Mass Shifts through Re-scattering

Ron S. Longacre

Brookhaven National Laboratory, Upton, NY 11973, USA

Abstract

In this note we present a model that can produce a mass shift in a resonance due to interference between a scattering amplitude and that amplitude having rescattering through the resonance.

1 Starting point

For the first part of this story we will define what a scattering cross section is. We will only consider in this work elastic scattering of pions. Two pions can scatter at a certain energy which we will call \( M_{\pi\pi} \). The differential cross section \( \sigma \) at a given \( M_{\pi\pi} \) is

\[
\frac{d\sigma}{d\phi d\theta} = \frac{1}{K^2} \left| \sum_\ell (2\ell + 1) T_\ell P_\ell(\cos(\theta)) \right|^2
\]

where \( \phi \) and \( \theta \) are the azimuthal and scattering angles, respectively. \( T_\ell \) is a complex scattering amplitude and \( \ell \) is the angular momentum. \( P_\ell \) is the Legendre polynomial, which is a function of \( \cos(\theta) \). \( K \) is the flux factor equal to the pion momentum in the center of mass. For this note we only consider \( \ell = 0 \) and \( \ell = 1 \). The \( T_0 \) and \( T_1 \) elastic scattering amplitudes are complex amplitudes described by one real number which is in units of angles. The form of the amplitude is

\[
T_\ell = e^{i\delta_\ell} \sin(\delta_\ell)
\]

We note that \( \delta \) depends on the value of \( \ell \) and \( M_{\pi\pi} \). We will use the \( \ell = 0 \) and \( \ell = 1 \) \( \delta \)'s given in [1].

---

\[^1\text{This research was supported by the U.S. Department of Energy under Contract No. DE-AC02-98CH10886}\]
2 Defining A

Let us consider two pions scattering in the final state of the heavy ion collision. The scattering will be either $\ell = 0$ or $\ell = 1$ partial waves. The $M_{\pi\pi}$ of the scattering di-pion system will depend on the probability of the phase space of the overlapping pions. The pions emerge from a close encounter in a defined quantum state with a random phase. We will call this amplitude $A$ and note that the absolute value squared of the amplitude is proportional to the phase space overlap. The emerging pions can re-interact or re-scatter through the quantum state of the pions, which is a partial wave or a phase shift. We have amplitude $A$ plus $A$ times the re-scattering of pions through the phase shift consistent quantum state of $A$. The correct unitary way to describe this process is given by [2] equation (4.5)

$$T = \frac{V_1 U_1}{D_1} + \left( \frac{V_2 + \frac{D_{\rho \pi \pi}}{D_1}}{D_2 - \frac{D_{\rho \pi \pi}}{D_1}} \right) \left( U_2 + \frac{D_{\rho \pi \pi}}{D_1} \right)$$

(3)

In the above equation we have two terms, 1 and 2. The first term denoted by 1 is the $\pi\pi$ scattering through p-wave which will become the amplitude $A$ mentioned above, where $V$ is the incoming and $U$ is the outgoing $\pi\pi$ system. The second term denoted by 2 is the direct production of the $\rho$ meson with $V$ being the production, the propagation being $D$ and the decay being $U$. We see that there are terms $D_{12}$ which involves a loop of pions between scattering pions and the formation of a $\rho$ by the pions.

3 Final equation

The complete derivation is in the appendix. From the appendix we get two terms, one being the direct production of the $\rho$ or $\pi\pi$ p-wave phase shift and the second being the $\rho$ from re-scattering. The final equation number 21 has two important factors, one is two-body phase space and the other is a coefficient $\alpha$. This coefficient is related to the real part of the $\pi\pi$ rescattering loop and is given by equation 14. When the pions rescatter or interact at a close distance or a point $\alpha$ has its maximum value of one. While if the pions rescatter or interact at a distance determined by the diffractive limit the value of $\alpha$ is zero.

In the equation 21 $|T|^2$ is the cross section for $\rho$ production, where $D$ is the direct production amplitude and $A$ is the amplitude introduced above
for the re-scattering pions into the \( \rho \) meson. \( \delta \) is the \( \pi \pi \) phase shift \([1]\). \( q \) is the \( \pi \pi \) center of mass. At a given \( p_T \) and \( y \) bin, \( D \) will be a constant as a function of \( M_{\pi\pi} \). The \( \alpha \) factor is one for rescattering coming from a point source, but goes to zero when rescattering is diffractive. The dependence of \( A \) is calculated by the phase space overlap of di-pions added as four vectors and corrected for proper time, with the sum having the correct \( p_T \) and \( y \) for a given \( M_{\pi\pi} \).

Finally we must use the correct two body phase space. For a two body system of pions, phase space goes to an constant as \( M_{\pi\pi} \) goes to infinity. Let us choose this constant to be unity. Phase space which is denoted by PS is equal to

\[
PS = \frac{2qB_\ell(q/q_0)}{M_{\pi\pi}}
\]

where \( B_\ell \) is a Blatt-Weisskopf-barrier factor \([3]\) for \( \ell \) angular momentum quantum number. The \( q_0 \) is the momentum related to the range of interaction of the \( \pi\pi \) scattering. 1 fm is the usual interaction distance which implies that \( q_0 = 200 \text{ MeV/c} \). For the \( \rho \) meson \( \ell = 1 \) and \( B_1 = \frac{(q/q_0)^2}{(1+(q/q_0)^2)} \).

