TESTING SUPERGRAVITY GRAND UNIFICATION AT FUTURE ACCELERATOR AND UNDERGROUND EXPERIMENTS

R. Arnowitt\textsuperscript{a}) and Pran Nath\textsuperscript{b})

\textsuperscript{*a}) Center for Theoretical Physics, Department of Physics
Texas A&M University, College Station, TX 77843-4242
\textsuperscript{a}) Physics Research Division, Superconducting Super
Collider Laboratory, Dallas, TX 75237
\textsuperscript{b}) Department of Physics, Northeastern University
Boston, MA 02115

ABSTRACT

The full parameter space of supergravity grand unified theory with $SU(5)$ type $p \to \bar{\nu}K$ proton decay is analysed using renormalization group induced electroweak symmetry breaking under the restrictions that the universal scalar mass $m_o$ and gluino mass are $\leq 1$ TeV (no extreme fine tuning) and the Higgs triplet mass obeys $M_{H_3}/M_G < 10$. Future proton decay experiments at SuperKamiokande or ICARUS can reach a sensitivity for the $\bar{\nu}K$ mode of $(2 - 5) \times 10^{33}$ yr allowing a number of predictions concerning the SUSY mass spectrum. Thus either the $p \to \bar{\nu}K$ decay mode will be seen at these experiments or a chargino of mass $m_\tilde{W} < 100$ GeV will exist and hence be observable at LEP2. Further, if $(p \to \bar{\nu}K) > 1.5 \times 10^{33}$ yr, then either the light Higgs has mass $m_h \leq 95$ GeV or $m_\tilde{W} \leq 100$ GeV i.e. either the light Higgs or the light chargino (or both) would be observable at LEP2. Thus, the combination of future accelerator and future underground experiments allow for strong experimental tests of this theory.

\textsuperscript{*} Permanent address
1. INTRODUCTION

The observation that, for a supersymmetric mass spectrum, the three coupling constants of the Standard Model measured at the scale $Q = M_Z$, $\alpha_1(M_Z) \equiv (5/3)\alpha_Y(M_Z)$, $\alpha_2(M_Z)$ and $\alpha_3(M_Z)$, unify at the Gut scale $Q = M_G$ to a common value $\alpha_G$ [1] has lead to considerable effort to deduce additional consequences of supergravity grand unification models [2,3]. Unification takes place at $M_G \sim 10^{16}$ GeV with the SUSY particles having mass at $M_S \sim 10^{2.5}$ GeV provided only one pair (the minimal number) of Higgs doublets exist. It is possible that the fact that the three coupling constants meet at a point at $10^{16}$ GeV is merely a numerical accident without significance. If, however, one accepts this result as a guide to new physics, it suggests first the validity of grand unification, and second that the particle spectrum above the electroweak scale and up to the Gut scale is that of the supersymmetrized Standard Model with only one pair of Higgs doublets. (More pairs of Higgs doublets leads to too small a value of $M_G$ and hence too rapid $p \rightarrow e^+\pi^0$ proton decay.)

It is of interest that the unification of the couplings is not a property of low energy supersymmetry alone. Thus the 5/3 factor relating $\alpha_1$ to $\alpha_Y$ is needed to achieve unification, and this reflects on how $\alpha_Y$ is embedded into a grand unification group $G$ at the Gut scale. (Examples of acceptable choices of $G$ are $SU(5)$, $O(10)$, $E_6$ etc. but not [$SU(3)]^3$). There is no reason in low energy SUSY theory to insert the 5/3 factor, and hence no reason to expect coupling constant unification from purely low energy considerations. Further, supersymmetry must break spontaneously, and no phenomenologically acceptable way of doing this in low energy global supersymmetry has been constructed. In supergravity, however, spontaneous symmetry breaking of supersymmetry in the “hidden” sector occurs naturally, either at the tree level [4] or via condensates [5].

