Non supersymmetric femion boson symmetry

Vaibhav Wasnik

Indian Institute of Technology, Goa

In this work we present a symmetry relating bosons to fermions which cannot be represented as a supersymmetric algebra. We present an action made up of a complex scalar and a fermion in four dimensions that respects the symmetry quantum mechanically. We next invoke gauge symmetry by adding a gauge field and a corresponding fermion and show that the constructed action obey’s the fermion-boson symmetry. Just like supersymmetry, this symmetry allows the quadratic divergence of the Higgs mass to smooth to a logarithmic one in certain cases, however other phenomenological as well as theoretical possibilities could be different.

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Introduction

Since its discovery supersymmetry [1] has been an important area of research. Supersymmetry among other things has been touted as a solution to the Higgs hierarchy problem [2]. Because the Higgs in a supersymmetric theory comes with its superpartners in the supersymmetry representation, its divergence goes from being quadratic to being logarithmic [3]. The Coleman Mandula theorem [5] restricted the possible symmetries of the S-matrix. Because the generators of the allowed symmetries followed commutation relations, supersymmetry involving anti-commutators was an extension to the symmetries allowed by the Coleman Mandula theorem. If $Q_\alpha$’s ($\alpha = 1, 2$ in 4d) are the generators of this symmetry they obey the anticommutation relationships.

$$\{Q_\alpha, Q_\beta^\dagger\} = 2i\sigma^{\mu}_{\alpha,\beta}\partial_\mu$$
$$\{Q_\alpha, Q_\beta\} = 0$$ (1)

If we start with a bosonic vaccum $|0\rangle$ corresponding to a complex scalar field $\phi$ in four dimensions, which is made up of two real components. It is paired with a chiral spinor $\psi_{\alpha}$ corresponding to $Q_\alpha^\dagger|0\rangle$, where each $\psi_{\alpha}$ is complex. Hence, to balance things one has to consider a complex scalar $F$ corresponding to $\epsilon_{\beta,\alpha}Q_\beta^\dagger Q_\alpha^\dagger|0\rangle$. The supersymmetry is realized in this simple model as

$$\delta\phi = \epsilon\psi$$
$$\delta\psi = i\sigma^{\mu}\bar{\epsilon}\partial_\mu\phi + \epsilon F$$
$$\delta F = i\epsilon\bar{\sigma}\partial_\mu\psi$$ (2)

The conventions regarding what bars on $\epsilon, \bar{\sigma}^\mu$ mean will be explained below.

A simple question to ask is whether other symmetries that take bosons in to fermions and vice versa are allowed, but which do not have the form as above. The simplest such symmetry we can think of is the one where $F$ is absent.

$$\delta\phi = \epsilon\psi$$
$$\delta\psi = i\sigma^{\mu}\bar{\epsilon}\partial_\mu\phi$$ (3)

One can easily see that this symmetry cannot be simplisticly represented by a fermionic generator of the form $Q_\alpha$, unless there is some extra constraint to disallow states of the form $\epsilon_{\beta,\alpha}Q_\beta^\dagger Q_\alpha^\dagger|0\rangle$. This eq(3) is known to be a symmetry of the action [4]

$$S = \int d^4x[\partial_\mu\phi^\dagger\partial^\mu\phi - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi]$$ (4)

In this work we show that the above is also a symmetry of the extended action below quantum mechanically as long as certain constraints are obeyed.

$$S = \int d^4x[\partial_\mu\phi^\dagger\partial^\mu\phi - i\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + W_1(\phi, \phi^*) + W_2(\phi, \phi^*)\psi\psi + h.c.]$$ (5)
The supersymmetry transformation eq. (2) would have been a symmetry of the above if $W_2$ was functions of only $\phi$ and not $\phi^\ast$ along with a certain relationship between $W_1$ and $W_2$. However, we will see that in order for eq. (3) to be a symmetry we can have that $W_2$ is a function of both $\phi$ and $\phi^\ast$, along with certain relationship between $W_1$ and $W_2$. We later add gauge fields and the corresponding fermions, which are related by a symmetry like eq. (3) to the above Lagrangian and show that the theory possesses a non-supersymmetric boson-fermion symmetry.

