Estimation the Shape Parameter of (S-S) Reliability of Kumaraswamy Distribution

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Abstract

In this paper dealt with estimating the reliability in the (S-S) stress-strength of Kumaraswamy function distribution using different estimation methods, Maximum likelihood, Moment method, Shrinkage method depend on to Monte Carlo simulation Comparisons between estimation methods have been using mean square error criteria.

Keywords: Reliability, Stress-Strength (S-S), Kumaraswamy distribution, Maximum likelihood estimator, Moment estimator and Shrinkage estimator.

I. Introduction

The Kumaraswamy distribution like the Beta distribution [IV], but has the important feature of an invertible closed form cumulative distribution function The Kumaraswamy distribution was suggested by Poondi Kumaraswamy (1930 - 1988) [III]. The Kumaraswamy distribution is suitable for many natural phenomena that the results have minimum and upper limits in the biomedical and epidemiological research. Several studies have dealt with the Kumaraswamy distribution, in [IV] they produced a study in Estimation for parameters of the Kumaraswamy distribution based on general progressive type II censoring, in [II] they introduce and study the size-biased form of Kumaraswamy distribution, in [V] In this paper, the problem of estimating P[Y<X] for the kumaraswamy generalized class of distributions has been addressed.

The probability density function of the Kumaraswamy distribution is

\[ f(x; \alpha, \theta) = \begin{cases} \alpha \theta x^{\alpha-1} (1 - x^\alpha)^{-1} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \quad (1) \]
Where $\alpha, \theta > 0$ are the two shape and scale parameters respectively. 

And the cumulative probability function is

$$F(x; \alpha, \theta) = 1 - (1 - x^\alpha)^\theta$$ \hspace{1cm} (2)

as a special case, when $\alpha = 1$, the cumulative probability function will be as bellow

$$F(x, \theta) = 1 - (1 - x)^\theta$$ \hspace{1cm} (3)

and the probability density function of the Kumaraswamy distribution will be as bellow:

$$f(x; \theta) = \theta (1 - x)^{\theta-1}$$ \hspace{1cm} (4)

The stress $(Y)$ and the strength $(X)$ in stress-strength (S-S) model will be considered as random variables, for a detailed study of the possible applications of the reliability parameter, it is suggested to read the interested reader [V].

Reliability function of (4) will be

$$R = P(Y < X) = \int f(x) f(y) \, dy \, dx$$

$$= \int_0^1 \int_0^x \theta_1 (1 - x)^{\theta_1-1} \theta_2 (1 - y)^{\theta_2-1} \, dx \, dy$$

$$R = \int_0^{\theta_2} \theta_1 (1 + x)^{-(\theta_1+1)} \, dx \int_0^x \theta_2 (1 + y)^{-(\theta_2+1)} \, dy$$

$$= \int_0^{\theta_2} \left( - \theta_1 (1 + x)^{-(\theta_1+1) - \theta_2} + \theta_1 (1 + x)^{-(\theta_1+1)} \right) \, dx$$

$$R = \frac{\theta_2}{\theta_1 + \theta_2}$$ \hspace{1cm} (5)

The aim of this paper is estimating the reliability $(R)$ in the (S-S) stress-strength of Kumaraswamy when the stress and the strength are not identically independent follows the Kumaraswamy distribution (KD) using viruses methods, shrinkage estimation methods, Maximum likelihood and Moment method. We compare between the proposed estimation methods through Monte Carlo simulation depend on mean square error (MSE) criterion.

