Sparticles at the LHC

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ABSTRACT: Sparticle mass hierarchies will play an important role in the type of signatures that will be visible at the Large Hadron Collider. We analyze these hierarchies for the four lightest sparticles for a general class of supergravity unified models including nonuniversalities in the soft breaking sector. It is shown that out of nearly $10^4$ possibilities of sparticle mass hierarchies, only a small number survives the rigorous constraints of radiative electroweak symmetry breaking, relic density and other experimental constraints. The signature space of these mass patterns at the Large Hadron Collider is investigated using a large set of final states including multi-leptonic states, hadronically decaying $\tau$s, tagged $b$ jets and other hadronic jets. In all, we analyze more than 40 such lepton plus jet and missing energy signatures along with several kinematical signatures such as missing transverse momentum, effective mass, and invariant mass distributions of final state observables. It is shown that a composite analysis can produce significant discrimination among sparticle mass patterns allowing for a possible identification of the source of soft breaking. While the analysis given is for supergravity models, the techniques based on mass pattern analysis are applicable to wide class of models including string and brane models.

KEYWORDS: Sparticles, SUGRA, Lepton and Jet Signatures, LHC.
1. Introduction

Supersymmetry (SUSY) remains a leading candidate to describe new physics beyond that of the Standard Model (SM). Recently, an approach for identifying supersymmetric particles (sparticles) was proposed involving sparticle mass hierarchies, or sparticle mass patterns. Such patterns could yield distinct identifiable signatures at the Fermilab’s Tevatron and at the CERN Large Hadron Collider (LHC)[1, 2]. At the same time, the hierarchical mass patterns
are model dependent and the determinations of such patterns could be helpful in extrapolating the data back to the theoretical model. This new approach has been investigated within the framework of gravity mediated breaking of supersymmetry\cite{3, 4, 5} and specifically within the minimal supergravity grand unified model, the mSUGRA model\cite{3} (for a review see \cite{6}). The analysis of \cite{1, 2} was a rather brief introduction to the technique. Here we carry out a more in depth analysis within models with both universal and nonuniversal soft supersymmetry breaking\cite{8, 9, 10}. Thus, in the minimal supersymmetric extension of the Standard Model (MSSM) there are 32 supersymmetric masses. We list them here to set notation. There are 4 Higgs boson masses, of which three \((h, H, A)\) are neutral, the first two being CP even and the third CP odd, and one charged Higgs \(H^\pm\). In the gaugino-Higgsino sector there are two charged mass eigenstates (charginos) \(\tilde{\chi}^\pm_{i=1,2}\), four charge neutral states (neutralinos) \(\tilde{\chi}^0_{i=1,4}\), and one strongly interacting gluino \(\tilde{g}\). In the sfermion sector, before diagonalization, there are 9 scalar leptons (sleptons) which are superpartners of the leptons with left and right chirality and are denoted as: \(\{\tilde{e}_{L,R}, \tilde{\mu}_{L,R}, \tilde{\tau}_{L,R}, \tilde{\nu}_{eL}, \tilde{\nu}_{\mu L}, \tilde{\nu}_{\tau L}\}\). Finally there are 12 squarks which are the superpartners of the quarks and are represented by: \(\{\tilde{u}_{L,R}, \tilde{c}_{L,R}, \tilde{t}_{L,R}, \tilde{d}_{L,R}, \tilde{s}_{L,R}, \tilde{b}_{L,R}\}\). Mass diagonal slepton and squark states will in general be mixtures of \(L, R\) states.

If the 32 masses are treated as essentially all independent, aside from sum rules on the Higgs, chargino and neutralino masses, then without imposition of any phenomenological constraints, the number of hierarchical patterns for the sparticles could be as many as \(O(10^{28})\) or larger. This represents a mini landscape reminiscent of the string landscape (which, however, is much larger with as many as \(O(10^{1000})\) possibilities). The number of possibilities can be reduced by very significant amounts in supergravity models with the imposition of the constraints of radiative electroweak symmetry breaking (REWSB), and other phenomenological constraints. This was precisely what was accomplished in the analysis of \cite{1, 2}. The analysis of Ref.\cite{1, 2} focused on the mass hierarchies for the first four lightest sparticles, and found the residual number of hierarchies to be 22 in mSUGRA. Here, the possible signatures from some of the patterns were also discussed along with the prospects for direct detection of dark matter within various mass hierarchies.

The phenomenology of supergravity (SUGRA) models has been discussed since their inception and there exists a considerable amount of literature regarding the implications of SUGRA (for early works see \cite{11, 12}, for more recent works see \cite{13, 14}, for works with nonuniversalities see \cite{15}, and for works with hierarchical breaking and with \(U(1)\) gauge extensions see \cite{13, 17, 18}). While many analyses of the mSUGRA parameter space have been limited to the case of vanishing trilinear couplings, several works\cite{19, 20, 21, 22, 14, 23, 24} have appeared relaxing this assumption, and new portions of the parameter space have been found consistent with all known experimental constraints on the model.

In this paper we give a more exhaustive analysis of sparticle mass hierarchies for SUGRA models including nonuniversalities and also carry out a more detailed analysis of the signatures arising from these patterns. We further focus on ways in which patterns can be distinguished from each other using the relevant distinctive features of the signature space. It is found...
that for some model points one encounters the phenomenon where two distinct points in the parameter space of soft breaking may yield the same signatures within a $2\sigma$ error bar. We also discuss in this paper how such signature degeneracies can sometimes be lifted by an increased integrated luminosity. Finally, we discuss the issue of how well the soft parameters $m_0$ and $m_{1/2}$ (where $m_0$, $m_{1/2}$ are the mass parameters in mSUGRA models defined in Sec.(2)) may eventually be determined at the LHC which allows one to obtain an estimate on the resolution of these parameters using optimal LHC luminosities.

The outline of the rest of the paper is as follows: In Sec.(2) we give a discussion of the sparticle landscape. Specifically, the landscape for the 4 lightest sparticles (in addition to the lightest Higgs boson) for the mSUGRA case is discussed in Sec.(2.1) and the landscape for the 4 lightest sparticles for the nonuniversal SUGRA case is discussed in Sec.(2.2). This includes cases with nonuniversalities in the Higgs sector, nonuniversalities in the third generation sector, and nonuniversalities in the gaugino sector. In Sec.(2.3), we also discuss the possible number of mass patterns that can arise for the above case, as well as for the case when all 32 sparticle masses are taken into account. An analysis of the patterns and their origin in the space of soft breaking parameters is given in Sec.(3). An analysis of the benchmarks for the landscape of 4 sparticle patterns is given in Sec.(3.2). Sec.(4) is devoted to the discussion of the sparticle signatures at the LHC. In Sec.(4.1) we give a discussion of the various SUSY tools that are utilized in this analysis. We discuss technical details of the analysis of the LHC signatures we have investigated in Sec.(4.2). We then move on to discuss how one can distinguish sparticle mass patterns arising in mSUGRA in Sec.(4.3), and sparticle mass patterns in SUGRA with nonuniversalities in Sec.(4.4). The trileptonic signal as a tool to distinguish patterns is discussed in Sec.(4.5). We utilize both event counting signatures and kinematical signatures in our analysis, the latter being discussed in Sec.(4.6). In Sec.(4.7), a method for distinguishing patterns utilizing a large set of signatures is also given. We discuss the signature space degeneracy among different models and how to lift it in Sec.(5.1), and then we generalize our analysis to investigate the resolving power of the LHC with regards to its ability to probe the soft parameter space in Sec.(5.2). Conclusions are given in Sec.(6). Some of our longer tables have been relegated to the Appendix.

2. The Sparticle Landscape

The analysis proceeds by specifying the model input parameters at the GUT scale ($M_G \sim 2 \times 10^{16}$ GeV) and using the renormalization group equations (RGEs) to predict the sparticle masses and mixing angles at the electroweak scale. The RGE code used to obtain the mass spectrum is SuSpect 2.34 [25], which is the default RGE calculator in MicrOMEGAs version 2.0.7 [24]. We have also investigated other RGE programs including ISASUGRA/ISAJET [27], SPheno [28] and SOFTSUSY [29]. We have cross checked our analysis using different codes and find significant agreement in most regions of the parameter space. The largest sensitivity appears to arise for the case of large $\tan \beta$ and the analysis is also quite sensitive to the running bottom mass and to the top pole mass (we take $m_b^{\overline{MS}}(m_b) = 4.23$ GeV and
$m_t(\text{pole}) = 170.9 \text{ GeV}$ in this analysis). Such sensitivities and their implications for the analysis of relic density calculations are well known in the literature and a detailed comparison for various codes can be found in Refs. (31, 32, 33, 34).

Below we give the relevant constraints from collider and astrophysical data that are applied throughout the analysis unless stated otherwise.

1. WMAP 3 year data: The constraint on the relic abundance of dark matter under the assumption the relic abundance of neutralinos is the dominant component places the bound: \( 0.0855 < \Omega_{\chi_1^0} h^2 < 0.1189 \) (2\( \sigma \)) [35].

2. As is well known sparticle loop exchanges make a contribution to the FCNC process \( b \rightarrow s\gamma \) which is of the same order as the Standard Model contributions (for an update of SUSY contributions see [36]). The experimental limits on \( b \rightarrow s\gamma \) impose severe constraints on the SUSY parameter space and we use here the constraints from the Heavy Flavor Averaging Group (HFAG) along with the BABAR, Belle and CLEO experimental results: \( \mathcal{B}(b \rightarrow s\gamma) = (355 \pm 24^{+9}_{-10} \pm 3) \times 10^{-6} \). A new estimate of \( \mathcal{B}(\bar{B} \rightarrow X\gamma) \) at \( \mathcal{O}(\alpha_s^2) \) gives [38] \( \mathcal{B}(b \rightarrow s\gamma) = (3.15 \pm 0.23) \times 10^{-4} \) which moves the previous SM mean value of \( 3.6 \times 10^{-4} \) a bit lower. In the analysis we use a 3.5\( \sigma \) error corridor around the HFAG value. The total \( \mathcal{B}(\bar{B} \rightarrow X\gamma) \) including the sum of SM and SUSY contributions are constrained by this corridor.

