DISORIENTATION OF SUPRATHERMALLY ROTATING GRAINS AND THE GRAIN ALIGNMENT PROBLEM

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ABSTRACT

We discuss the dynamics of dust grains subjected to torques arising from H$_2$ formation. In particular, we discuss grain dynamics when a grain spins down and goes through a “crossover” event. As first pointed out by Spitzer & McGlynn, the grain angular momentum before and after a crossover event are correlated, and the degree of this correlation critically affects the alignment of dust grains by paramagnetic dissipation. We calculate the correlation including the important effects of thermal fluctuations within the grain material. These fluctuations limit the degree to which the grain angular momentum $J$ is coupled with the grain principal axis $a_i$ of maximal inertia. We show that this imperfect coupling of $a_i$ with $J$ plays a critical role during crossovers and can substantially increase the efficiency of paramagnetic alignment for grains larger than 0.1 $\mu$m. As a result, we show that for reasonable choices of parameters, the observed alignment of $a_i \geq 0.1$ $\mu$m grains could be achieved by paramagnetic dissipation in suprathermally rotating grains, if radiative torques caused by starlight were not present. We also show that the efficiency of mechanical alignment in the limit of long alignment times is not altered by the thermal fluctuations in the grain material.

Subject headings: dust, extinction — ISM: magnetic fields — polarization

1. INTRODUCTION

Understanding the observed alignment of interstellar grains is a challenging problem of nearly a half-century’s standing (see Roberge 1996). Lacking a proper understanding of the alignment processes, we can only tentatively interpret polarimetric data in terms of the magnetic field. Indeed, polarizing grains can be aligned with long axes either perpendicular or parallel to the magnetic field, depending on what causes the alignment (see Lazarian 1994); they can also be not aligned at all (see Goodman 1996).

One of the essential features of grain dynamics in the diffuse interstellar medium (ISM) is suprathermal rotation (Purcell 1975, 1979). Originally, three separate causes of suprathermal rotation were suggested: inelastic scattering of impinging atoms when the gas and grain temperatures differ, photoelectric emission, and H$_2$ formation on grain surfaces. The latter was shown to dominate the other two for typical conditions in the diffuse ISM (Purcell 1979). More recently, radiative torques owing to starlight have been identified as an important mechanism driving suprathermal rotation (Draine & Weingartner 1996, 1997).

Which torques (H$_2$ or radiative) dominate depends on the grain environment and the number of H$_2$ formation sites on the grain surface. The latter parameter is highly uncertain, and in this paper we consider H$_2$ torques only, which is appropriate if a single grain has $\lesssim 10^3$ active sites (Draine & Lazarian 1997). A more general study that includes both H$_2$ and radiative torques is planned for the future.

Alignment of grains rotating suprathermally differs considerably from the alignment of thermally rotating grains. The theory of paramagnetic alignment of suprathermal grains was discussed by Purcell (1979) and Spitzer & McGlynn (1979) and has been elaborated on by Lazarian (1995b, 1995c, 1996). Until recently, mechanical alignment, i.e., alignment caused by a gaseous flow, was thought not to be applicable to suprathermally rotating grains, as rapid rotation makes the grains not susceptible to such a process. However, two new mechanisms of mechanical alignment of suprathermally rotating grains, namely, the “crossover” and “cross section” mechanisms, have been suggested recently (Lazarian 1995d) and have been shown to be effective in interstellar regions with gas-grain streaming (Lazarian & Efroimsky 1996; Lazarian et al. 1996).

The crossover event is the most important period in the dynamics of suprathermally rotating grains. The H$_2$ formation sites on a grain surface have a finite “lifetime” $t_I$, which may be determined by the “resurfacing” of the grain by accreted atoms (Purcell 1979) or poisoning of active sites by oxygen (Lazarian 1995c). Because of the changes in the resulting torque, the spin-up has a finite duration, and this limits the paramagnetic alignment attainable. In the case of mechanical alignment, it is during the crossovers that the grain is susceptible to alignment caused by gaseous bombardment.

Spitzer & McGlynn (1979, hereafter SM) carried out a pioneering analysis of the dynamics of grains during the crossover process and showed that the direction of angular momentum before and after crossover are correlated. However, they found that the correlation was insufficient for the paramagnetic mechanism to achieve significant alignment within the model they adopted.

Recent progress in the understanding of certain subtle issues concerning grain dynamics has led us to reexamine the crossover process. SM assumed that during suprathermal rotation, the grain angular momentum $J$ is perfectly aligned with the axis of major inertia, $a_i$. This

$^1$ This paper is dedicated to the memory of Lyman Spitzer, Jr.

$^2$ See Davis & Greenstein (1951), Jones & Spitzer (1967), Mathis (1986), Roberge, DeGraff, & Flaherty (1993), and Lazarian (1995a) for paramagnetic alignment of thermally rotating grains and Gold (1951), Lazarian (1994), Roberge, Hanany, & Messinger (1995), and Lazarian (1997) for mechanical alignment of thermally rotating grains.

$^3$ For brevity, we refer to the principal axis of largest moment of inertia as the “axis of major inertia.”
coupling arises from internal dissipation which, as is known from theoretical mechanics, causes a spinning solid body to rotate about its $a_1$-axis, which is the state of minimum rotational kinetic energy for fixed angular momentum. The assumption of perfect relaxation seems natural, as the timescale for internal relaxation for supra-thermally rotating grains is many orders of magnitude less than the timescale of the spin-up, but it is not exact: it disregards thermal fluctuations within the grain body.

In fact, Lazarian (1994) showed that owing to thermal fluctuations, the coupling mentioned above is never perfect (the quantitative theory of this phenomenon is presented in Lazarian & Roberge 1997). The component of $J$ perpendicular to the $a_1$-axis, although tiny compared to $|J|$ during suprathermal rotation, is very important in the course of a crossover. We therefore reconsider the SM theory of crossovers in order to allow for the effects of thermal fluctuations.

In § 2 we pose the problem and present the necessary facts concerning incomplete internal relaxation. In § 3 we derive the “disorientation parameter” $F$, accounting both for thermal fluctuations within the grain material and for the effects of gaseous bombardment; the latter effect is of secondary importance in diffuse clouds but may be important in molecular clouds. The consequences of the incomplete disorientation on paramagnetic and mechanical alignment are discussed in § 4, and the conclusions are presented in § 5.

2. THE PROBLEM

A crossover is the event that occurs between two sequential spin-ups when the component of $J$ parallel to the axis of major inertia $a_1$ passes through zero. This is a critical period for grain dynamics, and during the crossover the grain is susceptible to disorientation, which will limit the effectiveness of paramagnetic alignment. If the grain is situated in a region with substantial gas-grain streaming, the grain is susceptible to mechanical alignment during crossovers. Our task in the present paper is to describe the evolution of grains through crossover events while accounting for the effects of thermal fluctuations within the grain material.

To understand the crossover, one needs to recall certain basic features of suprathermal rotation. Here we assume that grains are spun-up by torques arising from H$_2$ formation and consider a “brick” with dimensions $b \times b \times a$ and density $\rho_s$. The ratio $r = b/2a$ determines the degree of grain oblateness; $r = 1$ for the 2:2:1 grain discussed in Purcell (1979). It is possible to show (SM) that the components of the torque perpendicular to the axis of major inertia average out and therefore only the component of the torque parallel to this axis matters. We direct the $z$-axis along the axis $a_1$. We let

$$ n_{H} = n(H) + 2n(H_2), $$

where $n(H)$ and $n(H_2)$ are the concentrations of atomic and molecular hydrogen, respectively; the H$_2$ fraction we denote by $y = 2n(H_2)/n_{H}$.

