On the fundamental structure of nature and the unification of forces

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Abstract

A model for the fundamental structure of nature is presented. It is based on two fundamental fermions moving with the velocity of light and differing from each other by the projection of the spin on the momentum vector. The energy of both fermions is proportional to the momentum, which is scaled inversely with the size of a length quantum. All the known forces are a manifestation of one elementary interaction, the spin exchange or spin flip-flop which takes place when two different elementary fermions come together in the same space cell. At this stage the model can explain the properties of the photon as a two-fermion particle and it can be shown that the Dirac theory for relativistic fermions could be deduced from this model. The model predicts that particles like the electron or the quark are stable combinations of a large number of the fundamental fermions, but proof of that prediction has not been given.

introduction

Today’s theories and models of elementary particles and interactions are based in part on ad hoc assumptions and have internal inconsistencies which are unaccounted for. One of the main difficulties is based on the fact that particles like electrons or quarks, which have a relatively low rest energy, appear in all experiments to be pointlike. The combination of low energy and confinement to a very small region in space should be impossible because basic laws tell us that confinement goes together with high momentum components and that high momenta give a high contribution to the energy. This difficulty arises also for the photon which has zero rest mass and is so small in the plane perpendicular to its momentum vector that it has virtually no cross section for collisions with other photons.

The contradiction between confinement and low energy can not be discarded by an ad hoc introduction of negative potential energy as is done for systems of interacting particles. Negative potential energy is presented as resulting from the
presence of force-carrying particles like gravitons or photons or gluons. These carriers of the interactions, if confined, should give themselves a high momentum contribution to the energy. In a consistent model all particles should be treated equal with respect to the general laws of physics.

A second point which has not been treated satisfactorily is the symmetry of negative and positive eigenvalues of the Dirac Hamiltonian for relativistic fermions. The positive values have been attributed to real particles whereas the negative ones are piled up in an infinite sea of ghost particles and only the defects in that sea of particles are thought to be real.

The model of the material world which is presented here deals with those difficulties by introducing two fundamental fermions as universal building blocks for all the particles, including those which carry the interactions. The combination of high momenta with low energy is made possible by accepting negative energy states as real. Essential features of this model are a simplification of the Dirac Hamiltonian by elimination of the rest mass term and a reinterpretation of the Hamiltonian, making the square of it responsible for the evolution of a system of interacting particles.

1 the elementary fermions

To preserve the antisymmetry which is manifested in so many ways in natural phenomena, the fundamental building blocks should be fermions. The uncertainty relation

$$\Delta E \Delta t \geq \hbar$$

(1)

couples $\Delta E$, the uncertainty in the energy of a particle, to the time span $\Delta t$ during which the energy is measured. A similar relation holds between the uncertainty of the momentum $p$ and the dimension of the space cell in which it is evaluated. It is therefore meaningless to define the value for the momentum or the energy of a fundamental building block at one instant or at one point in space. The closer a building block of a particle with spherical symmetry comes to the center, the higher the value of the azimuthal momentum component. The maximum value depends on the choice of a quantum of space, conveniently introduced to avoid infinities which result from taking $r$, the distance from the center, to zero. If we choose a quantum $\Delta r_o$ and a corresponding $\Delta t_o$, related to it through $\Delta r_o = c \Delta t_o$ where $c$ is the velocity of light, we set the maximum momentum and energy of a fundamental building block equal to $p_o = \hbar/\Delta r_o$ and $E_o = \hbar/\Delta t_o$.

A spin 1/2 fermion has two orthogonal spin states. The spin up and the spin down variant can therefore be different particles. They can be transformed into one another by a spin flip-flop. If the transformation occurs every half cycle of a cyclic motion, an averaged value of the momentum and the energy can be calculated by integration over the period of the cycle. It is a well known phenomenon in atomic physics that in the presence of strong coupling a new energy level halfway between two levels is formed as a result of fast switching between these levels. A quantity measured by integration over a period which
is much longer than the switching time is determined by the properties of the new intermediate level. If this fast switching occurs between a positive and a negative energy state, the intermediate level can have an energy close to zero, even if the unperturbed energy levels are far from zero.

