FINAL STATE INTERACTIONS IN $WW$ PRODUCTION

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Abstract

It is shown that colour transparency causes nonperturbative colour-singlet final-state interactions to have a negligible effect on the production rate and the dijet mass spectra in $e^+e^- \rightarrow WW$. However, the same cannot be said of nonperturbative colour-octet exchange, for which we show that there are indications of observable effects, though we are unable to present precise estimates.

Introduction

An important aim at LEP2 is to measure the mass of the $W$-boson to high accuracy, perhaps to within 50 MeV[1]. A preferred method, because it offers the prospect of the most statistics, is to produce a pair of $W$’s, each of which decays into a pair of quark jets. The invariant mass of each jet pair is then measured.

An obvious question is whether final-state interactions among the jets will cause a problem: the $W$-particles have a short lifetime, so the quarks are close together when they are produced and interactions among them could well be significant. There have been several calculations of colour exchange among the quarks[2][3], mostly reaching the conclusion that the effect is quite small.

In this paper, we first examine the effect of colour-singlet exchange. Because the relative energies of the quarks are large, we use soft-pomeron phenomenology[4] to model this exchange. While this phenomenology is well-established and has had many successes, its extension to the present problem does involve uncertainty. Nevertheless, our conclusion that there is a negligibly small effect is probably reliable. This result comes about because colour-transparency effects[5] suppress colour-singlet exchange.

We then go on to perform a similar calculation of colour-octet exchange. We model this exchange using Cornwall’s solution to the Schwinger-Dyson equations[6], which provides a well-motivated way to handle the nonperturbative region of gluon exchange. In the case of perturbative gluon exchange, there is an infrared divergence, corresponding to soft exchange, which is cancelled by adding in the contribution from soft gluon emission. However, these infrared divergences are not genuine, because the nonperturbative corrections to the propagator, which have their origin in confinement, remove them. The Cornwall formalism does not provide a method of calculating the gluon emission, but since
it effectively gives the gluon a mass, any emission that does occur cannot be soft. Hence there is nothing to cancel the correction to the cross section from soft gluon exchange. We therefore calculate soft nonperturbative gluon exchange and maintain that it is likely to provide a lower bound to the true correction to the cross section from colour-octet exchange. We find that this lower bound is far from small.

**Formalism**

We consider the differential cross-section \(d^2\sigma/dM_1 dM_2\) for the \(e^+e^-\) annihilation into \(WW\) and the decay of each \(W\) into a quark-antiquark pair, \(WW \to q\bar{q}q\bar{q}\), with \(M_1\) and \(M_2\) the invariant masses of the jet pairs. One of the Born diagrams for this process is shown in figure 1a. We concentrate on the corrections to this diagram; we do not expect any significant difference in the result for the other Born diagram, in which the \(e^+e^-\) transform into \(WW\) through neutrino exchange.

We suppose that the interactions among the final quarks are two-body interactions, as depicted in figure 1b. We suppose further that they do not flip the helicities of the quarks. Then the ratio of the amplitude of figure 1b to that of figure 1a is independent of the spin states of the leptons and the quarks and is

\[
R(P_1, P_2, p_1, p_2) = \frac{i}{2(2\pi)^4} \int d^4\Delta \frac{T_{qq}(p_1, p_2, \Delta)}{((P_1 + \Delta)^2 - M_W^2)((P_2 - \Delta)^2 - M_W^2)}
\]

(1)

Here the \(W\) mass is complex, \(M_W = M - i\Gamma\). The quark-quark scattering amplitude \(T_{qq}\) includes the two off-shell lines \(\bar{p}_1 = p_1 + \Delta\) and \(\bar{p}_2 = p_2 - \Delta\). It is averaged over the quark spin states. We have to sum (1) over the four different ways of attaching the interaction to the quark and antiquark lines.

