Relativistic Calculation of Hadronic Baryon Decays

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The description of baryon resonance decays represents a major challenge of strong interaction physics. We will report on a relativistic approach to mesonic decays of light and strange baryon resonances within constituent quark models. The calculations are performed in the point-form of relativistic quantum mechanics, specifically focussing on the strange sector. It is found that the relativistic predictions generally underestimate the experimental data. The nonrelativistic approximation of the approach leads to the decay operator of the elementary emission model. It is seen that the nonrelativistic reduction has considerable effects on the decay widths.

There is already a long tradition in studying mesonic decays of baryon resonances within constituent quark models (CQMs). However, most of the studies hitherto have been performed within nonrelativistic or so-called relativised models [1, 2, 3, 4, 5]. Recently, the Graz group has presented relativistic CQM calculations for $\pi$ and $\eta$ decays of $N$ and $\Delta$ resonances employing a decay operator along the point-form spectator model (PFSM) in the framework of relativistic (Poincaré-invariant) quantum mechanics [6]. A similar relativistic study following a Bethe-Salpeter approach has been reported by the Bonn group [7, 8]. In this contribution we report results for $\pi$ decays of strange baryon resonances by the relativistic Goldstone-boson exchange (GBE) and one-gluon exchange (OGE) CQMs of Refs. [9] and [5], respectively. The nonstrange decays of strange baryon resonances have not found much attention in the past. So far there are no covariant results but only the studies in Refs. [10, 11, 12]. In addition to the relativistic predictions we also present the decay widths resulting from the nonrelativistic reduction of the PFSM decay operator, which corresponds to the elementary emission model (EEM).

The decay width of a baryon resonance is defined by the expression

$$
\Gamma_{i \to f} = \frac{|q|}{4M^2} \frac{1}{2J+1} \sum_{M_J,M_{J'}} \frac{1}{2T+1} \sum_{M_T,M_{T'},M_{Tm}} |F_{i \to f}|^2
$$

with the transition amplitude $F_{i \to f}$ given by the matrix element of the reduced (four-momentum conserving) decay operator $\hat{D}^m_{rd}$ between incoming and outgoing baryon states

$$
F_{i \to f} = \langle V', M', J', M_{J'}, T', M_{T'} | \hat{D}^m_{rd} | V, M, J, M_{J}, T, M_{T} \rangle.
$$

Here, the index $m$ refers to the particular mesonic decay mode and $q_\mu = (q_0, \mathbf{q})$ denotes the four-momentum of the outgoing meson in the rest-frame of the decaying baryon resonance $|V, M, J, M_{J}, T, M_{T} \rangle$; the latter is characterized by the eigenvalues of the velocity $V$, mass $M$, intrinsic spin $J$ with $z$-component $M_{J}$, and isospin $T$ with $z$-projection $M_{T}$. 

1
Correspondingly the outgoing baryon state is denoted by the primed eigenvalues. Representing the baryon eigenstates in a suitable basis, the matrix element in Eq. (2) leads to the integral

\[
\langle V', M', J', M_J', T', M_T' | \hat{D}^{m}_r d | V, M, J, M_J, T, M_T \rangle = \frac{2}{M M'} \sum_{\sigma, \sigma_1', \mu, \mu_1'} \sum_{\sigma_2, \sigma_3} \int d^3k_2 d^3k_3 d^3k'_2 d^3k'_3 \\
\times \sqrt{\left(\sum_i \omega_i^3\right)} \frac{1}{\prod_i 2\omega_i'} \Psi_{M'J'M_J'M_T'M_T'}(k'_i; \mu'_i) \prod_{\sigma'_i} D^{J'}_{\sigma'_i \mu'_i} \{R_W[k'_i; B(V')]\} \\
\times \langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{D}^{m}_r d | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle \\
\times \prod_{\sigma_i} D^{J}_{\sigma \mu_i} \{R_W[k_i; B(V)]\} \sqrt{\left(\sum_i \omega_i^3\right)} \frac{1}{\prod_i 2\omega_i} \Psi_{MJMJMTMT}(k_i; \mu_i),
\]

where \( \Psi_{MJMJMTMT}(k_i; \mu_i) \) is the rest-frame wave function of the incoming baryon and analogously \( \Psi_{M'J'M_J'M_T'M_T'}(k'_i; \mu'_i) \) the one of the outgoing baryon. Both wave functions result from the velocity-state representation of the baryon eigenstates. The momentum representation of the decay operator follows from the PFSM construction [6, 13], where one assumes that only one of the quarks directly couples to the emitted meson:

