Effective field theory: A complete relativistic nuclear model

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(Dated: August 19 2003)

We analyzed the results for finite nuclei and infinite nuclear and neutron matter using the standard \( \sigma - \omega \) model and with the effective field theory. For the first time, we have shown here quantitatively that the inclusion of self-interaction of the vector mesons and the cross-interaction of all the mesons taken in the theory explain naturally the experimentally observed softness of equation of state without loosing the advantages of standard \( \sigma - \omega \) model for finite nuclei. Recent experimental observations support our findings and allow us to conclude that without self- and cross-interactions the relativistic mean field theory is incomplete.

PACS numbers: 21.10.Dr, 21.10.Tg, 21.60.-n, 21.60.Fw

In the quest for an unified model describing both finite nuclei and nuclear matter, the quantum hadrodynamics (QHD) has been a successful tool for the past few decades. QHD is the field theory of the nuclear many-body problem using hadron degrees of freedom. Models based on relativistic QHD takes care \textit{ab initio} of many natural phenomena which are practically absent or have to be included in an \textit{ad hoc} manner in the non-relativistic formalism. One of the first successful models based on QHD with relativistic mean field (RMF) was constructed by Walecka \cite{1} with vector and scalar meson fields. Later on, to get reasonable incompressibility and to get good results for finite nuclei, cubic and quartic nonlinearities of the \( \sigma \) meson (standard nonlinear \( \sigma - \omega \) model) were added \cite{2}. These models were proposed to be renormalizable and that constraint limited the scalar interactions to a quartic polynomial and disallowed the scalar-vector cross interactions and vector-vector self-interactions. However, the coupling constants are not assigned with their bare (experimental) values but with some effective value to have proper results for finite nuclei. Hence the renormalizability of the Lagrangian gets compromised by the use of effective coupling constants.

Inspired by effective field theory (EFT), Furnstahl, Serot and Tang \cite{3} abandoned the idea of renormalizability and extended the RMF theory by allowing other nonlinear scalar-vector and vector-vector interactions in addition to tensor couplings \cite{4, 5, 6, 7, 8, 9}. The EFT contains all the non-renormalizable couplings consistent with the underlying symmetries of QCD. The effective Lagrangian is obtained by employing suitable expansion scheme to truncate the infinite number of terms. In such scheme the ratios \( \Phi/M, W/M, |\nabla \Phi|/M^2 \) and \( |\nabla W|/M^2 \) are the useful expansion parameters \cite{3, 4, 5, 6, 7}. where \( \Phi \) and \( W \) are scalar and vector meson fields respectively and \( M \) is the nucleon mass. With the help of the concept of naturalness (i.e., all coupling constants are of the order of unity when written in appropriate dimensionless form), it is then possible to compute the contributions of the different terms in the expansion and to truncate the effective Lagrangian at a given level of accuracy \cite{4, 6, 7}.

None of the couplings should be arbitrarily dropped out to the given order without a symmetry argument. References \cite{6, 7, 8, 9} have shown that it suffices to go to fourth order in the expansion. At this level one recovers the standard nonlinear \( \sigma - \omega \) model plus a few additional couplings, with thirteen free parameters in all. These parameters have been fitted (parameter sets G1 and G2) to reproduce some observables of magic nuclei \cite{2}. The fits display naturalness, and the results are not dominated by the last terms retained. This evidence confirms the utility of the EFT concepts and justifies the truncation of the effective Lagrangian at the first lower orders.

Recent applications of the models based on EFT include studies of pion-nucleus scattering \cite{10} and of the nuclear spin-orbit force \cite{11}, as well as calculations of asymmetric nuclear matter at finite temperature with the G1 and G2 sets \cite{12}. In a previous work \cite{8} we have analyzed the impact of each one of the new couplings introduced in the EFT models on the nuclear matter saturation properties and on the nuclear surface properties. In Ref. \cite{8} we have looked for constraints on the new parameters by demanding consistency with Dirac-Brueckner-Hartree-Fock (DBHF) \cite{13} calculations and the properties of finite nuclei. Using EFT we successfully explained the properties of the drip-line nuclei as well as the symmetric and asymmetric infinite nuclear matter including the neutron star and compared with other theoretical calculations \cite{14}. Very recently \cite{15}, the flow of matter in heavy-ion collisions is analyzed to determine the pressures attained at densities ranging from two to five times the saturation density of nuclear matter. This experimental determination of the equation of state (EOS) of dense matter motivated us to study the applicability of various relativistic models at extreme conditions. In this letter we present the observations based on the results of our EFT calculations, the standard nonlinear \( \sigma - \omega \) model and some of the recent experiments.

