Abstract. Fairness aware data mining (FADM) aims to prevent algorithms from discriminating against protected groups. The literature has come to an impasse as to what constitutes explainable variability as opposed to discrimination. This distinction hinges on a rigorous understanding of the role of proxy variables; i.e., those variables which are associated both the protected feature and the outcome of interest. We demonstrate that fairness is achieved by ensuring impartiality with respect to sensitive characteristics and provide a framework for impartiality by accounting for different perspectives on the data generating process. In particular, fairness can only be precisely defined in a full-data scenario in which all covariates are observed. We then analyze how these models may be conservatively estimated via regression in partial-data settings. Decomposing the regression estimates provides insights into previously unexplored distinctions between explainable variability and discrimination that illuminate the use of proxy variables in fairness aware data mining.
Impartial Predictive Modeling and the Use of Proxy Variables

Kory D. Johnson\textsuperscript{1}, Dean P. Foster\textsuperscript{2}, and Robert A. Stine\textsuperscript{3}

\textsuperscript{1} TU Wien, 1040 Vienna, Austria
kory.johnson@tuwien.ac.at

\textsuperscript{2} Amazon, New York, New York, USA
dean.foster@gmail.com

\textsuperscript{3} University of Pennsylvania, Philadelphia, Pennsylvania, USA
stine@wharton.upenn.edu

1 Introduction

Machine learning has been a boon for improved decision making. The increased volume and variety of data has led to a host of data mining tools for knowledge discovery; however, automated decision making using vast quantities of data needs to be tempered by caution. One goal in FADM is to provide suitably “fair” estimates of a response $Y$, given legitimate covariates $x$, sensitive covariates $s$, and suspect covariates $w$. The primary distinctions between these categories concerns the ability of an individual to be morally responsible for their value: covariates in $s$ are considered to be outside of one’s control, e.g. race and gender, while $x$ are those features for which an individual can be held accountable. The $w$ group contains those covariates for which it is uncertain whether or not one ought to be held responsible for their value.

These covariate groups and the response $Y$ are connected through an unknown, joint probability distribution $P(Y, x, s, w)$, and our data consists of $n$ iid draws from this joint distribution. The standard statistical goal is to estimate the conditional expectation of $Y$ given the covariates:

$$Y = E[Y | x, s, w] + u$$

where $u$ has mean zero and is uncorrelated with all functions of the covariates.

Our goal is to estimate a conditional expectation that is “fair” with respect to the sensitive covariates. Others have argued for penalizing discrimination during estimation \cite{11, 10, 26}, modifying the input data before supervised training takes place \cite{33, 18, 5}, or modifying objective functions with fairness criteria \cite{24, 39, 19, 20}. Conceptually closest to our work is \cite{20}, which explicitly discusses economic models of equality of opportunity and presents an optimization problem of maximizing utility subject to a constraint on prediction error. Their formulation incorporates the concept of “effort-based” utility to encode the effects of legitimate covariates.

From a technical perspective, \cite{27, 29, 13, 30} use similar path diagrams as those presented in Section 2 to describe fair estimation methods. Our focus, however,
is different, as we concretely describe heretofore unknown distinctions between covariate groups. This understanding is crucial to correctly determining these groups, which are considered to be externally given in previous analyses. Other papers which address this issue include [34,1], though the present paper covers more settings (none, some, or all suspect covariates), broader use cases (extending to “black-box” models), and defends the disentanglement and partial omission of the proxy signal.

A simple example clarifies the issue of fairness. Consider a bank that wants to estimate the risk of a loan applicant via a credit score model. Such automated eligibility or pricing systems are ubiquitous both in banking and public assistance offices [14]. The concern is that personalized credit pricing could provide results which either reflect discrimination which is present in the training data or either intentionally or unintentionally discriminate based on race or gender etc. Fair lending law attempts to address this through “input scrutiny”, in which “sensitive” or “protected” covariates such as race, gender, and age are barred from use [10]. The goal of input scrutiny is to model prices solely on the remaining “legitimate” covariates, i.e., the log of historical credit use. The use of big data in pricing models, however, has opened the door to non-standard data sources such as purchase history and online activities, which could allow for predatory or targeted pricing [22]. This highlights a third covariate group of “suspect” or “potentially discriminatory” covariates whose information content for establishing creditworthiness is uncertain and potentially serves only to establish “creditworthiness by association,” i.e., by associating an applicant to a protected covariate [22]. The canonical example of such a proxy variable is the applicant’s address. While location is not a protected characteristic such as race, it is often barred from use given the ability to discriminate using it.

