Unifiable Supersymmetric Dark Left-Right Gauge Model

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Abstract

The recently proposed dark left-right gauge model, with $Z'$ and $W_R^\pm$ bosons at the TeV scale, is shown to have a simple supersymmetric extension which is unifiable. Its one-loop gauge-coupling renormalization-group equations are shown to have identical solutions to those of the minimal supersymmetric standard model. It also has a rich dark sector, with at least three stable particles.
**Introduction**: In the conventional left-right gauge extension of the Standard Model (SM) of particle interactions, the $SU(2)_R$ fermion doublet $(\nu, e)_R$ pairs up with the usual $SU(2)_L$ fermion doublet $(\nu, e)_L$ through a Higgs bidoublet, so that both the electron $e$ and the neutrino $\nu$ obtain Dirac masses. Remarkably, this situation is not compulsory. It is in fact possible to have a symmetry such that $\nu_R$ is not the Dirac mass partner of $\nu_L$. It becomes another particle entirely, call it $n_R$, and the same symmetry makes it a dark-matter fermion (scotino). This intriguing scenario has been elaborated in some recent papers [1, 2, 3, 4]. The left-right structure itself was discussed already 23 years ago [5, 6] in the context of superstring-inspired $E_6$ models. Called the Alternative Left-Right Model (ALRM), it has the important property of no tree-level flavor-changing neutral currents. This makes it possible for the $SU(2)_R$ breaking scale to be as low as a TeV, allowing both its charged $W_R^\pm$ and $Z'$ gauge bosons to be observable at the large hadron collider (LHC). However, its relevance to dark matter was not considered until one year ago [1].

In this paper, the latest version [4] of the Dark Left-Right Model (DLRM) is shown to have a simple supersymmetric extension with gauge-coupling unification. The resulting one-loop renormalization-group equations turn out to have solutions identical to those of the Minimal Supersymmetric Standard Model (MSSM), as well as two previously proposed left-right extensions [7, 8]. It also has a rich dark sector, with at least three stable particles [9].

**Model**: Consider the gauge group $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$. Following Ref. [4], a new global $U(1)$ symmetry $S$ is imposed so that the spontaneous breaking of $SU(2)_R \times S$ will leave the combination $L = S + T_{3R}$ unbroken. Under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1) \times S$, the superfields transform as shown in Table 1. Because of supersymmetry, the Higgs sector is doubled, in analogy to the transition from the SM to MSSM. Another set of Higgs doublet superfields $\eta$ and a new set of charged Higgs singlet superfields $\chi$ are added to obtain gauge-coupling unification [7, 8].
Table 1: Particle content of proposed model.

| Superfield | $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)$ | $S$ |
|------------|-----------------------------------------------|-----|
| $\psi = (\nu, e)$ | (1, 2, 1, $-1/2$) | 1 |
| $\psi^c = (e^c, n^c)$ | (1, 1, 2, $1/2$) | $-3/2$ |
| $\nu^c$ | (1, 1, 1, 0) | $-1$ |
| $n$ | (1, 1, 1, 0) | 2 |
| $Q = (u, d)$ | (3, 2, 1, $1/6$) | 0 |
| $Q^c = (h^c, u^c)$ | (3*, 1, 2, $-1/6$) | 1/2 |
| $d^c$ | (3*, 1, 1, $1/3$) | 0 |
| $h$ | (3, 1, 1, $-1/3$) | $-1$ |
| $\Delta_1$ | (1, 2, 2, 0) | 1/2 |
| $\Delta_2$ | (1, 2, 2, 0) | $-1/2$ |
| $\Phi_{L1}$ | (1, 2, 1, $-1/2$) | 0 |
| $\Phi_{L2}$ | (1, 2, 1, $1/2$) | 0 |
| $\Phi_{R1}$ | (1, 1, 2, $-1/2$) | $-1/2$ |
| $\Phi_{R2}$ | (1, 1, 2, $1/2$) | 1/2 |
| $\eta_{L1}$ | (1, 2, 1, $-1/2$) | $-2$ |
| $\eta_{L2}$ | (1, 2, 1, $1/2$) | 2 |
| $\eta_{R1}$ | (1, 1, 2, $-1/2$) | $-3/2$ |
| $\eta_{R2}$ | (1, 1, 2, $1/2$) | 3/2 |
| $\chi_1$ | (1, 1, 1, $-1$) | $-2$ |
| $\chi_2$ | (1, 1, 1, 1) | 2 |

