Is CP a Gauge Symmetry?

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Abstract

Conventional solutions to the strong CP problem all require the existence of global symmetries. However quantum gravity may destroy global symmetries, making it hard to understand why the electric dipole moment of the neutron (EDMN) is so small. We suggest here that CP is actually a discrete gauge symmetry, and is therefore not violated by quantum gravity. We show that four dimensional CP can arise as a discrete gauge symmetry in theories with dimensional compactification, if the original number of Minkowski dimensions equals $8k + 1$, $8k + 2$ or $8k + 3$, and if there are certain restrictions on the gauge group; these conditions are met by superstrings. CP may then be broken spontaneously below $10^9$ GeV, explaining the observed CP violation in the kaon system without inducing a large EDMN. We discuss the phenomenology of such models, as well as the peculiar properties of cosmic “CP strings” which could be produced at the compactification scale. Such strings have the curious property that a particle carried around the string is turned into its CP conjugate. A single CP string renders four dimensional spacetime nonorientable.
1. Motivation: the strong CP problem

We still do not know why CP is violated by the weak interactions but not by the strong. In order to satisfy the upper bound of $1.2 \times 10^{-25}$ e-cm on the neutron electric dipole moment \(1\) the strong CP violating parameter \(\bar{\theta}\)

\[
\bar{\theta} \equiv \theta_{\text{QCD}} + \arg \det m_q
\]

must satisfy \(\bar{\theta} \lesssim 2 \times 10^{-10}\) for the usually quoted up to down quark ratio \(m_u/m_d = 0.56\) \(2\). The absence of strong CP violation violates ‘t Hooft’s naturalness condition that a parameter is only allowed to be very small if setting it to zero increases the symmetry of the theory \(3\). Simply setting \(\bar{\theta}\) to zero requires fine tuning \(\theta_{\text{QCD}}\), the coefficient of \(\frac{\alpha_{\text{QCD}}}{8\pi} G \tilde{G}\), to cancel \(\arg \det m_q\). Since CP is broken by the weak interactions, the fine tuning does not increase the symmetry of the standard model. This is the strong CP problem.

There are presently three possible solutions to the strong CP problem, all of which involve imposing global symmetries on the low energy world\(1\). One possible solution is that the up quark is massless. In this case there is an anomalous \(U(1)\) symmetry at the QCD scale, rendering \(\theta_{\text{QCD}}\) unphysical. A massless up quark is not necessarily in conflict with current algebra \(6\) since an effective up quark mass may be mimicked by instanton effects \(7\) or new low energy interactions \(8\). Such theories require that one impose a chiral global symmetry (which is anomalous) to eliminate the up quark’s Yukawa coupling to the Higgs.

A second solution is that of Peccei and Quinn, who realized that an anomalous global \(U(1)_{\text{PQ}}\) symmetry could be realized nonlinearly at low energies, still solving the strong CP problem while allowing all the quarks to get masses from the Higgs \(9\). This nonlinear realization of \(U(1)_{\text{PQ}}\) leads to a light pseudo Goldstone boson, the axion \(10\). The axion couplings may be adjusted \(11\) to avoid laboratory \(12\), astrophysical \(13\) and cosmological \(14\) constraints. Neither the axion nor the massless up quark solutions are natural in the \(’t\) Hooft sense since in either case the global \(U(1)\) symmetry is only approximate—being anomalous—and significant \(U(1)\) violating instanton effects can occur that lead to an unacceptably large \(\bar{\theta}\) in the Peccei-Quinn case \(15\).

The third solution to the strong CP problem differs from the first two in that instead of an anomalous \(U(1)\) symmetry, one imposes an exact discrete symmetry—either CP or

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\(1\) In addition there has been the proposal that there is no strong CP problem—see \(4\); we do not see how this solution could avoid leading to a light \(\eta'\) meson. There is also an argument that wormhole physics may set the effective \(\bar{\theta}\) to the CP conserving value of \(\pi\) \(3\).
P—which is broken spontaneously. The parameter $\bar{\theta}$ is then finite and calculable, and in a special class of models can be small enough to be consistent with experiment [16,17].

