Chromospheric thermal continuum millimetre emission from non-dusty K and M red giants

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ABSTRACT
We examine the thermal free–free millimetre fluxes expected from non-dusty and non-pulsating K through mid-M giant stars based on our limited understanding of their inhomogeneous chromospheres. We present a semi-analytic model that provides estimates of the radio fluxes for the mm wavelengths [e.g. Combined Array for Research in Millimetre-wave Astronomy (CARMA), Atacama Large Millimetre/submillimetre Array (ALMA) and Karl G. Jansky Very Large Array (JVLA) Q band] based on knowledge of the effective temperatures, angular diameters and chromospheric Mg II h & k emission fluxes. At 250 GHz, the chromospheric optical depths are expected to be significantly less than unity, which means that fluxes across the mm and submm range will have a contribution from the chromospheric material that gives rise to the ultraviolet emission spectrum, as well as the cool molecular material known to exist above the photosphere. We predict a lower bound to the inferred brightness temperature of red giants based on heating at the basal flux limit if the upper chromospheres have filling factor ≃ 1. Multifrequency mm observations should provide important new information on the structuring of the inhomogeneous chromospheres, including the boundary layer, and allow tests of competing theoretical models for atmospheric heating. We comment on the suitability of these stars as mm flux calibrators.

Key words: stars: activity – stars: chromospheres – stars: late-type – radio continuum: stars.

1 INTRODUCTION
Convective motions beneath the photospheres of late-type (cool) stars agitate the plasma, leading to the excitation of acoustic and magnetic disturbances. These waves induce non-radiative heating of the upper photosphere and chromosphere. Essentially all cool stars possess material heated above the prediction of radiative equilibrium (RE), which emits a chromospheric spectrum that is most conspicuous in the ultraviolet (UV). The specific roles of acoustic and magnetic waves in heating the quiet solar chromosphere and the role of magnetic fields in structuring its atmosphere are the subject of considerable debate (Kalkofen, Ulmschneider & Avrett 1999; Ayres 2002; Vecchio, Cauzzi & Reardon 2009). For the Sun and red giants the chromospheric plasma has a detectable signature in the millimetre (mm) thermal continuum (Altenhoff, Thum & Wendker 1994; Loukitcheva et al. 2004).

Stellar chromospheric surface fluxes measured from dominant emission lines, such as Ca ii h & k and Mg ii h&k, show a wide scatter at a given effective temperature (Teff), just as found for regions of different activity on the Sun. However, there is a well-defined empirical lower bound known as the basal flux, which is a strong function of effective temperature, Fbasal ∼ Teff 7 2, but is only a weak function of surface gravity (g∗). The chromospheric cooling from the quietest solar regions, in the centres of supergranules, is close to the basal flux for a Teff = 5770 K star (e.g. Schrijver & Zwaan 2000). Stars that have chromospheric emission above the basal flux are thought to have additional contributions from areas of enhanced surface magnetic activity, analogous to solar active regions (sunspots and associated plage). However, no consensus has been reached about the origin of the basal flux. One line of thought is that it arises from the deposition of purely acoustic shock energy generated in the subphotospheric convection zone (Ulmschneider 1991), while the absence of appropriate spectroscopic shock signatures in inactive basal-flux red giants (Judge & Carpenter 1998) suggests a magnetic origin. In fact, on the Sun both mechanisms are seen to operate: small-scale transient shock heating in the so-called ‘K grains’ and more steady magnetic heating in the large-scale supergranulation network.

The physical structures of chromospheres from these two mechanisms are likely to be quite different. The acoustic wave picture would lead to a 3D network of shocks that is highly time variable, and at a given position the gas temperature would fluctuate from very cool to very hot with the mean temperature being cool

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Millimetre emission from non-dusty red giants

(Wedemeyer-Böhm et al. 2007). This mechanism leads to localized intermittent chromospheric emission but, which when averaged over the stellar disc, would not be apparent in the mm radio. The presence of magnetic fields can lead to longer lived structures and enhanced atmospheric heating. Time-independent 1D semi-empirical models for the Sun that represent regions of different activity are well established (Vernazza, Avrett & Loeser 1976, 1981; Fontenla, Avrett & Loeser 1990). Similar but spatially unresolved models have been constructed for a few cool stars including in active red giants. These semi-empirical models are designed to reproduce the temporally and spatially averaged UV and optical emission-line profiles and fluxes. A characteristic of these models is that they require a gradual temperature rise from the top of the photosphere to the upper chromosphere, and then a very steep rise through the chromosphere–corona transition region (Kelch et al. 1978; Harper 1992; Luttermoser, Johnson & Eaton 1994; McMurry 1999).

One might expect that acoustic models, whose mean temperatures do not increase with height, would fail to predict the same UV fluxes as semi-empirical models with an outward temperature increase, but this is not necessarily the case. In the UV, $h\nu/kT \gg 1$ and the source functions and emissivities $\propto e^{-h\nu/kT}$ are very sensitive to temperature. In this situation, the shock peaks can dominate the temporally and spatially averaged emission of the dynamic atmosphere. Thus, it is possible for acoustic shock and semi-empirical models to produce the same chromospheric fluxes even though the temperature structures are profoundly different (Carlsson & Stein 1995).

However, in the mm radio, the thermal continuum source (Planck) function depends linearly on $T_{\nu}$, so both solar and stellar mm observations can probe differences between intermittent and long-term structured atmospheres and potentially differentiate between acoustic and magnetically heating models. A comparison between UV and radio diagnostics could test the symbolic inequality:

$$\left\langle e^{-h\nu/kT} \right\rangle \neq e^{-h\nu/kT},$$

where the averages are over space and time. The left-hand side exemplifies UV emission diagnostics, while the right-hand side reflects the same source function but with the mean temperature, ($T$), as inferred, for example, from mm radio emission. In practice, UV and mm-radio fluxes calculated from different atmospheric models can be compared to observations. The hydrogen ionization determines the electron densities which, in part, determine both the mm-radio opacity and the UV collisional excitation rates. The remains difference in the temperature dependence for formation leads to the test of equation (1).

Here we analyse published thermal mm (250-GHz) fluxes (Altenhoff et al. 1994) from single non-dusty and non-pulsating K and M giants (luminosity class III) to investigate their chromospheric signatures and explore the role of future multifrequency observations in revealing the nature of chromospheric structure. Future solar Atacama Large Millimetre/submillimetre Array (ALMA) imaging will be focused on obtaining sufficient spatial resolution to separate the magnetic and non-magnetic regions (Louchkina, Solanki & White 2008), whereas studies of inactive basal-flux red giants may allow us to study the contamination from magnetic active regions by observing giants that are covered with inactive chromospheric regions analogous to quiet solar supergranule centres.