For these reasons, activity over the past two years has centered around obtaining predictions of different supergravity grand unified models [6-15]. In these models, our lack of knowledge of the physics of the hidden sector can be parameterized in terms of four parameters: $m_o$ (universal scalar mass), $m_{1/2}$ (universal gaugino mass), $A_o$ (cubic soft breaking parameter) and $B_o$ (quadratic soft breaking parameter). Refs. [6,7] treat the general $SU(5)$ supergravity, while Ref. [10] examines this in the special case where
$B_o = A_o - m_o$. The constraints of proton decay $p \to \bar{\nu} + K^+$ are included in Refs. [6,7]. The No-Scale models ($m_o = A_o = 0$ and also $B_o = 0$) are examined without proton decay (as would be the case for the flipped $SU(5)$ model [16]) in Refs. [11,14] and with the constraint of proton decay in Ref. [12]. Cosmological constraints are discussed in Ref. [13]. Models with $O(10)$ symmetry were examined in Ref. [15].

In this paper we will consider a general class of supergravity $Gut$ models (defined explicitly in Sec. 2 below) which allow for an “$SU(5)$-type” proton decay in the $p \to \bar{\nu} + K^+$ mode. [As discussed in Sec. 2, it is difficult to prevent this type of proton decay for $SU(5)$-type models, except for the case of flipped $SU(5)$.] Present proton decay data [17] significantly restricts the parameter space. Thus in previous work it was shown that it leads generally to a lower bound on $m_o$ and an upper bound on the gluino mass $m_\tilde{g} = (\alpha_3/\alpha_G)m_{1/2}$, with both squarks and gluino probably requiring the SSC or LHC to be seen [6,7]. Since $m_o$ is large, radiative breaking [18] of $SU(2) \times U(1)$ at the electroweak scale generally implies that the $\mu$ parameter (which scales the coupling of the two Higgs doublets $H_1$ and $H_2$ in the superpotential) obeys $\mu^2 >> M_Z^2$. This leads to a number of scaling laws between the charginos ($\tilde{W}_i, i = 1, 2$), neutralinos ($\tilde{Z}_i, i = 1 \cdots 4$) and the gluino:

\begin{align}
2m_{\tilde{Z}_1} & \cong m_{\tilde{W}_1} \cong m_{\tilde{Z}_2} \quad (1.1a) \\
m_{\tilde{Z}_{3,4}} & \cong m_{\tilde{W}_2} \gg m_{\tilde{Z}_1} \quad (1.1b) \\
m_{\tilde{W}_1} & \cong \frac{1}{3} m_{\tilde{g}}(\mu < 0); \quad m_{\tilde{W}_1} \cong \frac{1}{4} m_{\tilde{g}}(\mu > 0) \quad (1.1c)
\end{align}

(where $m_{\tilde{Z}_i} < m_{\tilde{Z}_j}$ for $i < j$ etc.) Bounds on the Higgs masses ($h, H = CP$ even states, $A = CP$ odd and $H^\pm = charged$ Higgs) also are obtained [6,7]:

\begin{equation}
m_h \lesssim 110 GeV; \quad m_A \cong m_H \cong m_{H^\pm} \gg m_h \quad (1.2)
\end{equation}

Further, there arises an upper bound on the top quark mass, $m_t \lesssim 175$ GeV with the first two generations of squarks and all three generations of sleptons approximately degenerate. The third generation of squarks are highly split. Finally we mention that bounds exist on
\[ \tan \beta \equiv \frac{< H_2 >}{< H_1 >} \text{ of } [6] \tan \beta \lesssim 7 \text{ and on } A_t \text{ (top quark } A \text{ parameter at the electroweak scale) of } |A_t| \lesssim 1.5. \]

In this paper we examine additional constraints that can be expected to arise from future proton decay experiments such as SuperKamiokande and ICARUS, and from future accelerator experiments at LEP200 and the Tevatron. We will see that, with the expected sensitivities, when one combines the results of underground and accelerator experiments one can obtain strong tests of this class of models. While each type of experiment by itself can limit the allowed parameter space, together they can test the validity of supergravity models with proton decay.

