THE N − ∆ WEAK AXIAL-VECTOR AMPLITUDE C_5^A(0)

Milton Dean Slaughter *

Department of Physics, University of New Orleans, New Orleans, LA 70148

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Abstract

The weak N − ∆ axial-vector transition amplitude ⟨Δ | A_{π+}^{μ} | N⟩ —important in N* production processes in general and in isobar models describing ν_μ N → μΔ processes in particular —is examined using a broken symmetry algebraic approach to QCD which involves the realization of chiral current algebras. We calculate a value for the form factor C_5^A(0) in good agreement with experiment.

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*E-Mail address (Internet): mslaught@uno.edu

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I. INTRODUCTION

The $N - \Delta$ weak axial-vector transition matrix element is important when one considers: Neutrino quasielastic scattering ($\nu_\mu + n \rightarrow \mu^- + p$); $\Delta^{++}$ production reactions ($\nu_\mu + p \rightarrow \mu^- + \Delta^{++}$); Hyperon semi-leptonic decays; Increased understanding of higher symmetries and relativistic and non-relativistic quark models; Models involving isobars; Dynamical calculations involving QCD; Dispersion relations; and Current algebras [1,2].

II. THEORY: THE $\Delta^{++}$ PRODUCTION PROCESS

The $\Delta^{++}$ production process can be studied in our model by considering the low energy matrix element

$$\langle \mu^- | \Delta^{++} | \nu p \rangle = \frac{G}{\sqrt{2}} J^h_\mu J^\mu_l = \frac{G}{\sqrt{2}} \langle \Delta^{++} | V_\mu - A_\mu | p \rangle J^\mu_l,$$

where $J^h_\mu \equiv$ hadronic weak current and $J^\mu_l \equiv$ leptonic current.

$V_\mu$ ($A_\mu$) is the hadronic vector (axial) current.

With our normalization, [1] nucleon–nucleon hadronic matrix elements may be written as:

$$\langle B_2(p_2, \lambda_2) | J^h_\mu | B_1(p_1, \lambda_1) \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_1 m_2}{E_1 E_2}} \bar{u}_2(p_2, \lambda_2) [\Gamma_\mu] u_1(p_1, \lambda_1)$$

where

$$\Gamma_\mu = f_1(q'^2) \gamma_\mu + (i f_2(q'^2)/m_1) \sigma_{\mu\nu} q'^\nu + (i f_3(q'^2)/m_1) q'^\mu$$

$$+ \{ g_A(q'^2) \gamma_\mu + (i g_P(q'^2)/m_1) \sigma_{\mu\nu} q'^\nu + (ig_3(q'^2)/m_1) q'^\mu \} \gamma_5$$

1 We normalize physical states according to $< \not{p} | \not{p'} > = \delta^3(\not{p} - \not{p'})$. Dirac spinors are normalized by $\bar{u}(p)u(p) = 1$. Our conventions for Dirac matrices are $\{ \gamma^\mu, \gamma^\nu \} = 2g^{\mu\nu}$ with $\gamma_5 \equiv i \gamma^0 \gamma^1 \gamma^2 \gamma^3$, where $g^{\mu\nu} = \text{Diag} \{ 1, -1, -1, -1 \}$. The Ricci-Levi-Civita tensor is defined by $\varepsilon_{0123} = -\varepsilon^{0123} = 1 = \varepsilon_{123}$. 

2
In Eqs. (1) and (2), \( u_1(p_1, \lambda_1) \) is the Dirac spinor for the initial state (octet) baryon which has mass \( m_1 \), four momentum \( p_1 \), and helicity \( \lambda_1 \). Similarly, \( u_2(p_2, \lambda_2) \), is the Dirac spinor for the final state (octet) baryon which has mass \( m_2 \), four momentum \( p_2 \), and helicity \( \lambda_2 \), and \( q^\prime\prime \equiv p_2 - p_1 \).

