Analysis the Characteristic of Energy and Damage of Coal-Rock Composite Structure Under Cycle Loading

Tan Li · Guangbo Chen · Zhongcheng Qin · Qinghai Li

Received: 19 March 2021 / Accepted: 25 June 2021 / Published online: 1 July 2021
© The Author(s), under exclusive licence to Springer Nature Switzerland AG 2021

Abstract The stability of coal-rock composite structures is of great significance to coal mine safety production. The uniaxial cyclic loading tests of the coal-rock composite structures with different coal-rock height ratios were carried out to study the stability and deformation failure characteristics of the coal-rock composite structure. The coal-rock height ratio play a vital role in the energy dissipation of coal-rock composite structures. The higher the coal-rock height ratio, the greater the average elastic energy and dissipated energy produced per cycle of coal-rock composite structures, the smaller the total elastic energy and dissipated energy produced in the process of cyclic loading. A more sensitive joint calculation method for calculating damage variables was proposed based on the difference of damage variables calculated by dissipated energy method and acoustic emission method. The joint damage variable calculation method can more accurately and sensitively reflect the damage of coal-rock composite structure under cyclic loading. The macroscopic crack first appears in the coal specimen in the coal-rock composite structure. The degree of broken coal specimens in the composite structure is inversely proportional to the coal-rock height ratio. The strength and deformation characteristics of the coal-rock composite structure are mainly affected by the coal sample in the composite structure.

Keywords Coal-rock height ratio · Coal-rock composite structures · Damage variable · Energy characteristic · Cyclic loading

1 Introduction

With the increase of coal mining depth, engineering disasters such as underground roadway deformation, roof fall, and rock burst frequently occur (Maleki and Lawso 2017; Islavat et al., 2020). To ensure the safety and efficient mining of coal resources, a large number of coal pillars need to be reserved around the stope, such as isolation coal pillars, section coal pillars, waterproof coal pillars, fault protection coal pillars (Yang et al., 2019). These reserved coal pillars play the role of natural support, boundary, and isolation (Frith and Reed 2018; Wang et al. 2019). The stability of the composite structure of the coal pillar and roof determines the safety of the entire stope and the overlying rock. Once the composite structure of coal pillar and roof instability and damage, it will bring
many catastrophic consequences, such as damage to the surface buildings or the surface suddenly "collapses" causing heavy casualties and property losses (Mark 2018; Du et al., 2019a, b).

The disasters in coal mining are affected by the coal and rock themselves and the result of the combined action of the coal-rock composite structure. In addition to the ground stress, the coal-rock composite structure is also affected by cyclic loading of roadway driving, chamber blasting, and working face mining (Yao et al., 2020). Under this similar cyclic loading, the coal-rock composite structure will inevitably cause damage, reduce the carrying capacity, and cause the instability and destruction of roadways and coal pillars (Mathey 2018; Waclawik et al., 2018; Zhang et al., 2018a, b).

At present, a large number of scholars have done a lot of research on the damage characteristics of coal specimens under cyclic loading and unloading. Zhang et al. (2019a, b) establishes the axial and radial anisotropic models of coal samples in Triaxial Cyclic Loading and unloading and uses the digital evaluation model (DEM) to study the damage mechanism of coal samples. Shkuratnik et al. (2006) studies the acoustic emission characteristics and strain memory effect of coal samples under cyclic loading and reveals the formation and performance law of coal’s acoustic emission memory effect. Huang et al. (2021) studies the influence of strain on damage deformation of coal samples and divides the strain damage curve of coal samples into three typical damage stages: early deceleration damage, middle constant velocity damage, and late sudden acceleration damage. Guo et al. (2018) carried out systematic experimental research on rock-coal-rock specimens with different coal thicknesses and deeply analyzed the mechanical failure behavior in a conventional compression test. The overall elastic modulus and peak stress of rock-coal-rock specimens are between rock and coal. The deformation and strength behavior of rock-coal-rock specimens are related to the thickness of the coal seam and related to the confining pressure. Du et al. (2019a, b) obtained the stress–strain data and acoustic emission signals of coal samples susceptible to impact. The degradation rate of elastic modulus is defined to describe the degradation of mechanical properties during loading. The input energy density increases nonlinearly with the load increase, and the dissipated energy density has a linear relationship with the axial load. Bai et al. (2019) study the progressive failure characteristics and mechanism of different gangue coal rock specimens. According to the distribution of acoustic emission events and the development of local strain, the local failure characteristics of coal or rock elements in coal rock composite structure are revealed. Duan et al. (2020) used the cycle response ratio ($LURR$) to study the coal damage. At the beginning of the cycle, $LURR$ is larger. With the loading and unloading of the cycle, $LURR$ decreases sharply at first, decreases slowly to 1, and then increases slowly from 1. After stabilization, the increase of $LURR$ value can be used as the precursor information to judge the failure and instability of coal and rock. Zhang et al. (2018a, b) measured the acoustic emission response characteristics of coal samples under cyclic loading, and there is a good positive correlation between acoustic emission parameters and stress. The acoustic emission signals of coal samples under cyclic loading show a noticeable Kaiser effect. Sabapathy et al. (2019) use the impact energy coefficient to evaluate the mine damage instability, and the impact energy coefficient can be used to predict the rock burst disaster.

The coal-rock composite system is accompanied by energy accumulation, transformation, and release in the process of damage and failure (Yu et al., 2020; Liu et al., 2019; Pan et al., 2020; Gao et al., 2020). Therefore, studying the characteristic of energy and damage of the coal-rock composite system under cyclic loading is of great significance to preventing coal mine disasters.

2 The Characteristic of Energy and Damage

2.1 Energy Characteristic

Rock deformation and failure are accompanied by the input, accumulation, dissipation, and release of energy. Analyzing the energy transformation in rock deformation and failure can better find the internal essence of rock instability and failure.

