FINITELY GENERATED MODULES OVER QUASI-EUCLIDEAN RINGS

LUC GUYOT

Abstract. Let $R$ be a unital commutative ring and let $M$ be an $R$-module that is generated by $k$ elements but not less. Let $E_n(R)$ be the subgroup of $GL_n(R)$ generated by the elementary matrices. In this paper we study the action of $E_n(R)$ by matrix multiplication on the set $Um_n(M)$ of unimodular rows of $M$ of length $n \geq k$. Assuming $R$ is moreover Noetherian and quasi-Euclidean, e.g., $R$ is a direct sum of finitely many Euclidean rings, we show that this action is transitive if $n > k$. We also prove that $Um_k(M)/E_k(R)$ is equipotent with the unit group of $R/a_1$ where $a_1$ is the first invariant factor of $M$. These results encompass the well-known classification of Nielsen non-equivalent generating tuples in finitely generated Abelian groups.

1. Introduction

In this paper rings are supposed unital and commutative. The unit group of a ring $R$ is denoted by $R^\times$. Let $M$ be a finitely generated $R$-module. We denote by $\text{rk}_R(M)$ the minimal number of generators of $M$. For $n \geq \text{rk}_R(M)$, we denote by $Um_n(M)$ the set of unimodular rows of $M$ of length $n$, i.e., the set of elements in $M^n$ whose components generate $M$. We consider the action of $GL_n(R)$ on $Um_n(M)$ by matrix right-multiplication. Let $E_n(R)$ be the subgroup of $GL_n(R)$ generated by the elementary matrices, i.e., the matrices which differ from the identity by a single off-diagonal element. Two unimodular rows $m, m' \in Um_n(M)$ are said to be $E_n(R)$-equivalent if there exists $E \in E_n(R)$ such that $m' = mE$. Our chief concern is the description of the orbit set $Um_n(M)/E_n(R)$ when $R$ enjoys a cancellation property shared by Euclidean rings, namely: for every $n \geq 2$ and every $r = (r_1, \ldots, r_n) \in R^n$.

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there exist \( E \in \mathbb{E}_n(R) \) and \( d \in R \) such that
\[
(d, 0, \ldots, 0) = rE.
\]

Rings with the above property are known as quasi-Euclidean rings in the sense of O’Meara and Cooke [O’M65, Coo76] (see [AJLL14, Theorem 11] for equivalence of definitions). A Noetherian quasi-Euclidean ring \( R \) is therefore a principal ideal ring (PIR), i.e., a ring whose ideals are principal. It is moreover an elementary divisor ring (see definition below) so that every finitely generated \( R \)-module admits an invariant factor decomposition, that is a decomposition of the form
\[
R/a_1 \times R/a_2 \times \cdots \times R/a_k \text{ with } R \neq a_1 \supset a_2 \supset \cdots \supset a_k
\]
where \( k \) is necessarily equal to \( \text{rk}_R(M) \). If \( R \) is both Noetherian and quasi-Euclidean, properties (1) and (2) make the study of the action of \( E_n(R) \) on \( \text{Um}_n(R) \) particularly amenable. Indeed, this assumption implies that \( E_n(R) \) acts transitively on \( \text{Um}_n(M) \) for every \( n > \text{rk}_R(M) \) and enables us to exhibit a complete invariant of \( E_n(R) \)-equivalence when \( n = \text{rk}_R(M) \). These two claims are corollaries of Theorem A below.

**Theorem A.** Let \( R \) be a Noetherian quasi-Euclidean ring and let \( M \) be a finitely generated \( R \)-module. Let \( R/a_1 \times \cdots \times R/a_k \) be the invariant factor decomposition of \( M \). Then every row in \( \text{Um}_n(M) \) with \( n \geq k \) is \( E_n(R) \)-equivalent to a row of the form \((\delta e_1, e_2, \ldots, e_k, 0, \ldots, 0)\) with \( \delta \in (R/a_1)^\times \) and where \( e_i \in M \) is defined by \((e_i)_i = 1 \in R/a_i \) and \((e_i)_j = 0 \) for \( j \neq i \). If \( n > k \), then \( \delta \) can be replaced by the identity.

**Corollary B.** Let \( M \) be as in Theorem A. The action of \( E_n(R) \) on \( \text{Um}_n(M) \) by matrix multiplication is transitive for every \( n > \text{rk}_R(M) \).

