CP violation in supersymmetric theories

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We review the present status of CP violating problem in supersymmetric extensions of the standard model. We analyze the constraints imposed by the experimental limits of the electron, neutron, and mercury electric dipole moments on the supersymmetric CP phases and show that only the scenarios with flavour-off-diagonal CP violation remain attractive. These scenarios require hermitian Yukawa matrices which naturally arise in models with left–right symmetry or a SU(3) flavour symmetry. In this case, $\varepsilon_K$ and $\varepsilon'/\varepsilon$ can be saturated by a small non-universality of the soft scalar masses through the gluino and chargino contributions respectively. The model also predicts a strong correlation between $A_{CP}(b \to s\gamma)$ and the neutron electric dipole moment. In this framework, the standard model gives a the leading contribution to the CP asymmetry in $B \to \psi K_S$ decay, while the dominant chargino contribution to this asymmetry is $< 0.2$. Thus, no constraint is set on the non-universality of this model by the recent BaBar and Belle measurements.

1. Introduction

The understanding of the origin of CP violation is one of the remaining open questions in particle physics. In the standard model (SM), CP violation and flavour transition arise from the complex Yukawa couplings which have a Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix with physical $\delta_{CKM}$ phase of order unity. At present, CP violation is observed in the $K$ and $B$ systems and the experimental results are consistent with the SM. However, CP violation can be tested from the existing bounds on the electric dipole moment (EDM) of the neutron and the electron. It is remarkable that the SM contribution to the EDM of the neutron is of order $10^{-30}$ e.cm, while the experimental limit is of order $10^{-26}$ e.cm\textsuperscript{1}. We expect that with the further improvements of experimental precision the EDM will provide a crucial test of CP violation. Also more experimental information on the CP violation will be obtained from the B-factory soon, which would be a crucial test of the CP violation in SM. Furthermore, SM model is unable to explain the cosmological baryon asymmetry of our universe, the presence of new sources of CP violation is required for this explanation.

In supersymmetric (SUSY) extensions of the SM there are additional sources of CP violation, due to the presence of new CP violating phases which arise from the complexity of the soft SUSY breaking terms and the SUSY preserving $\mu$-parameter. These new phases have significant implications and can modify the SM predictions in CP violating phenomena. In particular, they would give large contributions to
the electric dipole moment (EDM) of the electron, neutron and mercury atom, to CP violating parameters \((\varepsilon_K, \varepsilon'/\varepsilon)\) of \(K - \bar{K}\) system, and to the CP asymmetries in the \(B - \bar{B}\) system. These phases can be classified into two categories. The first category includes flavor-independent phases such as the phases of the \(\mu\)-parameter, \(B\)-parameter, gaugino masses and the overall phase of the trilinear couplings. The other category includes the flavor-dependent phases, i.e. the phases of the off-diagonal elements of \(A_{ij}\) after the overall phase is factored out and phases in the squark mass matrix \(m_{ij}^2\). Two of the flavor-independent phases can be eliminated by the \(U(1)_R\) and \(U(1)_{PQ}\) transformations under which these parameters behave as spurions. The Peccei–Quinn transformation act on the Higgs doublets and the right–handed superfields in such a way that all the interactions but which mix the two doublets are invariant. The Peccei–Quinn charges are \(Q_{PQ}(\mu) = Q_{PQ}(B\mu)\), \(Q_{A}(\mu) = Q_{PQ}(m_i) = 0\). The \(U(1)_R\) transforms the Grassmann variable \(\theta \to \theta e^{i\alpha}\) and the fields in such a way that the integral of the superpotential over the Grassmann variables is invariant, \(i.e.,\) the \(U(1)_R\) charge of the superpotential is 2. As a result, \(Q_R(B\mu) = Q_R(\mu) - 2\), \(Q_R(A) = Q_R(m_i) = -2\). The six physical CP–phases of the theory are invariant under both \(U(1)_R\) and \(U(1)_{PQ}\), and can be chosen as

\[
\text{Arg } (A_i^* m_i), \quad \text{Arg } ((B\mu)^* \mu A_{\alpha}), \quad (1)
\]

where \(i = 1, 2, 3\) and \(\alpha = d, u, l\). All other CP–phases can be expressed as linear combinations. If the \(A\)–terms and the gaugino masses are universal, there are two physical phases \(\text{Arg } (A^* m), \ \text{Arg } (B^* A)\).

However, the non–observation of EDMs imposes a stringent constraint on flavor–independent SUSY phases. Putting these new phases to zero is not natural in the sense of ‘t Hooft since the Lagrangian does not acquire any new symmetry in the limit where these new phases vanish. This is the so-called SUSY CP-problem. In this article we will review the constraints imposed by the EDMs on the flavor-diagonal CP-phases and the possible scenarios allowing to suppress the EDM contributions; small SUSY CP phases, heavy sfermions, EDM cancellation, and flavor off–diagonal CP violation. Also we will discuss aspects of CP violation in the \(K\) and \(B\) systems due to the flavor-off-diagonal phases from non-degenerate \(A\)–terms.

This article is organized in the following way. In section 2 we briefly discuss the origin of the soft SUSY breaking terms. Section 3 is devoted to the study of the constraints from the EDMs of the electron, neutron, and mercury atom in generic SUSY models. We present the possible ways to avoid overproduction of EDMs and explain that, in SUSY models with flavor off–diagonal phases, the EDMs can be kept sufficiently small while these phases unconstrained. The effect of the off–diagonal phases in the CP violation process in the kaon system is given in section 4. In section 5 we analyze the CP violation in the \(B\)–sector and show that in this framework the large CP asymmetry in the \(B \to \psi K_S\) decay is given by the SM contribution while the SUSY contribution is very small. In contrast, the SUSY contribution to the CP asymmetry in the \(B \to X_{S}\gamma\) decay can be as large as \(\pm 10\%\). Finally, the conclusions are presented in section 6.
2. The origin of the soft terms

It is clear that the SUSY CP violating problem is a problem of SUSY breaking since the relevant phases originate from SUSY breaking terms. In this section we briefly discuss the possible mechanisms which may give rise to the SUSY soft breaking terms. The general structure one has in mind includes three sectors:

i) The observable sector which comprises all the ordinary particles and their SUSY partners,

ii) a “hidden” or “secluded” sector where the breaking of SUSY occurs,

iii) the messengers of the SUSY breaking from the hidden to observable sector.

The two most explored alternatives that have been studied in this context are:

a) SUSY breaking supergravity where the mediators are gravitational interactions, the scale of SUSY breaking is \( M_S \approx \sqrt{M_P M_W} \) and the mass of the fermionic partner of the graviton, the gravitino, is \( m_{3/2} \propto M^2_S/M_P \).

b) SUSY broken at a much lower scale with messengers provided by some gauge interactions. In this case \( m_{3/2} = M_S^2/M_P \) is generally very small, the gravitino being the most likely candidate for the LSP of the theory.

We start the former and still more popular alternative. As well known that the invariance under local SUSY transformation implies invariance under local coordinate change. Thus, local SUSY (supergravity) naturally includes gravity. The effective action, up to two derivatives, is completely specified in terms of three functions which depend on the chiral superfields of the theory. The Kähler potential \( K(\Phi, \Phi^*) \), the analytic superpotential \( W(\Phi) \) and the gauge kinetic functions \( f^a(\Phi) \) which are also analytic functions of the chiral superfields. The tree level supergravity scalar potential is given by

\[
V = e^G \left[ G^i (G^{-1})^j G^j - 3 \right] + \frac{1}{2} f^a D^a D^a, \tag{2}
\]

where \( G(\Phi, \Phi^*) = K(\Phi, \Phi^*) + \log |W(\Phi)|^2 \), \( G^i = \frac{\partial G}{\partial \phi^i} \) and \( G_i^j = \frac{\partial G}{\partial \phi^i \phi^j} \). The gravitino mass is given by Eq.\ref{eq:gravitino_mass} shows that unlike the case of the global supersymmetry one can simultaneously have \( \langle G_i \rangle \neq 0 \) (which breaks SUSY) and vanishing cosmological constant.

Supergravity is broken in a “hidden sector”, namely a sector of the theory that couples to the “observable sector” of quarks, leptons, gauge fields, Higgs and their supersymmetric partners, only through gravitational interactions. The simplest model of a supersymmetry breaking hidden sector is known as “Polonyi model”. In this model the minimal form of the Kähler potential \( K = z^i z_i + y^r y_r \) is assumed (\( z_i \) and \( y^r \) denote the fields of hidden and observable sectors respectively). The resulting soft breaking terms are obtained taking the so-called flat limit, \( i.e. \) sending \( M_P \to \infty \) while keeping the ratio \( M_S^2/M_P = m_{3/2} \) fixed ( \( M_S \) denotes the scale of supergravity breaking). One obtains a common mass term for all the scalar

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* The auxiliary fields of the chiral and vector supermultiplets are given in terms of \( G_i \) see Ref.\ref{ref:kahler}.

One can see that if at least one of the \( G_i \) VEV’s is non-vanishing SUSY is broken.
particles in the observable sector which is equal to the gravitino mass. Also in this model we get a universal $A$ term \( i.e. A_U = A_D = A_L = A \).

However, it is possible to obtain effective potentials in which this universality is absent by taking the kinetic terms for the chiral superfields to non-minimal. As recently stressed, the soft supersymmetry breaking parameters may be non-universal in the effective theories which are derived from the superstring theories. In general supergravity, the soft scalar mass and the $A$-parameter are given by

\[
m^2_{\alpha} = m^2_{3/2} - \bar{F} m \partial_\alpha \partial_n \ln \tilde{K}_\alpha,
\]

\[
A_{\alpha\beta\gamma} = F^m \left[ \tilde{K}_m + \partial_m \ln Y_{\alpha\beta\gamma} - \partial_m \ln (\tilde{K}_\alpha \tilde{K}_\beta \tilde{K}_\gamma) \right],
\]

where $m$ refers to the SUSY breaking fields and $\tilde{K}_\alpha = \tilde{K}_{\bar{\alpha} \alpha}$. Note that in Eq.(3) we have diagonal soft scalar masses due to the assumption that the Kähler metric is diagonal. General supergravity models allow for a non-vanishing off-diagonal Kähler metric $K_{\alpha \bar{\beta}}$. However, such mixing typically does not appear in superstring models at the leading order due to additional “stringy” symmetries beyond those of the Standard Model. The gaugino masses are given in terms of the gauge kinetic function $f_a$, where $\text{Re} f_a = 1/g^2_a$ and the subscript $a$ represents the corresponding gauge group,

\[
M_a = \frac{1}{2} (\text{Re} f_a)^{-1} F^m \partial_m f_a.
\]

Apart from $m_\alpha$, $A_{\alpha\beta\gamma}$, and $M_a$ soft parameters, soft bosonic bilinear can be also present. In case of the MSSM, all the symmetries of the low energy allow for a superpotential term coupling the two Higgs doublets of the form $W = \mu H_1 H_2$. The associated soft breaking term in the scalar potential will have the form $B \mu H_1 H_2 + h.c.$, where $B$ is a dimensionful parameter of order gravitino mass. It is well known that in order to get appropriate $SU(2)_L \times U(1)_Y$ breaking, the $\mu$ has to be of the same order of magnitude as the SUSY breaking soft terms.

Now, we turn to the alternative mechanism of breaking supersymmetry at low energies and that gauge interactions are the “messenger” of supersymmetry breaking.\footnote{The gauge mediated supersymmetry breaking (GMSB) models have several attractive features. As gauge interactions are flavor blind, squark and slepton masses are universal. In the minimal model of this kind, the messanger fields transform as a single flavor of $5 + \bar{5}$ of $SU(5)$. Hence, there are $SU(2)_L$ doublets $l$ and $\bar{l}$ and $SU(3)_C$ triplets $q$ and $\bar{q}$. In order to introduce supersymmetry breaking into the messanger sector, these fields may be coupled to a gauge singlet spurion, $S$, through the superpotential}

\[
W = \lambda_1 S H + \lambda_2 S q \bar{q},
\]

due to interactions with some supersymmetry breaking sector of theory. The field $S$ has a non-zero expectation value both for its scalar and auxiliary component, $\langle S \rangle$.
and $\langle F_S \rangle$. Integrating out the messenger sector give rise to gaugino masses at one loop.

$$m_{\lambda_i} = c_i \frac{\alpha_i}{4\pi} \Lambda,$$

(7)

where $\Lambda = \frac{\langle F_S \rangle}{\langle S \rangle}$, $c_1 = 5/3$, $c_2 = c_3 = 1$ and $\alpha_1 = \frac{\alpha}{\cos^2 \theta_W}$. For the scalar masses one has

$$\tilde{m}^2 = 2\Lambda^2 \left[ C_3 \left( \frac{\alpha_3}{4\pi} \right)^2 + C_2 \left( \frac{\alpha_2}{4\pi} \right)^2 + \frac{5}{3} \left( \frac{\gamma}{2} \right)^2 \left( \frac{\alpha_1}{4\pi} \right)^2 \right],$$

(8)

where $C_3 = 4/3$ for color triplets and zero for singlets, $C_2 = 3/4$ for weak doublets and zero for singlets, and $\gamma = 2(Q - T_3)$ is the ordinary hypercharge. In this model, the $A$-terms are high order effects. The $\mu$ and $B$ parameters can arise through interactions of the Higgs field with various singlets. This model has few new parameters relative to the minimal standard model. These parameters are $\Lambda$, $\mu$ and $B$. Since the scalar masses are functions of only gauge quantum numbers, these models also automatically solve the supersymmetric flavor problem.

