Quantum projection ghost imaging: a photon-number-selection method [Invited]

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We establish a quantum theory of computational ghost imaging and propose quantum projection imaging where object information can be reconstructed by quantum statistical correlation between a certain photon number of a bucket signal and digital micromirror device random patterns. The reconstructed image can be negative or positive, depending on the chosen photon number. In particular, the vacuum state (zero-number) projection produces a negative image with better visibility and contrast-to-noise ratio. The experimental results of quantum projection imaging agree well with theoretical simulations and show that, under the same measurement condition, vacuum projection imaging is superior to conventional and fast first-photon ghost imaging in low-light illumination.

Keywords: ghost imaging; quantum projection; image reconstruction.
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1. Introduction

In recent years, low-photon flux imaging technology has attracted extensive attention due to its wide application in various fields, such as remote sensing, biological science, and medical inspection [1-17]. The light source illuminating the object in most imaging systems is a classical one, such as coherent light or thermal light. Under low-light illumination, only one or a few photons transmitted or reflected by the object can reach the detector with a certain probability. There are two major challenges. First, the zero-count of photons dominates the measurement process. In the intensity correlation, only detected photons act as a valid signal, and all vacuum signals are discarded. This greatly reduces the detection efficiency in imaging. Second, the shot noise in the classical source will lower the signal-to-noise ratio (SNR) of the image. Quantum illumination in the imaging system, such as a single-photon source or a two-photon entangled source, would avoid these problems [16].

The development of the single-photon detector device and photon-counting techniques provides favorable conditions for low-light imaging. Kirman et al. [12] proposed a low-flux imaging technique, called first-photon imaging, where for each pixel the number of illumination pulses prior to the first photon detection is used as an initial reflectivity estimate. They claimed that this avoids the Poisson noise inherent in low-flux operation. A similar way of first photon detection, called fast first-photon ghost imaging (FFPGI), can be applied to computational ghost imaging [8]. Sonnleitner et al. [3] utilized an image retrodiction approach that provides the full probability treatment for high-quality imaging data. Morris et al. [4] investigated how to obtain higher-quality images with a small number of photons. The authors attempted to find the answer to the question of how many photons it takes to form an image. Using compressive techniques [14,15], they acquired the reconstructed image of a wasp wing with an average photon-per-pixel ratio of 0.45. A recent paper by Johnson et al. asked the same question as the title of their paper [10]. They pointed out a basic fact that “for intensity images, it seems that one detected photon per image pixel is a realistic guide, but this may be reduced by making further assumptions on the sparsity of an image in a chosen basis, such as spatial frequency.” In any case, experimental results and experience tell us that under low-light conditions, increasing a certain number of photons will significantly improve the image quality.

Under low-light illumination of a classical source, the quantum effects of light are revealed. In this paper, we set up the
quantum theory of computational ghost imaging and propose quantum projection ghost imaging (QPGI). In the quantum projection scheme, the image is constructed by a certain number of photons of the object beam correlated with the random patterns of the digital micromirror device (DMD), and it thus can essentially avoid Poisson noise and improve image quality. Of particular interest is that quantum projection of the vacuum state can greatly increase imaging efficiency in the low-photon situation and still obtain a high-quality negative image as well.

2. Theory

2.1. General quantum theory for computational imaging

We first consider the quantum theory of computational ghost imaging. As shown in Fig. 1, computational ghost imaging usually relies on a DMD, which is an array of many micromirrors that flip independently. Each micromirror has two states, 0 and 1, which indicate that incident light is blocked or reflected. While a computer makes independently random control to all micromirrors, the probability function of each micromirror follows a Bernoulli distribution,

$$p(\alpha) = \begin{cases} 1 - q, & \alpha = 0 \\ q, & \alpha = 1 \end{cases}$$  \hspace{1cm} \text{(1)}$$

where $\alpha$ is the binary variable and $q$ is the probability that the micromirror pixel is opened ($\alpha = 1$).

