The formation of chondrules is one of the oldest unsolved mysteries in meteoritics and planet formation. Recently an old idea has been revived: the idea that chondrules form as a result of collisions between planetesimals in which the ejected molten material forms small droplets that solidify to become chondrules. Pre-melting of the planetesimals by radioactive decay of $^{26}$Al would help produce sprays of melt even at relatively low impact velocity. In this paper we study the radiative cooling of a ballistically expanding spherical cloud of chondrule droplets ejected from the impact site. We present results from numerical radiative transfer models as well as analytic approximate solutions. We find that the temperature after the start of the expansion of the cloud remains constant for a time $t_{\text{cool}}$ and then drops with time $t$ approximately as $T \approx T_0[(3/5)t/t_{\text{cool}} + 2/5]^{-5/3}$ for $t > t_{\text{cool}}$. The time at which this temperature drop starts $t_{\text{cool}}$ depends via an analytical formula on the mass of the cloud, the expansion velocity, and the size of the chondrule. During the early isothermal expansion phase the density is still so high that we expect the vapor of volatile elements to saturate so that no large volatile losses are expected.

**Key words:** meteorites, meteors, meteoroids – radiative transfer

### 1. INTRODUCTION

The formation of chondrules is one of the fundamental questions of meteoritics and planet formation. Chondrules are the 0.1–1 millimeter-size once-molten silicate spherules found abundantly in primitive meteorites known as chondrites (Jones et al. 2005; Sears 2004; Davis et al. 2014). Most of these chondrites in fact consist predominantly of these chondrules, so the melt-producing events that created them must have been extremely common during the first few million years of the solar system. Yet there is confusing and conflicting evidence as to what these events might have been. Boss (1996) published an overview of the status of the discussion at that time, though significant new developments have occurred since then.

Many theories have been put forward over the last half century. One theory involves impact melt sprays. Put forward by Urey & Craig (1953) and refined by Kieffer (1975) this model states that high-velocity impacts ($\gtrsim 3–5$ km s$^{-1}$) could lead to sprays of impact melt that produce droplets which solidify into chondrules. While these required collision velocities are high, a small fraction of impacts might acquire such velocities (Bottke et al. 1994). Perhaps the so-called “Grand Tack” scenario of Walsh et al. (2011), in which Jupiter temporarily entered the asteroid belt region before migrating back outward, might produce such high velocities. However, Taylor et al. (1983) put forward a number of arguments against the impact-melt scenario, some of which were based on the differences between chondrites and lunar impact regolith.

Zook (1980) suggested that if the interiors of the colliding bodies are already in a molten state due to $^{26}$Al decay, then the required impact velocities to create sprays of melt are much lower and thus more consistent with expectations of the average relative velocities between planetesimals. Sanders & Taylor (2005), Hevey & Sanders (2006), and Sanders & Scott (2012) followed up on this idea of pre-molten impactors and back it up with models and meteoritic evidence. Sanders & Scott (2012) argued that this scenario is hard to avoid, given that in the first 2.5 million years most planetesimals were internally nearly fully molten by $^{26}$Al decay, and that collisions between planetesimals were extremely common. Each collision would almost certainly release substantial amounts of molten rock into the nebula in the form of sprays of lava droplets, and it is natural to assume that these may be chondrules. They argue that the near-solar composition of chondrules can be explained by the vigorous convection inside the molten planetesimals that may slow down iron/nickel-core formation, thus keeping the melt solar. Some degree of differentiation would then in fact explain the low iron-content L and LL chondrites.

Recently, Asphaug et al. (2011) performed smoothed particle hydrodynamics (SPH) simulations for such impacts to demonstrate the dynamics of this scenario. In addition they proposed a simple way to calculate the melt droplet size by equating the released enthalpy after the collision to the surface energy of the droplets.

There are many alternative scenarios proposed in the literature. Perhaps the most popular model is the flash heating by nebular shocks (Hood & Horanyi 1991). Desch & Connolly (2002) and Ciesla & Hood (2002) developed detailed one-dimensional (1D) radiative shock models with dust particles and chondrules interacting both radiatively and frictionally with the gas. They showed how the radiative shocks exhibit temperature spikes of mere tens of seconds (with cooling rates $>10^4$ K hr$^{-1}$) that would be good candidates for chondrule-forming events. After the main shock temperature spike their model exhibits a further cooling at intermediate cooling rate ($\sim 50$ K hr$^{-1}$), which cools the chondrules to sufficiently low temperature in a sufficiently short amount of time. This model seems to produce the flash-heating events required for turning “dust bunnies” into chondrules. However, so far the shock scenario still has several unresolved issues (e.g., Desch et al. 2012; Boley et al. 2013; Stammel & Dullemond 2014).

Some issues have also been raised about whether large-scale shocks in the optically thick solar nebula can explain the time scales involved in chondrule formation (Stammel & Dullemond 2014). It is noteworthy that there exist several other nebular flash heating models, most notably flash heating by nebular lightning (Gibbard et al. 1997) and flash heating by energy dissipation.
in current sheets forming in MHD turbulence (Hubbard et al. 2012).

One important constraint on chondrule formation models is that these heating events were very short lived compared to any other nebular time scale. By comparing textures of chondrules to those obtained in furnace experiments, it can be inferred that chondrules must have cooled from the liquidus temperature down to below the solidus temperature in a matter of hours (Hewins 1983; Hewins & Connolly 1996; see also references in Morris & Desch 2010 and the excellent review paper by Desch et al. 2012). In other words: the chondrule-forming events must have been flash events. On the other hand, under optically thin conditions a molten chondrule would radiatively cool in about a second, which would be too fast.

Another constraint that a chondrule formation model must fulfill is the retention of volatile elements such as Fe, Mg, Si, Na, and K. Chondrules are not observed to have low abundances of these elements. In particular for the highly volatile elements Na and K this is a puzzle because any heating event that heats a (proto-)chondrule above $\sim 1700$ K and keeps these chondrules above that temperature for more than a few minutes will cause most of the Na and K to evaporate out of the chondrule. Alexander et al. (2008) argue that this means that chondrules must have formed in regions that are extremely dense in solids, much more so than typical dust concentration mechanisms in the protoplanetary disk can achieve. According to their calculations such regions are so dense in solids that they must be self-gravitating. Morris et al. (2012) instead propose that chondrules formed behind the bow shock of a fast-moving planetary embryo, and that the embryo’s atmosphere is rich in such volatile elements caused by outgassing from the embryo’s interior, thus providing the necessary vapor pressure to keep the chondrules volatile-rich.

In this paper we will focus on the impact splash hypothesis, either the low-velocity pre-molten planetesimal version or the high-velocity impact-melt version. We assume that after the collision between two planetesimals a cloud of molten droplets of magma was released. Because soon after the ejection of this cloud of magma droplets the internal pressure of the cloud would have dropped to near-zero (there is, to good approximation, only vacuum between the droplets), the cloud would simply expand ballistically. Initially the density of the cloud is so high that the cloud is completely optically thick and no radiation can escape from its interior. There is also no adiabatic cooling because of the lack of pressure. So the temperature of the magma droplets will initially stay roughly constant in time. As the cloud expands, however, the optical depth drops and eventually the cloud will start to cool radiatively. During the early stages the density is very high and the evaporation of volatile elements will be saturated (as was argued by Alexander et al. 2008). Once the temperature drops below the solidus, volatiles can no longer escape.

