Supertube domain-walls and elimination of closed time-like curves in string theory

Nadav Drukker

The Niels Bohr Institute, Copenhagen University
Blegdamsvej 17, DK-2100 Copenhagen, Denmark.
drukker@nbi.dk

Abstract:
We show that some novel physics of supertubes removes closed time-like curves from many supersymmetric spaces which naively suffer from this problem. The main claim is that supertubes naturally form domain-walls, so while analytical continuation of the metric would lead to closed time-like curves, across the domain-wall the metric is non-differentiable, and the closed time-like curves are eliminated. In the examples we study the metric inside the domain-wall is always of the Gödel type, while outside the shell it looks like a localized rotating object, often a rotating black hole. Thus this mechanism prevents the appearance of closed time-like curves behind the horizons of certain rotating black holes.

Keywords: domain-walls, closed time-like curves, supertubes, DLCQ.
1. Introduction

Many solutions of general relativity and supergravity exhibit closed time-like curves, though they have sources that seemingly are physical. One example is the Gödel universe [1], which describes a rotating space with negative cosmological constant. Some supersymmetric solutions of supergravity were found in [2], which are similar in many ways to the original metric of Gödel, but without a cosmological constant, as the metric of Som and Raychaudhuri [3].

These constructions raised the question if string theory can live on spaces with closed time-like curves, and some interesting ideas related to holography were raised in [4]. Many other papers, including [5] - [21] looked at these spaces, as well as other metrics with closed time-like curves in string theory and supergravity, and studied the dynamics of different objects in them. This is a new twist on the question if spaces that violate causality are good backgrounds for physics, see for example [22, 23, 24].

The metrics of the Gödel type describe homogeneous spaces, which can be represented as rotating about any given point. Choosing a preferred point there is a certain region around it which does not contain closed time-like curves. This region is bounded by a surface made up of closed null curves, usually called the “velocity of light surface.” The physics restricted only to this region is totally causal, and causality violation requires travelling outside this domain.
Thus closed time-like curves may be prevented if the metric is changed beyond the velocity of light surface, which can be done by putting a domain-wall there. In a previous paper \cite{25} it was shown that this happens naturally in a certain example of a supersymmetric Gödel universe with rotation in a single plane\(^1\). The purpose of this paper is to prove that this mechanism is quite general, and in many examples supertubes will form domain-walls and prevent the appearance of closed time-like curves.

The Gödel like universes are quite sick, as they have very peculiar asymptotics. But there are many solutions of general relativity and supergravity which are asymptotically flat, but contain closed time-like curves. When examining such rotating metrics it’s quite common to find that the metric near the source rotates fast enough to create closed time-like curves. Again one may define a “velocity of light surface”, outside of which causality is protected. The simplest examples of such metrics are rotating black holes, where the velocity of light surface is at the inner-horizon.

Applying our mechanism we will show that in certain examples a domain-wall of supertubes at the velocity of light surface will source these metrics. This will preserve the asymptotic form of the metric, but will change it beyond the velocity of light surface. Across the domain-wall the metric is continuous, but not differentiable, so looking inside the domain-wall one would find a metric that has no sources, is rotating, and whose analytical continuation would have closed time-like curves beyond the domain-wall. Thus the metric inside is just the Gödel like spaces mentioned above.

Our main claim is thus that the regions with closed time-like curves in Gödel’s space, as well in the rotating black holes are the wrong analytical continuation of the metric. In fact one must include a source that forms a domain-wall, thus the metric will not be analytic but will continue from a piece of Gödel universe to a rotating asymptotically flat space.

The obvious question that would arise, is how can a supertube, which is a three-dimensional object, a tubular D2-brane\(^2\) fill out nine-dimensions and form a domain-wall? Usually in string theory D-branes are localized objects, but we will show that a supertube becomes delocalized as it approaches a velocity of light surface.

The statement that a D-brane is localized is based on the fact that the scalars in the world-volume theory on the brane get a VEV. The location of the D-brane, is just this VEV, and all possible locations are described by the moduli space of the relevant gauge theory. We will show that the entire velocity of light surface, though it has six dimensions transverse to the supertube is actually a single point in the supertube moduli space. Thus a supertube cannot sit at a given location on the velocity of light surface, rather is smeared over it.

There is another example of such an effect in string theory, dubbed the enhançon mechanism \cite{28}. There the naive metric of the D6-brane wrapping a K3 surface has a naked singularity, but by looking at the moduli space of the gauge theory living on the brane one realizes that it actually cannot be brought too close to the origin, and forms a spherical domain-wall. Thus the naked

\(^1\)This same idea was employed in general relativity for the original Gödel metric in \cite{26}.

\(^2\)We will also look at other examples, which are dual to the regular supertube, as well as the three charge one constructed recently in \cite{27}, which is a tubular D6-brane. But none of those objects are nine-dimensional, as a domain-wall should be.
singularity is replaced by the domain-wall.

Our example is a very close analogy, with the D6-brane replaced by the supertube (or one of its duals). In both examples we have the same amount of supersymmetry, eight supercharges (or \( \mathcal{N} = 4 \) in three dimensions), but while the gauge theory on the D6-brane is well known, and the enhançon locus is the \( SU(2) \) point, the physics on the supertube is a non-commutative gauge theory, with a compact direction that becomes light-like (DLCQ) as the velocity of light surface is approached. As far as we know this field theory has never been studied.

In the next section we will study the supertube as a probe of certain spaces with closed time-like curves. We will make some comments on the field theory living on the brane, and calculate the metric on moduli space. As we shall see the velocity of light surface will correspond to a single point in moduli space, providing the justification for smearing the supertube on that locus. In the following section we apply this mechanism to several metrics, finding families of metrics which have the Gödel form behind the domain-wall and are asymptotically flat rotating metrics outside. There are many such metrics and we will not try to provide a complete survey of all of them, instead illustrating the technique on certain interesting examples. Perhaps the most interesting being the BMPV black hole \([29]\). Finally we end with a discussion on the relevance of this mechanism to causality protection in string theory and to the physics of black holes.