The number of H$_2$ molecules ejected per second from an individual site is $\gamma a^2 n_{H} v_{\oslash} (1 - y) (r + 1)^{-1}$, where $\gamma$ is the fraction of H atoms (with mean speed $v_{\oslash}$ and mass $m_{H}$) adsorbed by the grain and $v$ is the number of active sites over the grain surface. The mean square torque from H$_2$ formation is (see the Appendix)

$$ \langle L_z^2 \rangle \approx \frac{2}{3} \gamma^2 a^6 (1 - y) n_{H} m_{H} v_{\oslash}^2 E v^{-1} r^4 (r + 1), $$

where $E \approx 0.2$ eV is the kinetic energy of a nascent H$_2$ molecule. The fluctuating torque $L_z$ spins up grains to an rms angular velocity

$$ \langle \omega^2 \rangle^{1/2} = \langle L_z^2 \rangle^{1/2} \frac{t_d}{t_L + t_d} \left( \frac{t_L}{t_L + t_d} \right)^{1/2} $$

(Purcell 1979), where $I_z = (8/3) \rho_{s} a_{s}^5 r_0^4$ is the $z$ component of the momentum of inertia, $t_d$ is the rotational damping time (see the Appendix)

$$ t_d = \frac{2r}{(r + 2)} \frac{a_{\oslash}^2}{n_{H} m_{H} v_{\oslash} (1.2 - 0.293 y)} $$

and $t_L$ is the lifetime of an active site.

To obtain both characteristic numerical values and functional dependencies, we will use quantities normalized by their standard values (see Table 1). We denote the normalized values by symbols with hats, e.g., $\hat{a} = a/(10^{-5} \text{ cm})$, with $10^{-5} \text{ cm}$ as the standard value of grain size. We consider an H$_2$ formation efficiency $\gamma = 0.2$ and $v_{\oslash} = 1.5 \times 10^5 \text{ cm s}^{-1}$ to be “standard.” For diffuse clouds, we assume that all hydrogen is atomic and therefore $y = 0$. Note that sometimes the choice of standard values is somewhat arbitrary,

| Notation | Meaning |
|----------|---------|
| $y \equiv 2n(H_2)/n_{H}$ | H$_2$ fraction |
| $\gamma \equiv 0.2$ | H recombination efficiency |
| $v_{H} \equiv (8K_{t} / \pi m_{H})^{1/2} = 1.5 \times 10^5 \text{ cm s}^{-1}$ | Thermal velocity |
| $\hat{a} \equiv 10^{-5} \hat{a} \text{ cm}$ | Grain size |
| $r \equiv b/2a$ | Grain axis ratio divided by 2 |
| $E \equiv 0.2E \text{ eV}$ | Kinetic energy of nascent H$_2$ |
| $\sigma \equiv 10^{-2} \hat{a} \text{ cm}^{-2}$ | Surface density of recombination sites |
| $v \equiv 90(r + 1)\hat{a}^2$ | Number of recombination sites |
| $T_c \equiv 15\hat{T}_c \text{ K}$ | Grain temperature |
| $T \equiv 85\hat{T} \text{ K}$ | Gas temperature |
| $n_{H} \equiv 20\hat{n}_{H} \text{ cm}^{-3}$ | Density of H nucleon |
| $\hat{n}_{H} \equiv 3\hat{n}_{H} \text{ g cm}^{-3}$ | Solid density |
| $B \equiv 5\hat{B} \times 10^{-6} \text{ G}$ | Magnetic field |

TABLE 1
PARAMETERS OF GRAINS AND AMBIENT MEDIUM ADOPTED IN THIS PAPER

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4 The most important internal dissipation process is Barnett relaxation (Purcell 1979).
e.g., for the time being we assume the density of active sites\(^5\) \(\alpha\) to be \(10^{11}\) cm\(^{-3}\), so that \(v = 80\alpha^2\tau^2(r+1)\).

Using standard values of the parameters, we obtain the following expression for the angular velocity:

\[
\langle \omega^2 \rangle^{1/2} \approx \left( \frac{3E}{\text{am}_H} \right)^{1/2} \frac{\gamma}{2a^2} \frac{(1-y)}{(1-0.293y)} \times \frac{1}{r^{3/2}(r+2)} \frac{t_L}{(t_d+t_L)}^{1/2} \approx (2.0 \times 10^9) \frac{\hat{E} \gamma}{\alpha^2} \frac{(1-y)}{(1-0.24y)} \times \frac{1}{r^{3/2}(r+2)} \frac{t_L}{(t_d+t_L)}^{1/2} \text{s}^{-1}.
\]

The lifetime of an active site of \(\text{H}_2\) formation is limited by both accretion of a monolayer of refractory material (SM) and poisoning by atomic oxygen (Lazarian 1995c). Further, on we use the term “resurfacing” to refer to the fastest mechanism of the two. The component of the mean torque along the axis of major inertia \(a_1\) before and after resurfacing may be directed either in the same direction as before the process or in the opposite direction. In the latter case, the grain undergoes a spin-down.

The mean interval between crossovers is (Purcell 1979)

\[
\tau_c \approx (t_L t_d)^{1/2}.
\]

Although the ratio \(t_a/\tau_c \approx t_a/t_d\) can be as small as \(10^{-5}\) (see Table 2), the alignment of \(J\) with the axis of major inertia is not perfect. The deviations of \(a_1\) from \(J\) arise from thermal fluctuations within the grain material (Lazarian 1994; Lazarian & Roberge 1997). To estimate the value of such deviations, recall that rotation about \(a_1\) corresponds to the minimum of the grain kinetic energy for fixed \(J\) (internal dissipation does not alter \(J\)). For a symmetric oblate grain with \(I_z > I_x = I_y \equiv I_\perp\) (i.e., our brick with \(r > 0.5\)), the grain kinetic energy is

\[
E_\perp(\beta) = \left( \frac{J_z^2}{2I_z} \right) \left[ 1 + \sin^2 \left( \frac{\beta}{I_\perp} \right) \right],
\]

where \(\beta\) is the angle between \(J\) and \(a_1\). In thermodynamic equilibrium the fluctuations of the kinetic energy should have a Boltzmann distribution

\[
f(\beta) d\beta = \text{const} \times \sin \beta \exp \left[ -\frac{E_\perp(\beta)}{kT_\perp} \right] d\beta,
\]

where \(T_\perp\) is the dust temperature. It follows from equation (8) that when \(J_z^2 \gg (I_z I_\perp kT_\perp)/(I_z - I_\perp)\), the fluctuating component of angular momentum perpendicular to \(a_1\) can be approximated as

\[
\langle J_\perp^2 \rangle \approx \langle J_z^2 \rangle_0 \approx \left( \frac{I_z I_\perp kT_\perp}{I_z - I_\perp} \right)^{1/2}.
\]

We may define the “thermal transverse angular velocity”

\[
\omega_\perp \equiv \langle J_\perp^2 \rangle^{1/2} = \left[ \frac{I_z I_\perp kT_\perp}{I_z - I_\perp} \right]^{1/2} = (7.43 \times 10^9) \frac{\hat{T}_d^{1/2}}{\alpha^{3/2} \beta^{5/2}} \left( \frac{15}{16\beta^2 - 1} \right)^{1/2} \text{s}^{-1}.
\]

As we will see below, \(\omega_\perp\) is the characteristic value for the minimum value of the grain angular velocity during a crossover.

