Creep motions of flux lines in type II superconductors with point-like defects

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We simulated the creep motions of flux lines subject to randomly distributed point-like pinning centers. It is found that at low temperatures, the pinning barrier \( U \) defined in the Arrhenius-type \( F = F - U \) characteristics increases with decreasing force \( F \propto F^{-\mu} \), as predicted by previous theories. The exponent \( \mu \) is evaluated as \( 0.28 \pm 0.02 \) for the vortex glass and \( 0.5 \pm 0.02 \) for the Bragg glass (BrG). The latter is in good agreement with the prediction by the scaling theory and the functional-renormalization-group theory on creep, while the former is a new estimate. Within BrG, we find that the pinning barrier is suppressed when temperature is lifted to approximately half of the melting temperature. Characterizations of this new transition at equilibrium are also presented, indicative of a phase transition associated with the replica-symmetry breaking.

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Introduction – An elastic medium immersed into random environment is a generic system modeling a wealth of physical situations. In spite of the considerable efforts expended and impressive progress, many crucial issues of the underlying physics remain unresolved, and understanding the statics and dynamics of such systems is one of the major challenges of condensed matter physics.

Intensive activities have taken place in vortex states of type II superconductors since the discovery of high-\( T_c \) superconductivity in cuprates \cite{1, 2, 3, 4, 5, 6, 7}. Initiated by the collective pinning theory \cite{8}, theoretical understanding has been advanced \cite{2, 4, 9, 10, 11, 12}. The competition between the elastic force and the random force builds a complex potential landscape governing vortex dynamics. An important notion has been established, namely the typical energy barrier felt by the system becomes diversifyingly large \( U(F) \sim F^{-\mu} \) at small driving forces \cite{2, 4, 11, 12}, which results in an extremely small velocity of the system following the Arrhenius law.

The degree of divergence \( \mu \) is governed by the large-scale elastic properties of the system and properties of disorder \cite{10}. The value of exponent \( \mu \) is however not easy to evaluate accurately, since the static and dynamic properties of the system are intervened in a very complex way. The situation is even more severe when the randomness is large and thus the order is destroyed fully. In most theory restoring forces assume elastic behaviors of the system involved, which is not the case in strongly disordered systems, where plastic deformations become important (See discussions in Ref. \cite{10}).

We tackle this problem by computer simulations on three-dimensional (3D) flux lines subject to randomly distributed point-like pinning centers (for a parallel work on 1D domain wall in 2D space see Refs. \cite{13, 14}). Tuning the strength of random pinning force, we can reach the Bragg glass (BrG) \cite{12, 15} at weak pinning and vortex glass (VG) \cite{10} at relatively strong pinning, at equilibrium. Langevin dynamics then permits us to explore the dynamics of the system subject to driving force at various temperatures without any uncontrollable approximation.

The main results are as follows: The creep law of flux lines predicted by previous theories is clearly reproduced. The exponent is estimated as \( \mu = 0.5 \pm 0.02 \) for BrG and \( \mu = 0.28 \pm 0.02 \) for VG, both universal for the respective class. While the former one is very close to the expected value, the latter one is a new estimation. For weak pinning where BrG is stable below the melting temperature \( T_m \), we find an unpinned state at \( T_g < T < T_m \) in addition to the pinned one at \( T < T_g \), indicating a replica-symmetry breaking transition at \( T_g \approx T_m/2 \).

Model and simulation details – The model system is a stack of superconducting planes of thickness \( d \) with period \( s \) of the layer structure, with the magnetic field perpendicular to the layers. Each plane contains \( N_v \) vortices and \( N_p \) quenched pins. The overdamped equation of motion of the \( i \)th vortex at position \( \mathbf{r}_i \) is

\[
\eta \mathbf{\dot{r}}_i = -\sum_{j \neq i} \nabla_i U^{VV}(\mathbf{r}_{ij}) - \sum_p \nabla_i U^{VP}(\mathbf{r}_{ip}) + \mathbf{F} + \mathbf{F}_{th}. \tag{1}
\]

