Localization of a random heteropolymer onto a surface

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Abstract

We study the localization of a random heteropolymer onto an homogeneous surface, the problem which is equivalent to the wetting of an interface at disordered substrate in two dimensions, via replica trick by using the Green’s function technique. The exact treatment of one- and two-replica binding states is used to compute the free energy of the random heteropolymer. We present analytical results for two particular cases: (i) nearly statistically symmetric copolymer in the vicinity of the threshold of the annealed problem, and (ii) the asymmetric polymer with the interaction part of the annealed Hamiltonian being nearly zero. In both cases the localization is due to two-replica binding states. In the case (i) the two-replica binding state exists both above and below of the one-replica binding state. In the case (ii) the energy of the two-replica binding state at the transition is finite. A schematic phase diagram of the localization-delocalization transition of the random heteropolymer is suggested.

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Adsorption of a polymer chain onto a surface is of large practical interest ranging from biological physics to technological applications \([1]-[3]\) (and citations therein). The adsorption of a polymer with heterogeneous sequence structure is relevant in connection with the study of the behavior of proteins near surfaces. The problem of wetting in two dimensions \([10]-[14]\) is closely related to the adsorption of a symmetric heterogeneous polymer onto a surface. The polymer adsorption has been studied in connection with denaturation of double-stranded DNA in solution \([15]-[16]\) and growth problems \([17]\).

The role of disorder on the absorption of the random heteropolymer remains a subject of controversy. This question has been addressed recently by several groups \([11–14]\). The quenched part of disorder is concluded to be irrelevant in \([11,12]\), while on the contrary, the work \([13]\) shows that the quenched part of disorder shifts the transition temperature of the localization. Very recently the similar problem was considered in \([14]\) by using a functional renormalization group. These authors predict that the localization transition for the symmetric heteropolymer is of Kosterlitz-Thouless type.

In this Paper we study the adsorption of a random asymmetric heteropolymer via replica trick by using the Green’s function technique. In contrast to the previous studies the Green’s function method allows an exact consideration of one-replica (1p) and two-replica (2p) binding states. Our analysis shows that for statistically symmetric heteropolymer the 2p binding state exists at the localization transition of the 1p binding state. The exact solution of the two-replica problem at the point where the interaction part of the annealed Hamiltonian is zero, indicates that the localization transition is first-order. We incorporate the one-replica and two-replica localized states into a novel and heuristic procedure to compute the quenched free energy.

The partition function of a polymer containing \(N\) segments interacting with the surface is

\[
Z = \int Dz(s) \exp \left[ -\frac{1}{2l^2} \int_0^N ds \left( \frac{dz(s)}{ds} \right)^2 - \int_0^N ds V_0(z(s)) - \int_0^N ds \zeta(s) V_{int}(z(s)) \right],
\]

where \(V_0(z) = (\infty, z \leq 0; 0, z > 0)\) is the repulsive interaction potential with the wall, \(l\) is the statistical segment length. The heterogeneity of the polymer is described by random Gaussian variables \(\zeta(s)\), which are characterized by the moments \(\zeta(s) = \zeta, \zeta(s)\zeta(s') = \zeta^2 + \Delta \delta(s - s')\). If \(\zeta = 0\), then the number of the monomers which are attracted or repelled from the well is on average the same, so that the heteropolymer is statistically symmetric. On the contrary, if \(\zeta \neq 0\), there is an excess of the monomers, which are repelled from \((\zeta > 0)\) or attracted to \((\zeta < 0)\) the well. In this case the heteropolymer is asymmetric \([4]\). The attractive interaction with the surface will be modelled by the potential \(V_{int}(z) = u\delta(z - z^0)\), where \(z^0\) is small but nonzero (see below). This choice of the potential allows the exact treatment of the problem with two replicas, as will be shown below.

