Noncontact diagnostics of remote anisotropic plasmas

A S Mustafaev and A Y Grabovskiy

Department of General and Technical Physics, Saint Petersburg Mining University, Saint Petersburg 199106, Russia

Abstract. A new noncontact method for diagnostics of nonequilibrium plasma that integrates technologies of optical-and-magnetic technique of the Hanley effect and those of probe investigations is proposed. The method mentioned has been experimentally tested in plasmas with cardinally dissimilar structure of the electron velocity distribution function. As a model plasma object the positive column of glow discharge has been studied and as a distant object a short low-voltage beam discharge. The following potentialities of this method were illustrated, viz. in the distant object the complete electron velocity distribution function was determined and the diagram of electron pressure angular anisotropy was constructed. Verification of the method accuracy was carried out using the technique of flat single-sided probe.

1. Introduction

Progress of plasma high technologies in many ways is provided by achievements in development of new diagnostic methods, their efficiency, reliability and possibility of application in realistic conditions of plasma facilities [1, 2]. At present, optical techniques are represented widely by noncontact diagnostics however none of them feed researchers with information on local characteristic of nonequilibrium plasma, its electron velocity distribution function (EVDF) and angular data.

The flat single-sided probe technique [2] differs by microcausality and ensures EVDF detection even in such severe environment as near-electrode discharge area, hostile plasma environment of thermionic energy convertors and switching devices, etc. And still, it makes it impossible to study remotely distant plasma objects. Moreover, a flat probe is difficult to make. That is why cylindrical probes are most commonly used in world experimental practice.

In [3] the authors give evidence of a cylindrical probe to restore even components $f_2$ of EVDF expansion in terms of Legendre polynomials only and show the developed technique of reconstruction of a complete EVDF by a joint use of experimental data of the cylindrical probe and a system of “linking” equations found as a result of the kinetic Boltzmann equation.

The authors of the present paper propose a new method of remote diagnosis of nonequilibrium plasma that combines the merits of microcausality and simplicity of the cylindrical probe technique [3] and remotability of the optical one [4].

2. Experimental facilities and technique

To carry out experimental research the set-up described in [3] (Figure 1) has been used.

The glow discharge (block I, Figure.1) was generated in a quartz tube (1) with 3 cm in diameter and 30 cm in length between a flat cathode 10 mm in diameter (2) and anode of the same size (3). Barium-impregnated cathode was heated by an electron gun (4) whereas a spiral made from tantalum-niobium wire 0.5 mm in diameter served as a heater. Then through a 1 mm wide gash a cylindrical probe, made of molybdenum wire with 1 mm in length and 0.07 mm in diameter was inlet into a
discharge (5). The probe was strengthened on a special 3D micrometer system that supported its spatial and angular motion with an accuracy of ±0.01 mm. The noneffective part of the probe was insulated by a capillary and ceramic tube made of beryllium oxide. The surface of probe holder was insulated by alundum ceramics.

A low-voltage beam discharge (LVBD) was generated in a diode device [5] the discharge gap of which was formed between a flat cathode with 1.5 mm in thickness and 10 mm in diameter and a flat molybdenum cathode of a similar geometry. The interelectrode gap could change within the range of 1-80 mm. The discharge lateral face was bounded to a metallic shield with adjustable potential, its diameter being 11 mm.

All experimental devices were place into a vacuum chamber (block II, Figure 1) with an inner diameter of 160 mm and length of 0.5 m. Smooth supplying of the spectrally pure gas into the chamber was provided by a system of reducers and a pin inlet. The vacuum in the system was made by turbomolecular pumps (block III, Figure 1) which provided a limit rarefaction of 5×10⁻⁹ Torr.

The block IV illustrates of optical system.

To obtain the values of the second derivative of the probe current with respect to the probe potential the PC-based measuring and computing complex was used, where 100% modulated voltage \( \Delta U=U_0(1+\cos \omega t)\cos \omega t \) was used as a differentiating signal (demodulation method) [6].