When the initial baryon is a decuplet state and the final state baryon is an octet state we have (in the notation of Mathews [3]):

\[
\langle B_2(p_2, \lambda_2) | J^h_\mu | B_1'(p_1, \lambda_1) \rangle = \frac{1}{(2\pi)^3} \sqrt{\frac{m_1 m_2}{E_1' E_2}} \bar{u}_2(p_2, \lambda_2) [\Gamma_{\mu\beta}] u_1(p_1, \lambda_1) \]

\[
\Gamma_{\mu\beta} = (f_1'(q^2) + g_1'(q^2) \gamma_5) g_{\mu\beta} + (f_2'(q^2) + g_2'(q^2) \gamma_5) \gamma_\mu q_\beta \\
+ (f_3'(q^2) + g_3'(q^2) \gamma_5) q_\mu q_\beta + (f_4'(q^2) + g_4'(q^2) \gamma_5) p_1\mu q_\beta
\]

Where \( u_1^\beta(p_1, \lambda_1) \) is a Rarita-Schwinger spinor and where the \( f_i' \) and \( g_i' \) are axial-vector and vector form factors respectively. (Or in the notation of C. H. Llewellyn-Smith [1] when we write only the axial-vector part, we obtain):

\[
\Gamma_{\mu\beta}^{Axial} = C_A^5(q^2) g_{\mu\beta} \\
+ C_A^6(q^2) q_\mu q_\beta/m^2 \\
+ C_A^4(q^2) \{(p_1 \cdot q/m^2) g_{\mu\beta} + q_\beta (p_1 + p_2)_\mu/(2m^2) + q_\mu q_\beta/(2m^2)\} \\
+ C_A^3(q^2) \{((m^* - m)/m) g_{\mu\beta} + q_\beta \gamma_\mu/m\}
\]

The usual Cabibbo assumptions (extended to acknowledge the existence of \( c, b, \) and \( t \) quarks—i.e. one utilizes the Kobayashi and Maskawa (K-M) matrix) are then invoked in order to reduce the number of form factors in Eq.(2) from six to four—namely \( f_1(q^2), f_2(q^2), g_1(q^2), \) and \( g_P(q^2) \).

These assumptions are:

- Universality of the coupling of the leptonic current to hadronic current;
- \( J^h_\mu \) components transform like the charged members of the \( SU(3) \) \( J^P = 0^- \) octet;
• Generalized CVC (isotriplet current hypothesis) holds—implies that \( f_1 \) and \( f_2 \) can be calculated from the known proton and neutron electromagnetic form factors and that \( f_3 = 0 \);

• No second class currents exist—implies that \( g_3 = 0 \).

With those assumptions, Eq.(2) then effectively reduces to:

\[
\Gamma_\mu = f_1(q''^2)\gamma_\mu + (if_2(q''^2)/m_1)\sigma^{\nu\mu}q''^\nu + f_3\langle q''^2\rangle\gamma_\mu\gamma_5 + (ig_P(q''^2)/m_1)\sigma^{\mu\nu}q''_{\nu\delta}^\gamma_5.
\]

(6)

For example the well known nucleon weak axial-vector form factor \( g_A(q''^2) \) can be parametrized by

\[
g_A(q''^2)/g_A(0) \approx \left[1 - q''^2/m_A^2\right]^{-2},
\]

where

\[
\langle p, p_2| A_{\pi+}^\mu(0) | n, p_1 \rangle \approx (2\pi)^{-3}\sqrt{(mm_n)/(E_{p_1}E_{p_2})}\bar{u}_p(p_2)\left[g_A(q''^2)\gamma^{\mu\nu}\gamma_5\right]u_n(p_1),
\]

\( m_n = \) neutron mass, and \( q''^2 = (p_2 - p_1)^2 \).

