} e^{-ikz_0}, \]  
(29) 
where \( z_0 \) is the initial central peak position of the pulse and \( k_0 \) is the wavevector corresponding to the central frequency of the photon pulse. When \( k_0 > 0 \) \( (k_0 < 0) \) the pulse is propagating to the right (left). The average photon number \( \bar{n} = \sum_k \bar{n}_k = \int_{-\infty}^{\infty} |\alpha_k|^2 dk \). For the right propagating incident pulse (i.e., \( k_0 > 0 \)), the incident photon pulse is given by
\[ \alpha_j^R(t) = \frac{\sqrt{\bar{n}\Delta v_g}}{\pi^{1/4}} e^{-\Delta^2(z_{j0} - v_g t)^2/2} e^{ik_0 z_{j0}} e^{i\Delta k(z_{j0} - v_g t)}, \]  
(30) 
For the left propagating incident pulse (i.e., \( k_0 < 0 \)), the incident photon pulse is then given by
\[ \alpha_j^L(t) = \frac{\sqrt{\bar{n}\Delta v_g}}{\pi^{1/4}} e^{-\Delta^2(z_{j0} + v_g t)^2/2} e^{-ik_0 z_{j0}} e^{-i\Delta k(z_{j0} + v_g t)}, \]  
(31)
where $\delta_{j0} = z_j - z_0$ and $\Delta k = |k_0| - k_a$ is the detuning between the center frequency of the pulse and the emission transition frequency.

The numerical results for the coherent state input are shown in Fig. 2 where the coherent state is scattered by two emitters. We assume that the distance between these two emitters is 0.125$\lambda_a$ where $\lambda_a = 2\pi/k_a$. The excitations of the two emitters as a function time for two different incident average photon number ($n = 1$ and $n = 30$) are shown in Fig. 2(a). When the average incident photon number is small, e.g. $n = 1$, both emitters are first excited and then deexcited as the coherent pulse passing through. However, when the average incident photon number is large, e.g. $n = 20$, the excitations of both emitters can have oscillations which is the signature of Rabi oscillations.

The corresponding reflected and transmitted photon pulse shapes are shown in Fig. 2(b). When the average photon number is small, the reflected pulse (red solid line) has a single peak and the transmitted pulse (blue dashed line) has two peaks due to the interference between the incident photon and the reemitted photon. When the average photon number is large, most photons are transmitted (olive dashed line) and only a very small part of the photons are reflected (orange solid line). This is because the large pulse can saturate the emitter excitation and only a very small part of photons can be absorbed. Here, the reflection photon pulse can have two peaks instead of one peak due to the Rabi oscillations which does not occur when the photon number is small.

For a coherent pulse with finite time duration, the average photon number reflected by the emitters may be saturated. Here, we also study the average reflected photon number $\bar{n}_R$ as a function of average incident photon number $\bar{n}_R$ for two fixed pulse spectrum widths ($\Delta = \Gamma$ and $\Delta = \Gamma/5$) and the results are shown in Fig. 2(c) where the distance between the two emitters is 0.125$\lambda_a$. When the pulse width is about $\Gamma$, the average reflected photon number increases quickly first as $\bar{n}_R$ increases but then it increases extremely slowly when $\bar{n}_R$ is large due to the saturation effect (blue line with open circles in Fig. 2(c)). It is also noted that when the incident photon number is large, the average reflected photon number can be larger than two despite that there are only two emitters. This is because the incident pulse is not short enough to saturate the emitters immediately. When the incident pulse duration is shorter, i.e., the incident pulse has broader spectrum (e.g., $\Delta = 5\Gamma$), $\bar{n}_R$ first increases and then oscillates as $\bar{n}_R$ increases (red line with solid circles in Fig. 2(c)) due to the stimulated emission. The average reflected photon number is obviously less than 2 because the shorter pulse can saturated the emitters quickly. For comparison, we also plot the results when there is only a single emitter in the system (blue dashed line and the red dashed-dotted line). We can see that their behaviors are similar but the average reflected photon number for two emitters is larger than that of the single emitter. When the pulse width is much smaller than the decay rate of the emitters, the average photon being reflected by a single emitter is less than 1, which can be used to produce single photon sources [72].

### B. Single photon wavepacket

Compared with the coherent state input, the calculation of Fock state input is more involved mostly because of its quantum nature. The theory developed in Sec. II can be also applied for the arbitrary Fock state input. In this subsection, we consider the simplest case where only single photon pulse is incident. For the single photon pulse case, we have developed a dynamical transport theory for calculating the real-time evolution of the system based on the wavefunction approach 38, 39. Here, we show that the master equation developed here can be also applied to the single photon pulse case.

Suppose that the incident photon is a single photon wavepacket described by the wavefunction

$$|\Psi_F\rangle = \int_{-\infty}^{\infty} \alpha(k) a_k^{\dagger}|0\rangle dk,$$

where $\int_{-\infty}^{\infty} |\alpha(k)|^2 dk = 1$. Since $a_k|\Psi_F\rangle = \alpha_k|0\rangle$, the
\( \rho_j(t) \) term in Eq. (13) is then given by

\[
\rho_j(t) = Tr[R(U(t)\rho_j(0)U^+(t))] = \alpha_j(t)\rho_{01}^S(t),
\]

where \( \alpha_j(t) \) is given by Eq. (27) and we define a new operator \( \rho_{01}^S(t) = Tr[R(U(t)\rho_J(0) \otimes |0\rangle\langle 0|)U^+(t)] \). If we define \( \rho_{11}^S(t) = Tr[R(U(t)\rho_J(0) \otimes |\Psi_F\rangle\langle \Psi_F|U^+(t)] \), we have from Eq. (13) that

\[
\dot{\rho}_{11}^S(t) = -i\sum_{j=1}^{N_a} \varepsilon_j(t)[\sigma_j^+\rho_{11}^S(t)]
\]

\[
- i\sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j}{2}}[\alpha_j(t)\sigma_j^+(t),\rho_{01}^S(t)]
\]

\[
- i\sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j}{2}}[\alpha_j(t)\sigma_j^-(t),\rho_{01}^S(t)]
\]

\[
- i\sum_{j=1}^{N_a} \text{Im}(\Lambda_{j1})[\sigma_j^+\sigma_j^-] - \mathcal{L}[\rho_{11}^S(t)],
\]

where \( \mathcal{L}[\rho_{11}^S(t)] = \sum_{j} \text{Re}(\Lambda_{j1})[\sigma_j^+\sigma_j^- - 2\alpha_j\rho_{11}^S] \) is the collective dissipation term, \( \rho_{00}^S(t) \) is not a valid density matrix because it is traceless but it satisfies \( \rho_{00}^S(t) = \rho_{11}^S(t) \). Since a new operator \( \rho_{01}^S(t) \) appears, Eq. (34) is itself not a closed equation and we need to derive an extra equation for \( \rho_{01}^S(t) \).