3. The process \( B_s \rightarrow \mu^+\mu^- \) can become significant for large \( \tan \beta \) since the decay has a \( \tan^6 \beta \) dependence and thus large \( \tan \beta \) could be constrained by the experimental limit \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 1.5 \times 10^{-7} \) (90\% CL), \( 2.0 \times 10^{-7} \) (95\% CL) [39]. This limit has just recently been updated [40] and gives \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 1.2 \times 10^{-7} \) (95\% CL). Preliminary analyses have reported the possibility of even more stringent constraints by a factor of 10. We take a more conservative approach in this analysis and allow model points subject to the bound \( \mathcal{B}(B_s \rightarrow \mu^+\mu^-) < 9 \times 10^{-6} \) (for a review see [42]).

4. Additionally, we also impose a lower limit on the lightest CP even Higgs boson mass. For the Standard Model like Higgs boson this limit is \( \approx 114.4 \) GeV [13], while a limit of 108.2 GeV at 95\% CL is set on the production of an invisibly decaying Standard Model like Higgs by OPAL [44]. For the MSSM we take the constraint to be \( m_h > 100 \) GeV. We take the other sparticle mass constraints to be \( m_{\tilde{\chi}_1^\pm} > 104.5 \) GeV [15] for the lighter chargino, \( m_{\tilde{t}_1} > 101.5 \) GeV for the lighter stop, and \( m_{\tilde{\tau}_1} > 98.8 \) GeV for the lighter stau.

In addition to the above one may also consider the constraints from the anomalous magnetic moment of the muon. It is known that the supersymmetric electro-weak corrections to \( g_\mu - 2 \) can be as large or larger than the Standard Model corrections [46]. The implications of recent experimental data has been discussed in several works (see, e.g. [47]). As in [21], here we use a rather conservative bound \( -11.4 \times 10^{-10} < g_\mu - 2 < 9.4 \times 10^{-9} \).
2.1 The mSUGRA landscape for the 4 lightest sparticles

One mSUGRA model is a point in a 4 dimensional parameter space spanned by $m_0$, $m_{1/2}$, $A_0$, $\tan \beta$, and the sign of $\mu$, where $m_0$ is the universal scalar mass, $m_{1/2}$ is the universal gaugino mass, $A_0$ is the universal trilinear coupling, $\tan \beta$ is the ratio of the two Higgs VEVs in the minimal supersymmetric Standard Model (MSSM), and $\mu$ is the Higgs mixing parameter that enters via the term $\mu H_1 H_2$ in the superpotential. Typically scans of the parameter space are done by taking a vanishing trilinear coupling, and/or by looking at fixed values of $\tan \beta$ while varying ($m_0$, $m_{1/2}$). In this work we carry out a random scan in the 4-D input parameter space for fixed signs of $\mu$ with Monte Carlo simulations using flat priors under the following ranges of the input parameters

$$0 < m_0 < 4 \text{ TeV}, \quad 0 < m_{1/2} < 2 \text{ TeV} \quad |A_0/m_0| < 10, \quad 1 < \tan \beta < 60.$$  \hspace{1cm} (2.1)

Since SUGRA models with $\mu > 0$ are favored by the experimental constraints much of the analysis presented here focuses on this case. Specifically for the $\mu > 0$ mSUGRA case, we perform a scan of the parameter space with a total of $2 \times 10^6$ trial parameter points. We delineate the patterns that emerge for the first four lightest sparticles. Here we find that at least sixteen hierarchical mass patterns emerge which are labeled as mSPs (minimal SUGRA Pattern). These mSPs can be classified according to the particle which is heavier than the LSP, and we find four classes of patterns in mSUGRA: the chargino patterns (CP), the stau patterns (SUP), the stop patterns (SOP), and the Higgs patterns (HP), as exhibited below

1. Chargino patterns (CP) : mSP1, mSP2, mSP3, mSP4
2. Stau patterns (SUP) : mSP5, mSP6, mSP7, mSP8, mSP9, mSP10
3. Stop patterns (SOP) : mSP11, mSP12, mSP13
4. Higgs patterns (HP) : mSP14, mSP15, mSP16.

The hierarchical mass patterns mSP1-mSP16 are defined in Table 1. We note that the pattern mSP7 appears in the analyses of [48, 49, 50]. We also performed a similar scan for the mSUGRA with $\mu < 0$ case using the Monte Carlo simulation with flat priors and the same parameter ranges as specified in Eq.(2.1). Most of the mSP patterns that appear in the $\mu > 0$ case also appear in the $\mu < 0$ case (see Table 1). However, in addition one finds new patterns shown below

1. Stau patterns (SUP) : mSP17, mSP18, mSP19
2. Stop patterns (SOP) : mSP20, mSP21
3. Neutralino patterns (NP) : mSP22.

We note that the analysis of Ref.[51] has a sparticle spectrum which corresponds to mSP11 and contains light stops. Light stops have also been discussed recently in [52, 23].
While the earlier works which advocated benchmark points and slopes made good progress in systematizing the search for supersymmetry, we find that they do not cover the more broad set of possible mass hierarchies we discuss here. That is, many of the mSP patterns do not appear in the earlier works that advocated benchmark points for SUSY searches. For example the Snowmass mSUGRA points (labeled SPS) and the Post-WMAP benchmark points of , make up only a small fraction of the possible mass hierarchies listed in Table 1. The CMS benchmarks classified as Low Mass (LM) and High Mass (HM) (for a recent review see ) does a good job covering the mSP1 pattern which appears as the most dominant pattern in our analysis, but there are no Higgs patterns or stop patterns discussed in the CMS benchmarks as well as in SPS or in Post-WMAP benchmarks. We exhibit the possible mass hierarchies we discuss here. That is, many of the mSP patterns do not appear in the earlier works that advocated benchmark points for SUSY searches. For example the Snowmass mSUGRA points (labeled SPS) and the Post-WMAP benchmark points of , make up only a small fraction of the possible mass hierarchies listed in Table 1. The CMS benchmarks classified as Low Mass (LM) and High Mass (HM) (for a recent review see ) does a good job covering the mSP1 pattern which appears as the most dominant pattern in our analysis, but there are no Higgs patterns or stop patterns discussed in the CMS benchmarks as well as in SPS or in Post-WMAP benchmarks. We exhibit the possible mass hierarchies we discuss here. That is, many of the mSP patterns do not appear in the earlier works that advocated benchmark points for SUSY searches. For example the Snowmass mSUGRA points (labeled SPS) and the Post-WMAP benchmark points of , make up only a small fraction of the possible mass hierarchies listed in Table 1. The CMS benchmarks classified as Low Mass (LM) and High Mass (HM) (for a recent review see ) does a good job covering the mSP1 pattern which appears as the most dominant pattern in our analysis, but there are no Higgs patterns or stop patterns discussed in the CMS benchmarks as well as in SPS or in Post-WMAP benchmarks. We exhibit the possible mass hierarchies we discuss here. That is, many of the mSP patterns do not appear in the earlier works that advocated benchmark points for SUSY searches. For example the Snowmass mSUGRA points (labeled SPS) and the Post-WMAP benchmark points of , make up only a small fraction of the possible mass hierarchies listed in Table 1. The CMS benchmarks classified as Low Mass (LM) and High Mass (HM) (for a recent review see ) does a good job covering the mSP1 pattern which appears as the most dominant pattern in our analysis, but there are no Higgs patterns or stop patterns discussed in the CMS benchmarks as well as in SPS or in Post-WMAP benchmarks. We exhibit the possible mass hierarchies we discuss here. That is, many of the mSP patterns do not appear in the earlier works that advocated benchmark points for SUSY searches. For example the Snowmass mSUGRA points (labeled SPS) and the Post-WMAP benchmark points of , make up only a small fraction of the possible mass hierarchies listed in Table 1. The CMS benchmarks classified as Low Mass (LM) and High Mass (HM) (for a recent review see ) does a good job covering the mSP1 pattern which appears as the most dominant pattern in our analysis, but there are no Higgs patterns or stop patterns discussed in the CMS benchmarks as well as in SPS or in Post-WMAP benchmarks. We exhibit the possible mass hierarchies we discuss here. That is, many of the mSP patterns do not appear in the earlier works that advocated benchmark points for SUSY searches. For example the Snowmass mSUGRA points (labeled SPS) and the Post-WMAP benchmark points of , make up only a small fraction of the possible mass hierarchies listed in Table 1. The CMS benchmarks classified as Low Mass (LM) and High Mass (HM) (for a recent review see ) does a good job covering the mSP1 pattern which appears as the most dominant pattern in our analysis, but there are no Higgs patterns or stop patterns discussed in the CMS benchmarks as well as in SPS or in Post-WMAP benchmarks. We exhibit the possible mass hierarchies we discuss here. That is, many of the mSP patterns do not appear in the earlier works that advocated benchmark points for SUSY searches. For example the Snowmass mSUGRA points (labeled SPS) and the Post-WMAP benchmark points of , make up only a small fraction of the possible mass hierarchies listed in Table 1. The CMS benchmarks classified as Low Mass (LM) and High Mass (HM) (for a recent review see ) does a good job covering the mSP1 pattern which appears as the most dominant pattern in our analysis, but there are no Higgs patterns or stop patterns discussed in the CMS benchmarks as well as in SPS or in Post-WMAP benchmarks. We exhibit the
Figure 1: Distribution of the surviving hierarchical mass patterns in the landscape for the mSUGRA model with $\mu > 0$ (light) and $\mu < 0$ (dark), under various constraints as discussed in the text.

| Snowmass          | mSP  | Post-WMAP3         | mSP  |
|-------------------|------|--------------------|------|
| SPS1a, SPS1b, SPS5| mSP7 | $A', B', C', D', G', I', J', M'$ | mSP5 |
| SPS2              | mSP1 | $I', L'$           | mSP7 |
| SPS3              | mSP5 | $E'$               | mSP1 |
| SPS4, SPS6        | mSP3 | $K'$               | mSP6 |

Table 2: Mapping between the mSPs and the Snowmass, Post-WMAP3, and CMS benchmark points. The points $B' = LM1$, $I' = LM2$, $C' = LM6$. HM1 in SuSpect has $m_{\chi^0} > m_{\tilde{\chi}_1}$, but this is not the case for ISAJET, SPheno, and SOFTSUSY. Only LM1, LM2, LM6, and HM1, HM2 are capable of giving the correct relic density. Thus the mapping above applies only to the mass pattern, while all of our mSP and NUSP benchmark points satisfy the relic density constraints from MicrOMEGAs with SuSpect. The CMS test points do a better job of representing mSP1 which is the dominant pattern found in our analysis. There are no HP test points or SOP test points in any of the previous works.

mapping of mSPs with other benchmarks points in a tabular form in Table (2).

