Multiphonon decay of strong mode in quantum lattice

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Abstract

A nonperturbative theory of multiphonon anharmonic decay of strongly excited local mode is developed whereby the mode is considered classically and phonons, quantum mechanically. The decay rate of the mode is expressed via the negative frequency parts of the phonon pair correlation functions. In the case of two-phonon decay the later satisfy the linear integral equations while in the case of two- and more-phonon decay they satisfy the nonlinear integral equations. As a result, the processes mentioned differently depend on the mode amplitude \( A \): two-phonon processes smoothly diminish if \( A \to \infty \) while three- and more-phonon processes are fully switched-off at large amplitudes and they abruptly switch-on if the amplitude approaches the critical value. At that the decay rate gets rather high value (of the order of the mode quantum per period). The final stage of the relaxation is well described by the perturbation theory.

I. INTRODUCTION

The problem of the time evolution of strong vibrational excitations in crystals nowadays attracts a remarkable attention. One of the essential results in this field is the demonstration of the existence of localized modes (ILM, intrinsic local mode or SLS, self-localized soliton).
in perfect crystals. The results are obtained by applying equations of classical nonlinear
dynamics. The central point concerning the ILMs is their stability. In the classical limit
these modes are stable if all their harmonics are out of resonance with the phonon spectrum.
This condition is fulfilled e.g. for the local modes with high frequency (the basic frequency
of the mode is above the phonon band). On the contrary, the quantum mechanics allows
the distinction of the high frequency quanta (due to the processes of creation of two or
more phonons). Therefore, the question of the stability of the modes mentioned should
be considered on the basis of the quantum theory. However, the problem mentioned can
not be solved by applying standard quantum perturbation theory, commonly used for the
description of quantum transitions since one cannot assume that the perturbation parameter
is small (note that one of the essential conditions of existence of ILMs in a nonlinear lattice
is a large amplitude of vibration).

Recently we proposed a nonperturbative theory of the two-phonon anharmonic decay of
strong local modes associated with lattice defects and an analogous theory of the decay of
the ILMs in the perfect monatomic chain. We have found that the decay law is highly
nonexponential: at definite "critical" amplitudes (typically of the order of 0.2 - 0.4 Å)
the relaxation jumps take place being accompanied by generation of phonon bursts. The
process considered in takes into account cubic anharmonicity. The interaction term which
is responsible for the process is \( \sim \sum_{i_1i_2} W_{i_1i_2}^{(3)} Q(t) \hat{x}_{i_1} \hat{x}_{i_2} \), where \( W_{i_1i_2}^{(3)} \) are the parameters of
the cubic anharmonicity, \( Q(t) \) is the time-dependent classical coordinate of the strong local
mode, \( \hat{x}_{i_1} \) and \( \hat{x}_{i_2} \) are the coordinate operators of phonons. Such interaction leads to the
quadratic (with respect to phonon operators \( \hat{a}_i, \hat{a}_i^+ \)) time-dependent phonon Hamiltonian.
The latter can be diagonalized exactly by means of the linear transformation of phonon operators

\[
\hat{b}_j(t) = \sum_i (\mu_{ij} \hat{a}_i + \nu_{ij} \hat{a}_i^+) ;
\]  

(1)

the terms \( \sim \hat{a}_i^+ \) result from fast (oscillatory) change in time of the phonon Hamiltonian. Due to these terms the initial zero-point state \( |0\rangle \) is not the zeroth state of the transformed
destruction operator $\hat{b}_j$, i.e. phonons are created in the lattice. This transformation allows one to find energy of generated phonons and, thus the energy loss of the mode for any cubic anharmonicity and at any time moment.

Let us mention that an analogous transformation of the field operators takes place in the field of a gravitationally collapsing star (black hole), causing thermal radiation (so-called quantum emission of black holes)\cite{11,12,13}. Note also an analogy of emission of phonon pairs by a strong mode with superconducting transition: in both cases mean values of destruction (and creation) operators (of phonons or electrons (holes), respectively) equal to zero but mean values of products of two destruction operators (as well as mean values of products of two creation operators) differ from zero.