Here \( \eta \) is the viscosity coefficient. The intraplane vortex repulsion is given by the modified Bessel function \( U^{VV}(\rho_{ij}, z_{ij} = 0) = \epsilon_0 K_0(\rho_{ij}/\lambda_{ab}) \), and the interplane vortex attraction is \( U^{VV}(\rho_{ij}, z_{ij} = s) = (s_0/\sigma)[1 + \ln(\lambda_{ab}/s)]/[\rho_{ij}/(2\gamma s)] - 1 \) for \( \rho_{ij} \leq 2\gamma s \) and \( U^{VV}(\rho_{ij}, z_{ij} = s) = (s_0/\sigma)[1 + \ln(\lambda_{ab}/s)]/[\rho_{ij}/(2\gamma s) - 2] \) otherwise, between two vortices belonging to the same flux line and sitting on adjacent planes, where \( \epsilon_0 = \phi_0^2/2\pi\mu_0\lambda_{ab}^2 \) with \( \lambda_{ab} \) the magnetic penetration depth of the superconducting layer, \( r_g = \gamma s \) with \( \gamma \) the anisotropy parameter. The pinning potential is \( U^{VP}(\rho_{ip}) = -\alpha A_p \exp[\rho_{ip}/R_p] \), where \( A_p = (\epsilon_0 d/4) \ln[1 + (R_p^2/2\xi_{ab}^2)] \) with \( \xi_{ab} \) the in-plane coherence length and \( \alpha \) the dimensionless pinning strength. Finally, \( \mathbf{F} \) is the uniform Lorentz force, and \( \mathbf{F}_{th} \) is the thermal noise force with zero mean and a correla-
proximately to 60 m/s lowered temperature, the dynamics is to be governed by the zero-temperature depinning force \[18\], where the velocity \(v\) Refs. [18, 20]. Figure 1 presents the average velocity at small for which an amorphous VG is realized at low tempera-
ture [18]. The prefactor \(U\), for weak pinning strengths for which BrG is realized at low temperatures, we obtain \(U = 0.018 \pm 0.003\) and \(\mu = 0.50 \pm 0.05\). The above results indicate that the exponent \(\mu\) for \(\alpha \leq 0.1\) falls into another class, universal for weak pinning strengths for which BrG is realized at low temperatures.

Our estimate on the exponent \(\mu\) is in good agreement with the theoretical prediction \[11\]
\[
\mu = \frac{D - 2 + 2\zeta_{eq}}{2 - \zeta_{eq}} = \frac{1}{2}
\]
with \(\zeta_{eq} = 0\) the roughness exponent for BrG.

As pointed out in Ref. [3], previous theories on vortex creep started from the hypothesis of elastic energy, which is established only for elastic deformations, and thus work at best for BrG. The exponent \(\mu\) should be reformulated for strong pinnings associated with VG where deformations are plastic [3]. It was not clear yet that whether \(\mu\) is larger in BrG or in VG [3, 21]. The present simulation results suggest a stronger divergence of energy barrier in BrG.

**New phase boundary in BrG** – So far, we have concentrated on low temperatures, where the thermal energy only activates flux lines from pinning centers, resulting in an Arrhenius-type motion with a well defined energy barrier which depends on the driving force. As temperature is lifted, thermal fluctuations become more important, which makes the competition between the randomness and the intervortex forces very subtle. Here we focus on the weak pinning case \(\alpha = 0.05\), for which the ground state is a BrG with \(T_{m} \approx 0.075\).

Figure 3 presents the \(v - F\) characteristics at several typical temperatures. At low temperatures, the \(v - F\) curves are nonlinear at low forces, similar to Figs. 1 and 2. However, above \(T_{g} \approx 0.035\), the \(v - F\) characteristics is linear down to the small force limit. This indicates that...
FIG. 3: $v - F$ characteristics for $\alpha = 0.05$ at different temperatures. A transition temperature $T_g \simeq 0.035$ is defined which separates the nonlinear and linear $v - F$ curves. The solid lines are for eye-guide. Inset: a schematic phase diagram of vortex states.

The potential barrier sensed by BrG is smeared to zero in an intermediate temperature regime $T_g < T < T_m$, where the crystalline order is still preserved. A similar change is observed for relative strong pinning where VG is realized at low temperatures, for which the details will be reported elsewhere.

We have found that even at equilibrium, i.e. for zero driving force, the system behaves in qualitatively different ways for temperatures below and above $T_g$. In Fig. 4, we display two-dimensional vortex trajectories and the corresponding structure factors. At $T = 0.03 (< T_g)$ (Fig. 4(a)), vortices are trapped in cages formed by the random pinning potential and the intervortex repulsions (even for time much longer than that shown in Fig. 4); a profound BrG order is clearly seen. At $T = 0.09 (> T_m)$ (Fig. 4(c)), the system is in a liquid state where vortices diffuse freely and randomly. At $T = 0.06$, the behaviors of the vortices are different as evidenced in Fig. 4(b): Over an intermediate time scale, vortices are trapped at local positions in a way such that the BrG order is established. Vortices then move quickly to another set of localized positions, which also establishes the BrG order, and stay in the new positions over another intermediate time duration. The intermittent trapped-and-moving motions continue, and eventually vortices can travel over distances larger than the vortex lattice constant $a_0$.

In order to capture the vortex motions better, we have monitored the number $N(t)$ of vortices that move over distances smaller than $a_0$ after time $t$. As shown in Fig. 4(d), $N(t)$ decreases exponentially with $t$ in a large time scale at $T = 0.06$ since vortices move over distances larger than the lattice constant. The exponential decay occurs even in a small time scale at $T = 0.09$ as the system is random. At $T = 0.03$, $N(t)$ remains unity during the time evolution because all the vortices are caged.

