Wideband Small-Signal Model of Common-Mode Inductors Based on Stray Capacitance Estimation Method

Shotaro Takahashi* a) Member, Sari Maekawa* Member

(Manuscript received Jan. 00, 20XX, revised May 00, 20XX)

This study presents a wideband small-signal model for common-mode inductors (CMIs) based on the stray capacitance estimation method. The proposed model can be used to calculate the small-signal characteristics of CMIs over a wide band of frequencies by substituting the stray capacitance values in the model are considering the frequency dependence of the complex permeability. The common-mode (CM) impedances of the fabricated manganese zinc ferrite and nanocrystalline CMIs are measured using an impedance analyzer to validate the proposed model. Comparisons between the results of the measured and calculated CM impedances for the fabricated CMIs show that the proposed model enables the calculation of the CM impedances of CMIs over a wide frequency range from 1 kHz to 100 MHz.

Keywords: common-mode inductors, EMI, MnZn ferrites, nanocrystalline, stray capacitance

1. Introduction

As a side effect of high-frequency switching of power converters, the frequency band of electromagnetic interference (EMI) generated by the switching of power semiconductor devices becomes higher (1)-(5). Generally, EMI filters composed of passive components are installed into the power line to reduce the conducted EMI and satisfy the EMI standards (6)-(10). However, the attenuation characteristics of common-mode inductors (CMIs) degrade in the high-frequency range due to the frequency dependence of complex permeability and winding stray capacitance (11)-(16). Therefore, to avoid the trial-and-error process of EMI filter design, it is necessary to model the frequency characteristics of CMI over a wide bandwidth at the design stage.

The frequency dependence of the complex permeability is modeled using the frequency-dependent R-L series-connected circuit, and stray capacitance is modeled as the parallel-connected capacitor. Heldwein et al. have presented a comprehensive physical characterization and modeling procedure for three-phase CMIs (18). In (18), the authors deals with the common-mode (CM) and the differential-mode (CM) frequency characteristics of CMIs and describes the effect of magnetic core saturation on the equivalent series core resistance. However, a more accurate analytical model of stray capacitance is necessary to model the capacitive behavior of the CM impedance in the high-frequency range.

Several previous studies have proposed methods for estimating stray capacitance of inductors (19)-(20). Massarini et al. proposed an analytical model for the stray capacitance of multilayer inductors (19). This model calculates the stray capacitance from several simple equations by approximating the electric force lines as straight lines. On the other hand, this method overestimates the

turn-to-turn capacitance because it does not consider the winding pitch between adjacent windings. Pasko et al. proposed an analytical model for the stray capacitance of CMIs (24). Since this model is based on the equations presented in (19), the error increases as the winding pitch increases. Ayachit et al. modified the equations proposed in (19) to calculate the turn-to-turn capacitance with the winding pitch (25). Since air-core solenoid inductors have been studied in (25), the turn-to-core capacitance has not been considered. Moreover, the winding pitches of toroidal inductors are different in three different regions: inner, lateral, and outer. Therefore, this method cannot be simply applied to inductors with magnetic cores. Based on the analytical model proposed in (25), a method for estimating stray capacitance in three-phase CMIs has been proposed (26). In (26), CMIs fabricated with a toroidal core of nickel-zinc ferrites with very high resistivity are investigated. Thus, the turn-core capacitance is neglected for simplicity of calculation. Therefore, this method cannot be applied to inductors fabricated with other magnetic materials. Kovacic et al. performed a detailed analysis of stray capacitance of CMIs (22). The turn-to-turn capacitance and turn-to-core capacitance are calculated by assuming that the electric force lines between conductors are circular arcs. Furthermore, each turn of CMIs is modeled independently as RLC lumped circuits. The analytical model proposed in (22) allows calculating the CM impedance of CMIs by taking the capacitive coupling and the inductive coupling between turns into account. However, as the number of turns increases, the number of model components increases, which results in heavy simulation using this model.

