The non-linear evolution of jet quenching

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arXiv: 1403.1996

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closely related work:
Liou, Mueller, Wu (arXiv:1304.7677)
Blaizot, Mehtar-Tani (arXiv:1403.2323)
E.I., Triantafyllopoulos (arXiv:1405.3525)
Hard probes in heavy ion collisions

- Hard particle production in nucleus–nucleus collisions (RHIC, LHC) can be modified by the surrounding medium (‘quark–gluon plasma’).

The ensemble of these modifications: ‘jet quenching’
- energy loss, transverse momentum broadening, di–jet asymmetry ...
- cf. the review talks by Federico Antinori and Jean–Paul Blaizot

- Assuming the coupling to be weak, can one understand these phenomena from first principles (perturbative QCD)?
A ubiquitous transport coefficient

- In pQCD, all such phenomena find a common denominator:
  - incoherent multiple scattering off the medium constituents
  - random kicks leading to Brownian motion in $k_\perp$: $\langle k^2_\perp \rangle \sim \hat{q} \Delta t$
  - acceleration causing medium induced radiation (BDMPSZ, LPM)
  - multiple branchings leading to many soft quanta at large angles

- At leading order in $\alpha_s$, only one transport coefficient:
  - the jet quenching parameter $\hat{q}$
A ubiquitous transport coefficient

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- Will this universality survive the quantum (‘radiative’) corrections?
  - if so, how will these corrections affect the value of $\hat{q}$?
An energetic quark acquires a **transverse momentum** $p_\perp$ via collisions in the medium, after propagating over a **distance** $L$.

Quark energy $E \gg$ typical $p_\perp \implies$ small deflection angle $\theta \ll 1$.
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The quark transverse position is unchanged: eikonal approximation

$$V(x) = P \exp \left\{ i g \int dx^+ A^-_a(x^+, x)t^a \right\}$$

The quark is a ‘right mover’: $x^+ \equiv (t + z)/\sqrt{2} \sim \sqrt{2}t$ is its LC time
Transverse momentum broadening (2)

- Direct amplitude (DA) $\times$ Complex conjugate amplitude (CCA):

$$ y_p x_p = 0 \quad 8L_0 x = +0L $$

The $p_\perp$–spectrum of the quark after crossing the medium ($r = x - y$)

$$ \frac{dN}{d^2p} = \frac{1}{(2\pi)^2} \int e^{-ip\cdot r} \langle S_{xy} \rangle, \quad S_{xy} \equiv \frac{1}{N_c} \text{tr}(V_x V_y^\dagger) $$

Average over $A^-_a$ (the distribution of the medium constituents)
Formally, $\langle S_{xy} \rangle$ is the average $S$–matrix for a $q\bar{q}$ color dipole

- ‘the quark at $x$’ : the physical quark in the DA
- ‘the antiquark at $y$’ : the physical quark in the CCA

Quark cross–section $\leftrightarrow$ dipole amplitude

The dipole $S$–matrix also controls the rate for medium–induced gluon branching (energy loss, jet fragmentation)
The tree–level approximation

- At zeroth order, $\langle S_{xy} \rangle$ is fully specified by one parameter: $\hat{q}_0$
- Weakly coupled medium $\Rightarrow$ quasi independent color charges
  - Gaussian distribution for the color fields $A^-$, local in time ($x^+$)
  - multiple scattering series exponentiates (Glauber, McLerran–Venugopalan)

$$\langle S_{xy} \rangle = e^{-T_2 g} \approx \exp \left\{ -\frac{1}{4} L \hat{q}_0 (1/r^2) r^2 \right\}$$

- $T_2 g$: scattering amplitude via two–gluon exchange (single scattering)

Diagram:

- $x_\perp$
- $y_\perp$
- $x^+ = 0$
- $L$
The tree–level approximation

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$$

$T_2g$ : scattering amplitude via two–gluon exchange (single scattering)
The tree–level jet quenching parameter

\[ \hat{q}_0(Q^2) \equiv n \int Q^2 \frac{d^2 k}{(2\pi)^2} k^2 \frac{g^4 C_F}{(k^2 + m_D^2)^2} \simeq 4\pi \alpha_s^2 C_F n \ln \frac{Q^2}{m_D^2} \]

\[ \hat{q}_0 : \text{density of the medium constituents; } m_D : \text{Debye mass} \]

