Sea polarisation and the Drell-Hearn-Gerasimov sum-rule(s)

S. D. Bass

Institut für Kernphysik, KFA-Jülich, D-52425 Jülich, Germany

ABSTRACT

The Drell-Hearn-Gerasimov sum-rule is really two sum-rules: one for each of the valence and the sea/glue contributions to the nucleon wavefunction. The convergence of these sum-rules follows from the Froissart bound for spin dependent processes in QCD and is necessary for the consistency of the constituent quark model of low energy QCD. Some challenges for future polarised photoproduction experiments, for example at ELFE, are discussed.
1 Introduction

The EMC spin effect [1] has inspired a great deal of interest in QCD spin physics — both at high and at low energies. Polarised deep inelastic scattering experiments are presently underway at CERN, DESY and SLAC to measure the spin dependent nucleon structure functions at high energy and high $Q^2$ [2]. Experiments will soon begin at ELSA and MAMI [3] to measure the spin dependent photoproduction cross sections with a view to testing the Drell-Hearn-Gerasimov sum-rule [4] (for reviews see [5,6]). This paper is about the Drell-Hearn-Gerasimov sum-rule. We explain why the no-subtraction hypothesis, which underlies the dispersion relation derivation of this sum-rule, is correct because of the unitarity bound in QCD. We then explain why the validity of this sum-rule is necessary for the constituent quark model of low energy QCD. Finally, we discuss some challenges for future spin photoproduction experiments, for example at the proposed future ELFE facility [7,8].

The Drell-Hearn-Gerasimov sum-rule [4,5] relates the difference in the spin dependent inclusive cross sections for a real transverse photon scattering from a longitudinally polarised nucleon target to the anomalous magnetic moment of the target nucleon. Using standard dispersion relation theory and assuming no need for subtractions, we consider

$$\text{Re} f_2(\nu) = \frac{\nu}{4\pi^2} \int_0^\infty d\nu' \frac{\nu'}{(\nu'^2 - \nu^2)(\sigma_{\frac{1}{2}} - \sigma_{\frac{3}{2}})(\nu')},$$

where $\nu$ is the photon energy and $f_2(\nu)$ is the spin dependent part of the forward Compton scattering amplitude

$$f(\nu) = \chi_f^* \left[ f_1(\nu) \varepsilon_f \cdot \varepsilon_i + i f_2(\nu) \vec{\sigma} \cdot (\varepsilon_f^* \times \varepsilon_i) \right] \chi_i$$

Photoproduction spin sum-rules are obtained by taking the odd derivatives (with respect to $\nu$) of $\text{Re} f_2(\nu)$ in equ.(1), viz. $\left( \frac{d}{d\nu} \right)^{2n+1} \text{Re} f_2(\nu)$, and then evaluating the resulting expression at $\nu = 0$. For small photon energy $\nu \to 0$,

$$f_1(\nu) \to -\frac{\alpha}{m} + (\vec{\pi}_N + \vec{\beta}_N)\nu^2 + O(\nu^4)$$

and

$$f_2(\nu) \to -\frac{\alpha \kappa^2_N}{2m^2} \nu + \gamma_N \nu^3 + O(\nu^5)$$

Here $\kappa_N$ is the anomalous magnetic moment of the target nucleon, $m$ is the nucleon mass, and $\gamma_N$ and $(\vec{\pi}_N + \vec{\beta}_N)$ are the charge parity $C = +1$ spin and Rayleigh polarisabilities of the nucleon respectively. The Drell-Hearn-Gerasimov sum-rule is derived when we evaluate the
first derivative \((n = 0)\) of \(\text{Re} f_2(\nu)\) in equ.(1) at \(\nu = 0\), viz.
\[
\int_0^\infty \frac{d\nu'}{\nu'} \left( \sigma_{1/2}^N - \sigma_{3/2}^N \right) (\nu') = -\frac{2\pi^2 \alpha}{m^2 - \kappa^2_N}
\]
(5)

The third derivative of \(\text{Re} f_2(\nu)\) yields a second sum-rule for the spin polarisibility \(\gamma_N\)
\[
\int_0^\infty \frac{d\nu'}{\nu'^3} \left( \sigma_{1/2}^N - \sigma_{3/2}^N \right) (\nu') = \frac{1}{4\pi^2 \gamma_N}
\]
(6)

These photoproduction sum-rules have a parallel in high \(Q^2\) deep inelastic scattering where
the odd moments of the spin dependent structure function \(g_1\) project out the nucleon matrix
elements of gauge-invariant local operators, each of which is multiplied by a radiative coefficient.

