We discuss the “odderon” exchange at high energy within the framework of the Color Glass Condensate (CGC).

1 General strategies in the Color Glass Condensate

In this talk, we provide the description of $C$-odd odderon exchange within the framework of the Color Glass Condensate (CGC). The theoretical framework for the CGC is intended to describe hadronic scatterings at very high energies. Below, we briefly explain its general strategies by using the scattering of a simple projectile in a reference frame where most of the total momentum of the scattering is carried by the target ($\approx$ the infinite momentum frame of the target). The same strategies are applicable for the odderon exchange as we will discuss later.

1. When the scattering energy is large enough, one can treat the target as the CGC which is a dense gluonic state on two dimensional transverse plane. The CGC provides a strong gauge field $\alpha^a(x) = A^a_T(x)$ which is random due to random motion of the valence partons. The randomness of the gauge field is governed by the weight function $W_\tau(\alpha)$, whose energy (or rapidity $\tau$) dependence is determined by the so-called JIMWLK equation.

2. One has to determine the scattering operator which is associated with the specific scattering process. Since the scattering energy is high, one can use the eikonal approximation where the transverse positions of incoming partons do not change after the scattering. Thus, one can easily obtain the $S$-matrix for a fixed configuration of the gauge field $\alpha$ by computing the matrix element $\langle \text{out; } \alpha, x, y, z, \cdots | \text{in; } \alpha, x, y, z, \cdots \rangle$ where $x, y, z, \cdots$ stand for the transverse positions of the partons in the projectile. The outgoing parton differs from the incoming parton up to the gauge factor which is picked up while traversing the external gauge field. To obtain the final result, one further has to take an average of this $S$-matrix over the possible configurations of the gauge field with the weight function $W_\tau(\alpha)$.

3. Once the relevant operator is specified, one is able to compute the evolution equation for it from the JIMWLK equation. This evolution equation is a first order differential equation with respect to the rapidity $\tau$. In particular, if the operator is gauge invariant, the JIMWLK equation can be made simple so that infrared finiteness becomes manifest. The use of this simplified version of the JIMWLK equation greatly reduces the manipulations to obtain the

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The simplest physical example is the dipole-CGC scattering where the incoming and outgoing states are $q\bar{q}$ dipoles: $|\text{in};\alpha\rangle \sim \psi^\text{in}_i(x)\psi^\text{in}_i(y)|0\rangle$, $|\text{out};\alpha\rangle \sim \psi^\text{out}_i(x)\psi^\text{out}_i(y)|0\rangle$. The relation between $\psi^\text{in}_i$ and $\psi^\text{out}_i$ is given by $\psi^\text{out}_i = (V^\dagger_i)_{ij}\psi^\text{in}_j$ with $V^\dagger_i$ being the Wilson line (in the fundamental representation) along the trajectory of the quark:

$$V^\dagger_x = \mathcal{P} \exp \left\{ ig \int dx^- \alpha^a(x^-,x)t^a \right\}. \quad (1.1)$$

The physical $S$–matrix is obtained after averaging over the random external field:

$$S_\tau(x, y) = \int \mathcal{D}\alpha\, W_\tau[\alpha] \langle \text{out};\alpha|\text{in};\alpha \rangle = \frac{1}{N_c} \langle \text{tr}(V^\dagger_x V^\dagger_y)\rangle_\tau. \quad (1.2)$$

This implies that the relevant operator for the dipole-CGC scattering is given by $\frac{1}{N_c}\text{tr}(V^\dagger_x V^\dagger_y)$. The evolution equation for this operator is obtained by the JIMWLK equation, and the result is called the Balitsky equation.

## 2 Odderon operators in the CGC

We follow the general strategies mentioned above to describe C-odd ”odderon” exchange at high energies. This can be done by adding the information of the C parities to the incoming and outgoing states. Below we discuss two simple projectiles: a color dipole and 3-quark state.