FADM asks whether or not the estimates the bank constructs are fair and perhaps even what effect enforcing fairness has on profit [28]. This is different than asking if the data are fair or if the historical practice of giving loans was fair. It is a question pertaining to the estimates produced by the bank’s model, and thus necessarily would imply a shift in lending law to outcome-focused analysis [16]. This generates several questions. First, what does fairness mean for this statistical model? Second, what should the role of the sensitive covariates be in this estimate? Third, how do legitimate and suspect covariates differ? Lastly, how do we constrain the use of the sensitive covariates in black-box algorithms? These questions are addressed in the remainder of the paper.

A primary hurdle for FADM is to separate explainable variability from discrimination. Many authors have argued that input scrutiny is insufficient to achieve fairness [26,25,16]. Due to the relationships between race and other covariates, merely removing race can leave lingering discriminatory effects that permeate the data and potentially perpetuate discrimination. The use of covariates associated with sensitive race to discriminate is called “redlining”, which originated in the United States to describe maps that were color-coded to represent areas in which banks would not invest. While policies were facially neutral and race blind, they were in-effect discriminatory. The advent of big data has given rise to what
has been called technological or digital redlining \[31\], wherein similar demarca-
tions can be made by, for example, only looking for housing close to high-quality
schools. Conceptually, the core issue is the misuse of available information.

We improve upon previous discussions by providing a simple, tractable for-
mulation of impartiality that addresses issues often encountered in real data;
namely, that sensitive features often do not appear to be unrelated to the re-
response. In what follows, we argue that this task is accomplished by creating
impartial estimates. Intuitively, impartiality requires that the sensitive covari-
ates do not influence estimates. For clarity, we will refer to this as the fairness
assumption:

*Fairness Assumption:* Sensitive covariates ought not be a relevant source of vari-
ability or merit.

A clear objection to this assumption is that it is often not observed in the
data, but this is precisely the point. To be explicit, consider a circumstance in
which $Y$ is an observed quantity realized by an agent in a “free” (non-coerc-
ed) way, e.g. credit history. To play devil’s advocate in this case, a most funda-
mental question for FADM is: if models are intended to describe the world, and the
fairness assumption is often inaccurate, why compromise predictive accuracy for
fairness’ sake? Instead of addressing this philosophically, we specify models in
which fairness is the accurate statistical description of the world. Our fairness
assumption is specified as an “ought” statement, as it embodies this often unre-
alized ideal.

Impartial estimates are fair because the covariate groups are chosen to be
normatively relevant and are assumed to be provided externally. As such, the
statistical task is disjoint from the normative task of identifying covariate groups.
This project uses the term “impartial” to describe the statistical goal. This is
done in order to separate our task from normative complications. That being
said, the different covariate groups have normative significance and need to be
differentiated.

The construction of our impartial estimates is motivated by distinctions in
the philosophical literature on equality of opportunity, which analyzes the way
in which benefits are allocated in society \[2\]. One way of understanding equal-
ity of opportunity is formal equality of opportunity (FEO), which requires an
open-application for benefits (anyone can apply) and that benefits are awarded
to those of highest merit. Merit will of course be measured differently depending
on the benefit in question. There may, however, be cause for concern if discrim-
ination exists in the analysis of merit or the ability of some individuals to be of
high merit.

Substantive equality of opportunity (SEO) contains the same strictures as
above, but is satisfied only if everyone has a genuine opportunity to be of high
merit. In particular, suppose there are social restrictions or benefits that only
allow one group to be of high merit. In this case, proponents of SEO claim
that true equality of opportunity has not been achieved. While many countries
lack a formal system to enforce this, some may argue that cycles of poverty and
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wealth lead to a similar regress in the reasons for the disparity between protected groups.

Identifying impartial estimates and fairness constructively has both philosophical and empirical support. The philosopher John Rawls discusses fair institutions as the method of achieving fairness [38]. Similarly, [9,17] explain the importance of process fairness as opposed to merely outcome fairness. Process fairness focuses on how people are treated throughout the process of a decision. The authors identify several examples in which firms attempt to layoff workers in a fair manner. Workers often feel that the decisions are fair when they are consulted frequently and the process is transparent, even if their severance packages are far worse. This points out the importance of fair treatment as fair use of information, not merely a measure of the outcome.

