\[ \mathcal{N} = 1 \] Supersymmetric Theory of Higher Spin Gauge Fields in \( AdS_5 \)
at the Cubic Level

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Abstract

We formulate gauge invariant interactions of totally symmetric tensor and
tensor-spinor higher spin gauge fields in \( AdS_5 \) that properly account for higher-
spin-gravitational interactions at the action level in the first nontrivial order.

1 Introduction

Study of the higher spin theory in \( AdS \) background is of interest due to its potential
relationship (see e.g. [1, 2, 3, 4] and reference therein) with a symmetric phase of a
theory of fundamental interactions presently identified with M theory. An additional
motivation for the study of higher spin gauge theories came recently [5, 6, 7, 8, 9, 10]
from somewhat different arguments based on \( AdS/CFT \) correspondence [11, 12, 13],
pointing at the same direction. From this perspective the case of \( AdS_5 \) is of particular
importance, because higher spin gauge theories in \( AdS_5 \) are dual to 4\( d \) superconformal
theories. The case of \( \mathcal{N}=4 \) supersymmetry is most interesting as the corresponding 4\( d \)
superconformal model is \( \mathcal{N}=4 \) SYM.

In [14] it has been shown that totally symmetric bosonic higher spin gauge fields
propagating on \( AdS_5 \) admit consistent higher-spin-gravitational interactions at least in
the cubic order. The corresponding action was constructed in the first nontrivial order.
The system exhibits higher spin symmetries associated with certain higher spin algebra
originally introduced in [15] and called \( cu(1,0|8) \) in [7] and requires \( AdS \) geometry rather than the flat one thus extending \( 4d \) results of [16] to \( d = 5 \). One difference compared to the \( 4d \) case is that the \( 5d \) higher spin algebra \( cu(1,0|8) \) contains non-trivial center freely generated by the central element \( N \) [15]. As a result, \( cu(1,0|8) \) gives rise to the infinite sets of fields of any spin. The factorization of the algebra \( cu(1,0|8) \) with respect to the maximal ideal generated by \( N \), that gives rise to the reduced higher spin algebra \( hu_0(1,0|8) \) in which every integer spin appears in one copy, was shown to admit consistent interactions as well [14].

In this paper we continue the study of higher spin interactions of totally symmetric massless fields in \( AdS_5 \), extending the analysis of [14] to the model with fermions that exhibits the higher spin symmetries associated with the simplest \( AdS_5 \) higher spin superalgebra \( cu(1,1|8) \). The totally symmetric higher spin gauge fields originating from \( cu(1,1|8) \) are arranged into an infinite sequence of supermultiplets \( \{s\}^{(k)}, 0 \leq k < \infty \), with a spin content \((s, s - \frac{1}{2}, s - 1)^{(k)}\) determined by an integer highest spin \( s = 2, 3, ..., \infty \). Strictly speaking, the theory we consider is not fully supersymmetric because we truncate away all lower spin fields with \( s \leq 1 \) (in particular, the spin 1 field from the spin 2 supermultiplet). This truncation is done to simplify analysis because lower spin fields require special formulation while our goal is to check consistency of the higher-spin-gravitational interactions. By analogy with the \( 4d \) analysis (see second reference in [16]) it is not expected to be a hard problem to extend our analysis to the case with lower spin fields included. Note that a truncation of lower spin fields is only possible at the cubic level\(^1\) and these fields (in particular, scalar fields) have necessarily to be introduced in the analysis of higher-order corrections. Correspondingly, we will refer to the theory under consideration as to \( 5d \) supersymmetric higher spin gauge theory.

We consider both unreduced model based on \( cu(1,1|8) \) with all fields appearing in infinitely many copies and the reduced model based on the superalgebra \( hu_0(1,1|8) \), in which every supermultiplet appears just once. For these particular models we build higher spin actions that describe properly, both at the free field level and at the level of cubic interactions, the systems of totally symmetric boson and fermion \( 5d \) higher spin gauge fields with spins \( s \geq 3/2 \), interacting with gravity. Let us note that the constructed higher-spin cubic vertices do not necessarily exhaust all possible interactions in the order under consideration. The full structure of the cubic action can only be fixed from the analysis of higher orders, which problem is beyond the scope of this paper.

Let us note that, our formulation operates in terms of appropriate auxiliary and extra fields identified with particular higher spin connections. These auxiliary variables

\(^{1}\)At the cubic level such an incomplete system remains formally consistent because one can switch out interactions among any three elementary (i.e., irreducible at the free field level) fields without spoiling the consistency at this order. This is a simple consequence of the Noether current interpretation of the cubic interactions: setting to zero some of the fields is always consistent with the conservation of currents.
simplify the formulation enormously, being expressed in terms of derivatives of the particular physical higher spin fields (modulo pure gauge degrees of freedom) by virtue of appropriate constraints [17, 18]. This is analogous to the formulation of gravity by requiring the metric postulate to be true to define connection in terms of derivatives of the metric tensor instead of rewriting the Einstein action directly in terms of the metric tensor. In this paper we impose the generalized “higher spin metric postulate” constraint conditions. The explicit expressions for the auxiliary fields in terms of the physical ones are not discussed here because, as is clear from the corresponding 4d analysis of [1], the particular expressions are not very illuminating. It is however straightforward to figure out a form of some particular cubic vertex for physical fields by solving appropriate constraints which have a form of linear algebraic equations on the auxiliary variables (see section 3).

As argued in [14], it is not straightforward to incorporate an extended supersymmetry with $\mathcal{N} \geq 2$ in the present construction of cubic higher spin couplings. This is because $\mathcal{N} \geq 2$ supermultiplets originated from $cu(2^{\mathcal{N}-1}, 2^{\mathcal{N}-1}|8)$ require mixed symmetry higher spin fields in $AdS_5$ to be included. However, Lagrangian formulation of such fields in $AdS$ spacetime, is not yet elaborated in full details even at the free field level, although a significant progress was achieved recently in [20, 21, 22].

The paper is organized as follows. In section 2 we recall the construction of $\mathcal{N} = 1$ $AdS_5$ higher spin superalgebra $cu(1,1|8)$ in terms of star product algebras of superoscillators and define appropriate reality conditions. Gauging of $cu(1,1|8)$ is studied in section 3. The construction of the $AdS_5$ higher spin action functional is the content of section 4 where, at first, in section 4.1 we discuss general properties of the higher spin action and give the final output of our analysis, and then explicitly derive the quadratic (section 4.2) and cubic (section 4.3) higher spin actions possessing necessary higher spin symmetries. Reduction to a higher spin gauge theory associated with the reduced algebra $hu_0(1,1|8)$, in which every integer spin supermultiplet appears in one copy, is performed in section 5. Section 6 contains conclusions. Some technicalities are collected in two Appendices.

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2 To avoid misunderstandings, let us note that what we call $\mathcal{N}$ extended $AdS_5$ supersymmetry in this paper in some other works (see e.g. [19] and references therein) is referred to as $2\mathcal{N}$ extended $AdS_5$ supersymmetry.

3 The situation with the equations of motion for mixed-symmetry higher spin fields is simpler. The gauge invariant equations of motion for all types of massless fields in $AdS_d$ for even $d$ were found in [23]. Lorentz covariant equations of motion for some particular higher spin fields in $AdS_5$ with special values of energy $E_0$ were studied in [24].
2 \hspace{1cm} 5d \textit{Higher Spin Superalgebra}

Consider the associative Weyl-Clifford algebra with (anti)commutation generating relations
\[
[a_\alpha, b^\beta]_\text{\ast} = \delta^\alpha_\beta, \quad [a_\alpha, a_\beta]_\ast = [b^\alpha, b^\beta]_\ast = 0, \quad \alpha, \beta = 1, \ldots, 4, \tag{2.1}
\]
with respect to Weyl star product
\[
(F \ast G)(a, b, \psi, \bar{\psi}) = F(a, b, \psi, \bar{\psi}) (\exp \triangle) G(a, b, \psi, \bar{\psi}), \tag{2.2}
\]
where
\[
\triangle = \frac{1}{2} \left( \frac{\partial}{\partial a_\alpha} - \frac{\partial}{\partial b^\alpha} - \frac{\partial}{\partial \psi} - \frac{\partial}{\partial \bar{\psi}} - \frac{\partial}{\partial a_\alpha} + \frac{\partial}{\partial b^\alpha} + \frac{\partial}{\partial \psi} + \frac{\partial}{\partial \bar{\psi}} \right). \tag{2.3}
\]

The generators
\[
T^{\alpha \beta} = a_\alpha b^\beta \equiv \frac{1}{2} (a_\alpha \ast b^\beta + b^\beta \ast a_\alpha),
\]
\[
Q_\alpha = a_\alpha \bar{\psi}, \quad \bar{Q}^\beta = b^\beta \psi, \tag{2.4}
\]
\[
U = \psi \bar{\psi} \equiv \frac{1}{2} (\psi \ast \bar{\psi} - \bar{\psi} \ast \psi)
\]
close to the superalgebra $gl(4|1; \mathbb{C})$ with respect to the graded Lie supercommutator
\[
[F, G)_\ast = F \ast G - (-1)^{\pi(F)\pi(G)} G \ast F, \tag{2.5}
\]
where the $Z_2$ grading $\pi$ is defined by
\[
F(-a, -b, \psi, \bar{\psi}) = (-1)^{\pi(F)} F(a, b, \psi, \bar{\psi}), \quad \pi(F) = 0 \text{ or } 1. \tag{2.6}
\]
The set of generators (2.4) consists of $gl(4; \mathbb{C})$ generators $T$, supersymmetry generators $Q$ and $\bar{Q}$ and $u(1)$ generator $U$. The central element in $gl(4|1; \mathbb{C})$ is
\[
N = a_\alpha b^\alpha - \psi \bar{\psi}. \tag{2.7}
\]
The generators of $sl(4|1; \mathbb{C})$ are
\[
t^{\alpha \beta} = a_\alpha b^\beta - \delta^{\alpha \beta} \psi \bar{\psi}, \quad q_\alpha = a_\alpha \bar{\psi}, \quad \bar{q}^\beta = b^\beta \psi. \tag{2.8}
\]
The $AdS_5$ superalgebra $su(2,2|1)$ [25] is a real form of $sl(4|1; \mathbb{C})$ singled out by the reality conditions defined below.
A natural higher spin extension of $su(2,2|1)$ introduced in [15] under the name $shsc^{\infty}(4|1)$ and called $cu(1,1|8)$ in [7] is associated with the star product algebra of all polynomials $F(a,b,\psi,\bar{\psi})$ satisfying the condition

$$[N,F]_\star = 0 .$$

(2.9)

In other words, the 5d higher spin superalgebra $cu(1,1|8)$ is spanned by star-(anti)com-mutators of the elements of the centralizer of $N$ in the star product algebra (2.1). As a corollary, every $F$ satisfying (2.9) has the form

$$F(a,b,\psi,\bar{\psi}) \equiv A(a,b) + B(a,b)\psi + D(a,b)\bar{\psi} + E(a,b)\psi\bar{\psi}$$

(2.10)

where we use notations

$$a_{\alpha(k)} \equiv a_{\alpha_1} \ldots a_{\alpha_k} , \quad b^{\beta(k)} \equiv b^{\beta_1} \ldots b^{\beta_k}$$

(2.11)

and $A_{\beta(k)}, B_{\beta(k+1)}, D_{\beta(k)}^{\alpha(k)}$ and $E_{\beta(k)}^{\alpha(k)}$ are arbitrary multispinors totally symmetric in lower and upper indices. Note that $F \in cu(1,1|8)$ is even in superoscillators.

To single out an appropriate real form of the complex higher spin algebra $cu(1,1|8)$ we impose reality conditions in the following way. Introduce an involution $\dagger$ defined by the relations

$$(a_{\alpha})^\dagger = ib^{\beta} C_{\beta\alpha} , \quad (b^{\alpha})^\dagger = iC^{\alpha\beta} a_\beta ,$$

(2.12)

$$\psi^\dagger = \bar{\psi} , \quad (\bar{\psi})^\dagger = \bar{\psi} ,$$

(2.13)

where $C_{\alpha\beta} = -C_{\beta\alpha}$ and $C^{\alpha\beta} = -C^{\beta\alpha}$ are some real antisymmetric matrices satisfying

$$C_{\alpha\gamma} C^{\beta\gamma} = \delta_\alpha^\beta .$$

(2.14)

An involution is required to reverse an order of product factors

$$(F \ast G)^\dagger = G^\dagger \ast F^\dagger$$

(2.15)
and to conjugate complex numbers
\[(\mu F)^\dagger = \bar{\mu} F^\dagger, \quad \mu \in \mathbb{C}, \quad (2.16)\]
where the bar denotes complex conjugation. The involution \(\dagger\) leaves invariant the defining relations (2.1) of the star product algebra and satisfies \((\dagger)^2 = Id\). By (2.15) the action (2.12), (2.13) of \(\dagger\) extends to an arbitrary element \(F\) of the star product algebra. Since the star product we use corresponds to the totally (anti)symmetric (i.e. Weyl) ordering of the product factors, the result is
\[(F(a_\alpha, b^\beta, \psi, \bar{\psi}))^\dagger = \bar{A}(ib^\gamma C_{\gamma\alpha}, iC^{\beta\gamma} a_\gamma) + \bar{D}(ib^\gamma C_{\gamma\alpha}, iC^{\beta\gamma} a_\gamma) \psi
+ \bar{B}(ib^\gamma C_{\gamma\alpha}, iC^{\beta\gamma} a_\gamma) \bar{\psi} + \bar{E}(ib^\gamma C_{\gamma\alpha}, iC^{\beta\gamma} a_\gamma) \psi \bar{\psi}.\]  
(2.17)
The involution \(\dagger\) (2.17) allows us to define a real form of the Lie superalgebra built by virtue of graded commutators of elements (2.10) by imposing the condition (for more details see e.g. [26])
\[F^\dagger = -i\pi(F) F.\]  
(2.18)
This condition defines the real higher spin algebra \(cu(1, 1|8)\) [7]. It contains the \(\mathcal{N} = 1\) \(AdS_5\) superalgebra \(su(2, 2|1)\) as its finite-dimensional subalgebra. In fact, the reality condition (2.18) guarantees that \(cu(1, 1|8)\) admits massless unitary representations with energy bounded below [27].

