CAN LISA RESOLVE DISTANCE TO THE LARGE MAGELLANIC CLOUD?  

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ABSTRACT  

The Laser Interferometer Space Antenna (LISA) is expected to detect $N \sim 22 \times 10^{\pm 1}$ close white dwarf binaries in the Large Magellanic Cloud (LMC) through their gravitational radiation with signal-to-noise ratios greater than $\sim 10$ in observational durations of 3 years or more. In addition to chirp mass, location on the sky, and other binary parameters, the distance to each binary is an independent parameter that can be extracted from an analysis of gravitational waves from these binaries. Using a sample of binaries, one can establish the mean distance to the LMC as well as the variance of this distance. Assuming no confusion noise at frequencies above 2 mHz, LISA might determine the LMC distance to $\sim 4.5 \sqrt{N/22}{\%}$ and the line of sight extent of LMC to $\sim 15(N/22)^{1/4}{\%}$, relative to its distance, at the one-sigma confidence. These estimates are competitive to some of the proposed direct geometric techniques to measure LMC distance in future with missions such as SIM and GAIA.  

Subject headings: gravitational waves — gravitation — binaries

1. INTRODUCTION  

The distance to the Large Magellanic Cloud (LMC) is the first-step in the extragalactic distance scale. Its distance has been estimated using over 80 different techniques, among which red clump stars, Cepheids, RR Lyrae stars, cluster main sequence fitting, and eclipsing binaries are the well-known (see, a summary in Benedict et al. 2002). While a combined average of recent results from selected “best” techniques indicate a distance modulus of $18.50 \pm 0.02$ (Alves 2003) or a few percent accurate distance to LMC, no single technique has reached the precision to be a reliable determination on its own. When considered as a whole, various techniques suggest the existence of two distinct distance scales: a “short-scale” with a modulus at the low end below 18.5, and a “long-scale” above this mean (Jensen et al. 2003).

The HST Key Project calibrated the Cepheids to a LMC distance modulus of 18.5 with a systematic uncertainty of 0.13 mag (Freedman et al. 2001). However, it is suggested that the measurement scatter would be larger than 0.13 mag even for best results from techniques that give a modulus close to 18.5 (Jensen et al. 2003). While the uncertainty to the LMC distance can be reduced with better data and with elimination of systematic effects, it is still useful to consider new methods that have the potential to make significant improvements in the future and are less affected by systematics and uncertain calibrations.

The biggest improvement is expected for methods that can produce a direct geometric distance measurement to LMC without any need to cross-calibrate against techniques that depend on the distance ladder. Several examples are discussed in Gould (2000) and involve the trigonometric parallax and estimates based on kinematic arguments. The LMC parallax, at the level of 20 $\mu$as, can be established with the Space Interferometer Mission (SIM) to an accuracy of 2 $\mu$as to 8 $\mu$as depending on the number of repeated observations. This leads to, at best, a 10% distance determination to the LMC. Among the kinematic method, the geometric distance based on the light travel time across the SN 1987A ring is now well known (Panagia et al. 1991), but differences at the 10% level in distance still remain on the interpretation of the same data (Gould 1995; Somebome et al. 1996). The second kinematic method based on the radial velocity gradient technique (e.g., Detweiler et al. 1984) requires measurements of the LMC radial velocity field, its mean proper motion, and the position angle of LMC photometric nodes (Gould 2000). The limitation remains with the LMC proper motion measurement. While an estimate accurate to 2% was expected with the Full-Sky Astrometric Mapping Explorer (FAME), due to the cancellation of this project by NASA, the next opportunity relies on the launch of GAIA.$^1$

Around the same time scale as results from GAIA would be available, there is another technique to establish the distance to LMC directly. This technique involves the analysis of gravitational waves from a sample of close white dwarf binaries (CWDBs) in LMC using observations with the Laser Interferometer Space Antenna (LISA)$^2$ mission. As studied in the literature (Hils et al. 1990; Nelemans et al. 2001), the LISA’s frequency coverage between $10^{-1}$ Hz and $10^{-4}$ Hz is ideal for a direct detection of CWDBs in our galaxy, in addition to a large number of possible extragalactic binary sources such as merging massive black holes (Hughes 2002). While a variety of Galactic binaries could be expected, including those involving neutron stars, solar-mass scale black holes or some interacting binaries, we will only consider the sample related to binary white dwarfs as these are expected to be the major fraction of GW emitting binaries with relatively large amplitude and simple evolution.

