Axion couplings in grand unified theories

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ABSTRACT: We show that the couplings of axions to gauge bosons are highly restricted in Grand Unified Theories where the standard model is embedded in a simple 4D gauge group. The topological nature of these couplings allows them to be matched from the UV to the IR, and the ratio of the anomaly with photons and gluons for any axion is fixed by unification. This implies that there is a single axion, the QCD axion, with an anomalous coupling to photons. Other light axion-like particles can couple to photons by mixing through the QCD axion portal and lie to the right of the QCD line in the mass-coupling plane. Axions which break the unification relation between gluon and photon couplings are necessarily charged under the GUT gauge group and become heavy from perturbative mass contributions. A discovery of an axion to the left of the QCD line can rule out simple Grand Unified models. Axion searches are therefore tabletop and astrophysical probes of Grand Unification.

KEYWORDS: Axions and ALPs, Grand Unification

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1 Introduction

Topological couplings provide a unique opportunity to connect the far UV and IR dynamics of theories. In general, even if the UV theory is simple, the IR dynamics can be very complicated, as exemplified by the rich structure of nuclear physics that arises from the relatively simple QCD Lagrangian. Topological couplings, however, are unaffected by such dynamics and are invariant under renormalisation group flow. If the UV theory is simple then this provides restrictive constraints on the topological interactions in the IR with direct experimental consequences. The axion-photon coupling is extremely important for experimental searches (see [1–4] for a review) and gets a UV contribution from a quantised anomaly coefficient which is a topological quantity [5]. There are additional mixing contributions to the coupling that come from working in a canonical basis of mass eigenstates, but this mixing is calculable in the IR given a mass generation mechanism for the axion [6, 7]. In this paper we study the restrictions on axion-photon couplings in Grand
Unified Theories (GUTs). The axion-photon coupling is particularly relevant as the majority of experimental searches for axions aim to detect axions through this coupling [8–21].

Axions are compact bosons with a discrete gauged shift symmetry. In this context it is very useful to interpret axions as “0-form” gauge fields, and the shift symmetry as a large gauge transformation. This language highlights the topological nature of axions. The axion couplings to gauge fields are Chern-Simons like couplings which are topological quantities, and we show in section 2 that these couplings are quantised. Axions are compelling new physics candidates, motivated from the bottom-up perspective by their role in solving the strong CP problem [6, 7, 22–26] and dark matter [27–29], as well as from top-down constructions in string theory where they are ubiquitously present [30–32].

The QCD axion [6, 7] is a particularly well-motivated axion, which couples anomalously to QCD. If QCD instantons dominate over the other contributions to the axion potential, then by the Vafa-Witten theorem [33] the axion potential is minimised when the CP violating phase \( \bar{\theta} \) is set to zero, dynamically solving the strong CP problem. This also leads to a relationship between the mass of the QCD axion and its coupling to gauge bosons. In particular the coupling of the QCD axion to photons is [34]

\[
\frac{g_{a\gamma\gamma}}{m_a} = \frac{\alpha_{em}}{2\pi} \frac{\sqrt{z} + \frac{1}{\sqrt{z}}}{m_{\pi} f_{\pi}} \left( \frac{E}{N} - 1.92 \right), \quad z = \left( \frac{m_u}{m_d} \right).
\] (1.1)

The rational number \( E/N \) which represents the ratio of the anomaly coefficients to photons and gluons. This line in \( g_{a\gamma\gamma}-m_a \) space is known as the QCD line. For the purposes of experiments which search for the axions via the anomalous photon coupling, the QCD axion is effectively a one parameter model.

In addition to the QCD axion there could be other (ultra)light axions that can couple to photons, referred to as axion-like particles (ALPs). ALPs generally do not have low-energy instanton contributions as in the case of QCD axion, and the dominant contribution to their mass arises from the breaking of their continuous shift symmetry by UV effects [35]. Such effects are thought to be unavoidable in theories of quantum gravity [36], but they can be exponentially small [37]. ALPs may therefore have much smaller masses than the QCD axion and have a coupling to photons which is independent of their mass. This has motivated a broad search for ALPs with photon coupling and mass off the QCD line [38], often in the low-mass region of parameter space. We show that if the Standard Model (SM) is unified in the UV, however, the ALPs coupling to photons is generated by mechanisms that are correlated with the mass for the axion and points to a preferred region of ALP parameter space where \( g_{a\gamma\gamma}/m_a \) is smaller than the QCD axion expectation.

Grand Unification remains one of the most compelling UV completions that explains some of the features of the SM by embedding the SM gauge fields in a simple group at some high energy scale [39, 40]. This framework provides an explanation for the SM quantum numbers and the relative size of the gauge couplings in the IR. As we will show in this paper, it also has strong implications for axion phenomenology. The underlying reason for this is that the couplings of axions to gauge bosons are determined by anomaly coefficients, which are necessarily quantised and dictated by gauge quantum numbers in the UV.
The requirement that all of the SM gauge groups unify in the UV requires that the axion must couple anomalously to all gauge bosons or none at all. In the limit where mass mixing between axions can be ignored there is a single axion — the QCD axion — which couples to photons as well as gluons, and all other ALPs are fully decoupled (barring derivative couplings to fermions, as we discuss below). The QCD axion anomalously coupled to the GUT gauge group follows equation (1.1) with the ratio $E/N$ fixed by the structure of the GUT gauge group.\footnote{The minimal GUT prediction $E/N = 8/3$ was noted long ago [41]. The anomaly ratio can take more general values in GUT theories where the SM is non-trivially embedded. See appendix A.} We discuss the GUT prediction for the phenomenology of the QCD axion in section 3.

Any axion with mass and coupling not on the QCD line we refer to as an ALP. Once effects of mass mixing are considered an ALP can inherit a coupling to the photon through the ‘QCD axion portal’. In this case, in the absence of tuning, a light ALP $b$ will have couplings to gauge bosons which are suppressed by $m_b^2/m_{QCD}^2$. Thus we see that through a completely different mechanism, an ALP in these theories still has a correlation between its mass and photon coupling. If an ALP does not couple through mixing with the QCD axion, it can pick up a coupling to the photon if it is itself charged under the GUT gauge group (but neutral under the surviving SM group), as is the case in composite axion models [42–45]. In this case it will pick up a perturbative mass from GUT interactions, analogous to the electromagnetic contribution to the charged pion masses. In section 4 we determine the photon coupling of ALPs generated through each of these two effects as well as the implications of additional dark photons mixing with the SM photon. We show that the coupling of ALPs to photons is necessarily weaker than that of the QCD axion for the same mass, in the parameter space to the right of the QCD line.

ALP-fermion couplings are not quantised and are therefore not suppressed in GUTs, implying that experiments that search for ALPs through their fermion couplings [46, 47] may be promising methods to search for ALPs in the context of GUTs, which are discussed in section 5. Assuming flavour-conserving couplings, the ALP coupling to electrons in astrophysical systems turns out to give the most stringent bounds on the ALP parameter space. Flavour-violating couplings, if present, can also place strong constraints on the ALP decay constant in laboratory experiments [48–50]. We show that, remarkably, while axion-mediated forces are unsuppressed for the QCD axion, for light ALPs the monopole-dipole or the monopole-monopole interaction is strongly suppressed. The experimentally interesting monopole-dipole forces can distinguish between the QCD axion and ALPs in GUTs.

The main result our work highlights is that the phenomenology of the QCD axion and additional ALPs is strongly constrained in GUT theories. Axions therefore give us a low-energy handle to probe grand unification in table-top experiments [47, 51, 52] and in the sky [31, 53]. The traditional experimental strategy to look for GUTs is to search for the decay of protons into mesons and anti-leptons. The current limit on the life-time of the proton: $\tau > 2.4 \times 10^{34}$ yr [54] constrains the GUT scale to be $M_{GUT} \gtrsim 2 \times 10^{16}$ GeV. The bound on the life-time is expected to be improved by a factor of around 10 at Hyper-K [55], which results in only a factor of $\sim 2$ in the reach for $M_{GUT}$. Our results imply that a
discovery of a light ALP with $g_{a\gamma\gamma}/m_a$ larger than the QCD prediction can potentially determine if the SM gauge groups are unified in the UV. ALP searches can then give us a novel experimental handle on GUTs.

In the literature the term GUT applies more widely to theories which only partially unify the SM gauge groups. We discuss these theories in section 6 and find that the possibility of finding a light ALP in these theories is correlated with sacrificing some of the predictions of GUTs — namely, charge quantisation, the prediction of the weak mixing angle and non-existence of exotic fractionally-charged states — although this is somewhat model dependent. Another important class of related theories are GUTs in higher dimensions, where unification and compactification of extra dimensions have interesting interplay. These theories, known as orbifold GUTs [56–61] and related string theoretic scenarios will be studied in more detail in upcoming work.

It is worth comparing the results in this work with previous literature on axions and GUTs, which has focused on the dynamical aspects of unification such as $\beta$ functions, matter representations, the GUT scale and axion mass. In [62], the emphasis was on possible representations of the SM that can lead to perturbative unification. In the models of [63–68], the GUT scale was tied to the axion decay constant.\footnote{See also [69–71] for connections of PQ symmetry to GUTs in a different context.}

Our work focuses on the topological aspect of axion couplings, and is directly tied to charge quantisation in GUTs. Thus it is complementary to the dynamical details of the theory such as the scale of PQ breaking or patterns of GUT symmetry breaking.

2 Quantisation of axion couplings

The central idea underlying our analysis is the fact that axion couplings to gauge bosons are quantised, and can be anomaly matched between the UV and the IR [72, 73]. In this section we review the arguments that show that the couplings of an axion to a gauge boson are quantised.\footnote{In [74] the question of quantisation of axion couplings with photons is revisited, however the coupling to non-Abelian gauge bosons is still assumed to be quantised. We will not have anything to add to the discussion in this paper.}

The effects of possible kinetic and mass mixing will be studied in subsequent sections and form the bulk of our analysis which shows that the coupling and mass for ALPs are correlated in unified theories.

The axion is defined as a compact scalar $a$, or equivalently a pseudoscalar field with a discrete gauge symmetry $a \simeq a + 2\pi F_a$. In many examples, the compactness of the axion follows from its role as the Goldstone boson when a compact global Peccei-Quinn symmetry $U(1)_{PQ}$ is spontaneously broken. In other cases it arises from dimensional reduction of a gauge theory with a compact gauge group. It is useful to think of the discrete shift of the axion as a large gauge transformation for a 0-form gauge field. The quantisation of axion couplings then follows similar logic to the quantisation of $U(1)$ gauge charges.

Anticipating our discussion of GUTs, we start with an example of a single axion coupled to a gauge field $\mathcal{G}$ associated with a simple group $\mathcal{G}$, and review the quantisation of axion couplings to gauge groups. The axion-photon coupling is intimately related to
the Wess-Zumino-Witten term in the chiral Lagrangian, or the Chern-Simons couplings of gauge bosons in odd-dimensions. The Lagrangian describing an axion anomalously coupled to the gauge group $G$ is

$$\mathcal{L} = -\frac{1}{4} G_{\mu\nu}^a G^{a\mu\nu} + \frac{1}{2} \partial_\mu a \partial^\mu a + A \frac{\alpha_{\text{GUT}}}{F_a} G^{a\mu\nu} \tilde{G}^{a\mu\nu}$$  \tag{2.1}$$

where $\tilde{G}^{a\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} G_{\alpha\beta}^a$. The parameters $\alpha$ and $F_a$ are not topological and are sensitive to geometry and dynamical effects such as RG running. The anomaly coefficient, $A$, however, is quantised and by the usual arguments of anomaly matching transmits information from the UV to the IR unpolluted by intervening physics.\footnote{Equation (2.1) is defined in a basis where the determinant of the fermion mass matrix is real. It can be defined in more physical terms using the axion to photon decay amplitude, see discussion in [75].}

In this section we review one general argument that shows this quantisation of the axion-photon coupling.

A concise way to see the quantisation of the axion-photon coupling is by analogy with Chern-Simons theory in odd dimensions. If we put the 4D theory in Euclidean space on $S_4$ by adding a point at infinity to $\mathbb{R}_4$, on a constant axion background the action is proportional to the winding number of the gauge field configuration

$$\int_{S_4} \frac{\alpha_{\text{GUT}}}{8\pi} G^{a\mu\nu} \tilde{G}^{a\mu\nu} = n \in \mathbb{Z}. \tag{2.2}$$

The hallmark of a Chern-Simons action is that it is not gauge-invariant (in this case under the discrete gauge symmetry of the axion, $a \rightarrow a + 2\pi F_a$),

$$I[a + 2\pi F_a] = I[a] + 2\pi n A. \tag{2.3}$$

However, it is gauge invariant modulo $2\pi$, and hence can be used as a quantum action, $\exp(iI)$ in the path integral for $A \in \mathbb{Z}$.

