Analysis of an Agriculture Data Using Markov Basis for Independent Model

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Abstract: Algebra statistical is one of the recent topics that has seen rapid development, it is considering as the consequence of the convergence of ideas between statistics and algebra. Moreover, Algebraic statistic has many applications in many different fields, one of these important field is agriculture. In this paper, we analyse a 3x2 contingency table which contains agriculture teaching experience and perception of secondary school agriculture. The method of algebraic statistic they will be used for a mainly Markov Basis of independent model and the computation of Entropy for variance alternative contingency tables based on Tsallis Entropy. In addition, the usefulness of this paper is to illustrate agriculture data. Finally, we will find Toric ideal to generate the elements of Markov Basis.

Keywords: Contingency table, Markov Basis for independent Model, Toric ideal, Tsallis Entropy.

1. Introduction
Algebra has seen many applications in statistics, recently that computational algebraic geometry and related techniques in algebra and combinatorics have been used to study statistical models and inference problems. This helped the rapid growth of the new field, it is the algebraic statistic. In 1996 was the first meeting between algebra and statistics as one term known as algebra statistics, this was done by G. Piston and H. Wynn [1]. Algebraic statistics have many applications in many fields, one of these important field is agriculture. Agriculture is the rudder, which leads the countries as they have a great effect on human population growth as the growth in agriculture has a direct impact in allowing the population to continue to grow. Therefore, agriculture teachers are implemented to instill values and knowledge in their teaching agricultural methods. It can help to define the prosperity of students necessary to ameliorate agricultural output and build conscious society educated is estimated real value of the advancement of agricultural reality in any country. Due to the importance of agriculture, we choose the data represent the connection between teaching practices and comprehension of teaching agricultural in secondary school [2].

One of the important topics in algebra Statistical is Markov Basis for independent model, which is used to analysis data using algebra method. In 2012 S. Aoki and el at, introduce the concept of Markov Basis for the independent model with hypothesis tests for contingency tables [7]. In addition, we can apply Markov Basis to Fisher's exact test for contingency tables if the cells with frequencies less than 5. Generally Markov Basis is not unique, even if we suppose the Markov basis is minimal, however, the contingency tables Markov Basis is unique for two way contingency tables [1]. Moreover, in 2000 R. Yoshida introduced an important relationship in toric ideal that connected the moves and binomials in the fundamental theorem of Markov Basis [4]. Beside that, the definition of the Entropy is the measure of an uncertainty associated with random variables. Moreover, the Entropy has many applications in many fields such as physics, thermodynamics, statistics [5]. In addition, there are many formulas to find the value of information such that: Shannon Entropy, Tsallis Entropy [5 -6].

In this work, we execute the Markov Basis for independent model using as method to analyze the agricultural data in two way contingency table. Moreover, find all elements of the fiber with fixing the rows and the the columns as equal sum alternatives tables. According to the independence
determined by the Tsalli’s Entropy value. Besides, generating the elements of Markov Basis for independent model depend on Toric ideal according to the fundamental theorem of Markov Basis.

section 2, introduced some basic definitions and the notation of the Markov Basis for independent model and the Entropy.

section 3, represented the real an agricultural data as well as we will set the application of Markov Basis for independent model and Tsalli’s Entropy. It’s a new method to find the best alternative table in term of independent.

2. Preliminaries
This section provides a review of some of the basic concepts involved in this research, such as Markov Basis and Entropy. Suppose Ω= |π|=q×μ, where ω∈Ω. Consider a q-dimensional sufficient statistic defined by c=Σs∈Ψ ωs, where aω∈Ω is a μ-dimensional vector for i=1,⋯,μ. Define a matrix (ω)=|ω| of degree q×μ, where ωij is the jth element of ai. Moreover, let ω be another sufficient statistic such that ω=|ω|∈Ω.

According to define a set of fiber and denoted by Fω=|ω|∈Ω. To define the toric ideal, first its worth to kwon that the toric ideal is homogeneous, which is mean that there exists Y∈Ω such that Y’⋯=1, i=1,⋯,μ, and directly leads to |ω|=|ω|, s∈Fω. The size of the fiber Fω is denoted by |Fω|.

