Tunable storage of optical pulses in a tailored Bragg-grating structure

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Scenarios for controllable creation, trapping and holding of single and multiple solitons in a specially designed nonlinear Bragg grating (BG) are proposed. The setting includes a chirped BG segment, which is linked via a local defect to a uniform BG with a built-in array of defects. A parabolic relation between the trapping position of the incident soliton and its power is obtained. Simultaneous trapping of two and three solitons at different locations is demonstrated too.

I. INTRODUCTION

A possibility to dramatically reduce the velocity of ultrashort light pulses in various optical materials \cite{1-3} has drawn a great deal of attention, since the observation of slow \cite{4} and completely halted light \cite{5} in experiments. However, the controlled storage of this ultrashort light pulse has been a challenge, which hinders their use in broadband optical signal processing, such as ultrafast coding, decoding and multiplexing. Different settings were proposed to realized the slow light, such as electromagnetically induced transparency \cite{1,6,7}, simulated Brillouin \cite{8} and Raman \cite{9} scattering, coherent population oscillations \cite{2}, dispersion engineering in photonic-crystal waveguides \cite{3,11}, and others. Among these techniques, Bragg-grating (BG) structures, which induce very strong dispersion in a vicinity of the photonic bandgap \cite{11}, are used too, for the generation the slow light due to its tunability and cascadability in practical applications.

The relation between the delay and bandwidth of pulses was previously experimentally demonstrated in a linear resonant systems \cite{12}. To support the slow propagation of non-spreading pulses in the BG, the Kerr nonlinearity may be used, as it is not subject to the limitation of the trade-off between the pulse delay and bandwidth, which is inherent in the linear settings \cite{12}. Standing and moving robust light pulses (BG solitons), whose velocity may be orders of magnitude smaller than the velocity of light in the host material, were studied in detail theoretically \cite{13,14}. Producing quiescent pulses by collisions of counterpropagating BG solitons \cite{24} was predicted too.

In experiments, BG solitons, with the velocity corresponding to 76% of the speed of light in the uniform medium, were first produced in Ref. \cite{21}. Subsequently, a number of experiments \cite{11,22,24} were carried out to investigate slow-light effects in BG structures. In particular, high intensity of the pulse leads to a change of the local refractive index via the Kerr nonlinearity, which helps to enhance the retardation of pulses propagating in the BG structure. Recently, large femtosecond Kerr nonlinearity was experimentally demonstrated in cholesteric liquid crystals \cite{25}. Its large Kerr coefficient would considerably reduce the high intensity. The lowest experimentally demonstrated velocity of solitons in BGs is 0.16$c_0$ \cite{11}, where $c_0$ is the speed of light in vacuum.

However, there remain challenging problems impeding the use of the slow light pulses in BGs. First, the creation of very slow or standing optical modes in BG structures has not yet been reported in experiments. Second, to the best of our knowledge, no work has demonstrated, as yet, controllable trapping and storage of single and multiple pulses at different positions in BGs, a reason being that it is difficult to create suitable input pulses for this purpose, using uniform BGs. Indeed, as the optical fields inside the BG include forward- and backward-traveling waves, generating such very slow or quiescent pulses requires to couple forward and backward waves with nearly equal powers into the BG waveguide, doing which for the backward wave being obviously difficult.

Recently, we have proposed a specially tailored structure, built of a linearly chirped BG segment linked, through a local defect, to a uniform grating \cite{26}. This setting demonstrates a possibility to create very slow or standing stable pulses, the advantage being that the pulse coupled into the uniform grating may be manipulated by means of the chirped segment. This scheme makes it possible to prepare the right mix of forward- and backward-traveling fields. Using it, the creation of BG solitons with extremely small and zero velocities was predicted, offering an essential improvement of previously published results \cite{24,27}.

In this work, we aim to develop the above-mentioned setting by introducing a periodic set of defects into the uniform grating, to achieve controllable trapping and storage of stable single and multiple pulses at different positions in the BG. In section II, we present the necessary model, which is based on nonlinear coupled-mode

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equations (CMEs) for the BG. In section III, the operation of the system is demonstrated by means of CME simulations. The paper is concluded by Section IV.

