Abstract—We present a semianalytical scheme for the design of broad-angle multichannel metagratings (MGs), sparse periodic arrangements of loaded conducting strips (meta-atoms), embedded in a multilayer printed circuit board (PCB) configuration. By judicious choice of periodicity and angles of incidence, scattering off such an MG can be described via a multiport network, where the input and output ports correspond to different illumination and reflection directions associated with the same set of propagating Floquet–Bloch (FB) modes. Since each of these possible scattering scenarios can be modeled analytically, constraints can be conveniently applied on the modal reflection coefficients (scattering matrix entries) to yield a diffractive response, which, when resolved, produce the required MG geometry. We show that by demanding a symmetric MG configuration, the number of independent S parameters can be dramatically reduced, enabling satisfaction of multiple such constraints using a single sparse MG. Without any full-wave optimization, this procedure results in a fabrication-ready layout of a multichannel MG, enabling retroreflection suppression and diffrusive scattering from numerous angles of incidence simultaneously. This concept, verified experimentally via a five-channel MG prototype, offers an innovative solution to both monostatic and bistatic radar cross section (RCS) reduction, avoiding design and implementation challenges associated with dense metasurfaces used for this purpose.

Index Terms—Diffusers, Floquet–Bloch (FB), metagratings (MGs), multichannel, multifunctionality, radar cross section (RCS), scattering.

I. INTRODUCTION

The radar cross section (RCS) of an object is a measure for the ability to detect it by the power scattered off it in a given direction when illuminated by a radar [1]. Since controlling such scattering is crucial for many defense applications, means for RCS reduction have been studied extensively over the years. The RCS depends on the respective positions of the radar antennas and the scatterer; it is customary to distinguish between bistatic radar scenarios, where the transmitter and receiver are separated by a distance comparable to their distance to the target, and monostatic radar scenarios, where the transmitter and receiver are colocated. While in the former case, the angle of incidence upon the target generally differs from the reflection angle that would be intercepted by the receiver, in the latter, the detection is based on the retroreflection from the target. Therefore, and considering that the angle of illumination is unknown in general, the ideal RCS reduction cover would need to diminish scattered fields in as many directions as possible, and perform equally well for multiple excitation scenarios.

In recent years, metasurfaces (MSs), thin sheets of closely packed subwavelength polarizable particles (meta-atoms), have been extensively investigated as a means to tackle such challenges [2]. Being low profile and conformal, and with their proven ability to control wave scattering for various applications, they have the potential to serve as effective target coatings, acting to reduce their radar signature. One approach to achieve this is to design absorbing MSs [3], [4], which could be used to suppress both monostatic and bistatic RCS at the same time. However, such devices require engineering lossy materials within thin sheets, which is often nontrivial from a practical standpoint [5].

Alternatively, one can avoid such complexities while still obtaining substantial RCS reduction with ultrathin covers by using diffusive MSs. One such common design relies on the checkerboard configuration, as proposed in [6], [7], and [8]. These structures are composed of alternating perfect electric conductor (PEC) and artificial magnetic conductor (AMC) reflective unit cells; when excited by a normally incident plane wave, the π phase shift between the PEC and AMC reflected waves cause destructive interference in that direction. While these surfaces are relatively simple to design and manufacture, they aim to reduce the monostatic RCS for normal-direction excitation scenarios (where the specular reflection coincides with the retroreflection), yet expose the target for detection from other angles.

An evolution of this idea, presented in [9], [10], [11], [12], [13], [14], and [15], uses diffusive MSs composed of elements exhibiting varying reflection phases, thereby purposefully deflecting the specular reflection toward (one or more) different directions. However, since the reflection phases correspond to normally incident wave excitations, wide-angle response is generally not guaranteed. Extending the angular response is typically obtained via brute-force optimization methods, not as an inherent part of the design, which is usually time consuming and lacks physical insight [5].

Further to the challenges described in the previous paragraphs, utilizing MSs for RCS reduction faces an additional, more fundamental difficulty, resulting from the need to obey the homogenization approximation. To form the equivalent surface constituents yielding the generalized sheet transition conditions (GSTCs) governing most of the synthesis procedures [16], dense closely packed meta-atom arrangements

Yarden Yashno, Graduate Student Member, IEEE, and Ariel Epstein, Senior Member, IEEE
Fig. 1. (a) Physical configuration of a symmetric $M = (2P + 1)$-port PEC-backed PCB MG diffuser. Front view of the MG, featuring $K$ loaded copper strips distributed within the medium in the planes $z = t_n$, with horizontal and vertical offsets $d_k$ and $h_k$, respectively, with respect to the reference meta-atom at $d_1 = h_1 = 0$. The structure is excited from below by a plane wave, with an angle of incidence $\theta_{in} = \theta_p$ associated with one of the input ports $\rho = -P, \ldots, P$. The denoted output ports correspond to the propagating Floquet–Bloch (FB) modes scattered from the MG toward the angles $\theta_q$ (see also Section II-B).

(b) Loaded wire meta-atom (top view). The loads repeat periodically with separation distance $L_x = \lambda / 10 \ll \lambda$ along the $x$-axis, effectively forming a distributed per-unit-length impedance $\tilde{Z}_k$. (c) Load detailed geometry. The width $W_k$ of the copper printed capacitor controls the effective distributed impedance $\tilde{Z}_k$.

(d) Multiangle diffusive RCS reduction functionality illustration for a five-port multichannel MG [corresponding to (a) with $P = 2$]. For each excitation scenario, the retroreflection vanishes and the incident power is split equally between the remaining four ports.

should be used. Such a requirement, however, tends to complicate the design and fabrication processes due to the need to devise and realize a large number of particles with deep subwavelength features and small separation distances. In addition, following the typical GSTC-oriented synthesis approach results in abstract MS constituent distribution, which generally implements the desired surface response for a given (single) excitation$^2$ [17]. Thus, even if one succeeds translating these constituents into a practical MS prototype, usually via full-wave simulations utilized to design the physical geometries associated with the various meta-atoms, attributing multifunctional properties to the MS (e.g., accommodating multiple angles of incidence) forms yet another nontrivial challenge.

To address these issues, we present herein an alternative approach, utilizing the concept of metagratings (MGs) to devise effective diffusive covers with broad acceptance angle. Similar to MSs, MGs are composed of small polarizable elements, arranged periodically; however, in contrast to MSs, the meta-atoms in MGs are sparsely distributed [18]. Due to their sparsity, MGs are not governed by homogenization; instead, to design them, the detailed interactions between the meta-atoms are considered based on a suitable analytical model, tying the available geometrical degrees of freedom (DOFs) to the scattered fields. Subsequently, by judicious tailoring of the meta-atom distribution and their detailed dimensions, the scattering from individual elements can be engineered to form a desired interference pattern. This yields efficient high-fidelity practical designs, avoiding the extensive full-wave optimization associated with MS synthesis while featuring simpler and easier-to-fabricate layouts [19], [20], [21], [22].

Indeed, in recent years, we have developed a semi-analytical scheme to synthesize multilayer printed-circuit-board (PCB) MGs based on capacitively loaded conducting wires as meta-atoms [see Fig. 1(a)–(c)], demonstrating the ability to manipulate beams at microwave frequencies in versatile manners with high efficiencies [23], [24], [25], [26], [27]. Similar configurations have been subsequently used in a wide variety of scenarios, allowing diverse static and dynamic beamforming [28], [29], [30], [31], [32], enhancing waveguide systems [33], [34], and alternative antenna devices [35], [36], [37].

