A Fundamental Instability for the Solar Wind

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Abstract

As has been known nearly since the beginning of space research with satellites and rockets that the temperature of the atmosphere of our Sun rises rapidly from the photosphere at about 6000 K to the order of $10^6$ K. The major heating of the solar wind apparently occurs in a narrow region, the transition region, just above the chromosphere, a region where remote sensing of atomic energy levels shows a temperature of $10^6$ deg. However, since the early days of the recognition of the solar wind it has been recognized that there must also be further heating as the solar wind escapes the Sun, to overcome adiabatic cooling, and it is this heating that is the subject of the Parker Solar Probe mission, and of this work. As is well known, the solar wind is turbulent, which suggests that plasma instabilities play an important role in its behavior. The role of instabilities in shaping the solar wind was clearly shown by Kasper et al. and Hellinger et al. As shown in Figure 4 of Kasper or Figure 1 of Hellinger, the distribution function of the ions is limited by well-known instabilities. It seems that there ought to be an instability that is common and depends on omnipresent plasma characteristics. In this work it is assumed that such may be provided by the expansion of the solar wind magnetic field as it leaves the Sun.

1. Introduction

As has been known nearly since the beginning of space research with satellites and rockets that the temperature of the atmosphere of our Sun rises rapidly from the photosphere at about 6000 K to the order of $10^6$ K, resulting in the outflow referred to as the solar wind. The major heating of the solar wind apparently occurs in a narrow region, the transition region, just above the chromosphere, a region where remote sensing of atomic energy levels shows a temperature of $10^6$ deg. It seems that no in situ sensing will ever be possible in that region. However, since the early days of the recognition of the solar wind (e.g., Hartle & Sturrock 1968) it has been recognized that there must also be further heating as the solar wind escapes the Sun, to overcome adiabatic cooling, and it is this heating that is the subject of the Parker Solar Probe mission, and of this work.

As is well known, the solar wind is turbulent, which suggests that plasma instabilities play an important role in its behavior. In fact, Klein et al. (2019) have shown the observed distribution functions of solar wind particles are commonly unstable but the nature of the instability was not provided. The Klein work is based on observations of temperature anisotropies and other measured properties of the solar wind. These do not include the distribution considered in the present work. The role of instabilities in shaping the solar wind was clearly shown by Kasper et al. (2006) and Hellinger et al. (2006). As shown in Figure 4 of Kasper et al. or Figure 1 of Hellinger et al., the distribution function of the ions is limited by well-known instabilities.

It seems that there ought to be an instability that is common and depends on omnipresent plasma characteristics. In this work it is assumed that such may be provided by the expansion of the solar wind magnetic field as it leaves the Sun.

2. This Work

2.1. Calculation of Growth and Growth Rate

As the solar wind escapes the Sun the density is decreased, as is the magnetic field. This must cause some change in the distribution function of the particles. In this work, the changes in the distribution function will be investigated under the assumption that the first plasma invariant, $|B|/W_{\perp}$, and the kinetic energy are conserved, where $|B|$ is the magnitude of the magnetic field and $W_{\perp}$ is the perpendicular (to $B$) kinetic energy of any particle. Although it is known that these are not conserved on a grand scale, one may think that it may be conserved on a shorter scale (Marsch & Goldstein 1983; Marsch 2012; Matteini et al. 2012). Matteini et al. (2012) especially considered the conservation of the first invariant on the sort of shorter timescale that might be common in the turbulent solar wind.

Some support for this view is provided by the anisotropy. The observed anisotropy of the solar wind is generally that the parallel temperature is larger than the perpendicular temperature, (Kasper et al. 2006; Hellinger et al. 2006), which is in accordance with what would be expected from reduction of the perpendicular energy with the change being fed into the parallel energy by conservation of energy.

