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Back to the roots: the concepts of force and energy

https://doi.org/10.1515/zpch-2021-3122
Received August 31, 2021; accepted October 28, 2021; published online November 29, 2021

Abstract: The concepts of force and energy are analyzed in the context of state and process equations. In chronological order, the application of the cause-effect principle in process equations is studied in mechanics, thermodynamics, special relativity, general relativity, and quantum theory. The differences in the fundamental approaches to nature and the significance of a consistent physical interpretation of formulas and state variables are emphasized. It is shown that the first origins for the crisis of modern theoretical physics are to be found in the concepts of force and energy in mechanics, which partly violate the cause-effect principle. This affects all theories based on mechanics and underlines their historical conditionality. The systematic application of driving forces and the cause-effect principle in process equations suggests a return to causal realistic physics. It meets the wave character of matter, is compatible with the experiment, and allows a unified description of interaction.

Keywords: force and energy; mechanics; quantum physics; special and general relativity; state and process equations; thermodynamics.

1 Introduction

The concepts of force and energy are inseparably linked with those of mass and time. Since forces and energies are omnipresent and the basis of physics, scientists have been trying for decades with great effort to unify the so-called fundamental forces described in the standard models, which has been successful only to a limited extent.

In his comprehensive analysis “Concepts of force”, the physicist and historian Max Jammer describes as early as 1957, on the one hand, the continuing desire for a
unified understanding of interaction and, on the other hand, the mystical concepts of force and potential energy:

“Force,” so to say, was the common denominator of all physical phenomena and seemed thereby to be a promising instrument to reduce all physical events to one fundamental law. [1, p. 242]

It may be contended, and in fact has been contended, that the concept of potential energy is not less mystical than the concept of force. [1, p. 243]

The situation is still unchanged to this day. After initially achieving many great successes with the standard models, physicists note i) the increasing discrepancy between experiment and theory, and ii) the continuing inconsistency of different physical theories. This mainly affects the current concepts of force and energy, as the following examples show:

1. Cosmological observations suggest an energy density of the vacuum that is smaller by a factor of $10^{120}$ than suggested by the Standard Model of particle physics, as Steven Weinberg already formulated in 1989:

   As everyone knows, the trouble with this is that the energy density $\langle \rho \rangle$ of empty space is likely to be enormously larger than $10^{-47}$ GeV [2].

2. None of the approaches to quantum gravity, which attempt to unify the three fundamental forces of the Standard Model of particle physics with the gravitational force described by the theory of general relativity, has been successful so far [3].

3. Gravitational potential energy $E_{\text{pot}}$ cannot be captured in the energy-momentum tensor of general relativity, although this was tried again and again [4, 5].

4. Dark Matter and Dark Energy, postulated in the $\Lambda$CDM theory, the current standard model of cosmology, have remained undetected for decades. Dark Energy might be based on misinterpreted data [6–8]. The predictions do not agree with astronomical observations, e.g., of dwarf galaxies [9].

Due to the persistent problems that cannot be solved even by mathematically sophisticated enhancements, the deep crisis of fundamental theoretical physics is admitted [10, 11].

Unlike much other work that looks for advancements for existing theories, the present work is dedicated to the fundamental principles. The starting point is a strict distinction between state equations and process equations. Their application in mechanics, thermodynamics, special and general relativity, and finally quantum mechanics is presented and discussed. The historical conditionality of the physical
theories and of the standard models is emphasized, which are originally based on
the idealizations of relativistic mechanics. The importance of the cause-effect
principle and of a consistent physical interpretation of formulas and state variables
is highlighted, which is crucial for the formation and development of theories. The
implications of a rigorous application of process equations are outlined.

2 State equations and process equations

The differences between state considerations and process equations will be
highlighted.

2.1 State equations

Nature is characterized by processes. Therefore, states are idealizations. This
applies not only to the so-called equilibrium state, which can only be approxi-
mated, but also to any momentary snapshot describing a state of matter under a
given set of conditions.

In physics, the properties of matter at a given moment are abstracted via state
variables such as volume, interface area, position, charge, energy density, pres-
sure, mass, and temperature. The name equation of state (EOS) is often limited to
thermodynamic relations such as the ideal gas law, or the Van der Waals equation
of state. Such equations describe the functional relationship between the state
variables of a many-body system at a given time \( t \).

In a broader sense, the notion state equation is used in this paper for the
functional relationship between the state variables of any system that can be a one-,
two- or many-body system. State equations in that general sense describe how state
variables of a system are interrelated at \( t \):

\[
X_i = f (X_j \neq X_i, \xi, \ldots) ; \quad \xi = f (\xi_j \neq \xi_i, X_i, \ldots) ; \quad \Delta t = 0.
\]  

(1)

\( X_i \) are extensive state variables (volume, momentum, mass, etc.) as quantity mea-
sures (How much?) that represent values for the whole system. \( \xi_i \) are intensive state
variables (density, temperature, pressure, etc.) as quality measures (How strong?)
that describe the conditions within a system. \( \xi_i \) can be expressed by the quotient or
differential quotient of two \( X_i \).

As state equations refer to states, the following features are missing: i) process
variables \( Y_i \), ii) an energy transfer in time, iii) the different roles that \( \xi_i \) and \( X_i \) play in
processes, and iv) a statement about the cause-effect principle. Examples are
\[ E_{\text{kin}} = \frac{m}{2}v^2 \] and \[ E_{\text{pot}} = mgh \] for a body, Newton’s law \[ F_g = \frac{G m M}{r^2} \] for a two-body system, the ideal gas law for a many-body system, and Einstein’s energy-mass relation \[ E = mc^2 \] for any kind of system.

### 2.2 Process equations

Processes represent dynamic changes of matter over time. In contrast to states, which describe a being without temporal development, processes describe the becoming. This irrefutable experience was already expressed by Heraclitus’ panta rhei. A natural process is complex because different properties (state variables) of matter change simultaneously.

Physical process equations are abstractions that have proven themselves for centuries. Each equation describes the change of only one state variable \( X_i \) of a system in time. The general characteristics is the transfer of energy \( E \) in time, and an analog causality structure:

\[
Y_i = \Delta E(X_i) = \int_{X_i,1}^{X_i,2} \xi_i \, dX_i = \int_{X_i,1}^{X_i,2} \left( \frac{\partial E}{\partial X_i} \right)_{X_j \neq X_i} \, dX_i; \tag{2}
\]

\[
\delta Y_i [J] = \xi_i [J/a] \, dX_i [a]; \quad \Delta t > 0,
\]

where \( \xi_i \) is the generalized force, \( dX_i \) the generalized effect, and \( Y_i \) the process variable such as volume work. Equation (2) refers to a so-called equilibrium process, an expression paradoxical in itself, because in equilibrium, where for the local gradient \( d\xi_i/dx = 0 \) holds, no process takes place. A process is caused by \( d\xi_i/dx \neq 0 \). Here, equilibrium process means that the infinitesimally small process steps \( \delta Y_i \) run so slowly that the existing local gradient of \( \xi_i \) can be neglected (“equilibrium”), while the changes of \( X_i \) accumulate over \( t \) in \( Y_i \). For non-equilibrium processes, \( d\xi_i/dx \) is explicitly written down (see Section 4).

Since only an intensive state variable \( \xi_i \) can have a local gradient, only \( \xi_i \) can be a force. The effect is always the temporal change of an extensive state variable \( X_i \) whose value after the process is the measure of the new energetic state of a system. Thinking in terms of quality \( \xi_i \) and quantity \( X_i \) can be traced back to Aristotle and was systematized by Immanuel Kant, who described intensive state variables as the cause and changes in the extensive ones as the effect of a process [12, p. 120ff.]. While \( Y_i \) exhibits the unit of energy [J], \( \xi_i \) has the unit of energy related to the unit of that state variable \( X_i \) that is changed, i.e. [J/a]. \( \xi_i \) and \( X_i \) in one process equation are energetically conjugated.
The rules of the cause-effect principle in Equation (2) are:

**Rule 1** Each abstracted process equation describes an energy transfer \( dE \) in time and has its own force \( \xi_i \) and effect \( dX_i \).

**Rule 2** The force is an *intensive* state variable because only \( \xi_i \) can have a local gradient \( d\xi_i/dx \) causing a change in \( X_i \) over time.

**Rule 3** \( dX \) is the effect of a process. The value of the *extensive* state variable \( X_i \) after a process is a measure of the new energetic state of a system. \( E \) depends on \( X_i \), i.e. \( E(X_i) \).

**Rule 4** \( dX_i/dt \) can be accompanied by simultaneous changes in local gradients \( d\xi_i/dx \) because state variables are interconnected via state equations.

**Rule 5** An extensive state variable \( X_i \) can neither have a local gradient nor play the role of a force.

Rule 5 is a mandatory consequence of the nature of extensive state variables \( X_i \), such as mass \( m \), volume \( V \), interface area \( A \), energy \( E \), momentum \( P \), and entropy \( S \). As they refer to the object as a whole, they say nothing about the conditions at specific points in space and do not exhibit local gradients. There is no spatial intensity grade, no gradation of \( X_i \).

The abstraction content of process equations is to be considered. As several \( X_i \) of an object change simultaneously, a natural process needs to be described by several process equations, whose changes in energy add up. The total change in energy is then given by \( k \) combined equations and corresponds, in terms of its mathematical structure, to the total differential:

\[
dE = \sum_{i=1}^{k} \delta Y_i = \sum_{i=1}^{k} \xi_i dX_i = \sum_{i=1}^{k} \left( \frac{\partial E}{\partial X_i} \right)_{X_1, \ldots, X_k (\neq X_i)} dX_i. \tag{3}
\]

The main features of state equations and process equations are listed in Table 1.

| **Table 1**: State equations versus process equations. |
|--------------------------------------------------------|
| **State equation** | **Process equation** |
| Subject | State | Dynamic process |
| Time-related reference | Momentary snapshot (the *being*) | Temporal development (the *becoming*) |
| Meaning | Functional relationship between state variables | Energetic change of a system in the course of time according to the cause-effect principle |
| **Cause** | Local gradient \( d\xi_i/dx \) of the Generalized Force \( \xi_i \) |
| **Effect** | Temporal change \( dX_i/dt \) of the Generalized Effect \( dX_i \) |
| State variable | Intensive | Extensive |
3 Mechanics

3.1 Process equations of mechanics and their current interpretation

Displacement work $W_x$ is the most famous process equation of mechanics:

$$ W_x = \Delta E_{\text{pot}} = F \int_{x_1}^{x_2} dx = F (x_2 - x_1); \quad \delta W_x = dE_{\text{pot}} = F dx, \quad (4) $$

where $F$ is the force, $x$ the spatial coordinate, $E_{\text{pot}}$ the potential energy, and $\Delta x$ the distance. Other types of mechanical work, where $E_{\text{pot}}$ is transferred, can simply be derived mathematically, e.g. lifting work $W_h = F_g \Delta h$ with the gravitational force $F_g$, spring tensioning work $W_s = F_s \Delta x$ with the spring tension force $F_s$, volume work $W_v = -p \Delta V$ with the pressure $p$, or interfacial work $W_A = \sigma \Delta A$ with the interfacial tension $\sigma$.

The concept of force has a millennial history in natural philosophy [1]. In classical mechanics, it originates from Isaac Newton who called the mass $m$ the “quantity of matter”, the momentum $P$ the “quantity of motion”, and defined force as the flux of momentum [13]:

$$ F = \frac{d (mv)}{dt} = \frac{dP}{dt} = \dot{P}. \quad (5) $$

In 1876, nearly 200 years later, Gustav Kirchhoff described a moving “material point” and introduced the mass $m$ as a constant coefficient [14]:

$$ F = m \frac{dv}{dt} = ma = m \ddot{x}. \quad (6) $$

Force as the product of $m$ and acceleration $a$, also expressed by Ernst Mach and Gustav Hertz, allowed quite accurate calculations of motion and initiated the triumphal march of mathematical physics. In Mach’s mechanics, $F$ and $m$ are “purely mathematical expressions relating certain measurements of space and time” [1, p. 221]. According to Kirchhoff, $F = ma$ applies to both the force that causes a displacement and the “accelerative force”:

Motional forces that act simultaneously on a point are composed just like accelerative forces. [14, p. 23]

This interpretation is still used today. Analogous to Equation (4), acceleration work $\delta W_v$, in which the kinetic energy $E_{\text{kin}}$ is transferred and the velocity $v$ is changed, is expressed by:

$$ \delta W_v = dE_{\text{kin}} = F dx; \quad F = \dot{P} \quad \text{or} \quad F = ma. \quad (7) $$
Newton’s second axiom $F = \dot{P}$ (or $F = ma$) is interpreted as a cause-effect principle between the force $F$ and the change in momentum (or velocity):

While according to Newton’s law of motion, the force is the cause of the change in momentum […]. [15, p. 2]

Some physicists transform Equation (7) using $F = ma$ to show that the effect of $W_v$ is a change in velocity. Then, the kinetic energy $E_{\text{kin}}$ can be obtained via integration from zero to $v$:

$$\delta W_v = F dx = m \frac{dv}{dt} dx = m v dv; \quad W_v = E_{\text{kin}} = m \int_0^v v\,dv = \frac{m}{2} v^2 = \frac{P^2}{2m}. \quad (8)$$

The interpretation $F = \dot{P}$ is also used today in the one- and three-dimensional versions of Newton’s law of friction that describes an internal transport process:

$$F_f = \dot{P} = -\eta A \frac{dv}{dx}; \quad \dot{P} = -\eta \vec{\nabla} v, \quad (9)$$

where $F_f$ is the friction force, $\eta$ the dynamic viscosity, $A$ the area between streaming layers, $\dot{P}$ the momentum flux density, and $v$ the velocity in $y$-direction perpendicular to the gradient.

The immense achievement of mechanics to abstract $E_{\text{kin}}$ and $E_{\text{pot}}$ of a body allows simple calculations and is used all the time today. If, for example, the conservation of energy is to be demonstrated by means of free fall, it is common practice to compare two states of a body and to equate the amounts of energy of a body:

$$E_{\text{pot}} = E_{\text{kin}}; \quad mgh = \frac{m}{2} v^2. \quad (10)$$

Please note: To show the energy conservation, $E_{\text{kin}}$ and $E_{\text{pot}}$ are intrinsic properties, i.e. they belong to the body.

