Numerical Solution of Nonlinear Schrödinger Approaches Using the Fourth-Order Runge-Kutta Method for Predicting Stock Pricing

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Abstract. Stocks are a certificate that shows the book of ownership of a company. The certainties of stock price are important for investors. This study aims to predict changes in stock prices. The predicting model of the stock prices in this study uses the non-linear Schrödinger equation. Because this equation has no known analytical solution, so a numerical solution that calculated using the fourth-order Runge-Kutta method to determine the stock prices. This research will also develop an algorithm of numerical solutions in the fourth-order Runge-Kutta method. The analogy of parameters between the nonlinear Schrödinger equation with economic parameters that affect stock prices is obtained based on the hypothesis and the economic theory. The assumed main parameters influence stock prices include movement or growth in average stock prices that is volatility, strike price, speed of average stock returns, adaptive market potential, and current stock price. This present method has been validated to predict the real stock prices of two companies. The prediction results obtain the value of mean absolute percentage error (MAPE) of Astra Agro Lestari Tbk. (AALI) is 0.4633 % and Polychem Indonesia Tbk. (ADMG) is 3.48678 %. Based on these MAPE results, the non-linear Schrodinger equation has shown that good agreement with the real stock price.

1. Introduction
In the mid-nineties, Stanley and co-workers [1] have introduced the term econophysics to explain that economic and financial phenomena can be described by the physical phenomena that have been researched by physicists. Econophysics has been categorized as an interdisciplinary research field because of applying theories and methods of physical science to solve problems in economic and financial phenomena. The econophysics field can also be assumed as the physics of finance since this field can be described as the global dynamics of financial markets. The econophysics will involve much statistical physics to understand market dynamics. This is because many similarities have been found between data on stock markets and statistical physics.

The real market is not an ideal system. The real market only approaches an approximately ideal system. This fact is unfamiliar with physicists because they are already familiar with the study of ideal systems. The ideal systems are commonly used in a scientific investigation because it has as instrumental in the development of physics as a discipline. Physicists have used their imaginations to develop new theories which then used to design experiments or vice versa. However, physicists always understand those ideal systems only an approach to real systems and the behavior of real systems will not always be the same from an ideal system. A system approach can also be used in the study of economic and
financial systems. These systems can be assumed as realistic 'ideal' conditions. The validity results often depend on the assumptions made [1].

One of the interesting systems of economics and finances is the stock market because it is influenced by all varying degrees of the tradings and all tradings can decide the behavior change of the value of the stock index. Each stock trading is following certain well-defined rules. These certain rules are not known in detail the connection between the microscopic system and the macroscopic system [2]. Therefore, this study will try to observe a macroscopic variable with microscopic inputs, how these microscopic variables take effect to set the actual sales of a stock company.

Stock pricing based on the Black-Scholes theory in the economic approach is assumed that the stock price depends on the stock price in previous which is represented as a variable depending on time. The Black-Scholes model is a statistical physics equation that is derived from the stochastic differential equation. This model can also describe the time evolution of the probability density function for a position occupied by a particle [3, 4]. The market applications of the Black-Scholes model have indicated that this model is not capable of providing stock price data of real markets. Therefore, different alternative stock pricing models are being developed besides that complexity stock pricing problems are also challenging problems in the field of computational physics.

Physics always try to involve how macroscopic system effects can be explained by the amount number of interactions of microscopic systems, some of the tools used are statistical physics. Therefore, this tool can also be applied to understand stock market dynamics using micro-economic parameters that affect stock prices more accurately. This concept is expected to help economists better understand the predicting of the stock market. Based on the following problem, many papers [5-7] that try to describe the economy using the statistical physics of quantum mechanics. One of the most interesting works is based on the Black-Scholes that transformed into the time-dependent Schrödinger equation [8].

Stocks are financial instruments that are pieces of paper to represent ownership over economic assets. The value changing of stock price strongly depends on time due to uncontrollable exogenous and endogenous factors. Quantum mathematics is the general approach form of quantum mechanics that describe the expectation values of random quantities. However, quantum mathematics can be used as an interpretation that is very different from quantum mechanics when applied to other disciplines such as economics and finance. The symbols' interpretation of quantum mathematics in such other disciplines have no fixed prescription. Therefore, an analogy of the symbols' interpretation between the quantum mathematics equation with economic factors that affect stock prices needs to be connected based on the hypothesis of the economic theory. To predict an uncertainty of the financial system in the future, quantum mathematics has been applied to the modeling of financial instruments and then these models are tested by comparing the real market prices [9].

