Induced nucleation in weak first order phase transitions

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Abstract

We study induced nucleation by considering the accumulation rate of shrinking subcritical bubbles. We derive the probability for a collection of subcritical bubbles to form a critical bubble, and argue that this mechanism could well play a role in electroweak phase transitions if the Higgs is heavy.
Cosmological phase transitions have recently received much attention, in particular in the context of electroweak baryogenesis [1]. First order phase transition and bubble dynamics in the Standard Model have been studied in detail, and it has become increasingly clear [2, 3, 4] that for realistic Higgs masses, much heavier than 60 GeV, the electroweak phase transition appears to be only weakly first order. If the Higgs mass $m_H \gtrsim 100$ GeV, both lattice studies and perturbative calculations run into technical troubles. It is however conceivable that for such Higgs masses the electroweak transition is close to a second order one, and that it proceeds not by critical bubble formation but via thermal fluctuations. These would give rise to a considerable phase mixing at the critical temperature.

In the context of the EW phase transition, subcritical bubbles were first discussed in [5]. In [6] it was shown that phase equilibrium can be reached provided the transition is weak enough. In these papers, however, the disappearance of the subcritical bubbles was not accounted for. This happens in two ways: the subcritical bubbles, being unstable configurations, tend to shrink; the bubbles are also a subject of constant thermal bombardment so that they may disappear simply because of thermal noise. The thermalization rate of small-amplitude configurations near the critical temperature has been estimated in the EW theory in [7], where it was found that compared with typical first order transition times, thermalization is rather fast. Kinetics of subcritical bubbles has been investigated by Gelmini and Gleiser [8], who found, with a specific assumption about the form of the destruction rate due to thermal noise, that thermal noise becomes subdominant as the Higgs mass is increased. Their study deals with phase mixing above the critical temperature, and the possibility that pre-tansitional phenomena might play a role in the EW phase transition. Phase mixing has also recently been simulated numerically by Gleiser [9] in a 2+1 –dimensional model, with some interesting results. We shall return to the issue of bubble shrinking by termal noise at the end of the paper.

In the case of a very weak first order phase transition subcritical bubbles may coalesce and form large regions of broken phase at and below the critical temperature, and effectively trigger the phase transition. This happens if the probability of subcritical fluctuation is large enough, in which case a collection of subcritical bubbles can form a region of the size of a critical bubble before they have time to shrink away. This might be called induced nucleation. Although it is clear that some dynamics is also involved, in the present paper we study the formation and growth of subcritical bubbles in a probabilistic approach. We derive an expression for the random growth of subcritical regions and compare the rate with the formation rate of critical bubbles. Interestingly enough, it turns out that for a large set of the parameter values induced nucleation is indeed possible, although for an extremely weak phase transition its completion is prevented because of the large size of the critical bubble.
Conventionally two possibilities for critical bubble formation in $d + 1$ dimensions is considered. These correspond to the formation rates

\[ \Gamma_{d+1}/V \sim e^{-S_{d+1}[\phi_{cr}]/\hbar}, \]  
\[ \Gamma_d/V \sim e^{-\beta S_d[\phi_{cr}]} . \]  

Eq. (1) describes the situation where the critical bubble is formed by quantum tunneling, and Eq. (2) by large over-the-barrier thermal fluctuations. Note the absence of $\hbar$ in Eq. (2). $S_d[\phi_{cr}]$ is $d$–dimensional euclidean action for the critical bubble configuration $\phi_{cr}$. Such critical bubbles are a result of a single, instantenous fluctuating event through or over the potential barrier. The timescale $\tau_\omega$ of such a critical bubble of radius $R_c$ is determined by the thermal mass $m(T)$ and wave number $k_c \simeq 1/R_c$:  
\[ \tau_\omega = 1/\omega = [m(T)^2 + k_c^2]^{-1/2}. \]

In a very weakly first order phase transition, however, the two phases are very much mixed already at the critical temperature, and thermal processes could create critical bubbles from small, unstable subcritical bubbles of broken phase. Instead of quantum tunneling or large fluctuations, the phase transition could be triggered by small semiclassical thermal fluctuations. This is called induced nucleation.

