ND-POR: A POR Based on Network Coding and Dispersal Coding*

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SUMMARY Nowadays, many individuals and organizations tend to outsource their data to a cloud storage for reducing the burden of data storage and maintenance. However, a cloud provider may be untrustworthy. The cloud thus leads to a numerous security challenges: data availability, data integrity, and data confidentiality. In this paper, we focus on data availability and data integrity because they are the prerequisites of the existence of a cloud system. The approach of this paper is the network coding-based Proof of Retrievability (POR) scheme which allows a client to check whether his/her data stored on the cloud servers are intact. Although many existing network coding-based PORs have been proposed, most of them still incur high costs in data check and data repair, and cannot prevent the small corruption attack which is a common attack in the POR scheme. This paper proposes a new network coding-based POR using the dispersal coding technique, named the ND-POR (Network coding - Dispersal coding POR) to improve the efficiency in data check and data repair and to protect against the small corruption attack.

**key words**: data integrity, data availability, proof of retrievability, network coding, dispersal coding, cloud storage

1. Introduction

1.1 Background

Since amount of data is increasing exponentially, data storage and management become increasingly troublesome tasks for the data owners. To reduce the burdens for the data owners, the concept of remote storage known as cloud has been proposed. A cloud is considered as a service through which the clients can use to publish, access, manage and share their data remotely and easily from anywhere via the Internet. However, the shortcoming of this system is that a cloud storage provider could not be necessarily trusted. The cloud system thus introduces three security challenges: data availability, data integrity and data confidentiality. Because data availability and data integrity are the pre-conditions for the existence of a system, they are more important than data confidentiality. Therefore, this paper focuses on data availability and data integrity rather than data confidentiality.

**Proof of Retrievability.** To assist the client in checking whether cloud servers satisfy data availability and data integrity, researchers proposed Provable Data Possession (PDP) [1], [2] and Proof of Retrievability (POR) [3]–[5] which are challenge-response protocols between a client and cloud servers. Both protocols support data check. However, only the POR can ensure that the data are always retrievable. Thus, the POR is considered to be a stronger tool. The POR protocol consists of four phases: keygen, encode, check and repair.

**Approaches.** Based on the POR protocol, the following two approaches are commonly used. In the first approach, the data are only stored on a single server [3], [4]. The client periodically checks the data and can thus detect data corruption. However, this approach does not allow data repair when a corruption is detected. In the second approach, the client stores data redundantly on multiple servers. Many papers focus on this approach, e.g., [5]–[14]. When a corrupted server is detected, the client can use the remaining healthy servers to repair the data stored on the corrupted server. In this approach, there are three common techniques: replication, erasure coding and network coding.

- **Replication.** Curtmola et al. [9] proposed this technique which allows the client to store a file replica on each server. When a corruption is detected, the client uses one of healthy replicas to repair the corruption. The drawback of this technique, however, is high storage cost because the client must store a whole file on each server.

- **Erasure coding.** Because replication has high storage cost, erasure coding is applied to outsourced data [10] to provide space-optimal data redundancy. In this technique, each server stores file blocks (portions of the file) instead of file replica (copy of the whole file) like replication. The size of the file blocks stored on each server is less than the size of the whole file. Thus, erasure coding can reduce storage cost of replication. However, the drawback of this technique is that to repair a corrupted data, the client must reconstruct the entire file before generating new coded blocks. Therefore, this technique increases computation cost and communication cost in data repair.

- **Network coding.** To improve the efficiency in data repair, researchers apply network coding technique to such outsourced data [11]. Unlike erasure coding, the...
The client does not need to reconstruct the entire file before generating new coded blocks. Instead, the coded blocks which are retrieved from the healthy servers are used to generate new coded blocks. Therefore, this paper focuses on the network coding technique.

The data cannot be checked without embedded information, i.e., Message Authentication Code (MAC) or digital signature. The MAC is sometimes called tag. The MAC is used in the symmetric key setting while the digital signature is used in the asymmetric key setting. Because this paper is based on the symmetric key setting, the MAC is used in the proposed scheme.

**Network Coding-based POR.** Based on the POR, many schemes have been proposed, for instance, [8], [9] using replica, [5]–[7] using erasure coding. However, we are only aware of a few network coding-based POR schemes. Dimakis et al. [12] was the first to apply the network coding to achieve a remarkable reduction in the communication overhead of the repair component. Li et al. [13] proposed a tree-structure data regeneration with the linear network coding to achieve an efficient regeneration traffic and bandwidth capacity by using an undirected-weighted maximum spanning tree. Chen et al. [14] presented the Remote Data Checking for Network Coding-based distributed storage system (RDC-NC) scheme which provides a decent solution for efficient data repair by recoding encoded blocks on the healthy servers during the repair procedure. Le et al. [15] introduced the NC-Audit scheme for efficient check and repair using a new homomorphic MAC technique called SpaceMac. Recently, Chen et al. [16] have proposed the NC-Cloud scheme to improve the cost-effectiveness of repair using the functional minimum-storage regenerating (FMSR) codes, which lightens the encoding requirement of storage nodes during repair.

In these network coding-based POR schemes, the most notable scheme is the RDC-NC scheme [14]. It, unlike the other previous schemes, not only focuses on the efficiency, but also considers how to prevent the three common attacks of the POR: **replay attack, pollution attack** and **large corruption attack**. However, the RDC-NC scheme has some shortcomings: (i) the corruption check is still inefficient because only one server can be checked per challenge and (ii) it cannot prevent another common attack of the POR: **small corruption attack**. The small corruption attack is defined in [6], [17]–[19]. In this attack, the adversary tries to corrupt the data with a small data unit to hide data loss incidents. Protecting against the small corruption attack protects the data itself, not just the storage resource. Modifying a single bit may destroy an encrypted file or invalidate authentication information. The difference between the large and small corruption attacks is that the small corruption attack corrupts at most $t$-fraction of the file while the large corruption attack corrupts more than $t$-fraction of the file, where $t$ is a parameter. These are described more details in the adversarial model (Sect. 3).

To address the small corruption attack, the common solution is to use the Error-Correcting Code (ECC) [20], which allows the data to be checked for errors and corrected even one bit on the fly. The ECC has several types, i.e., Hamming code, Golay code, Reed-Muller code, Reed-Solomon code, etc. However, this paper uses the Reed-Solomon code because the Universal Hash Function can be constructed using the Reed-Solomon code. Bowers et al. [6] then proposed the dispersal coding using the Reed-Solomon code in order to prevent the small corruption attack and to ensure the file integrity with high probability. However, [6] uses the erasure coding instead of the network coding.

### 1.2 Contribution

This paper proposes a new POR scheme using the network coding and the dispersal coding, called ND-POR. To the best of our knowledge, the ND-POR scheme is the first POR to apply both the dispersal coding and the network coding. The contribution of the ND-POR scheme is described as follows:

**Security.** The ND-POR scheme, unlike the RDC-NC scheme, can prevent the small corruption attack.

**Efficiency.**

- The RDC-NC scheme allows the client to check one server for each challenge. Meanwhile, the ND-POR scheme allows the client to check all servers simultaneously for each challenge.
- In the RDC-NC scheme, the number of MACs is $n$, where $n$ denotes the number of servers, $\alpha$ denotes the number of coded blocks stored on a server and $s$ denotes the number of segments in a coded block. In the ND-POR scheme, the number of MACs is only $\frac{l}{n}$, where $l$ denotes some servers out of $n$ servers ($l < n$) and is far less than the dominant parameter $s$.
- In data repair, the RDC-NC scheme uses the network coding to repair the corruptions. Meanwhile, the ND-POR scheme performs two phases: if the number of corruptions is smaller than the ECC boundary, the ECC is used to repair the corruptions; otherwise the network coding is used to repair the corruptions. Thus, the corruptions are repaired with an overwhelming probability. Furthermore, the ECC uses the parity information on the server itself to repair without the other healthy servers as the network coding.

The dispersal coding is constructed based on UMAC (MAC obtained from Universal Hash Function) which is closely related to the network coding-based schemes [21], [22]. Hence, the network coding and the dispersal coding can be suitably combined together in the ND-POR scheme.

