Numerical study on various vortices in pump sump based on LBM-LES

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Abstract. The complex flow pattern in the intake sump of pump station affects the safe operation of the pump unit, and therefore it is particularly important to analyse the causes and the evolution of various vortex. In this paper, the combined model based on LBM (Lattice Boltzmann Method) and LES (Large Eddy Simulation) is used to investigate the flow pattern, where the rigid-lid assumption is employed to deal with the free surface in the intake sump. The numerical results show that, the free surface vortex forms behind the pump bell and rotates in anti-clockwise direction, there exists one vortex separately near the bottom parts of both side walls and there is a low velocity recirculation zone near the bottom of the center of the back wall. The velocity variation of the vortex core of subsidence vortex at different depths below the free surface demonstrated that the peripheral velocity has the minimum value at the center of the vortex core. The predicted flow pattern is in accordance with the actual rule, and the location, shape and size of the free surface vortex in the water intake are captured well, all the analyses and comparisons showed that the LBM-LES model and the rigid-lid assumption method is more accurate to simulate the free surface vortex, and can provide guidance for structural improvement and performance optimization of the pump sump.

Key words: pump sump; LBM-LES; free-surface vortex; numerical simulation

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1. Introduction

The intake pool is a hydraulic structure from which the pump suction pipe takes water directly, and the internal flow inside it is complicated⁴. In order to ensure that the intake pool provides a good incoming condition for the pump, the flow pattern inside the inlet pool are mainly observed by means of physical model test. Therefore, it is of great significance to study the flow law in the suction tank of a pumping station. The study shows¹ that there are various vortexes in the inlet pool of an open pumping station under most working conditions. These swirling vortexes will affect the flow pattern into the pump, and
further affect the operating efficiency of the pump unit, and even cause pump cavitation, vibration, and noise, which probably lead to the consequence that the pump unit can not work normally\[2\].

This paper mainly introduces D3Q19 model and LBM-LES coupling model, and then the gas-liquid interface is treated by using the rigid lid. The various vortexes in the inlet pool is numerically simulated, and the evolution of the free-surface vortex behind the horn tube in the region is captured well\[3\]. It is easy to form a low-velocity reflux zone near the center of the back wall, and the flow pattern is basically the same on the left and right sides of the wall.

2. Lattice Boltzmann method

2.1. The D3Q19 lattice model

Lattice Boltzmann equation (LBE) viewed as a special ‘discretization’ form of continuous Boltzmann equation is the evolution equation of the distribution function with discrete time,

\[
f_i(x + e_i \Delta t, t + \Delta t) - f_i(x,t) = -\frac{1}{\tau} (f_i(x,t) - f_i^{(eq)}(x,t))
\]  

(2.1)

where \( f_i \) is the distribution function for particles with discrete velocity \( e_i \) at position \( x \) and time \( t \) along the \( i \)th direction of velocity \( e_i \), \( \Delta t \) is the time step, \( f_i^{(eq)} \) is the corresponding local equilibrium distribution, and \( \tau \) is the single relaxation time.

The left-hand terms of the Eq. (2.1) model a streaming step for fluid particles while the right-hand terms express the collisions process through relaxation

\[
f_i(x,t) = f_i(x,t) - \frac{1}{\tau} (f_i(x,t) - f_i^{(eq)}(x,t))
\]  

(2.2)

Collision:

\[
f_i(x + e_i \Delta t, t + \Delta t) = f_i(x,t)
\]  

(2.3)

Streaming:

where \( f_i \) is the post-collision distribution function.

A cubic lattice with 19 particle discrete velocity directions (D3Q19 model\[5\]) adopted here is illustrated in Fig. 1.

![Fig.1. D3Q19 lattice model](image)

The particle discrete velocity \( e_i \) is defined as

\[
e(c) =\begin{bmatrix}
0 & -1 & 0 & 0 & -1 & -1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & -1 & 0 & -1 & 1 & 0 & 0 & -1 & -1 & 1 & 0 & 1 & 1 & -1 & 0 & 1 & 1 & -1 & 0 & 0 & 1 & 0 & 1 & -1 & 1 & -1
\end{bmatrix}
\]  

(2.4)

where \( c = \Delta x / \Delta t \), \( \Delta x \) is the lattice spacing.