Conventions

This section is taken from [6]. $SO(1, 3)$ is locally isomorphic to $SL(2, C)$ group which is made up of $2 \times 2$ matrices with complex numbers as elements, whose determinant is 1. To see this consider a co-ordinate in spacetime $(t, x, y, z)$. $SO(1, 3)$ acts to keep the interval $t^2 - x^2 - y^2 - z^2$ invariant. Now consider the following matrix.

$$ X = \begin{bmatrix} t + z & x - iy \\ x + iy & t - z \end{bmatrix} = t + It + \sigma_x x + \sigma_y y + \sigma_z z \tag{6} $$

We have that $Det(X) = t^2 - x^2 - y^2 - z^2$. $Det(X)$ is invariant under $X \to ZXZ^\dagger$, if $Det(Z) = 1$, i.e. $Z$ being any matrix in $SL(2, C)$.

If we write the elements of such a matrix as $Z^b_a$, then we can show that

$$ \epsilon_{ab} Z^c_c Z^d_d = \epsilon_{cd} \tag{7} $$

Hence $\epsilon_{ab}$ could play the role of a metric to lower indices. If one representation of this $SL(2, C)$ is $\psi_\alpha$ for $\alpha = 1, 2$, another representation is $\psi'_\alpha$. We write this as $\psi_\alpha$, to distinguish indices. Given the sigma matrices $\sigma^\mu$, if we consider a vector $p_\mu$, we can see that $Det(p_\mu \sigma^\mu) = \bar{p} \cdot p$. If we write $p'^\mu \sigma_\mu = Z^\mu \sigma^\mu Z^\dagger_\mu$, then we see that $\bar{p} \cdot \bar{p} = p' \cdot p'$. This implies that $\sigma^\mu$ should be written as $\sigma^\mu_{\alpha, \bar{\alpha}}$. If we consider $\eta_{\mu\nu} = diag(1, -1, -1, -1)$. Then we have that

$$ \sigma^\mu \sigma^\nu + \bar{\sigma}^\nu \sigma^\mu = 2\eta_{\mu\nu} \tag{8} $$

where $\bar{\sigma}^\mu = [1, -\sigma^k]$. This would imply that we can write $(\sigma^\mu)^{\bar{\alpha}, \alpha}$

Requirements of symmetry

If an action $S = \int d^4x L(\phi, \psi)$ is invariant under a symmetry transformation

$$ \phi' = \phi + \epsilon \psi $$
$$ \psi' = \psi + i\sigma^\mu \bar{\epsilon} \partial_\mu \phi \tag{9} $$

Then if $\epsilon$ is made a function of $x$, we should have that the action would change by $S \to S + \int d^4x \partial_\mu \epsilon(x) J^\mu(x)$. So

$$ \int D\psi D\phi e^{\frac{\phi}{\epsilon}} \int d^4x L(\phi, \psi) = \int D\phi' D\phi e^{\frac{\phi'}{\epsilon'}} \int d^4x L(\phi', \psi') = \int D\psi D\phi e^{\frac{\phi}{\epsilon}} \int d^4x L(\phi, \psi) + \frac{\epsilon}{\epsilon'} \int d^4x \partial_\mu J^\mu(x) \tag{10} $$

where first equality above is a variables change and in second equality we use the fact that measure is invariant. Expanding to order $\epsilon$ we get

$$ \int D\psi D\phi e^{\frac{\phi}{\epsilon}} \int d^4x L(\phi, \psi) \partial_\mu J^\mu(x) = 0 \tag{11} $$

So a symmetry comes with a conserved current.