The most important difference between the models of the gravity mediated SUSY breaking and the models of gauge mediation is that in the later models the LSP is the gravitino and its mass is of order $10^{-10}$ GeV. Thus leads to a very different phenomenology from the formare model. It allows the lightest neutralino $\chi_0^1$ to decay to photon plus gravitino and this gives a signal for SUSY as a missing energy.

3. SUSY CP violating phases and constraints from the EDMs

As mentioned in the introduction, in supersymmetric theories there are several possible sources for CP violating phases in addition to the CKM phase. These CP violating phases have an important impact on the phenomenology of CP violation. They can induce an EDM of quarks and leptons at one-loop level. The EDMs of the neutron, electron, and mercury atom impose severe constraints on the flavor-diagonal phases, the so-called SUSY CP problem.

$$d_n < 6.3 \times 10^{-26} \text{ e cm},$$

(9)

$$d_e < 4.3 \times 10^{-27} \text{ e cm},$$

(10)

$$d_{Hg} < 2.1 \times 10^{-28} \text{ e cm}.$$  

(11)

With the expected improvements in experimental precision, the EDM is likely to be one of the most important tests for physics beyond the Standard Model for some time to come, and EDMs will remain a difficult hurdle for supersymmetric theories. Indeed it is remarkable that the SM contribution to the EDM of the neutron is of order $10^{-30}$ e cm, whereas the “generic” supersymmetric value is $10^{-22}$ e cm.

In this section we review EDM constraints in the context of supersymmetric theories as well as known mechanisms to suppress the EDMs. We also study to what extent different scenarios rely on assumptions about the neutron structure,
i.e. chiral quark model vs parton model. Let us first summarize the contributions to the three most significant EDMs, beginning with the most reliable, the electron EDM.

The electron EDM is defined by the effective CP-violating interaction

\[ \mathcal{L} = -\frac{i}{2} d_e \bar{e} \sigma_{\mu\nu}\gamma_5 e F^{\mu\nu}, \]  

(12)

where \( F^{\mu\nu} \) is the electromagnetic field strength. The experimental bound on the electron EDM is derived from the electric dipole moment of the thallium atom and is given by

\[ d_e < 4 \times 10^{-27} e \text{ cm}. \]  

(13)

In supersymmetric models, the electron EDM arises due to CP-violating 1-loop diagrams with the chargino and neutralino exchange:

\[ d_e = d_e^+ + d_e^0. \]  

(14)

Since the EEDM calculation involves little uncertainty it allows to extract reliable bounds on the CP-violating SUSY phases.

The neutron EDM has contributions from a number of CP-violating operators involving quarks, gluons, and photons. The most important ones include the electric and chromoelectric dipole operators, and the Weinberg three-gluon operator:

\[ \mathcal{L} = -\frac{i}{2} \bar{q} \sigma_{\mu\nu}\gamma_5 q F^{\mu\nu} - \frac{i}{2} \bar{q} \sigma_{\mu\nu}\gamma_5 T^a q G^{\mu\nu}_a \]

\[ - \frac{1}{6} d_g f_{abc} G_{a\mu\rho} G_{b\nu}^0 G_{c\lambda\sigma} \epsilon^{\mu\nu\lambda\rho}, \]  

(15)

where \( G_{a\mu\nu} \) is the gluon field strength, \( T^a \) and \( f_{abc} \) are the SU(3) generators and group structure coefficients, respectively. Given these operators, it is however a nontrivial task to evaluate the neutron EDM since assumptions about the neutron internal structure are necessary. In what follows we will study two models, namely the quark chiral model and the quark parton model. Neither of these models is sufficiently reliable by itself, however a power of the combined analysis should provide an insight into implications of the bound on the neutron EDM. Another approach to the neutron EDM based on the QCD sum rules has appeared in earlier work. We note that in any case the NEDM calculations involve uncertain hadronic parameters such as the quark masses and thus these calculations have a status of estimates. The major conclusions of the present work are independent of the specifics of the neutron model.

The chiral quark model is a nonrelativistic model which relates the neutron EDM to the EDMs of the valence quarks with the help of the SU(6) coefficients:

\[ d_n = \frac{4}{3} d_d - \frac{1}{3} d_u. \]  

(16)

The quark EDMs can be estimated via Naive Dimensional Analysis as

\[ d_q = \eta^E d_q^E + \frac{\eta^C}{4\pi} d_q^C + \frac{\eta^G \epsilon}{4\pi} d_q^G, \]  

(17)
where the QCD correction factors are given by $\eta^E = 1.53$, $\eta^C \simeq \eta^G \simeq 3.4$, and $\Lambda \simeq 1.19$ GeV is the chiral symmetry breaking scale. We use the numerical values for these coefficients as given in [14]. The parameters $\eta^C, G$ involve considerable uncertainties stemming from the fact that the strong coupling constant at low energies is unknown. Another weak side of the model is that it neglects the sea quark contributions which play an important role in the nucleon spin structure.

The supersymmetric contributions to the dipole moments of the individual quarks result from the 1-loop gluino, chargino, neutralino exchange diagrams

$$d^{E,C}_q = \tilde{d}^{(E,C)}_q + d^{+\mp (E,C)}_q + d^{0\mp (E,C)}_q,$$

and from the 2-loop gluino-quark-squark diagrams which generate $d^G$.

The parton quark model is based on the isospin symmetry and known contributions of different quarks to the spin of the proton [15]. The quantities $\Delta^q$ defined as

$$\langle n|\frac{1}{2}\bar{q}_\mu \gamma_5 q|n\rangle = \Delta^q S_\mu,$$

where $S_\mu$ is the neutron spin, are related by the isospin symmetry to the quantities $(\Delta^q)_p$ which are measured in the deep inelastic scattering (and other) experiments, i.e. $\Delta_u = (\Delta_d)_p$, $\Delta_d = (\Delta_u)_p$, and $\Delta_s = (\Delta_s)_p$. To be exact, the neutron EDM depends on the (yet unknown) tensor charges rather than these axial charges. The main assumption of the model is that the quark contributions to the NEDM are weighted by the same factors $\Delta^i$, i.e.

$$d_n = \eta^E(\Delta dd^E + \Delta ud^E + \Delta sd^E).$$

In our numerical analysis we use the following values for these quantities $\Delta_d = 0.746$, $\Delta_u = -0.508$, and $\Delta_s = -0.226$ as they appear in the analysis of Ref. [16]. As before, we have

$$d^{E}_q = \tilde{d}^{(E)}_q + d^{+\mp (E)}_q + d^{0\mp (E)}_q.$$ 

The major difference from the chiral quark model is a large strange quark contribution (which is likely to be an overestimate [17]). In particular, due to the large strange and charm quark masses, the strange quark contribution dominates in most regions of the parameter space. This leads to considerable numerical differences between the predictions of the two models.

The EDM of the mercury atom results mostly from T-odd nuclear forces in the mercury nucleus, which induce the effective interaction of the type $(I \cdot \nabla)\delta(r)$ between the electron and the nucleus of spin $I$ [18]. In turn, the T-odd nuclear forces arise due to the effective 4-fermion interaction $\bar{q}pi\gamma_5 n$. It has been argued that the mercury EDM is primarily sensitive to the chromoelectric dipole moments of the quarks and the limit

$$d_{Hg} < 2.1 \times 10^{-28} \text{e cm}$$

can be translated into

$$|d^C_d - d^C_u - 0.012d^C_s|/g_s < 7 \times 10^{-27} \text{cm},$$

where $g_s$ is the strong coupling constant. As in the parton neutron model, there is a considerable strange quark contribution. The relative coefficients of the quark
contributions in (22) are known better than those for the neutron, however the overall normalization is still not free of uncertainties.

Below we list formulae for individual supersymmetric contributions to the EDMs due to the Feynman diagrams in Fig. 1. In our presentation we follow the work of Ibrahim and Nath.

\[ \sim g, \sim \chi^0, \sim \chi^+ \]

Fig. 1. Leading SUSY contributions to the EDMs. The photon and gluon lines are to be attached to the loop in all possible ways.

Neglecting the flavor mixing, the electromagnetic contributions to the fermion EDMs are given by:

\[
\begin{align*}
\bar{d}_q^{(E)} / e & = -\frac{2 \alpha_s}{3 \pi} \sum_{k=1}^{2} \text{Im}(\Gamma_1^{kq}) \frac{M_3^2}{M_{\tilde{q}_k}^2} Q_q B \left( \frac{M_3^2}{M_{\tilde{q}_k}^2} \right), \\
\bar{d}_u^+ / e & = -\frac{\alpha_{em}}{4 \pi \sin^2 \theta_W} \sum_{k=1}^{2} \sum_{i=1}^{2} \text{Im}(\Gamma_{uik}) \frac{m_{\chi^+}^2}{M_{\tilde{d}_k}^2} \left[ Q_d B \left( \frac{m_{\chi^+}^2}{M_{\tilde{d}_k}^2} \right) + (Q_u - Q_d) A \left( \frac{m_{\chi^+}^2}{M_{\tilde{d}_k}^2} \right) \right], \\
\bar{d}_d^+ / e & = -\frac{\alpha_{em}}{4 \pi \sin^2 \theta_W} \sum_{k=1}^{2} \sum_{i=1}^{2} \text{Im}(\Gamma_{dik}) \frac{m_{\chi^+}^2}{M_{\tilde{d}_k}^2} \left[ Q_\bar{d} B \left( \frac{m_{\chi^+}^2}{M_{\tilde{d}_k}^2} \right) + (Q_d - Q_\bar{d}) A \left( \frac{m_{\chi^+}^2}{M_{\tilde{d}_k}^2} \right) \right], \\
\bar{d}_e^+ / e & = \frac{\alpha_{em}}{4 \pi \sin^2 \theta_W} \sum_{i=1}^{2} \frac{m_{\chi^+}^2}{m_{\tilde{\nu}_i}^2} \text{Im}(\Gamma_{ei}) A \left( \frac{m_{\chi^+}^2}{m_{\tilde{\nu}_i}^2} \right), \\
\bar{d}_f^0 / e & = \frac{\alpha_{em}}{4 \pi \sin^2 \theta_W} \sum_{k=1}^{2} \sum_{i=1}^{4} \text{Im}(\eta_{fik}) \frac{m_{\chi^0}^2}{M_{\tilde{f}_k}^2} Q_f B \left( \frac{m_{\chi^0}^2}{M_{\tilde{f}_k}^2} \right). \tag{23}
\end{align*}
\]

Here

\[ \Gamma_1^{1q} = e^{-i \phi_3} D_{q2k} D_{q1k}^* \], \tag{24}

with \( \phi_3 \) being the gluino phase and \( D_q \) defined by \( D_q^1 M_q^2 D_q = \text{diag}(M_{\tilde{u}_1}^2, M_{\tilde{d}_1}^2) \). The sfermion mass matrix \( M_{\tilde{f}}^2 \) is given by

\[
M_{\tilde{f}}^2 = \left( \frac{M_L^2 + m_f^2 + M_{\tilde{f}_f}^2}{2} Q_f \sin^2 \theta_W \cos 2\beta \right) \frac{m_f(A_f - \mu R_f)}{m_f(A_f - \mu R_f)} \frac{m_f(A_f - \mu R_f)}{M_{\tilde{R}}^2 + m_f^2 + M_{\tilde{f}_f}^2 Q_f \sin^2 \theta_W \cos 2\beta}, \tag{25}
\]
where \( R_f = \cot \beta \) (\( \tan \beta \)) for \( I_3 = 1/2 \) \((-1/2)\). The chargino vertex \( \Gamma_{fik} \) is defined as

\[
\Gamma_{uik} = \kappa_u V_{i2}^* D_{d1k} (U_{i1}^* D_{d1k}^* - \kappa_d U_{i2}^* D_{d2k}^*) ,
\]

\[
\Gamma_{dik} = \kappa_d U_{i2}^* D_{u1k} (V_{i1}^* D_{u1k}^* - \kappa_u V_{i2}^* D_{u2k}^*)
\]  \( (26) \)

and analogously for the electron; here \( U \) and \( V \) are the unitary matrices diagonalizing the chargino mass matrix: \( U^* M_{\chi}^+ V^{-1} = \text{diag}(m_{\chi_1^+, \chi_2^+}). \) The quantities \( \kappa_f \) are the Yukawa couplings

\[
\kappa_u = \frac{m_u}{\sqrt{2} m_W \sin \beta} , \quad \kappa_{d,e} = \frac{m_{d,e}}{\sqrt{2} m_W \cos \beta} .
\]  \( (27) \)

The neutralino vertex \( \eta_{fik} \) is given by

\[
\eta_{fik} = \left[ -\sqrt{2} \{ \tan \theta_W (Q_f - I_3) \} X_{1i} + I_3 X_{2i} \} D_{f1k}^* - \kappa_f X_{bi} D_{f2k}^* \right]
\times \left[ \sqrt{2} \tan \theta_W Q_f X_{1i} D_{f2k} - \kappa_f X_{bi} D_{f1k} \right] ,
\]  \( (28) \)

where \( I_3 \) is the third component of the isospin, \( b = 3 \) \((4)\) for \( I_3 = -1/2 \) \((1/2)\), and \( X \) is the unitary matrix diagonalizing the neutralino mass matrix: \( X^T M_{\chi} \eta X = \text{diag}(m_{\chi_1^0}, m_{\chi_2^0}, m_{\chi_3^0}, m_{\chi_4^0}). \) In our convention the mass matrix eigenvalues are positive and ordered as \( m_{\chi_1^0} > m_{\chi_2^0} > \ldots \) (this holds for all mass matrices in the paper).

