Calculation of The Charged Leptons’ Anapole Moment in The Extended Standard Model

C. Aydın
Department of Physics, Karadeniz Technical University, 61080, Trabzon, Turkey

Abstract: Using Feynman-'t Hooft gauge and dimensional regularization, the static parity-violating coupling of the charged leptons to an external electromagnetic field is calculated in the minimal supersymmetry standard model.

Key words: Standard model, minimal supersymmetric model, lepton, form factors, anapole moment.

PACS: 14.60.-z, 11.10.ef, 12.60.Jv, 12.10.-g, 14.80.Nb, 13.40.Gp

1 Introduction

The Standard Model (SM) is a theory concerning the electromagnetic, weak and strong interactions. The SM of elementary particle is a gauge quantum field theory. The internal symmetry that defines the SM is the local SU(3) × SU(2) × U(1) gauge symmetry and the most general Lagrangian that describes the dynamics of the fields[1,2]. The main point of these theories is that they are renormalizable so that the useful higher-order calculations can be done. One of the major triumphs of the unifiy theory is the calculations to high order in α, μ and r² and their successful comparison with experiment. Although the SM is a beautiful model with predictive power, few unknowns prevail as well as some problems. For example, one of them is to determine to absolute scale of the neutrino mass or the nature of the neutrino, either it is a Dirac or a Majorana particle. Physics beyond the SM has drawn physicist attention for a long time. One of the most appealing theories to describe physics at the TeV scale is the minimal supersymmetric extensions of the SM (MSSM) [3,4]. It also besides giving a solution to the hierarchy problem, it provides us with a good candidate for cold dark matter (CDM), namely, the lightest neutralino which is a Majorana particle.

It is known that in quantum field theory (QFT), the interaction vertex between a single photon and a fermion can be characterized in terms of four electromagnetic (EM) form factors. In the limit of vanishing momentum transfer between the photon and fermion, the form factors encode the static EM properties of the particle, namely, its charge, magnetic dipole moment, electric dipole moment and anapole moment. After the discovery of violation of parity in the weak interaction Zeldovich showed that there is the static coupling which the called the anapole moment of the particle with a propertional to the Fermi constant \( G_F \). In the late 1960’s and early 1970’s, Dubovik connected the quantum description of the anapole to classical electrodynamics [5,12]. The existence of an anapole moment has been experimentally verified in nuclei of heavy atoms, in 1997, it was measured experimentally in the nucleus of Cesium-133 and Ytterbium-174 [14]. In nucleus, the spin and the circulating orbital motion of the external nucleus generate the nuclear spin I and an associated effective current. The weak force introduces a small toroidal component to this orbital and spin current, thus generating a parity violating anapole moment [15,19].

A fermion in a P-violating, CP-conserving theory (in this situation, the particle can not have an electric dipole moment) has the electromagnetic current given as [20–23,25]

\[
\langle \ell(p')|J_\mu(p)|\ell(p)\rangle = ie\bar{\psi}_\ell(p_f)\left[\gamma_\mu F_1(q^2) + \mu(q'^2/2m)\sigma_{\mu\nu}F_2(q^2) + \frac{q^2a}{16\pi^2M_W^2}(\gamma_5\gamma_\mu - g_\mu)F_3(q^2)\right]u(p_i) \tag{1}
\]

where \( q^2 \to 0 \). The physical significance of the form factors is easy to understood by considering in the nonrelativistic limit. In this limit, the interaction energy with an

\[
F_1(q^2) = 1 + \frac{1}{6}q^2 < r^2 > + O(q^4) \tag{2}
\]

\[
F_2(q^2) = 1 + O(q^4), \ F_3(q^2) = 1 + O(q^4) \tag{3}
\]
external electromagnetic field takes the form
\[ H_{\text{int}} = -\mu(\vec{\sigma} \cdot \vec{B}) - d(\vec{\sigma} \cdot \vec{E}) - a(\vec{\sigma} \cdot \vec{j}) \]  
(4)

where \( \vec{B} \) and \( \vec{E} \) are the magnetic and electric fields, \( \vec{\sigma} \) is the Pauli spin matrix, and \( \vec{j} = (\vec{\nabla} \times \vec{B} - \vec{E}) \) is the electric current density at the point where the particle is situated.

