GUP inspired asymptotic safety in the radiation process of thin accretion disk around a Schwarzschild-like black hole

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We study quantum gravity effects on radiation process of the thin accretion disks around Schwarzschild-like black-hole. The quantum gravity correction is invoked through the framework of generalization of uncertainty which is equivalent to renormalization group improved quantum gravity and maintain the limit of the asymptotically safe proposition of gravity. It admits a free parameter that encodes the quantum effects on the spacetime geometry. It allows us to study how the thermal properties of the disk are modified in the quantum regime. We explicitly make estimations of quantum correction to the time averaged energy flux, the temperature of the disk, the differential luminosity, and the conversion efficiency of accreting mass into radiation. We observe a conspicuous shifting of the radius of the innermost stable circular orbit (ISCO) toward small values together with an enhancement of the maximum of the average thermal radiation and greater conversion efficiency of accreting mass into radiation compared to the classical gravity scenario.

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I. INTRODUCTION

Black hole is a remarkable solution to the classical field equations thanks to Einstein within the context of general relativity theory. In astrophysical sense it is the regions of spacetime that gets deformed undergoing gravitational collapse. The solutions of the equation are inflected with singularities inside the event horizon. Therefore it loses its prophetic character and it is no longer possible to extend classical formulation of spacetime inside the event horizon. This forces to think about general relativity as an effective theory of gravity that remains valid solely up to certain energy scales. At high energy scales, such as the Planck scale, it is expected that a full theory of quantum gravity will resolve the unphysical singularities in the spacetime manifold. So the predictively be rehabilitated. However, attempts of describing gravity within the framework of quantum theory face the danger of the perturbative non-renormalizable character of general relativity. As a consequence, different approaches have been surfaced, namely a loop quantum gravity spin foams [1–5], string theory [6–8], etc.. Another promising proposal to deal with this downside is that the Asymptotic Safety scenario which uses the useful techniques of the functional renormalization group. The existence of a non-Gaussian fixed point of the gravitational renormalization group flow that controls the behavior of the idea at trans-Planckian energies is that the main speculation within this construction. The physical degrees of freedom interact predominantly anti-screening within the neighborhood of the non-Gaussian fixed point which renders physical quantities safe from unphysical divergences within the vicinity of the Plank scale which renders physical quantities safe from unphysical divergences [9]. It is the fact that asymptotic safety defines a harmonious and prognostic scientific proposition among the frame of quantum field theory we couldn’t ignore its eventuality, still, it
stands as a vaticination since a rigorous evidence for the existence of the non-Gaussian fixed point continues to be lacking. It is the fact that AS defines a consistent and prognostic scientific theory for gravity among the framework of quantum theory we could not ignore its potential, however it stands as a prediction since a rigorous existence proof for the NGFP continues to be lacking. There is, however, substantial evidence supporting the existence of the non-trivial renormalization group fixed point at the center of this construction [10, 11]. The importance of black holes as testing ground for gravity theories within the strong field regime has motivated numerous studies on the implications of asymptotic safety gravity for black hole physics, most of them geared toward determining quantum corrections to the classical metrics.

Generalization of uncertainty is a fascinating extension which has its origin in string theory and loop quantum gravity. In the article [12], the author analyzes quantum gravity corrections to the accretion onto black holes within the context of asymptotically safe gravity. The asymptotic safety was maintained there invoking running Newton constant proposition. The identical asymptotic safety could be contemplated within the Generalize Uncertainty Principle (GUP) framework which we will attempt to explore and study the physical quantities connected to accretion process maintaining AS during this composition. The usage of the generalised scientific theory based on GUP, which renders asymptotic safety, is used here to account for quantum gravity corrections to the Schwarzschild metrics. Quantum spacetime with running gravitational coupling that included into the scenario gets manifested in effective mass.

Investigating the gravity-induced quantum interference pattern, followed by the Experiment with Gedanken-experiment to determine the weight of photon it has been established that the running Newton’s law applies to photons. Examining the quantum interference pattern caused by gravity, and then the thought Gedanken-experiment for determining the weight of a photon, it has been established that the running Newton gravitational constant can be stimulated by the principles of generalised uncertainty, which ends up in quantum gravity corrections to Schwarzschild region metrics [13]. The enhanced quantum corrected metric is used here cherish the Schwarzschild metric indeed and will be used to study the quantum corrected thermal behavior of thin accretion disc.

II. INSERTION OF QUANTUM GRAVITY EFFECT INTO THE SCHWARZSCHILD METRIC

Before jumping into the formulation of having a modified black hole endowed with quantum gravity effect through GUP perspective it would be beneficial to give a brief description of GUP. Let us turn into that.