In Fig. (1) we give the relative distribution of these hierarchies found in our Monte Carlo
The most common patterns found are CPs and SUPs, especially mSP1 and mSP5. However there exists a significant region of the parameter space where SOPs and HPs can be realized. The percentages of occurrence of the various patterns in the mSUGRA landscape for both $\mu$ positive and $\mu$ negative are exhibited in Fig. (1). The analysis of Fig. (1) shows that the chargino patterns (CP) are the most dominant patterns, followed by the stau patterns (SUP), the stop patterns (SOP), and the Higgs patterns (HP). In contrast, most emphasis in the literature, specifically in the context of relic density analysis, has focused on the stau patterns, with much less attention on other patterns. Specifically the Higgs patterns have hardly been investigated or discussed. The exceptions to this, in the context of the Higgs patterns, are the more recent works of Refs. [1], [2], and similar mass ranges for the Higgs bosons have been studied in [58] (see also [59]).

2.2 The landscape of the 4 lightest sparticles in NUSUGRA

Next we discuss the landscape of the 4 lightest sparticles for the case of nonuniversal supergravity models. Here we consider nonuniversalities in the Higgs sector (NUH), in the third generation sector (NU3), and in the gaugino sector (NUG). Such nonuniversalities appear quite naturally in supergravity models with a non-minimal Kähler potential, and in string and D-Brane models. The parametrization of the nonuniversalities is given by

\begin{align}
\text{NUH} : M_{H_u} &= m_0(1 + \delta_{H_u}), \quad M_{H_d} = m_0(1 + \delta_{H_d}), \\
\text{NU3} : M_{q_3} &= m_0(1 + \delta_{q_3}), \quad M_{d_3} = m_0(1 + \delta_{d_R}), \\
\text{NUG} : M_1 &= m_1/2, \quad M_{2,3} = m_1/2(1 + \delta_{M_{2,3}}).
\end{align}

In the above $\delta_{H_u}$ and $\delta_{H_d}$ define the nonuniversalities for the up and down Higgs mass parameters, $M_{q_3}$ is the left-handed squark mass for the 3rd generation, and $M_{u_3} (M_{d_3})$ are the right-handed u-squark (d-squark) masses for the 3rd generation. The nonuniversalities in the gaugino sector are parameterized here by $\delta_{M_2}$ and $\delta_{M_3}$. We have carried out a Monte Carlo scan with flat priors using $10^6$ model points in each of the three types of NUSUGRA models, taking the same input parameter ranges as specified in Eq. (2.1) and $-0.9 \leq \delta \leq 1$.

Almost all of the mSP patterns seen for the mSUGRA cases were found in supergravity models with nonuniversal soft breaking, as the mSUGRA model is contained within the nonuniversal supergravity models. In addition we find many new patterns labeled NUSPs (nonuniversal SUGRA pattern), and they are exhibited in Table (3). As in the mSUGRA case one finds several pattern classes, CPs, SUPs, SOPs, and HPs as exhibited below. In addition, we find several Gluino patterns (GP) where the gluino is the NLSP.

1. Chargino patterns (CP) : NUSP1, NUSP2, NUSP3, NUSP4
2. Stau patterns (SUP) : NUSP5, NUSP6, NUSP7, NUSP8, NUSP9
3. Stop patterns (SOP) : NUSP10, NUSP11
4. Higgs patterns (HP) : NUSP12
5. Gluino patterns (GP) : NUSP13, NUSP14, NUSP15.
Table 3: New 4 sparticle mass patterns that arise in NUSUGRA over and above the mSP patterns of Table (1). These are labeled nonuniversal SUGRA patterns (NUSP) and at least 15 new patterns are seen to emerge which are denoted by NUSP1-NUSP15.

| NUSP  | Mass Pattern | NU3 | NUG |
|-------|--------------|-----|-----|
| NUSP1 | $\tilde{\chi}_1^0 < \tilde{\chi}_1^\pm < \tilde{\chi}_2 < t_1$ | Y   | Y   |
| NUSP2 | $\tilde{\chi}_1^0 < \tilde{\chi}_1^0 < A \sim H$ | Y   |     |
| NUSP3 | $\tilde{\chi}_1^0 < \tilde{\chi}_1^\pm < \tilde{\tau}_1 < \tilde{\chi}_2^0$ |     | Y   |
| NUSP4 | $\tilde{\chi}_1^0 < \tilde{\chi}_1^\pm < \tilde{\tau}_1 < \tilde{l}_R$ | Y   |     |
| NUSP5 | $\tilde{\chi}_1^0 < \tilde{\tau}_1 < \tilde{\nu}_\tau < \tilde{\tau}_2$ | Y   |     |
| NUSP6 | $\tilde{\chi}_1^0 < \tilde{l}_1 < \tilde{\nu}_\tau < \tilde{\chi}_1^\pm$ | Y   |     |
| NUSP7 | $\tilde{\chi}_1^0 < \tilde{l}_1 < \tilde{l}_1 < A/H$ |     | Y   |
| NUSP8 | $\tilde{\chi}_1^0 < \tilde{l}_1 < \tilde{l}_R < \tilde{\nu}_\mu$ |     | Y   |
| NUSP9 | $\tilde{\chi}_1^0 < \tilde{l}_1 < \tilde{\chi}_1^\pm < \tilde{l}_R$ |     | Y   |
| NUSP10| $\tilde{\chi}_1^0 < t_1 < g < \tilde{\chi}_1^\pm$ | Y   |     |
| NUSP11| $\tilde{\chi}_1^0 < \tilde{l}_4 < A \sim H$ | Y   |     |
| NUSP12| $\tilde{\chi}_1^0 < A \sim H < \tilde{g}$ | Y   |     |
| NUSP13| $\tilde{\chi}_1^0 < \tilde{g} < \tilde{l}_1 < \tilde{\chi}_2^0$ | Y   |     |
| NUSP14| $\tilde{\chi}_1^0 < \tilde{l}_1 < \tilde{\chi}_1^\pm$ | Y   |     |
| NUSP15| $\tilde{\chi}_1^0 < \tilde{g} < A \sim H$ | Y   |     |

It is interesting to note that for the 4 sparticle landscape we find saturation in the number of mass hierarchies that are present. For example, for the case $\mu > 0$ in mSUGRA, increasing the soft parameter scan from $1 \times 10^6$ parameter model points to $2 \times 10^6$ model points does not increase the number of 4 sparticle patterns. In this context it becomes relevant to examine as to what degree the relic density and other experimental constraints play a role in constraining the parameter space and thus reducing the number of patterns. This is exhibited in Table (1) where we demonstrate how the relic density and the other experimental constraints decrease the number of admissible model points in the allowed parameter space for the mSUGRA models with both $\mu > 0$ and $\mu < 0$, and also for the cases with nonuniversality in the Higgs sector, nonuniversality in the third generation sector, and with nonuniversality in the gaugino sector. In each case we start with $10^6$ model points at the GUT scale, and find that the electroweak symmetry breaking constraints reduce the number of viable models to about 1/4 of what we started with. We find that the allowed number of models translates into SUGRA mass patterns which are typically less than 100. The admissible set of parameter points reduces drastically when the relic density constraints are imposed and are then found to typically reduce the set of viable mass patterns by a factor of about 200 or more, with a reduction in the number of allowed patterns by a factor of 2 or more. Inclusion of all other experimental constraints further reduces the number of admissible points by a factor between 30% and 50%, with a corresponding reduction in the number of patterns by up to 40%. The above analysis shows that there is an enormous reduction in the number of
admissible models and the corresponding number of hierarchical mass patterns after the constraints of radiative breaking of the electroweak symmetry, relic density constraints, and other experimental constraints are imposed.

| Model Type | Trial Models | Output Models | No. of Patterns | Relic Density Constraints | No. of Patterns | All Constraints | No. of Patterns |
|------------|--------------|---------------|----------------|--------------------------|----------------|----------------|----------------|
| mSUGRA($\mu > 0$) | $10^5$ | 265,875 | 55 | 1,360 | 22 | 902 | 16 |
| mSUGRA($\mu < 0$) | $10^5$ | 226,991 | 63 | 1,000 | 31 | 487 | 18 |
| NUH($\mu > 0$) | $10^5$ | 222,023 | 59 | 1,024 | 24 | 724 | 15 |
| NU3($\mu > 0$) | $10^5$ | 229,928 | 73 | 970 | 28 | 650 | 20 |
| NUG($\mu > 0$) | $10^5$ | 273,846 | 103 | 1,788 | 36 | 1,294 | 28 |

Table 4: An analysis of mass patterns for the four lightest sparticles. Exhibited in the table are the model type, the number of trial input points for each model, the number surviving the radiative electroweak symmetry breaking scheme as given by SuSpect (column 3), the number surviving when the relic density constraints are applied with MicrOMEGAs (column 5), the number surviving with inclusion of all experimental collider constraints (column 7), along with the corresponding number of hierarchical mass patterns in each case (column 8).

