Comparing the group theoretical approaches to the description of dipole and quadrupole collectivity in nuclei

A I Georgieva

Institute of Nuclear Research and Nuclear Energy, Bulgarian Academy of Sciences, 72 bul. Tzarigradsko Chaussee, Sofia 1784, Bulgaria
E-mail: anageorg@inrne.bas.bg

Abstract. The comparison of the two types of collectivity in nuclear dynamics is based on the equal realizations of their basic symmetries, represented by the algebra-chain \( u(3) \supset su(3) \supset so(3) \). We illustrate this on the example of two phenomenological algebraic models: the semimicroscopic algebraic cluster model (SACM) with one of the clusters considered as a scalar and the interacting two vector boson model (IVBM). In both of them the root group structure that leads to the above chain is based on the coupling of two different \( U(3) \) representations, with different physical interpretations. These similarities allow for the use of the same mathematical tools in building the theory, but there are important differences that come from the different higher symmetries in which this equal substructures are embedded.

1. Introduction

The nucleus is a unique and sophisticated many-body system which at the present stage is impossible to be described with the direct application of the quantum mechanics. For this reason the accumulated experimental data is interpreted in the framework of different models, focusing its basic assumptions on specific properties of the nuclei. These models are based on different physical pictures. The shell model describes it as a small atom, considering microscopically the individual movements of the nucleons on their specific orbits. The collective model considers the nucleus as a liquid drop, introducing the quadrupole collectivity, which describes the deformation of the nuclear shape. The cluster model makes it similar to a molecule formed by two smaller systems and is based on the dipole degrees of freedom. Therefore, in order to really understand the nuclear structure it is of utmost importance to find the exact connection of these models.

In this contribution we discuss the interrelation of the two different types of collectivity, by comparing two models, exploring each of them: the Interacting Two Vector Boson model (IVBM) [1] and the Semimicroscopic Algebraic Cluster Model (SACM) with one of the clusters considered as a scalar [2]. This investigation is based on the comparison of the common algebraic structures used in both models, and on the comparison with the observed experimental spectra. As an example of the physical applications, the possible clusterizations of the shape isomers (superdeformed, hyperdeformed etc.) states of some nuclei are presented [3, 4].
2. Algebraic structures

2.1. The basic SU(3) symmetry

The basic connection between the quadrupole shape and clusterization was found in the fifties, via the bridge of the shell model. This relation is based on the reduction chain:

\[
U(3) \supset SU(3) \supset SO(3) \supset SO(2)
\]

introduced by Elliott [5], and relates the invariant operators of the algebras in (1) to the observables – quadrupole-quadrupole interaction \( Q_m \) and angular momentum \( L_m \), of the described system. The chain (1) also gives the harmonic oscillator basis, hence in this way the quadrupole deformation and collective rotation can be derived from the spherical shell model: the states belonging to a collective band are determined by their specific \( SU(3) \) symmetry. Elliott’s model has the advantage of allowing for a geometrical analysis of the eigenstates of a nuclear system via the relations between the microscopic parameters \( (\lambda, \mu) \) (1) and the parameters \( (\beta, \gamma) \) of the collective model [6, 7]:

\[
\beta^2 = \lambda^2 + \mu^2 + 3(\lambda + \mu) + 2\lambda \mu \quad \text{and} \quad \gamma = \tan^{-1}\left(\frac{\sqrt{3}\mu}{2\lambda + \mu + 3}\right).
\]

Wildermuth and Kanellopoulos [8] established the relation between the shell and cluster models. They have shown that the Hamiltonians of the two models can be rewritten into each other exactly in the harmonic oscillator approximation. This relation results in a close connection between the corresponding eigenvectors, too: the wave function of one model is a linear combination of those of the other, which belong to the same energy. Later on this relation was interpreted by Bayman and Bohr [9] in terms of the \( SU(3) \) symmetry. As a consequence, the cluster bands can be considered from the shell model viewpoint, like a set of states having special \( SU(3) \) symmetry, just like the collective rotational bands.

2.2. Direct product coupling

The further elaboration of the algebraic collective nuclear models is based on the consideration of more realistic assumptions, considering two interacting systems of particles. The models presented in this contribution – the IVBM [1] and the SACM [2] are exactly of this type.

