Classification of customer lifetime value models using Markov chain

Dony Permana¹, Udjianna S. Pasaribu², Sapto W. Indratno², and Suprayogi³

1 Statistics Study Program, Faculty of Mathematics and Natural Sciences Universitas Negeri Padang, Indonesia
2 Statistics Research Division, Faculty of Mathematics and Natural Sciences Institut Teknologi Bandung, Indonesia
3 Industrial System and Techno-Economics, Research Group, Faculty of Industrial Technology Institut Teknologi Bandung, Indonesia

E-mail: donypermana27@gmail.com, {udjianna,sapto, yogi}@math.itb.ac.id, yogi@mail.ti.itb.ac.id

Abstract. A firm’s potential reward in future time from a customer can be determined by customer lifetime value (CLV). There are some mathematic methods to calculate it. One method is using Markov chain stochastic model. Here, a customer is assumed through some states. Transition inter the states follow Markovian properties. If we are given some states for a customer and the relationships inter states, then we can make some Markov models to describe the properties of the customer. As Markov models, CLV is defined as a vector contains CLV for a customer in the first state. In this paper we make a classification of Markov Models to calculate CLV. Start from two states of customer model, we make develop in many states models. The development a model is based on weaknesses in previous model. Some last models can be expected to describe how real characters of customers in a firm.

1. Introduction

In a marketing system, a consumer of a goods or services can be seen to has some states that passed. Transition a consumer from one state to another is assumed to follow Markovian [4, 9]. Relationship inter states have meaning characteristics of the consumer. Consumers who are loyal to a product of goods or services are marked with the transition from customer state to itself. Such transition is called retention. The Retention need a kind of marketing cost. A consumer is called former customer if he or she don’t buy again the product. But transition from former customer state to customer state is possible and it is called acquisition of former. The acquisition also need another kind of marketing cost. Model describes characteristics of a consumer based on relationship inter states can be made by Markov Model.

In previous research, the Markov model is used to determine customer lifetime value (CLV) [6, 8, 10]. It is called models of CLV. Another research, it is used to determine an inverse problem of CLV model [5]. Model of CLV is used to expenditure decision management [3], segmentation of consumer, data mining in an auto-repair and maintenance company [2], etc. There are some models of CLV, but not yet classified.
The objective of this paper are to (1) classify Markov models, create a naming system and describing characters of a customer in a marketing system, and (2) make transition chart for every model. Procedures of classifying are (1) analyzing the number of state of customer in a firm that is needed, (2) analyzing relationships inter the states, (3) constructing a transition chart, (4) setting a transition matrix, (5) developing another model, and (6) creating a naming system.

2. Result
We make two types of model of CLV. The first is CF (Customer-Former) model, contains customer state and former customer state. The second is PCF (Prospect-Customer-Former) model, contains three states are prospect, customer, and former customer. Indexes number in brackets have meaning the number of each state. Example, $CF(1,2)$ have one customer state and two levels former customer states, $PCF(1,2,1)$ have one prospect state, two levels customer state, and one former customer state. Here is diagram of classify of model of CLV and the naming system of it.

![Diagram of classify of model CLV and its naming system](image)

Figure 1. Diagram of classify of model CLV and its naming system

Look at Figure 1. Actually, we can make some another model from CF, example $CF(3,.)$, $CF(4,.)$ and so on. For PCF model, we can also make $PCF(1,3,.)$, $PCF(1,4,.)$ and so on. It depends on the reality of the characteristics of customer in a firm or needs in the field. Figure 1 show how we classify the model and make a naming system of it. The next figures show diagram transition of every model that written in Figure 1.

Figure 2 shows transition diagram of models of $CF(1,.)$. There are three models are $CF(1,1)$, $CF(1,2)$, and $CF(1,3)$. The $CF(1,1)$ model has no absorbent state. It isn’t realistic in the field of a product marketing. Because models without absorbent will take a lot of marketing costs. The $CF(1,2)$ is better than $CF(1,1)$. Here, absorbent state is saved in former customer level 2. But the weakness of this model is former customer level 1 only have one offer to come back to the customer state.

Thus, we make $CF(1,3)$ that is better than $CF(1,2)$. We may think that $CF(1,4)$ is better than $CF(1,3)$. That may be true, but it must be remembered that the more of former customer state will need a lot of cost for marketing.

