Evidence for a Cosmological Phase Transition From the Dark Energy Scale

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Abstract

A finite vacuum energy density implies the existence of a UV scale for gravitational modes. This gives a phenomenological scale to the dynamical equations governing the cosmological expansion that must satisfy constraints consistent with quantum measurability and spatial flatness. Examination of these constraints for the observed dark energy density establishes a time interval from the transition to the present, suggesting major modifications from the thermal equations of state far from Planck density scales. The assumption that a phase transition initiates the radiation dominated epoch is shown under several scenarios to produce fluctuations to the CMB of the order observed. Quantum measurability constraints (e.g., uncertainty relations) define cosmological scales bounded by luminal expansion rates. It is shown that the dark energy can consistently be interpreted as being due to the vacuum energy of collective gravitational modes which manifest as the zero-point motions of coherent Planck scale mass units prior to the UV scale onset of gravitational quantum de-coherence for the cosmology.

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1 Introduction

Recent evidence for the acceleration of the universe suggests that the dominant component of today’s cosmology is in the form of dark energy. Many feel that understanding the nature of this dark energy ranks among the most compelling of all outstanding problems in physical science. This presentation will suggest conceptual frameworks for investigating this intriguing problem.

For this treatment, it is assumed that the behaviors of the known particles do not qualitatively change up to energies in the TeV range. It is further assumed that the current understanding of general relativity as a gravitational theory with thermal energy content is adequate over the same range, and consequently that the cosmological Friedman-Lemaitre (FL) dynamical equations are reliable guides once the observational regime has been reached where the homogeneity and isotropy assumptions on which those equations are based become consistent with astronomical data to requisite accuracy. The elementary particle behaviors are assumed to apply locally on coordinate backgrounds with cosmological curvature. There is direct experimental evidence that quantum mechanics does apply in the background space of the Schwarzschild metric of the Earth from experiments by Overhauser and collaborators[1, 2]. These experiments show that the coherent self-interference pattern of single neutrons changes as expected when the plane of the two interfering paths is rotated from being parallel to being perpendicular to the “gravitational field” of the Earth. These experiments provide a verification of the principle of equivalence for quantum systems (at least in stationary geometries).

It is expected that during some period in the past, quantum coherence of gravitating systems should have qualitatively altered the dynamics of the cosmology. Often, the onset of the importance of quantum effects in gravitation is taken to be at the Planck scale. However, as is the case with Fermi degenerate stars, this need not be true of the cosmology as a whole. For the same reason that neutron star densities are not necessary for quantum coherence effects to manifest in superfluid helium, Planck scale densities need not be necessary for macroscopic coherence effects to manifest for Planck scale masses. Quantum coherence refers to the entangled nature of quantum states for space-like separations. This is made evident by superluminal correlations (without the exchange of signals) in
the observable behavior of such quantum states. Note that the exhibition of quantum coherent behavior for gravitating systems does not require the quantization of the gravitation field.

The equations which govern the (spatially flat) cosmological expansion satisfy spatial scale invariance, but not temporal scale invariance, due to the behavior of the intensive energy densities which drive the dynamics. However, a specific scale for gravitational modes can be defined by the onset of a microscopic critical density defining a phase transition. This density couples the macroscopic cosmological dynamics to the microphysical phenomena associated with particle dynamics. Collective gravitational modes which satisfy quantum measurability constraints of higher energies than that defined by the phase transition microscopically thermalize once this threshold is reached, while super-horizon modes satisfy those measurability constraints at later times. The consequences of the existence of this coherent gravitational UV scale is what will be explored in this presentation.

As mentioned, the Friedmann-Lemaitre (FL) equations for an ideal fluid governing cosmological evolution is spatially scale invariant unless there is spatial curvature, with the time scale determine by intensive energy densities. The oldest galactic clusters are of the order 12Gyrs, so that any estimations of the age of the expansion must be greater than this. Using current observations, the period of present expansion has been estimated to have a duration of about 13.7 ± 0.2Gyrs. The cosmological critical density (defined in terms of the present Hubble expansion rate as the maximum energy density which would not close the universe, resulting in eventual collapse) is of the order \( \rho_{\text{crit}} \approx 0.9 \times 10^{-29} \text{ g} \cdot \text{c}^2/\text{cm}^2 \approx 0.5 \times 10^{-5} \text{ GeV/cm}^3 \). The relative densities of the various constituent components are of the order \( \Omega_{\gamma} \approx 4.9 \times 10^{-5} \), \( \Omega_{\text{baryons}} \approx 0.04 \), \( \Omega_{\text{dark matter}} \approx 0.22 \), and \( \Omega_{\text{dark energy}} \approx 0.73 \). The radiation component corresponds to about 413 photons/cm\(^3\) with a cosmological microscopic entropy density dominated by these photons \( s_{\gamma} \approx 2905k_B/\text{cm}^3 \). The universe is evidently very hot with regards to the baryons, with an entropy per baryon of the order \( 10^{10}k_B \) for the universe (compared with \( 10^{-2}k_B \) for the sun and of the order \( k_B \) for neutron stars). This large entropy per baryon is related to the baryon-antibaryon asymmetry which ultimately results in the small component of dust which planets and stars are made of.

The observed cosmic microwave background radiation is extremely uni-
form. When variations due to motions of our galaxy and known sources are subtracted, the primordial variations are seen to be of the order $10^{-5}$ (Figure 1). This early remnant from the big bang is responsible for a few percent of the thermal noise appearing as “snow” in broadcast television. The surface of last scattering imaged by the Cosmic Microwave Background (CMB) radiation is a snapshot of the universe during the period of the formation of hydrogen atoms from the prior hot plasma of the constituent particles. Because a hot hydrogen plasma is essentially opaque to electromagnetic radiation, this region of last scattering acts as a fog to our (electromagnetic) view of the universe at earlier times. It was formed when the universe was about 300K years old, at a black body temperature $k_B T_{LS}$ corresponding to about 0.3 eV (about 3000°K), at a time when cosmological scales were about $z_{LS} \sim 1100$ times smaller than today. The temperature has been suppressed by this expansion factor, resulting in the relatively cold, dark night sky observed today, which glows as a hot, hydrogen plasma cooled to the present 2.74°K microwave background. The fluctuations from uniformity in the CMB map of the sky are of the order $\frac{\delta \rho_{CMB}}{\rho_{LS}} \sim 10^{-5}$. For comparison, typical fluctuation scales observed today are given by $\frac{\delta \rho_{stars}}{\rho_0} \sim 10^{30}$ for stars, $\frac{\delta \rho_{galaxies}}{\rho_0} \sim 10^5$ for galaxies, $\frac{\delta \rho_{clusters}}{\rho_0} \sim 10^2$ for clusters, and $\frac{\delta \rho_{superclusters}}{\rho_0} \sim 1$ for superclusters. The spectrum of the CMB radiation and power spectrum of the fluctuations are represented in Fig. 2.

The small fluctuations are believed to have propagated as primordial acoustic waves prior to the de-coupling of those photons from ionized hydrogen. In the early universe, after many Compton scatterings, charges and photons would reach statistical equilibrium, with the photons effectively having a non-vanishing chemical potential since Comp-
Figure 2: Figure on left demonstrates extreme agreement of observed CMB intensity spectrum with blackbody curve. Figure on right demonstrates observed fluctuation power spectrum vs angular mode.

Photon scattering conserves photon number. The wave equation results from combining photon number conservation $n'_\gamma + \vec{\nabla} \cdot (n_\gamma \vec{v}_\gamma) = 0$ to Euler's equation $(\rho_\gamma + P_\gamma)\vec{v}_\gamma = -c^2 \vec{\nabla} P_\gamma$, where the temporal derivatives indicated by primes are with respect to conformal time (which gives the proper distance a photon will have traveled when multiplied by the dimensional cosmological scale factor). For a photon gas, the number density is proportional to the cube of the temperature, while the energy density is proportional to the fourth power of the temperature. The equation of state of a photon gas satisfies $P_\gamma = \frac{1}{3} \rho_\gamma$. Since the dominant component of the observed photon velocity will be in the direction of the propagation vector $k$ of the modes, small dimensionless temperature fluctuations $\Theta$ will satisfy $3\Theta_k'' + k^2 c^2 \Theta_k = 0$, which indicates that the (conformal) speed of sound in the acoustic wave was given by $c/\sqrt{3}$.

The (luminal) horizon problem examines the large scale homogeneity and isotropy of the observed universe. Examining the distance photons can have traveled during the evolution of the universe prior to decoupling from ionized matter, which turns out to be about $1/100$ the distance they have traveled to present time, the subsequent expansion should image light in the cosmic microwave background from $100^3 = 10^6$ luminally disconnected regions. Yet, uniformity of temperature and angular correlations of the fluctuations across the whole sky have been accurately measured by several
experiments. Even more intriguing, the observation of these correlations provides evidence for a space-like coherent phase associated with the cosmological fluctuations that produced the CMB. It is commonly assumed that a period of inflation in the early universe is necessary to explain these acausal phenomena. However, a classical inflation cannot explain space-like coherence of a macroscopic phase. Indeed, only quantum phenomena exhibit properties of space-like phase coherence of otherwise stochastic processes.

Using the usual vacuum state in Minkowski space-time, the equal time correlation function $\langle \text{vac}|\Psi(x, y, z, t)\Psi(x', y', z', t)|\text{vac}\rangle$ of a quantum field $\Psi(x)$ does not vanish for space-like separations. For example, for massless scalar fields, $\langle \text{vac}|\Psi(x)\Psi(y)\Psi(x')\Psi(y')|\text{vac}\rangle = \frac{1}{4\pi^2s^2}$, with space-like phase coherence, where the proper distance satisfies $s^2 = |x - y|^2 - (x^0 - y^0)^2$, which falls off with the inverse square of the distance between the points. Since the vacuum expectation value of the field $\Psi$ vanishes in the usual case, this clearly requires space-like correlations, ie

$$\langle \text{vac}|\Psi(x)\Psi(y)\Psi(x')\Psi(y')|\text{vac}\rangle \neq 2\langle \text{vac}|\Psi(x)|\text{vac}\rangle\langle \text{vac}|\Psi(y)|\text{vac}\rangle.$$ (1.1)

However, since the commutator of the field does vanish for space-like separations, a measurement at $y$ cannot change the probability distribution at $x$. In the approach here taken, global gravitational coherence solves (or defers) the horizon problem because the correlations of a macroscopic quantum system are space-like; it is hypothesized that the same will be true of any type of quantum dark energy or phases of gravitational significance. Quantum systems additionally satisfy measurability constraints resulting in the usual uncertainty relations. It will be argued that the equilibration of microscopic interactions can only occur on cosmological scales consistent with quantum measurement constraints.