**Corollary C.** Let \( R \) be a Noetherian quasi-Euclidean ring. Let \( b_1, \ldots, b_k \) be proper ideals of \( R \) and set \( b = b_1 + \cdots + b_k \). Denote by \( M \) the \( R \)-module \( R/b_1 \times \cdots \times R/b_k \) and for \( m = (m_i) \in M^k \) denote by \( \det(m) \) the determinant of the matrix whose coefficients are the images in \( R/b \) of the \((m_i)_j \)'s via the natural maps \( R/b_j \to R/b \). Then \( m, m' \in \text{Um}_k(M) \) are \( E_k(R) \)-equivalent if and only if \( \det(m) = \det(m') \).

Suppose \( k = \text{rk}_R(M) \). Then the ideal \( b \) of Corollary C coincides with the first invariant factor \( a_1 \) of \( M \) (cf. proof). Putting Theorem A and Corollary C together, we see that \( m \in \text{Um}_k(M) \) is \( E_k(R) \)-equivalent to \((\det(m)e_1, e_2, \ldots, e_k)\) where \((e_i)_i \) is as in Theorem A and \( \det \) as in Corollary C. Therefore the map \( g \mapsto \det(g) \) induces a bijection from \( \text{Um}_k(M)/E_k(R) \) onto \((R/a_1)^\times \). The latter bijection endows \( \text{Um}_k(M)/E_k(R) \) with an Abelian group structure. The subject whether \( \text{Um}_k(R)/E_k(R) \) has a group structure for \( R \) a commutative ring...
of finite Krull dimension is well studied [VS76, vdK83, vdK89, Rao98, Fas11] but we don’t know of any similar results for modules. By analogy with [MR87, Definition 11.3.9], we can define the elementary rank of a finitely generated $R$-module $M$, for any associative ring $R$ with identity, as the least integer $e$ such that the action of $E_n(R)$ on $U_{n_1}(M)$ is transitive for all $n > e$. This rank is not less than $rk_R(M) - 1$ and not greater than $rk_R(M) - 1 + sr(M)$, where $sr(M)$ is the stable rank of $M$, a natural generalization of the Bass stable rank of rings to modules [MR87, Definition 6.7.2]. We showed in our situation that the elementary rank is $rk_R(M) - 1$ if $R/a_1$ has 2 elements and coincides with $rk_R(M)$ otherwise.

Specifying the above results to $R = \mathbb{Z}$ yields the characterization of Nielsen equivalent generating tuples in finitely generated Abelian groups. This characterization was obtained in part by several authors [NN51, LM93, DG99] and reaches its complete form in [Oan11]. In order to present it, we introduce the following definitions. Given a finitely generated group $G$, denote by $rk(G)$ the minimal number of generators of $G$. For $n \geq \text{rk}(G)$, let $V_n(G)$ be the set of generating $n$-vectors of $G$, i.e., the set of elements in $G^n$ whose components generate $G$. Two generating $n$-vectors are said to be Nielsen equivalent if they can be related by a finite sequence of transformations of $G^n$ taken in the set $\{L_{ij}, I_i; 1 \leq i \neq j \leq n\}$ where $L_{ij}$ and $I_i$ replace the component $g_i$ of $g = (g_1, \ldots, g_n) \in G^n$ by $g_j g_i$ and $g_i^{-1}$ respectively and leave the other components unchanged.

**Corollary D.** Let $G$ be a finitely generated Abelian group whose invariant factor decomposition is $\mathbb{Z}_{d_1} \times \cdots \times \mathbb{Z}_{d_k}$ with $1 \neq d_1 \mid d_2 \mid \cdots \mid d_k$, $d_i \geq 0$ and where $\mathbb{Z}_{d_i}$ stands for $\mathbb{Z}/d_i\mathbb{Z}$ (in particular $\mathbb{Z}_0 = \mathbb{Z}$). Then every generating $n$-vector $g$ with $n \geq k$ is Nielsen equivalent to $(\delta e_1, e_2, \ldots, e_k, 0, \ldots, 0)$ for some $\delta \in (\mathbb{Z}_{d_i})^\times$ and where $e_i \in G$ is defined by $(e_i)_i = 1 \in \mathbb{Z}_{d_i}$ and $(e_i)_j = 0$ for $j \neq i$.

