On Mass Formulas for Charm and Beauty Baryons

T. M. Aliev\textsuperscript{a} \textsuperscript{*†}, A. Ozpineci\textsuperscript{a} \textsuperscript{‡}, V. Zamiralov\textsuperscript{b} \textsuperscript{§}
\textsuperscript{a}Middle East Technical University, Ankara, Turkey
\textsuperscript{b}Institute of Nuclear Physics, M. V. Lomonosov MSU, Moscow, Russia

October 22, 2009

Abstract

Possible mixing and its consequences of heavy cascade baryons $\Xi - \Xi'$ is analyzed and its importance in the analysis of their characteristics is shown within the non-relativistic quark model and QCD sum rules.
1 Introduction

During the last few years, very intriguing observations appeared in charm and beauty baryon spectroscopy[1]. It would be interesting to discuss how successfully various models describe the mass of the observed baryons. The central problem of all studies on this subject is establishing the structure of new baryons within the quark model, in particular, of the cascade baryons with new flavors (see [2]-[14] and references therein).

On the other hand, many characteristics of the baryons are successfully determined in the framework of QCD sum rules method [15](see [16] about the modern status of this method). The main issue in the applications of sum rules is the choice of an interpolating current with the same quantum numbers of the corresponding baryon.

In the present work, we discuss some interesting points changing considerably the usual way of constructing mass formulae for the heavy baryons either within the quark model or in the QCD sum rules method.

The plan of this work is as follows. In section II, we discuss the problem of mixing of the baryon wave functions in the quark model [2] and the obtained results are used to construct wave functions of the baryons which are used to calculate the mass as well as the magnetic moments of the baryons. In the last section, we apply the same method to construct interpolating QCD currents and modify the sum rules. In conclusion, we discuss our results briefly.

2 Mixing of the cascade baryons $\Xi_c$- $\Xi'_c$ in the quark model

Let us consider standard non-relativistic quark model (NRQM) wave functions of the charm cascade baryons $\Xi_c$'s of the $SU(4)$ 20'-plet, having in their content three different quarks $u$, $s$, $c$. Upon reduction to $SU(3)$ these baryons occur in the sum of sextet and triplet representations of $SU(3)$ ($20'_4 = 8_3 + 6_3 + \bar{3}_3 + 3_3$). One can choose the wave functions of the $\Lambda$- like baryons with the quark content ($usc$) as

$$\sqrt{6}\Xi_c([us]c) = -c_1u_1s_2 - u_1c_1s_2 + s_1c_1u_2 + c_1s_1u_2$$

$$\sqrt{6}\Xi_c([uc]s) = +s_1u_1c_2 + u_1s_1c_2 - s_1c_1u_2 - c_1s_1u_2$$

$$\sqrt{6}\Xi_c([sc]u) = -s_1u_1c_2 - u_1s_1c_2 + s_1u_1s_2 + u_1c_1s_2$$

Since the sum of these states is equal to zero, only two of the states are linearly independent. But any two of them are not orthogonal to each other, so we construct to every state of the Eq.(3) three possible orthogonal combinations, corresponding to $\Sigma$ like baryons:

$$2\sqrt{3}\Xi'_c([us]c) = 2s_1u_1c_2 + 2u_1s_1c_2 - c_1u_1s_2 - u_1c_1s_2 - s_1c_1u_2 - c_1s_1u_2$$

$$2\sqrt{3}\Xi'_c([uc]s) = 2c_1u_1s_2 + 2u_1c_1s_2 - s_1u_1c_2 - u_1s_1c_2 - s_1c_1u_2 - c_1s_1u_2$$

$$2\sqrt{3}\Xi'_c([sc]u) = 2c_1s_1u_2 + 2s_1c_1u_2 - s_1u_1c_2 - u_1s_1c_2 - s_1u_1s_2 - u_1c_1s_2$$

