Extracting $\gamma$ from three-body $B$-meson decays

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To date, the weak-phase $\gamma$ has been measured using two-body $B$-meson decays such as $B \to DK$ and $B \to D\tau$, whose amplitudes contain only tree-level diagrams. But $\gamma$ can also be extracted from three-body charmless hadronic $B$ decays. Since the amplitudes for such decays contain both tree- and loop-level diagrams, $\gamma$ obtained in this way is sensitive to new physics that can enter into these loops. The comparison of the values of $\gamma$ extracted using tree-level and loop-level methods is therefore an excellent test for new physics. In this talk, we will show how U-spin and flavor-SU(3) symmetries can be used to develop methods for extracting $\gamma$ from $B \to K\pi\pi$ and $B \to KK\bar{K}$ decays. We describe a successful implementation of the flavor-SU(3) symmetry method applied to BABAR data.

The observed baryon-to-photon ratio of the universe [1] is several orders of magnitude larger than that predicted by the Standard Model (SM). In order to explain this, we need new sources of CP violation beyond the standard Cabibbo-Kobayashi-Maskawa (CKM) paradigm. Of the three angles of the CKM unitarity triangle, $\gamma$ is the least well-measured. A more precise determination might reveal a deviation from the SM expectations, thus providing a hint of new sources of CP violation.

The standard ways of measuring $\gamma$, the so-called GLW [2, 3], ADS [4], and GGSZ [6] methods, provide a theoretically clean way of measuring the SM value of $\gamma$ [6]. All of these methods use the interference between tree diagrams in $B$ decays, and so they are sensitive only to tree-level new physics (NP) [6]. However, the low-energy effects of most types of NP appear only at loop level, so that it is useful to find alternative methods for extracting $\gamma$ that involve decays whose amplitudes include contributions from loop diagrams. A discrepancy between the values of $\gamma$ extracted using tree-level and loop-level methods will point to the presence of NP. This will be particularly useful as experiments become more and more precise [8–10]. Here we present two alternative methods for extracting $\gamma$ using charmless three-body $B$ decays.

The first method uses a pair of $B \to PPP$ decays ($P = \pi, K$) related by U-spin symmetry. It was proposed in Ref. [11] as an extension of a method involving two-body U-spin pairs [12]. Examples of pairs of three-body decays to which this method can be applied are (i) $B_c^0 \to KS\pi^+\pi^- (b \to d)$ and $B_s^0 \to KS\pi^+K^- (b \to s)$, and (ii) $B_c^0 \to KS\pi^+K^- (b \to d)$ and $B_s^0 \to KS\pi^+\pi^- (b \to s)$. These decays have both tree- and loop-level contributions. It was shown that, using the time-dependent Dalitz-plot analyses of each of the U-spin-related three-body $B$ decays, there are enough observables to extract $\gamma$ from a fit. This is done as follows.

In every Dalitz-plot bin, one can construct the equivalent of a CP-averaged branching ratio, a direct CP asymmetry, and an indirect CP asymmetry. Now, U-spin symmetry implies one relationship involving the branching ratios and CP asymmetries, so there are effectively five independent observables. However, if one includes the measurement of the $B^0\bar{B}^0$ mixing phase as an external input, there are only four theoretical parameters: three hadronic parameters (two magnitudes of amplitudes and one relative strong phase), and $\gamma$. With more observables than unknown parameters, $\gamma$ can be extracted. In addition, one can measure the size of U-spin breaking by using the U-spin relationship between branching ratios and CP asymmetries.

Now, the hadronic parameters are momentum dependent, i.e., their values vary from point to point, or bin to bin, over the Dalitz plot. The extraction of $\gamma$, as well as measuring the size of U-spin breaking, can therefore be performed in local regions and averaged over the entire Dalitz plot. The observables defined in this method become exact when the bins are pointlike. Of course, in practice, finite bin sizes must be used, and this introduces a systematic error, since smaller (larger) bin sizes imply more (fewer) bins in the Dalitz plane, but fewer (more) data points within each bin. For a given dataset, the optimal bin size minimizes the error associated with these competing effects.

The second method, proposed in Ref. [13], extracts $\gamma$ using information from the Dalitz plots of $B \to K\pi\pi$ and $B \to KK\bar{K}$ decays, which are related by flavor SU(3) symmetry. There are several ingredients. First, the method relies on diagrammatic analyses of three-body final states [14, 15]. Second, under SU(3), the final state has an $S_3$ symmetry, which is a symmetry under the interchange of the three identical final-state particles. As a result, one can split a three-body decay amplitude into a fully-symmetric, a fully-antisymmetric and four mixed-symmetric states. In Ref. [16], it was shown...
that flavor-SU(3) diagrams are equivalent to flavor-SU(3) matrix elements for the fully-symmetric three-body final state. (Note that this equivalence includes rescattering effects to all orders in $\alpha_s$.) Third, in Ref. [14], it was shown that, for a given decay amplitude, one can construct the fully-symmetric amplitude by performing an isobar analysis of the Dalitz plot for the decay.

An implementation of this method was first presented in Ref. [17]. It was shown that, in the SU(3) symmetry limit, there are four effective diagrams that contribute to $B^0 \to K^+\pi^0\pi^-$, $B^0 \to K^0\pi^+\pi^-$, $B^0 \to K^+K^0K^-$, and $B^0 \to K^0K^0K^0$. The BABAR data for these decays are given in Ref. [18]. By performing an amplitude analysis using the isobar model for each of the four Dalitz plots, nine observables were constructed. But there are only eight theoretical unknowns: seven hadronic parameters (four magnitudes of diagrams and three relative strong phases) and $\gamma$. As before, with more observables than unknown parameters, one can extract $\gamma$. And since both observables and hadronic parameters in three-body decays are momentum dependent, $\gamma$ could be determined independently from different local regions of the Dalitz plots.

The analysis of Ref. [17] found a fourfold discrete ambiguity in the determination of $\gamma$. One of the four values agreed quite well with the independent measurements of $\gamma$ from tree decays, while the other three solutions did not.

In order to address the issue of SU(3) breaking, in Ref. [17] it was assumed that there is a single SU(3)-breaking parameter $\alpha_{SU(3)}$ relating $B \to K\pi\pi$ and $B \to KKK$ decays ($\alpha_{SU(3)} = 1$ corresponds to the flavour-SU(3) limit). It was found that, by adding the decay $B^+ \to K^+\pi^+\pi^-$ to the analysis, $\alpha_{SU(3)}$ could be extracted. Averaged over the kinematically-allowed Dalitz regions, it was found that $\alpha_{SU(3)} = 0.97\pm0.05$, indicating the absence of significant SU(3) breaking.

Now, the original analysis presented in Ref. [17] did not include all the errors. In particular, systematic uncertainties due to correlations among the various points in the Dalitz plot were not taken into account. To be fair, in Ref. [17] a hope was expressed that the analysis could be redone by physicists (experimentalists?) with more expertise (and computational power), so that these effects could be included. Fortunately, a new analysis presented in this conference has now taken into account the missing uncertainties [19]. Although the details of the results change somewhat – a sixfold discrete ambiguity is found in the determination of the weak phase – the analysis demonstrates that $\gamma$ can indeed be extracted from three-body charmless hadronic $B$ decays.

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