A. Description GUP with minimum measurable length

In recent times it has been noticed that various approaches to quantum gravity including string theory [14, 15], noncommutative geometry [18], loop quantum gravity [20] predict the existence of a minimum measurable length of the order of Planck length. It leads to different generalizations of usual uncertainty relation in the context of quantum gravity [16, 20–22, 29], although Heisenberg uncertainty principle is the cornerstone of the formulation of quantum theory that puts a fundamental limit on the precision of measuring the position and momentum. In the Heisenberg uncertainty principle, $\Delta x \rightarrow 0$ leads to $\Delta p \rightarrow \infty$, therefore, the standard Heisenberg uncertainty relation $\Delta x \Delta p \geq \hbar$ becomes skinny to explain the existence of a minimum measurable length. Thus, it necessitates the replacement of Heisenberg uncertainty principle by the Generalized Uncertainty Principle (GUP) [23] to accommodate the possibility of minimum measurable length. In the article [23] Kempf showed that a generalized uncertainty relation could be defined by

$$\Delta x_k \Delta p_l \geq \hbar \delta_{kl} \left( 1 + \eta \left[ (\Delta p)^2 + \langle p \rangle^2 \right] \right),$$

(1)

where $\eta$ is GUP parameter which has the the definition $\eta = \frac{\eta_0}{M_{PL} c^2}$, where $M_{PL}$ is the Planck mass and $\eta_0$ is parameter of the order of unity. The expression (1) has the potential to accommodate the minimum measurable length within the revised principle. It is straightforward to see that for the above GUP (1), the minimum non-zero length is found out to be

$$\langle \Delta x \rangle_{Min} = \hbar \sqrt{\eta} \sqrt{1 + \eta \langle p \rangle^2},$$

(2)

where setting $\langle p \rangle = 0$ results to the absolute minimal measurable length:

$$\langle \Delta x \rangle_{Min} = \hbar \sqrt{\eta} = \sqrt{\eta_0} M_{PL}.$$
where \( l_{Pl} = \left( \frac{G \hbar}{c^3} \right)^{\frac{1}{2}} \approx 10^{-35} \text{m} \) \([23, 25]\) is the Planck length. This generalized uncertainty \([1]\) corresponds to the following deformed commutation relation between position and momentum \([23]\)

\[ [x_k, p_l] = i\hbar \delta_{kl} (1 + \eta p^2), \]  

(4)

where \( p^2 = \sum_k p_k^2 \). What follows next is an attempt to have a modification of the relation \([1]\) and \([3]\) in one dimension by

\[ \Delta x \Delta p \geq \hbar \left( 1 + \eta (\Delta p)^2 \right), \]  

(5)

and

\[ [x, p] = i\hbar \left( 1 + \eta p^2 \right) = i\hbar \zeta, \]  

(6)

where \( \zeta = 1 + \eta p^2 \). Hence, Eqn. \([5]\) can be rewritten as

\[ \Delta x \Delta p \geq \hbar \zeta. \]  

(7)

III. SCHWARZSCHILD METRIC ENDOWED WITH QUANTUM GRAVITY CORRECTION

Let us now formulate a Schwarzschild-like spacetime metric where quantum gravity correction gets induced through generalized uncertainty principle keeping in sight on the Gedanken-experiment initially proposed by Einstein. Einstein made a trial to exhibit the violation of the indeterminacy principle through a Gedanken-experiment which was purported to measure the load of photons \([26–28]\). He assumed a box containing photon gas with a totally reflective wall was suspended by a spring scale. There was a system inside the box that caused the shutter to open and shut at moment \( \tau \) for the time interval \( \Delta t \), which allowed to passing out just one photon. A clock capable of showing extremely high precision measurement of your time could be accustomed measure the interval \( \Delta \tau \) and at the same time the mass difference of the box would determine the energy of the emitted photon as per Einstein’s assumption, the amount required for photon radiation is precisely \( \Delta \tau \rightarrow 0 \) which may be lead to the violation of the uncertainty relation for energy and time, i.e. \( \Delta E \Delta \tau \rightarrow 0 \). However Bohr argued that \([27]\), Einstein’s deduction wasn’t flawless since he neglected the time-dilation effect which might play a significant role due to the difference of gravitational potential. Based on general relativity, when altitude changes, the speed of your time flow also changes thanks to the change in their gravitational potential. Thus, for the put down the box, the time uncertainty \( \Delta \tau \), in terms of the vertical position uncertainty \( \Delta x \), would be expressed as \([27, 28]\)

\[ \Delta \tau = \frac{g \Delta x}{c^2 \tau}, \]  