As the system expands, the magnetic field and particle density are proportional to $1/R^2$. Conservation of total energy and the first invariant imply a new distribution function. After an expansion resulting in a change in the magnitude of $B$ from $B_0$ to $B_1$, the first adiabatic invariant implies that distribution function will have

\[ V_{1\parallel}^2 = V_{0\parallel}^2 + V_{0\perp}^2(1 - B_1/B_0) \]  

and conservation of energy implies

\[ V_{1\parallel}^2 = V_{0\parallel}^2 + V_{0\perp}^2(1 - B_1/B_0) \]  

There is also a difference in the limits. Even a particle with zero parallel energy then acquires

\[ V_{1\parallel}^2 = V_{0\perp}^2(1 - B_1/B_0). \]  

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Figure 1. A one-dimensional distribution function for the unstable plasma. The red curve shows the limits of the empty region calculated according to the two conservation laws.

Figure 2. Calculated relation between the small parallel temperature, $T_{\text{par}}[2]$, of the subtracted distribution and the expansion $B_1/B_0$, of the magnetic field.
There are then no particles left with zero perpendicular energy except those initial zero energy. There is therefore an empty region in the distribution.

It is assumed that the electrons are sufficiently mobile that they are little affected and have refilled, on the timescale of the ions, the missing volume.

This new distribution function is not Maxwellian. In this work it is desired to use a dispersion relation solver for sums of Maxwellians to determine growth rates, so the new distribution function has been approximated by subtracting a very narrow (in $T_{par}$) Maxwellian to represent the empty part. The resulting function can then be given by a sum of the slightly modified original parallel distribution minus a distribution to cancel the empty region. This canceling distribution must then satisfy two requirements: (1) the number of particles at $V_{par}=0$ must equal the number of the original distribution to make the sum vanish at $V = 0$. Denoting the slightly changed original distribution by $1$ and the negative cancellation distribution by $2$, $V_0$ becomes $V_1$, where

$$
\text{Dens}1 = \sqrt{\frac{M_p}{2\pi k T_{par}}} \exp\left(\frac{V_{perp}^2}{2kT_1}\right) dV_{perp} dV_{par}
$$

(4)

and further, (2) the total particle number must remain unchanged.

$$
\text{Dens}[1] + \text{Dens}[2] = 1,
$$

(6)

where the total distribution has been normalized to 1.

The result is

$$
\text{Dens}[1] = \frac{1}{1 - \sqrt{kT_2/kT_{par}}},
$$

(7)

$$
\text{Dens}[2] = 1 - \text{Dens}[1].
$$

(8)

This approximation is denoted by the black curve in Figure 1. The red curve represents the edges of the empty part of the true distribution due to the two conservation laws as discussed above. The expansion is 2%.

It will be seen that the canceling distribution is not a very good approximation, as the slope is opposite. However, the resulting function is a sort of giant bump-on-tail distribution, and it is assumed here that the distribution depends more on the existence of this empty region than on its shape. However, there is an empty region in both. The magnitude of the parallel temperature of the chosen canceling region is chosen by equating the two at the point where integration of the distribution function includes half of the total number of

![Growth rate](image)

**Figure 3.** Growth rate (imaginary part of the frequency) as a function of the effective parallel temperature of the canceling distribution.
particles, as shown in the figure by the red diamonds. The relation between the true and the parallel temperature of the subtraction Maxwellian derived in this way is shown in Figure 2. A better approximation must await simulations as Hellinger et al. (2019).

It is apparent from a visual inspection and from Penrose (1960) that the red distribution is indeed unstable due to its positive slope. Calculations using the subtracted Maxwell distribution also give the instability. Figure 3 displays the imaginary part of the frequency for solutions obtained from a warm plasma dispersion relation based on Stix (1962). In this figure, the wavevector, \( k \), is \( k c / \omega_{pe} - 0.0177 \) for all calculations. For a typical astronomical unit density of 10 cm\(^{-3}\), this would give a wavelength of about 1000 m. The plasma parameters for these calculations have been taken from observations of the Parker Solar Probe (FIELDS and SWEAP experiments) as available on CDAWeb for the time. The relevant parameters are density 240 cm\(^{-3}\) and magnetic field 88 nT. Isotropic plasma temperature (55 eV) has been used. These data were taken from CDAWeb data shortly after the first perihelion, on 2018 November 8. They lead to an Alfvén speed of 124 km s\(^{-1}\) and ion sound speed of 145 km s\(^{-1}\). Note that the fast mode is the highly damped mode. The real part of the frequency is zero to within the accuracy of the computer calculations. The frequency has been scaled to the ion cyclotron frequency as an estimate of the high frequency end of the inertial part of the solar wind cascade. The parallel temperature of the subtraction distribution is given in dimensionless form \( kT_{\parallel}/M_pC^2 \).

