NUMERICAL SIMULATIONS OF RELATIVISTIC SHOCK ACCELERATION

Michał Ostrowski
Obserwatorium Astronomiczne, Uniwersytet Jagielloński, ul. Orla 171, 30-244 Kraków, Poland
(E-mail: mio@oa.uj.edu.pl)

We review the present status of the cosmic ray acceleration theory in mildly relativistic shock waves. Due to the involved substantial particle anisotropies analytical methods can tackle only simple situations involving weakly turbulent conditions near the shock. The numerical Monte Carlo methods are used to study the acceleration process in more general conditions. Contrary to non-relativistic shocks, the cosmic ray spectra at relativistic shock waves depend substantially on the magnetic field configurations and turbulence spectra. Depending on these conditions both very flat and very steep spectra can be created, with the characteristic acceleration time scales changing non-monotonously with the turbulence amplitude. Thus, the discussed theory is not able to uniquely explain the observed synchrotron spectra of relativistic shocks. We also mention an interesting possibility of particle acceleration at incompressible flow discontinuities (shear layers) at boundaries of relativistic jets.

1 Introduction

Relativistic plasma flows are detected or postulated to exist in a number of astrophysical objects, ranging from a mildly relativistic jet of SS433, through the-Lorentz-factor-of-a-few jets in AGNs and galactic ‘micro-quasars’, up to ultra-relativistic outflows in sources of gamma ray bursts and, possibly, in pulsar winds. As nearly all such objects are efficient emitters of synchrotron radiation and/or high energy photons, our attempts to understand the processes generating cosmic ray particles are essential for understanding the fascinating phenomena observed. Below we will discuss the work carried out over the last 15 years in order to understand the cosmic ray acceleration processes acting at relativistic shocks. We limit the discussion to mildly relativistic flows with Lorentz factors up to, say, 10. The acceleration processes acting at non-relativistic and ultra-relativistic shocks are discussed in this proceedings by Gallant. We also shortly discuss a possibility for cosmic ray acceleration at shear layers accompanying relativistic jets.
2 Cosmic ray acceleration at relativistic shock waves

2.1 The Fokker-Planck description of the acceleration process

In the case of the shock velocity reaching values comparable to the light velocity, the particle distribution at the shock becomes anisotropic. It complicates to a great extent both the physical picture and the mathematical description of particle acceleration. The first attempt to consider the acceleration process at the relativistic shock was presented in 1981 by Peacock, however, no consistent theory was proposed until a paper of Kirk & Schneider (1987a). Those authors considered the stationary solutions of the relativistic Fokker-Planck equation for the case of a parallel shock wave. For the applied momentum pitch-angle diffusion operator, \( \partial / \partial \mu (D_{\mu \mu} \partial f / \partial \mu) \), they generalised the diffusive approach to higher order terms in particle distribution anisotropy and constructed general solutions at both sides of the shock which involved solutions of the eigenvalue problem. By matching two solutions at the shock, the spectral index of the resulting power-law particle distribution can be found by taking into account a sufficiently large number of eigenfunctions. The same procedure yields the particle angular distribution and the spatial density distribution. The low-order truncation in this approach corresponds to the standard diffusion approximation and to a somewhat more general method described by Peacock. The above analytic approach (or the ‘semi-analytic’ one, as the mentioned matching of two series involves numerical fitting of the respective coefficients) was verified by Kirk & Schneider (1987b) by the method of particle Monte Carlo simulations.

An application of this approach to more realistic conditions – but still for parallel shocks – was presented by Heavens & Drury (1988), who investigated the fluid dynamics of relativistic shocks (cf. also Ellison & Reynolds 1991) and used the results to calculate spectral indices for accelerated particles. They considered the shock wave propagating into electron-proton or electron-positron plasma, and performed calculations using the analytic method of Kirk & Schneider for two different power spectra for the scattering MHD waves. In contrast to the non-relativistic case, they found that the particle spectral index \( \alpha \), depends on the form of the wave spectrum. The unexpected fact was noted that the non-relativistic expression \( \alpha = 3R / (R - 1) \) (\( R \) - a shock compression ratio) provided a quite reasonable approximation to the actual spectral index (see the initial points of curves at Fig. 1, the ones for \( \Psi_1 = 0 \), or \( U_1 = U_{B,1} \)).