(8)

where \( \tau \) represents the time period of weighing the photon. As it is known, according to the quantum theorem, the uncertainty relation in energy and time of the photon is express as \( \Delta E \Delta \tau \geq \hbar \) which after substituting Eqn. \([8]\) turns into

\[ \Delta E \geq \frac{\hbar c^2}{g \tau} \Delta x, \]  

(9)

Let us now look at the relation between the weight of the photon in the Gedanken-experiment and the corresponding quantities in quantum mechanics in the GUP framework \([9]\). If we focus on the original position of the pointer on the box before opening the shutter, we will find that after releasing photon in the box the pointer moves up with reference to its original position. To get back the pointer in its original position in a period of time \( \tau \), some weights equal to the weight of the photon must be added to the box. If we now use Eqn. \([8]\), having accuracy in measuring the position \( \Delta x \) as marked by the indicator of the clock, the minimum uncertainty in momentum \( \Delta P_{Min} \) will be

\[ \Delta P_{Min} = \frac{\zeta \hbar}{\Delta x}, \]  

(10)

since the quantum weight limit of a photon is \( g \Delta m \), so in a period of time \( \tau \) the smallest photon weight will be equal to \( \zeta \hbar/\tau \Delta x = \Delta P_{Min}/\tau \leq g \Delta m \). Now, using Eqn. \([10]\) we also find that

\[ \zeta \hbar = \Delta x \Delta P_{Min} \leq g \tau \Delta x \Delta m. \]  

(11)

If the relation \( \Delta E = c^2 \Delta m \), is used for \( \Delta m \) the Eqn. \([11]\) can be written down as

\[ \Delta E \geq \frac{\zeta \hbar c^2}{g \tau} \Delta x. \]  

(12)
where $\zeta$ refers to the the GUP effects. Note that in the absence of GUP framework, i.e. when $\eta \to 0$, the standard energy-time uncertainty relation Eqn. (9) is reobtained.

A careful look on the standard uncertainty relation between energy and time (9) and the generalized uncertainty relation (17) gives rise to an interpretation that gravitational field strength $g$ is modified to $\frac{g}{\zeta}$. Then, replacing $g$ by $\frac{g}{\zeta}$ we have

$$\tilde{g} = \frac{g}{\zeta} = \frac{G_0 M}{\zeta R^2}. \quad (13)$$

Hence, using (13), the modified Schwarzschild metric turns into

$$ds^2 = -\left(1 - \frac{2G_0 M}{\zeta c^2 r}\right)c^2 dt^2 + \left(1 - \frac{2G_0 M}{\zeta c^2 r}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (14)$$

where $G_0$ stands for universal gravitational constant. On the other hand, as stated in some other literature [24, 30], when two virtual particles with energies $\Delta E$ are at a distance $\Delta S$ from each other, the tidal force between them is obtained by

$$F = \frac{2G_0 M \Delta E}{r^3} \Delta x. \quad (15)$$

So, the uncertainty in momentum is given by

$$\Delta p = F \Delta r = \frac{2G_0 M \Delta E}{r^3} \Delta r \Delta r, \quad (16)$$

where $\Delta t$ represents the life time of the particle.

If virtual particles turns into real particles having the exposure of tidal force, the uncertainty relations $\Delta p \Delta x \geq \hbar$ and $\Delta E \Delta t \geq \hbar$ can be used with reasonably well justifiable manner. Therefore, using these uncertainty relations in (16), we find that

$$(\Delta p)^2 \geq \frac{2\hbar^2 G_0 M}{r^3}. \quad (17)$$

Accordingly, it can be written that $p^2 \approx (\Delta p)^2 \approx \frac{2\hbar^2 G_0 M}{r^3}$. Hence applying this modified uncertainty relation, we obtain the Schwarzschild metric in the presence of minimal measurable length as

$$ds^2 = -\left(1 - \frac{2G_0 M r^2}{c^2 \left(r^3 + \eta \frac{2\hbar^2 G_0 M}{c^2}\right)}\right)c^2 dt^2 + \left(1 - \frac{2G_0 M r^2}{c^2 \left(r^3 + \eta \frac{2\hbar^2 G_0 M}{c^2}\right)}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2). \quad (18)$$

This metric specifies a set of spacetime that depends on different scales of momentum via the modified mass of the black hole. For dimensionless case, the modified Schwarzschild metric (18) reads

$$ds^2 = -\left(1 - \frac{2Mr^2}{r^3 + 2\eta M}\right)dt^2 + \left(1 - \frac{2Mr^2}{r^3 + 2\eta M}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2). \quad (19)$$

