Correlation functions in the Coulomb branch of
\[ N = 4 \text{ SYM} \] from AdS/CFT correspondence

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Abstract

We study \( SU(N) \to S(U(N/2) \times U(N/2)) \) symmetry breaking in \( N = 4 \) SYM via AdS/CFT correspondence. Two stacks of \( \frac{N}{2} \) parallel D3 branes are separated by a distance \( 2d \) in the transverse directions. This leads to an interaction between different \( l \)-wave dilatonic KK modes when expanded in \( S^5 \) harmonics. We calculate certain two point correlation functions of the dual gauge theory in the decoupling limit \( d\alpha' \ll 1 \). Due to mode mixing, the diagonal correlation functions have \( \frac{1}{N} \) conformal-like correction as well as deformation terms. The off-diagonal correlators are also nonvanishing and their leading order is \( \frac{1}{N} \). We discuss briefly the spectrum of the glueball excitations.

1 Introduction

A recent development in the understanding of the string theory was a conjecture by Maldacena according to which, in some appropriate low energy limit, there is a duality between \( \mathcal{N} = 4 \) supersymmetric \( SU(N) \) gauge theory and type II strings on \( AdS_{d+1} \times \mathcal{M} \) background. This is nothing but a manifestation of the old ideas involving the deep connection between gauge theories and strings. Therefore, from this point of view it is clear that many problems in large \( N \) gauge theories can be studied via supergravity in AdS spaces. The precise meaning and the explicit rules
for relating AdS supergravity and the type IIB string theory were given in \cite{2} and \cite{3}. Namely, the explicit form of the correspondence between the correlation functions of the operators in conformal field theory (CFT) living on the asymptotic conformal boundary of the $d + 1$ dimensional anti-de Sitter space and type II supergravity action evaluated on the classical solutions $\Phi^i$ was proposed to be:

$$\langle exp \int_{S^d} \Phi^i_0 \mathcal{O}_i \rangle_{CFT} = exp \left( -S[\Phi^i] \right)$$

where $\mathcal{O}_i$'s are conformal operators in CFT and $\Phi^i_0 = \Phi^i|_{\partial AdS}$ is the boundary value of the classical solutions which serve as a source for the primary operators in CFT. The proposed correspondence has been examined in a number of calculations of the correlation functions of operators in CFT using the classical solutions of various fields from the spectrum of AdS supergravity. These, even incomplete proofs of the validity of the holographic principle, supply us with a powerful tool for studying strongly coupled gauge theories. Most of the intensive investigations of AdS/CFT correspondence however are for the case of supergravity in pure AdS background, i.e. at the origin of the moduli space where there is an enhanced superconformal symmetry and many problems can be managed exactly even at strong coupling.

Departure from the origin of the moduli space means allowing scalar fields to have non-zero expectation values and therefore to move to the Coulomb branch of the theory. The vacua in the Coulomb branch are maximally supersymmetric but some of the gauge symmetry is broken by the expectation value of scalar fields. These cases correspond to certain solutions of the supergravity field equations in which the bulk space geometry approaches AdS near the boundary, but differs from it in the interior. From holographic correspondence point of view such geometries are related either to a relevant deformation of the conformal field theories, or to existence of states of the CFT with vacuum expectation value which sets the energy scale. In a recent paper by Klebanov and Witten \cite{4} a more general investigation of the symmetry breaking is given and applied to several examples. There are recent papers studying the Coulomb branch of the gauge theories in four dimensions \cite{14, 10, 11, 8, 16} which are based on continuous spherically symmetric distributions of large number of $D3$ branes. They, in fact, extend the results of \cite{12, 13} by investigating different probes in the bulk of the AdS space and their ”holographic images” on the boundary. Since the effective background is asymptotically $AdS_5 \times S^5$ one can expand the fields in spherical harmonics and solve the supergravity equations of motion for the Kaluza-Klein (KK) modes. In this case, due to spherical symmetry of the distribution of the $D3$ branes the different KK modes decouple, thus enabling one to find exact
solutions of the field equations using AdS/CFT correspondence to obtain the two point correlation functions.

In the present paper we will consider the simplest example of two stacks of equal number coincident $D3$ branes separated by a distance $2\vec{d}$, which corresponds to a vacuum state of the gauge theory where the gauge $SU(N)$ symmetry is broken down to $S(U(N/2) \times U(N/2))$ by the expectation value of scalar fields.

In Section 2 we consider the supergravity solutions in the presence of two center $D3$ brane geometry and the reduction of the 10 dimensional dilaton action to five dimensional one by integrating over $S^5$. It turns out that the equations of motion for the dilatonic KK modes are coupled. This coupling modifies the two-point correlation functions, as discussed, in Section 3. The diagonal correlation functions of the operators $O^n$ in the gauge theory are modified by a $1/N$ correction term to the conformal-like part as a consequence of the coupling between the different modes. In contrast to the conformal case and the case of uniformly distributed $D3$-branes, the correlation functions $\langle O^n(\vec{x})O^{n+2}(\vec{y}) \rangle$ are nonvanishing and are of order $1/N$ in our approximation. In addition, all correlators have a deformation term which, after a Wick rotation, exhibits a pole structure. We interpret the appearance of double poles as a degenerate spectrum of glueballs. All these questions are subject of consideration in Section 4. We summarize, interpret and comment on our results in the Concluding section.

2 Two stacks of $D3$ branes: equations and solutions

Let us consider the case of two center $D3$ branes. In this case, a large number $N$ of $D3$ branes are separated into two parallel stacks of $N/2$ branes each. Their separation distance introduces an energy scale in the theory. The $SU(N)$ gauge symmetry is broken down to $S(U(N/2) \times U(N/2))$. From the gauge theory side this corresponds to Higgsing of the gauge theory by scalar fields:

$$\vec{X} = \begin{pmatrix} \vec{d}_1.I & 0 \\ 0 & \vec{d}_2.I \end{pmatrix}$$  \hspace{1cm} (1)

where $I$ is the $(N/2) \times (N/2)$ identity matrix and $\vec{d}_i$ is the position of the i-th stack in the transverse direction. Let us parametrize the moduli space in terms of gauge invariant fields as in [15, 8]:

$$O^I = C_{i_1 \ldots i_k}^I Tr \left( X^{i_1} \ldots X^{i_k} \right)$$  \hspace{1cm} (2)
where \( C_{i_1...i_k} \) is a totally symmetric traceless rank \( k \), \( SO(6) \) tensor and \( X_i \)'s are as in (4). The Higgsing gives expectation values for the chiral operators \( O^I \) which can be expressed in terms of order parameters \( d_i \) [14, 15]:

\[
\langle O^I \rangle \sim \prod_{k=1}^{n} (d_1)_{i_k} + \prod_{l=1}^{n} (d_2)_{i_l} - \text{traces}
\]

To describe correlation functions on the Coulomb branch let us start with the ten dimensional Type II supergravity in the two-centered \( D_3 \) brane background:

\[
ds^2 = H^{-\frac{1}{2}} (-dt + dx_1 + dx_2 + dx_3) + H^{\frac{1}{2}} \sum_{j=1}^{6} dy_j^2
\]

where \( H \) is the Green’s function with two separated sources of equal charge:

\[
H = \frac{L^4}{2} \left( \frac{1}{|\vec{y} - \vec{d}_1|^4} + \frac{1}{|\vec{y} - \vec{d}_2|^4} \right)
\]

In (4), \( \vec{y} \) are the coordinates normal to the branes [14].

