Lepton-Flavor Violation in the Supersymmetric Standard Model with Seesaw-Induced Neutrino Masses

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Abstract

We examine the lepton-flavor violation caused by a Yukawa coupling matrix \( y_{\nu,ij} \) for right-handed neutrinos in the supersymmetric standard model. We stress that decay rates for \( \tau \to \mu\gamma \) and \( \mu \to e\gamma \) may reach the range to be accessible to near future experiments if left-right mixing terms in the slepton mass matrix are substantially large.
A small, but non-vanishing neutrino mass, if any, is regarded as an important indication of a new physics beyond the standard model. The most interesting candidate is the well-known seesaw model \[1\] that explains very naturally the small mass for neutrino in terms of a large Majorana mass for a right-handed neutrino \(N\). This model has attracted many authors not only because of its natural structure but also because the presence of \(N\) would illuminate some of the deep questions in particle physics.

From the phenomenological point of view, on the other hand, introduction of three families of the right-handed neutrinos \(N^i\) (where \(i\) and \(j\) are flavor indices) brings two new ingredients to the standard model; one is a new scale of the Majorana masses for \(N^i, M_I\), and the other a new matrix for Yukawa coupling constants of \(N^i, y_{\nu,ij}\). Thus, we have two independent Yukawa matrices in the lepton sector as in the quark sector. In general, a simultaneous diagonalization of the both matrices is quite accidental and then the addition of the new Yukawa coupling \(y_{\nu,ij}\) causes a lepton-flavor violation. In the standard model, however, the amplitudes for the lepton-flavor violating processes at low energies are suppressed by an inverse power of \(M_I\) at least and hence we do not expect sizable rates for such processes as far as \(M_I\) is very large. The neutrino oscillation is a famous exception in this point of view.

It has been, however, noted by several authors \[2\] that in the supersymmetric standard model (SSM) the Yukawa coupling \(y_{\nu,ij}\) of \(N^i\) generates off-diagonal entries in the mass matrices for sleptons through the renormalization effects, which leads to unsuppressed lepton-flavor violations such as \(\tau \to \mu \gamma, \mu \to e \gamma\), and so on. The predicted rates for the individual processes depend on the unknown Yukawa matrix \(y_{\nu,ij}\). If off-diagonal elements of \(y_{\nu,ij}\) are very small like in the quark sector, however, the reaction rates for these processes are predicted too small to be accessible to the near future experiments.

The purpose of this letter is to point out that left-right mixing terms in the slepton mass matrix may give larger contributions to the lepton-flavor violating processes, which have not been considered in the previous literatures \[2\]. We show, as a consequence, that the decay rates for \(\tau \to \mu \gamma\) and \(\mu \to e \gamma\) can reach indeed the range close to the present experimental upper limits \(\text{Br}(\tau \to \mu \gamma) \leq 4.2 \times 10^{-6}\) and \(\text{Br}(\mu \to e \gamma) \leq 4.9 \times 10^{-11}\) \[4\] even

\footnote{It has been pointed out very clearly \[3\] that the similar lepton-flavor violation occurs in the SUSY grand unified theories (GUTs).}
when the off-diagonal elements of $y_{\nu,ij}$ are small as $y_{\nu,23}/y_{\nu,33} \sim 0.04$ and $y_{\nu,13}/y_{\nu,33} \sim 0.01$.

In the SSM with three families of right-handed neutrinos $N^i$ ($i = 1 - 3$), the superpotential $W$ for the lepton sector is given by,

$$ W = y_{e,ij} \bar{E}^i L^j H_1 + y_{\nu,ij} N^i L^j H_2 + \frac{1}{2} M_{I,ij} N^i N^j, $$

where $L^i$, $\bar{E}^i$ and $N^i$ are the chiral multiplets for left-handed lepton doublets, right-handed lepton singlets and right-handed neutrinos, and $H_1$ and $H_2$ those for the Higgs doublets.