The dynamical equation for \( \rho_{01}^S(t) \) can be derived using similar procedures as deriving \( \rho_{S}(t) \) shown in Sec. II and it is given by (see Appendix C)

\[
\dot{\rho}_{01}^S(t) = -i\sum_{j=1}^{N_a} \varepsilon_j(t)[\sigma_j^-\rho_{01}^S(t)]
\]

\[
- i\sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j}{2}}[\alpha_j(t)\sigma_j^+(t),\rho_{00}^S(t)]
\]

\[
- i\sum_{j=1}^{N_a} \text{Im}(\Lambda_{j1})[\sigma_j^+\sigma_j^-\rho_{01}^S(t)] - \mathcal{L}[\rho_{01}^S(t)],
\]

where \( \rho_{00}^S(t) = Tr[R(U(t)\rho_J \otimes |0\rangle\langle 0|U^+(t)] \) is another density matrix describing the evolution of the system when initially there is no photon. Using similar procedure, it is not difficult to obtain that

\[
\dot{\rho}_{00}^S(t) = -i\sum_{j=1}^{N_a} \varepsilon_j(t)[\sigma_j^-\rho_{00}^S(t)]
\]

\[
- i\sum_{j=1}^{N_a} \text{Im}(\Lambda_{j1})[\sigma_j^+\sigma_j^-\rho_{00}^S(t)] - \mathcal{L}[\rho_{00}^S(t)],
\]

where we see that no new density operator appears, so the equations are now closed.

Hence, the master equation for the single photon state input consists of three cascaded equations as given by Eqs. (34-36) where a single equation is needed in the coherent state input. The dynamics of the emitters for arbitrary single photon pulse input can then be calculated from these three equations. The time evolution of the average value of an arbitrary emitter operator \( O(t) \) can be then calculated as \( \langle O(t) \rangle = Tr[S(O\rho_{01}^S(t))] \).

One numerical example is shown in Fig. 3 where we consider a single photon wavepacket interacting with two emitters. Here, we assume that the single photon wavepacket has a Gaussian spectrum as shown in Eq. (29) and the distance between emitters is \( \lambda_a/8 \). The emitter excitation as a function of time is shown in Fig. 3(a). Due to the collective interaction, the first emitter can have much higher excitation probability than that of the second one and the excitation of the second emitter has a Rabi-like oscillations which does not occur when the incident photon pulse is in a coherent state with \( \bar{n}_m = 1 \). This is due to the interference between two excitation channels, i.e., the excitation of the incident photon and the excitation by the first excited emitter. In the coherent state input, this interference is concealed.

The reflected and transmitted photon pulse shapes are shown in Fig. 3(b) from which we can see that the transmitted pulse has multiple peaks due to the quantum interference between the incident photon and the reemitted photons by the two emitters. The visibility of the oscillation is much larger than that in the coherent state input. The reflected and transmitted spectra when the incident single photon is a plane wave are shown in Fig. 3(c) where we can see the asymmetric Fano-like structure. This is caused by the interference between two emitters.
are consistent with the results we calculated based on energy shifts and decay rates. The results shown in Fig. m, n ≤ ρ where

FIG. 4: Two photon wavepacket interacting with two emitters. (a) emitter excitation as a function of time. ∆/Γ = 1. (b) Reflected and transmitted pulse shapes after the scattering for the same parameters as (a). In both figures, λa/8.

emission channels, i.e., the emission from the two collective excited states √2(eg) ± |ge⟩) which have different energy shifts and decay rates. The results shown in Fig. 3 are consistent with the results we calculated based on the wavefunction approach [38].

C. N-photon wavepacket

In addition to the single photon Fock state, we can also derive generalized master equations for the multi-photon Fock state input. Compared with the single-photon input, the calculation of multi-photon Fock state input is more complicated. We first consider a relative simple subset which is the direct generalization of the single photon wavepacket, i.e.,

$$|N_\alpha\rangle = \frac{1}{\sqrt{N!}} \left[ \int_{-\infty}^{\infty} dk\,\alpha(k) a_k^+ \right]^N |0\rangle,$$  \hspace{1cm} (37)

where we have the normalization condition \( \int_{-\infty}^{\infty} |\alpha_k|^2 dk = 1 \). A general N-photon wavepacket can be always decomposed to the superposition of the wavefunction shown in Eq. (37). For the wavepacket shown in Eq. (37), we can have

$$a_k |N_\alpha\rangle = a_k \frac{1}{\sqrt{N!}} \left[ \int_{-\infty}^{\infty} dk'\,\alpha(k') a_{k'}^+ \right]^N |0\rangle = \sqrt{N\alpha(k)} |N - 1_\alpha\rangle.$$  \hspace{1cm} (38)

In general, we have the relation \( a_k |m_\alpha\rangle = \sqrt{n\alpha(k)} |m - 1_\alpha\rangle \) and therefore

$$a_k \rho_s(0) \otimes |m_\alpha\rangle \langle n_\alpha| = \sqrt{n\alpha(k)} \rho_s(0) \otimes |m - 1_\alpha\rangle \langle n_\alpha|.$$  \hspace{1cm} (39)

Using the similar procedure to derive Eq. (13), we can derive a ladder set of dynamical equations for the N-photon wavepacket input which is given by

$$\hat{\rho}_{mn}^S(t) = \frac{i}{2} \sum_{j=1}^{N_\alpha} \varepsilon_j(t) [\sigma_j^z, \rho_{mn}^S(t)] - i \sum_{j=1}^{N_\alpha} \sqrt{\frac{\Gamma_j}{2}} \left[ \sqrt{n\alpha_j(t)} [\sigma_j^+, \rho_{mn-1,n}(t)] + \sqrt{n\alpha_j^*(t)} [\sigma_j^-(t), \rho_{mn-1,1}(t)] \right]$$

$$- i \sum_{jl} \text{Im}(\Lambda_{jl}) [\sigma_j^+ \sigma_l^-, \rho_{mn}^S(t)] - \mathcal{L}[\rho_{mn}^S(t)],$$  \hspace{1cm} (40)

where \( \rho_{mn}^S(t) = T_{RR}[U(t) \rho_S(0) \otimes |m\rangle \langle n| U^+(t)] \) and 0 ≤ m, n ≤ N. Considering that \( \rho_{mn}^S = \rho_{nm}^S \), \( (N + 1)(N + 2)/2 \) master equations are required to make the equations closed where N is the total incident photon number. For example, three master equations are needed for single-photon input, while six master equations for two-photon input. For the general N-photon input state, we can always decompose it into the superposition of the form of Eq. (37) and then we can follow the same procedure to derive a set of closed master equations.