### 3 5d Higher Spin Gauge Fields

The \(AdS_5\) totally symmetric higher spin gauge fields can be described [17, 18, 6, 14, 29, 24] in terms of 1-form gauge fields \(\Omega(a, b, \psi, \bar{\psi}|x) = dx \Omega_n(a, b, \psi, \bar{\psi}|x)\) (\(n = 0, \ldots, 4\)) of \(cu(1, 1|8)\)

\[\Omega(a, b, \psi, \bar{\psi}|x) = \Omega_{E_1}(a, b|x) + \Omega_{O_1}(a, b|x) \psi + \Omega_{O_2}(a, b|x) \bar{\psi} + \Omega_{E_2}(a, b|x) \psi \bar{\psi},\]  
(3.1)
where
\[\Omega_{E_1}(a, b|x) = \sum_{k=0}^{\infty} (\Omega_{E_1}(x))^{\alpha(k)}_{\beta(k)} a_\alpha(k) b^\beta(k),\]  
(3.2)
\[\Omega_{E_2}(a, b|x) = \sum_{k=0}^{\infty} (\Omega_{E_2}(x))^{\alpha(k)}_{\beta(k)} a_\alpha(k) b^\beta(k),\]  
(3.3)
with commuting multispinors \((\Omega_{E_{1,2}}(x))^{\alpha(m)}_{\beta(m)})\) (label E means ”even”) and
\[\Omega_{O_1}(a, b|x) = \sum_{k=0}^{\infty} (\Omega_{O_1}(x))^{\alpha(k)}_{\beta(k+1)} a_\alpha(k) b^\beta(k+1),\]  
(3.4)
\[\Omega_{O_2}(a, b|x) = \sum_{k=0}^{\infty} (\Omega_{O_2}(x))^{\alpha(k+1)}_{\beta(k)} a_\alpha(k+1) b^\beta(k),\]  
(3.5)
with anticommuting multispinors \( (\Omega_{\alpha_1,2}(x))^{(m)}_{\beta_{(n)}} \), \(|m - n| = 1\) (label O means "odd"). We require the component gauge fields \( \Omega^{(m)}_{\beta_{(n)}}(x) = dx_0 \Omega^{\alpha_{(m)}}_{\alpha_{(n)}}(x) \), \(|m - n| \leq 1\) to commute with the basis elements of \( cu(1,1|8) \) (i.e. with the superoscillators \( a_\alpha, b_\beta, \psi \) and \( \bar{\psi} \)).

The higher spin field strength \( R(a,b,\psi,\bar{\psi}|x) \equiv R \)

\[
R = d\Omega + \Omega \wedge \ast \Omega , \quad d = dx_0 \frac{\partial}{\partial x_0}
\]  

admits an expansion analogous to (3.1)-(3.5). Infinitesimal higher spin gauge transformations are

\[
\delta \Omega = D\epsilon , \quad \delta R = [R, \epsilon]_*,
\]  

where 0-form \( \epsilon = \epsilon(a,b,\psi,\bar{\psi}|x) \) is an arbitrary infinitesimal higher spin gauge symmetry parameter and

\[
DF = dF + [\Omega, F]_* .
\]  

To analyse interactions we will use the perturbation expansion with the dynamical fields \( \Omega_1 \) treated as fluctuations above the appropriately chosen background \( \Omega_0 \)

\[
\Omega = \Omega_0 + \Omega_1 , \quad (3.9)
\]

where the vacuum gauge fields \( \Omega_0 = \Omega_0^\alpha_{\beta}(x) a_\alpha b_\beta \) correspond to background \( AdS_5 \) geometry described by virtue of the zero-curvature condition \( R(\Omega_0) \equiv d\Omega_0 + \Omega_0 \wedge \ast \Omega_0 = 0 \) (for more details see Appendix A of this paper and [3, 14]). Since \( R(\Omega_0) = 0 \), we have \( R = R_1 + R_2 \), where

\[
R_1 = d\Omega_1 + \Omega_0 \ast \wedge \Omega_1 + \Omega_1 \ast \wedge \Omega_0 , \quad R_2 = \Omega_1 \ast \wedge \Omega_1 .
\]  

The Abelian lowest order part of the transformation (3.7) has the form

\[
\delta_0 \Omega_1 = D_0 \epsilon , \quad \delta_0 R_1 = 0
\]  

with the covariant derivative \( D_0 \) (3.8) evaluated with respect to the background field \( \Omega_0 \).

The higher spin gauge fields of the real higher spin algebra \( cu(1,1|8) \) singled out by the conditions (2.18), satisfy the reality conditions [26, 7]

\[
\Omega^\dagger = -i \pi(\Omega) \Omega .
\]  

In fact, this condition implies that the odd component fields \( (\Omega_{\Omega_1})^{(s)}_{\beta_{(s+1)}}(x) \) and \( (\Omega_{\Omega_2})^{(s+1)}_{\alpha_{(s)}}(x) \) are conjugated to each other while the even component fields \( (\Omega_{E_1,2})^{(s)}_{\beta_{(s)}}(x) \) are self-conjugated.

In accordance with the analysis of [17, 18, 6, 14, 29, 24] 5d totally symmetric higher spin fields can be described by 1-forms \( \Omega^{(m)}_{\beta_{(n)}}(x) \equiv dx_0 \Omega^{\alpha_{(m)}}_{\alpha_{(n)}}(x) \), \(|m - n| \leq 1\), being
traceless multispinors symmetric separately in the upper and lower indices. The case of \( m = n = s \) corresponds to the bosonic spin \( s' = s + 1 \) field while the cases of \( n = s, m = s + 1 \) and \( n = s + 1, m = s \) correspond to the fermionic spin \( s' = s + 3/2 \) field. Thus, even and odd multispinors in (3.2)-(3.5) are identified with bosonic and fermionic totally symmetric higher spin fields, respectively. As shown in [18], the number of on-shell degrees of freedom \( \text{deg}(m, n) \) described by \( \Omega^{\alpha(m)}_{\beta(n)}(x), |m - n| \leq 1 \), is given by

\[
\text{deg}(m, n) = \begin{cases} 
2s + 3, & n = m = s, \\
4(s + 2), & m = s + 1 \text{ or } n = s + 1,
\end{cases}
\]

being precisely the (real) dimensionalities of the corresponding (spin)-tensor irreps of the little group \( SO(3) \). The multiplet \((s, s - 1/2, s - 1)\) therefore contains equal numbers of boson and fermion degrees of freedom.

The multispinors in (3.2)-(3.5) are not traceless and, therefore, each of them decomposes into a sum of irreducible traceless components. Namely, for any fixed \( n \) and \( m \), tensor \( \Omega^{\alpha(m)}_{\beta(n)}(x) \) decomposes into the set of irreducible traceless components \( \Omega^{\alpha(k)}_{\beta(l)}(x), (\Omega^{\alpha(k-1)}_{\beta(l)}(x) = 0) \) with all \( k + l \leq n + m, k - l = n - m, k \geq 0, l \geq 0 \). As a result, a field of every spin appears in infinitely many copies in the expansion (3.1)-(3.5)

\[
\Omega = \sum_{k=0}^{\infty} \sum_{s=2}^{\infty} D^{(k)}(s) \oplus D^{(k)}(s - \frac{1}{2}) \oplus D^{(k)}(s - \frac{1}{2}) \oplus D^{(k)}(s - 1),
\]

where \( D^{(k)}(s) \) denotes a \( k \)-th copy of spin \( s \) \( su(2, 2) \) irreducible representation carried by traceless multispinors in the 1-form \( \Omega^{\alpha(m)}_{\beta(n)} \), \( |m - n| \leq 1 \).

The origin of this infinite degeneracy can be traced back to the fact that the algebra \( cu(1, 1|8) \) is not simple but contains infinitely many ideals \( I_{P(N)} \), where \( P(N) \) is any star-polynomial of \( N \), spanned by the elements of the form \( \{ x \in I_{P(N)} : x = P(N) \ast F, \ F \in cu(1, 1|8) \} \) [15]. One may consider quotient algebras \( cu(1, 1|8)/I_{P(N)} \). The most interesting reduction is provided by the algebra \( hu_0(1, 1|8) = cu(1, 1|8)/I_N \), where \( I_N \) is the ideal spanned by the elements \( x = N \ast F = F \ast N \). The higher spin model with spectra of spins associated with \( hu_0(1, 1|8) \) is built in section 5.

For the future convenience we introduce the two sets of the differential operators in the auxiliary variables

\[
T^+ = a_\alpha b^\alpha, \quad T^- = \frac{1}{4} \frac{\partial^2}{\partial a_\alpha \partial b^\alpha}, \quad T^0 = \frac{1}{4} (N_a + N_b + 4)
\]

and

\[
P^+ = T^+ - \psi \bar{\psi}, \quad P^- = T^- + \frac{1}{4} \frac{\partial^2}{\partial \psi \partial \bar{\psi}}, \quad P^0 = T^0 + \frac{1}{4} (N_\psi + N_{\bar{\psi}} - 1),
\]

where

\[
N_a = a_\alpha \frac{\partial}{\partial a_\alpha}, \quad N_b = b^\alpha \frac{\partial}{\partial b^\alpha},
\]

\[
N_\psi = \psi \frac{\partial}{\partial \psi}, \quad N_{\bar{\psi}} = \bar{\psi} \frac{\partial}{\partial \bar{\psi}}.
\]
These operators form the $sl_2$ algebras

\[
[T^0, T^\pm] = \pm \frac{1}{2} T^\pm, \quad [T^-, T^+] = T^0, \quad (3.18)
\]

\[
[P^0, P^\pm] = \pm \frac{1}{2} P^\pm, \quad [P^-, P^+] = P^0. \quad (3.19)
\]

Expansion coefficients of an element $\Omega(a, b, \psi, \bar{\psi}|x)$ are supertraceless iff $P^- \Omega(a, b, \psi, \bar{\psi}|x) = 0$. As a result, the operators $P^-$ and $P^+$ allow one to write down the decomposition of an arbitrary element $\Omega(a, b, \psi, \bar{\psi}|x)$ of $cu(1, 1|8)$ into irreducible $su(2, 2|1)$ supermultiplets as

\[
\Omega(a, b, \psi, \bar{\psi}|x) = \sum_{k=0}^{\infty} \sum_{s=1}^{\infty} \chi(k, s) (P^+)^k \Omega^{k, s+1}(a, b, \psi, \bar{\psi}|x), \quad (3.20)
\]

where $\chi(k, s)$ are some non-zero normalization coefficients, $s + 1$ denotes highest integer spin in a supermultiplet and $\Omega^{k, s+1}$ defined by $P^0 \Omega^{k, s+1} = (2s + 3)/4 \Omega^{k, s+1}$ are supertraceless

\[
P^- \Omega^{k, s+1}(a, b, \psi, \bar{\psi}|x) = 0. \quad (3.21)
\]

The condition (3.21) solves explicitly as

\[
\Omega^{k, s+1}(a, b, \psi, \bar{\psi}|x) = \tilde{\Omega}^{k, s+1}_{E_1}(a, b|x) - \frac{1}{(2s + 2)} T^+ \tilde{\Omega}^{k, s}_{E_2}(a, b|x) \quad (3.22)
\]

\[+ \tilde{\Omega}^{k, s+\frac{1}{2}}_{O_1}(a, b|x) \psi + \tilde{\Omega}^{k, s+\frac{1}{2}}_{O_2}(a, b|x) \bar{\psi} + \tilde{\Omega}^{k, s}_{E_2}(a, b|x) \psi \bar{\psi}, \]

where all $su(2, 2)$ multispinors are traceless

\[
T^- \tilde{\Omega}^{k, s'}_{E_{1,2}}(a, b|x) = T^- \tilde{\Omega}^{k, s'}_{O_{1,2}}(a, b|x) = 0, \quad s' = s, s + \frac{1}{2}, s + 1. \quad (3.23)
\]

Thus, the gauge fields originating from $cu(1, 1|8)$ are arranged into an infinite sequence of supermultiplets $\{s'\}^{(k)}$, $0 \leq k < \infty$, with a spin content $(s', s'-\frac{1}{2}, s'-1)^{(k)}$ determined by an integer highest spin $s' = 2, 3, \ldots, \infty$.

The decomposition (3.20) can be rewritten in the $su(2, 2)$ basis with all multispinors being traceless rather than supertraceless. The two bases are related by a finite field redefinition. The final result derived in Appendix B is

\[
\Omega_{E_{1,2}}(a, b|x) = \sum_{n, s=0}^{\infty} v_{E_{1,2}, n} (T^0) (T^+)^n \Omega^{n, s+1}_{E_{1,2}}(a, b|x), \quad (3.24)
\]

\[
\Omega_{O_{1,2}}(a, b|x) = \sum_{n, s=0}^{\infty} v_{O_{1,2}, n} (T^0) (T^+)^n \Omega^{n, s+3/2}_{O_{1,2}}(a, b|x), \quad (3.25)
\]

where $v_{E_{1,2}, n}$ and $v_{O_{1,2}, n}$ are some non-zero normalization coefficients and

\[
T^0 \Omega^{n, s+1}_{E_{1,2}}(a, b|x) = \frac{1}{2} (s + 2) \Omega^{n, s+1}_{E_{1,2}}(a, b|x), \quad (3.26)
\]
\[ T^0 \Omega_{O_{1,2}}^{n,s+3/2}(a,b|x) = \frac{1}{4} (2s + 5) \Omega_{O_{1,2}}^{n,s+3/2}(a,b|x), \quad (3.27) \]

\[ T^- \Omega_{E_{1,2}}^{n,s+1}(a,b|x) = T^- \Omega_{O_{1,2}}^{n,s+3/2}(a,b|x) = 0. \quad (3.28) \]

For the future convenience, we fix the normalization coefficients in the form

\[ v_{E_{1,2},n}(s) = (2i)^n \sqrt{\frac{(2s+3)!}{n!(2s+3+n)!}}, \quad (3.29) \]

\[ v_{O_{1,2},n}(s) = (2i)^n \sqrt{\frac{(2s+4)!}{n!(2s+4+n)!}}, \quad (3.30) \]

where the factor of \( i^n \) is introduced because the operator \( T^+ \) is antihermitian.

It is worth noting that unlike the supertrace decomposition (3.20), the fields carrying the same label \( n \) in (3.24)-(3.25) may belong to different supermultiplets.

In addition to (3.28) fields \( \Omega(a,b|x) \) satisfy the conditions

\[ (1 + N_a - N_b)\Omega_{O_1}(a,b|x) = 0, \quad (1 + N_b - N_a)\Omega_{O_2}(a,b|x) = 0, \]

\[ (N_b - N_a)\Omega_{E_{1,2}}(a,b|x) = 0, \quad (3.31) \]

that express the condition (2.9).

The operators \( T^i \) (3.15) are \( su(2,2) \) invariant. As a result

\[ D_0(T^i) = 0, \quad (3.32) \]

which relations have to be understood in the sense that \( D_0(X(F)) = X(D_0(F)) \), where \( X \) is one of the operators \( T^i \), while \( F \) is an arbitrary element of the higher spin algebra.

