Very Large Intermediate Breaking Scale
In The Gepner Three Generation Model

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Abstract

A detailed study of the intermediate symmetry breaking scale, via the renormalization group equations, for a three generation heterotic string model arising from the N=2 superconformal construction is reported. The numerical study shows that the model admits a very large intermediate breaking scale \( \gtrsim 1.0 \times 10^{16} \) GeV. The role of the gauge singlets in this model is studied, and it is found that these fields play a crucial role in determining the directions and the scale of the intermediate symmetry breaking. The importance of the mixing in generation space is also studied. The generation mixing terms are found to have special effects in the intermediate symmetry breaking. Remarkably these terms can produce some new Yukawa couplings (not present at the Planck scale) through loops. These couplings are in general very small compared to the ones with non-vanishing tree level values and thus offer a new mechanism to solve the lepton/quark mass hierarchy problem.
1. Introduction

Since the work of Green and Schwarz [1], the superstring theory has become the best, if not the unique, candidate for the theory of the fundamental interactions. The heterotic string [2] compactified on the Calabi-Yau manifolds [3] emerged as the first phenomenologically viable theory in which the internal degrees of freedom are given geometrical meaning. Here the ten dimensional manifold $M_{10}$ is compactified to $M_4 \times K_6$, where $M_4$ is the Minkowskian spacetime, and $K_6$ is a three dimensional Kähler manifold with vanishing first Chern class (the Calabi-Yau manifold). This programme provides us with an effective four dimensional field theory of $N = 1$ supergravity coupled to Yang-Mills theory with an $E_6 \times E_8$ gauge group [4]. The matter fields are in the fundamental 27 representations of $E_6$.

On the other hand, superstring theories may be viewed as theories on the two dimensional world sheet manifold whose properties are governed by two dimensional superconformal field theories. Here one does not need to adopt the geometrical point of view. From the four dimensional Minkowski spacetime point of view, one can equally well think of the theory as being constructed on $M_4 \times S$, where $S$ is taken to be the tensorial products of discrete series of $N = 2$ minimal superconformal field theories. According to Gepner [5], one can take as many copies of $N = 2$ models as one likes, subject to the condition that the total central charge of the tensorial products adds up to 9, to fulfill the anomaly cancellation requirement for the heterotic string.

The Gepner construction of four dimensional string theories is closely related to the Calabi-Yau models [6]. In particular, there is strong evidence that a superconformal construction corresponds to a mirror pair of Calabi-Yau models with the betti numbers $b_{11}$ and $b_{12}$ of the internal manifolds interchanged, i.e., the family in one model is actually the mirror family in the other and \textit{vis versa} [7]. Although most work done along this line is in so-called weighted complex projective manifolds of dimension four ($\text{WCP}^4$), and the Tian-Yau manifold [3], the most studied three generation Calabi-Yau model [8], has yet to be studied in superconformal field theory terms. There does appear, however, a three generation model which admits constructions both in terms of Calabi-Yau manifold [9] and in terms of Gepner’s language [10]. Hence, it is of interest to study this model from both
viewpoints.

What makes Calabi-Yau models distinguished from the Standard Model and Grand Unified Theories is that not only the number of chiral fermion generations but also the Yukawa couplings among them are determined by the topology of the internal Calabi-Yau manifold, though subject to some still unknown normalizations of the fields [11]. The Yukawa couplings among the 27’s are not renormalized by any $\sigma$-model loop corrections (hence are exact) due to a number of strong non-renormalization theorems [12]. The calculability of Yukawa couplings greatly improves the predictive power of the Calabi-Yau models compared to the Standard Model or the Grand Unified Theories in which one inserts Yukawa couplings by hand to produce the masses appropriate for leptons and quarks. Thus, even though there are still great problems in determining the true vacuum of the heterotic string, the low-energy spectrum and couplings are fixed once one chooses one’s favourite model.

What makes Gepner’s construction distinguished from Calabi-Yau models is that not only the Yukawa couplings among the 27’s of $E_6$, but also those among the $\overline{27}$’s of $E_6$ (corresponding to (1,1)-forms of Calabi-Yau models) are calculable [13]. Moreover, all the nonrenormalizable interactions as well as the $E_6$ singlet couplings and the Yukawa couplings among them and chiral fields, can be calculated, at least in principle, thanks to the intrinsic connection between the $N = 2$ minimal models [14] and the parafermion models [15], and in turn, the SU(2) WZW models [16] which has been studied in great detail [17]. The $E_6$ singlet couplings are less tractable in Calabi-Yau models because the $E_6$ singlets belong to $H^1(\text{End } T)$, the group of deformations of moduli, and its dimension differs depending on the manifold. $E_6$ singlets play a very important role in extracting low energy phenomenology [18]. As we shall see later, they also play a unique role in mediating the intermediate breaking of gauge symmetry in the Gepner/Schimmrigk model which we will study in this paper.

It should be emphasized that both these constructions or any other approaches to four dimensional string theories still share a big drawback with respect to supergravity theories in that the mechanism leading to the breaking of supersymmetry is not yet understood. This drawback strongly restricts the predictive power of the model.
This paper is organized as follows. In section 2, we briefly discuss the construction of the Gepner three generation model [10] and its Calabi-Yau counter construction as given by Schimmrigk [9], mainly to set up notations and conventions. Section 3 is devoted to the discussion of the renormalization group equations of the model and the intermediate breaking scale. Section 4 contains conclusions and possible future work that can be done. A note about our notation: in this paper, we shall not distinguish between the spinor and the scalar components in the same matter multiplet, e.g., \( L_8 \) denotes both spinor and scalar components of the eighth lepton generation.