In this paper we will argue that only the third solution, spontaneously broken CP (or P) may be viable when the effects of quantum gravity are considered. Quantum gravitational effects arising from wormholes [18], virtual black hole formation and evaporation [19] or nonperturbative effects in string theory [20], may lead to violation of some or all global symmetries in the effective theory below the Planck mass [21]. Presently it is unknown how large the symmetry violating quantum gravitational effects are or whether it is possible to escape the conclusion that there are no exact global symmetries. In any case, both continuous gauge symmetries and unbroken discrete subgroups of gauge symmetries (“discrete gauge symmetries”) are thought to be preserved by Planck scale physics, as they are violated neither by wormholes nor black holes [22,23]. Approximate continuous global symmetries may arise as accidental symmetries (symmetries which are automatically respected by all gauge invariant renormalizable Lagrangians containing some set of fields), but are otherwise expected to be violated by higher dimension operators suppressed by inverse powers of the Planck mass $m_P$.

This leads one to contemplate a modified version of ’t Hooft’s naturalness principle—which we will call “strong naturalness”—namely that all small parameters in the world must be protected by gauge symmetries.

Is the standard model consistent with strong naturalness? Both lepton and baryon number in the standard model are examples of accidental global symmetries, enforced by $SU(3) \times SU(2) \times U(1)$ invariance. The lowest dimension allowed operators that violate $B$ and $L$ are dimension six, which cause no problem if suppressed by $m_P^2$. In the supersymmetric standard model there are dimension four and five operators which cause problems such as proton decay, but they can be eliminated by imposing discrete gauge symmetries [24]. The approximate global flavor symmetries that arise in the standard model due to the smallness of some of the Yukawa couplings could also be protected from gravity by discrete gauge symmetries which are spontaneously broken below the Planck mass. The small value of the Higgs mass looks unnatural in the standard model, but in supersymmetric (SUSY) models, the soft SUSY violating terms responsible for the Higgs potential could be protected from gravitational corrections by local supersymmetry at short distances (although there remains the “$\mu$-problem”, which must be solved by other means). This leaves the cosmological constant (about which we have nothing new to say) and $\bar{\theta}$ as the remaining, curiously small parameters in the standard model.
At first sight, the strong CP problem does not appear to be consistent with strong naturalness. A vanishing up quark mass cannot be protected by a discrete or continuous gauge symmetry, as any symmetry which protects only the up quark mass is anomalous in the standard model—and the up quark Yukawa coupling would have to be less than $10^{-14}$ for the EDMN to be unobserved. The axion solution cannot be protected by a gauge symmetry either, since the $U(1)_{PQ}$ symmetry is also anomalous (by construction) and cannot be gauged. PQ violating operators arising from Planck mass physics can spoil the axion solution to the strong CP problem [25-27] (though also see ref. 3, for a discussion of the effects of wormholes on $\theta$ and the axion potential). If Planck scale physics induces PQ-violating gauge invariant operators of dimension $d$ with coefficients $\mathcal{O}(1/m_P)^{(d-4)}$, then the axion decay constant $f_a$ must be well below the Planck mass or there will still be a strong CP problem even if axions exist. If $f_a$ is above the astrophysical bound of $10^{10}$ GeV then the dimension of the induced operators must be greater than ten. Models with an accidental anomalous $U(1)$, respected by all terms up to any desired dimension can be built [28], but are complicated and severely constrained. In particular, such models frequently have many colored particles and strong QCD interactions at short distances, which means that small QCD instantons may interfere with the axion’s ability to solve the strong CP problem [13].

Models with spontaneously broken CP can solve the strong CP problem without requiring that any global symmetries other than CP itself be imposed on the Lagrangian [29]. While this scenario satisfies ’t Hooft’s original naturalness criterion, it fails under strong naturalness as CP could be violated by gravitational effects. For example, black hole no hair theorems imply that a black hole which has swallowed a CP-odd particle cannot be distinguished from one which swallows a CP-even particle of the same mass. However if CP were a discrete gauge symmetry then black holes would carry discrete CP gauge hair [22,23], and virtual black holes could not violate CP. Discrete gauge symmetries are also not violated by wormholes [22,23], and should be preserved by all quantum gravitation effects. In the next section we will show that CP can arise as a discrete gauge symmetry, provided that there are more than four spacetime dimensions. In §3 we discuss the spontaneously broken CP solution to the strong CP problem and restrictions on the scale of spontaneous CP violation due to quantum gravity. In §4 we examine one of the most amusing possible consequences of CP as a gauge symmetry: cosmic CP strings. Space containing a CP string has the curious property that a particle can be transported around an uncontractible loop and transformed into its CP conjugate, and that a four dimensional universe containing a single string is nonorientable, although the higher dimensional manifold is orientable. In §5 we comment briefly on the possibility of parity as a discrete gauge symmetry.
2. CP as a discrete gauge symmetry

