Complex Acquisition of the Fourier Transform Imaging of an Arbitrary Object

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A scheme to a complex-valued acquisition of the Fourier transform imaging was proposed. The main idea is to project the real and the imaginary parts of a diffraction field to intensity distributions respectively. The whole procedure was algorithm independent and needs no a priori knowledge of an arbitrary object. An example was demonstrated with a numerical modeling and its results.

Keywords: Complex Acquisition; Arbitrary Object; Fourier transform imaging.

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When talk about the image forming, the process by instruments like the eye, the camera, the reflecting and refracting telescope, and the microscope etc., were generally refers to a point to point correspondence between two real spaces [1]. If a situation requires full information about the objects rather than only a magnitude transmittance or reflectance, such procedures would be invalid because what they mapped was only intensity relations, and phase information would thus lost. A possible way to obtain the phase knowledge of the object’s transmittance is to convert it into a spatially varying diffraction pattern with form of Fourier transform (supposed in Fraunhofer region), and then, invert it to its object function. This procedure is based on the fact that Fourier transform keeps unitary relations between real and reciprocal spaces. Unfortunately, the diffraction fields are also only recordable by an intensity-sensitive detector and this awkward fact would lead phase loss to occur in the reciprocal space again. To solve the phase problem, efforts have been paid by utilizing oversampling methods [2] with iterative algorithms [3, 4, 5]. The modulus of Fourier transform can thus be phased and then inverting into an object functions. This method was reported recently in X-ray diffraction microscopy [6] and has been extended from x-ray crystallography [2] to the imaging of noncrystalline materials [8] and single cells [9]. Its potential for imaging of single protein complexes by using ultra-short X-ray pulses with extreme intensity were also discussed [10]. The excellent works as mentioned above have now achieved the imaging of single virion even with a resolution of 22nm [11].

Anyway, the phase-less magnitude alone would not be able to sustain the unitary property of Fourier transform. Therefore, iterative algorithms to recover the phase encoded in the diffraction pattern have to rely on a priori knowledge of the objects more or less. If a object was arbitrary i.e. not purely absorptive or not purely phased, the ambiguity would arise [12]. The difficulties of the complex-valued acquisitions without any original information seemed to have already been predisdestined by quantum mechanics. i.e., the complex amplitude can not be specified exactly in a single measurement [13]. On the other hand, for coherent sources are not obtainable in wave length like hard X-ray region, potential advantages [14] might be jeopardized when using the ultra-short, intense X-ray pulses[10].

In this paper, we theoretically propose a methods and a scheme to a complex-valued acquisition of the Fourier transform imaging of the arbitrary object’s transmittance with incoherent light. The main thought is to project the real and the imaginary parts of a Fourier-transformed field to intensity distributions respectively. The whole procedure is algorithms free and needs no a priori knowledge of an arbitrary object.

The proposed scheme for set up is shown in Fig 1. Fields from the thermal source is split by a 50/50 beam splitter BS1 to form a two-arm optical system with equal distance from source plane to planes η1, η2, where full wave information will be registered. At the cross section of both arms before η1 and η2, the other 50/50 beam splitter BS2 would insert. As for BS2, we must manage to ensure that the half wave loss only occurs when the light reflects only on one of its surfaces. When a situation needs to introduce a phase factor shift of j, a phase plate J would be inserted into the upper part of the system. The total scheme differs mainly from the Mach-Zehnder interferometer is that it uses a prismatic lens P rather than a plane mirror to guide the optical path in its upper part. The object with complex transmittance of \( f(\xi) \) and

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FIG. 1: The proposed scheme for set up
is placed at plane $\xi$ in lower part of the setup. The distance from plane $\xi$ to plane $x$, and to plane $\eta_1$, and from plane $x$ to plane $\eta_2$ are $d_1$, $d_2$ and $d$ respectively. The equal length of the two arms requires that

$$d = d_1 + d_2.$$  \tag{1}

Among the setup, an optional phase plate $P'$ might insert at the lower part of the scheme to form a fixed optical path difference between two arms of a phase factor of $1/\sqrt{7}$, whereas, it was not a necessary component.

