Abstract—We introduce a modulation for unsourced massive random access whereby the transmitted symbols are rank-1 tensors constructed from Grassmannian sub-constellations. The use of a low-rank tensor structure, together with tensor decomposition in order to separate the users at the receiver, allows a convenient uncoupling between multi-user separation and single-user decoding. The proposed signaling scheme is designed for the block fading channel and multiple-antenna settings, and is shown to perform well in comparison to state-of-the-art unsourced approaches.

I. INTRODUCTION

Massive random access, whereby a large number of devices communicate with a single receiver, constitutes a key design challenge for future generations of wireless systems. The considered scenarios typically consider sporadic traffic with small payloads; furthermore, only a fraction of the transmitters are active at a given time (random access). In that context, it is desirable to let users transmit to the base station without any prior resource request (grant-free). At the physical layer, this requires a departure from the design assumptions prevailing in current cellular systems [1].

The grant-free random access problem has recently been revisited taking massive connectivity into account (see [2]). A classical approach is to rely on a functional split at the receiver between activity detection and channel estimation on one hand (typically, based on user-specific pilot sequences) and multi-user equalization and demapping of the information-bearing symbols on the other hand. Another related approach in this context lies in ALOHA schemes and their extensions, such as slotted coded ALOHA [3].

More recently, the emergence of unsourced random access [4] has sparked a renewed interest in the problem. In this paradigm, the identity of the active transmitters is not associated with a specific waveform at the physical layer. Theoretical analysis of the unsourced scenario has been done in [4] in the single-in-single-output (SISO) case for an additive white Gaussian noise (AWGN) channel and extended to quasi-static Rayleigh fading in [5]. Several practical schemes have been proposed for this scenario. For the SISO AWGN case, [6] proposed a scheme close to sparse regression codes proposed in [7] where the idea is to see the unsourced access as a very large compressed sensing problem where messages are encoded through sparse vectors. In order to enable reasonable decoding complexity, the linear compressed sensing mapping is split into blocks while the messages are encoded by a binary outer code. In [8], the authors combined the compressed sensing approach with a multi-user coding scheme, also allowing a low-complexity decoder. Relaxing the AWGN hypothesis and assuming Rayleigh fading, [5] proposes a scheme based on a low-density parity check (LDPC) code using a belief propagation decoder. The MIMO case with Rayleigh fading is addressed in [9] as well as in [10] wherein encoders inspired from compressed sensing were used while adapting the decoder to the MIMO setup.

In this article, we propose a scheme for unsourced transmission where multi-user multiplexing is handled through the use of a tensor formulation. Specifically, each user transmits a sequence that is associated to a rank-one tensor, while the BS receives the linear combination of these signals weighted by the respective channel realizations, which itself can be interpreted as a tensor summing a number of rank-one components equal to the number of active users. This structure allows the receiver to separate the users without requiring a separate activity detection or channel estimation step; the channels are estimated jointly with the data by the receiver. The benefits of the proposed approach are:

- user separation can be performed by the receiver without relying on pilot sequences, thus circumventing the difficult problem of pilot sequence design for grant-free access, and without involving the discrete nature of the modulation;
- the proposed scheme applies to a broad class of channel models (including AWGN and block-fading) and generalizes gracefully to multiple-antenna channels while benefiting from spatial diversity, without critically relying on any assumption on the fading distribution.

II. UNSOURCED MASSIVE RANDOM ACCESS BACKGROUND AND CHANNEL MODEL

We consider uplink transmission from \( K \) single-antenna user equipments (UE) to a base station (BS) equipped with \( N \) antennas. Let us consider a block of \( T \) channel uses, during which we assume that only \( K_a \ll K \) users are active (where \( K_a \) is still typically large) and simultaneously transmit a payload of \( B \) information bits each. The set of active users is a random subset of the \( K \) users (hence its cardinality \( K_a \) is a random variable) and, following the unsourced random access paradigm [4], we will work under the assumption that all users use the same constellation \( \mathcal{C} = \{c_1, \ldots, c_{2^B}\} \) containing \( 2^B \) elements. Clearly, under these assumptions, the receiver can only decode the messages up
to a permutation over the user indices, and the error event is defined accordingly (see Section V). 1

