Cosmological constraints on non-adiabatic dark energy perturbations

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The exact nature of dark energy is currently unknown and its cosmological perturbations, when dark energy is assumed not to be the cosmological constant, are usually modelled as adiabatic. Here we explore the possibility that dark energy might have a non-adiabatic component and we examine how it would affect several key cosmological observables. We present analytical solutions for the growth-rate and growth index of matter density perturbations and compare them to both numerical solutions of the fluid equations and an implementation in the Boltzmann code CLASS, finding they all agree to well below one percent. We also perform a Monte-Carlo analysis to derive constraints on the parameters of the non-adiabatic component using the latest cosmological data, including the temperature and polarization spectra of the Cosmic Microwave Background as observed by Planck, the Baryon Acoustic Oscillations, the Pantheon type Ia supernovae compilation and lastly, measurements of Redshift Space Distortions (RSD) of the growth-rate of matter perturbations. We find that the amplitude of the non-adiabatic pressure perturbation is consistent with zero within 1σ. Finally, we also present a new, publicly available, RSD likelihood for MontePython based on the “Gold 2018” growth-rate data compilation.

I. INTRODUCTION

Recent observations of Type Ia supernovae (SnIa) at the end of the previous century have indicated that on cosmological scales the Universe is undergoing a phase of accelerated expansion, usually attributed to the cosmological constant Λ [1, 2]. Since then, this finding has been confirmed via a plethora of different observations something which, in conjunction with theoretical developments, has led to the creation of a robust description of the evolution of the Universe on cosmological scales within the framework of General Relativity (GR). This paradigm is known as the standard Λ cold dark matter model (ΛCDM) and it contains just six free parameters, which describe the dark energy (DE) and matter contents of the cosmos. Currently, the ΛCDM model is our best phenomenological description of the data [3].

However, since the first detection of DE, several alternatives to the ΛCDM model have also been developed, which roughly fall under the umbrella of two main categories. First, there are the so-called Modified Gravity (MG) models [4], which assume that GR is modified on large scales, the so-called Infrared (IR) modifications, in order to accommodate current observations [5]. However, certain modifications of GR are fraught with difficulties, such as the Ostrogradsky instability, that arises when a non-degenerate Lagrangian with time derivatives higher than second order, leads to an unstable Hamiltonian [6, 7]. Furthermore, several tests with cosmological data seem to be in very good agreement with GR [8–19].

The second category of theories that are serious contenders to ΛCDM are DE models [20] with as yet unobserved scalar fields that dominate over the other matter species at late times, while at the same time, avoiding fine-tuning [21, 22]. Most of these DE models also exhibit perturbations, which will affect the large scale structure (LSS) of the Universe, however they tend to be subdominant at late times and on scales of interest. As a result, in order to constrain the cosmological parameters to a percent level and discriminate between the various theories, DE perturbations should be well understood as they are expected to play an important role in the near future [23–25].

These two categories seem at a first glance quite dissimilar, however it is possible to unify them within the same framework. One way to do this is to map the MG models, to linear order, to some DE fluid via the effective fluid approach. Then, MG models can be interpreted as DE fluids described by an equation of state w(a), a pressure perturbation δP(k, a), and an anisotropic stress σ(k, a) [24–26]. Hence, the evolution of the background is determined by w(a), while the evolution of the perturbations is governed by δP(k, a) and σ(k, a), both of which are time and scale dependent. In this case however, the effective fluid DE pressure perturbation δP(k, a) could also be interpreted, as we will do in Sec. II as containing both an adiabatic and a non-adiabatic contribution.

On the other hand, the presence of DE anisotropic stress has the interesting side-effect that the DE sound speed c^2_{DE} can in general be negative, without sacrificing the overall stability of the perturbations. This is true as long as the effective sound speed, which is the sum of the DE sound speed and the anisotropic stress, is always positive [32]. Moreover, it can be shown that a varying adiabatic sound speed of DE perturbations can mimic anisotropic stresses [33, 34].

In this paper we will consider an holistic approach and also consider non-adiabatic DE perturbations, motivated by the following reasons. First, in Ref. [35] it was shown with a machine learning approach, based on the
Genetic Algorithms, that current data seem to give hints for the existence of DE anisotropic stress, thus going beyond simple DE models within GR. This could also leave open the possibility for a non-adiabatic DE component, as then the DE component could originate from a higher energy model, usually of the MG type. Second, the previous observation is crucial since, as mentioned earlier and will be seen in detail in the following sections, when MG models are described by the effective fluid approach, equivalently they can also be modelled as a DE fluid with a non-adiabatic component. Hence, we conclude that a non-adiabatic DE component could arise naturally in a wide class of models.

Finally, Primordial Black Holes (PBH) can be a significant component of Dark Matter \(^{66}\) and give rise to entropy perturbations at early times on very small scales. They grow like isocurvature energy density perturbations and may eventually generate a significant component on large scales \(^{37}\). Note that PBH as dark matter behaves as an adiabatic component on very large scales, since it follows the large scale curvature perturbations just like baryons and photons. It is only on small scales that it has an isocurvature component, which is also highly non-Gaussian and can grow to become relevant at late times, around vacuum energy domination. While the PBH entropy perturbations happen in very different scales from those of DE, this clearly provides another mechanism for giving rise to a non-adiabatic component in the dark sector.

Here we consider the effects of the non-adiabatic DE perturbations on LSS of the Universe, as the latter is directly affected by the underlying gravitational theory, something which allows us to easily search for deviations from GR. A main probe of LSS is the matter density perturbations, which in linear theory can be parameterized through the growth parameter \(\delta_m = \frac{\delta\rho_m}{\bar{\rho}_m}\), and the growth-rate \(f \equiv \frac{\delta\rho_m}{\frac{\partial\ln m}{\partial\ln a}}\), which is the former’s logarithmic derivative while \(\bar{\rho}_m\) is the background matter density and \(\delta\rho_m\) its perturbation to linear order. The growth-rate can also be parameterized via the growth-index \(\gamma\) parameter \(^{38}\), which in the \(\Lambda\)CDM model is equal to \(\gamma \approx 6/11\), hence making it easier to look for deviations from GR. The growth-index is defined as the exponent of the growth-rate \(f(z) = \Omega_m^1(z)\), and, as in the \(\Lambda\)CDM model the growth rate is scale-invariant on large scales, this makes \(\gamma\) a useful discriminator of DE models \(^{39}\).

One of the main advantages of the growth-rate is that it encodes information about how gravity affects the LSS, as the latter requires only linear physics, which is well understood. This means the growth can be a particularly useful probe \(^{40}\). Similarly, the growth-index can help discriminate models both between DE and MG, see Ref. \(^{16, 17}\), but also between \(\Lambda\)CDM \(^{14}\) and MG models that are fully degenerate at the background level \(^{27}\) \(^{31}\) \(^{41}\).