Taking the two-photon input as an example, the master equations are given by
between the emitters is 

and we have $\rho_{nm}^{S} = \rho_{mn}^{S*}$. Hence, for two-photon wavepacket, six cascaded master equations are required to calculate the dynamics of the system.

A numerical example is shown in Fig. 4 where we consider two-photon interacting with two emitters. Similar to the single-photon case, we also assume that the two-photon pulse has a Gaussian spectrum and the distance between the emitters is $\lambda_s/8$. Compared with the single photon case, the emitter excitation in the two-photon case is larger and both excitations increase first and then decrease which is similar to the coherent state input (Fig. 4(a)). Different from the single photon case, the emitter 2 does not have Rabi-oscillation like structure. This is mainly because the double excited state $|ee\rangle$ can also be populated in the two photon cases and it can cover the interference effect like in the single photon case. The corresponding reflected and transmitted pulse shapes are shown in Fig. 4(b) from which we can see that they are similar to those in the single photon case but the transmitted pulse has only a small oscillation in the two photon case.

D. The effects of pulse width

In the stationary scattering theory, the incident field is usually assumed to be a plane wave. In practical experiments, the incident light is always a pulse with finite duration and finite bandwidth. Here, our theory allows us to study the effects of the pulse widths.

Taking the single emitter as an example, we investigate the average reflected photon number as a function of pulse widths for different input photon states. The results are shown in Fig. 5(a). For all four incident pulses, $\bar{n}_R$ decreases when the pulse spectrum width increases (i.e., the pulse duration becomes shorter) due to the saturation effects. When the pulse has a white spectrum (i.e., the pulse duration is extremely short), almost no photon will be reflected for both the coherent state inputs and the Fock state inputs because most photon frequencies are far detuned from the resonance frequency. In contrast, when the pulse spectrum is extremely narrow (i.e., the pulse is at the plane wave limit) and its frequency is in resonance with the emitter transition frequency, almost all of the incident photons will be reflected for both the Fock state inputs and the coherent state inputs. When the pulse spectrum width is finite, the Fock state input can have larger reflectivity than that of the coherent state input with the same average incident photon number. For the same pulse width, the pulse with $\bar{n}_{in} = 1$ has larger reflectivity than that of the pulse with $\bar{n}_{in} = 2$ due to saturation effects.

The reflectivity and transmissivity by a single emitter as a function of detuning frequency for the Fock state
input and the coherent state input at the plane wave limit are shown in Fig. 5(b). It is seen that the reflectivity and transmissivity are exactly the same for the Fock state input and the coherent state input at the plane wave limit. When the incident frequency is resonant with the emitter transition frequency, it will be completely reflected due to quantum interference. When the photon frequency is large detuned from the emitter frequency, it can pass through the emitter without being scattering. The widths of the reflectivity and transmissivity depend on the emitter decay rate. Therefore, the reflectivity and transmissivity for a certain frequency is a property of the waveguide-QED system, and it does not depend on the photon statistics of the incident photons. However, for an incident photon with finite spectrum width, the reflectivity and transmissivity can strongly depend on the pulse width and the photon statistics of the incident photons.

E. Multiple emitters and frequency modulation

In addition to the one or two emitters, our theory can be applied to calculate the interacting of photon pulse with arbitrary number of emitters until the computation power is saturated. Here, we take five emitters with nearest neighbor distance 0.25λ0 as an example. The excitation probability for the five emitters as a function of time is shown in Fig. 6(a) where we assume that the incident photon pulse is in a coherent state with average photon number 1. We can see that the first emitter has the largest excitation probability, but it is quickly deexcited and can transfer its energy to the other emitters. The other emitters have smaller excitation probabilities, but they can oscillate and last for a period of time much longer than the decay time of single emitter and the incident pulse duration. This is because the collective subradiant states can be formed due to the emitter interactions and they can be populated by the incident photon pulse. The corresponding reflected and transmitted photon pulses are shown in Fig. 6(b). Most energy is reflected and the reflected pulse has a major peak. In contrast the transmitted pulse has multiple peaks due to quantum interference between the incident field and the reemitted fields by the emitters.

Our theory also allow us to calculate the transport dynamics when the emitter frequencies are externally modulated. As an example, we consider a single emitter interacting with a coherent photon pulse. The emitter’s frequency is modulated such that ε(t) = 10Γ sin(10Γt).
The emitter excitation as a function of time is shown as the red solid line in Fig. 6(c). For comparison, the result without frequency modulation is also plotted as the black dashed line. It is seen that the excitation with modulation is smaller and has some small oscillations. The corresponding scattering pulses are shown in Fig. 6(d) where the red solid line is the reflected pulse and the blue dashed line is the transmitted pulse. We can see that the reflected pulse has small modulations, while the transmitted pulse has very significant modulations. In comparison, the scattering pulses without modulations have smooth shapes (black solid and dashed lines). Hence, by frequency modulation we can realize complicated photon pulse shaping.

**IV. SUMMARY**

In this article we derive master equations for multiphoton interacting with multiple emitters coupled to 1D waveguide. Our theory can be applied to calculate the transport of arbitrary incident photon wavepackets with very general states of light such as coherent state, Fock state and their superpositions. It can also be used to calculate the scattering of multiple emitters with random distribution and even with external frequency modulation. We compare the dynamics of emitters and scattering pulse shapes when the incident photon pulses are coherent state, single photon state or multiple photon states. With finite incident pulse width, different states of light can induce different system dynamics and different scattering properties. The average reflected photon number by a single emitter decreases when the incident pulse duration is shorter for both the coherent state input and the Fock state input, but the Fock state input can have higher average reflected photon number than that of the coherent state input with the same average photon number. This result can be useful for single photon generation. At the plane wave limit, the reflectivity and transmissivity of the waveguide-QED system for a certain frequency are the same and do not depend on the statistics of the incident photons. Our theory also allows to study the scattering properties of a photon pulse by emitters with frequency modulations which can be used for photon pulse shaping. Thus, the theory developed here can become an important basics for studying the many-body physics and quantum information applications in the waveguide-QED system.

