Strong long-range spin-spin coupling with Kerr magnon interface

Wei Xiong,1, 2 Miao Tian,1 Guo-Qiang Zhang,2 Mingfeng Wang,1 Jiaojiao Chen,1, 3, * and J. Q. You2, †

1 Department of Physics, Wenzhou University, Zhejiang 325035, China
2 Interdisciplinary Center of Quantum Information and Zhejiang Province Key Laboratory of Quantum Technology and Device, Department of Physics and State Key Laboratory of Modern Optical Instrumentation, Zhejiang University, Hangzhou 310027, China
3 Hefei Preschool Education College, Hefei 230013, China
(Dated: December 2, 2021)

Strong long-range coupling between distant spin qubits is crucial for solid-state quantum information processing. However, achieving such a strong spin-spin coupling remains a well-known challenge. To this end, we propose an efficient method to realize a strong coupling between two distant spins via the Kerr effect of the magnons in a yttrium-iron-garnet sphere. By applying a strong microwave field on the magnon, the Kerr effect of the magnon is converted to its squeezing effect characterized by the squeezing parameter. Thus, the coupling between the spins and the squeezed-magnon can be exponentially enhanced, which in turn allows the spin-magnon distance greatly improved from nanometer to micrometer. By considering the virtual excitation of the squeezed-magnon in the dispersive regime, strong spin-spin coupling mediated by the squeezed-magnon can be achieved. As applications, remote quantum state transfer, nonlocal two-qubit iSWAP gate with high fidelity and remote quantum charger-battery device are investigated. Our proposal provides a potential platform to perform distant quantum information tasks and build the thermodynamic device with weakly coupled spin-magnon systems.

I. INTRODUCTION

Nitrogen-vacancy center spins in diamond [1, 2], with high controllability [2] and long coherence time [3–5], are key elements in the development of solid-state quantum information science. The essential question is how to achieve the strong long-distance spin-spin coupling, which is still a challenge to date. This is because the direct spin-spin coupling is usually weak and limited by their spatial separation [6–8]. Thus, seeking a good quantum interface for realization of the indirect spin-spin coupling is significant. In general, such an indirect coupling is not limited by the spin-spin separation.

Among various quantum interfaces including superconducting circuits [9, 10], phonons [11–20], critical polaritons [21, 22], squeezed photons [23, 24] and optomechanics [25–27], the emerged magnons [28–32] (i.e., the energy quanta of spin waves) in magnetical materials have played an essential role in quantum physics [33–45], with different types of magnets such as sphere magnets [46–49] and film layer magnets [50–53]. Due to the small mode volume and high spin density of the Kittel mode in a yttrium-iron-garnet (YIG) sphere magnet, magnons can be strongly coupled to other quantum systems [54–63]. With these advantages, magnons in the YIG sphere have been employed as the potential interfaces for realizing strong spin-spin [64, 65] interactions. For instance, the nanosphere, analogy with the plasmonics [66], is employed for concentrating magnetic fields in the near-infrared range of the electromagnetic spectrum [67, 68]. Thus, the local magnetic environment of spin qubits in the microwave domain is strengthened, leading to the strong spin-magnon and spin-spin couplings. However, the spins are required to be placed near the surface of the sphere, which leads to difficulty in manipulating the single spin qubits. In Ref. [64], the intrinsic nonlinearity of the magnons is taken into account for enhancing the spin-magnon coupling, but such the nonlinearity is usually weak in fact. Besides, magnons in bulk magnetic materials [69] and well-designed magnon waveguide [70] can also be used to mediate the strong spin-spin coupling.

To improving the limitations in previous studies [64, 65], we propose an efficient approach to realize strong long-range spin-magnon and spin-spin coupling with Kerr magnons in the YIG nanosphere, where the spin qubits, located at the micrometer distance from the surface of the YIG nanosphere, are weakly coupled to the magnons driven by a strong microwave field. Experimentally, the strong and tunable Kerr nonlinearity of the magnons, originated from the magnetocrystalline anisotropy in the YIG sphere [71], has been demonstrated [72]. Such nonlinear effects have been used to study bistability [71, 73] and tristability [74, 75], nonreciprocity properties [76], quantum entanglement [77] and quantum phase transition [78]. The strong microwave field acting on the Kerr magnons gives rise to the strong magnon squeezing effect. In the squeezed-magnon presentation, the spin-magnon coupling is exponentially enhanced. With reasonable parameters, the enhanced spin-magnon coupling can enter the strong or ultrastrong coupling regime. This strong coupling allows quantum state transfer between the spin qubit and the squeezed-magnon, and realization of the quantum charger-battery device. We further consider the enhanced spin-magnon system in the dispersive regime, thus the strong and long-range spin-spin coupling can be obtained by adiabatically eliminating the degree of free-
The Kerr magnon driven by a strong microwave field with frequency \( \omega \) gives rise to the anharmonicity of the magnon, where \( \gamma/2\pi = g_e \mu_B / \hbar \) is the gyromagnetic ratio with the g-factor \( g_e \) and the Bohr magneton \( \mu_B \), and the Kerr coefficient \( K/\hbar \), so the Kerr effect will be more and more important for the small YIG sphere [see Fig. 1(b)]. For example, when \( R = 50 \) nm, \( K/2\pi = 128 \) Hz, which is much larger than that of the microsphere experimentally considered. With the Kerr effect, spin-magnon coupling enhancement can be predicted for the small sphere, leading to strong spin-magnon coupling even for long distance between them. Moreover, the Kerr coefficient can be tuned to be positive or negative by respectively adjusting the crystallographic axis [100] or [110] of the YIG sphere along the biased magnetic field [72]. To see coupling enhancement induced by the Kerr effect, we directly apply a strong microwave field (MWF) with amplitude \( \Omega_d \) and frequency \( \omega_d \) to drive the magnon, which can be characterized by the Hamiltonian \( H_D = \Omega_d (m^\dagger e^{-i\omega_d t} + m e^{i\omega_d t}) \). Thus, the total Hamiltonian of the hybrid system is \( H_{total} = H_{NL} + H_D \). By rewriting the magnon operator as the expectation value plus its fluctuation, i.e., \( m \rightarrow \langle m \rangle + m \), the Kerr Hamiltonian \( H_K \) in Eq. (1) can be linearized via neglecting the high order fluctuation terms, guaranteed by the strong driving MWF [79]. At the rotating frame with respect to \( \omega_d \), Eq. (1) reduces to [79]