The discussion above can easily be generalised to include multiple massless axions. If there is a set of global $U(1)$ symmetries anomalous\footnote{Technically these global symmetries are broken since the associated current has an ABJ anomaly with a dynamical gauge field. We will continue to use the term anomalous for such symmetries, since we are interested in matching the anomalies in weakly coupled regimes for the SM gauge fields. Similarly, breaking of the global $U(1)$ symmetries by quantum gravitational effects will be negligible for the cases of interest.} under $G$ that are realized non-linearly by axions, the $U(1)$ symmetries can always be redefined so that only one of them is anomalous under $G$. We will call this symmetry $U(1)_{\text{PQ}}$. This sort of redefinition is very similar to the case of the baryon and lepton numbers in the SM. The $U(1)_B$ and $U(1)_L$ symmetries are each anomalous with respect to $SU(2)_{\text{ew}}$, but can be redefined into $U(1)_{B-L}$ and $U(1)_{B+L}$ symmetries, of which only $U(1)_{B+L}$ is anomalous. In a similar way, we can choose a linear combination of the $U(1)$ symmetries such that only $U(1)_{\text{PQ}}$ will be anomalous under $G$. When $U(1)_{\text{PQ}}$ is spontaneously broken the corresponding axion will saturate the anomaly and other axions will remain decoupled from gauge fields. This is consistent with our discussion above of quantisation of couplings — only one axion $a$ couples to $G$ with a non-zero integer $A \in \mathbb{Z}$, and all other axions $b_i$ couple with the integer $A_{b_i} = 0$.

This is a rather strong statement, so it is worth emphasizing the assumptions under which this is true.
• We have not included the effects of axion mass terms or kinetic mixing, which can generate mixing between axions. When these effects are included an ALP $b$ with vanishing anomaly coefficient may couple to $G \tilde{G}$ through mixing with the state $a$.

• The axion is assumed to be neutral under the full GUT gauge group $G$. If an axion is charged under $G$ there may be multiple axions coupled to photons. The GUT charge however implies that the axion picks up a perturbative mass from GUT dynamics.

• In our analysis in this paper, we assume a simple unified gauge group $G$. We study the implications of relaxing this assumption in section 6 within 4D field theories. The interplay of axions and grand unification in higher dimensional GUTs will be studied in a follow-up paper.

In each of these cases, we find that the ALP mass and coupling to photons are correlated, and in specific cases lead to concrete targets in the $m_a - g_a \gamma \gamma$ plane. In particular, we find that

$$\frac{g_b \gamma \gamma}{m_b} \gg \frac{\alpha_{em}}{2\pi} \frac{1}{m_\pi f_\pi} \quad (2.4)$$

is not possible in 4D GUT theories unless there is very finely tuned cancellation in contributions to the axion mass.

3 GUT predictions for the QCD axion

In this section we review the predictions for the couplings of the axion $a$ anomalously coupled to a simple GUT group $G$ in the UV which contains the SM. This corresponds to the well-known case of the QCD axion which solves the strong CP problem. After spontaneous symmetry breaking (SSB) of $G$ and $SU(2)_{ew}$ the axion will have couplings to gauge bosons given by:

$$A^{GUT} \frac{a}{8\pi} G_{\mu\nu} \tilde{G}^{\mu\nu} \rightarrow \frac{a}{F_a} \left[ \frac{\alpha_{em}}{8\pi} E F_{em,\mu\nu} \tilde{F}_{em}^{\mu\nu} + \frac{\alpha_s}{8\pi} N G_{\mu\nu}^{QCD} \tilde{G}_{QCD,\mu\nu} \right]. \quad (3.1)$$

The anomaly coefficients $E$ and $N$ are rational numbers that set the axion coupling to photons and gluons respectively and are fixed by the embedding of the SM into $G$. In the low-energy theory the Lagrangian is parameterised as

$$\mathcal{L} = \frac{a}{F_a} \frac{\alpha_s}{8\pi} G_{QCD,\mu\nu}^{a} \tilde{G}^{a,\mu\nu}_{QCD} + \frac{1}{4} g_{a\gamma\gamma} a F_{em,\mu\nu} \tilde{F}_{em}^{\mu\nu}, \quad (3.2)$$

where

$$g_{a\gamma\gamma} = \frac{\alpha_{em}}{2\pi f_a} \left( \frac{E}{N} - 1.92 \right). \quad (3.3)$$

For the simplest embedding — where the SM gauge group is contained in an SU(5) subgroup of $G$ — the ratio is fixed to be $E/N = 8/3$ (see appendix A for the more general case). Thus simple GUTs make a sharp prediction for the relative couplings of the QCD axion.
to photons and gluons. The axion-gluon coupling produces a mass for the axion through QCD effects

\[ m_{\text{QCD}} = t, \quad z = (m_u/m_d). \]

(3.4)

Requiring a solution to the strong CP problem implies that this dominates the mass of the axion, giving a one-to-one relationship between the axion-photon coupling and the axion mass, defining the QCD line for the GUT axion.

The coupling in equation (3.3) is a combination of an anomaly coefficient and a calculable component from mixing of the axion with mesons and is independent of whether the PQ spontaneous symmetry breaking scale is above or below the GUT scale. The \( U(1)_{\text{PQ}} \) current divergence above the scale of PQ and GUT breaking is

\[ \partial_{\mu} j_{\mu}^{\text{PQ}} = A \alpha_{\text{GUT}} \frac{1}{8\pi} \tilde{G}^{\mu\nu} = A \alpha_{\text{GUT}} \left( \alpha_3 k_3 \tilde{G} + \alpha_2 k_2 \tilde{W} + \alpha_1 k_1 \tilde{B} + \ldots \right), \]

(3.5)

where in the last step we have isolated the SM gauge boson contributions using the broken phase notation above the GUT scale. The coefficients \( k_i \) stand for the index of embedding of the \( i \)-th group. In the spontaneously broken phase, the anomalous variation of the effective action under the PQ symmetry arises from the shift of the axion in the terms in equation (3.1). This is analogous to the WZW term in the QCD chiral Lagrangian that matches the EM anomaly of the neutral component of the chiral current. Note that the running of the coupling constants, \( \alpha_i \), does not affect the quantized coefficients \( k_i \). Thus, it is unavoidable that the GUT singlet axion — that is, an axion coming from a \( U(1)_{\text{PQ}} \) commuting with \( G \) — couples to both photons and gluons in either case, \( M_{\text{GUT}} > f_a \) or \( M_{\text{GUT}} < f_a \).

The prediction of a one-to-one correspondence between the axion mass and photon coupling described by equation (3.3) can in general be altered by introducing new sources of \( U(1)_{\text{PQ}} \) symmetry breaking. However these same terms will generically spoil the solution to the strong CP problem. QCD generates a potential for the axion of the form

\[ V_{\text{QCD}}(a) \simeq f_a^2 m_a^2 \left( 1 - \cos \left( \frac{a}{f_a} + \theta \right) \right). \]

(3.6)

This potential explicitly breaks the shift symmetry of the axion and dynamically sets the effective \( \theta \)-angle, defined as

\[ \theta_{\text{eff}} \equiv \frac{\langle a \rangle}{f_a} + \tilde{\theta}, \]

(3.7)

to zero. If there are additional terms which break the \( U(1)_{\text{PQ}} \) symmetry they will generically lead to \( \theta_{\text{eff}} \neq 0 \), and therefore must be strongly suppressed relative to the QCD contribution to the potential in order to satisfy the severe constraints from experimental searches of neutron EDMs [76]:

\[ \theta_{\text{eff}} \lesssim 10^{-10}. \]

(3.8)

For the axion solution to the strong CP problem to be retained, any \( U(1)_{\text{PQ}} \) breaking contributions to the potential must either be strongly subdominant to the QCD contribution.

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or aligned with the QCD vacuum. In subsections 3.1 and 3.2 we discuss possible sources of additional $U(1)_{\text{PQ}}$ breaking but ultimately argue that these terms must be negligible. For this reason we neglect such terms in the remainder of this work.

### 3.1 Additional instantons

One way additional sources of PQ breaking can arise is if there are additional confining gauge groups $G'$ embedded in $G$. The QCD axion will couple to the $G'$ gauge group through the term

$$L \supset N_h \frac{a}{F_a} G' \tilde{G}' ,$$

(3.9)

where $N_h$ is an anomaly coefficient. After confinement, $G'$ instantons will generate a potential for the axion of the form which will generically be offset by an angle $\delta$ from the QCD vacuum:

$$\Delta V(a) = \frac{N}{N_h} \Lambda^4 \left( 1 - \cos \left( \frac{N_h a}{F_a} + \theta_h \right) \right) ,$$

(3.10)

$$\delta = \frac{N}{N_h} \theta_h - \bar{\theta} ,$$

where the factors of $N, N_h$ in the potential and definition of $\delta$ are included for convenience. The bounds on $\theta_{\text{eff}}$ require that

$$\frac{\Lambda^4 \sin(\delta')}{m_a^2 f_a^2 + \frac{N_h}{N} \Lambda^4 \cos(\delta')} \lesssim 10^{-10} ,$$

(3.11)

where $\delta' = N_h \delta / N$, requiring any contributions to the axion potential that are not aligned with the QCD vacuum to be exceptionally small relative to the QCD terms. Assuming a misalignment angle $\delta \sim \mathcal{O}(1)$ the quality problem can be restated in terms of the contributions to the axion mass as:

$$\frac{m^2}{m_{\text{QCD}}^2} \lesssim 10^{-10} .$$

(3.12)

If $\delta$ is sufficiently small then these additional contributions can be consistent with $\theta_{\text{eff}} \simeq 0$ while giving a large mass to the QCD axion. This has the effect of moving the QCD axion off the QCD line towards the right while preserving the ratio of couplings to gluons and photons. The misalignment angle $\delta$ vanishes above the GUT scale due to the unification of the SM with $G'$, but can run and become $\mathcal{O}(1)$ at low energies depending on the matter content of the model.

### 3.2 Perturbative contributions

One can also consider the effect of explicit PQ-breaking operators [78]

$$\Delta V_n(a) \sim c_n \frac{F_n}{M_{\text{P}}^4} e^{in/F_a} + \text{h.c.} .$$

(3.13)

\^Similar potentials are also generated if $G'$ is spontaneously broken at a high scale [77] with a possible chiral and exponential suppression due to a weak gauge coupling.
These operators generate a mass for the axion of order

$$m^2 \sim c_n \frac{F_a^{n-2}}{M_P^{n-4}}.$$  \hfill (3.14)

If the potential terms (3.13) have a minimum which is misaligned with respect to the QCD vacuum by an angle $\delta \sim 1$ then the mass generated by these operators must satisfy the bound (3.12), meaning these contributions must be forbidden up until large $n$. One way to forbid the lower $n$ operators is to impose a $\mathbb{Z}_N$ symmetry under which $\Phi$ is charged non-trivially [78, 79]. This forbids all operators (3.13) for $n \leq N$ and therefore improves the ‘quality’ of the global PQ symmetry. In any case, we again retain the prediction of the ratio of gluon-photon couplings.

PQ-violating operators may also alter the predictions above by shifting the ratio $E/N$. At dimension 6 there is an operator of the form:

$$\frac{c}{\Lambda^2} \Phi \text{Tr}[\Sigma \tilde{G} G] + \text{h.c.} \quad (3.15)$$

where the scalar $\Sigma$ is an adjoint field taking a non-zero VEV in the hypercharge direction, and $\Phi$ is the complex scalar, the phase of which is the axion. We expect that these operators are suppressed by $\Lambda$ at the Planck scale or perhaps below if the fundamental scale lies between $M_{\text{GUT}}$ and the $M_P$ (see for example [80]). As a concrete example, for SU(5) the adjoint VEV reads

$$\langle \Sigma \rangle \propto \text{diag}(-1/3, -1/3, -1/3, 1/2, 1/2).$$  \hfill (3.16)

This operator induces a shift to the previously obtained predictions by inducing an axion coupling to photons of size $g_{\alpha \gamma \gamma} \sim c_\alpha \text{em} \frac{\langle \Sigma \rangle}{\Lambda^2}$. For axions that do not couple to GUT bosons through an anomaly, this could lead to an observable photon coupling. However, this operator necessarily breaks the axion shift-symmetry and induces a perturbative mass for the axion which can be estimated as:

$$m^2 \sim c_\alpha^2 \frac{\Lambda^3}{F_a}.$$  \hfill (3.17)

where we have assumed the cutoff of the theory at $\Lambda$. Different UV completions can affect this estimate, but not change the overall conclusion that the ALP gets a large perturbative mass contribution. For the ALP to be light $c$ must be very small, severely suppressing the contribution to the axion-photon coupling. This could happen naturally if the operator in equation (3.15) is generated by non-perturbative effects and comes with an exponential suppression factor.

### 3.3 Discrete symmetries, mirror worlds and clockwork

The strong-CP problem may also be solved using discrete symmetries. The simplest example of this is the Nelson-Barr mechanism [81, 82] using $P$ and $C$ discrete symmetries. In this case, the axion can be heavy and decoupled without affecting the solution to the strong-CP problem.