We have also studied how vortices change their nearest neighbors (NNs). At $t = 0$, we identify the NNs of each vortex in terms of the Delaunay triangulation method. Due to random motions, some vortices defined as NNs at $t = 0$ diffuse away. The quantity $C_{NN}(t)$ is defined as the number of NNs identified at $t = 0$ remaining as NNs up to time $t$, with a trivial normalization to unity at $t = 0$. The quantity $C_{NN}$ varies with time.
coordinated vortices

\[ P \sim \text{constant} \]

reciprocal lattice vector

coordinated, even though the individual vortices are mobile in a stochastic way. Compared with thermal fluctuations, vortex-vortex interactions are strong while pinning is weak, resulting in an unpinned BrG. The pinned BrG at \( T = 0.03 \) is well ordered as captured by large values of \( S(Q) \) and \( P_b \), while they are both small at \( T = 0.09 \) corresponding to the vortex liquid.

Recently an experimental finding of a second-order phase boundary within the solid phase of the vortex system in BSCCO was reported \cite{22}; the transition line \( B_g(T) \) dividing the BrG phase domain into two parts. This transition was discussed as a transition between the two different types of glasses, the high-temperature pre-glass or marginal glass phase, with the nearly linear response behavior, and the low-temperature true glass domain. The transition between the two phases manifests itself in a replica-symmetry breaking within the replica description of disordered vortex system \cite{23}. It is tempting to associate our observation of the change in dynamic vortex behavior with this transition. Indeed, at low temperatures \( T < T_g \), vortices remain indefinitely near their equilibrium positions since on top of the elastic forces keeping them there, vortices are additionally immobilized by disorder. At intermediate temperatures, \( T_g < T < T_m \), pinning of single vortices is not efficient any more and they may switch between their equilibrium positions due to thermal diffusion. However, putting this conclusion on a firm quantitative basis requires a more detailed study of the processes of vortex relaxation in both phases and will be a subject of a forthcoming publication.

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[1] G. Blatter et al., Rev. Mod. Phys. 66, 1125 (1994).
[2] L. B. Ioffe and V. M. Vinokur, J. Phys. C 20, 6149 (1987).
[3] J. Kierfeld, H. Nordborg, and V. M. Vinokur, Phys. Rev. Lett. 85, 4948 (2000).
[4] P. Chauve, T. Giamarchi, and P. Le Doussal, Phys. Rev. B 62, 6241 (2000).
[5] T. Nattermann and S. Scheidl, Adv. Phys. 49, 607 (2000).
[6] T. Giamarchi and S. Bhattacharya, High Magnetic Fields: Applications in Condensed Matter Physics, Spectroscopy (Springer, New York, 2002), p.314.
[7] S. Brazovskii and T. Nattermann, Adv. Phys. 53, 177 (2004).
[8] A. I. Larkin and Yu. N. Ovchinnikov, J. Low Temp. Phys. 34, 409 (1979).
[9] T. Nattermann, Europhys. Lett. 4, 1241 (1987).
[10] M. P. A. Fisher, Phys. Rev. Lett. 62, 1415 (1989).
[11] M. V. Feigel’man et al., Phys. Rev. Lett. 63, 2303 (1989).
[12] T. Nattermann, Phys. Rev. Lett. 64, 2454 (1990).
[13] S. Lemerle et al., Phys. Rev. Lett. 80, 849 (1998).
[14] A. B. Kolton, A. Rosso, and T. Giamarchi, Phys. Rev. Lett. 94, 047002 (2005).
[15] T. Giamarchi and P. Le Doussal, Phys. Rev. Lett. 72, 1530 (1994); Phys. Rev. B 52, 1242 (1995).
[16] D. S. Fisher, M. P. A. Fisher, and D. A. Huse, Phys. Rev. B 43, 130 (1991).
[17] E. H. Brandt, Phys. Rev. Lett. 50, 1599 (1983); *ibid* J. Low Temp. Phys. 53, 41 (1983); S. Ryu et al., Phys. Rev. 85, 6241 (2000).
Lett. 68, 710 (1992); C. Reichhardt, C. J. Olson, and F. Nori, Phys. Rev. Lett. 78, 2648 (1997); A. van Otterlo, R. T. Scalettar, and G. T. Zimányi, Phys. Rev. Lett. 81, 1497 (1998); E. Olive et al., Phys. Rev. Lett. 91, 037005 (2003).

[18] M. B. Luo and X. Hu, Phys. Rev. Lett. 98, 267002 (2007).
[19] L. N. Bulaevskii et al., Phys. Rev. B 50, 3507 (1994).
[20] G. Blatter, V. B. Geshkenbein, and J. A. G. Koopmann, Phys. Rev. Lett. 92, 067009 (2004).
[21] T. Giamarchi and P. Le Doussal, Phys. Rev. B 55, 6577 (1997).
[22] H. Beidenkopf et al., Phys. Rev. Lett. 95, 257004 (2005).
[23] D. Li, B. Rosenstein, and V. Vinokur, J. Supercond. Nov. Magn. 19, 369 (2006).