Solutions of Faddeev-Yakubovsky equations in configuration space for the 4N scattering states

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Abstract. The Faddeev-Yakubovsky equations in configuration space have been solved for the four nucleon system with special interest in their scattering states. We present results concerning the structure of the first $^4\text{He}$, scattering states in different (T,S) channels including the first inelastic threshold $\text{N}+\text{3N} \rightarrow \text{NN}+\text{NN}$ and the n-t cross section using realistic potentials.

1 Introduction

The theoretical description of the A=4 continuum constitutes, in addition to its technical difficulty, a serious challenge for the NN interaction models. The remarkable success encountered in the description of the 3N system and in the 4N bound state \cite{1} is largely based on the three nucleon interaction (TNI). The two body dynamics alone loses $\approx 10\%$ in the 3N binding energy and $\approx 15\%$ in the 4N. TNI compensate this defect but, a part from an energy scaling, are found to have small influence in the low energy scattering states \cite{2}. Contrary to the A=3 case, these states present several nearthreshold structures, which can be hardly sensible to the TNI and should be reproduced with the two-body forces only. A first example is provided by the n+t cross section which manifests a much more vivid behaviour than the flat n-d one, with a resonant structure at $T_{lab} \approx 3$ MeV (figure \ref{fig:1}). A second interesting accident is the existence of quasidegenerate n-$^3\text{He}$ and p-$^3\text{H}$ thresholds with, in their middle, the first $^4\text{He}$ excitation and all that in an energy gap of 0.76 MeV. The position of such a state, which dominates the inter-threshold dynamics, has to be ensured with an accuracy better than 100 KeV in order to reproduce the experimental cross sections, what is a strong requirement for an excited state.

2 Formalism

In the case of four identical particles interacting via a two-body potential V, the Faddeev-Yakubovsky (FY) equations result into two integrodifferential equa-
tions, coupling two amplitudes denoted K and H:

\[
\begin{align*}
(E - H_0 - V)K &= V \left[ (P_{23} + P_{13}) (\varepsilon + P_{34}) K + \varepsilon (P_{23} + P_{13}) H \right] \\
(E - H_0 - V)H &= V \left[ (P_{13} P_{24} + P_{14} P_{23}) K + P_{13} P_{24} H \right]
\end{align*}
\]

(1)
in which \(P_{ij}\) are the permutation operators and \(\varepsilon = \pm 1\) depending on we deal with bosons or fermions. The total wavefunction is reconstructed according to:

\[
\begin{align*}
\Psi &= \Psi_{1+3} + \Psi_{2+2} \\
\Psi_{1+3} &= \left[ 1 + \varepsilon (P_{13} + P_{23}) \right] \left[ 1 + \varepsilon (P_{14} + P_{24} + P_{34}) \right] K \\
\Psi_{2+2} &= \left[ 1 + \varepsilon (P_{13} + P_{23} + P_{14} + P_{24}) + P_{13} P_{24} \right] H
\end{align*}
\]

(2) (3)
Each amplitude \(F=K,H\) considered as a function of its own set of Jacobi coordinates, is expanded in the corresponding angular variables according to

\[
\langle xyz | F \rangle = \sum_{\alpha} F_\alpha(xyz) Y_\alpha(\hat{x}, \hat{y}, \hat{z}).
\]

(4)
\(F_\alpha\) are the unknowns and \(Y_\alpha\) are tripolar harmonics containing spin, isospin and angular momentum variables. Label \(\alpha\) holds for the set of quantum numbers defined in a given coupling scheme and includes the type of amplitude K or H.

Boundary conditions for scattering states are of Dirichlet-type. In the 1+3 elastic case e.g. they are implemented by imposing at large enough value of \(z\)

\[
\begin{align*}
K(x, y, z) &= t(x, y) \\
H(x, y, z) &= 0
\end{align*}
\]

t(\(x, y\)) being the triton Faddeev amplitudes previously determined. FY equations ensure a solution which, for n+t S-waves, behaves asymptotically like

\[
K(x, y, z) \sim t(x, y) \sin (qz + \delta)
\]

where \(\delta\) is the phaseshift and \(q\) is related to the center of mass kinetic energy \(T_{cm}\) and to the physical momentum \(k\) by \(T_{cm} = \frac{\hbar^2}{2m} q^2 = \frac{2}{3} \frac{\hbar^2}{m} k^2\). A more detailed explanation of the formalism and numerical methods used can be found in [3].