The described process of phonon generation by a strong mode is allowed if $\omega_l < 2\omega_M$, where $\omega_l$ is the frequency of the local mode, $\omega_M$ is the top band frequency. If $\omega_l > 2\omega_M$, then one needs to account for three- (or more) phonon processes. These processes arise from the interaction terms $\sim \sum_k W_{i_1i_2\ldots i_{n-1}}^{(n+k)} Q^k(t)\hat{x}_{i_1}\hat{x}_{i_2}\ldots\hat{x}_{i_{n-1}}$ with $n \geq 4$. The phonon Hamiltonian with such terms cannot be diagonalized. Therefore, the method of\cite{9} is not directly applicable for $n \geq 4$. Below we present another method which is based on the asymptotical behaviour of a pair correlation function and which is applicable for arbitrary $n$.

II. GENERAL FORMULAS

We take into account that according to the Coleman theorem\cite{14} the vacuum (zero-point) state of bosons in the classical field is not invariant with respect to the transformation which changes the classical field. This means that destruction operators are different for different values of the classical field. Therefore, if one describes the effect of a classical mode, anharmonically interacting with phonons in the quasi-harmonic (Hartree-Fock, or mean-field) approximation

$$\hat{H}_{ph}(t) = \sum_j \hbar \omega_j (\hat{b}_j^+(t)\hat{b}_j(t) + 1/2) \quad (2)$$

3
then the destruction operators linearly change with the mode amplitude. For slow relaxation (as compared to the mode frequency \( \omega_l \)) the transformation is diagonal in the phonon space:

\[
\hat{b}_i(t) \simeq \mu_i(t) \hat{a}_i + \nu_i(t) \hat{a}_i^+
\]

(\( \mu_{ij} \simeq \mu_i \delta_{ij}, \nu_{ij} \simeq \nu_i \delta_{ij} \); see (1)). Here \( \hat{a}_i \) and \( \hat{a}_i^+ \) are phonon operators before excitation of the local mode (which is supposed to take place at time \( t = 0 \)); \( |\mu_i|^2 - |\nu_i|^2 \) = 1; the constant term on the right hand side of (2) is considered to be small and neglected. In this approximation all multiphonon decay processes are described in the same way: the relaxation rate of the mode

\[
\frac{dE_l(t)}{dt} = -\frac{dE_{ph}(t)}{dt} = -\sum_i \hbar \omega_i \frac{d|\nu_i(t)|^2}{dt} \tag{3}
\]

is determined by the negative frequency terms in the large time asymptotics of the two-time correlation function

\[
< 0 | \hat{x}_i(t + \tau) \hat{x}_i(t)|0 > \simeq \frac{\hbar}{2\omega_i} [ |\mu_i(t)|^2 e^{-i\omega_i \tau} + |\nu_i(t)|^2 e^{i\omega_i \tau}], \quad t \gg \tau \sim \omega_i^{-1} \tag{4}
\]

(here fast oscillating terms \( \sim \exp (\pm i\omega_i(2t + \tau)) \) are averaged out).

\( |\nu_i(t)|^2 \) can be found as follows. Presenting coordinate operators by means of initial creation and destruction operators \( \hat{x}_i(t) = \sqrt{\hbar/2\omega_i} (g_i(t) \hat{a}_i + g_i^*(t) \hat{a}_i^+) \) one gets

\[
< 0 | \hat{x}_i(t + \tau) \hat{x}_i(t)|0 > = (\hbar/2\omega_i) g_i(t + \tau) g_i^*(t). \tag{5}
\]