### 1.3 Organization

The background of the Proof of Retrievability, the network
coding and the dispersal coding are described in Sect. 2. The adversarial model is given in Sect. 3. The ND-POR scheme is proposed in Sect. 4. The security and efficiency analyses are discussed in Sect. 6. The conclusion and future work are drawn in Sect. 7.

2. Preliminaries

2.1 System Model

The ND-POR scheme has two entities. The first entity is the client who can be individuals or organizations. The client outsources his/her data to a cloud storage and relies on the cloud storage for data storage and maintenance. The second entity is the cloud servers which are managed by a cloud provider. The cloud servers store the data of the clients and have responsibility to prove to the client that the stored data are always available and intact.

2.2 Proof of Retrievability

To check whether the availability and integrity of the data stored on the cloud servers are satisfied, researchers proposed Proof of Retrievability (POR) [3]–[5] which is a challenge-response protocol between a client (verifier) and a server (prover). A POR protocol has the following phases:

- **keygen(1^λ):** Given a security parameter λ, the client generates a secret key (sk) and a public key (pk). For symmetric key setting, pk is set to be null.
- **encode(sk, F):** The client encodes an original file (F) to an encoded file (F'), then stores F' on the server.
- **check(sk):** The client uses sk to generate a challenge (c) and sends c to the server. The server computes a response (r) and sends r back to the client. The client then verifies r to determine whether F is available and intact.
- **repair():** If a corruption is detected in the check phase, the client will execute this phase to repair the corrupted data. The technique of this phase depends on the each specific scheme, e.g., replication, erasure coding or network coding.

2.3 Network Coding

The network coding [11], [21] offers a good trade-off in terms of redundancy, reliability, and repair bandwidth. The network coding is firstly proposed in the network scenario, and is then applied in the distributed storage system scenario.

**Fundamental Concept.** In the network scenario, suppose that a source node wants to send a file F to a receiver node via the network. The source node firstly divides F into m blocks: \( F = v_1 \| \cdots \| v_m \), \( v_k \in \mathbb{F}_p^k \) where \( k \in \{1, \cdots, m\} \) and \( \mathbb{F}_p^k \) denotes a \( k \)-dimensional finite field of a prime order \( p \). The source node then augments \( v_k \) with a vector of length \( m \) which consists of a single ‘1’ in the \( k \)-th position and ‘0’ elsewhere. Let \( \{b_1, \cdots, b_m\} \) denote the augmented blocks. \( b_k \) has the following form:

\[
b_k = (v_k, 0, \cdots, 0, 1, 0, \cdots, 0) \in \mathbb{F}_p^{z + m}
\]

The source node sends \( \{b_1, \cdots, b_m\} \) as packets to the network. Suppose that an intermediate node in the network receives \( \theta \) packets \( \{b_{i_1}, \cdots, b_{i_\theta}\} \). The intermediate node generates \( \theta \) coefficients \( \alpha_1, \cdots, \alpha_\theta \in \mathbb{F}_p \) and linearly combines the received packets and transmits the resulting linear combination to the adjacent nodes. Therefore, each packet carries \( m \) accumulated coefficients which produce that packet as a linear combination of all \( m \) augmented blocks. The receiver node can retrieve the augmented blocks from any set of \( m \) combinations. If \( y \in \mathbb{F}_p^{z + m} \) is a linear combination of \( b_1, \cdots, b_m \in \mathbb{F}_p^{z + m} \), then the file blocks \( v_1, \cdots, v_m \) can be calculated from the first coordinate of \( y \) using the coefficients that contained in the last \( m \) coordinates of \( y \).

**Application in Distributed Storage System.** In the network scenario, there are multiple entities: source node, intermediate nodes and receiver node. However, when the network coding is applied in the distributed storage system scenario, there are only two entities: a client and cloud servers. Let \( w = z + m \) where \( z \) and \( m \) are introduced in the fundamental concept. From the original file \( F = \{v_1, \cdots, v_m\} \), the client firstly creates \( m \) augmented blocks \( \{b_1, \cdots, b_m\} \in \mathbb{F}_p^{z + m} \). The client then chooses \( m \) coefficients \( \{\alpha_1, \cdots, \alpha_m\} \in \mathbb{F}_p \) and linearly combines \( m \) augmented blocks to create the coded blocks as \( c = \sum_{k=1}^m \alpha_k \cdot b_k \in \mathbb{F}_p^w \). The client stores the coded blocks on the servers. \( \{\alpha_1, \cdots, \alpha_m\} \) are chosen such that the matrix which consists of all the coefficients of the coded blocks has full rank. Koetter et al. [23] proved that if the prime \( p \) is chosen large enough and the coefficients are chosen randomly, the matrix will have full rank with a high probability. When a corruption is detected, the client retrieves the coded blocks from the healthy servers and linearly combines them to regenerate new coded blocks. For example, in Fig. 1, from the augmented blocks \( \{b_1, b_2, b_3\} \), the client chooses the coefficients to compute six coded blocks. The client stores two coded blocks on each of the servers \( \{S_1, S_2, S_3\} \). Suppose that \( S_1 \) is corrupted, the client requests \( S_2 \) and \( S_3 \) to compute the aggregated coded blocks by themselves using the linear combinations. The client finally mixes the aggregated coded blocks to obtain two new coded blocks for the new server.

2.4 Dispersal Coding

To prevent the small corruption attack and to allow the client to repair the data with a high probability, the dispersal coding is proposed [6] with a minimal additional storage over-
Universal Hash Function (UHF). A UHF [24] is a function $h : \mathcal{K} \times I^l \rightarrow I$ where $I$ denotes a field with operations $(+, \times)$. This UHF compresses a message $m \in I^l$ into a compact digest based on a key $\kappa \in \mathcal{K}$ such that the hash of two different messages is different with an overwhelming probability over keys. A common UHF is almost XOR universal (AXU) which satisfies:

- $h$ is an $\epsilon$-UHF family if $\forall x \neq y \in I^l : \Pr_{\kappa \sim \mathcal{K}}[ h_\kappa(x) = h_\kappa(y) ] \leq \epsilon$.
- $h$ is an $\epsilon$-AXU family if $\forall x \neq y \in I^l$, and $\forall z \in I$: $\Pr_{\kappa \sim \mathcal{K}}[ h_\kappa(x) \oplus h_\kappa(y) = z ] \leq \epsilon$.
- If a UHF is linear, for any message pair $(m_1, m_2)$: $h_\kappa(m_1) + h_\kappa(m_2) = h_\kappa(m_1 + m_2)$.

Error-Correcting Code (ECC). An ECC [20] is used to express a sequence of the original data and the parity data such that any errors can be detected and corrected. An ECC has two parameters: $(n, l)$ where $l$ denotes the number of the original blocks, and $n$ denotes the number of blocks after adding $(n - l)$ redundant blocks. There exists $(n, l)$-ECC codes that can correct up to $t = \frac{n - l}{2}$ errors. The Reed-Solomon code (RS) [25] is a kind of ECC which uses a special polynomial: $g(x) = (x - a_1)(x - a_2) \cdots (x - a_l)$. The codeword of the RS code is $c(x) = g(x) \cdot i(x)$ where $g(x)$ is the generator polynomial over $\mathbb{F}_p$, $i(x)$ is the information block, and $a$ is a primitive element of the field.

Encoder: The $2t$ parity symbols are given by: $p(x) = i(x) \cdot x^{n-t} \pmod{g(x)}$.

Decoder: Given a codeword $r(x) \pm$ which is the original codeword $c(x)$ plus errors: $r(x) = c(x) + e(x)$, the RS decoder identifies the position and magnitude of up to $t$ errors.

- Calculating the syndrome: The RS codeword has $2t$ syndromes which depend on errors. The syndromes are calculated by substituting the $2t$ roots of $g(x)$ into $r(x)$.
- Finding the symbol error locations: This involves solving the equations with $t$ unknowns. The first step is to find an error locator polynomial using the Berlekamp-Massey algorithm or the Euclid algorithm. The second step is to find the roots of this polynomial using the Chien search algorithm.