Let $h_k \in \mathbb{C}^N$ denote the channel from user $k$ to the receiver. We assume in this paper a block-fading model whereby the channel realizations remain constant over the considered block of $T$ channel uses, and is a priori unknown to both the transmitters and the receiver. Let us further assume without loss of generality (w.l.o.g.) that the active users are indexed by $1, \ldots, K_d$. We let $s_k \in \mathcal{C} \subset \mathbb{C}^T$ denote the symbol sent by user $k$ over the $T$ channel uses and $W \in \mathbb{C}^{T+N}$ the noise realization. The users are assumed block-synchronous, therefore the signal $Y \in \mathbb{C}^{T+N}$ received by the $N$ antennas over the $T$ channel accesses can be written as $Y = \sum_{k=1}^{K_d} s_k h_k^T + W$. Let $y, w \in \mathbb{C}^{TN}$ denote the respective vectorized versions of $Y$ and $W$. We can rewrite the received signal using the Kronecker product operator (denoted by $\otimes$) as

$$y = \sum_{k=1}^{K_d} s_k \otimes h_k + w. \quad (1)$$

### III. Tensor-Based Modulation

#### A. Tensor Structure

In order to facilitate user separation from the sum of the received signals (1) at the receiver, we propose to structure the constellation $\mathcal{C}$ based on a tensor construction. Here, we merely consider tensors to be multi-dimensional data structures, which can be seen as the generalization of matrices to dimensions greater than 2. Specifically, let us consider a complex-valued tensor of order $d$ (which can be construed as $d$-dimensional array of complex scalars) of respective dimensions $T_1, \ldots, T_d$. Note that $\Pi_{i=1}^d T_i$ scalars forming the tensor can also be stored sequentially in a vector (see [11, Sec. 2.4]). The corresponding vectorization operator defines an isomorphism between the space of $(T_1, \ldots, T_d)$-dimensional tensors and the space of $(\Pi_{i=1}^d T_i)$-dimensional vectors, endowed with the respective sum operations. For notational convenience, throughout the paper we will use the vector representation while referring to algebraic arguments applying in the space of tensors.

In the proposed approach, we assume that the blocklength $T$ can be factored as $T = \prod_{i=1}^d T_i$ for some $d \geq 2$ and $T_1, \ldots, T_d \geq 2$, and that the sequence of $T$ complex baseband symbols transmitted by user $k$, denoted by $s_k \in \mathbb{C}^{T_i}$, is the vector representation of a rank-1 tensor of dimensions $T_1, \ldots, T_d$, characterized by the existence of vectors $x_{i,k} \in \mathbb{C}^{T_i}$ for $1 \leq i \leq d$, such that

$$s_k = x_{1,k} \otimes \cdots \otimes x_{d,k} \in \mathbb{C}^{T_1 \times \cdots \times T_d} = \mathbb{C}^T. \quad (2)$$

Here, we assume that each $x_{i,k}$ is an element of a sub-constellation $\mathcal{C}_{i,T}$ defined as a discrete subset of $\mathbb{C}^{T_i}$, i.e.

$$x_{i,k} \in \mathcal{C}_{i,T} \subset \mathbb{C}^{T_i}. \quad (3)$$

The resulting vector constellation $\mathcal{C}$ is a discrete subset of $\mathbb{C}^T$ comprised of all possible combinations of elements of the sub-constellations, i.e.

$$\mathcal{C} = \{ x_1 \otimes \cdots \otimes x_d, x_1 \in \mathcal{C}_{1,T}, \ldots, x_d \in \mathcal{C}_{d,T} \}. \quad (3)$$

Substituting (2) in (1), the received signal becomes

$$y = \sum_{k=1}^{K_d} x_{1,k} \otimes \cdots \otimes x_{d,k} \otimes h_k + w \in \mathbb{C}^{TN}, \quad (4)$$

where we let $y_0$ denote the noise-free received signal.