The quantum computation can be applied to financial problems is potential prospects and also is interesting topics [10]. Mathematical modeling of quantum mechanics of stock price dynamics in a financial market has been proposed to describe behavioral of financial factors by using the Bohmian model of quantum mechanics. This quantum model of the financial market has been studied in the problem of the smoothness of stock price-trajectories [11]. A new Black–Scholes–Schrödinger model based on the stock pricing formulation has been proposed to derive a more general quantum model of stock pricing. This new model has an associated potential to the random dynamics of the underlying stock price and could be seen as a more general formulation [12, 13].

Numerical methods form an important part of stock pricing prediction due to many cases that do not have a closed-form analytic solution. Many numerical methods for stock pricing prediction such as the Binomial model, Finite difference methods and Monte Carlo method. The best convergence of those methods to apply on the analytic Black-Scholes of the stock pricing is the finite difference method, called Crank Nicolson. This method is unconditionally stable, more accurate and converges faster than the Binomial model and Monte Carlo method [14].

A nonlinear Schrödinger equation is a quantum physics that based on relations between the linear Schrödinger and the Black-Scholes partial differential equations. This study aims to apply a nonlinear Schrödinger equation to predict the stock pricing that solved numerically using a fourth-order Runge-Kutta method with a boundary value problem [15]. The obtained numerical results of stock pricing will be compared to real stock pricing.
2. Method
This study will focus on the numerical calculation for predicting stock pricing using an ordinary differential equation of a nonlinear Schrödinger as the stock pricing model. A nonlinear Schrödinger of the stock pricing model has assumed no contained external potential energy forms [6]. Stock pricing problems are often discussed in the econophysics using the quantum probability concept and their probability density function is obtained using the solution to a time-dependent linear Schrödinger equation. The linear Schrödinger equation can govern the evolution of the complex wave function $\psi$ where the square value of wave function $|\psi|^2$ defines the probability density function. Stock pricing models based on linear Schrödinger equations and their relation to Black-Scholes models have been studied in many works of literature. Therefore, the complexity of stock pricing problems is challenging in the field of computational mathematics or physics.

2.1. Schrödinger equation
The time-dependent Schrödinger equation is a linear partial differential equation where consists of the first-order in time and the second-order in the spatial variables. This equation is commonly represented as follow:

$$\hat{H}\psi(x,t) = i\hbar \frac{\partial \psi(x,t)}{\partial t},$$

where

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \hat{U}(x,t)$$

$\hat{H}$ is an abbreviation of Hamiltonian and describe the system energy consists of the kinetic and potential energy of the particles constituting the system $U(x,t)$ represents the potential energy operator. The Schrödinger equation in quantum physics has the same concept as Newton’s second law of motion in classical mechanics. This equation declares how a physical system will change over time. In classical mechanics, the positions and momenta of all particles are a function of each time $t$ so that the physical system can be described fully. In quantum mechanics, the solution to Schrödinger’s equation is a wave function $\psi$ that contained the information about the physical system fully. For finding the probability density of the particle at position $x$ can calculate the square of the absolute value of the wave function, $|\psi(x,t)|^2$. Besides that, Schrödinger’s equation for many-particle systems also possibly obtain wave functions for finding other observable quantities as the momenta of the particles. This study is possible to explain an economic system in the same way as a physical system, so it is also possible Schrödinger equation can describe economic systems using the solution of the wave function. Because of the wave function can explain the behavior of economic forms, for example, stock pricing.

2.2. Nonlinear Schrödinger equation without potential external potential as a stock pricing model
In quantum physics, the nonlinear Schrödinger equation is a nonlinear variation of the Schrodinger equation where this equation is a classical field equation with the main application being to propagate light in nonlinear optical fiber and planar waveguides. In general, the nonlinear Schrödinger equation appears as one of the universal equations that describe the evolution of slow quasi-monochromatic wave packets in weak nonlinear media that have dispersion. The nonlinear Schrödinger equation in classical field theory can be described as the following:

$$i\partial_t \psi = -\frac{1}{2} \partial_x^2 \psi + \kappa |\psi|^2 \psi,$$

where $\psi(x,t)$ is complex fields. This equation emerges from the Hamiltonian as follows:

$$H = \int dx \left[ \frac{1}{2} \partial_x \psi \right]^2 + \kappa \frac{|\psi|^4}{2},$$
with the Poisson equation is:

\[
\{\psi(x), \psi(y)\} = \{\psi^*(x), \psi^*(y)\} = 0 \tag{5}
\]

\[
\{\psi^*(x), \psi(y)\} = i\delta(x - y) \tag{6}
\]

This classical field equation never illustrates the evolution of time from a quantum state.