To harness this concept to obtain broad-angle reduction of both monostatic and bistatic RCS, we utilize a multichannel scattering perspective [38], [39]. Since MGs are periodic composites, they must comply with the FB theorem. In other words, when illuminated by a plane wave, real power can only be scattered into a finite set of discrete directions (associated with the various FB modes), determined by the angle of incidence and the period length. Accordingly, we describe the MG as a multiport network, where the input and output channels are defined by the angles of incidence and the corresponding scattering angles, respectively [38], [40] [see Fig. 1(a)]. We leverage this method to design an MG, which for each excitation scenario, evenly scatters the incident power...
between the output channels, while specifically suppressing the coupling to the retroreflection channel [see Fig. 1(d)]; thus, wide-angle monostatic and bistatic RCS reduction is achieved within a single passive device.

Specifically, to design a suitable multielement multilayer MG, we harness the model presented in [23], [24], and [26], invoking first the superposition principle to evaluate separately the scattering off the given PCB dielectric stack in the absence of the MG grid (external fields), and the contribution of the secondary fields produced by the currents induced on the MG [20], [41]. Next, we utilize Ohm’s law to relate these currents to the electric fields acting on the loaded wires, forming a set of coupled equations associating the MG properties (wire positions and capacitive loads) with the induced currents and overall FB coupling coefficients. This analytical formulation, in turn, can be used to evaluate the scattered fields for given MG configuration and incident beam angle (the forward problem), serving as the basis for the inverse problem solution: retrieving the meta-atom constellation and dimensions that would generate the desired interference patterns when illuminated from each of the multiple considered excitation angles (input channels).³

Importantly, by formulating the desired scattering requirements for the possible angles of incidence as a set of constraints imposed on the structure’s scattering matrix (S-matrix), it is possible to evaluate and possibly reduce the number of DOFs needed to realize the multifunctional MG. In general, since we strive to devise a passive and lossless design, reciprocity and power conservation are enforced, identifying the minimal set of independent scattering coefficients. However, herein we impose an additional requirement, demanding that the MG configuration would be symmetric. Aligned with the broad-angle RCS reduction scenario, we show that such a symmetry requirement also reduces the number of overall independent S-parameters, translated into a reduced number of constraints. In contrast to other recently reported multifunctional MG designs, e.g., [32], [42], [43], this symmetry-oriented approach we follow allows dramatic minimization of the required elements per period, leading to a compact and sparse formations. This methodology, verified via full-wave simulations and demonstrated experimentally, yields a general and reliable approach for designing multichannel MGs for versatile and broad angle beam-manipulation applications. In particular, it paves the path to enhanced low-profile RCS reduction covers, which are simple to fabricate (spare) and design (require no full-wave optimization), successfully suppressing both monostatic and bistatic RCS for multiple angles of incidence.

³As will be laid out in detail in Section II, this design methodology differs from the one employed in [23], [24], and [26], where effectively only the wire coordinates were used directly as DOFs for the synthesis process, and the load impedances were set automatically once these were determined as to satisfy the linear set of constraints on the coupling coefficients (see (17)–(20) of [26]). Herein, we use both the load impedances and the wire coordinates as independent DOFs and rely on inverse solution of the overall (nonlinear) set of constraints (taking into account also multiple excitations), which is essential for achieving the multichannel functionality with a minimal number of meta-atoms per period.

II. THEORY

A. Scattering off a PCB MG (Analysis)

We consider a 2-D (\(\partial/\partial x = 0\)) \(\Lambda\)-periodic structure, composed of \(N\) metallization layers embedded in a dielectric substrate of permittivity \(\varepsilon_2 = \varepsilon_{sub}\) bounded within the region \(z \in [0, t_N]\), backed by a PEC⁴ at \(z = t_N\) and surrounded by a medium (air, by default) with permittivity \(\varepsilon_1\) occupying the half-space \(z < 0\) [see Fig. 1(a)]. The wavenumbers and wave impedances for each medium are given, respectively, by \(k_i = \omega(\mu_i\varepsilon_i)^{1/2}\), \(\eta_i = (\mu_i/\varepsilon_i)^{1/2}\), where \(\mu_i\) is the permeability of the \(i\)th medium; the subscript \(i\) refers to the medium where the fields are evaluated (\(i = 1, 2\)). Within a period, \(K\) wires with impedances per-unit-length \(Z_k\) are distributed among the layers, forming the MG. To maximize the available DOFs, the wires can be vertically and horizontally offset with respect to one another, and we denote the \(k\)th wire position as \((y, z) = (d_k, h_k)\), where \(d_k \in (-\Lambda/2, \Lambda/2), h_k \in \{t_1, t_2, \ldots, t_{N-1}\}\). The bottom metal layer is situated at the substrate–air interface, defined as the \(z = t_1 = 0\) plane.

The structure is excited from below by a transverse electric (TE) polarized \((E_z = E_y = H_x = 0)\) plane wave, \(E^{inc}(y, z) = E_{in}e^{-jky \sin \theta_{inc}} e^{-j\omega \beta_{inc}}\), with amplitude \(E_{in}\) and angle of incidence \(\theta_{inc}\). According to the FB theorem, the scattered fields may couple only to a discrete set of FB modes, with a finite number of them being propagating (and the rest evanescent). Specifically, the transverse and longitudinal wavenumbers of the \(m\)th FB mode in the \(i\)th medium are determined by the period length and the incidence angle via [41]

\[
k_{m,i} = k_1 \sin \theta_{inc} + \frac{2\pi m}{\Lambda}, \quad \beta_{m,i} = \sqrt{k_1^2 - k_{m,i}^2} \tag{1}
\]

with the square-root branch chosen such that \(\Im[\beta_{m,i}] \leq 0\) to satisfy the radiation condition.

To find these scattered fields, we follow the formulation presented in [26], applying it to the specific case considered herein, of a metal-backed multielement MG with a single dielectric substrate material. Correspondingly, for a given MG configuration, this is achieved by: 1) evaluating the scattered fields in the absence of the MG grid (external fields); 2) evaluating the secondary fields produced by the currents induced on the MG wires; and 3) assessing the actual amplitudes of these induced currents and summing over all field contributions.

1) External Field Contribution: As in [26], due to the field discontinuity caused by the presence of the wires, we divide the problem domain into \(N-1\) regions, where the \(n\)th region is confined between the \(z = t_n-1\) and \(z = t_n\) metallization layers. However, since the PCB MG herein features a single type of dielectric, for calculating the external field it is sufficient to treat separately only two spatial sections; specifically, we distinguish between the fields in the observation region \((n = 1)\) and within the dielectric substrate \((n > 1)\).