The loop functions \( A(r), B(r), \) and \( C(r) \) are defined by

\[
A(r) = \frac{1}{2(1 - r)^2} \left( 3 - r + \frac{2 \ln r}{1 - r} \right) ,
\]

\[
B(r) = \frac{1}{2(r - 1)^2} \left( 1 + r + \frac{2r \ln r}{1 - r} \right) ,
\]

\[
C(r) = \frac{1}{6(r - 1)^2} \left( 10r - 26 + \frac{2r \ln r}{1 - r} - \frac{18 \ln r}{1 - r} \right) .
\]  \( (29) \)

The chromoelectric contributions to the quark EDMs are given by

\[
d_q^\eta(C) = \frac{g_s N_c}{4\pi} \sum_{k=1}^2 \text{Im}(\Gamma_{qik}) \frac{M_{\tilde{\chi}_k^0}}{M_{\tilde{\chi}_k}^2} C \left( \frac{M_{\tilde{\chi}_k}^2}{M_{\tilde{\chi}_k}^2} \right) ,
\]

\[
d_q^{\chi^+}(C) = \frac{g^2 g_s}{16\pi^2} \sum_{k=1}^2 \sum_{i=1}^2 \text{Im}(\Gamma_{qik}) \frac{m_{\chi_i^+}}{M_{\tilde{\chi}_k}^2} B \left( \frac{m_{\chi_i^+}}{M_{\tilde{\chi}_k}^2} \right) ,
\]

\[
d_q^{\chi^0}(C) = \frac{g^2 g_s}{16\pi^2} \sum_{k=1}^2 \sum_{i=1}^2 \text{Im}(\eta_{qik}) \frac{m_{\chi_i^0}}{M_{\tilde{\chi}_k}^2} B \left( \frac{m_{\chi_i^0}}{M_{\tilde{\chi}_k}^2} \right) .
\]  \( (30) \)

Finally, the contribution to the Weinberg operator \( \bar{b} q \) from the two-loop gluino-top-stop and gluino-bottom-sbottom diagrams reads

\[
d^\theta = -3a_s m_t \left( \frac{g_s}{4\pi} \right)^3 \text{Im}(\Gamma_{qik}^2) \frac{z_1 - z_2}{(M_3)^2} H(z_1, z_2, z_t) + (t \rightarrow b) ,
\]  \( (31) \)
where \( z_i = \left( \frac{M_i}{M_3} \right)^2 \), \( z_t = \left( \frac{m_t}{M_3} \right)^2 \). The two-loop function \( H(z_1, z_2, z_t) \) is given by

\[
H(z_1, z_2, z_t) = \frac{1}{2} \int_0^1 dx \int_0^1 du \int_0^1 dy x(1-x)u \frac{N_1 N_2}{D^4},
\]  

(32)

where

\[
N_1 = u(1-x) + z_t x(1-x)(1-u) - 2ux[z_1 y + z_2 (1-y)],
\]

\[
N_2 = (1-x)^2(1-u)^2 + u^2 - \frac{1}{9}x^2(1-u)^2,
\]

\[
D = u(1-x) + z_t x(1-x)(1-u) + ux[z_1 y + z_2 (1-y)].
\]  

(33)

The numerical behaviour of this function was studied in \[\[\]. We emphasize that the b-quark contribution is significant and often exceeds the top one.

In addition to the Weinberg two-loop diagram, there is another (Barr-Zee) two-loop contribution which originates from the CP-odd Higgs exchange \[\[\]. Its numerical effect is however negligible \[\[\].

Let us now discuss possible ways to avoid overproduction of EDMs in supersymmetric and string-inspired models. There are four known scenarios allowing to suppress the EDM contributions: small SUSY CP-phases, heavy sfermions, EDM cancellations, and flavor-off-diagonal CP violation.

1. **Small SUSY CP-phases:**

For a light (below 1 TeV) supersymmetric spectrum, the flavor–independent SUSY CP phases have to be small in order to satisfy the experimental EDM bounds \[\[\].

---

Fig. 2. EDMs as a function of \( \phi_A \) (left) and \( \phi_\mu \) (right). 1 – electron, 2 – neutron (chiral model), 3 – mercury, 4 – neutron (parton model). The experimental limit is given by the horizontal line. Here \( \tan \beta = 3, m_0 = m_{1/2} = A = 200 \text{ GeV} \).
In Figs. 2 we illustrate the EDMs behaviour as a function of the CP-phases in the mSUGRA-type models, where we have set \( m_0 = m_{1/2} = A = 200 \text{ GeV} \). At low \( \tan \beta \), the EDM constraints impose the following bounds (at the GUT scale):

\[
\begin{align*}
\phi_A &\lesssim 10^{-2} - 10^{-1}, \\
\phi_\mu &\lesssim 10^{-3} - 10^{-2}, \\
\phi_{M_i} &\lesssim 10^{-2}.
\end{align*}
\]

(34)

We note that \( \phi_A \) is less constrained than \( \phi_\mu \) and \( \phi_{M_i} \). There are two reasons for that: first, \( \phi_A \) is reduced by the RG running from the GUT scale down to the electroweak scale and, second, the phase of the \((\delta_{11}^d)_{LR}\) mass insertion which gives the dominant contribution to the EDMs is more sensitive to \( \phi_\mu \) and \( \phi_{M_i} \) due to \( |A| < \mu \tan \beta \). We note that the bounds (34) stay practically the same if we allow for an “uncertainty” factor 2-3 in the overall EDM normalization.

Generally it is quite difficult to explain why the soft CP-phases have to be small. In principle, small CP-phases could appear if CP were an approximate symmetry of nature \(^{20}\). However, recent experimental results show that CP violation in the \( B - \bar{B} \) mixing is large \(^{21}\) and thus the approximate CP hypothesis cannot be motivated.

2– Heavy sfermions:

This possibility is based on the decoupling of heavy supersymmetric particles. Even if one allows \( O(1) \) CP violating phases, their effect will be negligible if the SUSY spectrum is sufficiently heavy \(^{22}\). Generally, SUSY fermions are required to be lighter than the SUSY scalars by, for example, cosmological considerations. So the decoupling scenario can be implemented with heavy sfermions only. Here the SUSY contributions to the EDMs are suppressed even with maximal SUSY phases because the squarks in the loop are very heavy and the mixing angles are small. It was shown \(^{23}\) that in order to satisfy the EDM of the mercury atom the sfermion masses have to be of order 10 TeV, which leads to a large hierarchy between SUSY and electroweak scales. In Fig. 3 we display the EDMs as functions of the universal scalar mass parameter \( m_0 \) for the mSUGRA model with maximal CP-phases \( \phi_\mu = \phi_A = \pi/2 \) and \( m_{1/2} = A = 200 \text{ GeV} \). We observe that all EDM constraints except for that of the electron require \( m_0 \) to be around 5 TeV or more. The mercury constraint is the strongest one and requires \( m_0 \approx 10 \text{ TeV} \).

This leads to a serious fine-tuning problem. Recall that one of the primary motivations for supersymmetry was a solution to the naturalness problem. Certainly this motivation will be entirely lost if a SUSY model reintroduces the same problem in a different sector, i.e., for example a large hierarchy between the scalar mass and the electroweak scale. Also note that the dependence of the decoupling scale on the EDMs is quite slow. For instance, if we relax the mercury bound by a factor of 2
the decoupling scale changes from 10 TeV to about 8 TeV. Thus our conclusions are insensitive to the exact values of the EDMs.

3—**EDM cancellations:**

The cancellation scenario is based on the fact that large cancellations among different contributions to the EDMs are possible in certain regions of the parameter space which allow for $O(1)$ flavour-independent CP phases. However, a recent analysis has shown that this possibility is practically ruled out if in addition to the electron and neutron EDM constraints, the mercury constraint is imposed.

For the case of the electron, the EDM cancellations occur between the chargino and the neutralino contributions. For the case of the neutron and mercury, there are cancellations between the gluino and the chargino contributions as well as cancellations among contributions of different quarks to the total EDM. It has been shown that the parameters allowing the EDM cancellations strongly depend on the neutron model. For example, in the parton model, it is more difficult to achieve these cancellations due to the large strange quark contribution. Therefore, one cannot restrict the parameter space in a model-independent way and caution is needed when dealing with the parameters allowed by the cancellations.

In mSUGRA, the EDM cancellations can occur simultaneously for the electron, neutron, and mercury along a band in the $(\phi_A, \phi_\mu)$ plane (Fig.4). However, in this case the mercury constraint requires the $\mu$ phase to be $O(10^{-2})$ and the magnitude of the A-terms to be suppressed ($\sim 0.1m_0$) which results in only a small effect of the A-terms on the phase of the corresponding mass insertion. Thus the fact that the phase of the A-terms is unrestricted should not be attributed to the cancellations, but rather to the suppressed contribution of the A-terms.

If the gluino phase is turned on, simultaneous EEDM, NEDM, and mercury
EDM cancellations are not possible. The gluino phase affects the NEDM cancellation band while leaving the EEDM cancellation band almost intact. An introduction of the bino phase $\phi_1$ qualitatively has the same “off-setting” effect on the EEDM cancellation band as the gluino phase does on that of the NEDM. (Fig. 5). Note that the bino phase has no significant effect on the neutron and mercury cancellation bands since the neutralino contribution in both cases is small. When both the gluino and bino phases are present (and fixed), simultaneous electron, neutron, and mercury EDMs cancellations do not appear to be possible along a band. This conclusion has been also obtained in type I string inspired models.

The cancellation scenario involves a significant fine-tuning. Indeed, restricting the phases to the band where the cancellations occur does not increase the symme-
try of the model and thus is unnatural according to the 'tHooft’s criterion.

### 4– Flavor-off-diagonal CP violation:

Finally, we consider the possibility that the SUSY CP violation have a flavor off-diagonal character just as in the SM. In the SM, CP violation appears in flavor-changing processes which is one of the reasons why the predicted EDMs are small. A similar situation may occur in supersymmetric models. In this case, the origin of the CP violation is assumed to be closely related to the origin of flavor structures rather than the origin of SUSY breaking. Thus the flavor blind quantities as the $\mu$-term and gaugino masses are real. As mentioned, the EDMs of the neutron, electron, and mercury atom impose severe constraints on the flavor-diagonal phases. Indeed, these are the only phases that enter into the EDM calculations. However, this leaves the possibility that the flavor-off-diagonal phases are non-zero. Of course, one has to specify the basis in which the considered quantity is classified as off-diagonal.

