Inflation and Leptogenesis

in a $U(1)$-enhanced supersymmetric model

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Abstract

Motivated by the flavored Peccei-Quinn symmetry for unifying flavor physics and string theory, we investigate a supersymmetric extension of standard model (SM) for a lucid explanation of inflation and leptogenesis by introducing $U(1)$ symmetries such that the $U(1)-[\text{gravity}]^2$ anomaly-free condition together with the SM flavor structure demands additional sterile neutrinos as well as no axionic domain-wall problem. Such additional neutrinos may play a crucial role as a bridge between leptogenesis and new neutrino oscillations along with high energy cosmic events. In the model gravitational interactions explicitly break supersymmetry (SUSY) down to SUSY$_{\text{inf}} \times$ SUSY$_{\text{vis}}$, where SUSY$_{\text{inf}}$ corresponds to the supergravity symmetry with its goldstino (mainly as inflatino) eaten by gravitino, while the orthogonal SUSY$_{\text{vis}}$ is approximate global symmetry with its corresponding uneaten goldstino giving masses to all the supersymmetric SM superpartners. In a realistic moduli stabilization, we show that the moduli backreaction effect on the inflationary potential leads to the energy scale of inflation with the inflaton mass in a way that the power spectrum of the curvature perturbation and the scalar spectral index are to be well fitted with the latest Planck observation. We suggest that a new leptogenesis scenario could naturally be implemented via Affleck-Dine mechanism. So we show that the resultant baryon asymmetry, constrained by the sum of active neutrino masses and new high energy neutrino oscillations, crucially depends on the reheating temperature $T_{\text{reh}}$. And we show that the model has a prediction $T_{\text{reh}} \approx (59 - 84)$ TeV, which is compatible with the required $T_{\text{reh}}$ to explain the baryon asymmetry of the Universe.

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I. INTRODUCTION

The standard model (SM) of particle physics has been successful in describing properties of known matter and forces to a great precision until now, but we are far from satisfied since it suffers from some problems or theoretical arguments that have not been solved yet, which follows: inclusion of gravity in gauge theory, instability of the Higgs potential, cosmological puzzles of matter-antimatter asymmetry, dark matter, dark energy, and inflation, and flavor puzzle associated with the SM fermion mass hierarchies, their mixing patterns with the CP violating phases, and the strong CP problem. The SM therefore cannot be the final answer. It is widely believed that the SM should be extended to a more fundamental underlying theory. If nature is stringy, string theory should give insight into all such fundamental problems or theoretical arguments\(^1\). As indicated in Refs. \([1, 2]\)\(^2\), such several fundamental challenges strongly hint that a supersymmetric hybrid inflation framework with new gauge symmetries as well as higher dimensional operators responsible for the SM flavor puzzles may be a promising way to proceed.

Since astrophysical and cosmological observations have increasingly placed tight constraints on parameters for axion, neutrino, and inflation including the amount of reheating, it is in time for a new scenario on axion and neutrino to mount such interesting challenges, see also Ref. \([1, 4]\). In a theoretical point of view axion physics including neutrino physics requires new gauge interactions and a set of new fields that are SM singlets. Thus in extensions of the SM, sterile neutrinos and axions could naturally be introduced, \(e.g.,\), in view of \(U(1)\) symmetry. As a new paradigm to explain the aforementioned fundamental challenges, in this paper we investigate a minimal and economic supersymmetric extension of SM for a lucid explanation of inflation and leptogenesis, which can be realized within the framework\(^3\) of \(G \equiv SM \times U(1)_X \times A_4\). All renormalizable and nonrenormalizable operators allowed by such gauge symmetries, non-Abelian discrete symmetry, and \(R\)-parity exist in the superpo-

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1. In Ref. \([2]\) a concrete model is designed to bridge between string theory as a fundamental theory and low energy flavor physics.
2. Ref. \([1]\) introduces a superpotential for unifying flavor and strong CP problems, the so-called flavored PQ symmetry model in a way that no axionic domain wall problem.
3. Here the flavored Peccei-Quinn (PQ) symmetry \(U(1)_X\) embedded in the non-Abelian \(A_4\) finite group \([3]\) could economically explain the mass hierarchies of quarks and leptons including their peculiar mixing patterns as well as provide a neat solution to the strong CP problem and its resulting axion \([4]\).
tential as in Ref. [4]. Here we stress that, as shown in Ref. [4] by numerical analysis for the active neutrinos based on the present model, the values of atmospheric ($\theta_{23}$) and Dirac CP phase ($\delta_{CP}$) together with well-fitted solar $\theta_{12}$ and reactor $\theta_{13}$ mixing angles have a remarkable coincidence with the most recent data by the NOνA [5] and/or T2K [6] experiments. We assume throughout that the model can be derived as consistent type IIB string vacuum. In such a vacuum, as shown in Ref. [4] the $U(1)_X$-mixed anomalies such as $U(1)_X-[U(1)_Y]^2$, $U(1)_X-[SU(2)_L]^2$, $U(1)_X-[SU(3)_C]^2$, and $U(1)_Y-[U(1)_X]^2$ have been cancelled by appropriate shifts of Ramond-Ramond axions in the bulk [7]. And since non-perturbative quantum gravitational effects spoil the axion solution to the strong CP problem [8, 9], in order to eliminate such breaking effects of the axionic shift symmetry by gravity the author in Ref. [4] has imposed an $U(1)_X \times [gravity]^2$ anomaly cancellation condition [4] in a way that no axionic domain-wall problem occurs, thereby additional sterile neutrinos are introduced. Such sterile neutrinos are light or heavy and do not participate in the weak interaction. Moreover, the latest results [10] from Planck and Baryon Acoustic Oscillations (BAO) show that the contribution of light sterile neutrinos to $N_{\nu_{\text{eff}}}$ at the Big-Bang Nucleosynthesis (BBN) [11] era is negligible; such neutrinos may play a crucial role as a bridge between leptogenesis and new neutrino oscillations along with high energy cosmic events. As demonstrated in Ref. [2], by introducing two gauged $U(1)$ symmetries in the context of supersymmetric moduli stabilization based on type IIB string theory, three size moduli and one axionic partner with positive masses are stabilized while leaving two axions massless. The two massless axion directions are gauged by the $U(1)$ gauge interactions, and such gauged flat directions are removed through the Stuckelberg mechanism, leaving behind low energy symmetries which are anomalous global $U(1)_X$.

Supergravity (SUGRA) is a theory with local super-Poincare symmetry. As addressed in Ref. [4] where two $U(1)$ symmetries are imbedded, the $U(1)_{X_1}$ and $U(1)_{X_2}$ breaking scales are separated by the Gibbons-Hawking temperature, $T_{\text{GH}} = H_I/2\pi$, and both of which are to be much above the electroweak scale

$$\langle \Phi_1 \rangle < \frac{H_I}{2\pi} < \langle \Phi_2 \rangle,$$

(1)

where $H_I$ is the inflationary Hubble constant, and the fields $\Phi_1 = \{\Phi_S, \Theta\}$ and $\Phi_2 = \{\Psi, \tilde{\Psi}\}$ are charged under the $U(1)_{X_1}$ and $U(1)_{X}$ symmetries, respectively. Here we have assumed that the electroweak symmetry is broken by some mechanism, such as radiative effects.
when SUSY (supersymmetry) is broken. So we can picture two secluded SUSY breaking sectors by the inflationary sector and by the visible sector in the present Universe, i.e., SUSY=SUSY$_{\text{inf}}$×SUSY$_{\text{vis}}$, respectively. Both sectors interact non-gravitationally via inflaton field as well as gravitationally. In the absence of direct interactions, gravitational or non-gravitational, the $U(1)_{X_2}$-charged chiral superfields $\Phi_2$ have a two-fold enhanced SUSY$_{\text{inf}}$×SUSY$_{\text{vis}}$ Poincare symmetry, while the $U(1)_{X_1}$-charged chiral superfields $\Phi_1$ have a SUSY$_{\text{vis}}$ Poincare symmetry. However, gravitational interactions explicitly break the SUSY down to true SUSY$_{\text{inf}}$×SUSY$_{\text{vis}}$, where SUSY$_{\text{inf}}$ corresponds to the genuine SUGRA symmetry, while the orthogonal SUSY$_{\text{vis}}$ is approximate global symmetry. In each sector, spontaneous breakdown of $F$-term occurs at a scale $F_i$ ($i=$ inf, vis) independently, producing a corresponding goldstino. Hence, in the presence of SUGRA, the SUSY$_{\text{inf}}$ is gauged and thus its corresponding goldstino is eaten by the gravitino via super-Higgs mechanism, leaving behind the approximate global symmetry SUSY$_{\text{vis}}$ which is explicitly broken by SUGRA and thus its corresponding the uneaten goldstino as a physical degree of freedom. During inflation and the beginning of reheating (preheating) the SUSY$_{\text{inf}}$ is mainly broken by the inflaton implying the goldstino produced is mainly inflatino; the gravitino produced non-thermally is effectively massless as long as the Hubble parameter is larger than the gravitino mass, $H > m_{3/2}$. However, this correspondence does not necessarily hold at late times, since the SUSY$_{\text{vis}}$ is broken by other fields in the true vacuum implying that the corresponding uneaten goldstino gives masses mainly to all the supersymmetric SM superpartners in the visible sector; gravitinos are produced non-thermally by the decay of the inflaton.

In this paper, in order to provide a lucid explanation for inflation we present a realistic moduli stabilization, which is essential for the flavored PQ axions to be realized at low energy scale [4]. Such moduli stabilization has moduli backreaction effects on the inflationary potential, which provides a lucid explanation for the cosmological inflation at high energy scale. The inflaton as a source of inflation is displaced from its minimum and whose slow-roll dynamics leads to an accelerated expansion of the early universe. During inflation the universe experiences an approximately de Sitter (dS) phase with the inflationary Hubble constant $H_I \sim 2 \times 10^{10}$ GeV. Quantum fluctuations during this phase can lead to observable signatures in cosmic microwave background (CMB) radiation temperature fluctuation, as the form of density perturbation, in several ways [12], when they become much bigger than the Hubble radius long after inflation has been completed. When interpreted in this way,
inflation provides a causal mechanism to explain the observed nearly-scale invariant CMB spectrum. In the present inflation model which provides intriguing links to ultraviolet (UV)-complete theories like string theory, the PQ scalar fields $\Psi(\bar{\Psi})$ play a role of the waterfall fields, that is, the PQ phase transition takes place during inflation such that the PQ scale $\mu_{\Psi}(t_f)$ is fixed by the amplitude of the primordial curvature perturbation and turns out to be around $0.7 \times 10^{16}$ GeV which is smaller than string moduli mass $m_T$. In the present model, since SUSY breaking is transmitted by gravity, all scalar fields acquire an effective mass of the order of the expansion rate during inflation. So we expect that the inflaton acquires a mass of order the Hubble constant $H_I$, and which in turn indicates that the soft SUSY breaking mass (the inflaton mass $m_{\Psi_0}$) during inflation strongly depends on the scale of waterfall fields by VEVs $v_{\Psi}(t_f)$ and/or $v_{\bar{\Psi}}(t_f)$ induced by tachyonic SUSY breaking masses. Thus such moduli stabilization with the moduli backreaction effects on the inflationary potential leads to the energy scale of inflation with the inflaton mass, $m_{\Psi_0} = \sqrt{3} H_I$, in a way that the power spectrum of the curvature perturbation and the scalar spectral index are to be well fitted with the latest Planck observation. Since the moduli masses are much larger than inflaton mass and accordingly are quickly stabilized to their minima at finite moduli fields values separated by high barrier from the runaway direction during inflation without perturbing the inflaton dynamics, the height of the barrier protecting metastable Minkowski space ($\approx$ dS space) are independent of the gravitino mass hence the Hubble scale $H_I$ during inflation is also independent of the gravitino mass. The moduli-induced slope partially cancels the slope of the Coleman-Weinberg potential, which flattens the inflationary trajectory and reduces the distances in field space corresponding to the $N_e \sim 50$ e-folds of inflation. The number of e-foldings depends on the amount of reheating which in turn depends on the decay rate of the inflaton and waterfall field field into relativistic particles. And the amount of reheating could be strongly correlated with both baryogenesis via leptogenesis and the yield of gravitinos. Note that after inflation the inflaton and waterfall fields get mixed almost maximally to form mass eigenstates, and the universe is dominated by both the inflaton and one of waterfall fields, while the other waterfall field gives negligible contribution to the total energy of the universe. And at the reheating epoch the inflation and waterfall field release their energy into a thermal plasma by their decays, and the universe is reheated.

Now, we suggest, interestingly enough, a new leptogenesis scenario which could natu-
rally be implemented through Affleck-Dine (AD) mechanism for baryogenesis \cite{16} and its subsequent leptonic version so-called AD leptogenesis \cite{17}. The interaction between the AD fields and inflaton generates the potential for $D$-flat direction, and which in turn produces coherent oscillations along the supersymmetric flat directions, leading to dynamics in field space that ultimately breaks CP and baryon number. Interestingly enough, the pseudo-Dirac mass splittings, suggested from the new neutrino oscillations along with high energy cosmic events \cite{4}, strongly indicate the existence of lepton-number violation which is a crucial ingredient of the present leptogenesis scenario. Then the AD fields have large VEVs along the flat directions during inflation in the early universe, in turn which together with $H_I$ provides a lower bound on the pseudo-Dirac mass splittings for the new neutrino oscillations \cite{4}. The AD fields start their coherent oscillations after the inflation ends and they create a large net lepton number asymmetry, which is finally transferred to matter particles when they eventually decay. So the resultant baryon asymmetry is constrained by the cosmological observable \textit{(i.e.} the sum of active neutrino masses) with the new high energy neutrino oscillations, and crucially depends on the reheating temperature which depends on gravitational and non-gravitational decays of the inflaton and waterfall field. Since all the particles including photons and baryons in the present universe are ultimately originated from the inflaton and waterfall field decays, it is crucial to reveal how the reheating proceeds. We show that the reheating temperature is mainly determined by the non-gravitational decay of the waterfall field, leading to a relatively low reheating temperature $T_{\text{reh}} \simeq (59 - 84)$ TeV which is consistent with that for explaining the right value of the baryon asymmetry of the universe (BAU), $Y_{\Delta B} \simeq 8 \times 10^{-11}$ \cite{13}, together with the pseudo-Dirac mass splittings responsible for new oscillations $\Delta m_i^2 \simeq O(10^{-12-13})$ eV$^2$. And we show that, even the gravitational coupling is universal, it is too weak to cause the reheating with gravity in the present model. Thus, the present model is very attractive in that with the predictive reheating temperature almost at around 70 TeV scale we can have the right value of the BAU which constrains the pseudo-Dirac mass splittings for new neutrino oscillations. In addition, since gravitinos are present in the supersymmetric model we are going to address gravitino overabundance problem. We consider direct decays of the inflaton to gravitinos competing with the thermal production in the thermal plasma formed after reheating when setting limits on the couplings governing inflaton decay, see Eq. (145). We stress that in the present model the gravitino mass $O(100)$ TeV is given by the process of supersymmet-
ric moduli stabilization in the Kallosh and Linde (KL)-type model \cite{14}, whose value gives suitable large gaugino masses \cite{14}. Since the yield of gravitinos is proportional to $T_{\text{reh}}$ and inversely proportional to $|g_{\Psi_0}|^2$, i.e., $Y_{3/2} \simeq 2.3 \times 10^{-17} (T_{\text{reh}}/70 \text{TeV}) (2.5 \times 10^{-10}/g_{\Psi_0})^2$, a lower bound on the Higgs-inflaton coupling can be derived as $2.5 \times 10^{-11} \lesssim g_{\Psi_0}$ in Eq. (145) by the BBN constraints $Y_{3/2}^{\text{BBN}}$ \cite{18} with the reheating temperature $T_{\text{reh}} \sim 70 \text{ TeV}$ for the successful leptogenesis.

The rest of this paper is organized as follows. In Sec. II we setup and review the model based on $A_4 \times U(1)_X$ symmetry in order to investigate an economic SUSY inflationary scenario and a new leptogenesis via AD mechanism. In Sec. III, first we study a realistic moduli stabilization in type IIB string theory with positive vacuum energy, which is essential for the flavored PQ axions at low energy as well as a lucid explanation for cosmological inflation at high energy scale. And we investigate how the size moduli stabilized at a scale close to $\Lambda_{\text{GUT}}$ significantly affect the dynamics of the inflation, as well as how the $X$-symmetry breaking scale during inflation is induced and its scale is fixed at $\sim 0.7 \times 10^{16}$ GeV by the amplitude of the primordial curvature perturbation and the spectral index. The main focus on Sec. IV is to show that a successful leptogenesis scenario could be naturally implemented through AD mechanism, and subsequently estimate the reheating temperature that is required to generate sufficient lepton number asymmetry following the hybrid $F$-term inflation. In turn, we show that the successful leptogenesis is closely correlated with the neutrino oscillations available on high- and low-energy neutrinos, and how the amount of reheating could be strongly correlated with the successful leptogenesis and the yield of gravitinos. And we show that it is too weak to cause the reheating with gravity in the present model even the gravitational coupling is universal. Moreover, we discuss that it is reasonable for the reheating temperature $T_{\text{reh}} \simeq (59 - 84)$ derived from the non-gravitational decays of the inflaton and waterfall field to be compatible with the required reheating temperature for the successful leptogenesis. Finally, we discuss briefly on dynamics of the waterfall fields and how the uneaten goldstino gives masses to all the supersymmetric SM superpartners. What we have done is summarized in Sec. V, and we provide our conclusions.
II. FLAVOR $A_4 \times U(1)_X$ SYMMETRY AND ITS REVIEW

Unless flavor symmetries are assumed, particle masses and mixings are generally undetermined in the SM gauge theory. In order to provide an elegant solution to the strong CP problem and describe the present SM flavor puzzles associated with the fermion mass hierarchies including their mixing patterns, the author in Ref. [1, 4] has introduced the non-Abelian discrete $A_4$ flavor symmetry [19, 20] which is mainly responsible for the peculiar mixing patterns, as well as an additional continuous symmetry $U(1)_X$ which is mainly for vacuum configuration as well as for describing mass hierarchies of leptons and quarks. Along with Ref. [1] in a way that no axionic domain wall problem occurs, which plays a crucial role in cosmology when the $X$-symmetry breaking occurs after inflation, this $U(1)$ symmetry is referred to as “flavored-PQ symmetry”. Then the symmetry group for matter fields (leptons and quarks), flavon fields and driving fields is $A_4 \times U(1)_X$, whose quantum numbers are assigned in TABLE I and II. In addition, the superpotential $W$ in the model (see, Eqs. (3) and (5)) is uniquely determined by the $U(1)_R$ symmetry, containing the usual R-parity as a subgroup: 

{$\{\text{matter fields} \rightarrow e^{i\xi/2} \text{matter fields}\}$ and $\{\text{driving fields} \rightarrow e^{i\xi} \text{driving fields}\}$},

with $W \rightarrow e^{i\xi}W$, whereas flavon and Higgs fields remain invariant under an $U(1)_R$ symmetry. As a consequence of the $R$ symmetry, the other superpotential term $\kappa_\alpha L_\alpha H_u$ and the terms violating the lepton and baryon number symmetries are not allowed.$^6$

We take the $U(1)_{X_1}$ breaking scale corresponding to the $A_4$ symmetry breaking scale and the $U(1)_{X_2}$ breaking scale to be separated by Gibbons-Hawking temperature, $T_{GH} = H_I/2\pi$, and both of which are to be much above the electroweak scale in our scenario,$^7$ that is,

$$\langle H_{u,d} \rangle \ll \langle \Phi_T \rangle, \langle \Phi_1 \rangle < \frac{H_I}{2\pi} < \langle \Phi_2 \rangle \tag{2}$$

where $H_I$ is the inflationary Hubble constant, and the fields $\Phi_1 = \{\Phi_S, \Theta\}$ and $\Phi_2 = \{\Psi, \tilde{\Psi}\}$

$^4$The flavon fields are responsible for the spontaneous breaking of the flavor symmetry, while the driving fields are introduced to break the flavor group along required vacuum expectation value (VEV) directions and to allow the flavons to get VEVs, which couple only to the flavons.

$^5$It is likely that an exact continuous global symmetry is violated by quantum gravitational effects [21]. Here the global $U(1)_X$ symmetry is a remnant of the broken $U(1)_X$ gauge symmetry which connects string theory with flavor physics [2, 4].

$^6$In addition, higher-dimensional supersymmetric operators like $Q_i Q_j Q_k L_l$ ($i, j, k$ must not all be the same) are not allowed either, and stabilizing proton.

$^7$See the symmetry breaking scales from the astrophysical constraints [3], and in more detail Sec. III D on the PQ symmetry breaking scale during inflation.
are charged under the $U(1)_X$ and $U(1)_X$ symmetries, respectively. Here we assume that the electroweak symmetry is broken by some mechanism, such as radiative effects when SUSY is broken. So we can picture two secluded SUSY breaking sectors by the inflationary sector and by the visible sector in the present Universe, \textit{i.e.}, $\text{SUSY} = \text{SUSY}_{\text{inf}} \times \text{SUSY}_{\text{vis}}$, respectively. Both sectors interact non-gravitationally via inflaton field as well as gravitationally. Since the Kahler moduli superfields putting the GS mechanism into practice are not separated from the SUSY$_{\text{inf}}$ during inflation, the $U(1)_X^2$-charged matter fields develop a large VEV during inflation by taking tachyonic SUSY breaking scalar masses $m_{\Phi^2}^2 \sim -H^2_I$, induced ‘dominantly’ by the $U(1)_X^2$-$D$-term, compared to the Hubble induced soft masses generated by the $F$-term SUSY breaking. On the other hand, in the present Universe both the $U(1)_X^i$-charged matter fields $\Phi_1$ and $\Phi_2$ develop large VEVs by the soft-SUSY breaking mass. So, in the absence of direct interactions, gravitational or otherwise, the $U(1)_X^i$-charged chiral superfields $\Phi_2$ have a two-fold enhanced $\text{SUSY}_{\text{inf}} \times \text{SUSY}_{\text{vis}}$ Poincare symmetry. However, gravitational interactions explicitly break the SUSY down to \textit{true} $\text{SUSY}_{\text{inf}} \times \text{SUSY}_{\text{vis}}$, where SUSY$_{\text{inf}}$ corresponds to the genuine SUGRA symmetry, while the orthogonal SUSY$_{\text{vis}}$ is only approximate global symmetry. In each sector, spontaneous breakdown of $F$-term occurs at a scale $F_i$ $(i = \text{inf, vis})$ independently, producing a corresponding goldstino. In the presence of SUGRA, SUSY$_{\text{inf}}$ is gauged and thus its corresponding goldstino is eaten by the gravitino via super-Higgs mechanism, leaving behind the approximate global symmetry SUSY$_{\text{vis}}$ which is explicitly broken by SUGRA and thus its corresponding the uneaten goldstino as a physical degree of freedom. During inflation and the beginning of reheating (preheating) the SUSY$_{\text{inf}}$ is mainly broken by the inflaton implying the goldstino produced is mainly inflatino; the gravitino produced non-thermally is effectively massless as long as $H > m_{3/2}$. However, this correspondence does not necessarily hold at late times, since the SUSY$_{\text{vis}}$ is broken by other field in the true vacuum implying that the corresponding uneaten goldstino gives masses mainly to all the supersymmetric SM superpartners in the visible sector.

The $U(1)_X$ invariance forbids renormalizable Yukawa couplings for the light families, but would allow them through effective nonrenormalizable couplings suppressed by $(F/\Lambda)^n$ with $n$ being positive integers $[22, 23]$. Even with all couplings being of order unity, hierarchical masses for different flavors can be naturally realized. The flavon field $\mathcal{F}$ is a scalar field which acquires a vacuum expectation value (VEV) and breaks spontaneously the flavored-PQ symmetry $U(1)_X$. Here $\Lambda$, above which there exists unknown physics, is the scale of flavor
dynamics, and is associated with heavy states which are integrated out. The effective theory below $\Lambda$ is rather simple, while the full theory will have many heavy states. We assume that the cut-off scale $\Lambda$ in the superpotential (5) is a scale where the complex structure and axio-dilaton moduli are stabilized through fluxes. So, in our framework, the hierarchy $\langle H_{u,d} \rangle = v_{u,d} \ll \Lambda$ is maintained, and below the scale $\Lambda$ the higher dimensional operators express the effects from the unknown physics. Since the Yukawa couplings are eventually responsible for the fermion masses they must be related in a very simple way at a large scale in order for intermediate scale physics to produce all the interesting structure in the fermion mass matrices. On the other hand, cosmological observables, such as power spectrum of curvature perturbations and spectral index, do not generically receive significant contributions from possible higher-dimensional non-renormalizable operators, as these are suppressed by the Planck mass $M_P$. So inflationary dynamics is mainly governed by a few renormalizable operators which might have observable implications for laboratory experiments.

