Low-cost automatic device for obtaining the coefficient of static friction

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Abstract. Considering that the coefficient of friction between materials is one of the important characteristics in the design, either by wear or by requiring friction for power transmission, and this value is usually achieved in tables and these values come out of experimentation or results with tests using devices. Furthermore, these devices are not often found in the laboratories of the universities. For this reason, it was decided to adopt one of the methods of calculation, in this case the inclined plane. This consists of finding the angle of inclination in which one material begins to slide on another, since calculating the tangent of this angle determines the coefficient of friction. To this end, a device was built, consisting of a platform that by means of a mechanism is tilted little by little, until the object under study begins to move. In addition, to exclude human error, the device has sensors and an adequate control, in such a way that the process is completely automatic. Finally, the device is based on open technology, such as Arduino, managing to build a device to obtain the coefficient of friction at low cost. Its development and the results obtained are shown in this paper.

1. Introducción

The following article describes the construction of a low-cost automatic device for obtaining the coefficient of static friction. Initially, the theory used is shown. In this case, the inclined plane method. Then, the elements used for the construction of the device are presented, taking into account even the way in which the angular position sensor is adjusted. In such a case, a linear potentiometer describes how the data is taken in order to convert the readings of the potentiometer into an equation that results in the angle of inclination. After this step, we proceed to the connection of the electronic and mechanical elements of the device, giving to the reader a clear idea of doing so in case of wanting to replicate this device. Finally, the program is attached to the Arduino IDE that controls the system.

This article seeks to supply the laboratories of the universities at low cost and with their own developments. In the case of this equipment, the cost is approximately US $ 50, and from here a lot of possibilities arise. The readers are left to discuss the functionality because at least for the Technological Foundation Antonio de Arévalo is contributing in the laboratory of applied physics, where the student does the experiment manually and then the results are contrasted with the automated equipment to get their own conclusions of the calculation of the coefficient of static friction.
2. Calculation of the coefficient of static friction

The static friction force which is produced by the interaction between the irregularities of two surfaces (see Figure 1) is what prevents any relative movement between the two bodies to a limit where the movement already begins. That threshold of movement is characterized by the coefficient of static friction. The coefficient of static friction is typically greater than the coefficient of kinetic friction. [1].

![Figure 1. Irregularities between surfaces.](image1)

By making the free body diagram of the case of figure 1, we obtain the forces that intervene on the block (see figure 2), and from there we can attain the friction force.

![Figure 2. Forces exerted on the block.](image2)

Where:
- **F**: External force or force applied by man on the block.
- **W**: block weight.
- **N**: normal strength of the block.
- **fr**: Frictional force.

From here, it is obtained:

\[ N = w \quad (1) \]

\[ F = fr \quad (2) \]

The relation between **fr** y **N** is:

\[ fr = \mu \cdot N \quad (3) \]

In conclusion, to determine the friction force is required knowing the value of \( \mu \), and this depends on the materials that are in contact. The coefficient of friction (\( \mu \)) is a dimensionless magnitude that...
expresses the proportionality existing between the force of friction \( (fr) \) and the normal force \( (N) \), which exerts a body that rests or moves on another. [2]

In this way, it is clear that it is necessary to have the value of \( \mu \), which is found in tables whose values have been found by experimentation. Since it cannot be said with certainty what the coefficient of friction is, taking into account that it plays an important role the polishing of the materials or if there is a lubricant between them. In table 1, it can be seen some and the most common data of friction's coefficients.

| Surfaces in contact     | \( \mu \). Static friction coefficient |
|------------------------|--------------------------------------|
| Wood on wood           | 0,5                                  |
| Steal on ice           | 0,03                                 |
| Teflon on teflon       | 0,04                                 |
| Rubber on dry cement   | 1                                    |
| Glass on glass         | 0,9                                  |
| Ski (waxed) on snow \( (0^\circ C) \) | 0,1                                 |
| Wood on leather        | 0,5                                  |
| Aluminum on steel      | 0,61                                 |
| Human joints           | 0,02                                 |

In this way, whenever it is required to calculate the friction force, it is required to know the \( \mu \), and the question is, what if the materials in contact are not in a table? Or what if the finish of the surfaces in contact is special? Here, the need to find a method to calculate the coefficient of static friction arises. There are several methods but the one chosen for this case it is the inclined plane method (see figure 3), from which you can see the internal and external forces that appear on the block. For this case, any external force will be applied.