A substantial progress in understanding the acceleration process with a highly anisotropic particle distribution at the shock brought the work of Kirk & Heavens (1989; see also Ostrowski 1991a and Ballard & Heavens 1991), who considered particle acceleration at subluminal \( (U_{B,1} < c) \) relativistic shocks with oblique magnetic fields. They assumed the magnetic momentum conservation, \( p_\perp^2 / B = \text{const} \), at particle interaction with the shock and applied the Fokker-Planck equation discussed above to describe particle transport along the field lines outside the shock, while excluding the possibility of cross-field diffusion. In the cases when \( U_{B,1} \) reached relativistic values, they derived very flat energy spectra with \( \gamma \approx 0 \) at \( U_{B,1} \approx c \) (Fig. 1). In such conditions, the particle density in front of the shock can substantially – even by a few orders of magnitude – exceed the downstream density (see the curve denoted ‘-8.9’ at Fig. 2). Creating flat spectra and great density contrasts is due to the effective reflections of anisotropically distributed upstream particles from the region of compressed magnetic field downstream of the shock.

As stressed by Begelman & Kirk (1990), in relativistic shocks one can often find the superluminal conditions with \( U_{B,1} > c \), where the above presented approach is no longer valid.

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*We use here a phase space distribution function index \( \alpha \) and the synchrotron spectral index \( \gamma = (\alpha - 3) / 2 \). An index ‘1’ (‘2’) is used for quantities measured in the plasma rest frame upstream (downstream) of the shock. In the text, \( U \) is a shock velocity, \( \Psi \) is an inclination of the magnetic field, \( B \), with respect to the shock normal, \( U_{B,1} \equiv U_1 / \cos \Psi_1 \) is the shock velocity projected at the magnetic field. An amplitude of the magnetic turbulence, \( \delta B \), is often characterized with the ratio of the particle cross-field diffusion coefficient, \( \kappa_\perp \), to the parallel diffusion coefficient, \( \kappa_\parallel \).
Then, it is not possible to reflect upstream particles from the shock and to transmit downstream particles into the upstream region. In effect, only a single transmission of upstream particles re-shapes the original distribution by shifting particle energies to larger values. The energy gains in such a process, involving a highly anisotropic particle distribution at the shock, can be quite significant, exceeding the value expected for the adiabatic compression.

The approach proposed by Kirk & Schneider (1987a) and Kirk & Heavens (1989), and the derivations of Begelman & Kirk (1990) are valid only in case of weakly perturbed magnetic fields. However, if finite amplitude MHD waves are present in the medium, both the Fokker-Planck approach is no longer valid and the approximate magnetic momentum conservation no longer holds for particles interacting with oblique shocks. Then numerical methods have to be used.

### 2.2 Particle acceleration in the presence of finite amplitude magnetic field perturbations

The first attempt to consider the acceleration process at a parallel shock wave propagating in a turbulent medium was presented by Kirk & Schneider (1988), who included into the kinetic transport equation the Boltzmann collision operator describing the large angle scattering. By solving the resulting integro-differential equation they demonstrated the hardening of the particle spectrum for increasing contribution of the large-angle scattering. The reason for such a spectral change is the additional isotropization of particles interacting with the shock, leading to an increase in the particle mean energy gain. In oblique shocks, this simplified approach cannot be used because the character of individual particle-shock interaction – reflection and transmission characteristics – depends on the magnetic field perturbations. Let us additionally note that application of the point-like large-angle scattering model in relativistic shocks does not provide a viable physical representation of the scattering at MHD waves (Bednarz & Ostrowski 1996).

To handle the problem of the particle spectrum in a wide range of background conditions, the Monte Carlo particle simulations were widely used (e.g. Kirk & Schneider 1987b; Ellison et al. 1990; Ostrowski 1991a, 1993; Ballard & Heavens 1992, Naito & Takahara 1995, Bednarz...
The energetic particle density across the relativistic shock with an oblique magnetic field (Ostrowski 1991b). The shock with $U_1 = 0.5c$, $R = 5.11$ and $\psi_1 = 55^\circ$ is considered. The curves for different perturbation amplitudes are characterized with the value of $\log \kappa_\perp/\kappa_\parallel$ given near the curve. The data are vertically shifted for picture clarity. The value $X_{\text{max}}$ is the distance from the shock at which the upstream particle density decreases to $10^{-3}$ part of the shock value.