Assume that each micromirror is illuminated by an independent and identical statistical source with the probability distribution $p_1(n)$ of photon number $n$. The joint-probability distribution between the outgoing photon number $n$ and micromirror pixel value $\alpha$ for each micromirror is given by

$$p(\alpha, n) = \begin{cases} (1-q)\delta(n), & \alpha = 0 \\ q \times p_1(n), & \alpha = 1 \end{cases}$$  \hspace{1cm} \text{(2)}$$

where $\delta(n) = 1$ for $n = 0$ and $\delta(n) = 0$ for $n = 1, 2, \cdots$. It reflects a statistical correlation between the two random variables, $\alpha$ and $n$. If any number of photons join in the bucket detection, the micromirror must be opened. Otherwise, the micromirror is closed with the probability of $1 - q$. This is the key to computational ghost imaging. The photon probability distribution of outgoing light for each micromirror is obtained to be

$$p^{(1)}(n) = \sum_{\alpha} p(\alpha, n) = (1-q)\delta(n) + qp_1(n).$$  \hspace{1cm} \text{(3)}$$

In comparison with $p_1(n)$ of incident light, the probability of the vacuum state is increased, since the 0-state of the micromirror is taken into account. For $M$ independent micromirrors, the photon probability distribution can be derived by the convolutions

$$p^{(M)}(n) = p^{(1)}(n_1) \otimes p^{(1)}(n_2) \otimes \cdots \otimes p^{(1)}(n_M)$$

$$= (1-q)^M \left[ \delta(n) + \sum_{l=1}^{M} C_M^l \left( \frac{q}{1-q} \right)^l p_1(n) \right].$$  \hspace{1cm} \text{(4)}$$

where $n = n_1 + n_2 + \cdots + n_M$ and $C_M^l = M!/(l!(M-l)!)$.

The function $p_l(n) = p_1(n_1) \otimes p_1(n_2) \otimes \cdots \otimes p_1(n_l)$, where $n = n_1 + n_2 + \cdots + n_l$. Equation (4) is also valid for the case when a uniform source illuminates all DMD mirrors, where $p_l(n)$ is the photon probability distribution of any $l$ mirrors illuminated. For the common case of coherent light illumination, the two cases are equivalent. If $p_1(n)$ is a Poisson distribution $p_1(n) = N_1^n e^{-N_1}/n!$ with mean photon number $N_1$, $p_l(n)$ also satisfies the Poisson distribution with mean photon number $lN_1$. For simplicity, we consider a binary object, and all the photons transmitted or reflected from the object are received by the detector, the so-called bucket detection. Suppose the object is illuminated by $M$ micromirrors, i.e., the object has $M$ pixels. Equation (4) is the photon probability distribution for the bucket signals.

In a computational imaging scheme where the input light photons are in Poisson statistics (see details in Section 3), Figs. 2(a1) and 2(a2) show the probability distribution of the bucket photons for the two cases. The probability of the vacuum state is clearly separated far from the other photon states because the 0-level of micromirrors only provides the probability of the vacuum state. The experimental results fit well with the theoretical curve, except for the 0 and 1 photon states, due to the stray light and dark counting of the single-photon avalanche detector (SPAD).

According to the mechanism of computational ghost imaging, for a given binary object all micromirror pixels are divided into two sets: on-micromirror-pixels are imaged onto the object and off-micromirror-pixels never do. The correlation of the bucket signal with the random variable of micromirrors will
The on-micromirror-pixel is given by

\[ P_{\text{on}}(\alpha, n) = \sum_{n_1=0}^{n} p^{(M-1)}(n-n_1)p(\alpha, n_1) \]

\[ = \begin{cases} 
(1-q)p^{(M-1)}(n), & \alpha = 0 \\
q^{(M)}(n) - (1-q)p^{(M-1)}(n), & \alpha = 1. 
\end{cases} \]  

The average photon number of the outgoing beam from the pixel is \( qN_1 = \sum n_n^{(1)}(n) \), and the average photon number of the bucket signal for \( M \) pixels is \( \langle n \rangle_M = \sum n_n^{(M)}(n) = MqN_1 \).

