The Real Significance of the Electromagnetic Potentials*

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Abstract

The importance of the potential is revealed in a newly discovered effect of the potential. This paper explore the same issue introduced in quant-ph/9506038 from several different aspects including electron optics and relativity. Some people fail to recognize this effect due to a wrong application of gauge invariance.

I. METAPHYSICS: CAN WE QUESTION THE CODE WE LIVE BY?

This paper is about an effect which does not even have an appropriate name. We can name it potential effect, effect of potential, effect of potential in simply connected region, or, Liu effect. I attach my name to the effect for the reason that, As far as I know, I am the only one who advocate this effect. Besides, there was no real obstacle to the discovery of this effect any-time in the last fifty years.

We have existed in this world for only a limited time. We accept knowledge at a limited speed. So our total knowledge is finite. Many of us believe that the information in the world is infinite, or nearly infinite when compared to what we have already received. We find rules, just like the animals do, only much better. Beside our brain size, we are doing much better largely because we have a very good education system which can pass most of the knowledge from generation to generation. A could-be experiment is to observe an individual human who is isolated from all human knowledges.

The very personal knowledge we have is never accurate. What is the safe distance between a lion and its prey? Neither party has an accurate number for this. When the knowledge is recorded so as to be transmitted from individual to individual, it must be coded on finite media for practical reasons. At this point, knowledge becomes knowledges. The most basic code is our language. There is obvious difference between the real knowledge and the knowledges that piped through the code we use. We tend to regard the knowledges as the knowledge most of the times, however, there are times when the difference can not be ignored. The case I discuss here is one of such cases.

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We have other codes in science and technology beside language. Physics formulas is an obvious example. Others are slightly difficult to explain in a paragraph so I resort to an example. Newton said: “F=ma, so ...”, while he actually meant “if F=ma, then we have ...”. In this case, F=ma is the code, or gauge, to confine the way we record our knowledge of physics. It turned out to be an efficient code, and thus remained “correct” for centuries, until the discovery of relativity and quantum mechanics at the beginning of this century. The problem is, people used to regard things like F=ma as sacred, something to be taken for granted. We can record physics with p=mv or G(a hypothetical quantity) = m(da/dt) as well as F=ma. F=ma is chosen because it is more efficient to record the knowledge we know. Notice that F=ma introduces two new quantities, F and m, not just one. Both F and m are meaningless (or mean something else, depend on the way you look at it,) before the introduction of F=ma. Let me explain further for some readers. The acceleration caused by gravity on most objects is (nearly) a constant on the surface of the earth. The accelerations caused by a spring on most objects are (nearly) constants. After the introduction of F=ma, we find it possible to define m and F of most of the objects familiar to us as constants. This in turn provides a concise language, or code, or gauge, to record our knowledge of this world. There is nothing sacred about it. It is just the way science is.

With the advantage of a coding system there comes disadvantage. People tend to ignore cases when descriptions in accepted codes become complicated. This is to some degree inevitable. We only have limited times. There are two kinds of works in physics. The first kind is to spend time on recording the world into knowledges under the current codes. The second kind is to spend time on finding more efficient recording codes. Most of the great discoveries belong to the second kind. However, there also seems to be a third kind of works recently. It is about finding “structurally satisfactory” recording codes which is not necessarily efficient in recording our world. They also spend enormous amount of time in recording and converting existing data into these codes, because, the codes are indeed “not necessarily efficient.” There seems to be a misunderstanding between the simplicity of the code, and its efficiency in recording our world.

The code relevant to this paper is electromagnetic gauge degree of freedom. It is usually regarded as sacred and untouchable by some physicists. What is this electromagnetic gauge degree of freedom? It is a degree of freedom which has no influence on observation. Different “values” of it means different ways of recording the same thing. To use a technical word, physics is invariant under electromagnetic gauge degree of freedom. Conversion between one way of recording to the other is called a gauge transformation.

So far so good. But what if we do see something which changes with respect to the gauge choices? To some people, this is simply out of the scope of interest. They put the factor to unity before a second thought. Therefore, we do not even see enough explicitly expressed opinion about this. One explicitly expressed opinion about this, which I would like to quote here, is the opinion of Peshkin and Lipkin (1995) [6]. They are both well-known scientists in this curriculum. According to them, only gauge-invariant quantities have physical meaning. Quantities that can not be made gauge invariant have no physical meaning and therefore should be get rid of. This opinion is quite common. Then they face the question of how to get rid of these quantities. Different values of the gauge mean different physical predictions. Which value should be chosen? The following opinions of theirs is critical. They believe that the gauge can be, and should be chosen in such a way that the quantity in question becomes
not measurable. A theoretical technique which is sometimes coined as “gauge-away” is
normally used to deal with these gauge-dependent quantities.

