Continuous Monitoring of Pareto Frontiers on Partially Ordered Attributes for Many Users

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ABSTRACT
We study the problem of continuous object dissemination—given a large number of users and continuously arriving new objects, deliver an object to all users who prefer the object. Many real-world applications analyze users’ preferences for effective object dissemination. For continuously arriving objects, timely finding users who prefer a new object is challenging. In this paper, we consider an append-only table of objects with multiple attributes and users’ preferences on individual attributes are modeled as strict partial orders. An object is preferred by a user if it belongs to the Pareto frontier with respect to the user’s partial orders.

Users’ preferences can be similar. Exploiting shared computation across similar preferences of different users, we design algorithms to find target users of a new object. In order to find users of similar preferences, we study the novel problem of clustering users’ preferences that are represented as partial orders. We also present an approximate solution of the problem of finding target users which is more efficient than the exact one while ensuring sufficient accuracy. Furthermore, we extend the algorithms to operate under the semantics of sliding window. We present the results from comprehensive experiments for evaluating the efficiency and effectiveness of the proposed techniques.

1 INTRODUCTION
Many applications serve users better by disseminating objects to the users according to their preferences. User preferences can be modeled via a variety of means including collaborative filtering [19], top-k ranking [7, 8], skyline [2], and general preference queries [5, 12]. In various scenarios, users’ preferences may or may not change occasionally, while the objects keep coming continuously. Such scenarios warrant the need for a capability of continuous monitoring of preferred objects. While previous studies have made notable contributions on continuous evaluation of skyline [14, 27] and top-k queries [28], we note that two important considerations are missing from prior works:

• Many users: There may be a large number of users and the users may have similar preferences. Prior works focus on the query needs of one user and thus their algorithmic solutions can only be applied separately on individual users. A solution can potentially attain significant query performance gain by leveraging users’ common preferences.

• Partially ordered attributes: Prior works focus on top-k and skyline queries. In multi-objective optimization, a more general concept than skyline is Pareto frontier. Consider a table of objects with a set of attributes. An object is Pareto-optimal (i.e., it belongs to the Pareto frontier) if and only if it is not dominated by any other object [1, 13]. Object y dominates x if and only if y is better than or equal to x on every attribute and is better on at least one attribute. In defining the better-than relations, most studies on skyline queries assume a total order on the ordinal or numeric values of an attribute, except for [17, 29] which consider strict partial orders. The psychological nature of human’s preferences determines that it is not always natural to enforce a total order. Oftentimes real-world preferences can only be modeled as strict partial orders [5, 12, 17].

Consider the following motivating applications which monitor Pareto frontiers on partially ordered attributes for many users.

• Social network content and news delivery: It is often impossible and unnecessary for a user to keep up with the plethora of updates (e.g., news feeds in Facebook) from their social circles. When a new item is posted, if the item is Pareto-optimal with respect to a user, it can be displayed above other updates in the user’s view. Similar ideas can be adopted by mass media to ensure their news reaches the right audience. User preferences can be modeled on content creator, topic, location, and so on. Enforcing total orders on such attributes is both cumbersome and unnatural.

• Publication alerts: Bibliography servers such as PubMed and Google Scholar can notify users about newly published articles matching their preferences on venues and keywords. Such attributes do not welcome total orders either.

• Product recommendation: When a new product becomes available, a retailer can notify customers who may be interested. It can distill customers’ preferences on product specifications (e.g., brand, display and memory for laptops) from profiles, past transactions and website browsing logs. Example 1.1 discusses this application more concretely.

Example 1.1. Consider an inventory of laptops in Table 1 and customers’ preferences on individual product attributes (display, brand and CPU) modeled as strict partial orders in Table 2. For an attribute, the corresponding strict partial order is depicted as a directed acyclic graph (DAG), more specifically a Hasse diagram. Given two values x and y in the attribute’s domain, the existence of a path from x to y in the DAG implies that x is preferred to y. With respect to customer c1 and attribute brand, the path from Lenovo to Toshiba implies that c1 prefers Lenovo to Toshiba. There is no path between Toshiba and Samsung, which indicates c1 is indifferent between the two brands.

The strict partial orders on various attributes together represent a customer’s preferences on objects. For instance, c1 prefers o2={14, Apple, dual} to o1={12, Apple, single}, since they prefer 13−15.9 to 10−12.9 on display and dual to single on CPU. With regard to o1 and o3={15, Samsung, dual}, c1 does not prefer one over the other because, though they prefer 13−15.9 to 10−12.9 and dual to single, they prefer Apple to Samsung on brand.

According to the data in Tables 1 and 2, if the existing products are o1 to o16 (ignore o15 and o16 for now), the Pareto frontiers of c1 and c2 are {o2} and {o2, o3}, respectively. Suppose o15={16.5, Lenovo, quad} just becomes available. For c1, o15 does not belong to the Pareto frontier. It is dominated by o2, because c1 prefers 14-inch display over 16.5-inch, Apple over Lenovo, and dual-core

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This paper formulates the problem of continuous monitoring of Pareto frontiers: given a large number of users and continuously arriving new objects, for each newly arrived object, discover all users for whom the object is Pareto-optimal. Users’ preferences are modeled as strict partial orders, one for each attribute domain of the objects.

It is key to devise an efficient approach to this problem. The value of a Pareto-optimal object diminishes quickly; the earlier it is updated the more relevant it is for the user. The brute-force approach is subject to a clear drawback—repeated and wasteful maintenance of Pareto frontier for every user.

Sharing computation across users To tackle the aforementioned drawback, we partly resort to sharing computation across users. The challenge lies in the diversity of corresponding partial orders—a Pareto-optimal object with respect to one user may or may not be in the Pareto frontier for another user. Nonetheless, users have common preferences. In Table 2, both $c_1$ and $c_2$ prefer 13–15.9 inch display the most. Both prefer Apple and Lenovo to Toshiba and Samsung, and they both prefer single-core CPU the least. In Table 2, $U$ is a virtual user whose partial orders depict the common preferences of $c_1$ and $c_2$. Intuitively, users having similar preferences can be clustered together.

We thus design algorithms to mitigate repetitive computation via sharing computation across similar preferences of users. To intuitively understand the idea, consider two example scenarios. i) If $o$ is dominated by $o'$ with respect to the common preferences of a set of users, then $o$ is disqualified in Pareto-optimality for all users in the set. In Example 1.1, consider $o_{16}=(16, Toshiba, single)$ as the new object. With respect to $U$, $o_{16}$ is dominated by both $o_2=(14, Apple, dual)$ and $o_{15}=(16.5, Lenovo, quad)$. Therefore, $o_{16}$ belongs to the Pareto frontier of neither $c_1$ nor $c_2$. ii) Before the arrival of $o_2$, obviously $o_1=(12, Apple, single)$ is the only Pareto-optimal object for $U$, $c_1$ and $c_2$. Now consider the emergence of $o_2$. As $o_1$ is dominated by $o_2$ with respect to $U$, $o_1$ is replaced by $o_2$ in the Pareto frontier. This comparison is sufficient to decide that $o_1$ is dominated by $o_2$ for both $c_1$ and $c_2$.

Clustering users To find users sharing similar preferences, we study the novel problem of clustering strict partial orders, which are used to model the preferences of both users and clusters. We measure the similarity between clusters and users by their common preferences. Such similarity measures factor in the different significance of preferences at various levels of the partial orders. Table 3 depicts six customers’ preferences on brand, in which $c_4$, $c_5$, and $c_6$ prefer Lenovo to all other brands except that $c_4$ prefers Samsung over Lenovo. Consider the objects in Table 1. For both $c_1$ and $c_6$, the Pareto frontiers contain $\{o_9, o_{10}, o_{15}\}$, while $c_4$ has $\{o_1, o_2, o_{12}\}$ as its Pareto frontier. We can say that $c_5$ and $c_6$ are more similar than $c_4$ and $c_5$ or $c_4$ and $c_6$.

Approximation The clustering algorithm may produce clusters that comprise few users, due to diverse preferences. With small clusters, the shared computation mentioned above may not pay off its overhead. Our response to this challenge is to use

| brand  | CPU  | display |
|-------|------|---------|
| Apple | dual | 13–15.9 |
| Lenovo| quad | 13–15.9 |
| Toshiba| single | 13–15.9 |

Table 1: User preferences. $U_2=\{c_1, c_2\}$, $U_3=\{c_3, c_4\}$.
approximation. As in many data retrieval scenarios, insisting on exact answers is unnecessary and answers in close vicinity of the exact ones can be just good enough. Specifically, given a set of users, if a sizable subset of the users agree with a preference, the preference can be considered an approximate common preference. This relaxation eases the aforementioned concern regarding small clusters as more approximate common preferences lead to larger clusters. As an example, in Table 2, while \( c_2 \) does not share with \( c_3 \) the preference of Apple over Samsung, its preference does not oppose it either. We can consider “Apple over Samsung” as an approximate common preference. A possible set of approximate common preferences of \( c_1 \) and \( c_2 \) form the strict partial orders in the row for virtual user \( \bar{U} \).

**Alive objects** Objects can have limited lifetime. The trends in social networks and news media change rapidly. Similarly, in any inventory, products become unavailable over time. In these scenarios users look for alive objects only. To meet this real-world requirement, we further extend our algorithms to operate under the semantics of a sliding window and thus to disseminate an object only during its lifespan.

In summary, the contributions of this paper are as follows:

- We study the problem of continuous object dissemination and formalize it as finding Pareto-optimal objects regarding partial orders. Given a large number of users and continuously arriving objects, our goal is to swiftly disseminate a newly arrived object to a user if the user’s preferences—modeled as strict partial orders on individual attributes—approve the object as Pareto-optimal.

- We devise efficient solutions exploiting shared computation across similar preferences of different users.

- We study the novel challenge of clustering user preferences represented as strict partial orders. Particularly we design similarity measures that achieve high efficiency and accuracy of answers.

- We extend our proposed solutions to deal with Pareto frontier maintenance under sliding window.

- We conduct extensive experiments using simulations on two real datasets (a movie dataset and a publication dataset). The results demonstrate clear strengths of our solutions in comparison with baselines, in terms of execution time and efficacy.

## 2 RELATED WORK

Pareto-optimality is a subject of extensive investigation. Its study in the computing fields can be dated back to **admissible points** [1] and **maximal vectors** [13]. Börzsönyi et al. [2] introduced the concept of skyline—a special case of Pareto frontier—in which all attributes are numeric and amenable to total orders. Kriegling [12] defined preferences as strict partial orders on which preference queries operate. After that, several studies specialized on skyline query evaluation over categorical attributes [3,17,18,29], among which [17, 18, 29] particularly considered query answer maintenance and only [17, 29] allow partial orders on attribute values. Nevertheless, they all consider only one user and none utilizes shared computation across multiple users’ partial orders.

