Transformation Of Non-Commitment Decision Risk Of Three-Way Decision Based on Limited Time Observation

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Abstract. In order to avoid or reduce the loss of direct decision-making caused by insufficient information, on the basis of traditional two-way decision, the non-commitment decision is added in three-way decision. However, in most practical applications, the final decision needs to be made within a limited time, the risk of non-commitment decision will increase with time and even be equivalent to the risk of commitment decision. Therefore, from the perspective of multi-time decision points, the method of transformation of non-commitment decision risk of three-way decision based on limited time observation is proposed. First, the non-commitment decision risk is analyzed and the time risk coefficient is introduced to explain the law of non-commitment risk changing with time. Then, the subjective and objective factors in the decision process are analyzed so as to make the decision process more in line with human cognition and thinking logic. Finally, an example is given to verify the effectiveness and practicability of the method.

1. Introduction
Three-way decision (3WD), proposed by professor Yao Yiyu of university of regina, Canada, is a decision model based on human cognition, which is used to deal with decision problems under the condition of uncertain and incomplete information[1-2]. Compared with the traditional decision methods, the three-way decision theory increases the boundary domain and gives the non-commitment decision, which is an extension of the two-way decision theory. Three-way decision theory’s core idea is to divide a universe into three disjoint regions, and formulate corresponding decision strategies for each region. When the available information is sufficient, the decision to accept or reject is generally made directly; when the information is insufficient, the decision to make further observation will be made without commitment, and new decisions will be made after more information is obtained. The whole decision process follows Bayesian minimum risk criterion, so the risk and loss of making wrong decisions due to uncertainty can be reduced, and the decisions made are more in line with human thinking logic and practical application.

In recent years, scholars at home and abroad have made in-depth research on three-way decision theories, methods, models and applications[3]. The existing extended models and modified models include game rough sets, interval sets [4] and fuzzy interval sets [5], etc. Three-way decision theory emphasizes increasing the non-commitment decision and setting a buffer between the known information and the final decision, so as to reduce and avoid the loss and risk caused by the direct decision due to uncertainty. In some practical applications, the decision strategies of the three domains are equally important, and delay decision can reduce the loss when the information is insufficient. However, in some applications, the non-commitment decision is equivalent to rejection. It also bears
considerable risks. For example, when the medical diagnosis is faced with an emergency, it is easy to delay the patient’s condition and even cause irreparable loss if the treatment is delayed. Li et al.[6] made a risk analysis on the non-commitment decision, and based on the principle of minimum misjudgment cost, studied and constructed the transformation process from the non-commitment decision to the two-way decisions. Shao et al.[7] studied the transformation of non-commitment risk in coordinated decision information system with decision tree method. The existing researches on the risk transformation of non-commitment decision are based on the principle of maximizing profits[8] and set pair connection entropy[9], etc.

The above-mentioned research on risk transformation of non-commitment decision mainly considers the loss or gain of economic and time, but in practice, most decision problems are dynamic and complex, and non-commitment decision is also due to insufficient information to make decisions, while information collection is a process that circulates and changes with time. The known amount of information determines the conditional probability of the target, and the change of time promotes the urgency of the decision process. If more sufficient information cannot be obtained in a limited time, the risk of non-commitment decision will gradually increase, and the possible loss will be greater than the intended loss. For example, in the process of medical diagnosis, if treatment plan is not given within a limited time, it will be delayed in the boundary domain due to some reasons such as economic and family, which will lead to the deterioration of the disease and bring life risks due to long-term conservative treatment. The loss reduced by collecting information is less than the loss caused by placing the object in the boundary domain, and the risk will change from decreasing trend to increasing. Therefore, it is necessary to analyze and transform the risk of non-commitment decision in boundary areas from the time scale. From the perspective of limited time observation, this paper considers the influence of different time decision points on three-way decision boundary areas, analyzes the risk of non-commitment decision, and introduces time risk coefficient to assist decision-makers to make decisions in limited time observation environment, thus reducing excessive losses caused by excessive delay.