2 The validity of the no-subtraction hypothesis

Besides the no-subtraction hypothesis, the Drell-Hearn-Gerasimov sum-rule is derived assuming
only very general principles: electromagnetic gauge invariance, Lorentz invariance, causality
and unitarity. The no-subtraction hypothesis is the only “QCD input” to the sum-rule. It
is correct because of the Froissart unitarity bound for \(\Delta \sigma = (\sigma_{1/2}^N - \sigma_{3/2}^N)\) at large \(\nu \to \infty\).

We consider the Regge theory [9], which works for both real and also complex energy. It is
well known [10,11] from spin dependent Regge theory that the iso-triplet contribution to \(\Delta \sigma\)
behaves as
\[
(\Delta \sigma)_3(\nu) \sim \nu^{\alpha_{a_1} - 1}
\]
(7)
at large \(\nu\). Here \(\alpha_{a_1}\) is the intercept of the \(a_1\) Regge trajectory \((-0.5 \leq \alpha_{a_1} \leq 0\) [12]) . The
iso-singlet contribution is still somewhat controversial and depends on the spin structure of the
short range part of the exchange potential [13]. The soft pomeron exchange, which dominates
the large \(\nu\) (centre of mass energy greater than about 10GeV) behaviour of the spin averaged
cross section, does not contribute to \(\Delta \sigma\) [11]. However, the physics which gives us the soft
pomeron can contribute to \(\Delta \sigma\) at large photon energy \(\nu\). If the pomeron were a scalar, then
we would find
\[
(\Delta \sigma)^{[1]}_0(\nu) \sim 0
\]
(8)
at large \(\nu\). In the Landshoff-Nachtmann approach [14], the soft pomeron is modelled by the
exchange of two non-perturbative gluons and behaves as a \(C = +1\) vector potential with a
correlation length of about 0.1fm. This vector pomeron gives a contribution [13,15]
\[
(\Delta \sigma)^{[2]}_0(\nu) \sim \frac{\ln \nu}{\nu}
\]
(9)
to $\Delta\sigma$ at large $\nu$. It has been suggested [16] that there may be a negative signature two pomeron cut contribution to $\Delta\sigma$ at large energy, viz.

$$\langle \Delta\sigma \rangle_0^{[3]}(\nu) \sim \frac{1}{\ln^2 \nu}$$

although the coefficient of this term in the Regge calculus was found to be zero in a recent re-analysis of the theory [17]. It is an important challenge to determine the strength of the three possible large $\nu$ contributions equ.(7-10) in future photoproduction experiments and make contact with the small $x$ behaviour of the high $Q^2$ deep inelastic spin structure function $g_1$ [13,15,18] which is also determined by Regge theory. Each of the possible contributions in equ.(7-10) give a convergent integral in equ.(1,5) and support the no-subtraction hypothesis that went into the derivation of the sum-rule. The Regge theory works for all complex energy [9] so one finds a vanishing contribution to the Drell-Hearn-Gerasimov sum-rule from “the half circle” at infinite complex momentum which appears when we close the contour in the dispersion relation derivation of the sum-rule. We would have needed a subtraction if the sea were generated all with the same polarisation, which is not what happens in QCD. (In this case $\Delta\sigma$ would grow to reach the Froissart bound for the spin averaged cross section ($\sigma_T \sim \ln^2 \nu$) and the integral in equs.(1,5) would not converge.) Henceforth, we accept the Drell-Hearn-Gerasimov sum-rule as valid.