When the rotation is suprathermal, \(\langle J_\perp^2 \rangle^{1/2}\) is negligible compared to \(J\) and angle \(\beta\) is very close to zero. However, as the component of angular momentum \(J_\parallel\) parallel to \(a_1\) decreases during crossovers, the angle \(\beta\) increases.

When the angular velocity decreases sufficiently, internal relaxation becomes less efficient and the value of \(\langle J_\perp^2 \rangle\) rises.

\(5\) The density \(\alpha\) of active sites is the most uncertain quantity. It depends on the interplay of the processes of photodesorption and poisoning (Lazarian 1995c, 1996).

\(6\) If \(J\) does change because of \(\text{H}_2\) torques, one should substitute its value averaged over the time of internal relaxation in equation (7).
as a result of the stochastic character of H$_2$ formation and impacting gas atoms. At some point, the component of $J$ parallel to $a_i$ passes through zero and the grain flips over (SM). Our task is to calculate the correlation of the grain angular momentum before and after the crossover. This is done in the next section.

3. CROSSOVERS

In our treatment below, we repeat the reasoning introduced in SM but include an allowance for thermal fluctuations. The zeroth approximation, following SM, is the dynamics of a grain subjected to continuous torques only. The dynamical effects of the stochastic torques can be evaluated by an approximate theory based on small perturbations of the zeroth-order solution.

3.1. Zeroth Approximation

Let $xyz$ be a coordinate system frozen into the grain, with $z$ along the grain axis of major inertia $a_i$. Let $x_0, y_0, z_0$ be an inertial coordinate system, with $z_0 \parallel J$ (at some initial time). Let $\beta$ be the angle between the $z$-axis and $J$: $J_z = J \cos \beta$ (see Fig. 1). If no external torques act, then $J = \text{const}$ and the $z$-axis and the angular velocity $\omega$ will each precess around $J$ (or $z_0$) at a frequency $\omega_p = [I_z - I_\perp]/[I_z I_\perp]$, where $I = I_z$ is the moment of inertia.

Now consider the effect of a (weak) torque $L$ which is fixed in body coordinates $xyz$. On time scales long compared to $\omega_p^{-1}$, the rotation of the grain around $\omega$ and the precession of $\omega$ around $z_0$ imply that the only torque component that does not average to zero is that owing to $L_z$, the component of $L$ along the $z$-axis. After this averaging, we obtain

$$\frac{dJ_z}{dt} = L_z \cos \beta \frac{J}{|J|}. \quad (11)$$

From the Euler equations (see SM), we find the components of $\omega$ in body coordinates

$$\omega_z = \frac{J_z}{I_z} = \frac{L_z t}{I_z}, \quad (12)$$

$$\omega_\perp = J_\perp/I_\perp = \text{const}, \quad (13)$$

where $t = 0$ at the moment of crossover. Equation (11) can be rewritten

$$\frac{dJ_z}{dt} = L_z \cos \beta \frac{J_z}{J_{z0}}. \quad (14)$$

Since $J_z = L_z t$, we obtain

$$J_{z0}^2 = L_z^2 t^2 + J_\perp^2. \quad (15)$$

According to equation (11), the direction of $J$ does not change—the torque $L$ acts only to change its magnitude (see eq. [15]). Thus the zeroth approximation predicts a perfect correlation between the angular momentum directions prior to and after the crossover. The stochastic torques make the story more involved.

3.2. Crossovers and Barnett Fluctuations

Our considerations given above ignored the fluctuations associated with the Barnett effect. As discussed by Lazarian & Roberge (1997), angle $\beta$ fluctuates on the Barnett relaxation time (Purcell 1979)

$$t_{\beta}(\omega) = \frac{A_\beta}{\omega^2}, \quad (16)$$

where

$$A_\beta = (7.1 \times 10^{-7}) d^2 \tilde{\epsilon} \tilde{T}_d \tilde{K}_0(r) \text{ s}^{-1} \quad (17)$$

and

$$\tilde{K}_0(r) = \frac{3}{125} \frac{(4r^2 + 1)^3}{r^4(4r^2 - 1)}. \quad (18)$$

Note that the ratio $r = \frac{1}{2}$ corresponds to a cubical grain, for which no internal relaxation is expected, in agreement with equation (17).

The fluctuations in $\beta$ span the interval $(0, \pi)$ when $J \rightarrow J_{\perp}$ (Lazarian & Roberge 1997). SM showed that during crossovers, $J \sim J_{\perp}$, and therefore such fluctuations must be accounted for provided that $t_{\epsilon} > t_{\beta}(\omega_{\perp})$, where the crossover time is

$$t_{\epsilon} = \frac{2J_{\perp}}{J}, \quad (19)$$

in which $J$ is the time derivative of $J$.

When $t_{\beta}(\omega_{\perp}) \ll t_{\epsilon}$, Barnett fluctuations will cause $\beta$ to range over the interval $(0, \pi)$, which results in frequent reversals of the torque in inertial coordinates. Conseque-
ly, the actual time spent during the crossover will be increased. Quantitative analysis of this regime is beyond the scope of the present paper; it does appear clear, however, that each crossover will be accompanied by substantial disalignment when \( t_B \approx t_c \).

In the present study, we confine ourselves to the other limiting case, namely, \( t_c/t_B \ll 1 \), in which case the Barnett fluctuations during a crossover can be disregarded and the initial distribution of \( J_1 \) with the mean value given by equation (9) is produced by the Barnett fluctuations during the long time interval between crossovers.

We shall prove below that for typical interstellar conditions, the value of \( J_1 \) mostly arises from thermal fluctuations within the grain material during intervals of suprathermal rotation and therefore is given by equation (9). Thus we can estimate \( t_c \):

\[
t_c \approx 2\frac{J_{10}}{J} \approx 2.6 \times 10^7 \left( \frac{b_T}{E} \right)^{1/2} \frac{1}{\tilde{h} \hat{h}} (1 - y) \tilde{Z}_0(r) \,
\]

where

\[
\tilde{Z}_0(r) = \left[ \frac{3r(1 + 4r)}{5(4r^2 - 1)} \right]^{1/2}.
\]

The ratio

\[
t_B = \frac{a_c}{a_c}^{1/3},
\]

where \( a_c \) is the critical radius, which for \( \omega \approx J_1/J_2 \) is equal to

\[
a_c \approx (1.47 \times 10^{-5}) \left[ \frac{\hat{h}}{(1 - y)^2 \tilde{h}^2 \hat{h}_E \hat{p}^2} \tilde{K}_1(r) \right]^{1/13} \text{cm},
\]

where

\[
\tilde{K}_1(r) \approx 234375r^5 \left( \frac{4r^2 - 1}{(4r^2 + 1)^3} \right).
\]

It follows from our discussion above that we attempt to deal only with the case \( a > a_c \), while leaving the more complex regime \( a < a_c \), where Barnett fluctuations during the crossover are important, to be dealt with elsewhere. We use the inequality \( a > a_c \) rather than \( a \ll a_c \) because of the strong dependence of the time ratio \( t_c/t_B \) on \( a : t_c > 12t_B \) for \( a = 10^{-5} \) cm, while \( t_B > 7t_c \) for \( a = 2.0 \times 10^{-5} \) cm. Below we analyze the implications of the critical size \( a_c \) in the context of the variations of alignment with grain size.

