Efficient Dynamic Mesh Refinement Technique for Simulation of HPM Breakdown-Induced Plasma Pattern Formation

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Abstract—Numerical simulation of the complex plasma dynamics associated with high power, high-frequency microwave breakdown at high pressures, leading to the formation of filamentary plasma structures, is a computationally challenging problem. The widely used 2-D electromagnetic (EM)—plasma fluid model, which accurately captures the experimental observations, requires a runtime of several days to months to simulate standard problems due to stringent numerical requirements in terms of cell size and time step. This article presents a self-aware mesh refinement (MR) algorithm that uses a coarse mesh and a fine mesh that dynamically expand based on the plasma profile topology to resolve the sharp gradients in the E-fields and plasma density in the breakdown region. The dynamic MR (DMR) technique is explained in detail, and its performance has been evaluated using a standard benchmark microwave breakdown problem. We observe a speedup of 8 (of the order of \( O(r^2) \)), when the refinement factor \( r \) is 2) compared with a traditional single uniform fine-mesh-based simulation. The technique is scalable and performs better when the problem size increases. We also present a comprehensive spatio-temporal visual analysis to explain the complex physics of high-power microwave (HPM) breakdown, leading to self-organized plasma filaments as an application of the DMR technique.

Index Terms—Mesh refinement (MR), microwave breakdown, multiscale modeling, plasma simulation.

I. INTRODUCTION

HIGH-power microwave (HPM) breakdown has been studied theoretically and experimentally since the 1950s for a wide variety of applications such as aerodynamic flow control, combustion, precision radar systems, energy deposition in supersonic and hypersonic gas dynamic flows, beamed energy propulsion, deep-space communications applications, electromagnetic (EM) warfare, and ultra-wideband (UWB) HPM transmission [1], [2], [3], [4], [5], [6], [7], [8]. However, only recently, detailed experimental investigations of the plasma dynamics during breakdown have been possible with the use of sophisticated high-speed cameras [9], [10], [11], [12], [13], [14], [15], [16], [17]. Different types of gas discharges have been reported experimentally such as streamer, overcritical, subcritical, volumetric, and attached to an initiator [2], [15], [17]. To completely understand the properties of each type of discharge, it is crucial to further improve our current understanding of the EM wave–plasma interaction, plasma formation, the subsequent energy exchange between wave and plasma, and afterward between the gas and the plasma. Different semi-analytical models and computational techniques have been used to study this problem [18], [19], [20], [21], [22], [23], [24], [25], [26], [27], [28], [29], [30], [31], [32], [33], [34]. The fluid [20], [23], [24], [25], [26], [31] and kinetic [33], [34] models are generally used for investigating such problems. However, computationally expensive kinetic simulations are required for weakly collisional and collisionless plasmas, whereas the fluid models can accurately capture the physics of collisional plasmas. In the case of high-pressure plasmas, which are highly collisional, the mean free path is much smaller than the size of the plasma and the plasma can be treated as a continuum (fluid).

Recent experiments [2], [5], [9], [10], [11], [12], [13], [14], [15], [16], [17] reveal the formation of self-organized plasma structures, either fish-bone-like filaments (in overcritical) or comb-shaped and branching (in subcritical), which occur during air breakdown from focused (MW/cm²), or nonfocused (kW/cm²) high-frequency microwave under atmospheric pressure, respectively. The plasma structures propagate toward the microwave source. The filaments elongate in the direction of the microwave electric field [20], [25]. The high-density plasma filaments enhance the scattered EM field at the antinode of standing waves that result from the continuous ionization diffusion mechanism and sustain the self-organized plasma pattern formation [20]. This problem has been used as a benchmark problem by several researchers to perform new investigations or validate the computational techniques [25], [26], [27], [31], [35], [36], [37], [38].

Accurate 2-D simulations of the HPM breakdown experiments mentioned above have been performed using
the well-established fluid model involving the coupling of Maxwell’s equations and plasma continuity equation [20], [24], [25], [29], [39]. It is a complex multiphysics multiscale model due to the presence of different space and time scales [24], [27] which needs to be resolved accurately. Most of the previous works [8], [20], [22], [24], [25], [26], [30], [31], [32], [37], [38], [39], [40] used the finite difference time domain (FDTD) method in the Maxwell–plasma fluid model-based simulations. The simulations quite well reproduce the experimental observations but at the cost of high computation, and therefore, most of the past 2-D numerical investigations using the fluid model have been carried out to hundreds of nanoseconds. The excessive computational cost is due to stringent restrictions on the grid spacing and time steps [20], [24], [25], [31]. Therefore, it is challenging to simulate large problem sizes over longer timescales (tens of microseconds) using homogeneous mesh having the finest resolution of grid size to capture the gradients in plasma density and the secondary $E$-field that originates from it. Different time steps (femtoseconds to nanoseconds) are required by the solvers [38] that are coupled with each other to simulate the Maxwell–plasma fluid model. Unless the solver that requires frequent updates gets speedup, the higher time scale phenomenon such as gas heating [25] cannot be realized efficiently. The simulation of this complex phenomenon at longer timescales will further help understand the underlying physics for various applications in microwave rockets [5], [37], [41], aerospace research [2], [17], [24], [42], high-speed combustion [43], safe operation of HPM devices [29], etc.

