Emergence of simple patterns in many-body systems: from macroscopic objects to the atomic nucleus*

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Abstract. Strongly correlated many-body systems often display the emergence of simple patterns and regular behaviour of their global properties. Phenomena such as clusterization, collective motion and appearance of shell structures are commonly observed across different size, time, and energy scales in our universe. Although at the microscopic level their individual parts are described by complex interactions, the collective behaviour of these systems can exhibit strikingly regular patterns. This contribution provides an overview of the experimental signatures that are commonly used to identify the emergence of shell structures and collective phenomena in distinct physical systems. Examples in macroscopic systems are presented alongside features observed in atomic nuclei. The discussion is focused on the experimental trends observed for exotic nuclei in the vicinity of nuclear closed-shells, and the new challenges that recent experiments have posed in our understanding of emergent phenomena in nuclei.

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1 Introduction

Our understanding of the universe is intimately related to our description of many-body systems. The knowledge of the fundamental particles and forces of nature is as important as our ability to understand how these building blocks are organized to form complex systems. Remarkably, the emergence of simple and regular patterns are common features observed in strongly correlated many-body systems \textsuperscript{[1–9]}. At the microscopic level, the individual parts of different physical systems can be described by fundamentally different interactions, however, their collective behaviour can exhibit similar patterns. These seemingly simple regularities of certain properties of a physical system tend to suggest the existence of underlying symmetries and allows simple models to provide a good description of the observed data \textsuperscript{[9–13]}. However, the link between these models and their microscopic interactions is an open question in many fields of physics.

The recent progress of both experimental and theoretical developments have allowed an unprecedented connection between reductionist and emergent views of nature. Advances in many-body methods and the rapid development of computing power have provided new paths towards the ab initio description of macroscopic phenomena. Theoretical developments are motivated by the ambition of a first principles description of emergent phenomena, yet this reductionist approach is deeply motivated by empirical observations \textsuperscript{[14,15]}. The emergence of unexpected phenomena is uniquely accessed through experiments. A deeper understanding of the microscopic origin of the observed physical phenomena is achieved through systematic experimental studies confronted with the theoretical descriptions.

This article presents a short overview of experimental signatures that are commonly used to characterize the emergence of phenomena in different physical systems. Various examples of objects from the human size scale down to the femtometer scale are presented. The discussion is centered on the observables that are used to indicate the emergence of phenomena such as shell structures and “magic” numbers - integer number of constituents with notably different properties. Albeit not exhaustive, an effort is made to include citations that could be useful to direct interested readers to the relevant literature.

The manuscript is divided in two main parts: The first part provides a brief description of selected examples that illustrate the emergence of regular patterns in macroscopic systems and their relation with similar patterns observed
in the atomic nucleus. The second part is focused on the experimental signatures used to discuss the emergence of collective phenomena and shell structures in nuclei. Commonly discussed properties such as binding energies, nuclear charge radii, excitation energies and transition probabilities are presented. The discussion is expanded using recent experimental results obtained for the ground-state properties of nuclei in the neighborhood of nuclear shell closures. Finally, an emphasis is made on the trends and open questions that the new observations pose for our current understanding of nuclear structure in different regions of the nuclear chart.

2 Emergence of simple patterns in many-body systems

Throughout nature, driving forces give rise to optimization problems for the arrangement of constituents in many-body systems at almost every size scale. On biological scales, this manifests in a variety of collective phenomena and pattern formation. Such as in the phyllotaxis of plants, where simply growth patterns appear in the arrangement of leaves or flowers around a plant step. One particularly striking example is observed in the growth of seeds in a sunflower head, in which the number of spirals of seeds follows the Fibonacci sequence. A large variety of patterns emerge in smaller systems as a consequence of the optimal arrangement of their constituents, from the clustering in framboidal pyrite to the crowding of molecules in cells, where simply growth patterns appear in the arrangement of leaves or flowers around a plant step. One particularly striking example is observed in the growth of seeds in a sunflower head, in which the number of spirals of seeds follows the Fibonacci sequence. A large variety of patterns emerge in smaller systems as a consequence of the optimal arrangement of their constituents, from the clustering in framboidal pyrite to the crowding of molecules in cells.

The energetic efficiency of these more stable magic number configurations leads to their prevalence in biological systems also, such as with the magic number of capsid proteins which appear to form virus capsid structures. A summary of different systems in nature which exhibit these magic numbers of stability is shown in Table 1. Experimental signatures and typical size of the different systems are shown in the same table. For non-deformable constituents with their packing constrained by symmetric polyhedral shapes, the magic numbers that appear can be determined for a system with constituents of any size using purely geometric considerations, and they appear in nature with these numbers when this is the case. However, the identification of naturally occurring magic numbers from a

distribution becomes more challenging for greater length scales, as the likelihood of finding ensembles with a similar number of homogenous constituents decreases.