We claim that even if an action under a transformation

$$ \phi' = \phi + \epsilon \psi $$
$$ \psi' = \psi + i\sigma^\mu \bar{\epsilon} \partial_\mu \phi \tag{12} $$

changes so that \( S \rightarrow S + \int d^4x \epsilon f(\phi, \phi^*) \frac{dL}{d\psi} \), we would still get a conserved current, where \( f(\phi, \phi^*) \) is a function. To see this, again make \( \epsilon \) a function of \( x \). Then we should have that the action would change by \( S \rightarrow S + \int d^4x \epsilon f(\phi, \phi^*) \frac{dL}{d\psi} + \int d^4x \partial_\mu \epsilon(x) J^\mu(x) \). Then we get

\[
\int D\psi D\phi e^{-\frac{i}{\hbar} \int d^4x L(\phi, \psi)} = \int D\psi' D\phi' e^{-\frac{i}{\hbar} \int d^4x L(\phi', \psi')} = \int D\psi D\phi e^{-\frac{i}{\hbar} \int d^4x L(\phi, \psi)} [1 - \int d^4x \frac{i}{\hbar} \epsilon f(\phi, \phi^*) \frac{dL}{d\psi} + \frac{i}{\hbar} \int d^4x \epsilon(x) \partial_\mu J^\mu(x)]
\]

or

\[
\int D\psi D\phi e^{-\frac{i}{\hbar} \int d^4x L(\phi, \psi)} \int d^4x \epsilon f(\phi, \phi^*) \frac{dL}{d\psi} = \int D\psi D\phi e^{-\frac{i}{\hbar} \int d^4x L(\phi, \psi)} [\int d^4x \epsilon(x) \partial_\mu J^\mu(x)]
\]

Now, for any function \( f(x, y) \)

\[
\int \Pi_y d\phi(y) d\psi(y) e^{-\frac{i}{\hbar} \int d^4x f(\phi(z), \phi^*(z)) \frac{dL}{d\psi(z)}} = -\frac{\hbar}{i} \int \Pi_y \frac{d\phi(y) d\psi(y)}{d\psi(z)} x d\phi(z) \int dRe(\psi(z)) dIm(\psi(z)) \left[ \frac{d}{dRe(\psi(z))} - i \frac{d}{dIm(\psi(z))} \right] (e^{-\frac{i}{\hbar} \int d^4x f(\phi(z), \phi^*(z))}) = 0
\]

The reason being that the integral over time in the action \( S = \int d^4x L \) is over \( t(1 - i\epsilon) \) and we are talking of path integrals of total derivatives.  

Hence we should have a conserved current

\[
\int D\psi D\phi e^{-\frac{i}{\hbar} \int d^4x L(\phi, \psi)} \partial_\mu J^\mu(x) = 0
\]

implying a symmetry quantum mechanically. We can similarly show that the action change by \( S \rightarrow S + \int d^4x \epsilon f(\phi, \phi^*) \frac{dL}{d\psi} \), implies a symmetry quantum mechanically. We will use this exact feature for actions that change as \( S \rightarrow S + \int d^4x \epsilon f(\phi, \phi^*) \frac{dL}{d\psi} + \int d^4x \epsilon \psi \frac{dL}{d\psi} \), instead of simply closing off shell, as implying a quantum symmetry to derive constraints on \( W_1 \) and \( W_2 \) in the work below.

**Complex scalar and fermion**

Consider the Lagrangian

\[
L = -i \bar{\psi} \gamma^\mu \partial_\mu \psi + i \bar{\psi} \gamma^\mu \gamma^5 \psi + \partial_\mu \phi^* \partial_\mu \phi + W_1(\phi, \phi^*) + W_2(\phi, \phi^*) \psi \psi + h.c.
\]

we have

\[
\frac{dL}{d\phi} = \frac{dW_1}{d\phi} + \frac{dW_2}{d\phi} \bar{\psi} \psi
\]

\[
\frac{dL}{d\phi^*} = \frac{dW_1}{d\phi^*} + \frac{dW_2}{d\phi^*} \bar{\psi} \psi
\]

\[
\frac{dL}{d\psi} = i \epsilon \partial_\mu \bar{\psi} \gamma^\mu + W_2(\phi, \phi^*) \epsilon \psi
\]

\[
\frac{dL}{d\psi} = -i \epsilon \gamma^\mu \partial_\mu \psi + W_2(\phi, \phi^*) \bar{\psi}
\]
Let $W_2 = \frac{d^2 W(\phi, \phi^*)}{d\phi^*}$, then under e.q. 3 the action change