For simplicity, we assume the mass matrix $M_{I,ij}$ is proportional to the unit matrix as

$$ M_{I,ij} = M_I \begin{pmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{pmatrix} $$

The analysis with the general case $M_{I,ij} = M_I \delta_{ij}$ is straightforward, but as long as $M_{Ii} \sim M_{Ij}$ the conclusion will not be much different from that in the simplest case (2).

We choose a basis on which $y_{e,ij}$ is diagonal at the electroweak scale $m_Z$,

$$ y_{e,ij}(\mu = m_Z) = y_{e,i} \delta_{ij}. $$

To demonstrate our main point, we assume tentatively that the Yukawa couplings of $N$ are identical to those of the up-type quarks, similar to the case of SO(10) unification.

$$ y_{\nu,ij} = y_{u,ij}. $$

We suppose that these relations hold at the gravitational scale $\mu = M_G \simeq 2.4 \times 10^{18}\text{GeV}$. With the basis where the Yukawa couplings for the down-type quarks are flavor-diagonal at the electroweak scale, $y_{d,ij} = y_{d,i} \delta_{ij}$, the $y_{u,ij}$ is written as

$$ y_{u,ij} = V_K^T \begin{pmatrix} y_u \\ y_c \\ y_t \end{pmatrix} V_{KM}, $$

where $V_{KM}$ is the Kobayashi-Maskawa matrix. The meaning of the Yukawa couplings of the up-type quarks $y_u$, $y_c$ and $y_t$ is the obvious one.

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2We do not use the other SO(10)-like relation $y_{e,ij} = y_{d,ij}$ for the down-type quarks, since it gives wrong results on the masses for the first and second families.
The seesaw mechanism \([1]\) induces small neutrino masses as\(3\)
\[
m_{\nu,ij} \simeq \frac{1}{M_{I,k}} y_{\nu,ik} y_{\nu,kj} \langle H_2 \rangle^2
\]
\[
= \frac{(H_2)^2}{M_I} V_{v}^T V_{v} V_{v}^T V_{l} \left( \begin{array}{ccc}
 y_u & y_c & y_t \\
 y_u & y_c & y_t \\
 y_u & y_c & y_t \\
\end{array} \right) V_{l}
\]  
\(6\)

We see that the mixing matrix appearing in the neutrino oscillation is approximately identical with \(V_{KM}\), due to the large hierarchy \(y_u \ll y_c \ll y_t\) \(5\). For a given \(M_I\), we determine the mass eigenvalues and the mixing angles for the neutrinos. We adjust \(M_I \sim 10^{13}\)GeV so that the mass for \(\nu_\tau\) is predicted to be \(m_{\nu_\tau} = 10\)eV \(\text{[2]}\) which lies in an interesting range in cosmology \(\text{[6]}\). To estimate the Yukawa matrix \(y_{u,ij}(\mu = M_G)\) we use the observed values \(m_u(1\text{GeV}) = 4.5\text{MeV}, m_c(1\text{GeV}) = 1.27\text{GeV}\) \(\text{[9]}\), \(m_t = 174\text{GeV}\) \(\text{[10]}\) and \(V_{KM}\). In our numerical calculations, we use the central value of each matrix element \(V_{KM,ij}\) given in ref. \(\text{[4]}\).

We are now in a position to discuss the lepton-flavor violation. In the minimal SUSY standard model (MSSM), soft SUSY-breaking mass terms for sleptons have the general form,
\[
L_{\text{soft}} = -m_{E,ij}^2 \tilde{E}^j \tilde{E}^i - m_{L,ij}^2 \tilde{L}^i \tilde{L}^j - \left( A_{e,ij} \tilde{E}^j \tilde{L}^i H_1 + h.c. \right).
\]  
\(7\)

The lepton-flavor conservation is easily violated by taking non-vanishing off-diagonal elements of each matrices and the sizes of such elements are strongly constrained from experiments. In the SUSY standard model based on the supergravity \(\text{[11]}\), it is therefore assumed that the mass matrices \(m_E^2\) and \(m_L^2\) are proportional to the unit matrix, and \(A_{e,ij}\) is proportional to the Yukawa matrix \(y_{e,ij}\). With these soft terms, the lepton-flavor number is conserved exactly.