\[
\langle p'_1, p'_2, p'_3; \sigma'_1, \sigma'_2, \sigma'_3 | \hat{D}^{m}_r d | p_1, p_2, p_3; \sigma_1, \sigma_2, \sigma_3 \rangle = -3 \left( \frac{M}{\sum_i \omega_i} \frac{M'}{\sum_i \omega_i'} \right) \frac{1}{2m} \frac{1}{\sqrt{2\pi}} \bar{u}(p'_1, \sigma'_1) \gamma_5 \gamma^\mu F^\mu u(p_1, \sigma_1) q_{\mu} 2p_{20} \delta^3(p_2 - p'_2) \delta_{\sigma_2 \sigma'_2} 2p_{30} \delta^3(p_3 - p'_3) \delta_{\sigma_3 \sigma'_3}. \quad (4)
\]

Here, \( g_{qgm} \) is the quark-meson coupling constant, \( m_1 \) the mass of the active quark, \( F^\mu \) the flavor-transition operator specifying the particular decay mode, and \( u(p_1, \sigma_1) \) the quark spinor. All details of the formalism and the notation can be found in Ref. [14]. The form of the decay operator is congruent with the calculations in Ref. [6] and also consistent with the baryon charge normalisation and time-reversal invariance of the electromagnetic form-factors [15]. The nonrelativistic approximation of the PFSM decay operator leads to the traditional EEM [14].

In Table 1 we present the direct predictions of the GBE and OGE CQMs for the \( \pi \) decay modes of the strange baryon decay resonances and compare with the latest compilation of the PDG [16]. Both the covariant PFSM results as well as the nonrelativistic EEM results have been calculated with theoretical and experimental masses as input. It is immediately evident that the relativistic predictions usually underestimate the experimental data or at most reach them from below. A similar finding was already made for the \( \pi \) decay widths of \( N \) and \( \Delta \) resonances [6]. Here, there appear only two exceptions, namely the widths of \( \Lambda(1405) \) and \( \Lambda(1670) \). In case of the former it is caused by the (theoretical) mass, which is far too high for both CQMs; the overprediction disappears when the experimental mass is used. On the other hand the resonance mass of the \( \Lambda(1670) \) is more or less well reproduced in accordance with experiment. In this case we may suspect the large decay width to result from another reason, possibly a coupling of resonance states.

For the \( \Sigma(1750) \) the CQMs offer three states that can be identified with this resonance. In Table 1 we present the decay widths of all theoretical levels (in the entries distinguished by the superscripts 1, 2, and 3). It is seen that the decay width of the third state

\[ \text{Table 1: Decay widths of the \( \Sigma(1750) \) resonance.} \]
Table 1: Theoretical predictions for $\pi$ decay widths by the GBE and OGE CQMs in comparison to experiment [16]. The relativistic calculations follow from the PFSM, while the EEM results represent their nonrelativistic limits.

