Kilohertz Quasi-Periodic Oscillations, Magnetic Fields and Mass of Neutron Stars in Low-Mass X-Ray Binaries

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ABSTRACT

It has recently been suggested that the maximum observed quasi-periodic oscillation (QPO) frequencies, $\nu_{\text{max}}$, for several low-mass X-ray binaries, particularly 4U 1820-30, correspond to the orbital frequency at the inner-most stable orbit of the accretion disk. This would imply that the neutron stars in these systems have masses $\gtrsim 2M_\odot$, considerably larger than any well-measured neutron star mass. We suggest that the levelling off of $\nu_{\text{QPO}}$ may be also understood in terms of a steepening magnetic field which, although possibly dipolar at the stellar surface, is altered substantially by disk accretion, and presents a “wall” to the accretion flow that may be outside the innermost stable orbit. General relativistic effects add to the flattening of the $\nu_{\text{QPO}} - \dot{M}$ relation at frequencies below the Kepler frequency at the innermost stable orbit. We offer two other possible ways to reconcile the low value of $\nu_{\text{max}}$ ($\approx 1060$ Hz for 4U 1820-30) with a moderate neutron star mass, $\approx 1.4M_\odot$: at sufficiently large $\dot{M}$, either (i) the disk terminates in a very thin boundary layer near the neutron star surface, or (ii) $\nu_{\text{QPO}}$ is not the orbital frequency right at the inner edge of the disk, but rather at a somewhat larger radius, where the emissivity of the disk peaks.

Subject headings: accretion, accretion disks – stars: neutron – X-rays: stars – gravitation – stars: magnetic fields

1. Introduction

Recent observations with Rossi X-ray Timing Explorer (RXTE) have revealed kilohertz quasi-periodic oscillations (QPOs) in at least eighteen low-mass X-ray binaries (LMXBs; see Van der Klis 1998a,b for a review; also see Eric Ford’s QPO web page at http://www.astro.uva.nl/ecford/qpos.html for updated information). These kHz QPOs are characterized by their high levels of coherence (with $\nu/\Delta \nu$ up to 100), large rms amplitudes (up to 20%), and wide span of frequencies (500 – 1200 Hz). In almost all sources, the X-ray power spectra show twin kHz peaks moving up and down in frequency together as a function of photon count rate, with the separation frequency roughly constant (The clear exceptions are Sco X-1 and
4U 1608-52, van der Klis et al. 1997, Mendez et al. 1998a; see also Psaltis et al. 1998. In Aql X-1, only a single QPO has been detected.). Moreover, in several sources, a third, nearly coherent QPO has been detected during one or more X-ray bursts, at a frequency approximately equal to the frequency difference between the twin peaks or twice that value. (An exception is 4U 1636-53, Mendez et al. 1998b.) The observations suggest a generic beat-frequency model where the QPO with the higher frequency is associated with the orbital motion at some preferred orbital radius around the neutron star, while the lower-frequency QPO results from the beat between the Kepler frequency and the neutron star spin frequency. It has been suggested that this preferred radius is the magnetosphere radius (Strohmayer et al. 1996) or the sonic radius of the disk accretion flow (Miller, Lamb and Psaltis 1998; see also Kluzniak et al. 1990). The recent observational findings (e.g., the variable frequency separations for Sco X-1 and 4U 1608-52) indicate that the “beat” is not perfect, so perhaps a boundary layer with varying angular frequencies, rather than simply the neutron star spin, is involved.

This paper is motivated by recent RXTE observation of the bright globular cluster source 4U 1820-30 (Zhang et al. 1998), which has revealed that, as a function of X-ray photon count rate, $\dot{C}$, the twin QPO frequencies increase roughly linearly for small photon count rates ($\dot{C} \approx 1600 - 2500$ cps) and become independent of $\dot{C}$ for larger photon count rates ($\dot{C} \approx 2500 - 3200$ cps). (The QPOs become unobservable for still higher count rates.) It was suggested that the $\dot{C}$ – independent maximum frequency ($\nu_{\text{max}} = 1060 \pm 20$ Hz) of the upper QPO corresponds to the orbital frequency of the disk at the inner-most stable circular orbit (ISO) as predicted by general relativity. This would imply that the NS has mass of $2.2M_\odot$ (assuming a spin frequency of 275 Hz). It has also been noted earlier (Zhang et al. 1997), based on the narrow range of the maximal QPO frequencies ($\nu_{\text{max}} \approx 1100 - 1200$ Hz) in at least six sources (which have very different X-ray luminosities), that these maximum frequencies correspond to the Kepler frequency at the ISO, which then implies that the neutron star masses are near $2M_\odot$ (see also Kaaret et al. 1997).

The neutron star masses inferred from identifying $\nu_{\text{max}}$ with the Kepler frequency at the ISO would, if confirmed, be of great importance for constraining the properties of neutron stars and for understanding the recycling processes leading to the formation of millisecond pulsars. However, while it is tempting to identify $\nu_{\text{max}}$ with the orbital frequency at the ISO, this seemingly natural interpretation may not be true. One clue that this identification may not be correct is that the inferred neutron star masses are substantially above the masses of those neutron stars for which accurate determinations are available (Thorsett & Chakrabarty 1999) even though spin-up to $\nu_s \sim 300$ Hz only requires accretion of a very small amount of material ($\ll M_\odot$; §2). The cause of the flattening of the $\nu_{\text{QPO}} - \dot{C}$ correlation, and the value of the maximum frequency, are still not understood (and the existence of a plateau in $\nu_{\text{QPO}}$ with increasing $\dot{M}$ is debatable; e.g. Mendez et al. 1998c). We suggest in §3 that the steepening of the magnetic field, expected near the accreting neutron star, together with general relativistic effect, naturally leads to the flattening in the $\nu_{\text{QPO}} - \dot{M}$ correlation. In §4 we advocate two alternative interpretations of the
maximum QPO frequency without invoking excessively large neutron star masses.

