Analysis on Inductance and Torque of PMSM Considering Magnetic Saturation

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Abstract. This paper analyses the surface-mounted PMSM which controlled by Id=0 vector control based on Ansoft in which finite element simulation of 2D static magnetic field can be operated on, then calculating and analysing the data with MATLAB, and then operating on the analysis of the change law of torque and inductance under different load conditions, and then paying more attention on the impact of magnetic saturation to torque and inductance. With the analysis of magnetic saturation, this paper puts forward a scheme of control and design used by PMSM.

1. Introduction
Permanent magnet motor comparing with the traditional electric field motor, not only more flexible and diverse on shape and size, but also has less loss, higher effectiveness and the strength of high power factor, high Power density on performance [1]. However, Permanent magnet motor is easy to appear the magnetic saturation phenomenon when it is working on low speed but large torque, which lead hard control [2]. So, Permanent magnet motors designed on all walks of life especially crane, are always have a larger maximum torque than they are needed actually. Although the design of having too much magnetic circuit margin can effectively prevent the occurrence of magnetic saturation, it results in waste of materials and energy which is neither economic nor energy saving when contrary to the country's sustainable development strategy. So, the study of permanent magnet synchronous motor’s characteristics on magnetic saturation can not only promote the actual application on more occasions, but also can provide a theoretical basis on the design optimization and research on control strategy of modern permanent magnet motors [3].

2. Calculation principle

2.1. Calculation principle of three-phase current excitation in simulation analysis
Stator d, q straight axis current change into A,B,C three-phase current [4]:

\[
\begin{bmatrix}
i_d \\
i_q \\
i_c
\end{bmatrix} = \sqrt{3} \begin{bmatrix}
\cos \theta & \sin \theta \\
\cos(\theta - \frac{2}{3} \pi) & \sin(\theta - \frac{2}{3} \pi) \\
\cos(\theta + \frac{2}{3} \pi) & \sin(\theta + \frac{2}{3} \pi)
\end{bmatrix} \begin{bmatrix}
i_d \\
i_q \\
i_c
\end{bmatrix}
\]

(1)
Where: \( i_d, i_q \) respectively, for the two-phase rotating coordinate system in the vertical axis current and cross-axis current with unit A. \( i_a, i_b, i_c \) is three-phase current, unit A. \( \theta \) is the angle (electrical angle) between the d axis and the A axis during rotation of the rotor.

2.2. The Calculation Principle of Right Angle Inductor
Since the d and q axes are imaginary virtual coordinate systems, the fluxes cannot be directly detected and therefore need to be detected indirectly. In the d-axis calculation, assuming that the d-axis and A-axis coincidence, then the d-axis flux( \( \varphi_d \) ) and A-axis flux( \( \varphi_A \) ) are equal, so you can get [5]:

\[
L_d = \frac{\psi_d}{i_d} = \frac{\varphi_d N_2}{i_d} = \frac{\varphi_d \times \frac{3}{2} N_1}{i_d} = \frac{\varphi_A N_1}{i_d} = \frac{\psi_A}{i_d}
\]  

(2)

Similarly, when calculating the q-axis inductance, the q-axis coincides with A and is calculated by detecting the flux value of the A-axis:

\[
L_q = \frac{\psi_q}{i_A}
\]  

(3)

Where: \( \psi_A, \psi_d \) respectively is the flux chain of A-axis, d-axis. \( L_d, L_q \) respectively is the d-axis and q-axis inductance. Respectively, \( \varphi_A, \varphi_d \) is the A-axis and d-axis flux. \( N_1, N_2 \) is turns of three-phase coordinate and turns of direct-axis and quadrature-axis.

2.3. Calculation principle of torque
In this paper, as using surface mount permanent magnet synchronous motor and \( i_d = 0 \) vector control. Combined with three-phase coordinate system to convert the two-phase rotating coordinate system after the principle of constant power, the stator current space vector( \( i_s \) ) is \( \sqrt{3} \) times than the effective value of the phase current( \( I_e \) ), so:

\[
T_{cm} = \frac{3}{2} p(\psi_a i_q - \psi_q i_d) = \frac{3}{2} p(\psi_a i_d + (L_d - L_q) i_d i_q) = \frac{3}{2} p \psi_a i_q = \frac{3}{2} p \psi_a \times i_d \cos \theta
\]  

(4)

Where: \( T_{cm} \) is the electromagnetic torque; \( \psi_a \) is the permanent magnet flux; \( p \) is the differential operator.

3. Motor parameters and finite element simulation and analysis based on Ansoft
In order to study the variation of torque and direct axis inductance with different loads, in this paper, a three-phase four-pole surface-mount permanent magnet synchronous motor is designed, with the \( i_d = 0 \) vector control adopted. Then, the permanent magnet motor is modeled and analyzed by finite element simulation based on 2d static magnetic field of Ansoft [6,7]. The specific motor model parameters are shown in Table 1.

|                | 4 |
|----------------|---|
| stator outer diameter/ (mm) | 120 |
| stator inner diameter/(mm)    | 75  |
| number of stator slots        | 24  |
| rotor outer diameter/(mm)     | 62  |
3.1. The establishment of simulation model

Combine table 1 parameters in Ansoft to establish two dimensional simulation model, as shown in figure 1, figure 2.