Subcritical bubbles will of course disappear very rapidly, but if their production rate is large enough, the total volume in the broken phase may actually grow because of the presence of fluctuating subcritical bubbles. Our aim is to determine the timescale in which a critical bubble is thermally built up from subcritical bubbles by calculating the probability for the appearance of a growing (spherical) subcritical bubble. Following [8], we assume that thermal noise plays a subdominant part, and that the disappearence of the bubbles can be described in terms of shrinkage only. As we shall argue, this seems to be a reasonable approximation in our case.

Let us assume that the subcritical bubble is a gaussian, spherically symmetric configuration with diameter $l$, i.e.

\[ \phi_l(r) = v(T)e^{-2r^2/l^2}, \]  
where subcritcality implies that $l \ll R_c$. At finite temperature we need the $d$–dimensional action

\[ S_d[\phi_l] = \int d^d x [\frac{1}{2}(\nabla \phi_l)^2 + V(\phi_l)], \]  
which essentially determines the formation rate of subcritical bubbles per unit volume:

\[ \Gamma_V(l) \equiv \Gamma[\phi_l]/V \simeq T^{d+1} \left[ \frac{\beta S_d[\phi_l]}{2\pi} \right]^{d/2} e^{-\beta S_d[\phi_l]} . \]  

In order to be specific, we shall deal with a phenomenological potential for the order parameter $\phi$ suitable for a simple description of a first order phase transition,
given by
\[ V(\phi) = \frac{1}{2} m(T)^2 \phi^2 - \frac{1}{3} \alpha T \phi^3 + \frac{1}{4} \lambda \phi^4. \] (6)

Most of the analysis takes place at the critical temperature \( T_c \) (or slightly below), given by the condition
\[ m(T_c)^2 = \frac{2 \alpha^2 T_c^2}{9 \lambda}. \] (7)

At the critical temperature the non-zero minimum \( v(T_c) \) of the potential reads
\[ v(T_c) = \frac{2 \alpha}{3 \lambda} T_c \] (8)

and the correlation length is
\[ l_c = \frac{1}{m(T_c)}. \] (9)

We shall let the dimension of the space–time to be a free parameter; the number of spatial dimensions is denoted by \( d \). Note that this means that the parameters \( \alpha \) and \( \beta \) are, in general, dimensional; only when \( d = 3 \) they are dimensionless.

Given the phenomenological potential Eq. (6), we can find out the action for the subcritical configuration Eq. (3) at \( T_c \). The result is
\[ \beta S_d = \frac{2 \pi \alpha^2 T d}{9 \lambda^2} \left[ \frac{\sqrt{\pi} l}{2m l_c} \right]^{(d-2)} \left[ 1 - 2 \left( \frac{1}{2} \right)^{\frac{d}{2}} + \left( \frac{1}{2} \right)^{\frac{d}{2}} \left( \frac{l}{l_c} \right)^2 \right]. \] (10)

The formation rate for subcritical bubbles is then given according to Eq. (5).

In the cosmic soup there should exist all sizes of subcritical bubbles. We shall, however, use in our calculations a mean subcritical bubble \( l = l_c \) with an associated nucleation rate \( \Gamma V \). With such an average subcritical bubble we are able to write down the probability for a process in which a large number of small subcritical bubbles are created adjacent to each other so that they form a critical bubble.