The numerical value \( a_c \approx 1.5 \times 10^{-5} \) cm is quite robust; the most uncertain grain parameter is the surface density of active sites \( a \), but even varying \( a \) by a factor of \( 10^2 \) changes \( a_c \) by only a factor of \( 10^{0.13} \approx 1.36 \).

Although we have considered only suprathermal rotation driven by \( H_2 \) formation so far, the existence of a critical size for which \( t_B \approx t_c \) seems to be generic to the problem of disorientation in the course of crossovers.

In our treatment above, we disregarded gaseous friction. This is a good approximation in the diffuse medium since \( t_s \ll t_c \) (see Table 2). In molecular clouds, as \( y \rightarrow 1 \), the two timescales may become comparable (e.g., if the atomic hydrogen fraction \( [1 - y] \) drops below \( 10^{-5} \)) and gas drag should be included. We also assume \( t_s \gg t_c \); in the case of \( t_s \ll t_c \), it becomes important to allow for variations in the time-averaged torque \( L_c \) during the crossover.

### 3.3. Random Torques

We consider the dynamical evolution given by equation (15) as a zeroth-order solution of the problem and the dynamical effects produced by stochastic torques as perturbations of this solution.

Each torque event produces an impulsive change of angular momentum \( \Delta J = \Delta J_2 z + \Delta J_1 \). The angular deviations of \( J \) in the \((J, z)\)-plane are given by

\[
\Delta \eta_\parallel = -\frac{\Delta J_2 \sin \beta + \Delta J_{11} \cos \beta}{J}
\]

and in the transverse direction by

\[
\Delta \eta_\perp = \frac{\Delta J_{12}}{J},
\]

where \( \Delta J_{11} \) and \( \Delta J_{12} \) are the components of \( \Delta J \) in the plane parallel and perpendicular to the \((J, z)\)-plane, respectively. The grain is subject to the action of various torques. Here we discuss only torques arising from \( H_2 \) formation (\( \Delta J_\parallel \)) and gaseous bombardment (\( \Delta J_\perp \)). Let \( N_1 \) be the rate of \( H_2 \) formation and \( N_2 \) be the rate of gas-grain collisions. To simplify our notation, we denote the mean square change in angular momentum per \( H_2 \) formation event

\[
(\Delta J)\sin^2 \beta + (\Delta J)\cos^2 \beta
\]

where \( N_2/N_1 \) is the number of gas-grain collisions per \( H_2 \) formation event. The mean quadratic deviation of \( J \) per \( H_2 \) formation is

\[
\langle (\Delta J)\sin^2 \beta + (\Delta J)\cos^2 \beta \rangle = \frac{1}{2} \langle (\Delta J) \rangle^2,
\]

owing to rotation around the axis \( a_1 \). Thus

\[
\langle (\Delta \eta)^2 \rangle = J^{-2} \times \left[ \langle (\Delta J_2)\sin^2 \beta + (\Delta J)\cos^2 \beta \left( \frac{1 + \cos^2 \beta}{2} \right) \right].
\]

The cumulative deflection caused by \( i + 1 \) random impulses may be expressed as

\[
\cos \eta_{i+1} = \cos \eta_i \cos (\Delta \eta) + \sin \eta_i \sin (\Delta \eta) \cos \phi_i,
\]

where \( \phi_i \) is the angle between the deviation \( \Delta \eta \) and the great circle measured by \( \eta_i \). Averaging provides us with the result (see SM)

\[
\langle \cos \eta_i \rangle = \Pi_i \langle \cos \Delta \eta \rangle.
\]

As \( \Delta \eta \) is small, we can expand \( \cos \Delta \eta \) to obtain

\[
\Pi_i \langle \cos \Delta \eta \rangle \approx \Pi_i \left[ 1 - \frac{1}{2} \langle \cos \Delta \eta \rangle^2 \right] \approx \Pi_i \exp \left[ -\frac{1}{2} \langle \cos \Delta \eta \rangle^2 \right].
\]

Hence

\[
\langle \cos \eta_i \rangle \approx \exp (-F),
\]
where $F$ is the disorientation parameter

$$F \equiv \frac{1}{2} \int_{-\infty}^{+\infty} N_1 \langle \Delta \eta^2 \rangle dt \quad (35)$$

and $N_1 = L_z / \langle \Delta J_z \rangle$ is the number of H$_2$ torque events per second. From equations (27) and (30), it is seen that both H$_2$ torques and gaseous bombardment are included in equation (35).

To evaluate $F$, we obtain $\beta(t)$ from the zeroth-order grain dynamics with only regular torques ($J_z = \text{const}$, $J_z = L_z$); from then $\beta = J_z / J_z$ we obtain

$$\frac{dt}{d\beta} = - J_z \sin^2 \beta = - J_z^2 \sin^2 \beta. \quad (36)$$

Substituting this into equation (35) and integrating from $\beta = 0$ to $\beta = \pi$, one obtains

$$F = \frac{\pi}{4} J_1 \frac{\langle \Delta J_z^2 \rangle}{\langle \Delta J_z \rangle} \left[ 1 + \frac{3}{2} \frac{\langle \Delta J_z^2 \rangle}{\langle \Delta J_z \rangle^2} \right]. \quad (37)$$

Using equation (27) and equation (A6)–(A10), we find

$$F = \frac{179\pi}{224} \frac{\langle \Delta J_z^2 \rangle}{J_1 \langle \Delta J_z \rangle} \tilde{K}_2(r), \quad (38)$$

where

$$\tilde{K}_2(r) = \frac{56}{179} \left\{ 1 + \frac{3}{8} \left[ \frac{16r^2(r+1) + 6r + 3}{2r^2+5} \right] \right\}. \quad (39)$$

Equation (37) was derived assuming $J_z = \text{const}$; in fact, it is the value of $J_z$ when $\beta \approx \pi/2$ that should be used in equation (37). In the pioneering study by SM, it was assumed that $J_z$ is initially zero. This assumption is valid only for grain temperatures approaching absolute zero. For nonzero grain temperatures, the mean value of this component squared cannot be less than $J_z^2$ given by equation (9). To account for this nonzero component while avoiding solving the corresponding Fokker-Planck equation, in our simplified treatment, we consider the evolution of the difference $\langle J_z^2 \rangle - J_z^2$ using the lucid approach suggested by SM.

