Calibration for CHAMP Accelerometer Data Based on Crossover Points of the Satellite

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ABSTRACT  The German CHAlleging Minisatellite Payload (CHAMP) was launched in July 2000. It is the first satellite that provides us with position and accelerometer measurements, with which the gravity field model can be determined. One of the most popular methods for geopotential recovery using the position and accelerometer measurements of CHAMP is the energy conservation method. The main aim of this paper is to determine the scale and bias parameters of CHAMP accelerometer data using the energy conservation method. The basic principle and mathematical model using the crossover points of CHAMP orbit to calibrate the accelerometer data are given based on the energy balance method. The rigorous integral formula as well as its discrete form of the observational equation is presented. This method can be used to estimate only one of the scale and bias parameters or both of them. In order to control the influence of outliers, the robust estimator for the calibration parameters is given. The results of the numerical computations and comparisons using the CHAMP accelerometer data show the validity of the method.

KEYWORDS  energy conservation; scale factor; bias parameter; robust estimation

CLC NUMBER  P228.41; P207

Introduction

Since its launch on 15 July 2000, data from the CHAMP gravity mission have demonstrated notable improvement in the modeling of Earth’s gravity field. The CHAMP carrying a GPS receiver and a three-axes accelerometer is the first satellite that provides us with orbit and accelerometer data simultaneously. Using these data, a global gravity field can be recovered. The accelerometer data and precise orbits of CHAMP are released by GFZ, given in a 10 seconds interval. Although the accelerometer data have been pre-processed by GFZ, they need to be re-calibrated for the unknown bias and scale factor\(^{1,2}\). A lot of methods of calibration for CHAMP accelerometer data are proposed by many scholars\(^{3,4}\). One of the most efficient and simple methods for calibrating the accelerometer data is based on energy conservation equation. Using the principle of energy conservation for gravity field recovery from satellite observation has been considered since the early satellite era\(^{5,6}\). Because of the technique and quality of orbit determination at that time, the results were pessimistic and the method has not been used operationally until today. With the launch of CHAMP and GRACE, the energy conservation approach can be usefully applied. Recently the feasibility of the method was tested using the real CHAMP orbits and acceleration data by many scholars\(^{7,8}\).

The ISDC of GFZ provides two types of accelerometer production: one is the data file with no scale and bias parameter (or scale equals unit one, bias equals zero), which is the version of * . 91; the other is with constant scale and bias, which is the version of * . 12 (or maybe the ver-
sion of *. A lot of research has shown that the accelerometer data need to be calibrated due to unknown bias and scale factor, which can make great influence on the determination of orbit and gravity field. On the other hand, scale and bias cannot be constant in time\(^{6,7}\), so the unchanged parameters provided by GFZ is not suit for user, especially for those who requires high accuracy of the accelerometer data. In most of the existant literatures, it is a common method to estimate only one parameter out of scale and bias, while fixing the other to a constant. A detailed and rigorous mathematical model for calibrating accelerometer data using crossover points of CHAMP tracks is rarely met with.

1 Energy conservation equation

The energy conservation equation for CHAMP satellite in the inertial system can be expressed as \(^{6,7,10}\):

\[
T = \frac{1}{2} \|v\|^2 - V_{\text{sun}} - V_{\text{moon}} - \omega(xv_y - yv_x) - F - E_0 - U_0
\]

(1)

where \(T\) is the disturbing potential; \(\frac{1}{2} \|v\|^2\) is the kinetic energy per unit mass of the satellite; \(\|v\|^2 = (v_x, v_y, v_z)^T\) is the velocity vector; \(V_{\text{sun}}\) and \(V_{\text{moon}}\) are the tidal potential of the sun and the moon respectively; \(\omega\) is the rotation angular velocity of the earth; \(x = (x, y, z)^T\) is the position vector; \(F\) is the energy loss or dissipation resulting from the non-conservative force; \(E_0\) is an integration constant; \(U_0\) is the normal potential without centrifugal term of Geodetic Reference System 1980 (GRS80).

The frictional energy \(F\) can be calculated as :

\[
F = \int v \cdot a dt
\]

(2)

where \(a = (a_x, a_y, a_z)^T\) is the acceleration vector due to non-conservative force measured by accelerometer on board of CHAMP.

