Jets and Quantum Field Theory

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Abstract

We discuss quantum–field–theoretic interpretation of the family of observables (the so–called C–algebra) introduced in \cite{12} for a systematic description of multijet structure of multiparticle final states at high energies. We argue that from the point of view of general quantum field theory, all information about the multijet structure is contained in the values of a family of multiparticle quantum correlators that can be expressed in terms of the energy–momentum tensor.

Introduction

The concept of hadron jets is a cornerstone of the modern high energy physics (see e.g. the reviews \cite{1}, \cite{2}): It would simply be impossible to discuss the experimental procedures employed e.g. in the recent discovery of the top quark \cite{3}, \cite{4} without using the language of hadron jets. Yet apart from the early discussion of the issue of perturbative IR safety in connection with perturbative calculability \cite{3}, \cite{4}, remarkably little (if anything at all) has been done to integrate the jet paradigm into the framework of Quantum Field Theory. This is despite the fact that perturbative QFT is the only systematic calculational framework for obtaining theoretical predictions about jets. The conventional theory of jets was developed by trial and error within experimental and phenomenological communities and is based on the notion of jet definition algorithm which is foreign to QFT. On the other
hand, theoretical studies of jet algorithms and the related issues of Sudakov factorization etc. (cf. e.g. [7], [8] and refs. therein) simply accepted the conventional jet paradigm without attempting to bring it into conformity with QFT. This would be understandable should QFT be a phenomenological scheme of limited applicability. But QFT is a fundamental framework to describe mechanics of the most elementary bits of matter known to date that is capable of yielding predictions that agree with experiments to unprecedented precision [9], and there are no reasons whatsoever to doubt its basic nature.

The eclectic nature of the theory of jets did not fail to result in practical difficulties. These are manifest in what is known as ambiguities of jet definition (see e.g. (1)) that limit the precision of experimental results obtained using the conventional data processing techniques. For instance, it is expected that the dominant error of determination of the top mass at the LHC will be limited by the systematic error due to ambiguities of jet algorithms [10]. The difficulties of a different nature arise in theoretical studies: jet cross sections are impossible to compute analytically even in the simplest cases, while studies of such issues as power corrections are unduly cumbersome (as compared with the non–jet case).

It has recently been argued ([11], [12]; see also [13]) that a systematic description of jet–related features of hadronic final state in high energy physics can be achieved in a QFT–compatible manner within the so–called formalism of C–algebra (C is from ‘calorimetric’, here and below). The C–algebra consists of a basic class of observables — the so–called C–correlators that have a rather simple form of multiparticle correlators with their dependence on particles’ energy rigidly fixed (see below) — plus a few rules to construct new observables from those already available. The resulting observables possess optimal stability properties with respect to data errors and statistical fluctuations, and can be computed from events bypassing jet finding algorithms, thus avoiding the notorious problem of ambiguities of jet definition. On the other hand, the examples given in [12] demonstrate that the expressive power of the C–algebra is sufficient to express practically any jet–related property studied in high energy physics (such as ‘n–jet fractions’ and mass spectra of ’multijet substates’).

As was pointed out in [12], the central role of the observables of corre-

\footnote{There is, of course, gravity but the fact has no bearing on the high energy physics yet.}
lator form in the C–algebra opens a prospect of a systematic QFT study of the theory of jets. However, the issue of compatibility of the C–algebra with QFT was only touched upon in \[12\]. There are two points that remain to be clarified. First, we will show that all information about multijet structure is contained in true quantum correlators (the arguments of \[12\] are incomplete in this respect). Second, we will express such correlators in terms of the energy momentum tensor so that the resulting definition acquires a fundamental non–perturbative tensor aspect.

**Setup**

An overview of the arguments that went into the construction of the C–algebra is given in \[13\]. Here we only summarize the formulas needed below.

