A New Approach in the Study of Oscillation Criteria of Even-Order Neutral Differential Equations

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Received: 17 January 2020; Accepted: 31 January 2020; Published: 5 February 2020

Abstract: Based on the comparison with first-order delay equations, we establish a new oscillation criterion for a class of even-order neutral differential equations. Our new criterion improves a number of existing ones. An illustrative example is provided.

Keywords: even-order differential equations; neutral delay; oscillation

1. Introduction

In the last decade, many studies have been carried out on the oscillatory behavior of various types of functional differential equations, see [1–24] and the references cited therein. As a result of numerous applications in technology and natural science, the issue of oscillation of nonlinear neutral delay differential equation has caught the attention of many researchers, see [1,3–5,8,12,17,19,22–24]. For instance, they are frequently used for the study of distributed networks containing lossless transmission lines, see [11].

In this paper, we are concerned with improving the oscillation criteria for the even-order neutral differential equation of the form

\[ r(t) \left( \left( z^{(n-1)}(t) \right)^{\frac{1}{\alpha}} \right) + p(t) x^{\sigma}(\tau(t)) = 0, \quad (1) \]

where \( t \geq t_0, n \geq 4 \) is an even natural number and \( z(t) := x(t) + p(t) x(\tau(t)) \). In this work, we assume that \( \alpha \) is a quotient of odd positive integers, \( r \in C[t_0, \infty), r(t) > 0, r'(t) \geq 0, \int_{t_0}^{\infty} r^{-1/\alpha}(s) \, ds = \infty, p, q \in C[t_0, \infty), q(t) > 0, 0 \leq p(t) < p_0 < \infty, q(t) \) is not identically zero for large \( t, \tau \in C[t_0, \infty), \sigma \in C[t_0, \infty), \tau'(t) > 0, \tau(t) \leq t \) and \( \lim_{t \to \infty} \tau(t) = \lim_{t \to \infty} \sigma(t) = \infty \).

By a solution of (1) we mean a function \( x \in C^3[t_y, \infty), t_y \geq t_0 \), which has the property \( r(t) \left( \left( z^{(n-1)}(t) \right)^{\frac{1}{\alpha}} \right) \in C^1[t_y, \infty), \) and satisfies (1) on \([t_y, \infty)\). We consider only those solutions \( x \) of (1) which satisfy \( \sup \{ |x'(t)| : t \geq T \} > 0 \), for all \( T \geq t_y \). A solution \( x \) of (1) is said to be non-oscillatory if it is positive or negative, ultimately; otherwise, it is said to be oscillatory.

A neutral delay differential equation is a differential equation in which the highest-order derivative of the unknown function appears both with and without delay.

In the following, we briefly review some important oscillation criteria obtained for higher-order neutral equations which can be seen as a motivation for this paper.
In 1998, based on establishing comparison theorems that compare the \(n\)th-order equation with only one first-order delay differential equations, Zafer [23] proved that the even-order differential equation
\[
z^{(n)}(t) + q(t)x(\sigma(t)) = 0
\]  
(2)
is oscillatory if
\[
\liminf_{t \to \infty} \int_{\sigma(t)}^{t} Q(s) \, ds > \frac{(n-1)!}{e},
\]  
(3)
or
\[
\limsup_{t \to \infty} \int_{\sigma(t)}^{t} Q(s) \, ds > (n-1)! \sigma'(t) \geq 0.
\]
where \(Q(t) := q^{n-1}(t) (1 - p(\sigma(t))) \hat{q}(t)\). In a similar approach, Zhang and Yan [24] proved that (2) is oscillatory if either
\[
\liminf_{t \to \infty} \int_{\sigma(t)}^{t} Q(s) \, ds > \frac{(n-1)!}{e},
\]  
(4)
or
\[
\limsup_{t \to \infty} \int_{\sigma(t)}^{t} Q(s) \, ds > (n-1)! \sigma(t) \geq 0.
\]
It’s easy to note that \((n-1)! < (n-1)! 2^{(n-1)(n-2)}\) for \(n > 3\), and hence results in [24] improved results of Zafer in [23].