The difference between this model and Dirac’s theory for elementary fermions, which also uses the option of positive and negative energies, is that we start here with elementary fermions moving at the velocity of light and therefore without rest mass, and construct localised particles with them. In our model the rest energy is a consequence of internal motion and it is very small compared to the relativistic energies of the constituents. In Dirac’s theory the rest energy is introduced ad hoc.

We introduce two elementary fermions called ”ef” and use the name particles for stable or quasi-stable combinations of efs. One ef is righthanded (ref), the other one lefthanded (lef). They move with the velocity of light and both have spin 1/2. For the ref the momentum and spin vector are parallel and for the lef they are antiparallel:

ref : momentum →, spin →
lef : momentum →, spin ←

As will become clear by analysis of the velocity \( d\vec{r}/dt \), the direction of motion of each ef is along the spin vector rather than the momentum vector. A ref and a lef ”meet” each other by entering the same space cell. In that cell the spin exchange leads to a scattering of the motion and this exchange is at the origin of the fast ref-lef transitions which lower the energy of every fundamental fermion bound in a particle. The fundamental interaction between efs can be defined as a spin flip-flop. The levels between which the fast switching takes place are \( \pm E_0 \) for a particle with the size of an elementary space cell. Combinations of equal numbers of refs and lefs form bosons. Fermionic particles are formed by combining \( n \) refs with \( m \) lefs where \( m \) and \( n \) differ by an uneven number. This number difference is probably only 1 for all the known fermions. Fermions in stable form are therefore bosons combined with one extra ref or lef.

2 short recapitulation of the Dirac theory for fermions

The Dirac Hamiltonian \( H_D \) is a four by four matrix of the form:

\[
H_D = c(\hat{\alpha}\cdot\vec{p}) + mc^2\hat{\beta}
\]  

(2)

where \( \hat{\alpha} \) is a matrix composed of two Pauli spin matrices \( \hat{\sigma} \) in off-diagonal positions and \( \hat{\beta} \) has two unit matrices with different sign on the diagonal:

\[
\hat{\alpha} = \begin{pmatrix}
0 & \hat{\sigma} \\
\hat{\sigma} & 0
\end{pmatrix}
\]  

(3)
and

\[ \hat{\beta} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]  

\[ \hat{p} \] is the momentum operator \(-i\hbar \vec{\nabla} = -i\hbar \nabla\). The Pauli spin matrices \(\sigma_x, \sigma_y, \sigma_z\) for spin quantisation in the z-direction are given by:

\[ \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]  

(5)

\[ \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \]  

(6)

\[ \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \]  

(7)

\(H_D\) operates on a four-vector as shown in the following equation:

\[ H_D = \begin{pmatrix} mc^2 & 0 & -i\hbar \frac{\partial}{\partial \tau} & -i\hbar \left( \frac{\partial}{\partial x} + i\frac{\partial}{\partial \eta} \right) \\ 0 & -mc^2 & -i\hbar \left( \frac{\partial}{\partial \zeta} + i\frac{\partial}{\partial \eta} \right) & i\hbar \frac{\partial}{\partial \zeta} \\ -i\hbar \left( \frac{\partial}{\partial \zeta} + i\frac{\partial}{\partial \eta} \right) & i\hbar \frac{\partial}{\partial \zeta} & 0 & -mc^2 \\ -i\hbar \left( \frac{\partial}{\partial \tau} + i\frac{\partial}{\partial \eta} \right) & i\hbar \frac{\partial}{\partial \zeta} & -ic \hbar \left( \frac{\partial}{\partial x} - i\frac{\partial}{\partial \eta} \right) & mc^2 \end{pmatrix} \]

\(\text{operating on}\)

\[ \begin{pmatrix} f_1 | + > \\ g_1 | - > \\ f_2 | + > \\ g_2 | - > \end{pmatrix} \]  

(8)