In deriving equation (38), we assumed $J_z = \text{const}$. Recognizing now that $J_z$ will be time-dependent, we note that disorientation of the grain depends primarily on the value of $J_z^{-1}$ near the time of crossover. We therefore seek to establish $\langle J_z^2 \rangle(0)^{1/2}$, the value at the moment $t = 0$ of crossover. For $t_c \ll t_B \ll t_d$, one can write the phenomenological equation

$$\frac{d\langle J_z^2 \rangle - J_z^2}{dt} = N_1 \langle \Delta J_z^2 \rangle - J_z^2 / t_B, \quad (40)$$

which generalizes equation (37) of SM. All the time during a crossover, apart from a short interval when the grain actually flips over, $\omega \gg \omega_z$, and therefore it is possible to assume that $\omega_z \approx \omega$ (see SM). According to our initial assumption, regular torques dominate the zero-order dynamics. Thus $dt = (I_z / L_z) \times d\omega_z = (I_z / N_z \langle \Delta J_z \rangle) d\omega_z$ follows from equation (36). Substituting

$$\zeta = \frac{\langle \Delta J_z \rangle}{I_z \langle \Delta J_z \rangle + N_z / N_1 \langle \Delta J_z \rangle} \left( \frac{2I_z}{A_u N_1 \langle \Delta J_z \rangle} \right)^{1/3}$$

$$x \langle J_z^2 - J_z^2 \rangle \quad (41)$$

and

$$u = \left( \frac{2I_z}{A_u N_1 \langle \Delta J_z \rangle} \right)^{1/3} \quad (42)$$

into equation (40) gives (SM)

$$\frac{d\zeta}{du} = 1 - \zeta u^2. \quad (43)$$

Therefore, for negative $\omega_z$ increasing to zero for $t = 0$, one gets

$$\langle J_z^2 \rangle(0) = J_z^2 \text{torque} + J_z^2, \quad (45)$$

where

$$J_z^2 \text{torque} = \left( \frac{3^{1/3} \Gamma(\frac{4}{3})}{2} \right) \left\{ \frac{N_z}{N_1} \frac{\langle \Delta J_z \rangle}{\langle \Delta J_z \rangle^2} \right\}$$

$$\times (A_u N_1)^{1/3} \langle \Delta J_z \rangle |^{-2/3} I_z^{1/3} \langle \Delta J_z \rangle^2 (1 + \chi), \quad (46)$$

$$\chi \equiv \left( \frac{A_u N_1}{I_z} \langle \Delta J_z \rangle \right)^{1/3} \quad (47)$$

$$(\chi)$$ is the gamma function, and

$$\omega_z \approx \left( \frac{A_u N_1}{I_z} \langle \Delta J_z \rangle \right)^{1/3} \quad (48)$$

which is the value of $\omega_z$ such that the crossover time $I_z \omega_z / (N_1 \langle \Delta J_z \rangle)$ equals the relaxation time. During this time, the number of torque events is $\langle \omega_z \rangle I_z \langle \Delta J_z \rangle$, and the product of this number over the mean squared increment of angular momentum per torque event, i.e., $[\langle \Delta J_z \rangle^2 / N_1 \langle \Delta J_z \rangle]$, provides the estimate for $\langle J_z^2 \rangle - J_z^2$ in accordance with equation (46).

Comparison of our equation (46) and equation (42) in SM shows that the mean value of $J_z^2$ arising from stochastic torques is increased by a factor $(1 + \chi)$, where

$$\chi \approx \frac{2k(T + T_d)}{\gamma E} \left( \frac{1.2 - 0.293 y}{1 - y} \right)$$

$$\approx \frac{0.52}{\gamma E} \left( \frac{T + T_d}{100 K} \right) \left( \frac{1 - 0.24 y}{1 - y} \right) \quad (49)$$

and $T$ is the gas temperature. In molecular gas with $1 - y \ll 1$, $\chi$ can be large, but in H I regions, $\chi \lesssim 1$. We also note that $\chi$ does not depend on grain geometry.

Supersonic drift causes mechanical alignment, which we briefly discuss in § 4.2. Here we limit discussion to the case where gaseous bombardment is isotropic during the crossover event.

3.4. Relative Importance of Barnett Torques

For typical interstellar conditions, the $J_z^2$ term in equation (45) is much more important than the term owing to
gaseous bombardment. The importance of the Barnett fluctuations relative to the stochastic torques is measured by

\[ R^2 = \frac{J_{\perp,0}^2}{J_{\perp,\text{torque}}^2} \approx 395 \left[ \frac{\hat{T}_d}{\bar{\nu}(1 - y)E_{\perp}E_{\parallel}^2} \right]^{1/3} \frac{1}{1 + \chi} \hat{K}_s(r), \tag{50} \]

where

\[ \hat{K}_s(r) = \left( 4r^2 - 1 \right)^{2/3} \left[ 16r^2(1 + 6r + 3) \right]. \tag{51} \]

The fact that the latter function tends to infinity for cubic grains \((r = \frac{1}{2})\) is a consequence of the simplifications within our model. In fact, for cubic grains, the perpendicular component of the grain angular moment will be of the order of the overall angular momentum, as pointed out in § 2. It is easy to see that for moderately oblate grains, however, \(\langle J_{\perp}^2(0) \rangle\) is dominated by the term \(J_{\perp,0}^2\) arising from thermal fluctuations. As \(\chi \sim (1 - y)^{-1}\), for small concentrations of atomic hydrogen, \(J_{\perp,\text{torque}}^2\) may become important.

One of the problems with considering very small concentrations of atomic hydrogen is that disorientation then happens not only during crossovers but also during spin-ups (see Lazarian 1995d), and this requires the theory of suprathermal alignment to be modified. Moreover, according to equation (23), for \(y \rightarrow 1\) only very large grains obey the theory we discuss here.

On estimating the critical size \(a_c\) in equation (23), we assumed that \(\langle J_{\perp}^2 \rangle = J_{\perp,0}^2\). In general, this is not true, and our estimate of \(a_c\) should be multiplied by \((1 + R^{-2})^{1/2}\). The latter value, however, is \(\sim 1\) according to equation (50); therefore, our estimate of the critical size given by equation (23) stays essentially unaltered.

Although in the grain frame of reference \(J_{\perp,\text{torque}}\) and \(J_{\perp,0}\) appear very similar, their difference is obvious in the inertial frame. \(J_{\perp,0}\) arises from thermal fluctuations within the grain material that do not alter the direction of \(J\). On the contrary, \(J_{\perp,\text{torque}}\) arises from gaseous bombardment and stochastic \(H_2\) formation events that do directly affect the direction of \(J\).

As seen from equation (50), we expect to have \(J_{\perp,0}^2 \gg J_{\perp,\text{torque}}^2\); in this limit, the disorientation parameter \(F\) (see eq. [38]) can be obtained:

\[ F \approx (9.0 \times 10^{-3}) \hat{a}^{1/2} \hat{a}^{1/2} \hat{T}_d^{1/2} \hat{\rho}_s^{1/2} \hat{\nu}^{1/2}(1 + \frac{\chi}{2}) \hat{K}_d(r), \tag{52} \]

where

\[ \hat{K}_d(r) = \frac{8(2r + 5)}{179} \left[ \frac{5(4r^2 - 1)}{3r(4r + 1)} \right]^{1/2} \times \left\{ 1 + \frac{3[16r^2(1 + 6r + 3)]}{8r^2(2r + 5)} \right\}. \tag{53} \]

The disorientation decreases as \(r \rightarrow \frac{1}{2}\), which corresponds to a cubic grain.

For the case of \(J_{\perp,\text{torque}}^2 \gg J_{\perp,0}^2\), we can also obtain an estimate of \(F\):

\[ F \approx (0.27) \hat{a}^{1/3} \hat{a}^{1/2} \hat{T}_d^{1/2} \hat{\rho}_s^{1/2} \hat{\nu}^{1/6} \sqrt{1 + \chi} \hat{K}_s(r), \tag{54} \]

where

\[ \hat{K}_s(r) = \frac{8}{179} \sqrt{205(2r + 5)(4r^2 - 1)^{1/2}} \times \left\{ 1 + \frac{3[16r^2(1 + 6r + 3)]}{8r^2(2r + 5)} \right\}. \tag{55} \]

This estimate coincides with that in Lazarian (1995c) in the limit of negligible contribution from the gaseous bombardment.

