Giant Planar Hall Effect in Topological Metals

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Much excitement has been generated recently by the experimental observation of the chiral anomaly in condensed matter physics. This manifests as strong negative longitudinal magnetoresistance and has so far been clearly observed in Na$_3$Bi, ZrTe$_5$, and GdPtBi. In this work we point out that the chiral anomaly must lead to another effect in topological metals, that has been overlooked so far: Giant Planar Hall Effect (GPHE), which is the appearance of a large transverse voltage when the in plane magnetic field is not aligned with the current. Moreover, we demonstrate that the GPHE is closely related to the angular narrowing of the negative longitudinal magnetoresistance signal, observed experimentally.

The recent theoretical [1] and experimental [9,14] discovery of Dirac and Weyl semimetals has extended the notions of nontrivial electronic structure topology to metals. It has also reinforced the connection that exists between the physics of materials with topologically nontrivial electronic structure and the physics of relativistic fermions. Chiral anomaly, which refers to nonconservation of the chiral charge in the presence of collinear external electric and magnetic fields, is a particularly important example that highlights such a connection. Discovered theoretically by Adler [15] and by Bell and Jackiw [16] in the relativistic particle physics context, it provided the explanation for the observed fast decay of a neutral pion into two photons, naively not allowed by the conservation of the chiral charge. We will use $\bar{\hbar} = \hbar$ in the relativistic particle physics context, unless specified otherwise.

As was argued in Refs. [20–22, 24], transport in topological (both Weyl and Dirac) metals is distinguished by the existence of an extra (nearly) conserved quantity, the chiral charge, which is coupled to the electric charge in the presence of an external magnetic field. The hydrodynamic transport equations for the electric and the chiral charge have the following form [21,22]

$$\frac{\partial n}{\partial t} = D \nabla^2 (n + gV) + \Gamma \mathbf{B} \cdot \nabla (n_e + gV_e),$$

$$\frac{\partial n_c}{\partial t} = D \nabla^2 (n_e + gV_e) - \frac{n_e + gV_e}{\tau_c} + \Gamma \mathbf{B} \cdot \nabla (n + gV).$$

(1)

Here $-en$ is the electric charge density, $-en_c$ is the chiral charge density (defined as the difference between the total right handed and total left handed charge); $D$ is the diffusion coefficient (we take the diffusion coefficients, corresponding to the electric and the chiral charges to be the same for simplicity, although they may in general be different due to electron-electron interaction effects); $g$ is the density of states at the Fermi energy; $\Gamma = e^2/2\pi^2g$ is a transport coefficient, which characterizes the chiral anomaly induced coupling between the electric and the chiral charge in the presence of an applied magnetic field $\mathbf{B}$; and $\tau_c$ is the chiral charge relaxation time, which is taken to be long, reflecting the near conservation of the chiral charge. We will use $\hbar = c = 1$ units throughout this paper, except in some of the final results.

In this paper we both explain the angular narrowing phenomenon and connect it with another effect, which has so far not been mentioned in relation to the chiral anomaly, the Giant Planar Hall Effect (GPHE). Note that a closely related explanation of the angular narrowing effect has already been proposed by us in Ref. [23], but the connection with the GPHE was not understood there. We argue that the presence of both the negative longitudinal magnetoresistance with a characteristic dependence on the angle between the current and the magnetic field, and the GPHE may be regarded as a smoking gun signature of the chiral anomaly.

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The presence of the electrostatic potential $V$ and the “chiral electrostatic potential” $V_c$ (this is a hypothetical external potential that couples antisymmetrically to the right- and left-handed charge) reflects the presence of both diffusion and drift contributions to the electric and chiral currents correspondingly. In equilibrium the two contributions must cancel each other, which, in particular, implies $n_e + gV_c = 0$ in this case. We note that a time-independent spatially-uniform $V_c$ will always be present in a noncentrosymmetric topological metal [25].

Let us consider an experimental setup, shown in Fig. 1. A.A. Burkov
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The chiral anomaly manifests in Weyl and Dirac semimetals as a very unusual large negative longitudinal magnetoresistance, quadratic in the applied magnetic field, as predicted theoretically [20–22]. While the existence of the effect, and most of its observed features are in qualitative agreement with the theory, one puzzling feature has remained unexplained. As was first pointed out in [17], the observed dependence of the magnetoresistance on the angle $\theta$ between the current and the applied magnetic field, is against the expectations, drawn from the existing theory. Namely, the theory of Refs. [20–22] naively predicts a $\cos^2 \theta$ dependence, due to the quadratic dependence of the chiral anomaly contribution to the conductivity on the magnetic field, but the observed angular dependence appears to be much stronger.

