Hydrodynamic Modeling and Design of Robotic Fish using Slender Body Theory

Palmani Duraisamy¹, Manigandan Nagarajan Santhanakrishnan*¹

¹Department of Electronics and Instrumentation Engineering, SASTRA Deemed University, Thanjavur, Tamil Nadu-613401, India.

E-mail: manigandanns@eie.sastra.edu

Abstract. Design of bio-inspired robotic fish has been an important area of research for applications such as water quality monitoring, aquatic animal behavioral study and underwater exploration. The modeling of robotic fish requires high dimensional analysis, which increases the design complexity. To address this complexity, approximate models are preferred and frequently used in the design of robotic fishes. The accuracy of these models mainly relies on various hydrodynamic parameter identification of which drag coefficient estimation is highly challenging. This paper utilizes a simple deceleration motion model to estimate the drag coefficient effectively so that the linearization error is reduced. The non-linearity in the deceleration model is simplified and mapped as a linear model to predict the drag coefficient. The estimated drag coefficient is used for dynamic modeling of robotic fish using slender body theory. The dynamic model is validated through experimental setup using accelerometer and visual sensor data. The maximum swimming speed of 0.32 ms⁻¹ is achieved at 1.2 Hz caudal fin oscillations.

Keywords: robotic fish, dynamic modeling, deceleration model, drag estimation, speed tracking.

1. Introduction

Underwater vehicles propelled by natural methods (bioinspired) has been an interesting area of research which provides greater performance in terms of maneuverability, stealth, noise, and energy efficiency compared to other engineered designs. Among various bioinspired designs, robotic fish is the most remarkable one to achieve agile motions. Dynamic models of robotic fish are essential to achieve desired motion response often necessary in many real-time applications. Generally, dynamic model utilizes Newtonian [1] or Lagrangian mechanics [2] and Lighthill’s models [3-4] frequently for modeling the kinematics and hydrodynamics of the robotic fish. Lagrangian models require higher order dimensional analysis, which increases the system complexity. On the other hand, Newtonian models are simple, computationally efficient and frequently used for modeling a robotic fish [5]. Among different approaches of Lighthill’s model, slender body theory [6] effectively models the hydrodynamics of a swimming fish, if the dimensions and oscillations of the fish are small compared to its physical length.

The parameters such as caudal fin thrust and drag coefficient are necessary for modeling a robotic fish. Without a physical model, its highly challenging to determine these parameters. The drag
coefficients for objects with simple geometry can be directly calculated using numerical computations, but for arbitrary shapes it is estimated using experimental approaches [7-9]. Numerical method uses CFD like tools [10], whereas on the other hand analytical methods use experimental design for its estimation. In model-based estimation [1,3], the robotic fish swims at a constant speed so that the thrust generated by robotic fish is exactly balanced by drag force. The overall drag coefficient is estimated using least mean square method through graphical approach, which demands high-speed motion capture camera for velocity and displacement measurement. In constant speed methods [11-15], a robotic fish is towed at different speeds using a motion control setup. The towing force is measured using force sensor and the slope of velocity versus squared speed plot gives us the drag coefficient. The requirement of suitable force sensors and motion control unit complicates the system design. Wain Leung Chan et al [7] presented an experiment that simultaneously estimates drag coefficient and added mass, which also requires a high-resolution optical encoder setup and timing precision unit for displacement measurement. The proposed work introduces a simple deceleration motion model of robotic fish for drag coefficient estimation. The principle behind the experiment is inverse velocity-time relation to estimate the drag coefficient to mass ratio. The least mean square fit of experimental data estimates drag coefficient to mass ratio. Next, the thrust generated by a robotic fish is modeled using slender body approach by considering caudal fin oscillations. The Newtonian dynamic model relates the thrust and drag to estimate the speed of a robotic fish at given instant. The closed form control system of robotic fish requires this speed estimation to follow the predefined trajectories to achieve the desired motion.

2. Modeling of Robotic Fish

2.1. Kinematic Model
The proposed robotic fish design is inspired from carangiform swimmers. Its oscillation is confined to rear on-third of the body to produce swimming thrust. These swimmers are efficient in terms of speed and maneuverability because of its streamlined structure [4]. The free body diagram of proposed robotic fish and its equivalent kinematic model is shown in figure 1. Oscillation of tail is modeled [3] using the expression as follows,

\[ h(x, t) = x \tan(\theta(t)) \]

where, \( x \) is the length of the oscillating body of the robot, \( \theta(t) \) is oscillation angle sinusoidally varies with respect to time.

\[ \theta(t) = \theta_m \sin(2\pi ft) \]

where, \( \theta_m \) is oscillation amplitude and \( f \) is the frequency of oscillation.