A discrete gauge symmetry arises when a continuous gauge symmetry G is spontaneously broken to a subgroup H which has disconnected components. A classic example \[30-32,23\] is the symmetry breaking pattern \(SO(3) \to O(2)\), which can be realized by giving the traceless symmetric tensor representation of \(SO(3)\) a vev proportional to the matrix \(\text{diag}(1,1,-2)\). Such a vev leaves a gauged \(U(1)\) symmetry and a discrete charge conjugation symmetry unbroken. Now charge conjugation is an element of the \(SO(3)\) gauge group, which exchanges particles with antiparticles carrying the opposite \(U(1)\) charge.

Can CP arise as such a discrete gauge symmetry? A major difference between C and CP is that CP exchanges a left handed Weyl spinor with its right handed complex conjugate. Since left and right handed Weyl spinors are in different irreducible representations of the Lorentz group, CP does not commute with Lorentz transformations. If CP is to be an element of a continuous local symmetry which does not commute with the four dimensional Lorentz group, then the Lorentz group must be extended by introducing extra dimensions. CP may then be embedded in this extended group.

Another apparent obstacle to imposing CP as a local symmetry is that the usual CP transformation given by

\[
\begin{align*}
\psi_L(x_\mu) &\to -\sigma_2 \psi_L^*(x^\mu)
\psi_R(x_\mu) &\to \sigma_2 \psi_R^*(x^\mu) \\
\phi(x_\mu) &\to \pm \phi^*(x^\mu) \\
A_\mu(x_\mu) &\to \pm A^\mu(x^\mu)
\end{align*}
\]

(2.1)

reverses all three spatial dimensions as well as exchanging all fields with their complex conjugates. (In the above expression \(\phi, \psi,\) and \(A_\mu\) are (pseudo) scalar, fermion, and (pseudo) vector fields respectively, and \(\eta_{\mu\nu} = \text{diag}(+, -, -, -)\)). The reflection of the spatial dimensions is not a symmetry of the Lagrangian, although it can be a symmetry of the action; furthermore, it cannot be obtained by any local transformation.

An alternative is to define a passive CP—denoted as \(\tilde{\text{CP}}\) — which does not involve global spatial reflections. It is defined as

\[
\begin{align*}
x_\mu &\to \bar{x}_\mu = x^\mu \\
\psi_L(x_\mu) &\to -\sigma_2 \psi_L^*(\bar{x}^\mu) = -\sigma_2 \psi_L^*(x_\mu) \\
\psi_R(x_\mu) &\to \sigma_2 \psi_R^*(\bar{x}^\mu) = \sigma_2 \psi_R^*(x_\mu) \\
\phi(x_\mu) &\to \pm \phi^*(\bar{x}^\mu) = \pm \phi^*(x_\mu) \\
A_\mu(x_\mu) &\to \pm A_\mu(\bar{x}^\mu) = \pm A_\mu(x_\mu)
\end{align*}
\]

(2.2)

where the first transformation is an orientation changing general coordinate transformation. Note that derivatives of fields transform as

\[
\partial_\mu \phi(x_\mu) \xrightarrow{\tilde{\text{CP}}} \bar{\partial}_\mu(\pm \phi^*(\bar{x}^\mu)) = \partial^\mu(\pm \phi^*(x_\mu)).
\]

(2.3)
Whereas under the usual CP we have $\mathcal{L}(x_\mu) \to \mathcal{L}(x^\mu)$, under $\hat{\text{CP}}$ $\mathcal{L}(x_\mu) \to \mathcal{L}(x_\mu)$ for a CP invariant Lagrangian. However any action which is invariant under the usual definition of CP is invariant under $\hat{\text{CP}}$ and vice versa. For example, the parity violating term $\int d^4 x \, \bar{G} \hat{G}$ is eliminated from the action since although this term is invariant under proper general coordinate transformations, it is odd under coordinate transformations with negative Jacobian, hence odd under $\hat{\text{CP}}$. For the remainder of this paper we will drop the hat and simply refer to the passive transformation (2.2) as “CP”.

