Shock propagation and neutrino oscillation in supernova

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Abstract

The effect of the shock propagation on neutrino oscillation in supernova is studied paying attention to evolution of average energy of $\nu_e$ and $\bar{\nu}_e$. We show that the effect appears as a decrease in average $\nu_e$ (in case of inverted mass hierarchy, $\nu_\tau$) energy at stellar surface as the shock propagates. It is found that the effect is significant 2 seconds after bounce if $3 \times 10^{-5} < \sin^2 \theta_{13} < 10^{-2}$.

1 Introduction

Recently effects of shock propagation on neutrino oscillation in supernova was studied [1, 2] and it was shown that some characteristic signatures emerge as the shock propagates through the regions where matter-enhanced neutrino flavor conversion occurs.

There have been many studies on neutrino oscillation in supernova: extracting information of neutrino parameters from the observation of SN1987A neutrinos [3, 4, 5, 6] or a future supernova neutrinos [7, 8, 9, 10, 11], and probing supernova physics from observed neutrinos of a future supernova [12]. But all of them are done without the effect of the shock propagation.

In this paper the effect of the shock propagation is studied paying attention to evolution of average energies of $\nu_e$ and $\bar{\nu}_e$. We show when and with which parameter ($\sin^2 2\theta_{13}$) the effect is significant or can be neglected safely.

2 Neutrino oscillation and shock propagation

If mixing angle is small, dynamics of flavor conversions is well described by resonant oscillation, which occurs at density,

$$\rho_{\text{res}} \simeq 1.4 \times 10^6 \text{g/cc} \left( \frac{\Delta m^2}{1 \text{eV}^2} \right) \left( \frac{10 \text{MeV}}{E_\nu} \right) \left( \frac{0.5}{Y_e} \right) \cos 2\theta,$$

(1)
where $\Delta m^2$ is the mass squared difference, $\theta$ is the mixing angle, $E_\nu$ is the neutrino energy, and $Y_e$ is the mean number of electrons per baryon. Flavor conversion probabilities are determined by adiabaticity parameter $\gamma$:

$$\gamma \equiv \frac{\Delta m^2 \sin^2 2\theta}{2E_\nu \cos 2\theta \left| \frac{dn_e}{dr} \right|}.$$ (2)

Here $\Delta m^2$ and $\theta$ are

$$\theta_{13} \text{ and } \Delta m_{13}^2 \text{ at H – resonance},$$

$$\theta_{12} \text{ and } \Delta m_{12}^2 \text{ at L – resonance},$$

where mixing matrix is taken as:

$$U = \left( \begin{array}{ccc} c_{12} c_{13} & s_{12} c_{13} & s_{13} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} & c_{12} c_{23} - s_{12} s_{23} s_{13} & s_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} & -c_{12} s_{23} - s_{12} c_{23} s_{13} & c_{23} c_{13} \end{array} \right),$$ (3)

where $s_{ij} = \sin \theta_{ij}, c_{ij} = \cos \theta_{ij}$ for $i,j = 1,2,3(i < j)$. When $\gamma \ll 1$, the resonance is nonadiabatic and the fluxes of the two involved mass eigenstates are completely exchanged. On the contrary, when $\gamma \gg 1$, the resonance is adiabatic and the conversion between mass eigenstates does not occur.