This class of models requires hermitian Yukawa matrices and A-terms, which forces the flavor-diagonal phases to vanish (up to small RG corrections) in any basis. To see this, first note that the trilinear couplings
\[ \hat{A}_{ij}^\alpha = A_{ij}^\alpha Y_{ij}^\alpha \]
are also hermitian. The quark Yukawa matrices are diagonalized by the unitary transformation
\[ q \rightarrow V^q q , \quad Y^q \rightarrow (V^q)^T Y^q (V^q)^* = \text{diag}(h_1^q, h_2^q, h_3^q) , \]
such that the CKM matrix is $V_{CKM} = (V^u)^d$. If we transform the squark fields in the same manner, which defines the super-CKM basis, we will have
\[ \hat{A}^q \rightarrow (V^q)^T \hat{A}^q (V^q)^* . \]
As a result, the trilinear couplings remain hermitian and the flavor-diagonal CP-phases inducing the EDMs vanish. Note that this argument would not work if in the original basis the trilinear couplings but not the Yukawas were hermitian since the diagonalization would require a biunitary transformation which would generally introduce the diagonal phases.

The hermiticity is generally spoiled by the renormalization group (RG) running from the high energy scale down to the electroweak scale (see, e.g. [24]). In models under consideration we have the following setting at high energies:
\[ Y^\alpha = Y^{\alpha \dagger} , \quad A^\alpha = A^{\alpha \dagger} , \quad \text{Arg}(M_k) = \text{Arg}(B) = \text{Arg}(\mu) = 0 . \]
Generally, the off-diagonal elements of the A-terms can have $O(1)$ phases without violating the EDM constraints. Due to the RG effects, large phases in the soft trilinear couplings involving the third generation generate small phases in the flavour-diagonal mass insertions for the light generations, and thus induce the EDMs. How-
ever, the amount of generated “non-hermiticity” depends on $[Y^u, Y^d]$, $[\hat{A}^u, \hat{A}^d]$, etc. and thus is suppressed by the off-diagonal elements of the Yukawa matrices. In particular, the size of the flavor-diagonal phases is quite sensitive to $Y_{13}$. For $Y_{13} \leq \mathcal{O}(10^{-3})$, the RG effects are unimportant and the induced EDMs are below the experimental limits.

In supergravity models, the trilinear parameters are given in terms of the Kähler potential and the Yukawa couplings

$$A_{\alpha\beta\gamma} = F^m \left[ \hat{K}_m + \partial_m \ln Y_{\alpha\beta\gamma} - \partial_m \ln(\hat{K}_\alpha \hat{K}_\beta \hat{K}_\gamma) \right],$$

(39)

where the Latin indices refer to the hidden sector fields that break SUSY and the Greek indices refer to the observable fields. According to our previous assumption, the $F^m$ is real. Also $\hat{K}_\alpha$ and $\hat{K}_m$ are always real, thus the $A$–terms are Hermitian if the derivatives of the Kähler potential are either generation–independent or the same for the left and right fields of the same generation, i.e., if $\hat{K}_{Q_L, R_i} = \hat{K}_{Q_L, \tilde{U}_{R_i}}$. These conditions are usually satisfied in string models. It is interesting to note that although the SUSY breaking does not bring in new source of CP violating, the trilinear soft parameters involve off–diagonal CP violating phases of $\mathcal{O}(1)$. This stems from the contribution of the term $\partial_m \ln Y_{\alpha\beta\gamma}$, which has been found to be significant and sometimes even dominant in string models. In what follows, we will show that these phases are unconstrained by the EDMs and will study their phenomenological implications in the $K$ and $B$ systems.

The relevant quantities appearing in the soft Lagrangian are $(Y_q^A)_{ij} = (Y_q)_{ij} (A_q)_{ij}$ (indices not summed) which are also Hermitian at the GUT scale. For the sake of definiteness, we consider the following Hermitian Yukawa matrices at the GUT scale

$$Y^u = \lambda_u \begin{pmatrix} 5.94 \times 10^{-4} & 10^{-3} i & -2.03 \times 10^{-2} \\ -10^{-3} i & 5.07 \times 10^{-3} & 2.03 \times 10^{-5} i \\ -2.03 \times 10^{-2} & -2.03 \times 10^{-5} i & 1 \end{pmatrix} ,$$

(40)

$$Y^d = \lambda_d \begin{pmatrix} 6.84 \times 10^{-3} & (1.05 + 0.947 i) \times 10^{-2} & -0.023 \\ (1.05 - 0.947 i) \times 10^{-2} & 0.0489 & 0.0368 i \\ -0.023 & -0.0368 i & 1 \end{pmatrix} ,$$

(41)

where $\lambda_u = m_t / v \sin \beta$ and $\lambda_d = m_b / v \cos \beta$. These matrices reproduce, at low energy, the quark masses and the CKM matrix. The renormalization group (RG) evolution of Yukawa couplings and the $A$ terms slightly violate the Hermiticity. Therefore, the resulting $Y_q^A$ at the electroweak scale has very small non–zero phases in the diagonal elements (due to the large suppression from the off–diagonal entries of the Yukawa). However, what matters is the relevant phases appearing in the squark mass insertions in the super-CKM basis, i.e., the basis where the Yukawa matrices are diagonalized by a unitary transformation of the quark superfields $\bar{U}_{L,R}$ and $\bar{D}_{L,R}$ (Note that since the Yukawas are Hermitian matrices they are diagonalized by one unitary transformation, i.e., $V_L^q = V_R^q$). Accordingly the trilinear terms $Y_q^A$ transform as $Y_q^A \rightarrow V_{Y}^T Y_q^{AV} V_{Y}^T$. Thus the $Y_q^A$ stay Hermitian to a very good
degree in the super-CKM basis. Therefore, the imaginary parts of the flavor conserving mass insertions

\[
(\delta_{ii}^{d(u)})_{LR} = \frac{1}{m_{\tilde{q}}^2} \left[ \left( V^T q^T A V q \right)_{ii} v_1(2) - \mu Y_t^{d(u)} v_2(1) \right],
\]

that appear in the EDM calculations are suppressed. In the above formula the \(m_{\tilde{q}}\) refers to the average squark mass and \(v_i = \langle H^0_i \rangle / \sqrt{2}\). The current experimental bound \(\text{Im}(\delta_{11}^{d(u)})_{LR} \lesssim 10^{-7}\).

We start our analysis by revisiting the EDM constraints on the flavor off diagonal phases of SUSY models with Hermitian Yukawa as in Eq.(40) and the following Hermitian \(A\)-terms:

\[
A_d = A_u = \begin{pmatrix}
A_{11} & A_{12} e^{i \phi_{12}} & A_{13} e^{i \phi_{13}} \\
A_{12} e^{-i \phi_{12}} & A_{22} & A_{23} e^{i \phi_{23}} \\
A_{13} e^{-i \phi_{13}} & A_{23} e^{-i \phi_{23}} & A_{33}
\end{pmatrix}.
\]

We also assume that the soft scalar masses and gaugino masses \(M_a\) are given by

\[
\begin{align*}
M_a &= m_{1/2}, \quad a = 1, 2, 3, \\
m_0^2 &= m_{h_1}^2 = m_{h_2}^2 = m_0^2, \\
m_Q^2 &= m_{\tilde{Q}}^2 = m_{\tilde{D}}^2 = m_0^2 \text{ diag}\{1, \delta_1, \delta_2\},
\end{align*}
\]

where the parameters \(\delta_i\) and \(A_{ij}\) can vary in the ranges \([0, 1]\) and \([-3, 3]\) respectively. Note that in most string inspired models, the squark mass matrices are diagonal but not necessary universal. The non–universality of the squark masses is not constrained by the EDMs. However, this non–universality (specially between the first two generations of the squark doublets) is severely constrained by \(\Delta M_K\) and \(\epsilon_K\).

In Fig. 6 we display scatter plots for the neutron and mercury EDMs versus the phase \(\phi_{12}\) for \(\tan \beta = 5\), \(m_0 = m_{1/2} = 200\) GeV, \(A_{ij}\) are scanned in the range \([-3, 3]\), and the phases \(\phi_{13}\) and \(\phi_{23}\) are randomly selected in the range \([0, \pi]\). As stated above, the parameters \(\delta_i\) are irrelevant for the EDM calculations and we
set them here to one. Finally, since $\mu$ is real the EDM results display very little dependence on $\tan \beta$.

It is important to mention that we have also imposed the constraints which come from the requirement of absence of charge and colour breaking minima as well as the requirement that the scalar potential be bounded from below \[^{29}\]. These conditions may be automatically satisfied in minimal SUSY models, however in models with non–universal $A$–terms they have to be explicitly checked. In fact, sometimes these constraints are even stronger than the usual bounds set by the flavor changing neutral currents \[^{30}\].

As can be seen from Fig. 6, the EDMs do not exceed the experimental bounds for most of the parameter space. Generally, they are one or two order of magnitude below the present limit, and the flavor–off diagonal phases of the $A$–terms can be $\mathcal{O}(1)$ without fine–tuning. The points that lead to mercury EDM above the experimental bound correspond to $\varphi_{23} \simeq \pi/2$. This phase induces a considerable contribution to the chromoelectric EDM of the strange quark $C_{S}^{2}$. Thus the compatibility with mercury EDM experiment requires that the phase $\varphi_{23}$ should be slightly smaller than $\pi/2$.

4. CP violation in the Kaon system

We have shown in the previous section that the Hermiticity of the Yukawa couplings and $A$–terms allows the existence of large off–diagonal SUSY CP violating phases while keeping the EDMs sufficiently small. However, the important question to address is whether these “EDM–free” phases can have any implication on other CP violation experiments. In this section, we will concentrate on possible effects in the kaon system.

4.1. Indirect CP violation

In the kaon system and due to a CP violation in $K^{0} \rightarrow \bar{K}^{0}$ mixing, the neutral kaon mass eigenstates are superpositions of CP–even ($K_{S}$) and CP–odd ($K_{L}$) components. However, the CP–odd $K_{L}$ decays into two pions through its small CP–even component. This decay, $K_{L} \rightarrow \pi \pi$, was the first observation of CP violation. The measure for the indirect CP violation is defined as

$$
\varepsilon_{K} = \frac{A(K_{L} \rightarrow \pi \pi)}{A(K_{S} \rightarrow \pi \pi)}.
$$

The experimental value for this parameter is $\varepsilon_{K} \simeq 2.28 \times 10^{-3}$. Generally, $\varepsilon_{K}$ can be calculated via

$$
\varepsilon_{K} = \frac{1}{\sqrt{2} \Delta M_{K}} \text{Im}(K^{0}|H_{\text{eff}}^{\Delta S=2}|K^{0})
$$

(48)
Here $H_{\rm eff}^{\Delta S=2}$ is the effective Hamiltonian for the $\Delta S = 2$ transition. It can be expressed via the operator product expansion as

$$H_{\rm eff}^{\Delta S=2} = \sum_i C_i(\mu) Q_i + h.c.$$  \hspace{1cm} (49)

In the above formula, $C_i(\mu)$ are the Wilson coefficients and $Q_i$ are the EH local operators.

The relevant operators for the gluino contribution are

$$Q_1 = \bar{d}_L \gamma_\mu s_L \bar{d}_L^\alpha \gamma^\mu s_L^\alpha, \quad Q_2 = \bar{d}_R \gamma_\mu s_L \bar{d}_R^\alpha \gamma^\mu s_L^\alpha, \quad Q_3 = \bar{d}_R \gamma_\mu s_L \bar{d}_R^\alpha \gamma^\mu s_L^\alpha,$$

$$Q_4 = \bar{d}_R s_L \bar{d}_L s_R, \quad Q_5 = \bar{d}_R s_L \bar{d}_L s_R$$

as well as the operators $Q_{1,2,3}$ that are obtained from $Q_{1,2,3}$ by the exchange $L \leftrightarrow R$. In the latter equations, $\alpha$ and $\beta$ are $SU(3)_c$ colour indices, and the colour matrices obey the normalization $\text{Tr}(t^a t^b) = \delta^{ab}/2$. Due to the gaugino dominance in the chargino–squark loop, the most significant $\tilde{\chi}^\pm$ contribution is associated with the operator $Q_1$.