3.2 Analysis of the equations and their interpretation

In Table 2, the different variants for interpreting mechanical forces and energies are listed with debatable interpretations marked in red. With reference to the Rules 1–5 of the cause-effect principle, the variants will be analyzed below.

3.2.1 Displacement work $W_x$ (transfer of $E_{\text{pot}}$)

The force $\xi_i$ of displacement work $W_x$ is described by two different terms:

$$\frac{d(mv)}{dt} \neq m \frac{dv}{dt}. \quad (11)$$
Two variants for one and the same state variable \( F \) are an expression of uncertainty. **Rule 1** of the cause-effect principle, according to which each \( Y_i \) is related to only one \( \xi_i \), is violated.

Newton’s \( F = \ddot{P} \) is applied when it fits, mostly in fluid dynamics [16, 17]. The proponents of \( F = \ddot{P} \) want to explain force, i.e. not only define it as an irreducible or purely relational concept without its own ontological status [18–20]. \( F = \ddot{P} \) represents an intensive state variable \( \xi_i \) whose local gradient \( d\ddot{P}/dx \) can cause a displacement \( dx \) in time. **Rule 2** is fulfilled.

The opponents of \( F = \ddot{P} \) defend the achievements of modern theoretical physics [15]. Kirchhoff’s \( F = ma \) is mostly used and already taught in school. The analogy to \( F_g \approx mg \) is obvious, where it is known that Newton’s law of gravity is sufficient for practical purposes, but represents an approximation. For strong fields, the more accurate description of general relativity is used today.

The transferred energy \( E_{\text{pot}} \) manifests itself in the new position, if, for instance, a spring is tensioned or a body is lifted. It holds \( E_{\text{pot}}(x) \) or \( E_{\text{pot}}(h) \) of a body, i.e. \( E_{\text{pot}} \) belongs to the body. The amount of energy transferred depends on extensive state variables. **Rule 3** is fulfilled.

### 3.2.2 Acceleration work \( W_v \) (transfer of \( E_{\text{kin}} \))

The force \( \xi_i \) of acceleration work \( W_v \) is described by three different terms (cf. Equations (7) and (8) and variants 4–6 in Table 2):

\[
\frac{d(mv)}{dt} \neq m \frac{dv}{dt} \neq mv.
\]  

#### Variants 4 and 5

In Equation (7), \( Fdx \) is attributed to acceleration work. This violates **Rule 1** of the cause-effect principle, according to which each process equation has its own...
force $\xi_i$ and effect $dX_i$. $dx$ is the effect of displacement work, whereas the effect of acceleration work should be $dv$.

Newton’s $F = \dot{P}$ and Kirchhoff’s, Mach’s and Hertz’s $F = ma$ are functional relationships between state variables at a given time $t$, i.e. state equations. They correspond to Equation (1). If $F = \dot{P}$ and $F = ma$ are interpreted as showing that $F$ causes $\dot{P}$ [15] or $a$, Newton’s 2nd axiom is misinterpreted as process equation. Each dynamic process equation represents an energy transfer that takes time (Rule 1). Therefore, the identities $F = \dot{P}$ or $F = ma$ do not represent a cause-effect relationship:

$$F = \dot{P} \ (\neq \delta Y_i = dE); \ \ dt = 0. \quad (13)$$

If instead the literal interpretations “$F$ causes the change in momentum” [15] or “$F$ causes acceleration” are translated into process equations, i.e. $FdP$ or $Fdv$, contradictions occur. Neither the product of $F$ and $P$, nor that of $F$ and $v$ has the unit of energy. Furthermore, the effect $dv$ contradicts Rules 2 and 3 of the cause-effect principle because $v$ is an intensive state variable $\xi_i$. Just as $dx$ is not the effect of acceleration work, $F$ cannot be the force of acceleration work. In $Fdx$, cause and effect of acceleration work are not specified.

If $Fdx$ is assigned to both $W_x$ and $W_v$, not even energy conservation during the free fall can be demonstrated by means of process equations. Lifting work $W_h$ is negative because $h$ decreases, whereas $W_v$ is positive because $v$ increases. With $F_g dh$ for both process equations, the following contradiction arises:

$$-\delta W_h = \delta W_v; \quad -F_g dh \neq F_g dh. \quad (14)$$

Today, we are accustomed to consider state equations via $mgh = \frac{1}{2}mv^2$, which leads to correct results. However, since the fall is a process in time, especially the process equations $Y_i$ must be able to show energy conservation.

**Variant 6**

Acceleration means that $v$ is changed during the dynamic process. Transforming $Fdx$ with constant mass $m$ in this sense, $mvdv$ follows. Here, $P = mv$ as extensive state variable $X_i$ is interpreted as force $\xi_i$. This violates the Rules 2, 3, 5 of the cause-effect principle. Since $X_i$ is a quantity measure referring to the whole system (how much?) and not to the conditions at one point, it cannot have a local gradient (Rule 5) and thus cannot act as force $\xi_i$ (Rule 2). Equation (8) does not represent a process equation.

That $P$ cannot be a force $\xi_i$ will be illustrated by the following example: Let two bodies 1 and 2 with $P_1 \neq P_2$, $m_1 \neq m_2$, and $v_1 = v_2$ (flying in the same direction) contact each other. The momenta differ, but no collision and no acceleration work $W_v$ will take place because no local gradients exist. Let two bodies with $P_1 = P_2$, $m_1 \neq m_2$, and
\( v_1 \neq v_2 \) contact each other. A local gradient \( dv/dx \) exists, and a process is initiated. Decisive is the local gradient of an intensive state variable \( \xi \) such as \( v \), or \( F \).

3.2.3 Friction work (transfer of \( E_{\text{kin}} \))

In fluid mechanics, the intensive state variable \( \dot{P} = dP/dt \) is called friction force \( F_t \) (cf. Equation 9). This designation takes Newton’s second axiom \( F = \ddot{P} \) literally, but undervalues the fact that there must be a local gradient of the force for a process to occur (Newton’s first axiom). In Equation (9), there is no local gradient \( d\dot{P}/dx \). In this way, the name friction force violates Rules 2 and 3. Even if this is only a problem of naming, it shows the variety of interpretations in mechanics and the lack of awareness for the cause-effect principle in process equations. In thermodynamics of irreversible transport processes, it is well known that \( dv/dx \) between adjacent layers of a streaming fluid causes a momentum flux \( \dot{P} \) between the layers, which represents the effect of the process (see Section 4).

3.3 Historical reference and interim conclusion

The current state of mechanics can only be understood by considering the historical conditionality of theories. Newton was brilliant, but had only limited information in the seventeenth century [13]. His first and second axioms and acceleration work remained underinterpreted. However, even if Newton’s law of gravity is interpreted today as a direct action at a distance, already Newton and Leonhard Euler rejected an infinitely fast interaction.

In the course of industrialization in the nineteenth century, mathematical physics began to dominate, while natural philosophy receded into the background and Kant was hardly considered. Kirchhoff’s suggestion \( F = ma \) [14] and the state thinking of mechanics allowed fast, simple and sufficiently accurate calculations, and pragmatic solutions were needed. As with \( m = \text{const.} \) process equations can be easily transformed into each other, they were transformed without paying attention to the cause-effect principle in acceleration work.

The mathematical transformations are correct. However, physical process equations are not pure mathematics. By misinterpreting Newton’s and Kirchhoff’s state equations \( F = \ddot{P} \) and \( F = ma \) as process equations and by mixing displacement and acceleration work in \( Fdx \), the differences between i) force and energy, ii) state and process equations, iii) individual process equations, and iv) \( E_{\text{pot}} \) and \( E_{\text{kin}} \) were blurred very early. This agrees with Max Jammer’s assessment that acceleration
“may be regarded as a miracle” because “there is no ‘bridge’ that leads from configuration to accelerated motion” [1, p. 258].

Indeed, one of the first equations we learn in school is Kirchhoff’s equation $F = ma$. We grew up with its interpretation and got used to it. Therefore, it seems almost impossible to be able to question it. And yet, if the causality remains unsettled, one uses a mathematical theory whose physical content is not fully understood. The mixing of processes and the blurring of state variables makes mechanics to a conceptually inconsistent theory. The key results of the analysis in this section are:

The interpretation of acceleration work fundamentally violates the cause-effect principle.
Newton’s second axiom $F = ma$ is misinterpreted as process equation.
The differences between i) force and energy, ii) state and process equations, iii) individual process equations, and iv) $E_{\text{pot}}$ and $E_{\text{kin}}$ remain undervalued.
The concepts of force and energy in mechanics are unsettled.

4 Thermodynamics

4.1 Process equations of thermodynamics (TD) and their current interpretation

In TD, processes are described. One distinguishes exchange processes (index e) between a thermodynamic system and its surroundings, and internal processes (index i). Exchange processes such as heat transfer $\delta Q$, substance exchange $\delta W_n$, and different kinds of work $\delta W_i$ change the amount of internal energy $U$ of a system [21, 22]:

$$dU = \delta Q + \delta W_n + \sum \delta W_i = TdS + \mu dn - pdV + \sigma dA + \ldots - Td_i S,$$

(15)

where $\delta Q = Td_c S = TdS - Td_i S$. Equation (15) goes back to the empirically confirmed first and second laws of TD formulated in 1850 and 1865 by Rudolf Clausius [23, 24]. The first one describes the principle of energy conservation, the second one the increase in entropy $S$ for any natural irreversible process.

Irreversible transport processes in an isolated system ($dU = 0$) are connected with entropy production $dS/dt > 0$ [25]. The distribution of the energy and thus its quality is changed. Near equilibrium, linear transport equations can be derived, which possess an analogous, empirically confirmed structure:

$$\frac{1}{A} \frac{dX_i}{dt} = -L_{ii} \frac{d\xi_i}{dx}; \quad X_i = -L_{ii} \overrightarrow{\nabla} \xi_i; \quad \overrightarrow{\nabla} \xi_i = \frac{\partial \xi_i}{\partial x} \overrightarrow{e_x} + \frac{\partial \xi_i}{\partial y} \overrightarrow{e_y} + \frac{\partial \xi_i}{\partial z} \overrightarrow{e_z},$$

(16)
where \( F_i = \frac{d\xi_i}{dx} \) is the thermodynamic force (the local gradient of an intensive state variable \( \xi_i \)), \( J_i = \frac{dX_i}{dt} \) the thermodynamic flux (the temporal transport of an extensive state variable \( X_i \)) through an area \( A \), \( \dot{X}_i \) the flux density, and \( L_{ii} \) the linear phenomenological coefficient that represent a material property. In equilibrium, \( F_i \) and \( J_i \) are zero. Examples are Fick’s first law of diffusion and Newton’s law of friction (cf. Equation (9)):

\[
\dot{n} = -DA \frac{dc}{dx}; \quad \dot{\mathbf{n}} = -D \nabla c; \quad \dot{P} = -\eta A \frac{d\mathbf{v}}{dx}; \quad \dot{P} = -\eta \nabla \mathbf{v},
\]

(17)

where \( D \) is the diffusion coefficient, \( c \) the concentration, \( \dot{n} \) the flux of amount of substance, and \( \dot{\mathbf{n}} \) the corresponding flux density.

If one applies the cause-effect principle in process equations, volume work \( W_V \) and interfacial work \( W_A \), where \( E_{\text{pot}}(V, A) \) is transferred, change spatial properties \( X_i \), i.e. the volume \( V \) and the interface area \( A \) of a system. The internal energy \( U(V, A) \) of the system changes, but not its mass if heat transfer \( Q \) and substance exchange \( W_n \) are excluded [26, 27]:

\[
dU = -p \, dV + \sigma \, dA \neq 0; \quad dm_0 = 0; \quad dS, dn = 0.
\]

(18)

The Euler form of \( U \) for a resting system is then given by:

\[
U = TS + \mu n - p \, V + \sigma \, A + \cdots = m_0 c^2 + E_{\text{pot}} = E_0 + E_{\text{pot}},
\]

(19)

where \( m_0 \) is the so-called rest mass of a system (that is not moved as a whole). This implies that the so-called rest energy \( E_0 \) does not cover the whole energy content of a resting system. Equation (19) opens the way towards an extension of the 2nd law and a justification for the fundamental validity of the laws of TD [28–30].

### 4.2 Analysis of the equations and their interpretation

The concept of a thermodynamic system implies an observer-independent objective reality in the Euclidean space. The force and energy concepts listed in Table 3 will be discussed below.

**Table 3:** The interpretation of process equations in thermodynamics.

| \( \delta Y_i \) | Force \( \xi_i \) | Effect \( dX_i \) | Transferred in time |
|-----------------|-----------------|-----------------|-------------------|
| Heat transfer \( \delta Q \) | Temperature \( T \) | \( dS \) | \( E_{\text{kin}}, \, h\nu \) | Ponderable |
| Substance exchange \( \delta W_n \) | Chemical potential \( \mu \) | \( dn \) | \( N \) | Particles |
| Volume work \( \delta W_V \) | Pressure \( p \) | \( dV \) | \( E_{\text{pot}} \) | Not ponderable |
| Interfacial work \( \delta W_A \) | Interfacial tension \( \sigma \) | \( dA \) | | |

\( S \), entropy; \( n \), amount of substance; \( V \), volume; \( A \), interface area.
In Equation (15) that is called Gibbs fundamental equation for $U$, the Rules 1–5 of the cause-effect principle are fulfilled. It corresponds to Equation (3) and represents a linear differential equation of the Pfaffian form:

$$dz(x, y, ...) = P(x, y, ...)dx + Q(x, y, ...)dy + \cdots;$$

where $z$, $P$, and $Q$ each depend on all independent state variables $x, y, \ldots$. Mathematically this represents a total differential, which means that there is nothing more behind it than elementary trigonometry in Euclidean space. Nevertheless, mathematics is far from being sufficient in physics. To do justice to the physical content, distinguishing between intensive and extensive state variables and respecting the cause-effect principle are crucial.

From $U(S, V)$, J. Willard Gibbs derived three further energetic functions of a system such as the Gibbs free energy $G(T, p)$ [21]. By means of Legendre transformation, the energetically conjugated state variables $P$ and $x$, or $Q$ and $y$ in Equation (20) are interchanged. This practice-orientated idea allows a more accurate description of processes under controlled conditions because $S$ and $V$ are more difficult to control and adjust than $T$ and $p$. Moreover, it pays tribute to the fact that natural processes are complex processes in which several extensive state variables $X_i$ change simultaneously.