Imaging \(s\) to be an axis perpendicular to \(z\) we interpret Eq.\((1)\) as a partition function of a directed line interacting with a heterogeneous substrate at \(z = 0\), which is the wetting problem. The random variables \(\zeta(s)\) are now attributed to the substrate. The wetting interpretation of the polymer problem is a particular case of the relation between the polymer in \(d\) dimensions and directed polymer in \(d+1\) dimensions. In the case of wetting the condition \(\zeta \neq 0\) means that on average the interface interacts with the substrate. If \(\zeta > 0\), then the interface is repelled from the substrate.
Performing the average over $\zeta(s)$ by using the replica trick we obtain the replica partition function as

$$Z_n = \overline{Z^n} = \int Dz_a(s) \exp\left(-\sum_{a=1}^{n} \int_0^N ds \left(\frac{1}{2l^2} \left(\frac{dz_a(s)}{ds}\right)^2 + V_0(z_a(s)) + \zeta V_{int}(z_a(s))\right) + \frac{\Delta}{2} \sum_{a,b=1}^{n} \int_0^N ds V_{int}(z_a(s)) V_{int}(z_b(s))\right).$$

(2)

The free energy has to be computed as follows

$$-\beta F = \frac{\partial Z_n}{\partial n} \bigg|_{n=0}. \quad (3)$$

Due to the fact that (2) contains only the one-fold integral over $s$, the partition function $Z_n$ can be interpreted as the probability amplitude of a quantum mechanical system of $n$ particles associated with the Hamiltonian

$$H_n = \sum_{a=1}^{n} \left(-D \nabla_a^2 + V_0(z_a) - \beta \delta(z_a - z^0)\right) - \Delta u^2 \sum_{a<b}^{n} \delta(z_a - z^0) \delta(z_b - z^0), \quad (4)$$

where $D = l^2/2$, $\beta = \Delta u^2/2\delta_0 - \zeta$, with $\delta_0 = 1/\delta(0)$ being of the order of magnitude equal to the width of the potential well, and $\zeta = \zeta u$. In the following we will speak of particles instead of replicas. The first term in (4) is associated with the annealed average of the free energy. The annealed part of $H_n$ is equivalent to the localization problem of a quantum mechanical particle in the vicinity of the wall, and can be solved exactly. The binding problem for (4) with $n = 2$ can also be solved exactly. The Hamiltonian (4) at given $n$ can be interpreted as a Hamiltonian of $n$ polymers interacting with the surface. The 2nd term in the first sum describes the attraction of the monomers to the surface, which is independent of each other. The second sum in (4) gives an additional attractive interaction, if the monomers belonging to different polymers contact the surface simultaneously. We are not aware, if such an interaction can be realized in reality. Eq.(4) at $n = 2$ and for $\beta = 0$ and $V_0(z_a) = 0$ is exactly the Hamiltonian of a quantum mechanical particle in two dimensional delta potential. As it is well-known from text books the binding state in this case exists for infinitesimally weak potential. While $\sum_{a=1}^{n} V_0(z_a)$ does not possess the radial symmetry, the Hamiltonian (4) at $n = 2$ and $V_0(z_a) \neq 0$ does not correspond to a quantum mechanical problem in a radial symmetric two dimensional potential well with an impenetrable core at the origin. The Hamiltonian $H_n$ at $n = 2$, $V_0(z_a) = 0$ and $\beta = 0$ is related to the Poland-Sheraga model [15]. The essential difference to the Poland-Sheraga model consists in the fact that all contacts occur at $z = z_0$, i.e. the model (4) neglects the wiggling of the zipped polymer pair. It is evident that the last term in (4) favors the localization of replicas.

To this end it is convenient to consider the one-replica Green’s function, $G(z_1, N; z_0^1) \equiv \langle \delta(z(N) - z_1) \delta(z(0) - z_0^1) \rangle$, associated with the annealed part of the Hamiltonian (4). The Laplace transform with respect to $N$ (the variable $N$ plays the role of the imaginary time for the quantum mechanical particle) of the perturbation expansion of $G(z_1, N; z_0^1)$ in powers of the attraction strength $\beta$ is a geometric series, which is summed as
\[ G(z_1, p; z_1^0) = G_0(z_1, p; z_1^0) + \beta \frac{G_0(z_1, p; z_0)G_0(z_0, p; z_1^0)}{1 - \beta G_0(z_0, p; z_0)}, \]  

where \( G_0(z_1, p; z_1^0) = (\exp(-|z_1 - z_1^0|\sqrt{p/D}) - \exp(-|z_1 + z_1^0|\sqrt{p/D}))/\sqrt{4Dp} \) is the Laplace transform of the Green’s function of the diffusion equation in the half space \((z \geq 0)\) with the Dirichlet boundary condition at \(z = 0\). The equation

\[ 1 - \beta G_0(z_0, p; z_0) = 1 - \beta (4Dp)^{-1/2}(1 - \exp(-2z_0\sqrt{p/D})) = 0, \]

