Application of regression in algorithm of nonlinear stochastic adaptation of unstable multidimensional objects

S I Kolesnikova

St. Petersburg State University of Aerospace Instrumentation, 67, Bolshaya Morskaia str., St. Petersburg, 190000, Russia

E-mail: skolesnikova@yandex.ru

Abstract. The results of a study of applicability of kernel estimation in the synergetic control systems for the objects unstable in an open-loop state (without a stabilizing control) have been presented. The effectiveness of kernel estimates has been shown for four nonlinear objects with unstable limiting states. The estimate the effectiveness of embedding the kernel predictive estimate of the state variables of a nonlinear object, subjected to disturbances of an unknown nature, into the system of synergetic control is demonstrated.

1. Introduction

At present time, in the methods of nonlinear control the focus is shifted towards synthesizing the regulators for the objects with predetermined target manifolds (see, e.g., [1-4]), which, based on the principles of the synergetic control theory (SCT), allow for an analytical regulator synthesis without a preliminary manipulations with a complex object. The conditions for application of these methods are natural [1, 4]:

- analytical description of the aim of control (target manifold) and its consistency with the physical properties of a controlled object;
- boundedness of solutions and existence of a stable target system for the ‘object–target manifold’ pair.

The merit of SCT consists in implementation of the principle of self-organization, resulting in minimization of the efforts to achieve the required target qualities including the intrinsic reactions (physically common for the object) to external impacts [1]. This peculiarity of SCT guarantees the development of an energy-saving control (if any).

On the other hand, by setting the desirable target manifolds, testing their physical attributes and achievability one can design new objects with unexpected properties (new materials, in particular, if the analytical formula of their initial description is known).

The purpose of this work is to demonstrate a possibility and to estimate the effectiveness of embedding the kernel predictive estimate [5] of the state variables of a nonlinear object, subjected to disturbances of an unknown nature, into the system of synergetic control. It is clear that an interpretation of the disturbances can be quite diverse: from the inaccuracy of a nonlinear model (which is nearly evident due to the object complexity) to the noise in the measurement of state variables.

To begin with, let us briefly describe the essence of two algorithms based on SCT, whose outputs are the control systems correcting systematic and random disturbances, respectively. We assume that the objects under study are set by the systems of ordinary differential equations or difference stochastic
equations, and the disturbances, as the unknown functions of time, enter the right-hand part of the description.

![Diagram](image)

**Figure 1.** General structure of a control system on a target manifold with a kernel estimation and forecast of the state of a complex object.

A positive feature of the two algorithms for synthesizing adaptive control systems, which are implemented below, is their analytical justification [6].

2. **Algorithm of nonlinear adaptation**

The main points of the algorithm of nonlinear adaptation are the following:

- perform extension of the phase space via simulating derivative models of disturbances as a linear function of the target macrovariables;
- design control on the basis of the analytical design of aggregated regulators (ADAR) as the principal method of SCT as applies to the closed system obtained in Step 1;
- select auxiliary macrovariables in a special form, whose attainment automatically results in the compensation of disturbances (in the final synthesis stage).

**Remark 1.** The purpose of any control system is to maintain the standard condition. In an ideal case, an error as a deviation from this condition would be zero. If the system is subjected to random disturbances, even under the standard condition it is impossible to ensure a zero error.

Since any detailed study invariably requires a transition to a discrete description [7], the significance of the algorithms for discrete synthesis of stochastic nonlinear regulators becomes evident.

The main points of the algorithm of nonlinear stochastic adaptation (NAS) are the following.

1. Perform a classical discrete ADAR-synthesis at fixed random functions.
2. Determine the conditional mathematical expectation from the control found in Stage 1. The satisfiability of this operation is ensured by the peculiarity of the ADAR-synthesis.
3. Find the dependence for random functions as the functions measuring the object’s state relying on a substitution of the control found in Stage 2 into the main functional Euler-Lagrange equation having a linear form for the predetermined quality functional.

Let us exemplify two applications of the algorithms of nonlinear adaptation in the solution of new control problems as an illustration of their implementation and then present the results of modeling of the control systems obtained with and without the kernel estimates (figure 1).
Remark 2. Not any initial data smoothing can result in a ‘good’ form of a controlled process. The point is that “one can throw the baby with the bathwater”, since in the systems of deterministic chaos it is not always clear what we are cutting out (smoothing): a useful signal or its measurement noise.