⁴The same methodology can be applied to affect the radar signature of nonmetallic objects, which would require suppression of the fields transmitted through the MG as well [27], and consideration of the dielectric properties of the target. However, to demonstrate the concept, we chose to focus on cases where the MG cover is attached to a metallic body, which requires a smaller number of DOFs while providing solution to many common RCS reduction scenarios [5].
Consequently, the external field (i.e., in the absence of the MG wires) in the nth region can be written as a sum of forward and backward propagating plane waves adhering Snell’s law

\[ E_n^{\text{ext}}(y, z) = A_{0,n} e^{-j k_0 y - j eta_{n0} z} + B_{0,n} e^{-j k_0 y - j eta_{n0} z} \]  

where the subscript i refers to the medium of the nth layer (i = 1 for n = 1 and i = 2 for n > 1) [see (1)]. The amplitudes \( A_{0,n} \) and \( B_{0,n} \) are calculated by imposing the relevant boundary conditions, namely, continuity of the tangential fields at the air–dielectric interface (\( z = 0 \)) and vanishing of the tangential electric field at the PEC interface (\( z = t_N \), leading to

\[
A_{0,1}^{\text{ext}} = E_{\text{m}} \\
A_{0,n>1}^{\text{ext}} = E_{\text{m}} \frac{1 + \Gamma_0}{1 - \Gamma_0 e^{-2j eta_{n0} t_N}} \\
B_{0,1}^{\text{ext}} = E_{\text{m}} \Gamma_0 + e^{-2j eta_{n0} t_N} \\
B_{0,n>1}^{\text{ext}} = A_{0,n>1}^{\text{ext}} e^{-2j eta_{n0} t_N} \tag{3}
\]

where the local reflection coefficient is defined as \( \Gamma_m = (Z_{m,2} - Z_{m,1})/(Z_{m,2} + Z_{m,1}) \), with the TE wave impedance of the nth mode in the ith medium being \( Z_{m,i} = k_i \eta_i / j \beta_{n,i} \).

2) Grid-Induced Field Contribution: Once excited by the incident field, currents would be induced on the MG wires, giving rise to secondary fields. To evaluate these grid-originated fields, we superimpose the contributions of the various wires in the MG, acting as a \( \Lambda \)-periodic array of electric line sources. Specifically, the electric field in the nth layer due to the (yet to be evaluated) currents \( I_k \) developing on the kth wire in each position can be written as an infinite series of FB modes

\[ E^{(k)}_m(y, z) = \sum_{m = -\infty}^{\infty} E^{(k)}_{m,n}(y, z) \tag{4} \]

where the field corresponding to the nth mode can, once more, be expressed as a sum of forward and backward propagating plane waves

\[ E^{(k)}_{m,n}(y, z) = A^{(k)}_{m,n} e^{-j k_{m,n} y - j \beta_{m,n} z} + B^{(k)}_{m,n} e^{-j k_{m,n} y + j \beta_{m,n} z} \tag{5} \]

To find the amplitudes \( A^{(k)}_{m,n} \) and \( B^{(k)}_{m,n} \), we use the recursive formalism scheme described in [26] and [44], applying the source conditions (tangential field discontinuity) due to the current-carrying wires in the configuration while considering multiple reflections within the grounded dielectric substrate; for brevity, we provide here only the final results.

Referring to the specific configuration considered herein [see Fig. 1(a)], we once again distinguish between the wires positioned at the air–dielectric interface (external wires), and wires located within the substrate (internal wires). For the external wires, we evaluate separately the coefficients in two regions: above the wire (\( 1 < n < N \)), and below it (\( n = 1 \))

\[
\begin{align*}
A^{(k)}_{m,1} &= 0 \\
B^{(k)}_{m,1} &= -\frac{Z_{m,1} I_k}{2 \Delta} \frac{\Gamma_0 + e^{-2j \beta_{m,n} t_N}}{1 - \Gamma_0 e^{-2j \beta_{m,n} t_N}} \\
A^{(k)}_{m,n>1} &= \frac{Z_{m,1} I_k}{2 \Delta} \frac{1 + \Gamma_0}{1 - \Gamma_0 e^{-2j \beta_{m,n} t_N}} \\
B^{(k)}_{m,n>1} &= A^{(k)}_{m,n>1} e^{-2j \beta_{m,n} t_N} . \tag{6}
\end{align*}
\]

For the internal wires located at some \( z = t_n \), three regions are taken into account separately: above the wire (\( n_k < n < N \)), below the wire within the substrate (\( 1 < n < n_k \), and below the MG (\( n = 1 \))

\[
\begin{align*}
A^{(k)}_{m,1} &= 0 \\
A^{(k)}_{m,1 < n \leq n_k} &= -\frac{Z_{m,1} I_k}{2 \Delta} \frac{\Gamma_0 + e^{-2j \beta_{m,n} t_N}}{1 + \Gamma_0 e^{-2j \beta_{m,n} t_N}} \\
A^{(k)}_{m,n > n_k} &= -\frac{Z_{m,2} I_k}{2 \Delta} \frac{e^{j \beta_{m,n} t_N} - \Gamma_0 e^{-j \beta_{m,n} t_N}}{1 - \Gamma_0 e^{-2j \beta_{m,n} t_N}} \\
B^{(k)}_{m,1 < n \leq n_k} &= \frac{B^{(k)}_{m,n > n_k}}{2 \Delta} \frac{1 - \Gamma_0 e^{-2j \beta_{m,n} t_N}}{1 - \Gamma_0 e^{-2j \beta_{m,n} t_N}} \\
B^{(k)}_{m,n > n_k} &= A^{(k)}_{m,n > n_k} e^{-2j \beta_{m,n} t_N} . \tag{7}
\end{align*}
\]

Subsequently, for a given MG configuration and given induced currents \( I_k \), the total field in the nth layer can be deduced by summing the external field (2) with the MG contribution (4), reading

\[ E_n^{\text{tot}}(y, z) = E_n^{\text{ext}}(y, z) + \sum_{k = 1}^{\infty} \sum_{m = -\infty}^{\infty} E^{(k)}_{m,n}(y, z) . \tag{8} \]

3) Evaluation of Induced Currents: To enable actual evaluation of (8), we need to assess the currents \( I_k \) induced on the loaded wires in the given configuration when illuminated by \( E^{\text{ext}} \). To this end, we utilize Ohm’s law, relating the total field applied on a wire to the current flowing through it via the distributed load impedance \( \bar{Z}_k \) [see Fig. 1(c)]. Explicitly, for the kth wire, this yields [41]

\[ I_k \bar{Z}_k = E_{n_k}^{\text{ext}}(d_k, h_k) + \sum_{k' \neq k} \sum_{m = -\infty}^{\infty} E^{(k')}_{m,n}(y \rightarrow d_k, z \rightarrow h_k) + \sum_{m = -\infty}^{\infty} E^{(k)}_{m,n}(y \rightarrow d_k, z \rightarrow h_k) \tag{9} \]

where the first term of the RHS represents the excitation field in the absence of the MG, the second term corresponds to the fields produced by all wires other than the wire itself (\( k' \neq k \)) at its position, and the last term corresponds to the fields produced by the wire itself, forming together the total field acting on the kth wire.

The first and second terms are calculated directly by substituting the reference wire position into (2)–(7). For the third term, due to the singularity of the Hankel function at the origin, it is not possible to use the precalculated terms for the grid-induced fields (4)–(7). Instead, we follow the technique presented in [24] and [26], and write the kth wire self-induced field as a summation of the field generated by the reference wire on its shell (using the flat wire approximation [41]), and the field produced by the other strips at the position of the reference wire (interpreted as a series of image sources), yielding

\[ \tilde{E}^{(k)}_f = \frac{I_k}{2} \left\{ \sum_{n = -\infty}^{\infty} \frac{k_i \eta_i}{2 \Lambda \beta_{n,i}} I_k + \sum_{m = 0}^{\infty} \frac{k_i \eta_i j}{\pi} \left[ \log \frac{2 \Lambda}{\pi w} + \frac{1}{2} \sum_{m = 0}^{\infty} \left( \frac{2 \pi}{m} - \frac{1}{m} \right) \right] \right\} \]

\[ + \sum_{n = -\infty}^{\infty} \frac{k_i \eta_i}{2 \Lambda \beta_{m,i}} I_k + \sum_{m = 0}^{\infty} \left[ \frac{k_i \eta_i j}{\pi} \left[ \log \frac{2 \Lambda}{\pi w} + \frac{1}{2} \sum_{m = 0}^{\infty} \left( \frac{2 \pi}{m} - \frac{1}{m} \right) \right] \right] \left[ A^{(k)}_{m,n} e^{-j \beta_{m,n} t_N} + B^{(k)}_{m,n} e^{j \beta_{m,n} t_N} \right] , \tag{10} \]