\[
H_L = \frac{1}{2} \Delta_q \sigma_z + H_K + g(\sigma_+ m + m^\dagger \sigma_-)
\]

with \( H_K = \Delta_m m^\dagger m - (K/2)(m^2 + m^\dagger 2) \), where \( \Delta_m = \omega_m + 2KNm - \omega_d \) with \( N_m = \langle m \rangle^2 \) is the magnon-number-dependent frequency detuning, and \( \Delta_q = \omega_q - \omega_d \) is the spin qubit frequency detuning. \( K = (m^2)^2 \) is the enhanced coefficient induced by the linearization of the Kerr magnon. As \( K \) can be positive (negative), so \( K > 0 ( < 0) \). Note that the terms \( m^\dagger m^\dagger \) corresponds to simultaneous creation (annihilation) of two degenerate magnons with respect to a pumping field, which yields the total number of excitations not preserved in the hybrid system. Also, such square terms can give rise to magnon squeezing effect.

FIG. 1. (a) Schematic diagram of the proposed hybrid system consisting of an individual NV center spin weakly coupled to a Kerr magnon in a YIG sphere with the radius \( R \). The NV center spin, as a qubit with two states \( |g\rangle \) and \( |e\rangle \) shown in pink box, is located \( d \) distance from the sphere surface, and the magnon is driven by a strong microwave field with frequency \( \omega_d \) and amplitude \( \Omega_d \). (b) The coefficient of the magnon Kerr nonlinearity \( K/2\pi \) versus the radius \( R \) of the YIG sphere. (c) The spin-magnon coupling versus the radius \( R \) of the YIG sphere. (d) Sketch map of two single spins weakly coupled to the Kerr magnon driven by a strong microwave field.
III. STRONG AND ULTRASTRONG LONG-RANGE SPIN-MAGNON COUPLING

Long-range light-matter interaction is important for quantum information processing. Previous proposals [63, 65] show that strong spin-magnon coupling can only be attained when the spin is close to the surface of the YIG sphere, i.e., \( d \sim nm \). To realize strong long-range spin-magnon interaction, we operate the whole system at the squeezed-magnon frame via Bogoliubov transform \( m = m_s \cosh r_m - m_i \sinh r_m \). Substituting this expression into Eq. (2) and setting the squeezing parameter \( r_m = (1/4) \ln(\Delta_m + K)/(\Delta_m - K) \), Eq. (2) becomes [79]

\[
H_S = \frac{1}{2} \Delta_m \sigma_z + \Delta_s m_s^\dagger m_s + G (m_s^\dagger + m_s) (\sigma^+ + \sigma^-),
\]