Some of the first constraints on the sound speed of DE were reported in Ref. \(^{45}\) by using WMAP data. However, given the data at the time, no significant sensitivity on the adiabatic sound-speed was reported. On the other hand, non-adiabatic perturbations were studied within the context of a decaying vacuum cosmology in Ref. \(^{69}\), where they were found to only have an effect on larger scales. Constraints on non-adiabatic DE models using only growth RSD data were reported in Ref. \(^{47}\), which used a particular parameterization for the non-adiabatic DE perturbations based on a linear combination of the intrinsic and entropy perturbations \(\Gamma(a)\) and \(S(a)\). Using a conjoined analysis of the \(f\sigma_8\) and \(H(z)\) data no deviations from \(\Lambda\)CDM were found. In the next sections we will present a broader approach by considering a general ansatz for the non-adiabatic DE perturbations and we will use the latest cosmological data, including Planck 18, BAO and RSD measurements to constrain its model parameters.

The structure of our paper is as follows: in Sec. \(\text{II}\) we present the theoretical background of our analysis and a realistic parameterization for the non-adiabatic DE pressure perturbations, along with analytic solutions for the growth of matter density perturbations and the growth index \(\gamma\), while in Sec. \(\text{III}\) we compare our numerical and analytical solutions against an implementation of the non-adiabatic perturbations in the Boltzmann code CLASS. In Sec. \(\text{IV}\) we present our results from a Monte-Carlo Markov Chain (MCMC) analysis using the latest cosmological data, while in Sec. \(\text{V}\) we discuss our conclusions. Finally, in Appendix \(\text{A}\) we present an implementation of the redshift space distortions (RSDs) likelihood for MontePython.

**II. THEORY**

We will consider a spatially flat Universe and assume that the scalar perturbations of the metric can be described by the perturbed Friedmann-Robertson-Walker metric in the conformal Newtonian gauge

\[
ds^2 = a^2 \left[ -\left(1 + 2\psi\right)d\tau^2 + \left(1 - 2\phi\right)dx_1dx_1 \right],
\]

where \(a = a(\tau) = \frac{1}{1 + z}\) is the scale factor, \(z\) is the redshift and \(d\tau = dt/a\) is the conformal time in terms of the cosmic time \(t\).

We assume that a DE fluid is responsible for the accelerated expansion of the Universe and that its background evolution can be described by an equation of state \(w = P/\bar{\rho}\), while its fluctuations can be described by a pressure perturbation \(\delta P\) and anisotropic stress \(\sigma\). The energy momentum tensor of the fluid can be written

\[
T^\mu_\nu = P\delta^\mu_\nu + \left(\rho + P\right)U^\mu U_\nu,
\]

where the overhead bar \(\bar{\rho}\) denotes a background quantity, \(U^\mu = dx^\mu/\sqrt{-ds^2}\) is the four velocity, given to linear order by \(U^\mu \approx \frac{1}{a}(1 - \psi, u^i)\) for \(u^i = dx^i/d\tau\) and the density and pressure include both background and perturbations, i.e. \(\rho = \bar{\rho} + \delta\rho\) and \(P = \bar{P} + \delta P\). The components...
of the energy momentum tensor are then given by

\begin{align}
T^0_0 &= -\left(\dot{\rho} + \delta\rho\right), \\
T^i_0 &= (\dot{\rho} + \dot{P})u_i, \\
T^i_j &= (\dot{P} + \dot{\rho})\delta^i_j + \Sigma^i_j,
\end{align}

where \(\Sigma^i_j\) is the anisotropic stress tensor, which is traceless \(\Sigma^i_i = 0\) and can also be written via the \(\sigma\) parameter as \((\dot{\rho} + \dot{P})\sigma \equiv -(k^i k_j - \frac{1}{3}\delta^i_j)\Sigma^i_j\).

The evolution equations of the fluid variables \(\delta = \frac{\delta\rho}{\rho}\) and velocity of the DE fluid \(\theta = ik^j u_j\) can be found by the conservation of the energy momentum tensor \(T^\mu_{\nu,\nu} = 0\) and are given by \([25, 48]\):

\[
\dot{\delta} = -(1 + w)\left(\theta - 3\dot{\phi}\right) - 3H\left(\frac{\delta P}{\dot{\rho}} - w\delta\right),
\]

\[
\dot{\theta} = -H(1 - 3w)\theta - \frac{\dot{w}}{1 + w}\theta + \frac{\delta P}{\dot{\rho}}\bar{k}^2 - k^2\theta + k^2\psi, (7)
\]

where \(H \equiv \frac{\dot{a}}{a}\) is the conformal Hubble parameter and \(k\) is the wavenumber of the Fourier mode of the perturbations, which in GR are decoupled.

In general, is is most convenient to describe the DE pressure perturbation in the rest-frame \(\delta P\), denoted here by a hat, which is defined as the frame where the fluid is at rest, i.e. \(\dot{\theta} = 0\). Then, the pressure perturbation in the rest-frame can be expressed in terms of the energy density \(\rho\) and entropy \(S\) as \(\hat{P} = P(\rho, S)\) as \([49]\): \n
\[
\hat{\delta}P = \frac{\partial P}{\partial\rho}\bigg|_s \delta\rho + \frac{\partial P}{\partial S}\bigg|_{\rho} \delta S,
\]

where the DE density and entropy perturbations at the rest-frame are given by \(\delta\rho\) and \(\delta S\) respectively. In principle, the non-adiabatic contribution may come from some internal degrees of freedom, as for example happens in the quintom model \([23]\). We can straight-forwardly identify the DE rest-frame sound speed as \(c^2_s \equiv \frac{\partial P}{\partial\rho}\bigg|_s\),

\[
(9)
\]

which is equal to one for quintessence, but is in the range \(c^2_s \in [0, 1]\) for K-essence or other models \([50]\). For modified gravity models it can even be negative, in which case one would presume that a negative value would cause instabilities in the perturbations, unless there is anisotropic stress to stabilize them \([32]\).

We can now decompose the pressure perturbation in terms of the sound speed \(c^2_s\) and a non-adiabatic part \(\hat{\delta}P_{\text{nad}}\) as

\[
\hat{\delta}P = \hat{c}_s^2\hat{\rho}\dot{s} + \hat{\delta}P_{\text{nad}}, \quad (10)
\]

where both quantities are defined in the DE rest-frame and the non-adiabatic contribution at the rest-frame can be identified as

\[
\hat{\delta}P_{\text{nad}} = \frac{\partial P}{\partial S}\bigg|_{\rho} \delta S. \quad (11)
\]

In order to use the aforementioned expressions for the pressure perturbation in any other frame besides the DE rest-frame, we have to change gauge by considering a general coordinate transformation between the hatted (DE rest-frame) and the unhatted (general) frame \([23, 48]\):

\[
x^\mu = \hat{x}^\mu + d^\mu, \quad (12)
\]

where \(d^\mu = \alpha(x, \tau) \vec{\nabla}\beta(x, \tau) + \vec{c}(x, \tau)\), for some functions \(\alpha, \beta\) and \(\epsilon\). Then, the perturbation variables transform as \([48]\)

\[
\delta = \hat{\delta} - \alpha\hat{\rho}, \quad (13)
\]

\[
\theta = \hat{\theta} - \alpha k^2, \quad (14)
\]

\[
\delta P = \hat{\delta}P - \alpha \hat{\dot{P}}, \quad (15)
\]

where in the rest frame we have that \(\dot{\theta} = 0\). We can use Eq. \((14)\) to eliminate \(\alpha\), as \(\dot{\theta} = 0\), thus finding from Eq. \((15)\)

\[
\hat{\delta}P = \hat{\delta}P - 3Hc^2_s\rho \frac{(1 + w)\theta}{k^2}, \quad (16)
\]

where \(c^2_s = \frac{\dot{\rho}}{\rho} = w - 3\frac{\dot{w}}{1 + w}\) is the so called adiabatic sound speed and we have used the background conservation equation

\[
\hat{\dot{\rho}} + 3H(1 + w)\rho = 0. \quad (17)
\]

