Written mathematical communication accuracy on linear equation and inequality

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Abstract. Written mathematical communication is the process of sharing idea and mathematics understanding in written form. One of the qualified communication indicators is accuracy. Therefore, accuracy is considered to be one of the essential characteristics that pre-service mathematics teacher should have. This research implemented qualitative in nature aiming to describe the pre-service teachers’ written mathematical communication accuracy in explaining one variable linear equation and inequality. The subjects were 20 pre-service mathematics students who were given mathematical communication task. Then, two subjects were chosen based on their preparation which classified as quite complete. The data was collected from lesson plan, presentation material, and teacher’s note on whiteboard during the learning process. The data were analysed from each sentence containing mathematical language. The result of this research showed that both subjects could do written mathematical communication towards one variable linear equation and inequality accurately. This accuracy could be described by considering the accuracy of mathematical notation and terminology, the correct concept explanation, the procedure with clear and logical algebra manipulation, the calculation process, and also rigorous final result. However, the subjects could not explain logically the reason why the sign would change, if both sides of an inequality were multiplied by negative number.

1. Introduction
Communication is a process of delivering information, both verbally and non-verbally, which is conducted by communicator and interlocutor through certain media which could be manifested into symbol, sign, or certain behavior [1,2]. Communication is one of activities which are commonly occurred in daily life. Communication occurs between two people or more, with various context and purpose. One of them is learning situation context. In 21st century education framework, one of competencies which are critical to develop is the ability to communicate [3]. This thing is also in line with one of the aims in learning mathematics that is to make the students be able to express their idea, thought, and knowledge to others [4]. Furthermore, communication can be divided into two, those are oral and written communication.

In regard to communication ability, listening and speaking ability should be as good as reading and writing ability, so that the aim to establish a good teacher can be realized [5]. Great communicator is for those who are able to receive, comprehend, compose, and express it into their own language with excellent quality. This is one of the requirements how to be a qualified teacher, since it will allow them to transfer knowledge, skill, and value at the same time. The ability to communicate effectively will
influence the learning quality because the material which is delivered by the teacher is easy to understand, and it can also make the learning situation become more attractive, so the students will be more enthusiastic [6]. The learning interaction between teacher and student, student and student, and student and learning material occurs through the communication process. Effective communication ability is really important while delivering the lesson, managing the class, and also during classroom interaction process to achieve the students’ success [7, 6]. Mastering subject knowledge is not enough to be a qualified teacher, but organizing, managing, and communicating ability are needed to design a lesson, to arrange appropriate assessment, and to develop an objective test [8]. It can be assumed that it is important for every teacher to master an effective communication ability to achieve the learning success.

Mathematics can be seen as a language, namely a symbolic language, which is characterized on the logic and term used based on agreement or clear definition [9]. By considering mathematics as a language, the communication process during the mathematics learning will be different compared with another communication on another subject since mathematical object is abstract [10]. Communication during mathematics learning is called as mathematical communication. Mathematical communication can be defined as a process of delivering message containing mathematical material, idea, and mathematical knowledge, both orally and in written [11]. Written mathematical communication is the process of revealing thought and interpreting mathematics in written, and it is considered to be one of the most decisive competences for pre-service teacher’s student [12]. For teacher, written mathematical communication is equally important to oral mathematical communication during the teaching and learning process [13]. It is a way to bridge the interaction between teacher and students during the learning process which is done in written. This communication is also possible to be delivered into the lesson plan, student’s worksheet, presentation material, or writing in the whiteboard. A teacher can deliver the mathematical knowledge into a concept, notation or symbol, formula, table, and procedure or algorithm. Written mathematical communication should be presented accurately, clearly, carefully, and in detail, especially while using the language, the lexicon, the symbol, mathematical terminology, and mathematical calculation [14-16].