Recently, advanced parallelization strategies for emerging many-core architectures have been proposed which significantly reduces the simulation time, but this requires sophisticated computing facilities [24]. To address the computational challenges associated with the 2-D FDTD-based EM–plasma fluid model, alternative numerical techniques have been developed for both the structured meshes [25] and unstructured meshes [27]. Although the techniques are capable of balancing the tradeoff between accuracy and computational cost, they require complex mathematical formulation, proper choice of higher order basis functions, and modification of the existing model. Recently, a static mesh refinement (MR) technique for the well-established FDTD-based fluid model has been presented for studying evolving plasma dynamics [38]. Although the static MR-based technique is accurate and relatively fast compared with single uniform fine mesh, the overall performance is restricted by the size of the fixed preset refined mesh that restricts bigger and longer simulations. Therefore, a self-aware dynamic MR (DMR) technique that generates fine mesh on demand, based on the plasma evolution, is proposed in this article. The DMR technique will particularly aid in carrying out comprehensive parametric studies to investigate the influence of different parameters such as the microwave $E$-field strength, frequency of the microwave, pressure, and different gas species on the HPM-induced plasma evolution at a significantly lower computational cost. The key contributions of this article are as follows.

1) Development and implementation of the DMR technique for the Maxwell-plasma fluid model for investigating complex plasma dynamics during the HPM breakdown.
2) Validation and performance analysis of the proposed DMR technique against published results.
3) To understand and visualize the plasma pattern formation and its spatio-temporal evolution under real experimental conditions using the DMR-based code.

The remainder of the article is organized as follows. Section II provides a brief introduction to the physical model, its numerical implementation, and the computational challenges associated with a benchmark simulation. In Section III, we discuss the theory and implementation of the DMR technique. In Section IV, we report the accuracy and efficiency of the proposed technique, and subsequently, we use the simulation results to provide a detailed spatio-temporal analysis of the complex plasma dynamics during the HPM breakdown followed by conclusions in Section V.

II. PHYSICAL AND COMPUTATIONAL MODEL

A. Physical Model

The microwave breakdown in air/gases at high pressure leading to complex plasma dynamics is a highly collisional and nonlinear process [12], [20], [21], [24], [25], [29], [32], [36], [37], [40]. The well-established EM–plasma fluid model used by several researchers [19], [20], [21], [25], [26], [27], [29], [31], [32], [40] to reproduce the experimental observations [9], [10], [14] primarily comprises a solution of Maxwell’s and plasma continuity equations [20], [21]. The electron current density ($J$) couples both the sets of equations (Maxwell’s and plasma) [20]. The plasma is assumed to be quasi-neutral, and the ion contribution to the current density is negligible. The fluid continuity equation governs the temporal evolution of the plasma density averaged over one period of the EM wave [24]. Under the local electric field or effective $E$-field ($E_{\text{eff}}$) approximation, the gain of energy and momentum from the electric field is balanced by the losses through collisions. This equilibrium results in the local electric field governing the ionization and attachment processes. Also, the diffusion mechanism dominates over drift for plasma propagation in the high-frequency regime. Therefore, only the diffusive term appears in the flux divergence term in the continuity equation [24]. The effective diffusion coefficient acts as a transition between the bulk ambipolar diffusion and the free-electron diffusion in the plasma front [20], [21]. The following four equations are primarily solved for investigating the spatio-temporal evolution of the plasma during the gas breakdown:

\[
\frac{\partial E}{\partial t} = \frac{1}{\epsilon_0} (\nabla \times \vec{H}) - \frac{1}{\epsilon_0} (\vec{J}) \tag{1}
\]

\[
\frac{\partial H}{\partial t} = -\frac{1}{\mu_0} (\nabla \times \vec{E}) \tag{2}
\]

\[
\frac{\partial \vec{v}_e}{\partial t} = -\frac{e}{m_e} \vec{E} - \nu_m \vec{v}_e \tag{3}
\]

\[
\frac{\partial n_e}{\partial t} - \nabla \cdot (D_{\text{eff}} \nabla n_e) = n_e (v_i - v_a) - r_e n_e^2 \tag{4}
\]
where $\mu_0$ and $\varepsilon_0$ stand for the magnetic permeability and electrical permittivity of vacuum, respectively, $J$ is the plasma current density ($J = -e n_e \vec{u}_e$) in (A m$^{-2}$), $n_e$ is the plasma density (here electron density) in (m$^{-3}$), $\vec{u}_e$ is the electron velocity in (m/s), $V_m$ is the electron-neutral collision frequency in (s$^{-1}$) (for air, and $V_m = 5.3 \times 10^8 p$, where $p$ is the ambient pressure in (torr) [20]. The details related to the effective diffusion coefficient ($D_{eff}$) in (m$^2$/s) can be found in [21] and [20]. In (4), $v_i$, $v_a$, and $v_0$ are the ionization and attachment frequencies, respectively, which are used to calculate the effective ionization, $v_i - v_a = \gamma v_d$, in (s$^{-1}$), where $v_d$ is the electron drift velocity in (m/s). The ionization coefficient in terms of $E_{eff}/p$ is given by $\gamma = A p [\exp(-B p / E_{eff})]$ in (m$^{-3}$), and the coefficients $A$ and $B$ correspond to air whose values are decided based on $E_{eff}/p$, in (V/cm.torr) [20]. The recombination coefficient ($r_{ei}$) in (m$^{-3}$s$^{-1}$) is given by $r_{ei} = \beta \times 10^{-13} (300/T_e)^{1/2}$, and $\beta$ varies between 0 and 2. $D_{eff}$, $v_i$, $v_a$, and $r_{ei}$ form the transport coefficients for the plasma continuity equation [38].