In principle, one can choose Eqs.(1,4), or (2,5) or (3,6) as a pair of charm cascade baryons $\Xi_c, \Xi'_c$. Rotating the Eqs.(1,4) by $60^\circ$ one obtains (up to a sign) the states (2,5); while rotating them by $120^\circ$ one obtains the states (3,6) (up to a sign).
Upon reduction of the \( SU(4) \) \( 20' \)-plet to the \( SU(3) \) multiplets along the values of charm, \( C = 0, 1, 2 \), one can see that \( \Lambda([ud][c^+] \rightleftarrows \Xi_c^+([us][c]) \) and \( \Xi_c^0([ds][c]) \) form an anti-triplet while \( \Sigma([ud][c]), \Xi_c^+([us][c]) \) and \( \Xi_c^0([ds][c]) \) enter the \( SU(3) \) sextet.

However there is no reason to expect that the experimentally observed charm cascade baryons \( \Xi_c, \Xi'_c \) should belong to the pure anti-triplet or pure sextet states. Upon choosing another pair of the states \( \Xi_c, \Xi'_c \) from the Eqs.\((1-6)\) these anti-triplet and sextet states mix similar to mixing of the initial pure states of the octet \( \omega \) and unitary singlet \( \phi_0 \) state yielding the observed vector nonet mesons \( \omega \) and \( \phi \).

As an example, let us choose a pair of charm cascade baryons \( \Xi_c, \Xi'_c \) with the quark content \( (usc) \). Formally, masses of the \( \Xi_c \) are defined as

\[
M_{\Xi_c} = \langle \Xi_c | \hat{m} | \Xi_c \rangle, \quad M_{\Xi'_c} = \langle \Xi'_c | \hat{m} | \Xi'_c \rangle
\]

(7)

where \( \hat{m} \) is a mass operator in NRQM \([2]\).

Using \([18]\) and the pair of states given in Eqs.\((1) \) and \((4)\), masses of all \( (usc) \) cascade baryons are related to each other as

\[
M_{\Xi_c([us][c])} + 3M_{\Xi_c([us][c])} = 2M_{\Xi_c([uc][s])} + 2M_{\Xi_c([cs][u])},
M_{\Xi_c([us][c])} + 3M_{\Xi_c([us][c])} = 2M_{\Xi_c([uc][s])} + 2M_{\Xi_c([cs][u])}.
\]

(8)

Analogous relations exist for other choices of the cascade pair:

\[
M_{\Xi_c([uc][s])} + 3M_{\Xi_c([uc][s])} = 2M_{\Xi_c([us][c])} + 2M_{\Xi_c([cs][u])},
M_{\Xi_c([uc][s])} + 3M_{\Xi_c([uc][s])} = 2M_{\Xi_c([us][c])} + 2M_{\Xi_c([cs][u])}.
\]

(9)

and

\[
M_{\Xi_c([cs][u])} + 3M_{\Xi_c([cs][u])} = 2M_{\Xi_c([us][c])} + 2M_{\Xi_c([us][c])},
M_{\Xi_c([cs][u])} + 3M_{\Xi_c([cs][u])} = 2M_{\Xi_c([us][c])} + 2M_{\Xi_c([us][c])}.
\]

(10)

In general, the off-diagonal terms are not zero. Upon using relations from \([18]\) non-diagonal mass terms of the pair \( \Xi_c([us][c]) \) \( \Xi'_c([uc][s]) \) could be written in terms of other states of the Eqs.\((1-6)\) as

\[
\sqrt{3}M_{\Xi_c([us][c])}\Xi'_c([uc][s]) = M_{\Xi_c([uc][s])} - M_{\Xi'_c([cs][u])} = -M_{\Xi_c([uc][s])} + M_{\Xi_c([cs][u])},
\]

and similarly for the other two pairs of the heavy cascade baryons:

\[
\sqrt{3}M_{\Xi_c([us][c])}\Xi'_c([uc][s]) = -M_{\Xi'_c([us][c])} + M_{\Xi_c([us][c])} = M_{\Xi_c([us][c])} - M_{\Xi_c([cs][us])},
\]

\[
\sqrt{3}M_{\Xi_c([us][c])}\Xi'_c([uc][s]) = M_{\Xi'_c([us][c])} - M_{\Xi_c([us][c])} = -M_{\Xi_c([us][c])} + M_{\Xi_c([uc][s])}.
\]

It is rather obvious that only for some particular choice of parameters the non-diagonal mass terms are equal to zero. For example, isotopic invariance leads to the vanishing of these non-diagonal mass terms of the baryons \( \Lambda([ud][h]) \) and \( \Sigma([ud][h]), \Sigma([ud][h]), \ h = s, c, b \).