II. THE MODEL

The purpose of introducing a chirped BG before the uniform grating with a periodic array of defects is to provide a suitable setting for the generation of slow pulses, which can be subsequently trapped in the main section of the system. First, the setting without the defect lattice is defined by means of the spatially modulated profile of the local refractive index [26]:

\[ n_0(1 + 2\Delta n \cos(2\pi z (1 + Cz)/\Lambda_0)) \]

for \( 0 \leq z < L_c \), and \( n_0(1 + 2\Delta n \cos(2\pi z (1 + CL_c)/\Lambda_0)) \) for \( (L_c \leq z \leq L) \). Here \( z \) is the propagation distance, \( L \) the total length of the setting, \( L_c \) the length of the chirped Bragg gratings, and \( C \) the chirp coefficient. Further, \( \Lambda_0 \) is the BG period at the input edge, \( n_0 \) the average refractive index, and \( \Delta n \) the amplitude of the refractive-index modulation.

The nonlinear CME system is an accurate model for the propagation of optical pulses in BGs with the Kerr nonlinearity and inhomogeneity of the refractive index [15, 28–30]:

\[ \pm i \frac{\partial E_{f,b}}{\partial z} + \frac{i}{v_g} \frac{\partial E_{f,b}}{\partial t} + \delta(z)E_{f,b} + \kappa(z)E_{b,f} \]

\[ + \gamma |E_{f,b}|^2 + 2|E_{b,f}|^2 E_{f,b} = 0, \] (1)

where \( t \) is time, \( E_f \) and \( E_b \) are amplitudes of the forward-traveling and backward-traveling waves, respectively, \( v_g = c_0/n_0 \) is the group velocity in the material of which the BG is fabricated, and \( \gamma = 2\omega/c \) is the nonlinearity coefficient, with \( \omega \) being the carrier frequency and \( n_2 \) the Kerr coefficient. The strength of the coupling between forward and backward waves is \( \kappa = \pi\Delta n/\Lambda_0 \), while \( \delta(z) \), which represents the local wavenumber detuning, is

\[ \delta(z) = \begin{cases} 
\delta_0 - 2\pi Cz/\Lambda_0, & \text{for } 0 \leq z < L_c, \\
\delta_0 - 2\pi CL_c/\Lambda_0, & \text{for } L_c \leq z \leq L,
\end{cases} \] (2)

where \( \delta_0 = 2\pi n_0/\lambda - \pi/\Lambda_0 \) is the detuning at the left end of chirped segment. The nonlinear terms in Eqs. (1) account for the self- and cross-phase modulation, respectively. We denote the chirped segment of length \( L_c \) as chirped-BG (the chirped Bragg grating), and the uniform one as uniform-BG (the uniform Bragg grating), the length of which is \( L - L_c \). The setting implies that the local BG period decreases with \( z \) in the chirped-BG, hence the nominal Bragg wavelength is gradually shifted to smaller values for pulses running through the chirped grating. In other words, the wavelength of the input pulse, originally taken near the blue edge of the bandgap, drifts into the depth of the bandgap in the course of the propagation of the pulse [26]. As a consequence, the power of the forward-propagating wave is gradually converted into the backward wave, helping to introduce the balance between the powers of the two waves, which is necessary for the creation of a very slow BG soliton in the uniform-BG [26–28]. The soliton’s velocity is determined by the remaining imbalance between the powers:

\[ V_{sol} = v_g(1 - f)/(1 + f) \] (3)

\[ f(t) = \frac{\int_0^L |E_{b,f}(z,t)|^2 dz}{\int_0^L |E_f(z,t)|^2 dz}. \]

By introducing a defect at the junction between the chirped-BG and uniform-BG segments, one can create a standing soliton stably trapped by the defect [26]. Here, we introduce an array of defects in the uniform-BG, as multiple positions at which the stable trapping should be possible. For this purpose, we define the local detuning in the uniform-BG region as

\[ \delta_d(z) = \delta(z) \times \left[ 1 + \varepsilon \sum_{n=1}^N \exp \left\{ - \frac{[z - (1 + (n - 1)S)]^2}{d_w^2} \right\} \right], \] (4)

with \( \varepsilon \) being the depth of each defect, \( N \) the total number of defects, \( S \) the spacing between them, and \( d_w \) the width of defect. The array is displayed in Fig. 1, which shows \( \delta_d(z) \) as a function of \( z \). Thus, the defect array is determined by a parameter set, \( (S, d_w, \varepsilon) \). The respective shape of the effective potential for solitons, \( V(z) \), which is opposite to \( \delta_d(z) \) [31–32], predicts that, when the pulse’s kinetic energy, \( K \), is smaller than the local height of the effective potential, \( V_0 \), the pulse cannot pass the local potential maximum. If \( K \) slightly exceeds \( V_0 \), it is expected that the pulse may pass the maximum into the adjacent potential well at a small velocity, and get trapped there [32]. Below, this possibility is confirmed by systematic simulations of Eqs. (1).
III. NUMERICAL ANALYSIS