B. Computational Modeling and Benchmark Problem

The solution to the EM–plasma fluid model described above can be numerically achieved using two coupled computational solvers, an EM wave solver and a plasma solver. We use the Yee [44] cell-based FDTD scattered field formulation to solve Maxwell’s equation in the EM solver. The plasma solver uses a finite difference (FD) scheme to solve the plasma continuity equation computationally. The plasma solver requires the diffusion and growth or decay-associated terms that use the effective $E$-field provided by the EM wave solver [20]. The plasma density and EM fields are evaluated at specific locations on the overlapped Cartesian grids [24]. The evaluation involves two events of different timescales, the fast-evolving EM wave that requires frequent E- and H-field updates with smaller time steps and the slow-evolving plasma density that requires less frequent updates with bigger time steps [38].

Before the DMR implementation is explained in detail, we describe the benchmark simulation setup on which the proposed DMR has been applied and evaluated. We consider the HPM-breakdown-induced filamentary plasma propagation problem as reported in [20], [21], and [24]. Fig. 1(a), shows a schematic of our computational domain. A linearly polarized plane wave (110 GHz) propagating in air at atmospheric pressure is incident from the left-hand side of the domain. The electric ($E$) field is $Y$-directed in the plane of the domain, and the wave propagation vector ($k$) is along $X$. The incident field is larger than the breakdown field. The initial plasma density with a Gaussian profile is considered at a small region centered at $(x_0, y_0)$, which eventually evolves into a self-organized plasma filamentary structure and propagates from right to left toward the HPM source as simulation progresses [14], [20], [21], [24].

Fig. 1(b) shows the presence of sharp gradients in the plasma density along the $x$- and $y$-directions during the evolution of the plasma structure. For accurate simulations, $n_e$ and EM fields need to be calculated on a very fine mesh which can resolve the sharp gradients in $n_e$ and $E$ [24], [40]. The

**Fig. 1.** (a) Schematic of the computational domain. $\{c_{kx}, c_{ky} \in \mathbb{Q}^+\}$, and $\chi_0$ and $\chi_y$ are the fractions in $[0, 1)$ of $L_x$ and $L_y$, respectively. The MUR outer radiation boundary condition has been used for scattered field formulation. (b) Formation of self-organized plasma filaments during the HPM breakdown (snapshot at $t = 45$ ns, $E_0 = 5.5$ MV/m, freq = 110 GHz). The maximum density (max) is $6 \times 10^{21}$ m$^{-3}$ and (min) is 0.

### III. Self-Aware Mesh Refinement Technique

The proposed self-aware MR technique or DMR technique is developed on the basic framework of static MR. MR hierarchically decomposes the computational domain into a coarse mesh and fine mesh that are overlapped and logically connected to maintain the continuity between the evaluated quantities on both the meshes [38], [45], [46], [47]. In the case of static MR, the fine mesh region remains static/preset along with the coarse mesh.

The static MR cannot be used very efficiently for a system that is evolving in time and space, as in the case of plasma evolution during the HPM breakdown. Also, if the preset fixed refinement region is large, it leads to unnecessarily large computations. The proposed DMR technique considers a fixed coarse mesh and an expanding fine mesh based on the evolving plasma profile. Fig. 2 shows how the initial fine mesh (confined within a small fraction of the total computational domain) expands in the proposed DMR technique. The mesh expansion varies with the amount of refinement region as a function of time ($t$), denoted by $R(t)$. The fine mesh expansion procedure is explained below in detail.
Fig. 2. Self-aware expansion of fine mesh in DMR to capture the 2-D distribution of plasma density as the filamentary pattern evolves.

Fig. 3. Spatial variation in $\epsilon_n$ and $\sigma$ for a 1-D plasma density ($n_e$) distribution. The EM wave of frequency 110 GHz is considered here.

A. Initiation of Fine Mesh Expansion

The initiation of fine mesh expansion requires the detection of sharp variations in evaluated quantities (e.g., $E$ - and $H$ -fields, plasma density, and velocity) on the discretized grids and using a suitable fine mesh to capture such variations. Generally, the threshold criteria based on gradients in overall and instantaneous energy [47] are used widely to determine the grid size. The criteria hold for EM-scattering problems, where improper resolution of scattering geometry results in higher scattered $E$-fields. For the HPM breakdown, when the medium properties continuously evolve, we adopt alternative criteria that decide the initiation of mesh expansion and the grid size to resolve sharp variations.

