Impact of Interactive Vertical Overlap of Cumulus and Stratus on Global Aerosol, Precipitation, and Radiation Processes in the Seoul National University Atmosphere Model Version 0 With a Unified Convection Scheme (SAM0-UNICON)

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Abstract The previously proposed parameterization for the integrated vertical overlap of cumulus and stratus is generalized to handle both conventional exponential-random stratus overlap and nonconventional (i.e., other than exponential-random) cumulus overlap in a simultaneous way. With the parameterization of the decorrelation length scale of stratus as a function of vertical wind shear, our parameterization simulates various interactive feedback between vertical cloud overlap and other physical processes. This interactive vertical overlap parameterization of cumulus and stratus was implemented into all relevant physics parameterizations (i.e., convection, stratus microphysics, radiation, aerosol wet deposition, and aerosol activation at the base of stratus) of the Seoul National University Atmosphere Model version 0 with a Unified Convection Scheme (SAM0-UNICON) in a fully consistent way. It is shown that the overall performance of the interactive cloud overlap parameterization to simulate the observed mean climate is similar to that of the original overlap parameterization. Given that an intensive tuning has not yet been performed with the new overlap parameterization, this result is quite encouraging.

1. Introduction

Any general circulation model (GCM) with a coarse grid size computes a partial cloud fraction between 0 and 1 in each grid layer. Thus, a vertical cloud overlap parameterization that computes how much cloud fraction is overlapped vertically is an indispensable component of a GCM. The treatment of vertical cloud overlap in many GCMs, however, is rudimentary, and inconsistent vertical cloud overlaps are used in different physics parameterizations. For example, in the Community Atmosphere Model version 5 (Park et al., 2014), the stratus microphysics scheme uses a maximum stratus overlap regardless of the vertical separation distance and neglects the decrease of stratiform precipitation area associated with the complete evaporation of stratiform precipitation in clear portions; the shallow and deep convection schemes assume that the convective precipitation area is 1 for computing evaporation rates of convective precipitation; the aerosol wet deposition scheme computes convective (stratiform) precipitation area as the vertical integral of cumulus (stratus) fractions in the layers above, weighted by the net production rate of convective (stratiform) precipitation; the aerosol activation scheme assumes a maximum vertical overlap between adjacent stratus by neglecting the existence of cumulus for computing aerosol activation at the base of stratus; and the radiation scheme uses a maximum vertical overlap between adjacent total clouds that combine stratus, cumulus, and stratiform snow, and a random overlap between total clouds separated by a clear layer. To some degree, this inconsistent treatment was inevitable because, instead of being developed as a stand-alone scheme, vertical cloud overlap has historically been treated as a small internal portion of individual physics parameterizations and a single modeler could not develop an entire set of physics parameterizations. However, for more accurate climate prediction with various anthropogenic forcings, it is likely that the integrated parameterization of aerosol-cloud-turbulence-precipitation-radiation interactions becomes more important. Hence, it may be necessary to use a single consistent cloud overlap parameterization in all relevant physics schemes.
As a first step to achieve this goal, Park et al. (2017) (referred to as P2017 hereafter) developed a heuristic parameterization to handle an integrated vertical overlap of cumulus and stratus in various physics schemes. Following previous studies (e.g., Hogan & Illingworth, 2000; Shonk et al., 2010; Tompkins & Giuseppe, 2007), P2017 assumes that cumulus and stratus have an exponential-random vertical overlap; that is, vertically continuous cumulus (stratus) have an overlap that varies exponentially between maximum and random according to a decorrelation length scale, \( \Delta z_c \) (\( \Delta z_s \)), whereas vertically separated cumulus (stratus) are randomly overlapped vertically. One of the unique aspects of P2017 is its capability to handle multiple cloud types (i.e., cumulus and stratus) in an integrated way as opposed to a single cloud type. By extending P2017, Park (2018) (referred to as P2018 hereafter) suggested a set of economical and analytical equations for computing cloud overlap areas; these equations are more accurate and much more efficient than the heuristic equations suggested by P2017, which suffered from the truncation errors in association with the digitalization of the overlap areas. More recently, Park et al. (2019) (referred to as P2019 hereafter) implemented the integrated vertical overlap parameterization of cumulus and stratus developed by P2017 and P2018 into the convection, stratus microphysics, and radiation schemes of the Seoul National University Atmosphere Model version 0 with a Unified Convection Scheme (SAM0-UNICON, Park et al. (2017), Park, Shin, et al., 2019) in an online mode and examined how the global climate is influenced by the integrated vertical overlap of cumulus and stratus.

In P2019, the decorrelation length scale of cumulus and stratus are simply set to \( \Delta z_c = \infty \) (i.e., maximum overlap) and \( \Delta z_s = 2 \, \text{km} \), respectively. However, numerous studies noted that the decorrelation length scale is not a constant but, rather, a function of location, season, and large-scale environmental conditions, such as the vertical shear of horizontal wind and atmospheric stability (Barker, 2008; Li et al., 2015; Lin & Mapes, 2004; Mace et al., 2009; Mace & Benson-Troth, 2002; Naud et al., 2008; Oreopoulos & Norris, 2011). Thus, for a more complete treatment, it is necessary to parameterize the decorrelation length scales as a function of appropriate environmental variables. As a parameterization for the decorrelation length scale, Shonk et al. (2010) suggested a latitude-dependent formula, and more recently, Giuseppe and Tompkins (2015) (referred to as GT15, hereafter) derived a wind-shear dependent empirical parameterization by analyzing satellite-observed cloud data. Most of the previous studies, however, are based on the analysis of a single merged cloud not being segregated by cloud types (e.g., cumulus and stratus), such that the implementation of the findings of previous studies into the integrated vertical overlap parameterization of P2017 and P2018 for multiple clouds types needs additional consideration.

The Unified Convection Scheme (UNICON, Park, 2014a, 2014b) in SAM0-UNICON has the capability to diagnose the vertical overlap of cumulus by explicitly calculating vertical changes in the radius and center coordinate of the convective updraft plumes as a function of the updraft vertical velocity and vertical shear of environmental horizontal winds. Conceptually, this explicit but nonconventional diagnosis of vertical cumulus overlap is a more advanced approach than the conventional exponential-random cloud overlap. However, the integrated overlap parameterizations of P2017 and P2018 are only designed to handle the conventional exponential-random overlap. In this paper, we will generalize the integrated overlap parameterization of P2017 and P2018, such that it can handle both nonconventional cumulus overlap and conventional exponential-random stratus overlap. Subsequently, following GT15, the decorrelation length scale of stratus, \( \Delta z_s \), will be parameterized as a function of the vertical shear of environmental horizontal winds.

As explained above, P2019 implemented the integrated vertical overlap parameterization of cumulus and stratus into cloud microphysics and radiation schemes. However, aerosol wet deposition and activation schemes are also directly influenced by the assumed vertical cloud overlap. In order to impose full consistency, we will also implement the same overlap parameterization into the aerosol wet deposition and activation schemes. As a result, all relevant physics schemes will operate on a single integrated cloud overlap parameterization in a fully consistent and unified way.

The structure of this paper is as follows. Section 2 provides a set of new integrated cloud overlap equations designed to handle both the explicit nonconventional cumulus overlap diagnosed by UNICON and the conventional exponential-random stratus overlap. Section 3 provides a brief description of SAM0-UNICON and various simulation settings, including how the new interactive vertical cloud overlap parameterization is implemented into aerosol wet deposition and aerosol activation schemes (further details are provided in the appendices). Section 4.1 compares two global simulations with a set of fixed decorrelation length scales.
First, we define the following variables in each layer, below. All other equations in P2017 and P2018 are used without any changes. The impact of interactive vertical tilting implemented on aerosol wet deposition and aerosol activation schemes will be discussed in section 4.3. A summary and conclusions are provided in section 5.

### 2. Vertical Overlap Equations for Nonconventional Cumulus Overlap and Conventional Stratus Overlap

One of the assumptions in the integrated vertical cloud overlap scheme of P2017 and P2018 is that both cumulus and stratus have vertical overlaps parameterized by the linear combination of the maximum and random overlaps. Instead of using this conventional exponential-random overlap, however, UNICON computes the vertical overlap of cumulus in an explicit way by diagnosing vertical variations of the center coordinate \((x, y)\) and the radius of the convective updraft plume as a function of the convective updraft vertical velocity and vertical shear of environmental horizontal winds. The production and evaporation rates of convective precipitation are computed in accordance with this nonconventional geometric vertical tilting of cumulus updraft plumes, which is one of the unique aspects of UNICON that does not exist in other convection schemes. An additional merit of this geometric tilting overlap compared to the exponential-random overlap is that it provides information on the vertical cumulus overlap, not only between two adjacent layers but also among remotely separated multiple layers. To improve computational efficiency, however, UNICON simply assumes that the convective precipitation area at the model interface has a single disk shape, with its center coordinate computed by the weighted averages of the center coordinates of the precipitating convective updraft plume in the current layer and the precipitation flux falling from the top interface of the current layer (equation (86) of Park, 2014a). Indeed, this is an unrealistic assumption because for a given vertical tilting of precipitating convective updraft plumes in multiple layers, convective precipitation area at a certain height is likely to have a complex shape that cannot be described by a single disk. Presumably, the ideal approach is to combine this nonconventional geometric tilting overlap for cumulus by relaxing the assumption of a single disk shape for convective precipitation area with the conventional exponential-random overlap for stratus by parameterizing \(\Delta_z\) as a function of relevant environmental variables. Unfortunately, we could not derive complete integrated overlap equations suited for this ideal approach, mainly owing to the difficulty in incorporating the vertical cumulus overlap among remotely separated multiple layers into the integrated overlap equations. Consequently, in our study, we only consider vertical cloud overlap between adjacent layers, similar to previous studies. However, vertical cumulus overlap between adjacent layers is computed directly from the nonconventional geometric tilting of convective updraft plumes, while the vertical stratus overlap is computed from the conventional exponential-random assumption. For this alternative approach, we generalize the vertical overlap equations of P2018 for cumulus and stratus in the adjacent layers \((O(a, a_c)), \) where \(\uparrow\) denotes the value in the layer directly above the current layer), as shown below. All other equations in P2017 and P2018 are used without any changes.