Let us discuss briefly the geometry of this configuration and the decoupling limit. Each of the two stacks develops a throat (geometry of which is AdS) but they are located at different points of \( S^5 \), separated by a distance \( 2\vec{d} \). In each throat the dilaton field can be expanded in \( S^5 \) scalar spherical harmonics which will have non vanishing overlap with the corresponding expansion around the other throat. For small \( d \) (compared to the radius of AdS) and large \( r \) the two throats meet and in the limit \( r \to \infty \) share a flat four dimensional boundary. In this case it is natural to make an appropriate shift in the, \( \vec{y} \)-direction, choosing the origin so that \( \vec{d}_1 = -\vec{d}_2 = \vec{d} \) and therefore:

\[
H = \frac{L^4}{2} \left( \frac{1}{|\vec{y} + \vec{d}|^4} + \frac{1}{|\vec{y} - \vec{d}|^4} \right)
\]

which allows us to consider the problem as effectively single centered. Due to interaction between the different KK modes in the bulk, it is expected that their images on the boundary result in a mixing between the correlation functions of operators

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4We call these operators chiral in the sense of [15].

5It is clear that at large \( r \) the space is asymptotically \( AdS_5 \times S^5 \). Since we consider the large \( N \) case we have neglected the one in \( H \). \( L^4 = 4\pi g_s \alpha'^2 N \).
of different conformal dimensions. We would like to point out that starting from Type IIB string theory and taking the decoupling limit $\alpha' \to 0$ while keeping $d/\alpha'$ fixed, we obtain an $\mathcal{N} = 4$, 4-dimensional SYM theory with gauge group $SU(N)$ spontaneously broken down to $S(U(N/2) \times U(N/2))$ by Higgs scalar expectation value in the adjoint representation of $SU(N)$. The Higgs scalars parametrize the positions of the stacks. We use the supergravity approximation which is valid for $d \ll L^2$. In general one must distinguish between three different regions: the first one is near the location of the branes. Here the throats are well separated. The second is where the throats come together, but this region is still far from the boundary. The last region is near the boundary where the two throats effectively coincide. Since our coordinate system is symmetric with respect to the positions of the stacks, the expansion over spherical harmonics qualitatively changes on the boundary $r = |\vec{d}|$. There we will apply the matching condition technique in order to ensure well defined smooth solutions in the bulk. Therefore, using this choice of the origin of the coordinate system one can expand the harmonic function $H$ in Taylor series in the form:

$$H = \left( \frac{L}{r_>} \right)^4 \left[ 1 + \sum_{l=1}^{\infty} 2^{2l} (2l + 1) \left( \frac{r_<}{r_>} \right)^{2l} Y^{2l}(\Omega_5) \right]$$

(6)

where $Y^{2l} = \sum C_{i_1 \ldots i_{2l}} y^{i_1} \ldots y^{i_{2l}}$ and $r_>, r_<$ are the larger or smaller of $r = |\vec{y}|$ and $\vec{d}$ respectively.

Let us now consider the kinetic part of the 10D dilaton action:

$$S_{10}^\Phi \sim \int d^{10} X \sqrt{G} G^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi + \ldots$$

(7)

After integration by parts the above action takes the form:

$$S_{10}^\Phi = S_{10}^{kin} + S_B$$

(8)

where:

$$S_{10}^{kin} \sim \int_{AdS} d^{10} X \left( H\Phi \nabla^2_\parallel \Phi + \Phi \nabla^2_\perp \Phi \right)$$

(9)

and

$$S_B \sim \int_{\partial AdS} d\Sigma^\mu \Phi \nabla_\mu \Phi$$

(10)
One can substitute for the dilaton field its expansion in $S^5$ scalar spherical harmonics $Y^I(\Omega_5)$:

$$\Phi(\vec{x}, \vec{y}) = \sum_{I=0}^{\infty} \Phi^I(\vec{x}, r) Y^I(\Omega_5)$$  \hspace{1cm} (11)

where $r = |\vec{y}|$, and to obtain the five dimensional action in the background metric we integrate over $S^5$. As it was noted in a number of papers (see for instance [17]), we have an infinite tower of KK modes $\Phi^I$ and there are no fundamental physical reasons to truncate. The resulting boundary action takes the form:

$$S_B = \int_{r \to \infty} d^4 x r^5 \left[ A_\geq \Phi^0 \frac{\partial}{\partial r} \Phi^0 + \sum_{I,J=1}^{\infty} A_{\geq IJ} \Phi^I \frac{\partial}{\partial r} \Phi^J + \sum_{I=0}^{\infty} A_{\geq I} \frac{\partial}{\partial r} (\Phi^0 \Phi^I) \right]$$  \hspace{1cm} (12)

where:

$$A_\geq = \int d\Omega_5 H_\geq = V_5$$  \hspace{1cm} (13)

$$A^I_\geq = \int d\Omega_5 H_\geq Y^I = V_5 \left( \frac{L}{r} \right)^4 \frac{1}{l+1} \left( \frac{d}{r} \right)^{2l} \delta^I,2l$$  \hspace{1cm} (14)

$$A_{\geq IJ} = \int d\Omega_5 H_\geq Y^I Y^J = V_5 \left( \frac{L}{r} \right)^4 2^{IJ+1} \delta^I^J \frac{2^{I+1} (I+1)(I+2)}{r^4}$$

$$+ V_5 \frac{L^4}{r^4} \sum_{l=1}^{\infty} 2^{2l} (2l+1) \left( \frac{d}{r} \right)^{2l} a(2l, I, J) \langle C_{2l} C^I C^J \rangle$$  \hspace{1cm} (15)

As in [15] we use the following normalization:

$$a(L, I, J) = \frac{1}{(\Sigma + 2)!2^{\Sigma-1} \cdot \alpha_1! \alpha_2! \alpha_3!}$$

$$\Sigma = \frac{1}{2} (k_L + k_I + k_J); \quad \alpha_1 = \Sigma - k_L; \quad \alpha_2 = \Sigma - k_I; \quad \alpha_3 = \Sigma - k_J$$  \hspace{1cm} (16)

We will see that the last term in (12) produces only local terms on the boundary and therefore is irrelevant for our considerations.

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6We will use the notations and normalization of the spherical harmonics as in [15].
The kinetic term can be easily reduced to five dimensional effective action in the form (8) (from now on we will denote $\Phi_0$ by $\Phi$):

$$S_{kin} = \int d^4x \, d^5r \, \left\{ \Phi \left[ \left( \int d\Omega_5 H \right) \nabla_\parallel^2 + V_5 \frac{1}{r^5} \partial_r \Phi \right] \Phi + \left( \int d\Omega_5 H Y^I \right) \Phi^I \nabla_\parallel^2 \Phi^J + \left( \int d\Omega_5 Y^I Y^J \right) \Phi^I \partial_r \Phi^J + \frac{1}{r^2} \left( \int d\Omega_5 Y^I \nabla_\parallel^2 Y^J \right) \Phi^I \Phi^J \right\} \tag{17}$$

It is straightforward to derive the equations of motion following from (17) in the form:

$$V_5 \Delta_r \Phi + \left( \int d\Omega_5 H \right) \nabla_\parallel^2 \Phi + \left( \int d\Omega_5 Y^I \right) \nabla_\parallel^2 \Phi^I = 0 \tag{18}$$

$$\left( \int d\Omega_5 Y^I Y^J \right) \Delta_r \Phi^I + \frac{1}{r^2} \left( \int d\Omega_5 Y^I \nabla_\parallel^2 Y^J \right) \Phi^I + \left( \int d\Omega_5 H Y^I Y^J \right) \nabla_\parallel^2 \Phi^I + \left( \int d\Omega_5 H Y^I \right) \nabla_\parallel^2 \Phi = 0, \tag{19}$$

where $\Delta_r = (1/r^5) \partial_r (r^5 \partial_r)$.

