A New GNSS Relative Positioning Algorithm Based on Alternative Use of the Positions of Reference Receivers

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ABSTRACT

In this paper, we present a new relative positioning algorithm based on GNSS Regression measurement models (GR models) by using multiple antennas based on alternative use of the positions of reference receivers. We show the algorithms of estimating all unknown antenna positions by applying the double difference (DD). Then we derive updating algorithms by using true positions of the reference receivers. Finally, we carried out the experiments using real GNSS data and show the positioning results for the the proposed method as well as the conventional method.

1 INTRODUCTION

It is well known that relative positioning can give accurate results of positioning as a relative position by using at least two receivers. For estimating a relative position, you need GNSS measurements including the true position of the reference receivers on the premise that receivers observe GNSS signals of the same satellites. If the baseline length between the reference receiver (known) and the rover receiver (unknown) is not long, GNSS measurements from satellites observed by receivers include similar observation errors; therefore, you can cancel observation errors by using GNSS measurements including the true reference position and calculate the accuracy position.

In this paper we present the new relative positioning algorithm based on alternative use of the positions of reference receivers. To estimate all receiver positions including reference receiver, GNSS measurements are applied the DD without true position of the reference receiver. Then, rover position are estimated again by using constraint condition of true position of the reference receiver. In other words, this method updates the rover position already estimated with the constraint condition of true reference position. This paper is aimed at verifying the performance of this method.

2 GNSS REGRESSION MODELS AND DD MEASUREMENT EQUATIONS

First of all, similarly to [1]-[7], we formulate all observed positioning data consisting of the L1 carrier-phase and pseudoranges based on C/A code, by using the GNSS regression models. The natural extensions of GNSS regression models for multiple frequencies of GPS, Galileo, Compass/BeiDou, GLONASS, US-GPS and QZSS modernization are also similarly formulated. Namely, we consider the following fundamental measurements of the pseudoranges $\rho_{\text{CA},u}(t)$ based on the C/A code and L1 band carrier-phases $\varphi_{\text{L1},u}(t)$ (equivalently, $\Phi_{\text{L1},u}(t)$ as the unit of length), respectively, as follows [8]-[15]:

$$\rho_{\text{CA},u}(t) = r_u(t, t - \tau_u^p) + \delta \Phi_{\text{CA},u}(t) + \delta T_{\text{CA}}^u(t) + \varepsilon_{\text{CA},u}(t)$$

$$\Phi_{\text{L1},u}(t) = \lambda_1 \varphi_{\text{L1},u}(t)$$

where $\lambda_1$ is the wavelength of the carrier wave and calculated by $c/f_1$. $c \approx 3.00592458 \times 10^8 [\text{m/s}]$ denotes the speed of light, and $f_1$ is the central frequency of the L1 carrier wave

$$f_1 = 2 \times 77 \times 10.23 [\text{MHz}] = 1575.42 [\text{MHz}]$$

In Eqs. (1)-(2), the so-called receiver biases, $\delta \Phi_{\text{CA},u}$, $\delta \Phi_{\text{L1},u}$, and the satellite biases, $\delta T_{\text{CA}}^u$, $\delta T_{\text{L1}}^u$, are contained in the usual observed positioning data consisting of pseudorange based on the C/A code and the L1 carrier-phase [16]. $r_u(t, t - \tau_u^p)$ is the geometric distance between the receiver $u$ at the time $t$ and the satellite $p$ at the time $t - \tau_u^p$. $\tau_u^p$ denotes the travel time from the satellite $p$ to the receiver $u$. Namely,

$$r_u(t) = r_u(t, t - \tau_u^p)$$

$$= \left( |x_u(t) - x_p^p| + |y_u(t) - y_p^p| + |z_u(t) - z_p^p| \right)^2$$

$$= \|u(t) - s_p(t - \tau_u^p)\|$$

$$= \|u(t) - s_p(t - \tau_u^p)\|$$

where $u \equiv [x_u, y_u, z_u]^T$ and $s_p \equiv [x_p, y_p, z_p]^T$ are a user (unknown) and satellite positions, respectively. $n_s$ denotes the number of the observable satellites. $\delta \Phi_{\text{CA},u}$ and $\delta T_{\text{L1}}^u(t)$ in Eqs. (1)-(2) reflect the delay or the advance associated with the transmission of the L1 signal through the ionosphere and the troposphere, respectively. $\delta \Phi_{\text{CA},u}(t)$ and $\delta T_{\text{L1}}^u(t)$ are the
clock errors of the receiver \( u \) at the time \( t \) and the satellite \( p \) at the time \( t - \tau_p^u \). \( N_p^u \) denotes integer ambiguity between the satellite \( p \) and the receiver \( u \), and \( e_{CA,u}(t) \), \( e_{L1,u}(t) \) denote measurement errors.