Einstein’s equation is assumed to accurately describe the evolution of the thermal universe. The equation relates a geometrically conserved combination of curvature tensors to the dynamically conserved energy-momentum tensor:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G_N}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu}. \quad (1.2)$$

The cosmological constant term, originally considered a blunder included to justify a stationary state universe, has recently been resurrected in an
effort to explain an apparent acceleration of the expansion rate of the universe. For the present discussion, it will be placed on the right hand side of Einstein’s equation, implying a closer connection to the microphysics of the energy-momentum dynamics rather than an inherent aspect of the geometry.

Type 1a supernovae are standard candles with a 12% dispersion in brightness and a known temporal profile. These supernovae form as the mass from a binary companion of a white dwarf accretes onto the white dwarf until it reaches the Chandrasekhar limit associated with gravitational saturation of the Fermi degeneracy pressure of the electrons, when it explodes in a standard way. High redshift type 1a supernovae are about 25% fainter than would be expected in a decelerating universe. If this faintness were due to macroscopic “gray” dust, the finite grain size would not look gray in the infrared (similarly the sunset looks red). No sign of such an effect has been observed. Also, dust effects would be expected to modify more distant galaxies even more so than the nearer galaxies, yet these galaxies do appear to decelerate. Thus, the luminosities of distant type 1a supernovae show that the rate of expansion of the universe has been accelerating for about 6 giga-years. Figure demonstrates that nearer galaxies seem to be accelerating their recession, whereas further galaxies are decelerating. One implies from this that the scale was decelerating in the more distant past, and is presently accelerating. This conclusion is independently supported by analysis of the Cosmic Microwave Background (CMB) radiation. The CMB indicates a spatially flat universe with the total density relative to the cosmological critical density given by \( \Omega_{\text{total}} \approx 1 \). When examining the features of the anisotropies in the CMB, features in an open (closed) universe later appear to be closer (further) than they actually were at the time observed. However, the observed location of the first acoustic peak appropriate to the expected sound velocity constrains the space to be very nearly flat. The height of the first acoustic peak is also sensitive to the matter density. Careful measurements including totally unrelated processes (such as the deuterium/hydrogen ratio) give relativistic radiation, baryonic, and total pressureless mass (including dark matter) relative densities adding up to about 0.27 < 1. This means that the missing relative energy density must be of the order 0.73. Thus both the standard candle luminosity and CMB structure results are independently in quantitative agreement with a (positive) cosmological constant.
Figure 3: Plot of observed scale expansion rate vs scale. The theoretical plots indicate expected behaviors for cosmologies with $\Omega_m=0, 0.16, 0.32, 0.48, 0.64, 0.80, 1$.00
fit to the data. The existence of a finite cosmological constant / dark energy density defines a length scale that should be consistent with those scales generated by the microscopic physics, and must be incorporated in any description of the evolution of the universe.

The density of states for quantized modes can be shown to be independent of the shape of the boundary region: \[ \rho_\Lambda = \frac{g_e}{b_\Lambda \frac{1}{2\pi^2}} \int_{k_{IR}}^{k_{UV}} \hbar k v_p k^2 dk \approx \frac{g_e}{b_\Lambda \frac{1}{8\pi^2}} \frac{\hbar v_p}{k^4} k_{UV}^4 \] (1.3)

where \( v_p \) is the phase velocity of the modes, \( b_\Lambda \) is a factor relating how much of the vacuum energy makes up the dark energy (\( b_\Lambda = 2 \) if all vacuum energy is dark energy) and \( v_p \) is the phase velocity of the modes.

Quantum measurement and flatness arguments will later be made for quantized energy units \( \mathcal{E} \) requiring that the scales associated with those energies, \( R_\mathcal{E} \), must satisfy \( \dot{R}_\mathcal{E} \leq c \), which relates \( k_{UV} \) to the cosmological expansion. The gravitationally coherent cosmological dark energy modes begin to decouple from the thermalized energy density in the Friedmann-Lemaître(FL) equations when the FRW UV scale expansion rate is no longer supra-luminal. A key assumption in this presentation is that this decoupling corresponds to a phase transition from some form of a macroscopic coherent state to local states understood in terms of late time observations. If there is a UV scale associated with the gravitational modes, the dark energy can be associated with the vacuum energy of those modes.

We will briefly discuss the nature of vacuum energy. Vacuum energy is not due to the background fluctuations of the basis of a particular perturbative expansion (like vacuum polarization or mass renormalization). The physical manifestations must be independent of any basis of expansion of the physical states. The question then is, dark energy corresponds to the vacuum energy of what? The known particle spectrum would have far too large vacuum energy to correspond with observations. Rather than focusing on vacuum energy, we will examine the zero-point motions of quantum correlated gravitating sources. The basic assumption is that the quantum zero-point energies whose effects on the subsequent cosmology are fixed by the phase transition are to be identified with the cosmological constant, or “dark energy”.

Since we identify “dark energy” as a particular “vacuum energy” driven by zero-point motions, it might be illuminating to examine the physics
behind other systems that manifest vacuum energy. One such physical system is the Casimir effect\[10\]. Casimir considered the change in the vacuum energy due to the placement of two parallel conducting plates separated by a distance $a$. He calculated an energy per unit area of the form

$$\frac{1}{2} \left( \sum_{\text{modes}} \hbar c k_{\text{interior}} - \sum_{\text{modes}} \hbar c k_{\text{exterior}} \right) = -\frac{\pi^2 \hbar c}{720 a^3}$$

resulting in an attractive force given by

$$\frac{F}{A} = -\frac{\pi^2 \hbar c}{240 a^4} \approx -0.013 \text{ dynes} \left( \frac{\text{a/micron}}{\text{cm}^2} \right)$$

independent of the charges of the sources. Although the effect does not depend on the electromagnetic coupling strength of the sources, it does depend on the nature of the interaction and configuration. Lifshitz and his collaborators\[11\] demonstrated that the Casimir force can be thought of as the superposition of the van der Waals attractions between individual molecules that make up the attracting media. This allows the Casimir effect to be interpreted in terms of the zero-point motions of the sources as an alternative to vacuum energy of the associated quanta. At zero temperature, the coherent zero-point motions of source currents on opposing plates correlate in a manner resulting in a net attraction, whereas if the motions were independently random, there would be no net attraction. On dimensional grounds, one can determine that the number of particles per unit area undergoing zero-point motions that contribute to the Casimir result vary as $a^{-2}$. Boyer\[12\] and others subsequently derived a repulsive force for a spherical geometry of the form

$$\frac{1}{2} \left( \sum_{\text{modes}} \hbar c k_{\text{interior}} - \sum_{\text{modes}} \hbar c k_{\text{exterior}} \right) = 0.92353 \hbar c \frac{a}{a}.$$ 

This shows that the change in electromagnetic vacuum energy is dependent upon the geometry of the boundary conditions, although it does not depend on the detailed couplings of the involved interactions. Both predictions have been confirmed experimentally. It is important to note that this energy grows inversely with the geometric scale $E_{\text{Casimir}} \sim \frac{\hbar c}{a}$.

Others have likewise noted that correlated zero-point motions can be used to describe vacuum energy effects in the Casimir effect (see Fig.\[4\]). As expressed by Daniel Kleppner\[13\],
The van der Waals interaction is generally described in terms of a correlation between the instantaneous dipoles of two atoms or molecules. However, it is evident that one can just as easily portray it as the result of a change in vacuum energy due to an alteration in the mode structure of the system. The two descriptions, though they appear to have nothing in common, are both correct.

Figure 4: The first diagram explains the Casimir pressure in terms of a fewer number of modes (denumerably infinite) between the plates as compared to the number of vacuum modes in the external region. The second diagram demonstrates an attraction due to space-like correlated Van der Waals induced polarizations resulting in net attraction between the sources. Uncorrelated polarizations demonstrated in the third diagram result in no net attraction.

Additionally, as pointed out by Wheeler and Feynmann[14], and others[15], one cannot unambiguously separate the properties of fields from the interaction of those fields with their sources and sinks. Since there are no manifest boundaries in the description of early cosmology presented here, it is more convenient to examine the effects of any gravitational “vacuum energy” in terms of the correlated zero-point motions of the sources of those gravitational fields. In what follows, the zero-point motions of coherent sources will be considered to correspond to the vacuum energies of the associated quanta.
Another system which manifests physically measurable effects due to zero-point energy is liquid $^4$He. One sees that this is the case by noting that atomic radii are related to atomic volume $V_a$ (which can be measured) by $R_a \sim V_a^{1/3}$. The uncertainty relation gives momenta of the order $\Delta p \sim h/V_a^{1/3}$. Since the system is non-relativistic, one estimates the zero-point kinetic energy to be of the order $E_o \sim (\Delta p)^2 \sim \frac{h^2}{2m\mu V_a^{2/3}}$. The minimum in the potential energy is located around $R_a$, and because of the low mass of $^4$He, the value of the small attractive potential is comparable to the zero-point kinetic energy. Therefore, this bosonic system forms a low density liquid at densities much less than those associated with the nuclear masses involved (The lattice spacing for solid helium is expected to be even smaller than the average spacing for the liquid, which means that a large external pressure is necessary to overcome the zero-point energy in order to form solid helium).

Applying this reasoning to relativistic gravitating mass units with quantum coherence within the volume generated by a Compton wavelength $\lambda_m^3$, the zero-point momentum is expected to be of order $p \sim \frac{h}{\sqrt{2\pi} \lambda_m} \sim \frac{h}{\lambda_m}$. This gives a zero point energy of order $E_0 \approx \sqrt{2mc^2}$. If we estimate a mean field potential from the Newtonian form $V \sim -\frac{Gm^2}{\lambda_m} = -\frac{m^2}{M_p^2}mc^2 << E_0$, it is evident that the zero point energy would dominate the energy of such a system.