In the following sections, for the sake of simplicity, we use the modified Schwarzschild metric (18) in the form of

$$ds^2 = -f(\eta c^2) c^2 dt^2 + \frac{1}{f(\eta c^2)} dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2), \quad (20)$$

where

$$g(r\eta) = f(\eta) = 1 - \frac{2G_0 M r^2}{c^2 \left(r^3 + \eta \frac{2\hbar^2 G_0 M}{c^2}\right)} = 1 - \frac{2MG_0}{c^2 r} \frac{r^3}{r^3 + 2\eta \hbar^2 \left(\frac{M G_0}{c^2}\right)} = 1 - \frac{2M}{c^2 r} G(\eta), \quad (21)$$

where

$$G(\eta) = \frac{G_0 r^3}{r^3 + 2\eta \hbar^2 \left(\frac{M G_0}{c^2}\right)}. \quad (22)$$
So for \( \eta \rightarrow 0 \), \( G(\eta \eta) \rightarrow G_0 \) and the quantum correction disappears and we get back to the Schwarzschild metric. If we consider the metric in equation in natural unit setting \( c = 1 \) and \( G_0 = 1 \) in (19) to find out the position of the Horizon we need to have the solution of the equation

\[
1 - \frac{2Mr^2}{r^3 + 2\eta M} = 0,
\]

which has the solution

\[
r_H = \frac{2}{3} M + \frac{4}{3} M \cos^{-1} \left( \frac{1}{3} \frac{8}{M^2} \left(1 - \frac{27}{8} \frac{\eta}{M^2}\right)\right),
\]

provided the mass of the black hole satisfy the condition \( M > M_c \), where \( M_c = \frac{27}{16} \eta \).

When \( M \leq M_c \) it fails to describe any horizon in the spacetime geometry since equation (23) can not provide any positive solution in that situation. Plots of the improved metric coefficient \( f(r) \) for different value of mass \( M \) with \( \eta = \frac{16}{27} \) and \( \eta = 0 \) (classical) are given below.

**FIG. 1:** Plots of the improved metric coefficient \( f(\eta \eta) \) for \( M = 0.4 < M_c \) (yellow), \( M = M_c \) (red), and \( M = 1.8 > M_c \) (blue), with \( M_c = 1 \) corresponding to \( \eta = \frac{16}{27} \). The black line shows the classical \( f_0(r) \) for \( M = 1.8 \) corresponding to \( \eta = 0 \) and green line shows the effect with \( M = 0 \).

**IV. DESCRIPTION OF THE GEODESIC IN THE GUP INSPIRED QUANTUM CORRECTED SPACETIME GEOMETRY**

In this article we are intended to study the accretion phenomena onto a spherically symmetric Schwarzschild black hole employing a modified uncertainty relation that admits a quantum gravity correction that finds its place holding the hand of the concept of minimal measurable length. There are other models which are associated with the GUP correspond to the concept of minimum measurable length and maximum observable momentum simultaneously and it is a fascinating problem which is amenable for application for all kinds of available generalization of uncertainty relation [31–41]. We will only consider the generalization related to the existence of a minimal length. During this context we consider steady, accretion onto a modified static and spherically symmetric Schwarzschild black hole. We obtain the critical point, critical fluid velocity, temperature, mass accretion rate, and observed total integrated flux within the proposed GUP framework.

The line element in Eqn. (19) can be written down within the form of a Schwarzschild-like metric by introducing an effective mass \( M_{\text{eff}}(r) \) which could be a function of the radial coordinate and depends on the free parameter \( \xi \), that is

\[
ds^2 = -\left(1 - 2\frac{M_{\text{eff}}(r)}{r}\right)dt^2 + \left(1 - 2\frac{M_{\text{eff}}(r)}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2),
\]

with

\[
M_{\text{eff}}(r) = \frac{M}{1 + 2Mr^2/\eta}.
\]
Putting $\tilde{\eta} = \frac{M}{M^3} = \frac{\eta}{M^2}$, and $x = \frac{\eta}{M}$, Eqn. (26) can be rewritten down as
\[
M_{\text{eff}}(r) = \frac{M}{1 + \frac{2}{x^3}}.
\] (27)

Plots of the effective mass per unit mass as a function of $x$ is shown below.

FIG. 2: Plots of the effective mass as a function of $x$ for $\eta = \tilde{\eta}c = 16/27$ (red) and $\tilde{\eta} = 0.3$ (blue). The vertical dashed lines indicate the ISCO for the same values of $\tilde{\eta}$ respectively.