The phase space factor PS as a function of \( q \) near the \( \pi\pi \) threshold is given by \( q^{2\ell+1} \). Thus in the appendix we use \( q^{2\ell+1} \) for the factor PS except for equation 21 which is the final equation.

A  Appendix

Starting with equation (4.5) from \([2]\)

\[
T = \frac{V_1U'_1}{D_1} + \frac{\left(V_2 + D_{12}U'_1\right)\left(U'_2 + D_{12}U'_1\right)}{D_2 - D^2_{12}}
\]

In order to have the correct threshold kinematics, we define

\[
U'_1 = U_1\sqrt{q^{2\ell+1}}
\]

\[
U'_2 = U_2\sqrt{q^{2\ell+1}}
\]

where \( q \) is the \( \pi\pi \) center of mass momentum and \( \ell \) is the value of the angular momentum. The amplitude \( A \) of the text is given by

\[
\frac{V_1U_1}{D_1} = A
\]
Thus we have

\[
\frac{V_1 U'_1}{D_1} = A \sqrt{q^{2\ell+1}}
\]  

(9)

The phase shift for the \(\ell^{th}\) partial wave will be given by \(\delta_\ell\), where

\[
\frac{U'_2 U'_2}{D_2} = e^{i\delta_\ell} sin(\delta_\ell)
\]  

(10)

The above equality is true if the \(D_1\) mode plays no role in the \(\pi\pi\) scattering in the \(\ell^{th}\) partial wave. But in the initial state there is a large production of \(D_1\). The \(U'\)'s are the basic coupling of the \(D\)'s to the \(\pi\pi\) system. In order to decouple \(D_1\) from the \(\pi\pi\) system \(U_1\) must go to zero. We can maintain a finite production of \(D_1\) if we define

\[
V_1 = \frac{1}{U_1}
\]  

(11)

Thus the first term in the equation becomes

\[
\frac{V_1 U'_1}{D_1} = \frac{1}{U_1} \sqrt{q^{2\ell+1}} = \sqrt{q^{2\ell+1}} D_1 = A \sqrt{q^{2\ell+1}}
\]  

(12)

The form of \(D_{12}\) is given by

\[
D_{12} = \alpha U_1 U_2 + i q^{2\ell+1} U_1 U_2
\]  

(13)

\(D_{12}\) is the real and imaginary part of the two pion loop from state 1 to state 2. The \(U'\)'s are the \(\pi\pi\) couplings and the imaginary part goes to zero at the \(\pi\pi\) threshold. The \(\alpha\) factor is one for rescattering coming from a point source, but goes to zero when rescattering is diffractive. A simple form for \(\alpha\) is given by

\[
\alpha = (1.0 - \frac{r^2}{r_0^2})
\]  

(14)

where \(r\) is the radius of rescattering in fermis and \(r_0\) is 1.0 fermi or the limiting range of the strong interaction. The second term of the first equation is

\[
\frac{\left( V_2 + \frac{D_{12} V_1}{D_1} \right) \left( U'_2 + \frac{D_{12} U'_1}{D_1} \right)}{D_2 - \frac{D_{12}^2}{D_1}}
\]  

(15)
Rewriting

\[
\left( V_2 + \frac{a U_1 V_1}{D_1} + i q^{2\ell+1} \frac{U_2 V_1}{D_1} \right) \left( U_2' + \frac{D_1 U_1'}{D_1} \right) \frac{D_2 - \frac{D_1^2}{D_1}}{D_2}
\]  

(16)

Let us make substitutions

\[
V_1 = \frac{1}{U_1}, U_2 = \frac{U_2'}{\sqrt{q^{2\ell+1} D_1}}, D_1 = A, D_{12} = 0
\]

(17)

The second term becomes

\[
\frac{\left( V_2 + \frac{a U_2'}{\sqrt{q^{2\ell+1}}} + i \sqrt{q^{2\ell+1}} A U_2' \right) U_2'}{D_2}
\]

(18)

The first term is

\[
\frac{V_1 U_1'}{D} = A \sqrt{q^{2\ell+1}}
\]

(19)

Adding the first and the second terms and substituting the phase shift,

\[
T = \frac{V_2}{U_2} \frac{e^{i\delta_\ell \sin(\delta_\ell)}}{\sqrt{q^{2\ell+1}}} + A \left( \frac{e^{i\delta_\ell \alpha \sin(\delta_\ell)}}{\sqrt{q^{2\ell+1}}} + \sqrt{q^{2\ell+1}} e^{i\delta_\ell \cos(\delta_\ell)} \right)
\]

(20)

The term with the factor \( \frac{V_2}{U_2} \) is the direct production of the di-pion system. We shall call this amplitude \( D \). The re-scattered amplitude is \( A \) and is modified by the di-pion phase shift. These two amplitudes have some random phase and are not coherent. Thus the cross section is

\[
|T|^2 = |D|^2 \frac{\sin^2(\delta_\ell)}{PS} + |A|^2 \frac{\alpha \sin(\delta_\ell) + PS \cos(\delta_\ell)}{PS}^2
\]

(21)

References

[1] G. Grayner et. al., Nucl. Phys. B 75 (1974) 189.

[2] R. Aaron and R. S. Longacre, Phys. Rev. D 24 (1981) 1207.

[3] F. von Hippel and C. Quigg, Phys. Rev. 5 (1972) 624.