That vacuum energy can be thought of as resulting from the zero-point motions of the sources is also supported by the calculation of Bohr and Rosenfeld[16], who minimized the effect of a classical measurement of an electric field averaged over a finite volume on the value of a magnetic field at right angles averaged over (i) a non-overlapping volume, and (ii) an overlapping volume, and vice versa. When this minimum disturbance of sources is put equal to the minimum uncertainty which the uncertainty principle allows for the measurement they reproduce the result of averaging the quantum mechanical commutation relations over the corresponding volumes. Such arguments can be extended to the corresponding case when the sources and detectors are gravitational.
2 Cosmological Scale De-coherence From Dark Energy

The Friedman-Robertson-Walker (FRW) metric for a homogeneous isotropic cosmology is given by

\[ ds^2 = c^2 dt^2 + R^2(t) \left( \frac{dr^2}{1 - \kappa r^2} + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 \right). \]  

(2.1)

Here the FRW scale factor \( R \) is taken to have dimensions of length (not to be the dimensionless scale relative to the present scale \( a(t) \equiv R(t)/R_0 \)), with \( dr \) being dimensionless.

Our approach will be to start from well understood macrophysics, assume that the physics of a cosmological phase transition defines an FRW scale parameter, and examine cosmological physics at that time with regards to the physical consistency of the thermal state of the cosmology. For times after that transition there is general confidence that well understood micro- and macro-physics are valid. The FRW scale parameter must be expressed in terms of scales relevant to microscopic physics.

2.1 Quantum Measurability Constraints on Scale Expansion

Substitution of the FRW metric Eq. (2.1) into the Einstein Field equation (1.2) driven by an ideal fluid result in the Friedmann-Lemaître (FL) equations. The FL equations, which relate the rate and acceleration of the expansion to the fluid densities, are given by

\[ H^2(t) = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N}{3c^2} (\rho + \rho_\Lambda) - \frac{\kappa c^2}{R^2}, \]  

(2.2)

\[ \frac{\ddot{R}}{R} = -\frac{4\pi G_N}{3c^2} (\rho + 3P - 2\rho_\Lambda), \]  

(2.3)

where \( H(t) \) is the Hubble expansion rate, the dark energy density is given by \( \rho_\Lambda = \frac{\Lambda c^4}{8\pi G_N} \), \( \rho \) represents the FL fluid energy density, and \( P \) is the pressure. These equations combine to give the 1st Law of Thermodynamics for adiabatic expansion \( d(\rho R^3) = -Pd(R^3) \). The term which involves the
spatial curvature $\kappa$ has explicit scale dependence on the FRW parameter $R$. The dark energy density makes a negligible contribution to the FL expansion during early times, but becomes significant as the FL energy density decreases due to the expansion of the universe.

The scale evolution can be explicitly solved for relevant cosmological energy content. It is convenient to define conformal time, which involves a coordinate transformation insuring that light cones have a slope of unity:

$$ds^2 = R^2 [-d\eta^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)]$$

$$d\eta = \frac{c dt}{R(t)}$$

(2.4)

Light travels on null geodesics, making the conformal time equivalent to the (dimensionless) distance traveled by photons in a given time interval.

Driven by constant energy density, an inflationary scale evolves according to

$$R(t) = R_I e^{H_I(t-t_I)} , \quad \frac{1}{R(\eta)} = \frac{1}{R_I} - \frac{H_I}{c}(\eta - \eta_I).$$

(2.5)

The energy density of radiation varies as $R^{-4}$, so a radiation dominated scale satisfies

$$R^2(t) = R_{RD}^2 + 2R_{RD}c(t - t_{RD}) , \quad R(\eta) = R_{RD}[1 + (\eta - \eta_{RD})].$$

(2.6)

Near radiation/matter equality the scale satisfies

$$\frac{t}{t_{eq}} = 2 + \frac{R}{R_{eq}} - 2 R_{eq}^2 - 1 \left( \frac{R}{R_{eq}} + 1 \right)^{\frac{1}{2}}.\quad \frac{2}{1 + 2}.$$ (2.7)

During matter domination, the density varies inversely with the spatial volume, so the scale satisfies

$$R^{3/2}(t) = R_{MD}^{3/2} + 2R_{MD}^{1/2}c(t - t_{MD}) , \quad R(\eta) = R_{MD}\left[1 + \frac{1}{2}(\eta - \eta_{MD})\right]^2.$$ (2.8)

The proper distance to the particle horizon satisfies

$$d_H = \int_0^{d_H} \sqrt{g_{rr}} dr = R(t) \int_0^t \frac{c dt'}{R(t')} + d_H(t_1).$$ (2.9)
During inflation this has the form
\[ d_H = \frac{c}{H_i} \left( e^{H_i(t-t_i)} - 1 \right) + d_I, \] (2.10)
during radiation domination it satisfies
\[ d_H(t) = R(t) \left( \frac{R(t) - R_{RD}}{R_{RD}} \right) + d_H(t_{RD}) \Rightarrow 2c(t - t_{RD}) + d_{RD}, \] (2.11)
and during matter domination
\[ d_H(t) = 2R(t) \left( \frac{R^{1/2}(t) - R_{MD}^{1/2}}{R_{MD}^{1/2}} \right) + d_H(t_{MD}) \Rightarrow 3c(t - t_{MD}) + d_{MD}, \] (2.12)
where the arrows demonstrate the functional behaviors well within the given epoch.

If \( \dot{R} \) is an arbitrary scale in the Friedmann-Lemaitre equations for a spatially flat space in the radiation-dominated epoch and a phase transition occurs at scale \( R_{PT} \) with expansion rate \( \dot{R}_{PT} \), then Eq. 2.2 gives
\[ \dot{\dot{R}} R = \dot{R}_{PT} R_{PT} = \text{const.} \] (2.13)
This form can be integrated to give typical time scales of the form \( \Delta t = \frac{R}{2 \dot{R}} \).

Any quantized energy scale \( E \) defines a length scale \( R_e \) by the relation
\[ E = \frac{\hbar c}{R_e}, \] (2.14)
Using a quantized energy scale of order \( E \) and cosmological time associated with this scale in the energy-time uncertainty relation defines a constraint on the expansion rate associated with that scale:
\[ \Delta E \Delta t \geq \frac{\hbar}{2} \Rightarrow \frac{\hbar c}{R_e} \frac{R_e}{2R_e} \geq \frac{\hbar}{2} \Rightarrow \dot{R}_e \leq c. \] (2.15)

We can utilize the scale invariance of the FL equations to examine the subsequent evolution of scales \( \dot{R} \) that progressively satisfy the measurability constraint \( \dot{R} = c \). During the radiation epoch, the density varies with the inverse square of the time \( t \):
\[ \left( \frac{\dot{R}}{R} \right)^2 = \frac{8\pi G_N}{3c^2} \rho_{UV} \left( \frac{t_{UV}}{t} \right)^2 = \left( \frac{c}{R_{UV}} \right)^2 \left( \frac{t_{UV}}{t} \right)^2. \] (2.16)
Thus the scales and modes satisfy the measurability condition \( \dot{R} = c \) at times
\[
\dot{R} = R_{UV} \left( \frac{\dot{t}}{t_{UV}} \right), \quad \ddot{k} = k_{UV} \left( \frac{t_{UV}}{\dot{t}} \right).
\] (2.17)

Substitution for \( t_{UV} = R_{UV}/2c \) demonstrates that the threshold for modes that satisfy the measurability condition follow the evolution of the horizon in Eq. 2.11.

Therefore, assuming that the scale factor at the time of the phase transition is defined by a quantized microscopic energy scale \( R_{PT} = R_E \), this scale must satisfy \( \dot{R}_{PT} = c \).

Setting the expansion rate to \( c \) in the Lemaitre equation 2.2 with \( \kappa = 0 \), the energy density during dark energy UV decoherence is given by
\[
\rho_{UV} = \frac{3c^2}{8\pi G_N} \left( \frac{c}{R_{UV}} \right)^2 - \rho_\Lambda.
\] (2.19)

For convenience, energy scales \( m_{UV} \) and \( \epsilon \) will be defined from these densities using
\[
\rho_{UV} \equiv \frac{(m_{UV}c^2)^4}{(\hbar c)^3}, \quad \rho_\Lambda \equiv \frac{\epsilon^4}{(\hbar c)^3}.
\] (2.20)

Similarly, the scale acceleration at the time of this transition can be determined:
\[
\frac{\ddot{R}_{UV}}{\dot{R}_{PT}} = -c^2 \left( \frac{1}{R_{UV}^2} + \frac{\Lambda}{3} \right) \Rightarrow \ddot{R}_{UV} \approx -\frac{c^2}{R_{UV}}.
\] (2.21)

### 2.2 Spatial Curvature Constraints

The energy density during UV decoherence \( \rho_{UV} \) can be directly determined from the Lemaitre equation 2.2 to satisfy
\[
H_{UV}^2 = \left( \frac{c}{R_{UV}} \right)^2 = \frac{8\pi G_N}{3c^2} \left( \rho_{UV} + \rho_\Lambda \right) - \frac{\kappa c^2}{R_{UV}^2}.
\] (2.22)

A so called “open” universe (\( \kappa = -1 \)) is excluded from undergoing this transition, since the positive dark energy density term \( \rho_\Lambda \) already excludes a solution with \( \dot{R} \leq c \). Likewise, for a “closed” universe that is initially radiation dominated, the scale factors corresponding to decoherence \( \dot{R}_{UV} = c \)
and maximal expansion $\dot{R}_{\text{max}} = 0$ can be directly compared. From the Lemaître equation

$$\frac{c^2}{R_{\text{max}}^2} = \frac{8\pi G_N}{3c^2} [\rho(R_{\text{max}}) + \rho_{\Lambda}] \cong \frac{8\pi G_N}{3c^2} \rho_{PT} R_{\text{PT}}^4 R_{\text{max}}^4 \Rightarrow R_{\text{max}}^2 \cong 2 R_{\text{PT}}^2. \quad (2.23)$$

Clearly, this closed system never expands much beyond the transition scale. Quite generally, the constraints consistent with quantum measurability for quantized energy scales ($\dot{R}_{\text{UV}} \leq c$) requires that all cosmologies which develop structure be spatially flat.