This study proposes a small-signal model of CMIs for CM based on the stray capacitance estimation method. Section 2 presents the small-signal model for CMIs based on the frequency dependence of complex permeability. The model parameters related to the complex permeability are discussed in Section 3. The stray capacitance estimation method proposed in the previous study (23) is modified to calculate the turn-to-turn and turn-to-core capacitance of CMIs in Section 4. The CM frequency characteristic of CMIs can be calculated over the wideband frequency range by substituting the

---

* Correspondence to: Shotaro Takahashi. E-mail: s.takahashi@st.seikei.ac.jp
  a) Seikei University,
  3-1-1, Kita-machi, Kichijoji, Musashino, Tokyo 180-8633, Japan

© 2001 The Institute of Electrical Engineers of Japan.
estimated stray capacitance into the model. In Section 5, the CM frequency characteristics of the fabricated CMIs made of manganese-zinc (MnZn) ferrite and nanocrystalline are measured using an impedance analyzer. The comparison between the measurement results and the calculation results from the proposed model shows that the proposed modeling method can calculate the CM frequency characteristic of CMIs over the wide frequency range at the design stage.

2. Small-Signal Model of CMIs

Fig. 1 shows the configuration of a single-phase CMI. The single-phase CMI consists of a toroidal core with two windings wound around the core in the same direction as the CM currents. $C_s$ is the stray capacitance of each winding, represented as a capacitance network of the turn-to-turn and turn-to-core capacitance. The CM impedance of the CMI $Z_{CM}$ is measured by using an impedance analyzer under the circuit connection shown in Fig. 1.

Fig. 2 shows a small-signal model of the CMI for CM. $L_{CM}$ is the CM inductance, $R_{CM}$ is the CM resistance for small-signal losses, and $C_{CM}$ is the winding stray capacitance for CM. $L_{CM}$ and $R_{CM}$ are related to the frequency dependence of the complex permeability and can be defined as functions of the frequency $f$. The CM capacitance $C_{CM}$ is the parallel connection of the total stray capacitance of each winding $C_s$ ($C_{CM} = 2C_s$). The dominant parameter is the winding DC resistance at the low-frequency range (e.g., below 1 kHz). As well known, the winding resistance is also the frequency-dependent component and increases with the frequency. However, the influence of the winding resistance on the small-signal characteristic of the CMI for CM is insignificant compared to that of the small-signal core loss resistance related to the imaginary part of the complex permeability of MnZn ferrites and nanocrystalline in the high-frequency range. Furthermore, the stray capacitance becomes the dominant parameter in the higher frequencies than several MHz. Thus, the reliability of the small-signal model of the CMI depends on the ability to model the complex permeability and stray capacitance accurately. For these reasons, the model neglects the winding resistance in this study.

The CM impedance of the model is given by

$$|Z_{CM}| = \sqrt{r^2 + x^2}$$

(1)

$$r_s = \frac{R_{CM}}{1 - \omega^2L_{CM}C_{CM}^2 + (\omega C_{CM}R_{CM})^2}$$

(2)

$$x_s = \frac{\omega L_{CM}(1 - \omega^2L_{CM}C_{CM}^2 - C_{CM}R_{CM}^2/L_{CM})}{(1 - \omega^2L_{CM}C_{CM}^2 + (\omega C_{CM}R_{CM})^2)}$$

(3)

where $\omega = 2\pi f$.

The following sections will describe the identification procedure of each model parameter.

3. Calculation of CM Inductance and CM Resistance

The magnetic properties of the core material are essential for modeling the inductor impedance. The complex permeability ($\mu_r = \mu_r' - j\mu_r''$) is essential for modeling the wideband frequency behavior of the inductor. Even if the winding resistance can be neglected in the inductor impedance, the inductor still indicates the complex impedance due to core losses. The impedance of an inductor with $N_m$ turns can be represented as a combination of an equivalent series inductance $L$ and an equivalent series resistance $R$ is given by

$$j\omega L = j\alpha_{\mu_0}(\mu_r' - \mu_r'' \omega^2) \frac{A_s}{I_s} N_m^2 = j\alpha L_s + R_s$$

(4)

where $A_s$ is the cross-sectional area, $I_s$ is the magnetic path length of the core, and $\mu_0$ is the permeability of free space ($\mu_0 = 4\pi \times 10^{-7}$ H/m).

According to (17) and (18), the equivalent series resistance can be calculated from an imaginary part of the complex permeability and represents small-signal losses. The core loss resistance for large-signal can be calculated by Steinmez equation is dependent on the inductor current and is larger than the equivalent resistance for small-signal. Typically, a CM filter is a combination of the CMI and Y-capacitors. Since almost CM current flows through Y-capacitors, it is sufficient to calculate the equivalent resistance for small-signal in the design of EMI filters.