2. Possible Problems with Neutron Star Masses $\gtrsim 2 \, M_\odot$

The most important concern for the inferred neutron star mass of $\gtrsim 2 M_\odot$ is an empirical one. LMXBs have long been thought (e.g., Alpar et al. 1982) to be the progenitors of binary millisecond radio pulsars. The recent discovery of binary X-ray pulsar SAX J1808-3658 (with spin period 2.5 ms and orbital period 2 hrs; Wijnands & van der Klis 1998; Chakrabarty & Morgan 1998) appears to confirm this link. Measurements of neutron star masses in radio pulsar binaries give values in a narrow range around $M \simeq 1.4 M_\odot$; the data are consistent with a neutron star mass function that is flat between $\gtrsim 1.1 M_\odot$ and $\lesssim 1.6 M_\odot$ at 95% CL (Thorsett & Chakrabarty 1999, Finn 1994). The masses of neutron stars in X-ray binaries are also consistent with $M \simeq 1.4 M_\odot$ (e.g., van Kerkwijk et al. 1995). Of particular interest is the 5.4 ms recycled pulsar B1855+09 with a white dwarf companion: this system is thought to have gone through a LMXB phase (Phinney & Kulkarni 1994), and contains a neutron star with $M = 1.41 \pm 0.10 M_\odot$ (Thorsett & Chakrabarty 1999; earlier Kaspi et al. 1994 estimated $M = 1.50 \pm 0.26 M_\odot$). The 23 ms pulsar PSR B1802-07, which is in a white dwarf binary that is also thought to have gone through the LMXB phase, has an inferred mass $M = 1.26 \pm 0.15 M_\odot$ (95% confidence; Thorsett & Chakrabarty 1999).

If $1.4 M_\odot$ is the mass of the neutron star immediately after its formation in core collapse, then to make a $2.2 M_\odot$ object would require accretion of material of at least $0.8 M_\odot$. Such large accretion mass may be problematic. If we neglect torques on the star due to the interaction of its magnetic field and the accretion disk, the added mass needed to spin up the NS to a spin frequency $\nu_s = \Omega_s/(2\pi)$ is

$$\Delta M \simeq \frac{I \Omega_s}{\sqrt{GMr_{in}}} \simeq 0.07 M_\odot \frac{I_{45}}{\sqrt{M_{1.4} r_{in}}} \left(\frac{\nu_s}{300 \, \text{Hz}}\right),$$

where $I = 10^{45} I_{45}$ g cm$^2$ is the moment of inertia, $M = 1.4 M_{1.4} M_\odot$ is the neutron star mass and and $r_{in} = 10 r_6$ km is the radius of the inner edge of the accretion disk, which could correspond to either the stellar surface (radius $R$) or the inner-most stable orbit (ISO) in the absence of a magnetic field strong enough to influence the flow substantially (see Cook et al. 1994). When the neutron star magnetic field is strong enough, the inner radius $r_{in}$ corresponds to the Alfvén radius. (We note that the positions of all known millisecond pulsars and binary pulsars in the $P - \dot{P}$ diagram for radiopulsars are consistent with spinup via accretion onto neutron stars with dipolar surface fields $\gtrsim 10^{8-9}$ G.) For magnetic accretion, we expect

$$I \dot{\Omega}_s = M \sqrt{GMr_{in}} f(\omega_s),$$

where $\omega_s = \Omega_s/\Omega_K(r_{in})$, with $\Omega_K(r_{in})$ the Kepler frequency at $r_{in}$. The dimensionless function $f(\omega_s)$ includes contributions to the angular momentum transport from magnetic stresses and accreting material. It is equal to zero at some equilibrium $\omega_s$, but the actual form of $f(\omega_s)$ depends
on details of the magnetic field–disk interaction. Treating $r_{\text{in}}$ as a constant, we find

$$\Delta M = \frac{I_45}{\sqrt{G M r_{\text{in}}}} \left[ \frac{1}{\omega_s} \int_0^{\omega_s} \frac{d\omega'_s}{f(\omega'_s)} \right]$$

$$= 0.07 M_\odot I_{45} \sqrt{M_1 r_6} \left[ \omega_s^{-1} \ln \left( \frac{1}{1 - \omega_s} \right) \left( \frac{\nu_s}{300 \text{ Hz}} \right) \right]$$

$$= 0.04 M_\odot I_{45} M_1^{2/3} \left[ \omega_s^{-4/3} \ln \left( \frac{1}{1 - \omega_s} \right) \left( \frac{\nu_s}{300 \text{ Hz}} \right)^{4/3} \right] \quad (3)$$

where, in the last two lines, we have adopted a simple functional form $f(\omega_s) = 1 - \omega_s$; generically,

$$\Delta M \approx 0.07 M_\odot I_{45} \sqrt{M_1 r_6} \left( \frac{\nu_s}{300 \text{ Hz}} \right) \psi(\omega_s), \quad (4)$$

where $\psi(\omega_s) \to 1$ for $\omega_s \ll \omega_{s,c}$, assuming that the torque tends to zero at a critical value $\omega_s = \omega_{s,c}$.

Large $\Delta M$ is possible if there is a lengthy phase of accretion with nearly zero net torque (e.g. accreting $0.8 M_\odot$ at a mean accretion rate of $\dot{M} = 10^{17} \text{ g s}^{-1}$ would require about 400 Myr) following a much shorter phase of spin-up to $\omega_s \to \omega_{s,c}$ (e.g. accreting $0.05 M_\odot$ at $\dot{M} = 10^{17} \text{ g s}^{-1}$ would require 30 Myr). If magnetic field decays during accretion (e.g. Taam & van den Heuvel 1986, Shibazaki et al. 1989), then the spin-up phase would have been even shorter. (Spin diffusion due to alternating or stochastic episodes of spin-up and spin-down [e.g. Bildsten et al. 1997, Nelson et al. 1997] might be allowed – but constrained – in such a picture.) To accommodate masses as large as $2 M_\odot$, these LMXBs must be rather old and must have spun up rapidly at first, and then not at all for $\geq 90\%$ of their lifetimes. Gravitational radiation might provide a mechanism for enforcing virtually zero net torque during the bulk of accretion (Bildsten 1998, Andersson et al. 1999). But equations (3) and (4) show that only very small $\Delta M$ is required to achieve $\nu_s \sim 300 \text{ Hz}$, irrespective of the mechanism responsible for halting spin-up at such frequencies.

