The Radiative Return: a review of experimental results

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The radiative return is a new method for hadronic cross section measurements at electron-positron colliders, which are operated at a fixed center-of-mass energy (so-called particle factories). In order to lower the effective hadronic mass $M_{\text{hadr}}$, only such events are taken, in which one of the electrons or positrons has emitted an initial state radiation photon. We present precision measurements of the pion formfactor from the Frascati $\phi$-factory DAΦNE with the KLOE experiment and measurements of higher particle multiplicities as well as a measurement of the timelike proton-antiproton formfactor from the BaBar experiment at the B-factory PEP-II. These radiative return measurements are compared to results, which are obtained by means of an energy scan, i.e. by means of a systematic variation of the beam energy of the collider. We also report on the impact of these measurements on the hadronic contribution of the anomalous magnetic moment of the muon, which is obtained via a dispersion integral using hadronic cross section data as input.

1. THE RADIATIVE RETURN AND ITS CONNECTION TO THE MUON ANOMALY

1.1. The radiative return method

Modern particle factories, such as the Frascati $\phi$-factory DAΦNE or the B-factories PEP-II and KEK-B are designed for a fixed center-of-mass energy $\sqrt{s}$. An energy scan for the measurement of hadronic cross sections is therefore not feasible. A new and complementary ansatz is given by the 'radiative return' method, which utilizes events, in which one of the electrons (positrons) has emitted a photon. The invariant mass $M_{\text{hadr}}$ of the hadronic system is reduced in this way and the hadronic cross section in the energy range $M_{\text{hadr}} < \sqrt{s}$ becomes accessible. In order to deduce from the measured radiative cross section $\sigma(e^+e^- \rightarrow \text{hadrons} + \gamma)$ the non-radiative cross section $\sigma(e^+e^- \rightarrow \text{hadrons})$, a theoretical radiator function is used via the relation:

$$M^2_{\text{hadr}} \cdot \frac{d\sigma_{\text{hadrons}+\gamma}}{dM^2_{\text{hadr}}} = \sigma_{\text{hadr}} \cdot H(M^2_{\text{hadr}})$$

The radiator function, which describes the initial state radiation (ISR) process, has been computed up to NLO by the Monte-Carlo generator PHOKHARA [1] with a precision of 0.5%. If the measured radiative cross section is divided by the radiative muon cross section $e^+e^- \rightarrow \mu^+\mu^-\gamma$, the ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$ can be deduced directly. In this case one does not need to normalize the data to the integrated luminosity, nor is the radiator function needed. However, an excellent separation of hadrons from muons is required.

In the following we present results from the KLOE experiment at the Frascati collider DAΦNE, which is operated on the $\phi(1020)$-resonance and allows to measure the pion formfactor below 1 GeV. As we will see in the next subchapter, this measurement is of crucial importance for the hadronic contribution to the anomalous magnetic moment of the muon and a precision $\approx 1\%$ or better is needed. After that we discuss a series of results from the BaBar experiment for the timelike proton-antiproton formfactor and for several exclusive final states with higher multiplicities in the mass range from threshold up to 4.5 GeV. The combination of KLOE and BaBar data allows to cover the hadronic cross section in the entire mass range below $\sim 4.5$ GeV. This is the relevant energy region needed for a significantly improved determination of the anomalous magnetic moment of the muon.
1.2. The anomalous magnetic moment of the muon

The recent measurement of the muon anomaly $a_\mu$ at the Brookhaven National Laboratory with a precision of $0.5$ ppm \[2\] has led to renewed interest in accurate measurements of the cross section for $e^+e^- \rightarrow$ hadrons. Hadronic contributions to the photon spectral functions due to quark loops are not calculable in the framework of perturbative QCD. It is well known, however, that the hadronic piece of the spectral function is connected by unitarity to the cross section for $e^+e^- \rightarrow$ hadrons. A dispersion relation can thus be derived, giving the contribution to $a_\mu$ as an integral over the hadronic cross section. Also the hadronic contribution to the running of the electromagnetic fine structure constant at the Z-pole $\Delta \alpha^{(5)}_{\text{had}}(s)$ is not calculable within perturbative QCD (pQCD) at low energies and is obtained in a similar way by means of a dispersion relation. Current electroweak precision tests at the scale $s = M_Z^2$ are unfortunately limited by the precision of the order of $1\%$ by which the hadronic piece to the running of $\alpha$ is known.