2. Supertube probe

2.1 Supertubes

Let us review the essential features of supertubes \([30]\). Those are bound states of D0-branes and fundamental strings with angular momentum that take up a cylindrical shape. The most convenient way to describe them is in terms of a cylindrical D2-brane, which carries magnetic and electric fluxes corresponding to the D0 and fundamental string charges respectively. Quite remarkably, by adjusting the magnetic and electric field the supertube can take any profile in the transverse space, and it will still be supersymmetric. The supersymmetry projectors are only those related to the D0-brane and fundamental string. The system preserves therefore 1/4 supersymmetry, or eight supercharges.

In flat space the supertube is described by the Dirac-Born-Infeld action

\[
\mathcal{L} = -\tau_2 \sqrt{-\det(g + 2\pi\alpha' F)} ,
\]

(2.1)

\(\tau_2 = 1/((2\pi)^2 \alpha'^3/2g_s)\) is the D2-brane tension, \(g_s\) is the string coupling, \(g\) is the induced metric, and \(F\) the field strength in the world-volume. As mentioned, we should turn on electric and magnetic fields, so taking the supertube extended in the \(t\) and \(y\) directions of space, and along a circle of radius \(R\) with an angular coordinate \(\varphi\), the ansatz is

\[
2\pi\alpha' F = E\, dt \wedge dy + B\, dy \wedge d\varphi ,
\]

(2.2)

the action for static configurations is

\[
\mathcal{L} = -\tau_2 \sqrt{R^2(1 - E^2) + B^2} .
\]

(2.3)
The integrals of the magnetic field and of the momentum conjugate to the electric field, \( \Pi \), are conserved quantities

\[
Q_s = 2\pi\alpha' \int d\varphi \Pi = 2\pi\alpha' \int d\varphi \frac{\delta L}{\delta E}, \quad Q_0 = \frac{1}{2\pi\alpha'} \int d\varphi B. \quad (2.4)
\]

The energy per unit length of the tube is given by the integral of the Hamiltonian density

\[
\tau = \tau_2 \int d\varphi R^{-1} \sqrt{(B^2 + R^2)(\Pi^2/\tau_2^2 + R^2)} \geq \tau_0 |Q_0| + \tau_s |Q_s|, \quad (2.5)
\]

where \( \tau_0 = 1/(2\pi g_s\alpha'^{1/2}) \) and \( \tau_s = 1/(2\alpha') \) are the D0-brane and string tensions. This bound is saturated for \( \tau_2 R^2 = Q_0 Q_s \), and from that one can also derive the condition \( E = 1 \). This condition is also necessary in order for the supertube to be supersymmetric.

The supertube carries angular momentum, which can be thought as the Pointing vector in the world-volume. Assuming the BPS bound is saturated, and all the D0-branes and string belong to the bound state, it is given by

\[
J = Q_0 Q_s = \tau_2 R^2. \quad (2.6)
\]

In general we do not have to assume that there is a single D2-brane, but any number \( N \). This means that from the D2-brane worldvolume point of view there is a \( U(N) \) gauge theory, and the relations above will be modified. For a given radius the angular momentum will be linear in the number of D2-branes and the total charge \( Q_0 Q_s \) will be \( N \) times larger still, because it is less efficient to get angular momentum out of the non-Abelian fields than from the Abelian ones. Thus we find

\[
J = \frac{Q_0 Q_s}{N} = \tau_2 N R^2. \quad (2.7)
\]

This issue is explained in detail in [31].

2.2 Probe analysis

As was shown in [25], supertube are interesting probes of certain type IIA supergravity solutions with closed time-like curves, specifically Gödel universes. We wish to study this probe further, and place it in some other geometries that have closed time-like curves, like some of the Gödel universes described in [5], or black hole solutions. Let us take the solutions of IIA supergravity of the supertube family given by [32, 33]

\[
ds^2 = - U^{-1}V^{-1/2}(dt - A)^2 + U^{-1}V^{1/2}dy^2 + V^{1/2} \sum_{i=1}^{8} (dx^i)^2 \\
B_2 = U^{-1}(dt - A) \wedge dy - dt \wedge dy, \\
C_1 = - V^{-1}(dt - A) + dt, \\
C_3 = - U^{-1} dt \wedge dy \wedge A, \\
e^\Phi = U^{-1/2} V^{3/4}.
\]
Here $U$ and $V$ are harmonic functions in the eight $x^i$ directions and $A$ is a one-form satisfying the Maxwell equation. Later we will concentrate on specific examples corresponding to Gödel universes with angular momentum in one or two planes, and to the two-charge black hole, but the probe analysis is actually much more general.

We consider a supertube in this background extended in the $y$ and $t$ directions and following some closed path in the transverse space. Let us label the periodic coordinate on the supertube by $\varphi$. In the world volume we turn on electric and magnetic fluxes given by

$$2\pi\alpha' F = E\, dt \wedge dy + B\, dy \wedge d\varphi.$$  \hspace{1cm} (2.9)

In addition there are the components of the NS-NS $B$-field pulled back to the worldvolume

$$\mathcal{P}[B_2] = (-1 + U^{-1})\, dt \wedge dy + U^{-1} f \, dy \wedge d\varphi,$$  \hspace{1cm} (2.10)

where $f$ is the pull-back of the one form in the periodic direction $f \, d\varphi = \mathcal{P}[A]$.