2.3 Hierarchical patterns for the full sparticle spectrum

We discuss now the number of hierarchical mass patterns for the full set of 32 sparticles in SUGRA models when the constraints of electroweak symmetry, relic density, and other experimental constraints are imposed. The result of the analysis is given in Fig. (2) and Table (3). Here one finds that increasing the number of model points in the scan does increase the number of patterns. However, the ratio of the number of patterns to the total number of models that survive all the constraints from the scan decreases sharply as shown in the right panel of Fig. (3). This means that although saturation is not yet achieved one is moving fast towards achieving saturation with a relatively small number of allowed patterns for all the 32

![Figure 2: Left panel: The number of hierarchical mass patterns for 32 sparticles vs the number of trial points for mSUGRA models which survive the electroweak symmetry breaking constraints, the relic density and all other experimental constraints. The number of hierarchical mass patterns show a trend towards saturation. Right panel: A similar phenomena is seen in the ratio between the number of patterns over the number of surviving trial points in mSUGRA models.](image-url)
| Models [No.] | No. after constraints | No. of patterns |
|--------------|-----------------------|-----------------|
| mSUGRA(µ > 0) $10^6$ | 902               | 505             |
| mSUGRA(µ < 0) $10^6$ | 487               | 268             |
| NUH(µ > 0) $10^6$    | 724               | 517             |
| NU3(µ > 0) $10^6$    | 650               | 528             |
| NUG(µ > 0) $10^6$    | 1294              | 1092            |
| All Above $5 \times 10^6$ | 4057         | 2557            |

Table 5: The table exhibits a dramatic reduction of the landscape from upward of $\sim O(10^{28})$ hierarchical mass patterns for the 32 sparticle masses to a much smaller number when the electroweak symmetry breaking constraints, the relic density constraints, and other experimental constraints are applied. Column 1 shows one million input parameter points for each of the models investigated, and the number surviving all the constraints are exhibited in column 2, while column 3 gives the number of hierarchical patterns.

sparticles within SUGRA models consistent with the various experimental constraints. The analysis of Table 5 shows that the number of allowed patterns for the 32 sparticles, which in the MSSM without the SUGRA framework can be as large as $O(10^{28})$ or larger, reduces rather drastically when various constraints are applied in supergravity models. We note that some patterns are repeated as we move across different model types listed in the first column of Table 5. Thus the total number of patterns listed at the bottom of the last column of this table is smaller than the sum of patterns listed above in that column. We note that the precise number and nature of the patterns are dependent on the input parameters such as the top mass and a significant shift in the input values could modify the pattern structure.

3. Sparticle Patterns and the Nature of Soft Breaking

3.1 Correlating mass hierarchies with the soft parameter space

It is interesting to ask if the patterns can be traced back to some specific regions of the parameter of soft breaking from where they originate. This indeed is the case, at least, for some of the patterns. The analysis illustrating the origin of the patterns in the parameter space is given in Fig.(3). Exhibited are the landscape of sparticle mass spectra in the planes (I) $\tan \beta$ vs $A_0/m_0$ and (II) $m_0$ vs $m_{1/2}$, when the soft parameters are allowed to vary in the ranges given in Eq.(2.1). Many interesting observations can be made from these spectral decompositions. For example, a significant set of the mSP1 (CP) models lie in the region $|A_0/m_0| < 2$ and correspond to the Hyperbolic branch (HB)/Focus Point (FP) regions, while most of the SOPs have a rather large ratio of $A_0/m_0$ with the satisfaction of REWSB. For the mSUGRA $\mu > 0$ case, the region around $\tan \beta = 50$ has a large number of models that can be realized, while the region around $\tan \beta = 30$ has far less model points. We also note that most of the HPs reside only in the very high $\tan \beta$ region in mSUGRA, but this situation can be changed significantly in the NUH case where HP points can be realized in the $\tan \beta$ region as low as $\tan \beta \sim 20$. In the $m_0$ vs $m_{1/2}$ plane, one finds that most of CPs and HPs have a larger universal scalar mass than most of the SUPs and SOPs.
Figure 3: Exhibition of the dispersion of mSPs arising in mSUGRA in the tan $\beta$ vs $A_0/m_0$ plane (left panels), and in the $m_0$ vs $m_{1/2}$ plane (right panels) for the $\mu > 0$ case (upper panels) and $\mu < 0$ case (lower panels). The analysis is based on a scan of $10^6$ trial model points with flat priors in the ranges $m_0 < 4 \text{ TeV}$, $m_{1/2} < 2 \text{ TeV}$, $1 < \tan \beta < 60$, and $|A_0/m_0| < 10$. mSP1 is seen to lie in the region where $|A_0/m_0| < 2$. For the case $\mu < 0$, no HPs are seen and no model points survive in the region where $\tan \beta > 50$ in contrast to the $\mu > 0$ case where there is a significant number for $\tan \beta \gtrsim 45$.

Often in the literature one limits the analysis by fixing specific values of $A_0$ and $\tan \beta$. For $A_0$ the value most investigated is $A_0 = 0$. However, constraining the values of $A_0$ or of $\tan \beta$ artificially eliminates a very significant part of the allowed parameter space where all the relevant constraints (the REWSB constraint as well as the relic density and the experimental constraints) can be satisfied as seen in Fig.(3). One can extract the familiar plots one finds in the literature where $A_0$ and $\tan \beta$ are constrained from a reduction of the top-right panel of Fig.(3). The results of this reduction are shown in Fig.(4) with a focused scan in specific regions of the soft parameter space. Specifically the bottom-left and top-right panels of Fig.(4) show the familiar stau coannihilation\cite{61, 62, 48} regions and the HB/FP branch, the bottom-right panel gives the stau coannihilation region and the stop coannihilation region because of the relatively large $A_0$ value, and the top-left panel is of the form seen in the works of
Figure 4: Dispersion of patterns in the $m_0$ vs $m_{1/2}$ plane for fixed values of $\tan \beta$ and $A_0/m_0$. The region scanned is in the range $m_0 < 4$ TeV and $m_{1/2} < 2$ TeV with a 10 GeV increment for each mass. Only a subset of the allowed parameter points relative to Fig. (3) remain, since the scans are on constrained surfaces in the mSUGRA parameter space.

Djouadi et al. [21] where the Higgs funnel plays an important role in the satisfaction of the relic density.

A similar analysis for the nonuniversal case is given in Fig. (5). Here in addition to the mSPs new patterns emerge which we label as nonuniversal sugra patterns or NUSPs. Among the NUSPs the dominant patterns are NUSP1 (CP) and NUSP13 (GP), which are seen to arise the model with nonuniversalities in the gaugino sector, i.e., the NUG model. In general, the NUG is dominated by the CP patterns whereas the NUH case is rather diverse offering the possibility of Higgs patterns at lower, less fine tuned values of $\tan \beta$.

3.2 Benchmarks for sparticle patterns

As discussed in Sec. (2.1), many of the sparticle mass patterns discussed in this analysis do not appear in the Snowmass, Post-WMAP, and CMS benchmark points. With some of these mSP and NUSP having a significant probability of occurrence, we therefore provide a
Figure 5: An exhibition of the NUSPs and mSPs for the NUH, NU3, and NUG models in the tan $\beta$ vs $A_0/m_0$ plane. The range of SUGRA parameters are the same as the case mSUGRA ($\mu > 0$). One may notice that the mSP1 points arising from NU models lie in a relatively larger $A_0/m_0$ region. Most of the models in NU cases are still mSPs, and among the NUSPs, only two patterns have a relatively large population, these being NUSP1 and NUSP13. One may also notice that in NUH case, the HPs can exist in a low tan $\beta$ region as opposed to the mSUGRA case where HPs can either exist in the large tan $\beta$ region ($\mu > 0$) or are totally eliminated ($\mu < 0$).

A larger set of benchmark points for the various patterns in different SUGRA scenarios. These benchmark points are exhibited in Tables (9,10,11,12,13) of the Appendix. Each of these benchmarks satisfies the relic density and other experimental constraints with SuSpect linked to MicrOMEGAs. We have explicitly checked that the first mSP benchmark point in each of the tables can be reproduced by using SPheno, and SOFTSUSY by allowing minor variations in the input parameter space. The benchmarks are chosen to cover wide parts of the SUGRA parameter space. We give these benchmarks, several for each mass pattern, as the search for SUSY from the point of view of mass patterns has important consequences for LHC experimental searches. Some of the patterns are correlated with certain well investigated phenomena such as the HB/FP branches of REWSB and the stau-neutralino co-annihilation regions. However, many of the patterns arise from multiple annihilation processes.
4. LHC Signatures for Mass Patterns

4.1 Event generation and detector simulation

Before moving to the discussion of the LHC signatures arising from various mSPs and NUSPs, we first give a detailed description of our LHC simulation procedure.

After the imposition of all the constraints mentioned in the previous sections, such as the relic density constraints from WMAP data, the constraints on the FCNCs, as well as mass limits on the sparticle spectrum, we are left with the candidate model points for the signature analysis. For each of these model points, a SUSY Les Houches Accord (SLHA) file is interfaced to PYTHIA 6.4.11 through PGS4 for the computation of SUSY production cross sections and branching fractions. In this analysis, for signals, we have generated all of PYTHIA’s $2 \rightarrow 2$ SUSY production modes using MSEL = 39. Leading order cross sections from PYTHIA and leading order cross sections from PROSPINO 2.0 were cross checked against one another for consistency over several regions of the soft parameter space. TAUOLA is called by PGS4 for the calculation of tau branching fractions as controlled in the PYTHIA parameter card (.pyt) file. With PGS4 we use the Level 1 (L1) triggers based on the Compact Muon Solenoid detector (CMS) specifications and the LHC detector card. Muon isolation is controlled by employing the cleaning script in PGS4.