2. Construction of the Model

Construction of the model

In constructing the three generation model, one starts by taking a tensorial product of one level 1 and three level 16 models from the discrete series of the \( N = 2 \) superconformal theories [10]. Each field in the \( 1 \times (E16)^3 \) model (where \( E \) means that we take \( E_7 \) invariant which exists for level 16 when constructing modular invariant partition function) consists of a product of four “atomic” fields which are primary fields of the \( N = 2 \) discrete series. These are labelled by

\[
\left( \begin{array}{cccc}
  l & q & s \\
  \bar{l} & \bar{q} & \bar{s}
\end{array} \right)_k,
\]

(1)

where \( k \) is the level, \( l (\bar{l}) \), the “isospin”, \( q (\bar{q}) \), the “magnetic” quantum number of the underlying left (right) \( SU(2) \) current algebra, and \( s (\bar{s}) \) denotes the sector: \( s (\bar{s}) = 0 \), NS sector; \( s (\bar{s}) = \pm 1 \), Ramond sector, and \( s (\bar{s}) = \pm 2 \) are the fields obtained by acting on NS or R states by the supercharges, \( G_{1/2}^\pm \), of the \( N = 2 \) superconformal algebra.

Each primary field in Eq. (1) at level \( k \) has a central charge \( c_k = 3k/(k+2) \). (Thus the \( 1 \times (E16)^3 \) model has a central charge 9, cancelling the anomaly.) The conformal dimension and \( U(1) \) charge are given by

\[
h = \frac{4(l+2) - \tilde{q}^2}{8(k+2)} + \frac{1}{8} s^2,
\]

(2)

\[
Q = -\frac{\tilde{q}}{k+2} + \frac{1}{2} s
\]

(3)

where \( \tilde{q} = q + s \).
The modular invariant partition function can be constructed, by using the $E_7$ invariant available at level 16, in a way that is consistent with $N = 1$ spacetime supersymmetry. One keeps only those composite states in tensorial products with odd integer $U(1)$ charge (the generalized GSO projection). The resultant model can be mapped onto a heterotic one.

The model so constructed has a very large discrete symmetry. This can be seen from the connection between the $N = 2$ discrete series at level $k$ and the $Z_k$ parafermion model — they differ from each other by a trivial scalar. This model has a $Z_{k+2}$ symmetry, which is just $Z_{18}$ for $k = 16$ with a $Z_2$ factor being trivial. Hence, each copy of $E_{16}$ field in the composite fields has a $Z_9$ symmetry.

The resultant model has 35 generations ($27$’s of $E_6$), 8 mirror generations ($27$’s of $E_6$) and 197 scalars (singlets of $E_6$), with a discrete symmetry group

$$\tilde{G} = S_3 \times G/Z_9', \quad (4)$$

where $Z_9'$ is the $Z_9$ subgroup of

$$G = Z_3 \times Z_9^3, \quad (5)$$
generated by the group element of $G$ carrying the $G$-charge $(1,1,1,1)$. $S_3$ is the permutation symmetry of three $E_{16}$ factors. This is a $35 - 8 = 27$ generation model. One can obtain a three generation model by “twisting” the model by a $Z_3 \times Z_3'$ subgroup of $\tilde{G}$, with $Z_3$ being the cyclic permutation subgroup of $S_3$ and $Z_3'$ generated by the $G$-charge $(0,3,6,0)$.

The above construction is related to a Calabi-Yau model due to Schimmrigk [9]. The Calabi-Yau manifold is defined by two polynomials in the ambient space $CP^3 \times CP^2$:

$$\sum_{i=0}^{3} z_i^3 = 0; \quad \sum_{i=1}^{3} z_i x_i^3 = 0 \quad (6)$$

where $z_i, i = (0,1,2,3)$, are the coordinates of $CP^3$, and $x_i, i = (1,2,3)$, are those of $CP^2$. This manifold has the Euler characteristic $\chi = -54$ and hence possesses 27 generations. As shown by Gepner, this model has exactly the same generation structure and discrete symmetry, Eq. (4), as the $1 \times E_{16}^3$ model constructed above. One can obtain a three
generation model by moding out two discrete groups $\tilde{Z}_3$ and $\tilde{Z}_3'$ of order three,

$\tilde{Z}_3 : \quad g : (z_0, z_1, z_2, z_3; x_1, x_2, x_3) \rightarrow (z_0, z_2, z_3, z_1; x_2, x_3, x_1);$

$\tilde{Z}_3' : \quad h : (z_0, z_1, z_2, z_3; x_1, x_2, x_3) \rightarrow (z_0, z_1, z_2; x_1, \alpha x_2, \alpha^2 x_3),$ (7)

where $\alpha^3 = 1$, $\alpha \neq 1$. Notice that $\tilde{Z}_3$ is freely acting, but $\tilde{Z}_3'$ leaves three tori

$$(z_0^3 + z_2^3 + z_3^3 = 0) \times (1, 0, 0);$$ (8)

$$(z_0^3 + z_1^3 + z_3^3 = 0) \times (0, 1, 0);$$

$$(z_0^3 + z_1^3 + z_2^3 = 0) \times (0, 0, 1)$$
invariant. A smooth Calabi-Yau manifold can be obtained by blowing up the invariant tori which will necessarily introduce new massless modes whose Yukawa couplings are not easy to calculate.