### 2.1 The dipole-CGC scattering

To single out C-even (‘pomeron’) or C-odd (‘odderon’) exchanges, we take the C-odd and C-even dipole states as the incoming and outgoing states, respectively. Then the relevant operator for C-odd exchanges in the dipole-CGC scattering is extracted and is given by

$$O(x, y) \equiv \frac{1}{2iN_c} \text{tr}(V^\dagger_y V^\dagger_x - V^\dagger_y V^\dagger_x) = -O(y, x). \quad (2.1)$$

Note that the C-odd contribution is the imaginary part of the $S$-matrix element: $\langle O(x, y) \rangle_\tau = 3m S_\tau(x, y)$. The corresponding C-even, pomeron exchange, amplitude, $N(x, y)$, is identified with the real part of the $S$-matrix: $N(x, y) \equiv 1 - \frac{1}{N_c} \text{tr}(V^\dagger_y V^\dagger_x + V^\dagger_x V^\dagger_y)$.

From perturbative QCD, we expect that the lowest order contribution to the odderon exchange is of the form $d^{abc}A^a_\mu(x)A^b_\mu(y)A^c_\mu(z)$ with $d^{abc} = 2\text{tr}(\{t^a, t^b\}t^c)$ being a totally symmetric tensor. The same structure indeed emerges from the CGC operator (2.1) in the weak-field limit. By expanding the Wilson lines (1.1) up to cubic order with respect to the field $\alpha$, one finds the expected structure ($\alpha_x^a = \int dx^- \alpha^a(x^-,x)$)

$$O(x, y) \simeq \frac{-g^3}{24N_c} d^{abc} \left\{ 3(\alpha_x^a \alpha_y^b \alpha_z^c - \alpha_y^a \alpha_z^b \alpha_x^c) + (\alpha_x^a \alpha_y^b \alpha_z^c - \alpha_y^a \alpha_x^b \alpha_z^c) \right\}. \quad (2.2)$$

Note that this combination of trilinear field operators is gauge invariant by construction.

### 2.2 The 3-quark–CGC scattering

We now turn to the 3-quark–CGC scattering at high energies. The 3-quark colorless state may be given by the ”baryonic” operator $\epsilon^{ijk}\psi^i(x)\psi^j(y)\psi^k(z)$, where $\epsilon^{ijk}$ is the complete antisymmetric symbol ($i, j, k = 1, 2, 3$). By using the same eikonal approximation as for the dipole-CGC scattering, one obtains the ”3-quark odderon operator”

$$O(x, y, z) = \frac{1}{3!2i} \left( \epsilon^{ijk}\epsilon^{lmn}V^\dagger_{jlm}(x)V^\dagger_{km}(y)V^\dagger_{kn}(z) - \text{c.c.} \right). \quad (2.3)$$

This somewhat unfamiliar operator can be made considerably simple if one multiplies the identity $\epsilon^{ijk}\epsilon^{lmn}V^\dagger_{jlm}(w)V_{jlm}(w)V_{jl}(w) = 3! \det V(w) = 3!$, and then chooses the arbitrary coordinate $w$ to be one of the quark coordinates, say $w = z$. Namely, one can equivalently rewrite (2.3) as

$$O(x, y, z) = \frac{1}{3!2i} \left[ \text{tr}(V^\dagger_x V^\dagger_y V^\dagger_z) - \text{tr}(V^\dagger_x V^\dagger_z V^\dagger_y) - \text{tr}(V^\dagger_y V^\dagger_z V^\dagger_x) - \text{c.c.} \right]. \quad (2.4)$$
Furthermore, when two of the coordinates are the same, the 3-quark odderon operator reduces to the dipole odderon operator, Eq. (2.1).

\[ O(x, z, z) = O(x, z) = -O(x, x, z). \quad (N_c = 3) \]

This is physically reasonable because the diquark state is equivalent to an antiquark as far as color degrees of freedom are concerned.