In cases where the axion and a discrete symmetry together solve the strong-CP problem, the axion mass may be larger or smaller than that predicted by QCD. In [83, 84] a $\mathbb{Z}_N$
symmetry relating different copies of the SM (mirror worlds) was shown to lead to a light QCD axion. In these models the axion mass scales as $m_a \sim \frac{1}{2^{\frac{N}{2}}}$, with $N$ the number of SM copies or GUT-like sectors. The gauge group in these models is

$$G_1 \times \ldots \times G_N$$

(3.18)

where the SM is embedded into one $G$ factor. A QCD axion which is much lighter than $m_{QCD}$ then requires a large number of SM copies and these copies must not be unified with the SM.\footnote{Further unification of the different $G_i$ factors in (3.18) into a simple group reveals the fact that an axion transforming non-trivially under the $\mathbb{Z}_N$ necessarily carries GUT charge. Such possibility leads to a (potentially large) perturbative mass for the axion, as will be shown in section 4.2.}

The contribution to the axion potential from new, UV instantons of the mirror world can also be aligned respect to QCD by using discrete symmetries. This has led to different models\cite{85–88} where the axion is substantially heavier than usual while still solving the strong CP problem.

A different mechanism that has been proposed to increase the QCD axion coupling to photons, mimicking an ALP, is the clockwork mechanism\cite{72, 89–91}. This mechanism works by introducing $n + 1$ complex scalars $\phi_i$ and $n + 1$ U(1) symmetries, with scalar $\phi_i$ charged under the symmetries (U(1)$_i$, U(1)$_{i+1}$) with charge (1, $q$). The scalars at either end of the chain, $\phi_0$ and $\phi_n$, couple to fermions charged under different gauge groups, e.g. QCD and EM respectively. The U(1) symmetries are broken to a diagonal $U(1)_{PQ}$ symmetry under which the $n^{th}$ scalar has PQ charge $q^n$. Since the fermions which generate the electromagnetic anomaly couple only to $\phi_n$ and the fermions which generate the QCD anomaly couple to $\phi_1$ then the ratio of anomalies scales like

$$\frac{E}{N} \propto q^n,$$

(3.19)

leading to an exponential hierarchy in the anomaly coefficients. This enhancement clearly relies on coupling the fermions that generate the electromagnetic and color anomalies to different scalars, which is not possible if the SM is unified and the fermions form complete GUT multiplets.

We see that while the examples provided in this subsection increase the ratio $g_{a\gamma\gamma}/m_a$ without breaking PQ symmetry, none of them is compatible with a simple gauge group in the UV.

## 3.4 Measuring the QCD axion couplings

A measurement of $g_{a\gamma\gamma}$ for the QCD axion will tell us the value of the ratio $E/N$, so can provide an indirect probe of grand unification. As above, for a level one embedding GUT theories predict $E/N = 8/3$. Extracting this ratio will be the most immediate target following any discovery. Measuring this ratio precisely is experimentally challenging, but a measured value far from the GUT prediction will be strongly at odds with unification, even with a large experimental uncertainty.

The measurement of this coupling is an interesting experimental question. If the axion decay constant is $f_a \simeq 10^{12}$ GeV, then the axion lies in the band of currently operating
cavity resonance haloscopes [92–94]. The sensitivity of axion haloscopes is set by the limited
time that the resonant cavity is tuned to a specific mass. Upon discovery, a large amount
of data at the axion mass can be gathered, which can measure the rate of axion-photon
conversion to a high precision. The rate is proportional to,

\[ P_{a \rightarrow \gamma} \propto \rho_a g_{a\gamma\gamma}^2, \]  

(3.20)

where \( \rho_a \) is the local axion density [9, 10]. The narrow bandwidth of the cavity means
that the resonance condition corresponds to a precise measurement of the axion mass.
Thus, haloscopes are in a position to measure the ratio \( g_{a\gamma\gamma}/m_a \) to test equation (1.1).
Unfortunately the quantity \( \rho_a \) is difficult to estimate even if we assume the axion makes up
all of dark matter (DM), as the local DM density is not known precisely [95–102]. This
presents a challenge to using QCD axion couplings in this way as a precision test of GUTs.
It will be interesting to consider the possibility of identifying a sub-component of DM with
a more accurate local density prediction, such as axions trapped within the Earth’s or Sun’s
gravitational basin [103, 104]. Even if it is a subdominant component, it is possible that it
can be differentiated from the halo dark matter through velocity measurements [105], and
that the haloscope sensitivity may be enough to extract a measurement of \( g_{a\gamma\gamma} \).

Another class of proposed haloscope experiments look for axion DM through its coupling
to gluons to measure a time-dependent oscillating electric dipole moment (EDM). These
experiments are a way to measure the axion abundance, but are limited by theoretical
uncertainties on nuclear matrix elements which are only known to about 30% [106]. Another
potential challenge, depending on the value of \( f_a \), is that the EDM experiments and the
photon experiments may not both be sensitive to the QCD axion however a region of overlap
does exist. If the axion exists in this overlap region more precise calculations of the matrix
elements can break the degeneracy between the couplings and the local DM density. Other
observables which use derivative axion couplings do not help since these couplings are not
quantized and undergo renormalization.

We see that GUTs provide a very precise target for the QCD axion, but utilizing the
relation between the photon and gluon coupling to test aspects of unification is experimentally
challenging. It will be interesting to develop other strategies that can measure and test the
quantized couplings after an initial discovery.

4 GUT prediction for axion-like particles

In this section we consider the possibilities for generating an axion-photon coupling for
axions which have vanishing anomaly with \( G \). This can occur through mass or kinetic
mixing of axions, axions charged under \( G \) or mixing between GUT gauge bosons and a
hidden gauge sector. For the case of axion mixing we find that while it is possible to
have a light ALP, the ALP coupling to photons is suppressed proportional to the squared
ALP mass in the low-mass limit. Charged ALPs receive contributions to their mass from
integrating out heavy gauge bosons or additional instantons and generically must be heavier
than \( m \gtrsim 1 \text{ GeV} \) to satisfy current experimental constraints. Kinetic mixing of gauge
bosons generates an ALP-photon coupling which is proportional to the (small) mixing parameter \(\epsilon \lesssim 10^{-8}\).

Unification of the SM in the UV therefore rules out the possibility of very light ALPs with experimentally accessible couplings to photons in the absence of tuning. This points to a preferred region of parameter space at masses greater than the QCD axion expectation for ALP searches which rely on the photon-coupling. ALP couplings to fermions are unsuppressed, however, so experiments [46, 47] which probe ALP couplings to SM fermions offer a promising avenue to search for light ALPs in GUT theories. In this way, GUT theories can naturally lead to an ALP phenomenology similar to the case of the photophobic ALP [107, 108].

4.1 ALPs through the QCD axion portal

If the \(U(1)_{PQ}\) symmetry\(^8\) mixes with other global \(U(1)\) symmetries then it is possible for some axions to couple to photons without obeying equation (1.1). This manifests itself as mass mixing in the axion potential, which for \(N\) axions \(a_i\) is given by

\[
V(a_i) = \left(\sum_{i=1}^{N} a_i f_i \right) G \tilde{G} + \frac{1}{2} M^2_{ij} a_i a_j. \tag{4.1}
\]

The axions may also mix kinetically, but after redefining fields to move to a basis where the axion kinetic terms are canonical the effects of kinetic mixing are included in the potential (4.1) if we allow arbitrarily large decay constants [73, 109]. In order to not introduce a quality problem there must be a \(U(1)\) subgroup of \(U(1)_{PQ} \times \prod_i U(1)\_i\) that remains unbroken except due to QCD effects, which is true if the \(M^2\) matrix has a zero eigenvalue.

An illustrative example is the case of two mixed axions with potential

\[
V(a_1, a_2) = \left(\frac{a_1}{f_1} + \frac{a_2}{f_2}\right) G \tilde{G} + \frac{1}{2} m^2_{2} a_2^2. \tag{4.2}
\]

After QCD confines the mass terms in the potential are:

\[
V_{\text{mass}} = \frac{f^2_\pi m^2_\pi}{2} \left(\frac{a_1}{f_1} + \frac{a_2}{f_2}\right)^2 + \frac{1}{2} m^2_{2} a_2^2, \tag{4.3}
\]

and the coupling to QCD induces mass mixing between \(a_1\) and \(a_2\). In this case we find that there is always a QCD-like axion \(a\) with a mass and photon coupling set by \(f_{\text{eff}}\), where

\[
\frac{1}{f^2_{\text{eff}}} = \frac{1}{f^2_1} + \frac{1}{f^2_2}, \tag{4.4}
\]

and a second mass eigenstate \(b\) which may be lighter or heavier depending on the value of \(m^2_2\).

In the limit \(m_2 \ll f_\pi m_\pi / \max(f_1, f_2)\), the QCD axion is identified with

\[
\frac{a}{f_{\text{eff}}} \simeq \frac{a_1}{f_1} + \frac{a_2}{f_2}. \tag{4.5}
\]

---

\(^8\)By \(PQ\) symmetry we refer to the \(U(1)\) which is anomalous with \(G\). In equation (4.1) the axion under this symmetry is the combination \(\sum_{i=1}^{N} \frac{a_i}{f_i}\).
and the orthogonal field $b$ is approximately massless with negligible coupling to photons. More precisely, in this limit the coupling of the light axion to photons is suppressed relative to the QCD axion by a factor

$$g_{b\gamma\gamma} \propto \frac{m_b^2 \times \max(f_1^2, f_2^2)}{f_\pi^2 m_\pi^2}. \quad (4.6)$$

In the opposite limit, when $m_2 \gg f_\pi m_\pi / \min(f_1, f_2)$, then the QCD axion is identified with $a \simeq a_1$ and the second axion $b \simeq a_2$ couples to photons with a coupling of similar magnitude but is much heavier than the QCD axion. When $m_2^2 \simeq m_{QCD}^2$ both mass eigenstates have a mass and photon coupling which falls close to the QCD line.

The photon coupling and mass of the two mass eigenstates are shown in figure 1 for multiple combinations of the parameters $f_1, f_2, m_2$. The grey points are generated by taking randomly generated values of the parameters $f_1, f_2$ logarithmically spaced in the range $[10^{10}, 10^{18}]$ GeV and $m_2$ logarithmically spaced in the range $[10^{-11}, 1]$ eV. Allowing for larger (smaller) values of $m_2$ simply increases the number of points away from the QCD line to the right (bottom) of the plot.

The coloured lines in figure 1 are generated by fixing $f_1$ and $f_2$ and varying $m_2$ in the range $[10^{-11}, 1]$ eV. The yellow line shows points where $f_1 = f_2 = 10^{12}$ GeV. In this case, there is always a QCD axion with mass given by equation (3.4) with $f_a = f_{\text{eff}}$ and a second eigenstate with the same coupling but larger mass (the horizontal line to the right of the point on the QCD line) or with coupling suppressed by the factor $m_b^2 / m_{QCD}^2$ (the line approaching the QCD line from below). The other coloured lines each show various cases for fixed $f_1 \neq f_2$ with $m_2$ scanned. In each case there is always a QCD axion with coupling and mass set by $f_{\text{eff}}$.

These results clearly point to an allowed range of parameter space for ALPs at masses heavier than the QCD axion (to the right of the QCD line). A light ALP with sizeable coupling to photons is not generated via mass mixing. This can be understood as follows: in order to have a sizeable coupling to photons the non-QCD axion must contain a sizeable admixture of the combination $a_1 f_1 + a_2 f_2$, giving a contribution to $m_b^2$ of order $m_{QCD}^2$.

The points to the right of the QCD axion, corresponding to the points where $m_2$ is large, form a flat line with coupling to photons given by:

$$g_{b\gamma\gamma} = \frac{\alpha_{\text{em}}}{2\pi f_2} \left( \frac{E}{N} - 1.92 \right), \quad (4.7)$$

and correspond to the case where $a_2$ is heavy and $a_1$ is identified with the QCD axion to a good approximation. The points below the QCD line correspond to the cases where $m_2$ is small and form a line described by

$$g_{b\gamma\gamma} \simeq g_{a\gamma\gamma} \frac{m_b^2 \times \max(f_1^2, f_2^2)}{f_\pi^2 m_\pi^2}, \quad (4.8)$$

approaching the QCD line at a point where the coupling and mass are set by $\max(f_1, f_2)$. When $f_1 > f_2$ (blue and green points) it is possible that there are two axions on the QCD line with different masses if $m_2$ is tuned appropriately.
Figure 1. Scatter plot showing the mass and photon coupling of the two mass eigenstates in the
model described by the potential (4.2). Shown in light grey are the points generated, the dashed
lines show the maximum and minimum masses for the QCD-like eigenstate and are set by the chosen
range of $f_1, f_2$. The thin black line shows the QCD axion line. The coloured lines show the points
generated for fixed decay constants: the red line shows the case where $f_1 = 10^{12}$ GeV, $f_2 = 10^{13}$ GeV,
the yellow line shows $f_1 = f_2 = 10^{12}$ GeV, the green line shows $f_1 = 10^{12}$ GeV, $f_2 = 10^{11}$ GeV and
the blue shows $f_1 = 10^{13}$ GeV, $f_2 = 10^{12}$ GeV.