A first estimate for the expected mm flux from a red giant is provided by the hard disc model, which is appropriate when the ionized density scale height, $H'_{\text{ion}}$, is much smaller than the stellar radius, $R_*$. The observed flux density ($F_\nu$) is given by

$$F_\nu = \left(\frac{\phi}{2}\right)^2 F_*'$$

where $F_*'$ is the average flux density at the surface, having angular diameter $\phi$ with respect to the distance observer. Working in terms of brightness temperatures $T_{\text{BB}}$, we have

$$F_\nu = \frac{\tau k h \nu^2 \phi^2}{2 e^2} \int_{0}^{1} T_{\text{BB}}(\mu) \mu \, d\mu = \frac{\pi k h \nu^2 \phi^2}{e^2} (T_{\text{BB}}),$$

where $\mu$ is the usual cosine of the angle between a ray and the normal to the emitting surface, and $T_{\text{BB}}$ is the weighted mean brightness temperature. In more convenient units, the flux in mJy

$$F_\nu = 13.8 \left(\frac{T_{\text{BB}}}{3900 \, \text{K}}\right) \left(\frac{\nu}{250 \, \text{GHz}}\right)^2 \left(\frac{\phi}{10 \, \text{mas}}\right)^2.$$

Many nearby red giants now have measured angular diameters, $\phi$, and estimated effective temperatures, $T_{\text{eff}}$ (Mozurkewich et al. 2003). Inserting $\phi$, and substituting $T_{\text{eff}}$ for $T_{\text{BB}}$ into equation (4) shows that many of these giants can in principle be observed at multiple frequencies with high signal-to-noise ratio (S/N) values when ALMA is completed given the expected sensitivity limits (Butler & Wooten 1999). Here we develop a more physically motivated semi-analytical model for mm fluxes that incorporates constraints on chromospheric structure from independent infrared (IR) and UV observations, which also allows us to include the effects of different levels of chromospheric heating.

In Section 2 we compile pertinent empirical constraints derived from semi-empirical models and other studies of red giants, and in Section 3 estimate the expected chromospheric mm optical depths. In Section 4 we develop a semi-analytic model for $T_{\text{BB}}$ based on these empirical constraints, which we then calibrate against direct measurements of the well-studied red giant α Tau (K5 III) (Aldebaran). A comparison with observations and specific predictions for ALMA frequencies are presented in Section 5, and a discussion of the diagnostic potential of mm observations in Section 6. Conclusions are drawn in Section 7.

2 EMPIRICAL CONSTRAINTS ON STELLAR CHROMOSPHERES

In this section we gather together nuggets of information from previous studies of red giant chromospheres, which will incorporate into our analytic model for the mm radio fluxes. The radio optical depth, $\tau_\nu$, is proportional to different powers of the atmospheric properties – electron density, temperature and path length – and we seek to estimate these for stars covering a range of $T_{\text{eff}}$ and different levels of stellar activity.

Historically, the usual lack of temporal and spatial resolution in observations of the chromospheres of distant stars partly motivated the development of 1D time-independent semi-empirical models. The earliest models used diagnostics observable from the ground – Ca II h & k, the Ca II IR triplet and Hα – and early satellite observations of Mg II h & k from Copernicus (Ayres, Linsky & Shine 1974; Ayres & Linsky 1975). These types of models are analogous to the venerable VAL series of solar models, although for the latter spatial resolution did permit construction of components representative of regions of different activity levels (Vernazza et al. 1976, 1981). More

1 mJy =10^{-29} \, \text{W m}^{-2} \, \text{Hz}^{-1} = 10^{-26} \, \text{erg s}^{-1} \, \text{cm}^{-2} \, \text{Hz}^{-1}.
recent variants of these solar component models are the FAL series (Fontenla et al. 1990), which are employed in solar irradiance studies (Fontenla et al. 2009), among other uses. Two notable features of these semi-empirical models are (1) the temperature inversion in the low chromosphere above a temperature minimum, \( T_{\text{min}}^{\text{UV}} \approx 0.75 T_{\text{eff}} \), and (2) the electron density, \( n_e \), is, to within a factor of \( \sim 2 \), approximately constant throughout the chromosphere (Ayres 1979; Harper 1992). At the temperature minimum, the electron density \( \sim 10^{-9} \text{cm}^{-3} \) is dominated by (photo)ionized metals; however, as the temperature increases through the chromosphere, hydrogen gradually becomes more ionized until at the top of the chromosphere \( \sim 10^4 \text{K} \), hydrogen is predominantly ionized and then \( n_e \sim n_{\text{H}} \).

Through the chromosphere, the increasing ionization of hydrogen counters the rapid outward decline of the density due to hydrostatic equilibrium, maintaining a more or less constant \( n_e \) with increasing altitude (Ayres 1979). However, once the hydrogen has become mostly ionized, there no longer is a ready source of bound electrons to be liberated, and so in the higher layers the electron density will fall off rapidly, following the hydrostatic decline in \( n_{\text{H}} \) itself (Ayres 1979).

While there is a debate on whether the Sun has a permanent chromospheric temperature rise or not – it should be noted that presently the semi-empirical models nevertheless are the best thermodynamic inventory of stellar chromospheric plasma available. They approximately reproduce the amounts of plasma at different electron temperatures. This is discussed further in Section 6.

Pre-IUE, Mg \( \text{H} \& \text{K} \) observations revealed a strong dependence of chromospheric heating with effective temperature, \( F_*^\text{Mg} \sim T_{\text{eff}}^4 \) (Linsky & Ayres 1978; Ayres 1979). Ayres (1979) developed a scaling law for the mass column density at the onset of the chromospheric temperature rise, which can be rephrased in terms of the chromospheric electron density, \( n_e \), where atmospheric heating is sufficient to overcome \( \text{H}^+ \) cooling, namely

\[
n_e \sim A_{\text{Be}}^{1/2} F^{1/2} n_{\text{Be}}^{1/2} T_{\text{eff}}^{5/2}.
\]

\( A_{\text{Be}} \) represents the abundance of low first ionization potential elements relative to solar, \( F \) is the possible enhancement of a particular star relative to the general \( T_{\text{eff}} \) scaling related to differences in intrinsic activity. In Ayres (1979), \( F = 1 \) was taken to represent levels appropriate to inactive stars or the quiet sun and \( F = 10 \) for solar plage regions or (fast rotating) stars showing enhanced activity. Some empirical evidence for this trend on \( n_e \) in red giants is given by Byrne et al. (1988) based on IUE C II \( 2325 \text{Å} \) intersystem flux ratios, and by Buchholz, Ulmschneider & Cuntz (1998) who compared time-dependent acoustically heated atmospheric models with the mass column density of an effective \( T_{\text{UV}} \). Chromospheric mm radio fluxes are sensitive to the opacity, which in turn depends on the square of the electron density. Here we assume the scaling in equation (5) provides an appropriate dependence of \( n_e \) on star specific parameters.