2. REVIEW OF FORMALISM

We summarize briefly here the formalism used in calculating consequences of supergravity Gut models. The class of supergravity Gut models we will consider are defined by the following assumptions:

(i) There exists a hidden sector which is gauge singlet with respect to the physical sector gauge group \( G \) which breaks supersymmetry. This breaking communicates to the physical sector only gravitationally. [Thus in the super Higgs mechanism, this condition is realized by an additive superpotential \( W = W_{\text{phys}}(z_a) + W_{\text{hidden}}(z) \) where supersymmetry is spontaneously broken by the VEVs \( < z >= O(\kappa^{-1}) \), \( \kappa^{-1} = M_{P\ell} \equiv (hc/8\pi G_N)^{1/2}(M_{P\ell} = 2.4 \times 10^{18} \text{ GeV}) \). Here, the \( \{z_a\} \) are the physical fields.]

(ii) A Gut sector exists which breaks \( G \) to \( SU(3)_C \times SU(2)_L \times U(1)_Y \) at scale \( M_G \).

(iii) After integrating out the super heavy fields (and eliminating the super Higgs fields) the only light particles remaining below \( M_G \) are the supersymmetric Standard Model particles with one pair of light Higgs doublets.

(iv) The super Higgs couplings in the Kahler potential are generation blind.

Conditions (ii) and (iii) are what is implied by the analysis of the coupling constant unification. Condition (i) is needed to maintain the gauge hierarchy and (i) and (iv) together guarantee the suppression flavor changing neutral interactions.

Conditions (i)-(iv), plus the requirement that the gauge kinetic function \( f_{\alpha\beta} \) and Kahler metric \( d_i^j \) can be expanded in a series scaled by \( \kappa \) \( (f_{\alpha\beta} = c_{\alpha\beta} + \kappa c_{\alpha\beta i} z^i + \cdots, \)
\[ d^i_j = c^i_j + \kappa c^i_{jk} z^k + \cdots, \{ z_i \} = \{ z_a - < z_a >, z - < z > \} \] then leads to the following general theorem [19]: The renormalizable interactions (arising equivalently from the \( \kappa \to 0 \) limit) of a general model is characterized at \( M_G \) by an effective superpotential with quadratic and cubic terms \( W = W^{(2)} + W^{(3)} \) given by

\[
W = \mu_o H_1 H_2 + [\lambda^{(u)}_{ij} q_i H_2 u^C_j + \lambda^{(d)}_{ij} q_i H_1 d^C_j + \lambda^{(e)}_{ij} \ell_i H_1 e^C_j], \tag{2.1a}
\]

an effective potential given by

\[
V = \left\{ \sum_a \left| \frac{\partial W}{\partial z_a} \right|^2 + V_D \right\} + \left[ m_o^2 \sum_a z^*_a z_a + (A_o W^{(3)} + B_o W^{(2)} + h.c.) \right] \tag{2.1b}
\]

and a universal gaugino mass term \( \mathcal{L}_\text{mass} = -m_1/2 \tilde{\lambda}^\alpha \lambda^\alpha \). In Eq. (2.1), \( q_i, \ell_i, H_1, H_2 \) are \( SU(2)_L \) quark, lepton and Higgs doublets \((i = 1, 2, 3 \) is the generation index\), \( u^C_i, d^C_i, e^C_i \) are conjugate singlets, \( V_D \) is the usual \( D \) term, \( \lambda^{(u,d,e)}_{ij} \) are the usual Yukawa coupling constants and \( \{ z_a \} \) now represents the scalar components of the light chiral multiplets. Thus aside from the Yukawa coupling constants of the Standard Model, the theory depends upon four soft breaking parameters \( m_o, m_{1/2}, A_o, B_o \) (which parameterize the properties of the hidden sector) and the parameter \( \mu_o \).