The evolution of the cosmology during the period for which the dark energy UV scale is in the microscopic thermal spectrum is expected to be accurately modeled using the FL equations. There is a period of deceleration, followed by acceleration towards an approximately De Sitter expansion. The rate of scale parameter expansion is sub-luminal during a finite period of this evolution, as shown in Figure 5. The particular value

![Figure 5: Log graphs of redshift $R_o/R$ and gravitational UV scale expansion rate vs time](image)

for the scale at de-coherence (which is determined by the microscopic dynamics of the dark energy during de-coherence) chosen for the graphs is given in terms of the measured dark energy density $\rho_{\Lambda} = \epsilon^4$. The present time since the “beginning” of the expansion corresponds to the origin on both graphs. The value of the expansion rate is by assumption equal to the speed of light for any particular value chosen for $R_{PT}$, as well as when this expansion scale reaches the de Sitter radius $R_{\Lambda} \approx 1.56 \times 10^{28} \text{cm}$ associated with the measured dark energy.
2.3 Gravitating quantum energy scales

The existence of a cosmological bound on the density attainable by non-coherent, thermal energies has implications on limits of applicability of classical relativity. This suggests that fundamental microscopic physics undergoes a macroscopic quantum transition for densities beyond this limit, modifying the assumptions of classical relativity in this regime. It is well known that coherent particles violate the predictions of classical relativity (just as the coherent physics of stationary atoms violate the predictions of classical electromagnetism). For instance, the horizon of a black hole with mass $M$, spin $S \equiv JMc$, and charge $q^2 \equiv Q^2c^4/G_N$ is given by

$$r_H = R_S \pm \sqrt{R_S^2 - (J^2 + Q^2)} , \quad R_S = \frac{2G_NM}{c^2}. \quad (2.24)$$

The term under the radical is unphysical for an electron,

$$R_S^2 - (J^2 + Q^2) = 4 \left( \frac{m_e}{M_P} \right)^2 L_P^2 - \left[ \left( \frac{\hbar}{2m_e c} \right)^2 + \alpha L_P^2 \right] \simeq - \left( \frac{\hbar}{2m_e c} \right)^2 < 0, \quad (2.25)$$

being dominated by the quantized angular momentum. Quantum behavior therefore modifies classical general relativity in the regions near the horizon.

A useful estimate can be obtained with regards to the expected mass and distance scales separating coherent from non-coherent behavior. Writing $M_{CL}c^2 \equiv \frac{1}{4\pi} R^3 \rho_{UV}$ as the mass scale representing this coherence limit in a Schwarzschild geometry, and introducing a microscopic critical mass scale $\rho_{UV} \equiv (m_{UV}c^2)^3/(\hbar c)^3$, an upper limit for a coherence radial scale is given by

$$R_{CL} = \left( \frac{3}{4\pi m_{UV}} \right)^{1/3} \lambda_{m_{UV}}. \quad (2.26)$$

Later, we will estimate the UV energy scale as $m_{UV}c^2 \simeq 3 \times 10^3 GeV$. Comparing this radial scale with the Schwarzschild radius for a classical black hole, coherence transition scales $R_{CL*}, M_{CL*}$ can be calculated:

$$R_{CL*} \simeq \left( \frac{M_{CL}}{M_{Sun}} \right)^{1/3} \times 42 cm,$$
$$R_{CL*} \simeq 3 \times 10^{-8} R_{S,sun} \simeq 9 \times 10^{-3} cm,$$
$$M_{CL*} \simeq 3 \times 10^{-8} M_{Sun} \simeq 3 \times 10^{30} M_P. \quad (2.27)$$
Thus, if the early cosmology is any indication, thermal energies with Schwarzschild radii less than about 0.01 cm should show macroscopic coherence effects.

For quantum thermal systems, typical thermal energies $k_B T_{\text{crit}}$ are given by kinetic energies for constituent particles of mass $m$, which define a thermal distance scale $R_{\text{thermal}} \approx \frac{hc}{k_B T_{\text{crit}}}$ that satisfies

$$R_{\text{thermal}} \lambda_m \sim (\Delta x)^2 \quad (2.28)$$

in terms of the Compton wavelength of the mass scale $\lambda_m \equiv h/mc$ and the scale of zero-point motions of those masses. This relationship just follows from the momentum-space uncertainty principle. For example, for a degenerate free Fermi gas, the number density relationship $n = \frac{2m}{\pi^2} \left(\frac{\hbar}{2m} \right)^{3/2}$ implies $R_{\epsilon} \lambda_m = \frac{1}{2} \left(\frac{6\pi^2}{3m} \right)^{2/3} (\Delta x)^2$ [19]. For a simple harmonic oscillator, the zero-point energy satisfies $\frac{(\Delta p)^2}{2m} + \frac{1}{2} m \omega^2 (\Delta x)^2 = \frac{1}{2} \hbar \omega$. The zero-point kinetic and potential energies each partition half of the vacuum energy $\frac{1}{4} \hbar \omega$, The resulting uncertainties $(\Delta x)^2 = \hbar/2m \omega$ and $(\Delta p)^2 = \hbar m \omega/2$ saturate the quantum measurability condition $\Delta x \Delta p = \hbar/2$. Thus, for simple harmonic motions $R_{h \omega} \lambda_m = 2(\Delta x)^2$. For systems with fixed microscopic coherence scales a condensate loses macroscopic coherence below a critical density as illustrated by Fig. 6.

Figure 6: Overlapping regions of coherence during expansion
2.4 Estimate of size of source masses for the zero-point energy

To estimate the energy scales associated with the zero-point motions, assume there are \( N \) such sources in a volume specified by \( R^3 \), and the energy parameter \( \epsilon \). On average, each coherent energy unit contributes zero-point energy of the order

\[
\frac{\epsilon}{N} \sim \frac{(\Delta P)^2}{2M} \geq \frac{\hbar^2}{8M(\Delta X)^2},
\]

(2.29)

where the uncertainty principle has been used in the form \( (\Delta P)(\Delta X) \geq \frac{\hbar}{2} \). Replacing the spatial uncertainty with the coherence scale \( \Delta X \sim R\epsilon \) relates the energy scale to the zero-point energy \( M \sim N\epsilon \). The cosmological density at the time of the phase transition is given by the ratio of total (non-relativistic) energy to the volume of coherence

\[
\rho_{UV} = \frac{NMc^2}{R_{UV}^3} \sim N^2 \frac{\epsilon^4}{(hc)^3} = N^2 \rho_\Lambda. \tag{2.30}
\]

The FL equations determine the density during the phase transition from Eq. 2.19

\[
\rho_{UV} \cong \frac{3}{8\pi} \frac{(Mpc^2\epsilon^2)^2}{(hc)^3} \tag{2.31}
\]

giving direct estimates of the coherent energy units involved in the zero-point motions

\[
N \sim \frac{Mpc^2}{\epsilon}; \quad M \sim M_P. \tag{2.32}
\]

That is, the sources of the vacuum energy must be at the Plank mass scale, each with zero point energy \( \sim \frac{\epsilon^2}{Mpc^2} \) on average. The coherent mass units undergoing zero-point behaviors have pairwise gravitational couplings \( G_N M^2 \) of order unity, whereas the de-coherent energy density during the FL expansion will consist of masses with considerably smaller gravitational couplings. Any microscopic mass \( m \) with the coherence scale defined in Eq. 2.28 will have zero-point energies of the order \( \frac{(\Delta P)^2}{2m} \sim \epsilon \) at the time of the phase transition, which is expected to red shift as the cosmology expands. If the space-time were Schwarzschild, the scale \( R_{UV} \) would be that associated with the Schwarzschild radius of that geometry. For brevity,
the collective modes of the Planck mass units will here be referred to as *gravons*. The zero-point motions of those coherent energy units correspond to the vacuum energy of the gravons.

## 3 Estimate of Density Fluctuations

The question arises as to why the dark energy scale freezes out of the subsequent cosmological expansion. This scale must be a microscopic (non-expanding) scale of gravitational relevance. The energy scale is expected to freeze out because the medium in some way dissolves (or more precisely, precipitates) at the gravitational UV scale. In this sense it is the FL energy density $\rho_{UV}$ that de-coheres from the dark energy $\rho_\Lambda$ (making our reference of this process as dark energy de-coherence somewhat a misnomer, since actually the thermal energy scales de-cohere). We will explore scenarios that might decouple the subsequent expansion from the dark energy scale. If the initial cosmology is inflationary, one expects the Hubble rate at the UV scale to determine $\rho_{UV}$ (which represents the vacuum energy density of the thermal state), assuming through energy conservation that the constant inflationary energy density thermalizes into the radiation energy density that drives the subsequent expansion. The dark energy of the present cosmology is due to the local thermal effects of the inflationary deSitter horizon prior to the end of inflation. If the microscopic density scale associated with the transition is $\rho_v$, then the fluctuations scale for a transition from an inflationary to a microscopic thermal state is expected to be of the order $\frac{\rho_v}{\rho_{UV}}$. The horizon is expected to dissolve due to the onset of the microscopic thermalizations. More on this will be discussed in Section 4.1.

A second scenario involves a pure quantum transition from an initial quasi-stationary state. As the zero-point motions locally de-cohere, the deviations from uniformity are expected to appear as fluctuations in the cosmological energy density. Since these motions are inherently a quantum effect, one expects the fluctuations to exhibit the space-like correlations consistent with a quantum phenomenon. Measurable effects of quantum mechanical de-coherence are expected to manifest stochastically. As an intuitive guide into how dark energy might freeze out as the system de-coheres, consider a uniform distribution of non-relativistic masses $M$ interacting pairwise through simple harmonic potentials at zero temper-
nature. If charges were used as field sources rather than masses, Bohr and Rosenfeld\cite{16} showed that the uncertainty relations associated with the positions and momenta of the charges result in averaged commutation relations between the electric and magnetic fields which are classically produced by those charges. Correlated zero-point motions of the oscillators with vanishing time averages give equal partitioning of energies $\frac{1}{4}\hbar \omega$ to the kinetic and potential components. If the masses evaporate, the kinetic component is expected to drive density fluctuations of an expanding gas, whereas the potential components remain in the springs (as zero point tensions) of fixed density, as illustrated in Figure 7. If the evaporation is rapid, the compressional vacuum energy is expected to be frozen in as dark energy in the background during the phase transition on scales larger than the de-coherence scale. The mass units which were undergoing zero-point motions no longer behave as coherent masses after evaporation, and only reflect their former state through the density inhomogeneities generated during the evaporation process, no longer coupling to the frozen-in potential component of the zero point energy other than through a fixed macroscopic background. Such an interpretation could represent the source of a cosmological constant as a frozen energy density, rather than as a geometric attribute.