A tensor is said to be rank-$r$ whenever $r$ is the smallest integer such that the tensor can be written as a sum of $r$ rank-1 tensors [11, Sec. 3.1]. Considering eqs. (2) and (4) above, note that each user transmits a rank-1 tensor of order $d$, while $y_0$ is the vector representation of a tensor of order $d+1$ and dimensions $T_1, \ldots, T_d, N$ having rank at most $K_d$.

#### B. Tensor Decomposition, Identifiability and User Separation

The use of the tensor interpretation as a basis for the proposed modulation is motivated by the fact that the decomposition of a rank-$r$ tensor into a sum of $r$ rank-1 tensors is often unique (up to a permutation over the $r$ indices). Such a decomposition is known as the canonical polyadic decomposition (CPD), and its unicity is an important question in tensor theory [12]. 2 Note that although the tensor decomposition problem can be seen a generalization of matrix decomposition (matrices are merely tensors of order 2), the conditions for tensors of order at least 3 to be uniquely decomposable differ significantly from the matrix case (see [11, Sec. 3.2] for details). For instance, at order 3 or more, there are large classes of rank-$r$ tensors admitting a unique CPD without imposing orthogonality constraints between the factors, even for cases where $r > T_i$ for all $i$.

Considering again the noise-free version of (4), we have the following definition:

**Definition 1 (Unicity of the CPD and rank condition):** The $(T_1, \ldots, T_d, N)$-dimensional tensor represented by $y_0$ admits a unique rank-$K_d$ CPD if for any set $\{ x_{1,k} \in \mathbb{C}^{T_1}, \ldots, x_{d,k} \in \mathbb{C}^{T_d}, h_k^T \in \mathbb{C}^N, 1 \leq k \leq K_d \}$ with $K_d' \leq K_d$ such that

$$\sum_{k=1}^{K_d'} x_{1,k} \otimes \cdots \otimes x_{d,k} \otimes h_k^T = y_0,$$

then it holds that $K_d' = K_d$ and there exists a permutation $\sigma$ such that $x_{1,\sigma(1)} \otimes \cdots \otimes x_{d,\sigma(1)} \otimes h_1^T = x_{1,\sigma(2)} \otimes \cdots \otimes x_{d,\sigma(2)} \otimes h_2^T$.

Note that if $y_0$ is uniquely decomposable into its $K_d$ rank-1 components $x_{1,k} \otimes \cdots \otimes x_{d,k} \otimes h_k$, the contributions of the $K_d$ transmitters can be separated by a CPD. In other words, the tensor structure together with the CPD provides a way to perform user separation at the receiver.

Note that Definition 1 considers the decomposition in the continuous domain (i.e. $x_{i,k} \in \mathbb{C}^{T_i}$). A definition more directly related to the communication problem at hand, in

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1Note that, if the identity of the users is included in the message (e.g. in the form of $\log K$ bits of data), the total number of users $K$ has an impact on the achieved spectral efficiency since the data payload is reduced to $B - \log_2 K$ per user – we ignore that aspect in this work and focus on the unsourced problem.

2It is unfortunately impossible to give a simultaneously concise and rigorous summary of the conditions under which the CPD is unique. We refer the interested reader to [13].
the sense that it takes into account the discrete nature of the \( \mathcal{C}_i \), is as follows:

**Definition 2 (Discrete identifiability):** The noise-free received tensor is identifiable in the discrete case if for any set \( \{ \mathbf{x}_{1,k} \otimes \cdots \otimes \mathbf{x}_{d,k} \in \mathcal{C}, \mathbf{h}_k^i \in \mathbb{C}^{N_i}, 1 \leq k \leq K_a \} \) with \( K_a \leq K_a' \) and

\[
\sum_{k=1}^{K_a'} \mathbf{x}_{1,k} \otimes \cdots \otimes \mathbf{x}_{d,k} \otimes \mathbf{h}_k^i = \mathbf{y}_0
\]

then it holds that \( K_a' = K_a \) and there exists a permutation \( \sigma \) such that \( \mathbf{x}_{1,k} = \mathbf{x}_{\sigma(k)} \) and \( \mathbf{h}_k^i = \mathbf{h}_{\sigma(k)} \).