A. Controllable trapping of the single pulse

In this section, systematic simulations of the model are reported for physical parameters of silicon, whose Kerr coefficient and average refractive index are $4.5 \times 10^{-14}$ cm$^2$/W and $n_0 = 3.42$ [34]. The amplitude of the refractive index-modulation is taken as $\Delta n = 0.006$, a typical chirp coefficient in the chirped-BG segments $C = 2.5064 \times 10^{-4}$ cm$^{-1}$, and the BG period at the input edge of the sample is fixed to be $\Lambda_0 = 154.1$ nm. Simulations based on Eqs. (1) and (4) were carried out with the boundary condition corresponding to the Gaussian pulse,

$$E_f(z = 0, t) = E_0 \exp \left[ - (t/t_0)^2 \right],$$  

with temporal width $t_0 = 16$ ps, launched into the system at carrier wavelength 1053 nm, which is placed near the blue edge of the photonic bandgap. Further, the total length of the system and the length of the chirped-BG segment are fixed as $L = 1.54$ cm and $L_c = 0.193$ cm, respectively (below, the influence of $L_c$ on the pulse storage is discussed), thus the length of the uniform-BG segment is 1.347 cm. The number of defects in the array is $N = 10$, with individual defect characterized by parameters $(S, d_w, \varepsilon) = (0.132$ cm, 50 $\mu$m, 0.04), i.e., the space between the defects, $S$, is much larger than the width of each one, $d_w$. In comparison with the spatial width of the pulse (the temporal duration of 16 ps corresponds to 4.8 mm in space) the defect of width 50 $\mu$m may indeed be considered as a point-like object.

Simulations of Eqs. (11) were performed, by means of the fourth-order Runge-Kutta method. The intensities $I_P$ used in simulations are below the damage threshold [34] $|I_P|$ is the peak intensity of pulse, defined as $I_P = |E_0|^2$, see Eq. (14). First, setting the initial pulse intensity $I_P$ to 2.07 GW/cm$^2$, trapping of a stable pulse by the first defect (i.e., the formation of a standing optical soliton) is observed, as shown in Figs. (a,b). These results demonstrate that the Kerr nonlinearity in the BG, at the present (quite realistic) intensity level not only compensates the pulse’s dispersion in the vicinity of the edge of photonic bandgap, securing the formation of the gap soliton, but also shifts the photonic bandgap to a longer wavelength, which enables the pulse to pass the chirped-BG segment, before getting trapped in the uniform-BG section. Indeed, when the pulse reaches the junction between chirped-BG and uniform-BG, the power of the forward and backward components approximately equal, giving rise to a nearly halted pulse, as seen from Eq. (3), which may be readily captured by the first potential well.

The evolution of the pulse’s overall velocity, $V_{sol}$, defined according to Eq. (9), is additionally shown in the inset of Fig. (a). It is seen that $V_{sol}$ rapidly drops at $t < 3$ ns, due to the transfer from forward component to the backward one in the course of passing the chirped-BG segment. After that, $V_{sol}$ approaches zero, suggesting the trapped of the soliton in the potential well.

With the incident-pulse intensity increased to $I_P = 2.72$ GW/cm$^2$, trapping of a stable soliton in the second potential well is observed in Figs. (c,d). These results reveal that the pulse with this level of the intensity passes the first defect, and then keep moving at a small velocity, eventually being trapped by the second well. The further increase of the intensity to $I_P = 3.36$ GW/cm$^2$, as shown in Figs. (e,f), leads to the trapping of a stable soliton in the sixth potential well.