For nonlinear equation, Xing et al. [22] proved that (1) is oscillatory if
\[
\left(\sigma^{-1}(t)\right)' \geq \sigma_0 > 0, \quad \tau'(t) \geq \tau_0 > 0, \quad \tau^{-1}(\sigma(t)) < t
\]
and
\[
\liminf_{t \to \infty} \int_{\tau^{-1}(\sigma(t))}^{t} \frac{\hat{q}(s)}{r(s)} \left(s^{-1}\right)^{\alpha} \, ds > \left(\frac{1}{\sigma_0} + \frac{\hat{p}_0}{\sigma_0 \tau_0}\right) \frac{(n-1)!^{\alpha}}{e},
\]  
(5)
where \(\hat{q}(t) := \min\{q(\sigma^{-1}(t)), q(\sigma^{-1}(\tau(t)))\}\).

If we apply the previous results to the equation
\[
\left(x(t) + \frac{7}{8}x\left(\frac{1}{e}t\right)\right)'' + \frac{q_0}{\tau^2} x\left(\frac{1}{e^2}t\right) = 0, \quad t \geq 1,
\]  
(6)
then we get that (6) is oscillatory if

\[
\begin{array}{c|c|c|c}
\text{The condition} & (3) & (4) & (5) \\
\hline
\text{The criterion} & q_0 > 113,981.3 & q_0 > 3561.9 & q_0 > 3008.5 \\
\end{array}
\]

Hence, Xing et al. [22] improved the results in [23,24].

By establishing a new comparison theorem that compare the higher-order Equation (1) with a couple of first-order delay differential equations, we improve the results in [22–24]. An example is presented to illustrate our main results.

In order to discuss our main results, we need the following lemmas:

**Lemma 1** ([13]). If the function \(x\) satisfies \(x^{(i)}(t) > 0, \ i = 0, 1, \ldots, n, \) and \(x^{(n+1)}(t) < 0, \) then
\[
\frac{x(t)}{t^n/n!} \geq \frac{x'(t)}{t^{n-1}/(n-1)!}.
\]
Lemma 2 ([2] Lemma 2.2.3). Let \( x \in C^n ([t_0, \infty), (0, \infty)) \). Assume that \( x^{(n)} (t) \) is of fixed sign and not identically zero on \([t_0, \infty)\) and that there exists a \( t_1 \geq t_0 \) such that \( x^{(n-1)} (t) x^{(n)} (t) \leq 0 \) for all \( t \geq t_1 \). If \( \lim_{t \to \infty} x (t) \neq 0 \), then for every \( \mu \in (0, 1) \) there exists \( t_\mu \geq t_1 \) such that

\[
x (t) \geq \frac{\mu}{(n-1)!} t^{n-1} |x^{(n-1)} (t)| \quad \text{for} \quad t \geq t_\mu.
\]

Lemma 3 ([3] Lemmas 1 and 2). Assume that \( u, v \geq 0 \) and \( \beta \) is a positive real number. Then

\[
(u + v)^\beta \leq 2^{\beta-1} (u^\beta + v^\beta), \quad \text{for} \quad \beta \geq 1
\]

and

\[
(u + v)^\beta \leq u^\beta + v^\beta, \quad \text{for} \quad \beta \leq 1.
\]

2. Main Results

Here, we define the next notation:

\[
P_k (t) = \frac{1}{(\tau - 1) (t)} \left( 1 - \frac{(\tau - 1) (t)}{(\tau - 1) (t)} \right), \quad \text{for} \quad k = 2, n,
\]

\[
R_0 (t) = \left( \frac{1}{r (t)} \int_t^{\infty} q (s) P_n (\sigma (s)) \, ds \right)^{1/\alpha}
\]

and

\[
R_m (t) = \int_t^{\infty} R_{m-1} (s) \, ds, \quad m = 1, \ldots, n - 3.
\]

Lemma 4 ([20] Lemma 1.2). Assume that \( x \) is an eventually positive solution of (1). Then, there exist two possible cases:

