Research on Fitness Function of Tow Evolution Algorithms Using for Neutron Spectrum Unfolding

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Using evolution algorithms to unfold the neutron energy spectrum, fitness function design is an important fundamental work for evaluating the quality of solution, but it has not attracted much attention. In this work, we investigated the performance of 8 fitness functions attached to genetic algorithm (GA) and differential evolution algorithm (DEA) used for unfolding four neutron spectra of IAEA 403 report. Experiments show that the fitness functions with a maximum in GA can limit the ability of population to percept the fitness change but the ability can be made up in DEA, and the fitness function with a feature penalty item help to improve the performance of solutions, and the fitness function using the standard deviation and the Chi-Squared shows the balance between algorithm and spectra. The results also show that the DEA has good potential for neutron energy spectrum unfolding. The purpose of this work is to provide evidence for structuring and modifying the fitness functions, and some genetic operations that should be paid attention were suggested for using the fitness function to unfold neutron spectra.

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I. INTRODUCTION

Bonner multi-sphere neutron energy spectrometer (BSS) was introduced by Bramblett et al. [1], which consists of multiple moderator spheres with different thicknesses and a thermal neutron detector [2,3]. However, the neutron spectrum cannot be read from the detector directly but need the neutron spectrum unfolding method to calculate, and the unfolding process is expressed by the first kind Fredholm integral equation, could be represented in a discrete form as Eq. (1) [4].

\[ C_j = \sum_{i=1}^{n} R_{ij} \varphi_i + \varepsilon_j \quad j = 1, 2, 3, \ldots m \]  

Where \( C_j \) is neutron count reading from the \( j \)th detection unit, and \( R_{ij} \) is the response of \( j \)th detection unit to neutrons of \( i \)th energy group simulated from Monte Carol code generally, and \( \varphi_i \) is the neutron fluence of \( i \)th energy group, and \( \varepsilon_j \) is the detect error of \( j \)th detection unit. \( C_j \) and \( R_{ij} \) will be the input of the unfolding algorithm, and \( \varphi_i \) be the output. Normally, the number of detect units is far smaller than the number of energy groups (i.e., \( m \ll n \), in this work, \( m=15, n=53 \)).

To obtain an approximate neutron spectrum, some artificial intelligence optimization methods can be used, such as neural networks method [5], particle swarm optimization [6, 7], genetic algorithm [8] and differential evolution algorithm in this work. The quality of the solution and the algorithm performance is often strongly related to fitness function in GA and DEA [9]. In the current literature, however, a few detailed studies have been carried out on the fitness function for neutron unfolding problem. In this work, the structural characteristics of different fitness functions and their effects on the quality of the solution were investigated in GA and DEA frame.

II. MATERIAL AND METHODS

1. Genetic algorithm

GA [10] imitates the natural evolution of biology which has been introduced by Darwin (i.e., "survival of the fittest"). Unlike the common linear optimization technique using for neuron spectrum unfolding, the GA optimization is capable of finding the global optima in a large multi-dimensional solution space [13].
**Mutation**, in this work, a single point mutation was performed to introduce new genes into the individual. Firstly, randomly select an individual from the current population following the mutation probability, then, randomly select a locus from the selected individual to mutate to a real number $\in (0, \min(C_i/R_i))$.

**Crossover** is a way for the excellent genes of parents can be passed on to the offspring. The whole arithmetic crossover operator was performed [11]

$$
\begin{align*}
o^1 &= u \cdot p^1 + (1-u) \cdot p^2 \\
o^2 &= (1-u) \cdot p^1 + u \cdot p^2
\end{align*}
$$

where $o^1$ and $o^2$ are the offspring, $p^1$ and $p^2$ are the parents selected randomly from the population, $u$ is a rand number $\in (0,1)$.

**Selection** is the elimination mechanism in GA. Russian roulette method was used in this work, which results in individuals with higher fitness have more chance to be carried forward to the next generation. Before the selection, individuals were evaluated by the fitness function.

2. **Differential evolution algorithm**

DEA was first proposed by Storn et al. [12], also consists of mutation, crossover, and selection, but different from specific evolution strategy.