Eq. (3) contains the satellite orbital errors. The estimated satellite orbits are obtained from the navigation messages which are decoded from the transmitted L1 signal. Let us denote \( \tilde{s}^p \) as the estimated position of the satellite \( s^p \) at the time \( t - \tau_p^u \).

We use the following relations of the derivatives
\[
\begin{align*}
\frac{\partial r_p^u}{\partial x_1} &= \frac{x_n - x_p}{r_n^u}, \\
\frac{\partial r_p^u}{\partial y_1} &= \frac{y_n - y_p}{r_n^u}, \\
\frac{\partial r_p^u}{\partial z_1} &= \frac{z_n - z_p}{r_n^u}, \quad (p = 1, 2, \ldots, n_s),
\end{align*}
\]
and also
\[
\begin{align*}
\frac{\partial r_p^u}{\partial x_p} &= \frac{x_n - x_p}{r_n^u}, \\
\frac{\partial r_p^u}{\partial y_p} &= \frac{y_n - y_p}{r_n^u}, \\
\frac{\partial r_p^u}{\partial z_p} &= \frac{z_n - z_p}{r_n^u}, \quad (p = 1, 2, \ldots, n_s).
\end{align*}
\]

Then we have the relation:
\[
\frac{\partial r_p^u}{\partial t} = \frac{\partial r_p^u}{\partial s^p}
\]

Thus the 1st order Taylor series approximation of Eq. (3) around the previous estimated value \( u = \hat{u} \) and \( s^p = \tilde{s}^p \) is given by
\[
\begin{align*}
r_p^u &= \hat{r}_p^u + \nabla \hat{r}_p^u \cdot (\hat{u} - \tilde{s}^p) \\
&= \frac{(\hat{u} - \tilde{s}^p)^T}{||\hat{u} - \tilde{s}^p||} (\hat{u} - \tilde{s}^p)
\end{align*}
\]

for \( p = 1, 2, \ldots, n_s \), where gradient vectors are as follows:
\[
g_{\tilde{s}^p} = \left[ \frac{\partial r_p^u}{\partial u} \right]_{u = \hat{u}, s^p = \tilde{s}^p} = \frac{(\hat{u} - \tilde{s}^p)^T}{||\hat{u} - \tilde{s}^p||}
\]

The GR models utilize the extended Kalman filter based on the geometric distance linearized by the gradient vectors [6], and provide appropriate positioning performances. The DD-based PPP methods employ the same estimation method as that of previous PPP methods.

The following parameters contained in the usual observed positioning in Eq. (1-2) are canceled by DD-based GR models:
\[
\delta t_{1,u}, \delta P, I_{p}, T_{p}, \delta \theta_{CA,u}, \delta \theta_{L1,u} : \text{Receiver and satellite clock errors}, \quad P_{p}, T_{p} : \text{Ionospheric and tropospheric delays}, \quad \delta \theta_{CA,u}, \delta \theta_{L1,u} : \text{Satellite hardware biases}, \delta \theta_{CA,u}, \delta \theta_{L1,u} : \text{Receiver hardware biases}.
\]

Let us assume that the estimated values: \( s^p, p = 1, \ldots, n_s \), of the satellite positions: \( s^p, p = 1, \ldots, n_s \), are practically obtained by using ephemeris parameters broadcast by GPS satellites, or precise ephemerides of all satellites provided by the International GNSS Service (IGS), as follows [1]-[6]:
\[
\tilde{s}^p = s^p + e_{s^p}, \quad p = 1, \ldots, n_s,
\]
where we assume \( e_{s^p} \) are Gaussian white noises.