Fig. 1 (from ref. \[5\]) shows the contributions of different energy intervals of hadronic cross section data to the absolute value and to the squared error for $a_\mu^{\text{hadr}}$ and $\Delta \alpha^{(5)}_{\text{hadr}}(M_Z^2)$. We see that in the case of the muon anomaly, the radiative return program described above covers the entire energy range of interest. The two-pion cross section (pion formfactor) is the by far dominating channel, since below 1 GeV it contributes to $\sim 60\%$ to $a_\mu^{\text{hadr}}$. In the case of the fine structure constant $\sim 30\%$ of the absolute value of $\Delta \alpha^{(5)}_{\text{hadr}}(M_Z^2)$ are covered by the mass range $< 4.5$ GeV. If pQCD is applicable down to lower energies or in case novel theoretical techniques can be used in the evaluation of the dispersion integral, the impact of the hadronic cross section data below 4.5 GeV is very much enhanced. For further details concerning the evaluation of the dispersion integrals we refer to ref. \[3\] \[4\] \[5\].

2. THE KLOE MEASUREMENT OF THE PION FORMFACTOR

KLOE has published a radiative-return analysis of the pion formfactor, in which the ISR-photon is required to be emitted at small (large) polar angles $\theta_\gamma < 15^\circ$ ($\theta_\gamma > 165^\circ$) \[6\]. No photon tagging is possible in such an approach. The high momentum resolution of the KLOE drift chamber allows a separation of signal from background with very high precision even without the kinematic closure of the event. A new and complementary analysis at large photon angles ($50^\circ < \theta_\gamma < 130^\circ$) and with tagged photons is under study now and will allow to cover the threshold region, which was kinematically forbidden before. Moreover, the beforementioned normalization to radiative muon pairs will allow an important cross-check of the radiator function approach.

2.1. Small angle analysis

The small angle analysis provides high statistics for ISR-photons and suppresses the relative amount of events, in which the photon is emitted from the final state pions (final state radiation, FSR). FSR is an irreducible background to the
radiative return analysis. The relative amount of FSR-events is well below 1\% for the chosen selection cuts and has been estimated by means of the Monte-Carlo-generator PHOKHARA, which uses the model of scalar QED (pointlike pions) as a model for FSR. Special attention has been given to events, in which simultaneously an ISR- and FSR-photon are emitted (NLO-FSR) since those events must not be considered as a background and are also not suppressed by the acceptance cuts. The relative contribution of NLO-FSR events is known with a precision of 0.3\%.

A total experimental error of 0.9\% has been achieved for the \( e^+e^- \rightarrow \pi^+\pi^-\gamma \) cross section measurement. The error consists of the individual contributions of the selection efficiencies and of the precision, with which the residual background after all selection cuts is known; further details can be found in ref. [6].

The KLOE measurement of the pion formfactor has been used to compute the contribution of the two-pion cross section to \( a^{\text{had}}_\mu \) in the energy interval 0.35 < \( M_{\pi\pi}^2 \) < 0.95 GeV\(^2\). The value \( a^{\pi\pi}_\mu = (388.7 \pm 0.8_{\text{stat}} \pm 3.5_{\text{syst}} \pm 3.5_{\text{theo}}) \times 10^{-10} \) covers about 60\% of the total contribution. The KLOE result agrees within 0.5 standard deviations \(^\text{1}\) with values for \( a^{\pi\pi}_\mu \) computed from the data sets of the experiments CMD-2 [8] and SND [10], which were operated in the last years at the VEPP-2M-collider in Novosibirsk. The relative difference of the mass spectra is shown in fig. 3. For this comparison the KLOE data points have been interpolated and the measured data points of CMD-2 and SND are used in the plot. We observe relatively large deviations of up to some percent between KLOE and SND at high and low masses, while the overall agreement is better with CMD-2. Please notice that SND very recently has updated [9] its previous measurement ref. [10], taking into account a new treatment of radiative corrections. The disagreement with KLOE has diminished with new data, but a clear trend of difference is still visible. The good agreement in the dispersion integral is partly due to a compensation effect at lower and higher energies.