Using Eqs. \((10), (13)\) and \((17)\) in Eq. \((16)\) we can write the pressure perturbation in any gauge as

\[
\delta P = c^2_s\rho\dot{s} + \delta P_{\text{nad}} + 3H \left( c^2_s - c^2_{\alpha} \right) \rho \frac{(1 + w)\theta}{k^2}, \quad (18)
\]

which is in agreement with Ref. \([23]\). Thus, our final expressions for the evolution equations for the DE perturbations in the conformal Newtonian gauge are given by

\[
\dot{\delta}_{DE} = -(1 + w)\left(\theta_{DE} - 3\dot{\phi}\right) - 3H \left( c^2_s - c^2_{\alpha} \right) \rho \frac{(1 + w)\theta_{DE}}{k^2} - 3H \hat{\delta}P_{\text{nad}} \rho, \quad (19)
\]

\[
\dot{\theta}_{DE} = -H(1 - 3c^2_s)\theta_{DE} + \frac{k^2c^2_s}{1 + w} \delta_{DE} - k^2\sigma \rho, \quad (20)
\]

Compared to Refs. \([48\) and \([25\], the last terms in Eqs. \((19)\) and \((20)\) are new. The latter, ignoring any non-adiabatic contributions, are commonly used in the Boltzmann codes to model the behavior of the DE perturbations. In order to include them in the aforementioned codes, we will henceforth assume that the DE fluid at the rest frame also has a non-adiabatic component \(\hat{\delta}P_{\text{nad}}\).

This extra component however, can in principle destabilize the perturbations. To demonstrate this, we follow
Ref. [32] and we eliminate θ from Eqs. [19]–[20], resulting in a second order equation for the growth of DE perturbations $\delta_{DE}$:

$$
\ddot{\delta}_{DE} + (\cdots) \dot{\delta}_{DE} + (\cdots) \delta_{DE} = -k^2 \left( (1 + w)\psi + \dot{c}_s^2 \delta_{DE} + \delta P_{nad}/\bar{\rho} - \frac{2}{3} \pi \right) + \cdots, \tag{21}
$$

where the dots (\cdots) indicate the presence of complicated expressions and we have redefined the anisotropic stress parameter of the DE fluid as $\pi = \frac{3}{2} (1 + w) \sigma$. Here we focus solely on the last $k^2$ term, which as discussed in Ref. [23], it will act as a source driving the perturbations. However, since the potential scales as $\psi \sim 1/k^2$ in matter domination, the only terms that matter are the sound speed, the non-adiabatic perturbation and the anisotropic stress. Therefore, we can define an effective sound speed as

$$
c_s^{2,\text{eff}} = c_s^2 + \frac{\delta P_{nad}}{\bar{\rho} \delta_{DE}} - \frac{2}{3} \pi / \delta_{DE}, \tag{22}
$$

which has to be positive for the perturbations to be stable at all scales.

In order to solve Eqs. [19] and [20], we need to choose a parameterization for the DE non-adiabatic pressure perturbations. Here we will consider a case which is motivated by the effective fluid approach of Refs. [27], [28], and as an example we will consider the designer $f(R)$ model, see Ref. [27], which is constructed so that the background expansion corresponds exactly to ΛCDM but to linear order, it can have perturbations [41]. This implies that $w = -1$ and from Eq. [18] we have that for the designer $f(R)$ model

$$
\frac{\delta P_{nad,des}}{\bar{\rho}} = \frac{\delta P}{\bar{\rho}} - c_s^2 \delta_{DE} - 3 \mathcal{H} \left( c_s^2 - c_a^2 \right) \frac{V_{DE}}{k^2}, \tag{23}
$$

where $\delta P/\bar{\rho}$ and $\delta_{DE}$ are given by Eqs. (42) and (43) of Ref. [27]. $V_{DE} = (1 + w) \theta_{DE}$, while $c_s^2 = 1$ for $f(R)$. Note that for this model, in general we have $V_{DE} \neq 0$ even if $w = -1$ [27].

We plot this function for the designer $f(R)$ model for $\Omega_{m0} = 0.3$, $f_{R0} = -10^{-4}$ and $w = -1$ in Fig. 1 where we see that at both early and late times, the non-adiabatic component evolves as a power law of the form $\delta P_{nad}/\bar{\rho} \simeq c_0 a^n k^2/H_0^2$. Specifically, we find that [27]

$$
n = \frac{9}{4} + \frac{\sqrt{73}}{4} \simeq 4.386, \tag{24}
$$

$$
c_0 = \frac{-5 + \sqrt{73}}{36} g(\Omega_{m0}) f_{R0}, \tag{25}
$$

where

$$
g(\Omega_{m0}) \simeq \frac{\Omega_{m0}^{17/12} - \frac{\sqrt{73}}{12}}{\sum_{m=0}^{\infty} \frac{n^{m}}{\Gamma(1 - n) \Gamma(1) \Gamma(2) \cdots \Gamma(n)}}, \tag{26}
$$

Inspired from this functional form, in what follows we will assume the rather general ansatz

$$
\frac{\delta P_{nad}}{\bar{\rho}} = c_0 a^n k^2/H_0^2, \tag{27}
$$

where $(c_0, n)$ are parameters to be determined, however the exponent $n$ has to be positive so as to ensure the non-adiabatic DE perturbation vanishes at early times, thus we will assume the prior $n \in (0, \infty)$. In the next sections we will present constraints on the parameters $(c_0, n)$ in the case of $w = \text{const}$ and of no DE anisotropic stress ($\sigma = 0$).

### A. The initial conditions

Here will now discuss the initial conditions for the DE perturbations in both gauges and in two different regimes, in matter and radiation domination. First, we consider the initial conditions in the conformal Newtonian gauge in matter domination, for which we follow Ref. [25]. In a similar vein we consider two regimes, first that the DE perturbations are larger than the sound horizon, $k \ll aH/\dot{c}_s$ or equivalently that $c_s^2 = 0$, and second we also consider the small scales solutions $k \gg aH/\dot{c}_s$, which implies the terms scaling as $k^2$ dominate of over the rest.

In any case, the initial conditions for matter and the potential (assuming no anisotropic stress) in matter domi-
inflation are unchanged and given by [25]
\[
\delta_m(a) = \delta_0 \left( a + \frac{3H_0^2\Omega_{m0}}{k^2} \right),
\]
\[
V_m(a) = -\delta_0 H_0 \sqrt{\Omega_{m0}} a^{1/2},
\]
\[
\phi = -\frac{3}{2}\delta_0 \frac{H_0^2\Omega_{m0}}{k^2},
\]
where \(\delta_0\) is a normalization set at early times from inflation, while \(V_i \equiv (1 + w_i)\theta_i\).

