The robust recovery model of distribution network considering correlation between wind speed and load

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Abstract. In order to consider the correlation between distributed power output and load, this paper uses the value-at-risk theory to quantitatively calculate the uncertainty of wind speed and load demand, and establishes a wind speed-load joint probability distribution model using bivariate normal distribution, which will take into account the wind speed uncertainty. The robust decision-making problem of reliability and load participation is characterized as a mixed-integer linear programming problem, which takes the minimization of the equivalent load loss as the optimization goal, establishes the load recovery path as the optimization variable, and takes the wind speed and load as the disturbance variable to account for the wind speed-load dependent robust restoration model of the distribution network is solved by the Gurobi high-efficiency solver. Finally, the IEEE69-bus power distribution system with wind power was used to verify the effectiveness of this model and solution method.

1. Introduction

In recent years, wind power generation has made great progress in its operation and control technology. With the research and implementation of technologies such as Micro-Grid (MG) and Active Distribution Network (ADN), the penetration rate of wind power in the distribution network has gradually increased\textsuperscript{[1]}. How to use DG to make a reasonable recovery plan after a major blackout in the distribution network to minimize the loss of power outage is a research hotspot in the current distribution network load recovery\textsuperscript{[2]}. However, the output of DG is affected by natural factors, which brings many uncertain factors to the restoration of the distribution network.

In order to solve the uncertainty of DG output, existing studies have used scene classification\textsuperscript{[3]}, interval estimation\textsuperscript{[4]} and Latin hypercube sampling\textsuperscript{[5]} to convert the DG output uncertainty model into a deterministic model for solution. However, due to the limitation of data, the above methods cannot completely simulate the problems caused by uncertainty. In addition to the uncertainty of DG's output, the uncertainty of load also has an influence. Therefore, scholars have done a lot of research on DG output and load uncertainty\textsuperscript{[6]–[7]}.

However, the above robust recovery model does not consider the correlation between the DG output and the load \textsuperscript{[8]}. In addition, the robust control parameters are only given by experience and there is no specific value rule. Once the actual fluctuation of the uncertain variable exceeds the adjustment range of the robust control parameter, the robust optimization recovery scheme will not be applicable.

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To solve the above problems, this paper uses wind speed and load normal predictions to characterize wind speed and load respectively, and uses the value at risk (VaR) method to calculate their uncertainty. With the minimum equivalent load loss as the optimization objective, this paper establishes a robust recovery model of the distribution network that takes wind speed and load demand as disturbance variables and takes into account the wind speed-load correlation. This model ensures that when the wind speed and load fluctuate within a given interval, the obtained scheme can operate reliably; when it exceeds the given interval, a backup recovery scheme can also be obtained, which provides a new idea for the formulation of a robust recovery scheme.

2. Robust distribution network recovery model
In order to account for the uncertainty of wind power and load, this paper adopts a robust optimization model.

2.1. Objective function
This paper takes the minimum equivalent load loss in the distribution network as the optimization goal:

$$\min \max \sum_{s=1}^{M} C_s (L_s - L_{rs})$$

(1)

Where: \(W\) is the set of load restoration paths in the distribution network; \(v, p\) are the wind speed and the load demand respectively; \(M\) is the number of load types; \(C_s\) is the restoration weight of various loads; \(L_s\) is the total load of this type load in the distribution network, \(L_{rs}\) is the total amount of this type load restored in the restoration plan.

2.2. Constraints
1) Wind speed and load fluctuation constraints
The general fluctuation constraint of wind speed and load is expressed as:

$$\begin{align*}
&v_j = \bar{v}_j + \mu_{DGj} \cdot \sigma_j, \quad j = 1, ..., T_{DG} \\
&-1 \leq \mu_{DGj} \leq 1, \quad j = 1, ..., T_{DG} \\
&\sum_{j=1}^{T_{DG}} \mu_{DGj} \leq \Gamma_{DG}, \quad \Gamma_{DG} \in [0, T_{DG}] \\
&p_r = \bar{p}_r + \mu_{P} \cdot \sigma_r, \quad r = 1, ..., T_r \\
&-1 \leq \mu_{P} \leq 1, \quad r = 1, ..., T_r \\
&\sum_{r=1}^{T_r} \mu_{P} \leq \Gamma_T, \quad \Gamma_T \in [0, T_r]
\end{align*}$$

(2)

Where: \(v_j, \bar{v}_j, \mu_{DGj}, \sigma_j\) are the actual wind speed, expected wind speed, wind speed fluctuation control amount, and maximum wind speed deviation value of each DG, respectively; \(T_{DG}\) is the number of DGs; \(\Gamma_{DG}\) is the robust control parameter for DG fluctuations; \(p_r, \bar{p}_r, \mu_{P}, \sigma_r\) are the actual load demand, expected load demand, load fluctuation control amount, and maximum load deviation value of each node, respectively; \(T_r\) is the load number included in the distribution network; \(\Gamma_T\) is robust to load fluctuations control parameter.