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electric current density is then given by
\[ \mathbf{j} = \frac{\sigma}{e} \nabla \mu + e g \Gamma \mu_c \mathbf{B}, \]
where \( \sigma = e^2 g D \) is the Drude conductivity. The second term in Eq. (2) expresses the chiral magnetic effect (CME) [26]. Since \( \mu_c = 0 \) in equilibrium, CME vanishes in equilibrium as it should [27, 28].

We assume that in the steady state the current only flows in the \( x \) direction, which means
\[ j_x = \frac{I}{L_y}, \quad j_y = j_z = 0. \]

We will also assume that the magnetic field is only rotated in the \( xy \) plane (\( xz \) plane is identical). This implies that we may take both electrochemical potentials \( \mu \) and \( \mu_c \) to be independent of \( z \). Then Eqs. (2), (3) allow us to express \( \nabla \mu \) in terms of the current and the chiral electrochemical potential \( \mu_c \) as
\[ \frac{\partial \mu}{\partial x} = \frac{eI}{\sigma L_y^2} - \frac{\mu_c}{L_a} \cos \theta, \]
\[ \frac{\partial \mu}{\partial y} = -\frac{\mu_c}{L_a} \sin \theta, \]
where \( \theta = \arctan(B_y/B_x) \) is the angle between the applied magnetic field and the current and we have introduced a magnetic field related length scale
\[ L_a = \frac{D}{\sqrt{eB}}, \]
which will play a crucial role in what follows. Note that this length scale is distinct from the usual magnetic length \( \ell_B = 1/\sqrt{eB} \) and appears due to the chiral anomaly [hence the subscript \( a \)]. \( 1/L_a \) quantifies the strength of the chiral anomaly induced coupling between the electric and the chiral charge. The existence of this new length scale was first pointed out in Ref. [29]. Substituting Eq. (4) into the equation for the chiral electrochemical potential, we obtain
\[ \frac{\partial^2 \mu_c}{\partial x^2} + \frac{\partial^2 \mu_c}{\partial y^2} - \frac{\mu_c}{\lambda^2} = -\frac{eI \cos \theta}{\sigma L_a L_y^2}, \]
where
\[ \lambda^2 = \frac{L_y^2 L_c^2}{L_a^2 + L_c^2}. \]
and we have introduced another important length scale \( L_c = \sqrt{\Gamma D \tau_c} \), which has the meaning of the chiral charge diffusion length. Transport effects due to the chiral anomaly may be expected to be significant only when \( L_c \) is a macroscopic length scale, i.e., when the chiral charge is a nearly conserved quantity. More specifically, as will be shown below, the parameter that determines the strength of the chiral anomaly induced magnetotransport effects in topological metals is the ratio of the two length scales \( L_c/L_a \).

We may further simplify Eq. (6) by noticing that the condition of no charge current in the \( y \) direction, expressed by the second of Eqs. (4), may always be satisfied by taking \( \mu_c \) to be independent of \( y \), which then implies uniform gradient of the electrochemical potential \( \mu \) in the \( y \) direction. Eq. (6) then simplifies to
\[ \frac{d^2 \mu_c}{dx^2} - \frac{\mu_c}{\lambda^2} = -\frac{eI \cos \theta}{\sigma L_a L_y^2}. \]

Equation (8) needs to be solved with the appropriate boundary conditions. These are naturally not universal and depend on the details of the experimental setup being modelled. We will assume that the sample is attached to uniform current carrying leads across the whole width of the sample cross-section, and the lead material is a normal non-topological metal. In this case, chiral charge must rapidly relax upon entering the normal leads and the most appropriate boundary condition is thus of the Dirichlet type
\[ \mu_c(x = \pm L_x/2) = 0. \]
From Eq. (13), we have
\[ \rho_{xx} = \rho_{\parallel} - \Delta \rho \cos^2 \theta, \]
\[ \rho_{yx} = -\Delta \rho \sin \theta \cos \theta, \] (15)
where \( \Delta \rho = \rho_{\perp} - \rho_{\parallel} \) is the chiral anomaly induced resistivity anisotropy.