\( w \) being the copper trace width [see Fig. 1(c)].
Since the forward and backward grid-induced field amplitudes of (5) are linearly proportional to the currents, (9) can be rewritten as

$$I_k \bar{Z}_k = E^\text{ext}_{n_k}(d_k, h_k) + \sum_{q \neq k} \varepsilon_q I_q + \varepsilon_{self} I_k$$  \hspace{1cm} (11)

where the coefficients $\varepsilon_q$ and $\varepsilon_{self}$ can be directly extracted from (5)–(7) and (10), respectively. Thus, the currents can be calculated by a simple matrix inversion $I_{K \times 1} = \Psi_{K \times K}^{-1} E^\text{ext}_{K \times 1}$, where $I_{K \times 1}$ is a vector composed of the induced currents $I_k$; $E^\text{ext}_{K \times 1}$ is the excitation vector, containing the external field values in the wire positions $E^\text{ext}_{n_k}(d_k, h_k)$; and the matrix $\Psi_{K \times K}$ is defined by

$$\Psi = \begin{bmatrix} \varepsilon_{1}^{(1)} - \varepsilon_{self} & \ldots & -\varepsilon_{1}^{(K)} \\ \ldots & \ldots & \ldots \\ -\varepsilon_{1}^{(K)} & \ldots & \varepsilon_{K}^{(K)} \end{bmatrix}.$$  \hspace{1cm} (12)

4) Evaluation of Load Impedances: We should recall at this stage that the MG loaded-wire meta-atoms are physically realized using copper traces in a conventional PCB configuration, featuring printed capacitors as loads [see Fig. 1(a)–(c)]. Thus, to enable assessment of $\Psi$ of (12), evaluate the various currents $I_k$, and subsequently solve the MG scattering problem, it is required to relate the effective distributed impedance $\bar{Z}_k$ of the various loads to the respective printed capacitor widths $W_k$ used in practice. Following semianalytical formulas developed in previous work, we approximate the reactive part of $\bar{Z}_k$ as $\Im\{\bar{Z}_k\} \approx -2.85 K_{corr}/(\omega L_x W_k \varepsilon_{eff,r})$ [45], where $L_x$ is the periodicity along the $x$-axis [see Fig. 1(b)], and $K_{corr}$ is a frequency-dependent correction factor (estimated as $K_{corr} \approx 0.947$ mill/F for $\omega = s = 4$ mil at the working frequency of $f = 20$ GHz used herein [26]). The effective relative permittivity for the external wires is given by $\varepsilon_{eff,r} = (\varepsilon_1 + \varepsilon_2)/(2\varepsilon_0)$ for meta-atoms at the dielectric–air interface ($n = 1$), and $\varepsilon_{eff,r} = \varepsilon_2/\varepsilon_0$ for the internal wires ($n > 1$), where $\varepsilon_0$ is the vacuum permittivity. The resistive part of the distributed impedances, related to the conductor (copper) losses, was estimated using skin depth and cross section considerations as $\Re\{\bar{Z}_k\} \approx 14.5 \times 10^{-3} \, [\eta/\lambda]$ for the same operating conditions [23].

This completes the forward problem analysis: for given incident plane wave and MG parameters (i.e., the wire positions $(y, z) = (d_k, h_k)$ and the capacitor widths $W_k$), one may follow the above formalism to deduce the distributed impedances from the capacitor widths, evaluate the currents from (12) with (2), (3), (5)–(7), (10), and (11), and calculate the fields scattered off the multilayer MG toward the observer via (8) with $n = 1$.

B. Multichannel Diffusive MG (Synthesis)

Once the analytical model relating a given MG configuration to the fields scattered off it when illuminated from a given $\theta_m$ is established as in Section II-A above, we may proceed to tackle the synthesis problem at hand, defining suitable constraints on the coupling coefficients to the various reflected FB modes (for multiple excitations simultaneously in our multichannel scenario), as to facilitate the desired functionality.

As mentioned in Section I, we wish to exploit symmetry properties of the problem to reduce the overall number of constraints (leading to simpler designs with fewer DOFs). To obtain a symmetric multichannel network (about $\theta = 0^\circ$), we choose the $2P + 1$ angles of incidence spanning the effective acceptance angle range to be $\theta_m \in \Theta \triangleq [\arcsin(\rho \lambda/\Lambda) \mid p = -P, \ldots, P]$ [see Fig. 1(a) and (d)]. The advantage of such a choice, beyond manifesting the symmetry of the scattering scenario, is that for each of these incidence angles, the propagating scattered FB modes will be reflected to angles of the same set $\theta_{out} \in \Theta$. This would form a symmetric $M = (2P + 1)$-port network [38], where the number of channels (propagating modes) is set by $\Lambda$ via $\Lambda = |\Lambda/\lambda|$ [26], promoting a broad-angle response.

Following this idea, and along the lines of [26], we can calculate the amplitude of the field $E_{q,p}$ scattered toward the $q$th port when the MG is excited from the $p$th port using (8) with $n = 1$, reading:

$$E_{q,p} = B_{q,p} = B_{q,p}^\text{ext} - \delta_{q,-p} + \sum_{k=1}^{K} B_{q,p}^{(k)}.$$  \hspace{1cm} (13)

where $B_{q,p}^{(k)}$ is the downward propagating wave amplitude $B_{m,p}$ of (5) in Section II-A, evaluated in the observation region $n = 1$ for a scenario in which $\theta_m = \theta_p$, and the FB mode order $m$ corresponds to the output angle defined by $\theta_q = -\arcsin(k_{m,p}/\lambda)$; $K_{q,p}$ is the amplitude of the reflected external field $B_{q,p}^\text{ext}$ of (2) for the same excitation scenario $\theta_m = \theta_p$, which only contributes in case that the $q$th port corresponds to the specular reflection, namely, $q = -p$.

With these definitions, we describe the multichannel MG system using an $M \times M$ scattering matrix (S-matrix), where the input and output ports correspond to the FB modes associated with the excitation angles $\theta_p \in \Theta$ and the scattering angles $\theta_q \in \Theta$, respectively. Specifically, the fraction of power coupled to the $q$th port by the MG when excited from the $p$th port can be evaluated via the corresponding scattering coefficient $S_{q,p}$ using (13), reading [24]

$$|S_{q,p}|^2 = \frac{|E_{q,p}|^2}{E_{in}} \frac{2 \cos \theta_q}{\cos \theta_p}.$$  \hspace{1cm} (14)