which is the standard Hamiltonian of the Rabi model with the exponentially enhanced coupling strength \( G = g e^{r_m}/2 \) characterizing the interaction between the spin qubit and the squeezed-magnon with the controllable frequency \( \Delta_s = \sqrt{\Delta_m^2 - K^2} \). Since parameters \( \Delta_m \) and \( K \) are both tunable, the squeezing parameter \( r_m \) can be arbitrarily large in principle when the instability threshold approaches, i.e., \( \Delta_m = K \). This directly leads to the enhanced spin-magnon coupling several orders of magnitude larger than the original coupling. Such a enhancement allows the distance between the spin and the magnon to be improved from the nm scale to \( \mu m \) scale. To address this, the exponentially enhanced spin-magnon coupling versus the spin-magnon separation is plotted in Fig. 2(a). Without magnon squeezing \( (r_m = 0) \), the spin-magnon coupling sharply decreases to zero when \( d \) is increased to dozens of nanometer. With the magnon squeezing \( (r_m = 10) \), we find that the enhanced coupling \( G \) can be \( \sim \) MHz even for \( d \sim \mu m \), i.e., \( G = 2\pi \times 0.7 \sim 4 \) MHz for \( d = 1 \) \( \mu m \). Compared to the previous proposals [63, 65], the spin-magnon separation is enhanced by three orders of magnitude while large spin-magnon coupling is still kept. For NV centers and magnons, the typical decay rates are respectively \( \gamma_s \sim 1 \) KHz and \( \kappa_m \sim 1 \) MHz. Thus, the enhanced spin-magnon system can enter the strong coupling regime, which is justified by the Rabi oscillation in Fig. 2(b), where the population of the magnon and the spin qubit as a function of the evolution time at \( \Delta_m = \Delta_s = 10G \) is plotted. Obviously, quantum state exchange between the magnon and the spin qubit can occur, where the squeezed-magnon is initially excited and the spin qubit is in the ground state. As \( \Delta_s \) and \( \Delta_m \) are both controllable via tuning the frequency of the MWF, \( G \) can be comparable with or exceed \( \Delta_s/\alpha \). This suggests that the enhanced spin-magnon system can also be in the ultrastrong coupling regime with the assistance of the MWF. Note that the undesired terms [79] in Eq. (3) have been neglected due to the fact \( e^{-r_m} \rightarrow 0 \) when \( e^{r_m} \rightarrow +\infty \). This can be achieved in our proposal by tuning the crystallographic axis \([100]\) of the YIG sphere along the biased magnetic field, i.e., \( K > 0 \).

FIG. 2. (a) The spin-magnon coupling strength versus the distance between the spin qubit and the surface of the YIG sphere with the squeezing parameter \( r_m = 0 \) and \( r_m = 10 \). (b) The population of the magnon and spin qubit versus the evolution time with \( G = 4 \) MHz and \( \Delta_q = \Delta_s = 10G \).

FIG. 3. (a) The occupation of the spin 1, spin 2 and the magnon versus the evolution time by numerically solving Eq. (5) when dissipations are included. (b) The fidelity of two-qubit iSWAP gate versus the evolution time by considering the effect of dissipations from the spins and the magnon. Here \( \kappa_m = 1 \) MHz, \( \gamma_q = 1 \) KHz, \( G_{\text{eff}} = 70 \) KHz.

FIG. 4. (a) The energy stored in s single spin battery and (b) the corresponding power as functions of the evolution time with different magnon excitation number \( m = 1 \) and \( m = 5 \). Here \( \Delta_s = \Delta_q = 10G \) and \( G = 4 \) MHz. (c) The maximum energy stored in n spin batteries and the corresponding power as functions of the spin number \( n \). Here \( \omega_{\text{eff}} = 10G_{\text{eff}} \) and \( G_{\text{eff}} = 70 \) KHz.

FIG. 4. (a) The energy stored in s single spin battery and (b) the corresponding power as functions of the evolution time with different magnon excitation number \( m = 1 \) and \( m = 5 \). Here \( \Delta_s = \Delta_q = 10G \) and \( G = 4 \) MHz. (c) The maximum energy stored in n spin batteries and the corresponding power as functions of the spin number \( n \). Here \( \omega_{\text{eff}} = 10G_{\text{eff}} \) and \( G_{\text{eff}} = 70 \) KHz.
IV. LONG-RANGE SPIN-SPIN COUPLING MEDIATED BY THE SQUEEZED MAGNONS

Now we consider the case that the squeezed magnon is coupled to two identical spins with the effective frequency $\Delta_s$, where two spins are placed at $d \sim 1 \mu m$ distance from the YIG sphere surface [see Fig. 1(d)]. As $d \gg R \sim 30$ nm, the size of the magnon can be ignored and thus the separation between two spins is $\sim 2 \mu m$. When the condition $G \ll \Delta_+ \equiv \Delta_s + \Delta_\parallel$ satisfies, RWA is allowed for Eq. (3) and thus the counter-rotating terms are neglected, so we have

$$H_R = \Delta_s m_i^+ m_s + \sum_{i=1}^{2} \left[ \frac{1}{2} \Delta_\parallel \sigma_i^{(j)} + G (m_i^j \sigma_i^{(j)} + h.c.) \right], \quad (4)$$

which is a typical Tavis-Cummings model. We further consider the spin-magnon system is in the dispersive regime, i.e., $G \ll |\Delta_+ - \Delta_\parallel|$, which allows to obtain the effective coupling between two remote spins. This can be achieved by adiabatically eliminating the degrees of freedom of the squeezed magnons via Fröhlich-Nakajima transformation [80, 81]. The effective spin-spin coupling Hamiltonian can be governed by [79]

$$H_{\text{eff}} = \frac{1}{2} \omega_{\text{eff}} (\sigma_i^{(1)} + \sigma_i^{(2)}) + G_{\text{eff}} (\sigma_{\perp i}^{(1)} \sigma_{\perp i}^{(2)} + h.c.), \quad (5)$$