$\text{d}G$, for example, is described by the Gibbs fundamental equation:

$$dG = d(U - TS + pV) = \left(\frac{\partial G}{\partial T}\right)_{p,n}dT + \left(\frac{\partial G}{\partial p}\right)_{T,n}dp + \left(\frac{\partial G}{\partial n}\right)_{T,p}dn$$

$$= -SdT + Vdp + \mu dn \delta W_n.$$  \hspace{1cm} (21)

Here it should be noted that mathematical operations cannot change the physical content or the causal structure. The terms $-SdT$ and $Vdp$ in Equation (21) do not represent cause-effect principles or process equations. For example, the local gradient of the volume $V$ does not cause $dp/dt$. The volume $V$ is an extensive state variable $X_i$, i.e. there is no local gradient of $V$ (Rule 5). The filling of $V$ is unknown. The only statement that can be made about two different adjacent volumes is: 5 L and 2 L make 7 L.

$\mu$ changes simultaneously when processes such as $TdT$ and $-pdV$ take place because $\mu$ is linked to the other state variables via state equations (Rule 4). In a process equation, $\xi_i$ and $X_i$ cannot be interchanged. The difference between $\xi_i$ and $X_i$ is also evident in the fact that equilibrium conditions require the equality of $\xi_i$.
such as $p$, $T$, $\mu$ of spatially adjacent phases or within a system, whereas $X_i$ such as $S$, $U$, $G$ becomes maximal or minimal.

In Equations (16) and (17), the Rules 1–5 of the cause-effect principle are fulfilled, too. Since not $\xi_i$, but directly $d\xi_i/dx$ is called thermodynamic force $F_i$, the designation is even more tailored to the cause-effect principle. If the local gradient of $\xi_i$ and thus $F_i$ is zero, there is no cause of a process and consequently no measurable flux $J_i$ within the system. As described above, the naming of $\dot{P}$ in mechanics (M) and TD of irreversible processes is opposite, which does not change anything in the equation or calculations:

\[
\dot{P} \quad \text{(M: Friction force $F_i$ (cause))} = -\eta A \frac{dv_z/dx}{\text{TD: Flux $J_i$ (effect)}}. \quad \text{(22)}
\]

If the force $F_i = d\xi_i/dx$ causes the flux $J_i = dX_i/dt$ as described in Equations (16) and (17), this is called a direct process. The magnitude of $J_i$ depends not only on $F_i$, but also on material properties $L_{ii}$ such as the dynamic viscosity $\eta$, or the diffusion coefficient $D$. In addition to direct processes, cross processes are possible, where $F_i$ causes $J_k$ and $L_{ik} = L_{ki}$ [31]. Besides linear transport equations, there are also nonlinear ones, but the cause-effect principle is always preserved, and the forces $d\xi_i/dx$ and fluxes $dX_i/dt$ are unambiguous.

In this context, it is beyond question that intensive state variables $\xi_i$ can also change with time. However, since $\xi_i$ is an indication of quality, not of quantity, $\xi_i$ cannot be transported through an area $A$ with time. $\xi_i$ is changed simultaneously because a flux $J_i$ as effect of a transport process is accompanied by changed gradients $d\xi_i/dx$ (Rule 4). This is described for instance by Fick’s second law of diffusion:

\[
\dot{c} = D \frac{d^2c}{dx^2}; \quad \dot{c} = D \nabla^2c = D\Delta c. \quad \text{(23)}
\]

In TD, just as in mechanics, $E_{\text{pot}}$ belongs to the body. Each spatially extended system contains $E_{\text{pot}}$. Today, mechanics (in addition to the theory of general relativity) is used to describe the positional energy $E_{\text{pot}}$ of a body as a whole, while TD is used to describe the positional energy $E_{\text{pot}}$ of the components of a system (body, liquid, etc.)

Unlike mechanics, TD is characterized by an uncompromising process thinking that i) includes an indication of the direction of processes, and ii) describes matter far from Kirchhoff’s material points. If, for example, the volume $V$ of a body is decreased by $W_V = \Delta E_{\text{pot}} = -p \Delta V$, while all other state variables $X_i$ are kept constant, then only $E_{\text{pot}}$ is added. Equations (18) and (19) fulfill Rule 3 of the cause-effect principle. Since $E_{\text{pot}}(V)$ is only a function of the spatial property $V$, the mass of the body remains unaffected, i.e. $E_{\text{pot}}$ does not satisfy $E = mc^2$. 

By contrast, if photons, particles or molecules are added to a system, $m$ increases accordingly. If energy is added by heat conduction $\delta Q = T \, \delta S$ at $V$, $n = \text{const.}$, the mass of the system increases, too. The molecules are accelerated. $E_{\text{kin}}$ and $T$ increase. That the mass of the system increases with $T$, as already described by Friedrich Hasenöhrl and Max Planck [32, 33], means that not only the particles but also their kinetic energy $E_{\text{kin}}$ in the system is ponderable in real terms.

### 4.3 Historical reference and interim conclusion

The current state of TD is historically conditioned. While TD had dominated physics alongside mechanics in the 2nd half of the nineteenth century, its reputation declined in twentieth century due to at least two reasons:

1. Within the kinetic theory of gases, Ludwig Boltzmann failed to fundamentally justify irreversibility, i.e. the “arrow of time”. His failure, which he admitted in 1895, was regarded as final, although the kinetic theory of gases represents a strong idealization.

2. In 1905 and 1915, partly based on the force concepts of mechanics, Albert Einstein developed his theories of special and general relativity, which basically contradict TD’s process approach.

By accepting special relativity, the force concept of mechanics and the energy concept of relativistic mechanics became the foundations of theoretical physics. TD continued to be used in applied sciences. However, it was no longer understood as fundamental, but limited to statistical phenomena. The term phenomenological TD (as distinct from statistical TD) was used somewhat pejoratively in the sense that this theory applies only to macroscopic many-body systems. The preference for mechanics led to the undervaluation of process equations. By partially abandoning causality, quantum mechanics was developed.

Again and again, there were scientists who tried to justify the causality and the arrow of time [11, 34, 35]. They had to face logical contradictions by presupposing $E = mc^2$ (including $E_{\text{pot}}$) as secured. The first chapter in Prigogine’s book “Thermodynamics of Irreversible Processes”, for instance, is titled “Conservation of Mass in Closed and Open Systems” [25, p. 3ff]. Chemical reactions and substance exchange are described by $\delta m_i = M_i \delta n_i$ with the molar mass $M_i$ of component $i$. Since $M_i$ is a constant that cannot have a local gradient, Rule 2 of the cause-effect principle is violated. Trying to establish consistency with $E = mc^2$, Prigogine avoided describing natural processes by means of Equation (3), i.e. by $k$ combined process equations such as $\delta W_n = \Sigma \mu_i \delta n_i$ and $\delta W_V = -p \, dV$. 

Here it should be noted that the state variable amount of substance $n$ does not represent matter including $E_{\text{pot}}$. By definition $n$ represents a countable number of particles or smallest constituents without paying tribute to their changing position. There is no question that particles are ponderable (cf. Table 3). And yet, volume work and interfacial work, which are performed simultaneously during chemical reactions in closed and open systems because the system’s components change their positions, are not taken into account. In this way, the relation between positional energy $E_{\text{pot}}$ and mass remained unexamined.

The conceptual difference of the three theories mechanics, general relativity (that is partly based on the concepts of mechanics), and thermodynamics led to the fact that $E_{\text{pot}}$ could never be treated uniformly. The key results of the analysis in this section are:

---

The empirically confirmed thermodynamic process equations satisfy the cause-effect principle. The process equations refer to an objective reality in the Euclidian space. They suggest the non-ponderability of $E_{\text{pot}}$ and the ponderability of $E_{\text{kin}}$ in real terms. The concepts of force and energy in thermodynamics are unambiguous.

## 5 Special relativity

### 5.1 Equations of special relativity (SR) and their current interpretation

The theory of special relativity [36–40] describes inertial frames of reference moving against each other in empty space ($E_{\text{pot}} = 0$). Time $t$, length $L$ and other variables are interpreted as dependent from observations in moving or resting inertial frames, while the light velocity $c$ is set invariant. Instead of $m/2v^2$, the kinetic energy $E_{\text{kin}}$ of a “material point” is described by [39, p. 30]:

$$E_{\text{kin}}(v) = E_0(y - 1) = m_0c^2\left(\frac{1}{\sqrt{1 - v^2/c^2}} - 1\right); \quad y \geq 1,$$

(24)

where $E_0$ and $m_0$ are the rest energy and rest mass measured in the rest system, and $y$ is the Lorentz factor. His famous equation [40, p. 49]:

$$E_0 = m_0c^2$$

(25)
Albert Einstein interprets as follows:

The mass of a body is a measure of its energy content. [37, p. 641]
Thus, any energy in the gravitational field has energy of position which corresponds to the
energy of position of a “ponderable” mass of magnitude $E/c^2$. [38, p. 462]
The principle of mass conservation coincides with the principle of energy conservation. [39, p. 32]
Mass and energy are thus equal in essence [...]. The mass of a body is not a constant, but changes
with its energy changes. [40, p. 49]

Textbooks describe the total energy of a moving point mass by the relativistic Pythagoras:

$$E(P) = E_0 + E_{\text{kin}} = mc^2 = \gamma E_0 = \sqrt{E_0^2 + (cP)^2} \geq E_0,$$

(26)
where $m(v)$ is the so-called relativistic mass measured in a moved inertial frame.
Some textbooks omit an explicit equation for $m(v)$. Most textbooks give an equa-
tion, but caution that it should be used with care. Some textbooks calculate with
$m(v)$:

$$m(v) = \gamma m_0 = \frac{1}{\sqrt{1 - v^2/c^2}} m_0 \geq m_0.$$

(27)
In 1905, Einstein introduces the force acting on a “slowly accelerated electron” [36,
p. 919]:

"Mass $\times$ Acceleration = Force".

(28)
In 1907, Einstein defines force as the rate of change in momentum [38, p. 435]:

$$F = \dot{P}.$$

(29)
According to Einstein’s interpretation that the mass of a body changes if its energy
changes, many textbooks write the following equations and use them to confirm
$E = mc^2$:

$$dE_0 = c^2 dm_0; \quad dE = c^2 dm.$$

(30)

5.2 Analysis of the equations and their interpretation

Special relativity idealizes matter even more than mechanics. Like in the ideal gas,
neither mechanical energy $E_{\text{pot}}$ nor electromagnetic interaction exists. Further-
more, not a single process equation can be found as will be discussed below.
5.2.1 Equations (24)–(27)

Equations (24)–(27) are state equations as defined in Equation (1). They describe the interrelation of \( E_{\text{kin}}, E_0 \) and \( E \) at a given time \( t \). The total energy \( E = E_0 + E_{\text{kin}} \) is larger than the rest energy \( E_0 \). Using the respective mass expressions, \( m \) has to be larger than \( m_0 \) in real terms if each kind of energy is ponderable as Einstein suggests:

\[
mc^2 = m_0c^2 + E_{\text{kin}}.
\]

(31)

Thus \( m = \gamma m_0 \) in Equation (27) is mathematically correct, logically and fully analog to the expressions of total energy \( E = yE_0 \), length \( L = L_0/\gamma \), etc. However, it remains unknown which mass of a moved object is described. Unlike the Lorentz theory [41] that attempts to describe the dynamic effects of the ether on a moved object at different \( v \) by means of \( \gamma \), SR describes observations from inertial frames moving in empty space. In kinematics, no kind of mechanical work is allowed. Based on a purely relational thinking of Mach and the mixing of displacement and acceleration work in \( Fdx \), a definitive departure from dynamic processes takes place. The special feature is here the assertion of real changes in energy, i.e. processes – without a single process equation \( Y_t = \Delta E \) with \( \Delta t \neq 0 \). How does the energy change without processes?

The unsolvable interpretation problem of \( m(v) \) is well-known [26]; whole books have been written about it. In a pragmatic way, \( m(v) \) is called a “limited recipe” [42]. The irresolvable conflict is: Equation (27) is indispensable to describe the real mass increase of elementary particles with \( v \) measured in accelerators. However, the mass of a body cannot depend on the coordinate system from which the body is observed. Therefore, today mostly \( m_0 \) is called real mass, sometimes also \( m \). But everybody, who decides for one of the variants within the framework of SR, must remain wrong. It often goes unnoticed that \( E = yE_0, L = L_0/\gamma \), etc. within SR involve the same interpretation problem.

5.2.2 Equations (28) and (29)

Equations (28) and (29) are state equations as defined in Equation (1). Einstein keeps the force concept of mechanics in SR. This is twice impossible because:

i) \( F = ma \) or \( F = \dot{P} \) represent the force \( \xi_i \) of displacement work where \( E_{\text{pot}} \) is transferred, not of acceleration work as there exists no process \( Fdv \) (cf. Table 2).

In SR, \( E_{\text{pot}} \) is zero.

ii) Observations are logically incompatible with dynamic forces \( \xi_r \).
Hence Equations (28) and (29) have no place in SR. In his first paper on SR in 1905, Einstein vaguely introduces the force concept of mechanics with Equation (28) by avoiding the denotation $m_0$ or $m$. A force is described that is “observed from a system moving at the same velocity as the electron at that moment” [36, p. 919]. Often, what matters is not what is asserted, but what is omitted or avoided. Within SR, it is impossible to commit to $m_0$ or $m$. An acting force cannot be described in good conscience from the point of view of a differently moved inertial system. If just observations are concerned, there exist neither dynamic process nor force $\xi_i$. The result is a dead relational world without time and without any real change in energy. A world that is even less real than that of the ideal pendulum in mechanics, which also does not know time, but still knows energy conversion and $E_{\text{pot}}$.

Consequently, the concept of force in SR is even more internally inconsistent than that in mechanics. The vague concept of force manifests itself also in the variable dependencies $E_{\text{kin}}(v)$ and $E(P)$ in textbooks. According to Rule 5, energy has to be a function of an extensive state variable $X_i$. The desire exists, but $E(P)$ is not realizable within SR.

5.2.3 Equation (30)

Is there really no process equation in SR? According to Einstein’s interpretation

Since mass and energy are equal according to the results of special relativity. [43, p. 241]
The mass of a body […] changes with its energy changes. [40, p. 49],

$dE = c^2 dm$ is used today to confirm $E = c^2 m$ based on processes such as nuclear reactions.