Based on this above study, a nonlinear Schrödinger equation will be proposed as a nonlinear and stochastic stock pricing model. For defining the stock price wave function, the wave function is defined again as \(\psi(S, t)\) where absolute square \(|\psi(S, t)|^2\) define the probability density function for the stock price, so terms of the stock price and time have the form:

\[
i \frac{\partial \psi(S, t)}{\partial t} = -\frac{1}{2}\sigma \frac{\partial^2 \psi(S, t)}{\partial S^2} + V(x)\psi(S, t) + \beta|\psi(S, t)|^2\psi(S, t) \tag{7}
\]

where \(i = \sqrt{-1}\) and \(V(S) + \beta|\psi(S, t)|^2\) describe the total potential energy while \(V\) express the external potential.

Equation (7) will be solved using an expansion method of the Jacobi elliptic function provided that the interest rate is low or \(\beta(r) \ll 1\), so equation (7) obtains a solution by changing into the form as follows:

\[
\psi(S, t) = \phi(\xi) \exp(i(kS - \omega t)), \tag{8}
\]

where \(\phi(\xi) \in \mathbb{R}\) is a function that depends on \(\xi = S - \sigma kt\), and \(k, \omega \in \mathbb{R}\) is a constant parameter. The parameter \(k\) is interpreted as the number of waves and the parameter \(\omega\) is interpreted as an angular frequency. Equation (8) will get the derivative and form of the equation as below:

\[
i \frac{\partial \psi(S, t)}{\partial t} = e^{i(kS - t\omega)} \left(\omega\phi(\xi) - ik\sigma \frac{\partial \phi(\xi)}{\partial \xi}\right) \tag{9}
\]

\[
-\frac{1}{2}\sigma \frac{\partial^2 \psi(S, t)}{\partial S^2} = \frac{1}{2}e^{i(kS - t\omega)} \sigma \left(k^2\phi(\xi) - 2ik \frac{\partial \phi(\xi)}{\partial \xi} - \frac{\partial^2 \phi(\xi)}{\partial \xi^2}\right) \tag{10}
\]

\[
\beta|\psi(S, t)|^2\psi(S, t) = e^{i(kS - t\omega) - 2Im(kS - t\omega)}\beta|\phi(\xi)|^2\phi(\xi) \tag{11}
\]

The symbol \(Im\) define an imaginary part of a complex number while \(k, S, \omega, t\) are real constants, therefore:

\[
Im(kS - t\omega) = 0 \tag{12}
\]

when \(\phi(S) > 0\) then the result is:

\[
|\phi(\xi)|^2\phi(\xi) = \phi(\xi)^3 \tag{13}
\]

Substitute equation (12) and (13) into equation (11) to obtain:

\[
\beta|\psi(S, t)|^2\psi(S, t) = e^{i(kS - t\omega)}\beta\phi(\xi)^3 \tag{14}
\]

After substitute equations (9), (10) and (14) became equation (7), the final equation is obtained as follows:

\[
\frac{\partial^2 \phi(\xi)}{\partial \xi^2} + \left(\omega - \frac{1}{2}\sigma k^2\right)\phi(\xi) - \beta\phi(\xi)^3 = 0
\]
The form of the nonlinear Schrödinger equation in above will be calculated using the fourth-order Runge-Kutta method for a second-order ordinary differential equation that it becomes:

\[
\frac{\partial^2 \phi(\xi)}{\partial \xi^2} = \beta \phi(\xi)^3 - \left( \omega - \frac{1}{2} \sigma k^2 \right) \phi(\xi)
\]  

(16)

In this study, equation (16) is considered as the factors that affect stock prices. The parameters of the nonlinear Schrödinger equation are analogous to economic parameters, where \( \phi \) is the movement of stock prices or the growth of average stock prices, \( \zeta \) (particle position on the quantum) is the price of basic assets or prices current stock, \( \omega \) is the average speed of stock returns, \( \sigma \) is volatility, \( k \) is the agreed stock price, and \( \beta \) is the adaptive market potential that depends on interest rates. Furthermore, the nonlinear Schrödinger equation is programmed using the Matlab programming language. The input values are determined by the initial to the final value of the stock pricing in the last week to predict the stock price in the following week. Thus, the \( \frac{\partial^2 \phi(\xi)}{\partial \xi^2} \) function is obtained as a result of the predicted stock price every week.