The complex relative permittivity for a collisional plasma is defined as [20], [40]

$$
\epsilon_r = \left(1 - \frac{\omega_p^2}{\omega^2 + \nu_m^2}\right) - i \left(\frac{\omega_p^2}{\omega^2 + \nu_m^2}\right) \left(\frac{\nu_m}{\omega}\right)
$$

where $\omega_p = (n_e e^2/m\epsilon_0)^{1/2}$ is the electron plasma frequency. $\text{Re}(\epsilon_r)$ decides the propagation of the EM wave in the plasma, whereas the conductivity of the plasma depends on $\text{Im}(\epsilon_r)$. The cutoff density ($n_{\text{cutoff}}$), in the context of EM propagation in plasma is given by $n_{\text{cutoff}} = n_{\text{critical}}(\nu_m/\omega)$, where $n_{\text{critical}} = (m/\epsilon_0 e^2)\omega$, When the plasma density crosses the cutoff density, plasma starts reflecting the EM wave [20].

The decision to initiate the fine mesh expansion depends on the occurrence of a fixed plasma density (we refer to it as threshold density). The threshold density criteria are well-validated based on the EM propagation characteristics (or dispersion relation) in plasma [48]. Threshold density in our method is determined by performing a convergence study described in the next section, when the occurrence of minimum variation in permittivity ($\epsilon$) and conductivity ($\sigma$) occurs for a given plasma density profile and incident EM wave frequency, as observed in Fig. 3. For deciding the grid size of coarse and fine meshes, we take an inverse approach where the finest grid size is predecided based on valid assumptions [24], and the coarse grid size is decided based on the MR factor. The cell size remains fixed for both the fine and coarse meshes during the simulation.

B. Amount of Fine Mesh Expansion

Fig. 1(b) shows the presence of sharp gradients in the plasma density during the HPM breakdown and is an important parameter in deciding the amount of fine mesh expansion. We have considered two gradient length scales corresponding to plasma density and rms $E$-field (scattered field from plasma), $l_{\text{den}}$ and $l_{E_{\text{rms}}}$, respectively. Mathematically, $l_{\text{den}} = n_e/|\nabla n_e|$ and $l_{E_{\text{rms}}} = E_{\text{rms}}/|\nabla E_{\text{rms}}|$, where $|\nabla n_e|$ and $|\nabla E_{\text{rms}}|$ are the magnitudes of the gradient in density and $E$-field respectively. For the expansion of the fine mesh, the growth of filaments and the associated density gradients along $y_{\text{central}}$ and $x_{\text{central}}$ as shown in Fig. 1(a) and (b) have been considered. Due to the symmetric nature of plasma propagation, the region above $y_{\text{central}}$ starting at $y_0$ is only considered for analysis. In Fig. 4(a)–(d), the length of the initial fine mesh and the amount of fine mesh expansion along $x$ and $y$ are represented by $m_{\text{it}}$ and $m_{\text{iy}}$, and $m_{\text{ex}}$ and $m_{\text{ey}}$, respectively. Fig. 4(a) and (b) shows the distribution of plasma density, rms $E$-field, and the corresponding gradient length scales along the $y$-direction. The $l_{\text{den}}$ has a sharp transition from a high value, $l_{\text{den}} \gg \lambda$ ($\nabla n_e \rightarrow 0$), to a low value, $l_{\text{den}} < (1/100)\lambda$. There is a high gradient in the rms $E$-field that exists from $0.75\lambda$ to $0.85\lambda$; see Fig. 4(b), and as a result the gradient length scale ($l_{E_{\text{rms}}}$) transits from low ($l_{E_{\text{rms}}} < \lambda \sim (1/5)\lambda$) to high ($l_{E_{\text{rms}}} \gg \lambda$, when ($\nabla E_{\text{rms}} \rightarrow 0$)). The initial fine mesh centered around initial density can capture the sharp gradients in both density and rms $E$-field along $y_{\text{central}}$ provided, $m_{\text{iy}}$ is large enough to cover the occurrence of smallest, $l_{\text{den}}$ and $l_{E_{\text{rms}}}$, and the fine mesh grid size satisfies $\Delta S \sim \min(l_{\text{den}}, l_{E_{\text{rms}}})$. Next, for the fine mesh expansion along $y_{\text{central}}$, the amount of expansion, $m_{\text{iy}}$, must be able to capture the occurrence of smallest, $l_{\text{den}}$ and $l_{E_{\text{rms}}}$, as specified above.

Similarly, Fig. 4(c) and (d) presents the distribution of plasma density and rms $E$-field and their corresponding gradient length scales in the $x$-direction (along $x_{\text{central}}$, the plasma front propagation direction). The amount of mesh expansion is determined by the presence of gradient length scales. By referring Fig. 4(a)–(d), it can be observed that $\min(l_{\text{den}}) < \min(l_{E_{\text{rms}}})$. Thus, $l_{\text{den}}$, instead of $l_{E_{\text{rms}}}$, decides the fine mesh grid size. Based on the gradients shown in Fig. 4(a)–(d), $m_{\text{ex}} > m_{\text{ey}}$. Therefore, a different amount of mesh expansion is required in the $x$- and $y$-directions.