3. Application example of the NAS-synthesis of a scalar regulator for the Lorentz model

Consider a discretized (according to Euler) description with a scalar control with respect to the 3-rd variable and the random disturbance given by \( \xi[k+1] = c\xi[k] \) of the sum of the current value and the previous value with the attenuation factor of \( 0 < c < 1 \):

\[
X_1[k+1] = F_1[k] = X_1[k] + \tau_0 (a X_2[k] X_3[k] - \gamma X_3[k]),
\]

\[
X_2[k+1] = F_2[k] = X_2[k] + \tau_0 \left( \mu \left( X_1[k] + X_3[k] \right) - \beta X_1[k] X_3[k] \right),
\]

\[
X_3[k+1] = F_3[k] + \tau_0 \left( \xi[k+1] + c\xi[k] \right), F_3[k] = X_3[k] + \tau_0 (\delta X_2[k] - \lambda X_3[k]).
\]

Let the aim of control be set in the form of sustaining a balance between the variables \( X_2[k], X_3[k] \) with the proportionality coefficient \( \rho \)

\[
\psi[k] = X_1[k] + \rho X_2[k] \quad \text{as} \quad k \to \infty \to 0
\]

Following the steps of the NAS-algorithm, obtain, respectively the following:

1) ADAR-control at the fixed random functions \( \xi[k], k = 0, 1, \ldots \) for (1) and (2):

\[
\psi[k+1] + T \psi[k] = X_1[k+1] + \rho X_2[k+1] + T \psi[k] = F_1[k] + \tau_0 u[k] + \tau_0 \left( \xi[k+1] + c\xi[k] \right) + \rho F_2[k] + T \psi[k] = 0 \Rightarrow
\]

\[
u^t[k] = -\tau_0^{-1} \left( \tilde{F}_1[k] + \rho \tilde{F}_2[k] - (\tilde{\xi}[k+1] + c\tilde{\xi}[k]) - \tau_0 \tilde{T} \psi[k].
\]

2) Conditional mathematical expectation from the control found in Stage 1

\[
u[k] = \mathbb{E} \{ u^t[k] / \xi^t \} = -\tau_0^{-1} \left( \tilde{F}_1[k] + \rho \tilde{F}_2[k] - c\tilde{\xi}[k] - \tau_0 \tilde{T} \psi[k].
\]

3) Dependence for the random functions as the functions for measuring the object’s states for the found control (3) \( \tau_0 \xi[k] = \psi[k] + T \psi[k-1] \).

The final form of the system of control:

\[
X_1[k+1] = F_1[k],
\]

\[
X_2[k+1] = F_2[k],
\]

\[
X_3[k+1] = F_3[k] + \tau_0 \left( \xi[k+1] + c\xi[k] \right),
\]

\[
u[k] = -\tau_0^{-1} \left( \tilde{F}_1[k] + \rho \tilde{F}_2[k] + (T + c) \psi[k] + cT \psi[k-1] \right) \psi[k] = X_1[k] + \rho X_3[k].
\]

Let us present the results of numerical simulation of the system (4). The simulations were performed at the following parameter values:

\[
X_1[0] = 1, X_2[0] = 1, X_3[0] = 1, \tau_0 = 0.001, u_{\min} = 0, u_{\max} = 100,
\]

\[
c = 0.1, T = 0.05, M \{ \xi[k] \} = 0, \sigma_{\xi[k]} = D^{1/2} \{ \xi[k] \} = 3;
\]

\[
\alpha = 5; \beta = 8; \gamma = 1; \delta = 0.6; \lambda = 2.46; \mu = 2.1.
\]
Figure 2. Plots of transition processes in a controlled Lorentz model with (c) and without kernel estimation (b).

Results of simulating a system of the NAS-control in accordance with the algorithm of nonlinear stochastic adaptation (figure 2).

4. Additional examples of controlled objects on the base NAS-algorithm and kernel regression

4.1.1. Example 1. Immunological model.

Let us look at a couple of simulation results (figures 3 and 4) for an immunological model with the hepatitis data [8, 9]. The mathematical description of this model suggests that the first two equations ‘resemble’ those of the Verhulst model of deterministic chaos.

Figure 3. Comparative plots of transition processes in the immunological model (fatal outcome) using kernel estimation and without it; the variable $V$ is concentration of antigens in the affected organ.

Figure 4. Dynamics of RMS $\sigma_{V[k]}^2$ of the controlled index $V$ in the immunological model using kernel estimation (green lines) and without it (ADAR - red and NAS - blue).
4.1.2. Example 2. Predator–prey model.
Look at the results of simulation of NAS-control for the model of deterministic chaos – a discretized predator–prey model with an additional random noise (figure 5) and NAS-control:

\[ x[k+1] = x[k] + \tau_o(ax[k] - bx[k]y[k]), \]
\[ y[k+1] = y[k] + \tau_o(-dy[k] + mx[k]y[k] + u[k]) + \xi[k+1] + c\xi[k], \]
\[ x[0] = x_0 > 0, y[0] = y_0 > 0, \]

where \( E\{\xi[k]\} = 0, \) \( D\{\xi[k]\} = \sigma^2_k, k \geq 0 \).