An interacting sea of the quantum fluctuations due to zero point motions should exhibit local statistical variations in the energy. Statistical arguments can be made\cite{17,18} which infer that the dark energy $E_\Lambda$ is

Figure 7: Early quantum stage undergoing zero-point fluctuations during de-coherence, with evaporation and subsequent expansion of thermal energy density associated with masses which were previously coherently attached via the springs. Potential energy density gets frozen in during evaporation, whereas kinetic energy density drives density fluctuations.
expected to have uniform density, and to drive fluctuations of the order

\[ < (\delta E)^2 > = E_\Lambda^2 \frac{d}{dE_\Lambda} < E > \quad (3.1) \]

Given an equation of state \( < E > \sim (E_\Lambda)^b \), the expected fluctuations satisfy

\[ \frac{< (\delta E)^2 >}{< E >^2} = b \frac{E_\Lambda}{< E >} \quad (3.2) \]

The specific equation of state depends on the details of the macrosopic quantum system. For a gravon gas with energy \( (N_g + \frac{1}{2})h\hbar c = (2N_g + 1)e \sim \rho_{UV}V_c \), the exponent in Eq. [3.2] has the value \( b = 1 \). In terms of the densities, one can directly write \( \frac{< (\delta E)^2 >}{< E >^2} = \frac{< (\delta \rho)^2 >}{\rho^2} = b \frac{\rho_\Lambda}{\rho} \). At the time of the formation of the fluctuations, this means that the amplitude \( \delta \rho / \rho \) is expected to be of the order

\[ \delta_{PT} = \left( b \frac{\rho_\Lambda}{\rho_{PT}} \right)^{1/2} = \sqrt{b} \frac{R_{PT}}{R_\Lambda} \quad (3.3) \]

where \( \rho_{PT} \) is the cosmological energy density at the time of the phase transition that decouples the dark energy, and \( \Lambda = 8\pi G_N/\rho_\Lambda c^4 = 3/R_\Lambda^2 \) is the cosmological constant.

To determine how any fluctuations grow, consider a spatially flat universe with negligible dark energy satisfying \( H^2 = \frac{8\pi G_N}{c^2} \rho \). A small positive (negative) fluctuation in energy density will close (open) the universe, resulting in a positive (negative) curvature \( \kappa \), with consistent expansion rate, giving \( H^2 = \frac{8\pi G_N}{c^2} \rho - \frac{c^2}{R^2} \). Subtracting these equations, and solving for the dimensionless density perturbations, these perturbations are found to grow as follows:

\[ \frac{\delta \rho}{\rho} = \frac{3c^4 \kappa}{8\pi G_N \rho R^2} \propto \begin{cases} R^2 & \text{radiation dominated} \\ R & \text{matter/dust dominated} \end{cases} \quad (3.4) \]

Thus, for adiabatic perturbations (those that fractionally perturb the number densities of photons and matter equally), the energy density fluctuations grow according to \[ \delta = \begin{cases} \delta_{PT} \left( \frac{R(t)}{R_{PT}} \right)^2 & \text{radiation-dominated} \\ \delta_{eq} \left( \frac{R(t)}{R_{eq}} \right)^2 & \text{matter/dust-dominated} \end{cases} \quad (3.5) \]
which gives an estimate for the scale of fluctuations at last scattering from a radiation epoch phase transition expressed by

\[ \delta_{LS} \approx \frac{(1+z_{PT})^2 \sqrt{b \Omega_{\Lambda o}}}{(1+z_{eq})(1+z_{LS})} \left( \frac{\rho_{\Lambda o}}{\rho_{PT}} \right)^{1/2} \]

where a spatially flat cosmology has been assumed. If the transition occurs during dust/plasma domination,

\[ \delta_{LS} \approx \frac{1}{1+z_{LS}} \sqrt{\frac{b \Omega_{\Lambda o}}{(1-\Omega_{\Lambda o})(1+z_{eq})}} \approx 2.5 \times 10^{-5} \sqrt{b}, \]

which varies from \(2 \times 10^{-5}\) if the phase transition occurs at radiation-matter equality, to \(4 \times 10^{-5}\). These estimates are only weakly dependent on the density during the phase transition \(\rho_{PT}\), and is of the order observed for the fluctuations in the CMB (see [5] section 23.2 page 221). Fluctuations of a scale larger than \(\delta_{PT}\) would be correspondingly larger at last scattering.

4 Scenarios Prior to Phase Transition

The period of transition from prior coherence to radiation is typically referred to as reheating. Prior to reheating, one expects the energy of the universe to be in the form of coherent modes of the condensate, which have available degrees of freedom partitioned differently from the later thermal state. During reheating, the energy in the condensate modes precipitates into a multitude of particle modes, which thermalize.

The assumption of radiation dominance during the phase transition corresponds to a thermal temperature of

\[ \eta(T_{PT}) (k_B T_{PT})^4 \approx \frac{90}{8 \pi^3} (M_P c^2)^2 \left( \frac{\hbar c}{R_{PT}} \right)^2, \]

where \(\eta(T_{PT})\) counts the number of degrees of freedom associated with particles of mass \(mc^2 < k_B T_{PT}\), and \(M_P = \sqrt{\hbar c/G_N}\) is the Planck mass. Here we have used Eq. 2.19 and the energy density for relativistic thermal energy \(\rho_{\text{thermal}} = \eta(T) \frac{\pi^4 (k_B T)^4}{30 (\hbar c)^3}\).
4.1 Inflationary prior state

If both classical general relativity driven by the total energy density and quantum mechanics can be reliably used to describe the cosmology prior to the time of the decoupling of the dark energy, quantum measurability constraints on the gravitational interactions (which have couplings of order unity) suggests a change in the state of the energy density. One scenario demands that this energy density be in the form of vacuum energy (or zero-point energy) with respect to the forms of matter/energy prevalent in the cosmology shortly after the transition. Inflation then ends when the UV gravitational scale crosses the deSitter horizon associated with the inflation, initiating microscopic thermalizations.

Assuming a transition that conserves energy, if the energy density of the present cosmology during the phase transition is set by the inflationary energy density, the deSitter scale of the inflation \( \Lambda_I = \frac{3}{R_{\Lambda_I}^2} \) is defined in terms of the dark energy scale of the present cosmology using \( \rho_{\Lambda_I} = \rho_{UV} \), which gives the UV scale:

\[
\rho_{\Lambda_I} = \frac{\Lambda_I c^4}{8\pi G_N} = \frac{3c^2}{8\pi G_N R_{UV}^2} \Rightarrow R_{\Lambda_I} = R_{UV}.
\]

The time scale for microscopic thermalizations could be considerably more rapid than the Hubble time \( \tau \ll \frac{R_{UV}}{c} \). We will assume the microscopic transition behavior to be described by a functional form \( F(\zeta) \), where

\[
F(\zeta) \Rightarrow \begin{cases} 
0 & \text{if } \zeta \to 0 \\
1 & \text{if } \zeta \to \infty 
\end{cases}
\]

so that during the transition from inflation to radiation, the energy density is expected to be of the form

\[
\rho(t) = \rho_{UV} \left\{ 1 + \left[ \left( \frac{R_{UV}}{R(t)} \right)^4 - 1 \right] F\left( \frac{t - t_s}{\tau} \right) \right\}. \tag{4.4}
\]

Here \( t_s \) represents the onset of thermalization. Substitution into the LeMaitre equation allows estimation of the temporal behavior of the scale during thermalization. During the initial onset of thermalization the expansion is slightly modified from inflation by

\[
R(t) \approx R_{UV} \left[ 1 + F'(0) \frac{c}{4R_{UV}} \frac{t^2 - t_s^2}{\tau} \right] e^{c(t-t_I)/R_{UV}}, \tag{4.5}
\]
whereas the behavior is dominated by radiation when $\Delta t > \tau$, as seen in Fig. 8. The temporal scale is set by both the Hubble rate $c/R_{UV}$ and the microscopic scale $\tau$. The change in the cosmological scale during thermalization $\frac{\Delta R}{R_{UV}} \approx \frac{c\tau}{R_{UV}}$ depends on the relative time scales of microscopic vs macroscopic rates. The figure shows that the initial inflation settles into a radiation dominated cosmology with a crossover given by $\tau$.

Since the energy density does not red-shift during the inflationary period, thermalization must represent the onset of the vacuum modes of the thermal epoch. This then implies that the microscopic UV modes $k_{\text{micro}}$ satisfy

$$\rho_{UV} \approx \frac{\eta_{\text{micro}}}{2} \frac{\hbar c}{8\pi^2} k_{\text{micro}}^4$$

(4.6)

where $\eta_{\text{micro}}$ represent the degeneracy of microscopic states. This implies that in order for the inflation to consistently conserve energy,

$$\hbar k_{\text{micro}} c = \frac{\sqrt{4\pi}}{\eta_{\text{micro}}} m_{UV} c^2 \quad , \quad \text{microscopic UV scale}$$

$$\hbar k_{UV} c = \frac{\sqrt{4\pi}}{g_{\text{eff}}} \epsilon \quad , \quad \text{gravitational UV scale}$$

(4.7)

A transition from an inflationary epoch requires the onset of thermalizations from the microphysics background, implying the existence of UV scales defined by the transition.
If the energy scale of the microscopic fields is given by the electro-weak symmetry breaking scale $v$, the expected fluctuations would be of the order

$$\Delta I \approx \left( \frac{v}{m_{UV}c^2} \right)^4 \approx 4.5 \times 10^{-5}. \quad (4.8)$$

This means that if the Higgs field defines microscopic energy scales, the super-horizon modes should have amplitudes of the order observed in the CMB.