The form factors of the charged lepton and the neutrinos (uncharged leptons) have been calculated in gauge field theories. The questions of whether neutrinos have masses and if so, whether they are Dirac or Majorana particles are two of the most important issues in both particle physics and astrophysics. It is known that the neutrinos are massless that it is Weyl neutrinos in the SM. There are many possible extensions of SM which give rise massive neutrino. While a Dirac particle has four form factors, a Majorana particle has only one which it is the anapole moment [23-24].

The cross section for scattering of electrons by polarized protons had been evaluated by Zeldovich and Perelemov [24], and polarized muon by Sarkar [27] taking into account the effect of anapole term. In 1980, the anapole moment of the electron was calculated by Dombey and Kennedy [25] in the SM. In 1987, H.Czyz et al. [28] discussed the anapole moment of charged leptons in the context of the SM. In 2017, Whitcomb and Latimer performed a scattering calculations which probes the anapole moment with a spinless charged particle. They showed that, in the non-relativistic limit, the cross sections agree with a quantum mechanical computation of the cross section for a spinless current scattered by an infinitesimally thin toroidal solenoid [29]. As a result, they have obtained the effect of the anapole term on the scattering.

Motivated by this study, we have been studying the anapole moment, one of the least studied electromagnetic properties of a particle since 1987. Lately, there had been a surge of interest in the study of anapole moment from the astrophysical as well as the particle physics point of view, and also serves as basis to explain the dark matter [30-32].

Neutrino oscillation results imply that the flavor neutrino fields \( \nu_L(x) \) are the mixtures of the left-right handed components of the fields of the neutrinos, with define masses as
\[ \nu_{\ell L}(x) = \frac{1}{2}(1 - \gamma_3)\nu_\ell(x) = \sum_{i=1}^{3} U_{\ell i} \nu_{i L}(x), \quad \ell = e^-, \mu^-, \tau^- \]  
(5)

where \( U \) is the the unitary PMNS mixing matrix relating the neutrino mass eigenstates to the weak eigenstates and \( \nu_\ell(x) \) is the field of neutrino (Majorana or Dirac) with mass \( m_\ell \). Flavor fields \( \nu_L(x) \) enter into the SM charged current (CC)
\[ L_i^{\text{CC}}(x) = -\frac{g}{2\sqrt{2}} \sum_{\ell=e,\mu,\tau} \bar{\nu}_\ell \gamma_\alpha \ell_L(x) W^\alpha(x) + h.c \]  
(6)

and neutral current (NC) interactions
\[ L_i^{\text{NC}}(x) = -\frac{g}{2\cos\theta_W} \sum_{\ell} \left[ \bar{\nu}_\ell \gamma_\alpha \nu_{\ell L} + \bar{\nu}_\ell (g_V + g_A \gamma_5) \ell \right] Z_\alpha(x) \]  
(7)

is the neutrino NC. \( W^\alpha(x) \) and \( Z^\alpha(x) \) are the fields of \( W \) and \( Z \) vector bosons, \( g \) is the electroweak interaction constant and \( \theta_W \) is the weak(Weinberg) angle [33]. The vector and axial-vector couplings are
\[ g_V \equiv t_{3L}(\ell) - 2q_\ell \sin^2 \theta_W, \quad g_A \equiv t_{3L}(\ell) \]
where \( t_{3L}(\ell) \approx -\frac{1}{2} \) is the weak isospin charged leptons \( \ell \) and \( q_\ell \) is the charge of \( \ell \) in units of \( e \), which is equal to \( gy \sin \theta_W \), is the positron electric charge.

The interactions of the charged lepton and sneutrino with chargino (lepton-sneutrino-chargino vertices), and the charged lepton and the charged slepton with the neutralino (lepton-slepton-neutralino vertices) are correspondingly given by Lagrangian as [10, 11]
\[ L_i^{\text{SUSY}}(x) = \sum_{\ell} \left[ \bar{l}(N^L P_L + N^R P_R) \chi^\ell \ell + (C^L P_L + C^R P_R) \chi^\ell \ell \right] + h.c \]  
(8)