\(3\)This equation holds only at tree-level. In our numerical analysis, we take into account the renormalization effects.

\(4\)In this case, the mass for \(\nu_\mu\) is predicted as \(\sim 10^{-3}\)eV, which is the right value for the MSW solution to the solar neutrino problem \(\text{[7]}\). However, the predicted mixing angle \(\theta_{\nu_e,\nu_\mu} \simeq \theta_{\text{Cabibbo}}\) is much larger than that in the small angle solution of MSW \(\text{[3]}\). Therefore, to account for the solar neutrino deficit, we must use smaller value for \(y_{\nu,12}\). However, our main conclusions for \(\tau \rightarrow \mu \gamma\) and \(\mu \rightarrow e \gamma\) given in the text are unchanged, since the Yukawa coupling \(y_{\nu,12}\) is almost irrelevant for these processes as noted in ref. \(\text{[3]}\).
It is, however, not true if the effects of the right-handed neutrinos are taken into account \[^2\]. The Yukawa coupling of neutrino \( y_{\nu,ij} N^i L^j H_2 \) in eq. (1) and the soft SUSY-breaking terms such as

\[
\mathcal{L}_{\text{soft},\nu} = -m_{N,ij}^2 \bar{N}^i \tilde{N}^j - \left( A_{\nu,ij} \bar{N}^i \tilde{L}^j H_2 + \text{h.c.} \right)
\]

induce off-diagonal elements of \( m_L^2 \) through the radiative corrections. The renormalization group equation (RGE) for \( m_L^2 \) is given by

\[
\frac{d m_{L,ij}^2}{d \ln \mu} = \left( \frac{d m_{L,ij}^2}{d \ln \mu} \right)_{\text{MSSM}} + \frac{1}{16\pi^2} \left[ (m_L^2 y_{\nu}^\dagger y_{\nu})_{ij} + (y_{\nu}^\dagger y_{\nu} m_L^2)_{ij} + 2(y_{\nu}^\dagger y_{\nu})_{ij} m_{H_2}^2 \right. \\
\left. + (y_{\nu}^\dagger m_{N,ij}^2 y_{\nu})_{ij} + 2(A_{\nu}^\dagger A_{\nu})_{ij} \right],
\]

where \( (d m_{L,ij}^2 / d \ln \mu)_{\text{MSSM}} \) represents the \( \beta \)-function in the MSSM \[^4\] and \( m_{H_2}^2 \) is the soft SUSY breaking mass for the Higgs doublet \( H_2 \). As one can see easily in eq. (9), off-diagonal elements of \( m_{L,ij}^2 \) are induced by the renormalization effects if non-vanishing off-diagonal elements of \( y_{\nu} \) and \( A_{\nu} \) exist.

In order to obtain the slepton mass matrix \( m_{L,ij}^2 \) at the electroweak scale, we solve the RGEs for the full relevant parameters numerically. At the energy scale \( M_I \leq \mu \leq M_G \) we use the RGEs derived from the MSSM with the right-handed neutrinos, and below the energy scale \( M_I \) the MSSM-RGEs without the right-handed neutrinos. For the soft SUSY-breaking parameters, we assume the following boundary conditions suggested in the minimal supergravity \[^1\], \[^3\];

\[
m_{f,ij}^2(\mu = M_G) = m_0^2 \delta_{ij},
\]

\[
A_{f,ij}(\mu = M_G) = a m_0 y_{f,ij},
\]

where \( m_{f,ij}^2 \) is the soft SUSY-breaking mass matrix for sfermion \( \tilde{f} \) (with \( f = u, d, e \) and \( \nu \)), \( A_{f,ij} \) the so-called \( A \)-parameter, \( y_{f,ij} \) the Yukawa coupling constants for the fermion \( f \), \( m_0 \) the universal SUSY-breaking mass, and \( a \) is a free parameter of \( O(1) \). For gaugino masses, we use tentatively the relation implied by the SUSY GUTs \[^3\]

\[
\frac{m_{G3}}{g_3^2} = \frac{m_{G2}}{g_2^2} = \frac{3}{5} \times \frac{m_{G1}}{g_1^2},
\]

See ref. \[^2\] for explicit formulae.
where $m_{G3}$, $m_{G2}$, $m_{G1}$, and $g_3$, $g_2$, $g_1$ are the gaugino masses and gauge coupling constants for the gauge groups $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$, respectively.

