Increasing the Flexibility of Combined Heat and Power Systems through Optimal Dispatch with Variable Mass Flow Rate

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Abstract—To increase the flexibility of combined heat and power systems, we consider its optimal dispatch problem with variable mass flow rate, which is assumed to be fixed in most of the literature. This paper adopts a novel heat system model to remove binary variables from the original optimal dispatch. The resulting non-convex problem is further decomposed into a convex sub-problem with fixed mass flow rate and a master problem which updates the mass flow rate. Then a solution method is developed for the decomposed model, which calculates sensitivity of the sub-problem objective function with respect to the given mass flow rate, then applies gradient projection to update mass flow rate in the master problem. Compared with existing methods with fixed mass flow rate, the proposed mechanism can exploit more flexibility to reduce system’s overall cost without any additional devices.

Index Terms—combined heat and power system, economic dispatch, decomposition method, flexibility.

I. INTRODUCTION

A. Motivation

In northern Europe [1] and northern China [2], combined heat and power generation accounts for 30%-50% of total power generation, and the Energy Information Administration forecasts that the capacity of combined generation will grow at 5% per year in the U.S. [3]. Such a rapid growth of combined heat and power systems globally is motivated by the complementary property of the two energy sectors: the electric power system requires real-time power balance whereas the temperature in a heat system has a much higher inertia. Thus, if operated properly, heat systems can serve as storages for electric power systems, which may in turn provide alternative heat sources to heat systems. Therefore, it is of crucial importance to exploit such a complementary property with all possible adjustment means.

In the optimization of combined heat and power systems, temperature and mass flow rates are two most important types of control variables in heat systems [4][5]. However, due to the non-convexity caused by adjusting mass flow rate and temperature simultaneously, many research papers only consider varying temperature adjustment, which limits the flexibility of combined heat and power systems [6]. In order to further increase the adjustable range of heat power and improve the flexibility of combined heat and power systems, it is crucial to adjust both mass flow rate and temperature of heat systems in combined heat and power dispatch problems.

B. Related Works

Recently, on the dispatch of combined heat and power systems, a large part of existing literature is based on fixed mass flow rate in heat systems. The authors of [7] propose a convex model of the combined heat and power dispatch for Energy Hubs, but the detailed electric and heat networks still need to be considered to reflect transmission limits. To address the issue of modeling electric and heat networks, Ohm’s Law is extended to heat systems, and the flexibility from heat storages and heat exchangers is investigated in [8]. Paper [9] adopts an optimal dispatch model which fixes mass flow rate and only adjusts temperature. To further utilize the flexibility from the heat dynamic process, authors of [10] and [11] formulate convex combined heat and power dispatch programs to accommodate more renewables and reduce costs. Paper [12] considers distributed energy resources (DERs) in combined heat and power dispatch. Although [8]-[12] consider the inertia of the heat dynamic process for more flexibility, the mass flow rate is fixed in heat systems. As a result, the flexibility of the combined heat and power system is still limited and can be further improved by varying mass flow rate.

For more flexibility and accuracy, other researchers adopt variable mass flow rate adjustment in their optimal dispatch models. Since adjusting mass flow rate can cause strong non-convexity, two kinds of methods, data-driven methods and model-based methods, have been developed.

The data-driven methods are proposed in [13] and [14], which treat the non-convex optimization problems of combined heat and power systems as black-boxes and use historical data to predict future states. Although the data-driven methods in [13] and [14] are effective to deal with non-convex optimization models, they may suffer from problems of tractability, reliability, and accuracy, especially under different operation strategies. Heuristic algorithms can be embedded in data-driven methods to improve reliability and accuracy. Paper [15] applies a mixed integer nonlinear program (MINLP) to model the combined heat and power dispatch and solves it by a heuristic iterative method. Authors of [16] and [17] develop heuristic

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methods to solve the non-convex optimal dispatch considering DERs which introduce nonlinear constraints. These heuristic methods can relieve reliability and accuracy problems caused by black-box methods and overcome the non-convexity caused by varying mass flow rate, however, they may suffer from problems in convergence and tractability. Besides, paper [15] does not consider multiple DERs.

