COSMOLOGICAL THEORIES OF SPECIAL AND GENERAL RELATIVITY - II

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Abstract

Astronomers measure distances to faraway galaxies and their velocities. They do that in order to determine the expansion rate of the Universe. In Part I of these lectures the foundations of the theory of the expansion of the Universe was given. In this part we present the theory. A formula for the distance of the galaxy in terms of its velocity is given. It is very simple: \[ r(v) = \frac{c \tau}{\beta} \sinh \frac{\beta v}{c}, \] where \( \tau \) is the Big Bang time, \( \beta = \sqrt{1 - \Omega_m} \), and \( \Omega_m \) is the mass density of the Universe. For \( \Omega_m < 1 \) this formula clearly indicates that the Universe is expanding with acceleration, as experiments clearly show.

1 Gravitational field equations

In the four-dimensional spacevelocity the spherically symmetric metric is given by

\[ ds^2 = \tau^2 dv^2 - e^\mu dr^2 - R^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right), \tag{1.1} \]

where \( \mu \) and \( R \) are functions of \( v \) and \( r \) alone, and comoving coordinates \( x^\mu = (x^0, x^1, x^2, x^3) = (\tau v, r, \theta, \phi) \) have been used. With the above choice of coordinates, the zero-component of the geodesic equation becomes an identity, and since \( r, \theta \) and \( \phi \) are constants along the geodesics, one has \( dx^0 = ds \) and therefore \( u^\alpha = u_\alpha = (1, 0, 0, 0) \). The metric (1.1) shows that the area of the sphere \( r = \text{constant} \) is given by \( 4\pi R^2 \) and that \( R \) should satisfy \( R' = \partial R / \partial r > 0 \). The possibility that \( R' = 0 \) at a point \( r_0 \) is excluded since it would allow the lines \( r = \text{constants} \) at the neighboring points \( r_0 \) and \( r_0 + dr \) to coincide at \( r_0 \), thus creating a caustic surface at which the comoving coordinates break down.

As has been shown in Part I the Universe expands by the null condition \( ds = 0 \), and if the expansion is spherically symmetric one has \( d\theta = d\phi = 0 \). The metric (1.1) then yields \( \tau^2 dv^2 - e^\mu dr^2 = 0 \), thus

\[ \frac{dr}{dv} = \tau e^{-\mu/2}. \tag{1.2} \]
This is the differential equation that determines the Universe expansion. In the following we solve the gravitational field equations in order to find out the function $\mu(r,v)$.

The gravitational field equations, written in the form

$$R_{\mu\nu} = \kappa(T_{\mu\nu} - g_{\mu\nu}T/2),$$

where

$$T_{\mu\nu} = \rho_{\text{eff}} u_{\mu} u_{\nu} + p (u_{\mu} u_{\nu} - g_{\mu\nu}),$$

are now solved. One finds that the only nonvanishing components of $T_{\mu\nu}$ are $T_{00} = \tau^2 \rho_{\text{eff}}$, $T_{11} = c^{-1} \tau p$, $T_{22} = c^{-1} \tau p R^2$ and $T_{33} = c^{-1} \tau p R^2 \sin^2 \theta$, and that $T = \tau^2 \rho_{\text{eff}} - 3 c^{-1} \tau p$.

One obtains three independent field equations (dot and prime denote derivatives with $v$ and $r$)

$$e^{\mu} \left(2 R \ddot{R} + \dot{R}^2 + 1\right) - R^2 = - \kappa \tau c^{-1} e^{\mu} R^2 p,$$

$$2 \dot{R} - R' \dot{\mu} = 0,$$

$$e^{-\mu} \left[\frac{1}{R} R' \mu' - \left(\frac{R'}{R}\right)^2 - \frac{2}{R} R''\right] + \frac{1}{R} \dot{R} \dot{\mu} + \left(\frac{\dot{R}}{R}\right)^2 + \frac{1}{R^2} = \kappa \tau^2 \rho_{\text{eff}}.$$

**2 Solution of the field equations**

The solution of (1.6) satisfying the condition $R' > 0$ is given by

$$e^{\mu} = R^2 / (1 + f(r)),$$

where $f(r)$ is an arbitrary function of the coordinate $r$ and satisfies the condition $f(r) + 1 > 0$. Substituting (2.1) in the other two field equations (1.5) and (1.7) then gives

$$2 R \ddot{R} + \dot{R}^2 - f = - \kappa \tau c^{-1} \tau R^2 p,$$

$$\frac{1}{R} \ddot{R} \left(2 \dot{R} R' - f'\right) + \frac{1}{R^2} \left(R^2 - f\right) = \kappa \tau^2 \rho_{\text{eff}},$$

respectively.

