Zero-dimensional model for the critical condition for the L–H transition in the flux–gradient relation

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Abstract

In this paper, the physics of the L- to H-mode transition is discussed, focusing on the mechanism induced by the localized mean radial electric field. In addition to the neoclassical effect, an additional source of the electric field is introduced. The influence of turbulence-driven Reynolds stress is taken into account here. The case where the zonal flows are not excited is analysed. Even if the spontaneous zonal flows by turbulence cannot be excited, the bifurcation of the mean radial electric field can occur, if the electric field is strong enough \(e\varrho_p E_r/T_i \sim -1\). Hard transition of the electric field takes place. The enhancement of electric field above this critical value can induce a bifurcation in the gradient–flux relation, i.e. the disappearance of the L-mode branch. The critical condition of the heat flux for the onset of transition is obtained.

Keywords: L–H transition, threshold, H-mode

(Some figures may appear in colour only in the online journal)

1. Introduction

After the discovery of transition to the H-mode [1], intensive work has been done to understand the mechanisms that induce the transition to the improved confinement state. Central idea to model the H-mode transition is the fundamental role of the radial electric field \(E_r\) in the transition and the suppression of turbulence [2, 3]. Although the strong and localized radial electric field in the transport barrier has been confirmed experimentally [4, 5] and the suppression of turbulence has also been observed [6–9], the physics mechanisms of H-mode has not yet been quantitatively understood. In particular, the threshold condition has not yet been explained. The fast response of the central plasma after the transition at the edge [10, 11] has not yet been understood either. These facts indicate that our understanding of the L- to H-mode transition is still far from satisfactory.

As the number of observations increases, varieties in the possible mechanisms that work at the L–H transition have also become noticeable. In addition to the role of the mean radial electric field [2, 12], mechanisms that include zonal flows (which are constant on a magnetic surface but oscillating in radius [13]) have attracted considerable attention [14]. The transition occurs without being associated with zonal flows [15]. Thus, the mechanism for transition that is induced by the mean radial electric field needs to be investigated. The study in this line is also motivated by the recent experimental observation and data analysis. The JFT-2M and HL-2A experiments have shown that the change from a limit cycle state to the H-mode occurs if the normalized radial electric field reaches a critical value \(X = e\varrho_p E_r/T_i \sim O(-1)\) [15, 16]. The observations on ASDEX-U indicated that the states of a discharge (L-mode, LCO and H-mode) are separated by the minimum radial electric field near the edge [17]. In a wide range of plasma densities, the critical value of the radial electric field was reported to be \(E_r \sim -15 \text{ kV m}^{-1}\). For the ion temperature of \(T_i \sim 200 \text{ eV}\), the normalized radial electric field satisfies the relation \(e\varrho_p E_r/T_i \sim -1\). This magnitude is close to the condition where a bifurcation of the radial electric field can occur [2, 12, 18, 19]. Along this line of thought, an emphasis is made on the importance of ion pressure gradient in [17]. This is because the ion pressure gradient is one of the strong driving forces for the mean electric field. It should be noted that the radial electric field is not necessarily proportional to the ion pressure gradient [2, 12, 20, 21]. Measurement on JT-60U has also shown that the electric field can change without
substantial change in the ion pressure gradient [22]. Such a sudden change of radial electric field was reported to occur near the condition $e_p E_r / T_i \sim 1$.

In this paper, the physics of the L- to H-mode transition is discussed, focusing on the mechanism induced by the localized mean radial electric field. In addition to the neoclassical effect, an additional source of the electric field is introduced. The influence of turbulence-driven Reynolds stress is taken into account here. We analyse the case where this additional drive is weak so that the zonal flows are not excited. Even if the spontaneous zonal flows by turbulence cannot be excited, the bifurcation of the mean radial electric field takes place, if the electric field is strong enough, $X \sim -1$. A hard transition of the electric field can occur. The enhancement of the electric field above the critical value $X \sim -1$ can induce the bifurcation in the gradient–flux relation, i.e. the disappearance of the L-mode branch. The critical condition for the heat flux for the onset of transition is obtained.

