Rotation of D-brane and Non-commutative Geometry

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Abstract

Our motivation is to find the relationship between the commutator of coordinates and uncertainty relation involving only the coordinates. The boundary condition with constant background field is connected with the rotation of D-brane at general angle. And the mode expansions of D-brane we found is more reasonable than those appeared in literature. The partition functions and scattering amplitudes are also discussed.

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1 Introduction

The non-commutative geometry is one of the most important and most interesting idea in the recent string theory. [1] In an excellent paper Seiberg and Witten express the commutator of coordinates in terms of a constant Neveu-Schwarz B-field (see [2] and references therein). On the other hand an uncertainty relation involving only the coordinates was appeared in the Polchinski’s wonderful book,[3] which is developed from some discussions about dynamics of D-branes.[4,5,6] As we know from ordinary quantum mechanics the uncertainty relation and the commutator between coordinate and momentum both involve a Planck constant (over 2π)h and henceforth are related each other. Naturally, we are interested in knowing if there is any relationship between the commutator of coordinates and the uncertainty relation in the coordinates in the domain of non-commutative geometry.

We will find in this paper that the boundary conditions with constant background B-field, which is in a special form, are equivalent to the rotation of D-branes (including their analytic continuation, the relative motion of D-branes). So we may use these properties of D-brane to seek the relationship mentioned above.

The rotations of D-branes are discussed by several authors.[3,7] However, it seems to us that the mode expansions are hard to reduce into the parallel D-branes (including NN, DD, ND, DN coordinates) when the rotated angles vanish (or π/2 , in case where vanishing angle leads to perpendicular D-branes ). By using the above equivalence we find new mode expansions proposed in this paper is more convincing. That is also the main part of our positive results.

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Given the relevant mode expansions we can calculate the partition functions easily. In concrete, we study D0-D4 and D4-D4 open strings. They have different partition functions but same non-vanishing potential. By means of analytic continuation we find the scattering amplitudes of relative motion of D-branes identical with one given by Polchinski. But it is difficult to interpret consistently the relationship between commutator of pure coordinates and uncertainty relation in small separation.

2 Non-commutative Geometry and the Rotation of D-brane

Following Seiberg and Witten [2] the commutators of coordinates can be written as

\[ [X^i, X^j] = i\theta^{ij} \] (1)

Under a nonzero constant background the \( \theta \) can be expressed in terms of B-field. Especially in the zero slope limit ( \( \alpha' \sim \epsilon^{1/2} \rightarrow 0, g_{ij} \sim \epsilon \rightarrow 0 \) ) we have \( \theta = 1/B \), but we will not restrict ourselves in this limit. In this paper, we set \( g_{ij} = \epsilon\delta_{ij} \), and B in a special form

\[ B = \frac{\epsilon}{2\pi\alpha'} \begin{pmatrix} 0 & b_1 & 0 & 0 \\ -b_1 & 0 & 0 & 0 \\ 0 & 0 & 0 & b_2 \\ 0 & 0 & -b_2 & 0 \end{pmatrix} \] (2)

Now, let us consider an open string stretched between two D-branes. The first example is D0-D4 open string which has following boundary conditions ( \( i, j = 1, \ldots, 4 \) )

\[ \partial_\sigma X^0|_{\sigma=0,\pi} = \partial_\tau X^{5\cdots 9}|_{\sigma=0,\pi} = \partial_\tau X^{1\cdots 4}|_{\sigma=0} = g_{ij} \partial_\tau X^j + 2\pi\alpha' B_{ij} \partial_\tau X^j|_{\sigma=\pi} = 0 \] (3)

in which we write \( X' \) for dimensions \( 5\cdots 9 \), because of the fact that from the last boundary condition the D4 brane is not perpendicular to the open string directions at \( \sigma = \pi \) (it represents that the brane has sloped or deformed under interaction with background field), so in general, \( X^{5\cdots 9} \) may not orthogonalize \( X^{1\cdots 4} \). To get a mode expansions for\( X^{1\cdots 4} \), Seiberg and Witten introduced following "complex" scalar fields: \( Z_1 = X_1 + iX_2, Z_2 = X_3 + iX_4 \) and \( \bar{Z}_1 = X_1 - iX_2, \bar{Z}_2 = X_3 - iX_4 \). But \( \bar{Z}_{I=1,2} \) are not the complex conjugate fields of \( Z_{I=1,2} \), because that \( X_{1\cdots 4} \) can have their own complex mode expansions unless \( X_{1\cdots 4} \) are hermitian, and then \( b_{1,2} \) are pure imaginary. In this regime the boundary condition at \( \sigma = \pi \) changes to [2]

\[ \partial_\sigma Z_I + b_I \partial_\tau Z_I = 0 \quad , \quad \partial_\sigma \bar{Z}_I - b_I \partial_\tau \bar{Z}_I = 0 \] (4)

