Estimated relative permittivity of contaminated laterite soil: An empirical model for GPR waves

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Abstract. Estimated relative permittivity performed on soil is essential for forecasting the performance of Ground Penetrating Radar (GPR) in an in-depth manner. This study investigated and verified the empirical relationship model between relative permittivity and volumetric water content in soil to predict the relative permittivity of contaminated laterite soil. In this study, a 24-hour measurement involving 800 MHz shielded antenna GPR was carried out in a concrete simulation field tank filled with Terap Red soil (1.5 m x 2.6 m x 1.5 m) at UiTM Perlis, Malaysia. Embedded moisture content probe was simultaneously measured to monitor the response of volumetric water content in contaminated soil in order to formulate an empirical relationship between relative permittivity and moisture content. The GPR data were pre-processed and filtered with Reflexw 7.5, while regression analysis was performed to evaluate the empirical relationship model. The model outcomes were retrieved from a number of cross-validation schemes, including correlation analysis ($R^2$), root mean square error (RMSE), and calibrated Agilent Technologies Automated Vector Analyser (VNA). A third-order polynomial for analysis of variance (ANOVA) best fitted the model with positively strong correlation ($R^2=0.989$, N=24, P < 0.01) and RMSE 0.003 < RMSEpredicted < 0.19. Verification of the proposed model using calibrated VNA displayed exceptional agreement between 0.06% comparisons.

Keywords: Empirical model, prediction, relative permittivity, VNA, GPR

1. Introduction

Ground Penetrating Radar (GPR) has been reckoned as an effective tool to monitor non-destructive soil contamination [1-2]. The function of GPR, which is dictated by electromagnetic (EM) properties response, is reliant on relative permittivity ($\varepsilon_r$). Relative permittivity refers to the ratio of electric-field storage capacity for a material that is highly related to soil moisture content [3]. The value of relative...
permittivity differs across soil type as it depends on volumetric moisture content. The presence of diesel fuel in soil substitutes the water retained in soil and gets trapped in the soil particle, thus affecting the permittivity value indirectly. Typically, relative permittivity has a crucial role in determining the velocity values for depth accuracy control of a material at subsurface level. Additionally, it is common to apply models for estimating the relative permittivity of soil to forecast the performance of EM sensors, such as GPR [4].

A number of outstanding models for estimation of relative permittivity are based on empirical relationship model stemming from field-based correlation, apart from volumetric and phenomenological methods [5]. It is noteworthy to highlight that most of the empirical models are initiated in Time Domain Reflectometry (TDR), particularly in light of velocity and soil water content. The models discussed include Topp’s model [6] and Complex Refractive Index Model (CRIM) [7]. Topp’s model uses the third-order polynomial to define the correlation between volumetric water content and relative permittivity of the bulk material using a range of materials [6]. Topp’s model is derived from three types of fine grain, namely sandy, loam, and clay.

Karim, Kamaruddin, and Hasan proposed the third-order polynomial to develop an empirical relationship model derived from GPR measurement in order to estimate water content in peat soil [8]. The model prediction of relative permittivity, nonetheless, is applied for soil with similar deposit. In this present study, the empirical relationship model was enhanced to estimate the relative permittivity of laterite soil contaminated by diesel fuel using GPR. The model outcomes were recovered using the following cross-validation schemes: correlation analysis ($R^2$), root mean square error, and calibrated Agilent Technologies Automated Vector Analyser (VNA).

2. Materials and methods

2.1. Materials
Moisture content test and GPR measurements were carried out on a laterite soil mixture in dedicated test-bed located at Universiti Teknologi MARA Perlis (UiTMPs). Ambient temperature at a monthly average of 30°C/24°C was set with non-controlled moisture. A larger scale was constructed from a concrete block with the following dimension: 1.5 m x 2.6 m x 1.5 m, and 5 cm thickness. The concrete block served as a high conductivity material [9] after weighing in the reduction of EM wave reflection effects, and the controlled propagation of EM wave over the boundary of simulation site that distinguishes the areas between simulation sites. Laterite soil filling was obtained from an area located in the Harum Manis cultivation of Agrotechnology Farm, UiTMPs (6.2659N, 100.1648E). Particle sizes that ranged from 2.000 to 0.015 mm were gained from the sieved analysis of grain size distribution by adhering to British Standard (BS) 1377-2:1990 [10].

A test-bed was prepared by burying a PVC pipe (diameter: 4.5 cm) at 50 cm depth. Next, 30-litre diesel leak was released at once from a hole made in the middle of a PVC pipe with a diameter of 0.5cm.

