Critical thinking and communication skills of 10th grade students in trigonometry

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Abstract. This study aims to determine the mathematical critical thinking and communication skills of high school students in trigonometry materials. This research uses descriptive qualitative approach. Subjects were randomly selected from 10th grade as many as five students. The instrument used was a test of critical thinking skills, communication, and non-tests in the form of an attitude scale questionnaire of 16 statements. The results showed that students had critical thinking skills and communication in the low category. Students are not able to complete trigonometry questions completely. This is because students are poorly trained in solving trigonometric problems with non-routine questions. Based on the findings, it can be concluded that: (1) mathematical critical thinking skills of five students of class X on trigonometry material are still relatively low. (2) Mathematical communication skills of five students of class X on trigonometry are still relatively low. (3) There is a relationship between critical thinking skills and mathematical communication of students. (4) And students' attitudes toward mathematics learning, tests of mathematical critical thinking, and communication skills on trigonometric material are good. So the critical thinking skills and mathematical communication of five students in trigonometry material are still lacking because students have difficulty in modeling trigonometry problems, and stuck with modification questions because many students are not able to complete the answers to the questions given.

1. Introduction

There are seven basic capital that all students must have in order to be able to play a good role in the future including problem solving, collaboration, adaptability, initiative and entrepreneurship, effective communication, access to information, and a sense of curiosity [1]. Then according to Reeve, E. M said "... the following" key "21st Century Skills were reviewed: Science, Technology, Engineering, Mathematics (STEM); Problem-Solving; and the 4Cs: Critical Thinking, Communication, Collaboration, & Creativity" [2]. In the United States, the National Education Association (NEA) believes that every child must have strong content mastery, as well as Four Cs. Based on these opinions state that the abilities that must be mastered in the future include critical thinking and communication.

Mathematical Critical thinking is intellectually disciplined process or ability of actively and skillfully conceptualizing, applying, analyzing, synthesizing, and/or evaluating mathematical situations in a reflective way, observation, cognitive strategies to generalize, prove, experience, reflection, reasoning, involves prior mathematical knowledge, or communication, as a guide to belief and action [3,4]. Other experts say critical thinking allows one to make reliable and valid decisions, ideas, arguments, or research and determine whether certain conclusions are true or false, act ethically, and can adapt to changes in a particular environment [5,6]. While mathematical communication is the ability to explain algorithms and unique ways of solving problems and constructing real-world phenomena graphically, words, equations, tables, and other forms of mathematical representation [7]. So, communication skills...
are the ability to express ideas and understanding mathematics verbally and in writing using numbers, symbols, images, graphics, diagrams or words. Critical thinking skills and mathematical communication must complement each other in order to survive in learning mathematics. Communication skills are the most important thing for developing students' thinking skills and knowing student achievement. This was said by Bruner that communication skills play an important role in the development of cognitive structures, and that language is a means, not only to represent experience, but also to change or transform ideas [8]. In line with that, according to Sumarmo, U "mathematical communication is an important component in learning mathematics, a tool for learning ideas, and clarifying mathematical understanding"[9]. So to find out critical thinking and mathematical understanding of students, students need to communicate it well.

The results of the study showed that the middle school students' mathematical critical thinking ability was still low. In particular, students failed to understand the periodic system in the trigonometric functions [10]. Not only that, students' mathematical communication skills still do not meet indicators, especially in regulating and accommodating mathematical thinking and for evaluating mathematical thinking in solving algebraic problems [11]. The reason of students have low critical and communications skills is that students have difficulties with properties of periodicity and the fact that trigonometric functions are not one-to-one [12]. In addition, there is poor understanding of radian measure and a lack of its connection to the unit circle. So as to make students unable to expand high-level thinking skills especially critical thinking. Already several times for students who are fooled by modifying the questions given by the teacher, this can slightly indicate that students are less critical in reading and solving problems. Not only that, students' mathematical communication skills still do not meet indicators, student is able to explain mathematical ideas in writing with drawings, diagrams, tables or algebra 4 (13.33%) [13]. Mathematical communication skills of students in classes IX-1 SMP Negeri 3 Bilah Hulu Labuhan Batu is still low [13]. The results of other observations showed that students who were given a story question that was modified to Higher Order Thinking Skills (HOTS) had not been able to understand, criticize, and pour the results well.

The results of the observation before the study showed that the completion of trigonometry problems was often found by many students who answered incorrectly and did not even answer at all. The obvious cause is still unknown, but the underlying cause is that students are unable to answer questions that are varied by their teacher and make mathematical models of story problems. So that the trigonometry material needs to be studied why students cannot answer optimally the trigonometric questions given. The cause can be from students' abilities, delivery of material, or student attitudes towards the subject matter. For this reason, this study focuses on knowing students 'communication and critical thinking abilities and students' attitudes towards mathematics in trigonometry. Correspondingly, the cause of trigonometric problem solving is that students do not think of other ideas, are less careful about understanding the problem, and less skilled in doing algebraic manipulation [11]. So the purpose of this study was to find out the process and the level of mathematical critical thinking and communication skills of class X students on trigonometry materials.

