Application of wavelet analysis to differences in modal rotations for damage identification

A Katunin, J V Araújo dos Santos and H Lopes

1Institute of Fundamentals of Machinery Design, Silesian University of Technology, Konarskiego 18A, 44-100 Gliwice, Poland
2IDMEC, Instituto Superior Técnico, Universidade de Lisboa, Av. Rovisco Pais, 1049-001 Lisboa, Portugal
3DEM, ISEP, Instituto Politécnico do Porto, Rua Dr. António Bernardino de Almeida, 431, 4249-015 Porto, Portugal

E-mail: andrzej.katunin@polsl.pl

Abstract. Shearography is an effective non-destructive testing method which already found numerous applications in structural damage identification and for various industrial branches. This method, however, consider the detection and localization based on the irregularities found on rotation fields. This paper presents the concept of non-destructive damage identification in structural elements using modal rotation together with advanced signal processing for the enhancement of its sensitivity, thus allowing for overcoming the problem of small damage detectability. The presented approach uses a set modal rotations fields of a beam, initially processed from shearographic phase maps, for undamaged and damage states. The difference in modal rotations, between the two damages states, is initially calculated and afterward is subjected to a wavelet transform. The presented results show the effectiveness of the presented approach in tasks of structural damage identification.

1. Introduction
Shearography is a very efficient non-destructive testing (NDT) method in numerous industrial applications due to its outstanding properties, because it allows non-contact and full-field measurements, very fast measurement of large areas, and the highest possible resolution among the widely used NDT methods for damage identification. The shearography is an interferometric technique that allows to measure the rotation field, being used in industrial environment due to its good robustness to external perturbations. However, shearographic testing, which is based on speckle phenomenon, results in acquisition of shearographic phase maps, where the damage detection and localization is performed based on observed disturbances of the rotations, extracted from these phase maps. The problem of application of this method arises when a small damage needs to be detected in a structure, which often produces very small irregularities in the rotations, causing the damaged site to be overlooked. In some industrial branches, however, damage oversight is unacceptable. This is the case of inspections of aircraft structures, which has been inducing the development of processing methods that allow for the increase of the sensitivity of shearography to various types of damage.

Numerous studies reported several techniques used for enhancement of damage detectability of shearography (see e.g. [1-3]), which are based on filtering out the noise and computing the rotation field. However, the filtering solves the problem only partly, since the detection of small damage sites remains impossible. Recent attempts in enhancement of damage detectability using shearography have been made by Lopes at al. [4] and Minnini et al. [5], where the authors proposed a multi-step procedure of initial processing of the shearographic phase maps, which consists of filtering and unwrapping, giving a smooth continuous filtered rotation field ready to further post-processing. In [5] the optimal spatial sampling was used to obtain the higher order spatial derivatives of the modal rotation fields, which were used for localization of small damage, while further studies presented in
[6] used the discrete wavelet transform (DWT) for enhancing the sensitivity of the approach. These two latter approaches allow to a precise damage localization and the identification of its shape.

In this paper, a concept of further improvement of the sensitivity of the developed method is presented. Considering the previously published works of Solis et al. [7,8], who used differences in wavelet coefficients of the mode shapes, i.e. modal displacements, subjected to continuous wavelet transform (CWT) of undamaged and damaged beams for enhancement of damage identification, the developed method can be modified in a similar way. The concept of enhancement of damage identification based on differences in modal rotations subjected to CWT for an undamaged and damaged structure was presented and verified on experimental data of a damaged aluminum beam. The obtained results reveal and increase the sensitivity of the developed method to the presence of damage.

2. Experimental

The experimental studies were performed on an aluminum beam, being the damage introduced by creating a slot along the beam width and with the depth of 0.41 mm, using a milling machine. The scheme of the damage beam with specific dimensions is presented in figure 1.