If it is known the position vector \(x\), velocity vector \(v\) and the acceleration vector \(a\) of the non-conservative force, then the disturbing potential \(T\) can be determined. With the relationship between the disturbing potential and spherical harmonic coefficients, the Earth gravity field model can be recovered using time-wise or space-wise method.

It should be pointed out that large errors occur in the measurements of the radial component, due to a failure in one of the electrodes of the accelerometer\(^{6, 7, 10}\). So the energy dissipation calculated from Eq. (2) will be distorted if the radial component is not dealt with reasonably. An approximate formula is proposed as follows\(^{6,7}\):

\[
F_{\text{drag}} = \int |v| |a_y| dt
\]

(3)

where \(|v|\) is the module of the velocity vector.

Eq. (3) seems acceptable because the main part of the non-conservative forces is from air drag, which is measured by the along-track component \(a_y\) of the accelerometer. For this reason, the scale and bias parameters of the along-track component are only re-calibrated.

2 Determination of the scale and bias parameters based on crossover points of the satellite

Eq. (1) can be re-written as

\[
T = \bar{T} - \int |v| |a_y| dt - E_0
\]

(4)

where

\[
\bar{T} = \frac{1}{2} \|v\|^2 - V_{\text{sun}} - V_{\text{moon}} - \omega(xv_y - yv_x) - U_0
\]

(5)

At crossover points of the orbit ascending and descending tracks, the potential values must be identical disregarding temporal variations in the gravity field. Potential difference will be show up if there are systematic errors such as scale factor and bias in the accelerometer data. At crossover points, the following equation can be written:

\[
\bar{T}_{\text{ascend}} - \int_{t_0}^{t_{\text{ascend}}} |v| |a_y| dt - E_0 =
\]

\[
\bar{T}_{\text{descend}} - \int_{t_0}^{t_{\text{descend}}} |v| |a_y| dt - E_0
\]

(6)

After a brief derivation, Eq. (6) can be written as

\[
\int_{t_0}^{t_{\text{ascend}}} |v(t)| a_y(t) \cdot dt = \Delta T
\]

(7)

where \(\Delta T(t) = \bar{T}_{\text{ascend}} - \bar{T}_{\text{ascend}}\), if \(t_{\text{ascend}} < t_{\text{descend}}\).
Both sides of Eq. (7) should plus minus if $t_{\text{ascend}} > t_{\text{descend}}$. For the convenience of discussion, assuming that $t_{\text{ascend}} < t_{\text{descend}}$. It doesn't influence the subsequent derivation.

Let bias parameter and scale factor be represented by $k_0$ and $k_1$ respectively, and there are $n$ crossover points, then the observational equations can be written as:

$$
\int_{t_i}^{t_i^{\text{ascend}}} |v(t)| (k_0 + k_1 a_y(t)) \cdot dt = \Delta T_1
$$

$$
\int_{t_i}^{t_i^{\text{descend}}} |v(t)| (k_0 + k_1 a_y(t)) \cdot dt = \Delta T_2
$$

$$
\vdots
$$

$$
\int_{t_{n-1}^{\text{ascend}}}^{t_n^{\text{ascend}}} |v(t)| (k_0 + k_1 a_y(t)) \cdot dt = \Delta T_n
$$

where $t_i^{\text{ascend}}$ and $t_i^{\text{descend}}$ represent the time of ascending track and descending track of the $i$th crossover point respectively.

Since the above formulae are in the integral form, it should be transformed into discrete form according to the sampling interval of accelerometer data. Supposing there are $m$ epochs between $t_{n-1}^{\text{ascend}}$ and $t_n^{\text{ascend}}$ indicated by the time $t_m(1), t_m(2), \ldots, t_m(m)$, then the $i$th observational equation can be written in discrete form as:

$$
|v(t_i)| (k_0 + k_1 a_y(t_i^{\text{ascend}})) \cdot \Delta t + \sum_{j=1}^{m_n} |v(t_m(j))| (k_0 + k_1 a_y(t_m(j))) \cdot \Delta t + |v(t_m(m))| (k_0 + k_1 a_y(t_m(m))) \cdot (t_i^{\text{ascend}} - t_n(m)) = \Delta T_i
$$

where $\Delta t$ is the sampling interval.

