Study and Application on Production Decline Law of Water Drive Characteristic Curves

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Abstract. Water drive characteristic curves and production decline curves are two important methods to evaluate the recoverable reserves of oil field and predict development index. In this paper, we begin with deducing of Arps decline rules, and concluded that if the relationship of production decline rate of the same or similar model, they have the same or similar development index. According to this law, four kinds of commonly used water drive curve was deduced with oil production decline rate under the condition of fixed liquid. And Using this relationship, we established the method to prediction each year (month) development index of four commonly water drive curve. Through example, with the industry standard of recoverable reserves, compared to the results of the calculation formula of achieved higher prediction accuracy, and precision increases along with the time step narrowing of set, and the precision can be controlled. The research of this paper provides a basis for the prediction of water drive characteristic curve development index, and a basis for the selection of the recoverable reserve, and a method and reference for the further study of the water characteristic drive curve.

Keywords: Water drive characteristic curves; production decline curves; production decline law; recoverable reserves; development index.

1. Introduction
When the water flooding reservoir enters the middle and high water cut period, the dynamic method should usually be used to predict the development index and evaluate the recoverable reserves. The water flooding characteristic curve and the decline curve are frequently used methods.

The water flooding characteristic curve reflects the change law of oil-water two-phase. Since the water cut change macroscopically reflects the water flooding characteristic of the oil field, we simply refer to it as the displacement characteristic, which can reflect the water flooding characteristic of the oil field macroscopically [1]. The theory of production decline is proposed by J.J.Arps in 1945 [2]. The Arps decline curve has become an important method for reserves evaluation and production prediction during the production decline stage of the reservoir, and it has been widely used at home and abroad. Literature 1 uses the Taylor series expansion to discuss the relationship between the water flooding curve and the decline curve. From the perspective of seepage mechanics, literature 8 establishes the relationship between Arps and Logistic production decline equations under different conditions and different water flooding characteristic curves through the relative permeability curve. The corresponding
relationship between. In this paper, through the derivation analysis of Arps declining equation, this paper proposes an analysis method based on the declining rate-production curve to study the declining law of water drive characteristic curve, which will provide reference for further understanding and application of water drive characteristic curve in the future.

2. Deduction and analysis of decreasing Arps

2.1. Expression of decreasing rate

According to the decline theory of J.J. Arps, the definition of decline rate:

\[ d = -\frac{dq_o}{q_o dt} = k q_o^n \]  

(1)

Where: k is a constant, and d and qo are both functions of time t.

In the Arps decline theory formula (Equation 1), d is the instantaneous decline rate, and oil production qo is the instantaneous production, which are continuous functions that change with time, and there is no meaning of the cumulative amount at a certain stage (one year / month). In [3] there is an explanation of the effective decline rate and the instantaneous decline rate. In our practice, the effective annual decline rate (Equation 2) is used, and the meaning of its output is the cumulative output over a certain period of time (for example, one year). In practical applications, formula (1) and formula (2) are related. For details, see [3].

\[ D(\text{year}) = \frac{\text{Last years output} - \text{This years output}}{\text{Last years output}} \]  

(2)

2.2. Derivation of Arps decreasing equation

There are generally three types of Arps decreasing curve: exponential decreasing, hyperbolic decreasing and harmonic decreasing [4-9].

If you make a simple modification of formula (1) (when n ≠ 0):

\[ \frac{-dq_o}{q_o^{1+n}} = kdt \]  

(3)

Solve the differential equation:

\[ q_o = \left(nC + nk t\right)^{-\frac{1}{n}} \]  

(4)

Bring in initial conditions: \[ q_{oi} = \left(nC\right)^{-\frac{1}{n}}, \] \[ d_i = k q_{oi}^n \]:

\[ q_o = q_{oi} \left(1 + nd_i t\right)^{-\frac{1}{n}} \]  

(5)

That is: when \(0 < n < 1\), it is hyperbolic decreasing, when \(n = 1\), it is harmonic decreasing.