The metrics of the supertube type may have closed time-like curves when the space is rotating, so $A$ has an angular component. If the supertube is circular, and in a rotating plane, $f$ will be non-zero. The angular component of the metric is $V^{1/2}(r^2 - f^2/(UV))$, and it can be negative. For example, the Gödel metrics are given by constant $U$ and $V$ and $f = cr^2$, so when $r^2 > UV/c^2$, this angular direction becomes time-like.

At low energies the dynamics of the supertube will be described by some gauge theory. The simplest description of the system, as was done in [33] for a supertube in flat space, is in terms of a non-commutative gauge theory, where one absorbs the effect of the magnetic and electric fields in the open string metric and non-commutativity parameters [34]. To that end first apply a gauge transformation that sets the world-volume gauge fields to zero at the expense of the NS-NS $B$-field. Effectively the total $B$ field induced on the world volume is (after setting $E = 1$)

$$\hat{B}_2 = \mathcal{P}[B_2] + 2\pi\alpha' F = (U^{-1})\, dt \wedge dy + (U^{-1} f + B) \, dy \wedge d\varphi.$$  \hspace{1cm} (2.11)

This gives the open string metric and noncommutative parameter [35] (the lines and columns correspond to the $t$, $\varphi$ and $y$ directions)

$$G^{ab} + \Theta^{ab} = \left(\frac{1}{g + \hat{B}_2}\right)^{ab} = \frac{1}{B} \begin{pmatrix} -m & -V^{1/2} f + Vr^2/B \\ -V^{1/2} f + Vr^2/B & 0 & 1 \\ -(f + Vr^2/B) & -1 & V^{1/2}r^2/B \end{pmatrix},$$  \hspace{1cm} (2.12)

where

$$m = V^{1/2} \left(UB + 2f + \frac{Vr^2}{B}\right).$$  \hspace{1cm} (2.13)

With this we can immediately write the low energy action for the supertube, it’s a non-commutative $U(1)$ gauge theory in three dimensions on a manifold with the open string metric (2.12). The bosonic part of the action will include the Yang-Mills part, and in addition seven scalars $X^i$ parameterizing the transverse fluctuations of the supertube with metric $\gamma_{ij}$

$$\mathcal{L} = -\tau_2 e^{-\Phi} \sqrt{-g + \hat{B}_2} \left(\frac{1}{4} G^{ab} G^{cd} F_{ae} F_{bd} + \frac{1}{2} G^{ab} \gamma_{ij} D_a X^i D_b X^j\right).$$  \hspace{1cm} (2.14)
We can write the non-commutativity parameter as $\Theta = B^2/(Vr^2)(-dt + (BU + f)d\varphi) \wedge dy$. The coordinate $y$ is always a space-like direction, the other direction is space-like for $B^2 > r^2V/U$ and time-like for $B^2 < r^2U/V$.

In the open string metric (2.12) the square of the norm of the angular vector $\partial_{\varphi}$ is given by $m$, which we may rewrite as

$$G_{\varphi\varphi} = m = \frac{Vg_{\varphi\varphi}}{B} + \frac{V^{1/2}U^2}{B} \left( B + \frac{f}{U} \right)^2 .$$

(2.15)

This is the sum of the angular term in the closed string metric and a positive definite term. As long as the supertube wraps a space-like circle, $m$ is positive, but when the circle is a closed time-like curve, then for certain values of the magnetic field $m$ may be negative. This will correspond to a field theory on a manifold with a compact time.

Thus we have a two dimensional phase diagram with $r$ and $B$, and two curves on it $B = (V/U)^{1/2}r$ (light-like non-commutativity) and $m = 0$ (light-like compact direction).

Let us look at it in more detail in the case of Gödel’s universe, where $U = V = 1$ and $f = -cr^2$ and is depicted in Figure 1.

- For $B < 1/(2c)$ as one varies $r$, the curve $m = 0$ is not crossed, and there are two phases, for $r < B$ it’s space-like noncommutativity and for $r > B$ time-like noncommutativity.

- For $1/(2c) < B < 1/c$ at small $r$ there is space-like noncommutativity, then one crosses the $r = B$ curve to the time-like noncommutative phase, and finally at $m = 0$ one crosses over to a phase with compact time-like direction.

- For $B > 1/c$ one starts at small $r$ with space-like noncommutativity, then one reaches $m = 0$, beyond which there is a compact time-like direction, and finally one reaches time-like noncommutativity.

![Figure 1: The phases of the theory on the supertube probing Gödel’s space.](image-url)
We are quite accustomed to gauge theories with space-like non-commutativity, and when the non-commutativity parameter is time-like it’s also possible to make sense of the theory by adding extra open strings (NCOS theory) \footnote{the DLCQ of some six dimensional non-commutative theories were studied in \cite{40,41}.}. So it seems like as long as \( r < \frac{1}{c} \) all the probes are well behaved, with either space-like or time-like non-commutativity. None of our probes develops any sickness in that region, thus we think this region is safe and a reasonable string background.

At larger radii there are certain probes that will cross the \( m = 0 \) line, or have effectively a compact time-like direction. We do not know how to make sense of those theories, at most we can have theories with compact light-like directions (DLCQ, or discreet light-cone quantization). The problem first occurs at \( f^2 = U V r^2 \) for the supertube with \( B = -V^{1/2} U^{-1/2} r \), which has light-like non-commutativity in this compact direction. This case is particularly simple, because all the non-commutativity is in the \((\varphi, y)\) plane.

One approach to understand what happens to the supertube as it nears the velocity of light surface, or the point of DLCQ is to study the DLCQ limit of non-commutative gauge theories. In regular field theories this limit is a very subtle one, because an infinite number of zero-modes appear. This issue is much less severe for non-commutative field theories.