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Figure 1. Comparison between the nd and nt elastic cross sections (in barns)
3 Results

We have been interested in the first $0^+$ excitation of $^4\text{He}$, experimentally manifested as a $\Gamma=0.5$ MeV resonant state, 0.4 MeV above the $\alpha\alpha$ threshold \cite{4}. Using MT I-III potential \cite{5} we got \cite{3} the binding energies (B) and r.m.s. radius (R) displayed in table 1, what means a bound state with $B_{^4\text{He}^*}=0.26$ MeV below the N+NNN threshold. Using AV14 \cite{6} and Nim II \cite{7} potentials we found close results and similar conclusions were also reached using separable interactions \cite{8}. To investigate the structure of this state, we looked at the two-body correlation functions. The results (figure 2) show a superposition of two structures with different length scales, the short distance part being similar to the triton one. This suggests – rather than a breathing mode as it is widely considered in the literature – a 3+1 structure formed by an almost unperturbed triton plus a fourth nucleon orbiting around. This state, with a r.m.s. $R \approx 5$ fm, constitutes actually the first "halo-nucleus". It seems difficult for a strong interaction to generate a first excitation in the continuum and the open question is whether or not Coulomb forces, when added to the existing models, would be able to put it in the right place with the required accuracy.

Within the same model we have calculated (table 2) the parameters of the effective range expansion

$$g(k) \equiv k \cot(\delta) = \left[1 - \left(\frac{k}{k_0}\right)^3\right]^{-1} \left[-\frac{1}{a} + \frac{1}{2} r_0 k^2 + v_0 k^4 + o(k^6)\right]$$  \hspace{1cm} (5)

for N+NNN S-states. Expansion (5) provides an accurate representation of the below breakup scattering although the existence of the $^4\text{He}^*$ pole makes necessary the explicit inclusion of a singular term for the $T=S=0$ channel. It is worth noticing the coherence between the $^4\text{He}^*$ binding energy and the scattering parameters from table 2. If expansion (5) is inserted in the S-wave scattering amplitude $f_0^{-1}(k) = g(k) - ik$, 2 imaginary poles follow and the nearest to threshold $k_- = 0.095$ fm$^{-1}$ corresponds to $E_- = -0.25$ MeV, in agreement with the direct calculations of the $^4\text{He}^*$ binding.

The 4N scattering states with two open channels N+NNN→NN+NN has been calculated for the S=T=0 final state \cite{3}. Of particular interest is the extraction of the imaginary part of the strong NN+NN scattering length which controls the fusion rate in the process d+d → n+$^3\text{He}$. We found $a_R = +4.91 \pm$

### Table 1. Binding energy (MeV) and r.m.s. radius (fm)

|       | $^4\text{He}$ | $^4\text{He}^*$ | $^3\text{H}$ |
|-------|---------------|-----------------|-------------|
| B     | 30.30         | 8.79            | 8.53        |
| R     | 1.44          | 4.95            | 1.72        |

### Table 2. Low energy N+3N parameters (fm)

| S   | T | a   | $r_0$ | $v_0$ | $k_0$ |
|-----|---|-----|-------|-------|-------|
| 0   | 1 | 14.75 | 6.75   | 0.462 | -     |
| 1   | 1 | 3.25  | 1.82   | 0.231 | -     |
| 0   | 0 | 4.13  | 2.01   | 0.308 | 0.505 |
| 1   | 0 | 3.73  | 1.87   | $\simeq$ 0 | - |
0.02 fm and a very small value of \( a_I = -0.0115 \pm 0.0001 \) fm which should be only slightly modified once the Coulomb interaction is switched on. This value is due to the small overlapping between the K and H configurations which respectively govern the N+NNN and NN+NN asymptotic states. Other calculations of the dd cross section at very low energy exist \([9]\) but no values of \( a_I \) were given.