Looking at $E = c^2 \cdot m$, one recognizes a proportionality of two extensive state variables $X_i$ of an object at a given moment $t$, no equivalence or equality. $E = c^2 \cdot m$ is analogous to other proportionalities such as $m = M \cdot n$ with the molar mass $M$. Both equations are state equations according to Equation (1). It also applies $dm = M \cdot dn$.

However, interpreting $dE = c^2 \cdot dm$ or $dm = M \cdot dn$ as process equations means a strong violation of the cause-effect principle. $c^2$ and $M$ are constants and can never act as a force $\xi_i$. Rules 2 and 4 are violated because a local gradient $d\xi_i/dx$ is impossible. In addition, Rule 1 is violated because attempts are made to reduce a complex natural process described by Equation (3), in which many $X_i$ change simultaneously (each $dX_i$ associated with $dE$), to one process equation. $dE = c^2 dm$ and $dm = M dn$ are misinterpreted as process equations.

The correct process equation for reactions and particle transport is $\mu dn$ (cf. Table 3), where $\mu$ is variable. A resting body with the same particle number $n$
(and thus the same mass \( m \)) may very well have more energy if it is compressed for example. \( \text{d}E = c^2 \text{d}m \) disregards that the position itself is an energetic value. Although \( E_{\text{pot}} \) is assumed to be zero, Einstein concluded in SR that mechanical \( E_{\text{pot}} \) was ponderable. One of the reasons for this might be the state thinking of mechanics and the historical interpretation of Newton’s second axiom (cf. Table 2). In 1907, Einstein writes:

the fact that an amount of energy \( E \) has a mass of \( E/c^2 \) is valid [...] not only for the inertial but also for the gravitational mass. [38, p. 462]

Knowing only \( F \text{d}x \) and accustomed to use \( m/2 \cdot v^2 = mgh \), he assumed that both \( E_{\text{kin}} \) and \( E_{\text{pot}} \) are ponderable if the inertial mass \( m_i \) and the gravitational mass \( m_g \) are equal. This is a fatal fallacy because \( E_{\text{pot}}(r) \) only depends on the distance \( r \) between bodies, not on \( m \), as lifting work shows. Energy conservation and \( m_i = m_g \) can only be explained by means of process equations with ponderable \( E_{\text{kin}} \) and non-ponderable \( E_{\text{pot}} \) [30].

Due to the high suggestive power of the state equations of mechanics and \( E = mc^2 \), all “elementary derivations” of \( E = mc^2 \) (cf. [44, pp. 62–89]) underestimate the necessity of using process equations. Thinking purely mathematically, the logical paradoxes of SR remain ignored: Special relativity asserts processes in the absence of processes.

Please note: If Einstein writes “The mass of a body is a measure of its energy content.” [37, p. 641], he interprets \( E_{\text{pot}} \) as an intrinsic property because a real body always possesses \( E_{\text{pot}} \), e.g. tensional energy.

### 5.3 Historical reference and interim conclusion

At the end of the nineteenth century, scientists started to describe a velocity-dependent mass. In his ether theory, Hendrik Antoon Lorentz derived the transverse mass \( m_t(v) = y m_0 \) for the real mass increase when accelerating an electron in the direction of the path (and similar state equations to match not only \( F_{\parallel} \), but also \( F_{\perp} \) and angular dependencies between \( F \) and \( v \)) [41, p. 820]. In doing so, he proposed, to some extent, a return to \( F = \dot{P} \) instead of \( F = ma \). To explain the results of the Michelson-Morley experiment, he proposed a dynamic length contraction \( L(v) = L_0/\gamma \) of an electron in the direction of the motion:

I shall now suppose that the electrons, which I take to be spheres of radius \( R \) in the state of rest, have their dimensions changed by the effect of a translation. [41, p. 818]
Since the ether could not be detected, physics got into a crisis. In 1905, Einstein proposed a solution [36]. Influenced by Mach’s relational thinking and Kirchhoff’s idealizing concept of mass, he defined the ether away and reinterpreted Lorentz’ equations by means of observations from inertial frames. In terms of realism, SR was a significant regression compared to Lorentz’ theory: Instead of spatially extended particles, point-like particles and light signs move through empty space. Instead of real directed masses \( m(v) \) and lengths \( L(v) \), apparent mass increases and length contractions are described. However, the so-called *time dilation* \( dt = \gamma dr \) is described as real [36]. The conflicting interpretation within the same formalism violates the scientific method and blurs kinematics and dynamics [45, 46]. Whereas energetic changes are intended to be described, e.g. by \( dE = c^2 dm \), the absence of processes and forces is a hallmark of the theory. In fact, SR even was a big step backwards compared to mechanics, which knows process equations.

But a solution was needed. SR attracted with promises such as: i) Link between mechanics and electrodynamics, ii) Accessibility of the total energy content of a system, iii) Application of the Lorentz formalism to each state variable [38], regardless of its intensive or extensive nature, iv) Assertion of \( F = ma \) while allowing also \( F = \dot{P} \) to a certain extent, v) Openness to purely geometric considerations, and vi) Experimental evidence.

Generalizations are tempting, all the more so when they are proposed by physicists almost simultaneously with the light quantum hypothesis. In 1907, Max Planck preferred the interpretation of Einstein to that of Lorentz. Accepting the paradox force concept of SR, partly due to the unsettled force concept of mechanics, he defined the entropy \( S \) that changes in TD with each natural process as Lorentz-invariant [47]. Irreversibility was put aside.

More realistic proposals like those of the physical chemists Gilbert Newton Lewis, who derived \( m(v) = \gamma m_0 \) without relativity [48], Walther Nernst, who suggested to maintain the ether [49], or Wilhelm Ostwald, who never gave up the idea of continuous energy, even after accepting the atomic hypothesis, were rejected, while Minkowski’s spacetime idea, who suggested a formal equivalence of the spatial coordinates and the time coordinate [50] was welcomed. The positivist and pragmatic approach to physics promoted a mathematical theory with logical and conceptual inconsistencies that claimed to best describe the experiments. This cannot be detached from the triumph of general relativity.

The empirical basis of SR is invoked again and again. What is measured, however, are real changes of matter: real mass increases and length contractions of elementary particles, real changes in the oscillation transitions of atomic clocks.
In special relativity there are neither acting forces nor process equations. The theory is characterized by the claim of forces and processes in their absence. The Lorentz-transformed state variables are misinterpreted as observer-dependent. $\text{d}E = c^2 \text{d}m$ violates the cause-effect principle and does not represent a process equation. It wrongly replaces the diversity of process equations for describing a complex natural process. Special relativity has been experimentally disproved countless times.

### 6 General relativity

#### 6.1 Equations of general relativity (GR) and their current interpretation

Einstein’s field equations of gravitation [51, p. 783] are often written in the following form:

\[
G_{\mu \nu} = R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} = \kappa T_{\mu \nu}; \quad \kappa = \frac{8 \pi G}{c^4},
\]

(32)

where $G_{\mu \nu}$ denotes the **Einstein tensor**, $R_{\mu \nu}$ the **Ricci tensor**, $R$ the **curvature scalar**, $g_{\mu \nu}$ the **metric tensor**, $T_{\mu \nu}$ the **stress-energy tensor** or **energy-momentum tensor** and $\kappa$ a constant proportionality factor containing the Newtonian gravitation constant $G$, and the light velocity $c$. Sometimes, the term $\Lambda g_{\mu \nu}$ with the **cosmological constant** $\Lambda$ is added to $G_{\mu \nu}$, sometimes the term $(-\Lambda/\kappa)g_{\mu \nu}$ is considered as a universal contribution to $T_{\mu \nu}$.

$T_{\mu \nu}$ replaces in a certain way the mass in Newton’s law of gravitation. The analogy becomes obvious if the mass density $\mu = m/V$ in the Poisson equation for the Newtonian gravitational potential $\phi$ is replaced by the **energy density** $\rho$:

\[
\Delta \phi(\vec{r}) = 4 \pi G \mu(\vec{r}) = (4 \pi G/c^2) \rho(\vec{r}); \quad \rho = \mu c^2.
\]

(33)

Since “in Special Relativity $\rho$ is not a scalar but rather just one component of a tensor” [5, p. 182], the generalized tensor $T_{\mu \nu}$ is used that refers to matter including radiation and non-gravitational force fields, but excluding the gravitational field as Einstein writes:

We distinguish in the following between “gravitational field” and “matter” in the sense that everything except the gravitational field is called “matter”, i.e. not only “matter” in the usual sense, but also the electromagnetic field. [52, p. 802]
The ultimate form of $T_{\mu\nu}$ has not yet been found and is the subject of current research. There are covariant, contravariant and mixed forms of $T_{\mu\nu}$ that possesses different components, e.g. those of the energy density and flux, momentum density, and stress [4, 5]. $T_{\mu\nu}$ is interpreted as physical source of the gravitational field that is described as space-time curvature on the left side of Equation (32). In their famous book “Gravitation”, Misner, Thorne and Wheeler write:

“I weigh all that’s here” is the motto of spacetime curvature. No physical entity escapes this surveillance. [4, p. 475]

For the perihelion shift $\epsilon$, Einstein derives the formula [52, 53]:

$$\epsilon = 24\pi^3 \cdot \frac{a^2}{T^2 c^2} (1 - e^2),$$  \hspace{1cm} (34)

where $e$ denotes the eccentricity, $a$ the semi-major axis, and $T$ the orbital period. He interprets Equation (34) as confirming GR.

### 6.2 Analysis of the equations and their interpretation

In GR, the force and energy concepts of SR are transferred to gravity by using Minkowski’s spacetime idea [50]. Important conceptual basics of GR are:

i) the formal equivalence $Fdx$ of acceleration work and displacement work,

ii) the observer-dependency of state variables based on the invariance of $c$,

iii) “the formal equivalence of the spatial coordinates and the time coordinate” [52, p. 669],

iv) and the total energy $E = mc^2$.

Based on i–iii), gravitation is described – in analogy to acceleration in SR – by means of observations from inertial frames. With i–ii), all internal contradictions of mechanics and SR (see Sections 3 and 5) are adopted. Point iii) reinforces the departure from dynamic processes. According to Equations (2) and (3), the “formal equivalence of the spatial coordinates and the time coordinate” is only mathematical, not physical. There is a large physical distinction between $x$ and $t$, for instance because only intensive state variables $\xi_i$ can have spatial gradients $d\xi_i/dx$, while only extensive state variables $X_i$ can be transported with time $(dX_i/dt)$. Already the premises of GR make clear that there is no process equation in GR.

### 6.2.1 Equation (32)

Just like $F = ma$, $F = mGm/r^2$, and $\Delta \phi = 4\pi G \cdot \mu$, the equation $G_{\mu\nu} = \kappa \cdot T_{\mu\nu}$ with the constant $\kappa$ is a state equation as defined by Equation (1). It is well-known that
gravity is the expression for a specific abstracted force, not for a process. If Einstein interprets his field equations as process equations and if $T_{\mu\nu}$ is interpreted as causing the spacetime curvature:

The field equations of gravitation [...] provide the equations of the process completely. [52, p. 810]

The field equation shows how the stress-energy of matter generates an average curvature (Einstein = G) in its neighborhood. [4, p. 42]

[...] particles and other sources of mass-energy cause curvature in the geometry. [4, p. 47],

process and state equations are confused. Rules 1–5 of the cause-effect principle are violated. Equation (32) describes no energy transfer $dE[J] = \delta Y_i$ with $dt \neq 0$. $\kappa T_{\mu\nu}$ represents nothing else than a substitution of Newton’s gravitational potential in Equation (33), which is claimed to be equal to the curvature $G_{\mu\nu}$ of spacetime. The identity $G_{\mu\nu} = \kappa T_{\mu\nu}$, however, openly conflicts with Einstein’s idea of the spatially separated entities “gravitational field and matter” [52, p. 802]. Interaction between separated entities needs time. Thus, Equation (32) represents a physical impossibility with serious internal contradictions (cf. Table 4).

In the following, selected conceptual problems connected with $G_{\mu\nu} = \kappa T_{\mu\nu}$ and the separation of “matter” and “gravitational field” are discussed:

$T_{\mu\nu}$ refers to real systems such as “liquid bodies” [52, p. 771]. The tensor components include kinetic energy $E_{\text{kin}}$ and radiation $hv$. This approach is more realistic than the point idealization of mechanics and contradicts SR because now is admitted that both $E_{\text{kin}}$ and $hv$ contribute in real terms to the gravitational mass of a system. Whereas $m(v)$ in SR is only observed, $m(v)$ in $T_{\mu\nu}$ is real like Lorentz suggested [41].

| Table 4: The conceptual ambiguity of Einstein’s field equations. |
| --- |
| $T_{\mu\nu}$ | $G_{\mu\nu}$ |
| **Interpretation** | Matter (including radiation) | Gravitational field = spacetime curvature |
| Source of gravity [5] | Spacetime reacts (effect) |
| All is ponderable. | **I weigh all that’s here** [4, p. 475]. |
| **Internal conflicts** | $T_{\mu\nu}$ and $G_{\mu\nu}$ are separated entities. |
| Process and cause-effect principle are claimed, but only a state equation is given. A state equation refers to the state variables of one system. | $T_{\mu\nu}$ describes “real matter” that already contains $E_{\text{pot}}$. |
| A gravitational field is not nothing. It has to react to and to weight itself. |
However, the greater realism of $T_{\mu\nu}$ is thwarted by the unrealistic term $G_{\mu\nu}$:

1. $G_{\mu\nu}$ is intended to replace the classic gravitational field. In the desire to keep SR with $E_{\text{pot}} = 0$ as a boundary case, Einstein subsequently introduces $E_{\text{pot}}$ in GR by assuming a gravitational field $G_{\mu\nu}$ next to matter. Roger Penrose speaks of “disembodied gravitational energy” [54, p. 464] that is interpreted as interaction energy. This interpretation of field theories has already become commonplace today. And yet, is represents a fundamental reinterpretation. In mechanics and TD, $E_{\text{pot}}$ is positional energy that belongs to the body. Energy conservation can be only demonstrated by means of embodied $E_{\text{pot}}$ [30].