At frequencies above $\sim 2$ mHz, most CWDBs will be spectrally resolved (Cornish & Larson 2003), and one expects a detection of $\sim 2200 \times 10^{d1/2} (f/2$ mHz)$^{-8/3}$ galactic binaries (e.g., Hils et al. 1990; Bender & Hils 1997; Seto 2002 and references therein) from the disk alone. Note that there is an order of magnitude uncertainty in the ex-

$^1$http://astro.estec.esa.nl/GAIA  
$^2$http://lisa.jpl.nasa.gov
pected number of binaries in either direction (Bender & Hils 1997). Scaling to a LMC mass of \((8.7 \pm 4.3) \times 10^9 M_\odot\) (van der Marel et al. 2002), assuming a mass ratio of 1\% relative to the Milky Way \((\sim 5 \times 10^{11} M_\odot;\) Kochanek 1996), we expect \(\sim 22\) CWDBs in the LMC to be resolved by LISA above 2 mHz. This number is again uncertain by an order of magnitude and the fraction relative to Milky Way is further complicated by the fact that the star-formation histories of the two disks are different. While the current LMC star-formation rate is a factor of 2 to 5 higher than the Milky Way, it is unlikely that this will substantially increase the number of CWDBs in LMC relative to the Galaxy as the binary formation traces the past history with a time-lag of order a few Gyrs or more.

The gravitational waves from each binary allow one to establish certain parameters related to that binary, such as the location, chirp mass, distance, period, and the orientation or the direction of the binary with respect to LISA. Here, we focus on the radial distance measurement, but our analysis considers measurement of all these parameters from the data streams as these parameters are correlated with each other. In the case of the Galactic structure, Ioka et. al. (1998) performed an analysis similar to the current approach.

The discussion is organized as following: In the next section, we consider the distance measurement with LMC CWDBs in the LISA data stream and how it can be applied in the context of general LMC studies. While with LISA distance information can be directly extracted from the gravitational-wave signal, the location information on the sky one can obtain is limited due to relatively small signal-to-noise ratios (SNRs). This could result in a substantial uncertainty in locating each binary within the LMC. It would, however, still be easily possible to distinguish LMC binaries from Galactic ones through information on the estimated distances, as the Galactic scale \(~10\) kpc is much smaller than the distance to LMC \(~50\) kpc. In addition to the mean distance estimated from identified CWDBs in LMC, one can also determine the radial line-of-sight extent of LMC. The line-of-sight distance through LMC is needed when interpreting microlensing observations (e.g., Mancini et al. 2004) and previous modeling shows the existence of a number of Cepheids both behind and front of the main disk at distances \(>7\) kpc (Nikolaev et al. 2004). Given that one expects LMC to be tidally disrupted, three-dimensional information is also useful when modeling the origin and nature of Magellanic clouds in general (e.g., Bekki & Chiba 2005). We conclude with a summary in \(\S\) 3.

2. LISA STUDIES OF LMC

First, we will review the CWDB detections with LISA and the concentrate on the distance measurement related to LMC.

A CWDB is expected to have a circular orbit due to tidal interaction in its early evolutionary stage. The characteristic amplitude of its gravitational waves is given by (Thorne 1987)

\[
A = 2 \frac{G^{5/3}}{c^3} \left( \frac{\pi f}{3} \right)^{2/3} M_c^{5/3}
\]

(1)

where \(D\) is the distance to the source, and \(M_c\) is the chirp mass defined by \(M_c = (M_1 M_2)^{3/5}(M_1 + M_2)^{-1/5}\) with two masses \(M_1\) and \(M_2\) of the binary. We can also estimate the chirp mass through observation of the time evolution of the frequency \(\dot{f}\) as it is given by

\[
\dot{f} = \frac{96\pi^{8/3} G_{5/3}}{5c^5} f^{11/3} M_c^{5/3}
\]

(2)

\[
= 7.9 \times 10^{-19} \left( \frac{f}{10^{-3}\text{Hz}} \right)^{11/3} \left( \frac{M_c}{0.3 M_\odot} \right)^{5/3} \text{sec}^{-2}.
\]

The resolution \(\Delta \dot{f}\) depends strongly on the observational period \(T_{\text{obs}}\) as \(\Delta \dot{f} \propto T_{\text{obs}}^{-5/2}\). From eqs.(1) and (3) we can estimate the distance \(D\) from the measured amplitude \(A\), frequency \(f\), and its time derivative \(\dot{f}\) as

\[
D = \frac{5c\dot{f}}{48\pi^2 A f^3}.
\]

(3)

This important fact related to the distance measurement was pointed out by Schutz in 1986 (Schutz 1986). In the case of an adequate sample of GW emitting binaries in the LMC, one can determine both the mean distance as well as the width.

