Distortion-Invariant Pattern Recognition with Adaptive Correlation Filters

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1. Introduction

Pattern recognition based on correlation is one of the most useful techniques for many applications. Since the pioneer work of VanderLugt (1964), correlation filters have gained popularity thanks to their shift-invariance property, good mathematical basis, and easy implementation by means of digital, optical or hybrid optical/digital systems. However, conventional correlation filters are sensitive to intensity signal degradations (blurring and noise) as well as to geometrical distortions of an object of interest. Basically, blurring is owing to image formation process, and it can be produced by imperfection of capturing devices, relative motion between a camera and an input scene, propagation environment, etc. An observed input scene always contains noise produced by an imaging system (i.e. imperfection of imaging sensors) or by a recording medium (i.e. quantization errors) (Bertero & Boccacci, 1998; Perry et al., 2002). On the other hand, geometric distortions change the information content and, therefore, affect the accuracy of recognition techniques.

Two types of geometric distortions are distinguished: internal and external distortions. The internal distortions are produced by the geometrics of a sensor; they are systematic and can be corrected by a calibration. External distortions affect the sensor position or the object shape; they are unpredictable (Starck et al., 1998).

This chapter treats the problem of distortion-invariant pattern recognition based on adaptive composite correlation filters. The distinctive feature of the described methods is the use of an adaptive approach to the filters design (Diaz-Ramirez et al., 2006; González-Fraga et al., 2006). According to this concept, we are interested in a filter with good performance characteristics for a given observed scene, i.e., with a fixed set of patterns or a fixed background to be rejected, rather than in a filter with average performance parameters over an ensemble of images. Specifically, we treat two problems: reliable recognition of degraded objects embedded into a linearly degraded and noisy scene (Ramos-Michel & Kober, 2007) and adaptive recognition of geometrically distorted objects in blurred and noisy scenes (Ramos-Michel & Kober, 2008).

The first problem concerns with the design of optimum generalized filters to improve the recognition of a distorted object embedded into a nonoverlapping background noise when the input scene is degraded with a linear system and noise. The obtained filters take into account explicitly information about an object to be recognized, background noise, linear system degradation, linear target distortion, and sensor noise. For the filter design, it is
assumed that this information is available or can be estimated from the nature of degradations. Therefore, the proposed filters establish upper bounds of patterns recognition quality among correlation filters with respect to the used criteria when the input scene and the target are degraded. The second problem is to decide on presence or absence of a geometrically distorted object embedded on a degraded and noisy scene. Since the performance of conventional correlation filters degrades rapidly with object distortions, one of the first attempts to overcome the problem was the introduction of synthetic discriminant functions (SDFs) (Casasent, 1984). However, conventional SDF filters often possess a low discrimination capability. New adaptive SDF filters for reliable recognition of a reference in a cluttered background designed on the base of optimum generalized filters are presented. The information about an object to be recognized, false objects, and background to be rejected is utilized in the proposed iterative training procedure. The designed correlation filter has a prespecified value of discrimination capability. The synthesis of adaptive filters also takes into account additive sensor noise by training with a noise realization. Therefore, the adaptive filters may possess a good robustness to the noise. Computer simulation results obtained with the proposed filters are compared with those of various correlation filters in terms of recognition performance.

2. Generalized correlation filters for pattern recognition in degraded scenes

In pattern recognition two different tasks are distinguished: detection of objects and estimation of their exact positions (localization) in images. Using a correlation filter, these tasks can be done in two steps. First, the detection is carried out by searching the highest correlation peak at the filter output, then, this coordinate is taken as the position estimation of a target in the input scene. The quality of detection and localization of a target may be limited by: (i) presence of additive and disjoint background noise in observed scenes, (ii) scene intensity degradations owing to image formation process, and (iii) geometric distortions of a target. Next, we design generalized optimum filters which are tolerant to intensity degradations of input scenes.

2.1 Design of generalized optimum filters

The detection ability of correlation filters can be quantitatively expressed in terms of several criteria, such as probability of detection errors, signal-to-noise ratio, peak sharpness, and discrimination capability (Vijaya-Kumar & Hassebrook, 1990). Optimization of these criteria leads to reducing false recognition errors. After the detection task has been solved, we still are faced with small errors of target position estimation that are due to distortions of the object by noise. The coordinate estimations lie in the vicinity of their actual values. Therefore the accuracy of the target location can be characterized by the variance of measurement errors along coordinates (Kober & Campos, 1996; Yaroslavsky, 1993). The variance minimization depends on a mathematical model of the input scene. Basically, two models are considered: overlapping and nonoverlapping models. Many correlation filters were proposed. For instance, if an input scene contains a reference object corrupted by additive noise (overlapping model), the matched spatial filter (MSF) (VanderLugt, 1964) is optimal with respect to the signal-to-noise ratio. Horner and Gianino (1984) suggested the phase-only filter (POF) that maximizes the light efficiency. For the overlapping model, the optimal filter (OF) was proposed by minimizing the probability of anomalous errors (false alarms) (Yaroslavsky, 1993). If an input scene contains a reference object embedded into a disjoint
background (nonoverlapping model) and additive noise, the following correlation filters were derived: the generalized matched filter (GMF) maximizes the ratio of the expected value of the squared correlation peak to the average output variance (Javidi & Wang, 1994), the generalized phase-only filter (GPOF) maximizes the light efficiency (Kober et al., 2000), and the generalized optimum filter (GOF) maximizes the ratio of the expected value of the squared correlation peak to the average expected value of the output signal energy (POE) (Javidi & Wang, 1994). Other generalized filters were also introduced (Goudail & Réfrégier, 1997; Javidi et al., 1996; Réfrégier, 1999; Réfrégier et al., 1993; Towghi & Javidi, 2001).