### Table 1. Analogy parameters between the nonlinear Schrödinger equation with economic parameters that affect stock prices based on the hypothesis and the economic theory

| The parameters of the Schrodinger equation | The parameters of economic |
|-------------------------------------------|-----------------------------|
| Schrodinger wave function (\( \phi \))     | the movement of stock prices or the growth of average stock prices |
| Standard Deviation (\( \sigma \))         | volatility                  |
| Wavenumber (\( k \))                      | the agreed stock price      |
| Angle frequency (\( \omega \))            | the average speed of stock returns |
| Coefficient of Landau (\( \beta \))       | the adaptive market potential that depends on interest rates |
| Position variable (\( \zeta \))           | the price of basic assets or prices current stock |

2.3. Numerical solution of the nonlinear Schrödinger equation

To solve the second-order ordinary differential equations of the nonlinear Schrödinger equation, these equations will be reduced to first-order ordinary differential equations that assumed by the function value \( \phi'(\xi = 0) = \phi_0 \):

\[
\phi(\xi) = f(\phi(\xi), \xi).
\]  

(17)

then a new function is introduced as follow:

\[
z(\xi) = \phi'(\xi).
\]  

(18)

From equation (17) and (18) can be defined as follow:

\[
z'(\xi) = f(\phi(\xi), \xi).
\]  

(19)

To solve the final step of the numerical solution can be defined as follow:
\[ \phi'(\xi) = z(\xi). \]  

(20)

3. Simulations and Results

The stock can be found in the world of capital markets where the stock of a company can change in price at any time. The movement of stock prices fluctuates greatly, the more fluctuations that occur in the stock price of a company then most likely the movement of stock prices will increase or decrease in the company. This fluctuation in stock price movements is the same as the nonlinear Schrodinger wave function because this wave function \( \phi \) has an irregular and freely moving particle-wave. This wave function can describe the movement or growth of average stock prices and also can predict the factors that affect stock prices.

The volatility is one of the factors that affect stock prices. The volatility is a statistical measurement for price fluctuations during a certain period. The volatility shows the measurement of the decline or increases in stock prices in a short time without measuring the price level. The wider the fluctuation range of stock prices, the higher the volatility and the possibility of rising or falling stock prices is very large, so that volatility greatly affects the development of stock prices. This is the same as the standard deviation \( \sigma \) which is one of the nonlinear Schrodinger equation variables that give effect to other parameters to get the constant value of the nonlinear Schrodinger equation so that the second-order derivative function of the Schrodinger equation can be generated.

In addition to volatility, there is a strike price or deal price as one of the factors affecting stock prices. The strike price is the price of an agreement or the price of an agreement between a seller and a buyer of shares. The lower the strike price in a company's stock, the greater the premium to be paid. A strike price can be analogous to wave numbers \( k \) which means scaling constants in the nonlinear Schrodinger equation. The scaling constant is a fixed variable that has been determined in the Yukawa potential. The Yukawa potential is a quantum potential that reflects the fact that the strong nuclear force carried by particles is about 200 times the mass of the electron mass.

The stock return is the result obtained from an investment. The stock return can be considered as a level of profit because it gets a return obtained from investment returns in a company. The average speed of stock returns greatly affects the returns of a company because the greater the speed of the company's average return it will get a big profit so that the stock price will increase. The average speed of stock returns is equal to the angular frequency \( \omega \) in the nonlinear Schrodinger equation because the angular frequency \( \omega \) is a scalar measure of the rotation rate and is expressed in radians per second. Angular frequency as the angular velocity that can determine the average angular acceleration. The averages angular acceleration is obtained from the ratio of the angular velocity change \( \Delta \omega \) to time. The angular frequency is very influential on the average angular acceleration or the average angle value. The higher the angular frequency or angular velocity, the greater the angular average value.

