Short-range Gravity experiment using digital image analysis

Kazufumi Ninomiya\textsuperscript{1,2}, Reiko Kishi\textsuperscript{1}, Haruna Murakami\textsuperscript{1}, Hironori Nishio\textsuperscript{1}, Naruya Ogawa\textsuperscript{1}, Atsushi Taketani\textsuperscript{2} and Jiro Murata\textsuperscript{1}

\textsuperscript{1}Department of Physics, Rikkyo University, Nishi-Ikebukuro, Tokyo 171-8501, JAPAN
\textsuperscript{2}RIKEN Nishina Center, 2-1 Hirosawa, Wako, Saitama 351-0198, JAPAN
E-mail: kazufumi@rikkyo.ac.jp

Abstract. According to a large extra dimension model, a deviation from Newton’s inverse square law is expected at below sub-millimeter range. We have developed an experimental set up using a torsion balance pendulum and an online digital-image analysis system, aiming to test the Newtonian inverse square law at below millimeter scale. In addition, composition dependence of gravitational constant $G$ is also tested at a millimeter scale, motivated to test the weak equivalence principle. In this paper, current status and results are described.

1. Introduction

1.1. Newtonian inverse square law

Extreme weakness of the gravitational force comparing to the other three gauge interactions is considered as one of the most severe problem in theoretical physics, which is known as the hierarchy problem. In order to build a unified theory including all four interactions, we cannot avoid resolving this problem. According to a large extra-dimension model which is known as the ADD model \cite{1}, the hierarchy problem can be naturally resolved in a geometrical understanding by introducing additional spatial dimensions outside of our four dimensional world in the brane world scenario. Astonishingly, the ADD model predicts that the size of the extra-dimension can be as large as a millimeter scale in order to set the high dimensional Planck energy to be around 1TeV. Existence of such additional spatial dimensions requires a modification of the gravitational inverse square law below the size of the extra dimension scale. For example, if we assume $n$ large extra dimensions with their size of $\lambda$, the gravitational potential can be expressed in a modified form because of Gauss’s law as followings;

\begin{align}
V_{r \geq \lambda} &= -G\frac{Mm}{r} \\
V_{r \leq \lambda} &= -G'\frac{Mm}{r^{1+n}},
\end{align}

here, higher dimensional gravitational constant is defined as $G' = \lambda^n G$. To compare with experimental tests of various gravitational models, a modified gravitational potential is often expressed in a Yukawa interaction form.
\[ V(r) = -\frac{GmM}{r^2} \left(1 + \alpha e^{-\frac{r}{\lambda}}\right), \]  

(2)

here, \(\alpha\) is coupling constant and \(\lambda\) is the range of the new interaction. Numbers of experiments have been tested the inverse square law, excluding the non-zero value of \(\alpha\) in various length scales. However, as shown in Figure 1, high precision tests have been performed in a very limited region around astronomical scales. Little is known at below millimeter range, where deviation from the inverse square law is predicted by the ADD model.

![Figure 1](image.png)

**Figure 1.** Summary of experimental search of the Yukawa term. Excluded region is illustrated as the shaded area [2].

1.2. Weak equivalence principle

We have developed a new type of experimental device using video imaging sensor to perform an experiment using a torsion balance pendulum to test the inverse square law at below millimeter scale. Our device can investigate not only the inverse square law, but also can test weak equivalence principle (WEP) using the same experimental technique. The weak equivalence principle is expressed as that, ratios between gravitational mass and inertia mass for every material are universal for any compositions. From the definition of the inertial mass \(a = F/m_I\) and that of gravitational mass \(F \propto m_g\), gravitational acceleration satisfies \(a \propto m_g/m_I\). If the ratio between gravitational mass and inertia mass is constant for every material, the universality of free fall must be kept. Therefore, test of the universality of free fall can be regarded as a test of the weak equivalence principle.