Let us first find out how large a part of the space is occupied by the broken phase before the possible formation of critical bubbles, i.e. by subcritical bubbles in general. This problem has also been analyzed, in the case of critical bubbles, in [10] and in [11], however, without the shrinking effects (see also [8]). Let the radius of the subcritical bubble be \( R_{sc} \) and its life–time
\[ t_{sc} = R_{sc}/v, \] (11)

where \( v \) is the (average) speed of the shrinking bubble wall. Let us consider a volume \( V \) and, for the moment, discretize the time into infinitesimal intervals of length \( \delta t \). We denote the volume occupied by the broken phase at the instance \( n \delta t \) \( (n \in \mathbb{Z}_+) \) by \( V_b(n) \). Between the times \( (n-1)\delta t \) and \( n \delta t \) \( N_n = \Gamma V \delta t (V - V_b(n-1)) \) new bubbles of broken phase have been fluctuated, each having the volume \( \pi^{d/2} R_{sc}^d/\Gamma (d/2 + 1) \).
Similarly, at \((n-1)\delta t\) there were \(N_{n-1}\) bubbles but at \(t = n\delta t\) their volume has shrunk to \(\pi^{d/2}(R_{sc} - v\delta t)^d/\Gamma(d/2 + 1)\). This procedure can be continued until all coexisting bubbles have been counted. Correspondingly, in the broken phase \(\bar{N} = \Gamma'_V \delta t V_b(n-1)\) bubbles of the symmetric phase are created, and so on. Note that at the critical temperature \(\Gamma_V = \Gamma'_V\) and \(V = V'\). Taking into account both effects, and taking the limit \(\delta t \to 0\), we obtain an integral equation for the \(V_b/V\) ratio

\[
\frac{V_b}{V} \equiv s(x) = k \min\{x, 1\} \int_0^{\min\{x, \rho\}} dy \left[1 - s(x - y)\right]^d \left[1 - y\right]^d - \frac{k'}{\rho} \int_0^{\min\{x, \rho\}} dy \left[1 - s(x - y)\right]^d \left[1 - y\right]^d
\]

with the boundary condition \(s(0) = 0\), where \(x\) is the scaled time \(x = tv/R_{sc}\), \(\rho = (vR_{sc})/(v' R_{sc})\) and

\[
k = \Gamma' V \frac{\pi^{d/2} R_{sc}^{d+1}}{\Gamma(d/2 + 1)v} ; \quad k' = \Gamma'_V \frac{\pi^{d/2} (R_{sc}')^{d+1}}{\Gamma(d/2 + 1)v'}.
\]

Here \(v'\) is the velocity of the unbroken phase bubble wall and \(R_{sc}'\) is the radius of the subcritical bubble of symmetric phase in broken phase. We have not taken into account the thermal noise effect described in [8]. It would modify the definitions of \(k\) and \(k'\) but here we assume that the resulting changes are small. From the integral equation Eq. (12) the asymptotic, equilibrium value of \(s\) can be easily solved:

\[
\lim_{x \to \infty} s(x) \equiv s_{eq} = \frac{k}{d + 1 + k + k'}. \tag{14}
\]

The equilibrium value at the critical temperature (where \(k = k'\)) has the intuitively natural property that \(s_{eq} \to \frac{1}{2}\) when \(k \to \infty\) (\(\Gamma_V \to \infty\)). Equation Eq. (14) tells us how large a part of the spatial volume is occupied by the broken phase after the system has relaxed to equilibrium in an unorganized way so that no bubbles of critical size form. Note also that when the temperature decreases, \(k\) increases and \(k'\) decreases so that \(s_{eq}\) increases, too.

The issue at hand is then, when are the circumstances such that the subcritical bubbles can form a critical bubble? If this happens, the phase transition can be triggered by subcritical bubbles and completed by the expansion of the created critical bubble. Here we have to be slightly under the critical temperature because no bubble dynamics is really present exactly at the critical temperature. We assume that the critical bubble is built up layer by layer and consider spherical bubbles only, because they minimize the number of the subcritical bubbles needed for each layer and thus maximize the probability. Let us say that the radius of a bubble, still subcritical, which exists at the time \(t\), is \(R\). In order to that the bubble is really growing, in the time \(2R_{sc}/v\) new subcritical bubbles have to fill a layer of volume \(V_R = \pi^{d/2}[(R +
Because the process is Poisson distributed, the probability that \( N \) bubbles are nucleated is

\[
p_N = \frac{1}{N!} \lambda^N e^{-\lambda} \tag{15}\]

where

\[
\lambda = \Gamma_V V_R \frac{2R_{sc}}{\nu} = 2k[\left(\frac{R}{R_{sc}} + 2\right)^d - (\frac{R}{R_{sc}})^d]. \tag{16}\]