In this first paper we intend to compute how the temperature behaves as a function of time after the impact. To do this we set up a simple model: that of a homologous ballistically expanding homogeneous spherical cloud of lava droplets. We compute the temperature of the chondrules as a function of time $t$ and comoving radial location inside the cloud $r/r_{cloud}$ by solving the time-dependent radiative transfer equation. We also present an analytic approximate solution. While the spherical cloud model is not an accurate model of the complex shape of an impact splash, it can be regarded as a model of part of the impact splash. As such we believe that it will give reasonable estimates of the radiative cooling behavior of the impact splash as a function of model parameters such as the total mass of the cloud and its expansion velocity. The total mass of the cloud tells something about the masses of the two colliding bodies while the expansion velocity of the cloud tells us something about the impact velocity.

2. EXPANDING CLOUD MODEL

When lava droplets are produced in a collision between two planetesimals, they will disperse away from the impact site in a ballistic way. Let us call this the “impact splash.” If the impact velocity is larger than the escape velocity of the two colliding planetesimals this ballistic (pressureless) expansion will be linear (i.e., the velocity of expansion will not change with time). We regard the impact splash as an expanding cloud of lava droplets (chondrules-to-be). This cloud is not necessarily centered on the impact site; it can also move away from the impact site. The radius of the cloud increases linearly with time $t$ and thus the density of the cloud will drop as $1/r^3$. Depending on the complexity of the geometry of the impact splash (see, e.g., Asphaug et al. 2011) we can also consider the splash to be multiple smaller clouds. In either case, each of these clouds will expand linearly. The scenario is pictographically shown in Figure 1.

If the impact is at low velocity, then the gravitational pull of the surviving (or merged) body can cause some (or most) of the material to re-accrete. In this case the $1/r^3$ expansion will cease and turn into recompression. Initially, however, the linear expansion and $1/r^3$ density drop will still be a good description.

To compute the temperature of the droplets in these clouds as a function of time after impact we must solve the time-dependent equation of radiative transfer. This is a very non-local problem because a cooling droplet at location $x_1$ radiates away some heat into the form of infrared radiation, which can then be absorbed by another droplet at location $x_2 \neq x_1$, which is then heated. The droplets are thus radiatively coupled over large distances. Solving this problem in 3D for complex cloud geometries is somewhat challenging.

We believe, however, that much can already be learned from a simple spherical cloud model, for which the radiative transfer problem can be solved to high precision and reliability with the 1D tangent-ray variable Eddington factor method. We will describe the method in Section 4.

The three main parameters of the model are

$$M_{cloud}, T_0, v_{exp}$$

where $M_{cloud}$ is the total mass worth of chondrules (lava droplets) in our spherical cloud, $T_0$ is the initial temperature of the chondrules, and $v_{exp}$ is the expansion velocity of the outer radius of the cloud. To get a feeling for the numbers it is more convenient to express the mass of the cloud in terms of the radius $R_{melt,0}$ of a ball of magma of the same mass $M_{cloud}$:

$$M_{cloud} = \frac{4\pi}{3} \xi_{chon} R_{melt,0}^3$$

where $\xi_{chon}$ is the material density of the chondrule droplets and thus, by definition, the material density of the hypothetical ball of magma. For a linearly expanding cloud the radius of the cloud is

$$R_{cloud}(t) = v_{exp} t$$

where the time $t$ is the time since the impact and the ejection of the cloud of melt droplets, assuming perfectly ballistic
Let us consider chondrules (lava droplets) of radius $r$ that $R_{\text{cloud}}(t) \gg R_{\text{melt},0}$, so that the droplets are clearly separated from each other. The velocity profile inside the cloud is

$$v(r, t) = \frac{r}{R_{\text{cloud}}(t)},$$

which is of course only valid for $r \leq R_{\text{cloud}}(t)$.

Let us define a coordinate $r$ centered on the center of the spherical cloud. We assume that the density within the cloud is constant and outside of the cloud is zero. The density of the cloud as a function of time is then

$$\rho_{\text{cloud}}(r, t) = \begin{cases} \frac{3M_{\text{cloud}}}{4\pi R_{\text{cloud}}^3(t)} & \text{for } r \leq R_{\text{cloud}}(t) \\ 0 & \text{for } r > R_{\text{cloud}}(t). \end{cases}$$

Hereafter we write only $\rho_{\text{cloud}}(t)$ instead of $\rho_{\text{cloud}}(r, t)$. The time-dependence of $\rho_{\text{cloud}}(t)$ is

$$\rho_{\text{cloud}}(t) = \frac{3M_{\text{cloud}}}{4\pi v_{\text{exp}}^3 t^2}.$$  

Let us consider chondrules (lava droplets) of radius $a_{\text{chon}}$ and material density $\bar{\rho}_{\text{chon}}$. We take $\bar{\rho}_{\text{chon}} = 3.3$ g cm$^{-3}$ for our model. The mass of a chondrule is then

$$m_{\text{chon}} = \frac{4\pi}{3} \bar{\rho}_{\text{chon}} a_{\text{chon}}^3.$$  

The number density of chondrules is then

$$n_{\text{chon}}(t) = \frac{\rho_{\text{cloud}}(t)}{m_{\text{chon}}}.$$  

The geometric cross section of a chondrule is $\pi a_{\text{chon}}^2$. Let us assume that the chondrule has zero albedo. Then the absorption cross section equals the geometric cross section. The opacity (i.e., the cross section per gram) is then

$$\kappa = \frac{\pi a_{\text{chon}}^2}{4\pi \bar{\rho}_{\text{chon}} a_{\text{chon}}^3} = \frac{3}{4} \frac{1}{\bar{\rho}_{\text{chon}} a_{\text{chon}}^2}.$$  

We will assume that the opacity of any possible vapor between the chondrules will be low compared to that of the chondrules so that it can be ignored. The optical depth from the center of the cloud to the edge is

$$\tau(t) = \rho_{\text{cloud}}(t) \kappa R_{\text{cloud}}(t) = \frac{3}{4\pi} \frac{M_{\text{cloud}}}{v_{\text{exp}}^3 t^2}.$$  

The rate of cooling will strongly depend on this optical depth.

In this paper we will present our results based on a set of fiducial models as well as parameter scans. The parameters of the fiducial models are listed in Table 1.

### 3. ANALYTIC ESTIMATE OF THE COOLING BEHAVIOR OF THE CHONDRULE CLOUD

As the cloud expands it can start to radiate away energy. A fully fledged time-dependent radiative cooling computation will yield temperature as a function of time and space: $T(r,t)$. We will compute this in Section 4.

A first estimate of the cooling behavior of the cloud can already be made with pen and paper. The cloud will initially be...
for the moment, as an isothermal cloud) is thermal energy stored in the chondrule cloud (approximating it, Equation (12) we must use the heat capacity formula. The total of the temperature as a result of the radiative loss given by that factor to account for the lower temperature of the surface optically thick:

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Equation (15) yields

\[ T \simeq \frac{4 \pi R^2_{\text{cloud}} \sigma T_{\text{eff}}^4}{1 - \tau} \]  

where \( T \) is now the central temperature and \( 1 / \tau \) is the correction factor to account for the lower temperature of the surface \( T_{\text{eff}} \), and is based on radiative diffusion theory which states that \( T \simeq \tau^{1/4} T_{\text{eff}} \) (for \( \tau \gg 1 \)). To compute the change of the temperature as a result of the radiative loss given by Equation (12) we must use the heat capacity formula. The total thermal energy stored in the chondrule cloud (approximating it, for the moment, as an isothermal cloud) is

\[ E = M_{\text{cloud}} c_m T \]  

where \( c_m \) is the mass-weighted specific heat of the lava droplet. A value of \( c_m = 10^7 \text{ erg g}^{-1} \text{ K}^{-1} \) is a reasonable value, which we will adopt here. The radiative-loss time scale can now be defined as

\[ t_{\text{rad}}(t) = \frac{E}{L} = \frac{M_{\text{cloud}} c_m T}{4 \pi R^2_{\text{cloud}} \sigma T_{\text{eff}}^4}. \]  

This time scale varies with time: it is very long at early times because the cloud is then still extremely optically thick. At late times the cloud is optically thinner and the radiation can more freely escape (Equation (12)) and \( t_{\text{rad}} \) becomes smaller. It is therefore to be expected that at early times the temperature of the chondrules remains constant. Note that since the cloud consists of liquid drops that stay at a constant volume and move away from each other, there is no adiabatic cooling involved here. Any potential vapor may adiabatically cool, but we will ignore this effect in this paper, assuming that the total mass in vapor is always small compared to the mass in droplets.