2) The network radial constraint
The network radial constraint expressed by formula (3):

$$\begin{align*}
&\sum_{i=1}^{n} u_i \leq 1, \quad u_i \in [0, 1], \forall v \\
&\sum_{i,j} u_{ij} < L_{w}, \quad \forall v, \forall \lambda \\
&\left(1 - u_{ij}\right) \sum_{s=1}^{T} |L_{w} - L_{s} e^{-\lambda (1 - u_{ij})} | < 0, \forall v, \forall \lambda
\end{align*}$$

(3)
Where: $w^n_i$, $\pi^n_i$, and $\pi'^n_i$ are the restoration path 0/1 decision variable, the set of branches included, and the number of branches contained of node $n$, respectively; $T_n$ is the number of available restoration paths for the node $n$; $A^j_{ik}$ is the variable indicates the current direction of the branch $(j,k)$ in the restoration path $i$.

3) Current constraints
The line power flow constraint is expressed by the following formula (4):

\[
\begin{align*}
\min & \quad P_{ij} \leq P_{ij}^m \\
\max & \quad Q_{ij} \leq Q_{ij}^m \\
\min & \quad P_{ij} + Q_{ij} \leq (S_{ij}^m)^2
\end{align*}
\]

Where: $P_{ij}$, $Q_{ij}$, and $S_{ij}^m$ are the active power, reactive power, and maximum allowable power passing through the branch, respectively. The power constraint of the restoration network as follows:

\[
\sum_{i=1}^{T_n} P_i \leq \sum_{d=1}^{P_{DGd}(d)}
\]

Where: $P_{DGd}(d)$ is the active and reactive power output of the DG $d$ connected to the restored load.

4) Voltage constraints

\[
\begin{align*}
\left| V_i - V_i^\alpha \right| &= 2(p_i R_i + Q_i X_i) - M_v \\
\left| V_i - V_i^\alpha \right| &= 2(p_i R_i + Q_i X_i) - M_v \\
\left| (V_i^m) \right| &\leq V_i^m
\end{align*}
\]

Where: $V_i$ is the voltage amplitude of the node $i$; $V_i^\max$ and $V_i^\min$ are the upper and lower limits of the voltage amplitude respectively; if the branch $(i,j)$ is selected as the restoration path, $M_v$ is 0. Otherwise, it is a large positive number.

In order to make up for the shortcomings of the robust control parameters of the above model that are subjective and conservative, the VaR method is used to quantitatively describe the uncertainty of wind speed and load, so as to overcome the subjectivity of the value and reduce the conservativeness of the robust optimization model.

3. Wind speed-load correlation model based on VaR

3.1. Method of value at risk
In the restoration of the distribution network, the less the actual load demand exceeds the expected value, the better it is for the network recovery, and the load exceeding the expectation will lead to an upper deviation. While the larger the wind speed, the better, and the value is not enough to expect to cause a lower deviation. Based on the above analysis, the uncertainty of forecasted wind speed and forecasted load are described as equations (7) and equations (8):

\[
\begin{align*}
\rho_{0-w}^{\alpha}(s,\alpha) &= \max\left\{ s \in [s_1, s_2], \frac{\int_{t=s_1}^{s_2} f(t) dt}{\int_{t=s_1}^{s_2} f(t) dt} - \alpha \right\} \\
C_{0-w}^{\alpha}(s,\alpha) &= \frac{\int_{s_1}^{s_2} f(t) dt}{\int_{t=s_1}^{s_2} f(t) dt} - \alpha \\
C_{0-l}^{\alpha}(s,\alpha) &= \frac{\int_{s_1}^{s_2} f(t) dt}{\int_{t=s_1}^{s_2} f(t) dt} - \alpha
\end{align*}
\]
\[
\begin{align*}
\mathcal{V}_{\alpha}(x, \alpha) &= \min \left\{ x \in [x_l, x_h] : \int_{x_l}^{x} \frac{f(x)dx}{\alpha} \geq \alpha \right\} \\
C_{\alpha}(x, \alpha) &= \int_{x}^{\mathcal{V}_{\alpha}(x, \alpha)} f(x)dx - \int_{x}^{\mathcal{V}_{\alpha}(x, \alpha)} f(x)dx
\end{align*}
\]
(8)