Let us recall the problem \footnote{the DLCQ of some six dimensional non-commutative theories were studied in \cite{40,41}.}, if the periodic direction \( \varphi \) becomes light-like, any field configuration that depends only on \( \varphi \) will be a zero mode. It’s just a free wave, and it can have arbitrary \( \varphi \) dependence, it is then hard to control the perturbative expansion with all those zero modes running around loops. If this compact direction is non-commutative we may use the operator formalism, then instead of our fields being functions of this direction, they are operators on a Hilbert space where \( \varphi \) and another direction (in our case \( y \)) are conjugate variables. This means that the \( y \) dependence and \( \varphi \) dependence of our field are correlated, so there are no longer the zero-modes that are functions only of \( \varphi \), but do not depend on \( y \).

One would therefore expect there to be at most a single zero mode for every field, and not an infinite degeneracy of them, and the perturbative expansion will be well defined.

For practical purposes, though, the operator formalism is not very convenient, and it is much easier to take the fields to be regular functions, but add the necessary phase in the Feynman diagrams to represent the Moyal product. This way the zero modes will show up again, but they will still not lead to divergent graphs. In non-commutative gauge theory the phases are always odd function of the momentum, typically \( \sin(p_\mu \Theta^{\mu\nu} k_\nu / 2) \) \footnote{the DLCQ of some six dimensional non-commutative theories were studied in \cite{40,41}.}. Since the zero modes do not carry momentum in one of the non-commutative directions, this phase factor kills the terms that would otherwise be divergent.

This very peculiar type of theory, the DLCQ limit of non-commutative field theory, should represent very interesting physics, but we will not pursue it here.\footnote{the DLCQ of some six dimensional non-commutative theories were studied in \cite{40,41}.} Instead we will study the moduli space of supertubes near this point where the compact direction becomes light-like. This will turn out to be sufficient for out purposes.

2.3 Moduli space

Our main claim is that when supertubes approach the velocity of light surface, they delocalize,
forming a domain-wall on the velocity of light surface. This is very similar to the enhançon mechanism, where a D6-brane wrapping a K3 surface becomes delocalized over a 2-sphere, again forming a domain-wall \cite{28}. In that example the gauge theory was quite well understood, and the enhançon point corresponds to a point of enlarged gauge symmetry.

The problem of a non-commutative gauge theory in the limit as the compact direction becomes light-like is not as well understood, and as stated above, will not be studied here in detail. But this will not be required for what we are trying to prove, it will be enough to look at the moduli space near this point of compact time.

Recall that the position of a D-brane is described in the world-volume theory by the vacuum expectation value of some scalar fields. It is usually the case that D-branes are localized, and cannot be smeared (if their dimensionality is not too small), and this corresponds to the fact that the scalar fields should always get a VEV. But this in fact breaks down in the case of the enhançon, where the scalar fields are W-bosons, and clearly do not get VEVs anymore in the enhanced symmetry point.

This can be seen by studying the moduli space of the theory on the D-brane. One may deform the shape of the supertube without breaking supersymmetry \cite{12, 13, 14}, but such deformations might change the total charges carried by the supertube, those of D0-branes, fundamental strings and angular momentum. It is still possible to consider those deformations, assuming the supertube is in a bath of other supertubes, D0-branes and strings that can absorb the charge difference, but we will try to avoid this, and focus only on real moduli.

If the supertube carries the maximal angular momentum for given charges, its shape cannot be deformed. In the case of a single angular momentum it will have a circular geometry (also in curved space it will be a circle in the coordinates \( x^i \) we are using). Then the only moduli are rigid translations of the supertube, which we will focus on.

The distance between two D-branes in space-time is given by the difference between the scalar VEVs, but the distance in field space is given by the metric on moduli space. This metric can be readily extracted from the low energy effective action (2.14), and is just proportional to the kinetic term for the scalars

\[
ds^2 = M_{ij} dX^i dX^j. \tag{2.16}
\]

Here \( X^i \) represent the center of mass of the tube and the coefficient \( M \) is the average value of \( m \)

\[
M_{ij} = \frac{1}{2\pi} \int d\varphi \frac{m}{B} e^{-\varphi} \gamma_{ij} \sqrt{-(g + \hat{B}_2)}. \tag{2.17}
\]

As a first example let us look at some Gödel-like spaces. In that case \( U = V = 1 \) and \( A = -c_{ij} x^i dx^j \) with a skew-symmetric matrix \( c_{ij} \). We take the supertube to lie in the (7, 8) plane, centered at \((X^7, X^8)\), and with a radius \( R \), or explicitly we use the embedding

\[
x^7 = X^7 + R \cos \varphi, \quad x^8 = X^8 + R \sin \varphi. \tag{2.18}
\]

The pullback of \( A \) to the world-volume is then

\[
f = -c_{78}(R^2 + RX^7 \cos \varphi + RX^8 \sin \varphi). \tag{2.19}
\]
This gives the kinetic term for the zero modes

$$M_{ij} = \frac{1}{2\pi} \int d\varphi e^{-\Phi} \delta_{ij} \left( B + \frac{R^2}{B} + 2f \right) = e^{-\Phi} \left( B + \frac{R^2}{B} - 2c_{78}R^2 \right) \delta_{ij}. \quad (2.20)$$

Thus the value of $M$ is independent of the position of the supertube, which is just a manifestation of the homogeneity of those Gödel metrics. It seems like no interesting phenomena can emerge here, but still, as discussed above, $M$ may vanish or turn negative for certain values of $c_{78}$, $R$ and $B$. For all positive values of $M$ the moduli space is just a copy of $\mathbb{R}^8$, like in flat space [43], but when $M = 0$ the metric on moduli space vanishes, so the moduli space shrinks to a single point.