To evaluate the estimation error for distance of each binary with LISA, we have to determine at least 8 parameters (including, \(A, f\) and \(\dot{f}\)) concurrently from the LISA data stream that is affected by the complicated motions of three LISA satellites. These parameters are the direction of the binary (two parameters), orientation of the binary (two parameters) and a phase constant. We refer the reader to the literature (Cutler 1998; Takahashi & Seto 2002; Rubbo et al. 2004; Królak et al. 2004; Vecchio & Wickham 2004) for detailed studies related to parameter estimation of nearly monochromatic binaries.

In this paper we used a code based on Seto (2004) that was originally written for super massive black hole binaries. In this code, the parameter estimation errors are evaluated with the Fisher matrix approach for LISA's three orthogonal data streams used for a data analysis technique based on the time delay interferometry (TDI; Armstrong et al. 1999; Prince et al. 2002). This code includes the complicated effects caused by the finiteness of the arm-length of the detectors (see, Seto 2002; Cornish & Rubbo 2003; Rubbo et al. 2004; Vecchio & Wickham 2004). For the instrumental noise curve of LISA, we use the standard values given in Prince et al. (2002) and do not include the binary Galactic confusion noise as we are dealing with binaries at \(f \geq 2\) mHz. In the last part of this section, we will return to the issue of confusion noise and will discuss its role in changing our basic results.

To obtain how well LISA can establish distance from a sample of CWDBs at the distance to LMC, taken to be 50 kpc, we performed Monte-Carlo analysis at five frequencies between 2.5 mHz and 6.5 mHz with each frequency containing a sample of 100 binaries with fixed chirp mass at 0.3 \(M_\odot\). If we normalize the total number of binaries to be 22, we expect 15 binaries at 2.5 mHz, 4 at 3.5 mHz, 2 at 4.5 mHz, and 1 at 5.5 mHz.
In Fig. 1, we summarize our results with respect to the distance measurement and signal detection. With a one year integration, we have a typical SNR value of $\lesssim 10$ and it would not be easy to detect CWDBs in the LMC. Furthermore, the distance estimation error is considerably large. In the top panel of the left figure, the distance error is given in a relative form $\sigma_D/D$ with respect to the mean distance $D$. For one year data, this ratio is larger than 1 and no constraints on the distance possible. This 1 year result should be regarded as a reference given that the Fisher matrix approach we use here is based on the linear response of the fitting parameters to the data. The error $\sigma_D/D$ becomes significantly smaller with $T_{\text{obs}} = 3\, \text{yr}$. This is partly due to the decrease in the correlation between parameters when $T_{\text{obs}} \gtrsim 2\, \text{yr}$, and partly due to the rapid improvement on the estimation of $\dot{f}$ as a function of $T_{\text{obs}}$ (Takahashi & Seto 2002).

In the right panel of figure 1, the histogram of the distribution of the distance estimation errors are given for 100 binaries with $T_{\text{obs}} = 10\, \text{yr}$ at two specific frequencies. We can observe a tail at large $\sigma_D/D$. It is made by nearly face-on binaries. In this highly symmetric configuration, it is difficult to determine the two parameters for the direction of the angular momentum accurately. These parameters have a large correlation with the GW amplitude $A$ under the Fisher matrix formalism (Takahashi & Seto 2002), though the SNRs for binaries at a given location become maximum at face-on configuration.

