Continuum QRPA in the coordinate space representation

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We formulate a quasi-particle random phase approximation (QRPA) in the coordinate space representation. This model is a natural extension of the RPA model of Shlomo and Bertsch to open-shell nuclei in order to take into account pairing correlations together with the coupling to the continuum. We apply it to the $^{120}$Sn nucleus and show that low-lying excitation modes are significantly influenced by the pairing effects although the effects are marginal in the giant resonance region. The dependence of the pairing effect on the parity of low-lying collective mode is also discussed.

1. Introduction

The random phase approximation (RPA) has provided a convenient and useful method to describe the excited states of many-fermion systems. There are a number of ways to formulate the RPA.$^{1-9}$ In a practical point of view, we particularly mention here a configuration space formalism and a response function formalism. The configuration space formalism diagonalizes a non-hermitian matrix denoted often by $A$ and $B$ matrices which are constructed in the model space of one particle-one hole (1p-1h) states. In contrast, the response function formalism is based on the linear response theory and solves a Bethe-Salpeter equation for the response function formalism is based on the linear response space of one particle-one hole (1p-1h) states. In contrast, the configuration space.

The response function formalism of RPA becomes simple when the interaction is a zero-range contact force. In that case, the excitations to particle continuum states can be treated exactly by solving the single-particle Green function in the coordinate space. This method was developed for the first time by Shlomo and Bertsch$^{5}$ and subsequently applied to self-consistent calculations of nuclear giant resonances with the Skyrme interaction by Liu and Van Giai.$^{10}$ Recently, it has been extensively used by Hamamoto and her collaborators to discuss giant resonances of neutron-rich nuclei, where continuum effects play an essential role due to a much lower threshold energy than that of $eta$-stable nuclei.$^{11,12}$ An extension to the three-dimensional space has also been carried out recently by Nakatsukasa and Yabana for the study of atomic clusters.$^{13}$ The coupling between particle-particle continuum was also studied in $^{11}$Li by Bertsch and Ebseben.$^{14,15}$

Although it has been well known that the pairing correlation plays an important role in the ground state of open-shell nuclei, it has been neglected in applying the response function formalism to describe excited states of atomic nuclei until very recently.$^{16}$ So far, a simple filling approximation has been employed in open-shell nuclei in order to simulate the pairing correlations. However, it is important to take into account the pairing effects consistently for the study of excited states of open-shell nuclei, and thus the quasi-particle RPA (QRPA) should be used instead of the RPA.$^{17}$

In this contribution, we generalize the formalism of Shlomo and Bertsch to the QRPA and discuss the effects of pairing correlations on excited states of open-shell nuclei.$^{18}$ We shall study only cases where the BCS approximation works well and thus all states in the pairing active space are bound, leaving out as a future work the self-consistent treatment of the pairing effects in the continuum using the Hartree-Fock-Bogoliubov + RPA theory.$^{19-21}$ Our method is closely related to that in Ref. 16, although the details are somewhat different. This paper is organized as follows. In Sec. 2, we briefly review the formalism of Shlomo and Bertsch and extend it to the QRPA. In Sec. 3, we apply the formalism to the isoscalar (IS) monopole, quadrupole, and octupole modes as well as the isovector (IV) dipole mode of excitation of the $^{120}$Sn nucleus and discuss the effects of pairing correlations on the excited states. A summary is given in Sec. 4.

2. Linear response theory for open-shell nuclei

We begin with the configuration space formalism of the RPA theory and then make a connection to the response function method. The RPA equation can be given in the compact form

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X \\ Y \end{pmatrix} = \omega \begin{pmatrix} X \\ Y \end{pmatrix},$$

(1)

where $X$ and $Y$ are the forward and backward amplitudes, respectively. For a residual interaction of the delta-type contact force, $v_{res}(r_1, r_2) = v((r_1 + r_2)/2) \delta(r_1 - r_2)$, the matrix elements of $A$ and $B$ for the $L$-multipole mode read

$$A_{ph,p'h'} = (\epsilon_p - \epsilon_h) \delta_{ph,p'h'} = B_{ph,p'h'},$$

$$\frac{I(p(h))}{2L + 1} (|j_p||Y_L||j_h,h') (|j_{p'}||Y_L||j_{h'}),$$

(2)

where $p(h)$ denotes a particle (hole) state and $(|j||Y_L||j')$ is the reduced matrix element. The radial integral $I$ is given by
\[ I(\phi ph') = \int_0^\infty \frac{dr}{r} v(r) \phi_\beta(r) \phi_\alpha(r) \phi_\alpha'(r), \tag{3} \]

where \( \phi(r) \) is the single-particle radial wave function. We have absorbed the overall factor \((-1)^k\) in front of the \( B \) matrix by redefining the sign of the backward \( Y \) amplitudes.