A. Superpotential dependent on driving fields

To impose the $A_4$ flavor symmetry on our model properly, apart from the usual two Higgs doublets $H_{u,d}$ responsible for electroweak symmetry breaking, which are invariant under $A_4$ (i.e. flavor singlets $1$ with no $T$-flavor), the scalar sector is extended by introducing two types of new scalar multiplets, flavon fields $\Phi_T, \Phi_S, \Theta, \tilde{\Theta}, \Psi, \tilde{\Psi}$ that are $SU(2)$-singlets and driving fields $\Phi_T^0, \Phi_S^0, \Theta_0, \Psi_0$ that are associated to a nontrivial scalar potential in the symmetry breaking sector: we take the flavon fields $\Phi_T, \Phi_S$ to be $A_4$ triplets, and $\Theta, \tilde{\Theta}, \Psi, \tilde{\Psi}$ to be $A_4$ singlets with no $T$-flavor (1 representation), respectively, that are $SU(2)$-singlets, and driving fields $\Phi_T^0, \Phi_S^0$ to be $A_4$ triplets and $\Theta_0, \Psi_0$ to be an $A_4$ singlet. Under $A_4 \times U(1)_X \times U(1)_R$, the driving, flavon, and Higgs fields are assigned as in TABLE I. The superpotential dependent on the driving fields, which is invariant under $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X \times A_4$, is given at leading order by

$$W_v = \Phi_T^0 (\bar{\mu} \Phi_T + \tilde{g} \Phi_T \Phi_T) + \Phi_S^0 \left( g_1 \Phi_S \Phi_S + g_2 \tilde{\Theta} \Phi_S \right) + \Theta_0 \left( g_3 \phi_S \phi_S + g_4 \Theta \Theta + g_5 \tilde{\Theta} \tilde{\Theta} + g_6 \tilde{\Theta} \tilde{\Theta} \right) + g_7 \Psi_0 \left( \bar{\Psi} \Psi - \mu_\Psi^2 \right), \quad (3)$$

where the fields $\Psi$ and $\tilde{\Psi}$ charged by $-q, q$, respectively, are ensured by the $U(1)_X$ symmetry extended to a complex $U(1)$ due to the holomorphy of the superpotential. SUSY hybrid infla-
TABLE I: Representations of the driving, flavon, and Higgs fields under $A_4 \times U(1)_X$. Here $U(1)_X \equiv U(1)_{X_1} \times U(1)_{X_2}$ symmetries which are generated by the charges $X_1 = -2p$ and $X_2 = -q$.

| Field | $\Phi^T_0$ | $\Phi^S_0$ | $\Theta_0$ | $\Psi_0$ | $\Phi_S$ | $\Phi_T$ | $\Theta$ | $\tilde{\Theta}$ | $\Psi$ | $\tilde{\Psi}$ | $H_d$ | $H_u$ |
|-------|-------------|-------------|------------|----------|-----------|-----------|----------|--------------|--------|--------------|-------|-------|
| $A_4$ | 3           | 3           | 1          | 1        | 3         | 3         | 1        | 1            | 1      | 1            | 1     | 1     |
| $U(1)_X$ | 0           | 4$p$        | 4$p$       | 0        | $-2p$     | 0         | $-2p$    | $-2p$       | $-q$   | $q$          | 0     | 0     |
| $U(1)_R$ | 2           | 2           | 2          | 2        | 0         | 0         | 0        | 0            | 0      | 0            | 0     | 0     |

The PQ scale $\mu \equiv \sqrt{v_\Psi v_\tilde{\Psi}}/2$ corresponding to the scale of the spontaneous symmetry breaking scale sets the energy scale of inflation during inflation, see Eq. (77), as well as the energy scale at present in Ref. [4]. Since there is no fundamental distinction between the singlets $\Theta$ and $\tilde{\Theta}$ as indicated in TABLE II, we are free to define $\tilde{\Theta}$ as the combination that couples to $\Phi^S_0 \Phi_S$ in the superpotential $W_v$ [19]. Due to the assignment of quantum numbers under $A_4 \times U(1)_X \times U(1)_R$ the usual superpotential term $\mu H_u H_d$ is not allowed, while the following operators driven by $\Psi_0$ and $\Phi^T_0$ are allowed by

$$g_{\Psi_0} \Psi_0 H_u H_d + \frac{g_T}{\Lambda} (\Phi^T_0 \Phi_T)_1 H_u H_d,$$

which is to promote the $\mu$-term $\mu_{\text{eff}} \equiv g_{\Psi_0} \langle \Psi_0 \rangle + g_T \langle \Phi^T_0 \rangle v_T/(\sqrt{2}\Lambda)$ of the order of $m_S$ and/or $m_S v_T/\Lambda$ (here $\langle \Psi_0 \rangle$ and $\langle \Phi^T_0 \rangle$: the VEVs of the scalar components of the driving fields, $m_S$: soft SUSY breaking mass). The inflaton field $\Psi_0$ can predominantly decay into Higgses (and Higgsinos) through the first term after inflation $^9$, which is important for inflation and leptogenesis (see Sec. [IV]), while the second term is crucial for relating the sizable $\mu$-term with the low energy flavor physics. Here the supersymmetry of the model is assumed broken by all possible holomorphic soft terms which are invariant under $A_4 \times U(1)_X \times U(1)_R$ symmetry, where the soft breaking terms are already present at the scale relevant to flavor dynamics.

$^8$ See the details in Sec. [III].

$^9$ As will be discussed in Sec. [IV] the size of the renormalizable superpotential coupling of the inflaton to particles of the SM is severely restricted by the reheating temperature, $T_{\text{reh}}$, and in turn a successful leptogenesis. Consequently, we have $\mu_{\text{eff}} \approx g_T \langle \Phi^T_0 \rangle v_T/\Lambda$ as in Ref. [4], which can describe the correct CKM mixing matrix with $v_T/\Lambda \sim 0.04 \approx \lambda^2/\sqrt{2}$.
And it is evident that at leading order the scalar supersymmetric $W(\Phi_T\Phi_S)$ terms are absent due to different $U(1)_X$ quantum number, which is crucial for relevant vacuum alignments in the model to reproduce the present large leptonic mixing and small quark mixing \cite{ref1, ref4}. It is interesting that at the leading order the electroweak scale does not mix with the potentially large scales $\langle \Phi_S \rangle, \langle \Phi_T \rangle, \langle \Theta \rangle$ and $\langle \tilde{\Psi} \rangle$.

**B. Review of Lepton sector**

Before discussing a leptogenesis scenario, we briefly review the lepton part addressed in Ref. \cite{ref4}. Under $A_4 \times U(1)_X$, the matter fields are assigned as in TABLE II. Because of the chiral structure of weak interactions, bare fermion masses are not allowed in the SM. Fermion masses arise through Yukawa interactions\footnote{Since the right-handed neutrinos $N^c$ ($S^c$) having a mass scale much above (below) the weak interaction scale are complete singlets of the SM gauge symmetry, they can possess bare SM invariant mass terms. However, the flavored-PQ symmetry $U(1)_{X}$ guarantees the absence of bare mass terms $M N^c N^c$ and $\mu_s S^c S^c$.}. Recalling that $v_{\Psi}/\Lambda = v_{\tilde{\Psi}}/\Lambda \equiv \lambda$ in Eq. (A9) is used when the $U(1)_X$ quantum numbers of the SM charged fermions are assigned.

| Field | $L_e, L_\mu, L_\tau$ | $e^c, \mu^c, \tau^c$ | $N^c$ | $S^c_e, S^c_\mu, S^c_\tau$ |
|-------|----------------------|------------------------|-------|------------------------|
| $A_4$ | $1, 1', 1''$         | $1, 1'', 1'$          | 3     | 1, 1'', 1'             |
| $U(1)_X$ | $-9q - p$           | $p + 15q, p + 13q, p + 11q$ | $p$   | $p + 25q$             |
| $U(1)_R$ | $1$                 | $1$                   | $1$   | $1$                   |

In the lepton sector, based on the field contents in TABLE II and III the superpotential for Yukawa interactions under $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_X \times A_4$ reads at leading order

$$W_\ell \nu = y_1^s L_e S^c_e H_u + y_2^s L_\mu S^c_\mu H_u + y_3^s L_\tau S^c_\tau H_u$$

$$+ \frac{1}{2} \left( y_1^{ss} S^c_e S^c_e + y_2^{ss} S^c_\mu S^c_\mu + y_3^{ss} S^c_\tau S^c_\tau \right) \tilde{\Psi}$$

$$+ y_1^l L_e (N^c_1 \Phi_T) \frac{H_u}{\Lambda} + y_2^l L_\mu (N^c_1 \Phi_T) \frac{H_u}{\Lambda} + y_3^l L_\tau (N^c_1 \Phi_T) \frac{H_u}{\Lambda}$$

$$+ \frac{1}{2} \left( y_4^e \Theta + y_6^e \Theta \right) (N^c_1 \Phi_S) \frac{H_u}{\Lambda}$$

$$+ y_e L_e e^c H_d + y_\mu L_\mu \mu^c H_d + y_\tau L_\tau \tau^c H_d.$$  (5)
In the above leptonic Yukawa superpotential\textsuperscript{11}, $W_{\text{\ell\nu}}$, charged lepton sector has three independent Yukawa terms at the leading: apart from the Yukawa couplings, each term does not involve flavon fields. The left-handed lepton doublets $L_e, L_\mu, L_\tau$ transform as $1, 1'$, and $1''$, respectively; the right-handed leptons $e^c \sim 1, \mu^c \sim 1''$, and $\tau^c \sim 1'$. In neutrino sector, two right-handed Majorana neutrinos $S$ and $N$ are introduced, in a way that no axionic domain-wall problem occurs and the mixed $U(1)_X$-[gravity]\textsuperscript{2} anomaly is free\textsuperscript{4}, to make light neutrinos pseudo-Dirac particles and to realize tribimaximal (TBM) pattern\textsuperscript{12}, respectively; $S^c_e, S^c_\mu, S^c_\tau$ and $N$ transform as $1, 1'', 1'$, and $3$ under $A_4$ symmetry, respectively. They compose two Majorana mass terms; one is associated with an $A_4$ singlet $\bar{\Psi}$, while the other one is associated with an $A_4$ singlet $\Theta$ and an $A_4$ triplet $\Phi_S$, in which all flavon fields associated with the Majorana mass terms are the SM gauge singlets. The two different assignments of $A_4$ quantum number to Majorana neutrinos guarantee the absence of the Yukawa terms $S^c N^c \times$ flavon fields. Correspondingly, two Dirac neutrino mass terms are generated; one is associated with $S^c$, and the other is $N^c$. Imposing the continuous global $U(1)_X$ symmetry in TABLE\textsuperscript{II} explains the absence of the Yukawa terms $LN^c\Phi_S$ and $N^cN^c\Phi_T$ as well as does not allow the interchange between $\Phi_T$ and $\Phi_S$, both of which transform differently under $U(1)_X$, so that bi-large $\theta_{12}, \theta_{23}$ mixings with a non-zero $\theta_{13}$ mixing for the leptonic mixing matrix could be obtained after seesawing\textsuperscript{25} (as will be shown later, the effective mass matrix achieved by seesawing contributes to TBM mixing pattern and pseudo-Dirac mass splittings, except for active neutrino masses. Such pseudo-Dirac mass splittings are responsible for very long wavelength, which in turn connect to an axion decay constant\textsuperscript{4}, see Eqs. (16) and (91).

Since the $U(1)_X$ quantum numbers are assigned appropriately to the matter fields content as in TABLE\textsuperscript{II} it is expected that the SM gauge singlet flavon fields derives higher-dimensional operators, which are eventually visualized into the Yukawa couplings of charged leptons as a function of flavon field $\Psi$, \textit{i.e.}, $y_{e,\mu,\tau} = y_{e,\mu,\tau}(\Psi)$:

$$y_e = \hat{y}_e \left( \frac{\Psi}{\Lambda} \right)^6, \quad y_\mu = \hat{y}_\mu \left( \frac{\Psi}{\Lambda} \right)^4, \quad y_\tau = \hat{y}_\tau \left( \frac{\Psi}{\Lambda} \right)^2. \quad (6)$$

On the other hand, the neutrino Yukawa couplings in terms of the flavons $\Psi(\tilde{\Psi})$ and $\Theta$ are

\textsuperscript{11} Direct NG (Nambu-Goldstone) mode couplings to ordinary leptons through Yukawa interactions are discussed in Ref. \textsuperscript{4}.

\textsuperscript{12} See Eq. (89) the exact TBM mixing \textsuperscript{24}.
given as
\begin{align*}
y_{i}^{s} &= \hat{y}_{i}^{s} \left( \frac{\Psi}{\Lambda} \right)^{16}, \\
y_{i}^{ss} &= \hat{y}_{i}^{ss} \left( \frac{\Psi}{\Lambda} \right)^{51} \Theta, \\
y_{i}^{\nu} &= \hat{y}_{i}^{\nu} \left( \frac{\tilde{\Psi}}{\Lambda} \right)^{9}, \\
\hat{y}_{\Theta} &\approx \hat{y}_{\tilde{\Theta}} \approx \hat{y}_{R} \approx \mathcal{O}(1). \quad (7)
\end{align*}

Here the hat Yukawa couplings $\hat{y}$ are complex numbers and of order unity, i.e. $1/\sqrt{10} \lesssim |\hat{y}| \lesssim \sqrt{10}$. We note that the flavon fields $\Phi_{S}$ and $\Phi_{T}$ derive dimension-5 operators in the Dirac neutrino sector, apart from the Yukawa couplings, while the flavon fields $\Psi$ and $\tilde{\Psi}$ derives higher dimensional operators through the Yukawa couplings with the $U(1)_{X}$ flavor symmetry responsible for the hierarchical charged lepton masses as shown by Eqs. (6) and (7).

C. A direct link between Low and High energy Neutrinos

Once the scalar fields $\Phi_{S}, \Theta, \tilde{\Theta}, \Psi$ and $\tilde{\Psi}$ get VEVs, the flavor symmetry $U(1)_{X} \times A_{4}$ is spontaneously broken And at energies below the electroweak scale, all leptons obtain masses. Since the masses of Majorana neutrino $N_{R}$ are much larger than those of Dirac and light Majorana ones, after integrating out the heavy Majorana neutrinos, we obtain the following effective Lagrangian for neutrinos

\begin{align*}
-L_{W}^{\nu} &\simeq \frac{1}{2} \left( \nu_{L}^{c} S_{R} \right) M_{\nu} \left( \nu_{L}^{c} S_{R} \right) + \frac{1}{2} N_{R} M_{R} N_{R}^{c} + \ell_{R} M_{\ell} \ell_{L} + \frac{g}{\sqrt{2}} W_{\mu} \ell_{L} \gamma^{\mu} \nu_{L} + h.c. \quad (8)
\end{align*}

with

\begin{align*}
M_{\nu} &= \begin{pmatrix}
-m_{D}^{T} M_{R}^{-1} m_{D} & m_{D}^{T} S_{D} \\
M_{S} & m_{S}
\end{pmatrix}, \quad (9)
\end{align*}

Here the Majorana neutrino mass terms $M_{\nu\nu}$ and $M_{S}$, and the Dirac mass term $m_{DS}$ are given (see Appendix[A1]) by

\begin{align*}
M_{\nu\nu} &= U_{L}^{\star} \hat{M}_{\nu\nu} U_{L}^{\dagger} = -m_{D}^{T} M_{R}^{-1} m_{D}, \quad M_{S} = U_{R}^{\star} \hat{M}_{S} U_{R}^{\dagger}, \quad m_{DS} = U_{R}^{\star} \hat{M} U_{L}^{\dagger}, \quad (10)
\end{align*}

where “hat” matrices represent diagonal mass matrices of their corresponding leptons, and $U_{L(R)}$ are their diagonal left(right)-mixing matrix. Since $m_{DS}$ is dominant over $M_{\nu\nu}$ and $M_{S}$ due to Eqs. (A15-A18), the low energy effective light neutrinos become pseudo-Dirac particles. Keeping terms up to the first order in heavy Majorana mass, in the mass eigenstates
\[ W^T \nu M_{\nu}^\dagger W_{\nu}^* = \begin{pmatrix} |\hat{M}| |\delta| & 0 \\ 0 & |\hat{M}| - |\delta| \end{pmatrix} \equiv \text{diag}(m_{\nu_1}^2, m_{\nu_2}^2, m_{\nu_3}^2, m_{s_1}^2, m_{s_2}^2, m_{s_3}^2), \]

where \( \hat{M} \equiv \text{diag}(m_1, m_2, m_3) \). Here the pseudo-Dirac mass splitting, \( \delta \), can be given by

\[ \delta \equiv \hat{M}_{\nu\nu} + \hat{M}_{s}^\dagger \simeq \hat{M}_{\nu\nu}, \]

where the second equality is due to \(|\hat{M}_{\nu\nu}| \gg |\hat{M}_{s}|\). As is well-known, because of the observed hierarchy \(|\Delta m_{\text{Atm}}^2| = |m_{\nu_3}^2 - (m_{\nu_1}^2 + m_{\nu_2}^2)/2| \gg \Delta m_{\text{Sol}}^2 \equiv m_{\nu_2}^2 - m_{\nu_1}^2 > 0\), and the requirement of a Mikheyev-Smirnov-Wolfenstein resonance for solar neutrinos, there are two possible neutrino mass spectra: (i) the normal mass ordering (NO) \( m_{\nu_1}^2 < m_{\nu_2}^2 < m_{\nu_3}^2, m_{s_1}^2 < m_{s_2}^2 < m_{s_3}^2 \), and (ii) the inverted mass ordering (IO) \( m_{\nu_3}^2 < m_{\nu_1}^2 < m_{\nu_2}^2, m_{s_3}^2 < m_{s_1}^2 < m_{s_2}^2 \), in which the mass-squared differences in the \( k \)-th pair \( \Delta m_k^2 \equiv m_{\nu_k}^2 - m_{s_k}^2 \) are small enough that the same mass ordering applies for the both eigenmasses, that is,

\[ \Delta m_k^2 = 2m_k|\delta_k| \ll m_{\nu_k}^2 \]  

for all \( k = 1, 2, 3 \). It is anticipated that \( \Delta m_k^2 \ll \Delta m_{\text{Sol}}^2, |\Delta m_{\text{Atm}}^2| \), otherwise the effects of the pseudo-Dirac neutrinos should have been detected. But in the limit that \( \Delta m_k^2 = 0 \), it is hard to discern the pseudo-Dirac nature of neutrinos. The pseudo-Dirac mass splittings could be limited by several constraints, that is, the active neutrino mass hierarchy, the BBN constraints on the effective number of species of light particles during nucleosynthesis, the solar neutrino oscillations: we roughly estimate a bound for the tiny mass splittings

\[ 6 \times 10^{-16} \lesssim \Delta m_k^2/eV^2 \lesssim 1.8 \times 10^{-12}, \]

where the upper bound comes form the solar neutrino oscillations [26], and the lower bound comes from the inflationary (Sec. III) and leptogenesis (Sec. IV) scenarios by assuming \( m_{\nu_i} \sim 0.01 \text{ eV}. \)

\[ m_{\nu_1} \sim 0.01 \text{ eV}. \]
Letting the mass of active neutrino $m_{\nu_k} = m_k$, then the sum of light neutrino masses given by

$$\sum_k m_{\nu_k} = \frac{1}{2} \left( \frac{\Delta m^2_1}{\delta_1} + \frac{\Delta m^2_2}{\delta_2} + \frac{\Delta m^2_3}{\delta_3} \right)$$

(15)
is bounded by $0.06 \lesssim \sum_i m_{\nu_i}/\text{eV} < 0.194$; the lower limit is extracted from the neutrino oscillation measurements, and the upper limit\(^1\) is given by Planck Collaboration\([10]\) which is subject to the cosmological bounds $\sum_i m_{\nu_i} < 0.194 \text{ eV}$ at 95% CL (the CMB temperature and polarization power spectrum from Planck 2015 in combination with the BAO data, assuming a standard $\Lambda$CDM cosmological model).

The masses of the active neutrinos, $m_{\nu_i}$, are determined in a completely independent way that the neutrino mixing angles are obtained through the seesaw formula in Eq. (16) (see also Eq. [87]), but they are tied to each other by the tiny mass splittings in Eq. (11). From the basis rotations of weak to mass eigenstates, one of Majorana neutrino mass matrices, $M_{\nu\nu} = -m_D^T M_R^{-1} m_D$ in Eq. (9), can be diagonalized as

$$\hat{M}_{\nu\nu} = U^T \hat{M}_{\nu\nu} U_L = -U^T m_D^T M_R^{-1} m_D U_L \approx \delta .$$

(16)
The three neutrino active states emitted by weak interactions are described in terms of the six mass eigenstates as

$$\nu_\alpha = U_{ak} \xi_k \quad \text{with} \quad \xi_k = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & i \end{pmatrix} \begin{pmatrix} \nu_k \\ S^c_k \end{pmatrix} ,$$

(17)
in which the redefinition of the fields $\nu_k \rightarrow e^{i \pi/4} \nu_k$ and $S^c_k \rightarrow e^{-i \pi/4} S^c_k$ is used. Since the active neutrinos are massive and mixed, the weak eigenstates $\nu_\alpha$ (with flavor $\alpha = e, \mu, \tau$) produced in a weak gauge interaction are linear combinations of the mass eigenstates with definite masses, given by $|\nu_\alpha\rangle = \sum_k N_{\nu} W^*_k W_{ak} |\xi_k\rangle$ where $W_{ak}$ are the matrix elements of the explicit form of the matrix $W_L$. Note that even the number $N_{\nu}$ of massive neutrinos can be larger than three, in the present model the light fermions $S_\alpha$ do not take part in the standard weak interaction and thus are not excluded by LEP results according to which the

\(^1\) Massive neutrinos could leave distinct signatures on the CMB and large-scale structure at different epochs of the universe’s evolution\([27]\). To a large extent, these signatures could be extracted from the available cosmological observations, from which the total neutrino mass could be constrained.
The number of active neutrinos are coupled with the $W^\pm$ and $Z$ bosons is $N_\nu = 2.984 \pm 0.008$ \cite{28}. The charged gauge interaction in Eq. (9) for the neutrino flavor production and detection is written in the charged lepton basis as
\[ -\mathcal{L}_{\text{c.c.}} = \frac{g}{\sqrt{2}} W^-_\mu \bar{\nu}_\alpha \frac{1 + \gamma_5 \gamma^\mu}{2} U_{\nu k} \xi_k + \text{h.c.}, \tag{18} \]
where $g$ is the SU(2) coupling constant, and $U \equiv U_L$ is the $3 \times 3$ Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix $U_{\text{PMNS}}$. Thus in the mass eigenstate basis the PMNS leptonic mixing matrix \cite{29} at low energies is visualized in the charged weak interaction, which is expressed in terms of three mixing angles, $\theta_{12}, \theta_{13}, \theta_{23}$, and three CP-odd phases (one $\delta_{CP}$ for the Dirac neutrino and two $\varphi_{1,2}$ for the Majorana neutrino) as
\[ U_{\text{PMNS}} = \begin{pmatrix} c_{13} c_{12} & c_{13} s_{12} & s_{13} e^{-i \delta_{CP}} \\ -c_{23} s_{12} - s_{23} c_{12} s_{13} e^{i \delta_{CP}} & c_{23} c_{12} - s_{23} s_{12} s_{13} e^{i \delta_{CP}} & s_{23} c_{13} \\ s_{23} s_{12} - c_{23} c_{12} s_{13} e^{i \delta_{CP}} & -s_{23} c_{12} - c_{23} s_{12} s_{13} e^{i \delta_{CP}} & c_{23} c_{13} \end{pmatrix} P_\nu, \tag{19} \]
where $s_{ij} \equiv \sin \theta_{ij}$, $c_{ij} \equiv \cos \theta_{ij}$ and $P_\nu$ is a diagonal phase matrix what is that particles are Majorana ones. After the relatively large reactor angle $\theta_{13}$ measured in Daya Bay \cite{30} and RENO \cite{31} including Double Chooz, T2K and MINOS experiments \cite{32}, the recent analysis based on global fits \cite{33, 34} of the neutrino oscillations enters into a new phase of precise determination of mixing angles and mass squared differences, indicating that the exact TBM \cite{24} for three flavors should be corrected in the lepton sector. As shown in Ref. [4] by numerical analysis based on the present model, together with well-fitted solar $\theta_{12}$ and reactor $\theta_{13}$ mixing angles the values of atmospheric ($\theta_{23}$) and Dirac CP phase ($\delta_{CP}$) have a remarkable coincidence with the most recent data by the NO\nuA \cite{5} and/or T2K \cite{6} experiments.