Optical measurements were made with the use of optical-and-magnetic technique of the Hanley effect [4], based on dependence of the degree of polarization of spontaneous emission on the intensity value of the transverse magnetic field. The intensity was selected in such a way so that particle kinetic characteristics in plasma remained unaltered, but it was sufficient to break up transverse components of alignment of atom quantum states.

3. The method of cylindrical probe

Let us consider a cylindrical probe arbitrarily oriented in axisymmetric plasma. Let \( \lambda \) be the angle between the plasma symmetry axis and the probe axis. We introduce an auxiliary spherical coordinate system in which the polar axis is perpendicular to the plane of the probe axis and plasma symmetry axis (Figure 2). The direction of the plasma symmetry axis in the auxiliary coordinate system is characterized by the polar and azimuth angles \( \theta = \pi / 2 \) and \( \phi = 0 \); the direction of the normal of a certain element of the probe surface, by the angles \( \theta_2 \) and \( \phi_2 = (\lambda \pm \pi / 2) \). The angle between these directions is designated as \( \Phi_0 \).

\[
\cos \Phi_0 = \sin \theta_2 \cos \phi_2.
\]  

Supposing that the EVDF \( f(\varepsilon, \theta) \) and second derivative of the probe current with respect to the probe potential \( I''_U(qU, \Phi_0) \) are twice differentiable with respect to the angle, we expand them into series in terms of the Legendre polynomials:

\[
f(\varepsilon, \theta) = \sum_{j=0}^{\infty} f_j(\varepsilon)P_j(\cos \theta),
\]  

\[
I''_U(qU, \Phi_0) = \frac{2\pi q^3 S}{m^2} \sum_{j=0}^{\infty} F_j(qU)P_j(\cos \Phi_0).
\]

Using relation (1), we integrate expression (3) over the surface \( S \) of the cylindrical probe:

\[
I''_U(qU, \Phi_0) = \frac{2q^3 S}{m^2} \sum_{j=0}^{\infty} F_{2j}(qU) \int_{0}^{\pi} P_{2j}(\sin \theta_2 \sin \lambda) d\theta_2,
\]  

where \( S \) is the surface of the probe area.

It is seen from equation (4) that components with odd indices come out when integrating over the probe surface. This is connected with the fact that one can always find for a cylindrical probe two surface elements the normals to which constitute an angle of 180° by virtue of the probe symmetry.
For Legendre polynomials with odd indices, $P_{2j+1}(\cos \Phi_0) = -P_{2j+1}(\cos(180^\circ + \Phi_0))$, for this reason, only components with even indices $F_{2j}$ remain in expression (4).

Therefore, the cylindrical probe makes it possible to determine only even components of the EVDF expansion and the number of even components determined is connected with the number of independent orientations of the probe. Thus, measurements with a cylindrical probe do not permit one to determine the Legendre component $f_1$ and make conclusions about the current anisotropy of the plasma.

To solve this problem, authors proposed the cylindrical probe method, based on joint use of experimental data and the kinetic Boltzmann equation which has the following form for the electron distribution function in an electric field

$$
\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} + \frac{eE}{m} \cdot \frac{\partial f}{\partial \mathbf{v}} + S = 0,
$$

where $e$ and $m$ are the electron charge and mass, respectively, $\mathbf{E}$ is the electric field strength, and $S$ is the collision integral.

Let us substitute expansion (2) into equation (5):

$$
\sum_{j=0}^{\infty} P_j(\cos \theta) \frac{\partial f_j}{\partial t} + \mathbf{v} \cos \theta \sum_{j=0}^{\infty} P_j(\cos \theta) \frac{\partial f_j}{\partial \mathbf{r}} - \frac{eE}{m} \cos \theta \times

\times \sum_{j=0}^{\infty} P_j(\cos \theta) \frac{\partial f_j}{\partial \mathbf{v}} - \frac{eE}{mv} \sin^2 \theta \sum_{j=0}^{\infty} f_j \frac{\partial P_j}{\partial (\cos \theta)} + S = 0. \tag{6}
$$