- Finding symbol error values: This involves solving the equations with $t$ unknowns using the Forney algorithm.

Universal Hash Function which is constructed using the Reed-Solomon code (RS-UHF). A RS-UHF [26] is constructed in a way as follows. Suppose that a message $m$ is a vector $\vec{m} = (m_1, \cdots, m_l)$ where $m_i \in I$ and suppose that an $(n, l)$-RS code over $I$ is used. $\vec{m}$ is viewed as a polynomial representation of the form $p_m(x) = m_1 x^{n-1} + m_{l-1} x^{n-2} + \cdots + m_1$. A RS code can be defined as a vector $\vec{r} = (k_1, \cdots, k_n)$. The codeword of a message $\vec{m}$ is the evaluation of polynomial $p_m(x)$ at point $(k_1, \cdots, k_n)$. A UHF is $h_\kappa(m) = p_m(\kappa)$ where $\kappa$ is the key.

Message Authentication Code (MAC). A MAC [27] is used to authenticate a message and to detect message tampering and forgery. A MAC is a tuple of $(\text{MGen}, \text{MTag}, \text{MVer})$:

- $	ext{MGen}(1^l)$: generates a secret key $\kappa$ given a security parameter $\lambda$.
- $\text{MTag}_\kappa(m)$: computes a tag $\tau$ for the message $m$ with the key $\kappa$.
- $\text{MVer}_\kappa(m, \tau)$: outputs 1 if $\tau$ is a valid tag, and 0 otherwise.

Pseudo-random Function (PRF). A PRF [28] is used to generate exponentially many random bits in a way that behaves like a random function. A PRF is a keyed family of a function $g : \mathcal{K}_{\text{PRF}} \times L \rightarrow I$ which is indistinguishable from a random family of functions from $L$ to $I$. A PRF can be constructed from any pseudorandom generator as follows. Let $G : \{0, 1\}^n \rightarrow \{0, 1\}^{2n}$ be a length-doubling pseudorandom generator. Define $G_0 : \{0, 1\}^n \rightarrow \{0, 1\}^n$ such that $G_0(x)$ is the first $n$ bits of $G(x)$. Define $G_1 : \{0, 1\}^n \rightarrow \{0, 1\}^n$ such that $G_1(x)$ is the last $n$ bits of $G(x)$. For the key $\mathcal{K}_{\text{PRF}} \in \{0, 1\}^n$ and an input $x \in \{0, 1\}^n$, the PRF is constructed as: $F_{\mathcal{K}_{\text{PRF}}}(x) = G_1(G_0(x_1) \cdots G_0(x_n)) \cdots G_0(x_1) \cdots G_0(x_n)$ where $x_i \in \{0, 1\}$ ($i = 1, \cdots, n$) are the elements of $x$. 

Fig. 1 An example of the network coding
\(x = \{x_1, \ldots, x_n\} \in \{0, 1\}^n\).

**MAC based on Universal Hash Function (UMAC).** A UMAC [26] can be constructed as the composition of a UHF with a PRF. Given a UHF family \(h: \mathcal{K}_{\text{UHF}} \times \mathcal{I} \rightarrow \mathcal{I}\) and a PRF family \(g: \mathcal{K}_{\text{PRF}} \times \mathcal{L} \rightarrow \mathcal{I}\), the UMAC is a tuple of UMAC = (UGen, UTag, UVer):

- UGen(1\(^l\)): generates key \((k, \kappa')\) uniformly at random from \(\mathcal{K}_{\text{UHF}} \times \mathcal{K}_{\text{PRF}}\).
- UTag\(_{k,\kappa}(m)\): works in space \(\mathcal{K}_{\text{UHF}} \times \mathcal{K}_{\text{PRF}} \times \mathcal{I} \rightarrow \mathcal{L} \times \mathcal{I}\), outputs \((r, h_s(m) + g_s(r))\) in which a unique counter \(r \in \mathcal{L}\) is increased in each execution.
- UVer\(_{k,\kappa'}(m, (c_1, c_2))\): works in space \(\mathcal{K}_{\text{UHF}} \times \mathcal{K}_{\text{PRF}} \times \mathcal{I} \times \mathcal{L} \times \mathcal{I}\), outputs 1 if and only if \(h_s(m) + g_s(c_1) = c_2\).

**Dispersal Coding.** The dispersal coding [14] is constructed as follows: To tag a message, the message is encoded under a MAC based on Universal Hash Function (PRF). Each server is given a unique counter \(\lambda_i\) that is increased in each execution.

- \(KGenECC(1^n)\): selects key \(\mathcal{K} = \{\kappa_i\}_{i=1}^{\alpha}, \{\kappa'_{i,n+s+1}\}_{i=1}^{\alpha}\) randomly from space \(\mathcal{K} = I^n \times (\mathcal{K}_{\text{PRF}})^\alpha\). The keys \(\kappa_i\) are used for the RS code. The keys \(\kappa_{i,n+s+1}\) are used for the PRF in the UMAC.
- \(MTagECC(m_1, \ldots, m_l)\): outputs \((c_1, \ldots, c_n)\) in which \(c_i = \text{RS-UFH}_s(m_i)\) when \(i \in [1, \ldots, n-s]\) and \(c_i = \text{UTag}_{k,\kappa'}(m_1, \ldots, m_l) + g_s(r_i)\) when \(i \in [n-s+1, \ldots, n]\).
- \(MVerECC(c_1, \ldots, c_n)\): strips off the PRF from \(c_{n+s+1}, \ldots, c_n\) as: \(c'_i = c_i + g_s(r_i)\) where \(i \in [n-s+1, \ldots, n]\), and then decodes \((c_1, \ldots, c_{n-s}, c'_{n+1}, \ldots, c'_n)\) using the RS decoder to obtain the message \(\overline{m} = (m_1, \ldots, m_l)\). If the RS decoder fails at the point \(\kappa_{i,n+s+1}\), \(MVerECC\) outputs \((\perp, 0)\). If one of the last \(s\) symbols of \((c_1, \ldots, c_n)\) is a valid MAC on \(\overline{m}\) under UMAC, \(MVerECC\) outputs \((\perp, 1)\), otherwise it outputs \((\perp, 0)\).

Because the dispersal coding uses the RS code in MTagECC to tag the message and uses the RS decoder in MVerECC to verify, the dispersal coding can prevent the small corruption attack.

### 3. Adversarial Model

This paper considers an adversary \(\mathcal{A}\) as follows. \(\mathcal{A}\) may control the servers by corrupting the servers and robbing all the privileges of the servers. If \(\mathcal{A}\) has not corrupted a server, \(\mathcal{A}\) cannot do anything because that all the data and the keys between the client and the servers are assumed to be transmitted via a secure channel. After \(\mathcal{A}\) corrupts a server, \(\mathcal{A}\) can modify/replace/forge data stored on that server and pretend to be a healthy server by providing a fake valid MAC tag to the client, can prevent the client from recovering the original file, and can perform the below four attacks (small corruption attack, large corruption attack, replay attack and pollution attack). A restriction of \(\mathcal{A}\) is that \(\mathcal{A}\) can control at most \((n-h)\) of the \(n\) servers within any time step (called epoch). More concretely, after corrupting a server, \(\mathcal{A}\) can perform as follows:

1. \(i\) access to the encode and check phases to output a codeword \(c\) such that \(\mathcal{A}\) can pass the verification without being detected with an advantage defined as: \(\text{Adv}_{\mathcal{A}}^{\text{ND-\_POR}}(\mathcal{A}) = \Pr[^{\mathcal{A}}\rightarrow \text{KGenECC}(1^n); c \leftarrow \text{MTagECC}(\cdot), MVerECC(\cdot) | (\cdot), (\cdot)\)\) the large corruption attack but cannot prevent the small corruption attack, large corruption attack and replay attack.

**Small corruption attack.** \(\mathcal{A}\) corrupts at most a \(t\)-fraction of the file of a small data unit, where \(t = \frac{c-s-1}{2}\), in order to hide the data loss incidents. This applies to the servers that want to preserve their reputation. To prevent the small corruption attack, the ECC is used to detect and correct errors [5, 6].