For the filter design process, the best way is to use the data from the manufacturer. However, it is a rare case that the complex permeability of the core material over the wide frequency range up to 100 MHz is available from the manufacturer's datasheets. Therefore, the data obtained from the measured impedance of an inductor is used for modeling in this study. In the frequency range where the effect of the stray capacitance is neglected, the real and imaginary parts of the complex permeability can be calculated from the following equations:

$$\mu_r' = \frac{l_s}{\mu_0 A_s N_m^2}$$

(5)
The equivalent series inductance and the equivalent series resistance can be measured by using the impedance analyzer.

Fig. 3 shows the measured complex permeabilities of MnZn ferrite (part number: B64290L0048X830, EPCOS) and nanocrystalline (part number: T-60006-L2030-W358, VAC). The measurements are carried out by using the impedance analyzer (E4990A, Keysight) in the frequency range from 1 kHz to 100 MHz. The real part of the complex permeability of MnZn ferrite shown in Fig. 3(a) begins to increase slightly at around 100 kHz and then converges to zero. The imaginary part rises sharply. Furthermore, it indicates that there are stray capacitances between the turn and the nonadjacent turn.

The slopes of the increase and decrease of the imaginary part are typically unavailable due to the high effective permittivity. In most cases, the manufacturers measure the frequency characteristics of the magnetic core solenoid inductor proposed in Fig. 2 are calculated based on the measured complex permeability

\[
\mu_r(f) = R_e(f) \cdot \frac{l_c}{2 \pi \mu_0 A_c N_m^2} \tag{6}
\]

Fig. 4 depicts the winding stray capacitance for \( N \) turns. The per-turn-based model proposed in (22) can simulate multiple resonances of CMIs.

4. Estimation of Stray Capacitance

4.1 Turn-to-Turn Capacitance

The CM inductance and the CM resistance of the model shown in Fig. 2 are calculated based on the measured complex permeability

\[
L_{CM}(f) = \mu_0 \mu_r(f) \cdot \frac{A_c}{l_c} N^2 \tag{7}
\]

\[
R_{CM}(f) = 2 \pi f \mu_0 \mu_r(f) \cdot \frac{A_c}{l_c} N^2 \tag{8}
\]

where \( N \) is the turn number of each winding of the CMI.
in (25) is modified to calculate the turn-to-turn capacitance and turn-to-core capacitance for the toroidal CMI.

Fig. 5 shows the analytical model of the turn-to-turn capacitance presented in (25). The model includes the winding pitch of the adjacent turns. According to (25), the turn-to-turn capacitance $C_{i,i}$ can be calculated as

$$C_{i,i} = \frac{2\pi l_{w,i}}{\sqrt{\varepsilon - 1}} \cdot \tan^{-1}\left(\sqrt{1 + \frac{2}{\varepsilon - 1}}\right)$$  \hspace{1cm} (9)

$$\alpha = \frac{1}{\varepsilon} \ln \frac{d_o}{d_w} + \frac{p}{d_o}$$  \hspace{1cm} (10)

where $\varepsilon$ is the permittivity of free space, $\varepsilon$ is the relative permittivity of the wire insulation, $l_{w,i}$ is the winding length, $d_o$ is the diameter of the wire including the insulation, $d_w$ is the diameter of the conductor, and $p$ is the winding pitch.

Typically, the mean length per turn $l_{t}$ is used as $l_{w}$ in the above equation. The mean length per turn of toroidal inductors is assumed as shown in Fig. 6, and is given by

$$l_{t} = OD \cdot ID + 2HT + 4d_o$$  \hspace{1cm} (11)

where $OD$ is the outer diameter, $ID$ is the inner diameter, and $HT$ is the height of the core, respectively.

Due to the winding pitch increases from the inner to the outer region of the toroidal core, as shown in Fig. 7, the turn-to-turn capacitance has to be calculated for three different regions: inner, lateral, and outer. For the inner and outer regions, the winding pitches $p_i$ and $p_o$ are represented as functions of $\varphi$

$$p_i = (ID - d_o) \cdot \sin \frac{\varphi}{2(N - 1)}$$  \hspace{1cm} (12)

$$p_o = (OD + d_o) \cdot \sin \frac{\varphi}{2(N - 1)}$$  \hspace{1cm} (13)

The winding pitch for the lateral region $p_l$ is obtained as the average value of $p_i$ and $p_o$, $p_l = (p_i + p_o)/2$.