At some point in time, however, \( t_{\text{rad}} \) becomes smaller than \( t \) and the chondrules start to cool. Based on the results of the true radiative transfer calculations of Section 4 it turns out that near the center of the cloud the transition from the initial constant temperature phase to the temperature-decline phase occurs roughly at a time \( t_{\text{cool}} \) defined by

\[ t_{\text{rad}}(t_{\text{cool}}) = 5 t_{\text{cool}}. \]  

Inserting Equations (14), (12), (13), and (10) into Equation (15) yields

\[ t_{\text{cool}} = \left( \frac{3}{5} \frac{4 \pi R^2_{\text{cloud}} c_m k}{(4 \pi)^2 R^4_{\text{cloud}} \sigma T_{\text{eff}}^4} \right)^{1/5}. \]  

Note that in Equation (15) the factor 5 is purely empirical. The estimate we make here is essentially a dimensional analysis in which proportionality factors have to be calibrated against more exact calculations (in our case the full radiative transfer calculation).

Roughly for \( t < t_{\text{cool}} \) the temperature stays constant while for \( t > t_{\text{cool}} \) the temperature drops with time. From now on we shall define the \( t_{\text{cool}} \) as the one calculated for the center of the cloud. Near the surface the cooling sets in earlier.

The temperature decline with time can also be estimated, at least up until the point where the cloud becomes optically thin, after which our approximation breaks down. The way to obtain this estimate is to solve the cooling equation

\[ \frac{d T(t)}{d t} = - \frac{T(t)}{t_{\text{cool}}(t)} \]  

where \( t_{\text{cool}}(t) \) is the cooling time given by Equation (16) but with \( T_0 \) replaced by \( T(t) \). The reasoning behind this is that if we would instead use \( t_{\text{rad}}(t) \) (which might look more reasonable at first sight) we will quickly cool to temperatures for which condition \( t_{\text{rad}} \lesssim t \) is again no longer fulfilled. This is because \( t_{\text{rad}} \propto T^{-3} \) and thus very rapidly rises with declining temperature. If we insert Equation (16) with \( T_0 \) replaced by \( T \) into Equation (17) we obtain a differential equation that can be solved by separation of variables. The solution is

\[ T(t > t_{\text{cool}}) = T_0 \left[ \frac{3}{5} \frac{t}{t_{\text{cool}}} + \frac{2}{5} \right]^{-5/3}, \]  

where here \( t_{\text{cool}} \) is again the original one with \( T_0 \). We assume that for \( t < t_{\text{cool}} \) we have

\[ T(t < t_{\text{cool}}) = T_0. \]  

The solution Equations (18) and (19) holds true for the central temperature. But most of the mass of a homogeneous sphere resides in the outer parts. For the temperature at \( r = 0.8 R_{\text{cloud}} \) and \( r = 0.9 R_{\text{cloud}} \) (roughly the radii of half mass and of 75% mass, respectively) the best fitting solution to the real solutions of Section 4 are similar to Equation (18) but with the term \( 2/5 \) replaced by \( 3/5 \) and \( 3.8/5 \), respectively. These analytic estimates of the temperature as a function of time are plotted in Figure 2. As one can see, although the time after the impact at which the cooling stars is different depending on whether you look at the center or at the edge of the cloud, the typical cooling rate after the cooling begins is similar throughout the cloud.
It is important to remind ourselves that the cooling normally starts well before the cloud becomes optically thin. Optically thin cooling would be extremely fast: a single chondrule would cool within about one second. The protracted cooling over a time scale of hours occurs because the high optical depth (even after \( t = t_{\text{cool}} \)) acts as a kind of blanket that keeps the chondrules warm, albeit a blanket that is thinning over time as the optical depth drops. To get a feeling for this we can calculate the optical depth at the time \( t = t_{\text{cool}} \) from Equation (16):

\[
\tau_{\text{cool}} \equiv \tau(t_{\text{cool}}) = \frac{3^{3/5}5^{2/5}M_{\text{cloud}}^{1/5}3^{3/5}2^{2/5}10^{6/5}}{(4\pi)^{1/5}v_{\exp}^{2/5}r_{m}^{5/2}}. \tag{20}
\]

So Equations (18) and (19), which are based on the assumption of the “blanket effect” of high optical depth, are expected to be valid for the case \( \tau_{\text{cool}} \gg 1 \). In Table 1 one can see that \( \tau_{\text{cool}} \) is well above 1 for all models, except the most extreme model F4, which anyway has a much-too-small cooling time to be valid for the case.

The analytic model of Equations (18) and (19) tells us that for \( t > t_{\text{cool}} \) the temperature drops off suddenly on a typical time scale of \( t_{\text{cool}} \). The temperature decay rate at \( t = t_{\text{cool}} \) is

\[
\dot{T}(t = t_{\text{cool}}) \equiv \left. \frac{dT}{dt} \right|_{t=t_{\text{cool}}} = -\frac{T_{0}}{t_{\text{cool}}}. \tag{21}
\]

This is plotted, together with \( t_{\text{cool}} \), in Figure 3. It is also useful to define another kind of cooling time scale \( t_{1400} \), which is the time between the start of the cooling at \( T = T_{0} \) and the time when the temperature has dropped below the solidus temperature 1400 K. To first order we can write

\[
t_{1400} \simeq \frac{T_{0} - 1400 \text{ K}}{\dot{T}(t = t_{\text{cool}})} = \left( 1 - \frac{1400 \text{ K}}{T_{0}} \right) t_{\text{cool}}. \tag{22}
\]

The quantities \( \dot{T} \) and \( t_{1400} \) can be compared to the constraints coming from the analysis of textures of chondrules, which put time limits on the cooling process on the order of hours, or more precisely: cooling rates in the range 10 to \( \sim \)3000 K hr\(^{-1}\). (see Morris & Desch 2010 and Desch et al. 2012, and references therein, though see also Miura & Yamamoto 2014 for a different view based on theoretical modeling of crystallization), indicated by the gray area in Figure 3. According to Equation (16) \( t_{\text{cool}} \) goes as \( M_{\text{cloud}}^{2/5} \) and as \( v_{\exp}^{-4/5} \). There is not very much freedom of choice of \( T_{0} \): it must lie somewhere between 1770 and 2120 K (see Morris & Desch 2010, and references therein).

