The masses and shadows of the black holes Sagittarius A* and M87 in modified gravity (MOG)

J. W. Moffat* and V. T. Toth*

*Perimeter Institute for Theoretical Physics, Waterloo, Ontario N2L 2Y5, Canada

We present calculations for the anticipated shadow sizes of Sgr A* and the supermassive black hole in the galaxy M87 in the context of our modified theory of gravity (MOG, also known as Scalar–Tensor–Vector–Gravity, or STVG). We demonstrate that mass estimates derived from stellar dynamics in the vicinity of these black holes are the Newtonian masses of the black holes even in the MOG theory. Consequently, shadow sizes increase as a function of the key MOG parameter α, and may offer an observational means to distinguish the MOG theory from standard general relativity.

Following up on previous investigations [1–6], we examine the angular size of the shadow cast by the photon spheres of supermassive black holes such as Sagittarius A* (Sgr A*) in the central region of the Milky Way in the context of our modified theory of gravity (MOG) [7], and the dependence of this apparent size on the theory’s parameters.

The theory combines a tensorial gravitational field $g_{\mu\nu}$, and a variable gravitational coefficient $G = G_N(1 + \alpha)$ with a massive vector field $\phi_\mu$ associated with a source charge $Q_\phi = \sqrt{\alpha}G_N M$ that is proportional to mass and yields a repulsive force, which cancels out excess gravity at short range ($< O(\text{kpc})$), making the short-range behavior Newtonian. The test particle equation of motion, derived from the field equations of the theory, in the gravitational field of a compact mass $M$, is given by [7]:

$$\ddot{r} = -\frac{G_N M}{r^3} \left[1 + \alpha - \alpha c^{-\mu r}(1 + \mu r)\right] r,$$

where $\alpha = O(1)$ and $\mu = O(\text{kpc}^{-1})$ are parameters related to the theory’s scalar fields, and $G_N$ is Newton’s constant of gravitation. This equation of motion remains valid everywhere so long as $v^2/c^2 \ll 1$ and $GM/c^2 r \ll 1$.

The source mass $M$ that appears in this modified Newtonian acceleration law is the gravitating object’s “Newtonian” mass. This is to be distinguished from the so-called Arnowitt–Deser–Misner (ADM) mass, $M_{\text{ADM}} = (1 + \alpha)M$ [5].

In the case when $r \ll \mu^{-1}$, the MOG equation of motion reduces to the Newtonian form:

$$\ddot{r} = -\frac{G_N M}{r^3} r.$$

The conditions of validity for Eq. (2) are manifestly satisfied for stars in tight orbit around the Milky Way’s supermassive black hole, Sgr A*. The closest known star to Sgr A*, S2 (aka. S0-2), reached an estimated speed of 7650 km/s when it approached Sgr A* within 120 astronomical units (AU) [8]. Even during this close approach, the Newtonian and MOG equations of motion differ only by much less than one part in a thousand and therefore, the orbits are accurately characterized using the Newtonian equation of motion (2).

The immediate consequence of this is that the mass of Sgr A*, as determined from these stellar orbits, is the same Newtonian mass that one would use in Newtonian gravity or general relativity.

Given that the mass of Sgr A* is known with reasonable accuracy, the question arises: what is the anticipated size of its event horizon, or more precisely, what is the apparent size of its photon sphere, which determines its “shadow”, in the context of MOG? More specifically, how does the shadow change as a function of the MOG parameter $\alpha$?