The soft SUSY breaking mass matrix $m^2_{L,ij}$ is determined by solving the coupled RGEs for all relevant parameters. Since $\tan \beta (\equiv \langle H_2 \rangle / \langle H_1 \rangle)$ dependence of the result is relatively mild, we take $\tan \beta = 3$ for the time being. The result for the larger $\tan \beta$ will be given later for comparison. For the case $0 \leq m_{G2} \leq m_0$, the mass matrix $m^2_{L,ij}$ at the electroweak scale is given by

$$m^2_L(\mu = m_Z) \simeq \begin{pmatrix} (1.0 - 2.0) & -(0.9 - 1.1) \times 10^{-4} & -(2.3 - 2.7) \times 10^{-3} \\ -(0.9 - 1.1) \times 10^{-4} & (1.0 - 2.0) & -(1.0 - 1.2) \times 10^{-2} \\ -(2.3 - 2.7) \times 10^{-3} & -(1.0 - 1.2) \times 10^{-2} & (0.75 - 1.69) \end{pmatrix}$$

where the magnitude of each entry becomes larger as $m_{G2}$ larger. However, for the case $m_{G2}(\mu = m_Z) > m_0$, ratios of the off-diagonal elements to the diagonal ones become smaller than those in eq.(13), which give a more suppression of the lepton-flavor violation as a result.

We are now ready to calculate the reaction rates for various lepton-flavor violating processes. Let us first consider the diagrams considered in ref.[4] contributing to the process $l_i \to l_j \gamma$. In this paper, we ignore the mixings in the neutralino and chargino sectors and consider that the bino and the winos are mass eigenstates. This approximation is justified if the SUSY-invariant higgsino mass $\mu_H$ is large, which is nothing but a situation we will consider in the present paper.

From the diagrams in fig. 4, we obtain the amplitude for this process as

$$\mathcal{M}(l_i \to l_j \gamma) = C_{LL} \tilde{T}_j P_R[\tilde{f}, f] l_i,$$

where $q$ and $\epsilon$ are the momentum and polarization vector of the emitted photon. Here, the coefficient $C_{LL}$ is given by

$$C_{LL} = \frac{1}{16\pi^2} \frac{e m_l}{m_\ell} \left\{ \frac{m^2_{L,ij}}{m^4_{\tilde{e}_L}} g^2 S_e (m^2_{G1}/m^2_{\tilde{e}_L}) \right. \right.$$  

$$\left. + \frac{m^2_{L,ij}}{m^4_{\tilde{\nu}_L}} g^2 S_\nu (m^2_{G2}/m^2_{\tilde{\nu}_L}) + \frac{m^2_{L,ij}}{m^4_{\tilde{\nu}_L}} g^2 S_\nu (m^2_{G2}/m^2_{\tilde{\nu}_L}) \right\}. \tag{15}$$
where $m_{l_i}$, $m_{\tilde{e}_L}$ and $m_{\tilde{\nu}_L}$ represent masses for the lepton $l_i$, charged $\tilde{e}_L$ and neutral sleptons $\tilde{\nu}_L$, respectively, and the functions $S_e$ and $S_\nu$ are given by

$$S_e(x) = \frac{1}{48(x-1)^5} \{17x^3 - 9x^2 - 9x + 1 - 6x^2(x+3)\ln x\}, \quad (16)$$
$$S_\nu(x) = -\frac{1}{12(x-1)^5} \{-x^3 - 9x^2 + 9x + 1 + 6x(x+1)\ln x\}. \quad (17)$$

Then, the decay rate can be obtained from the amplitude (14) as

$$\Gamma(l_i \rightarrow l_j \gamma) = \frac{1}{4\pi} |C_{LL}|^2 m_{l_i}^3, \quad (18)$$