A similar conclusion holds for multiple axions coupling to gauge bosons through the
QCD axion portal. For the general case of $N$ axions the mass terms in the potential can be
written as
\[
V_{\text{mass}} = \frac{f^2 m^2}{2} \left( \frac{a_0^2}{f_0^2} \right) + \frac{1}{2} a_j^T M^2 a_i ,
\]
for $i, j \in 0, \ldots, N - 1$, where $a_0$ is the axion of the anomalous PQ symmetry. In this basis,
the only axion which couples to photons is $a_0$ with coupling constant
\[
g_{\alpha \gamma \gamma} = \frac{\alpha_{\text{em}}}{2\pi f_0} \left( \frac{E}{N} - 1.92 \right) .
\]
We can then rotate to the basis of mass eigenstates $b_i$ with an orthogonal matrix $R$:
\[
a_i = R_{ij} b_j .
\]
The mass and coupling to photons of the $i$'th eigenstate is
\[
m^2_i = \frac{f^2 m^2}{f_0^2} R_{0i}^2 + R_{ij}^T M^2_{jk} R_{ki} ,
\]
\[
g_{b \gamma \gamma} = \frac{\alpha_{\text{em}}}{2\pi f_0} \left( \frac{E}{N} - 1.92 \right) R_{0i} ,
\]
where there is no sum over $i$ in the first term. This basis makes it clear that the QCD line
provides an upper bound on the ALP parameter space in GUTs.

The results from the two-axion case continue to apply qualitatively in the general case
as well. This is evident in the $a$ basis where we further rotate $a_i$ for $i \geq 1$ to diagonalise
the components of $M_{ij}^2$ with $i \geq 1, j \geq 1$, leaving only pairwise mixing between $a_0$ and the orthogonal states. As in the two axion case if the mixing term is large the corresponding ALP can be integrated out and corresponds to the heavy ALPs on the horizontal lines shown in figure 1. In the light mass regime the ALP will again have a coupling to photons suppressed by $m_{\text{light}}^2/m_{\text{QCD}}^2$.

4.2 GUT charged axions

The discussion of section 3 presumed that the axion was a GUT singlet and therefore coupled to $G$ only through the anomaly coefficient. In the low energy theory there may be additional pseudo Nambu-Goldstone bosons (pNGBs) which are singlets under the SM but not under $G$. The simplest possibility is an elementary scalar in the adjoint representation of $G$. However, in this case GUT gauge boson loops will generate a perturbative mass proportional to the UV cutoff. Supersymmetry may reduce this mass further, but scalar masses parametrically below $m_{\text{susy}}$ seem to be unlikely.

A potentially interesting ALP candidate with mass below the UV cutoff can occur in composite axion models [42–45]. Just as pions emerge as pNGBs after QCD confinement breaks the flavour symmetry of the quark sector, the axion can emerge as the pNGB of a broken flavour symmetry of the hidden sector. After the spontaneous breaking of the GUT symmetry this axion gets an irreducible mass from integrating out the heavy GUT gauge bosons. In this case the axion can couple to a different linear combination of the SM gauge groups than the QCD axion. Its mass is protected by compositeness but there remains an irreducible contribution to the axion mass from gauge boson loops which rules out a light axion. It is also possible to include elementary axions in addition to the composite axion(s), but the phenomenology of the singlet states is not changed from the discussion in previous sections.

We now describe an example model which illustrates the physics of the composite axion. The gauge group we consider is

$$G \times \text{SU}(N),$$

(4.13)

with $G$ a simple group containing the SM. We take all the SM matter fields to be singlets under SU($N$), with SU($N$) confining at a scale $\Lambda_N < M_{\text{GUT}}$. The additional matter content of the model consists of fermion fields with charges under $(G, \text{SU}(N))$ given by:

$$\Psi \sim (\square, N), \quad \bar{\Psi} \sim \left(\bar{\square}, \bar{N}\right) \quad \psi \sim (1, N) \quad \bar{\psi} \sim \left(1, \bar{N}\right),$$

(4.14)

where the $\square$ indicates the fundamental representation of $G$. Taking $G = \text{SU}(5)$ as the canonical example, there is a flavour symmetry

$$\text{U}(6)_L \times \text{U}(6)_R = \text{SU}(6)_L \times \text{SU}(6)_R \times \text{U}(1)_V \times \text{U}(1)_A.$$

(4.15)

$\text{U}(1)_V$ is exact and is the equivalent of baryon number under the hidden sector gauge group and $\text{U}(1)_A$ is explicitly broken by $\text{SU}(N)$ instantons. After confinement the fermions

---

9Note that if $G$ and SU($N$) are unified at a UV scale, then instantons of the new sector may reintroduce the PQ quality problem (see section 3.1).
condense and get an expectation value
\[ \langle \Psi\bar{\Psi} \rangle = \langle \psi\bar{\psi} \rangle = \Lambda_N^3, \] (4.16)
spontaneously breaking the flavor symmetry down to
\[ \text{SU}(6)_L \times \text{SU}(6)_R \times \text{U}(1)_V \times \text{U}(1)_A \rightarrow \text{SU}(6)_V \times \text{U}(1)_V. \] (4.17)
The SU(6)_V flavour symmetry is explicitly broken by the weak gauging of the GUT group.
The pNGBs arising after spontaneous breaking of the flavor symmetry transform in the adjoint representation of \( \text{SU}(6)_V \), 35, and decompose into fields with GUT SU(5) charges:
\[ 35 \rightarrow 24 + 5 + \bar{5} + 1. \] (4.18)
The pNGBs in the fundamental, 5 + \bar{5}, all have SM charges and get mass contributions from SM gauge boson loops of order:
\[ m \sim g_{\text{sm}} \Lambda_N, \] (4.19)
where \( g_{\text{sm}} \) is a SM coupling\(^{10}\) [43] (see [110] for a recent discussion).

The pNGB transforming as a GUT adjoint, 24, which we call \( \varphi^a \), splits under the SM gauge group as:
\[ \varphi^a \sim (8, 1)_0 + (1, 3)_0 + (3, 2)_{5/6} + (3, 2)_{-5/6} + (1, 1)_0. \] (4.20)
As for the 5 + \bar{5} above, those states with SM charge get a large mass induced by SM gauge boson loops, as in equation (4.19). The SM singlet \( \varphi^{24} \), on the other hand, is identified as a composite axion and appears massless at this level. However, after integrating out the heavy GUT gauge bosons our EFT contains a 4-fermion operator:
\[ \frac{\alpha_{\text{GUT}}}{M_{\text{GUT}}^2} \langle \Psi\bar{\Psi} \rangle^2. \] (4.21)
Expanding around the vev (4.16) leads to mass terms\(^{11}\) for \( \varphi^{24} \) which scale as
\[ m_{\varphi^{24}} \sim \frac{\alpha_{\text{GUT}}}{10^{10} \text{ GeV}} \left( \frac{10^{16} \text{ GeV}}{M_{\text{GUT}}} \right)^2 10 \text{ TeV}, \] (4.22)
where the decay constant for the composite axion is \( f_a \sim \Lambda_N \). This result coincides with the estimate of [113, 114], where a similar flavor symmetry appears in a different context.

The composite axion field \( \varphi^{24} \) couples to a different linear combination of the SM gauge bosons than the QCD axion. \( \varphi^{24} \) parametrises SU(5) transformations in the hypercharge

\(^{10}\)The coupling, or combination of couplings, appearing in equation (4.19) depends on the charges of the pNGB under the SM gauge group.

\(^{11}\)One could ask if SUSY can further protect the mass of the ALP. However, in this kind of scenarios SUSY may be dynamically broken by fermion condensates of the new confining interaction [111]. Indeed, as shown in [112], it is generically expected that SUSY is dynamically broken unless there are flat directions for the potential. The study of specific models is interesting by itself and will not be pursued here.
direction, and the couplings to GUT gauge bosons are set by the anomaly coefficients 
Tr[T^{24}\{T_a, T_b\}]. In particular the coupling to photons is given by

$$L = \frac{\phi^{24}}{F_a} N_{\text{em}} \frac{4}{8\pi} \frac{\sqrt{3}}{5} F_{\mu\nu} \tilde{F}^{\mu\nu},$$  
(4.23)

which comes from the anomaly coefficient with the W bosons of the SM.

The composite axion therefore gives a very different prediction for the relationship
between the axion-photon coupling and the axion mass. In particular, $g_{a\gamma\gamma} \propto m_a^{-1/2}$, so
lighter axions couple more strongly to photons. As shown in figure 2 much of the low mass
parameter space is therefore ruled out, limiting the composite axion mass to be at least of
order $m_a \gtrsim 1 \text{ GeV}$ with decay constant $f_a \gtrsim 10^5 \text{ TeV}$. Note that when the QCD axion is
mostly composite, its decay constant and that of the $\phi^{24}$ field almost coincide. In this case,
one can also indirectly constrain the mass $m_{\phi^{24}}$ from QCD axion bounds.

The example above can be generalized for larger flavor symmetries including multiple
species in different representations, however, this does not qualitatively change the overall
picture. As the flavor symmetry is enlarged multiple copies of GUT-charged and GUT-
singlet pNGBs may appear. The charged states get a perturbative mass as detailed above.
The singlets couple diagonally to the full $G$ and follow the discussion in section 4.1.

### 4.3 Dark photon-photon mixing

We consider now the possible effects of photon mixing with a massless dark photon
where $\text{U}(1)_D$ is not unified with $G$ in the UV.\footnote{\textsuperscript{12}The situation where $\text{U}(1)_D$ is ultimately
embedded in the UV GUT group does not modify the result of
section 3.}

Assuming that the GUT and dark sector couple
to different axions and neglecting axion mixing the axion couplings to gauge bosons are described by the Lagrangian

$$\mathcal{L} = \alpha_{\text{GUT}} \frac{a}{f_a} G^a \tilde{G}^a + \alpha_D \frac{b_D}{f_b} F_D \tilde{F}_D. \quad (4.24)$$

A tree-level crossed mixing term between the dark photon and the GUT gauge bosons is forbidden by gauge invariance. Higher dimensional operators such as:

$$O_{\text{mix}} = \frac{c}{M_P} F_D \Sigma G,$$

where $\Sigma$ is a scalar field in the adjoint representation, will induce mixing between the dark photon and hypercharge boson after GUT symmetry breaking. For example, when this operator is generated at one loop, the dimensionless coefficient $c$ is computed by evaluation of the loop diagram in figure 3:

$$c \sim \frac{g_{\text{GUT}} g_D y \Sigma}{16\pi^2} \langle \Sigma \rangle M_{\text{Pl}}, \quad (4.26)$$

and includes the appropriate gauge and Yukawa couplings. After symmetry breaking, and redefining the photon and dark photon field so that they are canonically normalized, the induced dark axion coupling to photons will be:

$$\frac{\epsilon^2}{8\pi} \alpha_D \frac{b_D}{f_b} F_{\text{em}} \tilde{F}_{\text{em}}, \quad \text{with: } \epsilon^2 = \left( \frac{g_{\text{GUT}} g_D y \Sigma}{16\pi^2} \frac{\langle \Sigma \rangle}{M_{\text{Pl}}} \right)^2. \quad (4.27)$$

Even if we assume a large VEV for the adjoint field, $\langle \Sigma \rangle \sim M_{\text{GUT}}$, and order one couplings, $g_{\text{GUT}} \sim g_D \sim y \Sigma \sim O(1)$, we see that the kinetic mixing parameter is approximately given by $\epsilon^2 \lesssim 10^{-8}$. The coupling of $b_D$ to photons is suppressed by the small parameter $\epsilon^2$, so deviations from the results of section 3 are small.

Although mixing effects proportional to $\epsilon^2$ are tiny, it might happen that in extreme situations we can get a light ALP to the left of the QCD line. This occurs because when $U(1)_D$ is totally hidden from the visible sector the mass of $b_D$ is a free parameter (although a contribution to the mass may be generated by loops of dark magnetic monopoles [115] or other UV instantons) and the constraints on $f_b$ are weak. The requirements for a light ALP to be generated in this way are a large hierarchy between decay constants, $f_b \sim O(1) \text{ TeV} \ll f_a$, a totally hidden $U(1)_D$ and the absence of kinetic and mass mixing.
between $a$ and $b_D$. When the latter conditions are not satisfied the $b_D$ ALP may get a coupling to fermions (these couplings are not topological and can be generated by renormalisation effects). Astrophysical probes typically require decay constants $f_b \gg \text{TeV}$ for ALPs with couplings to SM fermions [4]. In addition, we expect dark charged states at the $f_b$ scale which will appear as millicharged states in the canonical basis. These states are phenomenologically interesting for the hierarchical situation where $f_b \sim \text{TeV}$ [116–119].

5 ALP phenomenology

The results of section 4 indicate that in GUTs the expectation is that ALPs will have ratio $g_{a\gamma\gamma}/m_a$ smaller than the QCD prediction — falling to the right of the QCD line in $g_{a\gamma\gamma} - m_a$ space. The lack of experimental access to this region of parameter space highlights the need for experimental approaches which do not rely on the ALP-photon coupling. Of particular interest in this direction are the axion couplings to fermions. As can be seen from the operator

$$c_{ij} \frac{\partial a}{f_a} \bar{\psi}_i \gamma^\mu \gamma^5 \psi_j,$$  \hspace{1cm} (5.1)

the fundamental difference with respect to the coupling to gauge bosons is that the fermion couplings are always shift-symmetric. Derivative couplings therefore have no topological protection and can be generated by renormalisation effects even if set to zero at a given energy scale. The phenomenology of ALPs in GUT theories is therefore similar to the photophobic [108] or gluophobic [107] ALP models, which also have suppressed couplings to gauge bosons but no suppression of fermion couplings.

