Earth Rotation Parameter Estimation by GPS Observations

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ABSTRACT The methods of Earth rotation parameter (ERP) estimation based on IGS SINEX file of GPS solution are discussed in detail. There are two different ways to estimate ERP: one is the parameter transformation method, and the other is direct adjustment method with restrictive conditions. By comparing the estimated results with independent copyright program to IERS results, the residual systemic error can be found in estimated ERP with GPS observations.

KEYWORDS Earth rotation parameter; parameter transformation; systemic error; daily SINEX file

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Introduction

The orientation of the earth in space is described by three kinds of movement. The first of which is the change of the earth axis’ direction relative to the space. The second is the change of the earth axis’ direction relative to the earth itself. The third is the velocity change of the axial rotation of the earth. The periodic part of the first item is called as nutation, and the secular part is called as precession. The second item is called as polar motion. The third item is called as the velocity change of axial rotation of the earth or the change of the daily length. In generalization, the parameters which are used to describe the three quantificational movements of the earth rotation above are called Earth orientation parameter (EOP). In narrow sense, only the UT1 which describes the velocity change of the earth rotation (the polar motion correction has been added) and the polar coordinate \((x_p, y_p)\) which describes the polar motion can be called Earth rotation parameter (ERP). So EOP is equal to ERP adding nutation and precession.

Now the research of ERP estimation is mainly by the measurement of very long base interveining (VLBI), global positioning system (GPS), satellite laser ranging (SLR) and other special technology to determine the ERP, the movement of the plate, the deformation of the earth crust, the mutual action between the earth rotation and the earth shell, and the rule and mechanism of the change of the earth rotation, the physical and geometrical change, movement and reason of the earth’s surface.

In recent year, compared with VLBI and SLR, GPS has inestimable effect in determining the ERP for its time resolution has improved greatly. With GPS observations, the change of daily length and the polar motion value \((x_p, y_p)\) can be obtained. IGS and its Analysis Center (AC) participate in the maintenance of the world coordinate reference frame and the release of the earth rotation parameter by the International Earth Rotation Service (IERS). The domestic research institution in this field is mainly Shanghai Astronomical Observatory of the Chinese Academy of Sciences. As one of the world data processing center of VLBI, SLR and LLR, it...
provides EOP sequence and other analysis result to IERS before March 15th each year. The precision of EOP with VLBI is better than ±0.3 mas.

In this paper, the methods of ERP estimation with IGS daily GPS SINEX files are studied emphatically. For the algorithm realization of ERP estimation based on SINEX files, parameter transformation method is used.

1 Theory of parameter transformation

Parameter transformation can also be comprehended as “re-parameterization” or “parameter reconfiguration”. The essential of which is to express the parameter to be estimated as linear function of the new parameter, and then transform the corresponding normal equation system of the old parameter to the corresponding normal equation system of the new parameter through linear computation. By computing the new normal equation, the new parameter can be obtained.

Suppose that the error equation is:

\[ v = B\xi - l \]  (1)

Its normal equation is:

\[ N\xi = b \]  (2)

where \( N = B^T P B \), \( b = B^T P l \).

The normal equation can be transformed to the new form expressed by the new parameter \( \hat{x} \). Suppose that the transformation relation between the new parameter \( \hat{x} \) and the old parameter \( \xi \) is as follows:

\[ \hat{x} = C\xi + dx \]  (3)

Eq. (3) is the parameter transformation equation, so the relevant error equation Eq. (1) can be written as:

\[ v = B\hat{C}\xi \]  (4)

The corresponding normal equation is:

\[ C^T B^T P B\xi = C^T B^T P(I - Bdx) \]  (5)

Let \( \bar{N} = C^T B^T P B\xi = C^T N\xi, \bar{b} = C^T B^T P(I - Bdx) = C^T (b - Ndx) \), then

\[ \bar{N}\hat{x} = \bar{b} \]  (6)

After parameter transformation, the unknown parameter in new NEQ system \( \hat{x} \) is obtained.