The integral (3) can be computed by discretizing it as

\[ I(\phi ph') \approx \sum_k \frac{\Delta r}{r_k} v(r_k) \phi_\beta(r_k) \phi_\alpha(r_k) \phi_\alpha'(r_k), \tag{4} \]

where \( \Delta r \) is the spacing of the radial coordinate. Since the interaction is then given as a sum of separable form, the RPA frequencies \( \omega \) can be obtained by solving the generalized RPA dispersion relation \( det(1 - \Pi_0(\omega)\chi) = 0 \), where the matrices \( \Pi_0(\omega) \) and \( \chi \) are given by

\[ \Pi_0(i; j; \omega) = -\sum_{pn} D_{ph}(i) D_{ph}(j) \times \left( \frac{1}{\epsilon_p - \epsilon_h - \omega - i\eta} + \frac{1}{\epsilon_p - \epsilon_h + \omega + i\eta} \right) \tag{5} \]

\[ \chi(i, j) = \chi(i) \delta_{i,i} = \frac{\Delta r}{r_i^2} v(r_i), \tag{6} \]
respectively, in the coordinate space representation. Here, \( \eta \) is an infinitesimal real number, and \( D_{ph} \) is given by

\[ D_{ph}(i) = \phi_\beta(r_i) \phi_\alpha(r_i) \langle j_i | p | Y_{LM} | j_h l_h \rangle \frac{1}{\sqrt{2L+1}} \tag{7} \]

Note that \( \Pi_0(\omega) \) is nothing but the unperturbed response function in the linear response theory. Here we introduce the RPA response function which obeys the Bethe-Salpeter equation

\[ \Pi_{RPA} = \Pi_0 + \Pi_0 \chi \Pi_{RPA}. \tag{8} \]

This equation can be solved in the coordinate space by matrix inversion as \(^7\)

\[ \Pi_{RPA}(i; j; \omega) = \sum_k (1 - \Pi_0(\omega) \chi)^{-1} \Pi_0(k; j; \omega) \tag{9} \]

The response of the system to an external field \( V_{ex}(r) = V_{ex}(r) Y_{LM}(\hat{r}) \) is then given by \(^7\)

\[ S(\omega) = \sum_f |\langle f | V_{ex} | 0 \rangle|^2 \delta(E_f - E_0 - \omega) \tag{10} \]

\[ = \frac{1}{\pi} \int dm \int dr \int dr V_{ex}(r) \Pi_{RPA}(i; j; \omega) V_{ex}(r). \tag{11} \]

The exact treatment of the continuum effects can be achieved by eliminating the sum of the particle states in the free response function (5) using the complete set of the wave function.\(^5\) This leads to

\[ \Pi_0(i; j; \omega) = -\sum_h \frac{\phi_\beta(r_i) \phi_\alpha(r_j)}{2L+1} \sum_{j_p l_p} \langle j_p l_p | Y_{LM} | j_h l_h \rangle^2 \times \left( \frac{1}{h - \epsilon_h - \omega - i\eta} + \frac{1}{h - \epsilon_h + \omega - i\eta} \right), \tag{12} \]

where \( \hat{h} \) is the single-particle Hamiltonian, and the single-particle Green function is given by

\[ \left\langle r_i \left| \frac{1}{\hat{E}_h - \epsilon_h + \omega - i\eta} \right| r_j \rightangle = \frac{2m u_{r_i} u_{r_j}}{\hat{W}}, \tag{13} \]

Here, \( u \) and \( w \) are the regular and irregular solutions of the Hamiltonian \( \hat{h} \) at energy \( \epsilon_h + \omega \), and \( \hat{W} \) is the Wronskian given by \( W = uw' - uw' \).