Number of experimental tests confirmed that ratios between inertial mass for different compositions are same for the ratio between gravitational mass, using gravitational force from the Earth and the Sun. On the other hand, there are no experimental tests of this weak equivalence principle at a short range below cm scale. On this motivation, we have performed an experiment to test the weak equivalence principle by testing the universality of the gravitational constant \(G\) for different compositions [3]. There are several theoretical models which predict violation of the weak equivalence principle [4, 5, 6, 7]. Baryon number coupling force is one of such models. If a new interaction which couples to baryon number, the weak equivalence principle seems to be violated. Because nuclear mass is very similar to the baryon number, it is difficult to
distinguish between them. However, the gravitational mass is slightly different from the baryon number time nucleon mass, because of the mass defect and proton/neutron mass difference. Therefore, existence of the new baryon number coupling force can be examined by measuring the gravity-like force between different substances.

2. Principle

2.1. Torsion pendulum

In our experiment, a gravitational force is measured using a torsion pendulum with an image analysis system to monitor the pendulum movement. Gravitational torque can be measured as an angular twisting displacement of the torsion balance bar, which can be associated with the torque in a form of Hooke’s law with a spring constant \( \kappa \).

\[
\tau = -\kappa \cdot \Delta \theta,
\]

here, \( \tau \) is torque and \( \kappa \) is torsional spring constant. The torsional spring constant \( \kappa \) is estimated using the inertia moment and periodic oscillation of the torsion pendulum. The torsion pendulum is composed of two columns suspended by an aluminum bar. This column is called as "target". The torsion pendulum is hanged by \( \phi 40 \mu m \) thin wire. A typical measuring procedure is shown in Figure 2. When the attractor position changes, balanced position between the spring force and gravitational force of the torsion pendulum is moved, resulting the angular displacement. The angular displacement of the pendulum is measured by a video camera viewing from the top of the apparatus. Figure 3 shows a typical time sequence data in such measurement, which clearly indicates the gravitational signal.

Figure 2. A typical measurement procedure, rotating the attractor position monitoring the angular displacement of the torsion pendulum.
2.2. Digital image analysis

We have developed a digital image analysis system for the position monitoring, which was originally developed for monitoring motion of the geometrical deformation of a large radiation detector which was used at a high energy collider experiment [8]. As shown in Figure 4, the video image is captured with a video camera, and stored as movie files recording the motion of the torsion pendulum. After then, image analysis is performed for all the static image frames extracted from the movie file. The angle information of the torsion pendulum can be extracted by performing a linear least square fitting on the pixel intensity data [9].

Angular resolution of around 1 micro degree is achieved using the image analysis system. This system can determine the angle and the offset position of the torsion pendulum at the same time, using the two dimensional information. It is a huge advantage comparing to a laser interferometer measurement which cannot distinguish the twisting motion from a parallel swinging motion without twisting, causing a big systematic error.

Figure 3. A typical time sequence data of the measurement like Figure 2.

Figure 4. Digital image analysis technique determining the position and the angle of a line shaped object.
3. Experiment

3.1. Test of the Newtonian inverse square law

Figure 5 shows the experimental setup of Newton-SC experiment, which aims to test the inverse square law at centimeter scales. In this study, we are aiming to explore the existence of the extra dimension based on the ADD model. In this sense, we are aiming to perform the highest precision test of the inverse square law at below millimeter scale. Therefore, Newton-SC experiment is a performance test of our experimental devices towards sub-millimeter test. All the components of the device are electrically connected and made of non-magnetic metals. In addition, all the apparatus are set inside a vacuum chamber. In order to suppress mechanical vibration noises, the apparatus is set on a basement room at Rikkyo University.

Unlike the measurement shown in Figure 2, the attractors are continuously rotating around the target in a constant angular velocity in this measurement. Twisted angle of the torsion balance bar is measured as a function of the setting angle of the rotating attractor position relative to the target. Result from the Newton-SC experiment is shown in Figure 6. As shown in the Figure 6, the experimental data were obtained as continuous points, which are compared with theoretical predictions. Red line shows result of a numerical calculation supposing the Newtonian gravity. Other lines show examples of numerical calculations supposing modified gravitational potentials which include the strong Yukawa potential with different parameter settings. As a result, our data are consistent with Newtonian gravity in this measurement within experimental precision. We also set an excluded area on the $\alpha - \lambda$ plot in Figure 7. The yellow line is set using our experimental data. We succeeded to test the Newton’s inverse square law at cm scale with 5% accuracy in this experiment.