In the presence of supersymmetric ($\tilde{g}$ and $\tilde{\chi}^\pm$) contributions, the following result for the amplitude $M_{12}$ is obtained:

$$M_{12} = M_{12}^{\text{SM}} + M_{12}^{\tilde{g}} + M_{12}^{\tilde{\chi}^\pm}. \hspace{1cm} (51)$$

The SM contribution can be written as

$$M_{12} = \frac{G_F^2 M_W^2}{12\pi^2} f_K^2 m_K \hat{B}_K \mathcal{F}^*, \hspace{1cm} (52)$$

where $\hat{B}_K$ is the renormalization group invariant which is given by 0.85 ± 0.15, $f_K = 160$ MeV is the K–meson decay constant, and $m_K = 490$ MeV the K–meson mass. The function $\mathcal{F}$ can be give as

$$\mathcal{F} = \eta_1 \lambda_1^2 S_0(x_c) + \eta_2 \lambda_2^2 S_0(x_t) + 2\eta_3 \lambda_c \lambda_t S_0(x_c, x_t), \hspace{1cm} (53)$$

and $\lambda_1 = V_{ts}^\ast V_{td}$, $\eta_i$ are QCD correction factors ($\eta_1 = 1.38 \pm 0.20$, $\eta_2 = 0.57 \pm 0.01$, $\eta_3 = 0.47 \pm 0.04$ - NLO values), and $S_0(x_i)$ are the loop functions given by

$$S_0(x_c) = x_c, \quad S_0(x_t) = 2.46 \left(\frac{m_t}{170 \text{GeV}}\right)^{1.52}$$

$$S_0(x_c, x_t) = x_c \left(\log \frac{x_t}{x_c} - \frac{3x_t}{4(1 - x_t)} - \frac{3x_t^2 \log x_t}{4(1 - x_t)^2}\right) \hspace{1cm} (54)$$

The supersymmetric term $M_{12}^{\tilde{g}}$ is given by

$$M_{12}^{\tilde{g}} = \frac{-\alpha_S}{216 m^2} \frac{1}{3} m_K f_K^2 \left\{ (\delta_{12}^d)^2_{LL} + (\delta_{12}^d)^2_{RR} \right\} B_1(\mu) \left( 24 x f_0(x) + 66 \tilde{f}_0(x) \right)$$

$$+ (\delta_{12}^d)^2_{LL} (\delta_{12}^d)^2_{RR} \left( \frac{M_K}{m_3(\mu) + m_d(\mu)} \right)^2 \left[ B_4(\mu) \left( 378 x f_0(x) - 54 \tilde{f}_0(x) \right) + B_5(\mu) \left( 6 x f_0(x) + 30 \tilde{f}_0(x) \right) \right]$$

$$+ (\delta_{12}^d)^2_{LL} (\delta_{12}^d)^2_{RR} \left( \frac{M_K}{m_3(\mu) + m_d(\mu)} \right)^2$$
\[
\begin{aligned}
(-\frac{255}{2}B_2(\mu) - \frac{9}{2}B_3(\mu)) x f_6(x) + (\delta_{12}^d)_{LR}(\delta_{12}^d)_{RL} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 \\
[-99B_4(\mu) - 49B_5(\mu)] \tilde{f}_6(x)
\end{aligned}
\]

where \( x = (m_3/m_q)^2 \) and the functions \( f_6(x) \), \( \tilde{f}_6(x) \) are given by

\[
\begin{aligned}
f_6(x) &= \frac{6(1 + 3x) \ln x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^5}, \\
\tilde{f}_6(x) &= \frac{6(1 + 3x) \ln x + x^3 - 9x^2 - 9x + 17}{6(x - 1)^5}.
\end{aligned}
\]

The matrix elements of the operators \( Q_i \) between the \( K \)-meson states in the vacuum insertion approximation (VIA), where \( B = 1 \), can be found in Ref.\(^6\). The VIA generally gives only a rough estimate, so other methods, e.g. lattice QCD, are required to obtain a more realistic value. The matrix elements of the renormalized operators can be written as

\[
\begin{aligned}
\langle \bar{K}^0 | Q_1(\mu) | K^0 \rangle &= \frac{1}{3} M_K f_K^2 B_1(\mu), \\
\langle \bar{K}^0 | Q_2(\mu) | K^0 \rangle &= -\frac{5}{24} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_2(\mu), \\
\langle \bar{K}^0 | Q_3(\mu) | K^0 \rangle &= -\frac{1}{24} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_3(\mu), \\
\langle \bar{K}^0 | Q_4(\mu) | K^0 \rangle &= -\frac{5}{24} \left( \frac{M_K}{m_s(\mu) + m_d(\mu)} \right)^2 M_K f_K^2 B_4(\mu),
\end{aligned}
\]

where \( Q_i(\mu) \) are the operators renormalized at the scale \( \mu \). The expressions of the matrix elements of the operators \( Q_1-3 \) are valid for the operators \( \bar{Q}_1-3 \), and for \( \mu = 2 \text{ GeV} \) we have

\[
\begin{aligned}
B_1(\mu) &= 0.60, \\
B_2(\mu) &= 0.66, \\
B_3(\mu) &= 1.05, \\
B_4(\mu) &= 1.03, \\
B_5(\mu) &= 0.73.
\end{aligned}
\]

Using these values, the gluino contribution to the \( K^0 - \bar{K}^0 \) can be calculated via Eq.\(^{13}\). As mentioned above, for universal soft scalar masses the LL and RR insertions are generated only through the RG running and can be neglected. The LR and RL mass insertions appear at the tree level and may have tangible effects. It is worth mentioning that, the RL and LR mass insertions contribute with the same sign in Eq.\(^{17}\) and for Hermitian A-terms they are almost equal, so no cancellation between these two contributions occurs.

In Fig.\(^7\) we plot the values of \( |\varepsilon_K| \) versus the phase \( \varphi_{12} \) for \( \delta_i = 1 \) and the other parameters are chosen as in Fig.\(^3\). From this figure, we conclude that the SUSY contribution with Hermitian Yukawa and universal soft scalar masses can not account for the experimentally observed indirect CP violation in the Kaon system.
The gluino contribution to $|\varepsilon_K|$ as a function of $\varphi_{12}$ for $\delta_i = 1$, $\tan \beta = 5$, and $m_0 = m_{1/2} = 200$ GeV.

In Ref.\textsuperscript{7} values for $\varepsilon_K \sim 10^{-3}$ have been obtained but these values require light gaugino mass ($m_{1/2} \sim 100$ GeV) which is now excluded by the new experimental limits on the mass of the lightest Higgs. Also it requires that the magnitude of the off–diagonal entries of the $A$–terms should be much larger (at least five times larger) than the diagonal ones, which looks unnatural.

The chargino contribution to the $K^0 - \bar{K}^0$ mixing is given by\textsuperscript{32}

$$M_{\tilde{\chi}^\pm} = \frac{g^2}{768\pi^2m_{\tilde{q}}} \frac{1}{3} m_K f_K B_1(\mu) \left( \sum_{a,b} K^*_{a2}(\delta^u_{LL})_{ab} K_{b1} \right)^2 \sum_{i,j} |V_{i1}|^2 |V_{j1}|^2 H(x_i, x_j), \quad (62)$$

where $x_i = (m_{\tilde{\chi}^+}/m_{\tilde{q}})$. Here $K$ is the CKM matrix, $a, b$ are flavour indices, $i, j$ label the chargino mass eigenstates, and $V$ is one of the matrices used for diagonalizing the chargino mass matrix. The loop function $H(x_i, x_j)$ is given in Ref.\textsuperscript{32}. However, the chargino contribution can be significant only if there is a large LL mixing in the up–sector, namely $\text{Im}(\delta^u_{LL})_{21} \sim 10^{-3}$ and $\text{Re}(\delta^u_{LL})_{21} \sim 10^{-2}$\textsuperscript{32}. Such mixing can not be accommodated with the universal scalar masses assumption (i.e., $\delta_i = 1$). In this case, the values of the $\text{Im}(\delta^u_{LL})$ are of order $10^{-6}$ which leads to a negligible chargino contribution to $\varepsilon_K$. With non–universal soft scalar masses ($\delta_i \neq 1$) a possible enhancement in the chargino contribution is expected however, as we will show, this non–universality also leads to a larger enhancement in the gluino contribution. So, the dominant SUSY contribution to the $K - \bar{K}$ mixing in this class of models will be provided by the gluino exchange diagrams.

A possible way to enhance $\varepsilon_K$ is to have non–universal soft squark masses at GUT scale\textsuperscript{28}. As mentioned above, the soft scalar masses are not necessarily universal in generic SUSY models and their non–universality is not constrained by
the EDMs. Thus for $\delta_i \neq 1$ the mass insertion $(\delta_{12}^d)_{RR}$ is enhanced and we can easily saturate $\varepsilon_K$ through the gluino contribution. To see this more explicitly, let us consider the LL and RR squark mass matrices in the super-CKM basis

$$(M_2^d)_{LL} \sim V^d M_Q^2 V^d,$$

$$(M_2^d)_{RR} \sim V^d (M_D^2)^T V^d. \quad (63)$$

Due to the universality assumption of $M_Q^2$ at GUT scale, the matrix $(M_2^d)_{LL}$ remains approximately universal and the mass insertions $(\delta_{12}^d)_{LL}$ are sufficiently small ($\text{Im}(\delta_{12}^d)_{LL} \sim 10^{-5}$). However, since the masses of the squark singlets $M_D^2$ are not universal, Eq. (63), sizeable off-diagonal elements in $(M_2^d)_{RR}$ are obtained.

We find that $\text{Re}(\delta_{12}^d)_{RR}$ is enhanced from $\sim 10^{-7}$ in the universal case ($\delta_i = 1$) to $\sim 10^{-3}$ for $\delta_i \sim 0.7$ while the imaginary part remains the same, of order $10^{-7}$. Thus, in this case, we have

$$\sqrt{\text{Im}((\delta_{12}^d)_{LL} (\delta_{12}^d)_{RR})} \simeq \sqrt{\text{Re}((\delta_{12}^d)_{RR} \text{Im}(\delta_{12}^d)_{LL})} \simeq 10^{-4}$$

which is the required value in order to saturate the observed result of $\varepsilon_K$.\[\Box\]

In Fig. 8 we show the dependence of $|\varepsilon_K|$ on the parameters $\delta_i$. There, the two curves, from top to bottom, correspond to the values of $|\varepsilon_K|$ versus $\delta_1$ (for $\delta_2 = 1$) and $\delta_2$ (for $\delta_1 = 1$) respectively. As explained above, in this scenario the main contribution of $\varepsilon_K$ is due to LL and RR sectors and the LR sector has essentially no effect on $\varepsilon_K$. We also see that any non-universality between the soft scalar masses of the third and the first two generations can not lead to significant contribution to $\varepsilon_K$ and some splitting between the scalar masses of the first two generations is
necessary. This stems from the fact that the effect of the third generation on the mass insertion ($\delta_{12}^d$) is suppressed by $V_{13} \sim O(10^{-2})$ while $V_{12} \sim \sin \theta_C$. Finally, as we can see from this figure, in order to avoid over saturation of the experimental value of $\varepsilon_K$, the parameter $\delta_1$ should be of order 0.8.

4.2. Direct CP violation

Next let us consider SUSY contributions to $\varepsilon'/\varepsilon$. The ratio $\varepsilon'/\varepsilon$ is a measure of direct CP violation in the $K \to \pi\pi$ decays and is given by

$$\varepsilon'/\varepsilon = -\frac{\omega}{\sqrt{2} |\varepsilon|} \frac{\text{Re} A_0 - \frac{1}{\omega} \text{Im} A_2}{\text{Re} A_0},$$

where $A_{0,2}$ are the amplitudes for the $\Delta I = 1/2, 3/2$ transitions, and $\omega \equiv \text{Re} A_2/\text{Re} A_0 \simeq 1/22$ reflects the strong enhancement of $\Delta I = 1/2$ transitions over those with $\Delta I = 3/2$. Experimentally it has been found to be $\text{Re}(\varepsilon'/\varepsilon) \simeq 1.9 \times 10^{-3}$ which provides firm evidence for the existence of direct CP violation. The $\text{Im} A_{0,2}$ are calculated from the general low energy effective Hamiltonian for $\Delta S = 1$ transition,

$$H_{\text{eff}}^{\Delta S = 1} = \sum_i C_i(\mu) O_i + h.c.,$$

where $C_i$ are the Wilson coefficients and the list of the relevant operators $O_i$ for this transition is given in Ref.\cite{37,38,39}. Let us recall here that these operators can be classified into three categories. The first category includes dimension six operators: $O_{1,2}$ which refer to the current-current operators, $O_{3-6}$ for QCD penguin operators and $O_{7-10}$ for electroweak penguin operators.\cite{37} The second category includes dimension five operators: magnetic- and electric-dipole penguin operators $O_g$ and $O_\gamma$ which are induced by the gluino exchange.\cite{38} The third category includes the only dimension four operator $O_Z$ generated by the $\bar{s}dZ$ vertex which is mediated by chargino exchanges.\cite{39} In addition, one should take into account $\tilde{O}_i$ operators which are obtained from $O_i$ by the exchange $L \leftrightarrow R$.