### 3. Steepening Magnetic Fields Near the Accreting Neutron Star

We shall adopt, as a working hypothesis, that the upper QPO frequency is approximately equal to the Kepler frequency at a certain critical radius of the disk (Strohmayer et al. 1996; Miller, Lamb & Psaltis 1998; van der Klis 1998) that is determined by the combined effects of general relativity and stellar magnetic field. For sufficiently strong magnetic fields, the disk may be truncated near this radius, where matter flows out of the disk and is funneled toward the neutron star. This critical radius then corresponds to the usual Alfvén radius (Strohmayer et al. 1996). Even if the fields are relatively weak ($10^7 - 10^8 \text{ G}$) and the field geometry is such that matter remains in the disk, the magnetic stress can still slow down the orbital motion in the inner

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1 In the model of Titarchuk et al. (1998), the QPO corresponds to vertical oscillation of the disk boundary layer, but the oscillation frequency is equal to the local Kepler frequency. Even in the “non-beat” frequency model of Stella and Vietri (1998), the upper QPO frequency still corresponds to the orbital frequency.
disk by taking away angular momentum from the flow, and accreting gas then plunges toward the star at supersonic speed – a process that is also accelerated by relativistic instability. In this case, the critical radius would correspond to the sonic point of the flow (Lai 1998). We neglect the possible role of radiative forces discussed by Miller et al. (1998). As emphasized by van der Klis (1998a), the fact that similar QPO frequencies ($500 - 1200$ Hz) are observed in sources with vastly different average luminosities (from a few times $10^{-3} L_{\text{Edd}}$ to near $L_{\text{Edd}}$) suggests that radiative effects cannot be the only factor that induces the correlation of the QPO frequency and the X-ray flux for an individual source.

Despite many decades of theoretical studies (e.g., Pringle & Rees 1972; Lamb et al. 1973; Ghosh & Lamb 1979; Arons 1987; Spruit & Taam 1990; Aly 1991; Sturrock 1991; Shu et al. 1994; Lovelace, Romanova & Bisnovatyi-Kogan 1995, 1999; Miller & Stone 1997), there remain considerable uncertainties on the nature of the stellar magnetic field – disk interactions. Among the issues that are understood poorly are the transport of magnetic field in the disk, the configuration of the field threading the disk, and the nature of outflows from the disk. To sidestep these complicated questions, we adopt a simple phenomenological prescription for the vertical and azimuthal components of the magnetic field on the disk,

$$B_z = B_0 \left( \frac{R}{r} \right)^n, \quad B_\phi = -\beta B_z,$$

where $B_\phi$ is evaluated at the upper surface of the disk, and $\beta$ is the azimuthal pitch angle of the field. If we neglect the GR effect, the critical radius $r_{\text{in}}$ is located where the magnetic field stress dominates the angular momentum transport in the disk, and it is approximately given by the condition

$$\dot{M} \frac{d\sqrt{GMr}}{dr} = -r^2 B_z B_\phi;$$

using the ansatz equation (5), and assuming Keplerian rotation (which may break down in a boundary layer near $r_{\text{in}}$; e.g. Lovelace et al. 1995), we find

$$r_{\text{in}} = R \left( 2\beta \frac{B_0^2 R^3}{M\sqrt{GM R}} \right)^{2/(4n-5)}.$$

and the Kepler frequency at $r_{\text{in}}$ is

$$\nu_K(r_{\text{in}}) \propto \dot{M}^{3/(4n-5)}.$$  

For a “dipolar” field configuration, $n = 3$ and $\nu_K(r_{\text{in}}) \propto \dot{M}^{3/7}$, as is well-known, but for smaller values of $n$, the dependence steepens; for example, $\nu_K(r_{\text{in}}) \propto \dot{M}$ for a “monopole” field, $n = 2$. The observed correlation $\nu_{\text{QPO}} \propto \dot{C}$ may require $n < 3$, although the relationship between $\dot{C}$ and $\dot{M}$ is unclear (Mendez et al. 1998c).

Unusual field topologies are possible as the disk approaches the surface of the neutron star. Values of $n \neq 3$ (and even violation of power-law scaling) might occur naturally, for open field configurations, which may be prevalent because of differential rotation between the star and
the disk (e.g. Lovelace et al. 1995). MHD winds driven off a disk could also result in \( n \neq 3 \) (e.g. Lovelace et al. 1995, Blandford & Payne 1982). Disks that are fully (Aly 1980, Riffert 1980) or partially (Arons 1993) diamagnetic will also have non-dipolar variation in field strength near their inner edges (see also §3.1 below). None of these possibilities requires the field to be substantially non-dipolar at the stellar surface, although for disks that penetrate close to the star (at \( r_{\text{in}} - R \ll R \)) any non-dipolar field components, if strong enough, would be significant.

A particular field configuration that could explain the observed variation of \( \nu_{\text{QPO}} \) with \( \dot{C} \) might have \( n < 3 \) at moderate values of \( r_{\text{in}} \), leading to a strong correlation between \( \nu_{\text{QPO}} \) and \( \dot{M} \) (and hence \( \dot{C} \)). As \( \dot{M} \) rises, the disk approaches the star, and the field topology could become more complex, resulting in additional, non-power-law radial steepening of the field strength. As is argued below, this could happen even if the field is dipolar at the surface of the star, particularly if the disk is diamagnetic. This steepening of the field results in a flattening of the \( \nu_{\text{QPO}} - \dot{M} \) relation. Additional flattening results from incipient general relativistic instability at the inner edge of the disk.

### 3.1. A Specific Ansatz: Diamagnetic Disk

An illustration of the field steepening discussed above is as follows. Consider a vacuum dipole field produced by the star \( |B_z| = \mu/r^3 \) (in the equatorial plane perpendicular to the dipole axis). Imagine inserting a diamagnetic disk in the equatorial plane with inner radius \( r_{\text{in}} \). Flux conservation requires \( \pi(r_{\text{in}}^2 - R^2)|\bar{B}_z| = 2\pi\mu/R \), which gives the mean vertical field inside between \( R \) and \( r_{\text{in}} \):

\[
|\bar{B}_z(r_{\text{in}})| = \frac{2\mu}{R(r_{\text{in}}^2 - R^2)}. \tag{9}
\]

This field has scaling \( |\bar{B}_z| \propto 1/r^2 \) for large \( r \), which would result in \( \nu_K(r_{\text{in}}) \propto \dot{M} \) (see eq. [8]), and stiffens as the disk approaches the stellar surface.