We define \(\nu = P_1 \cdot P_2\). It is useful to introduce linear combinations \(R_1\) and \(R_2\) of \(P_1\) and \(P_2\) satisfying \(R_1^2 = 0 = R_2^2\):

\[
P_1 = R_1 + \lambda_1 R_2 \quad \quad P_2 = R_2 + \lambda_2 R_1
\]

(2a)

with

\[
\lambda_1 = \frac{\nu - \sqrt{\nu^2 - M_1^2 M_2^2}}{M_2^2} \quad \quad \lambda_2 = \frac{\nu - \sqrt{\nu^2 - M_1^2 M_2^2}}{M_1^2}
\]

(2b)

Then \(\nu_R = R_1 \cdot R_2\) satisfies \(\nu_R = \nu(1 + \lambda_1 \lambda_2)\). We parametrise

\[
\Delta = \frac{\alpha}{2\nu_R} R_1 - \frac{\beta}{2\nu_R} R_2 + \delta
\]

(3a)
where $\delta \cdot R_1 = 0 = \delta \cdot R_2$, so that $\delta$ is a two-dimensional anti-euclidean vector. Then

$$
\int d^4 \Delta = \frac{1}{4\nu R} \int d\alpha d\beta d^2 \delta \quad (3b)
$$

We find that it is consistent to assume that most of the contribution to the $\Delta$ integration will arise from values of $\alpha$ and $\beta$ that are much less than $\nu R$, so that

$$
\hat{t} = \Delta^2 \sim \delta^2 \quad (4a)
$$

and the squared 4-momenta of the $W$'s are

$$
\bar{P}_1^2 = (P_1 + \Delta)^2 \sim A_1 - \beta + \lambda_1 \alpha \\
\bar{P}_2^2 = (P_2 - \Delta^2) \sim A_2 - \alpha + \lambda_2 \beta \quad (4b)
$$

with

$$
A_1 = M_1^2 + \delta^2 \quad A_2 = M_2^2 + \delta^2 \quad (4c)
$$

With

$$
p_1 = xR_1 + y'R_2 + \pi_1 \quad p_2 = x'R_1 + yR_2 - \pi_2 \quad (5a)
$$

where $\pi_1$ and $\pi_2$ are each transverse to both $R_1$ and $R_2$ and so again are two-dimensional anti-euclidean vectors,

$$
\int d^4 p_1 \delta^+(p_1^2) \delta^+((P_1 - p_1)^2) = \frac{1}{4\nu R} \int dxdy' d^2 \pi_1 \delta(y' - \lambda_1 (1 - x)) \delta(\pi_1^2 + \lambda_1 x (1 - x)) \\
\int d^4 p_2 \delta^+(p_2^2) \delta^+((P_2 - p_2)^2) = \frac{1}{4\nu R} \int dx'dy d^2 \pi_2 \delta(x' - \lambda_2 (1 - y)) \delta(\pi_2^2 + \lambda_2 y (1 - y)) \quad (5b)
$$

Further, the energy variable for the $qq$ interaction is

$$
\hat{\nu} = p_1 \cdot p_2 = \nu R (xy + \lambda_1 \lambda_2 (1 - x)(1 - y)) - \pi_1 \cdot \pi_2 \quad (6a)
$$

and the squared 4-momenta of the virtual quarks are

$$
\bar{p}_1^2 = (p_1 + \Delta)^2 \sim B_1 - \beta x + \lambda_1 \alpha (1 - x) \\
\bar{p}_2^2 = (p_2 - \Delta)^2 \sim B_2 - \alpha y + \lambda_2 \beta (1 - y) \quad (6b)
$$

with

$$
B_1 = \delta^2 - 2\pi_1 \cdot \delta \quad B_2 = \delta^2 - 2\pi_2 \cdot \delta \quad (6c)
$$