C. Quantity Updates on Mesh and Synchronization

The schematic in Fig. 5(a) represents the MR region (in dashed). Here, a single level of refinement is shown, having overlapped coarse and fine meshes, with a grid refinement factor ($r$) of 2, such that the coarse-to-fine grid size ratio is $2:1$. As depicted in Fig. 5(b), two meshes are overlapped, the coarse and embedded fine. $E$, $H$, density, and velocity (synchronized with $E$), as shown in Fig. 5, are updated as discussed in [24]. The fields and density are updated simultaneously on the coarse and fine meshes, maintaining the proper sequence of
Fig. 4. One-dimensional distribution of (a) plasma density and (b) rms $E$-field and their corresponding gradient scale lengths $l_{\text{den}}$ and $l_{\text{Erms}}$, respectively, along the upper half of the central $y$-axis ($y_{\text{central}}$) through initial plasma density. Similarly, in (c) and (d) along the central $x$-axis ($x_{\text{central}}$) through the initial plasma density. The initial plasma density is located at $(x_0, y_0) = [0.85L_x, 0.5L_y]$ = [1.2, 0.75], $L_x = L_y = 1.5\lambda$. $m_{1x}$ and $m_{1y}$, and $m_{2x}$ and $m_{2y}$ are the length of the initial refinement region and the mesh expansion along $x$ and $y$, respectively. The $\lambda \approx 0.0027$ m corresponds to frequency ($f$) = 110 GHz.

Fig. 5. (a) Mesh refined (the dashed region with a refinement factor of 2) discretized computational grid showing locations for computation of EM fields and density. (b) Expanded view of the overlapped coarse and fine meshes. The different data transfer of the updated $E$-field, $H$-field, and plasma density from coarse mesh to its corresponding fine mesh locations on both coarse–fine boundary (cfb) and fine boundary (fb) is shown. (c) Schematic of interpolation techniques, for $r = 2$. The locations $(x_1, y_1)$ are as follows: I: $\{1/2r, 1/r\}$, II: $\{1/2r, 1/r\}$, and III: $\{1/r, 1/r\}$. Here, I: $E$-field (and velocity), II: $H$-field, and III: plasma density represent different interpolations for the coarse mesh data.

frequent FDTD and less frequent (on each period of EM wave) update of plasma continuity equation [24]. The different time steps are based on the respective Courant–Friedrichs–Lewy (CFL) conditions. The time step associated with the EM wave solver is much smaller compared with the plasma solver [20], [24].

The updates must be synchronized between the two meshes in space and time to maintain continuity. Let the grid size, time step, total cells, and total iterations for the benchmark case (uniform fine mesh) be represented as $\Delta S_f$, $\Delta t_f$, $N_f$, and $I_f$, respectively. In the case of DMR, the coarse mesh cell size and time step are represented as $\Delta S_c$ and $\Delta t_c$, and for fine mesh, they are represented as $\Delta S_f$ and $\Delta t_f$, respectively. $\Delta t_c$ and $\Delta t_f$ are associated with Maxwell’s updates (which primarily determine the total execution time). The subscripts $c$ and $f$ correspond to coarse and fine meshes, respectively. Here, $\Delta S_c = \lambda/N_x$ and $\Delta S_f = r\Delta S_f$. Similarly, $\Delta t_c = r\Delta t_f$. In one coarse mesh update, $\Delta t_c$, the fine mesh performs $r$ updates with $\Delta t_f$. This follows for all the quantities. The total fine mesh cells depend on the amount of refinement region ($R(t)$); here, $R(t)$ is a fraction of the total computation domain, and $R(t) \in [0, 1]$.

The MR algorithm transfers the evaluated quantities (fields, velocity, and plasma density) from the coarse mesh to both the coarse–fine boundary (cfb) and the fine boundary (fb) for subsequent fine mesh updates. This transfer of evaluated quantities supports the nearest neighborhood dependence of FDTD and FD-based fields and density updates, respectively [20]. Finally, the fine mesh updated quantities are transferred back to the coarse mesh. Both the data transfer processes must occur within the coarse mesh update interval. The process avoids discontinuity in the obtained results due to a mismatch between the coarse and fine mesh values. The two boundaries (fb and cfb) and the subgrids are shown in Fig. 5(b).

The data transfer uses a direct copy or interpolation process, depending on the location (coinciding or noncoinciding) of the quantities on the overlapped grids. The interpolation is either a linear interpolation on cfb or a bilinear interpolation on fb as indicated by the direction of arrows in Fig. 5(b). Quantities $\Omega_i$, $i = 1, 2, 3$, and 4, shown in Fig. 5(c), represent $E$-field (and velocity), $H$-field, or plasma density on the coarse mesh. The dotted square represents the interpolation domain. The vertices
represent the coarse data and the desired fine data, either on edge (1-D) or inside the bounded area (2-D plane), can be obtained using the equation as follows:

\[
\begin{align*}
BL_{\text{free}} &= (1 - \frac{\text{frac}}{})(\Omega_{1\text{old}} A_3 + \Omega_{2\text{old}} A_4) + \Omega_{1\text{old}} A_1 + \Omega_{2\text{old}} A_2) + (\frac{\text{frac}}{})(\Omega_{1\text{new}} A_3 + \Omega_{2\text{new}} A_4) + \Omega_{1\text{new}} A_1 + \Omega_{2\text{new}} A_2) \\
L_{\text{free}} &= (1 - \frac{\text{frac}}{})(\Omega_{1\text{old}} (1 - x_p) + \Omega_{2\text{old}} (x_p)) + (\frac{\text{frac}}{})(\Omega_{1\text{new}} (1 - x_p) + \Omega_{2\text{new}} (x_p))
\end{align*}
\]

where suffixes old and new denote the previous and updated quantities, respectively, on coarse (c) mesh that requires interpolation on fine (f) mesh. \(A_i, i = 1, 2, 3, 4\), represents the area inside the interpolation domain (dotted square) calculated in terms of \(x_p\) and \(y_p\) represented in terms of \(r\). The frac and \((1 - \text{frac})\) are the ratios in which the old and the updated coarse mesh data values must be taken to obtain a smoothed interpolated quantity on the fine mesh. \(BL_{\text{free}}\) and \(L_{\text{free}}\) represent the bilinear and linear interpolated fine mesh data, respectively, from coarse mesh data (E- or H-field or velocity or plasma density).