In the first case \((k \ll aH/\dot{\epsilon}_s)\) we find that the initial conditions for the DE density and velocity perturbations are given by
\[
\delta_{DE}(a) = \delta_0 (1 + w) \left( \frac{a}{1 - 3w} + \frac{3H_0^2\Omega_{m0}}{k^2} \right)
- \frac{c_0 k^2 a^n}{\sqrt{\Omega_{m0}} a^{1/2}},
\]
\[
V_{DE}(a) = -\delta_0 (1 + w) H_0 \sqrt{\Omega_{m0}} a^{1/2}
+ \frac{c_0 k^2 a^n}{\sqrt{\Omega_{m0}}}.
\]

The second case \((k \gg aH/\dot{\epsilon}_s)\) we find that the initial conditions for the DE density and velocity perturbations are given by
\[
\delta_{DE}(a) = \frac{3}{2} (1 + w) \delta_0 H_0^2\Omega_{m0} \frac{c_s^2 k^2}{c_s^2 H_0^2}
- \frac{c_0 k^2 a^n}{c_s^2 H_0^2},
\]
\[
V_{DE}(a) = -\frac{9}{2} (1 + w) (\dot{\epsilon}_s - w) \frac{H_0^2\Omega_{m0}^{3/2}}{c_s^2 k^2} a^{-1/2}
+ \frac{c_0 k^2 \sqrt{\Omega_{m0}} a^{n-1/2} (n - 3w)}{c_s^2 H_0}
+ \frac{81H_0^2\Omega_{m0}(\dot{\epsilon}_s - w)^2}{a^2 k^4}. \tag{34}
\]

We find that in both cases the last terms containing \(c_0\), are new compared to Ref. [25] and correspond to the contribution of the non-adiabatic term.

For the simpler case of a constant adiabatic DE sound-speed \(c_s^2\), the initial conditions in the synchronous gauge in radiation domination where first derived in Ref. [51] as a series expansion in terms of \(k\tau\). Here we generalize this approach by also considering the non-adiabatic pressure perturbation and we follow Refs. [48, 51]. Since we have to expand in terms of \(k\tau\) we find that in this case it more convenient to consider the different regimes for the index \(n\) of the power law of our ansatz given by Eq. (27). Specifically, as we have already mentioned, \(n\) has to be positive in order for the non-adiabatic pressure perturbation to vanish at early times, so we will consider the regimes \(n \in (0, 1), n \in [1, 2), n \in [1, 2)\) and \(n \geq 3\), since then the scalar factor dominates differentially at early times.

Then, by expanding the Einstein and fluid equations in terms of \(k\tau\), following Refs. [48, 51], we find the initial conditions for the DE density \(\delta_{DE}\) and velocity \(\theta_{DE}\) perturbations for \(n \in (0, 1)\)
\[
\delta_{DE}(a) = \frac{\delta_0 (3c_s^2 - 4)(k\tau)^2(w + 1)}{6c_s^2 - 12w + 8}
+ \frac{c_0 k^2}{4H_0^2(3c_s^2 - 6w + 4)(\dot{\epsilon}_s - w)}
\cdot \left[ 4 (w ((k\tau)^2 - 9w + 12) - 4)
- 3\dot{\epsilon}_s (((k\tau)^2 - 6) w + 4) \right], \tag{35}
\]
\[
\theta_{DE}(a) = \frac{-\delta_0 c_s^2 k(k\tau)^3}{6c_s^2 - 12w + 8}
+ \frac{c_0 k^2 (k\tau)(\dot{\epsilon}_s ((k\tau)^2 - 6) + 12w - 8)}{4H_0^2(w + 1)(3c_s^2 - 6w + 4)(\dot{\epsilon}_s - w)}. \tag{36}
\]

For \(n \in [1, 2)\) we have that
\[
\delta_{DE}(a) = \frac{\delta_0 (3c_s^2 - 4)(k\tau)^2(w + 1)}{6c_s^2 - 12w + 8}
+ \frac{3a c_0 k^2 (w - 1)}{H_0^2(2c_s^2 - 3w + 1)}, \tag{37}
\]
\[
\theta_{DE}(a) = \frac{-\delta_0 c_s^2 k(k\tau)^3}{6c_s^2 - 12w + 8}
+ \frac{ac_0 k^2 (k\tau)(3w - 1)}{3H_0^2(w + 1)(-2\dot{\epsilon}_s^2 + 3w - 1)}. \tag{38}
\]

For \(n \in [1, 2)\) we have that
\[
\delta_{DE}(a) = \frac{\delta_0 (3c_s^2 - 4)(k\tau)^2(w + 1)}{6c_s^2 - 12w + 8}
+ \frac{3a c_0 k^2 (3w - 4)}{H_0^2(6c_s^2 - 12w + 8)}, \tag{39}
\]
\[
\theta_{DE}(a) = \frac{-\delta_0 c_s^2 k(k\tau)^3}{6c_s^2 - 12w + 8}
+ \frac{ac_0 k^2 (k\tau)(3w - 2)}{2H_0^2(w + 1)(-3\dot{\epsilon}_s^2 + 6w - 4)}. \tag{40}
\]

while for \(n \geq 3\) the contribution from the non-adiabatic pressure perturbation of Eq. (27) is subdominant and we recover the results of Ref. [51].

B. Approximate solutions and the growth index

Here we present analytic solutions to the evolution equations Eqs. (19) and (20), but also analytic expressions for the growth index \(\gamma\) at late times. We note that the forthcoming approximations are only used to gain insight and intuition on the effects of the non-adiabatic term on the growth and the LSS and are not used in CLASS or the MCMC analysis later on, for which we solve the corresponding equations numerically.

One way to determine how the non-adiabatic DE pressure perturbation, and DE in general, affects the growth of matter density perturbation \(\delta_m \equiv \frac{\delta m}{\rho_m}\), is to rewrite
the fluid equations for matter and DE as a second order differential equation for $\delta m$. To do so, we assume homogeneity, isotropy and neglect neutrinos, which is a viable approximation since our data is not in such small scales affected by them. Then, the growth of matter can be followed with the second order differential equation

$$\delta''(a) + \left[ \frac{3}{a} + \frac{H'(a)}{H(a)} \right] \delta'(a) - \frac{3\Omega_{m}H_{0}^{2}G_{\text{eff}}(a)}{2a^{5}H(a)^{2}G_{N}} \delta(a) = 0,$$

where the effects of DE or a modified gravity theory, such as $f(R)$, at the perturbations level can be taken into account by the effective Newtonian constant $G_{\text{eff}}(a)$. 

