A Novel Graphical Method for Dual-Frequency Two Sections Transformer

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Abstract: The most useful transmission-line construct is used to realize impedance matching at dual-frequency. The usual algebraic method is to solve the transmission equation which is precise but lack of intuition. A novel graphical method which is simple, intuitive and has explicit physical meaning is proposed to solve the matching problem. The parameters can be determined by simple geometrical relationship. Simulation and experimental results show that the proposed method is convenient, precise and can be used to extend bandwidth as well.

Keywords: Graphical method, dual-frequency, impedance matching, transformer, transmission lines

Classification: Microwave and millimeter-wave devices, circuits, and modules

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1 Introduction

Dual-frequency impedance transformer is widely used in microwave devices, such as antenna [1], power divider [2] and power amplifier [3]. Research on how to match the impedance at dual-frequency simultaneously has been carried out in [4]-[9]. In [4], a dual-frequency method is carried out but the application is only a frequency and its first harmonic. In [5]-[6], resonators are used to achieve dual-band impedance matching with the drawback of complex circuit configuration. In [7], two arbitrary complex impedances are matched at two frequencies however four sections transmission-line is used. The number of transmission line sections can decrease to two [8]-[9], however the parameters of the above transformers are solved by algebraic method which is lack of intuition.

In the above dual-frequency matching method, we aim at the transformer in [9] which has a simple two sections transmission-line in structure. A novel graphical way is proposed to solve the problem. In Section 2, the matching process is described in graphical way and the parameters are worked out by simple graphic relationship. In Section 3, three groups of numerical results are given to present the matching characteristics and prove the validity of the proposed graphical method. Finally, conclusion is given in Section 4.

2 Proposed Graphical Method

The simple two sections transmission-line transformer is shown in Fig. 1. The impedance is transformed from $Z_L$ to $Z_S$ at two frequencies ($f_1$, $f_2$) through two transmission-lines ($TL_1$, $TL_2$) whose characteristic impedance and length are $Z_1$, $L_1$ and $Z_2$, $L_2$ respectively. The problem is how to determine the above four parameters ($Z_1$, $L_1$, $Z_2$, $L_2$) by the four given variables ($Z_L$, $Z_S$, $f_1$, $f_2$). Although four transmission equations (real and imaginary parts at two frequencies) can be listed [9], the solving process is a little intricate and not intuitive.

Fig. 1. Two sections dual-frequency transmission-line transformer. $Z_L$ is transformed to $Z_S$ by $TL_1$ and $TL_2$ at $f_1$ and $f_2$. 

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At Smith Chart plane, the matching problem becomes simple and intuitive. A transmission line transforms the impedance along the circle, and a quarter wavelength line corresponds to a half circle. As shown in Fig. 2, a certain point, for instance L, represents an impedance $Z_L$ and a reflection coefficient $\Gamma_L$ at the same time. The value can be read directly in standard Smith Chart. The relationship is [10]:

$$\Gamma_L = \frac{Z_L/Z_0 - 1}{Z_L/Z_0 + 1}$$

$$Z_L = Z_0 \frac{1 + \Gamma_L}{1 - \Gamma_L}$$

where $Z_0$ is the system impedance of 50 Ohm. For matching at dual-frequency, we assume $f_1 < f_2$, and introduce a new variable, the center frequency $f_c$:

$$f_c = \frac{f_1 + f_2}{2}$$

The process of the proposed graphical method is shown in Fig. 2:

Firstly, at the center frequency $f_c$, a quarter transmission line $TL_1$ (electrical length is $\pi/2$) with the characteristic impedance of $Z_1$ transforms point L to point C. A circle which has the diameter of LC is painted. For a certain transmission line, electrical length is proportional to frequency. So for $f_1$ and $f_2$, the electrical length of transmission line $TL_1$ is:

$$\theta_1 = \frac{f_1 \pi}{f_c} < \frac{\pi}{2}$$

$$\theta_2 = \frac{f_2 \pi}{f_c} > \frac{\pi}{2}$$

So at low frequency $f_1$, $TL_1$ transforms point L to point A (the red solid arc), and the arc angle ($\angle AEL$) is twice of electrical length ($\theta_1$) [10]. At high frequency $f_2$, $TL_1$ transforms point L to point B (the blue solid arc). As relationship of (4) can be derived by (3), point A and B are symmetry thus the impedances are conjugate.

$$\frac{\pi}{2} - \theta_1 = \frac{\pi}{2} - \frac{f_1 \pi}{f_c} = \frac{\pi}{2} - \frac{f_1 - f_1}{f_c} = \frac{\pi}{2} - \frac{f_2 - f_1}{f_c} = \theta_2 - \frac{\pi}{2}$$

Secondly, a same size circle is painted through point A, B and S. It presents the transmission line $TL_2$ with the same electrical length and different characteristic impedance of $Z_2$. As the radius of the two circles is the same, the two red arcs are equal (AE=AF, so $\angle AEG = \angle AFG$, therefore $\angle AEL = \angle AFS$) while two blue arcs are equal. So $TL_2$ with the same electrical length transforms point A to S (the red dot arc) at $f_1$ while it transforms point B to S (the blue dot arc) at $f_2$.

Thus this constructs realize dual-frequency matching from L to S. The transforming process is shown in Fig. 1 and Fig. 2. In a word, solid arcs are transforming process of $TL_1$ and dot arcs are process of $TL_2$. Red arcs represent the transforming process at $f_1$ while blue arcs represent process at $f_2$.