The i th coordinate vector is denoted by rω=(0,⋯,0,1,0,⋯,0), where i is in the (ωi)th position.

Define the polynomial ring in q as K[u1,⋯,un]=K[u] with u1,⋯,un over the field K, where uω∈K with the cell i∈1 For a μ-dimensional column vector s∈Ω, let uω=us,⋯,us∈K[u] denote a monomial. Since c=(c1,⋯,cμ)’, then k[c]=k[c1,⋯,cμ]=|ω|∈Ω. c=ωs could be represented as mapping s:k[u]→k[c]. Asume that Gω=k(σ) which is known as Toric ideal connected with ω. Let us define the move as μ-dimensional vector m∈Zμ; in another word m∈Gω, then m is considering as a move of ω. Let m+=(m1+,⋯,mμ+)’ and m=−(m1−,⋯,mμ−)’ symbolize as the positive and negative part of m given by m1+=max(m1,0), m1=−=−min(m1,0) respectively. We can add m to any vector s without change it, ω(s+m)=ωs. m is usable to s if and only if s+m∈Fω, and i.e, s≥m, i.e, s−m∈Ω.

Particularly m is applicable to m−. We can call that a move m contains vector s if m+s=s or m−=s. The size of (m+) or (m−) is called a degree of m and denoted by deg(m)=|m+|=|m−|, such that |m|=Σi=1|m|=2deg(m).

Let β=|m1,⋯,mμ| and consider two vectors d and v be in the same fiber, i.e.,(d,v)∈Fωd=|Fω|v is accessible from d by β if ξ=m1,⋯,mμ from β and ϵj∈{−1,1}, j=1,⋯,k, where v=d+Σj=1jmi and d+Σj=1jmi∈|Fωd|L=1,⋯,k. We can write d∼v (mod β) if v is accessible from d by β. These[β] are equivalence classes of Fω, because of symmetry, if d∼v (mod β) called d and v are mutually accessible by β.

To defined the Markov basis., let β=|m1,⋯,mμ| is a Markov basis if for all c, the fiber Fωc itself forms one β-equivalence class.. In statistical applications, a Markov basis makes it possible to construct a connected Markov chain over Fωd for any observed frequency data d.

These results were obtained by showing the fact that β=|m1,⋯,mμ| is a Markov basis if and only if the set of binomials \{umk−umk, k=1,⋯,L\}. Is a generator of the Toric ideal Gω, associate to (ω) [6], as a basis for generalizing the standard statistical mechanisms. In this paper, we will introduce the effect of the Tsalli’s Entropy on the fiber elements in the Markov basis for independent model in term of independently [8-9].

3. Presented method
This section, explained method which banned on Markov basis for independent model.
Step(1): Construct data as in table(1) to represent the contacts between Agriculture Teaching Experience and Perception. Table (1): represents the links between Agriculture Teacher Teaching Experience and Perception.

| Agriculture Teaching Experience | Low | High | Total |
|-------------------------------|-----|------|-------|
| 1 – 2 years                   | 3   | 4    | 7     |
| 3 – 5 years                   | 5   | 6    | 11    |
| 3 – 5 years                   | 5   | 8    | 13    |
| Total                         | 13  | 18   | n =31 |

Step(2): testing the data for the independent model using hypothesis tests depending on logliner model with odds ratio test.

Step (3): According to table(1), to organize Markov basis elements employing the definition of the movie [7]

$$m_{ij} = \begin{cases} +1 & (i,j) = (i_1,j_1), (i_2,j_2) \\ -1 & (i,j) = (i_1,j_2), (i_2,j_1) \\ 0 & \text{otherwise} \end{cases}$$

Where, \( m = m(i_1,j_1,i_2,j_2) = \{m_{ij}\} \).