Although chaotic light fields fluctuates randomly, the relations among instantaneous values of optical fields $E$ in plane $\eta_1$, $\eta_2$ and $\xi$ are deterministic according to Fresnel diffraction theory. To illustrate the theoretical bases for this complex-valued retrieval procedure clearly, first we suppose, that we did not facilitate the beam splitter $BS_2$, phase plate $P'$, and $J$ in Fig.1. In this situation, the Optical fields on $\eta_1$, and $\eta_1$ fulfill

$$E(\eta_2) = \frac{\text{e}^{jkd}}{\sqrt{\lambda d}} \int \text{e}^{j\frac{(\xi - x)^2}{2d_1}} d_1 \quad (2)
$$

and

$$E(\eta_1) = \frac{\text{e}^{jkd_2}}{\sqrt{\lambda d_2}} \int \text{e}^{j\frac{(\xi - x)^2}{2d_1}} d_2 \quad (3)
$$

under the Fresnel approximation. Let the thermal source is totally chaotic with evenly distributed intensity, i.e.,

$$\langle E(x)E(x') \rangle = I\delta(x-x'), \quad (4)$$

in which $\langle \cdots \rangle$ stands for the assemble average. Then, the derivation of the mutual intensity between plane $\eta_1$ and $\eta_2$ can be derived from Eq. 11 to Eq.14 to be

$$\langle E(\eta_1)E^*(\eta_2) \rangle = \int \text{e}^{j\frac{(\xi - \eta)^2}{2d_1}} F \{ f(\xi) \} \quad (5)
$$

In which $F \{ \cdots \}$ refers to $F \{ f(x) \}$, the Fourier transform of $f(x)$. Now with the beam splitter $BS_2$ facilitated, the fields in plane $\eta_1$, and $\eta_2$ turn out to be:

$$E_1(\eta_1) = \frac{1}{\sqrt{2}} \left[ E(\eta_1) - E(\eta_2) \right], \quad (6)
$$

and

$$E_2(\eta_2) = \frac{1}{\sqrt{2}} \left[ E(\eta_1) + E(\eta_2) \right], \quad (7)
$$

when half wave loss only on one side of $BS_2$ was considered. Their corresponding intensity distribution registered on plane $\eta_1$, and $\eta_2$ are

$$I_1(\eta_1) = \langle E_1(\eta_1)E_1^*(\eta_1) \rangle = \frac{1}{2} \left( |E(\eta_1)|^2 - 2 \text{Re} \langle E(\eta_1)E^*(\eta_2) \rangle + |E(\eta_2)|^2 \right), \quad (8)
$$

and

$$I_2(\eta_2) = \langle E_2(\eta_2)E_2^*(\eta_2) \rangle = \frac{1}{2} \left( |E(\eta_1)|^2 + 2 \text{Re} \langle E(\eta_1)E^*(\eta_2) \rangle + |E(\eta_2)|^2 \right), \quad (9)
$$

respectively. The fact can be easily seen from two equations above that the real part of Eq.5 is embedded in the intensity registration on both plane $\eta_1$, and $\eta_2$, and can be extracted by subtracting the two equal backgrounds by means of

$$\text{Re} \langle E(\eta_1)E^*(\eta_2) \rangle = \frac{I_2(\eta_2) - I_1(\eta_1)}{2}. \quad (10)
$$

Further more, the linearity Eq.5 indicates that if a phase shift of $\phi$ is introduced in the upper or the lower part of the scheme shown as Fig.1 a phase factor of $e^{-j\phi}$ or $e^{j\phi}$ would be multiplied on the right side of the equation consequently. Following the thought, if we inserted a phase plate $J$ into the upper part of the system to introduce a phase shift of $\pi/2$, the complexes vector stands for Eq.5 would rotate an angle of $-\pi/2$ consequently. So the imaginary part of Eq.5 can also be retrieved in a way similar to eq.(10) by:

$$\text{Im} \langle E(\eta_1)E^*(\eta_2) \rangle = \frac{I_2(\eta_2) - I_1(\eta_1)}{2}. \quad (11)
$$

since a factor of $-j$ had been brought into Eq.5. In Eq.11, $I_2(\eta_2)$ and $I_1(\eta_1)$ are intensity registration on plane $\eta_2$, and $\eta_1$, after a phase plate $J$ was inserted into the upper part of the system as Fig.1 shows. Note that we use a prismatic lens $P$ rather than a plane mirror to guide the optical path in upper part of the system. This arrangement leads to a bilateral symmetry between coordinates in plane $\eta_1$, and $\eta_2$, i.e.,

$$\eta_1 = -\eta_2 (= \eta). \quad (12)
$$

Comparing Eq.5, Eq.11, Eq.11, and Eq.12, we proposed the complex-valued acquisition of an object’s Fourier transform imaging. The procedure can be written in one equation as:

$$F \{ f(\xi) \} = F \left( \frac{2\eta}{\lambda d_2} \right) \propto \frac{1}{\sqrt{2}} \left( I_2(\eta_2) - I_1(\eta_1) \right) + j \frac{I_2(\eta_2) - I_1(\eta_1)}{2}. \quad (13)$$
To give an example of the retrieval, we conceived an object with a complex-valued transmittance of:

$$f(\xi) = \left\{ (1 + \cos 0.05\xi) + j \left[ \text{rect} \left( \frac{\xi + 150}{105} \right) - \text{rect} \left( \frac{\xi - 150}{105} \right) \right] \right\} \text{rect} \frac{\xi}{1000}. \quad (14)$$