**Large corruption attack.** \(\mathcal{A}\) corrupts more than a \(t\)-fraction of the file of a large data unit, where \(t\) is the same parameter as in the small corruption attack, to discard a significant fraction of the data. This applies to the servers who want to sell the storage resource to multiple clients. To prevent the large data corruption, the spot check method is proposed [1], [4] in which the client randomly samples small portions of the data. Then, the server returns a computation over these portions of the data to the client. The results are checked by MACs. The spot check can only prevent the large corruption attack but cannot prevent the small corruption attack [1, 3].

**Replay attack.** \(\mathcal{A}\) tries to prevent the client from repairing the corruption by re-using the old coded blocks instead of the current coded blocks and providing these old coded blocks to the client in the repair phase. For example, the client encodes the augmented blocks \((b_1, b_2, b_3)\) into six coded blocks: \(c_{11} = b_1\) and \(c_{12} = b_2 + b_3\) (stored on the server \(S_1\)), \(c_{21} = b_3\) and \(c_{22} = b_1 + b_2\) (stored on the server \(S_2\)), \(c_{31} = b_1 + b_2\) and \(c_{32} = b_2 + b_3\) (stored on the server \(S_3\)). In epoch 1, suppose that \(S_3\) is corrupted. In epoch 2, the client repairs \(S_3\) by two new coded blocks: \(c'_{31} = b_1 + b_2 + b_2\) and \(c'_{32} = 2b_1 + b_2\). In the end of epoch 2, suppose that \(S_1\) is corrupted. In epoch 3, \(S_1\) is repaired by two new coded blocks: \(c'_{11} = 3b_1 + 3b_2\) and \(c'_{12} = 3b_2 + 3b_3\). At this time, \(\mathcal{A}\) re-uses the old coded blocks \(c_{11}\) and \(c_{22}\) of \(S_3\) instead of \(c'_{31}\) and \(c'_{32}\). Thus, if \(S_2\) is corrupted in epoch 4, the linear combination between the coded blocks of \(S_1\) and \(S_2\) is unable to repair \(S_2\).

**Pollution attack.** \(\mathcal{A}\) uses a valid data to avoid detection in the check phase, but provides an invalid data in the repair phase. For example, the client encodes the augmented blocks \((b_1, b_2, b_3)\) into six coded blocks: \(c_{11} = b_1\) and
$c_{12} = b_2 + b_3$ (stored on the server $S_1$), $c_{21} = b_3$ and $c_{22} = b_1 + b_2$ (stored on the server $S_2$), $c_{31} = b_1 + b_3$ and $c_{32} = b_2 + b_3$ (stored on the server $S_3$). In the check phase, suppose that the corrupted server $S_3$ is detected. In the repair phase, $S_3$ is repaired by two new coded blocks: $c_{31} = b_1 + 2b_3$ and $c_{32} = 2b_1 + b_2$. At this time, $A$ corrupts $S_1$ without detection because this time is the repair phase, not the check phase. To repair $S_3$, suppose that the client requests coded blocks from $S_1$ and $S_2$. $S_1$ then provides invalid coded blocks to the client.

One of the contributions is to prevent the small corruption attack. The other three attacks are still prevented in this paper by using the same solution as the RDC-NC scheme. These are discussed in the security analysis (Sect. 5).

## 4. The Proposed ND-POR Scheme

Throughout this paper, the notations described in Table 1 are used.

In the ND-POR scheme, $n$ servers are employed. The first $l$ servers $\{S_1, \ldots, S_l\}$, called NC-servers, store the coded blocks $\{c_{ij}\}_{i \in \{1, \ldots, l\}, j \in \{1, \ldots, \beta\}}$. The last $(n-l)$ servers $\{S_{l+1}, \ldots, S_n\}$, called DC-servers, store the dispersal coding parity blocks $\{d_{ij}\}_{i \in \{l+1, \ldots, n\}, j \in \{1, \ldots, \beta\}}$. The structure of the ND-POR scheme is depicted in Fig. 2.

### 4.1 Keygen

$C$ generates the secret key: $K = \{K_{tag}, K'_{tag}, \{K_i, K_{i, l}^{\text{enc}}\}_{i \in \{1, \ldots, n\}}\}$ which are randomly chosen in $\{0, 1\}^*$.

### 4.2 Encode

$F = \{v_1, \ldots, v_m\}$, $v_k \in \mathbb{F}_p$ where $k \in \{1, \ldots, m\}$. $C$ creates $m$ augmented blocks $\{b_1, \ldots, b_m\}$ in which $b_k \in \mathbb{F}_p^w$ ($k \in \{1, \ldots, m\}$) has the form as Eq. 1 in Sect. 2.3, and where $w = z + m$. Given a set of $m$ augmented blocks, $C$ computes $\beta$ coded blocks $c_{ij}$ using the linear combinations and stores them on the NC servers $\{S_1, \ldots, S_l\}$. Then encodes $c_{ij}$ using the dispersal coding into a dispersal coding parity block $d_{ij}$ for each row and stores them on the DC-servers $\{S_{l+1}, \ldots, S_n\}$. Namely, the encode phase is described as follows.

1. $C$ computes coded blocks from $m$ augmented blocks:

   For $\forall i \in \{1, \ldots, l\}$, $\forall j \in \{1, \ldots, \beta\}$:
   
   - Generate $m$ coefficients $\alpha_{ijk} \leftarrow \mathbb{F}_p$ where $k \in \{1, \ldots, m\}$.
   - Compute coded blocks: $c_{ij} = \sum_{k=1}^{m} \alpha_{ijk} b_k$.

   Therefore, a matrix $\{c_{ij}\}_{i \in \{1, \ldots, l\}, j \in \{1, \ldots, \beta\}}$ is constructed.

2. $C$ computes dispersal coding parity blocks in each row:

   For $\forall i \in \{l+1, \ldots, n\}$, $\forall j \in \{1, \ldots, \beta\}$, $C$ computes $d_{ij} = \text{MTagECC}_{K_i, K_{i, l}^{\text{enc}}}(c_{i, l}, \ldots, c_{i, \beta})$.

3. $C$ computes metadata for coded blocks:

   For $\forall i \in \{1, \ldots, l\}$:
   
   - Generate $w$ values $\{\xi_1, \ldots, \xi_w\}$: $\xi_u = f_{\text{tag}}(i|u)$ where $u \in \{1, \ldots, w\}$.
   - For $\forall j \in \{1, \ldots, \beta\}$, $c_{ij} \in \mathbb{F}_p^w$ is viewed as a column vector of $w$ symbols: $c_{ij} = (c_{ij_1}, \ldots, c_{ij_w})$ with $c_{ij_u} \in \mathbb{F}_p$ where $u \in \{1, \ldots, w\}$. $C$ computes...
a repair tag for \( c_{ij} \): 
\[
T_{ij} = f_{\mathcal{K}_{\text{na}}}(l||j)||\alpha_{ij1}|| \cdots ||\alpha_{ijm}) + \sum_{w=1}^w \xi_w c_{ijw} \quad \text{(mod } p)\]

- \( C \) encrypts the coefficients: \( \epsilon_{ijk} = \text{Enc}_{\mathcal{K}_{\text{nu}}} (\alpha_{ijk}) \) where \( k \in \{1, \ldots, m\} \). This encryption is used to prevent the replay attack.

4. \( C \) distributes data to the servers:

- \( C \) sends the coded blocks \( \{c_{ij}\}_{e \in \{1, \ldots, \ell \}, j \in \{1, \ldots, \beta\}} \), the encrypted coefficients \( \{\epsilon_{ijk}\}_{e \in \{1, \ldots, \ell \}, j \in \{1, \ldots, \beta\}, k \in \{1, \ldots, m\}} \) and the repair tags \( \{T_{ij}\}_{e \in \{1, \ldots, \ell \}, j \in \{1, \ldots, \beta\}} \) to \( S_i \) where \( i \in \{1, \ldots, l\} \).
- \( C \) sends the dispersal coding parity blocks \( \{d_{ij}\}_{e \in \{p, \ldots, n\}, j \in \{1, \ldots, \beta\}} \) to \( S_i \) where \( i \in \{l+1, \ldots, n\} \).