2. Model

2.1. Processes that are taken into account

The equation of the evolution of the mean radial electric field has been discussed in the literature [2, 12, 14, 18–20, 23]. In this paper, we consider the case where the neoclassical term and the driving force by turbulence are dominant. This is because the role of ion orbit loss has been discussed in the literature [2, 12], and because the subject of this paper is to investigate the threshold power for the onset of transition. This study is aimed at deriving the relation between the realized gradient for a given flux so as to establish the condition where the L-mode branch disappears. The complex roles of turbulence (i.e. the direct induction of transport in L-mode as well as the indirect suppression of transport through electric field generation) must be considered simultaneously in the analysis of L-to-H transition. Based on these considerations, a simple model that governs the evolution of the radial electric field is employed as

\[ \dot{\epsilon}_X \equiv \dot{X} / \dot{t} = - \left( \frac{G + X}{1 + X} \right) - \alpha \mod X'' I \]  

(1a)

where

\[ X = e_p E_r / T_i \]  

(1b)

is the normalized radial electric field, and the parameter

\[ G = -\rho_p (\gamma_t^2 / n + \gamma_T^2 / T_i) \]  

(1c)

denotes the normalized gradient of the mean ion pressure that controls the neoclassical radial electric field [23, 24]. Here, $\gamma$ is the specific heat ratio of ions and is of the order unity. In this paper, we do not discriminate the density gradient and ion temperature gradient, so that the gradient of the mean plasma parameter is characterized by the parameter $G$. The coefficient $\epsilon_x$ on the lhs indicates the enhancement factor of the dielectric constant of the magnetized plasma owing to toroidicity [20, 25, 26]. ($\epsilon_x = 1 + 2q^2$ and $\epsilon_x = 1 + 1.6q^2 / \sqrt{q}$ for the plateau region and the banana regime, respectively, where $q$ is the safety factor and $e = r / R$ is an inverse aspect ratio.) In equation (1a), the first term on the rhs stands for the neoclassical relaxation via transit time damping. The explicit form is given in [23] by using the plasma dispersion function with complex argument, showing that the poloidal friction is suppressed when $X$ is of the order unity. In order to have an analytic insight for the bifurcation condition, an algebraic model form is employed here, where a tail in the damping rate at a large value of $X$ is considered. The coefficient $\nu$ is given as $\nu \sim q^2 n_o$ in the plateau regime, which is relevant for present experimental conditions ($n_o = n / q R$, the transit angular frequency, $\nu_{th}$: ion thermal velocity [23]). The second term on the rhs of equation (1a) indicates the drive of radial electric field by the Reynolds stress of turbulence, and the coefficient $\alpha \mod$, i.e. the coefficient for modulational coupling with background turbulence, is given as

\[ \alpha \mod = c_i^2 \nu_{drift}^2 \rho_p^2 \gamma_p^2 k^2 \]  

(2)

where $c_i$ is the sound velocity, $\gamma_p$ is the characteristic time scale of drift wave turbulence, $\rho_p$ is the ion gyroradius at sound velocity, and $k_i$ and $k_p$ are the poloidal and perpendicular wave number of drift wave fluctuations, respectively [13]. In the present model analysis, the coefficient $\alpha \mod$ is treated as a constant. The normalized turbulence intensity $I$ is introduced here as

\[ I = (ek_i \rho_p T_i^{-1})^2, \]  

(3)

where $\phi$ is the potential perturbation. The fluctuating electric field is also normalized to $T_i / \rho_p$, as is the case of the mean radial electric field. It is noted that the rate of energy transfer from drift wave turbulence to meso-scale and macro-scale electric field (which appears in the last term on the rhs of equation (1a)) is proportional to the second derivative of the electric field, $X''$, as is explained in the literature (see, e.g., section 3.2.2 of the review [13]). As a result of this, the spatial modulation of turbulence intensity, which is in phase with the curvature (not gradient) of the radial electric field, is predicted. Such a spatial modulation has been confirmed by experiments [27, 28].