### 2.1 Basal fluxes

We begin by considering the ratio of a star’s measured Mg \( \text{H} \& \text{K} \) flux to the basal flux, i.e. a basal-flux star has \( F = 1 \). We recall the usual implicit assumption that Mg \( \text{H} \& \text{K} \) line fluxes represent a fixed fraction of the total chromospheric radiative losses (Linsky & Ayres 1978). The other major coolants are H Ly\( \alpha \), Ca II and Fe II (e.g. Judge & Stencel 1991).

For this work we take IUE Mg \( \text{H} \& \text{K} \) fluxes from Pérez-Martínez, Schröder & Cuntz (2011), which have typical uncertainties of \( \pm 10 \text{ per cent} \), and we also adopt their \((V-K)\)-based expression for the basal flux (erg cm\(^{-2}\) s\(^{-1}\)),

\[
\log F_{\text{basal}} = 6.78 \log T_{\text{eff}} - 19.74,
\]

which has a very similar \( T_{\text{eff}} \) dependence to Linsky & Ayres (1978). We have chosen specifically their \((V-K)\) form of the basal flux, because the implied angular diameters are more reliable than the \( B-V \) form at lower \( T_{\text{eff}} \). Judge & Stencel (1991) presented Mg \( \text{H} \& \text{K} \) surface fluxes for a sample of low- and intermediate-mass giants, and their lower bound drops below equation (6) for \( T_{\text{eff}} < 3300 \text{K} \).

### 2.2 Inhomogeneous chromospheres

Observations of the 4.6-\( \mu \text{m} \) CO fundamental bands have demonstrated that on the Sun and late-type stars, there exist regions of cool molecular gas with temperatures well below the \( T_{\text{min}}^{\text{UV}} \) derived from UV and optical diagnostics (Ayres & Testerman 1981; Wiedemann et al. 1994). On the sun the area filling factor of the CO material has been proposed to be up to 80 per cent in the low chromosphere (Ayres 2002), but that factor is controversial. For the red giants the filling factor is close to unity with the temperature scaling as \( T_{\text{CO}} \sim 0.60 T_{\text{eff}} \) (which is close to RE values when molecular cooling is included). At present, there is a vigorous debate as to the exact nature of the CO clouds given the highly dynamic, time-dependent nature of the solar atmosphere (Kalkofen et al. 1999; Ayres 2002; Leenaarts et al. 2011). The presence of the CO material must be accounted for in chromospheric models, particularly when these regions are probed by mm continuum emission. For example, recent spectral interferometry of the much later spectral type M giant BK Vir (M7 III) has revealed the presence of spatially extended CO with an inner extended component at \( R = 1.2 R_\star \), with \( T_{\text{CO}} \sim 0.65 T_{\text{eff}} \) (Ohnaka et al. 2012).

At lower effective temperatures, other molecules, such as water vapour (Tsuji 2008), can play a role in further reducing the effective boundary layer temperature. Quirrenbach et al. (1993) found evidence from the Mark III interferometer that the TiO continuum becomes significantly geometrically extended by M5 III (R Lyr), for which the ratio of TiO to continuum diameters is \( 1.12 \pm 0.02 \). These extended molecular components possibly are related to the ‘MOLsphere phenomenon phenomenon’ that is more common in the Mira and lower gravity M stars (Tsuji et al. 1997; Perrin et al. 2004).

### 2.3 Chromospheric extent

The apparent angular size of a stellar radio source clearly is an important determinant of the radio flux. Here we consider the factors that control the angular diameter in the mm band, and in particular its ratio, \( G \), to the photospheric angular diameter, i.e. \( \varphi_{\text{min}} = G \varphi_\star \).

The ratio of pressure scale height, \( H_{\text{pres}} \), to stellar radius, \( R_\star \), is

\[
\frac{H_{\text{pres}}}{R_\star} \sim \frac{T_{\text{eff}} R_e}{M_\star},
\]

so that red giants with their much larger radii, but near solar masses, have relatively thicker chromospheres than the Sun.

Eclipse observations of the \( \zeta \) Aurigae systems, which have intermediate-mass K Ib primaries, show large plasma turbulence and direct evidence for atmospheric extensions greater than the thermal pressure scale height (e.g. Eaton 1993). This indicates that \( H_{\text{pres}} \) is controlled by both thermal and non-thermal motions. Here
we describe the increase in $H_{\text{pres}}$ from non-thermal flows by a quasi-
hydrostatic equilibrium,

$$H_{\text{pres}} = \frac{v^2 + 2kT}{2(\mu)m_{\text{H}}kg_c},$$

where $v$ is a characteristic velocity. For $\zeta$ Aur systems, $v$ is
consistent with the turbulence inferred from spectra, and possibly cor-
related on spatial scales comparable to the density/pressure scale
height. ($\mu$) is the chromospheric average of the mean mass per par-
ticle in units of the hydrogen mass $m_{\text{H}}$, and $g_c$ is the surface gravity.
We adopt ($\mu$) = 1.2 corresponding to a partially ionized hydrogen
($n_e \approx 0.1n_{\text{H}}$) and surface helium abundance of $n_{\text{H}} = 0.1n_{\text{H}}$.

Semi-empirical models typically include this non-thermal term in
the equations of quasi-hydrostatic equilibrium. For $\zeta$ Aur systems,
the implied turbulence is slightly greater than the hydrogen sound
speed, and in our model we set $v = \sqrt{2kT}$. While estimates of stel-
lar radii are available from interferometry (e.g. Baines et al. 2010)
and nearby red giants also have Hipparcos parallaxes, the stellar masses
remain uncertain for the single stars. Typical estimates are
1–2 $M_\odot$, leading to some uncertainty in the surface gravity in equa-
tion (8).

The semi-empirical one-component model of $\alpha$ Tau (McMurry
1999) predicts a 250-GHz (1.2-mm) radial optical depth of unity
at $R \simeq 1.03R_\star$, but the apparent angular diameter (half-central
intensity) corresponds to $\phi_{\text{mm}} \simeq 1.1\phi_e$. This is a result of the larger
column density of the tangential sight-lines that define the 1.2-mm
limb. The apparent angular diameter of $\alpha$ Boo predicted from a
semi-empirical model of Drake (1985) is $\phi_{\text{mm}} \simeq 1.06\phi_e$, and for $\gamma$
Her (M6 III) $\phi_{\text{mm}} \simeq 1.08\phi_e$ (Luttermoser et al. 1994). These models
were constructed with different diagnostics and assumptions about
the contribution of turbulent pressure. Therefore, although there is
a trend of increasing fractional chromospheric extension with later
spectral type, the lower $T_{\text{eff}}$, and thus lower chromospheric heating,
leads to lower $n_e$ and reduced mm optical thickness. In the light
of these competing factors, we adopt a typical extension of 1.08$\phi_e$
(i.e. $G = 1.08$), and accept an additional uncertainty of $\sim 2$ per cent
in $G$ which is consistent with the level of approximations adopted
here. In summary, here ($T_{\text{Br}}$) for the mm radio is defined as