where $\omega_{\text{eff}} = (1 + 2 (m_i^j m_s)) \Delta_\parallel / \Delta_\perp$ is the magnon-number-dependent frequency of the each spin induced by the spin-magnon dispersive coupling. $G_{\text{eff}} = G \Delta_\perp / \Delta_\parallel$ is the effective spin-spin coupling strength. To estimate this strength, we take $G \sim 2 \pi \times 0.7$ MHz for $d \sim 1 \mu m$ and $\Delta_\perp \sim 10$ GHz, so $G_{\text{eff}} / 2 \pi \sim 70$ KHz $\gg \gamma_\parallel \sim 1$ KHz. This strong long-range spin-spin coupling enable quantum state transfer between two spins mediated by the magnon and two-qubit iSWAP gate to be achieved. The dynamics can be governed by

$$\dot{\rho} = i [\rho, H_{\text{eff}}] + \kappa_m D[m_i^j] \rho + \gamma_\parallel D[\sigma_{\perp i}^{(1)}] \rho, \quad (6)$$

where $D[x] = xpx^{-1} - (x^+xp + px^+x)$ with $x = m_i^j$, $\sigma_i^{(1)}$, $\sigma_i^{(2)}$. Here $\gamma_\parallel$ denotes the transversal relaxation rate of the NV spin. Experimentally, the longitudinal relaxation rate is much smaller than $\gamma_\parallel$ [82], so we ignore its effect on the system dynamics.

In Fig. 3, we plot the occupation probability and gate fidelity versus the evolution time, where we initially prepare spin 1 in the excited state, spin 2 in the ground state and the magnon in the ground state. From Fig. 3(a) we see that in the presence of dissipations ($\kappa_m = 1$ MHz and $\gamma_\parallel = 1$ KHz) quantum state from spin 1 (2) can be transferred to spin 2 (1) while the magnon is kept at its initial state, due to the dispersive spin-magnon coupling. Thus, the fidelity of the two-qubit iSWAP gate is robust against the decay rate of the magnon [see Fig. 3(b)]. Besides, it is also insensitive to the decay rate of the spin qubit because of the achieved strong spin-spin coupling (i.e., $G_{\text{eff}} \gg \gamma_\parallel$)[see Fig. 3(b)].

V. LONG-RANGE QUANTUM BATTERY

Quantum battery (QB) [83-86] is one of the most prominent micro-devices in the rapid-developing quantum thermodynamics, which can transfer energy from the charger to the battery. Below we investigate this device with the proposed setup. Firstly, we consider the QB consists of a magnon coupled to a spin qubit, where the magnon acts as the charger and the spin qubit acts as the battery. Initially, the charger is in the state $|m\rangle$ with $m$ excitations, and the battery is in the ground state $|g\rangle$. Governed by the Hamiltonian (3) within the RWA, the energy stored in the QB and its power oscillate with the evolution time [see Fig. 4(a)]. At a certain time, i.e., $t = \pi / (2G)$, the QB is fully charged by the magnon and it has the maximum power [see Figs. 4(a) and (b)]. More importantly, we find the excitations in the magnon mode can speed up the charging process by the factor $\sqrt{m}$ [see Fig. 4(a), where we take $m = 5$ for example] and the power is hence greatly improved by $\sqrt{m}$. This can be interpreted as the QB is simultaneously charged by $m$ chargers with energy $h\Delta_s$. The situation of $n = 2$ and the effect of dissipations from the magnon and the spin qubit on the QB are also discussed [see [79] for details].

Then we consider another QB consisting of $n + 1$ spin qubits, in which the 0th spin qubit acts as the charger, and other $n$ identical spin qubits act as the battery. Such a system can be realized by dispersively coupling the magnon to $n + 1$ spins at the same distance $d$ from the YIG surface, thus the system Hamiltonian can be governed by

$$H_{\text{eff}}^{(n+1)} = \frac{1}{2} \omega_{\text{eff}} \sum_{i=0}^{n} \sigma_i^{(1)} + G_{\text{eff}} \sum_{i, j=0}^{n} (\sigma_{\perp i}^{(1)} \sigma_{\perp j}^{(2)} + h.c.), \quad (7)$$

Initially, the charger is in the excited state $|e_0\rangle$ and other $n$ spins are all in the ground states $|g_1 g_2 g_3 ... g_n\rangle$. Evolved by Eq. (7), the evolved state at time $t$ is $|\psi_{0\rightarrow n} = A_n |e_0\rangle |g_1 g_2 g_3 ... g_n\rangle + B_n (t) |g_1 g_2 e_3 ... g_n\rangle + |g_1 g_2 g_3 ... g_{n-1} e_n\rangle + ... + |g_1 g_2 g_3 ... g_n e_0\rangle$ with

$$A_n = \frac{1}{\pi n \pi} e^{(-nG_{\text{eff}} t)} (1 + n \nu (n+1) G_{\text{eff}}^2 t) \quad \text{and} \quad B_n = \frac{1}{\pi n} e^{(-nG_{\text{eff}} t)} (-1 + e^{[(n+1)G_{\text{eff}} t]}).$$