Where: \(x_l\) and \(x_h\) are the endpoints of the variable \(x\) prediction interval; \(f(x)\) is the probability density of \(x\) obeys the distribution; \(\alpha\) is the confidence level.

The probability density of the normal distribution that predicts the wind speed and load obeys is \(v \sim N(\mu_1, \sigma_1^2)\) and \(p \sim N(\mu_2, \sigma_2^2)\). Then the deviation caused by the wind speed and load forecast interval at the confidence level is expressed as \(V_{\alpha}^{down}(v, \alpha)\), \(C_{\alpha}^{down}(v, \alpha)\) and \(V_{\alpha}^{up}(p, \alpha)\), \(C_{\alpha}^{up}(p, \alpha)\) according to equations (7)–(8).

3.2. Wind speed-load correlation model based on VaR

Considering that wind speed and load are both affected by a variety of climatic factors, there is a certain correlation. In this paper, the bivariate normal distribution function that takes into account the correlation is used to construct the analytic expression of the wind speed-load bivariate joint distribution. Substituting \(v \sim N(\mu_1, \sigma_1^2)\), \(p \sim N(\mu_2, \sigma_2^2)\) into the bivariate normal distribution joint probability density expression, the wind speed-load bivariate normal joint probability density function is shown in equation (9):

\[
f_{(v, p)}(v, p) = \frac{1}{2\pi \sigma_v \sigma_p \sqrt{1-\rho_{vp}}} \exp \left[ \frac{1}{2(1-\rho_{vp})} \left\{ \frac{(v-\mu_v)^2}{\sigma_v^2} - 2\rho_{vp} \frac{(v-\mu_v)(p-\mu_p)}{\sigma_v \sigma_p} + \frac{(p-\mu_p)^2}{\sigma_p^2} \right\} \right]
\]
(9)

Where: \(\rho_{vp}\) is the correlation coefficient between wind speed \(v\) and load \(p\).

In the scenario of confidence level \(\alpha\), according to the relationship between wind speed forecast accuracy and load forecast accuracy, a variable with high forecast accuracy first uses one-dimensional normal distribution data to determine its risk value and conditional risk value, and then according to the two-variable normal conditional probability density formula, The corresponding risk value and conditional risk value of another variable can also be determined by preference. Based on this, the wind speed fluctuation interval \([V_{\alpha}^{down}(v, \alpha_{\rho_{vp}}), v_h]\) and the load fluctuation interval \([p_l, V_{\alpha}^{up}(p, \alpha_{\rho_{vp}})]\) can be obtained, so that the constraint equation (2) can be transformed into the expression (10).

\[
\begin{align*}
V_{\alpha}^{down}(v, \alpha_{\rho_{vp}}) &\leq v \leq v_h \\
p_l &\leq p \leq V_{\alpha}^{up}(p, \alpha_{\rho_{vp}})
\end{align*}
\]
(10)

In summary, the robust restoration model of the distribution network that takes into account the wind speed-load correlation built in this paper is composed of objective function equation (1), constraint equation (10), and equation (3) ~ equation (6), which can be solved by YALMIP tool.

4. Results & Discussion

The calculation example uses the IEEE-69 node standard power distribution system which has 69 nodes and 73 branches. All branches are equipped with tie switches or section switches. Add DG at nodes 19, 36, and 67, and all devices have black start capability. The system diagram is shown in Figure 1. The recovery scenario is that the distribution network loses the main network power supply, and the DG is used to restore the load. The wind speed and load data are shown in Table 1.
Figure 1. The modified IEEE69 nodes power system

Table 1. Wind speed and load parameter

| Prediction interval | Predicted mean | Prediction accuracy | Actual accuracy |
|---------------------|----------------|--------------------|-----------------|
| $v$ [8.12]          | 10             | 0.8                | 1.55            |
| $p$ [0.9,1.1]       | 1              | 0.9                | 0.06            |