Given a set of objects, Wong et al. [24–26] identify the minimum set of preference relations that preclude an object from being in the Pareto frontier. This minimum set is the combination of each possible preference relation with regard to the values of all unique objects in the set. In case of any update in the object set, the minimum disqualifying condition must be recomputed. Hence, it is not designed for continuously arriving objects.

Vlachou et al. [22, 23] and Yu et al. [28] aimed at finding all users who view a given object as one of their top-k favourites, i.e., the results of a reverse top-k query. Delliis et al. [6] studied reverse skyline query—selecting users to whom a given object is in the skyline. These works consider only numeric attributes. There is no clear way to extend them for categorical attributes or even partial orders.

All these studies, while about object dissemination, focused on different aspects of the problem than ours. Particularly, no previous studies on Pareto frontier maintenance have exploited shared computation across users’ preferences. Besides, as Sec. 5 shall explain, no prior work studied similarity measures for partial orders or how to cluster partial orders.

### 3 PROBLEM STATEMENT

| \( \mathcal{O} \) | set of objects |
|---|---|
| \( d \in \mathcal{D} \) | attribute |
| \( c \in \mathcal{C} \) | user |
| \( \succ_c \) | binary relation over dom(\( d \)) with regard to \( c \)’s preference |
| \( \Pi(c) \) | the Pareto frontier with regard to \( c \) |
| \( \mathcal{O}_P \) | the target users of \( d \) |
| \( \mathcal{O}_C \) | all set of queries |
| \( \sim \) | the similarity measure between two clusters \( \mathcal{C}_L \) and \( \mathcal{C}_R \) |
| \( \mathcal{S} \) | the maximal values of \( \sim \) |
| \( \mathcal{B} \) | branch cut in dendrogram |

**Table 4: Notations**

This section provides a formal description of our data model and problem statement. Table 4 lists the major notations. Consider a set of users \( \mathcal{C} \) and a table of objects \( \mathcal{O} \) that are described by a set of attributes \( \mathcal{D} \). For each user \( c \in \mathcal{C} \), their preference regarding \( \mathcal{O} \) is represented by strict partial orders. For each attribute \( d \in \mathcal{D} \), the strict partial order corresponding to \( c \)’s preference on \( d \) is a binary relation over dom(\( d \))—the domain of \( d \), as follows.

**Definition 3.1 (Preference Relation and Tuple).** Given a user \( c \in \mathcal{C} \) and an attribute \( d \in \mathcal{D} \), the corresponding preference relation is denoted \( \succ_c \). For two attribute values \( x, y \in \text{dom}(d) \), if \( (x, y) \) belongs to \( \succ_c \), i.e., \( (x, y) \not\in \succ_c \), also denoted \( x \succ_c y \), it is called a **preference tuple**. It is interpreted as “user \( c \) prefers \( x \) to \( y \) on attribute \( d \)”. A preference relation is irreflexive (\( (x, x) \not\in \succ_c \)) and transitive (\( (x, y) \in \succ_c \land (y, z) \in \succ_c \implies (x, z) \in \succ_c \)), which together also imply asymmetry (\( (x, y) \in \succ_c \implies (y, x) \not\in \succ_c \)).

**Definition 3.2 (Object Dominance).** A user \( c \)’s preferences regarding all attributes induce another strict partial order \( \succ_c \) that represents \( c \)’s preferences on objects. Given two objects \( o, o' \in \mathcal{O} \), \( c \) prefers \( o' \) to \( o \) if \( o' \) is identical or preferred to \( o \) on all attributes and \( o' \) is preferred to \( o \) on at least one attribute. More formally, \( o \succ_c o' \) (called \( o \) dominates \( o' \)) if and only if (\( \forall d \in \mathcal{D} : o \succ_c o', o \succ_c o \lor o \succ_c d \succ_c o', o \succ_c d \land \exists d \in \mathcal{D} : o \succ_c d \lor o \succ_c d \succ_c o' \)). If \( \forall d \in \mathcal{D} : o \succ_c d' \), we say that \( o \) and \( o' \) are identical, denoted as \( o = o' \).

**Definition 3.3 (Pareto Frontier).** An object \( o \) is Pareto-optimal with respect to \( c \), if no other object in \( \mathcal{O} \) dominates it. The set of Pareto-optimal objects (i.e., the Pareto frontier) in \( \mathcal{O} \) for \( c \) is denoted \( \Pi(c) \), i.e., \( \Pi(c) = \{ o \in \mathcal{O} | \forall o' \in \mathcal{O} \land o' \succ_c o \} \). Note that the concept of skyline points [2] is a specialization of the more general Pareto frontier, in that the preference relations for skyline points are defined as total orders (with ties) instead of general strict partial orders.
Definition 3.4 (Target Users). Given an object o, the set of all users for whom o belongs to their Pareto frontiers are called the target users. The target user set is denoted \( C_o \), i.e., \( C_o = \{ c \in C | o \in P_c \} \).

Example 3.5. Consider Table 1 and Table 2. \( O = \{ o_1, o_2, \ldots , o_{15} \} \) (ignore \( o_{16} \) for now), \( C = \{ c_1, c_2 \} \), and \( D = \{ \text{display, brand, CPU} \} \). With respect to \( c_1 \), \( (10-12.9, 16-18.9, (\text{Apple, Samsung}) \) and \( \text{(dual, triple)} \) are some of the preference tuples on attributes display, brand and CPU, respectively. Similarly, for \( c_2 \), \( (16-18.9, 19-24, (\text{Toshiba, Sony}) \) and \( \text{(triple, dual)} \) are some sample preference tuples.

\[ P_{c_1} = \{ o_2 \}, \text{since all other objects are dominated by } o_2 \text{ with respect to } c_1. \ P_{c_2} = \{ o_2, o_3, o_{15} \}, \text{as } o_2, o_3 \text{ and } o_{15} \text{ dominate } \{ o_1, o_4, o_6, o_8, o_9, o_{13} \}, \{ o_4, o_6, o_8, o_{13} \} \text{ and } \{ o_4, o_5, o_7, o_{10}, o_{11}, o_{12}, o_{14} \}, \text{respectively. Therefore, } C_{o_2} = \{ c_1, c_2 \} \text{ and } C_{o_3} = C_{o_{15}} = \{ c_2 \}. \text{ Objects other than } o_2, o_3, o_{15} \text{ do not have target users in } C, \text{i.e., } C_o = \emptyset. \text{ \( V_o \in O \} - \{ o_2, o_3, o_{15} \}. \]

Problem Statement The problem of continuous monitoring of Pareto frontiers is, given a set of users \( C \), their preference relations on attributes \( D \), and a set of continuously growing objects \( O \) with the latest object \( o \), find \( C_o \) — the target users of \( o \).

4 SHARING COMPUTATION ACROSS USERS

Algorithm Baseline A simple method to our problem will check, for every user, whether a new object belongs to the corresponding Pareto frontier. The pseudo code of this approach, named Baseline, is shown in Alg. 1. Upon the arrival of a new object \( o \), for every user \( c \), it sequentially compares \( o \) with the current Pareto-optimal objects in \( P_c \). 1) If \( o \) is dominated by any \( o' \) or \( o \) is identical to \( o' \), further comparison with the remaining objects in \( P_c \) is skipped. The case of \( o \) being dominated by \( o' \), \( o \) is disqualified from being a Pareto-optimal object; if \( o \) is identical to \( o' \), then \( o \) is Pareto-optimal, i.e., it is inserted into \( P_c \). 2) If \( o \) dominates any \( o' \), \( o \) is discarded from \( P_c \). It can be concluded already that \( o \) belongs to \( P_c \), but the comparisons should continue since \( o \) may dominate other existing objects in \( P_c \). 3) If \( o \) is not dominated by any object in \( P_c \), it becomes an element of \( P_c \). Readers familiar with the literature on skyline queries may have realized that the gist of the algorithm is essentially the basic skyline query algorithm [2]. The crux of its operation is based on an important property, that it suffices to compare new objects with only the Pareto-optimal objects, since any new object dominated by a non-Pareto-optimal object must be dominated by some Pareto-optimal objects too.

For a user \( c \), suppose the aforementioned baseline approach on average takes time \( i \) to maintain the Pareto frontier upon the entrance of an object. Maintaining \( n \) objects for all users in \( C \) needs \( O(n\cdot |C| \cdot i) \) time. The drawback of Baseline is it repeatedly applies the same procedure for every user. In terms of computation efficiency, the approach may become particularly unappealing when there are a large number of users and new objects constantly arrive. To counter this drawback, our idea is to share computations across users that exhibit similar preferences. To this end, our method is simple and intuitive. If several users share a set of preference tuples, it is only necessary to compare two objects once, if they attain the attribute values in the preference tuples. If an object is dominated by another object according to these common preference tuples, it is dominated with respect to all users sharing the same preferences. This idea guarantees to filter out only “true negatives” for these users, and it only needs to further discern “false positives” for each individual user.

Definition 4.1 (Common Preference Tuple and Relation). Given a set of users \( U \subseteq C \), an attribute \( d \in D \), and two values \( x, y \in \text{dom}(d) \), if \( (x, y) \) belongs to preference relation \( \succ_\triangle \) for all \( c \in U \), then it is called a common preference tuple. The set of common preference tuples of \( U \) on attribute \( d \) is denoted \( \succ_\triangle^U \), i.e., \( \succ_\triangle^U = \bigcap_{c \in U} \succ_\triangle \). By definition, \( \succ_\triangle^U \) also represents a strict partial order (Theorem 4.2, proof omitted). We call it a common preference relation. It can be viewed as the preference of a virtual user that is denoted \( U \).

\[ \triangle \]

THEOREM 4.2. \( \succ_\triangle^U \) is a strict partial order. \[ \triangle \]

Since, for each \( d \), \( \succ_\triangle^U \) is a strict partial order, the set of users’ preferences (i.e., the virtual user \( U \)’s preferences) regarding all attributes in \( D \) induce another strict partial order \( \succ_\triangle^U \) on objects.