![Figure 1. Scheme of introduced damage.](image)

The modal rotation fields of the bending modes were measured using an in-house built shearographic system that can be seen in figure 2. The beam is suspended by two rubber bands and is excited harmonically by a loudspeaker at its natural frequencies. The illumination of the beam surface is made by a continuous-wave laser with a wavelength of 535 nm and output power of 1.3 W. An acousto-optic modulator is used to pulse the laser and synchronize it with the vibration of the beam. Intensity patterns are recorded by a digital camera with a spatial resolution of 4 million measurement points. A temporal phase shifting is applied in order to extract the phase map from the intensity patterns. A comprehensive description of the experimental setup can be found in [5].

The initial processing of modal rotation fields obtained from shearographic measurements with further phase shifting involved filtering using sine/cosine average filter and unwrapping using the Goldstein algorithm. The details on these procedures can be found in [5,6], and the results for each particular step are presented in figure 3. Note that the results presented in figure 3 are addressed to the damaged case. However, due to small damage none of disturbances of the modal rotation are visible, even after the filtering and the unwrapping procedures.
Figure 2. Experimental setup used for the measurement of modal rotation fields.

(a) Unfiltered discontinuous phase map

(b) Filtered discontinuous phase map

(c) Filtered continuous phase map

(d) Modal rotation

Figure 3. Diagram describing the determination of experimental modal rotation from shearographic phase maps.

3. Post-processing and results
The performed experimental studies and initial processing of shearographic phase maps allowed for acquiring the modal rotations of the beam in the undamaged and damaged states. These modal rotation differences are then subjected to post-processing using a CWT-based approach.

3.1. Post-processing using wavelet analysis
CWT is an integral transform which can be given by the following equation:

$$Wf\ a,b = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} f(x) \psi^{*} \left( \frac{x-b}{a} \right) dx,$$  \hspace{1cm} (1)
where a transformed signal \( f \) is multiplied by the complex conjugate of the wavelet kernel \( \psi^* \), dependent on scaling \( a \) and translation \( b \) parameters, which generate a family of elementary wavelet functions by scaling and translation operations:

\[
\psi_{a,b} x = \frac{1}{a} \psi \frac{x-b}{a} .
\]  

(2)

CWT holds the property of linearity:

\[
W f_1 + f_2 a,b = W f_1 a,b + W f_2 a,b ,
\]

(3)

where \( f_1 \) and \( f_2 \) are analyzed signals. This property can be used in the damage identification problem by determination of CWT of the modal rotation differences between the baseline (undamaged beam) and the damaged beam as follows:

\[
W \Delta f a,b = W f - f a,b ,
\]

(4)

where \( f \) is a baseline and \( f \) is the damage beam.

This procedure can be extended to 2D signals by considering extension of (1) to the following form:

\[
W f a,b_1,b_2,\vartheta = \frac{1}{a} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f x,y \psi^* \frac{x-b_1}{a},\frac{y-b_2}{a} dx dy ,
\]

(5)

where \( f \) is a 2D analysed signal, \( b_1, b_2 \) are the translation parameters in \( x \) and \( y \) directions, and \( \vartheta \) denotes rotation. According to the property of linearity (3) the difference of two sets of wavelet coefficients of two 2D signals takes the following form:

\[
W \Delta f a,b_1,b_2,\vartheta = W f - f a,b_1,b_2,\vartheta .
\]

(6)

For the analysis of 1D data, the Haar wavelet was selected due to its mathematical simplicity and effectiveness in damage identification among other popular wavelet bases. For 2D signals the Marr wavelet was selected due to its advantages with respect to other wavelets, namely good spectral resolution, and the highest sensitivity to small changes in a signal. The selected wavelets allowed for effective improvement of the sensitivity of the developed method.

As one can see, the resulting sets of wavelet coefficients are represented by 3 parameters in 1D case (4), and 5 parameters in 2D case (6), which leads to difficulties in the graphical representation of the results, especially for 2D case. In order to overcome these difficulties one can assume, similarly as the authors of [9,10], that \( \vartheta = 0 \), since the analyzed data are defined in Cartesian coordinates, and the applied 2D wavelet bases are isotropic, thus, being insensitive to changes of \( \vartheta \). Additionally, one can also assume that \( a = \text{const} \), where \( a \) is the best selected scale value. Such assumptions allow for a 3D representation of the results for 2D data.