II. Estimation methods of $R = P(Y < X)$

II.i. Maximum Likelihood Estimator (MLE)

Assume that $x_1, x_2, ..., x_n$ to be a random sample of KD $(1, \theta_1)$ and $y_1, y_2, ..., y_m$ to be a random sample of KD $(1, \theta_2)$ then, the likelihood function $L(\theta_1, x_i, y_1)$ of the mentioned sample can be obtained below:
\[ l = \prod_{i=1}^{m} f(x_i) \prod_{j=1}^{n} f(y_j) \quad (6) \]

\[ l = \prod_{i=1}^{m} \theta_1^i x_i (1 - x_i)^{\theta_1 - 1} \prod_{j=1}^{n} \theta_2^j y_j (1 - y_j)^{\theta_2 - 1} \quad (7) \]

Taking the logarithm of both sides in the equation (7) then implies
\[ \ln(l) = n \ln(\theta_1) + \sum_{i=1}^{m} (\ln x_i + (\theta_1 - 1) \ln(1 - x_i)) + m \ln(\theta_2) + \sum_{j=1}^{n} (\ln y_j + (\theta_2 - 1) \ln(1 - y_j)) \quad (8) \]

Derive the above equation w.r.t. \( \theta_i (i = 1, 2) \), and equating the result to zero, we conclude
\[ \hat{\theta}_{1,\text{mle}} = \frac{-m}{\sum_{i=1}^{m} \ln(1 - y_i)} \quad (9) \]
\[ \hat{\theta}_{2,\text{mle}} = \frac{\sum_{i=1}^{m} \ln(1 - y_i)}{\sum_{i=1}^{m} \ln(1 - y_i)} \quad (10) \]

By substituting \( \hat{\theta}_{i,\text{mle}} \) in equation (5), we get the reliability estimation for (S-S) model using the Maximum Likelihood method as in the following :-
\[ \hat{R}_{\text{mle}} = \frac{\hat{\theta}_{2,\text{mle}}}{\hat{\theta}_{1,\text{mle}} + \hat{\theta}_{2,\text{mle}}} \quad (11) \]

II.i. Moment Method (MOM)

The moment method will be treated as in this subsection to estimate the parameter \( \theta_i \), \( (i=1,2) \), the formula of rth moment about origin:[VI]
\[ E(X^r) = \theta_1^r \mu^r(\theta_1 + 1) \mu(\theta_1 + 1) \mu(\theta_1 + 2) \]

When \( r=1 \) then
\[ E(X) = \theta_1 \mu(\theta_1 + 2) \mu(\theta_1 + 1), \quad E(Y) = \theta_1 \mu(\theta_1) \mu(\theta_1 + 2) \]
\[ E(X) = \bar{X}, E(Y) = \bar{Y} \]
\[ \theta_1 = \bar{X} \mu(\theta_1 + 2) \mu(\theta_1 + 1) \]
\[ \theta_2 = \bar{Y} \mu(\theta_2 + 2) \mu(\theta_2 + 1) \quad (12) \]

We obtain the estimation of the unknown shape parameters \( \theta_1, \theta_2 \), from equations (12), (13) as follows:
\[ \hat{\theta}_{1,\text{MOM}} = \bar{X} \theta_1(\theta_1 + 1) \quad (14) \]
\[ \hat{\theta}_{2,\text{MOM}} = \bar{Y} \theta_2(\theta_2 + 1) \quad (15) \]
By substituting $\hat{\theta}_{\text{mom}}$ in equation (5), we get the reliability estimation for (S-S) model using the moment estimation method as in the following:--

$$\hat{R}_{\text{mom}} = \frac{\hat{\theta}_{2\text{mom}}}{\hat{\theta}_{1\text{mom}} + \hat{\theta}_{2\text{mom}}}$$  \hspace{1cm} (16)

**II.i.ii. Shrinkage Estimation Method (SHM)**

The method of estimation of deflation is the Bayesian method based on previous information on the value of the parameter specified from previous experiments or previous studies. However, in some cases, only previous information is available from the initial guess value $\theta_0$ [I]. Estimator $\hat{\theta}_{\text{mle}}$ through combine them by Shrinkage weight factor as bellow:

$$\hat{\theta}_{\text{sh}} = \varphi(\hat{\theta}_i)\hat{\theta}_{\text{mle}} + \left(1 - \varphi(\hat{\theta}_i)\right)\theta_0, \hspace{1cm} i = 1, 2$$  \hspace{1cm} (17)

Where $\varphi(\hat{\theta}_i), 0 \leq \hat{\theta}_i \leq 1$ represent shrinkage weight factor.