Definition 4.3 (Pareto Frontier for \( U \)). An object \( o \) is Pareto-optimal with respect to \( U \) if no other object dominates it according to \( \succ_\triangle^U \). The Pareto frontier of \( O \) for \( U \) is denoted \( P_U \), i.e., \( P_U = \{ o \in O | \forall o' \in O, s.t. o' \succ_\triangle^U o \} \).

\[ \triangle \]

Example 4.4. From Table 2, \( \succ_\triangle^\text{CPU} \) = \{ (\text{dual, single}), (\text{dual, quad}), (\text{triple, single}), (\text{triple, quad}) \} \) and \( \succ_\triangle^\text{CPU} \) = \{ (\text{dual, single}), (\text{triple, single}), (\text{quad, single}), (\text{quad, dual}), (\text{quad, triple}) \}. According to Def. 4.1, the common preference relation of \( c_1 \) and \( c_2 \) is \( \succ_\triangle^{(c_1,c_2)} \) = \{ (\text{dual, single}), (\text{triple, single}), (\text{quad, single}) \}. Similarly, we can derive \( \succ_\triangle^{(c_1,c_2)} \) = \{ (\text{dual, single}), (\text{triple, single}), (\text{quad, single}) \}.

\[ \triangle \]

THEOREM 4.5. Given any set of users \( U \), for all \( c \in U \), \( P_U \supseteq P_c \) and \( P_U \subseteq \bigcap_{c \in U} P_c \).

\[ \triangle \]

Proof: We prove by contradiction. Suppose that there exists \( c \in U \) such that \( P_U \not\supseteq P_c \), which would mean there exists \( o \in O \) such that \( o \in P_U \) and \( o \not\in P_c \). That implies the existence of an \( o' \in O \) such that \( o' \succ_\triangle^U o \) and \( o' \not\succ_\triangle c \). However, by Def. 4.1, \( o' \succ_\triangle^U o \) implies \( o' \succ_\triangle c \). Therefore, the existence of \( o' \) is impossible. This contradiction eventually leads to that \( P_U \supseteq P_c \). Hence, \( P_U \subseteq \bigcap_{c \in U} P_c \), which implies \( P_U \subseteq \bigcap_{c \in U} P_c \) according to De Morgan’s laws.
LEMMA 4.6. Given any set of users $U$, for all $c \in U$, $P_c = \{ o \in P_U | o' \in P_U \text{ s.t. } o' \succ c \}$.

Example 4.7. In Table 2, $P_U = \{ o_2, o_4, o_5, o_8, o_11 \}$ and $P_{c_5} = \{ o_2, o_3, o_{15} \}$. $P_U \supseteq P_{c_5} \supseteq P_{c_2}$. Moreover, $P_{\bar{U}} = \{ o_1, o_4, o_5, o_6, o_7, o_9, o_{12}, o_{13}, o_{14} \}$ and $P_{\bar{U}} \cap P_{c_5} = \{ o_1, o_4, o_5, o_6, o_7, o_9, o_{12}, o_{13}, o_{14}, o_{15} \}$. $P_{\bar{U}} \subseteq P_{\bar{c_5}} \cap P_{\bar{c_2}}$. △

Theorem 4.5 suggests an appealing quality of the common preference relations of $U$. By $P_{\bar{U}} \supseteq P_{\bar{c}_i}$, the Pareto frontier of $U$ subsumes the Pareto frontier of every user member in $U$. What it means is that, if we simply compute the Pareto frontier of $U$, we get to retain all the objects that we eventually look for. Consider $P_e$ as the ground truth and $P_{\bar{U}}$ as the predictions. The objects that are filtered out ($P_{\bar{U}}$) are all “true negatives” and there are no “false negatives”. The set $P_{\bar{U}}$ may contain “false positives”, which we just need to throw out after further verification, as Lemma 4.6 suggests.

This approach’s merit is the potential saving on object comparisons. For a cluster of users, many non Pareto-optimal objects may be filtered out altogether for all the users, without incurring the same comparisons repeatedly for each user.

To capitalize on the above ideas, our method must answer three questions. (1) How to find users sharing similar preferences? (2) For a set of similar users $U$, how to maintain the corresponding Pareto frontier $P_U$ based on their common preference relations $\succ^U_d$ for different attributes $d$? (3) For each user $c \in U$, how to discern the “false positives” in $P_U$ and thus find $P_c$. Note that the second and the last challenges need to be addressed for constantly arriving new objects.

For (1), our method is to cluster users based on the similarity between their preference relations. While many clustering methods have been developed for various types of data, none is specialized in clustering partial orders. Our clustering method is discussed in Sec. 5. For (2) and (3), our algorithm takes a filter-then-verify approach and is thus named FilterThenVerify, of which the pseudo code is displayed in Alg 2.

Alg. FilterThenVerify Upon the arrival of a new object $o$, for every cluster $U$, FilterThenVerify compares $o$ with the current members of $P_U$ based on the preference relations of the virtual user $U$. Various actions are taken, depending on the comparison outcomes, as follows:

1. If $o$ dominates any $o'$ in $P_U$ according to $\succ^U_d$ of all relevant $d$, $o'$ is removed from $P_U$ (Line 7 of Procedure updateParetoFrontier(U) in Alg. 2). For every $c \in C$ such that $o' \in P_c$, $o'$ is also discarded from $P_c$ (Line 6 of Procedure updateParetoFrontier(U)).
2. If $o$ is dominated by any $o'$ in $P_U$, then $o$ does not occupy the Pareto frontier of any user in $U$ (Theorem 4.5). Further operations involving $o$ are unnecessary (Line 8 of Procedure updateParetoFrontier(U)).
3. After comparing $o$ with all current objects in $P_U$, if it is realized that $o$ is not dominated by any $o'$, then $o$ becomes a member of $P_U$ (Line 9 of Procedure updateParetoFrontier(U)). Furthermore, for each $c \in U$, $o$ is further compared with the members of $P_c$ based on the preference relations of $c$, by using Procedure updateParetoFrontier of Alg. 1 (Line 6 of Alg. 2).

Example 4.8. In this example we explain the execution of FilterThenVerify on Table 1 and Table 2. Suppose users $c_1$ and $c_2$ form a cluster $U$, of which the preference relations are depicted in Table 2. The existing objects are $o_1$ to $o_{15}$, and $o_{15} = \{ 16', Lenovo, quad \}$ is the object that just becomes available. Before $o_{15}$ arrives, the Pareto frontier of $U$ is $P_U = \{ o_2, o_3, o_7, o_{10} \}$.

