Statistical laws in urban mobility from microscopic GPS data in the area of Florence

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Abstract. The application of Statistical Physics to social systems is mainly related to the search for macroscopic laws that can be derived from experimental data averaged in time or space, assuming the system in a steady state. One of the major goals would be to find a connection between the statistical laws and the microscopic properties: for example, to understand the nature of the microscopic interactions or to point out the existence of interaction networks. Probability theory suggests the existence of a few classes of stationary distributions in the thermodynamics limit, so that the question is if a statistical physics approach could be able to enroll the complex nature of the social systems. We have analyzed a large GPS database for single-vehicle mobility in the Florence urban area, obtaining statistical laws for path lengths, for activity downtimes and for activity degrees. We show also that simple generic assumptions on the microscopic behavior could explain the existence of stationary macroscopic laws, with a universal function describing the distribution. Our conclusion is that understanding the system complexity requires a dynamical database for the microscopic evolution, which allows us to solve both small space and time scales in order to study the transients.

Keywords: scaling in socio-economic systems, traffic and crowd dynamics

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1. Introduction

Any statistical analysis of real systems is based on the Ergodic Principle for the microscopic dynamics, which implies the relaxation towards steady states and the independence property of elementary components. Even if the existence of microscopic interactions is necessary for the system to evolve towards a statistical equilibrium, in this state any particle moves independently from the others and all the particles are statistically equivalent (any particle may be representative for the whole). The thermodynamics laws that are derived from a statistical mechanics approach concern some macroscopic observables of the system, evolving adiabatically with respect to the microscopic relaxation time (i.e. we can consider the whole system in an almost equilibrium state), so that the effects of single-particle dynamics are conveniently described by means of stochastic processes. As a consequence, there should exist a natural separation among macroscopic and microscopic spacetime scales. Indeed space and time scales are expected to be strictly correlated: to understand small-scale phenomena we need to solve short time scales and vice versa. Nevertheless the statistical mechanics has great success in describing the evolution of macroscopic systems and there is a strong effort to generalize the results for a non-equilibrium thermodynamics and for application to complex systems [1]. The statistical properties of social systems have been recently considered from a different point of view due to the possibility of recording large microscopic datasets [2,3]. The main problem is what are the macroscopic effects of cognitive behavior for ‘social particles’. Indeed the cognitive behavior would imply the existence of strong bidirectional interactions among the dynamics at different space and time scales of the system [4]. Emergence and self-organization characterize the macroscopic states, but the question is which macroscopic observables (if they exist) may enroll the complex nature of the system. These variables may also play an important role in the study of phase transitions and in the control parameters’ definition.

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In Italy GPS data on individual vehicle paths are currently recorded for insurance reasons over a sample $\approx 2\%$ of the whole private vehicle population \cite{5,6}. This dataset gives the opportunity to study the individual mobility demand in urban contexts. The GPS dataset contains the geographical coordinates, the time, the instantaneous velocity and the path length of individual trajectories at positions whose relative distance is of the order of 1–2 km. Special signals are recorded when the engine is switched on and off. We remark that the data refer mainly to private transportation mobility and that, due to privacy legal problems, we do not have any knowledge on the social features of individuals in the sample.

In this paper we analyze the statistical distributions of the path lengths of individual trajectories, of the activity downtime and the distribution of the monthly activity degree. Our aim is to point out the main macroscopic features of urban mobility studying their correlation with the idea of ‘asystematic mobility’ recently proposed by sociologists to explain the observational data in a modern metropolis. We consider GPS data recorded during March 2008 in the Florence urban area. We show that some simple assumptions on single particles, like the existence of an ‘individual mobility energy’ and of an ‘individual mobility time’ that define the daily agenda, may explain the statistical laws emerging from the GPS data. Moreover, in the equilibrium state individuals seem to minimize their interactions, behaving independently, so that the Maximum Entropy Principle of statistical mechanics can be applied \cite{7}. These results are consistent with the idea that the sprawling phenomenon of modern cities implies that citizens move as stochastic particles \cite{8}. Finally our analysis enlightens some average cognitive properties of individuals in urban mobility. This paper is organized as follows: in section 2 we briefly describe the GPS database for vehicle mobility; in the remaining sections we discuss the three statistical laws on path length distribution, on the activities downtime and on the activity degree that are inferred from the data.