The blowing-up procedure is analogous to the twisting in superconformal construction. But the twisting is nothing more than assigning new boundary conditions on massless modes and hence does not create any essential difficulties for the calculation of correlation functions. This is another advantage of Gepner’s construction over Schimmrigk’s.

**Massless modes and couplings**

The gauge group $E_6$ can be broken down to $[SU(3)]^3$ via flux-breaking. As shown in [19], of two possible embeddings of $Z_3 \times Z_3'$ into $E_6$, depending on whether $Z_3$ or $Z_3'$ correspond to a trivial embedding. The one corresponding to the trivial embedding of $Z_3$ introduces large intrinsic CP violations [19] which we will not consider in this paper. Thus the trivial embedding of $Z_3'$ generates 9 lepton, 6 mirror lepton, 3 quark, 3 anti-quark and no mirror quark or mirror anti-quark generations [20, 19, 21], which, in terms of $SU(3)_C \times SU(3)_L \times SU(3)_R$ quantum numbers, can be represented by:

$$9L(1, 3, \bar{3}) \oplus 6 \bar{L}(1, \bar{3}, 3) \oplus 3Q(3, \bar{3}, 1) \oplus 3Q^c(\bar{3}, 1, 3)$$ (9)

where $L \oplus Q \oplus Q^c$ furnish the 27’s, and $\bar{L} \oplus \bar{Q} \oplus \bar{Q}^c$, the $\overline{27}$’s, of $E_6$. In addition to the chiral generations, there are 61 gauge singlets, $\phi_i$, where $i = 1, \ldots, 35$, correspond to the 35 moduli fields, and $i = 36, \ldots, 61$, to the remaining singlets. In terms of the SM quantum numbers, the nonets in Eq. (9) can be denoted by:

$$\left( L_{\nu}^i \right)_i = \begin{pmatrix} h_0^0 & h^0 & e^c \\ h^+ & h^0 & \nu^c \\ e & \nu & N \end{pmatrix}_{i}, \quad i = 1, \ldots, 9;$$ (10)
The cubic superpotential depends on the following Yukawa couplings:

\[
\lambda \left( \begin{array}{ccc} h^0 & h^+ & \bar{e} \\ h^- & h^0 & \bar{\nu} \\ e^c & \bar{\nu}^c & \bar{N} \end{array} \right), \quad i = 1, \ldots, 6;
\]

where \( a = 1, 2, 3 \), is the SU(3)\(_C\) index, \( l = (e, \nu) \), \( q^a = \left( \begin{array}{c} u^a \\ d^a \\ \rho^a \end{array} \right) \) \( H = (h^0, h^+), \) and \( H' = (h'^-, h'^0) \) are the lepton, quark and Higgs doublets, \( D^a, D^c_a \) are the color Higgs triplets of the SU(5) supersymmetric grand unified theory, \( N \) and \( \nu^c \) are SU(5) singlets while \( N \) is also an SO(10) singlet. Finally, the fields with a bar are the corresponding mirror fields.

The Yukawa couplings of this model have been calculated by many groups [22, 23, 24]. Here and in what follows, we use the results of [22] which provides the most exhaustive list of Yukawa couplings as well as possible non-zero nonrenormalizable interactions. Our notations for gauge singlets also follow that of [22]. The most general \([SU(3)]^3\) invariant cubic superpotential depends on the following Yukawa couplings:

\[
W_3 = \lambda^1_{ijk} \text{det}(Q_i Q_j Q_k) + \lambda^2_{ijk} \text{det}(Q^c_i Q^c_j Q^c_k)
\]

\[+ \lambda^3_{ijk} \text{det}(L_i L_j L_k) + \lambda^4_{ijk} \text{Tr}(Q_i L_j Q^c_k)
\]

\[+ \lambda_{ijk} \text{det}(\bar{L}_i \bar{L}_j \bar{L}_k) + \eta_{ijk} \phi_i \text{Tr}(L_j \bar{L}_k) + \rho_{ijk} (\phi_i \phi_j \phi_k),
\]

where in terms of the Standard Model particles in Eq. (10) – (13):

\[
\text{det}(QQQ) = \epsilon_{aa'a''} d^a u^{a'} D^{a''};
\]

\[
\text{det}(Q^c Q^c Q^c) = \epsilon^{aa'a''} d^c_a u^{c_{a'}} D^{c_{a''}};
\]

\[
\text{det}(LLL) = H^\lambda H^\lambda_\bar{\lambda} N + H^\lambda \nu^\lambda \bar{\nu}_{\bar{\lambda}} + H^\lambda e^c \bar{e}_{\lambda};
\]

\[
\text{Tr}(QLQ^c) = D^{a} N D^{c}_{a} + D^{a} e^{c}_{a} u^{c}_{a} + D^{a} \nu^{c}_{a} d^{c}_{a}
\]

\[+ q^{a\lambda} l_{\lambda} D^{c}_{a} + q^{a\lambda} H^\lambda u^{c}_{a} + q^{a\lambda} H'^{\lambda} d^{c}_{a},
\]

here \( \lambda \) is the SU(2)\(_l\) index, \( a, \) the SU(3)\(_c\) index and \( i, j, k, \) the generation index. It turns out that there are 23 \((27)^3\), 3 \((\bar{27})^3\), 25 \([1(27\bar{27})]\) and 110 \((1)^3\) non-vanishing Yukawa couplings, and the structure of \((1)^3\)couplings reveals that the moduli fields do not couple among themselves [22]. Yukawa couplings of type \((27)^3\) and \((\bar{27})^3\), together with those
of type $[1(2727)]$ which are relevant for our discussion in the following sections are listed in Table 1. Notice that, in Table 1, we also listed the values of these couplings at one loop level, the results from running the renormalization group equations to be discussed in Section 3. The potentially non-vanishing non-renormalizable interactions which are not forbidden by selection rules were also studied in Ref. [23].