| Decay | $J^P$ | Exp. [MeV] | Theoretical Mass | Experimental Mass |
|-------|-------|-----------|------------------|------------------|
|       |       |           | Relativistic     | Nonrel. EEM      |
|       |       |           | GBE  | OGE | GBE | OGE | GBE | OGE | GBE | OGE |
| $\to \Sigma\pi$ | | | | | | | | | | |
| $\Lambda(1405)$ | $\frac{1}{2}^-$ | $(50 \pm 2)$ | 55 | 78 | 320 | 611 | 15 | 17 | 76 | 112 |
| $\Lambda(1520)$ | $\frac{3}{2}^-$ | $(6.55 \pm 0.16)^{+0.04}_{-0.04}$ | 5 | 9 | 5 | 8 | 2.8 | 3.1 | 2.1 | 2.3 |
| $\Lambda(1600)$ | $\frac{1}{2}^+$ | $(53 \pm 38)^{+60}_{-10}$ | 3 | 33 | 2 | 34 | 3 | 17 | 1.2 | 15 |
| $\Lambda(1670)$ | $\frac{3}{2}^-$ | $(14.0 \pm 5.3)^{+8.3}_{-2.5}$ | 69 | 103 | 620 | 1272 | 68 | 94 | 572 | 1071 |
| $\Lambda(1690)$ | $\frac{5}{2}^-$ | $(18 \pm 6)^{+4}_{-2}$ | 19 | 25 | 24 | 28 | 18 | 21 | 23 | 22 |
| $\Lambda(1800)$ | $\frac{1}{2}^-$ | seen | 68 | 101 | 473 | 1175 | 70 | 95 | 485 | 1095 |
| $\Lambda(1810)$ | $\frac{3}{2}^+$ | $(38 \pm 23)^{+10}_{-11}$ | 3.8 | 21 | 55 | 150 | 4.1 | 5.0 | 55 | 94 |
| $\Lambda(1830)$ | $\frac{3}{2}^-$ | $(52 \pm 19)^{+11}_{-12}$ | 14 | 19 | 16 | 24 | 16 | 20 | 22 | 24 |
| $\to \Lambda\pi$ | | | | | | | | | | |
| $\Sigma(1385)$ | $\frac{3}{2}^+$ | $(4.2 \pm 0.5)^{+0.7}_{-0.5}$ | 3.1 | 0.5 | 6.5 | 1.1 | 2.0 | 2.1 | 4.1 | 4.8 |
| $\Sigma(1660)$ | $\frac{1}{2}^+$ | seen | 10 | 24 | 2 | 15 | 12 | 14 | 2.4 | 6.9 |
| $\Sigma(1670)$ | $\frac{3}{2}^-$ | $(27 \pm 9)^{+12}_{-6}$ | 15 | 23 | 21 | 32 | 13 | 17 | 17 | 21 |
| $\Sigma(1750)^1$ | $\frac{3}{2}^-$ | $(3.6 \pm 3.6)^{+5.6}_{-6}$ | 58 | 102 | 480 | 1249 | 63 | 102 | 574 | 1402 |
| $\Sigma(1750)^2$ | $\frac{1}{2}^-$ | $(3.6 \pm 3.6)^{+5.6}_{-6}$ | 32 | 44 | 135 | 312 | 32 | 38 | 136 | 262 |
| $\Sigma(1750)^3$ | $\frac{3}{2}^-$ | $(3.6 \pm 3.6)^{+5.6}_{-6}$ | 10 | 1.0 | 116 | 34 | 10 | 0.9 | 110 | 32 |
| $\Sigma(1775)$ | $\frac{5}{2}^-$ | $(4.2 \pm 1.8)^{+0.8}_{-0.3}$ | 1.9 | 3.8 | 2.9 | 6.9 | 2.2 | 3.2 | 3.5 | 5.3 |
| $\Sigma(1940)$ | $\frac{3}{2}^-$ | seen | 2.2 | 3.7 | 0.5 | 1.1 | 4.9 | 5.8 | 1.6 | 2.4 |
| $\to \Xi\pi$ | | | | | | | | | | |
| $\Xi(1530)$ | $\frac{3}{2}^+$ | $(9.9)^{+1.7}_{-1.9}$ | 2.2 | 1.3 | 4.4 | 3.0 | 5.5 | 5.3 | 11.4 | 12.5 |
| $\Xi(1820)$ | $\frac{3}{2}^-$ | seen | 0.4 | 1.6 | 0.3 | 1.4 | 0.7 | 1.2 | 0.6 | 0.9 |
$\Sigma(1750)^3$ is pretty consistent with the magnitude of the experimental data and it should be identified with the measured $\Sigma(1750)$. The other two states can then be interpreted with lower lying resonances (such as the $\Sigma(1620)$ and $\Sigma(1560)$) not so well established by experiment. Regarding the classification of these states see also Ref. [17].

From the comparison of the PFSM results with experimental masses as input one learns that the effects from different hyperfine interactions generally play a minor role. Considerable influences are seen only in $\Sigma\pi$ and $\Lambda\pi$ decays of $\Lambda(1600)$, $\Sigma(1750)^3$, and $\Sigma(1660)$.

The nonrelativistic results corresponding to the EEM scatter below and above the experimental data. The effect of the nonrelativistic reduction is strongly dependent on the decaying resonance. It is governed essentially by the truncation in the spin couplings as well as the elimination of the Lorentz boosts.

We have reported the first covariant results for $\pi$ decays of strange baryon resonances within CQMs. Obviously the approach needs further improvements. In the first instance, one might think of a coupled-channel formulation. The importance of additional Fock components has already been seen in a PFSM calculation of mesons decays [18, 19] and also recent studies of baryon resonances [20, 21].

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