$$\langle T_{\text{Br}} \rangle = \frac{2e^2F_e}{\pi k v^2 (G\phi_e)^2}.$$

Very recently, single baseline, spectrally resolved, K giant chro-
mospheric Ca $\Pi$ IR-triplet visibilities from Center for High Angular
Resolution Astronomy (CHARA) have been presented by Berio
et al. (2011). The authors attempted to estimate the chromospheric
extent by using a correction procedure based on the Eriksson, Lin-
sky & Simon (1983) semi-empirical chromosphere of the coronal
star $\beta$ Cet (K0 III). Berio et al. (2011) derive chromospheric ex-
tents of 1.16–1.47$R_\star$, although these values are based on a model
whose actual spatial extent is only 1.02$R_\star$. The extended emission
observed in the cores of the IR-triplet lines depends on the presence
of Ca $\Pi$ ions which might not be related to the regions of higher elec-
tron density to which the radio is sensitive. The calculation of the IR-
triplet visibilities requires a careful treatment of the Ca $\Pi$ to Ca $\Pi$
photoionization by H Ly$\alpha$ (Linsky & Avrett 1970; Rowe 1992),
spherical geometry and possible impact of cross-redistribution of
photons scattering within the Ca $\Pi$ $h$ & $k$ resonance lines (Uiten-
broeek 1989). The new CHARA measurements are very important
diagnostics and one of us (NO’R) is undertaking these non-local
thermodynamic equilibrium radiative transfer calculations at the present
time.

3 CHROMOSPHERIC MILLIMETRE
OPTICAL THICKNESS

As mentioned earlier, the chromospheric electron density has con-
tributions from the low first ionization potential elements and from hy-
drogen. At low temperatures ($T < 6000$ K), ionization of hydrogen is
a two-stage process: excitation to the $n = 2$ level by electron colli-
sions or scattered H Ly$\alpha$ photons followed by photoionization by
the optically thin photospheric Balmer continuum. At higher tem-
peratures, direct collisional ionization becomes important. When
hydrogen is partially ionized, free–free (thermal bremsstrahlung)
opacity dominates at mm wavelengths. To examine the influence of
the stellar outer atmosphere on the mm thermal emission, we first
consider the optical thickness of the warm chromospheric compo-
ent. For typical conditions, the opacity corrected for stimulated
emission is given by Rybicki & Lightman (2004)

$$\kappa = \frac{0.17272 n_e n_{\text{ion}} Z_{\text{ion}}^2}{T_{\text{eff}}^{1.5} v^2} \frac{g_0}{g_\ast} \text{ cm}^{-1},$$

where the variables are in CGS units, $Z_{\text{ion}}$ is the charge of the ions,
n$_e$ and $n_{\text{ion}}$ are the number densities of the electrons and ions, re-
spectively, and $g_0$ is the free–free Gaunt factor. At cm wavelengths,
the familiar power-law approximation from Altenhoff et al. (1960)
is

$$g_0 = 11.96 T_e^{0.15} v^{-0.1},$$

but at ALMA frequencies, which range from 84 to 950 GHz (Band
3–10), the power-law dependence is slightly different, namely

$$g_0 \approx 24.1 T_e^{0.26} v^{-0.17},$$

derived using the accurate Gaunt factors from Hummer (1988).

Under typical chromospheric conditions, the majority of abundant
species are either neutral or single ionized, so that $Z_{\text{ion}} = 1$ and $n_e$
are. This leads to an expression for the mm opacity:

$$\kappa_e = \frac{0.427 n_e^2}{T_{\text{eff}}^{1.24} v^2} \text{ cm}^{-1}.$$

The optical thickness of a slab of depth, $\Delta L$, is then simply $\Delta \tau_e = \kappa_e \Delta L$.

The assumption that the electron density is approximately con-
stant, as the ionization $n_e/m_1$ goes from $10^{-3} \rightarrow 1$, implies that
the hydrogen density has declined by four orders of magnitude and
therefore the physical thickness of the layer $\Delta L \approx 9$ density scale
heights. To the level of approximation here we set $H_e \approx H_{\text{pres}}$.

For the radial optical depth of the chromosphere, we have

$$\Delta \tau_e \approx \frac{0.427 n_e^2}{T_{\text{eff}}^{1.24} v^2} \times 9 \times \frac{2kT_e}{\mu m_1g_e} \frac{n_e^2}{T_{\text{eff}}^{1.24} v^2} \frac{k}{m_1 g_e} \approx \frac{6.4 n_e^2}{T_{\text{eff}}^{1.24} v^2} \frac{k}{m_1 g_e}.$$

This expression reveals only a very weak dependence on tempera-
ture variations within the chromosphere which is a result of the
cancellation resulting from the combination of opacity and scale
height. In the following, we set the surviving temperature factor
to $T_{\text{eff}}^{1.24}$. Inserting into equation (14) typical values for the M2.5
III star $\beta$ Peg (Decin et al. 2003; $M_e = 2.0 M_\odot$, $R = 110R_\odot$, and
$T_{\text{eff}} = 3600$ K) and a typical UV-based value of $n_e \approx 6 \times 10^8$ cm$^{-3}$
(Byrne et al. 1988), we find for 1.2-mm (250-GHz) and 3-mm
(100-GHz) chromospheric optical thicknesses of $\Delta \tau_{250\text{GHz}} \approx 0.1$
and $\Delta \tau_{100\text{GHz}} \approx 0.8$, respectively. Thus, depending on the level
of stellar activity, the mm wavelength regime is one where chromo-
spheres can go from optically thick at mm and cm wavelengths to
optically thin in the submm.

If the optical thickness of the chromosphere is less than unity,
then the next emitting layer we must consider is the cool molecular
gas immediately above photosphere with its near-unity filling factor. In these cool, high-density layers, where the hydrogen density increases rapidly with depth, H\(^{-}\) free-free absorption can become an important opacity source (Harper, Brown & Lim 2001). Here we assume that the bifurcated molecular regions are opaque at these wavelengths. Thus, the optical depth unity is reached when \(T_e \approx T_{\text{CO}}\), which is reasonable if the temperatures in these outer RE layers change only slowly with optical depth.