The \( \kappa \) does not have too much wiggle room either: the radii of chondrules are known. This means that constraints on \( t_{\text{cool}} \) from textural analysis directly set limits on the ratio \( M_{\text{cloud}}/v_{\exp}^{2} \) or equivalently on the ratio \( R_{\text{melt},0}/v_{\exp}^{2} \). If, for example, we choose \( a_{\text{chond}} = 0.03 \) cm and \( T_{0} = 2000 \) K we obtain \( \kappa = 7.6 \) cm\(^{2}\) g\(^{-1}\). If we require, for instance, the cooling time scale to be \( t_{\text{cool}} = 10 \) minutes, then we find that \( M_{\text{cloud}}/v_{\exp}^{2} \approx 10^{17} \) g s\(^{-2}\) cm\(^{-2}\). This would be fulfilled, e.g., for \( v_{\exp} = 1 \) km s\(^{-1}\) and \( M_{\text{cloud}} = 10^{17} \) g (a mass corresponding to a ball of magma of roughly \( R_{\text{melt},0} = 2 \) km radius). It would also be fulfilled by a smaller mass and smaller velocity: e.g., for \( v_{\exp} = 10 \) m s\(^{-1}\) and \( M_{\text{cloud}} = 10^{13} \) g (\( R_{\text{melt},0} = 0.1 \) km radius). Such small mass could either mean that upon impact only a small fraction of the debris is in the form of chondrule droplets, or it could simply mean that the largest closed unit of the droplet splash (subcloud) is so small, but many such expanding cloudlets of chondrules are ejected. The elongated streams of debris found in Asphaug et al. (2011) could be regarded as a string of such smaller mass cloudlets.

This is shown more directly in Figure 4, which shows the parameter space of the model, with the gray area again showing the models that give cooling rates between 10 and 3000 K hr\(^{-1}\).

**4. Time-Dependent Radiative Transfer**

So far we have only made an estimate of the cooling behavior of the cloud of liquid chondrules. Let us now calculate the temperature profile with a full treatment of time-dependent radiative transfer. The time-dependence comes in only due to the time it takes the lava droplets to convert their heat into radiation, not in the form of light-travel time (which typically only plays a role for media at temperatures above \( 10^{5} \) K). This can be easily verified by comparing the radiative energy density \( aT^{4} \) to the material energy density \( \rho_{cm} T \), which in our case always satisfies \( aT^{4} \ll \rho_{cm} T \). Therefore the radiative transfer equation itself is stationary, and the time-dependence is in the equation for heating/cooling of the chondrules.
4.1. Equations of Time-dependent Radiative Transfer

The formal radiative transfer equation is (see, e.g., Mihalas & Mihalas 1999)

\[ n \cdot \nabla I(x, n) = \rho \kappa (B(x) - I(x, n)) \]  

(23)

where \( I \) is the frequency-integrated mean intensity in erg cm\(^{-2}\) ster\(^{-1}\), \( x \) is the position vector in space, \( n \) is the unit vector of direction of the radiation, and finally \( B \) the frequency-integrated Planck function given by

\[ B(T) = \frac{\sigma}{\pi} T^4. \]  

(24)

We will use frequency-integrated quantities because the opacity of a chondrule is expected to be roughly equal to the geometric cross section, independent of frequency, because the chondrules are much larger than the wavelength of infrared radiation. In this case the frequency-dependence of the radiative transfer does not need to be considered. Equation (23) is called the “formal transfer equation,” and describes the transport of radiation along rays of direction \( n \). It is easy to integrate if the value of \( B(x) = B(T(x)) \) is known everywhere, but since we want to calculate the temperature \( T(x) \), this function is part of the thing we want to solve.

In our 1D spherically symmetric setting Equation (23) can be written as

\[ \frac{dI_\mu(r)}{dr} + \frac{1 - \mu^2}{r} \frac{dI_\mu(r)}{d\mu} = \alpha(B(T(r)) - I_\mu(r)) \]  

(25)

where we shortened

\[ \rho \kappa =: \alpha. \]  

(26)

Here \( \mu = \cos(\theta) \) where \( \theta \) is the angle between the ray along which the radiation is followed and the radially outward pointing unit vector. The Eddington factor method of solving this equation relies on the definition of the first three angular moments of the intensity at each location \( r \):

\[ J(r) = \frac{1}{2} \int_{-1}^{+1} I_\mu(r) d\mu \]  

(27)

\[ H(r) = \frac{1}{2} \int_{-1}^{+1} I_\mu(r) \mu d\mu \]  

(28)

\[ K(r) = \frac{1}{2} \int_{-1}^{+1} I_\mu(r) \mu^2 d\mu. \]  

(29)

Here \( J \) is called the mean intensity and \( H \) is the flux divided by \( 4\pi \). By integrating Equation (25) over \( (1/2)d\mu \) after multiplying it by \( 1 \) and by \( \mu \), respectively, one obtains the first two moment equations:

\[ \frac{1}{r^2} \frac{d(r^2 H)}{dr} = \alpha(B(T) - J) \]  

(30)

\[ \frac{dK}{dr} + \frac{3f - 1}{r} J = -\alpha H. \]  

(31)

These are two equations with three unknowns. To close this set of equations we write

\[ K(r) = f(r) J(r) \]  

(32)

where \( f \) is called the “Eddington factor.” If \( f(r) \) is known, then the two moment equations are in closed form and can be solved for \( J(r) \) and \( H(r) \). From the moment equations alone we cannot know what \( f(r) \) is for all \( r \), but for certain limiting cases we know their values: for optically thick media \( f = 1/3 \). Outside of the cloud for \( r \rightarrow \infty \) we will get \( f \rightarrow 1 \), which is the free-streaming limit. However, for general \( r \) and for general values of the optical depth we must employ another method of calculating \( f(r) \). We do this by integrating, for the current value of \( T(r) \) (and thus \( B(T(r)) \)), the formal transfer equation Equation (25) using the “tangent ray method” (see, e.g., Mihalas & Mihalas 1999). By integrating the resulting intensities over the angle we calculate the current estimates of \( J(r) \) and \( K(r) \). Let us call these \( J_{\text{fe}}(r) \) and \( K_{\text{fe}}(r) \) (where “fe” stands for “formal transfer equation”). Our current estimate of \( f(r) \) is then

\[ f(r) = \frac{K_{\text{fe}}(r)}{J_{\text{fe}}(r)}. \]  

(33)

This is then the \( f(r) \) function we stick into Equation (32), so that the moment equations can be solved for \( J(r) \) and \( H(r) \). We also need to impose boundary conditions at the inner and outer edge. At the inner edge we take \( H = 0 \) (zero flux). At the outer edge we could set \( H = J \), which is valid if the outer edge of our computational domain is at \( r \rightarrow \infty \). For finite outer radius we set instead

\[ H(r_{\text{out}}) = \frac{H_{\text{fe}}(r_{\text{out}})}{J_{\text{fe}}(r_{\text{out}})} J(r_{\text{out}}). \]  

(34)

Note that we can choose to set the outer edge of our computational domain \( r_{\text{out}} \) at \( r_{\text{out}} = R_{\text{cloud}}, \) but we can also set it at \( r_{\text{out}} > R_{\text{cloud}}, \) as long as we properly set the boundary condition at \( r = r_{\text{out}} \) according to Equation (34).

We can now combine the two moment equations into the following form:

\[ \frac{1}{r} \frac{d}{dr} \left\{ \frac{r^2}{\alpha} \left[ \frac{d(f J)}{dr} + \frac{3f - 1}{r} J \right] \right\} = J - \frac{\sigma}{\pi} r^4. \]  

(35)

This can be solved, together with the boundary conditions, using a matrix equation. The details of this are discussed in the Appendix.

Next we must compute how the temperature \( T(r) \) reacts to the radiation field, or in other words, how \( T(r) \) radiatively cools with time. The energy equation is

\[ \rho c_m \frac{dT}{dt} = 4\pi \alpha \left( J - \frac{\sigma}{\pi} T^4 \right) \]  

(36)

where \( \rho \) is given by Equation (6). The \( d/dt \) operator is the convective derivative, i.e., the time derivative is computed for a given chondrule. Equation (36) is a local equation at each location. The non-locality of the heating/cooling is only through the non-locality of the equation for \( J(r) \) (Equation (35)).