In the four-vector on which the Hamiltonian \(H_D\) operates \(f_1, g_1, f_2, g_2\) are functions of space and time coordinates. This four-vector can be interpreted as two two-vectors \(\phi\) and \(\chi\) which are transformed into each other and the eigenvalue \(\epsilon\) of \(H_D\) for an isolated particle with rest mass \(m\) can be obtained from two matrix equations:

\[ c(\hat{\sigma}, \hat{p})\chi = (\epsilon - mc^2)\phi \]  

(9)

\[ c(\hat{\sigma}, \hat{p})\phi = (\epsilon + mc^2)\chi \]  

(10)

The operator \(c(\hat{\sigma}, \hat{p})\) is nondiagonal but the square of this operator is diagonal with an eigenvalue \((\epsilon^2 - (mc^2)^2)\) which is equal to \(c^2 p^2\). The eigenvalue \(\epsilon\) is interpreted as the energy of the particle. It can have a negative as well as a positive value, and there is no a priori reason to prefer one over the other.

### 3 the evolution operator in this model

An operator acting exclusively on efs, which have no rest mass, can have no \(mc^2\) term as in Dirac’s Hamiltonian.

The operator \(c(\hat{\sigma}, \hat{p})\) based on Pauli spin matrices \(\hat{\sigma}\) is a Hamilton operator when it is applied to isolated efs. Its plane wave eigenfunctions are:

\[ Ne^{ik.r} | + > \text{ with eigenvalue } chk \]

\[ Ne^{ik.r} | - > \text{ with eigenvalue } -chk \]

where \(N\) stands for a normalisation factor and the spin eigenstates \(| + >\) and \(| - >\) are for quantisation in the direction of \(\vec{k}\).
All functions of space coordinates and the spin states for interacting efs can be expanded in plane wave spinors. However, for localised particles which are centered around one point or around an axis, spherical or cylindrical spinors form a more natural basis. The only type of spinor which is completely worked out in this paper is that of the photon, with cylindrical symmetry. Therefore the examples given below are for cylindrical symmetry but the changes which have to be made for spherical symmetry are straightforward.

In cylindrical coordinates the radius \( r \) is defined in the xy plane

\[
\begin{align*}
\hat{\sigma}.\vec{p} &= -i\hbar(\sigma_z\partial_z + \sigma_+\partial_- + \sigma_-\partial_+) \\
\sigma_{\pm} &= \frac{\sigma_x \pm \sigma_y}{\sqrt{2}} \\
\partial_{\pm} &= e^{\pm i\varphi} \left( \frac{\partial}{\partial r} \pm \frac{i}{r} \frac{\partial}{\partial \varphi} \right)
\end{align*}
\]

The spinor wavefunction \( \psi \) of a particle with two or more efs can be written as a symmetrized combination of products of spinors, each spinor referring to the coordinates of one ef:

\[
\psi = \left( \begin{array}{c} u_1 \\ v_1 \end{array} \right) \left( \begin{array}{c} u_2 \\ v_2 \end{array} \right) \cdots \left( \begin{array}{c} u_i \\ v_i \end{array} \right) + \text{symmetric permutations}
\]

where the \( u_i \) and the \( v_i \) are fuctions of space and time coordinates.

The operator \( c(\hat{\sigma},\vec{p}) \), which we will denote by \( H \), although it is in general not diagonal and therefore not an Hamiltonian, must act consecutively on each of the spinors and could therefore be represented as a sum:

\[
H = \sum c(\hat{\sigma}_i,\vec{p}_i)
\]

The sum should run over all efs which are not excluded by antisymmetry from the space occupied by the particle. It should therefore include efs in vacuum states when they interact with the efs in a particle. \( H \) is a local operator and so is also \( H^*H = -H^2 \). The ij product terms in \( H^2 \) refer to different efs when \( i \neq j \) but the interaction must be local, referring to efs in the same space-time
cell. 

$H^2$ is the interaction operator in our model. The combination of raising and lowering which transforms a left in a right and vice versa, is induced by $H^2$, more specifically in the terms producing a spin exchange in $|+> - > or |-> + >$ combinations.