**III. PREVIOUS RESULTS AND METHODOLOGY**

We consider helicity states with \( \lambda = +1/2 \) (i.e. spin non-flip sum rules) and the non-strange \((S = 0) L = 0\) ground state baryons \((J^{PC} = 1^{+}, 3^{+})\). It is well-known that if one defines the axial-vector matrix elements:

\[
\langle p, 1/2| A_{\pi+}^\mu | n, 1/2 \rangle \equiv f = g_A(0),
\]

\[
\langle \Delta^{++}, 1/2| A_{\pi+}^\mu | \Delta^+, 1/2 \rangle \equiv -\sqrt{\frac{3}{2}}g,
\]

4
\[ \langle \Delta^{++}, 1/2 | A_{\pi^+} | p, 1/2 \rangle \equiv -\sqrt{6}h, \]

and applies asymptotic level realization to the chiral \( SU(2) \otimes SU(2) \) charge algebra
\[ [A_{\pi^+}, A_{\pi^-}] = 2V_3, \]

then
\[ h^2 = (4/25) f^2 \ (the \ sign \ of \ h = +(2/5) f) \quad \text{and} \quad g = (-\sqrt{2}/5) f. \]

If one further defines (suppressing the index \( \mu \)):
\[ < \Delta^+, 1/2, \vec{s} | j_3 | \Delta^+, 1/2, \vec{t} > \equiv a, \quad < p, 1/2, \vec{s} | j_3 | p, 1/2, \vec{t} > \equiv b, \]
\[ < n, 1/2, \vec{s} | j_3 | \Delta^0, 1/2, \vec{t} > \equiv c, \quad < \Delta^0, 1/2, \vec{s} | j_3 | n, 1/2, \vec{t} > \equiv d \]
(note that other required matrix elements of \( j_3 \) can then be obtained easily from the double commutator \([ [j_3^\mu(0), V_{\pi^+}], V_{\pi^-} ]] = 2 j_3^\mu(0) \))

and if one also inserts the algebra \([ j_3^\mu(0), A_{\pi^+} ] = A_{\pi^+}^\mu(0) (j_3^\mu \equiv j_3^\mu + j_3^S \), where \( j_3^S \equiv \) isovector part of \( j^\mu \) and \( j_3^S \) is isoscalar\) between the ground states \( \langle B(\alpha, \lambda = 1/2, \vec{s}) | \) and \( | B'(\alpha, \lambda = 1/2, \vec{t}) \rangle \) with \( |\vec{s}| \to \infty, |\vec{t}| \to \infty \), where \( \langle B(\alpha) | \) and \( | B'(\beta) \rangle \) are the following \( SU_F(2) \) related combinations: \( \langle p, n \rangle, \langle p, \Delta^0 \rangle, \langle \Delta^{++}, p \rangle, \langle n, \Delta^- \rangle, \langle \Delta^{++}, \Delta^+ \rangle, \langle \Delta^+, \Delta^0 \rangle, \langle \Delta^0, \Delta^- \rangle, \langle \Delta^+, n \rangle \), then one obtains (we use \( < N | j_S^\mu | \Delta >= 0 \) ) the constraint equations (not all independent):

\[ 2fb - \sqrt{2}h(c + d) = f^{L=0}(\lambda = 1/2) \langle p | A_{\pi^+}^\mu | n \rangle, \]
\[ \sqrt{2}h(a + b) + (-\sqrt{2}g - f)c = f^{L=0}(\lambda = 1/2) \langle p | A_{\pi^+}^\mu | \Delta^0 \rangle, \]
\[ \sqrt{6}h(-3a + b) + \sqrt{3/2}gd = f^{L=0}(\lambda = 1/2) \langle \Delta^{++} | A_{\pi^+}^\mu | p \rangle, \]
\[ \sqrt{6}h(3a - b) - \sqrt{3/2}gc = f^{L=0}(\lambda = 1/2) \langle n | A_{\pi^+}^\mu | \Delta^- \rangle, \]
\[ -\sqrt{6}ga + \sqrt{6}hc = f^{L=0}(\lambda = 1/2) \langle \Delta^{++} | A_{\pi^+}^\mu | \Delta^+ \rangle, \]
\[-2\sqrt{2} ga + \sqrt{2} h(c + d) = f_{L=0}(\lambda = 1/2) \langle \Delta^+ | A_{\pi^+}^\mu | \Delta^0 \rangle, \quad (12)\]