Those constituents between magic numbers, can also be said to belong to a ‘shell’, in analogy to the electronic shells of atoms or for nucleons in atomic nuclei. In some cases this is reflected by the spatial arrangement of the constituents. This has been observed for example in ‘dusty’ plasmas, where charged dust particles (on the micrometer scale) can self assemble into a plasma crystal arrangement with a radial spherical shell distribution of particles, with the total system on the scale of millimeters. Such mesoscopic systems are often called ‘artificial atoms’ due to their close resemblance with atomic systems. The examples enlisted in Table 1 for dust particles, occur for particular experimental conditions. These experiments have several highly tunable parameters, which can result in different sequences of magic numbers. Similar self arrangement and appearance of magic numbers has also been observed in 2-dimensional mesoscopic experiments using micrometer-sized superconducting disks.

At the scale of hundreds of micrometers, polystyrene spheres (colloidal particles) with diameters of around 200 nm have been observed to self assemble into colloidal clusters. While the interaction has a complicated description including surface chemistry, capillary forces and entropy maximization, and the presence of depletants, these clusters were also found to exhibit magic number configurations. These specific numbers of colloidal particles were found to result in a higher thermodynamic stability, as observed through decreased evaporation rates, similar to that of the total energy of an unstable nucleus as reflected in its half-life. Due to the absence of a repulsive force, these colloidal systems can range from a few particles to billions of particles (colloidal crystals).

Perhaps the systems with the most in common with the atomic nucleus are atomic clusters, an area of physics which has historically benefited from analogies with nuclear models. Clusters of atoms were observed to have magic numbers of enhanced stability reflected in their produced mass abundance spectra (see Table 1). The electronic structure of the constituent atom ultimately dictates the properties of the atomic clusters, however phenomenological models have been developed to provide a good description of the observed magic numbers, similar to the shell model of the atomic nucleus. A ‘wine-bottle’ shaped potential used to describe these atomic clusters was adapted from the Woods-Saxon potential. This potential predicted ‘super shells’ to appear as the number of atoms in the clusters approaches $N = 1000$, due to higher-order stabilizing effects, analogous to the predicted islands of stability of heavy nuclei. The predicted super-shell magic numbers were soon observed in sodium clusters. Deformation also has an analogous role in these clusters as in atomic nuclei, where the most stable clusters have spherical deformation and those between shell closures have
Table 1. Experimental signatures of the emergence of shell structures and “magic” numbers in different many-body systems. The size scales and the common observables that are used to characterize the properties of each systems are indicated. Here the ellipses (“...”) are used to denote that additional magic numbers have been omitted for space.

| Constituent          | Size System                       | Size Observable(s) | Magic numbers | Refs. |
|----------------------|-----------------------------------|--------------------|---------------|-------|
| Spheres              | Any Spherical confinement         | Any Density        | 6, 12, 21, 25, 38 ... | 1     |
| Sunflower seeds      | ∼ 1 cm Sunflower head             | 5-50 cm Number of spirals | 3, 5, 8, 13, 21, 34, ... | 2, 16, 17 |
| Dust particles       | µm 3D plasma crystal              | mm Radial distribution | 2, 21, 60, 107 .. | 3     |
| Superconducting disks| µm Vortex shells                  | 5 µm Radial distribution | 5, 16, 32 | 4     |
| Polystyrene spheres | 244 nm Colloidal cluster          | ∼ 2.8 µm Evaporation rate | 135, 297, 851, 801, 1283, 2583 ... | 5     |
| Spherical capsid protein | ∼ 5 nm Virus capsid structure   | ∼ 50 nm Abundance  | 15, 17, 18, 42 | 18, 20 |
| C atoms              | 170 pm Fullerenes                 | ∼ 6 nm Mass abundance | 60, 70, 72, 76, 78, 84 .. | 7, 21, 22 |
| H2O                  | 275 pm Electron-bound water clusters | ∼ 3-20 Å Mass abundance | 2, 6, 7, 11 | 23    |
| Xe atoms             | 216 pm Atom clusters              | ∼ 2-10 Å Mass abundance | 13, 16, 19, 25, 55, 71, 87, 147 ... | 24, 25 |
| Na atoms             | 227 pm Atom clusters              | ∼ 2-10 Å Mass abundance | 8, 20, 40, 58, 92 ... | 26, 28 |
| Electrons            | fm Atoms                          | 31-348 fm Ionization energy | 2, 10, 18, 36, 54 | 30    |
| Nucleons             | fm Nuclei                         | 1-10 fm Binding energies, t1/2, < r² >, B(E2), E2+, Q8, μ, Solar abundances, Neutron capture σ | 2, 8, 20, 28, 50, 126 .. | 31, 35, 36, 37, 38, 39, 40 |