We take the experimental nomenclature of lepton being defined only as electron or muon and thus distinguish electrons and muons from tau leptons. SM backgrounds have been generated with QCD multi-jet production due to light quark flavors, heavy flavor jets ($b\bar{b}$, $t\bar{t}$), Drell-Yan, single $Z/W$ production in association with quarks and gluons ($Z+$ jets / $W+$ jets), and $ZZ$, $WZ$, $WW$ pair production resulting in multi-leptonic backgrounds. Extraction of final state particles from the PGS4 event record is accomplished with a code SMART (= SUSY Matrix Routine) written by us which provides an optimized processing of PGS4 event data files. The criteria for the discovery limits of new signals is that the SUSY signals should exceed either $5\sqrt{N_{SM}}$ or 10, i.e., $N_{SUSY} > \text{Max} \{5\sqrt{N_{SM}}, 10\}$. We have also cross checked various results of our analysis with three CMS notes and we have found agreement with these works using SMART and PGS4 for signal and backgrounds.

We note that several works where sparticle signatures are discussed have appeared recently. However, the issue of hierarchical mass patterns and the correlation of signatures with such patterns has not been discussed which is what the analysis of this work investigates.

4.2 Post trigger level cuts and LHC signatures

Generally speaking, there are two kinds of LHC signatures: (i) event counting signatures, and (ii) kinematical signatures. We have investigated both of these for the purpose of discriminating the sparticle mass patterns. We list our event counting signatures in Table (6), where

\footnote{More specifically this choice generates 91 SUSY production modes including gaugino, squark, slepton, and SUSY Higgs pair production but leaves out singly produced Higgs production. For further details, see \[65\]. A treatment of singly produced Higgs production in the context of sparticle mass hierarchies was included in the analysis of Ref. \[6\].}
we have carried out analyses of a large set of lepton + jet signals. In our counting procedure, only electron and muon are counted as leptons, while taus are counted independently. For jet identification, we divide jets into two categories, b-tagged jets and jets without b-tagging, which we simply label as b-jets and non-b-jets. There are some counting signatures that only concern one class of measurable events, for example, the number of one b-jet events. There are also types of signatures of final state particles with combinations of two or three different species. For instance, one such example would be the number of events in which there is a single lepton and a single tau.

When performing the analysis of event counting, for each SUGRA model point, we impose global post trigger cuts to analyze most of our PGS4 data. Below we give our default post trigger cuts which are used throughout the paper unless stated otherwise.

1. In an event, we only select photons, electrons, muons, and taus that have transverse momentum \( P_T > 10 \text{ GeV} \) and \( |\eta| < 2.4 \).

2. For hadronic jets, only those satisfying \( P_T > 60 \text{ GeV} \) and \( |\eta| < 3 \) are selected.

3. We require a large amount of missing transverse momentum, \( P_T^{\text{miss}} > 200 \text{ GeV} \).

4. There are at least two jets that satisfy the \( P_T \) and \( \eta \) cuts.

Our default post trigger level cuts are standard and are designed to suppress the Standard Model background, and highlight the SUSY events over a broad class of models.

The different kinematical signatures we investigated for the purpose of discriminating among sparticle mass patterns are exhibited in Table (7). One may further divide the kinematical signatures into two classes: namely those involving transverse momentum \( P_T \) and those which involve invariant mass. For those involving \( P_T \), we have investigated missing \( P_T \) distributions and the effective mass, the latter being the sum of missing \( P_T \) and \( P_T \) of all jets contained within an event. For the kinematical variables using invariant mass, we reconstruct such quantities for four different cases, i.e., the invariant mass for all jets, for \( e^+e^- \) pair, for \( \mu^+\mu^- \) pair, and for \( \tau^+\tau^- \) pair.

4.3 Discrimination among mSPs in mSUGRA

We turn now to a discussion of how one may distinguish among different patterns. The analysis begins by considering the 902 model points that survive our mSUGRA scan with \( 10^6 \) trial points, and simulating their LHC signals with PGS4 using, for illustration, 10 fb\(^{-1}\) of integrated luminosity at the LHC. In our analysis we will focus mostly on the counting signatures. Here the most useful counting signature is the total number of SUSY events after trigger level cuts and post trigger level cuts are imposed. All other counting signatures are normalized with respect to the total number of SUSY events passing the cuts and thus appear as fractions lying between (0,1) in our figures. Further, to keep the analysis statistically significant, we admit only those points in the parameter space that generate at least 500 total SUSY events.
| Signature | Description | Signature | Description |
|-----------|-------------|-----------|-------------|
| 0L        | 0 Lepton    | 0T        | 0 $\tau$   |
| 1L        | 1 Lepton    | 1T        | 1 $\tau$   |
| 2L        | 2 Leptons   | 2T        | 2 $\tau$   |
| 3L        | 3 Leptons   | 3T        | 3 $\tau$   |
| 4L        | 4 Leptons and more | 4T | 4 $\tau$ and more |
| 0L1b      | 0 Lepton + 1 b-jet | 0T1b | 0 $\tau$ + 1 b-jet |
| 1L1b      | 1 Lepton + 1 b-jet | 1T1b | 1 $\tau$ + 1 b-jet |
| 2L1b      | 2 Leptons + 1 b-jet | 2T1b | 2 $\tau$ + 1 b-jet |
| 0L2b      | 0 Lepton + 2 b-jets | 0T2b | 0 $\tau$ + 2 b-jets |
| 1L2b      | 1 Lepton + 2 b-jets | 1T2b | 1 $\tau$ + 2 b-jets |
| 2L2b      | 2 Leptons + 2 b-jets | 2T2b | 2 $\tau$ + 2 b-jets |
| ep        | $e^+$ in 1L | em        | $e^-$ in 1L |
| mp        | $\mu^+$ in 1L | mm | $\mu^-$ in 1L |
| tp        | $\tau^+$ in 1T | tm | $\tau^-$ in 1T |
| OS        | Opposite Sign Di-Leptons | 0b | 0 b-jet |
| SS        | Same Sign Di-Leptons | 1b | 1 b-jet |
| OSSF      | Opposite Sign Same Flavor Di-Leptons | 2b | 2 b-jets |
| SSSF      | Same Sign Same Flavor Di-Leptons | 3b | 3 b-jets |
| OST       | Opposite Sign Di-$\tau$ | 4b | 4 b-jets and more |
| SST       | Same Sign Di-$\tau$ | TL | 1 $\tau$ plus 1 Lepton |

**Table 6:** The table gives a list of 40 signatures which are computed along with the signatures of Table(7) in this analysis for each point in the SUGRA model parameter space. $L = e, \mu$ signifies only electrons and muons.

| Kinematical signatures |
|------------------------|
| 1. $P_T^{miss}$        |
| 2. Effective Mass = $P_T^{miss} + \sum_j P_j^T$ |
| 3. Invariant Mass of all jets |
| 4. Invariant Mass of $e^+e^-$ pair |
| 5. Invariant Mass of $\mu^+\mu^-$ pair |
| 6. Invariant Mass of $\tau^+\tau^-$ pair |

**Table 7:** The table describes kinematical signatures which are used in addition to the signatures of Table(6) for each point in the SUGRA model parameter space.

We give now the details of the analysis. In Fig.(6), we investigate the signature space spanned by a variety of signature channels. The top left panel gives a plot with one signature consisting of events with one lepton and the second signature consisting of event with no leptons. It is seen that the stop patterns (SOPs) that survive the cuts are confined in a small
Figure 6: Top Left: An exhibition of the mSPs in the 1L vs 0L where the fraction of events to the total number of events in each case is plotted. The analysis shows that the Stop Patterns (SOP) appearing on the right-bottom corner are easily distinguished from other patterns. The analysis shows that SOP has few lepton signals. Top Right and Bottom Left: Plots in the signature space with fraction of events with 1b vs 0b and 2b vs 1b exhibiting the separation of CPs and HPs from SOPs and SUPs, with CPs and HPs occupying one region, and SOPs and SUPs occupy another in this signature space except for a very small overlap. Bottom Right: An exhibition of the mSPs in the signature space with the average missing $P_T$ for each parameter point in the mSUGRA parameter space along the y-axis and the fraction of events with 0b along the x-axis. The plot shows a separation of the CPs and HPs from SOPs and SUPs. Further, mSP4 appears isolated in this plot. Most of the CPs and HPs have less than 60% events without b-jet content. The Standard Model events are also plotted on the figures, and many regions of the signature space are clearly distinguishable from the SM.
Figure 7: An exhibition of how the mSPs can be discriminated within a given class, i.e., within CPs, SUPs, SOPs, and HPs. The analysis shows that patterns within a given class can be discriminated.

are well separated from the SOPs and the SUPs for much the same reason as in upper-right panel. Finally, a plot is given in the lower-right panel where one signature is the average missing $P_T$ while the other signature involves events with no tagged b-jets. Again in this plot
the CPs (which include mSP4) and HPs are well separated from the SOPs and SUPs.

The analysis of Fig. (6) exhibits the rather remarkable phenomenon in that the Standard Model background shown by the black dot in each of the plots is confined to a very specific region in each case in these plots. For some cases, e.g., for the patterns CP and HP in the upper right hand corner of Fig. (6), the separation between the SUGRA prediction and the Standard Model background is strikingly clear, allowing an easy identification not only of new physics but also of the nature of the pattern that leads to such a signature.

We discuss now the possibility of discriminating sub-patterns within a given pattern class. An analysis illustrating this possibility is given in Fig. (7). Here the top two panels illustrate how the sub-patterns mSP1, mSP2, mSP4 within the chargino class (CP) are distinguishable with appropriate choice of the signatures. A similar analysis regarding the discrimination for the sub-patterns in the stau class (SUP) is given in the two middle panels. The lower-left panel gives an analysis of how one may discriminate the stop sub-patterns mSP11, mSP12, mSP13 in the stop class (SOP), and finally the lower-right panel shows the plots that allows one to discriminate the Higgs patterns mSP14 and mSP16 from each other. There are a variety of other plots which allow one to discriminate among patterns. With 40 counting signatures one can have \(40 \choose 2\) such plots and it is not possible to display all of them. A global analysis where the signatures are simultaneously considered for a large collection of mSPs and NUSPs is discussed in Sec.(4.7).