Since the metric has different forms in the regions of $r < d$ and $r > d$ we will have two systems of equations - one for each region. Taking into account the expression for $H$ (9) and the orthogonality of the spherical harmonics (with normalization as in (12)) the equations of motion are given by:

a) $r > d$ region

$$\triangle_r \Phi(r, \vec{k}) - \frac{L^4}{r^4} k^2 \Phi(r, \vec{k}) = \frac{L^4}{r^4} k^2 \sum_{l=1}^{\infty} \frac{1}{l+1} \left( \frac{d}{r} \right)^{2l} \Phi^{2l}(r, \vec{k}). \tag{20}$$

$$\triangle_r \Phi^n(r, \vec{k}) - \frac{n(n+4)}{r^2} \Phi^n(r, \vec{k}) - \frac{L^4}{r^4} k^2 \Phi^n(r, \vec{k}) =$$

$$\frac{L^4}{r^4} k^2 2^n (n+1)(n+2) \sum_{l=1}^{\infty} 2^{2l} (2l+1) \left( \frac{d}{r} \right)^{2l} \sum_{m=|2l-n|}^{2l+n} A^{2l,n,m} \Phi^m(r, \vec{k})$$

$$+ \frac{L^4}{r^4} k^2 2^{2l} (2l+1) \left( \frac{d}{r} \right)^{2l} \delta^{2l,n} \Phi(r, \vec{k}). \tag{21}$$
b) $r < d$ region

\[
\Delta_r \Phi(r, \vec{k}) - \frac{L^4}{d^4} k^2 \Phi(r, \vec{k}) = \frac{L^4}{d^4} k^2 \sum_{l=1}^{\infty} \frac{1}{l+1} \left( \frac{r}{d} \right)^{2l} \Phi^{2l}(r, \vec{k})
\]  

\[
\Delta_r \Phi^n(r, \vec{k}) - \frac{n(n+4)}{r^2} \Phi^n(r, \vec{k}) - \frac{L^4}{d^4} k^2 \Phi^n(r, \vec{k}) = \frac{L^4}{d^4} k^2 2^{n-1} (n+1)(n+2) \sum_{l=1}^{\infty} 2^{2l} (2l+1) \left( \frac{r}{d} \right)^{2l} \sum_{m=|2l-n|}^{2l+n} A^{2l,n,m} \Phi^m(r, \vec{k}) + \frac{L^4}{d^4} k^2 2^{2l} (2l+1) \left( \frac{r}{d} \right)^{2l} \delta^{2l,n} \Phi(r, \vec{k})
\]  

In the above expressions we used the following notation:

\[
A^{L,I,J} = a(L, I, J) \langle C^L C^I C^J \rangle
\]

and $a(L, I, J)$ and $\langle C^L C^I C^J \rangle$ are as in eq. (16). We see that the equation for the different KK modes of the dilaton field $\Phi^n$ are coupled. In general, according to the AdS/CFT correspondence, the operators $\mathcal{O}^I$ are mapped to conformal fluctuations in the metrics of $AdS_5$ and $S^5$. In our case this implies that the dilaton action projected on the boundary will produce correlation functions of the operators $\mathcal{O}^I \sim tr (F^2 X^I)$. In contrast to the uniformly distributed $D3$ branes case, there is coupling between the different KK modes, which signals a nontrivial contribution to the corresponding correlations functions. The mixing between the KK modes means that, due to the fact that we have a broken conformal symmetry, the sources coming from gravity solutions will produce in general mixed correlation functions. We will see how it happens in the next section.

The field equations in the two regions can be transformed into Bessel equations by the following substitutions:

a) for $r > d$ let us define:

\[
\Phi^n(r, \vec{k}) = \xi^2 f_\alpha(\xi, \vec{k}); \quad \xi = \frac{d}{r}; \quad \hat{\xi} = \kappa \xi
\]

\[
\hat{\xi} = \kappa \xi; \quad \kappa^2 = \frac{L^4}{d^2 k^2}; \quad (\xi \leq 1)
\]
then the equations (20, 21) become:

\[ f''(\hat{\xi}) + \frac{1}{\hat{\xi}} f'(\hat{\xi}) - \left(1 + \frac{4}{\hat{\xi}^2}\right) f(\hat{\xi}) = \sum_{l=1}^{\infty} \frac{1}{(l + 1)\kappa^{2l}} \hat{\xi}^{2l} f_{2l}(\hat{\xi}) \]  

(25)

\[ f''_n(\hat{\xi}) + \frac{1}{\hat{\xi}} f'_n(\hat{\xi}) - \left(1 + \left(\frac{n + 2}{\hat{\xi}^2}\right)\right) f_n(\hat{\xi}) \]

\[ = 2^{n-1}(n + 1)(n + 2) \sum_{l=1}^{\infty} \frac{2^{2l}(2l + 1)}{\kappa^{2l}} \hat{\xi}^{2l} \sum_{m=|2l-n|}^{2l+n} A^{2l,n,m} f_m(\hat{\xi}) \]

\[ + \frac{2^{2l}(2l + 1)}{\kappa^{2l}} \hat{\xi}^{2l} \delta^{2l,n} f(\hat{\xi}). \]

(26)

b) For \( r < d \), defining:

\[ \Phi^n(r, \vec{k}) = \eta^{-2} f_n(\eta, \vec{k}); \]

\[ \eta = \frac{r}{d}; \quad \hat{\eta} = \kappa \eta \]

\[ \hat{\eta} = \kappa \eta; \]

\[ \kappa^2 = \frac{L^4}{d^2 k^2}; \quad (\eta \leq 1) \]

the equations (22, 23) become:

\[ f''(\hat{\eta}) + \frac{1}{\hat{\eta}} f'(\hat{\eta}) - \left(1 + \frac{4}{\hat{\eta}^2}\right) f(\hat{\eta}) = \sum_{l=1}^{\infty} \frac{1}{(l + 1)\kappa^{2l}} \hat{\eta}^{2l} f_{2l}(\hat{\eta}) \]  

(27)

\[ f''_n(\hat{\eta}) + \frac{1}{\hat{\eta}} f'_n(\hat{\eta}) - \left(1 + \left(\frac{n + 2}{\hat{\eta}^2}\right)\right) f_n(\hat{\eta}) \]

\[ = 2^{n-1}(n + 1)(n + 2) \sum_{l=1}^{\infty} \frac{2^{2l}(2l + 1)}{\kappa^{2l}} \hat{\eta}^{2l} \sum_{m=|2l-n|}^{2l+n} A^{2l,n,m} f_m(\hat{\eta}) \]

\[ + \frac{2^{2l}(2l + 1)}{\kappa^{2l}} \hat{\eta}^{2l} \delta^{2l,n} f(\hat{\eta}). \]

(28)

where primes on \( f \) denote differentiation with respect to its argument. The terms on the right hand side of the equations above involve coupling to KK modes. For small \( d \) and large \( N \), we can solve these equations perturbatively. One can see that the right hand side of both systems of equations is readily expanded in \( d^2/L^4 = \pi^2 m_W^2 \frac{\alpha^2}{\kappa^4} \).
which in the decoupling limit is small. Here, \( m_W = \frac{d}{\pi a} \) is the \( W \) boson mass, generated by symmetry breaking. This is precisely the region of validity of our approximation, i.e. for energies below the \( W \)-boson mass. In the zeroth order we ignore the coupling of the modes and these solutions are used as inhomogeneous terms in obtaining the first order solution.

Let us consider the zeroth order approximation. The solutions of the homogeneous system for \( r > d \) are:

\[
f_n(\hat{\xi}, \vec{k}) = \begin{cases} 
  I_{n+2}(\hat{\xi}) \
  K_{n+2}(\hat{\xi}) 
\end{cases} \quad n = 0, 1, \ldots \tag{29a}
\]

and therefore the general solution for the dilatonic modes is:

\[
\Phi_n(\kappa \xi) = \xi^2 \left[ K_{n+2}(\kappa \xi) \hat{\mu}_n^{(0)}(\vec{k}) + I_{n+2}(\kappa \xi) \mu_n^{(0)}(\vec{k}) \right] \tag{30}
\]

where \( \hat{\mu}_n^{(0)}(\vec{k}) \) and \( \mu_n^{(0)}(\vec{k}) \) are yet to be determined coefficients.