The KLOE measurement presented above refers to data taken in 2001 with a total integrated luminosity of \( \sim 140 \text{ pb}^{-1} \). KLOE is now performing an analysis using 2002 data (\( \sim 240 \text{ pb}^{-1} \)) [11], for which a total systematic error (experimental and theoretical) < 1\% is expected. The acceptance cuts of this analysis will be unchanged; the improvement will be due to better and more stable running conditions as well as due to modification in the online and offline environment, which will result in lower systematic errors associated to the
trigger and background-filter efficiencies. Concerning theory, a new version of BABAYAGA is available [7], which allows to reduce the luminosity error by about a factor 2. The final goal of the analysis is a measurement of $R$, which requires that $e^+e^- \rightarrow \mu^+\mu^-\gamma$ events are selected with equally high precision as $\pi^+\pi^-\gamma$ events.

2.2. Large angle analysis

The analysis described above, in which the ISR-photon is emitted at small polar angles, does not allow to cover the threshold region $M^2_{\pi\pi} < 0.35$GeV$^2$, since in this kinematical region the two pions are emitted essentially back-to-back to the ISR-photon and hence cannot be detected simultaneously in the fiducial volume defined for the pion tracks $50^\circ < \Theta_\pi < 130^\circ$ [12]. In order to measure the pion formfactor at threshold, KLOE is now performing a complementary analysis, in which the ISR-photon is tagged at large polar angles $50^\circ < \Theta_\gamma < 130^\circ$. Due to the $1/s^2$ dependence in the dispersion integral for $a_{\mu}^{\text{had}}$, the low mass region of the two-pion cross section is actually giving a $\sim 20\%$ contribution to the total integral and hence an improved determination of the cross section at threshold is needed.

At large photon angles background from $\phi \rightarrow \pi^+\pi^-\pi^0$ is huge and dedicated selection cuts, like a cut on the angle between the missing momentum and the tagged photon direction, as well as a kinematic fit in the background hypothesis with a cut on $\chi^2_{\pi\pi\pi}$ are needed to suppress this contribution. Moreover, irreducible background from events with the same $\pi^+\pi^-\gamma$ final state is not negligible anymore at large photon angles. This background category, which has to be subtracted relying on Monte-Carlo prediction, is given by FSR-events and by the $\phi$ radiative decay into the scalar $f_0(980)$ with $f_0(980) \rightarrow \pi^+\pi^-\gamma$. A possible model dependence of FSR (the model of scalar QED is used in PHOKHARA) and of the description of the scalar $f_0(980)\gamma$ amplitude, can be tested by means of the forward-backward asymmetry. In a recent KLOE publication [13] good agreement between data and simulation has been found for the forward-backward asymmetry, setting upper limits for the systematic errors associated with these model uncertainties.

The main limitation for the measurement of the pion formfactor at threshold will arise from the background channels $\phi \rightarrow \pi^+\pi^-\pi^0$ and $\phi \rightarrow f_0(980)\gamma \rightarrow \pi^+\pi^-\gamma$. In order to further reduce the systematic errors associated to these channels, the DAΦNE collider has taken data off-resonance at a center-of-mass energy of $\sqrt{s} = 1.00$ GeV in its last KLOE run (December 2005 to March 2006, 250pb$^{-1}$ integrated luminosity). The off-peak analysis will allow a considerably improved determination of the threshold region. Moreover, together with the data taken on-peak it will be possible to study the interference of the $f_0(980)$ amplitude with FSR.

![Figure 3. Relative difference of the pion formfactor measurements from the experiments CMD-2 (triangles) and SND (circles), relative to the KLOE measurement. The KLOE data points have been interpolated and the statistical (light grey) and the systematic (dark grey) error bands are shown in the plot.](image-url)
3. THE RADIATIVE RETURN AT PEP-II WITH THE BABAR DETECTOR