**V. ACKNOWLEDGEMENT**

This research is supported by startup grants (No. 74130-18841222 and No. 74130-31610033) from Sun Yat-sen University and the Key R&D Program of Guangdong Province (Grant No. 2018B030329001). The research of MSZ is supported by a grant from King Abdulaziz city for Science and Technology (KACST).
Appendix A: Derivation of Eq. (13)

On inserting Eqs. (7-10) in the main text into Eq. (2), we can obtain

\[
\dot{O}_S(t) = \frac{i}{2} \sum_{j=1}^{N_a} \varepsilon_j(t)[\sigma_j^+(t), O_S(t)] + i \sum_{j=1}^{N_a} g_{k_0}^j e^{ik_z t} e^{-i\Delta \omega t} [\sigma_j^+(t), O_S(t)] \left( a_k(0) - i \sum_{l=1}^{N_a} g_k^l e^{-ik_z t} \int_0^t \sigma_l^+(\tau') e^{i\Delta \omega t'} d\tau' \right)
\]

\[
+ \frac{i}{2} \sum_{j=1}^{N_a} \sum_{l=1}^{N_a} g_k^j g_l^* e^{-ik_z t} \int_0^t \sigma_l^+(\tau') e^{-i\Delta \omega t'} d\tau' [\sigma_j^+(t), O_S(t)]
\]

\[
+ \frac{i}{2} \sum_{j=1}^{N_a} g_{k_0}^j e^{ik_z t} e^{-i\Delta \omega t} \left( a_{\bar{k}}(0) - i \sum_{l=1}^{N_a} g_{\bar{k}}^l e^{-i\bar{k}_z t} \int_0^t \sigma_l^-(\tau') e^{i\Delta \omega t'} d\tau' \right)
\]

\[
+ \frac{i}{2} \sum_{j=1}^{N_a} \sum_{l=1}^{N_a} g_{\bar{k}}^j g_{\bar{l}}^* e^{-i\bar{k}_z t} \int_0^t \sigma_{\bar{l}}^-(-\tau') e^{i\Delta \omega t} \int_0^t \sigma_l^+(\tau') e^{-i\Delta \omega t'} d\tau' [\sigma_j^-(t), O_S(t)]
\]

\[
+ \sum_{j=1}^{N_a} \sum_{l=1}^{N_a} g_{\bar{k}}^j g_{\bar{l}}^* e^{ik_z t} e^{i\Delta \omega t} \left( a_{\bar{l}}(0) - i \sum_{l=1}^{N_a} g_{\bar{l}}^l e^{-i\bar{k}_z t} \int_0^t \sigma_l^-(\tau') e^{i\Delta \omega t'} d\tau' \right)
\]

\[
+ \sum_{j=1}^{N_a} \sum_{l=1}^{N_a} \int_0^t \sigma_l^+(\tau') e^{-i\Delta \omega t'} d\tau' [\sigma_j^-(t), O_S(t)]
\]

\[
= \frac{i}{2} \sum_{j=1}^{N_a} \varepsilon_j(t)[\sigma_j^+(t), O_S(t)]
\]

\[
+ \frac{i}{2} \sum_{j=1}^{N_a} g_{k_0}^j e^{ik_z t} e^{-i\Delta \omega t} [\sigma_j^+(t), O_S(t)] \left( a_k(0) - i \sum_{l=1}^{N_a} g_k^l e^{-ik_z t} \int_0^t \sigma_l^+(\tau') e^{i\Delta \omega t'} d\tau' \right)
\]

\[
+ \frac{i}{2} \sum_{j=1}^{N_a} \sum_{l=1}^{N_a} g_k^j g_l^* e^{-ik_z t} \int_0^t \sigma_l^+(\tau') e^{-i\Delta \omega t'} d\tau' [\sigma_j^+(t), O_S(t)]
\]

\[
+ \frac{i}{2} \sum_{j=1}^{N_a} g_{k_0}^j e^{ik_z t} e^{-i\Delta \omega t} \left( a_{\bar{k}}(0) - i \sum_{l=1}^{N_a} g_{\bar{k}}^l e^{-i\bar{k}_z t} \int_0^t \sigma_l^-(\tau') e^{i\Delta \omega t'} d\tau' \right)
\]

\[
+ \frac{i}{2} \sum_{j=1}^{N_a} \sum_{l=1}^{N_a} g_{\bar{k}}^j g_{\bar{l}}^* e^{-i\bar{k}_z t} \int_0^t \sigma_{\bar{l}}^-(-\tau') e^{i\Delta \omega t} \int_0^t \sigma_l^+(\tau') e^{-i\Delta \omega t'} d\tau' [\sigma_j^-(t), O_S(t)]
\]

According to the Weisskopf-Wigner approximation, we have

\[
\sum_k g_k^j e^{ik_z t} e^{-i\Delta \omega t} e^{i\Delta \omega t} = \sqrt{\frac{\Gamma_j}{2\pi}} e^{ik_z t} e^{i\Delta \omega t} e^{-i\Delta \omega t} \delta(t - \frac{|z_j|}{v_g}),
\]

\[
\sum_k g_k^j e^{-ik_z t} e^{i\Delta \omega t} e^{-i\Delta \omega t} = \sqrt{\frac{\Gamma_j}{2\pi}} e^{-ik_z t} e^{-i\Delta \omega t} e^{i\Delta \omega t} \delta(t + \frac{|z_j|}{v_g}),
\]

\[
\sum_{l=1}^{N_a} g_{\bar{l}}^j e^{-i\bar{k}_z t} e^{i\Delta \omega t} e^{-i\Delta \omega t} e^{i\Delta \omega t} = \Omega_{jl} e^{-i\bar{k}_z t} e^{i\Delta \omega t} \delta(t - \frac{|r_{jl}|}{v_g}),
\]

\[
\sum_{l=1}^{N_a} g_{\bar{l}}^j e^{i\bar{k}_z t} e^{-i\Delta \omega t} e^{-i\Delta \omega t} e^{i\Delta \omega t} = \Omega_{jl}^* e^{i\bar{k}_z t} e^{-i\Delta \omega t} \delta(t + \frac{|r_{jl}|}{v_g}),
\]

