An Alternative Method of Construction and Analysis of Asymmetrical Factorial Experiment of the type 6× 2^2 in Blocks of Size 12

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Abstract
This paper focuses on the construction and analysis of an extraordinary type of asymmetrical factorial experiment which corresponds to fraction of a symmetrical factorial experiment as indicated by Das (1960). For constructing this design, we have used 3 choices and for each choice we have used 5 different cases. Finding the block contents for each case we have seen that there are mainly two different cases for each choice. In case of analysis of variance, we have seen that, for the case where the highest order interaction effect is confounded in 4 replications, the loss of information is same for all the choices. Again for the case where the highest order interaction effect is confounded in 3 replications, the loss of information is also same for all the choices and one effect which is confounded due to fractionation has the same loss of information for all the choices.

Keywords: Asymmetrical Design; Confounding; Replications; Pseudo Factors; Fractionation; and Balanced Design.
1. Introduction

Construction and analysis of confounded factorial designs are not as straightforward as those of symmetrical factorial design. M. N. Das (1960) [1] developed a method of constructing asymmetrical factorial design by considering it as a fraction of symmetrical factorial design. In this investigation we have constructed a balanced confounded design of the type $6 \times 2^2$ in blocks of size 12 by a simple modification of the method of M. N. Das (1960) [1], we have introduced 3 choices according to the pseudo factors combinations and in every choice we have considered 5 cases. It has been seen that for the case 1, case 2, case 3 and case 4 in every choice, a mixed effect alone is confounded and the design has been balanced in 4 replications and for case 5 in every choice, one mixed effect is confounded and one real factor interaction effect is confounded due to fractionation. Here, the design has been balanced in 3 replications. Reviewing the existing literature we
have seen that a lot of works was done on construction and analysis of confounded asymmetrical factorial experiments. Several authors like R. H. E. Inskon (1963) [2], K. Kishen and B. N. Tyagi (1973) [3], C. P. Kartha (1967) [4], M. G. Sardana and M. N. Das (1965) [5], P. R. Sreenath and M. G. Sardana (1967) [6], M. N. Das and P. S. Rao (1967) [7], A. K. Banarjee and M. N. Das (1969) [8], B.V. Shah (1960b) [9], M. N. Das and N. C. Giri (1979) [10] gave the method of constructing and analyzing different types of confounded asymmetrical factorial experiments.

2. Materials and Methods

6 × 2² is an asymmetrical factorial design with 3 factors, one at 6 levels and other two factors each at 2 levels. We want to construct a confounded asymmetrical design in blocks of size 12.

For constructing the asymmetrical factorial experiment of the type 6× 2² in blocks of size 12 we have, P=6× 2²=24=total number of treatment combinations and R=12=block size.

Now, according to M. N. Das (1960) [1] we know that, \( \frac{P}{R} = N = s^k \), where s is a prime number and k is any positive integer.

Hence we have, \( \frac{P}{R} = N = \frac{24}{12} = 2 = 2^1 = s^k \). So, the factor at 6 levels is the factor of asymmetry and is denoted by X. The other 2 factors each at 2 levels are the 2 real factors and are denoted by A and B.
Here \( P_i > 6 \) so that is \( 6 > 2 \), so we can write \( 2^{3-1} < 6 < 2^3 \). So, we have to take 3 pseudo factors \( X_1, X_2 \) and \( X_3 \) corresponding to \( X \).

Therefore, \( M = t_0 + \sum n_i = 2 + 3 = 5 \), [where \( M \) = number of factors, \( t_0 \) = number of real factors, \( \sum n_i \) = number of pseudo factors corresponding to all the factors of asymmetry] and the corresponding symmetrical factorial experiment is \( s^M = 2^5 \) and the treatment combinations of this factorial experiment can be written as usual.

Now, since we have to consider 6 treatment combinations to define 6 levels of factor of asymmetry, we have to go for \( \frac{1}{4} \) fraction of \( 2^3 \) factorial experiment which can be obtained by considering any one of the following 3 defining contrasts:

1. \( I = X_1 = X_2 = X_1X_2 \)
2. \( I = X_1 = X_3 = X_1X_3 \)
3. \( I = X_2 = X_3 = X_2X_3 \)

From the defining contrast, we can easily understand the omitted combinations of the pseudo factors. In designs where the number of omitted combinations of the pseudo factors corresponding to a factor of asymmetry has a common multiple with the total number of combinations of the pseudo factors, different choices of the initial replication may lead to different designs.