Our main contribution is a framework which allows for rigorous analysis of how the use of covariates changes when moving between FEO and SEO. We address the complications produced by the various viewpoints by introducing and demonstrating the importance of the suspect covariate group $w$. After providing this framework, it will be clear how to both construct impartial estimates using simple procedures as well as correct black-box estimates.

Section 2 introduces various data generating models in order to construct mathematical models of impartiality. This section also draws connections to the literature on causal inference. Importantly, the claim that these are impartial requires a strict interpretation of the influence of sensitive covariates. Section 3 provides a simple procedure for constructing impartial estimates. The difference between achieving group fairness via FEO and SEO is demonstrated via a simple example in Section 4. Section 5 analyzes two data sets from criminology that consider the effect of race and sex. Impartial estimates are produced with an package R [35] that is available on github.com.

2 Mathematical Models of Impartiality

Fairness in modeling will be explained via path diagrams to conveniently represent conditional independence assumptions. While often used as a model to measure causal effects, we are explicitly not using them for this purpose. This stems from a different object of interest: in causal modeling, one cares about a casual parameter or direct effect of the covariate of interest whereas we care about the estimates produced by the model. Estimating a causal effect requires considering a counterfactual, such as a patient’s outcome under the treatment even though they were part of the control group. See, for example, [30].

Our goal is to create impartial estimates, whereas the estimation of a causal effect would attempt to answer whether the historical data are impartial. Hence, we do not require the same type of causal interpretation, which is fraught with difficulties for attributes such as race [215]. Counterfactuals can be easily computed because it only requires producing an estimate for a modified observation, regardless of whether it exists in the data set. We do not need recourse to the interventionist or causal components of standard causal models [32] and can
focus only on their predictive component. Path diagrams only represent the conditional independence assumptions made between variables as necessitated by the fairness assumption.

The rest of this section briefly introduces impartial estimates in stages via models in which the fairness assumption is tractable. We begin by enforcing FEO, which only uses sensitive and legitimate covariates. The goal in FEO is to have a best estimate of merit while satisfying the legal requirements of disparate treatment and disparate impact. Second, we consider a full SEO model, in which there are no legitimate covariates, only sensitive and suspect covariates. The total model case with sensitive, legitimate, and suspect covariates is considered last.

**Table 1.** Observationally Equivalent Data Generating Models: FEO (first row), SEO (second row), mixture (third row).

| OBSERVED | UNRESTRICTED | FAIR |
|----------|--------------|------|
|         |              |      |
| ![Diagram](image1.png) | ![Diagram](image2.png) | ![Diagram](image3.png) |
| ![Diagram](image4.png) | ![Diagram](image5.png) | ![Diagram](image6.png) |
| ![Diagram](image7.png) | ![Diagram](image8.png) | ![Diagram](image9.png) |

FEO is not concerned with potentially discriminatory covariates. Consider an idealized population model that includes all possible covariates: $x_o$ contains the observed, legitimate covariates, and $x_u$ contains the unobserved, legitimate covariates. Unobserved covariates could be potentially observable such as drug use, or unknowable such as future income. The data are assumed to have a joint distribution $\mathbb{P}(Y, s, x_o, x_u)$, from which $n$ observations are drawn. The fairness assumption requires that $s$ is not predictive for the response given full information:

$$\mathbb{P}(Y|s, x_o, x_u) = \mathbb{P}(Y|x_o, x_u).$$

It is important to posit the existence of both observed and unobserved legitimate covariates to capture the often observed relationship between sensitive covariates.
and the response. Specifically, observed data often show

\[ P(Y|s, x_o) \neq P(Y|x_o), \]

which violates the fairness assumption.

These assumptions are captured succinctly for various models using the path diagrams of Table 1. Single headed arrows show direct effects while dashed, double-headed arrows indicate correlations which are potentially unfair. Observed data are often only representable by a fully connected graph which contains no conditional independence properties. This observed distribution can be generated from multiple full-information models. The first possible representation of the full data is an unrestricted model. In this case, sensitive covariates are not conditionally independent of the response given full information. The fairness assumption is enforced by assuming that sensitive information is conditionally independent of the response given full information. Under the fair, full-information model, the apparent importance of sensitive information in the observed data is only due to unobserved covariates.

One objection to FEO is the assumption that all \( x \) covariates are legitimate. Thus, while the response may be explained in terms of \( x \) without recourse to \( s \), that is only because the covariates \( x \) are the result of structural discrimination. This class of “potentially illegitimate” or “suspect” covariates is denoted by \( w \) and can be used to estimate merit, but only in such a way that does not distinguish between groups in \( s \). This treats \( w \) as proxy variables for missing information. The setting for which all legitimate covariates are considered suspect is given in the second row of Table 1. Note again that the observed data model can be the result of multiple full-information models. The final row of Table 1 shows the fairness assumption in a model that includes both legitimate and suspect covariates.