Oblate or prolate deformation [8,90]. The Nilsson model of the atomic nucleus [91] has been adapted to describe axially deformed clusters, known as the Clemenger-Nilsson model [92]. Giant dipole resonances of atomic nuclei also have a counterpart in these cluster systems, in the form of plasma resonance frequencies [94,95]. Taking the example of the sodium clusters, many of the observables corroborate the same set of magic numbers [26,28,96,98] which are of electronic origin. A modified set of magic numbers was found in the melting temperatures of the clusters [99]. However, this required an additional interpretation considering the geometric shells of the positions of the atomic nuclei alongside the electronic shells, due to the importance of the positions of the atomic nuclei in the melting process [29]. This highlights how experimental observables can probe very different aspects of the same physical system, leading to different sequences of magic numbers for different properties of the same sys-
ties as a function of the neutron and proton numbers are
state, \( S \) open shell [108]. ii. a relatively large two-nucleon separa-
sures, and a larger increase through the filling of the new
of the charge radius as nucleons are added beyond a shell
= 20, 28, 50, 82 and 126, there is a pronounced change
N charge radius, \( \langle r^2 \rangle \).

3.1 Experimental signatures of shell structures

The signatures of nuclear shell structures are manifested in different observables [103,107,114,117]. The numbers of nucleons that completely fill nuclear closed-shells are the so-called “magic” numbers. Nuclei with a magic number of nucleons are commonly observed to have the following signatures: i. a relatively small mean-squared charge radius, \( \langle r^2 \rangle \). As seen in Figure 1 at nucleon number \( N = 20, 28, 50, 82 \) and 126, there is a pronounced change of the charge radius as nucleons are added beyond a shell closure (“kink”), with a smooth increase towards shell closures, and a larger increase through the filling of the new open shell [108]. ii. a relatively large two-nucleon separation energy, \( S_{2n} \); iii. a small quadrupole moment value, \( Q_s \); iv. a high excitation energy of the first \( 2^+ \) state, \( E_{2^+} \); and v. a small transition probability to the first \( 2^+ \) excited state, \( B(E2) \). A compilation of these experimental properties as a function of the neutron and proton numbers are shown in Figure 1 and Figure 2 respectively. The data corresponding to different isotones are shown in Figure 1 using bars of different colors to indicate the magnitudes of the observables for each isotope, the same is shown in 2 as a function of atomic number.

The changes of the mean square charge radii when two neutrons are added, \( \Delta \langle r^2 \rangle (2n) \), are presented in Figure 1. The analogous differences when two protons are added, \( \Delta \langle r^2 \rangle (2p) \), are shown in Figure 2 however the data in this case is relatively sparse as the charge radii of many elements have not yet been measured. At magic number of nucleons these differences exhibit a minimum value, with local maxima occurring after crossing the closed shell. As the magnitude of \( Q_s \), \( B(E2) \), and \( E_{2^+} \) scales with the atomic number and the nuclear size, these parameters were normalized in order to compare light and heavy nuclei on the same scale. The experimental values of \( Q_s \) and \( B(E2) \) were scaled to the dimensionless values \( Q_s/Z R \) and \( B(E2)/Z^2 R^2 \), with \( Z \) the proton number and \( R = 1.18 A^{1/3} \) the droplet-model radius. Normalized observables present minimum values around the nucleon numbers 28, 50, 82 and 126, with a clear correlation seen in the trends of all observables. For some isotopes, additional local minima appear around nuclear numbers 2, 8, 16, 20, and 40. Figure 1(iv), for example, shows bars of different color at \( N = 20 \), indicating that nuclei with the same number of neutrons, such as \( ^{24}\text{Mg} \) and \( ^{48}\text{Ca} \), have very different \( E(2^+) \) values [143]. The isotopes with magic nucleon numbers have relatively high binding energy, and their charge distribution exhibit smaller variations with respect to the spherical shapes (small quadrupole moments). The nuclear charge radius commonly increases with the number of nucleons, but the slope of the increase is notably smaller approaching the nuclear closed shells. These nuclei are more difficult to excite than their neighbors, which is evidenced by their relatively high excitation energies and low excitation probabilities.