The way we solve these equations numerically is by setting up a radial grid of \( N \) gridpoints \( r_i \) (with \( 1 \leq i \leq N \)) upon which we define the mean intensity \( J_i \) and the temperature \( T_i \). Since the cloud is expanding we let the grid points move along with the material (Lagrangian approach) so that the location of grid point \( i \) at time \( t \) is

\[ r_i(t) = r_i(t_0) + \eta_i v_{\text{exp}} t \]  

(37)

where \( \eta \) is the dimensionless radial coordinate

\[ \eta = \frac{r}{R_{\text{cloud}}} = \frac{r}{v_{\text{exp}} t}. \]  

(38)
Figure 5. Temperature evolution of the fiducial expanding cloud models F1 (top left), F2 (top right), F3 (bottom left), and F4 (bottom right). See Table 1 for the parameters. The temperatures are shown at three positions in the cloud: the center, at 80% of the radius and at 90% of the radius (near the surface of the cloud). The vertical line shows the time $t_{\text{cool}}$ of Equation (16).

The values of $\eta_i$ do not change with time. Each chondrule stays at constant $\eta_i$ and so the comoving time derivative $d/dt$ used in, e.g., Equation (36), becomes simply the time derivative of that quantity at a given $\eta_i$ grid point.

In principle one could imagine a simple time-dependent numerical integration method for the combination of Equations (35) and (36). We know $J_n^t$ and $T_n^t$ at time $t = t_n$, and we can solve for $T_n^{t+1}$ by taking an explicit Euler integration step of Equation (36). Then we can solve the moment equations (because now we have $B_n^{t+1} = B(T_n^{t+1})$) and obtain $J_n^{t+1}$. The problem with this scheme is that the required time steps can become very small due to numerical stiffness. A more robust method is to integrate the complete system Equations (35) and (36) using an implicit integration scheme. This is discussed in the Appendix.

4.2. Results of the Time-dependent Radiative Transfer Models

The results of the time-dependent radiative transfer calculations are shown in Figure 5. These results confirm the estimates made in Section 3. We find that the shape of the cooling curves is mostly the same for all models, except for the time scaling, which we know from the analytically expression of $t_{\text{cool}}$ (Equation (16)). Only for the rather extreme (and presumably unrealistic) case such as fiducial model F4 with $R_{\text{melt,0}} = 0.01$ km and $v_{\text{exp}} = 1000$ m s$^{-1}$ do we find from the full radiative transfer model that the cooling is slower than the analytic solution. This is because in that case the condition that $\tau_{\text{cool}}$ (Equation (20)) is $\gg 1$ is broken and the cooling time scale will then not be determined by the optical depth decline but by the effectiveness by which a chondrule can convert its heat into radiation. These cases, however, typically imply time scales of seconds rather than hours, and these are anyway too short to be consistent with chondrule textures.

It seems that the analytic solutions are fairly accurate when $\tau_{\text{cool}} \gtrsim 10$, which is always true for the interesting regime of parameter space.

From Figures 2 and 5 we see that the model systematically predicts a period of constant temperature with a duration similar to the later cooling phase. This is true at least near the center of the cloud. Near the surface this temperature plateau is shorter, but the cooling time is similar.

In Figure 6 we show, for model F1, how the temperature looks as a function of radial position in the cloud, for different times after the collision. Since the cloud is expanding, which would make it difficult to compare the temperature profiles at different times, we use the radial coordinate relative to the outer cloud radius. One sees that the outer regions start cooling earlier because the optical depth to the surface is smaller. Also one sees that a negative radial temperature gradient is produced, which causes the radiative diffusive energy to transport to the surface.

The cooling curve from our model is significantly different from the ones predicted from shock heating (e.g., Morris & Desch 2010; Morris et al. 2012). The shock heating models...
predict a radiative preheating phase, a temperature spike with rapid cooling, and depending on geometric conditions followed by a slower cooling phase. In our model the temperature is high from the start (or for high-speed collisions: upon impact), stays relatively constant for some time and then drops over a similar time scale.

5. DISCUSSION

The scenario of splashes of melt droplets originating from colliding pre-molten planetesimals has been discussed at length in several papers by Sanders and coworkers (e.g., Sanders & Scott 2012) and the paper by Asphaug et al. (2011). These papers discuss how this scenario holds up against the many meteoritic constraints. We will not repeat these arguments here, but refer for those discussions to those papers. Instead, we will focus on the discussion of aspects related to our splash cooling model.

5.1. Splash Geometry

Our model describes a simple spherical expanding cloud as a model for (part of) the splash resulting from colliding bodies. It is obvious that this is a drastic simplification. The SPH model of Asphaug et al. (2011) shows a much more realistic geometry involving, among other complexities, tube-like geometries. A proper 3D time-dependent radiative transfer simulation performed in concert with the gravito-hydrodynamic simulation of the impact splashing is required to know precisely what the cooling behavior is. Nevertheless we predict that our result can be used to estimate the results of those detailed computations: the tube-like geometry can be approximated as a homologously expanding cylinder, expanding not only in the two perpendicular directions but also in the longitudinal direction. Its properties are then not too much different from a chain of spherically expanding clouds with diameters similar to the diameter of the tube, except that these spherical cloud cannot cool in all directions ($4\pi$) but only in part of the sky (in the other part it will heat its neighbors and be heated by its neighbors). We would then expect that the cooling time $t_{\text{cool}}$ will be a bit longer but not by a large factor.

Also, one can expect some parts of the splash to be ejected at slow speed compared to the escape speed, so that it may fall back to the remaining (unsplashed) bodies and accrete on them. This evidently breaks the assumption of homologous expansion, and as a result will also deform any initially spherical sub-cloud into a more pancake-like shape. The idea of regarding the splash cloud as a collection of spherical sub-clouds is then clearly no longer valid, not even in an approximative way. In such cases a full 3D radiation-gravito-hydrodynamic simulation of the splash is unavoidable. However, multi-dimensional radiation-hydrodynamics is an enormously challenging problem, in particular for problems involving such extremely complex geometries. It is an interesting challenge for the future. For obtaining order-of-magnitude estimates we may be forced to stick, for the time being, with simple models such as the one presented in this paper.

5.2. Chondrule Sizes

According to the model by Asphaug et al. (2011), by which the lava drop size is calculated from the initial pressure under which the magma was in the pre-molten planetesimals before the collision, the planetesimal sizes must have been 10 km or larger. Since our cloud of lava droplets can be one of a number of such clouds ejected from the impact, the $R_{\text{melt,0}}$ of our model cannot be directly compared to that $>10$ km size lower limit from Asphaug. But it does give an indication that, if we adopt their model of the chondrule sizes, it seems reasonable to be looking to values of roughly the order of $R_{\text{melt,0}} \sim 10$ km or larger. To elucidate our reasoning: if we collide two fully molten planetesimals of 20 km radius, and we model the splash with 10 spherically expanding droplet clouds, then we would get $R_{\text{melt,0}} \sim 12$ km. In Figure 3 this means we are looking at the region of the diagram above the dotted line.

There is a caveat, however: What tells us that it is just 10 clouds, not 1000? This brings us to the complicated issue of the actual geometry of the splash. If we “decompose” the splash geometry into many small spherical clouds we must assure that these clouds can radiatively cool to the outside. In other words: we cannot consider a big spherical cloud as consisting of many smaller sub-clouds, because they will irradiate each other and not cool much. But if the cloud geometry is a long extended streak, it can be considered as consisting of a chain of spherical subclouds. We are here confronted with the limitation of our spherical expanding cloud model.