The above Eqs. (5) and (6) summarize the basic quantum description of computational ghost imaging. So now we can calculate the correlation coefficients for imaging. We first define the mean value and mean square value of the photon number injected onto each micromirror as

\[ N_1 = \sum_{n} n p_1(n), \]  

\[ N_2 = \sum_{n} n^2 p_1(n). \]  

Using Eq. (3), the corresponding mean values of outgoing photons for the micromirror are

\[ \langle n \rangle = \sum_{n} n p_1^{(1)}(n) = qN_1, \]  

\[ \langle n^2 \rangle = \sum_{n} n^2 p_1^{(1)}(n) = qN_2. \]  

Since all micromirrors are independent of each other, the mean value and mean square value of bucket photons are obtained to be

\[ \langle n \rangle_M = \sum_{n} n p_1^{(M)}(n) = MqN_1, \]  

\[ \langle n^2 \rangle_M = \sum_{n} n^2 p_1^{(M)}(n) = \left( n_1 + n_2 + \cdots + n_M \right)^2 \]

\[ = Mq\left[ N_2 + (M-1)qN_1^2 \right]. \]  

The first-order correlation coefficients between the binary variable and bucket photon number for off- and on-micromirror-pixels are calculated to be

\[ \langle an \rangle_{\text{off}} = \sum_{\alpha, n} an p_{\text{off}}(\alpha, n) = q^2 MN_1, \]  

\[ \langle an \rangle_{\text{on}} = \sum_{\alpha, n} an p_{\text{on}}(\alpha, n) \]

\[ = \sum_{\alpha, n} [np^{(M)}(n) - n(1-q)p^{(M-1)}(n)] \]

\[ = q^2 MN_1 + q(1-q)N_1. \]  

Hence, imaging pixels can be distinguished from non-imaging pixels. The imaging visibility is obtained as...
\[ V = \frac{\langle a | k \rangle_{\text{on}} - \langle a | k \rangle_{\text{off}}}{\langle a | k \rangle_{\text{on}} + \langle a | k \rangle_{\text{off}}} = \frac{1}{1 + 2Mq/(1-q)}. \] (12)

The expression is general, independent of the intensity and statistical nature of the incident light. When \( q = 1/2 \), \( V = 1/(1 + 2M) \) is the same as the visibility of ghost imaging with thermal light correlation. Therefore, computational ghost imaging can greatly improve visibility by decreasing the probability of \( q \). The price paid is a reduction in detection efficiency.

The second-order correlation coefficients for off- and on-micromirror-pixels are written as

\[ \langle a^2 n^2 \rangle_{\text{off}} = \sum_{\alpha} a^2 p(\alpha) \sum_{n} n^2 p^{(M)}(n) = q^2 \langle n^2 \rangle_M, \] (13)

\[ \langle a^2 n^2 \rangle_{\text{on}} = \sum_{n} n^2 p^{(M)}(n) - (1-q) \sum_{n} n^2 p^{(M-1)}(n) = \langle n^2 \rangle_M - (1-q) \langle n^2 \rangle_{M-1}. \] (14)

With these coefficients, we can calculate the contrast-to-noise ratio (CNR)\[^{[18]}\],

\[ \text{CNR} = \frac{|\langle a | n \rangle_{\text{on}} - \langle a | n \rangle_{\text{off}}|}{\sqrt{(\langle a^2 n^2 \rangle_{\text{off}} - \langle a^2 \rangle_{\text{off}}^2)(\langle a^2 n^2 \rangle_{\text{on}} - \langle a^2 \rangle_{\text{on}}^2)}}. \] (15)

### 2.2. Quantum projection imaging

Here we propose a novel imaging scheme using quantum projection detection. As shown in Figs. 2(a1) and 2(a2), the bucket photons contain great fluctuations. To bypass the fluctuations, we measure and pick a specific number of photons in the bucket signal and corresponding DMD pixel patterns to form the image.