$$
\delta S = \int d^4 x \left[ \frac{dW_1}{d\phi} \bar{\psi} \phi + \frac{dW_1}{d\phi^*} \bar{\psi} \phi + iW_2(\phi, \phi^*) \psi \sigma^\mu \partial_\mu \phi + \frac{dW_2}{d\phi^*} \bar{\psi} \phi + h.c. \right]
$$

$$
\delta S = \int d^4 x \left[ \frac{dW_1}{d\phi} \bar{\psi} \phi + \frac{dW_1}{d\phi^*} \bar{\psi} \phi + i\psi \sigma^\mu \partial_\mu \phi + \frac{dW_2}{d\phi^*} \bar{\psi} \phi + h.c. \right]
$$

$$
\delta S = \int d^4 x \left[ \frac{dW_1}{d\phi} \bar{\psi} \phi + \frac{dW_1}{d\phi^*} \bar{\psi} \phi - i \frac{dW_2}{d\phi^*} \bar{\psi} \phi \times \partial_\mu \psi \sigma^\mu \bar{\psi} + \frac{dW_2}{d\phi^*} \bar{\psi} \phi + h.c. \right]
$$

So

$$
\delta S = \int d^4 x \left[ \frac{dW_1}{d\phi} \bar{\psi} \phi + \frac{dW_1}{d\phi^*} \bar{\psi} \phi - \frac{dW_2}{d\phi^*} \bar{\psi} \phi \times W_2(\phi, \phi^*) \bar{\psi} + h.c. \right]
$$

So if

$$
\frac{dW_1}{d\phi} = - \frac{dW(\phi, \phi^*)}{d\phi^*} \times W_2(\phi, \phi^*)
$$

We see the eq 3 is a symmetry. If $W$ is holomorphic however we have that for any constant $c$

$$
\delta S + \int d^4 x \frac{dL}{d\phi} \frac{dW(\phi, \phi^*)}{d\phi} + c \int d^4 x \frac{dL}{d\phi^*} + h.c.
$$

So if

$$
\frac{dW_1}{d\phi} = - \frac{dW(\phi, \phi^*)}{d\phi^*} \times W_2(\phi, \phi^*)
$$

as long as

$$
\frac{d}{d\phi}[W_1 + (1 + c^*)W_1^*] = - \frac{dW(\phi)}{d\phi^*} \times W_2(\phi)
$$

which implies

$$
\frac{dW_1}{d\phi} = \frac{dW(\phi)}{d\phi^*} \times \frac{dW(\phi)}{d\phi}
$$

Hence we have that eq 3 is a symmetry of eq 17 if $W$ is holomorphic and the above equation is obeyed

**Gauge theory I**

We have that the following susy transformation

$$
\delta A_\mu^a = - \frac{1}{\sqrt{2}} \bar{\psi} \sigma_\mu \lambda^a + h.c.
$$

$$
\delta \lambda^a = \frac{1}{\sqrt{2}} \epsilon D^a + \frac{i}{2\sqrt{2}} \sigma_\mu \sigma^\nu \epsilon F_{\mu\nu}
$$

$$
\delta D^a = \frac{i}{\sqrt{2}} (\bar{\psi} \sigma_\mu \nabla^\mu \epsilon - \nabla^\mu \lambda^a \sigma_\mu \epsilon)
$$

$$
\delta \phi = \epsilon \psi
$$

$$
\delta \psi = i(\sigma_\mu \epsilon) \nabla^\mu \phi + c \epsilon F
$$

$$
\delta F = i \bar{\psi} \sigma_\mu \nabla^\mu \psi + \sqrt{2} (T^a \phi) \bar{\psi} \lambda^a
$$
is a symmetry \( \delta S \) of

\[
S = \int d^4x \left[ -\frac{1}{4g^2} F_{a\mu\nu}^a F_{a\mu\nu} - \frac{i}{g^2} \lambda^a \sigma^\mu \nabla_\mu \bar{\lambda}^a + \frac{1}{2g^2} D^a D^a \right.
+ \nabla_\mu \phi \nabla^\mu \phi^* - i\bar{\psi} \sigma^\mu \nabla_\mu \psi - D^a \phi^* T^a \phi - i\sqrt{2} \phi^* T^a \lambda^a \psi + FF^* + h.c. \]
\]