The actual field at \( r = r_{\text{in}} \) is difficult to calculate. Aly (1980) found the magnetic field of a point dipole in the presence of a thin diamagnetic disk (thickness \( H \ll r \) at radius \( r \)), and demonstrated that the field strength at \( r_{\text{in}} \) is enhanced by a factor \( \sim (r_{\text{in}}/H)^{1/2} \). (See also Riffert 1980 and Arons 1993.) However, the situation is different for a finite-sized dipole (a conducting sphere of radius \( R \)) in the presence of a diamagnetic disk. This can be seen by considering a simpler problem, where we replace the disk by a diamagnetic sphere (with radius \( r_{\text{in}} \)). The magnetic field at radius \( r \) (between \( R \) and \( r_{\text{in}} \)) is given by (in spherical coordinates with the

\[2\] In replacing the disk with a spherical surface, we lose the square-root divergence found by Aly (1980) for infinitesimal \( H/r \). But note that for small fields, the disk penetrates near the star, and \( H \) may not be very small compared with \( r_{\text{in}} - R \). In assuming a point dipole, Aly (1980) (and Riffert 1980) exacerbated the divergence, and their results probably apply only when \( r_{\text{in}} \gg R \).
magnetic dipole along the $z$-axis):

$$B_r(r, \theta) = \left( -\frac{2\mu}{r_{in}^3} + \frac{2\mu}{r^3} \right) \frac{\cos \theta}{1 - \alpha^3},$$

$$B_\theta(r, \theta) = \left( \frac{2\mu}{r_{in}^3} + \frac{\mu}{r^3} \right) \frac{\sin \theta}{1 - \alpha^3},$$

(10)

(11)

where $\alpha = R/r_{in}$. Thus the vertical magnetic field at the inner edge of the disk ($r = r_{in}$) is

$$|B_z(r_{in})| = \frac{3\mu}{r_{in}^3 - R^3}.$$  

(12)

We see that the magnetic field steepens as $r_{in}$ approaches the stellar surface. In reality, some magnetic field will penetrate the disk because of turbulence in the disk and Rayleigh-Taylor instabilities (Kaisig, Tajima & Lovelace 1992); however, some steepening of the field may remain.

Adopting the magnetic field ansatz (9) and $B_\phi = -\beta B_z$, we can use (6) to calculate $r_{in}$; this gives

$$2b^2 \frac{x_c^{2.5}}{(x_c^2 - 1)^2} = 1,$$

(13)

where $x_c = r_{in}/R$ and

$$b^2 = \frac{\beta B_0^2 R^3}{M \sqrt{GM} R} = 0.07 \left( M_{1.4}^{-1/2} R_{10}^{3/2} \right) \left( \frac{\beta B_7^2}{M_{17}} \right),$$

(14)

$\mu = B_0^2 R^3/2$ ($B_0 = 10^7 B_7$ G is the polar field strength at the neutron star surface), $M_{1.4} = M/(1.4 M_\odot)$, $R_{10} = R/(10 \text{ km})$, and $M_{17} = \dot{M}/(10^{17} \text{ g s}^{-1})$. Alternatively, if we adopt (12), we find

$$\frac{9}{2} b^2 \frac{x_c^{2.5}}{(x_c^2 - 1)^2} = 1.$$  

(15)

Figure 1 shows the Kepler frequency at $r_{in}$ as a function the scaled mass accretion rate, $\dot{M}_{17} M_{1.4}^{1/2} R_{10}^{-5/2} M_{17}/\beta B_7^2 = 0.07/b^2$. Clearly, for small $\dot{M}$, $\nu_K(r_{in})$ depends on $\dot{M}$ through a power-law, but the dependence weakens as $\dot{M}$ becomes large, in qualitative agreement with the observed $\nu_{QPO}-\dot{M}$ correlation. General relativistic effects also flatten the $\nu_{QPO}-\dot{M}$ relation, as we discuss next.

### 3.2. General Relativistic Effects

General relativity (GR) introduces two effects on the location of the inner edge of the disk. First, the space-time curvature modifies the vacuum dipole field. For example, in Schwarzschild metric, the locally measured magnetic field in the equatorial plane is given by

$$B^\theta = \frac{\mu}{r^3} \left[ 6y^3(1 - y^{-1})^{1/2} \ln(1 - y^{-1}) + \frac{6y^2(1 - y^{-1}/2)}{(1 - y^{-1})^{1/2}} \right],$$

(16)
where \( y = rc^2/(2GM) \) (Petterson 1974; Wasserman & Shapiro 1983). The GR effect steepens the field only at small \( r \). For \( r = 6GM/c^2 - 10GM/c^2 \), we find the approximate scaling \( B^\phi \propto r^{-3-\epsilon} \), with \( \epsilon \approx 0.3 - 0.4 \). We shall neglect such a small correction to the dipole field given the much larger uncertainties associated with the magnetic field – disk interaction.

A more important effect of GR is that it modifies the the dynamics of the accreting gas around the neutron star. Without magnetic field, the inner edge of the disk is given by the condition \( dl_K/dr = 0 \), where \( l_K \) is the specific angular momentum of a test mass:

\[
l_K = \left( \frac{GMr^2}{r - 3GM/c^2} \right)^{1/2}.
\]  

(17)

This would give the usual the ISO at \( r_{\text{iso}} = 6GM/c^2 \), where no viscosity is necessary to induce accretion\(^3\). Since magnetic fields take angular momentum out of the disk, we can determine the inner edge of the disk using an analogous expression\(^4\)

\[
\dot{M} \frac{dl_K}{dr} = -r^2 B_z B_\phi,
\]  

(18)

(see eq. [11]). In Lai (1998) it was shown that this equation determines the limiting value of the sonic point of the accretion flow (although a Newtonian pseudo-potential was used in that paper). Adopting the magnetic field ansatz (9), we find

\[
2b^2 \frac{x_c^{2.5}}{(x_c^2 - 1)^2} = \left( 1 - \frac{6GM}{c^2 r_{\text{in}}} \right) \left( 1 - \frac{3GM}{c^2 r_{\text{in}}} \right)^{-3/2}.
\]  

(19)

Similarly, using (12), we have

\[
\frac{9}{2} b^2 \frac{x_c^{2.5}}{(x_c^3 - 1)^2} = \left( 1 - \frac{6GM}{c^2 r_{\text{in}}} \right) \left( 1 - \frac{3GM}{c^2 r_{\text{in}}} \right)^{-3/2}.
\]  

(20)

It is clear that for \( b \gg 1 \), eq. (19) or (20) reduces to the Newtonian limit (see §3.1), while for \( b = 0 \) we recover the expected \( r_{\text{in}} = r_{\text{iso}} = 6GM/c^2 \). For small \( b \), the GR effect can modify the inner disk radius significantly. In Fig. 1 we show the orbital frequency at \( r_{\text{in}} \) (as a function of the “effective” accretion rate) as obtained from (19) and (20). We see that the GR effect induces additional flattening in the correlation between \( \nu_K(r_{\text{in}}) \) and \( \dot{M} \) as \( r_{\text{in}} \) approaches \( r_{\text{iso}} \).

\(^3\)When viscosity and radial pressure force is taken into account, the flow is transonic, with the sonic point located close to \( r_{\text{iso}} \).