& Ostrowski 1996). At first, let us consider subluminal shocks. The field perturbations influence the acceleration process in a few ways. As they enable the particle cross field diffusion, a modification (decrease) of the downstream particle escape probability allows larger number of particles to re-cross the shock upstream. This factor tends to harden the spectrum. Next, the perturbations decrease particle anisotropy, leading to an increase of the mean energy gain of reflected upstream particles, but – what is more important for oblique shocks – they also increase the particle upstream-downstream transmission probability due to less efficient reflections, enabling additional particles to escape from further acceleration. The third factor is due to perturbing particle trajectory during an individual interaction with the shock discontinuity and breakdown of the approximate conservation of $p^2_\perp/B$. Because reflecting a particle from the shock requires a fine tuning of the particle trajectory with respect to the shock surface, even small amplitude perturbations can decrease the reflection probability in a substantial way (cf. Ostrowski 1991). Simulations show (see Fig. 3 for $U_{B,1} \leq 1.0$, $\Psi_1 \leq 60^\circ$) that – until the wave amplitude becomes very large – the factors leading to efficient particle escape dominate with the resulting steepening of the spectrum to $\gamma \sim 0.2 - 0.8$ ($\alpha \geq 3.4$), and the increased downstream transmission probability lowers the cosmic ray density contrast across the shock (Fig. 2).

In parallel shock waves propagating in a highly turbulent medium, the effects discovered for oblique shocks can also manifest their presence because of the local perturbed magnetic field compression at the shock. The problem was considered using the technique of particle simulations by Ballard & Heavens (1992; cf. Ostrowski 1988 for a non-relativistic shock). They obtained very steep spectra in the considered cases, with the synchrotron spectral index growing from $\gamma \sim 0.6$ at medium relativistic velocities up to nearly 2.0 at $U_1 = 0.98c$. These results apparently do not correspond to the large-perturbation-amplitude limit of Ostrowski’s (1993; see the discussion therein) simulations for oblique shocks and the analytic results of Heavens & Drury (1988).

For large amplitude magnetic field perturbations the acceleration process in superluminal
shocks can lead to the power-law particle spectrum formation (cf. Fig. 3 for $\Psi_1 = 72.5^\circ$), against the statements of Begelman & Kirk (1990) valid at small wave amplitudes only. Such a general case was discussed by Ostrowski (1993) and by Bednarz & Ostrowski (1996, 1998).

2.3 The acceleration time scale

The shock waves propagating with relativistic velocities raise a question about the involved cosmic ray acceleration time scales, $T_{\text{acc}}$. A simple comparison to the values derived with the non-relativistic formula shows that $T_{\text{acc}}$ relatively decreases with increasing shock velocity for parallel (Quenby & Lieu 1989; Ellison et al. 1990) and oblique (Takahara & Terasawa 1990; Newman et al. 1992; Lieu et al. 1994; Quenby & Drolias 1995; Naito & Takahara 1995) shocks. However, the numerical approaches used in the listed papers, based on assuming particle isotropization for all scatterings, neglect or underestimate a significant factor affecting the acceleration process – the particle anisotropy. Ellison et al. (1990) and Naito & Takahara (1995) included also the more realistic derivations involving the pitch-angle diffusion approach. The calculations of Ellison et al. for parallel shocks show similar results to those they obtained for large amplitude scattering. For the shock with velocity 0.98$c$ the acceleration time scale is reduced by the factor $\sim 3$ with respect to the non-relativistic formula. Naito & Takahara considered shocks with oblique magnetic fields. They confirmed the reduction of the acceleration time scale with increasing inclination of the magnetic field, the fact derived earlier for non-relativistic shocks. However, their approach neglected effects of particle cross field diffusion and assumed the adiabatic invariant conservation at particle interactions with the shock, thus limiting the validity of their results to a small amplitude turbulence near the shock.

