Mathematical Model of Reliability Evaluation on Air Volume of Ventilation System in Volume Adjustment by Frequency Conversion

Zhehua Du¹*, Xin Lin²
¹ Wuhan Second Ship Design and Research Institute, Wuhan, Hubei, 430205, China
² Hubei Province Engineering Consulting Co., LTD., Wuhan, Hubei, 430071, China
*Corresponding author’s e-mail: Jackydhzh@163.com

Abstract. The reliability on air volume of ventilation system in volume adjustment by frequency conversion of fan was theoretically studied. Based on the law of large numbers, the probability could be transformed to frequency ratio with fully considering the fluctuation of air volume. Two conversion formulas for transforming air volume under normal working frequency into air volume under required frequency with or without considering natural ventilation pressure were deduced. On this basis, the reliability of air volume under required frequency can be predicted through measuring air volume under normal working frequency, then the unknown reliability can be predicted before volume adjustment by frequency conversion of fan. The reliability matrix and reliability value matrix on air volume of ventilation system were deduced and constructed. The relationship between the matrices and fan frequency was theoretically analyzed, as well as the forecast on reliability matrix and reliability value matrix of the required frequency based on the conversion formulas.

1. Introduction
Ventilation system reliability can be defined as the ability to provide a qualified, continuous, and stable ventilation environment. The so-called quality qualification means that the evaluation index of ventilation system must be kept within the range specified in the regulation. When fan is frequency-regulated, it will break original ventilation state and cause a large change in the overall air volume. This may cause air volume to exceed requirement specified in the regulations, and then cause unreliable ventilation.

Variable frequency fan has the advantages of convenient and simple operation, high degree of automation, fast, efficient, real-time, quantitative, energy saving and accurate adjustment. However, variable frequency fan is generally only used for fine-tuning, and it is difficult to effectively play the role. The reason is that the influence of variable frequency air conditioning on the reliability of ventilation system has become an important limiting factor. The first problem that needs to be solved in the application of variable frequency air conditioning is reliability. In view of this, the author makes a preliminary exploration of the reliability evaluation of variable frequency air conditioning from the perspective of qualified air volume.

2. Overview
For the space equipped with variable frequency fan, its working frequency is generally set to be less than the highest frequency of 50 Hz in normal operation state, and certain frequency conversion
adjustment space is reserved. Fan frequency in normal operation is called normal frequency, and fan frequency conversion regulation is the regulation from normal frequency to required frequency. Based on whether air volume meets the requirement of regulation and design, and taking the fluctuation of air volume into full consideration, reliable definition of air volume in the branch of variable frequency adjustable air is given as follow. When fan is adjusted from normal frequency $f_c$ to required frequency $f$, air volume of the $i$ branch is measured $j$ times. If measured air volume value $Q_{ijf}$ satisfies: $\forall j \in \mathbb{N}$, and there are: $Q_{\text{min}} \leq Q_{ijf} \leq Q_{\text{max}}$, it is said that air volume of the $i$ branch is reliable when variable frequency air conditioning is performed. Multiple measurements are performed to fully consider the fluctuation of air volume. $Q_{\text{max}}$ needs to be set according to the maximum air speed required by regulations. $Q_{\text{min}}$ is the maximum of the minimum air volume $Q_{\text{min}}$ specified in the regulation and basic air volume required for ventilation design.

Therefore, it is necessary to introduce the concept of reliability, which can be used to characterize the reliability level of the branch in the variable frequency air conditioning. Bbranch air volume reliability $R_i$ is defined as follow. Air volume of $i$ branch at required frequency $f$ is measured for $j$ times. The probability that measured air volume value $Q_{ijf}$ can be kept within a reasonable interval, that is, $Q_{\text{min}} \leq Q_{ijf} \leq Q_{\text{max}}$, is called $i$ branch air volume reliability.