The winding length for the inner, lateral, and outer regions: $l_{w,i}$, $l_{w,l}$, and $l_{w,o}$ are

$$l_{w,i} = l_{w,0} = HT + 2d_o$$  \hspace{1cm} (14)

$$l_{w,l} = \frac{OD - ID}{2}$$  \hspace{1cm} (15)

Here, the mean length per turn of toroidal inductors is obtained as $l_{t_i} = l_{w,i} + 2l_{w,l} + l_{w,o}$, and is equal to the equation (11).

The turn-to-turn capacitance for the inner, lateral, and outer regions: $C_{i,i}$, $C_{i,l}$, and $C_{i,o}$, are calculated by substituting the winding pitches and the winding lengths calculated from the equations (12)-(15) into the equation (9). The turn-to-turn capacitance $C_{i,i}$ is obtained as the sum of the calculated capacitance for three different regions ($C_{i,i} = C_{i,i} + 2C_{i,l} + C_{i,o}$).

### 4.2 Turn-to-Core Capacitance

For calculating the turn-to-core capacitance, the electric flux line between the conductor of the winding and the magnetic core is assumed, as shown in Fig. 8.
The turn-to-core capacitance is obtained as a series connection of the capacitance due to the insulation of the winding $C_{t-c}$, the capacitance of air gap $C_{t-c,g}$, and the capacitance due to the insulation of the core $C_{i-c}$.

The calculation starts from assuming the cylindrical elementary capacitance $dC_{t-c}$, as shown in Fig. 9, which is obtained as

$$dC_{t-c} = \varepsilon_0 \varepsilon_r \frac{r}{d} d\theta dl$$

(16)

By integrating the equation (16), the radius $r$ ranging from $r_w$ to $r_c$ and the length $l$ ranging from zero to the mean length per turn $l_r$, the elementary capacitance due to the insulation of the winding related to an elementary angle $d\theta$ for one turn is given by

$$dC_{t-c,i} = \varepsilon_0 \varepsilon_r d\theta \int_{r_w}^{r} \int_{0}^{l_r} \frac{r_0 r_1 \ln r_0}{2 \ln r_w} d\theta = \varepsilon_0 \varepsilon_r d\theta \int_{r_w}^{r} \frac{r \ln r_0}{2 \ln r_w} d\theta$$

(17)

The air gap length between the winding and the magnetic core $g$ is obtained as the function of the angle $\theta$

$$g(\theta) = \frac{d_g}{2}(1 - \cos \theta)$$

(18)

The elementary capacitance of air gap is obtained as

$$dC_{t-c,g} = \frac{\varepsilon_0 dS}{g(\theta)} = \frac{\varepsilon_0 d_l}{1 - \cos \theta} d\theta$$

(19)

where an elementary surface area $dS$ is

$$dS = \frac{d_{dl}}{2} d\theta$$

(20)

The elementary capacitance due to the insulation of the core $dC_{i-c}$ is calculated as

$$dC_{i-c} = \frac{\varepsilon_0 d_0 l_1}{2 t_c} d\theta$$

(21)

where $t_c$ is the thickness of the insulation and $\varepsilon_c$ is the relative permittivity of the insulation.

The elementary turn-to-core capacitance is obtained as a series connection of $dC_{t-c,i}$, $dC_{t-c,g}$, and $dC_{i-c}$

$$dC_{t-c} = \frac{\varepsilon_0 l_1}{\varepsilon_i \ln \frac{d_0}{2} + \frac{2 t_c}{\varepsilon_c d_0} + 1 - \cos \theta}$$

(22)

The turn-to-core capacitance is obtained by integrating the equation (22) for $\theta$ ranging from $\pi/2$ to $-\pi/2$. The following equation is useful for integrating the equation (22)

$$\int \frac{dx}{a - \cos x} = \frac{2}{\sqrt{a^2 - 1}} \tan^{-1} \left( \frac{a + 1}{\sqrt{a^2 - 1}} \tan \left( \frac{x}{2} \right) \right)$$

(23)

Using the equation (23), the turn-to-core capacitance is given by

$$C_{t-c} = \frac{4 \varepsilon_0 l_1}{\sqrt{\beta + 1}^2 - 1} \tan^{-1} \left( \frac{\sqrt{\beta + 1}}{1 + \sqrt{\beta}} \right)$$

(24)

$$\beta = \frac{1}{\varepsilon_i} \ln \frac{d_0}{2} + \frac{2 t_c}{\varepsilon_c d_0}$$

(25)