A simple closed set of equations is formed by considering the equations of normalized turbulence intensity, $I$, and mean pressure gradient $G$. The nonlinear decorrelation rate of turbulence $\Delta \omega$ is assumed to be proportional to $I$, and is written as $\Delta \omega = \omega_0 I$. The dynamical equation for turbulence intensity is written as [13, 29]

\[ \dot{\gamma} (G) I \sim \omega_0 I^2 + \alpha \mod \gamma X'' I, \]  

(4)

where the linear growth rate $\gamma$ may depend on the pressure gradient. The energy balance equation is written as

\[ \dot{G} / \gamma = (I + C) \frac{\partial \gamma}{\partial x} (I + C) G \]  

(5)

where $\chi_0$ indicates the normalized value: one may choose the evaluation $\chi_0 \sim B_p L_p^3 \rho_p (T / eB) (L$: characteristic scale length), for which the diffusion coefficient $\chi_0 G$ reduces to the gyro-Bohm diffusion coefficient when the fluctuation amplitude is evaluated by the mixing length estimate. The parameter $C$ in equation (5) indicates the residual mechanism (e.g. neoclassical process in energy transport) that induces cross field transport in the absence of drift wave turbulence.
2.2. Zero-dimensional model

We here take a more simplified, zero-dimensional model, in order to pursue the analytic insight for the transition condition. A characteristic scale length of the radial inhomogeneity is chosen here according to the consideration that the turbulence drive for the flow is the strongest for the particular scale as is given in [30, 31]. Following this idea, one may choose the estimate $X'' \sim -l^{-3}X$, where the scale length $l$ is assumed as prescribed. Then the set of equations for the turbulence intensity and the mean radial electric field takes a normalized form as

$$
\frac{\partial I}{\partial t} = \gamma(G)I - \omega_2 l^2 - \alpha X^2 I,
$$

(6)

$$
\nu \frac{\partial X}{\partial t} = -\frac{(G + X)}{1 + X^2} + 2\alpha X I
$$

(7)

where $\alpha = \alpha_{med} l^{-2}$. Considering the fact that the linear instability is driven by plasma inhomogeneity, one may choose the dependence of the growth rate of the gradient as

$$
\gamma(G) = \gamma_0 G \propto G,
$$

(8)

which naturally gives the gyro-Bohm dependence of turbulent transport (in the absence of the mean radial electric field).

2.3. Steady state

The subject of this paper is to investigate the critical condition for the onset of L–H transition. For this purpose, the stationary state is analysed, and the bifurcation condition is looked for. In a steady state, the balance equation for the turbulence intensity gives a steady-state solution

$$
I = I_0 \cdot (G - \hat{\alpha} X^2),
$$

(9a)

with

$$
I_0 = \omega_2^{-1} \gamma_0, \quad \text{and} \quad \hat{\alpha} = \alpha \gamma_0^{-1}.
$$

(9b)

Equation (9a) shows that the ratio $\hat{\alpha}$ is the control parameter that characterizes the impact of the radial electric field on the turbulence level. The momentum balance equation provides a condition

$$
\nu \frac{(G + X)}{1 + X^2} = 2\alpha X I,
$$

(10a)

and the parameter

$$
\hat{\beta} = \frac{\alpha}{\nu} I_0
$$

(10b)

denotes the magnitude of the turbulence drive of the mean electric field in comparison with the drive by the neoclassical process. In this paper, we analyse the case where the zonal flows are not excited, i.e.

$$
\hat{\beta} < 1.
$$

(11)

By substituting equation (9a) into the rhs of equation (10a), the balance between the neoclassical force and the turbulence drive in a steady state takes the form

$$
\frac{(G + X)}{1 + X^2} = \hat{\beta}(G - \hat{\alpha} X^2)X.
$$

(12a)

The relation between the ion temperature gradient and the mean radial electric field is derived from equation (12a) as

$$
G = -\frac{X^2 + \hat{\alpha} \hat{\beta}(1 + X^2)^2}{1 - \hat{\beta}(1 + X^2)X}.
$$

(12b)

The equation for the mean energy closes the set of dynamical equations. Introducing the normalized heat flux in the stationary state, $S = (\rho_0 T_i^{-1} \chi_i^{-1}) Q_i/n$, where $Q_i/n$ is the ion heat flux per particle, one has the relation between gradient, turbulent intensity and flux as

$$
S = -(I + C)G.
$$

(13)

From equations (9a), (12b) and (13), the variables $(I, X, G)$ are determined by the given source flux $S$.