3. Renormalization Group Equations and Large Intermediate Breaking Scale

If we are to make any predictions for low energy phenomena at the electroweak scale $M_{ew} \approx 100$ GeV, the gauge group $SU(3)_C \times SU(3)_L \times SU(3)_R$ must be broken to the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Moreover, supersymmetry must also be broken because of apparent asymmetry between bosons and fermions at the electroweak scale. The origin of the supersymmetry breaking remains a most challenging problem in elementary particle physics despite many efforts devoted to the issue. We do not attempt to make any conjecture about the solution of this problem in this paper. Thus we shall simply assume some mechanism exists to softly break supersymmetry at a very high scale close to that of compactification which is not much less than Plank scale, i.e., we assume that $M_{SUSY} \sim M_C \sim 2.4 \times 10^{18}$ GeV. Then the soft breaking of supersymmetry will introduce the standard soft breaking terms:

1. A common scalar mass for all spin zero modes;
2. A common mass for all gauginos;
3. Trilinear scalar couplings for all the Yukawa couplings.

To break the gauge symmetry, we adopt intermediate breaking mechanism by taking negative mass squared as the signal of the Higgs-like mechanism [25]. Thus, we want to run the renormalization group equations for (1) Gauge Couplings; (2) Yukawa Couplings; (3) Scalar Masses; (4) Gaugino Masses; (5) Trilinear Scalar Couplings, starting from the supersymmetry breaking scale down to some intermediate scale $M_I$ at which some scalar mass turns negative, and the gauge symmetry $[SU(3)]^3$ can be broken down to the standard model gauge group by allowing particles with $[SU(3)]^3$ quantum numbers (1,3,2) and (1,3,3), i.e., $\nu^c$ ($\bar{\nu}^c$) and $N$ ($\bar{N}$) in lepton (mirror-lepton) nonets, to grow large vacuum expectation values (VEV). After intermediate symmetry breaking, some zero modes will acquire superheavy masses, due to the large VEV growth along $\nu^c$ ($\bar{\nu}^c$) and $N$ ($\bar{N}$) direc-
tions, and hence will decouple from low energy spectrum. Therefore, it is possible to work out the low mass particles that will survive down to the electroweak scale and further running of the renormalization group equations down to the electroweak scale will give some definite predictions for the low energy phenomenology.

The renormalization group equations for SUSY with most general soft breaking terms and arbitrary gauge groups have been worked out at one and two loop levels by many authors [26, 27]. It is straightforward to calculate them for the model at hand. For example, the renormalization group equation for $\lambda_{123}$ at one loop level is:

$$\frac{d}{dt}\lambda_{123} = \frac{\lambda_{123}}{16\pi^2} \left\{ 12(\lambda_{123})^2 + 32(\lambda_{122})^2 + 32(\lambda_{133})^2 + 3(\lambda_{212})^2 + 6(\lambda_{123})^2 + 3(\lambda_{313})^2 + 3(\lambda_{222})^2 + 6(\lambda_{223})^2 + 6(\lambda_{432})^2 + 3(\lambda_{323})^2 + 6(\lambda_{333})^2 + 6(\lambda_{142})^2 + 6(\lambda_{143})^2 + 6(\lambda_{154})^2 + 6(\lambda_{162})^2 + 6(\lambda_{163})^2 + 6(\lambda_{172})^2 + 6(\lambda_{173})^2 - 8(gc^2 + gL^2) \right\},$$

(16)

where $t = \ln \frac{M}{M_c}$ and the notations for Yukawa couplings follow those of Eq. (14).

Some explanations about the coefficients in Eq. (16) are in order. Because supersymmetry is softly broken, the RG equations are dictated only by the possible wave function renormalizations which are governed by the renormalizable interactions. There are seven different types of Yukawa couplings as indicated by Eq. (14): $\lambda_{1ijk}$, $\lambda_{2ijk}$, $\lambda_{3ijk}$, $\lambda_{4ijk}$, $\bar{\lambda}_{ijk}$, $\eta_{ijk}$, and $\rho_{ijk}$. Thus, the wave function renormalization for, say, $Q_2^a$, will receive a contribution from, say, $\lambda_{123}$, with a coefficient determined by

$$(\lambda_{123})^2 \epsilon_{aa_2a_3} \epsilon_{ll_2l_3} \delta_{a_2a_2'} \delta_{a_3a_3'} \delta_{ll_2l_2'} \delta_{ll_3l_3'} \epsilon_{a'a'a_2'a_3'} \epsilon_{ll_2'l_2'l_3'l_3'} = 4(\lambda_{123})^2 \delta_{aa'} \delta_{ll'}. \quad (17)$$