This scattering matrix representation allows us to conveniently stipulate the constraints guaranteeing the desired MG functionality, namely, multistatic monostatic and bistatic RCS reduction. To achieve these goals simultaneously, we demand that for each excitation scenario, coupling to the retroreflection channel would vanish ($S_{p,p} = 0$), and the incident power would be uniformly scattered into the other available channels.
we restrict the number of channels considered herein to 2 and a lossless configuration [46] such as the one prescribed in (15); therefore, the number of equations (propagating FB modes), i.e., the number of unknowns (induced currents) in the linear system would be equal to or larger than the number of independent constraints in our problem. Specifically, we recall that the MG is composed of reciprocal, passive and (ideally) lossless elements (highly conducting strips and low-loss dielectrics), and that the ultimate configuration should be symmetric about the \( \bar{x}_2 \) plane. Reciprocity implies that imposing constraints over \( S_{p,q} \) immediately determines \( S_{q,p} \), removing the need to explicitly consider the latter (see Fig. 2). Furthermore, the symmetric nature of the MG requires that \( S_{p,q} = S_{-p,-q} \) [see Fig. 1(a) and (d)], making it sufficient to apply constraints only on one of these S-parameters. Lastly, power conservation implies that the S-matrix should be unitary, and, in particular, the absolute square of the S-parameters in each row or column should sum up to unity [46]. In other words, once all the S-parameters but one in a row \( q' \) are determined, the power coupled to this last element \( p' \) is bound by \( 1 - \sum_{p \neq p'} |S_{p,q'}|^2 \), and thus need not be constrained by our synthesis procedure\(^7\) (see Fig. 2). In summary, as illustrated in Fig. 2, for an \( M = 2P + 1 \) port network, the number of independent elements is reduced by reciprocity, symmetry, and passivity from the initial \( M \times M = (2P + 1)^2 \) to \( (P + 1)^2 - (P + 1) = P(P + 1) \), corresponding to a dramatic reduction in the required constraints.

With the number of independent constraints established, we may proceed to executing the synthesis procedure based on a suitable number of DOFs (elements per period and their coordinates). To this end, we use the forward problem analytical formulation of Section II-A and solve the inverse problem defined by the constraints (15) using the MATLAB library function \texttt{lsqnonlin}. Since the problem is nonlinear, different initial values of \((d_h, h, W_h)\) provided to the function result in convergence to different solutions. To search for an optimal configuration, we run the function with different random initial values 50 times (overall runtime \(< 1 \text{ min on a standard desktop computer}\)), and choose out of the resulting 50 sets of MG designs the one with the smallest residual [the square norm of the deviation from the desired S-matrix, as defined in (15)]. Since this process provides numerous options for detailed fabrication-ready PCB layouts, we may choose a configuration which will best match practical fabrication constraints (low profile, layer thicknesses that match a far less strict limitation, and may be satisfied with fewer DOFs (simpler designs).

In fact, we can decrease the number of required DOFs even further if we harness the S-matrix properties, which should be inherently satisfied by the MG as we design it, reducing the number of independent constraints in our problem. Specifically, we recall that the MG is composed of reciprocal, passive and (ideally) lossless elements (highly conducting strips and low-loss dielectrics), and that the ultimate configuration should be symmetric about the \( \bar{x}_2 \) plane. Reciprocity implies that imposing constraints over \( S_{p,q} \) immediately determines \( S_{q,p} \), removing the need to explicitly consider the latter (see Fig. 2). Furthermore, the symmetric nature of the MG requires that \( S_{p,q} = S_{-p,-q} \) [see Fig. 1(a) and (d)], making it sufficient to apply constraints only on one of these S-parameters. Lastly, power conservation implies that the S-matrix should be unitary, and, in particular, the absolute square of the S-parameters in each row or column should sum up to unity [46]. In other words, once all the S-parameters but one in a row \( q' \) are determined, the power coupled to this last element \( p' \) is bound by \( 1 - \sum_{p \neq p'} |S_{p,q'}|^2 \), and thus need not be constrained by our synthesis procedure\(^7\) (see Fig. 2). In summary, as illustrated in Fig. 2, for an \( M = 2P + 1 \) port network, the number of independent elements is reduced by reciprocity, symmetry, and passivity from the initial \( M \times M = (2P + 1)^2 \) to \( (P + 1)^2 - (P + 1) = P(P + 1) \), corresponding to a dramatic reduction in the required constraints.

With the number of independent constraints established, we may proceed to executing the synthesis procedure based on a suitable number of DOFs (elements per period and their coordinates). To this end, we use the forward problem analytical formulation of Section II-A and solve the inverse problem defined by the constraints (15) using the MATLAB library function \texttt{lsqnonlin}. Since the problem is nonlinear, different initial values of \((d_h, h, W_h)\) provided to the function result in convergence to different solutions. To search for an optimal configuration, we run the function with different random initial values 50 times (overall runtime \(< 1 \text{ min on a standard desktop computer}\)), and choose out of the resulting 50 sets of MG designs the one with the smallest residual [the square norm of the deviation from the desired S-matrix, as defined in (15)]. Since this process provides numerous options for detailed fabrication-ready PCB layouts, we may choose a configuration which will best match practical fabrication constraints (low profile, layer thicknesses that match

\[^6\text{As known, a three-port matched network could not be realized by a passive and lossless configuration [46] such as the one prescribed in (15); therefore, we restrict the number of channels considered herein to } M > 3.\]

\[^7\text{Since our aim is to reduce the RCS in each observation angle, enforcing an upper bound on the scattering is sufficient.}\]
commercially available dielectric laminates, etc.) and proceed toward implementation.

III. RESULTS AND DISCUSSION

A. Prototype

To verify the developed synthesis method, we follow the scheme described in Section II to design a multichannel MG diffuser for multangle monostatic and bistatic RCS reduction. In particular, we aim at realizing a five-channel MG \((M = 5)\) at \(f = 20\) GHz \((\lambda \approx 15\) mm\), corresponding to an S-matrix \((15)\) which manifests zero retroreflection, and equally divides the incident power among the four remaining channels \((|S_{q,p}|^2 = 0.25, \forall p \neq q)\). We set the periodicity to \(\Lambda = \lambda / \sin(29^\circ) \approx 31\) mm, leading to a set of five propagating FB modes, defining the input and output channel propagation angles as \(\theta_0 = 0^\circ, \theta_{\pm 1} = \pm \arcsin(\lambda / \Lambda) = \pm 29^\circ\), and \(\theta_{\pm 2} = \pm \arcsin(2\lambda / \Lambda) = \pm 75.84^\circ\) (see Section II-B). While the design method is applicable to any choice of scattering angles [adhering (1) and the special considerations detailed in Section II-B], these specific values were selected since they provide a sufficiently wide angular coverage under the considered number of channels. As discussed later toward the end of Section III, the number of channels can be increased and the angular coverage can be tuned to further reduce and mold the RCS signature of the target to the levels required by a given application.

We attempt to design the desired MG prototype based on a configuration composed of two metallization layers \((N = 3)\) of 0.5 oz copper traces \((\text{copper thickness } t = 18 \mu\text{m})\), embedded within a Rogers RO3003 \((\varepsilon_{\text{sub}} = 3.00, \tan \delta = 0.001)\) substrate, comprising three meta-atoms per period \((K = 3)\). The first meta-atom (loaded wire) is positioned in the origin at the air–dielectric interface \((d_1, h_1) = (0, 0)\), while the second and third meta-atoms are located at \((d_2, h_2)\) and \((d_3, h_3)\) within the substrate. As discussed in Section II-B above, to reduce the number of independent constraints and subsequently the required number of DOFs, we impose a symmetric configuration, namely, the internal wires are designed with identical capacitor widths \(W_2 = W_3\) (identical loads), and are positioned symmetrically about the origin \((d_2 = -d_3, h_2 = h_3)\), as depicted in Fig. 3.

