Estimating a nonparametric data-driven model of the lift on a pitching wing

M F Siddiqui, J Decuyper, P Z Csurcsia, J Ertveldt, T De Troyer, J Schoukens, M C Runacres

1 Thermo and Fluid Dynamics (FLOW), Vrije Universiteit Brussels, Pleinlaan 2, 1050, Brussels, Belgium
2 Department of Mechanical Engineering, Vrije Universiteit Brussel, Pleinlaan 2, 1050, Brussel, Belgium.

E-mail: muhammad.faheem.siddiqui@vub.be

Abstract. In aerodynamics, as in many engineering applications, a parametrised mathematical model is used for design and control. Often, such models are directly estimated from experimental data. However, in some cases, it is better to first identify a so-called nonparametric model, before moving to a parametric model. Especially when nonlinear effects are present, a lot of information can be gained from the nonparametric model and the resulting parametric model will be better. In this article, we estimate a nonparametric model of the lift force acting on a pitching wing, using experimental data. The experiments are done using the Active Aeroelastic Test Bench (AATB) setup, which is capable of imposing a wide variety of motions to a wing. The input is the angle of attack and the output is the lift force acting on the NACA 0018 wing. The model is estimated for two different types of input signal, swept sine and odd-random multisine signals. The experiments are done at two different pitch offset angles (5° and 20°) with a pitch amplitude of 6°, covering both the linear and nonlinear aerodynamic flow regime. In the case of odd-random multisines nonlinearity on the FRF is also estimated. We show that the level and characterisation of the nonlinearity in the output can be resolved through a nonparametric model, and that it serves as a necessary step in estimating parametric models.

1. Introduction

The determination of aerodynamic load on an aerofoil manoeuvring over a wide range of angles of attack and pitch rates is an important design problem for several aerodynamic applications. The problem is specifically important for helicopters and vertical-axis wind turbines (VAWT), whose performance is strongly influenced by unsteady flow separation. In both examples, blades may undergo large cyclic variations in the angle of attack relative to the flow. As long as the flow remains attached, the classical unsteady aerodynamic models (e.g. Wagner [1] or Theodorsen [2]) give a sufficiently accurate approximation of the aerodynamic loads. However, the load estimation for a high pitch rate, large-amplitude manoeuvres requires unsteady, nonlinear aerodynamic models.

An interesting aerodynamic phenomenon which occurs at a relatively high pitch rate and sufficiently high angle of attack is dynamic stall. The dynamic stall phenomenon is characterized by a periodic separation and reattachment of the flow over the wing where the stall angle is higher than the static stall angle [3][4]. Semi-empirical model such as [5] and [6] are specifically...
developed to capture dynamic stall. But these models require parameters which are determined experimentally or by numerical simulations [7]. These models are also specifically designed for periodic pitching manoeuvres and can not be directly applied to arbitrary flow conditions (e.g. gust).

Reduced-order modelling (ROM) has also shown promising results in developing unsteady nonlinear models for aerodynamic and aeroelastic applications [8][9]. In case of highly nonlinear aerodynamic systems which involve high amplitude oscillations, the performance of classical ROM approaches (e.g. POD and Volterra theory) does not give satisfactory results [10]. However, there is active ongoing research in developing ROM for dynamic stall [11].

Another approach to model nonlinear unsteady aerodynamics is the system identification approach in which data-driven (black-box) models are constructed from input and output data. Data-driven models have a wide range of model classes, each having their certain advantages [12]. One such model structure is the Polynomial Nonlinear State Space (PNLSS) model class [13]. Decuyper et al. [14] modelled the kinematics of an oscillating cylinder with the PNLSS technique using computational fluid dynamic (CFD) simulations data. In [15] the PNLSS model was developed for a pitching aerofoil using the experimental data in the attached flow regime. Before estimating a PNLSS model, it is paramount to estimate a nonparametric model first. This first step is often overlooked, but nonetheless essential for the construction of an accurate parametric PNLSS model. In this work, a nonparametric modelling framework is presented for an unsteady pitching wing in attached as well as separated flow regime. The input of the model is the wing angular displacement and the output is the resulting unsteady lift force. The nonparametric model is essentially a frequency response function (FRF) estimated by dividing the Fourier transform of output and input of the system. The FRF is known at the measured frequency lines (data points) and does not directly convey any information between those lines.