According to the same principle, when \(n = 0\), we can deduce:

\[ q = q_{oi} e^{-d_i t} \]  

(6)

That is: when \(n = 0\), the index is decreasing.

It can be seen from the derivation of the Arps decline equation: if the water flooding curve can derive the expression of decline rate-production, then the law of decline of the water flooding curve can be obtained and can be used to predict the development index.

3. The decreasing law of the four common types of water flooding curves

There are many types of water flooding curves. In this paper, four types used in industry standards are selected for research [4]. (Other types of dynamic method prediction models can also apply the method of this article to study its decreasing law)

In [10], the expressions of the decline rate of the A-type water drive curve with respect to water cut, fluid production and fitting parameters were derived. For the production process where the fluid production was basically stable, a prediction model for oil production was given. The relevant derivation of the water flooding curve in this paper is under the assumption of fixed liquid, that is, the liquid production \(q_l\) does not change with time. If this condition cannot be guaranteed, the conclusion in this paper is not true.
Table 1. Production decline rate - oil production relationship of four types water drive characteristic curves

| Type of water drive curve                  | Relation between decline rate and oil production |
|-------------------------------------------|--------------------------------------------------|
| Type A water flooding curve               | $d = \frac{b}{q_l} \left[\frac{1}{4} q_l^2 - \left(q_o - \frac{1}{2} q_l \right)^2 \right]$ |
| (Maximoff-Child Charter)                  |                                                  |
| Type B water flooding curve (Shadronov)   | $d = bq_o$                                        |
| Type C water drive curve (Sipachev)       | $d = 2b \sqrt{\frac{q_l}{a}} q_o^\frac{1}{2}$     |
| D-type water drive curve (Nazarov)        | $d = \frac{2b}{q_l \sqrt{a - 1}} q_o^\frac{1}{2} (q_l - q_o)^3$ |

3.1. Schematic diagram of decline rate of water flooding curve-oil production curve

For the sake of comparison, according to the relationship between the decline rate and production of the four water flooding curves (the maximum value of $d$ is 0.1, the maximum value of $q_o$ is 100, and $q_l = 100$), a schematic diagram of the decline rate and oil production curve is drawn to represent the fixed fluid. Under different conditions, the decline rate of different water drive curves varies with oil production (or water cut).

From the schematic diagrams of the four decline rate-oil production curves, we can draw:

1. The decreasing law of type B water drive curve (Shadzhonov) is equivalent to the harmonic decreasing curve at $n = 1$, and the decreasing law of type C water drive curve (Sipachev) is equivalent to the hyperbolic curve at $n = 0.5$ Decreasing curve. (The viewpoint in this article is consistent with the literature [1].)

2. According to the literature [6], the water-flooding characteristic curve is not applicable to the low water cut development stage and the extra high water cut development stage of the oil field. It can be seen from the above figure: in the middle and high water cut period, except for a prediction model of the D-type water drive curve, the decline rate begins to decline with the increase of water cut after 75% water cut, and the decline rates of the other three water drive curves are The water content rises and falls. Whether it can indicate that the D-type water flooding curve requires the water content to exceed 75% requires further demonstration.

3. If the block has a high water cut, use the A-type or D-type water flooding curve to predict the development index, and its decreasing law can be roughly equivalent to hyperbolic or harmonic decreasing.

4. Application of the relationship between the decline rate of water flooding curve and oil production

The relationship between the decline rate and oil production of the water flooding curve can be used to calculate the initial decline rate, decline type and other information in the prediction stage, and then the initial decline parameters determined by other methods can be compared to provide a reference for the selection of the prediction method.

The waterflood curve itself is the relationship between cumulative quantities, and it is easy to find the recoverable reserves, but it is not possible to directly predict the oil production in each future year like a decreasing curve. In this paper, based on the relationship between the decline rate of water flooding curve and oil production, a prediction method of oil production index of water flooding curve is established to predict the oil production of each year in the future and achieve precision control.

