Prediction of multi-inputs bubble column reactor using a novel hybrid model of computational fluid dynamics and machine learning

Amir Mosavi\textsuperscript{a,b}, Shahaboddin Shamshirband\textsuperscript{c,d}, Ely Salwana\textsuperscript{e}, Kwok-wing Chau\textsuperscript{f} and Joseph H. M. Tah\textsuperscript{b}

\textsuperscript{a}Institute of Automation, Kando Kalman Faculty of Electrical Engineering, Obuda University, Budapest, Hungary; \textsuperscript{b}School of the Built Environment, Oxford Brookes University, Oxford, United Kingdom; \textsuperscript{c}Department for Management of Science and Technology Development, Ton Duc Thang University, Ho Chi Minh City, Vietnam; \textsuperscript{d}Faculty of Information Technology, Ton Duc Thang University, Ho Chi Minh City, Vietnam; \textsuperscript{e}Institute of Visual Informatics, Universiti Kebangsaan Malaysia, Bangi, Selangor, Malaysia; \textsuperscript{f}Department of Civil and Environmental Engineering, Hong Kong Polytechnic University, Hong Kong, People’s Republic of China

**ABSTRACT**

The combination of machine learning and numerical methods has recently become popular in the prediction of macroscopic and microscopic hydrodynamics parameters of bubble column reactors. Such numerical combination can develop a smart multiphase bubble column reactor with the ability of low-cost computational time when considering the big data. However, the accuracy of such models should be improved by optimizing the data parameters. This paper uses an adaptive-network-based fuzzy inference system (ANFIS) to train four big data inputs with a novel integration of computational fluid dynamics (CFD) model of gas. The results show that the increasing number of input variables improves the intelligence of the ANFIS method up to $R = 0.99$, and the number of rules during the learning process has a significant effect on the accuracy of this type of modeling. Furthermore, the proper selection of model’s parameters results in higher accuracy in the prediction of the flow characteristics in the column structure.

**ARTICLE HISTORY**

Received 5 March 2019
Accepted 27 April 2019

**KEYWORDS**

Machine learning; computational fluid dynamics (CFD); hybrid model; adaptive neuro-fuzzy inference system (ANFIS); artificial intelligence; big data; prediction; forecasting; optimization; hydrodynamics; fluid dynamics; soft computing; computational intelligence; computational fluid mechanics

**Abbreviation**

$g$ Gravitational force (m s$^{-2}$)

$k$ Turbulent kinetic energy for modeling of dispersed phase (m$^2$ s$^{-2}$)

$M_I$ Interfacial force (N m$^{-3}$)

$M_D$ Drag force for modeling of dispersed phase (N m$^{-3}$)

$P$ The pressure in the reactor (N m$^{-2}$)

MFs Membership functions for ANFIS

$\tau_k$ Shear stress of phase k (Pa)

$\epsilon_g$ The volume of the dispersed phase (-)

**Subscripts**

$G$ Dispersed phase

$L$ Matrix/Continuous phase

**Greek Symbols**

$\epsilon$ Turbulent energy dissipation rate per unit mass (m$^2$ s$^{-3}$)

$\bar{\varepsilon}$ Average phase hold-up (-)

$\rho$ Density of phases (kg m$^{-3}$)

$\mu_T$ Turbulent viscosity (Pa s$^{-1}$)

1. Introduction

As multiphase contactors and reactors, bubble columns have an extensive application in chemical, biochemical and petrochemical industries (Masood & Delgado, 2014; Masood, Khalid, & Delgado, 2015; Rabha, Schubert, & Hampel, 2013; Şal, Gül, & Özdeminir, 2013). Bubble columns have various advantages including simple structure for phase interactions (liquid–gas or liquid–gas–solid interactions), high transfer rates of mass and heat and compactness during operation and maintenance and
the simple structure of sparging mechanism (Kumar, Degaleesan, Laddha, & Hoelscher, 1976; Pino et al., 1992; Shah, Kelkar, Godbole, & Deckwer, 1982). In reaction engineering, three-phase bubble column reactors have an extensive application. For instance, to manufacture industrially valuable bioproducts, gas–liquid–solid interaction reactors are the frequency used in biochemical applications (Essadki, Nikov, & Delmas, 1997; Lopez de Bertodano, Lahey, & Jones, 1994; Sokolichin & Eigenberger, 1994). To understand better about complex behavior of mass and heat transfer rate, hydrodynamic characteristics such as gas–liquid interactions, bubble coalescence, and break-up, it is required to investigate design parameters and optimization of the process in bubble column reactors (Dhote, Ekamba, & Joshi, 2004; Krishna & Van Baten, 2003; Maaelej, Benadda, & Otterbein, 2003; Wang et al., 2003).