2. GPS data for vehicle mobility

A sample of $\approx 2\%$ of the private vehicles in Italy has a GPS system for insurance reasons. Any vehicle is associated with an ID number, so that it is possible to follow its mobility over a long time. Each datum gives position, velocity, distance covered from the previous measure and quality of signal. The data give a sampling of individual trajectories each $\approx 2$ km, but a signal is also recorded any time the engine is switched on or off. The data suffer from the GPS limited precision, in particular when the GPS loses the satellite signal. These problems are especially relevant at starting points of the trajectories or when vehicles are parked inside a building, and short paths could be strongly affected by these pathologies. When the quality of signal is good the time precision of the recorded data is practically perfect, whereas the space precision is of the order of 10 m, usually sufficient to localize a vehicle on the road. Both the instantaneous velocity and the space covered are given with an adequate precision since they result from a calculation based on GPS data recorded each second, but not registered. We have developed several methods to clean the data of spurious effects in order to avoid possible bias in the sample.

In the present work we consider the GPS data in the Florence urban area recorded during March 2008: these data are related to 35000 vehicles moving in a circular area of radius $\approx 30$ km, around the historical center and defining $\approx 2.5 \times 10^6$ different trajectories.
Figure 1. Aggregated GPS position data in the Florence area recorded during March 2008; the red dots correspond to a recorded instantaneous velocity $\leq 30$ km h$^{-1}$, whereas the yellow dots correspond to velocities in the interval 30–60 km h$^{-1}$ and the green dots to a velocity $\geq 60$ km h$^{-1}$.

Figure 2. Empirical distribution of the daily mobility length $L$ (dots). The continuous line is an interpolation with a Maxwell–Boltzmann distribution with average value $\langle L \rangle \simeq 25$ km.
We have restricted our analysis to the trajectories which start inside a circle of 10 km around the historical town and remain inside the considered area, so that with a good probability we select people living and moving in Florence. Then we look for the vehicles which perform daily loops from starting points that we identify as ‘home’: in this case the number of trajectories reduces to $\approx 4 \times 10^5$. In figure 1 we show the considered area where we have plotted the aggregate position GPS data: the color refers to the different instantaneous velocities (red means less than 30 km h$^{-1}$, whereas yellow refers to a velocity in the interval 30–60 km h$^{-1}$ and green to a velocity $\geq 60$ km h$^{-1}$). We are completely ignorant of the social composition of the sample and on the specific drivers, but we expect that such individuals perform mobility related to the activities present in the Florence area and have a certain knowledge of the road network.

3. Path length distribution

The activity sprawling that characterizes the modern metropolis has certainly a strong influence on the individual mobility demand [9]. Even if the Florence historical center is a very special area full of artistic and tourist attractions, but forbidden to private traffic, nevertheless we assume the activities are randomly distributed in the urban system. This hypothesis is quite reasonable because we consider a large urban area and our sample is surely composed of inhabitants and not tourists. As a consequence, we expect that the citizen mobility agenda is influenced by individual features rather than by the city structure. In cities the stationary average traffic state should emerge as the result of individual interactions of cognitive particles which share the same spatial resources. In particular, we assume that drivers organize their mobility by applying a minimization strategy of the interactions with other individuals [10]. In this conceptual framework, the

$$a \approx 0.7$$
mobility can be seen as the realization of many independent individual agendas and the dynamical properties become similar to that of a Boltzmann gas. Even if it is obviously true that individuals are non-identical particles, path length and activity downtime can be considered common mobility features for all people, and good candidates for a statistical physics approach to describe the stationary state. For each vehicle we have recorded the total lengths $L$ of the daily round trips for the whole period considered. The length distribution is plotted in figure 2 where we point out the existence of an interpolation with a Maxwell–Boltzmann distribution

$$p(L) = L_0 \exp(-L/L_0)$$

with $L_0 \approx 25$ km the characteristic daily path length. The distribution (1) provides a very good fit of the experimental data, and it can be justified by the Maximum Entropy Principle under the assumptions that the individuals are independent particles and that there exists an average daily trip length in the population [7]. In such a case the distribution (1) is realized when any particle chooses its mobility energy randomly. It is straightforward to associate an individual ‘mobility energy’ to the daily path lengths. The assumption that citizens organize their mobility as they own an internal ‘mobility energy’ agrees with similar hypotheses discussed by Kolb and Helbing [11] to explain the daily travel-time distributions for different transport modes.