It is clear that the states of definite mass and their masses can be obtained by diagonalizing the mass matrix: of the chosen model \([2]\)

\[
\hat{M}_{\Xi} = \begin{pmatrix}
M_{\Xi'_c} & M_{\Xi_c}\Xi'_c \\
M_{\Xi'_c}\Xi_c & M_{\Xi_c}
\end{pmatrix}
\]

(11)
The corresponding secular equation yields the physical masses to be:

\[
M_{\Xi_c}^2 = \frac{1}{2}(M_{\Xi_c} + M_{\Xi_c}') \pm \frac{1}{2}\sqrt{(M_{\Xi_c} - M_{\Xi_c}')^2 + 4M_{\Xi_c}'^2},
\]

(12)

where the off-diagonal elements are assumed to be equal, i.e. \(M_{\Xi_c\Xi_c'} = M_{\Xi_c'\Xi_c}\). (Recently this formula was written also in [17]). It is these masses which one should compare with experiment. The values of these masses do not depend on which pair from Eqs. (1-6) is chosen for the \(\Xi_c\) and \(\Xi_c'\) states since the sum of \(M_{\Xi_c}\) and \(M_{\Xi_c'}\) as well as the square root in Eq. 12 are invariant under rotations in the flavor space by 60° and 120°.

Thus for a chosen representation of the \(\Xi_c\) and \(\Xi_c'\) baryons, if the off-diagonal entries are not zero, i.e. \(M_{\Xi_c\Xi_c'} = \langle \Xi_c | \hat{m} | \Xi_c' \rangle \neq 0\), then the corresponding wave functions do not describe observable particles. To obtain the representation of the observable particles, the \(\Xi_c\) and \(\Xi_c'\) should be rotated by some angle \(\alpha\),

\[
\Xi^\alpha_c = \Xi_c' \cos \alpha + \Xi_c \sin \alpha, \quad \Xi^\alpha_c = -\Xi_c' \sin \alpha + \Xi_c \cos \alpha.
\]

(13)

Requiring that the off-diagonal elements of the mass matrix for these newly defined states to be zero, the rotation angle \(\alpha\) should be chosen such that:

\[
\tan 2\alpha = \frac{2\langle \Xi_c' | \hat{m} | \Xi_c \rangle}{(M_{\Xi_c} - M_{\Xi_c})},
\]

(14)

where \(\Xi_c', \Xi_c\) are any pair (1,4), (2,5), or (3,6) from the Eqs.(1-6). It would be natural to define these diagonalized states as the physical ones. In this case a quark structure of the baryon (in the given model!) would be a superposition of the states \(\Xi_c\) and \(\Xi'_c\).

### 3 Quark model for masses of new baryons

Let us apply the approach presented in the previous section heavy baryons within the model of [2]. It is convenient to use the fact that already 20 years ago masses of heavy baryons with new quantum numbers were calculated within quark models (cf., e.g., [2]-[10]). It is of interest to note that the predictions on the masses of heavy baryons of these models are in surprisingly good agreement with the modern data. In this work, the mass operator of [2] is used due to its simplicity and clearness:

\[
M_B = m_0 + \sum_{i=1}^{3} m_i + \chi \sum_{i>j} \frac{\vec{S}_i \cdot \vec{S}_j}{m_i \cdot m_j},
\]