The $qq$-interaction amplitude $T$ is a function of the energy $\hat{\nu}$, the momentum transfer $\hat{t}$, and the squared 4-momenta $\bar{p}_1^2$ and $\bar{p}_2^2$ of the virtual quarks. According to standard analyticity properties$^7$, the singularities of $T$ are confined to the upper halves of the complex planes of the two variables (6b). This is conveniently expressed by the representation

$$
T = \int_0^\infty d\kappa_1 d\kappa_2 T(\hat{\nu}, \hat{t}, \kappa_1, \kappa_2) \exp i(\kappa_1 \bar{p}_1^2 + \kappa_2 \bar{p}_2^2) \quad (7a)
$$

We introduce the $W$ propagators

$$
\mathcal{P}_1 = \frac{1}{\bar{p}_1^2 - M_W^2} \quad \mathcal{P}_2 = \frac{1}{\bar{p}_2^2 - M_W^2} \quad (7b)
$$
Figure 2: Regions of the \((x, y)\)-plane for which the integral \((8a)\) is nonzero. The curve is \(E = 0\), with \(E\) given in \((8c)\).

Insert the expressions \((4b)\) and \((6b)\) into \((7)\) and perform the integrations over \(\alpha\) and \(\beta\) by closing each integration contour in the appropriate half of the complex plane:

\[
\int d\alpha d\beta T \mathcal{P}_1 \mathcal{P}_2 = -\int_0^\infty d\kappa_1 d\kappa_2 T(\hat{\nu}, \hat{t}, \kappa_1, \kappa_2) \frac{(2\pi)^2}{1 - \lambda_1 \lambda_2} \theta(f \kappa_1 + G \kappa_2) \theta(F \kappa_1 + g \kappa_2) \exp i(u_1 \kappa_1 + u_2 \kappa_2)
\]

where

\[
\begin{align*}
  u_1 &= B_1 + (A_1 - M_W^2) f + (A_2 - M_W^2) F \\
  u_2 &= B_2 + (A_1 - M_W^2) G + (A_2 - M_W^2) g \\
  f &= \lambda_1 \lambda_2 (1 - \Lambda x)/(1 - \lambda_1 \lambda_2) \\
  g &= \lambda_1 \lambda_2 (1 - \Lambda y)/(1 - \lambda_1 \lambda_2) \\
  F &= \lambda_1 (1 - 2x)/(1 - \lambda_1 \lambda_2) \\
  G &= \lambda_2 (1 - 2y)/(1 - \lambda_1 \lambda_2) \\
  \Lambda &= \frac{1 + \lambda_1 \lambda_2}{\lambda_1 \lambda_2}
\end{align*}
\]

The two \(\theta\)-functions in \((8a)\) give different limits to the \(\kappa_1\) and \(\kappa_2\) integrations for different ranges of values of \(x\) and \(y\), corresponding to different signs for \(f, g, F, G\). This is shown in figure 2, where the numbers denote the regions of the \((x, y)\)-plane for which the integral is non-zero. The region below the dashed curve is the region \(E < 0\), where

\[
E = fg - FG
\]

Suppose, as an example, that \(T(\hat{\nu}, \hat{t}, \vec{p}_1^2, \vec{p}_2^2)\) were to have the simple factorised form

\[
T(\hat{\nu}, \hat{t}, \vec{p}_1^2, \vec{p}_2^2) = A(\hat{\nu}, \hat{t}) \phi(\vec{p}_1^2, \mu_1^2) \phi(\vec{p}_2^2, \mu_2^2)
\]

\[
\phi(p^2, \mu^2) = \frac{1}{p^2 - \mu^2}
\]

Then the integral \((8a)\) is \((2\pi)^2 A(\hat{\nu}, \hat{t})/(1 - \lambda_1 \lambda_2)\) times the following:

**Region 1:** \(F, G > 0, \quad f, g < 0\)

\[
\frac{E \theta(-E)}{FG} \frac{1}{D_1 D_2}
\]
Region 2: \( F, G > 0, \ f > 0, \ g < 0 \) (so that \( E < 0 \))

\[-\frac{1}{U_1D_2}\]