These type of reactors are produced in different shapes such as cylindrical and rectangular and different sizes, and they are a suitable domain for phase interactions such as liquid–gas, liquid–gas, and solid reactors (Behkish, Men, Inga, & Morsi, 2002; Cho, Woo, Kang, & Kim, 2002; Li & Prakash, 2002; Michele & Hempel, 2002; Ruzicka, Zahradnik, Drahos, & Thomas, 2001). The gas distributors are also located at the bottom of the domain and sparge gas phase as a dispersed phase into the matrix phase as liquid phase or liquid–solid phase. When there are solid materials in the matrix (continuous phase), bubble column reactors are broadly called a slurry bubble column reactors (Bouaifi, Hebrard, Bastoul, & Rousseau, 2001; Deen, Solberg, & Hjertager, 2000; Luo, Lee, Lau, Yang, & Fan, 1999; Shimizu, Takada, Minekawa, & Kawase, 2000). Bubble columns have an extensive application in different industries such as chemical, biochemical and pharmaceutical, where the interaction of different phases are very crucial, or the chemical reactions during production are sometimes required (Degaleesan, Dudukovic, & Pan, 2001). For instance, they are also used in biochemical processes including biological wastewater treatment as well as fermentation (Prakash, Margaritis, Li, & Bergougnou, 2001; Shah et al., 1982). They also have an extensive application for large-scale aerobic fermentations in the bioprocessing industry (Doran, 1995; Masood & Delgado, 2014; Šal et al., 2013). Furthermore, they are utilized for performing a range of reactions in the chemical industry (Anabtawi, Abu-Eishah, Hilal, & Nabhan, 2003; Maaelej et al., 2003; Shah et al., 1982).

There has been a strong interest in modeling bubble columns by (CFD) since their industrial applications are diverse (Krishna, Urseanu, Van Baten, & Ellenberger, 1999; Rampure, Kulkarni, & Ranade, 2007; Sanyal, Vásquez, Roy, & Dudukovic, 1999). There have been various numerical, experimental, and mathematical approaches developed to estimate and measure the flow pattern and bubbles dynamics (Besbes, El Hajem, Ben Aissia, Champagne, & Jay, 2015; Islam, Ganesan, & Cheng, 2015; Li, Zhong, Jin, Lu, & He, 2014; Liu & Hinthrichsen, 2014; Masood et al., 2015; Masood & Delgado, 2014; McClure, Aboudha, Kavanagh, Fletcher, & Barton, 2015; Pourtousi, Sahu, & Ganesan, 2014; Wang et al., 2014; Xiao, Yang, & Li, 2013; Xing, Wang, & Wang, 2013; Ziegenhein, Rzehak, & Lucas, 2015; Ziegenhein, Rzehak, Krepper, & Lucas, 2013). Nevertheless, there are a number of difficulties in making a complete prediction for the fluid structure and the interaction between phases during the bubbling process (Chau & Jiang, 2002). Besides, the optimization of bubble column reactors for different operational conditions (superficial gas velocity, pressure, and temperature of continuous phase size of the reactor and time of mixing process) is required expensive computational time and efforts. The measurement of fluid properties for each node in a 3D bubble column requires very fine mesh in the computational methods and also causes the disturbance in experimental methods. Additionally, The significant disadvantage of the computational methods for simulating the large reactor (more than 2 m) with several operational parameters/inputs is computation time and computer capability (Chau, 2017; Faizollahzadeh Ardabili et al., 2018; Simonnet, Gentric, Olmos, & Midoux, 2007, 2008; Tabib, Roy, & Joshi, 2008). Due to these disadvantages, soft computing approaches, particularly the ANFIS method has been developed for estimating the fluid properties in the column for different conditions which have not been experimented in the lab or simulated by numerical methods (Burns, Frank, Hamill, & Shi, 2004; Cheng & Chau, 2002; Moazenzadeh, Mohammadi, Shamshirband, & Chau, 2018; Pfleger & Becker, 2001; Pourtousi, Sahu, Ganesan, Shamshirband, & Redzwanz, 2015; Taherei Ghazvinei et al., 2018; Yaseen, Sulaiman, Deo, & Chau, 2019). These algorithms are used to mimic the hydrodynamics of the bubble column reactor for a specific condition. However, they cannot feel the exact physics, and they are capable based on their understanding (training data) (Panella & Gallo, 2005; Pourtousi, Zeinali, Ganesan, & Sahu, 2015; Ryoo, Dragojlovic, & Kaminski, 2005; Schurter & Roschke, 2000).