We now contemplate the possibility that the transformation CP is an element of a continuous local symmetry $G$ of a higher dimensional theory. The group $G = G_L \times G_g \times G_{YM}$, where $G_L = \text{spin}(d-1,1)$, the $d$-dimensional Lorentz group, $G_g = d$-dimensional general coordinate transformations with positive Jacobian and $G_{YM}$ is the internal Yang-Mills group. In order to have a chiral fermion spectrum in four dimensions $G_{YM}$ must not be trivial [33]. We assume the four dimensional CP transformation is given by the product of $X_L(\in G_L)$, $X_g(\in G_g)$, $X_{YM}(\in G_{YM})$ i.e.

$$\text{CP} = X_L X_g X_{YM},$$

(2.4)

and enumerate the possibilities for $d$ and $G_{YM}$.

Note that alternative CP transformations are possible, which require enlarging the particle spectrum. Any symmetry which reverses all gauge charges, exchanges left and right handed fermions, and reverses the orientation of space can be called CP. For instance the fermion spectrum could be doubled by introducing mirror particles and CP could exchange each fermion with its mirror. The restrictions of the dimensionality of space and the gauge group which arise from requiring that CP be embedded in a continous gauge symmetry will depend on the definition of CP. Since the observed approximate CP exchanges all fields with their own complex conjugates, we will define CP by eq. (2.2).

In order for the CP transformation to be an element of the local symmetry group $G$ of the underlying $d$-dimensional ($d > 4$) theory we need all four dimensional fields to be in the same irreducible representation of $G$ as their complex conjugates. If both $G_L$ and $G_{YM}$ admit an inner automorphism (an automorphism which is an element of the group) which exchanges each element of a representation with its complex conjugate, then we identify these automorphisms as $X_L$ and $X_{YM}$ respectively. This is only possible if the Yang-Mills group $G_{YM}$ is one of the Lie groups $E_8$, $E_7$, $SO(2n+1)$, $SO(4n)$, $Sp(2n)$, $G_2$ or $F_4$ (or a product of these groups). Only these groups admit an inner automorphism.
which changes the sign of the entire Cartan subalgebra \[34\]. For the Lorentz group, the irreducible spinor representations must be Majorana. For an even number of spacetime dimensions, we require that the Majorana condition commutes with the Weyl condition, \textit{i.e.} we need Majorana-Weyl spinors. (If the Majorana and Weyl conditions do not commute then a Majorana spinor is the direct sum of a complex Weyl spinor and its conjugate, and a Lorentz transformation cannot exchange the Weyl spinors with their complex conjugates.) Majorana spinors are allowed for odd dimensional spacetimes of dimension \(d = 8k+1, 8k+3\) and Majorana-Weyl spinors are only possible in \(8k + 2\) dimensions \[35\], and for these dimensions the Lorentz group contains an inner automorphism which complex conjugates all representations. For \(d = 8k+1, 8k+2, \text{and} 8k+3\), and for the Yang-Mills groups specified above, we can define four dimensional CP to be the product of an inner automorphism of \(G_{YM}\), a \(G_g\) transformation \(X_g\) which reverses the orientations of both 4-D Minkowski space and the compactified manifold, and the \(G_L\) Lorentz transformation

\[
X_L = \begin{pmatrix}
1 & -1 & -1 & 0 \\
-1 & 1 & -1 & 0 \\
-1 & -1 & 1 & 0 \\
0 & 0 & 0 & K_L
\end{pmatrix}
\]

(2.5)

where \(K_L\) is a \((d-4)\) dimensional real matrix satisfying \(K_L K_L^T = 1\) and \(\det(K_L) = -1\). \(X_g\) must be chosen to include a compactified coordinate transformation \(\theta_i \rightarrow -\theta_i\), where the vector fields \(\partial/\partial \theta_i\) generate the Cartan subalgebra of the continuous isometry group of the compactified dimensions. Then the transformation \(CP = X_L X_g X_{YM}\) exchanges all gauge charges of four dimensional fields with the gauge charges of their antiparticles, including gauge interactions arising from the isometries of the compactified space. Both \(X_L\) and \(X_g\) are orientation preserving on the higher dimensional space and are elements of the local symmetry group of the full theory.