A useful consequence of this fact is

\[ R_1(T^j(\Omega)) = T^j(R_1(\Omega)), \quad (3.33) \]

where \( R_1 \) denotes the linearised higher spin curvature (3.10). Due to (3.33), the linearised curvatures admit the expansion analogous to (3.24)-(3.25):

\[ R_{1,E_{1,2}}(a,b|x) = \sum_{n,s=0}^{\infty} v_{E_{1,2},n}(T^0) (T^+)^n R_{1,E_{1,2}}^{n,s+1}(a,b|x), \quad (3.34) \]

\[ R_{1,O_{1,2}}(a,b|x) = \sum_{n,s=0}^{\infty} v_{O_{1,2},n}(T^0) (T^+)^n R_{1,O_{1,2}}^{n,s+3/2}(a,b|x), \quad (3.35) \]

where the curvatures on r.h.s.’s satisfy the irreducibility conditions analogous to (3.26)-(3.31).
The $su(2, 2)$ irreducible higher spin gauge field $Ω^{n,s}$ decomposes into a set of Lorentz covariant fields that form irreducible representations of the Lorentz algebra $so(4, 1) \subset su(2, 2)$. Different Lorentz gauge fields get different dynamical interpretation. For example, the $su(2, 2)$ irreducible field $Ω_{\beta}^{a}(x)$ in the adjoint of $su(2, 2)$ used to describe spin 2 field contains the frame field and Lorentz connection as the different irreducible Lorentz components. To decompose $su(2, 2)$ representations into Lorentz irreps we make use of the compensator formalism. (For more details on the compensator formalism and the decomposition procedure see [14] and Appendix A of this paper.) Namely, let $V^{\alpha \beta} = -V^{\beta \alpha}$ be a nondegenerate antisymmetric matrix. Then, the Lorentz subalgebra of $su(2, 2)$ can be defined as the stability algebra of $V^{\alpha \beta}$. In fact, this can be done locally with $V^{\alpha \beta}(x)$ being a field. We shall treat $V^{\alpha \beta}$ as a symplectic form that allows to raise and lower spinor indices in the Lorentz covariant way

$$A^\alpha = V^{\alpha \beta} A_\beta, \quad A_\alpha = A^\beta V_{\beta \alpha}.$$  \hspace{1cm} (3.36)

Let us introduce the operators

$$S^- = V^{\alpha \beta} a_\alpha \frac{\partial}{\partial b_\beta}, \quad S^+ = V^{\alpha \beta} b_\alpha \frac{\partial}{\partial a_\beta}, \quad S^0 = N_b - N_a,$$  \hspace{1cm} (3.37)

satisfying the commutations relations

$$[S^0, S^\pm] = \pm 2 S^\pm, \quad [S^-, S^+] = S^0, \quad [S^i, T^j] = 0.$$  \hspace{1cm} (3.38)

With the help of the operators (3.37) the decomposition into the higher spin Lorentz irreducible 1-forms is given by

$$Ω_{E_{1,2}}(a, b|x) = \sum_{t=0}^{s} (S^+)^t \omega_{e_{1,2}}^t (a, b|x),$$  \hspace{1cm} (3.39)

where

$$\omega_{e_{1,2}}^t (a, b|x) = \omega^{s+t+1, \beta(s-t)}_{e_{1,2}} (x) a^{s+t+1}_{\alpha} b^{s-t}_{\beta}$$  \hspace{1cm} (3.40)

are bosonic fields and

$$Ω_{O_{1}}(a, b|x) = \sum_{t=0}^{s} (S^+)^t \omega_{o_{1}}^t (a, b|x),$$  \hspace{1cm} (3.41)

$$Ω_{O_{2}}(a, b|x) = \sum_{t=0}^{s} (S^+)^t \omega_{o_{2}}^t (a, b|x),$$  \hspace{1cm} (3.42)

where

$$\omega_{o_{1}}^t (a, b|x) = \omega^{s+t+1, \alpha(s-t)}_{o_{1}} (x) a^{s+t+1}_{\alpha} b^{s-t}_{\beta},$$  \hspace{1cm} (3.43)

$$\omega_{o_{2}}^t (a, b|x) = \omega^{s+t+1, \beta(s-t)}_{o_{2}} (x) a^{s+t+1}_{\alpha} b^{s-t}_{\beta}$$  \hspace{1cm} (3.44)

are fermionic fields. With respect to their tangent indices Lorentz higher spin fields $\omega_{e_{1,2}}$ (3.40) and $\omega_{o_{1,2}}$ (3.43), (3.44) are described by the traceless two-row Young diagrams,
\[ S^{-\omega^t_{e1,2}}(a, b|x) = 0, \quad S^{-\omega^t_{e2}}(a, b|x) = 0, \quad S^{+\omega^t_{e1}}(a, b|x) = 0, \]
\[ T^{-\omega^t_{e1,2}}(a, b|x) = 0, \quad T^{-\omega^t_{e1,2}}(a, b|x) = 0. \] (3.45)

The Lorentz higher spin curvatures \( r^t(a, b|x) \) associated with the fields \( \omega^t(a, b|x) \) are defined by means of analogous procedure applied to \( R(a, b|x) \). Their form in terms of Lorentz gauge fields is as follows

\[ r^t_e = D\omega^t_e + \tau^{e} - \omega^{t+1} + \tau^{+} \omega^{t-1}, \] (3.46)
\[ r^t_\alpha = D\omega^t_\alpha + \mathcal{T}^{-\omega^{t+1}} + \mathcal{T}^{0} \omega^{t} + \mathcal{T}^{+} \omega^{t-1}, \] (3.47)

where \( D \) is the background Lorentz derivative. The explicit expressions for the operators \( \tau \) and \( \mathcal{T} \) are given in [14] and [29], respectively. The corresponding gauge transformation laws (3.11) have the form analogous to (3.46)-(3.47).

From the dynamical point of view bosonic \( \omega_e \) and fermionic \( \omega_o \) fields (3.40), (3.43), (3.44) with \( t = 0 \) are analogous to the frame field and gravitino and are treated as dynamical fields \( \omega^{ph} \) while all other fields with \( t > 0 \) play a role analogous to Lorentz connection. These are either auxiliary fields (\( t = 1 \) for bosons) or “extra” fields (\( t \geq 2 \) for bosons and \( t \geq 1 \) for fermions). Extra fields do not contribute into the free action functional since its variation w.r.t. extra fields is required to be zero identically. (This is the so called extra field decoupling condition; see section 4.) However, these fields do contribute at the interaction level. To make such interactions meaningful, one has to express the auxiliary and extra fields in terms of the physical ones modulo pure gauge degrees of freedom. This is achieved by imposing appropriately chosen constraints [17, 18] which have the form

\[ \Upsilon^+_{2,t} \wedge r^t_1 = 0, \quad 0 \leq t < s, \] (3.48)

where \( r^t_1 \) are Lorentz linearized curvatures (3.46)-(3.47) and

\[ \Upsilon^+_{2} = \begin{cases} \tau^0 \wedge \tau^+, & \text{for bosons,} \\ \mathcal{T}^0 \wedge \mathcal{T}^+, & \text{for fermions.} \end{cases} \] (3.49)

is such a 2-form operator that the number of independent algebraic conditions, imposed on the curvature components \( r^t_1 \) by (3.48) coincides with the number of components of the extra field \( \omega^{t+1} \) minus the number of its pure gauge components. For explicit expressions of the tau operators we refer the reader to [14, 29].

An important fact is that, by virtue of these constraints most of the higher spin curvatures \( r^t(a, b|x) \) vanish on mass-shell according to the following relationship referred to as the First On-Mass-Shell Theorem [17, 18, 6, 14, 24]:

\[ r^{\alpha(s+t), \beta(s-t)}(x) = X^{\alpha(s+t), \beta(s-t)}(\frac{\delta S_2}{\delta \omega^{ph}_e}), \quad \text{for } t < s, \]
\[ r^{\alpha(2s)}(x) = h^\alpha \wedge h^\gamma C_e^{\alpha(2s+2)}(x) + X^{\alpha(2s)}(\frac{\delta S_2}{\delta \omega^{ph}_e}), \quad \text{for } t = s \] (3.50)
for bosons and
\[ r^{\alpha(s+t+1),\beta(s-t)}(x) = Y^{\alpha(s+t+1),\beta(s-t)} \left( \frac{\delta S_2}{\delta \omega_p^m} \right), \quad \text{for } t < s, \] (3.51)

\[ r^{\alpha(2s+1)}(x) = h_{\alpha \gamma} \wedge h_{\gamma \beta} C^{\alpha(2s+3)}_o(x) + Y^{\alpha(2s+1)} \left( \frac{\delta S_2}{\delta \omega_p^m} \right), \quad \text{for } t = s, \]

(plus complex conjugate) for fermions. Here \( h_{\alpha \beta} \) denotes the background frame field (see Appendix A) and \( X \) and \( Y \) are some linear functionals of the r.h.s.’s of the free field equations. The 0-forms \( C_e \) and \( C_o \) on the l.h.s.’s of (3.50), (3.51) represent generalised Weyl tensors which are totally symmetric multispinors. The \( su(2,2) \) covariant version of (3.50)-(3.51) is

\[ R_E(a,b|x) \bigg|_{\text{m.s.}} = H_2^{\alpha \beta} \frac{\partial^2}{\partial a_\alpha \partial b_\beta} \text{Res}_\mu (C_E(\mu a + \mu^{-1} b|x)) \] (3.52)

for bosons and

\[ R_{O_1}(a,b|x) \bigg|_{\text{m.s.}} = H_2^{\alpha \beta} \frac{\partial^2}{\partial a_\alpha \partial b_\beta} \text{Res}_\mu (\mu C_{O_1}(\mu a + \mu^{-1} b|x)), \] (3.53)

\[ R_{O_2}(a,b|x) \bigg|_{\text{m.s.}} = H_2^{\alpha \beta} \frac{\partial^2}{\partial a_\alpha \partial b_\beta} \text{Res}_\mu (\mu^{-1} C_{O_2}(\mu a + \mu^{-1} b|x)) \] (3.54)

for fermions. Here \( H_2^{\alpha \beta} = h_{\alpha \gamma} \wedge h_{\gamma \beta} \), the label \( \bigg|_{\text{m.s.}} \) implies the on-mass-shell consideration \( \frac{\delta S_2}{\delta \omega_p^m} = 0 \) and \( \text{Res}_\mu \) singles out the \( \mu \)-independent part of Laurent series in \( \mu \). Note that a function of one spinor variable

\[ C(\mu a + \mu^{-1} b) = \sum_{k,l} \frac{\mu^{k-l}}{k! l!} C^{\alpha_1 \ldots \alpha_k \beta_1 \ldots \beta_l} a_{\alpha_1} \ldots a_{\alpha_k} b_{\beta_1} \ldots b_{\beta_l} \] (3.55)

has totally symmetric coefficients \( C^{\alpha_1 \ldots \alpha_k \beta_1 \ldots \beta_l} \) while \( \text{Res}_\mu \) in (3.52)- (3.54) singles out its part that belongs to \( cu(1,1|8) \) with the numbers of the oscillators \( a \) and \( b \) differing by at most 1.

4 \( \mathcal{N} = 1 \) Supersymmetric Higher Spin Action

The aim of this section is to formulate the action for the \( AdS_5 \) massless boson and fermion gauge fields of \( cu(1,1|8) \) that solves the problem of higher-spin-gravitational interactions in the first nontrivial order. The reported results extend the purely bosonic analysis (\( \mathcal{N} = 0 \)) of [14] to the \( \mathcal{N} = 1 \) supersymmetric case.
4.1 General properties

The action functional underlying the 5d non-linear higher spin dynamics in the cubic order has the following standard form [1, 16, 14]

\[ S = \int U_{12} \wedge R(\Omega_1) \wedge R(\Omega_2), \]  

(4.1)

which is a higher spin generalization of the MacDowell-Mansouri action for gravity [28]. \( U_{12} \) are some 1-form coefficients built from the frame field and the compensator. \( R(\Omega_{1,2}) \) are higher spin curvatures associated with higher spin gauge fields \( \Omega_{1,2} \) (3.1). Our goal is to find such coefficients \( U_{12} \) that account for the correct description of free higher spin dynamics and its consistent non-trivial interaction deformation. Note that if \( U_{12} \) would be an invariant tensor of the higher spin algebra, the action (4.1) would be a topological invariant thus describing no non-trivial dynamics. Of course, the main justification of the form (4.1) for the action is that it will be shown to describe correctly the higher spin dynamics at least in the cubic order.

Let us now discuss the structure of the action (4.1) in more detail. An appropriate ansatz is

\[ S(R, R) = \frac{1}{2} A(R, R), \]  

(4.2)

where the symmetric bilinear \( A(F, G) = A(G, F) \) is defined for any 2-forms \( F \) and \( G \)

\[ F = F_{E_1} + F_{O_1} \psi + F_{O_2} \bar{\psi} + F_{E_2} \psi \bar{\psi}, \]  

(4.3)

\[ G = G_{E_1} + G_{O_1} \psi + G_{O_2} \bar{\psi} + G_{E_2} \psi \bar{\psi} \]

as

\[ A(F, G) = B(F_{E_1}, G_{E_1}) + F(F_{O_1}, G_{O_1}) , \]  

(4.4)

where [14, 29]

\[ B(F_{E_1}, G_{E_1}) \equiv B'(F_{E_1}, G_{E_1}) + B''(F_{E_2}, G_{E_2}) , \]  

(4.5)

\[ B'(F_{E_1}, G_{E_1}) = \int_{\mathcal{M}^5} \hat{H}_{E_1} \wedge \text{tr}(F_{E_1}(a_1, b_1) \wedge G_{E_1}(a_2, b_2))|_{a_i = b_i = 0}; \]  

(4.6)

\[ B''(F_{E_2}, G_{E_2}) = \int_{\mathcal{M}^5} \hat{H}_{E_2} \wedge \text{tr}(F_{E_2}(a_1, b_1) \wedge G_{E_2}(a_2, b_2))|_{a_i = b_i = 0}; \]

\[ F(F_{O_1}, G_{O_1}) = \frac{1}{2} \int_{\mathcal{M}^5} \hat{H}_{O_1} \wedge \text{tr}(G_{O_1}(a_1, b_1) \wedge F_{O_1}(a_2, b_2))|_{a_i = b_i = 0} \]  

(4.7)

\[ + \frac{1}{2} \int_{\mathcal{M}^5} \hat{H}_{O_2} \wedge \text{tr}(F_{O_2}(a_1, b_1) \wedge G_{O_1}(a_2, b_2))|_{a_i = b_i = 0}. \]
1-forms $\hat{H}_E^1, \hat{H}_E^2, \hat{H}_O$ are the following differential operators

$$\hat{H}_E^i = \alpha_i(p, q, t) E_{\alpha\beta} \frac{\partial^2}{\partial a_{\alpha_1} \partial a_{\alpha_2}} \hat{b}_{12} + \beta_i(p, q, t) E_{\alpha\beta} \frac{\partial^2}{\partial b_{\beta_1} \partial b_{\beta_2}} \hat{a}_{12}$$

$$+ \gamma_i(p, q, t) (E_{\alpha\beta} \frac{\partial^2}{\partial a_{\alpha_2} \partial b_{\beta_1}} \hat{c}_{21} - E_{\alpha\beta} \frac{\partial^2}{\partial b_{\beta_2} \partial a_{\alpha_2}} \hat{c}_{12} ), \quad i = 1, 2,$$

$$\hat{H}_O = \alpha_3(p, q, t) E_{\alpha\beta} \frac{\partial^2}{\partial a_{\alpha_1} \partial a_{\alpha_2}} \hat{b}_{12}\hat{c}_{12} + \beta_3(p, q, t) E_{\alpha\beta} \frac{\partial^2}{\partial b_{\beta_1} \partial b_{\beta_2}} \hat{a}_{12}\hat{c}_{12}$$

$$+ \gamma_3(p, q, t) E_{\alpha\beta} \frac{\partial^2}{\partial a_{\alpha_1} \partial b_{\beta_2}} \hat{c}_{12}.$$  \hspace{1cm} (4.8)

Here $E_{\alpha\beta} = DV_{\alpha\beta}$ is the frame field (see Appendix A). The coefficients $\alpha, \beta, \gamma$, which parameterize various types of index contractions, depend on the operators:

$$p = \hat{a}_{12}\hat{b}_{12}, \quad q = \hat{c}_{12}\hat{c}_{21}, \quad t = \hat{c}_{11}\hat{c}_{22},$$  \hspace{1cm} (4.9)

where

$$\hat{a}_{12} = V_{\alpha\beta} \frac{\partial^2}{\partial a_{\alpha_1} \partial a_{\alpha_2}}, \quad \hat{b}_{12} = V_{\alpha\beta} \frac{\partial^2}{\partial b_{\beta_1} \partial b_{\beta_2}}, \quad \hat{c}_{ij} = \frac{\partial^2}{\partial a_{\alpha_1} \partial b_{\beta_2}}.$$  \hspace{1cm} (4.10)

In what follows we will use the notation $A_{\alpha, \beta, \gamma}(F, G)$ for (4.4) with the collective coefficients

$$\alpha = (\alpha_1, \alpha_2, \alpha_3), \quad \beta = (\beta_1, \beta_2, \beta_3), \quad \gamma = (\gamma_1, \gamma_2, \gamma_3).$$  \hspace{1cm} (4.11)

In our analysis the higher spin gauge fields will be allowed to take values in some associative (e.g., matrix) algebra $\Omega \rightarrow \Omega_{IJ}$. The resulting ambiguity is equivalent to the ambiguity of a particular choice of the Yang-Mills gauge algebra in the spin 1 sector. The classification of the higher spin gauge theories associated with the different Yang-Mills algebras is given in [7]. Therefore, the higher spin actions (4.6) and (4.7) are formulated in terms of the trace $\text{tr}$ in this matrix algebra (to be not confused with the trace in the star product algebra). As a result, only cyclic permutations of the matrix factors will be allowed under the trace operation. Remarkably, this property simplifies considerably the analysis of the gauge invariance of the cubic action. Note that the gravitational field is required to take values in the center of the matrix algebra, being proportional to the unit matrix. For this reason, the factors associated with the gravitational field are usually written outside the trace.