Using this definition (27) we can write down the Lagrangian from the modified metric endowed with quantum correction replacing $M$ by $M \rightarrow M_{\text{eff}}(r)$.
\[
2L = -\left(1 - 2 \frac{M_{\text{eff}}(r)}{r}\right)\dot{t}^2 + \left(1 - 2 \frac{M_{\text{eff}}(r)}{r}\right)^{-1} \dot{r}^2 + r^2 \dot{\phi}^2,
\] (28)

where the over dots referees to derivative with respect to the affine parameter. We have restricted ourselves to the equatorial plane setting ($\theta = \pi/2, \dot{\theta} = 0$). This indeed does not loose any generality. From Eqn. (28), the generalized momenta are computed as follows
\[
 p_t = -\left(1 - 2 \frac{M_{\text{eff}}(r)}{r}\right) \dot{t} = -K,
\] (29)
\[
 p_r = \left(1 - 2 \frac{M_{\text{eff}}(r)}{r}\right)^{-1} \dot{r},
\] (30)
\[
 p_\phi = r^2 \dot{\phi} = A.
\] (31)

The constants $K$ and $A$ are representing the energy and angular momentum per unit rest mass of the particle respectively, The canonical Hamiltonian is abstained by the Legendre transformation
\[
H = p_t \dot{t} + p_r \dot{r} + p_\phi \dot{\phi} - L,
\] (32)
and it reads
\[
2H = -K \dot{t} + \left(1 - 2 \frac{M_{\text{eff}}(r)}{r}\right)^{-1} \dot{r}^2 + A \dot{\phi} = -1.
\] (33)

Note that it is independencies of time so is a constant quantity. The particle is assumed at rest at infinity that allows to set the righthand side of the equation equal to $-1$. We obtain the equation corresponding to the energy plugging in Eqns. (29) and (31) into Eqn. (33)
\[
\frac{1}{2} \dot{r}^2 + V_{\text{eff}}(r) = \frac{1}{2}(K^2 - 1),
\] (34)
where $V_{eff}(r)$ is given by

$$V_{eff}(r) = -\frac{M_{eff}(r)}{r} + \frac{h^2}{2r^2} - \frac{M_{eff}(r)A^2}{r^3}. \quad (35)$$

This $V_{eff}(r)$ is nothing but the effective potential per unit mass in this situation. Let us now proceed to find out the equation of the orbit of the massive particle of effective mass $M_{eff}(r)$. To this end we define

$$\dot{r} = \frac{dr}{d\tau}. \quad (36)$$

By using Eqn. (35) we can write

$$\dot{r} = \frac{dr}{d\phi} = \frac{A}{r^2} \frac{dr}{d\phi}. \quad (37)$$

It is beneficial to make the change of variable $u = 1/r$ at this stage. In terms of $u$ Eqn. (34) can be written down as

$$\left(\frac{du}{d\phi}\right)^2 + u = \frac{(K^2 - 1)}{A^2} + \frac{2uM_{eff}(u)}{A^2} + 2u^3 M_{eff}(u), \quad (38)$$

using Eqn. (37). Eqn. (38) leads us to the following equation of the orbit after undergoing the differentiation with respect to $\phi$

$$\frac{d^2u}{d\phi^2} + u = \frac{M}{(1+2m\eta u^3)} \left[ \frac{1}{A^2} - \frac{6M\eta u^5}{A^2(1+2m\eta u^3)} + 3u^2 - \frac{6M\eta u^5}{(1+2m\eta u^3)} \right]. \quad (39)$$

A. Condition of circular orbits of the massive particles

To determine the fundamental equations describing the time-averaged radial disk structure, we first explicitly calculate the precise expression of angular momentum $A$, the energy $K$ per unit mass and the angular velocity $\omega$ of particles taking possession of the circular trajectories.

For circular orbits within the equatorial plane $\dot{r} = 0$. Therefore that $r$ is constant, and consequently $u = 1/r$ may be a constant indeed. We now define two dimensionless quantities $x = r/M$ and $\tilde{\eta} = \frac{M\eta}{M^3} = \frac{\eta}{M^2}$. From Eqn.(39) leads us we obtain an expression for the specific angular momentum (angular momentum per unit mass) $A$ and if we rewrite $A$ in terms of the dimensionless quantities $x$ and $\tilde{\eta}$ it reads

$$A = \frac{x^2 M \sqrt{(x^3 - 4\tilde{\eta})}}{\sqrt{(x^6 - 3x^4 + 4\tilde{\eta}x^3 + 4\eta^2)}}. \quad (40)$$

Putting $\bar{A} = \frac{A}{M}$, variation of $\bar{A}$ with $x$ is shown below.