Conventional filters are sensitive to intensity signal degradations. So particular cases of the degradations were taken into account in the filter design (Campos et al., 1994; Carnicer et al., 1996; Navarro et al., 2004; Vargas et al., 2003). However, it appears that the problem of detection and localization with correlation filters has not been solved when the target and the input scene are degraded with linear systems. In this section, we derive generalized filters which are tolerant to the degradations. The POE criterion is defined as the ratio of the square of the expected value of the correlation peak to the expected value of the output signal energy (Javidi & Wang, 1994):

$$\text{POE} = \frac{\mathbb{E} [y(x_0, x_0)^2]}{\mathbb{E} [y(x, x_0)]^2},$$

where $y(x, x_0)$ is the filter output when the target is located at the position $x_0$ in the input scene. $\mathbb{E} \{\cdot\}$ denotes statistical averaging, and the overbar symbol in the denominator denotes statistical averaging over $x$.

The second used criterion is referred to as the peak-to-average output variance (SNR). It is defined as the ratio of the square of the expected value of the correlation peak to the average output variance (Javidi & Wang, 1994):

$$\text{SNR} = \frac{\mathbb{E} [y(x_0, x_0)^2]}{\text{Var} [y(x, x_0)]},$$

where $\text{Var} \{\cdot\}$ denotes the variance. The light efficiency (Horner & Gianino, 1984) is important in optical pattern recognition. For the nonoverlapping model of the input scene, it can be expressed as

$$\eta_{\text{light}} = \int \mathbb{E} [y(x_0, x_0)^2] \, dx / \int \mathbb{E} [s(x, x_0)]^2 \, dx,$$

where $s(x)$ represents the input scene.

Next, we derive three generalized optimum filters by maximizing the criteria. For simplicity, one-dimensional notation is used. Integrals are taken between infinite limits. The same notation for a random process and its realization is used.

**A. Generalized correlation filters for object recognition in a noisy scene degraded by a linear system**

Let us consider the nonoverlapping signal model. The input scene $s(x)$ is degraded by a linear system $h_{\text{LD}}(x)$ and corrupted by additive sensor noise $n(x)$, and contains a target $t(x)$ located at unknown coordinate $x_0$ (random variable) and a spatially disjoint background noise $b(x, x_0)$:

$$s(x, x_0) = [t(x - x_0)] + b(x, x_0) \cdot h_{\text{LD}}(x) + n(x).$$
where “\(\bullet\)” denotes the convolution operation, and \(\int h_{LD}(x) \, dx = 1\). The following notations and assumptions are used.

1. The nonoverlapping background signal \(b(x,x_0)\) is regarded as a product of a realization \(b(x)\) from a stationary random process (with expected value \(\mu_b\)) and an inverse support function of the target \(w(x)\) defined as zero within the target area and unity elsewhere:
   \[
b(x,x_0) = b(x)w(x-x_0).
   \] (5)

2. \(B_0(\omega)\) is the power spectral density of \(b_0(x)=b(x)-\mu_b\).

3. \(n(x)\) is a realization from a stationary process with zero-mean and the power spectral density \(N(\omega)\).

4. \(T(\omega), W(\omega),\) and \(H_{LD}(\omega)\) are the Fourier transforms of \(t(x), w(x),\) and \(h_{LD}(x)\), respectively.

5. The filter output \(y(x)\) is given by \(y(x,x_0)=s(x,x_0)\bullet h(x)\), where \(h(x)\) is the real impulse response of a filter to be designed.

6. The stationary processes and the random target location \(x_0\) are statistically independent of each other.

Next, we derive optimum correlation filters. These filters are modified versions of the following generalized correlation filters: the GOF (Javidi & Wang, 1994), GMF (Javidi & Wang, 1994), and GPOF (Kober et al., 2000). The transfer functions of the designed filters are referred to as GOFLD, GMFLD, and GPOLFD, which are optimal with respect to the POE, the SNR, and the light efficiency, respectively (Ramos-Michel & Kober, 2007).