Region 3: \( F, G > 0, \ f < 0, \ g > 0 \) (so that \( E < 0 \))

\[-\frac{1}{D_1U_2}\]

Region 4: \( F, G > 0, \ f, g > 0 \) (so that \( E < 0 \))

\[-\frac{1}{U_1U_2}\]

Region 5: \( F > 0, \ G < 0 \ f > 0, \ g < 0 \)

\[-\theta(E)\frac{C}{U_1D_1} - \theta(-E)\frac{1}{U_1D_2}\]

Region 6: \( F < 0, \ G > 0 \ f < 0, \ g > 0 \)

\[-\theta(E)\frac{c}{U_2D_2} - \theta(-E)\frac{1}{D_1U_2}\]

(10a)

where

\[ U_1 = u_1 - \mu_1^2 \quad U_2 = u_2 - \mu_2^2 \]

\[ D_1 = U_1 + CU_2 \]

\[ D_2 = cU_1 + U_2 \]

\[ C = -f/G \quad c = -g/F \]

(10b)

This complexity is not unexpected. In coordinate space there are many different contributions, corresponding to the possible different time-orderings of the vertices in figure 1b. Although we cannot identify a direct correspondence between the different time-orderings and the various cases in (10), we speculate that there is a connection between the two.

For both singlet and octet exchange we perform the integrations over the two components of \( \delta \) numerically. In each case we find that there is an important contribution from values of \( x \) and \( y \) that lie near the curve \( E = 0 \). When \( E = 0 \),

\[ GD_1 = GB_1 - fB_2 \quad FD_2 = FB_2 - gB_1 \]

and so the dependence of \( D_1 \) and \( D_2 \) on \( M_W \) disappears. So, on the curve \( E = 0 \) in region 1, the amplitude of figure 1b no longer depends on the \( W \) mass; presumably this corresponds to the \( W \)'s decaying very quickly, before they can propagate, so that the quarks are still close together and their interaction is therefore enhanced\(^3\).

**Soft-pomeron exchange**

In the case of colour-singlet exchange, we find that the amplitude is so strongly peaked near to \( E = 0 \) that we do not need to consider interference between the different ways of attaching the exchange to the quark lines. We calculate \( R \), defined in (1) and appropriately summed over the different ways of attaching the exchanges to the quarks, and integrate \(|1 + R|^2\) over the angle between \( \pi \) and \( \pi' \). So we
include the square of figure 1b, and also the interference with the Born term of figure 1a. In this way we determine the ratio

\[ R(M_1, M_2, x, y) = \frac{d^4\sigma^{\text{CORRECTION}}}{dM_1dM_2dx dy} / \frac{d^4\sigma^{\text{BORN}}}{dM_1dM_2dx dy} \]  

(11)

where \( x \) and \( y \) are defined in (5a).

We now introduce the specific soft-pomeron-exchange form for the final-state interaction. Then[4]

\[ A(\hat{\nu}, \hat{t}) = 2\beta_0^2 (2\hat{\nu})^{\alpha(i)} \]

\[ \alpha(\hat{t}) = 1 + \epsilon_0 + \alpha' \hat{t} \]

\[ \beta_0 = 2 \text{ GeV}^{-1} \quad \epsilon_0 = 0.08 \quad \alpha' = 0.25 \text{ GeV}^{-2} \]  

(12a)

We need a form factor for the coupling of the pomeron to the off-shell quarks. The best available choice is[8] to use (9) and replace the function \( \phi(p^2, \mu^2) \) with

\[ \phi(p^2, 0) - \phi(p^2, \mu_0^2) \]  

(12b)

with \( \mu_0 \approx 1 \text{GeV} \).

The result of our numerical computation of \( R \), defined in (11), is that it is extremely small over almost the whole \((x, y)\) plane, at the one per mil level or less. This is to be attributed to colour transparency[5], which in our calculation manifests itself as a strong cancellation between the two terms in (12b).