By using the $m_{L,ij}^2$ given in eq.(13), we have calculated the branching ratios of the processes $\tau \rightarrow \mu \gamma$ and $\mu \rightarrow e \gamma$. We have checked that for $m_{G2} = 45\text{GeV}$, the branching ratios for each processes have the maximums $O(10^{-9})$ and $O(10^{-12})$, respectively, at the possible minimum value of the slepton mass $\simeq 45\text{GeV}$, which are, however, much smaller than the present experimental limits ($\text{Br}(\tau \rightarrow \mu \gamma) \leq 4.2 \times 10^{-6}$ and $\text{Br}(\mu \rightarrow e \gamma) \leq 4.9 \times 10^{-11}$ [4]).

So far, we have neglected the left-right mixings in the slepton mass matrix. However, if the SUSY invariant mass $\mu_H$ for Higgs doublets is so big that $m_\tau \mu_H \tan \beta \sim O(m_0^2)$, we have non-negligible left-right mixings in the slepton mass matrix. As we will show below, these mixing terms give rise to contributions to the $l_i \rightarrow l_j \gamma$ process larger than the previous estimate in eq.(18). The main reason for this is that the chirality flip in the fermion line occurs at the internal gaugino mass term (see fig. 2) while in the previous case it occurs at the external lepton mass term. Since the gaugino mass is much bigger than the lepton mass, these new diagrams may yield dominant contributions.

Before calculating the diagrams in fig. 2, we first diagonalize the charged slepton mass matrix given by the following $6 \times 6$ matrix;

$$\mathcal{L}_{\text{mass}} = -(\tilde{L}^\dagger, \tilde{E}) \left( \begin{array}{cc} m_{LL}^2 & m_{LR}^2 \\ m_{LR}^2 & m_{RR}^2 \end{array} \right) \left( \begin{array}{c} \tilde{L} \\ \tilde{E}^\dagger \end{array} \right), \quad (19)$$

with

$$m_{LL,ij}^2 = m_{L,ij}^2 + m_Z^2 \cos 2\beta \left( \sin^2 \theta_W - \frac{1}{2} \right) \delta_{ij}, \quad (20)$$
$$m_{RR,ij}^2 = m_{E,ij}^2 - m_Z^2 \sin 2\beta \sin \theta_W \delta_{ij}, \quad (21)$$
$$m_{LR,ij}^2 = m_{l_i} \delta_{ij} \mu_H \tan \beta + A_{E,ij} v \cos \beta / \sqrt{2}. \quad (22)$$

\(^6\text{We have assumed } m_{l_i} \gg m_{l_j}.\)
where $m_Z$ is the $Z$-boson mass and $\sin^2 \theta_W$ the Weinberg angle. Notice that as far as the $a$-parameter in eq. (11) is $O(1)$, the $A_{E,ij}$ is negligibly small in the case of $\mu_H \tan \beta \gg m_0$, which concerns us. $m_{E,ij}^2$ at the electroweak scale are determined by solving the RGEs in the same way of the previous calculation for $m_{L,ij}^2$, and are found to be $m_{E,ij}^2 \simeq (1.0 - 1.4)m_0^2 \delta_{ij}$.

With the diagonalization matrix $U$ for eq. (13) and the eigenvalues $m_{e,A}^2 (A = 1 - 6)$, we can write the amplitude for the process $l_i \rightarrow l_j \gamma$ as

$$M(l_i \rightarrow l_j \gamma) = C_{LR} U_l^T P_R [\hat{A}, \hat{f}] l_i,$$  

where the coefficient $C_{LR}$ is given by

$$C_{LR} = \frac{1}{16\pi^2} g_1^2 m_{G1} \sum_A U_{l,i} U_{l,j}^\dagger T_{LR}(m_{G1}^2/m_{e,A}^2),$$  

with

$$T_{LR}(x) = \frac{1}{4(x - 1)^3} (x^2 - 1 - 2x \ln x).$$

(24)