The transition to a radiation dominated cosmology likely involves a “latent heat” at the phase transition due to a change in the entropy per constituent quantum. If the cosmology transitions from an inflationary epoch, we can directly estimate the ratio of local thermal entropy to inflationary entropy. The entropy during inflation counts associated super-horizon states (information lost across the horizon) for given sub-horizon configurations, and is given by

$$S_{\text{inflation}} = \frac{A}{4G_N} \frac{k_Bc^3}{\hbar} = k_B\pi \left( \frac{R_{UV}M_Pc^2}{\hbar c} \right)^2, \quad (4.9)$$

while the deSitter temperature during the inflation is given by

$$k_B T_{UV} = \frac{\hbar c}{2\pi R_{UV}} \sim E_{\text{dark}} \approx \frac{\pi^2}{12} \hbar k_{UV} v_p. \quad (4.10)$$

For radiation, the entropy density is related to the energy density by $\sigma_{UV} = \frac{4}{3}\rho_{UV}$. Using the identification for $\rho_{UV}$ given in Eq. 2.22, the relative (sub-horizon) entropies during the transition is given by

$$\frac{S_{\text{thermal}}}{S_{\text{inflation}}} = 2 \frac{\hbar c}{3 R_{UV} k_B T_{UV}} \frac{1}{\epsilon} \frac{1}{m_{UV}c^2} \sim \epsilon \ll 1. \quad (4.11)$$

Thus, in this scenario the thermal entropy of the expanding energy density is smaller compared to that associated with the deSitter horizon of the prior inflation, although the temperature increases considerably. The information lost from the existence of an inflationary horizon is considerably larger than that lost due to thermalization of microscopic degrees of freedom.

The scale of the horizon temperature during the inflation is comparable to the scale of the dark energy today. The deSitter thermal energy is due
to super-horizon quantum correlations. It is plausible that this thermal energy which has its roots in quantum correlations across the horizon would be connected with the subsequent dark energy.

Using the FL equation for the acceleration Eq. 2.21 the acceleration just after the phase transition is given by \( \ddot{R}_e = -\frac{c}{\bar{h}} \epsilon \). The inflationary scale just prior to the phase transition has an acceleration given by \( \ddot{R}_e = +\frac{c}{\bar{h}} \epsilon \). The transition requires a change in the scale acceleration rate of the order of the dark energy in each scale region of the subsequent decelerating cosmology.

Others have used related arguments to examine the initial inflationary period. Choosing electro-weak symmetry restoration estimates of the early 1990’s, Ed Jones\cite{21, 22} predicted a cosmological constant with \( \Omega_\Lambda \approx 0.6 \) before the idea of a non-vanishing small cosmological constant was fashionable. This is one of the appeals of this particular scenario.

### 4.2 Quantum Transition cosmology

An alternative scenario connects the temporal progression in classical general relativity to gravitational de-coherence, implying a macroscopic coherent quasi-stationary quantum state of density \( \rho_{UV} \) prior to the phase transition. Since a stationary state is temporally extended, a stationary density is not expected to drive the cosmological dynamics in the FL equations during this “pre-coherent” period. One cannot ascertain temporal relations unless the proliferous time-like interrelations needed to construct the measurement space-time grid have occurred. Temporal progression begins only when de-coherent interrelations break the stationarity of the quantum state, and is described by the Friedman-LeMaitre equations with appropriate scale eventually taking the value \( R_{UV} \) as the transition proceeds. The measurement of time therefore begins with quantum mechanics, not classical general relativity. The macroscopic scale evolution is then due to thermal proliferation. The microscopic gravitational scale gets frozen when vacuum gravitational energy de-coheres from thermal energies. An initial quantum stationary state would transition from the (low) entropy associated with the degrees of freedom of the condensate to \( \sigma_{UV} = \frac{4}{3} \rho_{UV} \) during the thermalization transition.

In this description, there is no initial temporal singularity, or well-defined \( t=0 \) due to quantum uncertainty. A quantum fluctuation of a size
that produces dark energy $\epsilon$ with scale $R_{\epsilon}$ gives the same field dynamics as would an inflationary scenario with deSitter scale $R_{\epsilon}$ that matches the phase transition into the decelerating epoch.

5 What is Special About $\rho_{PT}$ in This Cosmology

The currently accepted values\textsuperscript{5} for the cosmological parameters involving dark energy and matter will be used for the reverse time extrapolation from the present:

$$h_0 \approx 0.72; \quad \Omega_{\Lambda} \approx 0.73; \quad \Omega_M \approx 0.27.$$

Here $h_0$ is the normalized Hubble parameter. Note that this value implies that the universe currently has the critical energy density $\rho_c \approx 5.5 \times 10^{-4} GeV \text{ cm}^{-3}$. The values of parameters for the observed cosmology are given by $\rho_{\Lambda} \approx 4.0 \times 10^{-6} GeV/cm^3$, $R_{\Lambda} \approx 1.5 \times 10^{28} cm \approx 1.6 \times 10^{10} ly$, $\epsilon \approx 2.4 \times 10^{-12} GeV \approx \hbar c/8.4 \times 10^{-3} cm$.

Using Eq. 1.3 connecting the dark energy to the UV mode $\rho_{\Lambda} \approx \frac{\rho_c}{b_{\Lambda}} \frac{h_{\nu_{\text{b}}} \bar{h} \nu_{\text{b}}}{8\pi^2} k_{UV}^4$, along with the flatness/measurability requirement

$$\left( \frac{c}{R_{UV}} \right)^2 \approx \frac{8\pi G_N}{2c^2} \rho_{UV},$$

one only needs to connect the UV mode scale $k_{UV}$ to the FRW scale parameter $R_{UV}$. The horizon scale mode is expected to satisfy $|\vec{k}| = \pi/d_H$. For generality, we will relate the horizon to the UV scale using

$$d_{H,UV} = b_R R_{UV}.$$  

Using Eqns. 1.3 and 5.2, numerical values can be given for the UV scale
parameters:

\[ d_{H,UV} = b_R R_{UV} \approx \left( \frac{g_{\epsilon}}{\Lambda_{\epsilon}} \frac{v_p}{c} \right)^{1/4} 9 \times 10^{-3} \text{cm} \]

\[ \hbar k_{UV} c \approx \left( \frac{b_R}{g_{\epsilon} v_p} \right)^{1/4} 6.9 \times 10^{-12} \text{GeV} \]

\[ \rho_{UV} \approx b_R^2 \left( \frac{b_R}{g_{\epsilon} v_p} \right)^{1/2} \frac{(3000 \text{GeV})^4}{(\hbar c)^4} \]

\[ m_{UV} c^2 \approx b_R^{1/2} \left( \frac{b_R}{g_{\epsilon} v_p} \right)^{1/8} 3000 \text{GeV}. \]

(5.4)

The microscopic scale for gravitational de-coherence is fixed by the critical density \( \rho_{UV} \). If this density corresponds to the vacuum mode for microscopic states, the microscopic ultraviolet cutoff can be directly calculated as in Eq. 4.7

\[ \hbar k_{\text{micro}} c = \sqrt{4\pi \eta_{\text{micro}}} m_{UV} c^2 \quad , \quad \text{particle modes,} \]

\[ \hbar k_{UV} c = \sqrt{4\pi} \left( \frac{c}{v_p g_{\epsilon}} \right)^{1/4} \epsilon \quad , \quad \text{gravitational modes,} \]

(5.5)

where again \( v_p \) is the phase velocity of the vacuum modes, and the particle modes are assumed to have phase velocity \( c \). This gives an estimate of the microscopic cutoff:

\[ \hbar k_{\text{micro}} c \approx b_R^{1/2} \left( \frac{b_R \Lambda_{\epsilon} c}{g_{\epsilon} v_p} \right)^{1/8} \eta_{\text{micro}}^{-1/4} 10600 \text{GeV}, \]

(5.6)

where \( \eta_{\text{micro}} \) is expected to be of the order \( \frac{427}{4} \). The UV scale temperature of the radiation can likewise be calculated:

\[ \rho_{UV} \sim \eta(T_{UV}) \frac{\pi^2 (k_B T_{UV})^4}{30 (\hbar c)^4} \Rightarrow k_B T_{PT} \sim b_R^{1/2} \left( \frac{b_R \Lambda_{\epsilon} c}{g_{\epsilon} v_p} \right)^{1/8} 1250 \text{GeV}, \]

(5.7)

which corresponds to a redshift \( z_{UV} \sim 1 \times 10^{16} \).

In most of our calculations we will assume scalar modes \( g_{\epsilon} = 1 \). For a transition from inflation, the horizon scale inherently satisfies \( \dot{R}_I = c \), and is expected to directly correspond to the UV scale, setting the parameter \( b_R = 1 \), and all vacuum energy will be assumed to constitute the
dark energy $b_\Lambda = 2$. For the quantum thermalization scenario, the causal region is taken to satisfy periodic boundary conditions $b_R = 1/2$, and the compressional vacuum energy is taken to constitute the dark energy $b_\Lambda = 4$.

One expects microscopic physics to fix the particular scale as a quantum phase transition associated with the UV energy scale for the gravitational modes, $\rho_{PT} \equiv (m_{UV}c^2)^{4}/(\hbar c)^3$. As a further illustration of the expectation of the manifestation of macroscopic quantum effects on a cosmological scale, consider Bose condensation. As mentioned in the introduction, since Compton scattering conserves photon number, the photons in the early universe should have a non-vanishing chemical potential, making even these massless particles susceptible to Bose condensation. A microscopic critical density is reached when thermal modes can no longer accommodate a distribution of all of the particles, forcing macroscopic occupation of the lowest energy state. For low mass bosons, the Planck distribution gives the thermal component of those particles, with macroscopic occupation of the zero mode for particles that are not thermally accommodated. For non-relativistic particles of mass $m$, this density satisfies

$$N_m = \frac{\zeta(3/2)\Gamma(3/2)}{(2\pi)^{2/3}}(2mk_BT_{crit})^{3/2}, \quad \rho_m \equiv \frac{N_m}{V}mc^2.$$

For instance, if the scale were associated with the density of thermal bosonic matter, the critical temperature is related to the (non-relativistic) mass by $k_BT_{crit} = \left(\frac{(2\pi)^2}{g_m\zeta(3/2)\Gamma(3/2)}\right)^{2/3} m_{UV}c^2 \approx 6.6\frac{(\hbar c)^2}{g_m}\frac{m_{UV}c^2}{(mc^2)^{5/3}} \approx 3.313(m_{UV}c^2)^{2/3}$. Since this temperature is comparable to the ambient temperature of thermal standard model matter at this density, bosonic matter at this density would have a significant condensate component. Bosons with mass $mc^2 > 2700GeV$ and densities comparable to $\rho_{UV}$ would be expected to have a condensate component. Using the relation connecting microscopic and macroscopic scale given previously by $R_\epsilon \lambda_m \sim (\Delta x)^2$, the zero-point energies of each of the UV energy units $m_{UV}$ is of the order of the dark energy $\frac{\Delta P^2}{2m_{UV}} \sim \epsilon$.