4.3 Check

\( C \) chooses a number of row indices to challenge the servers using the spot check method. The servers respond \( C \). \( C \) checks the responses using the \( \text{MVerECC} \) algorithm. All the servers operate over the same subset of rows. Because the responses of all the servers lie on a codeword, all the servers can be checked for each challenge.

1. \( C \) challenges the servers: \( C \) firstly chooses an integer \( v \sim \{1, \beta\} \). \( C \) then sends to each server a set of row indices \( D = \{j_1, \ldots, j_e\} \) where \( j_1, \ldots, j_e \sim \{1, \beta\} \) and a key \( k \in I \) where \( I \) is a field with operation \((+, \times)\).

2. The servers respond: \( R_i = \text{RS-UHF}_i (c_{ij_1}, \ldots, c_{ij_e}) \).

3. \( C \) verifies the servers: Because all servers operate over the same subset of rows \( D \), the combined response \( R = (R_1, \ldots, R_p) \) is a codeword of the dispersal coding. \( C \) firstly checks \( R \) by calling \( \text{MVerECC}(R_1, \ldots, R_p) \) of the dispersal coding to verify. It returns false if the responses are invalid, and return true otherwise. After checking \( R \), \( C \) checks the validity of each individual response \( R_i \) to detect which server is corrupted. For the \( N \) NC-servers \( \{S_1, \ldots, S_N\} \), \( R_i \) is a valid response if it matches the \( i \)-th symbol in \( \overline{n} \). For the \( (n-l) \) DC-servers \( \{S_{n+1}, \ldots, S_n\} \), \( R_i \) is a valid response if it is a valid MAC on \( \overline{n} \).

4.4 Repair

If a failure is detected in the check phase, \( C \) executes the repair phase with the following two sub-phases:

**Sub-phase 1.** The corruptions are firstly repaired by the \( \text{RS} \) decoder with the boundary number of corruptions \( t = \frac{n-m}{2} \).

If the number of corruptions is more than \( t \), \( C \) uses the sub-phase 2.

**Sub-phase 2.** The corruptions are repaired by the network coding. \( C \) firstly requires the healthy servers to compute the aggregated coded blocks. Then, \( C \) combines these coded blocks to generate \( \beta \) coded blocks for the new server. Suppose that \( S_y \) is the corrupted server and \( S_y' \) is the new server which is used to replace \( S_y \).

1. \( C \) requests \( h \) healthy servers \( \{S_{i_1}, \ldots, S_{i_h}\} \) to compute the aggregated coded blocks and the proofs of correct encoding:

   For \( \forall i \in \{i_1, \ldots, i_h\} \):
   - \( C \) generates the coefficients \( \{x_{i_1}, \ldots, x_{i_h}\} \) where \( x_{ij} \sim \mathbb{F}_p \) with \( j \in \{1, \ldots, \beta\} \).
   - \( C \) requests \( S_i \) to compute an aggregated coded block and a proof of correct encoding.
   - \( S_i \) computes \( \overline{a}_i = \sum_{j=1}^\beta x_{ij} c_{ij} \in \mathbb{F}_p^\mu \), then computes a proof of correct encoding: \( \theta = \sum_{j=1}^\beta x_{ij} T_{ij} \pmod p \) and sends \( \overline{a}_i, \theta, \{\epsilon_{ij_1}, \ldots, \epsilon_{ij_m}\}_{e \in \{1, \ldots, \beta\}} \) to \( C \).
   - \( C \) decysts the encrypted coefficients from \( S_i \) to get the raw coefficients: \( \{\alpha_{ij_1}, \ldots, \alpha_{ij_m}\}_{e \in \{1, \ldots, \beta\}} \).
   - \( C \) re-generates \( w \) values \( \{\xi_1, \ldots, \xi_w\} \) where \( \xi_u = f_{\mathcal{K}_{\text{na}}} (l||u) \) where \( u \in \{1, \ldots, w\} \).
   - \( C \) checks if \( \theta \neq \sum_{j=1}^\beta x_{ij} f_{\mathcal{K}_{\text{na}}}(l||\alpha_{ij1})\cdots||\alpha_{ijm})+\sum_{w=1}^w \xi_w \epsilon_{ijw} \pmod p \) where \( \{\alpha_{ij_1}, \ldots, \alpha_{ij_m}\} \) are the symbols of the block \( \overline{a}_i \). This verification is to ensure that \( S_i \) does not have the pollution attack.

2. \( C \) repairs \( S_y' \):
   - \( C \) generates \( w \) values \( \{\xi_1, \ldots, \xi_w\} \) where \( u \in \{1, \ldots, w\} \).
   - For \( \forall j \in \{1, \ldots, \beta\}, \forall y \in \{1, \ldots, h\} \), \( C \) generates the coefficients \( \alpha_{yj} \sim \mathbb{F}_p \), and computes the coded block: \( c_{yj} = \sum_{y=1}^h \xi_y c_{yj} \), \( \overline{a}_j \in \mathbb{F}_p^\mu \). By viewing \( c_{yj} \) as a column vector of \( w \) symbols: \( c_{yj} = (c_{yj_1}, \ldots, c_{yj_w}) \), \( C \) computes a repair tag for the block \( c_{yj} \): \( T_{yj} = f_{\mathcal{K}_{\text{na}}}(l||\alpha_{yj1})\cdots||\alpha_{yjm})+\sum_{w=1}^w \xi_w c_{yjw} \pmod p \), and encrytps the coefficient: \( \forall y \in \{1, \ldots, h\}, \epsilon_{yj} = \text{Enc}_{\mathcal{K}_{\text{nu}}} (\alpha_{yj}) \).

3. \( C \) sends to the new server \( S_y' \):
   - \( \{c_{yj}\}_{j \in \{1, \ldots, \beta\}}, \{\epsilon_{yj}\}_{j \in \{1, \ldots, \beta\}, y \in \{1, \ldots, h\}}, \{T_{yj}\}_{j \in \{1, \ldots, \beta\}} \).

5. Security Analysis

This section describes the advantage of the defined adversary and explains how the small corruption attack, large corruption attack, replay attack and pollution attack are prevented.

5.1 Adversarial Check and Repair

A MAC consists of three algorithms: \( \{\text{MGen}, \text{MTag}, \text{MVer}\} \).
Let $q_1$ denote the number of queries to $\text{MTag}$, and $q_2$ denote the number of queries to $\text{MVer}$. Let $t$ denote the running time. The boundary of the advantage of $\mathcal{A}$ on $\text{UMAC}$ [26] is given in the following fact:

**Fact 1:** Let $\text{Adv}_{\text{UMAC}}(q_1, q_2, t)$ denote the advantage of the adversary $\mathcal{A}$ on $\text{UMAC}$ making $q_1$ queries to $\text{MTag}$, $q_2$ queries to $\text{MVer}$, and running in the time $t$. Let $\text{Adv}_{\text{prf}}(q_1, q_2, t)$ denote the advantage of the adversary $\mathcal{A}$ making $(q_1 + q_2)$ queries to the oracle PRF and running in the time $t$. Suppose that the UHF is an $e^{\text{UHF}}$-AXU family of hash function. Then, the following inequality is obtained:

$$\text{Adv}_{\text{UMAC}}(q_1, q_2, t) \leq \text{Adv}_{\text{prf}}(q_1 + q_2, t) + e^{\text{UHF}}q_2$$

Furthermore, the boundary of the advantage of $\mathcal{A}$ on the dispersal coding codeword is given as follows:

**Theorem 1:** Let $\text{Adv}_{\text{codeword}}(q_1, q_2, t)$ denote the advantage of $\mathcal{A}$ on the dispersal coding making $q_1$ queries to $\text{MTagECC}$, $q_2$ queries to $\text{MVerECC}$, and running in the time $t$. If RS-UHF is constructed from an $(n, l)$-RS code, then the following inequality is obtained:

$$\text{Adv}_{\text{codeword}}(q_1, q_2, t) \leq 2[\text{Adv}_{\text{UMAC}}(q_1, q_2, t)]$$