- If $n > k$, then $\delta$ above can be replaced by the identity.
- If $n = k$ then $\delta$ must be $\pm \det(g)$ with $\det$ defined as in Corollary C.

In particular $G$ has only one Nielsen equivalence class of generating $n$-vectors for $n > k$ while it has $\text{max}(\varphi(d_1)/2, 1)$ Nielsen equivalence classes of generating $k$-vectors where $\varphi$ denotes the Euler totient function extended by $\varphi(0) = 0$.

Our results allow further applications to the study of Nielsen equivalence in split extensions of Abelian groups by cyclic or free Abelian groups. Consider for instance an infinite cyclic group $C$ and denote by $\mathbb{Z}[C]$ its integral group ring. Let $R$ be a quasi-Euclidean quotient of $\mathbb{Z}[C]$, e.g., $R = \mathbb{Z}_n[C]$ for $n$ a square-free integer, and let $M$ be a finitely generated $R$-module. Then the image of $C$ in
$R$ is a subgroup of $R^x$ so that $C$ acts naturally on $M$ by automorphisms. Let $G = M \rtimes C$ be the corresponding semi-direct product. Let $T$ be the subgroup of $R^x$ generated by the images of $-1$ and $C$, set $\Lambda = R/a_1$ where $a_1$ is the first invariant factor of $M$, and let $T_\Lambda$ be the image of $T$ in $\Lambda^x$. Our results allow us to show that the set of Nielsen equivalence classes of generating $k$-tuples of $G$ is equipotent with $\Lambda^x/T_\Lambda$ for $k = \text{rk}(G)$ [Guy16b]. (See also [Guy16a, Corollary 4.ii] for an application of Corollary B).

We introduce now definitions and preliminary results that will be used by the proofs of our statements in Section 2.

A ring $R$ is said to be an elementary divisor ring if every matrix $A$ over $R$ admits a diagonal reduction, i.e., if we can find invertible matrices $P, Q$ and elements $d_1, \ldots, d_n \in R$ such that $PAP = \text{diag}(d_1, \ldots, d_n)$ and $d_1 | d_2 | \cdots | d_n$. Principal ideal domains (PID) are elementary divisor rings [DF04, Theorem 4]. This classical result extends effortlessly to PIRs thanks to a theorem of Hungerford. Indeed, the class of elementary divisor rings is stable under taking quotients and direct sums. As a PIR is a direct sum of rings, each of which is the homomorphic image of a PID [Hun68, Theorem 1], our claim follows.

A Noetherian elementary divisor ring has moreover a unique invariant factor decomposition [Kap49, Theorems 9.1 and 9.3], thus we showed

**Lemma 1.** Let $R$ be a PIR. Then the following hold:

(i) $R$ is an elementary divisor ring.

(ii) Every finitely generated $R$-module $M$ has a unique invariant factor decomposition, i.e., a decomposition of the form $R/a_1 \times R/a_2 \times \cdots \times R/a_n$ with $R \neq a_1 \supset a_2 \supset \cdots \supset a_n$ where the factors $a_i$ are uniquely determined by the latter condition. For such decomposition, $n$ is the minimal number of generators of $M$, that is $n = \text{rk}_R(M)$.

The following result of Whitehead [Wei13, Example I.1.11] will come in handy in the proof of Theorem A. Let $R$ be a unital commutative ring. For $u \in R^x$, we denote by $D_i(u) \in \text{GL}_n(R)$ the diagonal matrix which coincides with the identity except possibly for its $(i, i)$-entry, which is set to $u$. Let $D_n(R^x)$ be the subgroup of $\text{GL}_n(R)$ generated by the matrices $D_i(u)$. Whitehead’s lemma implies that a matrix $D \in D_n(R^x)$ lies in $\text{SL}_n(R)$ if and only if it lies in $E_n(R)$.