(15)

where \(m_0\) is an overall constant, \(m_0 = 77\) MeV, and \(\chi = 22.05 \cdot 10^{-3} \text{ GeV}^3\) [2]; \(S_q\) is the spin operator of the quark \(q\). Quark masses are taken from [2]:

\[
m_u = m_d = 336 \text{ MeV}, \quad m_s = 510 \text{ MeV}, \quad m_c = 1680 \text{ MeV}, \quad m_b = 5000 \text{ MeV}.
\]

Masses of charm and beauty cascade and \(\Omega^-\) baryons were calculated using Eq.(15) in [2] for particular quark combinations of baryons. We have performed calculations for all the possible quark combinations in baryons with the same formula and put results into the Table 1.
As an example, the analysis for baryons with two different heavy quarks forming $SU(3)$ triplets ($\Xi^{+0,0}_c$, $\Omega^{0}_{cb}$) and ($\Xi^{+0,0}_c$, $\Omega^{0}_{cb}$) is presented below. We would write in detail calculations for the pair $\Xi^{+}_c$ and $\Xi^{+}_c$.

(1) We begin with the quark content proposed in [2], i.e. let $b$ be a single quark while the pair ($uc$) in (anti)symmetric state form a diquark. Then, for the diagonal elements of the mass matrix, we get

$$M_{\Xi_c(cub)} = m_0 + m_u + m_c + m_b + \chi(-\frac{3}{4}) \frac{1}{m_u \cdot m_c} = 7063.7; (7064[2])$$

$$M_{\Xi'_c(cub)} = m_0 + \sum_q m_q + \chi(-\frac{1}{2} \frac{1}{m_u m_b} - \frac{1}{2} \frac{1}{m_c m_b} + \frac{1}{4} \frac{1}{m_u m_c}) = 7094.9(7095[2])$$

and the non-diagonal matrix elements are:

$$|\sqrt{3}\langle \Xi'_c(cu)b | \Xi_c(cub) \rangle| = \chi \frac{3}{4} \left( \frac{1}{m_u} - \frac{1}{m_c} \right) \frac{1}{m_b} \sim 7.9 \text{ MeV}$$

Resolving the secular equation

$$x_{1,2} = 7079.3 \pm \frac{1}{2} \sqrt{31.2^2 + \frac{4}{3} 7.9^2} = 7079.3 \pm 16.4,$$

one obtains the mass eigenvalues as

$$x_1 = 7095.7 \quad x_2 = 7063.5$$

In order to go from the initial states $\Xi, \Xi'$ to those with the masses $x_{1,2}$ one should rotate them at the angle $\alpha = 16.3^\circ/2 = 8.15^\circ, \tan 2\alpha = 0.2924$.

(2) Now, let the quarks $c$ and $b$ form a diquark while the light quark $u$ is the single one. In this case

$$M_{\Xi_c(ucb)} = m_0 + m_u + m_c + m_b + \chi(-\frac{3}{4}) \frac{1}{m_u \cdot m_c} = 7091 \text{ MeV},$$

$$M_{\Xi'_c(ucb)} = m_0 + \sum_q m_q + \chi(-\frac{1}{2} \frac{1}{m_u m_c} - \frac{1}{2} \frac{1}{m_u m_b} + \frac{1}{4} \frac{1}{m_u m_c}) = 7067.6 \text{ MeV}.$$
\[ M \Xi'_cb(\{ub\}c) = m_0 + \sum_q m_q + (-\frac{1}{2} \frac{1}{m_u m_c} - \chi \frac{1}{2} \frac{1}{m_b m_c} + \frac{1}{4} \frac{1}{m_b m_u}) = 7075.3 \text{ MeV}; \]

The non-diagonal mass matrix element being
\[
|\sqrt{3} \langle \Xi'_cb(\{cb\}u) \Xi_{cb}(\{ub\}) \rangle| = \frac{3}{4} \left( \frac{1}{m_u} - \frac{1}{m_b} \right) \frac{1}{m_c} \sim 27.7 \text{ MeV}
\]

with the solutions to the secular equation
\[ x_1 = 7095.7 \text{ MeV} \quad x_2 = 7062.9 \text{ MeV}. \]

In order to go from the initial states \( \Xi, \Xi' \) to those with the masses \( x_{1,2} \) one should rotate them at the angle \( \alpha = 256.5^\circ / 2 = 128.25^\circ, \tan 2\alpha = 4.1539. \)

Similar calculations are done also for other charm and beauty baryons and the results are presented in Table 1.