4 DESCRIPTION OF THE SEMI-ANALYTIC MODEL

We now assemble the different aspects discussed above and construct a semi-analytic model to provide estimates of the mm, e.g. ALMA band, radio fluxes. We then calibrate against well-studied \(\alpha\) Tau (K5 III) to normalize all the approximation constants of the order of unity accumulated along the way. A schematic representation of the model inhomogeneous chromosphere is shown in Fig. 1. Above the UV temperature minimum \(T_{\text{UV Min}}\) lies the opaque cool molecular material with temperature \(T_{\text{CO}}\). Immediately above this, perhaps in an overlying magnetic canopy, is the base of the warm chromosphere with temperature \(T_{\text{Br}}\); \(T_e\) then rises linearly in mass column density to the upper chromosphere with temperature \(T_{\text{Chrom}}\) and has an optical thickness \(\Delta \tau\). In the model \(n_e\) is approximately constant (to within a factor of \(\pm 2\)) and \(T_e\), in the warm component, is approximately linear with height. The source function \(S_\nu \propto T_e\), and since the mm optical depth is \(\tau_w \propto n_e^2 \Delta L \cdot S_\nu\) is a linear function of \(\tau_w\). In this case the integration of the formal solution of the radiative transfer equation for the brightness temperature can be done analytically.

\[
T_{\text{Br}}(\mu) = \alpha T_{\text{CO}} + \beta T_{\text{Br}} + \gamma T_{\text{Chrom}},
\]

where \(T_{\text{CO}}, T_{\text{Br}}\) and \(T_{\text{Chrom}}\) are the key temperatures described above and weighting coefficients are obtained from the formal solution of the transfer equation (Olson & Kunasz 1987), which in our notation are

\[
\alpha = e^{-\Delta \tau},
\]

\[
\beta = 1 - (1 - \alpha) / \Delta \tau,
\]

\[
\gamma = 1 - \alpha - \beta.
\]

From Wiedemann et al. (1994), we adopt \(T_{\text{CO}} \approx 0.6 T_{\text{eff}}\) and \(T_{\text{Br}} \approx 4500 \text{ K}\). We take \(T_{\text{Chrom}} = 10000 \text{ K}\) where hydrogen has become 50 percent ionized (McMurty 1999), and any further increase in temperature now leads to a rapid decline in \(n_e\) and hence also \(\kappa_w\), creating a transparent boundary above \(T_{\text{Chrom}}\).

In this analytic model, the radio fluxes are dependent on \(T_{\text{eff}}\) and the optical depth of the warm chromosphere layer above the CO zone. If the emitting layers are not too thick compared to the stellar radius, the stellar flux can be described as the integral of \(T_{\text{Br}}(\mu)\) for a plane-parallel slab, where \(\mu = \cos \theta\) and \(\theta\) is the angle between the atmosphere normal and the ray. This allows us to write a weighted mean brightness temperature, e.g. in equation (4), as

\[
\langle T_{\text{Br}} \rangle = \frac{1}{2} \int_0^1 T_{\text{Br}}(\mu) \mu \, d\mu.
\]

In keeping with the simple nature of our analytic model, we replaced the integral by a single-point Gaussian quadrature \((n = 1, \mu_1 = 1/2\) and \(w_1 = 1)\). \(\langle T_{\text{Br}} \rangle\) is found by substituting \(\Delta \tau\) from equation (14), adjusted for the viewing angle \(\mu = 1/2\), into equations (16)–(18) to evaluate the weighting coefficients and then solve for equation (15).

4.1 Calibrating with \(\alpha\) Tau (K5 III)

To mitigate some of the inherent uncertainties in the model, we evaluate the chromospheric optical thickness (equation 14) for \(\alpha\) Tau using empirical constraints. We initially use the scheme outlined above to calculate the 250-GHz flux and compare it to the measured 1986 Institut de Radioastronomie Millimétrique (IRAM) flux of 51 \pm 6 mJy. \(\Delta \tau\) then is scaled by a constant to normalize the model (Altenhoff et al. 1994). We use the Altenhoff et al. (1994) compilation because it represents the largest stellar sample available. We later (Section 5.3) compare \(\alpha\) Tau to more recent interferometric observations by Cohen et al. (2005).

Robinson, Carpenter & Brown (1998) give estimates of \(n_e\) in \(\alpha\) Tau for three epochs (1990, 1994 and 1996) based on near-UV C \(\Pi\) spectra obtained with the Hubble Space Telescope (HST); \(n_e \approx 10^9 \text{ cm}^{-3}\). Unfortunately, there were no contemporaneous IUE observations in 1986 when the IRAM 250-GHz observations were made. Electron densities inferred from the C \(\Pi\) 2325 Å multiplet ratios are weighted towards regions of high \(T_e\) and \(n_e\). Because semi-empirical models show that \(n_e\) is constant to within a factor of \(\sim 2\), we adopt a mean \(n_e\) that is half the peak observed value, \((n_e^2)^{1/2} = n_e^2\text{(HST)}/4\).

The predicted flux is then obtained from equation (3), with \(G = 1.08\) and \(\langle T_{\text{Br}} \rangle\) given by the model. Combining the stellar parameters adopted in Table 1, we derive \(F_{250\text{GHz}} = 52\) mJy, likely a fortuitous agreement. The model predicts a radial optical depth of \(\Delta \tau_{\mu = 1} = 0.05\) and \(\Delta \tau_{\mu = 1/2} = 0.09\).
Table 1. Adopted properties of α Tau.

| Quantity | Value | Unit | Reference |
|----------|-------|------|-----------|
| Spectral type K3 III | φ_α | 21.1 ± 0.2 | mas | Mozurkewich et al. (2003) |
| T_\text{eff} | 3871 ± 48 | K | Mozurkewich et al. (2003) |
| R_\star | 44.56 ± 2.94 | \text{R}_\odot | Decin et al. (2003) |
| log g_\star | 1.5 ± 0.15 | cm s^{-2} | Decin et al. (2003) |
| M_\star | 2.30 ± 0.85 | \text{M}_\odot | Inferred (g_\star) |
| [Fe/H] | -0.15 ± 0.20 | | Decin et al. (2003) |
| F_{\text{Mg II}(h & k)} | 6.50 \times 10^{17} | \text{erg cm}^{-2} \text{s}^{-1} | Pérez Martínez et al. (2011)+φ_α |

4.2 Star-to-star variations in Δτ

α Tau’s Mg u h & k flux is about 1.7 times basal, and it might be slightly metal poor (see Tables 2 and 1, respectively), both of which affect the optical thickness. To scale this chromospheric thickness to other red giants, and remove explicit dependence on α Tau’s stellar parameters, we define an optical thickness \( Δτ_\alpha(μ) \) for a fictitious star with α Tau’s \( T_\text{eff} \) but now with basal-flux heating (\( F = 1 \)) and solar metallicity. Using the Ayres (1979) scaling of equation (5) gives \( Δτ_α(μ = 1) = 0.04 \).

Combining equations (5) and (6), the chromospheric optical thickness becomes

\[
Δτ_ν(μ) = \frac{0.04}{μ} \frac{A_\text{Fe}}{A_\text{Fe}_\odot} \frac{F_ν}{F_{\text{basal}}} \left( \frac{T_\text{eff}}{3871 \text{K}} \right)^{5.54} \left( \frac{ν}{250 \text{GHz}} \right)^{−2.17}.
\]

Note that the surface gravity term has now cancelled.

