The Fast Algorithm of Running Stability in Mechanical Analysis of High Speed Railway

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Abstract. The running stability of high-speed railway is the goal of operators and vehicle manufacturers, so the running stability of high-speed railway is an important part of vehicle dynamics analysis. The stability analysis involves a lot of working conditions and suspension parameters, and the analysis workload is huge. Therefore, in this paper, the random vibration and running stability of high-speed trains are taken as the research object, and the relevant fast algorithm of responding to the power spectrum of high-speed railway vehicles is derived through the virtual excitation method, so as to realize the running stability research of high-speed railway vehicles. In this paper, the vertical and horizontal response power spectrum and stability index of a high-speed train are analyzed. Finally, the stability index is used to analyze the dynamic performance of the train at the same time, and the line test results of the dynamic performance of the train are evaluated. The research results show that the simple and efficient analysis algorithm is helpful to improve the analysis efficiency and reduce the simulation calculation time.

Keywords: Running Stability, High-Speed Railway, Virtual Excitation Method, Response Power Spectrum

1. Introduction
With the development of high-speed trains, many countries have launched the research on the power of high-speed trains. Wu [1] proposed a new uncertain analysis method for vehicle dynamics with mixed uncertain parameters. Ling [2] reports a three-dimensional train track model that captures the flexible vibration characteristics of a high-speed train compartment based on a flexible multi-body dynamics approach. The finite element method and the multi-body dynamics (MD) method are used to model the flexible car body. The rigid body motion is obtained by MD theory, and the structural deformation is calculated by the finite element method and the modal superposition method. Gil gómez [3] built a more advanced vehicle model in VI-CarRealTime based on kinematics and flexibility data, measured values of shock absorbers and measured values of winter tires.

Based on multi-body dynamics software UM, a multi-body dynamics model of CRH2 vehicle was established by [4] CAO. The running stability of vehicle under different conditions was analyzed, and the influence of suspension parameters on running stability was studied.[5] An articulated monorail dynamic model together with a genetic algorithm is developed to optimise the curving dynamics of the...
articulated monorail vehicle by Jiang[6]. Focusing on safety, comfort and with an overall aim of the comprehensive improvement of a vision-based intelligent vehicle, a novel Advanced Emergency Braking System (AEBS) is proposed by Ugras [7] based on Nonlinear Model Predictive Algorithm. Yao [8] studies a multi-objective optimal design of three different frame vibration control configurations and compare their performances in improving the lateral stability of a high-speed train bogie. Liu [9] numerically explored the aerodynamic features of a train running through a rectangular windbreak transition region. Wang [10] studied the safety and stability of light rail train on multi span simple supported beam bridge, and experienced some settlement and beam deformation caused by foundation settlement and beam deflection / camber. The results show that under the condition of beam deformation, the speed is the main factor affecting the safety and stability of LRT.

The stability index is usually used to evaluate the dynamic performance of trains in China. The principle of virtual excitation is used to analyze the running stability of rail vehicles in this paper, and a fast algorithm of stability analysis is proposed, which transforms the stationary random vibration analysis into simple harmonic vibration analysis. Firstly, the virtual excitation analysis method of power spectrum analysis of rail vehicles is derived. The amplitude spectrum of system response is obtained by using the power spectrum of system response and the basic theory of spectrum analysis, and then the stability index can be obtained by the calculation method of stability. The vibration power spectrum, vibration response amplitude spectrum and Sperling stability index of a high-speed train are analyzed by using this method.

2. Method

2.1 Basic principle of virtual excitation method

When a linear system is excited by a single point stationary random excitation \( x(t) \) with a self power spectral density of \( S_{xx}(x) \), the self power spectrum of its response is:

\[
S_{yy}(\omega) = |H(\omega)|^2S_{xx}(x)
\]  

(1)

When the random excitation is replaced by the simple harmonic excitation \( e^{i\omega t} \), the corresponding simple harmonic response is \( y = H(\omega)e^{i\omega t} \). If we multiply the constant \( \sqrt{S_{xx}(x)} \) before the excitation \( e^{i\omega t} \) obviously, we construct a virtual excitation

\[
x(t) = \sqrt{S_{xx}(\omega)}e^{i\omega t}
\]  

(2)

Then its response should also be multiplied by the same constant. The corresponding dummy quantity of the corresponding random variable is represented by the variable with "*", and the conjugate variable of the corresponding variable is represented by the variable with "#":

\[
y * y = |y|^2 = |H(\omega)|^2S_{xx}(\omega) = S_{yy}(\omega)
\]  

(3)

There are simple and unified formulas for calculating self-power spectrum and cross power spectrum. As long as the relationship between response and excitation is linear, the virtual excitation method can be applied. No matter in the calculation of self-spectrum or cross spectrum, the virtual harmonic excitation factor \( e^{i\omega t} \) and its complex yoke \( e^{-i\omega t} \) always appear in pairs, and eventually multiply and cancel, which also reflects the non-time variation of self-spectrum and cross spectrum of stationary problems.