The BaBar experiment has previously published results of the exclusive $\pi^+\pi^-\pi^0$, $\pi^+\pi^-\pi^+\pi^-$, $K^+K^-\pi^+\pi^-$ and $K^+K^-K^+K^-$ final states with better coverage than all previous experiments and comparable or better precision, using 89 fb$^{-1}$ of data. New results are presented here for final states with 6 hadrons and for the timelike proton-antiproton formfactor. The new results contain a larger data set of ~ 240 pb$^{-1}$. The mass range of interest in all these measurements $M_{\text{had}} < 4.5$ GeV is rather distant from the center-of-mass energy of PEP-II ($\sqrt{s} = m_{\Upsilon(4S)} = 10.58$ GeV), requiring hence a high-energetic ISR-photon with an energy in the center-of-mass system of $E^*_\gamma > 3$ GeV. Such an event signature separates ISR-events from background from the $\Upsilon(4S)$-resonance (B-decays and their consecutive decay products), since photons originating from these events are in general much less-energetic. The event signature at the B-factory also implies the necessity for photon tagging in order to have high acceptance for the hadronic system. The mass resolution is improved by performing a kinematic fit in the signal hypothesis, requiring constraints on four-momentum and (in case of neutral pions) of the $\pi^0$-mass. A cut on the $\chi^2$ of the kinematic fit is the main tool for background subtraction and the shape of the $\chi^2$-distribution is used with a sideband-method to obtain the residual background after all cuts.

In this paper we report not only on the cross section measurements, but we also present studies of the internal structures. For all channels discussed here the $J/\psi$ signals were used to extract the associated branching fractions and in some cases this has been done also for the $\psi(2S)$ signals. BaBar has published 10 $J/\psi$ and 3 $\psi(2S)$ branching ratios; in 4 cases there has been no previous measurements, in 5 cases the measurements are better than the current world average. In the following we give a brief overview of the individual channels published so far by BaBar with special emphasis on the more recent analyses of 6 Hadrons and $e^+e^- \rightarrow p\bar{p}$.

3.1. $e^+e^- \rightarrow 3$ Pions

The $\pi^+\pi^-\pi^0$ mass spectrum has been measured from 1.05 GeV up to the $J/\psi$ mass region with a systematic error of ~ 5% below 2.5 GeV and up to ~ 20% at higher masses.[15]. The spectrum is dominated by the $\omega$, $\phi$ and $J/\psi$ resonances. The BaBar measurement could improve significantly on the world’s knowledge of the excited $\omega$ states. Fig. 4 shows the BaBar data points together with previous measurements in the energy range relevant for the exited states $\omega'$ and $\omega''$. A clear disagreement with DM2[15] is seen above 1.4 GeV. The BaBar spectrum has been fitted up to 1.8 GeV and the following results for the masses and widths of the $\omega'$ and $\omega''$ have been found[16]. $M(\omega') = (1350 \pm 20 \pm 20)$ MeV, $\Gamma(\omega') = (450 \pm 70 \pm 70)$ MeV, $M(\omega'') = (1660 \pm 10 \pm 2)$ MeV, $\Gamma(\omega'') = (230 \pm 30 \pm 20)$ MeV.

3.2. $e^+e^- \rightarrow 4$ Hadrons

The $\pi^+\pi^-\pi^+\pi^-$, $K^+K^-\pi^+\pi^-$ and $K^+K^-K^+K^-$ exclusive final states have been measured from threshold up to 4.5 GeV with systematic errors of 5%, 15% and 25%, respectively[16]. Fig. 5 shows the mass distribution of the events satisfying the $\pi^+\pi^-\pi^+\pi^-$ and $K^+K^-\pi^+\pi^-$ kinematics as well as $K/\pi$ identifi-

\[^2\text{In this fit } M_{\omega,\phi} \text{ and the phases between } \omega, \phi \text{ with respect to } \omega', \omega'' \text{ are fixed.}\]
culation constraints. We identify that the radiative return technique at BaBar allows to cover a wide energy range in one single experiment. The $K^+K^-K^+K^-$ measurement is the first measurement ever. Background is relatively low for all channels under study (e.g. few percent at 1.5 GeV for $\pi^+\pi^-\pi^+\pi^-$) and is dominated by ISR-events of higher multiplicities and of continuum non-ISR events at higher masses.

The $\pi^+\pi^-\pi^+\pi^-$ final state is dominated by the two-body $a_1(1260)\pi$ intermediate state; the $K^+K^-\pi^+\pi^-$ final state shows no significant two-body states, but rich three-body structure, including $K^*(890)K\pi$, $\phi\pi\pi$, $\rho KK$ and $K^*_2(1430)K\pi$.