The solutions in \( r < d \) region are:

\[
f_n(\hat{\eta}, \vec{k}) = \begin{cases} 
  I_{n+2}(\kappa \hat{\eta}) \
  K_{n+2}(\kappa \hat{\eta}) 
\end{cases} \quad n = 0, 1, \ldots \tag{31}
\]

or, taking into account that \( \eta = 1/\xi \):

\[
\Phi_n(\frac{\kappa}{\xi}) = \xi^2 \left[ K_{n+2}(\frac{\kappa}{\xi}) \hat{\alpha}_n^{(0)}(\vec{k}) + I_{n+2}(\frac{\kappa}{\xi}) \alpha_n^{(0)}(\vec{k}) \right] \tag{32}
\]

In general, our solution must be regular in the interior of AdS space so that the solutions for \( r > d \) must be extended in \( r < d \) region to a regular solution. The only regular solution in this region is:

\[
\Phi_n(\frac{\kappa}{\xi}) = \xi^2 I_{n+2}(\frac{\kappa}{\xi}) \alpha_n^{(0)}(\vec{k}) \tag{33}
\]

In addition, the two solutions (30) and (33) must be matched on the boundary of the two regions (\( \xi = 1 \)):

\[
\Phi_n(\frac{\kappa}{\xi}) \bigg|_{\xi=1} = \Phi_n(\kappa \xi) \bigg|_{\xi=1} \tag{34}
\]

\[
\left[ \Phi_n(\frac{\kappa}{\xi}) \right]' \bigg|_{\xi=1} = \left[ \Phi_n(\kappa \xi) \right]' \bigg|_{\xi=1} \tag{35}
\]
These conditions uniquely determine the coefficients $\mu_n^{(0)}$ and $\alpha_n^{(0)}$:

$$
\mu_n^{(0)}(\vec{k}) = -\frac{K_{n+2}(\kappa)I_{n+2}(\kappa) + K'_{n+2}(\kappa)I'_{n+2}(\kappa)}{2I_{n+2}(\kappa)I'_{n+2}(\kappa)}
$$

(36)

$$
\alpha_n^{(0)}(\vec{k}) = \frac{K_{n+2}(\kappa)I'_{n+2}(\kappa) - K'_{n+2}(\kappa)I_{n+2}(\kappa)}{2I_{n+2}(\kappa)I'_{n+2}(\kappa)}
$$

(37)

To determine the last unknown coefficient $\hat{\mu}_n^{(0)}$, note that the scalar field near the boundary behaves as

$$
Q(\xi, \vec{k}) \approx \xi^{\frac{d}{2} - \nu}Q_b(\vec{k}) + \ldots,
$$

where $Q_b(\vec{k})$ is the boundary value of $Q$ and $\nu$ is the spectral parameter in the equations of motion. Applying the above formula to the $n$-th dilatonic KK mode and using the behaviour of the Bessel functions of small argument, one can see that the only contribution to the above behaviour comes from $K_{n+2}$ and therefore:

$$
\Phi_n^b(\vec{k}) = \frac{(n + 1)!}{(n + 1)!} \left(\frac{\kappa}{2}\right)^{n+2} \hat{\mu}_n^{(0)}(\vec{k})
$$

or:

$$
\hat{\mu}_n^{(0)}(\vec{k}) = \frac{2}{(n + 1)!} \left(\frac{\kappa}{2}\right)^{n+2} \Phi_n^b(\vec{k})
$$

(38)

With this, all the coefficients in the zeroth order solutions are completely determined. We substitute the zeroth order solutions on the right hand side of (25,26) and (27,28) to find the first order correction. The equations for the first order ($l = 1$) are as follows:

a) $r > d$ region

$$
\begin{align*}
&f''(\hat{\xi}) + \frac{1}{\hat{\xi}} f'(\hat{\xi}) - \left(1 + \frac{4}{\hat{\xi}^2}\right) f(\hat{\xi}) = \frac{1}{2\kappa^2} \hat{\xi}^2 \left[K_4(\hat{\xi}) \hat{\mu}_2^{(0)}(\vec{k}) + I_4(\hat{\xi}) \mu_2^{(0)}(\vec{k})\right] \\
&f_n''(\hat{\xi}) + \frac{1}{\hat{\xi}} f_n'(\hat{\xi}) - \left(1 + \frac{(n + 2)^2}{\hat{\xi}^2}\right) f_n(\hat{\xi}) \\
&\quad = \frac{3^{2n+1}(n + 1)(n + 2)}{\kappa^2} \hat{\xi}^2 \left[A^{2,n,n-2} \left(K_n(\hat{\xi}) \hat{\mu}_{n-2}^{(0)}(\vec{k}) + I_n(\hat{\xi}) \mu_{n-2}^{(0)}(\vec{k})\right) + A^{2,n,n} \left(K_{n+2}(\hat{\xi}) \hat{\mu}_n^{(0)}(\vec{k}) + I_{n+2}(\hat{\xi}) \mu_n^{(0)}(\vec{k})\right) + A^{2,n,n+2} \left(K_{n+4}(\hat{\xi}) \hat{\mu}_{n+2}^{(0)}(\vec{k}) + I_{n+4}(\hat{\xi}) \mu_{n+2}^{(0)}(\vec{k})\right)\right] \\
\end{align*}
$$

(40)
b) $r < d$ region

\[
\begin{align*}
  f''(\hat{\eta}) + \frac{1}{\hat{\eta}} f'(\hat{\eta}) - \left(1 + \frac{4}{\hat{\eta}^2}\right) f(\hat{\eta}) &= \frac{1}{2\kappa^2} \hat{\eta}^2 I_4(\hat{\eta}) \alpha_2^{(0)}(\vec{k}) \\
  f''_n(\hat{\eta}) + \frac{1}{\hat{\eta}} f'_n(\hat{\eta}) - \left(1 + \frac{(n+2)^2}{\hat{\eta}^2}\right) f_n(\hat{\eta}) &= \frac{32n(n+1)(n+2)}{\kappa^2} \left[ A^{2,n,n-2} I_{n-1}(\hat{\eta}) \alpha_n^{(0)}(\vec{k}) + A^{2,n,n+2} I_{n+1}(\hat{\eta}) \alpha_n^{(0)}(\vec{k}) \right]
\end{align*}
\]

(41)

Solving these equations we find that the general solutions for $r > d$ are:

\[
\begin{align*}
  f_n(\hat{\xi}) &= K_{n+2}(\hat{\xi}) \hat{\eta}_n^{(1)}(\vec{k}) + I_{n+2}(\hat{\xi}) \mu_n^{(1)}(\vec{k}) \\
  &+ \frac{2n(n+1)(n+2)}{\kappa^2} \left\{ A^{2,n,n-2} \left( \hat{\xi}^3 I_{n-1}(\hat{\xi}) \mu_n^{(0)} - \hat{\xi}^3 K_{n-1}(\hat{\xi}) \hat{\eta}_n^{(0)} \right) + A^{2,n,n} \left[ \left( \hat{\xi}^3 I_{n-1}(\hat{\xi}) + 3(n+1)\hat{\xi}^2 I_n(\hat{\xi}) \right) \mu_n^{(0)} - \left( \hat{\xi}^3 K_{n-1}(\hat{\xi}) + 3(n+1)\hat{\xi}^2 K_n(\hat{\xi}) \right) \hat{\eta}_n^{(0)} \right] + A^{2,n,n+2} \left( \hat{\xi}^3 I_{n+1}(\hat{\xi}) \mu_n^{(0)} - \hat{\xi}^3 K_{n+1}(\hat{\xi}) \hat{\eta}_n^{(0)} \right) \right\}
\end{align*}
\]