**Mutation** in DEA does not send the offspring into the population immediately but will be temporarily stored for transferring to the crossover operation. Three individuals were randomly selected from the population, and the mutation was performed as the following Eq. (3) [12]

$$
\text{x}^{\text{tem}} = \text{x}^{i1} + F \cdot (\text{x}^{i2} + \text{x}^{i3})
$$

where $\text{x}^{\text{tem}}$ is the temporary individual, $F$ is a scale factor $\in (0,2)$, in this work, $F=0.5$, $\text{x}^{i1}$, $\text{x}^{i2}$ and $\text{x}^{i3}$ are selected individuals from the population, and $i1 \neq i2 \neq i3$.

**Crossover** operation is carried out based on mutation. A target individual $\text{x}^{\text{tar}}$ and the temporary individual $\text{x}^{\text{tem}}$ cross by sending the genes to offspring [12]:

$$
\text{x}^{o}_i = \begin{cases} \\
\text{x}^{\text{tem}}_i, & u < \text{pc} \\
\text{x}^{\text{tar}}_i, & \text{otherwise} 
\end{cases}
$$
where $x^o = [x_1^o, x_2^o, x_3^o, ..., x_n^o]^T$ is the offspring will be sent to the selection, $u$ is a random number $\in (0,1)$, $pc$ is the crossover probability.

**Selection** of DEA carries strictly the individual forward to the next generation with higher fitness between the target individual $x^{\text{tar}}$ and the offspring $x^o$, before the selection, both of them were evaluated by the fitness function.

From the above, both in GA and DEA, selection decides which individual can survive to the next generation based on the fitness. Therefore, the fitness function controls the evolution of the population.

### 3. Fitness function

In the application of neutron spectrum unfolding, the fitness function is used to estimate how close between the solution and the reference neutron spectrum. In this work, we found that the evolution behavior and the quality of solution varied significantly from different fitness functions, and the same fitness function in the DEA and GA also shows the makeable differences, the fitness functions are given in table 1.

| Fun No. | Fitness Function |
|---------|------------------|
| F1 [13] | $f_1 = \sum_{j=1}^{m} \left[ \beta_1 - \left( T_j / \sum_{j=1}^{m} R_j \phi_j^{\text{cal}} + C_j \right)^2 \right]$ |
| F2 [14, 15] | $f_2 = 1 \left( \frac{1}{\sum_{j=1}^{m} T_j^2 / C_j} \right)^2$ |
| F3 [14] | $f_3 = \max\left( \frac{\sum_{j=1}^{m} T_j^2 / C_j}{\sum_{j=1}^{m} T_j / C_j} \right) \cdot 2 - \frac{\sum_{j=1}^{m} T_j^2 / C_j}{\sum_{j=1}^{m} T_j / C_j}$ |
| F4 [16] | $f_4 = 1 \left( \frac{\sum_{j=1}^{m} T_j^2}{\sum_{j=1}^{m} C_j} + \beta_4 \cdot p_1 \right)$ |
| F5 | $f_5 = \delta\left( \frac{\sum_{j=1}^{m} R_j \phi_j^{\text{cal}}}{C_j} \right)$ |
| F6 [4] | $f_6 = \beta_6 + \sum_{j=1}^{m} \left( \frac{T_j}{C_j} \right)^2$ |
| F7 [17] | $f_7 = \sqrt{\frac{\sum_{j=1}^{m} C_j^2 / \sum_{j=1}^{m} T_j^2}{\sum_{j=1}^{m} T_j^2 / C_j}}$ |
| F8 | $f_8 = 1 \left( \frac{\sum_{j=1}^{m} T_j / C_j}{\sum_{j=1}^{m} T_j / C_j} \right)^2 + 0.5 \cdot \beta_8 \cdot (p_1 + p_2)$ |
Where \( T_j = (\sum_{j=1}^{m} R_j \phi_{ij}^{\text{cal}}) - C_j \), and \( \phi_{ij}^{\text{cal}} \) is the neutron fluence of \( i \)th energy group calculated from unfolding algorithm, and \( \beta_1 = 0.1 \cdot \beta_2 = \beta_3 = 100 \), \( p_1 = \sum_{i=2}^{n} (\phi_{i}^{\text{cal}} - \phi_{i-1}^{\text{cal}})^2 \), \( p_2 = \sum_{i=2}^{n} (\phi_{i+1}^{\text{cal}} - 2 \cdot \phi_{i}^{\text{cal}} + \phi_{i-1}^{\text{cal}})^2 \), \( p_1 \) and \( p_2 \) are used to calculate the continuity of energy spectrum [8, 16]. F5 and F8 are varieties from literature [8, 18].