### 3.3. $e^+e^- \rightarrow 6\text{ Hadrons}$

The 6-Hadrons final state has been measured in the exclusive channels $3(\pi^+\pi^-)$, $2(\pi^+\pi^-)2\pi^0$ and $K^+K^-2(\pi^+\pi^-)$ [17]. The cross section in the last case has never been measured before; the precision in the first two cases is $\sim 20\%$, which is a large improvement with respect to existing data. Again, the entire energy range from threshold up

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**Figure 5.** The energy dependence of the $e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-$ (upper plot) and the $e^+e^- \rightarrow K^+K^-\pi^+\pi^-$ (lower plot) cross section obtained by BaBar (filled circles) by radiative return in comparison with previous data.

**Figure 6.** The energy dependence of the $e^+e^- \rightarrow 3(\pi^+\pi^-)$ (upper plot) and the $e^+e^- \rightarrow 2(\pi^+\pi^-)2\pi^0$ (lower plot) cross section obtained by BaBar (filled circles) by radiative return in comparison with previous data.
to 4.5 GeV is measured in one single experiment. The distributions for the final states $3(\pi^+\pi^-)$ and $2(\pi^+\pi^-)2\pi^0$ are shown in fig. 6. A clear dip is visible at about 1.9 GeV in both modes. A similar feature was already seen by FOCUS \[19\] in the diffractive photoproduction of six charged pions. The spectra are fitted using the sum of

$$|G_{p\bar{p}}| = \sum_{i} G_{p_{i}\bar{p}}$$

where the factor $C$ accounts for the Coulomb interaction of the final state particles. The proton helicity angle $\theta_p$ in the $p\bar{p}$ rest frame can be used to separate the $|G_E|^2$ and $|G_M|^2$ terms. Their respective variations are approximately $\sim \sin^2 \theta_p$ and $\sim (1 + \cos^2 \theta_p)$. By fitting the $\cos \theta_p$ distribution to a sum of the two terms, the ratio $|G_E/G_M|$ can be extracted. This is done separately in six bins of $M_{p\bar{p}}$. The results are shown in fig. 7 and disagree significantly with previous measurements from LEAR \[21\] close to threshold. At larger values of $M_{p\bar{p}}$ the BaBar measurement finds $|G_E/G_M| \approx 1$.

In order to compare the cross section measurement with previous data ($e^+e^-\rightarrow p\bar{p}$ experiments), the effective form factor is introduced: $G = \sqrt{|G_E|^2 + 2m_p^2/s}|G_M|^2$. The BaBar measurement of $G$ is in good agreement with existing results, as can be seen in fig. 8. The structure of the formfactor is rather complicated; we make the following observations: (i) BaBar confirms an increase of $G$ towards threshold as seen before

Figure 7. BaBar measurement of the ratio of the electric and magnetic formfactor describing the cross section $e^+e^-\rightarrow p\bar{p}$. Filled circles depict BaBar data, the curve is a fit result. Open circles show data from the experiment PS170.
by other experiments; (ii) two sharp drops of the spectrum at $M_{p\bar{p}} = 2.25$ and 3.0 GeV are observed; (iii) data at large values $M_{p\bar{p}} > 3$ GeV is in good agreement with the prediction from perturbative QCD.

4. CONCLUSIONS

At the particle factories DAΦNE and PEP-II the radiative return method allows precision measurements of the hadronic cross section, which are of utmost importance for an improved determination of the hadronic contribution of the muon anomaly $a_{\mu}^{hadr}$ and of the running fine structure constant $\alpha(M_Z^2)$. At DAΦNE the KLOE experiment has measured the pion form factor with a precision of 1.3%. An update of the analysis will lead to a further reduction of the systematic error and a normalization to radiative muon pairs is foreseeable. Moreover, KLOE has collected data with the collider running off-resonance, which will allow an improved determination of the threshold mass region.

At the B-factory PEP-II a program to measure all exclusive hadronic channels from threshold up to 4.5 GeV is underway. In addition to the published results presented in this paper, there are ongoing analyses for further final states, including the important two-pion channel.

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