FIG. 3: The angular momentum $\bar{A} = \frac{A}{M}$ vs. $x$ for several values of $\tilde{\eta}$. From left to right: $\tilde{\eta} = \tilde{\eta}_C = \frac{16}{27}$ (red), $\tilde{\eta} = 0.3$ (blue). The black solid curve corresponds to the classical case $\tilde{\eta} = 0$. 
Making use of the condition of having circular orbit $\dot{r} = 0$ in Eqn. (34) we can have the expression of the specific energy (energy per unit mass) $K$, and substituting the expression for $\bar{A}$ the specific energy $K$ can be expressed as

$$K = \frac{\sqrt{(x^3 - 2x^2 + 2\eta)(x^6 - 2x^5 + 4\eta x^3 - 4\eta^2 x^2 + 4\eta^2)}}{\sqrt{(x^3 + 2\eta)(x^6 - 3x^5 + 4\eta x^3 + 4\eta^2)}}.$$  

(41)

Upon substituting Eqn. (40) into Eqn. (35), the effective potential acquires the following form

$$V_{\text{eff}} = -\frac{x^2}{(x^3 + 2\eta)} \left[ 1 - \frac{x^2(x^3 - 4\eta)}{(x^6 - 3x^5 + 4\eta x^3 + 4\eta^2)} - \frac{x^2(x^3 - 4\eta)}{2(x^6 - 3x^5 + 4\eta x^3 + 4\eta^2)} \right].$$  

(42)

Plot of $V_{\text{eff}}$ as function of $x$ for the same values of the free parameter $\tilde{\eta}$ in the range $0 \leq \tilde{\eta} \leq 16/27$ is shown in Figure 4.

FIG. 4: Plot of the effective potential for different values of $\tilde{\eta}$. From left to right: $\tilde{\eta} = \tilde{\eta}_C = 16/27$ (red), $\tilde{\eta} = 0.3$ (blue). The black solid curve refers to the classical case $\tilde{\eta} = 0$.

Using Eqns. (40) and (41) the angular velocity can be computed from Eqns. (29) and (31) as

$$\omega = \frac{\dot{\phi}}{\dot{t}} = \frac{\sqrt{(x^3 - 4\eta)(x^3 - 2x^2 + 2\eta)}}{M \sqrt{(x^3 + 2\eta)(x^6 - 2x^5 + 4\eta x^3 - 4\eta^2 x^2 + 4\eta^2)}}.$$  

(43)

Note that $V_{\text{eff}}, \bar{A}, K$ and $\omega$ reduce to the corresponding classical expressions in the limit $\tilde{\eta} \to 0$.

At the local minima of the effective potential the possession of circular orbits materialize. Thus, the radius of the innermost stable circular geodesic orbit $x_{\text{isco}}$ is found out from the condition

$$\frac{d^2 V_{\text{eff}}}{dx^2} = 0.$$  

(44)

Note that here it will appear in a dimension less manner. On the other hand ISCO can be calculated from the following condition [42].

$$\frac{dA}{dx} = \frac{dK}{dx} = 0.$$  

(45)

We are furnishing the ISCO of the thin disk for a set of selected values of $\tilde{\eta}$ in a tabular form

| $\tilde{\eta}$ | $x_{\text{isco}}$ |
|---------------|-------------------|
| $16/27$       | 5.58396           |
| 0.3           | 5.80455           |
| 0             | 6.0               |

TABLE I: values of $x_{\text{isco}}$ for different $\tilde{\eta}$.
FIG. 5: The sketch of effective potential for the values of $\tilde{\mathcal{A}}$, evaluated at ISCO. From left to right: $\tilde{\eta} = \tilde{\eta}_C = \frac{16}{27}$ (red), $\tilde{\eta} = 0.3$ (blue). The classical case is described by the black solid curve where $\tilde{\eta} = 0$.