It is quite obvious that this “gauge-away” treatment is always less than ideal. The
philosophy behind such treatment is: “we do not really know what its value should be, so
let us choose a value so as to keep the prediction of theory as simple as possible.” The reason
that it is less than ideal is, while simplicity is usually closer to the truth than complexity,
simplicity does not always equal to the truth in every segment of physics. Even if we honor
the simplification of physics as a whole, that still doesn’t mean we should always individually
simplify every segment of physics to the extremes.

This is our discussion from a metaphysical point of view. We must have unbiased opinions
about all possibilities. If we made a choice of convenience with the reason unknown, we
should try to find the reason later. If gauge invariance is true, yet we still see some quantities
varying with respect to gauge choices, then what are they? This question is obviously
reasonable. We used to say, there is reason behind everything. This everything obviously
also include the quantities which are gauged away. In the next section, we shift to a physics
perspective.

II. QUANTUM MECHANICS AS ELECTRON OPTICS

Aharonov-Bohm effect (AB effect) is a well-known effect in quantum mechanics. It
is so named because Aharonov and Bohm published a paper in 1959 titled: “Significance
of Electromagnetic Potentials in the Quantum Theory.” The movement of an electron is
influenced directly by potential, in a region where there is no electromagnetic field at all!

Let me first briefly describe a typical setting of this effect. The basic component is a
long-enough solenoid, shielded from the travelling electron by an impenetrable wall. The
purpose of the shield is to ensure that the electron only contact with the potential outside,
not the magnetic field inside the solenoid. This shielded solenoid is put in between the
double slits. The double slits would cast a typical sinusoidal interference pattern on the
screen. When a current is applied to the solenoid, the central maximum would shift to one
side. This effect, the AB effect, is caused by potential only.

In this essay, I am going to talk about two surprises about this effect. First, a small
surprise: this AB effect was actually first discovered by Ehrenberg and Siday in 1949. This is
a well-known fact among the specialists. Secondly, a big surprise: this AB effect is actually
a particular case of a more general “potential effect.” This was discovered by me.

Everybody knows that quantum mechanics is not as perfect as special relativity. Someone
says: “for quantum mechanics, Helium atom has one electron too many.” AB effect is
another good example of this. We can not predict AB effect from Schrödinger’s equation
of 1926. There was a long time-span of thirty-three years between the two. Yet, thirty-six
years have passed after the publication of AB effect, and we still see little change in quantum
mechanics due to this. There are some works about solving the AB problem from modeling
in quantum mechanics. But the success is limited. The two solenoids scattering problem,
which should be a direct expansion of the AB(one solenoid) scattering, is still under debate.

I must say a few words about electron optics in order to introduce the work of Ehrenberg
and Siday properly. Electron optics can be divided into classical and quantum parts. In
principle, everything should be governed by quantum mechanics through wave-like treatment

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of the electron. But in fact, the classical theory of electron optics still dominates. Traditionally, quantum mechanics solves two kinds of problems: bound-state problem and scattering problem. Approximation methods is almost always used. Typical methods are: perturbation expansion and variation principle. Electron optics, however, requires something more rigorous which standard quantum mechanics can not provide.

Among many other physicists, Ehrenberg and Siday (1949) tried to solve this problem. They defined electron refractive-index as a function of vector potential. Near the end of their paper, they discussed “a curious effect”, which is exactly the AB effect: On the two sides of a magnetic flux, vector potential has different values. This means different refractive index for two geometrically equivalent paths. This difference in refractive index would cause an observable phase-shift.

Now we are ready to get to the business of introducing potential effect [3].

The potential effect can be introduced in many ways. Here I choose to do it through electron optics. As we can see, wave electron-optics is, or should be, a copy of quantum mechanics. The physical foundation must be the same. In terms of electron optics, quantum mechanics is about how an electron propagates in electromagnetic media. A bound-state of quantum mechanics is a stationary propagation state, or, a standing-wave state, of electron optics. In another words, a bound-state is a state that scatters into itself. What about other interactions? First of all, quantum mechanics is primarily a theory about electromagnetic interaction. Secondly, other interactions can always be written in gauge formats similar to that of the electromagnetic interaction. So for the moment we don’t need to worry about the difference too much.