Recently, model-based methods have been proposed to address the trackability and convergence problems. One idea of realizing this method is to simplify the optimization model. For example, to reduce nonlinear equality constraints, heat dynamic process is not considered in [18] and [19]. Because there are still other nonlinear constraints, it is hard for [18] and [19] to ensure accuracy and convergence simultaneously. The authors of [20] have proposed a novel MINLP dispatch model and solved it by the modified Generalized Benders Decomposition method with convergence guarantee. But there are varying degrees of simplification and approximation in the optimization model and the solving method. Moreover, the large-scale binary variables in [20] may lead to the calculation inefficiency, and it does not consider multi-DERs, either.

C. Summary of Contributions

In this paper, we first model the optimal dispatch problem of combined heat and power systems as a non-convex program with convex objective function and nonlinear constraints. Innovatively, we utilize time-correlated and space-correlated nonlinear equations, which partition a heat pipeline into small sections, to reflect heat dynamic process [21][22]. As a result, integer variables in the original optimal dispatch models [15][20] can be removed without compromising on the accuracy [23].

Second, we propose an efficient solution method for the optimal dispatch problem of combined heat and power systems. Based on the framework of Generalized Benders Decomposition, we decompose the original non-convex problem with nonlinear constraints into a convex sub-problem with fixed mass flow rate and a master problem which uses the gradient projection to update mass flow rate. The application of Outer Approximation [24] accelerates the calculation by generating a cutting plane for the infeasible sub-problem. Moreover, the proposed method always converges to a local optimum that is no worse than the initial point.

The remainder of this paper is organized as follows. In Section II, the model of combined heat and power systems is presented. Section III illustrates the decomposition of the optimization model, and Section IV presents the solution method for the convex sub-problem and the master problem. In Section V, the proposed method is compared to existing benchmarks.

II. PROBLEM MODELING

In this section, we describe the physical model, operation model, and optimization model of combined heat and power systems, respectively.

A. Physical Model

Fig. 1 shows the structure of the combined heat and power system, in which the electric power system and the heat system are coupled with energy sources. Energy sources include thermal generator, combined heat and power (CHP) unit, electric boiler, electricity from the main grid, etc.

As presented in Fig. 1, the heat system consists of heat nodes, heat supply network and heat return network. In the heat system, heat energy is transmitted by media, which is assumed to be water in this paper with the thermal dynamic process. Unlike electric power systems, a heat node has to use heat exchangers to get heat from the heat network. The heat exchangers obtain heat from high-temperature water in the supply network and release the low-temperature water to the return network. For clarity, we use $\tau$ and $T$ to denote the pipe temperature and the node temperature in the following parts, respectively.

B. Operation Model

The operation model describes the electric and heat power flow of combined heat and power systems.

1) Electric Network Model

In the power system, the DC power flow model is adopted. The real-time electric power balance is required between the generation side and the load side:

$$\sum_{i \in E} p_{ij} = \sum_{i \in E} d_{ij}, \quad \forall i \in N, 1 \leq t \leq T,$$

where $p_{ij}$ denotes the electric power generation of electric node $i$ at time $t$. If a node does not have generators, then its $p_{ij} = 0$. Scalar $d_{ij}$ is the electric load demand of node $i$ at time $t$. Set $E$ is the set of electric nodes in the power system, and $T$ is the last scheduling period.

The electric power $l_{ij}$ of line $i$ at time $t$ is calculated by

$$l_{ij} = \sum_{j \in E} SF_{ij} \cdot (p_{ij} - d_{ij}) \quad \forall i \in L, 1 \leq t \leq T,$$

where $SF_{ij}$ indicates the shift factor of node $j$ to line $i$. Set $L$ is the set of power lines. In particular, we do not assume radial power networks in this paper.

2) Heat Pipeline Model

In heat systems, it is assumed that 1) the heat supply network and the heat return network are radial, respectively, and 2) the direction of mass flow rate is the same as the given direction. These assumptions hold in most practical heat systems and are
widely adopted in the literature [15][19][20].