Applying the constraints \((15)\) via the methodology described in Section II yields a suitable MG design, featuring meta-atoms at \((d_1, h_1) = (0, 0)\) and \((d_3, h_3) = (-d_2, h_2) = (0.3507\lambda, 0.1187\lambda) = (5.3\) mm, 1.8 mm\) below the metallic mirror at \(z = t_3 = 0.1935\lambda = 2.9\) mm, with printed capacitors of widths \(W_1 = 1.56\) mm and \(W_2 = W_3 = 1.18\) mm. Once the geometric parameters are thus set, we may readily evaluate the S-matrix of the MG by solving the forward problem \((8)–(14)\) with the finalized \((d_k, h_k, W_k)\), yielding

\[
|S|^2 = \begin{pmatrix}
0.02 & 0.24 & 0.23 & 0.25 & 0.23 \\
0.24 & 0.01 & 0.24 & 0.25 & 0.25 \\
0.23 & 0.24 & 0.04 & 0.24 & 0.23 \\
0.25 & 0.25 & 0.24 & 0.01 & 0.24 \\
0.23 & 0.25 & 0.23 & 0.24 & 0.02
\end{pmatrix}.
\] (16)

We note that the matrix agrees well with the desired scattering goal \((15)\) for our case \((M = 5)\). As expected, the matrix corresponds to a reciprocal, symmetric, and passive system.

To verify the theoretical predictions, a single period of the prototype MG was modeled in CST Microwave Studio and simulated under periodic boundary conditions. Since in practice, the desired layer configuration in the fabricated device was realized by cascading laminates of standard thicknesses bonded using 2 mil-thick Rogers 2929 bondply \((\varepsilon_{\text{sub}} = 2.94\varepsilon_0\) and \(\tan \delta = 0.003)\), we defined this actual structure in the full-wave solver for the final verification purposes. Ultimately, also due to limited availability in real time, the manufactured MG featured total thickness of \(t_3 = 111\) mil = 2.82 mm, with the meta-atoms positioned at \(h_1 = 0, h_2 = h_3 = 67\) mil = 1.70 mm, slightly away from the designated planes. The simulated S-matrix of the corresponding (actual) MG was found to be

\[
|S|^2 = \begin{pmatrix}
0.02 & 0.26 & 0.22 & 0.21 & 0.27 \\
0.26 & 0.00 & 0.21 & 0.28 & 0.24 \\
0.22 & 0.24 & 0.06 & 0.24 & 0.22 \\
0.24 & 0.28 & 0.21 & 0.00 & 0.26 \\
0.27 & 0.21 & 0.22 & 0.26 & 0.02
\end{pmatrix}. \] (17)

The simulated S-mat shows overall good agreement with the analytically predicted one \((16)\); the observed minor deviations can be attributed to the slight discrepancies between the actual fabricated model and the theoretical design discussed in the previous paragraph. Furthermore, Fig. 4 compares the analytically predicted scattered fields to the full-wave simulated ones.
for the three designated angles of incidence, namely, $\theta_{m} = 0^\circ$ [see Fig. 4(a) and (d)], $\theta_{m} = 29^\circ$ [see Fig. 4(b) and (e)], and $\theta_{m} = 75.84^\circ$ [see Fig. 4(c) and (f)], revealing excellent agreement. These results further establish the reliability of the analytical model, demonstrating its usefulness for the design of realistic multichannel PCB MGs.

B. Experiment

After this validation of the theoretical model, and without any further optimization, we used the corresponding PCB layout (see Fig. 3) to fabricate a $9'' \times 12''$ board (PCB Technologies Ltd., Migdal Ha’Emek, Israel); the manufactured prototype is presented in Fig. 5. The MG device under test (DUT) was characterized in an anechoic chamber at the Technion using a near-field measurement system (MVG/Orbit-FR Engineering Ltd., Emek Hefer, Israel). In the chamber, the MG was positioned on a foam holder in front of a Gaussian beam antenna (Millitech, Inc., GOA-42-S000094, focal distance of 196 mm $\approx 13\lambda$), illuminating the device with a quasi-planar wavefront (see Fig. 6). Due to blockage effects presented by the Gaussian beam antenna when scanning at wide angles, we have placed the MG at a larger distance from the Gaussian beam antenna, corresponding to 430 mm $\approx 29\lambda$. Nonetheless, as can be deduced from previous work [47], the focal region is large enough to provide a sufficiently collimated beam even at this distance, enabling proper characterization of the DUT.

To facilitate the manufacturing process, the MG circumference was partly sacrificed to accommodate metallic markings and drilled holes used for accurate alignment during fabrication, leaving an active area smaller than the overall $9'' \times 12''$ board size. Therefore, to properly calibrate our measurements and avoid interaction with these irrelevant scatterers, we attached to the foam holder a metallic frame covering these markings, allowing quantification of the effective reference excitation power which illuminates the MG area in reality [see Fig. 6(a)].

C. Characterization

By rotating the foam holder about its axis, the alignment between the Gaussian beam antenna and the MG was tuned such that the DUT scattering properties for different angles of incidence could be probed. In particular, measurements were performed for the excitation angles corresponding to the designated input ports of the multichannel MG [see Fig. 6(b)]. For each of these angles of incidence, a cylindrical near-field measurement was performed, recording the scattering pattern across the frequency range $f \in [18,22]$ GHz. The measurement was conducted by rotating together the Gaussian beam antenna and the MG (aligned to a certain illumination angle $\theta_{m}$) around a near-field probe situated at a distance of 850 mm $\approx 57\lambda$ away from the MG, collecting the scattered fields. The far-field patterns were then deduced by the system’s postprocessing software from the near-field measurements using the equivalence principle [48].

Fig. 7 shows such representative scattering patterns, recorded for normal incidence [input port $p = 0$ in Fig. 1(a)] at the operating frequency of $f = 20$ GHz. The measurements are conducted once in the absence of the MG [dash-dotted red, see Fig. 6(a)] and in the presence of the MG [solid blue, see Fig. 6(b)], with the former serving as a reference for the input power.
Fig. 8. Frequency response of the prototype MG diffuser of Fig. 3. The fraction of incident power coupled to each of the output ports for two different excitation scenarios [(a)–(e)] $\theta_{in} = 29^\circ$ and [(f)–(h)] $\theta_{in} = 0^\circ$ is plotted as a function of the frequency. The experimental results (red circles and magenta squares) are compared with the results obtained via full-wave simulation (solid blue) for the input and output ports associated with the angles (a) $\theta_{in} = 29^\circ$, $\theta_{out} = -75.84^\circ$; (b) $\theta_{in} = 29^\circ$, $\theta_{out} = -29^\circ$; (c) $\theta_{in} = 29^\circ$, $\theta_{out} = 0^\circ$; (d) $\theta_{in} = 29^\circ$, $\theta_{out} = 29^\circ$ (red circles), and $\theta_{in} = -29^\circ$, $\theta_{out} = -29^\circ$ (magenta squares); (e) $\theta_{in} = 29^\circ$, $\theta_{out} = 75.84^\circ$; (f) $\theta_{in} = 0^\circ$, $\theta_{out} = 29^\circ$ (red circles), $\theta_{out} = -29^\circ$ (magenta squares); (g) $\theta_{in} = 0^\circ$, $\theta_{out} = 75.84^\circ$ (red circles), $\theta_{out} = -75.84^\circ$ (magenta squares); and (h) $\theta_{in} = 0^\circ$, $\theta_{out} = 0^\circ$. Insets depict the characterization scenario, while dashed vertical lines mark the operating frequency $f = 20$ GHz.

output port angles $\theta_{\pm 1} \approx \pm 29^\circ$, $\theta_{\pm 2} \approx \pm 75.84^\circ$, a closer examination reveals that the exact maxima occur at $\theta_1 = 31^\circ$, $\theta_{-1} = -28^\circ$, $\theta_{2} = 73^\circ$, and $\theta_{-2} = -71^\circ$. These results lead to two observations. First, the asymmetry in the recorded peak angles implies that the limited (manual) alignment accuracy of the system may introduce angular deviations of up to around 1° in measurements. Second, for large deflection angles $\theta_{\pm 2}$, the main lobe maxima shift toward lower angular values, which is attributed to the reduced effective aperture size at these near-grazing angles [49].