The above discussion constructs the renormalizable interactions valid below \( M_G \). Since \( M_G \) is close to \( M_{P\ell} \), i.e. \( M_G/M_{P\ell} \approx 10^{-2} \), one may suspect the existence of additional “Planck slop” terms. Since the nature of these are unknown, we omit them in the following discussions. However, their possible existance implies that the models we are considering may have errors of order of a few percent.

Supergravity Gut models offer a natural origin of electroweak breaking. Thus from Eq. (2.1b), all spin zero particle have a \((\text{mass})^2 \) of \( m_o^2 > 0 \) at the Gut scale. Running the renormalization group equations (RGE) down to the electroweak scale, one finds that the \( H_2 \) \((\text{mass})^2 \) can turn negative triggering electroweak breaking [18]. The Higgs part of the effective potential is

\[
V_H = m_1^2(t) | H_1 |^2 + m_2^2(t) | H_2 |^2 - m_3(t)^2 (H_1 H_2 + h.c.)
+ \frac{1}{8} [g_2^2(t) + g_3^2(t)] | H_1 |^2 - | H_2 |^2 \tag{2.2}
\]
where \( t = \ln[M_G^2/Q^2] \) is the running parameter, \( m_i^2(t) = m^2_{H_i}(t) + \mu^2(t), \) \( i = 1, 2, \) and \( m_3^2(t) = -B(t)\mu(t) \) and \( \Delta V_1 \) is the one loop correction [20]. At \( Q = M_G \) (i.e. \( t = 0 \)) the running masses then obey the boundary conditions \( m_i^2(0) = m^2_o + \mu^2_o, m_3^2(0) = -B_o\mu_o \) and \( g_2^2(0) = (5/3)g_1^2(0) = 4\pi\alpha_G. \) Minimizing \( V_H \) with respect to \( v_i \equiv < H_i^o >, \) \( i = 1, 2 \) yields the equations:

\[
\frac{1}{2} M_Z^2 = \frac{\mu^2_1 - \mu^2_2 \tan^2 \beta}{\tan^2 \beta - 1}; \quad \sin 2\beta = \frac{2m^2_3}{\mu^2_1 + \mu^2_2} \tag{2.3}
\]

where \( \mu^2_i = m^2_i + \Sigma_i \) and \( \Sigma_i \) are the loop corrections:

\[
\Sigma_i = \Sigma_a(-1)^{2s_an_a}[M_a(v_i)]^2\ln[M_a^2/\sqrt{eQ^2}](\partial M_a^2/\partial v_i) \tag{2.4}
\]

\((M_a, s_a, n_a \) are the mass, spin, and number of helicity states of particle \( a. \)\) In practice, Eqs. (2.3) is insensitive to the value of \( Q \) in the electroweak scale [21] and one may set \( Q = M_Z. \) (Also, for most of the parameter space, the loop corrections are small.)

The RGE allow the parameters in Eqs. (2.3) to be expressed in terms of the Gut scale parameters of Eqs. (2.1). It is convenient to use Eqs. (2.3) to eliminate \( \mu^2_o \) and \( B_o \) in terms of \( \tan \beta \) and the other Gut parameters. Thus one is left with

\[
m_o, m_{1/2}, A_o, \tan \beta \text{ and } m_t \tag{2.5}
\]

as unknown constants since \( M_G \) and \( \alpha_G \) are determined by the unification analysis. (Using two loop RGE and neglecting all thresholds we find \( M_G = 10^{16.19 \pm 0.34} \text{ GeV}, \) \( \alpha_G^{-1} = 25.7 \pm 1.7 \) and the common SUSY particle mass is \( M_S = 10^{2.37 \pm 1.0} \text{ GeV}, \) where the error is due to the uncertainty in \( \alpha_3 \) which we take as \( \alpha_3(M_Z) = 0.118 \pm 0.007 \) [22].\) Since the sign of \( \mu_o \) is not determined by Eq. (2.3) there are two branches: \( \mu > 0 \) and \( \mu < 0. \)