Special attention was paid to the n+t scattering. We found \([3]\) that the description provided by the simple MT I-III model fits remarkably well the data, as it can be seen in figure \([3]\) where the differential cross sections at different energies are displayed. Using realistic potentials, we have performed \([10]\) the same calculations for the \( J^p = 0^+, 1^+, 0^-, 1^-, 2^- \) states with \( ^1S_0, ^3S_1, ^3D_1 \) waves in \( V_{NN} \) and expansion \([4]\) limited to \( l_x, l_y, l_z = 0, 1, 2 \). The singlet \((a_0)\), triplet \((a_1)\), coherent \((a_c)\) scattering length and zero energy cross section \((\sigma(0))\) are in table \([3]\) compared with experimental values taken from \([11]\). They are in very good agreement with those obtained by Pisa group \([12]\) \( a_0 = 4.32 \) and \( a_1 = 3.80 \) but the inferred \( a_c \) and \( \sigma(0) \) values are well far from experiment.

**Table 3.** n+t scattering length with realistic interactions

|        | \( a_0 \) | \( a_1 \) | \( a_c = \frac{1}{4}a_0 + \frac{3}{4}a_1 \) | \( \sigma(0) = \pi(a_0^2 + 3a_1^2) \) |
|--------|-----------|-----------|---------------------------------|---------------------------------|
| AV14   | 4.31      | 3.79      | 3.92                            | 194                             |
| Nijm II| 4.31      | 3.76      | 3.90                            | 192                             |
| MT I-III| 4.10      | 3.63      | 3.75                            | 177                             |
| exp    | –         | –         | 3.59\( \pm 0.02 \)              | 170\( \pm 3 \)                 |
Calculations have been pursued beyond zero energy. One can see from figure 4 (dashed curve), that two-body realistic potentials alone fail in describing the low energy cross section as they failed in giving the binding energies $B_3$ and $B_4$. We would like to emphasize that this failure is in fact already present in the n+d case, although somehow hidden because the sizeable disagreement concerns only the doublet scattering length which turns to be very small compared to the quartet one and it is furthermore suppressed by statistical factors.

![Figure 3. n-t differential cross section with MT I-III at different n laboratory energies](image)

![Figure 4. n-t total cross section with AV14 compared to experiment](image)

To improve this situation we introduced the phenomenological TNI:

$$W(\rho) = W_r e^{-\mu \rho} - W_a \rho e^{-\mu \rho}$$

Using the parameter set $W_r = 500$, $W_a = 174$, $\mu = 2.0$, we got an overall low energy agreement with the values $B_3 = 8.48$ MeV, $B_4 = 29.0$ MeV, $a_0 = 4.0$ fm, $a_1 = 3.53$ fm, $a_c = 3.65$ fm and $\sigma(0) = 168$ fm$^2$ and a satisfactory n+t S-wave cross section (solid line). We point out that some dispersion exists in the experimental results which makes difficult the comparison with theoretical values. If one assumes the two last measurements of $a_c$ to be valid, our calculations are not far from the proposed value $a_c = 3.60$. In this case, however, our scattering lengths are not compatible with those inferred from experiment, which are in their turn incompatible with each other. A more precise measurement of $\sigma(0)$ would be helpful. For that purpose the possibilities offered by the neutron beam facility, recently approved at CERN, could deserve some attention from the nuclear Few-Body community.

If the inclusion of TNI is determinant in the n+t S-wave region, it doesn’t improve the resonance peak, on the contrary. In a first tentative to explain this discrepancy we have included P-waves in the NN interaction. We found...
that although separated P-waves had significant influence in the cross section, their global effect was very small and they could not explain the disagreement. A recent work done by A. Fonseca [14] using AGS equations and 1-rank approximation, was able to push expansion (4) further than we could do and found a satisfactory description. It would be interesting to have independent confirmation of this result and go beyond the first obstacle of the A=4 continuum.

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