Discussion on Propositional Logic Incorporating Set Thought into Discrete Mathematics

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Abstract: Discrete mathematics is a basic core course for computer and big data majors. It includes four parts: mathematical logic, set theory, algebraic structure, graph theory. They are relatively independent branches respectively, of which propositional logic is the first part of discrete mathematics teaching materials in mathematical logic. This chapter is difficult for students in the second half of this chapter because the basic concepts are not easy to understand thoroughly. In order to solve this problem, this paper integrates the set theory into the understanding and mastery of the connectives, truth tables and important equivalent formulas in propositional logic, thus breaking through the learning difficulties in this chapter of propositional logic, enabling students to find a sense of achievement in learning from the first chapter of discrete mathematics and stimulating students' interest in learning this course.

Discrete mathematics is a mathematical discipline that studies the structure of discrete quantities and their interrelationships. It is an important branch of modern mathematics, as well as a basic core course for computer major and big data major. Its theory is strong [2]. Learning this course can cultivate students' logical reasoning ability, abstract thinking ability and meticulous generalization ability, and lay a foundation for other professional courses. Discrete mathematics includes four learning parts: mathematical logic, set theory, algebraic structure and graph theory. Among them, mathematical logic mainly studies the reasoning and formalization methods of discrete objects. Propositional logic is the first chapter in mathematical logic, and is also the first part of most discrete mathematics textbooks. This chapter is characterized by many concepts, abstraction, rigorous logic and many proof, which makes it difficult for students to learn [3]. Based on many years of teaching experience, this paper summarizes the root of the students' key difficulties in learning this chapter, integrates the collective thought to teach students to compare and understand, and then solves all the learning difficulties in this chapter.

1. The Importance of Learning Discrete Mathematics
Due to the limitation of computer storage space, the data in the computer can only be expressed as limited bits (32 bits or 64 bits), that is, the data is discrete. Due to the limitation of computer running time, the number of operations in the computer must be limited (Tianhe-1 is the fastest computer in the world, with thousand trillion operations per second), which means that the forms of operations are discrete. Discrete mathematics is a specialized course for computer and related majors. It is the theoretical basis for courses such as data structure, operating system, compilation principle, algorithm design and analysis.

The core of computer science is the treatment of discrete objects, and the science of studying discrete objects is discrete mathematics. Therefore, scholars who study computers and engage in
computer applications should not only learn discrete mathematics well, but also solve practical problems with discrete mathematics [1].

2. Incorporating Set Thoughts into Connectives

Compound proposition is a proposition formed by connecting simple propositions with connectives. Connectives is an important component of compound proposition. From the point of view of operation, connectives are an operation. In order to study the proposition, we must first define and symbolize the connectives. The definition of connectives is to integrate the set idea into the generated truth value [4]. Based on the completeness of connectives, this paper introduces the comparative understanding of negation “¬”, conjunctive “∧” and disjunctive “∨” in connectives. Firstly, the truth value 1 (or T) of the true proposition is regarded as the universal set U of the set, and the truth value 0 (or F) of the false proposition is regarded as the empty set φ of the set. The following is a detailed introduction to the collective understanding of the three binary connectives.

2.1 Negative connectives “¬”

The negative “¬” is understood as the set complement operation of “Δ”. The results of negative conjunctions “¬” and set complement operations “Δ” are shown in Table 1.

Table 1 Comparison of results between negative “¬” and set complement “Δ”

| p   | ¬p  | Δp  |
|-----|-----|-----|
| 1   | 0   | φ   |
| 0   | 1   | U   |

From the comparison results in Table 1, it can be seen that the definition of negation “¬” can be completely understood by the set complement operation “Δ”.

2.2 Conjunctive connectives “∧”

The conjunctive “∧” is understood as the intersection operation of the set “∩”. The conjunctive connectives “∧” is defined as the truth table in Table 2, and the intersection operation result with the set is as in Table 3. Among them, p and q are two propositions.

Table 2 Definition of conjunctive “∧”

| p | q | p ∧ q |
|---|---|-------|
| 1 | 1 | 1     |
| 1 | 0 | 0     |
| 0 | 1 | 0     |
| 0 | 0 | 0     |

Table 3 Results of intersection operation “∩”

| p   | q   | p ∧ q |
|-----|-----|-------|
| U   | U   | U     |
| U   | φ   | φ     |
| φ   | U   | φ     |
| φ   | φ   | φ     |

From the comparison results of Table 2 and Table 3, it can be seen that the definition of conjunction “∧” can be completely understood by the intersection operation of sets “∩”.