The other factors influence subsequent stock prices are adaptive market potential, this market potential depends on interest rates. The greater the interest rate, the stock price will go down or vice versa. The potential of adaptive markets is the analytical result of the Landau coefficient \( \beta \) in the nonlinear Schrodinger equation. There is a Landau Theory about the second-phase phase transition that emphasizes broken symmetry. If the symmetry is damaged or the composition of the symmetry is low, then the number of particles will be more organized and more particles will interact or vice versa. Factors affect the last stock price are the current stock price. The last stock price also affects the stock price in the company, because the position of the last stock price can determine the price of an upcoming stock to go up or down in price. This is the same as the position variable \( \xi \) in the nonlinear Schrodinger equation. This variable also acts as a "time" variable in numerical integration. This position variable can determine the exact position of particles in quantum.

The economic parameter constants in the two company shares in Table 2 and 3 explain that the constant value of the period January 30 - March 26, 2017, can change because it depends on the price of the company's shares each period. The interest rate is obtained from Bank Indonesia data. This value is determined at intervals of one year while the value of volatility is obtained from data of [www.investing.com](http://www.investing.com) [16]. This agreed value of the stock price is determined from calculations using a predetermined economic formula, then the value of the average speed of stock returns is determined to
start by calculating the value of stock returns on each company every day. The results of the calculation of the economic parameter constants are combined with the constant values of the nonlinear Schrödinger parameters obtained from the previous literature, these equations will be programmed using Matlab programming language for stock price predictions.

Table 2. Parameter value of economic constant on AALI company stocks period 30 January 2017 to 26 March 2017. Source: [http://investing.com](http://investing.com) [16]

| The parameters of economic | Value       |
|----------------------------|-------------|
| volatility                 | 265.625     |
| the agreed stock price     | 15391.026   |
| the average speed of stock returns | -0.001   |
| the adaptive market potential that depends on interest rates | 4.750 |

Table 3. Parameter value of economic constant on ADMG company stocks period 30 January 2017 to 26 March 2017. Source: [http://investing.com](http://investing.com) [16]

| The parameters of economic | Value       |
|----------------------------|-------------|
| volatility                 | 5.475       |
| the agreed stock price     | 141.461     |
| the average speed of stock returns | -0.002   |
| the adaptive market potential that depends on interest rates | 4.750 |

Tables 4 and 5 show the predicted value of the stock prices of the two companies for two months. The predicted value of the stock price is generated from the Matlab simulation using equation (16). The stock price predictions are conducted weekly, during the period of 30 January 2017 to 26 March 2017, where stock price data from Monday to Friday. The stock price data on Monday is considered the initial position of the stock price while data on Friday is considered as the last position of the stock price. On Saturday and Sunday, the stock price prediction is assumed to be the same as Friday because on those two days there was no economic activity especially stock trading. Therefore, every Saturday and Sunday the price value is considered the same as the price value on the last day, which is Friday. Besides, during holidays, the stock price will remain the same as the previous day's stock price.
Table 4. Prediction results of stock prices on Astra Agro Lestari Tbk. (AALI) for the period of 30 January 2017 to 26 March 2017 using the fourth-order Runge-Kutta method

| Period | Days          |                |                |                |                |                |                |
|--------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|
|        |               | February       |                |                |                |                |                |
|        |               | 1              | 2              | 3              | 4              | 5              | 6              | 7              |
| 1      | 15798.441     | 15796.882     | 15795.323     | 15793.764     | 15792.205     | 15792.205     | 15792.205     |               |
| 2      | 15998.441     | 15996.882     | 15995.323     | 15993.764     | 15992.205     | 15992.205     | 15992.205     |               |
| 3      | 15798.441     | 15796.882     | 15795.323     | 15793.764     | 15792.205     | 15792.205     | 15792.205     |               |
| 4      | 15223.441     | 15221.882     | 15220.323     | 15218.764     | 15217.205     | 15217.205     | 15217.205     |               |
|        |               | March          |                |                |                |                |                |                |
|        |               | 1              | 2              | 3              | 4              | 5              | 6              | 7              |
| 1      | 14973.441     | 14971.882     | 14970.323     | 14968.764     | 14967.205     | 14967.205     | 14967.205     |               |
| 2      | 15273.441     | 15271.882     | 15270.323     | 15268.764     | 15267.205     | 15267.205     | 15267.205     |               |
| 3      | 15198.441     | 15196.882     | 15195.323     | 15193.764     | 15192.205     | 15192.205     | 15192.205     |               |
| 4      | 15248.441     | 15246.882     | 15245.323     | 15243.764     | 15242.205     | 15242.205     | 15242.205     |               |