In order to facilitate the description of the method, a flow chart was developed:

1. Select the water drive curve, fit the sample points according to the selection, and fit the intercept $a$ and slope $b$;
2. According to the actual situation, fine-tune the fitting parameters intercept $a$ and slope $b$ (this step can also be omitted). But in this paper, in order to make the prediction line pass the end point of historical data, the method of adjusting the intercept $a$ is adopted without changing the slope $b$ value.
3. Determination of initial output:
   (1) Initial fluid production. Since the water flooding curves are all cumulative quantities, the larger the initial fluid production value, the shorter the predicted cutoff time. Therefore, the determination of the initial liquid production volume should analyze the trend of historical data. If the last point of the historical data is not an abnormal point, the liquid production volume at this point is usually the standard. In this paper, it is assumed that the fluid production does not change with time, so the fluid production is therefore constant.
   (2) Initial oil production. In the water flooding curve decline rate-production formula, because the oil production is an instantaneous value, it cannot be obtained immediately. Therefore, first analyze the trend of historical data. If the end point of historical data is not an abnormal point, take the oil production amount at the end point.

4. Set the time step $\Delta t$, which can be 1 year, 0.1 year ... Theoretically, the shorter the time step, the closer to the true value. First, use the relationship between water flooding curve decline rate and oil production to find the initial instantaneous decrease rate of the predicted time, and $\Delta t$, $2 \Delta t$ ... Oil production, instantaneous decline rate, and calculate the annual production based on this. Finally, the remaining recoverable reserves and recoverable reserves are obtained according to the oil production over the years.

5. When the water content of the block is high, a decline index $n$ can be calculated based on the recoverable reserves calculated in step 2 and the initial instantaneous oil production and instantaneous decline rate calculated in step 4. Change the exponential decrement formula in step 4 to hyperbolic / harmonic decrement. In this way, it can ensure higher accuracy even when the time step is large. If this step is not used, reducing the time step can also achieve higher accuracy.

6. According to the production sequence in step 4, draw the water flooding curve at the prediction stage, and calculate the intercept $a_1$, slope $b_1$ and correlation $R^2$. Properly adjust the initial instantaneous oil production in step 4 to maximize the correlation $R^2$ of the water flooding curve in the prediction stage. At this time, the annual production calculated in step 4 is saved as the predicted oil production output over the years.

5. Application examples
   One actual block is selected, and the fitting sample points are the same (see the table for data), and the calculation is carried out according to the four water drive curves.

| Time (year) | Oil production (10,000 tons) | Water production (10,000 tons) | Cumulative oil production (10,000 tons) | Accumulated water production (10,000 tons) |
|------------|-----------------------------|-------------------------------|----------------------------------------|------------------------------------------|
| 2007       | 6.07                        | 97.2                          | 666.79                                  | 2602.01                                  |
| 2008       | 6.63                        | 112.5                         | 673.42                                  | 2714.51                                  |
| 2009       | 6.08                        | 104.05                        | 679.5                                  | 2818.56                                  |
| 2010       | 5.82                        | 105.33                        | 685.32                                  | 2923.89                                  |
| 2011       | 5.72                        | 106.08                        | 691.04                                  | 3029.97                                  |
| 2012       | 5.7                         | 103.51                        | 696.74                                  | 3133.48                                  |
| 2013       | 5.08                        | 100.61                        | 701.82                                  | 3234.09                                  |
| 2014       | 4.98                        | 105.59                        | 706.8                                  | 3339.68                                  |
| 2015       | 4.45                        | 100.11                        | 711.25                                  | 3439.79                                  |