The first and last terms at left side of Eq. (9) are the discrete expressions due to the discrepancy of the crossover point time and the observation time, and the rest is the discrete form according to the sampling interval.

The approximate formulae of Eq. (8) can be written as:

$$
|v(t_1)| (k_0 + k_1 a_y(t_1)) \cdot \Delta t(t_1) = \Delta T_1
$$

$$
|v(t_2)| (k_0 + k_1 a_y(t_2)) \cdot \Delta t(t_2) = \Delta T_2
$$

$$
\vdots
$$

$$
|v(t_n)| (k_0 + k_1 a_y(t_n)) \cdot \Delta t(t_n) = \Delta T_n
$$

where $t_i = (t_i^{\text{ascend}} + t_i^{\text{descend}}) / 2$, and $\Delta t(t_i) = |t_i^{\text{ascend}} - t_i^{\text{descend}}|$. It should be pointed out that the approximate formulae are equivalent to substitute the integral equation by mid-point formula. Its accuracy relies on the characteristics of non-conservative acceleration change in the integral range. It is shown through a real numerical example that the calibration is not so well using the above approximate formulae. On the basis of these facts, the discrete forms of the rigorous Eq. (8) are used for determining the scale and bias parameters in this paper.

The following problems should be noticed when determining the scale and bias parameters using the crossover points of CHAMP.

1) As the time of crossover point is not always the same as that of observation, the disturbing potential should be up/down continued. Within the used period, CHAMP's altitude varies between 400-460 km. For such a height variation, the disturbing potential can be expanded in Taylor series (restricted to quadratic order) with sufficient accuracy. Then the disturbing potential $T$ reads:

$$
T = T_0 + \left( \frac{\partial T_0}{\partial r} \Delta h + \frac{1}{2} \frac{\partial^2 T_0}{\partial r^2} \Delta h^2 \right)
$$

2) In order to insure enough crossover points as well as to make computation simple, the disturbing potential can be radially continued to a constant orbit height such as 430 km. This makes them in a circle orbit and has crossover points as many as possible.

3) In the procedure of up/down continuation, the existing potential model EGM96, EING-1S or EING-2 can be used, which makes little difference on the disturbing potential.

4) The non-conservative acceleration at the crossover points can be interpolated or extrapolated from CHAMP accelerometer data.

3 Robust estimator of scale and bias parameters

The adjustment mathematical models based on Eq. (9) can be expressed as:
\[ V = A \hat{X} - L \]  

where

\[
A = \begin{bmatrix} c_1 & d_1 \\ c_2 & d_2 \\ \vdots & \vdots \\ c_n & d_n \end{bmatrix}, \quad \hat{X} = [k_0 \ k_1]^T, \quad L = \begin{bmatrix} \Delta T_1 \\ \Delta T_2 \\ \vdots \\ \Delta T_s \end{bmatrix}
\]

\[ c_i = | v(t_i^{\text{second}}) | \cdot (t_m(1) - t_i^{\text{second}}) + \sum_{j=1}^{n} | v(t_m(j)) | \cdot \Delta t + | v(t_m(m_i)) | \cdot \Delta t + \]

\[ + \sum_{j=1}^{n} | v(t_m(j)) | \cdot a_j(t_i^{\text{second}}) \cdot \Delta t + \]

\[ | v(t_m(m_i)) | \cdot a_j(t_m(m_i)) \cdot (t_i^{\text{second}} - t_m(m_i)) \]

\[ d_i = | v(t_i^{\text{second}}) | \cdot (t_m(1) - t_i^{\text{second}}) \cdot a_j(t_i^{\text{second}}) \]

The LS estimator based on Eq. (12) is

\[ \hat{X}_{LS} = (A^TPA)^{-1}A^TFL \]

where \( P \) is the weight matrix of observations.