Let \(i\) number the particles produced in an event. The event is seen by calorimetric detectors as a finite sequence of pairs \(P = \{E_i, \hat{p}_i\}\), where \(E_i\) and \(\hat{p}_i\) are the \(i\)-th particle’s energy and direction (which can be represented e.g. by a point of the unit sphere). Strictly speaking one should bear in mind the following two points. First, such an ”event” is an element of the factor space of the space of final states with respect to an equivalence relation (namely, collinear particles are calorimetrically indistinguishable). Second, \(E_i\) should be interpreted as \(|\vec{p}_i|\); at high energies one neglects particles’ masses and the two quantities are equivalent from the point of view of their use in the description of multijet structure.

Following \[12\], we describe the C–algebra using the language of particles. Regarding hadrons in final states as free massive particles we have to interpret the formulas of \[12\] in the context of the corresponding Fock space. The final formula has a substantially wider meaning: no assumptions about the structure of the space of final states will be necessary.

An observable is defined via a function on the events, \(f(P)\), and the value of the observable measured for a given initial state is obtained by averaging \(f(P)\) over the entire ensemble of events generated from that initial state. One can always formally define (without adding new information) the operator \(O_f\) such that

\[
O_f |P\rangle = f(P) |P\rangle
\]  

Then the measured value of the observable is

\[
\langle O_f \rangle \equiv \langle in | O_f | in \rangle,
\]  

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where \(|in\rangle\) is the corresponding initial state.

The C–algebra of \([12]\) consists of the so–called C–correlators that form its basis and a few rules to construct new observables. A C–correlator has the form

\[
F_N(P) = \sum_{i_1} \ldots \sum_{i_N} E_{i_1} \ldots E_{i_N} f_N(\hat{p}_{i_1} \ldots \hat{p}_{i_N}) ,
\]  

(2.3)

where \(N\) is any positive integer and \(f_N(\hat{p}_1 \ldots \hat{p}_N)\) can be any symmetric function of its \(N\) angular arguments; without loss of physical generality we take it to be \(C^\infty\). Notice that the energy dependence of such a correlator is fixed.

Among the rules to construct new elements of C–algebra are algebraic combinations as well as integration and certain forms of minimization with respect to a parameter (neither of which is of interest in the present context). A construction of a different type involves differential distributions which effectively reduces to allowing new observables of the form

\[
\delta(s - F(P)) ,
\]  

(2.4)

where \(s\) is a real parameter, \(F\) is an observable from the C–algebra, and \(\delta\) is the Dirac \(\delta\)–function. (This construction becomes phenomenologically useful only in combination with other rules but this is of no importance here).

**Expressing spectral observables via C–correlators 3**

The first point that has to be clarified is as follows. The spectral observable 2.4 is, of course, to be interpreted as a measure, i.e. it describes a family of numeric–valued observables, each corresponding to a continuous function \(\chi(s)\):

\[
\int ds \chi(s) \delta(s - F(P)) = \chi(F(P))
\]  

(3.1)

Although such an observable is indeed expressed in terms of a C–correlator, \(F(P)\), but only at the level of a single event. This, however, is not enough for an entirely meaningful QFT interpretation. What one would like to be able to say is that the observable after the averaging over all events could be interpreted in terms of C–correlators also taken after such averaging. The additional argument required for this is as follows.

According to the well–known Weierstrass approximation theorem (see e.g. [14, Theorem 802]), any continuous function \(\chi(s)\) can be approximated in the
uniform sense by polynomials of $s$ within a bounded interval. (Note that a $C$–correlator is always bounded by a constant.) So the observables of the form 3.1 can be approximated to arbitrary precision by finite linear combinations of $F^n(P)$, $n \geq 0$. If $F$ is an $N$–correlator then $F^n$ is an $n \times N$–correlator. Due to uniformity of the approximation, one can always change the order of taking the linear combination and averaging over all events for a given initial state. The conclusion is that any differential observable from the $C$–algebra as described in [12] can be regarded as appropriately approximated by $C$–correlators after the averaging over all events. So it becomes entirely meaningful to say that all physical information about the multijet structure of events corresponding to a given initial state is given by the average values of all $C$–correlators over the corresponding events. Therefore, in what follows we concentrate on operator QFT interpretation of the $C$–correlators 2.3.