### 3 The Drell-Hearn-Gerasimov sum-rule and the spin structure of the nucleon

There is a beautiful property of the Drell-Hearn-Gerasimov sum-rule that has not been noted previously. The inclusive spin dependent cross sections receive contributions from the photon’s coupling to sea quarks and gluonic degrees of freedom as well as to the valence quarks. At the same time, the anomalous magnetic moment $\kappa_N$ is a $C = -1$ (pure valence) quantity. (It is measured in the nucleon’s matrix element of the conserved, $C = -1$ vector current.) The constituent quark model expression for the magnetic moment $\mu_N = \kappa_N + 1$ is:

$$\mu_N = \sum_q e_q \frac{\sigma_q}{2m_q}$$

Here $e_q$ is the quark charge, $m_q$ is the constituent quark mass and

$$\sigma_q = (q - \bar{q})^\dagger - (q - \bar{q})^\dagger$$

(12)
is the fraction of the nucleon’s spin that is carried by the valence quarks; \( q^{↑(↓)} \) denotes the probability for finding a quark in the nucleon with flavour \( q \) and polarised in the same (opposite) direction as the nucleon’s polarisation; similarly \( \overline{q}^{↑(↓)} \) for the anti-quark. The valence spin content \( \sigma_\gamma \) is invariant under \( Q^2 \) evolution. (It is \( C \) odd and satisfies the same \( Q^2 \) evolution equation as the non singlet axial charge \( g_3^A \).) Therefore, it describes equally the constituent quarks of low energy QCD and also the valence current quarks which are measured in high \( Q^2 \) deep inelastic scattering. The fact that \( \kappa_N \) is a \( C = -1 \) pure valence quantity means that one does not need to know anything about the Dirac sea or about the gluonic components in the nucleon wavefunction in order to evaluate equ.(5). The inverse-energy weighted integral of the glue and Dirac sea contributions to the difference in the spin dependent cross sections at photoproduction is exactly zero. The vanishing of the polarised sea and gluonic contribution to the Drell-Hearn-Gerasimov integral, equ.(5), follows from the \( C \)—parity of the terms in the sum-rule and is derived assuming only that the no subtraction hypothesis is correct. In other words, the general principles of electromagnetic gauge invariance, Lorentz invariance, causality and unitarity imply that the polarised sea and gluonic contribution to the integral is either zero or infinite (so that one would need a subtraction). In the infinite scenario the sea and glue would be generated all with one polarisation, which is not QCD. The vanishing sea and gluonic contribution to the Drell-Hearn-Gerasimov sum-rule implies an exact cancellation between the spin structure of the long and short range parts of the \( C = +1 \) exchange potential. If we introduce a cut-off \( \nu_0 \), then

\[
\int_0^{\nu_0} \frac{d\nu'}{\nu'} \Delta \sigma|_{\text{sea+glue}}(\nu') = -\int_{\nu_0}^\infty \frac{d\nu'}{\nu'} \Delta \sigma|_{\text{sea+glue}}(\nu')
\]

independent of the choice of cut-off \( \nu_0 \). The long range and short range parts of the \( C = +1 \) exchange potential, which are described by chiral perturbation theory [19] and the Bonn [20] and Paris [21] potentials of nuclear physics and by Regge theory [10-17] respectively, “know about each other” as, indeed, they must in a consistent field theoretical description of the structure of the nucleon.

The pure valence nature of the Drell-Hearn-Gerasimov sum-rule allows us to understand why the sum-rule is nearly saturated by the \( N - \Delta \) transition [22,6], which is the leading valence quark spin excitation. It will be interesting to compare the relative saturation of the Drell-Hearn-Gerasimov and spin polarisability sum-rules, which are each given by the difference of two positive cross sections, in future photoproduction experiments. This requires first a direct and independent measurement of \( \gamma_N \) in a forward Compton scattering experiment. The
experimental question to be tested is at what energy $\nu$ do we obtain 80%, 90% of the each of the two sum-rules? The spin polarisability sum-rule, equ.(6), receives contributions from the photon’s coupling to polarised sea and gluonic excitations, as well as to the valence quarks, in the nucleon’s wavefunction.