If we allow the supertube to absorb charges, that would change $R$ and $B$ and we would find a much larger moduli space. A slice through this space with circular tubes of fixed values of the charges would be a copy of $\mathbb{R}^8$, except for a singularity where $M = 0$.

But since we are not assuming that $R$ and $B$ can be modified dynamically, those are regarded as parameters, not moduli. Still we can look at supertubes with different values of these parameters, and compare their dynamics. $M$ should be considered the inertial mass of the supertube, the smaller it is the easier it is to move the supertube around. When it vanishes, the supertube has no inertial mass and can no longer be localized. If the moduli space is a single point, one cannot consider supertubes at different positions, and the only possibility is that the tube is in a superposition all over space.

This establishes our main claim that there is only a single theory for a supertube on the velocity of light surface, and therefore the supertube is delocalized and smeared over it. This happens for a tube of radius $R = 1/c_{78}$, while for larger radii $M$ will be negative, which is unphysical. We conclude that a consistent background will have a domain-wall at this value of $R$ built of such smeared supertubes.\footnote{If one expands the action beyond the quadratic level there may be terms of higher order in $m$, which could in principle change the dynamics. The cases we will concentrate on below have $m = 0$ for all $\varphi$, so all the higher order terms will also vanish and will not remove the singularity in moduli space.}

### 2.4 Other backgrounds

Above we studied what happens to a supertube as it approaches the velocity of light surface in some metrics of the Gödel type, but the same analysis is valid for many other backgrounds with closed time-like curves. We saw that the low energy dynamics of the supertube are described by a non-commutative supersymmetric three dimensional gauge theory. The scalars in the gauge theory parameterized a space that depended on the background.

In other backgrounds a lot of the same will still apply, the supertube will be described by a non-commutative gauge theory with eight supercharges, and as it approaches the velocity of light surface the compact direction becomes light-like. The metric on moduli space will be different, but still will be proportional to $M$, as before. Thus we will arrive at the same conclusion that the supertube will delocalize on the velocity of light surface, forming a domain-wall of the appropriate geometry.
We will study supertubes in other geometries below. As opposed to the Gödel example, those metrics will not be homogeneous, and the moduli space will not be flat. Instead $M$ will vary and vanish only at some singular points. So everywhere in space supertubes can be localized, except when we reach the singular point, and they have to delocalize into a domain-wall.

2.5 Validity of probe approximation

It is important to establish the range of validity of the probe analysis. The main requirement is that the back-reaction of the supertube probe is small compared to the background. In all the discussion above we assumed that the supertube is a single D2-brane, carrying some D0 and fundamental string charges. If the velocity of light surface we are studying is large, the supertube will carry a macroscopic amount of charge, and therefore could back-react on the geometry.

As was shown in the case of the three dimensional Gödel space in \cite{25}, and will be shown in other examples below, the metric one gets from the back-reaction inside a shell of supertubes is one of those Gödel metrics. So we should really think of the probe calculation not inside a homogeneous space, instead inside one of those domain-wall solutions. For the probe approximation to be valid we have to assume that the domain-wall is much heavier than the probe, and the way to achieve that is by assuming the shell is made up of a large number of supertubes, while it’s probed only by a single one.

So we have shown rigorously that if we have a shell made up of a large number of supertubes, as an extra supertube approaches it, it will dissolve into it, spreading homogeneously over the entire domain-wall. Thus we wish to claim that the domain-wall is not an approximate notion from a distribution of a large number of supertubes. Each of the constituent supertubes is spread over the entire velocity of light surface forming a continuous object.

While we can make a rigorous statement only for a large number of supertubes, the same may be true for arbitrarily small numbers of supertubes, in particular one. So a single supertube smeared into a domain-wall may be a legitimate string theory background. While we cannot substantiate this claim, the analysis above seems to make it self consistent.

3. Domain wall metrics

3.1 Generalities

We will apply our procedure to a few examples of known spaces with closed time-like curves by adding domain-walls made out of smeared supertubes. In the examples we consider the domain-walls will have the topology $S^{k-1} \times \mathbb{R}^{10-k}$, with $k = 2, 4$.

The sources are invariant under translations in $\mathbb{R}^{10-k}$ and under rotations in $\mathbb{R}^k$ (if we ignore the angular momentum that breaks the rotational symmetry). Like in electro-magnetism the solution outside the source will not depend on the radius of the source, so the metric will look like that of a rotating point source in $\mathbb{R}^k$. Natural candidates are rotating black holes. Those spaces usually have closed time-like curves, and a velocity of light surface with the topology $S^{k-1} \times \mathbb{R}^{10-k}$. This is where we will place the supertube source.
Inside the supertube there will be another metric, with the same symmetries. Since the metric should be continuous across the domain-wall, the interior metric will also have a velocity of light surface exactly at the same location. So both the inside and outside metrics agree on the location of the domain-wall. The inside metric describes a space that by naive analytical continuation would have had closed time-like curves outside a certain radius. Natural examples of such metric are spaces of the Gödel type. If there was no rotation the metric inside the domain-wall would have been flat space (as in the case of the enhançon), but the angular momentum carried by the supertubes can be seen inside (like the magnetic field inside a solenoid). So the metric inside should be close to a rotating flat space, and Gödel fits this description quite well.

All our simple examples will follow this pattern, with an interior metric of the Gödel type, a domain-wall, and outside a rotating black hole metric.

One can consider more complicated examples, with concentric supertube shells, where the inner most region will look like Gödel, the outside will be a black hole, and the extra regions in the middle will look like black holes in Gödel spaces.

It is possible to consider other topologies for the domain-wall. One example is $S^1 \times S^2 \times \mathbb{R}^6$, which may be relevant for black ring metrics [46, 47, 48].