5 COMPARISON WITH OBSERVATIONS AND PREDICTIONS

To test our semi-analytic model for the mm thermal fluxes, we require a sample of single inactive mundane red giants with well-determined stellar parameters. The 250-GHz (1.2-mm) single 30-m dish IRAM survey of Altenhoff et al. (1994) presents the largest sample of measurements for these stars. We combined this survey with the accurate φ_α and \( T_\text{eff} \) sample from the Mark III optical interferometer (Mozurkewich et al. 2003). The stars in common are listed in Table 2. We also have added σ Lib (M3 III) from the smaller sample of Dehaes et al. (2011). This combination of data enables us to make initial tests of the model. In Fig. 2, we compare the observed \( ⟨T_{\text{Br}}⟩ \) (\( G = 1.08 \)) with the predicted basal limit for 250 GHz.

The spectral types of interest are single G, K and M giants. Unfortunately, there were no single G giant detections reported in the two stellar samples, so we do not consider these objects further here. Because we are interested in the chromospheric emission, we need to avoid stars with significant ionized wind opacity, as is likely

Table 2. Stellar properties. 250-GHz radio fluxes and chromospheric optical thicknesses. The stellar properties are from Mozurkewich et al. (2003), the mm fluxes, except σ Lib, are from Altenhoff et al. (1994) and the Mg II fluxes are from Pérez Martínez et al. (2011).

| Star | HR | HD | Spectral type | \( T_\text{eff} \) (K) | \( σ(T_\text{eff}) \) (K) | φ (mas) | σ(φ) (mas) | Flux (mJy) | σ(Flux) (mJy) | \( F/F_{\text{basal}} \) (Mg II h & k) | \( Δτ(μ = 1/2) \) |
|------|-----|----|--------------|-----------------|-----------------|--------|----------|-----------|-------------|----------------|-----------------|
| β And | 337 | 6860 | M0 III | 3763 | 46 | 13.75 | 0.14 | 25.0 | 4.0 | 2.2 | 0.15 |
| α Cet | 911 | 18884 | M1.5 IIIa | 3578 | 53 | 13.24 | 0.26 | 15.0 | 3.0 | 2.0 | 0.10 |
| ρ Per | 921 | 19058 | M4 III | 3281 | 40 | 16.55 | 0.17 | 28.0 | 3.0 | 1.7 | 0.05 |
| α Tau | 1457 | 29139 | K5 III | 3871 | 48 | 21.10 | 0.21 | 51.0 | 6.0 | 1.7 | 0.09 |
| ρ Aur | 1577 | 31398 | K3 III | 4086 | 50 | 7.50 | 0.07 | 13.0 | 3.0 | 1.7 | 0.18 |
| η Gem | 2216 | 42995 | M3 III | 3462 | 43 | 11.79 | 0.12 | 20.0 | 5.0 | 3.0 | 0.13 |
| υ Gem | 2286 | 44478 | M3 IIIb | 3483 | 43 | 15.12 | 0.15 | 31.0 | 6.0 | 1.3 | 0.06 |
| α Lyn | 3705 | 80493 | K7 III | 3836 | 47 | 7.54 | 0.07 | 6.0 | 1.0 | 1.9 | 0.14 |
| α Hya | 3745 | 81797 | K3 II-III | 4060 | 50 | 9.73 | 0.10 | 9.0 | 2.0 | 1.3 | 0.14 |
| μ UMa | 4069 | 89758 | M0 III | 3793 | 47 | 8.54 | 0.09 | 7.0 | 2.0 | 2.4 | 0.17 |
| α UMa | 4301 | 95689 | K0 III | 4637 | 62 | 6.74 | 0.10 | 6.0 | 2.0 | 1.0 | 0.22 |
| σ Boo | 5340 | 124897 | K2 III | 4226 | 53 | 21.37 | 0.25 | 78.0 | 8.0 | 2.7 | 0.11 |
| β UmI | 5563 | 131873 | K4 III | 3849 | 47 | 10.30 | 0.10 | 16.0 | 4.0 | 2.2 | 0.12 |
| γ Dra | 6705 | 164058 | K5 III | 4013 | 52 | 9.86 | 0.13 | 10.1 | 1.3 | 1.2 | 0.12 |
| δ Lyn | 7139 | 175585 | M4 III | 3330 | 44 | 11.53 | 0.16 | 13.0 | 4.0 | 1.9 | 0.07 |
| R Lyn | 7157 | 175665 | M5 III | 3174 | 41 | 18.02 | 0.22 | 14.0 | 4.0 | 1.5 | 0.04 |
| γ Aql | 7525 | 186791 | K2 III | 4099 | 50 | 7.27 | 0.07 | 13.0 | 2.0 | 2.2 | 0.24 |
| β Peg | 8775 | 217906 | M2.5 III | 3448 | 42 | 17.98 | 0.18 | 23.0 | 5.0 | 1.6 | 0.07 |
| σ Lib | 5603 | 133216 | M3.5 III | 3634 | 110 | 11.00 | 0.05 | 12.1 | 2.0 | 2.0 | 0.11 |
to occur at lower gravity (higher luminosity), e.g. the eclipsing binary ζ Aur (kK 1b + B5 Ve; Harper et al. 2005) and its spectral-type proxy λ Vel (K4 1b; Carpenter et al. 1999) and earlier spectral types. We also exclude the hybrid chromosphere stars γ Aql (K3 II) and ι Aur (K3 II) because the hybrid α TrA (K3 II) shows evidence for a warm ionized wind (Harper et al. 2005). We include α Boo (K2 III) even though it is thought to have a 10 000-K partially ionized wind, because the mass-loss rate $M = 2 \times 10^{-10}$ $M_\odot$ yr$^{-1}$ (Drake 1985) is small, although perhaps not entirely negligible. For the later spectral-type semiregular M giants, it is not clear when the energy and momentum deposited by small amplitude radial pulsations begin to dominate the sources present in earlier spectral-type red giants and begin to fundamentally alter the chromospheric structure (see e.g. Eaton, Johnson & Cadmus 1990). We thus include R Lyr (M5 III) while noting that Luttermoser et al. (1994) found that it was not possible to construct a time-independent semi-empirical model for g Her (M6 III).

For the sample, we have estimates and errors for $\phi$, $T_\alpha$, and $F$. In Fig. 3 we directly compare the model (with $G = 1.08$) to Altenhoff et al. (1994). The model is broadly consistent with the observed fluxes, but because of the large uncertainties ($\geq 15$ per cent) of the IRAM measurements it is not clear whether there is additional intrinsic scatter that is not accounted for in the model. Combined Array for Research in Millimetre-wave Astronomy (CARMA) and ALMA observations with flux $S/N$ values of $\sim 20$ will be a defining test of our simple model of time-independent but inhomogeneous red giant chromospheres.

