The Study on the Influence of Hanger’s Different Axial Deflection Based on Weibull Distribution

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Abstract: Based on the finite element analysis and tests on the strength of 7-wires steel strands under deflection, the bearing capacity of hangers of arch bridges considering deflection is studied. According to the results of finite element analysis and tests on steel strands that make of hangers, it is found that the decrease of bearing capacity of hangers is caused by the decrease of ultimate tensile strength of steel strands under the deflection. The ultimate tensile strength of steel stands will be weakened with the increase of extent of deflection and obeys Weibull distribution. Thus, the relationship between the ultimate tensile strength of steel strands and deflection angles are obtained in this paper by finite element analysis and tests. Furtherly, the ultimate tensile strength of steel strands obeying Weibull distribution under the deflection is used the to further research the bearing capacity of hangers.

1. Introduction
Since the appearance of through arch bridges, hangers has been one of the focuses of through arch bridge designers. Hangers are regarded as a kind of safe structures for a long time [1-3]. However, during the recent decades, many arch bridges failed owing to the sudden failure of hangers. Some scholars propose factors that may lead the sudden failure, such as fatigue and corrosion of hangers. In this paper, a new factor is proposed that may lead the sudden failure of hangers: deflection of hangers. Under the effect of temperature, all parts of arch bridges will produce deformation; different parts of bridges have different degrees of deformations. Along the longitudinal direction of the bridge, arch ribs are fixed by foundations at both ends of bridges and cannot produce deformation, however, bridge decks can produce significant deformation owing to the exist of expansion joints. The different deformation along the longitudinal direction of the bridge at both ends of hangers will make hangers deflect (shown in Figure 1). The deflection of cables in bridges also exists in suspension bridges and cable-stayed bridges. Luckily, the main cable of the suspension bridge does not have enough stiffness to force suspenders to become deformed significantly; the inclined setting way of cables in cable-stayed bridge mainly cause the change in axial force of the cable and the bending deformation can be ignored [4,5]. Therefore, the deflection of hangers is a unique phenomenon and may be a deep reason leading to the decrease of bearing capacity of hangers of arch bridges.
Therefore, the bearing capacity of hangers under deflection is studied in this paper. The probability distribution of ultimate tensile strength of steel strand under deflection is researched by finite element analysis and testified by tensile tests on steel strength. Furtherly, the method to calculated the bearing capacity of hangers under deflection is determined based on the Weibull distribution. The results of the researches could provide theoretical basement for design of bearing capacity of hangers.

Figure 1. Deflection of hangers

2. Calculation for deflection angles of hangers

There are two main types of supports for the bridge deck of through arch bridge [6]. One way is using the fixed support and sliding support for the bridge. At present, this kind of support is no longer applied, and it only exists in a small part of old bridges that have exceeded the service life. The other way is to use rubber supports in two ends of the bridge; In this case, the bridge deck would have uniform deformation at ends of bridges. The latter which is used in most conditions is discussed in this paper.

Under the effect of temperature, the deformation of the bridge in the longitudinal direction of the bridge is basically related to the temperature change of the whole bridge, and the distribution of the temperature gradient of the bridge deck can be ignored [7]. Since the deformation along the transverse direction of the bridge is not considered, the calculation model of the bridge can be simplified into a two-dimensional model (as shown in Figure 2). It is assumed that the bridge deck is simplified into beam elements along the longitudinal direction of the bridge and affected by the temperature change (Δt). The elastic deformation of the bridge deck under the resistance of the lower anchorage end of the hanger is ignored, and the half part of the bridge is taken for analysis.

Figure 2 Deflection bending of hangers caused by longitudinal deformation of bridge decks
Firstly, the rotation angle caused by the longitudinal deformation of bridge decks is discussed. The displacement of the lower anchorage zone of the i-th hanger on the right side of the bridge center is:

\[ \Delta L = \alpha \Delta t \]

(1)

where \( L_i \) is the distance between the bridge center and i-th hanger; \( \Delta t \) is the change in temperature of the whole bridge deck; \( \alpha \) is the thermal expansion coefficient of the bridge deck [8].

The deflection bending angle \( \theta \) of hangers caused by longitudinal deformation of bridge decks (as shown in Figure 2) of the i-th hanger on the right side of the center of the bridge is:

\[ \theta = \frac{\Delta L_i}{H_i} \]

(2)

where \( H_i \) is the length of the i-th hanger.

3. Model of bearing capacity of hangers

Fig 3. Mechanical model of hangers made of parallel seven-wire strands considering deflection

The parallel steel strands can be regarded as many parallel steel fiber bundles. Therefore, the bearing capacity of hangers made of parallel steel strands could be studied by the statistical strength theory based on fiber bundle strength. When the theory of fiber bundle strength is adopted, the parallel cable can be simplified as a series parallel model (as shown in Figure 3). The single steel wire is equivalent to a series system, and the whole hanger is equivalent to a parallel system composed of multiple steel strands. Assuming that the strength of each steel strand is an independent random variable with the same distribution and steel strands share the same load, the bearing capacity of the hanger is related to the number of steel strands. Assuming that the steel strand under most unfavorable loading conditions is the weakest system in hangers so the weakest system determines the bearing capacity of hangers.