In the present paper, the equilibrium distribution function \( f_i^{(eq)} \) is denoted as

\[
f_i^{(eq)} = \alpha \rho \left[ 1 + 3 \frac{e_i \cdot u}{2} + \frac{9}{2} (e_i \cdot u)^2 - \frac{3}{2} u^2 \right]
\]  

(2.5)
The equilibrium distribution function \( f^{(eq)} \) does not only depend on the local density \( \rho \) but also macroscopic velocity \( u \). The weighting coefficient \( \omega_i \) are given as follows

\[
\begin{align*}
\omega_i = \begin{cases} 
\frac{1}{3} & i = 0 \\
\frac{1}{18} & i = 1, \ldots, 6 \\
\frac{1}{36} & i = 7, \ldots, 18
\end{cases}
\end{align*}
\]  
(2.6)

Once the distribution functions in the LBE are solved, the macroscopic variables, such as density \( \rho \), macroscopic velocity \( u \) and pressure \( p \) can be yielded by statistics from the first and two moments of the distribution functions by

\[
\rho = \sum_{i=0}^{18} f_i, 
\rho \frac{u}{\rho} = \frac{1}{\rho} \sum_{i=0}^{18} e_i f_i, 
\rho = \frac{1}{3} c^2 = \rho c_s^2
\]  
(2.7)

where \( c_s^2 = c^2/3 \) is the sound speed of system.

### 2.2. LBM-LES coupling model

In turbulent flow, the viscosity coefficient of the fluid is affected by turbulence. LES introduces a turbulence influence factor to reflect the effect of turbulent flow. For the single relaxation model of LBM, this turbulence effect is directly reflected in the equivalent relaxation time \( \tau_e \), which is related to the equivalent viscosity coefficient including the molecular viscosity and the turbulent viscosity based on SGS model. The equivalent turbulent viscosity \( \nu_e \) is as follows,

\[
\nu_e = \nu + \nu_{sgr}
\]  
(2.8)

where \( \nu \) is the molecular viscosity coefficient and \( \nu_{sgr} \) is the subgrid vortex viscosity coefficient.

The SGS model is the most widely used in the subgrid scale stress model, and the subgrid viscosity coefficient \( \nu_{sgr} \) is defined as follows,

\[
\nu_{sgr} = (C_s \overline{\Delta})^2 |\tilde{S}| 
\]
(2.9)

where \( C_s \) is the Smagorinsky constant; \( \overline{\Delta} \) is the filter scale, and generally taken as the grid volume cubic root \( \Delta = \frac{1}{N} \sum_i \Delta_i \), \( |\tilde{S}| \) is the modulus of strain rate tensor. Its definition is as follows,

\[
|\tilde{S}| = \sqrt{2\tilde{S}_{ij} \tilde{S}_{ij}} 
\]
(2.10)

\[
\tilde{S}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) 
\]
(2.11)

The relationship between the equivalent viscosity coefficient \( \nu_e \) and the equivalent relaxation time \( \tau_e \),

\[
\nu_e = \frac{2\tau_e - 1}{6} c^2 \Delta t 
\]
(2.12)

The equivalent relaxation time \( \tau_e \) is replaced by the relaxation time \( \tau \) in the evolution equation (2-1) of the LBM single relaxation model. Then the LES is introduced into the LBM to construct the LBM-LES coupling model\(^\text{[10]}\), and the evolution equation of the coupled model is obtained.

\[
f_e(x+e\Delta t, t+\Delta t) - f_e(x, t) = -\frac{1}{\tau_e} (f_e(x, t) - f^{(eq)}(x, t)) 
\]
(2.13)

### 2.3. External force term treatment

The lattice Boltzmann model can deal with the gravitational force, i.e., external force. The corresponding LBE considering external forces is as follows \(^\text{[9]}\),
\[ f_i(x + e_i \delta t, t + \delta t) - f_i(x, t) = \omega \left[ f_i^n(x, t) - f_i(x, t) \right] + \frac{\delta t}{2} F_i \]  

where \( F_i \) is the force components in the \( i \)th direction, which can be derived from the external force item expressed in the following relationship,

\[ F_i = (2 - \omega) \left( \frac{e_i - u}{c_i^2} + \frac{e_i \cdot u}{c_i^9} \right) F \]  

At the same time, the flow velocity is modified as

\[ \rho u = \sum_i e_i f_i + \frac{1}{2} \delta t F \]  

The evolution equations and (2.14) equations and (2.16) are used to calculate the velocity of the flow.