\(^4\)Note that in the limit of perfect conductivity, it is possible to express the Maxwell stress tensor in terms of a magnetic field four-vector \( B \) that is orthogonal to the fluid velocity four-vector \( U \) (e.g. Novikov & Thorne 1973, pp. 366-367). The field components \( B_\phi \) and \( B_z \) in eq. (14) and below are actually the projections of \( B \) onto a local orthonormal basis (i.e. \( B_\phi \rightarrow \vec{e}_\phi \cdot B \) and \( B_z \rightarrow \vec{e}_z \cdot B \)) even though we have retained the nonrelativistic notation for these field components. No additional relativistic corrections are required with these identifications understood.
We emphasize the phenomenological nature of eqs. (18)-(20): they are not derived from a self-consistent MHD calculation, and take account of the dynamics of the disk under a prescribed magnetic field configuration. However, we believe that they indicate the combined effects of dynamically altered magnetic field and GR on the inner region of the accretion disk. By measuring the correlation between the QPO frequency and the mass accretion rate, one might be able to constrain the magnetic field structure in accreting neutron stars, and reach quantitative conclusions about the nature of the interaction of the accretion disk and magnetic field.

4. Where are the QPOs Produced?

Implicit in the discussion of magnetic fields and $\nu_{QPO}$ in the preceding sections were the assumptions that the QPO arises at a radius outside the star that coincides with the inner radius of the accretion disk. Here, we examine two ways in which these assumptions might be violated, and show how the relatively small measured values of $\nu_{\text{max}}$ might be consistent with neutron star masses near $1.4M_\odot$.

4.1. Disk Termination at the Neutron Star Surface

For the model discussed in §3, the steepening magnetic field and general relativity produce the flattening in the correction between the QPO frequency $\nu_{QPO} = \nu_K(r_{\text{in}})$ and the mass accretion rate $\dot{M}$. But $\nu_{QPO}$ becomes truly independent of $\dot{M}$ only when $r_{\text{in}}$ approaches $r_{\text{iso}}$ or the stellar radius $R$. It has been suggested (see §1) that the $\dot{M}$-independent QPO frequency corresponds to the Kepler frequency at $r_{\text{iso}}$. But it is also possible that the inner disk radius reaches the stellar surface, which is outside the ISO, as $\dot{M}$ increases. We note that observationally it is difficult to distinguish the flattening of $\nu_{QPO}$ and a true plateau. It is not clear that the flattening feature at $\nu_{QPO} \sim 1100$ Hz observed in 4U 1820-30 (Zhang et al. 1998) corresponds the maximum QPO frequency, but we shall assume it does and explore the consequences.

The maximum QPO frequency, $\nu_{\text{max}}$, is given by the orbital frequency at the larger of $r_{\text{iso}}$ and $R$. To linear order in $\nu_s$ (the spin frequency), the ISO is located at $r_{\text{iso}} = (6GM/c^2)(1 - 0.544a)$, and the orbital frequency at ISO is

$$\nu_K(r_{\text{iso}}) = \frac{1571}{\dot{M}_{1.4}}(1 + 0.748a) \, \text{Hz},$$

with the dimensionless spin parameter

$$a \simeq 0.099 \frac{R_{10}^2}{\dot{M}_{1.4}} \left( \frac{\nu_s}{300 \, \text{Hz}} \right),$$

where we have adopted $I = (2/5)\kappa MR^2$ for the moment of inertia of the neutron star, with $\kappa \simeq 0.815$ (appropriate for a $n = 0.5$ polytrope). The orbital frequency at the stellar surface can
be written, to linear order in \( \nu_s \), as

\[
\nu_K(R) = 2169 M_{1.4}^{1/2} R_{10}^{-3/2} \left[ 1 - 0.094 a \left( \frac{M_{1.4}}{R_{10}} \right) \right] \text{Hz.} \tag{23}
\]

Note that in the above equations, \( R \) refers to the equatorial radius of the (spinning) neutron star, which is related to the radius, \( R_0 \), of the corresponding nonrotating star by:

\[
\frac{R - R_0}{R_0} \simeq 0.4 \frac{\Omega^2 R_0^3}{GM} \simeq 0.0078 M_{1.4}^{-1} R_{10}^3 \left( \frac{\nu_s}{300 \text{Hz}} \right)^2, \tag{24}
\]

where we have again adopted the numerical parameters appropriate for a \( n = 0.5 \) polytrope (Lai et al. 1994). One may appeal to numerical calculations (e.g., Miller, Lamb & Cook 1998) for more accurate results, but the approximate expressions given above are adequate.

Figure 2 shows the contours of constant \( \nu_{\text{max}} = \min[\nu_K(r_{\text{iso}}), \nu_K(R)] \) in the \( M-R_0 \) plane. For large \( M \) and small \( R_0 \), the contours are specified by \( \nu_K(r_{\text{iso}}) \), while for larger \( R_0 \) and small \( M \), the contours are specified by \( \nu_K(R) \). We see that to obtain the maximum QPO frequency of order 1100 – 1200 Hz, one can either have a \( M \gtrsim 2 M_\odot \) neutron star (with \( R_0 \lesssim 16 \) Km), or have a \( M \approx 1.4 M_\odot \) neutron star with \( R_0 \approx 14 - 15 \) km. Here we focus on the latter interpretation, in which the accretion disk terminates at the stellar surface before reaching the ISO. A boundary layer forms in which the angular velocity of the accreting gas changes from near the Keplerian value (at the outer edge of the boundary layer) to the stellar rotation rate. Depending on the thickness of the boundary layer, the inferred the NS radius may be somewhat smaller. Moreover, the peak rotation frequency may be below \( \nu_K(R) \), which would also allow smaller values of \( R_0 \).

In addition to avoiding a large neutron star mass (see §2), the identification of \( \nu_{\text{max}} \) with the Kepler frequency near the stellar surface may allow a plausible explanation of the observed correlation between the QPO amplitude and the X-ray flux. While the mechanism of producing X-ray modulation in a kHz QPO is uncertain, in many models (e.g., Miller et al. 1998; see also Klużniak et al. 1990) the existence of a supersonic “accretion gap” between the stellar surface and the accretion disk is crucial for generating the observed the X-ray modulation. If we interpret \( \nu_{\text{max}} \) as the Kepler frequency at the ISO, which is always outside the stellar surface, then the “accretion gap” always exists, and there is no qualitative change in the flow behavior as the inner disk approaches ISO. It is therefore difficult to explain why the QPO amplitude decreases and eventually vanishes as the X-ray flux increases. The situation is different if \( \nu_{\text{max}} = \nu_K(R) \), since the gap disappears when the mass accretion rate becomes sufficiently large. At small \( \dot{M} \) there is a gap (induced by a combination of magnetic and GR effects) between the inner edge of the disk and the stellar surface. Since the impact velocity of the gas blob at the stellar surface is larger for a wider accretion gap, we expect the modulation amplitude to be larger for small accretion rates \( \dot{M} \). As \( \dot{M} \) increases, the inner disk edge approaches the stellar surface, and we expect the QPO amplitude to decrease. The maximum QPO frequency signifies the closing of the accretion gap

\[5\] When \( \dot{M} \) is too low (for a given \( B_0 \)) so that \( r_{\text{in}} \) is far away from the stellar surface, the accreting gas can be
and the formation of a boundary layer. Since there is no supersonic flow in this case, one might expect the QPO amplitude to vanish. In addition, there may be changes in the spectral properties of the system as the gap closes.