Now let us have a version of wave electron optics:

1. Electron propagates as a wave.

2. The electromagnetic potentials behaves as a media of the propagating electron. This media changes the local wavelength and frequency of the propagating electron.

I hope you can see the marvel of this theory. All quantum mechanics problems can be solved this way. Computer simulation of wave-front can be used as a handy method to solve practical problems this way.

Of course there are problems. Otherwise, it would be a popular method already. The problem is gauge invariance. To solve propagating electron problem this way, we need to know the “refractive index” of each point exactly. It seems that a gauge transformation would cause change of the media. So here we encounter another quantity which is gauge dependent.

The issue of gauge-dependent quantities is already discussed in section one. The prescription of “gauge away” obviously doesn’t work here, because that would result in plane-wave-only solution for all quantum problems of electron! This is in the blind point of some physicists. It is thus ignored as a possible method due to this impasse of gauge dependence. It is interesting to learn that, most basic problems in electron optics are solved using classical electromagnetics, in which the electron is represented as a particle.

Now let us revisit the AB effect. In AB effect, the electron goes through two different paths. Normal gauge transformation does not change the optical path-length difference between the two paths. Therefore, AB effect has no trouble with gauge invariance.
Potential effect [3] is different from AB effect in the sense that it means observability of the effect of potential in any regions, not just multiply connected regions. Thus, AB effect is just a special kind of potential effect [3]. The simply connected region is obviously the best example and testing ground of this effect. The notion of the potential effect also fixes the “refractive index” uncertainty in the above-mentioned problem of electron optics. Thus, the existence of the potential effect also means a fundamental change in our views of quantum mechanics.

How to observe the effect of potential in a simply connected region? It is actually very simple. The only additional instrument needed is an elongated toroidal solenoid which can provide a constant vector-potential region inside. When an electron travels inside, the local wave-length would change, and this change is measurable by any interference experiment. Wave-length is always the first observable quantity in any experiment that shows the wave-like nature of the electron. The experiment can be double slit, single slit or even holography.

People haven’t seen fundamental changes in physics for a long time. Therefore, it is quite understandable that they doubt my discovery when they see it for the first time. It has been three and a half years since my first attempt to publish the result. The following two opinions are typical: (1) “Of course the effect exists and there is no news here. It is a simple variation of AB effect.” (2) “The effect can not possibly exist because it violates the principle of gauge invariance.” We should notice that point one contradicts point two. So it is physically illogical for one to say:“either (1) is true, or (2) is true.” By very simple logic we can already reject one of them.

Of course both points are wrong. The effect exists and it is revolutionary. In the follow I discuss this problem from the perspective of Lorentz invariance. After that, I would like to direct you to my paper [3].

The theory of special relativity states that time and space can be combined to be called as space-time. They are exchangeable. The description of a wave-front propagating with respect to time is somehow non-relativistic. The relativistic description of this is, a “stationary” state existing in space-time.

The point is, phase comparison can be conducted between any two space-time points, rather than just between two spatial paths. This is another perspective to look at the nature of this effect of potential. It can be seen as a generalization of AB effect. An electron coheres with itself in time, and remembers where it comes from. When an electron travels into a region with different potential values, its wave-length and frequency change accordingly. This change in time is for real, as real as the phase difference between two different paths created by potential in multiply connected regions.

Most experts in this field do not believe this effect can exist (in simply connected region) because they think there is no reference for the electron to compare its phase with. And they therefore believe that gauge transformation means that the phase-factor is uncertain. But how they establish equality between an uncertain, gauge-dependent phase factor and a zero phase factor is something beyond my understanding. As mentioned above, I believe the existence of this effect because, among other reasons, time coherence is just as reasonable as spatial coherence. This is a consequence of Lorentz invariance.

AB effect can be easily derived from potential effect in the simply connected region. We simply regard the double path as the combination of two simply connected paths. The phase shift in the two paths will be different. The difference between the two is the AB phase shift.
The existence of this effect of potential means a brand-new version of quantum theory. This effect fixes the uncertainty in above-mentioned electron optics. So logically, all quantum problems can be defined and solved in this way. We no longer logically need any eigenstate equations. All we need is a proper handling of the quantity $p-eA$ while treating electron as a propagating wave. This is the real significance of the electromagnetic potentials.

III. CLASSICAL AND QUANTUM ELECTROMAGNETICS

When a particle is generated, its 4-momentum

$$i\hbar \partial_\mu + qA_\mu$$

should have unique expectation. The natural way of doing this, is define $qA_\mu$ to be zero at this point. These conditions pin-down the gauge degree of freedom. The potential thus becomes uniquely defined for this particular particle.