2. If $E_{\text{pot}}$ is “disembodied” interaction energy, it is assigned to $G_{\mu\nu}$ that replaces the gravitational field. With $E = mc^2$ of SR, $E_{\text{pot}}$ has to be ponderable. However, $G_{\mu\nu}$’s motto is “I weigh all that’s here.” It cannot be both co-cause and effect, i.e. it has to be, but cannot be ponderable. This is an irresolvable contradiction. Thus, physicists tried to include $E_{\text{pot}}$ in $T_{\mu\nu}$, which implies the unacknowledged concession that $E_{\text{pot}}$ belongs to the body (matter). However, all attempts have failed [4–6]. Until today $T_{\mu\nu}$ does not describe the energy-momentum of the gravitational field itself [5, p. 198].

   Should not the leading theory of gravity be able to describe $E_{\text{pot}}$?

   Please note minor inconsistencies: i) If the gravitational field is described by $G_{\mu\nu}$, but matter by $T_{\mu\nu}$, one separates $E_{\text{pot}}$ “outside the body” from $E_{\text{pot}}$ of its components, ii) If the gravitational field $G_{\mu\nu}$ and the electromagnetic field in $T_{\mu\nu}$ are spatially separated, their unification becomes impossible from the outset.

3. To increase the confusion, $G_{\mu\nu}$ is identified with the curvature of the non-Euclidean spacetime. The flat Minkowski spacetime of SR is abandoned:

   Euclidean geometry is not valid in the gravitational field even in first approximation. [52, p. 820]

   With the identity $G_{\mu\nu} = \kappa T_{\mu\nu}$, the Einstein tensor $G_{\mu\nu}$ is both a dynamic and a kinematic term. Therefore, the essence of spacetime has remained undecidable until today. Sometimes spacetime is interpreted as honey, other times as pure metrics. There are families of interpretive positions [55, 56], all of which must remain incorrect because already Minkowski’s assumptions on space and time are physically incorrect. The contradictio in adiecto “geometrodynamics” is an irresolvable conflict. By reducing all dynamic processes to kinematic translation, the dead world of SR is further potentiated in GR. Strictly speaking; real physical quantities are excluded if $G_{\mu\nu}$ is used.
4. $G_{\mu\nu}$ reduces nature to observer impressions and geometrical considerations. In this way, processes in time are denied and the 2nd law of TD is abandoned. Although GR claims to be a cosmological theory, it cannot describe evolution in one direction. There is no physical expression for the arrow of time. The becoming is openly denied:

Therefore, it seems more natural to think of the physically being as a four-dimensional being instead of, as before, as the becoming of a three-dimensional being. [39, p. 121]

The solution to the problems is: While $T_{\mu\nu}$ is real, $G_{\mu\nu}$ is not required. The gravitational field belongs to a body just like $m, E_{\text{pot}}$, and other abstracted state variables connected in state equations. It cannot be separated. $\kappa T_{\mu\nu}$ is an expression for the gravitational potential that lacks no mass because $E_{\text{pot}}(r, h, V, \ldots)$ is not ponderable if one admits the independent energetic relevance of spatial state variables. $T_{\mu\nu}$ describes the direct dynamic force effect on matter (including radiation) quite well, as already Steven Weinberg assessed:

It simply doesn’t matter whether we ascribe these predictions to the physical effect of gravitational fields on the motion of planets and photons or to a curvature of space and time. [57, p. 147]

6.2.2 Equation (34)

Equation (34) corresponds to Paul Gerber’s formula for the perihelion shift of 1898. Gerber emphasizes the importance of lifting work, recognizes that $E_{\text{pot}}$ is a function of distance (not of mass), and calculates the propagation of interaction with the finite velocity $c$ [58].

6.3 Historical reference and interim conclusion

In 1905, Einstein interpreted Planck’s formula of black-body radiation by means of his brilliant and realistic light quantum hypothesis [59]. Fascinated by the idea of discontinuous energy, he reinterpreted also Lorentz’ equations and $E = mc^2$ of Weber, Heaviside, Poincaré, and others (cf. [27]) in a theory of point-like particles moving in empty space [36, 37]. In 1907, the associated reinterpretation of space and time was welcomed by the mathematician Minkowski, who evoked the four-dimensional spacetime [50]. In 1909, Planck proclaimed a revolution in the world view [60, p. 117]. In 1915, Einstein reinterpreted Newton’s gravitational law and Paul Gerber’s formula [51–53]. While retaining some of the concepts of SR, such as
spatial separability of particles or of bodies, he offered a solution with the non-Euclidean, curved spacetime, which he celebrates as a triumph of mathematics:

The magic of this theory will hardly escape anyone who has really understood it: it means a veritable triumph of the method of the general differential calculus founded by GAUSS, RIEHANN, CHRISTOFFEL, RICCI and LEVI-CIVITER. [51, p. 779]

GR attracted with strong claims such as: i) Generalization, ii) Flexibility, iii) Openness to purely geometric considerations, iv) Experimental evidence, and even v) Simplicity:

We have now derived the most general laws, which the gravitational field and matter satisfy, by consistently using a coordinate system [...]. We achieved thereby a considerable simplification of formulas and calculations. [52, p. 816]

Focused on mathematics and biased interpreted experiments such as light deflection [61], GR was accepted. This reinforced the acceptance of SR. The decisive point was that $T_{\mu\nu}$ is indeed superior to the Newtonian approach. The logical conflicts of $G_{\mu\nu}$ that reduces physics to mathematics without respecting the laws of nature were tolerated. Critics of spacetime [45, 62] were first devalued and later ignored. GR was celebrated without considering that:

The correctness of the mathematical formalism is not sufficient to validate a scientific structure as coherent and free from contradiction [...] In reality the two relativity theories are brimming with paradoxes. [63, p. 248]

Geometrodynamics became the leading theory of gravity and the basis of the ΛCDM model, the standard model of cosmology. This led to sophisticated mathematical theories open to speculations such as Dark Matter, Dark Energy, quintessence, big bang, inflation, and high-dimensional spacetimes. The flexible tensor mathematics of GR with a great richness of variants for $T_{\mu\nu}$ and $G_{\mu\nu}$ [4, 5] and many adjustable parameters, the multiple meanings of spacetime, and correction factors such as $\Lambda g_{\mu\nu}$ allowed a wide range of interpretations.

Nevertheless, the ΛCDM model is in deep crisis today. Its basic hypotheses contradict experimental facts [6–9] and the Standard Model of particle physics [2]. Even if the elegance and beauty of GR is often invoked: GR cannot be “really understood” as Einstein demands [51, p. 779]. Logical absurdities and oscillations between kinematics and dynamics can be written down, but not understood.

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General relativity gives the illusion of processes without a single process equation. It both claims and violates the cause-effect principle.

Einstein’s field equations contain the realistic term $T_{\mu\nu}$ and the unrealistic term $G_{\mu\nu}$.

Spacetime does not exist and has been experimentally disproved countless times.

Gravitational energy $E_{pot}$ is not ponderable and an intrinsic property of a body.
7 Quantum theory

7.1 The equations of quantum mechanics (QM) and their current interpretation

The fundamental equation of QM is the Schrödinger equation [64] that is often given by:

$$H \psi(t) = i \hbar \frac{\partial}{\partial t} \psi(t), \quad (35)$$

where $\psi$ denotes the wave function, $\hbar = h/2\pi$, and $H$ the Hamiltonian, i.e. the operator of the total mechanical energy $E$ of a system. The form of $H$ depends on the system. For an elementary particle, the non-relativistic Hamiltonian is mostly expressed by the sum of the operators $T(P)$ of the kinetic energy $E_{\text{kin}}$ and $V(q)$ of the potential energy $E_{\text{pot}}$:

$$H(P, q) = T(P) + V(q) = \frac{P^2}{2m} + V(q) = -\frac{\hbar}{2m} \nabla + V(q), \quad (36)$$

where $P$ and $q$ are the generalized momenta and coordinates in configuration space, and $m$ is the particle mass. The relativistic $H$ of a “free particle” is expressed, for example, by:

$$H(P) = c \sqrt{(P_x^2 + P_y^2 + P_z^2)} + m_0^2 c^2, \quad (37)$$

where $m_0$ is the rest mass, and $V(q) = 0$.

The current interpretation of $\psi$ goes back to the standard formulation of QM of 1927, also referred to as Copenhagen interpretation. According to Werner Heisenberg and Niels Bohr [65, 66], Equation (35) has to be interpreted as a mathematical concept, i.e. $\psi$ does not represent an expression for a real wave. Quantum objects do not move along trajectories. Heisenberg postulated that his uncertainty relations:

$$\Delta P \Delta q \geq \hbar/2; \quad \Delta E \Delta t \geq \hbar/2 \quad (38)$$

mean that $P$ and $q$ as well as $E$ and $t$ of a particle are indeterminate in real terms. The uncertainty of $E$ is an essential conceptual basis of quantum field theories (QFT), such as quantum electrodynamics (QED) and quantum chromodynamics (QCD). Here, the fundamental forces are described via fields created by carrier particles.
7.2 Analysis of the equations and their interpretation

There is a long-standing debate in physics and philosophy about the current interpretation of quantum theory that is referred to as “non-causal”, “statistical”, “non-deterministic”, or “anti-realistic” [10, 67–73] and that describes nature as being absurd:

The theory of QED describes nature as absurd from the point of view of common sense. And it agrees fully with experiment. So I hope you can accept nature as she is – absurd [74].

7.2.1 Equation (35)

In terms of its formal structure, Equation (35) is a realistic differential equation, comparable, for example, to a first-order time law $kc = dc/dt$ for describing the time evolution of matter (such as the radioactive decay) in chemical kinetics. Solutions of such type of differential equation are exponential functions $y = A \exp(-kx)$ with the specific form of the factors $A$ and $k$ depending of the problem. In general, such equations are approximations.

Equation (35) was interpreted quite differently. According to the Copenhagen interpretation, wave properties and particle properties are two complementary sides of reality (principle of complementarity), which are mutually exclusive. Particles or waves only become real when they are observed (measured). This means that there exists no objective reality, which exists independently of the measuring process. This contradicts Erwin Schrödinger’s understanding of its own equation, who interpreted particles, starting from De-Broglie’s matter waves, as real waves (cf. Table 5):

It is necessary to ascribe to $\psi$ a physical, namely an electromagnetic, meaning [...] A definite $\psi$ distribution in configuration space is interpreted as a continuous distribution of electricity (and of electric current density) in actual space. [75, p. x]

| Table 5: Two interpretation variants of quantum mechanics. |
|---------------------------------------------------------|
| **Copenhagen interpretation (CI)**                     | **Schrödinger interpretation**                  |
| Basic statements                                      |                                             |
| Observer-dependency                                   | Objective reality                           |
| Indeterminism (no definite location)                  | Determinism (definite location)             |
| Incomprehensibility of nature                          | Comprehensibility of nature                 |
| Completeness                                           | Incompleteness                              |
| Hidden variables                                       | Hidden variables                            |
| Representatives                                       |                                             |
| Niels Bohr, Werner Heisenberg, Paul Jordan, Max Born,  | Erwin Schrödinger, Louis de Broglie,        |
| John von Neumann, and others [65, 66, 76]             | Paul Ehrenfest, David Bohm, Jean-Pierre Vigier, and others [64, 77, 78] |
Equation (35) is time-symmetrical, i.e. it suggests that all quantum processes are reversible. It is known that kinetics cannot describe the direction of processes because it is not an exact theory, but derived from TD under idealizing assumptions. Schrödinger described Equation (35) as not complete, which not only concerns the spin of particles:

Thus there still seems to be something lacking in the wave law for the $\psi$ function, – corresponding to the “reaction of radiation” of the classical electron theory, which may result in a dying away of the higher vibrations in favour of the lower ones [...] This necessary complement is still missing. [75, p. xi]

7.2.2 Equations (36) and (37)

The fundamentally new feature in Equation (36) is the extension of the concept of kinetic energy $E_{\text{kin}}$ by connecting the so-called “rest energy” $E_0$ of particles and the electromagnetic energy $h\nu$ with $E_{\text{kin}}$. Understanding his equation as “a step from classical point-mechanics towards a continuum-theory” [75, p. 45], Schrödinger describes a “free particle” as vibration:

In the first instance, this equation is stated for purely periodic vibrations sinusoidal with respect to time [...] It contains a “proper value parameter” $E$, which corresponds to the mechanical energy in macroscopic problems, and which for a single time-sinusoidal vibration is equal to the frequency multiplied by Planck’s quantum of action $h$. [75, p. ix]

However, since $H$ was not interpreted free from the accepted force and energy concepts of mechanics, SR and classical field theories, conceptual problems arise:

1. Formally, $T(P)$ corresponds to Rule 3 of the cause-effect principle. However, $T(P)$ is only written down, while $T(v)$ is used. While in 1926, Schrödinger still considered a variable mass $m(v)$ [64, p. 361], he corrected himself in 1928 in the English version of the same paper and neglected “the relativistic variation of mass”. [75, p. 1]

By taking SR as given, one adopts its concepts such as the unsolved problem of $m_0$ and $m(v)$, the blurring of kinematics and dynamics, the special role attributed to photons, and the idea of point-like particles – a concept that contradicts Schrödinger’s idea of (spatially extended) vibrations. Equation (37) is, for example, a priori unable to describe:

a. The real variable mass $m(v)$ of particles as detected in particle accelerators because $m(v)$ in SR is observer-dependent, and

b. The conversion of parts of $E_0$ into $E_{\text{kin}}$ and vice versa as measured for the oscillating mass of neutrinos.
2. \( V(q) \) formally satisfies **Rule 3** of the cause-effect principle. Nevertheless, the conceptual problem of \( V(q) \) in the Hamiltonian \( H \) is even more serious than that of \( T(P) \):

In mechanics and Hamiltonian mechanics, \( E_{\text{pot}} \) is the *positional energy of a body*. The operator for \( E_{\text{pot}} \) in \( H \), however, covers Coulomb energy \( V_C = q\Phi \) if a charged particle in an *electrostatic field* is described. The field is described as an entity next to matter and Coulomb energy is called \( E_{\text{pot}} \), an interpretation of quantum field theories (QFT) that has become commonplace today. And yet, the reading is conceptually questionable:

i) It represents an essential shift in meaning compared to mechanics.

ii) If electromagnetic energy of charges, which is released due to attractive binding, is interpreted as \( E_{\text{pot}}(r) \), this contradicts the statement of Schrödinger that electromagnetic vibrations are described via \( E_{\text{kin}}(P) \). Whereas \( E_0 \), \( h\nu \) and \( E_{\text{kin}} \) refer to motion, \( E_{\text{pot}}(r) \) refers to positional energy, i.e. just the opposite. Whereas released binding energy \( h\nu \) is ponderable and reduces the mass of a bound charge, \( E_{\text{pot}} \) is not ponderable.

iii) Coulomb’s law (just like Newton’s gravitation law or Einstein’s field equations) are state equations as defined in Equation (1). They describe how the state variables of one system are interrelated at \( t \). Since interaction between separated entities needs time, this conflicts the idea of separated entities *electromagnetic field – matter*.