We find that in the case of small $m_{LR}^2$, this amplitude (24) is well approximated by

$$C_{LR}|_{\text{mass-insertion}} = \frac{1}{16\pi^2} g_1^2 m_{G1} \sum_k U_{L,j} U_{R,k}^\dagger \left[ \frac{1}{m_{e,Ri}^2 - m_{e,LR}^2} T_{LR} \left( m_{G1}^2/m_{e,Ri}^2 \right) - \frac{1}{m_{e,LR}^2} T_{LR} \left( m_{G1}^2/m_{e,LR}^2 \right) \right],$$  

which is obtained by using the $m_{LR}^2$ mass insertion. Here, $U_L$ is the diagonalization matrix for the mass matrix of the left-handed sleptons (20), $m_{e,LR}^2$ its eigenvalues and $m_{e,Ri}^2 = m_{e,RR,ii}^2$ in eq.(21). From the amplitude (23), we get the decay rate as

$$\Gamma(l_i \rightarrow l_j \gamma) = \frac{1}{4\pi} |C_{LR}|^2 m_{l_i}^3,$$  

(27)

which should be compared with the result in eq.(18).

For a given set of $\tan \beta$ and $m_{G2}$, the decay rates depend basically on the parameters $\mu_H$ and $m_0$. However, it is convenient to express them in terms of two physical parameters, $m_{\bar{\tau}_1}$ and $m_{\bar{\tau}_2}$ ($m_{\bar{\tau}_1} < m_{\bar{\tau}_2}$), which are mass eigenvalues for the sleptons belonging mainly to the $\tau$-family. The results are shown in fig. 3 for $\tan \beta = 3$, with $m_{G2} = 45$GeV and
$m_{\tilde{\tau}_1} = 50\text{GeV}$ being fixed. For a comparison, we also show the results for the case of $\tan\beta = 30$. We easily see that the branching ratios for $\tau \to \mu\gamma$ and $\mu \to e\gamma$ are predicted as

$$\text{Br}(\tau \to \mu\gamma) \simeq 3 \times 10^{-8} - 4 \times 10^{-7},$$

(28)

$$\text{Br}(\mu \to e\gamma) \simeq 5 \times 10^{-12} - 2 \times 10^{-11},$$

(29)

for $m_{\tilde{\tau}_2} \simeq (100 - 250)\text{GeV}$. It should be stressed that the large branching ratios in eqs. (28) and (29) are never obtained in the case where the left-right mass-insertion is applicable ($\mu_H \tan\beta \lesssim m_0$). To see how the results depend on $m_{\tilde{\tau}_1}$ and the gaugino mass, we show our results in fig. 4 taking $m_{\tilde{\tau}_1} = 100\text{GeV}$ and $m_{G_2} = 90\text{GeV}$. From figs. 3 and 4, we see that the obtained branching ratios are much larger than the previous estimates from eq. (18) and they lie in the range which will be studied experimentally in no distant future.

We should note that our results shown in figs. 3 and 4 are also larger than the SUSY-GUT predictions [3]. This is because the authors in ref. [3] have not taken into account the left-right mixing effects. However, if these effects are induced, the similar conclusion may be obtained even in the SUSY SU(5) GUTs.

We now briefly discuss other lepton-flavor violating processes such as $\mu \to 3e$ and $\tau \to 3\mu$. With the parameter space discussed in the present paper, there also appears an enhancement factor $m_{G_2}/m_\tau$ in the Penguin diagrams, which will give dominant contributions to these processes. In this case, the decay rates of $\mu \to 3e$ and $\tau \to 3\mu$ have simple relations to those of the $\mu \to e\gamma$ and $\tau \to \mu\gamma$ processes as

$$\frac{\text{Br}(\mu \to 3e)}{\text{Br}(\mu \to e\gamma)} \simeq \frac{2\alpha}{3\pi} \ln \frac{m_\mu}{m_e} \simeq 0.8 \times 10^{-2},$$

(30)

$$\frac{\text{Br}(\tau \to 3\mu)}{\text{Br}(\tau \to \mu\gamma)} \simeq \frac{2\alpha}{3\pi} \ln \frac{m_\tau}{m_\mu} \simeq 0.4 \times 10^{-2},$$

(31)

where we have taken only the logarithmic contributions. As for the $\mu$-$e$ conversion in nuclei, the amplitudes of the box diagrams depend on the squark masses. Thus, in the large-squark-mass region, the box diagrams yields negligible contributions. The detailed

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7The minimum value ($m_{\tilde{\tau}_2} \simeq 100\text{GeV}$) comes from the constraint that the lightest sneutrino should be heavier than 41.7GeV [4].
analysis on various lepton-flavor violating processes including the \(\mu\)-\(e\) conversion will be given in the future publication [14].