Firstly, regression is carried out according to the four water drive curve formulas, and the fitting line is translated to make it pass through the end point.
Table 3. Regression parameters and adjusting parameters table

| Type of water drive curve | Regression parameter | Adjustment parameters |
|--------------------------|----------------------|-----------------------|
|                          | a        | b      | a'     | Recoverable reserves (10,000 tons) | Remaining recoverable reserves (10,000 tons) |
| lnwp-Np                  | 3.708220 | 0.006233 | 3.709721 | 843.844259 | 132.594259 |
| lnLp-Np                  | 4.532460 | 0.005338 | 4.534121 | 863.676049 | 152.426049 |
| Lp/Np-Lp                 | 1.453695 | 0.001056 | 1.453492 | 785.643397 | 74.393397  |
| Lp/Np-Wp                 | 2.012176 | 0.001112 | 2.010815 | 770.038792 | 58.788792  |

The initial output is based on 2015, and the liquid production is 1.0456 million tons / year. When the cut-off condition reaches 98% water content, that is, the annual oil production reaches 20.912 million tons / year; due to space limitations, the adjustment is directly given here. The best initial instantaneous oil production and initial instantaneous decline rate. Since the water content of the block is more than 95% and it is in a very high water content stage, a decrement index n is calculated according to the method in step 5, so that a higher prediction accuracy can be obtained in the case of a larger time step ( Readers can try to use exponential decrement, take a smaller time step to make predictions, or get higher prediction accuracy). Since the decreasing law of type B and type C is the same as that of harmonic and hyperbolic (n = 0.5) ARPS, the corresponding decreasing index can be directly used. The principle of the n value of type A and type D is: Table 3 The recoverable reserves (or remaining recoverable reserves) calculated by the parameters in accordance with the ARPS decreasing formula are equivalent to those in Table.

Table 4. Forecast period initial parameter table

| Type of water drive curve | Instantaneous oil production (10,000 tons) | Instantaneous annual decline rate (decimal) | Decreasing index n |
|--------------------------|--------------------------------------------|---------------------------------------------|--------------------|
| lnwp-Np                  | 4.659269                                   | 0.027749                                    | 0.9648             |
| lnLp-Np                  | 4.718360                                   | 0.025189                                    | 1                  |
| Lp/Np-Lp                 | 4.461789                                   | 0.037831                                    | 0.5                |
| Lp/Np-Wp                 | 4.331550                                   | 0.044186                                    | 0.45               |

Finally, the parameters are calculated according to the parameters in the table, and the time step is taken as 1 year. The calculated annual oil production by various methods is shown in Table 5. When the predicted cut-off condition (water content 98%) is reached, the last year may not necessarily be the output of the whole year, so the output of the last year tends to be abrupt.
Table 5. Predicted results table