The simplest solution of the above two equations, which satisfies the condition $R' = 1 > 0$, is given by $R = r$. Using this in Eqs. (2.2) and (2.3) gives $f(r) = \kappa \tau c^{-1} \tau r^2$, and $f' + f/r = - \kappa \tau^2 \rho_{\text{eff}} r$, respectively. Using the values of $\kappa = 8\pi G / c^2 \tau^2$ and $\rho_c = 3 / 8\pi G \tau^2$, we obtain

$$f(r) = (1 - \Omega_m) r^2 / c^2 \tau^2,$$

where $\Omega_m = \rho / \rho_c$. We also obtain

$$p = \frac{1 - \Omega_m}{\kappa c} \frac{1 - \Omega_m}{\tau} = 4.544 \times 10^{-2} g/cm^2,$$

(2.5)
\[ e^{-\mu} = 1 + f(r) = 1 + \tau c^{-1} \kappa p r^2 = 1 + (1 - \Omega_m) r^2/c^2 r^2. \]  

Accordingly, the line element of the Universe is given by

\[ ds^2 = \tau^2 dv^2 - \frac{dr^2}{1 + (1 - \Omega) r^2/c^2 r^2} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \]

or,

\[ ds^2 = \tau^2 dv^2 - \frac{dr^2}{1 + (\kappa \tau/c) p r^2} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \]  

This line element is the comparable to the FRW line element in the standard theory.

It will be recalled that the Universe expansion is determined by Eq. (1.2),

\[ \frac{dr}{dv} = \tau e^{-\mu/2}. \]  

The only thing that is left to be determined is the sign of \((1 - \Omega_m)\) or the pressure \(p\). Thus we have

\[ \frac{dr}{dv} = \tau \sqrt{1 + \kappa \tau^{-1} p r^2} = \tau \sqrt{1 + \frac{1 - \Omega_m}{c^2 r^2}}. \]  

### 3 Physical meaning

For \(\Omega_m > 1\) one obtains

\[ r(v) = \frac{ct}{\alpha} \sin \frac{v}{c}, \quad \alpha = \sqrt{\Omega_m - 1}. \]  

This is obviously a closed Universe, and presents a decelerating expansion.

For \(\Omega_m < 1\) one obtains

\[ r(v) = \frac{ct}{\beta} \sinh \frac{v}{c}, \quad \beta = \sqrt{1 - \Omega_m}. \]  

This is now an open accelerating Universe.

For \(\Omega_m = 1\) we have, of course, \(r = \tau v\).

### 4 The accelerating Universe

From the above one can write the expansion of the Universe in the standard Hubble form \(v = H_0 r\) with

\[ H_0 = h \left[ 1 - (1 - \Omega_m) v^2/6c^2 \right], \]  

where \(h = \tau^{-1}\). Thus \(H_0\) depends on the distance it is being measured. It is well-known that the farther the distance, the lower the value for \(H_0\) is measured. This is possible only for \(\Omega_m < 1\), i.e. when the Universe is accelerating. In that case the pressure is positive.

Figure 1 describes the Hubble diagram of the above solutions for the three types of expansion for values of \(\Omega_m\) from 100 to 0.245. The figure describes the three-phase evolution of the Universe. Curves (1)-(5) represent the stages of
Figure 1: Hubble's diagram describing the three-phase evolution of the Universe according to cosmological general relativity theory.

_decelerating expansion_ according to Eq. (3.1). As the density of matter $\rho$ decreases, the Universe goes over from the lower curves to the upper ones, but it does not have enough time to close up to a Big Crunch. The Universe subsequently goes over to curve (6) with $\Omega_m = 1$, at which time it has a constant expansion for a fraction of a second. This then followed by going to the upper curves (7) and (8) with $\Omega_m < 1$, where the Universe expands with _acceleration_ according to Eq. (3.2). Curve no. 8 fits the present situation of the Universe. For curves (1)-(4) in the diagram we use the cutoff when the curves were at their maximum.