Hence

\[
S = \int d^4x \left[ -\frac{1}{4g^2} F_{a\mu\nu}^a F_{a\mu\nu} - \frac{i}{g^2} \lambda^a \sigma^\mu \nabla_\mu \bar{\lambda}^a + \nabla_\mu \phi \nabla^\mu \phi^* - i\bar{\psi} \sigma^\mu \nabla_\mu \psi - i\sqrt{2} \phi^* T^a \lambda^a \psi + h.c. \right. 
\]

changes as

\[
\delta S = - \int d^4x \left[ \frac{1}{g^2} \delta D^a D^a + \frac{i}{\sqrt{2}} (\bar{\epsilon} \sigma_\mu \nabla^\mu \lambda^a - \nabla^\mu \bar{\lambda}^a \sigma_\mu \epsilon) \phi T^a \phi + \delta FF^* + F \delta F^* + h.c. \right] 
\]

This implies that under

\[
\delta A^a_\mu = - \frac{1}{\sqrt{2}} \bar{\epsilon} \sigma_\mu \lambda^a + h.c. \\
\delta \lambda^a = + \frac{i}{2\sqrt{2}} \sigma^\mu \sigma^\nu \epsilon F_{\mu\nu} \\
\delta \phi = \epsilon \psi \\
\delta \psi = i(\sigma_\mu \bar{\epsilon}) \nabla^\mu \phi
\]

with \( W_2 = \frac{\partial L}{\partial \phi \psi} \)

\[
S = \int d^4x L = \int d^4x \left[ -\frac{1}{4g^2} F_{a\mu\nu}^a F_{a\mu\nu} - \frac{i}{g^2} \lambda^a \sigma^\mu \nabla_\mu \lambda^a 
+ \nabla_\mu \phi \nabla^\mu \phi^* - i\bar{\psi} \sigma^\mu \nabla_\mu \psi - i\sqrt{2} \phi^* T^a \lambda^a \psi + W_1 + W_2 \psi \psi + h.c. \right]
\]

changes as

\[
\delta S = \int d^4x \left[ -\frac{i}{\sqrt{2}} (\bar{\epsilon} \sigma_\mu \nabla^\mu \lambda^a - \nabla^\mu \bar{\lambda}^a \sigma_\mu \epsilon) \phi T^a \phi + \frac{dW_1}{d\phi} \epsilon \psi + \frac{dW_1}{d\phi^*} \bar{\epsilon} \psi + \frac{dW_2}{d\phi} \bar{\epsilon} \psi \bar{\psi} \psi + i\frac{dW_2}{d\phi^*} \epsilon \psi \right. \\
\delta S = \int d^4x \left[ -\frac{i}{\sqrt{2}} (\bar{\epsilon} \sigma_\mu \nabla^\mu \lambda^a - \nabla^\mu \bar{\lambda}^a \sigma_\mu \epsilon) \phi T^a \phi + \frac{dW_1}{d\phi} \epsilon \psi + \frac{dW_1}{d\phi^*} \bar{\epsilon} \psi + \frac{dW_2}{d\phi} \bar{\epsilon} \psi \bar{\psi} \psi + i\frac{dW_2}{d\phi^*} \epsilon \psi \times \bar{\epsilon} \sigma_\mu \partial^\mu \psi - W_2 \bar{\epsilon} \sigma_\mu \psi T^a A^\mu \phi + h.c. \right] \\
\delta S = \int d^4x \left[ -\frac{i}{\sqrt{2}} (\bar{\epsilon} \sigma_\mu \nabla^\mu \lambda^a - \nabla^\mu \bar{\lambda}^a \sigma_\mu \epsilon) \phi T^a \phi + \frac{dW_1}{d\phi} \epsilon \psi + \frac{dW_1}{d\phi^*} \bar{\epsilon} \psi + \frac{dW_2}{d\phi} \bar{\epsilon} \psi \bar{\psi} \psi + i\frac{dW_2}{d\phi^*} \epsilon \psi \times \bar{\epsilon} \sigma_\mu \partial^\mu \psi - W_2 \bar{\epsilon} \sigma_\mu \psi T^a A^\mu \phi + h.c. \right]
\]