4.4 Sparticle signatures including nonuniversalities

In this subsection, we give an analysis including nonuniversalities in three different sectors: NUH, NU3, and NUG. In our analysis we simulate various models with the same constant number of events N which we take, as an example to be N = 10^4. To discriminate among the patterns in the signature space, we introduce another set of post trigger cuts, which we denote as ‘b jet cuts’, in addition to the default post trigger cuts specified in Sec.(4.2). The criteria in the b-jet cuts are the same as the default post trigger cuts, except that we change the condition ‘at least two hadronic jets in the event’ to ‘at least one b-tagged jet in the event’. We exhibit our analysis utilizing both the default cuts and the b jet cuts in Fig.(8). One can see that even with inclusion of a variety of soft breaking scenarios, some mSPs still have very distinct signatures in some specific channels.

Thus in the top-left panel we give a plot of mSP7 (SUP) and mSP11 (SOP) in the signature space 1L/N (b jet cuts) vs 0L1b/N, where 0L1b/N is obtained with the default post trigger cuts. Here we find that these two model types are clearly distinguishable as highlighted by shaded and unshaded regions. A similar analysis with signatures consisting of 1L1b/N (b jet cuts) vs 0L1b/N for mSP4 (CP) and mSP7 (SUP) is given in the top-right panel. The lower-left panel gives an analysis of mSP4 (CP) and mSP5 (SUP) also in the signature space consisting of 1L1b/N (b jet cuts) vs 0L1b/N. Finally, in the lower-right panel we give an analysis of mSP3 (CP) and mSP11 (SOP) in the signature plane e^+ /N vs 1b/N. These analyses illustrate that the patterns and often even the sub-patterns can be discriminated with the appropriate choice of signatures for a general class of SUGRA models including nonuniversalities.
4.5 The trileptonic signal as a pattern discriminant

The trileptonic signal is an important signal for the discovery of supersymmetry. For on-shell decays the trileptonic signal was discussed in the early days in [11, 79] and for off-shell decays in [80]. (For a recent application see [72]). Here we discuss the trileptonic signal in the context of discrimination of hierarchical patterns. In Fig. (9) we exhibit the dependency of the trilepton signal on the chargino mass. It is seen that mSP5 gives the largest number of events in this channel while the CP pattern (mSP1) and the HP pattern (mSP14) can also produce a large number of trilepton events above the discovery limit, while the chargino mass reach is extended for the mSP5 as opposed to the mSP1 and mSP14. The above observations hold for some of the other SUP patterns as well. A similar analysis shows that the light chargino mass can be discovered in this channel up to about 500 GeV in the SUP pattern. Another interesting display of the trileptonic signal is when this signal is plotted against some relevant mass splittings. Thus the left-panel of Fig. (10) gives an analysis for the trileptonic signal for two patterns: the Chargino pattern mSP1 and the Stau pattern mSP5 plotted against the...
Figure 9: A plot of the number of trilepton events versus the light chargino mass for three patterns, one from each class, CP, SUP and HP. The SUP pattern gives the largest trileptonic signal followed by the HP and CP patterns.

Figure 10: The number of tri-lepton events versus the sparticle mass splittings. The left panel shows clear separations for hierarchical mass patterns in the number of trilepton events produced with 10 fb$^{-1}$ as a function of the NLSP and the LSP mass splitting for the chargino (CP) pattern mSP1 and Stau (SUP) mSP5. The plot on the right shows a similar effect for the case where the mass splitting is taken to be the difference of the CP odd Higgs boson mass and the LSP for both the Higgs pattern mSP14 and the stau pattern mSP5. The Standard Model background is highly suppressed in this channel.

NLSP-LSP mass splitting with 10 fb$^{-1}$ of data. The analysis of the left-panel of Fig. (10) shows that the SUP pattern presents the best opportunity for discovering SUSY through the 3 lepton mode. The analysis also shows a clear separation among mass patterns and further a majority of the model points stand above the discovery limit which in this channel is $\approx 15$ events under the post trigger level cuts discussed in Sec.(4.2). The right-panel of Fig. (10)
gives an analysis of the trileptonic signal vs the mass splitting of the CP odd Higgs and the lightest neutralino LSP for patterns mSP5 and mSP14. Again, we see a clear separation of model points. We note that CP odd Higgs can sometimes be even lighter than the LSP, and thus the quantity $\Delta M = M_A - M_{\tilde{\chi}_1^0}$ plotted on the x-axis can sometimes become negative.

4.6 Kinematical distributions

In addition to the event counting signatures discussed above the kinematical signatures are an important tool for pattern discrimination. We illustrate this using the kinematical variables consisting of missing $P_T$ and the effective mass (see Table (7) for their definitions) and an illustration is given in Fig.(11). Specifically the analysis of Fig.(11) uses four mSUGRA points one each in the patterns CP, SUP, SOP and HP. The analysis of Fig.(11) shows that the distributions for the CP, HP, SOP and SUP are substantially different. It is interesting to note that in the missing $P_T$ distribution, the HP and SUP model points have a relatively flat distribution compared to the CP and SOP model points. The missing $P_T$ distribution and the effective mass distribution are useful when designing post trigger level cuts to optimize the signal over the background. For instance, one can take a 1 TeV effective mass cut to analyze the SUP and HP signals shown in Fig.(11), but this method will fail when it comes to the CP and SOP points since most of their events have a rather small effective mass.

We also investigate the invariant mass distribution for the opposite sign same flavor (OSSF) di-leptons ($e^+e^-, \mu^+\mu^-$) in Fig.(12). We applied the default post trigger cuts as in
Figure 12: A plot of the opposite sign same flavor (OSSF) di-lepton invariant mass distribution at LHC with 10 fb$^{-1}$ with the default post trigger cuts imposed for two different mSP points on top of the SM $t\bar{t}$ background. The two mSPs are clearly distinguishable from each other in the distribution.

4.7 A ‘global’ analysis, fuzzy signature vectors, and pattern discrimination

In the above we have given specific examples of how patterns can be differentiated from each other. In the previous sections we used only a few of the 40 signatures exhibited in Table 6. However, in the analysis we have carried out we have examined all of them. Thus for each parameter point we have analyzed 40 signatures. We now define correlations among these signatures. Thus consider an ordered set where the signatures are labeled $S_1, S_2, \ldots, S_{40}$ and let the number of events in each signature be $N_1, N_2, \ldots, N_{40}$. Define a signature vector for a given point $x_\alpha$ ($\alpha = 1, 2, \ldots, p$) in the parameter space

$$\xi^a = (\xi^a_1, \xi^a_2, \ldots, \xi^a_{40})$$

(4.1)

where $\xi_i = N_i^a/N$ and $N$ is the total number of SUSY events. As the parameter point $x_\alpha$ varies over the allowed range within a given pattern it generates a signature vector where the elements trace out a given range. Thus for a pattern X one generates a fuzzy pattern vector $\Delta\xi^X$ so that

$$\Delta\xi^X = (\Delta\xi^X_1, \Delta\xi^X_2, \ldots, \Delta\xi^X_{40})$$

(4.2)

where $\Delta\xi^X_i$ is the range traced out by the element $\xi_i^X$ as the parameter point $x_\alpha$ moves in the allowed parameter space of the pattern X. What makes the vector $\Delta\xi^X$ fuzzy is that its elements are not single numbers but a set which cover a range. We define now the inner
product of two such fuzzy pattern vectors so that

\[ C_{XY} \equiv (\Delta\xi^X|\Delta\xi^Y) = 0(1) \]  

(4.3)

where the inner product is 0 if the element \( \Delta\xi^X_i \) and \( \Delta\xi^Y_i \) overlap for all \( i (i = 1, \ldots, 40) \), and 1 if at least one of the elements of pattern \( X \), \( \Delta\xi^X_i \) does not overlap with \( \Delta\xi^Y_i \), the element for pattern \( Y \). Therefore, if for two patterns \( X \) and \( Y \) one finds there is no overlap at least for one signature component \( \Delta\xi_j \), then these two patterns can be distinguished in this specific signature and one obtains \( C_{XY} = 1 \). Otherwise \( C_{XY} = 0 \) which means that all components of \( \Delta\xi^X \) and \( \Delta\xi^Y \) have an overlap and cannot be distinguished under this criteria. We can generalize the above procedure for the signatures

\[ \zeta_{i,j} = \frac{N_i}{N_j}, \quad (i, j = 1, \ldots, 40). \]  

(4.4)

Repeating the previous analysis, one can construct another fuzzy signature vector for pattern \( X \) as

\[ \Delta\zeta^X = (\Delta\zeta^X_1, \ldots, \Delta\zeta^X_i, \ldots, \Delta\zeta^X_{39}, \Delta\zeta^X_{40}) \]  

(4.5)

where the elements have a range corresponding to the range spanned by the soft parameters \( x_\alpha \) as they move over the parameter space specific to the pattern. Further, the definition of the inner product Eq. (4.3) still holds for this new fuzzy signature vector. We have carried out a full analysis of such comparisons, using 40 different signatures, and their combinations.

**Figure 13:** A table exhibiting the discrimination of patterns using the criterion of Eq.(4.3) where various signatures with both the default post trigger cuts and b jet cuts are utilized. If the element of \( i \)th row and \( j \)th column is 1, i.e., \( C_{ij} = 1 \), one can distinguish the \( i \)th mass pattern from the \( j \)th one.
as defined in Eq. (4.4) and Eq. (4.5). An analysis based on this criterion is given in Fig. (13). The analysis of Fig. (13) shows that most of the patterns tested can indeed be distinguished from each other. The analysis exhibited in Fig. (8) are the special cases of the results in Fig. (13). For instance, the clear separation between mSP7 and mSP11 in the signature space shown in the top-left panel of Fig. (8) gives the elements $C_{45} = C_{54} = 1$ of Fig. (13). Finally we note that the analysis in Fig. (13) is done without statistical uncertainties. Inclusion of uncertainties in pattern analysis would certainly be worthwhile in a future work.