In order to investigate the concept of the mobility energy together with the Maximum Entropy Principle, we consider the relation between the daily path length and the single-trip path length, building up the rank distribution of individual daily activities. To define an activity from GPS mobility data, we apply a clustering procedure to vehicle stop positions, identifying the positions which lie in a circle of diameter $\approx 500$ m (this is...
considered an acceptable distance between the true destination and the parking place [12]).
Moreover we have associated an activity when the elapsed time before the next trip is
greater than 15 min. In figure 3 we plot the rank distribution for the daily activities
together with an exponential interpolation
\[ p(k) \propto a^k \]  
that provides a very good fit of experimental observations. Equation (2) is consistent with
the assumption that, on average, the individuals behave as independent random particles
which define their daily agendas in a random way. According to previous hypotheses, it is
possible to compute in an analytical way the single-trip length distribution, as the
distribution realized by uniformly spreading \( k \) points into a given segment of length \( L \). A simple calculation provides the single-trip length distribution in the form (see the
appendix)
\[ p_N(x) = \frac{c}{L} \sum_{k=1}^{N} (k + 1)k a^k (1 - x/L)^{k-1} \]  
where \( c \) is a normalizing factor and \( N \) is the maximum number of daily activities; we
remark that the choice of the points in the segment is contextual without any time-
ordering. It is quite natural to assume that there should exist a correlation between the
number of daily activities \( N \) and the daily mobility length \( L \), but the GPS data do not
suggest any correlation function, so that we decided to use an effective daily length in the
theoretical distribution (3) to make a comparison with the empirical one. It turns out
that, if we exclude the very short paths, the curve (3) fits very well with the experimental

\[ y = 0.09 x^{-0.97} \]
4. Activity downtime distribution

Time is the second fundamental individual variable of human mobility, directly related to the dynamical realization of the daily agenda. From the GPS database in the Florence area we have computed the downtime spent in each daily activity by a fixed individual; we discard from the activities the sleeping time linked to circadian rhythms. The distribution of the activity downtimes of the recorded individuals is plotted in figure 5, recovering the well-known Benford’s law [13]. In order to give a microscopical interpretation of the empirical downtime distribution, we assume that individuals cannot determine a priori each activity downtime, because this is varied depending on unpredictable circumstances. According to this hypothesis, on average, each particle has a finite mobility time at its disposal to perform the desired activities, and he consumes the time in successive random choices up to the end of the whole mobility time. If one computes the interval distribution that is obtained by choosing successively \( k \) points in a given segment, one gets analytically Benford’s distribution [13]

\[
p(t) \propto \sum_{k=1}^{N} \frac{(\ln t)(k-1)}{(k-1)!} \approx \frac{1}{t}.
\]
Briefly, the statistical results of the monthly mobility in the Florence area, recorded by the GPS data on vehicles, suggest that the macroscopic average properties are the same as those of Boltzmann’s particles moving in a homogeneous space with an average energy and a finite time at their disposal. The energy introduces a global constraint in the individual daily mobility, whereas the time can be seen as a local constraint in the activity planning since it is consumed step by step.

However, computing the distribution of the total activity downtime for the monitored vehicles, we can get an idea of the typology of urban mobility described by our sample. The results are plotted in figure 6. We remark on the presence of two peaks: one is related to short downtime activities, which probably corresponds to a specific use of the vehicle for a single trip, whereas the second peak centered at \( \simeq 10 \) h denotes people performing a more complex mobility agenda, which contains the working activities. Finally the presence of a long queue for the daily activity downtime (\( \geq 12 \) h) is probably due to business vehicles in our sample, which are used by different people.