The spinor function of a stable composite particle for which the left-right transformations form closed vertices must therefore be an eigenfuncion of $H^2$. The "Zitterbewegung" which was recognised shortly after the introduction of the Dirac Hamiltonian is a consequence of this cyclic raising and lowering. It is easily interpreted in our model because $d\vec{r}/dt$ can be evaluated by commuting $\vec{r}$ with $c(\vec{\sigma}.\vec{p})$, the result of this commutator being $c\vec{\sigma}$. A bound ef executes a "Zitterbewegung", a sort of spiraling motion which is guided by the spin precession.

For non-interacting efs the cross terms in $H^2$ are absent and then we have, by virtue of

$$(\vec{\sigma} \cdot \vec{A})(\vec{\sigma} \cdot \vec{B}) = \vec{A} \cdot \vec{B} + i\vec{\sigma} \cdot (\vec{A} \times \vec{B})$$

$$<HH>=\sum (c\vec{\sigma}_i\vec{p}_i)(c\vec{\sigma}_i\vec{p}_i)= - \sum c^2\vec{p}_i^2$$

where the $p_i$ are numbers and $<...>$ stands for integration in order to calculate the expectation value of the operator.

Then the operator $H^2$ measures minus the sum of the squares of the energies. The interaction terms give contributions which reduce the norm of $<H^2>$, the expectation value of $H^2$.

Efs are never really isolated. Even if they are subjected to strong localisation in particles the tails of the spinors will extend in space and overlap with other particles. The exchange of photons, which are described further on, is a form of interaction through vacuum polarisation and is much more efficient than an overlap by exponentially decaying tails. The products of spinors in the expression for the total wavefunction which describes several interacting particles should always be completed by including all the photons allowed by the available degrees of freedom. The interaction may reduce $<H^2>$, by increasing the overlap for a right-left combination or decreasing it for right-right or left-left combinations. It would leave $<H^2>$ unchanged if the particle would be isolated and in an eigenstate of $H^2$. But this is never realised. Any change in $<H^2>$ for the initial system of efs will give rise to the emission of new particles and thereby increase the entropy of the system, and this process goes on continuously.

The evolution of a system of particles is a consequence of a tendency towards maximum entropy.

The physical interpretation of the $H^2$ operator which emerges from these considerations is that it represents the negative square of the energy if applied to one particle and minus the sum of the squares of energies for several non-interacting particles. Creation of new particles, a process which goes on continuously, reduces $<-H^2>$ because the new particles are formed under conservation of total momentum and the sum of the squares of fractions is smaller than the square
of the whole. In this process the entropy of the system is increased. \(-H^2\) is therefore never really constant. It may be approximately constant for a given restricted time scale and for corresponding restricted regions of space, in which all the efs execute cyclic motions, being bound in well-defined particles. This corresponds to having a set of closed vertices in Feynman diagrams. \(-H^2\) changes when the vertices are opened up and new particles are formed.

A proposition which is essential to our model is that a fermionic particle, consisting of an ef with a cloud of refs and lefs in equal numbers bound to it, can interact with a similar particle and form a new quasi-stable eigenstate of \(H^2\). In that case the particles formed in the interaction are virtual particles, emitted but also reabsorbed in a closed interaction vertex. This is at the origin of the formation of structures like the proton with three fermionic particles or the hydrogen atom with a proton interacting with an electron and so on.

4 the link with the Dirac Hamiltonian

For a particle with the center of mass c.m. at a position \(\vec{r}_m\) and momentum \(\vec{p}_m\) corresponding to the motion of the c.m., and with internal position vector \(\vec{r}_i\) and corresponding momentum \(\vec{p}_i\) for the efs in the particle, the total momentum operator can be expressed as a sum \(\vec{p}_m + \vec{p}_i\). The operator \(H\) can consequently be split in two parts:

\[
H = \sum c \vec{\sigma} \vec{r}_i + \sum c \vec{\sigma} \vec{p}_m
\]  

(18)

The first part refers to an internal contribution to the energy of the particle and can therefore be identified with the rest energy \(mc^2\). The second part refers to the motion of the c.m. and is the kinetic term. The rest energy term is zero if the contributions of refs and lefs cancel out. It will be shown that this is the case for the photon in our model, and it can be expected from symmetry arguments that the same conclusion will hold for all single-particle bosons in the ground state.