1. **Generalized Optimum Filter (GOFLD)**

   The filter GOFLD maximizes the POE given in Eq. (1). From Eq. (4) the expected value of the filter output \(E[y(x,x_0)]\) can be expressed as

   \[
   E\{y(x,x_0)\} = \frac{1}{2\pi} \int \left[ T(\omega) + \mu_b W(\omega) \right] H_{LD}(\omega) \exp\left[ j\omega(x-x_0) \right] d\omega.
   \] (6)

   The square of the expected value of the output peak can be written as

   \[
   \left| E\{y(x_0,x_0)\} \right|^2 = \frac{1}{4\pi^2} \int \left| T(\omega) + \mu_b W(\omega) \right| H_{LD}(\omega) H(\omega) d\omega.
   \] (7)

   The denominator of the POE can computed as

   \[
   E\left\{ \left| y(x,x_0) \right|^2 \right\} = \text{Var}\{y(x,x_0)\} + \left| E\{y(x,x_0)\} \right|^2.
   \] (8)

   Here, the spatial averaging converts a nonstationary process at the filter output to a stationary process. It is supposed that the output-signal energy is finite (for instance, spatial extend of the filter output is \(L\) (Javidi & Wang, 1994)). The expressions for the average of the output-signal variance \(\text{Var}\{y(x,x_0)\}\) and the average energy of the expected value of the filter output \(\left| E\{y(x,x_0)\} \right|^2\) are given, respectively, by

   \[
   \text{Var}\{y(x,x_0)\} = \frac{1}{2\pi} \int \left[ \frac{\alpha}{2\pi} B_0(\omega) \bullet |W(\omega)|^2 \right] H_{LD}(\omega) d\omega + N(\omega) \left| H(\omega) \right|^2 \, d\omega,
   \] (9)
and

\[
E\left\{ |y(x,x_o)|^2 \right\} = \frac{1}{2\pi} \int \alpha |T(\omega) + \mu_0 W(\omega)|^2 \left| H_{LD}(\omega) \right|^2 \left| H(\omega) \right|^2 d\omega,
\]

where \( \alpha = 1/L \) is a normalizing constant (Kober & Campos, 1996). Substituting Eqs. (9) and (10) into Eq. (8), we obtain the average output energy:

\[
E\left\{ |y(x,x_o)|^2 \right\} = \frac{1}{2\pi} \int \left\{ \alpha \left| T(\omega) + \mu_0 W(\omega) \right|^2 + \frac{1}{2\pi} B_0(\omega) \cdot |W(\omega)|^2 \left| H_{LD}(\omega) \right|^2 + N(\omega) \right\} \left| H(\omega) \right|^2 d\omega.
\]

Using Eqs. (7) and (11) the POE can be written as

\[
\text{POE} = \frac{(2\pi)^{-1} \int \left| T(\omega) + \mu_0 W(\omega) \right|^2 H_{LD}(\omega) H(\omega) d\omega}{\int \left\{ \alpha \left| T(\omega) + \mu_0 W(\omega) \right|^2 + \frac{1}{2\pi} B_0(\omega) \cdot |W(\omega)|^2 \left| H_{LD}(\omega) \right|^2 + N(\omega) \right\} \left| H(\omega) \right|^2 d\omega}.
\]

Applying the Schwartz inequality, we obtain the optimum filter:

\[
\text{GOF}_{LD}(\omega) = \frac{\left\{ \left[ T(\omega) + \mu_0 W(\omega) \right] H_{LD}(\omega) \right\}^*}{\alpha \left| T(\omega) + \mu_0 W(\omega) \right|^2 + \frac{1}{2\pi} B_0(\omega) \cdot |W(\omega)|^2 \left| H_{LD}(\omega) \right|^2 + N(\omega)},
\]

where the asterisk denotes the complex conjugate. Note that the filter takes into account information about a linear image degradation and additive noise by means of \( H_{LD}(\omega) \) and \( N(\omega) \), respectively. Besides, the transfer function of the filter contains \( T(\omega) + \mu_0 W(\omega) \), which defines a new target to be detected. Therefore, the information about the target support function and the mean value of a background is important as well as the target signal itself.

2. **Generalized Matched Filter (GMF)\( _{LD} \)**

This filter maximizes SNR given in Eq. (2). Using Eqs. (7) and (9), the SNR can be expressed as follows:

\[
\text{SNR} = \frac{(2\pi)^{-1} \int \left[ T(\omega) + \mu_0 W(\omega) \right] H_{LD}(\omega) H(\omega) d\omega}{\int \left\{ \frac{\alpha}{2\pi} B_0(\omega) \cdot |W(\omega)|^2 \left| H_{LD}(\omega) \right|^2 + N(\omega) \right\} \left| H(\omega) \right|^2 d\omega}.
\]

Applying the Schwartz inequality, the optimum correlation filter is obtained:

\[
\text{GMF}_{LD}(\omega) = \frac{\left\{ \left[ T(\omega) + \mu_0 W(\omega) \right] H_{LD}(\omega) \right\}^*}{\frac{\alpha}{2\pi} B_0(\omega) \cdot |W(\omega)|^2 \left| H_{LD}(\omega) \right|^2 + N(\omega)}.
\]

One can observe that the filter contains information about the linear degradation system and additive noise.
3. Generalized Phase Optimum Filter (GPOF\textsubscript{LD})

Using Eq. (4), the light efficiency given by Eq. (3) can be expressed as

\[
\eta_H = \frac{\int \left[ (T(\omega) + \mu_\omega W(\omega)) H_{LD}(\omega) \right]^2 |H(\omega)|^2 d\omega}{\int \left[ (T(\omega) + \mu_\omega W(\omega)) H_{LD}(\omega) \right]^2 d\omega}.
\] (16)

Thus, the optimum correlation filter is given by

\[
\text{GPOF}_{LD}(\omega) = \frac{T(\omega) + \mu_\omega W(\omega)}{[T(\omega) + \mu_\omega W(\omega)]} \exp\left[-j\theta_{H_{LD}}(\omega)\right],
\] (17)

where \(\theta_{H_{LD}}(\omega)\) is the phase distribution of the linear degradation. It can be seen that the GPOF\textsubscript{LD} does not take into account the degradation by additive noise. Therefore, it is expected that this filter will be sensitive to the noise.