- The cross–section for \( p_{\perp} \)–broadening:

\[ \frac{dN}{d^2 p} = \frac{1}{(2\pi)^2} \int e^{-i p \cdot r} e^{-\frac{1}{4} L\hat{q}_0(1/r^2)} r^2 \simeq \frac{1}{\pi Q_s^2} e^{-p^2/Q_s^2} \]

- The saturation momentum: exponent of \( O(1) \) when \( r \sim 1/Q_s \)

\[ Q_s^2 = L\hat{q}_0(Q_s^2) = 4\pi \alpha_s^2 C_F n L \ln \frac{Q_s^2}{m_D^2} \propto L \ln L \]

- The physical jet quenching parameter: \( \hat{q}_0(Q_s^2) \propto \ln L \)

- N.B. \( p_{\perp} \)–broadening probes the dipole \( S \)–matrix near unitarity
Radiative corrections to $p_\perp$–broadening

- The quark ‘evolves’ by emitting a gluon (‘real’ or ‘virtual’)

- The ‘evolution’ gluon is not measured: one integrates over $\omega$ and $k$

- All partons undergo multiple scattering: non–linear evolution
Dipole evolution

Alternatively depicted as the evolution of the dipole $S$–matrix:

Exchange graphs between $q$ and $\bar{q}$, or self–energy graphs

This evolution needs not be restricted to a change in $\hat{q}$

\[ \langle S(r) \rangle \]

quantum corrections can change the functional form of $\langle S(r) \rangle$
The phase space

- The radiative corrections are suppressed by powers if $\alpha_s$ ...
  ... but can be enhanced by the phase–space for gluon emissions

- A ‘naive’ argument: bremsstrahlung in the vacuum

\[ dP = \frac{\alpha_s C_R}{\pi^2} \frac{d\omega}{\omega} \frac{d^2k}{k^2} \]

- The emission requires a formation time $\tau \simeq 2\omega/k^2_\perp$

- For our present purposes, better use $\tau$ instead of $\omega$
The phase space

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  ... but can be enhanced by the phase–space for gluon emissions

- A ‘naive’ argument: bremsstrahlung in the vacuum

\[ dP = \frac{\alpha_s C_R}{\pi} \frac{d\tau}{\tau} \frac{dk^2_\perp}{k^2_\perp} \]

- $\tau$ can take all the values between $\lambda \sim 1/T$ and $L$

- For a given $\tau$, $k^2_\perp$ should be larger than $\hat{q}\tau$ (multiple scattering) but smaller than $Q^2_s = \hat{q}L$ (dipole resolution $r \sim 1/Q_s$)
The phase space

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$$\Delta P(L) = \frac{\alpha_s C_R}{\pi} \int_\lambda^L \frac{d\tau}{\tau} \int_{\hat{q}\tau}^{\hat{q}L} \frac{dk^2_\perp}{k^2_\perp} = \frac{\alpha_s C_R}{\pi} \frac{1}{2} \ln^2 \frac{L}{\lambda}$$

- $\Delta P(L)$ is large, double–logarithmic, correction

$$\Delta P(L) \sim \mathcal{O}(1) \text{ for } L = 5 \text{ fm}, T = 500 \text{ MeV}, \alpha_s = 0.3$$
The previous argument is ‘naive’ as it ignores multiple scattering.