where \( \Gamma_i = \frac{2\Gamma_i}{v_g} |g_{k_0}^i|^2 \) with \( g_{k_0}^i = \sqrt{\frac{\Gamma_i}{2\pi}} |e^{ik_z t} e^{-i\Delta \omega t} e^{i\Delta \omega t} \delta(t - \frac{|z_j|}{v_g}) |^2 \), and \( \Omega_{jl} = \frac{3\sqrt{2\pi n_0}}{4} \sin^2 \phi \frac{-i}{k_{\bar{r}_{jl}}} + (1 - 3 \cos^2 \phi)(\frac{1}{(k_{\bar{r}_{jl}})^2} + \frac{i}{(k_{\bar{r}_{jl}})^2}) e^{ik_z r_{jl}} \) with \( r_{jl} = |r_{jl}| - |\bar{r}_{jl}| \).
To proceed, we assume that the emitters are close such that \(z_{ij}/v_g \ll 1/\Gamma\), we can approximate that \(\sigma_j^-(t - \frac{z_{ij}}{v_g}) \approx \sigma_j^-(t)\) in the rotating frame. Indeed, this is the usual case. For example, if \(v_g \sim 10^8 m/s\) and \(\Gamma \sim 10^8 Hz\), we require that the distance between the emitters \(z_{ij} \ll 1 m\) which is the usual case. By doing this approximation, Eq. (13) then becomes

\[
\dot{\rho}_S(t) = \frac{i}{2} \sum_{j=1}^{N_a} \varepsilon_j(t) [\sigma_j^+(t), O_S(t)] + i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j}{2}} [\sigma_j^+(t), \rho_S(t)] [\sigma_j^+(t) + b_j(t)] + \sum_{j} A_{ji} [\sigma_j^+(t), O_S(t)] \sigma_i^-(t) - \sum_{j} A_{ji}^* \sigma_j^+(t) [\sigma_j^-(t), O_S(t)],
\]

where \(a_j(t) = \sqrt{\frac{\sqrt{n_j}}{2\pi}} \int_0^\infty e^{ik_{j}z}a_k(0)e^{-i\omega_k t}dk\) is the absorption of the incident waveguide photons and \(b_j(t) = \sqrt{\frac{\sqrt{n_j}}{2\pi}} \int \int e^{ik_{j}r_j}a_{\lambda}(0)e^{-i\omega_{\lambda} t}d^3r_{\lambda}\) is the absorption of the incident nonguided photons. The collective interaction between the emitters is given by [39]

\[
\Lambda_{jl} = \sqrt{\frac{n_j}{2} n_l} e^{ik_{j}z_{jl}} + \frac{3}{4} \sqrt{\frac{n_j}{n_l}} \frac{i}{k_{a_{jl}}} (1 - 3 \cos^2 \phi) \left( \frac{1}{(k_{a_{jl}})^2} + \frac{i}{(k_{a_{jl}})^2} \right) e^{i k_{jl} |r_{jl}|}.
\]

From Eq. (A6), we can derive a corresponding master equation for the emitters. Since \(Tr_{S+R}[O_S(t)\rho] = Tr_S[O_{SPS}(t)]\) where \(\rho_S(t) = Tr_R[\rho(t)]\), we have

\[
Tr_S[O_S \dot{\rho}_S(t)]
= Tr_{S+R}[\dot{\rho}_S(t)]
= \frac{i}{2} \sum_{j=1}^{N_a} \varepsilon_j(t) Tr_{S+R}[\sigma_j^+(t), O_S(t)] \rho_S(t)
+ \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j}{2}} Tr_{S+R}[\sigma_j^+(t), O_S(t)] [\sigma_j^+(t) + b_j(t)] \rho_S(t)
+ \sum_{j} A_{ji} Tr_{S+R}[\sigma_j^+(t), O_S(t)] \sigma_i^-(t) \rho_S(t)
- \sum_{j} A_{ji}^* [\sigma_j^+(t), O_S(t)] [\sigma_j^-(t), \rho_S(t)]
- \sum_{j} Tr_S[O_S[\sigma_j^+ \sigma_i^- \rho_S(t) - \sigma_i^- \rho_S(t) \sigma_j^+]]
- \sum_{j} A_{ji} Tr_S[O_S[\sigma_j^+ \sigma_i^- - \sigma_j^- \rho_S(t) \sigma_i^+]]
= \frac{i}{2} \sum_{j=1}^{N_a} \varepsilon_j(t) Tr_S[O_S[\sigma_j^+ \rho_S(t)]]
+ \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j}{2}} Tr_S[O_S[\rho_{ji}^+(t), \sigma_j^+]]
+ \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j}{2}} Tr_S[O_S[\rho_{ji}^-(t), \sigma_j^-]]
- \sum_{j} A_{ji} Tr_S[O_S[\rho_{ji}^+(t), \sigma_j^+ - \sigma_j^- \rho_S(t) \sigma_i^+]]
- \sum_{j} A_{ji}^* Tr_S[O_S[\rho_{ji}^- \sigma_j^- - \sigma_j^+ \rho_S(t) \sigma_i^-]],
\]

where \(\rho_{ji}^+(t) = Tr_R[U(t)a_j(t) + b_j(t)]\rho(0)U^\dagger(t)\) is the contribution from the incident sources. In this paper, we consider that the incident photon is coming from the waveguide photons and the non-guided modes are initially in the vacuum. Since \(a_{\lambda}(0)|0\rangle = 0\), we have \(b_j(t)\rho(0) = 0\) and therefore \(\rho_{ji}^+(t) = Tr_R[U(t)a_j(t)\rho(0)U^\dagger(t)]\) is due to the contribution of the incident waveguide photons. Comparing both size of Eq. (A8), we can obtain the master equation for the system density matrix given by

\[
\dot{\rho}_S(t) = -\frac{i}{2} \sum_{j=1}^{N_a} \varepsilon_j(t) [\sigma_j^+, \rho_S(t)]
+ \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j}{2}} [\sigma_j^+, \rho_{ji}^+(t)]
+ \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j}{2}} [\sigma_j^-, \rho_{ji}^-(t)]
- \sum_{j} A_{ji} \{[\sigma_j^+ \sigma_i^- \rho_S(t) - \sigma_i^- \rho_S(t) \sigma_j^+],
\]

which is the master equation shown in Eq. (13) in the main text.
Appendix B: Derivation of the input-output relations

From Eqs. (15-19) in the main text, we can obtain

$$
a^R_{\text{out}}(t) = a^R_{\text{in}}(t - zN/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt' \int_0^\infty \frac{v_g}{2\pi} e^{i\delta k z_N} e^{-i k z_j} e^{i \Delta \omega_k(t'-t)} dk$$

$$= a^R_{\text{in}}(t - zN/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt' \int_0^\infty e^{i\delta k (zN-z_j)} e^{i k v_g (t'-t)} dk$$

$$= a^R_{\text{in}}(t - zN/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt' \int_{-\kappa_0}^\infty e^{i\delta k z_N} e^{i k v_g (t'-t)} d\kappa$$

$$= a^R_{\text{in}}(t - zN/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt' \int_{-\infty}^\infty e^{i\delta k z_N} e^{i k v_g (t'-t)} d\kappa$$

$$= a^R_{\text{in}}(t - zN/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt' \frac{2\pi}{v_g} \delta(t' - t + zN/v_g) dt'$$

$$= a^R_{\text{in}}(t - zN/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt'$$

$$\approx a^R_{\text{in}}(t - zN/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt'$$