2. Preliminary

The basic ideas and concepts of three-way decision is briefly reviewed in this section.

According to Bayesian decision criteria, the three-way decision model sets two state sets and three action sets to describe the decision process, suppose $\Omega = \{C, \neg C\}$ is a state set, which represents the object belongs to $C$ or not, respectively. The set of action is given by $A = \{a_p, a_b, a_N\}$, where $a_p, a_b, a_N$ represent three actions: acceptance, delay and rejection. Considering that different actions will bring different degrees of losses, suppose $\lambda_{pp}, \lambda_{bp}, \lambda_{np}$ ($\lambda_{pp} \leq \lambda_{bp} \leq \lambda_{np}$), respectively indicating the losses caused by taking three different actions at that time when $x \in C$, suppose $\lambda_{pn}, \lambda_{bn}, \lambda_{nn}$ ($\lambda_{nn} \leq \lambda_{bn} \leq \lambda_{pn}$), respectively indicating the losses caused by taking three different actions at that time when $x \not\in C$. To sum up, we can get the loss matrix corresponding to different actions taken under different conditions as shown in table 1.

|           | $C$    | $\neg C$ |
|-----------|--------|----------|
| $a_p$     | $\lambda_{pp}$ | $\lambda_{pn}$ |
| $a_b$     | $\lambda_{bp}$ | $\lambda_{bn}$ |
| $a_N$     | $\lambda_{np}$ | $\lambda_{nn}$ |

Usually, $Pr(C|x)$ represents the conditional probability of the object belongs to state $C$ through the characterization of equivalence class $[x]$, similarly, $Pr(\neg C|x)$ represents the probability of $[x]$ belongs to state $\neg C$, thus, the expected loss for each object $x$ can be calculated as follows:

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In the Bayesian decision-making process, the minimum risk criterion is followed, and the decision rules (P)-(N) are expressed as [11]:

\[
\begin{align*}
L(a_P | [x]) &= \lambda_{PP} \Pr(C | [x]) + \lambda_{PN} \Pr(\neg C | [x]) \\
L(a_B | [x]) &= \lambda_{BP} \Pr(C | [x]) + \lambda_{BN} \Pr(\neg C | [x]) \\
L(a_N | [x]) &= \lambda_{NP} \Pr(C | [x]) + \lambda_{NN} \Pr(\neg C | [x])
\end{align*}
\] (1)

3. Risk analysis of non-commitment decision in boundary domain

In the process of three-way decision-making, whether it is to accept, delay or reject, it is the decision-maker's choice. In reality, the decision-maker makes a choice according to the existing information. If the information is sufficient, he can directly make two-way decisions of acceptance or rejection. However, in the case of insufficient information, the decision-maker may bring unpredictable losses if he accepts or rejects the decision. Considering this uncertainty, the decision-maker chooses to give a non-commitment decision, that is, to delay the decision-making, and set a buffer before the final decision-making to get more information to support the decision.

In the general sense, the non-commitment decision is influenced by the subjective factors of the decision-maker. For example, the decision-maker is unwilling to make a decision due to the mood and other factors, but the delayed decision in the three-way decision is not a non-commitment decision in the general sense. From the analysis of the reasons, the delay is a passive choice caused by insufficient information, and this passive choice is objective, but the decision-maker wants to make a commitment decision but can't. From the result analysis, non-commitment decision means that when the object is in the boundary domain, there is no corresponding action, which is equivalent to maintaining the status, which is also risky. Therefore, it is necessary to consider the risks and losses brought by delay decision, and carry out reasonable quantitative analysis and transformation to reduce the excessive losses caused by excessive delay.