To fill the volume \( V_R \) at least \( N_R = \frac{V_R \Gamma(d/2+1)}{\pi^{d/2} R_{sc}^d} = (R/R_{sc} + 2)^d - (R/R_{sc})^d \) bubbles are needed. The probability that the layer is filled is thus

\[
p(R) = \sum_{N \geq N_R} p_N. \tag{17}\]

With a little algebraic acrobatics the formula Eq. (17) can be cast in the form

\[
p(R) = P(N_R, \lambda) \tag{18}\]

where

\[
P(N, \lambda) = \frac{1}{\Gamma(N)} \int_0^\lambda dt t^{N-1} e^{-t} \tag{19}\]

is a scaled incomplete \( \Gamma \)–function.

We have derived the probability \( p(R) \) that a new layer of ‘elementary’ subcritical bubbles is formed fast enough so that the subcritical bubble size increases. We have neglected processes where large subcritical bubbles merge. This might be justifiable because a growing subcritical bubble should be a rare event so that the distance between any such configurations is large. In any case, such processes would only make the growth rate larger.

When one uses a phenomenological potential of the type Eq. (6) there arises an extra complication because in realistic theories the order parameter (e.g. a Higgs field) often also has a phase. Then it is not enough that subcritical bubbles form next to each other but their phases should be correlated, too. Otherwise there should arise a domain wall between the two bubbles, and creating such a wall would require extra energy. In principle, one can easily account also for the phase by introducing a parameter \( \Omega \in [0, 1] \) which determines the probability that the phase of the nucleated subcritical bubble is correlated with the phase of the pre–existing bubble. The probability for formation of \( N \) bubbles is then modified to read

\[
p'_N = \frac{1}{N!} (\Omega \lambda)^N e^{-\lambda} \tag{20}\]

and thus the probability of layer formation is

\[
p(R, \Omega) = e^{-(1-\Omega)\lambda} P(N_R, \Omega \lambda). \tag{21}\]
The value of $\Omega$ is, however, a dynamical question which we are unable to address here. When the new subcritical bubble overlaps with the pre–existing bubble, the latter may influence the phase of the newly forming bubble and may even force the phases to be correlated. In such a case $\Omega = 1$. Subcritical bubbles, which could be considered as wave packets of elementary quanta, might also have a velocity relative to each other. In that case energetics would no longer hinder the removal of the domain walls, resulting in $\Omega = 1$. In what follows, we shall write down our general expressions for an arbitrary $\Omega$, but when applying the results, we shall merely assume that $\Omega = 1$.

We are now ready to write down the probability that a subcritical bubble is growing. It is simply the product over all layer probabilities,

$$p(k) = \prod_i p(R_i, \Omega) = \exp[\sum_i \ln P(R_i, \Omega)] \equiv \exp[j(R_c, \Omega, k)],$$

where $R_i = (2i + 1)R_{sc}$ and $i$ runs from $i = 1$ to $i = [1/2 + R_c/R_{sc}] + 1$, i.e. to the value where the bubble has reached the critical size. For large $R_c/R_{sc}$ the function $j$ can be approximated well by the integral

$$j \simeq \frac{1}{2} \int_0^{R_c/R_{sc}} dR \ln p(R, \Omega)$$

$$= \frac{1}{2} \int_0^{R_c/R_{sc}} du [\ln P(N(u), 2k\Omega N(u)) - 2k(1 - \Omega)N(u)],$$

where $N(u) = (u + 2)^d - u^d$. Now $j$ can be computed numerically once the ratio $R_c/R_{sc}$, $\Omega$, $d$ and $k$ are given.