A large number of studies have shown that the instability and failure of the coal-rock composite structure are caused by the elastic energy accumulated by rock and coal and the work done by external forces, which can be expressed as follows (Lu et al., 2013; Li et al., 2017; Ma et al., 2020):
\[ U_u + U_r^e + U_c^e = E_c + W_c \]  

(1)

where \( U_u \) is the energy input from the outside, \( U_r^e \) is the elastic energy accumulated by rock, \( U_c^e \) is the elastic energy accumulated by coal, \( E_c \) is the total kinetic energy, \( W_c \) is the surface energy.

To reveal the energy characteristics of a coal-rock composite structure in the damage process and describe it mathematically, the loading and unloading curve of rock in a coal-rock composite structure is used to explain, there is a relationship curve as shown in Fig. 1 during the loading and unloading of the rock.

The area under the unloading curve is elastic energy \( U_r^e \), and the area between the unloading and loading curves is the dissipated energy \( U_r^p \). When the rock is unloaded from strain \( \varepsilon_r \) to strain \( \varepsilon_r' \), the elastic energy accumulated before unloading can be expressed in integral form as,

\[ U_r^e = L_r S_r \int_{\varepsilon_r}^{\varepsilon_r'} \sigma d \varepsilon \]  

(2)

Similarly, the elastic energy accumulated by coal \( U_c^e \) can be expressed as,

\[ U_c^e = L_c S_c \int_{\varepsilon_c}^{\varepsilon_c'} \sigma d \varepsilon \]  

(3)

where the \( L_c \) and \( L_r \) are the heights of coal and rock in the coal-rock composite structure; \( S_c \) and \( S_r \) are the cross-sectional areas of the coal and rock, respectively; \( \varepsilon_c \) and \( \varepsilon_c' \) are the strain of rock and coal body when the coal-rock composite structure reaches its peak strength, respectively; \( \varepsilon_r \) and \( \varepsilon_r' \) are the residual strains of rock and coal respectively after rebounding.

For the unloading stage, the input energy \( U_u \) can be expressed in the integral form of the whole stress–strain curve,

\[ U_u = LS \int_{\varepsilon_1}^{\varepsilon_1} \sigma d \varepsilon \]  

(4)

where the \( L \) and \( S \) are the overall height and cross-sectional area of the composite structure respectively, the \( \varepsilon_1' \) is the axial strain of the whole composite structure when the peak stress is reached, it can be calculated from the overall strain recorded by the pressure machine or expressed as \( \varepsilon_1' = \frac{m-n}{m} \varepsilon_r + \frac{n}{m} \varepsilon_c \), where \( m \) is the overall height of the coal-rock composite structure, \( n \) is the height of the coal body, \( \varepsilon_r \) is the strain of the whole composite structure after the failure, which is calculated by the displacement of the pressure machine.

The total kinetic energy \( E_c \) of coal after failure can be expressed as:

\[ E_c = E_c^1 + E_c^2 = \frac{1}{2} \sum \Delta m_i v_i^2 \]  

(5)

The surface energy \( W_c \) consumed by coal crushing can be expressed as:

\[ W_c = W_c^1 + W_c^2 = L_s S_c C_j (r_2^{D-3} - r_1^{D-3}) \]  

(6)

In the test, the time-strain curve \( \varepsilon = g(t) \) can be obtained by a strain gauge, and there is a corresponding relationship \( \sigma = \phi(t) \) between stress and time when the load is shared by rock and coal. Therefore, there is the functional relationship \( \sigma = \phi[g^{-1}(t)] \) between strain and stress in rock and coal. There is a relationship between the height and the cross-sectional area of rock and coal:

\[ \begin{cases} 
L = L_c + L_r = \frac{n}{m} L + \frac{m-n}{m} L \\
S = S_c + S_r = m L SC_j
\end{cases} \]  

(7)

Therefore, formula (1) can be expressed as:

\[ LS \left\{ \int_{\varepsilon_1}^{\varepsilon_1} \sigma d \varepsilon + \frac{m-n}{m} \int_{\varepsilon_c}^{\varepsilon_c} \phi[g_3^{-1}(\varepsilon)] d \varepsilon + \frac{n}{m} \int_{\varepsilon_r}^{\varepsilon_1} \phi[g_4^{-1}(\varepsilon)] d \varepsilon \right\} \]

\[ = \frac{1}{2} \sum \Delta m_i v_i^2 + \frac{n}{m} LSC_j (r_2^{D-3} - r_1^{D-3}) \]  

(8)

Fig. 1  Loading and unloading curve
Formula (8) reflects the relationship between height of rock and coal composite coal-rock composite structure and energy dissipation law: with the increase of coal height \( n \), coal cumulative elastic \( U^e_c \) increases, rock cumulative elastic \( U^e_r \) decreases, and the increment of coal elastic energy is smaller than the decrease of rock elastic energy, the sum of the first three terms of the equation decreases, the surface energy term of the equation increases, the kinetic energy term decreases, the damage degree decreases, and the risk of rock burst decreases. On the contrary, the damage degree increases, the broken coal body has more kinetic energy, and the risk of rock burst increases.

Besides, the properties of rock and coal also have a significant effect on energy dissipation. When the rock is used as the energy accumulator, its strength and elastic modulus affect the strain energy density. When the coal body is destroyed, the rock rapidly releases energy to aggravate the coal body failure and the risk of rock burst increases. As the main failure part of coal rock composite structure, the strength and elastic modulus of coal affect the stress–strain curve of its part.

To sum up, the damage and instability of coal-rock composite structures are mainly affected by the proportion and the mechanical properties of rock and coal. It is necessary to analyze the characteristics of the energy and damage of the composite structure to ensure the stability of the composite structure of coal pillar and roof.