5. Activity degree distribution

Both the Boltzmann distribution for the mobility energy and Benford’s law for the activity downtime enroll stochastic features of the system, but they do not explain how such features can be related to the individual daily agenda, which are certainly the result of cognitive behaviors. In order to answer this question, we perform a statistical analysis of the downtime related to daily activities, considering the monthly degree \( k \) for the different individual activities (i.e. the number of times that a citizen repeats a certain activity during a month) [14]. Letting \( t \) the activity downtime, we introduce the joint probability \( p(t, k) \) to denote the probability of finding a \( k \)-degree activity associated to a
Figure 8. Empirical distributions of the $k$-degree activity downtime as a function of the normalized downtime $t/(t)_k$. The different symbols refer to the different activity degrees: $k = 3$ (circles), $k = 4$ (squares), $k = 5$ (rhombus), $k = 6$ (up triangles) and $k = 7$ (down triangles).

downtime $t$. Then by definition we have to recover Benford’s law (4) by summing over $k$:

$$\sum_k p(t, k) \propto \frac{1}{t}. \quad (5)$$

We also have the equality

$$p(t, k) = p(t \mid k)kp(k)$$

where $p(t \mid k)$ is the conditional probability for a downtime $t$ considering only the $k$-degree activities and $p(k)$ is the probability to detect a $k$ degree activity; the factor $k$ takes into account the multiplicity of the $k$ degree activities. The study of the conditional probability $p(t \mid k)$ can shed some light on understanding the mobility habits related to the use of private vehicles and to face the question of the relevance of repeated activities both in the mobility and in the use of time. Remarkably the experimental observation suggests the existence of a universal probability distribution $f(u)$ for the normalized downtime $t/(t)_k$:

$$p(t \mid k) = \frac{f(t/(t)_k)}{(t)_k} \quad (6)$$

where $(t)_k$ is the average downtime for the $k$-degree activities. We read this universal function as the signature of the fact that individuals organize their time, when performing a private car mobility, in a common way independently from the specific activity, i.e. the relative downtime fluctuations are the result of a stochastic universal mechanism. Moreover there should exist a common feature among the individuals, concerning how they manage the downtime related to the $k$-degree activities, since only the average value

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Figure 9. Empirical distributions of the $k$-degree activity downtime as a function of the normalized downtime $t/(\langle t \rangle_k)$. The different symbols refer to the different activity degrees: $k = 8$ (circles), $k = 9$ (squares), $k = 10$ (rhombus), $k = 11$ (up triangles) and $k = 12$ (down triangles).

$\langle t \rangle_k$ characterizes the $k$ dependence of the conditional probability $p(t | k)$. This universal character could be explained by thinking that the $t/(\langle t \rangle_k)$ variable is a generic ‘measure’ of the mobility actions, valid for every individual. More precisely, $t/(\langle t \rangle_k)$ can be considered the temporal norm for all the mobility related activities. From the empirical data we detect $\simeq 3 \times 10^5$ activity downtimes and we have computed the dependence of the average value $\langle t \rangle_k$ using the degree $k = 3, \ldots, 20$. It is evident from figure 7 that we have an almost linearly increasing behavior of $\langle t \rangle_k$ as the degree $k$ increases. This means the existence of a relation between the activity degree and the activity ‘use value’ (individual satisfaction, profit, etc.) that introduces an individual tendency to repeat and to spend time in the activities with a relevant added value [15]. A possible local interpolation of the empirical data is obtained by using the function (continuous line in figure 7)

$$\langle t \rangle_k \propto \exp(\gamma k^a)$$

where $a \simeq 0.3$ and $\gamma \simeq 0.7$. In figures 8–10 we plot the empirical probability densities for different degrees (from $k = 3$ to 20) to investigate the existence of the universal distribution $f(u)$. There is a decreasing of the data number as $k$ increases, but all the distributions are computed with a sample of the same order (from $4 \times 10^4$ to $1 \times 10^4$). The figures show three different features. There is a collapse of all the curves on a unique distribution: this is clear in figure 8 (the tail spread is consistent with statistical fluctuations) and in the first part of all the plotted distributions that contains the great majority of the data. All the distributions show a large contribution from the short-time activities and a fast decaying large tail for large $(t/(\langle t \rangle_k)) > 2$. There is a smooth rise of a ‘signal’ as $k$ increases, denoted by the appearance of two peaks at $t/(\langle t \rangle_k) \simeq 1$ and 3: this is clearly shown in the last figure 10. Therefore the empirical observation gives a strong indication for the existence of a universal distribution $f(u)$ for the normalized activity...
The empirical distributions of the $k$-degree activity downtime as a function of the normalized downtime $t/\langle t \rangle_k$. The different symbols refer to the different activity degrees: $k = 16$ (circles), $k = 17$ (squares), $k = 18$ (rhombus), $k = 19$ (up triangles) and $k = 20$ (down triangles).