Similarly, the contribution from, say, $\lambda_{4213}$ is

$$(\lambda_{4213})^2 (\delta_{aa_3} \delta_{a_3a'} \delta_{a_3'a'}) (\delta_{r_1r_3} \delta_{r_1' r_3'} \delta_{r_1' r_3'') (\delta_{ll_2} \delta_{ll_2'} \delta_{ll_3} \delta_{ll_3'} ) = 3(\lambda_{4213})^2 \delta_{aa'} \delta_{ll'}. \quad (18)$$

The coefficient $3 \times 4 = 12$ in front of $(\lambda_{123})^2$ in Eq. (16) arises because $\lambda_{123}$ receives contributions from the wave function renormalization of three fields, $Q_1$, $Q_2$ and $Q_3$. Analogously, $\eta_{ijk}$ will contribute a factor of 9 to the wave function renormalization of $\phi_i$ and a factor of 1 to that of $L_j$ or $\bar{L}_k$, respectively.
In our analysis of the intermediate scale, we have used the renormalization group equations at the one loop level, except for the gauge couplings where we have used RG equations at the two loop level (excluding the dependence on Yukawa couplings). The reason the more accurate treatment of the gauge couplings is needed arises from the fact that the SU(3)\textsubscript{C} coupling constant is not renormalized at the one loop level, \textit{i.e.}

\[
\frac{d}{dt} g_I = \left\{ \frac{1}{16\pi^2} a_I + \frac{1}{(16\pi^2)^2} \sum_J b_{IJ} g_J^2 \right\} g_I^3 ,
\]

where

\[
a_I = \begin{pmatrix} 0 \\ 18 \\ 18 \end{pmatrix} ; \quad b_{IJ} = \begin{pmatrix} 48 & 24 & 24 \\ 24 & 252 & 120 \\ 24 & 120 & 252 \end{pmatrix} ,
\]

and \( I, J = C, L, R \) are the indices for the color, left and right SU(3) subgroups. One also notices that \( g_L \) and \( g_R \) have the same values as the results of running the renormalization group equations, thus the Weinberg angle is \( \sin^2 \theta_w = 3/8 \) before the intermediate breaking, in accordance with the standard SU(5) grand unified theory.

One must be careful not to ignore tiny mixings among generations while running the renormalization group equations. Although the non-renormalization theorem guarantees that, aside from possible wave function renormalizations, there is no need for coupling constant or mass renormalizations, the potential mixings among generations can have tremendous effects on the running of the RG equations, and even the determination of the direction of VEV growth. In the presence of mixings, a scalar mass squared turning negative does not necessarily mean that we have symmetry breaking. One must diagonalize the scalar mass squared matrix in generation space and find the eigenvectors with negative eigenvalues of the scalar mass squared matrix. Only along these directions can scalars trigger the intermediate breaking by growing VEV’s. As it turns out, \textit{the mixing creates many new Yukawa couplings which would otherwise remain zero, and these new couplings are, in general, of order of} \( 10^{-2} \) \textit{and even} \( 10^{-4} \) \textit{compared to the usual ones which are} \( \mathcal{O}(1) \) (Table 1). \textit{The generation mixing may provide a natural avenue to solve the lepton/quark mass hierarchy problem}. Notice that, for most string theories in four dimensions, the

\* This idea is supported by a recently revived interest on possible \textit{ansatz} for fermion
presence of the generation mixing is a generic phenomenon\textsuperscript{**}.

In the Gepner three generation model, the structure of the Yukawa couplings reveals that there are many mixings occurring. For example, $\lambda_{3146}$ and $\lambda_{3156}$ give rise to a mixing between $L_4$ and $L_5$ (Figure 1(a)), and $\lambda_{3246}$ and $\lambda_{3267}$ produce a mixing between $L_4$ and $L_7$ (Figure 1(b)). A new coupling $\lambda_{3347}$ develops from this latter mixing (Figure 2). The renormalization group equation for $\lambda_{3347}$ is

$$\frac{d}{dt}\lambda_{3347} = \frac{1}{16\pi^2}(8\lambda_{3246}\lambda_{3267}\lambda_{3377} + \cdots), \quad (21)$$

where \cdots represents the many terms that are proportional to the many new couplings listed in Table 1 which have vanishing tree level values. One sees clearly that, even if one starts running this equation from the zero value for $\lambda_{3347}$, the nonzero values will develop from the first term on the right-hand side. In Table 1, we also list all the new $(27)^3$ couplings arising from mixing.

The scalar mass squared matrix takes a block diagonal symmetric form in generation space:

$$(M^2)_{ab} = (3 \times 3) \otimes (4 \times 4) \otimes (2 \times 2) \otimes (2 \times 2), \quad (22)$$

where the $(3 \times 3)$ matrix denotes the mixing among $L_1, L_2, L_3$; the $(4 \times 4)$ matrix represents that among $L_4, L_5, L_6$ and $L_7$; and the $(2 \times 2)$ matrices are those among $Q_2, Q_3$ and among $Q^c_2, Q^c_3$ which have the same form and values. In addition, these masses obey

$$(m^2)_{l_i} = (m^2)_{l_j}; \quad (m^2)_{l_4} = (m^2)_{l_5};$$

$$(m^2)_{l_4l_i} = (m^2)_{l_5l_i}, \quad \text{for} \quad i = 6, 7; \quad (23)$$

$$(M^2)_{q_iq_j} = (M^2)_{q^c_iq^c_j}, \quad \text{for} \quad i, j = 1, 2, 3.$$ 

Thus there are 14 independent mass parameters for leptons, 6 for mirror leptons and 4 for quarks and anti-quarks. Because there are far too many gauge singlets, we excluded all the mass matrices in SUSY GUTs [28]. The two parameters $\delta_u$ and $\delta_d$ in Eq. (8) of Ref. [28] arise actually from the generation mixing.