Another aspect that is highlighted by Fig. 7 is the inability of the described (bistatic) measurement setup to measure the scattered power for $\theta_{in} = \theta_{out}$ (manifested in the absence of measured data points around $\theta = 0$ in the plot when the MG is present). Since in our near-field system the probe and Gaussian beam antenna cannot be aligned to the same angle (otherwise they will physically collide), an alternative measurement (monostatic) should be used to evaluate retroreflection.

Specifically, we connect to this end the Gaussian beam antenna to a dedicated vector network analyzer (four-port Keysight E5080B ENA), functioning as both the transmitter and the receiver for this measurement. Subsequently, the recorded reflection coefficient ($S_{11}$) provides a measure for the amount of retroreflected power. To quantify the fraction of power coupled to this channel, we compare this value with a reference measurement conducted with a planar metal plate matching the dimensions of the MG.

Finally, for both bistatic and monostatic setups, we evaluate the power coupled to each channel by considering the ratio between the peak measured gain values of the MG scattering patterns $G_{MG}(\theta_{out})$ and the reference measurement $G_{ref}(\theta_{in})$ quantifying the effective incident power, taking into account the differences in the effective aperture size [22], [26], [47], [50]

$$\eta_{out} = \frac{G_{MG}(\theta_{out})}{G_{ref}(\theta_{in})} \frac{\cos \theta_{in}}{\cos \theta_{out}}$$

which for the retroreflection case $\theta_{in} = \theta_{out}$ simply reduces to the ratio of the measured reflection coefficients in the input of the Gaussian beam antenna.

Fig. 8 presents the measured frequency dependency of the power coupled to each propagating mode (output channels), for illumination from the predefined excitation angles (input channels5), compared to the simulation results. The graphs indicate good correspondence between the simulated and experimentally measured scattering patterns in the measured frequency range. The minor differences observed [mainly in Fig. 8(a)–(c) and (e)] between theoretical predictions and experimental estimations can be attributed to limited angular alignment accuracy in our setup (see discussion after Fig. 7), which may introduce small errors in the actual angle of incidence with respect to the desired $\theta_{in}$; possible fabrication inaccuracies and material parameter tolerances may also contribute to such deviations.

The measured frequency response is further affected by the periodic nature of the designed multichannel MG. Indeed, certain effects observed in Fig. 8 stem from the variation of $\theta_{in}$. 

5The measurement setup did not allow proper characterization of the MG when illuminated by large oblique angles of incidence, mainly due to the small effective aperture size for such angles (proportional to $\cos \theta_{in}$ [48]). Thus, Fig. 8 only considers $\theta_{in} = 0^\circ$, $\pm 29^\circ$, whereas the performance associated with $\theta_{in} = \pm 75.84^\circ$ is deduced from reciprocity considerations (see discussion surrounding Fig. 9 later on).
the particular propagation angle associated with the scattering toward the \( p \)th port, as dictated by (1). Specifically, when reducing the operating frequency, higher-order FB modes are driven into the invisible region, eventually becoming evanescent (and thus cannot outcouple real power). This explains the low-frequency behavior observed in Fig. 8(a), (e), and (g), where the scattering toward the presented port abruptly vanishes when a certain “cutoff” frequency is crossed.\(^9\) Such a crossing has an impact on the distribution of power among the other FB modes as well, as can be seen, for instance, in the substantial growth in the power coupled to the port around \( f = 19.2 \) GHz in Fig. 8(h), stemming from the cutoff identified for the same excitation scenario in Fig. 8(g). Similarly, abrupt trend changes can be spotted in Fig. 8(b) and (c) around the cutoff frequencies \( f = 18.8 \) GHz and \( f = 19.6 \) GHz, associated, respectively, with the FB modes characterized in Fig. 8(a) and (e) for the same angle of incidence \( \theta_{\text{in}} = 29^\circ \).

Another effect of the modal frequency dependency manifested by (1) is related to the retroreflection phenomena. While for \( \theta_{\text{in}} = 0^\circ \), retroreflection coincides with the specular reflection and thus always directed toward broadside [see Fig. 8(h)], for other illumination angles (e.g., \( \theta_{\text{in}} = 29^\circ \)) the multichannel scattering scenario would not include ports with \( \theta_{\text{out}} = \theta_{\text{in}} \) for measurements outside the designated operating frequency. In other words, significant power could be recorded at \( \theta_{\text{out}} = \theta_{\text{in}} \) only for a limited range of frequencies around the design working point \( f = 20 \) GHz (unless \( \theta_{\text{in}} = 0^\circ \)), whereas outside this range coupling toward this direction would not be allowed as per the FB theorem. Following this observation, we present the retroreflection data for \( \theta_{\text{in}} = 29^\circ \) only for the range \( f \in [19.5, 20.5] \) GHz [see Fig. 8(d)], in which the deviation of \( \theta_{\text{out}} \) from \( \theta_{\text{in}} \) is still mild such that the retroreflection measurement is still meaningful.\(^10\)

Overall, it can be seen that the MG performs well around the designated operating frequency, suppressing coupling to the retroreflection mode while distributing the scattered power among the other channels. For comparison, in the presence of a standard metallic surface (in the absence of the diffusive MG elements), scattered power would be fully coupled to the specular channel for the entire frequency range, resulting in \( \approx 4 \) times higher bistatic signature [e.g., in scenarios like Fig. 8(b)] and \( \approx 10 \) times higher monostatic scattering [e.g., compared to Fig. 8(h)], at the operating frequency. These properties are further apparent from Fig. 9, quantifying the fraction of power coupled to the various output ports for each of the considered excitations \( \theta_{\text{in}} = 0^\circ \) (left), \( \theta_{\text{in}} = 29^\circ \) (center), and \( \theta_{\text{in}} = 75.84^\circ \) (right) at \( f = 20 \) GHz. For comparison, four sets of data are presented for each scenario. The first row (Analytical) corresponds to the predictions of the analytical model as output by the synthesis procedure (8)-(14), in correspondence with (16); the second row (Full-wave: design) presents the scattered power as recorded in full-wave simulations for this chosen design; the third row (Full-wave: actual) indicates the expected effects on the MG performance when the actual laminate stack used for the fabricated prototype is considered in CST, previously reported in (17); and the fourth row (Experiment) documents the scattered power as measured in the anechoic chamber. As can be clearly seen, the collected data show good agreement between analytically computed, simulated and measured power coupling to each port. Even the constraints posed by laminate availability did not significantly deteriorate the MG operation, implying a certain robustness against fabrication errors. From an RCS reduction perspective, for all considered angles of incidence in this multichannel scenario, the measured retroreflection was found to be lower than 6% of the incident power, and the maximal power coupled to a single direction (output port) did not exceed 28%. Hence, the presented results verify the efficacy of the proposed MG in realizing intricate multiantenna scattering management, relying on a PCB fabrication-ready sparse configuration designed using a full-wave-optimization-free methodology.