In actual measurement, it is difficult to obtain the probability value accurately. Based on the large number theorem [1], the probability can be reflected by the frequency ratio. Therefore, the reliability of branch air volume is characterized to some extent by frequency ratio measurement. Based on this, a method to calculate the reliability of branch air volume is given. $N$ is measurement times of measured air volume $Q_{ijf}$ that meets $Q_{\text{min}} \leq Q_{ijf} \leq Q_{\text{max}}$, and $M$ is total measurement times.

$$\eta_i = \frac{N}{M} \times 100\% \quad (1)$$

3. Air volume conversion formula and reliability prediction

3.1. Air volume conversion formula

According to the proportional relationship between motor speed $n$ and frequency $f$ [2] and the proportional relationship between the air volume $Q$ and the speed $n$ [3], the wind volume of the fan is in a proportional relationship with the frequency, and the proportional coefficient is $k$.

$$Q = kf \quad (2)$$

Formula (2) is modified by natural wind pressure, and modification is formally formula (3), where $Q_N$ is air volume driven by natural wind pressure.

$$Q = kf + Q_N \quad (3)$$

It is assumed that fan air volume $Q$ is $y_i$ times air volume $Q_i$ of branch $i$, and $y_i$ does not change with frequency.

$$Q_i = y_i Q \quad (4)$$

Substituting equation (4) into equation (3), and making $ky_i = k_i$, $Q_Ny_i = Q_{Ni}$, following formula can be obtained.

$$Q_i = k_f + Q_{Ni} \quad (5)$$

Equation (5) is variation function of branch air volume with frequency, and $Q_{Ni}$ is called branch natural air volume driven by natural wind pressure. This function can realize the conversion between normal frequency air volume and required frequency air volume. Knowing normal frequency $f_c$ and air volume $Q_{nc}$ of $i$ branch, substituting them into the formula (5), and omitting branch natural air flow $Q_{Ni}$, the following formula is obtained.

$$k_i = \frac{Q_{nc}}{f_c} \quad (6)$$

Substituting required frequency $f$ and airflow value $Q_{ijf}$ into equation (5), omitting branch natural airflow $Q_{Ni}$, and combining equation (6), airflow value $Q_{ijf}$ at required frequency $f$ can be obtained to achieve normal frequency airflow and to be adjusted frequency air volume conversion.
\[ Q_{if} = \frac{Q_{ic} f}{f_c} \]  
(7)

If natural wind pressure has a greater impact in some areas, \( Q_{Ni} \) cannot be omitted. At this time, air volume of the frequency to be adjusted is shown in following formula.

\[ Q_{if} = \frac{f(Q_{ic} - Q_{Ni})}{f_c} + Q_{Ni} \]  
(8)

Formula (7) and (8) are both referred to as conversion formulas of normal frequency air volume and required frequency air volume. The difference is that formula (7) does not consider the influence of branch natural air volume, so formula (8) is more accurate than formula (7), but formula (8) is more complicated than formula (7).

3.2. Reliability prediction based on air volume conversion formula

Air volume conversion formulas (7) and (8) provide a bridge between normal frequency air volume and air volume whose frequency needs to be adjusted. Based on formula (7), reliability \( \eta_1 \) of arbitrary frequency \( f \) is as follows. \( N_i \) is the measurement times satisfying \( Q_{min} \leq \frac{Q_{ic} f}{f_c} \leq Q_{max} \). \( M_1 \) is the total number of measurements.

\[ \eta_1 = \frac{N_i}{M_1} \times 100\% \]  
(9)

Based on formula (8), normal frequency is unchanged, and then by measuring a large amount of air volume at normal frequency, the reliability \( \eta_2 \) of any frequency can be obtained. \( N_2 \) is the measurement times satisfying \( Q_{min} \leq \frac{f(Q_{ic} - Q_{Ni})}{f_c} + Q_{Ni} \leq Q_{max} \). \( M_1 \) is the total number of measurements.

\[ \eta_2 = \frac{N_2}{M_1} \times 100\% \]  
(10)