Non–linear evolution is well understood for a shock–wave target:

- proton–nucleus collisions at RHIC or the LHC

\[ \tau = x^+ - y^+ \gg \text{target width } L \implies \text{eikonal approx.} \]

- the ‘evolution’ gluon interacts at a fixed transverse coordinate \( z \)

Non–linear equations for correlators of Wilson lines, like \( \langle S_{xy} \rangle \): Balitsky, JIMWLK, BK (large \( N_c \))

- the functional form of \( \langle S(r) \rangle \) for \( r \sim 1/Q_s \) changes indeed.
Beyond the eikonal approximation

- The eikonal approximation **fails** for gluon emissions inside the medium
  - the fluctuation can scatter at any time \( t \) during its lifetime: \( y^+ < t < x^+ \)

- One needs to consider the **transverse diffusion** of the gluon fluctuations
  - \( D = 2 + 1 \) quantum mechanical problem in a random background field
  - formal solution in the form of a path integral

- Generalization of the JIMWLK (or BK) equations to an extended target (‘medium’) *E.I., arXiv: 1403.1996*
The BK equation for jet quenching

\[
\frac{\partial S_{L,0}(x, y)}{\partial \omega} = \times \left[ S_{L,t_2}(x, y) S_{t_2,t_1}(x, r(t)) S_{t_2,t_1}(r(t), y) S_{t_1,0}(x, y) - S_{L,0}(x, y) \right]
\]
\[
\frac{\partial S_{L,0}(x, y)}{\partial \omega} = \partial_{r_1}^i \partial_{r_2}^i \int_{r_1, r_2} [\mathcal{D}r(t)] e^{i \frac{\omega}{2} \int_{t_1}^{t_2} dt \dot{r}^2(t)} \\
\times \left[ S_{L,t_2}(x, y) S_{t_2,t_1}(x, r(t)) S_{t_2,t_1}(r(t), y) S_{t_1,0}(x, y) - S_{L,0}(x, y) \right]
\]
The BK equation for jet quenching

\[ \frac{\partial S_{L,0}(x, y)}{\partial \omega} = -\frac{\alpha_s N_c}{2\omega^3} \int_0^L dt_2 \int_0^{t_2} dt_1 \partial_{\mathbf{r}_1} \partial_{\mathbf{r}_2} \int \mathcal{D}\mathbf{r}(t) \, e^{i \frac{\omega^2}{2} \int_{t_1}^{t_2} dt \cdot \dot{\mathbf{r}}^2(t)} \times \left[ S_{L,t_2}(x, y) S_{t_2,t_1}(x, \mathbf{r}(t)) S_{t_2,t_1}(\mathbf{r}(t), y) S_{t_1,0}(x, y) - S_{L,0}(x, y) \right] \]

- A functional equation: path integral for \( \mathbf{r}(t) \)
  - likely, too complicated to be solved in the general case
- A starting point for controlled approximations
Only one scattering during the lifetime of the fluctuation

- enhanced by the infrared & collinear ‘divergences’ of bremsstrahlung

\[ S_{t_2,t_1}(z, y) \approx e^{-\frac{1}{4}(t_2-t_1) \hat{q} B_\perp^2} \]

\[ B_\perp^2 = |z - y|^2 \sim 1/p_\perp^2 \]

\[ t_2 - t_1 \sim \tau = \omega/p_\perp^2 \]

- External dipole ‘near saturation’: \( r \sim 1/Q_s \Rightarrow p_\perp^2 \lesssim Q_s^2 = \hat{q}L \)

- Weak scattering \( \iff \) small exponent \( \Rightarrow p_\perp^2 \gg \hat{q}\tau \)

- Large longitudinal (energy) phase–space: \( \lambda \ll \tau \ll L \)

\( \Rightarrow \) large transverse phase–space as well: \( \hat{q}\tau \ll p_\perp^2 \ll \hat{q}L \)
The phase–space for linear evolution

- $Q_s^2(\tau) \equiv \hat{q}\tau$ : the saturation line for gluons with lifetime $\tau$

- The longitudinal phase–space:
  \[
  \lambda \ll \tau \ll L
  \]

- ... and the transverse one:
  \[
  \hat{q}\tau \ll p^2_\perp \ll \hat{q}L
  \]

- ... increase equally fast!