3. Critical conditions

3.1. Electric field bifurcation

Equation (12b) describes the possible bifurcation of the mean radial electric field. In the absence of additional drive, $\hat{\beta} = 0$, or in the limit of the weak radial electric field, $|X| \ll 1$, equation (12b) provides that the radial electric field is proportional to the ion temperature gradient,

$$
X = -G.
$$

(14)

This indicates the mean radial electric field, which is driven by the neoclassical process. The influence of this field on the anomalous transport has been a basis for many models [32]. If the additional drive is present, a deviation of the mean radial electric field from the neoclassical relation (14) can occur.

When the feedback of the generated mean radial electric field on the turbulence intensity is weak, i.e. the parameter $\hat{\alpha}$ is small, a bifurcation of the radial electric field (for common ion temperature gradient) is possible. This is understood by noting the fact that the numerator of equation (12b) is approximated as $X$, i.e.

$$
G = \frac{|X|}{1 + \hat{\beta}(1 + X^2)|X|} \quad \text{for} \quad X < 0,
$$

(15)

when the condition

$$
\hat{\alpha} \hat{\beta}(1 + X^2)X^2 < 1 \quad \text{for} \quad X \sim -\hat{\beta}^{-1/3}
$$

(16)

is satisfied. The rhs of equation (15) is a decreasing function of $-X$ in the limit of a strong radial electric field, while equation (14) holds in the limit of a small electric field. Therefore, under this circumstance, the radial electric field, being closed as a function of the mean pressure gradient, $X(G)$, has the property of bifurcation.

In contrast, if the parameter $\hat{\alpha}$ is large and the condition $\hat{\alpha} \hat{\beta}(1 + X^2)X^2 > 1$ (which is opposite to equation (16)) is satisfied in the large value of $|X|$, $X \sim -\hat{\beta}^{-1/3}$, the rhs of equations (12a) and (12b) is an increasing function of a stronger negative radial electric field. Thus, $X(G)$ is a monotonic function, and equation (15) turns as, in the limit of a strong electric field,

$$
G \sim \hat{\alpha} X^2
$$

(17a)

so that the relation

$$
X \sim -\sqrt{G/\hat{\alpha}}
$$

(17b)

holds. Figure 1 illustrates the numerical solution of equations (12a) and (12b), illustrating $X(G)$ for various
values of the parameter $\hat{\alpha}$. In the limit of weak gradient, the neoclassical relation equation (14) is satisfied. When the electric field is strong, $X < -1$, a deviation from the neoclassical estimate appears. The bifurcation occurs when the parameter $\hat{\alpha}$ is small.

The condition that the strong bifurcation appears in the relation $X(G)$ is estimated analytically. The condition equation (16), under which equation (15) holds, is written as $\hat{\alpha} \hat{\beta}(1 + \hat{\beta}^{-2/3})\hat{\beta}^{-2/3} < 1$. In the limit of $\hat{\beta} \ll 1$, one has $\hat{\alpha} \hat{\beta}^{-1/3} < 1$, so that an asymptotic relation for the boundary is obtained as

$$\hat{\alpha} < O(1)\hat{\beta}^{1/3} \quad \text{for} \quad \hat{\beta} \ll 1. \quad (18a)$$

(The coefficient, which is written as $O(1)$ in equation (18a), is estimated as $9/\sqrt{74}/6/35$.) In another limit, $\hat{\beta} \gg 1$, the condition $\hat{\alpha} \hat{\beta}(1 + \hat{\beta}^{-2/3})\hat{\beta}^{-2/3} < 1$ is rewritten as $\hat{\alpha} \hat{\beta}^{1/3} < 1$. That is,

$$\hat{\alpha} < O(1)\hat{\beta}^{-1/3} \quad \text{for} \quad \hat{\beta} \gg 1. \quad (18b)$$

Figure 2 illustrates the upper bound of the parameter $\hat{\alpha}$, below which the solution $X(G)$ shows a bifurcation near $X \sim -1$.

### 3.2. Bifurcation in the gradient–flux relation

By eliminating the turbulence intensity from equations (9a) and (13), the flux is expressed as a function of the gradient holds in the small flux limit, which shows the L-mode property. When the gradient becomes large, the flux $S$ takes the maximum

$$S_{\text{max}} \simeq \frac{4}{27\hat{\alpha}^2} \quad \text{at} \quad G \simeq \frac{2}{3\hat{\alpha}}. \quad (22)$$

When the source exceeds this critical value, the L-mode branch disappears.