4. Air volume reliability matrix and prediction

4.1. Reliability prediction based on air volume conversion formula

The above mentioned air volume reliability refers to air volume reliability of a single branch, while variable frequency air volume refers to simultaneous change of air volume of whole ventilation system. Air volume reliability of ventilation system involves many branches, which can be studied by constructing matrix. In order to more intuitively measure air volume reliability of ventilation system during variable frequency air conditioning, an air volume matrix \( A(f) \) can be constructed. The number of rows is the number of branches, the number of columns is the number of measurements, and matrix element \( Q_{ij} \) is air volume value for \( i \) branch \( j \) measurement when fan frequency is \( f \). Fan frequency conversion adjustment is divided into fan frequency increase and fan frequency decrease. The following sections use fan frequency increase as an example to study the reliability of ventilation network air volume during fan frequency conversion. In order to study the reliability of ventilation system when fan is increasing frequency and increasing air volume, the maximum air volume value matrix \( B \) of the branch is defined [4].

\[
A(f) = \begin{bmatrix}
Q_{11} & Q_{12} & \cdots & Q_{1m} \\
Q_{21} & Q_{22} & \cdots & Q_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
Q_{n1} & Q_{n2} & \cdots & Q_{nm}
\end{bmatrix}
\]  
\[(11)\]

\[
B = \begin{bmatrix}
Q_{1\text{max}} & Q_{1\text{max}} & \cdots & Q_{1\text{max}} \\
Q_{2\text{max}} & Q_{2\text{max}} & \cdots & Q_{2\text{max}} \\
\vdots & \vdots & \ddots & \vdots \\
Q_{n\text{max}} & Q_{n\text{max}} & \cdots & Q_{n\text{max}}
\end{bmatrix}
\]  
\[(12)\]
According to regulations, the upper limit of air speed is different for different branches. Therefore, the maximum wind value matrix $B$ is identical to the elements in the same row, but the elements in different columns are different, and the matrix elements are independent of the frequency. We subtract $A(f)$ from the $B$ matrix to get the $C(f)$ matrix. By converting the elements of matrix $C(f)$ into a zero-one matrix, matrix $D(f)$ is obtained.

$$D(f) = \begin{bmatrix} d_{11} & d_{12} & \ldots & d_{1n} \\ d_{21} & d_{22} & \ldots & d_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ d_{n1} & d_{n2} & \ldots & d_{nm} \end{bmatrix}$$ (13)

Let the ratio of the sum of elements in column $j$ of matrix to the total branch number $n$ be $\xi_j$, which is called single measurement reliability of ventilation system.

$$\xi_j = \frac{\sum_{i=1}^{n} d_{ij}}{n} \times 100\%$$ (14)

$\xi_j$ values of $m$ times measurements are averaged to obtain $p$ value. The $p$ value is defined as the reliability of ventilation system air volume during variable frequency air conditioning. Therefore, air volume reliability of variable frequency adjustable air can be studied from the perspective of ventilation system, which can reflect air volume reliability level of ventilation system when fan is variable frequency.

$$p = \frac{\sum_{j=1}^{m} \xi_j}{m} \times 100\%$$ (15)

Matrix $D(f)$ is extended by the values of $\xi_j$, $\eta_i$, and $p$ to obtain matrix $P(f)$. Matrix $P(f)$ is called reliability matrix of ventilation system under variable frequency air regulation, through which the reliability of single branch and the whole of ventilation system can be intuitively seen.

$$P(f) = \begin{bmatrix} d_{11} & d_{12} & \ldots & d_{1m} & \eta_1 \\ d_{21} & d_{22} & \ldots & d_{2m} & \eta_2 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ d_{n1} & d_{n2} & \ldots & d_{nm} & \eta_n \\ \xi_1 & \xi_2 & \ldots & \xi_m & p \end{bmatrix}$$ (16)