The validity of the high energy no-subtraction hypothesis is also necessary for the spin structure of the constituent quark model at low momentum scales. The difference in the two spin cross sections can be written in terms of two spin dependent nucleon form factors $G_1(\nu, Q^2)$ and $G_2(\nu, Q^2)$, viz.

$$\sigma_{\frac{1}{2}}^N - \sigma_{\frac{3}{2}}^N = \frac{16m_\pi^2\alpha}{2m\nu - Q^2}\left(m\nu G_1(\nu, Q^2) - Q^2 G_2(\nu, Q^2)\right)$$

(14)

Following Anselmino et al.[23], we define the $Q^2$ dependent quantity

$$I(Q^2) = m^3 \int_{\frac{m}{Q^2}}^\infty \frac{d\nu}{\nu} G_1(\nu, Q^2)$$

(15)

The Drell-Hearn-Gerasimov sum-rule tell us that

$$I(0) = -\frac{1}{4}\kappa_N^2$$

(16)

At the high $Q^2$ of deep inelastic scattering experiments

$$m^2\nu G_1(\nu, Q^2) \rightarrow g_1(x, Q^2)$$

(17)

so that

$$I(Q^2) = \frac{2m^2}{Q^2} \int_0^1 dx g_1(x, Q^2).$$

(18)

The first moment of $g_1$ is

$$\int_0^1 dx g_1(x, Q^2) = \frac{1}{2} \sum_\gamma e_\gamma^2 \Delta q(Q^2) C_q(1, \alpha_s(Q^2))$$

(19)

Here

$$\Delta q(Q^2) = (q + \bar{q})^+(Q^2) - (q + \bar{q})^+(Q^2)$$

(20)

is the contribution to the nucleon’s spin from quarks and anti-quarks of flavour $q$ and $C_q(1, \alpha_s(Q^2))$ is the radiative QCD coefficient. (These $\Delta q$ include the effect of the axial anomaly [24].) We compare equ.(20) with the conserved $C = -1$ spin coupling in equ.(12) which is measured in the Drell-Hearn-Gerasimov sum-rule. The $\bar{q}$ sea polarisation contribution (which includes polarised glue via the anomaly) completely disappears at $Q^2 = 0$, where higher twist effects have built up an effective spin operator for low momentum scales. The $C = -1$ spin coupling
to \((q - \bar{q})\) measures a conserved quantity and so corresponds to the same spin operator for both current and constituent quarks. We find an a posteriori justification of the spin structure of the constituent quark model.

It is interesting to compare the physics of Drell-Hearn-Gerasimov with the spin averaged cross section. Here the no-subtraction hypothesis is not valid. If we try to use an unsubtracted dispersion relation in the spin averaged case, then we find an equation relating the total \(\gamma N\) cross section to the Thomson term in \(f_1\) (equ.(3)), which is given by the square of the forward matrix element of the \(C = -1\) vector current, which itself measures the number of valence quarks in the nucleon. If an unsubtracted dispersion relation were valid for the spin averaged cross-section then the nucleon would have to be elementary! Pomeron exchange, which tells us that total cross section grows as \(\ln^2 \nu\) at very large \(\nu\), implies that the number of sea quarks is infinite in high \(Q^2\) deep inelastic scattering. At low momentum scales the higher twist contributions become important and build up effective constituent quark operators. The infinite number of anti-quarks (and gluons) which are seen at high \(Q^2\), and which are manifest as the need for a subtraction in the dispersion relation for spin averaged photoproduction, form a condensate in the vacuum at some low scale leading to spontaneous symmetry breaking of chiral symmetry and pion physics. The formation of this condensate is responsible for the large constituent quark mass. Thus, photoproduction sum-rules illustrate the self-consistency of our understanding of the structure of the nucleon at low and at high energies.

## 4 Outlook

Finally, we consider the challenges for future experiments. The twin experiments at MAMI and ELSA will measure the spin dependent photoproduction cross sections up to a photon energy of 3GeV [3]. They will cover the resonance region and the domain where we expect Regge theory to start to work. (Regge theory provides a good description of the unpolarised \(\gamma p\) cross section starting at \(\sqrt{s} \sim 2.5\)GeV [25].) It will be important to extend these polarised photoproduction experiments to higher energies to extract the relative strengths of the iso-triplet and iso-singlet Regge exchange contributions in equ.(7-10). One could then make contact with the small \(x\) behaviour of the spin structure function \(g_1\) of polarised deep inelastic scattering at high \(Q^2\) – see [13,15,18] for theory and [26] for the latest data from SMC. ELFE [7,8] will provide polarised real photon beams with \(\sqrt{s} \sim 7\)GeV (well within the domain of Regge physics); a highly polarised proton beam at HERA energy would take us deep into the region of “pomeron
physics”. Secondly, it will be interesting to compare the relative saturation of the (in practice $C = -1$) Drell-Hearn-Gerasimov and $C = +1$ spin polarisation sum-rules in equs.(5,6) to test the spin structure of sea and gluonic excitations at photoproduction.