5.3. Partial Pre-melting

The reason why pre-melting seems to be necessary for the impact splash scenario to work is that very high-speed collisions ($>5$ km s$^{-1}$) needed to produce melt from cold planetesimals are presumably rare. However, pre-melting may lead to differentiation of the planetesimals, causing the iron and nickel to sink into the core and producing a basaltic mantle. Chondrules formed from such objects would be very non-solar in composition. Although Sanders & Scott (2012) argue that convection reduces the efficiency of the differentiation and that some amount of differentiation is in fact consistent with e.g., L and LL chondrites, it seems that this issue is not yet conclusively solved and more detailed and quantitative modeling of the differentiation process in these molten planetesimals may need to be done.

However, a middle way is if planetesimals were pre-heated to elevated temperatures below the melting point. This is to be expected in particular for later generation planetesimals (i.e., planetesimals formed at a time when a substantial fraction of the
26Al has already decayed). Lower velocity collisions might then be sufficient to add the remaining energy needed to produce melt. The problem with this scenario is that it requires fine-tuning: The planetesimal must have formed at a time when the remaining abundance of 26Al is low enough not to melt the planetesimal but high enough to heat it to temperatures only slightly below the melting temperature. This might be too much coincidence.

On the other hand, we know that some planetesimals were strongly differentiated: the parent bodies of iron and basaltic meteorites. If chondrules formed from collisions between planetesimals, one would expect some chondrules to have been formed from collisions with such differentiated planetesimals. Some chondrules should have compositions that are very iron poor, presumably more iron-poor than LL-chondrites. This raises the question: Where are these? One possible answer is that chondrules produced in these very early phases (<1 million years after calcium-aluminum-rich inclusions, CAI, formation) were either accreted into the sun or reincorporated into other bodies, where they were again molten. But if this process of elimination is so efficient, why did some CAIs survive and get incorporated into the (later) chondrites? This shows that while the impact splashing model is appealing, there is more work to be done to answer such questions.

### 5.4. Compound Chondrules

In our homologous expanding cloud model the melt droplets move away from their nearest neighbor at extremely low velocity (millimeters per minute). Small inhomogeneities (which are of course unavoidable in such a messy process as an impact) would lead to some droplets actually colliding. When they do so during the initial constant temperature phase, they presumably coalesce and form a larger melt droplet. However, when they collide during the cooling phase, they may have become cool enough to be plastic but not liquid anymore. They will then produce a compound chondrule.

It would be useful if we could quantify the percentage of chondrules that become compound in this way. However, at present we see no way how we can estimate the random relative velocities between adjacent chondrules in the expanding clouds.

Many compound chondrules, however, clearly show one chondrule having remained rigid, with the other having been indented. Therefore, clearly they were of different temperature. This is difficult to achieve in our simple spherical expanding cloud model. For this it will be necessary to have other debris (e.g., another cloud-fragment emerging from the impact site) to interdisperse with our cloud of droplet, so as to have chondrules from different thermal environments to mix. And this must happen on a time scale shorter than the time scale of thermal equilibration between the chondrules from these different origins. To make this more quantitative would be, however, a challenge. It would likely require us to contemplate complex splash geometries.

Metzler (2012) found that some chondrites in fact contain entire large-scale clusters of compound chondrules. A possible explanation could be that some of the impact debris may have fallen back onto the remainder of the biggest of the two colliding planetesimals. If this happens quickly enough, so that this re-accretion occurs before the chondrules have cooled below the temperature at which the chondrules are still sticky, then it is reasonable to assume that such clusters may form as chondrules simply fall onto each other.

### 5.5. What about Relict Grains and Rims?

Another problem with our simple spherically symmetric expanding plume model is that it does not account for any possible "pollution" of the melt droplets with other (non-molten) debris. Some chondrules contain relict grains (Jones 1996), which must have been inserted while the chondrule was still liquid, but the chondrule must have cooled very soon thereafter. As in the case with the compound chondrules (Section 5.4) this would require debris from a different part of the impact debris cloud to interdisperse with our cloud of droplets. This foreign debris must have originated from non-molten parts of the original planetesimals, e.g., the regolith or pristine dust covering the crust of the original planetesimals. Only those grains from that non-molten debris that entered late enough can enter the chondrules shortly before they cool below the solidus, and thus survive. As with the compound chondrules, however, obtaining quantitative estimates of this process is challenging.

### 5.6. Producing Chondritic Parent Bodies

The impact model may provide a plausible scenario for the formation of chondrules, but how do these chondrules (and matrix) accrete to form another planetesimal (the parent body of the later chondritic meteorite)? A nice aspect of the low-velocity collision of pre-molten planetesimals model is that much of the debris may, in fact, reaccrete onto one of the surviving original planetesimals (or what remains of it). The idea is that due to the high gas density in the protoplanetary disk the stochastic planetesimal motions are of low velocity so that when they collide, they collide essentially at velocities that are only just above their mutual escape speed. Since much of the energy is dissipated in the inelastic impact, most of the debris will not have enough kinetic energy to escape the system and will (after perhaps a flyby or two) eventually accrete onto a single body or a binary body. However, there is evidence that inside a single chondrite there exist chondrules of different ages (Villeneuve et al. 2009). And some chondrules experienced multiple heating events. Somehow chondrules produced in multiple chondrule-forming impacts must have mixed together and form a single chunk of rock that later is observed as a chondritic meteorite. The immediate re-accretion scenario may make this somewhat difficult because once the collision releases most of the magma, and much of this is reaccreted as chondrules (for the molten part) and matrix (for the non-molten part) the remaining body will no longer be a sphere of magma.

If, however, the collisions would be high-speed enough that much of the debris escapes, the newly formed chondrules will disperse into the protoplanetary disk. There they mix with the pool of other chondrules of various ages, and then later accrete into a chondrite parent body. An important question in this scenario is how aerodynamic drag and the resulting radial drift affects this: Will the chondrules stay long enough in the asteroid belt region before they drift away or not? Jacquet et al. (2012) study this question in detail.

Perhaps more likely is a combination of the two. In particular the cluster chondrites of Metzler (2012; see Section 5.4) can be easily understood in terms of the reaccretion scenario while the multiple ages seems to point more toward a dispersion and later accretion scenario.

### 5.7. Volatile Elements

One of the main characteristics of most chondrules is their "normal" abundance of volatile elements such as Na, K, etc.
If a chondrule would stay at a high temperature (e.g., 2000 K) for more than a few minutes, these elements would have evaporated out of the chondrule and leave a volatile-depleted chondrule behind. We believe that the impact splash scenario may naturally resolve this issue, because during the hot phases of the expanding impact splash the density of the cloud is so high that the vapor quickly reaches the saturation pressure and evaporation stops. However, this idea must be verified by actual modeling: Will the evaporation indeed saturate? Will the temperature drop below the evaporation temperature before the density of the expanding cloud becomes too low to saturate the pressure? Answering these questions with explicit calculations is the topic of paper II in this series.

5.8. What Kind of Chondrules Qualify for This Scenario?

So far we have not made any distinction between different kinds of chondrules or chondrites. Given that chondrule properties vary considerably between chondritic classes, the natural question arises: Do all chondrules form through the same mechanism or are different chondrule formation scenarios responsible for different kinds of chondrules? And for which kind, if at all, could the impact splash scenario of this paper be applicable?