The time dependence of the coordinate operators can be determined from the Lagrangian equations of motion. Inserting these equations to the correlation functions (5) and expressing multiphonon correlation functions via the Wick-type products of pair correlation functions one gets linear (for cubic anharmonicity) or nonlinear (for quartic and higher order anharmonicity) equations for the pair correlation functions. It is straightforward to show that for the multiphonon decay one obtains the same equations if to calculate \( g_i(t) \) from classical equations of motion for dimensionless normal displacements with "nonclassical" initial conditions \( g_i(t) = \exp (-i\omega_i t + \phi_i), t \leq 0 \) and then to average the correlation functions over the random phases \( \phi_i \) (the later averaging is denoted below as \( \langle ... \rangle \)).
We demonstrate the method for anharmonic interactions in the pair potential approximation with account of the central forces only. For the case of simplicity we suppose that the strong odd mode is fully localized on the site \( l = 0 \). Taking \( Q(t) = A_t \cos \omega_l t \) one gets the following interaction Hamiltonian

\[
\hat{H}_{an}^{(n+1)} \simeq \frac{2}{n} A_t \cos \omega_l t \sum_m V_m^{(n)} \hat{q}_m^n, \quad n \geq 2,
\]

where \( V_m^{(n)} \equiv V_{ml}^{(n)} = -\sum_k W_{0m}^{(n+k+1)} A_t^k / (2(n-1)!) \) and \( \omega_l \equiv \omega_{lt} \) depend on the mode amplitude, \( q_m(t) = \sum_i \epsilon_{mi} \hat{x}_i(t) \). Lagrangian equations for \( g_i(t) \) are the following:

\[
\ddot{g}_i(t) + \omega_i^2 g_i(t) + 2A_t \cos \omega_l t \sum_m \omega_i \bar{e}_{im} V_m^{(n)} g_m^{n-1}(t) = 0
\]

(\( \bar{e}_{mi} = e_{mi}/\omega_i^{1/2} \)). Corresponding integral equations read

\[
g_i(t) = e^{-i\omega_i t + \phi_i} + 2A_t \sum_m \bar{e}_{im} V_m^{(n)} \int_0^t dt_1 \sin(\omega_i(t-t_1)) g_m^{n-1}(t_1) \cos \omega_l t_1
\]

(7)

where \( g_m(t) = \sum_i \bar{e}_{im} g_i(t) \) satisfy the analogous equations

\[
g_m(t) = f_m(t) + 2A_t \sum_m \int_0^t dt_1 G_{mm'}(t-t_1) V_m^{(n)} g_{m'}^{n-1}(t_1) \cos \omega_l t_1,
\]

(8)

\( G_{mm'}(t) = \sum_i \bar{e}_{im} \bar{e}_{im'} \sin \omega_i t \) is the dynamical Green function. Substituting (7) to (5) and taking into account (3) and (4) one gets for \( t \gg \omega_l^{-1} \)

\[
\frac{dE_{ph}(t)}{dt} = \frac{\hbar A_t^2 (n-1)}{2\pi} \int_0^{\omega M} d\omega \operatorname{Sp}[\text{Im} G(\omega_l - \omega) V^{(n)}(t; \omega) V^{(3)}],
\]

(9)

where

\[
D_{mm'}^{(n-1)}(t; \omega) = \int_{-\infty}^{\infty} d\tau e^{i\omega \tau} D_{mm'}^{n-1}(t; \tau),
\]

(10)

is the positive frequency (\( \omega > 0 \)) part of the \( n-1 \)-phonon spectrum;

\[
D_{mm'}(t; \tau) = (g_m(t+\tau)g_{m'}^*(t)),
\]

(11)

\[
G_{mm'}(\omega) = \int_0^\infty e^{-i\omega t} G_{mm'}(t) dt
\]
is the phonon Green function in frequency representation. In (9) it is accounted that the main
correlation contribution to the relaxation rate is given by \( t_1, t'_1 \sim t \) and that the negative frequency part
of the correlator \( \langle g_i(t+\tau)g_i^*(t) \rangle \) comes from the negative frequency term \( A_t^2 \exp(i\omega_i(t_1-t'_1))/4 \)
of the \( Q(t_1)Q(t_{1'}) \) ( here \( t_1 \) and \( t'_1 \) are the integral variables in (7) and (8), \( t_1 > t'_1 \)).