When multiple DERs are integrated in heat systems, a heat node may have more than one injecting pipelines. Thus, the node temperature mixing equations (3)-(4) are applied to calculate node temperature from its injecting pipe end temperature:

\[
\pi_{i,j} + \sum_{m,j} T_{i,j}^m = (\pi_{i,j} T_{i,j}^{NS}) + \sum_{m,j} m_j \tau_{i,j}^m, \quad \forall i \in H, j \in P \cap I_s(i), 1 \leq t \leq T,
\]

\[
\pi_{i,j} + \sum_{m,j} T_{i,j}^m = (\pi_{i,j} T_{i,j}^{RP}) + \sum_{m,j} m_j \tau_{i,j}^m, \quad \forall i \in H, j \in P \cap I_s(i), 1 \leq t \leq T,
\]

where \( \tau_{i,j}^m \) and \( \tau_{i,j}^R \) are pipe end temperatures of pipe \( j \) at time \( t \) in heat supply and return networks, respectively. Scalar \( \pi_{i,j} \) is the node mass flow rate of node \( i \) at time \( t \), and \( m_j \) indicates the pipe mass flow rate of pipe \( j \) at time \( t \). Set \( H \) is the set of heat nodes in the heat system. Sets \( P_s(i) \) and \( P_r(i) \) indicate the sets of pipelines in heat supply and return networks, respectively. Sets \( I_s(i) \) and \( I_r(i) \) are sets of pipelines injecting into node \( i \) in heat supply and return networks, separately. As shown in Fig.1, \( T_{i,j}^{NS} \) and \( T_{i,j}^{RP} \) are the node supply and return temperatures of node \( i \) at time \( t \), respectively. Scalars \( T_{i,j}^{NS} \) and \( T_{i,j}^{RP} \) are the node temperatures of node \( i \) at time \( t \) in supply and return networks.

The pipe start temperature equals to the temperature of its connecting node:

\[
\tau_{i,j}^m = T_{i,j}^m, \quad j \in P \cap I_s(i), i \in H, 1 \leq t \leq T,
\]

\[
\tau_{i,j}^R = T_{i,j}^R, \quad j \in P \cap I_r(i), i \in H, 1 \leq t \leq T,
\]

where \( \tau_{i,j}^m \) and \( \tau_{i,j}^R \) denote pipe start temperatures of pipe \( j \) at time \( t \) in heat supply and return networks, respectively. Sets \( L_s(i) \) and \( L_r(i) \) are sets of pipelines leaving from node \( i \) in supply and return networks.

In existing research like [15] and [20], researchers formulate the combined heat and power dispatch with variable mass flow as MINLPs, where integers are used to track time delays of heat dynamic process. In this paper, we adopt the pipe temperature segment model (7)-(8) [21] to describe heat dynamic process using a series of time-related and space-related nonlinear equations. As a result, the complexity caused by integers is reduced without compromising on accuracy [22][23].

\[
(a, b, c) \tau_{i,j}^m(j, t) = c, \quad \tau_{i,j}^m(j, t - \Delta t) +
(b, m, j) \tau_{i,j}^m(j - 1, t - d), \quad \forall i \in P_s(j = 1, 2, \ldots, S_i),
\]

\[
(a, b, c) \tau_{i,j}^R(j, t) = c, \quad \tau_{i,j}^R(j, t - \Delta t) +
(b, m, j) \tau_{i,j}^R(j - 1, t - d), \quad \forall i \in P_r(j = 1, 2, \ldots, S_i),
\]

where \( \tau_{i,j}^m(j, t) \) and \( \tau_{i,j}^R(j, t) \) are pipe temperatures of pipe \( i \) in heat supply and return networks at segment \( j \) from pipe start point at time \( t \), respectively. Scalar \( \Delta t \) is the given time interval. Scalar \( S_i \) indicates the segment number of pipe \( i \), in which \( S_i = \lfloor x_i / \Delta x \rfloor \), where \( x_i \) is the length of pipe \( i \), and \( \Delta x \) is the given length of each pipe segment. We have \( \tau_{i,j}^m(0, t) = \tau_{i,j}^{NS} \) and \( \tau_{i,j}^R(S_i, t) = \tau_{i,j}^{RP} \) in the heat supply network. \( a, b, c, d \) are coefficients related to the characteristics of pipe \( i \):