For general coefficients, the quadratic part of the action (4.2) does not describe massless higher spin fields because of ghost-type degrees of freedom associated with
extra fields $\omega^{s,t}, t > 0$. To eliminate these extra degrees of freedom one should fix the operators $\hat{H}$ (4.8) and (4.9) in a specific way by requiring the variation of the quadratic action with respect to the extra fields to vanish identically [17, 18]. This condition is referred to as the extra field decoupling condition. Another restriction on the form of the action (4.2) comes from the requirement that its quadratic part should decompose into an infinite sum of free actions for different copies of fields of the same spin associated with the spinor traces. This factorization condition [14] fixes a convenient basis in the space of fields rather than imposes true dynamical limitations on form of the action. Both of these conditions on the form of the quadratic part of the action (4.4) are analyzed in section 4.2. Also, we introduce the $C$-invariance condition [14] which states that the action (4.4) possesses the cyclic property with respect to the central element of the higher spin superalgebra. Being imposed, this natural condition simplifies greatly the analysis of the dynamical system involving infinite sequences of supermultiplets of the same spin. We show that the factorization condition along with the extra field decoupling condition and the $C$-invariance condition fix the functions $\alpha, \beta, \gamma$ (4.12) up to the normalization coefficients in front of the individual free actions modulo some ambiguity associated with total derivative terms in the Lagrangian.

In the sequel we find a precise form of the cubic action (4.2) that describes properly higher-spin-gravitational interactions of spin $s \geq 3/2$ fields in the first nontrivial order. Note, that although this positive result indicates the existence of a full nonlinear higher spin action, the constructed cubic action is not expected to be complete even at the cubic level. As mentioned in Introduction one reason for this is that the full spectrum of fields in the appropriate higher spin supermultiplet also contains spin 0, 1/2 and 1 massless fields not included in the consideration of this paper. Our modest goal here is to show that, similarly to the 4$\text{d}$ case [16], the problem with (cubic) higher-spin-gravitational interactions in the flat background [30, 31] can be avoided in $AdS_5$.

As explained in [16, 14] the analysis of the gauge invariance in the cubic order is simplified greatly by using the First On-Mass-Shell Theorem. The condition that the higher spin action is invariant under some deformation of the higher spin gauge transformations is equivalent to the condition\(^6\) that the original (i.e. undeformed) higher spin gauge variation of the action is zero once the linearized higher spin curvatures $R_1$ are replaced by the Weyl tensors $C$ according to (3.52)-(3.54). As a result, the problem is to find such functions $\alpha, \beta$ and $\gamma$ (4.12) that

$$\delta S(R, R)\big|_{E=0, R=h\wedge hC} \equiv \mathcal{A}_{\alpha,\beta,\gamma}^h(R, [R, \epsilon], \epsilon)\big|_{R=h\wedge hC} = 0 \quad (4.13)$$

\(^6\)Note that terms resulting from the gauge transformations of the gravitational fields and the compensator $V^{\alpha\beta}$ contribute into the factors in front of the higher spin curvatures in the action (4.4) – (4.7). The proof of the respective invariances is given in [14] and is based entirely on the explicit $su(2,2)$ covariance and invariance of the whole framework under diffeomorphisms. Also, one has to take into account that the higher spin gauge transformation of the gravitational fields is at least linear in the dynamical fields and therefore has to be discarded in the analysis of $\Omega^2 \epsilon$ type terms under consideration.
for an arbitrary gauge parameter $\epsilon(a, b, \psi, \bar{\psi}|x)$. As shown in section 4.3 this condition, supplemented with the factorization condition along with the extra field decoupling condition and the $C$-invariance condition, fixes the coefficients in the form

$$\alpha_1(p, q, t) + \beta_1(p, q, t) = \Phi_0 \sum_{m,n=0}^{\infty} (-1)^{m+n} \frac{m+1}{2^{2(m+n+1)}(m+n+2)!m!(n+1)!} p^m q^n ,$$

(4.14)

$$\gamma_1(p, q, t) = \gamma_1(p + q) , \quad \gamma_1(p) = \Phi_0 \sum_{m=0}^{\infty} (-1)^{m+1} \frac{1}{2^{2m+3}(m+2)!m!} p^m ,$$

(4.15)

$$\alpha_2(p, q, t) + \beta_2(p, q, t) = \frac{1}{4} (\alpha_1(p, q, t) + \beta_1(p, q, t)) , \quad \gamma_2(p, q, t) = \frac{1}{4} \gamma_1(p, q, t) ,$$

(4.16)

$$\alpha_3(p, q, t) + \beta_3(p, q, t) = \Phi_0 \sum_{m,n=0}^{\infty} (-1)^{m+n+1} \frac{1}{2^{2m+3}(m+1)!(m+n+2)!n!} p^m q^n ,$$

(4.17)

$$\gamma_3(p, q, t) = \gamma_3(p + q) , \quad \gamma_3(p) = \Phi_0 \sum_{m=0}^{\infty} (-1)^{m+1} \frac{1}{2^{2m+1}m!(m+1)!} p^m ,$$

(4.18)

where $\Phi_0$ is an arbitrary normalization factor to be identified with the (appropriately normalized in terms of the cosmological constant) gravitational coupling constant.

### 4.2 Quadratic Action

The free part $S_2$ of the action is obtained by the substitution of the linearized curvatures and the background frame field into (4.2). The resulting action is manifestly invariant under the linearized transformations (3.11) because the linearized curvatures $R_1$ are invariant i.e., $\delta R_1 = 0$. We want the free action to be a sum of actions for the irreducible higher spin fields. This requirement is not completely trivial because of the infinite degeneracy of the algebra due to the traces.

The factorization condition requires

$$S_2 = \sum_{n, s=0}^{\infty} B_{2}^{s,n}(\Omega_{E_{1,2}}^{n,s+2}) + \sum_{n, s=0}^{\infty} F_{2}^{s+3/2,n}(\Omega_{O_{1,2}}^{n,s+3/2}) ,$$

(4.19)

i.e. the terms containing products of the fields $\Omega_{E_{1,2}}^{n,s}$ and $\Omega_{O_{1,2}}^{m,s}$ with $n \neq m$ in the trace decomposition (3.24)-(3.25) should all vanish. As follows from (3.24)-(3.28) this is true iff

$$A_{\alpha, \beta, \gamma}(F, (T^+)^k G) = A_{\alpha^{(k)}, \beta^{(k)}, \gamma^{(k)}}((T^-)^k F, G) , \quad \forall k$$

(4.20)
for some new parameters $\alpha^{(k)}$, $\beta^{(k)}$, $\gamma^{(k)}$ (4.12). The factorization condition for the bosonic action $B$ (4.5) was analyzed in [14], where it was shown that

$$ B_{\alpha, \beta, \gamma}(F_E, T^+ G_E) = B_{\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}}(T^- F_E, G_E) , $$

where the new parameters $\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}$ express unambiguously in terms of $\alpha, \beta, \gamma$:

$$\alpha^{(1)}_{1,2} = 4 \left( 2 + p \frac{\partial}{\partial p} \right) \left( 1 + q \frac{\partial}{\partial q} \right) + \left( 2p \frac{\partial}{\partial p} + 2q \frac{\partial}{\partial q} + t \frac{\partial}{\partial t} + 6 \frac{\partial}{\partial t} \right) \alpha_{1,2} , \quad (4.22)$$

$$\beta^{(1)}_{1,2} = 4 \left( 2 + p \frac{\partial}{\partial p} \right) \left( 1 + q \frac{\partial}{\partial q} \right) + \left( 2p \frac{\partial}{\partial p} + 2q \frac{\partial}{\partial q} + t \frac{\partial}{\partial t} + 6 \frac{\partial}{\partial t} \right) \beta_{1,2} , \quad (4.23)$$

$$\gamma^{(1)}_{1,2} = 4 \left( 1 + p \frac{\partial}{\partial p} \right) \left( 2 + q \frac{\partial}{\partial q} \right) + \left( 2p \frac{\partial}{\partial p} + 2q \frac{\partial}{\partial q} + t \frac{\partial}{\partial t} + 6 \frac{\partial}{\partial t} \right) \gamma_{1,2} \quad (4.24)$$

provided that the following relation is satisfied

$$(1 + p \frac{\partial}{\partial p}) (\alpha_{1,2} + \beta_{1,2}) + 2(1 + q \frac{\partial}{\partial q}) \gamma_{1,2} = 0 . \quad (4.25)$$

As observed in [14], (4.25) is automatically true for the coefficients $\alpha^{(1)}_{1,2}, \beta^{(1)}_{1,2}$ and $\gamma^{(1)}_{1,2}$ and, therefore, (4.25) guarantees (4.20) in the bosonic sector for all $k$.

In the fermionic sector one gets

$$ F_{\alpha_3, \beta_3, \gamma_3, (F_O, T^+ G_O)} = F_{\alpha_3^{(1)}, \beta_3^{(1)}, \gamma_3^{(1)}}(T^- F_O, G_O) $$

$$ + \frac{1}{2} \int_{M^5} Q_O(p, q, t) E_{\alpha_3} \frac{\partial^2}{\partial a_1 \partial b_1} \hat{e}_{12} \wedge \text{tr}(G_{O_2}(a_1, b_1) \wedge F_{O_1}(a_2, b_2))|_{a_i=b_i=0} \quad (4.26)$$

$$ + \frac{1}{2} \int_{M^5} Q_O(p, q, t) E_{\alpha_3} \frac{\partial^2}{\partial a_1 \partial b_1} \hat{e}_{12} \wedge \text{tr}(F_{O_2}(a_1, b_1) \wedge G_{O_1}(a_2, b_2))|_{a_i=b_i=0} , $$

where

$$ Q_O = (1 + p \frac{\partial}{\partial p})(\alpha_3 + \beta_3) + \frac{\partial}{\partial q} \gamma_3 \quad (4.27) $$

and

$$\alpha_3^{(1)} = 4 \left( 2 + p \frac{\partial}{\partial p} \right) \left( 1 + q \frac{\partial}{\partial q} \right) + \left( 2p \frac{\partial}{\partial p} + 2q \frac{\partial}{\partial q} + t \frac{\partial}{\partial t} + 7 \frac{\partial}{\partial t} \right) \alpha_3 , \quad (4.28)$$

$$\beta_3^{(1)} = 4 \left( 2 + p \frac{\partial}{\partial p} \right) \left( 1 + q \frac{\partial}{\partial q} \right) + \left( 2p \frac{\partial}{\partial p} + 2q \frac{\partial}{\partial q} + t \frac{\partial}{\partial t} + 7 \frac{\partial}{\partial t} \right) \beta_3 , \quad (4.29)$$

$$\gamma_3^{(1)} = 4 \left( 1 + p \frac{\partial}{\partial p} \right) \left( 1 + q \frac{\partial}{\partial q} \right) + \left( 2p \frac{\partial}{\partial p} + 2q \frac{\partial}{\partial q} + t \frac{\partial}{\partial t} + 5 \frac{\partial}{\partial t} \right) \gamma_3 , \quad (4.30)$$

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The factorization condition therefore requires

\[ Q_0 = (1 + p \frac{\partial}{\partial p})(\alpha_3 + \beta_3) + \frac{\partial}{\partial q}\gamma_3 = 0. \quad (4.31) \]

From (4.31) it follows that the same relation is true for the coefficients \( \alpha^{(1)} \), \( \beta^{(1)} \) and \( \gamma^{(1)} \) (4.28)-(4.30), and, therefore, (4.25), (4.31) guarantee (4.20) for all \( k \).

An important role in the analysis of [14] was played by the \( C \)-invariance condition requiring that \( B(T^+ \ast F_E, G_E) = B(F_E, G_E \ast T^+) \). In the purely bosonic case the operator \( T^+ \) coincides with the central element \( N \). The meaning of the \( C \)-invariance condition is that the bilinear form used for the construction of the action has the cyclic (trace) property with respect to elements of the center of the algebra. It simplifies greatly the analysis of interactions and, eventually, allows for elementary reduction to the quotient algebra with the ideal generated by the central element \( N \) factored out (see section 5).

The supersymmetric \( C \)-invariance condition has analogous form

\[ A(N \ast F, G) = A(F, G \ast N), \quad (4.32) \]

where \( F \) and \( G \) are any elements satisfying \( F \ast N = N \ast F, \ G \ast N = N \ast G \). Making use of the formula

\[ N \ast F = (P^+ - P^-)F \]

\[ = ((T^+ - T^-)F_E - \frac{1}{4}F_E) + (T^+ - T^-)F_O, \psi \]

\[ + (T^+ - T^-)F_O, \bar{\psi} + ((T^+ - T^-)F_E - F_E) \psi \bar{\psi} \]

(4.34)

and taking into account the factorization condition (4.20), we rewrite the \( C \)-invariance condition (4.32) as

\[ B_{\alpha, \beta, \gamma}(F_E, T^+ G_E) + B_{\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}}(F_E, T^+ G_E) \]

\[ + F_{\alpha, \beta, \gamma}(F_O, T^+ G_O) + F_{\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}}(F_O, T^+ G_O) \]

\[ = B_{\alpha, \beta, \gamma}(T^+ F_E, G_E) + B_{\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}}(T^+ F_E, G_E) \]

\[ + F_{\alpha, \beta, \gamma}(T^+ F_O, G_O) + F_{\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}}(T^+ F_O, G_O) \]

\[ - \frac{1}{4} B'_{\alpha, \beta, \gamma}(F_{E_1}, G_{E_2}) + B''_{\alpha, \beta, \gamma}(F_{E_1}, G_{E_2}) \]

\[ + \frac{1}{4} B'_{\alpha, \beta, \gamma}(F_{E_2}, G_{E_1}) - B''_{\alpha, \beta, \gamma}(F_{E_2}, G_{E_1}) \].