Clearly, the condition of Definition 2 is appropriate for the communications problem at hand\(^3\), while the CPD unicity condition is unnecessarily strong (there might be tensors which are discretely identifiable but do not admit a unique CPD). Definition 1 is nonetheless relevant, since i) it constitutes a sufficient condition for the discrete identifiability, and ii) it is simpler than Definition 2 since it is independent from the design of the discrete constellation \( \mathcal{C} \). Note also that if the unicity of the CPD can be ensured (at least with high probability) for the considered tensor dimensions, a simple multi-user decoding approach can be implemented, in the form of a CPD (performing the separation between the users) followed by parallel single-user decoders — we will return to this point in Section IV-B.

**C. Constellation Design**

The above discussion on identifiability highlights several important issues related to the design of the discrete constellation \( \mathcal{C} \). The first one is that the ability of the CPD to reliably separate the signals from the active users in the noise-free case hinges on the condition that \( \mathbf{y}_0 \) admits a unique rank-\( K_a \) CPD. The second one is that, at each transmitter, the information bits need to be encoded into a rank-1 tensor through the \( d \) discrete sub-constellations. We now discuss both aspects.

1) **Rank conditions on the received tensor \( \mathbf{y}_0 \):** Following [12], we define the expected generic rank of the set of tensors of size \( T_1, \ldots, T_d, N \) as

\[
R^0 = \left[ \frac{TN}{N + \sum_{i=1}^d (T_i - 1)} \right].
\]

For a general choice of the \( \mathcal{C}_i \), the expected generic rank naturally provides an upper bound to the number of separable users, since in the case \( K_a > R^0 \) the CPD will almost surely provide a rank-\( R^0 \) decomposition, i.e. the users will not be separable, while when \( K_a < R^0 \), the set of rank-\( K_a \) tensors with unique CPD is dense in the set of rank-\( K_a \) tensors for all but a few combinations of tensor sizes, whenever \( d \geq 2 \) and \( N, T_1, \ldots, T_d > 1 \), at least for \( T N \leq 15000 \) [13]. In the case where the constellation \( \mathcal{C} \) is structured, a rigorous analysis of the problem remains elusive (note that even for the simpler case of the non-coherent block-fading MAC, there is no generally accepted solution to the problem of designing a good constellation [14]).

2) **Design of the sub-constellations:** We now discuss the mapping of information to the rank-1 tensors \( \mathbf{s}_k \). Observe that \( \mathbf{x}_{1,k} \otimes \cdots \otimes \mathbf{x}_{d,k} \otimes \mathbf{h}_k \) cannot be distinguished from \( \alpha_1 \mathbf{x}_{1,k} \otimes \cdots \otimes \alpha_d \mathbf{x}_{d,k} \otimes \alpha_{d+1} \mathbf{h}_k \) with \( \alpha_1, \ldots, \alpha_{d+1} \in \mathbb{C} \) whenever \( \prod_{i=1}^{d+1} \alpha_i = 1 \). Therefore, for a given \( \mathbf{s}_k \in \mathcal{C} \) the components \( \mathbf{x}_{1,k}, \ldots, \mathbf{x}_{d,k} \) can only be retrieved up to a set of \( d \) complex scalar multiplicative coefficients. In order to account for this scalar indeterminacy, each sub-constellation \( \mathcal{C}_i \) can either i) embed at least one reference symbols, or ii) rely on Grassmannian codebook design [15], [16].

**IV. Multi-User Receiver**

**A. Maximum Likelihood (ML) Decoder**

Let us consider the joint multi-user ML detection problem under the assumption that the noise \( \mathbf{w} \) is Gaussian with i.i.d. coefficients. For the sake of clarity, we will first assume that the number of active users \( K_a \) is known to the receiver. The channel realizations \( \mathbf{h}_k \) can be considered as nuisance parameters here, and the ML estimator writes

\[
\{ \hat{\mathbf{x}}_{1,k}, \hat{\mathbf{h}}_k \} = \arg\min_{\{ \mathbf{x}_{1,k} \in \mathcal{C}_1, \ldots, \mathbf{h}_k \in \mathbb{C}^{N_i} \}} \left\| \mathbf{y} - \sum_{k=1}^{K_a} \mathbf{x}_{1,k} \otimes \cdots \otimes \mathbf{x}_{d,k} \otimes \mathbf{h}_k \right\|^2_2. \tag{8}
\]

Note that the optimization over \( \{ \mathbf{h}_k \} \) is a least squares problem, and therefore has a closed form solution. Solving the discrete problem in (8) via an exhaustive search, however, requires \( 2^{BK_a} \) evaluations of the objective function. Therefore, joint multi-user ML decoding of the active users is a task of formidable complexity.