An interesting case where CP can arise as a discrete local symmetry occurs in superstring theory \[20\]. The popular 10 dimensional superstring theories with gauge group \(SO(32)\) or \(E_8 \times E_8\) satisfy the conditions given above for four dimensional CP to be a gauge transformation. These are also examples of higher dimensional theories which can have an acceptable low energy particle spectrum and gauge group, for some compactifications. Furthermore, it is possible for CP to remain unbroken after compactification in realistic
models. For instance, as was noted by Strominger and Witten when the $E_8 \times E_8$ superstring has six dimensions compactified on an internal manifold of $SU(3)$ holonomy, with the spin connection embedded in the gauge connection, the existence of a discrete orientation reversing isometry of the internal manifold guarantees CP invariance of the four dimensional theory. Furthermore for some compactifications the four dimensional effective theory may also contain extra gauge symmetries (e.g. additional $U(1)$’s or a a four dimensional grand unified $E_6$ gauge symmetry) and fermion fields (additional quarks which are vector-like under the low energy $SU(3) \times SU(2) \times U(1)$ gauge symmetry with the quantum numbers of a right handed down quark plus its mirror). These additional fields and gauge symmetries could play the role of the new particles and symmetries introduced in refs. \cite{16,29,37} to constrain the form of the fermion mass matrices, as needed to solve the strong CP problem in spontaneously broken CP models.

Four dimensional C, P, and CP invariance arising from higher dimensional theories has been discussed before \cite{20,40,36} for the purpose of ruling out some higher dimensional theories. However this earlier work on CP invariance in higher dimensional theories has been motivated by a goal orthogonal to ours, namely it has been assumed that CP should \emph{not} be a symmetry of the effective four dimensional theory. What we are proposing is that CP remains an exact discrete gauge symmetry below the compactification scale and is spontaneously broken at much longer distances, accounting for the violation observed in the weak interactions. The CP violating vacuum of the full theory can be considered to be a perturbation from the CP conserving configuration. If the deformation is small enough and can be described by four dimensional scalar fields, then we can study the CP conserving effective four dimensional theory which arises from compactification to the CP conserving pseudo-vacuum. This effective theory includes CP odd scalar fields whose vevs are non zero in the ground state. Planck scale physics will not induce CP violating operators in the effective theory, although in general we do expect non renormalizable CP conserving operators, suppressed by powers of the compactification scale, which we take to be near the Planck mass. We discuss the possible effects of such operators in the next section.

\footnote{It is commonly thought that string theories can solve the strong CP problem by means of the axion mechanism \cite{38}, but it is far from clear that the mechanism survives nonperturbative contributions to the axion potential from hidden sector gauge interactions \cite{17} and string world sheet instantons \cite{39}.}
3. Spontaneously broken CP and the strong CP problem

If CP is spontaneously broken then the strong CP parameter $\theta$ is calculable, but not necessarily small. A class of models in which $\theta$ does come out to be naturally small, in the sense of 't Hooft, is described in refs. [16,37,41]. In these papers only renormalizable Lagrangians are considered, and the issue of strong naturalness is not discussed, since CP is assumed to be an exact global symmetry. However, as we have just shown, CP could be a discrete gauge symmetry which is left unbroken at the compactification scale, and then quantum gravitational effects could not induce CP violating operators in the effective four dimensional theory. However there could be nonrenormalizable terms arising from Planck scale physics and suppressed by powers of the Planck mass, which will lead to additional CP violating effects below the scale of spontaneous CP violation. Here we show that inclusion of such nonrenormalizable operators in the effective Lagrangian may spoil the spontaneously broken CP solution to the strong CP problem. We will argue that this solution should survive Planck scale effects if the scale of spontaneous CP violation is below $10^9$ GeV.

First let us review a class of models utilizing the spontaneously broken CP solution to the strong CP problem. Models which satisfy the following criteria can give the standard model with sufficiently small strong CP violation as a low energy effective theory, when the effects of Planck scale physics are ignored.

1. The ordinary Higgs doublet, $H$, acquires an $SU(2) \times U(1)$ breaking vev whose phase may be chosen to be zero (In multi Higgs doublet models if the relative phases of the Higgs doublets are nonzero the neutron electric dipole moment may come out too large [12]). The Higgs couples with real Yukawa couplings to $n_f$ families of type “F” quarks. The $F$ quarks are in ordinary chiral representations of $SU(3) \times SU(2) \times U(1)$.