ALP-fermion couplings possess some model dependence as they depend on the $U(1)$ charge assignments for fermions, so in this section we briefly describe what the general expectations for these couplings are in theories with several axion fields and an underlying GUT. In field theory language, the kind of ALPs that we discuss in this section are those where the mixed $[G]^2 \times U(1)$ anomaly cancels but the $U(1)$ symmetry still has a chiral charge assignment. In other words, we are interested in the chiral, anomaly-free part of the group

$$U(1)_{PQ} \times \Pi_i U(1)_i.$$  \hspace{1cm} (5.2)

For the sake of concreteness, let us consider a simple toy example. We assume a standard SO(10) GUT with SM fermions in 3 copies of the 16 spinor. In addition to the anomalous $U(1)_{PQ}$ symmetry giving us the QCD axion, one can consider a $U(1)$ symmetry with charges +1, 0, -1 for each spinor. One trivially sees that this ALP will not couple to photons or gluons (as the $[SO(10)]^2 \times U(1)$ anomaly cancels) but still couples at tree-level to fermions. This model has no special features but gives an example of how $O(1)$ ALP-fermion couplings can appear.

A generic ALP coupled to fermions through (5.1) has flavor conserving and flavor violating couplings at tree-level. In the case of flavor conserving couplings the strongest constraints come from astrophysical probes which, for example, can place restrictive bounds to the axion-electron coupling, $c_{ee}/f_a \lesssim O(10^{-9}) \text{GeV}^{-1}$ (see [4] for a review). Recently the XENON1T collaboration [120] has reported an excess on electron recoils that can be
accommodated with a non-anomalous PQ symmetry [121]. If confirmed it would provide an interesting signal of physics beyond the SM, possibly connected to the flavor structure of the SM [122, 123].

On the other hand, flavor violating couplings offer stringent constraints for the cases with off-diagonal coefficients, $c_{ij} \neq 0$ [48]. To cite some examples, the exotic decays $\mu \to e a$ and $K \to \pi a$ provide the bounds:

$$\frac{c_{\text{em}}}{f_a} \lesssim 1.2 \times 10^{-9} \text{ GeV}^{-1}, \quad \frac{c_{\text{sd}}}{f_a} \lesssim 1.2 \times 10^{-12} \text{ GeV}^{-1}. \quad (5.3)$$

We refer the reader to [49, 50] for a comprehensive analysis.

Light bosons may also mediate long-range forces which can produce observable effects in macroscopic objects. This is the case for scalar bosons mediating monopole-monopole (in the sense of multipole expansion) interactions between fermions, and pseudoscalar fields which lead to dipole-dipole spin-dependent forces between fermions. Axions mediate a new type of interaction which has a monopole-dipole nature [124]. This has lead to new experimental ideas to look for axions in the laboratory [47, 51, 52]. Being $\mathcal{P}$ and $\mathcal{C}$P violating this axion-mediated monopole-dipole interaction is necessarily proportional to the topological phase $\theta_{\text{eff}}$ and therefore involves axion-gauge boson couplings.

In the GUT framework we considered in previous sections only the QCD axion and ALPs with comparable (or larger) masses will lead to sizeable monopole-dipole interactions. Additional light ALPs ($m_b \ll m_{\text{QCD}}$) will give monopole-dipole, or monopole-monopole interactions which are generated by mixing with the QCD axion and are subsequently suppressed by $m_b^2/m_{\text{QCD}}^2$, $m_b^4/m_{\text{QCD}}^4$ respectively. For example, the scalar and dipole coupling constants that characterize the monopole-dipole interaction between nucleons for a light ALP $b$ with mass $m_b$ are roughly given by:

$$g_s^N g_p^N \sim \theta_{\text{eff}} \frac{m_q^2}{f_a^2} \frac{m_b^2}{m_{\text{QCD}}^2}, \quad (5.4)$$

with $m_q$ a light quark mass. In the same way the axion couplings to gauge bosons are highly suppressed for light axions, long-range forces other than dipole-dipole interactions are strongly screened for axions with large wavelength. Figure 4 shows the limits and projections for experiments looking for monopole-dipole interactions of nucleons. The QCD axion prediction is shown in green (taking $\theta_{\text{eff}}$ to lie in the range $10^{-20} < \theta_{\text{eff}} < 10^{-10}$) and sets an upper bound for the possible magnitude of the force for a given ALP mass in GUTs. A discovery of a force mediated by an axion above the QCD band is inconsistent with unification into a simple group in the UV. Experiments looking for long range monopole-dipole forces [47, 125] can therefore probe GUTs in a very similar way to experiments which aim to measure the axion-photon coupling.

A final aspect of ALP phenomenology that deserves mention is related to collider physics. The region of parameter space that can be probed at these facilities — that is, masses above the MeV scale and couplings of order $c_{ij}/f \sim O(1) \text{TeV}^{-1}$ — corresponds to the heavy axion regime and no specific prediction arises for ALP couplings in this region of parameter space. A standard analysis (see [127, 128], and [3] for a recent review) applies even in the presence of a simple GUT in the UV.
Figure 4. Limits on CP-violating monopole-dipole interactions for nucleons, adapted from [126]. Purple is different LAB exclusion limits, blue is astro-lab exclusion, gray is the projected limits for ARIADNE. The QCD axion prediction is in green color.

6 Non-simple groups

In this section we consider some well-motivated models where the SM is not embedded in a simple gauge group in the UV but some or all of the predictions of GUT theories are retained. These include the Pati-Salam model [129] based on the group $\text{SU}(4)_C \times \text{SU}(2)_L \times \text{SU}(2)_R$, Trinification [130] based on $\text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R$ and flipped SU(5) which is based on the gauge group $\text{SU}(5) \times U(1)_X$ [131, 132]. These scenarios also arise as intermediate steps in the symmetry breaking chain of more fundamental GUTs based on the gauge groups $\text{SO}(10)$ [39, 40] and $E_6$ [133], however, can be considered as UV complete models themselves. If the gauge group is ultimately simple in the UV then the results of sections 3 & 4 apply. If instead these models are not further unified in the UV then it is possible to find an axion coupled to photons without coupling to QCD. However, as we show in this section, this comes at the cost of losing the appealing GUT predictions of the weak mixing angle and the absence of fractionally charged states. The only way to preserve these predictions is to further unify the gauge groups, which reduces to the case considered above.

Flipped GUTs are well-motivated in certain string frameworks because they do not need adjoint fields to break the gauge group down to the SM [134, 135]. Additionally, in specific set-ups the doublet-triplet splitting can also be solved [134]. Unlike the minimal Georgi-Glashow SU(5) model, flipped SU(5) also includes small neutrino mass generation as a right-handed neutrino lies inside one of the representations. This particle gets a large Majorana mass after the spontaneous gauge symmetry breaking down to the SM, giving us the possibility of implementing the seesaw mechanism. Pati-Salam and Trinification, being manifestly left-right symmetric, are well-suited to explain the SM hypercharges and relate them to the SM baryon and lepton numbers. Assuming that the fermion content has non-exotic representations (as discussed below) this provides a way to address electric charge
quantisation in integers for isolated states. Additionally, one usually links the breaking of the left-right symmetry at a high scale to the generation of small neutrino masses.

6.1 Flipped GUTs

A well known situation where we have a partially unified theory is the case of the flipped SU(5) theory [131, 132], where the gauge group is given by SU(5) × U(1)_X. The SM fermions (plus right-handed neutrinos) are contained in the representations 5, 3, 5̄, 10, and 5, with the subscript denoting the U(1)_X charge. The scalar sector also differs from the standard SU(5) model where the initial GUT gauge symmetry is broken by a Higgs in the adjoint representation, 24. Instead, in flipped SU(5) the breaking down to the SM is carried by a Higgs transforming as 10. After properly normalizing the generators the gauge couplings of SU(5) × U(1)_X — that is, α_5 and α_X — obey the tree-level matching condition:

\[ \alpha_2(M_{\text{GUT}}) = \alpha_3(M_{\text{GUT}}) = \alpha_5(M_{\text{GUT}}), \quad \frac{25}{\alpha_1(M_{\text{GUT}})} = \frac{1}{\alpha_5(M_{\text{GUT}})} + \frac{24}{\alpha_X(M_{\text{GUT}})}. \] (6.1)

In flipped SU(5) the weak mixing angle is given at the GUT scale by:

\[ \sin^2 \theta_w(M_{\text{GUT}}) = \frac{\alpha_Y}{\alpha_2 + \alpha_Y} = \frac{3/8}{1 + 3/5 \left( \frac{\alpha_5}{\alpha_X} - 1 \right)}. \] (6.2)

When α_5 = α_X at the GUT scale the standard prediction \( \sin^2 \theta_w(M_{\text{GUT}}) = 3/8 \) is retained, which occurs naturally when the theory is embedded in a higher rank GUT. In the general case, however, the couplings are independent parameters.

If one abandons further unification of the U(1)_X factor there can exist an axion coupled to photons without receiving a QCD potential. This occurs if there are particles charged under U(1)_X generating an anomaly and can be seen in the Lagrangian:

\[ \mathcal{L} = \alpha_5 \frac{a}{f_a} G \tilde{G} + \alpha_X \frac{b_X}{f_b} F_X \tilde{F}_X. \] (6.3)

In this case a and b_X can be different linear combinations, corresponding to different U(1) symmetries each anomalous under SU(5) or U(1)_X. However, in this case where U(1)_X is not unified with SU(5), the usual GUT prediction for \( \sin^2 \theta_w \) is lost. This also implies giving up the explanation of the integer quantisation of electric charges, as there may exist fractionally charged isolated states — e.g. a fermion in the 1_1 representation would have an exotic electric charge \( q = 1/5 \).

6.2 Pati-Salam and trinification

Pati-Salam [129] and Trinification [130] are both well-known (partially) unified theories where the hypercharge comes from a diagonal generator of a non-abelian group. Both theories are manifestly left-right symmetric and are the maximal subgroups of SO(10) and E_6, respectively. We consider the Trinification model here but similar arguments apply for Pati-Salam.
In Trinification the UV gauge group is $SU(3)_C \times SU(3)_L \times SU(3)_R$ and the electric charge generator is given by:

$$Q_{em} = T_3^L + Y = T_3^L + T_3^R + \sqrt{\frac{1}{3}} \left( T_8^L + T_8^R \right).$$  \hspace{1cm} (6.4)$$

This means that, in terms of the Dynkin indices $t_r$ and the PQ charge $q_{r}^{PQ}$ of each representation $r$, the ratio of anomaly coefficients can be written as:

$$\frac{E}{N} = (1 + 1/3) \left( \sum_{r \in SU(3)_{L,R}} t_r q_{r}^{PQ} \right) / \left( \sum_{r \in SU(3)_C} t_r q_{r}^{PQ} \right).$$  \hspace{1cm} (6.5)$$

If there is a representation without color charge that is charged under PQ we may not only have $E/N \neq 8/3$, but also a light ALP coupled only to photons ($N = 0$).

The prediction of the weak mixing angle can be accommodated if there exists a $Z_3$ symmetry relating different gauge groups so that $\alpha_C = \alpha_L = \alpha_R$ in the UV. If all axions are singlets under the $Z_3$ symmetry then the three anomaly coefficients which contribute to $E/N$ must be equal. This corresponds to the results in previous sections where one axion has $E/N = 8/3$ while the anomaly coefficients for other axions are zero. Another possibility is that each SU(3) factor couples anomalously to a different axion and these three axions are permuted under the exchange symmetry in the same way as the gauge bosons. In this case there are three axions coupled to photons with equal SU(3) anomaly coefficients. In some sense this resembles the GUT-charged axion scenario in section 4.2. The main difference is that since the axions are now only charged under a discrete symmetry, there is no gauge boson generating a perturbative mass.

The group structure of the trinification model with a $Z_3$ is similar to the mirror matter models discussed in section 3.3, with the gauge group heuristically appearing as $G^N \rtimes Z_N$. The difference between the two is that in mirror matter models the SM is embedded in a single copy of the gauge group while in trinification the different SM gauge groups come from different SU(3) factors. This allows the possibility of an ALP coupled only to photons in trinification but not in the mirror world scenario.

Trinification can also generically have fractionally charged states. To study the absence of fractional charges it is useful to work with the charge $Q'$:

$$Q' = Q_{em} + \frac{T_{color}}{3}. \hspace{1cm} (6.6)$$

The operator $T_{color}$ denotes the SU(3)$_C$ colour triality of a representation; confinement implies that all isolated states are color singlets with $T_{color} = 0 \mod 3$. Even though quarks have a fractional $Q_{em}$ they have an integer $Q'$ charge, so that for hadrons both $Q'$ and $Q_{em}$ are integers. The requirement of having only integer charges determines the allowed matter representations. Together with the $Z_3$ symmetry this turns out to be quite constraining and the only possible set of representations that is chiral, anomaly free with integer electric charges is:

$$(3, \bar{3}, 1) + (\bar{3}, 1, 3) + (1, 3, \bar{3}), \hspace{1cm} (6.7)$$
plus other representations obtained by taking tensor products of this set. For example, additional representations of the form $(3, 1, 1) + (1, 3, 1) + (1, 1, 3)$ will lead to fractionally charged hadrons or leptons. If particles in these representations exist at all they must be well above any explored energy scale in order to satisfy collider and cosmological bounds [137].