4.3 Total Stray Capacitance for N Turns

Fig. 10 shows the stray capacitance network of two turns. The total stray capacitance for two turns $C_i(2)$ is obtained as

$$C_i(2) = C_{i-t} + \frac{C_{i-c}}{2}$$

(26)

Applying the $\Delta$-$Y$ transformation to the stray capacitance network shown in Fig. 11, the total stray capacitance for three turns $C_i(3)$ can be represented as

$$C_i(3) = \frac{C_{i-t}}{2} + \frac{C_{i-c}}{2}$$

(27)

Similarly, when the number of turns is set to four or five, total stray capacitances are determined using the $\Delta$-$Y$ transformation as shown in Figs. 12 and 13. The total stray capacitances for four and five turns are obtained as
As a result, the total stray capacitance for \( N \) turns \( C_t(N) \) is

\[
C_t(N) = \begin{cases} 
C_{lt} + \frac{C_{lt}}{2} & (N = 2) \\
C_{lt} + \frac{C_{lt}}{2} + \frac{C_{lt}}{2} + \frac{C_{lt}}{2} & (N = 3) \\
C_{lt}(N + 2) + \frac{C_{lt}}{2} + \frac{C_{lt}}{2} & (N \geq 4)
\end{cases}
\]

(30)

The CM capacitance \( C_{CM} \) is the parallel connection of \( C_t(N) \), hence, \( C_{CM} = 2C_t(N) \) is used in the model.

5. Validation of the Model

The validation of the model is carried out by comparing the measured and calculated results of the CM impedances of the single-phase CMIs for MnZn ferrite and nanocrystalline materials. Table 1 shows the specifications of the fabricated CMIs. The calculated stray capacitances for each CMI are listed in Table 2. Here, \( C_{lt} \) is the calculated turn-to-turn capacitance when the winding pitch is set to \( p_t \) (the winding pitch in the inner region of a toroidal core), and \( C_{lt} \) is the calculated result when the winding pitch is set to \( p_o \) (the winding pitch in the outer region). Due to the winding pitch being set to constant, the calculated result when the winding pitch is set to \( p_t \) is possibly overestimated, and the result when the winding pitch is set to \( p_o \) may be underestimated. On the other hand, the proposed method calculates the turn-to-turn capacitance for three different regions of toroidal cores: inner, lateral, and outer. Therefore, it can be confirmed from Table 2 that the calculated turn-to-turn capacitances from the proposed method are intermediate values between the calculated results when the winding pitch is assumed to be constant.

The measurements were performed using the impedance analyzer (E4990A, Keysight) in the frequency range from 1 kHz to 100 MHz. Fig. 14 shows a picture of the fabricated CMI (CMI-2). The angle \( \varphi \) that each phase winding covers the magnetic core is managed by mounting the CMI on the printed circuit board. The fabricated CMI and the impedance are connected by using copper tape with low impedance. The measured and calculated CM impedances of the fabricated CMIs are shown in Figs. 15 and 16.

![Figure 12: Stray capacitance network for four turns.](image)

![Figure 13: Stray capacitance network for five turns.](image)

| Table 1. Specifications of the fabricated CMIs. |
|-----------------------------------------------|
| material | CMI-1 | CMI-2 |
| MnZn ferrite | 35.5 mm | 32.8 mm |
| Nanocrystalline | 19.2 mm | 17.6 mm |
| parts number | T60606-2030- W358 |
| OD [mm] | 13.6 | 12.5 |
| ID [mm] | 0.55 | 1.25 |
| HT [mm] | 0.88 | 0.88 |
| \( d_t [mm] \) | 0.81 | 0.81 |
| \( N \) | 14 | 14 |
| \( \theta \) | \( 5\pi/6 \) | \( 5\pi/6 \) |
| insulation (core) | plastic case | plastic case |
| \( \varepsilon_{hi} \) | 3.5 | 3.5 |
| \( \varepsilon_{lo} \) | 3.6 | 3.0 |

| Table 2. Calculated results of stray capacitances. |
|-----------------------------------------------|
| CMI-1 | CMI-2 |
| \( C_{lt} [pF] \) | 0.30 | 0.32 |
| \( C_{lt} [pF] \) | 2.12 | 0.96 |
| \( C_{lt} [pF] \) | 0.46 | 0.49 |
| \( C_{lt} [pF] \) | 0.19 | 0.19 |
| \( C_{CM} [pF] \) | 2.39 | 1.21 |
| \( C_{CM,calc} [pF] \) | 16.6 | 15.4 |
To validate the accuracy of the proposed stray capacitance estimation method, Figs. 15 and 16 also show the results calculated using the CM capacitance $C_{CM,\text{conv}}$ estimated by the conventional method (24).