In practical calculation, we do not always have to start from a point source. We can also start from any intermediate wave-front. But similar conditions apply and the gauge is fixed. This is a quantum phenomena. The gauge enters the propagation through the phase factor of the wave-function only. The fact that phase is a constant on a wave-front means that the gauge can be treated as the phase factor. If a quantity has absolutely no impact on measurement, than that quantity should be recycled.

The potential effect [3] is a reflection of this significance of potential. The effect is detectable not just in some special multiply connected region, like the AB effect, but also in everywhere when the change of 4-wavelength becomes measurable. A typical and simplest example is its effect in a simply connected region. And a typical experimental realization of this example is any wavelength sensitive electron experiment inside an elongated toroidal solenoid. Since the gauge is fixed, the potential value inside the toroidal solenoid is directly related to the wave-vector $k$ of the electron when it is inside the toroidal solenoid. If we use the traditional double-slit setting, the change of current in the toroidal solenoid would cause change of fringe-spacing on screen.

We can in principle use this fixed gauge to define reflective index. After that, computer simulation becomes very handy. It could become the best option of solving complicated problems because the method is numerically not sensitive to the complexity.

A. Number of variables

We used to describe electromagnetic interaction on the classical level in terms of the fields $E$ and $B$. We should notice that all of the classical electrodynamics [7, for example] can be formulated in terms of the potentials $\Phi$ and $A$, instead of $E$ and $B$. We simply replace the fields using

$$F_{\mu\nu} = \frac{\partial A_\mu}{\partial x_\nu} - \frac{\partial A_\nu}{\partial x_\mu}.$$  

We can see that, on the classical level, the potential is a simpler code than the fields for the description of electromagnetic interaction. This is obvious in the sense that the potential has a total of four components while the fields have a total of six components.
The potential description on the classical level is almost perfect save one thing: the
gauge invariance. Due to the gauge invariance, there are only three independent physical
components on the classical level. This fourth variable, the gauge, is actually connected to
the wave-function of quantum mechanics. This gives an un-separable coupling:

\[ qA_\mu \to qA_\mu + \partial_\mu G, \quad \text{and} \quad \psi \to \psi \times \exp \left( \frac{i}{\hbar} G \right). \]

In some sense, this gauge is no longer \textit{invariant} in the quantum region. Of course, we can
always construct invariant quantities. In this case, it is

\[ (qA_\mu + i\hbar \partial_\mu) \psi. \]

This becomes known as the canonical momentum, or, the minimum coupling. One may
argue that we have the minimum coupling first then we construct \[ \square \] from it. How it come to
be is not important. The important thing is, this minimum coupling have been indirectly
proved through the very accurate experiments of QED.

Now let us count number of variables again. Since the wave-function \( \psi \) is complex, it
counts as two variables. One of them is coupled through the gauge \( G \). So, the number of
total independent variables is \( 4 + 2 - 1 = 5 \). This 5 may sound strange to some people.
We count both the potential which provides influence, and the quantum representation of
the traveling quantum particle. The reason they are counted together is because they are
coupled together.

\section*{B. Current-current interaction}

On a more fundamental level, we must understand interaction on the basis of direct
date-particle-particle interaction. In the case of minimum coupling, The potential is an indirect
representation of the currents - a lot of moving particles. We now express \( A_\mu \) in terms of
4-velocity of the potential-providing particles.

The notion of a conserved four canonical momentum is the back bone of the effect. We
can re-express it as:

\[ \text{const.} = p_\mu (x) - qA_\mu (x) \]
\[ = \hbar k_\mu (x) - q \sum_i q_i \int d\tau D_r (x - r_i (\tau)) V_{i,\mu} (\tau), \]

where \( V_\mu \) is the four-velocity, \( D_r (x - r_i (\tau)) \) is the retarded Green function \[ \square, p.654 \], summation is over all particles except the one which is represented as \( \hbar k_\mu (x) \). In the case of
electron-only, we have

\[ k_\mu (x) = k_{0,\mu} + \alpha \sum_i \int d\tau D_r (x - r_i (\tau)) V_{i,\mu} (\tau). \]

This interpretation means the influence of the electron is not bounded. It can affect other
particles through a propagator. That propagator determines all features of classical and
quantum theory of electromagnetics. We expect the result of higher order effects comes
from radiative corrections of both the potential-source and the selected particle.
The particles that provide the potential and the particle that receives the potential are not represented symmetrically here. If we consider the quantum uncertainty of the potential provider, we need another degree of freedom.