The properties of light nuclei (\( A<20 \)) exhibit different patterns with respect to the number of nucleons. These nuclei do not display the regular trends observed in heavier isotopes. The addition or removal of a single nucleon can produce drastic changes on the properties of these few-nucleus systems. Common patterns are more evident in heavy nuclei, with a few points outside the general trends. Some particular isotopes, as in the region around \( Z = 40, N = 60 \) and \( Z = 62, N = 90 \), are considered to present a rapid onset of deformation [103,114]. Interestingly, collective phenomena such as shape coexistence and phase transitions observed for nuclei in the region \( Z = 62, N = 90 \) have been suggested to exhibit analogous features as those for silicon clusters, which are governed by very different interactions [145].

3.2 Simple patterns in complex nuclei

Nuclear electromagnetic moments such as the magnetic dipole and electric quadrupole moment provide complementary insights into the microscopic and collective properties of nuclei [146,147]. In fact, electromagnetic moments...
played a key role in motivating the most basic models of nuclear physics: the nuclear shell model [111], and nuclear deformation [148–150]. Systematic experimental studies of isotopes around nuclear closed shells have revealed surprisingly simple trends in the evolution of nuclear ground-state electromagnetic properties as a function of the neutron number [115,118–120,146,151–153].

Nuclei in the vicinity of the tin isotopes give outstanding examples of simple patterns. The electromagnetic properties of these complex nuclei, with around 50 protons and more than 50 neutrons, seem to be described by a single particle in a nuclear orbital. The experimental nuclear $\gamma$-factor (the ratio between the dipole magnetic moment and the nuclear spin) and electric quadrupole moments of cadmium ($Z = 48$), indium ($Z = 49$), and tin ($Z = 50$) isotopes are shown in Figure 3, exhibiting simple trends as a function of neutron number. A simplified single-particle model provides a good description of these observations. In the shell model picture, the electromagnetic properties of odd-even indium isotopes are given by a single proton hole in the $\pi h_{11/2}$ orbit [154] [156]. This simple picture of nuclear structure seems to be supported by a rather constant value of their nuclear moments, which present very small variations when neutrons are added. For the even-proton nuclei, cadmium and tin, the naive shell-model expectation is that that the electromagnetic properties of even-odd isotopes are dominated by a single neutron occupying the $\nu h_{11/2}$ neutron orbit. This idea is also supported by a constant value of the magnetic moment, and a linear trend in the nuclear quadrupole moments. In this shell model picture, a particle occupying an orbit around closed shells has a negative quadrupole moment, which is interpreted as polarizing a spherical core towards an oblate deformation ($Q_s < 0$) [146]. If neutrons are added

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**Fig. 1.** Experimental nuclear properties as a function of the neutron number: i. mean-squared charge radii difference when two neutrons are added, $\langle r^2 \rangle(2n)$; ii. derivative of the two-neutron separation energy $dS(2n)$; iii. normalized spectroscopic quadrupole moments $Q_s/Z\sqrt{R}$; iv. scaled inverse of the excitation energy of the first $2^+$ state, $1/E_{2^+}/Z$; and v. normalized transition probability to the first $2^+$ excited state, $B(E2)/Z^2R^2$. Data taken from [31–33,36–38,103,115,118–142].
Fig. 2. Experimental nuclear properties as a function of the proton number: i. mean-squared charge radii difference when two protons are added, \( \langle r^2 \rangle (2p) \); ii. derivative of the two-proton separation energy \( dS(2p) \); iii. normalized spectroscopic quadrupole moments \( Q_s/ZR \); iv. scaled inverse of the excitation energy of the first \( 2^+ \) state, \( 1/E_{2}\); and v. normalized transition probability to the first \( 2^+ \) excited state, \( B(E2) / Z^2 R^2 \). Data taken from [31–33,36–38,103,115,118–142].

to the same orbit, the values of quadrupole moments cross zero when the orbit is half-filled, and take positive values when more than half of the orbit is occupied. This is interpreted as a “hole” polarizing the core towards prolate deformation \( (Q_s > 0) \). Similar trends have been observed in the calcium \( (Z = 20) \) [115], nickel \( (Z = 28) \) [157] and lead \( (Z = 82) \) [146] regions.