### 3 Creating Impartial Estimates

In this section, we demonstrate one simple way to estimate the models of Section 2. We define impartiality with respect to the linear projection of \( Y \) on a set of covariates \( v \):

\[ Y = v'\beta + \epsilon, \]

where the error term \( \epsilon \) satisfies

\[ \mathbb{E}[\epsilon] = 0, \quad \text{Cov}(v, \epsilon) = 0. \]

This allows core ideas to be fully explained in a familiar setting as well as clear decompositions of covariate effects. While our definition uses linear projections, estimates are not required to be linear. Section 5 uses the notion of suspect covariates to correct “black box” estimates.

For clarity, we introduce the full estimation procedure in stages. Collect the observations into matrices \( Y, S, X, \) and \( W \), and consider the estimated response...
given various subsets of covariate groups. Estimates will be decomposed into various components using projections. For a matrix $M$, $H_M$ is the projection matrix onto the column-span of $M$. We use bracket notation to indicate when two matrices are joined column-wise, e.g. $M = [X_o, W]$ contains the columns of both $X_o$ and $W$. All covariates are assumed to have mean 0 as we include an intercept in our model. Covariate matrices of each covariate type are separated into the portion correlated with the others and the component which is orthogonal to them. We will refer to these as the “shared” and “unique” components, respectively.

While the decomposition is standard, this is perhaps a non-standard presentation. It is important to note that the coefficient for each group is computed only from the unique component of that group in the model considered. For example, if all covariate groups are used, let $X_{o,a} = (I - H_{[S,W]})X_o$ be $X_o$ “adjusted” for the other covariates in the model. The least squares estimate of its parameter in this model is given by $\hat{\beta}_{x,f} = (X_{o,a}'X_{o,a})^{-1}X_{o,a}'Y$, provided the inverse exists.

In what follows, we put additional subscripts on parameters which depend on the model considered. This highlights that parameters are different depending on the model in which they are estimated. Lastly, note that “hats” are used to indicate estimated values.

First, consider a model with only $S$ and $X$ as predictors (1) and the resulting decomposition of the estimates (2). This is the FEO model as all non-sensitive covariates are considered legitimate.

\begin{align}
Y &= \beta_{0,f} + \beta_{s,f}^tS + \beta_{x,f}^tX_o + \epsilon_f \\
Y_f &= \hat{\beta}_{0,f} + H_{X_o}S\hat{\beta}_{s,f} + (I - H_{X_o})S\hat{\beta}_{s,f} + H_SX_o\hat{\beta}_{x,f} + (I - H_S)X_o\hat{\beta}_{x,f}
\end{align}

By decomposing the estimates as in equation (2), we can identify components which are of philosophical and legal interest. The term $dt$ captures the disparate treatment effect: it is the component of the estimate which is due to the unique variability of $S$. Given the data generating model in Table 1, we assume that the apparent importance of $S$ (signified by the magnitude of $\hat{\beta}_{s,f}$) is due to excluded covariates; however, it is identified by $S$ in the observed data. While this may be a “sufficiently accurate generalization” in that the coefficient may be statistically significant, this is considered to be illegal statistical discrimination [7]. The term $di$ captures the disparate impact effect. We refer to it as the informative redlining effect (as it is due to an legitimately informative covariate) in order to contrast it with an effect identified shortly. Intuitively, it is the misuse of informative covariates and is the result of the ability to estimate $S$ with other covariates. It is important that the adjustment is identified by variability in $S$ instead of $X_o$. If one merely ignores $S$ completely in estimation, the term $di$ will still be included in the final estimate, resulting in an estimate which is not impartial.

Previous discussions of redlining do not distinguish between the terms $di$ and $sd+$ in equation (2) [20,25], because they are both due to the correlation between $X_o$ and $S$. It is clear that they are different, however, as $H_{X_o}S$ is in the space spanned by $X_o$ and $H_SX_o$ is in the space spanned by $S$. Furthermore, the coefficients attached to these terms are estimated from different sources. Intuition
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may suggest we remove all components in the space spanned by $S$, but this is often incorrect. The term $sd+$ cannot be excluded in many settings because it accounts for the group means of $X$. Excluding $sd+$ implies that the level of $X$ is not important but that an individual’s deviation from their group mean is. This makes group membership a hindrance or advantage and is inappropriate for a legitimate covariate. Therefore, in this model $sd+$ must be included.