| Time (year) | Predicted annual oil production (10,000 tons) | inwp-Np | lnLp-Np | Lp/Np-Lp | Lp/Np-Wp |
|-------------|---------------------------------------------|---------|---------|----------|----------|
| 2016        |                                             | 4.595776| 4.659915| 4.378958 | 4.237854 |
| 2017        |                                             | 4.473200| 4.546799| 4.219335 | 4.058097 |
| 2018        |                                             | 4.356856| 4.439045| 4.068283 | 3.889103 |
| 2019        |                                             | 4.246287| 4.336280| 3.925300 | 3.730065 |
| 2020        |                                             | 4.141079| 4.238166| 3.789535 | 3.580245 |
| 2021        |                                             | 4.040857| 4.144394| 3.660784 | 3.438972 |
| 2022        |                                             | 3.945279| 4.054681| 3.538485 | 3.305629 |
| 2023        |                                             | 3.854035| 3.968771| 3.422214 | 3.179655 |
| 2024        |                                             | 3.766840| 3.886425| 3.311581 | 3.060533 |
| 2025        |                                             | 3.683433| 3.807427| 3.206228 | 2.947791 |
| 2026        |                                             | 3.603576| 3.731577| 3.105823 | 2.846095 |
| 2027        |                                             | 3.527049| 3.658690| 3.010062 | 2.739745 |
| 2028        |                                             | 3.453651| 3.585969| 2.918663 | 2.643676 |
| 2029        |                                             | 3.383197| 3.521137| 2.831364 | 2.552447 |
| 2030        |                                             | 3.315514| 3.456168| 2.747924 | 2.465748 |
| 2031        |                                             | 3.250443| 3.393553| 2.668120 | 2.383292 |
| 2032        |                                             | 3.187839| 3.333166| 2.591742 | 2.304812 |
| 2033        |                                             | 3.127565| 3.274891| 2.518597 | 2.230063 |
| 2034        |                                             | 3.069494| 3.218619| 2.448506 | 2.158817 |
| 2035        |                                             | 3.013509| 3.164248| 2.381300 | 1.039099 |
| 2036        |                                             | 2.959501| 3.111683| 2.316824 | 2.037005 |
| 2037        |                                             | 2.907367| 3.060837| 2.254931 | 1.962019 |
| 2038        |                                             | 2.857014| 3.011625| 2.195486 | 1.887179 |
| 2039        |                                             | 2.808352| 2.963971| 2.138361 | 1.814382 |
| 2040        |                                             | 2.761298| 2.917801| 0.745090 | 1.745090 |
| 2041        |                                             | 2.715774| 2.873048| 1.745090 | 1.676379 |
| 2042        |                                             | 2.671707| 2.829647| 1.676379 | 1.608769 |
| 2043        |                                             | 2.629030| 2.787537| 1.608769 | 1.541159 |
| 2044        |                                             | 2.587678| 2.746663| 1.541159 | 1.473550 |
| 2045        |                                             | 2.547591| 2.706970| 1.473550 | 1.405941 |
| 2046        |                                             | 2.508713| 2.668408| 1.405941 | 1.338332 |
| 2047        |                                             | 2.470989| 2.630929| 1.338332 | 1.270723 |
| 2048        |                                             | 2.434369| 2.594888| 1.270723 | 1.203114 |
| 2049        |                                             | 2.398007| 2.559043| 1.203114 | 1.135505 |
| 2050        |                                             | 2.364257| 2.524554| 1.135505 | 1.067906 |
| 2051        |                                             | 2.330677| 2.490982| 1.067906 | 0.990307 |
| 2052        |                                             | 2.298027| 2.458291| 0.990307 | 0.912708 |
| 2053        |                                             | 2.266270| 2.426447| 0.912708 | 0.835110 |
| 2054        |                                             | 2.235368| 2.395417| 0.835110 | 0.757511 |
| 2055        |                                             | 2.205290| 2.365171| 0.757511 | 0.679913 |
| 2056        |                                             | 2.176001| 2.335680| 0.679913 | 0.602314 |
| 2057        |                                             | 2.147473| 2.306914| 0.602314 | 0.524716 |
| 2058        |                                             | 2.119675| 2.278849| 0.524716 | 0.447118 |
| 2059        |                                             | 1.155486| 2.251459| 0.447118 | 0.369521 |
| 2060        |                                             | 2.224718| 2.224718| 0.369521 | 0.291923 |
| 2061        |                                             | 2.198606| 2.198606| 0.291923 | 0.214325 |
| 2062        |                                             | 2.173100| 2.173100| 0.214325 | 0.136727 |
| 2063        |                                             | 2.148178| 2.148178| 0.136727 | 0.059129 |
| 2064        |                                             | 2.123822| 2.123822| 0.059129 | 0.081531 |
| 2065        |                                             | 1.838660| 1.838660| 0.081531 | 0.103933 |

Remaining recoverable reserves (10,000 tons) 132.592192 152.426049 74.393397 58.786638

Length of time (years) 43.5506012 49.8749359 24.35466121 19.4930037

Comparing the remaining recoverable reserves calculated according to the annual oil production in Table 5 with the remaining recoverable reserves in Table 3: Type B and Type C water drive curves have
error of less than 100,000-600,000 tons, Type A and Type D water drives The curve error is less than 0.01 million tons (if the step size is reduced, the error can be further reduced). If the water flooding curve at the prediction stage can be drawn according to the parameters in Table 5, and the intercept a1, slope b1 and correlation R2 are calculated, the intercept a1, slope b1 and Table 3 are very close to a' and b, and the correlation R2 exceeds "10^9s" (verified by readers here).