Despite the occurrence of a large number of individuals with high degree activities ($k \geq 10$), some new features appear but with a small statistical weight. A possible interpolation of the distribution $f(u)$ is given by

$$f(u) \propto \frac{1}{u^{\alpha}} e^{-\alpha u} \quad (8)$$

where the coefficient $\alpha$ has a value $\approx 0.4$. The distribution (8) is singular at the origin so that the interpolation is certainly approximated at $u \to 0$ (see figure A.2 in the appendix). The exponential decay is the typical Boltzmann statistics as for the path length distribution, whereas the $u^{-1}$ behavior is consistent with Benford’s law for the downtime distribution. So this ‘universal distribution’ seems to mix both the features shown in figures 4 and 5. To explain the singular $f(u)$ trend at the origin, we guess that it can be a sign of a strong free-will in the individual behavior in the short-time activity range. Surely for a more precise justification further studies are required. We can use the interpolation (8) to extract the signal from the high degree activity distribution by computing the ratio between the empirical distribution data and the interpolation (8); in figure 11 we plot the ratio distribution results. As can be seen, the empirical data define two peaks centered at $t \approx 4$ h and at $t \approx 10$ that are common to all the distributions when $16 \leq k \leq 20$. Even if these peaks are not statistically relevant, they can be related to individuals that perform repeated activities linked to the canonical working time schedule. Clearly the working time schedule introduces further constraints to the individual mobility agenda which are not taken into account by the universal distribution $f(u)$. Now the existence of an universal distribution implies (see equation (6))

$$p(t, k) = f \left( \frac{t}{\langle t \rangle_k} \right) k \langle p(k) \rangle \langle t_k \rangle. \quad (9)$$

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Figure 11. Ratio between the empirical distributions of the $k$-degree activity downtime and the interpolation (8) as a function of time ($h$). The different symbols refer to the different activity degrees: $k = 16$ (circles), $k = 17$ (squares), $k = 18$ (rhombus), $k = 19$ (up triangles) and $k = 20$ (down triangles). Two peaks at a downtime $t \simeq 4$ h and at $t \simeq 10$ h that are common to all the distributions can be observed.

Then using the interpolation (7) and performing the change of variable $u(k) = t/\exp(\gamma k^a)$, we find that Benford’s law (5) implies a power law distribution for the activity degrees (see the appendix):

$$p(k) \simeq \frac{1}{k^{2-a}}.$$  

(10)

According to the estimate (7), we expect an exponent $\simeq -1.7$. In figure 12 we plot the empirical activity degree distribution with a numerical interpolation by a power law $k^{-b}$; the data provide $b \simeq 1.6$ which is consistent with the analytical estimate (10).

6. Conclusions

The citizens’ mobility is an interesting social phenomenon that involves a large number of ‘intelligent’ elementary components with a free-will individual property, so that we can speak of a cognitive dynamics. In this paper we analyze a lot of car mobility data for the Florence area, showing the emergence of three robust statistical laws for the path lengths, the activity downtime and degree. These laws can be explained as the direct consequence of some simple hypotheses as the particle independence, and above all assuming the existence of an individual mobility energy and of a finite individual time spent for the desired urban activities. Moreover, from the downtime distribution we deduce a universal function which fits with empirical observations and data. We think that this universal function can be interpreted as an indication of the cognitive time perception common to
all human beings, or at least surely common to car drivers. Finally the urban mobility is clearly complex, but our steady state statistics cannot point out the typical complex signatures such as, for instance, self-organized states. To detect complexity in mobility dynamics, it is necessary to investigate the transients, i.e. the states far from equilibrium.