The pseudo-Dirac mass splittings in Eq. (13) will manifest themselves through very long wavelength oscillations characterized by the $\Delta m^2_{\nu_k}$. Such new oscillation lengths far beyond the earth-sun distance will be provided by astrophysical neutrinos, which fly galactic and extra galactic distances with very high energy neutrinos. Once very tiny mass splittings are determined by performing astronomical-scale baseline experiments to uncover the oscillation effects of very tiny mass splitting $\Delta m^2_{\nu_k}$, the active neutrino mixing parameters ($\theta_{12}, \theta_{23}, \delta_{CP}$ and $m_{\nu_1}, m_{\nu_2}, m_{\nu_3}$) are predicted in the model due to Eqs. (13) and (16). Thus we can possibly connect the pseudo-Dirac neutrino oscillations with the low energy
neutrino properties as well as a successful leptogenesis in Eq. (107). With the help of the mixing matrix Eq. (A11), the flavor conversion probability between the active neutrinos follows from the time evolution of the state $\xi_k$ as,

$$P_{\nu_\alpha \rightarrow \nu_\beta} (W_\nu, L, E) = \left| \left( W^*_{\nu} e^{-i \hat{M}_\nu^2 L W_\nu^T} \right)_{\alpha \beta} \right|^2 = \frac{1}{4} \sum_{k=1}^{3} |U_{\beta k}|^2 \left| e^{i \frac{m_{k}^2 L}{2}} + e^{i \frac{m_{k}^2 L}{2}} \right| U^*_{\alpha k},$$

in which $L =$ flight length, $E =$ neutrino energy, and $\hat{M}_\nu \equiv W_\nu^T M_\nu W_\nu$, see Eq. (A12). For the baseline, $4\pi E/\Delta m^2_{\text{Sol,Atm}} \ll L$, the probability of neutrino flavor conversion reads

$$P_{\nu_\alpha \rightarrow \nu_\beta} = \sum_{k=1}^{3} |U_{\alpha k}|^2 |U_{\beta k}|^2 \cos^2 \left( \frac{\Delta m^2_k L}{4E} \right),$$

where the oscillatory terms involving the atmospheric and solar mass-squared differences are averaged out over these long distances. See the related experiments.

III. INFLATION

The inflation that inflated the observable universe beyond the Hubble radius, and could have produced the seed inhomogeneities needed for galaxy formation and the anisotropies observed by COBE, must occur at an energy scale $V^{1/4} \leq 4 \times 10^{16}$ GeV, well below the Planck scale. At this relatively low energies, superstrings are described by an effective $\mathcal{N} = 1$ supergravity theory. We work in the context of supersymmetric moduli stabilization, in the sense that all moduli masses are independent of the gravitino mass and large compared to the scale of any other dynamics in the effective theory, e.g., the scale of inflation, $m_{T_i} > H_I$ where $H_I = \sqrt{V/3M_P^2}$ is the Hubble scale during inflation. As in Refs. 2, 4, the size moduli with positive masses have been stabilized, while leaving two axions massless and one axion massive, i.e. $m_T \sim m_{\phi_{ST}} \gg m_{3/2}$. So we will discuss that such moduli stabilization has moduli backreaction effects on the inflationary potential, in particular, the spectral index of inflaton fluctuations, which provides a lucid explanation for the cosmological inflation at high energy scale. We are going to see how the size moduli stabilized at a scale close to $\Lambda_{\text{GUT}}$ significantly affect the dynamics of the inflation, as well as how the $X$-symmetry breaking scale during inflation is induced and its scale is fixed at $\sim 0.7 \times 10^{16}$ GeV, close to $\Lambda_{\text{GUT}}$, by the amplitude of the primordial curvature perturbation.
The model addressed in Refs. [1, 2] naturally causes a hybrid inflation\(^{15}\), in which the QCD axion and the lightest neutralino charged under a stabilizing symmetry could become components of dark matter. We work in a SUGRA framework based on type IIB string theory, and assume that the dilaton and complex structure moduli are fixed at semi-classical level by turning on background fluxes\(^{48}\). Below the scale where the complex structure and the axio-dilaton moduli are stabilized through fluxes as in Refs. [49, 50], in Einstein frame the SUGRA scalar potential is

\[
V = e^G M_P^4 \left(\sum G^a G_a - 3\right) + \frac{1}{2} f_{ij} D^i D^j, \quad (22)
\]

where \(G^a = G^{a\bar{\beta}} G_{\bar{\beta}}\) with \(G^{a\bar{\beta}} = M_P^2 K^{a\bar{\beta}}\), \(M_P = (8\pi G_N)^{-1/2} = 2.436 \times 10^{18}\) GeV is the reduced Planck mass with the Newton’s gravitational constant \(G_N\), and \(f_{ij}\) is the gauge kinetic function. And the \(F\)-term potential is given by the first term in the right hand side of Eq. (22); the \(D\)-term, the second term in the right hand side of Eq. (22), is quartic in the charged fields under the gauge group, and in the model it is flat along the inflationary trajectory so that it can be ignored during inflation\(^{16}\). The generalized Kahler potential, \(G\), is given by

\[
G = \frac{K}{M_P^2} + \ln \frac{|W|^2}{M_P^6}. \quad (23)
\]

Here the low-energy Kahler potential \(K\) and superpotential \(W\) for moduli and matter superfields, invariant under \(U(1)_X\) gauged symmetry, are given in type IIB string theory by\(^2\)

\[
K = -M_P^2 \ln \left\{ (T + \bar{T}) \prod_{i=1}^2 \left( T_i + \bar{T}_i - \frac{\delta_{GS}^i}{16\pi^2} V_{X_i} \right) \right\} + \tilde{K} + ... \quad (24)
\]

with \(\tilde{K} = \sum_{i=1}^2 Z_i \Phi_i e^{-X_i} V_{X_i} + \sum_k Z_k |\varphi_k|^2\),

\[
W = W_Y + W_v + W_0 + W(T), \quad (25)
\]

in which \(\Phi_1 = \{\Phi_S, \Theta, \tilde{\Theta}\}, \Phi_2 = \{\Psi, \tilde{\Psi}\}, \varphi_i = \{\Psi_0, \Phi_0^T, \Phi_T\}\), dots represent higher-order terms. \(W_0\) stands for the constant value of the flux superpotential at its minimum. Since

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\(^{15}\) Supersymmetric realizations of \(F\)-term hybrid inflation were first studied in Ref. [42]. And the hybrid inflation model in supergravity\(^{43, 44}\) and the \(F\)-term hybrid inflation in supersymmetric moduli stabilization\(^{45}\) were studied in detail. See also Refs. [46, 47].

\(^{16}\) Assuming the FI \(D\)-terms do not appear during inflation, \(\xi_0^{FI} = 0\), it is likely that \(D\) terms in the inflaton sector do not give a significant contribution to the inflaton potential. See Sec. [III.D]
the Kahler moduli do not appear in the superpotential \( W \) at leading order, they are not fixed by the fluxes. So a non-perturbative superpotential \( W(T) \) is introduced to stabilize the Kahler moduli \( T \), although \( W(T) \) in Eq. (25) is absent at tree level. The Kahler moduli in \( K \) of Eq. (24) control the overall size of the compact space,

\[
T = \rho + i\theta, \quad T_i = \rho_i + i\theta_i \quad \text{with} \quad i = 1, 2,
\]

where \( \rho(\rho_i) \) are the size moduli of the internal manifold and \( \theta(\theta_i) \) are the axionic parts. As can be seen from the Kahler potential above, the relevant fields participating in the four-dimensional Green-Schwarz (GS) mechanism are the \( \Phi_i \), the vector superfields \( V_{X_i} \) of the gauged \( U(1)_{X_i} \) which is anomalous, and the Kahler moduli \( T_i \). The matter superfields in \( K \) consist of all the scalar fields \( \Phi_i \) that are not moduli and do not have Planck sized VEVs, and the chiral matter fields \( \varphi_k \) are neutral under the \( U(1)_{X_i} \) symmetry. We take, for simplicity, the normalization factors \( Z_i = Z_k = 1 \), and the holomorphic gauge kinetic function \( f_{ij} = \delta_{ij} (1/g_{X_i}^2 + ia_{T_i}/8\pi^2) \), i.e., \( T_i = 1/g_{X_i}^2 + ia_{T_i}/8\pi^2 \) on the Kahler moduli in the 4-dimensional effective SUGRA where \( g_{X_i} \) are the four-dimensional gauge couplings of \( U(1)_{X_i} \). Actually, gaugino masses require a nontrivial dependence of the holomorphic gauge kinetic function on the Kahler moduli. This dependence is generic in most of the models of \( \mathcal{N} = 1 \) SUGRA derived from extended supergravity and string theory. And vector multiplets \( V_{X_i} \) in Eq. (24) are the \( U(1)_{X_i} \) gauge superfields including gauge bosons \( A_i^\mu \). The GS parameter \( \delta_{i GS} \) characterizes the coupling of the anomalous gauge boson to the axion.

Non-minimal SUSY hybrid inflation can be defined by the superpotential \( W_{inf} \) which is an analytic function, together with a Kahler potential \( K_{inf} \) which is a real function

\[
W \supset W_{inf} = g_7 \Psi_0 \left( \Psi \bar{\Psi} - \mu_\Psi^2 \right),
\]

\[
\tilde{K} \supset K_{inf} = |\Psi_0|^2 + |\Psi|^2 + |\bar{\Psi}|^2 + k_4 |\Psi_0|^4 + k_1 |\Psi_0|^2|\Psi|^2 + k_2 |\Psi|^4 + k_3 |\Psi_0|^6 + ... \tag{28}
\]

where \( \Psi_0 \) and \( \Psi(\bar{\Psi}) \) denote the inflaton and PQ fields, respectively. The PQ scalar fields play a role of the waterfall fields, that is, the PQ phase transition takes place during inflation such that the PQ scale \( \mu_\Psi = \mu_\Psi(t_f) \) sets the energy scale during inflation.

The kinetic terms of the Kahler moduli and scalar sectors in the flat space limit of the 4 dimensional \( \mathcal{N} = 1 \) supergravity are expressed as

\[
\mathcal{L}_{\text{kinetic}} = K_{TT} \partial_\mu T \partial^\mu T + K_{T_i \bar{T}_i} \partial_\mu T_i \partial^\mu \bar{T}_i + K_{\Phi_i \bar{\Phi}_i} \partial_\mu \Phi_i \partial^\mu \bar{\Phi}_i^\dagger \tag{29}
\]
Here we set $K_{\Phi_i\Phi_i} = 1$ for canonically normalized scalar fields. In addition to the superpotential in Eq. (25) the Kahler potential in Eq. (24) deviates from the canonical form due to the contributions of non-renormalizable terms scaled by an UV cutoff $M_P$, invariant under the both gauge and the flavor symmetries.

A. Supersymmetric Moduli Stabilization

In string theory, one must consider stabilization of the volume moduli to explain why our universe is 4-dimensional rather than 10-dimensional. Since the three moduli all appear in the Kahler potential Eq. (24), by solving the $F$-term equations the three size moduli and one axionic partner with positive masses are stabilized while leaving two axions massless through an effective superpotential $W(T)$ [2]. As will be seen later, the two massless axion directions will be gauged by the $U(1)$ gauge interactions associated with $D$-branes, and the gauged flat directions of the $F$-term potential will be removed through the Stuckelberg mechanism. The $F$-term scalar potential has the form

$$V_F = \frac{e^{K/M_P^2}}{(T + T_1)(T_1 + T_2)(T_2 + T_2)} \left\{ \sum_{I = T, T_1, T_2} K^{IJ} |D_I W|^2 - \frac{3}{M_P^2} |W|^2 + K^{i\bar{i}} |D_i W|^2 \right\}$$

for $V_{X_i} = 0$, where $K^{IJ} = 0$ for $I \neq J$, and $I, J$ stand for $T, T_i$ and $i, j$ for the bosonic components of the superfields $\Phi_i, \varphi_i$. Here the Kahler covariant derivative and Kahler metric are defined as $D_I W \equiv \partial_I W + W \partial_I K/M_P^2$ and $K^{IJ} \equiv \partial_I \partial_J K$, where $D_I \overline{W} = (\overline{D_I W})$, and $K^{I\bar{J}}$ is the inverse Kahler metric $(K^{-1})_{I\bar{J}}$. In order for the Kahler moduli $T$ and $T_i$ to be stabilized certain non-perturbative terms are introduced as an effective superpotential [2]

$$W(T) = A(\Phi_i) e^{-a(T + T_1 + T_2)} + B(\Phi_i) e^{-b(T + T_1 + T_2)} ,$$

where the coefficients $a = 2\pi$ or $2\pi/N$ and $b = 2\pi$ or $2\pi/M$ are the corrections arising from D3 instantons or gaugino condensation in a theory with a product of non-Abelian gauge groups $SU(N) \times SU(M)$. Here $A(\Phi_i)$ and $B(\Phi_i)$ are analytic functions of $\Phi_i$ transforming under $U(1)_{X_i}$ as

$$A(\Phi_i) \rightarrow A(\Phi_i) e^{i\frac{a}{16\pi^2} (\delta_{i1} \Lambda_1 + \delta_{i2} \Lambda_2)} , \quad B(\Phi_i) \rightarrow B(\Phi_i) e^{i\frac{b}{16\pi^2} (\delta_{i1} \Lambda_1 + \delta_{i2} \Lambda_2)} ,$$

and invariant under the other gauge group. Since there are two non-perturbative superpotentials of the form $W_{np} = Ae^{-aT}$, the structure of the effective scalar potential has two
non-trivial minima at different values of finite $T_i$. One corresponds to a supersymmetric Minkowski vacuum which could be done through the background fluxes $W_0$, while the other corresponds to a negative cosmological constant which gives rise to a supersymmetric Anti de Sitter (AdS) vacuum. So the height of the barrier separates the local Minkowski minimum from the global AdS minimum, and the gravitino mass vanishes at the supersymmetric Minkowski minimum. As will be seen in Eq. [(70)], inflaton mass ($m_{\Phi_0} \sim H_I$) is much smaller than the size moduli masses, and consequently the size moduli will be frozen quickly during inflation without perturbing the inflation dynamics. And it is expected that $H_I \ll \Lambda_{\text{GUT}}$ as a consequence of the enormous flatness of inflaton potential, where $\Lambda_{\text{GUT}} \simeq 2 \times 10^{16}$ GeV is the scale gauge coupling unification in the supersymmetric SM. The scalar potential of the fields $\rho$ and $\rho_i$ has local minimum at $\sigma_0, \sigma_i$ which is supersymmetric, i.e.,

$$W(\sigma_0, \sigma_i) = 0, \quad D_T W(\sigma_0, \sigma_i) = D_{T_i} W(\sigma_0, \sigma_i) = 0,$$

and Minkowski, i.e.,

$$V_F(\sigma_0, \sigma_i) = 0,$$

where $\sigma_0 = \sigma_i = \frac{1}{a-b} \ln \left( \frac{a A_0}{b B_0} \right)$. And $W_0$ is fine-tuned as

$$W_0 = -A_0 \left( \frac{a A_0}{b B_0} \right)^{-3 \frac{a}{a-b}} - B_0 \left( \frac{a A_0}{b B_0} \right)^{-3 \frac{b}{a-b}},$$

where $A_0$ and $B_0$ are constant values of order $O(1)$ of $A(\Phi_i)$ and $B(\Phi_i)$, respectively, at a set of VEVs $\langle \Phi_i \rangle$ that cancel all the $D$-terms, including the anomalous $U(1)_{X_i}$, see Ref. [4]. Here the constant $W_0$ is not analytic at the VEVs $\langle \Phi_i \rangle$, where the moduli are stabilized at the local supersymmetric Minkowski minimum. Moreover, since $W(T)$ is an effective superpotential its analyticity does not need to be guaranteed in the whole range of the $\Phi_i$ fields, and so, as will be shown later, the anomalous FI terms at the global supersymmetric AdS minimum can not be cancelled and act as uplifting potentials. Restoration of supersymmetry in the supersymmetric local Minkowski minimum implies that all particles whose mass is protected by supersymmetry are expected to light in the vicinity of the minimum. However, supersymmetry breaks down and all of these particles become heavy once one moves away from the minimum of the effective potential. This is exactly the situation required for the moduli trapping near the enhanced symmetry points [53].
The $F$-term equations $D_T W = D_{T_i} W = 0$, where we set the matter fields to zero, provide 
\[ \rho = \rho_i, \text{ and lead to} \]
\[ a A e^{-3a \rho} e^{-i a \theta_{\text{st}}} + b B e^{-3b \rho} e^{-i b \theta_{\text{st}}} + \frac{W_0 + A e^{-3a \rho} e^{-i a \theta_{\text{st}}} + B e^{-3b \rho} e^{-i b \theta_{\text{st}}}}{2 \rho} = 0 \]  
(36)
for $V_{X_i} = 0$, where $\theta_{\text{st}} \equiv \theta + \theta_1 + \theta_2$. This shows that the three size moduli $(\rho, \rho_i)$ and one axionic direction $\theta_{\text{st}}$ are fixed, while the other two axionic directions ($\theta_1 \equiv \theta - \theta_1$ and $\theta_2 \equiv \theta - \theta_2$) are independent of the above equation. So, without loss of generality, we rebase the superfields $T$ with $\theta_{\text{st}} = \text{Im}[T]$ and $T_i$ with $\theta_i \equiv \text{Im}[T_i]$ as 
\[ T = \rho + i \theta \rightarrow T = \rho + i \theta_{\text{st}}, \]
\[ T_i = \rho_i + i \theta_i \rightarrow T_i = \rho_i + i \theta_i. \]  
(37)
Then from the $F$-term scalar potential, while the gravitino mass in the supersymmetric local Minkowski minimum vanishes, the masses of the fields $\rho$, $\rho_i$, and $\theta_{\text{st}}$, respectively, are obtained as
\[ m_T^2 = \frac{1}{2} K^{T \bar{T}} \partial_T \partial_{\bar{T}} V_F \bigg|_{T = \bar{T} = \sigma_0} = \frac{3 \ln \left( \frac{a A_0}{b B_0} \right)}{M_p^4 (a - b)} \left\{ A_0 a^2 \left( \frac{a A_0}{b B_0} \right)^{-3 \frac{a^2}{a^2}} + B_0 b^2 \left( \frac{a A_0}{b B_0} \right)^{-3 \frac{b^2}{b^2}} \right\}^2, \]
\[ m_{\theta_{\text{st}}}^2 = \frac{1}{2} K^{T \bar{T}} \partial_{\theta_{\text{st}}} \partial_{\bar{\theta}_{\text{st}}} V_F \bigg|_{T = \bar{T} = \sigma_0} = \frac{3 W_0}{M_p^2} \left\{ - A_0 a^3 \left( \frac{a A_0}{b B_0} \right)^{-3 \frac{a^2}{a^2}} - B_0 b^3 \left( \frac{a A_0}{b B_0} \right)^{-3 \frac{b^2}{b^2}} \right\} + 6 \ln \left( \frac{a A_0}{b B_0} \right) \left\{ - A_0 B_0 (a - b)^2 \left( \frac{a A_0}{b B_0} \right)^{-3 \frac{a^2 + b^2}{a^2 + b^2}} \left( \frac{a^2 - b^2}{2 \ln \left( \frac{a A_0}{b B_0} \right) + a b} \right) \right\}. \]  
(38)
Here the mass squared of the size moduli fields $\rho_i$ at the minimum is given by $m_{T_i}^2 \equiv m_{\rho_i}^2 = m_{\rho_i}^2 = 3 \sigma_0 |W_{TT}(\sigma_0)|^2 / M_p^4$ where $W_{TT}|_{\text{all matter fields}=0} = \frac{1}{2} A e^{-a(T + T_1 + T_2)} + b^2 B e^{-b(T + T_1 + T_2)}$ with $W_{TT} \equiv \partial^2 W / (\partial T)^2$. With the conditions $a < 0$, $b > 0$ ($|a| < |b|$) and $A_0 > 0$, $B_0 < 0$ we obtain positive values of masses. Here $a, b$ are constants, while $A_0, B_0$ are constants in $M_p^2$ units. For a simple choice of parameters, $A_0 = 1.02$, $B_0 = -2.99$, $a = -0.022$ and
\[
\begin{align*}
\sigma_0 &\simeq 18.8 \quad \text{and} \\
m_T &\simeq 3.01 \times 10^{16} \text{GeV} \quad m_{\theta_{1}} \simeq 1.28 \times 10^{15} \text{GeV}.
\end{align*}
\]  
(39)

As will be seen in Sec. III and in TABLE III, the moduli stabilized at a scale close to \(\Lambda_{\text{GUT}}\) will significantly affect the dynamics of the inflation and well fit the cosmological observables.

**B. Supersymmetry breaking and Cosmological constant**

As discussed before, the supersymmetric local Minkowski vacuum at \(\rho = \sigma_0\) and \(\rho_i = \sigma_i\) is absolutely stable with respect to the tunneling to the vacuum with a negative cosmological constant because the Minkowski minimum is separated from a global AdS minimum by a high barrier. This vacuum state becomes metastable after uplifting of a AdS minimum to the dS minimum with \(\Lambda_c \sim 10^{-120} M_P^4\). The other supersymmetric global AdS minimum is defined by

\[
W(\tilde{\sigma}_0, \tilde{\sigma}_i) \neq 0 \quad D_T W(\tilde{\sigma}_0, \tilde{\sigma}_i) = D_T W(\tilde{\sigma}_0, \tilde{\sigma}_i) = 0,
\]

(40)
corresponding to the minimum of the potential with \(V_{\text{AdS}} < 0\). And at this AdS minimum one can set the value of the superpotential \(\Delta W \equiv \langle W \rangle_{\text{AdS}}\) by tuning \(W_0\) at values of finite \(\tilde{\sigma}_0, \tilde{\sigma}_i\). The existence of FI terms \(\xi_i^{\text{FI}}\) for the corresponding \(U(1)_X\) implies the existence of uplifting potential which makes a nearly vanishing cosmological constant and induces SUSY breaking. A small perturbation \(\Delta W\) to the superpotential \([14, 54]\) is introduced in order to determine SUSY breaking scale. Then the minimum of the potential is shifted from zero to a slightly negative value at \(\sigma_0 = \sigma_0 + \Delta \rho\) and \(\sigma_i = \sigma_i + \Delta \rho_i\) by the small constant \(\Delta W\). The resulting \(F\)-term potential has a supersymmetric AdS minimum and consequently the depth of this minimum is given by \(V_{\text{AdS}} = -3 e^{\hat{K}/M_P^2} |W|^2/M_P^2\); which can be approximated in terms of \(W(\sigma_0 + \Delta \rho, \sigma_i + \Delta \rho_i) \simeq \Delta W + \mathcal{O}(\Delta W)^2\) as

\[
V_{\text{AdS}}(\Delta W) \simeq -3 \frac{\langle \Delta W \rangle^2}{8 \sigma_0 \sigma_1 M_P^2} = -3 \frac{3}{8 M_P^2} \left( \frac{a - b}{\ln \frac{a}{b} \sigma_0} \right)^2 (\Delta W)^2.
\]

(41)

\(\text{These values ensure } m_T \sim 10^{16} \text{ GeV and } |\tilde{g}_T| = \mathcal{O}(1) \times 10^{-3} \text{ through } \tilde{g}_T^2 = \tilde{g}_T^2/(2 \sigma_0)^2 \text{ in Eq. (53), satisfying the two observables, } i.e., \text{ the scalar spectral index } n_s \text{ and the power spectrum of the curvature perturbations } \Delta_R^2(k_0) \text{ in TABLE III.}\)
At the shifted minimum SUSY is preserved, i.e. $D_TW(\sigma_0 + \delta \rho) = 0$ and $D_TW(\sigma_i + \delta \rho_i) = 0$, leading to $W_T(\sigma_0 + \delta \rho) = W_T(\sigma_0 + \delta \rho_i) \simeq 3\Delta W/2\sigma_0$. At this new minimum the displacements $\delta \rho \equiv \delta \rho_i$ are obtained as

$$\delta \rho_i \simeq \frac{3\Delta W}{2\sigma_0 W^{TT}(\sigma_0)} = \frac{3(a - b)\Delta W}{2 \ln \left(\frac{aA_0}{bB_0}\right) \left\{A_0 a^2 \left(\frac{aA_0}{bB_0}\right)^{\frac{a}{a-b}} + B_0 b^2 \left(\frac{aA_0}{bB_0}\right)^{\frac{b}{a-b}}\right\}}.$$ (42)

After adding the uplifting potentials SUSY is broken and then the gravitino in the uplifted minimum acquires a mass $m_{3/2} = \langle e^{K/M_P^2} \rangle |W|^2/M_P^4$:

$$m_{3/2} = \sqrt{\left|\frac{V_{AdS}}{3M_P^2}\right|} \simeq \frac{\Delta W}{M_P^2} \left(\frac{a - b}{2 \ln \frac{aA_0}{bB_0}}\right)^{\frac{3}{2}}.$$ (43)

The important point is that the masses $m_T$ and $m_{\theta^\ast}$ in Eq. (38), as well as the height of barrier from the runaway direction, do not have any relation to the gravitino mass, i.e., $m_T \sim m_{\theta^\ast} \gg m_{3/2}$. Thus we will consider the $F$-term hybrid inflation for $H_I \gg m_{3/2}$ in the Sec. III.