For each CWDB in the LMC, we determine the distance error $\sigma_{D,i}$. Under the hypothesis that all binaries are at the same distance, we estimate the variance on this distance as $\sigma_D^2 = \sum 1/\sigma_{D,i}^2$. To quote final errors, we renormalize the whole binary sample in our Monte Carlo calculation to an average number of LMC CWDBs of 22. Following this procedure, on average, we find that the mean distance can be established to $4.5\sqrt{N/22}$% at the one-sigma confidence level. The measurement is competitive with some of the suggested techniques to establish the distance scale to LMC based on a single technique and is likely to be at the same level as the one based on the radial velocity gradient technique using GAIA’s proper motion estimate. Incidentally, in addition to the mean distance, we can also determine the line-of-sight width across LMC based on our sample of CWDBs. This involves estimating the excess variance of the distance estimates. To simplify the procedure, we ignore complications resulting from the true three-dimensional structure of LMC and make use of the hypothesis that the width is zero. We estimate the standard deviation on the distance errors and quote this as the uncertainty to which the thickness, $\Delta D$, can be measured. The error is calculated following $\sigma_{\Delta D} = \sum 2/\sigma_{D,i}^2$. For the same sample of 22 CWDBs, we find the error on the width to be at the level of $15\sqrt{N/22}0.25$% at the one-sigma confidence level relative to the mean distance of LMC. With the mean distance at 50 kpc, this amounts to establishing the thickness at 7.5 kpc.
While the mean distance is useful to establish the cosmic distance scale, the thickness addresses an important cosmological problem. While several microlensing surveys have monitored LMC, the observational data, with a measured optical depth of $12^{+3}_{-2} \times 10^{-8}$ (Alcock et al. 2000), is inconsistent with various model expectations (see, for example, Sahu 2003) which indicate a lower optical depth. The differences can be reconciled if there is a significant stellar populations either in the background or foreground of LMC such that self-lensing, lensing of LMC stars by sources within LMC, become important. While there is no convincing evidence for such massive structures based on the data so far (see, van der Marel 2004), this possibility is still not ruled out. The three dimensional analysis of LMC by Nikolaev et al. (2004) suggest the presence of Cepheids both behind and in front of the main LMC disk at distances excess of 7 kpc. The width that can be determined from CWDBs in the LISA data is comparable to such a line of sight extent. Thus, LISA data may play a crucial role in further understanding the three-dimensional structure of LMC, especially if the sample of CWDBs detectable with LISA were to be higher than our estimate.

In addition to LMC, LISA data can also be used to constrain the structure of SMC. With a mass of $3 \times 10^5 \, M_\odot$ (Gardiner & Noguchi 1996), we renormalize to a binary fraction that is a third of LMC. Scaling our numbers to SMC distance of $\sim 60$ kpc with $\sim 7$ binaries, we find that the mean distance can be established to $\sim 9\%$ and the line of sight extent of SMC to the level of $\sim 30$ kpc, both at the one-sigma level. These constraints may allow one to study the complex SMC line of sight structures where Cepheids have been observed across a line-of-sight depth of $\sim 30$ kpc (Mathewson et al. 1986).

So far we have not included the effects of the astrophysical confusion noise. The binaries that mainly contribute to determine the mean distance or the width is relatively at high frequencies with $f > 4.5\text{mHz}$. Therefore, our result would not be significantly affected by the Galactic white-dwarf binary confusion noise (Nelemans et al. 2001). However, we note that the effective noise level at these high frequencies could increase by a factor $\sim 1.5$ due to the fitting residual of resolved binaries for certain model parameters (see e.g. figure 6 in Barack & Cutler 2004).

There is another source of confusion noise that is worth mentioning here. This is the cosmological GW background made by inspiral waves from extreme mass ratio binaries with systems made by compact objects (with mass $\sim 1M_\odot$) and a supermassive black holes. The gravitational wave signals from such binaries are highly complicated due to the effects of strong gravity though the amplitudes are weak as these binaries are found at cosmological distances. Barack & Cutler (2004) pointed out that such binaries, however, may contribute to an additional confusion noise and could increase the effective noise level of LISA, even in the high frequency range around 5 mHz. At present, there are large uncertainties in calculating the amplitude of the background made by the extreme mass-ratio binaries. If this background is fairly large, the detection of LMC binaries themselves could be impacted given that these binaries are detected with marginal SNRs ($\sim 10$) in the presence of no confusion.