The description of EFT and the field equations for nuclear matter and finite nuclei can be found in Refs. \cite{3, 4}. The field equations were derived \cite{3} from an energy density functional containing Dirac baryons and classical
scalar and vector mesons. According to Refs. [3, 4] the energy density for finite nuclei can be written as

\[
\mathcal{E}(\mathbf{r}) = \sum_\alpha \varphi_\alpha^\dagger \left\{ -i \mathbf{\alpha} \cdot \nabla + \beta(M - \Phi) + W + \frac{1}{2} \gamma_3 R + \frac{1 + \gamma_3}{2} A - \frac{i}{2M} \beta \mathbf{\alpha} \left( f_\rho \nabla W + \frac{1}{2} f_\rho \gamma_3 \nabla R + \lambda \nabla A \right) + \frac{1}{2M^2} (\beta_\alpha + \beta_\beta \gamma_3) \Delta A \right\} \varphi_\alpha + \left( \frac{1}{2} + \frac{\kappa_3}{3! M} + \frac{\kappa_4}{4! M^2} \right) \frac{m_\sigma^2}{g_\sigma^2} \Phi^2 - \frac{\gamma_0}{4! g_\gamma^2} W^4 + \frac{1}{2g_\rho^2} \left( 1 + \alpha_1 \frac{\Phi}{M} \right) (\nabla \Phi)^2 - \frac{1}{2g_\omega^2} \left( 1 + \alpha_2 \frac{\Phi}{M} \right) (\nabla W)^2 - \frac{1}{2} \left( 1 + \eta_1 \frac{\Phi}{M} + \eta_2 \Phi^2 \right) \frac{m_\omega^2}{g_\omega^2} W^2 - \frac{1}{2g_\rho^2} (\nabla R)^2 - \frac{1}{2} \left( 1 + \eta_\rho \frac{\Phi}{M} \right) \frac{m_\rho^2}{g_\rho^2} R^2 - \frac{1}{2c^2} (\nabla A)^2 + \frac{1}{3g_\gamma g_\omega} A \Delta W + \frac{1}{g_\gamma g_\rho} A \Delta R, \tag{1}
\]

where the index \( \alpha \) runs over all occupied states \( \varphi_\alpha(\mathbf{r}) \) of the positive energy spectrum, \( \Phi \equiv g_\sigma \phi_0(\mathbf{r}), W \equiv g_\omega V_0(\mathbf{r}), R \equiv g_\rho b_0(\mathbf{r}), A \equiv \varepsilon A_0(\mathbf{r}) \), \( g_\sigma, g_\omega, g_\rho \) and \( \epsilon \) are the coupling constants corresponding to the fields \( \phi_0(\mathbf{r}), V_0(\mathbf{r}), b_0(\mathbf{r}) \) and \( A_0(\mathbf{r}) \) respectively and \( \kappa_3, \kappa_4, \eta_1, \eta_2, \gamma_0, f_v \) and \( f_\rho \) are non-linear coupling constants.

The terms with \( g_\sigma, \lambda, \beta_\alpha \) and \( \beta_\beta \) take care of effects related with the electromagnetic structure of the pion and the nucleon (see Ref. [3]). Specifically, the constant \( g_\gamma \) concerns the coupling of the photon to the pions and the nucleons through the exchange of neutral vector mesons. The experimental value is \( g_\gamma^2/4\pi = 2.0 \). The constant \( \lambda \) is needed to reproduce the magnetic moments of the nucleons. It is defined by

\[
\lambda = \frac{1}{2} \lambda_\rho (1 + \tau_3) + \frac{1}{2} \lambda_\sigma (1 - \tau_3), \tag{2}
\]

with \( \lambda_\rho = 1.793 \) and \( \lambda_\sigma = -1.913 \) the anomalous magnetic moments of the proton and the neutron, respectively. The terms with \( \beta_\alpha \) and \( \beta_\beta \) contribute to the charge radii of the nucleon [3].

Variation of the energy density \( \mathcal{E} \) with respect to \( \varphi_\alpha^\dagger \) and the meson fields gives the Dirac equation fulfilled by the nucleons and the meson field equations [9]. The Dirac equation corresponding to the energy density \( \mathcal{E} \) and the mean field equations for \( \Phi, W, R \) and \( A \) can be found in Ref. [3, 14]. The meson fields can also be interpreted as Kohn–Sham potentials [11] in the relativistic case [12] and in this sense they include effects beyond the Hartree approach like three-body and many-body interactions through the nonlinear couplings [3, 4].