In which 0.05, 150, 105, and 1000 are space parameters with $\mu m$ unit in plane $\xi$. If the object were to be illuminated by a coherent light with a wavelength of $\lambda = 0.532\mu m$, the real and imaginary parts of its Fourier transform as a function of space coordinates $\eta$ would be:

$$\text{Re} \{ \mathcal{F} \{ f(\xi) \} \} = 500 \sin c \left[ 1000 \left( \frac{\eta}{\lambda d_2} + \frac{0.05}{2\pi} \right) \right]$$

$$+ 1000 \sin c \left( 1000 \frac{\eta}{\lambda d_2} \right)$$

$$+ 500 \sin c \left[ 1000 \left( \frac{\eta}{\lambda d_2} - \frac{0.05}{2\pi} \right) \right], \quad (15)$$

and

$$\text{Im} \{ \mathcal{F} \{ f(\xi) \} \} = 210 \sin c \left( \frac{105 \eta}{\lambda d_2} \right)$$

$$\times \cos \left( 300\pi \frac{\eta}{\lambda d_2} \right); \quad (16)$$

as Fig. 2 shows. In Eq. (15) and Eq. (16), $d_2 = 75,000\mu m$, refers to a distance the coherent light field with $\lambda = 0.532\mu m$ from the object propagates until it reaches the plane $\eta$.

Based on the theory of statistical optics, we numerically modeled the dynamic process of the whole retrieval procedure under the setup scheme of Fig. 1 by using the conceived object with transmittance of Eq. (14). An optional phase plate prism $P'$ was inserted at the lower part of the scheme to form a fixed optical path difference of $\pi/4$ between two arms to compensate the factor of $1/\sqrt{2}$ in Eq. (5).

In the modeling, the monochromatic thermal light featured circular Gaussian random process with zero mean [10], the wave length of the thermal light was selected to be $\lambda = 0.532\mu m$, the propagation of the fields among the setup fulfills the Fresnel diffraction integrations stated by Eq. (2) to Eq. (3). For the setup, $d_1$, $d_2$, and $d$ are set to 60,000$\mu m$, 75,000$\mu m$, and 135,000$\mu m$. After an accumulative intensity registration, which covers 20,000 times of independent coherent time, the averaged intensities in both plane $\eta_1$ and $\eta_2$ were brought about in Fig. 3. In which (a) and (a') are intensity registered in plane $\eta_1$ before and after a phase plate J of $\pi/2$ was inserted in the upper part of scheme in Fig. 1, Comparing (c) and (c') with (a) and (b) in Fig. 2, the numerical results shows that the real and imaginary parts of the Fourier imaging of a conceived object (Eq. (13)) were both retrieved in a sub-wave-length scale.

In summary, by using incoherent light, we theoretically proposed a scheme to a complex acquisition of the Fourier transform imaging. The whole procedure needs no a priori knowledge of an arbitrary object. The unitary property of the Fourier transform thus sustains by intensity recordable detectors. In recent years, coincidence imaging by classical thermal sources has been widely investigated both in real [17, 18] and reciprocal spaces [19, 20, 21]. In these works, images were carried out either by two photon coincident rate or by correlation functions of intensity fluctuations. They both stand for the modules of optical fields. Useful methods to retrieval a complex-valued objects with discrete variables [22] were also reported combined with oversampling methods [2] and iterative algorithms [3, 4, 5]. The authors...
of Ref. [12] reconstruct the complex object without \textit{a priori} knowledge, but still based on computer operations. Unlike these works, we map the complex valued object in real space to complex valued Fourier transform only by intensity registration [24]. As the unitary property remains, the retrieval procedure by this scheme equals to imaging an arbitrary complex valued object in a full sense. To our knowledge, it is the first physical proposal to sustain the unitary property of an arbitrary object when imaging without \textit{a priori} knowledge.

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