### 5 Theory versus experiment

To find out the numerical value of $\tau$ we use the relationship between $h = \tau^{-1}$ and $H_0$ given by Eq. (4.1)(CR denote values according to Cosmological Relativity):

$$H_0 = h \left[ 1 - (1 - \Omega_m^{CR}) \frac{z^2}{6} \right],$$

(5.1)

where $z = v/c$ is the redshift and $\Omega_m^{CR} = \rho_m/\rho_c$ with $\rho_c = 3h^2/8\pi G$. (Notice that our $\rho_c = 1.194 \times 10^{-29} g/cm^3$ is different from the standard $\rho_c$ defined with $H_0$.) The redshift parameter $z$ determines the distance at which $H_0$ is measured. We choose $z = 1$ and take for $\Omega_m^{CR} = 0.245$, its value at the present time (corresponds to 0.32 in the standard theory), Eq. (5.1) then gives $H_0 = 0.874h$. At $z = 1$
the corresponding Hubble parameter $H_0$ according to the latest results from HST can be taken [20] as $H_0 = 70\text{km/s-Mpc}$, thus $h = (70/0.874)\text{km/s-Mpc}$, or $h = 80.092\text{km/s-Mpc}$, and $\tau = 12.486 \times 10^{17}\text{s}$.

What is left is to find the value of $\Omega^{CR}_\Lambda$. We have $\Omega^{CR}_\Lambda = \rho^{ST}_c / \rho_c$, where $\rho^{ST}_c = 3H_0^2/(8\pi G)$ and $\rho_c = 3h^2/(8\pi G)$. Thus $\Omega^{CR}_\Lambda = (H_0/h)^2 = 0.874^2$, or $\Omega^{CR}_\Lambda = 0.764$.

As is seen from the above equations one has

$$\Omega_T = \Omega^{CR}_m + \Omega^{CR}_\Lambda = 0.245 + 0.764 = 1.009 \approx 1,$$ (5.2)

which means the Universe is Euclidean.

Our results confirm those of the supernovae experiments and indicate on the existence of the dark energy as has recently received confirmation from the Boomerang cosmic microwave background experiment [21][22], which showed that the Universe is Euclidean.

6 Comparison with general relativity

One has to add the time coordinate and the result is a five-dimensional theory of space-time-velocity. One can show that all the classical experiments predicted by general relativity are also predicted by CGR. Also predicted a wave equation for gravitational radiation. In the linear approximation one obtains

$$\left( \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \nabla^2 + \frac{1}{\tau^2} \frac{\partial^2}{\partial v^2} \right) \gamma_{\mu\nu} = -2\kappa T_{\mu\nu},$$ (6.1)

where $\gamma_{\mu\nu}$ is a first approximation term,

$$g_{\mu\nu} \approx \eta_{\mu\nu} + h_{\mu\nu} = \eta_{\mu\nu} + \gamma_{\mu\nu} - \eta_{\mu\nu} \gamma/2,$$ (6.2)

$$\gamma = \eta^{\alpha\beta} \gamma_{\alpha\beta}.$$ (6.3)

Hence CGR predicts that gravitational waves depend not only on space and time but also on the redshift of the emitting source.

7 New developments on dark matter

Using the theory presented here, John Hartnett has recently shown that there is no need for the existence of dark matter in spiral galaxies. We only give the references to this work by Hartnett [25][26][27].

References

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| Theory type | Spacevelocity | Spacetime |
|-------------|--------------|-----------|
| Expansion type | Tri-phase: decelerating, constant, accelerating | One phase |
| Present expansion | Accelerating (predicted) | One of three possibilities |
| Pressure | 0.034g/cm² | Negative |
| Cosmological constant | 1.934 × 10⁻³⁵ s⁻² (predicted) | Depends |
| Ωₜ = Ωₘ + Ωₐ | 1.009 | Depends |
| Constant-expansion occurs at | 8.5Gyr ago (Gravity is included) | No prediction |
| Constant-expansion duration | Fraction of second | Not known |
| Temperature at constant expansion | 146K (Gravity is included) | No prediction |

Table 1: Cosmological parameters in cosmological general relativity and in standard theory.

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