B. Generalized correlation filters for recognition of a linearly degraded object in a noisy scene degraded by a linear system

The input scene contains a linearly degraded target located at unknown coordinate \(x_0\) and a spatially disjoint background \(b(x,x_0)\). The scene is additionally degraded with a linear system and corrupted by additive noise \(n(x)\): \(s(x,x_0) = \left[t(x-x_0) \ast h_{TD}(x) + b(x) w_{TD}(x-x_0)\right] \ast h_{LD}(x) + n(x), \) (18)

where \(h_{TD}(x)\) is a real impulse response of target degradation, \(\int h_{TD}(x) dx = 1\), \(w_{TD}(x-x_0) = 1 - w_t(x-x_0) \ast h_{TD}(x), w_t(x)\) is a support function of the target (with unity within the target area and zero elsewhere). It is assumed that linear degradations of the target and the scene do not affect each other. In a similar manner, three generalized correlation filters are derived. The transfer functions of these filters are referred to as GOF\textsubscript{LD,TD}, GMF\textsubscript{LD,TD}, and GPOF\textsubscript{LD,TD}. They maximize the POE, the SNR, and the light efficiency, respectively (Ramos-Michel & Kober, 2007).

1. Generalized Optimum Filter (GOF\textsubscript{LD,TD})

From the model of the input scene given in Eq. (18), the expected value of the filter output is

\[
E[y(x,x_0)] = \frac{1}{2\pi} \int \left[ T(\omega) H_{TD}(\omega) + \mu_\omega W_{TD}(\omega) \right] H_{LD}(\omega) H(\omega) \exp\left[j\omega(x-x_0)\right] d\omega,
\] (19)

where \(H_{TD}(\omega)\) and \(W_{TD}(\omega)\) are the Fourier transforms of \(h_{TD}(\omega)\) and \(w_{TD}(\omega)\), respectively. The intensity correlation peak can be computed as follows:

\[
\left|E[y(x_0,x_0)]\right|^2 = \frac{1}{4\pi^2} \int \left[ T(\omega) H_{TD}(\omega) + \mu_\omega W_{TD}(\omega) \right] H_{LD}(\omega) H(\omega) d\omega.
\] (20)

\(\text{Var}\{y(x,x_0)\}\) and \(\left|E[y(x,x_0)]\right|^2\) can be obtained, respectively, as

\[
\text{Var}\{y(x,x_0)\} = \frac{1}{2\pi} \int \left[ \frac{\alpha}{2\pi} B_o(\omega) \ast W_{TD}(\omega) \right] H_{LD}(\omega) H(\omega) d\omega,
\] (21)
and

\[ \left[ E\{y(x,x_0)\} \right]^2 = \frac{1}{2\pi} \int \alpha |T(\omega)H_{TD}(\omega) + \mu_{b}W_{TD}(\omega)|^2 |H_{LD}(\omega)|^2 \mid H(\omega) \mid^2 d\omega . \]  

(22)

With the help of Eqs. (1), (8), and (20)-(22), the POE is given by

\[ \text{POE} = \frac{(2\pi)^{-1} \int \left[ |T(\omega)H_{TD}(\omega) + \mu_{b}W_{TD}(\omega)|H_{LD}(\omega)H(\omega)d\omega \right]^2}{\int \left[ \alpha |T(\omega)H_{TD}(\omega) + \mu_{b}W_{TD}(\omega)|^2 + \frac{1}{2\pi} B_{0}(\omega) \mid W_{TD}(\omega) \mid^2 |H_{LD}(\omega)\mid + N(\omega) \mid H(\omega) \mid^2 \right] d\omega} . \]  

(23)

Applying the Schwarz inequality, a generalized optimum filter is derived:

\[ \text{GOF}_{LD_{TD}}(\omega) = \frac{\left[ |T(\omega)H_{TD}(\omega) + \mu_{b}W_{TD}(\omega)|H_{LD}(\omega)\right]^*}{\alpha \left[ |T(\omega)H_{TD}(\omega) + \mu_{b}W_{TD}(\omega)|^2 + \frac{1}{2\pi} B_{0}(\omega) \mid W_{TD}(\omega) \mid^2 |H_{LD}(\omega)\mid + N(\omega) \mid H(\omega) \mid^2 \right]} . \]  

(24)

2. **Generalized Matched Filter (GMF\textsubscript{LD\_TD})**

From Eqs. (2), (20), and (21), the SNR can be expressed as

\[ \text{SNR} = \frac{(2\pi)^{-1} \int \left[ |T(\omega)H_{TD}(\omega) + \mu_{b}W_{TD}(\omega)|H_{LD}(\omega)H(\omega)d\omega \right]^2}{\int \left[ \alpha \mid B_{0}(\omega) \mid W_{TD}(\omega) \mid^2 |H_{LD}(\omega)\mid + N(\omega) \mid H(\omega) \mid^2 \right] d\omega} . \]  