Thus, the energy stored in the $n$ spin batteries is $E_{0\rightarrow n} = n |B_n (t)|^2 = \frac{4n}{(n+1)^2} \sin^2 (\frac{n+1}{2} G_{\text{eff}} t)$ and the power is $P_{0\rightarrow n} = \frac{4n}{(n+1)^2} \pi \sin^2 (\frac{n+1}{2} G_{\text{eff}} t)$. At time $t_n = \pi / [(n + 1) G_{\text{eff}}]$, the energy has the maximum value $E_{\text{max}} = \omega_{\text{eff}} 4n / (n + 1)^2$ and the maximum power is $P_{\text{max}} = 4n G_{\text{eff}} / [(n + 1) \pi]$. We plot them as functions of the spin number in Fig. 4(c) with $G_{\text{eff}} = 70$ KHz. Obviously, the maximum energy stored in the batteries decrease with increasing $n$ while the maximum power increases. This is because each spin battery obtains the energy from the definite energy in the charger decreases with increasing spin numbers. However, each spin battery can interact with other $(n - 1)$ spin batteries and the charger due to the indirect coupling mediated by the magnon, which can be equivalent regarded as one of spin batteries is charged.
by the charger and other \( n - 1 \) batteries at the same time, so power is improved with the spin number.

VI. CONCLUSION

In summary, we have proposed a feasible proposal to achieve strong tunable spin-spin coupling with the Kerr magnons in a YIG nanosphere. Assisted by the strong microwave field, the Kerr nonlinearity of the magnons can be converted to the squeezing effect, which gives rise to the exponentially enhanced strong spin-magnon coupling in the squeezed-magnon frame with the experimental parameters. This strong coupling allows the separation between the spins and the YIG sphere improved from nanometer to micrometer. In the dispersive regime, the virtual squeezed-magnon can be adiabatically eliminated to introducing the strong long-distant spin-spin coupling. With the strong spin-magnon and spin-spin couplings, some quantum information tasks such as remote quantum state transfer and high-fidelity two-qubit iSWAP gate are investigated. Also, long-distance quantum battery can be achieved. Our proposal can provide a potential platform to realize remote quantum information processing and construct the thermodynamic devices in a hybrid quantum system consisting of the Kerr magnons weakly coupled to the single spins.

ACKNOWLEDGMENTS

This work is supported by the National Natural Science Foundation of China (Grants No. U1801661, No. 11934010), the Postdoctoral Science Foundation of China (Grant No. 2020M671687).

[1] R. Schirhagl, K. Chang, M. Loretz, and C. L. Degen, Nitrogen-Vacancy Centers in Diamond: Nanoscale Sensors for Physics and Biology, Annu. Rev. Phys. Chem. 65, 83 (2014).
[2] M. W. Doherty, N. B. Manson, P. Delaney, F. Jelezko, J. Wrachtrup, L. C. L. Hollenberg, The nitrogen-vacancy colour centre in diamond, Phys. Rep. 528, 1 (2013).
[3] N. Bar-Gill, L. Pham, A. Jarmola, D. Budker, and R. Walsworth, Solid-state electronic spin coherence time approaching one second. Nat. Commun. 4, 1743 (2013).
[4] F. Jelezko, T. Gaebel, I. Popa, A. Gruber, and J. Wrachtrup, Observation of Coherent Oscillations in A Single Electron Spin, Phys. Rev. Lett. 92, 076401 (2004).
[5] P. Neumann, D. Twitchen, M. Markham, R. Koselov, N. Mizuochi, J. Isoya, J. Achard, J. Beck, J. Tissler, V. Jacques, P. R. Hemmer, F. Jelezko, and J. Wrachtrup, Ultralong spin coherence time in isotopically engineered diamond, Nat. Mater. 8, 383 (2009).
[6] Y. Kubo, F. R. Ong, P. Bertet, D. Vion, V. Jacques, D. Zheng, A. Dréau, J.-F. Roch, A. Auffeves, F. Jelezko, J. Wrachtrup, M. F. Barthe, P. Bergonzo, and D. Esteve, Strong Coupling of A Spin Ensemble to A Superconducting Resonator, Phys. Rev. Lett. 105, 140502 (2010).
[7] D. Marcos, M. Wubs, J. M. Taylor, R. Aguado, M. D. Lukin, and A. S. Sørensen, Coupling Nitrogen-Vacancy Centers in Diamond to Superconducting Nitrogen Qubits, Phys. Rev. Lett. 105, 210501 (2010).
[8] X. Zhu, S. Saito, A. Kemp, K. Kakuyanagi, S. Karimoto, H. Nakano, W. J. Munro, Y. Tokura, M. S. Everitt, K. Nemoto, M. Kasu, N. Mizuochi, and K. Semba, Coherent coupling of a superconducting flux qubit to an electron spin ensemble in diamond Nature (London) 478, 221 (2011).
[9] W. Xiong, Y. Qiu, L. A. Wu, and J. Q. You, Amplification of the coupling strength in a hybrid quantum system, New J. Phys. 20, 043037 (2018).
[10] J. Twamley and S. D. Barrett, Superconducting cavity bus for single nitrogen-vacancy defect centers in diamond, Phys. Rev. B 81, 241202(R) (2010).
[11] O. Arcizet, V. Jacques, A. Siria, P. Poncharal, P. Vincent, and S. Seidelin, A single nitrogen-vacancy defect coupled to a nanomechanical oscillator, Nat. Phys. 7, 879 (2011).
[12] S. D. Bennett, N.Y. Yao, J. Otterbach, P. Zoller, P. Rabl, and M. D. Lukin, Phonon-Induced Spin-Spin Interactions in Diamond Nanostructures: Application to Spin Squeezing, Phys. Rev. Lett. 110, 156402 (2013).
[13] P. B. Li, Z. L. Xiang, P. Rabl, and F. Nori, Hybrid Quantum Device with Nitrogen-Vacancy Centers in Diamond Coupled to Carbon Nanotubes, Phys. Rev. Lett. 117, 015502 (2016).
[14] P. Rabl, P. Cappellaro, M. V. Gurudev Dutt, L. Jiang, J. R. Maze, and M. D. Lukin, Strong Magnetic coupling between an electronic spin qubit and a mechanical resonator, Phys. Rev. B 79, 041302(R) (2009).
[15] P. Rabl, S. J. Kolkowitz, F. H. L. Koppen, J. G. E. Harris, P. Zoller, and M. D. Lukin, A quantum spin transfucer based on nanoemometrical resonator arrays, Nat. Phys. 6, 602 (2010).
[16] S. Kolkowitz, A. C. Bleszynski Jayich, Q. P. Unterreithmeier, S. D. Bennett, P. Rabl, J. G. E. Harris, and M. D. Lukin, Coherent sensing of a mechanical resonator with a single-spin qubit, Science 335, 1603 (2012).
[17] B. Pigeau, S. Rohr, L. Mercier de Lepinay, A. G loophole, V. Jacques, and O. Arcizet, Observation of a phononic mallow triplet in a multimode hybrid spin-nanomechanical system, Nat. Commun. 6, 8603 (2015).
[18] B. Li, Y. Zhou, W. B. Gao, and F. Nori, Enhancing Spin-Phonon and Spin-Spin Interactions Using Linear Resources in a Hybrid Quantum System, Phys. Rev. Lett. 125, 153602 (2020).
[19] J. Teissier, A. Barfuss, P. Appel, E. Neu, and P. Maletinsky, Strain Coupling of A Nitrogen Vacancy Center Spin to A Diamond Mechanical Oscillator, Phys. Rev. Lett. 113, 020503 (2014).
[20] P. Ovartchaiyapong, K. W. Lee, B. A. Myers, and A. C. Bleszynski Jayich, Dynamic strain-mediated coupling of a single diamond spin to a mechanical resonator, Nat. Commun. 5, 4429 (2014).
[21] W. Xiong, Jiaojiao Chen, Baolong Fang, Mingfeng Wang, Liu Ye,3, and J. Q. You, Strong Tunable Spin-Spin Interaction in a Weakly Coupled Nitrogen Vacancy Spin-Cavity Electromechanical System, Phys. Rev. B 103, 174106 (2021).