**Proof:** Suppose that $\mathcal{A}$ is successful when $\mathcal{A}$ makes $q_1$ queries to the tagging oracle $\text{MTagECC}$, $q_2$ queries to the verification oracle $\text{MVerECC}$, and runs in time $t$. $\mathcal{A}$ outputs a codeword $(c_1, \ldots, c_n)$ which can be decoded to the message $\overline{m} = (m_1, \ldots, m_n)$ such that at least one of the last $s$ symbols in the codeword is a valid MAC on $\overline{m}$ computed with UHF. Another adversary $\mathcal{A}'$ is considered for the UMAC construction. $\mathcal{A}'$ is given access to a tagging oracle $\text{UTag}_{\text{e}}(\cdot)$ and a verification oracle $\text{UVer}_{\text{e}}(\cdot)$, and needs to output a new message and a tag pair. $\mathcal{A}'$ chooses a position $j \in [n - s + 1, \ldots, n]$ randomly, and generates keys $k_{\text{reg}}$ and $k_{\text{enc}}$ for $i \neq j$. $\mathcal{A}'$ runs $\mathcal{A}$. When $\mathcal{A}$ makes a query to tag $\overline{m} = (m_1, \ldots, m_n)$, $\mathcal{A}'$ computes $c_j \leftarrow \text{UTag}_{\text{e}}(\overline{m})$ for $i \in [1, \ldots, n - s]$, and $c_j \leftarrow \text{UTag}_{\text{e}}(\overline{m})$ for $i \in [n - s + 1, \ldots, n], i \neq j$. $\mathcal{A}'$ calls the UTag oracle to compute $c_j = \text{UTag}_{\text{e}}(\overline{m})$, $\mathcal{A}'$ then responds to $\mathcal{A}$ with $\overline{\sigma} = (c_1, \ldots, c_n)$. When $\mathcal{A}$ makes a query $\overline{\sigma} = (c_1, \ldots, c_n)$ to the verification oracle, $\mathcal{A}$ tries to decode $(c_1, \ldots, c_{j-1}, c_{j+1}, \ldots, c_n)$ into message $\overline{m}$. If the decoding fails (the number of errors in the codeword is more than $t = \frac{n - s + 1}{2}$), then $\mathcal{A}'$ responds to $\mathcal{A}$ with $(\perp, 0)$. Otherwise, let $\overline{m}$ be the decoded message. $\mathcal{A}'$ makes a query to the verification oracle $\alpha \leftarrow \text{UVer}_{\text{e}}(\overline{m}, c_j)$ and returns $(\overline{m}, \alpha)$ to $\mathcal{A}$. Assume that $\mathcal{A}$ outputs $\overline{\sigma} = (c_1, \ldots, c_n)$ under the codeword that can be decoded to $\overline{m}$, such that $\overline{m}$ was not an input to the tagging oracle and at least one of the last $s$ symbols in $\overline{\sigma}$ is a valid MAC for $\overline{m}$. Then $\mathcal{A}'$ outputs $(\overline{m}, \alpha)$. Because $t = \frac{n - s + 1}{2}$, the number of remaining correct blocks is at least $n - \frac{n - s + 1}{2} = \frac{n - s - 1}{2}$. The number of correct parity blocks is thus at least $\frac{n - s - 1}{2} - 1 = \frac{n - s - 3}{2}$. Furthermore, the number of the original parity blocks before errors is $(n - l)$. Therefore, the number of correct parity blocks is at least $\frac{(n - l - 1/2)^2}{n - l} \approx \frac{1}{2}$ of the number of the original parity blocks. In other words, the codeword $\overline{\sigma}$ can be decoded if at least a majority of its parity blocks are correct. Then, with probability at least $\frac{1}{2}$, $c_j$ is a correct MAC on $\overline{m}$. It follows that $\mathcal{A}'$ succeeds in outputting a correct message and MAC pair $(\overline{m}, \alpha)$ with probability at least half the success probability of $\mathcal{A}$.

Now the probability of $\mathcal{A}$ to prevent data recovery is given as the following theorem.

**Theorem 2:** $F$ can be recovered as long as in any epoch, at least $h$ out of $n$ servers are healthy and the matrix which consists of all the coefficients of the coded blocks has full rank, i.e., rank equals to $m$.

**Proof:** $m$ augmented blocks are $(b_1, \ldots, b_m)$ which are created from $m$ file blocks $(v_1, \ldots, v_m)$. The number of coded blocks is $n x (n$ servers, $x$ coded blocks per server). To compute a coded block $c_{ij}$ for the server $S_j$, $C$ chooses $m$ coefficients $[\alpha_{ij1}, \ldots, \alpha_{ijm}]$, and uses the linearly independent combination: $c_{ij} = \sum_{k=1}^{m} \alpha_{ijk} b_k$. $(b_1, \ldots, b_m)$ are viewed as the unknowns that need to be solved. After solving $(b_1, \ldots, b_m)$, the file blocks $v_1, \ldots, v_m$ can be obtained by picking the first coordinate of each $b_k$ where $k \in [1, \ldots, m]$. $F$ is finally recovered as $F = v_1 \cdots v_m$. To solve $m$ unknowns $(b_1, \ldots, b_m)$, at least $m$ coded blocks are required which make the matrix have full rank because the number of unknowns in an equation system has to be less than the number of equations. Let $(r_1, \ldots, r_m)$ denote such $m$ coded blocks which are required for file recovery. Let $(c_{\alpha1}, \ldots, c_{\alpha m})$ denote $m$ coefficients which are used to construct $c_{\alpha}$.

$$\begin{align*}
c_{r_1} &= \sum_{k=1}^{m} \alpha_{r_1} b_k \\
c_{r_2} &= \sum_{k=1}^{m} \alpha_{r_2} b_k \\
&\vdots \\
c_{r_m} &= \sum_{k=1}^{m} \alpha_{r_m} b_k
\end{align*}$$

Let $h$ be the number of healthy servers that collectively store $m$ coded blocks. In any epoch, $h = \frac{n}{x}$. In the RDC-NC scheme, there are $n$ servers and a $x$ coded blocks per server. Thus, the number of healthy servers in an epoch in the RDC-NC scheme is at least $h = \frac{n}{x}$. In the ND-POR scheme, there are also $n$ servers but such $n$ servers are divided into two types: $l$ NC-servers and $(n - l)$ DC-servers. Because $l < n$, each NC-server has $\beta = \frac{m}{cn}$ coded blocks. Therefore, the number of healthy servers in each epoch is at least $h = \frac{m}{\beta}$. If the theorem is satisfied, the probability for $\mathcal{A}$ to prevent recovering $F$ is negligible: $\Pr[F = \{v_1, \ldots, v_m\}]$ is not recovered] $\leq \epsilon$.
5.2 Small Corruption Attack

Theorem 3: The RS code in the dispersal coding is sufficient to prevent the small corruption attack.

Proof: Let $t_{\text{error}}$ denote the number of corruptions caused by $\mathcal{A}$ in an epoch. Firstly, because the RS code is constructed with the parameter $(n, l)$, the message is interpreted as the description of a polynomial $p$ of the degree less than $l$ which is evaluated at $n$ distinct points $\{a_1, \cdots, a_n\}$. The sequence of the values is the corresponding codeword $C: C = \{p(a_1), \cdots, p(a_n)\}$ (Sect. 2.4). Because any two different polynomials of the degree less than $l$ agree in at most $(l - 1)$ points, any two codewords of the RS code disagree in at least $n - (l - 1) = n - l + 1$ positions. Moreover, there are two polynomials that do agree in $(l - 1)$ points but are not equal. Hence, the distance of the RS code is:

$$d = n - l + 1$$

Secondly, because any two strings in $C$ differ in at least $d$ places, we have:

$$2t_{\text{error}} \leq d$$

From Eq. 4 and Eq. 5, $t_{\text{error}} \leq \frac{n - l + 1}{2}$. The inequality reflects the fact that, given any string $s$, there is at most one string $c \in C$ which is within the distance $d$ of $t_{\text{error}}$ from $s$. This means that the advantage of $\mathcal{A}$ is always bounded by the error resilience of the RS code. \hfill $\Box$

5.3 Large Corruption Attack, Replay Attack and Pollution Attack

The attacks are addressed in the RDC-NC scheme. This section briefly describes the key ideas as follows:

Large corruption attack. In the check phase, using the spot check method, $C$ periodically and randomly samples a set of indices of the coded blocks stored on each server. $C$ then checks whether these sampled blocks match with the embedded MAC. If the adversary corrupts a large fraction of the data stored on the server, $C$ easily detects the corruptions with an optimal computation and I/O at the server and communication between the server and the client.