Corollary C elaborates on Theorem A by showing that the unit $\delta$ of the theorem identifies with a natural invariant of $E_n(R)$-equivalence, namely the determinant in the largest quotient of $M$ that is a free module. This invariant extends Diaconis-Graham’s invariant defined for finitely generated Abelian groups [DG99]. It can be defined for any commutative unital ring $R$ and any finitely generated $R$-module $M$. Consider a generating set $m = (m_1, \ldots, m_n)$ of $M$ with minimal cardinality. We say that $r \in R$ is involved in a relation of
M with respect to \( \mathbf{m} \) if there is \((r_i) \in R^n\) such that \( \sum r_i m_i = 0 \) and \( r = r_i \) for some \( i \). Denote by \( \mathfrak{r}(M) \) the set of elements of \( R \) which are involved in a relation of \( M \) with respect to \( \mathbf{m} \). Clearly, \( \mathfrak{r}(M) \) is an ideal of \( R \) and it is easily checked that \( \mathfrak{r}(M) \) is independent of \( \mathbf{m} \). Let \( \mathfrak{m} = \pi(\mathbf{m}) \) be the image of \( \mathbf{m} \) by the natural map \( \pi : M \to M/\mathfrak{r}(M)M \) and let \( \mathbf{e} \) be the canonical basis of \((R/\mathfrak{r}(M))\). Then the map \( \mathfrak{m} \to \mathbf{e} \) induces an isomorphism \( \varphi_{\mathfrak{m}} \) from \( M/\mathfrak{r}(M)M \) onto \((R/\mathfrak{r}(M))\). For \( \mathbf{m}' \in \text{Um}_n(M) \), we define \( \det_m(\mathbf{m}') \) as the determinant of \( \varphi_{\mathfrak{m}} \circ \pi(\mathbf{m}') \) with respect to \( \mathbf{e} \). Let \( \varphi \equiv \varphi_{\mathfrak{m}} \circ \varphi_{\mathfrak{m}}^{-1} \). Because the identity \( \det_\mathfrak{m} = \det(\varphi) \det_m \) holds, we have shown the following

**Lemma 2.** Let \( n = \text{rk}_R(M) \) and let \( \mathbf{m}, \mathbf{m}' \in \text{Um}_n(M) \). There exists \( u \in (R/\mathfrak{r}(M))^\times \) such that \( \det_\mathfrak{m} = u \det_{\mathfrak{m}}' \).

It is straightforward to check that \( \det_\mathfrak{m} \) is an invariant of \( E_n(R) \)-equivalence, i.e., \( \det_{\mathfrak{m}}(\mathbf{m}'E) = \det_{\mathfrak{m}}(\mathbf{m}') \) for every \( E \in E_n(R) \). If \( \mathfrak{r}(M) = R \), then \( \det_{\mathfrak{m}} \) is trivial and hence useless. This doesn’t happen when \( R \) is an elementary divisor ring, for \( \mathfrak{r}(M) \) is then the first invariant factor \( a_1 \) of \( M \). The identity elements of each factor ring in a decomposition \( M \simeq R/a_1 \times \cdots \times R/a_n \) where \( n = \text{rk}_R(M) \) form a unimodular row of \( M \). We refer to this unimodular row as the *unimodular row naturally associated* to the given decomposition.

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## 2. Proofs

**Proof of Theorem A.** The case \( k = 1 \) follows straightforwardly from \( A \). We assume now that \( k \geq 2 \) and reverse the order of the sequence \( (a_i) \) for notational convenience, supposing that \( a_j \subset a_{j+1} \) for every \( j \). Let \( \mathbf{m} = (m_i) \in \text{Um}_n(M) \) with \( n \geq k \). We set \( m_{ij} \equiv (m_{ij}) \) for every \( 1 \leq i \leq n, 1 \leq j \leq k \) and identify \( \mathbf{m} \) with the matrix \( (m_{ij}) \). Applying \( A \) to every row \( m_i \equiv (m_{ij}), i \leq j \leq k \) we obtain a matrix \( E \in E_n(R) \) such that \( \mathbf{m}' = \mathbf{m} E \) is a lower-triangular matrix \( (m'_{ij}) \) with \( m'_{11} = 1 \). We claim that \( m'_{jj} \) is a unit of \( R/a_j \) for every \( 2 \leq j \leq k \). To see this, we consider the image \( \rho_l \) of \( (m'_{ij})_{1 \leq i, j \leq l} \) via the natural map induced by the projection \( R/a_l \to R/a_l \) for every \( 2 \leq l \leq k \). Since the rows of \( \rho_l \) generate \( (R/a_l)^l \), the matrix \( \rho_l \) is invertible \([\text{Mat89}, \text{Theorem 2.4}]\). Therefore \( \det(\rho_l) \in (R/a_l)^l \) and subsequently \( m''_{lj} \in (R/a_l)^l \). As a result we readily find \( E' \in E_n(R) \) such that \( \mathbf{m}'' \equiv \mathbf{m}' E' = \text{diag}(1, m''_2, \ldots, m''_k) \) with \( m''_j \equiv m'_{jj} \) for \( 2 \leq j \leq k \). If \( k = 2 \), we are done. Otherwise we consider the matrices
\[ E_j = D_j(1/m_j^u)D_k(m_j^u) \text{ for } 2 \leq j \leq k - 1. \] Since \( E_j \in E_k(R/a_j) \) for every \( j \) by Whitehead’s lemma, each matrix \( E_j \) has a lift in \( E_k(R) \) and the product of these lifts is a matrix \( E'' \in E_n(R) \) satisfying \( m''E'' = \text{diag}(1, \ldots, 1, m_k''') \) with \( m_k''' \in (R/a_k)^\times \). If \( n > k \), we can store \( 1 - m_k''' \) in the \((k, k + 1)\)-entry of \( m'' \), then turn \( m_k''' \) into 1 and eventually cancel the \((k, k + 1)\)-entry with obvious elementary column transformations. \[ \square \]