2.2 virtual excitation analysis method of rail vehicles

The basic algorithm of virtual excitation method is used to deduce the corresponding results on high-speed rail vehicles. For the rail vehicles running on the fixed track with multi-point input, when the system is excited by the multi-point hypothesis point out of phase stationary random excitation. The input of each point after the first input point can be regarded as a simple delay of the previous input point. Let each input component have the same form, but there is a constant factor of difference in action time, there is a time lag, then there are:
calculated different vehicle constant and evaluation comfort in many cases. The calculation of the vehicle constant is expressed by Equation (4):

$$
\begin{bmatrix}
F_1(t) \\
F_2(t) \\
\vdots \\
F_m(t)
\end{bmatrix} =
\begin{bmatrix}
a_1 F(t - t_1) \\
a_2 F(t - t_2) \\
\vdots \\
a_m F(t - t_m)
\end{bmatrix}
$$

(4)

Where $a_j (j = 1, 2, ..., m)$ is a real number, indicating the action intensity of each point. When the intensity of each input point is the same, let $a_j = 1$. $F(t)$ be the time history function of stationary random excitation, $t_j (j = 1, 2, ..., m)$ is a constant number, which is the excitation lag time at each excitation point. For high-speed railway vehicles, there is no time delay in the input of the first wheel set irregularity, set $t_1 = 0$. If $\{f(t)\}$ is regarded as a generalized single excitation, and if the self spectral density of $F$ is $S_{FF}(\omega)$, the corresponding virtual excitation is

$$
\tilde{F}(t) = \sqrt{S_{FF}(\omega)} e^{j\omega t}
$$

(5)

For the multi input rolling stock running on the fixed track, the input of each subsequent point is the simple delay of the first input point, and the input intensity of each point is the same, $a_j = 1, t_1 = 0$. The linear dynamic equation of the rolling stock is calculated and obtained:

$$
y = (-\omega^2 M + j\omega C + k)^{-1} (D_w + j\omega D_{dw}) e^{j\omega t = \{a_1 e^{-j\omega t_1}, a_2 e^{-j\omega t_2}, \ldots, a_m e^{-j\omega t_m}\} \sqrt{S_{FF}(\omega)} e^{j\omega t} = \tilde{Y} e^{j\omega t}
$$

(6)

$$
S_{yy}(\omega) = \tilde{Y} \tilde{Y}^T = \tilde{Y} \tilde{Y}^T
$$

Where: $\tilde{Y}$ and $\tilde{Y}^T$ are conjugate and transpose vectors of $\tilde{Y}$ respectively, and $S_{yy}(\omega)$ are power spectrum matrix of system response. When the vibration equation of vehicle system is established, the coordinate system is usually selected at the symmetrical position. When it is necessary to calculate the vibration at the asymmetrical position, $[S_{yy}] = [\Psi] [\tilde{Y}] [\tilde{Y}^T] = [\Psi] [\tilde{Y}] [\tilde{Y}^T] = [\Psi] [\tilde{Y}]$ the coordinate transformation matrix.

2.3 calculation method of stability index

Many different methods are used to evaluate the vibration and passenger comfort of railway vehicles in different countries. The research of evaluation method mainly includes two aspects: frequency weighting and comfort index application. Frequency weighting is only slightly different, while comfort index application is obviously different. According to the test results, Helberg and Sperling of Germany give the formula of $W_z$ and introduce the frequency correction term $f(f)$ to improve the evaluation accuracy, and separate the vertical vibration and the lateral vibration. At present, many countries still adopt it including China. The calculation formula of $W_z$ is as follows:

$$
W_z = 2.7^{10} \sqrt{a^3 f^5 F(f)} = 0.896^{10} \sqrt{a^3 f^5 F(f)}
$$

(7)

Where $W_z$ is the stability index, $a$ is the vibration acceleration, $f$ is the vibration frequency Hz, and $F(f)$ is the frequency correction coefficient. The above stability index is applicable to the constant amplitude vibration of a single frequency, but in fact, the vehicle vibration is random vibration. The acceleration measured from the vehicle body contains the whole natural frequency of vehicle vibration, which needs to be grouped according to frequency, and the stability index values of different accelerations in each frequency are calculated. Therefore, the total stability index is calculated by the following formula:

$$
W = (W_1^{10} + W_2^{10} + W_3^{10} + \ldots + W_6^{10})^{\frac{1}{10}}
$$

(8)

Where: $W_1, W_2, \ldots, W_6$ is to analyze the spectrum according to the frequency decomposition, and calculate the respective stability index according to the acceleration amplitude within each
frequency range. Table 1 is shown for specific evaluation criteria.

| Stability level | evaluate | ride index $W_x$ |
|-----------------|----------|------------------|
| Level 1         | excellent| <2.5             |
| Level 2         | Good     | 2.5~2.75         |
| Level 3         | qualified| 2.75~3.0         |

In this paper, starting from the running stability of high-speed trains, a fast algorithm based on the virtual excitation principle is proposed to analyze the stability of high-speed trains. The stationary random vibration analysis is transformed into simple harmonic vibration analysis and the stability index is obtained.

