Uncovering quirk signal via energy loss inside tracker

Wenxing Zhang

Institute of Theoretical Physics,
Chinese Academy of Sciences,
Beijing, China

November 27, 2019
Quirks are long lived exotic particles that are charged under both the Standard Model (SM) gauge group and a new confining gauge group with the confining energy equal to $\Lambda$.

Little Hierarchy Problem
Folded Supersymmetry
Quirky Little Higgs
Twin Higgs

$$\ell = 1_{\text{cm}} \times \left( \frac{1 \text{ keV}}{\Lambda} \right)^2 \times \left( \frac{m_Q}{100 \text{ GeV}} \right).$$

$\Lambda \ll m_Q$, $F_s \propto \Lambda^2$

$\Lambda \gtrsim \mathcal{O}(10 \text{ MeV}) \rightarrow$ Intensive oscillations
$\Lambda \in [10 \text{ keV}, 10 \text{ MeV}] \rightarrow$ Microscopic
$\Lambda \lesssim \mathcal{O}(10 \text{ eV}) \rightarrow$ Helical trajectory
$\Lambda \sim 100 \text{ eV}-10 \text{ keV} \rightarrow$ Macroscopic and Visible
Figure: The quirk pair trajectories inside the CMS tracker, with the confinement scale chosen as 1 eV, 1 keV and 2 keV (from left to right) for illustration. The quirk system in different panels have common initial momentum and quirk mass is 100 GeV. The cylinder segments indicate the tracking layer.
Quirks at the LHC

\[ \tilde{D}^c = (3, 3, 1, 2/3) , \quad (2) \]
\[ \tilde{E}^c = (1, 1, 1, -2) , \quad (3) \]
\[ D^c = (3, 3, 1, 2/3) , \quad (4) \]
\[ E^c = (1, 1, 1, -2) , \quad (5) \]

\( \tilde{D}^c \) and \( \tilde{E}^c \) are spin zero particles, and \( D^c \) and \( E^c \) are fermions. The electric charges of \( \tilde{D}^c/D^c \) and \( \tilde{E}^c/E^c \) are \( \frac{1}{3} \) and -1 respectively.

Due to the color confinement, one can only observe hadrons of quirk-quark bound state for \( \tilde{D}^c \) and \( D^c \). The probability for final state hadrons having charge \( \pm 1 \) is around 30%.

Figure: Production processes of quirks with different representations at the LHC.

Figure: The production cross sections for different quirks at 13 TeV LHC. We have also required an ISR jet with \( p_t(\text{jet}) > 100 \text{ GeV} \) in production processes.

- Initial state radiated (ISR) jet satisfy \( p_T > 100 \text{ GeV} \).
Quirk equation of motion: synchronism

The equation of motion for quirks is

\[
\frac{\partial \vec{P}}{\partial t} = \Lambda^2 \sqrt{1 - \vec{v}^2} \hat{s} - \Lambda^2 \frac{v_\parallel \vec{v}_\perp}{\sqrt{1 - \vec{v}^2}} + \vec{F}_{\text{ext}}
\]

\[
\vec{F}_{\text{ext}} = q\vec{v} \times \vec{B} - \langle \frac{dE}{dx} \rangle \hat{v}
\]  

Interaction between two particles is happening in the centre of mass (CM) frame simultaneously.
Suppose the two quirks are simultaneous in the CM frame at some point \( t_i^{CM} \), \( i = 1, 2 \).

\[
t_2^{CM} = \gamma (t_2^{Lab} - \vec{\beta} \cdot \vec{r}_2^{Lab})
\]

\[
t_1^{CM} = \gamma (t_1^{Lab} - \vec{\beta} \cdot \vec{r}_1^{Lab})
\]

\[
t_2^{CM} - t_1^{CM} = \gamma \left[ t_1^{Lab} - t_2^{Lab} - \vec{\beta} \cdot (\vec{r}_1^{Lab} - \vec{r}_2^{Lab}) \right] = 0
\]

Simultaneity condition thus translates into satisfying the eq. (10).

\[
t_1^{Lab} - t_2^{Lab} = \vec{\beta} \cdot (\vec{r}_1^{Lab} - \vec{r}_2^{Lab})
\]
Suppose at one point two quirks are simultaneous in the CM frame, and map the point from CM frame to Lab frame. All the variables below are defined in the Lab frame.