The number of particles originally with zero parallel velocity but having perpendicular velocity \( V_{0\perp} \), and which now have parallel velocity \( V_{1\parallel} \) above is

\[
N = \sqrt{M_p} \cdot (2p kT_{0\perp})^{-\frac{3}{2}} \cdot \exp \left( \frac{5 V_{0\perp}^2}{2kT_{0\perp}} \right). \tag{9}
\]

The real part of the frequency is zero but observations in the solar wind will see a wave carried by the solar wind, so that the observed frequency will not be zero but will be \( k V_{sw} \), where \( k \) is the wavenumber parallel to the solar wind. Figure 4 presents the wavenumber \( k \) calculated for varying values of \( kc/\pi \). Calculations have been carried out for two ranges. The blue curve starts at the maximum growth rate shown in Figure 3 and the red curve starts at 1% expansion.

The large imaginary frequency suggests a parallel with ion acoustic waves, which also have large damping. They have, additionally, other things in common in that the instability electrons and ions remain close to the same place so that their charges nearly cancel each other so that the resulting electric field is small compared to what it would be if the charges were separated. For ion acoustic waves, this characteristic is stronger the lower is the frequency. Therefore, a search of observational data for these as well as ion acoustic waves is most sensitive if it is based on density changes rather than electric fields. It is probably for this reason that the waves identified in the seminal paper Gurnett & Frank (1978) only identified ion acoustic waves above about 600 Hz, as they were detecting waves through their electric fields. At low frequencies, ion acoustic waves are better detected through their density changes. The ratio electric field to charge density is roughly proportional to the frequency \( \pi \).

Ion acoustic waves have been frequently found in the solar wind (Tu & Marsch 1995; Howes et al. 2012). It is difficult, in
view of the considerations above to distinguish between the results of this instability and proper ion acoustic waves of low frequency. However this remark does not apply to the higher frequency waves seen by Gurnett & Frank (1978).

It is apparent that this instability has much in common with the long known mirror instability (Gary et al. 1976; Tsurutani et al. 1982; Southwood & Kivelson 1993), but particularly with the analytical Vlasov treatment of Southwood and Kivelson, in that the real part of the frequency is zero and particles with zero parallel velocity play an important role. Here, however, the cause of the anisotropy is identified and connections to the cause are presented, together with other quantitative results. A quite different instability due to expansion has been investigated, using simulation, by Micera et al. (2021).

2.1.1. Connection to Expansion Rate

The parallel temperature of the subtraction distribution used must be related to the expansion ratio $B_1/B_0$. Because the shape of the empty zone is different for the subtraction distribution and the empty zone calculated above, a true fit is not possible. The relation between the artificial temperature $kT_2$ and the amount of expansion is obtained from the two conservation relations as expressed in Equations (1) and (2). The two empty zones have been equated at the points marked “x” in Figure 1, a point that corresponds to half of the total number of particles. The relation between the parallel temperature of the subtraction distribution found in this way is shown in Figure 5. Except for very small expansion the relation is $T_{\text{par}2}=2.2 \times 10^{-8} (1-B_1/B_0)$. Again the parallel temperature of the subtraction distribution is given in dimensionless units, $kT_{\text{par}}/(M_pC^2)$.