**Octet exchange**

According to Cornwall’s calculation[6], colour-octet exchange between quarks can be well approximated by

\[ A(\hat{\nu}, \hat{t}) = 16\pi \hat{\nu} \alpha_s(-\hat{t})D(-\hat{t}) \]  

(13a)

with

\[ D^{-1}(q^2) = q^2 + m^2(q^2) \]

\[ \alpha_s(q^2) = \frac{12\pi}{(33 - 2N_f) \log \left( \frac{q^2 + 4m_0^2(q^2)}{\Lambda^2} \right)} \]  

(13b)

where the running gluon mass is given by

\[ m^2(q^2) = m_0^2 \left[ \log \frac{q^2 + 4m_0^2}{\Lambda^2} \right]^{-12/11} \]  

(13c)

The fixed mass \( m_0^2 \) can be determined[9] from the condition that the simple exchange of a pair of gluons between quarks is the zeroth-order approximation to soft pomeron exchange at \( t = 0 \). This requires that the integral

\[ \beta_0^2 = \frac{4}{9} \int d^2q [\alpha_s(q^2)D(q^2)]^2 \]  

(14)

be about 4 GeV\(^{-2}\). With a choice of \( \Lambda = 200 \text{ MeV} \) this gives \( m_0 = 340 \text{ MeV} \).

While, strictly speaking, Cornwall’s analysis applies only to Euclidean \( q \), the form of (13) suggests that it effectively gives the gluon a mass which at low momentum values is close to \( m_0 \). Thus, while
Figure 3: The ratio $R$, defined in (11), at $\sqrt{s} = 175$ GeV and $M_1 = M$, for (a) $M_2 = M$, (b) $M_2 = M - \Gamma$, and (c) $M_2 = M + \Gamma$. 
there is uncertainty about how to calculate gluon emission, it seems that the confinement effects that Cornwall’s calculation reveals do not allow the emission of soft gluons. So, while the effect of gluon emission tends to cancel that of gluon exchange, in the case of soft exchange there is no such cancellation. For this reason, we calculate the contribution from exchange for which \( q^2 < m_0^2 \), and suggest that this provides a lower bound to the correction to the cross section from colour-octet exchange and emission. While we cannot be certain that this is a meaningful approach, we observe that the calculation is a very different one from that of the corresponding correction to the \( e^+e^- \) total cross section. In the \( WW \) calculation there are \( W \) propagators in addition to quark propagators, which damp the contribution to the integration (3b) from large values of \( \alpha \) and \( \beta \), so that the momentum transfer \( \Delta^2 \) carried by the exchanged gluon is confined to spacelike values, as is seen in (4a). In the \( e^+e^- \) case there is no such damping, and hence there are additional contributions from timelike \( \Delta^2 \), which we cannot calculate because the Cornwall form (13) of the gluon-exchange amplitude is valid only for spacelike \( \Delta^2 \).

We calculate the sum of the amplitudes of figure 1b with the soft-gluon exchange attached to the quarks in all of the four possible ways*. We square this amplitude: colour considerations forbid interference with the Born term of figure 1, and of course in the squared amplitude there appears a colour factor \( 2/9 \) because the two gluons together must form a singlet configuration. We use the form (13a) for \( A \) in (9). We simply set \( \mu_1 = \mu_2 = 0 \), because the nonperturbative quark propagator corresponding to the Cornwall gluon propagator is not available. The output for the ratio \( R \), defined in (11), is symmetric under \( x \to (1 - x) \) or \( y \to (1 - y) \) (or both). Figure 3 shows the contribution to \( R \) from the exchange of nonperturbative gluons with \( q^2 < 0.1 \) GeV\(^2 \). The energy is \( \sqrt{s} = 175 \) GeV and the plots are for \( M_1 = M \), with \( M_2 = M \) and \( M \pm \Gamma \). The rather violent dependence on \( x \) and \( y \) is striking.