[22] J. Chen, Z. Li, X. Q. Luo, W. Xiong, M. Wang, H. C. Li, Strong single-photon optomechanical coupling in a hybrid quantum system, Opt. Express 29, 32639 (2021).

[23] W. Qin, A. Miranowicz, P. B. Li, X. Y. Li, J. Q. You, and F. Nori, Exponentially Enhanced Light-Matter Interaction, Cooperativities, and Steady-State Entanglement Using Parametric Amplification, Phys. Rev. Lett. 120, 093601 (2018).

[24] C. Leroux, L. C. G. Govia, and A. A. Clerk, Enhancing Cavity Quantum Electrodynamics via Antisqueezing: Synthetic Ultrastrong Coupling, Phys. Rev. Lett. 120, 093602 (2018).

[25] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, Rev. Mod. Phys. 86, 1391 (2014).

[26] X. Y. Li, H. Jing, J. Y. Ma, and Y. Wu, PT-Symmetry Breaking Chaos in Optomechanics, Phys. Rev. Lett. 114, 253601 (2015).

[27] W. Xiong, Z. Li, Y. Song, J. Chen, G. Q. Zhang, and M. Wang, Higher-order exceptional point in a pseudo-Hermitian cavity optomechanical system, arXiv:2109.12232.

[28] D. D. Awschalom, C. H. R. Du, R. He, F. J. Heremans, A. Hoffmann, J. T. Hou, H. Kurebayashi, Y. Li, L. Liu, V. Novosad, J. Sklenar, S. E. Sullivan, D. Sun, H. Tang, V. Tiberkevich, C. Trevillian, A. W. Tsen, L. R. Weiss, W. Zhang, X. Zhang, L. Zhao, and C. W. Zollitsch, Quantum engineering with hybrid magnonics systems and materials. IEEE Transactions on Quantum Engineering 2, 5500836 (2021).

[29] B. Z. Rameshti, S. V. Kusminskiy, J. A. Haigh, K. Usami, D. Lachance-Quirion, Y. Nakamura, C. M. Hu, H. X. Tang, G. E. W. Bauer, and Y. M. Blanter, Cavity Magnonics, arXiv:2106.09312.

[30] X. Tang, G. E. W. Bauer, and Y. M. Blanter, Cavity Magnonics, npj Quantum Inf. 1, e1501286 (2016).

[31] X. Zhang, C. L. Zou, L. Jiang, and H. X. Tang, Strongly Coupled Magnons and Cavity Microwave Photons, Phys. Rev. Lett. 113, 083603 (2014).

