Higher Order Terms of Improved Mean Field Approximation for
IIB Matrix Model and Emergence of Four-dimensional Space-time

T. Aoyama\textsuperscript{1} and H. Kawai\textsuperscript{1,2}

\textsuperscript{1}Theoretical Physics Laboratory, RIKEN, Wako, 351-0198, Japan
\textsuperscript{2}Department of Physics, Kyoto University, Kyoto, 606-8502, Japan

Abstract

The spontaneous breakdown of SO(10) symmetry of the IIB matrix model has been studied by using the improved mean field approximation (IMFA). In this report, the eighth-order contribution to the improved perturbative series is obtained, which involves evaluation of 20410 planar two-particle irreducible vacuum diagrams. We consider SO(\(d\))-preserving configurations as ansatz (\(d = 4, 7\)). The development of plateau, the solution of self-consistency condition, is seen in both ansatz. The large ratio of the space-time extent of \(d\)-dimensional part against the remaining \((10 - d)\)-dimensional part is obtained for SO(4) ansatz evaluated at the representative points of the plateau. It would be interpreted as the emergence of four-dimensional space-time in the IIB matrix model.

PACS numbers: 02.30.Mv, 11.25.-w, 11.25.Yb, 11.30.Cp, 11.30.Qc
I. INTRODUCTION

Superstring theory is supposed to provide a unified microscopic description of the universe including gravity. Recent progress revealed that many perturbative vacua thus far examined are related to each other, which enforces us to pursue non-perturbative formulation of superstring theory. One of those constructive definitions is the IIB matrix model [1] formulated in a form of the large-$N$ reduced model of the ten-dimensional supersymmetric SU($N$) Yang-Mills theory. A characteristic feature of this model is that the eigenvalue distribution of ten bosonic matrices is interpreted as space-time, i.e., the space-time itself is treated as a dynamical variable of the model. It opens us a way toward explaining the origin of our four-dimensional space-time in the context of superstrings [2].

In this report, we consider the possibility of spontaneous breakdown of the original SO(10) symmetry through the dynamics of the IIB matrix model. It is examined by the anisotropy of eigenvalue distributions of the matrices. In this regard, the extents of space-time measured by the second moment of the matrices vary according to directions. It may occur that the $d$-dimensional subgroup of the rotational symmetry stays intact, which reflects that $d$-dimensional space-time emerges as the vacuum of the IIB matrix model.

Exploring non-perturbative dynamics of a model is not an easy task in general. Monte Carlo simulation, a powerful tool that has been successfully applied to various physical models, is plagued by the sign problem in this case, because the action has a complex phase originated from the fermionic part. Whilst it is suggested that the complex phase plays crucial role in the spontaneous breakdown of Lorentz symmetry [3].

Here, we instead exploit a technique called the improved mean field approximation (IMFA). It is one of variational methods capable of exploring non-perturbative solutions of the model. It was developed in Ref. [5], and first applied to IIB matrix model in Ref. [6], in which a systematic improvement to higher order terms was proposed and evaluated up to third order. The mechanism of spontaneous breakdown of rotational symmetry was further examined with the simplified models [7]. The analysis of the IIB matrix model by the IMFA method was proceeded to incorporate even higher-order contributions, up to fifth order in Ref. [8] and up to seventh order in Ref. [9]. In the latter work the automated calculation procedure was also developed.

---

1 A new technique called the factorization method is proposed to resolve the complex action problem in the Monte Carlo simulations [4].
The series of works showed that the spontaneous breakdown of Lorentz symmetry would occur to result in the emergence of the four-dimensional space-time. It was also observed that the extent of space-time of the four-dimensional part is larger than that of the remaining six-dimensional part, which is suggestive of the compactification scenario of string theory realized within the context of non-perturbative dynamics of the IIB matrix model.

In the IMFA prescription, the Gaussian terms with auxiliary parameters are introduced into the action of the original model, which may be considered as mean fields in the mean field approximation. The self-consistency condition for those parameters is given by the principle of minimal sensitivity \([10]\) as a guide line. It should be realized as a region of parameter space denoted as plateau. The development of plateau provides the indication whether or not the approximation is working well.