III. TWO-PHONON DECAY

In the case of two-phonon decay \((n = 2)\) the equations (7) are linear. These equations
have been solved in \[9\] which allowed us to derive the explicit expression for the relaxation rate. Here we present another derivation of the formula for the relaxation rate which is
based on the direct calculation of the pair correlation functions \( D^{(1)}(t; \omega) \equiv D(t; \omega) \) and
which can be extended for three- and more-phonon decay processes.

Taking into account (8) and (11) one gets the following equation for \( D(t; \tau) \):

\[
D(t; \tau) = d^*(t; -\tau) + 2A_t \int_0^{t_0} dt_1 G(t_0 - t_1)V^{(3)}d(t_1 - t)\cos \omega_1t_1, \tag{12}
\]

where \( t_0 = t + \tau, d(t; \tau) = \langle g(t + \tau)f^*(t) \rangle \). For \( t \gg \tau \gtrsim \omega_i^{-1} \) the matrix-function \( d(t; \tau) \)
satisfies the same equation as \( D(t; \tau) \) with \( d_0(\tau) = \langle f(\tau)f^*(0) \rangle \) instead of \( d^*(t; -\tau) \). This
equation (as well as (12)) can be solved as follows. First we present this equation in the form

\[
d(t; \tau) = d_0(\tau) + 2A_t \int_0^{t_0} dt_1 G(t_0 - t_1)V^{(3)}d_0(t_1 - t)\cos \omega_1t_1 + \\
4A_t^2 \int_0^{t_0} dt_1 \int_0^{t_1} dt_2 G(t_0 - t_1)V^{(3)}G(t_1 - t_2)V^{(3)}d(t; t_2 - t)\cos \omega_1t_1\cos \omega_1t_2. \tag{13}
\]

We are interested only in the positive frequency parts of \( D(t; \tau) \) and \( d(t; \tau) \). The linear (with
respect to \( A_t \)) term in the right-hand side of (13) gives only the fast oscillating contribution
to these parts and it averages out. Omitting also the fast oscillating part of the last two
terms in (13) one gets the following equation:

\[
d(t; \tau) \simeq d_0(\tau) + A_t^2 \int_0^{t_0} dt_1 \int_0^{t_1} dt_2 G(t_0 - t_1)V^{(3)}G(t_1 - t_2)V^{(3)}d(t; t_2 - t)e^{i\omega_1(t_2-t_1)} \tag{14}
\]

Introducing
\[ d_{(0)}(t; \omega) = \int_{-t}^{\infty} e^{i\omega \tau} d_{(0)}(t; \tau) \] (15)

one obtains

\[ d(t; \omega) = d_0(t; \omega) + A_t^2 \int_0^\infty dt_0 \int_0^{t_0} dt_1 \int_0^{t_1} dt_2 e^{i\omega(t_0-t)} G(t_0 - t_1)V^{(3)}G(t_1 - t_2)V^{(3)}d(t; t_2 - t) \exp(i\omega_l(t_2 - t_1)) \] (16)

Denoting \( \tilde{t}_0 = t_0 - t_1, \tilde{t}_1 = t_1 - t_2 \) and taking into account that

\[ \int_0^\infty dt_0 \int_0^{t_0} dt_1 \int_0^{t_1} dt_2 = \int_0^\infty dt_2 \int_0^\infty d\tilde{t}_1 \int_0^\infty d\tilde{t}_0 \]

one gets simple algebraic equation for \( d(t; \omega) \) with the following solution:

\[ d(t; \omega) = I - A_t^2 G(-\omega) V^{(3)} G(\omega_l - \omega) V^{(3)} \] \( d_0(t; \omega) \) (17)