2.2 Damage Characteristics

In continuous damage mechanics, all defects are considered continuous, and their effects on materials are expressed by one or several continuous internal field variables, which are called damage variables.

The common calculation and expression methods of damage variables are the effective cross-sectional area method, elastic modulus method, ultrasonic velocity method, dissipated energy method, strain method, etc. Because the process of rock damage and destruction is not only accompanied by the outward release of energy, it also generates acoustic emission signals, and this information contains the damage and destruction information inside the rock material. Therefore, this paper chooses two calculation methods: the dissipated energy method and the acoustic emission method to compare and analyze the damage variables of the coal-rock composite structure.

1. Dissipated energy method.

The damage and destruction of rock are accompanied by the transformation of energy. Energy transformation is an irreversible process of non-uniform dissipation. The evolution process of dissipated energy can reflect the irreversible deformation, damage, and failure characteristics of the rock. The damage variable \( D_i \) is defined as the ratio of the dissipated energy of each cycle to the cumulative total dissipated energy, and the expression is,

\[
D_i = \frac{U^d(i)}{U}
\]

where the \( D_i \) is the damage variable of the \( i \)-th cycle; \( U^d(i) \) is the dissipated energy produced by the \( i \)-th cycle; \( U \) is the cumulative total dissipated energy.

2. Acoustic emission method.

The deformation and failure of the rock are the results of the internal damage evolution of the rock. The internal damage of the rock will release the corresponding acoustic emission signals. These acoustic emission signals can reflect the instability and failure process of the rock. Therefore, it is feasible to use the acoustic emission method to characterize the rock damage.

Assuming that the strength of the micro-unit of the rock obeys the Weibull distribution, the damaged area of the cross-section of the rock is (Chen et al. 2018),

\[
S = S_m \int_0^\varepsilon \varphi(x) dx
\]

where the \( S_m \) is the cross-sectional area of the material without damage; \( \varphi(x) \) is the statistical distribution of the micro-element strength.

When the sample strain increment is \( \Delta \varepsilon \), the damaged area of the sample \( \Delta S \) is,

\[
\Delta S = S_m \int_0^\varepsilon \varphi(x) dx
\]

The rock damage variable can be expressed as,

\[
D_i = \frac{\Delta S}{S_m} = \int_0^{\Delta \varepsilon} \varphi(x) dx
\]
Assuming that the number of acoustic emission ringing produced by the damaged micro-unit area is \( n \), the number of acoustic emission ringing produced by the damaged area \( S \) is \( N \),

\[
n = \frac{N_m}{S_m} \tag{13}
\]

\[
N = n\Delta S = \frac{N_m\Delta S}{S_m} \tag{14}
\]

The ratio between the number of acoustic emission ringing \( N \) and the number of acoustic emission ringing \( N_m \) when completely damaged is as follows,

\[
\frac{N}{N_m} = \frac{\Delta S}{S_m} = \int_0^{\Delta e} \varphi(x)dx \tag{15}
\]

The relationship between the damage variable \( D_i \) and the number of acoustic emission ringing in each cycle can be expressed as,

\[
D_i = \frac{N}{N_m} \tag{16}
\]

3 Test System

3.1 Test System

The experiment system is shown in Fig. 2, including the loading system, acoustic emission system, and a digital video camera. The loading system is a RAW-2000kN microcomputer-controlled electro-hydraulic servo test system. The acoustic emission system is the SH-II acoustic emission system with 16 channels, the threshold value of the preamplifier is 40 dB and the resonance frequency is 150 ~ 750 kHz. The digital video camera is a Canon portable digital video camera, which records the development, expansion, and failure of the coal-rock composite structure. During the test, the loading system, the acoustic emission system, and the digital video camera are synchronized to ensure the same time parameters for data processing and test analysis.

3.2 Specimen Preparation

The coal and coarse sandstone used in the test came from the No. 3 coal seam and roof of Xin’an Coal Mine. Before the test, the coal and coarse sandstone were made into cylinders with a diameter of 50 mm.
and height of 100, 75, 67, 50, 33 and 25 mm respectively, and the upper and lower end faces of the sample were ground with a grinding machine to meet the test requirements. The coal and coarse sandstone specimens are bonded to form coal-rock composite structures with coal-rock height ratios of 1:3, 1:2, 1:1, 2:1, and 3:1 (\( \varphi 50 \times 100 \) mm). The coal-rock composite structure is marked as A, B, C, D, E according to the coal-rock height ratio from 1:3 to 3:1. The part of the specimens is shown in Fig. 3.

3.3 Experiment Scheme

The experiment adopts the stress loading method. Firstly, it is loaded at a speed of 1.5kN/s. When the load reaches 12kN (35–45% of the uniaxial compressive strength of coal sample), unload to 2kN at the same speed, this is the first cycle. The second cycle performs loading and unloading at the same speed, but the loading peak is 2kN higher than that of the first cycle. In this way, continue loading and unloading until the specimen is destroyed. The cyclic loading and unloading curve is shown in Fig. 4.

4 Damage Variable Calculation Method

The damage and destruction process of the coal-rock composite structure is accompanied by the change of energy characteristics and acoustic emission signals. Therefore, in the cyclic loading and unloading process, the acoustic emission method and the dissipated energy method are used to calculate the damage variables of the coal-rock composite structure. The two methods are used to calculate the damage variables of the same experiment. Observe the difference between the two damage variable calculation methods to obtain a more sensitive damage variable calculation method.

4.1 Dissipated Energy Method

Figure 5 shows the variation curve of elastic energy and cycles of coal-rock composite structure with different coal-rock height ratios. It can be seen that the elastic energy produced by the coal-rock composite structure increases gradually with the increase of the cycles. When the cycles are the same, the composite structure with a larger coal-rock height ratio produces more elastic energy. Because the higher the coal-rock height ratio is, the greater the elastic strain and the more elastic energy are stored in the coal-rock composite structure under the same load.