![Figure 3: The relation of $T_{\text{acc}}$ versus the particle spectral index $\alpha$ at different magnetic field inclinations $\psi_1$ given near the respective curves (from Bednarz & Ostrowski 1996). A turbulence amplitude grows along the presented curves. The minimum value of the model parameter $\kappa_1/\kappa_\parallel$ occurs at the encircled point of each curve and the wave amplitude monotonously increases along each curve up to $\delta B \sim B$. $r_{e,1}$ is the particle gyroradius in the effective (including perturbations) upstream magnetic field.](image)

A wider discussion of the acceleration time scale is presented by Bednarz & Ostrowski (1996), who apply numerical simulations involving the small angle particle momentum scattering. The approach treats properly the existing correlations between the particle energy gains and the respective particle diffusion times off the shock. It is also believed to provide a reasonable
description of particle transport in the presence of large turbulence amplitude and enables modelling of the cross-field diffusion effects. The resulting acceleration time scales given in the shock normal rest frame (cf. Begelman & Kirk 1990) are presented on Fig. 3 versus the respective spectral index $\alpha$. The results for varying turbulence amplitudes are given for several magnetic field inclinations. In parallel ($\Psi = 1^\circ$) shocks $T_{acc}$ diminishes with the growing perturbation amplitude and – not presented on the figure – the growing shock velocity. However, it is approximately constant for a given value of $U_1$, if we use the formal diffusive time scale, $\kappa_{\parallel,1}/(U_1c) + \kappa_{\parallel,2}/(U_2c)$, as the time unit. A new feature discovered in oblique shocks is a non-monotonic variation of $T_{acc}$ with the turbulence amplitude $\delta B$. The acceleration process leading to the power-law spectrum is possible in superluminal shocks only in the presence of large amplitude turbulence. Then, in contrast to the quasi-parallel shocks, $T_{acc}$ increases with increasing $\delta B$, accompanied with a substantial flattening of the spectrum. In the considered cases with the oblique field configurations one may note a possibility to have the extremely short acceleration time scale, comparable to the particle gyroperiod in the magnetic field upstream of the shock.

3 Energetic particle acceleration at relativistic shear layers

The acceleration processes acting in shocks formed in relativistic jets are not always able to explain the observed high energy electrons radiating far away from the shock. A natural but mostly unexplored possibility to explain such observations is the one involving particle acceleration at the interface between the jet and the surrounding ‘cocoon’. To date the knowledge of physical conditions within such velocity shear layers is very limited and only rough estimates for the considered acceleration processes are possible. Within the jet turbulent boundary layer with a small velocity shear the ordinary second-order Fermi acceleration, as well as the process of ‘viscous’ particle acceleration (cf. the review by Berezhko 1990 of the work done in early 80-th; also Earl et al. 1988, Ostrowski 2000) can take place. A mean particle energy gain in the viscous acceleration processes scales as $<\Delta E> \propto \left(\frac{<\Delta U>}{c}\right)^2$, where $<\Delta U>$ is the mean velocity difference between the ‘successive scattering acts’. It is proportional to the mean free path normal to the shear layer, $\lambda_n$, times the mean flow velocity gradient in this direction $\nabla n \cdot \vec{U}$. With $d$ denoting the shear layer thickness this gradient can be estimated as $|\nabla n \cdot \vec{U}| \approx U/d$. Because the acceleration rate in the Fermi II process is $\propto (V/c)^2$ ($V$ is the wave velocity $\approx$ the Alfvén velocity), the relative importance of both processes is given by a factor $(\lambda_n U dV)^2$. The relative efficiency of the viscous acceleration grows with $\lambda_n$ and in the formal limit of $\lambda_n \approx d$ it will dominate over the Fermi acceleration to a large extent. Because accelerated particles can escape from the accelerating layer only due to a relatively inefficient radial diffusion, the resulting particle spectra should be very flat, depending however on several unknown physical parameters of the boundary layer (Ostrowski 1990, 1998). Spectra of very high energy particles, with $\lambda_n \geq d$ (or $r_g > d$), do not require so detailed information about the boundary layer structure and can thus be derived with higher credibility.