For infinite nuclear matter all of the gradients of the fields in the energy density and field equations vanish. Due to the fact that the solution of symmetric and asymmetric nuclear matter in mean field depends on the ratios \( g_\sigma^2/m_\sigma^2 \) and \( g_\omega^2/m_\omega^2 \) [13], we have seven unknown parameters. By imposing the values of the saturation density, total energy, incompressibility modulus and effective mass, we still have three free parameters (the value of \( g_\rho^2/m_\rho^2 \) is fixed from the bulk symmetry energy coefficient \( J \)).

The baryon, scalar, isovector, proton and tensor densities are same as for finite nuclei in the nuclear matter limit, i.e., \( \rho = \frac{2}{(2\pi)^3} \int k^i d^3 k = \frac{2}{\sqrt{2}} k_f^2 \) and \( \rho_n = \frac{2}{(2\pi)^3} \int k^i d^3 k \frac{M^*}{\sqrt{(k^2 + M^*)^2}} \), and so on (here \( k_f \) is the Fermi momentum and \( k \) is the momentum at any density). The expressions for pressure and energy density are

\[
P = \frac{\gamma}{3(2\pi)^3} \int d^3 k E^*(k) + \frac{1}{4!} \gamma_0 g_\omega^2 V_0^4 + \frac{1}{2} \left( 1 + \eta_1 \frac{g_\sigma^2}{M} + \frac{\gamma_2}{2} \frac{g_\omega^2}{M^2} \right) m_\omega^2 V_0^2 - m_\sigma^2 \sigma^2 \left( \frac{1}{2} + \frac{\kappa_3 g_\sigma^2}{3! M} + \frac{\kappa_4 g_\omega^2 \sigma^2}{4! M^2} \right) + \frac{1}{2} \left( 1 + \eta_\rho \frac{g_\sigma^2}{M} \right) m_\rho^2 V_0^2, \tag{3}
\]

\[
\epsilon = \frac{\gamma}{(2\pi)^3} \int d^3 k E^*(k) - \frac{1}{4!} \gamma_0 g_\omega^2 V_0^4 - \frac{1}{2} \left( 1 + \eta_1 \frac{g_\sigma^2}{M} + \frac{\gamma_2}{2} \frac{g_\omega^2}{M^2} \right) m_\omega^2 V_0^2 + \frac{1}{2} g_\rho b_0 (\rho_p - \rho_n) + m_\sigma^2 \sigma^2 \left( \frac{1}{2} + \frac{\kappa_3 g_\sigma^2}{3! M} + \frac{\kappa_4 g_\omega^2 \sigma^2}{4! M^2} \right) + \frac{1}{2} \left( 1 + \eta_\rho \frac{g_\sigma^2}{M} \right) m_\rho^2 b_0 + g_\omega V_0 (\rho_p + \rho_n). \tag{4}
\]
Here $\gamma=2$ for pure neutron matter and $\gamma=4$ for symmetric nuclear matter. The asymmetry of nuclear matter is defined by the parameter $\alpha$. For the symmetric matter, $\alpha=0$ and for the neutron matter, $\alpha=1$.

While examining the effect of nonlinear coupling in nuclear matter using various nuclear force parameters, the NL3 parameter set [21] (considered as a representative of the standard nonlinear $\sigma-\omega$ parametrization) has been found to fail in following the DBHF results even at slightly higher densities. It is well known that the DBHF theory in relativistic framework explains well the nuclear matter at higher densities ($\sim 2\rho_0$) [20, 21]. In contrast to the standard $\sigma-\omega$ model, the EFT calculations at high density regimes yield results in accordance with DBHF. This scenario is depicted in Fig. 1, where we present the results of calculations with TM1 parameter set [21] also. In the calculation with TM1, only a quartic vector self-interaction term is included apart from the terms in NL3. This inclusion was done arbitrarily without considering the underlying QCD symmetries or the naturalness. However, with this self-interaction term, the TM1 give better results at higher densities. This demonstrates the importance of self-interactions at higher densities and exposes the inadequacy of the standard nonlinear $\sigma-\omega$ model. This argument is further supported by Fig. 2 in which the variation of binding energy per particle ($E/A$) is plotted as a function of $\rho/\rho_0$. The NL3 parameter set gives a much too stiff EOS whereas the other parameter sets give a softer EOS which is consistent with the observed neutron star masses $M$ and radii $R$ (See Table I) and measurements of kaon production in heavy-ion collisions [22].