In spite of the presence of this large number of operators that in principle can contribute to $\varepsilon'/\varepsilon$, it is remarkable that few of them can give significant contributions. As we will discuss below, this is due to the suppression of the matrix elements and/or the associated Wilson coefficients of most of the operators. The SM contribution to $\varepsilon'/\varepsilon$ is dominated by the operators $Q_6$ and $Q_8$, and can be expressed as

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)^{\text{SM}} = \frac{\text{Im} (\lambda_t \lambda^*_u)}{|\lambda_u|} F_{\varepsilon'},$$

where $\lambda_i = V_{ts}^* V_{td}$ and the function $F_{\varepsilon'}$ is given in Ref.\cite{33}. By using our Hermitian Yukawa in Eq. (40) we get

$$\varepsilon'/\varepsilon \simeq 7.5 \times 10^{-4}.$$
Again, the SM prediction is below the observed value. Nevertheless, this estimation cannot be considered as a firm conclusion for a new physics beyond the SM since there are significant hadronic uncertainties are involved.

The supersymmetric contribution to $\varepsilon'/\varepsilon$ depends on the flavor structure of the SUSY model. It is known that, in a minimal flavor SUSY model, it is not possible to generate a sizeable contribution to $\varepsilon'/\varepsilon$ even if the SUSY phases are assumed to be large. Recently, it has been pointed out that with non–degenerate $A$–terms the gluino contribution to $\varepsilon'/\varepsilon$ can naturally be enhanced to saturate the observed value. Indeed, in this scenario, the LR mass insertions can have large imaginary parts and the chromomagnetic operator $O_g$ gives the dominant contribution to $\varepsilon'/\varepsilon$.

\[
\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) \simeq \frac{11\sqrt{3}}{64\pi^2} \frac{m_s}{m_s + m_d} \frac{F_k^2}{F_{\pi}} m_k^2 m_\pi^2 m_\pi^2 \text{Im} \left[ C_g - \tilde{C}_g \right],
\]

where $C_g$ is the Wilson coefficient associated with the operator $O_g$, given by

\[
C_g = \frac{\alpha_s\pi}{m_\tilde{g}} \left[ \left( \delta_{12}^{d}_{LL} \left( -\frac{1}{3} M_3(x) - 3 M_2(x) \right) + (\delta_{12}^{d}_{LR}) \frac{m_\tilde{g}}{m_s} \left( -\frac{1}{3} M_1(x) - 3 M_2(x) \right) \right) \right],
\]

where the functions $M_i(x)$ can be found in Ref. 32 and $x = m_3^2/m_\tilde{g}^2$.

Using these relations, one finds that in order to saturate $\varepsilon'/\varepsilon$ from the gluino contribution the imaginary parts of the LR mass insertions for $x \simeq 1$ should satisfy $\text{Im}(\delta_{12}^{d}_{LR}) \sim 10^{-5}$. Such values can easily be obtained in this class of models. However, as mentioned above, in the case of Hermitian $A$–terms and Yukawa couplings we have $\delta_{12}^{d}_{LR} \simeq (\delta_{12}^{d}_{RL})$, hence we get $\text{Im} \left[ C_g - \tilde{C}_g \right] \simeq \text{Im} \left[ (\delta_{12}^{d}_{LR}) - (\delta_{12}^{d}_{RL}) \right] \simeq 10^{-6}$ which leads to a negligible gluino contribution to $\varepsilon'/\varepsilon$. It is worth noticing that, due to the universality assumption of $M_{Q_3}$, the imaginary part of the mass insertion $\delta_{12}^{d}_{LL}$ is of order $10^{-5}$. So its contribution to $C_g$ is negligible with respect to the LR one which is enhanced by the ratio $m_\tilde{g}/m_s$. To achieve the required contribution to $\varepsilon'/\varepsilon$ from the LL sector, one has to relax this universality assumption to get $\text{Im}(\delta_{12}^{d}_{LL}) \sim 10^{-2}$. However, as we will discuss below, in this case the chargino contribution is also enhanced and becomes dominant.

Now we turn to the chargino contributions. The dominant contribution is found to be due to the terms proportional to a single mass insertion $K_{ab}$.

\[
\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) \simeq \text{Im} \left[ \sum_{a,b} K_{ab}^\ast (\delta_{ab}^u)_{LL} K_{ab} \right] F_{\varepsilon'}(x_{\chi^\pm}).
\]

The function $F_{\varepsilon'}(x_{\chi^\pm})$, where $x_{\chi^\pm} = m_{\chi^\pm}/m_\tilde{g}$, is given in Ref. 32. We find that the contributions involving a double mass insertion, like those arising from the supersymmetric effective $sdZ$, can not give any significant contribution, however we take them into account. The above contribution is dominated by $(\delta_{12}^{u}_{LL})$ and in order to account for $\varepsilon'/\varepsilon$ entirely from the chargino exchange the up sector has to employ a large LL mixing. Again, with universal $M_{Q_3}$, $\text{Im}(\delta_{12}^{u}_{LL}) \sim 10^{-6}$ and the chargino contributions (as the gluino one) to $\varepsilon'/\varepsilon$ is negligible.
Finally, we consider another possibility proposed by Kagan and Neubert to obtain a large contribution to $\varepsilon'/\varepsilon$. It is important to note that in the previous mechanisms to generate $\varepsilon'/\varepsilon$ one is tacitly assuming that $\Delta I = 1/2$ transitions are dominant and that the $\Delta I = 3/2$ ones are suppressed as in the SM. However, in Ref. it was shown that it is possible to generate a large $\varepsilon'/\varepsilon$ from the $\Delta I = 3/2$ penguin operators. This mechanism relies on the LL mass insertion ($\delta_{12}^d$)$_{LL}$ and requires isospin violation in the squark masses ($m_\tilde{u} \neq m_\tilde{d}$). In this case, the relevant $\Delta S = 1$ gluino box diagrams lead to the effective Hamiltonian

$$H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{i=3}^{6} \left( C_i(\mu)Q_1 + \tilde{C}_i\tilde{Q}_1 \right) + \text{h.c.} \quad (71)$$

where

$$Q_1 = (\bar{d}_\alpha s_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V+A}, \quad Q_2 = (\bar{d}_\alpha s_\alpha)_{V-A} (\bar{q}_\beta s_\alpha)_{V+A}, \quad (72)$$

$$Q_3 = (\bar{d}_\alpha s_\alpha)_{V-A} (\bar{q}_\beta q_\beta)_{V-A}, \quad Q_4 = (\bar{d}_\alpha s_\alpha)_{V-A} (\bar{q}_\beta s_\alpha)_{V-A}, \quad (73)$$

and the operators $\tilde{Q}_i$ are obtained from $Q_i$ by exchanging $L \leftrightarrow R$. It turns out that the SUSY $\Delta I = 3/2$ contribution to $\varepsilon'/\varepsilon$ is given by

$$\text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right)_{\Delta I=3/2} \approx 19.2 \left( \frac{500 \text{GeV}}{m_\tilde{g}} \right)^2 B_{\delta_{12}^d}^{(2)}(x_u^L, x_u^R, x_d^R) \text{Im}(\delta_{12}^d)_{LL}. \quad (74)$$

Here, $x_u^L, x_d^R = \left( \frac{m_\tilde{u}_L}{m_\tilde{u}_R} \right)^2, x_d^L, x_u^R = \left( \frac{m_\tilde{d}_R}{m_\tilde{d}_L} \right)^2, B_{\delta_{12}^d}(m_c) \simeq 1$ and the function $K(x, y, z)$ is given by

$$K(x, y, z) = \frac{32}{27} \left[ f(x, y) - f(x, z) \right] + \frac{2}{27} \left[ g(x, y) - g(x, z) \right], \quad (75)$$

where $f(x, y)$ and $g(x, y)$ are given in Ref. It is clear from the above equation that for $m_{\tilde{d}_R} = m_{\tilde{u}_R}$, the function $K(x_d^L, x_u^R, x_d^R)$ vanishes identically. Thus, a mass splitting between the right–handed squark mass is necessary to get large contributions to $\varepsilon'/\varepsilon$ through this mechanism. Furthermore, the $\text{Im}(\delta_{12}^d)_{LL}$ has to be of order $O(10^{-3} - 10^{-2})$ to saturate the observed value of $\varepsilon'/\varepsilon$. It is clear that, with universal $M_Q^2$, this contribution can not give any significant value for $\varepsilon'/\varepsilon$.

Now we relax the universality assumption of $M_Q^2$ at GUT scale to enhance the mass insertion ($\delta_{12}^d$)$_{LL}$ and saturate the experimental value of $\varepsilon'/\varepsilon$. As mentioned above, the non–universality between the first two generation of $M_Q^2$ is very constrained by $\Delta M_K$ and $\varepsilon_K$. Therefore we just assume that the mass of third generation is given by $\delta_3 m_0$ while the masses of the first two generations remain universal and equal to $m_0$. This deviation from universality provides enhancement to both $\varepsilon_K$ and $\varepsilon'/\varepsilon$. We have chosen the parameters $\delta_i$ so that the total contributions of $\varepsilon_K$ from chargino and gluino are consistent with the experimental limits.

In Fig. 2 we present the different gluino and chargino contributions to the $\varepsilon'/\varepsilon$ and also the total contribution versus the imaginary part of the mass insertion $(\delta_{12}^d)_{LL}$. As explained above, there are two sources for the gluino contributions to
Fig. 9. The $\varepsilon'/\varepsilon$ contributions versus the Imaginary part of the mass insertion $(\delta_{12}^d)_{LL}$.

$\varepsilon'/\varepsilon$: the usual $\Delta I = 1/2$ chromomagnetic dipole operator and the new $\Delta I = 3/2$ penguin operators. Additionally, there are two sources for the chargino contribution to $\varepsilon'/\varepsilon$: the usual gluon and electroweak penguin diagrams with single mass insertion and the contribution from the SUSY effective $sdZ$ vertex. As can be seen from this figure, the dominant contribution to $\varepsilon'/\varepsilon$ is due to the chargino exchange with one mass insertion. It turns out that the chargino contribution with two mass insertions is negligible. As expected the gluino contribution via the chromomagnetic operator is also negligible due to the severe cancellation between the LR and RL contributions. The contribution from the $\Delta I = 3/2$ operators does not lead to significant results for $\varepsilon'/\varepsilon$. It even becomes negative (opposite to the other contributions) for $\text{Im}(\delta_{12}^d)_{LL} > 2.5 \times 10^{-3}$.

From this figure we conclude that a non-universality among the soft scalar masses is necessary to get large values of $\varepsilon'/\varepsilon$ and $\varepsilon_K$.

5. CP violation in the $B$–system

5.1. Flavour–dependent SUSY phases and CP asymmetry in $B \to J/\psi K_s$

Recent results from the $B$–factories have confirmed, for the first time, the existence of CP violation in the $B$–meson decays. In particular, the measurements of the CP asymmetry in the $B_d \to \psi K_s$ decay [2] have verified that CP is significantly violated in the $B$–sector. In $B_d - \bar{B}_d$ mixing, the flavor eigenstates are $B_d = (\bar{b}d)$ and $\bar{B}_d = (bd)$.

It is customary to denote the corresponding mass eigenstates by $B_H = pB_d + q\bar{B}_d$ and $B_L = p\bar{B}_d - qB_d$ where indices H and L refer to heavy and light mass eigenstates respectively, and $p = (1 + \bar{\varepsilon}_B)/\sqrt{2(1 + |\bar{\varepsilon}_B|)}$, $q = (1 - \bar{\varepsilon}_B)/\sqrt{2(1 + |\bar{\varepsilon}_B|)}$ where $\bar{\varepsilon}_B$
is the corresponding CP violating parameter in the $B_d - \bar{B}_d$ system, analogous of $\varepsilon$ in the kaon system. Then the strength of $B_d - \bar{B}_d$ mixing is described by the mass difference

$$\Delta M_{B_d} = M_{B_H} - M_{B_L}. \quad (76)$$

whose present experimental value is $\Delta M_{B_d} = 0.484 \pm 0.010 \ (ps)^{-1}$.

The CP asymmetry of the $B_d$ and $\bar{B}_d$ meson decay to the CP eigenstate $\psi K_S$ is given by

$$a_{\psi K_S}(t) = \frac{\Gamma(B^0_d(t) \to \psi K_S) - \Gamma(\bar{B}^0_d(t) \to \psi K_S)}{\Gamma(B^0_d(t) \to \psi K_S) + \Gamma(\bar{B}^0_d(t) \to \psi K_S)} = -a_{\psi K_S} \sin(\Delta m_{B_d} t). \quad (77)$$

The most recent measurements of this asymmetry are given by

$$a_{\psi K_S} = 0.59 \pm 0.14 \pm 0.05 \quad \text{(BaBar)},$$
$$a_{\psi K_S} = 0.99 \pm 0.14 \pm 0.06 \quad \text{(Belle)}. \quad (78)$$

where the second and third numbers correspond to statistically and systematic errors respectively, and so the present world average is given by $a_{\psi K_S} = 0.79 \pm 12$. These results show that there is a large CP asymmetry in the $B$ meson system. This implies that either the CP is not an approximate symmetry in nature and that the CKM mechanism is the dominant source of CP violation or CP is an approximate symmetry with large flavour structure beyond the standard CKM matrix. Generally, $\Delta M_{B_d}$ and $a_{\psi K_S}$ can be calculated via

$$\Delta M_{B_d} = 2|\langle B^0_d|H_{\text{eff}}^{\Delta B=2}|\bar{B}^0_d\rangle|, \quad (79)$$

$$a_{\psi K_S} = \sin 2\beta_{\text{eff}}, \quad \text{and} \quad \beta_{\text{eff}} = \frac{1}{2} \arg(\langle B^0_d|H_{\text{eff}}^{\Delta B=2}|\bar{B}^0_d\rangle). \quad (80)$$

where $H_{\text{eff}}^{\Delta B=2}$ is the effective Hamiltonian responsible of the $\Delta B = 2$ transitions.

