NLO Exclusive Evolution Kernels.

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Abstract

We outline a formalism used for a construction of two-loop flavor singlet exclusive evolution kernels in the $\overline{\text{MS}}$ scheme. The approach is based on the known pattern of conformal symmetry breaking in $\overline{\text{MS}}$ as well as constraints arising from the superconformal algebra of the $\mathcal{N} = 1$ super Yang-Mills theory.

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We outline a formalism used for a construction of two-loop flavor singlet exclusive evolution kernels in the $\overline{\text{MS}}$ scheme. The approach is based on the known pattern of conformal symmetry breaking in $\overline{\text{MS}}$ as well as constraints arising from the superconformal algebra of the $\mathcal{N} = 1$ super Yang-Mills theory.

1 $Q^2$ evolution of SPD

Exclusive processes provide an indispensable information for a construction of a unique picture of hadron wave functions, $\Psi$. Its lowest Fock components (integrated over different transverse momentum configurations of partons) go under the name of distribution amplitudes, $\phi(x)$. Being a fundamental characteristic, $\Psi$ defines all other inclusive and exclusive observables. A product of a wave function and a complex conjugate with fixed transverse momentum

$$\phi(x, \eta, \Delta_\perp) \sim \int d^2k_\perp \Psi^* \left( \frac{x+\eta}{2}, k_\perp + \frac{\Delta_\perp}{2} \right) \Psi \left( \frac{x-\eta}{2}, k_\perp - \frac{\Delta_\perp}{2} \right),$$

define a correlation function called skewed parton distribution (SPD). It generalizes its predecessor, — conventional inclusive density well known from DIS, to non-zero values of skewedness $\eta$ and $\Delta_\perp$. A peculiar feature of the SPDs is that they have a very different behaviour depending on the kinematical regime, i.e. an interplay of $x$ and $\eta$. Depending on the difference in the momentum fractions between the left- and right-hand-side of the parton ladder the SPDs behave like a regular parton distribution or like a distribution amplitude. Particular Mellin moments w.r.t. momentum fraction $x$ give hadron (and real Compton scattering) form factors and angular momenta of constituents.

In QCD the leading twist SPD is defined as a Fourier transform to the momentum fraction space of a light-ray operator constructed from $\varphi$-parton fields and sandwiched between hadronic states non-diagonal in momenta, schemati-
cally given by (Δ = \( p' - p \))

\[
\phi(x, \eta, \Delta_\perp |Q) = \frac{1}{2\pi} \int d\varphi(z) e^{izx} \langle h(p') | \varphi^\dagger(-z_-/2) \varphi(z_-/2) |Q | h(p) \rangle
\] (2)

The logarithmic \( Q \)-scale dependence of \( \phi \) arises due to a light-like separation of partons and is governed by a renormalization group equation. The generalized skewed kinematics for corresponding perturbative evolution kernels can unambiguously be restored from the conventional exclusive one \( \eta = 1 \).

\[
\frac{d}{d \ln Q^2} \phi(x|Q) = V(x, y|\alpha_s(Q)) \otimes \phi(y|Q),
\] (3)

where \( \tau_1 \otimes \tau_2(x, y) \equiv \int_0^1 dz \tau_1(x, z) \tau_2(z, y) \) defines the exclusive convolution and \( \phi = (\phi^q, \phi^g) \) is the vector of the quark and gluon distributions and \( V \) is a matrix of evolution kernels. Thanks to conformal invariance of classical QCD Lagrangian the leading order kernels having the structure \( V^{(0)}(x, y) = \theta(y-x) f(x, y) \pm \theta(x-y) f(\bar{x}, \bar{y}) \) can be diagonalized in the basis spanned by Gegenbauer polynomials \( C^{\nu} j(x) \otimes V^{(0)}(x, y) = \gamma^{(0)} j j C^{\nu} j(y) \) with forward anomalous dimensions (ADs) \( \gamma_j^{(0)} \). Beyond this level conformal symmetry is violated by quantum corrections and a diagonal AD matrix \( \gamma_j \) gets promoted to a triangular one \( \gamma_{jk}, k \leq j \). Thus \( V = V^D + V^{ND} \) with \( V^{ND} \propto \mathcal{O}(\alpha_s^2) \). An efficient formalisms to tackle the problem which eludes explicit multi-loop exercise and is based on the use of special conformal anomalies which produce the non-diagonal part, \( k < j \), of \( \gamma_{jk} \), converted into exclusive kernels \( V^{ND} \), and relations resulting from \( \mathcal{N} = 1 \) SUSY Ward identities which connect diagonal part of the kernels, \( V^D \), and allows to reconstruct all channels from a given \( QQ \) sector deduced by explicit evaluation of two-loop graphs.