This mathematical symbol or notation functions as a means of mathematical communication, which represents the thought and ideas in mathematics to ease the mathematical manipulation process [17]. The use of symbol or notation in written mathematical communication should be presented accurately, clearly, and appropriately [14-16]. Therefore, the use of written symbol in mathematical communication is very critical to maintain the accuracy. An effective written mathematical communication accuracy done by the teachers can also be seen from how the way they deliver their ideas, concept, and mathematical model [18]. On the other hand, poor written mathematical communication will make the students get difficulty to comprehend the concept or make a mistake when utilizing the concept [8]. Consequently, it is important for the pre-service teacher’s students to be able to present the concept, idea, and mathematical model clearly, so that the students are able to receive the mathematical idea and knowledge well [3]. So, in this communication, one of the important characteristics is the existence of accuracy [18].

Based on those considerations, this research has set the written mathematical communication accuracy framework towards linear equation and inequality in one variable which is focused on the accuracy of using the notation or symbol, explaining the concept, using mathematical terminology, and mathematical calculation. This framework will be used as a scaffolding to analyze the written mathematical communication accuracy.

2. Method
This paper is a descriptive research using qualitative approach. Qualitative approach was chosen as the research primary data was in the form of descriptive data, which is written mathematical communication. It is called as descriptive research since this paper aims to arrange the written mathematical communication accuracy descriptively. The subjects of this research were 20 fourth-batch teacher’s
students. The subjects were chosen based on the consideration that they had already had an experience in teaching practice. To collect the data, those 20 subjects were given mathematical communication task on linear equation and inequality in one variable. The task was in the form of lesson plan and presentation material which was also provided by the notes written on the whiteboard during the learning process. Among 20 subjects, 2 students were chosen as the focus of the research by considering the completeness of the task which was given. From the written communication task data, the researcher transcribed every sentence which contained mathematical language. The data were analyzed by transcribing the written data on the lesson plan, presentation material, and note on the whiteboard, and also doing data reduction to lessen the irrelevant data, identifying the written mathematical communication accuracy from the data which had already been reduced using written mathematical communication accuracy framework. To get the valid data, the researcher did a task based in-depth interview. Moreover, to achieve a reliable data, the analysis process was conducted on the second subject using the same stage. Finally, the collected data of written mathematical communication accuracy would be interpreted descriptively.

3. Results and discussion

The result of written mathematical task which was given to 20 subjects showed that 19 students (95%) could finish the task as it was instructed, whereas another student (5%) was only able to fulfill the small part of the task because the subject was ill. The written mathematical communication data was classified based on the task completeness, then it was found that there were 3 students (15%) classified as the most complete, 12 students (60%) classified as quite complete, and 5 students (25%) as less complete. Among all of the data, the two data which were classified as quite complete were taken to be analyzed in order to obtain the description of written mathematical communication accuracy.

3.1. Written Mathematical Communication Accuracy towards One Variable Linear Equation

The task of written mathematical communication towards linear equation in one variable which was given to the first subject (S1) consisted of equation understanding, equation in one variable, linear equation, one variable linear equation, solving roots linear equation in one variable, and three steps of solving linear equation in one variable. Based on the example form $2x + 3 = 15$, the first subject (S1) wrote some interpretations: (1) open sentence with hyphen “=” (equal sign), which is called as equation, (2) the equation which only has one variable, namely $x$, these equation is called as one variable equation, (3) an equation in which the highest exponent of the equation variable is one, is called as linear equation, (4) linear equation is an open sentence which contains equal sign, it only has one variable, and the highest exponent of the equation variable is one, (5) the substitute of literal (variable), an equation which makes the open sentence be true is called as square-root or the solution set. Take a close look at this equation, $2x + 3 = 15$. If $x$ is substituted into 6 or $x = 6$, the equation will change into $2 \times 6 + 3 = 15$, then the sentence is considered as true, and $x = 6$ is called as root or solution from the equation.