which is the denominator of the 2nd term in the r.h.s. of Eq.(5), is the energy eigenvalue condition for \(1p\) (one-replica or one-particle) localized state. Identifying \(D\) as \(h^2/2m\), Eq.(5) coincides exactly with the eigenenergy condition for the localization of a quantum mechanical particle in an attractive Delta-potential placed at the distance \(z_0\) from the wall. The localized state corresponds to the solution of (5) \(p_c > 0\). The energy of the localized state is given by \(E_{1,0} = -p_c\). It is easy to see from (5) that the localized state exists for \(\beta > \beta_c = D/z_0\). The inverse Laplace transform of \(G(z_0, p; z_0)\) for weak binding \((2z_0\sqrt{p/D} \leq 1)\) and \(\beta > \beta_c\) is obtained from (5) as

\[ G(z_0, N; z_0) \simeq \frac{\sqrt{D}}{p_c} \exp(Np_c) + \frac{1}{\sqrt{\pi N}} \exp(Np_c) + \frac{p_c z_0}{\sqrt{D}} \exp(Np_c) - \frac{p_c z_0}{\sqrt{D}} \exp(Np_c) \text{erf}(\sqrt{Np_c}), \]

where \(p_c = (\beta/\beta_c - 1)^2 D^3/(\beta^2 z_0^4)\). In computing (8) we neglected the term proportional to \(\delta(N)\) i.e. we consider \(N\) in (5) to be positive. Taking into account the delta function in the case \( N = (z_0^2 - s_i^2) \) leads to non significant changes.

To study the effect of the non-diagonal part of \(H_n\) in the case \(n = 2\) we will consider the connected part of the two-replica Green’s function \(G_{2,c}(z_1, z_2, N; z_1^0, z_2^0, 0) \equiv \langle \delta(z_1 - z_a(N))\delta(z_2 - z_b(N))\delta(z_1^0 - z_a(0))\delta(z_2^0 - z_b(0)) \rangle_c\), where \(a\) and \(b\) denote the replica indices \((a \neq b)\). The perturbation expansion of the Green’s function \(G_{2,c}(z_1, z_2, N; z_1^0, z_2^0, 0)\) in powers of the interaction (third and fourth terms in (6)) is represented graphically in Fig.1. The dotted lines are associated with the two-replica interaction given in Eq.(4). The ends of the dotted lines are associated with \(z_0\) and the arc length \(s_i\), which are ordered from the left to the right. An integration over \(s_i\) has to be performed. Each part of the continuous line between two consecutive dotted lines is associated with the one-replica Green’s function \(G(z^0, s_i; z^0, s_{i-1})\).

The left (right) external lines are associated with \(G(z, N; z^0, s_i) (G(z_0, s_i; z^0, 0)) \) \((z = z_1, \text{ or } z_2, \text{ while } z^0 = z_1^0, \text{ or } z_2^0)\). The graphical expansion in Fig.1, which visualizes the effect of the two-replica interaction in \(H_2\) in terms of space-time \((N = -it)\) trajectories, shows that both trajectories contact the surface at the same time. Thus, the return probability to have two consecutive contacts is the square of that for one particle. This suggests that the localization of two particles interacting according to (4) is closely related to the localization of one particle in two dimensions. The integral associated with a graph in Fig.1 is a folding, so that the Laplace transform with respect to \(N\) reduces the perturbation expansion in Fig.1 to a geometrical series, which is summed as

\[ G_{2,c}(z_1, z_2, p; z_1^0, z_2^0) = \alpha \frac{G_2(z_1, z_2, p; z_0, z_0)G_2(z_0, z_0, p; z_1^0, z_2^0)}{1 - \alpha G_2(z_0, z_0, p; z_0, z_0)}, \]  

where \(G_2(z_1, z_2, p; z_0, z_0) \equiv \langle \delta(z_1 - z_a(N))\delta(z_2 - z_b(N))\delta(z_1 - z_0)\delta(z_2 - z_0) \rangle_c\).
where \( \alpha = \Delta u^2 \), and \( \tilde{G}_2(z_1, z_2^2, p; z_1^0, z_2^0) \) is the Laplace transform of the product of two one-replica Green’s functions \( G(z_1, N; z_1^0)G(z_2, N; z_2^0) \). The denominator on the r.h.s. of Eq. (8) gives the eigenvalue condition for the two-replica (particle) bound state. We did not succeed to analyze the latter analytically, so that now we will consider the following particular cases: (i) approximately statistically symmetric polymer in the vicinity of the localization threshold of the annealed problem, and (ii) the asymmetric polymer under condition that the interaction part of the annealed Hamiltonian is approximately zero. The latter case can be realized by tuning the asymmetry parameter \( \zeta \). The two-replica partition function for directed polymers with random interactions was previously studied in [21]-[22].