The wind speed-load bivariate normal distribution model established above is combined with the value-at-risk method to solve the wind speed and load confidence threshold and the out-of-bounds expected value, as shown in Table 2:

Table 2. The value of wind speed and load VaR with CVaR

| $V_{\alpha}^w(v,\alpha_v)$ | $C_{\alpha}^w(v,\alpha_v)$ | $V_{\alpha}^p(v,\alpha_v)$ | $C_{\alpha}^p(v,\alpha_v)$ |
|-----------------------------|---------------------------|-----------------------------|---------------------------|
| 8.99                        | 8.5452                    | 1.045                       | 1.0677                    |

According to Table 2, when the actual wind speed value exceeds the confidence level, the expected value of the out-of-bounds is less than the confidence threshold, while the actual load value exceeds the confidence level, and the expected value of the out-of-bounds is greater than the confidence threshold, which conforms to the actual scenario. The wind speed, load confidence threshold and cross-boundary expected value in Table 2 are respectively brought into the model solution and the recovery results are shown in Table 3.

Table 3. The recovery results of expected scenario and cross boundary scenario

| Loss of load | Expected scenario | Cross-border scenarios |
|--------------|-------------------|------------------------|
| $l_{\alpha}^w$ | 0                 | 0                      |
| $l_{\alpha}^p$ | 2.0744            | 2.1474                 |
| $l_{\alpha}^w$ | 0.4338            | 2.648                  |

It can be seen from Table 3 that when the wind speed and load values are within the expected scenario, the power supply of the first type loads is restored, and priority is given to ensuring that the important loads are supplied with power. When the wind speed and load exceed the expected scenario, the loss of load is more than the recovery result of the expected scenario, but the primary load continues to maintain full power supply.

In order to verify the effectiveness of taking into account the wind speed-load correlation, different confidence levels and correlation coefficients are selected to analyze the confidence thresholds and out-of-bounds expectations of wind speed and load, as shown in Table 4:

Table 4. Analysis of correlation between wind speed and load.

| $\alpha$ | $\rho_{wp}$ | $V_{\alpha}^w(v,\alpha_v)$ | $C_{\alpha}^w(v,\alpha_v)$ | $V_{\alpha}^p(p,\alpha_p)$ | $C_{\alpha}^p(p,\alpha_p)$ |
|----------|-------------|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| 0.85     | 0           | 8.79                        | 1.0545                      | 8.4293                      | 1.0737                      |
| 0.85     | 0.1         | 8.85                        | 1.0545                      | 8.4677                      | 1.0737                      |
| 0.85     | 0.2         | 8.93                        | 1.0545                      | 8.5206                      | 1.0737                      |
| 0.9      | 0           | 8.57                        | 1.0655                      | 8.3042                      | 1.0805                      |
| 0.9      | 0.1         | 8.63                        | 1.0655                      | 8.3405                      | 1.0805                      |
| 0.9      | 0.2         | 8.71                        | 1.0655                      | 8.3906                      | 1.0805                      |
| 0.95     | 0           | 8.31                        | 1.0795                      | 8.1611                      | 1.0889                      |
It can be seen from Table 4 that when the correlation coefficient is fixed, as the confidence level increases, to ensure the confidence level of the wind speed value, $v_{\text{down}}^{\alpha}$ shows a decreasing trend, and the actual wind speed exceeds the confidence level, it also shows a downward trend. Therefore, $\alpha$ should be reasonably given that if the value is too large, it is not conducive to the use of wind power, and in contrast, a load shedding accident may occur due to wind speed drops during the recovery process. Corresponding to this, $v_{\text{up}}^{\alpha}$ shows an increasing trend with the increase of $\alpha$, and when the load exceeds the confidence level $C_{\alpha}$ also shows an increasing trend.

It is worth noting that when the value of the confidence level is fixed, as the correlation coefficient increases, the positive correlation between wind speed and load becomes stronger, and the severity of the confidence interval formed by the two risk values gradually improves, which will moderately reduce the conservativeness of the robust model solution, and will have a certain impact on improving the recovery effect of the formulated plan.

5. Conclusions
In this paper, the restoration model analyzes the changes in the robust restoration results of changing the wind speed-load correlation coefficient, and verifies the effectiveness of taking into account the wind speed-load positive correlation to improve the robustness of the robust model. It is more in line with the recovery plan of the actual recovery scenario, and provides guidance for the dispatcher to implement the plan.

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