\textsuperscript{**} The treatment of the renormalization group equations for the Tian-Yau model in Ref. [29] ignored all the mixings. We have checked their results in the $\overline{27}$ sector and obtained the same results, but the breaking takes place along different directions when the mixings are present.
gauge singlets but $\phi_{45}$ and $\phi_{58}$ into the running of the RG equations, just to demonstrate their effects on RG equations. In the actual running, we also excluded the gaugino masses and the trilinear scalar couplings for simplicity.

Let’s briefly summarize our results for the running of the renormalization group equations. Our analysis was performed in two steps: first, the gauge singlets were ignored. The system then decouples into two sectors: the 27 and $\overline{27}$ sectors. We found that the symmetry breaking occurs at very high scales:

$$M_I \approx 4.6 \times 10^{16} \text{ (GeV)}$$ (24)

for the 27 sector, and

$$M_I \approx 1.7 \times 10^{16} \text{ (GeV)}$$ (25)

for the $\overline{27}$ sector. But the 27 sector breaking is along the wrong direction — the lower eigenvalue of the scalar mass squared between $Q_2$ and $Q_3$ (thus $Q^c_2$ and $Q^c_3$ also) turns negative before everything else*. Notice that the intermediate scale for 27 sector is about three times higher than that of $\overline{27}$ sector. Hence the VEV growth at intermediate scale will automatically be along quark or anti-quark sector, and this means that $SU(3)_C$ breaks — a phenomenological disaster.

A scan of the singlet [1(27$\overline{27}$)] interactions reveals that many gauge singlets couple to chiral fermions more strongly than the chiral fermions couple among themselves. Thus it is necessary to consider these couplings in the analysis. Another remarkable fact is that, due to the chosen embedding of the discrete group $Z_3 \times Z'_3$ into $E_6$, no mirror generations of quarks and anti-quarks come into play. It is therefore not totally unreasonable for gauge singlets to shift the symmetry breaking direction. For example, the gauge singlets $\phi_{45}$, $\phi_{58}$, $\phi_{57}$, $\phi_{60}$ and $\phi_{61}$ have the following interactions with leptons and mirror leptons:

$$\eta_{38\bar{7}4} = 15.018, \quad \eta_{45\bar{2}4} = 6.087,$$

$$\eta_{57\bar{6}1} = 5.230, \quad \eta_{60\bar{4}6} = 1.424, \quad \eta_{61\bar{5}5} = 1.424.$$ (26)

* This is in disagreement with the results obtained in Ref. [20] where a breaking along $L_3$ direction was found. However, this analysis ignored the contributions from the generation mixing we discussed before.
in the notations of Eq. (14). Hence, if one introduces two gauge singlets $\phi_{45}$ and $\phi_{58}$ into the running of the renormalization group equations, one can reasonably expect that the mass squared of the scalar component of $\bar{L}_4$ will turn negative first, and due to the absence of the mixing in the $\overline{27}$ sector and the C-even property of $\bar{L}_4$, $\bar{N}_4$ allows VEV growth and thus triggers the symmetry breaking.

There are still some subtleties arising due to the unknown origins of supersymmetry breaking. As mentioned earlier, we assume some mechanism to break the supersymmetry which in turn introduces a common mass for all the scalar particles. But because of the uncertainty involved in SUSY breaking, we may assign different masses to scalar particles belonging to different representations of the gauge group, i.e., the gauge singlets may acquire masses different from those of $27$ or $\overline{27}$ scalars. Therefore, in running the renormalization group equations, we kept all the gauge nonsinglet masses fixed, and gauge singlet masses was entered as a free parameter. Defining $R = m_{\text{singlet}}^2 / m_{\text{non-singlet}}^2$, we considered the range $0 < R \leq 100$. For $R \sim O(1)$, the gauge singlet mass turns negative rather rapidly, at a scale very close to the compactification scale

$$M_{\text{singlet}} \sim 2.35 \times 10^{18} \text{ GeV},$$  \hspace{1cm} (27)

because of the large values of $[1(27\overline{27})]$ couplings. For $R \geq 7$, the $\bar{L}_4$ mass squared turns negative at a scale of

$$M'_{\text{singlet}} \sim 2.19 \times 10^{18} \text{ GeV},$$  \hspace{1cm} (28)

For $R > 50$, the $\bar{L}_4$ mass squared turns negative at a scale larger than that of the case when $R \sim O(1)$. Of course, one may suspect the validity of the renormalization group equation approach since the scalar masses turn negative almost immediately due to the large value of $[1(27\overline{27})]$ couplings. We think that some non-perturbative techniques must be developed before one can properly deal with this kind of problems.