2.2. Oblique $k$ Vector and Magnetic Field

The largest damping, i.e., growth rate, is for a wavevector, $k$, parallel to $B_0$, but damping continues for most angles of propagation. Figure 5 shows the imaginary part of the frequency as a function of the angle between the wavevector and $B_0$. It is to be noted that the damping is only reduced to half of its parallel value at an angle of 40°. The real part of the frequency is still zero to within the accuracy of the computer calculations for this work. For wavevector parallel to the magnetic field, the field generated by the instability is purely electrostatic but at oblique angles the result has some magnetic field. Figure 6 shows the ratio of $E$ to $B$ as a function of angle of propagation. Also shown, in colors, are the ratios for Alfvén

\begin{figure}[h!]
\centering
\includegraphics[width=0.8\textwidth]{growth_at_oblique_angles}
\caption{Growth rate for an oblique $k$ vector.}
\end{figure}
waves and for the slow wave for the plasma parameters used for this calculation. As the real part of the frequency of this disturbance is zero, the speed is taken from the ratio of $E$ to $B$. The electromagnetic field is incompressible—there is no component of the field in the direction of the wavevector. The field is elliptically polarized. It had been hoped that the instability would generate Alfvén waves and possibly fast and slow mode waves. The phase velocity of the Alfvén wave (red) has been taken to be $V_A / \cos(\theta)$ and that of the ion sound wave (green) likewise from $V_s / \cos(\theta)$. Temperature has been taken from $V_{th} = \sqrt{2kT / M_p}$ where $V_{th}$ is the most probable speed. It will be seen that the velocities are in the same range but do not show any connection. As separate electron and ion temperatures were not available, the sound speed has been taken as $\sqrt{4kT / M_p}$. It will be seen that there is no connection between the speed of the instability and either of the mode speeds.

2.3. The Twist

The generation of a magnetic field indicates some twisting of the fields. Because the twist of the eigenmodes in the ambient fields is different for the different modes, it has been suggested that the direction of the twist might be different according to the ratios of the fast and slow mode speeds. As is well known, if the terms fast and slow mode indicate phase speed, the properties of these modes are interchanged when the ratio $V_A / V_S$ changes from less than 1 to greater than 1. When they are elliptically polarized as for oblique propagation, they have the opposite directions of rotation. This was investigated using data from STEREO. On 2010 March 19, there were periods when the Alfvén speed was greater than the ion acoustic speed and vice versa. The growth rate equations were solved for oblique propagation at $30^\circ$ in two such differing cases, at 0800 where the ratio $V_A / V_S$ was 30 and at 1815 when the ratio was 0.53. No change in the

![Figure 6. Magnetic field for oblique propagation.](image)
direction of the twist was found. This is in accord with the lack of connection with the mode speeds, above.

### 2.4. Energy and Heating

It is clear that thus instability will grow until other processes return the distribution to a stable configuration. These must convert the expanded magnetic field to heating and probably filling the empty region, thus leaving a distribution enriched in parallel moving particles. It is generally accepted that the energy for such heating comes from the magnetic turbulence. In Kellogg (2020), however, it was shown that the acoustic energy available, calculated from Equation (3) of that publication and demonstrated in Figure 1, is generally larger than the magnetic turbulence energy. In this work, because the energy in the generated field is concentrated in magnetic field for even slightly oblique wavevector and thus quite variable, no calculation of the heating is attempted here, but heating must exist. Heating from the acoustic energy is even more uncertain so again no calculation is attempted.

### 3. Summary and Conclusions

On the assumption that the first adiabatic plasma invariant and that kinetic energy are conserved on a short timescale, an attempt at approximating the ion distribution resulting from expansion shows that expansion of the solar wind and its magnetic field causes an instability. For a wavevector parallel to the ambient magnetic field the resulting instability is purely electrostatic and has zero real part of the frequency. For an oblique wavevector, the instability has an elliptically polarized incompressible magnetic field. For increasing obliquity, the magnetic energy increases and dominates the electric energy. The instability grows rather rapidly even for quite small expansions. This suggests that such an instability has a part in forming the expanding winds of all stars with convection zones reaching the surface. As this disturbance is quickly damped, it is to be expected that it acts to move the distribution toward a more isotropic temperature distribution and also to provide some conversion of magnetic field into particle energy, though an accurate answer to these quantities must await simulation (Liewer et al. 2001).

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