![Figure 5](image_url)

**Figure 5.** The Newton-SC experimental setup. Electrostatic shield is set between the attractors and the targets.
Figure 6. Experimental result of the Newton-SC experiment.

Figure 7. $\alpha - \lambda$ plot at below centimeter scale. Present result is also shown. [10, 11, 12, 13, 14, 15]

Unfortunately, our experimental sensitivity was not enough to set a new limit comparing to the past measurements. It is mainly because our apparatus is aiming to confirm its ability to observe gravitational signal using the simple video image analysis system, which was not designed to achieve the highest precision measurement. We are now performing a next generation...
experiment using a Null-type principle in order to maximize the sensitivity on the deviation from the inverse square law suppressing the Newtonian gravity.

3.2. Test of the weak equivalence principle

Test of the weak equivalence principle can be performed as a test of composition dependence of the gravitational constant, which requires measurements with different attractors. In order to suppress systematic errors caused by attractor exchanging, a next generation device, Newton-II, is developed. As shown in Figure 8, Newton-II is designed to be able to move the attractors rotating outside the torsion balance bar position, without changing the attractor materials by opening the vacuum chamber. The result is shown in Figure 9. Present configuration is designed to suppress the systematic error and maximize the sensitivity to the relative strength of gravitational force for different materials. For example, target zero angle position and attractor angular phase can be determined from the measured data without dominated by mechanical alignment precision.

![Figure 8. The Newton-II experimental setup](image)

The Newton-II experiment is performed with continuously rotating attractors as same as for the Newton-SC experiment. Experimental results from the Newton-II experiment is shown in Figure 9. Contributions from the copper attractors and aluminum attractors are clearly seen as different nodes, which can be compared with the theoretical calculation assuming known gravitational constant shown as a green line in the Figure 9 obtained as superimpose from the two compositions. A relative strength between different attractor materials provides precision information about the composition dependence of the gravitational constant. The obtained results are consistent with the universal gravitational constant which is independent of the compositions.
Figure 9. Experimental result from the Newton-II experiment.

Figure 10. Allowed parameter region of the gravitational constant in the least square analysis.

In Figure 10, allowed region of the two parameters $G_{Al-W}$ (G between aluminum and tungsten) and $G_{Cu-W}$ (G between copper and tungsten) obtained in a least square analysis is shown. Best values from the analysis are obtained as:

$$G_{Al-W}/G_{Cu-W} = 1.009 \pm 0.010_{stat} \pm 0.048_{syst}$$
$$G_{Al-W}/G_{Newton} = 1.001 \pm 0.007_{stat} \pm 0.021_{syst},$$

here, $G_{Newton}$ is the known gravitational constant obtained from the value on the particle data book [16]. This result is consistent with the composition independence of the gravitational
constant within the present experimental precision. The obtained result also shows that the absolute value is consistent with the known gravitational constant. As same as for the new Yukawa interaction testing, we can set a similar constraint on a composition dependent new interaction. A new interaction coupling to the baryon number which violates the weak equivalence principle only at short range, can be parameterized in a new Yukawa term:

\[ V(r) = -\frac{GmM}{r^2} \left( 1 + \frac{\xi}{\mu_i \mu_j} e^{-\frac{r}{\lambda}} \right), \]

(4)

here, \( B_A \) is baryon number for material \( A \), and \( \mu_A \) is gravitational mass in hydrogen mass unit. Existence of the baryon number coupling force can be constrained by the present study as shown in Figure 11 as a \( \xi - \lambda \) plot. The upper limit of the new coupling constant \( \xi \) parameter is shown there. Figure 11 shows that we have succeeded to set the tightest upper limit of the \( \xi \) in a direct WEP test at the shortest range of below cm.