where
\[ P_{L,R} = \frac{1}{2}(1 \mp \gamma_5), \]
\[ N^L = -\frac{g}{\sqrt{2}} \sum_{AX} \left\{ \frac{m_\ell}{M_W \cos \beta} N_{A3} R'_{X1} + 2N_{A1} \tan \theta_W R'_{X1} \right\}, \]
\[ N^R = -\frac{g}{\sqrt{2}} \sum_{AX} \left\{ [-N_{A2} - N_{A1} \tan \theta_W] R'_{X1} + \frac{m_\ell}{M_W \cos \beta} N_{A3} R'_{X1} \right\}, \]
\[ C^L = g \sum_{X} \frac{m_\ell}{M_W \cos \beta} V_{AX} R'_{X1}, \quad C^R = g \sum_{X} U_{AX} R'_{X1}, \]

where \( P_{L,R} \) are the project operators. \( N \) and \( U, V \) are the diagonalized matrices of neutralino and chargino mass matrix, respectively. \( R \) is the diagonalized matrix of slepton or sneutrino mass matrix. The indices \( A \) (1...4 for neutralinos; 1, 2 for charginos) and \( X \) (1...6 for sleptons; 1, 2, 3 for sneutrinos) runs over the dimensions of the respective matrices, whereas \( i \) as usual runs over the generations, \( m_\ell \) is the mass of the \( i^{th} \) lepton and rest of the parameters carry the standard definitions.

This paper is structured as follows: In Section II, we present the calculations of the anapole moment of charged leptons in the framework of the MSSM. Finally, we present our discussion and conclusion in Section III.
2 Calculation

The basic diagrams which contribute to a parity-violating interactions are shown in Fig. 1.

The calculation here of the charged lepton anapole moment parallels the calculation by Sakakibara [46] of the electron anapole moment. The simplest gauge to choose to calculate these diagrams is Feynman-'t Hooft gauge, and the calculations are performed using the dimensional regularization produce which preserves gauge in variance. This calculation is done, using above interactions Lagrangians, in framework of the minimal supersymmetric extension of the Standard Model (MSSM)

We choose the Breit frame in which the charged lepton has momentum initially (p − q/2) and (p + q/2), then we find the matrix elements for the MSSM in Fig. 1 respectively.

\[ M_L = \frac{e^3}{4} \int \frac{d^2 k}{(2\pi)^2} \frac{\gamma^\alpha (g_\nu + g_A \gamma_5)(-\vec{k} + \vec{p} + \frac{q}{2} + m_\nu)\gamma^\mu (-\vec{k} + \vec{p} - \frac{q}{2} + m_\nu)\gamma_\alpha (g_\nu + g_A \gamma_5)}{[(k + p + \frac{q}{2})^2 - m_\nu^2][(k - p + \frac{q}{2})^2 - m_\nu^2][k^2 - M_Z^2]} \]  

\[ M_{ii} = \frac{e^2 g^2}{8} \int \frac{d^2 k}{(2\pi)^2} \frac{\gamma_\beta (1 - \gamma_5)(\vec{k} - m_\nu)\gamma_\alpha V_{\alpha \beta \mu}(k - p + \frac{q}{2} - k + p + \frac{q}{2}, q)}{[k^2 - m_\nu^2][(k + p + \frac{q}{2})^2 - M_Z^2][(-k + p + \frac{q}{2})^2 - M_Z^2]} \]  

\[ M_{ii} = \frac{i e}{2} \gamma^\mu (g_\nu + g_A \gamma_5) \frac{1}{q^2 - M_Z^2} \pi^{Z\gamma}_{\mu\nu} (q^2) \]  

where \( \pi^{Z\gamma}_{\mu\nu} \) is the mixing tensor for the \( \gamma - Z^0 \) mixing diagram evaluated numerically by Dombey and Kennedy [28]. They obtained

\[ \pi^{Z\gamma}_{\mu\nu} (q^2) = (3.83 \times 10^{-3}) e^2 q^2 g_{\mu\nu} + O(q^4) \]