Equation (12) can be solved in the same way. As a result one finds

\[ D^{(1)}(t; \omega) = \frac{2}{\pi} |I - A_t^2 G(-\omega) V^{(3)} G(\omega_l - \omega) V^{(3)}|^{-2} \Im G(\omega) \] (18)

and

\[ \frac{dE_{ph}(t)}{dt} = -\frac{dE_l(t)}{dt} \simeq \frac{\hbar \omega_l A_t^2}{2\pi} \int_{\omega}^{\omega_l} d\omega \text{Sp}[V^{(3)} \Im G(\omega_l - \omega)] \frac{V^{(3)}|I - A_t^2 G(\omega_l - \omega) V^{(3)} V^{(3)}|^{-2} \Im G(\omega)|.} \] (19)

Within notations and cyclic permutation of the matrices under Sp this formula coincides with analogous formula derived in \( ^9 \) by another method. Note that diagonal elements of the resolvent-matrix \( |I - A_t^2 G(\omega_l - \omega) V^{(3)} G(\omega) V^{(3)}|^{-1} \) are symmetric with respect to the change \( \omega \leftrightarrow \omega - \omega_l \). Taking also into account that \( G(-\omega) = G^*(\omega) \) one finds that the resolvent \((|I - A_t^2 G(-\omega_l/2) V^{(3)} G(\omega_l/2) V^{(3)}|^{-1})_{mm} \) is real. Therefore, the function under integral (19) for some critical values of \( A_t \) diverges at \( \omega = \omega_l/2 \). This leads to sharp peaks of the relaxation rate near \( A_{cr} \) of the shape \( |A_t - A_{cr}|^{-1} \sim |t - t_{cr}|^{-1/2} \) where time moment \( t_{cr} \) of the jump corresponds to \( A_t = A_{cr} \). The relaxation jumps are accompanied by explosion-like emission of phonon bursts.
IV. THREE- AND MORE-PHONON DECAY

Let us consider now decay processes with creation of more than two phonons, caused by quartic and higher order anharmonicities. Substituting (8) to (11) one obtains after averaging over $\phi_i$ the following nonlinear integral equations for the matrix-function $D(t; \tau)$:

$$D(t; \tau) = d_0(\tau) + 4(n - 1) \int_0^t dt_1 \int_0^t dt_2 Q(t_1) Q(t_2) G(t_0 - t_1) V^{(n)} D^{(n-1)}(t; t_1 - t_2) V^{(n)} G(t - t_2),$$

(20)

where $D^{(n-1)}(t; \tau)$ is matrix with the elements $(D^{(n-1)}(t; \tau))_{mm'} = D^{n-1}_{mm'}(t; \tau)$ (in (20) it is accounted that terms $\sim \langle f(t + \tau)(g^{n-1}(t'))^* \rangle$ and $\langle g^{n-1}(t') f^*(t) \rangle$ turn to zero for $n \geq 3$ when averaging over random phases $\phi_i$). The Green functions $G(\tau)$ essentially differ from zero only for small $|\tau| \lesssim \omega_i^{-1}$. Therefore, in the limit $t \gg \tau \lesssim \omega_i^{-1}$ one can replace the lower limits of integration by $-\infty$. Neglecting also fast oscillating terms $\sim \cos (\omega_l(t_1 + t_2))$ one gets

$$D(t; \tau) \simeq d_0(\tau) + 2(n - 1) A^2 \tau \int_{-\infty}^{\infty} d\tau_1 \int_{-\tau_1}^{\infty} dx \cos (\omega_l(\tau - x))$$

$$G(\tau_1 + x)V^{(n)} D^{(n-1)}(t; \tau - x) V^{(n)} G(\tau_1)$$

(21)

where $\tau_1 = t - t_2$, $x = t_0 - t_1 - \tau_1$). Taking into account that

$$\int_{-\infty}^{\infty} d\tau_1 \int_{-\tau_1}^{\infty} dx = \int_{-\infty}^{\infty} dx \int_{0}^{\infty} d\tau_1 + \int_{-\infty}^{0} dx \int_{-x}^{\infty} d\tau_1$$