D. Implementation of DMR Algorithm

First, an initial fine mesh region, \(a^{11\text{-}b^{11\text{-}c^{11\text{-}d^{11}}}}\), shown in Fig. 6, is considered as indicated by the black solid lines. It is located inside a coarse mesh region that covers the overall computational domain, A-B-C-D. The initial fine mesh contains the initial plasma density profile located at \(x_0\) and \(y_0\). As discussed in the previous section, the \(E\)-field, \(H\)-field, \(v_e\), and \(n_e\) are updated in both the coarse mesh and the initial fine mesh. Next, the fine mesh generation proceeds with two steps, the self-aware initiation and the expansion of the initial fine mesh. Different threshold lines parallel to \(x_{\text{central}}\) and \(y_{\text{central}}\), indicated by forward threshold \((X_{Fi})\), backward threshold \((X_{Bi})\), upper threshold \((Y_{ui})\), and lower threshold \((Y_{dj})\), where \(i = 1, 2, \ldots, n\) and \(j = 1, 2, \ldots, m\), where \(\{(n, m) \in \mathbb{N}\}\), are considered as shown by dotted lines. The threshold lines coincide with the fine mesh’s respective x and y boundaries (happens to be cbf). Before the fine mesh expansion initiates, it is checked whether \(n_e\) is greater than the threshold density on either \(X_{Fi}\) and \(X_{Bi}\) or \(Y_{ui}\) and \(Y_{dj}\). Based on whichever threshold line meets the threshold density criteria, the fine mesh is expanded along \(x_{\text{central}}\) or \(y_{\text{central}}\) resulting in a y-expanded: \(a^{1^{\text{i}}\text{-}b^{1^{\text{i}}\text{-}c^{1^{\text{i}}\text{-}d^{1^{\text{i}}}}}}\) or x-expanded: \(a^{1^{\text{i}}\text{-}b^{1^{\text{i}}\text{-}c^{1^{\text{i}}\text{-}d^{1^{\text{i}}}}}}\) fine mesh region, shown in Fig. 6. During fine mesh expansion, first, the \(E\)-, \(H\)-field \(v_e\), and \(n_e\) data on the entire initial fine mesh are transferred to similar locations on the expanded fine mesh to maintain continuity from the initial mesh. Next, for the remaining regions in the expanded fine mesh, the entire overlapped coarse data are interpolated on the fine mesh. The mesh expansion continues as and when required in a self-aware manner based on the spatio-temporal evolution of plasma.

IV. RESULTS AND DISCUSSION

For performance analysis, we consider the same computational setup as described in Fig. 1(a), where the size of the computational domain is represented by \(L_x = c_{k_x} \lambda\) and \(L_y = c_{k_y} \lambda\). We have taken different \(c_{k_x}\) and \(c_{k_y}\) for different computational experiments. The initial 2-D Gaussian plasma density is \(n_e(x, y) = n_0 \exp((-((x-x_0)^2/\sigma_x^2 + (y-y_0)^2/\sigma_y^2))\), where \(x_0\) and \(y_0\) are the location of \(n_0\), and \(\sigma_x\) and \(\sigma_y\) control the plasma width along x and y, respectively. We consider \(n_0 = 10^{16} \text{ m}^{-3}\), the incident E-field, \(E_0 = 5.5 \text{ MV/m}\), and frequency \((f) = 110 \text{ GHz}\). All the computations are carried out on a computer with an Intel Xeon CPU E5-2640 processor with 32-GB RAM.

A. Threshold Density for Mesh Expansion

The optimal choice of threshold density is obtained through a convergence study for a 1-D plasma density distribution along \(x_{\text{central}}\) for different threshold densities, as shown in Fig. 7. The failure in the convergence, for density \(\geq 10^{18} \text{ m}^{-3}\), can be observed by the presence of sudden spikes in plasma distribution along \(x_{\text{central}}\) in Fig. 7 (highlighted using circles). The optimal chosen density, \(n_e \leq 10^{18} \text{ m}^{-3}\) \(\approx 10^{16}\) to \(10^{17} \text{ m}^{-3}\), where both the values of \(\epsilon\) and \(\sigma\) have minimum variations as shown in Fig. 3 satisfies lower scattered \(E\)-field and avoids sharp gradient in the \(E\)-field or energy. The amount of mesh expansion is arrived at by referring to Fig. 4, to ensure that the fine mesh resolves the minimum gradient scale lengths along the \(y\)- and \(x\)-axes, respectively.