Let us write the final results of the double difference measurement equations as follows [10]-[12]:
\[
\begin{align*}
\rho^{\tilde{p}}_{CA,\hat{u}_i} &= \rho^{\tilde{p}}_{CA,\hat{u}_i} + (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T s^p - (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T \tilde{s}^p \\
&= (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T u_j - (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T u_i + (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T e_{s^p} - (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T e_{s^p} + e_{\rho_{CA,\hat{u}_i}}^{\tilde{p}}
\end{align*}
\]
\[
\Phi^{\tilde{p}}_{L1,\hat{u}_i} &= \Phi^{\tilde{p}}_{L1,\hat{u}_i} + (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T s^p - (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T \tilde{s}^p \\
&= (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T u_j - (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T u_i + (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T e_{s^p} - (\tilde{g}_{\hat{u}_i}^{\tilde{p}})^T e_{s^p} + \lambda_1 e_{\Phi_{L1,\hat{u}_i}}^{\tilde{p}}
\end{align*}
\]

where
\[
\begin{align*}
\tilde{g}_{\hat{u}_i}^{\tilde{p}} &= g_{\hat{u}_i}^{\tilde{p}} - g_{\hat{u}_i}^{\tilde{p}}, \quad \tilde{g}_{\hat{u}_i}^{\tilde{p}} = g_{\hat{u}_i}^{\tilde{p}} - g_{\hat{u}_i}^{\tilde{p}} \\
\rho_{CA,\hat{u}_i}^{\tilde{p}} &= \rho_{CA,\hat{u}_i}^{\tilde{p}} - \rho_{CA,\hat{u}_i}^{\tilde{p}} - (\rho_{CA,\hat{u}_i}^{\tilde{p}} - \rho_{CA,\hat{u}_i}^{\tilde{p}}) \\
\Phi_{L1,\hat{u}_i}^{\tilde{p}} &= (\Phi_{L1,\hat{u}_i}^{\tilde{p}} - \Phi_{L1,\hat{u}_i}^{\tilde{p}} - (\Phi_{L1,\hat{u}_i}^{\tilde{p}} - \Phi_{L1,\hat{u}_i}^{\tilde{p}})) \\
e_{\rho_{CA,\hat{u}_i}}^{\tilde{p}} &= e_{\rho_{CA,\hat{u}_i}}^{\tilde{p}} - e_{\rho_{CA,\hat{u}_i}}^{\tilde{p}} \\
e_{\Phi_{L1,\hat{u}_i}}^{\tilde{p}} &= e_{\Phi_{L1,\hat{u}_i}}^{\tilde{p}} - e_{\Phi_{L1,\hat{u}_i}}^{\tilde{p}}
\end{align*}
\]

For the case of \( p = 1, q = 2, \ldots, n_s \) and \( u_i = u_1, u_j = u_2 \) \( (n_s = 2) \), we have the following measurement equation for antennas of \( u_1 \) and \( u_2 \) and for \( n_s \) satellites [7, 17]:
\[
y_{u_2u_1}^{n_s} = C_{u_2u_1}^{n_s} q_{u_2u_1}^{n_s} + v_{u_2u_1}^{n_s}
\]

where
\[
y_{u_2u_1}^{n_s} = y_{u_2u_1}^{n_s} - y_{u_2u_1}^{n_s} - y_{u_2u_1}^{n_s} - y_{u_2u_1}^{n_s}
\]
\[
C_{u_2u_1}^{n_s} = \begin{bmatrix} C_{u_2u_1}^{n_s} & C_{u_2u_1}^{n_s} \\ C_{u_2u_1}^{n_s} & C_{u_2u_1}^{n_s} \end{bmatrix}
\]
\[
g_{u_1}^{n_s} = \begin{bmatrix} g_{u_1}^{n_s} \\ g_{u_1}^{n_s} \\ g_{u_1}^{n_s} \end{bmatrix}
\]
\[
\lambda_1 \mathbf{I}
\]
\[
G_{u_1}^{n_s} = \begin{bmatrix} \hat{u}_1^{n_s} & \hat{u}_1^{n_s} & \hat{u}_1^{n_s} \end{bmatrix}^{T} : (n_s - 1) \times 3.
\]
\[
y_{u_2u_1}^{n_s} \text{ and } y_{u_2u_1}^{n_s} \text{ are } ((n_s - 1) \times 1) \text{ vectors which are DD observables from raw C/A code pseudoranges and L1 carrier phases, respectively. } s^p(p = 1) \text{ is the reference satellite. } N_{u_2u_1}^{n_s} \text{ and } v_{u_2u_1}^{n_s} \text{ are DD-based integer ambiguity and observation noise vector, respectively. } G_{u_1}^{n_s} \text{ is the known } ((n_s - 1) \times 3) \text{ matrix which consists of gradient vectors } g_{u_1}^{n_s}. \]
2.1 State Equations for the Static Case