From the probability functions [Eqs. (5) and (6)] of computational imaging, we calculate the corresponding conditional probability distributions for a given photon number \( k \),

\[ P_{\text{off}}^{(\alpha)}(a|k) = p(a), \] (16)

\[ P_{\text{on}}^{(\alpha)}(a|k) = P_{\text{on}}(a,k)/p^{(M)}(k) = \begin{cases} \frac{(1-q)p^{(M-1)}(k)}{p^{(M)}(k)}, & \alpha = 0 \\ 1 - \frac{(1-q)p^{(M-1)}(k)}{p^{(M)}(k)}, & \alpha = 1 \end{cases}. \] (17)

With the fact \( \alpha = a^2 \), the first- and second-order conditional correlation coefficients are obtained equally to be

\[ \langle a | k \rangle_{\text{off}} = \langle a^2 | k \rangle_{\text{off}} = \sum_{\alpha} a p(\alpha) = q, \] (18)

\[ \langle a | k \rangle_{\text{on}} = \langle a^2 | k \rangle_{\text{on}} = \sum_{\alpha} a P^{(\alpha)}(a|k) = 1 - (1-q)p^{(M-1)}(k)/p^{(M)}(k). \] (19)

The visibility of quantum projection imaging depends on the measurement of \( k \) photons,

\[ V(k) = \frac{\langle a | k \rangle_{\text{on}} - \langle a | k \rangle_{\text{off}}}{\langle a | k \rangle_{\text{on}} + \langle a | k \rangle_{\text{off}}} = \frac{(1-q)|p^{(M)}(k) - p^{(M-1)}(k)|}{[p^{(M)}(k) - p^{(M-1)}(k)] + q[p^{(M)}(k) + p^{(M-1)}(k)]}. \] (20)

The significant feature of quantum projection imaging compared to conventional imaging is its potential to produce negative images. The condition for the formation of a negative image is

\[ p^{(M)}(k) < p^{(M-1)}(k). \] (21)

In this case, the object signal \( \langle a | k \rangle_{\text{on}} \) is less than the background \( \langle a | k \rangle_{\text{off}} \). The similar effects of positive and negative ghost imaging by conditional collection of certain range of bucket intensities have been reported both experimentally and theoretically\[^{[19–23]}\].

Visibility of quantum projection imaging can be expressed as the similar form to Eq. (12),

\[ V(k) = \frac{1}{1 + 2\mu(M,k)q/(1-q)} \] (22)

where the effective value of object pixels is defined as

\[ \mu(M,k) \equiv \frac{1}{1 - p^{(M-1)}(k)/p^{(M)}(k)}. \] (23)

The positive and negative values of \( \mu(M,k) \) correspond to the positive and negative images, respectively. For \( 0 < \mu(M,k) < M \), quantum projection positive imaging has better visibility than the conventional one. As for the negative values of \( \mu(M,k) \), however, under the condition of

\[ (1-q)/q < -\mu(M,k) < M + (1-q)/q, \] (24)

negative imaging has better visibility than the conventional one. The visibility reaches the perfect value when \( -\mu(M,k) \to (1-q)/q \).

The CNR of quantum projection imaging is defined as

\[ \text{CNR} = \frac{|\langle a | k \rangle_{\text{on}} - \langle a | k \rangle_{\text{off}}|}{\sqrt{(\langle a^2 | k \rangle_{\text{off}} - \langle a | k \rangle_{\text{off}}^2)(\langle a^2 | k \rangle_{\text{on}} - \langle a | k \rangle_{\text{on}}^2)}}. \] (25)