- The conditions for a double logarithmic approximation (DLA)
The phase–space for linear evolution

- \( Q_s^2(\tau) \equiv \hat{q}\tau \): the saturation line for gluons with lifetime \( \tau \)

- The longitudinal phase–space:
  \[ \lambda \ll \tau \ll L \]

- ... and the transverse one:
  \[ \hat{q}\tau \ll p_{\perp}^2 \ll \hat{q}L \]

- ... increase equally fast!

- The conditions for a double logarithmic approximation (DLA)

- Very different from the respective evolution for a shock wave:
  stronger dependence of \( Q_s^2 \) upon \( \tau \) (or \( 1/x \))

▶ see the talks by D. Triantafyllopoulos and K. Kutak
To DLA, the dipole $S$–matrix $S_L(r)$ preserves the same functional form as at tree–level, but with a renormalized $\hat{q}$:

$$S_L(r) \simeq \exp \left\{ -\frac{1}{4} L \hat{q}(L) r^2 \right\}$$

Universality: $\hat{q}_0(L) \to \hat{q}(L)$ in all the quantities related to $S$

- $p_\perp$–broadening, radiative energy loss, jet fragmentation ... 

BK equation reduces to a relatively simple, linear, equation for $\hat{q}(L)$

$$\hat{q}(L) = \hat{q}_0 + \bar{\alpha} \int_{L} \frac{d\tau}{\tau} \int_{\hat{q}_\tau}^{L} \frac{dp_{\perp}^2}{p_{\perp}^2} \hat{q}(\tau, p_{\perp}^2)$$

- Liou, Mueller, Wu (arXiv: 1304.7677) [$p_\perp$–broadening]
- Blaizot, Mehtar–Tani (arXiv: 1403.2323) [radiative energy loss]
- E.I. (arXiv: 1403.1996) [evolution of the dipole $S$–matrix]
To DLA, the dipole $S$–matrix $S_L(r)$ preserves the same functional form as at tree–level, but with a renormalized $\hat{q}$:

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$$\hat{q}(L) = \hat{q}_0 + \bar{\alpha} \int_\lambda^L \frac{d\tau}{\tau} \int_{\hat{q}\tau}^{\hat{q}L} \frac{dp_\perp^2}{p_\perp^2} \hat{q}(\tau, p_\perp^2)$$

Not the standard DLA limit of the DGLAP or BFKL eqs. : different boundary conditions (multiple scattering) $\implies$ different solutions

Predicts a strong dependence of $\hat{q}$ upon the medium properties: $L, T$
See the talk by Dionysis Triantafyllopoulos for

- details of the solution
- running coupling effects
- physical implications
Multiple scattering is tantamount to **gluon saturation in the target**

\[ L \sim \frac{1}{x} + \Delta L \]

\[ Q_s^2(x) \] is proportional to the width of the region where a gluon (with longitudinal fraction \( x \)) can overlap with its sources

- for a shockwave, this region is the SW width \( L \) (fixed and small)
- for a gluon in the medium, this is the gluon longitudinal wavelength:

\[ \tau \equiv \Delta x^+ = \frac{1}{p^-} \propto \frac{1}{x} \]

The \( x \)-dependence of \( Q_s^2(x) \) is further amplified by the evolution
Fixed coupling

- Use logarithmic variables, as standard for BFKL, or BK:
  \[ Y \equiv \ln \frac{\tau}{\lambda} \text{ (‘rapidity’)} \quad \text{and} \quad \rho \equiv \ln \frac{p_{T}^{2}}{q^{2}} \text{ (‘momentum’)} \]

  \[
  \hat{q}(Y, \rho) = \hat{q}^{(0)} + \bar{\alpha} \int_{0}^{Y} dY_{1} \int_{Y_{1}}^{\rho} d\rho_{1} \hat{q}(Y_{1}, \rho_{1}) \quad \text{with} \quad \rho \geq Y
  \]

- Not the standard DLA (as familiar from studies of DGLAP, or BFKL)!
  \[ \text{saturation boundary: } \rho_{1} \geq Y_{1} \text{ (multiple scattering)} \]