4. IMPLICATIONS FOR THE ALIGNMENT

4.1. Paramagnetic Alignment

Let \(\theta\) be the angle between \(J\) and the interstellar magnetic field \(B\). Paramagnetic alignment of suprathermally rotating grains can be described using the equation (Purcell 1979)

\[ \frac{d\theta}{dt} = -\frac{\sin \theta \cos \theta}{t_r}, \tag{56} \]

where \(t_r\) is the time of relaxation of a grain with volume \(V\) in the ambient field \(B\):

\[ t_r = \frac{I_z}{B^2 VK} = (6.7 \times 10^{13}) \frac{\hat{\nu} \hat{a}^2 \hat{T}_d}{B^2} s, \tag{57} \]

where \(K \approx 1.2 \times 10^{-13} \hat{T}_d^{-1} s\) (Draize 1996).

The solution of the differential equation above is trivial:

\[ \tan \theta = \tan \theta_0 \exp \left(-\frac{t}{t_r}\right). \tag{58} \]

If the grains are randomly oriented at \(t = 0\), Purcell (1979) showed that after time \(t\),

\[ Q(t) = \frac{3}{2} \left( \left[ \cos^2 \theta - \frac{1}{3} \right] \right) \]

\[ = \frac{3}{2} \frac{1 - (e^\delta - 1)^{-1/2} \arctan \sqrt{e^\delta - 1} - 1}{e^\delta - 1}, \tag{59} \]

where \(\delta \equiv 2t/t_r\) and square brackets denote averaging over initial grain orientations.

Now suppose that grains are randomly oriented following crossovers, and let \(P(t)dt\) be the probability that a randomly selected grain will have gone a time \(t_b \in [t, t + dt]\) since its last crossover event. To obtain the Rayleigh reduction factor (Greenberg 1968),

\[ \sigma = \frac{\int_0^\infty Q(t)P(t)dt}{\int_0^\infty P(t)dt}. \tag{61} \]

For our simplified treatment, we will assume that for any particular grain in the ensemble, the crossovers happen

\[ \sigma = \frac{\int_0^\infty Q(t)P(t)dt}{\int_0^\infty P(t)dt}. \tag{61} \]
periodically with period $t_{\text{max}}$. Then
\[
P(t_{\text{max}}) = \begin{cases} 
  t_{\text{max}}^{-1}, & t < t_{\text{max}} \\
  0, & t > t_{\text{max}}. 
\end{cases}
\] (62)

For this distribution, the mean time between crossovers (or zero-crossings) is $t_{\text{c}} = t_{\text{max}}$. Integrating equation (61), we get
\[
\sigma = 1 + \frac{3}{\delta_{\text{max}}} \left( \frac{\arctan \sqrt{\delta_{\text{max}}^2 - 1}}{\sqrt{\delta_{\text{max}}^2 - 1}} - 1 \right),
\] (63)

where $\delta_{\text{max}} = 2t_{\text{c}}/t_{r}$. For small $\delta_{\text{max}}$, equation (63) can be expanded
\[
\sigma \approx \frac{\delta_{\text{max}}}{10} + \frac{\delta_{\text{max}}^2}{210} + \frac{\delta_{\text{max}}^3}{840} + \cdots.
\] (64)

Up to this point, we assumed complete disorientation in the course of a crossover. It is evident from Table 2 that $t_{r} \gg t_{d}$ for typical interstellar conditions. As the mean “time back to crossover” for short-lived spin-up (e.g., for $t_{L} \lesssim t_{d}$) is of the order of $t_{d}$, paramagnetic alignment is marginal unless the proper order of paramagnetic alignment is marginal unless the directions of $J$ before and after crossovers are strongly correlated (SM).

To account for incomplete disorientation, SM adopted the following reasoning: consider crossovers that occur at intervals $t_{\text{max}}$; then, in a time $t_{\text{max}}/F$, the disorientation decreases $\cos \eta$ by $1/e$. Thus, according to SM, the effects of incomplete disorientation during crossovers may be approximated by replacing $t_{c}$ by $t_{c}/\min \langle F \rangle$. Hence, we use $\langle \ldots \rangle_{J}$ to denote averaging over the distribution of $J_{z}$. We remind our reader that up to now we evaluated the following reasoning: consider crossovers that occur at correlated (SM).

We conjecture that replacing $\delta_{\text{max}}$ in equation (63) with
\[
\delta_{\text{eff}} = \frac{2t_{\text{c}}}{[1 - \exp (-\langle F \rangle_{J})]}.
\] (65)

to obtain
\[
\sigma \approx 1 + \frac{3}{\delta_{\text{eff}}} \left( \frac{\arctan \sqrt{\delta_{\text{eff}}^2 - 1}}{\sqrt{\delta_{\text{eff}}^2 - 1}} - 1 \right).
\] (66)

may give a better fit than the SM approximation above, as it allows for residual correlation for $\langle F \rangle_{J} \gtrsim 1$. It is evident that for $\langle F \rangle_{J} \gg 1$ and $\langle F \rangle_{J} \ll 1$, our approximation coincides with that in SM.

To study the effects of incomplete disorientation below, we use Monte Carlo simulations to calculate $\sigma$ for different ratios of $t_{r}/t_{c}$ and $\langle F \rangle_{J}$.

To obtain $\langle F \rangle_{J}$, we require $\langle 1/J_{z}(0) \rangle_{J}$. To estimate this, we note that $\langle J_{z}(0) \rangle_{J} = \langle J_{z}^{2}(0) \rangle^{1/2}$ and, because of the symmetry inherent to the problem,
\[
\langle J_{z}^{2}(0) \rangle_{J} = \langle J_{z}^{2}(0) \rangle = 0.5 \langle J_{z}^{2}(0) \rangle_{J}.
\] (67)

For a Gaussian distribution with $\sigma_{z}^{2} = \langle J_{z}^{2}(0) \rangle_{J}$, one gets
\[
\langle J_{z}^{2}(0) \rangle_{J} = \frac{1}{2\pi \sigma_{z}^{2}} \int_{0}^{\infty} 2\pi r \exp \left( -\frac{r^{2}}{2\sigma_{z}^{2}} \right) dr = \frac{\sqrt{\pi}}{2 \sigma_{z}} = \frac{\sqrt{\pi}}{\sqrt{\langle J_{z}^{2}(0) \rangle_{J}^{1/2}}},
\] (68)

Thus, from equation (38), we get
\[
\langle F \rangle_{J} \approx \frac{\pi^{3/2}}{4} K_{1}(t) \frac{\langle (\Delta J_{z}^{2}) \rangle}{\langle (\Delta J_{z}^{2})(0) \rangle^{1/2}}
\]
\[
= \left( 1.60 \times 10^{-2} \right)^{1/2} \left[ \frac{\hat{F}}{\hat{F} T_{J} \hat{B} (1 + R^{-2})} \right]^{1/2} (1 + \chi) \hat{K}_{J}(t).
\] (69)