3.2. Results of damage identification for 1D data

The first tests were performed on the profiles of modal rotations of the structure for the undamaged and damaged states, which were measured with shearography technique. The results obtained for the CWT-based algorithm, using the rotation profile of the damage beam, and obtained for CWT differences-based, using the differences of modal rotation profile of undamaged and damage beam, are presented in figures 4 and 5, respectively. These results are presented for the beam first mode shape. In this study, it was assumed that \( a = 100 \) is the best scale parameter selected in the trial-and-error method.

From the obtained results one can observe that in both cases the damage signature is recognizable at the distance of 284 mm from the origin. However, during the application of the CWT-based algorithm the results are highly biased by the boundary effect, which appears in the form of high peaks at both
ends of the signal. In contrast to this, the results obtained using CWT differences-based algorithm significantly lower the boundary effect, thus allowing for better visibility of the peak related to damage, as can be seen in figure 5.

Figure 4. Results of damage identification using the CWT-based post-processing algorithm of the modal rotation profile.

Figure 5. Results of damage identification using the CWT differences-based post-processing algorithm of the modal rotation profile.

3.3. Results of damage identification for 2D data

Similar studies were performed on the 2D fields of modal rotations. Due to the mentioned necessity of limiting some of the parameters in order to make a visual representation of the results possible it was assumed that $\theta = 0$ and $\alpha = 15$. The obtained results for the beam first mode shape is presented in figure 6.

Figure 6. Results of damage identification using the post-processing algorithm CWT coefficients based on differences of the modal rotation field.

Although the boundary effect remains strong, one can observe a very well visible damage signature at the distance of 284 mm from the origin. The extended approach for modal rotation fields based on the CWT coefficients of differences of modal rotation creates new possibilities for improvement of damage identification in terms of its geometry.

4. Discussion and conclusions

Shearography is a NDT method with proven effectiveness in various industrial applications for damage identification, which is based on the disturbance analysis of the rotation field. However, this method fails in the case of small damage. The developed processing algorithm allows to significantly increase the damage detectability, primarily for small damage, which remains undetectable when the processing algorithms are not applied. Moreover, the developed algorithms based on CWT allowed for a significant reduction of the boundary effect, which appears in all cases when wavelet analysis is performed. It was shown that the algorithm based on the CWT coefficients of the rotation differences allows for emphasizing the damage signature, making damage identification results easier. The developed algorithm was also extended to the 2D domain, allowing the analysis of modal rotation fields instead of 1D modal rotation profiles. The presented results show high sensitivity of the extended algorithm to damage, and creates new possibilities for detection, localization, and identification of damage geometry, even with geometrically complex contours.
Acknowledgments
This publication is supported within the framework of the Rector’s grant no. 10/060/RGJ19/0108 in the area of scientific and development research of the Silesian University of Technology, Poland (AK), and LAETA, Project UID/EMS/50022/2019 through IDMEC and INEGI/UP, Portugal (JVAS, HL).

References

[1] Aebischer H A and Waldner S 1999 Opt. Commun. 162 205-10
[2] Kemao Q, Soon S H and Asundi A 2003 Opt. Laser Technol. 35 649-54
[3] Huang Y, Janabi-Sharifi F, Liu Y and Hung Y Y 2011 Opt. Express 19 606-15
[4] Lopes H M R, Araújo dos Santos J V, Mota Soares C M, Miranda Guedes R J and Pires Vaz M A 2011 Comput. Struct. 89 1754-70
[5] Minnini M, Gabriele S, Lopes H and Araújo dos Santos J V 2016 Mech. Syst. Signal Pr. 79 47-64
[6] Katunin A, Lopes H and Araújo dos Santos J V 2019 Mech. Syst. Signal Pr. 116 725-40
[7] Solis M, Algaba M and Galvin P 2013 Mech. Syst. Signal Pr. 40 645-66
[8] Solis M, Ma Q and Galvin P 2018 Strain 54 e12266
[9] Fan W and Qiao P 2009 Int. J. Solids Struct. 46 4379-95
[10] Huang Y, Meyer D and Nemat-Nasser S 2009 Mech. Mater. 41 1096-107