In the presence of the additional drive, $\hat{\beta} \neq 0$, the electric field is amplified near the critical condition $X \sim -1$. In order to investigate the transition of transport owing to this nonlinear link between the gradient and the radial electric field, we take a simplified limit of $C = 0$. From equations (12b) and (19), one has the relation between the gradient and flux $S(G)$. Figure 3 illustrates the flux as a function of the ion temperature gradient (thick line). The radial electric field is shown by the chained line. Owing to the bifurcation in the $X(G)$ relation, the flux $S$ becomes the multi-valued function of the ion temperature gradient.

![Figure 1](image1.png) **Figure 1.** Radial electric field as a function of the ion temperature gradient for various values of $\hat{\alpha}$. Other control parameter is set as $\hat{\beta} = 0.5$.

![Figure 2](image2.png) **Figure 2.** Threshold condition for the bifurcation in the relation between the ion gradient and the mean radial electric field, $X(G)$. Dotted lines indicate the slope of $\hat{\beta}^{1/3}$ and $\hat{\beta}^{-1/3}$.

![Figure 3](image3.png) **Figure 3.** Flux as a function of the ion temperature gradient (thick line). The radial electric field is shown by the chained line. Owing to the bifurcation in the $X(G)$ relation, the flux $S$ becomes the multi-valued function of the ion temperature gradient.

Gand the electric field $X$ as

$$S = (G - \hat{\alpha}X^2 + C)G. \quad (19)$$

Combining equations (12b) and (19) to eliminate the radial electric field $X$, one has the relation between the gradient and flux $S(G)$.

In the limit of weak gradient, one has the neoclassical relation $X = -G$, so that the gradient–flux relation takes the form

$$S = (G - \hat{\alpha}G^2 + C)G. \quad (20)$$

In this case, the relation

$$G \sim \sqrt{S} \quad (21)$$

When the source exceeds this critical value, the L-mode branch disappears.
The electric field becomes strong enough, from the neoclassical value, equation (15), occurs when the deviation of the electric field is large, the critical value of the flux starts to decrease following equation (22). The dependence on parameters is weakly dependent on the parameter \( \beta \). When the source intensity reaches the critical value, the L-mode branch disappears and the transition to the H-mode state occurs.

4. Summary and discussion

In this paper, the physics of the L- to H-mode transition is discussed, based on the mechanism induced by the localized mean radial electric field. In addition to the neoclassical effect, an additional source of the electric field is introduced. The influence of turbulence-driven Reynolds stress is taken into account here. We analyse the case where this additional drive is weak so that the zonal flows are not excited. Even if the spontaneous flow by turbulence cannot be excited, the bifurcation of the mean radial electric field can occur, if the electric field is strong enough, \( X \sim -1 \). A hard transition of the electric field can occur if the parameter \( \alpha \) is small. This parameter \( \alpha \) is related to the linear growth rate, and is large near the stability condition as was discussed in [33]. Thus, the jump of the mean radial electric field for constant ion temperature gradient is expected to occur in the parameters, which are far from the linear stability boundary. The enhancement of the electric field above the critical value \( X \sim 1 \) can induce a bifurcation in the gradient–flux relation, i.e. the disappearance of the L-mode branch. The critical condition for the heat flux for the onset of transition is a decreasing function of \( \beta \), i.e. the higher the neoclassical damping, the higher the threshold power. On the other hand, the threshold power is weakly dependent on the parameter \( \alpha \), except for the case where the neoclassical process induces the transition. This means that the dependence of the threshold condition on the type of linear instability and its growth rate appear through the combination of equation (10b). (Note that the turbulent transport is prescribed in [34], where the threshold power for the L–H transition has been discussed based on the electric field bifurcation model.) As is indicated in equation (25), the threshold condition is influenced by multiple mechanisms. The additional drive of the electric field by the ion orbit loss can reduce the threshold power. The charge exchange loss of momentum enhances the threshold condition, and can

\[
S_{\text{max}} \simeq \frac{4}{27} \alpha^{-2} \frac{4}{9} \beta^{-2/3}.
\]
We choose information for the understanding of H-mode physics. Observation of dynamics at the transition will provide valuable insights. At the onset of the L–H transition, while the threshold power is weak, the radial electric field is weakly dependent on the density. This behavior has been observed in the parameter range of JT-60U. Rapid transitions of the radial electric field have been observed on JT-60U. A variety of the soft and hard transitions of the radial electric field have been observed in the parameter range of $X \sim -1$ [22]. The observations on ASDEX-U reported that the radial electric field is weakly dependent on the density at the onset of the L–H transition, while the threshold power increases as the density increases [17]. Future tests by the observation of dynamics at the transition will provide valuable information for the understanding of H-mode physics.