First, we define the following variables in each layer,

\[
\begin{align*}
    m_c &= O(a, a_c), \\
    m_s &= \min[\min(a_s, a_c), 1 - P_s], \\
    P_c &= a + a_c - m_c, \\
    a_{cs} &= a_c - m_c, \\
    a_{cs+} &= a_{cs} - m_c, \\
    a_{ss} &= a_s - m_s, \\
    a_{s+} &= a_{ss} - m_s, \\
    a_{ms} &= a_{ms} - m_s, \\
    a_{m+} &= a_{ms} - m_s, \\
    \mu &= \frac{a_{ss}}{a_{ss} + a_c}, \\
    \mu_s &= \frac{a_{ss}}{a_{ss} + a_s}, \\
    \mu_c &= \frac{a_{ss}}{a_{ss} + a_c}, \\
    \mu_{c+} &= \frac{a_{ss}}{a_{ss} + a_c},
\end{align*}
\]

where the subscripts \(c, s, \) and \(r\) denote cumulus, stratus, and clear portions in each layer, respectively (i.e., \(a_c + a_s + a_r = 1\)); \(a\) is the fractional area; \(m_c\) is the vertical overlap area between two adjacent cumulus provided by UNICON from the computation of explicit geometric tilting of convective updraft plumes as a function of convective updraft vertical velocity and vertical shear of environmental horizontal winds; \(m_s\) is the vertical overlap area between two maximally overlapped adjacent stratus in the cumulus-free region; \(a_c\) and \(a_s\) are the cumulus areas excluding the overlap area with adjacent cumulus; \(a_s\) and \(a_s+\) are the stratus areas excluding the overlap area with adjacent stratus; and \((\mu, \mu_s, \mu_c, \mu_{c+})\) are the fractions occupied by stratus among the sum of stratus and clear portions. Here, \(P_c\) is the projection area of cumulus in two adjacent layers. Because cumulus is assumed to have a higher occupancy priority than stratus, mainly owing to the contrasting nature of associated turbulent eddies (i.e., vertically protruding nonlocal asymmetric turbulent
eddy within cumulus, and horizontally spread local symmetric turbulent eddies within stratus, \( m_s \) should be smaller than \( 1 - P_c \).

In the case that two adjacent cumulus have an overlap fraction \( O(a_{c1}, a_c) = m_c \), and two adjacent stratus are maximally overlapped with an overlap fraction \( O(a_{s1}, a_s) = m_s \), we can compute the other overlap areas by proportionally partitioning \( a_{c1}, a_{c2}, a_{c3} \), and \( a_{s1} \) into the remaining available spaces. For example, with the predetermined \( m_c \) and \( m_s \), it becomes \( O(a_{c1}, a_c) = O(a_{c2}, a_c) \) and \( O(a_{s1}, a_s) = O(a_{s2}, a_s) \). Because \( O(a_{c1}, a_{s1}) + O(a_{c2}, a_{s2}) = a_c \) and \( a_c \) is randomly overlapped with \( a_{c1} \) and \( a_{c2} \), it becomes \( O(a_{c1}, a_{s1}) = (a_{c1} \cdot a_s)/(a_{s1} + a_s) \) and \( O(a_{c2}, a_{s2}) = (a_{c2} \cdot a_s)/(a_{s2} + a_s) \), such that \( O(a_{c1}, a_s) = \mu_{c1} \cdot a_c \) and \( O(a_{c2}, a_s) = (1 - \mu_{c1}) \cdot a_c \). The resulting vertical overlap areas \( O(a_{l1}, a_{ml}) \) for \( l = c, s, r \) and \( m = c, s, r \) between the cloud fractions in the adjacent layers become

\[
\begin{align*}
O(a_{c1}, a_c) &= m_c, \\
O(a_{c1}, a_s) &= \mu_{c1} \cdot a_c, \\
O(a_{c2}, a_s) &= (1 - \mu_{c1}) \cdot a_c, \\
O(a_{c1}, a_r) &= \mu_{c1} \cdot a_{c1s}, \\
O(a_{c2}, a_r) &= m_s, \\
O(a_{c1}, a_r) &= a_s - \mu_{c1} \cdot a_{c1s}, \\
O(a_{c2}, a_r) &= \mu_{c2} \cdot a_c, \\
O(a_{c1}, a_r) &= a_{c1s} - \mu_{c1} \cdot a_c, \\
O(a_{c2}, a_r) &= a_{c2s} - \mu_{c2} \cdot a_c, \\
O(a_{c1}, a_r) &= a_r - a_{c1s} + \mu_{c1} \cdot a_s - (1 - \mu_{c1}) \cdot a_{c1s}, \\
O(a_{c2}, a_r) &= a_r - a_{c2s} + \mu_{c2} \cdot a_s - (1 - \mu_{c2}) \cdot a_{c2s},
\end{align*}
\]

which satisfy the required consistency relationships of \( \sum_l O(a_{l1}, a_{ml}) = a_{ml} \) and \( \sum_m O(a_{l1}, a_{ml}) = a_{l1} \) for \( l = c, s, r \) and \( m = c, s, r \). These equations are derived based on the consideration that \( a_{c1} \) and \( a_s \) are not overlapped; each of \( a_c \) and \( a_s \) is randomly overlapped with \( a_{c1s} \), and each of \( a_{s1} \) and \( a_{c2s} \) is randomly overlapped with \( a_c \). Note that if \( \min(a_{c1}, a_s) \leq 1 - P_c \), either \( a_{c1} \) or \( a_{s1} \) becomes 0, while if \( \min(a_{c1}, a_s) > 1 - P_c \), the equation becomes \( a_{c1}^* = a_{c1}^* + a_{c2s} \) and \( a_{s1}^* = a_{s1}^* + a_s \). As a result, the use of the proportional factors, \( \mu_{c1} \) and \( \mu_{c2} \), to proportionally partition the overlap fractions of stratus and clear portions under a maximum stratus overlap is correct.

In the case that two adjacent cumulus have an overlap fraction \( m_c \) and two adjacent stratus are randomly overlapped, we can compute the other overlap areas by proportionally partitioning \( a_{c1s}, a_{c2s}, \) and \( a_s \) into the remaining available spaces. The resulting vertical overlap areas \( O(a_{l1}, a_{ml}) \) become

\[
\begin{align*}
O(a_{c1}, a_c) &= m_c, \\
O(a_{c1}, a_s) &= \mu_{c1} \cdot a_c, \\
O(a_{c2}, a_s) &= (1 - \mu_{c1}) \cdot a_c, \\
O(a_{c1}, a_r) &= \mu_{c1} \cdot a_{c1s}, \\
O(a_{c2}, a_r) &= m_s, \\
O(a_{c1}, a_r) &= a_s - \mu_{c1} \cdot a_{c1s}, \\
O(a_{c2}, a_r) &= \mu_{c2} \cdot a_c, \\
O(a_{c1}, a_r) &= a_{c1s} - \mu_{c1} \cdot a_c, \\
O(a_{c2}, a_r) &= a_{c2s} - \mu_{c2} \cdot a_c, \\
O(a_{c1}, a_r) &= a_r - a_{c1s} + \mu_{c1} \cdot a_s - (1 - \mu_{c1}) \cdot a_{c1s}, \\
O(a_{c2}, a_r) &= a_r - a_{c2s} + \mu_{c2} \cdot a_s - (1 - \mu_{c2}) \cdot a_{c2s},
\end{align*}
\]

which also satisfies the required consistency relationships of \( \sum_l O(a_{l1}, a_{ml}) = a_{ml} \) and \( \sum_m O(a_{l1}, a_{ml}) = a_{l1} \) for \( l = c, s, r \) and \( m = c, s, r \).
For each combination of $K = C, S, M, R$ (each of which denotes convective, stratiform, mixed, and clear precipitation, respectively) and $l = c, s, r$, the overlap area $a^K_l$ between the precipitation area at the top interface, $a^K$, and the cloud area at the layer midpoint, $a_l$, is computed by

$$a^K_l = O(a_{c1}, a_l) \cdot (a^K_{c1}/a_{c1}) + O(a_{s1}, a_l) \cdot (a^K_{s1}/a_{s1}) + O(a_{r1}, a_l) \cdot (a^K_{r1}/a_{r1}),$$

and for each of $m = c, s, r$, we compute

$$O(a_{m1}, a_l) = \lambda_m \cdot O(a_{m1}, a_{l_{max,s}}) + (1 - \lambda_m) \cdot O(a_{m1}, a_{l_{ran,s}}),$$

with the weighting factors for stratus ($\lambda_s$) parameterized by

$$\lambda_s = \exp(-\Delta z/\Delta z_s).$$

where $\Delta z$ is the vertical separation distance between two adjacent grid layers, and $\Delta z_s$ is the decorrelation length scale for stratus. Above $a^K$ satisfies the mandatory consistency conditions of $a_l = \sum_K a^K_l$ and $a^K = \sum_m a^K_m$.