### 5.1 A connection to microscopic physics

Motivated by this approach, examine symmetry breaking in the early
universe \cite{23} with density scale $\rho_{\text{UV}}$: 

$$
\mathcal{L} = \sqrt{-g} \left[ -\frac{1}{2} (D_{\mu} \Phi_b) g^{\mu \nu} (D_{\nu} \Phi_b) + \frac{1}{4} (m_b^2 + 2 \mu^2) \Phi_b^2 - \frac{1}{8} f^2 \Phi_b^4 - \rho_{\text{UV}} \right] + \mathcal{L}_{\text{particle}},
$$

(5.9)

where for an FRW cosmology $\sqrt{-g} = R^3$, the particle Lagrangian includes the gauge boson field strength contribution $-F_{\mu \nu} F^{\mu \nu}/16 \pi$, and any vacuum energy subtractions are appropriately included in $\rho_{\text{UV}}$. In late times, as usual, the classical solution where the gauge fields vanish results in a non-vanishing vacuum expectation value for one of the field components

$$
\langle \Phi_1 \rangle = 0, \quad \langle \Phi_2 \rangle = v = \frac{\sqrt{m_b^2 + 2 \mu^2}}{f}.
$$

(5.10)

The energy scale $v$ is expected to be the microscopic energy scale in late times, when thermal energy states have cooled to the point that the standard particles have vanishing ground state expectation values. The free energy density $\mathcal{L}$ contains the term $\rho_{\text{UV}}$ which is the initial energy density that thermalizes into the observed cosmological content. The strategy is to use the energy density due to the symmetry breaking field (Higgs \cite{23}) prior to the thermalization of its excitations relative to the background given in Eq. (5.10) and the vector bosons into the microscopic particulate states whose remnants persist today. The action corresponding to this Lagrangian

$$
W_{\text{matter}} = \int \mathcal{L} d^4 x
$$

(5.11)

generate the conserved energy-momentum tensor in the Einstein equation

$$
T_{\mu \nu} = -\frac{2}{\sqrt{-g}} \frac{\delta W_{\text{matter}}}{\delta g_{\mu \nu}}.
$$

(5.12)

There is every indication that the cosmology will have extreme spatial homogeneity during the phase transition, so that for the present, spatial gradients will be neglected. For an FRW cosmology, the Jacobian factor can be calculated using $g = g_{00} g_{xx} g_{yy} g_{zz}$. Using Eq. (5.12) and $\delta \sqrt{-g} = \frac{1}{2} \sqrt{-g} g^{\mu \nu} \delta g_{\mu \nu}$, the energy density can then be calculated as

$$
T_{00} = \frac{1}{2} (\dot{\Phi} + B_0 \Phi)^2 - \frac{1}{4} (m_b^2 + 2 \mu^2) \Phi_b^2 + \frac{1}{8} f^2 \Phi_b^4 + \rho_{\text{UV}} + T_{00}^{\text{particle}}.
$$

(5.13)
Note that as previously mentioned, the initial stationary state has energy density $T_{00} = \rho_{UV}$. When particulate degrees of freedom are negligible, the general temporal equation of motion for the field is given by

$$\frac{1}{R^3} \frac{d}{dt} \left( R^3 [\dot{\Phi} + B_0 \Phi] \right) - \frac{1}{2} \left( m^2_{\Phi} + 2\mu^2 - f^2\Phi^2 \right) \Phi = 0. \tag{5.14}$$

It is interesting to explore the pre-thermal state of the quantum fields. Eq. 5.13 suggests a gravitational energy scale of the order $\rho_{UV}^{1/4} = m_{UV} c^2$. For independent fields, the density matrix formalizes a basis independent representation of the state of the system:

$$D = \sum_{j=1}^{\eta} w_j |\phi_j^R><\phi_j^R|. \tag{5.15}$$

Here, the fields $|\phi_j^R>$ have unit norm, $w_j$ represent the weights of the independent states, and $\eta$ is the number of independent states. For equal a-priori probabilities, the weights satisfy $w_j = \frac{1}{\eta}$. The microscopic fields in Eq. 5.13 are expected to have energy scales normalized by the vacuum expectation value $v$,

$$\frac{|\phi_j^R|}{\rho_{UV}^{1/4}} = \left( \frac{v}{m_{UV} c^2} \right) |\phi_j^R| >. \tag{5.16}$$

This means that the normalized density matrix for the pre-thermal physical states is given by

$$D = \sum_{j=1}^{\eta} \left( \frac{v}{m_{UV} c^2} \right)^2 |\phi_j^R><\phi_j^R|, \tag{5.17}$$

suggesting that with an equipartition of low mass modes in the pre-thermal cosmology, the degeneracy is

$$\eta = \left( \frac{v}{m_{UV} c^2} \right)^2 \approx \frac{427}{4}, \tag{5.18}$$

where the value has been calculated for a transition from a quantum quasi-stationary initial state, with vacuum compressional energy constituting the dark energy.
If both classical general relativity and quantum properties hold in the earliest stages, the FL equations have an initial inflationary period with the scale parameter satisfying
\[
\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N}{3}\rho_{tot},
\]
\[
\rho_{tot} \approx \left[\frac{(m^2 c^4 + 2\mu^2)^2}{8 f^2} + \frac{1}{2}(\Phi + B_0 \Phi)^2 + \frac{1}{8} f^2 \Phi^4 - \frac{1}{4} (m^2 c^4 + 2\mu^2) \Phi^4 \right] + \rho_{particles}.
\]

The dynamical equation relating the time derivatives of the component densities can be obtained from energy conservation
\[
T_{\mu \nu} = 0 = \dot{\rho}_{tot} + 3\frac{\dot{R}}{R}\rho_{tot}. 
\]
This equation describes the detailed thermalization of energy into the particulate states of present day cosmology, and gives the dynamical equation for the early phase transition. For the present, assuming a negligible contribution from the gauge boson, the field evolves prior to particulate fields using
\[
\frac{1}{R^3} \frac{d}{dt} (R^3 \dot{\Phi}) - \frac{1}{2} (m^2 c^4 + 2\mu^2 - f^2 \Phi^2) \Phi = 0. 
\]

If there is inflation to the thermal transition, Eq. (5.20) generates a field of the form
\[
\Phi = v_{UV} (1 - e^{-H_I (t - t_I)}),
\]
where \(v_{UV}\) is the UV scale vacuum expectation value of the field.

If the temporal progression defining the dynamics of the FL equations only begins after there is de-coherent energy density present, the initial stationary quantum state only becomes dynamical once the Higgs field begins to take a non-vanishing vacuum expectation value. For a stationary quantum to thermal transition, the time evolution begins as the stationary state evaporates
\[
\left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G_N}{3c^2} \rho_{thermal} \approx \begin{cases} 0 & t \to 0 \\ \frac{1}{4\pi^2} & t \to t_{UV}. \end{cases}
\]

Suppose the thermal density consists of that proportion of the energy in the broken symmetry state:
\[
\rho_{thermal} = \frac{\rho_{UV}}{v_{UV}^4 (hc)^3} \Rightarrow \frac{\dot{R}}{R} = \sqrt{\frac{8\pi}{3h^2 M_P c^2}} \frac{|\Phi|^2}{M_P c^2}. 
\]
The initial and end stage of thermalization in Eq. 5.22 can be determined:

\[
\Phi \approx \begin{cases} 
\frac{\hbar}{\mu} \sinh \frac{\mu t}{\hbar} & t \to 0 \\
 v_{\text{UV}} (1 - e^{-3 H_{\text{UV}} t}) & t \to t_{\text{RD}}.
\end{cases}
\]  

(5.24)

As can be seen, the late stage thermalization looks like inflation, but without the same early time behavior. The general temporal behavior is demonstrated in Fig. 9. The initial quasi-stationary state eventually undergoes

\[\text{Figure 9: Higgs field and boson field}\]

dynamics on microscopic time scales \(\tau \sim \frac{\hbar}{\mu}\), macroscopically damping to stationary behavior in times related to the cosmological expansion \(H_{\text{UV}}^{-1}\).

We will next briefly examine the thermodynamics of the transition. Taking an approach motivated in Reference [24], the energy density in Eq. 5.13 is taken as

\[
\mathcal{H} = \rho_{\text{UV}} + \frac{1}{2} (\Pi^2 + |\nabla \Phi|^2) - \frac{1}{4} (m_{\Phi}^2 + 2 \mu^2) \Phi^2 + \frac{1}{8} f^2 \Phi^4.
\]  

(5.25)

The Noether currents for a global symmetry with generators \(\{G_r\}\)

\[
\sqrt{-g} J^\mu_r = \frac{\partial L}{\partial (\partial_\mu \psi)} G_r \psi
\]  

(5.26)

satisfy \(\sqrt{-g} \frac{\partial}{\partial x^\mu} (\sqrt{-g} J^\mu_r) = 0 = J^\mu_r \mu\). Through co-moving surfaces with vanishing flux of this current, the charges \(N_r = \int J^\mu_r \sqrt{-g} d^3 x\) are conserved. The density \(\mathcal{H}\) has conserved charges with generators corresponding to \(2 \times 2\) identity and antisymmetric matrices. Energy conservation across the
transition requires that only a small constant energy density \( \rho_\Lambda \approx 0 \) remain after the symmetry is fully broken, giving the identification

\[
\rho_{UV} = \frac{1}{8} \left( m^2 + 2\mu^2 \right)^2 f^2.
\] (5.27)

The chemical potential is expected to vary through the values \( \mu^4 : 0 \to 2f^2(m_{UV}c^2)^4 \to 0 \) for the quasi-stationary state through thermalization to the electro-weak scale.