For fermionic particles with only a few refs and lefs the lowest possible value for the rest mass term will be huge on account of the huge value of each \(p_i\). Only when large numbers of efs combine together and lead to almost complete cancellation will the rest mass term be relatively small with respect to the energies of the efs. With respect to the energy in the c.m. motion however, the rest mass term of the particles studied in the laboratory is very big in most experiments. That is the typical situation to which the Dirac theory is applied.

The Dirac Hamiltonian shown in equation (8) hides the internal coordinates of a fermionic particle. The transformation from positive to negative energy states refers to the elementary interaction with the raising and lowering operators which are combined with spin flips of the efs. Each elementary interaction causes a switching between these states. The total spin of the particle however may or may not be changed by the interaction.

\[\partial/\partial z\] transforms \(f_1\) in (8) into \(f_2\) and leaves the spin unchanged, whereas \[\partial/\partial x + i\partial/\partial y\] transforms \(f_1\) into \(g_2\) with a spin-flip and \[\partial/\partial x - i\partial/\partial y\] transforms
g_1 \text{ into } f_2 \text{ with a reverse spin-flip. The spins refer here to the total fermion, not to the individual electrons in it. Some non-diagonal terms in the Dirac Hamiltonian, non-diagonal terms which are responsible for the coupling between positive and negative rest energies, cause spin flips whereas other terms leave the spin unchanged. Therefore the sign of the scalar spin-momentum product and the sign of the energy may be different for the states described by a four-vector, in contrast with electrons for which these signs are identical. A fermionic particle therefore has different quantum numbers for the spin and the charge, the latter discriminating between particles and antiparticles. An electron on the contrary can be characterised by one quantum number, the sign of the projection of the spin on the momentum vector. A particle has the same charge as the extra electron in it and differs from the corresponding antiparticle by the fact that one has a right electron and the other a left electron combined with an uncharged cloud.}

5 the photon

In its ground state the photon has zero energy and it must consist of an equal number of right and left electrons in very strong interaction, giving a complete cancellation of positive and negative terms in the energy. Excitation gives the photon an energy equal to c times its momentum in the direction of propagation. When a photon is exchanged between particles, the total angular momentum, orbital plus spin, which is transmitted is equal to \( \hbar \). The photon exhibits cylindrical symmetry, being delocalised in the direction of its angular momentum. It is therefore practical to use cylindrical spinor functions to describe the wave function of the photon. These spinors must be eigenfunctions of \( H^2 \), and therefore also of the \( \nabla^2 \) operator in cylindrical coordinates. This operator is:

\[
\left( \frac{\partial}{\partial z} \right)^2 + \partial_z \partial_r = \left( \frac{\partial}{\partial z} \right)^2 + \left( \frac{\partial}{\partial r} \right)^2 + \frac{1}{r} \frac{\partial}{\partial r} + \left( \frac{1}{r} \right)^2 \left( \frac{\partial}{\partial \phi} \right)^2
\] (19)

The following cylindrical spinor function \( \zeta \) is an eigenfunction with angular momentum \( \hbar \), non-singular for \( r=0 \), and zero momentum in the z-direction:

Spinor \( \zeta: u(r, \phi) \mid + > + v(r, \phi) \mid - > \)

where \( r \) and \( \phi \) are cylindrical coordinates, the spin states are quantised in the z-direction and \( u \) and \( v \) are given by:

\[
u(r, \phi) = N \frac{\cos(k_o r)}{\sqrt{k_o r}} - \frac{\sin(k_o r)}{\sqrt{k_o r}} e^{3i\phi/2}
\] (21)

It is interesting to note that in cylindrical symmetry the orbital angular momenta are formed by half odd integer quantum numbers, as opposed to integers
for spherical symmetry.  
The separate transformations of the raising and lowering operators are:

\[ \sigma_+ \partial_- v(r, \varphi) \mid - > = - k_o u(r, \varphi) \mid + > \]  
(22)

\[ \sigma_- \partial_+ u(r, \varphi) \mid + > = k_o v(r, \varphi) \mid - > \]  
(23)