The pattern of a neural network for the learning process and the fuzzy logic framework for decision are both combined in the ANFIS structure (Jang, 1996; Panella & Gallo, 2005). One of the most remarkable characteristics of this structure is its capability for learning complex relationships according to the pattern data (Chau & Albermani, 2002; Chau & Albermani, 2003; Chen & Chau, 2016; Lei, He, Zi, & Hu, 2007; Nabavi-Pesaraeai, Bayat, Hosseinzadeh-Bandbafha, Afraeyabi, & Chau, 2017; Schurter & Roschke, 2000; Yun et al., 2008).
This model categorizes the domain into different regions for modeling nonlinear and complex case studies (Ben-Nakhi, Mahmoud, & Mahmoud, 2008; Lei et al., 2007; Varol, Avci, Koca, & Oztop, 2007; Varol, Koca, Oztop, & Avci, 2008). A general local model is then extended for each local region according to linear functions or even adjustable factors (Jang, 1993, 1996; Jang, Sun, & Mizutani, 1997). This feature of the model enables the method to thoroughly learn the process and predict the missing local nodes in the prediction domain (Avila & Pacheco-Vega, 2009; Yun et al., 2008).

The ANFIS approach has been employed in several papers to learn data from CFD database and then predict the bubbling flow including flow pattern, amount of gas, and turbulent kinetic energy (Abd Fatah et al., 2015; Azwadi, Zeinali, Saidari, & Kazemi, 2013; Wang, Chau, Qiu, & Chen, 2015; Zeinali, Mazlan, Fatah, & Zamzuri, 2013). Moreover, this model was applied for predicting the microscopic parameters including bubble formation, detachment and rising. Pourtousi et al. (Pourtousi, 2012; Pourtousi, Sahu et al., 2015) recommended the new integration of the CFD data-set with artificial algorithms such as ANFIS method for prediction of the fluid flow recognition in the bubble column reactor. They trained their CFD database and simulated the new flow pattern, including turbulent kinetic energy liquid pattern and the interface of the dispersed and continuous phase in the reactor (Pourtousi, Sahu et al., 2015; Pourtousi, Zeinali et al., 2015). This study aims to use the methodology of Pourtousi et al. (Pourtousi, 2012; Pourtousi, Sahu et al., 2015) in the prediction of gas velocity in the bubble column reactor. Additionally, the different pattern of input parameters has been examined for various tuning parameters of the ANFIS method.

2. Methodology

2.1. Geometrical structure

In this research, an industrial two-phases reactor with 2.6 m was utilized. The ring sparger is embedded at the end of the bubble column reactor, and the diameter of the orifice hole is 0.7 mm.

2.2. CFD

In CFD, the single size eulerian-eulerian approach has been employed for simulating the homogeneous bubble column reactor hydrodynamics. The continuity equation is the first equation to be considered which is employed for calculating the volume of available gas or volume of the available liquid. The continuity equation is presented as:

\[
\frac{\partial}{\partial t}(\rho_k \epsilon_k) + \nabla (\rho_k \epsilon_k u_k) = 0
\]

The momentum transfer calculation is provided, and the amount of gas and liquid phase can be calculated by this equation. The momentum transfer calculation is written as:

\[
\frac{\partial}{\partial t}(\rho_k \epsilon_k u_k) + \nabla (\rho_k \epsilon_k u_k u_k) = -\nabla (\epsilon_k p_k) - \epsilon_k \nabla p + \epsilon_k \rho_k g + M_{I,k}
\]

For interactions between the main liquid and gas phase, the total interfacial force defines the main forcing scheme for the accurate dynamics of bubbles, and the total force between bubbles and matrix phase is expressed as:

\[
M_{I,LL} = -M_{I,L} = M_{D,L} + M_{TD,L}
\]

All forcing schemes between gas bubbles and liquid phase and the \(k - \epsilon\) turbulence model are consistent with Tabib et al (Tabib et al., 2008).

2.2.1. Grid

In this study, the non-uniform meshes are used for CFD analysis in the bubble column reactor. This mesh structure is similar to that of the study conducted by Laborde-Boutet, Larachi, Dromard, Delsart, and Schweich (2009).

2.3. ANFIS

The ANFIS approach is a useful tool which can be used to predict physical and biological phenomena that are found in nature. In various studies such as a study conducted by Takagi and Sugeno, the ANFIS approach has been described (Takagi & Sugeno, 1985). To start the learning process, learning data is first categorized at several levels of membership formations (MFs). As indicated in Figure 1, according to AND law, the first feedback from the learning step multiplies. The function \(i\) rule can be defined as follows:

\[
w_i = \mu_{Ai}(x) \mu_{Bi}(y) \mu_{Ci}(z) \mu_{Di}(v_{as})
\]

Where \(w_i\) refers to the output of learning feedback and \(\mu_{Ai}, \mu_{Bi}, \mu_{Ci}\) and \(\mu_{Di}\) also express the input of learning feedback.