Finally, following GT15, we parameterize $\Delta z_s$ as a function of the vertical shear of mean horizontal winds,

$$\Delta z_s = \Delta z_0 - \gamma \cdot \sqrt{\left[\frac{u(z + \Delta z) - u(z)^2}{\Delta z} + \left[\frac{v(z + \Delta z) - v(z)^2}{\Delta z}\right]^2\right]},$$

where $u$ and $v$ are the grid mean zonal and meridional velocities, respectively. From the analysis of satellite observations summarized in Table 4 of GT15 and by considering that the analysis of GT15 is based on all cloud types and that the decorrelation length scale of stratus is smaller than that of cumulus (e.g., see Figure 8 of GT15 and Pincus et al., 2005), we use $\Delta z_0 = 5 \text{ km}$ and $\gamma = 0.6 \text{ km}/(\text{m s}^{-1} \text{ km}^{-1})$ as the default values of the decorrelation length scale of stratus. The degree of vertical tilting of cumulus within UNICON is controlled by the nondimensional adjustment factor of the plume vertical velocity, $c_m$ (its default value is 0.9), which measures the degree to which the horizontal momentum of the convective plumes adjusts to the environment during vertical motion without mass exchange with the environment (equation (41) of Park, 2014a). We will explore the sensitivity to $\gamma$ and $c_m$.

### 3. Model Description and Simulation Setting

First, we implemented the interactive vertical overlap parameterization of cumulus and stratus developed in section 2 into the cumulus and stratus microphysics and radiation schemes of SAM0-UNICON (Park et al., 2017). Our implementation uses an independent precipitation approximation (IPA, P2017) for convective and stratiform precipitations, such that mixed precipitation is not generated. Our study uses the same radiation scheme as in SAM0-UNICON (Iacono et al., 2008) with the Monte Carlo Independent Column Approximation (Pincus et al., 2003) that constructs a set of stochastic subcolumns in which cloud fraction is either 0 or 1 in each grid layer. Instead of using a single merged cloud, however, we construct the subcolumns with multiple cloud types based on the vertical overlap probabilities between $a^K_{c1}$ and $a^K_{s1}$ explicitly computed from P2017, P2018, and P2019 for individual combinations of $K = S, R$ and $l = c, s, r$. As explained in Appendix B of P2019, we used a random number with an equal probability between 0 and 1 to determine which $a^K_l$ occupies each grid layer in a given subcolumn, moving from the top to the lowest model layer. At the top interface of the top model layer, the precipitation type is assumed to be a clear precipitation (i.e., $K = R$), such that there are three possible configurations in the top model layer, $a^K_C$, $a^K_S$, and $a^K_R$, which are summed to 1. We generate a random number in the top model layer. If $a^K_C$ is chosen by the random number in the top model layer, we compute $O(a^K_C, a^K_{c1})$ for all combinations of $K = S, R$ and $l = c, s, r$, such that the sum of $O(a^K_C, a^K_{c1})/a^K_C$ is 1. Then, a new random number generated in the second layer is used to determine which $a^K_{c1}$ occupies the second grid layer in the given subcolumn. As in the original radiation scheme, our implementation also treats the radiative impact of stratiform snow. However, for simplicity, we neglect the radiative impact of convective precipitation and so the precipitation type index $K$ does not include the one for convective precipitation, $C$. 

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The simulation setting is identical to the one used in P2019. A set of stand-alone simulations were conducted with the original and interactive overlap parameterizations forced by observed climatological sea surface temperature (SST) and sea-ice fraction, with an annual cycle for 10 years at a horizontal resolution of 0.98 latitude × 1.258 longitude with 30 vertical layers and a model integration time step of Δt = 30 min, as described in Park et al. (2014). As default parameter values for vertical cloud overlap, we use the decorrelation length scales, Δzl = ∞ (maximum cumulus overlap) and Δzs = 2 km for the original integrated overlap parameterization following P2019, while the convective momentum adjustment factor, cr = 0.9, and Δzs = 5 km and γ = 0.6 km/(m s⁻¹ km⁻¹) (both Δzl and γ are defined for stratus, as shown in equation (7)), are used for the new interactive overlap parameterization developed in the current paper. The simulation with the original and interactive overlap parameterizations will be referred to as OLD and NEW, respectively. Thus, NEW is the simulation with the new interactive cloud overlap parameterization implemented into convection, stratus microphysics, and radiation schemes. In addition, two sensitivity simulations are performed with cr = 0.1 (i.e., reduces the impact of vertical wind shear on the vertical tilting of cumulus, i.e., enhances vertical coherency of cumulus overlap) and γ = 1.0 (i.e., enhances the randomness of vertical stratus overlap).

The aerosol wet deposition scheme in the original SAM0 is structurally separated from the cumulus and stratus microphysics schemes, and it assumes its own vertical cloud overlaps different from the ones used in the proceeding cumulus and stratus microphysics schemes. For example, convective (stratiform) precipitation area in the aerosol wet deposition scheme is simply set to the vertical integral of cumulus (stratus) fractions in the layers above, weighted by the net production rate of convective (stratiform) precipitation, and for the purpose of computing wet deposition of nonactivated interstitial aerosols, both convective and stratiform precipitation areas are assumed to be randomly overlapped with clouds in the layer below. Given that convective and stratiform precipitations scavenge aerosols, this inconsistency in the assumed vertical overlaps of clouds and precipitations between cloud microphysics and aerosol wet deposition schemes is highly undesirable. With our implementation of the new integrated vertical cloud overlap parameterization, the aerosol wet deposition by convective and stratiform precipitations are treated within UNICON and stratus microphysics schemes, respectively, such that a single consistent cloud overlap parameterization is used both for cloud microphysics and aerosol wet deposition processes. The details on various aerosol wet deposition processes are provided in Appendices A and B. The aerosol activation at the base of stratus occurs in the overlap area (aC↑) between the clear portion (aC) and the stratus fraction just above the clear portion (aC↑). Based on the maximum stratus overlap assumption, the original SAM0 computes aC↑ = max(aC↑ − aC, 0) without considering the contribution of the cumulus fraction. As detailed in Appendix B, we recomputed aC↑, such that it is fully consistent with the integrated cloud overlap parameterization implemented into cumulus and stratus microphysics, radiation, and aerosol wet deposition schemes. The simulations with the new overlap parameterization implemented consecutively into the aerosol wet deposition and aerosol activation schemes on top of NEW will be referred to as WED and ACT, respectively. Thus, ACT is the final simulation with the interactive cloud overlap parameterizations implemented into all relevant physics parameterization schemes.

4. Results
4.1. Impact of Interactive Vertical Tilting of Cumulus and Stratus on Global Precipitation and Radiation Processes

Figure 1 shows the annual zonal mean cross sections of the overlap areas between cloud fractions in the adjacent layers (equation (5)) obtained from the NEW and OLD simulations and the differences between NEW and OLD. Consistent with the enhanced vertical tilting of cumulus from OLD to NEW, O(aC↑, aC) decreases from OLD to NEW (Figure 1b). In turn, the probability for the cumulus to be overlapped with noncumulus areas increases, resulting in the increases of O(aC↑, aC) (Figure 1e) and O(aC↑, aC) (Figure 1h). Compared to the cumulus components, the changes in O(aC↑, aC) are rather complicated, reflecting complex spatial variations of the vertical shear of horizontal wind on which the computation of O(aC↑, aC) is explicitly dependent in the NEW simulation (equation (7)). The similarity between ΔO(aC↑, aC) and ΔaC shown in Figure 2b also indicates that some of ΔO(aC↑, aC) are due to the changes in aC from OLD to NEW.

Figure 2 shows the annual zonal mean cross sections of cumulus fraction (aC), convective precipitation area (aC↑), overlap area between aC and aC (ac↑), and overlap area between aC and clear portion (aC↓) obtained from
Figure 1. Annual zonal mean overlap areas between cumulus fractions in the adjacent layers (a–c, O(\(a_{ac}\), \(a_{ac}\))), cumulus and stratus in the adjacent layers (d–f, O(\(a_{ac}\), \(a_{as}\))), stratus and cumulus in the adjacent layers (d–i, O(\(a_{as}\), \(a_{ac}\))), and stratus fractions in the adjacent layers (j–l, O(\(a_{as}\), \(a_{as}\))) from (left column) NEW, (right column) OLD, and (middle column) the differences between NEW and OLD. The global mean value is shown at the top left of each plot.
Figure 2. Annual zonal mean cumulus fraction (a–c, \(\alpha_c\)), convective precipitation area (d–f, \(\alpha^c\)), overlap area between the convective precipitation area and cumulus fraction (g–i, \(\alpha_c^c\)), and overlap area between the convective precipitation area and clear fraction (j–l, \(\alpha_c^r\)) from (left column) NEW, (right column) OLD, and (middle column) the differences between NEW and OLD. The global mean value is shown at the top left of each plot.
the NEW and OLD simulations and the differences between NEW and OLD. The ratio of global mean \( \Delta a_c \) (\( \Delta a_c^C, \Delta a_c^S, \Delta a_c^C \)) to the global mean \( a_c (a_c^C, a_c^S, a_c^V) \) of OLD is 0.010 (0.127, −0.117, 0.210) (we note that the numbers in the figures are truncated, while these values are calculated at higher precision). Because the evaporation rate of convective precipitation is proportional to \( a_c^V \) (e.g., equations (80)–(82) in Park, 2014a), this indicates that enhanced vertical cumulus tilting can increase evaporation of convective precipitation up to 20%, which is substantial even though global mean \( a_c^C \) is only 1.5% (Figure 2l). With an explicit treatment of vertical tilting of cumulus as a function of updraft vertical velocity and vertical shear of grid mean horizontal wind, \( a_c^C \) in NEW is larger than that in OLD, which assumes a maximum cumulus overlap. With enhanced tilting, however, \( a_c^C \) decreases and accretion of in-cumulus condensate by convective precipitation, if any, is also likely to decrease (we note that the current version of UNICON does not handle accretion processes). Conversely, \( a_c^S \) and associated evaporation of convective precipitation increase. The nonzero \( \Delta a_c \), shown in Figure 2b is the result of complex interactions among cumulus tilting, subgrid cold pool, mesoscale organized flow, convective activity, and vertical transport of horizontal momentum by convective updrafts and downdrafts. Enhanced evaporation within the planetary boundary layer (PBL) will strengthen the subgrid cold pool and mesoscale organized flow, which in turn is likely to foster stronger and wider convective updrafts within UNICON (Park, 2014a, 2014b). Stronger convective updrafts, however, may decrease vertical shear of mean horizontal winds by enhancing convective momentum transport, resulting in the decrease of vertical tilting of cumulus (i.e., self-restoring negative feedback). Positive \( \Delta a_c \) indicates that overall, enhanced cumulus tilting strengthens convective activity.