According to the basic model setting and conditions of three-way decision:

\[
\begin{align*}
\lambda_{PP} &\leq \lambda_{BP} \leq \lambda_{NP} \\
\lambda_{NN} &\leq \lambda_{BN} \leq \lambda_{PN} \\
0 &\leq \beta < \alpha \leq 1
\end{align*}
\]

Literature [12] points out “≤” shows that the risks brought by non-commitment decision will be equivalent to commitment decision under certain conditions, which mainly comes from the uncertainty of decision-making process. Then we can obtain two conclusions about the risk estimation of non-commitment decision:

1) Existence of risks: \( \lambda_{BP} \geq 0, \lambda_{BN} \geq 0 \)

2) Importance of risks: When the equal sign in the condition holds, the risk brought by not committing to a decision is equal to accept or reject decision.

In the decision-making process, the risk of dividing objects belonging to positive domain into positive domain or negative domain and objects belonging to negative domain into positive domain or negative domain, that is, the risk of commitment decision is relatively certain and has little change, so it is set as a fixed value. Meanwhile, according to the actual situation, the risk of dividing objects belonging to negative domain into boundary domain is set as a fixed value, mainly considering the change of \( \lambda_{BP} \). For example, in the face of medical emergency, set the state set \( \Omega = \{C, \neg C\} \) to \( \Omega = \{\text{surgery}, \text{no surgery}\} \), according to the definition of the loss function of three-way decision, putting patients who should have been operated in the boundary domain all the time to make non-commitment
decision will increase the life risk of patients with the urgency of time. That is $\lambda_{BP}$ will increase with time. However, the risk of placing patients who should not have surgery in the boundary domain will not change with time. Therefore, $\lambda_{BN}$ remains unchanged. In other cases, the meaning of state set can be adjusted according to actual problems, transform the problem to the change of $\lambda_{BP}$. Therefore, the risk changes in the boundary domain considered in this paper mainly refer to $\lambda_{BP}$. Then two supplementary conditions are put forward on the basis of the above:

3) Timeliness of risk: With the approach of decision-making time, the risk brought by non-commitment decision is increasing, that is, $\lambda_{BP}$ is a monotone increasing function related to time.

4) Upper bound of risk: The risk of non-commitment decision gradually approaches the risk of commitment decision in the process of increasing. But limited by conditions $\beta < \alpha$, $\lambda_{BP}$ should have an upper bound.

4. Transformation of non-commitment decision risk

4.1. Transformation from non-commitment decision to two-way decision

In the three-way decision, the division of acceptance domain, delay domain and rejection domain is determined by a pair of thresholds $(\alpha, \beta)$. The thresholds can be calculated by the values of each parameter in the loss function matrix. Then, according to the range of the conditional probability of the object, the decision-making on the target can be decided. There are three main directions in the research of further classification of objects in boundary domain, one is to obtain more sufficient information to get more accurate conditional probability for classification, the other is to adjust the size of boundary domain according to the actual situation, and the third is to integrate the two.

In this paper, from the second and third perspectives, time risk coefficient is introduced to adjust the threshold and narrow the scope of boundary domain in a limited time observation, which promotes the transformation of delayed decision of objects into two-way decision.

According to analysis in section 2, suppose $\lambda_{PP}$, $\lambda_{NP}$, $\lambda_{PN}$, $\lambda_{BN}$, $\lambda_{NN}$ as fixed value, analysis threshold calculation formula:

$$
\alpha = \frac{(\lambda_{PN} - \lambda_{BN})}{(\lambda_{PN} - \lambda_{BN}) + (\lambda_{BP} - \lambda_{PP})}
$$

$$
\beta = \frac{\lambda_{BN} - \lambda_{NN}}{(\lambda_{BN} - \lambda_{NN}) + (\lambda_{NP} - \lambda_{BP})}
$$

We can find that with the increase of $\lambda_{BP}$, $\alpha$ will decreases and $\beta$ will increases. Under the restriction of $\beta < \alpha$, the size of the boundary domain can be gradually reduced, so that the original object in the boundary domain can be transformed into positive domain or negative domain.