**II.i.iii.a. Shrinkage weight function (sh1)**

The shrinkage weight factor as a function of n and m respectively will be considered in the equation (16) as below

$$\varphi(\hat{\theta}_1) = K_1 = \frac{\sin(n)}{n} \hspace{1cm} \text{and} \hspace{1cm} \varphi(\hat{\theta}_2) = K_2 = \frac{\sin(m)}{m}$$

$$\hat{\theta}_{1\text{sh1}} = 1 \hat{\theta}_{1\text{mle}} + (1 - 1) \hat{\theta}_1$$  \hspace{1cm} (18)

$$\hat{\theta}_{2\text{sh1}} = 2 \hat{\theta}_{2\text{mle}} + (1 - 2) \hat{\theta}_2$$  \hspace{1cm} (19)

The identical (S-S) reliability using above shrinkage method $\text{sh1}$ will be

$$\hat{R}_{\text{sh1}} = \frac{\hat{\theta}_{2\text{sh1}}}{\hat{\theta}_{1\text{sh1}} + \hat{\theta}_{2\text{sh1}}}$$  \hspace{1cm} (20)

**2.3.2 Constant shrinkage factor (Sh2)**

In this subsection the constant shrinkage weight factor will be assumed as

$$\varphi(\hat{\theta}_1) = k_3 = 0.01, \hspace{1cm} \text{and} \hspace{1cm} \varphi(\hat{\theta}_2) = k_4 = 0.01$$

and so on, the following shrinkage estimators

$$\hat{\theta}_{1\text{sh2}} = k_3 \hat{\theta}_{1\text{mle}} + (1 - k_3) \hat{\theta}_{1\text{mom}}$$  \hspace{1cm} (21)

$$\hat{\theta}_{2\text{sh2}} = k_4 \hat{\theta}_{2\text{mle}} + (1 - 4) \hat{\theta}_{2\text{mom}}$$  \hspace{1cm} (22)

This involves the following estimates of shrinkage (Sh2) in reliability (S-S) in equation (5) using constant shrinkage factor:
II.iii.b. Beta shrinkage factor (sh3)

The Beta shrinkage weight factor will be supposed as
\[ \varphi(\bar{\theta}_1) = k_5 = \text{beta}(n, m), \text{ and } \varphi(\bar{\theta}_2) = k_6 = \text{beta}(n, m) \]
and implies the following estimates of shrinkage
\[ \hat{\theta}_{1sh3} = k_5 \hat{\theta}_{1mom} + (1 - k_5 \hat{\theta}_{1mle}) \]
(24)
\[ \hat{\theta}_{2sh3} = k_6 \hat{\theta}_{2mom} + (1 - k_6 \hat{\theta}_{2mle}) \]
(25)

By substituting \( \hat{\theta}_{i_{sh3}} \) in equation (5), we get the reliability estimation for (S-S) model using the Beta weight factor as in the following :--
\[ \hat{R}_{sh3} = \frac{\hat{\theta}_{2sh3}}{\hat{\theta}_{1sh3} + \hat{\theta}_{2sh3}} \]  
(26)

III. Simulation Study

In this section, Monte Carlo simulation method has been used and the obtained results are compared to the numerical results that previously obtained in the section 2. The simulation process were done using unlike sample size = (30, 50 and 100) and built on 1000 replications by MSE measures to check the performance as in the following:-

Step1: the random sample generated for X and Y according to the uniform distribution over the interval (0,1) as \( u_1, u_2, \ldots, u_n \) and \( w_1, w_2, \ldots, w_m \).