Applying Eqs. (18) and (19) to it, we obtain

\[ \text{CNR} = \sqrt{1-q} \left| 1 - \frac{p^{(M-1)}(k)}{p^{(M)}(k)} \right| \sqrt{q + \left[ 1 - \frac{(1-q)p^{(M-1)}(k)}{p^{(M)}(k)} \right] \frac{p^{(M-1)}(k)}{p^{(M)}(k)}} = \sqrt{\frac{1-q}{q(2\mu^2 - 2\mu + 1) + \mu - 1}}, \] (26)

where \( \mu \) has been defined by Eq. (23).
In the same experiment, Figs. 2(a1), 2(a2) and 2(b1), 2(b2) show the effective value of the object pixels $\mu(M, k)$ versus the projection photon number $k$ of the bucket signal, and Figs. 2(c1), 2(c2) and 2(d1), 2(d2) show the corresponding visibility and CNR of quantum projection imaging, respectively. In Fig. 2(b1), we can see that the positive imaging and negative imaging exist when the projection photon number $k \geq 14$ and $k \leq 13$, respectively. In the case of positive imaging, $\mu < M$ is valid for a larger $k$ ($k > 23$), and it gets the better visibility and CNR [see Figs. 2(c1) and 2(d1)]. However, the vacuum and small photon number ($k < 10$) projections in the negative imaging case also have the better visibility and CNR than the conventional imaging, as long as Eq. (24) is satisfied. Especially for the vacuum projection, $-\mu(M, k = 0) \to (1 - q)/q$, the visibility is almost perfect and the CNR is much higher than the others.

According to Eq. (4), it can be proved that when $Mq < 1$, the pixel number of the illuminated object is less than 1, and all the quantum projection images are positive except the vacuum projection imaging. Figures 2(a2)–2(d2) show the experiment for the case $M = 394$ and $q = 0.001$, which satisfies $Mq < 1$. The vacuum projection imaging has much better visibility than other ways. In most cases, however, the CNR of quantum projection imaging is better than that of conventional imaging.

It is worth pointing out that in computational ghost imaging, when some frames are selected for imaging, the remaining frames can be superimposed to form a complementary image [22]. In quantum imaging of the $k$-photon projection, the corresponding complementary image of non-$k$-photon projection is formed by projecting all other photon numbers except the $k$-photon. The probability of bucket detection of non-$k$-photon projection is given by $p^{(M)}(k) = 1 - p^{(M)}(k)$. Replacing $p^{(M)}(k)$ with $p^{(M)}(k)$ in Eqs. (17), (22), (25), and (26), we obtain the visibility and CNR for the complementary image. According to the mathematical formulas, there is in general no simple relationship between a pair of complementary images in terms of the visibility and CNR.

### 2.3. Vacuum projection imaging

In the quantum projection scheme above, a very peculiar option is the vacuum projection, where no photons are detected in the bucket signal. Let $p_i(0)$ be the probability of the vacuum state for the incident light on each micromirror. The corresponding vacuum probability for the outgoing light of the micromirror is $p_i^{(1)}(0) = 1 - q + q p_i(0)$. Since all micromirrors are statistically independent of each other, for $M$ micromirrors it has $p_i^{(M)}(0) = [p_i^{(1)}(0)]^M = [1 - q + q p_i(0)]^M$. Therefore, the condition of negative image $p_i^{(M)}(0) < p_i^{(M-1)}(0)$ is always satisfied for vacuum projection detection. According to Eq. (20), the visibility of vacuum projection imaging is obtained to be

$$V(k = 0) = \frac{(1 - q)(1 - p_i(0))}{1 - q + (1 + q) p_i(0)}.$$  \hspace{1cm} (27)

The corresponding CNR for vacuum projection imaging is written as

\begin{equation}
\text{CNR} = \frac{1}{\sqrt{\frac{(\alpha^2(0))_{\text{off}} - (\alpha(0))_{\text{off}}^2}{(\alpha^2(0))_{\text{on}} - (\alpha(0))_{\text{on}}^2}}} = [1 - p_i(0)] \sqrt{\frac{q (1 - q)}{1 - q + q p_i(0)^2 + p_i(0)}}. \hspace{1cm} (28)
\end{equation}