Latter we refer it the twisted condition. The mode expansions have been expressed by them as following

\[ Z_I = i \sqrt{\frac{\alpha'}{2}} \sum_n \left( e^{(n+\nu_I)(\tau+i\sigma)} - e^{(n+\nu_I)(\tau-i\sigma)} \right) \frac{\alpha_{n+\nu_I}}{n+\nu_I} \] \[ \bar{Z}_I = i \sqrt{\frac{\alpha'}{2}} \sum_n \left( e^{(n-\nu_I)(\tau+i\sigma)} - e^{(n-\nu_I)(\tau-i\sigma)} \right) \frac{\alpha_{n-\nu_I}}{n-\nu_I} \] (5)

They satisfy

\[ \partial_\tau Z_I|_{\sigma=0} = \partial_\tau \bar{Z}_I|_{\sigma=0} = 0 \] (6)
\[
\begin{align*}
\frac{b_I}{=} & - \frac{\partial_\sigma Z_I}{\partial_\tau Z_I} |_{\sigma=\pi} = \frac{\partial_\sigma \overline{Z}_I}{\partial_\tau \overline{Z}_I} |_{\sigma=\pi} \\
& = -i \frac{e^{i\pi \nu_I} + e^{-i\pi \nu_I}}{e^{i\pi \nu_I} - e^{-i\pi \nu_I}} = -\cot(\pi \nu_I) \equiv -\cot(\varphi_I) 
\end{align*}
\]

In an alternative description we can rewrite the boundary conditions (4) as follows
\[
\begin{align*}
(\cos(\varphi_I) \partial_\tau - \sin(\varphi_I) \partial_\sigma) Z_I |_{\sigma=\pi} &= \partial_\sigma' Z_I |_{\sigma=\pi} = 0 \\
(\cos(\varphi_I) \partial_\tau + \sin(\varphi_I) \partial_\sigma) \overline{Z}_I |_{\sigma=\pi} &= \partial_\sigma' \overline{Z}_I |_{\sigma=\pi} = 0 
\end{align*}
\]
in which we denote new world sheet coordinates for reference, which are in parallel and vertical directions of the D4 brane
\[
\begin{align*}
\sigma' &= \cos(\varphi_I) \tau - \sin(\varphi_I) \sigma , \quad \tau' = \sin(\varphi_I) \tau + \cos(\varphi_I) \sigma , \\
\sigma'' &= \cos(\varphi_I) \tau + \sin(\varphi_I) \sigma , \quad \tau'' = -\sin(\varphi_I) \tau + \cos(\varphi_I) \sigma 
\end{align*}
\]

To obtain the partition function of full string we should consider the contributions from another independent dimensions which are orthogonalized with above \( Z_I \) and \( \overline{Z}_I \), but not from \( X'_5 \cdots 9 \). To this end let us define
\[
Z_M = X_{2M-1} + iX_{2M} \quad \overline{Z}_M = X_{2M-1} - iX_{2M} \quad (M = I; J = 1, 2; 3, 4) 
\]

Except \( Z_I \) and \( \overline{Z}_I \), we introduce their counterparts \( Z_J \) and \( \overline{Z}_J \) respectively. Hence they observe boundary conditions
\[
\partial_\tau Z_J |_{\sigma=0} = \partial_\tau \overline{Z}_J |_{\sigma=0} = 0 
\]

and
\[
\begin{align*}
(\sin(\varphi_I) \partial_\tau + \cos(\varphi_I) \partial_\sigma) Z_J |_{\sigma=\pi} &= \partial_\sigma' Z_J |_{\sigma=\pi} = 0, \\
(- \sin(\varphi_I) \partial_\tau + \cos(\varphi_I) \partial_\sigma) \overline{Z}_J |_{\sigma=\pi} &= \partial_\sigma' \overline{Z}_J |_{\sigma=\pi} = 0. 
\end{align*}
\]