In the framework of the standard model (SM), $a_{\psi K_S}$ can be easily related to one of the inner angles of the unitarity triangles and parametrized by the $V_{CKM}$ elements as follows

$$a_{\psi K_S}^{\text{SM}} = \sin 2\beta, \quad \beta = \arg \left( \frac{-V_{cd} V^*_{cb}}{V_{td} V^*_{tb}} \right), \quad (81)$$

In supersymmetric theories the effective Hamiltonian for $\Delta B = 2$ transitions, can be generated, in addition to the $W$ box diagrams of SM, through other box diagrams mediated by charged Higgs, neutralino, gluino, and chargino exchanges. The Higgs contributions are suppressed by the quark masses and can be neglected. The neutralino exchange diagrams are also very suppressed compared to the gluino and chargino ones, due to their electroweak neutral couplings to fermion and sfermions. Thus, the dominant SUSY contributions to the off diagonal entry in the $B$-meson mass matrix, $M_{12}(B_d) = \langle B^0_d|H_{\text{eff}}^{\Delta B=2}|\bar{B}^0_d\rangle$, is given by

$$M_{12}(B_d) = M_{12}^{\text{SM}}(B_d) + M_{12}^g(B_d) + M_{12}^{\tilde{\chi}^+}(B_d). \quad (82)$$
where $\mathcal{M}^{\text{SM}}_{12}(B_d)$, $\mathcal{M}^{\tilde{g}}_{12}(B_d)$, and $\mathcal{M}^{\tilde{e}^+}_{12}(B_d)$ indicate the SM, gluino, and chargino contributions respectively. The SM contribution is known at NLO accuracy in QCD and as well as the leading SUSY contributions\(^3\) and it is given by

$$
\mathcal{M}^{\text{SM}}_{12}(B_d) = \frac{G_F^2}{12\pi^2} \eta_B \hat{B}_{B_d} f_{B_d} M_{B_d} M_W^2 (V_{td} V_{tb}^*)^2 S_0(x_t),
$$

(83)

where $f_{B_d}$ is the B meson decay constant, $\hat{B}_{B_d}$ is the renormalization group invariant $B$ parameter (for its definition and numerical value, see Ref. and reference therein) and $\eta = 0.55 \pm 0.01$. The function $S_0(x_t)$, connected to the $\Delta B = 2$ box diagram with $W$ exchange, is given by

$$
S_0(x_t) = \frac{4x_t - 11x_t^2 + x_t^3}{4(1 - x_t)^2} - \frac{3x_t^3 \ln x_t}{2(1 - x_t)^3},
$$

(84)

where $x_t = M_t^2/M_W^2$.

The effect of supersymmetry can be simply described by a dimensionless parameter $r_d^2$ and a phase $2\theta_d$ defined as follows

$$
r_d^2 e^{2i\theta_d} = \frac{\mathcal{M}_{12}(B_d)}{\mathcal{M}^{\text{SM}}_{12}(B_d)},
$$

(85)

where $\Delta M_{B_d} = 2|\mathcal{M}^{\text{SUSY}}_{12}(B_d)| r_d^2$. Thus, in the presence of SUSY contributions, the CP asymmetry $B_d \to \psi K_S$ is modified, and now we have

$$
a_{\psi K_S} = \sin 2\beta_{\text{eff}} = \sin(2\beta + 2\theta_d).
$$

(86)

Therefore, the measurement of $a_{\psi K_S}$ would not determine $\sin 2\beta$ but rather $\sin 2\beta_{\text{eff}}$, where

$$
2\theta_d = \arg \left( 1 + \frac{\mathcal{M}^{\text{SUSY}}_{12}(B_d)}{\mathcal{M}^{\text{SM}}_{12}(B_d)} \right),
$$

(87)

and $\mathcal{M}^{\text{SUSY}}_{12}(B_d) = \mathcal{M}^{\tilde{g}}_{12}(B_d) + \mathcal{M}^{\tilde{e}^+}_{12}(B_d)$.

The most general effective Hamiltonian for $\Delta B = 2$ processes, induced by gluino and chargino exchanges through $\Delta B = 2$ box diagrams, can be expressed as

$$
H^{\Delta B=2}_{\text{eff}} = \sum_{i=1}^5 C_i(\mu) Q_i(\mu) + \sum_{i=1}^3 \tilde{C}_i(\mu) \tilde{Q}_i(\mu) + h.c.,
$$

(88)

where $C_i(\mu)$, $\tilde{C}_i(\mu)$ and $Q_i(\mu)$, $\tilde{Q}_i(\mu)$ are the Wilson coefficients and operators respectively renormalized at the scale $\mu$, with

$$
Q_1 = \bar{d}_L^\gamma \gamma_L b_L^\alpha, \quad P_2 = \bar{d}_R^\beta \gamma_L b_L^\beta, \quad Q_3 = \bar{d}_R^\beta \gamma_L b_L^\beta,
$$

$$
Q_4 = \bar{d}_R^\beta \gamma_L b_L^\beta, \quad Q_5 = \bar{d}_R^\beta \gamma_L b_L^\beta.
$$

(89)

In addition, the operators $\tilde{Q}_{1,2,3}$ are obtained from $Q_{1,2,3}$ by exchanging $L \leftrightarrow R$. In the case of the gluino exchange all the above operators give significant contributions.
and the corresponding Wilson coefficients are given by

\[ C_1(M_S) = -\frac{\alpha^2}{216m^2_q} (24x f_6(x) + 66\tilde{f}_6(x)) (\delta^d_{13})_{LL}, \]

\[ C_2(M_S) = -\frac{\alpha^2}{216m^2_q} 204x f_6(x)(\delta^d_{13})_{RL}, \]

\[ C_3(M_S) = \frac{\alpha^2}{216m^2_q} 36x f_6(x)(\delta^d_{13})_{RL}, \]

\[ C_4(M_S) = -\frac{\alpha^2}{216m^2_q} \left[ \left( 504x f_6(x) - 72\tilde{f}_6(x) \right) (\delta^d_{13})_{LL}(\delta^d_{13})_{RR} - 132\tilde{f}_6(x)(\delta^d_{13})_{LR}(\delta^d_{13})_{RL} \right], \]

\[ C_5(M_S) = -\frac{\alpha^2}{216m^2_q} \left[ \left( 24x f_6(x) + 120\tilde{f}_6(x) \right) (\delta^d_{13})_{LL}(\delta^d_{13})_{RR} - 180\tilde{f}_6(x)(\delta^d_{13})_{LR}(\delta^d_{13})_{RL} \right], \]

where \( x = m^2_{\tilde{q}}/\tilde{m}^2 \) and \( \tilde{m}^2 \) is an average squark mass. The expression for the functions \( f_6(x) \) and \( \tilde{f}_6(x) \) can be found in Ref. [42]. The Wilson coefficients \( \tilde{C}_{1,2,3} \) are simply obtained by interchanging \( L \leftrightarrow R \) in the mass insertions appearing in \( C_{1,2,3} \).

Now we consider the chargino contribution to the effective Hamiltonian in Eq. (88) in the mass insertion approximation. The dominant chargino exchange can give significant contributions only to the operators \( Q_1 \) and \( Q_3 \) in Eq. (89). At the first order in the mass insertion approximation, the Wilson coefficients \( C_1^\chi(M_S) \) (by taking different the mass of the stop–right from the average squark mass) take the form

\[ C_1^\chi(M_S) = \frac{g^4}{768\pi^2\tilde{m}^2} \sum_{i,j} \left\{ |V_{i1}|^2 |V_{j1}|^2 \left( (\delta^u_{LL})_{31}^2 + 2\lambda(\delta^u_{LL})_{31}(\delta^u_{LL})_{32} \right) L_2(x_i, x_j) - 2Y_{i1}V_{i1}V_{j1}V_{j2}^* \left( (\delta^u_{LL})_{31}(\delta^u_{RL})_{31} + \lambda(\delta^u_{LL})_{32}(\delta^u_{RL})_{31} + \lambda(\delta^u_{LL})_{31}(\delta^u_{RL})_{32} \right) R_2(x_i, x_j, z) + Y_{i2}V_{i2}V_{i1}V_{j2}^* \left( (\delta^u_{RL})_{31}^2 + 2\lambda(\delta^u_{RL})_{31}(\delta^u_{RL})_{32} \right) \tilde{R}_2(x_i, x_j, z) \right\}, \]

\[ C_3^\chi(M_S) = \frac{g^4\gamma^2}{192\pi^2\tilde{m}^2} \sum_{i,j} U_{i2}U_{j2}V_{i1}V_{i1} \left( (\delta^u_{LL})_{31}^2 + 2\lambda(\delta^u_{LL})_{31}(\delta^u_{LL})_{32} \right) L_0(x_i, x_j), \]

where \( x_i = m^2_{\tilde{e}_i}/\tilde{m}^2 \), and the functions \( L_0(x, y) \) and \( L_2(x, y) \) are given by

\[ L_0(x, y) = \sqrt{y} \left( x h_0(x) - y h_0(y) \right), \quad h_0(x) = \frac{-11 + 7x - 2x^2}{(1-x)^3} - \frac{6\ln x}{(1-x)^4} \]

\[ L_2(x, y) = \frac{x h_2(x) - y h_2(y)}{x - y}, \quad h_2(x) = \frac{2 + 5x - x^2}{(1-x)^3} + \frac{6x\ln x}{(1-x)^4} \]

and

\[ R_2(x, y, z) = \frac{1}{x - y} (H_2(x, z) - H_2(y, z)), \quad \tilde{R}_2(x, y, z) = \frac{1}{x - y} (\tilde{H}_2(x, z) - \tilde{H}_2(y, z)) \]

\[ H_2(x, z) = \frac{3}{D_2(x, z)} \left\{ (-1 + x)(x - z)(-1 + z)(-1 - x - z + 3xz) \right\} \]
\[
H_2(x, z) = \frac{-6}{D_2(x, z)} \left\{ (-1 + x)(x - z)(-1 + z)(x + (-2 + x)z) + 6x^2(-1 + z)^3 \log(x) - 6(-1 + x)^3z^2 \log(z) \right\}.
\]

Here \(D_2(x, z) = (-1 + x)^3(x - z)(-1 + z)^3\) and \(\tilde{D}_2(x, z) = (-1 + x)^2(x - z)^2(-1 + z)^3\). Notice that in the limit \(z \to 1\), both the functions \(R_2(x, y, z)\) and \(\tilde{R}_2(x, y, z)\) tend to \(L_2(x, y)\). As in the gluino case, the corresponding results for \(\tilde{C}_1\) and \(\tilde{C}_1\) coefficients are simply obtained by interchanging \(L \leftrightarrow R\) in the mass insertions appearing in the expressions for \(C_{1,3}\).

In order to connect \(C_i(M_S)\) at SUSY scale \(M_S\) with the corresponding low energy ones \(C_i(\mu)\) (where \(\mu \simeq \mathcal{O}(m_h)\)), one has to solve the renormalization group equations (RGE) for the Wilson coefficients corresponding to the effective Hamiltonian in (88). Then, \(C_i(\mu)\) will be related to \(C_i(M_S)\) by the following relation:

\[
C_r(\mu) = \sum_i \sum_s \left( b_i^{(r,s)} + \eta_i^{(r,s)} \right) \eta^{a_i} C_s(M_S),
\]

where \(M_S > m_4\) and \(\eta = \alpha_S(M_S)/\alpha_S(\mu)\). The values of the coefficients \(b_i^{(r,s)}\), \(c_i^{(r,s)}\), and \(a_i\) appearing in (83) can be found in Ref.\[2\].