### 3.2 Metrics with one angular momentum

In [25] it was shown that the three dimensional Gödel solution of supergravity can be regarded as the interior of a supertube domain-wall. Let us briefly recall the construction.

To form a piece of three dimensional Gödel universe we have to take a source that forms a domain-wall of geometry $S^1 \times \mathbb{R}^8$. So we use polar coordinates $r$ and $\phi$ in the $(x^7, x^8)$ plane. With this source the metric for $r > R$ is described by (2.8) with

$$U = 1 + Q_s \ln \frac{r}{R}, \quad V = 1 + Q_0 \ln \frac{r}{R}, \quad A = -R d\phi. \quad (3.1)$$

$Q_s$ and $Q_0$ are respectively the fundamental string and D0-brane charge densities. The angular momentum was set so at the domain wall, at $r = R$, the angular direction is null.

Inside the domain-wall, for $r < R$, this is continuous to

$$U = 1, \quad V = 1, \quad A = -\frac{r^2}{R} d\phi. \quad (3.2)$$

which gives a metric that looks like the three dimensional Gödel universe

$$ds^2 = -\left(dt + \frac{r^2}{R} d\phi\right)^2 + dr^2 + r^2 d\phi^2 + dy^2 + \sum_{i=4}^9 (dx^i)^2 \quad (3.3)$$

$$H_3 = 2cr \, dr \wedge d\phi \wedge dy, \quad F_2 = -2cr \, dr \wedge d\phi, \quad F_4 = 2cr \, dt \wedge dr \wedge d\phi \wedge dy.$$

Since the full metric does not have closed time-like curves we are guaranteed that no supertube will suffer from the problem discussed before, of having a compact time direction in the three-dimensional gauge theory. At most we can find a compact time-like direction if the supertube coincides with the domain wall and has $B = R$. So let us consider the moduli space of a supertube
with this radius and magnetic field, it can be translated in the transverse six directions as well as the plane it’s in. We use the same embedding as before \((2.18)\)

\[
x^7 = X^7 + R \cos \varphi, \quad x^8 = X^8 + R \sin \varphi.
\] (3.4)

The metric on moduli space is given by

\[
ds^2 = M \left[ (X^7)^2 + (X^8)^2 \right] \sum_{i=1}^{8} (dX^i)^2.
\] (3.5)

The coefficient \(M\) is a function of the distance of the supertube from the domain-wall, and is given by the integral

\[
M = \frac{1}{2\pi} \int d\varphi \frac{BV}{U} \left( UR + V \frac{r^2}{R} + 2f \right).
\] (3.6)

When the supertube is far away from the origin it will not cross the domain wall and to perform the integral one uses the external metric and the relations \(r = (x^7)^2 + (x^8)^2\) and \(f = -(R + X^7 \cos \varphi + X^8 \sin \varphi)R^2/r^2\). The resulting expression is quite complicated, and not illuminating.

It is more interesting what happens to the supertube as it nears the origin. To simplify the expressions let us set \(X^8 = 0\), then if \(X^7 \ll R\) we may expand outside the domain-wall

\[
U \sim 1 + Q_s \frac{X^7}{R} \cos \varphi, \quad V \sim 1 + Q_0 \frac{X^7}{R} \cos \varphi, \quad f \sim -R + X^7 \cos \varphi.
\] (3.7)

Recall that the same supertube inside a homogeneous Gödel had \(M = 0\), so we only have to integrate the difference between the above and the naive continuation of the Gödel metric outside the wall. At the linear approximation half the tube is inside and half outside. So we get

\[
M \sim \frac{R}{2\pi} \int_{-\pi/2}^{\pi/2} d\varphi (Q_s + Q_0 + 4) X^7 \cos \varphi = \frac{R}{\pi} (Q_s + Q_0 + 4) X^7.
\] (3.8)

As expected it vanishes at the origin, so the supertube becomes smeared in the transverse six dimensions, implying the construction was consistent. Instead of studying this system further we turn now to the case with two angular momenta, which is much more interesting.

### 3.3 Metrics with two angular momenta

The three dimensional example was already studied in \([25]\), and it suffers from the fact that the metric outside the domain-wall is not asymptotically flat. Let us therefore shift to another example, with rotation in two planes. There is a simple geometry that is asymptotically flat, carries D0-brane and fundamental string charges and angular momentum in two planes. It is dual to the 2-charge rotating black hole made out of D1 and D5-branes, and is given by the general ansatz \((2.8)\) with

\[
U = 1 + \frac{Q_s}{r^2}, \quad V = 1 + \frac{Q_0}{r^2}, \quad A = -\frac{J}{2r^2 \sigma_L^3}.
\] (3.9)

\(Q_s\) and \(Q_0\) are the fundamental string and D0-brane charge densities, as can easily be verified from Gauss’s law. Since the metric in nontrivial in five space-time directions, \(r\) is the radial coordinate
in $\mathbb{R}^4$, and in the gauge field $\sigma_3^L$ is a left-invariant one-form on $S^3$. If we use the polar coordinates $(\eta, \phi)$ in the $(X^5, X^6)$ plane and $(\zeta, \psi)$ in the $(X^7, X^8)$ plane, so $r^2 = \eta^2 + \zeta^2$, we can write

$$\sigma_3^L = 2\eta^2/r^2 d\phi + 2\zeta^2/r^2 d\psi.$$

The solution above carries angular momentum in the two planes and from the asymptotic form of the metric we can read off its value $J_L = -J$.