To proceed further we have to calculate the average formation time $\tau$ and the thickness $\delta$ of a layer. Generally $\delta < 2R_{sc}$ because the bubble is shrinking simultaneously as the layer gets filled. $N$ bubbles in the volume $N\pi^{d/2}R_{sc}^d/\Gamma(d/2 + 1)$ are nucleated during the time $t$ with the probability

$$\frac{1}{N!} \left( v k N \frac{t}{R_{sc}} \right)^N e^{-v k N \frac{t}{R_{sc}}}$$

and at least $N$ bubbles with probability

$$\sum_{n \geq N} \frac{1}{n!} \left( v k N \frac{t}{R_{sc}} \right)^n e^{-v k N \frac{t}{R_{sc}}} = P(N, v k N \frac{t}{R_{sc}}).$$

Thus the average filling time of a layer is

$$\langle t \rangle = \int_{t=0}^{t=\infty} t dP(N, v k N \frac{t}{R_{sc}}) = \frac{R_{sc}}{v k}.$$
That is, the average time to form a new layer is $R_{sc}/(vk)$. Because during the same time the bubble shrinks an amount $R_{sc}/k$, the bubble radius after $i^{th}$ layer can be solved recursively. The result is $R_i = i(2 - 1/k)R_{sc}$. The critical bubble size has been reached when $i = i_c \equiv (2 - 1/k)^{-1}R_c/R_{sc}$ and thus the formation time of critical bubble is

$$\tau = i_c \frac{R_{sc}}{vk} = \frac{R_c/v}{2k - 1}. \quad (28)$$

The average layer thickness is given by

$$\delta \equiv R_{i+1} - R_i = (2 - 1/k)R_{sc}. \quad (29)$$

To complete the calculation we have to write down the formula for the probability $p_c$ that a subcritical bubble grows up to a critical one. Here we have to use the layer formation time $\tau$ and thickness $\delta$ as parameters, that is $\lambda = \Gamma V(t) \pi^{d/2}[(R + \delta)^d - R^d]/\Gamma(d/2 + 1)$. The calculation results in the expression

$$p_c(k) = e^{jc}, \quad (30)$$

where

$$jc = \frac{1}{2 - 1/k} \int_0^{R_c/R_{sc}} \! \! du \left[ \ln P(N_c(u), \Omega N_c(u)) - (1 - \Omega)N_c(u) \right] \quad (31)$$

and $N_c(u) = (u + 2 - 1/k)^d - u^d$. Inspection shows that in the case $\Omega = 1$ the value of $jc$, as a function of $k$, converges rapidly to the asymptotic function $jc(R_c/R_{sc}) \equiv j_c(R_c/R_{sc}, \Omega = 1, k \to \infty)$. A plot of $j_c(R_c/R_{sc})$ is presented in Figure 1. For $\Omega = 1$ a good fit for any $k > 1$ is

$$j_c = \frac{R_c}{R_{sc}}; 1, k = -10^{-0.49+0.27/k^{1.15}} \left( \frac{R_c}{R_{sc}} \right)^{1.01+0.008/(k-0.2)^{1.20}}. \quad (32)$$

For decreasing $\Omega$ $j_c$ increases, however, rapidly even for as small ratios as $R_c/R_{sc} = 10$. In Figure 2 we demonstrate this behaviour by presenting $j_c$ as a function of $\Omega$ for $k \to \infty$ and $R_c/R_{sc} = 10$.