The average elastic energy and total elastic energy of composite structure with different coal-rock height ratios during cyclic loading are calculated. The results are shown in Fig. 6. It can be seen that the higher the coal-rock height ratio is, the greater the average elastic energy per cycle of the coal-rock composite structure
is, but the lower the total elastic energy is. The smaller the coal-rock height ratio is, the smaller the average elastic energy per cycle is, but the higher the total elastic energy is. Because the composite structure with a small coal-rock height ratio has higher strength, and the average elastic energy produced in each cycle is less, but the cycles are more, which makes the accumulated total elastic energy is more; the composite structure with a large coal-rock height ratio has lower strength, and the average elastic energy produced in each cycle is more, but the cycles are less, which makes the accumulated total elastic energy is less.

The coal-rock height ratio is fitted with the average elastic energy per cycle and the total elastic energy respectively, the functional relationship between coal-rock height ratio and average elastic energy per cycle is as follows:

\[
y = -0.0014x^2 + 0.0071x + 0.026 \quad (R^2 = 0.78)
\]  

(17)

The functional relationship between the coal-rock height ratio and total elastic energy is as follows:

\[
y = 0.0079x^2 - 0.12x + 0.55 \quad (R^2 = 0.99)
\]  

(18)

Figure 7 shows the relationship between the dissipated energy of the coal-rock composite structure with different coal-rock height ratios and the cycles. The dissipated energy of the coal-rock composite structure increases with the increase of cycles. When the cycles are the same, the higher the coal-rock height ratio is, the more dissipated energy is produced.

Because there are lots of discontinuous structures such as pores, cracks, and joints in the coal body, the distribution is extremely uneven, the cracks in the coal body develop and expand, resulting in more dissipated
energy. On the other hand, there are relatively few pores, fissures, and joints in the rock, and the dissipated energy produced by cyclic loading and unloading is less. Therefore, with the increase of coal-rock height ratio, the dissipative energy of coal-rock composite structure increases.

The average dissipated energy per cycle and total dissipated energy of coal-rock composite structure are counted, the results are shown in Fig. 8. The larger the coal-rock height ratio is, the larger the average dissipated energy per cycle is, and the lower the total dissipated energy is; the smaller the coal-rock height ratio is, the smaller the average dissipated energy per cycle is, and the higher the total dissipated energy is. Because the smaller the coal-rock height ratio, the greater the compressive strength of the composite structure, the smaller the strain produced under the same load so that the average dissipated energy produced by each cycle is less; the smaller the coal-rock height ratio is, the larger the load required for the failure of the coal-rock composite structure is, the more the cycles is, and the more total dissipated energy is.

The coal-rock height ratio is fitted with the average dissipated energy per cycle and the total dissipated energy respectively, the functional relationship between coal-rock height ratio and average dissipated energy per cycle is as follows:

\[
y = -0.00152x^2 + 0.0072x + 0.00502 \quad (R^2 = 0.89)
\]

The functional relationship between the coal-rock height ratio and total dissipated energy is as follows:

\[
y = -0.00822x^2 + 0.00688x + 0.14609 \quad (R^2 = 0.97)
\]
The damage and failure of rock are accompanied by energy transformation, which is an irreversible process of non-uniform dissipation. The evolution process of dissipated energy can reflect the irreversible deformation, damage, and failure characteristics of the rock. Therefore, the evolution process of rock damage can be revealed through the change of dissipated energy. The damage variable $D_i$ is defined as the ratio of the dissipated energy per cycle to the cumulative total dissipation energy, the expression is

$$D_i = \frac{U^d(i)}{U}$$

(21)

where the $D_i$ is the damage variable of the $i$-th cycle; $U^d(i)$ is the dissipated energy of the $i$-th cycle; the $U$ is the cumulative total dissipated energy.

According to the dissipated energy method, the damage variables of the coal-rock composite structure under the cyclic loading are calculated, the calculation results are shown in Fig. 9. It can be seen that the damage variable presents a “ladder” growth with the increase of the cycles. With the increase of loading and cycles, the micro-cracks formed in the early stage begin to connect and form macro cracks. The micro-cracks continue to develop under the effect of cyclic loading and gradually form more macro-cracks. When the macro-cracks connect with each other, the damage variable increases rapidly. When the macro-cracks penetrate and expand each other under the cyclic loading, which finally leads to the failure of the composite structure, and the cumulative damage variable reaches 1 (Yang et al. 2020; Alneasan et al. 2019; Luo et al. 2020).

Figure 10 shows the relationship between cumulative damage variables and cycles of composite structures with different coal-rock height ratios. When the cycles are the same, the larger the coal-rock height ratio is, the greater the cumulative damage variable of the composite structure is. When the coal-rock height ratio is 1:3, 1:2, 1:1, 2:1, 3:1, the average cumulative damage variable growth per cycle is 0.0522, 0.0615, 0.0753, 0.0968, and 0.1188, respectively.

4.2 Acoustic Emission Method

The deformation and failure of the rock are accompanied by the outward release of energy and generate acoustic emission signals (Zarate et al., 2012; Aldahdooh et al., 2013; Gu et al., 2018). These acoustic emission signals contain the damage and failure information of rock materials (Moradian et al., 2016; Patricia and Celestino 2018; Zhang et al., 2019a, b). Therefore, it is feasible to use the acoustic emission method to characterize rock damage.

Figure 11 is the change curve of acoustic emission ringing and energy produced by the composite structure with different coal-rock height ratios under the action of cyclic loading and unloading. It can be seen that the acoustic emission ringing number and energy show different changes with the increase of the cycle time. In the early stage of cyclic loading and unloading, the acoustic emission ringing number and energy are large. In the middle stage, the acoustic emission ringing number and energy develop fluctuating; in the late stage, the acoustic emission ringing number and energy increase again. During the entire cycle of loading and unloading, the acoustic emission ringing number and energy generally show the “U” shaped change trend.