with the use of slender body theory, the thrust generated by carangiform robotic fish can be expressed as follows,

\[ F_T(t) = \frac{\rho A(l)}{2} \left[ \left( \frac{\partial h(x,t)}{\partial t} \right)^2 - v^2 \left( \frac{\partial h(x,t)}{\partial x} \right)^2 \right]_{x=l} \]

where, \( F_T(t) \) is instantaneous thrust force, \( \rho \) is density of water, \( A(l) \) is area of the circle calculated using overall dimension of tail as a diameter, \( v \) is horizontal speed of robotic fish along -x direction, and \( l \) is length of the tail. The squares of the derivative values are averaged for one cycle of oscillation and it is given by,

\[ G_1(\theta_m, f) = \frac{\rho A(l)}{2} \frac{1}{T} \int_0^T \left( \frac{\partial h(x,t)}{\partial t} \right)^2 dt \]

\[ G_2(\theta_m, f, v) = \frac{\rho A(l)}{2} v^2 \frac{1}{T} \int_0^T \left( \frac{\partial h(x,t)}{\partial x} \right)^2 dt \]

Therefore, the thrust generated by the robotic fish is a function of amplitude of oscillation (\( \theta_m \)) and frequency of oscillation (\( f \)).

\[ F_T(t) = G_1(\theta_m, f) - G_2(\theta_m, f, v) \]
2.2. Dynamic Modeling
Dynamic modeling of robotic fish follows the Newton’s law of motion,

\[ m \ddot{v} = F_T(t) - F_D(t) \]  

where, \( m \) is the mass of the physical robotic fish, \( \ddot{v} \) is derivative of velocity, and \( F_D(t) \) is horizontal drag force acting opposite to the thrust \( (F_T(t)) \). Drag force acting on the robotic fish is expressed as follows,

\[ F_D(t) = \frac{1}{2} \rho A C_d v^2(t) \]  

where, \( A \) is surface area of the robotic fish, \( C_d \) is drag coefficient of robot body and \( v \) is speed of the robotic fish.

An estimation of drag coefficient utilizes decelerated motion of a robotic fish. Under undisturbed decelerating motion, the generated thrust is zero and speed of the robotic fish decreases gradually during which at any instant the force of inertia \( (m \ddot{v}) \) counter balances the body drag \( (F_D(t)) \) [8].

\[ m \ddot{v} = -F_D(t) = -\frac{1}{2} \rho A C_d v^2(t) \]  

The non-linear differential equation (10) can be transformed into linear equation as follows,

\[ \frac{d\dot{v}(t)}{dx} = -k v^2(t) \]  

with the drag coefficient to mass ratio,

\[ k = \frac{\rho A C_d}{2m} \]  

the equation (11) is integrated and simplified as follows,

\[ x(t) = k t + x_0 \]  

where, \( x(t) = \frac{1}{v(t)} \) is reciprocal function of speed and \( \frac{1}{x_0} = v_0 \) is the speed at time \( t=0 \) (initial speed of the robotic fish). The reciprocal function is plotted using decelerated robotic fish data. The slope of the function estimates the overall drag coefficient. The drag coefficient \( C_d \) is computed from the overall drag coefficient by substituting area, body mass and density of water. The slender body theory assumes that this drag coefficient is constant throughout the motion.

3. Physical Prototype Design
Propulsion mechanism of a real fish utilizes the tail part effectively to generate required thrust for motion. The proposed design of robotic fish utilizes the same principle and generates required thrust using tail oscillations. The prototype was built with four discrete parts namely head, body, lateral body and tail to ensure the streamlined structure. Each of this part was fabricated using MakerBot 3D printer using polyactic acid (PLA) filament to provide moderate rigidity in the structure. Parts of the robotic fish are connected by fixed joints except the tail. The tail is attached by revolute joint using
servomotor to produce oscillatory motion. The complete prototype design is shown in figure 2. The prototype is designed such a way that the center of mass point is fixed at the body center of the robotic fish so that its aligned with respect to the body axis horizontally. Any corrections in the center of mass was compensated by adding weights inside of the robotic parts so as to balance the buoyancy force such that more than 95% of body parts are immersed into water and hence, ensures surface swimming. A high torque dynamixel servomotor AX-12 is used to produce oscillation motion of the tail, which has an inbuilt closed loop control system to reach the desired position accurately at rated speed. Prototype is completely sealed using waterproof skin made of silicone material. The rotary motion of the servomotor was programmed using Matlab software and interfaced via CM-530 controller to produce oscillation pattern of the tail. The physical parameters of robotic fish prototype are detailed in table 1.

Table 1. Details of physical parameters of robotic fish.

| Symbols | Parameters       | Value with units |
|---------|------------------|------------------|
| L       | Length of the robotic fish | 0.59 m          |
| l       | Length of the tail     | 0.16 m          |
| C       | Span of the tail      | 0.096 m         |
| A       | Surface area          | 0.153 m²        |
| m       | Mass                | 5.5 kg          |
| ρ       | Density of fluid     | 998.2071 kg m³  |