By selecting an appropriate model structure and tuning the parameters of the structure, a parametric model can be identified. The parametric model should be in good agreement with the measured data points and also holds information in between the lines.

The accuracy of the parametric model mainly depends on model complexity (number of poles and zeros) and noise in the data. These questions can be addressed by estimating the best linear approximation (BLA) of the nonlinear system along with the nonlinear distortions and disturbing noise in the presence of well-designed input signals [16, 17]. Developing a parametric nonlinear model may become computationally expensive due to a larger number of parameters to be optimised during the nonlinear optimisation process. Thus, it is always a relevant question whether to go for a nonlinear model or to use a linear model instead. In many practical cases, a system has weak nonlinear behaviour and a good linear model is sufficient. In the presence of a proper input signal, the nonparametric model can provide information about the level and type of nonlinearity of the system. This information can be used to reduce the number of parameters in the nonlinear optimisation step, thus reducing the cost of a parametric nonlinear model.

2. Experimental setup

The experiments are done using the active aeroelastic test bench (AATB) setup shown in Figure 1. The AATB setup allows to excite the wing with a variety of input signals, e.g. single sines, swept sines and multisines, at different pitch amplitudes and offset angles [18, 19]. The input is the angle of attack given to the wing by the AATB setup. The output is the lift force on the wing and is calculated by measuring the pressure on the pressure taps on the wings surface using the Scanivalve® pressure measurement system.

2.1. Post Processing

The control of the AATB is performed by a dSpace® real-time controller. The dSpace controls the motion of the wing. The dSpace is running at 2 kHz to accurately perform the position
control while the angle of attack is logged at 200 Hz. Reference (desired) input signals and actual input signals are both measured and the difference between the reference and the actual signals reveals the level of nonlinearity in the actuation mechanism. The output can be corrected for the nonlinearities of the AATB setup as explained in Section 3.2, provided that the reference and actual angle of attack are both measured.

The pressure is measured using the ZOC33/64Px Scanivalve® measurement system. The pressure and the resulting output signal is also sampled at 200 Hz. The input and the output signals are synchronised by measuring the lag between the signals. The lag is measured by estimating the cross-correlation between the two signals.

3. Nonparametric Model

In this work, we estimate the Frequency Response Function (FRF), which is obtained for the periodic signal by the division of the output spectrum \( L \) and the input spectrum \( U \),

\[
G = \frac{L}{U}.
\]

where \( U \) and \( L \) are the Fourier transform of the angle of attack \( \alpha(t) \) and the lift coefficient. The experiments are performed at low as well as high angle of attack with high enough pitch amplitude and a range of pitch rates. At the high angle of attack, a significant nonlinear contribution in the output data is expected due to the boundary layer separation and reattachment. The FRF measurement in the presence of nonlinearities strongly depends upon the choice of the input signal [20]. In this work, the FRF is estimated using the swept sine signal and the odd-random multisine signal. These signals are not commonly used in aerodynamics for estimating the FRF, therefore, will be briefly discussed below.

The swept sine signal is a periodic signal that continuously varies its frequency between two limits. The linear swept sine signal is given by

\[
\alpha(t) = A \sin \left[ 2\pi \left( \frac{c}{2} t^2 + f_0 t \right) \right], \quad c = \frac{f_1 - f_0}{T}
\]  

where \( T \) is the time it takes to sweep from the base frequency \( f_0 \) to the final frequency \( f_1 \). \( A_k \) is the amplitude of the swept sine.

The multisine signal is composed out of \( N \) harmonically related sines, with frequencies that are integral multiples of a base frequency \( f_0 \)

\[
\alpha(t) = \sum_{k=1}^{N} A_k \sin(2\pi k f_0 t + \phi_k)
\]

The amplitude \( A_k \) of each harmonic can be explicitly selected by the user. The phases \( \phi_k \) are randomly chosen between \([0 - 2\pi]\). In contrast to the swept sine signal, where all of the frequency
lines of the excitation band are excited, only odd lines are excited in the odd-random multisine signal i.e. \( k = [1, 3, ..., N] \). Some of the odd lines are left unexcited \( (A_k = 0) \), referred to as the detection lines.

The FRF, which is the linear response of the system, is estimated only for the excited frequency lines. The output on the non-excited lines will be due to the nonlinear response of the system or noise in the data. In the case of odd-random multisine input signal, we can use the Best Linear Approximation (BLA) framework which allows us to distinguish between the stochastic nonlinearities and noise in the data.