2.2. Data collection
The GPR measurement was performed before and 24 hours of continuous scanning after diesel fuel injection using a high 800 MHz shielded antenna. The shielded antenna was operated in a common offset (CO) GPR survey method with MALA GroundVision 2 acquisition software system and ProEx control unit. Notably, the acquisition software system was built by using the Mala Geoscience AB of Guideline Geo. The measurement was conducted on five profile lines along a single transect at the site with 50 cm space lines (see Figure 1 (a) and (b)). In order to yield optimum collection of GPR data, the 2.6 m trace, along with each of the four rows set apart by 0.5 m, was nominally traced. The measurement was performed on a single-offset 2D section using the setting parameters prescribed by the manufacturer (see Table 1):
Figure 1. The GPR measurement (a) gridline with 0.5 m interval for each grid and scanning direction, (b) gridline marked on the concrete block and filled with laterite soil, and (c) moisture content probe test.

Table 1. GPR processing parameters used for laterite soil scanning.

| Measurement Setting    | Parameter       |
|------------------------|-----------------|
| Antenna Separation     | 0.14 m          |
| Sample                 | 512             |
| Time Window            | 52.4128 ns      |
| Vertical Stack         | 8 folds         |
| Sampling Frequency     | 9600 Mhz        |
| Trig Interval          | 0.01 m          |
| First Arrival          | 10 sample       |
| Velocity               | 100 m/μs        |

The accuracy of locational for GPR measurement had been maintained by calibrating the distance measuring tools called wheel calibration and depth calibration. The calibration line was 10 m long on the ground surface, while the survey wheel distance error must not exceed 2%. The depth calibration was performed after setting the velocity parameter in a scale setting prior to the measurement. The radar velocities were calculated based on the speed of EM wave, \( V_m \) (equation 2).

Soil moisture content \( (\theta_w) \) was measured on laterite soil using a soil moisture probe that was mounted on PMS710 (see Figure 2c). The probe was inclosed at the contaminated area to optimise the affected soil moisture to GPR reflection measurement. Prior to moisture content test, the probe was calibrated with \( \theta_w \) laboratory analysis using the gravimetric water content (GWC) technique based on the standard procedure of classification test stipulated in BS 1377-2:1990. The \( \theta_w \) calculation of the soil specimen, \( w \), as dry soil mass \( (m) \) percentage nearest to 0.1%, is presented in the following equation:

\[
\frac{m_w - m_l}{m_s - m_l} \times 100(\%)
\]

2.3. Data processing
Prior to production of time/depth section and data analysis, the data were filtered during data processing by using REFLEXW™ software. This process not only enhanced the features of hyperbola, but also eliminated background and ambient noise. Table 1 lists the processing parameters applied for GPR data. The initial basic data processing of the GPR dataset began by editing the header file. Prior to processing, the header files of each section were viewed in sequence to verify the consistency of the survey parameters. The flow of post-acquisition processing operation is as follows: 1) time zero correction, 2) dynamic correction, 3) background removal, 4) dewow filtering, 5) automatic gain function, 6) bandpass filtering, and 7) hyperbola fitting.
Table 2. GPR processing parameters applied in Reflexw Software.

| Process                        | Parameter      |
|--------------------------------|----------------|
| Time zero                      | -2.040 ns      |
| S/R Distance                   | 0.18 m         |
| Dewow                          | 52.4128 ns     |
| Bandpass Butterworth Filter    | 600-1200       |
| Gain function                  | 7 db/m         |
| Hyperbola fitting              | 0.0639 ns      |

2.4. Estimation of relative permittivity

The determination of relative permittivity is also identified as the association between velocity of EM wave in medium ($v$) and velocity of EM wave in vacuum, which is based on the speed of light in free space ($c = 0.3$ m/ns). This correlation is described in the following equation:

$$V_v = \frac{c}{\sqrt{\varepsilon \mu}}$$  \hspace{1cm} (2)

Based on the assumption of low-loss material in Terap Red soil, the simplified version of equation 2 was obtained from the following equation:

$$V_v = \frac{c}{\sqrt{\varepsilon \mu r} + \left(\left(1 + P^2\right) + 1\right)}$$  \hspace{1cm} (3)

From the equation presented above, relative magnetic permeability ($\mu_r = 1$ for non-magnetic materials) that is close to unity [11] and loss factor, $P$, was considered as 0 value ($P \approx 0$).

2.5. Empirical analysis

The equation of the empirical model to predict relative permittivity is described by the linear regression empirical relationship between $\theta_v$ and $\varepsilon_r$. Estimated relative permittivity, $\varepsilon_r$, was obtained from GPR data. The model was assessed by using a number of statistical tools, including analysis of variance (ANOVA), and t-statistics for model parameter significance testing.