A special class of chondrites and chondrules is the CB and CH class. It has been recognized for some time that these chondrules may indeed have originated from colliding planetesimals or planets (see Desch et al. 2012 for a discussion). The evidence for this includes (1) the cryptocrystalline structure of CB chondrules, (2) their strong depletion of volatile elements, (3) zoned metal spherules that appear to have been condensed out of the gas phase in a highly energetic single-staged process (Krot et al. 2005), and (4) their young age, 5 Myr after CAIs. One can interpret these properties as meaning (1) that these chondrules must have cooled more rapidly (seconds to minutes) than “normal” chondrules (hours) to explain their textures, (2) that the vapor-melt plume of the impact must have rapidly become of low enough density to allow volatile elements to get out of the chondrules and not recondense, (3) the impact must have been of very high velocity (several km s⁻¹) to produce metal vapor, (4) which appears consistent with the young age, which means low gas densities in the protoplanetary disk and thus allows for high-speed impacts. Arguments (1) and (2) seem to require a rapid reduction of the density and optical depth, which can be achieved by a high impact velocity (also required for (3) and consistent with (4)) and a low mass of the impactor. Krot et al. (2005) argue, instead, for a very high mass collision to explain CB chondrules. Perhaps the adiabatic cooling of the impact-shock-compressed rock is enough to explain the fast cooling, or only a fraction of the mass from the giant impact actually gets converted into a plume of chondrules.

In contrast to the case for the CB/CH chondrules our model may explain the properties of “normal” chondrules: (1) barred or porphyritic textures indicating slower cooling (hours), (2) little depletion of volatiles (see Section 5.7), (3) possibly lower temperatures, and (4) earlier times after CAIs. The slower cooling rates come naturally out of our model because the optical depth of the cloud keeps the chondrules warm for a while. The little depletion of volatiles may be achieved by the high densities (though a quantitative analysis has to wait until Paper II in this series). And the lower temperatures are natural for lower impact velocities which are expected in the earlier phases of the protoplanetary disk. The melt is the molten interior of the 26Al-heated planetesimals, which also is expected during the earlier phases.

We might also be able to apply our radiatively cooling expanding cloud model to the high impact velocity scenario for CB/CH chondrules, but it would require us to include more physics, in particular a proper equation of state for the shock-heated rock and the subsequent adiabatic expansion and adiabatic cooling.

Among the “normal” chondrules there is also the issue of radial and barred textures versus porphyritic textures. The latter are the majority (about 85% of chondrules, Gooding & Keil 1981). Radial textures require that no nucleation sites were present in the melt, so that crystallization in the supercooled melt droplet starts from a single site and progresses radially outward from that site. In the splash scenario this means that the melt in the original pre-molten planetesimals was well above the liquidus, so that any previously existing crystals were molten. Conversely, for porphyritic chondrules numerous nucleation sites must have been present in the melt, and thus must have presumably remained present in the magma of the colliding planetesimals, meaning that that magma was likely slightly below the liquidus temperature. The impact may have heated the melt through shock-heating above the liquidus, but if the cooling sets in quickly enough some of these nucleation sites may survive. The question of porphyritic textures and the origin of the nucleation sites in the splash scenario remains, however, a tricky one. Sanders & Scott (2012) propose instead that these nucleation sites may have entered into the melt droplets as pollution from e.g., the surface regolith that may have been mixed in with the chondrules. The problem with this (as with the relict grain issue of Section 5.5) is that the timing of mixing must be ideal: too early and the temperature is still above the liquidus and these regolith particles will also melt; too late and the chondrules have already formed radial or barred textures.

Finally, there is the issue of Fe-content of chondrules: the H, L, and LL classes. If the pre-molten planetesimals have experienced strong differentiation, it is possible that the mantles of these planetesimals are poor in iron and nickel. If the masses of the planetesimals are low enough and convection might be present, then perhaps all too strong loss of Fe in the mantle can be avoided. The issue of H, L, and LL chondrites in the context of the splash scenario is further discussed in Sanders & Scott (2012).

6. SUMMARY AND CONCLUSION

This paper studies the radiative cooling of an expanding cloud of lava droplets originating from the low-velocity collision between two partly or fully pre-molten planetesimals. The model is also valid for high-speed collisions between cold planetesimals, in which the melt is produced by the collision itself.

The main result of this paper is a self-consistent cooling profile for chondrules based on the initial conditions of the expanding plume of melt droplets (chondrules). These cooling profiles are characterized by a time scale t_{cool} given by Equation (16) and are well approximated by a constant temperature for \( t < t_{cool} \) and a powerlaw dropoff (Equation (18)) for \( t > t_{cool} \). The cooling rate is given by Equation (17). The cooling time scale t_{cool} depends on the total mass of the plume and on its expansion velocity. Both parameters are related to the conditions of the collision between the two planetesimals.

A large total mass of the expanding cloud of chondrules means that the colliding planetesimals must have been at least of that mass, but presumably considerably more massive since not all of the matter is likely to end up in a single expanding plume.
The expansion velocity of the droplet cloud is related to the collision velocity of the two planetesimals: it is unlikely that it is higher than the impact velocity. Note that the expansion velocity is different from (and presumably much less than) the velocity by which the cloud moves away from the impact site. To relate both parameters (\(M_{\text{cloud}} \) and \(v_{\text{exp}}\)) to parameters of the colliding planetesimals it is necessary to perform 3D hydrodynamic simulations of the impact, such as those performed by Asphaug et al. (2011).

The results of our model, when compared to known cooling time scales of chondrules, put constraints on \(M_{\text{cloud}} \) and \(v_{\text{exp}}\), and thus on the conditions under which planetesimal collisions can produce chondrules.

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**APPENDIX**

**TIME-DEPENDENT RADIATIVE TRANSFER: DISCRETIZATION AND IMPLICIT INTEGRATION**

Solving the radiative transfer moment equations coupled to the time-dependent heating/cooling equation requires an implicit integration scheme. The two coupled equations to solve are

\[
F_1(J, T) := \frac{1}{\alpha} \frac{d}{dr} \left[ \frac{d(f J)}{dr} \frac{3 f - 1}{r} \right] - J + \frac{\sigma}{\pi} T^4
\]

and

\[
F_2(J, T) := \rho c_e \frac{dT}{dt} - 4 \pi \alpha \left( J - \frac{\sigma}{\pi} T^4 \right) = 0.
\]

The inner boundary condition is:

\[
F_1(J, T, r = r_{\text{in}}) := H(r = r_{\text{in}}) = 0
\]

where \(r_{\text{in}}\) is the smallest radius of our grid, which we take \(r_{\text{in}} \ll R_{\text{cloud}}\). This condition translates into

\[
\left[ \frac{d(f J)}{dr} \frac{3 f - 1}{r} \right]_{r = r_{\text{in}}} = 0.
\]

The outer boundary condition is

\[
F_1(J, T, r = r_{\text{out}}) := H(r = r_{\text{out}}) - J = 0
\]

with \(H = H_{\text{fe}}(r_{\text{out}})/J_{\text{fe}}(r_{\text{out}})\), which translates into

\[
F_1(J, T, r = r_{\text{out}}) := \left[ \frac{d(f J)}{dr} \frac{3 f - 1}{r} J + h \alpha J \right]_{r = r_{\text{out}}} = 0.
\]

The only time-derivative in the equations is the one on the temperature. The radiation field is assumed to immediately adapt.