Far away from the origin this space is perfectly causal, but close to the origin there are closed time-like curves. The purely angular part of the metric is given by

$$-U^{-1}V^{-1/2}A^2 + V^{1/2} \left( d\eta^2 + \eta^2 d\phi^2 + d\zeta^2 + \zeta^2 d\psi^2 \right),$$

and at the radius $R$ where $UV = J^2/R^6$ it is proportional to $d\eta^2 + d\zeta^2 + \eta^2\zeta^2/r^2(d\phi - d\psi)^2$. This metric is degenerate, so there are null curves on the $S^3$ at this radius, given by the direction $\partial_\phi + \partial_\psi$. This direction is the fibre of the Hopf fibration, so at the critical radius the $S^3$ degenerates to an $S^2$, while at smaller radii this direction becomes time-like.

We found therefore that this geometry has a velocity of light surface of geometry $S^3 \times \mathbb{R}^6$. According to our general prescription we should not trust the metric beyond this radius, instead we should put the sources on this sphere. One may think of the source as supertubes that follow the null curves on this domain-wall, or the Hopf fibres. Thus the domain-wall is constructed from supertubes with geometry $S^1 \times \mathbb{R}^2$ fibred over $S^2 \times \mathbb{R}^4$.

There are a few tests to check this construction. First compute what the interior metric is. Then see that this domain-wall does indeed give the above desired geometry. Finally we will probe this metric with different supertubes and see that they will delocalize as stated.

Since there are no more sources in the interior it should be a vacuum solution that is continuous to the one above across the domain-wall. Such a solution exists, and is given by the ansatz (2.8) with

$$U = 1 + \frac{Q_s}{R}, \quad V = 1 + \frac{Q_0}{R}, \quad A = -\frac{J r^2}{2R^4 \sigma_3^L}.$$

(3.11)

This is a deformation of the five dimensional Gödel universe of type IIA (eq (2.46) of [5]). The only difference being that $U$ and $V$ are not unity, but other constants.

The general supertube ansatz is described by harmonic functions and a Maxwell field, so all the solutions have simple analogs in electromagnetism. The three dimensional example in the previous section is analogous to a charged solenoid. The harmonic functions outside the domain-wall are the same as one would take for a charged cylinder in four dimensions. The gauge field gives a constant magnetic flux inside. The example at hand also has an analog, in electromagnetism in five dimensions.

Since there are nowhere vanishing vector fields on $S^3$, one may put an electric current on it. We will choose the current to run along the fibres of the Hopf fibration, and the current density will be proportional to the dual one-form $\sigma_3^L$. For our analogy the shell should also carry a uniform charge distribution. The solution of this five-dimensional electromagnetic problem is given by the expressions (3.9) and (3.11). Inside the domain-wall the scalar potentials are constant, but there is a magnetic field, and outside there are scalar potentials, as well as a decaying electromagnetic field.
We should take some care to normalize the charges of this solution. The location of the domain-wall is at the velocity of light surface at a radius \( r = R \), and if we assume that \( Q_s, Q_0 \gg R^2 \) this gives the condition \( J^2 = R^2 Q_s Q_0 \). Recall (2.7) that if the branes make up \( N \) bound states (i.e. \( N \) supertubes), the bound on the angular momentum is \( J \leq Q_0 Q_s / N \), or \( J \leq R^2 N \). We see that our solution saturates this bound.

We may look at a more general solution, where we put the domain-wall at a larger radius. Let us still take the same form for the exterior metric, and by continuity find the same form for the metric inside. The only difference is that the radius \( R \) will be greater than before. Note that in the interior metric the rotation is proportional to \( A = -J r^2 / (2 R^4) \), and due to the negative power of \( R \), this Gödel space in the interior has a smaller rotation parameter than the case above. This solution will be totally causal, with no closed time-like curves either inside or outside of the shell. But now there are also no closed null curves, and the supertubes are no longer at a velocity of light surface, so the bound above is not saturated, instead \( J^2 < R^2 Q_s Q_0 \). This means that not all the constituent D0-branes or fundamental strings are part of the bound state. Also we cannot justify this construction by the analog of the enhançon mechanism, and one should think of the supertubes as localized objects and are smeared only if one takes a continuum limit.

Given the general ansatz (2.8), it’s quite obvious that the metric we have written down is a solution, still it’s instructive to check the junction conditions across the domain-wall [49] to verify that we have the correct source. In the case of the enhançon this was done in [51], and we follow their calculation. One first calculates the jump in the extrinsic curvature, given by

\[
\gamma_{\mu\nu} = \frac{1}{2 \sqrt{g_{rr}}} \left( \partial_r g^{-}_{\mu\nu} - \partial_r g^{+}_{\mu\nu} \right).
\]

Here \( g^{\pm} \) is the metric in the Einstein frame outside (+) and inside (-) the shell. The stress-energy tensor across the junction is simply

\[
S_{\mu\nu} = \frac{1}{\kappa^2} \left( \gamma_{\mu\nu} - g_{\mu\nu} \gamma^p_p \right).
\]

The coupling in front, \( 2\kappa^2 = (2\pi)^7 \alpha'^4 g_s^2 \), is the Newton constant in ten dimensions. Note that the tensors \( \gamma \) and \( S \) are defined on the domain-wall, so the indices \( \mu, \nu, \rho \) take only nine values (all the directions excluding \( r \)).