Evolution of density profile is shown in Fig. 1. This is calculated by our numerical supernova model with progenitor mass, $18M_\odot$. (For detail, see [13].) As the shock passes through the matter outside the neutrinosphere, densities will typically increase by approximately a factor of two. As neutrinos subsequently heat the matter behind the shock, a high entropy region will develop which accelerates matter behind the shock, causing a density buildup between this high entropy bubble and the outgoing shock. The density gradient caused by this mechanism is typically several orders of magnitude greater than that caused by the shock passage. As seen in Fig. 1, the shock is barely visible on the outer part of the density gradient at approximately 0.1 solar radius. This "snowplowed" density gradient is typical of neutrino driven core collapse supernovae, and is expected to be generic of a range of progenitor masses.
Densities which correspond to H- and L-resonance are also shown (Note that the resonance density depends on neutrino energy). As can be seen, the shock reaches the resonant region about 2 seconds after bounce and the density behind the shock is lower than that forward the shock. In the late phase \((t > \text{several seconds})\), a neutrino experience three times of H-resonances (and/or L-resonances) and the radii of resonance points \((r_{\text{res}})\) are in general much smaller than the early phase. Since \(\gamma \propto r_{\text{res}}\) if the density profile is approximated to be power-law, in general the three resonances have different adiabaticity parameters which are much smaller than that of the early phase. Therefore the average energy of the observed neutrinos is expected to depend on time not only due to the evolution of the neutrinosphere and the protoneutron star but also due to the evolution of the shock.

We can estimate the above effect by calculating adiabaticity parameter. We consider the case where H-resonance occurs only one time and assume that the neutrino mass hierarchy is normal. In this case the adiabaticity parameter is, assuming \(\rho \propto r^{-n}\),

\[
\gamma_H \approx 2 \times 10^2 n^{-1} \left( \frac{\sin^2 2\theta_{13}}{10^{-2}} \right) \left( \frac{r_{\text{res}}}{3 \times 10^{-2} R_\odot} \right) \left( \frac{\Delta m^2}{3 \times 10^{-3}} \right) \left( \frac{10\text{MeV}}{E_\nu} \right).
\]

(4)

Note that the index \(n\) is almost independent of time as can be seen in Fig. 1. From this H-resonance is expected to be adiabatic if \(\sin^2 2\theta_{13} > 10^{-4}\). Adiabaticity parameter, however, becomes smaller as the shock propagates and two order smaller at 15 sec than in the early phase. Thus the H-resonance becomes less adiabatic unless \(\sin^2 2\theta_{13} > 10^{-2}\) as is in the model LMA-L \((\sin^2 2\theta_{13} = 0.043)\) of \([8]\). This will cause decreasing of \(\nu_e\) average energy at the stellar surface as the shock propagates. On the other hand, if H-resonance is non-adiabatic even in early phase as is in the model LMA-S \((\sin^2 2\theta_{13} = 10^{-6})\) of \([8]\), average energy will not change. Here it should be noted that almost half the neutrinos are emitted after 2 seconds after bounce \([14]\).

With inverted mass hierarchy, H-resonance occurs at anti-neutrino sector. In this case evolution of average energy at stellar surface is seen in \(\bar{\nu}_e\). Its qualitative feature is expected to be the same as that of \(\nu_e\) with normal mass hierarchy because it is determined by the behavior of the shock.

As to L-resonance, the adiabaticity parameter is, substituting the LMA (Large Mixing Angle) solution of the solar neutrino problem \([15, 16, 17, 18]\),

\[
\gamma_L \approx 4 \times 10^3 n^{-1} \left( \frac{\sin^2 2\theta_{12}}{1} \right) \left( \frac{r_{\text{res}}}{0.3 R_\odot} \right) \left( \frac{\Delta m^2}{7 \times 10^{-5}} \right) \left( \frac{10\text{MeV}}{E_\nu} \right).
\]

(5)

Thus L-resonance will remain to be adiabatic even if the radius of the resonance point becomes two order smaller than the early phase.

In the next section we study the above effect quantitatively by numerical calculation and obtain time evolution of the average energies of observed neutrinos (\(\nu_e\) and \(\bar{\nu}_e\)).