Second, consider a model with only $S$ and $W$ as predictors (3) and the resulting decomposition of the estimates (4):

$$Y = \beta_0,p + \beta_s,p^T S + \beta_{w,p}^T W + \epsilon_p,$$

$$\hat{Y}_p = \hat{\beta}_0,p + H_0 \left[ \begin{array}{c} \beta_s,p + (I-H_{W,S}) \hat{\beta}_{s,p} + H_{S} W \hat{\beta}_{w,p} + (I-H_{S}) W \hat{\beta}_{w,p} \end{array} \right].$$

In equation (4), $sd−$ addresses the concern that group differences in covariate $W$ are potentially discriminatory, as contrasted with $sd+$ in (2). For example, if $W$ is location, $sd−$ measures racial differences between neighborhoods. Given that proxy variables $W$ are not considered directly informative, it is unclear what these differences can legitimately contribute. If there is racial bias in neighborhood demographics, using this information would perpetuate this discrimination. Ensuring that this does not occur requires removing $sd−$ from the estimates of $Y$. This identifies a new type of redlining effect that we call uninformative redlining (as it is due to a proxy variable): it is the sum of $di$ and $sd−$. Uninformative redlining can be identified visually using the graphs in Table I. Fairness constrains the information contained in the arrow $s \rightarrow Y$ as well as information conveyed in the path $s \rightarrow w \rightarrow Y$. This is because $s \rightarrow w$ is potentially discriminatory. Therefore, impartial estimates with suspect or proxy variables only use the unique variability in $W$. An important consequence of this SEO model is that average estimates are the same for different groups of $s$. This is an alternate construction of the initial estimates used by [10].

Lastly, the final model includes all covariate groups $S$, $X$, and $W$ as predictors (5) and the resulting estimates (6):

$$Y = \beta_0,t + \beta_s,t^T S + \beta_{x,t}^T X_o + \beta_{w,t}^T W + \epsilon_t,$$

$$\hat{Y}_t = \hat{\beta}_0,t + H_0 \left[ \begin{array}{c} \beta_s,t + (I-H_{S,W}) \hat{\beta}_{s,t} + H_{S} W \hat{\beta}_{w,t} + (I-H_{S}) W \hat{\beta}_{w,t} \end{array} \right].$$

While the majority of the terms in the above display mirror the previous discussion, the suspect covariates display a more complex behavior. The unique component can still be considered additional information orthogonal to the sensitive covariates, but the component correlated with other covariates is labeled...
both \( sd^+ \) and \( sd^- \) to indicate that this combines both legal and illegal forms of statistical discrimination. The notation \( H_{X_o,S}W \) indicates that the shared component is the best linear estimate of \( W \) given both \( S \) and \( X \). As \( W \) is a suspect covariate, we can remove \( sd^- \) while retaining \( sd^+ \) by producing an impartial estimate of \( W \). In this case, \( W \) has taken the place of \( Y \) as the response in an FEO model that only contains \( S \) and \( X_o \).

Removing the components labeled \( di \), \( dt \), and \( sd^- \) can be accomplished through the following procedure.

**Definition 1 (Impartial Estimate)** Linearly impartial estimates are created with the following multi-step procedure, where “estimate” means “estimate via least-squares:”

1. Estimate the model (5) to produce \( \hat{\beta}_0, \hat{\beta}_s, \hat{\beta}_x, \) and \( \hat{\beta}_w \).
2. Create an impartial estimate of each element of \( w \).
   a. Estimate \( w = \lambda_0 + \Lambda_s s + \Lambda_x x_o + \eta \).
   b. Set \( \hat{w} = \hat{\lambda}_0 + \hat{\Lambda}_s s + \hat{\Lambda}_x x_o \).
   c. Collect the estimates, \( \hat{w} \), as \( \hat{W} \).
3. Set \( \hat{Y} = \hat{\beta}_0 + X_o \hat{\beta}_x + \hat{W} \hat{\beta}_w + (I - H_{X_o,S}) \hat{W} \hat{\beta}_w \).