Here, it was taken into account that

$$
\frac{\partial E}{\partial \mathbf{v}} = E \cos \theta \frac{\partial f}{\partial \mathbf{v}} + \frac{E \sin^2 \theta}{\mathbf{v}} \frac{\partial f}{\partial (\cos \theta)}.
$$
Multiplying equation (6) by the corresponding polynomial $P_j(\cos \theta)$ and integrating with respect to angular coordinates in the velocity space $d\Omega = \sin \theta d\theta d\phi$, we obtain the following equation for the functions $f_j$:

$$\frac{\partial f_j}{\partial t} + v \frac{\partial}{\partial z} \left\{ \frac{j + 1}{2j + 3} f_{j+1} + \frac{j}{2j - 1} f_{j-1} \right\} - eE \frac{1}{m} \frac{\partial}{\partial v} \left( v^{j+2} f_{j+1} \right) + \frac{j}{2j - 1} \frac{1}{v^j} \frac{\partial}{\partial v} \left( v^{-j} f_{j-1} \right) = v^\prime_{ea} f_j, \quad j = 0, 1, 2, \ldots, \tag{7}$$

where the collision integral takes into account the predominating role of elastic electron-atom collisions in terms of the frequency of elastic collisions $v^\prime_{ea}$.

In (7), the orthogonality of the polynomials

$$\int_{-1}^{+1} P_j(x) P_l(x) dx = \frac{2}{2j + 1} \delta_{jl}$$

and recurrence relations

$$xP_j(x) = \frac{j + 1}{2j + 1} P_{j+1} + \frac{j}{2j + 1} P_{j-1},$$

$$(1 - x^2) \frac{dP_j}{dx} = \frac{j(j + 1)}{2j + 1} \left[ P_{j-1}(x) - P_{j+1}(x) \right]$$

are taken into account. Subsequently setting $j = 0, 1, 2, \ldots n$ we obtain the following chain of equations:

$j = 0$

$$\frac{\partial f_0}{\partial t} + v \frac{\partial f_1}{\partial z} - \frac{eE}{3m v^2} \frac{\partial}{\partial v} \left( v^2 f_1 \right) + v^\prime_{ea} f_0 = 0; \tag{8}$$

$j = 1$

$$\frac{\partial f_1}{\partial t} + v \left( \frac{\partial f_0}{\partial z} + \frac{2}{5} \frac{\partial f_2}{\partial z} \right) - \frac{eE}{m} \left[ \frac{\partial f_0}{\partial v} + \frac{2}{5v^3} \frac{\partial}{\partial v} \left( v^3 f_2 \right) \right] + v^\prime_{ea} f_1 = 0; \tag{9}$$

and so on.

This chain of equations can be truncated at the first two if the function $f_2$ is negligible as compared to $f_0$, i.e., if

$$\frac{\partial f_0}{\partial v} \gg \frac{2}{5v^3} \frac{\partial}{\partial v} \left( v^3 f_2 \right), \quad \frac{\partial f_0}{\partial z} \gg \frac{2}{5} \frac{\partial f_2}{\partial z}.$$  

Solution of kinetic equations (8) and (9) allows one to reconstruct Legendre EVDF components with odd indices; for this purpose, it is necessary to measure even components experimentally.

Even components of the EVDF expansion are determined using the mathematical apparatus of the method [2, 6] permitting one to connect Legendre EVDF components with values of $I_U^*(qU, \Phi_0)$:

$$f_{2j}(qU) = F_{2j}(qU) + \int_{qU}^{\infty} f_{2j}(\varepsilon) \frac{\partial}{\partial (qU)} P_{2j} \left( \sqrt{\frac{qU}{\varepsilon}} \right) d\varepsilon. \tag{10}$$

Expression (10) is a Volterra integral equation of the second kind [7]. Using the resolvent method, it can be resolved with respect to functions $f_{2j}$:
\[
f_{2j}(qU) = F_{2j}(qU) + \int_{qU}^{\infty} F_{2j}(\varepsilon) R_{2j}(qU, \varepsilon) d\varepsilon, \quad j=0,1,2,\ldots, (11)\]

where \( R_j(qU; \varepsilon) = \frac{2-(j+1) \left( \frac{j}{qU} \right)}{qU} \sum_{k=0}^{j-2k-1} d_{kj} \left( \frac{\varepsilon}{qU} \right)^k \), \( a_{kj} = (-1)^k \frac{(2j-k)!(j-k)}{k!(j-k)!} \), for \( j=0, 2, 4 \ldots \)