### 3. Rigid-lid assumption

The free surface inside open pump intake is treated based on rigid-lid assumption with a regular fixed surface. The free surface of the computational domain is defined as a symmetric boundary, which is impenetrable. In most cases, the general symmetric boundary condition is given according to the solid wall condition of complete slip, that is,

\[ \frac{\partial \phi}{\partial n} = 0, \quad w \big|_{\text{free surface}} = 0 \]  

where \( \phi = u, v, p \), which represent the velocity components in the \( x \) and the \( y \) coordinate directions, and the pressure; \( w \) represents the velocity component in the \( z \) coordinate direction.

### 4. Numerical investigations

In this paper, the flow fields were carried out numerically inside the model of pumping station intake. The detailed parameters of the model are shown in the Fig. 2, in which the inner diameter of horn tube is \( d=100\text{mm} \).

**Fig.2.** Pump sump geometric model and main parameters

**4.1. Free surface vortex**

The streamlines in the Fig. 3 clearly show the vortex size and distribution. When it hits the back wall, the flow forms two non-symmetrical free surface vortexes behind the horn tube and a small vortex in the corner.
Fig. 3. Free surface vortex

(a) X-Y plane
(b) X-Z plane

Fig. 4. Submerged vortex on free surface

It can be seen from the Fig. 4 that there is a submerged vortex related to the free surface on the side of the horn tube. The vorticity distribution of X-Z plane is intercepted with the center of the vortex as the coordinate. The velocity distribution law of vortex in x direction at different depths is obtained.

The distribution of velocity $u$ at different depths is demonstrated in Fig. 5 based on vortex core shown in Fig. 4. The depths below the free liquid surface are taken as $Z=315\text{mm}$, $Z=295\text{mm}$, $Z=265\text{mm}$, $Z=245\text{mm}$ respectively. The velocity $u$ distribution is used to confirm the tangential velocity of the subsidence vortex at different depths, where the center of the vortex core is taken as the origin and the maximum radius of the vortex core on the free surface along the y axis can be obtained at the maximum of the velocity $u$. The variation law of circumferential velocity along the radius direction and the law of the development of the maximum value at different depths are similar.

As can be seen from the above figures, due to the suction by the subsidence vortex, the velocity value extends downward underneath the free surface, and the velocity reaches the minimum value at the bottom of the vortex. From the point of view that the vortex center increases along the radius, the flow velocity $u$ increases gradually with the increase of radius, but decreases gradually after increasing to a certain extent, and there is a maximum velocity in the process of this change.

4.2. Submerged vortex
The two monitoring sections is taken from the cross section of 0.35d from both side wall of the inlet pool, and the monitoring section near the back wall is taken from the section 0.35d from the back wall. The series of streamline distributions at different time are analysed in the following context, where the different time moment is taken as \( T = 1 \times 10^4 \Delta t \) and \( \Delta t = 0.5 \times 10^{-6} \text{s} \).

The Fig. 6 shows the evolution of the vortex streamline near the back wall from \( t = 10T \) to \( t = 12T \). It can be seen that the vortexes begin to form due to the flow impingement to the back wall and bottom walls, and finally a symmetrical return region appears gradually in the center of the back wall, and some small swirls are captured near the side wall. The streamline form is chaotic and the flow pattern is complex.

The Fig. 7 shows the vortex streamlines distributions near the side wall-1 from \( t = 10T \) to \( t = 12T \), it can be seen that as the flow gradually moves backward, many small vortexes forms near the bottom wall, and the swirl region change obviously. Finally, a large vortex appears near the back wall.

The side wall vortex appears near the lower side of the back wall, which is affected by the impact. It can be seen from Fig. 7 and Fig. 8 that the left and right wall-attached vortexes have good symmetric characteristics, and the shape and position of the vortex are basically identical.

**5. Conclusion**

In this paper, based on the analyses of the numerical simulation results of the combined LBM-LES coupling model, it was found that all the predicted results are in a good agreement with the actual situation. The location and size of the free surface vortex in practice are well captured. The vortex region near the side wall and the back wall easily forms near the bottom wall, and there is a low velocity secondary flow zone near the bottom wall in the center of the back wall. The velocity variation of the
vortex core at different depths of the free surface subsidence vortex demonstrates that the velocity \( u \) reach to a minimum value at the center of the vortex core due to the influence of swirl surface rotation. And the variation law of velocity value at different depth is basically similar.

In general, the combination LBM-LES model can well capture the position and shape of various vortexes in the inlet pool in the unsteady calculation of the flow field. Therefore, LBM-LES can be further used to optimize the structure and to analyse the performance of the pump sump.

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