The recent experimental observations [12, 22] rule out any strongly repulsive nuclear EOS and has confirmed the predictions made above. The zero-temperature EOS for symmetric nuclear matter derived experimentally is shown in Fig. 3 along with the results obtained from different calculations. In Fig. 3 we can see that the calculations based on NL3 deviate drastically from the experimental observation and the EFT calculations with G1 up to some extent and G2 to an excellent extent, agree with the experiment. A similar situation prevails in the EOS of neutron matter and can be seen in Fig. 4. From the figures it is very clear that NL3 calculations are not consistent with the observed neutron star masses $M$ and radii $R$ (in km) and spin-thickness $t$ (in fm). Table I: Upper panel: The surface energy coefficient $E_s$ and surface thickness $t$ (in fm). Middle panel: Energy per nucleon $E/A$ (in MeV), charge radius $r_{ch}$ (in fm) and spin-orbit splittings $\Delta E_{SO}$ (in MeV) of the least-bound nucleons. Lower panel: the neutron star radius $R$ (in km) and the mass ratio $M/M_{\odot}$.

|          | TM1 | NL3 | G1 | G2 | Exp. |
|----------|-----|-----|----|----|------|
| $E_s$    | 15.0| 16.2| 16.1| 16.4| 16.0–21.0 |
| $t$      | 1.91| 1.99| 1.98| 2.08| 2.2–2.5 |
| $^{16}$O | $E/A$| $-8.15$| $-8.08$| $-7.97$| $-7.98$ |
| $r_{ch}$ | 2.66| 2.73| 2.72| 2.72| 2.73 |
| $\Delta E_{SO}$ (n,1p) | 5.6| 5.6| 5.6| 5.9| 5.9 |
| (p,1p)   | 5.6| 6.3| 5.9| 5.9| 6.3 |
| $^{48}$Ca | $E/A$| $-8.65$| $-8.64$| $-8.67$| $-8.68$ |
| $r_{ch}$ | 3.46| 3.48| 3.44| 3.44| 3.47 |
| $\Delta E_{SO}$ (n,1d) | 5.6| 6.1| 5.8| 5.6| 3.6 |
| (p,1d)   | 5.2| 6.3| 6.2| 6.2| 4.3 |
| $^{90}$Zr | $E/A$| $-8.71$| $-8.69$| $-8.71$| $-8.71$ |
| $r_{ch}$ | 4.27| 4.28| 4.28| 4.28| 4.26 |
| $\Delta E_{SO}$ (n,2p) | 1.4| 1.6| 1.8| 1.8| 0.5 |
| $^{208}$Pb | $E/A$| $-7.87$| $-7.87$| $-7.87$| $-7.87$ |
| $r_{ch}$ | 5.54| 5.52| 5.5| 5.5| 5.5 |
| $\Delta E_{SO}$ (n,3p) | 0.7| 0.8| 0.9| 0.9| 0.9 |
| (p,2d)   | 1.4| 1.6| 1.8| 1.8| 1.3 |

$R$: The neutron star radius $R$ (in km) and the mass ratio $M/M_{\odot}$.
the calculations of Akmal et al.\cite{23} which employs the Argonne $v_{18}$ interaction. Such an interaction is not applied successfully in the case of finite nuclei. The EFT calculations are proved to give very good results for finite nuclear properties\cite{3} as well as for infinite nuclear matter including neutron star\cite{4}. In Table I we present some of the sample results of EFT calculations. More results for finite nuclei and nuclear matter can be found in Refs. \cite{9,13}. In choosing between the parameter sets G1 and G2 for further calculations we prefer G2. It is worth to note that G2 presents positive values of $\Phi^4$ coupling constant ($\kappa_4$), as opposed to G1 and to many of the most successful RMF parametrizations, such as NL3. Actually the negative value of $\kappa_4$ is not acceptable because the energy spectrum then has no lower bound\cite{24}. However such negative value is necessary in the standard $\sigma-\omega$ model to get the results closer to the experimental values. On the other hand to have positive value for $\kappa_4$ it is not necessary to make two parameter sets as was done in Ref.\cite{21}.

In conclusion, with the new experimental values for EOS, the predictions of EFT are proved to be true. With the inclusion of self- and cross-interactions and without forcing any change of parameters or modifying the formalism the EFT calculations with G2 parameter set explain finite nuclei and infinite nuclear matter in a unified way with commendable level of accuracy in both the cases. Any Lagrangian without all types of self- and cross-interactions is incomplete and at present EFT can be considered as a complete unified theory for finite nuclei as well as for infinite nuclear matter.

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