**Operator form of $C$–correlators**

One can formally rewrite 2.3 similarly to 2.2:

$$
\langle \sum_{i_1} \cdots \sum_{i_N} E_{i_1} \cdots E_{i_N} f_N(\hat{p}_{i_1} \cdots \hat{p}_{i_N}) \rangle_P = 
\int dn_1 \cdots \int dn_N \langle in | \varepsilon(n_1) \cdots \varepsilon(n_N) | in \rangle \times f_N(n_1, \ldots, n_N),
$$

where all $n$ are unit 3–vectors (points of unit sphere), $\varepsilon(n)$ is an operator–valued distribution on the unit sphere. It is a QFT interpretation of $\varepsilon(n)$ that we wish to obtain.

In the context of the asymptotic Fock space, one has

$$
\varepsilon(n) = \int \frac{dp}{2p^0} a^+(p) \ a^-(p) \times (\hat{p} \cdot n) \ \delta(\hat{p}, n),
$$

where the last factor is the $\delta$–function on the unit sphere localized at the point $\hat{p} = n$. One notices that in presence of such $\delta$–function, $(\hat{p} \cdot n) = |\hat{p}| \approx E$ (recall that any definition of jets and jet–related observables is valid only in the limit of high energies where all particles are regarded as massless). The operators $a^\pm$ are to be interpreted as free–field operators corresponding to the asymptotic states at $t = +\infty$. 


\( \varepsilon(n) \) in terms of fields. A heuristic derivation.

Without loss of generality we take all particles to be identical Lorentz-scalar bosons. We also limit ourselves in this first heuristic derivation to particles with non-zero mass \( m \). Consider the quantum field corresponding to \( a^\pm \):

\[
\varphi(x, t) = \int \frac{d^3k}{2k^0} \left( e^{ikx} a^+(k) + H.C. \right). \tag{5.1}
\]

(We dropped irrelevant normalization factors.) Take \( x = \frac{p^0}{p} t, \ t \to +\infty \), and formally use the stationary phase approximation. Then:

\[
\varphi \left( \frac{p}{p_0} t, t \right) \approx \frac{p^{3/2}}{t^{3/2}} \frac{1}{2m} \times \left( a^+(p)e^{i\frac{m^2}{2p_0}t+i\frac{3\pi}{4}} + H.C. \right) \tag{5.2}
\]

The combination \( a^+(p) a^-(p) \) can be extracted as follows:

\[
\partial_0 \varphi \partial_i \varphi \left( \frac{p}{p_0} t, t \right) = t^{-3} \left\{ \frac{p_i p_0^4}{2m^2} a^+(p) a^-(p) + O \left( e^{\pm 2i\frac{m^2}{2p_0}t} \right) \right\} + o(t^{-3}) \tag{5.3}
\]

Noticing that the combination of field derivatives on the l.h.s. is exactly \( T_{0i} \), the energy–momentum tensor of the free field \( \varphi \), and formally neglecting the oscillating terms, one obtains:

\[
\varepsilon(n) = m^2 t^3 \int \frac{dp}{p_0} \delta(p, n) T_{0i} \left( \frac{p}{p_0} t, t \right). \tag{5.4}
\]

The power of mass can be eliminated by rescaling \( p \to mp \). There are two subtle points that have to be clarified in the above derivation:

- The derivation has to be valid for massless field, and for \( m = 0 \) one should be able to take into account the rather complex singularity of the field near the light cone, namely,

\[
\varphi(x, t) \sim \begin{cases} t^{-1}, & |x| = t, \\ t^{-2}, & |x| < t. \end{cases} \tag{5.5}
\]

- How to accurately neglect the oscillating terms on the r.h.s. of 5.3?
These are the two issues that we are now going to consider.

**A more accurate derivation**

Motivated by the above derivation, consider the following expression (a smearing over $n$ is implicit):

$$\lim_{t \to +\infty} t^3 \int_0^1 \rho^2 d\rho \partial_0 \varphi \partial_i \varphi (n \rho t, t)$$

(6.1)

Substituting 5.1 one obtains the following expression for the coefficient of $a^\pm (p) a^\pm (q)$:

$$\int_0^1 \rho^2 d\rho \int \frac{d^3p}{2 p_0} \frac{d^3q}{2 q_0} e^{it[\pm(p_0-mp\rho)\pm(q_0-nq\rho)]} p_0 q_i a^\pm (p) a^\pm (q).$$

(6.2)

The asymptotics of this expression for $t \to +\infty$ can be found using the stationary phase method.