2. Method
This type of research is descriptive research with a qualitative approach. Descriptive research aims to describe precisely the characteristics of an individual, state, symptom or certain group. Bogdan and Taylor state that qualitative research is a research procedure that produces descriptive data in the form of written or oral words from people and observed behavior [14]. The subjects in this study were class X high school students as many as 5 randomly selected people. The main sources are informants or respondents, key informants and documentation. While secondary sources are literature, journals, previous research results, and books. Meanwhile, the research instrument consisted of two types, namely 1) auxiliary instruments in the form of description tests to obtain data from respondents in the form of tests of critical thinking skills and mathematical communication. And 2) instruments to clarify information that has been obtained, namely in the form of questionnaire guidelines and literature. As for the description test, students are given 2 mathematical problems to determine mathematical critical thinking skills and 2 mathematical problems for mathematical communication skills. The explanatory auxiliary instrument in the form of questionnaire to determine students' attitudes towards mathematics learning, especially in trigonometric material. The questionnaire used is a Likert scale with 16 statements and 4 answer choices. The choice of answers is, Strongly Agree (SS), Agree (S), Not Agree (TS) and
Strongly Disagree (STS). The statement is divided into 8 positive statements and 8 negative statements. Data analysis in this study through steps 1) data reduction, 2) data presentation, and 3) conclusion.

3. Results and discussion

3.1. Mathematical critical thinking and communication skills

In this section we discuss the results and analysis of student answers, each of which is given 4 trigonometric questions. The questions are as follows:

| No | Question of Critical Mathematical Thinking Ability |
|----|--------------------------------------------------|
| 1 | Adit plans to cross the river, if it is known that the width of the river is 63 m and the speed of the water current is \(2\sqrt{3} \text{ m/s}\), while the speed of the Adit motorboat is 6 m/s perpendicular to the river current. |
| 2 | Make a graph of the trigonometric function \(f(x) = 2 \sin(2x - 45°)\) and provide an explanation, process, and how to make it. |
| 3 | Two triangles are known, they are \(\triangle ABC\) and \(\triangle DEF\), \(\angle B\) = \(\angle D = 30°\), and rib length \(AB = 9\), \(BC = 2\sqrt{3}\), \(DE = 27\), and \(EF = 6\sqrt{3}\). Consider the following picture: |
| 4 | Lala tried to formulate trigonometry statements by conducting various experiments. Lala then formulated the statement as follows: |

- \(\sin x \cdot (\sec x + \cot x) = \tan x + \sin x\)
- \(1 + \cot^2 x = \csc^2 x\)

Then Lala tells you to check the statement. Is the statement that Lala formulated correct? If appropriate, prove the statement. But if it is wrong, show the location of the error and correct it so that the statement becomes true.

Critical thinking ability can be used to assess the validity of statements, ideas, arguments or research, interpretation, analysis, evaluation, inference, explanation, and self-regulation, and determine whether certain conclusions are true or false based on existing facts [5,15]. While communication skills play an important role in the development of cognitive structures, and language is a means, not only to represent experience, but also for changes or transformation of ideas [8]. The indicators used to compile the question are: (1) Expressing daily events in mathematical symbols or models; (2) Representing mathematical ideas given in the form of pictures or graphics; (3) Validating a statement or answer accompanied by relevant reasons; and (4) Evaluating and considering arguments, processes, or answers. The answers of several students will be analyzed based on the indicators of critical thinking skills according to Ennis [9], indicators of communication skills according to NCTM and Sumarmo [9], and the answer steps. To verify data, triangulation was carried out in the form of interviews and questionnaires. The results are formed in the category of students’ ability in critical thinking and communication in solving trigonometry questions. The following are the results.