Fig. 15 shows that the calculated CM impedance of CMI-1 using the stray capacitance estimated from the conventional method does not follow the measured result in the higher frequencies than 1 MHz. This is because the conventional stray capacitance estimation method does not include the information of the winding pitch and thus overestimates the turn-to-turn capacitance. On the other hand, the proposed stray capacitance estimation method considers the winding pitch in each region of the toroidal core; thus, the stray capacitance is estimated accurately. Note that the decrease of the CM impedance from around 1.5 MHz also depends on the complex permeability. As already shown in Fig. 3(a), the real part of the complex permeability of MnZn ferrite is negative from 1.5 MHz to 10 MHz. Even if the effect of the stray capacitance is neglected, the CM impedance decreases in the frequency range where the real part of the complex permeability is negative (27). Fig. 15 shows that the calculated CM impedance from the proposed model agrees well with the measurement results over a wide range of frequencies from 1 kHz up to 30 MHz by modeling the frequency dependence of the complex permeability and the stray capacitance accurately. The measured CM impedance of the CMI-1 has the second resonance due to the transmission line effect of the winding beyond 100 MHz. The calculated result from the proposed model does not follow the measured result from around 30 MHz since the proposed model does not include the transmission line effect. However, the ability to estimate the first resonance frequency of the CM impedance is beneficial in practice.

Fig. 16 indicates an excellent correlation between the calculated CM impedance of CMI-2 from the proposed model and the measured result in the wideband frequencies from 1 kHz to 100 MHz. However, the calculated CM impedance based on the conventional stray capacitance estimation method does not follow the measured result in the higher frequencies than 2 MHz due to the overestimated stray capacitance. The measured CM impedance increases with a slope of +20 dB per decade up to 50 kHz. As shown in Fig. 3(b), the real part of the complex permeability is approximately constant in this frequency range, and the core losses for small-signal can be neglected. From 50 kHz to 7 MHz, the CM impedance increases with a slope of around +10 dB per decade due to the increase of the imaginary part of the complex permeability. The CMI made of nanocrystalline behaves as the resistive component in this frequency range. This is a good characteristic for EMI filtering because the core losses of nanocrystalline well attenuate the high-frequency CM noise (28). The effect of the stray capacitance starts to appear from 7 MHz, and the measured CM impedance starts to decrease with a slope of ~20 dB per decade. Due to the high resistive component related to the imaginary part of the complex permeability, the resonance peak is highly damped. The measured results validate the importance of the accurate modeling of the complex permeability for modeling the small-signal characteristic of CMIs. If the complex permeability of the magnetic material is neglected, completely different calculated results would appear, filtering performance of CMIs could not be adequately evaluated over the wide frequency range.

6. Conclusion

This study proposes the modeling method for calculating the small-signal characteristic of CMIs for CM. The model includes information on the frequency dependence of complex permeability and stray capacitance. Every model parameter can be obtained at the design stage of the CMI. The stray capacitance estimation method proposed in the previous study is modified to calculate the stray capacitance of CMIs made by winding the coils around toroidal cores. The model can calculate the small-signal characteristic of CMIs for CM over the wideband frequency range by substituting the estimated stray capacitance into the model. The CM impedances of the CMIs made of MnZn ferrite and nanocrystalline are measured using the impedance analyzer. The comparison results between the measured and calculated CM impedances show that the proposed modeling method can calculate the CM small-signal characteristic of CMIs over the wide frequency range.

Acknowledgment

This study was supported by JSPS Grant-in-Aid for Early-Career Scientists Grant Number JP21K14145.
References

(1) D. Han, S. Li, Y. Wu, W. Choi, and B. Sarlioglu: “Comparative analysis on conducted CM EMI emission of motor drives: WBG versus Si devices”, *IEEE Trans. on Industrial Electronics*, Vol. 64, No. 10, pp. 8353–8363 (2017)

(2) G. Engelmann, A. Sewergin, M. Neubert, and R. W. De Doncker: “Design challenges of SiC devices for low- and medium-voltage DC-DC converters”, *IEEE Journal of Emerging and Selected Topics in Power Electronics*, Vol. 8, No. 1, pp. 505–511 (2019)