(4.35)

The condition is true iff

\[ B_{\alpha, \beta, \gamma}(F_E, G_E) = -B_{\alpha^{(1)}, \beta^{(1)}, \gamma^{(1)}}(F_E, G_E), \quad (4.36) \]
\[ \mathcal{F}_{\alpha,\beta,\gamma}(F_O, G_O) = -\mathcal{F}_{\alpha^{(1)},\beta^{(1)},\gamma^{(1)}}(F_O, G_O), \quad (4.37) \]

\[ \frac{1}{4} \mathcal{B}'_{\alpha,\beta,\gamma}(F_E, G_E) = \mathcal{B}''_{\alpha,\beta,\gamma}(F_E, G_E), \quad (4.38) \]

i.e.

\[ \alpha_i(p, q, t) = -\alpha_i^{(1)}(p, q, t), \quad \beta_i(p, q, t) = -\beta_i^{(1)}(p, q, t), \quad i = 1, 2, 3, \]

\[ (4.39) \]

and

\[ \gamma_j(p, q, t) = -\gamma_j^{(1)}(p, q, t), \quad \gamma_j(p, q, t) = -\gamma_j^{(1)}(p, q, t), \quad j = 1, 2, 3 \]

\[ (4.40) \]

The conditions (4.36)-(4.37) are equivalent to the requirement that the operators \( T^- \) and \( T^+ \) satisfy the following conjugation rules

\[ \mathcal{B}(T^\pm F_E, G_E) = -\mathcal{B}(F_E, T^\mp G_E), \quad (4.41) \]

\[ \mathcal{F}(T^\pm F_O, G_O) = -\mathcal{F}(F_O, T^\mp G_O). \quad (4.42) \]

It is worth to note that the relations (4.41)-(4.42) may be equivalently represented in the form

\[ \mathcal{B}(T^+ \star G_E, F_E) = \mathcal{B}(G_E, F_E \star T^+), \quad (4.43) \]

\[ \mathcal{F}(T^+ \star G_O, F_O) = \mathcal{F}(G_O, F_O \star T^+), \quad \mathcal{F}(G_O \star T^+, F_O) = \mathcal{F}(G_O, T^+ \star F_O), \quad (4.44) \]

as one can easily see using that

\[ T^+ \star F_{E_{1,2}} = \left( T^+ - T^- \right) F_{E_{1,2}}, \quad [T^+, F_{E_{1,2}}] = 0, \quad (4.45) \]

\[ T^+ \star F_{O_{1,2}} = \left( T^+ - T^- + \frac{1}{2} S^0 \right) F_{O_{1,2}}, \quad (4.46) \]

\[ F_{O_{1,2}} \star T^+ = \left( T^+ - T^- - \frac{1}{2} S^0 \right) F_{O_{1,2}}, \quad (4.47) \]

\[ [T^+, F_{O_{1,2}}] = F_{O_1}, \quad [T^+, F_{O_{2}}] = -F_{O_2}. \quad (4.48) \]

Using (4.45)-(4.48) along with (3.18) it is elementary to compute the relative coefficients for the different copies of fields in the decomposition (3.24)-(3.25). The normalization coefficients (3.29)-(3.30) are chosen so that the linearized actions have the same form for different copies of the higher spin fields parameterized by the label \( n \)

\[ S_2 = \sum_{n, s=0}^{\infty} \mathcal{B}_2^n \left( \Omega_{E_{1,2}}^{n,s+2} \right) + \sum_{n, s=0}^{\infty} \mathcal{F}_2^{s+3/2} \left( \Omega_{O_{1,2}}^{n,s+3/2} \right). \quad (4.49) \]

In the linearized approximation it is therefore enough to analyze the situation for any fixed \( n \). We confine ourselves to the case of \( \Omega' = \Omega^{0,s'} \), i.e. we will assume in the rest of this section that \( T^- \Omega' = 0 \).
The extra field decoupling condition requires
\[
\frac{\delta B_2}{\delta \omega_{e_{1,2}}^t} \equiv 0, \quad \text{for} \quad t \geq 2, \quad \text{and} \quad \frac{\delta F_2}{\delta \omega_{o_{1,2}}^t} \equiv 0, \quad \text{for} \quad t \geq 1.
\] (4.50)

It was analysed in [14] for the bosonic sector and in [29] for free fermions. For the reader’s convenience we sketch here the main steps of this analysis. The generic variation of \( S_2 \) is schematically
\[
\delta S_2 = \frac{1}{2} \int_{M^5} D_0 \dot{H}_O \wedge \delta \Omega \wedge R_{1,o} + \frac{1}{2} \int_{M^5} D_0 \dot{H}_O \wedge R_{1,o} \wedge \delta \Omega \\
+ \int_{M^5} D_0 \dot{H}_E \wedge R_{1,e} \wedge \delta \Omega_E .
\] (4.51)

According to (3.39) and (3.41), (3.42) generic variation of the extra fields has the form
\[
\delta \Omega^{ex}_{E_{1,2}}(a,b) = (S^+)^2 \xi_{E_{1,2}}(a,b),
\] (4.52)
with an arbitrary \( \xi_{E_{1,2}}(a,b) \) satisfying \((N_a - N_b - 4)\xi_{E_{1,2}}(a,b) = 0\), and
\[
\delta \Omega^{ex}_{O_2}(a,b) = S^+ \xi_{O_2}(a,b), \quad \delta \Omega^{ex}_{O_1}(a,b) = S^- \xi_{O_1}(a,b),
\] (4.53)
with arbitrary \( \xi_{O_{1,2}}(a,b) \) satisfying \((N_b - N_a - 3)\xi_{O_1}(a,b) = 0\) and \((N_a - N_b - 3)\xi_{O_2}(a,b) = 0\). The condition \( \delta S_2 = 0 \) with respect to the extra field variations (4.52), (4.53) requires
\[
\alpha_i(p,q,0) + \beta_i(p,q,0) = -2 \int_0^1 du \left( 1 + q \frac{\partial}{\partial q} \right) \rho_i(pu + q,0), \quad i = 1, 2,
\]
\[
\alpha_3(p,q,0) + \beta_3(p,q,0) = - \int_0^1 du \frac{\partial}{\partial p} \rho_3(pu + q,0),
\] (4.54)
\[
\gamma_i(p,q,0) = \rho_i(p + q,0), \quad i = 1, 2, 3.
\]

Here the functions of one variable \( \rho_i(p + q), i = 1, 2, 3 \) parameterize the leftover ambiguity in the coefficients in front of the free actions of fields with different spins.

As observed in [14, 29], at the free field level, there is an ambiguity in the coefficients
\( \alpha_i(p, q, t) \) and \( \beta_i(p, q, t), \ i = 1, 2, 3 \) due to the freedom in adding a total derivative

\[
\delta S_2 = \frac{1}{2} \sum_{j=1}^{2} \int_{M^5} d \left( \Phi_j(p, q, t) \text{tr}(R_{E_j}(a_1, b_1| x) \wedge R_{E_j}(a_2, b_2| x)) \right|_{a_i=b_i=0}
\]

\[
+ \frac{1}{2} \int_{M^5} d \left( \Phi_3(p, q) \hat{c}_{12} \text{tr}(R_{O_2}(a_1, b_1) \wedge R_{O_1}(a_2, b_2)) \right|_{a_i=b_i=0}
\]

\[
= \frac{1}{2} \sum_{j=1}^{2} \int_{M^5} \frac{\partial \Phi_j(p, q, t)}{\partial p} \left( h^{\alpha \beta}_{1} \frac{\partial^2}{\partial b_1^\alpha \partial b_2^\beta} \hat{a}_{12} - h^{\alpha \beta}_{2} \frac{\partial^2}{\partial a_{1\alpha} \partial a_{2\beta}} \hat{b}_{12} \right)
\]

\[
\wedge \text{tr}(R_{E_j}(a_1, b_1| x) \wedge R_{E_j}(a_2, b_2| x)) \right|_{a_i=b_i=0}
\]

\[
+ \frac{1}{2} \int_{M^5} \frac{\partial \Phi_3(p, q)}{\partial p} \left( h^{\alpha \beta}_{1} \frac{\partial^2}{\partial b_1^\alpha \partial b_2^\beta} \hat{a}_{12} - h^{\alpha \beta}_{2} \frac{\partial^2}{\partial a_{1\alpha} \partial a_{2\beta}} \hat{b}_{12} \right)
\]

\[
\wedge \text{tr}(R_{O_2}(a_1, b_1) \wedge R_{O_1}(a_2, b_2)) \right|_{a_i=b_i=0}.
\]

As a result, the variation of the coefficients

\[
\delta \alpha_i(p, q, t) = \epsilon_i(p, q, t), \quad \delta \beta_i(p, q, t) = -\epsilon_i(p, q, t), \quad i = 1, 2, 3
\]

(4.56)
does not affect the physical content of the quadratic action, i.e., in accordance with (4.54), only the combination \( \alpha_i(p, q, t) + \beta_i(p, q, t) \) has invariant meaning at the free field level.

Thus, the factorization condition (4.20) along with the extra field decoupling condition (4.50) fix the functions \( \alpha, \beta, \gamma \) (discarding the trivial ambiguity (4.56)) up to arbitrary functions \( \rho(p) \) parameterizing the ambiguity in the normalization coefficients in front of the individual free bosonic and fermionic actions. Remarkably, the analysis of the gauge invariance in the cubic order fixes the functions \( \rho(p) \) unambiguously.

### 4.3 Cubic Interactions

Now we are in a position to analyze the condition (4.13) to prove the existence of a deformation of the higher spin gauge transformation that leaves the cubic part of the action (4.2) invariant up to higher-order corrections. The undeformed higher spin transformation of the curvatures \( \delta R = [R, \epsilon] \), with

\[
R(a, b, \psi, \bar{\psi}|x) = R_{E_1}(a, b|x) + R_{O_1}(a, b|x)\psi + R_{O_2}(a, b|x)\bar{\psi} + R_{E_2}(a, b|x)\psi \bar{\psi}, \quad (4.57)
\]

\[
\epsilon(a, b, \psi, \bar{\psi}|x) = \epsilon_{E_1}(a, b|x) + \epsilon_{O_1}(a, b|x)\psi + \epsilon_{O_2}(a, b|x)\bar{\psi} + \epsilon_{E_2}(a, b|x)\psi \bar{\psi} \quad (4.58)
\]
gives
\[\delta R_{E_1} = [R_{E_1}, \epsilon_{E_1}] + \frac{1}{4} [R_{E_2}, \epsilon_{E_2}] + \frac{1}{2} [R_{O_1}, \epsilon_{O_2}] + \frac{1}{2} [R_{O_2}, \epsilon_{O_1}], \tag{4.59}\]
\[\delta R_{O_1} = [R_{E_1}, \epsilon_{O_1}] + [R_{O_1}, \epsilon_{E_1}] - \frac{1}{2} [R_{O_1}, \epsilon_{E_2}] + \frac{1}{2} [R_{E_2}, \epsilon_{O_1}], \tag{4.60}\]
\[\delta R_{O_2} = [R_{E_1}, \epsilon_{O_2}] + [R_{O_2}, \epsilon_{E_1}] + \frac{1}{2} [R_{O_2}, \epsilon_{E_2}] - \frac{1}{2} [R_{E_2}, \epsilon_{O_2}], \tag{4.61}\]
\[\delta R_{E_2} = [R_{E_1}, \epsilon_{E_2}] + [R_{E_2}, \epsilon_{E_1}] + \{R_{O_1}, \epsilon_{O_2}\} - \{R_{O_2}, \epsilon_{O_1}\}. \tag{4.62}\]
where \([f, g] = f \star g - g \star f\) and \([f, g] = f \star g + g \star f\) for \(f = f(a, b)\) and \(g = g(a, b)\). As argued in [14], the gauge transformation deforms to
\[\delta \Omega = \delta^g \Omega + \Delta(R, \epsilon), \tag{4.63}\]
where \(\Delta(R, \epsilon)\) denotes some \(R\)-dependent terms such that \(\Delta(0, \epsilon) = 0\) and \(\delta^g\) denotes the gauge transformation (3.7). The transformations (4.63) can be rewritten as
\[\delta \Omega_E = (\delta^g \Omega)_E + \tilde{\Delta}_E (R_E, \epsilon_E) + \tilde{\Delta}_E (R_O, \epsilon_O), \tag{4.64}\]
\[\delta \Omega_O = (\delta^g \Omega)_O + \tilde{\Delta}_O (R_O, \epsilon_E) + \tilde{\Delta}_O (R_E, \epsilon_O).\]
Our aim is to find an action \(S\) that admits a consistent deformation of the gauge transformation guaranteeing that
\[\delta^g S + \frac{\delta S}{\delta \omega^{ph}} \Delta \omega^{ph} = O(\Omega^3 \epsilon), \tag{4.65}\]
where \(\Delta\) is some deformation of transformation law of the physical fields to be found. Taking into account (4.64), the second term gets the form
\[\frac{\delta S}{\delta \omega^{ph}} \Delta \omega^{ph} = \frac{\delta B_2}{\delta \omega^{ph}} \tilde{\Delta}_E (R_E, \epsilon_E) + \frac{\delta B_2}{\delta \omega^{ph}} \tilde{\Delta}_E (R_O, \epsilon_O) \]
\[+ \frac{\delta F_2}{\delta \omega^{ph}} \tilde{\Delta}_O (R_O, \epsilon_E) + \frac{\delta F_2}{\delta \omega^{ph}} \tilde{\Delta}_O (R_E, \epsilon_O). \tag{4.66}\]
Note that a deformation of the gauge variation of the extra fields does not contribute into the variation to the order under consideration because of (4.50). The first term on the l.h.s. of (4.65) has the structure \(R_E R_E \epsilon_E + R_E R_O \epsilon_O + R_O R_O \epsilon_E\), where all curvatures are linearized. Imposing the constraints on the extra fields proposed in [17, 18] which imply that the First-On-Mass-Shell Theorem is satisfied we use the representation (3.50)-(3.51) for the linearized curvatures and rewrite schematically the
first term in (4.65) as

\[ C_E C_E \epsilon_E + C_E C_O \epsilon_O + C_O C_O \epsilon_E \]

\[ + \tilde{\mathcal{H}}_E \left( R_E, \frac{\delta B_2}{\delta \omega_E^{ph}}, \epsilon_E \right) + \tilde{\mathcal{H}}_E \left( R_O, \frac{\delta B_2}{\delta \omega_E^{ph}}, \epsilon_O \right) \]

\[ + \tilde{\mathcal{H}}_O \left( R_O, \frac{\delta F_2}{\delta \omega_O^{ph}}, \epsilon_E \right) + \tilde{\mathcal{H}}_O \left( R_E, \frac{\delta F_2}{\delta \omega_O^{ph}}, \epsilon_O \right) , \tag{4.67} \]

where \( \mathcal{H}_E \) and \( \mathcal{H}_O \) are some trilinear functionals. Clearly, all terms in \( \mathcal{H}_E \) and \( \mathcal{H}_O \) can be compensated by the appropriate deformations \( \tilde{\Delta} \) and \( \tilde{\Delta} \). The terms bilinear in the higher spin Weyl tensors \( C \) cannot be compensated this way. The condition that the higher spin action is invariant under some deformation of the higher spin transformations is therefore equivalent to the requirement that the \( C^2 \) terms cancel out. This is expressed by (4.13).