To find the effects of the non-adiabatic pressure perturbation we follow Ref. [25], where it was shown that for a DE fluid with constant equation of state $w$ during matter domination $Q \equiv G_{\text{eff}}(a)/G_{N}$ is given by

$$Q - 1 \equiv 0 \left(1 + \frac{\Omega_{m}}{\Omega_{m} - \delta P_{\text{nad}}/\rho} a^{-3w} \right).\tag{42}$$

To find a similar expression of $Q$ during dark energy domination, which is a solution on small scales $k \gg aH/c_{s}$, that takes into account the non adiabatic component $\delta P_{\text{nad}}/\rho$ we do the following. Defining the scalar velocity perturbation as $V \equiv ik_{j}T_{j}^{i}/\rho = (1 + w)\delta\theta$, Eqs. [19] and [20] can be rewritten, in the conformal gauge, as

$$\delta_{DE}' = -\frac{V_{DE}}{Ha^{2}} \left(1 + \frac{9a^{2}H(a)^{2}(c_{s}^{2} - w)}{k^{2}} \right) - \frac{3}{a} (c_{s}^{2} - w) \delta_{DE}$$

$$+ 3(1 + w)\phi' - \frac{3}{a} \delta P_{\text{nad}}/\rho,\tag{43}$$

$$V_{DE}' = -(1 - 3c_{s}^{2})V_{DE}/a + \frac{k^{2}}{Ha^{2}}c_{s}^{2}\delta_{DE} + (1 + w)\frac{k^{2}}{Ha^{2}}c_{s}^{2}\delta_{DE}$$

$$+ \frac{2\delta P_{\text{nad}}}{\rho} - \frac{2k^{2}}{Ha^{2}}c_{s}^{2}\phi,\tag{44}$$

where the prime ' is the derivative with respect to the scale factor $a$ and we are assuming there is no DE anisotropic stress, i.e $\sigma = 0$, hence $\phi = \psi$. In Eq. [44], in order to not have large velocity perturbations it is expected that the terms that scale as $k^{2}$ cancel out, hence

$$\delta_{DE}' = \frac{3}{2}(1 + w) H_{0}^{2} \Omega_{m} \frac{a^{2}}{c_{s}^{2}k^{2}} \delta_{0} - \frac{3}{2} \delta P_{\text{nad}}/\rho,\tag{45}$$

where we have used that $k^{2}\phi = -\frac{3}{2} \delta P_{\text{nad}}/\rho \Omega_{m}$, which is the solution for the perturbation equations in matter domination [25]. Then using Eqs. [43] and [45] we find

$$V_{DE}' = -3Ha(c_{s}^{2} - w)\delta - 3Ha \frac{\delta P_{\text{nad}}}{\rho}.\tag{46}$$

Now we can compute $Q$ in the dark energy domination regime as

$$Q - 1 = \frac{\rho_{DE}\Delta_{DE}}{\rho_{m}\Delta_{m}},\tag{47}$$

where $\Delta \equiv \delta + \frac{3aH_{0}^{2}c_{s}^{2}}{a^{2}}$ is the gauge invariant density perturbation. In matter domination we have that $\Delta_{m} = \delta_{0}a$, while for DE we have that

$$\Delta_{DE} \equiv \frac{3}{2} (1 + w) H_{0}^{2} \Omega_{m} \frac{a^{2}}{c_{s}^{2}k^{2}} \delta_{0} - \frac{\delta P_{\text{nad}}/\rho}{c_{s}^{2}},\tag{48}$$

which is similar to the initial condition given by Eq. [33]. From Eq. [48] we see that the dominant term comes from the contribution of the non adiabatic part, as the latter scales as $k^{2}$, see Eq. [27], hence $Q$ can be expressed as

$$Q - 1 \sim -\frac{1 - \Omega_{m}}{\Omega_{m}} \frac{\delta P_{\text{nad}}/\rho a^{-1-3w}}{c_{s}^{2}}.\tag{49}$$

1. Analytic solutions for the growth

Modeling the non-adiabatic pressure perturbation as in Eq. [27], $Q$ can be written as

$$Q(k, a) = 1 - \frac{1 - \Omega_{m}}{\Omega_{m}} \frac{c_{0}k^{2}}{\delta_{0}c_{s}^{2}H_{0}^{2}} a^{n-1-3w}.\tag{50}$$

In order to solve Eq. [41] with $Q \equiv G_{\text{eff}}(k, a)/G_{N}$ given by Eq. [50] we need to make an approximation due to the appearance of the term $a^{n-1}$, which makes it difficult to find analytic solutions. As we expect that $n \sim O(1)$ at late times, see Fig. 1 then we make a series expansion of the term $a^{n-1}$ around $n = 1$ of the form

$$a^{n-1} \simeq 1 + (n - 1) \ln a + \cdots$$

$$\simeq 1 - (n - 1) \ln (1 + z) + \cdots,\tag{51}$$

where in the second step we used that $a = 1/(1 + z)$. Since we are interested at the evolution of the growth at low redshifts, we replace the term $\ln (1 + z)$ with an average $b_{0} = \langle \ln (1 + z) \rangle$, which in the range $z \in [0, 2]$ is approximately $b_{0} \approx 0.6479$. Hence, under this approximation Newton’s constant becomes

$$Q(k, a) \simeq 1 - \frac{1 - \Omega_{m}}{\Omega_{m}} \frac{c_{0}k^{2}}{\delta_{0}c_{s}^{2}H_{0}^{2}} (1 - b_{0} (n - 1)) a^{-3w}.\tag{52}$$

Then, by making the following change of variables $a^{-3w} \equiv x$ and inserting Eq. [49] in Eq. [41] we find

$$\delta_{m}(a) = a_{2}F_{1} \left[ \frac{1}{4} - \frac{5}{12w} + B \frac{1}{4} - \frac{5}{12w} - B, 1 - \frac{5}{6w}; -1 - \frac{1}{\Omega_{m}} a^{-3w} \right],\tag{53}$$

where

$$B = \frac{1}{12w} \sqrt{(1 - 3w)^{2} + 24\delta B},\tag{54}$$

$$\delta B = -\frac{c_{0}k^{2}}{\delta_{0}c_{s}^{2}H_{0}^{2}} (1 - b_{0} (n - 1)).\tag{55}$$

To compare our analytical results with the full numerical solution from the evolution equations Eq. [19]-[20]
in the next sections we will use the combination $f\sigma_8(a)$ which is a measurable quantity and is defined as

$$ f\sigma_8(a) \equiv f(a) \cdot \sigma(a) = \frac{\sigma_{8,0}}{\delta_m(1)} \delta_m'(a), \quad (56) $$

where $\sigma(a) = \sigma_{8,0} \frac{\delta_m'(a)}{\delta_m(1)}$ is the redshift-dependent rms fluctuations of the linear density field at $R = 8h^{-1}\text{Mpc}$ while the parameter $\sigma_{8,0}$ is its value today.