3. Results and Discussion

Case 1: Here we consider \( X_1X_2X_3AB \) for confounding to get the replication in every choice.
Case 2: Here we consider $X_1X_3AB$ for confounding to get the replication in choice 1 and $X_1X_2AB$ for choice 2 and choice 3.

Case 3: Here we consider $X_2X_3AB$ for confounding to get the replication in choice 1 and choice 2 and $X_1X_3AB$ for choice 3.

Case 4: Here we consider $X_3AB$, $X_2AB$ and $X_1AB$ for confounding to get the replication in choice 1, choice 2 and choice 3 respectively.

Case 5: Here we consider $X_1X_2AB$, $X_1X_3AB$ and $X_2X_3AB$ for confounding to get the replication in choice 1, choice 2 and choice 3 respectively.

3.1 Choice 1:

For this choice we have considered defining contrast $I= X_1 = X_2 = X_1X_2$ and for case 1, case 2, case 3, case 4 and case 5 we have confounded the higher order interaction effects $X_1X_2X_3AB$, $X_1X_3AB$, $X_2X_3AB$, $X_3AB$ and $X_1X_2AB$ respectively to get the replications.

For case 1 where only one effect XAB corresponding to asymmetrical type is confounded in 4 replications, following the usual method of finding the block content and after recoding we get the block content for this case which is shown in Table 3.1.

Similarly, using the usual method of block content and after recoding we get same block content as like as case 1 for case 2, case 3 and case 4. But for case 5 where one effect XAB corresponding to asymmetrical type is confounded in 3 replications and one effect AB corresponding to asymmetrical type is confounded due to fractionation we get different block content which is shown in Table 3.2.
Table 3.1:

| Block-1 | Block-2 | Replication-1 | Replication-2 | Replication-3 | Replication-4 |
|---------|---------|---------------|---------------|---------------|---------------|
| 000     | 001     | 000           | 001           | 000           | 001           |
| 011     | 010     | 011           | 010           | 011           | 010           |
| 101     | 100     | 100           | 101           | 101           | 100           |
| 110     | 111     | 111           | 110           | 110           | 111           |
| 201     | 200     | 201           | 200           | 200           | 201           |
| 210     | 211     | 210           | 211           | 211           | 210           |
| 301     | 300     | 301           | 300           | 301           | 300           |
| 310     | 311     | 310           | 311           | 310           | 311           |
| 400     | 401     | 400           | 401           | 400           | 401           |
| 411     | 410     | 410           | 411           | 411           | 410           |
| 500     | 501     | 500           | 501           | 500           | 501           |
| 511     | 510     | 511           | 510           | 511           | 510           |

Table 3.2:

| Block-1 | Block-2 | Replication-1 | Replication-2 | Replication-3 |
|---------|---------|---------------|---------------|---------------|
| 000     | 001     | 000           | 001           | 000           |
| 011     | 010     | 011           | 010           | 011           |
| 101     | 100     | 100           | 101           | 101           |
| 110     | 111     | 111           | 110           | 110           |
| 201     | 200     | 201           | 200           | 200           |
| 210     | 211     | 210           | 211           | 211           |
| 300     | 301     | 300           | 301           | 300           |
| 311     | 310     | 311           | 310           | 311           |
| 401     | 400     | 400           | 401           | 400           |
| 410     | 411     | 411           | 410           | 411           |
| 501     | 500     | 500           | 500           | 501           |
| 510     | 511     | 510           | 511           | 510           |
In choice 1, since the same block content is obtained for case 1, case 2, case 3 and case 4 where the higher order interactions corresponding to symmetrical factorial have no common letters as a whole corresponding to pseudo factors in the defining contrast and balanced design is obtained by confounding only one effect XAB in 4 replications. Therefore, we can proceed with analysis considering any one of these 4 cases.

On the other hand in case 5, the higher order interaction corresponding to symmetrical factorial has common letters corresponding to pseudo factors in the defining contrast and for that a balanced design is obtained in 3 replications confounding XAB and AB is confounded due to fractionation.