The uplifting of the AdS minimum to the dS minimum can be achieved by considering non-trivial fluxes for the gauge fields living on the D7 branes [55], which can be identified as field-dependent FI $D$-terms in the $\mathcal{N} = 1$, 4D effective action [56]. As shown in Refs. [55], uplifting of the AdS minimum induces SUSY breaking and is achieved by adding to the potential two terms $\Delta V_i \approx |V_{AdS}|\sigma_i^2/\rho^2$ if the uplifting term occurs due to a $D$-term. Similarly, we can parameterize the uplifting terms as

$$\Delta V_i = \frac{1}{2}(\xi_i^{FI})^2 g_{X_i}^2 \simeq \frac{1}{2}|V_{AdS}| \left(\frac{\sigma_i}{\rho_i}\right)^3.$$ (44)

such that the value of the potential at the new minimum become equal to the observed value of the cosmological constant. So, the anomalous FI terms can not be cancelled, and act as uplifting potential. And expanding the Kahler potential $K$ in components, the term linear in $V_{X_i}$ produces the FI factors $\xi_i^{FI} = \frac{\partial K}{\partial V_{X_i}}|_{V_{X_i}=0} \Delta \rho_i$ as

$$\xi_i^{FI} = M_P^2 \frac{\delta^G S}{8\pi^2 \sigma_i} \Delta \rho_i.$$ (45)

Here the displacements $\Delta \rho_i \equiv \rho_i - \sigma_i$ in the moduli fields are induced by the uplifting terms,

$$\Delta \rho_i \simeq \frac{3M_P^2|V_{AdS}|\sigma_i}{W^{TT}(\sigma_0) \rho_i},$$ (46)
which are achieved by \( \partial_{\rho_i} (V_F + \Delta V_i) = 0 \). Since the uplifting terms by \( \Delta \rho_i \) making the dS minimum induce SUSY breaking, all particles whose mass is protected from supersymmetry become massive. With the choice of parameters above Eq. (39), the gravitino mass corresponds to

\[
m_{3/2} \simeq 104 \text{ TeV}
\]

implies \(|\Delta W| \simeq 10^{-11} M_P^2\), and which in turn means that the FI terms proportional to \( |V_{\text{AdS}}|/m_P^2 \) are expected to be strongly suppressed.

The cosmological constant \( \Lambda_c \) has the same effect as an intrinsic energy density of the vacuum \( \rho_{\text{vac}} = \Lambda_c M_P^2 \). The dark energy density of the universe, \( \Omega_\Lambda = \rho_{\text{vac}}/\rho_c \), is expressed in terms of the critical density required to keep the universe spatially flat \( \rho_c = 3 H_0^2 M_P^2 \) where \( H_0 = 67.74 \pm 0.46 \text{ km s}^{-1} \text{ Mpc}^{-1} \) is the present Hubble expansion rate \(^{[13]}\). Using the dark energy density of the universe \( \Omega_\Lambda = 0.6911 \pm 0.0062 \) of Planck 2015 results \(^{[13]}\), then one finds the cosmological constant \( \Lambda_c \sim 7.51 \times 10^{-121} M_P^4 \). From Eqs. (41) and (44), one can fine-tune the value of the potential in its minimum, \( V_{\text{min}} \), to be equal to the observed tiny values \( 7.51 \times 10^{-121} M_P^4 \),

\[
V_{\text{min}} = |V_{\text{AdS}}| \left\{ -1 + \frac{1}{2} \left( \frac{\sigma_1}{\rho_1} \right)^3 + \frac{1}{2} \left( \frac{\sigma_2}{\rho_2} \right)^3 \right\}.
\]

The positive vacuum energy density resulting from a cosmological constant implies a negative pressure, and which drives an accelerated expansion of the universe, as observed.

**C. Moduli backreaction on inflation**

Since in general the interference between the moduli and inflaton sectors generates a correction to the inflationary potential we consider the effect of string moduli backreaction on the inflation model which is linked to SUSY breaking scale\(^{18}\). In small-field inflation, such as hybrid inflation, this produces a linear term in the inflaton at leading order as in Ref. \(^{[57]}\). This is analogous to the effect of supersymmetry breaking which induces a linear term proportional to the gravitino mass. Depending on its size such a linear term can have

\(^{18}\) There are many studies \(^{[57, 58]}\) on the moduli backreaction effect on the inflation and its link to SUSY breaking.
a significant effect on inflationary observables well fitted in CMB data, in particular, the spectral index of scalar fluctuations.

At \( T(i) = \bar{T}(i) = \sigma_0 \) due to \( W(\sigma_0) = 0 = W_T(\sigma_0) \) one can obtain

\[
V_F|_{\sigma_0} = \frac{V_{\inf}}{(2\sigma_0)^3} + \frac{3e^{K/M_P^2}}{(2\sigma_0)^3 M_P^2} |W_{\inf}|^2, \tag{49}
\]

where \( V_{\inf} \) is the inflation potential in the absence of moduli sectors

\[
V_{\inf} = e^{K/M_P^2} \left\{ K^{ij} D_j W_{\inf}^2 - \frac{3}{M_P^2} |W_{\inf}|^2 \right\}. \tag{50}
\]

Since all powers of \( 2\sigma_0 \) in Eq. (49) can be absorbed by a redefinition of \( W_{\inf} \) the potential is rescaled as \( V_F|_{\sigma_0} \to V_{\inf} + \frac{3e^{K/M_P^2}}{M_P^2} |W_{\inf}|^2 \), indicating that there is no backreaction to the inflation on the moduli sector. However, due to the effect of the inflationary large positive energy density, see Eq. (57), the minimum of the moduli are shifted by \( \delta T \) and \( \delta T_i \), and at this new shifted position the potential is minimized. The displacements are obtained by imposing \( \partial_T V|_{\sigma_0+\delta T} = 0 \) and \( \partial_T V|_{\sigma_0+\delta T_i} = 0 \), and the expression for \( \delta T \) and \( \delta T_i \) can be expanded in powers of \( H_I/m_T \),

\[
\delta T(i) \simeq \frac{W_{\inf} \sqrt{3}}{2\sqrt{\sigma_0} m_T M_P^2} + \frac{1}{2(2\sigma_0)^2 m_T^2 M_P^2} \left\{ K^{ij} D_j W_{\inf} \partial_j W_{\inf} - \frac{3}{M_P^2} |W_{\inf}|^2 \right\} - \frac{W_{\inf}^2}{M_P^2} \left( \frac{3}{2} + \frac{3\sigma_0}{2} \frac{2}{M_P^2 m_T} \right) + O \left( \frac{H_I^2}{m_T^2} \right). \tag{51}
\]

This implies that there is a supersymmetry breakdown by the inflaton sector during inflation

\[
D_{T(i)} W|_{\sigma_0+\delta T(i)} = \frac{1}{\sqrt{6}(2\sigma_0)^{3/2} m_T} K^{ij} D_j W_{\inf} \partial_j W_{\inf} + O \left( \frac{H_I^2}{m_T^2} \right), \tag{52}
\]

i.e., \( D_{T(i)} W|_{\sigma_0+\delta T(i)} \) are suppressed by one power of \( m_T \), which vanish in the limit of infinitely heavy moduli.

Since the moduli are very heavy they stabilize quickly to their minima and the inflationary potential get corrected after setting \( T \) and \( T_i \) to their minima as follows

\[
V_F|_{\sigma_0+\delta T(i)} = \frac{V_{\inf}}{(2\sigma_0)^3} - \frac{5}{2(2\sigma_0)^5 W_{TT}(\sigma_0)} \left[ W_{\inf} \left\{ V_{\inf} + \frac{e^{K/M_P^2}}{5} K^{ij} \partial_j W_{\inf} D_j W_{\inf} \right\} + h.c. \right] + O \left( \frac{H_I^3}{m_T^3} \right). \tag{53}
\]

Using \( |W_{TT}(\sigma_0)| = \sqrt{3/2} M_P^2/m_T \), and rescaling as \( V_{\inf}/(2\sigma_0)^3 \to V_0(t_I) \) and \( W_{\inf}/(2\sigma_0)^{3/2} \to W_{\inf}(t_I) \), it is evident that the inflationary potential due to the moduli backreaction induces
a linear term in the inflaton potential

\[ V_{F|\sigma_0+\delta T(0)} = V_0(t_I) \left\{ 1 - \frac{5\sqrt{3}}{2\sqrt{2}} \frac{1}{m_T M_P^2} (W_{\text{inf}} + \text{h.c.}) \right\} + \mathcal{O} \left( \frac{|\Psi_0|^3}{m_T^3} \right) \]  

(54)

Clearly, as we can see here, in the limit \( m_T \to \infty \) the interference term between string moduli and inflaton sectors is disappeared.

D. Scale of PQ-symmetry breakdown during inflation

In the following, let us consider the PQ phase transition scale during inflation. Due to Eq. (2) during inflation we have

\[ v_{\Theta}(t_I) = v_S(t_I) = v_T(t_I) = 0. \]  

(55)

And the Kahler moduli fields we consider are stabilized during inflation and their potential has a local minimum at finite moduli fields values separated by a high barrier from the runaway direction. Since the moduli masses are much larger than the inflaton mass and accordingly will be frozen quickly during inflation without perturbing the inflaton dynamics, the height of barrier protecting metastable Minkowski (\( \simeq \text{dS} \)) space are independent of the gravitino mass hence the inflationary Hubble constant is also independent of the gravitino mass \[14\].

We consider the PQ symmetry breaking scale, \( \mu_\Psi(t_I) \), during inflation. In the global SUSY minima where \( V_{\text{SUSY}} = 0 \), all the flavon and driving fields have trivial VEVs, while the waterfall fields \( \Psi(\bar{\Psi}) \) can have non-zero VEVs. The FI D-terms must then be zero, i.e. \( \xi_1^{\text{FI}} = \xi_2^{\text{FI}} = 0 \). During inflation, if \( |\Psi_0| \) takes a large value the waterfall fields stay at the origin of the field space (the local minimum appears at \( \langle \Psi \rangle = \langle \bar{\Psi} \rangle = 0 \)); and the superpotential is effectively reduced to

\[ W_{\text{inf}}(t_I) = -\bar{g}_7 \Psi_0 \mu_\Psi^2(t_I), \]  

(56)

with \( \bar{g}_7^2 \equiv g_7^2/(2\sigma_0)^3 \) and \( \bar{g}_7 < 0 \), which gives a positive contribution to the inflation energy

\[ V_0(t_I) = 3H_I^2 M_P^2 \simeq \left| \frac{\partial W_{\text{inf}}(t_I)}{\partial \Psi_0} \right|^2 = \bar{g}_7^2 \mu_\Psi^4(t_I), \]  

(57)

and in turn drives inflation. Since the potential for \( |\Psi_0| \gg |\Psi_0^c| \equiv \mu_\Psi(t_I) \) with \( \langle \Psi \rangle = \langle \bar{\Psi} \rangle = 0 \) is flat before the waterfall behavior occurs, inflation takes place there. And the waterfall
behavior is triggered, when the inflaton $\Psi_0$ reaches the critical value $|\Psi_0^c|$. Once $|\Psi_0|$ rolls down from a large scale and approaches its critical value $|\Psi_0^c|$, the inflaton and waterfall fields get almost maximally mixed to form mass eigenstates:

$$
\Psi_0' \simeq \frac{1}{\sqrt{2}} (\Psi_0 \pm \bar{\Psi}), \quad \Psi' \simeq \frac{1}{\sqrt{2}} (\Psi - \Psi_0^\perp), \quad \bar{\Psi}' \simeq - \frac{1}{\sqrt{2}} (\bar{\Psi} + \Psi_0^\perp),
$$

(58)

where $\Psi_0^\perp \simeq (\pm \Psi_0 - \bar{\Psi})/\sqrt{2}$ is orthogonal to $\Psi_0'$. And their corresponding mass eigenvalues are given by

$$
m_{\Psi_0'} \simeq |\bar{g}_I| \mu_{\Psi}(t_I), \quad m_{\Psi'} \simeq |\bar{g}_I| \mu_{\Psi}(t_I), \quad m_{\bar{\Psi}'} \simeq 0.
$$

(59)

Let us schematically see this is the case. The potential at global SUSY limit

$$
V_{\inf}^{\text{global}} = \bar{g}_I^2 |\Psi\bar{\Psi} - \mu_{\Psi}^2(t_I)|^2 + 2|\Psi_0|^2 (|\Psi|^2 + |\bar{\Psi}|^2)
$$

$$
= \begin{pmatrix}
\Psi^* & \bar{\Psi}' \\
\bar{\Psi}'^* & \bar{\Psi}'
\end{pmatrix}
\begin{pmatrix}
\bar{g}_I^2 (|\Psi_0|^2 - \mu_{\Psi}^2(t_I)) & 0 \\
0 & \bar{g}_I^2 (|\Psi_0|^2 + \mu_{\Psi}^2(t_I))
\end{pmatrix}
\begin{pmatrix}
\Psi' \\
\bar{\Psi}'
\end{pmatrix} + ... 
$$

(60)

implies that (i) when $|\Psi_0| < \mu_{\Psi}(t_I)$, one of the mass eigenstates, $\Psi'$, becomes tachyonic: the waterfall fields fixed at $\langle \Psi \rangle = \langle \bar{\Psi} \rangle = 0$ is not stable since $\Psi(\bar{\Psi})$ have an opposite sign of $U(1)_{X_2}$ charges. As can be seen from Eq. (24) since the Kahler moduli superfields putting the GS mechanism into practice are not separated from the SUSY breaking by the inflaton sector during inflation, by taking tachyonic SUSY breaking scalar masses $m_{\Psi}^2 \sim -H_f^2$ induced dominantly by the $U(1)_{X_2} D$-term, the waterfall field $\Psi'$ rolls down its true minimum from a large scale. (ii) The other $\bar{\Psi}'$ stays positive definite throughout the inflationary trajectory up to a critical value $|\Psi_0^c| \approx \mu_{\Psi}(t_I)$. (iii) After inflation the universe is dominated by both the inflaton $\Psi_0'$ and one of waterfall fields, $\bar{\Psi}'$, while the other waterfall field $\Psi'$ gives negligible contribution to the total energy of the universe. (iv) After inflation and the waterfall transition mechanism has been completed $\Psi_0'$ approaches to zero and $\Psi'(\bar{\Psi}')$ relax to the flat direction of the field space given by $\Psi'\bar{\Psi}' = \mu_{\Psi}^2(t_I)$: the positive false vacuum of the inflaton field breaking the global SUSY spontaneously gets restored once inflation has been completed.

Now, we discuss how the inflation could be realized explicitly. The $F$-term scalar potential, the first term in the right hand side of Eq. (22), can be expressed as

$$
V(\phi_\alpha) = e^{K/M_p^2} \left\{ \sum_\alpha K^{\alpha\bar{\alpha}} D_\alpha W_{\inf} D_{\alpha\bar{\alpha}} W_{\inf}^* - 3 \frac{|W_{\inf}|^2}{M_p^4} \right\}
$$

(61)
with $\alpha$ being the bosonic components of the superfields $\hat{\phi}_\alpha \in \{ \hat{\Psi}_0, \hat{\Phi}_0^T, \hat{\Theta}_0, \hat{\Psi}, \hat{\Phi}_S, \hat{\Theta}, \hat{\Phi}_T \}$, and where the Kahler covariant derivative and Kahler metric are defined as

$$D_\alpha W_{\inf} \equiv \frac{\partial W_{\inf}}{\partial \phi_\alpha} + M_P^2 \frac{\partial K}{\partial \phi_\alpha} W_{\inf}, \quad K_{\alpha\beta} \equiv \frac{\partial^2 K}{\partial \phi_\alpha \partial \phi_\beta^*}$$

and $D_\alpha W_{\inf}^* = (D_\alpha W_{\inf})^*$ with $\tilde{K}_{\alpha\beta} \equiv (\tilde{K}_{\alpha\beta})^{-1}$. The lowest order (i.e. global supersymmetric) inflationary $F$-term potential $V_{\inf}^{\text{global}}$ receives corrections for $|\phi_\alpha| \ll M_P$. During inflation, working along the direction $|\Psi| = |\tilde{\Psi}| = 0$, from Eqs. (28) and (61) a small curvature needed for the slow-roll can be represented by the inflationary potential $V_{\inf}$

$$V_{\inf} = V_{\inf}^{\text{tree}} + V_{\text{sugra}} + \Delta V_{\inf}^{1-\text{loop}}. \tag{63}$$

The leading order potential, corrected by the interference term induced by the moduli back-reaction, can be written in Eq. (54) as

$$V_{\inf}^{\text{tree}} = V_0(t_I) \left\{ 1 - \frac{5\sqrt{3}}{2\sqrt{2}} \frac{\sqrt{V_0}}{m_T M_P^2 (\Psi_0 + \Psi_0^*)} \right\}, \tag{64}$$

where $V_0(t_I)$ is the rescaled vacuum energy during inflation, see Eq. (54). Substituting $K_{\inf}$ and $W_{\inf}$ in Eq. (28) into $V_F^{\inf}$ in Eq. (50), and minimizing with respect to $\Psi$ and $\tilde{\Psi}$ for $|\Psi_0| > \mu_\Psi(t_I)$ gives

$$V_F^{\inf} = \tilde{g}_\Psi^2 \mu_\Psi^4(t_I) \left\{ 1 - k_s \frac{|\Psi_0|^2}{M_P^2} + \gamma_s \frac{|\Psi_0|^4}{2 M_P^4} + \mathcal{O} \left( \frac{|\Psi_0|^6}{M_P^6} \right) \right\}, \tag{65}$$

where $\gamma_s \equiv 1 - 7k_s/2 - 3k_3$. Such a supergravity induced mass squared is expected to have the same form as the $\Psi_0$ mass squared, namely $\tilde{g}_\Psi^2 \mu_\Psi^4(t_I)/M_P^2 = V_0(t_I)/M_P^2$, which is the order of the Hubble constant squared $H_I^2 = V_0(t_I)/3 M_P^2$. Then the SUGRA contribution $V_{\text{sugra}}$ to $V_{\inf}$ leads to

$$V_{\text{sugra}} = -c_H^2 H_I^2 |\Psi_0|^2 + V_0 \gamma_s \frac{|\Psi_0|^4}{2 M_P^4} + \mathcal{O} \left( \frac{|\Psi_0|^6}{M_P^6} \right). \tag{66}$$

The inflaton $\Psi_0$ also receives 1-loop radiative correction in the potential due to the mismatch between masses of the scalar and fermion components of $\Psi(\tilde{\Psi})$, which are non-vanishing since SUSY is broken by $\partial W_{\inf}/\partial \Psi_0 \neq 0$. The corresponding 1-loop correction to the scalar potential is analytically calculated as

$$\Delta V_{1-\text{loop}} = \sum_i (-1)^i \frac{m_i^4}{64\pi^2} \ln \frac{m_i^2}{Q^2} = \tilde{g}_\Psi^4 \mu_\Psi^4(t_I) \frac{8\pi^2}{8\pi^2} F(x) \tag{67}$$
where \( F(x) = \frac{1}{4} \left\{ (x^2 + 1) \ln \frac{x^2 - 1}{x^2 + 1} + 2x^2 \ln \frac{x^2 + 1}{x^2 - 1} + 2 \ln \frac{\sqrt{2} \mu_\Psi^2 x^2}{Q^2} - 3 \right\} \) and the sum is taken over the field degrees of freedom and \( f = 0 \) for scalar and \( f = 1 \) for fermion. Here the \( Q \) is a renormalizable scale, \( x \) is defined as \( x \equiv |\Psi_0|/\mu_\Psi(t_I) = \varphi/(\sqrt{2} \mu_\Psi(t_I)) \) where \( \varphi \) is the normalized real scalar field. In the limit \( x \gg 1 \), i.e. \( \varphi \gg \sqrt{2} \mu_\Psi(t_I) \), this is approximated as

\[
\Delta V_{1-\text{loop}} \simeq \frac{\tilde{g}_I^4 \mu_\Psi^2(t_I)}{16\pi^2} \ln \frac{\tilde{g}_I^2 \varphi^2}{2Q^2}.
\]

(68)

If we let the inflaton field \( \Psi_0 \equiv \varphi e^{i\theta}/\sqrt{2} \), and during the inflation period, taking into account the radiative correction, supergravity effects, and moduli backerction effects, the inflationary potential is of the following form

\[
V_{\text{inf}}(\varphi) = V_0(t_I) \left\{ 1 - \frac{5\sqrt{3}}{2} \varphi \frac{\sqrt{V_0}}{m_T M_P^2} \varphi \cos \theta + \gamma_s \frac{\varphi^4}{8 M_P^4} + \frac{\tilde{g}_I^2}{16\pi^2} \ln \frac{\tilde{g}_I^2 \varphi^2}{2Q^2} \right\}
+ \frac{\varphi^2}{2} \left( m_{\Psi_0}^2 - k_s \frac{V_0}{M_P^2} \right). \tag{69}
\]

The moduli-induced slope partially cancels the slope of the Coleman-Weinberg potential, which flattens the inflationary trajectory and reduces the distance in field space corresponding to the \( N_e \sim 50 \) e-folds of inflation. And the inflaton mass \( m_{\Psi_0} \) can be given for \( k_s = 1 \) by

\[
m_{\Psi_0} = |\tilde{g}_I| \frac{\mu_\Psi^2(t_I)}{M_P}; \tag{70}
\]

since the inflaton acquires a mass of order the Hubble constant, \( m_{\Psi_0} = H_I \sqrt{3} \), agreement of theory’s prediction for spectral index \( n_s \) with observation strongly suggests the presence of a negative Hubble-induced mass-term, and the \( k_s \) parameter term vanishes identically. This inflaton mass (\( \gg m_{3/2} \)) can directly be obtained from Eqs. (27) and (28) as

\[
m_{\Psi_0} = |M_P^2 (\epsilon^G \nabla_{\Psi_0} G_{\Psi_0})|^\frac{1}{2} = \sqrt{3} H_I, \tag{71}
\]

where \( \nabla_k G_\alpha = \partial_k G_\alpha - \Gamma^j_{k\alpha} G_j \) with the Christoffel symbol \( \Gamma^j_{k\alpha} = G^j{}^{\kappa\tau} G_{\kappa\alpha} \) and \( \nabla_{\Psi_0} G_{\Psi_0} \simeq -(W_{\Psi_0}/W)^2 \) is used. This inflaton mass is in agreement with the above prediction in Eq. (70).

Inflation stops at \( |\Psi^*_0| \simeq \mu_\Psi(t_I) \), where the mass of \( \Psi \) becomes negative and the field acquires a non-vanishing expectation value. In order to develop the VEV of the waterfall field \( \Psi \), we destabilize the waterfall field \( \Psi \) by taking tachyonic Hubble induced masses of the PQ-breaking waterfall field, i.e., \( m_{\Psi_0}^2 \sim -H_I^2 < 0 \). Then, the VEV of the waterfall field could
be determined by considering both the SUSY breaking effect and a supersymmetric next leading order term. The next leading Planck-suppressed operator invariant under $A_4 \times U(1)_X$ is given by

$$\Delta W_v \simeq \frac{\hat{\alpha}}{M_P^2} \Psi_0 \Psi^2 \bar{\Psi}^2,$$

(72)

where we set the VEVs of all other matter fields to zero except the waterfall field and neglected their corresponding trivial operators. Note that the constant $\hat{\alpha} = \mathcal{O}(\alpha/8\pi)$ with a constant $\alpha$ being of order unity. Since the soft SUSY-breaking terms are already present at the scale relevant to inflation dynamics, the scalar potential for the waterfall field $\Psi$ at leading order reads

$$V_\Psi(t_I) \simeq \frac{1}{2} D_{X_2}^2 + \hat{\alpha}_\Psi \hat{m}_\Psi^2 |\Psi|^2 + \hat{\alpha}_\bar{\Psi} \hat{m}_{\bar{\Psi}}^2 |\bar{\Psi}|^2 + |\hat{\alpha}|^2 \frac{|\Psi|^4 |\bar{\Psi}|^4}{M_P^2} + ...,$$

(73)

where $|\hat{\alpha}_\Psi \hat{m}_\Psi^2|, |\hat{\alpha}_\bar{\Psi} \hat{m}_{\bar{\Psi}}^2| \ll |D_{X_2}(t_I)|$ with $|\hat{\alpha}_\Psi, \bar{\Psi}| \ll 1$ are taken. Here $\hat{m}_\Psi, \bar{\Psi} \sim |\Psi_0^X| \sim \mathcal{O}(|F_{\Psi_0}^X|/M_P)$ with $F_{\Psi_0} = K_{\Psi_0} \Psi_0 W_{\text{inf}} \simeq \sqrt{3} H_I M_P$ represents the Hubble induced soft scalar masses generated by the $F$-term SUSY breaking, during inflation. If the tachyonic SUSY breaking scalar masses are dominantly induced by the $U(1)_X$ $D$-term, $D_{X_2}(t_I) \sim \mathcal{O}(H_I^2)$, compared to the Hubble induced soft masses generated by the $F$-term SUSY breaking, the soft SUSY breaking mass of $\Psi$ during inflation are approximated by

$$m_{\Psi}(t_I) = \hat{\alpha}_\Psi \hat{m}_\Psi^2 + D_{X_2}(t_I) \simeq -\hat{\beta}_\Psi H_I^2,$$

(74)

with $\hat{\beta}_\Psi > 0$.