\[
a_i = \frac{1}{\Delta t} + \frac{1}{\rho c_i A_i R_i}, \quad b_i = \frac{1}{\Delta x \rho A_i}, \quad c_i = \frac{1}{\Delta t}, \quad d_i = \frac{T_{i,j}^{NS}}{\rho c_i A_i R_i},
\]

where \( \rho \) is the density of water, and \( c_p \) denotes the capacity of water. Scalars \( A_i \) and \( R_i \) are the cross-sectional area and the thermal conductive coefficient of pipe \( i \), respectively. Scalar \( T_{i,j}^{NS} \) is the ambient temperature of pipe \( i \) at time \( t \).

3) Heat Node Model

The mass flow rate should satisfy hydraulic Kirchhoff’s law: the difference of pipe mass flows injecting into a node and leaving from the node equals to the node consumed mass flow:

\[
A m_t = \pi_t, \quad 1 \leq t \leq T,
\]

where \( m_t \) and \( \pi_t \) are pipe and node mass flow vectors at time \( t \), respectively. \( A \) is the node-branch incidence matrix defined as [18], in which

\[
\begin{cases}
+1, \text{ the mass flow of pipe } j \text{ comes into node } i \\
-1, \text{ the mass flow of pipe } j \text{ leaves from node } i \\
0, \text{ no connection from pipe } j \text{ to node } i
\end{cases}
\]

The heat nodes exchange heat power with the heat supply and return networks through heat exchangers:

\[
h_{i,j} = c_p \pi_{i,j} (T_{i,j}^{NS} - T_{i,j}^{RP}) \quad \forall i \in H, 1 \leq t \leq T,
\]

where \( h_{i,j} \) is the heat power absorbed by node \( i \) at time \( t \). In the heat supply network, for load nodes \( T_{i,j}^{NS} = T_{i,j}^{NS} \), and for source nodes \( T_{i,j}^{NS} \) is set by operators. Similarly, in the heat return network, for load nodes \( T_{i,j}^{RP} \) is calculated by (4), and for source nodes \( T_{i,j}^{RP} = T_{i,j}^{RP} \).

C. Optimization Model

1) Objective Function

The objective function of the optimal dispatch is to minimize the total cost of all energy sources in all time intervals:

\[
\min f = \sum_{p,j} \sum_{i \in H} C_{i,j}(p, j, h_{i,j}),
\]

where \( C_{i,j} \) is the cost function of energy source \( i \) at time \( t \) [18][25]. Set \( G \) is the set of energy sources.

The cost of energy sources is expressed using a quadratic function of electricity and heat productions [15]:

\[
C_{i,j} = \eta_{i,j,0} + \eta_{i,j,1} p_{i,j} + \eta_{i,j,2} p_{i,j}^2 + \eta_{i,j,3} h_{i,j} + \eta_{i,j,4} h_{i,j}^2,
\]

where \( \eta_{i,j,0} - \eta_{i,j,4} \) are time-varying cost coefficients of source \( i \) at time \( t \), which are given by generation costs and time-of-use. For example, for thermal generators which only generate electricity, coefficients of heat-related terms are zero. For electric boilers, electricity-related coefficients are negative and heat-related coefficients are positive.

2) Constraints

a) Energy Source Constraints

The feasible regions of different kinds of electric and heat sources in Fig.1 are described by polytopes [10][15][28]:

\[
B_{i,j} \cdot p_{i,j} + K_{i,j} \cdot h_{i,j} \leq v_{i,j} \quad \forall i \in G, 1 \leq t \leq T,
\]

where \( B_{i,j} \), \( K_{i,j} \), and \( v_{i,j} \) are coefficients of the \( k \)th boundary of the feasible operating region of source \( i \). For example,

1) As shown in Fig.2 (a), if a source generates electricity and heat simultaneously such as a combined heat and power (CHP) unit, its polytope is in the first quadrant, where \( p_{i,j} \geq 0 \) and \( h_{i,j} \geq 0 \). Similarly, the polytope of an electric
boiler resembles that of the CHP unit, but it is in the fourth quadrant.