Algorithm 2: FilterThenVerify

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\textbf{Input: } U_1, U_2, \ldots, U_m \text{; clusters of users; } O \text{; existing objects; } o \text{; a new object }
\textbf{Output: } C_o \text{; target users of } o
1 \text{ foreach } U \in \{ U_1, U_2, \ldots, U_m \} \text{ do }
2 \quad \text{isPareto} \leftarrow \text{updateParetoFrontier}(U, o);
3 \quad \textbf{if } \text{isPareto} \text{ then }
4 \quad \quad \textbf{foreach } c \in U \text{ do }
5 \quad \quad \quad \text{if } \text{isPareto} \text{ then }
6 \quad \quad \quad \quad \text{P_c} \leftarrow \text{P_c} \setminus \{ o' \};
7 \quad \quad \quad \quad \text{P_U} \leftarrow \text{P_U} \setminus \{ o \};
8 \quad \quad \text{else if } o \succ U \text{ then } \text{isPareto} \leftarrow \text{false};
9 \quad \textbf{if } \text{isPareto} \text{ then } P_U \leftarrow P_U \cup \{ o \};
\textbf{return } \text{isPareto} ;
```

The algorithm starts by comparing $o_{15}$ with each element in $P_U$. As $o_{15}$ dominates $o_7 = \{ 9.5'', Lenovo, quad \}$ according to $U$’s preference relations, $o_7$ is discarded from $P_U$. Before $o_{15}$ arrives, $o_7$ also belongs to $P_{c_2}$. Therefore, $o_7$ is removed from $P_{c_2}$ as well. $o_{15}$ does not dominate any other object in $P_U$. It is not dominated by any either. Hence, it is inserted into $P_U$.

$o_{15}$ is further compared with the existing members of $P_{c_1}$ and $P_{c_2}$. It is dominated by $o_{12} = \{ 14'', Apple, dual \}$ according to $c_1$’s preference relations. Thus it is not part of $P_{c_1}$. According to $c_2$’s preferences, $o_{15}$ does not dominate any existing Pareto optional object (except the aforementioned $o_7$ which by now is already discarded). Therefore $P_{c_2}$ is not further changed and $o_{15}$ becomes part of $P_{c_2}$. Overall, $C_{o_{15}} = \{ c_2 \}$.

Moreover, consider the arrival of $o_{16} = \{ 16'', Toshiba, single \}$ after $o_{15}$. In the process of comparing $o_{16}$ with $P_U = \{ o_2, o_3, o_{15}, o_{16} \}$, it is realized that $o_{16}$ is dominated by $o_{12}$ according to $U$’s preference relations. Therefore, it does not belong to $P_U$. It is thus unnecessary to further compare $o_{16}$ with $P_{c_2}$, or $P_{c_2}$. $C_{o_{16}} = \emptyset$. Thereby, updateParetoFrontier(U) acts as a sieve to filter out non Pareto-optimal objects such as $o_{16}$. In this way FilterThenVerify reduces computation cost by avoiding repeated comparisons with such objects. △

Complexity Analysis of Algorithm 2 According to previous study, given the number of attributes, the computation of Pareto frontier is polynomial with respect to the number of objects. Therefore, given a set of attributes $D$, the complexity of Pareto frontier maintenance for a user $c$ or a cluster $U$ is polynomial with regard to the corresponding candidate Pareto-optimal objects.

For a cluster $U$, suppose the aforementioned filter-then-verify approach takes $t'$ time on average to maintain the Pareto frontier upon the entrance of an object. Consider $k$ as the number of clusters. Therefore, maintaining $n$ objects for all clusters in $C$ needs $n \cdot k \cdot t'$ time. With regard to each cluster, consider the Pareto frontier includes $m$ object on average, i.e., $m$ objects occupy the candidate Pareto-optimal object for each users in the
corresponding cluster. Now with regard to a particular user \( c \), suppose FilterThenVerify takes \( t'' \) time on average to maintain the Pareto frontier for each objects in \( m \). Hence, all users in \( C \) needs \( O(n \cdot (|C| \cdot t'')) \) time to maintain \( m \) objects. In summation, FilterThenVerify needs \( O(n \cdot k \cdot t'') + (m \cdot |C| \cdot t'') \) time to find the target users for all objects. Now we compare FilterThenVerify and Baseline in terms of time complexity. We can assume that \( t \approx t' \) and \( k < |C| \). Therefore, \( n \cdot k \cdot t' \leq n \cdot |C| \cdot t \). Besides, it is intuitive that \( m < n \) and thus \( t'' < t \). Hence, \( m \cdot |C| \cdot t'' \leq n \cdot |C| \cdot t \). In summation, \((n \cdot k \cdot t'') + (m \cdot |C| \cdot t'') < n \cdot |C| \cdot t \).

Now the question is: how to maximize \( \sum_{c \in C} n \cdot |C| \cdot t' \)? The key is to minimize the number of clusters \( k \) as well as the filtered objects \( m \). As we discussed earlier, larger clusters tend to share fewer preference tuples which may leave a number of objects as candidate Pareto-optimal objects, i.e., \( m \approx n \). On the contrary, the presence of lots of tiny clusters, i.e., \( k \approx |C| \) makes the filter-then-verify ineffective. Apparently, there exists a tradeoff between \( k \) and \( m \). In conclusion, in order to optimize \( k \) and \( m \), the clustering should be effective.

### 5 SIMILARITY MEASURES FOR CLUSTERING USER PREFERENCES

This section discusses how to cluster users based on their preference relations. Our focus is on the similarity measures rather than the clustering method. The method we adopt is the conventional hierarchical agglomerative clustering algorithm [9]. At every iteration, the method merges the two most similar clusters. The common preference relation of the merged cluster \( U \) on each attribute \( d \), i.e., \( >^d_U \), is computed. It then calculates the similarity between \( U \) and each remaining cluster. Given two clusters \( U_1 \) and \( U_2 \), their similarity \( sim(U_1, U_2) \) is defined as the summation of the similarities between their preference relations on individual attributes, as follows. This resembles the high-level idea of using \( L_1 \) norm distance between centroids for measuring inter-cluster similarity in conventional hierarchical clustering.

\[
sim(U_1, U_2) = \sum_{d \in D} \simsim(U_1, U_2)
\]

Individual users’ and clusters’ preference relations on attributes are strict partial orders. No prior work studied clustering approaches or similarity measures for partial orders. Similarity measures commonly used in clustering algorithms assume numeric or categorical attributes. Kamishima et al. [10, 11] and Ukkonen et al. [21] cluster total orders but not partial orders. Given two totally ordered attributes, these works use the comparative ranks of the corresponding values to measure similarity. Clearly, such similarity measures are not applicable for partially ordered attributes.

In this section we propose four different similarity functions for defining \( sim(U_1, U_2) \).

1) **Intersection size** This is simply the size of the intersection of \( >^d_{U_1} \) and \( >^d_{U_2} \), i.e., the number of common preference tuples of all users in the two clusters \( U_1 \) and \( U_2 \). It is defined as

\[
\simsim(U_1, U_2) = | >^d_{U_1} \cap >^d_{U_2} |
\]

**Example 5.1.** Table 3 shows three clusters \( U_1 \) (\( c_1, c_2 \)), \( U_2 \) (\( c_3, c_4 \)), and \( U_3 \) (\( c_5, c_6 \)) and the common preference relation associated with each cluster on attribute brand. \( U_1 \) and \( U_2 \) do not share any preference tuple and thus \( sim(U_1, U_2) = 0 \). \( U_1 \) and \( U_3 \) have \( sim(U_1, U_3) = 2 \). Similarly, \( U_2 \) and \( U_3 \) share \( sim(U_2, U_3) = 2 \) (Lenovo, Apple) and \( sim(U_2, U_3) = 2 \) (Lenovo, Toshiba).

2) **Jaccard similarity** The measure \( sim_1 \) captures the absolute size of the intersection of two preference relations. It does not take into account their differences. Consider three clusters \( U_1, U_2 \), and \( U_3 \) such that \( sim_1(U_1, U_2) = sim_1(U_1, U_3) \) (i.e., \( | >^d_{U_1} \cap >^d_{U_2} | = | >^d_{U_1} \cap >^d_{U_3} | \) and \( | >^d_{U_1} \cup >^d_{U_2} | < | >^d_{U_1} \cup >^d_{U_3} | \). We can argue that the similarity between \( U_1 \) and \( U_2 \) should be higher than (instead of equal to) that between \( U_1 \) and \( U_3 \), because \( U_1 \) and \( U_2 \) have a larger percentage of common preference tuples than \( U_1 \) and \( U_3 \). To address this limitation of \( sim_1 \), we define the Jaccard similarity between two preference relations as their intersection size over their union size, i.e., the ratio of common preference tuples to all preference tuples in the two preference relations. Formally, \( Jaccard(U_1, U_2) = \frac{| >^d_{U_1} \cap >^d_{U_2} |}{| >^d_{U_1} \cup >^d_{U_2} |} \).

**Example 5.2.** Continue Example 5.1. \( >^d_{U_1}\) and \( >^d_{U_2}\) have 6 preference tuples in total while \( >^d_{U_1} \) and \( >^d_{U_3} \) have 7. Thus, \( Jaccard(U_1, U_2) = 6/7 \) and \( Jaccard(U_1, U_3) = 7/8 \).

3) **Weighted intersection size** Intersection size and Jaccard similarity are based on the cardinalities of intersection and union sets of preference relations. In counting the cardinalities, they both treat all preference tuples equal. We argue that this is counter-intuitive. Values at the top of a partial order matter more than those at the bottom, in terms of their impact on which objects belong to the Pareto frontier. Accordingly we introduce **weighted intersection size**, a modified version of intersection size \( sim_1 \). In counting the common preference tuples of two preference relations, it assigns a weight to each preference tuple. Formally,

\[
\simsim_{w}(U_1, U_2) = \sum_{(c, c') \in >^d_{U_1} \cap >^d_{U_2}} \min(D(c, c')) + \min(D(c, c'))
\]

In the above equation, with regard to an attribute \( d \), the similarity between two clusters’ preference relations is a summation over their common preference tuples. For each common preference tuple \( (c, c') \), it computes the average weight of the better value \( c \) with respect to \( U_1 \) and \( U_2 \), respectively. Given a cluster \( U \), \( S_D^U \) is the set of **maximal values** in the partial order \( >^d_U \) and \( D(s, c') \) for each \( s \in S_D^U \) is the shortest distance from \( c' \) to \( c \) in \( >^d_U \). The weight of \( c' \) in \( U \) is the inverse of the minimal distance from any maximal value to \( c' \) (plus 1, to avoid division by zero). The concept of maximal value is defined as follows.

**Definition 5.3 (Maximal Value).** With regard to \( >^d_U \), value \( x \in dom(d) \) is a maximal value if no other value in \( dom(d) \) is preferred over \( x \). The set of maximal values for \( >^d_U \) is denoted \( S_D^U \). Formally, \( S_D^U = \{ x \in dom(d) \mid \exists y \in dom(d) s.t. (y, x) \in >^d_U \} \).

**Example 5.4.** Continue Example 5.1. The maximal values in \( >^d_{U_1} \) are \( \{Apple, Toshiba\} \) and \( >^d_{U_2} \) are \( \{Samsung, Apple\} \). In the partial order corresponding to \( >^d_{U_3} \), the minimal shortest distances to Apple, Lenovo, Samsung, and Toshiba from the maximal values (Apple, Toshiba) are \( 0, 1, 1 \) and \( 0 \), respectively. The corresponding weights are \( 1, 1, 2, 1 \) and \( 1, 1, 2, 1 \). Similarly, in the partial order corresponding to \( >^d_{U_2} \), the weights of Apple, Lenovo, Samsung and Toshiba are \( 1/3, 1/2, 1 \), and \( 1/3, 1/2, 1 \) and \( 1/3, 1/2, 1 \).
respectively. In $>_\text{brand}_{U_i}$, the corresponding weights are $1/2$, $1$, $1/3$ and $1/2$, respectively.

$U_1$ and $U_3$ have (Apple, Samsung) and (Lenovo, Samsung) as common preference tuples. For the two better-values in these preference tuples—Apple and Lenovo, the average weights are both $3/4$. The similarity $\text{sim}_{\text{brand}}^d(U_1, U_3) = \frac{1}{2} + \frac{1}{5} = \frac{1}{2}$. Similarly, $U_2$ and $U_3$ have (Lenovo, Apple) and (Lenovo, Toshiba) as common preference tuples. In $U_2$ and $U_3$, the average weight of Lenovo—the better-value in both common preference tuples—is $3/4$. The similarity $\text{sim}_{\text{brand}}^d(U_2, U_3) = \frac{1}{2} + \frac{1}{5} = \frac{1}{2}$.

4) Weighted Jaccard similarity This measure is a combination of the last two ideas—Jaccard similarity and weighted intersection size. As in Jaccard similarity, weighted Jaccard similarity computes the ratio of intersection size to union size. Similar to weighted intersection size, the values in a preference relation are assigned weights corresponding to their minimal distances to the preference relation’s maximal values. The measure’s definition is as follows.

$$\text{sim}^d(U_1, U_2) = \left\{ \begin{array}{ll} \frac{1}{\sum_{(u,v) \in U_1 \cap U_2} \min \{ D(u,v) \}} + \frac{1}{\sum_{(u,v) \in U_1 \cap \neg U_2} \min \{ D(u,v) \}} & \text{if } |U_1| + |U_2| > 0 \\ \frac{1}{|U_1| + |U_2|} & \text{otherwise} \end{array} \right.$$  \hspace{1cm} (5)

Example 5.5. Continue Example 5.4. Now $\text{sim}_{\text{brand}}^d(U_1, U_3) = \frac{1}{\left(1/3\right)\left(1/3\right) + \left(1/5\right)\left(1/5\right)} = \frac{1}{2}$. Since $>_\text{brand}_{U_1} = \{(\text{Apple}, \text{Lenovo}), (\text{Toshiba}, \text{Samsung})\}$ and $>_\text{brand}_{U_3} = \{(\text{Lenovo}, \text{Apple}), (\text{Lenovo}, \text{Toshiba})\}$, the similarity $\text{sim}_{\text{brand}}^d(U_1, U_3) = \frac{1}{2}$. Similarly, $\text{sim}_{\text{brand}}^d(U_2, U_3) = \frac{1}{2}$.

6 APPROXIMATE USER PREFERENCES

Two conflicting factors have crucial impacts on the effectiveness of FilterThenVerify. One is the size of the common preference relations. The other is the size of the clusters. Specifically, the more preference tuples a cluster’s users share, the more objects can be filtered out and thus the less verifications need to be done for individual users. On the contrary, the more users a cluster contains, the more repeated comparisons are avoided for these individual users. There is a clear tradeoff between these two factors, since larger clusters (i.e., more users in each cluster) naturally leads to smaller common preference relations.

Our approach to this challenge is approximation. As discussed in Sec. 1, it suffices for many applications to approximately identify target users. In this section, we show that we can find such approximation through a relaxed notion of common preference tuple, namely approximate common preference tuple. For a set of users, it allows a preference tuple to be absent from a tolerably small subset. If a sizable subset of the users agree with the preference tuple, it is considered an approximate common preference tuple. This relaxation addresses the aforementioned concern, since more approximate common preferences lead to larger clusters.

6.1 Approximate Common Preference Tuples and Relations

Definition 6.1 (Approximate Common Preference Tuple and Relation). Given a set of users $U \subseteq C$, an attribute $\theta \in D$ of which $|\text{dom}(\theta)| = m$, consider $A_1, \ldots, A_m$ which is an ordered permutation of all possible preference tuples $(x, y) \in \text{dom}(\theta) \times \text{dom}(\theta) | x \neq y$ such that $\text{freq}(A_i) \geq \text{freq}(A_{i+1})$ for $i \in [1, P^m]$, in which $\text{freq}(A_i)$ denotes the percentage of users in $U$ whose preference relations contain preference tuple $A_i$. The approximate common preference relation $\triangledown_U^d$ is defined as $\triangledown_U^d = \{ (A_i) \} \cap \neg \theta_1$ by $\text{freq}(A_i) > \theta_2$ and $\text{freq}(A_1) = 1$, where $R_i$ is defined as