In the next step, we have:

\[
\begin{align*}
t'_i &= t_i + \epsilon_i \\
\vec{P}'_i &= \vec{P}_i + \epsilon_i \vec{F}_i \\
E'_i &= \sqrt{m^2 + \vec{P}'_i^2} \\
\vec{r}'_i &= \vec{r}_i + \frac{\epsilon_i}{2} (\vec{v}_i + \frac{\vec{P}'_i}{E'_i}) \\
\beta' &= \frac{\vec{P}'_1 + \vec{P}'_2}{E'_1 + E'_2} \\
i &= 1, 2.
\end{align*}
\] (11)

If in the next step the two quirks are also simultaneous in CM frame, they should satisfy

\[
t'_1 - t'_2 = t_1 + \epsilon_1 - (t_2 + \epsilon_2) = \beta' \left( \vec{r}_1 - \vec{r}_2 + \frac{\epsilon_1}{2} (\vec{v}_1 + \frac{\vec{p}'_1}{E'_1}) - \frac{\epsilon_2}{2} (\vec{v}_2 + \frac{\vec{p}'_2}{E'_2}) \right)
\] (12)

Taylor-expand Eq(12) to \( O(\epsilon_i) \), we have

\[
\begin{align*}
\epsilon_1 [1 - \vec{v}_1 \cdot \vec{\beta} - \frac{\vec{r}_1 - \vec{r}_2}{E_1 + E_2} \cdot (\vec{F}_1 - \vec{v}_1 \cdot \vec{F}_1 \vec{\beta})] &= \\
\epsilon_2 [1 - \vec{v}_2 \cdot \vec{\beta} - \frac{\vec{r}_2 - \vec{r}_1}{E_1 + E_2} \cdot (\vec{F}_2 - \vec{v}_2 \cdot \vec{F}_2 \vec{\beta})]
\end{align*}
\] (13)
Quirk equation of motion: direction of $\mathbf{s}$

Boost the system from the CM frame to the lab frame

\[ F_x' = \frac{F_x}{1 + \beta v_y} \sqrt{1 - \beta^2} \]
\[ F_y' = F_y + \frac{\beta v_x F_x}{1 + \beta v_y} \]  \hspace{1cm} (14)

\[ F_{||}' = -\sqrt{1 - v_{\perp}'^2} \wedge^2 = \frac{F_x}{1 + \beta v_y} \sqrt{1 - \beta^2} \]
\[ F_{\perp}' = -\frac{v_{\perp}' v_{||}'}{\sqrt{1 - v_{\perp}'^2}} \wedge^2 = F_y + \frac{\beta v_x F_x}{1 + \beta v_y} \]  \hspace{1cm} (15)

\[ F_{||}' = F_x' \sin \alpha + F_y' \cos \alpha \]
\[ F_{\perp}' = -F_x' \cos \alpha + F_y' \sin \alpha \]  \hspace{1cm} (16)

\[ \tan \alpha = \frac{\tan \theta - \beta v_x + \beta v_y \tan \theta}{\sqrt{1 - \beta^2}} \]  \hspace{1cm} (17)

\[ \vec{r}_{s1} = (1 - \beta^2)[(\vec{r}_1 - \vec{r}_2) - \vec{\beta}(t_1 - t_2)] + [(\vec{r}_1 - \vec{r}_2) - \vec{\beta}(t_1 - t_2)] \cdot \vec{\beta}(\vec{\beta} - \vec{v}_1) \]
\[ \vec{r}_{s2} = (1 - \beta^2)[(\vec{r}_2 - \vec{r}_1) - \vec{\beta}(t_2 - t_1)] + [(\vec{r}_2 - \vec{r}_1) - \vec{\beta}(t_2 - t_1)] \cdot \vec{\beta}(\vec{\beta} - \vec{v}_2) \]  \hspace{1cm} (18)
Quirk equation of motion: $\langle \frac{dE}{dx} \rangle$

The average ionization energy loss of charged particle can be expressed as function of velocity in the Bethe-Bloch (BB) and Lindhard-Scharff (LS) regions.

$$\langle \frac{dE}{dx} (v)_{\text{LS}} \rangle = A_1 v$$

$$\langle \frac{dE}{dx} (v)_{\text{BB}} \rangle = A_2 \frac{q^2}{v^2} \ln \left( \frac{A_3 v^2}{1 - v^2} - v^2 \right)$$

$$A_1 = (3.1 \times 10^{-11} \text{ GeV}^2) \frac{\rho}{g/cm^3} \frac{q^{7/6} Z}{A(q^{2/3} + Z^{2/3})^{3/2}}$$

$$A_2 = (6.03 \times 10^{-18} \text{ GeV}^2) \frac{\rho}{g/cm^3} \frac{Z}{A}$$

$$A_3 = \frac{102200}{Z}$$

with the $A$, $Z$ and $\rho$ correspond to the relative atomic mass, atomic number and density of material, respectively.