4 Simple Example: FEO vs SEO

This section provides a simplified example to compare the estimates implied by FEO and SEO. Without a proper data-generation narrative, “fair” estimates can appear decidedly unfair. Consider a simple example of credit score modeling that has only two covariates: education level, \( x \), and sensitive group, \( s \). Suppose the data is collected on individuals who took out a loan of a given size, that higher education is indicative of better repayment, and that education is split into two categories: high and low. To see the relevant issues, \( s \) and \( x \) need to be associated. The two sensitive groups will be written as \( s^+ \) and \( s^- \) to indicate which group, on average, has higher education: the majority of \( s^- \) have low education and the majority of \( s^+ \) have high education. The response is the indicator of default, \( D \).

Synthetic data and estimates are provided in Table 2, in which there exist direct effects for both \( s \) and \( x \). This is consistent with the observed data graphs in Table 1. Five possible estimates are compared in Table 2: the full OLS model, the restricted regression which excludes \( s \), the FEO model in which education is considered a legitimate covariate, the SEO model in which education is considered a suspect covariate, and the marginal model which estimates the marginal probability of default without any covariates. Estimates are presented along with the in-sample RMSE from estimating the true default indicator and the discrimination score (DS) [11]. While we have argued that “discrimination score” is at times a misnomer since it does not separate explainable from discriminatory variation, it provides a useful perspective given its widespread use in the literature. Furthermore, as one often discusses discrimination toward groups, it is nevertheless helpful to gauge the difference between estimates between groups.
Table 2. Simplified loan repayment data and estimated default probabilities.

| Education (P(x)) | Low (.6) | High (.4) |
|------------------|----------|-----------|
| Group (P(s|x))   | s− (0.75) s+ | s− (0.25) s+ |
| Default Yes      | 225      | 60        | 20        | 30        | 335       |
| Default No       | 225      | 90        | 80        | 270       | 665       |

| P( Default YES|x, s) | DS | RMSE |
|-----------------------|----|------|
| Full Model            | .5 | .4   | .2   | .1   | -.25   | 13.84  |
| Exclude s             | .475 | .475 | .125 | .125 | -.17   | 13.91  |
| FEO                   | .455 | .455 | .155 | .155 | -.15   | 13.93  |
| SEO                   | .39  | .535 | .09  | .235 | 0.00   | 14.37  |
| Marginal              | .35  | .35  | .35  | .35  | 0.00   | 14.93  |

Since education is the only covariate that can measure similarity, one would expect that estimates should be constant for individuals with the same education. This is easily accomplished by the legal prescription of input scrutiny by excluding s. If the information is not observed, it cannot lead to disparate treatment directly related to group membership. The FEO model satisfies this as well. As seen in equation (2), the only difference between the two estimates is the coefficient on x. Said differently, the term di in equation (2) lies in the space spanned by x. Therefore its removal only changes estimates for education groups. Excluding s permits redlining because it increases the estimated disparity between low- and high-education groups. This disproportionately affects those in s− as they constitute the majority of the low education group. The FEO estimates result in some average differences between groups, but this is acceptable if the association between x and s is benign. This accurately measures the proportional differences desired by [4] for fair treatment.

The SEO estimates appear counter-intuitive: although s+ performs better in our data set even after accounting for education, these estimates predict the opposite. Understanding this requires accepting the world view implicit in the SEO estimates: average education differences between groups are the result of structural discrimination. Members of s− in the high education group have a much higher education than average for s−. Similarly, members of s+ who are in the high education group have a higher education than average for s+, but not by as much. The magnitude of these differences is given importance, not the education level.

As an example where these deviations are given importance, consider the college admissions process in the United States and the two explanatory covariates “class rank” and “SAT score.” The SAT is a standardized test commonly used in the United States to assess students’ readiness for university, and it is under scrutiny for doing more to measure disparities in students’ learning opportunities, e.g. wealth and race, than college preparedness [2315]. The SAT score provides information on where an applicant lies in the national test score distribution, whereas class rank specifies their location in the local grade distribution. Considering class rank is equivalent to placing importance on the deviation between
a student and others much more likely to be in a comparable situation. Similarly, the SAT score could be used to only measure differences within a group. Conceptually, this is what was done in the “Strivers” proposal, wherein students were termed “strivers” when they significantly outperformed the expected score based on socioeconomic and structural factors [12].

In our example, the SEO model balances the differences in education distributions, resulting in both groups having the same average estimated default. This is seen in the discrimination score of 0. Without claiming that education is partially the result of structural differences, the SEO estimates discriminate against \( s_+ \). Other methods to achieve group fairness produce estimates relevantly similar to SEO in this regard. Furthermore, if the structural effects are not such that all \( s_+ \) individuals are given a benefit or not all \( s_- \) individuals receive a detriment, then these models are merely approximations of the fair correction. An ideal protected or sensitive covariate \( s \) is exactly that which accounts for differences in the opportunity of being high merit. This is in line with Rawls’ conception of equality of opportunity [37].