$$R_i = \left\{ \begin{array}{ll} (A_1) & \text{if } i = 1 \\ (R_{i-1} \cup \{ A_i \})^+ & \text{if } R_{i-1} \cup \{ A_i \} \text{ is a strict partial order} \\ R_{i-1} & \text{otherwise} \end{array} \right.$$  \hspace{1cm} (6)

and $\theta_1$ and $\theta_2$ are two given thresholds. $\theta_1$ limits the size of the resulting $\triangledown_U^d$ while $\theta_2$ excludes infrequent preference tuples from $\triangledown_U^d$.

By this definition, the resulting approximate preference relation always includes the common preference tuples. The remaining possible preference tuples are considered in descending order of their frequencies, since preference tuples with higher frequencies are shared by more users. A preference tuple is included into $\triangledown_U^d$ only if its reverse tuple is not included. This guarantees asymmetry. Furthermore, when a preference tuple is included into $\triangledown_U^d$, the transitive closure of the updated $\triangledown_U^d$ is also included. This guarantees transitivity. Irreflexivity is guaranteed too since $A_1, \ldots, A_m$ does not include preference tuples in the form of $(x, x)$. These altogether assure $\triangledown_U^d$ is a strict partial order. Given an append-only database of objects, a strict partial order ensures that the preference query results are independent of the order by which objects are appended to the database. Therefore, $\triangledown_U^d$ can be viewed as the preference of a virtual user (denoted $U$) on attribute $d$.

Moreover, we denote the Pareto frontier of $O$ for $U$ as $P_U$. $\theta_1$ and $\theta_2$ regulate the size of $\triangledown_U^d$. A pair of large $\theta_1$ and small $\theta_2$ allows $\triangledown_U^d$ to include infrequent preference tuples. In such a case the approximate common preference relation becomes ineffective, since Procedure updateParetoFrontierU in Alg. 2 may retain a large number of candidates that must be verified for each $c \in U$. On the other hand, a pair of small $\theta_1$ and large $\theta_2$ may limit $\triangledown_U^d$ to contain only $\triangledown_U^d$ in which case the concern regarding small common preference relation remains.

As Def. 6.1 itself is procedural, it naturally corresponds to a greedy algorithm for constructing approximate preference relation $\triangledown_U^d$. The pseudo code GetApproxPreferenceTuples is in Alg. 3. First, all the common preference tuples are included (Lines 2-3). After that, preference tuples are considered in the order of frequency, as long as the two thresholds are satisfied (Line 4). For each preference tuple in consideration, if it together with all chosen tuples hitherto do not violate the properties of a strict partial order, their transitive closure is included into the approximate preference relation (Lines 6-7).
Algorithm 3: GetApproxPreferenceTuples

Input: \( A_i \); ordered permutation of all possible preference tuples, defined on \( \text{dom}(d) \), in descending order of their frequencies among users \( U \), \( \theta_1 \) and \( \theta_2 \) thresholds.

Output: \( \succsim^d_U \): approximate common preference relation of \( U \) on attribute \( d \).

1. for \( i = 1 \) to \( \text{golddom}(d) \) do
2. if \( \text{freq}(A_i) = 1 \) then \( \succsim^d_U \leftarrow \succsim^d_U \cup \{A_i\}; \) continue;
3. if \( \succsim^d_U \not\subseteq \theta_1 \) or \( \text{freq}(A_i) \leq \theta_2 \) then break;
4. if \( \succsim^d_U \) is a strict partial order then \( \succsim^d_U \leftarrow (\succsim^d_U \cup \{A_i\})^*; \)
5. return \( \succsim^d_U \).

Example 6.2. We use Figure 1 to explain the execution of GetApproxPreferenceTuples. Figure 1a depicts three users’ preference relations on brand. Suppose together these three users form a cluster. Assume \( \theta_1 = 7 \) and \( \theta_2 = 60\% \).

Table 5 shows the frequencies of all possible preference tuples after sorting. For instance, since all users prefer Apple to Toshiba, the corresponding frequency is 3/3; the frequency of \( \{\text{Apple}, \text{Samsung}\} \) is 2/3 as two of these three users prefer Apple to Samsung. At first GetApproxPreferenceTuples includes the common preference tuple \( \{\text{Apple}, \text{Toshiba}\} \) to \( \succsim^d_U \). It then includes \( \{\text{Apple}, \text{Samsung}\} \), \( \{\text{Lenovo}, \text{Toshiba}\} \), and \( \{\text{Toshiba}, \text{Samsung}\} \) as approximate preference tuples too. Furthermore, upon the addition of \( \{\text{Toshiba}, \text{Samsung}\} \), GetApproxPreferenceTuples includes \( \{\text{Lenovo}, \text{Samsung}\} \) as well since \( \{\text{Lenovo}, \text{Toshiba}\} \) and \( \{\text{Toshiba}, \text{Samsung}\} \) transitively induce it. The algorithm then considers \( \{\text{Samsung}, \text{Lenovo}\} \), which is disqualified since its reverse tuple \( \{\text{Lenovo}, \text{Samsung}\} \) is already included. The tuples will not form a strict partial order. The algorithm stops at \( \{\text{Apple}, \text{Lenovo}\} \) because its frequency is below the threshold 60\%.

Figure 1b illustrates the sequence of the included tuples and Fig.1c depicts the output approximate preference relation in the form of a Hasse diagram.

6.2 False Positives and False Negatives due to Approximation

FilterThenVerify (Alg.2) is extended to use approximate preference tuples and thus we rename it FilterThenVerifyApprox. The algorithm itself remains the same. Procedure updateParetoFrontierU maintains \( \tilde{P}_U \) as the candidate Pareto frontier. The algorithm eventually returns \( P_c \) for each user \( c \in U \), in which \( \tilde{P}_c = \{o \in P_U | \exists o' \in \tilde{P}_U \text{ s.t. } o' >_c o\} \), i.e., \( \tilde{P}_U \supseteq P_c \). Thus, \( C_\theta = \{c \in C | o \in P_c\} \). We use the example below to explain its execution over approximate preference relations.

Example 6.3. Consider Example 4.8, but use the approximate preference relations associated with virtual user \( \tilde{U} \) in Table 2. Upon the arrival of \( o_{15} \), it is compared with the elements in \( \tilde{P}_U = \{o_2, o_7\} \). \( \tilde{P}_U \) becomes \( \{o_2, o_7\} \) since \( o_{15} \) dominates \( o_2 \). \( o_2 \) is then also removed from \( \tilde{P}_c, o_{15} \) is further compared with \( \tilde{P}_c = \{o_7\} \) and \( \tilde{P}_c = \{o_7\} \), which does not lead to any further change. Overall, \( C_{o_{15}} = \{o_7\} \). The target users using approximate preference relations remain identical to the exact ones, i.e., no loss of accuracy in this case.

The rest of this section focuses on the accuracy of FilterThenVerifyApprox. It produces false positives if there exists such an \( o \) that \( o \in P_c \) but \( o \notin \tilde{P}_U \). It produces false negatives if there exists such an \( o \) that \( o \notin P_c \) but \( o \in \tilde{P}_U \). Below we present Theorems 6.5 and 6.7 to analyze how \( \tilde{P}_U \) and \( P_c \) relate to \( P_U \) and \( P_c \).

Theorem 6.5. Given a set of users \( U \) and an attribute \( d \), the common preference relation \( \succsim^d_U \) and an approximate common preference relation \( \succsim^d_U \) satisfy the following properties:

1) The approximate preference tuples are a superset of the common preference tuples, i.e., \( \succsim^d_U \supseteq \succsim^d_U \).
2) If any preference tuple along with its reverse tuple do not belong to the approximate common preference relation, neither of them belongs to the common preference relation either, i.e., \( (x,y) \notin \succsim^d_U \land (y,x) \notin \succsim^d_U \Rightarrow (x,y) \notin \succsim^d_U \land (y,x) \notin \succsim^d_U \).

Proof: We prove by contradiction. Suppose \( \tilde{P}_U \not\subseteq \tilde{P}_U \), which would mean there exists \( o \in O \) such that \( o \in \tilde{P}_U \) and \( o \notin \tilde{P}_U \). That leads to the existence of an \( o' \) such that \( o' >_U o \) and \( o' \notin \tilde{P}_U o \).
However, $o' \succ \bar{o}$ implies $o' \succ_\emptyset \bar{o}$ because $\Sigma_o^d \geq \Sigma_{\bar{o}}^d$ for every $d$ (Lemma 6.4). Therefore, the existence of $\bar{o}'$ is impossible. This contradiction proves that $\hat{P}_U \subseteq P_U$.

**LEMMA 6.6.** Given any set of users $U$, for all user $c \in U$, $\hat{P}_U \supseteq P_c$.

**THEOREM 6.7.** Given any set of users $U$, for all user $c \in U$, $\hat{P}_U \cap P_c \subseteq \hat{P}_c$.

**Proof:** We prove by contradiction. Suppose $\hat{P}_U \cap P_c \not\subseteq \hat{P}_c$, which would mean there exists $o \in O$ such that $o \in \hat{P}_U \cap P_c$ and $o \not\in \hat{P}_c$. $o \not\in \hat{P}_c$ implies the existence of an $o' \in O$ such that $o' \in \hat{P}_c$ and $o' \succ_c o$ (since $o \in \hat{P}_U \cap P_c$ and thus $o \in \hat{P}_U$ which means $o' \not\in \hat{P}_U$). Since $o' \succ_c o$, $o \not\in \hat{P}_c$ (Def. 3.3) and thus $o \not\in \hat{P}_U \cap P_c$. In other words, the existence of $\bar{o}'$ is impossible. This contradiction proves that $\hat{P}_U \cap P_c \subseteq \hat{P}_c$.

Consider a cluster $U$ and a user $c \in U$. The Venn diagram in Fig. 2 shows the effect of approximation through depicting $O$ (rectangle), $\hat{P}_U$ (outer blue circle), $P_c$ (inner blue circle), and $\hat{P}_c$ (inner red ellipse). Beside, Table 6 elaborates the area covered by these sets while Table 7 shows the confusion matrix for $c$. Note that using approximate common preference relations results in false negatives (III). Mislabeled declaring III as not Pareto-optimal further allows false positives (V) to sneak in.

With these notations in place, we are ready to quantify the accuracy of FilterThenVerifyApprox using standard evaluation measures in information retrieval. Specifically, **precision** is the fraction of objects found by FilterThenVerifyApprox that are truly Pareto-optimal, i.e., $\frac{\sum_{o \in \hat{P}_U \cap P_c} \sum_{c \in \hat{P}_c} \hat{P}_o}{\sum_{c \in \hat{P}_c} \hat{P}_o}$. Recall is the fraction of Pareto-optimal objects that are correctly found by FilterThenVerifyApprox, i.e., $\frac{\sum_{o \in \hat{P}_U \cap P_c} \sum_{c \in \hat{P}_c} \hat{P}_o}{\sum_{c \in \hat{P}_c} \hat{P}_o}$. With regard to a specific user $c$, the algorithm’s precision, recall and accuracy can be represented using the areas in Fig. 2, as follows.

$$\text{precision} = \frac{| IV \cap \bar{V} |}{| IV \cap \bar{V} |}$$

$$\text{recall} = \frac{| IV |}{| IV |}$$

$$\text{accuracy} = \frac{| U \cap IV \cap \bar{V} |}{| U \cap IV \cap \bar{V} |}$$

### 6.3 Similarity Functions

To make the clustering solution in Sec. 5 compatible with approximate preference relations, we extend the similarity measures, using ideas inspired by the Jaccard similarity for non-negative multidimensional real vectors [4].

1. **Jaccard Similarity** Consider an attribute $d$ with $|\text{dom}(d)| = m$. For each cluster $U$, construct a vector $U = (U(1), U(2), \ldots, U(P_U^m))$. For $i \in [1, P_U^m]$, $U(i)$ represents the frequency of $A_i$ (Definition 6.1) in $U$. Given two clusters $U$ and $V$, their Jaccard similarity on attribute $d$ is

$$\text{sim}_d(U, V) = \frac{\sum_{i, j} \min(U(i, j), V(i, j))}{\sum_{i, j} \max(U(i, j), V(i, j))}$$

**Example 6.8.** Consider $U_1$ and $U_3$ in Table 3. Suppose $A_i$ for $i \in [1, P_U^m]$ are (Apple, Lenovo), (Apple, Samsung), (Apple, Toshiba), (Lenovo, Apple), (Lenovo, Samsung), (Lenovo, Toshiba), (Toshiba, Apple), (Toshiba, Lenovo), (Toshiba, Samsung), (Samsung, Apple), (Samsung, Lenovo), (Samsung, Toshiba). The two vectors are $U_1 = (2/2, 2/2, 0/2, 0/2, 2/2, 0/2, 0/2, 1/2, 2/2, 0/2, 0/2)$ and $U_3 = (0/2, 2/2, 1/2, 2/2, 0/2, 0/2, 0/2, 1/2, 0/2, 0/2)$. For instance, $U_1$ has 1/2 on the $8^{th}$ dimension since only one of the two users’ preference relations contains (Toshiba, Lenovo). Hence, $\text{sim}_d(U_1, U_3) = 0.36$.

2. **Weighted Jaccard Similarity** This measure, denoted as $\text{sim}_d^w$, extends the namesake measure in Sec. 5 with the idea above. Its definition is the same as Eq. 9 except that a value $U(i)$ in a vector represents the frequency of $A_i$ in $U$ that takes into consideration the weights explained in Sec. 5. Consider $A_i$ as the preference relation $(A_i(x), A_i(y))$. Its definition is as follows.

$$\text{sim}_d^w(U, V) = \sum_{i} \min\left(\frac{1}{|U|}, \frac{1}{|V|}, \frac{1}{\min|S_d^U|}, \frac{1}{\min|S_d^V|}\right) \frac{\min|S_d^U|}{\max|S_d^U|}$$

**Example 6.9.** In Table 3, in the partial order depicting $\preceq_{\text{brand}}$, the distance to Apple from the maximal value Lenovo is 1, i.e., the weight of Apple is 1/2. Since only one of the two users in $U_2$ has (Apple, Toshiba) in their preference relation, $U_2$ has $\frac{2}{4} = \frac{1}{2}$ on the $3^{rd}$-dimension. In this way, we get $U_1 = (2/2, 2/2, 0/2, 0/2, 1/2, 0/2, 2/2, 0/2, 0/2, 0/2)$ and $U_3 = (0/2, 1/2, 0/2, 0/2, 1/2, 0/2, 1/4, 0/2, 0/2, 0/2)$. Therefore, $\text{sim}_d^w(U_1, U_3) = 0.19$.

### 7 ALIVE OBJECT DISSEMINATION

In Sec. 1, we discussed motivating applications such as social network content dissemination, news delivery and product recommendation. The significance of a particular social network content (e.g. a post in Facebook) or a piece of news diminishes eventually. Similarly, in any inventory, products are consumed and perishable products expire over time. In other words, objects can have limited lifetime. Thus, upon the arrival of a new object, it needs to compete only with the alive objects. To meet this requirement, we extend our problem as continuous monitoring of Pareto frontiers over alive objects for many users and formalize it as finding Pareto frontiers over sliding window.

Suppose $O = \{o_1, o_2, \ldots, o_n\}$ is a stream of objects, in which the subscript of each object is its timestamp. We consider a sliding window as a sequence of $w$ recent objects. Upon the arrival of an incoming object $o_{in}$, an object $o_{out}$ expires if $in - out = w$. Specifically, the sliding window contains objects whose timestamps are in $[\text{out}, \text{in})$, i.e., an object $o_{in}$ is alive during $(\text{out}, \text{in})$ if $i \in (\text{out}, \text{in})$. Given the concept of sliding window, we extend the definition of Pareto frontier in Def. 3.3 and the problem statement in Sec. 3.

**Definition 7.1 (Pareto Frontier).** An alive object $o$ is Pareto-optimal with respect to $c$, if no other alive object dominates it. $\hat{P}_c = \{o_i \in O | o_j \not\in O \text{ s.t. } o_j \succ_o o_i \land i, j \in (\text{out}, \text{in})\}$. The target users of $o_{in}$ is $C_{o_{in}} = \{c \in C | o_{in} \in \hat{P}_c\}$ (Def. 3.4).

**Problem Statement** The problem of continuous monitoring of Pareto frontiers over sliding window is, given a set of users $C$, their preference relations on attributes $O$, and a stream of objects
We note that no prior work studied Pareto frontier maintenance with regard to strict partial orders over sliding window. [15, 16] any succeeding object. This observation is formalized as Theorem 7.2. By Theorem 7.2, we extend our algorithms to maintain a Pareto frontier buffer.

**Algorithm 4: BaselineSW**

**Input:** C: all users; P: Pareto frontier; \( P^B \): Pareto frontier buffer; \( o_{in} \): incoming object; \( o_{out} \): outgoing object

**Output:** \( C_{o_{in}} \): target users of \( o_{in} \)