That not only \( E_{\text{pot}} = m\phi(r) \), but also \( V_C = q\Phi(P) \) depends on \( r \) between two objects, points the way to a unified understanding of interaction (see Section 8).

### 7.2.3 Equation (38)

Equation (38) provides the lower limit for the measurement accuracy of quantum properties. Heisenberg’s reading that a quantum object is indeterminate *in real terms*, i.e. does not has a definite position, energy, etc. because we cannot measure them, represents a positivistic shift in meaning, which foregrounds observations, denies the reality of quantum processes, and allows to violate energy conservation as long as Equation (38) is fulfilled.

By keeping the empty space of SR, the real existing quantum fluctuations were interpreted as emerging from nothing and vanishing into nothing. This interpretation allowed to maintain the conventional separation *matter – force field* in line with the point mechanics and \( E = mc^2 \) of SR. And yet, if in QFT three of four abstracted force fields are quantized by means of carrier particles (gauge bosons),
this implies a complete departure from continuous energy and deeply contradicts Schrödinger’s wave approach.

7.3 Historical reference and interim conclusion

In 1926, Schrödinger formulated his famous wave equation [64]. The Copenhagen interpretation (CI) of 1927 [65, 66] and its acceptance can only be understood in a historical context. In 1928, Schrödinger emphasized the “extraordinary differences” [75, p. 45] between his own theory and the matrix formalism of Heisenberg. But he was influenced by the concepts of relativity and classical field theories, which made a realistic interpretation of the Hamiltonian $H$ difficult. Thus, Schrödinger temporarily accepted CI as an “effective interim solution” [71, p. 20]. Not only de Broglie and Ehrenfest, but also Einstein and Planck supported Schrödinger’s interpretation, which underlines the multifaceted nature of scientists.

However, the interim solution prevailed. CI attracted with strong claims such as: i) Flexibility, ii) Mathematical accuracy, iii) Experimental evidence, and vi) Completeness. Its acceptance was supported by John von Neumann, who’s mathematical proof for the correctness and completeness of CI [76] was regarded as irrefutable for decades, although it was disproved by Grete Hermann already in 1932 [79]. In 1966, John Steward Bell recognized Neumann’s proof as seriously wrong [80].

In the meantime, the state of conviction in the scientific community had already been reached. CI became the leading interpretation of quantum mechanics. Quantum field theories were developed since the late 1920s, and the Standard Model of particle physics was under development. Many scientists recognized the epistemological problems of CI such as the “measuring problem” [67–73] and criticized the “mathematical phenomenology” [67]. But the Standard Model was successful. The formalism of matrices with arbitrarily adjustable parameters proved to be flexible and many experimental facts could be described correctly. Mathematical procedures called renormalization were applied to eliminate infinite values of variables occurring in the calculations. The weak and electromagnetic forces were unified and justified Nobel prizes were awarded because creative ideas were developed within the given framework of SR, CI, and QFT. Like in the case of GR, this led to sophisticated theories open to speculations such as super symmetry, Higgs boson field, and string theory. Whereas CI dominated the textbooks, realistic interpretations of quantum mechanics, e.g. those of David Bohm [78] and Louis de Broglie who developed causal quantum physics
with vivid ideas of spin, charge, etc. [77], were ignored and remain conceptually ambiguous as long as they accept SR.

Even though it is well known that Bohmian mechanics [81, 82] reproduces all predictions of CI, it is rarely found in textbooks today. The deep crisis of the Standard Model is partly admitted [11, 12], partly not [83]. Some physicists still consider it to be the most accurate theory and simply apply the formalism often taught and learned without criticism. However, there are many reasons to admit that the desire to preserve selected concepts of SR and classic field theories has led to a strong metaphysics and turning away from reality.

The Schrödinger equation describes the real wave behavior of matter. The anti-realistic interpretation of quantum mechanics (Copenhagen interpretation) is historically conditioned and represents a pragmatic and positivistic interim solution. Quantum mechanics is incomplete.

A realistic interpretation of the Schrödinger equation is possible and necessary.

8 Back to the roots

The main results of the previous analysis are listed in Table 6, where interpretations, which are incompatible with the cause-effect principle, are marked in red. While already the interpretations in mechanics partly contradict causality, the general insight is that the follow-up theories have moved further and further away from the process approach, culminating in possible time travel into the

The Schrödinger equation describes the real wave behavior of matter. The anti-realistic interpretation of quantum mechanics (Copenhagen interpretation) is historically conditioned and represents a pragmatic and positivistic interim solution.

Quantum mechanics is incomplete.

A realistic interpretation of the Schrödinger equation is possible and necessary.

Table 6: The current concepts of force and energy in fundamental theoretical physics.

| Work W | Force $\xi$ | Effect $dX_i$ | Energy $E$ (?) | Embodied |
|--------|-------------|---------------|---------------|----------|
| **M**  | $F = \dot{P}$ | $dx$          | $E_{\text{pot}}$ (x) | Embodied |
|        | $F = ma$    |               |               |          |
| **SR** | No process  | No force      | $E_{\text{pot}} = 0$ | Ponderable |
| **GR** | Realistic $T_{\mu\nu}$ | $d\xi$ | $E_{\text{pot}}$ = Field energy Gauge bosons | Ponderable |
|        | Unrealistic $G_{\mu\nu}$ | $d\xi$ |               | Disembodied |
| **QM** | Realistic Schrödinger equation | $d\xi$ | $T(P), (\nu)$ | Ponderable |
|        | Unrealistic Copenhagen interpretation | $d\nu$ |               | Embodied |
| **QG** | Quantization of the curved (higher-dimensional) spacetime | $d\nu$ |               |          |

M, mechanics; SR, special relativity; GR, general relativity; QM, quantum mechanics; QG, quantum gravity.
future and high-dimensional spacetimes. With the advancement of TD in the 19th
and 20th century, the force and energy concepts of mechanics could have been
reviewed, what did not happen.

In the following, the process equations of mechanics are reinterpreted by
respecting the cause-effect principle. After that, first plausible consequences of a
rigorous process approach in quantum physics are proposed. It will be shown that
interaction can be understood in a unified way.

8.1 The consistent application of the cause-effect principle in
mechanics

In line with Equation (2), displacement work \( W_x \) is expressed by:

\[
dW_x = dE_{\text{pot}}(x) = F \, dx = \left( \frac{\partial E}{\partial x} \right)_{x \rightarrow x'} \, dx ; \quad F \equiv \dot{P} = \frac{d(mv)}{dt},
\]

where the force \( \xi_i \) is the intensive state variable \( F \). Equation (39) means what it says: \( F \) and \( \dot{P} \) are identical, i.e. \( F \) can be traced back to \( P \) that is more elementary. An
analogous equation can be formulated for any kind of mechanical work, where \( E_{\text{pot}} \)
is transferred, such as spring tensioning, lifting or volume work, which allows first
insights into the nature of interaction and processes (cf. Table 7 with the new
interpretations are marked in blue):

i) A local gradient of the real momentum flux \( \dot{P} \) causes any kind of process, where
\( E_{\text{pot}} \) is transferred. This means that action at a distance is excluded, or in other
words: \( F \) is a contact force.

ii) \( E_{\text{pot}}(X_i) \) of an object depends on a spatial state variable \( X_i \) that is independent of
\( m \) and whose value refers to the energetic state. Thus, \( E_{\text{pot}} \) is not ponderable.

iii) Even if their position as a whole is neglected, extended objects always possess
\( E_{\text{pot}} \) due to the positional energy of their components. Gravitational energy is
thus one among many kinds of \( E_{\text{pot}} \). Its separate treatment in GR is artificial.

iv) Since each kind of \( E_{\text{pot}} \) such as gravitational or tensional energy follows the
same principle, \( E_{\text{pot}} \) is embodied in each case. It represents positional energy,
no “interaction energy”, no “binding energy”.

Table 7: The interpretation of mechanical work in line with the cause-effect principle.

| \( \delta Y_i \) | Force \( \xi_i \) | Effect \( dX_i \) | Energy \( E(X_i) \) |
|-----------------|-----------------|-----------------|----------------|
| Lifting work \( \delta W_h \) | \( F = \dot{P} \) | \( \Delta h \) | \( (h) \) |
| Spring tensional work \( \delta W \) | \( F = \dot{P} \) | \( \Delta x \) | \( (k) \) |
| Volume work \( \delta W_v \) | \( p = \dot{P}/A \) | \( \Delta V \) | \( E_{\text{pot}} \) |
| Interfacial work \( \delta W_A \) | \( \sigma = \dot{P}/L \) | \( \Delta A \) | \( (A) \) |
| Momentum work \( \delta W_P \) | \( v \) | \( \Delta P \) | \( E_{\text{kin}} \) | Not ponderable |
| Momentum work \( \delta W_P \) | \( m \) | | | Ponderable, embodied |
Please note: *Embodied* does not mean *localized* if the idea of point-like particles is abandoned.

In line with Equation (2), momentum work $W_P$ is expressed by:

$$\text{d}W_P = \text{d}E_{\text{kin}} = v \text{d}P = \left( \frac{\partial E}{\partial P} \right)_{X_i \neq P} \text{d}P; \quad v = \frac{\text{d}x}{\text{d}t},$$  \tag{40}

where the cause-effect principle requires that the force $\xi_i$ is the *intensive* state variable $\nu$, whereas $\text{d}(mv)$ is the effect of the process (Rules 1–3). $X_i = \text{const.}$ at $\partial E/\partial P$ means that no kind of $E_{\text{pot}}$ is transferred. The new interpretation (marked in blue in Table 7) allows further insights into the nature of interaction and processes:

i) A local gradient of the velocity $v$ (that represents the differential quotient of the two extensive state variables $x$ and $t$ and can be expressed by $\partial E/\partial P$) causes any kind of process, where $E_{\text{kin}}$ is transferred. This means again that action at a distance is excluded, i.e. $v$ is a contact force.

ii) $E_{\text{kin}}(P)$ of an object depends on the momentum $mv$ that contains $m$ and refers to the new energetic state. Thus, $E_{\text{kin}}$ is *ponderable*.

iii) If $v$ of an object remains constant, its mass remains constant. If $v$ of an object changes, also $m$ changes. If $v$ of two adjacent objects moving in the same direction is equal, no dynamic exchange process is initiated between them. Otherwise, momentum work is performed. Mass is linked to velocity, i.e. motion [26–30].

iv) $m$ and $v$ are connected in both the force of $W_x$ and the effect of $W_P$. The reciprocity of the forces and effects explains why energy conservation, the equivalence of gravitational and inertial mass and other principles are fundamental laws [30].

Just as $E_{\text{pot}}$ (the energy of rest) and $E_{\text{kin}}$ (the energy of movement) are counterparts, the two driving forces $F$ and $v$ can be interpreted as counterparts (cf. Table 8),

| Force | $F = \dot{\rho}$ | $v$ |
|-------|------------------|-----|
| Meaning | Attraction/Repulsion modifying $v$ | Spatial propagation |
| Reason for | Aggregation | Disaggregation |
| | Order | Disorder |
| Interaction mechanism | No collisions | Collisions |
| Directed towards transfer of | $E_{\text{pot}}$ | $E_{\text{kin}}$ |
which remains hidden if one assigns $F$ to both displacement work and acceleration work.

Compare, for example, the ideal gas ($E_{\text{pot}} = 0$) and real matter. The point masses of an ideal gas move with $v$ in all directions and interact by means of collisions, i.e. perform $W_p$. This kind of interaction is not treated in the standard models. If there are no walls, the points fly away without performing $W_V$, described by $(\partial U/\partial V)_{T,n} = 0$, or any kind of work, where $E_{\text{pot}}$ is transferred. If $F$ (that is modeled in the standard models) is present, the velocity $v$, the direction of movement and the number and strength of collisions are influenced, e.g. by gravitational attraction or by electromagnetic attraction/repulsion. While $v$ means translational propagation and is associated with disaggregation and disorder, $F$ is associated with aggregation and order, up to the ordered structure of matter. Since $F$ and $v$ are different forces and disaggregation and (ordered) aggregation are opposing principles, they cannot both be due to collisions. This leads to the important insight that gravitation (and other fundamental forces) are mediated by another mechanism than collisions.

Equation (19) can now be extended. For the total energy $E$ of a moved system holds:

\[
E = E_0 + E_{\text{kin}} + E_{\text{pot}} = m c^2 + E_{\text{pot}} ,
\]

where $E_0$ covers the energy of all particles including electromagnetic energy $\hbar \nu$ and is reduced by electromagnetic binding energy emitted to the outside. $E_0$ does not cover $E_{\text{pot}}$ that represents the positional energy of the system as a whole and of its components. Since matter has more energy-equivalent state quantities $X_i$ than mass, Equation (41) is called energy-matter equivalence [27]. Tables 3 and 7, and Equation (41) mean a closing of ranks between mechanics, TD, and electrodynamics and offer a new starting point for physical thinking.

---

The driving force $\xi_i$ of work, where $E_{\text{pot}}$ is transferred, is the momentum transfer $\dot{P}$. The driving force $\xi_i$ of work, where $E_{\text{kin}}$ is transferred, is the velocity $v$. $\dot{P}$ and $v$ are different forces associated with different interaction mechanisms.

### 8.2 First implications of a consistent application of the cause-effect principle

A new starting point is not a new beginning because one can draw on the work of many scientists, such as Hendrik Antoon Lorentz, Henry Poincaré, Walther Nernst, Gilbert Newton Lewis, Erwin Schrödinger, Louis de Broglie, David Bohm, and Jean-
Marc Lévy-Leblond, whose realistic approaches to quantum mechanics have been so far ignored as they were not compatible with special relativity and the spacetime concept of general relativity.