The symmetry $S$ is used here to distinguish $\psi, \Phi_{L1}, \eta_{L1}$ from one another, as well as $\psi^c, \Phi_{R2}, \eta_{R2}$. The bilinear terms allowed by $S$ are

$$\Delta_1 \Delta_2, \; \Phi_{L1} \Phi_{L2}, \; \Phi_{R1} \Phi_{R2}, \; \eta_{L1} \eta_{L2}, \; \eta_{R1} \eta_{R2}, \; \chi_1 \chi_2.$$  \hspace{1cm} (1)

The trilinear terms are

$$\psi \psi^c \Delta_1, \; QQ^c \Delta_2, \; Qd^c \Phi_{L1}, \; \psi \nu^c \Phi_{L2}, \; n \psi^c \Phi_{R1}, \; hQ^c \Phi_{R2},$$

$$\Phi_{L1} \Phi_{R2} \Delta_2, \; \Phi_{L2} \Phi_{R1} \Delta_1, \; \eta_{L1} \eta_{R2} \Delta_1, \; \eta_{L2} \eta_{R1} \Delta_2,$$

$$\Phi_{L1} \eta_{L1} \chi_2, \; \Phi_{R1} \eta_{R1} \chi_2, \; \Phi_{L2} \eta_{L2} \chi_1, \; \Phi_{R2} \eta_{R2} \chi_1.$$ \hspace{1cm} (4)
Hence \( m_e \) comes from the \( I_{3L} = 1/2 \) and \( I_{3R} = -1/2 \) component of \( \Delta_1 \) with \( L = 1/2 - 1/2 = 0 \), \( m_u \) from the \( I_{3L} = -1/2 \) and \( I_{3R} = 1/2 \) component of \( \Delta_2 \) with \( L = -1/2 + 1/2 = 0 \), \( m_d \) from \( \phi^0_{L1} \), \( m_\nu \) from \( \phi^0_{L2} \), \( m_n \) from \( \phi^0_{R1} \), and \( m_h \) from \( \phi^0_{R2} \). This structure guarantees the absence of tree-level flavor-changing neutral currents [10].

**Dark matter**: As it stands, both the neutrino \( \nu \) \((L = 1)\) and the scotino \( n \) \((L = 2)\) are Dirac fermions, and lepton number \( L \) is conserved. If we now introduce a mass term \( \nu^c \nu^c \) which breaks \( L \) by two units, then \( \nu \) gets a small Majorana mass through the canonical seesaw mechanism, as is usually assumed. As for \( n \), it remains a Dirac fermion, being still protected by a global U(1) symmetry. This can be understood by noting that with the addition of the \( \nu^c \nu^c \) term, the same allowed bilinear and trilinear terms in Eqs. (1) to (4) are obtained, if an odd matter parity \( M \) is assumed for \( \psi, \psi^c, \nu, Q, Q^c, d^c, h \) and the \( S \) assignments of \( \psi, \psi^c, \nu^c \) and \( n \) are changed to \( 0, -1/2, 0, \) and \( 1 \) respectively. There are thus two conserved quantities: the usual \( M \) (or its resulting \( R \)) parity and a global U(1) number \( L' = S + T_{3R} \), with \( L' = 0 \) for the usual quarks and leptons and \( L' = 1 \) for the scotino \( n \). Because of their \( S \) assignments, the \( \eta \) and \( \chi \) superfields appear always in pairs, so there is another parity \( H \) which is conserved. Hence there are at least three stable particles. Note that \( \eta \) and \( \chi \) communicate with the quarks and leptons only through \( \Delta \) and \( \Phi \), i.e. the so-called Higgs “portals”.

The various superfields of this model under \( L' \), \( M \), and \( H \) are listed in Table 2. The usual \( R \) parity is then defined as \( R \equiv MH(-1)^{2j} \). A possible scenario for dark matter is to have the following three coexisting stable particles: the lightest neutralino \((L' = 0, H = +, R = -)\), the scotino \( n \) \((L' = 1, H = +, R = +)\), and the exotic \( \eta^0_{R2} \) fermion \((L' = 1, H = -, R = +)\). However, there may be additional stable particles due to kinematics. For example, if the scalar counterpart of \( n \) cannot decay into \( n \) plus the lightest neutralino, then it will also be stable. There may even be several exotic stable \( \eta \) and \( \chi \) particles. The dark sector
may be far from just the one particle that is usually assumed, as in the MSSM.