The large neutron star radius (15 km for a 1.4M⊙ star) required if ν_{\text{max}} = ν_{K}(R) is only allowed for a handful of very stiff nuclear equations of state (see Fig. 2); most recent microscopic calculations give R₀ ∼ 10 km (e.g., Wiringa et al. 1988). Is such a large radius consistent with observations? No neutron star radii are known with the accuracy that has been achieved for numerous neutron star mass determinations, but several methods have been tried:

1. Observations of X-ray bursts have been used to determine empirical \( M - R \) relations, but these are hampered by the need for model-dependent assumptions regarding the total luminosity and its time history, anisotropy of the emission, radiated spectrum and surface composition, even when the source distance is known (e.g. van Paradijs et al. 1990, Lewin, van Paradijs & Taam 1995).

2. X-ray and optical observations of the (apparently nonrotating) isolated neutron star RX J185635-3754 (Walter, Wolk & Neuhäuser 1996, Walter & Matthews 1997), combined with limits on the source distance, \( D \), imply a blackbody radius \( R(1 + z) < 14(D/130\text{ pc}) \) km, where \( z \) is the surface redshift of the star.

3. Ray tracing and lightbending may be used to derive limits on \( R/M \) for periodically modulated X-ray emission. For two isolated neutron stars (PSR B1929+10 and B0950+08; Yancopoulos, Hamilton & Helfand 1994, Wang & Halpern 1997) and one millisecond pulsar (J0437-4715; Zavlin and Pavlov 1997, Pavlov & Zavlin 1998), the results are broadly consistent with \( Rc^2/2GM \approx 2.0 - 2.5 \), but the results depend on geometry (angles between rotation and magnetic axes, and rotation axis and the line of sight) as well as on the spectrum and (energy-dependent) anisotropy of the polar cap emission. The rather large observed pulsed fractions appear to rule out two polar cap hot spots unless \( Rc^2/2GM \) is rather large (e.g. \( \approx 4.3 \) for PSR B1929+10; Wang & Halpern 1997). Similar considerations may prove fruitful for periodically modulated flux from X-ray bursts (e.g. Miller & Lamb 1998); the pulse fractions observed so far are large, suggesting non-compact sources (e.g. Strohmayer et al. 1999, who find \( Rc^2/2GM \approx 5 \) for 4U1636-54, corresponding to an implausibly large radius of 21 km for \( M = 1.4M_{\odot} \)).

4. Burderi & King (1998) have argued that requiring the Alfvén radius to be intermediate between \( R \) and the corotation radius, \( R_{co} = (GM/\Omega_s^2)^{1/3} \), for the 2.5 ms pulsating source SAX J1808.4-3658 (discovered by Wijnands & van der Klis 1998) implies an upper bound

channeled out of the disk plane by the magnetic field toward the magnetic poles. The detail of the channeling process depends on the magnetic field geometry in the disk (such as the radial pitch angle of the field line). This may quench the kHz QPOs and give rise to X-ray pulsation (as in X-ray pulsars). The pulsating X-ray transient system SAX J1808.4-3658 may just be such an example.
of $R < 13.8(M/M_\odot)^{1/3}$ km, since the pulsations are detected at the same frequency for X-ray count rates spanning an order of magnitude. However, their bound depends on the model-dependent assumptions that the count rate is strictly proportional to $\dot{M}$ and the field strength in the disk is dipolar ($B \propto r^{-3}$). (See also Psaltis & Chakrabarty 1999.)

Taken together, the evidence neither supports nor excludes the possibility that $R \simeq 15$ km for $M \simeq 1.4M_\odot$ (or $R_c^2/2GM \simeq 3.6$) definitively, although most of the estimates listed above favor more compact models ($R \simeq 10$ km for $M \simeq 1.4M_\odot$) nominally.

### 4.2. QPOs from $r > r_{\text{in}}$?

QPOs are identified in the Fourier spectra of photon counts from X-ray sources, so it may be that most of the spectral power comes from radii outside $r_{\text{in}}$, possibly from the disk radius at which the differential photon emission rate is maximum. For example, if the QPO arises from a radius $r = (1 + \lambda)r_{\text{in}}$, then $\nu_{\text{QPO}} = (1 + \lambda)^{-3/2}\nu_K(r_{\text{in}})$. As $r_{\text{in}} \to 6GM/c^2$, the ISO in the slow-rotation limit, $\nu_{\text{QPO}} \to 2200 \text{Hz}/(M/M_\odot)(1 + \lambda)^{3/2}$, so observations that give $\nu_{\max} \simeq 1060 \text{Hz}$ asymptotically may actually require $M(1 + \lambda)^{3/2} = 2M_\odot$, or $1 + \lambda \approx 1.3$ if $M \approx 1.4M_\odot$, rather than $M \approx 2.1M_\odot$.

To obtain a simple realization of this idea, consider a Shakura-Sunyaev (1973) disk, for which the emitted flux from one face is

$$F(r) = \sigma SB T_e^4(r) = \frac{3GM\dot{M}f(r)}{8\pi r^3};$$

(25)

in the Newtonian limit (which we shall employ here for giving a simplified illustration). The function $f(r) = 1 - \beta \sqrt{r_{\text{in}}/r}$, where $\beta \leq 1$ parametrizes the rate of accretion of angular momentum from the disk onto the star relative to $\dot{M}/\sqrt{GMr_{\text{in}}}$ (e.g. Shapiro & Teukolsky 1983. eq. [14.5.17]; see also Frank, King & Raine 1992, §5.3); if “imperfect” fluid stresses vanish at $r_{\text{in}}$, then $\beta = 1$ (as in black hole accretion; see Page & Thorne 1974, Novikov & Thorne 1973). If the color temperature of the emission equals the effective temperature $T_e(r)$, then the “bolometric flux” of photons is $\sim F(r)/kT_e(r)$ at radius $r$, and the rate at which photons are emitted from radii between $r$ and $r + dr$ is of order

$$\frac{2\pi r F(r)}{kT_e(r)} \sim \frac{\dot{M}^{3/4}[f(r)]^{3/4}}{r^{5/4}}.$$  

(26)