In the third step of learning, the relative firing strengths of each rule are defined according to the following formula. The weight fraction of each layer is specified by:

\[
\bar{w}_i = \frac{w_i}{\sum(w_i)}
\]

Where \(\bar{w}_i\) is normalized firing strengths. In the fourth step of learning, Takagi and Sugeno (1985) used the
if–then rule function. The mesh formula in the ANFIS can be modified as follows:

$$\bar{w}_i f_i = \bar{w}_i (p_i x + q_i y + r_i z + s_i v_{as} + t_i)$$  \hspace{1cm} (6)

In the above formula $p_i$, $q_i$, $r_i$, $s_i$, and $t_i$ are parameters related to ‘if–then rules’.

3. Result and discussion

Simulation of a cylindrical bubble column (BCR) reactor by CFD resulted in various fluid parameters as the output of the CFD. From among such CFD outputs, coordinates in x, y, and z-direction, as well as superficial air velocity and air velocity could be mentioned. In this study, the information generated by the CFD will be investigated using ANFIS method.

In the study implementing the ANFIS method, part of the CFD output is used as input and the rest as output. The description of the system studied here is as follows; there are four inputs used in this study with coordinates in x-direction used as input 1, coordinates in y-direction used as input 2, and coordinates in z-direction used as input 3, while superficial air velocity was taken as input 4. This is while air velocity is the only output studied in this research. The following conditions are presumed for
the initiation of the learning process by machine learning (ANFIS):

- A maximum of 600 for an epoch.
- A total of 5000 data.
- 65% as the value for p which indicates the percentage of data (from the whole data) used in the training process.
- 65% of the data used in training, and 100% of the data used in the testing process.
- gbellmf type chosen as the type of membership functions (MFs)

With the abovementioned assumptions and considering one input, being coordinates in the x-direction, and air velocity as output, training, and testing processes were
carried out for each number of membership functions (2, 4, 6, and 8) separately. As shown in Figure 2 (a and b), R(Regression) amounts to 0.52 at best which indicates ANFIS methods are devoid the proper intelligence and changing the number of members functions led to no significant improvement in the intelligence of ANFIS method.

Increasing the number of inputs was studied as a way of increasing the system intelligence, and coordinates in x and y directions were taken as inputs and air velocity as output, meanwhile, the testing and training processes were carried out separately for numbers of membership functions (2, 4, 6, 8). Figure 3(a and b) shows an increase in the value of R from 0.52 to 0.76 which is an indication
Figure 7. (a) Compare CFD output and ANFIS method prediction using inputs 1 and 2. (b) Compare CFD output and ANFIS method prediction using inputs 1 and 3. (c) Compare CFD output and ANFIS method prediction using inputs 1 and 4. (d) Compare CFD output and ANFIS method prediction using inputs 2 and 3. (e) Compare CFD output and ANFIS method prediction using inputs 2 and 4. (f) Compare CFD output and ANFIS method prediction using inputs 3 and 4.

Figure 8. Points of the bubble column that were in the ANFIS learning process.
of improvement in the ANFIS method. When the number of membership functions is 4, the best value for R (R = 0.76) is reported, which is a proper rise but still not adequate, and there is a need for more investigation. That is why changing the membership functions; including gbellmf, gaussmf, gauss2mf, trimf, dsigmf, psigmf, pimf, with the number of membership functions being 4 was studied.

Training and testing processes were conducted separately for each type of MFs, and in the training process, 65% of the data was used. For the testing process, however, the sum of all data used in the training process plus the remaining 35% were evaluated by the ANFIS method.

According to Figure 4 (a and b), there was no considerable improvement in system intelligence.

Considering the fact that two inputs ultimately resulted in an increase of R to 0.75, it was decided to increase the number of inputs from 2 to 3 in order to enhance the system intelligence. Coordinates in x, y, and z directions were considered inputs while air velocity was the output.

Having two as the number of membership functions, the learning process was also carried out. This increase in the number of inputs led to a substantial enhancement in the intelligence of the ANFIS method, and R-value rose to 0.92. The appropriate increase in the intelligence of the ANFIS method took place when the number of (MFs) was 2. Moreover, increasing the number of membership functions to 4 also demonstrated acceptable results and R rose to 0.992 (see Figure 5 (a and b)).