### 3.1. Best Linear Approximation (BLA)

The BLA is well known technique to obtain the FRF in the system identification community. The methodology of BLA is described in detail in \[21\][22][17]. The Best Linear Approximation (BLA) of a nonlinear system is an improved technique to obtain the FRF which minimises the mean squared error between the output of the nonlinear system and the output of the linear model.

In the case of odd-random multisine signal, Equation (2), it is possible to control the amplitude and phase of each excited frequency. The BLA framework requires multiple phase realisations of the odd-random multisines. The realisation is referred to as the repetition of the input signal with the same excited lines but randomly selected phases. Since the phases of the excited lines are randomly selected, the nonlinear contribution on the excited lines will also appear in a stochastic manner. Averaging the FRF of different realisations will bring the level of stochastic nonlinear distortions at a given frequency line to its expected zero mean value, referred to as the BLA. The variance of the FRF over different phase realisations will provide an estimate of the stochastic nonlinear distortions. In the case of noise in the measurement data, the stochastic nonlinear distortions will also contain noise. The noise can be separated from the nonlinearities by using multiple periods of the input signal. The amplitude and phase of the excited frequencies in each period for a given realisation is exactly the same. The FRF is only obtained on the excited lines, thus averaging the input signal over different periods improves the signal to noise ratio and the variance over different periods indicates the noise on the FRF. This multidimensional averaging \[23\] procedure to obtain the BLA is shown in the Figure 2. Using the multidimensional averaging procedure, the output can be written as

\[
L = (\hat{G}_{BLA} + \hat{G}_S + \hat{G}_N)U
\]  

where, \( G_{BLA} \) is the linear model, \( G_S \) is the stochastic nonlinearities and \( G_N \) is the noise. In case of multiple periods, transient contaminated periods can be simply discarded before the multidimensional averaging procedure.

### 3.2. Estimating nonlinearity in the actuator

The basic idea of using odd random-phase multisines is to detect even and odd nonlinear contributions in the BLA. Power at the unexcited even lines will indicate even nonlinearities while power at the unexcited odd lines (detection lines) stems from odd nonlinear behaviour. However, the interpretation of the nonlinearity on the excited lines at the output can be jeopardized, if the actuator is imperfect. If there is power on the detection lines of the input signal, then the output signals will contain not only the nonlinearities due to the system but also the nonlinear contributions of the actuator. In the case of nonlinearity in the actuator, the indirect method \[24\] may be used to estimate the \( \hat{G}_{BLA} \). In the indirect method, without changing the multidimensional averaging procedure, the FRF is calculated as \( \hat{G} = S_{LR}/S_{UR} \), where \( S_{LR} = L/R \) and \( S_{UR} = U/R \), where \( R \) is the Fourier transform of the reference signal.

The output can be corrected for the nonlinearity in the actuator by first simulating the output on the non excited lines using the interpolated BLA and the input signal \( L_{simulated} = \hat{G}_{BLA}U \).
The output is corrected at the non-excited lines as follow: \( \hat{L}_{\text{corrected}} = \hat{L}_{\text{measured}} - \hat{L}_{\text{simulated}} \).

The proposed framework is suitable to clean up the output spectrum from the impurities of the actuator.

4. Estimating the FRF

The experiments are performed on a NACA-0018 wing with a chord of 0.3 m and a span of 0.53 m. The experiments are performed at a Reynolds number of 285000. This is representative of small VAWTs [25], but requires CFD or field test for up-scaling. However, the given Reynolds number is relevant to study the important lift characteristics of the wing. The excited frequency band for swept sine and multisine is from 0.01 - 3 Hz which corresponds to a reduced frequency \( k \) of 0.0001-0.19. The reduced frequency is defined as

\[
k = \frac{\pi fc}{U_\infty}
\]

where \( U_\infty \) is the free stream velocity. As mentioned earlier, the choice of the input signal is very important in the estimation of a nonparametric model. In modelling unsteady aerodynamics, usually, a monosine input signal is used. While estimating a model that is valid for the range of frequencies of interest, a swept sine is preferred as it is essentially a combination of monosines with increasing/decreasing frequencies. The design of a swept sine signal is much more restricted in amplitude and phase compared to multisine signal. The multisine signal appears similar to a swept sine signal in the frequency domain but it resembles an arbitrary signal in the time domain. Arbitrary signals are considered more robust in estimating the frequency response of nonlinear systems. This gives the multisine signal some advantages over a swept sine signal. However, both of the signals will invoke a different physical response of unsteady pitching wing and it will be interesting to compare the output of the two signals. Both of the signals are excited in the same reduced frequency range, pitch offset and pitch amplitude.