There exists the possibility that these singlets grow VEV’s first and give superheavy masses to some particles, and this may prevent us from running into the afore-mentioned problem of having the color SU(3) subgroup broken. In order to explore this possibility, we assume a four step scenario in which the gauge singlets grow VEV’s in the following order:
(i) $\phi_{58}$ grows VEV, then $L_7$ and $\bar{L}_4$ pair up and decouple;
(ii) $\phi_{57}$ grows VEV, then $L_6$ and $\bar{L}_1$ pair up and decouple;
(iii) $\phi_{60}$ grows VEV, then $L_4$ and $\bar{L}_6$ pair up and decouple;
(iv) $\phi_{61}$ grows VEV, then $L_5$ and $\bar{L}_5$ pair up and decouple.

Note that the last two steps may be interchanged. Our choice of having $\phi_{58}$ grow VEV rather than $\phi_{45}$ in (i) is because the survey on this model suggested that the possible light Higgs, after including the loop corrections, should necessarily lie in a direction along which the first three lepton generations mix [30]. Hence we do expect $L_4$, $L_5$, $L_6$ and $L_7$, but not $L_2$, to become superheavy and decouple. We also notice that in the scenario suggested above, four pairs of leptons and mirror leptons decouple. The four decoupled pairs are all C-even states (Sec. 4), except $\bar{L}_1$ which is C-odd. In view of the results we have already obtained, we can reasonably expect (i) – (iv) to happen at a very large scale $M_{\text{singlet}} \approx 5.0 \times 10^{17}$ GeV. After the decoupling of four generations of lepton-mirror lepton pairs, we are left with 5 lepton ($L_i$, $i = 1, 2, 3, 8, 9$), 2 mirror lepton ($\bar{L}_i$, $\bar{i} = 2, 3$), and 3 intact quark and conjugate quark generations. Of all 52 independent Yukawa couplings, there are only 15 survived (Table 1). The values of these Yukawa couplings in the second column in Table 1 are the values obtained from the running of the renormalization group equations when only two gauge singlets are involved, which give a rough picture about their sizes. We can run the renormalization group equations with these nonets and Yukawa couplings and it can be expected that the gauge symmetry breaking will take place at a scale, say, greater than $1.0 \times 10^{16}$ GeV. The low-energy spectrum and the proton stability will be studied in a subsequent paper.

4. Conclusions

We have analysed here the Gepner three generation heterotic string model, examining via the renormalization group equations the intermediate scale symmetry breaking. As expected, the model does demonstrate a very large intermediate breaking scale of $O(1.0 \times 10^{16}$ GeV) or larger. Actually, a very large intermediate scale may be expected to be a generic feature of the four dimensional effective field theory of heterotic string theory. This is because this class of models in general have a very large number of renormalizable Yukawa couplings which can rapidly turn a $(\text{mass})^2$ negative.
There are a few lessons that we have learned which we briefly discuss: In analysing the renormalization group equations of a coupled system as complex as the one we have studied and the one studied in [29], one must be very careful not to discard the presumably small mixing terms like those shown in Fig.(1). In fact, from our study of the Gepner three generation model, we conclude that these mixing terms in generation space play a very important role in extracting the phenomenological implications of the model. They can trigger the direction of scalar field VEV growth. They also offer the possibility of solving the generation mass hierarchy problem by producing additional very small Yukawa couplings among chiral families, which are zero at tree level but grow due to loop corrections. The origin of these is the presence of mixing among generations in the wavefunction renormalizations. We have also seen the importance of the gauge singlet couplings. We found that they provide almost a unique source to prevent the very early breaking of color gauge group in the Gepner three generation model.

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Tables and Figures

Table 1. Yukawa Couplings

This table contains all the Yukawa couplings we considered in the text when analyzing the renormalization group equations. The first and fourth columns are the couplings whose notations follow those of Eq. (14), the second and fifth columns are the tree level values of the corresponding couplings and the third and sixth are their one-loop level values. In obtaining these values, we have taken the following initial conditions: the common gauge coupling constant, \( g_0 = 0.70 \), the compactification scale, \( M_c = 2.4 \times 10^{18} \) GeV, and the gauge singlet and the gauge non-singlet mass ratio, \( R = 10.0 \). Then, it was found that \( M_1 \sim 1.91 \times 10^{18} \) GeV and the \( \bar{L}_4 \) mass turns negative first.