Eq. (15) has the form of a standard relation between the anisotropic magnetoresistance (AMR), represented by the first equation, and the planar Hall effect (PHE), which is expressed by the second equation [30–32]. Note that the name PHE is a bit of a misnomer: the off-diagonal resistivity does not satisfy the antisymmetry property of a true Hall effect \( \rho_{xy} = -\rho_{yx} \), since it does not originate from the Lorentz force. It is, however, the standard name for this phenomenon in the literature and we will thus use it as well. Both AMR and PHE are well known phenomena in ferromagnetic metals, originating in this case from the interplay of the magnetic order and the spin-orbit interactions. Both are typically very weak, but can be much stronger in ferromagnets with significant spin-orbit interactions, such as doped magnetic semiconductors [32]. What is remarkable about our result is that neither AMR nor PHE in a topological material require magnetic order, originating instead from the chiral anomaly, and their magnitude can be extremely large (in fact, approaching the theoretical upper limit at increasing magnetic field), as we show below. The sign of the effect in our case is also opposite to what is typically observed in ferromagnets: \( \Delta \rho \) as defined in Eq. (15), is positive in our case, but would typically be negative in a metallic ferromagnet [31].

The parallel resistivity \( \rho_{\parallel} \) exhibits a nontrivial dependence on the two intrinsic length scales \( L_{a,c} \) of the material and on the sample size \( L_x \). Let us first consider the regime of weak magnetic fields, corresponding to \( L_a \gg L_x \). In this case, taking the limit of large sample size \( L_y \gg L_x \), we obtain
\[ \rho_{\parallel} \approx \frac{1}{\sigma} \left[ 1 - \left( \frac{L_x}{L_a} \right)^2 \right], \] (16)
which corresponds to a small negative quadratic magnetic field dependent correction to the longitudinal resistivity. The PHE in this case is also small and given by
\[ \rho_{yx} = -\frac{1}{\sigma} \left( \frac{L_c}{L_a} \right)^2 \sin \theta \cos \theta. \] (17)

A more interesting regime is the regime of stronger magnetic field, corresponding to \( L_a \ll L_c \). Assuming the sample size \( L_x > L_a \), we obtain
\[ \rho_{\parallel} = \frac{L_a^2}{\sigma L_x^2} \left[ 1 + \frac{2L_x^2}{L_a L_c} \right], \] (18)
Eq. (18) exhibits an interesting and nontrivial scale dependence. Indeed, suppose that \( L_a < L_x < L_c^2/L_a \) the
upper limit is a third nontrivial length scale in this problem!). In this case

\[ \rho_\parallel \approx \frac{2L_a}{\sigma L_x} = \frac{4\pi^2 e_B^2}{c^2 L_x}. \]  

(19)

To understand the meaning of this result it is convenient to evaluate the corresponding conductance

\[ G_\parallel = \rho_\parallel \frac{L_y}{L_x} = \frac{e^2 N_\phi}{2\pi}, \]  

(20)

where \( N_\phi = L_y^2/2\pi e_B^2 \) is the number of magnetic flux quanta, penetrating the sample cross section. This is identical to the result of Ref. \[29\], obtained by a different method. Physically, this corresponds to a regime, in which the sample conductance is dominated by the chiral lowest Landau level, which is where the chiral anomaly contribution to Eqs. \[1, 2\] comes from \[21, 22\]. \( G_\parallel \) then corresponds to a conductance of \( e^2/h \) per lowest Landau level orbital state, i.e. is identical to the conductance of an effective one-dimensional system with \( N_\phi \) conduction channels.

Most importantly, the resistivity anisotropy and thus the magnitude of the PHE in this regime is given by

\[ \Delta \rho = \frac{1}{\sigma} \left( 1 - \frac{2L_a}{L_x} \right). \]  

(21)

Thus \( \Delta \rho \) is starting to approach its maximal possible value of \( 1/\sigma \) when the sample size is increased. We thus call this Giant Planar Hall Effect (GPHE) (this name was first used in relation to PHE in the context of magnetic semiconductors in Ref. \[32\]).

For larger sample sizes, when \( L_x > L_y^2/L_a \), the magnetic field dependence of the longitudinal resistivity crosses over from \( 1/B \) to \( 1/B^2 \)

\[ \rho_\parallel \approx \frac{1}{\sigma} \left( \frac{L_a}{L_c} \right)^2. \]  

(22)

The GPHE magnitude in this case becomes independent of the sample size and has reached its maximal magnitude

\[ \Delta \rho = \frac{1}{\sigma} \frac{(L_c/L_a)^2}{1 + (L_c/L_a)^2}, \]  

(23)

which converges to \( 1/\sigma \) as the magnetic field is increased.