5. Signature Degeneracies and Resolution of Soft Parameters

5.1 Lifting signature degeneracies

It may happen that two distinct points in the soft parameter space may lead to the same set of signatures for a given integrated luminosity within some predefined notion of indistinguishability. Thus consider two parameter points $A$ and $B$ and define the ‘pulls’ in each of their signatures by

$$P_i = \frac{|n^A_i - n^B_i|}{\sigma_{AB}},$$

$$\sigma_{AB} = \sqrt{(\delta n^A_i)^2 + (\delta n^B_i)^2 + (\delta n_i^{SM})^2}. \quad (5.1)$$

Here $\delta n^A_i \sim \sqrt{n^A_i}$ is the uncertainty in the signature events $n^A_i$, and we estimate the SM uncertainty as $\delta n_i^{SM} \sim \sqrt{y(\delta n^A_i + \delta n^B_i)}/2$. Here the parameter $y$ parameterizes the effect of the SM events, and for the analysis in this section, we take $y = 1$. We will adopt $5\sigma$ criteria for differentiating model points. In other words, if the pulls in each of the signatures is less than 5, then the two SUGRA parameter space points are essentially indistinguishable in the signature space. In such a situation one could still distinguish model points either by including more signatures, or by an increase in luminosity. Thus, for example, inclusion of the Higgs production cross sections, $B_s \rightarrow \mu^+\mu^-$ constraints, as well as the inclusion of neutralino proton annihilation cross sections constraints tend to discriminate among the model parameter points as shown in Ref. [2]. Here we point out that in some cases increasing the luminosity can allow one to lift the degeneracies enhancing a subset of signatures in one case relative to the other. For illustration we consider the following two sets of points in the pattern mSP5 in the mSUGRA parameter space in the following order $(m_0, m_{1/2}, A_0, \tan\beta, \text{sign}\mu)$.

Point A (192.6, 771.3, 1791.1, 8.8, +1),
Point B (163.0, 761.3, −775.8, 4.7, +1); (5.2)

Point A’ (159.3, 732.3, −783.1, 5.6, +1),
Point B’ (163.5, 753.3, −918.2, 3.3, +1). (5.3)

In Table (8) we compare the pulls for the pairs of points $(A, B)$ and $(A’, B’)$ at an integrated luminosity of 10 fb$^{-1}$ and 500 fb$^{-1}$. For points $A$ and $B$, one finds that the pulls are all less
Table 8: An exhibition of lifting the degeneracy of two points in the mSUGRA parameter space using luminosity. Two pairs of points (A, B) and (A', B') are indistinguishable under the 2 sigma criteria at 10 fb⁻¹ luminosity (column 3-8), but can be clearly separated when the luminosity increases to 500 fb⁻¹ (column 9-14). The Standard Model uncertainty is estimated as \( \delta n_i^{SM} = (\delta n_i^A + \delta n_i^B)/2 \).
500 fb$^{-1}$, and the signatures $(0, 1, 2, 6, 36)$ give pulls greater than 5, with the largest pulls being in excess of 12, allowing one to discriminate between the parameter points $A'$ and $B'$.

Thus we conclude that some model pairs, which appear degenerate in the signature space at one luminosity can become distinct from each other at a larger luminosity.

### 5.2 Resolving soft parameters using LHC data

Figure 14: An analysis showing the resolutions in $m_0$ and $m_{1/2}$ that can be reached with 1000 fb$^{-1}$ of integrated luminosity under the REWSB constraints. The two left panels give the number of SUSY events vs $m_0$ (top left panel) and vs $m_{1/2}$ (lower left panel) for 1000 fb$^{-1}$ of integrated luminosity. The right panels give the resolutions in $m_0$ (top right panel) and in $m_{1/2}$ (lower right panel) using the left panels.

We discuss now the issue of how well we can resolve the points in the parameter space $x_{\alpha}$ ($\alpha = 1, .., p$) for a given luminosity. Consider Eq.(5.1) and set $\delta N = \sqrt{N}$, and parameterize the standard model uncertainty by $\delta N^{SM} = \sqrt{y\delta N}$. Next we set the criterion for the resolution of two adjacent points in the SUGRA parameter space so that

$$\frac{\delta N}{\sqrt{2N + yN}} = 5.$$  (5.4)
Since $N = \sigma_{\text{susy}}(x_\alpha)\mathcal{L}_{\text{LHC}}$, where $\sigma_{\text{susy}}$ is the cross section for the production of sparticles, and $\mathcal{L}_{\text{LHC}}$ is the LHC integrated luminosity, the resolution achievable in the vicinity of SUGRA parameter point $x_\alpha$ at that luminosity is given by

$$\delta x_\alpha = \frac{5}{2}(2 + y)\mathcal{L}_{\text{LHC}}^{-\frac{1}{2}}\left(\frac{\delta \sigma_{\text{susy}}(x)}{\delta x_\alpha}\right)^{-1}. \quad (5.5)$$

In Fig.(14) we give an illustration of the above when $m_0$ varies between 500 GeV and 2000 GeV while $m_{1/2} = 500$ GeV, $A_0 = 0$, $\tan \beta = 30$, and $\mu > 0$. From Fig.(14) one finds that the resolution in $m_0$ strongly depends on the point in the parameter space and on the luminosity. Quite interestingly a resolution as small as a few GeV can be achieved for $m_0$ in the range 500-1000 GeV with 1000 fb$^{-1}$ of integrated luminosity. A similar analysis varying $m_{1/2}$ in the range 500~900 GeV for the case when $m_0 = 500$ GeV, $A_0 = 0$, $\tan \beta = 30$ and $\mu > 0$, shows that a resolution in $m_{1/2}$ as low as 1 GeV can be achieved with 1000 fb$^{-1}$ of integrated luminosity.

6. Conclusion

The minimal supersymmetric Standard Model has 32 sparticle masses. Since the soft breaking sector MSSM is arbitrary, one is led to a landscape of as many as $10^{28}$ or more possibilities for the sparticle mass hierarchies. The number of possibilities is drastically reduced in well motivated models such as supergravity models, and one expects similar reductions to occur also in gauge and anomaly mediated models, and in string and brane models. In this work we have analyzed the mass hierarchies for the first four lightest sparticle (aside from the lightest Higgs boson) for supergravity models. Specifically, in Sec.(2) we analyzed the mass hierarchies for the mSUGRA model and for supergravity models with nonuniversalities in the soft breaking in the Higgs sector, nonuniversalities in the soft breaking in the third generation sector, and nonuniversalities in the soft breaking in the gaugino sector. It is found that in each case only a small number of mass hierarchies or patterns survive the rigorous constraints of radiative breaking of the electroweak symmetry, relic density constraints on cold dark matter from the WMAP data, and other experimental constraints from colliders. These mass hierarchies can be conveniently put into different classes labeled by the sparticle which is next heavier after the LSP. For the SUGRA models we find six different classes: chargino patterns, stau patterns, stop patterns, Higgs patterns, neutralino patterns, and gluino patterns. Benchmarks for each of these patterns were given in Sec.(3). In Sec.(4) we discussed the techniques for the analysis of the signatures and the technical details on simulations of sparticle events. In this section we also discuss the backgrounds to the SUSY phenomena arising from the Standard Model processes. Additionally we discussed here the identification of patterns based on 40 event identification criteria listed in Fig.(13). It is found that these criteria allow one to discriminate among most of the patterns. An analysis of how one may lift degeneracies in the signature space, and how accurately one can determine the soft parameters using the LHC luminosities is given in Sec.(4). It is hoped that the analysis
of the type discussed here would help not only in the search for supersymmetry but also allow one to use the signatures to extrapolate back to the underlying supersymmetric model using the experimental data when such data from the LHC comes in. In the above our analysis was focused on supergravity unified models. However, the techniques discussed here have a much wider applicability including models based on gauge and anomaly mediated breaking, as well as string and brane based models.