Then the scalar potential in Eq. (73) for the waterfall field $\Psi$ is good approximated as

$$V_\Psi(t_I) \simeq -\hat{\beta}_\Psi H_I^2 |\Psi|^2 + |\hat{\alpha}|^2 \frac{|\Psi|^4 |\bar{\Psi}|^4}{M_P^2}.$$

(75)

Here the constant $\hat{\beta}_\Psi$ are of order unity, while $\hat{\alpha} = \alpha/(8\pi)$ with $\alpha$ being of order unity. We find the minimum as

$$v_\Psi(t_I) = \sqrt{\frac{2\hat{\beta}_\Psi}{|\hat{\alpha}|^2}} H_I \left( \frac{M_P}{v_\Psi} \right)^2,$$

(76)

leading to $M_P \gg \mu_{\Psi}(t_I) \gg H_I$ and the PQ breaking scales during inflation

$$\mu_{\Psi}^2(t_I) \equiv \frac{v_\Psi(t_I) v_\bar{\Psi}(t_I)}{2} = \sqrt{\frac{\hat{\beta}_\Psi}{2|\hat{\alpha}|^2}} \left( \frac{H_I}{v_\Psi(t_I)} M_P^2 \right).$$

(77)

In supersymmetric theories based on SUGRA, since SUSY breaking is transmitted by gravity, all scalar fields acquire an effective mass of the order of the expansion rate during inflation.

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So, we expect that the inflaton acquires a mass of order the Hubble constant, and which in turn indicates that the soft SUSY breaking mass (the inflaton mass $m_{\Psi_0}$) during inflation strongly depends on the scale of waterfall (or PQ) fields by the above Eq. (77); for example, for $\mu(t_I) \sim 10^{16}$ GeV one obtains

$$H_I \sim 2 \times 10^{10} \text{ GeV}$$

for $\hat{\beta}_i \sim 1$ and $\hat{\alpha} \sim 1/(8\pi)$, see TABLE III.

E. Cosmological observables

The inflaton as a source of inflation is displaced from its minimum and whose slow-roll dynamics leads to an accelerated expansion of the early universe. During inflation the universe experiences an approximately dS phase with the Hubble parameter $H_I$. Quantum fluctuations during this phase can lead to observable signatures in CMB radiation temperature fluctuation, as the form of density perturbation, in several ways [12], when the quantum fluctuations are crossing back inside the Hubble radius long after inflation has been completed. When interpreted in this way, inflation provides a causal mechanism to explain the observed nearly-scale invariant CMB spectrum. (i) Quantum fluctuations of the inflaton field during inflation give rise to fluctuations in the scalar curvature and lead to the adiabatic fluctuations\(^{19}\) that have grown into our cosmologically observed large-scale structure much bigger than the Hubble radius and then eventually got frozen. Adiabatic density perturbations seeded by the quantum fluctuations of the inflaton have a nearly scale-invariant spectrum, $\Delta^2_R(k_0)$, which is a cosmological observable of the curvature perturbations. The power spectrum of the curvature perturbations, $\Delta^2_R(k_0)$, reads in the Planck 2015 result at 68% CL (for the base $\Lambda$CDM model)\(^{13}\)

$$\Delta^2_R(k_0) = (2.141^{+0.050}_{-0.049}) \times 10^{-9},$$

at the pivot scale $k_0 = 0.002$ Mpc\(^{-1}\) (wave number), which is compatible with the one suggested for the COBE normalization\(^{59}\). (ii) Fluctuations of the metric lead to tensor-

\(^{19}\) These correspond to fluctuations in the total energy density, $\delta \rho \neq 0$, with no fluctuation in the local equation of state, $\delta (n_i/s) = 0$. On the other hand, isocurvature perturbations correspond to fluctuations in the local equation of state of some species, $\delta (n_i/s) \neq 0$, with no fluctuation in the total energy density, $\delta \rho = 0$\(^{12}\).
B mode fluctuations in the CMB radiation. Primordial gravitational waves are generated with a nearly scale-invariant spectrum, $\Delta^2_h(k_0)$, which reads in the Planck 2015 result $\Delta^2_h(k_0) < 1.97 \times 10^{-10}$. (iii) Quantum fluctuations are imprinted into every massless scalar field in dS space during inflation, with an approximately scale-invariant spectrum, $\langle|\delta\phi(k)|^2\rangle = (H_I/2\pi)^2/(k^3/2\pi^2)$ for a canonically normalized scalar field $\phi$, which is essentially a thermal spectrum at Gibbons-Hawking temperature $T_{GH} = H_I/2\pi$. The other important cosmological observables imprinted in the CMB spectrum are followings: the BAU (which will be discussed in Sec. IV), the fractions of relic abundance $\Omega_{DM}$ (see Ref. [4]) and dark energy $\Omega_\Lambda$ (see Sec. III B).

The slow-roll condition [60] is well satisfied up to the critical point $\varphi^c = \sqrt{2}\mu_\varphi(t_I)$, beyond which the waterfall mechanism takes place. Here the slow-roll parameters, $\epsilon$ and $\eta$, are approximately derived as

\[
\epsilon \equiv \frac{M_P^2}{2} \left(\frac{V_\varphi}{V}\right)^2 \approx \frac{1}{2} \left(\frac{\tilde{g}_{\varphi}^2}{8\pi^2} \frac{M_P}{\varphi}\right)^2 \left\{1 + \frac{3}{2} \frac{\gamma_s}{\tilde{g}_{\varphi}^2} \left(\frac{\varphi}{M_P}\right)^4 - \frac{5}{2} \frac{\sqrt{3}}{g_{\varphi}} \frac{\mu_\varphi}{m_T} \frac{\mu_\psi}{M_P} \frac{\varphi}{M_P} \cos \theta\right\}^2 \ll 1,
\]

\[
\eta \equiv \frac{M_P^2 V_{\varphi\varphi}}{V} \approx \frac{\tilde{g}_{\varphi}^2}{8\pi^2} \left(\frac{M_P}{\varphi}\right)^2 \left\{3 \frac{\gamma_s}{2} \frac{8\pi^2}{\tilde{g}_{\varphi}^2} \left(\frac{\varphi}{M_P}\right)^2 - 1\right\}, \quad |\eta| \ll 1,
\]

where $V_\varphi$ denotes a derivative with respect to the inflaton field $\varphi = \sqrt{2} \text{Re}\Psi_0$, and $M_P \gg |\Psi_0| \gg |\Psi_0|$ (or $M_P \gg |\varphi| \gg |\varphi^c|$) is assumed. Recalling that $\tilde{g}_{\varphi}^2 = g_{\varphi}^2/(2\sigma_0)^3$. The above equations clearly show that the curvature of the inflationary potential is dominantly affected by the moduli backreaction in Eq. (54) as well as the 1-loop radiative correction. In the slow-roll approximation, the number of $e$-foldings after a comoving scale $l$ has crossed the horizon is given by the inflationary potential through

\[
N(\varphi) = \int^{\varphi_l}_{\varphi(\varphi^c)} H_I dt = \frac{1}{M_P^2} \int^{\varphi_l}_{\varphi(\varphi^c)} \frac{V(\varphi)}{V_\varphi(\varphi)} d\varphi,
\]

where $\varphi_l$ is the value of the field at the comoving scale $l$, and $\varphi^c$ is the one at the end of inflation. The field value $\varphi^c$ is determined from the condition Max{$\epsilon(\varphi^c), |\eta(\varphi^c)|$} = 1 [61]. The power spectrum $\Delta^2_{R}(k_0)$ sensitively depends on the theoretical parameters of the
inflationary potential,

\[
\Delta_R^2(k_0) \simeq \frac{1}{12\pi^2 M_P^6} \frac{V^3(\varphi_l)}{|V_\varphi(\varphi_l)|^2}
\]

\[
\simeq \frac{2}{3} \frac{8\pi^2}{g_7^2} \left( \frac{\mu_\Psi}{M_P} \right)^4 \left( \frac{\varphi_l}{M_P} \right)^2 \left\{ 1 + \frac{5\sqrt{3}}{2} \frac{8\pi^2}{g_7} \left( \frac{\mu_\Psi}{m_T} \right) \left( \frac{\mu_\Psi}{M_P} \right) \cos \theta \right\}^{-2}
\]  

(82)

where the potential \( V(\varphi_l) \) and its derivative \( V_\varphi(\varphi_l) \) are evaluated at the epoch of horizon exit for the comoving scale \( k_0 \). It should be compared with the Planck 2015 result Eq. (79).

With the definition of the number of e-folds after a comoving scale \( k_0 \) leaves the horizon, we can obtain the corresponding inflaton value \( \varphi_l/M_P \) from Eq. (81). And the number of e-folds \( N_e \) corresponding to the comoving scale \( k_0 \) is around 50 depending on the energy scales \( H_I \) and \( T_{\text{reh}} \)

\[
N_e = 49.1 + \ln \left( \frac{0.002 \text{ Mpc}^{-1}}{k_0} \right) + \frac{1}{3} \ln \left( \frac{T_{\text{reh}}}{10^4 \text{ GeV}} \right) + \frac{1}{3} \ln \left( \frac{H_I}{10^{10} \text{ GeV}} \right)
\]  

(83)

where \( T_{\text{reh}} \) represents the maximal temperature of the last radiation dominated era, so-called the reheating temperature. The tensor and scalar modes have spectrum \( A_t = 2H_I^2/(\pi^2 M_P^2) \) and \( A_s \equiv \Delta_R^2(k_0) \) [13], respectively. In the supergravity F-term inflation we consider, the tensor-to-scalar ratio \( r = A_t/A_s \simeq 16\epsilon(\varphi_l) \) is much lower than the Planck 2015 bound \( r_{0.002} < 0.09 \), i.e. well below \( 10^{-2} \), and the running of the spectral index \( dn_s/d\ln g_7 \) is always smaller than \( 10^{-3} \) and so unobservable. And the scalar spectral index \( n_s \) is approximated as

\[
n_s \simeq 1 - 6\epsilon(\varphi_l) + 2\eta(\varphi_l) \simeq 2\eta(\varphi_l).
\]  

(84)

We can compare this quantity with the results of the Planck 2015 observation [13]

\[
n_s = 0.967 \pm 0.004.
\]  

(85)

In order for the power spectrum of the curvature perturbation and the spectral index to be well fitted with the Planck 2015 observation, the four independent parameters \( m_T, \mu_\Psi(t_I), \gamma_s, \) and \(|\tilde{g}_7|\) are needed and those parameters have predictions, \( m_T = \mathcal{O}(10^{16}) \gg \mu_\Psi(t_I) = \mathcal{O}(10^{15}) \) GeV, \( \gamma_s = \mathcal{O}(1) \) and \(|\tilde{g}_7| = \mathcal{O}(1) \times 10^{-3} \) as in TABLE III, where we have set \( \cos \theta = 1 \) in Eqs. (80) and (82). This table shows that the cosmological observables can be well fitted where both the moduli stabilized at a scale close to \( \Lambda_{\text{GUT}} \) and the PQ symmetry breaking scale induced at \( \mu_\Psi(t_I) \simeq 0.7 \times 10^{16} \) GeV < \( m_T \). As shown in TABLE III the number of
TABLE III: Four independent input parameters $m_T$, $\mu \Psi(t_I)$, $\gamma_s$, and $|\tilde{g}_7|$ provide predictions on $n_s$, $N_e$, $\Delta_R(k_0)/10^{-9}$, and $T_{\text{reh}}$/GeV.

| $m_T$ (10^16 GeV) | $\mu \Psi(t_I)$ (10^16 GeV) | $\gamma_s$ | $|\tilde{g}_7|$ (10^{-3}) | $H_I$ (10^10 GeV) | $\phi_l$ (10^10 GeV) | $\phi_c$ (10^10 GeV) | $n_s$ | $N_e$ | $\Delta_R(k_0)/10^{-9}$ | $T_{\text{reh}}$/GeV |
|------------------|-------------------|---------|----------------|---------|----------------|---------|-----|-----|----------------|-------------------|
| 7.173            | 0.673             | 0.544   | 1.189          | 1.281   | 1.051          | 0.952   | 0.968| 53.210| 2.095           | 1.807 x 10^9     |
| 5.592            | 0.685             | 0.568   | 0.963          | 1.074   | 1.047          | 0.969   | 0.969| 51.285| 2.187           | 6.694 x 10^6     |
| 9.057            | 0.672             | 0.448   | 1.077          | 1.155   | 1.027          | 0.950   | 0.968| 50.010| 2.149           | 1.357 x 10^5     |
| 9.381            | 0.674             | 0.321   | 1.160          | 1.254   | 1.024          | 0.954   | 0.967| 49.408| 2.121           | 2.562 x 10^4     |
| 9.287            | 0.673             | 0.671   | 1.025          | 1.102   | 1.046          | 0.951   | 0.968| 48.726| 2.154           | 3.021 x 10^5     |

$e$-foldings depends on the amount of reheating temperature which in turn depends on the decay rate of the inflaton $\Psi'_{0}$ and waterfall field $\bar{\Psi}'$ into relativistic particles. In the following section we will see how the amount of reheating, $T_{\text{reh}}$, could be strongly correlated with both baryogenesis via leptogenesis and the yield of gravitinos.

IV. LEPTOGENESIS

Let us discuss on how the matter-antimatter asymmetry of the universe could be realized in the context of the present model. In order to account for a successful leptogenesis, we introduce the AD mechanism for baryogenesis \cite{16} and its subsequent leptonic version so-called AD leptogenesis \cite{17}. In the global SUSY limit, i.e. $M_P \rightarrow \infty$, as well as in the energy scale where $A_4 \times U(1)_X$ is broken, some combinations of scalar fields do not enter the potential, composing flat directions of the scalar potential. So, taking the flat directions $H_u = L_i = \zeta_i / \sqrt{2}$ (a generation index $i = 1, 2, 3$), then the AD flat directions for leptogenesis \cite{17} are $\zeta_i = (2 \tilde{L}_i H_u)^{1/2}$ where $\tilde{L}_i$ are scalar components of the chiral multiplets $L_i$ of $SU(2)_L$-doublet leptons. After integrating out the heavy Majorana neutrinos, $N_R$, the relevant superpotential \cite{15} induces the effective operator at low energies

\[
W_{\text{eff}} \supset \frac{1}{2 M_i} (\tilde{L}_i H_u)^2, \quad \text{with } M_i \equiv \frac{v_u^2}{(M_{\nu\nu})_{ii}}. \tag{86}
\]

where $(\tilde{M}_{\nu\nu})_{ii} = (U_{\text{PMNS}}^T M_{\nu\nu} U_{\text{PMNS}})_{ii} \simeq \delta_i$ in Eq. \cite{16}. Recalling that the $3 \times 3$ mixing matrix $U_L = U_{\text{PMNS}}$ diagonalizing the mass matrix $M_{\nu\nu} = -m_D^T M_R^{-1} m_D$ participates in the charged weak interaction, the active neutrino mixing angles $(\theta_{12}, \theta_{13}, \theta_{23}, \delta_{CP})$ and the
pseudo-Dirac mass splittings $\delta_k$ responsible for new wavelength oscillations characterized by the $\Delta m^2_k$ could be obtained from the mass matrix $M_{\nu\nu}$ formed by seesawing. Then, from Eqs. (A17) and (A18) we obtain the $\mu-\tau$ powered mass matrix as in Refs. [1, 62]

$$M_{\nu\nu} = m_0 e^{i\pi} \begin{pmatrix}
1 + 2F & (1 - F) y_2 & (1 - F) y_3 \\
(1 - F) y_2 & (1 + \frac{F + 3G}{2}) y_2^2 & (1 + \frac{F - 3G}{2}) y_2 y_3 \\
(1 - F) y_3 & (1 + \frac{F - 3G}{2}) y_2 y_3 & (1 + \frac{F + 3G}{2}) y_3^2
\end{pmatrix}$$

$$= U_{PMNS}^* \hat{M}_{\nu\nu} U_{PMNS}^\dagger,$$

where

$$m_0 \equiv \left| \frac{y^2_1 y^2_2}{3M} \left( \frac{\nu_T}{\sqrt{2\Lambda}} \right)^2 \left( \frac{\nu_v}{\sqrt{2\Lambda}} \right)^{18} \right|,$$

$$F = (\tilde{\kappa} e^{i\phi} + 1)^{-1}, \quad G = (\tilde{\kappa} e^{i\phi} - 1)^{-1}. \quad (88)$$

In the limit $y^\nu_2 = y^\nu_3$ ($y_2, y_3 \to 1$), the mass matrix (87) gives the tri-bimaximal mixing (TBM) angles [24] and their corresponding mass eigenvalues $|\delta_k|$:

$$\sin^2 \theta_{12} = \frac{1}{3}, \quad \sin^2 \theta_{23} = \frac{1}{2}, \quad \sin \theta_{13} = 0,$$

$$|\delta_1| = \frac{\Delta m^2_1}{2m_1} = 3 m_0 |F|, \quad |\delta_2| = \frac{\Delta m^2_2}{2m_2} = 3 m_0, \quad |\delta_3| = \frac{\Delta m^2_3}{2m_3} = 3 m_0 |G|. \quad (89)$$

These $|\delta_k|$ are disconnected from the TBM mixing angles. It is in general expected that deviations of $y_2, y_3$ from unity, leading to the non-zero reactor mixing angle [30, 31], i.e. $\theta_{13} \simeq 8.5^\circ$ at 1$\sigma$ best-fit [34], and in turn opening a possibility to search for CP violation in neutrino oscillation experiments. These deviations generate relations between mixing angles and eigenvalues $|\delta_k|$. Therefore Eq. (87) directly indicates that there could be deviations from the exact TBM if the Dirac neutrino Yukawa couplings in $m_D$ of Eq. (A17) do not have the same magnitude, and the pseudo-Dirac mass splittings are all of the same order

$$|\delta_1| \simeq |\delta_2| \simeq |\delta_3| \simeq O(m_0). \quad (90)$$

As shown in Ref. [4] by numerical analysis, together with well-fitted $\theta_{12}$ and $\theta_{13}$ the values of atmospheric ($\theta_{23}$) and Dirac CP phase ($\delta_{CP}$) have a remarkable coincidence with the recent data by the NO$\nu$A [3] and/or T2K [6] experiments. From the overall scale of the mass matrix in Eq. (88) the pseudo-Dirac mass splitting, $\delta_2$, is expected to be

$$|\delta_2| \simeq 2.94 \times 10^{-11} \left( \frac{4.24 \times 10^9 \text{GeV}}{M} \right) \left| \frac{\nu_T}{\sqrt{2\Lambda}} \right|^2 \sin^2 \beta \text{ eV}, \quad (91)$$

$$\begin{array}{|c|c|}
\hline
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\hline
\end{array}$$
in which the scale of the heavy neutrino, $M$, can be estimated from Eq. (A19) through the astrophysical constraints as $M = |\hat{y}_\Theta| \times 2.75^{+1.50}_{-1.25} \times 10^9$ GeV which is connected to the PQ symmetry breaking scale via the axion decay constant in Ref. [4]. Eq. (91) shows that the value of $\delta_2$ depends on the magnitude $\hat{y}_\nu |v_T|/\Lambda$ since $M$ is constrained by the axion decay constraints: the smaller the ratio $v_T/\Lambda$, the smaller becomes $|\delta_2|$ responsible for the pseudo-Dirac mass splittings\(^{20}\). However, the value of $|\delta_2|$ is constrained from Eq. (14); for example, using $\tan \beta = 2$ and $v_T/\Lambda \simeq \lambda^2/\sqrt{2}$ we obtain\(^{21}\)

$$|\delta_2| \simeq 1.50 \times 10^{-14} |\hat{y}_1|^2 \text{eV}. \quad (92)$$

Since the potential is (almost) flat in these directions $\zeta_i$, they have large initial VEVs in the early universe, see Eq. (97). Such flat directions are lifted by some effective operators in a later epoch, receiving soft-masses in the SUSY breaking vacuum. Then the potential of the flat directions, $\zeta_i$, is directly written as

$$V_0(\zeta_i) = m_{\zeta_i}^2 |\zeta_i|^2 + \frac{m_3/2}{8M_i}(a_m \zeta_i^4 + \text{h.c.)} + \frac{|\zeta_i|^6}{4M_i^2}. \quad (93)$$

Here in the mass terms $m_{\zeta_i}^2$, we have included soft scalar masses generated by the $F$-term SUSY breaking, that is, the contribution from the effective $\mu$-term, $W \supset \mu_{\text{eff}} H_u H_d$, which gives mass terms $\mu_{\text{eff}}^2 |\zeta_i|^2/2$. Since our model lies in the gravity-mediated SUSY breaking mechanism it is expected that $m_{\zeta_i} \sim m_3/2$ and $|a_m| \sim 1$ in the $A$-term\(^{22}\). The potential for $\zeta_i$ in Eq. (93) is $D$-flat, $|\zeta_i| = 0$, and also $F$-flat in the limit of $\delta_i (or \Delta m_i^2) \to 0$. So, the AD fields $\zeta_i$ can develop large VEVs during inflation. As discussed before, during inflation the energy density of the universe is dominated by the inflaton $\Psi_0$, that is, $V_0(t_I) = 3H_I^2 M^2_P$. The potential for $D$-flat direction is generated from the coupling between the AD fields $\zeta_i$ and the inflaton $\Psi_0$, which generically takes the form

$$K \supset K_{\text{AD}} = |\Psi_0|^2 + |\zeta_i|^2 + \left( k_{\zeta_i} |\Psi_0|^2 \frac{|\zeta_i|^2}{M_P} + \text{h.c.)} + \gamma_{\zeta_i} \frac{|\Psi_0|^2 |\zeta_i|^2}{M^2_P} + \ldots, \quad (94)$$

\(^{20}\) Moreover, the overall scale of the heavy neutrino mass $M$ is closely related with a successful leptogenesis (see the details in Sec. IV), constraints of the mass-squared differences in Eq. (13), and the CKM mixing parameters, therefore it is very important to fit the parameters $v_T/\Lambda$ and $M$.

\(^{21}\) The value of $v_T/\Lambda$ is also related to the $\mu$-term in Eq. (4).