```plaintext
1. foreach \( c \in C \) do
2.   if \( o_{out} \in P_c \) then
3.     foreach \( o \in P_{B_c} \) do
4.       if \( o\preceq o \) then
5.         mendParetoFrontierSW(c, o);
6.       \( P_{B_c} \leftarrow P_{B_c} \setminus \{o_{out}\} \);
7.       if \( o_{in} \) not dominated by \( P_c \) then
8.         updateParetoFrontierSW(c, o);
9.       refreshParetoBufferSW(c, \( o_{in} \));
10. return \( C_{o_{in}} \);
```

**Procedure:**

- \( isPareto \leftarrow true \)
- foreach \( o \in P_c \) do
  - if \( o \succ o \) then \( isPareto \leftarrow false \); break;

- Procedure: refreshParetoBufferSW(c, \( o_{in} \))

- \( P_{B_c} \leftarrow P_{B_c} \cup \{o_{in}\} \);
- foreach \( o \in P_{B_c} \) do
  - if \( o \succ o \) then \( P_c \leftarrow P_c \setminus \{o\} \);

**Algorithm 5: FilterThenVerifySW**

**Input:** \( \{U_1, U_2, ..., U_n\} \): all clusters; \( P \): Pareto frontier; \( P^B \): Pareto frontier buffer; \( o_{in} \): incoming object; \( o_{out} \): outgoing object

**Output:** \( C_{o_{in}} \): target users of \( o_{in} \)