From our analysis, it follows that the predictions on the mass of the physically observed baryons does not depend on the particular quark construction of the baryons in the given model.

**Magnetic moments of the double-flavored baryons**

As another application of our approach, let us consider the magnetic moments of the double-flavored baryons \( \Xi^+_cb \) and \( \Xi'^+_cb \).

Magnetic moment of the baryon with the quark content \( \Xi^+_cb(\{cb\}u) \), which is used often in the modern works (cf., e.g., [11]) in the NRQM, is equal to the magneton of just the \( u \) quark, \( \mu_u \), while that of the baryon \( \Xi'^+_cb(\{cb\}u) \) is equal to \( (2\mu_c + 2\mu_b - \mu_u)/3 \).

However baryons of such quark content in the model of [2] have large non-diagonal mass terms, so one should choose as the wave function of the baryon a linear combination of \( \Xi'^+_cb(\{cb\}u) \) and \( \Xi^+_cb(\{cb\}u) \) with the mixing angle \( \alpha = 68.13^\circ \) (cf. Table 1). Then for the magnetic moments we have:

\[
\langle \Xi'^+_cb | \hat{\mu} | \Xi'^+_cb \rangle = \frac{1}{3} \left( 2\mu_c + 2\mu_b - \mu_u \right) \sin^2 \alpha + \mu_u \cos^2 \alpha - \frac{2}{\sqrt{3}} \sin \alpha \cos \alpha (\mu_c - \mu_b) = \left( 1 - \frac{4}{3} \sin^2 \alpha \right) \mu_u = -0.148 \mu_u \quad (16)
\]

\[
\langle \Xi^+_cb | \hat{\mu} | \Xi^+_cb \rangle = \frac{1}{3} \left( 2\mu_c + 2\mu_b - \mu_u \right) \cos^2 \alpha + \mu_u \sin^2 \alpha + \frac{2}{\sqrt{3}} \sin \alpha \cos \alpha (\mu_c - \mu_b) = \left( -\frac{1}{3} + \frac{4}{3} \sin^2 \alpha \right) \mu_u = 0.815 \mu_u \quad (17)
\]

where we have neglected \( \mu_b \) and \( \mu_c \) in comparison to \( \mu_u \). These predictions do not depend on the particular choice of the baryon pair \( \Xi_{cb} - \Xi'_{cb} \) provided one takes the mixing angle \( \alpha \) corresponding to the given combination, and differ considerably from the predictions given by the "pure" states \( \Xi^+_cb(\{cb\}u) \) and \( \Xi'^+_cb(\{cb\}u) \) (cf. Table 2).
4 Mixing of the states $\Xi - \Xi'$ in QCD sum rules

The problem of the $\Xi - \Xi'$ mixing can also be analyzed within the QCD sum rules framework. The main difference in this case is that, rather than working with the mass matrix, one deals with correlation functions of interpolating currents, i.e. the role of matrix elements (7) is played by correlators [15]

$$\Pi^{\Xi\Xi'} = i \int d^4x e^{i px} \langle 0 | T \{ \eta(x) \bar{\eta}(0) \Xi(0) \Xi'(0) \} | 0 \rangle,$$

which are calculated first in QCD using OPE, and secondly by inserting a complete set of physical states. Performing Borel transformation to suppress high–excited states and equating both expansions one obtains QCD sum rules [15].

In this case, mixing causes the non-diagonal correlation functions to have non-zero values. Prior to performing calculations of the physical properties, such as the magnetic moments, meson couplings etc., of the baryons, one should make sure that the corresponding interpolating currents have zero non-diagonal correlators.