Step(4): \( \beta = \{m_1, m_2, m_3, m_4, m_5, m_6\} \), [7,9] such as:

\[
m_1 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad m_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad m_3 = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}, \quad m_4 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},
\]

\[
m_5 = \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}, \quad m_6 = \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}
\]

Step(5): Depending on the summation of matrices, it collects the results from the previous step with data elements to find alternative data, and repeating the process with stop when contingency tables are equal, or getting negative cells. Figure(1) shows all the construction of these tables.

Step(6): To measure independence, Tsallis Entropy is applied as follows:

$$S_q = \frac{1}{q-1}(1 - \sum p_i^q)$$

The value of \( q \) is either more or less than one, but if \( q \to 1 \), Tsallis entropy reduces to the Boltzmann –Gibbs entropy [10]. \( q \) is called the Tsallis index by proponents of the theory. In this work we choose \( q>1 \).

Application

In this section, we introduce an application on the Markov Basis for the independent model with Tsallis Entropy as a measure as to choose the best element of the of the fiber (alternative table) in term of independent.

Example

This example, introduce the real data between two variables Agriculture Teaching Experience and Perception as represent as in table(1)[2]. At the first, testing connected between Agriculture Teaching Experience and Perception are independent or not by using hypothesis tests depending on logliner model with odds ratio test since the expected value of some cells are less than 5 [11]. Therefore, suppose that \( H_0: \text{odds ratio } = 1 \), then, \( \log(OR) = 0 \), thus the link of Agriculture Teaching Experience with Perception of Agriculture Secondary school is independent. \( H_1: \text{If } (OR) \neq 1, \text{then the } \log(OR) \neq 0, \) therefore, Agriculture Teaching Experience with Perception of Agriculture Secondary school is not independent. We can note that the expected frequency \( E_{r,c} = (n_r \times n_c)/n \), where \( n_r \) is the sum of rows, \( n_c \) is the sum of columns and \( n \) is the size of the sample. In this example:
Because that the expected frequency of the $E_{11}, E_{12}, E_{21}$ are less than 5, therefore, the odds ratio, it gives as:

$$\frac{E_{ij}^2}{E_{ii}E_{jj}} = 1, \text{ where } 1 \leq i < k \leq r \text{ and } 1 \leq j < l \leq c.$$  

Hence, there are three possible cases such as:

1. (OR)$E_{1122} = \frac{18.75}{18.75} = 1, \text{ (OR)} = 1, \log(OR) = 0 \text{ .................(1)}$

2. (OR)$E_{1211} = \frac{22.30}{22.30} = 1, \text{ (OR)} = 1, \log(OR) = 0 \text{ .................(2)}$

Since (OR) for all possibilities is 1, therefore, $\log(1) = 0$, hence, we expect $H_0$ and reject $H_1$. After that, construct Markov basis elements of the independent model by moves. Then, we get the elements of the Markov basis of independent model $\beta = (m_1, m_2, m_3, m_4, m_5, m_6)$ [10,11] as such in step(4).

Beside that, we add these elements of the Markov basis of independent model to the original table to get the set of the elements of the fiber (81 alternative tables) [12,13] such as:
Let \( A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \end{bmatrix}, \quad b = \begin{bmatrix} 4 \\ 3 \end{bmatrix} \). 
\( u^b - u^a = u_1^4 u_2^3 u_3^4 u_4^5 u_5^6 u_6^8 - u_1^3 u_2^4 u_3^5 u_4^6 u_5^7 u_6^8 = u_1^4 u_2^3 u_3^4 u_4^5 u_5^6 u_6^8 (u_4 u_5 - u_2, u_3) \) 
\( A \) is the configuration matrix of dimension \((r+c) \times r c\). 

\[ \begin{array}{cccccc} 
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 
\end{array} \]

\[ \begin{array}{cccccc} 
3 & 4 & 7 & 11 & 13 & 18 \\
5 & 6 & 7 & 11 & 13 & 18 \\
8 & 13 & 10 & 10 & 10 & 10 \\
0 & 1 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 1 
\end{array} \]

\[ \begin{array}{cccc} 
1 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 
\end{array} \]

\[ \begin{array}{cccc} 
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 
\end{array} \]

\[ \begin{array}{cccc} 
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 \\
0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 
\end{array} \]

\[ \begin{array}{cccc} 
1 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 \\
0 & 0 & 1 & 1 
\end{array} \]