In GA, 200 individuals were initialized randomly, and the maximum iteration was 3000, and the mutation probability was 0.1, and the crossover probability was 0.9. The parameters in the DEA were set the same as in GA but without mutation probability.

4. Reference spectra and Quality factors

To make the results more representativeness in this work, as shown in fig. 1, we selected four reference neutron spectra with different shape and continuity, and response matrix reported by IAEA 403 [18]. In the interval of \( 10^{-9} \sim 15.8 \text{MeV} \), 53 energy groups are divided, and 15 detection units are used. The result of the convolution of reference spectra and response matrix simulating neuron counts reading from detection units, and the random noise with a standard deviation of 5% was intentionally added.

![Fig. 1. Four reference spectra](image)

Spectrum quality factor (Qs) [4] was used to evaluate the shape difference between the solution and the reference energy spectrum (true value) in this work.

\[
Qs(\%) = 100 \cdot \sqrt{\frac{\sum_{i=1}^{n} (\phi_{i}^{\text{ref}} - \phi_{i}^{\text{cal}})^2}{\sum_{i=1}^{n} (\phi_{i}^{\text{cal}})^2}}
\] (5)
Where $\varphi_i^{ref}$ is the fluence of the $i$th energy group of the reference energy spectrum. A perfect solution produces $Q_s=0$.

**III. RESULTS AND DISCUSSION**

Static distribution is shown in fig. 2, Taking the fluence of each energy group of Spec 4 as the center, random and uniform sampling was performed within $(0, \min(C_j/R_j))$. Each experiment of fitness function sampled 200 thousand energy spectra to build a static population, simulating different quality solutions during the evolution process.

![Qs-fitness distribution of a static population](image)

Fig. 2. Qs-fitness distribution of a static population, for illustration, all the fitness values from different functions were mapped into the $(0, 1)$.

The individual with highest fitness produced the best $Q_s$ means that all the fitness functions were appropriate under ideal evolution, and the distribution of fitness functions varies from the form of functions. Specifically, $F_5$ is different from other functions with a bulge ($Q_s \approx 600$). The span of fitness function values of $F_2$, $F_8$, $F_4$, and $F_7$ increases successively, $Q_s$ is about close to 0, the distribution is about steep. In other words, in a collection of individuals close to the true value ($Q_s \approx 0$), a slight change in the shape of the spectrum can cause a drastic change in the value of this fitness functions. Conversely, the distribution of $F_1$, $F_3$, and $F_6$ are flatter, because there is a constant term in each function that limits the maximum value of the fitness. The fitness value of group 2 ($F_4$, $F_7$, $F_8$, and $F_2$) and group 3 ($F_1$, $F_3$, and $F_6$) can be described as $G_3 \approx \text{Max-G}_2$. 
Fig. 3. Qs-fitness distribution of dynamic population in GA using Spec4, during evolving, the fitness was ranked after each iteration, and 10% of individuals in the current population are uniformly extracted from the best to the worst.

For each fitness function in fig. 3, the same fitness level has a certain range of the Qs, that is to say, one value of fitness map to several solutions with different spectrum shapes. In the same way, individuals with the same Qs can produce a wide range of fitness. In this case, selection operation cannot distinguish the difference between the better individuals and the inferior individuals based on fitness. Moreover, inferior individuals with high fitness can deceive the algorithm, therefore, the optimal individual (Qs closer to 0) in the F2, F7, F8, and F5 are on the right side of the highest fitness.