Several comments are in order.

i) We have considered the parameter space where \(m_{\tilde{\tau}_1} \simeq (50 - 100)\text{GeV}\) \(m_{\tilde{\tau}_2} \simeq (100 - 250)\text{GeV}\) and \(\tan \beta = 3 - 30\). These parameters correspond to the original parameters \(\mu_H\) and \(m_0\) as

\[
\mu_H = \begin{cases} (1 - 5)\text{TeV} & : \tan \beta = 3, \\ (100 - 500)\text{GeV} & : \tan \beta = 30, \end{cases}
\]

and

\[
m_0 = (100 - 250)\text{GeV} : \tan \beta = 3 - 30.
\]

With such large \(\mu_H\), the radiative electroweak symmetry breaking [15] hardly occurs as long as the universal soft SUSY-breaking mass in eq. (10) is imposed. A solution to this difficulty is to abandon the GUT-like relation among the gaugino masses in eq. (12) so that the gluino gives rise to much larger masses for squarks than those for sleptons at the electroweak scale.

ii) One may expect that the hierarchy in the stau masses \((m_{\tilde{\tau}_1} \ll m_{\tilde{\tau}_2})\) will give a large contribution to \(\rho\)-parameter. We find, however, that their contribution is not substantial as far as the mass of the heavier stau is less than 500 GeV for \(m_{\tilde{\tau}_1} = 50\text{GeV}\).

iii) The large \(\mu_H\) \(\tan \beta\) induces also a large left-right mixing in the sbottom sector. We find that in some of the parameter space in eqs. (32) and (33), the lighter sbottom mass \(m_{\tilde{b}_1}\) may come in the region excluded experimentally \((m_{\tilde{b}_1} \lesssim 45\text{GeV})\). However, this problem can be easily solved by giving the gluino a mass larger than expected from the GUT-like relation (12), since in this case the lighter sbottom can be lifted above 45 GeV through the radiative corrections. This is also favorable for suppressing the \(b \rightarrow s\gamma\) decay substantially as will be discussed in ref. [14].

iv) As pointed out in ref. [3], the similar lepton-flavor violation may also occur in the framework of SUSY SU(5) GUTs. It should be noted here that in the present model the soft SUSY breaking masses \(m_{L,ij}^2\) for left-handed sleptons receive significant radiative corrections from the new Yukawa couplings \(y_{\nu,ij}\), while in the SUSY SU(5) case, those for right-handed sleptons, \(m_{E,ij}^2\), are subject to the large renormalization effects. Thus, in
the present model, the \( i = j = 3 \) element of \( m_{L,ij}^2 \) becomes smaller than other diagonal elements, \( m_{L,11}^2 \) and \( m_{L,22}^2 \), by amount of \( O((10 - 30)\%) \) [16] as was shown in eq.(13), whereas in the SUSY SU(5) GUTs the similar mass shift appears in the right-handed slepton sector. Therefore, if the soft SUSY-breaking parameters are precisely determined in future experiments, these two scenarios may be distinguished.

**Note added**

After completing this work we became aware that in the very recent paper [17] the lepton-flavor violation due to the Yukawa couplings \( y_{\nu,ij} \) are also examined in the context of SUSY SO(10) grand unification. The \( l_i \rightarrow l_j \gamma \) process is calculated there by using the \( m_{LR}^2 \) mass-insertion, and our formula in eq. (26) is consistent with their result of the \( y_{\nu,ij} \) Yukawa coupling effects. This is, however, only an approximation of our full formula (24) in the small \( m_{LR}^2 \) region.
References

[1] T. Yanagida, in Proceedings of the Workshop on Unified Theory and Baryon Number of the Universe, eds. O. Sawada and A. Sugamoto (KEK, 1979) p.95;
M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D. Freedman (North Holland, Amsterdam, 1979).