Interestingly, there exists a direct connection between the GPHE and the angular narrowing of the negative longitudinal magnetoconductivity signal \[17\], as we will now demonstrate. Using Eq. \[15\], we obtain

\[ \rho_{xx}^{-1}(B) - \rho_{xx}^{-1}(0) = \frac{\sigma(\Delta \rho/\rho_\parallel) \cos^2 \theta}{1 + (\Delta \rho/\rho_\parallel) \sin^2 \theta} \]

\[ = \frac{\sigma(L_c/L_a)^2 \cos^2 \theta}{1 + (L_c/L_a)^2 \sin^2 \theta}. \]  

(24)

What is missing in the standard expressions for the chiral anomaly induced magnetoconductivity \[20\] is the angular dependence in the denominator. Different curves correspond to different values of the ratio \( L_c/L_a \): 2 (blue, solid), 3 (orange, dashed) and 5 (green, dotted). (b) Dependence of the resistivity anisotropy \( \Delta \rho \) and thus the magnitude of the GPHE on \( L_c/L_a \).

![Graph](image)

FIG. 2. (Color online) (a) Inverse longitudinal magnetoresistivity as a function of the angle between the current and the magnetic field. Different curves correspond to different values of the ratio \( L_c/L_a \): 2 (blue, solid), 3 (orange, dashed) and 5 (green, dotted). (b) Dependence of the resistivity anisotropy \( \Delta \rho \) and thus the magnitude of the GPHE on \( L_c/L_a \).

In order to relate our results to the existing experimental data \[17\]-\[18\], it is useful to express the ratio \( L_c/L_a \) explicitly in terms of the magnetic field. We obtain

\[ \frac{L_c}{L_a} = \Gamma B \sqrt{\frac{\tau_c}{D}} \sim \left( \frac{h v_F/L_B}{\epsilon_F} \right)^2 \sqrt{\frac{\tau_c}{\tau}}, \]  

(25)

where \( v_F \) is the Fermi velocity of the Weyl (Dirac) fermions, \( \epsilon_F \) is the Fermi energy and \( \tau \) is the momentum relaxation time. Taking the values for these parameters from Ref. \[17\], we have \( v_F \approx 3.5 \times 10^7 \text{cm/s}, \epsilon_F \approx 30 \text{meV}, \) and \( \tau_c/\tau \approx 50 \). Assuming the density of states to be
\[ g = \frac{e^2}{2m^* \hbar^2} \frac{\tau_{\perp}}{\tau}, \]
and substituting the above values in Eq. (25), we obtain \( L_e / L_a \approx 0.7 B / 1 \text{ Tesla}. \)

In Fig. 2, we plot both the angular dependence of the inverse longitudinal magnetoresistivity and the magnitude of the GPHE using \( L_e / L_a \) values, which correspond to the range of magnetic fields of up to about 10 T, which was used in Refs. [17, 18]. Qualitatively, the behavior of the magnetoresistivity appears to agree with the experimental data. In particular, at low magnetic fields the magnetococonductivity peak follows a \( B^2 \) dependence, while the angular width goes roughly as \( 1/B \), consistent with Eqs. (24) and (25). At higher fields both appear to saturate, which is likely explained by the fact that in both experiments the quantum regime \( \epsilon_F < h \nu_F / \ell_B \) is reached at magnetic fields of just a few Tesla. In this regime we expect \( g \sim B \) and \( 1/\tau \sim B \), \( 1/\tau_e \sim B \), thus making the ratio \( L_e / L_a \) independent of the magnetic field. Fig. 2 also shows that the magnitude of the GPHE may approach its maximal value of \( \Delta \rho = 1/\sigma \) for experimentally accessible values of the magnetic field.

In conclusion, we have described a magnetotransport effect, related to the chiral anomaly, that has not been noticed before, the Giant Planar Hall Effect. We have also connected this effect to the angular dependence of the longitudinal magnetococonductivity, explaining its magnetic field dependent narrowing, pointed out in Ref. [17]. Observation of both the negative longitudinal magnetoresistance and the GPHE, with a specific relation between the angular dependence of the magnetoresistance and the GPHE magnitude, given by Eq. (24), constitutes a smoking gun evidence for the chiral anomaly. We note that the GPHE may have already been observed in a recent experiment on ZrTe5. [33]

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