Figure 3 shows similar annual zonal mean cross sections of stratus fraction \( (a_s) \), stratiform precipitation area \( (a_s^C) \), the overlap area between \( a_s \) and \( a_s^C \), and the overlap area between \( a_s \) and clear portion \( (a_s^V) \) obtained from NEW and OLD simulations and the differences between NEW and OLD. The ratio of global mean \( \Delta a_s \) (\( \Delta a_s^C, \Delta a_s^S, \Delta a_s^V \)) to the global mean \( a_s (a_s^C, a_s^S, a_s^V) \) of OLD is 0.0008 (−0.0015, 0.003, −0.008, respectively), which is much smaller than those of cumulus components. We think this indicates that the specified decorrelation length scale of stratus, \( \Delta z_s = 2 \) km in OLD represents the global mean value of the parameterized \( \Delta z_s \) in equation (7) reasonably well, rather than implying that vertical stratus overlap is less important than vertical cumulus overlap. The positive global mean \( \Delta a_s^S \) and negative global mean \( \Delta a_s^C \) imply that the simulation with a fixed \( \Delta z_s = 2 \) km slightly overestimates the randomness of the vertical stratus overlap compared to the simulation using the parameterized \( \Delta z_s \) in equation (7). If equation (7) appropriately describes the real world, it would be better to use a higher value for \( \Delta z_s \) than 2 km in the fixed \( \Delta z_s \) simulation. Although global mean differences are tiny, the differences in regional and vertical variations are not negligible, illustrating the importance of parameterizing \( \Delta z_s \) rather than using a fixed value. We note that equation (7) is obtained from a rigorous analysis of the observation data.

To better understand how vertical cloud overlap influences various physical processes, we plotted the changes in several cloud microphysical tendencies (Figure 4) and radiative heating rates (Figure 5) from OLD to NEW. The tendency equations for evp_RV:con (Figures 4a–4c: evaporation of convective rain in the clear environment, equations (81) and (82) in Park, 2014a), evp_SV (Figures 4d–4f: evaporation of stratiform snow in the clear environment, Morrison and Gettelman, 2008), and acc_LR (Figures 4g–4i: accretion of stratus liquid droplets by stratiform rain; equation (28) in Morrison and Gettelman, 2008) are explicit functions of \( a_c^C, a_c^S, a_c^V \), respectively, such that they are directly influenced by vertical cloud overlap. On the other hand, the tendency equations for aut_IS (Figures 4j–4l: autoconversion of stratus ice crystals into stratiform snow, equation (29) in Morrison and Gettelman, 2008), cme_VL (Figures 4m–4o: net condensation of water vapor into stratus liquid droplets, equation (A9) in Park et al., 2014), and cme_VI (Figures 4p–4r: net deposition of water vapor into stratus ice crystals, equation (23) in Morrison and Gettelman, 2008) are not explicitly dependent on the vertical overlap areas, such that they are only indirectly influenced by the changes in the grid mean input state variables (e.g., temperature, water vapor, cloud condensate, and cloud fraction) induced by different vertical cloud overlap at the previous time step. The differences of cloud microphysics and radiation tendencies between NEW and OLD shown in Figures 4 and 5 are the combined results of the direct and indirect effects of vertical cloud overlap.

The most distinct change in the cloud microphysics processes is a substantial increase of evp_RV:con (Figure 4b) in the tropical region: On a global average, it increases by 15%, which is roughly consistent with
Figure 3. Annual zonal mean stratus fraction ($a_s$, $a_s^s$), stratiform precipitation area ($\Delta a_s$, $\Delta a_s^{\text{NEW-OLD}}$), overlap area between the stratiform precipitation area and stratus fraction ($\Delta a_s^s$, $\Delta a_s^s^{\text{NEW-OLD}}$), and overlap area between the stratiform precipitation area and clear fraction ($\Delta a_s^c$, $\Delta a_s^c^{\text{NEW-OLD}}$) from (left column) NEW, (right column) OLD, and (middle column) the differences between NEW and OLD. The global mean value is shown at the top left of each plot.
Figure 4. Annual zonal mean tendencies of several cloud microphysics processes obtained from NEW (left column), OLD (right column) and the differences between NEW and OLD (middle column) for (a–c) evaporation of convective rain in the clear environment (evp_RV:con), (d–f) evaporation of stratiform snow in the clear environment (evp_SV), (g–i) accretion of stratus liquid droplets by stratiform rain (acc_LR), (j–l) autoconversion of stratus ice crystals into stratiform snow (aut_IS), (m–o) net condensation of water vapor into stratus liquid droplets (cme_VL), and (p–r) net deposition of water vapor into stratus ice crystals (cme_VI). The global mean value is shown at the top left of each plot.
Figure 5. Annual zonal mean radiative heating rate obtained from NEW (left column), OLD (right column), and the differences between NEW and OLD (middle column) for (a–c) SW radiation and (d–f) LW radiation. The global mean value is shown at the top left of each plot.

The global increase of $\alpha_C$ shown in Figure 2k. These increases of $\alpha_C$ and associated evaporation of convective precipitation remedy some of the dry biases in the tropical troposphere in the OLD simulation, as shown in Figure 6. Overall, the changes in stratiform microphysics tendencies are much weaker and noisier than $\Delta$evp_RV_con. In addition, the spatial coherency between stratiform microphysics tendencies and associated overlap areas (e.g., Figure 4e vs. Figure 3k; Figure 4h vs. Figure 3h) are weaker than that of evaporation of convective precipitation (i.e., Figure 4b vs. Figure 2k), implying that the changes in stratiform microphysical processes from OLD to NEW are more strongly influenced by the indirect effects of vertical cloud overlap. Vertical cloud overlap also influences radiative heating rate. From OLD to NEW, shortwave (SW) radiative heating rate increases (decreases) in the Southern (Northern) Hemisphere. The lower troposphere becomes more unstable by the changes of LW cooling rate, which seems to be associated with the increase of stratus fraction there (Figure 3b). However, overall changes of radiative heating rates are much weaker than those of cloud microphysics tendencies.

Figure 7 shows the differences of annual-mean total precipitation flux at the surface (PRECT), shortwave cloud radiative forcing (SWCF), and longwave cloud radiative forcing (LWCF) between NEW, OLD, and observations. The observations are from the Global Precipitation Climatology Project from January 1979 to December 2009 for PRECT (GPCP; Alder et al., 2003) and the Clouds and Earth's Radiant Energy Systems (CERES) Energy Balanced and Filled (EBAF) during March 2000 to February 2013 for SWCF and LWCF (CERES EBAF; Loeb et al., 2009). Other than slightly reducing the bias of global mean SWCF, implementation of interactive vertical tilting of cumulus and stratus into the cloud microphysics and radiation schemes
is not particularly helpful in reducing the biases of PRECT, SWCF, and LWCF in the OLD simulation. Given that our stand-alone simulations are driven by the climatological forcing data without any interannual variations, it may be hard to interpret the differences between NEW and OLD as an improvement or degradation, particularly, when compared with the observations in different periods. The spatial patterns of the differences between NEW and OLD are quite noisy, which may be due in part to compensating effects between the enhanced randomness of vertical cumulus overlap and the reduced randomness of vertical stratus overlap from OLD to NEW.

4.2. Sensitivity to $c_m$ and $\gamma$

Figure 8 shows the changes of cloud fraction, precipitation area, and overlap areas in response to the changes of $c_m$ and $\gamma$. The parameter $c_m$ measures the sensitivity of vertical cumulus tilting to the vertical shear of grid mean horizontal wind. If $c_m \to 0$, cumulus approaches to a maximum vertical overlap, such that $a_C^\gamma$ and $a_C^r$. 