In the application of the formalism to nuclear systems, we use the proton-neutron formalism which can take into account the couplings between the IS and IV modes of excitation.\(^11\) The Bethe-Salpeter equation given by Eq.(8) is then generalized to

\[ \left( \begin{array}{c} \Pi_{RPA}^{(p)} \\ \Pi_{RPA}^{(n)} \end{array} \right) = \left( \begin{array}{c} \Pi_0^{(p)} \\ \Pi_0^{(n)} \end{array} \right) + \left( \begin{array}{cc} \Pi_0^{(p)} \chi_{pp} & \Pi_0^{(n)} \chi_{pn} \\ \Pi_0^{(n)} \chi_{np} & \Pi_0^{(n)} \chi_{nn} \end{array} \right) \left( \begin{array}{c} \Pi_{RPA}^{(p)} \\ \Pi_{RPA}^{(n)} \end{array} \right), \tag{14} \]

For simplicity of notation, we do not present below the formalism in the two-component form, although we use it in the actual calculations given in the next section.

In the presence of pairing correlations, the elementary excitation is a two quasi-particle excitation, rather than a particle-hole excitation. A generalization of the RPA to the QRPA is given in Ref. 3 together with relevant \( A \) and \( B \) matrices in the QRPA equation. A free response function in the QRPA in the configuration space representation can be constructed in a manner similar to the RPA,

\[ \Pi_0(i; j; \omega) = -\sum_{\alpha \leq \beta} D_{\alpha \beta}(i) D_{\alpha \beta}(j) \times \left( \frac{1}{E_\alpha + E_\beta - \omega - i\eta} + \frac{1}{E_\alpha + E_\beta + \omega - i\eta} \right), \tag{15} \]

with

\[ D_{\alpha \beta}(i) = \phi_\alpha(r_i) \phi_\beta(r_i) \langle j_\alpha | l_\alpha | Y_{LM} | j_\beta l_\beta \rangle \times \frac{u_{\alpha} v_{\beta} + (-)^{\delta_{\alpha \beta}} v_{\alpha} u_{\beta}}{\sqrt{2L+1}} (1 + \delta_{\alpha \beta})^{-1/2}, \tag{16} \]

where \( v_\alpha^2 \) is the BCS occupation probability and \( u_\alpha^2 = 1 - v_\alpha^2 \). \( E_\alpha \) is the quasi-particle energy given by \( E_\alpha = \sqrt{(\epsilon_\alpha - \lambda)^2 + \Delta^2} \), where \( \lambda \) and \( \Delta \) are the chemical potential and the pairing gap, respectively. In the BCS approximation, \( \phi_\alpha \) is an eigenfunction of the single-particle Hamiltonian \( \hat{h} \) with an eigenenergy \( \epsilon_\alpha \). Since the quasi-particle energy \( E_\alpha \) is not an eigenvalue of \( \hat{h} \) in general, it is not straightforward to introduce the single-particle Green function (13) in the QRPA free response function. However, when the pairing gap is zero for states \( k \) outside the pairing active space, the quasi-particle energy becomes an eigenvalue of the single-particle Hamiltonian, i.e., \( E_k = \epsilon_k - \lambda, v_k = 0 \), and \( u_k = 1 \). We therefore consider separately excitations among states within the pairing active space and those from the inside to the outside of the active space. To the latter model space, we apply the same procedure as the RPA response function. The free response function in the BCS approximation (15) thus becomes
functions

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3. Continuum QRPA excitations in $^{120}$Sn

We now apply the continuum QRPA formalism to $^{120}$Sn and make a comparison between the QRPA and the RPA. We choose this system because it is a typical open-shell nucleus and also because it is a sub-shell closure nucleus where the RPA can be applied unambiguously without using the filling approximation for valence nucleons. The single-particle wave functions $\phi$ and the single-particle energies $\epsilon$ are obtained by solving the Schrödinger equation with a Woods-Saxon mean-field potential. As a residual interaction $v_{\text{res}}$, we use the $t_0$ and $t_3$ parts of the Skyrme residual interaction, which is obtained from the second derivative of the Skyrme energy functional with respect to proton and neutron densities. The ground-state density to be used in the density-dependent $t_3$ part of the residual interaction is generated from the single-particle wave functions $\phi$. We use the same parameters as those in Ref. 5 for the mean-field potentials and the residual interaction, i.e., $t_0 = -1100\text{MeV fm}^3$, $t_3 = 16000\text{MeV fm}^6$, $x_0 = 0.5$, $x_3 = 1$, and $\alpha = 1$ in the standard notation of the Skyrme functional. Since this model is not self-consistent, we renormalize $v_{\text{res}}$ for QRPA and RPA calculations, respectively, so that the spurious IS dipole mode appears at zero excitation energy. The resultant renormalization factors are $\kappa = 0.638$ and 0.660 for RPA and QRPA, respectively. For the pairing of neutrons, we use a schematic state-independent constant pairing gap $\Delta_n = \Delta = 1.392\text{MeV}$, which is estimated from the experimental binding energies of neighboring nuclei. The pairing active space which we adopt includes the levels up to the $N = 50$ major shell as well as the $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ levels. The single-particle energies for the valence levels are shown in Table 1, together with the BCS occupation probabilities. The proton pairing gap is set to be zero due to the magic number $Z = 50$.