Differentiating equation (26) implies a maximum emission rate at $\sqrt{r/r_{\text{in}}} = 1.3\beta$, consistent with $r > r_{\text{in}}$ provided that $\beta > 0.77$. Assuming that the QPO frequency is the Kepler frequency at the radius of peak (bolometric) photon emission,

$$\nu_{\text{QPO}} = \frac{\nu_K(r_{\text{in}})}{(1.3\beta)^3} \to \frac{1000 \text{Hz}}{\beta^3 M/M_\odot},$$

(27)
where the limiting result is for \( r_{\text{in}} \to 6GM/c^2 \). In order for the maximum value of \( \nu_{QPO} \) to be \( \nu_{\text{max}} \simeq 1060 \text{ Hz} \), we require \( M = 1.4M_\odot/(\beta/0.88)^3 \).

Real disk emission profiles for small \( r_{\text{in}} \), and the determination of \( \nu_{QPO} \), are not this simple for several reasons. A detailed calculation of the X-ray spectrum is needed, since the QPOs are found for counts in particular energy bands; the bolometric count rate is not a good approximation in general. (But note that Comptonization by hot coronal gas above the disk conserves photon number, so the approximation may be better than it appears at first sight.) In particular, the color temperature is not usually the same as the effective temperature, since electron scattering is the dominant opacity at relevant disk radii. The composition of the disk is also important; at low enough \( \dot{M} \), the disk will be matter-dominated, but at larger \( \dot{M} \), radiation-dominated. (Less important, but still significant, is the dependence of opacity on the element abundances in the accreting gas.) In addition, relativistic effects alter \( f(r) \) (e.g. Page & Thorne 1974, Novikov & Thorne 1973), and hence \( \nu_{QPO} \). Moreover, the angular momentum carried away by photons may not be insignificant once \( r_{\text{in}} \) approaches the ISO (Page & Thorne 1974, Epstein 1985). Instabilities associated with the transition from matter to radiation domination (Lightman & Eardley 1974) or the inner boundary layer (e.g. Epstein 1985) might also play a role in determining \( \nu_{QPO} \). These and other issues associated with the termination of disks at \( r_{\text{in}} \) and QPOs will be explored more fully elsewhere. However, the simplified example presented here indicates that \( \nu_{QPO} \) might plausibly arise from \( r > r_{\text{in}} \).

5. Conclusion

In this paper we have presented a phenomenological model of the inner region of the accretion disk for weakly magnetized neutron stars such as those in LMXBs. A notable feature of these systems is that both magnetic field and general relativity are important in determining the inner disk radius. Our result suggests that the combined effects of a steepening magnetic field – which is likely for disk accretion onto a neutron star – and general relativity can produce the flattening of the QPO frequency \( \nu_{QPO} \) as the mass accretion rate \( \dot{M} \) increases. If the field steepens fast enough with decreasing inner disk radius, \( \nu_{QPO} \) may vary little over a fairly substantial range of \( \dot{M} \) at values considerably below the Kepler frequency at the ISO due to general relativity. Observationally, the correlation between \( \nu_{QPO} \) and the RXTE photon count rate has been well-established, but the scaling between \( \nu_{QPO} \) and \( \dot{M} \) is ambiguous (Mendez et al. 1998c). An observational or phenomenological determination of this scaling would be quite useful in constraining the magnetic field structure in LMXBs.

Currently it is not clear whether the plateau behavior in the QPO frequency has been observed. But even if \( \nu_{QPO} \sim 1100 - 1200 \text{ Hz} \) represents the maximum possible QPO frequency, we argue that a massive neutron star \( (M \gtrsim 2M_\odot) \) is not necessarily implied. Instead, a \( M \simeq 1.4M_\odot \), \( R_0 \sim 14 - 15 \text{ km} \) neutron star may be a better solution, and is within the range allowed by some nuclear equations of state. If this is the case, the maximum QPO frequency signifies the closing of
the accretion gap and the formation of a boundary layer. Alternatively, the QPO frequency might be associated with the Kepler frequency at a radius somewhat larger than the inner radius of the disk, thus allowing lower mass for the accreting neutron star. In either case, better theoretical and phenomenological understanding of the termination of magnetized accretion disks is needed before observations of maximal kHz QPOs can be interpreted as purely general relativistic in origin, and used to deduce neutron star masses.

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REFERENCES

Alpar, A., Cheng, A. F., Ruderman, M., & Shaham, J. 1982, Nature, 300, 728.

Aly, J. J. 1980, A&A, 86, 192.

Aly, J. J. 1991, ApJ, 375, L61.

Andersson, N., Kokkotas, K. D., & Stergioulas, N. 1999, ApJ, in press [astro-ph/9806089].

Arons, J. 1987, in “The Origin and Evolution of Neutron Stars” (IAU Symp. No. 125), ed. D. J. Helfand & J.-H. Huang (D. Reidel Pub.: Dordrecht).

Arons, J. 1993, ApJ, 408, 160.

Bildsten, L. 1998, ApJ, 501, L89.

Bildsten, L. et al. 1997, ApJS, 113, 367.

Blandford, R. D., & Payne, D. G. 1982, MNRAS, 199, 883.

Burderi, L., & King, A. R. 1998, ApJ, 505, L135.

Chakrabarty, D., & Morgan, E. H. 1998, Nature, 394, 346.

Cook, G. B., Shapiro, S. L., & Teukolsky, S. A. 1994, ApJ, 423, L117.

Epstein, R. I., 1985, ApJ, 291, 822.

Finn, L. S. 1994, Phys. Rev. Lett., 73, 1878.

Frank, J., King, A., & Raine, K., 1992, Accretion Power in Astrophysics (Cambridge: Cambridge University Press), pp. 71-75.

Ghosh, P., & Lamb, F. K. 1979, ApJ, 232, 259.
Kaaret, P., et al. 1997, ApJ, 480, L27.

Kaisig, M., Tajima, T., & Lovelace, R. V. E. 1992, ApJ, 386, 83.

Kaspi, V. M., Taylor, J. H., & Ryba, M. F. 1994, ApJ, 428, 713.

Kluźniak, W., Michelson, P., & Wagoner, R. V. 1990, ApJ, 358, 538.

Lai, D. 1998, ApJ, 502, 721.

Lai, D., Rasio, F. A., & Shapiro, S. L. 1994, ApJ, 423, 344.