To answer this question, we turn to the Kerr–MOG metric. Inferred from the MOG gravitational field equations, this metric has the form in Boyer–Lindquist coordinates $r, \theta, \phi$ [1, 2]:

$$ds^2 = \left(1 - \frac{r_s r - r_Q^2}{\rho^2}\right) dt^2 - \left[r^2 + a^2 + a^2 \sin^2 \theta \left(\frac{r_s r - r_Q^2}{\rho^2}\right)\right] \sin^2 \theta d\phi^2 + 2 \sin^2 \theta \left(\frac{r_s r - r_Q^2}{\rho^2}\right) dtd\phi - \frac{\rho^2}{\Delta} dr^2 - \rho^2 d\theta^2,$$

where $\rho^2 = r^2 + a^2 \cos^2 \theta$, $\Delta = r^2 - r_s r + a^2 + r_Q^2$, $a$ is the angular momentum per unit mass, the Schwarzschild–MOG radius is given by $2(1 + \alpha)G_N M$ and the length scale associated with the vector charge is $r_Q = \sqrt{\alpha(1 + \alpha)G_N M}$.

While recognizing that rotation ($a > 0$) distorts the shape of the photon sphere and the black hole shadow, these distortions remain small and likely unobservable even for near extremal rotation. Therefore, in the following we consider only the non-rotating case for simplicity.
FIG. 1: The expected size of the shadow of the photon sphere of Sgr A* for different values of the MOG $\alpha$ parameter: a) $\alpha = 0$, b) $\alpha = 1$, c) $\alpha = 10$, assuming no rotation. For comparison, d) shows near extremal rotation at $\alpha = 10$ and $a = 0.99GM$.

In the non-rotating case $a = 0$, the photon sphere radius is given by

$$r_\gamma = \frac{1}{2}(1 + \alpha)G_NM\left(3 + \sqrt{\frac{9 + \alpha}{1 + \alpha}}\right) = \frac{1}{2}(1 + \alpha)G_NM X, \quad (4)$$

where, following [6] we introduced the shorthand $X = 3 + \sqrt{(9 + \alpha)/(1 + \alpha)}$. Furthermore, at $r = r_\gamma$, we have

$$\Delta_\gamma = r_\gamma^2 - rsr_\gamma + r_Q^2 = \frac{1}{4}(1 + \alpha)^2G_N^2M^2X^2 - (1 + \alpha)^2G_N^2M^2X + (1 + \alpha)G_N^2M^2. \quad (5)$$

The shadow radius, in turn, is determined by

$$r_{\text{shadow}} = \frac{r_\gamma^2}{\sqrt{\Delta_\gamma}} = \frac{(1 + \alpha)^2X^2}{2\sqrt{(1 + \alpha)^2X^2 - 4(1 + \alpha)^2X + 4\alpha(1 + \alpha)}}G_NM. \quad (6)$$

In Fig. 1 we show the apparent size of the shadow of Sgr A* for different values of $\alpha$. For this, we adopt the value of $M_{\text{SgrA*}} = 4 \times 10^6 M_\odot$ for the Newtonian mass of Sgr A*, and assume a distance of 7.86 kpc from the Earth [9]. For comparison, we also include a near extremal rotating case, using the formulation presented in [2] (not reproduced in the present paper; see also [4].)

As this figure demonstrates, and as it can also be seen from Eq. (6), the shadow radius for $\alpha > 1$ scales approximately proportionately to $\sim (1 + \alpha)$.

In addition to Sgr A*, we also computed the anticipated shadow size for the supermassive black hole in M87. As in the case of stars orbiting Sgr A*, the velocity dispersion of stars in the vicinity of this black hole are expected to follow the Newtonian prediction to first order. Therefore, mass estimates for the M87 black hole, determined from the dynamics of stars in its vicinity, are estimates of its Newtonian mass even in the MOG theory.

To compute the size of the M87 shadow, we adopted the values of $M = 7.22 \times 10^9 M_\odot$ [11], at a distance of 16.4 Mpc [11]. The results of this calculation are shown in Fig. 2.
FIG. 2: The expected size of the shadow of the photon sphere of the supermassive black hole in M87. Subplots are labeled as in Fig. 1.

TABLE I: Expected values of the angular diameters of Sgr A* ($\delta_{\text{Sgr A}^*}$) and M87 ($\delta_{\text{M87}}$) as functions of the MOG parameter $\alpha$.