(B1)

where $z_{N,j} = z_N - z_j$. Similarly, we have

$$a^L_{\text{out}}(t) = a^L_{\text{in}}(t + z_1/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt' \int_0^\infty \frac{v_g}{2\pi} e^{-i\delta k z_N} e^{-i k z_j} e^{i \Delta \omega_k(t'-t)} dk$$

$$= a^L_{\text{in}}(t + z_1/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt' \int_0^\infty e^{i\delta k z_N} e^{i k v_g (t'-t)} d(-k)$$

$$= a^L_{\text{in}}(t + z_1/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt' \int_{-\kappa_0}^\infty e^{i\delta k z_N} e^{i k v_g (t'-t)} d\kappa$$

$$= a^L_{\text{in}}(t + z_1/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt' \int_{-\infty}^\infty e^{i\delta k z_N} e^{i k v_g (t'-t)} d\kappa$$

$$= a^L_{\text{in}}(t + z_1/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt' \frac{2\pi}{v_g} \delta(t' - t + z_1/v_g) dt'$$

$$= a^L_{\text{in}}(t + z_1/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt'$$

$$\approx a^L_{\text{in}}(t + z_1/v_g) - i \sum_{j=1}^{N_a} \sqrt{\frac{\Gamma_j v_g}{4\pi}} \int_0^{t_f} \sigma_j^{-}(t') dt'$$

(B2)

Eqs. (B1) and (B2) are the input-output relations of the system from which we can calculate the field scattering properties of this system.
Appendix C: Derivation of Eq. (35)

We can derive a dynamical equation for $\rho_{01}(t)$ using similar method as deriving $\rho_S(t)$. Since $Tr_{S+R}[O_S \rho_{01}(t)] = Tr_{S+R}[O_S \rho_{01}(0)]$ where $\rho_{01}(0) = \rho_S(0) \otimes |0⟩⟨\Psi_F|$, we have

$$Tr_{S+R}[O_S \rho_{01}(t)] = Tr_{S+R}[O_S \rho_{01}(0)]$$

$$= i \frac{N_a}{2} \sum_{j=1}^{N_a} ε_j(t) Tr_{S+R}[\{σ_j^+(t), O_S\} \rho_{01}(0)]$$

$$+ i \frac{N_a}{2} \sum_{j=1}^{N_a} \sqrt{\frac{Γ_j}{2}} Tr_{S+R}[\{σ_j^+(t), O_S\} a_{in}^*(t - z_j/v_g) \rho_{01}(0)] + i \frac{N_a}{2} \sum_{j=1}^{N_a} \sqrt{\frac{Γ_j}{2}} Tr_{S+R}[a_{in}^*(t - z_j/v_g) [σ_j^-(t), O_S(t)] \rho_{01}(0)]$$

$$+ \sum_{jl} Α_j l Tr_{S+R}[\{σ_j^+(t), O_S(t)\} σ_l^-(t) \rho] - \sum_{jl} Α_j l Tr_{S+R}[\{σ_l^-(t), O_S(t)\} σ_j^-(t) \rho]$$

$$= - i \frac{N_a}{2} \sum_{j=1}^{N_a} ε_j(t) Tr_{S+R}[\{σ_j^+, O_{01}(0)\} - i \frac{N_a}{2} \sqrt{\frac{Γ_j}{2}} a_{in}^*(t - z_j/v_g) Tr_{S+R}[O_S \{σ_j^- , O_{00}(0)\}]$$

$$- \sum_{jl} Α_j l Tr_{S+R}[O_S \{σ_j^+, O_{01}(0)\} - σ_l^- O_{01}(0) \{σ_j^-\}] - \sum_{jl} Α_j l Tr_{S+R}[O_S \{σ_l^- O_{01}(0) \} σ_j^- - σ_j^- O_{01}(0) \{σ_j^-\}]]$$

\(\text{(C1)}\)

where $\rho_{00}(t) = U(t) \rho_S \otimes |0⟩⟨0| U^\dagger(t)$. Comparing both sides, we have

$$\dot{ρ}_{01}^S(t) = i \frac{N_a}{2} \sum_{j=1}^{N_a} ε_j(t) [σ_j^+, ρ_{01}^S(t)] - i \frac{N_a}{2} \sqrt{\frac{Γ_j}{2}} v_g a_{in}^*(t) [σ_j^-, ρ_{00}^S(t)] - i \sum_{jl} Im(Α_j l) [σ_j^+ σ_l^-, ρ_{01}^S(t)] - \mathcal{L}[ρ_{01}^S(t)],$$

\(\text{(C2)}\)

where $ρ_{00}^S(t) = Tr_R[ρ_{00}(t)]$. 

\[\]