Figure 6. Annual zonal mean (a) relative humidity (RH) from the ERAI observation and the differences of RH between (b) NEW and OLD, (c) NEW and OBS, and (d) OLD and OBS. The global mean value is shown at the top left of each plot. The pattern correlation and root-mean-square error (RMSE) between the simulation and the observation are shown at the top center and the top right of (c) and (d), respectively.
Figure 7. Annual mean (left column) PRECT, (middle column) SWCF, and (right column) LWCF from the (a–c) observations (OBS) and (m–o) NEW and the differences between (d–f) OLD and OBS, (g–i) NEW and OLD, and (j–l) NEW and OBS. The global mean value is shown at the top left of each plot. The pattern correlation and root-mean-square error (RMSE) between the simulation and the observation are shown at the top center and the top right of an individual difference plot, respectively. A similar convention will be applied to the following figures.
Figure 8. The differences of annual zonal-mean cloud, precipitation, and overlap areas between the simulation (a–d) with $c_m=0.1$ (i.e., less tilted cumulus overlap) and the NEW that uses $c_m=0.9$ and (e–h) with $\gamma = 1$ (i.e., more random stratus overlap) and the NEW that uses $\gamma = 0.6$. 
Figure 9. The differences of annual-mean (left) PRECT, (center) SWCF, and (right) LWCF between the simulation (a–c) with $c_m=0.1$ and NEW that uses $c_m=0.9$ and (d–f) with $\gamma =1$ and NEW that uses $\gamma =0.6$.

decrease but $a_c^C$ increases. Qualitatively, these responses are similar to the ones shown in Figure 2 but with an opposite sign. As explained in the previous section, enhanced cumulus tilting strengthens convective activity and increases $a_c$. Thus, with a smaller $c_m$ and reduced cumulus tilting, global mean $\Delta a_c$ becomes negative, but it shows a somewhat noisier pattern than the one shown in Figure 2b. This is presumably due to the aforementioned negative feedback of vertical cumulus overlap in our model: reduced cumulus tilting suppresses evaporation of convective precipitation within PBL, cold pool and mesoscale organized flow, convective activity, and convective momentum transport, which in turn enhances cumulus tilting by increasing vertical shear of mean horizontal winds. As $\gamma$ increases, stratus becomes more randomly overlapped vertically, such that both $a_s^S$ and $a_{sr}^S$ increase but $a_s^S$ decreases. Enhanced evaporation of stratiform precipitation and an associated increase of grid mean relative humidity is likely to increase $a_s$, which is parameterized as an increasing function of grid mean relative humidity in our model (Park et al., 2014). Although qualitatively identical in terms of the signs of global mean anomalies, stratus components show much noisier patterns than cumulus components. We speculate that this is due to the negative feedback involving stratus macrophysics and microphysics processes. For example, if precipitating stratus in two adjacent layers with an identical partial cloud fraction (e.g., $a_s = 0.5$) are tilted by enhanced wind shear, $a_s^S$ increases in the lower layer. Stratiform precipitation falling into clear areas will be evaporated, grid mean relative humidity will increase, more stratus will form, and thus some of $a_{sr}^S$ will be ultimately converted to $a_s^S$.

Figure 9 shows the changes of annual mean PRECT, SWCF, and LWCF in response to the changes in $c_m$ and $\gamma$. In contrast to negative $\Delta a_c^C$ (Figure 8d), global mean $\Delta$PRECT is negative (Figure 9a), which is not negligible, even though it is much smaller than the regional changes in magnitude. This result is also different from the previous sensitivity test using noninteractive cumulus tilting (Figure 10 in Park et al., 2019), in which global mean PRECT increases as cumulus is more maximally overlapped vertically. Not only the global mean values but also the spatial patterns of $\Delta$PRECT in Figure 9a differ substantially from that of Figure 10a of Park, Shin, et al. (2019), and the magnitude of regional $\Delta$PRECT with interactive tilting is smaller than that of noninteractive tilting. It is speculated that the aforementioned self-regulating negative feedback of interactive cumulus tilting involving the change of vertical wind shear is responsible for these differences. The spatial patterns of $\Delta$SWCF (Figure 9b) and $\Delta$LWCF (Figure 9c) are congruent with those of $\Delta$PRECT, and the magnitudes of global mean $\Delta$SWCF and $\Delta$LWCF are smaller than those with...
Figure 10. The differences of annual-mean (a) mesoscale convective organization, (b) radius of convective updraft plume at the surface, and (c) cumulus top height between the simulations with \( c_m = 0.1 \) and NEW. The global mean value is shown at the top left of each plot. The pattern correlations between these plots and Figure 9a within 30°S-30°N are shown at the top center of each plot.
Figure 11. The differences in annual mean (a, d) PRECT, (b, e) SWCF, and (c, f) AOD (upper) between WED and NEW and (middle) between WED and OBS. Also shown are the annual mean (g) AOD from OBS, (h) AOD from WED, and (i) the difference of AOD between NEW and OBS.

noninteractive cumulus tilting. Another interesting aspect is the northward shift of the Intertropical Convergence Zone and the southeastward shift of the South Pacific convergence zone. Detailed analysis on the responsible physical mechanisms is a future research subject. However, we speculate that these changes are induced by the changes in the vertical heating profile and subgrid cold pools driven by the evaporative cooling and subsequent nonlocal feedback processes (e.g., changes in large scale flows). Figure 10 shows that ΔPRECT, ΔSWCF, and ΔLWCF are closely associated with the changes in the mesoscale convective organization (defined as the fractional area of subgrid cold pools within UNICON), updraft plume radius at the surface (which is parameterized as a linearly increasing function of mesoscale convective organization), and cumulus top height with significant pattern correlations between ΔPRECT and Figure 10. In contrast to the case of cumulus, it is hard to imagine an apparent negative feedback of interactive stratus tilting involving the changes of vertical wind shear. With a larger $\gamma$, stratus becomes more randomly overlapped, and both global mean SWCF and LWCF are amplified. Although $\gamma$ controls a vertical stratus overlap and convective precipitation dominates over stratiform precipitation in the tropical regions (Park, 2017), $\gamma$ exerts nonnegligible impacts on tropical precipitation over the western equatorial Pacific and Indian Oceans, presumably through the modifications of atmospheric vertical stability (by changing evaporative cooling of stratiform precipitation and radiative heating) and associated convective activity.
4.3. Impact of Interactive Vertical Overlap of Cumulus and Stratus on Aerosol Wet Deposition and Aerosol Activation Processes

Figure 11 shows the differences of PRECT, SWCF, and aerosol optical depth (AOD) between WED and NEW and between WED and observations. The observed AOD is from the multisatellite composite observations (Kinne et al., 2006). With the integrated cloud overlap parameterization implemented into the aerosol wet deposition scheme, AOD increases substantially (approximately 10% of global mean AOD), particularly in the vicinity of tropical continents and the eastern equatorial Pacific and Atlantic Oceans. Although smaller than these, AOD also increases over the eastern United States, China, and eastern Europe, some of which contributes to reducing strong negative AOD biases over the northern continents. The spatial correlation between the simulated and observed AOD is slightly improved from NEW to WED (from 0.908 to 0.917), but the main biases (i.e., too small AOD over the continents and too large AOD over the eastern equatorial Pacific and Southern-Hemisphere (SH) Oceans) are still persisting. Global mean SWCF is strengthened by 1.2 W/m², while global mean PRECT decreases by 0.02 mm/day, which are, in fact, considerable changes.

Figure 12 shows the annual zonal-mean cross sections of the mass and number concentrations of interstitial aerosols (nonactivated aerosols existing outside of cloud liquid droplets and ice crystals) and stratus-borne aerosols (activated aerosols existing inside of cloud liquid droplets and ice crystals). The change in AOD, shown in Figure 11c, is a combined result of the changes in the masses and numbers of interstitial aerosols, because in our model, AOD is computed with interstitial aerosols only. Aerosols with larger sizes (i.e., aerosols in coarse mode) contribute more to aerosol mass, while aerosols with smaller sizes (i.e., aerosols in Aitken or accumulation modes) contribute more to aerosol numbers. Owing to the gravitational settling of larger aerosols, aerosol masses are concentrated in levels lower than those of aerosol numbers. The integrated cloud overlap parameterization implemented into the aerosol wet deposition scheme decreases aerosol masses but increases the number of interstitial aerosols, particularly in the tropics and northern hemispheric subtropical regions. Strong positive ΔAOD, shown in Figure 11c, is mainly due to the increase of aerosol numbers. These opposite variations of aerosol masses and numbers indicate that large aerosols in the coarse mode are scavenged more while small aerosols in the Aitken or accumulation modes are scavenged less with the new cloud overlap parameterization.

Figure 13 shows annual zonal-mean cross sections of various aerosol scavenging and resuspension tendencies described in Appendices A and B. The processes denoted by $P$ and $E$ denote aerosol scavenging and resuspension processes, respectively. Aerosol scavenging and resuspension processes associated with convective precipitation are concentrated in the tropics and midlatitude lower troposphere, while those associated with stratiform precipitation are concentrated in the midlatitudes and tropical upper troposphere. Globally, the strongest process is $P_{\text{dn}}$ (Figures 13f and 13n, the scavenging of stratus-borne aerosols by stratiform precipitation, equation (B1)), and the comparison with Figures 12e and 12g indicates that the global mean life time of stratus-borne aerosols is about 1.6 hr for number (the global mean value of Figure 12g divided by the global mean value of Figure 13n) to 2.3 hr for mass (the global mean value of Figure 12e divided by the global mean value of Figure 13f), which is much shorter than the lifetime of interstitial aerosols. The AOD shown in Figure 11 is solely from the interstitial aerosols without the contribution of stratus-borne aerosols. The AOD shown in Figure 11 is solely from the interstitial aerosols without the contribution of stratus-borne aerosols. For the mass of interstitial aerosols, the most dominant scavenging process is $P_{\text{dn}}$ (Figure 13g, scavenging by stratiform precipitation, equation (B2)), followed by $P_{\text{des}}$ (Figure 13b, scavenging by convective precipitation within a convective downdraft, equation (A2)) and $P_{\text{dn,env}}$ (Figure 13d, scavenging by convective precipitation within the environment, equation (A4)), while the most dominant resuspension process is $E_{\text{up}}$ (Figure 13h, evaporation of stratiform precipitation, equation (B3)). For the number of interstitial aerosols, the most dominant scavenging process is $P_{\text{dn}}$ (Figure 13i, scavenging by convective precipitation within convective updraft, equation (A1)), and the most dominant resuspension process is $E_{\text{up}}$ (Figure 13p). The pronounced positive biases of AOD over the eastern equatorial Pacific and SH Oceans shown in Figure 11f may be alleviated by enhancing $P_{\text{dn}}$ or reducing $E_{\text{up}}$. The efforts to reduce the remaining biases of AOD are a topic for future research, possibly in combination with the incorporation of the accretion process for convective precipitation and double-moment cumulus microphysics scheme.