Before we conclude, we should note that although the device was designed to equally divide the power between \( M - 1 = 4 \) ports only when excited from one of the \( M = 5 \) designated angles of incidence \( \Theta \), it actually reduces the maximal scattering for other excitation angles as well. Specifically, examining the angular response by full-wave simulations over the entire angular span (see Fig. 10) reveals that the power coupled to a single direction (FB mode) does not exceed 45% of the incident power, for all excitation angles in the range

\(^9\)In Fig. 8(a) and (e), finite power is recorded at frequencies lower than the theoretical “cutoff” frequency calculated as \( f = 18.8 \) GHz for (a), and \( f = 19.6 \) GHz for (e). This may be attributed again to small alignment errors due to the manual mechanism used in the experiment (see discussion after Fig. 7), correspondingly causing minor deviations in the angle of incidence. Due to the high angular sensitivity for this extreme deflection toward near-grazing angles, even such minor deviations would be sufficient to shift the “cutoff” frequency below \( f = 16 \) GHz [see (1)].

\(^{10}\)Recall that for the retroreflection measurement the Gaussian beam is static while respecting the MG, relying on the assumption that the main scattering would be toward \( \theta_{\text{out}} = \theta_{\text{in}} \) (see Section III).
of $|\theta_{\text{in}}| \leq 80^\circ$ excluding a small fragment around $\theta_{\text{in}} = 25^\circ$. Even such a conservative figure of merit shows significant improvement in scattering performance across a wide range of angles with respect to the reference specular reflection of a metallic plane. Clearly, increasing the periodicity $\Lambda$ beyond $P\lambda$ for some $P \in \mathbb{N}$ would correspondingly increase the “sampling” resolution of the angular domain with $M = 2P + 1$ symmetric channels, while at the same time reduce the power fraction scattered to each channel $\propto 1/(M-1)$. In that regard, and taking into account that an RCS reduction of 10 dB is considered as a typical limit in terms of acceptable performance [1], one may deduce that for practical purposes the MG diffuser should feature at least $M = 11$ scattering channels. Naturally, this would require more DOFs (meta-atoms per periods) to meet the increased number of constraints (S parameters) [26]; on the other hand, it is expected that if the angular density of the channels is sufficiently high (the angular difference between adjacent angles $\theta_\phi$ would be sufficiently small), the RCS reduction performance would be maintained more evenly across the entire angular range [40].

IV. CONCLUSION

In this article, we introduced a rigorous semianalytical method for designing PCB-compatible multichannel MG diffusers, enabling reduction of monostatic and bistatic RCS for multiple angles of incidence simultaneously. By adopting a symmetric and passive MG construct, along with a synthesis procedure that clearly identifies the minimal required number of DOFs and effectively exploits them, a highly sparse configuration is obtained. Specifically, the demonstrated prototype achieves multiangle functionality spreading over a wide angular range with only three subwavelength loaded wires within a two-wavelength period, compared to the typical number of 10–20 polarizable particles per period that would be used in a conventional (single-functionality) MS with the same periodicity. Moreover, the presented model, verified experimentally, enables producing a complete fabrication-ready design without relying on time consuming full-wave optimization. Since the analytical scheme is general, the methodology can be used, in principle, to exercise control over an arbitrary number of diffraction modes (by including additional meta-atoms per period) and meet the requirements of versatile multifunctional scattering scenarios. In the context of the explored RCS reduction problem, this generality can be harnessed to further enhance the diffuser performance by increasing the number of channels, correspondingly improving the angular response as well as the bistatic RCS reduction. These results and conceptual observations establish a reliable foundation for the design of sparse multiaxial MGs for a variety of electromagnetic applications.

ACKNOWLEDGMENT

The authors wish to thank Rogers Corporation for providing part of the laminates used in work; and the Keysight Team, Israel, for providing the four-port VNA used for the retroreflection measurements.

REFERENCES

[1] E. F. Knott, J. F. Schaeffer, and M. T. Tulley, Radar Cross Section. Rijeka, Croatia: SciTech, 2004.
[2] S. B. Glybovskiy, S. A. Tretyakov, P. A. Belov, Y. S. Kivshar, and C. R. Simovski, “Metasurfaces: From microwaves to visible,” Phys. Rep., vol. 634, pp. 1–72, Apr. 2016.
[3] Y. Ra’di, C. R. Simovski, and S. A. Tretyakov, “Thin perfect absorbers for electromagnetic waves: Theory, design, and realizations,” Phys. Rev. A, Gen. Phys., vol. 3, no. 3, Mar. 2015, Art. no. 037001.
[4] D. Lim and S. Lim, “Ultrawideband electromagnetic absorber using sandwiched broadband metasurfaces,” IEEE Antennas Wireless Propag. Lett., vol. 18, no. 9, pp. 1887–1891, Sep. 2019.
[5] A. Murugesan, K. T. Selvan, A. Iyer, K. V. Srivastava, and A. Alphones, “A review of metasurface-assisted RCS reduction techniques,” Prog. Electromagn. Res. B, vol. 94, pp. 75–103, 2021.
[6] M. Paquay, J. C. Irarritxe, I. Ederra, R. Gonzalo, and P. D. Maagt, “Thin AMC structure for radar cross-section reduction,” IEEE Trans. Antennas Propag., vol. 55, no. 12, pp. 3630–3638, Dec. 2007.
[7] J. C. I. Galarrregui, A. T. Pereda, J. L. M. de Falción, I. Ederra, R. Gonzalo, and P. de Maagt, “Broadband radar cross-section reduction using AMC technology,” IEEE Trans. Antennas Propag., vol. 61, no. 12, pp. 6136–6143, Dec. 2013.
[8] A. Ghayekhloo, M. Afshari, and A. A. Orouji, “An optimized checkerboard structure for cross-section reduction: Producing a coating surface for bistatic radar using the equivalent electric circuit model,” IEEE Antennas Propag. Mag., vol. 60, no. 5, pp. 78–85, Oct. 2018.
[9] J. Chen, Q. Cheng, and J. Zhao, “Reduction of radar cross section based on a metasurface,” Prog. Electromagn. Res., vol. 146, pp. 71–76, 2014.
[10] T. J. Cui, M. Q. Qi, X. Wan, J. Zhao, and Q. Cheng, “Coding metamaterials, digital metamaterials and programmable metamaterials,” Light. Sci. Appl., vol. 3, no. 10, p. e218, Oct. 2014.
[11] L.-H. Gao et al., “Broadband diffusion of terahertz waves by multi-bit coding metasurfaces,” Light Sci. Appl., vol. 4, no. 9, p. e324, Sep. 2015.
[12] K. Chen et al., “Geometric phase coded metasurface: From polarization dependent directive electromagnetic wave scattering to diffusion-like scattering,” Sci. Rep., vol. 6, no. 1, Dec. 2016, Art. no. 35968.
[13] M. Moccia et al., “Coding metasurfaces for diffuse scattering: Scaling laws, bounds, and suboptimal design,” Adv. Opt. Mater., vol. 5, no. 19, Oct. 2017, Art. no. 1700455.
[14] M. Feng et al., “Two-dimensional coding phase gradient metasurface for RCS reduction,” J. Phys. D, Appl. Phys., vol. 51, no. 37, Sep. 2018, Art. no. 375103.
[15] Y. Azizi, M. Soleimani, and S. H. Sedighy, “Ultra-wideband radar cross section reduction using amplitude and phase gradient modulated surfaces,” J. Appl. Phys., vol. 128, no. 20, Nov. 2020, Art. no. 205301.
[16] E. F. Kuester, M. A. Mohamed, M. Piker-May, and C. L. Holloway, “Averaged transition conditions for electromagnetic fields at a metallfilm,” IEEE Trans. Antennas Propag., vol. 51, no. 10, pp. 2641–2651, Oct. 2003.