![Figure 3. Block on inclined plane](image)

Where:

- \( N \): normal
- \( fr \): it is the friction force
- \( Px \): component of block weight on the X axis
Py: component of the weight of the block on the Y axis
Θ: is the angle of inclination of the plane

It is determined from figure 3 that:

\[ N = m \cdot g \cdot \cos \Theta \]  \hspace{1cm} (4)
\[ fr = m \cdot g \cdot \sin \Theta \]  \hspace{1cm} (5)

Thus, by replacing the values of (4) and (5) in the equation (3), it is obtained:

\[ \mu \cdot m \cdot g \cdot \cos \Theta = m \cdot g \cdot \sin \Theta \]  \hspace{1cm} (6)

From (6), it is determined that:

\[ \mu = \tan \Theta \]  \hspace{1cm} (7)

And this is how the coefficient of friction \( \mu \) can be determined practically. In figure 3, the block is not moving due to the frictional force. However, if the angle continues to increase, it will reach a value in which the block will begin to slip, and with that value of \( \Theta \) it is how the static \( \mu \) is calculated, which it is the case of this study.

### 3. Development of the device to calculate the static \( \mu \)

To calculate the static \( \mu \), the method of the inclined plane was taken into account.

It can be done manually (see figure 4) and to determine the angle at which the object begins to move and calculating the tangent of that angle, it is obtained friction coefficient.

![Figure 4](image)

**Figure 4.** Calculation of the friction coefficient experimentally. [4]

This method is very easy to execute because it is only to have a goniometer, a plate that can be tilted and the two study surfaces. Nevertheless, the disadvantage is that the assignment of the value of the angle \( \Theta \) is made by observation. Some errors can be presented by the human factor, and seeking to
eliminate this error factor, the low-cost automatic device for obtaining the coefficient of static friction arises.

3.1. Operation of the low-cost automatic device to obtain the static coefficient of friction

Before explaining the operation of the device, the main elements of the device and its function will be explained.

There are some devices in the market to determine the $\mu$, and they cost a couple of thousand dollars, like the Friction Coefficient Meter Mod. LI-COF-P01 of Figure 5, which is automatic. Besides, it has the necessary sensors to angle calculation, motor with variable speed, RS-232 communication, network port and weighs up to 20 Kg. [5]

![Figure 5: Friction Coefficient Meter Mod. LI-COF-P01 [5]](image)

Based on the basic operation, the prototype of figure 6 was reached. It consists of a platform that pivots on an axis, and it is lifted by a rope that is wound on a reel that moves a gearmotor with variable speed. Moreover, a linear potentiometer is responsible for measuring the angle of inclination. There is a presence sensor, an Arduino UNO card, an LCD screen among others.

![Figure 6: Low cost automatic device for obtaining the coefficient of static friction. where:
A: Control card Arduino UNO
B: LCD screen of 16x2
C: Platform
D: Linear potentiometer
F: Infrared motion sensor
E: Variable speed gearmotor](image)
The size of the device can vary according to the requirement. For this case, the base is 12 cm x 30 cm, and does not weigh more than 1.5 kg, which makes it very easy to move. The body of the device is made of acrylic which it is an economic material and easy to manufacture. Additionally, the connections of electronic devices are practically found in the network. However, the adjustment of the linear potentiometer will be shown in detail in the results stage of this article. In figure 7, the operating logic is clearly observed, even when the device is in operation.