5.1 Spectral indices

Spectral indices, $\alpha$ (where $F_\nu \propto \nu^\alpha$), are an empirical measure to help interpret multifrequency radio observation. For a constant temperature source with an angular diameter independent of frequency, $\alpha = 2$, reflecting the frequency factor in the Planck function. Many $\alpha$ have been obtained at quite different frequencies where the flux can arise from different atmospheric layers (e.g. at cm wavelengths, these can reflect, say, the wind acceleration at 6 cm but the chromosphere at 2 cm), which limits their utility. With the greater sensitivity of the Karl G. Jansky Very Large Array (JVLA) and ALMA, a finer sweep of frequencies can be made, and the resulting spectral energy distributions should provide more powerful diagnostic information.

Using the model above we see that the 100-GHz flux is predicted to come from the upper portion of the chromosphere: by 250 GHz the chromosphere is becoming transparent; while at 900 GHz the chromosphere can become quite transparent, revealing the cold molecular layers above the photosphere, as seen in the CO fundamental bands (Wiedemann et al. 1994). For the sample in Table 2, we find spectral indices using ALMA Band 3 (100 GHz) and Band 9 (660 GHz) lie in the range $\sim 1.60 \pm 0.15$, indicating that the mean temperatures are declining with increasing frequency.

5.2 Variability

The mm optical depths scale with chromospheric heating, so that observed changes in major cooling channel fluxes in the UV can reasonably be expected to induce corresponding changes in $\Delta T$, i.e. $F \propto f(\Delta T)$. Few quantitative studies exist of the temporal chromospheric flux variability of red giants, although both α Boo and α Tau have been well observed with IUE. For α Tau, Cuntz et al. (1996) find a peak-to-peak Mg ii & h & k flux variation of $\pm 4$ per cent on day time-scales, whereas Robinson et al. (1998) find a slightly larger variation of $\sim 15$ per cent from three HST observations over 7 yr. McClintock et al. (1978) found little evidence for variability in α Boo's chromospheric emissions above Copernicus's instrumental noise: 25 per cent over tens of hours and 20 per cent on year timescales. Ayres et al. (1995) reported a 6 per cent rms variation in the Mg ii & h & k line observed by IUE over nearly a decade, although a smaller variability on day time-scales was potentially masked by a long-term $\sim 10$ per cent secular trend. Brown, Gray & Baliunas (2008) find long-term Ca ii h & k variations at the 10 per cent level with a suggestion of a $>14$ yr cycle. α Boo has a significant Mg ii & h & k emission above the basal flux, and it is possible that the excess magnetic heating is responsible for changes in the chromosphere and mm radio emission (see also discussion in Sennhauser & Berdyugina 2011). Judge et al. (1993) have examined the chromospheric flux variations of M giants, $\rho$ Per (M4 II; $\pm 10$ per cent), R Lyra (M5 III; $\pm 4$ per cent) and g Her (M6 III; $\pm 10$ per cent). These changes do not follow the variations in V magnitude. Perturbing the $\Delta T$ in the analytic model by $\pm 15$ per cent imposes a fractional change in $(T_{\text{bol}})$ (or 250-GHz flux) of $<5$ per cent.

On the basis of these studies, we do not expect significant changes in the mm and submm radio fluxes if the chromospheric material is uniformly distributed, such as may result from a high filling factor of the warm chromosphere and/or spatial-averaged acoustic heating.

5.3 Comparison with recent observations

Red giants are feeble thermal radio emitters, and obtaining accurate and precise fluxes is difficult. Some more recent observations have been made of red giants by Cohen et al. (2005) (interferometric) and Dehaes et al. (2011) (single-dish bolometer arrays). Cohen et al. (2005) have reported Berkeley-Illinois-Maryland Association (BIMA) mm fluxes for α Tau and α Boo obtained at 217.8 GHz (1.38 mm) and 108.4 GHz (2.77 mm), and compared against predictions based on photospheric RE values. Cohen et al. (2005) reported evidence for excess mm emission from α Boo, which is consistent with the observed presence chromospheric heating. Further, the BIMA fluxes imply little change in temperature with frequency, whereas the present model reflects the decline of temperature as the higher frequencies probe closer to the photosphere. The 217.8-GHz flux for α Tau (when scaled) remains significantly lower than...
the IRAM 250-GHz flux. A comparison of the BIMA, RE and the present model is given in Table 3.

Dehaes et al. (2011) present 1.2-mm SIMBA and MAMBO-II bolometer array observations for a sample of red giants. We compare these measurements to the model in Table 4 and in Fig. 4. In general the agreement is quite good, except for α Cet (M2 III). The SIMBA flux of 58.3 ± 4.1 mJy is much greater than their MAMBO-II value of 23.6 ± 2.5 mJy, which the authors discount owing to unstable weather conditions. The latter, nevertheless, is much closer to the IRAM and model fluxes. The SIMBA flux for α Boo is close to the Cohen et al. (2005) value (when adjusted for the different frequencies), whereas our prediction is closer to IRAM. These measurements were made at different epochs, however.

The limited data hint at variability at mm wavelengths that is larger than one might anticipate based on the small observed chromospheric variability i.e., a 15 per cent peak-to-peak flux variability which implies a 4 per cent change in mm fluxes.

6 DISCUSSION

The time-independent inhomogeneous model appears to be consistent with most existing mm radio observations. However, because most measurements were made at a single frequency and have sizeable flux uncertainties, these data are only just beginning to provide insight into the chromospheres of inactive K and M giants. The characteristic temperatures of stellar chromospheres: $T_{\text{Min}}$, $T_{\text{CO}}$, $T_{\text{int}}$ and $T_{\text{from}}$ all are within 50 per cent of the mean, and the unfortunate reality is that there is limited thermal dynamic range in this problem. The errors on $\phi_*$ and $T_{\text{eff}}$ are at the few per cent level, so to test the limits of this model it will require multifrequency observations with S/N ~20 and as good an absolute calibration as possible (~5 per cent), both of which are challenging. Multiple observations at closely spaced frequencies are highly desirable, because each flux would have a contribution from overlapping spatial regions, and provide a much better measure of the mean temperature gradient than spectral indices based on widely separated frequencies. On this point we note that using $T_{\text{eff}}$ and $\phi_*$ to estimate the mm radio flux in equation (4) can give reasonable values even though it is a physically unsound model.

We have not explored the G giants, but the model could be extended to include a filling factor of hotter material not at the boundary layer temperature (Wiedemann et al. 1994) by setting, for example, $T_{\text{CO}} = T_{\text{Min}}$ for that component.