Based on the answers of students, students make sketches or models that illustrate the problem of the matter of the story. But students have not been able to understand, explain, and solve the problem well. In this first question, all the students observed have not been able to answer, according to them the reason is because the problem is difficult and do not know the steps and processes that must be done. Broadly speaking students cannot associate other concepts with trigonometry in this matter. Students can only draw sketches of the story. But there is a student who is able to find the speed of the boat, and cannot continue the answer. Students are less skilled in representing events in the form of drawings or sketches, but are slightly capable of completing algebraic calculations. From these answers we can know that there are some students who can represent events but cannot write down calculations and there are students who are less skilled at representing events but are slightly able to do calculations.
In general, the five students in this experiment were able to draw and graph trigonometric functions, but none of them provided explanations or reasons for making graphs in full. Student 1 (S1) slightly explains the graph making process by giving a brief calculation related to the value of amplitude and period of the graph, but the drawback is not explaining why the curve cuts the x-axis at 45° or $\pi / 4$. To simplify the drawing process of S1 make a graph $y = \sin x$. Although students do not fully communicate the reason, but students fulfill one indicator of communication skills, it can be seen that students can represent mathematical symbol shapes in the form of mathematical graphics. Student 2 (S2) cannot draw the requested graph but only draws the graph $y = \sin x$. Here students can represent the form of mathematical symbols in the form of images but are limited to certain graphs. Student 2 (S2) cannot represent $y = 2 \sin 2 (x - 45°)$ into a graphical form. So that S2 does not fulfill two indicators of communication skills, namely explaining the process and providing relevant reasons and representing mathematical symbols in the form of graphics. In the rightmost image that is done by S3, only the graph is obtained while the explanation is not written. Student 3 (S3) can represent mathematical symbols in graphical form, but does not communicate reasons, explanations, and processes.

Some students only determine the ratio of length and do not explain the comparison of the area. Student 1 (S1) calculates the comparison of the length of the lines of the three triangles and gets the wrong answer, here students miscalculate and do not calculate the length of AC and DF. Then S1 cannot answer incorrectly, is incomplete and does not provide a reason for the answer. So that S1 does not meet one of the indicators of critical thinking. In the next picture, S2 does not show a comparison of the sides of the triangle, but only shows the ratio of the area of the triangle. Student 2 (S2) is less thorough in reading questions, but can think critically because it is able to verify a conclusion with relevant reasons. Student 2 (S2) shows that the calculation gets the right ratio of the area of the triangle, so that it becomes the verification material for the wrong conclusions. The answers given by S2 are correct and critical, but the student's answer is not complete, because there is a statement that has not been verified. In the next picture, S3 only gives the reason, and does not do the calculation. The reason students use is the concept of congruence. Students' answers to give reasons for comparison are correct but not complete, so students can validate a statement and then provide relevant reasons. But students cannot validate statements regarding the comparison of the area of a triangle. According to the interview results, S3 forgets how to determine any triangle area so that it cannot answer the triangle area comparison.
In Figure 3 the answers of students 1, 2, and 3 directly prove this statement. But in the proof, some students have a simple mistake. Student 1 (S1) gives an answer but does not indicate the location of the error in the process of proof which causes a false statement. Student 1 (S1) only proves the statement and does not provide a solution. Proof that S1 is done is correct, but does not evaluate the process of proof in the wrong statement. So the process that should be detected where the error has not been explained by the student, and suggestions for improvement are not included. Student 2 (S2) answer at the first proof there was a step error, causing the final proof to experience an error. Whereas in the second verification, there are important steps that are passed, but the final answer is appropriate. In the first proof S2 cannot prove the statement, does not indicate the location of the error process, and does not provide a solution to the process or false statement. While S3 are not much different from S2, making a step error that causes evidence to be incorrect. The mistake that is made by S3 is to do a scratch or improper distribution. Then S3 does not mention the location of possible errors, arguments, and solutions so that the statements and proof processes are correct. At the second proof, S3 only changes one form, because S3 remembers trigonometric identities and forgets proof in detail. So it can be concluded that S1 fulfills one of the indicators by showing proof of a false statement, but does not give details of the error, gives no reason, and does not provide a solution. For students 2 and 3 do not fulfill the indicator because they cannot evaluate the statement or do not show proof, reason, and the right process. Seen from the answer, students are used to doing proof directly. Students have not been fully trained to evaluate, criticize, and provide arguments or solutions. But it is only a process and looks for right and wrong answers. Some students are not able to give reasons and answer correctly related to true or false statements. Students are still less critical and stuck with the concept of comparison, so it is still not right in giving the answer. Students are used to working on problems by looking at examples first so they cannot think critically optimally. There are students who have answered the question correctly, but are still incomplete in giving reasons. In addition, students pay less attention to the trigonometric concepts used, which causes a small error in answering. Maknun L. C., et.al said that most of the students answered correctly on the given problem, but they lacked appropriate argument to explain why it is true [10]. They also said that the algebraic ability is the most common error found in students’ answers.