8.2.1 Return to the ether in a new old form

The wave nature of matter, even atoms, has long been experimentally confirmed [84]. Until today, the intuitive idea has persisted that particles are not point-like and not separate from what surrounds them:

Many physicists think that particles are not things at all but excitations in a quantum field, the modern successor of classical fields such as the magnetic field. But fields, too, are paradoxical. [72, p. 42]

The field as an entity separated from matter, in which an abstracted state variable of an object is reified and materialized, is indeed questionable. It represents an artificial and paradoxical concept, because interaction between separated entities takes time, which is not described in state equations such as $G_{\mu\nu} = \kappa T_{\mu\nu}$ or $\Delta \phi = 4\pi G \rho$. The idea of a separate field is based on the misinterpretation of state equations as process equations, as already done in the case of Newton’s second axiom $F = ma$.

Not paradoxical, however, is the ether, which Einstein defined away in 1905 in favor of a theory of point-like particles in empty space. What is meant here is not the unacceptable idea of a medium next to matter like in the Maxwell or the Lorentz theory. Meant is the ether as a kind of primordial matter in which elementary particles represents different forms of excitation. Already Immanuel Kant understood matter as ether condensed in different degrees. In 1902, Henri Poincaré summarized related viewpoints of physicists:

One often goes even further and regards ether as the only original matter or even as the only real matter. […] According to Lord Kelvin, for example, what we call matter is only the location of points in which the ether is agitated by vortex-like movements; […]. For other more recent authors, Wiechert or Larmor, it is the place where the ether undergoes a kind of torsion of a very particular nature. [85, p. 169]

While this conception is new compared to contemporary physics, where the ether has fallen into disrepute, it actually represents a self-evident fact. If particles are created by excitations, they are not spatially separated from the ether, but represent ether energy in a special form. Their excitation energy distinguishes them from the unexcited ether medium that can be understood as an absolute reference system. This directly leads to Walther Nernst’s idea of the ether as a “supply of connected, continuous energy” [49] with natural constants:
PLANCK’s constant $h$, similar to the speed of light, now becomes a parameter characteristic of the light ether. [49, p. 116]

Here it can be extended that also $c^2$ is a material constant of the ether, namely the mass-specific excitation energy. To distinguish this ether notion from the Lorentz ether, the terms *quantum ether* [86] or *quanton ether* are proposed, following a suggestion of Jean-Marc Lévy-Leblond, who called all fermions and bosons *quantons* [87, p. 22].

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The ether represents a supply of connected, continuous energy.
Quantons are non-independent entities created by excitation of the ether.
Natural constants like $h$ and $c$ are characteristic parameters of the quanton ether.

8.2.2 New particle concept

If Augustin Jean Fresnel (1788–1827) attributed light to ether vibrations, now all quantons are attributed to ether excitations. Photons no longer play a special role. By abandoning the ideas of point-like particles and empty space, each quanton occupies a spatial area:

a quanton is an object that cannot be located in a point, [...] *a priori*, a quanton occupies the entire space available to it – in understandably very specific, particular forms. [87, p. 22]

From Equation (41) follows that the total energy $E_Q$ of a quanton $Q$ is given by its ponderable excitation energy $E_{ex,Q}$ and its non-ponderable energy $E_{pot,Q}$:

$$E_Q = E_{0,Q} + E_{kin,Q} + E_{pot,Q} = \frac{E_{ex,Q}}{mc^2} + E_{pot,Q}, \quad (42)$$

where $E_{0,Q} = m_0 c^2$ is the intrinsic excitation energy. Thus, quantons are discrete and countable, but also represent spatially continuous energy. $E_{pot,Q}$ is their positional energy in the medium, no binding energy. If Schrödinger describes a “free particle” as vibration:

It contains a “proper value parameter” $E$, [...] which for a single time-sinusoidal vibration is equal to the frequency multiplied by Planck’s quantum of action $h$. [75, p. ix],

this means that the Hamiltonian $H$ only exhibits $E_{ex,Q}$, while non-ponderable $E_{pot}$ is missing. There are further points deviating from the Copenhagen interpretation (CI):

i) All quantons are real, no matter how stable or unstable they are. As Schrödinger suggested, $\psi$ represents an expression for a real wave in the real space.
ii) Quantons are compact wave packets ("particles") connected with long-range spatial de Broglie waves that become weaker with increasing distance. Quantons are not either "particle" or wave (like in CI), but both, and both are real matter waves.

iii) A quanton propagates with $v$ in space. Its trajectory can be followed directly by considering also its quantum potential as suggested by David Bohm and others [71, 88].

iv) The uncertainty relations describe the lower limit for the measurement accuracy of quantum properties. They do not justify any statement about the non-reality of quantum phenomena or the violation of energy conservation.

v) The properties of quantons such as "rest mass", spin, charge, or color charge, are caused by their (mostly) hidden internal dynamics that can be described by non-linear wave equations [77]. Whereas $m_0$ is proportional to the quantity of $E_{0,0}$, other properties are attributed to the spatial quality of $E_{0,0}$. This is connected with concrete vividness.

vi) As emergent entities, quantons are neither "elementary" nor "particles" nor independent nor timeless. Irreversibility is to be found also on the quantum level [28]:

Thus there still seems to be something lacking in the wave law for the $\psi$ function, – [...] a dying away of the higher vibrations in favor of the lower ones. [75, p. xi]

As already Schrödinger admitted, his equation is an approximation. QM is incomplete.

Please note: The realistic interpretation of Schrödinger, de Broglie, Bohm, and others [64, 71, 75, 77, 78, 81, 82] cannot only describe all quantum phenomena, but is far superior to CI, because it does not suffer from logical and epistemological problems (cf. Section 7) and can explain the phenomena.

The Schrödinger equation and Heisenberg’s uncertainty relations can be interpreted realistically. A quanton exhibits ponderable excitation energy $E_{ex}$ and non-ponderable positional energy $E_{pot}$. Mass is linked to the amount of excitation energy $E_{ex}$, i.e. to motion. The properties of quantons are caused by their internal dynamics in Euclidian space.

8.2.3 New interaction concept – unified description of interaction

The above particle concept leads to an understanding of interaction far away from quantum field theories. QFTs and the new interaction concepts are contrasted in Figure 1.
The main conceptual idea of QFTs is the classic separation of particles (matter) from the field that mediates interaction. This implies that $E_0$ and $E_{\text{kin}}$ (of objects) and $E_{\text{pot}}$ (of the field) are spatially separated (cf. Figure 1a) and corresponds to Einstein’s idea of a “clean distinction” [89] between two distant objects $P_1$ and $P_2$. Since Einstein additionally assumed an empty space and $E = mc^2$ for the total energy, the classical field could not remain continuous either. Therefore, the idea arose to quantize the four abstracted force fields by means of four different carrier particles (gauge bosons), which mediate the interaction by momentum exchange. The electromagnetic, strong, and weak forces were quantized by virtual particles or quantum fluctuations of the vacuum that emerge from nothing and vanish into nothing. The quantization of gravity, however, remained impossible. As discussed above, the spatial separation of particle and field is based on a misinterpretation of state equations. The particle–field concept, the additional idea of four different carrier particles, and the focus on state equations deprive modern physics of the ability to describe interaction in a unified way.

The cause-effect principle and non-ponderable $E_{\text{pot}}$ suggest another understanding (cf. Figure 1b). Knowing that $E_{\text{kin}}$ can be generalized as done by Schrödinger and that quants $Q$ (and quanton clusters) are far extended wave entities as suggested by de Broglie, the spatial separation between $E_0/E_{\text{kin}}$ and $E_{\text{pot}}$, which contradicts the structure of state equations, can no longer be maintained. The quants and the fields are located in the same spatial area. This means that force, field lines, equipotential lines, etc. symbolize the quanton’s own properties. The new interpretation satisfies i) mechanics and thermodynamics, ii) the wave character of matter, and iii) the fact that nature is characterized by processes:

![Figure 1: Quantum field theories (a), and the new interaction concept (b).](image-url)
1. Interaction is realized via matter waves. By knowing that $\dot{p}$ and $v$ are different driving forces, there are more degrees of freedom for describing interaction. The matter waves of quantons interact twice: Firstly, by means of their long-rang De Broglie waves (of two or more binding partners), where the momentum flow $\dot{p}$ becomes real and causes their ordered aggregation. Secondly, by means of collisions due to their propagation with $v$ as a whole in the medium, which causes the disaggregation of quantons. In each case, the interaction represents a transfer of energy, i.e. a process, and reflects the antagonism of rest ($E_{\text{pot}}$) and movement ($E_{\text{kin}}$), where one kind of energy is dialectically preserved in the other and can be transformed into each other. Here, the standard models attempt to describe only the differently abstracted forces $\dot{p}$. They neglect i) the processes $\delta Y_i = dE_{\text{pot}}$, and ii) interaction by means of collisions, i.e. $\delta W_v = dE_{\text{kin}} = vdmv$.

2. As quantons are open systems, every natural quantum process is complex, i.e. several $X_i$ of the quanton change simultaneously. Equation (3) is to be applied.

Example $\alpha$: Electromagnetic binding $W_{e,1}$

If two charged quantons or clusters (such as an electron and a proton) begin to attract, they change their matter wave network in an internal binding reaction $W_{e,1}$ changing the quality of $E_{\text{ex}}$ in the two-body system. Simultaneously, photons $h\nu$ are emitted; the momenta $P$ and the positions $r$ of the binding partners change, etc. Thus, $W_{e,1}$ is combined with exchange processes such as heat transfer $Q$, momentum work $W_P$, lifting work $W_r$, and others. For the quantitative change in the total energy $E$ of the two-body system holds:

$$dE = \delta Q + \delta W_P + \delta W_r + \cdots = d(h\nu) + v\,dP + F_gdr + \cdots$$  \hspace{1cm} (43)

Since during the exothermic reaction, electromagnetic binding energy $E_{\text{ex},c} = h\nu$ (heat $Q$) is emitted and the momenta $P$ are changed, this is connected with a mass defect describing the energetic effects of $Q(P)$ and $W_P(P)$, but not that of $W_r(r)$:

$$\Delta m = m_{\text{bound}} - \sum_i m_{i,\text{free}} < 0.$$ \hspace{1cm} (44)

Example $\beta$: Gravitational binding $W_{e,2}$

If a non-charged quanton or cluster (such as a body) is lifted, the changed position $r$ is accompanied by a simultaneous change in the matter wave network of the binding partners. This internal binding reaction $W_{e,2}$ can be accompanied by the emission of gravitational waves $gw$ (a process $Q_g$ that is analogous to $Q$), momentum work $W_P$, a changing space filling, etc. For the change in the total energy $E$ of the two-body system holds:
\[ dE = \delta Q_g + \delta W_P + \delta W_r + \cdots = d\left(gw\right) + v\,dP + F_g\,dr + \cdots, \quad (45) \]

where \( Q_g(P) \) and \( W_P(P) \) cause a change in mass, while \( W_r(r) \) does not. Here it is to be noted that also the quanton photon \( \nu \) is subject to gravity. When it moves away from earth, it reduces its dynamic mass \( m_\nu = h\nu/c^2 \) by means of momentum work \( W_P \), which is known as gravitational red shift [90]. Since simultaneously the non-ponderable positional energy \( E_{\text{pot}} \) of the photon increases by means of \( W_r \), energy conservation is ensured [29, 30]. The result is: While state equations only allow a separate description of gravitational or electromagnetic forces, process equations allow a unified description of interaction. The binding reaction in the examples \( \alpha \) and \( \beta \) is always realized via De Broglie waves.

Lifting work (without changing \( m \) of a two-body system) and emission of waves (with changing \( m \)) both accompany the binding, which explains the analogous form of Coulomb’s and Newton’s law. The differences between the binding processes are i) the type of the De Broglie waves and their entanglement, superposition, etc., and ii) the type of the waves released (electromagnetic or gravitational). This leads to the task to model the special types of de Broglie waves of quanta and their interaction in the Euclidean space in order to explain the different binding strengths, and attraction and repulsion.

3. \( E_{\text{pot}} \) can be treated uniformly and explained. If the distance \( r \) between two quanta \( Q_1 \) and \( Q_2 \) is increased, lifting work is performed, and \( E_{\text{pot}} \) of the two-body system increases, quite analogous to the tensioning of a spring (cf. Table 7). Here only changes in \( E_{\text{pot}} \) are described, not absolute energies. \( E_{\text{pot}} \) can becomes minimal or optimal, but never zero. Each quanton is subject to a ground tension in the condensed ether medium (cf. Equation (42)). Although measurable positional \( E_{\text{pot}} \) is caused by the quality and quantity of the partially hidden and partially not hidden excitation energy in the ether medium, it represents the opposite, namely the tensional energy in a continuous wave network.

With Figure 1b, basic concepts of modern physics change:

1. Today, concurrent process equations, i.e. \( Y_i \) that run side by side, are mixed together. For example: The binding reaction \( W_{e,i} \) is accompanied by the release of \( h\nu \) via \( Q \) and lifting work \( W_r \). In potential curves (Lennard–Jones potential, etc.), released electromagnetic binding energy \( E_{\text{ex,C}} = h\nu \) is interpreted as potential energy. However, \( h\nu \) is the opposite: excitation energy. If non-ponderable \( E_{\text{pot}}(r) \) and ponderable \( E_{\text{ex,C}}(P) \) are called both potential energy, conceptual clarity is lost. It becomes necessary to stop calling \( E_{\text{ex,C}} \) potential
energy. It also becomes obvious that it is not the force that is quantized in the quantization efforts of the standard models, but rather the process \( Q \), i.e. the emission of waves such as photons \( h\nu \), that occurs simultaneously during binding.

2. Quantons are countable and discrete due to their excitation energy. This also applies to weakons, gluons, photons, and Higgs bosons. It is unquestionable that these “particles” exist. However, the role attributed to them in the Feynman diagrams and the Standard Model of particle physics becomes obsolete. As weakons, gluons, and photons, just like fermions, are created by excitation, they are neither only potential energy nor carrier particles of the force. Weakons, for example, can be interpreted – like in chemical kinetics – as an intermediate product of a physical-chemical reaction. In case of the Higgs bosons, their mass is to be explained by the ether like that of any other quanton. The Higgs boson field of 1964 represents a subsequently introduced, insufficient “ether substitute concept” [26].

3. It is out of question that quantum fluctuations, i.e. very short-lived, instable quantons, exist. They are an expression of the activity of the condensed medium, real in each case, do not violate energy conservation, and contribute to the limited accuracy in measuring the trajectories of stable quantons. Fluctuations can affect the stable quantons, if they appear and disappear in the spatial area occupied by them, and can occur as intermediate products in binding or decay reactions. However, they are not “carrier particles” of the force.