The IVBM is based on the introduction of two kinds of vector bosons (called \( p \) and \( n \) bosons), that ’build up’ the collective excitations in the nuclear system. The creation operators of these bosons are assumed to be \( SO(3) \) vectors and they transform according to two independent fundamental representations \((1,0)\) of the group \( SU(3) \). These bosons form an ‘F-spin’ doublet of the \( U(2) \) group and differ in its projection. The introduction of this additional degree of freedom leads to the extension of the \( SU(3) \) symmetry to \( U(6) \supset U_p(3) \oplus U_n(3) \supset SU(3) \supset SO(3) \). The bilinear products of the creation and annihilation operators of the two vector bosons generate the noncompact symplectic group \( Sp(12, R) \) [1]. The symplectic extension of the \( u(6) \) algebra with the dynamical symmetry \( Sp(12, R) \supset U(6) \supset U(2) \otimes U(3) \) [1] allows the change in the number of phonons needed to build the collective states, which results in larger model spaces, that can accommodate the more complex structural effects observed in the contemporary experiment.

The basic assumption of all the cluster models is that the relevant degrees of freedom of the nucleus are classified into two categories. Some of them account for the relative motion of the clusters, while others are related to the internal degrees of freedom of the clusters. We consider here binary cluster systems, with the interactions of the relative motion described by the vibron model of \( U(4) \supset U(3) \) group structure [10]. The internal structure of the clusters is described by the Elliott model (1) [5], where \( U(3) \) refers to the orbital part. From the viewpoint of the interaction the group structure simplifies to \( U_{C_i}(3) \otimes U_{C_i}(3) \otimes U_R(3) \supset U(3) \supset O(3) \), where \( C_i \) stands for the \( i \)’th cluster, and \( R \) indicates relative motion. The Hamiltonian built up from the invariant operators of this chain has a \( U(3) \) dynamical symmetry. The \( U(3) \) limit of the algebraic cluster models corresponds to a soft vibrational-rotational motion in the collective model terms.
When one of the clusters is a scalar, like an $\alpha$-particle for example, the $U_{C2}(3)$ is neglected and practically the direct product coupling $U_{C1}(3) \otimes U_{R}(3) \supset U(3) \supset O(3)$ is explored. From the microscopic viewpoint the $U(3)$ dynamical symmetry describes shell-model-like clusterization, when the cluster model state can be expressed in terms of a few (or a single) shell model basis states [11]. Furthermore, this symmetry is also the matching point with the quadrupole collective IVBM.

3. Application

The connection between the quadrupole and dipole (cluster-like) collectivity is revealed by the common symmetry characters, as discussed above. Both features are observed when the SACM is applied in the $SU(3)$ dynamical symmetry approximation. Then the cluster structure is evident from the model assumption, and the quadrupole shape of the state is uniquely determined by the $SU(3)$ quantum numbers. It was found that this dynamical symmetry gives a reasonable description of the detailed spectra in several cases, and it can reproduce the gross features especially well [11].

A very interesting phenomenon is the existence of the shape isomers, e.g. superdeformed (SD) and hyperdeformed (HD) states, and their possible clusterization [12, 13]. Symmetry-based methods can be very fruitful in their studies. Here we refer to an approach, which is closely related to the algebraic cluster model mentioned before. It is a two-step procedure [14]. First the shape isomers are obtained from a Nilsson-calculation by requiring the self-consistency of the quadrupole shape [15, 16, 17]. When doing so, the quasi-dynamical $U(3)$ symmetry is applied, which coincides with the real $U(3)$ symmetry in the simple cases when this latter one is valid too [18]. In the second step the $U(3)$ selection rule is used for the determination of the allowed clusterizations [19, 20, 21]. The model predictions can be compared directly to the experimental observation, and in some cases, like for the SD state of $^{28}\text{Si}$ [17] and for the HD state of the $^{36}\text{Ar}$ [22, 23, 24], they have been justified.

In conclusion, in this contribution it was established that the interrelation of the two approaches are due to the basic foundation of each of them by the microscopic shell model of nuclei, which also has an algebraic analogue: the $SU(3)$ model of Elliott. The comparison of the two models of nuclear dynamics is due to the common realization of their basic symmetries. The common intersection of the three basic structure models is the $U(3) \otimes U(3)$ dynamical symmetry. This finding is a generalization of the previously known result for the case of the single shell problem, when the intersection is the $U(3)$ dynamical symmetry.