Figure 14 shows the differences of PRECT, SWCF, and AOD between ACT and WED. It is somewhat surprising that a slight change in the vertical overlap area between the stratus fraction and clear portion ($a_{\tau}^D$) from $a_{\tau}^D = \max(a_{\tau}^C - a_{\tau}, 0)$ to equation (C1) induces substantial changes in PRECT, SWCF, and AOD.
Figure 12. Grid mean concentrations of the (a) mass of interstitial aerosols, \( \xi_{m,\text{int}} \), (c) number of interstitial aerosols, \( \xi_{n,\text{int}} \), (e) mass of stratus-borne aerosols, \( \xi_{m,\text{sb}} \), and (g) number of stratus-borne aerosols, \( \xi_{n,\text{sb}} \), from WED and (b, d, f, h) the differences between WED and NEW. The grid mean value is computed by excluding the cumulus portion, and the interstitial aerosols are assumed to be horizontally homogeneous within each grid layer, excluding the cumulus portion.
Figure 13. Annual zonal mean cross sections of various aerosol wet deposition and resuspension tendencies of (first and second columns) aerosol mass concentration and (third and fourth columns) aerosol number concentration explained in Appendices A and B: (a, i) scavenging rate of cumulus-borne and interstitial aerosols within convective updrafts by convective precipitation \((P_{C,up}^\xi)\), (b, j) scavenging rate of interstitial aerosols within convective downdrafts by convective precipitation \((P_{C,dn}^\xi)\), (c, k) resuspension rate of aerosols within convective downdrafts by the evaporation of convective precipitation \((E_{C,dn}^\xi)\), (d, l) scavenging rate of interstitial aerosols in the environment by convective precipitation \((P_{C,env}^\xi)\), (e, m) resuspension rate of aerosols by the evaporation of convective precipitation in the environment \((E_{C,env}^\xi)\), (f, n) scavenging rate of stratus-borne aerosols by the production of stratiform precipitation \((P_{S,up}^\xi)\), (g, o) scavenging rate of interstitial aerosols by stratiform precipitation \((P_{S,iso}^\xi)\), and (h, p) resuspension rate of aerosols by the evaporation of stratiform precipitation \((E_{S}^\xi)\).
Figure 14. The differences of annual-mean (left) PRECT, (middle) SWCF, and (right) AOD (a–c) between ACT and WED and (d–f) between ACT and OBS.

globally. In Figure 14b, the sign of $\Delta$SWCF in the marine stratocumulus decks over the eastern subtropical and midlatitude oceans is opposite to that in Figure 11e, and the RMSE of global SWCF is reduced from 10.15 to 9.79 W/m$^2$. This indicates that the simulation of low-level clouds and associated cloud radiative forcing in our model are sensitive to how aerosol activation is treated. Not only SWCF, through indirect feedback, PRECT decreases and AOD increases substantially. $\Delta$AOD shows some spatial coherences with $\Delta$SWCF but not a very strong coherence with $\Delta$PRECT. It is speculated that due to the decrease of aerosol activation rate, the optical thickness of low-level clouds decreases, while the relative concentration of interstitial aerosols increases (note that AOD is defined using interstitial aerosols only). It is interesting to note that in contrast to Figures 11b and 11c, the signs of $\Delta$AOD and $\Delta$SWCF shown in Figures 14b and 14c are the same.

As of this, we finished the implementation of the interactive cloud overlap parameterization of cumulus and stratus developed by P2017, P2018, P2019a, and our current study into all relevant physics parameterizations in SAM0-UNICON: cloud microphysics for convective and stratiform precipitation, SW and LW radiation, aerosol wet deposition by convective and stratiform precipitation, and aerosol activation at the base of stratocumulus. It is not unusual that the implementation of a new scheme degrades the overall performance of the model, owing to the conflict with the preexisting tuning parameters which are optimized with an old scheme. Figure 15 shows a Taylor diagram (Taylor, 2001) summarizing the performance of the models in reproducing the observed global mean climate. The observation data used for this diagram are the Interim ECMWF Reanalysis product (ERAI) (Simmons et al., 2007), CERES-EBAF, GPCP, the Willmott-Matsuura surface air temperature (Willmott) (Willmott & Matsuura, 1995), and the European Remote Sensing Satellite Scatterometer (ERS) (Bentamy et al., 1999). In this diagram, we compared the performance of SAM0-UNICON with the interactive cloud overlap parameterization implemented into all relevant physics schemes (ACT simulation) to that of the original SAM0-UNICON with the original cloud overlap parameterization (OLD simulation in P2019a, which is denoted by SAM0 in Figure 15). The overall performance of ACT to simulate the observed global mean climate is similar to the original SAM0 (the RMSEs of the ACT versus the observations relative to those of SAM0 are 0.98), which is a very encouraging result given that we have not yet performed any intensive tuning on the ACT. Compared to SAM0, the ACT degrades the simulations of SWCF but slightly improves the simulations of relative humidity. The global mean biases of net downwelling radiation at the top of the atmosphere, PRECT, SWCF, and AOD are 1.6 W/m$^2$, 0.325 mm/day, 0.9 W/m$^2$, and 0.023, respectively, for ACT; and 0.4 W/m$^2$, 0.331 mm/day, $-2.9$ W/m$^2$, and 0.001, respectively, for the original SAM0. Although SWCF decreases, the column-integrated cloud liquid water path over the
Figure 15. A space-time Taylor diagram from the ACT and original SAM0 simulations. The Taylor diagram shows the global performance of ACT (green) relative to the original SAM0 (black) versus the observations measured by the correlation and standardized deviation of 10 semi-independent climate variables indicated on the lower left portion of the figure. For each variable, using the monthly climatology for all available grid points, we compute the correlation with the observations and standardized deviation as the ratio of the simulated spatiotemporal standard deviation to the observed standard deviation, including the annual cycle. Any variable with a correlation of 1 and a standardized deviation of 1 indicates a perfect simulation of that variable. The RMSE=0.998 of ACT is the average of the relative RMSE of a simulated individual variable versus the observation with respect to the RMSE of SAM0, i.e., $RMSE(\text{ACT}) = \frac{1}{10} \sum_{i=0}^{9} \left( \frac{\text{RMSE}_i(\text{ACT})}{\text{RMSE}_i(\text{SAM0})} \right)$, where $i$ is a variable index. The Bias=1.001 of ACT is the average of the relative annual mean of an individual variable with respect to SAM0, i.e., $\text{Bias(ACT)} = \frac{1}{10} \sum_{i=0}^{9} \left( \frac{\text{mean}_i(\text{ACT})}{\text{mean}_i(\text{SAM0})} \right)$, where $\text{mean}_i$ is a global annual mean of the $i$th variable.

ocean increases substantially from 31.4 to 41.0 g/m² from SAM0 to ACT (not shown), which is an improvement when compared with the observed cloud liquid water path of 87.1 (O’Dell et al., 2008). We expect that additional tuning (e.g., reducing the critical relative humidity for the liquid stratus fraction, $RH_{c, l}$ from the current value of 0.9) may further reduce the biases in SWCF and improves the overall model performance, which, however, is left for future work, possibly with the implementation of aerosol activation, ice nucleation, and double moment cloud microphysics schemes in UNICON. Finally, we note that the computation cost of ACT is similar to that of SAM0 since the interactive overlap parameterization is formulated in an analytical way.

5. Summary and Conclusions

The integrated vertical overlap parameterization of cumulus and stratus developed by Park et al. (2017) and Park (2018) assumed that vertically adjacent cumulus (stratus) have an overlap that varies exponentially between maximum and random according to a decorrelation length scale, $\Delta z_c (\Delta z_s)$, whereas vertically separated cumulus (stratus) are randomly overlapped. However, the unified convection scheme (UNICON) in SAM0 explicitly computes vertical overlap of cumulus in a nonconventional way by calculating vertical changes in the radius and center coordinate of the convective updraft plumes as a function of the updraft vertical velocity and vertical shear of environmental horizontal winds. This is a more advanced approach than the conventional exponential-random vertical cloud overlap. In this paper, we generalized the integrated
overlap parameterization of P2017 and P2018, such that the parameterization can simultaneously handle
both the nonconventional cumulus overlap computed by UNICON and conventional exponential-random
stratus overlap. In addition, the decorrelation length scale of stratus, $\Delta z_s$, is parameterized as a function
of the vertical shear of environmental horizontal winds. The resulting overlap parameterization can sim-
ulate interactive feedback between vertical cloud overlap and other physical processes. We implemented
the interactive cloud overlap parameterization into the cloud microphysics and radiation schemes of SAM0
and compared the simulated climate (which is referred to as NEW) with that based on the conventional
exponential-random overlap with fixed decorrelation length scales for cumulus ($\Delta z_c = \infty$, maximum
cumulus overlap) and stratus ($\Delta z_s = 2$ km), respectively (which is referred to as OLD).