In the static case, we utilize the static state vector $\eta_{b_1u_1}$ in Eq. (12) for antennas of $u_1$ and $u_2$ ($n_1 = 2$) and for $n_s$ satellites. In order to simplify the expression, superscripts $s$, 1 and subscripts $u_1,u_2$ are omitted hereafter. Then in the case of $\eta$, the state equation is described by

$$\eta_{t+1} = \eta_t, \quad \eta_t : n' \times 1, \quad n' = 5 + n_s,$$

and the measurement equation is

$$y_t = C_t\eta_t + v_t,$$

Therefore the positioning algorithms based on the Kalman filter for Eqs. (14) and (15) are given as follows

$$\hat{\eta}_{t+1|t} = \hat{\eta}_{t|t} + K_t\nu_t,$$

$$\nu_t = y_t - C_t\hat{\eta}_{t|t-1},$$

(16)

(17)

(18)

(19)

(20)

(21)

(22)

(23)

(24)

(25)

Then we have the following quadratic form for the power term of the Eq.(28):

$$\frac{1}{2}(\eta - \hat{\eta})^T\Sigma^{-1}_\eta(\eta - \hat{\eta}) + \frac{1}{2}(\tilde{u}_1 - u_1)^TR_{u_1}^{-1}(\tilde{u}_1 - u_1)$$

$$= \frac{1}{2}\left\{\eta^T\Sigma^{-1}_\eta \eta - \eta^T\Sigma^{-1}_\eta \hat{\eta} - \hat{\eta}^T\Sigma^{-1}_\eta \eta + \hat{\eta}^T\Sigma^{-1}_\eta \hat{\eta}\right\} + \tilde{u}_1^TR_{u_1}^{-1}\tilde{u}_1 - c_{u_1}^T\eta + \eta^Tc_{u_1} + c_{u_1}^TM_{u_1}\eta$$

$$= \frac{1}{2}\left\{\eta^T(S_{\eta}^{-1} + M_{u_1})\eta - \eta^T(S_{\eta}^{-1}\hat{\eta} + c_{u_1})
\right.$$\n
$$\left. - c_{u_1}^T(S_{\eta}^{-1} + M_{u_1})\eta + \hat{\eta}^T(S_{\eta}^{-1}\eta + \tilde{u}_1^TR_{u_1}^{-1}\tilde{u}_1)\right\}$$

$$= \frac{1}{2}\left\{\eta^T(S_{\eta}^{-1} + M_{u_1})\eta - \eta^T(S_{\eta}^{-1}\hat{\eta} + c_{u_1})
\right.$$\n
$$\times\left[\eta^T(S_{\eta}^{-1} + M_{u_1})\eta - \eta^T(S_{\eta}^{-1}\hat{\eta} + c_{u_1})\right]$$

$$- (S_{\eta}^{-1}\hat{\eta} + c_{u_1})^T(S_{\eta}^{-1} + M_{u_1})^{-1}(S_{\eta}^{-1}\hat{\eta} + c_{u_1})$$

$$\times(S_{\eta}^{-1}\hat{\eta} + c_{u_1})^T(S_{\eta}^{-1} + M_{u_1})^{-1}(S_{\eta}^{-1}\hat{\eta} + c_{u_1})$$

$$+ \hat{\eta}^T(S_{\eta}^{-1} + M_{u_1})\hat{\eta} + \tilde{u}_1^TR_{u_1}^{-1}\tilde{u}_1$$

(29)

where $M_{u_1}$ and $c_{u_1}$ are given as follows.

$$c_{u_1} \equiv \begin{bmatrix} R_{u_1}^{-1}\tilde{u}_1 \\ \ast_{n_u+2}^{-1} \end{bmatrix},$$