Because in the early stage, the composite structure is subjected to the external load, the holes and cracks inside the specimen are compacted, and a large
number of acoustic emission signals are generated in this stage. In the middle stage, cracks are generated in the fragile parts of the coal-rock composite structure, and the cracks begin to expand at the tips of the original cracks. During this process, a small acoustic emission signal was generated. Through the accumulation of crack initiation and propagation in the middle stage of cyclic loading and unloading, the cracks were penetrated and expand, formed larger cracks, and a large number of acoustic emission signals was generated at the later stage of cyclic loading and unloading (Kim et al. 2019; Khazaei et al. 2015; Girard et al. 2012).

Figure 12 is the variation curves of the maximum acoustic emission ringing number and energy of coal-rock composite structures with different coal-rock height ratios. The maximum acoustic emission ringing number and energy both decrease with the increase of coal-rock height ratio, the acoustic emission ringing number is reduced by 70.71%, and the energy is reduced by 41.35%. Among them, when the coal-rock height ratio is increased from 1:3 to 1:1, the acoustic emission ringing number is reduced by 56.33%, and
the energy is reduced by 15.07%; when the coal-rock height ratio is increased from 1:1 to 3:1, the acoustic emission ringing number was reduced by 32.93%, and the energy was reduced by 30.94%. The reduction rate of acoustic emission ringing number when the coal-rock height ratio is increased from 1:3 to 1:1 is greater than the reduction rate when the coal-rock height ratio increased from 1:1 to 3:1. The energy reduction rate when the coal-rock height ratio is increased from 1:3 to 1:1 is smaller than that when the coal-rock height ratio is increased from 1:1 to 3:1.

It is assumed that the acoustic emission ringing number produced in the loading stages is \(N_{i+}\), the acoustic emission ringing number produced in the unloading stages is \(N_{i-}\), and the acoustic emission ringing number produced in each cycle is \(N_i\), the acoustic emission ringing number when the composite structure is destroyed is \(N\). The “\(i_+\)” means the \(i\) th loading stage, “\(i_-\)” means the \(i\) th unloading stage.

The damage variables in the loading stages and the unloading stages are \(D_{i+}\) and \(D_{i-}\) respectively; the damage variable in each cycle is \(D_i\), and the cumulative damage variable is \(D\), then the damage variables \(D_{i+}\) and \(D_{i-}\) produced in each loading and unloading stage can be expressed as,

\[
D_{i+} = \frac{N_{i+}}{N} \tag{22}
\]

\[
D_{i-} = \frac{N_{i-}}{N} \tag{23}
\]

The damage variable \(D_i\) generated by each cycle loading and unloading stage can be expressed as,

\[
D_i = D_{i+} + D_{i-} \tag{24}
\]

According to formula (1)-(3), the damage variables of composite structures with different coal-rock height ratios during cyclic loading and unloading are calculated, as shown in Fig. 13. The damage variable generated in the loading stage is greater than that in the unloading stage in the whole cycle loading and unloading stages. It indicates that the fracture degree of the coal-rock composite structure is obvious in the loading stage, and the energy is released by the crack opening in the unloading stage.

The damage variable generated per cycle calculated by the acoustic emission method is shown in Fig. 14. It can be seen that the damage is larger in the initial and later stages of the cyclic loading stage, and smaller in the middle of the cyclic loading stage. The relationship between the damage variable and the cycles was an approximate “U” shape.

In the initial stage, the damage variable decreases with the increase of cycles. The micro-cracks and micro-holes in the coal-rock composite structure are gradually compacted, resulting in a large damage variable. In the middle stage, the damage variable fluctuates with the increase of cycles, and there is no obvious decrease or increase. The internal micro-cracks of the coal-rock composite structure continue to develop, the new micro-cracks are gradually formed in the weak parts of the composite structure, the damage variable shows a fluctuating development. In the later stage, the internal fissures expand and penetrate, the damage is aggravated, the structure gradually loses stability and damage, the damage variable increases (Ebrahimian et al. 2019; Jiang et al. 2021; Moradian et al. 2016).

The cumulative damage variables of coal-rock composite structures are shown in Fig. 15. When the cycles are the same, the greater the coal-rock height ratio, the greater the cumulative damage variable. Because the greater the coal-rock height ratio, the lower the strength of the composite structure, the greater the deformation under the same loading, the greater the damage.

The cumulative damage variable growth rate and the average cumulative damage variable increment per cycle are shown in Fig. 10. When the coal-rock height ratio is 1:3, 1:2, 1:1, 2:1, and 3:1, the average cumulative damage variable growth per cycle is 0.046, 0.054, 0.065, 0.078 and 0.106, respectively.

Based on the above analysis, the damage variable calculated by the dissipated energy method shows the “ladder” growth, and the damage variable calculated by the acoustic emission method shows the “U” shape. The main reason for the difference between the two calculation methods is the source of the damage signal is different. The dissipated energy method is to characterize the damage variable through the dissipated energy, and the dissipated energy is obtained by calculating the area enclosed by the loading curve and the unloading curve of the coal-rock composite structure during the cyclic loading and unloading. The dissipated energy generated by the composite structure increases in the ladder under cyclic loading, and the damage variable calculated from the dissipated energy also increases in the ladder. The acoustic emission method is to calculate the damage variable...
Fig. 13  The variation curve of damage variable during cyclic loading and unloading

Fig. 14  The variation curve of damage variables calculated by the acoustic emission method
by counting the acoustic emission ringing number during the cycle loading and unloading. In the early stage, the acoustic emission ringing number is larger, and the calculated damage variable is also larger; in the middle stage, the acoustic emission ringing number is less, and the calculated damage variable is also smaller; in the later stage, the acoustic emission ringing number is larger, and the calculated damage variable is also larger. So the damage variables calculated by the acoustic emission method form the “U” shape.