(42)

For the region $r < d$, the regular solutions are:

\[
\begin{align*}
  f_n(\hat{\eta}) &= I_{n+2}(\hat{\eta}) \alpha_n^{(1)}(\vec{k}) + \frac{2n+1(n+1)(n+2)}{\kappa^2} \left[ A^{2,n,n-2} \hat{\eta}^3 I_{n-1}(\hat{\eta}) \alpha_n^{(0)} + A^{2,n,n} \left( \hat{\eta}^3 I_{n-1}(\hat{\eta}) + 3(n+1)\hat{\eta}^2 I_n(\hat{\eta}) \right) \alpha_n^{(0)} + A^{2,n,n+2} \hat{\eta}^3 I_{n+1}(\hat{\eta}) \alpha_n^{(0)} \right] \\
  f(\hat{\eta}) &= I_2(\hat{\eta}) \alpha_2^{(1)}(\vec{k}) + \frac{1}{12\kappa^2 \hat{\eta}^3 I_5(\hat{\xi}) \alpha_2^{(0)}}
\end{align*}
\]

(43)

See Appendix for details.
Once again we will use the required behaviour of the solutions near the boundary. Expanding the Bessel functions in (43) and (44) for small argument we find that:

\[ \hat{\mu}_n^{(1)}(\kappa) = \frac{2}{(n+1)!} \left[ \Phi_n(b(\vec{k})) + 2^{n+1} \frac{(n+1)!}{n!} A^{2n,n+2} \Phi_{n+2}(b(\vec{k})) \right] \left( \frac{\kappa}{2} \right)^{n+2} \]  

(47)

\[ \hat{\mu}_0^{(1)}(\kappa) = 2 \left[ \Phi_0(b(\vec{k})) + \frac{2}{3} \Phi_2(b(\vec{k})) \right] \left( \frac{\kappa}{2} \right)^2 \]  

(48)

As for the zeroth order solutions, the coefficients \( \hat{\mu}_n^{(1)}(\kappa) \) are determined by the boundary data. There is, however, an important difference - these coefficients are determined by a linear combination of \( \Phi_n(b(\vec{k})) \) and \( \Phi_{n+2}(b(\vec{k})) \). The solutions for \( n \)-th dilatonic KK mode contain two as yet undetermined coefficients, but they can be determined by the matching conditions on the boundary of the two regions. We will see later that these coefficients will play an important role in interpreting the correlation functions and therefore on the physics of the Coulomb branch.

To conclude this section let us summarize the results we have found.

Starting from 10D dilaton action in 2 stacks \( D3 \) brane background we derived the coupled equations of motion for the KK harmonics. We have obtained explicit solutions to first order in \( \frac{1}{\kappa^2} \). The arbitrary coefficients in the solutions were determined by applying the boundary conditions at \( r = d \) as well as from the appropriate asymptotic properties. The most interesting results are contained in eqns(47) and (48) which reflect the coupling of the \( n \) and \( n+2 \), KK modes. This leads to non conformal correction terms to the correlation functions.

### 3 Correlation functions

As mentioned in the Introduction two stacks of parallel, separated \( D3 \) branes preserve supersymmetry, since the Poincare supersymmetries of the gauge theory are maintained but superconformal invariance is broken by the Higgsing. In the case under consideration, the \( SU(N) \) symmetry is broken down to \( SU(N/2) \times SU(N/2) \). This will modify the correlation functions which we study in this section.

From the gauge theory side, according to AdS/CFT correspondence, evaluation of the action on the classical solutions will produce the correlation functions of the operators \( tr \left(F^2X^I\right) \) (we use the notations of [13]: \( X^I = C^{I}_{i_1...i_k}X^{i_1}...X^{i_k} \)). As shown in the Introduction, the nontrivial contributions will come from the terms of
the following form:

\[ S_B = -\frac{1}{2} \lim_{\varepsilon \to 0} \int d^4x \varepsilon^{-3} \Phi' \partial_\xi \Phi' |_{\xi=\varepsilon} \]  (49)

We saw in the previous Section that there are two undetermined coefficients \( \mu^{(1)}_n(\kappa) \) and \( \alpha^{(1)}_n(\kappa) \) for every \( n \). In order to proceed with the derivation of the correlation functions in the gauge theory we first determine these. For this purpose, we have to impose on the first order solutions the matching conditions (34,35) on the boundary of the two regions (\( \xi = 1 \)) (note that these are imposed now on the first order solutions). After tedious but straightforward calculations we find the following expression for \( \mu^{(1)}_n(\kappa) \):

\[
\mu^{(1)}_n(\kappa) = \beta_{n,n-2}(\kappa) \Phi^{n-2}_b(\vec{k}) + \beta_{n,n}(\kappa) \Phi^n_b(\vec{k}) + \beta_{n,n+2}(\kappa) \Phi^{n+2}_b(\vec{k}) \]  (50)

where:

\[
\beta_{n,n}(\kappa) = \beta^{(1)}_{n,n}(\kappa) + \beta^{(2)}_{n,n}(\kappa) \]  (51)

\[
\beta^{(1)}_{n,n} = \left\{ -\left( K_{n+2}I_{n+2} + K'_{n+2}I_{n+2} \right) \right. \\
\left. + 2^n(n+1)(n+2)A^{2,n,n} \left[ (\kappa K_{n-1} + 3(n+1)K_n) I_{n+2} \right. \\
\left. + \left( 3K_{n-1} + \kappa K'_{n-1} + \frac{6(n+1)}{\kappa^2}K_n + 3(n+1)K'_n \right) I_{n+2} \right] \right\} \times \\
\times \frac{1}{(n+1)!I_{n+2}I'_{n+2}} \]  (52)

\[
\beta^{(2)}_{n,n} = \frac{2^n(n+1)(n+2)A^{2,n,n}}{(n+1)!} \left[ (\kappa I_{n-1} + 3(n+1)I_n) I'_{n+2} \right. \\
\left. + \left( 3I_{n-1} + \kappa I'_{n-1} + \frac{6(n+1)}{\kappa}I_n + 3(n+1)I'_n \right) I^2_{n+2} \right] \times \\
\times \frac{1}{I^2_{n+2}I'_{n+2}} \]  (53)

\[
\beta_{n,n+2}(\kappa) = \beta^{(1)}_{n,n+2}(\kappa) + \beta^{(2)}_{n,n+2}(\kappa) \]  (54)

\(^8\)One can easily check that the terms which survive in the limit \( \xi \to 0 \) coming from \( A_I \Phi^0 \partial_\xi \Phi' \) contain only positive integer powers of \( k \) and hence are contact terms.
\[ \beta_{n,n+2}^{(1)} = -\frac{2^n (n + 1) (n + 2) A^{2,n,n+2}}{(n + 1)!} \left[ -2(n + 4) \left( K_{n+2} I'_{n+2} + K'_{n+2} I_{n+2} \right) \left( \frac{\kappa}{2} \right)^2 + \right. \\
\left. \frac{K_{n+5} I'_{n+2} + (3K_{n+5} + \kappa K'_{n+5}) I_{n+2}}{(n + 2)(n + 3)} \right] \frac{1}{I_{n+2} I'_{n+2}} \] (55)

\[ \beta_{n,n+2}^{(2)} = \frac{2^n (n + 1) (n + 2) A^{2,n,n+2}}{2(n + 3)!} \left[ \kappa I_{n+5} I'_{n+2} + K_{n+4} I'_{n+4} + \right. \\
\left. (3I_{n+5} + \kappa I'_{n+5}) I_{n+2} K'_{n+4} I_{n+4} + \right] \frac{1}{I_{n+2} I'_{n+2} I_{n+4} I'_{n+4}} \] (56)

\[ \beta_{n,n-2}(\kappa) = \beta_{n,n-2}^{(1)}(\kappa) + \beta_{n,n-2}^{(2)}(\kappa) \] (57)

\[ \beta_{n,n-2}^{(1)} = \frac{2^n (n + 1) (n + 2) A^{2,n,n-2}}{(n - 1)!} \times \\
\times \left[ (3K_{n-1} + \kappa K'_{n-1}) I_{n+2} + \kappa K_{n-1} I'_{n+2} \right] \frac{1}{I_{n+2} I'_{n+2}} \] (58)

\[ \beta_{n,n-2}^{(2)} = \frac{2^n (n + 1) (n + 2) A^{2,n,n-2}}{2(n - 1)!} \left[ \kappa I_{n-1} I'_{n+2} K_n I'_{n} + \right. \\
\left. (3I_{n-2} + \kappa I'_{n-2}) I_{n+2} K'_{n} I_{n} \right] \frac{1}{I_{n+2} I'_{n+2} I_{n} I'_{n}} \] (59)