The accelerometer data is preprocessed from the electronic signal measured by onboard accelerometry instrument. It is inevitably influenced by the spatial environment, so outliers may occur in the accelerometer observations. As known to us, LS will be distorted if outliers exist in the observations. An efficient way to control them is robust estimation. The robust estimator of scale and bias parameters can be expressed as

\[ \hat{X}_{\text{robust}} = (A^TPA)^{-1}A^TFL \]

where \( P \) is called equivalent weight matrix, and its elements are defined as

\[ p_{ij} = p_0 \cdot w_{ij} \]

\[ w_{ij} = \begin{cases} 1, & | \tilde{v}_i | = | v_i / \sigma_{v_i} | \leq k_0 \\ \frac{k_0}{| \tilde{v}_i |} \times \left[ (k_1 - | \tilde{v}_i |) / (k_1 - k_0) \right]^2, & k_0 < | \tilde{v}_i | \leq k_1 \\ 0, & | \tilde{v}_i | > k_1 \end{cases} \]

The LS estimator based on Eq. (12) is

\[ \hat{X}_{\text{LS}} = (A^TPA)^{-1}A^TFL \]

where \( P \) is valued as a unit matrix in this paper. The scheme of robust estimation is chosen as IGGIII. A detailed procedure about it can be seen in Reference[13]. If only one parameter such as bias, needs to be estimated, we can value scale factor in the observation equations as 1 or that provided by GFZ. If only scale factor needs to be estimated, the bias can be chosen as what is provided by GFZ.

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### 4 Computation and comparison

In order to test the above algorithm, a numerical example is made using the post science orbit (PSO) and accelerometer data (with the version of *.9) provided by ISDC of GFZ. We select 12 days’ data with equivalent interval from January 3 to February 6, 2002. The sampling interval is 10. The scale and/or bias parameters of the along-track component are to be determined. The comparisons are made between the estimated parameters and those provided by GFZ. Three schemes with LS and robust estimation are carried out. The calibration parameters of LS solution are shown in Fig. 1 and Fig. 2. Those of robust solution are shown in Fig. 3 and Fig. 4. The statistical results are listed in Table 1.

1) only estimating scale factor (bias is valued as \(-3.555 \times 10^{-6} \text{m/s}^2\), provided by GFZ).
2) only estimating bias parameter (scale factor is valued as 0.833, provided by GFZ).
3) estimating scale factor and bias parameter simultaneously.

| Scheme | Scale factor | Bias parameter \((10^{-6})\) |
|--------|--------------|--------------------------|
|        | LS \( \hat{X}_{\text{LS}} \) | Robust \( \hat{X}_{\text{robust}} \) | LS \( \hat{X}_{\text{LS}} \) | Robust \( \hat{X}_{\text{robust}} \) |
| Scheme 1 (only scale) | 0.826 5 | 0.826 6 | | | |
| Scheme 2 (only bias) | | | -3.574 6 | 3.574 1 |
| Scheme 3 (both) | 0.879 7 | 0.855 5 | | | | -3.539 5 | 3.571 9 |
| Parameters provided by ISDC of GFZ | 0.833 0 | | -3.555 \(0 \times 10^{-6}\) |

Conclusions based on the above results can be drawn as follows:
1) The method of calibration developed in this paper can efficiently estimate the scale and bias parameters or only one of them. The statistical means of the estimated parameters are almost the same as those provided by ISDC of GFZ. It indicates that the parameters provided by ISDC of GFZ are the average values in the whole, and cannot reflect the actually change of them per day. The difference is even smaller when fixing one parameter and estimating the other. The reason may be that there exists a little correlation between scale and bias parameters when estimating them simultaneously, which makes their accuracy decrease to some extent.

2) The scale factor and bias are not constant in the domain of time, but show a liner or piece-wise liner change. For the user of high accuracy, it is recommended to estimate the scale and bias parameters per day, or half-day, or even a few hours.

3) There is almost no difference between the results of LS and robust estimation for scheme 1 and scheme 2, and just a little difference for scheme 3 from the view of statistics. It indicates that there are no obvious anomalies in the observations used in this paper.

5 Summary

An algorithm of calibration for CHAMP accelerometer data is developed using the crossover points of satellite tracks based on energy conservation equation. It seems to be a very complex procedure because it includes the seek of the crossover points, the interpolation or extrapolation of the non-conservative acceleration, and the up/down continuation of disturbing potential. The advantage of this method is that it poorly relies on a prior gravity field model. Another efficient method to calibrate the accelerometer data is based on the known gravity field model using energy conservation equation, which will be studied in another paper.

ACKNOWLEDGEMENTS

Our thanks to the ISDC of the GeoForschungszentrum Potsdam (GFZ) for providing the data.

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Acknowledgements

Our thanks to the suggestion and help of prof. Zhang Chuanding.

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