(3) B. Zhang, and S. Wang: “A survey of EMI research in power electronics systems with wide-bandgap semiconductor devices”, *IEEE Journal of Emerging and Selected Topics in Power Electronics*, Vol. 8, No. 1, pp. 626–643 (2020)

(4) Y. Zhang, S. Wang, and Y. Chu: “Analysis and comparison of the radiated electromagnetic interference generated by power converters with Si MOSFETs and GaN HEMTs”, *IEEE Trans. on Power Electronics*, Vol. 35, No. 8, pp. 8050–8062 (2020)

(5) S. Takahashi, K. Wada, H. Ayano, S. Ogasawara, and T. Shimizu, “Review of modeling and suppression techniques for electromagnetic interference in power conversion systems”, *IEEE Journal of Industry Applications*, Vol. 11, No. 1 (2022)

(6) F. Y. Shih, D. Y. Chen, Y. P. Wu, and Y. T. Chen: “A procedure for designing EMI filters for AC line applications”, *IEEE Trans. on Power Electronics*, Vol. 11, No. 1, pp. 170–181 (1996)

(7) T. Naussauer, M. L. Heldwein, and J. W. Kolar: “Differential mode input filter design for a three-phase back–back PWM rectifier based on modeling of the EMC test receiver”, *IEEE Trans. on Industrial Electronics*, Vol. 53, No. 5, pp. 1649–1661 (2006)

(8) M. J. Nave: “Power line filter design for switched mode power supplies,” 2nd Edition. Mark Nave Consultants (2010)

(9) M. Hartmann, H. Erf, and J. W. Kolar: “EMI filter design for a 1 MHz, 10 kW three-phase-level PWM rectifier”, *IEEE Trans. on Power Electronics*, Vol. 26, No. 4, pp. 1192–1204 (2011)

(10) B. Toare, J. L. Schanen, L. Gerbaud, T. Meynard, J. Roudet, and R. Ruelland: “EMC modeling of drives for aircraft applications: modeling process, EMI filter optimization, and technological choice”, *IEEE Trans. on Power Electronics*, Vol. 28, No. 3, pp. 1145–1156 (2013)

(11) A. Roč, H. Bergsma, D. Zhao, B. Ferreira and F. Lefèvre: “A new behavioural model for performance evaluation of common mode chokes”, 2007 18th International Zurich Symposium on Electromagnetic Compatibility, pp. 501–504 (2007)

(12) C. R. Sullivan and A. Muezte: “Simulation model of common-mode choke for high-power applications”, *IEEE Trans. on Industry Applications*, Vol. 46, No. 2, pp. 884–891 (2010)

(13) C. Cuellar, N. Idir and A. Benabou: “High-frequency behavioral ring core inductor model”, *IEEE Trans. on Power Electronics*, Vol. 31, No. 5, pp. 3763–3772 (2016)

(14) M. Kački, M. S. Rylko, J. G. Hayes and C. R. Sullivan: “Magnetic material selection for EMI filters”, 2017 IEEE ENERGY Conversion Congress and Exposition (ECCE), pp. 2350–2356 (2017)

(15) K. Nomura, T. Kojima, and Y. Hattori: “Straightforward modeling of complex permeability for common mode chokes”, *IEEE Journal of Industry Applications*, Vol. 7, No. 6, pp. 462–472 (2018)

(16) S. Takahashi and S. Ogasawara: “A novel simulation model for common-mode inductors based on permeance-capacitance analogy”, 2020 IEEE Energy Conversion Congress and Exposition (ECCE), pp. 5862–5869 (2020)

(17) M. Bartoli, A. Reatti and M. K. Kazimierczuk: “Modelling iron-powder inductors at high frequencies”, *Proceedings of 1994 IEEE Industry Applications Society Annual Meeting*, pp. 1225–1232 (1994)

(18) M. L. Heldwein, L. Dalesandro and J. W. Kolar: “The Three-Phase Common-Mode Inductor: Modeling and Design Issues”, *IEEE Trans. on Industrial Electronics*, Vol. 58, No. 8, pp. 3264–3274 (2011)

(19) A. Massarini and M. K. Kazimierczuk: “Self-capacitance of inductors”, *IEEE Trans. on Power Electronics*, Vol. 12, No. 4, pp. 671–676 (1997)

(20) G. Grandi, M. K. Kazimierczuk, A. Massarini and U. Reggiani: “Stray capacitances of single-layer solenoid air-core inductors”, *IEEE Trans. on Industry Applications*, Vol. 35, No. 5, pp. 1162–1168 (1999)