In the rest of this research, one of the air superficial velocity parameters was particularly added to the system as input number 4. Under the new circumstances, with the position of meshes (nodes) and superficial air velocity as input parameter and air velocity as an output parameter, the learning process was performed separately with the number of (MFs) being 2 and 4 (see Figure 6(a and b)).

With 2 as the number of membership functions, R amounted to 0.97, whereas with the number of MFs being 4 R rose to 0.998 which is perfectly suitable for the ANFIS method, and represents a proper agreement between the ANFIS outputs and CFD outputs (See Figure 7(a–f)).
The use of air superficial velocity as input led to particularly suitable results, and with this intelligence in the ANFIS method, parts of BCR can also be predicted (Figure 8).

Points can be predicted that had no participation in the learning process, and this indicates the considerable ability of the machine learning in prediction (see Figure 9(a–f)).

Combining machine learning (ANFIS method) and CFD means a substantial reduction in the time required for making calculations, and also obviates the need for complex CFD.

4. Conclusions

In this study, the machine learning method of ANFIS is combined with CFD data to predict the macroscopic parameters such as gas velocity in the multiphase reactor. Four input parameters are elected as inputs of the multiphase reactor for the learning process, and then one output such as gas velocity is also considered in the input parameters. To understand the behavior of AI in learning CFD data, the different number of inputs, number of rules and membership functions have been examined. This study shows that one of the main advantages of artificial intelligent modeling is a combination of input with output parameters, and also replacement of outputs with inputs matrix. This replacement does not feel with smart method as it is data-based modeling, but we can understand the effect of outputs parameters on the input variables. The number of inputs has a significant impact on the accuracy of the method to capture the whole behavior of Fluid flow in the column.

Additionally, the combination of numerical methods and AI algorithms enable us to reduce the computational time and number of simulation time during the optimization process. However, this type of modeling should be considered as an assistance tool besides the numerical method. This framework is also limited to the amount of data, and it can only show the process behavior based on the input data. For future study, we will specifically use more input data based on the clustering algorithm and parallel code implementation.

Disclosure statement

No potential conflict of interest was reported by the authors.

References

Abd Fatah, A. Y., Mazlan, S. A., Koga, T., Zamzuri, H., Zeinali, M., & Imaduddin, F. (2015). A review of design and modeling of magnetorheological valve. International Journal of Modern Physics B, 29(04), 1530004.

Anabtawi, M., Abu-Eishah, S., Hilal, N., & Nabhan, M. (2003). Hydrodynamic studies in both bi-dimensional and three-dimensional bubble columns with a single sparger. Chemical Engineering and Processing: Process Intensification, 42(5), 403–408.

Avila, G., & Pacheco-Vega, A. (2009). Fuzzy-C-Means-Based Classification of Thermodynamic-Property data: A Critical Assessment. Numerical Heat Transfer, Part A: Applications, 56(11), 880–896.

Azzawi, C. S. N., Zeinali, M., Saldari, A., & Kazemi, A. (2013). Adaptive-network-based fuzzy inference system analysis to predict the temperature and flow fields in a lid-driven cavity. Numerical Heat Transfer, Part A: Applications, 63(12), 906–920.

Behkish, A., Men, Z., Inga, J. R., & Morsi, B. I. (2002). Mass transfer characteristics in a large-scale slurry bubble column reactor with organic liquid mixtures. Chemical Engineering Science, 57(16), 3307–3324.

Ben-Nakhi, A., Mahmoud, M. A., & Mahmoud, A. M. (2008). Inter-model comparison of CFD and neural network analysis of natural convection heat transfer in a partitioned enclosure. Applied Mathematical Modelling, 32(9), 1834–1847.

Besbes, S., El Hajem, M., Ben Aissa, H., Champagne, J.-Y., & Jay, J. (2015). PIV measurements and eulerian–lagrangian simulations of the unsteady gas–liquid flow in a needle sparger rectangular bubble column. Chemical Engineering Science, 126, 560–572.

Bouafi, M., Hebrard, G., Bastoul, D., & Roustan, M. (2001). A comparative study of gas hold-up, bubble size, interfacial area and mass transfer coefficients in stirred gas–liquid reactors and bubble columns. Chemical Engineering and Processing: Process Intensification, 40(2), 97–111.

Burns, A. D., Frank, T., Hamill, I., & Shi, J.-M. (2004). The Favre averaged drag model for turbulent dispersion in Eulerian multi-phase flows. Paper presented at the 5th international conference on multiphase flow, ICMF.

Chau, K.-w. (2017). Use of meta-heuristic techniques in rainfall-runoff modelling: Multidisciplinary Digital Publishing Institute.