![Flowchart](image)

**Figure 7.** Operation of the low-cost automatic device to obtain the coefficient of static friction.

4. **Calculation of the angle with a linear potentiometer.**
To determine the angle of inclination in which the object begins to move, and which serves as input for the calculation of the coefficient of static friction $\mu$, a linear potentiometer is used, which is coupled to the axis of the platform that is tilted. See figure 8.
This potentiometer gives as input to the Arduino card values between 0 and 1223, since the analog input of the Arduino is 10 bits, it must be determined the value of the angle of inclination with this input. For this case, measurements were made which can be seen in table 2, where in the first column is the value given by the linear potentiometer and in the second column the value of the angle of inclination measured with an external device. For this sense, the sensor of a Samsung Galaxy A7 cell phone was used.

![Potentiometer coupled to the axis of the platform.](image)

**Figure 8.** Potentiometer coupled to the axis of the platform.

**Table 2.** Relationship between the value given by the potentiometer and the Samsung Galaxy A7.

| Analog input | Grades (A7) |
|--------------|-------------|
| 370          | 0           |
| 395          | 8.4         |
| 413          | 14.5        |
| 439          | 21.9        |
| 465          | 30.6        |
| 479          | 36          |

Figure 9 shows the assembly to determine the data in Table 2, which lead to determine the equation that will govern the value of the angle of inclination.
Figure 9. Assembly to determine the relationship between sensor value Vs tilt angle.

With the data in table 2, the graph of figure 10 is made, and with the help of Excel, the equation that will govern the calculation of the angle of inclination can be determined.

![Graph](image)

**Figure 10.** Value of the potentiometer Vs the angle of the Samsung Galaxy A7.

Using a linear adjustment in Excel, it can be found the function that will determine the angle of inclination. This equation is the one that is used for programming the device.

\[
\theta = 0.3247 . VP - 120.0
\]  

(8)

Where

- \( \theta \): Angle of inclination given by the device
- \( VP \): Value given by the Potentiometer

This adjustment gave a determination coefficient of:

\[
R^2 = 0.9989
\]  

(9)
The equation (9) is a good indicator that this linear adjustment has a very good approximation [6]. Furthermore, it was decided to leave the equation (8) as the function that will govern the system for calculating the coefficient of static friction.

5. Connections and Programming

5.1. Connections

The first test that is done is to connect the screen, since the I2C adapter will be entered to the Arduino UNO card and the 16x2 LCD screen. This adapter uses only 4 connectors which are, Vcc, GND and two more that go to the A4 and A5 pins, as it is shown in figure 11.

![Diagram of connection between Arduino UNO, adapter for LCD I2C PCF8574 screen and LCD screen](image1)

Figure 11. Diagram of connection between Arduino UNO, adapter for LCD I2C PCF8574 screen and LCD screen [7]

Having this connection working and bearing in mind that the A4 and A5 pins are reserved for the I2C LCD PCF8574 screen adapter, the rest of the elements of the device are connected. (See figure 12)

![Low-cost automatic device connections for obtaining the coefficient of static friction](image2)

Figure 12. Low-cost automatic device connections for obtaining the coefficient of static friction
5.2. Programming

The programming of the low-cost automatic device for obtaining the coefficient of static friction will be presented below. This programming was done in the IDE of Arduino. However, the use of it, is left to the consideration of the readers

```c
#include <Arduino.h>
#include <Wire.h>
#include <SoftwareSerial.h>
#include "LiquidCrystal_I2C.h"

double angle_rad = PI/180.0;
double angle_deg = 180.0/PI;
double pwm;
double ang;
void stop();
int conta;
int entrada;
int s;
int start;
double u;
int reset;

LiquidCrystal_I2C lcd_I2C_0x27(0x27, 16, 2);

void setup(){
    lcd_I2C_0x27.init();
    lcd_I2C_0x27.backlight();
    lcd_I2C_0x27.clear();
    lcd_I2C_0x27.setCursor( (1) - 1, (1) - 1 );
    lcd_I2C_0x27.print( "CALCULATION OF" );
    lcd_I2C_0x27.print( "THE u STATIC" );
    _delay(2);
    lcd_I2C_0x27.clear();
    lcd_I2C_0x27.setCursor( (1) - 1, (1) - 1 );
    lcd_I2C_0x27.print( "H. RODRIGUEZ" );
    lcd_I2C_0x27.setCursor( (2) - 1, (2) - 1 );
    lcd_I2C_0x27.print( "TECNAR-2018" );
    _delay(2);
    pwm = 30;
    baja();
    stop();
    lcd_I2C_0x27.clear();
```
void loop(){