Following the usual method of ANOVA, for the case where balanced design is obtained in 4 replications we get the loss of information for XAB is \((1 - \frac{4}{5}) = \frac{1}{5}\) and for AB is \((1 - \frac{8}{9}) = \frac{1}{9}\).

### 3.2 Choice 2:

For this choice we have considered defining contrast I= \(X_1 = X_3 = X_1X_3\) and for case 1, case 2, case 3, case 4 and case 5 we have confounded the higher order interaction effects \(X_1X_2X_3AB, X_1X_2AB, X_2X_3AB, X_2AB\) and \(X_1X_3AB\) respectively to get the replications.

For case 1 where only one effect XAB corresponding to asymmetrical type is confounded in 4 replications, following the usual method of finding the block
content and after recoding we get the block content for this case which is shown in Table 3.3.

Table 3.3:

| Replication-1 | Replication-2 | Replication-3 | Replication-4 |
|---------------|---------------|---------------|---------------|
| Block-1       | Block-2       | Block-1       | Block-2       |
| 000           | 001           | 000           | 001           |
| 011           | 010           | 011           | 010           |
| 101           | 100           | 101           | 100           |
| 110           | 111           | 110           | 111           |
| 201           | 200           | 201           | 200           |
| 210           | 211           | 210           | 211           |
| 300           | 301           | 300           | 301           |
| 311           | 310           | 311           | 310           |
| 401           | 400           | 401           | 400           |
| 410           | 411           | 410           | 411           |
| 500           | 501           | 500           | 501           |
| 511           | 510           | 511           | 510           |

Similarly, using the usual method of block content and after recoding we get same block content as like as case 1 for case 2, case 3 and case 4. But for case 5 where one effect XAB corresponding to asymmetrical type is confounded in 3 replications and one effect AB corresponding to asymmetrical type is confounded due to fractionation we get different block content which is shown in Table 3.4.

In choice 2, since the same block content is obtained for case 1, case 2, case 3 and case 4 where the higher order interactions corresponding to symmetrical factorial have no common letters as a whole corresponding to pseudo factors in the defining contrast and balanced design is obtained by confounding only one effect XAB in 4
replications. Therefore, we can proceed with analysis considering any one of these 4 cases.

Table 3.4:

| Replication-1 | Replication-2 | Replication-3 |
|---------------|---------------|---------------|
| Block-1       | Block-2       | Block-1       | Block-2       | Block-1       | Block-2       |
| 000           | 001           | 000           | 001           | 000           | 001           |
| 011           | 010           | 011           | 010           | 011           | 010           |
| 101           | 100           | 100           | 101           | 101           | 100           |
| 110           | 111           | 111           | 110           | 110           | 111           |
| 200           | 201           | 200           | 201           | 200           | 201           |
| 211           | 210           | 211           | 210           | 211           | 210           |
| 301           | 300           | 300           | 301           | 301           | 300           |
| 310           | 311           | 311           | 310           | 310           | 311           |
| 401           | 400           | 401           | 400           | 400           | 401           |
| 410           | 411           | 410           | 411           | 411           | 410           |
| 501           | 500           | 501           | 500           | 500           | 501           |
| 510           | 511           | 510           | 511           | 511           | 510           |

On the other hand in case 5, the higher order interaction corresponding to symmetrical factorial has common letters corresponding to pseudo factors in the defining contrast and for that a balanced design is obtained in 3 replications confounding XAB and AB is confounded due to fractionation.

Following the usual method of ANOVA, for the case where balanced design is obtained in 4 replications we get the loss of information for XAB is \((1 - \frac{4}{5})=\frac{1}{5}\) and
for the case where balanced design is obtained in 3 replications we get the loss of information for XAB is \((1 - \frac{5}{7}) = \frac{2}{7}\) and for AB is \((1 - \frac{8}{9}) = \frac{1}{9}\).

### 3.3 Choice 3:

For this choice we have considered defining contrast \(I = X_2 = X_3 = X_2 X_3\) and for case 1, case 2, case 3, case 4 and case 5 we have confounded the higher order interaction effects \(X_1 X_2 X_3\), \(X_1 X_2\), \(X_1 X_3\), \(X_1 AB\) and \(X_2 X_3 AB\) respectively to get the replications.