Chau, K., & Albermani, F. (2003). Knowledge-based system on optimum design of liquid retaining structures with genetic algorithms. Journal of Structural Engineering, 129(10), 1312–1321.

Chau, K.-W., & Albermani, F. (2002). Expert system application on preliminary design of water retaining structures. Expert Systems with Applications, 22(2), 169–178.

Chau, K., & Jiang, Y. (2002). Three-dimensional pollutant transport model for the Pearl River Estuary. Water Research, 36(8), 2029–2039.

Chen, X. Y., & Chau, K. W. (2016). A hybrid double feedforward neural network for suspended sediment load estimation. Water Resources Management, 30(7), 2179–2194.

Cheng, C. T., & Chau, K.-W. (2002). Three-person multi-objective conflict decision in reservoir flood control. European Journal of Operational Research, 142(3), 625–631.

Cho, Y. J., Woo, K. J., Kang, Y., & Kim, S. D. (2002). Dynamic characteristics of heat transfer coefficient in pressurized bubble columns with viscous liquid medium. Chemical Engineering and Processing: Process Intensification, 41(8), 699–706.

Deen, N. G., Solberg, T., & Hjertager, B. H. (2000). Numerical simulation of the gas-liquid flow in a square cross-sectional bubble column. Paper presented at the Proceedings of 14th
Int. Congress of Chemical and Process Engineering: CHISA (Praha, Czech Republic, 2000).

Degaleesan, S., Dudukovic, M., & Pan, Y. (2001). Experimental study of gas-induced liquid-flow structures in bubble columns. AIChE Journal, 47(9), 1913–1931.

Dhotre, M., Ekambara, K., & Joshi, J. (2004). CFD simulation of sparger design and height to diameter ratio on gas hold-up profiles in bubble column reactors. Experimental Thermal and Fluid Science, 28(5), 407–421.

Doran, P. M. (1995). Bioprocess engineering principles. Elsevier.

Essadki, H., Nikov, I., & Delmas, H. (1997). Electrochemical probe for bubble size prediction in a bubble column. Experimental Thermal and Fluid Science, 14(3), 243–250.

Faizollahzadeh Ardabili, S., Najafi, B., Shamshirband, S., Minaei Bidgoli, B., Deo, R. C., & Chau, K. -w. (2018). Computational intelligence approach for modeling hydrogen production: A review. Engineering Applications of Computational Fluid Mechanics, 12(1), 438–458.

Islam, M. T., Ganesan, P., & Cheng, J. (2015). A pair of bubbles’ rising dynamics in a xanthan gum solution: A CFD study. RSC Advances, 5(11), 7819–7831.

Jang, J.-S. (1993). ANFIS: Adaptive-network-based fuzzy inference system. IEEE Transactions on Systems, man, and Cybernetics, 23(3), 665–685.

Jang, J.-S. (1996). Input selection for ANFIS learning. Paper presented at the Fuzzy Systems, 1996., Proceedings of the Fifth IEEE International Conference on.

Jang, J.-S. R., Sun, C.-T., & Mizutani, E. (1997). Neuro-fuzzy and soft computing; a computational approach to learning and machine intelligence.

Krishna, R., Urseanu, M., Van Baten, J., & Ellenberger, J. (1999). Influence of scale on the hydrodynamics of bubble columns operating in the churn-turbulent regime: Experiments vs. Eulerian simulations. Chemical Engineering Science, 54(21), 4903–4911.

Krishna, R., & Van Baten, J. (2003). Mass transfer in bubble columns. Catalysis Today, 79-80, 67–75.

Luo, X., Lee, D., Lau, R., Yang, G., & Fan, L. S. (1999). Maximum stable bubble size and gas holdup in high-pressure slurry bubble columns. AIChE Journal, 45(4), 665–680.

Maejil, S., Benadda, B., & Otterbein, M. (2003). Interfacial area and volumetric mass transfer coefficient in a bubble reactor at elevated pressures. Chemical Engineering Science, 58(11), 2365–2376.

Masood, R., & Delgado, A. (2014). Numerical investigation of the interphase forces and turbulence closure in 3D square bubble columns. Chemical Engineering Science, 108, 154–168.

Masood, R., Khalid, Y., & Delgado, A. (2015). Scale adaptive simulation of bubble column flows. Chemical Engineering Journal, 262, 1126–1136.

McClure, D. D., Aboudha, N., Kavanagh, J. M., Fletcher, D. F., & Barton, G. W. (2015). Mixing in bubble column reactors: Experimental study and CFD modeling. Chemical Engineering Journal, 264, 291–301.