Therefore we find their mode expansions easily
\[
\begin{align*}
Z_J &= i \frac{\alpha'}{2} \sum_n \left( e^{(n+\nu_I+1/2)\omega} - e^{(n+\nu_I+1/2)\overline{\omega}} \right) \alpha_n^{l} \frac{\alpha_l^{\nu_I+1/2}}{n + \nu_I + 1/2} \\
\overline{Z}_J &= i \frac{\alpha'}{2} \sum_n \left( e^{(n-\nu_I-1/2)\omega} - e^{(n-\nu_I-1/2)\overline{\omega}} \right) \alpha_n^{l} \frac{\overline{\alpha}_l^{\nu_I+1/2}}{n - \nu_I - 1/2} 
\end{align*}
\]

where \( \omega = \sigma + i\tau, \overline{\omega} = \sigma - i\tau \) It is very interesting that the boundary conditions (8) and (12) are equivalent to the rotation of D4 brane
\[
\begin{align*}
\partial_\sigma (- \sin(\varphi_I) Z_I + \cos(\varphi_I) Z_J) |_{\sigma=\pi} &= \partial_\sigma Z'_I |_{\sigma=\pi} = 0, \\
\partial_\sigma \left( \sin(\varphi_I) \overline{Z}_I + \cos(\varphi_I) \overline{Z}_J \right) |_{\sigma=\pi} &= \partial_\sigma \overline{Z}'_I |_{\sigma=\pi} = 0, \\
\partial_\tau (\cos(\varphi_I) Z_I + \sin(\varphi_I) Z_J) |_{\sigma=\pi} &= \partial_\tau Z'_J |_{\sigma=\pi} = 0, \\
\partial_\tau \left( \cos(\varphi_I) \overline{Z}_I - \sin(\varphi_I) \overline{Z}_J \right) |_{\sigma=\pi} &= \partial_\tau \overline{Z}'_J |_{\sigma=\pi} = 0. 
\end{align*}
\]

In fact if we assume that
\[
Z_I = -SZ'_I + CZ'_J \quad Z_J = CZ'_I + SZ'_J 
\]
in which 

\[ \partial_\sigma Z'_I = \partial_\tau Z'_J = 0 \]

From Eq. (8) and Eq. (12), we have

\[ \cos(\varphi_I) S \partial_\tau Z'_I + \sin(\varphi_I) C \partial_\sigma Z'_J = 0 \]

and

\[ \sin(\varphi_I) C \partial_\tau Z'_J + \cos(\varphi_I) S \partial_\sigma Z'_I = 0 \]

Eliminating \( \partial_\tau Z'_I \) from these two equations we obtain

\[ \cos^2(\varphi_I) S^2 \partial_\sigma Z'_J = \sin^2(\varphi_I) C^2 \partial_\sigma Z'_J. \]

The solution of this equation is

\[ S = \sin(\varphi_I), \quad C = \cos(\varphi_I) \]

In a similar method we can show that

\[ Z''_I = \sin(\varphi_I) Z_I + \cos(\varphi_I) Z_J, \quad Z''_J = \cos(\varphi_I) Z_I - \sin(\varphi_I) Z_J \]

Like Eq.(10) we suppose that \( Z'_I, Z''_I, Z'_J, Z''_J \) are combinations from orthogonal systems \( X'_m \) and \( X''_m \) \((m = 0, 1, \ldots, 9)\) with \( X'_{0,9} = X''_{0,9} = X_{0,9} \). That is to say systems \( X'_m \) and \( X''_m \) are different from \( X_m \) by a relative angle around \( X_9 \).

## 3 D4-D4 Open String

Similarly we can also consider two D4 branes with relative rotation angles. The boundary conditions for this system are

\[ \partial_\sigma Z|_{\sigma=0} = \partial_\sigma Z'|_{\sigma=0} = \partial_\tau Z|_{\sigma=0} = \partial_\tau Z'|_{\sigma=0} = 0 \]

and conditions (14) at \( \sigma = \pi \). Thus, the mode expansions of \( Z_J \) and \( Z_J' \) are still given by Eq.(13). As for \( Z_I \) and \( Z'_I \) we have following expressions

\[ Z_I = i \sqrt{\frac{\alpha'}{2}} \sum_n \left( e^{(n+\nu_I+1/2)\omega} + e^{(n+\nu_I+1/2)\omega'} \right) \frac{\alpha_n^{I} + \nu_I + 1/2}{n + \nu_I + 1/2} \]

\[ Z'_I = i \sqrt{\frac{\alpha'}{2}} \sum_n \left( e^{(n-\nu_I-1/2)\omega} + e^{(n-\nu_I-1/2)\omega'} \right) \frac{\alpha_n^{I} - \nu_I - 1/2}{n - \nu_I - 1/2} \]