Having completely determined these coefficients, we simply have to substitute the explicit form of our first order solutions in the boundary term action and take the limit \( \varepsilon \to 0 \). To see which terms survive this limit, we must use the expansion of the Bessel functions for small argument. After some lengthy but straightforward calculations we find the following expression for the diagonal correlation functions in momentum space (we have omitted terms of even integer powers of \( k \) since they produce in the position space \( \delta \)-function and its derivatives):

\[
\langle O^n(k)O^n(k') \rangle = -\delta(\vec{k} + \vec{k}') \left[ (-1)^n \frac{2^3}{(n + 1)! (n + 2)!} \left( \frac{\kappa}{2} \right)^{2n+4} \ln k \right. \\
\left. + (-1)^n 2^{n+1} \frac{4(n + 1)^2(n + 2)(n + 3)}{(n + 1)!^2} A^{2,n,n} \left( \frac{\kappa}{2} \right)^{2n+2} \ln k \right. \\
\left. - (-1)^n 2^{2n+2} \frac{(n + 2)(5n^2 + 13n + 12)}{(n - 1)! (n - 2)!} (A^{2,n,n})^2 \left( \frac{\kappa}{2} \right)^{2n} \ln k \right. \\
\left. + D_{n,n}(\kappa) \right] 
\]

(60)
Since we have mixing between the different KK dilatonic modes, the off-diagonal correlation functions are also nonvanishing. We find for them the following expression:

\[
\langle O_n(k)O_{n+2}(k') \rangle = -\delta(k + k') \left[ (-1)^n 2^{n+4}(n+2)(n+4) \frac{A^{2,n,n+2}(\frac{\kappa}{2})^{2n+4}}{(n+1)!^2} \ln k \right.
\]
\[
+ (-1)^n 2^{2n+4}(n+1)(n+2)^2(n+2)(n+3)(n+4) \frac{(n)!}{(n)!^2} \times
\]
\[
\left. \times A^{2,n,n} A^{2,n,n+2}(\frac{\kappa}{2})^{2n+2} \ln k + D_{n,n+2}(\kappa) \right] \tag{61}
\]

In addition to the conformal-like parts (integer powers of \(\kappa\) times \(\ln k\)) the correlation functions contain deformations coming from the coefficients entering the solutions. Substituting the expressions for \(\mu_n^{(1)}\) into the boundary action and keeping the terms that survive \(\xi \to 0\) limit one can obtain the deformation functions \(D_{n,m}\). To simplify the final expressions we introduce the following notations:

\[
a_n(\kappa) = \frac{2}{(n+1)!} \left( \frac{\kappa}{2} \right)^{n+2}; \quad \beta_n(\kappa) = -\frac{K_{n+2}I'_{n+2} + K'_{n+2}I_{n+2}}{2I_{n+2}I'_{n+2}}
\]
\[
b_n(\kappa) = 2^{n+2} \frac{(n+2)(n+4)}{n!} A^{2,n,n+2}
\]

In these notations, we find for \(D_{n,m}\) the expressions:

\[
D_{n,n}(\kappa) = \frac{2}{n+2} \left[ a_n \beta_{n,n} \left( \frac{\kappa}{2} \right)^{n+2} - 2^{n-2}(n-1)(n+1)(n+2)(5n+2)A^{2,n-2,n} a_n \beta_n \left( \frac{\kappa}{2} \right)^n \beta_{n-2,n} + 2^{n+1}(n+1)^2(n+2)(5n+3)A^{2,n,n} a_n^2 \beta_n \left( \frac{\kappa}{2} \right)^{-2} + b_{n-2} \beta_{n-2,n} \left( \frac{\kappa}{2} \right)^{n+2} \right] \tag{62}
\]
\[
D_{n,n+2}(\kappa) = \frac{2}{n+2} \left[ a_n \beta_{n,n+2} \left( \frac{\kappa}{2} \right)^{n+4} + b_n \beta_{n,n} \left( \frac{\kappa}{2} \right)^{n+2} \right] \
- 2^n(n+1)(n+3)(n+4)(5n+12)A^{2,n,n+2} a_{n+2} \beta_{n+2} \beta_{n,n} \left( \frac{\kappa}{2} \right)^{n+2} + 2^{n+1}(n+1)^2(n+2)(5n+3)A^{2,n,n} a_n^2 b_n \beta_n \left( \frac{\kappa}{2} \right)^{-2} + a_n a_{n+2} \beta_n \left( \frac{\kappa}{2} \right)^{n+4} \tag{63}
\]
Fourier transforming to position space we obtain the following for the two point correlation functions:

\[ \frac{\langle \mathcal{O}^n(\vec{x}) \mathcal{O}^n(\vec{y}) \rangle}{(L^2/d)^{2n+4}} = \frac{4(n+2)(n+3)}{\pi^2} \cdot \frac{1}{|\vec{x} - \vec{y}|^{2n+8}} + 2^{n+1} \frac{d^2}{4\pi g_s \alpha'^2 N} \cdot \frac{(n+1)^2(n+2)^2(n+3)}{\pi^2} \cdot \frac{1}{|\vec{x} - \vec{y}|^{2n+6}} + \hat{D}_{n,n}(dL^2|\vec{x} - \vec{y}|) + O(1/N^2) \quad (64) \]

\[ \frac{\langle \mathcal{O}^n(\vec{x}) \mathcal{O}^{n+2}(\vec{y}) \rangle}{(L^2/d)^{2n+6}} = 2^{n+3} \frac{d^2}{4\pi g_s \alpha'^2 N} \cdot \frac{(n+1)^2(n+2)^2(n+3)(n+4)}{\pi^2} \times \]

\[ \times A^{2,n,n+2} \frac{1}{|\vec{x} - \vec{y}|^{2n+8}} + \hat{D}_{n,n+2}(dL^2|\vec{x} - \vec{y}|) + O(1/N^2) \quad (65) \]

In above, we have used \( L^4 = 4\pi g_s \alpha'^2 N \), and the deformation of the conformal part \( \hat{D}_{n,m}(\frac{d}{L^2}|\vec{x} - \vec{y}|) \) is the Fourier transform of \( D_{n,m}(\kappa) \). It is well known that \( \mathcal{O}^n \sim \text{tr}(F^2X^n) \). In contrast to the case of spherically symmetric, continuously distributed D3-branes, the mixed correlators are nonvanishing. Keeping the first order terms in the solutions we have found that the correlation functions contain \( \frac{1}{N} \) corrections. The leading term in the mixed correlators, which comes from the first order solutions, is proportional to \( 1/N \). One can see directly from the explicit form of the correlation functions that in the conformal limit \( d \to 0 \) (or equivalently \( <X> \to 0 \)) the \( \frac{1}{N} \) correction term vanishes (the deformations \( D_{n,m} \) also vanish). In that limit the mixed correlation functions also vanish and the conformal symmetry will be restored.