Michele, V., & Hempel, D. C. (2002). Liquid flow and phase holdup—measurement and CFD modeling for two-and three-phase bubble columns. Chemical Engineering Science, 57(11), 1899–1908.

Moazen zadeh, R., Mohammadi, B., Shamshirband, S., & Chau, K.-w. (2018). Coupling a firefly algorithm with support vector regression to predict evaporation in northern Iran. Engineering Applications of Computational Fluid Mechanics, 12(1), 584–597.

Nabavi-Pelesaraei, A., Bayat, R., Hosseinzadeh-Bandbafha, H., Afrasyabi, H., & Chau, K.-w. (2017). Modeling of energy consumption and environmental life cycle assessment for incineration and landfill systems of municipal solid waste management—A case study in Tehran Metropolis of Iran. Journal of Cleaner Production, 148, 427–440.

Panella, M., & Gallo, A. S. (2005). An input-output clustering approach to the synthesis of ANFIS networks. IEEE Transactions on Fuzzy Systems, 13(1), 69–81.

Pfleger, D., & Becker, S. (2001). Modelling and simulation of the dynamic flow behaviour in a bubble column. Chemical Engineering Science, 56(4), 1737–1747.

Pino, L., Solari, R., Siquier, S., Antonio Estevez, L., Yepez, M., & Saez, A. (1992). Effect of operating conditions on gas holdup in slurry bubble columns with a foaming liquid. Chemical Engineering Communications, 117(1), 367–382.

Pourtousi, M. (2012). Simulation of particle motion in incompressible fluid by Lattice Boltzmann MRT model. Johor Bahru: Universiti Teknologi Malaysia.

Pourtousi, M., Sahu, J., & Ganesan, P. (2014). Effect of interfacial forces and turbulence models on predicting flow pattern inside the bubble column. Chemical Engineering and Processing: Process Intensification, 75, 38–47.

Pourtousi, M., Sahu, J., Ganesan, P., Shamshirband, S., & Redwan, G. (2015). A combination of computational fluid dynamics (CFD) and adaptive neuro-fuzzy system (ANFIS) for prediction of the bubble column hydrodynamics. Powder Technology, 274, 466–481.

Pourtousi, M., Zeinali, M., Ganesan, P., & Sahu, J. N. (2015). Prediction of multiphase flow pattern inside a 3D bubble column reactor using a combination of CFD and ANFIS. RSC Advances, 5(104), 85652–85672. doi:10.1039/c5ra11583c

Prakash, A., Margaritis, A., Li, H., & Bergougnou, M. (2001). Hydrodynamics and local heat transfer measurements in a
bubble column with suspension of yeast. *Biochemical Engineering Journal*, 9(2), 155–163.

Rabha, S., Schubert, M., & Hampel, U. (2013). Intrinsic flow behavior in a slurry bubble column: A study on the effect of particle size. *Chemical Engineering Science*, 93, 401–411.

Rampure, M. R., Kulkarni, A. A., & Ranade, V. V. (2007). Hydrodynamics of bubble column reactors at high gas velocity: Experiments and computational fluid dynamics (CFD) simulations. *Industri & Engineering Chemistry Research*, 46(25), 8431–8447.

Ruzicka, M., Zahradnik, J., Drahoš, J., & Thomas, N. (2001). Homogeneous–heterogeneous regime transition in bubble columns. *Chemical Engineering Science*, 56(15), 4609–4626.

Ryoo, J., Dragojlovic, Z., & Kaminski, D. A. (2005). Control of convergence in a computational fluid dynamics simulation using ANFIS. *IEEE Transactions on Fuzzy Systems*, 13(1), 42–47.

Sal, S., Gülb, ÖF, & Özdemir, M. (2013). The effect of sparger geometry on gas holdup and regime transition points in a bubble column equipped with perforated plate spargers. *Chemical Engineering and Processing: Process Intensification*, 70, 259–266.

Sanyal, J., Vásquez, S., Roy, S., & Dudukovic, M. (1999). Numerical simulation of gas–liquid dynamics in cylindrical bubble column reactors. *Chemical Engineering Science*, 54(21), 5071–5083.

Schurter, K. C., & Roschke, P. N. (2000). Fuzzy modeling of a magnetorheological damper using ANFIS. Paper presented at the Fuzzy Systems, 2000. FUZZ IEEE 2000. The Ninth IEEE International Conference on.

Shah, Y., Kelkar, B. G., Godbole, S., & Deckwer, W. D. (1982). Design parameters estimations for bubble column reactors. *AIChE Journal*, 28(3), 353–379.

Shimizu, K., Takada, S., Minekawa, K., & Kawase, Y. (2000). Phenomenological model for bubble column reactors: Prediction of gas hold-ups and volumetric mass transfer coefficients. *Chemical Engineering Journal*, 78(1), 21–28.