\[-\sqrt{6} ga + \sqrt{6} h d = f_{L=0}(\lambda = 1/2) \langle \Delta^0 | A_{\pi^+}^\mu | \Delta^- \rangle, \quad (13)\]

\[-\sqrt{2} h(a + b) + (f + \sqrt{2} g) d = f_{L=0}(\lambda = 1/2) \langle \Delta^+ | A_{\pi^+}^\mu | n \rangle. \quad (14)\]

 Applying asymptotic level symmetry, Eqs. (7)–(14) immediately imply that

\[d = c \quad (15)\]

and

\[a = b + \left[-\frac{1}{4} g h - \frac{1}{2\sqrt{2}} f \right] c. \quad (16)\]

One can calculate \( f_{L=0}(\lambda = 1/2) \) easily by setting \( \mu = 0 \), restoring the \( x \) dependence to the matrix elements and integrating over \( d\vec{x} \).

We find that \( f_{L=0}(\lambda = 1/2) = 1 \). Eqs. (7)–(14) then relate the weak matrix elements \( \langle \Delta^+ , 1/2 , \vec{s} | A_{\pi^+}^\mu (0) | \Delta^0 , 1/2 , \vec{t} \rangle \), \( \langle \Delta^+ , 1/2 , \vec{s} | A_{\pi^+}^\mu (0) | n, 1/2, \vec{t} \rangle \), and \( \langle p, 1/2, s | A_{\pi^+}^\mu (0) | \Delta^0, 1/2, \vec{t} \rangle \) to the matrix elements \( \langle p, 1/2, s | j_3^\mu (0) | p, 1/2, \vec{t} \rangle \) and \( \langle p, 1/2, \vec{s} | A_{\pi^+}^\mu (0) | n, 1/2, \vec{t} \rangle \).

In fact, one discovers the important relation

\[ (8h - 2f)b = 2\sqrt{2} \langle p | A_{\pi^+}^\mu | \Delta^0 \rangle - \langle p | A_{\pi^+}^\mu | n \rangle. \quad (17)\]

IV. RESULTS

We now choose \( \mu = 0 \) and take the limit \( |\vec{s}| \to \infty \) and \( |\vec{t}| \to \infty \) (\( \vec{s} \) and \( \vec{t} \) are both taken along the \( z \)-axis), then \( q^2 = q''^2 = q^0_\pi = 0 \). We find that

\[(2\pi)^3 < p, 1/2, \vec{s} | j_3^0 | p, 1/2, \vec{t} >= [1 - q^2/(4m^2)]^{-1} G_E^V(q^2) \]

\[(2\pi)^3 \langle p | A_{\pi^+}^\mu | n \rangle \to g_A(q''^2) \quad (18)\]

\[(2\pi)^3 \langle p | A_{\pi^+}^\mu | \Delta^0 \rangle \to \sqrt{\frac{2}{3}} \frac{(m + m^*)}{2m^*} C_{5\pi}^A(q^2) \]
Thus, Eq. (17) and Eq. (18) then predict that

$$C_5^A(0) = \frac{4}{5} \sqrt{3} \frac{m^*}{(m + m^*)} g_A(0).$$  \hspace{1cm} (19)$$

Numerically Eq. (19) reads (using $g_A(0) = 1.25$, $m^* = 1.232 \text{ GeV/c}^2$)

$$C_5^A(0) = 0.98$$  \hspace{1cm} (20)

This value is consistent with PCAC and yields the value $\Gamma(\Delta) \approx 100 \text{ MeV}$.