The above results suggest that the number of the trapping effective potential well increases with the increase of the incident-pulse’s intensity. To clearly demonstrate this trend, simulations were carried out with different intensities $I_P$, varying from 2 GW/cm$^2$ to 4.05 GW/cm$^2$, for two different defect strengths, $\varepsilon = 0.04$ and $\varepsilon = 0.06$. The outcome is depicted in Fig. (a), which shows the trapping position, $n$, as a function of the intensity. In both cases, the relation between $n$ and $I_P$ can be ap-

![FIG. 2. (Color online) The nonlinear propagation of pulses in the system with the defect array with $(S, d_w, \varepsilon) = (0.132$ cm, 50 $\mu$m, 0.04). The first and second columns demonstrate, severally, the intensity evolutions and the corresponding profiles at $t = 12$ ns, for three different injection intensities: (a,b) 2.07 GW/cm$^2$, (c,d) 2.72 GW/cm$^2$, (e,f) 3.36 GW/cm$^2$. Insets of (a), (c), (e) represent the evolutions of the pulse’s velocity $V_{sol}$, calculated by Eq. (3).]
proximated by

\[ n = 1 + a(I_P - I_{cr})^2, \]

where \( I_{cr} \) is a critical value of the injected intensity, \( I_P \), for trapping the pulse in the first potential well, and \( a \) is obtained by fitting the numerical data. For instance, \( a = 2.55 \text{cm}^4/(\text{GW})^2 \) for \( \varepsilon = 0.04 \), and \( a = 1.23 \text{cm}^4/(\text{GW})^2 \) for \( \varepsilon = 0.06 \). Although the critical intensities are practically identical in both these cases, \( I_{cr} = 2.07 \text{GW/cm}^2 \), see Fig. 3(a), there is a trend to decrease of \( I_{cr} \) with the decrease of \( \varepsilon \). Indeed, in the limit of \( \varepsilon \to 0 \) any pulse is able to pass the defect. Naturally, pulses stay trapped in a particular potential well when \( I_P \) takes values in some interval \( [20] \). For example, at \( \varepsilon = 0.06 \), the pulse is trapped by the first well for \( 2.07 \text{GW/cm}^2 < I_P < 2.72 \text{GW/cm}^2 \).

The quadratic dependence displayed by Eq. 6 can be explained, making use of the soliton’s equation of motion. In the framework of the CME system \([19]\), the soliton’s momentum is defined as (see, e.g., Ref. 19)

\[ M = i \int_{-\infty}^{+\infty} \left( \frac{\partial E_+^*}{\partial z} E_f + \frac{\partial E_+^*}{\partial z} E_b \right) dz. \]

As follows from the boundary conditions \([4]\), the initial momentum of the injected pulse scales as \( M_0 \sim I_P \). On the other hand, for a relatively slow soliton, its velocity is proportional to the momentum, \( V_{sol} \sim M \), see Eq. 4 \([19]\). Further, the radiative braking force acting on the soliton, \( F_{br} \), can be estimated as a loss of the momentum by the soliton passing a single defect, \( \Delta M \) times the number of defects passed in a unit of time, \( \approx V_{sol} / S \), see Eq. 4 \([4]\). The perturbation theory \([35]\) shows that \( \Delta M \) is proportional to the time necessary for the soliton to pass the defect, i.e., \( \Delta M \sim d_w / V_{sol} \). Thus, we obtain \( F_{br} \sim d_w / S \), which does not depend on velocity \( V_{sol} \), and the corresponding equation of motion, \( dM/dt = -F_{br} \), predicts that the soliton will come to a halt, \( M = 0 \), at time \( t_{halt} \sim M_0 \). Finally, an elementary mechanical analysis demonstrates that the total distance passed by the soliton under the action of the constant braking force is \( Z_{halt} \sim M_0^2 / F_{br} \sim I_P^2 \), which explains the quadratic fit provided by Eq. 6.

The soliton relaxing to the eventually trapped stationary state loses a part of its power through emission of linear waves (radiation). Accordingly, the final value of the total power of the trapped optical pulse,

\[ P_t = \int_{\text{trap}} \left[ |E_f(z, t \to \infty)|^2 + |E_b(z, t \to \infty)|^2 \right] dz \]

(the integral is computed over a region where the soliton is eventually trapped), normalized to the total input power, \( P(0) \), is displayed, as a function of the input intensity, \( I_P \), in Fig. 3(b). A trend of \( P_t / P(0) \) to decrease with the increase of \( I_P \) is explained by the fact that the pulse with larger power passes a larger number of defects, hence it is subject to stronger radiation losses. This curve also shows a weak oscillatory behavior with the increase of \( I_P \), ratio \( P_t / P(0) \) staying nearly constant in narrow intervals of \( I_P \). This phenomenon can be explained: pulses with \( I_P \) taking values in some intervals, which indeed are narrow, stay eventually trapped in the same potential well. Past these intervals, the ratio \( P_t / P(0) \) again decreases with the increase of \( I_P \).