2. The growth-rate index $\gamma$

Finally, we briefly discuss the growth index $\gamma$ in the presence of DE perturbations. The latter affect the evolution of the matter density contrast $\delta_m = \frac{\delta_m}{\bar{\rho}_m}$ and its growth rate $f(a) = \frac{d\ln \delta_m}{d\ln a}$. When we ignore DE perturbations, the latter can be approximated as $\Omega_{m,0}$ [56, 57]

$$ f(a) = \Omega_{m}(a) \gamma(a), \quad (57) $$

where the growth index $\gamma$ is given by

$$ \gamma(a) = \gamma_m(a) = \frac{\ln f(a)}{\ln \Omega_m(a)} \approx \frac{3(1 - w)}{5 - 6w} + \cdots, \quad (58) $$

which for $\Lambda$CDM reduces to $\gamma \sim \frac{6}{11}$ and by $\gamma_m$ we denote the contribution to the growth index coming from the CDM and the background evolution only. When we include the DE perturbations assuming they are sourced from an anisotropic stress, the growth index picks up a correction [57]

$$ \gamma = \gamma_m + \gamma_{DE}, \quad (59) $$

where the contribution coming from the DE perturbations is given by

$$ \gamma_{DE} \approx -\frac{3(1 + w)}{18w^2 - 21w + 5} + \cdots. \quad (60) $$

From now on we will refer to $\Omega_m(a)$ as $\Omega$ as a shorthand. If we include DE perturbations the growth index for the matter can be written to first order as

$$ \gamma = \frac{\ln(f(\Omega))}{\ln(\Omega)} = \frac{3(\delta B + w - 1)}{6w - 5} \quad \frac{3(\Omega - 1)((\delta B + w - 1)(9\delta B(4w - 3) - 3w + 2))}{2 ((5 - 6w)^2 (12w - 5)) + \cdots, \quad (61)$$

We can split the growth index into two parts, the first being the contribution from the CDM component and the background expansion denoted as $\gamma_m$, while the second being the contribution from the non-adiabatic component, denoted as $\gamma_{DE}$, then we have:

$$ \gamma = \gamma_m + \gamma_{DE}. \quad (62) $$

and we find from Eq. (61) that

$$ \gamma_m = \frac{3(w - 1)}{6w - 5} + \frac{3(3w - 2)(w - 1)(\Omega - 1)}{2(5 - 6w)^2 (12w - 5)} + \cdots, \quad (63) $$

$$ \gamma_{DE} = \frac{3\delta B}{6w - 5} + (\Omega - 1) \left( \frac{3\delta B(6w(6w - 11) + 29)}{2((5 - 6w)^2 (12w - 5))} - \frac{27\delta B^2(4w - 3)}{2((5 - 6w)^2 (12w - 5))} \right) + \cdots, \quad (64) $$

where $\delta B$ is given by Eq. (59). These expressions are similar to those when DE perturbations are included, originally derived in Ref. [57], but now the extra contribution comes instead from the non-adiabatic pressure perturbation.

III. COMPARISON WITH CLASS AND NUMERICAL SOLUTIONS

Here we present in detail how the non-adiabatic pressure perturbation, given by the ansatz of Eq. (27), affects several key cosmological quantities, such as the scale-dependent growth $f\sigma_8(k, z)$, the matter power spectrum $P(k, z)$ and the CMB TT power spectrum $C_l^{TT}$.

To do this, we implemented the non-adiabatic pressure perturbation as given by Eq. (27), along with the initial conditions in radiation domination in the synchronous gauge, given by Eqs. (35)-(40), in the Boltzmann code CLASS [58, 59]. To test our modifications, we also compare the numerical results from CLASS with the numerical solution in Mathematica of the evolution equations [19] and [20], but also with the analytical solutions of Sec. [11B].

We should note that there is a difference between the normalization used in CLASS, which uses units of Mpc thus affecting the initial values of the perturbations $\delta_0$, and in the numerical solution of the evolution equations [19] and [20] in Mathematica, where we set $\delta_0 = 1$, so that $\delta_m(a) \sim a$ in matter domination, but also $k$ is expressed in units of $H_0$. For example, a wavenumber of $k = 0.1\text{hMpc}^{-1}$ in CLASS corresponds to $k = 0.1 \frac{300}{H_0} = \frac{300}{H_0} H_0$ in our notation. Then, the coefficient $c_0$ is rescaled by a factor of $c_0_{\text{CLASS}} \rightarrow c_0_{\text{Math}} \left(\frac{300}{h}\right)^2 \frac{\delta_{0,\text{CLASS}}}{\delta_{0,\text{Math}}}$ between the two frameworks. In what follows, we will express all values of $c_0$ in the dimensionless picture, i.e. $c_0 = c_0_{\text{Math}}$, as that is easier to test numerically with any ODE solver, not only CLASS.

\footnote{Similarly, a wavenumber of $k = 0.1\text{hMpc}^{-1}$ is equivalent to $k = 0.1 \cdot 3000 H_0 = 300 H_0$.}
First, we show the dependence of the growth-rate \( f\sigma_8(k, z) \) and the matter power spectrum \( P(k, z) \) on the parameter \( c_0 \) keeping \( n \) fixed, see Fig. 2. In the left panel we show the evolution of the scale dependent growth rate \( f\sigma_8(k, z) \) for various values of \( c_0 \) and \( n = 0.5 \). In this case the growth was calculated with CLASS via \( \delta(k, z) = \sqrt{P(k, z)/P(k, 0)} \) for \( k = 0.1 \text{Mpc}^{-1} \). The points correspond to the “Gold 2018” growth-rate \( f\sigma_8 \) compilation shown in Table 1. Right panel: The matter power spectrum \( P(k, z) \) at \( z = 0 \), for various values of \( c_0 \) and \( n = 0.5 \). In both cases we assumed \( \Omega_m = 0.3 \), \( w = -0.8 \), \( h = 0.67 \).

On the other hand, on the right panel of Fig. 2 we show the power spectrum \( P(k, z) \) at \( z = 0 \), for various values of \( c_0 \) and \( n = 0.5 \). In both cases we assumed \( \Omega_m = 0.3 \), \( w = -0.8 \), \( h = 0.67 \). As can be seen, the effect of the non-adiabatic perturbations in this case is to suppress or enhance power, depending on the sign of \( c_0 \), an effect similar to that observed in Ref. [60] for a mixed DE-DM model but also as observed in Ref. [61] for a similar ansatz. Note that in general the matter power spectrum \( P(k, z) \) at scales \( k \sim 0.1 - 10 \text{ h/Mpc} \) can be constrained by Lyman alpha data [62], however as those observations are beyond the scope of this work we do not consider them in this analysis.

Next, we compare the results for the growth rate between CLASS, Mathematica and the analytical approximation to the growth equation. In Fig. 3 we show the absolute percentage difference of \( f\sigma_8(z) \) between the numerical solution from the evolution equations Eq. (20) (green line, denoted “ODE”) and the analytical approximation of Eq. (53) (denoted “Approx.”) with respect to the numerical solution from our CLASS implementation for \( c_0 = 2 \cdot 10^{-7} \). Right panel: Same as the left panel, but for \( c_0 = -2 \cdot 10^{-7} \). For both plots we assume \( \Omega_m = 0.3 \), \( w = -0.8 \), \( c_s^2 = 1 \), \( h = 0.67 \), \( k = 0.1 \text{Mpc}^{-1} \) and \( \sigma_8 = 0.8 \).
IV. MCMC RESULTS

Here we discuss how MontePython [63, 64] was used to place constraints via an MCMC approach on the parameters of the ansatz for the non-adiabatic pressure perturbation given by Eq. (27). We used the Planck 2018 CMB data and in particular we used the parameters of the ansatz for the non-adiabatic model for two data combinations each: CMB+BAO+SnIa and CMB+BAO+SnIa+RSD in order to assess the constraining power of the new RSD likelihood. For the wCDM model we ran four chains with roughly 200,000 points, while for the non-adiabatic model we ran 19 chains with roughly 2,000,000 points in total in order to make sure all the parameters, described below, were well converged.