The SEO estimates show another important property: their RMSE is lower than that of the marginal estimate of default while still minimizing the discrimination score. Therefore, if a bank is required to minimize differences between groups in the interest of fairness, it would rather use the SEO estimates than the marginal estimate. SEO still acknowledges that education is an informative predictor and contains an education effect. Furthermore, equality of opportunity is not satisfied when marginal estimates are used because all merit information is ignored. See [2] for a more detailed discussion.

5 Data Illustrations with Black-Box Estimates

This section presents results on two criminology data sets. For a detailed discussion of the trade-offs between different fairness measures in criminology see [6]. We show the performance of various procedures using the discrimination score (average predicted difference between groups), the root mean squared prediction error, as well as the positive and negative residual differences (PRD and NRD, respectively):

\[
PRD := \left| \frac{1}{n_1} \sum_{i \in S_1} \max\{0, Y_i - \hat{Y}_i\} - \frac{1}{n_0} \sum_{i \in S_0} \max\{0, Y_i - \hat{Y}_i\} \right|
\]

\[
NRD := \left| \frac{1}{n_1} \sum_{i \in S_1} \min\{0, Y_i - \hat{Y}_i\} - \frac{1}{n_0} \sum_{i \in S_0} \min\{0, Y_i - \hat{Y}_i\} \right|
\]

These measures play the role of false positive and false negative rates for regression problems [10]. All statistics shown in the following subsections are computed out-of-sample using 5-fold cross-validation and averaged over 12 repetitions.

Lastly, to correct a “black-box” estimate, suppose that we have an estimate of the response, \( Y^\dagger \), given by an unknown model with unknown inputs. The model
may use sensitive information to be intentionally or unintentionally discriminatory. While potentially informative, there is no guarantee that the estimates are impartial. This identically matches the description of suspect covariates. Therefore, if we treat $Y^+$ as a suspect covariate, its information can be used but not in a way that makes distinctions between protected groups. For simplicity, we only include estimates from a support vector regression [3] or random forest algorithm [8] as they are high-performing, off-the-shelf “black boxes.” Furthermore, they are allowed to use $s$ and $w$ in order to demonstrate that estimates can be easily corrected.

We compare our models with the propensity score stratification methods of [10] as well as the convex optimization approaches of [19,20]. [19] is motivated by social welfare considerations, places an upper bound on prediction error, and controls the PRD and NRD. [20] requires a prespecified utility function to be defined for all groups and explicitly tries to maximize the utility for both groups while placing an upper bound on prediction error. These three methods will be referred to as propensity, welfare, and utility, due to the motivations for the frameworks. These methods are compared to impartial estimates that use various covariate specifications for $s$, $x$, and $w$. The exclude model merely excludes $s$ during estimation, the $FEO$ model treats all non-sensitive features as legitimate, the $SEO$ model treats all non-sensitive features as suspect, and the mixed model treats some as legitimate and some as suspect. We separately consider each model when they are given additional black-box estimates which are treated as a suspect covariate ($rf$ or $svr$).

In all data cases, $Y$ is rescaled to lie in $[0,1]$ and larger values correspond to better estimates, i.e. lower estimates of crime and fewer days jailed. This is required to use the utility function specified in [20]. As acknowledged therein, the choice of this function has large impact on estimates as seen below. The final example uses the same data set used in [20] to promote easier comparison. Rescaling in this way yields small baseline values for the mean difference in $Y$ as well as the standard deviation of $Y$. As such, small reductions in prediction error can still be sizable on a more natural scale.

The first data set contains information from parolees. The guiding question is whether men are more likely to do hard time holding constant age, the neighborhood in which the live, and prior record. In particular, the goal is to provide estimates of number of days in jail which are impartial with respect to sex. This data set contains information from approximately 83,000 individuals on probation after excluding observations that with zero previous jail days as well as minors. Covariate information includes age, sex, and prior record information such as the number of violent priors as an adult or juvenile. We also consider adding a random forest estimate that is not constrained in its use of covariate information. Lastly, as all covariates and the response exhibit long right tails, all variables are log transformed. The only covariate treated as suspect is age, as beyond attributing to prior record, it is unclear what direct information this could contain.
For this data set, the standard deviation of the transformed response is 0.227 with a mean difference of 0.05 between groups. Therefore, in general, men have served longer sentences without conditioning on prior record. Table 3 shows that the random forest estimates do not appreciably improve prediction in this setting. That being said, the models already perform as well as all competitors in terms of prediction error. While the differences are slight, we can see that driving the DS to zero again results in small increases in the PRD and NRD.