Step2: transforming the above Kumaraswamy distribution KD with using (c.d.f.) as
\[ F(x, \theta) = 1 - (1 - F(x))^{1/\theta} \]

And, by applying the same way, we get
\[ y_j = [1 - (1 - F(y_j))^{1/\theta}] , j = 1, 2, 3, \ldots, m \]
Step 3: Calculate $\hat{\theta}_1^{\text{mle}}$, $\hat{\theta}_2^{\text{mle}}$, $\hat{\theta}_1^{\text{mom}}$, $\hat{\theta}_2^{\text{mom}}$, $\hat{\theta}_1^{\text{sh}1}$, $\hat{\theta}_2^{\text{sh}1}$, $\hat{\theta}_1^{\text{sh}2}$, $\hat{\theta}_2^{\text{sh}2}$, $\hat{\theta}_1^{\text{sh}3}$, $\hat{\theta}_2^{\text{sh}3}$ via equations (9), (10), (14), (15), (18), (19), (21), (22), (24) and (25) respectively.

Step 4: Calculate $\hat{R}^{\text{mle}}$, $\hat{R}^{\text{mom}}$, $\hat{R}^{\text{sh}1}$, $\hat{R}^{\text{sh}2}$ and $\hat{R}^{\text{sh}3}$ through equations (11), (16), (20), (23) and (26) respectively.

Step 5: Using $L=1000$ Replication, the MSE. Where, $\bar{R}$ mention to suggested estimators of Reliability.

The outcomes are put it in the tables (1), (2), (3), (4), (5) and (6) below.

**Table (1):** Estimation when $\theta_1 = 1$, $\theta_2 = 1$

| n  | m  | Mle     | Mom     | Sh1     | Sh2     | Sh3     |
|----|----|---------|---------|---------|---------|---------|
|    |    | $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ |
| 30 | 50 | 1.0343306 | 1.0318790 | 0.998736 | 0.999036 | 0.9998904 | 0.99998904 | 1.0033220 | 0.9991626 | 1.0035110 |
| 50 | 100| 1.0324465 | 1.0235297 | 0.9971103 | 1.0064888 | 0.9972490 | 1.0062418 | 0.9973745 | 1.0066154 | 0.9973873 |

**Table (2):** Estimation when $R = 0.5$, $\theta_1 = 1$, $\theta_2 = 1$

| n  | m  | $\bar{R}^{\text{mle}}$ | $\bar{R}^{\text{mom}}$ | $\bar{R}^{\text{sh}1}$ | $\bar{R}^{\text{sh}2}$ | $\bar{R}^{\text{sh}3}$ |
|----|----|------------------|------------------|------------------|------------------|------------------|
| 30 | 50 | 0.4996039 | 0.5011581 | 0.5010844 | 0.5011355 | 0.5011644 |
| 50 | 100| 0.4993901 | 0.4983379 | 0.4979474 | 0.4983095 | 0.4984112 |
| 100| 100| 0.4950075 | 0.5020290 | 0.501825 | 0.5018807 | 0.5020355 |

**Table (3):** MSE values when $R = 0.5$, $\theta_1 = 1$, $\theta_2 = 1$

| n  | m  | MSE $^{\text{mle}}$ | MSE $^{\text{mom}}$ | MSE $^{\text{sh}1}$ | MSE $^{\text{sh}2}$ | MSE $^{\text{sh}3}$ | Best |
|----|----|--------------------|--------------------|--------------------|--------------------|--------------------|------|
| 100| 30 | 0.4996416 | 0.4996416 | 0.4996416 | 0.4996416 | 0.4996416 | 0.4996575 |
| 50 | 100| 0.4986776 | 0.5007312 | 0.5007312 | 0.5007312 | 0.5007312 | 0.5007312 |
### Table 4: Estimation when $R = 0.5, \theta_1 = 2, \theta_2 = 2$