There is a maximum in F1 and F6, in the mid-late evolution, the fitness individuals is close, the opportunity between the better individuals and others to survive is close. In other words, the algorithm cannot accurately identify and keep high-quality individuals, resulting in an optimal individual with poor quality after 3000 iterations.

The maximum within F3 is tow times of the fitness of the best individual. Therefore, the fitness of individuals depending on the best one and themselves, and the fitness value cannot directly reflect the change of individuals between t generation and t+1 generation. The algorithm outputs the optimal solution of the last iteration instead of history optimal.
For a fitness function, we also hope that the fitness function can show the same excellent performance for different spectra. The statistical results of $Q_s$ of history optimal individuals are shown in Fig. 4, and each result was coming from 20 independent runs.

![Fig. 4. Qs of history optimal individuals in GA](image)

Most neutron energy spectra have good continuity, unless in some extreme cases [8, 16]. As the continuity (calculated as $\rho_1$ in F4 and F8, the continuity of Spec1 to Spec4 were 1.2, 0.1, 0.01, and 0.001) of the reference spectrum becomes better from Spec1 to Spec4, $Q_s$ of each solution becomes better in F4. Because F4 maintains a great continuity penalty factor to eliminate individuals with poor continuity in time. The ability of F2, F5, F7, and F8 to perceive the shape shake are weaker than F1, F3, and F6, but better results can be obtained.

To investigate the application of the fitness function in different algorithm frameworks, this work proposed to use the DEA to unfold the neutron energy spectrum. It can be considered as the feasibility verification of DEA for neutron spectrum unfolding.
As shown in Fig. 5, each result was also coming from 20 independent runs. The quality and the robustness of results except F3 has been improved in the DEA, and there is a surprise that F1 and F6 have the greatest performance improvement in DEA compared with the GA, because in DEA, only the individual with higher fitness will be remain, which eliminated the impact of inferior individuals on selection operation. All those improvements in DEA may provide some ideas for modifying GA’s genetic operation strategy, that eliminates inferior individuals in time or increase the proportion of better individuals.

IV. CONCLUSION

This work investigated the evolution behavior of the eight different fitness functions used in the GA and the DEA, and four different spectra in IAEA 403 report were selected to perform, and three conclusions can be obtained:

1. The fitness functions are designed for GA can also be used in DEA, and confirmed that the good potential of DEA for neutron energy spectrum unfolding.

2. Specifically, the fitness functions such as F1 and F6 with a upper are not friendly to GA, which will weaken the ability of perception and response to fitness change, therefore, we recommend that pay attention to increase the selection pressure of GA when using; F2, F5, F7 and F8 show that a good balance between the algorithm frameworks and the energy
spectra; For F4, a function with a large continuity penalty item shows the best solution, which means the fitness function designed for the characteristics of the spectrum can improve the performance; For F5 shows the best robustness in DEA; For F3, A dynamic maximum makes the function to output the worst solutions, so, there is not enough courage to use in most cases.

3. The fitness increase greatly, while the quality of the energy spectrum may not improve, which results in a waste of computing time. "shape change" as the condition for stopping iteration or restart is recommend, it is significant for applications of online and fast unfolding.

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| F4 [16] | \( f_4 = \frac{1}{\left( \sum_{j=1}^{m} T_j^2 + \beta_4 \cdot p_1 \right)} \) |
| F5 | \( f_5 = \delta \left( \frac{\sum_{j=1}^{m} R_y \phi_i^{cal}}{C_j} \right) \) |
| F6 [4] | \( f_6 = \beta_6 + \sum_{j=1}^{m} - \left( T_j / C_j \right)^2 \) |
| F7 [17] | \( f_7 = \sqrt{\sum_{j=1}^{m} C_j^2 / \sum_{j=1}^{m} T_j^2} \) |
| F8 | \( f_8 = \frac{1}{\left( \sum_{j=1}^{m} T_j / C_j \right)^2 + 0.5 \cdot \beta_8 \cdot \left( p_1 + p_2 \right)} \) |

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Fig. 4. Qs of history optimal individuals in GA
Fig. 5. Qs of history optimal individuals in DEA