Specifically, for the MCMC runs of the wCDM model we consider the following parameters: the DE equation of state parameter $w$; assuming it is constant, the baryon and cold dark matter density parameters $\Omega_{b} h^{2}$ and $\Omega_{c} h^{2}$ respectively, the angular scale of the acoustic oscillations $\theta$, the optical depth to Thomson scattering from reionization $\tau$ and the two parameters of the primordial power spectrum $A_{s}$ and $n_{s}$. In a nutshell, our parameter vector for the wCDM model is then $p_{\text{wCDM}} = (w, \Omega_{b} h^{2}, \Omega_{c} h^{2}, \theta, A_{s}, n_{s})$. On the other hand, for the MCMC runs of the non-adiabatic model, we include the parameters of the wCDM model, along with the two non-adiabatic parameters $c_{0}$ and $n$ of Eq. (27). Then, our parameter vector for the non-adiabatic model is $p_{\text{non-ad}} = (w, \Omega_{b} h^{2}, \Omega_{c} h^{2}, \tau, A_{s}, n_{s}, c_{0}, n)$.

In Fig. 6 we show the confidence contours for wCDM using CMB+Lens+BAO+SnIa (green contours) and the CMB+Lens+BAO+SnIs+RSD (blue contours), while in Tables [11] we present the 68% mean values and 95% confidence regions, for some of the parameters of the model. As can be seen, the contours are a bit shifted to higher values of $\sigma_{8}$ and $w$ when the RSD data included. This is consistent with the well-known tension for $\sigma_{8}$ between low and high redshift probes [18].

Next, in Fig. 7 we present the constraints for the non-adiabatic model. In particular we show the confidence contours using CMB+Lens+BAO+SnIa (green contours) and the CMB+Lens+BAO+SnIs+RSD (blue
contours), while in Tables III-IV we present the 68% mean values and 95% confidence regions, for some of the parameters of the model. As can be seen, the amplitude of the non-adiabatic perturbation $c_0$ is consistent with zero, while $n$ is very close to $n \sim 1/2$ as expected from the toy model based on the $f(R)$ designer model.

V. CONCLUSIONS

In this work we have explored the effects of a non-adiabatic DE pressure perturbation on the CMB and LSS. First, we derived the extra contribution of this non-adiabatic component on the DE perturbation equations, given by the last terms in Eqs. [19] and [20]. Since currently it is unknown if DE has a non-adiabatic component and, even if it does, the behavior of $\Delta P_{\text{had}}$ is unknown, we took advantage of the effective fluid approach of Refs. [27-28] in order to construct a realistic ansatz.

In particular, using the designer $f(R)$ model, we derived the expected behavior of this non-adiabatic component both at early and late times, finding that in either era it can be modeled as a power-law. Inspired from this, we then assumed the ansatz given by Eq. [27], where from the $f(R)$ model we expect $n \sim 0.5$. We then solved the fluid equations but also implemented it into the Boltzmann code CLASS. Moreover, using an approach similar to that of Ref. [67], we were able to find analytical approximations to the growth-rate of matter

| Param | best-fit | mean±σ | 95% lower | 95% upper |
|-------|----------|---------|-----------|-----------|
| $n_s$ | 0.9622   | 0.965±0.0039 | 0.9571    | 0.973    |
| $w$   | −1.024   | −1.03±0.033 | −1.095    | −0.9673   |
| $\Omega_{m,0}$ | 0.3109 | 0.3058±0.0076 | 0.2003    | 0.3215    |
| $10^{+9}A_s$ | 2.086 | 2.104±0.03 | 2.041    | 2.169    |
| $\sigma_{8,0}$ | 0.8152 | 0.8191±0.011 | 0.7973    | 0.8412    |

TABLE I. The best-fit, mean, 1σ errors and 95% confidence limits for the $w$CDM model for the data combination CMB+Lens+BAO+SnIa. In this case the minimum was found for $\chi^2 = 3890$.

$\chi^2$ = 3826.

| Param | best-fit | mean±σ | 95% lower | 95% upper |
|-------|----------|---------|-----------|-----------|
| $n_s$ | 0.966    | 0.965±0.0039 | 0.9571    | 0.973    |
| $w$   | −0.9978  | −1.027±0.027 | −1.086    | −0.9743  |
| $10^{+7}c_0$ | −0.3492 | −0.2056±0.400 | −1.000    | 1.000    |
| $n$   | 0.4127   | 0.5019±0.083 | 0.200    | 0.800    |
| $\Omega_{m,0}$ | 0.3083 | 0.3063±0.074 | 0.2914    | 0.3211   |
| $10^{+9}A_s$ | 2.098 | 2.104±0.032 | 2.041    | 2.168    |
| $\sigma_{8,0}$ | 0.8078 | 0.819±0.011 | 0.7983    | 0.8402   |

TABLE II. The best-fit, mean, 1σ errors and 95% confidence limits for the $w$CDM model for the data combination CMB+Lens+BAO+SnIa+RSD. In this case the minimum was found for $\chi^2 = 3826$.

$\chi^2$ = 3809.

| Param | best-fit | mean±σ | 95% lower | 95% upper |
|-------|----------|---------|-----------|-----------|
| $n_s$ | 0.9638   | 0.9669±0.004 | 0.9582    | 0.9742   |
| $w$   | −1.023   | −1.015±0.031 | −1.071    | −0.9608  |
| $10^{+7}c_0$ | −0.08274 | 0.001678±0.004 | −0.7133    | 0.7427    |
| $n$   | 0.5417   | 0.483±0.12 | 0.200    | 0.800    |
| $\Omega_{m,0}$ | 0.3041 | 0.305±0.0077 | 0.291    | 0.3206   |
| $10^{+9}A_s$ | 2.097 | 2.099±0.03 | 2.037    | 2.162    |
| $\sigma_{8,0}$ | 0.8136 | 0.812±0.0097 | 0.7925    | 0.832    |

TABLE III. The best-fit, mean, 1σ errors and 95% confidence limits for the non-adiabatic model for the data combination CMB+Lens+BAO+SnIa. In this case the minimum was found for $\chi^2 = 3809$.

$\chi^2$ = 3827.

| Param | best-fit | mean±σ | 95% lower | 95% upper |
|-------|----------|---------|-----------|-----------|
| $n_s$ | 0.9692  | 0.9663±0.0039 | 0.9584    | 0.9742   |
| $w$   | −1.005  | −1.013±0.028 | −1.07    | −0.9554  |
| $\Omega_{m,0}$ | 0.3061 | 0.3064±0.0082 | 0.2917    | 0.3217   |
| $10^{+9}A_s$ | 2.112 | 2.101±0.032 | 2.037    | 2.164    |
| $\sigma_{8,0}$ | 0.8114 | 0.8115±0.0099 | 0.7911    | 0.8314   |

TABLE IV. The best-fit, mean, 1σ errors and 95% confidence limits for the non-adiabatic model for the data combination CMB+Lens+BAO+SnIa+RSD. In this case the minimum was found for $\chi^2 = 3827$. 

FIG. 5. The effect of the non-adiabatic pressure perturbation, given by Eq. (27), on the TT CMB spectrum (left) and its low multipoles (right). As can be seen, the effect is either to enhance or to suppress power on small multipoles, with the rest of the TT spectrum remaining unchanged. In both cases we assumed $\Omega_m = 0.3$, $w = -0.8$, $h = 0.67$. 