Lamb, F. K., Pethick, C. J., & Pines, D. 1973, ApJ, 184, 271.

Lewin, W. H. G., Van Paradijs, J., & Taam, R. E. 1995, in X-ray Binaries, ed. W. H. G. Lewin, J. Van Paradijs & E. P. J. Van den Heuvel (Cambridge Univ. Press).

Lightman, A. P., & Eardley, D. M., 1974, ApJ, 187, L1.

Lovelace, R. V. E., Romanova, M. M., & Bisnovatyi-Kogan, G. S. 1995, MNRAS, 275, 244.

Lovelace, R. V. E., Romanova, M. M., & Bisnovatyi-Kogan, G. S. 1999, ApJ, in press [astro-ph/9811369].

Méndez, M., et al. 1998a, ApJ, 494, L65 [astro-ph/9807281].

Méndez, M., van der Klis, M., & van Paradijs, J. 1998b, ApJ, 505, L23 [astro-ph/9808281].

Méndez, M., van der Klis, M., Ford, E. C., Wijnands, R., & van Paradijs, J. 1998c, ApJ, submitted [astro-ph/9811261].

Miller, M. C., & Lamb, F. K., 1998, ApJ, 499, L37.

Miller, M. C., Lamb, F. K., & Cook, G. B. 1998, ApJ, 509, 793.

Miller, M. C., Lamb, F. K., & Psaltis, D. 1998, ApJ, 508, 791.

Miller, K. A., & Stone, J. M. 1997, ApJ, 489, 890.

Nelson, R. W. et al. 1999, ApJ, in press [astro-ph/9708193].

Novikov, I. D., & Thorne, K. S., 1973, in Black Holes, C. DeWitt & B. S. DeWitt, eds. (New York: Gordon and Breach), pp. 347-450.

Page, D. N., & Thorne, K. S., 1974, ApJ, 191, 499.

Pandharipande, V. R., & Smith, R. A. 1975, Nucl. Phys., A237, 507.

Pavlov, G. G., & Zavlin, V. E. 1998, ApJ, 490, L91.
Petterson, J. A. 1974, Phys. Rev., D10, 3166.
Phinney, E. S., & Kulkarni, S. R. 1994, ARA&A, 32, 591.
Pringle, J. E., & Rees, M. J. 1972, A&A, 21, 1.
Psaltis, D., et al. 1998, ApJ, 501, L95 (astro-ph/9805084).
Psaltis, D., & Chakrabarty, D. 1999, ApJ, submitted (astro-ph/9809335).
Riffert, H. 1980, ApSS, 71, 195.
Shakura, N. I., & Sunyaev, R. A. 1973, A&A, 24, 337.
Shapiro, S. L., & Teukolsky, S. A., 1983, Black Holes, White Dwarfs and Neutron Stars (New
York: J. Wiley & Sons), pp. 434-435.
Shibazaki, N., Murakami, T., Shaham, J., & Nomoto, K. 1989, Nature, 342, 656.
Shu, F. H., et al. 1994, ApJ, 429, 781.
Spruit, H. C., & Taam, R. E. 1990, A&A, 229, 475.
Stella, L., & Vietri, M. 1998, astro-ph/9812124.
Strohmayer, T., et al. 1996, ApJ, 469, L9.
Strohmayer, T. E., Zhang, W., Swank, J. H., & White, N. E., 1998, ApJ, in press (astro-
ph/9803110).
Sturrock, P. A. 1991, ApJ, 380, 655.
Taam, R. E., & van den Heuvel, E. P. J. 1986, ApJ, 305, 235.
Thorsett, S. E., & Chakrabarty, D. 1999, ApJ, 512, 288
Titarchuk, L., Lapidus, I., & Muslimov, A. 1998, ApJ, 499, 315.
Van der Klis, M. 1998a, in “The Many Faces of Neutron Stars” (Proc. NATO ASI) (astro-
ph/9710010).
Van der Klis, M. 1998b, astro-ph/9812395.
Van der Klis, M., Wijnands, R. A., Horne, K., & Chen, W. 1997, ApJ, 481, L97.
Van Kerkwijk, M. H., van Paradijs, J., & Zuiderwijk, E. J. 1995, A&A, 303, 497.
van Paradijs, J., Dotani, T., & Tanaka, Y. 1990, PASJ, 42, 633.
Walter, F. M., Wolk, S. J., & Neuhäuser, R. 1996, Nature, 379, 233.
Walter, F. M., & Matthews, L. D. 1997, Nature, 389, 358.
Wang, F. Y.-H., & Halpern, J. P., 1997, ApJ, 482, L159.
Wasserman, I., & Shapiro, S. L. 1983, ApJ, 265, 1036.
Wijnands, R. A. D., et al. 1997, ApJ, 479, L141.
Wijnands, R., & van der Klis, M. 1998, Nature, 394, 344.
Wiringa, R. B., Fiks, V., & Fabrocini, A. 1988, Phys. Rev. C38, 1010.
Yancopulos, S., Hamilton, T. T., & Helfand, D. J. 1994, ApJ, 429, 832.
Zavlin, V. E., & Pavlov, G. G., 1998, A&A, 329, 583.
Zhang, W., Strohmayer, T. E., & Swank, J. H. 1997, ApJ, 482, L167 [astro-ph/9703151].
Zhang, W., et al. 1998, ApJ, 500, L171.

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Fig. 1.— The orbital frequency of at the inner radius of the disk as a function of the “effective” mass accretion rate. The inner disk radius, $r_{in}$, is obtained by solving eq. (13) (light solid line) or (15) (dashed solid line), corresponding to different magnetic field structure. The heavy solid lines incorporate the effect of general relativity based on eq. (19) (heavy solid line) or (20) (heavy dashed line).
Fig. 2.— Constraints on the mass-radius ($M - R_0$) relation of neutron star from the maximum orbital frequency $\nu_{\text{max}}$ outside the star. Each closed curve shows the $\nu_{\text{max}} = \text{constant}$ contour in the $M - R_0$ plane, with the upper boundary $\nu_{\text{max}} = 1100$ Hz, and the lower boundary $\nu_{\text{max}} = 1200$ Hz. (the solid heavy lines correspond to spin frequency $\nu_s = 300$ Hz, and the heavy dashed lines $\nu_s = 0$). Note that in the case of rotating neutron star, $R_0$ is the radius of corresponding nonrotating stellar model with the same mass. The light solid curves depict two representative equations of state: TI is the very stiff tensor interaction model of Pandharipande & Smith (1975), and UV14+UVII is from model of Wiringa et al. (1988).