\(^{22}\) In the context of Kallosh-Linde (KL) type models the dominant contributions to $A$-term arise from loop corrections\([63]\) because at tree level $A$-terms are strongly suppressed by $m_3/2/m_T$, hence one needs relatively large $O(100)$ TeV gravitino mass in order to get properly large $A$-terms\([64]\).
where $k_{\zeta_i}$ and $\gamma_{\zeta_i}$ are complex and real constants, respectively, and the dots represent higher order terms which are irrelevant for our discussion. Then, due to the finite energy density of the inflaton $\Psi_0$ during inflation the AD fields $\zeta_i$ receive additional SUSY breaking effects. And such SUGRA contribution reads

$$V_{\text{sugra}}(\zeta_i) = -\tilde{c}_H H_I^2 |\zeta_i|^2 + \frac{H_I}{8M_i^4}(a_H \zeta_i^4 + \text{h.c.}).$$  \hspace{1cm} (95)

Here by taking $\tilde{c}_H > 0$ with $\tilde{c}_H$ being of order unity we assume that the AD fields $\zeta_i$ can obtain negative Hubble-induced mass terms. From Eq. (93) and (95) the total effective potential for the AD fields $\zeta_i$ relevant to the leptogenesis reads

$$V(\zeta_i) = V_0(\zeta_i) + V_{\text{sugra}}(\zeta_i).$$  \hspace{1cm} (96)

Then the minima of the potential are given by

$$\langle |\zeta_i| \rangle \simeq \left( \frac{4}{3} \tilde{c}_H \right)^{1/4} \left( \frac{m_i}{\Delta m_i^2} H_I v^2 \sin^2 \beta \right)^{1/2} \lesssim M_P,$$  \hspace{1cm} (97)

and $\arg(a_H) + 4 \arg(\zeta_i) \simeq \pi(2n + 1)/2$ with $n = 0, 1$, in which we have used $m_{\zeta_i}, m_{3/2}|a_m| \ll H_I$. The AD fields $\zeta_i$ at the origin are unstable due to the negative Hubble mass terms in Eq. (95), and so roll down toward their global SUSY minima of the potential in Eq. (96) during inflation. Thus, the AD fields $\zeta_i$ have large scales of $\sim \sqrt{v_u^2 H_I / |\delta_i|} \lesssim M_P$ in Eq. (97) during inflation. This is compatible with the fact that the Planck scale, $M_P$, sets the universe’s minimum limit, beyond which the laws of physics break. If we set the initial minima of the AD fields to the (almost) Planck scale, the ratios $m_i/\Delta m_i^2$ responsible for the neutrino mass splittings $\delta_i$ (relevant to the low energy neutrino oscillation as well as the high energy neutrino at the IceCube telescope) could be restricted as

$$\frac{1}{\delta_i} = \frac{2 m_i}{\Delta m_i^2} \lesssim \frac{M_P^2}{H_I v^2 \sin^2 \beta \left( \frac{3}{\tilde{c}_H} \right)^{1/2}}.$$  \hspace{1cm} (98)

Using $H_I \simeq 10^{10}$ GeV, $\sin \beta \simeq 1$, and $\tilde{c}_H \simeq 1$ we can obtain the lower bound

$$\delta_i \gtrsim 2.95 \times 10^{-14} \text{ eV}$$  \hspace{1cm} (99)

which is well compatible with the constraints from the neutrino data in Eq. (14) as well as a successful leptogenesis in Eq. (107).

After inflation ends, the inflaton $\Psi_0'$ and waterfall field $\tilde{\Psi}'$ (see Eq. (58)) begin to oscillate around their VEVs, $\langle \tilde{\Psi}' \rangle = \mu_\Psi$ and $\langle \Psi_0 \rangle \simeq 0$ (the VEV of $\Psi_0$ deviates from zero because
of the supergravity effect: $\langle \Psi_0 \rangle \sim m_{\frac{3}{2}}/|\tilde{g}_7|$ at the true minimum, see Eq. (119) and their decays produce a dilute thermal plasma formed by collisions of relativistic decay products. Since the energy density of the universe is still dominated by the inflaton $\Psi'_0$ and waterfall field $\Psi'_0$ during the inflaton and waterfall field oscillations epoch, the AD fields potential is still governed by the Hubble-induced mass terms in Eq. (95) together with $V_0(\zeta_i)$ in Eq. (93) at the first stage of oscillation. Thus, the AD fields $\zeta_i$ are trapped in the minima determined mainly by the Hubble $A$-term as in Eq. (97) because the curvatures around the minima along both radial and angular directions are of the order of $H_I$ also in this period. However, after inflation the values of $\zeta_i$ in Eq. (97) gradually decrease to the order of $\zeta_i$ masses as the Hubble parameter $H(T)$ decreases, then the negative Hubble-induced mass terms are eventually exceeded by the Hubble parameter at $T \sim \sqrt{m_{\zeta_i} v^2 \sin^2 \beta/\Delta m_i^2}$, i.e., $\dot{c}_H H(T)^2 \lesssim m_{\zeta_i}^2$ in the potential Eq. (96). And the AD fields begin to oscillate around the potential minima $\langle \zeta_i \rangle \simeq 0$ (actually, $m_{\zeta_i}$) with $H(T) = H_{osc}$ when the Hubble parameter $H(T)$ of the universe becomes comparable to the SUSY breaking mass $m_{\zeta_i}$. (Hereafter “osc” labels the epoch when the coherent oscillations commence.) Then the interactions of dimension-5 operators create lepton number.

Now we see how the lepton number is created. At the beginning of the oscillation, the AD fields have the initial values

$$|\zeta_i(t_{osc})| \simeq \left( \frac{4}{3} \tilde{c}_H \right)^{1/4} \left( \frac{m_{\zeta_i} m_i}{\Delta m_i^2} v^2 \sin^2 \beta \right)^{1/2} \ll M_P,$$

in which $m_{\zeta_i} \simeq H_{osc}$ is used. The evolution of the AD fields $\zeta_i$ after $H \simeq H_{osc}$ is described in a Friedmann-Robertson-Walker (FRW) universe by the equation of motion with the potential $V(\zeta_i)$ as

$$\ddot{\zeta}_i + 3H(T) \dot{\zeta}_i + \frac{\partial V(\zeta_i)}{\partial \zeta_i^*} \simeq 0,$$

where $H(T) = (\pi^2 g_*(T)/90 M_P^2)^{1/2} T^2$ $\approx 1.66(8\pi g_*(T) T^2/M_P^2)$ is the Hubble rate for a radiation-dominated era with the total number of effective degrees of freedom $g_*(T)$ at a temperature $T$. $\partial V(\zeta_i)/\partial \zeta_i^*$ $\simeq m_{\zeta_i}^2 \zeta_i$, and dot indicates time derivative. It is clear that the AD fields $\zeta_i$ oscillate around the origin ($\langle \zeta_i \rangle \simeq 0$, the VEVs of $\zeta_i$ deviate from zero due to the SUGRA effect) and the amplitude of the oscillation damps as $|\zeta_i| \propto H \propto t^{-1}$.

Since the AD fields $\zeta_i$ carry lepton number, the baryon number asymmetry will be created during coherent oscillation of the AD fields. The number density of the AD fields is related
to the lepton number density \( n_{L_i} \) as

\[
n_{L_i} = \frac{1}{2} i (\frac{\partial \zeta_i^*}{\partial t} - \zeta_i \frac{\partial \zeta_i}{\partial t^*}),
\]

then from Eq. (101) the evolution of \( n_{L_i} \) are given by

\[
\frac{\partial n_{L_i}}{\partial t} + 3 H n_{L_i} - \frac{m_{3/2}}{2 M_i} \text{Im}(a_m \zeta_i^4) - \frac{H}{2 M_i} \text{Im}(a_H \zeta_i^4) \approx 0.
\] (102)

Since the Hubble parameter \( H(T) \) decreases as temperature decreases, the relative phase between \( a_m \) and \( a_H \) changes with time when the AD fields \( \zeta_i \) trace the valleys determined mainly by the Hubble A-term\(^{23}\). And during their rolling towards the true minima, the contribution of \( \text{Im}(a_H \zeta_i^4) \) is suppressed compared with \( \text{Im}(a_m \zeta_i^4) \). Then the motion of \( \zeta_i \) in the angular direction generating lepton number is expressed as

\[
\frac{\partial n_{L_i}}{\partial t} + 3 H n_{L_i} \approx \frac{m_{3/2}}{2 M_i} \text{Im}(a_m \zeta_i^4),
\] (103)

where \( H = \dot{R}(t)/R(t) \), and \( R(t) \) stands for the scale factor of the expansion universe with cosmic time \( t \). The produced lepton number asymmetry at a time \( t \) can be obtained by integrating the above equation \( \partial(R^3 n_{L_i})/\partial t \approx \frac{m_{3/2}}{2 M_i} R^3 \text{Im}(a_m \zeta_i^4) \) where \( R = R(t) \). After the end of inflation, the inflaton field \( \Psi' \) and waterfall field \( \tilde{\Psi}' \) begin to oscillate around the potential minimum such that the universe is effectively matter dominated, which scales as \( R^3 \propto H^{-2} \propto t^2 \). And before the beginning of the \( \zeta_i \) oscillation, due to \( |\zeta_i| \propto H^{1/2} \propto t^{-1/2} \), the net lepton number generated keeps constant for the period \( t < t_{\text{osc}} \). During matter dominated epoch the Hubble parameter is related to the expansion time by \( H_{\text{osc}} = (2/3)t_{\text{osc}}^{-1} \). Then using Eq. (100) the generated lepton number at this stage \( (t = t_{\text{osc}}) \) is given approximately by

\[
n_{L_i}(t_{\text{osc}}) \approx \frac{\bar{c}_H}{9} m_i v^2 \sin^2 \beta \left( \frac{m_{3/2}}{2 M_i} |a_m| \right) H_{\text{osc}} \delta_{\text{eff}},
\] (104)

where \( \delta_{\text{eff}} \approx \sin(4 \text{arg} \zeta_i + \text{arg} a_m) \) represents an effective CP violating phase. It is expected that the production of net lepton asymmetry occurs before the reheating process completes, i.e., \( \Gamma_{\text{all}} = \Gamma_{\Psi_0} + \Gamma_{\tilde{\Psi}} < H_{\text{osc}} \), c.f., see Eq. (125) ; the production of lepton number is strongly suppressed after the AD fields \( \zeta_i \) start their oscillations, because \( \text{Im}(a_m \zeta_i^4) \) change their sign rapidly due to the oscillation of \( \zeta_i \) as well as the amplitude of \( \zeta_i \) oscillation is damped with expansion (see below Eq. (101)). Thus after inflation \( R^3 n_{L_i} \big|_{t = t_{\text{osc}}} = R^3 n_{L_i} \big|_{t = t_R} \approx \)

\(^{23}\) If there are no true minima, i.e. \( m_{3/2} = 0 \), the AD fields get eternally trapped in the minima Eq. (98) and there is no motion of \( \zeta_i \) changing with time along the angular direction, leading to no lepton number production.
\( n_{L_i}(t_R) / \rho_{\text{rad}}(t_R) \) stays constant until the inflaton \( \Psi'_0 \) and waterfall field \( \tilde{\Psi}' \) decays into light particles. Here \( \rho_{\text{rad}}(t_R) = 3 M_P^2 \Gamma_{\text{all}}^2 \) is the energy density of the inflaton. Then the generated lepton number when the reheating process completes \( (t = t_R, H \simeq \Gamma_{\text{all}}) \) is given by

\[
n_{L_i}(t_R) = n_{L_i}(t_{\text{osc}}) \left( \frac{\Gamma_{\Psi'_0}}{H_{\text{osc}}} \right)^2.
\]

(105)

The inflaton decays reheats the universe producing entropy \( s \) of radiation such that \( \rho_{\text{rad}}(t_R) = 3 T_{\text{reh}}^4 s(t_R) / 4 \). Then the lepton number asymmetry is approximately expressed as

\[
\frac{n_{L_i}(t_R)}{s} = \frac{\tilde{c}_H m_i v^2 \sin^2 \beta}{36 M_P^2 \Delta m_i^2} T_{\text{reh}} \left( \frac{m_{3/2}|a_m|}{H_{\text{osc}}} \right) \delta_{\text{eff}}
\]

(106)

when the reheating process of inflaton completes. Later, we will discuss the reheating temperature, see Sec. IV B, and its related gravitino problem, see Sec. IV A. Recalling that the \( H_{\text{osc}} \) depends on \( M_i \) as \( H_{\text{osc}} \simeq m_{\xi_i} / \sqrt{2} \). Since \( M_i \) is directly related to the pseudo-Dirac mass splittings \( \delta_i \) as \( M_i = \langle H_u \rangle^2 / \delta_i \) in Eq. (12) in addition to \( O(\delta_1) \simeq O(\delta_2) \simeq O(\delta_3) = O(m_0) \) in Eq. (90), there are three flat directions corresponding to the almost degenerate neutrino pairs \( \xi_i \) in Eq. (17), i.e., the three generation AD fields \( \zeta_i / \sqrt{2} = \tilde{L}_i = H_u \) with \( i = 1, 2, 3 \).

The lepton asymmetries in Eq. (106) are converted into the baryon asymmetry through non-perturbative sphaleron processes. We are in the energy scale where \( A_4 \times U(1)_X \times \text{SUSY} \) is broken but the SM gauge group remains unbroken. So the baryon number produced is thermalized in a hot plasma into real baryons at a relatively low temperature. Therefore, the present baryon asymmetry can be expressed by

\[
\frac{n_B}{s} \simeq 0.35 \sum_{i=1,2,3} n_{L_i} / s \\
\simeq 8.61 \times 10^{-11} \times \frac{\sum_{i=1}^3 m_i^{2}}{1.24 \times 10^{11} \text{eV}^{-1}} \times \left( \frac{\delta_{\text{eff}}}{0.1} \right) \times \left( \frac{T_{\text{reh}}}{70 \text{TeV}} \right) \left( \frac{m_{3/2}|a_m|}{H_{\text{osc}}} \right) .
\]

(107)

where \( n_B \) is the baryon number density and \( s \) is the entropy density, and we have used \( \sin \beta \simeq 1 \) and \( \tilde{c}_H \simeq 1 \). Considering \( |a_m| \simeq 1 \) and \( H_{\text{osc}} \simeq m_{3/2} \simeq m_{\xi_i} \), the resultant baryon asymmetry only depends on the neutrino parameters \( m_i \) and \( \Delta m_i^2 \), the reheating temperature \( T_{\text{reh}} \), and an effective \( CP \) violating phase \( \delta_{\text{eff}} \). Quantitatively, the value of BAU is inferred from the two observations, \( m_i (\simeq m_{\nu_i}) \) and \( \Delta m_i^2 \), independently: from Eqs. (14), (15), and (98) the following quantity could be extracted as

\[
10^{10} \text{eV}^{-1} \lesssim \sum_i \frac{m_{\nu_i}}{\Delta m_i^2} = \frac{1}{2} \left( \frac{1}{\delta_1} + \frac{1}{\delta_2} + \frac{1}{\delta_3} \right) \lesssim 5 \times 10^{13} \text{eV}^{-1} ,
\]

(108)
in which the upper bound is derived from an initial condition of the AD fields in Eq. (98); the lower bound comes from the neutrino data in Eqs. (14) and (15). In terms of $Y_{\Delta B} \equiv (n_B - n_{\bar{B}})/s_{\text{today}}$ (which is conserved throughout the thermal evolution of the universe) the BBN results [70] and the CMB measurement [13] read at 95% CL

$$Y_{\Delta B}^{\text{BBN}} = (8.10 \pm 0.85) \times 10^{-11}, \quad Y_{\Delta B}^{\text{CMB}} = (8.67 \pm 0.05) \times 10^{-11}.$$  \hspace{1cm} (109)

Taking into account $\delta_{\text{eff}} \simeq \mathcal{O}(0.1)$ and $\sum_i m_{\nu_i}/\Delta m_i^2 \simeq \mathcal{O}(10^{11}) \text{eV}^{-1}$, for the baryon asymmetry in Eq. (107) to satisfy the BBN results and CMB measurement the reheating temperature should be

$$T_{\text{reh}} \sim \mathcal{O}(70) \text{ TeV}.$$ \hspace{1cm} (110)

Later, we will show that the bound of Eq. (110) could be consistent with the bound from that predicted from Sec. IV B.

A. Gravitino production

It is well known that thermal leptogenesis in supersymmetric framework, which is one of attractive mechanism for origin of matter, requires a large reheating temperature in the early universe, $T_{\text{reh}} \sim M_1 > 10^9 \text{ GeV}$, where $M_1$ is a lightest heavy neutrino mass. The gravitino, which appears in all models with local supersymmetry, is the superpartner of the graviton. Gravitino is produced thermally [71] or non-thermally [72–76] in the cosmological history. The excessive production of gravitinos in the early universe may destroy the nucleosynthesis of the light elements for unstable gravitinos or overclose of universe for stable gravitinos [77]. Since the gravitino is present in the supersymmetric model, we are going to address (unstable) gravitino overabundance problem.

After the inflation ends, the inflaton $\Psi'_0$ and waterfall field $\bar{\Psi}'$ release their energy into a thermal plasma by the decays, and the universe is reheated. Since all the particles including photons and baryons in the present universe are ultimately originated from the decays, it is crucial to reveal how the reheating proceeds. For simplicity, hereafter we treat the mixed mass eigenstates as the single field eigenstates,

$$\Psi'_0 \rightarrow \Psi_0, \quad \Psi' \rightarrow \Psi, \quad \bar{\Psi}' \rightarrow \bar{\Psi}.$$ \hspace{1cm} (111)
As mentioned in the introduction, there are two secluded SUSY breaking sectors, i.e., SUSY=SUSY_{inf}×SUSY_{vis}. Gravitational interactions explicitly break the SUSY down to true SUSY_{inf}×SUSY_{vis}, where SUSY_{inf} corresponds to the genuine SUGRA symmetry, while the orthogonal SUSY_{vis} is approximate global symmetry. In each sector, spontaneous breakdown of $F$-term occurs at a scale $F_i$ ($i = \text{inf, vis}$) independently, producing a corresponding goldstino. Hence, in the presence of SUGRA, the SUSY_{inf} is gauged and thus its corresponding goldstino is eaten by the gravitino via super-Higgs mechanism, leaving behind the approximate global symmetry SUSY_{vis} which is explicitly broken by SUGRA and thus its corresponding the uneaten goldstino as a propagating degree of freedom.

During inflation and the beginning of reheating (preheating) when SUSY is spontaneously broken there are possible productions of fermionic quanta which are strongly coupled to the inflaton field. During this stage the SUSY_{inf} is mainly broken by the inflaton implying that the goldstino produced is mainly inflatino (instead of the gravitino in the low energy); the gravitino produced non-thermally$^{24}$ is effectively massless as long as the Hubble parameter is larger than the gravitino mass, $H > m_3/2$ $^{75}$. However, this correspondence does not necessarily hold at late times, since the SUSY_{vis} is broken by other fields in the true vacuum.

In SUGRA framework, with the linear Kahler potential in Eq. (28) the inflaton field $\Psi_0$ has a non-vanishing auxiliary field $G_{\Psi_0}$. Such non-vanishing auxiliary field allows the inflaton decay into a pair of the gravitinos, whose decay process is crucial in the reheating process $^{73}$. The constraint on the inflaton potential $G_{\Psi_0}$ depending on the gravitino mass must be satisfied to avoid an overproduction of the gravitino keeping the success of the standard cosmology. In the unitary gauge in the Einstein frame, the goldstino (the longitudinal component of the gravitino) can be gauged away through the super-Higgs mechanism leading to vanishing of the gravitino-goldstino mixing. Then the relevant interactions for the inflaton decay into a pair of gravitinos reads $^{73}$

$$-e^{-1} \mathcal{L} = \frac{1}{8} \epsilon_{\mu\rho\sigma} (G_{\Psi_0} \partial_\rho \Psi_0 - G_{\Psi_0} \partial_\rho \Psi_0^*) \bar{\psi}_\mu \gamma_\nu \psi_\sigma$$
$$+ \frac{e^{G/2}}{8} M_\text{P} (G_{\Psi_0} \Psi_0 + G_{\Psi_0} \Psi_0^*) \bar{\psi}_\mu [\gamma^\mu, \gamma^\nu] \psi_\nu$$

$^{24}$ The inflatinos produced during inflation and preheating may be partially converted to the gravitinos in the low energy, since $G_{\Psi_0}$ is generically non-zero in the true minimum $^{78}$. At this stage, since the inflationary sector and the sector responsible for the low energy effective SUSY breaking are distinct, the gravitinos generated non-thermally are produced with a sufficiently low abundance.
where $\psi_{\mu}$ is the gravitino field. The real and imaginary components of the inflaton field have the same decay rate at leading order \[74\]

$$
\Gamma_{3/2} = \Gamma(\Psi_0 \rightarrow \psi_{3/2} + \psi_{3/2}) \approx \frac{1}{288\pi} \frac{M_P^2}{K_{\Psi_0 \Psi_0}} |\langle G_{\Psi_0} \rangle|^2 \left( \frac{m_{\Psi_0}}{m_{3/2}} \right)^2 m_{\Psi_0}
$$

(113)

in the limit of $m_{\Psi_0} \gg m_{3/2}$ after canonical normalization $\hat{\Psi}_0 = \sqrt{K_{\Psi_0 \Psi_0}} \Psi_0$. The decay rate is enhanced by the gravitino mass in the denominator, which comes from the goldstino (mainly as the inflaton) in the massless limit. The decay into the gravitinos only proceeds at the stage $H < m_{3/2}$, when the SUSY breaking contribution of the inflaton is subdominant \[73\]. Thus, the gravitinos produced at the reheating epoch by the inflaton decay through the interaction \[112\] should coincide with those in the low energy.

Now, we estimate how much the gravitinos are produced at the reheating epoch. After the inflation ends both the inflaton $\Psi_0$ and waterfall field $\tilde{\Psi}$ oscillate around the potential minimum and dominate the universe until the reheating. We express the superpotential \[27\] relevantly

$$
W \supset W(z) + \tilde{g}_{7}(\tilde{\Psi} \Psi - \mu_{\Psi}^2)
$$

(114)

where $W(z)$ is introduced to determine SUSY breaking scale, see Sec. \[113\] and $\tilde{g}_{7}^2 = g_{7}^2/(2\sigma_0)^3$ corrected by the string moduli backreaction. Then the scalar potential in Eq. (22) is extremized in the true vacuum if $\langle \partial_i V \rangle = 0$, and the resulting cosmological constant should vanish if $\langle V \rangle = 0$. Together with, these conditions are satisfied if

$$
\langle G^a G_\alpha \rangle = 3, \quad \langle G^a \nabla_k G_\alpha + G_k \rangle = 0.
$$

(115)

Then the condition of the potential minimum read

$$
\langle M_P^2 \{ G_{\Psi_0 \Psi_0} G_{\Psi_0} + G_{\Psi \Psi_0} G_{\Psi} + G_{\tilde{\Psi} \Psi_0} G_{\tilde{\Psi}} + G_{z \Psi_0} G_z + G_{\Psi_0} \} + G_{\Psi_0} \rangle = 0,
$$

(116)

$$
\langle M_P^2 \{ G_{\Psi \Psi} G_{\tilde{\Psi}} + G_{\Psi_0 \Psi} G_{\Psi_0} + G_{\tilde{\Psi} \Psi_0} G_{\tilde{\Psi}} + G_{z \Psi_0} G_z + G_{\Psi_0} \} + G_{\Psi} \rangle = 0,
$$

(117)

and the minimization condition for $\tilde{\Psi}$ is the same as for $\Psi$. The inflaton mass ($\gg m_{3/2}$), after the inflation, is given by

$$
m_{\Psi_0} \approx \left| M_P^4 \langle e^{G_{\Psi_0} \nabla_{\Psi_0}} \nabla_{\Psi_0} G_{\tilde{\Psi}} \rangle \right|^{1/2} \approx |\tilde{g}_{7}| \mu_{\Psi} (t_I),
$$

(118)

where $\nabla_{\Psi_0} G_{\tilde{\Psi}} \simeq W_{\tilde{\Psi} \Psi_0}/W$ is used, which is almost equal to the mass of waterfall field $\tilde{\Psi}$. This inflaton mass is in agreement with Eq. (59). Since the $z$ field is responsible for the SUSY
The production of gravitinos after inflation has been studied in some detail \cite{81}. Assuming \( |G_\Psi| \simeq |G_{\tilde{g}}| \lesssim |\Psi|/M_P^2 \), one obtains \( G_\Psi \simeq W_\Psi/W \), leading to \( W_\Psi/W \simeq |\tilde{\Psi}|/M_P^2 \) and \( W_\Psi/W \simeq \tilde{\Psi}/M_P^2 \). Using \( W_\Psi \simeq \tilde{g}_7 \Psi_0 \tilde{\Psi} \) in Eq. (27) we obtain
\[
\langle \Psi_0 \rangle \simeq \frac{m_{3/2}}{|\tilde{g}_7|}.
\] (119)

Using \( |G_{\Psi_0}| \lesssim |\Psi_0|/M_P^2 \) one obtains \( W_{\Psi_0}/W \simeq \Psi_0/M_P^2 \). Inserting \( G_{\Psi_0}\Psi_0 = -W_{\Psi_0}^2/W^2 \), \( G_{\Psi_0} \simeq -W_{\Psi_0}/(WM_P^2) + \tilde{g}_7 \tilde{\Psi}/(m_{3/2}M_P^2) \), and \( G_{\Psi_0} \simeq \sqrt{3}W_{\Psi_0}/(WM_P) \) into Eqs. (116) and (117)
\[
\langle G_{\Psi_0} \rangle \sim \frac{3\langle \Psi_0 \rangle}{M_P^2} \simeq 3\frac{m_{3/2}}{|\tilde{g}_7|M_P^2}, \quad \langle G_\Psi \rangle \sim \frac{3m_{3/2}^2}{2|\tilde{g}_7|^2 M_P^4} \langle \Psi \rangle,
\] (120)
which indicates \( \langle G_{\Psi_0} \rangle \) is much larger than \( \langle G_\Psi \rangle \). Then, from Eqs. (113) and (118) the inflaton decay width is roughly given by
\[
\Gamma_{3/2} \simeq \frac{1}{32\pi} \left( \frac{m_{\Psi_0}}{M_P} \right)^4 \left( \frac{\mu_\Psi(t_f)}{M_P} \right)^2 \frac{m_{\Psi_0}}{W^2}.
\] (121)

At the reheating epoch, gravitinos are produced by the non-thermal inflaton decay process \( Y_{3/2}^{\Psi_0} \) (the yield of the gravitinos by the inflaton decay) as well as by the thermal scattering \( Y_{3/2}^{\text{th}} \) (the yield of the gravitinos produced by thermal scatterings); the ratio of gravitino-to-entropy density is given by \( Y_{3/2} = Y_{3/2}^{\Psi_0} + Y_{3/2}^{\text{th}} \), which remains constant as the universe expands as long as there is no additional entropy production. Gravitinos thermally produced in the early universe, predominantly via \( 2 \to 2 \) inelastic scatterings of gluons and gluinos by QCD process, have a potential problem for the thermal history of the universe. However, since their relic density, \( \Omega_{3/2}^{\text{th}}h^2 \), and contribution to the energy density, \( Y_{3/2}^{\text{th}} \), grow with the reheating temperature after inflation, the yield of the gravitinos thermally produced is estimated as \( Y_{3/2}^{\text{th}} \sim 10^{-19} (T_{\text{reh}}/1 \text{ TeV})^2 \) \[71, 80\], which is harmless for the reheating temperature satisfying the successful leptogenesis in Eq. (110). Hence the total abundance can be given for \( Y_{3/2}^{\Psi_0} \gg Y_{3/2}^{\text{th}} \) by
\[
Y_{3/2} \simeq Y_{3/2}^{\Psi_0}.
\] (122)

Here the gravitino yield produced by the inflaton decay process \( \Psi_0 \to \Psi_{3/2} + \Psi_{3/2} \) via the interaction Eq. (112) is
\[
Y_{3/2}^{\Psi_0} \equiv \frac{n_{3/2}^{\Psi_0}}{s} \simeq \frac{2}{\Gamma_{3/2}} \frac{3}{4} \frac{T_{\text{reh}}}{\Gamma_{\Psi_0}} \frac{3}{4} \frac{m_{\Psi_0}}{4 m_{\Psi_0}},
\] (123)

\[25\] The production of gravitinos after inflation has been studied in some detail \[81\].
where \( n_{\Psi_0}^{3/2} \) is the number density of gravitinos by the inflaton decay, and \( s = (2\pi^2/45)g_{ss}(T)T^3 \) is the entropy density with \( g_{ss}(T) \) being the effective number of the massless degrees of freedom at the temperature \( T \).