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8. Appendix

### Chargino Patterns (CPs)

| SUGRA Pattern | $m_0$ (GeV) | $m_{1/2}$ (GeV) | $A_0$ (GeV) | $\tan \beta$ ($v_u/v_d$) | $\mu$ (sign) | NUH ($\delta_{H_u}, \delta_{H_d}$) | NU3 ($\delta_{\tilde{q}_3}, \delta_{\tilde{b}_R}$) | NUG ($\delta_{M_2}, \delta_{M_3}$) |
|---------------|------------|----------------|-------------|-------------------------|-------------|--------------------------------|--------------------------------|--------------------------------|
| mSP1          | 2001       | 411            | 0           | 30.0                    | +           | (0,0)                         | (0,0)                           | (0,0)                           |
| mSP1          | 2366       | 338            | -159        | 9.8                     | -           | (0,0)                         | (0,0)                           | (0,0)                           |
| mSP1          | 1872       | 327            | -1893       | 14.9                    | +           | (0.107,0.643)                  | (0,0)                           | (0,0)                           |
| mSP1          | 1041       | 703            | 1022        | 11.6                    | +           | (0,0)                         | (-0.524,-0.198)                 | (0,0)                           |
| mSP1          | 1361       | 109            | 1058        | 14.4                    | +           | (0,0)                         | (0,0)                           | (0.920,0.850)                   |
| mSP2          | 1125       | 614            | 2000        | 50.0                    | +           | (0,0)                         | (0,0)                           | (0,0)                           |
| mSP2          | 2365       | 1395           | 3663        | 42.2                    | -           | (0,0)                         | (0,0)                           | (0,0)                           |
| mSP2          | 1365       | 595            | 3012        | 35.1                    | +           | (0.116,-0.336)                 | (0,0)                           | (0,0)                           |
| mSP2          | 1166       | 507            | -904        | 59.6                    | +           | (0,0)                         | (0.325,0.458)                   | (0,0)                           |
| mSP2          | 1414       | 221            | -551        | 54.3                    | +           | (0,0)                         | (0,0)                           | (0.156,0.968)                   |
| mSP3          | 741        | 551            | 0           | 50.0                    | +           | (0,0)                         | (0,0)                           | (0,0)                           |
| mSP3          | 1585       | 1470           | 3133        | 39.1                    | -           | (0,0)                         | (0,0)                           | (0,0)                           |
| mSP3          | 694        | 674            | -1564       | 27.0                    | +           | (0.922,-0.293)                 | (0,0)                           | (0,0)                           |
| mSP3          | 570        | 559            | 1042        | 41.3                    | +           | (0,0)                         | (-0.482,-0.202)                 | (0,0)                           |
| mSP3          | 392        | 312            | 320         | 41.3                    | +           | (0,0)                         | (0,0)                           | (-0.404,0.908)                 |
| mSP4          | 1674       | 137            | 1985        | 18.6                    | +           | (0,0)                         | (0,0)                           | (0,0)                           |
| mSP4          | 1824       | 127            | -1828       | 6.4                     | -           | (0,0)                         | (0,0)                           | (0,0)                           |
| mSP4          | 1021       | 132            | -638        | 6.6                     | +           | (0,0)                         | (-0.020,0.963)                  | (0,0)                           |
| mSP4          | 2181       | 127            | -3859       | 3.9                     | +           | (0,0)                         | (0,0)                           | (0.836,-0.248)                 |
| NUSP1         | 2738       | 1689           | -4243       | 42.4                    | +           | (0,0)                         | (-0.828,-0.899)                 | (0,0)                           |
| NUSP1         | 540        | 1190           | 2516        | 13.9                    | +           | (0,0)                         | (0,0)                           | (-0.408,-0.660)                |
| NUSP2         | 845        | 726            | 48.4        | +                       | (0,0)       | (-0.694,-0.400)               | (0,0)                           | (0,0)                           |
| NUSP3         | 396        | 1018           | -179        | 18.3                    | +           | (0,0)                         | (0.250,-0.452)                  | (0,0)                           |
| NUSP4         | 400        | 1558           | 2511        | 5.9                     | +           | (0,0)                         | (0,0)                           | (-0.607,-0.209)                |

Table 9: Benchmarks for the class CP where the chargino $\tilde{\chi}^{\pm}$ is the NLSP in mSUGRA and in NUSUGRA models. Benchmarks are computed with $m_0^{\overline{MS}}(m_0) = 4.23$ GeV, $\alpha_s^{\overline{MS}}(M_Z) = .1172$, and $m_t(pole) = 170.9$ GeV with SuSpect 2.34 interfaced to micrOMEGAS 2.07.

### Gluino Patterns (GP)

| SUGRA Pattern | $m_0$ (GeV) | $m_{1/2}$ (GeV) | $A_0$ (GeV) | $\tan \beta$ ($v_u/v_d$) | $\mu$ (sign) | NUH ($\delta_{H_u}, \delta_{H_d}$) | NU3 ($\delta_{\tilde{q}_3}, \delta_{\tilde{b}_R}$) | NUG ($\delta_{M_2}, \delta_{M_3}$) |
|---------------|------------|----------------|-------------|-------------------------|-------------|--------------------------------|--------------------------------|--------------------------------|
| NUSP13        | 2006       | 1081           | -2027       | 21.1                    | +           | (0,0)                         | (0,0)                           | (0.207,-0.844)                 |
| NUSP14        | 3969       | 1449           | -6066       | 29.3                    | +           | (0,0)                         | (0,0)                           | (0.611,-0.834)                 |
| NUSP15        | 1387       | 695            | 2781        | 50.5                    | +           | (0,0)                         | (0,0)                           | (0.136,-0.827)                 |

Table 10: Benchmarks for the class GP where the gluino $\tilde{g}$ is the NLSP. Such a pattern was only seen to appear in NUSUGRA models with non universal gaugino masses.
### Table 11: Benchmarks for the class SUP where the stau $\tilde{\tau}_1$ is the NLSP in mSUGRA and in NUSUGRA.
### Stop Patterns (SOPs)

| SUGRA Pattern | $m_0$ (GeV) | $m_{1/2}$ (GeV) | $A_0$ (GeV) | $\tan \beta$ | $\mu$ (sign) | NUH $(\delta_{H^+,\delta_{H^0}})$ | NU3 $(\delta_{33,\delta_{3R}})$ | NUG $(\delta_{M_2,\delta_{M_3}})$ |
|---------------|-------------|----------------|-------------|--------------|-------------|-------------------------------|-------------------------------|-------------------------------|
| mSP11         | 871         | 1031           | -4355       | 10.0         | +           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP11         | 1053        | 909            | 7574        | 5.9          | -           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP11         | 1391        | 1089           | 8192        | 14.9         | +           | (0.470,0.632)                 | (0.0)                         | (0.0)                         |
| mSP11         | 2204        | 963            | -1144       | 35.6         | +           | (0.0)                         | (0.642,-0.400)                | (0.0)                         |
| mSP11         | 1406        | 1471           | -2078       | 8.3          | +           | (0.0)                         | (0.0)                         | (0.130,-0.690)                |
| mSP12         | 1371        | 1671           | -6855       | 10.0         | +           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP12         | 1054        | 1372           | -5754       | 13.7         | -           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP12         | 915         | 927            | -3993       | 20.7         | +           | (0.078,0.833)                 | (0.0)                         | (0.0)                         |
| mSP12         | 826         | 1016           | -3926       | 12.8         | +           | (0.0)                         | (-0.630,-0.490)               | (0.0)                         |
| mSP12         | 1706        | 1287           | -4436       | 29.7         | +           | (0.0)                         | (0.0)                         | (0.416,-0.260)                |
| mSP13         | 524         | 800            | -3315       | 15.0         | +           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP13         | 765         | 1192           | -4924       | 12.0         | -           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP13         | 1055        | 1601           | -6365       | 13.6         | +           | (0.277,-0.820)                | (0.0)                         | (0.0)                         |
| mSP13         | 1073        | 1664           | -6528       | 11.6         | +           | (0.0)                         | (0.728,0.060)                 | (0.0)                         |
| mSP13         | 540         | 774            | -2432       | 5.3          | +           | (0.0)                         | (0.0)                         | (0.705,-0.201)                |
| mSP14         | 1040        | 560            | 450         | 53.5         | +           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP14         | 760         | 515            | 2250        | 31.0         | +           | (0.255,-0.500)                | (0.0)                         | (0.0)                         |
| mSP14         | 740         | 620            | 840         | 33.1         | +           | (0.0)                         | (-0.350,-0.249)               | (0.0)                         |
| mSP14         | 1205        | 331            | -710        | 55.9         | +           | (0.0)                         | (0.0)                         | (-0.490,0.450)                |
| mSP15         | 1110        | 760            | 1097        | 51.6         | +           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP15         | 1395        | 554            | -175        | 59.2         | +           | (0.0)                         | (-0.040,0.918)                | (0.0)                         |
| mSP15         | 905         | 500            | 1460        | 54.8         | +           | (0.0)                         | (0.0)                         | (-0.350,-0.260)               |
| mSP16         | 520         | 455            | 620         | 55.5         | +           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP16         | 282         | 464            | 67          | 43.2         | +           | (0.912,-0.529)                | (0.0)                         | (0.0)                         |
| NUSP10        | 718         | 467            | 1657        | 19.0         | +           | (0.0)                         | (0.0)                         | (0.0.23,-0.810)               |

Table 12: Benchmarks for the class SOP where the stop $\tilde{t}_1$ is the NLSP in mSUGRA and in NUSUGRA models.

### Higgs Patterns (HPs)

| SUGRA Pattern | $m_0$ (GeV) | $m_{1/2}$ (GeV) | $A_0$ (GeV) | $\tan \beta$ | $\mu$ (sign) | NUH $(\delta_{H^+,\delta_{H^0}})$ | NU3 $(\delta_{33,\delta_{3R}})$ | NUG $(\delta_{M_2,\delta_{M_3}})$ |
|---------------|-------------|----------------|-------------|--------------|-------------|-------------------------------|-------------------------------|-------------------------------|
| mSP14         | 1040        | 560            | 450         | 53.5         | +           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP14         | 760         | 515            | 2250        | 31.0         | +           | (0.255,-0.500)                | (0.0)                         | (0.0)                         |
| mSP14         | 740         | 620            | 840         | 33.1         | +           | (0.0)                         | (-0.350,-0.249)               | (0.0)                         |
| mSP14         | 1205        | 331            | -710        | 55.9         | +           | (0.0)                         | (0.0)                         | (-0.490,0.450)                |
| mSP15         | 1110        | 760            | 1097        | 51.6         | +           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP15         | 1395        | 554            | -175        | 59.2         | +           | (0.0)                         | (-0.040,0.918)                | (0.0)                         |
| mSP15         | 905         | 500            | 1460        | 54.8         | +           | (0.0)                         | (0.0)                         | (-0.350,-0.260)               |
| mSP16         | 520         | 455            | 620         | 55.5         | +           | (0.0)                         | (0.0)                         | (0.0)                         |
| mSP16         | 282         | 464            | 67          | 43.2         | +           | (0.912,-0.529)                | (0.0)                         | (0.0)                         |
| NUSP12        | 2440        | 475            | -2490       | 48.0         | +           | (0.0)                         | (0.0)                         | (-0.225,-0.845)               |

Table 13: Benchmarks for the class HP where the Higgs boson $(A, H)$ is the next nearest heavy particle after the LSP in mSUGRA and in NUSUGRA. The LSP and $(A, H)$ sometimes are seen to switch.
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