We may now apply the rate Eq. (34) to cosmology. Because Eq. (34) states that $\Gamma_{V,f} \propto e^{-(\beta S_{d}^{[\phi_t]} - j_c)}$, we have to compare the combination $\beta S_{d}^{[\phi_t]} - j_c \equiv S_{eff}$ with the Hubble rate. An induced critical bubble is obtained when

$$S_{eff} \approx \ln \left( \frac{M_{Pl}}{T} \right)^4. \quad (33)$$

The probability $p_c(R)$ is, in general, very small. The number of subcritical bubbles during the phase transition, i.e. the number of possible seeds, can, however, be very large. In this case the critical bubble formation rate is given by

$$\Gamma_{V,f} = p_c(k) \Gamma_V. \quad (34)$$
where $\Gamma_V$ can be computed by using Eq. (11). To get some idea of the magnitudes of the parameters needed for induced nucleation, we compute the rate Eq. (34) in the thin wall approximation using the potential Eq. (6) and assume that $v$ is of the order of unity.

First we need the size of the critical bubble. Assuming that there is only little supercooling, the bounce action can be written as [6]

\[
S/T = \frac{\alpha}{\lambda^{3/2}} \frac{2^{9/2} \pi}{3^5} \frac{\bar{\lambda}^{3/2}}{(\lambda - 1)^2} \simeq 150, \tag{35}
\]

where

\[
\bar{\lambda} \equiv 1 - 0.0442 \frac{\alpha^{1/2}}{\lambda^{3/2}} \equiv 1 - \delta. \tag{36}
\]

Small supercooling, or $1 - \bar{\lambda} \ll 1$ thus implies that $\alpha \ll 500 \lambda^3$. Solving for $\bar{\lambda}$ yields the transition temperature $T_f$. It then follows that at the transition temperature

\[
m^2(T_f) = \frac{2 \alpha^2}{9 \lambda} (1 - \delta) T_f^2. \tag{37}
\]

Expanding the potential at the broken minimum $\phi = v(T) = \alpha T (1 + \sqrt{1 - 8\bar{\lambda}/9})/2\lambda$ we find

\[
-\epsilon \equiv V(v, T_f) = \frac{1}{6} m^2(T_f) v^2 - \frac{1}{12} \lambda v^4 = -0.00218 \left( \frac{\alpha}{\lambda} \right)^{9/2} + O(\delta^2). \tag{38}
\]

The height of the barrier is situated at $\phi_{\text{max}} \simeq v/2$ with $V(\phi_{\text{max}}, T_c) \equiv V_{\text{max}} = \alpha^4 T_c^4/(144 \lambda^3)$. As $T_c \simeq T_f$ we may conclude that the thin wall approximation is valid if $-\epsilon/V_{\text{max}} = 0.314 \alpha^{1/2}/\lambda^{3/2} \ll 1$, or $\alpha \ll 10 \lambda^3$, which is in accordance with our assumption of small supercooling.

To get the size of the critical bubble we still need the surface tension. One easily finds

\[
\sigma = \int_0^\infty d\phi \sqrt{2V(T_c)} = \frac{2 \sqrt{2} \alpha^3}{91 \lambda^{5/2}} T_c^3. \tag{39}
\]

Thus we finally obtain

\[
\frac{R_c}{R_{sc}} = \frac{2\sigma}{\epsilon R_{sc}} \simeq 26.9 \frac{\lambda^{3/2}}{\alpha^{1/2}}. \tag{40}
\]

Here $R_{sc} \simeq 2/m(T_f)$. Since $\alpha \ll 10 \lambda^3$ we see that $R_c/R_{sc} \gg 1$ as it should.

Combining Eq. (10), Eq. (32), Eq. (33) and Eq. (40) at the critical temperature we obtain a condition for inducing a critical bubble:

\[
2.06 \frac{\alpha}{\lambda^{3/2}} + 9.10 \left( \frac{\lambda^3}{\alpha} \right)^{0.506} \simeq 150. \tag{41}
\]
This implies $\alpha \gtrsim 4.0 \times 10^{-3}\lambda^3$, that is, perhaps surprisingly, induced nucleation does not work for a too weak transition. The result can be interpreted so that in the range $1 \lesssim \lambda^3/\alpha \lesssim 250$ where the phase transition is very weak, it is triggered by subcritical bubble formation immediately below the critical temperature. If the phase transition is weaker, the critical bubble radius becomes too large to be induced by subcritical bubble growth. On the other hand, whenever $\lambda^3/\alpha \lesssim 1$ the phase transition is not weak any longer: the radius of a critical bubble decreases and bubble dynamics becomes important so that our calculation loses its validity. The correlation problem connected to the parameter $\Omega$ also becomes more important when the “seed bubble” is of same size as the newly produced ones.