TABLE II: The energy per unit mass at the ISCO and the efficiency $\epsilon$ of the conversion of the accreted mass into radiation for several values of $\tilde{\eta}$.

| $\tilde{\eta}$ | $A_{\text{ISCO}}$ | $\epsilon$ |
|---------------|----------------|---------|
| $\frac{16}{27}$ | 0.939752 | 6.0248 |
| 0.3           | 0.941393 | 5.8607 |
| 0             | 0.942809 | 5.7191 |

where, due to the fact that $A$ is constant for circular orbits, the derivatives must be calculated for $V_{\text{eff}}$ as given by Eqn. (35). This yields

$$\frac{d^2 V_{\text{eff}}}{dx^2} = - \frac{(x^6 - 28\tilde{\eta}x^3 + 8\tilde{\eta}^2)}{(x^3 + 2\tilde{\eta})^3} + \frac{3(x^3 - 4\tilde{\eta})}{(x^6 - 3x^5 + 4\tilde{\eta}x^3 + 4\tilde{\eta}^2)} - \frac{6x^5(x^3 - 4\tilde{\eta})}{(x^3 + 2\tilde{\eta})^2(x^6 - 3x^5 + 4\tilde{\eta}x^3 + 4\tilde{\eta}^2)} + \frac{18x^8(x^3 - 4\tilde{\eta})}{(x^3 + 2\tilde{\eta})^3(x^6 - 3x^5 + 4\tilde{\eta}x^3 + 4\tilde{\eta}^2)}$$

(46)

V. MASS ACCRETION RATE FOR THIN ACCRETION DISK

A general relativistic treatment of an accretion disk around a black hole was reported in the pioneering articles [42, 43]. Let us consider a simple non-relativistic model of an accretion disk around a compact central object. Recent investigation on this issue in different perspective are [44–47]. Here it is assumed that matter spirals inwards by losing angular momentum which, because of turbulent viscous density be transferred outward through the disk. As the gas moves inwards, it loses gravitational energy and heat over the surroundings by emitting thermal radiation [48]. This model assumes that disk is in a quasi-steady state lying in the equatorial plane of an stationary, axially-symmetric spacetime back ground. The disk material is assumed to be moving in nearly geodesic circular orbit. The disk is so thin that its maximum thickness $D$ satisfies $D/2R << 1$ where $R$ refers to the characteristic radius of the disk. The heat produced by stress and dynamical friction is efficiently emitted in the form of radiation substantially from the surface of the disk. The amounts describing the thermal characteristics of the disk are averaged over the azimuthal angle $\phi = 2\pi$, over the thickness $D$, and over the time scale $\Delta \tau$. Here $\tau$ is the time that the gas takes to flow inward through a distance $2D$. With these propositions, the time-averaged radius of the disk is attained from the laws of conservation of rest mass energy and angular momentum. The integration of the equation of mass conservation reveals
that the mass accretion rate remains constant for this process

\[ \dot{M} = \frac{dM}{d\tau} = -2\pi \Sigma(r)v^r = \text{constant}, \quad (47) \]

where \( v^r \) and \( \sigma \) are respectively the radial velocity and surface density of the accretion disk. Now the combined criteria of the energy and angular momentum conservation leads us to find out the expression of the differential of the luminosity \( dL_\infty \) at infinity \([42, 49]\).

\[ \frac{dL_\infty}{d\ln r} = 4\pi r \sqrt{-g} K F(r), \quad (48) \]

where \( F \) is the the flux of radiant energy emitted from the upper face of disk in the local frame of the accreting fluid. Let us call it as \( B(r) \equiv BMx \) for later convenience. It has the expressed in terms of the specific angular momentum \( A \), the specific energy \( K \) and the angular velocity \( \Omega \) which reads

\[ F(r) = -\frac{\dot{M}}{4\pi \sqrt{-g}} \frac{1}{(K - \Omega A)^2} \frac{d\Omega}{dr} \int_{r_{isc}}^r (K - \Omega A) \frac{dA}{dr} dr, \quad (49) \]

where \( \sqrt{-g} = r \) it remains the same for both for the GUP-improved quantum corrected metric and for the classical Schwarzschild metric. The numerical integration of Eqn. \([49]\) gets simplified by using the relation \( \frac{dK}{dr} = \Omega \frac{dA}{dr} \) \([42]\) and integrating by parts we have

\[ \int_{r_{isc}}^r (K - \Omega A) \frac{dA}{dr} dr = Kh - K_{isc} A_{isc} - 2 \int_{r_{isc}}^r A \frac{dA}{dr} dr. \quad (50) \]

Plot of energy flux per unit accretion rate \( \frac{F(x)}{\dot{M}} \) from a thin accretion disk around a GUP-improved Schwarzschild black hole as a function of \( x \) is shown below.

![Plot of energy flux per unit accretion rate from a thin accretion disk around a GUP-improved Schwarzschild black hole as a function of x](image_url)

**FIG. 6:** Energy flux per unit accretion rate from a thin accretion disk around a GUP-improved Schwarzschild black hole for \( \tilde{\eta} = \tilde{\eta}_c = \frac{16}{27} \) (red), \( \tilde{\eta} = 0.3 \) (blue). The black solid curve is the energy flux from the disk around a classical Schwarzschild black hole (\( \tilde{\eta} = 0 \)).