Table 2: Superfields under $L' = S + T_{3R}$, $M$, and $H$.

| $L'$ | $M$ | $H$ | Superfields                                      |
|------|-----|-----|--------------------------------------------------|
| 0    | −   | +   | $u, d, \nu, e$                                   |
| 0    | +   | +   | $g, \gamma, W^+_L, Z, Z'$                       |
| 0    | +   | +   | $\phi^0_{L1}, \phi^-_{L1}, \phi^0_{L2}, \phi^0_{R1}, \phi^0_{R2}$ |
| 0    | +   | +   | $\delta^0_{11}, \delta^-_{11}, \delta^+_{22}, \delta^0_{22}$ |
| 1    | −   | +   | $n, h^c$                                         |
| −1   | −   | +   | $n^c, h$                                         |
| 1    | +   | +   | $W^+_R, \phi^0_{R2}, \delta^+_{12}, \delta^0_{12}$ |
| −1   | +   | +   | $W^-_R, \phi^-_{R1}, \delta^-_{21}, \delta^0_{21}$ |
| 1    | +   | −   | $\eta^0_{R2}$                                    |
| −1   | +   | −   | $\eta^0_{R1}$                                    |
| 2    | +   | −   | $\eta^0_{L2}, \eta^0_{L1}, \eta^+_{R2}, \chi^+_{2}$ |
| −2   | +   | −   | $\eta^-_{L1}, \eta^0_{L1}, \eta^-_{R1}, \chi^-_{1}$ |

_Gauge-coupling unification:_ The one-loop renormalization-group equations for the gauge couplings of $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_X$ are given by

$$\frac{1}{\alpha_i(M_1)} - \frac{1}{\alpha_i(M_2)} = \frac{b_i}{2\pi} \ln \frac{M_2}{M_1},$$

where $\alpha_i = g_i^2/4\pi$ and the numbers $b_i$ are determined by the particle content of the model between $M_1$ and $M_2$. In the SM with two Higgs scalar doublets, these are given by

\[ SU(3)_C : \quad b_C = -11 + (4/3)N_f = -7, \]  
\[ SU(2)_L : \quad b_L = -22/3 + (4/3)N_f + 2(1/6) = -3, \]  
\[ U(1)_Y : \quad b_Y = (20/9)N_f + 2(1/6) = 7, \]

where $N_f = 3$ is the number of quark and lepton families. As such, the gauge couplings do not unify at a common mass scale, i.e. they do not satisfy the condition

$$\alpha_C(M_U) = \alpha_L(M_U) = (5/3)\alpha_Y(M_U) = \alpha_U.$$

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If the SM becomes the MSSM above $M_S$, the numbers $b_i$ change, i.e.

$$SU(3)_C : \quad b'_C = -11 + (2/3)(3) + (4/3 + 2/3)N_f = -3,$$

$$SU(2)_L : \quad b'_L = -22/3 + (2/3)(2) + (4/3 + 2/3)N_f + (2/3 + 1/3)2(1/2) = 1,$$

$$U(1)_Y : \quad (3/5)b'_Y = (4/3 + 2/3)N_f + (3/5)(2/3 + 1/3)(4)(1/4) = 33/5.$$  

Therefore

$$\ln \frac{M_U}{M_Z} = \frac{\pi}{2} \left( \frac{1}{\alpha_L(M_Z)} - \frac{1}{\alpha_C(M_Z)} \right),$$

$$\ln \frac{M_S}{M_Z} = \frac{\pi}{4} \left( \frac{3}{\alpha_Y(M_Z)} - \frac{12}{\alpha_L(M_Z)} + \frac{7}{\alpha_C(M_Z)} \right).$$

Now [11]

$$\alpha(M_Z)^{-1} = 127.953 \pm 0.049,$$

$$\sin^2 \theta_W(M_Z) = 0.23119 \pm 0.00014,$$  

$$\alpha_L(M_Z) = \alpha(M_Z)/\sin^2 \theta_W(M_Z) = 0.03381,$$  

$$\alpha_Y(M_Z) = \alpha(M_Z)/\cos^2 \theta_W(M_Z) = 0.01017,$$  

$$\alpha_C(M_Z) = 0.1215 \pm 0.0017.$$  

For $M_S > M_Z$, using Eq. (14),

$$\alpha_C < \frac{7\alpha_L\alpha_Y}{3(4\alpha_Y - \alpha_L)} = 0.1168,$$  

in disagreement with Eq. (19). However, this problem is usually fixed by going to two loops and spreading out the SUSY particle thresholds.