As an example let us present our results (Ostrowski 1998) of such numerical Monte Carlo derivations, comparing the ultra high energy particle spectrum generated at the jet terminal shock with the spectrum resulting from the jet boundary acceleration. The distributions of particles accelerated at the jet side-boundary, far from the shock, can be very flat. This feature results from the character of the acceleration process with particles having a chance to meet the accelerating surface and gain energy again and again for inefficient diffusive particle escape to the sides. Contrary to that, the shock acceleration process determines the spectrum inclination due to the joint action of the particle energization at the shock and the continuous escape due to particle advection with the downstream plasma. In the discussed simulations the escape probability grows with the particle energy providing a cut-off in the spectrum. For the shock...
spectrum, in the range of particle energies directly preceding the cut-off, the spectrum exhibits some flattening with respect to the inclination expected for the pure shock acceleration. There are two reasons for that flattening: additional particle transport from the downstream region to the upstream one through the cocoon surrounding the jet and inclusion of the flat spectral component resulting from the side boundary acceleration.

4 Final remarks

The work done to date on the test particle cosmic ray acceleration at mildly relativistic shocks do not provide results allowing for meaningful modelling of the observed astrophysical sources. The main reason for that deficiency is – in contrast to non-relativistic shocks – a direct dependence of the derived spectra on the conditions at the shock. Not only the shock compression ratio, but also other parameters, like the mean magnetic field inclination or the wave spectrum and amplitude, are significant here. Depending on the actual conditions one may obtain spectral indices as flat as $\alpha = 3.0$ ($\gamma = 0.0$) or quite steep ones with $\alpha > 5.0$ ($\gamma > 1.0$). The background conditions leading to the very flat spectra are probably subject to some instabilities; however, there is no detailed derivation describing the instability growth and the resulting cosmic ray spectrum modification. The situation may become simpler for ultra-relativistic shocks, as discussed by Gallant in this proceedings.

A true progress in modelling particle acceleration in actual sources requires a full plasma non-linear description, including feedback of accelerated particles at the turbulent wave fields near the shock wave, flow modification caused by the cosmic rays’ plasma pre-shock compression and, of course, the appropriate boundary conditions. A simple non-linear approach to the parallel shock case was presented by Baring & Kirk (1990), who found that relativistic shocks could be very efficient accelerators. However, it seems to us that in a more general case it will be very difficult to make any substantial progress in that matter. For very flat particle spectra the non-linear acceleration picture depends to a large extent on the detailed knowledge of the background and boundary conditions in the scales relevant for particles near the upper energy cut-off. The existence of stationary solutions is doubtful in this case. An important step toward considering detailed physics of the acceleration provides the work of Hoshino et al. (1993) for ultra-relativistic shocks, but there is a lack of analogous studies for mildly relativistic shocks.

One may note that observations of possible sites of relativistic shock waves (knots and hot spots in extragalactic radio sources), which allow for determination of the energetic electron spectra, often yield particle spectral indices close to $\alpha = 4.0$ ($\gamma = 0.5$). In order to overcome difficulties in accounting for these data Ostrowski (1994) proposed an additional ‘law of nature’ for non-linear cosmic ray accelerators. The particles within different energy ranges do not couple directly with each other and are supposed to form independent ‘degrees of freedom’ in the system. Our ‘law’ provides that nature prefers energy equipartition between such degrees of freedom, yielding the spectra with $\alpha \approx 4.0$.

The acceleration processes at astrophysical shear layers may provide a viable explanation for some ‘strange’ observational data in relativistic jets. For example it may have significant consequences for the relativistic jet structure and the high energy radiation of AGNs (Ostrowski 2000; Stawarz & Ostrowski, in preparation). Also, these mechanisms could be responsible for accelerating nuclei up to the ultra high energy scales (Ostrowski 1998). The discussed processes are particularly interesting because a simple inspection does not reveal any physical obstacles which could make them inefficient. Unfortunately, the resulting particle spectra depend to a large extent on the poorly known background physical conditions.
Acknowledgements

The present work was supported by the Komitet Badań Naukowych through the grant PB 258/P03/99/17.

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