All those five statements stated by the subject S1 are accurate, because those statements are interpreted clearly, correctly, completely, and preceded by an example as the illustration. Based on this mathematics form example $2x + 3 = 15$, is used consistently by S1 while explaining those five statements, so the explanation is easy-to-understanding. There are four symbols or notations which are used, such as number, variable, operation, and “=” sign. All of these symbols have been used appropriately. The terminologies used are: open sentence, variable, square, equation, linear equation, one variable linear equation, and square-root or solution. These seven terminologies have been used correctly and accurately.

Subject S2 started his/her written explanation by interpreting some definitions, such as: (1) presenting three examples of equation, those are $x + 8 = 15$, $3n - 7 = 20$, and $\frac{p}{5} + 9 = 12$. These open sentences used hyphen “=” (equal sign), and it is called as equation, (2) each equation is only has one variable, namely $x$, $n$, and $p$, then this equation is called as one variable equation, (3) each variable of this equation has the highest exponent of the equation variable is one. This equation is called as linear
equation, (4) so, this equation such as, \( x + 8 = 15 \), \( 3n - 7 = 20 \), and \( \frac{p}{5} + 9 = 12 \) is called as one variable linear equation.

Those four statements which were stated by subject S2 are equation, one variable equation, linear equation, one variable linear equation, have been presented accurately since all those explanations have been explained clearly, correctly, completely, and preceded by three examples as illustration. Those three equations are used consistently by S2 to ease him/her while explaining those four statements. Symbol or notation was used correctly, those symbols are number, \( x \), \( n \), and \( p \) variable, operations, and “=” sign. There were six terminologies used correctly and accurately, those are: open sentence, literal or variable, square, equation, linear equation, one variable linear equation. Subject S2 did not write the root or solution explanation of an equation clearly, but it was explained orally during the learning process with the help of a slideshow in powerpoint (look at figure 1).

Figure 1. S2 presentation material

Regarding to the one variable linear equation, S1 and S2 proposed three solving methods, those are: (1) substitution method, (2) adding or subtracting the two equation sides with the same number, (3) multiplying the two sides with the same number, or dividing both sides with the same non zero number. The substitution method written by S1 used the example of equation as follow, \( 2x - 1 = 5 \). For adding or subtracting the two sections method, S1 used the equation form, \( x + 2 = 4 \). Furthermore, for multiplying or dividing the two sections with the same number method, he/she used an example as follow, \( 3x = 18 \). Additionally, the substitution method written by S2 used an example as follow, \( 2x - 1 = 5 \). For the second method, S2 proposed an equation as an example as follow, \( x + 6 = 10 \). For the third method, S2 used \( 3x = 18 \) as an example. Written mathematical communication accuracy analysis for this material was focused on the procedure accuracy and the stage used to solve the equation. This accuracy of procedure-solving process of equation can be seen from the three stages, those are solving, algebraic manipulation, and calculation (look at figure 2).

Figure 2. Example of solving method
The figure 2 is a method to solve linear equation in one variable by reducing both sides with the same number. There are four solving steps, such as by adding some more information on the second stage. These solving steps have presented the logical sequence, done the correct algebraic manipulation, and calculated the calculations correctly. On the stage of algebraic manipulation and calculation, linear equation in one variable which was done using the other two methods has fulfilled the instructed criteria. Therefore, the written mathematical communication accuracy towards the linear equation in one variable task completion aspect is considered to be accurate.

3.2. Written Mathematical Communication Accuracy towards Linear Inequality in One Variable Material

Linear inequality in one variable lesson is the further lesson of linear equation in one variable. There are at least two differences between them. First, inequality is an open sentence which uses “>, <, ≥, ≤” signs. Second, root or solution from linear inequality in one variable is commonly consisted of one or more values. The data of written mathematical communication result towards linear inequality in one variable which is made by S1 consist of: inequality understanding, inequality in one variable, linear inequality, linear inequality in one variable, solving or linear inequality roots in one variable, and three solving steps of linear inequality in one variable.