2) As shown in Fig. 2 (b), if a source only generates electricity such as a thermal generator, the coefficient $K_{i,j}$ related to heat power output is zero, where $p_{i,j} \geq 0$ and $h_{i,j} = 0$.

3) As shown in Fig. 2 (c), if a source only generates heat such as a natural gas boiler, the coefficient $B_{i,j}$ related to electric power output is zero, where $p_{i,j} = 0$ and $h_{i,j} \geq 0$.

The master problem updates, revised mass flow rate or sensitivity or cutting plane is the line of mass flow vector $\pi$, according to (9). Therefore, the optimal dispatch problem of combined heat and power systems can be written as:

$$
\min f(x),
$$

s.t. $h_1(x, m) = 0$, $h_2(x) = \alpha_1^* x + \beta_0 = 0$,

$$
g_1(x) = \alpha_2^* x + \beta_2 \leq 0, \quad g_2(m) = \alpha_3^* m + \beta_3 \leq 0,
$$

where $h_1(x, m)$ denotes the nonlinear coupling constraints between $x$ and $m$, including (3)-(4), (7)-(8), and (10). The equality constraint $h_2(x)$ denotes the linear constraints on $x$ only, including (1)-(2) and (5)-(6). The inequality constraint $g_1$ represents the linear constraints on $x$ only, including (13)-(16) and (20). The inequality constraint $g_2(m)$ denotes the linear constraints on $m$ only, including (18)-(19). Since the equation (9) is applied to eliminate the node mass flow rate, we do not have equality constraints on $m$ only.

### III. Model Analysis and Decomposition

The challenge of solving optimization model (21) is that it is a non-convex program with bilinear constraints (3)-(4), (7)-(8), and (10). Although the problem (21) is non-convex, if $m$ is fixed, it will become a standard quadratic programming, which is easy and convenient to solve.

Based on the idea of Generalized Benders Decomposition, we treat $m$ as the complicating variable. Thus, as shown in Fig. 3, the problem (21) can be decomposed into a convex sub-problem with fixed $m$ and a master problem which optimizes $m$. If the sub-problem is feasible, the master problem updates $m$ based on the sensitivity calculated by the sub-problem; if the sub-problem is infeasible, the master problem revises the $m$ by removing infeasible $m$ according to cutting planes generated by sub-problems.
1) Convex Sub-Problem
The sub-problem is constructed by fixing $m$:

$$
\min_{x} f(x),
\text{s.t. } h_i(x, m^i) = 0, \quad h_2(x) = \alpha_i^x x + \beta_i = 0,
$$

where $m^i$ indicates the variable $m$ at $k$th iteration. The problem (22) is a standard convex problem because 1) the objective function is convex, and 2) all constraints are linear.

2) Master Problem
The master problem is formulated with fixed $x$:

$$
\min \ J^\ast(m),
\quad m \in M \cap \overline{FC},
$$

where $J^\ast$ is the optimal cost function of mass flow rate $m$ like the upper bound in Generalized Benders Decomposition. $M$ indicates the parameter space of pipe mass flow rate $m$ constructed by (18)-(19), and FC indicates the feasible cuts i.e., cutting planes.

The sub-problem and master problem are solved iteratively according to the following two steps:

First, the sub-problem (22) is solved. If the sub-problem is feasible, the envelope theorem is used to analyze the sensitivity of $m$. If the sub-problem is infeasible, Outer Approximation [24][26] is utilized to produce cutting planes.

Second, the master problem (23) uses gradient projection [28] to update $m$ if the sub-problem is feasible. And it removes infeasible $m$ from the original parameter space based on cutting planes if the sub-problem is infeasible.

IV. Solution Method

A. Sub-Problem
With fixed $m$, the sub-problem (22) is a standard convex problem, which can be solved conveniently by existing solvers. In this paper, CPLEX [29] is employed to solve the sub-problem.