In the weak-field approximation, one finds again a gauge invariant linear combination of trilinear field operators with the \( d \)-symbol:

\[ O(x, y, z) \approx \frac{g^3}{144} \varepsilon_{abc} \{ (\alpha^x_x - \alpha^a_z) + (\alpha^y_y - \alpha^a_z) \} \{ (\alpha^b_b - \alpha^b_x) + (\alpha^b_b - \alpha^b_x) \} \{ (\alpha^c_c - \alpha^c_y) + (\alpha^c_c - \alpha^c_y) \}. \]

(2.6)

3 Odderon evolution

Once we know the relevant operators for the C-odd scattering amplitudes, we can apply the JIMWLK equation to the operators to derive the evolution equations for them.

3.1 The dipole–CGC scattering

For the dipole-CGC scattering, the evolution equations obeyed by the average amplitudes \( \langle N(x, y) \rangle_\tau \) and \( \langle O(x, y) \rangle_\tau \) can be easily derived from the Balitsky equation because the operators \( N(x, y) \) and \( O(x, y) \) are, respectively, the real part and the imaginary part of the dipole-CGC scattering operator \((1/N_c)\text{tr}(V^1_kV^1_y)\) which satisfies the Balitsky equation. Therefore, the respective equations can be simply obtained by separating the real part and the imaginary part in the Balitsky equation. The result is \( (O_{xy} = O(x, y), N_{xy} = N(x, y)) \)

\[
\frac{\partial}{\partial \tau} \langle O_{xy} \rangle_\tau = \frac{\bar{\alpha}_s}{2\pi} \int d^2 z \ M_{xyz} \left( \langle O_{xz} + O_{zy} - O_{xy} - O_{yx}N_{zy} - N_{xz}O_{zy} \rangle_\tau \right), \quad (3.1)
\]

\[
\frac{\partial}{\partial \tau} \langle N_{xy} \rangle_\tau = \frac{\bar{\alpha}_s}{2\pi} \int d^2 z \ M_{xyz} \left( \langle N_{xz} + N_{zy} - N_{xy} - N_{yx}N_{zy} + O_{xz}O_{zy} \rangle_\tau \right), \quad (3.2)
\]

where we have defined the dipole kernel \( M_{xyz} = (x - y)^2/(x - z)^2(z - y)^2 \). As is the case with the Balitsky equations, the equations above do not close by themselves. In the weak-field limit, both of the evolution equations reduce to the (linear) BFKL equation. However, the BFKL equation for the odderon exchange must be solved with the antisymmetric condition \([2.1]\). Therefore, even if the evolution equations are the same, the respective solutions behave differently. In particular, it is known that the highest intercept of the C-odd BFKL solution is given by 1 which is smaller than the (hard) pomeron intercept \([4]\).

In the mean-field approximation, Eqs. (3.1)–(3.2) reduce to a closed system of coupled, non-linear, equations for \( \langle N \rangle_\tau \) and \( \langle O \rangle_\tau \) (which was named as WHIMIKS equation in Ref. [5])

\[
\frac{\partial}{\partial \tau} \langle O_{xy} \rangle_\tau = \frac{\bar{\alpha}_s}{2\pi} \int d^2 z \ M_{xyz} \left[ \langle O_{xz} \rangle_\tau + \langle O_{zy} \rangle_\tau - \langle O_{xy} \rangle_\tau - \langle O_{yx} \rangle_\tau - \langle N_{zy} \rangle_\tau - \langle N_{xz} \rangle_\tau \langle O_{zy} \rangle_\tau \right],
\]

\[
\frac{\partial}{\partial \tau} \langle N_{xy} \rangle_\tau = \frac{\bar{\alpha}_s}{2\pi} \int d^2 z \ M_{xyz} \left[ \langle N_{xz} \rangle_\tau + \langle N_{zy} \rangle_\tau - \langle N_{xy} \rangle_\tau - \langle N_{yx} \rangle_\tau - \langle N_{zx} \rangle_\tau \langle N_{zy} \rangle_\tau + \langle O_{xz} \rangle_\tau \langle O_{zy} \rangle_\tau \right].
\]

The first of these equations has been already proposed in Ref. [11] as a plausible non-linear generalization of the BFKL equation in the C-odd channel. The second equation is the Balitsky-Kovchegov equation supplemented by a new term describing the merging of two odderon.