7 Conclusion

Topological interactions are unique in that they are largely unchanged from the UV to the IR, offering a way to test the far UV dynamics of theories using low-energy experiments. The axion-photon coupling is an example of a topological quantity that is highly relevant for the large experimental program being conducted to search for axions. In this work we have studied in detail the restrictions on this coupling from the requirement that the SM is unified into a simple gauge group at a fundamental scale. This requirement means that any axion with a coupling to photons must also couple with a comparable strength to gluons and therefore receives a contribution to its mass from QCD instantons.

In the absence of mixing effects this implies that in a simple GUT theory there is only one axion with an anomalous coupling to both gluons and photons and corresponds to the well-studied case of the QCD axion. Once mixing effects are considered there may be other ALPs which couple to photons, but the restrictions imposed by unification in the UV imply that they have a coupling to mass ratio smaller than that of the QCD axion. If axions are charged under the GUT gauge group, they can couple preferentially to photons, but pick up a mass from GUT interactions. If the axions emerge as composite states their mass can be much lower than the GUT scale due to compositeness. In this case the ALP coupling to photons has a different dependence on the ALP mass and current experimental bounds require that $m_a \gtrsim 1$ GeV and $f_a > 10^5$ TeV.

There are also models which preserve some of the phenomenological predictions of GUT theories without unifying the SM into a simple gauge group in the UV. These theories were studied in section 6. Preserving the theoretical predictions of GUTs — coupling unification, predicting the weak-mixing angle and the absence of fractionally charged states — is still somewhat correlated with ALP phenomenology, but the details depend on the model.

These results mean that the discovery of a light ALP with an observable photon coupling is not consistent with a simple GUT UV completion of the SM. Low-energy searches for light ALPs through the ALP-photon coupling, cosmological signatures such as the rotation of CMB polarisation by light axions coupled to photons [31, 138] and their associated strings [53], and long-range monopole-dipole forces are therefore a novel way to test if the SM is unified at a fundamental level. A recent analysis of Planck and WMAP data has reported a hint of a non-zero cosmic birefringence angle $\beta = 0.342^{+0.094}_{-0.091}$, excluding $\beta = 0$ at 3.6$\sigma$ [139, 140]. This signal seems to be consistent with an ALP coupled to photons with a mass $H_{\text{CMB}} > m_a > H_0$. The fate of this signal will be decided in the future CMB experiments. ALP-fermion derivative couplings are not topological quantities so are not suppressed in the same way as the ALP-photon couplings. Searches for ALPs through these interactions offer a promising approach to search for light ALPs in GUT theories and can...
provide the strongest constraints on these models in the regime where the ALP-photon coupling is small.

It is intriguing that GUT theories defined in the far UV provide strong restrictions in IR physics with experimental implications. As we have shown in this work, if axion-like particles exist above the QCD line, ALP searches would be low-energy probe of grand unification.

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A Level of embedding

It is commonly assumed in the literature that the level of embedding of the different SM groups are level 1. This means that the SM descends from an SU(5) subgroup of the possibly larger, simple gauge group unifying interactions in the UV. While this is a compelling choice it is not fully general. Indeed, when the gauge group is sufficiently large there can be different non-trivial ways to obtain the SM at low energies, however it is very hard to find any concrete examples with a consistent low-energy spectrum.

A simple example illustrating a higher-level embedding is where one of the non-Abelian groups of the SM arises as a diagonal subgroup of a $k$-product group:

$$ G_1 \times \cdots \times G_k \rightarrow G_{\text{diag}}, \quad (A.1) $$

corresponding to the level of embedding of $G_{\text{diag}}$ being $k$. The generators of $G_{\text{diag}}$ are given as a linear combination of the original generators, $T^a = \sum_{i=1}^k T^a_i$, and the tree-level matching condition on the gauge coupling $1/\alpha = \sum_{i=1}^k 1/\alpha_i$ has to be satisfied at the SSB scale. One can also obtain higher-level embeddings by using gauge groups with non simply-laced algebras — that is, those which contain roots of different lengths [141].

In general, any possible embedding of the SM in a GUT can be labeled by 2 integers $(k_2, k_3)$ describing the level of embedding of the SU(2) and SU(3) sectors and the hypercharge normalisation $k_1$, which is a rational number. The regular embeddings in SU(5) [39], SO(10) [39, 40] and $E_6$ [133] correspond to $(k_2, k_3; k_1) = (1,1; 5/3)$. Several of the GUT predictions depend on these quantized numbers. For example, the weak mixing angle prediction at the unification scale is in general not fixed and depends on $k_1$, $k_2$ as [135]

$$ \sin^2 \theta_w = \frac{k_2}{k_1 + k_2}, \quad (A.2) $$

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and the ratio of anomaly coefficients for axion-photon coupling is given by

\[
\frac{E}{N} = \frac{k_2 + k_1}{k_3}.
\]  

(A.3)

The value of \( E/N \) coincides with the inverse of \( \sin^2 \theta_w \) at the unification scale if \( k_2 = k_3 \). We see that in the well-studied GUT models with standard embedding — e.g. SU(5) with \( \bar{5} + 10 \), SO(10) with \( 16 \) and \( E_6 \) with \( 27 \) — we get \( (k_2, k_3; k_1) = (1, 1, 5/3) \), and \( E/N = 8/3 \). While this appears to be not the most general possibility, we highlight below the severe model building challenges to find a different embedding of the SM in a 4D simple gauge group.

In higher embeddings, the action of UV instantons is reduced compared to the standard embedding [77, 142, 143].

\[
S_k \sim \frac{S_{k=1}}{k} \sim \frac{2\pi}{\alpha_{\text{GUT}} k}.
\]  

(A.4)

Small size gauge instantons (SSI) at the scale of symmetry breaking are not aligned with the QCD axion potential in general, and may reintroduce the strong CP problem for a higher level embedding for \( k \) as low as 2 [30].

Perhaps more importantly, chiral exotics and fractionally charged states are hard to avoid for generic choices of \( (k_2, k_3; k_1) \). As an example, let us consider the \( k \)-level embedding of SU(\( N \)) in a diagonal subgroup as above (similar arguments apply for other groups):

\[
\text{SU}(kN) \to [\text{SU}(N)]^k \to \text{SU}(N)_{\text{diag}}.
\]  

(A.5)

The fundamental of SU(\( kN \)) has the following branching rule:

\[
kN \to (N, 1, \ldots) + \ldots + (1, \ldots, N) \to k \times N,
\]  

(A.6)

yielding \( k \) copies of the fundamental of SU(\( N \)\( _{\text{diag}} \)). Any other representation — and its branching rules — can be obtained by taking tensor products of the fundamental. This has the important implication that given an anomaly-free, chiral (complex) set of SU(\( kN \)) representations, any complex representation \( R \) of SU(\( N \)\( _{\text{diag}} \)) will come in a number given by \( mk \), where \( m \) is an integer (in general we expect different \( m \) for different representations, \( R \)). Since the set of representations was complex with respect to SU(\( kN \)), this means that in general we expect to have

\[
\text{mk copies of } R + \text{mk copies of } \bar{R},
\]  

(A.7)

where \( \bar{R} \) is the complex conjugate of \( R \).

In terms of fermions, each \( \bar{R} \) can pair up with a \( R \) and get a large vector-like mass. Thus, at low energies, we expect to have a number of chiral fermions in the \( R \) representation of SU(\( N \)\( _{\text{diag}} \)) given by:

\[
(m - \bar{m})k.
\]  

(A.8)

For example, if SU(2)\( _L \) is embedded at level \( k_2 \), we expect at least \( k_2 \) lepton doublets and \( k_2 \) quark doublets (each representation with a different SU(3) or U(1) charge is a different
Since a chiral leptonic 4th family is excluded, we find the bound $k_2 \leq 3$. In addition, if $k_2 = 2$, we expect lepton doublets to come in multiples of 2, which is either too few or too many families. The only possibilities are:

$$k_2 = 1, \text{ and } k_2 = 3.$$  \hfill (A.9)

We can now use anomaly cancellation — which in the SM occurs family by family — to argue that we need the same number of lepton doublets as quark doublets. Not only that, to avoid $[SU(3)_C]^3$ anomalies we need the same number of triplets as anti-triplets. Therefore, in the absence of chiral exotics, it is easy to conclude that we get only 2 consistent possibilities:

$$k_2 = k_3 = 1, \text{ and } k_2 = k_3 = 3.$$  \hfill (A.10)

The criterion of anomaly cancellation does not give any hint about what the hypercharge normalisation is, as it is just a global factor in the anomalies that involve $U(1)$. However, from the definition of the electric charge operator, we find that in a situation where $k_1$ does not satisfy $k_1/k_2 = 5/3$, we expect isolated states with electric charge smaller than the electron charge in the spectrum.

While this is by no means a proof that generic embeddings are not theoretically consistent, it highlights the serious model building challenges that come with such non-standard situations. This suggests the striking result that $k_2 = k_3 = 1$ or $k_2 = k_3 = 3$, which taking a right value for hypercharge normalisation gives

$$\frac{E}{N} = \frac{8}{3}. \hfill (A.11)$$

It will be nice to build realistic 4 dimensional GUT models where the index of embedding is related to the replication of families.

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References

[1] P. Arias, D. Cadamuro, M. Goodsell, J. Jaeckel, J. Redondo and A. Ringwald, WISPy Cold Dark Matter, JCAP 06 (2012) 013 [arXiv:1201.5902] [inSPIRE].

[2] P.W. Graham, I.G. Irastorza, S.K. Lamoreaux, A. Lindner and K.A. van Bibber, Experimental Searches for the Axion and Axion-Like Particles, Ann. Rev. Nucl. Part. Sci. 65 (2015) 485 [arXiv:1602.00039] [inSPIRE].

[3] M. Bauer, M. Neubert and A. Thamm, Collider Probes of Axion-Like Particles, JHEP 12 (2017) 044 [arXiv:1708.00443] [inSPIRE].

[4] I.G. Irastorza and J. Redondo, New experimental approaches in the search for axion-like particles, Prog. Part. Nucl. Phys. 102 (2018) 89 [arXiv:1801.08127] [inSPIRE].

[5] G. ’t Hooft, Naturalness, chiral symmetry, and spontaneous chiral symmetry breaking, NATO Advanced Study Institutes Series 59, Springer Boston (1980), pp. 135–157 [DOI].
[6] S. Weinberg, *A New Light Boson?, Phys. Rev. Lett.* 40 (1978) 223 [inSPIRE].

[7] F. Wilczek, *Problem of Strong $P$ and $T$ Invariance in the Presence of Instantons, Phys. Rev. Lett.* 40 (1978) 279 [inSPIRE].

[8] F. Wilczek, *Two Applications of Axion Electrodynamics, Phys. Rev. Lett.* 58 (1987) 1799 [inSPIRE].

[9] P. Sikivie, *Experimental Tests of the Invisible Axion, Phys. Rev. Lett.* 51 (1983) 1415 [Erratum ibid. 52 (1984) 695] [inSPIRE].

[10] P. Sikivie, *Detection Rates for ‘Invisible’ Axion Searches, Phys. Rev. D* 32 (1985) 2988 [Erratum ibid. 36 (1987) 974] [inSPIRE].

[11] ALPS-II collaboration, *First sensitivity limits of the ALPS TES detector, in 10th Patras Workshop on Axions, WIMPs and WISPs, Geneva Switzerland, June 29–July 04 2014, pp. 63–66 [DOI] [arXiv:1409.6992] [inSPIRE].

[12] MADMAX Working Group collaboration, *Dielectric Haloscopes: A New Way to Detect Axion Dark Matter, Phys. Rev. Lett.* 118 (2017) 091801 [arXiv:1611.05865] [inSPIRE].

[13] CAST collaboration, *An Improved limit on the axion-photon coupling from the CAST experiment, JCAP* 04 (2007) 010 [hep-ex/0702006] [inSPIRE].

[14] CAST collaboration, *New CAST Limit on the Axion-Photon Interaction, Nature Phys.* 13 (2017) 584 [arXiv:1705.02290] [inSPIRE].

[15] ADMX collaboration, *A SQUID-based microwave cavity search for dark-matter axions, Phys. Rev. Lett.* 104 (2010) 041301 [arXiv:0910.5914] [inSPIRE].

[16] E. Armengaud et al., *Conceptual Design of the International Axion Observatory (IAXO), 2014 JINST* 9 T05002 [arXiv:1401.3233] [inSPIRE].

[17] MADMAX Working Group collaboration, *MADMAX: A new Dark Matter Axion Search using a Dielectric Haloscope, in 12th Patras Workshop on Axions, WIMPs and WISPs, Jeju Island South Korea, June 20–24 2016, pp. 94–97 [DOI] [arXiv:1611.04549] [inSPIRE].