(22)

and introducing the fourth-rank matrix-function

$$\tilde{G}_{mm_1; m'm_2}(x) = \int_{0}^{\infty} d\tau_1 G_{mm_1}(\tau_1 + x) G_{m'm_2}(\tau_1)$$

(23)

one gets the following set of nonlinear integral equations:

$$D_{mm'}(t; \tau) = D_{0,mm'}(\tau) +$$

$$2(n - 1) A^2 \sum_{m_1 m_2} V^{(n)} V^{(n)} \int_{-\infty}^{\infty} dx \cos (\omega_l x) D^{n-1}_{m_1 m_2}(t; x) \tilde{G}_{mm_1; m'm_2}(|\tau - x|)$$

(24)

($n \geq 3$). These equations can be solved numerically and, in some cases, analytically. Then, performing Fourier transformation (10) one can find $D^{(n-1)}(t; \omega)$ and then the relaxation rate (9).
V. SWITCH-OFF OF MULTIPHONON PROCESSES

As an example, let us consider a $n$-phonon decay ($n \geq 3$) of a high frequency local vibration, directed to the nearest neighbours. In this case only two coordinates $q_m$ of the nearest neighbours can be accounted. Besides, the high frequency local mode breaks the dynamical correlation (nonorthogonality) of these coordinates. Due to that the Green functions $G(\tau)$ and $\tilde{G}(|x|)$ become diagonal. Therefore the equation (24) takes the form

$$D(t; \tau) = d_0(\tau) + 2(n - 1)V^{(n)} A_t^2 \int_{-\infty}^{\infty} dx D^{n-1}(x)\tilde{G}(|\tau - x|) \cos \omega_l x$$  \hspace{1cm} (25)$$

(index $m$ is omitted). We consider the case of phonon spectrum with narrow single maximum at $\omega_0 = \omega_l/n$ of the Lorentzian shape with the width $\Gamma(\ll \omega_0)$.

In this case

$$G(\tau) = \omega_0^{-1} e^{-\Gamma|\tau|} \sin \omega_0 \tau; \quad d_0(\tau) = \omega_0^{-1} e^{-i\omega_0\tau - \Gamma|\tau|}; \quad \tilde{G}(x) = \frac{1}{4\Gamma \omega_0^2} e^{-\Gamma|x|} \cos (\omega_0 x)$$  \hspace{1cm} (26)$$

Approximate solutions of the integral equation (25) is

$$D(t; \tau) \approx \omega_0^{-1} [\alpha_t e^{-i\omega_0\tau} + \beta_t e^{i\omega_0\tau}] e^{-\Gamma|\tau|}$$  \hspace{1cm} (27)$$

($t \gg \tau \sim \omega_0^{-1}$), where $\beta_t = w_t \alpha_t^{n-1}$,

$$w_t = \frac{h^{n-2} V^{(n)} A_t^2 (n - 1)^2}{2\omega_0^{n-2} \Gamma^2 n(n - 2)}; \quad n \geq 3$$  \hspace{1cm} (28)$$

$\alpha_t$ is a solution of the equation

$$\alpha_t = 1 + w_t^n \alpha_t^{2(n-1)}.$$  \hspace{1cm} (29)$$

| $n$ | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|-----|---|---|---|---|---|---|---|----|
| $w_{n,cr}$ | 0.32 | 0.405 | 0.47 | 0.52 | 0.56 | 0.595 | 0.62 | 0.64 |
| $\alpha_{n,cr}$ | 1.25 | 1.173 | 1.127 | 1.09 | 1.068 | 1.059 | 1.042 | 1.031 |
This equation has real solutions only for $w_t$ being below the critical value $w_{n,cr}$ and, correspondingly for $A_t < A_{n,cr}$ and $\alpha_t < \alpha_{n,cr}$; values of $w_{n,cr}$ and $\alpha_{n,cr}$, see in Table 1).