2.6. Verification of soil relative permittivity

The performance displayed by the empirical model equation in predicting relative permittivity for contaminated laterite soil was verified via cross-validation schemes, which involved root-mean-squared error (RMSE) and in-situ measuring. A series permittivity analysis probe was conducted using VNA E8562B to measure the actual relative permittivity of the subsurface soil as a reference to the independent relative permittivity of GPR. In this sense, the VNA measurements yielded the actual permittivity values upon assessing the GPR data [12]. Besides, a number of published studies have proven the superior use of VNA in verifying relative permittivity values, such as that reported by Mishra, Bore, Jiang, Scheuermann, and Li [13]. They had placed focus on calibration via VNA for the calculated permittivity behaviour of kaolin suspensions in tap and deionised water using CRIM equation.
3. Results and discussion

3.1. Model fitting of empirical relative permittivity relationship

Empirical relationship appears to be the most accurate model to describe correlated $\theta_v$ and $\varepsilon_r$ based on TDR data [6]. Such relationship that describes $\varepsilon_r(\theta_v)$ can be established from four linear regression analyses: (i) simple linear, (ii) logarithmic, (iii) second-order, and (iv) third-order polynomial. A model that suggests higher-order regression is depicted in the following relationship:

$$\varepsilon_r = \alpha + \beta_1 \theta_v + \beta_2 \theta_v^2 + \beta_3 \theta_v^3$$  \hspace{1cm} (4)

where $\varepsilon_r$ is real relative permittivity element, while $\beta$ denotes constant coefficient for each predictor variable of $\theta_v$ (volumetric moisture content) and $\alpha$ (constant of intercept in $\varepsilon_r(\theta_v)$ plot) to represent the value of $\varepsilon_r$ at $\theta_v = 0$.

Figure 2(d) illustrates the best fitting model plotted for predictive relative permittivity, $\varepsilon_r$, of laterite soil contaminated by diesel. The exceptional agreement displayed by the third-order polynomial model, as presented in Table 3 for the best-fit model, exhibited a positively strong correlation with $R^2 = 0.989$, which implies 98.9% of the variation in volumetric moisture content, $\theta_v$, is ascribed to determine relative permittivity, $\varepsilon_r$. This best-fit model (third-order polynomial) outperformed the rest for goodness of fit with a standard error of 0.076212 (see Table 4).

![Figure 2](image-url)

**Figure 2.** Line of fit plot of the empirical relation model: (a) simple linear model, (b) logarithmic model, (c) second-order polynomial model, and (d) third-order polynomial model.
Table 3. Model summary of empirical relationship model.

| Type of Regression | Equation                                      | $R^2$  |
|--------------------|-----------------------------------------------|--------|
| Linear             | $\varepsilon_r = -3.695498 + 0.345464\theta_v$ | 0.959901 |
| Logarithmic        | $\varepsilon_r = -0.59852 + 3.39309\ln\theta_v$ | 0.935246 |
| 2nd Order Polynomial | $\varepsilon_r = 6.21647 + 0.16638\theta_v + 0.025066\theta_v^2$ | 0.974282 |
| 3rd Order Polynomial | $\varepsilon_r = -9.2058 + 4.58583\theta_v - 0.4518\theta_v^2 + 0.0156\theta_v^3$ | 0.989184 |

Table 4. A summary of empirical relationship model outputs and ANOVA analysis.

| Type of Regression | Significant F     | Standard Error | $R^2$  |
|--------------------|-------------------|----------------|--------|
| Linear Regression  | 7.38497E-17       | 0.139914       | 0.959901 |
| Logarithmic        | 1.45552E-14       | 0.177799       | 0.935246 |
| 2nd-order polynomial | 2.03007E-17     | 0.114688       | 0.974282 |
| 3rd-order polynomial | 8.06719E-20     | 0.076212       | 0.989184 |

The third-order polynomial model for ANOVA was computed (see Table 4) and yielded a $p$-value of 8.06719E-20, thus implying that null hypothesis, $H_0$, is rejected at a confident level of 0.01 or 1% level of significance. The significance of coefficient value was determined using $t$-statistics test, which gave hypothesis $H_0: \alpha = 0$ against $H_a: \alpha \neq 0$, and $H_0: \beta = 0$ against $H_a: \beta \neq 0$. Upon comparing this $p$-value with significance level, as tabulated in Table 5 to be lower than 0.01, it suggested at least 99% confidence level. Therefore, the null hypothesis is rejected, and the model is indeed significant. The model in yield appears to be in agreement the findings reported by Topp et al., Steelman and Endres, and Patriarca et al. [6,14-15] which highlighted that the volumetric water content of soil indirectly affected and had a significant relationship with the value of relative permittivity.