[18] Y. Kahn, B.R. Safdi and J. Thaler, *Broadband and Resonant Approaches to Axion Dark Matter Detection, Phys. Rev. Lett.* 117 (2016) 141801 [arXiv:1602.01086] [inSPIRE].

[19] A. Arvanitaki, S. Dimopoulos and K. Van Tilburg, *Resonant absorption of bosonic dark matter in molecules, Phys. Rev. X* 8 (2018) 041001 [arXiv:1709.05354] [inSPIRE].

[20] M. Baryakhtar, J. Huang and R. Lasenby, *Axion and hidden photon dark matter detection with multilayer optical haloscopes, Phys. Rev. D* 98 (2018) 035006 [arXiv:1803.11455] [inSPIRE].

[21] S. Chaudhuri, K. Irwin, P.W. Graham and J. Mardon, *Optimal Impedance Matching and Quantum Limits of Electromagnetic Axion and Hidden-Photon Dark Matter Searches, arXiv:1803.01627* [inSPIRE].

[22] R.D. Peccei and H.R. Quinn, *CP Conservation in the Presence of Instantons, Phys. Rev. Lett.* 38 (1977) 1440 [inSPIRE].

[23] J.E. Kim, *Weak Interaction Singlet and Strong CP Invariance, Phys. Rev. Lett.* 43 (1979) 103 [inSPIRE].

[24] M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, *Can Confinement Ensure Natural CP Invariance of Strong Interactions?, Nucl. Phys. B* 166 (1980) 493 [inSPIRE].
[25] M. Dine, W. Fischler and M. Srednicki, *A Simple Solution to the Strong CP Problem with a Harmless Axion*, Phys. Lett. B 104 (1981) 199 [InSPIRE].

[26] A.R. Zhitnitsky, *On Possible Suppression of the Axion Hadron Interactions* (in Russian), Sov. J. Nucl. Phys. 31 (1980) 260 [InSPIRE].

[27] J. Preskill, M.B. Wise and F. Wilczek, *Cosmology of the Invisible Axion*, Phys. Lett. B 120 (1983) 127 [InSPIRE].

[28] L.F. Abbott and P. Sikivie, *A Cosmological Bound on the Invisible Axion*, Phys. Lett. B 120 (1983) 133 [InSPIRE].

[29] M. Dine and W. Fischler, *The Not So Harmless Axion*, Phys. Lett. B 120 (1983) 137 [InSPIRE].

[30] P. Svrček and E. Witten, *Axions In String Theory*, JHEP 06 (2006) 051 [hep-th/0605206] [InSPIRE].

[31] A. Arvanitaki, S. Dimopoulos, S. Dubovsky, N. Kaloper and J. March-Russell, *String Axiverse*, Phys. Rev. D 81 (2010) 123530 [arXiv:0905.4720] [InSPIRE].

[32] M. Cicoli, M. Goodsell and A. Ringwald, *The type IIB string axiverse and its low-energy phenomenology*, JHEP 10 (2012) 146 [arXiv:1206.0819] [InSPIRE].

[33] C. Vafa and E. Witten, *Parity Conservation in QCD*, Phys. Rev. Lett. 53 (1984) 535 [InSPIRE].

[34] G. Grilli di Cortona, E. Hardy, J. Pardo Vega and G. Villadoro, *The QCD axion, precisely*, JHEP 01 (2016) 034 [arXiv:1511.02867] [InSPIRE].

[35] L. Hui, J.P. Ostriker, S. Tremaine and E. Witten, *Ultralight scalars as cosmological dark matter*, Phys. Rev. D 95 (2017) 043541 [arXiv:1610.08297] [InSPIRE].

[36] T. Banks and N. Seiberg, *Symmetries and Strings in Field Theory and Gravity*, Phys. Rev. D 83 (2011) 084019 [arXiv:1011.5120] [InSPIRE].

[37] R. Kallosh, A.D. Linde, D.A. Linde and L. Susskind, *Gravity and global symmetries*, Phys. Rev. D 52 (1995) 912 [hep-th/9502069] [InSPIRE].

[38] P. Agrawal et al., *Feebly-interacting particles: FIPs 2020 workshop report*, Eur. Phys. J. C 81 (2021) 1015 [arXiv:2102.12143] [InSPIRE].

[39] H. Georgi and S.L. Glashow, *Unity of All Elementary Particle Forces*, Phys. Rev. Lett. 32 (1974) 438 [InSPIRE].

[40] H. Fritzsch and P. Minkowski, *Unified Interactions of Leptons and Hadrons*, Annals Phys. 93 (1975) 193 [InSPIRE].

[41] M. Srednicki, *Axion Couplings to Matter. I. CP Conserving Parts*, Nucl. Phys. B 260 (1985) 689 [InSPIRE].

[42] J.E. Kim, *A Composite Invisible Axion*, Phys. Rev. D 31 (1985) 1733 [InSPIRE].

[43] K. Choi and J.E. Kim, *Dynamical Axion*, Phys. Rev. D 32 (1985) 1828 [InSPIRE].

[44] D.B. Kaplan, *Opening the Axion Window*, Nucl. Phys. B 260 (1985) 215 [InSPIRE].

[45] L. Randall, *Composite axion models and Planck scale physics*, Phys. Lett. B 284 (1992) 77 [InSPIRE].

[46] D. Budker, P.W. Graham, M. Ledbetter, S. Rajendran and A. Sushkov, *Proposal for a Cosmic Axion Spin Precession Experiment (CASPEr)*, Phys. Rev. X 4 (2014) 021030 [arXiv:1306.6089] [InSPIRE].
A. Arvanitaki and A.A. Geraci, *Resonantly Detecting Axion-Mediated Forces with Nuclear Magnetic Resonance*, Phys. Rev. Lett. **113** (2014) 161801 [arXiv:1403.1290] [SPIRE].

F. Wilczek, *Axions and Family Symmetry Breaking*, Phys. Rev. Lett. **49** (1982) 1549

J. Martin Camalich, M. Pospelov, P.N.H. Vuong, R. Ziegler and J. Zupan, *Quark Flavor Phenomenology of the QCD Axion*, Phys. Rev. **D 102** (2020) 015023 [arXiv:2002.04623] [SPIRE].

M. Bauer, M. Neubert, S. Renner, M. Schnubel and A. Thamm, *Flavor probes of axion-like particles*, JHEP **09** (2022) 056 [arXiv:2110.10698] [SPIRE].

G. Raffelt, *Limits on a CP-violating scalar axion-nucleon interaction*, Phys. Rev. **D 86** (2012) 015001 [arXiv:1205.1776] [SPIRE].

S. Okawa, M. Pospelov and A. Ritz, *Long-range axion forces and hadronic CP-violation*, Phys. Rev. **D 105** (2022) 075003 [arXiv:2111.08040] [SPIRE].

P. Agrawal, A. Hook and J. Huang, *A CMB Millikan experiment with cosmic axiverse strings*, JHEP **07** (2020) 138 [arXiv:1912.02823] [SPIRE].

Super-Kamiokande collaboration, *Search for proton decay via p → e+π0 and p → μ+π0 with an enlarged fiducial volume in Super-Kamiokande I-IV*, Phys. Rev. **D 102** (2020) 112011 [arXiv:2010.16098] [SPIRE].

K. Abe et al., *Letter of Intent: The Hyper-Kamiokande Experiment — Detector Design and Physics Potential —*, arXiv:1109.3262 [SPIRE].

Y. Kawamura, *Gauge symmetry breaking from extra space S1/Z2*, Prog. Theor. Phys. **103** (2000) 613 [hep-ph/9902423] [SPIRE].

A. Hebecker and J. March-Russell, *A Minimal S1/(Z2 × Z2′) orbifold GUT*, Nucl. Phys. **B 613** (2001) 3 [hep-ph/0106166] [SPIRE].

L.J. Hall and Y. Nomura, *Gauge unification in higher dimensions*, Phys. Rev. **D 64** (2001) 055003 [hep-ph/0103125] [SPIRE].

G. Altarelli and F. Feruglio, *SU(5) grand unification in extra dimensions and proton decay*, Phys. Lett. **B 511** (2001) 257 [hep-ph/0102301] [SPIRE].

L.J. Hall, H. Murayama and Y. Nomura, *Wilson lines and symmetry breaking on orbifolds*, Nucl. Phys. **B 645** (2002) 85 [hep-th/0107245] [SPIRE].

A. Hebecker and J. March-Russell, *The structure of GUT breaking by orbifolding*, Nucl. Phys. **B 625** (2002) 128 [hep-ph/0107039] [SPIRE].

G.F. Giudice, R. Rattazzi and A. Strumia, *Unificaxion*, Phys. Lett. **B 715** (2012) 142 [arXiv:1204.5465] [SPIRE].

H.M. Georgi, L.J. Hall and M.B. Wise, *Grand Unified Models With an Automatic Peccei-Quinn Symmetry*, Nucl. Phys. **B 192** (1981) 409 [SPIRE].

H.P. Nilles and S. Raby, *Supersymmetry and the strong CP problem*, Nucl. Phys. **B 198** (1982) 102 [SPIRE].

M.B. Wise, H. Georgi and S.L. Glashow, *SU(5) and the Invisible Axion*, Phys. Rev. Lett. **47** (1981) 402 [SPIRE].

L. Di Luzio, A. Ringwald and C. Tamarit, *Axion mass prediction from minimal grand unification*, Phys. Rev. **D 98** (2018) 095011 [arXiv:1807.09769] [SPIRE].
[67] A. Ernst, A. Ringwald and C. Tamarit, *Axion Predictions in SO(10) × U(1)_{PQ} Models*, JHEP 02 (2018) 103 [arXiv:1801.04906] [inSPIRE].

[68] P. Fileviez Pérez, C. Murgui and A.D. Plascencia, *The QCD Axion and Unification*, JHEP 11 (2019) 093 [arXiv:1908.01772] [inSPIRE].

[69] A. Davidson, V.P. Nair and K.C. Wali, *Peccei-Quinn Symmetry as Flavor Symmetry and Grand Unification*, Phys. Rev. D 29 (1984) 1504 [inSPIRE].

[70] A. Davidson, V.P. Nair and K.C. Wali, *Mixing Angles and CP Violation in the SO(10) × U(1)_{PQ} Model*, Phys. Rev. D 29 (1984) 1513 [inSPIRE].

[71] N. Chen, Y. Liu and Z. Teng, *Axion model with the SU(6) unification*, Phys. Rev. D 104 (2021) 115011 [arXiv:2106.00223] [inSPIRE].

[72] P. Agrawal, J. Fan, M. Reece and L.-T. Wang, *Experimental Targets for Photon Couplings of the QCD Axion*, JHEP 02 (2018) 006 [arXiv:1709.06085] [inSPIRE].

[73] K. Fraser and M. Reece, *Axion Periodicity and Coupling Quantization in the Presence of Mixing*, JHEP 05 (2020) 066 [arXiv:1910.11349] [inSPIRE].

[74] A.V. Sokolov and A. Ringwald, *Electromagnetic Couplings of Axions*, arXiv:2205.02605 [inSPIRE].

[75] M. Baryakhtar, E. Hardy and J. March-Russell, *Axion Mediation*, JHEP 07 (2013) 096 [arXiv:1301.0829] [inSPIRE].

[76] J.M. Pendlebury et al., *Revised experimental upper limit on the electric dipole moment of the neutron*, Phys. Rev. D 92 (2015) 092003 [arXiv:1509.04411] [inSPIRE].

[77] P. Agrawal and K. Howe, *Factoring the Strong CP Problem*, JHEP 12 (2018) 029 [arXiv:1710.04213] [inSPIRE].

[78] M. Kamionkowski and J. March-Russell, *Planck scale physics and the Peccei-Quinn mechanism*, Phys. Lett. B 282 (1992) 137 [hep-th/9202003] [inSPIRE].

[79] L.M. Krauss and F. Wilczek, *Discrete Gauge Symmetry in Continuum Theories*, Phys. Rev. Lett. 62 (1989) 1221 [inSPIRE].

[80] C.F. Kolda and J. March-Russell, *Low-energy signatures of semiperturbative unification*, Phys. Rev. D 55 (1997) 4252 [hep-ph/9609480] [inSPIRE].

[81] A. Hook, *Naturally Weak CP-violation*, Phys. Lett. B 136 (1984) 387 [inSPIRE].

[82] S.M. Barr, *Solving the Strong CP Problem Without the Peccei-Quinn Symmetry*, Phys. Rev. Lett. 53 (1984) 329 [arXiv:1802.10093] [inSPIRE].

[83] A. Hook, *Solving the Hierarchy Problem Discretely*, Phys. Rev. Lett. 120 (2018) 261802 [arXiv:1802.10093] [inSPIRE].

[84] L. Di Luzio, B. Gavela, P. Quilez and A. Ringwald, *An even lighter QCD axion*, JHEP 05 (2021) 184 [arXiv:2102.00012] [inSPIRE].

[85] V.A. Rubakov, *Grand unification and heavy axion*, JETP Lett. 65 (1997) 621 [hep-ph/9703409] [inSPIRE].