Replay attack. To avoid the adversary $\mathcal{A}$ replaying a coded block, the common solution is to use a counter which is incremented each time the coded block on a server is recreated due to server failure. However, in this solution, the client must store locally the latest value of the counters. Therefore, the RDC-NC scheme uses a different solution to mitigate the replay attack and to reduce the storage cost for the client. That is, the coefficients are encrypted and stored together with the coded blocks to prevent $\mathcal{A}$ from knowing how the original blocks were combined to obtain the coded block. The ability of $\mathcal{A}$ is negligible because $\mathcal{A}$ does not know which old coded blocks to replay.

Pollution attack. For each coded block $c_{ij}$ of the server $S_i$, a repair tag which is constructed from a MAC is embedded into the coded block. In the repair phase, $C$ requires a number of servers which are used for data repair to provide their aggregated coded blocks. Before computing the new coded blocks for the new server, $C$ uses the tag to check whether these servers combine the coded blocks correctly. Therefore, the servers cannot inject polluted blocks to $C$.

6. Efficiency Analysis

The Table 2 shows that the encode cost in the ND-POR scheme is more than that in the RDC-NC scheme. However, the encode phase is performed only one time in the beginning, but the check and repair phases are performed very often during the system lifetime. Therefore, the check and repair costs are more important than the encode cost. The Table 2 shows that the check and repair costs in the ND-POR scheme are less than these in the RDC-NC scheme.

Before analysing the costs in the RDC-NC and ND-POR schemes, recall that each coded block $c_{ij} \in \mathbb{F}_p^n$ where $w = m + z$. The coded block size is $w \log_2 p$. The unit of the below computational complexities is the number of operations over $\mathbb{F}_p$.

6.1 Encode Phase

The RDC-NC scheme computes $na$ coded blocks. Its encode computation cost is thus $O(na)$. The ND-POR scheme computes $\beta$ coded blocks, then computes $(n - l)\beta$ dispersal coding parity blocks, where $\beta = \frac{na}{\alpha}$. Its encode computation

| Table 2 | The comparison between the RDC-NC and ND-POR schemes |
|-----------------|-----------------|-----------------|
| Security       | RDC-NC          | ND-POR          |
| Small corruption attack | No              | Yes             |
| Large corruption attack   | Yes             | Yes             |
| Replay attack   | Yes             | Yes             |
| Pollution attack | Yes             | Yes             |
| Efficiency     | RDC-NC          | ND-POR          |
| Encode computation | $O(na)$         | $O(na \times \frac{z}{w})$ |
| Repair computation | $O(\frac{na}{n - l})$ | $O(n \log_2 n \log_2 \log_2 n) (\text{RS decode})$ |
|                  |                  | $O(\frac{\log_2 (n - l)}{\log_2 n}) (\text{Network coding})$ |
| The number of MACs | $n\alpha$        | $\alpha$        |
| Required healthy servers | $\lceil \frac{n}{\alpha} \rceil$ | $\lceil \frac{n}{\alpha} \rceil (l < n)$ |
| Storage server cost | $O(na \log_2 (w + s + 1))$ | $O(na \log_2 (w + s + 1))$ |
cost is thus $O\left(\frac{n^2}{\alpha p}\right)$. It is clear that the cost in the ND-POR scheme is more than $\frac{n}{\alpha}$ times that in the RDC-NC scheme. In an ECC, because the redundant blocks are chosen such that $n-l < l$, $\frac{n}{\alpha} > 2$. Because $n > l$, $\frac{n}{\alpha} > 1$. Therefore, $1 < \frac{n}{\alpha} < 2$. This means that although the cost in the ND-POR scheme is more than $\frac{n}{\alpha}$ times that of the RDC-NC scheme, it is less than double times.

**The number of MACs.** In the RDC-NC scheme, the number of MACs is $nas$ where $n$ is the number of servers, $a$ is the number of coded blocks stored on a server, $s$ is the number of servers out of $n$. This is because the MACs are embedded in the segments of the coded blocks. In the ND-POR scheme, the number of MACs is $la$ where $l$ is a number of servers out of $n$ servers ($l < n$). This is because the MACs are only required for the network coding coded blocks which are located in only $l$ servers.

### 6.2 Check Phase

The RDC-NC scheme challenges a subset of segment indices in each coded block of a server. C thus needs $na$ challenges to check all blocks stored on $n$ servers where $a$ is the number of blocks per server. In the ND-POR scheme, $C$ challenges a subset of row indices (the codewords of the dispersal coding). Each codeword lies on all $n$ servers. There are $\beta$ codewords. $C$ thus needs $\beta = \frac{na}{\alpha}$ challenges to check all blocks stored on $n$ servers. The cost in the RDC-NC scheme is more than $l$ times that in the ND-POR scheme. This is an advantage of the ND-POR scheme when multiple servers are checked per challenge instead of one server as the RDC-NC scheme.

### 6.3 Repair Phase

In the RDC-NC scheme, the cost for repairing a corrupted server is $O\left(\frac{3n}{\alpha^2} + \frac{l}{p} + \frac{l}{\alpha}\right)$ in which the cost of $h$ healthy servers is $h\frac{3n}{\alpha^2}$ and the cost of client-side is $\frac{l}{\alpha}$. Because $h = \frac{m}{\alpha}$, the cost is $O\left(n\frac{3n}{\alpha^2} + \frac{l}{p} + \frac{l}{\alpha}\right)$. In the ND-POR scheme, in the sub-phase 1, the corruptions are repaired by the RS decoder which has $O\left(n\frac{n}{\alpha}\right)$. Because $n$ is far less than the dominant parameter $|F|$, the cost of the RS decoder is less than the RDC-NC scheme. Let $n = 12$ as in the experimental evaluation of the RDC-NC scheme, the RS can decode in only 284 field operations. In the sub-phase 2, the corruptions are repaired by the network coding like the RDC-NC scheme. The cost is thus $O\left(h'\frac{3n}{\alpha^2} + \frac{l}{p} + \frac{l}{\alpha}\right)$. Because $h' = \frac{lm}{\alpha^2}$, the cost in the ND-POR scheme is thus $O\left(h'\frac{3n}{\alpha^2} + \frac{l}{p} + \frac{l}{\alpha}\right)$.

**Parameter Choice.** The parameter choice is now discussed for maximizing resilience of both cases simultaneously. Because an $(n,l)$-ECC can recover up to $t = \frac{n-l-1}{2}$ errors in each row, $A$ can win the ECC if the number of corruptions is more than $t$. Furthermore, because the number of healthy servers is at least $\frac{ml}{\alpha}$ (Theorem 2), $A$ can win if $A$ corrupts more than $\frac{ml}{\alpha}$. Let $f_1(l) = \frac{n-l+1}{2}$, $f_2(l) = \frac{m}{\alpha} \times \frac{l}{n}$, $l$ should be chosen such that the advantage of $A$ is reduced. In other words, $f_1(l)$ and $f_2(l)$ are increased. If $f_1(l)$ and $f_2(l)$ are considered separately, $f_1(l) = \frac{m}{2\alpha}$ increases if $l$ increases, $f_2(l) = \frac{m}{\alpha}$ increases if $l$ decreases. It is not synchronous. Hence, $l$ should be balanced between $f_1(l)$ and $f_2(l)$. Let $f_1(l) = f_2(l)$, we determine $l = \left\lceil \frac{m(n+1)}{2\alpha n^2} \right\rceil$.

**Healthy Servers For Data Repair.** As mentioned in Theorem 2, the RDC-NC scheme needs at least $\left\lceil \frac{mn}{\alpha} \right\rceil$ healthy servers for data repair while the ND-POR scheme only needs at least $\left\lceil \frac{lm}{\alpha} \right\rceil$ healthy servers for data repair ($l < n$). It is clear that the number of healthy servers in the RDC-NC scheme is more than $\frac{n}{\alpha}$ times that in the ND-POR scheme.