5 Joint Calculation Method of the Damage Variable

The reasonable definition of damage variable should meet the following basic requirements: (1) the physical meaning is clear; (2) the measurement is convenient and convenient for engineering application; (3) the law of damage evolution is consistent with the actual deterioration process of the material; (4) the initial damage can be considered (Voyiadjis et al., 2009).

At present, the calculation of damage variables is mainly based on a single parameter, such as elastic modulus, strain, dissipated energy, acoustic emission signal, etc., which has certain limitations, and there is a certain gap in the sensitivity of rock failure damage under cyclic loading. Because the rock failure releases not only energy but also acoustic emission signals, this information contains the internal damage information of composite structure. Therefore, the joint calculation method of damage variable is based on the dissipated energy method and the acoustic emission method to analyze the damage variables comprehensively.

Comparing the calculation results of the acoustic emission method and the dissipated energy method, the result is shown in Fig. 16. In the early stage of cyclic loading, the cumulative damage variable calculated by the acoustic emission method is larger than that calculated by the dissipated energy method; while in the later stage of cyclic loading, the cumulative damage variable calculated by the dissipated energy method is larger than that calculated by the acoustic emission method. There is a certain difference in the sensitivity of the two calculation methods to the damage variables, it shows that the different calculation methods have different sensitivity to damage variables.

The reason for the difference is that the frictional energy in the dissipated energy is mainly consumed by the friction between the internal structural surfaces, and is not consumed by the substantial damage of the internal effective load-bearing structure, which will directly affect the rationality and accuracy of the damage development analysis results. The proportion of friction energy consumption before the stress peak of the rock is relatively small, and the influence on the damage development analysis is not obvious. However, the proportion of friction energy consumption after the stress peak increases significantly, and the influence on the damage development analysis also increases. This explains the incorrect analysis results of the damage variable calculation method based on the dissipated energy at the later stage of the cycle.
loading and unloading (Wang et al. 2018). At the same time, the damage variable calculated by the dissipated energy method during the last cycle of cyclic loading and unloading, the loading curve and the unloading curve did not form a closed hysteresis loop, and the dissipated energy in the final process of cyclic loading and unloading could not be calculated.

For the acoustic emission method to calculate damage variables, many scholars believe that: in the initial stage of compression, the internal cracks and pores of the rock material will be closed and the damage will be reduced, and a large number of acoustic emission signals will also be sent out in this process. If the acoustic emission information at this stage is considered, the damage will increase, which is not consistent with the actual situation (Li et al. 2019; Wang and Liu 2011).

According to the above analysis, the damage variable calculated by the dissipated energy method is less sensitive in the later period of cyclic loading and unloading, and the damage variable calculated by the acoustic emission method is less sensitive in the early period of cyclic loading and unloading. Based on the characteristics of the two damage variable calculation methods, a new damage variable calculation method
considering the sensitivity of the cyclic loading and unloading in the early and late stages is proposed.

Therefore, the sensitivity of damage variables is analyzed by the sensitivity parameters in system analysis. When the analysis parameter \( a_k \) influences the characteristic \( P \), let the \( a_k \) vary within its possible range, and the system characteristic \( P \) is shown as:

\[
P = f(a_1, \ldots, a_{k-1}, a_k, a_{k+1}, \ldots, a_n)
\]

(25)

If a small change in \( a_k \) will cause a large change in \( P \), then \( P \) will be more sensitive to \( a_k \); if a small change in \( a_k \) will cause a small change in \( P \), then \( P \) will be less sensitive to \( a_k \).

It is assumed that when the cycles are less than the \( g(x) \), the cumulative damage variable calculated by the acoustic emission method is more sensitive; when the cycles are greater than the \( g(x) \) value, the cumulative damage variable calculated by dissipated energy method is more sensitive. Then the joint calculation method of the damage variable is:

\[
D = \begin{cases} 
\frac{N/N_m}{U^d(i)/U} & n \leq g(x) \\
U^d(i)/U & n > g(x)
\end{cases}
\]

(26)

where the \( N/N_m \) is the acoustic emission method and the \( U^d(i)/U \) is the dissipated energy method.

The \( g(x) \) of composite structures with different coal-rock height ratios is shown in Fig. 17. The \( g(x) \) decreases gradually with the increase of the coal-rock height ratio. The relationship between the \( g(x) \) and the coal-rock height ratio is fitted, and the function of the fitting curve is obtained as follows:

\[
g(x) = 1.08x^2 - 7.40x + 17.97
\]

(27)

According to the joint calculation method of damage variables, the cumulative damage variables of composite structures with different coal-rock height ratios under cyclic loading and unloading are calculated, the calculated results are shown in Fig. 18. It can be seen that the evolution process of cumulative damage variables generally shows a \( \alpha = \beta \) type growth, which can be divided into three stages: in the first stage, the joints and cracks in the coal-rock composite structure and the cracks at the coal-rock interface are compacted, the cumulative damage variable increases rapidly; in the second stage, the crack propagation at the crack tip of composite structures, or the micro crack produce at the weak part of the composite structure, the cumulative damage variable increase slowly; in the third stage, after the accumulation of previous damage, the cracks in the composite structure expand and penetrate, the cumulative damage variables increase rapidly. With the increase of cycles and loading, the cracks in coal rock structure continue to expand to form the large macro cracks, and finally lead to composite structural failure, the cumulative damage variable of the coal-rock composite structure reaches 1. The average cumulative damage variable per cycle is 0.04764, 0.05732, 0.0698, 0.08613 and 0.12357 respectively. The calculation result is more accurate than that of the acoustic emission method and dissipated energy method.