B. DMR Accuracy

For evaluating the accuracy of the DMR method, the consistency of shape and size of plasma filaments and
plasma front velocity are compared with uniform fine mesh implementation. We consider \( c_{kx}, c_{ky} = \{7.5, 1.5\} \) to investigate the size and shape of filaments. The initial Gaussian peak plasma density \( n_0 \) is located at \( x_0 \) and \( y_0 \), which is given by 0.85 \( L_x \) and 0.5 \( L_y \), respectively. Fig. 8(a) and (b) represents the distribution of plasma density and the corresponding scattered rms \( E \)-field at time \( t = 140 \) ns, obtained using the DMR technique with refinement factor \( r = 2 \). The observed results are in good agreement with the published results from [14], [20], [24]. The dynamic mesh could capture the nonuniform gradients in both density and scattered \( E \)-field using an optimal fine mesh region shown by dotted lines in Fig. 8(a) and (b). To validate the plasma front velocity, \( \{c_{kx}, c_{ky}\} = \{1.5, 1.5\} \) is considered for the simulation setup. Fig. 9, represents the temporal evolution of the plasma density along the central \( x \)-axis \( (x_{\text{central}}) \) in the computational domain for a uniform mesh (Uni) and DMR \( (r = 2, 4) \). The front velocity along \( x_{\text{central}} \) can be calculated by tracking the propagation of a specific plasma density level at the front (we have used \( 1 \times 10^{19} \) \( \text{m}^{-3} \)) with time. The calculated plasma front velocity \( (v_{\text{front}}) \) is \( \approx 30 \) km/s for both uniform mesh and DMR with \( r = 2 \). We found that the variation in front velocity and filament length lies within 1%–2% of the uniform fine mesh case (Uni). We observe that the velocities and lengths are consistent with the previously published experimental and simulated results [9], [14], [20], [24], [30], [40].

### C. Speedup and Efficiency of DMR Technique

For efficiency, subroutinewise execution time and overall execution time are compared between DMR and uniform fine mesh case (Uni) for a fixed problem size. Furthermore, performance is evaluated for different problem sizes \( P1, P2, \) and \( P3 \) to check the scalability of the proposed approach. From Fig. 10(a), it can be observed that the time required to execute the three constituent subroutines, the \( E \)-field \( (E) \), \( H \)-field \( (H) \), and plasma density \( (\text{Den}) \), is higher for the uniform fine mesh (Uni) in comparison to DMR for a fixed problem size \( P1 \). \( P1: 1.5\lambda \times 1.5\lambda \). The highest execution time is taken by \( E \) followed by \( H \) and \( \text{Den} \). DMR significantly reduces the overall execution time when compared with Uni, as observed in Fig. 10(b).

Fig. 10(a) and (b) shows that the DMR reduces the execution time with respect to the standard uniform mesh technique by a dynamic factor. This factor decreases as the simulation proceeds, and the fine mesh expands. Fig. 10(c) provides a better understanding of the contribution of DMR to the overall speedup and scalability for different problem sizes. For a fixed problem size, let \( P1: 1.5\lambda \times 1.5\lambda \), the initial speedup is of the order of 8 and drops to around 2 as the simulation proceeds. The result agrees with Fig. 10(b). For different problem sizes \( P2: 3.5\lambda \times 1.5\lambda \) and \( P3: 7.5\lambda \times 1.5\lambda \), the speedup transits from high to low and performs better as the problem size increases. For different problem sizes \( P1-P3 \), the overall speedup is \( 5, 7, \) and 8, respectively. The higher performance for bigger problem sizes is due to the small value of \( R(t) \) initially, which grows gradually as the plasma pattern spreads. For uniform fine mesh (Uni) implementation, the total cells \( \approx 10^5-10^6 \) and total computations per iteration are always fixed [Fig. 10(d)]. Whereas for DMR, the total number of cells and computations grow as the refinement region grows with simulation time, \( R(t) \). For uniform mesh, the total computational cost is proportional to \( N_f I_f \) (total cells \( \times \) total iterations). Whereas for DMR, the total computational cost is proportional to the sum of contributions from coarse and fine meshes. The coarse mesh has a fixed number of cells \( (N_c/r^2) \) and the updates are less frequent (by a factor of \( r \)) compared with the fine mesh, and therefore, the total cost for the coarse mesh is \( (1/r^3)N_c I_c \) (total coarse cells \( \times \) coarse iterations). For fine mesh, the computational cost is dynamic and is proportional to \( R(t) N_f I_f \) (total fine cells \( \times \) fine iterations). Therefore, for DMR, we obtain that the computational cost is proportional to \( N_f I_f (1/r^3 + R(t)) \) and primarily depends on the refinement factor and refinement region. The higher the refinement factor and the smaller the refinement region, the
better the speedup. Due to the smaller number of cells in the case of DMR [Fig. 10(d)], the size of the data structure is initially small, and the memory access performance is better compared with uniform fine mesh. Initially, the refinement region is very small, and the fine mesh data structure easily fits into cache memory, leading to higher cache hits. We observe a speedup better than the theoretical speedup of $r^3$ [Fig. 10(c)].