Let us recapitulate the results found in this Section. The physical picture that emerges in our study of separated two stack D3 brane background is qualitatively different from the case of continuously distributed D3 branes. As expected, due to the broken conformal symmetry, the correlation functions of the boundary SYM theory in its Coulomb phase have terms which vanish only in the conformal limit. There is a deformation term much like in the case of spherical D3 brane shell and conformal like correction which is of order \( \frac{1}{N} \). This correction does not appear in the case of uniformly distributed D3 branes. Futhermore, due to interaction between KK modes, off diagonal correlators are nonvanishing and are of order \( \frac{1}{N} \). All correction terms vanish in the conformal limit. The term proportional to \( \frac{1}{N} \) is not a string loop correction as we have used the classical supergravity limit.
4 Deformation terms and the spectrum

In the previous section it was shown that the correlation functions on Coulomb branch have deformation terms. Schematically, they have the following form:

$$D_{n,n}(\kappa) \sim (c_1)\beta_{n,n}(\kappa) + (c_2)\beta_n(\kappa)\beta_{n-2,n}(\kappa) + (c_3)\beta_n(\kappa)$$

$$D_{n,n+2}(\kappa) \sim (c_4)\beta_{n,n+2}(\kappa) + (c_5)\beta_n(\kappa) + (c_6)\beta_{n+2}$$

where the coefficients $c_i$ can be read off from the explicit form of $D_{n,m}$ and as before:

$$\beta_{n,m}(\kappa) = \beta_{n,m}^{(1)}(\kappa) + \beta_{n,m}^{(2)}(\kappa)$$

In the above formulae we have separated $\beta_{n,m}$ into $\beta_{n,m}^{(1)}$, the part has in the denominator a product of a Bessel function and its derivative, and $\beta_{n,m}^{(2)}$, which contains in the denominator the square of a Bessel function times its derivative. We note also that $\beta_n$ contains in the denominator $I_n^2I_n'$.

In order to analyze the structure of $D_{n,m}$ we perform a Wick rotation $\vec{k} \rightarrow -i\vec{k}$. One can use the transformation properties of the modified Bessel functions:

$$I_\nu(-ik) = e^{-i\pi \frac{\nu}{2}}J_\nu(k)$$

$$K_\nu(-ik) = \frac{i\pi}{2}e^{i\pi \frac{\nu}{2}}[J_\nu(k) + iN_\nu(k)]$$

to obtain the relevant expression for the case of Minkowski space time. Since the expressions are rather complicated we will analyze only the denominators. From the above formulae, one can see that there are two kinds of poles. Simple poles come from the zeroes of:

$$J_{n+2}(\kappa)J'_{n+2}(\kappa) = 0; \quad \text{or} \quad J_{n+2}(\kappa)J'_{n+2}(\kappa)J_n(\kappa)J'_n(\kappa) = 0$$

and poles of order two are determined by

$$[J_{n+2}(\kappa)J'_{n+2}(\kappa)]^2 = 0.$$ 

We interpret these as describing the spectrum of glueballs with double degeneracy. It is reasonable to expect that the two stacks might introduce some degeneracy in the spectrum since they contribute identically to the boundary. The spacing between the different resonant frequencies is given in units of $\frac{L^2}{d}$. It is interesting
to note that we obtain the same resonant frequencies as in the case of D3 brane shell. However the resonances are sharp. One possible way to resolve this puzzle, as proposed in [9] is to consider effect of absorption by the branes which introduces finite width to these resonances. It would be very interesting to study the stability of these resonances and their eventual decay into gauge bosons.

We show below that the deformation terms contain logarithmic corrections to the two point correlators due to the breaking of conformal symmetry. We will consider only the diagonal term in (64) (because it is leading order in $d L^2$) Then we find,

$$D_{0,0}(\kappa) \sim \left(\frac{\kappa}{2}\right)^2 \tilde{\beta}_{0,0} + \ldots$$

where ellipses stand for contributions from the mixing. In the expression for $\tilde{\beta}_{0,0}$ we perform a Wick rotation and we find

$$\tilde{\beta}_{0,0} = -\frac{N_2 J_2' + N_2' J_2}{J_2 J_2'} = -\frac{N_2'}{J_2'} - \frac{N_2}{J_2}$$

Thus we have to evaluate the expression

$$\frac{1}{(2\pi)^4} \int d^4k e^{-ik \cdot (\vec{x} - \vec{y})} \beta_{0,0}(\kappa) = \frac{1}{(2\pi)^2} \frac{1}{|\vec{x} - \vec{y}|} \int dk k^6 J_1(k |\vec{x} - \vec{y}|) \tilde{\beta}_{0,0}(\kappa)$$

Changing the variable $k \to k |\vec{x} - \vec{y}|$ one finds that

$$\frac{1}{(2\pi)^4} \int d^4k e^{-ik \cdot (\vec{x} - \vec{y})} \tilde{\beta}_{0,0}(\kappa) = \frac{1}{(2\pi)^2} \frac{1}{|\vec{x} - \vec{y}|^8} \int dk k^6 J_1(k) \tilde{\beta}_{0,0} \left(\frac{d}{L^2} \cdot \frac{k}{|\vec{x} - \vec{y}|}\right)$$

In order to obtain the leading order contribution, one can use the series representation for the Bessel function $N_2$ in $\tilde{\beta}_{0,0}$

$$\pi N_2(\kappa |\vec{x} - \vec{y}|) = J_2(\kappa |\vec{x} - \vec{y}|) log \frac{\kappa |\vec{x} - \vec{y}|}{2} + \ldots$$

In above the dots stand for terms of positive powers of $\kappa |\vec{x} - \vec{y}|$ which for a local theory is small ($|\vec{x} - \vec{y}|$ is finite, but infinitesimally small). Let us consider first the term $N_2 / J_2$. The integral becomes

$$\frac{1}{(2\pi)^4} \int d^4k e^{-ik \cdot (\vec{x} - \vec{y})} \tilde{\beta}_{0,0} = \frac{1}{(2\pi)^2} \frac{1}{|\vec{x} - \vec{y}|^8} \int dk k^6 J_1(k) (log k |\vec{x} - \vec{y}| + \ldots)$$

$^9$Since we want to demonstrate appearance of log corrections we will not keep track of numerical factors.
To compare our results with known results we will restrict to energies below the mass of the $W$-boson. In this case the integral over $k$ is finite and the logarithmic correction to the two point correlation function is reproduced. Analogous considerations hold in the case of $N'_2/J'_2$. We note one can repeat these steps for the non-diagonal part of $D_{0,0}$ which will produce a logarithmic correction to the mixed term. In the above range of energies, from the gauge theory side, the physics is as follows [18], [18]. Since the conformal symmetry is broken by Higgs scalar field, we will have a renormalization group flow from the conformal point of $SU(N)$ to IR fixed point with $SU(N/2)$. If we consider the case of a D3 brane probe, the corresponding picture on the gravity side is as a string stretched between the D3 brane and the horizon of AdS and can be described by the Born-Infeld action which is a Yang-Mills theory plus higher derivative terms. Then $d/\alpha'$ will correspond to the mass of a $W$-boson and the symmetry will be broken from $SU(N)$ to $S(U(N-1) \times U(1))$. Now, one can proceed by adding more and more D3 branes to the single one until the resulting theory becomes $S(U(N/2) \times U(N/2))$. All of this have correct gravity approximation when $g_sN \gg 1$, i.e. large N. In the large N limit only planar diagrams contribute. The logarithmic correction then has an interpretation as due to processes where a pair of virtual gauge particles is produced. The ultraviolet cut off is related to the gravity mass gap and in the strong coupling limit it leads to a condensate of gauge particles. It is shown in the above that our deformation term has poles in the Wick rotated $k$ plane that correspond to bound states of glueballs.