Subject S1 presented three examples of mathematical forms, such as x + 5 > 9, 2x < 8, - \frac{2}{3} y ≥ -6. From the example, x + 5 > 9, subject S1 solved the inequality by subtracting both sides with -5, so it results x + 5 - 5 > 9 - 5. The solution is x > 4. For inequality form, such as 2x < 8, subject S1 multiplies both sides inequality with \frac{1}{2}, so it brings solution form like x < 4. Hence, for inequality form, - \frac{2}{3} y ≥ -6, subject S1 multiplies both sides with -\frac{3}{2} and brings result y ≤ 9, sign ≥ was substituted by ≤ (look at Figure 3).

Subject S1 also explains the inequality of - \frac{2}{3} y ≥ -6, multiplies both sides multiplies with -\frac{3}{2} which results y ≤ 9, sign ≥ was substituted by ≤. The change of this inequality had been explained by subject S1 on the previous step through the illustration as follow:

\begin{align*}
8 > 2 & \\
-3 \times 8 > -3 \times 2 & \quad \text{(true sentence)} \\
-24 > -6 & \quad \text{(false sentence)}
\end{align*}

Subject S1 brings more understanding about this case (look at figure 4).
Figure 4. Explanation about the change of sign $>$ to $<$ in inequality

All explanations written by S1 had been presented correctly and accurately. Subject S1 used notation and mathematical terminology correctly and accurately. Some concepts related to the inequality were communicated in written form accurately. Likewise in the step, algebra manipulation, and calculation while solving the problem, all of them had already been communicated in written form accurately. There were two aspects which were not explained well by the subject: first, the subject was unable to argue using mathematical logic, such as: if both sides of a linear inequality are multiplied by negative number, why the sign must be changed. The subject only stated a notification that the sign should be changed, so that mathematical sentence could be formed correctly. Second, while concluding the root or solution, the subject only showed the fulfilled substitute variable values, and did not state the solution set.

Subject S2 also presented three inequality examples, such as $2n + 5 > 16$, $2x < 8$, and $-\frac{2}{3}y \geq -6$. Using the mathematical form, $2n + 5 > 16$, subject S2 solved the inequality by subtracting both sides using $-5$, so it brought the form $2n + 5 - 5 > 16 - 5$. The solution was $x > 5.5$. For inequality, $2x < 8$, subject S2 multiplied both inequality sides using $\frac{1}{2}$ and produced $x < 4$. Therefore, for inequality form, $-\frac{2}{3}y \geq -6$, subject S2 multiplied both sides with $-\frac{3}{2}$ and produced $y \leq 9$. The explanation which stated that sign $\geq$ substituted by $\leq$ was delivered on whiteboard while S2 taught the lesson during the learning process (look at figure 5).

Figure 5. Solving inequality on whiteboard

When the subject answers the interview, subject S2 stated that, if both sides of linear inequality were multiplied by the same negative number, the sign must be inversed in order to make it into a true sentence. Why? Subject just shook his head. This statement was only such a notification without giving mathematical argument logically. All explanations written by S2 while using notation, mathematical terminology, procedure, algebra manipulation, and calculation in solving the problem were communicated correctly and accurately. S2 did not explain correctly about the rationale of sign change, if both sides of linear inequality were multiplied by the same negative number, and did not state the inequality root as solution set.

4. Conclusions
Written mathematical communication is a process of sharing ideas and mathematical understanding in written form and is considered as one of the most crucial abilities for pre-service teacher’s student. The written mathematical communication had been revealed accurately by the students on linear equation and inequality in one variable. This accuracy could be described through the accuracy of mathematical notation and terminology, the correct and clear concept explanation, the use of procedure with clear
stage and logical algebra manipulation, the calculation process, and also rigorous final result. However, both subjects could not explain the rationale of inequality sign changing logically, if there were a case where both sides of an inequality were multiplied by the negative number. Moreover, the subject also did not state the equation and inequality root in the form of solution set.

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