![Figure 11. \( \xi - \lambda \) plot testing composition dependent new Yukawa interaction. [17, 18, 19, 20](image)](image)

4. Conclusions
In the Newton-SC experiment, we performed to test the Newtonian inverse square law at the centimeter scale. As a result, our data are consistent with Newtonian gravity in this measurement within experimental precision. In addition, we performed a direct measurement of the composition dependence of the gravitational constant \( G \) in Newton-II experiment. The obtained result consistent with the composition independence of the gravitational constant within the present experimental precision. This result can also be interpreted as a short-range test of WEP. We gives the most precise limit on the baryon number coupling \( \xi_B \) at the shortest scale. We are starting a new experiment named Newton-IV. In this experiment, we will be able to test the inverse square law at below millimeter scale with the highest precision by applying a null-type measurement.

Acknowledgments
Part of the present works are performed as undergraduate student experiments. The author thanks to Y. Miyano, M. Takahashi, T. Tsuneno, T. Amanuma, S. Danbara, T. Iino, S. Mizuno,
Y. Araki, T.Ohmori, Y. Sakurai, S. Yamaoka, R. Tsutsui, M. Hata, T. Akiyama, Y. Ikeda, Y. Sekiguchi and K. Watanabe for their vital efforts on this work.

References

[1] Arkani-Hamed N, Dimopoulos S and Dvali G 1998 Physics Letters B 429 263–272
[2] Fischbach E and Talmadge C 1999 The search for non-Newtonian gravity (Springer Verlag) ISBN 0387984909
[3] Hata M et al. 2009 Journal of Physics: Conference Series vol 189 (IOP Publishing) p 012019
[4] Lee T and Yang C 1955 Physical Review 98 1501–1501 ISSN 0031-899X
[5] Damour T 1996 Classical and Quantum Gravity 13 A33
[6] Zhang Y, Luo J and Nie Y 2000 Arxiv preprint gr-qc/0006075
[7] Damour T, Piazza F and Veneziano G 2002 Phys. Rev. Lett. 89 81601 ISSN 1079-7114
[8] Murata J 2005 IEEE Nuclear Science Symposium Conference Record vol 675
[9] Ninomiya K et al. 2009 Journal of Physics: Conference Series vol 189 (IOP Publishing) p 012026
[10] Yang S, Zhan B, Wang Q, Shao C, Tu L, Tan W and Luo J 2012 Physical Review Letters 108 81101
[11] Hoyle C, Kapner D, Heckel B, Adelberger E, Gundlach J, Schmidt U and Swanson H 2004 Phys. Rev. D 70 42004 ISSN 1550-2368
[12] Kapner D, Cook T, Adelberger E, Gundlach J, Heckel B, Hoyle C and Swanson H 2007 Physical review letters 98 21101
[13] Smullin S, Geraci A, Weld D, Chiaverini J, Holmes S and Kapitulnik A 2005 Physical Review D 72 122001
[14] Long J, Chan H, Churnside A, Gulbis E, Varney M and Price J 2003 Nature 421 922–925
[15] Hoskins J, Newman R, Spero R and Schultz J 1985 Phys. Rev. D 32 3084–3095 ISSN 1550-2368
[16] PDG 2001 Eur. Phys. J
[17] Smith G, Hoyle C, Gundlach J, Adelberger E, Heckel B and Swanson H 1999 Phys. Rev. D 61 22001 ISSN 1550-2368
[18] Su Y, Heckel B, Adelberger E, Gundlach J, Harris M, Smith G and Swanson H 1994 Phys. Rev. D 50 3614–3636 ISSN 1550-2368
[19] Schlamminger S, Choi K, Wagner T, Gundlach J and Adelberger E 2008 Phys. Rev. Lett. 100 41101 ISSN 1079-7114
[20] Adelberger E, Stubbs C, Heckel B, Su Y, Swanson H, Smith G, Gundlach J and Rogers W 1990 Physical Review D 42 3267