4. If real dynamic processes in time are described, the notion of spacetime loses its meaning. Higher-dimensional spacetimes are not required for a unified description of interaction.

Each quantum process is complex and to be described by \( k \) process equations. Interaction is realized via matter waves and can be understood in a unified way. Carrier particles and mass-providing Higgs bosons are non-realistic ideas.

8.2.4 Momentum work and the Lorentz transformations

With the developed quanton ether concept, the idea of inertial frames of reference becomes obsolete. Inertial systems, just like point particles, are conceivable idealizations within a thought experiment, but do not describe reality for at least two reasons:
1. A quanton is not independent of the ether medium, but represents itself ether medium in a new quality. As an emergent object it is permanently influenced by its “nutrient medium” and in turn affects it and its side-excitations. There are only *dynamic* processes in nature.

2. Since each kind of excitation energy $E_{\text{ex}}(P)$ is connected with waves, there are no uniform rectilinear, i.e. force-free movements in nature.

The reinterpretation of the Lorentz equations by inertial frames in SR is thus unrealistic. Consequently, all questions about Lorentz invariance or variance of state variables from the point of view of observers become obsolete. Lorentz’ own ether theory is based on realistic ideas. However, he assumed an ether “with a certain degree of substantiality” next to matter, i.e. separated from it [91, p. 230]. Therefore, the ether concept proposed here differs from his theory in several respects, but still harmonizes in basic equations, as will be shown below.

While unstable quantons almost instantaneously disappear, stable ones can propagate with $v$ within the medium. Each quanton propagates with its specific (optimal) self-velocity $v$ that correlates with the amount and spatial nature of its internal wave packets and the corresponding interaction with the continuous wave network. Here, a rule of thumb is that quantons with less *intrinsic excitation energy* $E_{0,Q}$ move faster.

If $v$ is to be changed, a process must take place that does not change $v$ alone. Firstly, the self-velocity $v$ of quantons (just as their internal velocity) is inseparably linked with their mass $m$ (cf. Table 7). For the momentum work $W_p$ that is performed, e.g. on an electron, applies:

$$\delta W_p = \delta E_{\text{kin}}(P) = v \delta (mv) = v^2 dm + vm \delta v.$$  \hspace{1cm} (46)

Secondly, $W_p$ is an abstraction. Several $X_i$ of the quanton change *simultaneously*, e.g. by displacement work $W_r$ against the continuous wave network, in the broadest sense also by volume work $W_V$, and interfacial word $W_A$; even excitation energy like $h\nu$ can be emitted. According to Equation (3), for the total energy of the electron applies:

$$dE = \delta W_p + \delta W_r + \delta W_V + \delta W_A + \delta Q + \cdots$$  \hspace{1cm} (47)

Equations (46) and (47) will be discussed below.

**Equation (46)**

Since $E_{\text{kin}}(P)$ is mass-proportional excitation energy $E_{\text{ex}}(P)$ (cf. Equation (42)), we can write:
\[ c^2 \text{dm} = v^2 \text{dm} + v \text{mdv}. \]  

(48)

On the one hand, Equation (48) can be transformed into:

\[ \frac{\text{dm}}{\text{dv}} = \frac{mv}{c^2 - v^2} > 0, \]  

(49)

which immediately shows that the slope of the function \( m = f(v) \) is positive because of \( v < c \). If \( v \ll c \) applies, the increase in \( m \) with \( v \) is infinitesimal and negligible due to the very large denominator, whereas with increasing \( v \), the slope increases.

On the other hand, integration of Equation (48)

\[ \int_{m_0}^m \text{d ln}m = \int_0^v \frac{\text{v}}{c^2 - v^2} \text{d}v \]  

(50)

leads to:

\[ \ln \frac{m}{m_0} = - \ln \left( \frac{(c^2 - v^2)^2 - \ln c^2}{2} \right) = \ln \left( 1 - \frac{v^2}{c^2} \right)^{-1/2}, \]  

(51)

\[ m = \frac{m_0}{\sqrt{(1 - v^2/c^2)}}. \]  

(52)

Equation (52) corresponds to Equation (27) for the so-called relativistic mass \( m(v) \).

Already in 1908, Equation (52) was derived from \( c^2 \text{dm} = v \text{d}(mv) \) for a “beam of radiation” without relativity by the physical chemist Gilbert Newton Lewis [48, p. 711]. He proposed “a method of distinguishing between absolute and relative motion” [48, p. 717], but adopted Einstein’s total energy \( E = mc^2 \) of a system:

We should then regard mass and energy as different names and different measures of the same quantity. [48, p. 708]

Lewis’ work was heavily criticized [92], the main criticism being, surprisingly, that he assumed “that all energy is of the same nature as radiant energy” [92, p. 657]. This criticism was justified, while that of \( W_p = v \text{d}(mv) \) not if the cause-effect principle is taken into account. However, while \( W_p = v \text{d}(mv) \) was rejected, the total energy \( E = mc^2 \) within SR was accepted by the scientific community. Lewis’ paper went largely unnoticed because i) no theoretical justification for \( v \text{d}(mv) \) was given, and ii) contradictions do indeed occur if momentum work \( W_p = \Delta E_{\text{kin}} \) is equated with changes in the total energy \( E \) of a system.

With Equation (42) that describes only excitation energy \( E_0(P) \) and \( E_{\text{kin}}(P) \) as ponderable, Equation (52) can be interpreted realistically:
i) The increase in mass with $v$ is real (as already suggested by Lorentz [41]) and experimentally confirmed for electrons and other particles in particle accelerators.

ii) Since the propagation of a quanton is wavelike, i.e. not uniform and rectilinear, the velocity vector components $v_x$, $v_y$, and $v_z$ are to be considered.

iii) The large mass of protons and neutrons (mass surplus compared to their constituents) is due to the increased velocity $v$ of quantons like gluons under confined conditions.

iv) Knowing that mass is linked to velocity and can be deduced from one principle (amount of excitation energy), $m_0$ and $m$ are no longer different variables. Since also $E_0$ is due to motion, $E_0$ and $E_{\text{kin}}$ are just as convertible into each other as $E_{\text{kin}}$ and $E_{\text{pot}}$. This can be used to explain the oscillating mass $m_0$ of the three neutrino types.

v) It becomes self-evident that there is no “rest-mass”, no “Lorentz-invariant mass” and no mass conservation, while energy conservation is always fulfilled (cf. [30]).

Analogous to Equation (52), the expression for $E_{\text{ex}}$ is:

$$E_{\text{ex}} = \gamma E_0.$$  \hfill (53)

**Equation (47)**

While $P$ is changed by momentum work $W_P = \Delta E_{\text{kin}}$, the position $r$ of a quanton like an electron is changed simultaneously by displacement work $W_r = \Delta E_{\text{pot}}$ (**Rule 1** of the cause-effect principle). In a non-force-free space, the spatial propagation of an electron represents work. Since a quanton (as a spacious deformable wave structure) is a complex open system, also volume work $W_V$ (the explanation of the Michelson–Morley experiment) takes place.

If $v$ and $m$ increase, the gravitational potential of the electron increases. It interacts more strongly with the continuous wave network or the medium, i.e. $\dot{P}$ becomes stronger – the resistance of the quanton ether to changing the self-velocity $v$. As early as 1906, Henri Poincaré, who preferred the Lorentz theory to Einstein’s interpretation his whole life, explained length contraction as volume work $W_V$ performed by the ether on the electron:

I tried to determine this force, I found that it can be considered as a constant external pressure, acting on the deformable and compressible electron, and whose work is proportional to the changes in the volume of this electron. [93, p. 130]
Thus, both the mass increase and the length contraction with $v$ are due to a real interaction with the ether medium. The increased $\dot{\mathcal{P}}$ also affects the internal excitation energy of an accelerated “clock” (quanton cluster), which is why atomic oscillation transitions slow down. The “clock” (matter) is changed, not time [46]. This interpretation, which was also prioritized by philosophers and proto-physicists, concedes the primacy of matter over geometry:

The Lorentz contractions or Einstein dilatations resulting from Lorentz metrics, on the other hand, can be interpreted – as in the case of Lorentz – as shortening of bodies or slowing down of movements. One does not need to speak of a revision of space and time. [94, p. 7]

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The Lorentz transformation for $m$ can be derived from the equation of momentum work. The propagation of a quanton in the ether has to be described by $k$ process equations. The mass increase, length contraction, process dilatation, etc. with increasing $v$ are real.

8.2.5 Quanton thermodynamics

The consistent application of the cause-effect principle leads to the insight that quantons, just as quanton clusters (bodies, liquids, etc.), are open systems, whose properties $\xi_i$ and $X_i$ are to be modeled. Isolated and closed systems are idealizations. While on the macroscopic level, several $X_i$ can be kept constant by applying suitable process conditions, this is not yet possible for quantum processes. In each natural quanton process, many $X_i$ change simultaneously. Just as there is no invariable (“Lorentz-invariant”) mass $m_0$, there is no invariable (“Lorentz-invariant”) entropy $S$. Unchangeable state variables are idealizations being valid only in the special case of low velocity $v$ and under the assumption of reversible processes.

As processes are the key to understanding, not states, thermodynamics extends not only mechanics at the macroscopic level, but also quantum mechanics at the microscopic level. This implies a unification of mechanics, thermodynamics, and quantum theory and a return of TD and irreversible processes to fundamental theoretical physics. Even the propagation of the (not timeless) quanton photon in the ether medium is a dynamic process because Schrödinger’s “dying away of the higher vibrations in favour of the lower ones” [75, p. xi] has to be considered.

In this context, the claim of completeness of equations, such as the Schrödinger equation and $E = mc^2$, has to be revised. In case of $G_{\mu\nu} = \kappa T_{\mu\nu}$, the spacetime interpretation must be abandoned. For each process, the real interaction via De Broglie waves and collisions is to be modeled by means of combined process equations.
9 Conclusions

The centuries-long attempt of physics to describe forces via state equations and separated fields has mathematical and idealizing reasons, while the conceptual content and the level of abstractions of the physical equations remained partly unconsidered.

The comprehensive analysis of the force and energy concepts in this paper shows that modern theoretical physics is based on idealizing state considerations that can describe processes only to a limited extent. The first reasons for the overestimation of state equations can already be found in mechanics, where the cause-effect principle is partly violated. Follow-up theories have moved further and further away from the process approach. Instead of causality, mathematical phenomenology and symmetry principles dominate today.

The consistent application of the cause-effect principle and of driving forces in process equations in mechanics leads to:
- a unification of mechanics, thermodynamics, and quantum physics,
- a unified description of interaction in Euclidean space,
- a particle concept that does justice to the wave character of matter,
- the explanation of the origin of mass and the mass increase of particles in accelerators.

The rediscovery of causality points the way to a realistic theoretical physics in Euclidean space and directed time.

List of used symbols and abbreviations

- $a$: Acceleration
- $A$: Interface area
- $c$: Velocity of light
- $c$: Concentration
- $D$: Diffusion coefficient
- $E$: Energy
- $E_0$: Rest energy
- $E_{pot}$: Potential energy
- $E_{kin}$: Kinetic energy
- $E_{ex}$: Excitation energy
- $E_{0,Q}$: Intrinsic excitation energy of a quanton $Q$
- $\eta$: Dynamic viscosity
- $F$: Force
- $F_i$: Thermodynamic force
- $\phi$: Newtonian gravitational potential
| Symbol | Meaning |
|--------|---------|
| $\Phi$ | Coulomb potential |
| $g$    | Gravitational acceleration |
| $G$    | Gravitational constant |
| $G$    | Gibbs free energy |
| $G_{\mu\nu}$ | Einstein tensor |
| $g_{\mu\nu}$ | Metric tensor |
| $\gamma \geq 1$ | Lorentz factor |
| $h$    | Planck's constant |
| $h$    | Height |
| $H$    | Hamiltonian |
| $j_i$  | Thermodynamic flux |
| $\kappa$ | Constant in Einstein's field equations |
| $L$    | Length |
| $L_{ii}$ | Phenomenological coefficient |
| $\Lambda$ | Cosmological constant |
| $m$    | Mass |
| $m_0$  | Rest mass |
| $M$    | Molar mass |
| $M_E$  | Mass of the Earth |
| $\mu$  | Chemical potential |
| $n$    | Amount of substance |
| $v$    | Frequency |
| $p$    | Pressure |
| $P$    | Momentum |
| $\rho$ | Flux of momentum |
| $\dot{\rho}$ | Momentum flux density |
| $Q$    | Heat exchange |
| $q$    | Generalized coordinate |
| $q$    | Charge |
| $R_{\mu\nu}$ | Ricci tensor |
| $\rho$ | Energy density |
| $r$    | Position, distance |
| $S$    | Entropy |
| $\sigma$ | Interface tension |
| $\psi$ | Wave function |
| $t$    | Time |
| $T$    | Temperature |
| $T_{\mu\nu}$ | Energy-momentum tensor |
| $T(P)$ | Operator of $E_{\text{kin}}$ |
| $U$    | Internal energy |
| $v$    | Velocity |
| $V$    | Volume |
| $V(q)$ | Operator of $E_{\text{pot}}$ |
| $V_C$  | Coulomb energy |
| $W$    | Work |
| $x$    | Spatial coordinate |
| $X$    | Extensive state variable |
Y  Process variable
ξ  Intensive state variable
Δ_i, d_i  Change by internal processes
Δ_e, d_e  Change by exchange processes
ΛCDM  Lambda Cold Dark Matter (the current standard model of cosmology)
TD  Thermodynamics
SR  Special relativity
GR  General relativity
QM  Quantum mechanics
QFT, QED, QCD  Quantum field theory/electrodynamics/chromodynamics
CI  Copenhagen interpretation
QG  Quantum gravity

Acknowledgements: The author would like to thank Dr. Steffen Arnrich, Dr. Mandy Klauck, Prof. Dr. Iris Römhild, Dr. Heiko Kalies, Prof. Dr. Gunther Göbel, and Dr. Matthias Fuhrland for fruitful discussions and the HTW University of Applied Sciences Dresden for its continuous support.

Author contributions: All the authors have accepted responsibility for the entire content of this submitted manuscript and approved submission.

Research funding: This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Conflict of interest statement: The authors declare no conflicts of interest regarding this article.

Ethical standards: The author declares no competing financial interests.

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