The formation of the gap soliton from the initial pulse is additionally illustrated in Fig. 4(a) by a typical example of the temporal evolution of its effective squared spatial width, defined as

\[ D_{eff}(t) = \frac{\int_0^L (|E_f(z, t)|^2 + |E_b(z, t)|^2) dz}{\int_0^L |E_f(z, t)|^4 + |E_b(z, t)|^4 dz}. \]

The width relaxes to a constant value by \( t = 4 \text{ ns} \), when the pulse is already trapped by a local potential well, and the soliton is well formed. Here, an estimate for the distance travelled by the pulse before it forms a trapped soliton is presented. Because \( V_{sol} \) oscillates around zero before the formation of a soliton completes [see the inset in Fig. 2(c)], we assume that the average velocity of the pulse is 0.015c, which corresponds to an estimate for the travelled distance \( \approx 1.8 \text{ cm} \), which is far larger than the effective nonlinearity length, \( L_{NL} = 1 / (\gamma I_F) \approx 0.14 \text{ cm} \). Length \( L_c \) of the chirped-BG segment strongly affects the subsequent pulse trapping in the uniform-BG section. To illustrate this feature, a relation between the trapping site, \( n \), and \( L_c \) is displayed in Fig. 4(b), for fixed parameters \( (S, d_w, \varepsilon) = (0.132 \text{ cm}, 50 \text{ \mu m}, 0.04) \), and fixed incident intensity, \( I_P = 2.72 \text{ GW/cm}^2 \). In this case, the trapping occurs when \( L_c \) takes values between 0.16 cm and 0.21 cm, with the trapping position, \( n \), decreasing with the increase of \( L_c \) in this interval. This is explained by the fact that the increase of \( L_c \) leads to a reduction to the pulse’s velocity, hence it is captured earlier, at smaller \( n \). However, the pulse cannot be trapped at \( L_c < 0.16 \text{ cm} \), and at \( L_c > 0.21 \text{ cm} \). In the former case, the pulse’s velocity remains too large, allowing it to pass the entire uniform-BG segment without being halted; in the latter case, the pulse cannot reach the junction between the chirped-BG and uniform-BG segments.
eventually suffering decay in the chirped-BG segment, due to the strong BG-induced dispersion.

B. Multi-pulse trapping

The present BG system is capable to trap several pulses too, in different potential wells, provided that the pulses are launched into the system successively. For instance, two-pulse trapping was demonstrated for the same sets of parameters as considered above for the single-pulse case, i.e., \((S, d_w, \varepsilon) = (0.132 \text{ cm}, 50 \mu\text{m}, 0.04)\). Specifically, the first pulse with intensity \(I_p = 2.72 \text{ GW/cm}^2\) [it is expected to be trapped at the second potential well, according to Figs. 2(c,d)], is launched into the system at \(t = 0\), which is followed by launching the second pulse with \(I_p = 2.07 \text{ GW/cm}^2\) at \(t = 3\) ns [Figs. 2(a,b)] suggests that it may be captured by the first potential well].

The outcome of the simulations demonstrates, in Figs. 5(a,b), that these two pulses are indeed stably trapped in two different wells.

An example of three-pulse trapping is shown in Figs. 4(c, d). The first pulse, with \(I_p = 3.04 \text{ GW/cm}^2\), entering the chirped-BG input edge at \(t = 0\), is trapped by the third well. The second pulse with \(I_p = 2.78 \text{ GW/cm}^2\) is launched at \(t = 3\) ns, and is captured by the second well. The third pulse with \(I_p = 2.07 \text{ GW/cm}^2\), launched at \(t = 6\) ns, is eventually trapped in the first well, as suggested by the above results for single pulses.

Although the above examples of the two- and three-pulse trapping seem as simple superpositions of the single-pulse dynamical processes considered above, it should be stressed that, loosing a considerable part of its power on its way to the final halt [see Fig. 2(b)], each pulse leaves a trace of emitted radiation in its wake. Nevertheless, Fig. 3 demonstrates that the passage of the areas “contaminated” by the radiation does not perturb the propagation of the secondary solitons, which additionally attests to the solitons’ stability.