With the enhanced vertical tilting of cumulus, the NEW simulates smaller overlap area between cumulus in
the adjacent layers ($O(a_{c1}, a_c)$) but larger overlap areas between cumulus and stratus in the adjacent layers. In
addition, the NEW simulates larger convective precipitation area ($a^c$) and overlap area between convective
precipitation and clear areas ($a^c_s$) but smaller overlap area between convective precipitation and cumulus
fraction ($a^c$) than the OLD. Enhanced evaporation of convective precipitation in association with larger $a^c$;
remedies some of the dry biases in the tropical troposphere in the OLD. The NEW also simulates a larger
cumulus fraction $a_c$ (i.e., stronger convective activity) than the OLD, presumably due to the strengthening of
subgrid cold pool and mesoscale organized flow forced by enhanced evaporation of convective precipitation,
which fosters stronger and wider convective updraft plume in the UNICON. The relative changes in the stra-
tus components ($a^c_s, a^c, a^c_s$) are smaller than those of the cumulus components, indicating that the
specified decorrelation length for stratus in the OLD represents the global mean value of the parameterized
decorrelation length of stratus reasonably well. Implementation of interactive vertical tilting of cumulus and
stratus into the cloud microphysics and radiation schemes does not reduce the biases of PRECT, SWCF, and
LWCF in the OLD simulation. We also examined the model sensitivity to the convective momentum adjust-
ment factor ($c_m$) and the slope parameter of the decorrelation length scale for stratus ($\gamma$), each of which
determines the degree of vertical tilting of cumulus and stratus with a larger $c_m$ ($\gamma$) denoting enhanced ran-
doneliness of vertical cumulus (stratus) overlap. With a smaller $c_m$, both $a^c_s$ and $a^c$ decrease, but $a^c_s$ increases,
which is qualitatively consistent with the changes from OLD to NEW. However, $a_s$ shows somewhat noisier
variations in space than the changes from OLD to NEW, presumably owing to the self-regulating negative
feedback of vertical cumulus overlap involving sequential changes of the evaporation of convective precip-
itation, subgrid cold pool, mesoscale organized flow, convective activity, convective momentum transport,
and vertical shear of mean horizontal wind. This self-regulating negative feedback also seems to explain the
differences in responses of PRECT, SWCF, and LWCF to enhanced vertical tilting of cumulus between the
simulations with interactive cloud overlap parameterization and a fixed decorrelation length scale. With a
larger $\gamma$, both $a^c_s$ and $a^c$ increase, but $a^c_s$ decreases; these are much noisier in space than those of cumulus
components, presumably due to the negative feedback in association with the evaporative enhancement of
grid mean relative humidity and the parameterization of stratus fraction as a function of grid mean relative
humidity. Although $\gamma$ controls the vertical stratus overlap, it also exerts nonnegligible impacts on the
tropical precipitation that is dominated by convective precipitation.

We also implemented the interactive cloud overlap parameterization into the aerosol wet deposition (WED)
and aerosol activation schemes (ACT), consecutively. From NEW to WED, global mean AOD increases sub-
stantially, and SWCF is also strengthened, which is largely due to the increases in the number of interstitial
aerosols with small sizes. The most dominant scavenging and resuspension processes for the numbers of
interstitial aerosols are the scavenging by convective precipitation within the convective updraft and evap-
oration of stratiform precipitation, respectively. The pronounced positive biases of AOD in the vicinity of
tropical continents and the eastern equatorial Pacific and Atlantic Oceans may be alleviated by adjusting
these two processes. From WED to ACT, SWCF is substantially reduced, particularly over the subtropical
and midlatitude marine stratocumulus deck, likely owing to the reduced aerosol activation rate. The ACT
is the final simulation with the interactive cloud overlap parameterization implemented into all relevant
physics parameterizations of SAM0 in a fully consistent way. Even without intensive tuning, the overall per-
formance of the ACT to simulate the observed global mean climate is similar to the original SAM0, which is
a very encouraging result. The column-integrated cloud liquid water path increases substantially from the
original SAM0 to ACT, which is also an improvement.

The main goal of our research is to develop a GCM with a strong interprocess consistency among various
physics and dynamics processes. With an integrated cloud overlap parameterization implemented into all

relevant physics parameterizations, this goal was partially achieved. To further improve the model, the following work will be necessary. First, the convection scheme should use a double moment cloud microphysics scheme with appropriate parameterizations for aerosol activation and ice nucleation, both at the base and lateral interface of convective updraft plumes. Current UNICON uses a single moment cumulus microphysics scheme without aerosol activation and ice nucleation, such that several physical quantities (e.g., the sizes of in-cumulus and detrained convective condensate, the fraction of cloud-borne aerosols among total aerosols within convective updrafts and downdrafts (see equations (A1) and (A2) in Appendix A), and the production and evaporation rates of convective precipitation) are specified or computed in a crude way. Second, the radiation scheme needs to be generalized, such that it can handle subgrid variations of water vapor, size, and amount of cloud condensates within cumulus and stratus. In addition to stratiform snow, the radiation scheme should handle the radiative properties of stratiform rain and convective precipitation. Third, more observational works are necessary to parameterize the decorrelation length scales of individual cumulus and stratus, rather than a single-merged cloud, if possible. The authors plan to implement this work, and related results will be reported in future manuscripts.

Appendices. Implementation of Interactive Vertical Overlap Parameterization of Cumulus and Stratus in Aerosol Wet Deposition and Aerosol Activation Schemes

Appendix A: Aerosol Wet Deposition by Convective Precipitation

Aerosol scavenging by convective precipitation consists of the following processes: 1) scavenging of cumulus-borne and interstitial aerosols within convective updraft; 2) scavenging of interstitial aerosols within convective downdraft; 3) resuspension of aerosols by the evaporation of convective precipitation within convective downdraft; 4) scavenging of interstitial aerosols within stratus (a,) and clear portions (a_r); and 5) resuspension of aerosols by the evaporation of convective precipitation within the clear portions (a_S).

The grid mean removal rates of cumulus-borne aerosols by the production of convective precipitation within cumulus and the interstitial aerosols within cumulus by the convective precipitation falling into cumulus, \( P_{C_{up}} \) (the superscript \( C \) denotes convective precipitation and the subscript \( up \) denotes convective updraft) are computed by

\[
P_{C_{up}} = \left( \frac{\dot{M}}{\rho \dot{w}} \right) \left[ \frac{\dot{p}_{C_{up}}}{\dot{q}_l} \left( \frac{\xi_c \dot{x}}{\tilde{q}_l + \tilde{q}_i} \right) + \left\{ \left( \frac{a_C}{a_c} \right) \alpha \beta  \bar{F}_C (1 - \xi_c) \right\} \right],
\]

where \( \dot{M} \) is the updraft mass flux; \( \dot{w} \) is the updraft vertical velocity; \( \dot{p}_{C_{up}} \) is the production rate of convective precipitation within convective updraft; \( \xi_c \) is total (a sum of cloud-borne and interstitial) aerosol concentration in the convective updraft; \( \xi_c \) is the fraction of cumulus-borne aerosols among total aerosols in the convective updraft; and \( \bar{F}_C \) is the in-precipitation (i.e., averaged over \( a_C \)) convective precipitation flux. Because the convective updraft rises from the surface without an iterative computation, the second term is 0, both in the new and old wet deposition codes. However, the first term is equally used both in the new and old wet deposition codes.

The grid mean removal rate of aerosols by the scavenging of interstitial aerosols within convective downdraft by convective precipitation, \( P_{C_{dn}} \) (the subscript \( dn \) denotes convective downdraft) is computed by

\[
P_{C_{dn}} = \left( \frac{\alpha \beta \bar{F}_C (1 - \xi_c) \dot{x}}{\rho \dot{w}} \right),
\]

where \( \alpha \) is a solubility factor; \( \beta \) is a scavenging coefficient with units of \( m^2/kg \); \( \bar{F}_C \) is the in-precipitation convective precipitation flux; \( \dot{x} \) is total aerosol concentration in the convective downdraft; and \( \xi_c \) is the fraction of cloud-borne aerosols among total aerosols within the convective downdraft. The old wet deposition code did not have this term.

The grid mean resuspension rate of aerosols by the evaporation of convective precipitation within convective downdraft, \( E_{C_{dn}} \), is computed by

\[
E_{C_{dn}} = \bar{E}_C \left( \frac{\dot{p}_{C_{dn}}}{\bar{F}_C} \right) \left( \frac{\dot{M}}{\rho \dot{w}} \right),
\]

where \( \bar{E}_C \) is the evaporation coefficient.
where $E^C$ is the evaporation rate of convective precipitation within the convective downdraft; $F^C$ is the grid mean convective precipitation flux; and $\tilde{F}^C_{\text{a}}$ is the grid mean aerosol flux in association with convective precipitation. Both the new and old wet deposition codes use the same formula.