| n   | m   | $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ |
|-----|-----|------------|------------|------------|------------|------------|------------|------------|------------|
| 30  | 30  | 0.0038593  | 0.0133667  | 0.0011326  | 0.0012926  | 0.0014510  | 0.0014510 |
|     | 50  | 0.0034016  | 0.0012137  | 0.0010791  | 0.0011503  | 0.0012270  | 0.0012270 |
|     | 100 | 0.0027744  | 0.0009137  | 0.0007836  | 0.0008639  | 0.0009438  | 0.0009438 |
| 50  | 30  | 0.0033794  | 0.0011234  | 0.0009934  | 0.0010633  | 0.0011352  | 0.0011352 |
|     | 50  | 0.0024290  | 0.0008451  | 0.0008218  | 0.0008011  | 0.0007815  | 0.0007815 |
|     | 100 | 0.0019024  | 0.0006483  | 0.0006305  | 0.0006142  | 0.0005992  | 0.0005992 |
| 100 | 30  | 0.0026452  | 0.0009002  | 0.0007731  | 0.0008513  | 0.0009311  | 0.0009311 |
|     | 50  | 0.0019474  | 0.0006325  | 0.0006155  | 0.0005999  | 0.0005848  | 0.0005848 |
|     | 100 | 0.0014011  | 0.0004735  | 0.0004612  | 0.0004493  | 0.0004378  | 0.0004378 |

### Table 5: Estimation when $R = 0.5, \theta_1 = 2, \theta_2 = 2$

| n   | m   | $R_{\text{MLE}}$ | $R_{\text{Mom}}$ | $R_{\text{Sh1}}$ | $R_{\text{Sh2}}$ | $R_{\text{Sh3}}$ |
|-----|-----|------------------|------------------|------------------|------------------|------------------|
| 30  | 30  | 0.5026585        | 0.4986342        | 0.4987321        | 0.4986636        | 0.4986134        |
|     | 50  | 0.4986102        | 0.5017386        | 0.5011409        | 0.5016421        | 0.5018254        |
|     | 100 | 0.4972914        | 0.5009116        | 0.5003044        | 0.5007698        | 0.5009221        |
| 50  | 30  | 0.5042406        | 0.497314         | 0.4979342        | 0.4974985        | 0.4973485        |
|     | 50  | 0.5037805        | 0.4977626        | 0.4977985        | 0.4978310        | 0.4978514        |
|     | 100 | 0.4989007        | 0.5002706        | 0.5002445        | 0.5002233        | 0.5002168        |
| 100 | 30  | 0.5018616        | 0.4996924        | 0.5002253        | 0.4998035        | 0.4996701        |
|     | 50  | 0.5021172        | 0.4992779        | 0.4993125        | 0.4993417        | 0.4993536        |
### Table (6): MSE values when $R = 0.5$, $\theta_1 = 2$, $\theta_2 = 2$

| $n$ | $m$ | $\text{mse}_{\text{mle}}$ | $\text{mse}_{\text{mom}}$ | $\text{mse}_{\text{sh1}}$ | $\text{mse}_{\text{sh2}}$ | $\text{mse}_{\text{sh3}}$ | Best |
|-----|-----|-----------------|-----------------|----------------|----------------|----------------|------|
| 30  | 30  | 0.0040076       | 0.0019746       | 0.0016502      | 0.0018724      | 0.0020171      | $\text{mse}_{\text{sh1}}$ |
|     | 50  | 0.0032384       | 0.0016401       | 0.0014462      | 0.0015559      | 0.0016267      | $\text{mse}_{\text{sh1}}$ |
|     | 100 | 0.0023506       | 0.0011962       | 0.0010473      | 0.0011377      | 0.0011977      | $\text{mse}_{\text{sh1}}$ |
| 50  | 30  | 0.0032942       | 0.0016181       | 0.0014325      | 0.0015370      | 0.0016054      | $\text{mse}_{\text{sh1}}$ |
|     | 50  | 0.0025377       | 0.0012956       | 0.0012625      | 0.0012330      | 0.0012142      | $\text{mse}_{\text{sh1}}$ |
|     | 100 | 0.0019983       | 0.0010303       | 0.0010043      | 0.0009805      | 0.0009653      | $\text{mse}_{\text{sh1}}$ |
| 100 | 30  | 0.0028032       | 0.0014855       | 0.0012986      | 0.0014143      | 0.0014902      | $\text{mse}_{\text{sh1}}$ |
|     | 50  | 0.0018519       | 0.0009279       | 0.0009043      | 0.0008827      | 0.0008690      | $\text{mse}_{\text{sh1}}$ |
|     | 100 | 0.0011985       | 0.0005861       | 0.0005717      | 0.0005579      | 0.0005490      | $\text{mse}_{\text{sh1}}$ |