For the coherent light with average photon number $N_1$, $p_i(0) = \exp(-N_1)$. In the vacuum projection cases, imaging visibility and CNR versus the probability of vacuum state are shown in Figs. 3(a1) and 3(b1), and versus the average photon number $N_1$ in Figs. 3(a2) and 3(b2), respectively. Figure 3 tells us that appropriate intensity of driving light ensures a good imaging result of vacuum projection. In the case of a larger average photon number $N_1$, a small probability $q$ can be selected to maintain a large probability of the vacuum state, as shown in Fig. 2.

As indicated in Eqs. (12) and (15) above, the visibility and CNR of computational ghost imaging are inversely proportional to the object pixels. In quantum projection imaging [23], however, the simple inverse relationship no longer holds. Interestingly, for vacuum projection imaging, Eqs. (27) and (28) show that both visibility and CNR are completely independent of object pixels. This feature will greatly improve the image quality and resolution in the face of large and complex objects in computational ghost imaging.

### 3. Experiment

The experimental setup is similar to that for conventional computational ghost imaging, but with a photon counting system replacing the intensity detection, as shown in Fig. 1. The photon counting system consists of an SPAD (Excelitas SPCM-AQRH-W6) and a time-correlated single-photon counting (TCSPC) module (PicoQuant PicoHarp 300) with a time resolution of 4 ps. The optical source is a supercontinuum pulsed laser SCPL (NKT, SuperK EXTREME) with a temporal pulse width...
of 20 ps and a frequency of 6.49 MHz. After passing through a filter, a beam of wavelength 660 nm is selected to illuminate the object. The laser beam is strongly attenuated by a neutral density filter (NDF, Daheng GCC-3010) before hitting the DMD (Xintong F4100). The object beam is reflected by the DMD and registered by a single-pixel SPAD with the help of a collecting lens, L. The DMD contains $1024 \times 768$ independently addressable micromirrors and is used to load random patterns and/or virtual objects (a music note and resolution bars).

The refresh time of the DMD frame can be set in the range of $5 \times 10^{-3}$ s to $2 \times 10^{-3}$ s. For example, if the refresh time is $10^{-3}$ s, the corresponding repetition rate of each frame is 6490 for the pulse frequency 6.49 MHz of the driving laser. We record the total number of photons in these 6490 pulses as the photon counts of the frame. For best experimental results, the photon number measured in the frame must be much smaller than the repetition rate of each frame.

We first create a virtual object in the DMD, a musical note with $M = 394$ pixels, and consider the two cases of the probabilities that the micromirror is opened, $q = 0.005$ and $q = 0.001$. We measure the photon statistical distributions of the bucket signal, where the average photon numbers are $\langle n \rangle_M = 14.9$ and $\langle n \rangle_M = 1.61$, as shown in Figs. 2(a1) and 2(a2), respectively. All the experimental results are shown with the circles and are consistent with the theoretical simulation of Eq. (4) for the Poisson photon distribution of the driving beam. In the experimental results, as we have already pointed out above, the decrease in 0-photon count and the increase in 1- and 2-photon counts are due to the stray light and dark counting of SPAD.

Figures 2(c1), 2(c2) and 2(d1), 2(d2) are the visibility and CNR for the two cases of quantum projection imaging, respectively. In Fig. 2(c1), the theoretical and experimental results show that negative images occur when the number of projection photons $k$ is not greater than 13; otherwise, positive images occur. The theoretical derivation manifests that when $Mq < 1$, negative imaging can only exist in vacuum projection, and this is verified in Fig. 2(c2). However, the experimental result shows the negative visibility for $k = 1$ photon. This inconsistency is due to the fact that many 0-photon counts have been added into the 1-photon case, as has been shown in Fig. 2(a2). The experimental results of the CNR are shown in Figs. 2(d1) and 2(d2). We can see that the CNR and visibility change synchronously. In general, all the experimental results in Fig. 2 are in good agreement with the theoretical curves.