(\text{I}_1) \quad z (t) > 0, z^\prime (t) > 0, z^{(n)} (t) > 0, z^{(n-1)} (t) > 0, z^{(n)} (t) < 0, \\
(\text{I}_2) \quad z (t) > 0, z^{(j)} (t) > 0, z^{(j+1)} (t) < 0 \quad \text{for all odd integer} \quad j \in \{1, 3, \ldots, n - 3\}, z^{(n-1)} (t) > 0, z^{(n)} (t) < 0,

for \( t \geq t_1 \), where \( t_1 \geq t_0 \) is sufficiently large.

Theorem 1. Let

\[
\frac{(\tau - 1) (t)^{n-1}}{(\tau - 1) (t)^{n-1} p (\tau - 1) (t))} \leq 1.
\]

Assume that there exist positive functions \( \eta, \zeta \in C^1 ([t_0, \infty), \mathbb{R}) \) satisfying

\[
\eta (t) \leq \sigma (t), \quad \eta (t) < \tau (t), \quad \zeta (t) \leq \sigma (t), \quad \zeta (t) < \tau (t), \quad \zeta^\prime (t) \geq 0 \quad \text{and} \quad \lim_{t \to \infty} \eta (t) = \lim_{t \to \infty} \zeta (t) = \infty.
\]

If there exists a \( \mu \in (0, 1) \) such that the differential equations

\[
\psi^\prime (t) + \left( \frac{\mu (\tau - 1) (\eta (t))^{n-1}}{(n-1)! (\tau - 1) (\eta (t))} \right)^{1/\alpha} q (t) P_n (\sigma (t)) \psi \left( \tau - 1 (\eta (t)) \right) = 0
\]

and

\[
\phi^\prime (t) + (\tau - 1) (\zeta (t)) R_{n-3} (t) \phi \left( \tau - 1 (\zeta (t)) \right) = 0
\]

are oscillatory, then Equation (1) is oscillatory.