(25)

Applying the Schwartz inequality, the optimum correlation filter is given by

\[ \text{GMF}_{LD_{TD}}(\omega) = \frac{\left[ |T(\omega)H_{TD}(\omega) + \mu_{b}W_{TD}(\omega)|H_{LD}(\omega)\right]^*}{\frac{\alpha}{2\pi} B_{0}(\omega) \mid W_{TD}(\omega) \mid^2 |H_{LD}(\omega)\mid + N(\omega) \mid H(\omega) \mid^2} . \]  

(26)

We see that the filter contains information about the linear system and additive noise.

3. **Generalized Phase Optimum Filter (GPOF\textsubscript{LD\_TD})**

By maximizing the light efficiency given in Eq. (3), the transfer function of the GPOF can be written as

\[ \text{GPOF}_{LD_{TD}}(\omega) = \left| \frac{T(\omega)H_{TD}(\omega) + \mu_{b}W_{TD}(\omega)}{\left| T(\omega)H_{TD}(\omega) + \mu_{b}W_{TD}(\omega) \right|} \right|^2 \exp\left[ -j\theta_{H_{LD}}(\omega) \right] . \]  

(27)

The filter does not take into account the degradation by additive noise. Therefore, it is expected that this filter will be sensitive to the noise.

2.2 Performance of optimum generalized filters

In this section the performance of the MSF (VanderLugt, 1964), the POF (Horner & Gianino, 1984), the OF (Yaroslavsky, 1993), the GMF (Javidi & Wang, 1994), the GOF (Javidi & Wang, 1994), the GPOF (Kober et al., 2000) and the proposed generalized filters is presented. The recognition of either a target or a moving object embedded into degraded test scenes is
evaluated in terms of discrimination capability (DC) and location accuracy. The DC is defined as the ability of a filter to distinguish a target from other different objects. If a target is embedded into a background that contains false objects, the DC can be expressed as

\[
DC = 1 - \frac{|C^B(0)|^2}{|C^T(0)|^2},
\]

where \( |C^B| \) is the maximum in the correlation plane over the background area to be rejected, and \( |C^T| \) is the maximum in the correlation plane over the area of target position. The area of the actual position is determined in the close vicinity of the actual target location. The background area is complementary to the area of target position. In our computer simulations the area of target position is chosen as the target area. Negative values of the DC indicate that a tested filter fails to recognize the target. The location accuracy can be characterized by means of the location errors (LE) defined as

\[
LE = \sqrt{(x_T - \hat{x}_T)^2 + (y_T - \hat{y}_T)^2},
\]

where \((x_T, y_T)\) and \((\hat{x}_T, \hat{y}_T)\) are the coordinates of the target exact position and the coordinates of the correlation peak taken as a target position estimation, respectively.

Fig. 1. (a) Test input scene, (b) objects used in experiments.

All correlation filters were implemented with the fast Fourier transform. To guarantee statistically correct results, 30 statistical trials of each experiment for different positions of a target and 20 realizations of random processes were carried out. The size of images used in experiments is 256×256. The signal range is [0-1]. Figure 1(a) shows a test input scene. The scene contains two objects with a similar shape and size (approximately 44×28 pixels) but with different gray-level contents. The target (upper butterfly) and the false object are shown in Fig. 1(b). The mean value and the standard deviation over the target area are 0.42 and 0.2, respectively. The spatially inhomogeneous background has a mean value and a standard deviation of 0.37 and 0.19, respectively.
Two scenarios of the object recognition are considered: (a) detection of a target in linearly degraded and noisy scenes, and (b) detection of a moving target in linearly degraded and noisy scenes.

A. Recognition of a target in linearly degraded and noisy scenes
First, the test input scene is homogenously degraded with a linear system. An example of the linear degradation is a uniform image defocusing by a camera.

Fig. 2. Test scenes corrupted by additive noise with $\sigma_n=0.12$ and defocused with: (a) $D=7$, and (b) $D=23$ pixels.

Fig. 3. Performance of correlation filters when the input scene is defocused with different values of $D$: (a) DC versus $D$, (b) LE versus $D$. 
Assume that the impulse response of the blurring is an impulse disk with a diameter $D$. The values of $D$ used in the experiments are 3, 7, 11, 15, 19, 23, and 31 pixels. Since additive sensor noise is always present, the test scene is additionally corrupted by additive zero-mean white Gaussian noise with the standard deviation $\sigma_n$. The values of $\sigma_n$ are equal to 0.02, 0.04, 0.08, 0.12, 0.16, and 0.17. Figures 2(a) and 2(b) show examples of the test scene linearly degraded with $D=7$ and 23 pixels, respectively, and corrupted by overlapping noise with $\sigma_n=0.12$. Figures 3(a) and 3(b) show the performance of the tested correlation filters with respect to the DC and the LE when the input is defocused with different values of $D$. It can be seen that the proposed filters GOF$_{LD}$ and GMF$_{LD}$ are always able to detect and localize exactly the target, whereas the GPOF$_{LD}$ is sensitive to the linear degradation. The performance of the other filters decreases as a function of $D$. The conventional GOF is able to detect the target; however, it yields large location errors. The MSF filter fails to recognize the target.