The gravitino yield is severely constrained by BBN, \( Y_{3/2} < Y_{3/2}^{BBN} \), in order to keep the success of the standard scenario of BBN [81]. Otherwise, the decay products of the gravitino would change the abundances of primordial light elements too much and consequently conflict with the observational data. Refs. [18, 82] shows that, when the hadronic branching ratio of the gravitino decay is of order unity, \( Y_{3/2}^{BBN} \sim 10^{-16} \) for \( m_{3/2} \sim 1 \text{ TeV} \) and \( Y_{3/2}^{BBN} \sim 10^{-15} - 10^{-13} \) for \( m_{3/2} \sim 10 \text{ TeV} \); for \( m_{3/2} \gtrsim 100 \text{ TeV} \) the constraint disappears. On the other hand, in the context of supersymmetric moduli stabilization where moduli are strongly stabilized, at tree level the gaugino masses and \( A \)-terms are strongly suppressed by \( m_{3/2}/m_T \) and as such effectively vanish [64], while the dominant contributions to the gaugino masses and \( A \) terms arise from loop corrections [63]: \( m_{1/2} = b_ag_a^2/(16\pi^2)(F^C/C_0) \) and \( A_{ijk} = -(\gamma_{ijk}/16\pi^2)(F^C/C_0) \) where \( b_a = 11, 1, -3 \) for \( a = 1, 2, 3 \) are the one-loop beta function coefficients, \( \gamma_{ijk} \) are the anomalous dimensions of the matter fields, and \( F^C/C_0 \sim m_{3/2} \).

Thus, in order to have suitably large gaugino masses, relatively large \(O(100) \text{ TeV} \) gravitino masses must be considered [64]. The relic abundance of neutralino LSP (lightest supersymmetric particle) as dark matter will be considered in future work, see also Ref. [4].

In order to estimate \( Y_{3/2} \) we have to calculate the decay width of the inflaton, \( \Gamma_{\Psi_0} \), at reheating epoch.

### B. Reheating temperature

Since inflation leaves the early universe cold and empty, the inflaton \( \Psi_0 \) and waterfall field \( \tilde{\Psi} \) where all energy resides in must transfer their energy to a radiation dominated plasma in local thermodynamic equilibrium at a temperature sufficient to allow standard nucleosynthesis \( T_{\text{reh}} > T(\text{BBN}) \). So the universe must be reheated after inflation. The energy of the inflaton \( \Psi_0 \) and waterfall field \( \tilde{\Psi} \) are transferred to the SM sector through their gravitational and/or non-gravitational decays once their fields acquire finite VEVs, which in turn produce SM matter. Their decay products thermalize.

We are in the case where the inflaton \( \Psi_0 \) and waterfall field \( \tilde{\Psi} \) dominate the energy of the universe when they decay. The reheating temperature \( T_{\text{reh}} \) resulting from the perturbative
decays of the inflaton $\Psi_0$ and waterfall field $\tilde{\Psi}$ may be estimated by using the relation

$$\Gamma_{\text{all}} = 3H(T_{\text{reh}}) \quad (124)$$

at the end of the reheating process, where the Hubble parameter $H(T)$ is given in the radiation dominated era of the universe. Inflaton $\Psi_0$ and waterfall field $\tilde{\Psi}$ decays reheat the universe, when $\Gamma_{\text{all}} \gtrsim 3H(T_{\text{reh}})$:

$$T_{\text{reh}} = \left(\frac{10}{\pi^2 g_*}\right)^{1/4} \sqrt{\Gamma_{\text{all}}} M_P, \quad \text{with } \Gamma_{\text{all}} = \Gamma_{\Psi_0}^\text{sugra} + \Gamma_{\tilde{\Psi}}^\text{sugra} + \Gamma_{\Psi_0}^\text{vis} + \Gamma_{\tilde{\Psi}}^\text{vis} \quad (125)$$

where $g_*(T)$ is the number of the relativistic degrees of freedom in the plasma$^{27}$, and $\Gamma_{\Psi_0}^\text{sugra} + \Gamma_{\tilde{\Psi}}^\text{sugra}$ and $\Gamma_{\Psi_0}^\text{vis} + \Gamma_{\tilde{\Psi}}^\text{vis}$ stand for gravitational and non-gravitational decay widths, respectively. Later, we will see that it is too weak to cause the reheating with gravity in the model even the gravitational coupling is universal.

As in Ref. [4] (see Eq. (4) for lepton sector), in the supersymmetric visible sector the inflaton $\Psi_0$ and waterfall field $\tilde{\Psi}$ couple to the SM particles via the following interactions dominantly

$$W \supset g_{\Psi_0} \Psi_0 H_u H_d + \hat{y}_c \left(\frac{\tilde{\Psi}}{\Lambda}\right)^2 Q_2 c^c H_u \quad (126)$$

where $g_{\Psi_0}$ is a real and positive coupling constant, while the hat Yukawa coupling $\hat{y}_c$ is of order unity complex number. Here $Q_2$ is the second generation left handed quark doublet, which transforms as $1''$ under $A_4$ symmetry; the right handed charm quark $c^c \sim 1'$ under $A_4$. The first term is also associated with the $\mu$-term in Eq. (4) since the VEV of $\Psi_0$ is given by $\langle \Psi_0 \rangle \sim m_{3/2}/|\tilde{g}_7|$. And so the inflaton with a non-zero VEV can decay into the visible sector through the non-gravitational coupling of the inflaton to matter with the decay rate

$$\Gamma_{\Psi_0}^\text{vis} = \Gamma(\Psi_0 \to 2 \text{ Higgsinos}) + \Gamma(\Psi_0 \to 2 \text{ Higgses}) \sim 2 \times \frac{|g_{\Psi_0}|^2}{16\pi} m_{\Psi_0}, \quad (127)$$

$^{26}$ The energy transfer from the inflaton and waterfall field to the SM fields in general proceeds both through non-perturbative effects and perturbative decays.

$^{27}$ We estimate the total number of effectively massless degree of freedom of the radiation, $g_*(T)$, at temperature of the order of the decay rate of the inflaton $\Gamma_{\Psi_0}$, i.e., there are 17 bosons and 48 Weyl fermions for $T_{\text{EW}} < T < m_{3/2}$: $g_*(T) = \sum_{j=\text{bosons}} g_j(T_j/T)^4 + (7/8) \sum_{j=\text{fermions}} g_j(T_j/T)^4 = 34 + (7/8)96 = 118$ where $T_j$ denotes the effective temperature of any species $j$. 

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where the masses of the final-states compared to that of the inflaton are neglected. For the second term in Eq. (126), expanding the waterfall field \( \tilde{\Psi} \) and the Higgs field \( H_u \), without loss of generality, as

\[
\tilde{\Psi} = \frac{1}{\sqrt{2}} \left( v_{\tilde{\Psi}} + \frac{h_{\tilde{\Psi}}}{\sqrt{2}} - i \phi \right), \quad H_u = \begin{pmatrix} v_u + \frac{h_u}{\sqrt{2}} \\ 0 \end{pmatrix},
\]

the second term in Eq. (126) is expressed in terms of Lagrangian form as

\[
-L = \hat{y}_c \left( \frac{v_{\tilde{\Psi}}}{\sqrt{2} \Lambda} \right)^2 v_u \left\{ 1 + \frac{h_u}{\sqrt{2} v_u} \left( h_{\tilde{\Psi}} - i \phi \right) \right\} \bar{c} L_c R + \text{h.c.}
\]

Putting Eqs. (130) and (127) into Eq. (125), the reheating temperature can be expressed as

\[
T_{\text{reh}} \simeq \left( \frac{10}{\pi^2 g_*} \right)^{1/4} \sqrt{m_{\psi_0} M_P (|g_{\psi_0}|^2 + |g_{\tilde{\Psi}}|^2)}.
\]

Since there is no information on the size of the renormalizable superpotential coupling \( g_{\psi_0} \) of the inflaton to the Higgses and Higgssinos, first we consider the case of \( \Gamma_{\text{reh}} \simeq \Gamma_{\psi_0}^\text{vis} \gg \Gamma_{\psi_0}^\text{sugra} \). In this case, that is, \( g_{\psi_0} \gg |g_{\tilde{\Psi}}| \), the size of the Higgs-inflaton coupling can severely restrict the lower limit on \( T_{\text{reh}} \) in Eq. (132) as

\[
T_{\text{reh}} \gtrsim 10^4 \text{TeV} \left( \frac{g_{\psi_0}}{10^{-9}} \right) \left( \frac{\tilde{g}_T}{0.94 \times 10^{-3}} \right)^{1/2} \left( \frac{\mu_{\psi} (t_I)}{6.7 \times 10^{15} \text{GeV}} \right)^{1/2}
\]

where we have used \( m_{\psi_0} = |\tilde{g}_T| \mu_{\psi} (t_I) \) in Eqs. (59) and (118). This lower limit on \( T_{\text{reh}} \) is conflict with the limit for the successful leptogenesis in Eqs. (107) and (110). Hence
we conclude that $|g_\Psi| \gtrsim g_{\Psi_0}$ for $\Gamma_{\text{reh}} \simeq \Gamma_{\Psi_0}^{\text{vis}} + \Gamma_{\Psi}^{\text{vis}} \gg \Gamma_{\Psi}^{\text{sugra}} + \Gamma_{\Psi_0}^{\text{sugra}}$; then the reheating temperature is in a good approximation given by

$$T_{\text{reh}} \simeq (59 - 84)\,\text{TeV}$$

(134)

for the successful letogenesis with Eqs. (107,110). As will be seen below Eqs. (144) and (145), the size of Higgs-inflaton coupling can have a lower bound with the given reheating temperature. And the first term in Eq. (126) does not contribute to the sizable $\mu$-term since the coupling $g_{\Psi_0}(\ll \tilde{g}_7)$ should be enormously suppressed for a successful leptogenesis to be satisfied, see Eqs. (107) and (134).

Next, we consider the gravitational effects on the reheating temperature. For example, the inflaton $\Psi_0$ with a non-zero VEV can also decay into the visible sector through the SUGRA effects [72]. Then the reheating can be induced by the inflaton decay through non-renormalizable interactions. The relevant interactions for the matter-fermion production are provided in the Einstein frame as [79]

$$e^{-1}L = \frac{i}{2}K_{ij}^* \chi^i \gamma^\mu \partial_\mu \chi^j + \frac{i}{8M_P^2}K_{ij}^* (K_\sigma \partial_\mu \phi^\sigma - K_\sigma \partial_\mu \phi^{*\sigma}) \chi^j \gamma^\mu \chi^i - \frac{i}{2M_P}K_{ij}^* \Gamma_{\sigma\rho}^i (\partial^\mu \phi^\sigma) \chi^j \gamma^\mu \chi^\sigma + \frac{1}{2}e^{K/2M_P^2}(D_i D_j W) \chi^i \chi^j + h.c.$$  

(135)

where $D_i D_j W = W_{ij} + \frac{K_i}{M_P} W + \frac{K_j}{M_P} D_j W + \frac{K_i K_j}{M_P^2} D_i W - \frac{K_i K_j}{M_P} W - \frac{\Gamma_i}{M_P} D_k W$. Here $\phi^i$ and $\chi^i$ stand for the matter fields, and $\chi^i$ collectively denotes on arbitrary filed including the inflaton $\Psi_0$. And the matter-scalar production is represented by the kinetic term and the scalar potential

$$-e^{-1}L = iK_{ij}^* \partial_\mu \phi^{i\sigma} \partial^\mu \phi^{*\sigma} + e^{K/M_P^2} \left\{ K_{ij}^* (D_i W)(D_j \bar{W}) - \frac{3}{M_P^2} |W|^2 \right\}.$$  

(136)

In the model superpotential the supersymmetric visible sector contains the following renormalizable interactions

$$W \ni y_t Q_3 t^c H_u + \frac{1}{2}M_R N^c N^c,$$  

(137)

where the first term is the top quark operator as in [1] and the second term comes from Eq. (5) after the $U(1)_X$ is spontaneously broken. The partial decay width of the inflaton through the neutrino Yukawa coupling is [72]

$$\Gamma_{\Psi_0}^{N(\text{sugra})} = \Gamma(\Psi_0 \to N^c N^c) + \Gamma(\Psi_0 \to \tilde{N}^c \tilde{N}^c)$$

$$\simeq 2 \times \frac{c_N}{32\pi} m_{\Psi_0} \left( 1 - \frac{4M^2}{m_{\Psi_0}^2} \right)^{1/2},$$  

(138)
where \( c_N \simeq e^{K/M_P^2} \left| \frac{\partial}{\partial \phi} W_{\phi^* \phi} - 2 \Gamma_{\phi^* \phi} \right|^2 \); (sum over \( k \)) and the heavy neutrino mass \( M \) given in Eq. (A19). For the minimal Kahler potential, for simplicity, using Eq. (119) the parameter \( c_N \) can be approximately given by

\[
c_N \approx \left( \frac{\langle \Psi_0 \rangle}{M_P} \right)^2 \left( \frac{M}{M_P} \right)^2 = \left( \frac{m_{3/2}}{m_{\Psi_0}} \right)^2 \left( \frac{\mu (t_I)}{M_P} \right)^2 \left( \frac{M}{M_P} \right)^2 \tag{139}
\]

where in the last equality the inflaton mass \( m_{\Psi_0} \) in Eq. (59) or Eq. (118) is used. And the partial decay width of the inflaton through the top quark Yukawa coupling is \( 72 \).

Similarly, the parameter \( c_t \) is approximately given by

\[
c_t \approx \left( \frac{\langle \Psi_0 \rangle}{M_P} \right)^2 |y_t|^2 = \left( \frac{m_{3/2}}{m_{\Psi_0}} \right)^2 \left( \frac{\mu (t_I)}{M_P} \right)^2 |y_t|^2. \tag{141}
\]

In addition, the decay rate into the visible sector through the top and neutrino Yukawa couplings is much larger than that into the gluons and gluinos via the anomalies of SUGRA \( 72 \).

Then, from Eqs. (138) and (140) the inflaton decay rate through the gravitational coupling of the inflaton to matter is approximately given by

\[
\Gamma_{\Psi_0}^{\text{sugra}} \simeq \Gamma_{\Psi_0}^{\text{(sugra)}} + \Gamma_{\Psi_0}^{N(\text{sugra})} \\
\simeq \frac{m_{\Psi_0}}{16 \pi} \left( \frac{m_{3/2}}{m_{\Psi_0}} \right)^2 \left( \frac{\mu (t_I)}{M_P} \right)^2 \left\{ \frac{2 |y_t|^2}{8 \pi^2} \left( \frac{m_{\Psi_0}}{M_P} \right)^2 + \left( \frac{M}{M_P} \right)^2 \left( 1 - \frac{4 M^2}{m_{\Psi_0}^2} \right)^2 \right\}. \tag{142}
\]

Given that \( m_{\Psi_0} \sim 10^{13} \text{ GeV}, \mu (t_I) \sim 10^{16} \text{ GeV}, M \sim 10^9 \text{ GeV}, \) and \( m_{3/2} \sim \mathcal{O}(100) \text{ TeV}, \) we clearly have \( \Gamma_{\Psi_0}^{\text{vis}} \gg \Gamma_{\Psi_0}^{\text{sugra}} \) for \( g_{\Psi_0} \sim |g_\phi|, \) and the total decay rate of the inflaton field in Eq. (124) is approximately given by

\[
\Gamma_{\Psi_0} \simeq \Gamma_{\Psi_0}^{\text{vis}} \tag{143}
\]

which is much larger than \( \Gamma_{3/2} \) in Eq. (121).

Inserting Eqs. (121) and (143) into Eq. (122), the production of the gravitinos, depending on the size of the Higgs-inflaton coupling, has a lower bound

\[
Y_{3/2} \simeq 2.3 \times 10^{-17} \left( \frac{2.5 \times 10^{-10}}{g_{\Psi_0}} \right)^2 \left( \frac{T_{\text{reh}}}{70 \text{ TeV}} \right) \left( \frac{|g_f|}{0.94 \times 10^{-3}} \right)^3 \left( \frac{\mu (t_I)}{6.7 \times 10^{15} \text{ GeV}} \right)^5. \tag{144}
\]
Since the total yield $Y_{3/2} \approx Y_{3/2}^{\Psi_0}$ is inversely proportional to $|g_{\Psi_0}|^2$ and proportional to $T_{\text{reh}}$ ($Y_{3/2}^{\Psi}$ is also proportional to $T_{\text{reh}}$), it can provide a lower bound on the size of the Higgs-inflaton coupling, $|g_{\Psi_0}|$, with the given reheating temperature for the successful leptogenesis;

$$2.5 \times 10^{-11} \lesssim |g_{\Psi_0}| \lesssim |g_{\tilde{\psi}}| \simeq 2.5 \times 10^{-10}.$$ (145)

We conclude that it is reasonable for the reheating temperature Eq. (110) derived from the successful leptogenesis to lie in the range $T_{\text{reh}} \simeq (59 - 84) \text{ TeV}$ in Eq. (134). The above yield of gravitino $Y_{3/2}$ in Eq. (144) with the reheating temperature for the successful leptogenesis is well constrained by the BBN constraints $Y_{3/2}^{\text{BBN}}$ in Ref. [18].

C. Dynamics of the waterfall fields and axino fields after inflation

Finally, we roughly describe the dynamics of the waterfall (PQ) fields after inflation. After the inflation ends, the inflaton $\Psi_0$ and waterfall field $\tilde{\Psi}$ start to oscillate and their decays produce a dilute thermal plasma formed by collisions of relativistic decay products. During the epoch when the energy density of the universe is still dominated by the oscillating inflaton and waterfall field, their oscillations behave as matter, so their amplitudes $|\Psi_0|$ and $|\tilde{\Psi}|$ decrease proportional to $H \propto 1/t \propto R^{-3/2}$. As described Sec. III D, the waterfall field $\Psi$ quickly rolls down to the flat direction (see Eq. (60)), $\tilde{\Psi} \Psi = \mu^2_{\Psi}$; actually, the flat direction is not flat at this stage, because the waterfall field $\Psi$ obtain mass of $m_{\Psi} \approx |\tilde{g}_7||\Psi_0|$ before the inflaton $\Psi_0$ and waterfall field $\tilde{\Psi}$ decay, see Eq. (59). Thus, the waterfall fields are stabilized at $\langle \Psi \rangle = \langle \tilde{\Psi} \rangle = \mu_\Psi$ at this epoch and both fields oscillate around $\mu_\Psi$. Since the gravitino mass is larger than $|\tilde{g}_7||\Psi_0|$, i.e. $m_{3/2} > |\tilde{g}_7||\Psi_0|$, the saxion $h_{\Psi}$ begins to move toward the true minimum; in addition, due to already $H(T) < m_{3/2}$, and hence the friction is not efficient, so the saxion adiabatically approaches to the true minima in Ref. [4] without oscillation. And also, even the inflaton $\Psi_0$ can decay into axinos, but its process can not be used as a reheating process since the produced axinos could not thermalize. In the following we will see this is the case.

In the gravity-mediated scenario, the axino mass is likely to be greater than the gravitino mass [86]. Since the gravitino mass serves as the order parameter for the spontaneous breaking of SUGRA when the cosmological constant is zero, one can estimate the axino mass. The goldstino field overlaps with a chiral superfield $C_i$ which acquires a VEV along
scalar \(c_i\) and auxiliary component \(F_i\). In the decoupling limit of \(\text{SUSY}_{\text{inf}} \times \text{SUSY}_{\text{vis}}\), when \(C_i\) acquires a VEV equal to \(F_i\), SUSY is spontaneously broken and thus there should exist a corresponding massless fermion, goldstino. The goldstino superfield \(C_i\) is non-linearly parameterized [86, 87] as

\[
C_i = e^{Q_{\theta_i} / \sqrt{F_i}} (c_i + \theta^2 F_i) \\
= c_i + \frac{\eta_i^2}{2F_i} + \sqrt{2} \theta_i + \theta^2 F_i, \tag{146}
\]

where \(Q = \partial / \partial \theta\) is the generator of SUSY transformations, \(\theta\) is a Grassmann variable, and all derivative couplings are neglected; \(\eta_i\) is the goldstino associated with the \(F\)-term breaking of SUSY \((i = \inf, \vis \text{ in the absence of direct gravitational couplings})\). The goldstino is electrically neutral, \(R = -1\), Majorana chiral fermion. When SUSY is broken in the presence of SUGRA, the \(\text{SUSY}_{\text{inf}}\) corresponding to the genuine SUGRA symmetry is gauged and thus its corresponding goldstino \(^{28}\) \(\chi\) (one linear combination of the fermionic component of the chiral superfield \(C_i\)) is eaten by the gravitino via the super-Higgs mechanism, leaving behind the approximate global symmetry \(\text{SUSY}_{\text{vis}}\) which is explicitly broken by SUGRA and thus its corresponding the uneaten goldstino \(\eta\) as a propagating degree of freedom. Here the physical state \(\chi(=\eta_{\inf \cos \theta + \eta_{\text{vis}} \sin \theta})\) and \(\eta(=\eta_{\inf} \sin \theta + \eta_{\text{vis}} \cos \theta)\) could be linear combinations of goldstinos, \(\eta_{\inf}\) and \(\eta_{\vis}\), generated in each secluded sector. The interaction state \(\eta_i' = (\eta_{\inf}, \eta_{\vis})^T\) can be expressed in terms of the physical state \(\eta_a = (\chi, \eta)^T\) with a \(2 \times 2\) unitary mixing matrix \(V_i{}^a\), i.e. \(\eta_i' = V_i{}^a \eta_a\).