[2] F. Borzumati and A. Masiero, Phys. Rev. Lett. 57 (1986) 961;
G.K. Leontaris, K. Tanvakis and J.D. Vergados, Phys. Lett. B171 (1986) 412.

[3] R. Barbieri and L.J. Hall, Phys. Lett. B338 (1994) 212; and references therein.

[4] Particle Data Group, Phys. Rev. D50 (1994) 1173.

[5] T. Yanagida and M Yoshimura, Phys. Lett. B97 (1980) 99;
G. Branco and A. Masiero, Phys. Lett. B97 (1980) 95.

[6] E.W. Kolb and M.S. Turner, The Early Universe (Addison-Wesley, 1990).

[7] S. Mikheyev and Y. Smirnov, Nuovo Cimento 9C (1986) 17;
L. Wolfenstein, Phys. Rev. D17 (1978) 2369.

[8] See for example, M. Fukugida and T. Yanagida, in Physics and Astrophysics of Neutrinos, eds. M. Fukugida and A. Suzuki (Springer Varlog, Tokyo, Japan), p.1.

[9] J. Gasser and H. Leutwyler, Phys. Rep. C87 (1982) 77.

[10] CDF Collaboration, Phys. Rev. D50 (1994) 2966.

[11] E. Cremmer, S. Ferrara, L. Grardello and A. van Proeyen, Nucl. Phys. B212 (1983) 413.

[12] S.P. Martin and M.T. Vaughn, Phys. Rev. D50 (1994) 2282.

[13] H.P. Nilles, Phys. Rep. 110 (1984) 1.

[14] J. Hisano, T. Moroi, K. Tobe and M. Yamaguchi, in preparation.

[15] K. Inoue, A. Kakuto, H. Komatsu and S. Takeshita, Prog. Theor. Phys. 68 (1982) 927.

[16] T. Moroi, Phys. Lett. B321 (1994) 56.
[17] R. Barbieri, L.J. Hall and A. Strumia, Preprint IFUP-TH 72/94, (January 1995, hep-ph/9501334).
Figure 1: Feynman diagrams which give rise to $l_i \rightarrow l_j \gamma$. In each diagram, the blob indicates the flavor-violating mass insertion of the left-handed slepton and at the cross mark the external lepton flips its chirality. The symbols $\tilde{e}_{Li}$, $\tilde{\nu}_{Li}$, $\tilde{B}$, $\tilde{W}_3$, and $W^-$ represent left-handed charged sleptons, left-handed sneutrinos, bino, neutral wino, and charged wino, respectively.

Figure 2: Feynman diagram which gives rise to $l_i \rightarrow l_j \gamma$. The blobs indicate insertions of the flavor-violating mass ($m^2_{L,ij}$) and the left-right mixing mass ($m^2_{LR,ii}$), and at the cross mark chirality flip of the bino ($\tilde{B}$) occurs. The symbols $\tilde{e}_{Li}$ and $\tilde{e}_{Ri}$ represent left-handed charged sleptons and right-handed charged sleptons, respectively. Notice, however, that we do not use the mass-insertion method in our calculation as stressed in the text.

Figure 3: Branching ratios for the processes $\tau \rightarrow \mu \gamma$ and $\mu \rightarrow e \gamma$ as functions of $m_{\tilde{\tau}_2}$. The solid lines correspond to $Br(\tau \rightarrow \mu \gamma)$ and the dashed lines to $Br(\mu \rightarrow e \gamma)$. Here, we have taken $m_{G2} = 45$GeV and $m_{\tilde{\tau}_1} = 50$GeV. We also show the present experimental upper bounds for each processes by the solid lines with hatches.

Figure 4: Same as fig. 3 except for $m_{G2} = 90$GeV and $m_{\tilde{\tau}_2} = 100$GeV.