1) Feasible Sub-Problem
If the sub-problem is feasible, the sensitivity of the optimal cost function with respect to the mass flow rate $m$, which denotes the gradient, is calculated by the envelope theorem:

$$
\frac{\partial J^\ast(m^i)}{\partial m_{i,t}} = \frac{\partial f^\ast(x)}{\partial m_{i,t}} \bigg|_{x=x^i, m=m^i} = \frac{\partial L(x,m)}{\partial m_{i,t}} \bigg|_{x=x^i, m=m^i},
$$

(24)

where $f^\ast$ is the optimal cost function, and $L(x,m)$ is the Lagrangian function of the sub-problem. $x^i$ is the variable $x$ at $k$th iteration.

2) Infeasible Sub-Problem
If the sub-problem is infeasible, the Outer Approximation is used to generate cutting planes for the master problem to remove corresponding infeasible $m$ from the original parameter space $M$. First, the relaxed sub-problem is solved:

$$
\min_{s,t} \sum_{i} s_i,
\text{s.t. } h_i(x, m^i) = 0, \quad h_2(x) = \alpha_i^x x + \beta_i = 0,\quad g_i(x) = \alpha_i^x x + \beta_i \leq s,
$$

(25)

where $s_i$ is the slack variable for the $i$th inequality constraint, and $s$ is the vector of slack variables.

Second, the cutting plane is calculated by Outer Approximation [24][26] since the problem (25) is a convex program with all constraints linear:

$$
(\lambda^i)^T [\nabla_m h_i(x^i, m^i)^T (m^i - m^i)] + (\bar{\mu}^i)^T g_i(x^i) \leq 0,
$$

(26)

where $\lambda^i$ and $\bar{\mu}^i$ are Lagrangian multipliers of $h_i$ and $g_i$ in the problem (25), respectively.

When (25) is solved, $\lambda^i$, $\bar{\mu}^i$, $\nabla_m h_i(x^i, m^i)^T$, and $g_i(x^i)$ are all fixed. Thus, inequality (26) is linear with $m$ as variables only, and it defines a cutting plane which removes infeasible $m$ from the original parameter space. Given by Outer Approximation, the cutting plane (26) is an over-estimation of the accurate cutting plane, which can accelerate the calculation by reducing iteration times [27].

B. Master Problem
The challenge of solving master problem (23) is that we do not have any close-form expression of $J^\ast$. Thus, the gradient projection [28] is applied to solve problem (23) iteratively.

1) Feasible Sub-Problem
If the sub-problem is feasible, the mass flow rate is updated by moving along the anti-gradient direction:

$$
m^{i+1} = m^i - \alpha^i \frac{\partial J^\ast(m^i)}{\partial m^i},
$$

(27)

where $\alpha^i$ is the step size at $k$th iteration, and we employ fixed step size in this paper. The gradient term in (27) is provided by the sub-problem as in (24). Matrix $P^i$ is the projection matrix at $k$th iteration which incorporates possible active boundary constraints (18)-(19) and cutting planes (26):

$$
P^i = I - H^i_{\lambda^i} \left( H^i_{\lambda^i} \right)^T \left( H^i_{\lambda^i} \right)^T,
$$

(28)

where $H^i_{\lambda^i}$ indicates the matrix of possible active constraints, for more details, see [30].

2) Infeasible Sub-Problem
If the sub-problem is infeasible, the master problem revises $m$ according to cutting planes, in which the revised mass flow rate $m^{i+1}$ in (29) denotes the intersection of the gradient direction and the cutting plane:

$$
m^{i+1} = m^i - \beta^i \frac{\partial J^\ast(m^i)}{\partial m^i},
$$

(29)

where $r$ indicates the last successful iteration, and $\beta^i$ indicates the revised step size:

$$
\beta^i = \left( \lambda^i \right)^T \left[ \nabla_m h_i(x^i, m^i)^T (m^i - m^i) \right] + (\bar{\mu}^i)^T g_i(x^i),
$$

(30)

Compared with the “optimal cut” in Generalized Benders Decomposition, when the sub-problem is feasible, the master problem in the proposed method does not cut the parameter space which has higher overall cost than the upper bound, but moves to the direction of reducing overall cost.