### Table (7): Estimation when $\theta_1 = 3$, $\theta_2 = 3$

| Mle | Mom | Sh1 | Sh2 | Sh3 |
|-----|-----|-----|-----|-----|
| $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ | $\Theta_1$ | $\Theta_2$ |
| 30  | 3.0906989 | 3.1394326 | 3.0083233 | 2.9804931 | 3.0110363 | 2.9857276 | 3.0091471 | 2.9820825 | 3.0092025 | 2.9812317 |
| 50  | 3.1297864 | 3.0858733 | 2.9816831 | 2.9837519 | 2.9865608 | 2.9842878 | 2.9831641 | 2.9847732 | 2.9825378 | 2.9848891 |
| 100 | 3.1245327 | 3.0264210 | 2.9968592 | 3.0010869 | 3.0010641 | 3.0012152 | 2.9981359 | 3.0013403 | 2.9978525 | 3.0013382 |
| 30  | 3.0493863 | 3.1231151 | 3.0076330 | 2.9867158 | 3.0078521 | 2.9912080 | 3.0080505 | 2.9880798 | 3.0080263 | 2.9875037 |
| 50  | 3.0747082 | 3.0782680 | 2.9882738 | 2.9868454 | 2.9887273 | 2.9873251 | 2.9891381 | 2.9877596 | 2.9892355 | 2.9878615 |
| 100 | 3.0665522 | 3.0315963 | 2.9985836 | 2.9982818 | 2.9990408 | 2.9984505 | 2.9992643 | 2.9986149 | 2.9992862 | 2.9986303 |
| 30  | 3.0226315 | 3.1279365 | 3.0021353 | 2.9835253 | 3.0022391 | 2.9882814 | 3.0023402 | 2.9849695 | 3.0023267 | 2.9843770 |
| 50  | 3.0292572 | 0.7299698 | 3.0026634 | 2.9932232 | 3.0027980 | 2.9936418 | 3.0029293 | 2.9940209 | 3.0029291 | 2.9940866 |
| 100 | 3.0357359 | 3.0497366 | 2.9960378 | 2.9871226 | 2.9962388 | 2.9874397 | 2.9964348 | 2.9877488 | 2.9964704 | 2.9878358 |

### Table (8): Estimation when $R = 0.5$, $\theta_1 = 3$, $\theta_2 = 3$

| $n$ | $m$ | $R_{\text{mle}}$ | $R_{\text{mom}}$ | $R_{\text{sh1}}$ | $R_{\text{sh2}}$ | $R_{\text{sh3}}$ |
|-----|-----|-----------------|-----------------|----------------|----------------|----------------|
| 30  | 30  | 0.5008083       | 0.5008083       | 0.5008083      | 0.5008083      | 0.5008083      |
|     | 50  | 0.4996686       | 0.4996686       | 0.4996686      | 0.4996686      | 0.4996686      |
|     | 100 | 0.4994749       | 0.4994749       | 0.4994749      | 0.4994749      | 0.4994749      |
| 50  | 30  | 0.4994811       | 0.4994811       | 0.4994811      | 0.4994811      | 0.4994811      |
|     | 50  | 0.4994857       | 0.4994857       | 0.4994857      | 0.4994857      | 0.4994857      |

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A. S. Mohammed et al
IV. Numerical Results

i- when $n = 30$, the minimum mean square error (MSE) for the (S-S) reliability estimators of Kumaraswamy distribution is holds using the shrinkage estimator based on shrinkage weight function $(\hat{h}_1)$ for $m = (30, 50, 100)$ and each $\alpha_1$ and $\alpha_2$ , this result indicates that, the shrinkage estimator of (S-S) reliability $(\hat{h}_1)$ is the best and follows by shrinkage estimator.