An explicit form of resolvent kernels for \( j \) is:

\[
\text{The second way is to restrict the number of summands in series (4) as follows:}
\]

Note that the integral over energy in (11) has a singularity at zero. For this reason, the integration is performed from a certain minimum value of energy \( U = U_{\text{min}} \) over the entire interval \([U_{\text{min}}, U_{\text{max}}]\) beyond which the quantity \( I_j^* \) turns out to be less than the sensitivity threshold of devices of the experimental setup and is taken to be zero.

Components \( F_{2j} \) can be determined in two ways. The first way directly involves the representation of components \( F_{2j} \) in the expansion of \( I_j^*(qU, \Phi_0) \) (4) from which we obtain

\[
F_{2j}(qU) = \frac{(2j+1)m^2}{4q^3 S} \int_{-1}^{1} S_N(x) P_{2j}(x) dx = \frac{(2j+1)m^2}{4q^3 S} \sum_{k=0}^{N} A_k^{(j)} I_j^*(x_k), (12)
\]

In so doing, the integral in (12) is replaced by a quadrature formula. If we consider \( P_{2j}(\cos \Phi_0) \) as a weight function and use a quadrature formula with an algebraic degree of accuracy that is not lower than the interpolation accuracy, we obtain

\[
F_{2j} \approx \frac{(2j+1)m^2}{4q^3 S} \int_{-1}^{1} S_N(x) P_{2j}(x) dx = \frac{(2j+1)m^2}{4q^3 S} \sum_{k=0}^{N} A_k^{(j)} I_j^*(x_k),
\]

where \( x \) is the angular variable, \( S_N(x) = \sum_{i=1}^{N} \frac{\omega(x)}{(x-x_i) \omega(x)} I_j^*(x_i) \) - is the interpolation Lagrange polynomial, and \( A_k^{(j)} = \int_{-1}^{1} \frac{\omega(x)}{(x-x_k) \omega(x)} P_{2j}(x) dx \).

Applying the same quadrature formula with a constant weight function equal to unity yields

\[
F_{2j} \approx \frac{(2j+1)m^2}{4q^3 S} \sum_{i=1}^{N} C_i I_j^*(x_i) P_{2j}(x_i), \quad \text{where} \quad C_i = \int_{-1}^{1} \frac{\omega(x)}{(x-x_i) \omega(x)} dx.
\]

The second way is to restrict the number of summands in series (4). In this case, measurements \( I_j^*(qU, \Phi_0) \) are performed for a definite number of angles \( \Phi_0 \). Then, to find the auxiliary functions \( F_{2j} \), it is necessary to solve a system of linear equations that can be formed from (4) due to using the chosen angles of probe orientation.

Thus, the method of reconstruction of the full EVDF by use of a cylindrical probe is as follows:

1. Values of \( I_j^*(qU, \Phi_0) \) are measured experimentally;
2. Values of \( F_{2j} \) and \( f_{2j} \) are subsequently calculated according to equations (12) and (11);
3. Then, odd EVDF components \( f_{2j+1} \) are reconstructed by solving corresponding kinetic equations (8) and (9);
4. The full electron velocity distribution function is finally reconstructed using equation (2).
4. Reconstruction of Anisotropic EVDF in Distant Plasma Objects

According to [4], the degree of linear polarization emission $P$ of atom ensemble is measured by polarization moments of this ensemble, viz

$$P = f\left(\rho_2 / \rho_0\right).$$

(13)

Moments of various classes $\rho_k$ are responsible for varied types of polarization of light emitted by atoms. Thus, linear polarization is determined by the second class moment, viz. by a spherical tensor of alignment $\rho_2$, components of which characterize atomic excitation anisotropy by direct electron impact from the ground state. The value of $\rho_2$ is determined by $f_2$ Legendre components of EVDF:

$$\rho_2 = \frac{N_n n_e}{\gamma_2^{\ast}} \int_{E_m}^{\infty} Q_2(e, e_{lim}) f_2(e) de,$$