The problem of stabilization of variable \( x[k] \) in the vicinity of the predetermined value of \( x^* \), which requires that the following conditions be fulfilled (here \( E\{\xi[k]\} \) is the sign of mathematical expectation for the random variable \( \xi[k] \)):

\[ E\{\psi(x[k])\} = E\{x[k] - x^*\} \rightarrow 0, \]
\[ D\{\psi[k+1] + T\psi[k]\} \rightarrow \min, k \rightarrow \infty; \]

is solved positively using the NAS algorithm, and the regulator acquires the following form:

\[ u[k] = \tau_o^{-1}(\phi[k+1] - \omega_1\psi^{(i)}[k] - y[k]) - mx[k]y[k] + \]
\[ dy[k] - c\tau_o^{-1}(\psi^{(i)}[k] + \omega_0\psi^{(i)}[k-1]), \]
\[ \phi[k] = (bx[k]\tau_o)^{-1}\psi^{*}[k](\alpha_2 + 1) + \frac{a}{b}\psi^{(i)}[k] = y[k] - \phi[k], \]

(5)

\( \tau_o, \omega_1, \omega_2 \) are the discretization parameters of the object and the regulator, respectively.

Figure 5. Dynamics of RMS of the controlled index in the predator–prey model using kernel estimation (green lines) and without it (ADAR - red and NAS - blue)

Shown in figures 4 and 5 are the results of simulation of control algorithms designed in accordance with two methods - classical ADAR [4] and stochastic adaptation (NAS [6]) with and without filtering, from which it follows that, firstly, starting from a certain level of noise both methods can have commensurable indices, while the use of kernel smoothing maintains the steady-state behavior of the controlled variable (judging by the output variable dispersion); secondly, the algorithm of stochastic adaptation NAS itself possesses a filtering property missing in the classical ADAR under the conditions of random disturbances, and the object with homogeneous disturbance is less prone to control than that with a normal disturbance.

Remark 3. Such parameters \( T, \omega_1, \omega_2 \) as a duration of reaching the vicinity of the target state require careful selection for the given individual characteristics of the model (4) and (5).
4.1.3. Example 3. SEHB model.

Let us apply the NAS-algorithm to find the stochastic regulator for self-energizing electrohydraulic brake (SEHB) [10]:

\[
\begin{align*}
  x_1[k+1] &= x_1[k] + \delta a_1 x_4[k], \quad E[k] = \sqrt{x_1[k] - x_i[k] - \alpha p} \\
  x_2[k+1] &= x_2[k] + \delta a_2 x_4[k] E[k], \\
  x_3[k+1] &= x_3[k] - \delta a_3 x_4[k] + \delta (n + 1 + c \xi[k]) + \delta g u[k], \\
  x_4[k+1] &= x_4[k] + \delta x_1[k], \quad k = 0, 1, 2, \ldots
\end{align*}
\]

(6)

Here the variables are: \(x_1\) – load pressure; \(x_2\) – supporting pressure; \(x_3\) – control valve spool velocity; \(x_4\) – control valve spool movement; \(u\) – input solenoid voltage of the sliding spool valve (control valve input) realizing the goal achievement \(\psi' = x_2 - x_* = 0\), \(t \to \infty\), where \(x_*\) is the target value; \(a_1, a_2, a_3, a_4, g, \alpha, p\) are the parameters.

![Figure 6. Phase portrait of the controlled index \(x_2[k]\) in SEHB-model using kernel estimation; normal noise \(\sigma = 0.1\).](image)

According to the NAS-algorithm, we obtain a regulator of the form (figure 6)

\[
\begin{align*}
  u[k] &= (\delta g)^{-1} (-F_1[k] - (\delta c + \omega_i) \psi_1[k] + \varphi_1[k] - \omega_i \delta c \psi_1[k] - 1), \\
  \psi_1[k] &= x_3[k] - \varphi_1[k], \quad \delta \varphi_1(n) = -x_4[k] + (1 + \omega_2) \varphi_2[k] - \omega_2 x_4[k], \\
  \varphi_2[k] &= -(1 + \omega_3) \psi' \varphi_2[k] (\delta a_2)^{-1} E^{\nu_2 [k]}, \quad |\omega| < 1, \quad i = 1, 2, 3.
\end{align*}
\]

(7)

Remark 3. Such parameters \(T, \omega_1, \omega_2, \omega_3\) as a duration of reaching the vicinity of the target state require careful selection for the given individual characteristics of the models (4)-(7).

5. Conclusion

The search for solutions to the problem of estimating the unmeasured state variables and predicting the state variables in the problems of control over nonlinear dynamic objects is given plenty of attention in foreign and Russian literature. The main advances in this direction are associated with linear objects or preliminary linearization of nonlinear objects. The latter approximation is a quite undesirable procedure for nonlinear objects with unstable operation modes, since a minor linear approximation error can give rise to unexpected changes in the future object’s behavior (the Lorentz butterfly effect).
Examples have been given for the cases where an application of kernel estimations does not affect the properties of the synergetic adaptive regulator due to its inherent robust properties.

Thus, an unambiguous solution pertaining the efficient application of kernel smoothing is highly unlikely, but can be quite reasonable for a concrete object of control (here: e.g., in an SEHB model).

However, in terms of constructing an observer of unmeasurable state variables the instrument under discussion shows promising results in the SCT-based stochastic control systems.

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