The excitations satisfy

\[
E^2_\pm = k^2 + 3\mu^2 + \frac{m^2}{2} \pm \left[ 4\mu^2k^2 + \left( 3\mu^2 + \frac{m^2}{2} \right)^2 \right]^{1/2}
\]

\[
\Rightarrow k < \mu \left\{ \begin{array}{cc}
\left( 1 + \frac{2\mu^2}{3\mu^2 + m^2/2} \right) k^2 + 6\mu^2 + m^2 & \text{massive mode} \\
\left( 1 - \frac{2\mu^2}{3\mu^2 + m^2/2} \right) k^2 & \text{Goldstone mode}.
\end{array} \right.
\] (5.28)

and the conserved charge in volume \( \Omega \) is given by

\[
\mathcal{N} = -\left. \frac{\partial V}{\partial \mu} \right|_{\langle \Phi \rangle = v} \Omega_{UV} \approx \frac{4\rho_{UV} \Omega_{UV}}{\mu_{UV}},
\] (5.29)

which corresponds to the number of particles. For small \( \mu \) (at low density) the charge is given by \( \mathcal{N} \approx \mu \nu^2 \Omega \), which implies that at later times \( \mu \approx \mu_{UV} \left( \frac{R_{UV}}{R} \right)^3 \). This demonstrates the expected late time vanishing of the chemical potential as a contribution to the Higgs field symmetry breaking scale.

We next explore arguments which should connect the scale of the microscopic fluctuations to macroscopic parameters. The fluctuation dissipation theorem in statistical physics connects the dissipation rate to stochastic fluctuations of the system. As brief demonstration examine a linear response from a stationary solution, with an additional stochastic gaussian variable driving fluctuations

\[
\frac{\partial \phi}{\partial t} = -\Gamma \frac{\delta L}{\delta \phi} + \zeta(\vec{x}, t),
\] (5.30)

where the stochastic variable satisfies \( < \zeta(\vec{x}, t) >= 0, < \zeta(\vec{x}, t) \zeta(\vec{x}', t') >= D \delta^3(\vec{x} - \vec{x}') \delta(t - t') \). The stationary, long time solution connects the RMS
fluctuation of the noise function to the dissipation of the order parameter and the temperature of the statistical bath $D = 2\Gamma k_B T$. The decays of the Higgs field are expected to drive the microscopic dissipation of the cosmological field. Ignoring the mass differences of the weak bosons, the decay of the Higgs field excitations into weak bosons are of the order

$$\Gamma_{H\rightarrow\text{weak bosons}} \approx \frac{e^2}{\hbar c} \frac{3}{128\pi \sin^2 \theta_W} \left( \frac{m_H}{m_W} \right)^2 m_W^2 c^2 \sim 2.5 \times 10^{-4} \left( \frac{m_H}{m_W} \right)^2 m_H^2 c^2$$

(5.31)

Setting fluctuations of the order of the microscopic vacuum expectation value $v$, if $k_B T_{UV} \approx 2100 GeV$, we expect $\Gamma \approx 14.5 GeV$. If this width corresponds to decays to the whole particle spectrum, we expect $m_H c^2 \approx 330 GeV$, whereas if the decays are just to weak bosons $m_H c^2 \approx 719 GeV$.

### 5.2 Power spectrum considerations

Finally, we explore the power spectrum of super-horizon scale fluctuations to examine the expected behavior of spatial modes as they satisfy measurability criteria. For a generic quantity $g$, the relation between the spatial modes $g_k$ and the power spectrum $P_g$ is given by

$$g(\vec{x}, t) = \int \frac{d^3 k}{(2\pi)^3/2} g_k(t) e^{i \vec{k} \cdot \vec{x}},$$

$$P_g(k) = \frac{k^3}{2\pi^2} |g_k|^2.$$

(5.32)

For massless scalar fields with an action given by

$$W = \frac{1}{2} \int \partial_\mu \psi \gamma^\mu \partial_\nu \psi \sqrt{-g} d^4 x$$

(5.33)

the modes of the fields $\psi_k$ satisfy

$$\ddot{\psi}_k + 3 \frac{\dot{R}}{R} \dot{\psi}_k + \frac{k^2}{R^2} \psi_k = 0.$$

(5.34)

Expanding the small perturbations using conformal time $\psi(x, \eta)$, the dynamics is given by

$$\frac{d^2 \psi_k(\eta)}{d\eta^2} + 2 \frac{R'(\eta)}{R(\eta)} \frac{d\psi_k(\eta)}{d\eta} + k^2 \psi_k(\eta) = 0.$$

(5.35)
In an inflation, Eq. 5.35 has a solution
\[ \psi_k = \Delta I \frac{e^{-ik\eta}}{\sqrt{2k}} \left( \frac{1}{R(\eta)} + \frac{iH_I}{kc} \right). \] (5.36)

The second term in the parenthesis is super horizon scale invariant. As previously mentioned, fluctuations generated by the electro-weak (microscopic) scale are expected to produce amplitudes of the order \( \Delta^2 \approx \left( \frac{\nu}{m_{UV}c^2} \right)^2 \sim 10^{-5} \). Next, scalar field evolution during radiation domination satisfies
\[ \psi_k = \tilde{\Delta}_{RD} e^{ik\tilde{R}(\eta)/\tilde{R}_{RD}} \frac{\tilde{R}_{RD}}{\sqrt{2k}} \left( \frac{4i + 4k - 2i}{3k} + 1 + \frac{1}{\eta - \eta_{MD}} \right), \] (5.37)

where \( \tilde{R} = \tilde{R}_{UV} [1 + (\eta - \tilde{\eta}_{RD})] \) independent of the scale of measurability \( \tilde{R}_{RD} \). The amplitude is scale invariant in wave number, but decreases with the inverse of the FRW scale factor. If the amplitude \( \Delta_{RD} \) varies as modes satisfy measurability constraints and is fixed by the scale of dark energy de-coherence in Eq. 3.3, the amplitude of the independent modes at last scattering grow to be of the same order in Eq. 3.6. Finally, consider the growth of field fluctuations during matter domination. A solution can be found that satisfies super-horizon (small k) and sub-horizon (large k) mode scales, to first order in the conformal time so that any scale invariant behavior can be extracted for a transition during matter domination. This solution takes the form
\[ \psi_k \Rightarrow_{\eta \rightarrow \eta_{MD}} \frac{\Delta_{MD}}{\sqrt{2k}} \left( \frac{4i + 4k - 2i}{3k} + 1 + \frac{1}{(\eta - \eta_{MD})/2} + e^{-k(\eta - \eta_{MD})} \right). \] (5.38)

The first term in the parenthesis is seen to contain a contribution to a scale invariant amplitude, precisely of the order measured for CMB fluctuations predicted by dark energy de-coherence in Eq. 3.4. The power spectrum of super-horizon scale fluctuations is dominated by a scale invariant term which, if driven by dark energy de-coherence as previously described, is of the size observed in CMB fluctuations
\[ P_\psi = \frac{k^3}{2\pi^2} |\psi_k|^2 \approx \frac{8}{9} \Delta_{MD}^2 \] (5.39)

This result is independent of whether there is an early inflationary stage. It only depends on a finite dark energy density which requires only the existence of a UV scale.
5.3 Conspiracy of coincidences

To end this discussion, we will briefly reiterate the numerical coincidences consistent with the UV dark energy scale assumption.

- The scale of the fluctuations generated in an inflationary era (Eq. 4.8), radiation era (Eq. 3.6), and matter/dust era (Eq. 3.7) are found to have an amplitude of the order observed in the CMB.

- The power spectra of the fluctuations generated during an inflationary era (Eq. 5.36) and matter/dust era (Eq. 5.38) are demonstrably scale invariant for super-horizon scale modes. Fluctuations generated during the radiation era (Eq. 5.37) are generated in a manner different from the others, but they grow to a flat amplitude at last scattering. However, one needs to question how any of the modes grow during epochs different from the one in which the fluctuations are produced.

- The decay width (conversion rate of condensate energy into thermal energy) during reheating is often argued to be the same as the expansion rate (see Riotto[20] Eq. 62). Using Riotto’s formula this gives a decay width essentially equal to the dark energy at a cosmological temperature corresponding to $T_{UV}$ (see Eq. 5.4).

- The number of microscopic degrees of freedom in the pre-thermal cosmology counted by equipartition of microscopic ground state expectation values for independent particle fields is of the observed magnitude in terms of the particle spectrum (Eq. 5.18).

- The fluctuations generated during the period of microscopic de-coherence which are related to the dissipation due to cosmological expansion are coincidental with the vacuum expectation value of the Higgs field for electro-weak symmetry breaking (Eq. 5.30).

6 Conclusions and Discussion

This presentation has demonstrated evidence that current cosmological observations can be accounted for by the hypothesis that at early times there was a phase transition from a macroscopically coherent state which
produced dark energy and fluctuations consistent with those observed today. We generally expect some form of cosmological quantum coherence at a scale far from the Planck scale, implying space-like correlations and phase coherence. Gravitational de-coherence at the UV scale defined by the dark energy gives evidence for the existence of a fixed microscopic scale of cosmological significance. Space-like/supraluminal correlations speak more to the quantum physics of the early universe than it does to a classical inflation.

The constraints of quantum measurement and flatness associates with any quantized energy scale a cosmological scale whose expansion rate is at most luminal. Using the measured value for the dark energy, some form of microscopic manifestation of gravitational physics is expected for densities on the TeV energy scale.

Since the results presented depend only on the physics of the transition, several prior scenarios have been explored which connect appropriately to the phase transition. In addition, numerical coincidences consistent with the various scenarios have been presented. In particular, an argument can be made that for the observed dark energy density, the cosmological density at de-coherence generates microscopic fluctuations of the order of the electro-weak symmetry scale, macroscopic damping of the order of the dark energy, and fluctuations that are fixed at (or grow to be of) the order observed at last scattering.

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