Proof. Let \( x \) be a non-oscillatory solution of (1) on \([t_0, \infty)\). Without loss of generality, we can assume that \( x \) is eventually positive. It follows from Lemma 4 that there exist two possible cases (I\(_1\)) and (I\(_2\)).
Assume that Case (I₁) holds. From the definition of \( z(t) \), we see that
\[
x(t) = \frac{1}{p(\tau^{-1}(t))} \left( z(\tau^{-1}(t)) - x(\tau^{-1}(t)) \right).
\]
By repeating the same process, we find that
\[
x(t) = \frac{z(\tau^{-1}(t))}{p(\tau^{-1}(t))} - \frac{1}{p(\tau^{-1}(t))} \left( z(\tau^{-1}(t)) - \frac{x(\tau^{-1}(t))}{p(\tau^{-1}(t))} \right).
\]
Using Lemma 1, we get \( z(t) \geq \frac{1}{(n-1)!} t^n z'(t) \) and hence the function \( t^{1-n} z(t) \) is nonincreasing, which with the fact that \( \tau(t) \leq t \) gives
\[
(\tau^{-1}(t))^{n-1} z(\tau^{-1}(t)) \leq (\tau^{-1}(\tau^{-1}(t)))^{n-1} z(\tau^{-1}(t)). \tag{12}
\]
Combining Equations (11) and (12), we conclude that
\[
x(t) \geq \frac{1}{p(\tau^{-1}(t))} \left( 1 - \frac{(\tau^{-1}(\tau^{-1}(t)))^{n-1}}{(\tau^{-1}(t))^{n-1} p(\tau^{-1}(\tau^{-1}(t)))} \right) z(\tau^{-1}(t)) = P_n(t) z(\tau^{-1}(t)). \tag{13}
\]
From Equations (1) and (13), we obtain
\[
\left( r(t) \left( z^{(n-1)}(t) \right)^{\alpha} \right)' + q(t) P_n^\alpha(\sigma(t)) z^\alpha(\tau^{-1}(\sigma(t))) \leq 0.
\]
Since \( \eta(t) \leq \sigma(t) \) and \( z'(t) > 0 \), we get
\[
\left( r(t) \left( z^{(n-1)}(t) \right)^{\alpha} \right)' \leq -q(t) P_n^\alpha(\sigma(t)) z^\alpha(\tau^{-1}(\eta(t))). \tag{14}
\]
Now, by using Lemma 2, we have
\[
z(t) \geq \frac{\mu}{(n-1)!} t^{n-1} z^{(n-1)}(t), \tag{15}
\]
for some \( \mu \in (0,1) \). It follows from (14) and (15) that, for all \( \mu \in (0,1) \),
\[
\left( r(t) \left( z^{(n-1)}(t) \right)^{\alpha} \right)' + \left( \frac{\mu (\tau^{-1}(\eta(t)))^{n-1}}{(n-1)!} q(t) P_n^\alpha(\sigma(t)) \left( z^{(n-1)}(\tau^{-1}(\eta(t))) \right)^\alpha \right) \leq 0.
\]
Thus, if we set \( \psi(t) = r(t) \left( z^{(n-1)}(t) \right)^{\alpha} \), then we see that \( \psi \) is a positive solution of the first-order delay differential inequality
\[
\psi'(t) + \left( \frac{\mu (\tau^{-1}(\eta(t)))^{n-1}}{(n-1)! \tau^\alpha(\tau^{-1}(\eta(t)))} \right) q(t) P_n^\alpha(\sigma(t)) \psi(\tau^{-1}(\eta(t))) \leq 0.
\]
It is well known (see [21] (Theorem 1)) that the corresponding Equation (9) also has a positive solution, which is a contradiction.
Assume that Case (I₂) holds. Using Lemma 1, we get that
\[ z(t) \geq tz'(t) \]  
(16)
and thus the function \( t^{-1}z(t) \) is nonincreasing, eventually. Since \( \tau^{-1}(t) \leq \tau^{-1}(\tau^{-1}(t)) \), we obtain
\[ \tau^{-1}(t) z \left( \tau^{-1} \left( \tau^{-1}(t) \right) \right) \leq \tau^{-1} \left( \tau^{-1}(t) \right) z \left( \tau^{-1}(t) \right). \]  
(17)
Combining (11) and (17), we find
\[ x(t) \geq \frac{1}{p(\tau^{-1}(t))} \left( 1 - \frac{\tau^{-1}(\tau^{-1}(t))}{(\tau^{-1}(t))} \right) \tau^{-1}(t) z \left( \tau^{-1}(t) \right) \]
\[ = p_2(t) z \left( \tau^{-1}(t) \right), \]
which with (1) yields
\[ \left( r(t) \left( \zeta^{(n-1)}(t) \right)^{\alpha} \right)' + q(t) p_2^\alpha(\sigma(t)) z^\alpha \left( \tau^{-1}(\sigma(t)) \right) \leq 0. \]
Since \( \zeta(t) \leq \sigma(t) \) and \( z'(t) > 0 \), we have that
\[ \left( r(t) \left( \zeta^{(n-1)}(t) \right)^{\alpha} \right)' \leq -q(t) p_2^\alpha(\sigma(t)) z^\alpha \left( \tau^{-1}(\zeta(t)) \right). \]  
(18)
Integrating the (18) from \( t \) to \( \infty \), we obtain
\[ z^{(n-1)}(t) \geq R_0(t) z \left( \tau^{-1}(\zeta(t)) \right). \]
Integrating this inequality from \( t \) to \( \infty \) a total of \( n - 3 \) times, we obtain
\[ z''(t) + R_{n-3}(t) z \left( \tau^{-1}(\zeta(t)) \right) \leq 0. \]  
(19)
Thus, if we set \( \phi(t) := z'(t) \) and using (16), then we conclude that \( \phi \) is a positive solution of
\[ \phi'(t) + \tau^{-1}(\zeta(t)) R_{n-3}(t) \phi \left( \tau^{-1}(\zeta(t)) \right) \leq 0. \]  
(20)
It is well known (see [21] (Theorem 1)) that the corresponding Equation (10) also has a positive solution, which is a contradiction. The proof is complete. □