Table 5. A summary of Best-Fit model by the third-order polynomial coefficient analysis.

| Coefficients | Standard Error | $t$ Stat | P-value  |
|--------------|----------------|----------|----------|
| Intercept    | -9.205796574   | 2.979595 | -3.08961 | 0.005779359 |
| 14.5         | 4.585829193    | 0.910753 | 5.035204 | 6.33911E-05  |
| 210.25       | -0.451848557   | 0.090982 | -4.96636 | 7.42582E-05  |
| 3048.625     | 0.015619194    | 0.002975 | 5.249368 | 3.88478E-05  |

3.2. Verification of the model

Both accuracy and precision of $\varepsilon_r$ predicted values for the contaminated laterite soil were retrieved from cross-validation involving RMSEpredicted, as prescribed by Roth, Malicki, and Plagge and Szyplowska et al. [7,17]. As illustrated in Figures 3 and 4, $\varepsilon_r$ predicted Value obtained from the empirical relationship model was generally closer to the $\varepsilon_r$ measured Value gained from GPR measurement that was calculated using Eq.3.
Figure 3. Frequency of $\varepsilon_r$ predicted value obtained from empirical relationship model compared to $\varepsilon_r(m)$ measured value from GPR measurement.

Figure 4. Comparison chart for values between $\varepsilon_r$ predicted and $\varepsilon_r$ measured for all type empirical relationship models.

As portrayed in Figure 4, the low value of volumetric water content in laterite soil had affected the difference of $\varepsilon_r$ predicted value with $\varepsilon_r$ measured value, thus corresponding to interpretation error as denoted in low amplitude signal of GPR radargram. The percentage of $\varepsilon_r$ predicted vs. $\varepsilon_r$ measured from third-order polynomial was better than the other regression analysis with differences ranging from 0.04% to 2.17% and an average of 0.69% (see Table 6). Similarly, Szyplowska et al. reported the deficiency of volumetric water content in soil that might be influenced by errors upon retrieving relative permittivity [17]. Nevertheless, the results obtained were still in consideration, particularly for the frequent quantitative estimations of RMSEpredicted values calculated for third-order polynomial, which was as low as $0.003 < \text{RMSE}_{\text{predicted}} < 0.19$. 
Table 6. Summary of comparison between $\varepsilon_r$ predicted and $\varepsilon_r$ measured for all regression analyses (fitted-model).

| Different (Fit model vs Measured) | 3rd-Order Polynomial | 2nd-Order Polynomial | Logarithmic | Linear Regression |
|----------------------------------|----------------------|----------------------|-------------|------------------|
| Diff (%)                         | Diff (%)             | Diff (%)             | Diff (%)    | Diff (%)         |
| Min                              | 0.00356              | 0.01183              | 0.00178     | 0.00675          | 0.10 |
| Max                              | 0.19473              | 0.28309              | 0.46546     | 0.37708          | 4.38 |
| Mean                             | 0.05438              | 0.09263              | 0.13609     | 0.10742          | 1.32 |

By referring to the boxplot displayed in Figure 5, the empirical relationship of third-order polynomial generated the most accurate prediction for $\varepsilon_r$ predicted of contaminated laterite soil ($\text{RMSE}_{\text{average}} = 0.05$), when compared to the other models. In addition, the empirical relationship model was verified with calibrated VNA to achieve good accuracy for $\varepsilon_r$ predicted.

**Figure 5.** Boxplot of comparison chart for RMSE predicted values for all empirical relationship models: (i) blue: third-order polynomial, (iii) grey: logarithmic, and (iv) yellow: simple linear.

**Figure 6.** Results of $\varepsilon_r$ values from calibrated VNA measurement.
Table 7. A comparison of $\varepsilon_r$ values between all empirical relationship models based on GPR and calibrated VNA measurements on contaminated laterite soil.

| Comparison $\varepsilon_r$ | 3rd-Order Polynomial | 2nd-Order Polynomial | Logarithmic | Linear Regression |
|---------------------------|----------------------|----------------------|-------------|------------------|
| Min                       | 0.509687999          | 0.341481536          | 0.615778391 | 0.507755653      |
| Max                       | -0.505792685         | -0.301827676         | -0.023537689 | -0.138712436     |
| Mean                      | 0.043512073          | 0.043512074          | 0.043512074 | 0.043512074      |

Figure 7. Percentage comparison of $\varepsilon_r$ values between all empirical relationship models with calibrated VNA measurement on contaminated laterite soil.