(30)

$$M_{u_1} \equiv \begin{bmatrix} R_{u_1}^{-1} \\ O_{(n_u+2)\times3} \end{bmatrix} \begin{bmatrix} O_{3\times(n_u+2)} \\ O_{(n_u+2)\times(n_u+2)} \end{bmatrix}$$

(31)

Then the update estimated vector $\tilde{\eta}$ and error covariance matrix $\Sigma_{\eta}$ of $\eta$ based on the minimum mean square estimate are given by

$$\tilde{\eta} = (\Sigma_{\eta}^{-1} + M_{u_1})^{-1}(\Sigma_{\eta}^{-1}\hat{\eta} + c_{u_1}),$$

(32)

$$\Sigma_{\eta} = (\Sigma_{\eta}^{-1} + M_{u_1})^{-1}.$$
3 EXPERIMENTAL RESULTS

In the experiments, two relative positionings of the conventional method [19], and the proposed method of updating by true positions were performed for static relative positioning, and the positioning accuracy was compared. The GPS data (see Table 1) were collected and provided by the GEONET (GNSS Earth Observation Network System) of Geospatial Information Authority of Japan (GSI).

Table 1: Experimental Conditions

| Date               | September 15, 2017 |
|--------------------|--------------------|
| GPS-Time           | 03:00:00 ~ 04:00:00 |
|                    | 11:00:00 ~ 12:00:00 |
| Reference station: | HAMAOKA 1           |
| Name               | TOPCON NETG3       |
| Receiver           | TPSCR.G5 GSI       |
| Rover station:     | HAMAOKA 2           |
| Name               | TRIMBLE NETR9      |
| Receiver           | TRM59800.80 GSI     |
| Baseline length    | 2.8813[km]         |
| Epoch interval     | ΔN = 30 [s]        |
| Elevation angle    | 15 [deg.]          |
| Measurement Data   | C/A code, L1 carrier-phase |
| Ionospheric delay estimation | No                 |
| Tropospheric delay estimation | No            |
| Ambiguity Resolution | Lambda method     |

In the experiment, we calculated HAMAOKA 2 as the rover position by applying true position (HAMAOKA 1) as the reference position during 03:00:00 ~ 04:00:00 and 11:00:00 ~ 12:00:00 on Sept.15, 2017. The results show the Eastward-Northward errors of estimated positions (see Fig. 1 and Fig. 3) and upper errors (see Fig. 2 and Fig. 4) of rover position. Table 2 and Table 3 show RMS errors of positioning in each direction.

Table 2: RMS errors (03:00:00~04:00:00)

|       | East [cm] | North [cm] | Up [cm] |
|-------|-----------|------------|---------|
| New method | 0.387    | 0.841      | 3.421   |
| Conventional method | 0.676    | 0.593      | 2.785   |

Fig. 1: EN Errors (03:00:00~04:00:00)

Fig. 2: Upper Positioning Errors (03:00:00~04:00:00)

Fig. 3: EN Errors (11:00:00~12:00:00)

Fig. 4: Upper Positioning Errors (11:00:00~12:00:00)
Table 3: RMS errors (11:00:00 ~ 12:00:00)

|                | East [cm] | North [cm] | Up [cm] |
|----------------|-----------|------------|---------|
| New method     | 0.339     | 0.455      | 2.331   |
| Conventional method | 0.658     | 0.233      | 2.041   |

From Figs. 1 and 3, by applying the new method, we have the positioning accuracy less than 1.0 cm E-N directional errors. It should be remarked that the unit of the vertical coordinate is centi-meter. The difference of results by both methods is very small on the horizontal E-N plane. In terms of upper position errors, differences between two method are less than 1 cm; (see Fig. 2 and Fig. 4).

4 CONCLUSIONS

We have presented the algorithm of relative positioning based on alternative use of the positions of reference receivers in short baseline. The positioning error terms of the observable mathematical models are cancelled by DD techniques, and the satellite positions (uncanceled terms) are obtained from navigation messages in consideration of the estimated errors.

The experiments were conducted for short baseline (baseline length 2.8813 [km]) with negligible effects of ionospheric/tropospheric delay. Proposed method of relative positioning gives the positioning accuracy in several cm level as with the conventional method. In the future, we will verify the performance in long baseline (baseline length over 10 [km]) and the kinematic environment.

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