For clarity, the calculation process of the master problem is illustrated in Fig.4. First, from $m^i$ to $m^{i+1}$, the master problem updates the mass flow rate $m$ based on the gradient with
projection according to (27), because if the projection is not considered, the \( m \) will break the constraints of \( g_2(m) \). Second, the process from \( m^{i+2} \) to \( m^{i+3} \) updates the \( m \) according to (27) in the gradient direction without projection, where \( P^i = I \). Next, after the process from \( m^{i+2} \) to \( m^{i+3} \), the mass flow rate \( m^{i+3} \) is not in the parameter space of \( m \), which causes the infeasible subproblem. Thus, a cutting plane is generated according to (26). Forth, the process from \( m^{i+3} \) to \( m^{i+5} \) indicates the process (29) of revising \( m \) based on the cutting plane, which removes infeasible \( m \) from original parameter space. The \( m^{i+5} \) is the intersection point of the gradient direction and the cutting plane. Last, the process from \( m^{i+4} \) to \( m^{i+5} \) finds the local optimum point \( m^{i+5} \); therefore, the iteration stops at \( m^{i+5} \).

![Diagram](image)

**Fig. 4. The iteration processes of the master problem.**

The convergence criterion \( \sigma^k \) is defined as the division of the difference of the objective function values between two iterations and the objective function value at the first iteration:

\[
\sigma^k = \frac{|J^k - J^1|}{J^1}.
\]

(31)

Given \( \delta \) as the maximum tolerance, if \( \sigma^k \leq \delta \), the iteration will stop at \( k \)th time with the optimal cost \( J^* \).

V. CASE STUDIES

In case studies, 1) a test bed with a 6-node electric power system and 6-node heat system, and 2) a practical combined power and heat system in Barry Island are presented to demonstrate the effectiveness of the proposed method. In both cases, the proposed method is tested and compared with two existing approaches:

1) The optimization model (21) directly solved by IPOPT [31] (direct method),

2) The traditional fixed mass flow optimization [11] solved by CPLEX (traditional method).

In the traditional method, mass flow rate is a given value rather than a decision variable. It is noticed that since traditional fixed mass flow optimization is a convex program, the results of IPOPT and CPLEX are same. The reason of choosing CPLEX is that it is easier for YALMIP to extract dual variables using CPLEX [32].

A. Simulation Based on the Test System

As shown in Fig.5, the test system [15][20] has a 6-node power system and a 6-node heat system with a CHP unit. From 11:00-13:00, we test the proposed method in the following two scenarios:

1) In the first scenario, the mass flow rate of the traditional method is set around a local optimum of the IPOPT with 5% disturbance. The initial values of direct method and the proposed method are the same as the value of the traditional method.

2) In the second scenario, the mass flow rate of the traditional method is 15 kg/s for all load nodes. The initial values of direct method and the proposed method are the same as the value of the traditional method.

Except for the mass flow rate, other data of the electric power system and the heat system in above three methods are same. The results of the two scenarios are shown in Table I.

![Diagram](image)

**Fig. 5. The topology of the first system.**

**TABLE I**

| Approach | Traditional method | Direct method | Proposed method |
|----------|-------------------|--------------|-----------------|
| Overall costs of the 1st scenario ($) | 594.40 | 594.19 | 594.31 |
| Overall costs of the 2nd scenario ($) | 635.44 | - | 612.40 |

As shown in TABLE I, in the first scenario, the proposed method had the optimal cost of $594.31, which was very close to the result $594.19 solved by the direct method. Admittedly, influenced by the step-size and convergence tolerance, there were small gaps between the solutions of the proposed method and the direct method. In the second scenario, the proposed method reduced 3.63% of the overall cost compared with the traditional method. However, the direct method failed to converge when the initial mass flow rate is changed.