To ensure the existence of Laplace transform of the one-replica Green’s function squared, we introduce a short-time cutoff by replacing \( 1/ \) by \( 1/(N + a) \). The cutoff \( a \) along the polymer can be eliminated in favor of the transversal length \( a_0 \) via \( a = a_0^2/4D \). The eigenvalue condition for the two-replica localized state, which consists in equality of the denominator of Eq. (8) to zero, is obtained for small \( p_c \), i.e. in the vicinity of the localization transition of the annealed problem, as

\[
0 = 1 - \frac{D}{\pi z_0^2} \alpha \exp(a_0^2 p/4D) \Gamma(0, a_0^2 p/4D) + \frac{\alpha}{\pi z_0^2 \beta^2} \left[ -2 \pi \sqrt{p} + \frac{2z_0}{\sqrt{D}} \exp(a_0^2 p/4D) \Gamma(0, a_0^2 p/4D) \right],
\]

(9)

where \( \Gamma(0, x) \) is the incomplete gamma function. Eq. (9) yields for \( p \) in the vicinity of the one-replica binding transition, i.e. for small \( p_c \)

\[
p_{2,c} = \frac{4D \exp(-\gamma)}{a_0^2} \exp\left(-\frac{\pi z_0^2 \beta^2}{D \alpha}\right),
\]

(10)

where \( \gamma \) is the Euler number. The energy \( E_{2,c} \) of the two-replica bound state is \(-p_{2,c}\). For the symmetric case, \( \alpha = 2\beta \delta_0 \), \( p_{2,c} \) decreases with decreasing \( \delta_0 \) at fixed \( \beta \sim \beta_c \) and \( z_0 \). This ensures the validity of the condition \( 2z_0 \sqrt{p/D} \leq 1 \) (weak binding) that we used to derive Eq. (10). A similar analysis slightly above the annealed threshold also results in Eq. (10), so that the two-replica bound state exists both below and above the threshold of the one-replica bound state. The result (10) shows that the two-replica bound state already exists at the one-replica localization transition. This is very reasonable and can be explained qualitatively as follows. The individual interactions with the surface contained in the annealed part of the Hamiltonian (11) result in an increase of the probability to find the monomers of the polymer pair in the vicinity of the surface. This compensates the decrease of the probability, which is due to the wall potential \( \sum_{a=1}^n V_0(z_a) \), and thus shifts the threshold to the lower values.

The asymmetric case (ii), \( \beta \rightarrow 0 \), where no one-replica localized states exist can be realized by tuning the asymmetry parameter \( \zeta \) as it is seen from the definition of \( \beta \). For small \( \beta \) the 1p Green’s function may be approximated by its bare value, so that we obtain

\[
\text{Notice that the case, when only the first term in (11) is present, corresponds to localization of a QM particle in a shallow 2d potential well. The above procedure gives an exact solution of the problem, if one identifies the length } a_0 \text{ with the width of the potential well.} \]

5
\[
G^2(z_0, N; z_0) = \frac{1}{4\pi DN}(1 - 2 \exp(-\frac{z_0^2}{2DN}) + \exp(-\frac{2z_0^2}{2DN})).
\]  

As above we replace \(1/N\) in the first term on the right-hand side of Eq. (11) by \(1/(N + a)\), where the cutoff \(a\) along the polymer can be eliminated in favor of the length \(a_0\) via \(a = a_0^2/4D\). The \(z_0\)-dependent terms in (11) are due to the boundary condition at the surface \(z = 0\). In the case of adsorption onto an interface only the term \(1/(4\pi DN)\) will appear in Eq. (11), so that in this case the two-replica bound state will exactly coincide with that in a shallow potential well in two dimensions. Using (11) we obtain from (8) the eigenvalue condition for the localization transition (= 0)

\[
1 + 4\alpha_1 K_0(4\sqrt{\tilde{p}/\sigma^2}) - 2\alpha_1 K_0(4\sqrt{2\tilde{p}/\sigma^2}) - \alpha_1 \exp(\tilde{p})\Gamma(0, \tilde{p}) = 0,
\]  