3.3 New trends in neutron-rich nuclei

Recent developments in both experimental and theoretical tools have provided a deeper insight in our understanding of nuclear properties at extreme proton-to-neutron ratios. Particular interest has been focused on the evolution of nuclear properties towards the suggested neutron-rich doubly magic nuclei: \(^{52,54}\)Ca \((Z = 20, N = 32,34)\) [103 116 117 123], \(^{78}\)Ni \((Z = 28, N = 50)\) [125 126 158 159], and \(^{132}\)Sn \((Z = 50, N = 82)\) [129 130]. These regions of the nuclear chart are being studied by several experimental techniques providing tests of theoretical descriptions at limits of the nuclear existence. While most of the measured experimental properties \( (S_{2n}, E(2^+), B(E2), \) and \( Q_s \) have been described by available nuclear models [115 117 159 160], the description of the nuclear size \( \langle r^2 \rangle \) has posed new challenges for modern nuclear theory [103 130 131 161 162]. This problem has been tackled with density functional theory, where satisfactory description of charge radii have been obtained in the calcium [131] and tin regions [130]. However, a description in the ab-initio framework has not been achieved yet [103 162].

Figure 3 shows the changes of the mean-squared charge radii around the calcium, nickel, and tin regions. The values for each isotopic chain are shown with respect to the value at the closed neutron shells. While a strong ele-
patterns seen in isotopes close to stability. For neutron-rich nuclei in the calcium region, the discontinuities seen in other observables such as $S_{2n}$, $E(2^+)$ values at neutron number $N = 32$, do not appear to be evident in the nuclear charge radii trends. A compilation of different properties measured in the calcium region is shown in Figure 3. The signatures of closed shells at $N = 20$ and $N = 28$ appears across all observables. For the nuclear charge radii (Figure 3) the signatures at $N = 20$ are present but less pronounced than for $N = 28$. At $N = 32$ and $N = 34$ the clear agreement for the signs of shell closures among the different observables breaks down, and distinct regular patterns appear for different observables.

Only very recently systematic measurements have been achieved for the nuclear charge radii in the vicinity of calcium and tin isotopes beyond $N = 28$ and $N = 82$. The charge radii and electromagnetic moments of $^{58,70}$Ni, $^{124−134}$Sn and $^{112−134}$Sb isotopes have been measured by the COLLEPS collaboration at ISOLDE-CERN. Moreover, results for $^{47−52}$K ($Z = 19$), $^{58−76}$Cu ($Z = 29$), $^{101−111}$Sn ($Z = 50$) and $^{101−133}$In ($Z = 49$) isotopes have been obtained by the CRIS collaboration at ISOLDE-CERN. Current efforts to extend these measurements to more exotic calcium, potassium, indium and tin isotopes are underway.

4 Conclusions

Despite the drastic difference in the interactions between their constituents, the collective properties of strongly correlated many-body systems exhibit surprisingly common features. A common feature is the appearance of shell structures and collective phenomena with seemingly simple trends. From dust particles governed by coulomb interactions, atomic clusters interacting by covalent bonds and inter-atomic potentials, up to nuclei governed by short-range nuclear forces. The interactions, length scale and dynamics are very different, but these systems present similar signatures of shell structures and collective phenomena. Perhaps the commonalities between these often seemingly disparate many-body systems may allow further mutual advancements in different fields, as for example was found in the field of atomic nanoclusters by the successful application of modified nuclear structure models.

The recent developments in many-body theory and the continuous increase in computing power have allowed an unprecedented reductionist insight of the emergent of physical phenomena. This connection between reductionist and emergence viewpoints is grounded uniquely through empirical observations.

The microscopic description of strongly correlated many-body systems demands different theoretical challenges. In contrast to other quantum systems, the atomic nucleus is formed by two different constituents (protons and neutrons) that interact mainly by the electromagnetic, strong and weak forces. Recent developments in many-body methods and higher computing power have provided great steps towards the understanding of the microscopic origin of collective phenomena in different regions of the nuclear
Fig. 4. Changes in the mean-square charge radii as a function of the neutron number: i. calcium ($Z = 20$), ii. nickel ($Z = 28$), and iii. tin region ($Z = 50$). Each isotopic chain is shown with respect to the isotope with neutron number at the closed shell. Experimental data were taken from [103,123–132].

Fig. 5. Experimental nuclear properties in the calcium region: i. mean-squared charge radii difference when two neutrons are added, $\langle r^2 \rangle (2n)$; ii. derivative of the two-nucleon separation energy $dS_{2n}$; iii. normalized spectroscopic quadrupole moments $Q_s/ZR$; iv. scaled inverse of the excitation energy of the first $2^+$ state, $1/E_{2^+}/Z^2$; and v. normalized transition probability to the first $2^+$ excited state, $B(E2)/Z^2R^2$. Experimental data were taken from [38,103,115,133,142].

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