In tables (1) and (2) we present the bounds on real and imaginary parts of mass insertions respectively, by taking into account a light stop–right mass. We considered two representative cases of \(\tilde{m}_{t_R} = 100, \ 200 \text{ GeV}\). Clearly, the light stop–right effect does not affect bounds on mass insertions containing LL interactions. From these results we could see that the effect of taking \(\tilde{m}_{t_R} < \tilde{m}\) is sizable. In particular, on the bounds of the mass insertions \((\delta_{RL}^u)_{31}(\delta_{RL}^u)_{3i}, \ (i = 1, 2)\) which are the most sensitive to a light stop–right.

From the results in tables (10) and (11), it is remarkable to notice that, in the limit of very heavy squark masses but with fixed right stop and chargino masses, the bounds on the mass insertions \((\delta_{RL}^u)_{31}(\delta_{RL}^u)_{3i}\) tend to constant values. This is indeed an interesting property which shows a particular non–decoupling effect of supersymmetry when two light right–stop run inside the diagrams in Fig. (1). This feature is related to the infrared singularity of the loop function \(\tilde{R}_2(x, x, z)\) in the limit \(z \to 0\). In particular, we find that \(\lim_{z \to 0} \tilde{R}_2(x, x, z) = f(x)/x\), where \(x = \tilde{m}_\chi^2/\tilde{m}^2\), and \(f(x)\) is a non-singular and non-null function in \(x = 0\). Then, in the limit \(\tilde{m} >> m_\chi\) the rescaling factor \(1/\tilde{m}^2\) in \(C_i^x\) will be canceled by the \(1/x\) dependence in the loop function and replaced by \(1/m_\chi^2\) times a constant factor.

This is a quite interesting result, since it shows that in the case of light right stop and charginos masses, in comparison to the other squark masses, the SUSY contribution (mediated by charginos) to the \(\Delta B = 2\) processes might not decouple and could be sizable, provided that the mass insertions \((\delta_{RL}^u)_{3i}\) are large enough. This effect could be achieved, for instance, in supersymmetric models with non–universal soft breaking terms.
In the case of SUSY models with Hermitian flavor structure, we find that in most of the parameter space the chargino gives the dominant contribution to $B_d - \bar{B}_d$ mixing and the CP asymmetry $a_{J/\psi K_S}$ and the gluino is sub-dominant. As we emphasized above, in order to have a significant gluino contribution for $\tilde{m} \sim m_g \sim 500$ GeV (i.e., $m_0 \sim M_{1/2} \sim 200$ at GUT scale), the real and imaginary parts of mass insertion $|\delta_{13}^d|_{LL}$ or $|\delta_{13}^u|_{LR}$ should be of order $10^{-1}$ and $10^{-2}$ respectively. However, with the above hierarchical Yukawas we find that these mass insertions are two orders of magnitude below the required values so that the gluino contributions are very small.

Concerning the chargino amplitude to the CP asymmetry $a_{J/\psi K_S}$, we find that the mass insertions $|\delta_{13}^d|_{RL}$ and $|\delta_{13}^u|_{LL}$ give the leading contribution to $a_{J/\psi K_S}$. However, for the representative case of $m_0 = m_{1/2} = 200$ and $\phi_{ij} \simeq \pi/2$ the values of these mass insertions are given by

\[
\sqrt{|\text{Re}[|\delta_{13}^d|_{31}]^2|} = 1.9 \times 10^{-1},
\]
\[
\sqrt{|\text{Re}[|\delta_{13}^u|_{31}]^2|} = 1.3 \times 10^{-1},
\]
\[
\sqrt{|\text{Re}[|\delta_{13}^u|_{31}]^2|} = 6 \times 10^{-4},
\]
\[
\sqrt{|\text{Im}[|\delta_{13}^d|_{31}]^2|} = 4 \times 10^{-3},
\]
\[
\sqrt{|\text{Im}[|\delta_{13}^u|_{31}]^2|} = 1 \times 10^{-4}.
\]

These results show that, also for this class of models, SUSY contributions cannot give sizable effects to $a_{J/\psi K_S}$. As expected, with hierarchical Yukawa couplings (where the mixing between different generations is very small), the SUSY contributions to the $B-\bar{B}$ mixing and the CP asymmetry of $B_d \to J/\psi K_S$ are sub-dominant.

| $\tilde{m}$ | $\tilde{m}_{IR}$ | $\sqrt{|\text{Re}[|\delta_{13}^d|_{31}]^2|}$ | $\sqrt{|\text{Re}[|\delta_{13}^u|_{31}]^2|}$ | $\sqrt{|\text{Re}[|\delta_{13}^u|_{31}]^2|}$ |
|----------|-----------------|-------------------------------|-------------------------------|-------------------------------|
| 400      | 100             | $1.9 \times 10^{-1}$          | $1.3 \times 10^{-1}$          | $6 \times 10^{-4}$           |
| 600      | 100             | $1.8 \times 10^{-1}$          | $1.9 \times 10^{-1}$          | $2.6 \times 10^{-4}$         |
| 800      | 100             | $1.8 \times 10^{-1}$          | $2.3 \times 10^{-1}$          | $2.9 \times 10^{-4}$         |

| $\tilde{m}$ | $\tilde{m}_{IR}$ | $\sqrt{|\text{Im}[|\delta_{13}^d|_{31}]^2|}$ | $\sqrt{|\text{Im}[|\delta_{13}^u|_{31}]^2|}$ | $\sqrt{|\text{Im}[|\delta_{13}^u|_{31}]^2|}$ |
|----------|-----------------|-------------------------------|-------------------------------|-------------------------------|
| 400      | 100             | $2.1 \times 10^{-1}$          | $1.8 \times 10^{-1}$          | $3.1 \times 10^{-4}$         |
| 600      | 100             | $2.0 \times 10^{-1}$          | $2.2 \times 10^{-1}$          | $3.0 \times 10^{-4}$         |
| 800      | 100             | $2.0 \times 10^{-1}$          | $2.6 \times 10^{-1}$          | $3.0 \times 10^{-4}$         |

Concerning the chargino amplitude to the CP asymmetry $a_{J/\psi K_S}$, we find that the mass insertions $|\delta_{13}^d|_{RL}$ and $|\delta_{13}^u|_{LL}$ give the leading contribution to $a_{J/\psi K_S}$. However, for the representative case of $m_0 = m_{1/2} = 200$ and $\phi_{ij} \simeq \pi/2$ the values of these mass insertions are given by
and the SM should give the dominant contribution.

5.2. Flavour–dependent SUSY phases and CP asymmetry in $B \to X_s \gamma$ decays

The Standard Model prediction for the CP asymmetry in rare $B$ decays $B \to X_s \gamma$ is very small, less than 1%. Thus, the observation of sizeable asymmetry in this decay would be a clean signal of new physics. The most recent result reported by CLEO collaboration for the CP asymmetry in these decays is

$$-9\% < A_{CP}^{b\to s\gamma} < 42\%,$$

and it is expected that the measurements of $A_{CP}^{b\to s\gamma}$ will be improved in the next few years at the $B$–factories.

Supersymmetric predictions for $A_{CP}^{b\to s\gamma}$ are strongly dependent on the flavour structure of the soft breaking terms. It was shown that in the universal case, as in the minimal supergravity models, the prediction of the asymmetry is less than 2%, since in this case the EDM of the electron and neutron constrain the SUSY CP–violating phases to be very small. Furthermore, it is also known that in this case, one can not get any sizeable SUSY contribution to the CP–violating observables, $\varepsilon$ and $\varepsilon'/\varepsilon$.

In this subsection, we explore the effect of these large flavour–dependent phases on inducing a direct CP violation in $B \to X_s \gamma$ decay. We will show that the values of the asymmetry $A_{CP}^{b\to s\gamma}$ in this class of models are much larger than the SM prediction in a wide region of the parameter space allowed by experiments, namely the EDM experimental limits and the bounds on the branching ratio of $B \to X_s \gamma$. The enhancement of $A_{CP}^{b\to s\gamma}$ is due to the important contributions from gluino–mediated diagrams, in this scenario, in addition to the usual chargino and charged Higgs contributions.

The relevant operators for $b \to s\gamma$ decay are given by

$$Q_2 = \bar{s}_L \gamma^\mu c_L \bar{c}_L \gamma^\mu b_L,$$

$$Q_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu},$$

$$Q_8 = \frac{g_s}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} T^a b_R G^a_{\mu\nu}.$$  (100)

The expression for the asymmetry $A_{CP}^{b\to s\gamma}$, corrected to next–to–leading order is given by

$$A_{CP}^{b\to s\gamma} = \frac{4\alpha_s(m_b)}{9 C_T^2} \left\{ \left[ \frac{10}{9} - 2z \left( v(z) + b(z, \delta) \right) \right] Im[C_2 C_7^*] + Im[C_7 C_8^*] \right\}$$

$$+ \frac{2}{3} z b(z, \delta) Im[C_2 C_8^*],$$  (101)

where $z = m_c^2/m_b^2$. The functions $v(z)$ and $b(z, \delta)$ can be found in Ref. The parameter $\delta$ is related to the experimental cut on the photon energy, $E_\gamma > (1 - \delta)m_b/2$, which is assumed to be 0.9.
The SUSY contributions to the Wilson coefficients $C_7, C_8$ are obtained by calculating the $b \to s \gamma$ and $b \to s g$ amplitudes at the electroweak scale respectively. The leading–order contributions to these amplitudes are given by the 1–loop magnetic-dipole and chromomagnetic dipole penguin diagrams respectively, mediated by charged Higgs boson, chargino, gluino, and neutralino exchanges. As pointed out above, SUSY models with non–universal $A$–terms may induce non–negligible contributions to the dipole operators $\tilde{Q}_7, \tilde{Q}_8$ which have opposite chirality to $Q_7, Q_8$. In the MSSM these contributions are suppressed by terms of order $O(m_s/m_b)$ due to the universality of the $A$–terms. However, in our case we should take them into account. Denoting by $\tilde{C}_7, \tilde{C}_8$ the Wilson coefficients multiplying the new operators $\tilde{Q}_7, \tilde{Q}_8$ the expression for the asymmetry in Eq.(101) will be modified by making the replacement

$$C_i C_j^* \to C_i C_j^* + \tilde{C}_i \tilde{C}_j^*. \quad (102)$$

The expressions for $\tilde{C}_7, \tilde{C}_8$ are given in the appendix and $\tilde{C}_2 = 0$ (there is no operator similar to $Q_2$ containing right–handed quark fields). Note that including these modifications may enhance the branching ratio of $B \to X_s \gamma$ and reduce the CP asymmetry, since $|C_7|^2$ is replaced by $|C_7|^2 + |\tilde{C}_7|^2$ in the denominator of Eq.(101). If so, neglecting this contribution could lead to an incorrect conclusion.

In the EDM-free models we are considering, we found that the flavor dependent phase $\phi_{23}$ gives a large contribution to the CP asymmetry. This can simply be understood by using the mass insertion; the gluino contributions to $C_7$ and $C_8$ (and also $\tilde{C}_7$ and $\tilde{C}_8$) are proportional to $(\delta d_{23})_{LR}$ which receives a dominant contributions from $A_{23}$ entry. The effect of the other flavor dependent phases on $A_{b \to s \gamma}^{\text{CP}}$ is found to be very small. In Fig. 11 we show the dependence of $A_{b \to s \gamma}^{\text{CP}}$ on the phase $\phi_{23}$ in the case of hermitian $A$-terms. We assume $\tan \beta = 10$, the diagonal elements $A_{ii} = m_0$, and consider three values for the off-diagonal elements $A_{ij}$, $(i \neq j)$: $|A_{ij}| = m_0$ (curve 1), $|A_{ij}| = 3m_0$ (curve 2), and $|A_{ij}| = 5m_0$ (curve 3). The conditions of the correct branching ratio has been automatically imposed.

We see that the CP asymmetry $A_{b \to s \gamma}^{\text{CP}}$ can be as large as 15%, which can be accessible at the $B$ factories. Also as emphasised in Ref.7, larger $\tan \beta$ is, the larger the CP asymmetry $A_{b \to s \gamma}^{\text{CP}}$ become. In the case of symmetric $A$-terms, the CP phase in $(\delta d_{23})_{LR}$ is due to the CKM mixing only; however, it implies a considerable CP asymmetry, $A_{b \to s \gamma}^{\text{CP}} \sim O(10\%)$.

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Fig. 10. CP asymmetry $A_{C\!P}^{b\rightarrow s\gamma}$ as a function of the flavor-dependent phase $\phi_{23}$ for $m_0 \simeq 150$ GeV, $\tan \beta = 10$, and $m_{\chi^+} \simeq 100$ GeV, $|A_{ij}| = m_0$. Curve 1: $|A_{ij}| = m_0 (i \neq j)$; curve 2: $|A_{ij}| = 3m_0$; curve 3: $|A_{ij}| = 5m_0$. 

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