2.3 Disjunctive connectives “∨”

The disjunction “∨” is understood as the union operation of the set “∪”. The disjunction connectives “∨” is defined as the truth table in Table 4, and the union operation result with the set is as in Table 5.
Table 4 Definition of disjunctive “∨”

| p | q | p ∨ q |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 0 | 1 |
| 0 | 1 | 1 |
| 0 | 0 | 0 |

Table 5 Results of Union operation “∪”

| p | q | p ∪ q |
|---|---|---|
| U | U | U |
| U | φ | U |
| φ | U | U |
| φ | φ | φ |

From the comparison results of Table 4 and Table 5, it can be seen that the definition of disjunctive “∨” can be completely understood by the union operation “∪” of sets.

3. Incorporating Set Thought into Truth Table

Given a propositional formula, the truth table can not only accurately give the true and false assignment of propositional formula, but also judge the type of formula. However, it is difficult to get the correct result of the truth table, which can be easily solved by using collective thinking.

eg. 1 Judging the Type of Propositional Formula $p \land ((p \lor q) \land \neg p) \rightarrow q$ with Truth Table

Answer: The truth table of the proposition formula is shown in Table 6, in which the corresponding operations of the set are written in small brackets.

| p | q | ¬p | $p \lor q$ | $(p \lor q) \land \neg p$ | $(p \land (p \lor q) \land \neg p) \rightarrow q$ | $p \land ((p \lor q) \land \neg p) \rightarrow q$ |
|---|---|----|---|---|---|---|
| U | U | 0  | 1 (U) | 0 (φ) | 1 (U) | 1 (U) |
| U | φ | 0  | 1 (U) | 0 (φ) | 1 (U) | 1 (U) |
| φ | U | 0  | 1 (U) | 0 (φ) | 1 (U) | 1 (U) |
| φ | φ | 0  | 1 (U) | 0 (φ) | 1 (U) | 0 (φ) |

With the operation idea of set, the result of truth table can be obtained accurately and quickly. As can be seen from Table 6, the propositional formula is a non-permanent true satisfiable formula.

4. Incorporating Set Thought into Important Equivalent Formulas

In the chapter of propositional logic, there are 24 important equivalent formulas, totaling 16 operation rules. The idempotent law, the absorption law, the zero law, the identity law, the implication equivalence and the inversion law are integrated into the understanding of the set thought, so that the other laws are easier to master.

1) The idempotent law: $p \lor p \Leftrightarrow p \land (p \lor p = p)$ (Disjunctive operation is regarded as union operation of set.)

$p \land p \Leftrightarrow p \land (p \land p = p)$ (Conjunctive operation is regarded as intersection operation of sets.)

2) The absorption law: $p \lor (p \land q) \Leftrightarrow p \land (p \lor (p \land q) = p)$, $p \land (p \lor q) \Leftrightarrow p \land (p \lor (p \land q) = p)$

3) The zero law: $p \lor T \Leftrightarrow T \iff p \land U = U$, $p \land F \Leftrightarrow F \iff p \land \phi = \phi$

4) The identity law: $p \lor T \Leftrightarrow p \iff p \land U = p$, $p \lor F \Leftrightarrow p \iff p \land \phi = p$

5) The implication equivalence and the inversion law: $p \rightarrow q \Leftrightarrow \neg q \rightarrow \neg p \Leftrightarrow \neg p \lor q \iff p \rightarrow q$ regard as $p \subseteq q$, equivalent to $\neg q \subseteq \neg p$, that is, $\neg q \rightarrow \neg p$; and $\neg p \lor q = U$, that is, $p \rightarrow q$ is true equivalent to $\neg p \lor q$ is true.
5. Conclusion
From the above analysis, it can be seen that the appropriate integration of set thought into the abstract concept of propositional logic can help students understand and master its definition, thus making it easier for them to learn.

We can further study the application of set in propositional logic. The operation law satisfied by set operation is similar to the equivalent operation law of propositional logic, such as commutative law, associative law, distributive law, idempotent law, double negation law, Dedekind completion, absorption law, law of exponentiation, contradiction law, etc. [5]. This kind of research can help students understand and master the concepts of strong reasoning in discrete mathematics more deeply.

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