The overlaying course of normal equation can be explained through the idea of parameter transformation. Suppose that in order to determine the common parameter \( \xi \), independent least square disposal has been carried out \( m \) times. Each independent result (\( i = 1, 2, \cdots, m \)) can be determined by the normal equation as follows:

\[ B^T_P B_i \xi = B^T_P l_i \]  (7)

The parameter \( \xi \) can be estimated by computing the normal equation. The matrix form of the equation is as follows:

\[ \begin{bmatrix} B^T_P & B_1 \\ B^T_P & B_2 \\ \vdots \\ 0 & B^T_P & B_m \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_m \end{bmatrix} = \begin{bmatrix} B^T_P l_1 \\ B^T_P l_2 \\ \vdots \\ B^T_P l_m \end{bmatrix} \]  (8)

Because each independent result \( \xi_i \) can be consolidated as \( \hat{x} \), in the final result, when \( \xi \) is treated as the fictitious observation of \( \hat{x} \), the parameter transformation equation correspond to \( \hat{x} \) is as follow:

\[ \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \\ \vdots \\ \hat{x}_m \end{bmatrix} = \begin{bmatrix} I \\ I \\ \vdots \\ I \end{bmatrix} \begin{bmatrix} \xi_1 \\ \xi_2 \\ \vdots \\ \xi_m \end{bmatrix} \]  (9)

where \( I \) is the unit matrix, its rank is the same as the number of rows of \( \hat{x} \). Then the combined normal equation is:

\[ \bar{N}\hat{x} = \bar{b} \]  (10)

where \( \bar{N} = \sum_{i=1}^{m} B^T_P B_i, \bar{b} = \sum_{i=1}^{m} B^T_P l_i \).

2 ERP estimation based on daily GPS SINEX file

The ERP form given by the IGS daily SINEX files are the ERP parameter value and its slope at the reference epoch (Fig. 1).

The continuity problem of the borderline for the daily ERP should be considered in the integrated data processing of continuous daily ERP. So it is necessary to transform the ERP value and its slope at the reference epoch to the ERP value of daily two borderlines. It is shown as follows (Fig. 2).
Therefore the parameter which expressed as $x = \left[x_{i+1/2}, \frac{dx}{dt}\right]^T$ must be transformed to the parameter expressed as $\tilde{x} = \left[x_i, x_{i+1/2}\right]^T$ by use of the parameter transformation method.

Considering Fig. 1 and Fig. 2, we have:

$$\tilde{x} = \begin{bmatrix} 1 & -\frac{t_2 - t_1}{2} \\ 1 & \frac{t_2 - t_1}{2} \end{bmatrix} \begin{bmatrix} x_{i+1/2} \\ \frac{dx}{dt} \end{bmatrix} = \begin{bmatrix} x_i \\ x_{i+1/2} \end{bmatrix}$$  \hspace{1cm} (11)

Corresponding to the parameter transformation equation $\tilde{x} = Cx + dx$, there is the following equation

$$C = \begin{bmatrix} 1 & -\frac{t_2 - t_1}{2} \\ \frac{t_2 - t_1}{2} & 1 \end{bmatrix}, \text{dx} = 0 \hspace{1cm} (12)$$

The parameter transformation equation above can be used to transform the daily ERP. The transformed ERP of back-to-back two days are illustrated as follows (Fig. 3).

For the integration of the ERP of multi-days, the following continuous condition at daily boundary of Earth rotation parameters should be added:

$$x_i - x_{i+1} = 0 \hspace{1cm} (13)$$

It follows that for the algorithm realization of ERP estimation two basic algorithm, the parameter transformation method and additional restrictive conditions method, are adopted. The algorithm includes following detailed steps.