2. CP is broken spontaneously by the vevs of some complex scalar fields $\phi_i$ which are singlets under $SU(3) \times SU(2) \times U(1)$. The ordinary gauge symmetries prevent any tree level Yukawa couplings of type $F-F-\phi_i$.

3. There is another set of quarks called $C$ and $\bar{C}$, distinguished from the $F$–type quarks by new symmetries. In ref. [16] additional global symmetries were used, but these may easily be gauged. The $C$-type quarks have ordinary $SU(3) \times SU(2) \times U(1)$ quantum numbers, and the $\bar{C}$’s have mirror quantum numbers, i.e. $SU(3) \times SU(2) \times U(1)$ allows a mass term connecting $C$ and $\bar{C}$. However this mass term must be real, i.e. any coupling of form $\bar{C}-C-\phi_i$ is forbidden by the new symmetries.
4. Yukawa couplings of type $\mathcal{C}$-$F$-$\phi_i$ are allowed, which is how CP violation is communicated to the ordinary quarks.

5. No Yukawa couplings of type $C$-$C$-$H$ are allowed by the new symmetries.

In this class of models at tree level the quark mass matrix has the form

\[
\begin{pmatrix}
F_R & C_R & \overline{C}_R \\
F_L & \text{real} & 0 & \text{complex} \\
C_L & 0 & 0 & \text{real} \\
\overline{C}_L & \text{complex} & \text{real} & \text{complex}
\end{pmatrix},
\] (3.1)

which has real determinant.

The masses involving the $C$ and $\overline{C}$ quarks do not break $SU(3) \times SU(2) \times U(1)$ and may be much larger than the weak scale. There will always be at least $n_f$ families of light quarks with weak scale masses, which may be complex linear combinations of $F$ and $C$-type quarks. All $\overline{C}$ quarks may get heavier than the weak scale by combining with the orthogonal combinations of $F$ and $C$ fields. At low energies the light quark mass matrix will be complex, and ordinary weak CP violation can occur via the usual unremovable phase in the Kobayashi-Maskawa weak mixing matrix. If the CP-violating $\phi_i$ vevs are big and the $\overline{C}$ quarks fairly heavy (at least $\mathcal{O}(\text{TeV})$) then the low energy effective theory can be the standard model and flavor changing neutral currents are suppressed.

For a discussion of radiative corrections to $\bar{\theta}$ in these models see ref. [41]; for the radiative corrections in the supersymmetric case see ref. [37]. These radiative corrections can easily be as small as $\mathcal{O}(10^{-11})$.

Note that the determinant of (3.1) is real only if the mass term connecting $C$ and $\overline{C}$ is real, which may be guaranteed by a gauge symmetry allowing $\overline{C}$-$C$ terms but forbidding $\overline{C}$-$C$-$\phi_i$ Yukawa couplings. In general, however, such a symmetry does not forbid $\overline{C}$-$C$-$\phi_i$-$\phi_j^*$ couplings. These couplings are nonrenormalizable, and so suppressed by $m_P$, but can give the $\overline{C}$-$C$ mass term a phase proportional to $\langle \phi_i \rangle / m_P$. Thus we expect the effective tree level $\bar{\theta}$ to receive a contribution of order

\[
\bar{\theta} = \mathcal{O} \left( \frac{\langle \phi_i \rangle}{m_P} \right),
\] (3.2)

which will be smaller than $10^{-10}$ if the scale of spontaneous CP violation is below $10^9$ GeV.
4. CP strings and walls

Imagine returning from a long celestial journey, only to find your loved ones to be made of antimatter, with their hearts on the wrong side. Apparently you travelled around a cosmic “CP string”. These CP strings could exist as defects if CP were an unbroken discrete gauge symmetry. Fortunately, CP strings cannot survive below the scale of spontaneous CP breaking, but they may play an important role in the early universe by eliminating cosmologically undesirable CP domain walls [43].

Topologically stable strings in four spacetime dimensions are possible whenever a simply connected internal gauge group G breaks to a group H containing disconnected components (stable strings are possible whenever the manifold G/H is not simply connected [31]). An amusing example is the “Alice string” [23,30,32] which arises when charge conjugation is an element of a spontaneously broken gauge group. In the presence of an Alice string, electric charge is double valued. Furthermore the relative charge between two particles can be changed by sending one of them around the string.