Table 5. Prediction results of stock prices on Polychem Indonesia Tbk. (ADMG) for the period of 30 January 2017 to 26 March 2017 using the fourth-order Runge-Kutta method

| Period | Days          |                |                |                |                |                |                |
|--------|---------------|----------------|----------------|----------------|----------------|----------------|----------------|
|        |               | February       |                |                |                |                |                |
|        |               | 1              | 2              | 3              | 4              | 5              | 6              | 7              |
| 1      | 147.441       | 145.88         | 144.323        | 142.764        | 141.205        | 141.205        | 141.205        |               |
| 2      | 146.441       | 144.88         | 143.323        | 141.764        | 140.205        | 140.205        | 140.205        |               |
| 3      | 135.441       | 133.88         | 132.323        | 130.764        | 129.205        | 129.205        | 129.205        |               |
| 4      | 138.441       | 136.88         | 135.323        | 133.764        | 132.205        | 132.205        | 132.205        |               |
|        |               | March          |                |                |                |                |                |                |
|        |               | 1              | 2              | 3              | 4              | 5              | 6              | 7              |
| 1      | 143.441       | 141.88         | 140.323        | 138.764        | 137.205        | 137.205        | 137.205        |               |
| 2      | 137.441       | 135.88         | 134.323        | 132.764        | 131.205        | 131.205        | 131.205        |               |
| 3      | 137.441       | 135.88         | 134.323        | 132.764        | 131.205        | 131.205        | 131.205        |               |
| 4      | 136.441       | 134.88         | 133.323        | 131.764        | 130.205        | 130.205        | 130.205        |               |

The results of stock price prediction using the nonlinear Schrodinger equation is calculated using the fourth-order Runge-Kutta method produces a prediction value that is not much different from the actual data and produces an error percentage of less than 10 %. Based on Table 6 and 7, the prediction results of Astra Agro Lestari Tbk's stock price (AALI) produces a value of MAPE of 0.463 %. Next, the prediction results of Polychem Indonesia Tbk's stock price (ADMG) produces the value of MAPE of 3.487 %. Based on the results of the error percentage in Tables 6 and 7, it can be concluded that the predicted value of the stock price is close to the actual data value using the nonlinear Schrodinger wave function ($\phi$).
Figure 1. Prediction results of the stock prices at Astra Agro Lestari Tbk. (AALI) period 30 January-26 March 2017.

Table 6. Comparison of the stock price prediction results with actual data of the stock price on Astra Agro Lestari Company's Tbk. (AALI) in February 2017 using the fourth-order Runge-Kutta method

| Date       | Prediction results | Actual data [16] | MAPE (%) |
|------------|--------------------|-----------------|----------|
| February 22| 15220.323          | 14775           | 0.463    |
| February 23| 15218.882          | 14700           | 0.463    |
| February 24| 15217.205          | 14975           | 0.463    |
| February 25| 15217.205          | 14975           | 0.463    |
| February 26| 15217.205          | 14975           | 0.463    |
| February 27| 14973.441          | 15600           | 0.463    |
| February 28| 14971.882          | 14950           | 0.463    |

Table 7. Comparison of the stock price prediction results with actual data of the stock price on Polychem Indonesia Tbk. (ADMG) in February 2017 using the fourth-order Runge-Kutta method

| Date       | Prediction results | Actual data [16] | MAPE (%) |
|------------|--------------------|-----------------|----------|
| February 22| 133.882            | 140             | 3.487    |
| February 23| 132.323            | 140             | 3.487    |
| February 24| 130.764            | 139             | 3.487    |
| February 25| 129.205            | 140             | 3.487    |
| February 26| 129.205            | 140             | 3.487    |
| February 27| 129.205            | 140             | 3.487    |
| February 28| 138.441            | 139             | 3.487    |

4. Conclusion
In this research, the nonlinear Schrödinger equation has been solved by the fourth-order Runge-Kutta method. This equation can be used to predict stock price fluctuations. The stock price simulation uses the numerical solution of the nonlinear Schrodinger equation yields good predictions. The accuracy of the stock price prediction results with actual data on the two company’s stock can be seen from the MAPE value. The stock price of Astra Agro Lestari Tbk (AALI) has a smaller MAPE value than the
The stock price of Polychem Indonesia Tbk. (ADMG), it is due to changes in the stock price of Astra Agro Lestari Tbk. (AALI) is very stable every week, so the predicted value approaches the actual value of the company's stock price.

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