Let us start our analysis with the variation with respect to an arbitrary bosonic higher spin transformation with the parameter \( \epsilon_{E_1}(a, b|x) = \epsilon_a^{(s)}(x) \alpha_a^{(s)} b^{(s)}. \) According to (4.13), our aim is to prove that there exist such coefficient functions \( \alpha, \beta \) and \( \gamma \) (4.12) satisfying the \( C \)-invariance condition, factorization condition and extra field decoupling condition that

\[ 2B^h \left( [\epsilon_{E_1}, R_{1, E_{1,2}}], [\epsilon_{E_1}, R_{1, E_{1,2}}] \right) \bigg|_{R=\hbar hC} \]

\[ + \mathcal{F}^h \left( \epsilon_{E_1}, R_{1, O_2} \right) \bigg|_{R=\hbar hC} \]

\[ + \mathcal{F}^h \left( \epsilon_{E_1}, R_{1, O_2} \right) \bigg|_{R=\hbar hC} = 0 \tag{4.68} \]

for arbitrary gauge parameter \( \epsilon_{E_1} = \epsilon_{E_1}(a, b|x) \) and arbitrary Weyl tensors \( C(a) \). Taking into account the decompositions (3.34)-(3.35), the condition (4.68) takes the form

\[ 2 \sum_{mn} B^h \left( \left( T^+ \right)^m v_{E_{1,2}, m}(T^0) R^m_{1, E_{1,2}}(a, b), \left[ \epsilon_{E_1}, \left( T^+ \right)^n v_{E_{1,2}, n}(T^0) R^n_{1, E_{1,2}}(a, b) \right] \right) \bigg|_{R=\hbar hC} \]

\[ + \sum_{mn} \mathcal{F}^h \left( \left( T^+ \right)^m v_{O_{1,2}, m}(T^0) R^m_{1, O_{1,2}}(a, b), \left[ \epsilon_{E_1}, \left( T^+ \right)^n v_{O_{1,2}, n}(T^0) R^n_{1, O_{1,2}}(a, b) \right] \right) \bigg|_{R=\hbar hC} \]

\[ + \sum_{mn} \mathcal{F}^h \left( \left[ \epsilon_{E_1}, \left( T^+ \right)^m v_{O_{1,2}, m}(T^0) R^m_{1, O_{1,2}}(a, b), \left[ \epsilon_{E_1}, \left( T^+ \right)^n v_{O_{1,2}, n}(T^0) R^n_{1, O_{1,2}}(a, b) \right] \right) \right) \bigg|_{R=\hbar hC} = 0 \tag{4.69} \]

with an arbitrary gauge parameter \( \epsilon_{E_1}(a, b|x) \) and arbitrary Weyl tensors \( C^m_{E_{1,2}}(a|x) \) and \( C^m_{O_{1,2}}(a|x) \) in the decomposition (3.52)-(3.54) for the linearized higher spin curvatures \( R^m_{1, E_{1,2}}(a, b|x) \) and \( R^m_{1, O_{1,2}}(a, b|x). \)

First of all, one observes that the dependence of \( v_n(T^0) \) on \( T^0 \) can be absorbed into (spin-dependent) rescalings of the Weyl tensors \( C^m(a) \) which are treated as arbitrary
field variables in this consideration. As a result it is enough to prove (4.69) for arbitrary constant coefficients \( v_n \). Now let us show that, once (4.69) is valid for \( m = n = 0 \), it is automatically true for all other values of \( m \) and \( n \) as a consequence of the relations (4.43) and (4.44) which follow from the factorization condition and the \( C \)-invariance condition. For the bosonic part this was shown in [14], where the proof was based on the bosonic \( C \)-invariance condition (4.43). Because the relation (4.43) is still valid, the proof remains the same. Thereby we focus on the fermionic part \( \mathcal{F} \).

Suppose that (4.69) is true for \( m_0 \geq m \geq 0 \), \( n_0 \geq n \geq 0 \). Consider the term with \( m = m_0 + 1 \). Then, from (4.46) it follows

\[
(T^+)^{m_0+1} R_{1,02}^{m_0+1}(a,b) = T^+ \ast ((T^+)^{m_0} R_{1,02}^{m_0+1}(a,b)) + T^- (T^+)^{m_0} R_{1,02}^{m_0+1}(a,b) - \frac{1}{2} (T^+)^{m_0} R_{1,02}^{m_0+1}(a,b) .
\]

(4.70)

After the substitution of this expression into (4.69) the term containing \( T^- \) gives zero contribution by the induction assumption since, taking into account that \( T^- R_{1,02}^{m_0+1}(a,b) = 0 \), \( T^- \) decreases a number of \( T^+ \). The last term in (4.70) does not contribute by the induction assumption as well. By virtue of the (4.44) along with the commutation relations (4.48) and the fact that \( T^+ \) commutes with bosonic elements of \( cu(1,1|8) \) the terms containing the star product with \( T^+ \) are

\[
\mathcal{F}^h \left( \left( (T^+)^{m_0} v_{O2,m_0} (T^0) R_{1,02}^{m_0} (a,b) \right)_{m.s.} \right) \ast \left( T^+ \ast \epsilon_{E1} , (T^+)^{m_0} v_{O1,n_0} (T^0) R_{1,01}^{m_0} (a,b) \right)_{m.s.} .
\]

(4.71)

which is zero by the induction assumption valid for any \( \epsilon_{E1} \). Analogously, one performs induction \( n_0 \to n_0 + 1 \) with respect to \( R_{1,01} \) with the help of (4.44)-(4.48).

Thus it is sufficient to find the coefficients satisfying the \( C \)-invariance condition and the factorization condition for traceless curvatures \( R = \mathcal{R} \equiv R^0 \) satisfying \( T^- (\mathcal{R}) = 0 \). In other words one has to prove that

\[
\mathcal{S}^h(\mathcal{R}, [\epsilon_{E1}, \mathcal{R}], \ast) = 0 ,
\]

(4.72)

where

\[
\mathcal{R}_{E1,2}(a,b) = H_2 \alpha^\beta \frac{\partial^2}{\partial a_\alpha \partial b_\beta} \text{Res}_\mu (C_{E1,2}(\mu a + \mu^{-1} b))
\]

(4.73)

for bosons and

\[
\mathcal{R}_{O1}(a,b) = H_2 \alpha^\beta \frac{\partial^2}{\partial a_\alpha \partial b_\beta} \text{Res}_\mu (\mu C_{O1}(\mu a + \mu^{-1} b)) ,
\]

(4.74)

\[
\mathcal{R}_{O2}(a,b) = H_2 \alpha^\beta \frac{\partial^2}{\partial a_\alpha \partial b_\beta} \text{Res}_\mu (\mu^{-1} C_{O2}(\mu a + \mu^{-1} b))
\]

(4.75)

for fermions. Note that because \( T^- (\mathcal{R}) = 0 \), the terms containing \( \hat{c}_{11} \) (4.11) and, therefore, \( t \) (4.10) do not contribute into the condition (4.72).
Consider the variation (4.69) of the fermionic action:

$$\delta F^h = \int_{M^5} \hat{H}_O \wedge \text{tr}(\delta R_{O_2}(a_1, b_1) \wedge R_{O_1}(a_2, b_2))|_{a_i=b_i=0}$$

$$+ \int_{M^5} \hat{H}_O \wedge \text{tr}(R_{O_2}(a_1, b_1) \wedge \delta R_{O_1}(a_2, b_2))|_{a_i=b_i=0}. \tag{4.76}$$

Substituting $\delta R_{O_1} = [R_{O_1}, \epsilon_{E_1}]_*$ and $\delta R_{O_2} = [R_{O_2}, \epsilon_{E_1}]_*$ one gets

$$\delta F^h = \int_{M^5} \hat{H}_O \wedge \text{tr}((R_{O_2} \ast \epsilon_{E_1})(a_1, b_1) \wedge R_{O_1}(a_2, b_2))|_{a_i=b_i=0}$$

$$- \int_{M^5} \hat{H}_O \wedge \text{tr}(\epsilon_{E_1} \ast R_{O_2})(a_1, b_1) \wedge R_{O_1}(a_2, b_2))|_{a_i=b_i=0}$$

$$+ \int_{M^5} \hat{H}_O \wedge \text{tr}(R_{O_2}(a_1, b_1) \wedge (R_{O_1} \ast \epsilon_{E_1})(a_2, b_2))|_{a_i=b_i=0}$$

$$- \int_{M^5} \hat{H}_O \wedge \text{tr}(R_{O_2}(a_1, b_1) \wedge (\epsilon_{E_1} \ast R_{O_1})(a_2, b_2))|_{a_i=b_i=0}. \tag{4.77}$$

Let us calculate explicitly the first term in (4.77). Making use of the star product (2.1) along with the identities (6.17)-(6.21) applied to the background fields, and rewriting (4.74), (4.75) as

$$R_{O_1}(a, b) = \text{Res}_\mu \mu^{-1} e^{\mu^{-1}a_\gamma \frac{\partial}{\partial c^\gamma} + \mu b_\gamma \frac{\partial}{\partial c^\gamma}} H_2^{\alpha\beta} \frac{\partial^2}{\partial c^\alpha \partial c^\beta} C_{O_1}(c)|_{c=0}, \tag{4.78}$$

$$R_{O_2}(a, b) = \text{Res}_\mu \mu^{-1} e^{\mu^{-1}a_\gamma \frac{\partial}{\partial c^\gamma} + \mu b_\gamma \frac{\partial}{\partial c^\gamma}} H_2^{\alpha\beta} \frac{\partial^2}{\partial c^\alpha \partial c^\beta} C_{O_2}(c)|_{c=0}, \tag{4.79}$$

one finds

$$\int_{M^5} \hat{H}_O \wedge \text{tr}((R_{O_2} \ast \epsilon_{E_1})(a_1, b_1) \wedge R_{O_1}(a_2, b_2))|_{a_i=b_i=0}$$

$$= -\frac{1}{30} \int_{M^5} H_5 \bar{k}^2 \text{Res}_\mu \left(\mu^{-1} e^{\frac{\bar{k}}{2}(\mu^{-1}a_2 - \mu a_2)}(\mu \bar{k} - \bar{u}_1) \Phi(Y)\right) \times$$

$$\times \text{tr}(C_{O_2}(c_2)C_{O_1}(c_1)\epsilon_{E_1}(a_3, b_3))|_{a=b=c=0}, \tag{4.80}$$

where $H_5$ denotes the vacuum 5-form defined in Appendix A,

$$\bar{k} = \frac{\partial^2}{\partial c_1 \partial c_2}, \quad \bar{u}_i = \frac{\partial^2}{\partial c_1^i \partial a_3}, \quad \bar{v}_i = \frac{\partial^2}{\partial c_1 \partial b_3}, \tag{4.81}$$

$$Y = (\mu^{-1}\bar{k} + \bar{v}_i)(\mu \bar{k} - \bar{u}_1) \tag{4.82}$$
\[ \Phi(Y) = Y(\alpha_3(Y, -Y) + \beta_3(Y, -Y)) + \gamma_3(Y, -Y). \]  

Calculating analogously the remaining terms in (4.77) one obtains for the whole variation (4.76)

\[ \delta \mathcal{F}^h = -\frac{1}{15} \int_{\mathcal{M}^5} H_5 \, \bar{k}^2 \, \text{Res}_\mu \left( \mu^{-1} e^{\frac{1}{2}(\mu^{-1} \bar{u}_2 - \mu \bar{u}_2)} (\mu \bar{k} - \bar{u}_1) \Phi(Y) \right) \times \]
\[ \times \text{tr} \left( C_{O_2}(c_2) C_{O_1}(c_1) \epsilon_{E_1}(a_3, b_3) \right) \left|_{a=b=c=0} \right. \]
\[ + \frac{1}{15} \int_{\mathcal{M}^5} H_5 \, \bar{k}^2 \, \text{Res}_\mu \left( \mu^{-1} e^{\frac{1}{2}(\mu^{-1} \bar{v}_1 - \mu \bar{v}_1)} (\mu \bar{k} - \bar{v}_2) \Phi(Z) \right) \times \]
\[ \times \text{tr} \left( C_{O_2}(c_2) C_{O_1}(c_1) \epsilon_{E_1}(a_3, b_3) \right) \left|_{a=b=c=0} \right., \]

where

\[ Z = (\mu \bar{k} - \bar{v}_2)(\mu^{-1} \bar{k} + \bar{u}_2). \]  

Introducing notations

\[ A = (\mu \bar{k} - \bar{u}_1), \quad B = (\mu^{-1} \bar{k} + \bar{v}_1), \]
\[ F = (\mu \bar{k} - \bar{v}_2), \quad D = (\mu^{-1} \bar{k} + \bar{u}_2), \]

the problem amounts to the search for a such function \( \Phi(Y) \) that

\[ \bar{k}^2 \, \text{Res}_\mu \left( \mu^{-1} A e^{\frac{1}{2}(\mu^{-1} \bar{u}_2 - \mu \bar{u}_2)} \Phi(AB) - \mu^{-1} F e^{\frac{1}{2}(\mu^{-1} \bar{v}_1 - \mu \bar{v}_1)} \Phi(FD) \right) \times \]
\[ \text{tr} \left( C_{O_2}(c_2) C_{O_1}(c_1) \epsilon_{E_1}(a_3, b_3) \right) \left|_{a=b=c=0} \right. = 0. \]  

Defining \( \hat{\Phi}(A, B) = A \Phi(AB) \) one rewrites (4.87) as

\[ \bar{k}^2 \, \text{Res}_\mu \left( \mu^{-1} e^{\frac{1}{2}(\mu^{-1} \bar{u}_2 - \mu \bar{u}_2)} \hat{\Phi}(A, B) - \mu^{-1} e^{\frac{1}{2}(\mu^{-1} \bar{v}_1 - \mu \bar{v}_1)} \hat{\Phi}(F, D) \right) \times \]
\[ \text{tr} \left( C_{O_2}(c_2) C_{O_1}(c_1) \epsilon_{E_1}(a_3, b_3) \right) \left|_{a=b=c=0} \right. = 0. \]  

Now one observes that the function \( \hat{\Phi}(A, B) = \Phi_0 \text{Res}_\nu (\nu^{-1} e^{\frac{1}{2}(\nu A + \nu^{-1} B)}) \), where \( \Phi_0 \) is some normalization constant, solves (4.88).

As a result, the condition (4.72) amounts to

\[ A(\alpha_3(A, -A) + \beta_3(A, -A)) + \gamma_3(A, -A) = \Phi_0 A^{-1} \text{Res}_\nu (\nu^{-1} e^{\frac{1}{2}(\nu A + \nu^{-1})}) \]
\[ = \frac{\Phi_0}{2} \int_0^1 du \, \text{Res}_\nu e^{\frac{1}{2}(\nu^{-1} + \nu u A)}. \]
Taking into account (4.54) this is solved by
\[ \gamma_3(p) = \frac{\Phi_0}{2} \int_0^1 du \, \text{Res}_\nu e^{\frac{1}{2}(-\nu^{-1} + \nu p u)} \tag{4.90} \]
and
\[ (\alpha_3 + \beta_3)(p, q) = \frac{\gamma_3(p + q)}{q} - \frac{\Phi_0}{2q} \int_0^1 du \, \text{Res}_\nu e^{\frac{1}{2}(-\nu^{-1} + \nu (pu + q))}. \tag{4.91} \]
With the aid of these expressions one can see that the following identities are true
\[ \left( p \frac{\partial^2}{\partial p^2} + 2 \frac{\partial}{\partial p} + \frac{1}{4} \right) \gamma_3(p) = 0, \tag{4.92} \]
\[ \left( \left( 2 + p \frac{\partial}{\partial p} \right) \frac{\partial}{\partial p} + \left( 2 + q \frac{\partial}{\partial q} \right) \frac{\partial}{\partial q} + \frac{1}{4} \right) \left( \alpha_3(p, q, 0) + \beta_3(p, q, 0) \right) = 0. \tag{4.93} \]
From these identities and relations (4.28)-(4.30) it follows then that the \( C^- \) invariance condition (4.32) is satisfied with
\[ \alpha_3(p, q, t) + \beta_3(p, q, t) = \alpha_3(p, q, 0) + \beta_3(p, q, 0), \quad \gamma_3(p, q, t) = \gamma_3(p, q, 0). \tag{4.94} \]
The power series expansion of the expressions (4.90)-(4.91) for \( \gamma_3(p) \) and \( \alpha_3(p, q, 0) + \beta_3(p, q, 0) \) gives (4.17) and (4.18).