Fig. 4. Tolerance to noise of correlation filters for pattern recognition in blurred scenes: (a) DC versus $\sigma_n$ with $D=7$, (b) LE versus $\sigma_n$ with $D=7$, (c) DC versus $\sigma_n$ with $D=11$, (d) LE versus $\sigma_n$ with $D=11$. 
Now we illustrate robustness of the filters to additive noise. Figure 4 shows the performance of the filters for pattern recognition in blurred (with $D=7$ and $D=11$) and noisy test scenes when the standard deviation of additive noise is varied. One can observe that the proposed filters GOF$_{LD}$ and GMF$_{LD}$ are always able to detect and to localize the object with small location errors, whereas the performance of the rest of the filters worsens rapidly as a function of $D$ and $\sigma_n$. 95% confidence intervals in the performance of the GOF$_{LD}$ are shown in Figs. 4 (c) and 4(d).

**B. Recognition of a moving target in linearly degraded and noisy scenes**

Let us consider a uniform target motion across a fixed background. For clarity and simplicity, we assume that the object moves from left to right with a constant velocity $V$ during a time capture interval of $[0, T]$. The impulse response of the target degradation can be expressed as follows (Biemond et al., 1990):

$$
h_{TM}(x) = \begin{cases} 
1/M, & \text{if } 0 \leq x \leq M = VT \\
0, & \text{otherwise}
\end{cases}.
$$

(30)

![Fig. 5. Illustration of a linear degradation by a uniform target motion in 3 pixels from left to right.](image)

The target motion leads to a partial (inhomogeneous) blur of the input scene. Figure 5 illustrates this degradation when a target and a background are one-dimensional discrete signals $(t_1, \ldots, t_9)$ and $(b_1, \ldots, b_9)$, respectively, and the target moves in 3 pixels from left to right. The input signal $s(r)$ is formed as the average of intermediate sequences $s'(r)$. In
experiments, a uniform target motion from left to right on $M$ pixels in test scenes is considered. The values of $M$ used in our experiments are 3, 5, 7, 9, 11, and 15 pixels. The test scene containing the moving target may be homogeneously degraded by a linear system with the parameter $D$. The values of $D$ used in our experiments are 3, 7, 9, 15, 19, and 23 pixels. Fig. 6(a) shows the test scene degraded with $M=9$ and $D=7$. Fig. 6(b) shows the degraded scene with $M=9$ and $D=7$, which is additionally corrupted by overlapping noise with $\sigma_n=0.12$.

Fig. 6. Test scene shown in Fig. 1(a) corrupted by: (a) target motion ($M=9$) and scene degradation ($D=7$), (b) target motion ($M=9$), scene degradation ($D=7$), and additive noise ($\sigma_n=0.12$).

Fig. 7. Performance of correlation filters for recognition of a moving target: (a) DC versus $M$, (b) LE versus $M$ for the scene.
Let us analyze the performance of the tested filters for recognition of a moving object in the still undistorted background. In this case, generalized optimum filters referred to as GMF_{TD}, GOF_{TD}, and GPOF_{TD} can be obtained from Eqs. (24), (26), and (27), respectively, by substituting into these equations $H_{LD}(\omega)=1$. Figures 7(a) and 7(b) show the performance of the correlation filters with respect to the DC and the LE when the input scene in Fig. 1(a) contains a moving target with different values of $M$.

![Graph showing recognition and localization performance](image)

Note that the proposed filters the GOF_{TD}, the GMF_{TD}, and the GPOF_{TD} are able to detect the target without location errors. On the other hand, one can see that the performance of the rest of the filters rapidly deteriorates in terms of the DC and the LE when the target displacement increases.

Next, the filters are tested for recognition of a moving object in defocused and noisy test scenes. The performance of the filters for $M=7, 11$, and $D=7$ in terms of the DC and the LE as
a function of the standard deviation of additive noise is shown in Fig. 8. Under these degradation conditions, the OF, the MSF, and the POF yield a poor performance. The GMFLD_TD is always able to detect and to localize the moving object for any tested values of $M$, $D$, and $\sigma_n$. However, it yields low values of the DC. The GOF_LD_TD provides the best performance in terms of the DC and the LE when the scene is corrupted by additive noise with $\sigma_n \leq 0.12$ and the target moves by $M \leq 15$ pixels.

3. Adaptive composite filters for recognition of geometrically distorted objects

3.1 Design of adaptive composite filters

In this section, we consider the task of recognition of geometrically distorted targets in input scenes degraded with a linear system and corrupted by noise. Various composite optimum correlation filters for recognition of geometrically distorted objects embedded in a nonoverlapping background have been proposed (Chan et al., 2000; Sjöberg & Noharet, 1998). However, there are no correlation-based methods for detection and localization of geometrically distorted objects in blurred and noisy scenes. We use a priori information about an object to be recognized, false objects, background noise, linear degradations of the input scene and target, geometrical distortions of the target, and additive sensor noise.