[1] H. J. Kimble, The quantum internet, Nature 453, 1023 (2008).
[2] H. Zheng, D. J. Gauthier, and H. U. Baranger, Waveguide-QED-based photonic quantum computation, Phys. Rev. Lett. 111, 090502 (2013).
[3] Z. Liao, X. Zeng, H. Nha, and M. S. Zubairy, Photon transport in a one-dimensional nanophotonic waveguide QED system, Phys. Scr. 91, 063004 (2016).
[4] D. Roy, C. Wilson, and O. Firstenberg, Strongly interacting photons in one-dimensional continuum, Rev. Mod. Phys. 89, 021001 (2017).
[5] M. Arcari, I. Söllner, A. Javadi, S. Lindskov Hansen, S. Mahmoodian, J. Liu, H. Thyrrestrup, E. H. Lee, J. D. Song, S. Stobbe, and P. Lodahl, Near-Unity Coupling Efficiency of a Quantum Emitter to a Photonic Crystal Waveguide, Phys. Rev. Lett. 113, 093603 (2014).
[6] A. Laucht, S. Pütz, T. Günther, N. Hauke, R. Saive, S. Frédérick, M. Bichler, M.-C. Amann, A. W. Holleitner, M. Kaniber, and J. J. Finley, A waveguide-coupled on-chip single-photon source, Phys. Rev. X 2, 011014 (2012).
[7] C. Schuck, W. H. P. Fernice, and H. X. Tang, Waveguide integrated low noise NbTiN nanowire single-photon detectors with milli-Hz dark count rate, Sci Rep. 3, 1893 (2013).
[8] L. Zhou, H. Dong, Y.-X. Liu, C. P. Sun, and F. Nori, Quantum supercavity with emitter mirrors, Phys. Rev. A 78, 063827 (2008).
[9] H. Dong, Z. Gong, H. Ian, L. Zhou, and C. P. Sun, Intrinsic cavity QED and emergent quasimode la modes for a single photon, Phys. Rev. A 79, 063847 (2009).
[10] D. E. Chang, L. Jiang, A. V. Gorshkov, and H. J. Kimble, Cavity QED with emitter mirrors, New J. Phys. 14, 063003 (2012).
[11] Z. Liao, X. Zeng, H. Nha, and M. S. Zubairy, Single-photon frequency-hop generation in a one-dimensional waveguide coupled to two emitter arrays, Phys. Rev. A 93, 033851 (2016).
[12] J. S. Douglas, H. Habibian, C.-L. Hung, A. V. Gorshkov, H. J. Kimble, and D. E. Chang, Quantum many-body models with cold atoms coupled to photonic crystals, Nat. Photon. 9, 326 (2015).
[13] P. Richerme, Z.-X. Gong, A. Lee, C. Senko, J. Smith, M. Foss-Feil, S. Michalakis, A. V. Gorshkov, and C. Monroe, Non-local propagation of correlations in quantum systems with long-range interactions, Nature 511, 198 (2014).
[14] Y. Yu, F. Ma, X. Y. Luo, B. Jing, P.-F. Sun, R.-Z. Fang, C.-W. Yang, H. Liu, M.-Y. Zheng, X.-P. Xie, W.-J. Zhang, L.-X. You, Z. Wang, T.-Y. Chen, Q. Zhang, X.-H. Bao, and J.-W. Pan, Entanglement of two quan-
tum memories via fibres over dozens of kilometres, Nature **578**, 240 (2020).

[15] T. van Leent, M. Bock, R. Garthoff, K. Redeker, W. Zhang, T. Bauer, W. Rosenfeld, C. Becher, and H. Weinfurter, Long-distance distribution of emitter-photon entanglement at telecom wavelength, Phys. Rev. Lett. **124**, 010510 (2020).

[16] K. Y. Bliokh and F. Nori, Transverse and longitudinal angular momenta of light, Phys. Rep. **592**, 1–38 (2015).

[17] H. Pichler, T. Ramos, A. J. Daley, and P. Zoller, Quantum optics of chiral spin networks, Phys. Rev. A **91**, 042116 (2015).

[18] P. Lodahl, S. Mahmoodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, Chiral quantum optics, Nature **541**, 473 (2017).

[19] J. Petersen, J. Volz, and A. Rauschenbeutel, Chiral nanophotonic waveguide interface based on spin-orbit interaction of light, Science **346**, 67 (2014).

[20] S. Mahmoodian, P. Lodahl, and A. S. Sørensen, Quantum networks with chiral-lightmatter interaction in waveguides, Phys. Rev. Lett. **117**, 240501 (2016).

[21] M.-T. Cheng, X.-S. Ma, J.-Y. Zhang, and B. Wang, Single photon transport in two waveguides chirally coupled by a quantum emitter, Opt. Express **24**, 19988 (2016).

[22] K. Xia, F. Nori, and M. Xiao, Cavity-free optical isolators and circulators using a chiral cross-Kerr nonlinearity, Phys. Rev. Lett. **121**, 203602 (2018).

[23] Y. Lu, S. Gao, A. Fang, Z. Liao, F. Li, Micro-scale FabryPerot interferometer with high spectral resolution and tunable transmission frequency via chiral waveguide-emitter coupling, Phys. Lett. A **382** 1823 (2018).

[24] J. Q. Grim, A. S. Bracker, M. Zalalutdinov, S. G. Carter, A. C. Kozen, M. Kim, C. S. Kim, J. T. Mlack, M. Yakes, B. Lee, and D. Gammon, Scalable in operando strain tunable and tunable transmission frequency via chiral waveguide-based Fabry-Perot interferometers and circulators using a chiral cross-Kerr nonlinearity, Phys. Rev. A **94**, 053842 (2016).

[25] Y. Shen and J.-T. Shen, Photonic-Fock-state scattering from a two-level system, Phys. Rev. Lett. **121**, 063816 (2018).

[26] Y. Lu, S. Gao, A. Fang, Z. Liao, F. Li, Micro-scale Fabry-Perot interferometer with high spectral resolution and tunable transmission frequency via chiral waveguide-emitter coupling, Phys. Lett. A **382**, 1823 (2018).

[27] J. Q. Grim, A. S. Bracker, M. Zalalutdinov, S. G. Carter, A. C. Kozen, M. Kim, C. S. Kim, J. T. Mlack, M. Yakes, B. Lee, and D. Gammon, Scalable in operando strain tunable and tunable transmission frequency via chiral waveguide-based Fabry-Perot interferometers and circulators using a chiral cross-Kerr nonlinearity, Phys. Rev. A **94**, 053842 (2016).

[28] Y. Shen and J.-T. Shen, Photonic-Fock-state scattering from a two-level system, Phys. Rev. Lett. **121**, 063816 (2018).

[29] Y. Shen and J.-T. Shen, Coherent frequency down-conversions and entanglement generation in a Sagnac interferometer, Opt. Express **25**, 16151 (2017).

[30] D. Roy, Two-photon scattering by a driven three-level emitter in a one-dimensional waveguide and electromagnetically induced transparency, Phys. Rev. Lett. **106**, 053601 (2011).

[31] D. Witthaut and A. S. Sørensen, Photon scattering by a three-level emitter in a one-dimensional waveguide, New J. Phys. **12**, 043052 (2010).

[32] I. Shomroni, S. Rosenberg, Y. Lovsky, O. Bechler, G. Guendelman, and B. Dayan, All-optical routing of single photons by a one-atom switch controlled by a single photon, Science **345**, 903–906 (2014).

[33] Y. Chen, M. Wubs, J. Mørk, and A. F. Koenderink, Coherent single-photon absorption by single emitters coupled to one-dimensional nanophotonic waveguides, New J. Phys. **13**, 103010 (2011).

[34] Z. Liao, X. Zeng, S. Y. Zhu, and M. S. Zubairy, Single-photon transport through an atomic chain coupled to a one-dimensional nanophotonic waveguide, Phys. Rev. A **92**, 023806 (2015).