Projecting out the axial parts proportional to

\[ \frac{i e}{16\pi^2} \gamma^\nu \gamma_5 \gamma^\mu \frac{g^2}{M_W^2} \]

and after long but straightforward calculations, neglecting the terms of order \((m_\nu/M_W)^2\), we obtain for the anapole part the following expressions as

\[ a_i = \frac{1}{3} (1 - 4\sin^2 \theta_W) \left[ 2\ln \left( \frac{M_Z}{m_\nu} \right) - \frac{7}{12} \right] \]  

\[ a_{ii} = -\frac{7}{12} \]  

\[ a_{iii} = \left( \frac{\cos \theta_W}{4} \right) 3.83 \times 10^{-3} \]  

\[ a_{iv} = \frac{1}{6} \left( \frac{M_W}{m_\nu} \right)^2 \left[ -\frac{7}{6} + 2\ln \left( \frac{m_\nu}{m_\chi} \right) - \left( C_L^2 - C_R^2 \right) \right] \]  

\[ a_v = \frac{1}{6} \left( \frac{M_W}{m_\nu} \right)^2 \left[ -\frac{1}{6g^2} \left( |N_L|^2 - |N_R|^2 \right) \right] \]

3 Conclusion

Anapole interaction is the least understood type of all possible between fermion and vector bosons. We calculated the anapole moment of the charged leptons in the framework of the MSSM. Even though this model is perhaps an unrealistically model, it is one of extending of the SM. No diagrams involving Higgs particle contribute to parity violation to this order. Constraints that can be obtained for this interaction from the entire body of available experiment data are investigated. We think that the most stringent constraints follow from experiments with polarized electrons. Because low energy scattering of polarized electron on electrons and other targets are very sensitive to parity violating effects that are proportional to \((1 - 4\sin^2 \theta_W)\).

The input parameters must be such as to satisfy a
number of experimental constraints [47]. The first criteria is that the computed Higgs boson mass must be consistent with the Higgs boson mass measurements by the ATLAS and the CMS collaboration [48]. The other important criteria is about the relic density of the dark matter given by the model that must be consistent with the measured by the Planck experiment and spartical spectrum of the model be consistent with the lower experimental limits on spartical masses. Additionally, the proper understanding of the nature of cold dark matter from experimental data require the precise analysis of all information that will become available from collider experiments, low-energy experiments astrophysical and cosmological observations [49].

There are many scenario in SUYS. However, the large \( \tan \beta \) scenario is an interesting issue that has been received attentions in MSSM. In these scenario, we obtain 

\[
a_T = \frac{1}{3} (1 - 4 \sin^2 \theta_W) \left[ 2 \ln \left( \frac{M_Z}{m_\ell} \right) - \frac{7}{12} \right] - \frac{7}{72} + \frac{1}{96} \left\{ - \frac{1}{6} \left( \frac{m_\ell}{m_\tilde{\ell}} \right)^2 + \left( \frac{m_\ell}{m_\tilde{\nu}} \right)^2 \left[ - \frac{7}{6} + 2 \ln \left( \frac{m_\tilde{\nu}}{m_\ell} \right) \right] \right\} \tan^2 \beta \tag{20}
\]

One can see that, if \( m_\tilde{\ell} \) and \( m_\tilde{\nu} \) are bigger than \( m_\ell \), \( m_\ell/m_\tilde{\ell} \) and \( m_\ell/m_\tilde{\nu} \) will be much smaller than 1. As a result, it seems unlikely that in the MSSM the charged lepton anapole moment is almost equal to the value given by the SM. This means that the terms come from SUSY are not effective due to the properties of neutralino and chargino.

The numerical values of the anapole moment for three different charged leptons are obtained as 

\[
a \simeq \begin{cases} 
0.56 & \text{for } e^- \\
0.27 & \text{for } \mu^- \\
0.10 & \text{for } \tau^-
\end{cases}
\]

and in terms of \( cm^2 \)

\[
\frac{g^2 a}{16\pi^2 M_W^2} \simeq \begin{cases} 
9.1 \cdot 10^{-35} cm^2 & \text{for } e^- \\
4.3 \cdot 10^{-35} cm^2 & \text{for } \mu^- \\
1.7 \cdot 10^{-35} cm^2 & \text{for } \tau^-
\end{cases}
\]

using the values given in Table 1.