6.1 Variability

Millimetre observations, such as those from JVLA $Q$ band, CARMA and ALMA, have the potential to distinguish between the debated acoustic or magnetic origin of chromospheric basal-flux heating. Numerical simulations show that granule sizes for main-sequence and giant stars scale with the surface pressure scale height (Nordlund & Dravins 1990; Freytag, Steffen & Dorch 2002) and the grain size then the spatially unresolved radio flux will have contributions from several shocks passing through each patch, so one might expect a basal-flux red giant to show quite small variability at mm wavelengths in this scenario. Barring limb projection effects, the rms fractional fluctuations of the total intensity would be of the order of $1/\sqrt{10^6} = 0.01$. However, if observations show significant variability, then it is likely that there is an additional, spatially more
correlated source of chromospheric heating, i.e. magnetic active regions or possibly non-radial pulsations. To explore this aspect, it would be important to observe giant stars whose Mg II h & k or Ca II h & k fluxes are as close to the basal limit as possible.

6.2 Molecular component

In the Sun the temperature minimum appears to be set by the balance between mechanical heating and cooling by H −. Above this temperature minimum there appears to be an unknown fraction, perhaps as much as 80 per cent, of the surface covered by CO clouds or columns (Ayres 2002). In the red giants the CO, which appears to be close to RE temperatures, appears to cover most of the stellar surfaces. g Her (30 Her, M6 III, φ = 19 mas, T eff = 3010 K) was not detected in the Altenhoff et al. (1994) survey (≪6 mJy), and the upper limit to its brightness temperature is only 465 K. The present model predicts 28 mJy, and this is very discrepant. The star has been observed with both JUE (Judge et al. 1993) and HST (Luttermoser et al. 1994), and it shows UV chromospheric emission. g Her and R Lyr both have similar φs and T effs, and have similar 2-cm radio VLA fluxes (Drake et al. 1991). In the model presented here we adopted T CO = 0.6T eff, but if the low fluxes for g Her and R Lyr are confirmed, then the molecular component of the outer atmosphere may be even cooler, perhaps dominated by an H2O MOLsphere; and M5 III may mark the onset of a cooler dominant component. However, even this would not seem enough to explain g Her’s low 1.2-mm flux, and multifrequency observations are urgently needed. γ Cru (M3 III) has been observed with the Australia Telescope Compact Array (ATCA) at 94 GHz. It has a large angular diameter (φ ≈ 25 mas, T eff = 3626 K; Cohen et al. 1996; Ireland et al. 2004) and the present model predicts ∼16 mJy, but the observed flux is significantly less (M. Cohen, private communication).

It would seem that K and early M giants at the basal limit might be suitable mm flux calibrators at the 5 per cent level. Random fluctuations in the brightness resulting from changes at the stellar photospheric granule scale should average down, and chromospheric heating fluctuations derived from optical and UV emission suggest small variations in the mm fluxes. For later spectral types, the uncertainties in this variable chromospheric heating is further reduced. That being said, the emission from the molecular material mixed in with warmer chromospheric plasma needs to be calibrated in some way, because it is challenging to establish directly from theoretical models or simulations of stellar atmospheres.

The semi-analytic model developed here is a time-independent 1D model with all the inherent assumptions. When the next generation of theoretical chromospheric heating models become available, one could analyse the thermodynamic structure and develop a new analytic representation. Until then, the present model, based on the interpretation of empirical evidence, should be regarded as a starting point. Even if real red giant chromospheres are highly time variable, the semi-empirical models upon which our mm radio predictor is based still represent a measure of the temperatures, electron densities and ionization required to produce the UV and optical spectral diagnostics. One could imagine an actual atmosphere as being a highly dynamic jumbled-up 1D model, in which case the temperature rise in the warm chromosphere no longer is monotonic. The implication for the radio fluxes would be to reduce them slightly as cool material is now present at greater altitudes. This systematic bias is being investigated.

A test of chromospheric models for the largest angular diameter giants may be possible with ALMA, at frequencies above 250 GHz, in its most extended (16-km maximum baseline) configuration. The chromospheres will be partially resolved, but they will provide constraints on the mm limb brightening or darkening.

7 CONCLUSIONS

We present a semi-analytic ‘toy’ chromospheric model to examine mm thermal continuum radio fluxes of non-dusty and non-pulsating red giants. Equation (15) shows, in a clear fashion, the contributions to the brightness temperature from the cool molecular layers at the base, and from the chromosphere itself, through weighting factors obtained from analytic solutions to the radiative transfer equation. At 3 mm, the chromospheres are semi-opaque, while at 1.2 mm the chromosphere becomes optically thin and the cool molecular material becomes visible. The model predicts a basal limit of radio emission that depends on the temperature of the boundary layers. Studies at (sub)mm wavelengths will provide new insights into these especially enigmatic molecular regions. The model also permits an analysis of radio flux variability in response to changes in chromospheric heating as observed in Mg II h & k or Ca II h & k. If the chromospheric heating rate is not known for a given red giant, the model can still predict the basal mm radio fluxes. Significant variations in radio emission for basal-flux red giants, if detected, would indicate a source of structured large-scale heating, possibly magnetic in nature. The physical arrangement, e.g. columns and sheets, of the cool and warm chromospheric plasma can be constrained by the mm radio limb brightening of red giants that can be spatially resolved when ALMA reaches its maximum baselines. A good test of this semi-analytic model would be to compare it against red giants with independently determined T effs, φs, and T COs, and that can be monitored for changes in chromospheric heating.

In summary, the present analytic model is consistent with most existing radio measurements and also highlights some possible anomalies. The sensitivity of mm fluxes to changes in chromospheric heating can be evaluated, and spectral indices are predicted for the existing thermal gradients. When a sample of multiwavelength (sub)mm radio observations with good S/N becomes available, these can be used to test the overall validity of the model. If the model passes this test, i.e. if there are no significant star-to-star discrepancies, the observations can then be used to refine and test the mean thermal gradient in the stellar chromospheres. The spectral shapes could be used to better describe the outward temperature gradient, or alternately they could invalidate the present model completely if there is no outward temperature increase. More importantly, these important new radio observations can test new 3D theoretical heating models when they become available.

There appear to be a few stars with significant 1.2-mm flux differences between widely separated epochs and different observing facilities that appear at first sight to be larger than one would expect based on observed changes in chromospheric heating. These would suggest significant magnetic contributions for stars at twice the basal UV flux levels. We also have identified a few of the late M giants which, while at the limit of applicability of the model, seem to have very low observed fluxes. However, we must await accurate new observations of these stars before we can draw any significant conclusions.

While optical angular diameters and effective temperatures for many giants are known to a precision and accuracy of a few per cent, we have shown that the physical dynamic range of T eff for the mm radio problem is limited. Future progress will require high S/N multiwavelength (sub)mm radio fluxes with as small absolute flux calibration errors as possible.
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