Plots of the differential luminosity at infinity per unit accretion rate from a thin disk around a GUP-improved Schwarzschild black hole as a function of \( x \) is shown below.
The radiation emitted can be considered as a black body radiation with the temperature given by

$$T(r) = \sigma^{-\frac{1}{4}}F(r)^{\frac{1}{4}},$$

(51)

as it is assumed that during the accretion process the disk is in thermodynamic equilibrium. Here $\sigma$ stands for Stefan-Boltzmann constant.

The radial profile of the temperature of the accretion disk (more precisely, the radial profile $\frac{1}{\sqrt{M_\eta \mathcal{F}}}$), is shown for different values of the dimensionless free parameter $\tilde{\eta}$ namely, for the critical value $\tilde{\eta}_c = 16/27$, for $\tilde{\eta} = 0.3$, and for the classical solution $\tilde{\eta} = 0$.

FIG. 7: Differential luminosity at infinity per unit accretion rate from a thin disk around a GUP-improved Schwarzschild black hole for $\tilde{\eta} = \tilde{\eta}_c = \frac{16}{27}$ (red), $\tilde{\eta} = 0.3$ (blue). The black solid curve is the energy flux from the disk around a classical Schwarzschild black hole ($\tilde{\eta} = 0$).

FIG. 8: Radial profiles of the temperature per unit accretion rate of a thin accretion disk around a GUP-improved Schwarzschild black hole for $\tilde{\eta} = \tilde{\eta}_c = \frac{16}{27}$ (red), $\tilde{\eta} = 0.3$ (blue). The black solid curve corresponds to the temperature of the disk around a Schwarzschild black hole in general relativity $\tilde{\eta} = 0$. 
| $\tilde{\eta}$ | $\frac{F(x)}{\dot{M}}$ | $B(x)$ | $T(x)$ |
|-----------|------------------|-------|--------|
|           | Maximum value Increase in max value in % | Maximum value Increase in max value in % | Maximum value Increase in max value in % |
| 0         | 0.00001367 – | 0.0233514 – | 0.0608144 – |
| 0.3       | 0.00001484 8.55 | 0.0238780 2.25 | 0.0620756 2.07 |
| 16/27     | 0.00001630 19.23 | 0.0244795 4.83 | 0.0635401 4.48 |

TABLE III: Table containing maximum value and increase in maximum value of the time averaged energy flux per unit accretion rate $\frac{F(x)}{\dot{M}}$, the differential luminosity per unit accretion rate $B(x)$ and the radial profile of the temperature of the accretion disk $T(x)$ at $\tilde{\eta} = 0, 0.3$, and $\frac{16}{27}$, respectively.

VI. SUMMARY AND CONCLUSION

In this article, we have studied quantum gravity corrections to the thermal properties of a relativistic thin accretion disk around a GUP improved Schwarzschild black hole within the IR-limit of the asymptotic safety situation for quantum gravity. We have calculated, precisely, the corrections to the time-averaged energy flux, the differential luminosity at infinity, the disk temperature, and conjointly the conversion potency of accreting mass into radiation in comparison to the predictions of classical general relativity theory.

We have found that an increase in the parameter $\eta$ that encodes the quantum effects within the GUP framework, not solely results in a shifting of the radius of the inner fringe of the disk and also the ISCO, toward smaller values, but, as a consequence, we have found a tendency to rising the energy radiated far off from the disk together with an increase in temperature of the disk. We have noticed a bent to conjointly rise in the differential luminosity reaching the observer far away at infinity along with a higher conversion potency of accreting mass into radiation. Besides, a shifting of the height of the radial profiles the thermal properties toward smaller values of the radial coordinate have conjointly been observed.

In [47], it has been shown that experimental knowledge reveals that this model is associated with nursing correct one at low luminosities however it is not so well appropriate at high luminosities, a regime that a skinny accretion disk provides a much better description [50, 51]. Throughout this text it has been reinstated everywhere once more. Once again, our investigation shows that quantum gravity within the physics of black hole manifests itself at distances even larger than the radius of the ISCO and do not seem to be restricted inside of the horizon so greatly to its immediate neighborhood. Our result encourages the study of quantum gravity effects on several realistic black holes environments, like accretion onto quantum improved Kerr, Kerr-Sen black hole as a varied or complementary path to confront the predictions of asymptotic safety with astronomical observation.

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