| Particle | Mass [GeV] |
|----------|------------|
| \( e^- \) | 5.11 \cdot 10^{-4} |
| \( \mu^- \) | 0.105 |
| \( \tau^- \) | 1.777 |
| \( Z^0 \) | 91.18 |
| \( W^\pm \) | 80.39 |
| \( H \) | 125 |
| \( \tilde{\chi}_1^0 \) | \( \geq 46 \) |
| \( \tilde{\chi}_2^\pm \) | \( \geq 94 \) |
| \( \tilde{\tau} \) | \( \geq 81.9 \) |
| \( \tilde{\mu} \) | \( \geq 94 \) |
| \( \tilde{\tilde{e}} \) | \( \geq 107 \) |
| \( \tilde{\nu} \) | \( \geq 41 \) |

Some constant values

| \( G_F \) | 1.16 \cdot 10^{-5} |
| \( 1/\alpha \) | 137 |
| \( \sin^2 \theta_W \) | 0.229 |
| \( \tan \beta \) | 20 |

The interesting point of these values is that all the above values are order 1/10 with neutrinos (neutral leptons). These results are very sensitive in the experiment point of view.
Fig. 1. Basic diagram of the anapole moment in the minimal supersymmetry standard model

References

1. S. Weinberg, Phys. Rev. Lett., 19: 1264 (1967)
2. A. Salam, Eighth Nobel Symposium. Stockholm: Almquist and Wiksell, 1968. 367
3. W. Greiner and B. Müller, Gauge Theory of Weak Interactions. Verlag: Springer, 2000
4. P. Langacher, The Standard Model and Beyond. Princeton, New Jersey: CRC Press, 2000
5. S. Weinberg, The Quantum Theory of Fields (Volume 2). Cambridge: Cambridge University Press, 2005
6. M. Abak and C. Aydm, Nuovo Cimento A, 101: 597 (1989)
7. C. Aydm, Modern Phys. Lett. A, 16: 1823 (2001)
8. Ya. B. Zeldovich and A. M. Perelomov, Sov. Phys. JETP, 12: 177 (1961)
9. S. Sarkar, Proc. Phys. Soc., 88: 788 (1966)
10. N. Domby and A. D. Kennedy, Phys. Lett. B, 91: 428 (1980)
11. H. Czyz, K. Kołodziej, and M. Zralek, Physica Scripta 37: 205 (1987)
12. H. Czyz, K. Kołodziej, M. Zralek, and P. Christova, Can. J. Phys., 66: 132 (1988)
13. M. J. Musolf and B. R. Holstein, Phys. Rev. D, 43: 2956 (1991)
14. K. M. Whitcomb and D. C. Latimer, Am. J. Phys., 85: 932 (2017)
15. C. M. Ho and R. J. Scherrer, Phys. Lett. B, 722: 344 (2013)
16. J. Kopp, L. Michalas, and J. Smirnov, JCAP 1404: 022 (2014)
17. L. G. Cabrel-Rosetti, M. Mondragon, and E. Reyes-Perez, Nucl. Phys. B, 907: 1 (2016)
18. A. Ibarra, C. E. Yaguna, and O. Zapata, Phys. Rev. D, 93: 035012 (2016)
19. P. Arias, J. Gamboa, and N. Tapia, arXiv: 1901.02946v2 (hep-ph) (2019)
20. G. Arcadi et al. arXiv: 1906.04755v1 (hep-ph) (2019)
21. J. Rosiek, Phys. Rev. D, 41: 3464 (1990); arXiv:9511250v3 (hep-ph) (1995)
22. T. Moroi, Phys. Rev. D, 53: 6565 (1996); Erratum: Phys. Rev. D, 56: 4424 (1997)
23. S. K. Vempati, arXiv: 1201.0334v1 (hep-ph) (2012)
24. X. X. Dong, S. M. Zhao, H. B. Zhang, F. Wang, and T. F. Feng, Chin. Phys. C, 40: 093103 (2016)
25. K. S. Sun, J. B. Chen, X. Y. Yang, and S. K. Cui, Chin. Phys. C, 43: 043101 (2019)
26. S. Sakakibara, Aachen preprint PITHA 79/17: (1979)
27. A. Abouabriham and P. Nath, arXiv: 1902.05538v2 (hep-ph) (2019)
28. A. Abouabriham and P. Nath, Phys. Rev. D, 98: 015009 (2019)
29. J. A. Aguilar-Saavedra et al., Eur. Phys. J. C, 46: 43 (2006)
30. M. Tanabashi et al., (Particle Data Group) Phys. Rev. D, 98: 030001 (2018) and (2019) update