In the unitary gauge where all terms proportional to \(\chi = G_i \eta^i\) vanish identically, the remaining fermions have a quadratic Lagrangian in the interaction basis \((\eta_1', \eta_2') \equiv (\eta_{\inf}, \eta_{\vis})\)

\[
-e^{-1} \mathcal{L} = iK_{ij} \bar{\eta}_i \bar{\psi}_i \sigma_\mu \mathcal{D}^\mu \eta_j + \frac{1}{2} m''_{ij} \bar{\eta}_i \eta_j \eta_j' + \frac{1}{2} m''_{ij} \bar{\eta}_i \eta_j' \eta_j' + m_{3/2} \left( \psi_\mu \sigma^{\mu
u} \psi_\nu + \bar{\psi}_\mu \bar{\sigma}^{\mu
u} \bar{\psi}_\nu - \epsilon^{\mu\rho\sigma} \bar{\psi}_\mu \bar{D}_\rho \psi_\sigma \right), \tag{147}
\]

where \(\mathcal{D}_\mu\) and \(\bar{\mathcal{D}}_\rho\) are general covariant derivatives and \(m''_{ij} = m_{3/2} M_P^2 \{ \nabla_i G_j + G_i G_j / 3 \}\) is the uneaten goldstino mass matrix [79]. The interaction state \(\eta_i'\) could be expressed in terms of the physical state \(\eta\) with a relation \(\tan \theta = F_{\text{vis}} / F_{\text{inf}}\): \(\eta_1' = -\eta \sin \theta\) and \(\eta_2' = \eta \cos \theta\). Since the direction \(\chi\) corresponding to the eaten goldstino has a zero mass eigenvalue, in a basis where \(\chi\) couples only derivatively its Lagrangian can be written as \(\mathcal{L}_{\text{eff}} = -\chi \partial_\mu \bar{J}_\mu / |F_Z| + \text{h.c.}\)

\(^{28}\) Note here the notation \(\chi\) is different from the one used in Eq. (135).

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where \( \tilde{J}^\mu \) is the supercurrent. And from Eqs. (43) and (114) since the \( z \) field is responsible for the SUSY breaking, from the condition for the vanishing cosmological constant (and hence flat space), \( i.e., \langle G^2 G_z \rangle = 3 \), we obtain \( |G_z| \simeq \sqrt{3}/M_P \) (see above Eq. (71)), leading to the effective SUSY breaking scale set by:

\[
|F_z| = \sqrt{F_{\text{inf}}^2 + F_{\text{vis}}^2} = \sqrt{|V_{\text{AdS}}|}
\]

\[
\simeq \sqrt{3} M_P m_{3/2}. \tag{148}
\]

Then the Lagrangian (147) for goldstino can be expressed in terms of the physical state \( \eta \)

\[ e^{-1} \mathcal{L} = i \bar{K}_{\bar{a}b} \bar{\eta}^\alpha D^\mu \eta^b - \frac{1}{2} m_{ab} \bar{\eta}^a \eta^b - \frac{1}{2} m_{ab} \bar{\eta}^a \eta^b, \tag{149} \]

where \( \bar{K}_{\alpha} = [V^\dagger_j K_{ij} V_i] \) is the Kahler metric with the true goldstino direction removed, and \( m_{ab} = [V^T m' V]_{ab} \). The remaining uneaten goldstino mass as axino mass can be determined by the physical mass-squared matrix

\[ m_{ab}^2 = m_a^\ell m_b^\ell, \tag{150} \]

with \( m_a^\ell = m_{ak} G^{\ell k} = m_{3/2} G^{\ell k} (\nabla_a G_k + \frac{1}{3} G_a G_k) \) and \( m_{b\ell} = m_{3/2} M_P^2 (\nabla_b G_\ell + \frac{1}{3} G_b G_\ell) \) in Ref. [79]. The condition for the potential minimum in Eq. (115) read

\[ \langle G^z (X_{za} - \frac{W_z W_a}{W^2}) + G_a \rangle = 0, \tag{151} \]

where \( X_{za} = K_{za} / M_P^2 + W_{za} / W - \frac{1}{3} G_{a} G_{j} \). Using \( G_z / W \simeq G_a / W \), and \( \langle G^z G_z \rangle = 3 \) in Eq. (115), we obtain

\[ X_{za} \simeq 2 \frac{G_a}{G^2}, \quad \nabla_a G_z \simeq \frac{2 G_a}{M_P^2 K^{a\bar{a}} G_{\bar{a}}} - G_z G_a, \tag{152} \]

and consequently, \( m_a^\ell = m_{3/2} (2 \delta_a^\ell - \frac{2}{3} M_P^2 K^{a\bar{a}} G_z G_a) \). Thus the mass matrix \( m_{ab} \) can be written as

\[ m_{ab} = m_a^\ell m_b^\ell \]

\[ \simeq m_{3/2} (2 \delta_{a b} - \frac{2}{3} G_a G^\ell \delta_{b\ell}). \tag{153} \]

Since the uneaten goldstino \( \eta \) is orthogonal to \( \chi = G_a \eta^a \), the second term in the bracket is irrelevant. We obtain that the axino mass is equivalent to \( m_{\bar{a}} \simeq 2 m_{3/2} \) as in Ref. [86].

Next we describe that, after the inflaton \( \Psi_0 \) and waterfall field \( \tilde{\Psi} \) decays, how the waterfall fields \( \Psi \) and \( \tilde{\Psi} \) could remain trapped in the true minima. The inflaton \( \Psi_0 \) and waterfall
field \( \tilde{\Psi} \) decays thermalize the universe and the decay products interact among others in the thermal bath. In the model, the waterfall fields \( \Psi \) and \( \tilde{\Psi} \) interact with the SM particles through the Yukawa couplings and QCD couplings. The PQ field \( \Psi \) decays as a result of scattering with the thermalized decay products of the inflaton \( \Psi_0 \) and waterfall field \( \tilde{\Psi} \). And the temperature of this dilute plasma behaves roughly as

\[
T \simeq \left( \frac{(T_{\text{reh}})^2 H M_P}{\alpha_s} \right)^{\frac{1}{4}}.
\]

Then, the effective potential for \( \tilde{\Psi} \) induced by thermal plasma (similarly for \( \Psi \)) is only provided by

\[
V_{\text{th}} = a_g \alpha_s^2 T^4 \ln \left( \frac{|\tilde{\Psi}|^2}{T^2} \right),
\]

where \( a_g \) is a constant and \( \alpha_s \equiv g_s^2/4\pi \), which lifts up the flat direction \( \Psi \tilde{\Psi} = \mu_\Psi^2 \). Then we obtain an effective thermal mass for \(|\tilde{\Psi}|\)

\[
m_{\text{th}}^2 = \frac{1}{2} \frac{\partial^2 V}{\partial \tilde{\Psi}^2} = \frac{\alpha_s^2 T^4}{|\Psi|^2}.
\]

The evolution of \( \Psi \) is now described by the equation of motion

\[
\ddot{\Psi} + 3H(T) \dot{\Psi} + \frac{\partial V_{\text{tot}}}{\partial \tilde{\Psi}^*} \simeq 0,
\]

where \( V_{\text{tot}} = V_0(t_I) + V_{\text{th}} \). After the reheating process finishes, both \( \Psi \) and \( \tilde{\Psi} \) fields stay at \( \mu_\Psi \). Since the thermal mass is larger than the Hubble parameter, \( m_{\text{th}} > H(T) \), \( \Psi \) rolls down the thermal potential and \(|\Psi|^2\) increases until \( m_{\text{th}} \sim H(T) \) with \( H_{\text{osc}} \simeq (a_g M_P)^{1/3}(\alpha_s T_{\text{reh}})^{2/3} \).

Then the \( \Psi \) stops rolling and gets trapped with the initial amplitude \(|\Psi|^2 \sim \alpha_s(T) M_P \). At this stage the gravitino mass is larger than the thermal mass \( m_{\text{th}} \sim a_g^{1/6} \alpha_s^{1/3}(T_{\text{reh}}^4/M_P)^{1/3} \), and hence the radial components of the fields \( \Psi \) and \( \tilde{\Psi} \) are stabilized at the true minima.

V. CONCLUSION

The model is based on the \( SM \times U(1)_X \times A_4 \) symmetry, which is essential for the flavored PQ axions at low energy. Note that the \( U(1)_X \)-charged Kahler moduli superfields put the GS anomaly cancellation mechanism into practice. As the \( U(1)_X \) breaking scales according to Ref. [4] are secluded by the Gibbons-Hawking temperature \( T_{\text{GH}} = H_I/2\pi \), the model is
designed in a way that gravitational interactions explicitly break supersymmetry (SUSY) down to SUSY$_{\text{inf}} \times$SUSY$_{\text{vis}}$, where SUSY$_{\text{inf}}$ corresponds to the supergravity symmetry, while the orthogonal SUSY$_{\text{vis}}$ is approximate global symmetry. Hence, in the presence of SUGRA, the SUSY$_{\text{inf}}$ is gauged and thus its corresponding goldstino is eaten by the gravitino via super-Higgs mechanism, leaving behind the approximate global symmetry SUSY$_{\text{vis}}$ which is explicitly broken by SUGRA and thus its corresponding the uneaten goldstino as a physical degree of freedom giving masses to all the supersymmetric SM superpartners.

In order to provide a lucid explanation for inflation we have considered a realistic supersymmetric moduli stabilization. Such moduli stabilization has moduli backreaction effects on the inflationary potential, in particular, the spectral index of inflaton fluctuations. During inflation the universe experiences an approximately dS phase with the inflationary Hubble constant $H_I \simeq 2 \times 10^{10}$ GeV. In the present inflation model which provides intriguing links to UV-complete theories like string theory, the PQ scalar fields $\Psi(\bar{\Psi})$ play a role of the waterfall fields, that is, the PQ phase transition takes place during inflation such that the PQ scale $\mu_{\Psi}(t_I)$ during inflation is fixed by the amplitude of the primordial curvature perturbation and turns out to be roughly $0.7 \times 10^{16}$ GeV. We have found that such moduli stabilization with the moduli backreaction effects on the inflationary potential could lead to the energy scale of inflation in a way that the power spectrum of the curvature perturbation and the scalar spectral index are to be well fitted with the Planck 2015 observation [13]. And we have driven that the inflaton mass during inflation is given by $m_{\Psi_0} = \sqrt{3} H_I$ which is much larger than the gravitino mass, and its mass is in agreement with its theory prediction for spectral index with observation.

Through the introduction of $U(1)_X$ symmetry in a way that the $U(1)_X-[\text{gravity}]^2$ anomaly-free condition together with the SM flavor structure demands additional sterile neutrinos as well as no axionic domain-wall problem, the additional neutrinos may play a crucial role as a bridge between leptogenesis and new neutrino oscillations along with high energy cosmic events. We have shown that a successful leptogenesis scenario could be naturally implemented through Affleck-Dine mechanism. The pseudo-Dirac mass splittings, which is suggested from new neutrino oscillations along with high energy cosmic events, strongly indicate the existence of lepton-number violation which is a crucial ingredient of the present leptogenesis scenario. The resultant baryon asymmetry is constrained by the cosmological observable (i.e. the sum of active neutrino masses) with the new high energy
neutrino oscillations. In addition, the resultant baryon asymmetry, which crucially depends on the reheating temperature, is suppressed for relatively high reheating temperatures. We have shown that the right value of BAU, $Y_{\Delta B} \simeq 8 \times 10^{-11}$ prefers a relatively low reheating temperature with the well constrained pseudo-Dirac mass splittings responsible for new oscillations $\Delta m_i^2$. Moreover, we have shown that it is reasonable for the reheating temperature $T_{\text{reh}} \simeq (59 - 84)$ TeV derived from the non-gravitational decays of the inflaton and waterfall field to be compatible with the required reheating temperature for the successful leptogenesis, leading to $\Delta m_i^2 \sim 10^{-12-13}$ eV$^2$. And we have shown that, even the gravitational coupling is universal, it is too weak to cause the reheating in the present model. We have stressed that the present model requires $m_{3/2} \simeq \mathcal{O}(100)$ TeV gravitino mass in order to have suitable large gaugino masses.
Appendix A: The $A_4$ Group

The group $A_4$ is the symmetry group of the tetrahedron, isomorphic to the finite group of the even permutations of four objects. The group $A_4$ has two generators, denoted $S$ and $T$, satisfying the relations $S^2 = T^3 = (ST)^3 = 1$. In the three-dimensional complex representation, $S$ and $T$ are given by

$$S = \frac{1}{3} \begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}. \quad (A1)$$

$A_4$ has four irreducible representations: one triplet $\mathbf{3}$ and three singlets $\mathbf{1}, \mathbf{1}'', \mathbf{1}''$. An $A_4$ singlet $a$ is invariant under the action of $S$ ($Sa = a$), while the action of $T$ produces $Ta = \omega a$ for $\mathbf{1}$, $Ta = \omega a$ for $\mathbf{1}'$, and $Ta = \omega^2 a$ for $\mathbf{1}''$, where $\omega = e^{2\pi i/3} = -1/2 + i\sqrt{3}/2$ is a complex cubic-root of unity. Products of two $A_4$ representations decompose into irreducible representations according to the following multiplication rules: $\mathbf{3} \otimes \mathbf{3} = \mathbf{3}_s \oplus \mathbf{3}_a \oplus \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$, $\mathbf{1}' \otimes \mathbf{1}'' = \mathbf{1}$, $\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}''$ and $\mathbf{1}'' \otimes \mathbf{1}'' = \mathbf{1}'$. Explicitly, if $(a_1, a_2, a_3)$ and $(b_1, b_2, b_3)$ denote two $A_4$ triplets, then we have Eq. (A2).

Four irreducible representations are $\mathbf{3}, \mathbf{1}, \mathbf{1}', \mathbf{1}''$ with $\mathbf{3} \otimes \mathbf{3} = \mathbf{3}_s \oplus \mathbf{3}_a \oplus \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}''$, and $\mathbf{1}' \otimes \mathbf{1}' = \mathbf{1}''$. The details of the $A_4$ group are shown in Appendix A. Let $(a_1, a_2, a_3)$ and $(b_1, b_2, b_3)$ denote the basis vectors for two $\mathbf{3}$’s. Then, we have

$$ (a \otimes b)_{\mathbf{3}_s} = \frac{1}{\sqrt{3}} (2a_1b_1 - a_2b_3 - a_3b_2, 2a_3b_3 - a_2b_1 - a_1b_2, 2a_2b_2 - a_3b_1 - a_1b_3), $$

$$ (a \otimes b)_{\mathbf{3}_a} = i(a_3b_2 - a_2b_3, a_2b_1 - a_1b_2, a_1b_3 - a_3b_1), $$

$$ (a \otimes b)_{\mathbf{1}} = a_1b_1 + a_2b_3 + a_3b_2, $$

$$ (a \otimes b)_{\mathbf{1}'} = a_1b_2 + a_2b_1 + a_3b_3, $$

$$ (a \otimes b)_{\mathbf{1}''} = a_1b_3 + a_2b_2 + a_3b_1. \quad (A2) $$

To make the presentation of our model physically more transparent, we define the $T$-flavor quantum number $T_f$ through the eigenvalues of the operator $T$, for which $T^3 = 1$. In detail, we say that a field $f$ has $T$-flavor $T_f = 0, +1, \text{ or } -1$ when it is an eigenfield of the $T$ operator with eigenvalue $1, \omega, \omega^2$, respectively (in short, with eigenvalue $\omega^{T_f}$ for $T$-flavor $T_f$, considering the cyclical properties of the cubic root of unity $\omega$). The $T$-flavor is an additive quantum number modulo 3. We also define the $S$-flavor-parity through the eigenvalues of
the operator $S$, which are +1 and -1 since $S^2 = 1$, and we speak of $S$-flavor-even and $S$-
flavor-odd fields. For $A_4$-singlets, which are all $S$-flavor-even, the 1 representation has no $T$-
flavor ($T_f = 0$), the 1' representation has $T$-flavor $T_f = +1$, and the 1'' representation has
$T$-flavor $T_f = -1$. Since for $A_4$-triplets, the operators $S$ and $T$ do not commute, $A_4$-triplet
fields cannot simultaneously have a definite $T$-flavor and a definite $S$-flavor-parity.

The real representation, in which $S$ is diagonal, is obtained through the unitary transforma-
tion

$$A \rightarrow A' = U_{\omega} A U_{\omega}^\dagger,$$  \quad (A3)

where $A$ is any $A_4$ matrix in the real representation and

$$U_{\omega} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}. \quad (A4)$$

We have

$$S' = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad T' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$  \quad (A5)

For reference, an $A_4$ triplet field with $T$-flavor eigenfields $(a_1, a_2, a_3)$ in the complex rep-
resentation can be expressed in terms of components $(a_{R1}, a_{R2}, a_{R3})$ as

$$a_{1R} = \frac{a_1 + a_2 + a_3}{\sqrt{3}}, \quad a_{2R} = \frac{a_1 + \omega a_2 + \omega^2 a_3}{\sqrt{3}}, \quad a_{3R} = \frac{a_1 + \omega^2 a_2 + \omega a_3}{\sqrt{3}}.$$  \quad (A6)

Inversely,

$$a_1 = \frac{a_{1R} + a_{2R} + a_{3R}}{\sqrt{3}}, \quad a_2 = \frac{a_{1R} + \omega^2 a_{2R} + \omega a_{3R}}{\sqrt{3}}, \quad a_3 = \frac{a_{1R} + \omega a_{2R} + \omega^2 a_{3R}}{\sqrt{3}}.$$  \quad (A7)

Now, in the $S$ diagonal basis the product rules of two triplets $(a_{R1}, a_{R2}, a_{R3})$ and $(b_{R1}, b_{R2}, b_{R3})$ according to $3 \otimes 3 = 3_s \oplus 3_a \oplus 1 \oplus 1' \oplus 1''$ are as follows

$$(a_R \otimes b_R)_{3_s} = (a_{R1} b_{R3} + a_{R3} b_{R1} + a_{R1} b_{R3} + a_{R1} b_{R2} + a_{R2} b_{R1}),$$
$$(a_R \otimes b_R)_{3_a} = (a_{R2} b_{R3} - a_{R3} b_{R2} - a_{R1} b_{R3} - a_{R1} b_{R2} + a_{R2} b_{R1}),$$
$$(a_R \otimes b_R)_1 = a_{1R} b_{1R} + a_{2R} b_{2R} + a_{3R} b_{3R},$$
$$(a_R \otimes b_R)_{1'} = a_{1R} b_{1R} + \omega^2 a_{2R} b_{2R} + \omega a_{3R} b_{3R},$$
$$(a_R \otimes b_R)_{1''} = a_{1R} b_{1R} + \omega a_{2R} b_{2R} + \omega^2 a_{3R} b_{3R}.$$  \quad (A8)
1. Lepton mass matrices

The model implicitly has two $U(1)_X \equiv U(1)_{X_1} \times U(1)_{X_2}$ symmetries which are generated by the charges $X_1 = -2p$ and $X_2 = -q$. The $A_4$ flavor symmetry along with the flavored PQ symmetry $U(1)_{X_1}$ is spontaneously broken by two $A_4$-triplets $\Phi_T, \Phi_S$ and by a singlet $\Theta$ in TABLE [I]. And the $U(1)_{X_2}$ symmetry is spontaneously broken by $\Psi, \tilde{\Psi}$, whose scales are denoted as $v_\Psi$ and $v_{\bar{\Psi}}$, respectively, and the VEV of $\Psi$ (scaled by the cutoff $\Lambda$) is assumed as

$$\langle \Psi \rangle \Lambda = \langle \tilde{\Psi} \rangle \Lambda = \lambda \sqrt{2}. \quad (A9)$$

Here the parameter $\lambda \approx 0.225$ stands for the Cabbibo parameter [29]. After getting VEVs $\langle \Theta \rangle, \langle \Phi_S \rangle \neq 0$ (which generates the heavy neutrino masses given by Eq. (A18)) and $\langle \Psi \rangle \neq 0$, the flavored PQ symmetry $U(1)_X$ is spontaneously broken at a scale much higher than the electroweak scale and is realized by the existence of the NG modes $A_{1,2}$ that couples to ordinary quarks and leptons at the tree level through the Yukawa couplings as in Ref. [4].

According to the simple basis rotation by Lim and Kobayashi [88], we perform basis rotations from weak to mass eigenstates in the leptonic sector,

$$\left( \nu_L \right) \rightarrow W_\nu^\dagger \left( \begin{array}{c} \nu_L \\ S^c_R \end{array} \right) = \xi_L . \quad (A10)$$

Here the transformation matrix $W_\nu$ is unitary, which is given by

$$W_\nu = \left( \begin{array}{cc} U_L & 0 \\ 0 & U_R \end{array} \right) \left( \begin{array}{cc} V_1 & iV_1 \\ V_2 & -iV_2 \end{array} \right) Z, \quad \text{with} \quad Z = \left( \begin{array}{ccc} e^{i \frac{\pi}{4}} \cos \theta & -e^{i \frac{\pi}{4}} \sin \theta \\ e^{-i \frac{\pi}{4}} \sin \theta & e^{-i \frac{\pi}{4}} \cos \theta \end{array} \right) \quad (A11)$$

where the $3 \times 3$ matrix $U_L$ participates in the leptonic mixing matrix, the $3 \times 3$ matrix $U_R$ is an unknown unitary matrix and $V_1$ and $V_2$ are the diagonal matrices, $V_1 = \text{diag}(1, 1, 1)/\sqrt{2}$ and $V_2 = \text{diag}(e^{i \phi_1}, e^{i \phi_2}, e^{i \phi_3})/\sqrt{2}$ with $\phi_i$ being arbitrary phases. Then the $6 \times 6$ light neutrino mass matrix in Eq. (9) is diagonalized as

$$W_\nu^T M_\nu W_\nu = Z^T \left( \begin{array}{cc} \tilde{M}_{\nu\nu} & \tilde{M} \\ \tilde{M}^T & \tilde{M}_S \end{array} \right) Z \equiv \text{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}, m_{s_1}, m_{s_2}, m_{s_3}) \quad (A12)$$

with

$$\tilde{M}_{\nu\nu} = U_L^T M_{\nu\nu} U_L, \quad \tilde{M}_S = U_R^T M_S U_R, \quad \tilde{M} = U_R^T m_{DS} U_L \equiv \text{diag}(m_1, m_2, m_3). \quad (A13)$$
The charged lepton mass term and the Dirac and Majorana neutrino mass terms read

\[
M_\ell = \begin{pmatrix}
y_e & 0 & 0 \\
0 & y_\mu & 0 \\
0 & 0 & y_\tau
\end{pmatrix}, \quad m_d = \begin{pmatrix}
y_e^0 & 0 & 0\\
0 & y_\mu^0 & 0\\
0 & 0 & y_\tau^0
\end{pmatrix} \frac{1}{\sqrt{2}} v_d,
\]

(A14)

\[
m_{DS} = \begin{pmatrix}
y_1^s & 0 & 0 \\
0 & y_2^s & 0 \\
0 & 0 & y_3^s
\end{pmatrix} \left( \frac{v_\Psi}{\sqrt{2} \Lambda} \right)^{16} v_u,
\]

(A15)

\[
M_S = \begin{pmatrix}
y_1^{ss} & 0 & 0 \\
0 & y_2^{ss} & 0 \\
0 & y_3^{ss}
\end{pmatrix} \left( \frac{v_\Psi}{\sqrt{2} \Lambda} \right)^{51} \frac{v_\Theta}{\sqrt{2} \Lambda},
\]

(A16)

\[
m_D = \begin{pmatrix}
y_1^\nu & 0 & 0 \\
0 & y_2^\nu & 0 \\
0 & y_3^\nu
\end{pmatrix} \frac{v_T}{\sqrt{2} \Lambda} \left( \frac{v_\Psi}{\sqrt{2} \Lambda} \right)^{9} v_u = \begin{pmatrix}
y_1^\nu \\
0 & y_2 \\
0 & y_3
\end{pmatrix} \frac{v_T}{\sqrt{2} \Lambda} \left( \frac{v_\Psi}{\sqrt{2} \Lambda} \right)^{9} v_u,
\]

(A17)

\[
M_R = \begin{pmatrix}
1 + \frac{2}{3} \kappa e^{i \phi} & -\frac{1}{3} \kappa e^{i \phi} & -\frac{1}{3} \kappa e^{i \phi} \\
-\frac{1}{3} \kappa e^{i \phi} & \frac{2}{3} \kappa e^{i \phi} & 1 - \frac{1}{3} \kappa e^{i \phi} \\
-\frac{1}{3} \kappa e^{i \phi} & 1 - \frac{1}{3} \kappa e^{i \phi} & \frac{2}{3} \kappa e^{i \phi}
\end{pmatrix} M,
\]

(A18)

where \( v_d \equiv \langle H_d \rangle = v \cos \beta / \sqrt{2} \), and \( v_u \equiv \langle H_u \rangle = v \sin \beta / \sqrt{2} \) with \( v \approx 246 \) GeV, and

\[
y_2 \equiv \frac{\hat{y}_2^\nu}{\hat{y}_1^\nu}, \quad y_3 \equiv \frac{\hat{y}_3^\nu}{\hat{y}_1^\nu}, \quad \kappa \equiv \sqrt{\frac{3}{2} \hat{y}_R v_S / M}, \quad \phi \equiv \arg \left( \frac{\hat{y}_R}{\hat{y}_\Theta} \right) \text{ with } M \equiv \left| \hat{y}_\Theta \frac{v_\Theta}{\sqrt{2}} \right|.
\]

(A19)

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