IV. EIKONAL APPROXIMATION

The complex wave-function is defined as

$$\psi(r, t) = a(r) \exp \left( \frac{i}{\hbar} (S(r) - Et) \right).$$

Notice that in this definition, energy eigenstate is automatically assumed, while in relativistic case this is not correct. Also, non-relativistic quantum mechanics

$$\frac{1}{2m} (-i\hbar \nabla + eA)^2 \psi - e\Phi \psi = i\hbar \frac{\partial \psi}{\partial t}$$

is usually adopted. After separation of real and virtual parts, the result is two real equations for \(a(r)\) and \(S(r)\):

$$\left( \nabla S + eA \right)^2 = 2mc\Phi + 2mE + \hbar^2 \frac{\nabla^2 a}{a},$$

and

$$\nabla \cdot \left[ a^2 \left( \nabla S + eA \right) \right] = 0.$$

These two equations are well-known, and can be interpreted as Hamilton-Jacobi equation and current-conservation equation. The eikonal approximation is good when the last term of (3) is relatively small. To put in word, the change of amplitude should be smooth enough. After the approximation,

$$\nabla S = \hat{t}(r) \sqrt{2mc\Phi + 2mE - eA}.$$

where \(\hat{t}(r)\) is unit vector which can be determined in principle, after the solution of Schrödinger equation has been found. If we have \(\hat{t}(r)\) beforehand, we can integrate it in the curl-free case.

\(\hat{t}(r)\) is usually assumed to be the classical trajectory of the electron. This means, in the curl-free case, \(\hat{t}(r) = \text{const.}\) We can also do the same for \(t(\mu)(x)\). But this is obviously an approximation. In cases like diffraction around sharp edges, \(\hat{t}(r)\) is obviously not constant. In this sense, the eikonal approximation is semiclassical. We expect it to be valid in the paraxial region in electron optics.

A. Lorentz invariance

Lorentz invariance requires equal treatments of all 4-variables. The mass of the electron is a scalar, but energy is not a scalar. Energy eigenstate is no longer a preferred description.
We must in principle allow frequency to change in space-time. This covariance argument leads to the following generalization of eikonal approximation.

The derivation should start from the Klein-Gordon equation if we do not consider spin. Also, the consideration of gauge invariance is taken into account. G in the follow is the gauge.

\[(i\hbar\partial^\mu + qA^\mu) (i\hbar\partial_\mu + qA_\mu) \psi = m^2\psi,\]  

where q is the charge. After letting

\[\psi (r, t) = a (r, t) \exp \left( \frac{i}{\hbar} G (r, t) \right),\]  

where a and G are real. After separation of real and virtual parts, we have

\[(qA^\mu - \partial^\mu G) (qA_\mu - \partial_\mu G) = m^2 + \hbar^2 \frac{\partial^\mu \partial_\mu a}{a}\]  

and

\[\partial^\mu \left[ a^2 (qA_\mu - \partial_\mu G) \right] = 0.\]  

These two are the relativistic version of basic equations. The eikonal approximation can be introduced as:

\[qA_\mu (x) - \partial_\mu G (x) = t_\mu (x) m.\]  

The m here is the electron rest-mass. This can be called covariant eikonal approximation. t_\mu (x) can also be identified as classical trajectory.

This is however not the one to be compared with the non-relativistic eikonal approximation. The energy appears in \[E = mc^2 \left( \frac{1}{\sqrt{1 - v^2/c^2}} - 1 \right)\].

The another choice of approximation using a three-dimensional t(x) is:

\[\nabla G - qA = \hat{t} (x) \left[ \sqrt{\left( \frac{\partial G}{\partial t} - q\Phi \right)^2 - m^2} \right].\]  

This also means that we see it as an energy eigenstate. Just like we can always have the third component (z) of angular momentum J_z determined simultaneously with J^2, we can have energy eigenstate together with the “mass eigenstate”. This way, it is comparable to the non-relativistic eikonal approximation. However, we should bear in mind that this is a choice, not a must.
V. APPLICATION

The discussion here is mostly conceptual. The effect is so straightforward that we need very little mathematics. The application of the effect is not discussed here because they are consequences of this prediction and are logically connected. The immediate application is the extension of AB effect. We can apply this effect of potential where AB effect is observed. The second application is in electron optics [4]. Beside these applications, we view this theory as a new way of solving general quantum problems. It reveals the real physical significance of the electromagnetic potential.
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