4.2. Reliability matrix prediction based on air volume conversion formula

The conversion formulas (7) and (8) of normal frequency air volume and other frequency air volume can greatly reduce the workload of calculating the reliability of ventilation system. If formula (7) is used for calculation, and branch air volume multiple measurement matrix at normal frequency is $A_c$, then ventilation network air volume matrix $A(f)$ at any frequency $f$ can be calculated by formula (17). Substituting $A(f)$ into above formulas (11) to (15), air volume reliability value $p$ of ventilation system can be obtained. If matrix is extended according to formula (16), air volume reliability matrix at any frequency can be predicted.

$$A(f) = \frac{A_c f}{f_c}$$ (17)

Similar to the prediction method in Section 3.2, this method can predict air volume reliability of ventilation system when fan is at the frequency to be adjusted. This can provide a preliminary reference for the influence of fan frequency modulation on the reliability of ventilation system.
5. Reliability value matrix of ventilation system

The above theory can better reflect the reliability of single branch air volume and the reliability of air volume of ventilation system during variable frequency air conditioning. However, the above reliability evaluation theory focuses on the frequency of air volume exceeding limit, which cannot reflect the degree of air volume exceeding limit. Under certain circumstances, the judgement results may be biased. For such cases, a new method should be selected to study the reliability of variable frequency air conditioning of ventilation system. Therefore, a reliable value matrix of ventilation system is constructed. Matrix $D'(f)$ is defined as formula (18), where "÷" means the corresponding element of the former matrix divided by the corresponding element of the latter.

$$D'(f) = C(f) \div B \quad (18)$$

Matrix element $d'_{ij}$ is represented by formula (19), which is the difference between air volume value of branch and the maximum value occupies the maximum value percentage.

$$d'_{ij} = \frac{q_{max} - q_{ij}}{q_{max}} \quad (19)$$

In the same way as above, matrix $P'(f)$ is constructed. To distinguish it from the above definition of reliability, it is called ventilation system reliability value matrix. Among them, $\eta'_i$, $\xi'_j$, and $p'$ values respectively represent air volume reliable value of i branch, the single measurement reliability value of ventilation system, and the reliable value of ventilation system. The reliability matrix emphasizes the number of times air volume exceeds the limit, while the reliability value matrix emphasizes the magnitude of air volume exceeds the limit. In the process of variable frequency air regulating, the reliability and the reliability value matrix should be measured at the same time. For the space with large fluctuation of air volume, more attention should be paid to the former, while for the space with small fluctuation of air volume, more attention should be paid to the former[5].

$$P'(f) = \begin{bmatrix}
    d'_{11} & d'_{12} & \cdots & d'_{1m} \\
    d'_{21} & d'_{22} & \cdots & d'_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    d'_{m1} & d'_{m2} & \cdots & d'_{mn} \\
    \xi'_{1} & \xi'_{2} & \cdots & \xi'_{m} & p' 
\end{bmatrix} \quad (20)$$

6. Conclusion

Based on the large number theorem, a definition and the measurement method of the reliability of air volume suitable for fan variable frequency air regulation are given. In this method, the fluctuation of air volume is fully considered and the range of qualified air volume is taken as the perspective.

Based on variable frequency air conditioning, two formulas for air volume conversion are given. One considers the effects of natural wind pressure, and the other ignores the effects of natural wind pressure. This formula can realize the conversion between normal frequency air volume and required frequency air volume. Based on this, it is possible to predict the reliability of the unknown but required adjustment frequency by measuring air volume of normal frequency. Furthermore, it is a reference for whether to perform frequency adjustment, which can avoid the situation that frequency is adjusted blindly and the reliability of air volume is poor.

In order to study the reliability of ventilation system, the reliability matrix and the reliability value matrix of ventilation system are constructed. The former emphasizes the number of air volume exceeding the limit, while the latter emphasizes the magnitude of air volume exceeding the limit. The two matrices can be used together to comprehensively evaluate the reliability of the ventilation system. At the same time, the unknown required frequency reliability / value matrix can also be predicted by the conversion formula.
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