To find the combination of $\eta_{\Xi}$ and $\eta_{\Xi'}$ that have zero non-diagonal correlator, consider the following interpolating currents that are obtained from $\eta_{\Xi}$ and $\eta_{\Xi'}$ after a rotation by $\alpha$:

$$\eta'_\alpha = \eta_{\Xi'} \cos \alpha + \eta_{\Xi} \sin \alpha, \quad \eta_\alpha = -\eta_{\Xi'} \sin \alpha + \eta_{\Xi} \cos \alpha,$$

where the mixing angle $\alpha$ should be chosen such that:

$$\Pi^{\Xi\Xi'}_\alpha = i \int d^4x e^{i px} \langle 0 | T \{ \eta_\alpha(x) \bar{\eta}_\alpha(0) \} | 0 \rangle = 0,$$

goes to zero.

Let us study this problem on a simplified toy-model using the QCD mass sum rules [19] written for the octet hyperons $\Sigma^0$ and $\Lambda$. Generalization to our case seems not to be without problems but is sufficient for our purposes.

Omitting the vacuum expectation values of the quarks $c$ and $b$ and neglecting mass of the $u$ quark in the QCD sum rules, we construct following [19] the QCD mass sum rule for the baryon $\Xi_{cb}(\{ub\}c)$

$$\frac{M^6}{8L^{4/9}} E_2 + \frac{bM^2}{32L^{4/9}} E_0 + \frac{M^2}{4L^{4/9}} a_u m_b E_0 - \frac{m^2_0}{48L^{26/27}} 3a_u m_b = \beta^2_{\Xi} e^{-\left(\frac{M^2_{\Xi}}{M^2}\right)},$$

where $a_q, b$ and $a_q m_0^2$ are defined as [15]:

$$a_q = -(2\pi)^2 \langle \bar{q}q \rangle, \quad b = \langle g_c G^2 \rangle, \quad L = \ln(M^2/\Lambda^2)/\ln(\mu^2/\Lambda^2)$$

$$a_q m_0^2 = (2\pi)^2 \langle g_c \bar{q} \sigma \cdot G q \rangle, \quad q = u, d, s,$$

$\mu$ being renormalization point, while $G$ is a gluon field with the coupling $g_c$ to quarks.

The corresponding sum rule for the $\Lambda$- like baryon $\Xi_{cb}(\{ub\}c)$ reads:

$$\frac{M^6}{8L^{4/9}} E_2 + \frac{bM^2}{32L^{4/9}} E_0 + \frac{M^2}{12L^{4/9}} E_0 a_u (2m_c - m_b)$$

$$- \frac{m^2_0}{48L^{26/27}} a_u (2m_c - 4m_b) = \beta^2_{\Xi} e^{-\left(\frac{M^2_{\Xi}}{M^2}\right)}.$$
The non-diagonal correlation function can also be written as:

$$\sqrt{3}\Pi^{\Xi\Xi'} = \left( \frac{M^2}{6L^{4/9}}E_0 - \frac{m_0^2}{24L^{26/27}} \right) \frac{3}{2} a_\alpha m_c. \quad (24)$$

Requiring that $\Pi^{\Xi\Xi'}_\alpha = 0$ leads to the value of the mixing angle given by:

$$\tan 2\alpha_{(ub)c} = \frac{\sqrt{3}m_c}{2m_b - m_c} \sim 0.342.$$  

For the other pair of heavy baryons $\Xi_{cb}^\prime\{uc\}b$ and $\Xi_{cb}(uc)b$ one should just change $c \to b$, wherefrom

$$\tan 2\alpha_{(uc)b} = \frac{\sqrt{3}m_b}{m_b - 2m_c} \sim 5.18.$$  

Finally for $\Xi_{cb}^\prime\{cb\}u$ one obtains

$$\frac{M^6}{8L^{4/9}}E_2 + \frac{bM^2}{32L^{4/9}}E_0 + \beta_\Xi^2 e^{-\left(M_{\Xi}^2/M^2\right)}, \quad (25)$$