Now

\[
\frac{dL}{d\lambda^a} = \frac{i}{g^2} \nabla_\mu \bar{\lambda}^a \sigma^\mu \epsilon \\
\frac{d\bar{\epsilon}}{d\lambda^a} = - \frac{i}{g^2} \bar{\epsilon} \sigma^\mu \nabla_\mu \lambda^a \\
\frac{d\bar{\epsilon} L}{d\psi} = -i\bar{\epsilon} \sigma^\mu \nabla_\mu \psi + W_2 (\phi, \phi^*)^* \bar{\psi} = -i\bar{\epsilon} \sigma^\mu \partial_\mu \psi - \bar{\epsilon} \sigma^\mu T^a A^\mu \psi + W_2 (\phi, \phi^*)^* \bar{\psi} \\
\frac{dW}{d\phi} \times \frac{dL}{d\psi} = -i\frac{dW}{d\phi} \times \bar{\epsilon} \sigma^\mu \partial_\mu \psi - \frac{dW}{d\phi} \times \bar{\epsilon} \sigma^\mu T^a A^\mu \psi + \frac{dW}{d\phi} \times W_2 (\phi, \phi^*)^* \bar{\psi}
\]

(33)
Hence under eq [30] if the action changes by $\delta S$ then

$$[-\delta S + \frac{1}{\sqrt{2}} \frac{dL}{d\lambda^a} \dot{\phi} T^a \phi + \frac{1}{\sqrt{2}} \frac{dL}{d\phi} \dot{\phi} T^a \phi - \frac{dW}{d\phi} \times \epsilon \frac{dL}{d\phi} + \epsilon \bar{\psi} \frac{dL}{d\phi^*}]
= \left[ + \frac{dW_1}{d\phi} \times \epsilon \bar{\sigma}^a T^a \phi A^a \psi \right] - \frac{dW_1}{d\phi} \epsilon \psi - \frac{dW}{d\phi} \epsilon \bar{\psi} \times W_2(\phi, \phi^*) \epsilon \bar{\psi} + h.c.]$$

(34)

as $W$ is gauge invariant, implies $\frac{dW}{d\phi^*} T^a \phi = 0$. So the eq[30] is a symmetry as long as

$$\frac{dW_1}{d\phi} = - \frac{dW^*}{d\phi^*} \times \frac{d^2W(\phi, \phi^*)}{d\phi^2}$$

(35)

Just as the previous section, we also have that eq[25] and eq[26] are a symmetry if $W$ is holomorphic and the equation below is obeyed

$$\frac{d}{d\phi} [W_1 + (1 + e^*) W_1] = - \frac{dW(\phi)^*}{d\phi^*} \times W_2(\phi)$$

(36)

**Gauge theory II**

Since eq[25] and eq[26] are a symmetry of

$$S = \int d^4x \left[ - \frac{1}{4g^2} F_{a\mu
u}^\mu F_{a\mu
u} - \frac{i}{g^2} \lambda^a \sigma^\mu \nabla_\mu \bar{\lambda}^a + \frac{1}{2g^2} D^a \bar{D}^a
+ \nabla_\mu \phi \nabla^\mu \bar{\phi}^* - i \bar{\psi} \sigma^\mu \nabla_\mu \psi - D^a \dot{\phi} T^a \phi - i \sqrt{2} \dot{\phi} T^a \phi + FF^* + h.c.]$$

(37)

It implies that

$$S = \int d^4x \left[ - \frac{1}{4g^2} F_{a\mu
u}^\mu F_{a\mu
u} - \frac{i}{g^2} \lambda^a \sigma^\mu \nabla_\mu \bar{\lambda}^a + \frac{1}{2g^2} D^a \bar{D}^a
+ \nabla_\mu \phi \nabla^\mu \bar{\phi}^* - i \bar{\psi} \sigma^\mu \nabla_\mu \psi - D^a \dot{\phi} T^a \phi - i \sqrt{2} \dot{\phi} T^a \phi + FF^* + h.c.]$$