(14)

where $\gamma_2$ is the alignment relaxation constant determined by relationships [4]. Constant $\gamma_2$ depends on the radiative lifetime of the given atomic state and on binary collisions; $e_{lim}$ is the threshold energy for the excitation of a given spectral line; $Q_2$ is the alignment cross-section of total angular moments of excited by electron impact helium atoms. The ground state population level $\rho_0$ is determined by component $f_0$:

$$\rho_0 = \frac{N_n n_e}{\gamma_0^{\ast}} \int_{E_m}^{\infty} Q_0(e, e_{lim}) f_0(e) de,$$

(15)

where $Q_0$ is the cross-section of excited by electron impact atoms; $\gamma_0$ is the natural decay constant, associated with the destruction of the helium $4\ensuremath{^4}\ensuremath{D}_2$ state population.

If components $f_0$ and $f_2$ have been measured and degrees of polarization $R$-th and $T$-th spectral lines are known, then the following system of equations is obtained for the plasma object with the known atomic structure, viz.

$$P_R = \int_{E_{th}^R}^{\infty} \int_{E_{lim}^R}^{\infty} Q_0(e, e_{lim}) f_0(e) de = \int_{E_{th}^R}^{\infty} \int_{E_{lim}^R}^{\infty} Q_2(e, e_{lim}) f_2(e) de,$$

$$P_T = \int_{E_{th}^T}^{\infty} \int_{E_{lim}^T}^{\infty} Q_0(e, e_{lim}) f_0(e) de = \int_{E_{th}^T}^{\infty} \int_{E_{lim}^T}^{\infty} Q_2(e, e_{lim}) f_2(e) de$$

(16)

Solution of the system (16) enables restoring excitation cross section $Q_0$ and alignment cross section $Q_2$ that are invariant by the type of plasma under investigation.

Accordingly, algorithm of the suggested method lies in the following, viz.

1. In the distant plasma object under study degrees of polarization $P_R$ and $P_T$ of two spectral lines of spontaneous emission are detected and its atomic structure is determined.
2. In laboratory conditions a model plasma with the same atomic structure as a distant object is produced and investigated as follows:
   - EVDF components $f_0$, $f_1$ and $f_2$ are reconstructed while using a cylindrical probe method [3];
   - simultaneously, the degrees of polarization of the same two spectral lines of spontaneous emission are measured as well as those of the distant object. Then from the system of equations (16) invariant atomic constants $Q_0$ and $Q_2$ are reconstructed.
3. Diagnostics of the plasma of the distant object is accomplished. Using the earlier determined atomic constants $Q_0$ and $Q_2$, the system of equations (16) and cylindrical probe method EVDF components $f_0$, $f_1$, $f_2$, $f_3$, ..., $f_n$ are restored, as well as relative angular anisotropy of electron pressure is defined too. The number of restored EVDF components depends on plasma anisotropy [2].

The proposed method has been put to an evaluation test with experiment. As a laboratory model object the positive column of helium glow discharge has been studied, the EVDF structure of which is well-known [6]. Using the cylindrical probe method, Legendre components $f_0$, $f_1$ and $f_2$ were reconstructed (Figure 3), in unison with spectrometric measurements of degrees of polarization of He
lines $\lambda_R = 4922 \text{ Å}$ and $\lambda_T = 6678 \text{ Å}$. As a result energy dependence of the alignment cross-section $Q_2$ was reconstructed (Figure 4).

**Figure 3.** Energy dependence of EVDF Legendre components in plasma of positive column (relative units). $P_{He} = 1$ torr, $i_p = 0.5$ A.

**Figure 4.** Energy dependence of the alignment cross-section $Q_2$, $P_{He}=0.5$ torr; $i=0.2$ A; $\lambda_R=4922$ Å.
The distant plasma object was represented by helium LVBD of low pressure, EVDF of which differed greatly from EVDF of the glow discharge by strong nonequilibrium and anisotropy [2, 3, 5]. The degrees of polarization of lines $\lambda_R$ and $\lambda_T$ were measured. The magnetic-polarization contours of the Hanle signal in the LVBD plasma at different currents are presented in Figure 5. Then the full distribution function for electrons of different energies (Fig. 6a, b) was restored with the use of invariant atomic constants $Q_0$ and $Q_2$.