[86] Z. Berezhiani, L. Gianfagna and M. Giannotti, *Strong CP problem and mirror world: The Weinberg-Wilczek axion revisited*, Phys. Lett. B 500 (2001) 286 [hep-ph/0009290] [inSPIRE].

[87] A. Hook, *Anomalous solutions to the strong CP problem*, Phys. Rev. Lett. 114 (2015) 141801 [arXiv:1411.3325] [inSPIRE].
[88] A. Hook, S. Kumar, Z. Liu and R. Sundrum, High Quality QCD Axion and the LHC, *Phys. Rev. Lett.* **124** (2020) 221801 [arXiv:1911.12364] [inSPIRE].

[89] K. Choi and S.H. Im, Realizing the relaxion from multiple axions and its UV completion with high scale supersymmetry, *JHEP* **01** (2016) 149 [arXiv:1511.00132] [inSPIRE].

[90] D.E. Kaplan and R. Rattazzi, Large field excursions and approximate discrete symmetries from a clockwork axion, *Phys. Rev. D* **93** (2016) 085007 [arXiv:1511.01827] [inSPIRE].

[91] M. Farina, D. Pappadopulo, F. Rompineve and A. Tesi, The photo-philic QCD axion, *JHEP* **01** (2017) 095 [arXiv:1611.09855] [inSPIRE].

[92] ADMX collaboration, A Search for Invisible Axion Dark Matter with the Axion Dark Matter Experiment, *Phys. Rev. Lett.* **120** (2018) 151301 [arXiv:1804.05750] [inSPIRE].

[93] ADMX collaboration, Extended Search for the Invisible Axion with the Axion Dark Matter Experiment, *Phys. Rev. Lett.* **124** (2020) 101303 [arXiv:1910.08638] [inSPIRE].

[94] ADMX collaboration, Search for Invisible Axion Dark Matter in the 3.3–4.2 µeV Mass Range, *Phys. Rev. Lett.* **127** (2021) 261803 [arXiv:2110.06096] [inSPIRE].

[95] M.T. Frandsen, F. Kahlhoefer, C. McCabe, S. Sarkar and K. Schmidt-Hoberg, Resolving astrophysical uncertainties in dark matter direct detection, *JCAP* **01** (2012) 024 [arXiv:1111.0292] [inSPIRE].

[96] J.I. Read, The Local Dark Matter Density, *J. Phys. G* **41** (2014) 063101 [arXiv:1404.1938] [inSPIRE].

[97] A.M. Green, Astrophysical uncertainties on the local dark matter distribution and direct detection experiments, *J. Phys. G* **44** (2017) 084001 [arXiv:1703.10102] [inSPIRE].

[98] S. Sivertsson, J.I. Read, H. Silverwood, P.F. de Salas, K. Malhan, A. Widmark et al., Estimating the local dark matter density in a non-axisymmetric wobbling disc, *Mon. Not. Roy. Astron. Soc.* **511** (2022) 1977 [arXiv:2201.01822] [inSPIRE].

[99] J. Buch, S.C.J. Leung and J. Fan, Using Gaia DR2 to Constrain Local Dark Matter Density and Thin Dark Disk, *JCAP* **04** (2019) 026 [arXiv:1808.05603] [inSPIRE].

[100] R. Guo, C. Liu, S. Mao, X.-X. Xue, R.J. Long and L. Zhang, Measuring the local dark matter density with LAMOST DR5 and Gaia DR2, *arXiv:2005.12018* [inSPIRE].

[101] J.-B. Salomon, O. Bienaymé, C. Reylé, A.C. Robin and B. Famaey, Kinematics and dynamics of Gaia red clump stars — Revisiting north-south asymmetries and dark matter density at large heights, *Astron. Astrophys.* **643** (2020) A75 [arXiv:2009.04495] [inSPIRE].

[102] A. Widmark, C.F.P. Laporte, P.F. de Salas and G. Monari, Weighing the Galactic disk using phase-space spirals — II. Most stringent constraints on a thin dark disk using Gaia EDR3, *Astron. Astrophys.* **653** (2021) A86 [arXiv:2105.14030] [inSPIRE].

[103] K. Van Tilburg, Stellar basins of gravitationally bound particles, *Phys. Rev. D* **104** (2021) 023019 [arXiv:2006.12431] [inSPIRE].

[104] W. DeRocco, S. Wegsman, B. Grefenstette, J. Huang and K. Van Tilburg, First Indirect Detection Constraints on Axions in the Solar Basin, *Phys. Rev. Lett.* **129** (2022) 101101 [arXiv:2205.05700] [inSPIRE].

[105] A.J. Millar, J. Redondo and F.D. Steffen, Dielectric haloscopes: sensitivity to the axion dark matter velocity, *JCAP* **10** (2017) 006 [Erratum ibid. **05** (2018) E02] [arXiv:1707.04266] [inSPIRE].
[106] M. Pospelov and A. Ritz, *Theta vacua, QCD sum rules, and the neutron electric dipole moment*, *Nucl. Phys. B* 573 (2000) 177 [hep-ph/9908508] [inSPIRE].

[107] G. Marques-Tavares and M. Teo, *Light axions with large hadronic couplings*, *JHEP* 05 (2018) 180 [arXiv:1803.07575] [inSPIRE].

[108] N. Craig, A. Hook and S. Kasko, *The Photophobic ALP*, *JHEP* 09 (2018) 028 [arXiv:1805.06538] [inSPIRE].

[109] P. Agrawal, J. Fan and M. Reece, *Clockwork Axions in Cosmology: Is Chrononatural Inflation Chrononatural?*, *JHEP* 10 (2018) 180 [arXiv:1803.07575] [IN_SPIRE].

[110] S. Dimopoulos, A. Hook, J. Huang and G. Marques-Tavares, *A collider observable QCD axion*, *JHEP* 11 (2016) 052 [arXiv:1606.03097] [IN_SPIRE].

[111] M. Dine, W. Fischler and M. Srednicki, *Supersymmetric Technicolor*, *Nucl. Phys. B* 189 (1981) 575 [IN_SPIRE].

[112] I. Affleck, M. Dine and N. Seiberg, *Dynamical Supersymmetry Breaking in Four-Dimensions and Its Phenomenological Implications*, *Nucl. Phys. B* 256 (1985) 557 [IN_SPIRE].

[113] L. Vecchi, *Axion quality straight from the GUT*, *Eur. Phys. J. C* 81 (2021) 938 [arXiv:2106.15224] [IN_SPIRE].

[114] R. Contino, A. Podo and F. Revello, *Chiral models of composite axions and accidental Peccei-Quinn symmetry*, *JHEP* 04 (2022) 180 [arXiv:2112.09635] [IN_SPIRE].

[115] J. Fan, K. Fraser, M. Reece and J. Stout, *Axion Mass from Magnetic Monopole Loops*, *Phys. Rev. Lett.* 127 (2021) 131602 [arXiv:2105.09950] [IN_SPIRE].

[116] E. Izaguirre and I. Yavin, *New window to millicharged particles at the LHC*, *Phys. Rev. D* 92 (2015) 035014 [arXiv:1506.04760] [IN_SPIRE].

[117] A. Berlin, N. Blinov, G. Krnjaic, P. Schuster and N. Toro, *Dark Matter, Millicharges, Axion and Scalar Particles, Gauge Bosons, and Other New Physics with LDMX*, *Phys. Rev. D* 99 (2019) 075001 [arXiv:1807.01730] [IN_SPIRE].

[118] A. Ball et al., *Search for millicharged particles in proton-proton collisions at \(\sqrt{s} = 13\) TeV*, *Phys. Rev. D* 102 (2020) 032002 [arXiv:2005.06518] [IN_SPIRE].

[119] D. Budker, P.W. Graham, H. Ramani, F. Schmidt-Kaler, C. Smorra and S. Ulmer, *Millicharged Dark Matter Detection with Ion Traps*, *PRX Quantum* 3 (2022) 010330 [arXiv:2108.05283] [IN_SPIRE].

[120] XENON collaboration, *Excess electronic recoil events in XENON1T*, *Phys. Rev. D* 102 (2020) 072004 [arXiv:2006.09721] [IN_SPIRE].

[121] F. Takahashi, M. Yamada and W. Yin, *XENON1T Excess from Anomaly-Free Axionlike Dark Matter and Its Implications for Stellar Cooling Anomaly*, *Phys. Rev. Lett.* 125 (2020) 161801 [arXiv:2006.10035] [IN_SPIRE].

[122] C. Han, M.L. López-Ibáñez, A. Melis, O. Vives and J.M. Yang, *Anomaly-free leptophilic axionlike particle and its flavor violating tests*, *Phys. Rev. D* 103 (2021) 035028 [arXiv:2007.08834] [IN_SPIRE].

[123] C. Han, M.L. López-Ibáñez, A. Melis, O. Vives and J.M. Yang, *Anomaly-free ALP from non-Abelian flavor symmetry*, *JHEP* 08 (2022) 306 [arXiv:2203.16376] [IN_SPIRE].

[124] J.E. Moody and F. Wilczek, *New Macroscopic Forces?*, *Phys. Rev. D* 30 (1984) 130 [IN_SPIRE].
[125] ARIADNE collaboration, Progress on the ARIADNE axion experiment, *Springer Proc. Phys. 211* (2018) 151 [arXiv:1710.05413] [inSPIRE].

[126] C.A.J. O’Hare and E. Vitagliano, Corrnering the axion with CP-violating interactions, *Phys. Rev. D 102* (2020) 115026 [arXiv:2010.03889] [inSPIRE].

[127] J. Jaeckel, M. Jankowiak and M. Spannowsky, LHC probes the hidden sector, *Phys. Dark Univ. 2* (2013) 111 [arXiv:1212.3620] [inSPIRE].

[128] J. Jaeckel and M. Spannowsky, Probing MeV to 90 GeV axion-like particles with LEP and LHC, *Phys. Lett. B 753* (2016) 482 [arXiv:1509.00476] [inSPIRE].

[129] J.C. Pati and A. Salam, Lepton Number as the Fourth Color, *Phys. Rev. D 10* (1974) 275 [Erratum ibid. 11 (1975) 703] [inSPIRE].

[130] K.S. Babu, X.-G. He and S. Pakvasa, Neutrino Masses and Proton Decay Modes in SU(3) × SU(3) × SU(3) Trinification, *Phys. Rev. D 33* (1986) 763 [inSPIRE].

[131] S.M. Barr, A New Symmetry Breaking Pattern for SO(10) and Proton Decay, *Phys. Lett. B 112* (1982) 219 [inSPIRE].

[132] J.P. Derendinger, J.E. Kim and D.V. Nanopoulos, Anti-SU(5), *Phys. Lett. B 139* (1984) 170 [inSPIRE].

[133] F. Gursey, P. Ramond and P. Sikivie, A Universal Gauge Theory Model Based on E6, *Phys. Lett. B 60* (1976) 177 [inSPIRE].

[134] I. Antoniadis, J.R. Ellis, J.S. Hagelin and D.V. Nanopoulos, The Flipped SU(5) × U(1) String Model Revamped, *Phys. Lett. B 231* (1989) 65 [inSPIRE].

[135] A. Font, L.E. Ibáñez and F. Quevedo, Higher Level Kac-Moody String Models and Their Phenomenological Implications, *Nucl. Phys. B 345* (1990) 389 [inSPIRE].

[136] J. Polchinski, *String theory. Vol. 2: Superstring theory and beyond*, Cambridge Monographs on Mathematical Physics, Cambridge University Press (2007) [DOI] [inSPIRE].

[137] M.L. Perl, E.R. Lee and D. Loomba, Searches for fractionally charged particles, *Ann. Rev. Nucl. Part. Sci. 59* (2009) 47 [inSPIRE].

[138] A. Lue, L.-M. Wang and M. Kamionkowski, Cosmological signature of new parity violating interactions, *Phys. Rev. Lett. 83* (1999) 1506 [astro-ph/9812088] [inSPIRE].

[139] Y. Minami and E. Komatsu, New Extraction of the Cosmic Birefringence from the Planck 2018 Polarization Data, *Phys. Rev. Lett. 125* (2020) 221301 [arXiv:2011.11254] [inSPIRE].

[140] J.R. Eskilt and E. Komatsu, Improved constraints on cosmic birefringence from the WMAP and Planck cosmic microwave background polarization data, *Phys. Rev. D 106* (2022) 063503 [arXiv:2205.13962] [inSPIRE].

[141] K.R. Dienes and J. March-Russell, Realizing higher level gauge symmetries in string theory: New embeddings for string GUTs, *Nucl. Phys. B 479* (1996) 113 [hep-th/9604112] [inSPIRE].

[142] J. Fuentes-Martín, M. Reig and A. Vicente, Strong CP problem with low-energy emergent QCD: The 4321 case, *Phys. Rev. D 100* (2019) 115028 [arXiv:1907.02550] [inSPIRE].

[143] C. Csiáki, M. Ruhdorfer and Y. Shirman, UV Sensitivity of the Azion Mass from Instantons in Partially Broken Gauge Groups, *JHEP 04* (2020) 031 [arXiv:1912.02197] [inSPIRE].