One finds that the stationary points are as follows. For the term $a^\pm (p) a^\pm (q)$ with opposite signs the stationary point is $p = q = \frac{mp}{\sqrt{1-\rho^2}}n$ with any $\rho \in [0, 1]$ for $m \neq 0$, and $p = 1, q = \omega n$ with any $\omega > 0$ for $m = 0$. For the term $a^\pm (p) a^\pm (q)$ with both signs equal one should take $\rho = 0$ and $p = q = 0$ for both $m \neq 0$ and $m = 0$, and the entire contribution is suppressed as compared with the previous case by an additional power of $t^{-1}$. This settles the problem of oscillating terms. Finally, one arrives at the following expression irrespective of the field’s mass:

$$\varepsilon (n) dn = \lim_{t \to +\infty} \int_0^1 \rho^2 d\rho n_i T_{0i} (\rho n, t) dn = \lim_{t \to +\infty} \int_{x \in Cone(t, n, dn)} dx n_i T_{0i} (x, t) dn$$

(6.3)

To help understand this expression, below is a graphical representation of the integration region $x \in Cone(t, n, dn)$ on the r.h.s.: Recall that $T_{0i}(x)$ is the space–time density of 3–momentum and is equal due to symmetry of the energy–momentum tensor to $T_{i0}(x)$, the density of energy flow. The convolution $n_i T_{0i}$ projects out the component of the flow in the direction $n$. The integration has to be performed over the entire cone including small
x (which is the least expected feature of the answer). This is apparently because, in general, particles in the final state may be arbitrarily slow.

Conclusions

We have demonstrated that all physical information about multijet structure of multiparticle events at high energies represented by the C–algebra of observables described in [12] is contained in the family of the so-called C–correlators of the form 4.1 where \( f_N \) are arbitrary smooth symmetric functions of their angular arguments, and the operator \( \varepsilon(n) \) is defined by 6.3. Such a definition of C–correlators retains physical meaning even if the theory does not admit a naive particle interpretation (e.g. QCD where the asymptotic states cannot contain quark and gluon fields; cf. e.g. the discussion in [15]; or QED where the structure of asymptotic states is rather complex due to massless photons). The definition of jet–related observables in terms of the C–correlators 4.1, 6.3 is non–perturbative which fills an important gap that remained in the theory of QCD jets.

To reiterate, we started with the assumption that hadrons can be regarded as free particles in final states. Then we showed that the C–correlators are expressed in terms of the energy momentum tensor of the corresponding
asymptotic free fields via formulas that are universal in the sense that they are independent of particles’ masses. Therefore, summing over all particles’ types yields the total energy momentum tensor. The latter, however, being related to the space–time symmetries of the theory, is independent of the set of fields used to formulate the theory. In particular, one can use its expression in terms of quark and gluon fields as well.

All this puts the issue of hadronization effects in jet cross sections into a clearer perspective. For instance, in the case of $e^+e^-$ annihilation into hadrons the bound states do not appear in the expressions 4.1, 6.3 at all. Likewise, the important issue of the structure of power corrections can be systematically studied (limiting the consideration by necessity to perturbation theory) following the pattern of [17] within the systematic formalism of asymptotic operation [18] appropriately modified for non–Euclidean asymptotic regimes [19, 20]. In particular, the form of 6.3 (cf. the integration over a half axis stretching from zero to infinity) seems to add plausibility to the recent hypothesis that string operators should play a role in power corrections to jet cross sections [21].

Lastly, it would be interesting to find out under what general assumptions one can prove existence of objects like C–correlators 4.1, 6.3 in the context of axiomatic QFT. It may be expected that this can be done under assumptions somewhat weaker than the usual one about a non-zero mass gap in the standard scattering theory (cf. [22]).

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