Here we find same exponentials in Eq.(13) and Eq.(16), so it is convenient to use

\[ \tilde{\nu}_I = \nu_I + 1/2 \quad \left( \tilde{\varphi}_I = \varphi_I + \pi/2 = \tan^{-1}(b_I) \right) \]

instead of \( \nu_I \) for D4-D4 string.
4 Branes at general angles

Till now we have shown that the twisted boundary conditions are equivalent to a rotation of D-brane: an angle $\varphi_1$ in the (1,5) and (2,6) planes; $\varphi_2$ in the (3,7) and (4,8) planes. As a matter of fact, we can still generalize it to more general case, that is a rotation with different angles in (1,5) and (2,6) planes and different angles in (3,7) and (4,8) planes. To show the generalized mode expansion we need only give an illustration: A rotation of angle $\varphi$ in one plane, say (1,5). We will give up to use the $Z_M$ fields, but utilize the original $X_m$ field. Reversing the above mentioned trick the boundary conditions

$$\partial_\sigma (-\sin(\varphi)X_1 + \cos(\varphi)X_5)|_{\sigma=\pi} = \partial_\tau (\cos(\varphi)X_1 + \sin(\varphi)X_5)|_{\tau=\pi} = 0 \quad (18)$$

are equivalent to the twisted one

$$(\cos(\varphi)\partial_\tau - \sin(\varphi)\partial_\sigma)X_1|_{\sigma=\pi} = (\sin(\varphi)\partial_\tau + \cos(\varphi)\partial_\sigma)X_5|_{\sigma=\pi} = 0. \quad (19)$$

Therefore the mode expansions of $X_1$ and $X_5$ are written as follows

$$X_1 = i\sqrt{\alpha'/2} \sum_n (e^{(n+\nu)\omega} - e^{(n+\nu+1/2)\omega}) \frac{\alpha_{n+\nu}^{1/2}}{n+\nu} \quad , \text{for } D_0 - D_4,$$

$$X_1 = i\sqrt{\alpha'/2} \sum_n (e^{(n+\nu)\omega} + e^{(n+\nu+1/2)\omega}) \frac{\alpha_{n+\nu+1/2}^{1/2}}{n+\nu+1/2} \quad , \text{for } D_4 - D_4. \quad (20)$$

$$X_5 = i\sqrt{\alpha'/2} \sum_n (e^{(n+\nu)\omega} - e^{(n+\nu+1/2)\omega}) \frac{\alpha_{n+\nu}^{1/2}}{n+\nu} \quad , \text{for } D_0 - D_4$$

$$X_5 = i\sqrt{\alpha'/2} \sum_n (e^{(n+\nu)\omega} - e^{(n+\nu+1/2)\omega}) \frac{\alpha_{n+\nu+1/2}^{1/2}}{n+\nu+1/2} \quad , \text{for } D_4 - D_4. \quad (21)$$

Obviously these expressions include the parallel branes as their special cases. Eq.(20a) reduces to DD open string when $\nu = 0$, DN when $\nu = 1/2$; and the case of Eq.(21) is just the opposite. Nevertheless, eq.(20b) will give NN string when $\tilde{\nu} = 0$ and ND when $\tilde{\nu} = 1/2$.

5 World sheet Supersymmetry

Let us go back to the issue in which there are given background fields $B$. The same conclusion about the equivalence between the twisted boundary conditions and rotation of D-brane can find from the boundary conditions of world sheet fermions. The latter are [2]

$$\bar{\psi}_i = R_{ij}(B)\psi^j$$

$$R(B) = (1 - 2\pi\alpha'g^{-1}B)B(1 + 2\pi\alpha'g^{-1}B)$$

$$= \begin{pmatrix}
-\cos(2\varphi_1) & -\sin(2\varphi_1) & 0 & 0 \\
\sin(2\varphi_1) & -\cos(2\varphi_1) & 0 & 0 \\
0 & 0 & -\cos(2\varphi_2) & -\sin(2\varphi_2) \\
0 & 0 & \sin(2\varphi_2) & -\cos(2\varphi_2)
\end{pmatrix} \quad (22)$$
One may consider a rotation so that