Thirdly, simple geometrical relationship is obtained:

$$LS = 2LG = 2r \left(1 + \cos \left(\angle AEG\right)\right) = 2r \left(1 + \cos \left(\pi - 2\theta_1\right)\right)$$

(5)
At reflection coefficient plane, point L and S are both on real axis, the length of segment LS can be presented by $\Gamma_S$ and $\Gamma_L$ which can be read from Smith Chart, and $\theta_1$ is shown in (3), the unique unknown parameter radius (r) can be solved:

$$r = \frac{[\Gamma_S - \Gamma_L]}{2(1 - \cos(2\theta_1))}$$  \hspace{1cm} (6)

So $\Gamma_C$ and $\Gamma_D$ can be expressed with r:

$$|\Gamma_C| = |\Gamma_L| - 2r$$
$$|\Gamma_D| = |\Gamma_S| + 2r$$  \hspace{1cm} (7)

If $Z_L < Z_S$, the points L, D, and C are on the left side of Smith Chart, the reflection coefficients are negative, otherwise the points are on the right side, and the reflection coefficients are positive.

At impedance plane, $Z_C$ and $Z_D$ can be obtained from $\Gamma_C$ and $\Gamma_D$ by (1). According to the quarter wavelength transformer, $Z_1$ and $Z_2$ are [10]:

$$Z_1 = \sqrt{Z_LZ_C}$$
$$Z_2 = \sqrt{Z_DZ_S}$$  \hspace{1cm} (8)

### 3 Numerical Examples

The impedance transforming process is intuitively shown in Fig. 2, and the results of the graphical solution are shown in (3) and (6)-(8). This section explains some simulation and experimental results to validate the method.

For the first case, the two frequencies are fixed: $f_1 = 10$ GHz and $f_2 = 20$ GHz. $Z_S$ is fixed to 50 Ohm, while $Z_L$ varies from 10 Ohm to 30 Ohm. The parameters are shown in Table I and simulated S11 by ideal transmission line model is shown in Fig. 3. The data shows that the proposed graphical method can transform different impedances at dual-frequency.
The next example illustrates $S_{11}$ as a function of $f_2$ for fixed $f_1=10$ GHz when $Z_L=20$ Ohm and $Z_S=50$ Ohm. For $f_2$ in the range of 12-20 GHz, the parameters obtained from (3) and (8) are listed in Table II and the corresponding simulated results are presented in Fig. 4. For any given two frequency, the proposed graphical method can realize matching in high precise.

Another case we consider here is the validation of frequency band extension which is always used in power amplifier design. Supposing the transistor load line is 10 Ohm ($Z_L=10$) and the system impedance is 50 Ohm ($Z_S=50$). The center frequency is fixed to 10 GHz and the frequency band is extended from 0 to 6 GHz. To verify the graphical method, the two sections transmission line transformers are fabricated on AlN ceramic substrate with a dielectric constant of 9.9. The parameters are shown in Table III, microstrip line simulation and measured results are shown in Fig. 5. Single frequency transformer is a special case where the two circles are tangency, so $TL_1$ and $TL_2$ are two quarter wavelength lines. As $f_1$ and $f_2$ keep away from each other, $S_{11}$ at center frequency is worse, but frequency band is extended. According to the different tolerance requirements, the bandwidth can be extended by tuning two frequency points.

### 4 Conclusion

A novel graphical method for dual-frequency transformer in two sections is proposed. The method not only describes the impedance transforming process in clear and intuitive way, but also solves the parameters by simple geometrical relationship. By simulations and experiments, the proposed graphical method is
verified. It is believed that the method can be used widely in dual-frequency matching and bandwidth extension.

**TABLE II**

| $f_1$ (GHz) | $f_2$ (GHz) | $Z_L$ (Ohm) | $Z_S$ (Ohm) | $Z_1$ (Ohm) | $Z_2$ (Ohm) | $\theta$ (°) |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 10          | 12          | 20          | 50          | 25.27       | 39.57       | 82          |
| 10          | 14          | 20          | 50          | 25.58       | 39.09       | 75          |
| 10          | 16          | 20          | 50          | 26.02       | 38.43       | 69          |
| 10          | 18          | 20          | 50          | 26.57       | 37.64       | 64          |
| 10          | 20          | 20          | 50          | 27.21       | 36.75       | 60          |

Fig. 4. Simulation results when $Z_L=20$ Ohm and $Z_S=50$ Ohm. $f_1$ is fixed to 10 GHz while $f_2$ varies from 12-20 GHz.

**TABLE III**

| $f_1$ (GHz) | $f_2$ (GHz) | $Z_L$ (Ohm) | $Z_S$ (Ohm) | $Z_1$ (Ohm) | $Z_2$ (Ohm) | $\theta$ (°) |
|-------------|-------------|-------------|-------------|-------------|-------------|-------------|
| 10          | 10          | 10          | 50          | 15.81       | 35.36       | 90          |
| 9           | 11          | 10          | 50          | 15.96       | 35.02       | 81          |
| 8           | 12          | 10          | 50          | 16.44       | 33.96       | 72          |
| 7           | 13          | 10          | 50          | 17.38       | 31.96       | 63          |

Fig. 5. Simulation and experimental results when $Z_L=10$ Ohm and $Z_S=50$ Ohm. The center frequency is fixed to 10 GHz. Bandwidth varies from 0-6 GHz.