We put \(J(r, t)\) and \(T(r, t)\) on a spatial grid \(\{r_1, \ldots, r_N\}\). So for time step \(n\) we have the values \(J^n_1, \ldots, J^n_N\) and \(T^n_1, \ldots, T^n_N\). The above equations are also to be evaluated at these grid points:

\[
F_{1,1} = 0, \ldots, F_{1,N} = 0 \quad \text{and} \quad F_{2,1} = 0, \ldots, F_{2,N} = 0.
\]

Let us define

\[
Q^n = (J^n_1, T^n_1, J^n_2, T^n_2, \ldots, J^n_N, T^n_N)^T
\]

and

\[
F(Q) = (F_{1,1}(Q), F_{2,1}(Q), F_{2,2}(Q), \ldots, F_{1,N}(Q), F_{2,N}(Q))^T.
\]

The objective of the time-integration of these equations is to find the values of \(Q^{n+1}\): the values of \(Q\) at the next time step.

For numerical stability we express all instances of \(Q\) in the equations \(F\) in their future form: \(Q^{n+1}\), except when a time-derivative of \(Q\) is used, which is written as \(\partial Q/\partial t = (Q^{n+1} - Q^n)/\Delta t\) and where the present value of \(Q^n\) is required. We thus obtain as our set of equations:

\[
F(Q^{n+1}) = 0
\]

which we need to solve for \(Q^{n+1}\). If \(F\) were linear in \(Q^{n+1}\) then one could write the vector \(F\) as a matrix multiplication with the vector \(Q^{n+1}\) plus a constant vector \(R\):

\[
F = MQ^{n+1} + R.
\]

Then the solution to \(F = 0\) would be just a matrix inversion:

\[
Q^{n+1} = -M^{-1} R.
\]

However, \(F\) is non-linear in \(Q^{n+1}\) because the Planck function \(B(T) = (\sigma/\pi)(T^4)^n\). Therefore we have a non-linear function \(F(Q^{n+1})\). So let us start with an initial guess \(Q^{n+1}_0\). We typically then have

\[
F(Q^{n+1}_0) \neq 0.
\]

Ideally we want to find a \(Q^{n+1}_k\) for which \(F(Q^{n+1}_k) = 0\), but we can only use Newton’s method by approximating \(F(Q^{n+1}_k)\) using first-order Taylor expansion:

\[
F(Q^{n+1}_k) \approx F(Q^{n+1}_0) + \frac{\partial F}{\partial Q} (Q^{n+1}_k - Q^{n+1}_0) = 0
\]

or more in general for the \(k\)th iteration:

\[
F(Q^{n+1}_k) \approx F(Q^{n+1}_{k-1}) + \frac{\partial F}{\partial Q} (Q^{n+1}_k - Q^{n+1}_{k-1}) = 0.
\]

If we define

\[
\Delta Q^{n+1}_k = Q^{n+1}_k - Q^{n+1}_{k-1};
\]

we can solve for \(\Delta Q^{n+1}_k\):

\[
\Delta Q^{n+1}_k = - \left( \frac{\partial F}{\partial Q} \right)^{\text{inv}} F(Q^{n+1}_k)
\]

and then we obtain the new

\[
Q^{n+1}_k = \Delta Q^{n+1}_k + Q^{n+1}_{k-1}.
\]

Now let us write out the discrete equations \(F_1(J, T)\) and \(F_2(J, T)\) explicitly:
For the gas equation we obtain:

$$F_{1,i}(J, T^{n+1}) = \frac{1}{\alpha_i r_{i}^{2} \Delta r_i} \left[ \frac{r_{i+1/2}^{2} f_i J_{i+1} - f_i J_i}{\alpha_{i+1/2} \Delta r_{i+1/2}} - \frac{r_{i-1/2}^{2} f_i J_i - f_i J_{i-1}}{\alpha_{i-1/2} \Delta r_{i-1/2}} \right] - \frac{(3 f_{i+1/2} - 1) (J_{i+1} + J_i) r_{i+1/2}}{2 \alpha_{i+1/2}} + \frac{(3 f_{i-1/2} - 1) (J_{i-1} + J_i) r_{i-1/2}}{2 \alpha_{i-1/2}} - J_i + \frac{\sigma}{\pi} (T_i^{n+1})^4 = 0. \quad \text{(A18)}$$

Boundary conditions only have to be applied to the first equation. The inner boundary condition:

$$F_{1,1}(J, T^{n+1}) = \frac{f_2 J_2 - f_1 J_1}{\alpha_{3/2} \Delta r_{3/2}} + \frac{3 f_3 J_3 - 1}{\alpha_{3/2}^2 r_{3/2}^2} = 0. \quad \text{(A20)}$$

The outer boundary condition:

$$F_{1,N}(J, T^{n+1}) = \frac{f_N J_N - f_{N-1} J_{N-1}}{\alpha_{N-1/2} \Delta r_{N-1/2}} + \frac{1}{2} (J_N + J_{N-1}) \times \left[ \frac{3 f_N - 1}{\alpha_{N-1/2}^2 r_{N-1/2}^2} + h \right] = 0. \quad \text{(A21)}$$

The energy equation can be rescaled to

$$F_{2,i}(J, T^{n+1}) = T_i^{n+1} - T_i^n - \frac{4 \pi \rho \alpha_i \Delta t}{\rho c_v} \times (J_i - \frac{\sigma}{\pi} (T_i^{n+1})^4 - q \Delta t \rho c_v = 0. \quad \text{(A22)}$$

The matrix coefficients then become

$$\left( \frac{\partial F_{1,i}}{\partial J_i} \right) = -\frac{1}{\alpha_i r_{i}^{2} \Delta r_i} \left[ \frac{r_{i+1/2}^{2} f_i J_{i+1} + r_{i-1/2}^{2} f_i J_{i-1}}{\alpha_{i+1/2} \Delta r_{i+1/2}} + \frac{r_{i+1/2}^{2} f_i J_i + r_{i-1/2}^{2} f_i J_i}{\alpha_{i-1/2} \Delta r_{i-1/2}} \right] - 1 \quad \text{(A23)}$$

$$\left( \frac{\partial F_{1,i}}{\partial J_{i+1}} \right) = \frac{1}{\alpha_i r_{i}^{2} \Delta r_i} \left[ \frac{r_{i+1/2}^{2} f_i J_{i+1} + (3 f_{i+1/2} - 1) r_{i+1/2}}{\alpha_{i+1/2} \Delta r_{i+1/2}} \right] - \frac{1}{\alpha_i r_{i}^{2} \Delta r_i} \left[ \frac{r_{i-1/2}^{2} f_i J_{i-1} + (3 f_{i-1/2} - 1) r_{i-1/2}}{\alpha_{i-1/2} \Delta r_{i-1/2}} \right] \quad \text{(A24)}$$

$$\left( \frac{\partial F_{1,i}}{\partial J_{i-1}} \right) = \frac{1}{\alpha_i r_{i}^{2} \Delta r_i} \left[ \frac{r_{i+1/2}^{2} f_i J_{i+1} + (3 f_{i+1/2} - 1) r_{i+1/2}}{\alpha_{i+1/2} \Delta r_{i+1/2}} \right] - \frac{1}{\alpha_i r_{i}^{2} \Delta r_i} \left[ \frac{r_{i-1/2}^{2} f_i J_{i-1} + (3 f_{i-1/2} - 1) r_{i-1/2}}{\alpha_{i-1/2} \Delta r_{i-1/2}} \right] \quad \text{(A25)}$$

$$\left( \frac{\partial F_{1,i}}{\partial T_i^{n+1}} \right) = \frac{4 \pi \alpha_i}{\rho_i c_v} (T_i^{n+1})^3 \quad \text{(A26)}$$

$$\left( \frac{\partial F_{1,i}}{\partial J_i} \right) = -\frac{4 \pi \alpha_i}{\rho_i c_v} \Delta t \quad \text{(A27)}$$

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