Consider now the dark left-right model. At $M_R$, there is the boundary condition

$$\frac{1}{\alpha_Y(M_R)} = \frac{1}{\alpha_R(M_R)} + \frac{1}{\alpha_X(M_R)} = \frac{1}{\alpha_L(M_R)} + \frac{1}{\alpha_X(M_R)}.$$
Above $M_R$, assuming the minimal supersymmetric content, without the exotic $\eta$ and $\chi$ superfields,

\begin{align}
    b_C &= -11 + (2/3)(3) + (2 + 1)N_f = 0, \\
    b_{L,R} &= -22/3 + (2/3)(2) + (4/3 + 2/3)N_f + (2/3 + 1/3)(6)(1/2) = 3, \\
    (3/2)b_X &= (2 + 1)N_f + (1 + 1/2)(8)(1/4) = 12.
\end{align}

As such, again the gauge couplings do not unify, i.e. they do not satisfy

$$\alpha_C(M_U) = \alpha_L(M_U) = \alpha_R(M_U) = (2/3)\alpha_X(M_U) = \alpha_U.$$ (25)

However, this may easily be changed with the addition of new particles \[12\]. With the $\eta$ superfields above $M_R$,

\begin{align}
    b'_C &= -11 + (2/3)(3) + (2 + 1)N_f = 0, \\
    b'_{L,R} &= -22/3 + (2/3)(2) + (4/3 + 2/3)N_f + (2/3 + 1/3)(8)(1/2) = 4, \\
    (3/2)b'_X &= (2 + 1)N_f + (1 + 1/2)(16)(1/4) = 15,
\end{align}

and the $\chi$ superfields above $M_X$,

$$ (3/2)b''_X = (2 + 1)N_f + (1 + 1/2)(16)(1/4) + (1 + 1/2)(2)(1) = 18. $$ (29)

The resulting solutions are

\begin{align}
    \ln \frac{M_U}{M_Z} &= \frac{\pi}{2} \left( \frac{1}{\alpha_L(M_Z)} - \frac{1}{\alpha_C(M_Z)} \right), \\
    \ln \frac{M_R^2}{M_X^2 M_Z^2} &= \pi \left( \frac{3}{\alpha_Y(M_Z)} - \frac{12}{\alpha_L(M_Z)} + \frac{7}{\alpha_C(M_Z)} \right).\end{align} (30)(31)

Note that Eqs. (30) and (31) are identical to Eqs. (13) and (14) of the MSSM respectively if we set $M_X = M_R = M_S$. Thus this model is no worse than the MSSM for gauge-coupling unification, and two-loop equations and particle thresholds may be invoked to fix it.
The same one-loop solutions are obtained in two other previously proposed supersymmetric left-right models [7, 8]. If the one-loop equations (30) and (31) are taken at face value, $M_U = 3.33 \times 10^{16}$ GeV and $M_R^{3/4} M_X^{-3/4} = 14.7$ GeV. Take for example $M_R = 500$ GeV, then $M_X = 55.2$ TeV. The additional singlets are thus much heavier, but since they do not affect either $SU(2)_R \times U(1)_X$ breaking or supersymmetry breaking, this is an acceptable scenario.

In terms of $SO(10)$ multiplets, $Q, Q^c, \psi, \psi^c$ belong to $16$; $h, d^c$ and $\Delta$ to $10$; $\Phi$ and $\eta$ to $16 + 16^*$; and $\chi$ to $120 + 120^*$. The chosen set of superfields is free of anomalies.

**Conclusion**: The dark left-right model, where the $SU(2)_R$ fermion doublet $(n, e)_R$ contains the dark-matter fermion (scotino) $n$ which is distinguished from the usual lepton $e$ by an $U(1)$ global symmetry, is extended to include supersymmetry. New superfields are added, resulting in one-loop renormalization-group equations for the gauge couplings with solutions identical to those of the MSSM. The dark sector is automatically extended to include at least three stable particles.

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