If one specifies the five parameters of Eq. (2.5) one may explicitly calculate the masses of all 32 SUSY particles (12 squarks, 9 sleptons, 1 gluino, 2 charginos, 4 neutralinos and 4 Higgs bosons). A characteristic example is given in Fig. 1. Note that the first two generations of squarks and all three generations of sleptons are nearly degenerate. However, the third generation of squarks is widely split, a feature that needs to be taken into account in phenomenological analyses. The charginos and neutralinos exhibit the scaling of Eqs.
(1.1) and the Higgs bosons the relations of Eq. (1.2). In general, there are 27 predictions available among the 32 SUSY masses (28 once the top mass is known), so the theory a priori has a great deal of predictive power.

All supergravity Gut models predict proton decay in the mode $p \rightarrow e^+ + \pi^0$, and most possess the SUSY mode $p \rightarrow \bar{\nu} + K^+$. We consider here models with $SU(5)$-type proton decay defined by the following:

(i) The Gut group $G$ contains an $SU(5)$ subgroup [or is $SU(5)$].

(ii) The matter that remains light after $G$ breaks to $SU(3)_C \times SU(2)_L \times U(1)_Y$ is embedded in the usual way in the $10 + \bar{5}$ representations of the $SU(5)$ subgroup.

(iii) After $G$ breaks, there are only two light Higgs doublets which interact with matter, and these are embedded in the $5$ and $\bar{5}$ of the $SU(5)$ subgroup.

(iv) There is no discrete symmetry or condition that forbids the proton decay amplitude. Under the above conditions (which can arise in a number models e.g. $G = SU(5), O(10), E_6$ etc.) there is a model independent amplitude for the $p \rightarrow \bar{\nu} + K^+$ decay arising from the exchange of the superheavy Higgsino color triplet of mass $M_{H_3}$ [23,24]. A characteristic diagram is shown in Fig. 2. (Diagrams with other gauginos can also enter, though these contributions are generally quite small.)

The total decay rate is $\Gamma(p \rightarrow \bar{\nu}K) = \Sigma_i \Gamma(p \rightarrow \nu_i K)$ where $i = e, \mu, \tau$. The CKM matrix elements enter at vertices in Fig. 2 allowing all three generations to enter in the loop integral. Thus one may write

$$\Gamma(p \rightarrow \bar{\nu}K) = \text{Const}(\beta_p/M_{H_3}) \sum_i |B_i|^2$$

where $B_i$ is the amplitude of the $\bar{\nu}_i K$ mode. $\beta_p$ is given by

$$\beta_p U_L^\delta = \varepsilon_{abc} \varepsilon_{\alpha \beta} < 0 \ | d_{aL}^\alpha u_{bL}^\beta u_{cL}^\delta | p >$$

where $U_L^\delta$ is the proton wave function. Lattice gauge calculations give [26] $\beta_p = (5.6 \pm 0.8) \times 10^{-3}$ GeV$^{-1}$. In general the first generation contributions to Eq. (2.6) are negligible and may be neglected. One has then [24]
\[ B_i = \frac{m_i^d V_{ii}^+}{m_s V_{21}} \left[ P_2 B_{i2} + \frac{m_t V_{31} V_{32}}{m_c V_{21} V_{22}} P_3 B_{i3} \right] \frac{L}{\sin 2\beta} \]  

(2.8)

where \( B_{ia} \) is the loop amplitude when generation a squarks (or sleptons) enter in the loop, \( V_{ij} \) is the CKM matrix and \( m_i^d, m_s \), etc. are quark masses. The \( P_a \) are additional CP violating phases arising in the dimension 5 operators. To minimize the constraints imposed by proton decay, we will assume in the following that \( P_2/P_3 = -1 \), i.e. second and third generation contributions destructively interfere. Detailed formulae for \( B_{ia} \) are given in Ref. [24].