In the present report, we proceed with the IMFA analysis of the IIB matrix model up to the eighth-order contribution. Since we are particularly interested in the spontaneous breakdown of SO(10) symmetry, we restrict ourselves to two particular choices of breaking patterns, namely, SO(4) ansatz and SO(7) ansatz, in which four- (seven-) dimensional rotational symmetry stays intact. By incorporating higher-order terms we expect a clearer signal for the emergence of plateau, as well as better estimates of the free energy and the ratio of the space-time extent in the true vacuum of the model.

The paper is organized as follows. In Section \(\text{II}\) we provide a description of the model and the application of the IMFA analysis in a concrete procedure. The particular choices of configurations to be examined are given in Section \(\text{III}\). In Section \(\text{IV}\) we present the results. Section \(\text{V}\) is devoted to the conclusion and discussions.

\section*{II. IIB MATRIX MODEL AND IMFA ANALYSIS}

\subsection*{A. IIB Matrix Model}

The model we are considering here is the IIB matrix model defined by the partition function,

\[ Z = \int dA d\psi e^{-S}, \quad S = \frac{1}{g_0^2} \text{Tr} \left\{ -\frac{1}{4} [A_\mu, A_\nu]^2 - \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right\}, \]  

(1)

where bosonic variables \(A_\mu\) and fermionic variables \(\psi_\alpha\) are both traceless \(N \times N\) Hermite matrices. \(A_\mu\) (\(\mu = 1, \ldots, 10\)) transforms as a vector under SO(10) rotation, while \(\psi_\alpha\) (\(\alpha = 1, \ldots, 16\))
transforms as a left-handed spinor. The action has symmetry under the U(N) matrix rotation, ten-dimensional Lorentz symmetry, and the type IIB supersymmetry.

The coupling constant $g_0$ may be absorbed by the rescaling of the fields. Then the action takes the following form,

$$ S = N \text{Tr} \left\{ -\frac{1}{4}[A_\mu, A_\nu]^2 - \frac{1}{2} \bar{\psi} \Gamma^\mu [A_\mu, \psi] \right\}, \quad (2) $$

where we have chosen $g_0^2N = 1$, and $A_\mu$ and $\psi_\alpha$ denote the rescaled fields. We will examine the large-$N$ behaviour of the model.

The extent of space-time along a certain direction $\mu$ is defined by the second moment of the matrix:

$$ R^2_{\mu} = \frac{1}{N} \langle \text{Tr} A^2_\mu \rangle. \quad (3) $$

### B. Improved Mean Field Approximation

The action of the IIB matrix model (2) does not have quadratic terms, and therefore the ordinary perturbation theory is not directly applicable. To such cases we employ a technique called the improved mean field approximation (IMFA) that is elucidated in the following.

We introduce a quadratic term $S_0$ with arbitrary parameters collectively denoted by $m_0$ and deform the original action as

$$ S \rightarrow S_0 + \lambda(S - S_0), \quad (4) $$

where $\lambda$ is a nominal parameter that should be taken to 1. The second term of the deformed action (4) may be considered as an interaction term with a coupling constant $\lambda$. So, we can formulate a perturbation theory and obtain a power series expansion with reference to $\lambda$. The series up to some finite order $n$ would provide an $n$th order approximation of the original model at $\lambda = 1$. By this procedure, a formal perturbative expansion of a model can be constructed even when the model does not have quadratic terms, for example, as in the case of the IIB matrix model.

The perturbative series thus obtained depends on the arbitrary parameters $m_0$. We have to determine the values of those parameters by some means. We adopt here the principle of minimal sensitivity [10] as a guiding principle; the true value of a physical quantity should be given when it depends least on the arbitrary parameters. It is because the deformation (4) of the action becomes trivial by construction if $\lambda$ is taken to 1, and the result is independent of those artificially introduced parameters. The dependence is brought in due to the truncation at the finite order of perturbation.
Therefore, if a region of parameter space exists in which the series becomes almost constant, there the parameter dependence vanishes effectively, and the true value should be reproduced. We call this region “plateau”. It has been tested and shown to work on a number of systems [5, 6, 7, 8, 9, 11, 12].

The concept of plateau above is rather obscure, and thus we need a more concrete criterion for the distinction of plateau in such a manner that the ambiguity due to subjectivity of recognition be excluded as much as possible. It is usually seen that the series as a function of the arbitrary parameters fluctuates on and near the plateau about the true value, and it is accompanied by a number of extrema of the function. This leads to a practical criterion for identifying plateau by the accumulation of extrema of the series with reference to the parameters.\(^2\) It has been adopted in the previous works [8, 9]. The values of the series are estimated at the extrema as the representatives of the plateau, which should give good approximations if the series is convergent. We adopt this criterion in the present analysis.