The spinor function \( \zeta \) can be interpreted either as a stable bosonic state with total angular momentum composed of a mixture of orbital- and spin contributions or as a function describing one fermion which alternates between ref and lef so quickly that it loses identity. The alternation is caused by spin tumbling through interaction with another ef with opposite spin and momentum, whose wave function \( \xi \) is:

Spinor \( \xi \) : \( \hat{u}(r, \varphi) \mid - > + \hat{v}(r, \varphi) \mid + > \)

\[ \hat{u}(r, \varphi) = N \frac{\sin(k_o r)}{\sqrt{k_o r}} e^{-i \frac{\varphi}{2}} \]  
(24)

\[ \hat{v}(r, \varphi) = N \left( \frac{\cos(k_o r)}{\sqrt{k_o r}} - \frac{\sin(k_o r)}{\sqrt{(k_o r)^3}} \right) e^{-i \frac{3 \varphi}{2}} \]  
(25)

In diagrammatic form the photon could be described as a string of closed vertices, the interaction points referring to the spin flip-flops, as shown in the next picture:

Each ef changes sign regularly and the energy is zero when integrated over a cycle. One single photon in a number state has a ref and a lef, with opposite spin but in an undefined phase of the cycle. Its spinor function is a combination of the up part of \( \zeta \) with the down part of \( \xi \) and vice versa:

\[ \Psi_{\text{zero energy photon}} = u(1) \mid + > \hat{u}(2) \mid - > + v(1) \mid - > \hat{v}(2) \mid + > \]  
(26)

where (1) and (2) refer to the spatial coordinates of the first respectively the second ef and the spin states are coupled to the preceding wavefunction. For this spinor combination the energy, the total momentum and the total angular momentum are zero. It is a spinor in the ”vacuum”. To transform it into a real photon which transmits energy the \( \zeta \) and the \( \xi \) part in it must have a momentum component in the z-direction:

\[ \Psi_{\text{photon}} = e^{ik_1 z} u(1) \mid + > e^{-ik_2 z} \hat{u}(2) \mid - > + e^{ik_1 z} v(1) \mid - > e^{-ik_2 z} \hat{v}(2) \mid + > \]  
(27)

transmitting an energy \( \hbar k c \) and an angular momentum \( \hbar \). Source-detector exchange of a photon is a coupling of the source to the detector by a diagram
as shown above. The source and the detector interact with the same photon, each on one end. When the source interacts with the $\zeta$ part, the detector takes the $\xi$ part and vice versa. This is shown symbolically in the next figure:

\[
\begin{array}{cccccc}
+ & - & + & - & + & - \\
\text{source} & \\
\text{detector} & \\
\end{array}
\]

For the photon the transmission event is simultaneous on both sides, the time being relativistically contracted. In this picture there is no "strange correlation at a distance", an expression which is frequently used in the literature when quantum correlations are examined. The detector and the source are on equal terms here and there is nothing strange about the fact that the source "knows" which detector, eventually far away from it, is going to be selected.

The appearance of half-integer quantum numbers for the orbital angular momenta of the spinors is not in contradiction with the bosonic symmetry of a photon. The spin is rotating synchronously with the orbiting ef and one turn over the orbit must therefore change the sign of the wavefunction, this change in sign being compensated by the spin rotation. The transformation formulae for change of spin-quantisation axis, from parallel to the momentum (\(| >_p \rangle \)) to parallel to z (\(| > \rangle \)) are given by:

\[
\begin{align*}
| + >_p &= e^{-i\frac{\theta}{2}} \cos\left(\frac{\theta}{2}\right) | + > + e^{i\frac{\theta}{2}} \sin\left(\frac{\theta}{2}\right) | - > \\
| - >_p &= -e^{-i\frac{\theta}{2}} \sin\left(\frac{\theta}{2}\right) | + > + e^{i\frac{\theta}{2}} \cos\left(\frac{\theta}{2}\right) | - >
\end{align*}
\]

where $\theta$ and $\varphi$ are the polar, resp. azimuthal angles of the spin vector with respect to the z-axis. For $\theta = \frac{\pi}{2}$, as in the cylindrical frame, and for the spin of an ef rotating in the xy plane with an azimuthal part of the wavefunction $\exp(i\varphi)$, we obtain a combination of azimuthal functions as in the spinor $\zeta$. Rotation described by $\exp(-i\varphi)$ leads to spinor $\xi$.