Thus it is shown that the coefficient functions (4.90) and (4.91) satisfy the factorization condition, \( C^- \) invariance condition, extra field decoupling condition and the condition (4.13) in the fermionic sector. The leftover ambiguity in the coefficients \( \alpha_3(p, q, t) + \beta_3(p, q, t) \) and \( \gamma_3(p, q, t) \) reduces to the overall factor \( \Phi_0 \) in front of the fermionic action \( \mathcal{F} \).

The explicit form of the coefficients of the bosonic action was fixed in [14] by the requirement of its invariance under the (appropriately deformed) higher spin transformations with the parameters \( \epsilon(a, b|x) = \epsilon_{\alpha(s)}^\alpha(x) a_{\alpha(s)} b_{\beta(s)} \). The results of [14] remain true in our model. The respective coefficient functions are
\[ \gamma_i(p) = \frac{\Phi_i}{4} \int_0^1 dv v \, \text{Res}_\nu \left( \nu e^{\frac{1}{2}(-\nu^{-1} + \nu v p)} \right), \quad i = 1, 2 \tag{4.95} \]
and
\[ \alpha_i(p, q, 0) + \beta_i(p, q, 0) = 2 \gamma_i(p + q) - \frac{\Phi_i}{2} \int_0^1 du \, \text{Res}_\nu \left( \nu e^{\frac{1}{2}(-\nu^{-1} + \nu (up + q))} \right), \quad i = 1, 2, \tag{4.96} \]
where \( \Phi_1 \) and \( \Phi_2 \) are arbitrary real constants.

The variation with respect to bosonic parameters \( \epsilon_{E_2}(a, b, \psi, \bar{\psi}|x) = \epsilon_{\alpha(s)}^\alpha(x) a_{\alpha(s)} b_{\beta(s)} \psi \bar{\psi} \) relates the coefficients \( \Phi_1, \Phi_2 \) as
\[ \Phi_2 = \frac{1}{4} \Phi_1 \tag{4.97} \]
and gives equations on the fermionic coefficients equivalent to those that follow from the variation with respect to $\epsilon_{E_1}(a, b|x)$ (4.77). Note that the condition (4.97) derived by virtue of the gauge symmetry gives the same relation between bosonic coefficients (4.38) as fixed by the $C$-invariance condition (4.32).

Consider now the variation of the full action with respect to fermionic transformation with an arbitrary gauge parameter $\epsilon_O(a, b, \psi, \bar{\psi}|x) = \epsilon_{O_1}(a, b|x)\psi + \epsilon_{O_2}(a, b|x)\bar{\psi}$. Taking into account (4.59)-(4.62), one obtains

$$
\delta A^h = \int_{M^5} \hat{H}_{E_1} \wedge \text{tr}(\mathcal{R}_{E_1}(a_1, b_1) \wedge (\mathcal{R}_{O_1} \star \epsilon_{O_2} - \epsilon_{O_2} \star \mathcal{R}_{O_1})(a_2, b_2))|_{a_i=b_i=0}
$$

$$
+ 2 \int_{M^5} \hat{H}_{E_2} \wedge \text{tr}(\mathcal{R}_{E_2}(a_1, b_1) \wedge (\mathcal{R}_{O_1} \star \epsilon_{O_2} + \epsilon_{O_2} \star \mathcal{R}_{O_1})(a_2, b_2))|_{a_i=b_i=0}
$$

$$
+ \int_{M^5} \hat{H}_{O} \wedge \text{tr}((\mathcal{R}_{E_1} \star \epsilon_{O_2} - \epsilon_{O_2} \star \mathcal{R}_{E_1})(a_1, b_1) \wedge \mathcal{R}_{O_1}(a_2, b_2))|_{a_i=b_i=0}
$$

$$
- \frac{1}{2} \int_{M^5} \hat{H}_{O} \wedge \text{tr}((\mathcal{R}_{E_2} \star \epsilon_{O_2} + \epsilon_{O_2} \star \mathcal{R}_{E_2})(a_1, b_1) \wedge \mathcal{R}_{O_1}(a_2, b_2))|_{a_i=b_i=0}
$$

$$
+ \text{analogous terms containing } \epsilon_{O_1}.
$$

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Proceeding analogously to the bosonic transformation one arrives at:

\[
\delta \mathcal{A}^h = \frac{1}{15} \int_{\mathcal{M}^5} H_5 \bar{k}^2 \text{Res}_\mu e^{\frac{1}{2}(\nu^{-1}a_2 - \nu b_2)} \left( \frac{1}{2} \Psi_i(Y) \frac{\partial}{\partial c_2^\sigma} - \mu (\nu^{-1}k + \bar{v}_i) \frac{\partial \Psi_i(Y)}{\partial \bar{c}_1^\sigma} \right)
\]

\[
\times \text{tr} (\epsilon_{O_2 \rho(s)} a_3 \gamma(s) \bar{b}_3^{(s)} C_{E_1(c_1)} C_{O_1(c_2)})|_{a=b=c=0}
\]

\[
+ \frac{1}{15} \int_{\mathcal{M}^5} H_5 \bar{k}^2 \text{Res}_\mu e^{\frac{1}{2}(\nu^{-1}a_1 - \nu \bar{u}_1)} \left( \left( Z \frac{\partial \Phi(Z)}{\partial Z} + \Phi(Z) \right) \frac{\partial}{\partial \bar{c}_2^\sigma} - \frac{\mu}{2} (\nu^{-1}k + \bar{u}_2) \Phi(Z) \frac{\partial}{\partial \bar{c}_1^\sigma} \right)
\]

\[
\times \text{tr} (\epsilon_{O_2 \rho(s)} a_3 \gamma(s) \bar{b}_3^{(s)} C_{E_1(c_1)} C_{O_1(c_2)})|_{a=b=c=0}
\]

\[
+ \frac{2}{15} \int_{\mathcal{M}^5} H_5 \bar{k}^2 \text{Res}_\mu e^{\frac{1}{2}(\nu^{-1}a_2 - \nu \bar{u}_2)} \left( \left( Z \frac{\partial \Phi(Z)}{\partial Z} + \Phi(Z) \right) \frac{\partial}{\partial \bar{c}_2^\sigma} - \frac{\mu}{2} (\nu^{-1}k + \bar{u}_2) \Phi(Z) \frac{\partial}{\partial \bar{c}_1^\sigma} \right)
\]

\[
\times \text{tr} (\epsilon_{O_2 \rho(s)} a_3 \gamma(s) \bar{b}_3^{(s)} C_{E_2(c_1)} C_{O_1(c_2)})|_{a=b=c=0}
\]

\[
+ \frac{1}{30} \int_{\mathcal{M}^5} H_5 \bar{k}^2 \text{Res}_\mu e^{\frac{1}{2}(\nu^{-1}a_1 - \nu \bar{a}_1)} \left( \left( Z \frac{\partial \Phi(Z)}{\partial Z} + \Phi(Z) \right) \frac{\partial}{\partial \bar{c}_2^\sigma} - \frac{\mu}{2} (\nu^{-1}k + \bar{a}_2) \Phi(Z) \frac{\partial}{\partial \bar{c}_1^\sigma} \right)
\]

\[
\times \text{tr} (\epsilon_{O_2 \rho(s)} a_3 \gamma(s) \bar{b}_3^{(s)} C_{E_2(c_1)} C_{O_1(c_2)})|_{a=b=c=0}
\]

\[
+ \text{ analogous terms containing } \epsilon_{O_1},
\]

where

\[
\Psi_i(Y) = \Phi_i \text{Res}_\nu e^{\frac{1}{2}(\nu^{-1}+\nu Y)}, \quad i = 1, 2,
\]

\[
\Phi(Z) = \Phi_0 Z^{-1} \text{Res}_\nu \left( \nu^{-1} e^{\frac{1}{2}(\nu^{-1}+\nu Z)} \right)
\]

and \(Y\) and \(Z\) are defined by (4.82) and (4.85). An important observation is that the functions (4.100) satisfy

\[
Z \frac{\partial \Phi(Z)}{\partial Z} + \Phi(Z) = \frac{\Phi_0}{2 \Phi_1} \Psi_i(Z), \quad i = 1, 2.
\]

Using notations (4.86) we get from (4.100)

\[
D \Phi(DF) = \Phi_0 \text{Res}_\nu \left( \nu^{-1} e^{\frac{1}{2}(\nu D + \nu^{-1}F)} \right),
\]

\[
B \partial \Psi_i(AB) = \frac{\Phi_i}{2} \text{Res}_\nu \left( \nu^{-1} e^{\frac{1}{2}(\nu B + \nu^{-1}A)} \right), \quad i = 1, 2.
\]
Assuming the relation between bosonic coefficients (4.97), with the help of (4.101) the problem is reduced to the search of a solution to the equations

\[
\frac{1}{2} k^2 \text{Res}_{\mu, \nu} \left( e^{\frac{1}{2}(\mu^{-1}u_2 - \mu^{-1}v_2)} \Psi_1(Y) + \frac{\Phi_0}{\Phi_1} e^{\frac{1}{2}(\mu^{-1}v_1 - \mu^{-1}u_1)} \Psi_1(Z) \right) \frac{\partial}{\partial \bar{c_2}} \times \\
\times \text{tr}(\epsilon_{O_2}^{\sigma(s)} a_3 \gamma(s) b_3^{\rho(s)} C_{E_{1,2}}(c_1) C_{O_1}(c_2)) \bigg|_{a=b=c=0} 
\]

\[
-\bar{k}^2 \text{Res}_{\mu, \nu} \left( \mu e^{\frac{1}{2}(\mu^{-1}u_2 - \mu^{-1}v_2)} B \partial \Psi_1(AB) + \frac{1}{2} \mu e^{\frac{1}{2}(\mu^{-1}v_1 - \mu^{-1}u_1)} D \Phi(FD) \right) \frac{\partial}{\partial \bar{c_1}} \times \\
\times \text{tr}(\epsilon_{O_2}^{\sigma(s)} a_3 \gamma(s) b_3^{\rho(s)} C_{E_{1,2}}(c_1) C_{O_1}(c_2)) \bigg|_{a=b=c=0} = 0 .
\]

Substituting the functions (4.100), (4.102) into (4.103) one gets

\[
\frac{1}{2} k^2 \text{Res}_{\mu, \nu} \left( (\Phi_1 + \Phi_0)e^{\frac{1}{2}(\mu^{-1}u_2 - \mu^{-1}v_2 + \nu^{-1}u_1 + \nu^{-1}v_1 - \nu^{-1}k + \nu^{-1}k - \nu^{-1}u_1)) \right) \frac{\partial}{\partial \bar{c_2}} \times \\
\times \text{tr}(\epsilon_{O_2}^{\sigma(s)} a_3 \gamma(s) b_3^{\rho(s)} C_{E_{1,2}}(c_1) C_{O_1}(c_2)) \bigg|_{a=b=c=0} 
\]

\[
-\frac{1}{2} \bar{k}^2 \text{Res}_{\mu, \nu} \left( (\Phi_1 + \Phi_0)\nu^{-1} \mu e^{\frac{1}{2}(\mu^{-1}u_2 - \mu^{-1}v_2 + \nu^{-1}u_1 + \nu^{-1}v_1 + \nu^{-1}k - \nu^{-1}u_1))} \right) \frac{\partial}{\partial \bar{c_1}} \times \\
\times \text{tr}(\epsilon_{O_2}^{\sigma(s)} a_3 \gamma(s) b_3^{\rho(s)} C_{E_{1,2}}(c_1) C_{O_1}(c_2)) \bigg|_{a=b=c=0} = 0 .
\]

This is true provided that

\[
\Phi_0 = -\Phi_1 .
\]

Analogous analysis of the terms with \( \epsilon_{O_1} \) in the higher spin transformation shows that the invariance condition (4.13) is satisfied provided that (4.97) and (4.105) are true. The leftover ambiguity in the coefficients (4.14)-(4.18) reduces to an overall factor \( \Phi_0 \) encoding the ambiguity in the gravitational constant.

Thus, the action (4.2) is shown to properly describe the higher spin \( N = 1 \) supersymmetric dynamics both at the free field level and at the level of cubic interactions provided that the coefficients of the bilinear form in (4.2) are fixed according to (4.14)-(4.18).

## 5 Reduced Model

So far we discussed the 5d higher spin algebra \( cu(1,1|8) \) being the centralizer of \( N \) in the star product algebra. This algebra is not simple as it contains infinitely many
ideals $I_{P(N)}$ spanned by the elements of the form $P(N) \star F$ for any $F \in cu(1,1|8)$ and any star-polynomial $P(N)$ [15]. In this section we focus on the algebra $hu_0(1,1|8)$ that results [7] from factoring out the maximal ideal corresponding to $P(N) = N$. As we show, elements of this algebra are spanned by the supertraceless multispinors. Thus $hu_0(1,1|8)$ describes the system of higher spin fields with every supermultiple t emerging once. Note that the algebra $hu_0(1,1|8)$ does not provide a maximal reduction of the original higher spin algebra. The higher spin algebras with maximally reduced spectra $ho_0(1,1|8)$ and its bosonic subalgebra $ho_0(1,0|8)$ were discussed in [6, 7].

We apply the approach elaborated for the pure bosonic system in [14] which consists of inserting a sort of projection operator $M$ to the quotient algebra into the action. Namely, let $M$ satisfy

$$N \star M = M \star N = 0. \quad (5.1)$$

Having specified the ”operator” $M$ we write the action for the reduced system associated with $hu_0(1,1|8)$ by replacing the bilinear form in the action with

$$\mathcal{A}(F,G) \rightarrow \mathcal{A}_0(F,G) = \mathcal{A}(F,M \star G), \quad (5.2)$$

where $\mathcal{A}(F,G)$ corresponds to the action describing the original (unreduced) higher spin dynamics. To maintain gauge invariance we require $M$ to commute with elements of $cu(1,1|8)$

$$F \star M = M \star F , \quad F \in cu(1,1|8). \quad (5.3)$$

In fact, this implies that $M$ should be some star-function of $N$. From the $C$-invariance condition it follows then

$$\mathcal{A}(F,M \star G) = \mathcal{A}(F \star M, G), \quad (5.4)$$

i.e. the bilinear form in the action with $M$ inserted remains symmetric.