### 6.4 Storage Cost

In the RDC-NC scheme, the size of $na$ coded blocks in $\mathbb{F}_p^n$ is $nw \log_2 p$. The size of $nas$ challenge tags in $\mathbb{F}_p$ is $nas \log_2 p$ where $s$ denotes the number of segments in a coded block of the RDC-NC scheme. The size of $na$ repair tags in $\mathbb{F}_p$ is $na \log_2 p$. Therefore, the storage cost in the RDC-NC scheme is $O(na \log_2 p(w + s + 1))$. In the ND-POR scheme, the size of $na$ coded blocks and $(n-l)\beta$ where $\beta = \frac{na}{\alpha}$ dispersal coding parity blocks in $\mathbb{F}_p^n$ is $w \log_2 p(na + (n-l)\beta) = \frac{wn}{\alpha} \log_2 p$. The size of $na$ repair tags in $\mathbb{F}_p$ is $na \log_2 p$. Thus, the storage cost in the ND-POR scheme is $O(na \log_2 p \left(\frac{\alpha}{\alpha} + 1\right))$. To make the cost in the ND-POR scheme better than the RDC-NC scheme, let $na \log_2 p \left(\frac{\alpha}{\alpha} + 1\right) < na \log_2 p(w + s + 1)$. As a result, the parameters should be chosen s.t. $w < \frac{\alpha}{\alpha}$. We now show the costs of the RDC-NC and ND-POR schemes.

**Example 1.** $n = 12$, $\alpha = 3$, $s = 5$, $l = 10$, $m = 7$, $w = 20$, $z = 13$, $p = 4099$, $|F| = 1092$. Suppose that all elements in $\mathbb{F}_p$ is less than or equal to 4095. This is to let the elements not exceed 12 bits length. These parameters satisfy the conditions which we stated in the manuscript:

- $1 < \frac{n}{\alpha} < 2$ as stated in Sect. 6.1.
- $l = \left\lceil \frac{mn+1}{2nw+\alpha} \right\rceil$ as stated in Sect. 6.3. (In this example, $l = \left\lceil \frac{12+1}{27+12} \right\rceil = 10$).
- $w < \frac{\alpha}{\alpha}$ as stated in Sect. 6.4.

We now show the costs of the RDC-NC and ND-POR schemes.

**The encode computation cost of the RDC-NC scheme is** $na = 12 \cdot 3 = 36$. Meanwhile, the encode computation cost of the ND-POR scheme is $na \frac{\alpha}{\alpha} = 12 \cdot 3 \cdot \frac{12}{10} = 43.2$. 

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The check computation cost of the RDC-NC scheme is $n\alpha = 12 \cdot 3 = 36$. Meanwhile, the check computation cost of the ND-POR scheme is $\frac{mn}{T} = \frac{12 \cdot 3}{10} = 3.6$.

The repair computation cost of the RDC-NC scheme is $\frac{3mn}{m+n} = \frac{3 \cdot 3 \cdot 1092}{7 + 3} = 2293.2$. Meanwhile, the repair computation cost of the ND-POR scheme is $n \log_2 n \log_2 \log_2 n = 284$ (in the case of the RS code), or $\frac{3mn}{m+n+\ell m} = \frac{3 \cdot 10 \cdot 1092}{12 + 3 + 10 \cdot 7} = 2163.4$ (in the case of the network coding).

The number of MACs in the RDC-NC scheme is $n = 12 \cdot 5 \cdot 3 = 180$. Meanwhile, the number of MACs in the ND-POR scheme is $l \alpha = 10 \cdot 3 = 30$.

The number of the required healthy servers for data repair in the RDC-NC scheme is $\lceil \frac{p}{n} \rceil = \lceil \frac{14}{12} \rceil = 3$. Meanwhile, in the ND-POR scheme is $\lceil \frac{p}{m} \rceil = \lceil \frac{15}{10} \rceil = 2$.

The storage cost in the RDC-NC scheme is $n \alpha \log_2 p(w + s + 1) = 12 \cdot 3 \cdot \log_2 4099 \cdot (20 + 5 + 1) = 11232$. Meanwhile, the storage cost in the ND-POR scheme is $n \alpha \log_2 p(m + 1) = 12 \cdot 3 \cdot \log_2 4099 \cdot (20 + 1) = 10800$.

Example 2. $n = 16, \alpha = 5, s = 4, l = 14, m = 10, w = 25, z = 15, p = 1031, |F| = 1500$. Suppose that all elements in $F_p$ is less than or equal to 1023. This is to let the elements not exceed 10 bits length. These parameters satisfy the conditions which we stated in the manuscript:

1. $1 < \frac{n}{T} < 2$ as stated in Sect. 6.1.
2. $l = \lceil \frac{n(a+1)}{2m+na} \rceil$ as stated in Sect. 6.3. (In this example, $l = \lceil \frac{15}{25 + 16} \rceil = 14$).
3. $w < \frac{1}{n}$ as stated in Sect. 6.4.

We now show the costs of the RDC-NC and ND-POR schemes.

The encode computation cost of the RDC-NC scheme is $n \alpha = 16 \cdot 5 = 80$. Meanwhile, the encode computation cost of the ND-POR scheme is $n \alpha \frac{\ell}{T} = 16 \cdot 5 \cdot \frac{14}{14} = 91.43$.

The check computation cost of the RDC-NC scheme is $n \alpha = 16 \cdot 5 = 80$. Meanwhile, the check computation cost of the ND-POR scheme is $\frac{mn}{T} = \frac{16 \cdot 5}{14} = 5.71$.

The repair computation cost of the RDC-NC scheme is $\frac{3mn}{m+n} = \frac{3 \cdot 10 \cdot 1092}{10 + 5} = 3000$. Meanwhile, the repair computation cost of the ND-POR scheme is $n \log_2 n \log_2 \log_2 n = 512$ (in the case of the RS code), or $\frac{3mn}{m+n+\ell m} = \frac{3 \cdot 14 \cdot 1092}{16 + 5 + 14 \cdot 10} = 2863.64$ (in the case of the network coding).

The number of MACs in the RDC-NC scheme is $n \alpha = 16 \cdot 4 \cdot 5 = 320$. Meanwhile, the number of MACs in the ND-POR scheme is $l \alpha = 14 \cdot 5 = 70$.

The number of the required healthy servers for data repair in the RDC-NC scheme is $\frac{mn}{T} = \frac{10}{5} = 2$. Meanwhile, that in the ND-POR scheme is $\frac{lm}{na} = \frac{14 \cdot 10}{16 \cdot 5} = 1.75$.

The storage cost in the RDC-NC scheme is $n \alpha \log_2 p(w + s + 1) = 16 \cdot 5 \cdot \log_2 1031 \cdot (25 + 4 + 1) = 24000$. Meanwhile, the storage cost in the ND-POR scheme is $n \alpha \log_2 p(m + 1) = 16 \cdot 5 \cdot \log_2 1031 \cdot (\frac{16 \cdot 25}{14} + 1) = 23657$.

In summary, although the ND-POR scheme combines the network coding and the dispersal coding, the ND-POR scheme is not worse than the RDC-NC scheme in totals.

7. Conclusion and Future Work

In this paper, the ND-POR scheme is proposed in which the network coding and the dispersal coding technique are combined to reduce the costs of two important phases, the check and the repair phases, and to prevent small corruption attack, replay attack, pollution attack and large corruption.

In future work, we focus on the following problem. The repair phase has not been optimized because the healthy servers need to provide the aggregated coded blocks to the client, then the client computes the new coded blocks and stores them on the new server. A new mechanism can be considered in which the healthy servers send their coded blocks directly to the new server without sending back to the client. This mechanism can reduce the burden for the client and also reduce the communication. To support this mechanism, a signature scheme can be employed such as [30], [31] that allows the new server to verify the coded blocks provided from the healthy servers instead of the client, and to construct the new coded blocks by itself.

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