An attractive approach to geometrical distortion-invariant pattern recognition is based on SDF filters (Casasent, 1984; Mahalanobis et al., 1987; Vijaya-Kumar, 1986). Basically, a conventional SDF filter uses a set of training images to generate a filter that yields prespecified central correlation outputs in the response to training images. It is able to control only one point at the correlation plane for each training image. This is why SDF filters often have a low discrimination capability. We are interested in a filter which is able to recognize geometrically distorted objects in a set of observed degraded scenes, i.e., with a fixed set of patterns and backgrounds to be rejected (Ramos-Michel & Kober, 2008), rather than in a filter with average performance parameters over an ensemble of images (Chan et al., 2000). The impulse response of the obtained filter is a linear combination of correlation filters optimized with respect to the peak-to-output energy and common matched filters. The optimum generalized filters are derived from a set of training images, whereas the matched filters are designed from the background to be rejected. With the help of an iterative training procedure, an adaptive composite filter is generated. The filter ensures high correlation peaks corresponding to versions of the target while suppressing possible false peaks. The proposed algorithm of the filter design requires knowledge of the background image. The background can be described either deterministically (typical picture) or stochastically (realization of a stochastic process).

Suppose that an input scene is homogenously degraded by a linear system and corrupted by additive noise. It contains geometrically distorted targets. For each object to be recognized, a generalized optimum filter (GOFLD) is designed [see Eq. (13)]. Each filter takes into account a priori information about the corresponding reference, background noise, linear degradation of the input scene, geometrical target distortions, and additive sensor noise. Let $\{t_i(x), i=1,2,...,N\}$ be a set of target images (linearly independent), each with $d$ pixels. This set is called the true class of objects. The set includes geometrically distorted versions of the references. For the $i$'th image the transfer function of the GOFLD filter is given by
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\[
\text{GOF}^{1D}(\omega) = \frac{\left[ T_i(\omega) + \mu_i \mathcal{W}_i(\omega) \right] H_{LD}(\omega)}{\alpha \left[ T_i(\omega) + \mu_i \mathcal{W}_i(\omega) \right]^2 + \frac{1}{2\pi} B_0(\omega) \cdot \mathcal{W}_i(\omega)^2 \left[ H_{LD}(\omega) \right]^2 + N(\omega)},
\]

(31)

where \( T_i(\omega) \) and \( \mathcal{W}_i(\omega) \) are the Fourier transforms of the \( i \)'th training object \( t_i(x) \) and its inverse support function \( w_i(x) \), respectively. We use the same notation and assumptions as in Section 2. Let \( h_i^G(x) \) be the inverse Fourier transform of the complex-conjugate frequency response of the generalized optimum filter for the \( i \)'th pattern. A linear combination of \{ \( h_i^G(x), i=1,2,\ldots,N \) \} can form a SDF filter for intraclass distortion-invariant pattern recognition. In this case the coefficients of a linear combination must satisfy a set of constraints on the filter output requiring a prespecified value for each training pattern.

Assume that there are various classes of objects to be rejected. For simplicity, a two-class recognition problem is considered. Thus, we are looking for a filter to recognize training images from one class and to reject images from another class, called the false class. Suppose that there are \( M \) training images from the false class \{ \( p_i(x), i=1,2,\ldots,M \) \}. Let us denote a set of training images formed from the input patterns as \( S = \{ t_1(x), \ldots, h_N(x), p_1(x), \ldots, p_M(x) \} \), and a new combined set of training images is defined as \( S_N = \{ h_1^G(x), \ldots, h_N^G(x), p_1(x), \ldots, p_M(x) \} \). According to the SDF approach (Casasent, 1984), the composite image is computed as a linear combination of training images belonging to \( S_N \), i.e.,

\[
h_{\text{SDF}}(x) = \sum_{i=1}^{N} a_i h_i^G(x) + \sum_{i=N+1}^{M+N} a_i p_i(x).
\]

(32)

Let \( R \) denote a matrix with \( N+M \) columns and \( d \) rows, whose \( i \)'th column is given by the vector version of the \( i \)'th element of \( S_N \). Using vector-matrix notation, Eq. (32) can be rewritten as

\[
h_{\text{SDF}} = Ra,
\]

(33)

where \( a \) represents the column vector of weighting coefficients \{ \( a_i, i=1,\ldots,M+N \) \}. We can set the filter output \{ \( u_i=1, i=1,\ldots,N \) \} for the true class objects and \{ \( u_i=0, i=N+1, N+2,\ldots,N+M \) \} for the false class objects, i.e. \( u = [1 \ 1 \ 1 \ \cdots \ 0 \ 0 \ \cdots \ 0]^T \). Here superscript \( T \) denotes the transpose. Let \( Q \) be a matrix with \( N+M \) columns and \( d \) rows, whose \( i \)'th column is the vector version of the \( i \)'th element of \( S \). The weighting coefficients are chosen to satisfy the following condition:

\[
u = Q^+ h_{\text{SDF}},
\]

(34)

where superscript \( + \) means conjugate transpose. From Eqs. (33) and (34) we obtain

\[
h_{\text{SDF}} = R \left[ Q^+ \right]^{-1} u.
\]

(35)

Using the filter given in Eq. (35), we expect that the central correlation peaks will be close to unity for all targets and it will be close to zero for false objects. It is important to note that this procedure lacks control over the full correlation output, because we are able to control only the output at the location of cross-correlation peaks. Therefore, other sidelobes may appear everywhere on the correlation plane. To achieve a good recognition, a modified
iterative algorithm (Diaz-Ramirez et al., 2006; González-Fraga et al., 2006) is proposed. At each iteration, the algorithm suppresses the highest sidelobe peak, and therefore the value of discrimination capability monotonically increases until a prespecified value is reached.