\[ \overline{\Psi}_i = (\rho^{-1}(B))_{ij} \overline{\psi}_j, \quad \Psi_i = \rho_{ij}(B) \psi_j, \]

\[ \rho(B) = \begin{pmatrix} \sin(\phi_1) & -\cos(\phi_1) & 0 & 0 \\ \cos(\phi_1) & \sin(\phi_1) & 0 & 0 \\ 0 & 0 & \sin(\phi_2) & -\cos(\phi_2) \\ 0 & 0 & \cos(\phi_2) & \sin(\phi_2) \end{pmatrix} = (R(B))^\frac{1}{2}. \tag{23} \]

Then, after the rotation \( \rho(B) \) the boundary condition will become a simple form

\[ \overline{\Psi}_i = \Psi_i \tag{24} \]

Once more, we have proven that the boundary condition with constant background field is equivalent to a rotation of D-brane. Rotation matrix is a special subgroup of \( U(4) \),

\[ \text{diag}(\exp(i\phi_1), \exp(i\phi_2), \exp(i\phi_3), \exp(i\phi_4)) \tag{25} \]

when \( \phi_1 = \phi_2 = \varphi_1 \), \( \phi_3 = \phi_4 = \varphi_2 \). As we know a D4 brane preserves super-charges of the form

\[ \epsilon_L Q^L + \epsilon_R Q^R \]

with

\[ \epsilon_R = \Gamma_0 \Gamma_1 \cdots \Gamma_4 \epsilon_L = \beta^\perp \epsilon_L, \tag{26} \]

in which Polchinski’s symbol \( \beta^\perp \) is also used. The supersymmetry unbroken by the rotation 4-brane requires

\[ \rho^{-1} \beta^\perp \rho = \beta^\perp \rho^2 = \beta^\perp, \tag{27} \]

and

\[ \rho^2 = \exp(2i \sum_{a=1}^{4} s_a \phi_a) \tag{28} \]

is the rotation in the usual spinor basis.\[3\] Since \( \phi_1 = \phi_2, \ \phi_3 = \phi_4 \) in our case, there are 1/4 unbroken supersymmetries of total 16, if \( \phi_a \neq 0 \).

Although there are unbroken supersymmetries the potential is nonzero.

### 6 Partition Functions

From mode expansions (5),(13),(16) we know

\[ [\alpha_{n+\nu}, \overline{\alpha}_{m+\nu}] = (n + \nu) \delta_{n,-m}, \quad \nu = \nu_I \text{ or } \nu_{\bar{I}} \tag{29} \]

It is easy to find the partition function for one such "complex" scalar (see for example[3])

\[ -i \frac{\exp(\pi \nu^2 t) \eta(it)}{\theta_{11}(i\nu t, it)} \tag{30} \]

On account of [8]

\[ Z^\alpha_\beta(\pi(\nu + \frac{1}{2}), it) = \frac{\theta_{\alpha\beta}(i(\nu + \frac{1}{2})t, it)}{\exp(\pi(\nu + \frac{1}{2})^2 t) \eta(it)} \]

\[ = \frac{\theta_{\alpha\beta}(i\nu t, it)}{\exp(\pi \nu^2 t) \eta(it)} \equiv Z^\alpha_\beta(\pi\nu, it) \tag{31} \]
The correspondence between two index pairs is

$$\begin{array}{c|cccc}
\alpha \beta & 00 & 01 & 10 & 11 \\
\overline{\alpha} \overline{\beta} & 01 & 00 & 11 & 10 \\
\end{array}$$