4. Experimental Setup
The robotic fish swimming trials were conducted in a circular tank of 1.8 m diameter. An overhead camera is fixed above the circular tank and an inertial measurement unit (IMU) sensor is fixed on the head of the robotic fish. The accelerometer data from the IMU sensor is recorded and is used for calculating speed of the swimming robotic fish. The overhead camera captures the motion of robotic fish at a speed of 25 frames per second. The decelerated motion of robotic fish is generated using a setup built inhouse as shown in figure 3. Robotic fish is towed using a string connected to servomotor. The servomotor is excited by Lego Mindstorms EV3 controller which is programmed to generate variable acceleration, so that it can produce constant speed during initial periods. After achieving constant speed of the robotic fish, the servomotor is stopped, so that the deceleration motion can be established. The decelerated robotic fish motion is captured using overhead camera for different initial speeds. The Kinovea software is used for calculating the speed of the robotic fish at different instants of time. The experimental trial of cruise swimming motion is shown in figure 4.
5. Results and Discussion
The speed of decelerated robotic fish motion is mapped to reciprocal function. The resultant reciprocal speed data is plotted and the slope of this line is estimated using least mean square method as shown in figure 5. Using this slope (k), the drag coefficient ($C_d$) is computed as 3.1032 using equation (11). Next, a simulation is performed to calculate speed of the robotic fish using the estimated drag coefficient. The swimming trials of robotic fish was conducted at different frequencies of oscillations. The speed of the robotic fish is tracked using overhead camera and accelerometer data from IMU sensor. Speed of robotic fish is plotted based on the simulation data, camera, and accelerometer data as shown in figure 6. The results of speed profile for different frequencies also produces similar relation with simulation and experiments. Due to the precise estimation of drag coefficient, it is seen that the real-time swimming speed of the robotic fish closely follows simulation results.
6. Conclusion and Future Scope
The model-based validation of speed profile for a robotic fish is accomplished in this work. An efficient estimation of drag coefficient was established by using deceleration motion model. Proposed method estimated the drag coefficient, which is closer to the experimental value, hence the accuracy of the dynamic model improved significantly. The dynamic model of the robotic fish was validated through simulation and experiments using camera and IMU sensor. In the future works, speed profile of proposed robotic fish can be utilized for the design of closed loop controllers in order to improve the speed tracking performance. This work may also be extended for two-dimensional maneuvering applications.
References
[1] Saurab Verma and Jian-Xin Xu 2018 Analytic modelling for precise speed tracking of multilink robotic fish IEEE Transactions on Industrial Electronics 65 5665–72
[2] Ozmen Koca G, Bal C, Korkmaz D, Bingol M C, Ay M, Akpolat Z H and Yetkin S 2018 Three-dimensional modeling of a robotic fish based on real carp locomotion Appl. Sci. 8 180
[3] Li X, Ren Q and Xu J X 2016 Precise Speed tracking control of a robotic fish via iterative learning control IEEE Transactions on Industrial Electronics 63 2221–28
[4] McMasters RL, Grey CP, Sollock JM, Mukherjee R, Benard A and Diaz AR 2008 Comparing the mathematical models of lighthill to the performance of a biomimetic fish Bioinsp. Biomim. 3, 247–69
[5] Palmani Duraisamy, Rakesh Kumar Sidharthan and Manigandan Nagarajan Santhanakrishnan 2019 Design, modeling, and control of biomimetic fish robot: a review J Bionic Eng. 16 967–93.
[6] Lighthill M J 1960 Note on the swimming of slender fish Journal of Fluid Mechanics 9 305
[7] Wai Leung Chan and Taesam Kang 2011 Simultaneous determination of drag coefficient and added mass IEEE Journal of Oceanic Engineering 36 422–30
[8] Bilo D and Nachtigall W 1980 A simple method to determine drag coefficients in aquatic animals Journal of Experimental Biology 87 357–59
[9] Wai Leung Chan, Taesam Kang, Young Jae Lee, Sang Kyung Sung and Kwang Joon Yoon 2007 Swimming study on an ostraciform fish robot International Conference on Control, Automation and Systems (Seoul) (South Korea/IEEE) 700–05
[10] Amit Tyagi and Debabrata Sen 2006 Calculation of transverse hydrodynamic coefficients using computational fluid dynamic approach Ocean. Eng. 33 789–809
[11] Li Wen, Guanhao Wu, Jianhong Liang and Jinlan Li 2010 Hydrodynamic experimental investigation on efficient swimming of robotic fish using self-propelled method Int. J. Offshore Polar Eng. 20 167–174
[12] Li Wen, Tianmiao Wang, Guanhao Wu, Jianhong Liang and Chaolei Wang 2012 Novel method for the modeling and control investigation of efficient swimming for robotic fish IEEE Trans. Ind. Electron. 59 3176–88
[13] Li Wen, Tianmiao Wang, Guanhao Wu and Jianhong Liang 2013 Quantitative thrust efficiency of a self-propulsive robotic fish: Experimental method and hydrodynamic investigation IEEE/ASME Trans.Mechatronics 18 1027–38
[14] Saurab Verma and Jian-Xin Xu 2017 Data-assisted modeling and speed control of a robotic fish IEEE Trans. Ind. Electron. 64 4150–57
[15] AC Fernandes and FPS Mineiro 2007 Assessment of hydrodynamic properties of bodies with complex shapes J. Appl. Ocean Res. 29 155–66

Acknowledgement
The authors would like to thank Dr. S. Rakesh Kumar, Assistant Professor, SASTRA Deemed University for his kind and helpful suggestions on our experimental apparatus and implementations.