$\chi^2$ = 3826.
perturbations $f\sigma_8(z)$ of better than 0.5% when compared with our numerical implementation in CLASS.

Since we expect the DE perturbations to have an effect, if at all, at late times when they are growing, we anticipate the non-adiabatic component will only affect the CMB at late times and on large scales. Equivalently, this implies it affects the low multipoles via the ISW effect and using our implementation in CLASS, we confirmed this. Furthermore, availing ourselves of the modifications in CLASS, we also make MCMC analyses using the latest cosmological data. Here, we used CMB, BAO, SnIa data, but also a new RSD likelihood for MontePython, which we present in this work for the first time. Doing this analysis we found that the parameter $c_0$ is consistent with zero at 1$\sigma$, while $n \sim 0.5$ is in agreement with the expectation from the designer $f(R)$ model.

In conclusion, we have shown that a non-adiabatic DE pressure perturbation could have measurable effects on the CMB and other key cosmological observables such as the growth-rate of matter density perturbations and the matter power spectrum. Using the latest cosmological data, including RSDs, and assuming a power-law for the non-adiabatic DE component given by Eq. (27), we constrained its amplitude and found it is consistent with zero and GR at 1$\sigma$. 

**FIG. 6.** The confidence contours for the wCDM model using the data combinations of Planck+Lensing+BAO+SnIa (green contours) and Planck+Lensing+BAO+SnIa+RSD (blue contours).
FIG. 7. The confidence contours for wCDM with non-adiabatic DE perturbations using the data combinations of Planck+Lensing+BAO+SnIa (green contours) and Planck+Lensing+BAO+SnIa+RSD (blue contours). Some of the contours appear to be truncated due to a peculiarity of the MontePython plotting routines and not due to our choice of the prior.

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NUMERICAL CODES

The RSD Montepython likelihood, introduced in this paper for the first time, for the growth-rate $f\sigma_8$ data set which is based on the compilation shown in Table V, can be found at https://github.com/snesseris/RSD-growth.
TABLE V. Compilation of the $f\sigma_8(z)$ measurements used in this analysis along with the reference matter density parameter $\Omega_{m0}$ (needed for the growth correction) and related references.

| $z$ | $f\sigma_8(z)$ | $\sigma_{\delta_{8}}(z)$ | $\Omega_{m0}^{\text{ref}}$ | Ref. |
|-----|----------------|--------------------------|--------------------------|------|
| 0.02 | 0.428 0.0465   | 0.3                       | [72]                     |
| 0.02 | 0.398 0.065    | 0.3                       | [73], [74]               |
| 0.02 | 0.314 0.048    | 0.266                     | [75], [74]               |
| 0.10 | 0.370 0.130    | 0.3                       | [76]                     |
| 0.15 | 0.490 0.145    | 0.31                      | [77]                     |
| 0.17 | 0.510 0.060    | 0.3                       | [78]                     |
| 0.18 | 0.360 0.090    | 0.27                      | [79]                     |
| 0.38 | 0.440 0.060    | 0.27                      | [79]                     |
| 0.25 | 0.3512 0.0783  | 0.25                      | [80]                     |
| 0.37 | 0.4602 0.0378  | 0.25                      | [81]                     |
| 0.32 | 0.384 0.095    | 0.274                     | [81]                     |
| 0.59 | 0.488 0.060    | 0.307115                  | [82]                     |
| 0.44 | 0.413 0.080    | 0.27                      | [83]                     |
| 0.60 | 0.390 0.063    | 0.27                      | [83]                     |
| 0.73 | 0.437 0.072    | 0.27                      | [83]                     |
| 0.60 | 0.550 0.120    | 0.3                       | [84]                     |
| 0.86 | 0.400 0.110    | 0.3                       | [84]                     |
| 1.40 | 0.482 0.116    | 0.27                      | [85]                     |
| 0.978 | 0.379 0.176 | 0.31                      | [86]                     |
| 1.23 | 0.385 0.099    | 0.31                      | [86]                     |
| 1.526 | 0.342 0.070   | 0.31                      | [86]                     |
| 1.944 | 0.364 0.106   | 0.31                      | [86]                     |

Appendix A: The RSD likelihood

Here we describe the RSD likelihood we used for the MCMC analysis done in the previous sections. In particular, we implement in python a likelihood for the “Gold 2018” growth-rate $f\sigma_8$ compilation with $N = 22$ data points given in Ref. [69] and shown in Table V with the corresponding references of each point.

The growth data used here are obtained from RSD measurements, which probe the LSS. In practice they correspond to the first and second moments of the line-of-sight projected density field.

We can then define the data vector $\mathbf{V}$ as:

$$\mathbf{V} = f\sigma_8^{\text{obs}, \text{th}} - f\sigma_8^{\text{th}, \text{th}}.$$  

The redshift correction for the Alcock-Paczynski effect, see Refs. [69], [18], and [89], while for earlier analyses see Refs. [15], [17], [19].

We give the values of the $\Omega_{m0}$ parameter for the fiducial flat $\Lambda$CDM model used in the fourth column of Table V.

The correlation data points are the three WiggleZ points from Ref. [83] and the four points from SDSS [86].

The covariance matrix of the WiggleZ data is given by

$$C_{\text{WiggleZ}} = 10^{-3} \begin{pmatrix} 6.400 & 2.570 & 0.000 \\ 2.570 & 3.969 & 2.540 \\ 0.000 & 2.540 & 5.184 \end{pmatrix},$$  

while the covariance matrix of the SDSS points is given by

$$C_{\text{SDSS-IV}} = 10^{-2} \begin{pmatrix} 3.098 & 0.892 & 0.329 & -0.021 \\ 0.892 & 0.980 & 0.436 & 0.076 \\ 0.329 & 0.436 & 0.490 & 0.350 \\ -0.021 & 0.076 & 0.350 & 1.124 \end{pmatrix}.$$  

The redshift correction for the Alcock-Paczynski effect as described in Ref. [18], is given in terms of a correction factor of

$$\text{fac}(z) = \frac{H(z')}{H_{\text{ref},i}(z')} \frac{dA_{\text{ref},i}(z')}{dA(z')},$$  

where the label “ref, $i$" stands for the fiducial cosmology used on each data point at the redshift $z'$. As a result, the now corrected growth-rate is $f\sigma_8^{\text{th}, \text{th}}$.

We can then define the data vector $\mathbf{V}$ as:

$$\mathbf{V} = f\sigma_8^{\text{obs}, \text{th}} - f\sigma_8^{\text{th}, \text{th}}.$$
and the chi-squared of our likelihood via
\[ \chi^2 = x^T C^{-1} x. \]  
\quad (A6)

Finally, in CLASS we can obtain the scale-dependent growth \( \delta(k, z) \) at each redshift via the matter power spectrum as \( \delta(k, z) = \sqrt{P(k, z)/P(k, 0)} \), where the matter power spectrum \( P(k, z) \) is obtained from the code itself via the function `cosmo_pk(k, z)`. Then, \( f \sigma_8(k, z) \) can be obtained with simple cubic interpolations and direct differentiation from Eq. (56).

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