Proton decay of this type is a characteristic feature of supergravity grand unification and one must do special things to avoid it. Thus the flipped \( SU(5) \times U(1) \) model [16] suppresses the \( p \to \bar{\nu} + K^+ \) mode by violating condition (ii) i.e. flipping the embedding of the quarks and leptons. Models that impose discrete symmetries to prevent this mode generally have more than one pair of light Higgs doublets, and sometimes relatively light Higgs color triplets [25]. This can produce problems with the unification of the coupling constants. One may construct models which fine tune the proton decay amplitude to zero. Thus consider a model with an arbitrary number of superheavy color Higgs triplets \( H_i, \bar{H}_i \) and chose the basis where only \( H_1, \bar{H}_1 \) couple to matter:

\[ \bar{H}_1 J + K H_1 + M_{ij} \bar{H}_i H_j \]  

(2.8)

Here \( J, K \) are bilinear matter sources and \( M_{ij} \) is the superheavy Higgs mass matrix. Eliminating the superheavy fields, the proton decay amplitude is then \(-K(M^{-1})_{11} J\), and if one five tunes the mass matrix so that \((M^{-1})_{11} = 0\), proton decay is suppressed. One must also arrange the Gut sector of the model so that only two Higgs doublets remain light. While it is possible to construct such models, we will not pursue them here as they are somewhat artificial.

3. CONSTRAINTS FROM FUTURE EXPERIMENTS

We examine now the constraints that can be obtained from proton decay and collider experiments. We allow the five parameters, \( m_o, m_{1/2}, A_o, \tan \beta \) and \( m_t \) to range over
the entire parameter space subject to: (i) there be no violation of current experimental bounds on the particle masses, (ii) radiative breaking of $SU(2) \times U(1)$ take place, (iii) proton decay constraints be obeyed, and (iv) $m_o, m_{\tilde{g}} < 1$ TeV (no extreme fine tuning) and $M_{H_3}/M_G < 10$ (where $M_G \simeq 1.5 \times 10^{16}$ GeV is grand unification mass neglecting heavy particle thresholds). The last condition takes into account the splitting that can arise in the superheavy particle spectrum. The upper bound on $M_{H_3}$ ($\simeq 2 \times 10^{17}$ GeV) is as large as is reasonable to assume and still not have to worry about Planck slope terms. (It is also what arises naturally in simple models [27] of the Gut sector.)

The current 90% C.L. experimental bounds are [17] $\tau(p \rightarrow e^+\pi^0) > 5.5 \times 10^{32}$ yr and $\tau(p \rightarrow \bar{\nu}K^+) > 1 \times 10^{32}$ yr. For SUSY theories one expects [28] $\tau(p \rightarrow e^+\pi^0) \sim 10^{31\pm1}$ $(M_V/6 \times 10^{14}$ GeV)$^4$ yr where $M_V$ is the superheavy vector boson mass. Super Kamiokande should be sensitive to the $e^+\pi^0$ mode up to a lifetime of $1 \times 10^{34}$ yr and up to $2 \times 10^{33}$ yr for the $\bar{\nu}K$ mode [29], while ICARUS expects to be sensitive to the $\bar{\nu}K$ mode up to a lifetime of $5 \times 10^{33}$ yr [30]. For Super Kamiokande to see the $e^+\pi^0$ decay mode would require $M_V \lesssim 6 \times 10^{15}$ GeV. On the other hand, the current Kamiokande data for $p \rightarrow \bar{\nu}K$ requires $M_{H_3} \gtrsim (0.8)M_G \simeq 1.2 \times 10^{16}$ GeV [6,7]. In simple models [27], one requires $M_{H_3} \lesssim 3M_V$, in order that the Gut physics remain treatable by perturbation theory. Thus it is not too likely that Super Kamiokande will be able to see the $p \rightarrow e^+\pi^0$ mode, and if it were observable there, the $p \rightarrow \bar{\nu}K$ decay would expected to be very copious.