[35] Z. Liao, H. Nha, and M. S. Zubairy, Dynamical theory of single-photon transport in a one-dimensional waveguide coupled to identical and nonidentical emitters, Phys. Rev. A **94**, 053842 (2016).

[36] Z. Liao and M. S. Zubairy, Quantum state preparation by a shaped photon pulse in a one-dimensional continuum, Phys. Rev. A **98**, 023815 (2018).

[37] Z. Liao, M. Al-Amri, and M. S. Zubairy, Measurement of deep-subwavelength emitter separation in a waveguide-QED system, Opt. Express **25**, 31997 (2017).

[38] F. Dinc, . Ercan, and A. M. Brańczyk, Exact Markovian and non-Markovian time dynamics in waveguide QED: collective interactions, bound states in continuum, superradiance and subradiance, Quantum **3**, 213 (2019).

[39] J. T. Shen and S. Fan, Strongly correlated two-photon transport in a one-dimensional waveguide coupled to a two-level system, Phys. Rev. Lett. **98**, 153003 (2007).

[40] T. Shi, Y.-H. Wu, A. González-Tudela, and J. I. Cirac, Bound states in Boson impurity models, Phys. Rev. X **6**, 012017 (2016).

[41] P. Facchi, M. S. Kim, S. Pascazio, F. V. Pepe, D. Pomarico, and T. Tufarelli, Bound states and entanglement generation in waveguide quantum electrodynamics, Phys. Rev. A **94**, 043839 (2016).

[42] G. Calajó, Y.-L. L. Fang, H. U. Baranger, and F. Ciccarello, Exciting a bound state in the continuum through multiphoton scattering plus delayed quantum feedback, Phys. Rev. Lett. **122**, 073601 (2019).

[43] H. Zheng, D. J. Gauthier, and H. U. Baranger, Waveguide QED: Many-body bound-state effects in coherent and Fock-state scattering from a two-level system, Phys. Rev. A **82**, 063816 (2010).

[44] Y.-L. L. Fang, H. Zheng, and H. U. Baranger, One-dimensional waveguide coupled to multiple qubits: photon-photon correlations, EPJ Quantum Technol. **1**, 3 (2014).

[45] Y. Shen and J.-T. Shen, Photonic-Fock-state scattering in a waveguide-QED system and their correlation functions, Phys. Rev. A **92**, 033803 (2015).
perfect elastic transmission, Phys. Rev. A 96, 013842 (2017).

[51] T. Shi and C. P. Sun, Lehmman-Symanzik-Zimmermann reduction approach to multiphoton scattering in coupled-resonator arrays, Phys. Rev. B 79, 205111 (2009).

[52] T. Shi, S. Fan, and C. P. Sun, Two-photon transport in a waveguide coupled to a cavity in a two-level system, Phys. Rev. A 84, 063803 (2011).

[53] M. Laakso and M. Pletyukhov, Scattering of two photons from two distant qubits: exact solution, Phys. Rev. Lett. 113, 183601 (2014).

[54] S. Fan, S. E. Kocabaş, and J.-T. Shen, Input-output formalism for few-photon transport in one-dimensional nanophotonic waveguides coupled to a qubit, Phys. Rev. A 82, 063821 (2010).

[55] K. Lalumière, B. C. Sanders, A. F. van Loo, A. Fedorov, A. Wallraff, and A. Blais, Input-output theory for waveguide QED with an ensemble of inhomogeneous emitters, Phys. Rev. A 88, 043806 (2013).

[56] S. Xu and S. Fan, Input-output formalism for few-photon transport: a systematic treatment beyond two photons, Phys. Rev. A 91, 043845 (2015).

[57] J. Combes, J. Kerckhoff, and M. Sarovar, The SLH framework for modeling quantum input-output networks, Advances in Physics: X 2, 784–888 (2017).

[58] A. Roulet and V. Scarani, Solving the scattering of N photons on a two-level emitter without computation, New J. Phys. 18, 093035 (2016).

[59] D. J. Brod, J. Combes, and J. Gea-Banacloche, Two photons co- and counterpropagating through N cross-Kerr sites, Phys. Rev. A 94, 023833 (2016).

[60] J. Combes and D. J Brod, Two-photon self-Kerr nonlinearities for quantum computing and quantum optics, Phys. Rev. A 98, 062313 (2018).

[61] P. Domokos, P. Horak, and H. Ritsch, Quantum description of light-pulse scattering on a single emitter in waveguides, Phys. Rev. A, 65, 033832 (2002).

[62] W. Konyk and J. Gea-Banacloche, One- and two-photon scattering by two emitters in a waveguide, Phys. Rev. A 96, 063826 (2017).

[63] T. Caneva, M. T. Manzoni, T. Shi, J. S. Douglas, J. I. Cirac, and D. E. Chang, Quantum dynamics of propagating photons with strong interactions: a generalized inputoutput formalism, New J. Phys. 17, 113001 (2015).

[64] G.-Z. Song, E. Munro, W. Nie, L.-C. Kwek, F.-G. Deng, and G.-L. Long, Photon transport mediated by an emitter chain trapped along a photonic crystal waveguide, Phys. Rev. A 98, 023814 (2018).

[65] T. Shi, D. E. Chang, and J. I. Cirac, Multiphoton-scattering theory and generalized master equations, Phys. Rev. A 92, 053834 (2015).

[66] K. M. Gheri, K. Ellinger, T. Pellizzari, and P. Zoller, Photon-wavepackets as flying quantum bits, Fortschr. Phys. 46, 401–415 (1998).

[67] B. Q. Baragiola, R. L. Cook, A. M. Brančzyk, and J. Combes, N-photon wave packets interacting with an arbitrary quantum system, Phys. Rev. A 86, 013811 (2012).

[68] J. You, Z. Liao, S.-W. Li, and M. S. Zubairy, Waveguide quantum electrodynamics in squeezed vacuum, Phys. Rev. A 97, 023810 (2018).

[69] J. You, Z. Liao, and M. S. Zubairy, Steady-state population inversion of multiple Ξ-type emitters by the squeezed vacuum in a waveguide, Phys. Rev. A 100, 013843 (2019).

[70] D.F. Walls and G. J. Milburn, Quantum Optics, (Springer, Verlag Berlin Heidelberg, 2008).

[71] M. O. Scully and M. S. Zubairy, Quantum Optics, (Cambridge University Press, Cambridge, 1997).

[72] P. Senellart, G. Solomon, and A. White, High-performance semiconductor quantum-dot single-photon sources, Nat. Nanotech. 12, 10261039 (2017).