1) Recover the daily normal equation. Considering the actual situation, suppose $x_i = \begin{bmatrix} x_c \\ \dot{x}_i \end{bmatrix}$ is the one-day earth rotation parameter, $i$ is the day number and $x_c$ is the earth rotation parameter at reference epoch (usually 12:00:00 at noon) of the $i$th day, $\dot{x}$ is the change rate of the earth rotation parameter on the $i$th day. Its normal equation system is:

$$\begin{bmatrix} N_{11} & N_{12} \\ N_{21} & N_{22} \end{bmatrix} \begin{bmatrix} x_c \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \hspace{1cm} (14)$$

This normal equation can be gained by transforming SINEX file, during this course the apriori restriction is eliminated. The earth rotation parameter can be estimated with other parameters or estimated alone after pre-eliminating other parameters. For convenience, only the disposal of normal equation corresponding to earth rotation parameter will be discussed.

2) Parameter transforming. Refer to Eq. (11), let $t_2 - t_1 = 1$ and transform the value of ERP $x_i$ and its change rate at reference epoch to the two boundary ERP values $\tilde{x}_i = \begin{bmatrix} x_a \\ x_b \end{bmatrix}$, then

$$\tilde{x}_i = \begin{bmatrix} 1 & -1/2 \\ 1/2 & 1 \end{bmatrix} \begin{bmatrix} x_c \\ \dot{x}_i \end{bmatrix} = \begin{bmatrix} x_a \\ x_b \end{bmatrix} \hspace{1cm} (15)$$

3) Form new normal equation. Referring to Eq. (3), we have $C = \begin{bmatrix} 1 & -1/2 \\ 1/2 & 1 \end{bmatrix}$ and $\text{dx} = 0$. Its new normal equation system is:
\[
\begin{bmatrix}
1 & 1 \\
-1 & 2
\end{bmatrix}
\begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix}
\begin{bmatrix}
1 \\
1
\end{bmatrix}
\].

(16)

Let \( \bar{N}_i \) be:

\[
\begin{bmatrix}
1 & 1 \\
-1 & 2
\end{bmatrix}X_i = \begin{bmatrix}
1 \\
1
\end{bmatrix}b_i
\].

Then

\[
\bar{N}_i X_i = b_i
\].

(17)

4) Overlap new normal equations of \( n \) days. Referring to Eq. (8), the \( n \)-days normal equation made up by daily new normal equations is:

\[
\begin{bmatrix}
1 & 1 \\
-1 & 2
\end{bmatrix}
\begin{bmatrix}
N_{11} & N_{12} \\
N_{21} & N_{22}
\end{bmatrix}
\begin{bmatrix}
1 \\
1
\end{bmatrix}
\].

(18)

5) Introduce into \( n-1 \) boundary continuous conditions. When comprehensively calculating the \( n \)-days ERP, \( n-1 \) boundary continuous conditions (i.e., the parameter value at the last epoch of the \( i \)th day is the same as the first one on the \((i+1)\)th day) of ERP should be added, and its form is

\[
\begin{bmatrix}
\bar{N}_1 \\
\bar{N}_2 \\
\vdots \\
\bar{N}_n
\end{bmatrix}X = \begin{bmatrix}
b_1 \\
b_2 \\
\vdots \\
b_n
\end{bmatrix}
\].

(19)

6) Integrated data processing of ERP. With the Eqs. (18) and (19), \( n \)-days ERP are integratively processed. There are two ways.

(1) Processing by indirect adjustment method with restrictive conditions.

To process by indirect adjustment method with restrictive conditions is treating Eq. (19) as additional parameter restriction. The common parameter restriction is \( \mathbf{C}' \mathbf{x} = \mathbf{w} \), which is corresponded to Eq. (19), then

\[
\mathbf{C}' = \begin{bmatrix}
0 & 1 & -1 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & 0 & \cdots & 0 & 1 & 0
\end{bmatrix}, \quad \mathbf{w} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0
\end{bmatrix}
\].