Acknowledgements:

It is a pleasure to thank N. d’Hose, P.V. Landshoff, D. Schütte and A.W. Thomas for helpful discussions. This work was supported in part by the Alexander von Humboldt Foundation.
References

[1] The European Muon Collaboration, J. Ashman et al., Phys. Lett. B206 (1988) 364; Nucl. Phys. B328 (1990) 1

[2] R. Windmolders, Int. J. Mod. Phys. A7 (1992) 639 and references therein

[3] Proposal to measure the DHG sum-rule at Bonn and Mainz, G. Anton et al.

[4] S. D. Drell and A. C. Hearn, Phys. Rev. Lett. 162 (1966) 1520
   S. B. Gerasimov, Yad. Fiz. 2 (1965) 839

[5] N. d’Hose, Lectures at ELFE Summer School, Cambridge 1995, to appear in the proceedings, edited by S.D. Bass and P.A.M. Guichon (Editions Frontières, 1996)

[6] D. Drechsel, Prog. Part. Nucl. Phys. 34 (1995) 181

[7] The ELFE PROJECT, edited by J. Arvieux and E. De Sanctis (Italian Physical Society, Bologna, 1993)

[8] Physics with ELFE, B. Frois and B. Pire, hep-ph/9512221, to appear in Proc. Int. Conf. Nucl. Phys., Beijing (1995)

[9] P.D.B. Collins, Regge Theory and High Energy Physics, Cambridge University Press (1978)

[10] R. F. Peierls and T. L. Trueman, Phys. Rev. 134 (1964) 1365
     A. H. Mueller and T. L. Trueman, Phys. Rev. 160 (1967) 1296, 1306

[11] R. L. Heimann, Nucl. Phys. B64 (1973) 429

[12] J. Ellis and M. Karliner, Phys. Lett. B213 (1988) 73

[13] F. E. Close and R. G. Roberts, Phys. Lett. B336 (1994) 257

[14] P. V. Landshoff and O. Nachtmann, Z Physik C35 (1987) 405
     P. V. Landshoff, Proc. Zuoz Summer School, PSI Proceedings 94-01 (1994) 135

[15] S. D. Bass and P. V. Landshoff, Phys. Lett. B336 (1994) 537

[16] L. Galfi, J. Kuti and A. Patkos, Phys. Lett. B31 (1970) 465
     F. E. Close and R. G. Roberts, Phys. Rev. Lett. 60 (1988) 1471
[17] J. Kuti, to appear in Proc. Erice Summer School on the Spin Structure of the Nucleon (1995)

[18] R. D. Ball, S. Forte and G. Ridolfi, Nucl. Phys. B444 (1995) 287, Erratum, Nucl. Phys. B449 (1995) 680

[19] V. Bernard et al., Nucl. Phys. B388 (1992) 315, Phys. Rev. D48 (1993) 3062

[20] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rep. 149 (1987) 1

[21] M. Lacombe et al., Phys. Rev. C21 (1980) 861

[22] I. Karliner, Phys. Rev. D7 (1973) 2717
   D. Drechsel and L. Tiator, J. Phys. G18 (1992) 44

[23] M. Anselmino, B. L. Ioffe and E. Leader, Yad. Fiz. 49 (1989) 214

[24] S. D. Bass and A. W. Thomas, Prog. Part. Nucl. Phys. 33 (1994) 449
   R. L. Jaffe and A. Manohar, Nucl. Phys. B337 (1990) 509

[25] P. V. Landshoff, private communication

[26] G. Baum, Bielefeld preprint, [hep-ex/9509007] (1995), to appear in Proc. Seventh Conf. of Perspectives in Nuclear Physics at Intermediate Energies, Trieste (1995)