We turn now to consider the $p \rightarrow \bar{\nu}K$ decay in detail. For $M_{H_3}/M_G = 3$ it was seen that current Kamiokande data requires [6] $m_o \gtrsim 500$ GeV, $m_{\tilde{g}} \lesssim 450$ GeV, $1.1 < \tan \beta < 5$, and $|A_t| \lesssim 1.2$. One can understand this qualitatively from the fact that in the limit of large $m_o$, the amplitude for the second generation contribution to $B_2$ is given approximately by $B_2 \simeq -2(\alpha_2/\alpha_3 \sin 2\beta)m_{\tilde{g}}/m_{\tilde{q}}^2$ where the squark mass is $m_{\tilde{q}}^2 \simeq m_o^2 + am_{\tilde{g}}^2$, $a \simeq 0.65$. Thus the proton decay constraint favors a heavy squark, a lighter gluino and a small $\tan \beta$. As $M_{H_3}/M_G$ is increased, the lower bound on $m_o$ decreases and the upper bound on $m_{\tilde{g}}$ increases so that at $M_{H_3}/M_G \gtrsim 7$, the current data can be satisfied for $m_{\tilde{g}} \lesssim 1$ TeV (i.e. for $m_o$ small, $B_2$ again decreases for very large $m_{\tilde{g}}$: $B_2 \sim 1/m_{\tilde{g}}$). The bands on $\tan \beta$ and $|A_t|$ also increase somewhat.

To see the reach of future proton decay experiments, we consider a fixed value of
$m_o$ and $m_t$, and calculate the maximum lifetime $\tau(p \to \bar{\nu}K)$ as all other parameters are varied over the entire allowed parameter space. This is shown in Figs. (3a,b,c) for $m_t = 125$ GeV, 150 GeV and 170 GeV ($\mu < 0$) for the three values $M_{H_3}/M_G = 3, 6,$ and 10. (The $\mu > 0$ lifetimes are slightly shorter, but show a similar behavior.) One sees that the entire parameter domain of $m_o < 1000$ GeV will be accessible to ICARUS for $M_{H_3}/M_G < 6$ (and accessible to Super Kamiokande for $m_o \lesssim 800$ GeV). Thus if $M_{H_3}/M_G < 6$, proton decay should be seen at ICARUS for this class of models. Fig. 4 shows the maximum value of $\tau(p \to \bar{\nu}K)$ for $M_{H_3}/M_G = 6$, $\mu < 0$ as a function of $m_t$ as all other parameters are varied over the entire allowed parameter space. One sees that the lifetime peaks at $m_t \simeq 145$ GeV. (This graph shows again that the domain $m_o \lesssim 800$ GeV, $M_{H_3}/M_G < 6$ will be accessible to Super Kamiokande.) Fig. 5 gives a plot of the maximum value of $\tau(p \to \bar{\nu}K)$ as a function of $m_o$ for $m_t = 150$ GeV, $\mu > 0$, but subject to the condition $m_{\tilde{W}_1} > 100$ GeV. One sees that here, proton decay is accessible to ICARUS for the entire parameter space with $M_{H_3}/M_G < 10$ (and to Super Kamiokande for $m_o \lesssim 950$ GeV). Thus for this class of models either proton decay will be seen at ICARUS or the Wino will be seen at LEP200 (or both) and also squarks and gluinos will be observable at the SSC or LHC.

While the $h$ Higgs boson is generally light, loop corrections can cause it to lie beyond the planned range of LEP200. However, by examining the full parameter space with $m_o$, $m_{\tilde{g}} < 1$ TeV and $M_{H_3}/M_G < 10$, one finds that if $\tau(p \to \bar{\nu}K) > 1.5 \times 10^{33}$ yr then either $m_h < 95$ GeV or $m_{\tilde{W}_1} < 100$ GeV. That is, if $\tau$ exceeds $1.5 \times 10^{33}$ yr (a condition that would be tested at both Super Kamiokande or ICARUS) then either the $h$ Higgs or the $\tilde{W}_1$ (and possibly both) would be observable at LEP200. Note also, since $m_{\tilde{Z}_1} \simeq \frac{1}{2} m_{\tilde{W}_1}$, one expects $m_{\tilde{Z}_1} \lesssim 50$ GeV, for this case, and hence the $\tilde{W}_1$, $\tilde{Z}_{1,2}$ should also be observable at the planned upgraded Tevatron when $m_{\tilde{W}_1} < 100$ GeV (via the process $p + \bar{p} \to W^* + X \to \tilde{W}_1 + \tilde{Z}_2 + X$, with a trileptonic plus missing $E_T$ signal).