One of the significant consequences of the nonlinear effects in the factorized evolution equation (the 2nd equation) is that the odderon amplitude \( \langle O \rangle_\tau \) will decay into zero with increasing energy. This is easily seen by noting that when the pomeron amplitude \( \langle N \rangle_\tau \) is close to 1 (deeply in saturation regime), the nonlinear terms in Eq. (3.3) cancel the first two terms on the r.h.s. and the resulting equation for \( \langle O \rangle_\tau \) simply implies decrease of the solution. Therefore, as one goes to higher energies, the odderon contribution becomes less and less important.
3.2 The 3-quark–CGC scattering

A straightforward application of the JIMWLK equation to the relevant operator \[2.3\] leads to the desired evolution equation. However, the resulting equation is highly complicated and it seems difficult to extract physical information from the result. Instead, in this talk, we rather show the evolution equation for the weak-field version \[2.6\] of the operator. This is indeed sufficient for the comparison of our result with the conventional description of the odderon exchange, the linear BKP equation \[6,7\]. After a straightforward but lengthy calculation, the following linear closed evolution equation for \( \langle O_{xyz} \rangle_\tau \equiv \langle O(x,y,z) \rangle_\tau \) is obtained

\[
\frac{\partial}{\partial \tau} \langle O_{xyz} \rangle_\tau = \frac{3\alpha_s}{4\pi} \int d^2w \, M_{xwy} \left( \langle O_{xwz} \rangle_\tau + \langle O_{wyz} \rangle_\tau - \langle O_{xyz} \rangle_\tau - \langle O_{wwz} \rangle_\tau - \langle O_{xxw} \rangle_\tau - \langle O_{yyw} \rangle_\tau - \langle O_{xyw} \rangle_\tau \right) 
+ \{ \text{2 cyclic permutations} \}.
\] (3.3)

Let us compare this result with the BKP equation. At first glance, our result \[3.3\] does not look equivalent to the BKP equation. In fact, within our framework, the BKP equation rather appears as the evolution equation for the 3-point Green’s function defined by \( f_\tau(x,y,z) \equiv d^{abc} \langle \alpha^a_x \alpha^b_y \alpha^c_z \rangle_\tau \).

Indeed, the evolution equation for this Green’s function reads

\[
\frac{\partial}{\partial \tau} f_\tau(x,y,z) = \frac{\alpha_s}{4\pi} \int d^2w \, M_{xwy} \left( f_\tau(x,w,z) + f_\tau(w,y,z) - f_\tau(x,y,z) - f_\tau(w,w,z) \right) 
+ \{ \text{2 cyclic permutations} \}.
\] (3.4)

Notice that this equation is nothing but the Fourier transform of the BKP equation which is usually written in the momentum space. Since the 3-quark odderon operator Eq. \[2.6\] can be represented as a linear combination of the 3-point Green’s functions, the equivalence between our result \[3.3\] and the BKP equation is essentially established. The apparent difference appeared because our operator partly contains the information of the impact factor of the projectile.

However, there is a caveat when we write \[3.3\]. In fact, since the Green function \( f_\tau(x,y,z) \) is not gauge invariant, if one applied the original JIMWLK equation to this operator, one would obtain a result which is different from \[3.3\] and is even ill-defined due to infra-red divergences. Instead of doing this, we have derived \[3.3\] from the simplified version of the JIMWLK equation which is free of any infra-red divergences and is justified for gauge invariant operators. This means that we can use the simplified JIMWLK equation to gauge variant operators as far as we finally consider gauge invariant quantities. In other words, the use of the simplified JIMWLK equation for the Green function corresponds to a kind of regularization.

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