where \(K_0(x)\) is the modified Bessel function of the second kind, \(\tilde{p} = \rho a_0^2/D\), \(\alpha_1 = \alpha/4\pi D\), \(\sigma = a_0/z_0\). Identifying the cutoff \(a_0\) with \(z_0\) gives \(\sigma = 1\). It appears that the results are not sensitive to the choice of \(a_0\). The numerical analysis of Eq. (12) for different \(\sigma\) (= 0.8, 1, 1.2) yields the critical value \(\alpha^*_c\) for the localization transition (= 0.5825, 0.7857, 1.085). It appears that at the transition \(\tilde{p}^0_{2,c}\) has the finite value (= 0.0001, 0.00195, 0.01963), i.e. the binding transition is a first-order transition. Above the transition, \(\alpha_1 > \alpha^*_c\), there are two solutions for \(\tilde{p}_{2,c}: \tilde{p}_{2,c} < \tilde{p}^0_{2,c}\) and \(\tilde{p}_{2,c} > \tilde{p}^0_{2,c}\). According to the ground state dominance argument the larger value governs the behavior of the polymer for large \(N\). The approximative consideration based on the Taylor expansion of the eigenvalue condition (12) for small \(\tilde{p}\)

\[
1 - \alpha_1(\gamma + \ln 2 - 2\ln\sigma) + \alpha_1\tilde{p}(-1 + \gamma + 8 \ln 2/\sigma^2 + \ln\tilde{p}) + ... = 0
\]  

is in agreement with the results of numerical consideration of Eq. (12). The reason of the unusual first-order transition is due to the term \(\tilde{p}\ln\tilde{p}\) in Eq. (13). The latter is responsible for that the l.h.s. of (13) has a minimum at finite \(\tilde{p}\), which leads to the first-order transition. The physical reason of the first-order transition is due to the boundary condition at the wall \(z = 0\), which results in a reduction of number of conformations, and drives the transition to be first-order. As we stressed above the problem associated with the Hamiltonian \(H_n\) given by Eq. (11) for number of replicas \(n = 2\) and \(\beta = 0\) is closely related to the Poland-Sheraga model [13], which was recently studied in [18]- [20] by taking into account the excluded-volume interaction between denaturated loops and the rest of the chain. The excluded volume interaction reduces the number of conformations and drives the transition to the first order. This is similar to the two-replica localized state, where the reduction of the number of conformations is due to the effect of the wall potential \(V_0(z)\).

We now will consider the computation of the free energy by using the replica formula (3) under taking into account the one-replica and two-replica binding states. In the case, if only \(1p\) (one-replica) binding states exist, Eq. (3) gives straightforwardly \(-\beta F = p_c\). However, the situation is nontrivial when a \(2p\) (two-replica) binding state exists. Taking into account the two-replica states in the two-pair approximation we obtain the partition function \(Z_n\) as

\[
Z_n = Z_1^n + \frac{n(n - 1)}{2}Z_2cZ_1^{n-2} + ... = \exp(np_cN) + \frac{n(n - 1)}{2}\exp(p_{2,c}N + (n - 2)p_cN) + ...,
\]  

(14)
where we have taken into account that $Z_1$ and $Z_{2,c}$ behave for large $N$ as $\exp(p_c N)$ and $\exp(p_{2,c} N)$, respectively, when both 1p and 2p bound states exist. If no 1p bound state exist, then $Z_1 = 1$. The next terms in (14) contain contribution of ternary and higher pairs. Using (3) and (14), the free energy is obtained to be proportional to $\exp((p_{2,c} - 2p_c) N)$, hence the free energy is not extensive quantity for $p_{2,c} - 2p_c > 0$. This shows that the two-pair approximation (14) is insufficient for computing the free energy. The problem is due to the fact that the exponentially increasing term in (14) that originates from the two-particle bound state does not contain the factor $n$, as it is the case for the 1p bound state. The factor $n$ in the exponential of the latter ensures that it disappears in the limit $n \to 0$.