NB: $\alpha = 0$ corresponds to general relativity.

| $\alpha$ | $\delta_{\text{Sgr A}^*}$ (\mu as) | $\delta_{\text{M87}}$ (\mu as) |
|----------|-----------------------------------|-------------------------------|
| 0        | 48.4                              | 45.3                          |
| 1        | 87.8                              | 82.1                          |
| 2        | 126.1                             | 118.0                         |
| 3        | 164.1                             | 153.5                         |
| 4        | 201.9                             | 188.9                         |
| 5        | 239.5                             | 224.1                         |
| 6        | 277.1                             | 259.2                         |
| 7        | 314.6                             | 294.3                         |
| 8        | 352.1                             | 329.4                         |
| 9        | 389.5                             | 364.4                         |
| 10       | 426.9                             | 399.4                         |

The value of $\alpha$ for these supermassive black holes is not known. A semi-empirical derivation [12] yields the approximate formula $\alpha = \alpha_\infty M/\sqrt{M + E}^2$ with $\alpha_\infty = O(10)$ and $E \sim 25000 M_\odot^{1/2}$, which seems to agree well with galaxy rotation data, but we do not know if this weak field approximation is valid for an object as compact as a black hole. If it is, we anticipate $\alpha = O(1)$ for Sgr A* and $\alpha = O(10)$ for M87.

Values for the anticipated angular diameters of the Sgr A* and M87 black holes for different values of $\alpha$ are shown in Table I.

The actual, observed size of these shadows may soon be revealed as much anticipated first results from the Event Horizon Telescope [13] project are made public. The angular resolution of this VLBI observational project is anticipated to be sufficient to resolve the shadow of Sgr A* and the M87 supermassive black hole. A larger than expected shadow
would be clearly in conflict with the predictions of general relativity, and point the way towards a modified gravity theory, such as MOG. Conversely, while a shadow size that agrees with the predictions of general relativity does not falsify the MOG theory, it puts severe constraints on the behavior of the $\alpha$ parameter.

[1] J. W. Moffat, European Physical Journal C 75, 175 (2015), 1412.5424.
[2] J. W. Moffat, European Physical Journal C 75, 130 (2015), 1502.01677.
[3] H.-C. Lee and Y.-J. Han, European Physical Journal C 77, 655 (2017), 1704.02740.
[4] M. Guo, N. A. Obers, and H. Yan, Phys. Rev. D 98, 084063 (2018), 1806.05249.
[5] P. Sheoran, A. Herrera-Aguilar, and U. Nucamendi, Phys. Rev. D 97, 124049 (2018), 1712.03344.
[6] H.-M. Wang, Y.-M. Xu, and S.-W. Wei, Journal of Cosmology and Astroparticle Physics 2019, 046 (2019), 1810.12767.
[7] J. W. Moffat, Journal of Cosmology and Astroparticle Physics 2006, 004 (2006), arXiv:gr-qc/0506021.
[8] Gravity Collaboration, R. Abuter, A. Amorim, N. Anugu, M. Bauböck, M. Benisty, J. P. Berger, N. Blind, H. Bonnet, W. Brandner, et al., Astron. Astrophys. 615, L15 (2018), 1807.09409.
[9] A. Boehle, A. M. Ghez, R. Schödel, L. Meyer, S. Yelda, S. Albers, G. D. Martinez, E. E. Becklin, T. Do, J. R. Lu, et al., Astrophys. J. 830, 17 (2016), 1607.05726.
[10] L. J. Oldham and M. W. Auger, Mon. Not. R. Astron. Soc. 457, 421 (2016), 1601.01323.
[11] S. Bird, W. E. Harris, J. P. Blakeslee, and C. Flynn, Astron. Astrophys. 524, A71 (2010), 1009.3202.
[12] J. W. Moffat and V. T. Toth, Class. Quant. Grav. 26, 085002 (2009), arXiv:0712.1796 [gr-qc].
[13] https://eventhorizontelescope.org