In the simulations, we use equation (34) to find $\langle \cos \eta_{J} \rangle$ for a fixed $J_{\perp}$ and then perform numerical averaging of $\langle \cos \eta_{J} \rangle$ over a Gaussian distribution of $J_{\perp}$. We require a distribution function for the stochastic jumps that correspond to $\langle \cos \eta_{J} \rangle$. We assume the distribution of $\eta$ has the form
\[
P_{\eta}(\eta)d\eta = C \sin \eta \exp \left[ -a \frac{\sin^{2} \eta}{2} \right] d\eta,
\] (70)

where $C$ is the normalization constant
\[
C = \frac{x^{2}}{2} \frac{1}{1 - \exp (-x^{2})}
\] (71)

and $x$ is the solution of the transcendental equation
\[
\langle \cos \eta_{J} \rangle = 1 + \exp \left( -a^{2} \right) \frac{2}{1 - \exp (-a^{2})}.
\] (72)

An individual jump over $\eta$ during a crossover event happens in a random direction, and we obtain the final value of $\theta_{t_{J}}$ after the $i$th crossover from
\[
\cos \theta_{t_{J}} = \cos \theta_{t_{i}} \cos \eta + \sin \theta_{t_{i}} \sin \eta \cos x,
\] (73)

where $x$ is a random variable uniformly distributed over $[0, 2\pi]$ and $\theta_{t_{i}}$ is the value of the alignment angle just before the $i$th crossover. Between crossovers, the dynamics of the alignment angle is determined by equation (58). Averaging over $P(t_{\text{max}})$ (see eq. [62]) is also performed to account for the distributions of the times since the last crossover.

The results of these calculations are shown in Figure 2, where we have plotted $\sigma$ versus $\delta_{\text{eff}}$ (defined by eq. [65]). For each value of $\delta_{\text{eff}}$, different symbols correspond to the alignment measures obtained for different values of $\langle F \rangle_{J}$. The solid line in the same plot corresponds to equation (66). In the limit $\langle F \rangle_{J} \to \infty$, we have complete disorientation, in which case equation (66) is an exact result (for periodic crossovers). However, it is evident that equation (66) provides a good approximation to the numerical results for finite $\langle F \rangle_{J}$, at least for periodic crossovers. More general models where the times between crossovers are determined through Monte Carlo simulations are studied in Draine & Lazarian (1997).

For typical values of interstellar parameters (see Table 2), one obtains $F \approx 0.014$ (see eq. [52]). Using our earlier estimate $t_{r} \approx 1.6t_{d} \tilde{a}^{1/2}$ (for assumed $t_{d}/t_{r} = 0.25/\tilde{a}$), we get $\delta_{\text{eff}} \approx 13.7$ (indicated by an arrow in Fig. 2), for which equation (66) gives $\sigma \approx 0.8$. This high degree of alignment is caused by the small estimated value of $\langle F \rangle$. In fact, it was argued in Lazarian (1995c) that $t_{d}$ is expected to be several times greater than $t_{r}$ for grains with $a > 10^{-5}$ cm and this, by increasing $t_{r}$, would further increase the expected alignment.

The assumed functional form (70) has the required behavior of $dP_{\eta}/d\eta = 0$ for $\eta = 0$, $\pi$; $P_{\eta} \sim \sin \eta$ for $\pi \to 0$; and $P_{\eta} \sim \eta$ for $\eta \ll 1$. 

\footnote{The theory of alignment for an arbitrary distribution of time intervals between crossovers is given in Draine & Lazarian (1997).}
Although we do not quantitatively discuss the alignment of grains with \( a < a_c \approx 1.5 \times 10^{-5} \) cm here, we conjecture that the alignment of such grains may be suppressed by the effective disalignment during each crossover (i.e. \( \langle F \rangle > 1 \)), owing to the variations in the angle \( \beta \) caused by Barnett fluctuations when \( t_B < t_c \) (see § 3.2). The strong dependence of \( t_B/t_c \) on \( a \) (see eq. [22]) suggests that this may account for the observed lack of alignment of interstellar grains with \( a \lesssim 10^{-5} \) (Kim & Martin 1995).

We see, then, that if the only important torques were those caused by \( \text{H}_2 \) formation, gas-grain collisions, and paramagnetic dissipation, we would expect paramagnetic grains in diffuse clouds to be substantially aligned for \( a > a_c \approx 1.5 \times 10^{-5} \) cm and probably minimally aligned for \( a < a_c \), at least qualitatively consistent with observations. It has recently been recognized, however, that starlight plays a major role in the dynamics of \( a > 0.1 \) \( \mu \)m grains: the torques exerted by anisotropic starlight drive suprathermal rotation (Draine & Weingartner 1996) and can directly act to align \( J \) with the interstellar magnetic field (Draine & Weingartner 1997).

The importance of radiative torques relative to \( \text{H}_2 \) formation torques can be measured by a “radiative torque” parameter \( \epsilon \), defined in Draine & Lazarian (1997). For the “typical” grain parameters considered in Draine & Lazarian (1997), \( \epsilon \approx 0.007 \) and radiative torques are sufficiently weak (relative to \( \text{H}_2 \) formation torques) that the present analysis, in which starlight torques are neglected, remains applicable. As discussed by Draine & Lazarian (1997), the existing uncertainties in grain properties—particularly the surface density \( \sigma \) of \( \text{H}_2 \) formation sites—are considerable, so that grains may well have \( \epsilon \gtrsim 1 \). A study of crossovers incorporating the effects of both \( \text{H}_2 \) formation and anisotropic starlight is planned. It appears to us highly likely that when both effects are included, the observed lack of alignment of \( a \lesssim 0.1 \) \( \mu \)m grains and substantial alignment for \( a \gtrsim 0.1 \) \( \mu \)m grains will be explained.

We recall that suprathermal rotation can also be driven by variations of the accommodation coefficient and photoelectric emissivity (Purcell 1979). In a molecular (\( y = 1 \)) region with no ultraviolet light, Purcell's estimate for the torque caused by variations in accommodation coefficient leads to \( a_c = 3.0 \times 10^{-5} \) cm as the radius for which \( t_B = t_c \). However, for this case we also find \( \langle F \rangle > 1 \) for \( a \approx a_c \), so that paramagnetic alignment will be ineffective unless \( t_c > t_c \).

4.2. Mechanical Alignment

It was previously thought that suprathermally rotating grains are not subject to mechanical alignment when the gas is streaming relative to the grain. However, two mechanisms of mechanical alignment of suprathermally rotating grains, namely, “cross section” and “crossover” alignment, were proposed by Lazarian (1995d). The first process is caused by the fact that the frequency of crossover events depends on the value of the cross section exposed to the gaseous flux (see Lazarian & Efroimsky 1996 for more details). The second mechanism arises from the substantial susceptibility of grains to alignment by gas-grain streaming during crossover events.

Both mechanical processes are related to the phenomenon of crossovers. Thus, our finding of reduced disorientation during crossovers is a new feature that should be incorporated into the discussion of mechanical alignment. As we mentioned earlier, this reduced randomization is valid only for grains with \( a > a_c \), where \( a_c \) is given by equation (23); therefore, no changes of the earlier results are expected for grains with \( a < 10^{-5} \) cm. Such grains can be aligned, for instance, by ambipolar diffusion, which favors small grains.

To start with, consider the cross section mechanism. In Lazarian (1995d), this mechanism was exemplified using a toy model, namely, a flat disk grain that randomly jumps in the course of a crossover from one position, where the surface of the disk is parallel to the flow, to the other position, where the disk surface is perpendicular to the flow. If the probability per unit time of a crossover is proportional to the rate at which atoms arrive at the grain surface, it is easy to see that the grain will spend more time at orientations where the cross section presented to the streaming gas is minimal. Within the toy model above, this corresponds to the position with the surface of the disk parallel to the flow.