3 Numerical calculation

We solve numerically evolution equations of neutrino wave functions along the density profiles shown in Fig. 1. From the wave functions, we obtain flavor conversion probabilities, from which neutrino spectra can be obtained by multiplying by the original neutrino fluxes. To make the shock propagation effect distinctive, the original energy spectra at each time are set to be the same as the
time-integrated spectra. Neutrino parameters are taken as:

\[
\sin^2 2\theta_{12} = 0.87, \quad \sin^2 2\theta_{23} = 1, \\
\Delta m^2_{12} = 7.0 \times 10^{-5}\text{eV}^2, \quad \Delta m^2_{13} = 3.2 \times 10^{-3}\text{eV}^2.
\]  (6)

As for \(\sin^2 2\theta_{13}\), we take various values including values corresponding to model LMA-L and LMA-S in [8, 9]. For detail of the calculational method and the original neutrino fluxes, see [8, 9].

Fig. 2 show average energy evolutions of \(\nu_e\) emitted at various times after bounce with normal mass hierarchy. Each figure differs in the value of \(\sin^2 2\theta_{13}\). The interesting behavior of the average energy in supernova (for example, 5 sec of the upper-left of Fig. 2) indicates that H-resonance occurs three times. As is discussed in the previous section, the average energy at the stellar surface does not change in time when \(\sin^2 2\theta_{13}\) is enough large (LMA-L, the upper-left of Fig. 2) or small (LMA-S, the lower-right of Fig. 2). On the other hand, average energy decrease by several MeV in the intermediate cases. By calculating with various values of \(\sin^2 2\theta_{13}\), we find that the shock propagation effects can be seen when \(3 \times 10^{-5} < \sin^2 2\theta_{13} < 10^{-2}\) but are absent till \(\sim 1\) second after bounce irrespective of \(\sin^2 2\theta_{13}\).

Thus average energy of observed \(\nu_e\) will change in time due to shock propagating effect. In fact neutrino average energies changes also due to evolution of protoneutron star and neutrinosphere. Fig. 3 shows evolutions of average energy of observed neutrinos taking intrinsic changes of neutrino average energies into account. \(\sin^2 2\theta_{13}\) is set to \(10^{-4}\). As can also be seen in the lower-left of Fig. 2, \(\nu_e\) energy with shock effect is lower by several MeV after about 2 sec after bounce than without shock effect.

As stated in the previous section, in case of inverted mass hierarchy it is \(\bar{\nu}_e\) that is affected by shock propagation. Features of the evolution of the average energy are almost the same quantitatively: values of \(\sin^2 2\theta_{13}\) and time after bounce, with which shock propagation effect is significant, difference between average energies of the early phase and late phase.

4 Discussion and summary

As we saw in the previous section, neutrino average energy decrease in general as the shock propagates. This is because the shock propagation cause decrease in the adiabaticity parameter of H-resonance and suppress the conversion between flavors. But it should be noted that the original spectra will change due to the evolution of the protoneutron star and the neutrinosphere. Thus we can not say about the value of \(\sin^2 2\theta_{13}\) and the mass hierarchy only from the evolution of the average energy of the observed neutrino. To do so, we need a model of supernova and spectrum evolution.

In this paper we studied the effect of the shock propagation paying attention to evolution of average energies of \(\nu_e\) and \(\bar{\nu}_e\). It is shown that the effect appears as a decrease in average \(\nu_e\) (in case of inverted mass hierarchy, \(\bar{\nu}_e\)) energy at stellar surface as the shock propagates. Further it is found that the effect is significant 2 seconds after bounce if \(3 \times 10^{-5} < \sin^2 \theta_{13} < 10^{-2}\).
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Figure 2: Average energy evolutions of $\nu_e$ emitted at various times after bounce with normal mass hierarchy. The values of $\sin^2 2\theta_{13}$ are that of the model LMA-L of [8, 9], $10^{-3}$, $10^{-4}$ and that of the model LMA-L of [8, 9], respectively.
Figure 3: Evolutions of average energy of observed neutrinos. Average energies of $\nu_e$ with and without shock effect are shown. Those of $\nu_e$ and $\nu_x = \nu_\mu, \nu_\tau$ without neutrino oscillation are also shown. $\sin^2 2\theta_{13}$ is set to $10^{-4}$. 