**Figure 5.** The contours of the Hanley signal for the He 4922Å line in a LVBD at the pressure $P_{He} = 0.32$ torr and different discharge currents $i_p$, A: 1 – 0.07; 2 – 0.12; 3 – 0.2; 4 – 0.3.

**Figure 6.** Energy dependence of the first three Legendre components of EVDF in plasma of LVBD (relative units). $P_{He}=1.5$ torr, $i_p=0.25$ A, $U_a=30$ V
Besides polarizing moments, a number of other essential plasma parameters [2] are connected with Legendre components of EVDF. In particular, $f_0$ and $f_2$ define quantities of scalar electron pressure

$$P_0 = \frac{8\pi \sqrt{2}}{3m^{3/2}} \int_0^\infty e^{3/2} f_0(e)de,$$

and those of anisotropic pressure component

$$P_2 = \frac{8\pi \sqrt{2}}{3m^{3/2}} \int_0^\infty e^{3/2} f_2(e)de,$$

that are diagonal elements of tensor of momentum flow density of electron $\Pi_{ij}$, viz.

$$\Pi_{ij} = m \int \mathbf{v}_i \mathbf{v}_j f(\mathbf{v})d\mathbf{v} = \begin{pmatrix} P_0 - \frac{1}{5} P_2 & 0 & 0 \\ 0 & P_0 - \frac{1}{5} P_2 & 0 \\ 0 & 0 & P_0 + \frac{2}{5} P_2 \end{pmatrix}.$$

While using reconstructed components $f_0$ and $f_2$ relative anisotropy of electron pressure $P_2/P_0$ has been defined in the plasma of a distant object and chart of the angular anisotropy (Figure 7) has been constructed.

![Figure 7](image_url)

**Figure 7.** Chart of angular anisotropy of electron pressure for electrons of different energies. $P_{He}=2$ torr, $i_p=0.15$ A, $U_a$ ranged between 2 and 30 V.

Component $f_1$ enables calculating electron current intensity $i_d$ in plasma:

$$i_d = \frac{4\pi}{3} n_e(0) \int_0^R \int_0^\pi \mu(r) \nu^3 f_1(\nu)drd\nu,$$

(17)
where \( \mu(r) \) - radial dependence of the electron relative density \( \mu(r) = \frac{n_e(r)}{n_e(0)} \); \( n_e(0) \) – electron density on the discharge axis.

We can to verify accuracy of the method if the values experimentally measured and those that have been estimated are congruent. Close fit has been made between estimated (17) and experimental data, the divergence between two results not exceeding 10%.

One of the most significant merits of the proposed method is recovery capability of complete EVDF in plasma which is impracticable for contact investigations.

References

[1] Mustafaev A S, Strakhova A A 3D-diagnostics of Function of Electron Distribution in Plasma 2017 Zapiski Gornogo instituta 226 462
[2] Mustafaev A S 2013 Electron Velocity Distribution Function in Anisotropic Plasmas (Saint-Petersburg: Saint Petersburg Mining University) p 171
[3] Mustafaev A S, Grabovskiy A Y New Possibilities of a Cylindrical Probe in Gas-Discharge Plasma 2015 High Temperature 53 347
[4] Kazantsev S A, Luchinkina V V, Mezentsev A P, Mustafaev A S, Rebane V N, Rys’ A G, Stepanov Y L Depolarization of the 4^1D_2 state of a helium atom by charged particles in beam plasma discharge 1994 Optics and Spectroscopy 76 809
[5] Mustafaev A S Dynamics of Electron Beams in Plasma 2001 Technical Physics 46 472
[6] Mustafaev A S, Grabovskiy A Y Probe Diagnostic of an Anisotropic Distribution Function of Electrons in Plasma (Review) 2012 High Temperature 50 785
[7] Volterra V 1959 Theory of Functionals and of Integral and Integro-Differential Equations (New York: Dover) p 288