while for $\Xi_{cb}(cb)u$

$$\frac{M^6}{8L^{4/9}}E_2 + \frac{bM^2}{32L^{4/9}}E_0 + \frac{2M^2}{12L^{4/9}}E_0 a_u(m_c + m_b)
- \frac{2m_0^2}{48L^{26/27}} a_u(m_c + m_b) = \beta_\Xi^2 e^{-\left(M_{\Xi}^2/M^2\right)}, \quad (26)$$

wherefrom

$$\tan 2\alpha_{(cb)u} = \frac{\sqrt{3}(m_c - m_b)}{(m_c + m_b)} \sim -0.872$$

These formulae are transformed from one into the other by shifting the angle $\alpha$ by 60° and 120°. At $m_c=1650$ MeV, $m_b=5$ GeV one obtains $\alpha_{(ub)c} \sim 9.5^\circ$, $\alpha_{(cb)u} \sim 69.5^\circ$, $\alpha_{(uc)b} \sim 129.5^\circ$.

It is of interest to note that mass relations of QCD in this approximation yield somewhat unexpected result that the minimal mixing angle favors the diquark pair $(ub)$ while others lead to large mixing angles.

Calculation of the magnetic moments of the double-flavored baryons in the quark model with the mixing angles from the QCD toy-model yield practically the same results as the quark model of [2].

## 5 Conclusion

We have tried to show the importance of mixing of heavy cascade baryons $\Xi - \Xi'$ in analysis of their characteristics. As an example, the non-relativistic quark model of [2] is used. The same approach is applied to the interpolating currents of these baryons in the framework of the QCD sum rules which is shown on the example of simplified mass QCD sum rules. The main conclusion is that in any given model of heavy baryons, one should first find the quark configuration leading to vanishing diagonal matrix elements. After finding the physical states, one should perform calculations of the various characteristics of these baryons. In the case of the QCD sum rules where there is no mass formulae in the common sense of the word, the problem of the truthful combination of interpolating currents deserves further study.
Acknowledgement

T.M.A. and A.O. acknowledge the support of the European Community-Research Infrastructure Integrating Activity Study of Strongly Interacting Matter (acronym Hadron-Physics2, Grant Agreement n. 227431) under the Seventh Framework Programme of EU.
Table 1. Baryon masses (in MeV) and mixing angles in the Ono model

| Baryon          | \{q_1q_2\}q_3 | [q_1q_2]q_3 | \sqrt{3}\langle \Xi'|\hat{m}|\Xi\rangle | x_1   | x_2   | α      |
|-----------------|----------------|-------------|------------------------------------------|-------|-------|--------|
| \(\Xi_c((us)c)\) | 2603           | 2504        | 9.9                                      | 2603  | 2503  | 3.3°   |
| \(\Xi_c((sc)u)\) | 2523.8         | 2558        | 69.3                                     | 2603  | 2503  | 63.25° |
| \(\Xi_c((uc)s)\) | 2534           | 2573.3      | 79.2                                     | 2603  | 2503  | 123.3° |
| \(\Xi_b((us)b)\) | 5945           | 5824        | 3.3                                      | 5945  | 5824  | 0.9°   |
| \(\Xi_b((sb)u)\) | 5852.6         | 5916.4      | 89.1                                     | 5944  | 5824  | 60.8°  |
| \(\Xi_b((ub)s)\) | 5855.9         | 5913.1      | 92.4                                     | 5945  | 5824  | 120.9° |
| \(\Xi_{cb}((uc)b)\) | 7094.9        | 7963.7      | 7.9                                      | 7095.7| 7063.5| 8.15°  |
| \(\Xi_{cb}((cb)u)\) | 7067.6        | 7091.0      | 19.4                                     | 7095.2| 7062.8| 68.13° |
| \(\Xi_b((ub)s)\) | 7075.3         | 7083.1      | 27.7                                     | 7095.9| 7062.9| 128.25°|
| \(\Omega_{cb}((sc)b)\) | 7267.9        | 7247.2      | 4.62                                     | 7268.24| 7246.86| 7.22°  |
| \(\Omega_{cb}((cb)s)\) | 7250.0        | 7265.0      | 13.2                                     | 7268.24| 7246.86| 67.27° |
| \(\Omega_{cb}((sb)c)\) | 7254.7        | 7260.4      | 17.82                                    | 7268.23| 7246.87| 127.26°|
Table 2. Magnetic moments of double-flavored baryons in NRQM for "pure" and mixed states (with index $\alpha$)