C. Application to IIB Matrix Model

Let us proceed to the IIB matrix model and evaluate the free energy of the model by the IMFA method. We introduce a quadratic term \(S_0\) of most generic form that preserves U(N) symmetry,

\[
S_0 = N \text{Tr} \left\{ \frac{1}{2} M_{\mu\nu} A_\mu A_\nu + \frac{1}{2} m_{\mu\nu\rho} \bar{\psi} \Gamma^{\mu\nu\rho} \psi \right\}.
\]  

Here, \(M_{\mu\nu}\) and \(m_{\mu\nu\rho}\) are arbitrary parameters. \(M_{\mu\nu}\) are symmetric with \(\mu\) and \(\nu\), while \(m_{\mu\nu\rho}\) are totally anti-symmetric with \(\mu\), \(\nu\) and \(\rho\).

The prescription of the IMFA method is alternatively formulated by starting with the “massive” theory defined by the action,

\[
S' = S_0(M, m) + S(\lambda),
\]  

where \(S(\lambda)\) is given by inserting a formal parameter \(\lambda\) to the original action [2] as

\[
S = N \text{Tr} \left\{ -\frac{\lambda}{4} [A_\mu, A_\nu]^2 - \frac{\sqrt{\lambda}}{2} \bar{\psi} \Gamma^\rho [A_\mu, \psi] \right\}.
\]  

\(^2\) An alternative approach for the identification of plateau from the profile of functions with the help of histograms is proposed in Ref. [12].
A physical quantity $f$ is evaluated by the perturbative series expansion in terms of the coupling constant $\lambda$ up to some finite order $n$:

$$f(\lambda; M, m) = \sum_{k=0}^{n} \lambda^k f_k(M, m).$$

(8)

Then, we perform the following substitution of parameters,

$$M_{\mu\nu} \rightarrow (1 - \lambda) M_{\mu\nu},$$
$$m_{\mu\nu\rho} \rightarrow (1 - \lambda) m_{\mu\nu\rho},$$

(9)

and rearrange the series in powers of $\lambda$. Finally, We disregard $O(\lambda^{n+1})$ terms and set the formal expansion parameter $\lambda$ to 1. Following those steps, we obtain the improved series $\tilde{f}(\lambda; m)$. It will be denoted as

$$f \rightarrow \tilde{f}(M, m) = f(\lambda; (1 - \lambda)M, (1 - \lambda)m)\bigg|_{\lambda \rightarrow 1}.$$  

(10)

As is seen in Eq. (9), the model is shifted to “massless” case at $\lambda = 1$, and thus the original model would be reproduced.

For a technical reason, we exploit the fact that the ordinary free energy is related to the two-particle irreducible (2PI) free energy by the Legendre transformation, the latter of which is expressed by the sum of 2PI vacuum diagrams [13]. It is because the number of diagrams incorporated reduces drastically.

We evaluate the sum of 2PI vacuum diagrams in terms of the exact propagators. Since we are interested in the large-$N$ limit, the diagrams to be evaluated are restricted to planar ones. The 2PI free energy $G$ is given by:

$$G(C, u)/N^2 = 3(1 + \log 2) + \left\{ -\frac{1}{2} \text{tr} \log C + \frac{1}{2} \text{tr} \log \hat{u} \right\} + \lambda \left\{ -\frac{1}{2} (\text{tr}(C^2) - (\text{tr}C)^2) - \frac{1}{2} C_{\mu\nu} \text{tr}(\hat{u} \Gamma^\mu \hat{u} \Gamma^\nu) \right\} + \ldots ,$$

(11)

where $C_{\mu\nu}$ and $\hat{u} = u_{\mu\nu\rho} \Gamma^{\mu\nu\rho}/3!$ are the exact propagators of $A_\mu$ and $\psi$, respectively:

$$\langle (A_\mu)_i (A_\nu)_j \rangle_{kl} = \frac{1}{N} C_{\mu\nu} \delta_{il} \delta_{jk},$$
$$\langle (\psi_\alpha)_i (\psi_\beta)_j \rangle_{kl} = \frac{1}{N^3} \frac{1}{3!} i u_{\mu\nu\rho} (\Gamma^{\mu\nu\rho} C^{-1})_{\alpha\beta} \delta_{il} \delta_{jk}.$$  

(12)