2 Using conformal symmetry

Conformal operators which are Gegenbauer moments of \( \phi \), \( C^{\nu} j(x) \otimes \phi(x) \sim \langle h'|\mathcal{O}_{jj}|h \rangle \), build an infinite dimensional irreps of the collinear conformal algebra so(2,1). Conformal Ward identities derived for the Green function with conformal operator insertion \( \mathcal{G} = \mathcal{O}_{jj} \prod_i \phi_i \) in the regularized QCD allows, by means of algebra of dilatation \( \mathcal{D} \) and special conformal transformation \( \mathcal{K} \), to prove a matrix constraint for ADs \( \gamma \) and special conformal anomaly \( \gamma^c \)

\[
[D, K_\perp] = iK_\perp \quad \Rightarrow \quad \left[ a + \gamma^c + 2 \frac{\beta}{\pi} b, \gamma \right]_\perp = 0,
\] (4)

with \( \alpha_s \)-independent matrices \( a \) and \( b \) and QCD beta function \( \beta = \frac{\alpha_s}{4\pi} \beta_0 + \cdots \). The solution of the above equation with available one-loop conformal anomalies

\( 2 \)
\( \gamma_c \) implies the following form of the nondiagonal part of the NLO kernel

\[
V_{\text{ND}}^{(1)}(x, y) = -(I - D) \left\{ \dot{V} \otimes \left( V^{(0)} + \frac{\beta_0}{2} \mathbb{I} \right) + \left[ g \otimes, V^{(0)} \right] \right\}(x, y), \tag{5}
\]

where \((I - D)\) projects out the diagonal part \(\gamma_j^{(1)}\). Here \(\dot{V}\) is given mostly by a logarithmic modification of LO kernels \(f \to f \ln \frac{x}{y}\) plus an addendum, while \(g\) is a kernel whose conformal moments are proportional to a \(w\) part of \(\gamma_c = -b\gamma^{(0)} + w\).

3 Using \(\mathcal{N} = 1\) SUSY

The last problem is to find \(V_D\). Although it seems straightforward to solve, since the Gegenbauer moments \(V_{jk}^{(1)} = \delta_{jk} \gamma_j\) coincide with forward ADs calculated to NLO presently, practical inversion is extremely hard to handle. The main difficulty being kernels stemming from crossed-ladder type diagrams which we called \(G\)-functions. Since the conformal symmetry breaking part has been previously fixed, we can assume conformal covariance for the ADs. If one puts (Majorana) quarks into adjoint representation of \(SU(N_c)\), the classical “QCD” Lagrangian enjoys \(\mathcal{N} = 1\) SUSY. In perturbative calculations (with SUSY preserving regularization) this simply means the following identification of Casimir operators: \(C_F = 2T_F = C_A \equiv N_c\). From the commutator of the dilatation and translational SUSY generators \([Q, D]\) = \(\frac{i}{2}Q\) applied to the Green functions \(\mathcal{G}\) one finds six constraints for eight \(G\)-functions for even and odd parity sectors. Since the \(QQ\)-function in the \(QQ\) channel is explicitly known the other ones can be unambiguously reconstructed and colour factors trivially restored. The full NLO kernel has now the following form

\[
V^{(1)} = -\dot{V} \otimes \left( V^{(0)} + \frac{\beta_0}{2} \mathbb{I} \right) - \left[ g \otimes, V^{(0)} \right] + G + D. \tag{6}
\]

4 Final reconstruction

The unknown remaining diagonal piece \(D\) can be reconstructed by forming the forward limit to splitting functions, e.g. \(QQP(z) = \text{LIM}_{z \to 0}^{QQ} V(x, y) \equiv \lim_{\xi \to 0}^{QQ} V(\frac{x}{\xi}, \frac{1}{\xi})/|\xi|\). Comparing it with the known two-loop DGLAP kernels one represents \(D\) as a convolution of simple kernels whose non-forward counterparts are easy to find. Since \(\text{LIM} \{V_1 \otimes V_2\} = \text{LIM}V_1 \otimes \text{LIM}V_2\) restoration of \(D\) from the forward case is simple and one gets a complete \(V^{(1)}\).

References

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