For the quadrupole mode shown in Fig. 1, the lowest RPA state is at $5.2\text{MeV}$, in comparison with the experimental value of $1.17\text{MeV}$. The transition strength is $B(E2:0^+ \rightarrow 2^+) = 0.031 e^2 b^2$ in the RPA calculation, while the experimental value is $0.2 e^2 b^2$. If one takes the pairing correlations into account, the low-lying $2^+$ RPA state goes substantially down in energy and the lowest QRPA state appears at $2.3\text{MeV}$ having a transition strength $B(E2:0^+ \rightarrow 2^+) = 0.107 e^2 b^2$, which is much closer to the experimental value. In both cases, the main peak of the IS giant quadrupole resonance (GQR) appears at approximately $13\text{MeV}$, exhaust-

\[
\Pi_0(i,j;\omega) = -\sum_{\alpha \in \beta} D_{\alpha\beta}(i) D_{\alpha\beta}(j) \times \left\{ \frac{1}{E_{\alpha} + E_{\beta} - \omega - i\eta} \right. \\
+ \left. \frac{1}{E_{\alpha} + E_{\beta} - \omega - i\eta} \right\} + \left\{ \frac{1}{E_{\alpha} + E_{\beta} - \omega - i\eta} \right. \\
+ \left. \frac{1}{E_{\alpha} + E_{\beta} - \omega - i\eta} \right\}, \quad (17)
\]

where the summations of $\alpha$ and $\beta$ are restricted to the states within the pairing active space. The last term in Eq. (17) is a correction for the double-counting of excitations within the pairing active space, which stems from the substitution of the completeness relation $\sum_{\alpha} |\phi_{\alpha}(i)|^2 = 1$ in the standard notation of the Schrödinger equation with a Woods-Saxon mean-field potential. As a residual interaction $v_{\text{res}}$, we use the $t_0$ and $t_3$ parts of the Skyrme residual interaction, which is obtained from the second derivative of the Skyrme energy functional with respect to proton and neutron densities. The ground-state density to be used in the density-dependent $t_3$ part of the residual interaction is generated from the single-particle wave functions $\phi$. We use the same parameters as those in Ref. 5 for the mean-field potentials and the residual interaction, i.e., $t_0 = -1100\text{MeV fm}^3$, $t_3 = 16000\text{MeV fm}^6$, $x_0 = 0.5$, $x_3 = 1$, and $\alpha = 1$ in the standard notation of the Skyrme functional. Since this model is not self-consistent, we renormalize $v_{\text{res}}$ for QRPA and RPA calculations, respectively, so that the spurious IS dipole mode appears at zero excitation energy. The resultant renormalization factors are $\kappa = 0.638$ and 0.660 for RPA and QRPA, respectively. For the pairing of neutrons, we use a schematic state-independent constant pairing gap $\Delta_n = \Delta = 1.392\text{MeV}$, which is estimated from the experimental binding energies of neighboring nuclei. The pairing active space which we adopt includes the levels up to the $N = 50$ major shell as well as the $1g_{7/2}$, $2d_{5/2}$, $2d_{3/2}$, $3s_{1/2}$, and $1h_{11/2}$ levels. The single-particle energies for the valence levels are shown in Table 1, together with the BCS occupation probabilities. The proton pairing gap is set to be zero due to the magic number $Z = 50$.