The zero energy photonic spinors introduced above can fill up all the available space in the vacuum. This representation of the vacuum is an alternative for a representation with plane wave spinors, but is better suited to treat the interaction between two particles. A system of particles can only be in a stable configuration if all interactions with the photons which are allowed in the region of space occupied by the system lead to closed vertices, i.e. to cyclic motion of all the efs in the system.

Eigenstates of the $\nabla^2$ operator in spherical symmetry are the spherical Bessel functions, quite similar to the functions in the cylindrical spinors but with integer rather than half integer powers of $k_\alpha r$ in the denominators, the lowest order function being $\sin(k_\alpha r)/k_\alpha r$. 


6 fermions

Combinations of ref-lef pairs and one extra ref or one extra lef which form stable localised particles are fermions. This follows from the rule for summing up an odd number of half integer spins and the fact that localised particles with spherical symmetry have an integer quantum number for the orbital angular momentum.

For a ref-lef-ref trio in a c.m. frame the three momentum vectors add up to zero, as shown in the next figure:

![Diagram of momentum vectors](image)

Projection of the momenta leads to: 
\[ -p \cos(0) + 2p \cos(\frac{\pi}{3}) = 0 \].

Projection of the spin vectors give components with cosinus and sinus functions of half angles. The combined spin of a ref-lef duo can not sum up with the spin of the remaining ref to a zero spin state.

A simple argument, based on the idea that the extra ref can interact with the photon-like ref-lef combination which has in itself acquired an energy of half the energy of an ef indicates that the resulting \( \hat{\sigma} \cdot \vec{\nabla} \) might be reduced by a factor three. This would still mean a huge rest energy for the ref-lef-ref trio.

As more efs are combined in one particle, the space needed to preserve antisymmetry, that is for the efs of the same type to avoid one another, must be larger. By analogy with examples in atomic and nuclear physics a "closed shell effect" can be expected, giving the largest energy reduction for efs in a closed packing within a certain radius. Starting with a given number of efs in interaction but not in the lowest energy state, the dynamics leading to a stable particle will involve the evaporation of photons or other particles. In this process the total energy of the particle plus the photons is constant and the dynamics are therefore not dictated by a tendency to a minimum energy.

The number of entities contributing to the energy, and through that number to the entropy of the system, is the driving factor for the evolution towards stable localised structures.

Particles like the electron, with a very low rest energy, must consist of a very large number of efs. A rough estimate of the number \( n \) could be based on the consideration that the ref-lef-ref trio of the example given above can in turn be treated as a particle with momentum smaller than \( p \), and coupled to new efs,
and so on. The trend of the energy as a function of $n$ should then follow some power law like $\kappa^n c p_o$, where $\kappa$ is smaller than 1. For $\kappa = \frac{1}{2}$ and $n=100$ the reduction would be of order $10^{-30}$ with respect to $c p_o$.

In the framework of this model a proton should consist of three particles of which one has a negative charge like the electron and two are positive like the positron and all have spin $\frac{1}{2}$. The Pauli exclusion principle is not violated by considering two fermions with the same charge and spin in close interaction because we are dealing with composite particles which need not be identical. They might differ by even numbers of efs which are exchanged between them. The forces binding the particles in a nucleus may be strong enough to disrupt the internal structure of the particles and cause massive inter-particle transfers of efs, giving rise to the weak and the strong nuclear interactions.

To include also neutrons and neutrinos in the model we must explain a combination of spin $\frac{1}{2}$ with zero charge. We identify the charge sign with the spin of the extra ef, taking one sign for the ref and the opposite sign for the lef. A fermion should therefore have a fixed charge on a very short time scale. However, although the sign of the charge is preserved in a spin flip-flop between two efs, a collective interaction involving all the efs in a particle and inducing a turnover of all the spins at once will change that sign. If this is repeated periodically and with a very short period, all measurements of electric interaction by integration over a longer period will show zero charge. Neutrinos could therefore be fermions in a state of rapid collective turnover from ref to lef character. Neutrons could differ from protons by having one fermion with the same rapid turnover.