The full contribution of scalars is

$$\prod_{I=1,2} (Z^1_1(\pi \nu_I, it)Z^1_0(\pi \nu_I, it))^{-1} , \text{ for } D0 - D4$$

and

$$\prod_{I=1,2} (Z^1_1(\pi \nu_I, it))^{-2} , \text{ for } D4 - D4$$

And the related fermionic partition function is

$$\frac{1}{2} \prod_{I=1,2} (Z^0_0(\pi \nu_I, it)Z^0_1(\pi \nu_I, it) - Z^0_1(\pi \nu_I, it)Z^0_0(\pi \nu_I, it))$$

$$- \prod_{I=1,2} Z^1_0(\pi \nu_I, it)Z^1_1(\pi \nu_I, it) - \prod_{I=1,2} Z^1_1(\pi \nu_I, it)Z^1_0(\pi \nu_I, it)]$$

$$= \prod_{I=1,2} Z^0_0(\pi \nu_I, it)Z^1_1(\pi \nu_I, it) , \text{ for } D0 - D4$$

and

$$\frac{1}{2} \left[ \prod_{I=1,2} (Z^0_0(\pi \nu_I, it))^2 - \prod_{I=1,2} (Z^0_1(\pi \nu_I, it))^2 - \prod_{I=1,2} (Z^1_0(\pi \nu_I, it))^2 \right]$$

$$= \prod_{a=1}^4 (Z^1_1(\pi \nu'_a, it))^2 = - \prod_{I=1,2} (Z^1_1(\pi \nu_I, it))^2 \text{ for } D4 - D4$$

in which

$$\nu'_1 = \frac{1}{2}(\nu_1 + \nu_1 + \nu_2 - \nu_2) = \nu_1$$

$$\nu'_2 = \frac{1}{2}(\nu_1 + \nu_1 - \nu_2 + \nu_2) = \nu_1$$

$$\nu'_3 = \frac{1}{2}(\nu_1 - \nu_1 + \nu_2 + \nu_2) = \nu_2$$

$$\nu'_4 = \frac{1}{2}(\nu_1 - \nu_1 - \nu_2 - \nu_2) = -\nu_2$$

from Riemann formula, and hence we have the last equality in (35). For the generalization to general rotated angle, the results are the same as ref.[3].

### 7 Discussion for Dynamics

In this section, let us focus on the D4-D4 case. By imitating an analysis made by Polchinski we may study the interaction of D-branes by means of analytic continuation. Since any $X^i$ on D4

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1. Because of the last term in relevant Riemann formula is positive [8], we must change sign for one of $\phi_a$ in (13.4.22) in reference[3].
brane has Neumann boundary condition just like the "time" component $X^0$. We can analytically continue one of $X^i$ to be the time, say $X^1 \to i\tilde{X}^0$ ($X^0$ continues to a space dimension). Then the rotation in $(1,5)$ plane will become a relative motion with velocity

$$ v = ib_1 = i \tan(\tilde{\varphi}_1) = \tanh(\tilde{u}), \quad \tilde{u} = i\tilde{\varphi}. \tag{37} $$

Now, the full answer is a combination of a rotation $(0, \tilde{\varphi}_1, \tilde{\varphi}_2)$, and a relative motion $(-i\tilde{u}, 0, 0, 0)$. According to ref[3] the scattering amplitude is expressed as follows

$$ A = -i \int_{-\infty}^{\infty} d\tau V(r(\tau), v), \quad r(\tau)^2 = y^2 + v^2\tau^2. \tag{38} $$

$$ V(r, v) = i \frac{2V_4}{(8\pi^2\alpha')^{2}} \int_0^{\infty} dt \frac{t^{1/2}}{2^{2i\alpha'}} \exp\left(-\frac{tr^2}{2\pi\alpha'}\right) \frac{\tanh(\tilde{u})\theta_{11}(\frac{i\tilde{u}}{\pi}, \frac{i}{t})}{\eta(it)^9\theta_{11}(\frac{i}{\pi}, \frac{i}{t})^{\frac{9}{8}}}. $$

Where $y$ is a separation along the 9-direction (here we use the same $\tau$ to denote Minkowski time, not the Euclidean one). Polchinski analyzed the small $v$ (so $v \approx \tilde{u}$) and even small $r$ case for nontrivial potential (see also [6]). That is $ut \approx \frac{x}{r^2} \approx 1$. Let $\delta x \approx r, \delta t \approx \frac{r}{v},$ he wrote down an uncertainty inequality

$$ \delta x\delta t \geq \alpha'. \tag{39} $$

Although non-commutators $\theta^{ij}$ are proportional to the string tension constant $\alpha'$, they depend on velocity too. This is quite different from the ordinary quantum mechanics except there were reason we could rescale coordinates in terms of $b_1$. It might be more questionary that the separation $r$ is not in 5-direction but essentially in the 9-direction. We expect that there will appear more convincing interpretation.

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