**B. Two-Step Decoder**

To circumvent the complexity issue, we propose to exploit the tensor structure of the proposed modulation, which allows a simpler two-step decoding process, as follows.

**Continuous multi-user separation:** First, an approximate CPD with \( K_a \) components is performed, in order to recover an approximate version (denoted by \( \hat{\mathbf{z}}_{1,k} \)) of the \( \mathbf{x}_{1,k} \) specifically, (8) is relaxed to yield the rank-\( K_a \) tensor approximation problem

\[
\{ \hat{\mathbf{z}}_{1,k}, \hat{\mathbf{h}}_k \} = \arg\min_{\{ \mathbf{z}_{1,k} \in \mathbb{C}^{T_1}, \ldots, \mathbf{z}_{d,k} \in \mathbb{C}^{T_d}, \mathbf{h}_k \in \mathbb{C}^{N_i} \}} \left\| \mathbf{y} - \sum_{k=1}^{K_a} \mathbf{z}_{1,k} \otimes \cdots \otimes \mathbf{z}_{d,k} \otimes \mathbf{h}_k \right\|^2_2. \tag{9}
\]

**Single-user demapping:** The second step consists in performing single-user demapping independently for each user, i.e. for each \( 1 \leq k \leq K_a \) solve the discrete problem

\[
\arg\min_{\{ \mathbf{x}_{1,k} \in \mathcal{C}, 1 \leq i \leq d, \mathbf{h}_k \in \mathbb{C}^{N_i} \}} \left\| \hat{\mathbf{z}}_{1,k} \otimes \cdots \otimes \hat{\mathbf{z}}_{d,k} \otimes \hat{\mathbf{h}}_k - \mathbf{x}_{1,k} \otimes \cdots \otimes \mathbf{x}_{d,k} \otimes \mathbf{h}_k \right\|^2_2. \tag{10}
\]

Solving (10) over \( \mathbf{h}_k \) as detailed in Appendix A shows that the problem is separable into \( d \) instances of the
minimum chordal distance demapping problem typical of non-coherent modulations:
\[
\hat{x}_{i,k} = \arg\max_{x_{i,k} \in C_i} \left| \frac{x_{i,k}^T \hat{z}_{i,k}}{\|x_{i,k}\| \|\hat{z}_{i,k}\|} \right|.
\]
Solving (11) can still be complex if an exhaustive search over the elements of \( C_i \) is performed. However, this complexity can be significantly decreased by the use of structured constellations such as [16] for \( C_i \).

C. Random user activation

In the random access scenario, the number of active users \( K_a \) is random and unknown to the base station. This can be addressed by performing the approximate CPD (9) using an upper bound \( K_a \) to \( K_a \). Assuming that the user activation probability is known, \( K_a \) can be chosen such that \( K_a \geq K_a \) is fulfilled with arbitrarily high probability. The subsequent demapping of the \( K_a \) rank-1 tensors resulting from the approximate CPD will yield \( K_a \) messages, among which at most \( K_a \) correspond to actually transmitted messages. One option is then to discard the messages based on power thresholding on \( |s_k| \); note however that, even if the noise level can be assumed low, this option is not well mathematically justified, because of the lack of a tensor equivalent to the Eckart–Young theorem [17]. Another option is to use a binary code to add redundancy at the transmitter, and to check whether the decoded binary sequences actually fulfill the code constraints. In this case, the choice of the code should emphasize the error detection capability, since it will be used by the receiver to discard messages that do not correspond to an active user, i.e. for which the demapper output is close to uniform i.i.d. bits.

V. SIMULATION RESULTS

The performance of the proposed tensor-based scheme was evaluated through numerical simulations.