(13)

$C$ is the charge conjugation matrix. The additive constants are adjusted according to the definition in Ref. [6]. We then perform the Legendre transformation to obtain the free energy $F$ by

$$F(M, m) = \left. \left( G(C, u) + \frac{1}{2} \sum_{\mu,\nu} M_{\mu\nu} C_{\mu\nu} - \frac{1}{2} \sum_{\mu,\nu,\rho} m_{\mu\nu\rho} u_{\mu\nu\rho} \right) \right|_{C=C(M,m),u=u(M,m)},$$

(14)
where $C(M, m)$ and $u(M, m)$ are determined by the solutions of the following relations:

$$
\frac{\partial G(C, u)}{\partial C_{\mu \nu}} + \frac{1}{2} M_{\mu \nu} = 0, \quad \frac{\partial G(C, u)}{\partial u_{\mu \nu \rho}} - \frac{1}{2} m_{\mu \nu \rho} = 0.
$$

(15)

The improved free energy $\tilde{F}$ is obtained by applying the procedure (10) to $F$. Then, we search for the extrema of $\tilde{F}(M, m)$ with reference to the parameters $M_{\mu \nu}$ and $m_{\mu \nu \rho}$, and identify the plateau by the accumulation of them. The values of free energy and other physical quantities are evaluated at the extrema.

III. ANSATZ

In the case of the IIB matrix model, a huge number of parameters are introduced along with the quadratic terms: 10 real numbers for $M_{\mu \nu}$ (assumed to be diagonalized), and 120 for $m_{\mu \nu \rho}$. It may demand enormous efforts to explore the whole parameter space for the solutions of plateau condition. Instead, we restrict ourselves to particular configurations in which SO($d$) subgroup of SO(10) stays intact. The explicit forms of configurations are chosen according to the guideline described in Ref. [8]. In this report we concentrate on $d = 4$ and $d = 7$ cases in particular.

Since we evaluate 2PI free energy in the first step, we specify the forms of exact propagators in each ansatz.

SO(7) ansatz. The fermionic propagators are represented by the rank three anti-symmetric tensor $u_{\mu \nu \rho}$. A non-zero element of $u$ accompanies three-dimensional block by considering the permutation of indices, and thus the SO(10) symmetry breaks down to SO(7) × SO(3).

In this case, the propagators take the following forms with three parameters, $c_1$, $c_2$, and $u$:

$$
C_{\mu \nu} = \begin{pmatrix}
\cdots & 7 & \cdots \\
\cdots & c_1 & \cdots \\
\cdots & c_2 & \cdots \\
\cdots & \cdots & \cdots
\end{pmatrix}, \quad \psi = u \Gamma^{8,9,10}.
$$

(16)

SO(4) ansatz. We assume that SO(4) symmetry is preserved. The remaining six-dimensional subspace is decomposed into two three-dimensional blocks. This leads to the pattern of breaking
FIG. 1: Distributions of extrema of the improved free energy in the parameter space for SO(4) ansatz (left) and SO(7) ansatz (right).

as SO(4) × SO(3) × SO(3)× Z₂. The extra Z₂ factor derives from the permutation among two SO(3) factors with the reversion of the first direction (to preserve parity in total).

In this case, the propagators are represented by the form given below with three parameters, c₁, c₂, and u:

\[
C_{\mu\nu} = \begin{pmatrix}
  c_1 & 4 \\
  & c_1
\end{pmatrix}, \quad \hat{u} = \frac{u}{\sqrt{2}}(\Gamma_{5.6.7}^{8.9.10}). \tag{17}
\]

IV. RESULTS

We evaluated the free energy of the IIB matrix model for SO(4) ansatz and SO(7) ansatz by the IMFA method up to eighth order of series expansion. The number of planar 2PI diagrams are shown in Table I. In this report, the eighth-order contribution consisting of 20410 distinct diagrams are newly evaluated. The automated procedure to generate and evaluate the set of diagrams has been developed in Ref. [9], which is used in the present study with slight extensions.

The improved free energy is obtained by following the procedure described in Sec. III through
the 2PI free energy $G$. Then, we search for the extrema of the improved free energy in the space of parameters $m_1$, $m_2$, and $m$ (dual to $c_1$, $c_2$, and $u$, respectively). The extrema of free energy for the eighth-order improved series are listed in Table II. We also confirmed that all the extrema of lower orders found in the former works [8, 9] are reproduced except one in seventh order of SO(4) ansatz.