Acknowledgments

This work is a result of a collaborative research with J. Cseh and J. Darai from the Institute of Nuclear Research of the Hungarian Academy of Sciences and was supported by a Joint RESEARCH PROJECT No. 7 ‘The atomic nucleus and neutron physics’ between the Bulgarian Academy of Sciences and the Hungarian Academy of Sciences and the Bulgarian National Foundation for Scientific Research through contract DID-02/16/17.12.2009.

References

[1] Georgieva A I, Ganev H G, Draayer J P and Garistov V P 2009 Physics of Particles and Nuclei 40 461
[2] Cseh J 1992 Phys. Lett. B 281 173
Cseh J, Lévai G 1994 Ann. Phys. (NY) 230 165
[3] Darai J, Cseh J, Lépine-Szily A, Algora A, Hess P O, Antonenko N V, Jolos R V and W. Scheid 2010 J. Phys.: Conf. Series 205 012022
[4] Darai J, Cseh J, Antonenko N V, Adamian G G and Georgieva A 2012 J. Phys.: Conf. Series 366 012009
[5] Elliott J P 1958 Proc. Roy. Soc. London A 245 128
Elliott J P 1958 Proc. Roy. Soc. London A 245 562
[6] Bohr A, Mottelson B R and Pines D 1958 Phys. Rev. 110 936
[7] Castaños O, Draayer J P and Leschber Y 1988 Z. Phys. A 329 33
   Leschber Y and Draayer J P 1987 Phys. Lett. B190 1
[8] Wildermuth K and Kanellopoulos Th 1958 Nucl. Phys. 7 150
[9] Bayman B F and Bohr A 1958/59 Nucl. Phys. 9 596
[10] Iachello F 1981 Phys. Rev. C 23 2778
[11] Lévai G, Cseh J and Scheid W 1992 Phys. Rev. C 46 548
   Cseh J, Lévai G and Scheid W 1993 Phys. Rev. C 48 1724
   Cseh J 1994 Phys. Rev. C 50 2240
   Cseh J, Lévai G, Ventura A and Zuffi L 1998 Phys. Rev. C 58 2144
   Cseh J, Gupta R K and Scheid W 1992 Phys. Lett. B 299 205
   Lévai G and Cseh J 1996 Phys. Lett. B 381 1
[12] Cseh J, Algora A, Darai J and Hess P O 2004 Phys. Rev. C 70 034311
[13] Algora A, Cseh J, Darai J and Hess P O 2006 Phys. Lett. B 639 451
[14] Darai J, Cseh J, Adamian G G and Antonenko N V 2012 Eur. Phys. J. Web of Conf. 38 16001
[15] Cseh J, Darai J, Sciani W, Otani Y, Lépine-Szily A, Benjamin E A, Chamon L C and Lichtenthäler Filho R 2009 Phys. Rev. C 80 034320
[16] Darai J, Cseh J, Antonenko N V, Royer G, Algora A, Hess P O, Jolos R V and Scheid W 2011 Phys. Rev. C 84 024302
[17] Darai J, Cseh J and Jenkins D G 2012 Phys. Rev. C 86 064309
[18] Hess P O, Algora A, Hunyadi M and Cseh J 2002 Eur. Phys. J. A 15 449
[19] Cseh J and Scheid W 1992 J. Phys. G 18 1419
[20] Cseh J, Darai J, Algora A, Yépez-Martinez H and Hess P O 2006 Rev. Mex. Fis. S. 52 1
[21] Cseh J and Darai J 2009 AIP Conf. Proc. 1098 (Fusion08) 225
[22] Cseh J, Darai J, Algora A, Yépez-Martinez H and Hess P O 2008 Rev. Mex. Fis. S. 54 30
[23] Sciani W, Otani Y, Lépine-Szily A, Benjamin E A, Chamon L C, Lichtenthäler Filho R, Darai J and Cseh J 2009 Phys. Rev. C 80 034319
[24] Cseh J, Darai J, Algora A, Antonenko N V and Adamian G G 2011 Eur. Phys. J. Web of Conf. 17 16001