The ionization energy loss function in the region between LS and BB are interpolated from experimental data.
Figure: The transverse velocity of the quirk-antiquirk system for different quirk masses (left panel). The reconstructed missing transverse energy for the background process and a few signal processes (right panel).
Numerical solution of the EoM

The equation of motion for quirks is

$$\frac{\partial \vec{P}}{\partial t} = \Lambda^2 \sqrt{1 - \vec{v}_\perp^2} \hat{s} - \Lambda^2 \frac{\vec{v}_\parallel \vec{v}_\perp}{\sqrt{1 - \vec{v}_\perp^2}} + \vec{F}_{\text{ext}}$$

$$\vec{F}_{\text{ext}} = q\vec{v} \times \vec{B} - \langle \frac{dE}{dx} \rangle \hat{v} \tag{19}$$

- Confinement Scale: $\Lambda = 1\text{keV}$.  
- $\Delta t \leq T \propto m_Q/\Lambda^2$, \n  $\Delta t \sim 10^{-4}\text{ns}$  
- Evolution time no longer than 25 ns.  
- Propagating outside HCal.

Figure: The total number hits for quirk system traveling through the tracker of CMS detector.
For quirk-antiquirk system propagates in the tracker, it leaves $N$ hits located at $\vec{h}_i$ ($i = 1, 2, \ldots, N$) when crosses the detector layers. Define

$$d(\vec{n}) = \sqrt{\frac{1}{N - 1} \sum_{i=1}^{N} (\vec{n} \cdot \vec{h}_i)^2} \quad (20)$$

$$d_{\text{plane}} = d(\vec{n})_{\text{min}} \quad (21)$$

$$d_{\text{min}} \sim 1.16 \left( \frac{m}{100 \text{ GeV}} \right) \left( \frac{\text{keV}}{\Lambda} \right)^4 \left( \frac{|q|}{e} \right) \left( \frac{B_{xy}}{T} \right) \mu\text{m}, \quad (22)$$

Figure: The reconstructed thickness of two quirks trajectories arises from the recorded hits.
Signal and BKG Analysis at the LHC

Simulation for Signal
- Confinement Scale: $\Lambda = 1\text{keV}$.  
- ISR Jet: $p_T \geq 100\text{GeV}$.  
- Pile Up: $\langle \mu \rangle = 50$.  
- Error of $\langle \frac{dE}{dx} \rangle$: $\delta \langle \frac{dE}{dx} \rangle = 0.05 \langle \frac{dE}{dx} \rangle$

Analyzation of BKG processes
- $Z(\rightarrow \nu \nu) + \text{jets}$: $p_T(\nu \nu) > 200\text{ GeV}$.  
  $\sigma_{Z(\rightarrow \nu \nu)jj} = 3.6\text{pb}$
- $Z(\nu \nu)e^+e^- + \text{jet}$: $p_T(\nu \nu) > 100\text{GeV}$,  
  $p_T(e^\pm) > 1\text{ GeV}$.  
  $\sigma_{Z(\rightarrow \nu \nu)e^+e^-j} = 2.5\text{fb}$
- Pile Up: $\langle \mu \rangle = 50$.  
- Error of $\langle \frac{dE}{dx} \rangle$: $\delta \langle \frac{dE}{dx} \rangle = 0.05 \langle \frac{dE}{dx} \rangle$

They will give $O(10^4)$ hits inside the tracker.
We have about $\mathcal{O}(10^4)$ hits inside the tracker for both signal and BKG.

a1. Remove all the hits with $\frac{dE}{dx} \leq 3.0$ MeV/cm.

a2. For those remaining hits, we divide all of them into different classes based on their $dE/dx$.

$$
\left( \frac{dE}{dx} \right)_\text{aver}^a = \frac{1}{N_a} \sum_{i=1}^{N_a} \left( \frac{dE}{dx} \right)_i, \quad (23)
$$