V. CONCLUSIONS AND SOME COMPARISONS WITH OTHER MODELS AND EXPERIMENT

- $SU(6)$ Symmetry Predicts

$$C_5^A(q^2 = 0)^{n \to \Delta^+} = \frac{2}{5} \sqrt{3} (g_A/g_V)^{n \to p}.$$  \hspace{1cm} (21)

This gives rise to the following dilemma: Does one use the pure $SU(6)$ result $(g_A/g_V)^{n \to p} = 5/3 \implies C_5^A(q^2 = 0)^{n \to \Delta^+} = 1.15$ or does one use the experimental value of $(g_A/g_V)^{n \to p} = 1.25 \implies C_5^A(q^2 = 0)^{n \to \Delta^+} = 0.87$?

Clearly, the value of $C_5^A(q^2 = 0)^{n \to \Delta^+}$ that one chooses to use can represent almost a factor of two in the predicted value of $\Gamma(\Delta)$.

- Non-Relativistic Conventional ($N$ and $\Delta$ space wave functions are identical) Quark Model

$$C_5^A(q^2 = 0)^{n \to \Delta^+} = \frac{2}{5} \sqrt{3} (g_A)^{n \to p} = 0.87.$$

- Static (Yukawa pion-nucleon coupling) Models

$$C_5^A(q^2 = 0)^{n \to \Delta^+} = \frac{g_{\Delta^+ p}}{\sqrt{6} g_N} (g_A)^{n \to p} = 1.11.$$

- Adler’s Model and the Feynman, Kislinger, Ravndal (FKR) Relativistic Quark Model
The predictions of the Adler and FKR models are quite similar. Both models predict (roughly) the same non-relativistic limits with the $q^2$ dependence of axial-vector transition amplitudes determined by $q^2$ dependence of nucleon axial-vector form factors, although the FKR model makes the stronger prediction that the dependence is the same. It is also true that in Adler and FKR models the low $q^2$ dependence is roughly the same as for the Static model.

- Experiment (CERN, Brookhaven, Argonne: by measuring the axial-vector mass $M_A$) generally favors the Adler model.

- PCAC

From the process $\nu n \rightarrow \Delta^+$ (i.e. $\nu + n \rightarrow \mu^- + \Delta^+$), PCAC predicts that

$$C_5^A(q^2 = 0)^{n \rightarrow \Delta^+} = \frac{f_\pi g_{\Delta^+ n \pi^+}}{m} = 1.2,$$

If the Goldberger-Treiman relationship $g_A/g_N = f_\pi/\sqrt{2}m$ is exactly satisfied, then the static model prediction and PCAC predictions are the same.

- We conclude that our broken symmetry algebraic approach to the calculation of $C_5^A(q^2 = 0)^{n \rightarrow \Delta^+}$ yields results consistent not only with experiment but also with the widely used Adler model. The broken symmetry approach also correctly gives the $\Delta$ width, and resolves mass and wave function degeneracy problems present in many widely-used quark models.
REFERENCES

[1] C. H. Llewellyn-Smith, *Phys. Rep.* **3C**, 264 (1972); E. Amaldi, S. Fubini, and G. Furlan, *Pion Electroproduction* (Springer-Verlag, Berlin, 1979).

[2] D. A. Dicus and J. R. Letaw, *Ann. Phys.* **126**, 32 (1980).

[3] J. Mathews, *Phys. Rev.* **137**, B444, (1964).

[4] Milton D. Slaughter, *Phys. Rev.* **C49**, R2894 (1994).

[5] M. Slaughter and S. Oneda, *Phys. Rev. Lett.* **59**, 1641(1987).

[6] M. Slaughter and S. Oneda, *Phys. Rev. D39*, 2062, (1989).

[7] S. Oneda, T. Tanuma, and M. D. Slaughter, *Phys. Lett.* **88B**, 343 (1979).

[8] S. Oneda and K. Terasaki, *Prog. Theor. Phys. Suppl.* **82**, 1 (1985).

[9] S. Oneda and Y. Koide, *Asymptotic Symmetry and Its Implication in Elementary Particle Physics* (World Scientific, Singapore, 1991).