(38)

changes under eq[25] and eq[26] as

$$\delta S = - \int d^4x [\delta FF^* + F \delta F^* + h.c.]$$

(39)

This implies that under

$$\delta A^a_\mu = - \frac{1}{\sqrt{2}} \epsilon \bar{\sigma}^a \lambda^a + h.c.$$  
$$\delta \lambda^a = \frac{1}{\sqrt{2}} \epsilon \bar{D}^a + \frac{i}{\sqrt{2}} \bar{D}^a \sigma^\mu \bar{\sigma}^\nu \bar{\epsilon} F_{\mu\nu}$$  
$$\delta D^a = \frac{i}{\sqrt{2}} (\bar{\epsilon} \bar{\sigma}^a \nabla^\mu \lambda^a - \nabla^\mu \bar{\lambda}^a \bar{\sigma}^a \bar{\epsilon})$$  
$$\delta \phi = \epsilon \psi$$  
$$\delta \psi = i (\sigma^a \epsilon) \nabla^\mu \phi$$

(40)

( with $W_2 = \frac{d^2W}{d\phi^2}$, $W$ will have to be a holomorphic function of $\phi$ in order to work towards a symmetric cancelation.)

$$S = \int d^4x \left[ - \frac{1}{4g^2} F_{a\mu
u}^\mu F_{a\mu
u} - \frac{i}{g^2} \lambda^a \sigma^\mu \nabla_\mu \bar{\lambda}^a + \frac{1}{2g^2} D^a \bar{D}^a
+ \nabla_\mu \phi \nabla^\mu \bar{\phi}^* - i \bar{\psi} \sigma^\mu \nabla_\mu \psi - D^a \dot{\phi} T^a \phi - i \sqrt{2} \dot{\phi} T^a \phi + W_1 + W_2 \psi \psi + h.c.]$$

(41)
changes as

\[ \delta S = \int d^4x \left[ dW_1 \epsilon \psi + \frac{dW_1}{d\phi^*} \bar{\epsilon} \overline{\psi} + iW_2(\psi \sigma_\mu \bar{\epsilon} \nabla^\mu \phi + h.c.) \right] \]

\[ \delta S = \int d^4x \left[ dW_1 \epsilon \psi + \frac{dW_1}{d\phi^*} \bar{\epsilon} \overline{\psi} - i \frac{dW}{d\phi} \times \partial^\mu \psi \sigma_\mu \bar{\epsilon} + W_2 \psi \sigma_\mu \bar{\epsilon} T^\alpha A^\alpha \mu \phi + h.c. \right] \]

\[ \delta S = \int d^4x \left[ dW_1 \epsilon \psi + \frac{dW_1}{d\phi^*} \bar{\epsilon} \overline{\psi} + i \frac{dW}{d\phi} \times \bar{\epsilon} \sigma_\mu \partial^\mu \psi - W_2 \epsilon \overline{\sigma}_\mu T^\alpha A^\alpha \mu \phi + h.c. \right] \]  

(42)

Now

\[ \bar{\epsilon} \frac{dL}{d\psi} = -i \bar{\epsilon} \sigma^\mu \nabla_\mu \psi + W_2(\phi)^* \bar{\epsilon} \overline{\psi} = -i \bar{\epsilon} \sigma^\mu \partial_\mu \psi - \bar{\epsilon} \overline{\sigma}^\mu T^\alpha A^\alpha \mu \psi + W_2(\phi, \phi^*)^* \bar{\epsilon} \overline{\psi} \]

\[ \frac{dW}{d\phi} \times \epsilon \frac{dL}{d\psi} = -i \frac{dW}{d\phi} \times \bar{\epsilon} \sigma^\mu \partial_\mu \psi - \frac{dW}{d\phi} \times \bar{\epsilon} \overline{\sigma}^\mu T^\alpha A^\alpha \mu \psi + \frac{dW}{d\phi} \times W_2(\phi)^* \bar{\epsilon} \overline{\psi} \]  

(43)