![Diagram](image)

**Fig. 6. Mass flow rate of pipe 3 (a) of the traditional method and the proposed method during iterations in the second scenario.**

In the second scenario, as presented in Fig.6 (a), in the traditional method, the mass flow rate of pipe 3 was 15 kg/s for all scheduling periods, while in the proposed method, the mass flow rate of pipe 3 was adjusted following the electricity price.
In Fig.6 (b), we can see the mass flow rate was iteratively updated; therefore, the heat pipeline storage ability can be better utilized, which improves the system’s flexibility. Thus, as shown in Fig.7, the CHP unit in the proposed method could generate more heat power during 11:50-12:20 than the traditional method. Considering the positive correlation of CHP electric and heat power outputs shown in Fig.7, the CHP unit can generate more electricity during 11:50-12:20 to reduce high-price electricity purchase from the grid. As a result, the efficiency of the combined heat and power system was improved using the flexibility from adjusting mass flow rate.

Fig. 7. The CHP electric and heat power outputs of the traditional method and the proposed method in the second scenario.

In summary, the proposed method can iteratively solve the optimization problem (21) and find a local optimum which is better than the traditional method. Under the small disturbance, the local optimums of the proposed method and the direct method are very close; Under different given initial values, the proposed method can avoid the divergence problem of the direct method.

B. Simulation Based on the Barry Island System

In Fig.8, the proposed method is investigated based on a real system in Barry Island, South Wales with a 9-node electric power system and a 33-node heat system [18].

Fig. 8. The topology of Barry Island system.

| TABLE II |
| Performance Comparison for Barry Island System |
|----------|----------------|----------------|----------------|
| Approach | Traditional method | Direct method | Proposed method |
| Overall costs ($) | $1.704 \times 10^4$ | - | $1.692 \times 10^4$ |

As shown in Table II and Fig.9, the proposed method reduced the overall cost by 0.95% compared with the traditional method. During iterations, the costs of CHP units and buying electricity from the grid were both reduced. However, the direct method failed to converge.

As shown in Fig.10, the mass flow rate was optimized by the proposed method during iterations, where the sub-problem had the optimal solution at each iteration.

Fig. 9. The cost of the proposed method during iterations.

Fig. 10. The mass flow rate of pipe 3 during iterations.

Fig. 11. The difference of heat power between generation side and load side.

Fig. 12. The (a) electric power and (b) heat power outputs of CHP unit 3.

On one hand, in the proposed method, the flexibility from adjusting mass flow rate can be used to reduce 1.57% of the cost of buying electricity from the grid compared with the traditional method. In the proposed method, because the mass flow rate was adjusted in Fig.10, the heat pipeline storage can be better utilized. Therefore, in Fig.11, the allowed unbalanced power between the generation side and the load side increased during 11:45 to 13:15. As a result, in the proposed method, the
CHP units can generate more heat power during 11:45 to 13:15. Due to the electric-heat coupling characteristics, CHP units can generate more electric power during 11:45 to 13:15 when electricity price is high. In this case, in Fig.12 (a) and (b), the electric and heat power outputs of CHP unit 3 in the proposed method have better followed the electricity price during 11:45 to 13:15 compared with the traditional method.

On the other hand, adjusting the mass flow rate can reduce the heat loss according to the heat pipeline model (7)-(8). In this case, the proposed method has reduced 3.43% heat energy loss compared with the traditional method. Thus, the combined heat and power system can satisfy the same load demand with higher efficiency compared with the traditional method.

Briefly, the proposed method can overcome the non-convergence problem of the direct method. Compared with the traditional method, the proposed method can reduce overall cost by increasing flexibility from adjusting mass flow rate.

In terms of calculation efficiency, the average solving time is 0.165s for each convex sub-problem, and the average time consumption of the master problem is 0.316s. Under the convergence tolerance $\delta=2\times10^{-4}$, the proposed method converged at 25 times, which can be use in day-ahead or intra-day dispatch.

VI. CONCLUSION

In this paper, the problem of increasing the flexibility of combined heat and power dispatch through adjusting mass flow rate is studied. Based on a nonlinear heat system model, the proposed non-convex optimization model reduces the complexity by removing the integers in existing MINLPs. The proposed solution method decomposes the non-convex optimization model into a convex sub-problem which calculates the sensitivity using envelope theorem and a master problem which updates the mass flow rate using gradient projection. In case studies, compared with traditional method, the proposed method has reduced 0.95%-3.63% of the overall cost. Compared with direct method solved by IPOPT, the proposed method can avoid divergence problems.

VII. REFERENCES

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