To appreciate the weakness of the transition required of induced nucleation, we may consider the 2-loop result for the electroweak effective potential calculated in [3]. Their result for $M_H = 87$ GeV is well fitted by the potential of the type Eq. (6) with $\alpha = 0.048$ and $\lambda = 0.061$. Thus in this case $\alpha \simeq 210\lambda^3$ is well outside of the induced nucleation range. As for larger Higgs masses $\alpha$ is likely to be smaller, it is conceivable that induced nucleation could play a role in EW phase transition. We should however emphasize that the result Eq. (11), and the subsequent estimates, should be considered as the most optimistic cases. Dynamics that correlates phases, and other issues related to the velocity of the subcritical bubbles or the surface tension between the bubbles, are likely to make induced nucleation more difficult, not facilitate it.

Regarding thermal noise, we have compared the thermalization rate of small amplitude configurations in the EW theory [7], given by $\Gamma \simeq 10^{-2}T$, with the formation time of an induced critical bubble, Eq. (28). We find that in the region where the thin wall approximation is valid, thermalization does not occur. This seems to indicate that thermal noise does not play an important role in induced nucleation.

It would be of interest to compare the present approach with the results of Gleiser [9], although it is not obvious to us how to do it. The problem appears to be a technical one only, though. Gleiser also employed a potential of the type Eq. (1) (in $d = 2$) but simulated the heat bath by white noise which introduced an additional parameter to the problem. It is not clear how the parameters in his simulation are related to the parameters (including $T_c$) here.
References

[1] For a review, see A.G. Cohen, D.B. Kaplan and A.E. Nelson, Ann. Rev. Nucl. Part. Phys. 43 (1993) 27.

[2] K. Kajantie, K. Rummukainen and M. Shaposnikov, Nucl. Phys. 407 (1993) 356; K. Farakos et al., Preprint CERN-TH.7244/94.

[3] W. Buchmüller et al., Preprint DESY-93-021; Z. Fodor et al., Preprint DESY-94-088.

[4] Z. Fodor, A. Hebeker, Preprint DESY-94-025.

[5] M. Gleiser, E. Kolb and R. Watkins, Nucl. Phys. B364 (1991) 411.

[6] K. Enqvist et. al., Phys. Rev. D45 (1992) 3415.

[7] P. Elmfors, K. Enqvist and I. Vilja, Nucl. Phys. 412 (1994) 459.

[8] G. Gelmini and M. Gleiser, Nucl. Phys. B419 (1994) 129.

[9] M. Gleiser, Preprint DART-HEP-94/01.

[10] A.H. Guth and S.-N. Tye, Phys. Rev. Lett. 44 (1980) 631, 963 (erratum). See also A.H. Guth and E.J. Weinberg, Phys. Rev. D23 (1981) 876.

[11] L.P. Csernai and J.I. Kapusta, Phys. Rev. Lett. 69 (1992) 737.
Figure captions

Figure 1. The probability function $j(r, \Omega = 1)$ defined in Eq. (23) in the asymptotic limit $k \to \infty$ as a function of the ratio of critical and subcritical bubble radii $r = R_c/R_{sc}$.

Figure 2. The function $j(r, \Omega)$ in the asymptotic limit $k \to \infty$ as a function of $\Omega$ for the fixed ratio $r = R_c/R_{sc} = 10$. 
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Fig. 1
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Fig. 2