If \( \delta S \) is change of action under eq. (40) then

\[ = [- \delta S - \frac{dW}{d\phi} \times \epsilon \frac{dL}{d\psi}] \]

\[ = \frac{d}{d\phi} \left[ \frac{dW}{d\phi} \times \bar{\epsilon} \sigma^\mu T^\alpha A^\alpha \mu \psi \right] - \frac{dW_1}{d\phi} \epsilon \psi - \frac{dW_1}{d\phi^*} \bar{\epsilon} \overline{\psi} - \frac{dW}{d\phi} \times W_2(\phi, \phi^*)^* \bar{\epsilon} \overline{\psi} + h.c. \]  

(44)

as \( W \) is gauge invariant, hence as before implies \( \frac{dW}{d\phi} T^\alpha \phi = 0 \). So eq. (43) is a symmetry as long as

\[ \frac{d}{d\phi} \left[ \frac{d(W_1 + W_1^*)}{d\phi} \right] = \frac{-dW^*}{d\phi^*} \times \frac{d^2W(\phi)}{d\phi^2} \]  

(45)

or

\[ (W_1 + W_1^*) = \frac{-dW^*}{d\phi^*} \times \frac{dW(\phi)}{d\phi} \]  

(46)

**Comparison with Supersymmetry**

If \( W(\phi) \) is holomorphic we had shown for the case of scalar and complex field as well as gauge theory I that

\[ W_2(\phi, \phi^*) = \frac{d^2W(\phi)}{d\phi^2} \]

\[ W_1 + (1 + c^*)W_1^* = \frac{-dW^*}{d\phi^*} \times \left[ \frac{dW(\phi)}{d\phi} \right] \]  

(47)

If \( c = -1 \) we get

\[ W_2(\phi, \phi^*) = \frac{d^2W(\phi)}{d\phi^2} \]

\[ W_1 + W_1^* = \frac{-2dW^*}{d\phi^*} \times \left[ \frac{dW(\phi)}{d\phi} \right] \]  

(48)
and in a supersymmetric theory we instead have that

\[ W_2(\phi) = -\frac{1}{2} \frac{d^2 W(\phi)}{d\phi^2} \]

\[ W_1(\phi) + W_1(\phi)^* = \frac{dW(\phi)}{d\phi} \frac{dW(\phi^*)}{d\phi^*} \]

(49)

We hence see that in these theories the quadratic divergence of Higgs is smoothed to a logarithmic divergence when

\[ W(\phi, \phi^*) = W(\phi) \text{ and } c = -1. \]

We however note that no choice of multiplying \( W \) by a constant of any kind in eq\(48 \) could make eq\(48 \) to match eq\(49 \). This would imply that the action given by eq\(17 \) with the constraint like eq\(48 \) would not be symmetric under supersymmetry \[7\].

In the previous section the potential of the theory is

\[ \frac{g^2}{2} (\phi^* T^a \phi)^2 - \frac{dW(\phi)}{d\phi} \frac{dW(\phi^*)}{d\phi^*} \]

This is in contrast to the potential for a supersymmetric theory which instead goes as

\[ \frac{g^2}{2} (\phi^* T^a \phi)^2 + \frac{dW(\phi)}{d\phi} \frac{dW(\phi^*)}{d\phi^*} \]

(51)

This would hence imply an opportunity for many possibilities to phenomenology as well as theory, given the different ways in which gauge symmetry could be broken.

The generator of symmetry eq\(40 \) is fermionic \( Q \). Quantum mechanically we have that \( \delta \lambda = i\{Q, \lambda\} \). Hence in order that vacuum conserves the symmetry we need that \( \delta \lambda = 0 \). This translates in to \( D^a \sim \phi^* T^a \phi = 0 \). This is less constraining than in a theory of supersymmetry unbreaking that also needs \( \frac{dW(\phi)}{d\phi} = 0 \). We hence see that the new symmetry in this paper could give rise to new theoretical and phenomenological directions that may not be possible in supersymmetric theories.

* Electronic address: wasnik@iitgoa.ac.in
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[8] See for example pg 34 in Polchinski, J. (1998). String theory: Volume 1, an introduction to the bosonic string. Cambridge university press, where the fact that path integral of total derivatives is zero is utilized to construct identities in 2d CFTs.