Here we consider the case where the action is invariant under a symmetry group G, which includes general coordinate transformations in $d > 4$ dimensions as well as internal symmetries. We assume the vacuum configuration spontaneously breaks G to a group H which includes CP as a discrete element. Since CP is an unbroken element of a spontaneously broken continuous symmetry then stable CP strings should exist [44].

A CP string is a defect in the spacetime manifold, as well as in the internal gauge group symmetry breaking order parameter. An object parallel transported around such a string could experience, for example, a $180^\circ$ rotation in a plane containing one compactified and one ordinary dimension, while having all gauge charges are conjugated as with the Alice string. When two explorers with right-handed coordinate systems start from the same point in space, pass around opposite sides of a CP string, and reencounter each other on the other side, each one will think that the other person has been CP conjugated, while claiming that they remain unchanged themselves. To bolster her claim that she remains unchanged, explorer B points to the right-handed reference frame she has brought with her; however explorer A sees her hold up a left-handed reference frame constructed out of antimatter, and isn’t convinced. In fact, to claim that one has changed and the other has not is not a gauge invariant statement. In order to avoid potentially dangerous physical conflict, the two explorers agree to set up an imaginary surface ending on the string, with the convention that anyone crossing this surface redefines “left-handed” to mean “right-handed”
and “matter” to mean “antimatter”. This is entirely analogous to the international date-line, where we reset our watches by 24 hours, and like the dateline, its position is a matter of convention. Now when the explorers meet, they can agree on the definitions of matter and antimatter, left-handed and right-handed. We will refer to this surface as the “CP dateline”\(^3\). Note that in the presence of such a string, the full manifold—including the compactified dimensions—is orientable, while the four-dimensional spacetime is not. Such strings have many peculiar properties \([45]\).

The CP string should be very heavy, having mass/length comparable to the compactification scale squared. Inside the string, at least some of the higher dimensional symmetries and extended gauge symmetry are realized. It is not clear whether such strings ever get formed in the early universe, since the compactification of the extra dimensions is not necessarily a phase transition, but if they are formed their cosmological behavior could be similar to other gauge strings which have been proposed to seed galaxy formation and large scale structure \([46]\), and, if they survive until recent epochs they must be characterized by a string tension less than \(10^{-5}m_p^2\) so that they do not contribute large inhomogeneities to the microwave background \([47]\). In many models of compactification it is difficult to see how strings could be produced without also producing heavy magnetic monopoles, which are troublesome cosmologically as they tend to contribute too much mass density to the universe \([48]\), however the monopoles may be eliminated by either inflation (which also eliminates the strings) \([49]\) or the Langacker-Pi mechanism \([50]\).

The era of CP strings ends when some complex scalars acquire CP violating vevs, and the strings become attached to domain walls. One must distinguish between the effects of the wall, and those of the CP dateline surface; they can be chosen to be coincident by convention, but it is not necessary. We will consider them not to be coincident, for pedagogical reasons. On traversing the domain wall but not the dateline, the CP violating phase in the scalar vev changes sign, and therefore so does the the CP violating phase in the low energy Hamiltonian. For example, consider an experiment involving kaons in the vicinity of the domain wall. When a \(K_L\) meson, with wavefunction \([51]\)

\[
|K_L\rangle = \frac{(1 + \epsilon)|K^0\rangle - (1 - \epsilon)|\bar{K}^0\rangle}{\sqrt{2(1 + |\epsilon|^2)}}
\]

\(^3\) The CP dateline is entirely analogous to the Preskill-Krauss construction for Alice strings \([23]\).
is transported rapidly (i.e. nonadiabatically) across the domain wall, its wavefunction is the same on the far side, but it is no longer a mass eigenstate, since on the far side of the wall there is a different Hamiltonian: $H(-\epsilon)$ instead of $H(\epsilon)$. What was a $K_L$ is now a superposition of the $K'_L$ and $K'_S$ mesons, which are the mass eigenstates of $H(-\epsilon)$. This is a physical effect: the $K'_S$ component decays away quickly and can be observed. The remaining $K'_L$ component will eventually decay as well, but one will notice that its CP violating leptonic decay mode favors electrons over the usual positrons. The experimenter who carried the meson through the domain wall will still be made of electrons and nucleons. If a second experimenter carries a second $K_L$ meson from the same initial point to the same final point, choosing a trajectory that does not pass through the domain wall, but instead circumnavigates the string at the wall boundary, she will pass through the CP dateline. At the dateline $\epsilon \rightarrow -\epsilon$ both in the hamiltonian and the meson wavefunction, by convention. The kaon will remain a mass eigenstate, and will also decay preferentially into electrons (with greater amplitude than the first meson, since there is no $K'_S$ component), but this experimenter is now made of positrons and antinucleons. That the two mesons behave differently is a gauge invariant fact: one had a component decay with the $K_S$ lifetime while the other didn’t. This is acceptable because the domain wall, unlike the CP dateline, is a physical barrier which one meson traversed (nonadiabatically) and the other did not.