The grid mean removal rate of interstitial aerosols within the environment by convective precipitation flux, $PC_{\text{env}}$ (the subscript env denotes the environmental area, which is defined as the sum of $a_s$ and $a_r$) is computed by

$$PC_{\text{env}} = (a_s^C + a_r^C) \alpha b F^C \tilde{\xi}_{\text{inst}}.$$  \hspace{1cm} (A4)

where $\tilde{\xi}_{\text{inst}}$ indicates the grid mean interstitial aerosols, excluding the cumulus portions. In the new code, the above equation is solved in an analytical way, such that the solution becomes $PC_{\text{env}} = (a_s^C + a_r^C)(\tilde{\xi}_{\text{inst}}/\Delta t)[1 - \exp(-a \beta F^C \Delta t)]$. In the old code, however, the above equation was solved in a numerical way with a certain limiter by replacing $a_s^C + a_r^C$ with $a^C$, such that the solution was $PC_{\text{env}} = a^C (\tilde{\xi}_{\text{inst}}/\Delta t) \min(b \beta F^C \Delta t, 1)$. Here, $a^C$ is a convective precipitation area computed by the vertical integral of $a_s$ in the layers above, weighted by the net production rate of convective precipitation. The old wet deposition code used $a^C$ instead of $a_s^C + a_r^C$ because it did not compute the vertical cloud overlap in an explicit way.

The grid mean resuspension rate of aerosols by the evaporation of convective precipitation in the clear portion, $E^C_{\text{env}}$ is computed by

$$E^C_{\text{env}} = a_s^C \tilde{E}^C \left(\frac{F^C}{\bar{F}^C}\right) \left(\frac{\bar{\Delta p}}{\Delta p_{\text{dry}}}\right).$$  \hspace{1cm} (A5)

where $\tilde{E}^C$ is the in-area (i.e., averaged over $a_s^C$) evaporation rate of convective precipitation; $F^C$ is the grid mean convective precipitation flux; and $\bar{F}^C$ is the grid mean convective aerosol flux that is obtained by the vertical integrals of $PC_{\text{up}}$, $PC_{\text{dn}}$, $EC_{\text{up}}$, $EC_{\text{dn}}$, $PC_{\text{env}}$, and $E^C_{\text{env}}$ from the top to the layer, just above the current layer. The old wet deposition code uses the same equation.

**Appendix B: Aerosol Wet Deposition by Stratiform Precipitation**

The stratiform precipitation area, $a^S$, can be overlapped with cumulus, $a_s$, stratus, $a_r$, and clear portion, $a^C$. In nature, stratiform precipitation falling into cumulus can scavenge both cumulus-borne and interstitial aerosols within $a^S$ (see Figure 2). In our implementation in the stratus microphysics scheme, however, these two processes are set to 0 because they should be treated in the convection scheme to conserve the sum of the column-integrated aerosol contents in the noncumulus portions and the surface aerosol flux falling with stratiform precipitation. Within the convection scheme, we neglect both the scavenging of cumulus-borne aerosols by stratiform precipitation to be consistent with IPA and also the scavenging of interstitial aerosols by stratiform precipitation within convective updrafts and downdrafts, owing to practical difficulty in treating stratiform precipitation within the convection scheme.

Aerosol scavenging by stratiform precipitation consists of the following three processes: 1) scavenging of stratus-borne aerosols by autoconversion in $a_s$ and accretion in $a^S_s$; 2) scavenging of interstitial aerosols in $a^S_s + a^S_r$ (note that scavenging in $a^S_s$ is neglected, as mentioned above); and 3) resuspension of stratiform aerosols by the evaporation of stratiform precipitation in $a^S_s$.

The grid mean removal rate of stratus-borne aerosols by the production of stratiform precipitation within stratus, $PS_{\text{str}}$ (the superscript $S$ denotes stratiform precipitation, and the subscript $\text{str}$ denote stratus-borne aerosols) is computed by

$$PS_{\text{str}} = \tilde{\xi}_{\text{str}} \begin{bmatrix} a_s PS_{\text{str}} + a^S_{\text{str}} \cdot PS_{\text{str}} \end{bmatrix} \left/ \tilde{q}_i + \tilde{q}_l \right.,$$  \hspace{1cm} (B1)

where $\tilde{\xi}_{\text{str}}$ is the in-stratus (i.e., averaged over $a_s$) concentration of stratus-borne aerosols; $\tilde{q}_i + \tilde{q}_l$ is the grid mean stratus condensate amount; $PS_{\text{str}}$ is the in-stratus production rate of stratiform precipitation by auto-conversion; and $PS_{\text{str}}$ is the in-area (i.e., averaged over $a^S$) production rate of stratiform precipitation by accretion. The old wet deposition code uses the same equation.

The grid mean removal rate of interstitial aerosols by stratiform precipitation, $PS_{\text{inter}}$ (the subscript is denotes the interstitial aerosols) is computed by

$$PS_{\text{inter}} = (a_s^C + a_r^C) \alpha b F^S \tilde{\xi}_{\text{inst}}.$$  \hspace{1cm} (B2)
where $\alpha$ is a solubility factor; $\beta$ is a scavenging coefficient with units of $m^2/kg$; $F^S$ is the in-precipitation (i.e., averaged over $a^S$) stratiform precipitation flux; and $\bar{z}_s$ is the grid mean concentration of interstitial aerosols, excluding the cumulus portion, which is assumed to be horizontally uniform within the grid layer. In the new code, the above equation is solved in an analytical way, such that the solution becomes $P^S_{z,\text{sh}} = \left(a^s_1 + a^s_2\right)\left(\bar{z}_s/\Delta t\right)\left[1 - \exp\left(-\alpha F^S/\Delta t\right)\right]$. In the old code, however, the above equation was solved in a numerical way with a certain limiter by replacing $a^s_1 + a^s_2$ with $a^s$, such that the solution was $P^S_{z,\text{sh}} = a^s a\left(z_s/\Delta t\right)\min\left(\beta F^S/\Delta t, 1\right)$. Here, $a^S$ is the stratiform precipitation area computed by the vertical integral of $a_s$ in the layers above, weighted by the net production rate of stratiform precipitation. The old wet deposition code used $a^S$ instead of $a^s_1 + a^s_2$ because it did not compute the vertical cloud overlap in an explicit way.

The grid mean resuspension rate of aerosols by the evaporation of stratiform precipitation in the clear portion, $E^S_{\text{clear}}$, is computed by

$$E^S_{\text{clear}} = a^S_{\text{clear}} E^S \left(\frac{P^S_{\text{clear}}}{F^S}\right) \left(\frac{\Delta p}{\Delta p_{\text{dry}}}\right),$$

where $\Delta p$ and $\Delta p_{\text{dry}}$ are the pressure thicknesses of the air and the dry air excluding water vapor, respectively; $E^S_{\text{clear}}$ is the in-area (i.e., averaged over $a^S_{\text{clear}}$) evaporation rate of stratiform precipitation; $F^S$ is the grid mean stratiform precipitation flux; and $P^S_{\text{clear}}$ is the grid mean stratiform aerosol flux that is obtained by the vertical integrals of $P^S_{z,\text{sh},1} - P^S_{z,\text{sh},1}$ and $E^S$ from the top to the layer, just above the current layer. The old wet deposition code uses the same equation, but in contrast to the new wet deposition code, parts of the resuspended aerosols are incorporated into the budget of stratus-borne aerosols instead of interstitial aerosols.

**Appendix C: Aerosol Activation at the Base of Stratus**

A UNICON in the current SAM0 uses a bulk single-moment cumulus microphysics scheme, such that it needs neither aerosol activation nor ice nucleation schemes. In nature, aerosol activation and ice nucleation within cumulus can occur when an unsaturated convective updraft reaches its lifting condensation level (LCL); when additional cloud condensates are generated within saturated convective updrafts by adiabatic expansion during upward motion; and when unsaturated environmental air is entrained into a cumulus updraft and saturated by the evaporative cooling of in-cumulus condensates. Ideally, the treatment of these aerosol activation and ice nucleation processes within cumulus should be accompanied by the implementation of a double moment cumulus microphysics scheme, which will be reported on in the future.

In SAM0, ice nucleation within stratus in each layer is a function of grid mean temperature, relative humidity, and subgrid vertical velocity derived from the eddy diffusivity provided by the moist PBL scheme assuming isotropic turbulences. In the current ice nucleation scheme for stratus, only dust and $SO_4$ can serve as an ice nuclei (IN); individual nucleated ice crystals can contain only one IN; and vertical cloud overlap does not have a direct impact on the ice nucleation process; thus, we have not made any modifications in the ice nucleation scheme.

Aerosol activation within stratus consists of two processes: one is the in-stratus activation associated with the shrinking and growing of stratus fraction with time, and the other is the aerosol activation at the base of stratus, which occurs when the aerosols in the unsaturated environmental air are transported into the overlying saturated stratus by local symmetric subgrid turbulent eddies and become saturated. The latter process is an explicit function of vertical stratus overlap, i.e., it should occur in the overlap area ($a^{T\text{sh}}$) between the clear portion ($a_c$) and the stratus fraction, just above the clear portion ($a^{T\text{sh}}$). The aerosol activation routine in the current SAM0 simply assumes a maximum stratus overlap and computes $a^{T\text{sh}} = \max(a^{T\text{sh}} - a_c, 0)$ in each layer without considering the contribution of the cumulus fraction. With the integrated vertical overlap parameterization of cumulus and stratus, however, we compute $a^{T\text{sh}}$ in each layer by combining equations (1)–(A3) of P2018 as follows:

$$a^{T\text{sh}} = O(a_{st}, a_r) = \lambda_{a} \cdot O(a_{st}, a_r)_{\text{max}, \lambda} + (1 - \lambda_{a}) \cdot O(a_{st}, a_r)_{\text{ran}, \lambda},$$

where $\lambda_{a}$ is obtained from equation (6) and the overlap areas $O(a_{st}, a_r)$ for individual vertical overlap configurations of stratus (i.e., $O(a_{st}, a_r)_{\text{max}, \lambda}$ for maximum stratus overlap and $O(a_{st}, a_r)_{\text{ran}, \lambda}$ for random stratus overlap) are obtained from equations (2) and (3).
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