To overcome the difficulty we suggest to take into account in the expansion of $Z_{2n}$ the term containing the maximal number of unconnected pairs. Then, instead of (14) we obtain

$$Z_{2n} = Z_1^{2n} + 2^{-n} \frac{\Gamma(2n + 1)}{\Gamma(n + 1)} Z_{2n/2}^{2n/2} + \ldots,$$

where we consider even $n$. The factor $2^{-n} \frac{\Gamma(2n + 1)}{\Gamma(n + 1)}$ in Eq.(15) is the analytical continuation of $(2n - 1)!!$ for arbitrary $n$. The free energy is now obtained from Eq.(3) as

$$-\frac{\beta F}{N} = p_c + \frac{1}{2} p_{2,c} + \ldots$$

The localization length $\xi_{loc}$ can be computed by using $p_{2,c}$ as $\xi_{loc} \simeq (p_{2,c}/D)^{-1/2}$. The regime (ii) considered above is obtained from (16), if one puts $p_c = 0$. Notice the plus sign in front of the 2p energy in (16). The computation of the 2p energy in the two-pair approximation (14) gives in the limit $n \to 0$ instead of (16) the minus sign in front of the second term. The inconsistency of this result is clearly demonstrated by the non extensiveness of the free energy computed by using Eq.(8). The procedure of taking into account the terms with maximal number of pairs can be justified by the following ground state dominance like argument: the term, $\exp(np_{2,c} N)$, dominates over $\exp(p_{2,c} N)$ for large $N$ and $n \geq 1$. The condition $n \geq 1$ is demanded in the procedure of introduction of the replica trick by considering the partition function $Z_n$ with $n$ being positive integer. For the peculiarities of the limits $N \to \infty$ and $n \to 0$ in the replica treatment of the directed polymer in disordered media see [23]. The extensiveness of the free energy is a posteriori justification of the above heuristic procedure.

We now will use the results of the study of 1p and 2p bound states of the replica Hamiltonian (4) to construct the phase diagram of the localization of the heteropolymer in variables $\zeta$ and $\alpha_1 = \Delta u^2 / 4\pi D$, which is shown schematically in Fig.2. We know the behavior at $\alpha_1^c$ and $\alpha_2^c$ from the study of the cases (ii) and (i). The latter corresponds to the localization transition of the annealed Hamiltonian (1p bound state) in the symmetric case ($\zeta = 0$). The dotted straight line is the localization line of the annealed Hamiltonian; $\zeta_c = -D/z_0$ is the value of the asymmetry parameter in the limit $\alpha_1 \to 0$. Since the random heteropolymer can arrange at the surface in such a way that pieces of the polymer which are attracted to the surface are in contact with the latter, while the pieces which are repelled are in the loops and tails (see [3] for a related discussion), the random quenched heteropolymer is expected to localize easier than the homopolymer or even easier than the annealed heteropolymer. Due to this the quenched part of the Hamiltonian (4), which is responsible for 2p bound states, shifts the critical temperature to higher values, so that the localization line will cut
the horizontal axis on the left from the point \( \alpha_1^a \). The continuous line cannot end in the dotted line, while the analysis carried out in \((i)\) in the vicinity of \( \alpha_1^a \) applies also along the dotted line, so that the 2p localized state exists there. Thus, the continuous line will be on the left of the dotted line, and it will end in \( \zeta_c \). In order to make our prediction that the localization transition at \( \alpha_1^c \) is first-order compatible with the prediction in [14] that the transition at \( \zeta = 0 \) is of Kosterlitz-Thouless type, one should assume that on the phase boundary in Fig. 2 there is a tricritical point separating the KT point at \( \zeta = 0 \) with the first-order transition at \( \alpha_1^c \).

In summary, we have considered adsorption of a random heteropolymer onto a homogeneous surface via replica trick by using the Green’s function technique. We use the exact treatment of one- and two-replica binding states of the replica Hamiltonian to compute the free energy of the random heteropolymer. We have considered analytically two particular cases: \((i)\) almost statistically symmetric polymer in the vicinity of the threshold of the annealed problem, and \((ii)\) the asymmetric polymer where the interaction part of the annealed Hamiltonian is nearly zero. In the former case we have obtained that the localization of the polymer is due to the 2p binding state, which exists both above and below of the 1p binding state, which corresponds to the localization transition of the annealed problem. We have obtained that in the case \((ii)\) the energy of the 2p binding state at the transition is finite, i.e. the localization is a first-order transition. A schematic phase diagram of the localization-delocalization transition of the random heteropolymer is suggested.

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Figure captions

Fig.1 The perturbation expansion of the connected part of the two-replica Green’s function.

Fig.2 The phase diagram of the localization-delocalization transition.