(20)

\[
\mathbf{x}_{2n+1} = [x_1, x_2, x_3, x_4, \ldots, x_m, x_n]^T, \quad \mathbf{w} = 0
\].

(20)

(2) Processing by parameter transformation method.

Eq. (19) including boundary continuous condition can also be achieved by parameter transformation. Corresponding to the parameter transformation equation \( \hat{\mathbf{x}} = \mathbf{C} \mathbf{x} + \mathbf{dx} \), there is

\[
\hat{\mathbf{x}}_{2n+1} = [x_1, x_2, x_3, x_4, \ldots, x_m, x_n]^T,
\]

\[
\begin{bmatrix}
1 & 0 & 0 & \cdots & 0 & 0 \\
0 & 1 & 0 & \cdots & 0 & 0 \\
0 & 0 & 1 & \cdots & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 1 \\
0 & 0 & 0 & \cdots & 0 & 0
\end{bmatrix}, \mathbf{dx} = \begin{bmatrix}
0 \\
0 \\
\vdots \\
0 \\
0 \\
0
\end{bmatrix}
\].

(21)

It should be noticed that the primary parameter is \( 2n \times 1 \) dimension, while the new one is \((n+1)\times 1 \) dimension. Eq. (6) can be adopted to compute parameters. In this paper the parameter transformation method is used to integratively process the ERP.

3 Example and analysis

The SINEX files of GPS daily solution in 2000 (sioigs10426, snx-sioigs10950, snx) supplied by http://garner.ucsd.edu/pub/combinations are used to estimate ERP with the software of generalized network adjustment based on coordinate pattern with independent property. Then the estimation result are compared with the result supplied by IGS Analysis Centers, and the difference is shown in Fig. 4 and Fig. 5.

In the figures, the dashed is the fitting
trendline. From Fig. 5, it is clear if only the GPS technology is used, the estimated polar motion parameter \( (y_p) \) has obvious systemic difference. Therefore, four parameters transformation are used to the estimated polar motion parameter to eliminate the systemic difference. The transformation equation is:

\[
\begin{bmatrix}
\Delta x_p \\
\Delta y_p
\end{bmatrix}_{\text{new}} = \begin{bmatrix}
\cos \alpha - \sin \alpha \\
\sin \alpha & \cos \alpha
\end{bmatrix} \begin{bmatrix}
x_p \\
y_p
\end{bmatrix}_{\text{old}} + (1 + m) \begin{bmatrix}
\cos \alpha - \sin \alpha \\
\sin \alpha & \cos \alpha
\end{bmatrix} \begin{bmatrix}
x_p \\
y_p
\end{bmatrix}_{\text{new}}
\]

(22)

where \( \begin{bmatrix}
\Delta x_p \\
\Delta y_p
\end{bmatrix}_{\text{new}} \) are translation parameters; \( m \) is the scale parameter; \( \alpha \) is revolving parameter; \( \begin{bmatrix}
x_p \\
y_p
\end{bmatrix}_{\text{new}} \) are results from by IERS; \( \begin{bmatrix}
x_p \\
y_p
\end{bmatrix}_{\text{old}} \) are results estimated only by GPS technology. After parameter transformation, the comparison of estimated result with IERS is shown in Fig. 6 and Fig. 7. As can be seen from Fig. 7, compared with Fig. 5, after four parameters transformation, the systemic error is well-eliminated.

Fig. 8 shows the comparison of the estimated UT1-UTC parameter with the IERS result, the mutually difference of two result is basically less than 0.1 ms.

4 Conclusions

The ERP can be estimated by use of IGS daily SINEX files produced by GPS tracking stations. The parameter transformation method can simplify the process. The processed result indicates that the systemic error will exist in the estimated ERP by only using GPS observations. As to the daily GPS SINEX files, why the distinct systemic error exist in the ERP, or whether this systemic error will affect other parameters estimation, and what its influenced magnitude will be, there is need to further study in the future.

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