Through the monitoring of the damage process of the coal-rock composite structure, it is found that the macro crack first appears in the coal specimen in the coal-rock composite structure and causes the axial splitting failure. The coal specimen is the first fractured body. When the coal specimen in the coal-rock composite structure is instability, the energy stored in the rock specimen will be released rapidly, which will further cause the destruction of the coal specimen in the composite structure.

The evolution law of damage variables obtained by the joint calculation method is in good agreement with the actual deterioration process of the composite structure and is more sensitive to the damage variables under cyclic loading, which can more effectively reflect the development and expansion of cracks of composite structure under cyclic loading.

The failure mode of the coal-rock composite structures with different coal-rock height ratios is shown in Fig. 19. The coal specimens in composite structures A and B have a large degree of fragmentation and are in the form of debris. The coal specimens in the C and D structures mainly undergo splitting failure, accompanied by fragments. The coal specimens in the E composite structures mainly
suffered an axial splitting failure, accompanied by external bulging of the coal wall. The rock specimen in the coal-rock composite structure did not break.

The high-definition camera shows that the crack propagation direction in the coal-rock composite structure mainly expands along the stress direction, and forms a crack distribution zone along the stress direction. The coal-rock composite structure is a tensile failure, and the broken coal body is granular.

Comparing the damage degree of the coal specimen in the coal-rock composite structures, it can be found that the smaller the coal-rock height ratio, the greater the damage degree of the coal specimen in the composite structure, the smaller the volume of debris after the coal specimen is destroyed. None of the rock specimens in the coal-rock composite structure was damaged, indicating that the strength and deformation characteristics of the coal-rock composite structure are mainly affected by the strength and deformation characteristics of the weaker coal in the composite structures.

Fig. 18 The curve of cumulative damage variable curve through the joint calculation method
6 Conclusions

(1) Lithology and coal-rock height ratio play an important role in the energy dissipation of coal-rock composite structure. The higher the coal-rock height ratio, the greater the average elastic energy and dissipated energy produced per cycle, the smaller the total elastic energy and dissipated energy produced in the process of cyclic loading.

(2) Because of the different calculation methods of damage variables, the calculated damage variable curves are also different. The damage variable calculated by the dissipated energy method shows the “ladder” growth, and the damage variable calculated by the acoustic emission method shows the “U” shape. The greater the coal-rock height ratio, the greater the average cumulative damage variable growth per cycle.

(3) Based on the sensitivity of the dissipated energy method and acoustic emission method to damage variables, the more sensitive joint calculation method is proposed. The cumulative damage variable curve obtained by the joint calculation method is more sensitive and can more effectively reflect the damage and instability process of the coal-rock composite structure under cyclic loading.

(4) The crack propagation direction in the coal-rock composite structure mainly expands along the stress direction and forms a crack distribution zone along the stress direction. The coal-rock composite structure is a tensile failure, and the broken coal body is granular. The smaller the coal-rock height ratio, the greater the damage degree of the coal specimen in the composite structure, the smaller the volume of debris after the coal specimen is destroyed.

Funding This study was supported by the National Natural Science Foundation of China [Grant numbers 51604164]; and by the program of youth teacher growth plan in Shandong province.

Declarations

Conflict of interest The authors declare they have no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

References

Aldahdooh MAA, Bunnori NM, Johari MAM (2013) Damage evaluation of reinforced concrete beams with varying thickness using the acoustic emission technique. Constr Build Mater 44:812–821. https://doi.org/10.1016/j.conbuildmat.2012.11.099

Alneasan M, Behnia M, Bagherpour R (2019) Analytical investigations of interface crack growth between two dissimilar rock layers under compression and tension. Eng Geol 259:105188. https://doi.org/10.1016/j.enggeo.2019.105188

Bai JW, Feng GR, Wang ZH, Wang SY, Qi TY, Wang PF (2019) Experimental investigations on the progressive failure characteristics of a sandwiched coal-rock system under uniaxial compression. Appl Sci. https://doi.org/10.3390/app9061195

Chen Y, Yang YG, Gao F, Zhang XX (2018) Researches on damage evolution and acoustic emission characteristics of rocks. Adv Civil Eng. https://doi.org/10.1155/2018/3108065

Du F, Wang K, Wang G, Huang Y, Yi L (2019) The mechanism of damage in gas-bearing coal–rock combination bodies and gas seepage in coals. Energy Sources, Part A: Recovery, Utilization and Environmental Effects. https://doi.org/10.1080/15567036.2019.1635665

Du W, Zhang K, Sun H (2019b) Mechanical properties and energy development characteristics of impact-prone coal specimens under uniaxial cyclic loading. AIP Adv 9(11):115114. https://doi.org/10.1063/1.5129733
Duan MK, Jiang CB, Xing HL, Zhang DM, Peng K, Zhang WZ (2020) Study on damage of coal based on permeability and load-unload response ratio under tiered cyclic loading. Arab J Geosci. https://doi.org/10.1007/s12517-020-5249-4

Ebrahimian Z, Ahmadi M, Sadri S, Li BQ, Moradian O (2019) Wavelet analysis of acoustic emissions associated with cracking in rocks - sciencedirect. Eng Fract Mech 217:106516–106516. https://doi.org/10.1016/j.enganenv.2019.106516

Frith R, Reed G (2018) Coal pillar design when considered a reinforcement problem rather than a suspension problem. Int J Min Sci Techno 28(1):11–19. https://doi.org/10.1016/j.ijmst.2017.11.013

Gao L, Gao F, Zhang ZZ, Xing Y (2020) Research on the energy evolution characteristics and the failure intensity of rocks. Int J Min Sci Technol 30(5):705–713. https://doi.org/10.1016/j.ijmst.2020.06.006