A. Modulation Parameters

Our set-up assumes a payload of \( B = 96 \) information bits at each transmitter, referred to as message, which are encoded into a BCH codeword of length \( B_{\text{tot}} = 110 \) bits; the BCH code can correct up to 2 bit errors thanks to the \( B_{\text{BCH}} = 14 \) bits of redundancy. Bit-to-symbol mapping is performed as depicted on Fig. 1, i.e. the \( B_{\text{tot}} \) coded bits are split into \( d \) sets of respectively \( B_1, \ldots, B_d \) bits, corresponding to the \( d \) tensor dimensions. The \( i \)-th set, comprised of \( B_i \) bits, is mapped to an element of the sub-constellation \( C_i \); in line with Section III-B, we use the Grassmannian constellation design from [16] for the \( C_i \), due to the availability of a low-complexity approximate demapper. Finally, the vector symbol \( s_k \) is formed by computing the Kronecker product (2). We consider two tensor dimensions for the tensor-based modulation (TBM), namely \( (T_1, T_2) = (64, 50) \) and \( (T_1, T_2, T_3, T_4, T_5) = (8, 5, 5, 4, 4) \). According to [16, Lemma 1], the minimum distance between the elements of \( C_i \) is minimized when the \( B_i \) are proportional to \( T_i - 1 \). In order to approximately fulfill this requirement, the \( B_{\text{tot}} = B + B_{\text{BCH}} = 110 \) bits are split according to \((B_1, B_2) = (62, 48)\) and \((B_1, B_2, B_3, B_4, B_5) = (37, 21, 21, 16, 15)\) respectively for the two considered cases. In both cases, the dimension of \( s_k \) is \( T = \prod_{i=1}^{d} T_i = 3200 \) channel uses.

B. Receiver details

Taking into account the non-convexity of the objective function of (9), the receiver used in the simulations consists of two iterations of the two-step decoder described in Section IV-B. In the first iteration, user separation (9), demapping (11), and inverse bit mapping (Fig. 1) are performed; then for each binary vector fulfilling exactly the BCH constraints, the corresponding message is deemed valid. Thus, at the first iteration, the BCH code is used for error detection only. In iteration 2, user separation (9) is performed a second time, during which the symbols corresponding to the messages decoded at iteration 1 are excluded from the optimization variables and replaced by their hard decision values. After user separation, demapping and inverse bit mapping, the binary vectors are BCH decoded: if decoding is successful (now using the error correction capability of the BCH code), the corresponding message is deemed valid; if decoding fails, the vector is discarded.

Let \( \hat{L} \) denote the list of messages deemed valid at either of the two iterations, while \( L \) denotes the list of messages actually transmitted by the active users. Since the messages can be decoded only up to a permutation over the users, the considered error metric is the average message error ratio (MER),
\[
\text{MER} = E \left[ \min \left( \frac{|\hat{L} \setminus L|}{\hat{L} \setminus L} \right) + \frac{|\hat{L} \setminus L|}{|\hat{L} \setminus L|} \right].
\]
(12)
The MER accounts for two types of error events: (i) a transmitted message was not detected; (ii) a detected message was not transmitted. The two error events may be correlated, therefore we limit the MER value to at most 1.

In our implementation, the approximate CPD step (9) is solved using the nonlinear least square algorithm combined with a preconditioner proposed in [18]. Furthermore, to facilitate comparison with the results available in the literature, we assume that the number of active users is known to the receiver through a genie, and therefore set \( K_a = K_a \).
C. Performance Results

The results in this section have been obtained for complex Gaussian i.i.d. (across the users and the $M$ receive antennas) fading channels with unit variance. The entries of the noise $\mathbf{w}$ are i.i.d complex Gaussian with variance $\sigma^2$, while the transmitted symbols are normalized according to $\|\mathbf{s}_k\|^2 = PT$ for all $k$, and $P$ is set according to the considered energy per bit to noise ratio $E_b/N_0 = \frac{PT}{4\sigma^2}$.

Fig. 2. Average per user MER vs. number of active users for a BS with $N = 50$ antennas and $E_b/N_0 = 0$ dB.