4. CONCLUSIONS

An analysis of the five dimensional parameter space of $m_o$, $m_{1/2}$, $A_o$, $\tan \beta$ and $m_t$ for supergravity models possessing $SU(5)$-type proton decay was carried out. The analysis was performed under the restrictions that (i) the electro-weak symmetry breaking is radiative,
(ii) there is no extreme fine tuning i.e. $m_o, m_{\tilde{g}} \leq 1\text{ TeV}$, and (iii) $M_{H_d}/M_G \leq 10$. It was then shown that the intersection of the experimental limits that can be achieved for the $\bar{\nu}K^+$ mode at SuperKamiokande and at ICARUS and the limits on the superparticle masses achievable at LEP200 and the Tevatron can exhaust the full parameter space of these supergravity models. Specifically it was shown that either the $\bar{\nu}K^+$ mode should be seen at SuperKamiokande and ICARUS or the lighter chargino should be observable at LEP200 provided it can achieve its optimum energy and detection efficiency. In this sense the proton decay experiments at SuperKamiokande and ICARUS and the LEP200 experiment are complimentary, and one needs both to check the full predictions of the $SU(5)$ Supergravity Model.

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FIGURE CAPTIONS

Fig. 1 SUSY mass spectrum for parameters $m_o = 600$ GeV, $m_{1/2} = 53$ GeV, $A_o = 0.0$, $\tan \beta = 1.73$, $m_t = 150$ GeV and $\mu < 0$. The first column shows the generation 1,2 squarks and all generations of sleptons. The second column is the third generation of squarks. The third column are the charginos and neutralinos and the last column the Higgs bosons.

Fig. 2 One of the diagrams contributing to proton decay mode $p \rightarrow \bar{\nu}_\mu + K^+$. The Wino converts quarks into squarks, and the baryon and lepton number violations occur at the $\tilde{H}_3$ vertex.

Fig. 3a The maximum value of $\tau(p \rightarrow \bar{\nu}K^+)$ vs $m_o$ for $m_t = 125$ GeV, $\mu < 0$. The maximum is calculated by allowing all other parameters except $m_o$ to vary over the entire allowed parameter space. The three curves are for $M_{H_3}/M_G = 3$, 6, and 10. The lower horizontal line is the upperbound for SuperKamiokande, i.e SuperKamiokande will be sensitive to lifetimes below this line. The higher horizontal line is for ICARUS.

Fig. 3b The same as Fig. 3a for $m_t = 150$ GeV.

Fig. 3c The same as Fig. 3a for $m_t = 170$ GeV.

Fig. 4 The maximum value of $\tau(p \rightarrow \bar{\nu}K^+)$ vs $m_o$ for $M_{H_3}/M_G = 6$ and $\mu < 0$. The solid line is for $m_o = 400$ GeV, the dashed line for $m_o = 800$ GeV, the dot-dashed line for $m_o = 1200$ GeV. The lower horizontal line is the bound that Super Kamiokande can detect, and the higher horizontal line is the upperbound for ICARUS.

Fig. 5 Maximum value of $\tau(p \rightarrow \bar{\nu}K^+)$ vs $m_o$ for $m_t = 150$ GeV, $\mu < 0$ when $m_{\tilde{W}_1}$ is constrained to be greater than 100 GeV. The solid line is for $M_{H_3}/M_G = 3$, the dashed line is for $M_{H_3}/M_G = 6$ and the dot-dashed line is for $M_{H_3}/M_G = 10$. The horizontal lines are as in Figs. 3 and 4.
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