Fig. 9. Block diagram of the iterative algorithm for the filter design.

The first step of the iterative algorithm is to carry out a correlation between a background (deterministic or stochastic) and the SDF filter given in Eq. (35). This filter is initially trained only with available versions of targets and known false objects. Next, the maximum of the filter output is set as the origin, and around the origin we form a new object to be rejected from the background. This object has a region of support equal to the union of those of all targets. The created object is added to the false class of objects. Now, the two-class recognition problem is utilized to design a new SDF filter. The described iterative procedure is repeated till a specified value of the DC is obtained. A block diagram of the procedure is shown in Fig. 9. The proposed algorithm consists of the following steps:

1. Design a basic SDF filter using available distorted versions of targets and known false objects [see Eq. (35)].
2. Carry out the correlation between a background and the filter, and calculate the DC using Eq. (28).
3. If the value of the DC is greater or equal to a desired value, then the filter design procedure is finished, else go to the next step.
4. Create a new object to be rejected from the background. The origin of the object is at the highest sidelobe position in the correlation plane. The region of support of the object is the union of the region of supports of all targets. The created object is added to the false class of objects.
5. Design a new SDF filter using Eq. (35) with the same true class and the extended false class of objects. Go to step 2.

As a result of this procedure, the adaptive composite filter is synthesized. The performance of this filter in the recognition process is expected to be close to that in the synthesis process.
3.2 Performance of adaptive composite filters

Now, we analyze the performance of the generalized optimum filter for pattern recognition in a linearly distorted scene (GOFLD) [given by Eq. (13)], the adaptive SDF filter (AMSF) (González-Fraga et al., 2006), the distortion-invariant minimum-mean-squared-error (MMSE) filter (Chan et al., 2000), and the proposed adaptive filter (AGOF) in terms of discrimination capability and location accuracy.

![Test scene](image)

Fig. 10. Test scene (degraded by motion blur with $M=5$ and additive noise with $\sigma_n=0.08$) contains the target: (a) rotated by 5 degree and scaled by factor of 0.8, (b) rotated by 10 degree and scaled by factor of 1.2.

We carried out experiments for recognition of scaled and rotated targets in blurred and noisy scenes. It is assumed that a camera moves from right to left on $M=5$ pixels. So, the input scene is degraded by the uniform motion blur given in Eq. (30). The scene also contains sensor noise with $\sigma_n=0.08$. Figures 10(a) and 10(b) show two examples of input scenes used in the experiment. To guarantee statistically correct results, 30 statistical trials for different positions of a target and 20 realizations of random processes were performed. For the filter design of the tested composite filters (AGOF, AMSF and MMSE) we used the same set of training images. The set contains versions of the target scaled by factors 0.8, 0.85, 0.9, 1.1, and rotated by 0, 3, 6, and 9 degrees (see Fig. 11). Besides, for the synthesis of the adaptive filters we used the background shown in Fig. 1(a) degraded with $M=5$ and $\sigma_n=0.08$.

Figure 12 shows the performance of the filters with respect to the DC and the LE when the target is scaled and rotated by 5 and 10 degrees. One can see that the proposed filter AGOF possesses the best average performance in terms of both criteria. The AMSF fails to recognize the distorted object when the target is scaled by a factor lower than 1.2. It is important to say that the number of iterations during the design process of the AGOF depends on a background and true and false objects. In our case, after 9 iterations in the design process the filter yields DC=0.93.
Fig. 11. Versions of the target distorted by rotation and scaling.

Fig. 12. Performance of correlation filters for recognition of rotated and scaled objects.
4. Conclusion

In this chapter we treated the problem of distortion-invariant pattern recognition based on adaptive composite correlation filters. First, we proposed optimum generalized filters to improve recognition of a linearly distorted object embedded into a nonoverlapping background noise when the input scene is degraded with a linear system and noise. The obtained filters take into account explicitly information about an object to be recognized, disjoint background noise, linear system degradation, linear target distortion and sensor noise. For the filter design, it is assumed that this information is available or can be estimated from the nature of degradations. Next, adaptive composite correlation filters for recognition of geometrically distorted objects embedded into degraded input scenes were proposed. The filters are a linear combination of generalized optimum filters and matched spatial filters. The information about an object to be recognized, false objects, and a background to be rejected is utilized in iterative training procedure to design a correlation filter with a prespecified value of discrimination capability. Computer simulation results obtained with the proposed filters are compared with those of various correlation filters in terms of recognition performance.

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