4.2. Time risk coefficient

In practical problems, because of the dynamic generation and change of information, the decision-making process becomes more and more dynamic and complex, and the dynamic change of information depends on time. Generally speaking, it is to set sampling points according to time and carry out dynamic sampling. Many scholars have studied that the closer to the present moment, the higher the importance of information. Literature [13-14] introduces time weight into the process of operational threat assessment, which expands the application of dynamic decision-making. Literature [15] considers the importance of new information combined with time weight and improves the accuracy of GM(1,1) model. This means that the closer it is to the present moment, the more urgent it is to make commitment-based decision, because the risk of non-commitment decision will gradually increase in a limited time.

According to the conditions limited in section 2, define $\lambda_{BP}$ as $\lambda_{BP}(t)$. The formula is as follows:
\[
\left\{ \begin{array}{c}
\lambda_{BP}(t) = \theta(t) \cdot \lambda_{NP} \\
(\lambda_{PN} - \lambda_{BN})[\lambda_{NP} - \lambda_{BP}(t)] > [\lambda_{BP}(t) - \lambda_{PP}]\lambda_{BN} - \lambda_{NN}
\end{array} \right.
\]

\(\theta(t)\) is defined as the time risk coefficient, which indicates the risk weights of observation points at different times within the limited time observation. According to the analysis in section 2, \(\theta(t)\) should meet the following three basic conditions:

1) \(\theta(t) > 0, \forall t = 1, 2, \ldots, n\). The time risk coefficient indicates that the risk of non-commitment decision increases with time, which should be a positive number.
2) When \(t \geq 1\), \(\theta(t)\) should increase monotonically. The time risk coefficient is based on the change of time, and the non-commitment decision increases with the increase of \(t\) value, so the time risk coefficient should increase monotonously in the interval \(t \geq 1\).
3) The value range of \(\theta(t)\) is \((0, 1)\). When \(t\) approaches infinity, \(\theta(t) = 1\), it means that when time approaches the present moment, the risk of non-commitment decision is equal to the risk of rejection. However, in practical application, due to the limitation of conditions \(\beta < \alpha\), the upper bound of \(\theta(t)\) is different due to the different values of the loss function of practical problems.

5. Numerical examples and discussion

In real life, the treatment method for patients is a dynamic decision-making process, and the treatment arrangement of patients is closely related to the development of time, doctors' diagnosis and treatment, and family conditions. According to different stages of illness, doctors will give suggestions for surgical treatment, conservative treatment or giving up treatment, then patients and their families as decision makers will consider what risks they can take according to the doctor's suggestions and family conditions, and then make decisions. Suppose that a patient is suffering from a disease and needs a certain time and process of diagnosis and examination. The advice given by the doctor is set as a state set \(\Omega = \{\text{surgery}, \text{no surgery}\}\). The action set for patients to make different decisions is set as \(A = \{a_P, a_B, a_N\}\), representing acceptance, delay and rejection, the losses caused by taking different actions are set as follows \(\lambda_{PP}, \lambda_{BP}, \lambda_{NP}, \lambda_{PN}, \lambda_{BN}, \lambda_{NN}\), according to section 3, suppose \(\lambda_{PP}, \lambda_{BP}, \lambda_{NP}, \lambda_{PN}, \lambda_{BN}, \lambda_{NN}\) as fixed value. \(\lambda_{BP}(t) = \theta(t) \cdot \lambda_{NP}\). Set the range of loss function as \((0, 1)\), the parameter values are set as shown in table 2.

| Decision     | surgery | no surgery |
|--------------|---------|------------|
| acceptance   | 0.1     | 0.8        |
| delay        | \(\theta(t) \cdot 0.9\) | 0.2        |
| rejection    | 0.9     | 0.1        |

The above loss function value has practical significance: patients who should undergo surgery decide to undergo surgery and patients who should not have surgery decide not to undergo surgery with less risk; Patients who should undergo surgery without surgery may cause irreparable loss or even life risk due to serious illness, so they bear a greater risk. If patients who do not need surgery undergo surgery, they will also cause the loss of misdiagnosis and misoperation, but the risk value is slightly smaller than that of patients who should have surgery but have no surgery.