Charge conjugation which transforms matter into antimatter is identified here with ref-lef transition. Given that particles are composed of ref-lef combinations, matter and antimatter are intimately mixed in all structures. The asymmetry in atoms here in our solar system, with all nuclei positive and all electron clouds negative, must follow from an initial fluctuation at the moment of formation of these structures.

7 electromagnetism and gravitation

Interaction between two particles, in the limit of large distance between the centers, can be treated by an approximation in which the energy of the separate particles is determined independently, and subsequently the overlap of the spinors from the two centers and the connection through photons from the vacuum is considered. We can try to identify the two well-known long distance interactions: electromagnetism and gravitation. The first is caused by interparticle correlation of the extra efs in each particle, the extra efs carrying the charge of the particle. The interaction between these extra efs is mediated through flip-flop interactions with the photon spinors in the vacuum and the interaction strength is proportional to the momentum of the extra efs in the interacting particles. The second one is due to the correlation of the total ref-lef unbalance in each particle, unbalance which is expressed in the rest energy plus
eventually the kinetic energy of the c.m. of each particle. This is independent of the charge sign and can be identified with gravitation. In the argument which was used to explain the reduction in energy in the ref-lef-ref trio a gravitational type of interaction was invoked. Calculation of the interaction strength between quasi-stable particles will be a formidable task, given that a very large number of ef{s moving under conditions of strong correlation are involved. In a crude model as indicated in the next figure the order of magnitude of the electromagnetic correction can be estimated:

\[
\text{momentum } p \quad \text{momentum } p
\]

The largest value of the momentum \( p \) of ef{s in the interacting particles depends on the size of the particles. The sizes of both particles are taken to be equal. We identify \( cp_{r_0}/r \), the energy correction at distance \( r \) obtained by switching the momentum \( p \) of the extra ef, with the electric energy \( e^2/4\pi\epsilon_0r \) and obtain \( pr_{r_0} = \frac{\hbar}{6\pi} \). The reduction by 137 of the momentum \( p \) with respect to the maximum \( p_0 = \hbar/r_0 \) might be an indication of the increased wavelength in a larger particle. 137 could be related to the number of ef{s in a typical particle like the electron. It might correspond to the number of cells, densely packed up for instance to the third nearest neighbour.

To derive an order of magnitude for \( p_0 \) we could take the gravitational interaction to the limit of interaction between two neighbouring ef{s and identify \( ch/r \) with \( G(p_0/c)^2/r \), where \( G \) is the universal constant for gravitation and the masses have been replaced by the full kinetic masses \( p_0/c \). The value for \( cp_0 \) comes out as \( 2 \times 10^9 \) Joule or \( \approx 10^{28} \) eV, a huge value corresponding to \( r_0 \) of the order of \( 10^{-35} \) meters. This makes the cross section of the photon so small that it escapes experimental verification.

Concluding remarks:
The model proposed here puts particles and force carrying particles in one category. It gives a simple and universal principle for all interactions: the spin flip-flop or exchange, which occurs when elementary fermions enter the same space cell. Elementary fermions have no rest mass and therefore the operator for the energy depends only on spin and momentum operators. Dirac’s relativistic Hamiltonian for fermions is a special case when composite one-center particles are considered and only the coordinates of the center of mass together with the total angular momentum components are taken as variables. The model can be applied successfully to explain the combination of angular momentum \( \hbar \) and zero rest mass and zero charge which we know as the photon. The photon
is the only particle which has been completely described in this model. An es-
timate of its cross-section is so small that it escapes experimental verification.
All this gives strong arguments for trying to transform this model into a theory
by calculating the ground state energy of a fermionic particle like the electron.

Final note by the author:
This paper was presented in slightly different form for publication to Annalen
der Physik in July 1998. The editors replied after a few months that two reviewers had left it without comment and that a third reviewer would be contacted. There was no further message up to now.