In Figure 2, the performance of TBM is compared to the compressed sensing–based approach proposed in [9], for $K_a$ ranging from 50 to 650, and $E_b/N_0 = 0$ dB. It can be observed that tensor-based methods maintain a low MER for $K_a$ ranging up to 650, while the design from [9] exhibits a large MER already for $K_a \approx 100$. In order to illustrate the role of the binary code, we also consider an uncoded set-up, not involving any binary code ($\text{BCH} = 0$). In this case, a single decoding stage (eqs. (9)-(11)) is performed at the receiver, and $\hat{\mathbf{L}}$ always consists of $K_a$ messages). Also observe that TBM with sizes $(8,5,5,4,4)$ yields consistently superior performance to what is achieved using tensors of sizes $(64,50)$, which seems to indicate that, for a fixed $T$, designs with higher $d$ are preferable.

Figure 3 depicts the $E_b/N_0$ required to achieve a target error rate, for $N = 50$ and $N = 1$ antennas. To facilitate comparison with the existing results, we now focus on the average per user probability of error (PUPE) [4], defined as

$$\text{PUPE} = E \left[ \frac{L \cdot \hat{L}}{|\hat{L}|} \right].$$

and depict the $E_b/N_0$ required to achieve a PUPE lower or equal to 0.1. For comparison, we also include the AWGN channel case. In the single-antenna receiver case, we compare to theoretical bounds, such as a Fano-type converse bound and a random coding achievability bound, both from [19, Appendix B] in the Rayleigh fading case, and the achievability bound of [4] for the AWGN case. We observe that the proposed scheme benefits from spatial diversity, in the sense that the maximum number of active users that can be successfully decoded increases with $N$.

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Fixing $x_{i,k} \in \mathcal{C}_i$ for all $i,k$, and solving (10) with respect to $h_k$, yields

$$h_k^* = \frac{(x_{1,k} \otimes \cdots \otimes x_{d,k})^H (\hat{z}_{1,k} \otimes \cdots \otimes \hat{z}_{d,k})}{\|x_{1,k} \otimes \cdots \otimes x_{d,k}\|^2}.$$  (14)

Substituting (14) into the objective function of (10), and applying the property that $a \otimes b = \text{vec}(ab^T)$ for arbitrary vectors $a$ and $b$ to both terms, we get

$$\|\hat{z}_{1,k} \otimes \cdots \otimes \hat{z}_{d,k} \otimes h_k - x_{1,k} \otimes \cdots \otimes x_{d,k} \otimes h_k^*\| = \|\text{vec}((\hat{z}_{1,k} \otimes \cdots \otimes \hat{z}_{d,k})h_k^T) - \text{vec}((x_{1,k} \otimes \cdots \otimes x_{d,k})(h_k^*)^T)\| = \|P_x(\hat{z}_{1,k} \otimes \cdots \otimes \hat{z}_{d,k})h_k^*\|_2^2,$$  (15)

where we define the projection matrix $P_x = I_T - \frac{(x_{1,k} \otimes \cdots \otimes x_{d,k})(x_{1,k} \otimes \cdots \otimes x_{d,k})^H}{\|x_{1,k} \otimes \cdots \otimes x_{d,k}\|^2}$. Using $\|ab^T\|_2^2 = \|a\|^2\|b\|^2$ and the fact that $P_x$ is a projector, the optimization problem (10) is equivalent to

$$\max_{x_{i,k} \in \mathcal{C}_i, \forall i} \frac{\|x_{1,k} \otimes \cdots \otimes x_{d,k}^H (\hat{z}_{1,k} \otimes \cdots \otimes \hat{z}_{d,k})\|_2^2}{\|x_{1,k} \otimes \cdots \otimes x_{d,k}\|^4}.$$  (16)

Finally, using $(a \otimes b)^H (a' \otimes b') = (a^H a')(b^H b')$ we obtain

$$\max_{x_{i,k} \in \mathcal{C}_i, \forall i} \prod_{l=1}^{d} \frac{|x_{l,k}^H \hat{z}_{l,k}|}{\|\hat{z}_{l,k}\| \|x_{l,k}\|}$$  (17)

which is clearly separable, and yields (11).