(21) L. Dalesandro, F. da Silveira Cavalcante and J. W. Kolar: “Self-capacitance of high-voltage transformers”, *IEEE Trans. on Power Electronics*, Vol. 22, No. 5, pp. 2081–2092 (2007)

(22) M. Kovacic, Z. Hanic, S. Stipetic, S. Krishnamurthy and D. Zarke: “Analytical wideband model of a common-mode choke”, IEEE Trans. on Power Electronics, Vol. 27, No. 7, pp. 3173–3185 (2012)

(23) L. Middelháıet, S. Skibin, R. Döbbelin and A. Lindemann: “Analytical determination of the first resonant frequency of differential mode chokes by detailed analysis of parasitic capacitances”, 2014 16th European Conference on Power Electronics and Applications, pp. 1–10 (2014)

(24) S. W. Pasko, M. K. Kazimierczuk and B. Grezski: “Self-capacitance of coupled toroidal inductors for EMI filters”, *IEEE Trans. on Electromagnetic Compatibility*, Vol. 57, No. 2, pp. 216–223 (2015)

(25) A. Ayachi and M. K. Kazimierczuk: “Self-capacitance of single-layer inductors with separation between conductor turns”, *IEEE Trans. on Electromagnetic Compatibility*, Vol. 59, No. 5, pp. 1642–1645 (2017)

(26) S. Takahashi, S. Ogasawara, M. Takemoto, K. Orikawa and M. Tamate: “A modeling technique for designing high-frequency three-phase common-mode inductors”, 2018 IEEE Energy Conversion Congress and Exposition (ECCE), pp. 6600–6606 (2018)

(27) S. Takahashi, S. Ogasawara, M. Takemoto, K. Orikawa and M. Tamate: “Experimental evaluation of the relationship between filter inductor impedances and dimensional resonances of MnZn ferrites”, *IEEE 4rd International Future Energy Electronics Conference and ECCE Asia (IFEEC 2019 - ECCE Asia)*, pp. 1–8 (2019)

(28) A. Roč and F. Lefèvre: “Nanocrystalline core material for high-performance common-mode inductors”, *IEEE Trans. on Electromagnetic Compatibility*, Vol. 54, No. 4, pp. 785–791 (2012)

Appendix

The conventional stray capacitance estimation method for CMIs proposed in (24) calculates the turn-to-turn capacitance $C_{lt}$ as follows:

$$C_{lt} = \frac{2\epsilon_{0}w_{i}}{\sqrt{\alpha^2 - 1}} \tan^{-1}\left(\sqrt{\frac{2}{\alpha^2 - 1}}\right)$$

(31)

When the number of turns is set to above 10, the total capacitance for $N$ turns $C_{lt}(N)$ is obtained as

$$C_{lt}(N) \approx 1.366 \cdot C_{lt} \quad (N \geq 10)$$

(33)
Shotaro Takahashi (Member) received Dr. Eng. degree from Hokkaido University, Sapporo, Hokkaido, Japan, in March 2020. From April 2020 to March 2021, he was a project assistant professor at Tokyo Metropolitan University, Hachioji, Tokyo, Japan. He has been an assistant professor with Seikei University, Kichijoji, Tokyo, Japan, since April 2021. His current research focuses on the electromagnetic compatibility of high-frequency switching power converters, active electromagnetic interference filters, and modeling of magnetic components. He was a recipient of the ICEMS Best Paper Award (Third Prize) in 2017 and the IEEJ Industry Applications Society Distinguished Transaction Paper Award in 2018. He has received four presentation awards from the IEEJ through 2021. He is a member of IEICE and IEEE.

Sari Maekawa (Member) received a master’s degree in electrical engineering from Hosei University. In April of the same year, he went to work for Toshiba. In April 2014, he received a Doctorate Degree from the Graduate School of Science and Technology at Meiji University. In 2019, he joined the Faculty of Science and Technology of Seikei University, as an associate professor. He is primarily engaged in the research and development of power electronics, EMI, and motor drives. In 2013, Dr. Maekawa received an IEEJ Best Paper Award (Award A) and an IEEJ Industry Applications Society Best Presentation Award. He also received the Ichimura Industry Award from the New Technology Development Foundation in 2014. His Doctorate degree is in engineering. Dr. Maekawa is a member of IEEE.