The results of discussions and interviews show that students have not fulfilled the expected indicators, but from these answers are composed of students who answer math questions. The findings in this study are summarized as follows: (1) Students cannot work on non-routine questions; (2) Students do not calculate the time to work on each item; (3) Students lack respect and critically see the questions given; (4) answer in succession (from the easiest question to the hardest question); (5) Every answer is not accustomed to answering reasons; (6) Accustomed to doing calculations and rarely concluding the results of calculations; (7) There is still fear of trigonometric material; (8) Students find it difficult to get out of difficulties in math problems; (9) Students like variety in mathematics learning activities; (10) Students say the biggest problem is working on the problem description forgetting the formula and steps; and (11) Step by step in answering based on compilation memories once working on the Related questions, and if the problem is modified it will become increasingly difficult for students.
3.2. Students’ attitudes toward tests of mathematical critical thinking and communication skills

These statements are prepared based on indicators: (1) Attitudes towards mathematics learning activities; (2) Attitudes toward questions of critical thinking skills and mathematical communication; and (3) Attitudes toward mathematics learning on trigonometry. The following are the results:

Table 2. Recapitulation of attitude scale 1.

| NO | POSITIVE STATEMENT | SS | S | TS | STS |
|----|--------------------|----|---|----|-----|
| 1  | Math is an interesting and challenging subject. (++) | 1  | 2 | 2  | 0   |
| 2  | Understanding mathematics is very helpful for learning other lessons (+) | 0  | 4 | 1  | 0   |
| 3  | Mathematics lessons about trigonometry have many benefits in everyday life. (++) | 1  | 3 | 1  | 0   |
| 4  | Mathematical essay questions on trigonometric material train me to communicate opinions (+) | 0  | 4 | 1  | 0   |
| 5  | I do the most difficult trigonometric tasks to improve critical thinking skills (+) | 0  | 1 | 3  | 1   |
| 6  | I enjoy learning mathematics on trigonometry material (+) | 0  | 1 | 2  | 2   |
| 7  | Questions about the ability to think critically mathematically on the topic of trigonometry make me more thorough in working on problems (+) | 2  | 3 | 0  | 0   |
| 8  | Math problems related to everyday life make me happy and excited in solving them (+) | 1  | 3 | 1  | 0   |

Table 3. Recapitulation of attitude scale 2.

| NO | NEGATIVE STATEMENT | SS | S | TS | STS |
|----|--------------------|----|---|----|-----|
| 1  | I am lazy to do my own problems with difficult trigonometry (-) | 1  | 2 | 2  | 0   |
| 2  | I was silent when I had difficulty learning trigonometry (-) | 2  | 1 | 2  | 0   |
| 3  | I have difficulty communicating my mathematical knowledge to teachers and classmates (-) | 0  | 2 | 2  | 1   |
| 4  | I feel my critical thinking skills are still lacking after studying trigonometry (-) | 0  | 3 | 2  | 0   |
| 5  | I avoid more difficult trigonometric problems (-) | 1  | 2 | 2  | 0   |
| 6  | Classroom learning activities that are varied and modified make it difficult for me to learn mathematics (-) | 1  | 0 | 4  | 0   |
| 7  | Creating a mathematical model of a given trigonometric problem is difficult for me (-) | 2  | 2 | 1  | 0   |
| 8  | The tasks in learning mathematics made me bored. (-) | 0  | 2 | 3  | 0   |

Note: the numbers in the table indicate how many students answered

Based on the responses of students who have filled out the questionnaire, students generally do not like math too much and also do not like difficult material, especially trigonometry. It seems that students already understand the importance of the role of mathematics in everyday life. Students realize that however difficult the learning of mathematics will become a provision for everyday life. Then the learning activities that occur so far make students happy but students prefer learning that varies or is modified so that it is more fun. Students feel that getting used to high-level thinking skills can strengthen their thinking skills to be more critical. In addition, students also like easy questions about the types of critical thinking skills and mathematical communication. But when students see a dead end in solving problems because there is no one else to consult, students feel all the mathematics material is difficult and finally do not want to work on difficult questions anymore and prefer to avoid. Then most students have difficulty in making mathematical models of a story problem. So it can be concluded that students have a positive attitude towards mathematics learning, tests of critical thinking and communication. But students still feel their mathematical critical thinking and communication are lacking.

4. Conclusion

Based on the results of the study, five students who were the subjects of the study had not been able to fulfill all indicators of critical thinking skills and mathematical communication. Students can only fulfill one or two indicators, so that students can say that they are still lacking in mastering these abilities. The reason is learning activities that have not provided opportunities for students to think critically and
communicatively. And there is no habituation of students to non-routine questions. So that math problems in trigonometry material are often considered difficult. It can be concluded that the mathematical critical thinking and communication skills of five students of class X on trigonometry are still relatively low. There is a relationship between mathematical critical thinking and communication skills of students. And students’ attitudes toward mathematics learning, tests of critical thinking skills, and mathematical communication on trigonometric material are good. So the critical thinking skills and mathematical communication of five students in trigonometry material are still lacking because students have difficulty in modeling trigonometry problems, and stuck with modification questions because many students are not able to complete the answers to the questions given.

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