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| level | energy (MeV) | occupation probability |
|-------|-------------|------------------------|
| $1g_{7/2}$ | $-11.184$ | $0.954$ |
| $2d_{5/2}$ | $-11.145$ | $0.953$ |
| $3s_{1/2}$ | $-0.948$ | $0.771$ |
| $1h_{11/2}$ | $-6.949$ | $0.173$ |

Table 1. Neutron single-particle levels near the Fermi surface for $^{120}$Sn obtained with a Woods-Saxon potential. The occupation probabilities $v^2$ are calculated in the BCS approximation with a constant pairing gap $\Delta_n = 1.392\text{MeV}$. The pairing active space includes those levels shown in this table as well as the levels up to the $N = 50$ major shell. The neutron Fermi energy $\lambda_n$ is $-8.149\text{MeV}$. |
Fig. 2. Same as Fig. 1, but for the IS octupole mode.

ing most of the sum rule strength. The experimental GQR is also observed at the excitation energy of approximately 13 MeV. As for the octupole mode of excitations in Fig. 2, the experimental value of the lowest 3\(^{-}\) state is at 2.4 MeV, while RPA and QRPA lead to 1.4 MeV and 3.0 MeV, respectively. The RPA without pairing correlations underestimates the lowest 3\(^{-}\) state energy and the QRPA is more satisfactory compared with the experimental value. Giant octupole resonances (GORs) are observed in Fig. 2 at an excitation energy of 23 MeV in both cases. We summarize in Table 2 the results of RPA and QRPA calculations together with the experimental data for the lowest 2\(^{+}\) and 3\(^{-}\) states. In contrast to the low-lying modes of excitations, in general, high-lying modes are much less sensitive to the pairing correlations.

Table 2. Comparison of the RPA and the QRPA calculations with the experimental data for the lowest-lying 2\(^{+}\) and 3\(^{-}\) modes of excitation of \(^{120}\)Sn.

|          | \(E_{2^{+}}\) (MeV) | \(B(E2)\) (e\(^{2}\)b\(^{2}\)) | \(E_{3^{-}}\) (MeV) | \(B(E3)\) (e\(^{2}\)b\(^{3}\)) |
|----------|------------------|-----------------|------------------|-----------------|
| RPA      | 5.2              | 0.031           | 1.4              | 0.159           |
| QRPA     | 2.3              | 0.105           | 3.0              | 0.0771          |
| Expt     | 1.17             | 0.2             | 2.4              | 0.09            |

Figures 3 and 4 show the strength function \(S(\omega)\) for IS energy and a larger transition strength. In contrast, such excitations are not allowed for odd parity excitations, e.g., the octupole mode. In that case, the dominant excitation is from one major shell to another regardless of the pairing correlation. Since the particle-hole excitations are weakened by the factor \(v^2\) in the presence of the pairing, the QRPA lowers the collectivity of low-lying collective excitations with odd parity. Another origin of the lower collectivity is the fact that a two quasi-particle energy is in general higher than the corresponding particle-hole energy. This can be seen by expanding a quasi-particle energy in terms of pairing gap and keeping only the leading term:

\[
E_p + E_h = \sqrt{(\epsilon_p - \lambda)^2 + \Delta^2} + \sqrt{(\epsilon_h - \lambda)^2 + \Delta^2}
\]

(18)

\[
\sim \epsilon_p - \epsilon_h + \frac{\Delta^2}{2 (\epsilon_p - \epsilon_h)(\lambda - \epsilon_h)}.
\]

(19)

In order to show these features more clearly, we show in Figs. 3 and 4 unperturbed strength functions for quadrupole and octupole modes, respectively.

Fig. 3. Same as Fig. 1, but for unperturbed response.

Fig. 4. Same as Fig. 3, but for the IS octupole mode.
monopole and IV dipole modes, respectively. For the monopole excitation shown in Fig. 5, the RPA does not show any low-lying mode, although the experimental second lowest $0^+$ state in the $^{120}$Sn nucleus is observed at an excitation energy of 1.874 MeV, which is attributed to a pairing vibration mode. The second lowest $0^+$ state in the QRPA is found at a rather low energy of 2.9 MeV, and may couple to the pairing vibrational mode. The IS giant monopole states are seen in the energy region of 17 to 28 MeV in both QRPA and RPA calculations. The transition densities for the IS monopole mode at two different energies, i.e., $\omega = 2.9$ MeV and 21.6 MeV, are shown in Fig. 7. We find that the former shows a characteristic behavior of the pairing vibration mode while the latter shows a compressional character. Results for the IV giant dipole resonance (GDR) are shown for RPA and QRPA in Fig. 6. As in Figs. 1, 2, and 5, the structure of GDR is not much disturbed by the effect of the pairing.