If the next hit satisfies

$$
\left( \frac{dE}{dx} \right)_\text{next} - \left( \frac{dE}{dx} \right)_\text{aver}^c < 1.0 \text{ MeV/cm} \quad (24)
$$

, the hit is assigned to the $c_{th}$ class.

a3. For classes containing less than 80 hits are selected out and considered as quirk hits candidate.

a4. For classes that have more than 80 hits, we remove the sets of hits which can be reconstructed as helix. The remaining hits are hits of these classes are considered as quirk hits candidate. This step is found to be very useful for suppressing pileup events.
For classes that have more than 80 hits, the location of the possible center of circles (COC) in transverse plane for any two hits in the class is given by

\[ x_0 = \frac{1}{2(k_1 - k_2)} \left[ k_1 x_2 \left( 1 + k_2^2 \right) - k_2 x_1 \left( 1 + k_1^2 \right) \right], \tag{25} \]

\[ y_0 = -\frac{x_0}{k_1} + \frac{x_1}{2k_1} + \frac{y_1}{2}, \tag{26} \]

\[ k_1 = \frac{y_1}{x_1}, \quad k_2 = \frac{y_2}{x_2}. \tag{27} \]

Find the center of circle.

Remove the hits located on the circle with \( \delta R \leq 0.1 \text{cm} \).

The plane normal vector can be determined for any two of the remaining hits:

\[ \hat{n} = \langle \vec{r}_1 \times \vec{r}_2 \rangle \tag{28} \]

Account the total number \( N_{\text{left}} \) of hits within the distance of 30 \( \mu \text{m} \) to the given plane.

\[ d_i = |\vec{r}_i \times \hat{n}| \tag{29} \]

| \( m_Q/\text{GeV} \) | 200 | 500 | 800 | BKG |
|------------------|-----|-----|-----|-----|
| \( \epsilon_{1500} \) | 0.903 | 0.899 | 0.871 | 0.153 |

Table: The signal and background reduction efficiencies \( (\epsilon_{1500} = N_{\text{sig}}^{\text{left}} / N_{\text{tot}}^{\text{sig}}) \) after iteration until the total hit number \( (N_{\text{sig}} + N_{\text{pile}}) \) less than 1500.
Based on the reconstructed quirk plane, define

\[ S_p = \sum_{i=0}^{N} e^{-d_i/d_0} \times \left( \frac{dE}{dx} \right)_i, \quad (30) \]

\[ \Delta \rho = \frac{|S_p - S_p'|}{S_p}, \quad (31) \]

The angle between \( S_p \) and \( S_p' \) is \( \frac{\pi}{12} \).

1. \( E_\text{miss} \geq 500 \text{ GeV}, \)
2. \( \Delta \rho \geq 0.8, \)
3. \( S_p \geq 50 \text{ MeV/cm}, \)
4. \( N_{\text{left}} \geq 10. \)

**Figure**: Distributions for number of hits within the \( d < 30 \mu \text{m} \) of quirk plane (upper) and the distance-weighted ionization energy loss \( S_p \) (lower).
Figure: The 95\% C.L. exclusion limit for quirks with different quantum numbers at varying integrated luminosity of 13 TeV LHC.
For the parameter space of interest (i.e. $\lambda \sim \mathcal{O}(100 - 1000)$ eV, $m_Q \sim \mathcal{O}(100)$ GeV, $p_T(Q \bar{Q}) \gtrsim 100$ GeV), most of the quirk pair can leave total number of $\sim 26$ hits inside the tracker. Those hits are lie on the plane with thickness $\lesssim \mathcal{O}(100)$ $\mu$m.

The main background for the quirk signal will be abundant pileup events (which we take $\langle \mu \rangle = 50$) as well as the SM $Z(\rightarrow \nu \nu) +$jets process.

Three discriminative variables can be defined:

- $N_{\text{left}}$ within the $d < 30$ $\mu$m
- distance-weighted ionization energy loss: $S_p$
- derivative of $S_p$: $\Delta_p$

$\tilde{D}/\mathcal{D}$ is excluded up to 2.1/1.1 TeV.
$\tilde{E}/\mathcal{E}$ is excluded up to 450/150 GeV.

If you plan to recruit postdoctors, please contact with me.

zhangwenxing@mail.itp.ac.cn