The main feature in our calculation is contained in the second term in (64) as well as in the non-vanishing values for the mixed correlators, i.e. correlators of conformal operators of different conformal weight. Note that these terms are proportional to $\frac{d^2}{\alpha'N^2} \sim \frac{m^2_{\text{char}}}{g_sN}$. If we integrate out all the massive particles in the gauge theory, we will obtain just the logarithmic correction found in [18] and reproduced in the above. If, however, we integrate out all the massive states above some characteristic mass scale ($m_{\text{char}} \ll \frac{d}{L}$), one can expect to have non-zero correlation functions between conformal operators of different conformal weight. This is also the structure of our mixed terms. Moreover, string loop corrections coming from the Born-Infeld action will be proportional to the above characteristic mass and will be negligible compared to the $\frac{1}{N}$ correction term for the correlators.

5 Conclusions

In the present paper, we have studied the Coulomb branch of SYM theory using AdS/CFT correspondence. Our considerations are based on the simplest example of
two stacks of $N/2$ parallel $D3$ branes separated by a distance $2d$ which break $SU(N)$ symmetry to $S(U(N/2) \times U(N/2))$. Inspite of its apparent simplicity, the problem has turned out to be complicated but interesting. This complication mainly comes from the coupling between different $n$-wave dilatonic KK modes in the equations of motion. These equations are solved in the leading $(1/N)$ order. We have shown that the coupling between KK dilatonic modes causes the appearance of mixed correlation functions which are of order $1/N$. According to AdS/CFT correspondence principle, the operators $\mathcal{O}^n$ correspond to $tr(F^2X^n)$. It is obvious from the explicit form of the correlation functions that in the conformal limit ($d \to 0$, or Higgs vev tends to zero) the $1/N$ correction term in the diagonal correlators and the mixed correlation functions vanish.

All the correlation functions have also deformation terms. The analysis in the phase space, after a Wick rotation, shows that these functions have a resonant structure - double poles at appropriate resonant frequencies. We interpret these double poles as the degenerate spectrum of glueballs.

It will be interesting to study the correlation functions beyond this level of approximation. We expect more conformal-like terms to appear in the correlations functions. We also have also to learn more about the spectrum of the glueballs and to look for a mechanism of deriving more realistic masses.

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6 Appendix: The particular solutions of (40).

In this Appendix we will find the particular solutions for the first order equations of motion. We have, in fact, to find particular solutions for three cases:

\[ f''_n(\hat{\xi}) + \frac{1}{\hat{\xi}} f'_n(\hat{\xi}) - \left( 1 + \frac{(n + 2)^2}{\hat{\xi}^2} \right) f_n(\hat{\xi}) = \begin{cases} \hat{\xi}^2 I_n(\hat{\xi}) \\ \hat{\xi}^2 I_{n+2}(\hat{\xi}) \\ \hat{\xi}^2 I_{n+4}(\hat{\xi}) \end{cases} \quad n = 1, \ldots 
\]  

\[ (67) \]
Let us consider the following expression:

$$D_n H \equiv H'' + \frac{1}{\xi} H' (\hat{\xi}) - \left(1 + \frac{(n+2)^2}{\xi^2}\right) H (\hat{\xi})$$

Since the right hand side of eq. (67) contain Bessel functions times powers of $\hat{\xi}$ it is instructive to consider the function $H$ of the form:

$$H = \hat{\xi}^a I_m (\hat{\xi}) \quad (68)$$

After simple transformations and using the Bessel equation for $I_m$ we find:

$$D_n H = \hat{\xi}^{a-2} \left[2a \hat{\xi} I_m' + (a^2 + m^2 - (n+2)^2) I_m \right] \quad (69)$$

If we substitute for $I_m'$ in the above equation:

$$\hat{\xi} I_m' = \hat{\xi} I_{m+1} + m I_m \quad (70)$$

we find:

$$D_n H = \hat{\xi}^{a-2} \left[2a \hat{\xi} I_{m+1} + (a + m)^2 - (n+2)^2) I_m \right] \quad (71)$$

Let us choose the parameters $a$ and $m$ in the following way:

$$a + m = n + 2$$

Therefore:

$$D_n H = \hat{\xi}^{n-m+1} 2(n - m + 2) I_{m+1} (\hat{\xi}) \quad (72)$$

For $m = n - 1$ one find:

$$\left[\frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \left(1 + \frac{(n+2)^2}{\xi^2}\right)\right] \frac{\hat{\xi}^3 I_{n-1} (\hat{\xi})}{6} = \hat{\xi}^2 I_n (\hat{\xi}) \quad (73)$$

which solves the first case in eq. (67). For $m = n$ we have:

$$\left[\frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \left(1 + \frac{(n+2)^2}{\xi^2}\right)\right] \frac{\hat{\xi}^2 I_n (\hat{\xi})}{4} = \hat{\xi} I_{n+1} (\hat{\xi}) \quad (74)$$

![Image: math equations with text]
To solve the second case one can use the properties of the Bessel function $I_{n+2}$:

$$\hat{\xi} I_{n+2}(\hat{\xi}) = \hat{\xi} I_n(\hat{\xi}) - 2(n + 2) I_{n+1}(\hat{\xi})$$

and therefore:

$$\hat{\xi}^2 I_{n+2}(\hat{\xi}) = \hat{\xi}^2 I_n(\hat{\xi}) - 2(n + 2) \hat{\xi} I_{n+1}(\hat{\xi})$$

Using the above we have the solution for the second case.

To obtain the solution for the third case instead of eq. (70) one can use the following property of Bessel functions:

$$\hat{\xi} I_m'(\hat{\xi}) = \hat{\xi} I_{m-1}(\hat{\xi}) - m I_m(\hat{\xi})$$

Then we will have:

$$D_n H = \hat{\xi}^{-2} \left[ 2a \hat{\xi} I_{m-1} + ((a - m)^2 - (n + 2)^2) I_m \right]$$

Let us choose:

$$a = m - n - 2; \quad m = n + 5$$

With this choice we find:

$$\left[ \frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \left( 1 + \frac{(n+2)^2}{\xi^2} \right) \right] \frac{\hat{\xi}^3 I_{n+5}(\hat{\xi})}{6} = \hat{\xi}^2 I_{n+4}(\hat{\xi})$$

(76)

Now, consider the system of equations containing the modified Bessel functions $K_n$:

$$f_n''(\hat{\xi}) + \frac{1}{\xi} f_n'(\hat{\xi}) - \left( 1 + \frac{(n+2)^2}{\xi^2} \right) f_n(\hat{\xi}) = \begin{cases} \hat{\xi}^2 K_n(\hat{\xi}), & n = 1, \ldots \end{cases}$$

(77)

we find:

$$\left[ \frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \left( 1 + \frac{(n+2)^2}{\xi^2} \right) \right] \left( -\frac{\hat{\xi}^3 K_{n-1}(\hat{\xi})}{6} \right) = \hat{\xi}^2 K_n(\hat{\xi})$$

(78)

$$\left[ \frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \left( 1 + \frac{(n+2)^2}{\xi^2} \right) \right] \left( -\frac{\hat{\xi}^2 K_{n+1}(\hat{\xi})}{4} \right) = \hat{\xi} K_{n+1}(\hat{\xi})$$

(79)

$$\left[ \frac{d^2}{d\xi^2} + \frac{1}{\xi} \frac{d}{d\xi} - \left( 1 + \frac{(n+2)^2}{\xi^2} \right) \right] \left( -\frac{\hat{\xi}^3 K_{n+5}(\hat{\xi})}{6} \right) = \hat{\xi}^2 K_{n+4}(\hat{\xi})$$

(80)
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