```plaintext
1. foreach \( i = 1 \) to \( n \) do
2.   if \( o_{out} \in P_{U_i} \) then
3.     foreach \( o \in P_{B_{U_i}} \) do
4.       if \( o \succ o \) then
5.         isPareto \leftarrow mendParetoFrontierUSW(U_i, o);
6.       if \( isPareto \) then
7.         foreach \( o \in U_i \) do
8.           mendParetoFrontierSW(c, o);
9.       refreshParetoBufferSW(U_i, o);
10. if \( o_{in} \) not dominated by \( P_{U_i} \) then
11.     updateParetoFrontierSW(C, \( o_{in} \)); //Algorithm 4
12. return \( C_{o_{in}} \);
```

**Procedure:**

- \( isPareto \leftarrow true \)
- foreach \( o \in P_{U_i} \) do
  - if \( o \succ o \) then return false;

- Procedure: refreshParetoFrontierSW(U, \( o_{in} \))

- \( P_{U} \leftarrow P_{U} \cup \{o_{in}\} \);
- foreach \( o \in P_{B_{U}} \) do
  - if \( o \succ o \) then \( P_{U} \leftarrow P_{U} \setminus \{o\} \);

Definition 7.4 (Pareto Frontier Buffer). With regard to user \( c \) and the sliding window \((out, in]\), an alive object \( o \) belongs to the Pareto frontier buffer if it is not dominated by any succeeding object. The Pareto frontier buffer is \( P_{B_c} = \{ o \in O | \forall o_j \in O s.t. o_j \succ c \land i<j \} \). By definition, \( P_{B_c} \succeq P_c \) (Def. 7.1).

**Theorem 7.5.** Given a set of users \( U \), for all \( c \in U \), i) \( P_{B_U} \supseteq P_U \) and ii) \( P_{B_U} \supseteq P_{B_c} \).

**Proof:** i) Together Def. 7.1 and 7.4 imply that \( P_{B_U} \supseteq P_U \).

ii) We prove by contradiction. Suppose that there exists \( c \in U \) such that \( P_{B_U} \nsubseteq P_{B_c} \), which would mean there exists \( o \in O \) such that \( o \in P_{B_c} \) and \( o \notin P_{B_U} \). That implies the existence of an \( o' \in O \) such that \( o' \succ u \) and \( o' \npreceq c \). However, by Def. 4.1, \( o' \succ c \) implies \( o' \succ c \). Therefore, the existence of \( o' \) is impossible. In conclusion, \( P_{B_U} \supseteq P_{B_c} \).

Note that, BaselineSW needs to maintain an exclusive Pareto frontier buffer per cluster \((P_{B_U})\) while a Pareto frontier buffer per user \((P_{B_c})\) is sufficient for FilterThenVerifySW.

**Example 7.6.** Continue Example 7.3. We get \( P_{B_{U_1}} = \{ o_9, o_9, o_{10} \} \). In this case, \( o_8 \) is the only element of \( P_{C_1} \). Since \( o_8 \) or \( o_7 \) could never been qualified in Pareto-optimality as they arrive before \( o_9 \), we do not need to store them. Nevertheless, upon the expiration of \( o_8 \), either \( o_9 \) or \( o_{10} \) could attain Pareto-optimality during their lifetime if they are not dominated by any following object. Therefore, \( o_8 \) and \( o_{10} \) are stored in \( P_{B_{C_1}} \). For instance, \( o_{10} \) acquires Pareto-optimality during \((8, 13]\) as it does not dominated by any following object.
Table 8: Product Table

| Display | Brand | CPU |
|---------|-------|-----|
| 21      | 17    | Lenovo dual |
| 22      | 9.5   | Sony single |
| 23      | 12    | Apple dual  |
| 24      | 16    | Toshiba quad |
| 25      | 15.5  | Samsung single |
| 26      | 14    | Apple dual  |

Table 9: Content of Pareto Frontiers and Pareto Buffers During 3 Different Phases of Window of BaselineSW

| Window | $P_{c_1}$ | $P_{c_2}$ | $P_{B_{c_1}}$ | $P_{B_{c_2}}$ |
|--------|-----------|-----------|----------------|----------------|
| 1      | {o_1, o_2} | {o_3, o_4} | {o_5, o_6}     | {o_7, o_8}     |
| 2      | {o_1, o_2} | {o_3, o_4} | {o_5, o_6}     | {o_7, o_8}     |
| 3      | {o_1, o_2} | {o_3, o_4} | {o_5, o_6}     | {o_7, o_8}     |

Table 10: Content of Pareto Frontiers and Pareto Buffers During 3 Different Phases of Window of FilterThenVerifySW

| Window | $P_{c_1}$ | $P_{c_2}$ | $P_{U}$ | $P_{B_{c_1}}$ | $P_{B_{c_2}}$ |
|--------|-----------|-----------|---------|----------------|----------------|
| 1      | {o_1, o_2} | {o_3, o_4} | {o_5}   | {o_6, o_7}     | {o_8}          |
| 2      | {o_1, o_2} | {o_3, o_4} | {o_5}   | {o_6, o_7}     | {o_8}          |
| 3      | {o_1, o_2} | {o_3, o_4} | {o_5}   | {o_6, o_7}     | {o_8}          |

Figure 3: Venn diagram depicting $P_{c_1}$, $P_{c_2}$, $P_{U}$, $P_{B_{c_1}}$, and $P_{B_{c_2}}$

In our sliding window framework, upon the expiration of an outgoing object $o_{out}$, for all $c \in C$, at first BaselineSW calls Procedure mendParetoFrontierSW to mend $P_c$. Because at this point, the alive objects those are exclusively dominated by $o_{out}$, acquire Pareto-optimality. While $o_{in}$ arrives, if $o_{in}$ belongs to $P_c$, then Procedure updateParetoFrontierSW in BaselineSW discards objects that are dominated by $o_{in}$, thereby updates $P_c$ (Line 8). After that, Procedure refreshParetoBufferSW in BaselineSW repairs $P_{B_c}$. Specifically, $o_{in}$ replaces the alive objects from $P_{B_c}$ that it dominates (Line 9). Thus $P_{B_c}$ remains concurrent with Def. 7.4.

On the contrary, in case of FilterThenVerifySW, upon the expiration of an outgoing object $o_{out}$, Procedures mendParetoFrontierUSW and mendParetoFrontierSW together mend $P_U$ and $P_c$ for all $U \subseteq C$, for all $c \in U$. While $o_{in}$ arrives, if $o_{in}$ belongs to $P_U$, then Procedure updateParetoFrontierUSW discards objects from $P_U$ that are dominated by $o_{in}$ (Line 11). Now for all $c \in U$, if $c$ approves $o_{in}$ as a Pareto-optimal object, then Procedure updateParetoFrontierSW finds out the objects in $P_c$ dominated by $o_{in}$ and thereby removes them (Line 14). Lastly, Procedure refreshParetoBufferUSW repairs $P_{B_U}$ so that it includes only the objects which have the potentiality to acquire Pareto-optimality over time with regard to $U$ (Def. 7.4 and Theorem 7.5) (Line 15).

Example 7.7. The executions of BaselineSW and FilterThenVerifySW on Table 8 and Table 2 are briefly explained here. Consider $W$, in and out as 6, 7 and 1, respectively.

While the sliding window is at $[1, 6]$, $P_{c_1} = \{o_1, o_2\}$, $P_{c_2} = \{o_3, o_4\}$, $P_{B_{c_1}} = \{o_1, o_2, o_4, o_6\}$ and $P_{B_{c_2}} = \{o_3, o_4, o_5, o_6\}$. Upon the expiration of $o_1 = \langle 17, Lenovo, dual \rangle$, the window is at $[1, 6]$. Now BaselineSW checks whether $o_1$ belongs to $P_{c_1}$ and $P_{c_2}$. Since $o_1$ belongs to $P_{c_1}$, $P_{c_1}$ is mended to $\emptyset$. Upon the arrival of $o_2 = \langle 14, Apple, dual \rangle$, the window includes objects correspond to $[1, 7]$. At this point BaselineSW starts checking whether $o_2$ is qualified as an element of $P_{c_1}$ and $P_{c_2}$, sequentially. In both $P_{c_1}$ and $P_{c_2}$, $o_2$ takes the place of $o_3$ (Line 8). After that, $o_1$ is stored to $P_{B_{c_1}}$ and $P_{B_{c_2}}$. Furthermore, for both $P_{B_{c_1}}$ and $P_{B_{c_2}}$, BaselineSW finds out the objects dominated by $o_1$ and discards them, i.e., $\{o_5, o_6\}$ and $\{o_3, o_5, o_6\}$, respectively. As these dominated objects arrives before $o_1$, they could never acquire Pareto-optimality. Now $P_{B_{c_1}} = \{o_4, o_7\}$ and $P_{B_{c_2}} = \{o_4, o_7\}$ (Line 9) (Def. 7.4). We get that $C_{P_{c_1}} = \{c_1, c_2\}$. The content of Pareto frontiers and Pareto buffers at 3 phases of window of BaselineSW is shown in Table 9.

In case of FilterThenVerifySW, while the sliding window is at $[1, 6]$, $P_{c_1} = \{o_1, o_3\}$, $P_{c_2} = \{o_3, o_4\}$, $P_{U} = \{o_1, o_3, o_4\}$ and $P_{B_{U}} = \{o_1, o_3, o_4, o_5, o_6\}$. Upon the expiration of $o_1$, the algorithm checks whether $o_1$ belongs to $P_{U}$. Therefore, $P_{U}$ becomes $\{o_3, o_4\}$ while the window is at $[1, 6]$. Upon the arrival of $o_2$, the window includes objects correspond to $[1, 7]$. Now FilterThenVerifySW starts checking whether $o_2$ can occupy $P_{U}$. With respect to $U$, $o_2$ dominates $o_3$, i.e., $o_2$ replaces of $o_3$ in $P_{U}$ (Line 11) as well as in both $P_{c_1}$ and $P_{c_2}$ (Line 14). After that, $o_1$ is stored in $P_{B_{U}}$. Moreover, $o_3$ dominates $o_5, o_6$ and $o_4$ in $P_{B_{U}}$. Since each of these dominated objects in $P_{B_{U}}$ arrives before $o_1$, they could never been qualified in Pareto-optimality. Therefore, FilterThenVerifySW discards them from $P_{B_{U}}$, i.e., $P_{B_{U}} = \{o_4, o_7\}$ (Line 15) (Def. 7.4). Finally we get $C_{P_{c_1}} = \{c_1, c_2\}$. The content of Pareto frontiers and Pareto buffers at 3 phases of window of FilterThenVerifySW are shown in Table 10. The Venn diagram in Fig.3 depicts $P_{c_1}$, $P_{c_2}$, $P_{U}$ and $P_{B_{U}}$. Note that, while BaselineSW needs to maintain individual Pareto frontier buffer per user ($P_{B_{c_1}}$ and $P_{B_{c_2}}$), a shared Pareto frontier buffer per cluster ($P_{B_{U}}$) suffices for FilterThenVerifySW. In conclusion, along with Pareto frontier maintenance, FilterThenVerifySW prunes comparisons in terms of Pareto buffer maintenance. △

8 EXPERIMENTS

8.1 Experiment Setup

The algorithms were implemented in Java. The maximal heap size of Java Virtual Machine (JVM) was set to 16 GB. The experiments were conducted on a computer with 2.0 GHz Quad Core 2 Duo Xeon CPU running Ubuntu 8.10.

Datasets Currently there exists no publicly available dataset that captures real users’ preferences in partial orders. We thus simulated such partial orders using two real datasets of users’ preferences.

Movie Dataset We joined the Netflix dataset (netflixprize.com) with data from IMDB (imdb.com). The Netflix dataset contains the ratings (ranging from 0 to 5) given by users to movies. From IMDB we fetched the movies’ attribute values, including actors, directors, genres, and writers. In this way, we found the attributes of 12,749 Netflix movies. The goal is to, for each particular movie, identify users who may like it according to their preferences on those attributes. The mapping from our problem formulation to...
this dataset is the following: (i) \( O \) is the set of 12,749 movies. (ii) \( C \) is the set of users. It includes the 1,000 most active users based on how many movies they have rated. The excluded less active users is the subject of the well-known cold-start problem in recommendation systems and is outside the scope of this work. (iii) \( D = \{ \text{actor, director, genre, writer} \} \). (iv) Given a user, the partial order on each attribute is simulated based on their preferences on the attribute values. The preference between two values on affiliation (and similarly author) is based on the number of collaborations between the user and the affiliation/author and the number of citations. For conference and keyword, the preference between two values is based on number of publications and number of citations. More specifically, consider a user \( c \) and an affiliation (or similarly another author) \( a \). Suppose \( c \) has \( p_a \) collaborations with \( a \) and has cited articles from \( a \) \( q_a \) times. If \( (p_a > p_b) \lor (q_a > q_b) \), then \((a, b) \in \succ_c \text{affiliation} \). (or \((a, b) \in \succ_c \text{author}\)). With regard to a conference (keyword) \( x \), suppose \( c \) has \( r_x \) publications associated with \( x \) and has cited publications associated with \( x \) \( s_x \) times. If \((r_x > r_y) \lor (s_x > s_y) \), then \((x, y) \in \succ_c \text{conference} \) (or \((x, y) \in \succ_c \text{keyword}\)).