We also examined the case when the second pulse was launched right after the first one, before the first pulse gets trapped, as in the example shown in Figs. 4(a-c). In this case, it is concluded that, when the time interval between these incident pulses is < 1 ns, the second pulse bounces back, see Figs. 4(a,b). On the other hand, if the time interval between the pulses exceeds 1 ns (allowing the first pulse to get well trapped), see Fig. 4(c) [also Fig. 4(a)], these pulses can end up being stably trapped in different potential wells simultaneously. In addition, the situation where the incident intensities of
The first and second pulses are $I_P = 2.07 \text{ GW/cm}^2$ and $2.72 \text{ GW/cm}^2$ is demonstrated in Figs. (d-f). In this case, the two pulses collide coherently in the course of the evolution. Figure (d) demonstrates that these pulses interact attractively, finally being trapped in the same potential well. On the contrary, Figs. (a,b,c), reveal repulsive interaction. Moreover, in Fig. (e), the repulsive interaction actually leads to release of an initially trapped soliton, which may be used for retrieval of data bits stored in the system, in terms of applications. The attractive and repulsive sign of the interaction is explained by the relative phase, namely, they attract or repel each other when they are in phase or out of phase, respectively [35, 36].

It is possible to introduce interaction between solitons trapped by adjacent defects, reducing the spacing, $S$, of the defect lattice [see Eq. (4)]. Denser lattices are of obvious interest too in terms of applications. In the Figs. (a,b), and (c), reduced $S$ is taken as $S = 0.044$ cm, $S = 0.066$ cm and, additionally, $S = 0.088$ cm, in comparison with $S = 0.132$ cm in Figs. (d) and (e) while $d_v$ and $\varepsilon$ keep the same values as before. Although this reduction of the spacing is not dramatic, it is sufficient to allow actual interactions between solitons originally trapped in close potential wells, due to significant overlapping between solitons’ tails. For these settings, simulations were again performed for the first pulse with $I_P = 2.72 \text{ GW/cm}^2$, injected into the system at $t = 0$, and the second pulse with $I_P = 2.07 \text{ GW/cm}^2$, injected at $t = 4.1$ ns. The large temporal delay allowed the first pulse to settle down into a trapped state before the second one would appear in the vicinity. Figures (a) and (b) reveal that the interaction between solitons, which are trapped, for a short time, by close defects, indeed occurs in these cases, and changes the results.

In the case of $S = 0.044$ cm, shown in Fig. (a), the second pulse is originally trapped at distance $2S$ from the first one. In this case, the attractively interacting pulses hop together into a higher potential well and eventually merge in it. For $S = 0.066$ cm, Fig. (b) demonstrates that the first soliton, originally trapped in the second well, is pulled by the incident second pulse back to the first well, where they merge into a single pulse staying in the first well. On the other hand, Fig. (c), corresponding to $S = 0.088$ cm, shows that the first pulse stays trapped in its original position, despite the repulsive interaction with the second incident pulse. As mentioned above, the attractive [Figs. (a,b)] or repulsive [Fig. (c)] sign of the interaction is determined by the relative phase of the two pulses [35, 36].

IV. CONCLUSIONS

We have introduced a system engineered as a concatenation of linearly chirped and uniform BGs (Bragg gratings), with the array of local defects embedded into the uniform grating. The system of CMEs (coupled-mode equations) have been used to simulate the evolution of single and multiple pulses injected into the system. The systematic analysis has demonstrated that, selecting parameters of the systems and the intensities of incident pulse, the conversion of the pulse into a well-formed BG soliton and, eventually, its trapping at a desired position, by one of the local defects, are provided by the system. The relation of the trapping position and input intensity was found, being close to a quadratic form. The co-trapping of two or several pulses may be complicated by the strong interaction between them, in the case when the temporal delay between the pulses, or the spacing between adjacent local defects, is relatively small.

As a further development of the analysis, it may be interesting to study in detail release of a trapped soliton by an incident one, an example of which is displayed in Fig. (e). It may also be interesting to consider an effects of a frequency shift added to the input pulse, which corresponds to multiplying input (6) by $\exp(-i\omega t)$, with constant frequency $\omega$. On the other hand, for very slow solitons, taking into account optoacoustic effects mediated by electrostriction [37, 38] may improve the accuracy of the model.

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