D. Physics of Complex Plasma Dynamics During HPM Breakdown: Spatio-Temporal Analysis

For the spatio-temporal analysis of plasma dynamics using DMR, we consider $\{c_{x_L}, c_{x_R}\} = (1, 1)$ and $n_0$ is located at $0.75L_x$ and $0.5L_y$. $E_0$ is $5.0$ MV/m, and the wave frequency is $110$ GHz. When a powerful millimeter-wave with a strong $E$-field interacts with the air (or gas) at high pressure, it delivers sufficient energy to the free electrons that accelerate and causes continuous ionization of the air/gas and leads to an avalanche breakdown (when ionization overcomes the attachment, diffusion, and recombination). The process results in the plasmoid formation [20], [25]. We can observe the plasmoid formation in Fig. 11 part A: (i), its growth into consecutive first, second, and third filaments, and the propagation of the plasma filamentary structure from right to left with time in Fig. 11 part A: (ii)–(iv) respectively.

The simulation results in Fig. 11 part B: (i)–(iv) show the temporal evolution of plasma density, rms $E$-field, diffusion coefficient, and rate of ionization along the central $x$-axis of the computational domain, whereas Fig. 11 part B: (v)–(viii) represents the same quantities along the $y$-direction ($y$-axis passing through the center of the rightmost filament). Fig. 11 helps us capture the plasmoid’s complete spatio-temporal evolution into filaments and its propagation during the first $100$ ns. This detailed scientific visualization further aids in understanding the complex plasma dynamics.

The initial plasmoid is located at $0.75L_x$ from the leftmost boundary as shown in Fig. 11 part A: (i), which is captured at $20$ ns. It remains Gaussian until the plasma density is small and cannot perturb the incoming microwave $E$-field as observed in Fig. 11 part B: (ii) [40]. The plasma density growth over time [Fig. 11 part B: (i)] can be attributed to the high incident rms $E$-field ($E_{\text{rms}} = 3.5$ MV/m) $> E_{cr}$, where critical (or breakdown) field $E_{cr} \approx 2.5$ MV/m $\text{rms}$ [20]. From Fig. 11 part A: (ii), the changes in the $E$-field distribution along the central $x$-axis during the time duration of $20–50$ ns can be observed, and during the same time interval, from Fig. 11 part B: (iv), we can observe the increase in the rate of ionization from $0.18 \times 10^9$ s$^{-1}$ to $1.3 \times 10^9$ s$^{-1}$ along the $x$-direction. From Fig. 11 part B: (ii) and (iv), we can see the strong correlation between the $E$-field peaks and the increased rate of ionization in space and time. The high ambient pressure results in multiple collisions between the electrons and neutral gas species. The ionization is a collision-assisted process governed by the local $E$-field or $E_{\text{eff}}$. In our study at $f = 110$ GHz, the wave angular frequency ($\omega$) is $2\pi f \approx 6 \times 10^{11}$ s$^{-1}$, $v_m \approx 4 \times 10^{12}$ s$^{-1}$; when $E_{\text{rms}} \geq 3.5$ MV/m, then $E_{\text{eff}} \approx 3.46$ MV/m $\geq E_{cr}$ also satisfies the breakdown criteria. Thus, high collisions result in much more ionization which in turn increases the plasma density. The increase in ionization rate causes an increase in the density of the plasma bulk, and the plasma starts modifying the total field like a dielectric. It is also important to consider the growth of the filament along the vertical ($y$)-axis, Fig. 11 part B: (v)–(viii). From Fig. 11 part A: (i) and part B: (v) between $t = 20$ and $50$ ns duration, we can see that the plasma density starts growing, and the filament stretches along the vertical cross section. The filament growth along the vertical direction can be attributed to the very large polar field that increases from $2.7$ MV/m to a maximum $\approx 5.42$ MV/m [Fig. 11 part B: (vi)].
due to higher ionization, and the free electrons in the filament fields stretch the plasma filaments along the vertical direction can be seen in Fig. 11 part B: (iv) and (viii). The high polar fields of the streamer. The filament tip has a higher density in comparison to the front, and the growth of the streamer tip gets modulated with the polar fields of the streamer.

V. CONCLUSION

In this article, a DMR technique has been proposed to solve the computationally challenging Maxwell–plasma
fluid model for simulating HPM-breakdown-induced plasma pattern formation and associated dynamics. The DMR technique leads to the generation of two meshes, a coarse mesh which is present throughout the computational domain and fine mesh which evolves with time in a self-aware fashion depending on plasma and electric field gradients. The implementation of the DMR technique has been described in detail. Furthermore, the technique has been evaluated in terms of accuracy and speedup by applying it to simulate and reproduce the experimental observations under similar conditions. The technique could accurately reproduce the complex plasma dynamics and plasma structures for different problem sizes and refinement factors. For a refinement factor of 2, we obtain an overall speedup of five to eight times for different case studies. The bigger problem sizes, involving the physical duration of $t > 100$ ns, which typically take months using a uniform fine mesh can be handled in a few days using the proposed DMR technique. Finally, through a novel spatio-temporal visual analysis, the complex physics involved during the high-power, high-frequency microwave breakdown and self-organized filamentation process have been discussed. The proposed DMR-based technique will be very helpful to investigate longer time-scale phenomena in the HPM breakdown (in the order of micro to milliseconds) such as gas heating which is sparsely reported in the existing literature. The DMR technique for self-aware mesh generation can be applied for any fluid-based plasma simulation (primarily involving spatio-temporal evolution of plasma topology) where fine resolution (using fine mesh) is required for accurate results. The DMR-aided simulations, at a much lower computational cost, will also help efficiently investigate and better understand the formation of different plasma structures (involving sharp gradients) observed in experiments.

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