Let \( s = \begin{bmatrix} 1 & 6 \\ 5 & 6 \\ 7 & 6 \end{bmatrix}, \ m = \begin{bmatrix} 1 & 6 \\ 6 & 5 \\ 6 & 7 \end{bmatrix} \)

\[ u^m - u^s = u_1^4 u_2^2 u_3^5 u_4^6 u_5^4 u_6^8 - u_1^4 u_2^3 u_3^4 u_4^5 u_5^6 u_6^8 = u_1 u_2 u_3 u_4 u_5 u_6 (u_4 u_5 - u_2, u_3) \]

\[ \text{As} = A m \text{ and } s, m \in N^n, \]

\[ m - s = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 \\
0 \\ 1 \\ -1 \\ 1 
\end{bmatrix} \in \text{Ker}_m A, \text{ since } A = \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 
\end{bmatrix} = 0, A \text{ is the configuration matrix.} \]

\[ u^d - u^c = u_1^4 u_2^3 u_3^4 u_4^5 u_5^6 u_6^8 - u_1^4 u_2^3 u_3^4 u_4^5 u_5^6 u_6^8 = u_1 u_2 u_3 u_4 u_5 u_6 (u_4 u_5 - u_2, u_3) \]

\[ \text{Ad} = A c \text{ and } c, d \in N^n, -d = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 1 
\end{bmatrix} \in \text{Ker}_d A, \text{ since } A = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 
\end{bmatrix} = 0, A \text{ is the configuration matrix.} \]

Let \( g'' = \begin{bmatrix} 5 & 2 \\ 8 & 5 \end{bmatrix}, \quad h'' = \begin{bmatrix} 4 & 3 \\ 0 & 11 \end{bmatrix} \)

\[ u^{g''} - u^{h''} = u_1^4 u_2^3 u_3^0 u_4^1 u_5^2 u_6^0 - u_1^4 u_2^3 u_3^0 u_4^1 u_5^2 u_6^0 = u_1^4 u_2^3 u_3^0 u_4^1 u_5^2 u_6^0 (u_2 u_5 - u_1 u_6) \]
\[ Ag'' = Ah'' \text{ and } g'', h'' \in N^n, \quad '' - h'' = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \in \text{Ker}_m A, \quad \text{since } A \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix} = 0, \text{ A is the configuration matrix.} \]

Let \( y = \begin{bmatrix} 0 \\ 0 \\ 8 \\ 8 \end{bmatrix}, \quad a' = \begin{bmatrix} 0 \\ 7 \\ 4 \\ 7 \end{bmatrix} \)
\[ u^{a'} - u^y = u_1^0 u_2^7 u_3^4 u_4^6 u_6 - u_1^0 u_2^0 u_3^7 u_4^3 u_5^0 u_6^8 = u_1^0 u_2^0 u_3^7 u_4^3 u_5^0 u_6^7 (u_4 u_5 - u_3 u_6) \]

\[ Aa' = Ay \text{ and } a', y \in N^n, \quad ' - y = \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ -1 \end{bmatrix} \in \text{Ker}_m A, \quad \text{since } A \begin{bmatrix} -1 \\ 0 \\ 1 \\ 1 \\ -1 \end{bmatrix} = 0, \text{ A is the configuration matrix.} \]

Let \( p = \begin{bmatrix} 3 \\ 4 \\ 3 \\ 7 \\ 6 \end{bmatrix}, \quad w = \begin{bmatrix} 4 \\ 3 \\ 2 \\ 9 \\ 7 \end{bmatrix} \)
\[ u^p - u^w = u_1^3 u_2^4 u_3^8 u_4^6 u_5^7 - u_1^3 u_2^4 u_3^2 u_4^5 u_5^7 = u_1^3 u_2^4 u_3^2 u_4^5 u_5^7 (u_2 u_3 - u_1 u_4) \]

\[ Ap = Aw \text{ and } p, w \in N^n, \quad w = \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} \in \text{Ker}_m A, \quad \text{since } A \begin{bmatrix} -1 \\ 1 \\ 1 \\ 0 \\ 0 \end{bmatrix} = 0, \text{ A is the configuration matrix.} \]