| Baryon                  | NRQM  | NRQM$\alpha$ |
|-------------------------|-------|--------------|
| $\Xi_{cb}(\{cb\}u)$    | $\mu_u$ | -0.148$\mu_u$ |
| $\Xi_{cb}(\{cb\}u)$    | $-1/3\mu_u$ | 0.815$\mu_u$ |
| $\Xi_{cb}(\{cu\}b)$    | $\mu_b \sim 0$ | -0.148$\mu_u$ |
| $\Xi_{cb}(\{cu\}b)$    | $2/3\mu_u$ | 0.815$\mu_u$ |
| $\Xi_{cb}(\{ub\}c)$    | $\mu_c \sim 0$ | -0.148$\mu_u$ |
| $\Xi_{cb}(\{ub\}c)$    | $2/3\mu_u$ | 0.815$\mu_u$ |
References

[1] C.Amsler et al.(Particle Data Group), Phys. Lett. B 667, 1 (2008), R. Mizuk, arXiv:0712.0310 (2007)

[2] Seiji Ono, Phys. Rev. D 17, 888 (1978).

[3] C.P.Singh, Phys. Rev. D 24, 2481 (1981).

[4] D.B.Lichtenberg et al., Zeitschr. fur Phys., C 19, 19 (1983); Phys. Rev. Lett. 48, 1653 (1982).

[5] S.Capstick and N.Isgur, Phys. Rev.D 34, 2809 (1986). Phys. Rev. D36, 2800 (1987).

[6] W.Y.P.Hwang and D.B.Lichtenberg, Phys. Rev. D35, 3526 (1987).

[7] R.Roncaglia, D.B.Lichtenberg, E.Predazzi, Phys. Rev. D52, 1722 (1995).

[8] L.Gelmi, V.S.Zamiralov, Moscow State University Physics bulletin, 26 5, 27 (1985).

[9] L.Gelmi, V.S.Zamiralov, S.N.Lepshokov, Moscow State University Physics bulletin, 28 6, 70 (1987).

[10] R.C.Verma and S.Srivastava, Phys. Rev. D38, 1623 (1988).

[11] D.Ebert, R.N.Faustov and V.O.Galkin, Phys. Rev. D 72, 034026 (2005).

[12] Armand Faessler, Th. Gutsche, M.A.Ivanov, J.C.Korner, V.E.Liubovitskij, D.Nicmorus, K.Pumsa-ard, Phys. Rev. D 73, 094013 (2006).

[13] B.Silvestre-Brac, F.Brau, and C.Semay, J.Phys.G29, 2685 (2003).

[14] N.Mathur, R.Lewis, and R.Woloshyn, Phys. Rev. D 66, 014502 (2002).

[15] B.L.Ioffe, Nucl. Phys. B 188, 317 (1981); V.M.Belyev, B.L.Ioffe, JETP 83, 876 (1982) (in Russian); V.M.Belyev, B.L.Ioffe, JETP 84, 1236 (1983) (in Russian).

[16] P. Colangelo and A. Khodjamirian, At the Frontier of Particle Physics / Handbook of QCD, ed. by M. Shifman (World Scientific, Singapore, 2001). Vol. 3, p1495

[17] M. Karliner, B. Kere–Zur, H. J. Lipkin, and J. L. Rosner, arXiv: 0706.2163 (2007).

[18] A.Ozpineci, S.B.Yakovlev, and V.S.Zamiralov, Mod. Phys. Lett. A 20, 243 (2005).

[19] S.-L. Zhu, W.-Y. P. Hwang, and Z.S.Yang, Phys. Rev. D 56, 7273 (1997).