The equation determining bearing capacity is:

\[ C = \min(C_1, C_2, \cdots, C_i, \cdots, C_n) \]

(3)

The ultimate tensile strength obeys Weibull distribution:

\[ F(c) = 1 - \exp\left[-\alpha \left(\frac{C}{u_c}\right)^k\right] \]

(4)

Mean value and variance are:

\[ V[C] = u_c^2 \alpha c^{-2/k_c} \left[ \Gamma(1 + 2/k_c) - \Gamma^2(1 + 1/k_c) \right] \]

(5)
Where $\Gamma$ is Gamma function; $u_c$ and $\alpha_c$ are respectively scale and shape parameters of Weibull distribution and could be obtained by tests. Thus the tensile tests on steel strands are conducted to determine related parameters.

Figure 4. Seven-wire strands and reaction frame in the tensile test

Steel strands and the reaction frame in test are shown in Figure 4. In the test, the anchor plate was remade to set deflection bending angles for seven-wire strands. The deflection bending angle of hangers of the Pingnan Third Bridge, which is the longest HTAB in construction in the world [9], could reach about 30 mrad. The deflection bending angle of the seven-wire strands in the test is set to 30 mrad at most. Table 1 shows the ultimate tensile strength of steel strands under deflection.

Table 1. Ultimate tensile strengths of the steel strands at different deflection angles in tests

| Deflection angle $\theta$ (mrad) | Ultimate tensile strength of steel strands $\sigma_u$ (MPa) |
|---------------------------------|----------------------------------------------------------|
|                                 | results of tests                                         | average value |
| 0                               | 1978.5                                                   | 1957.1        |
|                                 | 1957.1                                                   | 1928.5        |
|                                 | 1821.4                                                   | 1792.8        |
| 10                              | 1814.2                                                   | 1792.8        |
|                                 | 1792.8                                                   | 1621.1        |
|                                 | 1621.1                                                   | 1621.1        |
| 20                              | 1621.1                                                   | 1607.1        |
|                                 | 1607.1                                                   | 1471.4        |
| 30                              | 1435.7                                                   | 1445.2        |
|                                 | 1428.5                                                   |               |

According to the tests, the ultimate tensile strength of steel strand is:

$$u_c = -14.03x + 1800$$

(6)

where $u_c$ is the angle of deflection of steel strands; $y$ is ultimate tensile strength of steel strands under deflection. To testify the accuracy of results of tensile tests, finite element analysis of seven-wire strands was implemented. We set the different lateral bending angles for the finite element model of seven-wire strands and apply the tensile force on the seven-wire strand to make it breaks (shown in Figure 5). We got the Ultimate tensile strengths of seven-wire strands at different deflection angles by finite element analysis. The comparison of the results between tests and simulation is shown in Figure 6. According to the comparison, it is found that results of tests are similar to results of simulation, which indicates the accuracy of results of tests.
Hangers are made of some parallel steel strands [10]. The calculation model of hangers made of steel strands is shown as Figure 3. The bearing capacity of the hanger made of $m$ steel strands obeys asymptotic normal distribution of mean $E_m$ and standard deviation $D_m$ [11]:

$$E_m = m x_0 (1 - F_\xi(x_0)) + c_m$$

$$D_m = x_0 [m F_\xi(x_0)(1 - F_\xi(x_0))]^{1/2}$$

Where:

$$c_m = 0.966 m^{1/3} a$$

$$a^3 = \frac{f_\xi^2(x_0)x_0^4}{2 f_\xi(x_0) + x_0 f'_\xi(x_0)}$$

Where $f_\xi$ is the probability density function of ultimate tensile strength $C$ of single steel strand. $x_0$ is the solution of extremum problem of equation (9).

$$\max \{x(1 - F_\xi(x))\}$$

According to equation (5), the mean and standard deviation of bearing capacity is related to the number of steel strands in hangers and the bearing capacity obeys Weibull distribution [12]. When the length of hangers is smaller than 100 meters, $\alpha_\xi = 3, u_\xi = -14.03x + 1800, k_\xi = 51.02$. Based on the results, the bearing capacity of hangers made of 12 parallel steel strands is calculated under different deflection angles (as shown in Table 2).
Table 2. Results of bearing capacity of hangers

| Deflection angle \( \theta \) (mrad) | Bearing capacity (kN) |
|----------------------------------|------------------------|
| 0                                | 2880                   |
| 10                               | 2615                   |
| 20                               | 2439                   |
| 30                               | 2213                   |

4. Conclusion

Through the test and finite element analysis of steel strand with 1860 MPa ultimate tensile strength in deflection state, the influence of the deflection angle on the ultimate tensile strength in deflection bending state is obtained. The calculation formula for the bearing capacity of hangers based on Weibull distribution is proposed by the results above. According to the results, it is found that the deflection could weaken the bearing capacity of hangers significantly.

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