The distributions of extrema in the parameter space are plotted in Fig. II. It is observed that a
TABLE I: Number of planar 2PI vacuum diagrams of (“massive”) IIB matrix model.

| Order | Number of diagrams |
|-------|-------------------|
| 0th   | 2                 |
| 1st   | 2                 |
| 2nd   | 2                 |
| 3rd   | 4                 |
| 4th   | 12                |
| 5th   | 49                |
| 6th   | 321               |
| 7th   | 2346              |
| 8th   | 20410             |

small region exists for both ansatz in which a number of extrema are found close to each other (shown in a dashed circle). It is recognized that extrema of different orders as well as those of respective orders belong to this accumulation. There are also extrema forming a line which would be considered as overshoots that characteristically appear around the edge of a plateau.

In Fig. 2 the values of the free energy evaluated at the extrema are plotted for SO(4) and SO(7) ansatz against the order of the IMFA analysis. The bullets (●) correspond to the extrema that belong to the accumulation, while the circles (○) correspond to the other extrema. The cross marks (×) represent the unphysical extrema at which \( c_1 < 0 \) or \( c_2 < 0 \).

Next, we consider what shapes of configurations are realized as the solutions. It is examined by evaluating the extent of space-time defined as the moments of the eigenvalue distributions:

\[
R^2 = \frac{1}{N} \langle \text{Tr} A_1 A_1 \rangle = \frac{\partial F}{\partial m_1} \bigg|_{\text{improved}},
\]

\[
r^2 = \frac{1}{N} \langle \text{Tr} A_{10} A_{10} \rangle = \frac{\partial F}{\partial m_2} \bigg|_{\text{improved}}.
\]

Here, the notation \( \bigg|_{\text{improved}} \) denotes the application of the IMFA procedure to the series on the left.

The values of \( R, r \) and the ratio \( \rho = R/r \) evaluated at the extrema of free energy are also listed in Table II for each ansatz. The ratio is plotted in Fig. 3 for the order of approximation (the marks are the same as those of free energy). They show distinctive behaviour between SO(4) and SO(7) ansatz. In SO(7) case the ratio stays at \( \rho \approx 2.5 \), while in SO(4) case it grows larger as the order increases. It should be remarked that we obtained the ratio of the space-time extent in SO(4) ansatz
TABLE II: Numerical values.

| Ansatz | Free energy | $R^2$  | $r^2$  | $\rho$  |
|--------|-------------|--------|--------|--------|
| SO(4)  | 2.646862    | 11.400026 | 0.033561 | 18.430393 |
|        | -4.056151   | 11.234590 | 0.035111 | 17.887798 |
|        | -2.393029   | 6.430896  | 0.078981 | 9.023488  |
|        | 6.507018    | 6.249471  | 0.171968 | 6.028345  |
|        | -29774.524354 | 165.895937 | 8.280539 | 4.475985  |
|        | -3610.819690 | 23.506193 | 532.358893 | 0.210130 |
| SO(7)  | 2.885793    | 0.831046  | 0.123696 | 2.591994  |
|        | 2.931477    | 0.898519  | 0.148787 | 2.457429  |
|        | -52.619043  | 1.116045  | 9.571141 | 0.341475  |
|        | -108.828989 | 0.397967  | 6.643743 | 0.244747  |

at an order of magnitude larger than the isotropic configuration, which has not been seen until the present eighth-order calculation.

V.  CONCLUSION AND DISCUSSIONS

We applied the improved mean field approximation (IMFA) to the IIB matrix model and evaluated up to eighth order contribution for configurations preserving SO($d$) rotational symmetry as ansatz. We examined $d = 4$ and $d = 7$ cases in the present study.