Since the decreasing trend of type B and C water flooding curves is consistent with the decreasing APRS, it is not affected by the time step in the calculation; the decreasing trend of the type A and D water flooding curves is not exactly equal to the decreasing APRS, so the prediction accuracy influenced by time step.

From the fitting and prediction results of the four water drive curves: although the fitting and prediction accuracy of different methods are very high, the corresponding initial decline and decline types are different, which leads to the recoverable reserves (or remaining) calculated by different methods. Recoverable reserves) are different. If these sample points are used and the exponential declining method is used for fitting, the initial instantaneous annual declining rate is 0.041343, which is closest to the D-type water drive curve. To select a relatively reasonable result from the prediction results of different water flooding curve methods, it is necessary to combine the initial decline rate, decline index and prediction time length of various methods, and make a comprehensive judgment based on experience.

6. Conclusion
1. The declining rate-predictive model with the same oil production curve has the same trend of declining production;
2. Deduced the relationship between the decline rate of water drive curve and oil production under constant liquid conditions: the decline trend of type B water drive curve is consistent with the harmonic decline, and the decline trend of type C water drive curve is consistent with the hyperbolic decline of n = 0.5 The type A water drive curve and the type D water drive curve can only have a similar decline rule with the ARPS decline curve when the water content of the block is high;
3. The annual (monthly) production forecast under the condition of fixed water in the water drive curve is achieved, and a higher accuracy is achieved. Since the initial decline of different waterflood curves is very different, this index can be used as a reference for predicting the selection value;
4. During the high water cut period, the water flooding curve will have an upturn problem [5], and the water flooding characteristic curve is applicable to the development stage of medium and high water cut in the oil field. Different types of water flooding curves, even in the middle and high water cut period, the prediction results are very different. To determine whether the water drive curve result is reasonable, you can calculate the initial decline rate and the time it takes to reach the limit water cut according to the method provided in this article, and then make a comprehensive judgment based on the actual block of oilfield development.
5. The analysis method of decline rate-oil production curve in this paper is not limited to the study of water flooding curve, but it is still meaningful for other types of prediction models. For example: the decline curve of the attenuation curve type in the 89th edition of the industry standard ("Oilfield Recoverable Reserves Calibration Method" SY 5367-89), the expression is: \( Np t = at - b \), according to the derivation method of this article \( d = \frac{2}{\sqrt{b}} q^2 \), that is, the decreasing trend is consistent with n = 0.5 hyperbolic decreasing.

7. Explanation of symbols
- \( t \) ——time;
- \( d \) ——Instantaneous decline rate at time t;
- \( d_i \) ——Initial instantaneous decline rate (time t = 0);
- \( D \) ——Effective decline rate at time t;
- \( D_i \) ——Initial effective decline rate (time t = 0);
q_o —— Oil production at time t;
q_w —— Water production at time t;
q_l —— Fluid production at time t;
q_{oi} —— Initial oil production (time t = 0);
q_{oa} —— Ultimate oil production (when water content is taken as 98%);
n —— A decreasing index (0 ≤ n ≤ 1);
N_P —— Cumulative oil production at time t;
W_P —— Cumulative water production at time t;
L_P —— Cumulative fluid production at time t;
a, b —— The intercept and slope of the selected historical data fitting section of the water drive curve;
a_1, b_1 —— The intercept and slope of the water drive curve fitted according to the prediction results;
N_R —— Technical recoverable reserves, 98% of the limit water cut;

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