The calibrated VNA technique was employed in this study to measure the actual relative permittivity of subsurface soil as a reference to the independent relative permittivity of GPR. Hence, one can assume that the VNA measurements gave the actual permittivity values in evaluating the GPR data [12-13]. Figure 6 illustrates the outcomes yielded from the series of calibrated VNA measurement for laterite soil contaminated by diesel within 8 hours. The average relative comparison of the $\varepsilon_r$ predicted values calculated for all empirical relationship models between GPR and calibrated VNA measurements resulted in a slight difference that ranged from 0.043512073 to 0.043512074 (see Table 7), which was below 0.60% difference. The third-order polynomial exhibited better accuracy than the other models, as illustrated in Figure 7. Having reported that, the agreement of the series of $\varepsilon_r$ predicted yielded from third-order polynomial validation is deemed acceptable, which is also in agreement with Topp et al. [6].

4. Conclusion
This study is motivated by the need to ascertain the impact of volumetric water content in laterite soil on diesel fuel evaporate. Both GPR and soil moisture probe data were analysed for 24 hours to formulate the correlation of moisture soil condition with $\varepsilon_r$. By linear regression, the analysis outcomes portrayed the $\varepsilon_r$ significant reliance on volumetric water content in soil, which is in line with those reported by Topp et al. and Piuzzi et al. [6,18]. The empirical relationship model based on third-order polynomial regression best fit the proposed $\varepsilon_r$ prediction model for contaminated laterite soil by diesel fuel. Verification of the proposed model by measuring the actual $\varepsilon_r$ using calibrated VNA exhibited variances of 0.06% with positive correlation ($R^2 = 0.989$) and an accuracy of RMSE$_{average} = 0.05$. Model enhancement has been proven to improve the performance of estimating $\varepsilon_r$ contaminated laterite soil by volumetric moisture content for GPR measurement.

References
[1] Shaaban F, Habeebullah T M, Morsy E A and Gabr S 2016 Arab. J. Geosci. 9 754
[2] Shao S, Guo X and Ding H 2019 Temporal ground penetrating radar (GPR) imaging of an oil release within a porous medium: A description of anomalous GPR characteristics during the degradation process and a contaminated area determination method (Singapore: Springer)
[3] Martinez A, Byrnes A P, Survey K G and Avenue C 2001 *Modeling dielectric-constant values of geologic materials: An aid to ground-penetrating radar data collection and interpretation* vol 1

[4] Van Dam R L, Borchers B and Hendrickx J M H 2005 *Detect. Remediat. Technol. Mines. Minelike. Targets.* X. 5794 188

[5] Rubin A J and Ho C L 2018 A review of empirical methods to estimate relative permittivity 2018 17th Int. Conf. Gr. Penetrating. Radar. 2018.

[6] Topp G C, Davis J L and Annan A P 1980 *Water. Resour. Res.* 16 574

[7] Roth C H, Malicki M A and Plagge R 1992 *J. Soil Sci.* 43 1

[8] Karim N I A, Kamaruddin S A and Hasan R C 2018 *Int. J. Integr. Eng.* 10 177

[9] Wu T, Huang R, Chi M and Weng T 2013 *Comput. Concr.* 12 337

[10] British Standards Institution 1990 Soils for civil engineering purposes Part 8 BS 1377–4 40

[11] Van Dam R L 2001 *Causes of ground-penetrating radar reflections in sediment* (Amsterdam: de Vrije Universiteit Amsterdam)

[12] Pellinen T, Huuskonen-Snicker E, Eskelinen P and Olmos Martinez P 2015 *J. Traffic Transp. Eng.* 2 30

[13] Mishra P N, Bore T, Jiang Y, Scheuermann A and Li L 2018 *Meas. J. Int. Meas. Confed.* 121 160

[14] Steelman C M and Endres A L 2011 *Vadose Zo. J.* 10 270

[15] Patriarca C, Tosti F, Velds C, Benedetto A, Lambot S and Slob E 2013 *J. Appl. Geophys.* 97 81

[16] De Benedetto D, Castrignanò A and Quarto R 2013 *Procedia. Environ. Sci.* 19 436

[17] Szyplowska A, Szerement J, Lewandowski A, Kafarski M, Wilczek A and Skierucha W 2018 Impact of soil salinity on the relation between soil moisture and dielectric permittivity 2018 12th Int. Conf. Electromagn. Wave Interact. Water. Moist Subst. 2018 1

[18] Piuzzi E, Cannazza G, Cataldo A, De Benedetto E, De Giorgi L, Frezza F and Timpani F 2018 *Meas. J. Int. Meas. Confed.* 114 493