Simonnet, M., Gentric, C., Olmos, E., & Midoux, N. (2007). Experimental determination of the drag coefficient in a swarm of bubbles. *Chemical Engineering Science*, 62(3), 858–866.

Simonnet, M., Gentric, C., Olmos, E., & Midoux, N. (2008). CFD simulation of the flow field in a bubble column reactor: Importance of the drag force formulation to describe regime transitions. *Chemical Engineering and Processing: Process Intensification*, 47(9-10), 1726–1737.

Sokolichin, A., & Eigenberger, G. (1994). Gas—liquid flow in bubble columns and loop reactors: Part I. Detailed modelling and numerical simulation. *Chemical Engineering Science*, 49(24), 5735–5746.

Tabib, M. V., Roy, S. A., & Joshi, J. B. (2008). CFD simulation of bubble column—an analysis of interphase forces and turbulence models. *Chemical Engineering Journal*, 139(3), 589–614.

Taherei Ghazvinei, P., Hassanpour Darvishi, H., Mosavi, A., Yusof, K., b, W., Alizamir, M., . . . Chau, K.-w. (2018). Sugarcane growth prediction based on meteorological parameters using extreme learning machine and artificial neural network. *Engineering Applications of Computational Fluid Mechanics*, 12(1), 738–749.

Takagi, T., & Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE Transactions on Systems, Man, and Cybernetics*, SMC-15(1), 116–132.

Varol, Y., Avci, E., Koca, A., & Oztop, H. F. (2007). Prediction of flow fields and temperature distributions due to natural convection in a triangular enclosure using adaptive-network-based fuzzy inference system (ANFIS) and artificial neural network (ANN). *International Communications in Heat and Mass Transfer*, 34(7), 887–896.

Varol, Y., Koca, A., Oztop, H. F., & Avci, E. (2008). Analysis of adaptive-network-based fuzzy inference system (ANFIS) to estimate buoyancy-induced flow field in partially heated triangular enclosures. *Expert Systems with Applications*, 35(4), 1989–1997.

Wang, S., Arimatsu, Y., Koumatu, K., Furumoto, K., Yoshimoto, M., Fukunaga, K., & Nakao, K. (2003). Gas holdup, liquid circulating velocity and mass transfer properties in a mini-scale external loop airlift bubble column. *Chemical Engineering Science*, 58(15), 3353–3360.

Wang, W.-c., Chau, K.-w., Qiu, L., & Chen, Y.-b. (2015). Improving forecasting accuracy of medium and long-term runoff using artificial neural network based on EEMD decomposition. *Environmental Research*, 139, 46–54.

Wang, H., Jia, X., Wang, X., Zhou, Z., Wen, J., & Zhang, J. (2014). CFD modeling of hydrodynamic characteristics of a gas–liquid two-phase stirred tank. *Applied Mathematical Modelling*, 38(1), 63–92.

Xiao, Q., Yang, N., & Li, J. (2013). Stability-constrained multi-fluid CFD models for gas–liquid flow in bubble columns. *Chemical Engineering Science*, 100, 279–292.

Xing, C., Wang, T., & Wang, J. (2013). Experimental study and numerical simulation with a coupled CFD–PBM model of the effect of liquid viscosity in a bubble column. *Chemical Engineering Science*, 95, 313–322.

Yaseen, Z. M., Sulaiman, S. O., Deo, R. C., & Chau, K.-W. (2019). An enhanced extreme learning machine model for river flow forecasting: State-of-the-art, practical applications in water resource engineering area and future research direction. *Journal of Hydrology*, 569, 387–408.

Yun, Z., Quan, Z., Caixin, S., Shaolan, L., Yuming, L., & Yang, S. (2008). RBF neural network and ANFIS-based short-term load forecasting approach in real-time price environment. *IEEE Transactions on Power Systems*, 23(3), 853–858.

Zeinali, M., Mazlan, S. A., Fatah, A. Y. A., & Zamzuri, H. (2013). A phenomenological dynamic model of a magnetorheological damper using a neuro-fuzzy system. *Smart Materials and Structures*, 22(12), 125013.

Ziegenhein, T., Rzechak, R., Krepper, E., & Lucas, D. (2013). Numerical simulation of Polydispersed flow in bubble columns with the Inhomogeneous multi-size-group model. *Chemie Ingenieur Technik*, 85(7), 1080–1091.

Ziegenhein, T., Rzechak, R., & Lucas, D. (2015). Transient simulation for large scale flow in bubble columns. *Chemical Engineering Science*, 122, 1–13.