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Appendix

Let us consider $k$ stochastic variables uniformly distributed in the unit segment. The probability that a segment of length $\leq x$ is empty can be estimated according to

$$P(\leq x) = 1 - (1 - x)^k.$$ 

As a consequence the probability density that a certain segment $x$ is empty is given by

$$p(x) = \frac{dP}{dx} = k(1 - x)^{k-1}.$$ 

Therefore if one chooses randomly an integer number $k$ in the interval $[1, N]$, the probability density for a segment of length $x$ conditioned by the choice $k$ is

$$p_k(x) \propto (k + 1)k(1 - x)^{k-1} \quad x \in [0, 1]$$

(A.1)
since we have to take into account $k + 1$ possible segments. The probability (A.1) has to be weighted by the probability $p(k) \propto a^k$ to have $k$ points (see equation (2)) so that the probability density to detect a segment of length $x$ for any choice $k$ is

$$p_N(x) = \frac{(1 - a)^2}{(2 - a)a(1 - a^N) - Na^{N+1}(1 - a)} \sum_{k=1}^{N} (k + 1)k a^k (1 - x)^{n-1}$$  \hspace{1cm} (A.2)

where we have introduced a normalizing factor. In figure A.1 we show the comparison between equation (A.2) and a Monte Carlo distribution with $N = 7$. The distribution (A.2) allows an analytical approach to the single-trip length distribution (cf. figure 4 in this paper).

The empirical conditional distributions $p(t \mid k)$ for different degrees $k$ as a function of the normalized activity downtime $t/\langle t \rangle_k$ suggest that the statistical relevance can be described according to

$$p(t \mid k) = \frac{f(t/\langle t \rangle_k)}{\langle t \rangle_k}$$  \hspace{1cm} (A.3)

where we introduce a universal function $f(t/\langle t \rangle_k)$ which can be interpolated by

$$f(u) \propto \frac{1}{u} e^{-\alpha u}.$$  \hspace{1cm} (A.4)

The results are shown in figure A.2 for the activity degrees $k = 3, \ldots, 20$.

There is a strict relation between the activity degree distribution (see figure 12 in the paper) and the existence of a universal distribution probability $f(u)$ in equation (A.3).
Figure A.2. Empirical distributions for the conditional probabilities $p(t | k)$ for activity degrees $k = 3, \ldots, 20$ as a function of the normalized activity downtime $t / \langle t \rangle_k$ (different symbols). The continuous line refers to an interpolation with the function (4).

Indeed, taking advantage of the dependence of $\langle t \rangle_k$ on the degree $k$ pointed out by experimental observations (see figure 7 in the paper) we perform the change of variables

$$t = t \quad u = t / \langle t \rangle_k$$

in the joint probability distribution $p(k, t)$ of degree and downtime (see equation (9)). Using the definition (5), we get the new distribution

$$p'(u, t) = f(u)k(u)p(k(u)) \frac{dk}{du} = -f(u)kp(k) \langle t \rangle_k \left( \frac{d\langle t \rangle_k}{dk} \right)^{-1}$$

where $k$ has to be read as $k(u)$ in the lhs and $p(k)$ is the activity degree distribution. In the previous formula we approximately interpolate the discrete variable $k$ with a continuous variable. By integrating $u$ we have to recover Benford’s law $\propto 1/t$ for the global activity downtime distribution (see figure 5 in the paper). Since $f(u)$ is normalized as the probability distribution, this is possible if

$$kp(k) \langle t \rangle_k \left( \frac{d\langle t \rangle_k}{dk} \right)^{-1} = \text{const.}$$

(A.6)

According to the interpolation $\langle t \rangle_k \propto \exp \gamma k^a$ of the experimental data as shown in figure 7 in the paper, we explicitly have

$$\frac{d\langle t \rangle_k}{dk} \propto k^{a-1}e^{\gamma k^a} \propto k^{a-1}\langle t \rangle_k$$

therefore the condition (A.6) is

$$k^{2-a}p(k) = \text{const.}$$
i.e. a power law distribution of the activity degree with exponent $\leq 2$. This is consistent with the experimental observations as shown by figure 12 in the paper.

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