**Publication Dataset** We collected from the ACM Digital Library (dl.acm.org) 17,598 publications and their attributes, including affiliations, authors, conferences and topic keywords. The users are the authors themselves. The goal is to notify them about newly published articles. The recommendations are based on the users’ preference relations on the attributes. The mapping from our problem formulation to this dataset is the following: (i) \( O \) is the set of papers. (ii) \( C \) is the set of authors. It includes the 1,000 most prolific authors based on how many publications they have, similar to the 1,000 most active users in the movie dataset.

\( \mathcal{D} = \{ \text{affiliation, author, conference, keyword} \} \). The domain of attribute author is the same 1,000 authors in \( C \). (iv) Given a user, the partial order on each attribute is simulated based on their preferences on the attribute values. The preference between two values on affiliation (and similarly author) is based on the number of collaborations between the user and the affiliation/author and the number of citations. For conference and keyword, the preference between two values is based on number of publications and number of citations. More specifically, consider a user \( c \) and an affiliation (or similarly another author) \( a \). Suppose \( c \) has \( p_a \) collaborations with \( a \) and has cited articles from \( a \) \( q_a \) times. If \( (p_a > p_b) \lor (q_a > q_b) \), then \((a, b) \in \succ_c \text{affiliation} \). (or \((a, b) \in \succ_c \text{author}\)). With regard to a conference (keyword) \( x \), suppose \( c \) has \( r_x \) publications associated with \( x \) and has cited publications associated with \( x \) \( s_x \) times. If \((r_x > r_y) \lor (s_x > s_y) \), then \((x, y) \in \succ_c \text{conference} \) (or \((x, y) \in \succ_c \text{keyword}\)).

### 8.2 Baseline, FilterThenVerify, and FilterThenVerifyApprox

We conducted experiments to compare the performance of Base line, FilterThenVerify and FilterThenVerifyApprox. For FilterThenVerify (resp. FilterThenVerifyApprox), users are clustered by the conventional hierarchical agglomerative clustering algorithm [9] using the similarity functions in Sec. 5 (resp. Sec. 6.3) and, for each cluster, it extracts the common preference relation (resp. approximate common preference relation). The experiments use
three parameters which are number of objects (|O|), number of attributes (d), and branch cut (h). In hierarchical clustering, the branch cut h is a threshold that controls the number of clusters by governing the minimum pairwise similarity that two clusters must satisfy in order to be merged into one cluster. The sequential order of merging clusters is depicted as a tree called dendrogram. The branch cut thus controls where to cut the dendrogram. In Example 5.5, the set of clusters is \( \{\{c_1, c_2, c_5, c_6\}, \{c_3, c_4\}\} \) for \( h \in (0, \frac{1}{2}) \). This is because \( \text{sim}(U_4, U_2) = 0 \) where \( U_2 = \{c_3, c_4\} \) and \( U_4 \) is the cluster composed of \( c_1, c_2, c_5, \) and \( c_6 \).

Fig.4a shows, for each of the three methods on the movie dataset, how its cumulative execution time (by milliseconds, in logarithmic scale) increases while the objects (i.e., movies) are sequentially processed. Fig.5a depicts similar behaviours of these methods on the publication dataset. Fig.4b and Fig.5b, for the two datasets separately, further present the amount of work done by these methods, in terms of number of pairwise object comparisons (in logarithmic scale) for maintaining Pareto frontiers. The figures show that FilterThenVerify and FilterThenVerifyApprox beat Baseline by 1 to 2 orders of magnitude. The reason is as follows. With regard to a user \( c \), Baseline considers all objects as candidate Pareto-optimal objects and compares all pairs. On the contrary, FilterThenVerify eliminates an object \( o \) if the corresponding common preference tuples disqualify \( o \). FilterThenVerifyApprox incurs even less comparisons by benefiting from shared computations for clusters of users.

Fig.6a (Fig.7a) shows that the execution time of all these methods increased super-linearly by number of attributes (d). Fig.6b (Fig.7b) further reveals that the number of object comparisons also increases similarly. This is not surprising because more attributes result in larger Pareto frontiers, which makes it necessary for objects to be compared with more existing Pareto-optimal objects.

Table 11 reports the precision, recall and F-measure of FilterThenVerifyApprox on varying \( h \). We can observe that, when \( h \) got smaller, the recall slowly decreased. This is expected because smaller \( h \) results in larger clusters and potentially more approximate common preference tuples for each cluster. Those approximate common preference tuples cause false negatives—the domination and elimination of objects that are instead in the Pareto frontier under the true common preference tuples, which are a subset of the approximate common preference tuples. What can be more surprising is the almost perfect precision under the various \( h \) values in Table 11, i.e., almost no false positives were introduced into the results. For a user \( c \), an object \( o \) becomes a false positive if every single Pareto optimal object that dominates \( o \) becomes a false negative. As long as one of its dominating objects is not mistakenly filtered out, \( o \) will not be mistakenly introduced into the Pareto frontier. Therefore, an object is much less likely to become a false positive than a false negative. Overall, under the \( h \) values in Table 11, both precision and recall remain high. This may suggest that the thresholds \( \theta_1 \) and \( \theta_2 \) (Sec. 6.1) effectively ensure that the approximate common preference relation only includes frequent preference tuples and does not overgrow in size.

### 8.3 BaselineSW, FilterThenVerifySW, and FilterThenVerifyApproxSW

We further compare the performance of FilterThenVerifySW and FilterThenVerifyApproxSW with BaselineSW. In this regard, we simulated two data streams—movie and publication where \( O \) is composed of duplicated sequence of the corresponding dataset such that \( |O| = 1 \) million. Following [20], we experimented with windows of size 400, 800, 1,600, and 3,200, as well as report the cumulative execution times in milliseconds. In this direction, Fig.8a demonstrates the cumulative execution times (by milliseconds, in logarithmic scale) of the aforementioned methods on the movie stream. Fig.8a shows that the cumulative execution times increase super-linearly by \( W \) as wider window broadens the size of Pareto frontiers. These figures illustrate that both FilterThenVerifySW and FilterThenVerifyApproxSW outperformed BaselineSW by 1 to 2 orders of magnitude, which concur with the comparative behaviours of FilterThenVerify, FilterThenVerifyApprox and Baseline. This concurrence is also applicable for the publication stream (Fig.9a).

Fig.8b (Fig.9b) further reveals the amount of work done by these solutions, in aspect of compared objects (in logarithmic scale) to maintain Pareto frontiers over sliding window. Moreover, Fig.10a (Fig.11a) depicts the effectiveness of FilterThenVerifySW (FilterThenVerifyApproxSW) on varying \( d \). Fig.10b (Fig.11b) clarifies Fig.10a (Fig.11a) through illustrating the number of compared objects. The reason behind the comparative behaviour of Baseline, FilterThenVerify and FilterThenVerifyApprox is also applicable in this case. In addition, BaselineSW maintains exclusive Pareto buffer for each user (\( P^*B_c \)) while FilterThenVerifySW shares a Pareto buffer across users in a cluster (\( P^*B_L \)). Therefore, in sliding window protocol, the filter-then-verify approach attains the benefit of clustering in greater extent.

Table 12 demonstrates the precision, recall and F-measure of FilterThenVerifyApproxSW on varying \( W \) and \( h \). We can observe that the recall declines slowly by \( h \). Nevertheless, \( h \) does not have significant impact on the efficacy of FilterThenVerifyApproxSW. Besides, the loss of accuracy is due to false negatives rather than false positives. These behaviors concur with FilterThenVerifyApprox and the reasons behind are same as before. In addition, Table 12 reveals that \( W \) does not have noticeable impact on efficacy and FilterThenVerifyApprox remains effective on varying \( W \).

### 9 CONCLUSION

We studied the problem of continuous object dissemination, which is formalized as finding the users who approve a new object in Pareto-optimality. We designed algorithm for efficient finding of target users based on sharing computation across similar preferences. To recognize users of similar preferences, we studied the novel problem of clustering users where each user’s preferences are described as strict partial orders. We also presented an approximate solution of the problem of finding target users, further improving efficiency with tolerable loss of accuracy. Lastly, we performed a thorough experimental study to evaluate the efficiency and effectiveness of the proposed solutions.

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**REFERENCES**

[1] O. Barndorff-Nielsen and M. Sobel. On the distribution of the number of admissible points in a vector random sample. Theory of Probability & Its Applications, 11(2), 1966.

[2] S. Borosenyi, D. Kissmann, and K. Stocker. The skyline operator. In ICDE, 2001.

[3] C.-Y. Chan, P.-K. Eng, and K.-L. Tan. Stratified computation of skylines with partially-ordered domains. In SIGMOD, 2005.

[4] F. Cherichetti, R. Kumar, S. Pandey, and S. Vassilvitskii. Finding the jaccard median. In SODA, 2010.

[5] J. Chomicki. Preference formulas in relational queries. TODS, 28(4), 2003.

[6] E. Delic and B. Seeger. Efficient computation of reverse skyline queries. In VLDB, 2007.
Figure 8: Effect of window size on the movie dataset. Varying $W$, $|O| = 1M$, $h = 0.55$, $d = 4$.

Figure 9: Effect of window size on the publication dataset. Varying $W$, $|O| = 1M$, $h = 0.55$.

Figure 10: Comparison of BaselineSW, FilterThenVerifySW and FilterThenVerifyApproxSW on the movie dataset. Varying $d$, $W = 3,200$, $|O| = 1M$, $h = 0.55$.

Figure 11: Comparison of BaselineSW, FilterThenVerifySW and FilterThenVerifyApproxSW on the publication dataset. Varying $d$, $W = 3,200$, $|O| = 1M$, $h = 0.55$.

Table 12: The precision, recall and F-measure (in percentage) of FilterThenVerifyApproxSW. Varying $W$ and $h$, $|O| = 1M$, $d = 4$.