3. SUMMARY

The Laser Interferometer Space Antenna (LISA) is expected to detect a few to few hundreds close white dwarf binaries in the Large Magellanic Cloud (LMC) through their gravitational radiation. The distance to LMC is an independent parameter that can be extracted from an analysis of gravitational waves from these binaries. Taking a reasonable estimate on the number of CWDBs that can be resolved with LISA above a gravitational wave frequency of $2 \text{mHz}$ to be $\sim 20$, we find that LISA might determine the LMC mean distance to $\sim 4.5\%$ and the line of sight extent of LMC to the level of 7.0 kpc, both at the one-sigma level.

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REFERENCES

Alcock, C., et al. 2000, ApJ, 542, 281
Alves, D. R. 2003, New Astron. Rev., 48, 659
Armstrong, J. W., Estabrook, F. B., & Tinto, M. 1999, ApJ, 527, 814
Barack, L., & Cutler, C. 2004, Phys. Rev. D, 70, 122002
Bekki, K. & Chiba, M. 2005, MNRAS, 356, 680.
Bender, P. L., & Hils, D. 1997, Classical and Quantum Gravity, 14, 439
Benedit, G. F. et al. 2002, AJ, 123, 473
Cornish, N. J., & Larson, S. L. 2003, Phys. Rev. D, 67, 103001
Cornish, N. J., & Rubbo, L. J. 2003, Phys. Rev. D, 67, 022001
Cutler, C. 1998, Phys. Rev. D, 49, 2658
Detweiler, H. L., Yoss, K. M., Radick, R. R. & Becker, S. A. 1984, AJ, 89, 1058
Freedman, W. L., et al. 2001, ApJ, 553, 47
Gardiner, L. T., & Noguchi, M. 1996, MNRAS, 278, 191
Gould, A., 1995, ApJ, 452, 189
Gould, A., 2000, ApJ, 526, 156
Hils, D., Bender, P. L., & Webbink, R. F. 1990, ApJ, 360, 75
Hughes, S. A. 2002, MNRAS, 331, 805
Ioka, K., Tanaka, T., & Nakamura, T. 2000, ApJ, 528, 51
Jensen, J. B., Tonry, J. L., & Blakeslee, J. P. 2003, in Measuring the Dark Universe: Matter, Energy, and Gravity*, ed. M. Livio (Cambridge Univ. Press), astro-ph/0302325
Kochanek, C. S., 1996, AJ, 112, 248
Królak, A., Tinto, M., & Vallisneri, M. 2004, Phys. Rev. D, 70, 022003
Mancini, L., Calchi Novati, S., Jetzer, P., Scarletta, G., 2004, A&A, 427, 61
Mathewson, D. S., Ford, V. L., & Visvanathan, N. 1986, ApJ, 301, 664
Nelemans, G., Yungelson, L. R., Portegies Zwart, S. F., Verbunt, F., 2001, A&A, 365, 491
Nikolaev, S., Drake, A. J., Keller, S. C. et al. 2004, ApJ, 601, 260
Panagia, N., Gilmozzi, R., Macchetto, F., Adorf, H.-M. & Kirshner, R. P. 1991, ApJ, 380, L23
Prince, T. A., Tinto, M., Larson, S. L., & Armstrong, J. W. 2002, Phys. Rev. D, 66, 122002
Rubbo, L. J., Cornish, N. J., & Poujade, O. 2004, Phys. Rev. D, 69, 082003
Sahu, K. C. 2003, in "ark Universe: Matter, Energy, and Gravity", ed. M. Livio (Cambridge Univ. Press), astro-ph/0303235
Schutz, B. F. 1986, Nature, 323, 310
Seto, N. 2002, MNRAS, 333, 409
Seto, N. 2002, Phys. Rev. D, 66, 122001
Seto, N. 2004, Phys. Rev. D, 69, 022002
Sonneborn, G., Claes, F., Lundqvist, P., et al. 1996, ApJ, 477, 848
Takahashi, R. & Seto, N. 2002, ApJ, 575, 1039
Thorne, K. S., 1987, in Hawking S. W., Israel W., eds, 300 Years of Gravitation. Cambridge Univ. Press, Cambridge, p. 330
van der Marel, P. 2004, in “The Local Group as an Astrophysical Laboratory”, ed. M. Livio (Cambridge Univ. Press), astro-ph/0404192
van der Marel, P., Alves, D. R., Hardy, E. & Suntzeff, N. B. 2002, AJ, 124, 2639
Vecchio, A., & Wickham, E. D. 2004, Phys. Rev. D, 70, 082002