In conjunction with Fig. 2, Fig. 4 shows the experimental observation of reconstructed images for a virtual object (music note) of $M = 394$ pixels, and all values of imaging visibility and CNR are listed in the plots. For the three cases of average photon numbers $\langle n \rangle_M$ and probabilities $q$ of micromirror being opened, we measured the same number of 40,960 DMD frames and reconstructed the images in the four ways: vacuum projection ghost imaging (VPGI), quantum projection ghost imaging with $k$ photons (QPGI), conventional ghost imaging (CGI), and fast first-photon ghost imaging (FFPGI). In case (i) $q = 0.005$ and $\langle n \rangle_M = 14.8$, the frame numbers recorded in vacuum projection and $k = 31$ photons projection are 3729 and 432, respectively. In case (ii) $q = 0.001$ and $\langle n \rangle_M = 1.61$, the frame numbers recorded in vacuum projection and $k = 4$ photons projection are 22,968 and 2117, respectively. In case (iii) $q = 0.001$ and $\langle n \rangle_M = 1.02$, the frame numbers recorded in vacuum projection and $k = 3$ photons projection are 25,535 and 2549, respectively. Obviously, the negative images of vacuum projection are much better than all other positive images in terms of visibility and CNR.

In Fig. 5, the virtual objects are resolution bars (each contains five pixels) in various combinations. We use the same probability $q = 0.005$, the photon number $N_1 = 7.86$ of each micromirror, and 100,000 DMD frames for all the cases. The experimental results of VPGI, CGI, and non-vacuum projection ghost imaging (NVPGI) for the same objects are plotted in the first, second, and third columns, respectively. The corresponding photon number distributions of the bucket detection are plotted in the fourth column. It is clear that as the number of object pixels increases, both the visibility and CNR of VPGI can maintain high values unchanged, while both decrease in CGI and NVPGI (the complementary imaging of VPGI).

Two Chinese characters “zhēn” and “kōng” (vacuum) as real objects are also used in the experimental scheme of Fig. 1. The experimental results and relevant data are shown in Fig. 6. In all 40,000 DMD frames, 4696 and 7508 frames are assigned to the vacuum projection for “zhēn” and “kōng”, respectively, and two high-quality negative images are formed with these frames. The reconstructed images of the real objects also demonstrate that VPGI is the best choice with respect to CGI and FFPGI.

4. Discussion and Conclusion

Under strong enough light illumination, both conventional imaging and computational ghost imaging can easily produce the
signal and DMD random patterns, we propose quantum statistical correlation between the photon counts of the bucket theory of computational ghost imaging. Based on the quantum image for a given total number of photons?

The corresponding photon number distributions of bucket detection are plotted in the fourth column. \( F \) is the number recorded in vacuum projection.

![Fig. 5](image_url)

**Fig. 5.** Reconstructed images of resolution bars in various combinations with VPGI (first column), CGI (second column), and NVPGI (third column). The corresponding photon number distributions of bucket detection are plotted in the fourth column. \( F \) is the number recorded in vacuum projection.

![Fig. 6](image_url)

**Fig. 6.** Number of frames versus photon counts and reconstructed images of two Chinese characters with VPGI, CGI, and FFPGI in (a) “zhen” and (b) “kong.” The total number of frames is 40,000.

For low-light imaging application, we establish the quantum theory of computational ghost imaging. Based on the quantum statistical correlation between the photon counts of the bucket signal and DMD random patterns, we propose quantum projection imaging, in which the reconstructed image is formed by detecting a particular photon count in the bucket signal. The vacuum state and lower-photon counting projections yield negative imaging, while projections with higher photon counts yield positive imaging. Both theoretical and experimental results have shown that proper selection of quantum projection imaging can lead to better imaging results than CGI and fast first-photon ghost imaging. Quantum projection imaging, as a pure quantum version applied to computational ghost imaging, will attract much attention. In particular, vacuum projection imaging can achieve the best negative image, especially because its visibility and CNR are independent of object pixels. This important feature will greatly promote the wide application of quantum imaging technology in large and complex scenes.