For case 1 where only one effect XAB corresponding to asymmetrical type is confounded in 4 replications, following the usual method of finding the block content and after recoding we get the block content for this case which is shown in Table 3.5.

| Replication-1 | Replication-2 | Replication-3 | Replication-4 |
|---------------|---------------|---------------|---------------|
| Block-1       | Block-2       | Block-1       | Block-2       |
| 000           | 001           | 000           | 001           |
| 011           | 010           | 010           | 011           |
| 101           | 100           | 100           | 101           |
| 110           | 111           | 111           | 110           |
| 201           | 200           | 201           | 200           |
| 210           | 211           | 210           | 211           |
| 300           | 301           | 300           | 301           |
| 311           | 310           | 311           | 310           |
| 401           | 400           | 400           | 401           |
| 410           | 411           | 411           | 410           |
| 500           | 501           | 501           | 500           |
| 511           | 510           | 510           | 511           |

Table 3.5:
Similarly, using the usual method of block content and after recoding we get same block content as like as case 1 for case 2, case 3 and case 4. But for case 5 where one effect XAB corresponding to asymmetrical type is confounded in 3 replications and one effect AB corresponding to asymmetrical type is confounded due to fractionation we get different block content which is shown in Table 3.6.

Table 3.6:

| Replication-1 | Replication-2 | Replication-3 |
|---------------|---------------|---------------|
| Block-1 | Block-2 | Block-1 | Block-2 | Block-1 | Block-2 |
| 000 | 001 | 000 | 001 | 000 | 001 |
| 011 | 010 | 011 | 010 | 011 | 010 |
| 100 | 101 | 100 | 101 | 100 | 101 |
| 111 | 110 | 111 | 110 | 111 | 110 |
| 201 | 200 | 200 | 201 | 201 | 200 |
| 210 | 211 | 211 | 210 | 210 | 211 |
| 301 | 300 | 300 | 301 | 301 | 300 |
| 310 | 311 | 311 | 310 | 310 | 311 |
| 401 | 400 | 401 | 400 | 400 | 401 |
| 410 | 411 | 410 | 411 | 411 | 410 |
| 501 | 500 | 501 | 500 | 500 | 501 |
| 510 | 511 | 510 | 511 | 511 | 510 |

In choice 3, since the same block content is obtained for case 1, case 2, case 3 and case 4 where the higher order interactions corresponding to symmetrical factorial have no common letters as a whole corresponding to pseudo factors in the defining contrast and balanced design is obtained by confounding only one effect XAB in 4 replications. Therefore, we can proceed with analysis considering any one of these 4 cases.
On the other hand in case 5, the higher order interaction corresponding to symmetrical factorial has common letters corresponding to pseudo factors in the defining contrast and for that a balanced design is obtained in 3 replications confounding XAB and AB is confounded due to fractionation.

Following the usual method of ANOVA, for the case where balanced design is obtained in 4 replications we get the loss of information for XAB is \((1 - \frac{4}{5}) = \frac{1}{5}\) and for the case where balanced design is obtained in 3 replications we get the loss of information for XAB is \((1 - \frac{5}{7}) = \frac{2}{7}\) and for AB is \((1 - \frac{8}{9}) = \frac{1}{9}\).

4. Conclusions

Considering all the choices, we have seen that the alias structures as well as block contents in different choices are different. Here, XAB is confounded in 4 replications for the case where highest order interaction has no common letters corresponding to pseudo factors in the defining contrast and through the usual method of analysis we have observed that, the loss of information for XAB is \(\frac{1}{5}\) which is same for all the choices for this case. Again, XAB is confounded in 3 replications and AB is confounded due to fractionation for the case where highest order interaction has common letters corresponding to pseudo factors in the defining contrast and by the usual method of analysis we have seen that, the loss of information for XAB is \(\frac{2}{7}\) which is also same for all the choices and one effect AB is confounded due to fractionation has loss of information \(\frac{1}{9}\) for all the choices. As a result we can consider any one of the choices for constructing asymmetrical
factorial experiment of the type $6 \times 2^2$ in 12 plots block. Since the loss of information in case of confounded design in 4 replications is less as well as no two factor interactions is confounded due to fractionation. Therefore, considering this case we can get most efficient design of the type $6 \times 2^2$ in blocks of size 12 in 4 replications.

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