The cosmological properties of walls ending on strings have already been discussed \[14,52-55\]. Neither the strings nor the domain walls are topologically stable below the CP breaking scale. A single domain wall is metastable, since, although holes bounded by strings can appear in the wall, the probability of a hole appearing which is large enough to grow is exponentially suppressed by

$$\exp(-O(\mu^3/\sigma^2)) ,$$

where $\mu$ is the string tension and $\sigma$ is the wall tension. However if strings left over from a compactification transition are present at the scale of spontaneous CP violation, the resulting network of walls and strings soon vanishes, unless a period of inflation occurs between the two phase transitions.
5. Parity as a gauge symmetry

Spontaneously or softly broken parity has also been proposed as a solution to the strong CP problem \[17\]. The usual four dimensional parity transformation can be embedded in the local general coordinate invariance and Lorentz symmetries, provided there are five or more dimensions. However models with unbroken ordinary four dimensional parity are not phenomenologically viable since the observed fermions are in a chiral representation of the gauge group. This problem cannot be alleviated via spontaneous parity breaking. However the product of space-time parity and an internal symmetry transformation may also be called parity, as in left-right symmetric models \[56\] where the parity transformation also exchanges two inequivalent $SU(2)$ gauge groups. In these theories parity does not commute with internal gauge transformations and fermions can be in chiral representations of the gauge group. Such a four dimensional theory could conceivably arise from the compactification of a higher dimensions. Four dimensional parity is then a product of an inner automorphism of the Yang-Mills group, a local Lorentz transformation, and a general coordinate transformation of the higher dimensional theory. Obtaining fermions in chiral representations of the four dimensional group places severe restrictions on the number of extra dimensions and the higher dimensional theory \[33,40\]. An interesting feature of this sort of parity transformation is that for some choices of the spectrum and gauge groups there is no phenomenological requirement for parity to be broken \[57\]. In the unbroken parity models parity is a product of the usual parity and the exchange of the usual $SU(3) \times SU(2) \times U(1)$ gauge group with a “shadow” $SU(3) \times SU(2) \times U(1)$. If the parity transformation of such models is an element of a continuous gauge symmetry then it is possible that parity strings are topologically stable and could exist in our current universe. Circumventing such a parity string would be quite disconcerting, as not only would you find your loved ones with their hearts on the wrong side, but they would be invisible, since they would now be made of matter which doesn’t carry the same gauge interactions as the matter you are made of\[4\].

\[4\] J. Preskill, in \[32\], remarked on a similar possibility in connection with unbroken charge conjugation.
6. Summary

Planck mass physics effects such as wormholes may violate global symmetries, including the Peccei-Quinn symmetry usually invoked to solve the strong CP problem. However there exists a solution to the strong CP problem which does not involve any global symmetries—namely that CP (or P) could be a discrete gauge symmetry which is spontaneously broken below well the Planck mass. If the spontaneous CP violation scale is lower than $10^9$ GeV, then Planck scale physics should have little effect on this solution. With such a low scale for spontaneous CP violation one must worry about several cosmological problems. In general, the spontaneous breaking of a discrete symmetry produces cosmologically undesirable domain walls. Domain walls produced at the spontaneous CP or P violation scale will not be topologically stable, since the discrete symmetries are embedded in a continuous symmetry, but can be metastable. There are at least two possibilities for avoiding a cosmological domain wall problem; either inflation occurs and the reheating temperature after inflation is below the scale of spontaneous CP violation, or the domain walls can end on cosmic CP strings left over from an earlier stage of symmetry breaking and the entire network of walls and strings will disappear. Baryogenesis must take place during or after spontaneous CP violation; there are many possibilities for low energy baryogenesis \[58\].

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