    s = digitalRead(2);
    if(((s)==(1))){
        lcd_I2C_0x27.clear();
        lcd_I2C_0x27.setCursor( (4) - 1, (1) - 1 );
        lcd_I2C_0x27.print( "PRESS THE" );
        lcd_I2C_0x27.setCursor( (4) - 1, (2) - 1 );
        lcd_I2C_0x27.print( "START BUTTON" );
        start = digitalRead(3);
        if(((start)==(1))){
            while(!(((s)==(0)))){
                _loop();
                s = digitalRead(2);
                medir();
                lcd_I2C_0x27.clear();
                lcd_I2C_0x27.setCursor( (1) - 1, (1) - 1 );
                lcd_I2C_0x27.print( "TETA =" );
                lcd_I2C_0x27.setCursor( (4) - 1, (2) - 1 );
                lcd_I2C_0x27.print( ang );
            }
            stopp();
            u = tan(angle_rad*ang);
            lcd_I2C_0x27.clear();
            lcd_I2C_0x27.setCursor( (1) - 1, (1) - 1 );
            lcd_I2C_0x27.print( "u ESTATIC =" );
            lcd_I2C_0x27.setCursor( (4) - 1, (2) - 1 );
            lcd_I2C_0x27.print( u );
            reset = digitalRead(4);
        }
    }
}
if(((reset)==(1))){
    baja();
    lcd_I2C_0x27.clear();
    lcd_I2C_0x27.setCursor( (1) - 1, (1) - 1 );
    lcd_I2C_0x27.print( "CALCULATION OF" );
    lcd_I2C_0x27.setCursor( (4) - 1, (2) - 1 );
    lcd_I2C_0x27.print( "THE u STATIC" );
void sube()
{
    digitalWrite(8, 1);
    analogWrite(9, pwm);
}

void baja()
{
    while(!((ang)==(0)))
    {
        _loop();
        analogWrite(8, pwm);
        digitalWrite(9, 1);
        medir();
    }
}

void stopp()
{
    digitalWrite(8, 0);
    analogWrite(9, 0);
}

void medir()
{
    conta = 0;
    for(int __i__=0; __i__<10000; ++__i__)
    {
        entrada = analogRead(A0+0);
        conta = (conta) + (entrada);
    }
    entrada = (conta) / (10000);
    ang = ((entrada) * (0.3247)) - (120.01);
}

void _delay(float seconds){
    long endTime = millis() + seconds * 1000;
    while(millis() < endTime)_loop();
}

void _loop(){
}
6. Conclusions
It can be conclude that low-cost laboratory devices can be developed by using free technology such as Arduino and applying basic theoretical bases. It is important to note that complex calculations are not required for the construction of equipment. Although, this device automatically achieves a simple exercise if this calculation is done manually, it can involve human error when taking the data.

An equipment of less than 50 dollars was achieved, and in this way the goal of building low-cost equipment is fulfilled. Furthermore, it is recommended to use a multi-turn potentiometer to improve the accuracy. Also, it must be connected to a pinion box in such a way that by 90° of movement of the mobile platform, the potentiometer rotates for about 8 full turns.

Finally, at the moment the results can only be compared with the experiment done by hand. However, a certified device is being searching for the comparison of results. Currently, this accomplishes with the practical calculation applying the theory of the inclined plane.

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