ii- when $n = 50$, the minimum mean square error (MSE) for the (S-S) reliability estimators of the Kumaraswamy distribution is holds using the shrinkage estimator based on shrinkage weight function $(\hat{h}_3)$ for $m = (30, 50, 100)$ and each $\alpha_1$ and $\alpha_2$ , and the best estimator was a beta shrinkage estimator $(\hat{h}_1)$ when $m = 30$, this result indicates that, the shrinkage estimator of (S-S) reliability $(\hat{h}_2)$ was at most the best and follows by shrinkage estimator $(\hat{h}_3)$ and shrinkage estimator $(\hat{h}_1)$.

| $n$ | $m$ | $mse_{\text{mle}}$ | $mse_{\text{mom}}$ | $mse_{\hat{h}_1}$ | $mse_{\hat{h}_2}$ | $mse_{\hat{h}_3}$ | Best |
|-----|-----|------------------|------------------|------------------|------------------|------------------|------|
| 30  | 30  | 0.0041241        | 0.0025304        | 0.0021318        | 0.0024050        | 0.0025494        | $mse_{\hat{h}_1}$ |
|     | 50  | 0.0033056        | 0.0020807        | 0.0018553        | 0.0019798        | 0.0020465        | $mse_{\hat{h}_1}$ |
|     | 100 | 0.0029568        | 0.0017805        | 0.0015426        | 0.0016916        | 0.0017702        | $mse_{\hat{h}_1}$ |
| 50  | 30  | 0.0033048        | 0.0020101        | 0.0017889        | 0.0019127        | 0.0019786        | $mse_{\hat{h}_1}$ |
|     | 50  | 0.0023114        | 0.0014046        | 0.0013695        | 0.0013831        | 0.0013220        | $mse_{\hat{h}_3}$ |
|     | 100 | 0.0019048        | 0.0011815        | 0.0011525        | 0.0011261        | 0.0011124        | $mse_{\hat{h}_3}$ |
| 100 | 30  | 0.0028201        | 0.0017518        | 0.0015216        | 0.0016650        | 0.0017402        | $mse_{\hat{h}_1}$ |
|     | 50  | 0.0019100        | 0.0011323        | 0.0011046        | 0.0010792        | 0.0010660        | $mse_{\hat{h}_3}$ |
|     | 100 | 0.0012819        | 0.0007892        | 0.0007708        | 0.0007530        | 0.0007437        | $mse_{\hat{h}_3}$ |

Table (9): MSE values when $R = 0.5$, $\theta_1 = 3$, $\theta_2 = 3$
iii- when n=100, the minimum mean square error (MSE) for the (S-S) reliability estimators of the Kumaraswamy distribution is holds using the shrinkage estimator based on shrinkage weight function \((\alpha_1\), \(\alpha_2\)) for \(m=30\) and \((\alpha_3\)) for all \(m=(50,100)\) and the best estimator was a beta shrinkage estimator \((sh1)\) when \(m=30\), this result indicates that, the shrinkage estimator of (S-S) reliability \((sh2)\) was at most the best and follows by shrinkage estimator \((sh3)\) and shrinkage estimator \((sh1)\).

Some methods of goodness of fit analysis are employed here; the measurement give an indication of best method is mean square error (MSE) from tables for all.

1- For all \(n=(30,50,100)\) and \(m=(30,50,100)\) in this work for minimum mean square error (MSE) for the stress-strength reliability estimator of Kumaraswamy function distribution after noted the mean square error in tables , the result indicates that shrinkage estimator \((1)\) is the best .

2- For all \(n=(30,50,100)\) and \(m=(30,50,100)\) the minimum mean square error (MSE) for the stress- strength reliability estimator of power function distribution ,we noticed that the shrinkage estimator is the best and follows by maximum likelihood estimator (MLE), moment estimator (MOM) and least square estimator (LS).

3- Of the various cases when \((n=30\) and \(m=30)\), \((\alpha_1=1\) and \(\alpha_2=1)\) then be moment estimator (MOM) batter then maximum likelihood estimator (MLE).

V. Conclusion

In this absence of real data, we study the performance of the estimator obtained from simulated and the tables, that to estimate the reliability of shrinkage estimator method special constant type shrinkage (Sh1) and (Sh3) is the best performance.

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