As a result, all terms proportional to $N$ do not contribute to the action (5.2) which therefore is defined on the quotient subalgebra. The representatives of the quotient algebra $hu_0(1,1|8)$ are identified with the elements $F$ satisfying the supertracelessness condition

$$P^- F = 0. \quad (5.5)$$

This allows one to require all fields in the expansion (3.1)-(3.5) to be supertraceless. Indeed, by virtue of (4.33) any polynomial $\tilde{F}(a,b,\psi,\bar{\psi}|x) \in cu(1,1|8)$ is equivalent to some $F$ satisfying (5.5) modulo terms containing star products with $N$ which trivialize when acting on $M$. The star product $F \star G$ of any two elements $F$ and $G$ satisfying the supertracelessness condition does not necessarily satisfies the same condition, i.e. $P^-(F \star G) \neq 0$ (otherwise the elements satisfying (5.5) would form a subalgebra rather than the quotient algebra). However the difference is again proportional to $N$ and can be discarded inside the action built with the help of the bilinear form $A_0$. 

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To find explicit form of $M$, one observes that any star-function of $N$ is some (may be different) ordinary function of $N$, i.e.

$$\mathcal{M}(N) \equiv M(N) = M(a, b^\gamma) - M'(a, b^\gamma)\psi\bar{\psi},$$  \hspace{1cm} (5.6)

where $M'$ denotes a derivative of $M$. This is a simple consequence of the fact that any such function has to commute with the generators of $su(2, 2|1)$. The later condition imposes some first-order differential equations which are solved by an arbitrary function of $N$.

The substitution of (5.6) into (5.1) results in the second order differential equation

$$xM''(x) + 3M'(x) - 4xM(x) = 0, \quad x \equiv a\gamma b^\gamma,$$  \hspace{1cm} (5.7)

which admits a unique analytic solution (up to a factor)

$$M(x) = \int_0^1 d\tau \text{ Res}_\nu e^{-\frac{1}{4}(\nu^{-1} + 4\nu x^2\tau)}.$$  \hspace{1cm} (5.8)

Equivalently

$$M(x) = \sum_{n=0}^\infty \frac{x^{2n}}{n!(n+1)!}.$$  \hspace{1cm} (5.9)

Having found the operator $\mathcal{M}$ we define the action for the reduced system associated with $hu_0(1, 1|8)$ in the form (5.2). Note that $\mathcal{A}(F, G)$ with inserted $\mathcal{M}$ according to (5.2) is well-defined as a functional of polynomial functions $F$ and $G$ because for polynomial $F$ and $G$ only a finite number of terms in the expansion of $\mathcal{M}(a, b^\gamma, \psi\bar{\psi})$ contributes.

The modification of the action according to (5.2) does not contradict to the analysis of section 4.3 where the action (4.2) was claimed to be fixed unambiguously, because in that analysis we have imposed the factorization condition in the particular basis of higher spin fields thus not allowing the transition to the invariant action (5.2). The factorization condition is relaxed in this section. All other conditions, namely $C$-invariance condition, extra field decoupling condition and the condition (4.13) remain valid.

6 Conclusion

In this paper we have analysed cubic interactions in the theory of higher spin fields in $AdS_5$ for the particular case of $\mathcal{N} = 1$ supersymmetry. It is shown that free field abelian higher spin gauge transformations admit such a deformation that the constructed cubic action, that is general coordinate invariant and contains gravity, remains invariant up to higher order terms.

Our conclusions are valid both for unreduced model based on $cu(1, 1|8)$ (every supermultiplet $(s, s-\frac{1}{2}, s-1)$ determined by an integer highest spin $s = 2, 3, \ldots, \infty$ appears in infinitely many copies) and for reduced model based on $hu_0(1, 1|8)$ symmetry in which
every such supermultiplet appears only once. In this respect our conclusions are different from those of [32], where it was argued that only unreduced algebra $c^u(1,1|8)$ admits consistent dynamics in the framework of $4d$ higher spin conformal theory (although the two models are different since the model of [32], being a higher spin extension of the $4d C^2$ gravity, contains higher derivatives and ghosts, while our model in $AdS_5$ is unitary in the physical space at least at the free field level).

Note that the constructed higher-spin cubic vertices do not exhaust all possible consistent supersymmetric higher-spin interactions in $AdS_5$ in the order under consideration. One reason for that is that we discard low-spin ($s \leq 1$) interactions which truncation is consistent in the cubic order only. The study of the explicit form of cubic couplings of particular higher spins in terms of physical fields is the technically complicated problem requiring full-scale investigation which is beyond the scope of this paper. The developed technics contains, however, all necessary ingredients for the detailed analysis of the constructed interactions in terms of physical fields which may be of interest in the context of $AdS/CFT$ computations.

The generalization of the presented constructions to $\mathcal{N} \geq 2$ extended supersymmetry is not straightforward as it requires mixed symmetry higher spin fields to be included [7, 14]. The progress along this direction is hampered by lacking a manifestly covariant Lagrangian description of massless gauge fields of this type in $AdS_d$ with $d \geq 5$ even at the free field level. The method employed in the present paper for constructing higher spin cubic couplings is essentially based on the Lagrangian formulation of higher spin gauge field dynamics in terms of physical fields which is beyond the scope of this paper. The developed technics contains, however, all necessary ingredients for the detailed analysis of the constructed interactions in terms of physical fields which may be of interest in the context of $AdS/CFT$ computations.

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Appendix A. Compensator formalism in spinor notations

For the reader’s convenience in this appendix we collect some useful formulae on the compensator formalism in spinor notations developed in [14].

A \( o(6) \) complex vector \( V^A \) \((A = 0, ..., 5)\) is equivalent to the antisymmetric \( sl_4 \) bispinor \( V^{\alpha \beta} = -V^{\beta \alpha} \) having six independent components (equivalently, one can use \( V^{\alpha \beta} = \frac{1}{2} \varepsilon_{\alpha \beta \gamma \delta} V^{\gamma \delta} \) where \( \varepsilon_{\alpha \beta \gamma \delta} \) is the \( sl_4 \) invariant totally antisymmetric tensor (\( \varepsilon_{1234} = 1 \)). A \( o(4, 2) \) real vector \( V^A \) is described by the antisymmetric bispinor \( V^{\alpha \beta} \) satisfying the reality condition

\[
\nabla^{\gamma \delta} C_{\gamma \alpha} C_{\delta \beta} = \frac{1}{2} \varepsilon_{\alpha \beta \gamma \delta} V^{\gamma \delta} .
\]

One can see that the invariant norm of the vector

\[
V^2 = V^{\alpha \beta} V^{\alpha \beta}
\]

has the signature \((++---)\). The vectors with \( V^2 > 0 \) are time-like while those with \( V^2 < 0 \) are space-like. To perform a reduction of the representations of the \( AdS_5 \) algebra \( su(2, 2) \sim o(4, 2) \) into representations of its Lorentz subalgebra \( o(4, 1) \) we introduce a \( su(2, 2) \) antisymmetric compensator \( V^{\alpha \beta} \) with \( V^2 > 0 \). The Lorentz algebra is identified with its stability subalgebra. (Let us note that \( V^{\alpha \beta} \) must be different from the form \( C^{\alpha \beta} \) used in the definition of the reality conditions (2.12) - (2.18) since the latter is space-like and therefore has \( sp(4; R) \sim o(3, 2) \) as its stability algebra.)

Using that the total antisymmetrization over any four indices is proportional to the \( \varepsilon \) symbol, we normalize \( V^{\alpha \beta} \) so that

\[
V_{\alpha \beta} V^{\alpha \gamma} = \delta_{\beta \gamma}, \quad V^{\alpha \beta} = \frac{1}{2} \varepsilon_{\alpha \beta \gamma \delta} V^{\gamma \delta}, \quad \varepsilon_{\alpha \beta \gamma \delta} = V_{\alpha \beta} V^{\gamma \delta} + V_{\beta \gamma} V^{\alpha \delta} + V_{\gamma \alpha} V^{\beta \delta} .
\]

The gravitational fields are identified with the gauge fields taking values in the \( AdS_5 \) algebra \( su(2, 2) \)

\[
\Omega = \Omega^{\alpha \beta} a^{\alpha} b^{\beta} .
\]

The invariant definitions of the frame field and Lorentz connection for a \( x \)-dependent compensator \( V^{\alpha \beta}(x) \) are

\[
E^{\alpha \beta} = DV^{\alpha \beta} \equiv dV^{\alpha \beta} + \Omega^{\alpha \gamma} V^{\gamma \beta} + \Omega^{\beta \gamma} V^{\alpha \gamma} , \quad \Omega^{\alpha \beta} = \Omega^{\alpha \beta} + \frac{1}{2} E^{\alpha \gamma} V_{\gamma \beta} .
\]

The normalization condition (6.3) implies

\[
E_{\alpha \beta} = -DV_{\alpha \beta} , \quad E_{\alpha}^{\alpha} = 0 .
\]
AdS$_5$ background geometry is defined by zero-curvature condition

$$R^\alpha_\beta \equiv d\Omega^\alpha_\beta + \Omega^\alpha_\gamma \wedge \Omega^\gamma_\beta = 0,$$  (6.10)

which decomposes into Lorentz components as

$$R^{L\alpha}_\beta \equiv d\Omega^{L\alpha}_\beta + \Omega^{L\alpha}_\gamma \wedge \Omega^{L\gamma}_\beta + \frac{1}{4}E^\alpha_\gamma \wedge E^\gamma_\beta = 0,$$  (6.11)

$$T^{\alpha\beta} \equiv dE^{\alpha\beta} + \Omega^{\alpha}_\gamma \wedge E^{\gamma\beta} + \Omega^{\beta}_\gamma \wedge E^{\alpha\gamma} = 0.$$  (6.12)

(6.12) is the conventional zero-tension condition, while the equation (6.11) requires the geometry to be anti-de Sitter.

The nondegeneracy condition implies that $E^{\alpha\beta}$ spans a basis of the 5d 1-forms. The basis $p$-forms $E_p$ can be realized as

$$E^{\alpha\beta}_2 = E^{\beta\alpha}_2 = E^\alpha_\gamma \wedge E^\beta_\gamma,$$  (6.13)

$$E^{\alpha\beta}_3 = E^{\beta\alpha}_3 = E^\alpha_\gamma \wedge E^\beta_\gamma,$$  (6.14)

$$E^{\alpha\beta}_4 = -E^{\alpha\beta}_4 = E^\alpha_\gamma \wedge E^\beta_\gamma,$$  (6.15)

$$E^\gamma_5 = E^{\alpha\gamma}_4 \wedge E^\beta_\gamma.$$  (6.16)

The following useful relationships hold as a consequence of the facts that 5d spinors have four components and the frame field is traceless

$$E^{\alpha\beta} \wedge E^{\gamma\delta} = \frac{1}{2}(V^{\alpha\gamma} E^{\beta\delta}_2 - V^{\beta\gamma} E^{\alpha\delta}_2 - V^{\alpha\delta} E^{\beta\gamma}_2 + V^{\beta\delta} E^{\alpha\gamma}_2),$$  (6.17)

$$E^{\alpha\beta}_2 \wedge E^{\gamma\delta} = -\frac{1}{3}(V^{\alpha\gamma} E^{\beta\delta}_3 + V^{\beta\gamma} E^{\alpha\delta}_3 - V^{\alpha\delta} E^{\beta\gamma}_3 - V^{\beta\delta} E^{\alpha\gamma}_3 + V^{\gamma\delta} E^{\alpha\beta}_3),$$  (6.18)

$$E^{\alpha\beta}_4 \wedge E^{\gamma\delta}_3 = -\frac{1}{4}(V^{\alpha\gamma} E^{\beta\delta}_4 - V^{\beta\gamma} E^{\alpha\delta}_4 + V^{\alpha\delta} E^{\beta\gamma}_4 - V^{\beta\delta} E^{\alpha\gamma}_4),$$  (6.19)

$$E^{\alpha\beta}_4 \wedge E^{\gamma\delta}_5 = -\frac{1}{20}(2V^{\alpha\gamma} V^{\beta\delta} - 2V^{\alpha\delta} V^{\beta\gamma} - V^{\alpha\beta} V^{\gamma\delta}) E_5.$$  (6.20)

The following useful relationships hold as a consequence of the facts that 5d spinors have four components and the frame field is traceless

The background frame field and Lorentz connection are denoted $h = h^{\alpha}_a a^{\beta}_b$ and $\Omega_0^L = \Omega_0^{L\alpha}_a a^{\beta}_a$, respectively. Vacuum values of the $p$-forms $E_p^{\alpha\beta}$ (6.13)-(6.16) are denoted $H_p^{\alpha\beta}$. 

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Appendix B. Trace and supertrace decompositions in $cu(1,1|8)$.

In this appendix we show how to derive the expansions (3.24)-(3.25). We start with

$$\Omega(a,b,\psi,\bar{\psi}|x) = \sum_{k=0}^{\infty} \sum_{s=1}^{\infty} \chi(k,s)(P^+)^k \Omega^{k,s+1}(a,b,\psi,\bar{\psi}|x),$$

(6.22)

where $\chi(k,s)$ are some coefficients, $s+1$ denotes highest integer spin in a supermultiplet, $\Omega^{k,s+1}$ satisfy $P^0 \Omega^{k,s+1} = (2s+3)/4 \Omega^{k,s+1}$ and are supertraceless

$$P^- \Omega^{k,s+1}(a,b,\psi,\bar{\psi}|x) = 0.$$

(6.23)

The solution of (6.22)-(6.23) is given by (3.22), (3.23). Taking into account that

$$(P^+)^k = (T^+)^k - k(T^+)^{k-1}\psi\bar{\psi}$$

(6.24)

(the usual product here should not be confused with the star product) after some algebra one gets from (6.22)

$$\Omega(a,b,\psi,\bar{\psi}|x) = \sum_{k,s=0}^{\infty} \beta(k,s)(T^+)^k \Omega^{k,s+1}_E(a,b|x) + \sum_{k,s=0}^{\infty} \rho(k,s)(T^+)^k \Omega^{k,s+3/2}_E(a,b|x)\psi\bar{\psi}$$

$$+ \sum_{k,s=0}^{\infty} \chi(k,s+1)(T^+)^k \Omega^{k,s+3/2}_O(a,b|x)\psi + \sum_{k,s=0}^{\infty} \chi(k,s+1)(T^+)^k \Omega^{k,s+3/2}_O(a,b|x)\bar{\psi},$$

(6.25)

where

$$\beta(k,s)\Omega^{k,s+1}_E(a,b|x) = \theta(k-1)\chi(k-1,s+1)\Omega^{k-1,s+1}_E(a,b|x) + \theta(s-1)\chi(k,s)\Omega^{k,s+1}_E(a,b|x),$$

(6.26)

$$\rho(k,s)\Omega^{k,s+1}_E(a,b|x) = \chi(k,s+1)\Omega^{k,s+1}_E(a,b|x)$$

$$- \theta(s-1)(k+1)\chi(k+1,s)\Omega^{k+1,s+1}_E(a,b|x) + \frac{k}{2s+5}\chi(k,s+1)\Omega^{k,s+1}_E(a,b|x),$$

(6.27)

$$\Omega^{k,s+3/2}_O(a,b|x) = \Omega^{k,s+3/2}_O(a,b|x), \quad \Omega^{k,s+3/2}_O(a,b|x) = \Omega^{k,s+3/2}_O(a,b|x).$$

(6.28)

All multispinors on the l.h.s. of (6.25) are traceless as a consequence of (3.23). We see that supertraceless and traceless bases are related by a finite linear field redefinition.

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