Corollary 1. Assume that (7) holds and there exist positive functions \( \eta, \zeta \) such that (8) holds. If
\[ \liminf_{t \to \infty} \int_{\tau^{-1}(\eta(t))}^{t} \left( \frac{\tau^{-1}(\eta(s))}{\tau^{-1}(t)} \right)^{n-1} q(s) p_2^\alpha(\sigma(s)) ds > \frac{(n-1)!}{e} \]  
(21)
and
\[ \liminf_{t \to \infty} \int_{\tau^{-1}(\zeta(t))}^{t} \tau^{-1}(\zeta(s)) R_{n-3}(s) ds > \frac{1}{e}, \]  
(22)
then (1) is oscillatory.

Proof. It is well-known (see, e.g., [14] (Theorem 2)) that Condition (21) and (22) imply oscillation of (9) and (10), respectively. □
Example 1. Consider the equation

\[(x(t) + p_0 x(\delta t))^{(n)} + \frac{q_0}{p_0^n} x(\lambda t) = 0,\]  

(23)

where \(t \geq 1, q_0 > 0, \delta \in (p_0^{-1/(n-1)}, 1)\) and \(\lambda \in (0, \delta)\). We note that \(r(t) = 1, p(t) = p_0, \tau(t) = \delta, \sigma(t) = \lambda t\) and \(q(t) = q_0/t^n\). Thus, if we choose \(\eta(t) = \xi(t) = \lambda t\), then it’s easy to see that (7) and (8) are satisfied. Moreover, we have

\[P_k(t) = \frac{1}{p_0} \left(1 - \frac{\delta^{1-k}}{p_0}\right), \text{ for } k = 2, n,\]

\[R_0(t) = \frac{q_0}{p_0} \left(1 - \frac{1}{\delta p_0}\right) \frac{1}{(n-1)!},\]

and

\[R_{n-3}(t) = \frac{1}{(n-3)!} \frac{q_0}{p_0} \left(1 - \frac{1}{\delta p_0}\right) \frac{1}{(n-2) (n-1) t^2}.\]

Hence, Condition (21) and (22) become

\[q_0 \frac{1}{p_0} \left(\frac{\lambda}{\delta}\right)^n = \frac{1}{p_0^n} \left(1 - \frac{\delta^{1-n}}{p_0}\right) \ln \frac{\delta}{\lambda} > \frac{(n-1)!}{e},\]  

(24)

and

\[q_0 \frac{1}{p_0} \frac{\lambda}{\delta} \left(1 - \frac{1}{\delta p_0}\right) \ln \frac{\delta}{\lambda} > \frac{(n-1)!}{e},\]  

(25)

respectively. It’s easy to see that (24) implies (25). Therefore, by Corollary 1, we conclude that (23) is oscillatory if (24) holds.

Remark 1. For Equation (23), in particular case that \(n = 4, p_0 = 16, \delta = 1/2\) and \(\lambda = 1/3\), Condition (24) yields \(q_0 > 587.93\). Whereas, the criterion obtained from the results of [22] is \(q_0 > 4850.4\). Hence, our results improve the results in [22].

3. Conclusions

In this paper, our method is based on presenting a new comparison theorem that compare the higher-order Equation (1) with a couple of first-order equations. There are numerous results concerning the oscillation criteria of first order Equations (9) and (10) (see, e.g., [14,25–27]), which include various forms of criteria as Hille/Nehari, Philos, etc. This allows us to obtain also various criteria for the oscillation of (1). Further, we can try to obtain oscillation criteria of (1) if \(z(t) := x(t) - p(t) x(\tau(t))\) in the future work.

Author Contributions: The authors claim to have contributed equally and significantly in this paper. All authors read and approved the final manuscript.

Funding: The authors received no direct funding for this work.

Acknowledgments: The authors thank the reviewers for for their useful comments, which led to the improvement of the content of the paper.

Conflicts of Interest: There are no competing interests between the authors.

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