In other words, the cross section mechanism uses the fact that \( t_c \) is a function of the angle \( \phi \) between the grain axis of major inertia and the direction of the gaseous flow. Roughly speaking, our study above shows that crossovers with disorientation parameter \( \langle F \rangle \) and mean time between crossovers \( t_c \) are equivalent to crossovers with complete disorientation and the mean time between crossovers \( t_c/[1 - \exp (-\langle F \rangle)] \). If \( F \) is dominated by thermal fluctuations, its dependence on gaseous bombardment vanishes (see eq. [52]) as does its dependence on \( \phi \). Thus the only effect of incomplete disorientation during crossovers (as compared to full disorientation) is to increase the alignment time (the time to attain a steady state) by a factor of \([1 - \exp (-\langle F \rangle)]^{-1}\).

Crossover alignment depends on the ratio of the randomizing torques arising from \( \text{H}_2 \) formation and aligning torques caused by gaseous bombardment (see Lazarian 1995d). This ratio neither depends on the number of crossovers nor on the time of alignment. The fact that the
thermal fluctuations do not change the direction of $J$ is essential for understanding why this type of alignment is not suppressed in the presence of the incomplete disorientation during crossovers. It is possible to show, however, that the time of alignment increases by a factor of

$[1 - \exp (-\langle F \rangle)]^{-1}$.

In spite of the fact that the measure of the mechanical alignment does not change, our observation that the time required to reach steady state is increased by a factor of

$[1 - \exp (-\langle F \rangle)]^{-1}$ can be important. This is particularly important whenever grain alignment is caused by a transient phenomenon, e.g., a MHD shock. If $t_s/[1 - \exp (-\langle F \rangle)]$ is much longer than the time of streaming, the alignment of grains with $a > 1.5 \times 10^{-5}$ cm will be marginal.

5. CONCLUSION

We have shown that thermal fluctuations within the grain material limit the extent to which the axis $a_i$ of major inertia can be aligned with the angular momentum $J$ in suprathermally rotating grains. Although the fluctuating angle $\beta$ between $a_i$ and $J$ is tiny when the grain is rotating suprathermally, it becomes larger and of critical importance during periods of crossovers. We have proved that for grains with $a > a_c \approx 1.5 \times 10^{-5}$ cm, the nonzero component of $a_i$ perpendicular to $J$ arising from thermal fluctuations substantially diminishes the degree of the randomization of the angular momentum direction in the course of crossovers. If the only torques acting on a grain are those due to gas-grain collisions, $H_2$ formation, and paramagnetic dissipation, our estimates show that for large $(a \gtrsim a_c)$ grains, the grain alignment is close to perfect, while small grains $(a < a_c)$ would have only marginal alignment.

If there is gas-grain streaming, the thermal fluctuations increase the time for mechanical alignment for large suprathermally rotating grains, but they do not alter the limiting steady state measure of alignment. If the mechanical alignment is caused by Alfvénic waves, it acts in unison with the paramagnetic mechanism to enhance the alignment of large grains. For small grains, mechanical alignment caused by transient phenomena (e.g., ambipolar diffusion within MHD shocks) can be the dominant cause of alignment.

We are extremely grateful to the late Lyman Spitzer, Jr. for elucidating discussions of the crossover process as well as many other aspects of interstellar astrophysics. He will be profoundly missed.

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APPENDIX

SOME RESULTS FOR A SQUARE PRISM

Here we consider a square prism, with dimensions $b \times b \times a$, density $\rho$, mass $\rho b^2 a$, area $4ba + 2b^2$, and moments of inertia $I_z = (8/3) \rho a(b/2)^2$ and $I_y = I_x = (1/3) \rho a(b/2)^2$. We let $r \equiv (b/2a)$ and note that $r = 1$ corresponds to the prism grain discussed in Purcell (1979). We consider a hydrogen-helium gas with density $n_H \equiv n(H) + 2n(H_2)$, temperature $T$, molecular fraction

$$y \equiv \frac{2n(H_2)}{n_H},$$

and $n(He)/n_H = 0.1$. The square prism has

$$t_M = \frac{\rho a}{n_H v_H m_H (1.2 - 0.293y)} \frac{2r}{(r + 1)}, \quad t_d = \frac{(r + 1)}{(r + 2)} t_M,$$

where $t_M$ is the time for the grain to collide with its own mass of gas, $t_d$ is the rotational damping time (assuming incident atoms temporarily stick), and $v_H = (8kT/\pi n_H)^{1/2}$ is the mean speed of H atoms. If a fraction $\gamma$ of impinging H atoms are converted to $H_2$, then the $H_2$ formation rate is

$$N_1 = r(r + 1)\gamma(1 - y)n_H v_H a^2.$$

We assume the grain is spinning around the z-axis. The prism is assumed to have $\nu$ active sites of $H_3$ formation distributed randomly over the surface. Following Purcell, we assume that newly formed $H_2$ molecules depart from each recombination site at a rate $N_1/\nu$, with fixed kinetic energy $E$ but random directions ($dP/d\theta = 2 \sin \theta \cos \theta$, where $\theta$ is with respect to the local surface normal). The $\nu(1 + r)$ sites on the sides of the prism then produce a steady torque $L_\gamma$ with

$$\langle L_\gamma^2 \rangle^{1/2} = r^2(r + 1)\gamma(1 - y)n_H v_H a^2 \left(\frac{2m_H E}{3y}\right)^{1/2},$$

and a mean angular impulse per recombination event $\langle \Delta J_\gamma \rangle$, with

$$|\langle \Delta J_\gamma \rangle| \approx \frac{\langle L_\gamma^2 \rangle^{1/2}}{N_1} \frac{r}{(r + 1)^{1/2}} \left(\frac{2m_H a^2 E}{3y}\right)^{1/2}.$$

10 Purcell & Spitzer (1971) give $t_s/t_d$ for a square prism, but their equation (9) contains a typographical error: the factor $(5s + 1)$ should instead be $(4s + 1)$. Our $r$ is equal to $1/(2a)$ as defined in Purcell & Spitzer (1971).
Individual H recombination events, occurring at a rate $N_1$, contribute random angular momentum impulses with

$$\langle (\Delta J_z)^2 \rangle_H = \frac{1}{3} \frac{r^2(2r + 5)}{r + 1} m_H a^2 E,$$

(A6)

$$\langle (\Delta J_z)^2 \rangle_H = \frac{16r^2(r + 1) + 6r + 3}{12(r + 1)} m_H a^2 E.$$  

(A7)

Gas particles impinge at a rate

$$N_2 = 2r(r + 1)n_H v_H a^2 \left( 1.05 - y + \frac{y}{\sqrt{8}} \right);$$

(A8)

if impinging particles temporarily stick and then thermally desorb at temperature $T_d$, then these collision events produce

$$N_2 \langle (\Delta J_z)^2 \rangle_s = \frac{2}{3} r^3 (2r + 5) n_H m_H v_H a^4 k(T + T_d) \left( 1.2 - y + \frac{y}{\sqrt{2}} \right),$$

(A9)

$$N_2 \langle (\Delta J_z)^2 \rangle_s = \frac{1}{6} r [16r^2(r + 1) + 6r + 3] n_H m_H v_H a^4 k(T + T_d) \left( 1.2 - y + \frac{y}{2} \right).$$

(A10)