There are two such solutions but only the smaller one corresponds to slow relaxation (as compared to $\omega_l$) and, therefore only this solution should be accounted. The relaxation rate, then equals

$$- \frac{dE_t}{dt} = \frac{A_t^2 V^{(n)^2} \alpha_t^{n-1}}{(\hbar\omega_0)^{n-1}\Gamma} = \hbar\omega_0 \Gamma \alpha_t^{n-1} w_t \left[ \frac{n - 2}{n - 1} \right].$$

(30)

For small amplitude $A_t$, $\gamma = d \ln E_{ph}(t)dt$ does not depend on $A_t$, i.e. relaxation is exponential in accordance with perturbation theory\[3\]. For $A_t \leq A_{n,cr}$ the relaxation rate grows as $A_t \to A_{n,cr}$ and gets rather high value $\sim \Gamma\hbar\omega_l$ (of the order of the mode quantum per period). Absence of real solutions of the equation (29) for $w_t > w_{n,cr}$ ($A_t > A_{n,cr}^{(n)}$) means that $n$-phonon relaxation is fully switched-off. Note that in the case of two-phonon decay the switch-off effect also exists but it takes place only asymptotically with $A_t \to \infty$. The values of $A_{n,cr} \sim \hbar^{1-n/2} w_{n,cr}^{1/2} \Gamma\omega^{n/2}/|V^{(n)}|$ for $n \geq 3$ are much larger than the corresponding values of the critical amplitudes for two-phonon decay (for $n \geq 3 A_{n,cr} \to \infty$ if $\hbar \to 0$). Typically $A_{3,cr} \sim 1\AA \ll A_{4,cr} \ll A_{5,cr}$ etc. Note also that the the difference of the perturbative and nonperturbative treatments on the $n$-phonon decay for $n \geq 3$ is given by the factor $\alpha_t^{n-1}$ in (30). For all $A_t < A_{n,cr}$ this factor does not exceed 1.62. Therefore, the main results of the nonperturbative consideration of the multiphonon decay are the switching-off of the process if the mode amplitude exceeds the corresponding critical value, and the step-vise switching-on of the very fast relaxation when the amplitude reaches this value from above.

For not very large amplitudes ($< 1\AA$) three- and more- phonon decay processes are well described by the standard perturbation theory.

Thus, the general scenario of the relaxation of strongly excited local mode is the following. In the beginning the relaxation (which takes place due to creation of $n \geq 4$ phonons) is rather slow. Then, when the mode amplitude approaches the critical value for the $(n - 1)$-phonon decay processes, relaxation is jump-like enhanced (supposing these processes are allowed by the energy conservation law). After that the $(n - 1)$- phonon processes will slow-down
till the next critical amplitude (which corresponds to the \((n - 2)\)-phonon processes), then the relaxation process will be once more jump-like enhanced (supposing the \((n - 2)\)-phonon process is allowed) and so on. The relaxation rate can change very fast not only for a small change of the mode amplitude but also for a small change of the mode frequency. Final stage of the anharmonic relaxation is exponential and is well described by the perturbation theory.

In conclusion, we developed here a nonperturbative theory of multiphonon anharmonic decay of a strong local mode. We found that three- and more-phonon decay processes of the mode are described by essentially different equations than the two-phonon decay process. The main effect which is predicted by the nonperturbative theory is the switching-off of the \(n \geq 3\)-phonon process if the mode amplitude exceeds the corresponding critical value and sharp (step-wise) switching-on of the fast relaxation when the amplitude reaches this value from above.

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