[32] Y. P. Wang, J. W. Rao, Y. Yang, B. Yao, J. Dietrich, G. E. Bridges, X. L. Fan, D. S. Xue, and C. M. Hu, Interferometric control of magnon-induced nearly perfect absorption in cavity magnonics, Nat. Commun. 12, 1933 (2021).

[33] Z. B. Yang, X. D. Liu, X. Y. Yin, Y. Ming, H. Y. Liu, and R. C. Yang, Controlling Stationary One-Way Quantum Steering in Cavity Magnonics, Phys. Rev. Applied 15, 024042 (2021).

[34] O. O. Soykal and M. E. Flatté, Strong Field Interactions between a Nanomagnet and a Photonic Cavity, Phys.Rev.Lett. 104, 077202 (2010).

[35] Y. Tabuchi, S. Ishino, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Hybridizing Ferromagnetic Magnons and Microwave Photons in the Quantum Limit, Phys. Rev. Lett. 113, 083603 (2014).

[36] X. Zhang, C. L. Zou, L. Jiang, and H. X. Tang, Strong Coupled Magnons and Cavity Microwave Photons, Phys.Rev.Lett. 113, 156401 (2014).

[37] Y. Tabuchi, S. Ishino, A. Neguchi, T. Ishikawa, R. Yamazaki, K. Usami, and Y. Nakamura, Coherent coupling between a ferromagnetic magnon and a superconducting qubit, Science 349, 405 (2015).

[38] C. W. Sandweg, Y. Kajiwara, A. V. Chumak, A. A. Serga, V. I. Vasyuchka, M. B. Jungfleisch, E. Saitoh, and B. Hillebrands, Spin Pumping by Parametrically Excited Exchange Magnons, Phys.Rev.Lett. 106, 216601 (2011).

[39] H. Huebl, C. W. Zollitsch, J. Lotze, F. Hocke, M. Greiffenstein, A. Marx, R. Gross, and S. T. B. Goennenwein, High Cooperativity in Coupled Microwave Resonator Ferrimagnetic Insulator Hybrids, Phys. Rev. Lett. 111, 127003 (2013).

[40] X. Zhang, C. Zou, L. Jiang, and H. X. Tang, Superstrong coupling of thin film magnetostatic waves with microwave cavity, J. Appl. Phys. 119, 023905 (2016).

[41] Y. Li, T. Polakovic, Y. L. Wang, J. Xu, S. Lendinez, Z. Zhang, J. Ding, T. Khaire, H. Saglam, R. Divan, J. Pearson, W.-K. Kwok, Z. Xiao, V. Novosad, A. Hoffmann, and W. Zhang, Strong Coupling between Magnons and Microwave Photons in On-Chip Ferromagnet-Superconductor Thin-Film Devices, Phys.Rev.Lett. 123, 107701 (2019).

[42] S. V. Kusminskiy, H. X. Tang and F. Marquardt, Coupled spin-light dynamics in cavity optomagnonics, Phys. Rev. A 94, 033821 (2016).
[55] J. A. Haigh, S. Langenfeld, N. J. Lambert, J. J. Baumberg, A. J. Ramsay, A. Nunnenkamp, and A. J. Ferguson, Magneto-optical coupling in whispering-gallery mode resonators, Phys. Rev. A 92, 063845 (2015).

[56] H. Huebl, C. W. Zolitsch, J. Lotze, F. Hocke, M. Greifenstein, A. Marx, R. Gross, and S. T. B. Goennenwein, High cooperativity in coupled microwave resonator ferromagnetic insulator hybrids, Phys. Rev. Lett. 111, 127003 (2013).

[57] Y. Tabuchi, S. Ishino, A. Noguchi, T. Ishikawa, R. Ya-mazaki, K. Usami, Y. Nakamura, Coherent coupling between a ferromagnetic magnon and a superconducting qubit, Science 349, 6246 (2015).

[58] D. Lachance-Quirion, S. P. Wolski, Y. Tabuchi, S. Kono, K. Usami, and Y. Nakamura, Entanglement-based single-shot detection of a single magnon with a superconducting qubit, Science 367, 425 (2020).

[59] J. Li, S. Y. Zhu, and G. S. Agarwal, Squeezed states of magnons and phonons in cavity magnomechanics, Phys. Rev. A 99, 021801 (2019).

[60] R. F. Sabiryanov and S. S. Jaswa, Magnons and Magnon-Phonon Interactions in Iron, Phys. Rev. Lett. 83, 10 (1999).

[61] X. Zhang, C. L. Zou, L. Jiang, and H. X. Tang, Cavity magnomechanics, Sci. Adv. 2, e1501286 (2016).

[62] Y. P. Gao, C. Cao, T. J. Wang, Y. Zhang and C. Wang, Cavity-mediated coupling of phonons and magnons, Phys. Rev. A 96, 023826 (2017).

[63] X. L. Hei, X. L. Dong, J. Q. Chen, C. P. Shen, Y. F. Qiao, and P. B. Li, Enhancing spin-phonon coupling with a micromagnet, Phys. Rev. A 103, 043706 (2021).