4.1. Baseline Quasi-static results

The quasi-static lift coefficient is obtained by pitching the wing from -26° to 26° at a very slow pitch rate of 0.52 deg/s (\( k=0.006 \)). The pitch-up motion is shown by a black line and pitch-down motion is shown as a grey line in Figure 3. The lift curve shows static hysteresis between 14°
and 21° which is mainly due to the formation of the leading-edge separation bubble. The sudden drop in lift coefficient at 21° is due to the bursting of the laminar separation bubble near the leading edge of the wing. Beyond 21° the lift curve follows the grey line. When the wing is in pitch-down motion the leading-edge separation bubble does not appear as in the pitch-up motion, thus the boundary layer is reattached much later at around 14°. The static hysteresis is a characteristic of thick leading edge aerofoil. It depends on the history of the aerofoil position [26][27].

The multisine and swept sine signal used for the nonparametric model are at two different pitch offset angles 5° and 20° with the pitch amplitude of 6°. The 20° pitch offset signal is expected to contain significant aerodynamic nonlinearities compared to the 5° case.

Figure 3. Phase-averaged lift curve, pitching up (black), pitching down (grey).

4.2. Estimating the FRF for swept sine signal

The swept sine signal is commonly used to quickly estimate the frequency response of a system over a wide range of frequencies. In this work, a swept sine signal with a constant pitch amplitude of 6° is used at two different pitch offset angles of 5° and 20°. The frequency is swept from 0.01 Hz to 3 Hz (0.0001 < k < 0.19) with a sweep rate of 0.03 Hz/sec. Three periods are measured for the input signal at each pitch offset. The input, the output and the FRF spectrum of the swept sine signal at the two different pitch offset angles are shown in the Figure 4.

The input spectrum is flat with a slight overshoot at the edges of the excitation band (typical behaviour of a swept sine). The noise on the measurement is estimated by calculating the variance over the multiple periods. The noise level on the input signal is less than 1 %, indicating a good signal to noise ratio. The noise level on the output and the FRF increases with the increase in reduced frequency at 5° pitch offset case while the noise level is higher for 20° pitch offset throughout the frequency range. It is also interesting to note that the FRF for 5° pitch offset is smoother than 20° pitch offset case. The noisier FRF is indicative of the nonlinear distortion in the output data at a higher offset angle due to the flow separation on the wing surface. However, separating the nonlinear contribution from noise is not possible for swept sine signal. Therefore, in Section 4.3 we also used multisine signals.

The FRF spectrum at 5° pitch offset shows a decreasing trend with the increased reduced frequency. This is somewhat similar to a low pass behaviour. Similar characteristic is reported by [28] for the frequency response function of circulatory lift using the Theodorsen function. The FRF spectrum at 20° appears to be flat and noisier compared to 5° offset case, but the
response at lower reduced frequencies ($< 0.1$) shows poor signal to noise ratio. The bending frequency of the wing and the vortex shedding frequencies are much higher than the excited frequencies. Thus, resonances in the FRF are not expected. However, more detailed work is required to understand the physical nature of the FRF for the $20^\circ$ pitch offset case.

4.3. Estimating the FRF/BLA for a multisine signal
The multisine signal is excited from 0.01 to 3 Hz ($0.0001 < k < 0.19$) with a pitch amplitude of $6^\circ$ and at two different pitch offset angles $5^\circ$ and $20^\circ$, similar to the swept sine signal. There are seven phase realisations of the odd-random multisine signal with three periods in each realisation. In the case of the swept sine signal, all of the frequencies are excited while for the multisine signals only the odd frequency lines are excited. Some of the odd lines are left unexcited to distinguish between odd and even nonlinearities as explained in Section 3.

The input, output and the FRF spectrum for the two different pitch offset angles are shown in Figure 5. The input/output spectrum also contains the power on non-excited odd and even lines. This is the indication of nonlinearity in the actuation system which can be due to friction or free-play in the pitch bearing. This stochastic nonlinear distortion in the input spectrum can be estimated by identifying the BLA using the reference signal as input and actual input signal as the output. The estimated nonlinearity on the input is shown in Figure 6. The output can be corrected for the nonlinear distortion in the actuator using the technique explained in Section 3.2.