- Straightforward to solve via iterations (Liou, Mueller, Wu, 2013)

  \[
  \hat{q}_{s}(Y) = \hat{q}^{(0)} \frac{I_{1}(2\sqrt{\alpha} Y)}{\sqrt{\alpha} Y} = \hat{q}^{(0)} \frac{e^{2\sqrt{\alpha} Y}}{\sqrt{4\pi}(\sqrt{\alpha} Y)^{3/2}} \left[ 1 + \mathcal{O}(1/\sqrt{\alpha} Y) \right]
  \]

- Rapid increase at large \( Y \), with ‘anomalous dimension’ \( 2\sqrt{\alpha} \sim 1 \)

- The standard artifact of using a fixed coupling (recall e.g. BK)
One–loop QCD running coupling: 

$$\bar{\alpha} \rightarrow \bar{\alpha}(\rho_1) \equiv \frac{b}{\rho_1 + \rho_0}$$

$$\hat{q}(Y, \rho) = \hat{q}^{(0)} + b \int_0^Y dY_1 \int_{Y_1}^\rho \frac{d\rho_1}{\rho_1 + \rho_0} \hat{q}(Y_1, \rho_1)$$

The standard DLA with RC (no saturation boundary) would give

$$\hat{q}(Y, \rho) = \hat{q}^{(0)} I_1(2\sqrt{bY \ln \rho}) \propto e^{2\sqrt{bY \ln \rho}}$$

The actual solution is very different (and much more complicated!)

$$\ln \hat{q}_s(Y) = 4\sqrt{bY} - 3|\xi_1|(4bY)^{1/6} + \frac{1}{4} \ln Y + \kappa + O(Y^{-1/6})$$

$\triangleright \xi_1 = -2.338 \ldots$ is the rightmost zero of the Airy function

Surprisingly similar to the asymptotic expansion of $\ln Q^2_s(Y)$ for a SW

(Mueller, Triantafyllopoulos, 2003; Munier, Peschanski, 2003)
The enhancement factor $\hat{q}_s(Y)/\hat{q}^{(0)}$ as a function of $Y$:

Results are numerically similar up to $Y \approx 3$, but for larger $Y$, the rise is much faster with FC.
The enhancement factor $\hat{q}_s(Y)/\hat{q}^{(0)}$ as a function of $Y$:

Interestingly, the phenomenologically relevant values are $Y = 2 \div 3 \Rightarrow$ enhancement $= 2 \div 3$ with both FC and RC
Jet quenching

- Nuclear modification factor, di–hadron azimuthal correlations ...

- Energy loss & transverse momentum broadening by the leading particle
Additional energy imbalance as compared to p+p: 20 to 30 GeV

Compare to the typical scale in the medium: $T \sim 1$ GeV (average $p_\perp$)

Detailed studies show that the ‘missing energy’ is carried by many soft ($p_\perp < 2$ GeV) hadrons propagating at large angles
Radiative energy loss (1)

- Consider the radiation by a very energetic, eikonal, quark, for simplicity

\[
\begin{align*}
0 & \quad y^+ & \quad x^+ & \quad L \\
& \quad \infty & \quad L & \quad x^+ & \quad y^+ & \quad 0
\end{align*}
\]

- Once again, the cross-section can be related to (adjoint) dipoles:

\[
\begin{align*}
0 & \quad y^+ & \quad x^+ & \quad L \\
x_0 = 0
\end{align*}
\]
The only difference w.r.t. $p_{\perp}$–broadening:

the radiated gluon within the 1st dipole ($K$) is not eikonal anymore
However, the radiated gluon is relatively hard, $k^+ \sim \omega_c$, so the hierarchy is preserved between radiation and fluctuations: $\omega \ll k^+$

During the relatively short lifetime $t_2 - t_1 = \tau$ of the fluctuation ($\omega$), the radiated gluon ($k^+$) can be treated as eikonal.

Then the same arguments apply as in the case of $p_\perp$–broadening:

\[ \hat{q}^{(0)} \rightarrow \hat{q}_{\tau_f}(k_\perp^2) \quad \text{... in agreement with J.-P. Blaizot and Y. Mehtar-Tani} \]