**Proof of Corollary C.** Since \( \det \) is invariant under elementary row transformations, the ’only if’ part is established. To prove the converse, it suffices to show that there is \( u \in (R/b)^\times \) such that for every \( m \in M^k \), \( \det(m) = u\delta(m) \) where \( \delta = \delta(m) \) is the unit given by Theorem A. Let \( \pi, \pi' : M \to (R/b)^k \) be the \( R\)-epimorphisms naturally induced by the invariant factor decomposition and the decomposition \( R/b_1 \times \cdots \times R/b_k \) respectively. Our claim certainly holds if \( \pi' = \varphi \circ \pi \) for some automorphism \( \varphi \) of \((R/b)^k\).

To prove the latter fact, we first show that \( b \) is the first invariant factor in the decomposition of \( M \) if \( k = \text{rk}_R(M) \) and \( b = R \) if \( k > \text{rk}_R(M) \). As \( R \) is a PIR, we can write \( b_i = b_iR \) with \( b_i \in R \) for every \( i \). Let \( A = \text{diag}(b_1, \ldots, b_k) \), with \( A \) square of order \( k \). Since \( R \) is an elementary divisor ring, the matrix \( A \) admits a diagonal reduction \( \text{diag}(d_1, \ldots, d_k) \). As the ideal generated by the coefficients of \( A \) is invariant under this reduction, we have \( b = d_1R \). If \( d_1 \notin R^\times \), the ideals \( d_iR \) correspond to the invariant factors \( a_i \) of \( M \) and hence \( k = \text{rk}_R(M) \). Otherwise \( b = R \) and \( \det \) vanishes on \( \text{Um}_k(M) \) while rows in \( \text{Um}_k(M) \) are all \( E_k(R) \)-equivalent by Theorem A. Therefore we can assume that \( k = \text{rk}_R(M) \) and \( b = d_1R \) is a proper ideal of \( R \).

Considering the unimodular rows naturally associated to our two decompositions of \( M \) as a direct sum of cyclic factors, we see that the existence of \( \varphi \) is established by Lemma 2. This proves the claim and hence the result. \[ \square \]

**Proof of Corollary D.** The group \( G \) is a \( \mathbb{Z} \)-module and Um\(_n\)(\( G \)) naturally identifies with \( V_n(\mathbb{G}) \). Considering the matrix counterparts of the transformations \( L_{ij} \) and \( I_{i} \), it is easily checked that the Nielsen classes in \( V_n(\mathbb{G}) \) coincide with the orbits in \( \text{Um}_n(\mathbb{G}) \) of \( \text{GL}_n(\mathbb{Z}) = D_n(\{\pm 1\})E_n(\mathbb{Z}) \) where \( D_n(\{\pm 1\}) \) is the group of diagonal matrices with diagonal entries in \( \{\pm 1\} \). The result is then a straightforward consequence of Theorem A and Corollary C. \[ \square \]

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EPFL ENT CBS BBP/HBP. CAMPUS BIOTECH. B1 BUILDING, CHEMIN DES MINES, 9, GENEVA 1202, SWITZERLAND
E-mail address: luc.guyot@epfl.ch