The noise and stochastic nonlinear distortion in the output at low offset angle increases with the increase in the reduced frequency. At $20^\circ$ offset angle, nonlinear distortion level is higher at the low reduced frequencies and become flat at higher reduced frequencies. It is also interesting to note that the even nonlinearity is at the level of odd nonlinearity. This information is really important for developing a parametric nonlinear model. In the PNLSS model, the nonlinearity is modelled by monomial basis function and presence of even and odd nonlinearities in the output suggest that both even and odd basis function should be used to fully estimate the nonlinear response of the system [14].

The BLA estimates at both of the pitch offset angles is similar to that of the swept sine input signal. There is a 10 dB difference between the noise and nonlinear distortion on the estimated BLA for the two offset cases. The level of noise and nonlinearity for the higher offset angle at lower reduced frequency range ($k < 0.1$) is higher than that of lower offset case. This is due to nonlinear aerodynamic behaviour of a pitching wing caused by boundary layer separation. However, a similar response was expected for the higher reduced frequency range, but the noise and nonlinearities are at the same level for both of the offset angles cases. In the proposed BLA framework, the nonlinearity in the lift forces is assumed to be periodic, more specifically the system is considered to have periodic input for the same periodic output (PISPO). Thus any non-PISPO nonlinear contribution in the data will manifest itself as noise in the FRF. This issue can be resolved by using an input signal with a large number of periods and considering only those periods for the BLA which shows a similar time-domain response. This will result in further reducing the noise level of the FRF and a better estimate of nonlinear distortion.

4.4. The FRF comparison
The FRF estimates of the swept sine signal and multisine signal are compared in Figure 7 along with noise on the FRF. The FRF of both of the signals at the $5^\circ$ pitch offset is almost similar while the FRF of multisine signal at the $20^\circ$ pitch offset is significantly smoother than the FRF of swept sine signal. The FRF of the multisine signal is the averaged response of seven phase realisations, thus, any random nonlinear response is significantly eliminated from the FRF and the resulting FRF is the best linear approximation of the nonlinear system. The noise level shown in Figure 7 differs from the noise level shown in Figure 4 and 5. The total
Figure 4. The input spectrum, the output spectrum and the FRF spectrum of the swept sine signal with the pitch amplitude of 6° at the two different pitch offset angles 5° (left) and 20° (right).

The number of periods in both of the signals is different, thus, the noise level is normalized with a single period for comparison. The noise level for the two offset cases is similar for both of the signals. The noise for the swept sine signal at the higher offset case is more dispersed.
Figure 5. The input spectrum, the output spectrum and the FRF spectrum of the multisine signal with the pitch amplitude of 6° at the two different pitch offset angles 5° (left) and 20° (right).

as it also contains much more nonlinear contributions.
5. Conclusions

A framework is developed to estimate a nonparametric model of the lift force acting on a pitching wing using experimental data. The frequency response function (FRF) is estimated using multiple periods of swept sine and odd-random multisine signals. In the case of odd-random multisine signal, multiple phase realisations of the input signal are used. The phases of the excited frequencies are chosen at random for each realisation, thus, the nonlinear response of the system on the excited lines is also expected to be stochastic. Averaging the multiple realisations will give the best linear approximation (BLA) of the underlying linear system. The variance over multiple realisations gives the estimate of the nonlinear distortion in the system. The noise on the data is obtained by estimating the variance over multiple periods of the signal.

The FRF of the swept sine signal and the BLA of odd-random multisine signal for low pitch offset are similar and behave somewhat as a low pass filter. In case of the higher pitch offset angle, the nonlinear contributions are expected to be higher as the wing is pitching beyond static stall angle. The BLA of the multisine signal for higher pitch offset is smoother due to averaging...
over multiple phase realisations compared to the FRF of the swept sine signal. The frequency response of the two signals for the high offset case show a slight dip at lower reduced frequencies \((k < 0.1)\) and becomes flat at the higher reduced frequencies. This behaviour needs to be further investigated.

The associated computational cost of the nonparametric modelling, discussed in this work, is really low. It took less than a second to estimate the nonparametric model along with the nonlinear distortion and noise for the odd-random multisine signal. If the nonlinearity is very close to the FRF, then a nonlinear parametric model is necessary for accurate unsteady lift force estimation. The proposed nonparametric modelling framework also characterises the nonlinear distortion in the data as both odd and even. This information is useful in reducing the number of parameters to be optimised in the parametric modelling thereby reducing the computational cost of the parametric model. The finding of the nonparametric modelling and nonlinear distortion analysis will be used to develop a polynomial state space (PNLSS) model of the lift force for an unsteady pitching wing.