![Figure 1. 2d Simulation Model of Electric Machine](image1)

![Figure 2. Mesh after the model](image2)

Figure 1 is a two-dimensional simulation model studied in this paper, using distributed winding. In order to complete the finite element simulation analysis, we need to divide the model into the grid, figure 2 is the division of the grid after the model. In this paper, we use meshing based on the length of the internal unit of the model to divide the grid.

3.2. Simulation results analysis

According to the equation (4), this paper studies the inductance and torque variation under different loads by applying different phase current valid valued.

Through the current from 0~500A scan, we can get the inductance matrix curve in figure 3:

![Figure 3. Inductance matrix curve](image3)
In order to study the inductance of the quadrature axis, the inductance matrix obtained by the full model simulation on the 2d static magnetic field of Ansoft also needs to be transformed:

\[
\begin{bmatrix}
L_d \\
L_q
\end{bmatrix} = C^T L_{ABC} C
\]  

(5)

Where: 
\[
C = \begin{bmatrix}
\frac{2}{\sqrt{3}} & \cos\theta & \sin\theta \\
\cos(\theta - \frac{2}{3}\pi) & \sin(\theta - \frac{2}{3}\pi) \\
\cos(\theta + \frac{2}{3}\pi) & \sin(\theta + \frac{2}{3}\pi) 
\end{bmatrix}
\]

\(L_{ABC}\) is the inductance matrix.

Through the equation (5), combined with the simulation of the inductance matrix obtained under different load current under the axis of inductance, graph is drawn as shown in figure 4. As can be seen from figure 4: When the load current is applied between 0 and 175 A, \(L_d\) and \(L_q\) are equal, and basically remain unchanged, about 0.06mH; when the current is applied between 175 and 200A, \(L_d\) and \(L_q\) are both decline litter, and they are not equal anymore; when the load current is applied to more than 200A, \(L_d\) maintain a slow downward trend, and then \(L_q\) decline sharply.

![Figure 4. Curve of Inductance](image1)

Figure 5 shows the variation of the torque at different load currents obtained by Ansoft simulation. As shown in Figure 5, when the load current is applied between 0 and 75 A, torque shows a slow growth trend as the unit current increases the load is not high at this time; when the load current increases between 75 and 175A, Torque and phase current have a linear relationship, consistent with the formula (4), as the motor enters the torque high efficiency growth stage; when the current is added to 175 ~ 225A, torque with the load growth is no longer maintain a linear relationship, showing a slowing trend, as the motor begins to reach the magnetic saturation phase, so that the motor has passivation phenomenon with the load growth torque; when the current reaches 225A, the torque growth reaches the peak, that is, the maximum torque is reached; When the current is applied to 225A ~ 425A, the torque starts to decrease with the increase of current, as the motor torque is decreasing with the deepening of magnetic saturation; When the current is loaded to 425A, the magnetic circuit is fully saturated and the torque is no longer changed as the current increases.

![Figure 5. Variation Curve of Torque](image2)

3.3. Magnetic saturation analysis

Combined with the above simulation results, the direct axis inductance (\(L_d\)) shows a slow decline trend with the saturation of the magnetic saturation; quadrature axis inductance (\(L_q\)) shows a sharp decline trend with the deepening of magnetic saturation; With the deepening of magnetic saturation, the trend of linear growth is slowed down and the passivation phenomenon occurs. With the deepening of magnetic saturation, the torque reaches the maximum value and begins to descend until the last magnetic circuit is completely saturated and the torque is kept constant value unchanged.
In order to facilitate the proposed control scheme, the magnetic saturation is defined as three stages in conjunction with the change of torque in figure 5: The stage when the electromagnetic torque begins to passivate, and the torque continues to grow to the maximum is magnetic saturation first stage ($i_m \sim i_a$); The stage when the electromagnetic torque starts to decrease from the maximum value as the load current increases until the full magnetic saturation is magnetic saturation second stage ($i_a \sim i_b$); The stage when the torque maintains a constant value as full of magnetic saturation is magnetic saturation of the third stage ($\geq i_b$).

4. Conclusion
In order to improve the utilization of the magnetic circuit, it is not necessary to completely discard all the stages of magnetic saturation, we can still implement the appropriate control when permanent magnet synchronous motor gets magnetic saturation until the motor torque reaches the maximum torque combined with the law that the first phase of magnetic saturation torque will gradually increase with the increased load. Magnetic saturation of the second stage torque appears to decline, when the control has been lost meaning, thus, the magnetic saturation of the second and third stages is completely discarded. In short, the control scheme is to make full use of the first phase of magnetic saturation. Combined with the control scheme, when we design permanent magnet synchronous motor, we can design the maximum torque value for the torque limit, as it can greatly reduce the motor size.

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