| Model     | DS | PRD | NRD | RMSE |
|-----------|----|-----|-----|------|
| exclude   | 0.02| 0.02| 0.01| 0.14 |
| FEO       | 0.02| 0.02| 0.01| 0.14 |
| SEO       | 0.00| 0.03| 0.02| 0.14 |
| mixed     | 0.02| 0.02| 0.01| 0.14 |
| FEO_rf    | 0.02| 0.02| 0.01| 0.13 |
| SEO_rf    | 0.00| 0.03| 0.02| 0.13 |
| mixed_rf  | 0.02| 0.02| 0.01| 0.13 |
| propensity| 0.02| 0.01| 0.02| 0.22 |
| utility   | 0.07| 0.01| 0.01| 0.17 |
| welfare   | 0.02| 0.00| 0.02| 0.14 |

Our second illustration uses the Crime and Communities data set also considered in [20]. This allows for a more direct comparison as the authors specify a utility function in this case, removing the largest open input to using the method. The data set contains information on 1,994 communities such as demographic statistics (race, immigrant, and age distributions), law enforcement (budget, racial distribution of officers, etc), economic (income, unemployment, home-ownership, etc). In total, there are 100 predictive covariates which can be used to estimate the per capita number of violent crimes. Impartial estimates of crime in this case directly links back to the original redlining example presented in the introduction: lower estimates of crime provide more incentive for investment, etc.

Similar to [20], we label a community as a member of the protected group if more than 25% of the residents are black. Specifying the groups this way as opposed to merely non-Caucasian highlights an important concept behind the current work: it is possible to predict the proportion of black residents in a community using the other covariates. In fact, there are 14-20 covariates which can be used either separately or together to reduce the error sum of squares for predicting the proportion of black residents by 95%. As such, this data set provides our first sincere case with suspect covariates which fall into various categories. Examples of predictive economic indicators include the percentage of households with income from investments, social security, or from public assistance. Others include percentage of people born in the same state, or living in the same house.
as 5 years before, as well as number of people in shelters or on the street. The conceptual difficulty is that there may be some debate about whether these covariates are legitimate. For comparison, we consider all such covariates as suspect in the mixed model in Table 4.

| Model       | DS | PRD | NRD | RMSE |
|-------------|----|-----|-----|------|
| exclude     | 0.30 | 0.04 | 0.04 | 0.14 |
| FEO         | 0.28 | 0.02 | 0.05 | 0.14 |
| SEO 0.00    | 0.08 | 0.23 | 0.19 |
| mixed       | 0.23 | 0.00 | 0.08 | 0.14 |
| FEO_svr     | 0.28 | 0.01 | 0.04 | 0.11 |
| SEO_svr     | 0.00 | 0.08 | 0.23 | 0.17 |
| mixed_svr   | 0.23 | 0.01 | 0.07 | 0.11 |
| propensity  | 0.27 | 0.02 | 0.06 | 0.15 |
| utility     | 0.19 | 0.07 | 0.05 | 0.19 |
| welfare     | 0.30 | 0.01 | 0.02 | 0.14 |

We now see clear differences in performance of various impartial models. Importantly, the predictive performance of the mixed model matches that of the FEO model while reducing the mean difference between groups. Furthermore, there is also now a penalty in prediction for switching to a full SEO model which considers no legitimate covariates. As the mean difference in the transformed response is 0.31, it is clear that many models make little to no improvement in this regard. On the other hand, including the black box estimates improves predictive performance dramatically. While the welfare model does achieve good predictive performance, the mean difference is hardly changed. As a final note, we observe that the propensity model performs better, as it is no longer able to perfectly classify the protected groups based on the remaining legitimate covariates. That being said, if one wants the mean difference between groups to be small, the SEO models achieve this exactly.

6 Discussion

This paper provides a clear statistical theory of impartial estimation and explainable variability through the analysis of proxy variables. Covariate groups can only be specified for use in FADM if one has a clear understanding of the implications of the decisions. By considering full-data scenarios in which our fairness assumption is satisfied, we concretely describe the role of proxy variables and their allowable use in FADM. This also provides connections to legal concepts such as our newly identified “uninformative redlining” effect and distinctions between various types of statistical discrimination.
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