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**References**

1. B. Sun, M. P. Edgar, R. Bowman, et al., “3D computational imaging with single-pixel detectors,” Science 340, 844 (2013).
2. A. Kirmani, D. Venkatraman, D. Shin, et al., “First-photon imaging,” Science 343, 58 (2014).
3. M. Sonnleitner, J. Jeffers, and S. M. Barnett, “Image retrodiction at low light levels,” Optica 2, 950 (2015).
4. P. A. Morris, R. S. Aspden, J. E. C. Bell, et al., “Imaging with a small number of photons,” Nat. Commun. 6, 5913 (2015).
5. G. Gariepy, N. Krstajić, R. Henderson, et al., “Single-photon sensitive light-in-flight imaging,” Nat. Commun. 6, 6021 (2015).
6. D. Shin, F. Xu, D. Venkatraman, et al., “Photon-efficient imaging with a single-photon camera,” Nat. Commun. 7, 12046 (2016).
7. A. M. Pawlikowska, A. Halimi, R. A. Lamb, et al., “Single-photon three-dimensional imaging at up to 10 kilometers range,” Opt. Express 25, 11919 (2017).
8. X. Liu, J. Shi, X. Wu, et al., “Fast first-photon ghost imaging,” Sci. Rep. 8, 5012 (2018).
9. J. Tachella, Y. Altmann, N. Mellado, et al., “Real-time 3D reconstruction single-photon lidar data using plug-and-play point cloud denoisers,” Nat. Commun. 10, 4984 (2019).
10. S. D. Johnson, P. A. Moreau, T. Gregory, et al., “How many photons does it take to form an image?” Appl. Phys. Lett. 116, 260504 (2020).
11. X. Liu, J. Shi, L. Sun, et al., “Photon-limited single-pixel imaging,” Opt. Express 28, 8132 (2020).
12. Z. P. Li, J. T. Ye, X. Huang, et al., “Single-photon imaging over 200 km,” Optica 8, 344 (2021).
13. J. H. Shapiro, “Computational ghost imaging,” Phys. Rev. A 78, 061802 (2008).
14. D. L. Donoho, “Compressed sensing,” IEEE Trans. Inf. Theory 52, 1289 (2006).
15. O. Katz, Y. Bromberg, and Y. Silberberg, “Compressive ghost imaging,” Appl. Phys. Lett. 95, 131110 (2009).
16. P. A. Moreau, E. Toninelli, T. Gregory, et al., “Imaging with quantum states of light,” Nat. Rev. Phys. 1, 367 (2019).
17. M. Genovese, “Real applications of quantum imaging,” J. Opt. 18, 073002 (2016).
18. K. W. C. Chan, M. N. O’Sullivan, and R. W. Boyd, “Optimization of thermal ghost imaging: high-order correlations vs. background subtraction,” Opt. Express 18, 5562 (2010).
19. L. A. Wu and K. H. Luo, “Two-photon imaging with entangled and thermal light,” AIP Conf. Proc. 1384, 223 (2011).
20. R. E. Meyers, K. S. Deacon, and Y. Shih, “Positive-negative turbulence-free ghost imaging,” Appl. Phys. Lett. 100, 131114 (2012).
21. K. H. Luo, B. Q. Huang, W. Zheng, et al., “Nonlocal imaging by conditional averaging of random reference measurements,” Chin. Phys. Lett. 29, 074216 (2012).
22. H. Yang, S. Wu, H. B. Wang, et al., “Probability theory in conditional-averaging ghost imaging with thermal light,” Phys. Rev. A 98, 053853 (2018).
23. X. B. Song, S. H. Zhang, D. Cao, et al., “Inherent relation between visibility and resolution in thermal light ghost imaging,” Opt. Commun. 365, 38 (2016).