Figure Captions

Fig. 1(a) The mixing between \( L_4 \) and \( L_5 \).
Fig. 1(b) The mixing between \( L_4 \) and \( L_7 \).
Fig. 2 The new coupling \( \lambda^{3}_{347} \) arises from the mixing in Fig. 1(b).
| Coupling | Tree Value | One-loop Value | Coupling | Tree Value | One-loop Value |
|----------|------------|----------------|----------|------------|----------------|
| $\lambda_{122}^1$ | - | $-2.470 \times 10^{-3}$ | $\lambda_{122}^2$ | - | $-2.470 \times 10^{-3}$ |
| $\lambda_{122}^1$ | 0.654 | 0.629 | $\lambda_{123}^2$ | 0.654 | 0.629 |
| $\lambda_{133}^1$ | 0.537 | 0.511 | $\lambda_{133}^2$ | 0.537 | 0.511 |
| $\lambda_{144}^3$ | - | $-8.133 \times 10^{-4}$ | $\lambda_{155}^3$ | - | $-8.133 \times 10^{-4}$ |
| $\lambda_{146}^3$ | 0.577 | 0.559 | $\lambda_{156}^3$ | 0.577 | 0.559 |
| $\lambda_{147}^3$ | - | $1.330 \times 10^{-5}$ | $\lambda_{157}^3$ | - | $1.330 \times 10^{-5}$ |
| $\lambda_{145}^3$ | - | $-1.603 \times 10^{-3}$ | $\lambda_{166}^3$ | - | $-1.625 \times 10^{-3}$ |
| $\lambda_{167}^3$ | - | $-3.829 \times 10^{-3}$ | $\lambda_{177}^3$ | - | $1.543 \times 10^{-5}$ |
| $\lambda_{189}^3$ | - | $-1.868 \times 10^{-3}$ | $\lambda_{245}^3$ | - | $-2.605 \times 10^{-3}$ |
| $\lambda_{244}^3$ | - | $-6.438 \times 10^{-4}$ | $\lambda_{255}^3$ | - | $-6.438 \times 10^{-4}$ |
| $\lambda_{246}^3$ | 0.475 | 0.440 | $\lambda_{256}^3$ | 0.475 | 0.440 |
| $\lambda_{247}^3$ | - | $-9.075 \times 10^{-4}$ | $\lambda_{257}^3$ | - | $-9.075 \times 10^{-4}$ |
| $\lambda_{266}^3$ | - | $-1.286 \times 10^{-3}$ | $\lambda_{267}^3$ | 0.676 | 0.625 |
| $\lambda_{277}^3$ | - | $-8.628 \times 10^{-4}$ | $\lambda_{289}^3$ | 0.635 | 0.630 |
| $\lambda_{344}^3$ | - | $-2.460 \times 10^{-3}$ | $\lambda_{355}^3$ | - | $-2.460 \times 10^{-3}$ |
| $\lambda_{346}^3$ | - | $-1.888 \times 10^{-3}$ | $\lambda_{356}^3$ | - | $-1.888 \times 10^{-3}$ |
| $\lambda_{347}^3$ | - | $-3.198 \times 10^{-3}$ | $\lambda_{357}^3$ | - | $-3.198 \times 10^{-3}$ |
| $\lambda_{345}^3$ | 0.822 | 0.799 | $\lambda_{367}^3$ | - | $-1.039 \times 10^{-3}$ |
| $\lambda_{377}^3$ | 0.556 | 0.530 | $\lambda_{389}^3$ | - | $-1.038 \times 10^{-3}$ |
| $\lambda_{412}^4$ | - | $-6.040 \times 10^{-3}$ | $\lambda_{313}^4$ | - | $-5.408 \times 10^{-3}$ |
| $\lambda_{4213}^4$ | 0.577 | 0.560 | $\lambda_{312}^4$ | 0.577 | 0.560 |
| $\lambda_{4222}^4$ | 0.577 | 0.545 | $\lambda_{423}^4$ | 0.390 | 0.358 |
| $\lambda_{423}^4$ | - | $-5.189 \times 10^{-3}$ | $\lambda_{432}^4$ | - | $-5.189 \times 10^{-3}$ |
| $\lambda_{4232}^4$ | - | $-9.077 \times 10^{-4}$ | $\lambda_{433}^4$ | 1.054 | 1.005 |
| $\lambda_{4233}^4$ | - | $-3.925 \times 10^{-3}$ | $\lambda_{432}^4$ | - | $-3.925 \times 10^{-3}$ |
| $\lambda_{4241}^4$ | 0.577 | 0.564 | $\lambda_{425}^4$ | 0.577 | 0.564 |
| $\lambda_{4241}^4$ | - | $-3.526 \times 10^{-3}$ | $\lambda_{4152}^4$ | - | $-3.526 \times 10^{-3}$ |
| $\lambda_{4243}^4$ | - | $-4.630 \times 10^{-3}$ | $\lambda_{4351}^4$ | - | $-4.630 \times 10^{-3}$ |
| $\lambda_{4341}^4$ | 0.475 | 0.457 | $\lambda_{4153}^4$ | 0.475 | 0.457 |
| $\lambda_{4162}^4$ | - | $-3.601 \times 10^{-3}$ | $\lambda_{4261}^4$ | - | $-3.601 \times 10^{-3}$ |
| $\lambda_{4163}^4$ | 0.740 | 0.711 | $\lambda_{4361}^4$ | 0.740 | 0.711 |
| $\lambda_{4172}^4$ | - | $-9.608 \times 10^{-4}$ | $\lambda_{4271}^4$ | - | $-9.608 \times 10^{-4}$ |
| $\lambda_{4173}^4$ | - | $-7.709 \times 10^{-4}$ | $\lambda_{4371}^4$ | - | $-7.709 \times 10^{-4}$ |
| $\lambda_{134}$ | 1.153 | 1.072 | $\lambda_{244}$ | 0.556 | 0.490 |
| $\lambda_{256}$ | 0.822 | 0.812 | $\eta_{5414}$ | - | $-1.297 \times 10^{-2}$ |
| $\eta_{4524}$ | 6.081 | 3.804 | $\eta_{4534}$ | - | $-7.212 \times 10^{-3}$ |
| $\eta_{5844}$ | - | $-4.177 \times 10^{-3}$ | $\eta_{5854}$ | - | $-4.177 \times 10^{-3}$ |
| $\eta_{5864}$ | - | $3.803 \times 10^{-5}$ | $\eta_{5874}$ | 2.761 | 2.375 |