In order to solve the consistency condition for the arbitrary parameters called plateau condition, the stationary points (extrema) of the improved free energy are searched. It is observed that a region exists for each ansatz in which a number of extrema of eighth order as well as the extrema of different orders gather close to each other. It may be considered as a development of plateau. In the former works the discrepant behaviour was seen between even orders and odd orders of the improved series. There were no extrema found for SO(7) ansatz at even orders, and the free energy at extrema for SO(4) ansatz gave somewhat different sequence of values. Such discrepancy tends to be resolved with eighth-order terms taken into account. It would be consistent with the speculation that the result should be irrespective of the order of approximation, if the series is convergent and high enough orders are incorporated.
We evaluated the extent of space-time of $d$-dimensional part and that of $(10 - d)$-dimensional part in SO($d$) ansatz. They are given by the moments of the eigenvalue distributions at the extrema of free energy as the representative points of plateau. The deviation of the ratio of the extents from 1 represents the anisotropy of space-time realized as a non-perturbative vacuum of the IIB matrix model, and thus provides an indication of the spontaneous breakdown of Lorentz symmetry. In the present work, the ratio of an order of magnitude larger than the isotropic configuration was obtained for SO(4) ansatz. It might suggest that the actual ratio realized in the true vacuum of the theory may possibly be infinite, which implies the emergence of extended four-dimensional universe with the remaining six-dimensional part being compactified, as seen in the universe.

Still we do not have clear signal enough to infer the formation of plateau, and therefore no reasonable estimate for free energy nor other physical values would be assured at this stage. The configuration with smaller values of free energy should give dominant contribution in the whole configuration space, and it is supposed to be realized as our universe. In this regard, we can not yet definitely tell which of the ansatz would be plausible. More sophisticated scheme for identifying plateau and extracting the physical quantities would be required along with the effort to proceed to even higher order contributions.

Acknowledgments

The authors would like to thank M. Hayakawa, S. Kawamoto, T. Kuroki, T. Matsuo, J. Nishimura, and Y. Shibusa for valuable discussions and helpful comments. A part of calculation was performed by using the computational resources of the RIKEN Super Combined Cluster (RSCC).

[1] N. Ishibashi, H. Kawai, Y. Kitazawa and A. Tsuchiya, Nucl. Phys. B 498 (1997) 467.
[2] H. Aoki, S. Iso, H. Kawai, Y. Kitazawa, A. Tsuchiya and T. Tada, Prog. Theor. Phys. Suppl. 134 (1999) 47.
[3] T. Hotta, J. Nishimura and A. Tsuchiya, Nucl. Phys. B 545 (1999) 543.

J. Ambjørn, K. N. Anagnostopoulos, W. Bietenholz, T. Hotta and J. Nishimura, J. High Energy Phys. 0007 (2000) 011.

J. Ambjørn, K. N. Anagnostopoulos, W. Bietenholz, F. Hofheinz and J. Nishimura, Phys. Rev. D 65
(2002) 086001.

J. Nishimura and G. Vernizzi, Phys. Rev. Lett. 85 (2000) 4664.

J. Nishimura and G. Vernizzi, J. High Energy Phys. 0004 (2000) 015.

J. Nishimura, Phys. Rev. D 65 (2002) 105012.

[4] K. N. Anagnostopoulos and J. Nishimura, Phys. Rev. D 66 (2002) 106008.

J. Ambjørn, K. N. Anagnostopoulos, J. Nishimura and J. J. M. Verbaarschot, J. High Energy Phys. 0210 (2002) 062.

[5] D. Kabat and G. Lifschytz, Nucl. Phys. B 571 (2000) 419.

S. Oda and F. Sugino, J. High Energy Phys. 0103 (2001) 026.

F. Sugino, J. High Energy Phys. 0107 (2001) 014.

[6] J. Nishimura and F. Sugino, J. High Energy Phys. 0205 (2002) 001.

[7] J. Nishimura, T. Okubo and F. Sugino, Prog. Theor. Phys. 114 (2005) 487.

[8] H. Kawai, S. Kawamoto, T. Kuroki, T. Matsuo and S. Shinohara, Nucl. Phys. B 647 (2002) 153.

[9] H. Kawai, S. Kawamoto, T. Kuroki and S. Shinohara, Prog. Theor. Phys. 109 (2003) 115.

[10] P. M. Stevenson, Phys. Rev. D 23 (1981) 2916.

A. Dhar, Phys. Lett. B 128 (1983) 407.

[11] T. Aoyama, T. Matsuo and Y. Shibusa, arXiv:hep-th/0510105, accepted for publication in Prog. Theor. Phys. Vol. 115, No. 3.

[12] J. Nishimura, T. Okubo and F. Sugino, J. High Energy Phys. 0210 (2002) 043.

J. Nishimura, T. Okubo and F. Sugino, J. High Energy Phys. 0310 (2003) 057.

[13] R. Fukuda, M. Komachiya, S. Yokojima, Y. Suzuki, K. Okumura and T. Inagaki, Prog. Theor. Phys. Suppl. 121 (1995) 1.