[64] I. C. Skogvoll, J. Lidal, J. Danon, and A. Kamra, Tunable anisotropic quantum Rabi model via magnon—spin-qubit ensemble, arXiv: 2105.07430.

[65] T. Neuman, D. S. Wang, and P. Narang, Nanomagnonic Cavities for Strong Spin-Magnon Coupling and Magnon-Mediated Spin-Spin Interactions, Phys. Rev. Lett. 125, 247702 (2020).

[66] M. I. Stockman, Katrin Kneipp, S. I. Bozhevolnay, S. Saha, A. Dutta, J. Ndulaife, N. Kinsey, H. Reddy, U. Guler, V. M Shalaev, Roadmap on plasmonics, J. Opt. 20, 043001 (2018).

[67] A. García-Etxarri, R. Gómez-Medina, L. S. Fróuque-Pérez, C. López, L. Chantada, F. Scheffold, J. Aizpurua, M. Nieto-Vesperinas, and J. J. Sáenz, Strong magnetic response of submicron Silicon particles in the infrared, Opt. Express 19, 4815 (2011).

[68] M. K. Schmidt, R. Esteban, J. J. Sáenz, I. Suárez-Lacalle, S. Mackowski, and J. Aizpurua, Dielectric antennas—a suitable platform for controlling magnetic dipolar emission, Opt. Express 20, 13636 (2012).

[69] L. Trifunovic, F. L. Pedrocchi, and D. Loss, Long-Distance Entanglement of Spin Qubits via Ferromagnet, Phys. Rev. X 3, 041023 (2013).

[70] M. Fukami, D. R. Candido, D. D. Awschalom, and M. E. Flatté, Opportunities for Long-Range Magnon-Mediated Entanglement of Spin Qubits via On- and Off-Resonant Coupling, PRX Quantum 2, 040314 (2021).

[71] G. Q. Zhang, Y. P. Wang, and J. Q. You, Theory of the magnon Kerr effect in cavity magnonics, Sci. China-Phys. Mech. Astron. 62, 987511 (2019).

[72] Y. P. Wang, G. Q. Zhang, D. Zhang, X. Q. Luo, W. Xiong, S. P. Wang, T. F. Li, C. M. Hu, and J. Q. You, Magnon Kerr effect in a strongly coupled cavity-magnon system, Phys. Rev. B 94, 224410 (2016).

[73] Y. P. Wang, G. Q. Zhang, D. Zhang, T. F. Li, C. M. Hu, and J. Q. You, Bistability of Cavity Magnon-Polaritons, Phys. Rev. Lett. 120, 057202 (2018).

[74] J. M. P. Nair, Z. Zhang, M. O. Scully, and G. S. Agarwal, Nonlinear spin currents, Phys. Rev. B 102, 104415 (2020).

[75] M. X. Bi, X. H. Yan, Y. Zhang, and Y. Xiao, Tristability of cavity magnon polaritons, Phys. Rev. B 103, 104411 (2021).

[76] C. Kong, H. Xiong, and Y. Wu, Magnon-Induced Nonreciprocity Based on the Magnon Kerr Effect, Phys. Rev. Appl. 12, 034001 (2019).

[77] Z. Zhang, M. O. Scully, and G. S. Agarwal, Quantum entanglement between two magnon modes via Kerr nonlinearity driven far from equilibrium, Phys. Rev. Research 1, 023021 (2019).

[78] G. Q. Zhang, Z. Chen, W. Xiong, C. H. Lam, and J. Q. You, Parity-symmetry-breaking quantum phase transition in a cavity magnonic system driven by a parametric field, Phys. Rev. B 104, 064423 (2021).

[79] See Supplementary Material at xxx for detailed derivations of our main results.

[80] H. Fröhlich, Theory of the Superconducting State. I. The Ground State at the Absolute Zero of Temperature, Phys. Rev. 79, 845 (1950).

[81] S. Nakajima, Perturbation theory in statistical mechanics, Adv. Phys. 4, 363 (1953).

[82] A. Angerer, S. Putz, D. O. Krimer, T. Astner, M. Zens, R. Glattauer, K. Streltsov, W. J. Munro, K. Nemoto, S. Rotter, J. Schmiedmayer, and J. Majer, Ultralong relaxation times in bistable hybrid quantum systems, Sci. Adv. 3, e1701626 (2017).

[83] F. C. Binder, S. Vinjanampathy, K. Modi, and J. Goold, Quantacell: powerful charging of quantum batteries, New. J. Phys. 17, 075015 (2015).

[84] F. Campaiolo, F. A. Pollock, F. C. Binder, L. Céleri, J. Goold, S. Vinjanampathy, and K. Modi, Enhancing the Charging Power of Quantum Batteries, Phys. Rev. Lett. 118, 150601 (2017).

[85] D. Ferraro, M. Campisi, G. M. Andolina, V. Pellegrini, and M. Polini, High-power collective charging of a solid-state quantum battery, Phys. Rev. Lett. 120, 117702 (2018).

[86] S. Qi and J. Jing, Magnon-mediated quantum battery under systematic errors, Phys. Rev. A 104, 032606 (2021).