One conventional approximation to treat the continuum effect is to put a nucleus in a box and discretize the continuum states by imposing a boundary condition at the edge of the box. A model space of these states is usually truncated at the maximum single-particle energy $\epsilon_{\text{max}}$. Since our continuum QRPA method can treat the coupling to the continuum exactly, it is interesting to compare our method with the box discretization approximation. Figure 8 shows a convergence property of the box discretization method for the IS quadrupole response. We take the box size of $R_{\text{max}} = 10$ fm, and smear the strength function with $\eta = 0.5$ MeV for the purpose of representation. We have confirmed that the results do not change significantly even if we use a larger value of $R_{\text{max}}$. The blue line is the result of the continuum QRPA method with the exact treatment of the continuum effect, which is the same as in Fig. 6. The green, red, and light blue lines are results of the box discretization method with trun-
cation energy at $\epsilon_{\text{max}} = 10, 50,$ and $100\text{MeV}$, respectively. In the upper panel, we use the same residual interaction for all the calculations. As one can see, the convergence with respect to the maximum energy of continuum states is extremely slow: even with $\epsilon_{\text{max}} = 100\text{MeV},$ the peak positions are not reproduced. The peak height for the lowest energy peak is not reproduced either. We note that the truncation of the model space shifts the position of the spurious IS dipole state. It appears at 6.0, 4.4, and 3.7MeV for $\epsilon_{\text{max}} = 10, 50,$ and $100\text{MeV},$ respectively. In the lower panel, the renormalized residual interaction is used for each value of $\epsilon_{\text{max}}$ so that the IS dipole mode appears at zero energy. The renormalization factors $\kappa = 0.968, 0.776,$ and $0.732$ for $\epsilon_{\text{max}} = 10, 50,$ and $100\text{MeV},$ respectively. This procedure drastically improves the convergence. In the case of the truncation energy $\epsilon_{\text{max}} = 50\text{MeV}$ (red line), the exact solution is almost reproduced. The result converges at this energy and the shape of the strength function is not altered even when states up to $\epsilon_{\text{max}} = 100\text{MeV}$ are included. This study clearly indicates the importance of self-consistency between the space truncation and the renormalization of the residual interaction. If this self-consistency is imposed, the box discretization method will work well. It should be emphasized, however, that the escape width of resonances can be obtained only when the single-particle continuum states are treated exactly as we do in this article.

4. Summary and discussions

We proposed a QRPA model in the coordinate space to take into account the continuum effects. It is a generalization of the formalism of Shlomo and Bertsch for the continuum RPA to the QRPA. We treated separately the p-h excitations within the pairing active space and those between the active space and the non-active space. For the former, we explicitly used the two quasiparticle configurations in the response function in the coordinate space, while for the latter, we used the single-particle Green function in the coordinate space representation taking into account the coupling to the particle continuum properly. We applied the formalism to $^{128}\text{Sn}$ and showed that the pairing correlations enhance the collectivity of the positive parity low-lying states, while those of the negative parity low-lying states are hindered. We found that the QRPA is more satisfactory in terms of reproducing the experimental data of the energy of low-lying modes of excitation, while the giant resonances are not much affected by the pairing correlations. However, we would like to point out that these results do not justify the RPA model without the pairing correlations for the study of giant resonances in open-shell nuclei, especially near the drip lines. Since the single-particle energies and wave functions could be different in principle due to the density dependence of the mean field, the response properties even in the giant resonance region might be affected by the pairing correlations. We therefore advocate use of the QRPA in the entire excitation region in open-shell nuclei.

There are many possible applications of the continuum QRPA method presented in this paper. One of them is neutrino-nucleus scattering, e.g., $^{12}\text{C}(\nu,\nu ')^{12}\text{C}$ and $^{12}\text{C}(\nu,\bar{\nu})^{12}\text{N,}^{15}$ where the RPA and the QRPA have been one of the standard methods for theoretical investigations. Another application will be the excitations of drip-line nuclei. Because of a low energy threshold, the responses would be very sensitive to the pairing correlations. In these cases, the pairing active space includes particle continuum states and the BCS approximation should be carefully examined. Also, the residual interaction in the particle-particle channel, which we neglected in this paper, might have to be taken into account. A more sophisticated treatment of the pairing interaction is provided by the Hartree-Fock-Bogoliubov (HFB) theory, which consistently describes couplings between particle-hole and particle-particle channels. It would be an interesting future work to develop a continuum QRPA theory based on the HFB approximation.

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