For the two parameters of delayed decision-making, if patients who do not need surgery are always placed in the delay domain, that is, conservative treatment, with further diagnosis, it can be determined that they do not need surgery or undergo surgery, and the risk of delay is relatively small and stable. However, if patients who should undergo surgery are placed in the delay domain for conservative treatment for a long time due to the patient's family economic conditions, their illness will become more serious with the development of time, and the risk will gradually increase until they approach the serious losses incurred by refusing surgery. Therefore, this paper introduces the time risk coefficient to quantify this kind of risk.
Firstly, the time risk coefficient is constructed. In the first step, the upper limit value of the time risk coefficient needs to be solved, due to the limitation of conditions $\beta < \alpha$. Substitute the values of each parameter in table 2, and get $\lambda_{BP}(t) < 0.785$. Assume that there are 5 observation points in finite time, $\theta(t)$ is discretized into time series at equal intervals, it is verified that the discrete sequence can satisfy the three basic conditions given in section 3.2. In the second step, three-way decision thresholds need to be solved according to different $\lambda_{BP}(t)$ values, and the results are shown in table 3.

| $\theta(t)$ | $\lambda_{BP}(t)$ | $\alpha$ | $\beta$ |
|------------|------------------|----------|----------|
| 0.174      | 0.1566           | 0.914    | 0.119    |
| 0.348      | 0.3132           | 0.738    | 0.146    |
| 0.522      | 0.4968           | 0.602    | 0.199    |
| 0.696      | 0.6264           | 0.533    | 0.268    |
| 0.870      | 0.7830           | 0.468    | 0.461    |

When making the final decision, there are two situations:

1) The information is always limited or the newly acquired information is less, then the conditional probability of the object remains unchanged. At this time, the decision-making authority depends more on the subjective choice of the decision-maker on the basis of the existing objective information, and the three-way decision is in an auxiliary position. For example, according to the patient's condition and various factors, the probability of doctors recommending surgery is 50%. Set $\Pr(C | x) = 0.5$, according to table 3, at the first four time observation points, the subjects were always judged to be placed in the boundary domain, that is, conservative treatment was always taken. However, as time approached, when the final decision-making point was reached, the decision had to be made, the patients were judged to be in need of surgical treatment. Because for the probability 50%, which is the probability value with the highest uncertainty, it is more difficult to make a reliable decision, but the decision cannot be delayed because of the difficulty, but it will cause greater losses.

2) Information is gradually sufficient, then the conditional probability of objects will change. At this time, due to more abundant information, the patient's situation becomes clearer and the decision-making is more objective. For example, with the further clarification of the results of various examinations, the probability of doctors recommending surgery has changed from 50% to 65%, set $\Pr(C | x) = 0.65$, according to table 3, at the first three time observation points, the subject was always in the boundary domain, and was judged as accepted at the fourth decision point. Combined with the doctor's suggestion, the patient could get appropriate treatment earlier and recover to health faster.

6. Conclusions

In this paper, considering the dynamic process of information collection and decision-making, aiming at the problem that the risk of non-commitment decision in three-way decision will increase with time and lead to excessive delay and loss, a risk transformation method based on multi-time observation in the boundary region of three-way decision is proposed. The main contributions are as follows:

1) Considering the influence of different time observation points on the non-commitment decision of three-way decisions, this paper analyzes the risk of non-commitment decision, and puts forward four conclusions about the risk estimation of non-commitment.

2) The time risk coefficient and its three basic conditions are defined, and the influence of different time observation points on the risk of non-commitment decision is quantified.

3) Through an example, the influence of subjective and objective factors in the decision-making process is demonstrated and analyzed, which shows that the proposed method is more in line with human thinking logic and decision-making process, and has certain effectiveness and applicability.
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