3. Construction of High-speed Railway Mechanical Analysis Model based on Fast Algorithm of Running Stability

Running stability is one of the important evaluation indexes of the dynamic performance of high-speed vehicles, which is directly related to the operation quality of high-speed trains. Aiming at the running stability, this paper analyzes the running stability of a high-speed train by using the virtual excitation method. When the dynamic model is established, the vertical and lateral dynamic models are established respectively.

3.1 Construction of vertical dynamic model

The vertical dynamic model of the car body is 24 degrees of freedom, including: 2 DOF (sinking and floating, nodding), 1 DOF *8 (sinking and floating) of suspension frame, 2 DOF *7 (sinking and floating, nodding) of suspension magnet, combing the left and right together. According to the track spectrum obtained by fitting the excited power spectrum $S_{FF}(\omega)$ in formula 6, when the running speed of the vehicle is 400 km / h, the sinking and floating of the vehicle body, the nodal and the movement of the vehicle body floor above the first frame at the end of the vehicle body are taken as the objects of investigation.

3.2 Building lateral dynamic model

In this paper, the lateral dynamic model of high-speed train is established. The lateral dynamic model degrees of freedom of the vehicle body are 55 DOF, including: 3 DOF * 1 vehicle body (traverse, shake, roll), 3 DOF* 8 suspension frame (traverse, shake, roll), 2DOF*14 guide magnet (traverse, shake).

The vertical and horizontal displacement power spectra are obtained by formula (4) to formula (7), and the acceleration power spectra are obtained by formulas. The corresponding RMS value can be obtained by integrating them. The time domain signal and amplitude spectrum of displacement or acceleration response can be obtained by inversion by using power spectrum.

Because the high-speed high interference spectrum and the high-speed low-interference spectrum are only different in amplitude and the other forms are identical, the displacement power spectrum curve and the acceleration power spectrum curve are only different in amplitude and shape under the same vehicle running speed. On the other hand, the same spatial wavelength has different excitation frequency for the system due to the different driving speed, and the faster the driving speed, the higher the excitation frequency. Therefore, the response spectrum with higher running speed "lengthens" when the abscissa direction is lower.

4. Relationship between the Wavelength of Track Spectrum and the Running Stability of Vehicles

Track irregularity is one of the main causes of high-speed train vibration. Two important parameters of track irregularity are wavelength and amplitude. The effect of the irregularity of different wave length and amplitude is different.
4.1 Analysis for the Vertical model
Select the vibration at the floor above the first frame as the inspection point, and the parameters determining the index are amplitude and frequency. Formula (9) and formula (10) are used to calculate the ride comfort index of the car body under a single wavelength of irregularity excitation and its proportion in the overall ride comfort index of the car body. Finally, the irregularity wavelength that has the greatest impact on the vehicle ride comfort is obtained. According to the power spectrum obtained by the calculation method in this paper, the stability index \( w = 1.89 \), the RMS value of displacement vibration is 1.75mm, and the RMS value of acceleration vibration is 0.276m/s\(^2\). Taking the vehicle running speed as a variable, we can also get that the vehicle stability index is always less than 2.2 and the vehicle tends to be stable.

4.2 Analysis for the lateral model
Select the vibration above the first floor from the car body centerline as the inspection point. According to the power spectrum obtained by the calculation method in this paper, the stability index \( w = 1.95 \), the RMS value of displacement vibration is 0.29mm, and the RMS value of acceleration vibration is 0.396m/s\(^2\). The vehicle's lateral stability index is excellent. The vehicle's stability index is always less than 2.5, and the vehicle's stability is excellent.

![Figure 1 Stationary sensitive excitation wavelength at all operating speeds](image)

4.3 Analysis and evaluation of test results
The power spectrum of theoretical analysis can be obtained simply and rapidly by using the virtual excitation method, and the stability index can be obtained only by making a simple analysis. The analysis results are as shown in Figure 1. Although the sensitive excitation frequency at different speeds in the transverse direction is not as clear and consistent as that in the vertical direction, the peak frequency is basically the same as that in the transverse direction, indicating that the higher the running speed of the vehicle, the longer the sensitive wavelength affecting the running stability of the vehicle. The parameters that determine the size of the Sperling index are amplitude and frequency. That is to say, the larger the index is, the greater the vibration amplitude or the frequency weight caused by the vibration occurring at the sensitive frequency

5. Conclusion
The virtual excitation method in this study can be used to obtain the running stability index of rail vehicles after a simple analysis. By analyzing the influence of a single wavelength in the track spectrum on the running stability of vehicles, the range of line sensitive wavelength affecting the running stability can be determined. This method has the characteristics of simple expression and simple operation for the linear model. In this paper, the virtual excitation analysis method of high-speed train is transformed into simple harmonic vibration analysis, which greatly simplifies the calculation steps and keeps the theoretical accuracy, but it is necessary to linearize the non-linear model to use this method, which to a certain extent affects the accuracy of the analysis due to the
non-linear relationship between the suspension system and wheel rail in the high-speed train.

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