The interaction has a significant effect on the energy distribution of the jets, though the overall effect on the integrated cross section is not so large. To convert the plots of figure 3 to energy distributions, note that the energy of the jet \( p_1 \) is \( \frac{1}{2} X \sqrt{s} \), with

\[
X = \frac{x(1 - 2\lambda_1 + \lambda_1 \lambda_2) + \lambda_1 - \lambda_1 \lambda_2}{1 - \lambda_1 \lambda_2} \quad (15)
\]

* We do not consider diagrams where the gluon is exchanged simply between a pair of quarks associated with the same \( W \), since this is just the same as the familiar correction to \( R_{e^+e^-} \) and so is known to be small.
There are similar equations for the fractional energies of the other three jets. In the Born approximation, the probability distributions of the two pairs of jets are uncorrelated, and each is symmetric under $X \rightarrow (1 - X)$. Figure 4 shows the output, slightly smoothed, of a Monte Carlo calculation\textsuperscript{[10]} of the probability distribution $P(X)$ at energy $\sqrt{s} = 172$ GeV. We use this distribution to weight the output $\mathcal{R}$ defined in (11) and we then average it over $x$ and $y$. The result is shown in figure 5.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure5.png}
\caption{$\langle \mathcal{R} \rangle$ for $\sqrt{s} = 175$ GeV, $M_1 = M$, plotted against $M_2$.}
\end{figure}

**Conclusions**

We have found that colour transparency causes colour-singlet exchange to have very little effect on the cross section and spectra for the process $e^+e^- \rightarrow WW \rightarrow q\bar{q}q\bar{q}$. We have calculated this by modelling colour-singlet in terms of a phenomenological soft-pomeron exchange, in which the soft pomeron couples to quarks with a form factor that goes to 0 when the quarks go far off shell. We have shown elsewhere\textsuperscript{[8]} that when soft-pomeron exchange is modelled in terms of the exchange of a pair of nonperturbative gluons, this form factor arises from taking account of the various attachments of the gluons to the separate quarks of a colour-singlet system, that is it arises from colour-transparency effects.

For colour-octet exchange the situation is very different. We calculate the contribution to this from soft-nonperturbative-gluon exchange, which we argue to give a lower bound to the total colour-octet exchange. Figure 5 shows that this will make a small but noticeable change in the total rate for the $q\bar{q}q\bar{q}$ final state. In some regions of phase space the local effect can be significant, as can be seen from figure 3, and the energy distribution of jets will differ from that of the Born term alone. However there will be little impact on the mass determination due to the near symmetry of figure 5, although some increase in the width can be anticipated.

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References

1. *Determination of the mass of the W-boson*, Report of the Workshop on Physics at LEP2, CERN Yellow report CERN-96-01 [hep-ph/9602352]

2. J Ellis and K Geiger, Physical Review D54 (1996) 1967
   G Gustafson and J Häkkinen, Z Physik C64 (1994) 659
   G Gustafson, U Petersson and P Zerwas, Physics Letters B209 (1988) 90

3. T Sjöstrand and V Khoze, Z Physik C62 (1994) 281

4. P V Landshoff, Proc PSI school at zuoz (Villigen, 1994) [hep-ph/9410250]
   J R Forshaw and D A Ross, *Quantum chromodynamics and the pomeron*, Cambridge University Press (1997)

5. G Bertsch, S J Brodsky, A S Goldhaber and J F Gunion, Physical Review Letters 47 (1981) 297

6. J M Cornwall, Physical Review D26 (1982) 1453

7. R J Eden, P V Landshoff, D I Olive and J C Polkinghorne, *The analytic S-matrix*, Cambridge University Press (1966)

8. A Donnachie and P V Landshoff, Nuclear Physics B311 (1989) 509

9. A Donnachie and P V Landshoff, Nuclear Physics B311 (1988/89) 509
   M B Gay Ducati, F Halzen and A A Natale, Physical Review D48 (1993) 2324

10. D Eatough and T Wyatt, private communication