Figure 3 show transition diagram of models of $CF(2,.)$. There are three models too, $CF(2,1)$, $CF(2,2)$, and $CF(2,3)$. Model of $CF(2,.)$ is proposed for a firm whose two level customer. Example in health insurance, level 1’s customer has low premium price and level 2 customer...
Figure 2. Transition diagram of models of \( CF(1,.) \)

has high premium price. In a service product, level 1’s customer is for member and level 2’s customer is for non member.

Figure 4 show transition diagram of models of \( PCF(1,1,.). \) There are three models, \( PCF(1,1,1), PCF(1,1,2), \) and \( PCF(1,1,3). \) In this model, there is a prospect customer state. It is a state for a people that is offered to be a customer. But if he or she declined the offer, then he or she moved to former customer state. Such as \( CF(1,1), PCF(1,1,1) \) isn’t realistic in the field of a product marketing. So, we construct \( PCF(1,1,2) \) and \( PCF(1,1,3). \)

Based on transition diagrams, we can set matrices of transition. Size of matrix depends on the number of states. Model of \( CF(1,2) \) have three states, thus size of transition matrix is \( 3 \times 3. \) The following are the transition matrix of the models.

\[
P_{CF(1,1)} = \begin{pmatrix} r & 1-r \\ q & 1-q \end{pmatrix}
\]

\[
P_{CF(1,2)} = \begin{pmatrix} r & 1-r & 0 \\ q & 0 & 1-q \\ 0 & 0 & 1 \end{pmatrix}
\]
Figure 3. Transition diagram of models of $CF(2, .)$

\[
P_{CF(1,3)} = \begin{pmatrix} r & 1 - r & 0 & 0 \\ q_1 & 0 & 1 - q_1 & 0 \\ q_2 & 0 & 0 & 1 - q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix}
\]

(3)

\[
P_{CF(2,1)} = \begin{pmatrix} r_1 & s_{12} & 1 - r_1 - s_{12} \\ s_{21} & r_2 & 1 - r_2 - s_{21} \\ q_1 & q_2 & 1 - q_1 - q_2 \end{pmatrix}
\]

(4)

\[
P_{CF(2,2)} = \begin{pmatrix} r_1 & s_{12} & 1 - r_1 - s_{12} & 0 \\ s_{21} & r_2 & 1 - r_2 - s_{21} & 0 \\ q_{11} & q_{12} & 1 - q_{11} - q_{12} & 0 \\ q_{21} & q_{22} & 0 & 1 - q_{21} - q_{22} \end{pmatrix}
\]

(5)
Figure 4. Transition diagram of models of $PCF(1,1,.)$

\[
P_{CF(2,3)} = \begin{pmatrix}
    r_1 & s_{12} & 1-r_1-s_{12} & 0 & 0 \\
    s_{21} & r_2 & 0 & 1-r_2-s_{21} & 0 \\
    q_{11} & q_{12} & 0 & 0 & 1-q_{11}-q_{12} \\
    q_{21} & q_{22} & 0 & 0 & 1-q_{21}-q_{22} \\
    0 & 0 & 0 & 0 & 1
\end{pmatrix} \quad (6)
\]

\[
P_{PCF(1,1,1)} = \begin{pmatrix}
    0 & a & 1-a \\
    0 & r & 1-r \\
    0 & 0 & 1
\end{pmatrix} \quad (7)
\]

\[
P_{PCF(1,1,2)} = \begin{pmatrix}
    0 & a & 1-a & 0 \\
    0 & r & 1-r & 0 \\
    0 & q & 0 & 1-q \\
    0 & 0 & 0 & 1
\end{pmatrix} \quad (8)
\]
\[ P_{PCF(1,1,3)} = \begin{pmatrix} 0 & a & 1-a & 0 & 0 \\ 0 & r & 1-r & 0 & 0 \\ 0 & q_1 & 0 & 1-q_1 & 0 \\ 0 & q_2 & 0 & 0 & 1-q_2 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix} \]  

Furthermore, the transition matrix is used to calculate CLV[7] or will be estimated by inverse problem[1].

3. Conclusion
There is the naming system to classify Markov models for describing characters of a customer in a firm. The models are used to calculate CLV. The classification is made based on a number of state of customer that followed by a customer in a firm.

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