References
[1] Wagner H 1925 ZAMM—Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik 5 17–35
[2] Theodorsen T 1979 General Theory of Aerodynamic Instability and the Mechanism of Flutter Tech. rep. National Advisory Committee for Aeronautics; Washington, DC, United States
[3] McCroskey W J, Carr L W and McAlister K W 1976 AIAA Journal 14 57–63 ISSN 0001-1452
[4] Ericsson L and Reding J 1988 Journal of fluids and structures 2 1–33
[5] Leishman J G and Beddoes T S 1989 Journal of the American Helicopter Society 34 3–17
[6] Goman M and Khrabrov A 1994 Journal of Aircraft 31 1109–1115
[7] Sanchez Martinez M, Boutet J, Amantelese X, Terrapon V and Dimitriadis G 2019 Computation of Leishman-Beddoes model parameters using unsteady RANS simulations Proceedings of the AIAA SciTech 2019 Forum and Exhibition
[8] Balajewicz M and Dowell E 2012 Journal of Aircraft 49 1803–1812
[9] Huang R, Hu H and Zhao Y 2014 AIAA journal 52 1219–1231
[10] Silva W A 2007 Recent enhancements to the development of CFD-based aerelastic reduced-order models Proceedings of the 48th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics, and Materials Conference, April
[11] Glaz B, Friedmann P P, Liu L, Cajigas J G, Bain J and Sankar L N 2013 AIAA journal 51 910–921
[12] Schoukens J and Ljung L 2019 IEEE Control Systems Magazine 39 28–99
[13] Paduart J, Lauwers L, Swevers J, Smolders K, Schoukens J and Pintelon R 2010 Automatica 46 647–656 ISSN 0005-1098
[14] Decuyper J, De Troyer T, Runacres M C, Tiels K and Schoukens J 2018 Mechanical Systems and Signal Processing 98 209–230 ISSN 0888-3270
[15] Siddiqui M F, Ertveldt J Decuyper J, De Troyer T Schoukens J and Runacres M C 2018 Development of a nonlinear state space model of the forces acting on an aerfoil oscillating in pitch Proceedings of the International Conference on Noise and Vibration Engineering ISMA pp 547–558
[16] Pintelon R and Schoukens J 2012 System identification: a frequency domain approach (John Wiley & Sons)
[17] Csurcsia P Z 2013 Pollack Periodica 8 153–165
[18] Ertveldt J, Pintelon R and Vanlanduit S 2016 AIAA Journal 54 3265–3273 ISSN 0001-1452
[19] Ertveldt J, Schoukens J, Pintelon R, Vanlanduit S, De Pauw B and Rezayat A 2015 Design and testing of an active aerelastic test bench AATB for unsteady aerodynamic and aerelastic experiments 56th AIAA/ASCE/ASH/ASC Structures, Structural Dynamics, and Materials Conference p 1857
[20] Schoukens J, Pintelon R, Rolain Y and Dobrowiecki T 2001 Automatica 37 939–946
[21] Schoukens J, Swevers J, Pintelon R and Van Der Auweraer H 2004 Mechanical Systems and Signal Processing 18 727–738
[22] Schoukens J, Pintelon R and Rolain Y 2012 Mastering system identification in 100 exercises (John Wiley & Sons)
[23] Csurcsia P Z Peeters B and Schoukens J 2020 The best linear approximation of MIMO systems: First results on simplified nonlinearity assessment Nonlinear Structures and Systems, Volume 1 (Springer) pp 53–64
[24] Pintelon R and Schoukens J 2012 IEEE Transactions on Instrumentation and Measurement 62 1334–1345
[25] Tummala A, Velamati R K, Sinha D K, Indraja V and Krishna V H 2016 Renewable and Sustainable Energy Reviews 56 1351 – 1371
[26] Williams D R, Reißner F, Greenblatt D, Müller-Vahl H and Strangfeld C 2017 AIAA Journal 55 403–409
[27] Timmer W 2008 Wind engineering 32 525–537
[28] Taha H E and Rezaei A S 2020 Journal of Fluids and Structures 93 102868