In analysing the plasmas in the regime of $\epsilon = 0.2$ [22], the possibility of poloidal shock has been pointed out [36], for which explanation is developed in chapter 21 of [21]. The two-dimensional structure of the transport barrier has been studied, and the enhanced ion pinch has been discussed [37, 38]. A more detailed analysis on the toroidal effects is essential in understanding the physics of the L–H transition.

Recently, much attention has been paid to the effects of symmetry-breaking magnetic perturbations, being motivated to control the edge-localized modes (ELMs) [39]. The applied magnetic perturbation usually includes the resonant components, which have the same pitch as the magnetic field line near the edge. When applied on the H-mode plasma, the symmetry-breaking perturbation effectively suppresses type-I ELMs [39]. Judged from the fact that the plasma is rapidly rotating, and from the fact that the efficiency of the ELM suppression weakly depends on the relative amplitude of the resonant component that may induce the magnetic island (when the plasma rotation is slow [40]), it is conjectured that the amplitude of magnetic island, if it exists, is much smaller than what is predicted for the vacuum magnetic field. When magnetic islands are not formed, the variation of the magnetic field strength by the perturbation magnetic field is small. Then the influence on the mean radial electric field through the mechanism, which is discussed in this paper, is weak. Nevertheless, the effects of symmetry-breaking perturbation coils on the L–H transition condition need further considerations. This is because the resonant component can penetrate into the plasma so as to form the magnetic island, if applied as a static magnetic perturbation before the L–H transition happens. When a large magnetic island is formed in a collisionless plasma, $v_i/\omega_i < \epsilon^{1/2}$, in which the $E \times B$ rotation velocity is larger than the drift due to magnetic field inhomogeneity, an additional radial ion flux by magnetic islands, $\Gamma_i$, has been evaluated as [41]

$$\frac{\Gamma_i}{n} = C_i v_i \rho_i q_i \epsilon^{-3/2} \frac{\omega_i^2}{\omega_{E \times B}^2} \delta^2 (G + X),$$  

where $C_i$ is a numerical coefficient of order unity, $\delta$ is the island width, $\omega_{E \times B}$ is the $E \times B$ rotation frequency associated with the mean radial electric field, $\epsilon = r/R$ is the inverse aspect ratio at the mode rational surface, $v_i$ is the ion collision frequency. Noting the fact that the neoclassical ion flux in the banana regime is of the order of $\Gamma_i/n \sim v_i \rho_i q_i \epsilon^{-1/2} (G + X)$, the additional contribution by the island formation (equation (26a)) will be comparable to that in axisymmetric plasmas, in determining the first term on the rhs of equation (1a), if the condition

$$\epsilon^{1/2} g R^{-1} \omega_{E \times B}^{-1} \delta^2 \sim O(1)$$  

holds. The additional term enhances the damping term (the first term on the rhs of equation (1a)), and the mean radial electric field can be influenced. Under such a circumstance, however, both the additional transports of ions and electrons due to the magnetic islands affect the mean radial electric field. The preceding analysis [41] has shown that the new type of bifurcation can occur in the collisionless limit. This is analogous to the bifurcation to the ‘electron root’ branch in helical toroidal plasmas, the observation of which has been reported in [42]. Another influence on the radial electron current, which is induced by the change in magnetic topology, has been pointed out theoretically [43]. These additional terms for fluxes of electrons and ions are not included in the present analysis, so that the effect of the ELM control coil on the transition condition must be studied in a much wider context.

![Figure 5. Dependences of parameters at the critical condition. The dependences on $\hat{\alpha}$ are demonstrated in (a), and (b) shows those on $\hat{\beta}$.](image)
perspective than the focus of this paper. The extension of the present analysis to include the effect of ELM control coils is an urgent future task.

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