Hence, according to the example above, the Toric ideal is:
\[ I_A = < u_1 u_4 - u_2 u_3, u_3 u_6 - u_4 u_5, u_1 u_6 - u_2 u_5, u_2 u_5 - u_1 u_6, u_4 u_5 - u_3 u_6, u_2 u_3 - u_1 u_4 > \]

### 4. Effect of Tsallis Entropy of Markov Basis For Independent Model

The following example is showing the effect of Tsallis entropy of the Markov basis for independent model. The main idea is to choose the best table from the elements of the fiber compare with the basic data in term of independent. We motion above the formula of Tsallis entropy, hence, when \( q = 2 \), we apply the Tsallis entropy of the elements of the fiber (alternative tables) [6,10]. The result, as follows:

| A = 5.8178980229 | B = 5.813735715 | C = 5.8137356919 | D = 5.8095733612 |
|-----------------|-----------------|-----------------|-----------------|
| E = 5.817898023 | F = 5.8095733612 | G = 5.817898023 | H = 5.7408949011 |
| I = 5.7408949011 | J = 5.8095733612 | K = 5.7408949012 | L = 5.8012486993 |
| M = 5.8095733612 | N = 5.7929240376 | O = 5.7970863685 | P = 5.8095733612 |
| Q = 5.813735692 | R = 5.7929240375 | S = 5.8095733612 | T = 5.7804370448 |
| U = 5.7929240375 | V = 5.784599569 | W = 5.797086369 | X = 5.770437045 |
| Y = 5.7804370447 | E = 5.779594173 |

\[ \cdots, F' = 5.779979188, G' = 5.792924037, H' = 5.801248699, I' = 5.801248699, \]
\[ J' = 5.767950052, K' = 5.79292037, L' = 5.75130728, M' = 5.767950052, N' = 5.759625583, \]
\[ O' = 5.767274714, P' = 5.75962539, Q' = 5.776272282, R' = 5.75130728, S' = 5.75962539, \]
\[ T' = 5.75962539, U' = 5.75130728, V' = 5.734651405, W' = 5.747138398, X' = 5.75130728, \]

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$Y' = 5.747138398, Z' = 5.767950052, A'' = 5.780437045, B'' = 5.784599376, C'' = 5.780437045, D'' = 5.734651405, F'' = 5.734651405, G'' = 5.753610822, H'' = 5.747138398, I'' = 5.730489074, J'' = 5.726326743, K'' = 5.702393341, L'' = 5.693028096, M'' = 5.701242373, N'' = 5.718002080, O'' = 5.718002081, P'' = 5.70967742, Q'' = 5.734607493, R'' = 5.759625583, S'' = 5.75962539, U'' = 5.693028096, V'' = 5.684703434, W'' = 5.668064521, X'' = 5.71383975, Y'' = 5.726326743, Z'' = 5.730489074, A''' = 5.726326743, B''' = 5.693028096, C''' = 5.6472424558. We note that when q=2, there are two tables E, G more than the original table, hence, we take another value of q such as this table: Table (2) the effect of q in the (A,E,G) tables

| q  | Table | Value of Entropy |
|----|-------|------------------|
| 3  | A     | 2.9820583399     |
|    | E     | 2.9822597429     |
|    | G     | 2.9826641310     |
| 4  | A     | 1.9975091705     |
|    | E     | 1.9976131205     |
|    | G     | 1.9977408924     |
| 5  | E     | 1.4996206580     |
|    | G     | 1.4996625730     |
| 6  | E     | 1.1999344510     |
|    | G     | 1.1999454091     |

According to the result above in table (2), we note that table (G) is the best table to compare with the original table in term of independent.

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Figure 1: All the elements of fiber with original data (CT construction)