Girard L, Beutel J, Gruber S, Hunziker J, Weber S (2012) A custom acoustic emission monitoring system for harsh environments: application to freezing-induced damage in alpine rock-walls. Geoscientific Instrum Methods Data Syst Discuss 2(1):267–300. https://doi.org/10.5194/gid-2-267-2012

Gu QH, Ning JG, Tan YL, Liu XS, Ma QG, Xu Q (2018) Damage constitutive model of brittle rock considering the compaction of crack. Geomech Eng 15(5):1081–1089

Guo WY, Tan YL, Yu FH, Zhao TB, Hu SC (2018) Mechanical behavior of rock-coal-rock specimens with different coal thicknesses. Geomech Eng. https://doi.org/10.12989/gae.2018.15.4.1017

Huang P, Zhang J, Spearng A, Chai J, Dong C (2021) Experimental study of the creep properties of coal considering initial damage. Int J Rock Mech Min Sci 139(4):104629. https://doi.org/10.1016/j.ijrmms.2021.104629

Islavath SR, Deb D, Kumar H (2020) Development of a roof-to-floor convergence index for longwall face using combined finite element modelling and statistical approach. Int J Rock Mech Min. https://doi.org/10.1016/j.ijrmms.2020.104221

Jiang RC, Dai F, Liu Y, Li A, Feng P (2021) Frequency characteristics of acoustic emissions induced by crack propagation in rock tensile fracture. Rock Mech Rock Eng 54(4):2053–2065. https://doi.org/10.1007/s00603-020-0351-3

Khazaei C, Hazzard J, Chalaturnyk R (2015) Damage quantification of intact rocks using acoustic emission energies recorded during uniaxial compression test and discrete element modeling. Computers Geotech 67:94–102

Kim JS, Kim GY, Baik MH, Finsterle S, Cho GC (2019) A new approach for quantitative damage assessment of in situ rock mass by acoustic emission. Geomech Eng. 18(1):11–20

Li DX, Wang EY, Kong XG, Jia HS, Wang DM, Muhammad A (2019) Damage precursor of construction rocks under uniaxial cyclic loading tests analyzed by acoustic emission. Constr Build Mater 206:169–178. https://doi.org/10.1016/j.conbuildmat.2019.02.074

Li DY, Sun Z, Xie T, Li XB, Ranjith PG (2017) Energy evolution characteristics of hard rock during triaxial failure with different loading and unloading paths. Eng Geol 2018:270–281. https://doi.org/10.1016/j.enggeo.2017.08.006

Liu XX, Li Y, Zhao FJ, Zhou YM, Wang WW, Li SN (2019) Experimental research on mechanical and energy characteristics of reinforced rock under dynamic loading. Shock Vib 2:1–11. https://doi.org/10.1155/2019/4356729

Lu YN, Li XP, Xiao TL (2013) Analysis of mechanical characteristics and energy dissipation in fracture rock under triaxial loading. Rock Charact Model Eng Design Methods. https://doi.org/10.1201/b14917-25

Luo Y, Wang G, Li XP, Liu TT, Mandal AK, Xu MN, Xu K (2020) Analysis of energy dissipation and crack evolution law of sandstone under impact load. Int J Rock Mech Min Sci 132:104359. https://doi.org/10.1016/j.ijrmms.2020.104359

Ma Q, Tan YL, Liu XS, Gu QH, Li X (2020) Effect of coal thicknesses on energy evolution characteristics of roof rock-coal-floor rock sandwich composite structure and its damage constitutive model. Compos B Eng 198:108086. https://doi.org/10.1016/j.compositesb.2020.108086

Maleki H, Lawson H (2017) Analysis of geomechanical factors affecting rock bursts in sedimentary rock formations. Procedia Engineer 191.82–88. https://doi.org/10.1016/j.proeng.2017.05.157

Mark C (2018) Coal bursts that occur during development a rock mechanics enigma. Int J Min Sci Techno 28(1):35–42. https://doi.org/10.1016/j.ijms.2017.1280745

Mathey M (2018) Modelling coal pillar stability from mine survey plans in a geographic information system. J S Afr I Min Metall. 118(2):157–164

Moradian Z, Einstein HH, Ballivy G (2016) Detection of cracking levels in brittle rocks by parametric analysis of the acoustic emission signals. Rock Mech Rock Eng 49(3):785–800. https://doi.org/10.1007/s00603-015-0775-1

Pan B, Yu WJ, Shen WB (2020) Experimental study on energy evolution and failure characteristics of rock–coal–rock combination with different height ratios. Geotech Geol Eng 39(1):425–435. https://doi.org/10.1007/s10706-020-01501-4

Patricia R, Celestino TB (2018) Application of acoustic emission monitoring and signal analysis to the qualitative and quantitative characterization of the fracturing process in rocks. Eng Fract Mech 210:54–69. https://doi.org/10.1016/j.enganenv.2018.06.027

Raja S, Partha SP, Prabhat KM (2019) Evaluation of bump-proneness of underground coal mines using burst energy coefficient. Arab J Geosci 12(18):1–16. https://doi.org/10.1007/s12517-019-4746-9

Shkuratnik VL, Filimonov YL, Kuchurin SV (2006) Acoustic emission memory effect in coal samples under uniaxial cyclic loading. J Appl Mech Tech Phys 47(2):236–240. https://doi.org/10.1016/j.smp.2006.04.008-6

Voyiadjis GZ, Kattan PI (2009) A comparative study of damage variables in continuum damage mechanics. Int J Damage Mech. https://doi.org/10.1016/j.ijdamage.2018.02.006

Waclawik P, Kukutsch R, Konicek P, Kajzar V (2018) Monitoring of coal pillars yielding during room and pillar extraction at the great depth. Geomech Geodyn Rock Masses 1 and 2:711–716