The intermediate steps in the calculation are rather messy, but the final result is simple in particular if we choose to write the stress-energy tensor with upper indices. The only non-zero components are then

\[
S^{tt} = \frac{Q_s + Q_0 + 2(R^2 Q_s Q_0 - J^2) / R^4}{\kappa^2 g_r^{3/2} R^3},
\]

\[
S^{t3} = \frac{2J / R^2}{\kappa^2 g_r^{3/2} R^3},
\]

\[
S^{yy} = \frac{Q_s}{\kappa^2 g_r^{3/2} R^3}.
\]

The last line is proportional to the fundamental string density, which is smeared over the three-sphere whose volume is \( 2\pi^2 (R \sqrt{g_{rr}})^3 \) (and we are using the normalization where charges are divided
by the volume of a unit sphere). The second line corresponds to the angular momentum carried by the supertube, and the index 3 corresponds to the direction of \( \sigma_3 \), where the angular momentum is, but there is no component in the \( S^{33} \), or any of the other sphere directions. Finally the first line corresponds to the total energy density, which includes the contribution of the fundamental strings \( (Q_s) \) and D0-branes \( (Q_0) \). The other term vanishes when the supertube source carries maximal angular momentum \( J^2 = R^2 Q_0 Q_s \). To understand it note that the \( G^{tt} \) component of the open string metric (which is proportional to \( m \)) represents the extra mass a supertube had over the tension of its constituents. We wrote it as the sum of two terms, one a positive, and another one, \( UVR^2 - J^2 \). As the first term vanishes for supersymmetric configuration, we are left with the second one, which to leading order is the same as the access energy in the calculation above.

Let us now consider a supertube in this domain-wall geometry. First taking a planar tube in the \((X^5, X^6)\) plane, and we consider the most interesting case where the radius coincides with the size of the domain wall, and set the magnetic field to \( B^2 = R^2 (Q_0 + R^2) / (Q_s + R^2) \), which is the value where the open string metric becomes degenerate at the velocity of light surface. As before we will not write down the full expressions, just concentrate on the behavior near the domain-wall. In this case we may displace the supertube in the plane, but also in the \( X^7 \) and \( X^8 \) directions. Labeling \( \eta \) the displacement of the center of the supertube in the plane, and \( \zeta \) the displacement in the other direction a simple calculation yields

\[
m \sim \begin{cases} 
\sqrt{V} \left( \frac{R^2}{B} - \frac{Q_s B}{R^2} + \frac{2J}{R^2} \right) \left( \frac{2}{R} \frac{\eta}{R} \cos \varphi + \frac{\zeta^2}{R^2} \right) - \frac{J}{R^3} \cos \varphi , & \frac{\pi}{2} < \varphi < \frac{\pi}{2} \text{ (outside)} \\
- \frac{J}{R^3} \cos \varphi , & \frac{\pi}{2} < \varphi < \frac{3\pi}{2} \text{ (inside)} 
\end{cases}
\]

(3.15)

To get this we used the relation \( UVR^6 = J^2 \) at the domain wall. Now we can do the integration over \( \varphi \) and replace for \( B \), which gives

\[
M \sim \frac{Q_0 + R^2}{(Q_s + R^2)^2} \left( Q_s Q_0 + 2R^2(Q_0 + Q_s) + 3R^4 \right) \left( \frac{4\eta}{R} + \frac{\pi\zeta^2}{R^2} \right).
\]

(3.16)

If we took the near-horizon limit, where \( U = Q_s/r^2 \) and \( V = Q_0/r^2 \), the expression would simplify to

\[
M \sim \frac{Q_0^2}{Q_s} \left( \frac{4\eta}{R} + \frac{\pi\zeta^2}{R^2} \right).
\]

(3.17)

In both cases we find the expected result, that the metric on moduli space degenerates as the supertube reaches the velocity of light surface \( (\eta = \zeta = 0) \). So if we bring in a supertube from infinity, it will naturally dissolve into the domain wall, and smear in the transverse \( \mathbb{R}^4 \) direction. This is not enough to justify the domain wall, where we assumed that the supertubes are also smeared into an \( S^3 \), the reason being that our supertube carried angular momentum only in the \((X^5, X^6)\) plane. To rotate it into the other plane, we have to enlarge the moduli space to allow a change in the angular momentum by coupling to some external object. Otherwise we can consider more general supertubes that carry angular momentum in two planes.
One possibility is to consider the system comprised of a pair of supertubes, one in the $\{X^5, X^6\}$ plane and the other in the $\{X^7, X^8\}$ plane. The combined system carries both angular momenta, and for appropriate charges will have vanishing $M$ at the velocity of light surface. If the angular momenta in the two planes are equal, conservation of charge will not prevent us from rotating the two planes into each other and the combined system can be smeared into the $S^3$. But this rotation cannot take place smoothly as the two supertubes, which are disconnected, will have to exchange angular momentum.

Another possibility is to consider some objects constructed in [51]. The idea is for the supertube to follow a curve that winds circles in the two planes. One takes the ansatz

$$X^5 + iX^6 = \eta e^{im\varphi}, \quad X^7 + iX^8 = \zeta e^{im\varphi}.$$  \hspace{1cm} (3.18)

If $n = m$ this will be a circle in another plane, and carry only one independent angular momentum. But as long as $n \neq m$, this configuration carries two angular momenta. It has $J_1 = n\eta^2$ in the $(X^5, X^6)$ plane, and $J_2 = m\zeta^2$ in the second plane.

This tube wraps a curve on a two-torus inside a three-sphere, and there is a zero mode for rotating it around the torus by the modification $X^5 + iX^6 \rightarrow \eta e^{ip(\varphi + \varphi_0)}$, which will preserve the total angular momentum. In the open string metric for this supertube $M$ is not zero even if the tube is at the surface of light, but by taking $m$ and $n$ large and almost equal one can make $M$ arbitrarily small. If in addition we use a tube with $J_1 = J_2$, it would have an extra modulus of rotating one of the planes into the other. Then, when we average over the value of this modulus, this supertube will smear over the entire surface of the $S^3$ domain-wall.

There is a chain of dualities [52, 53] that takes the supertube to the D1-D5 system. If we apply this transformation to the above metric, the exterior would be the limit of the BMPV black hole with no linear momentum, which has eight supersymmetries. The duality replaces the fundamental strings with the D1-branes, so $U$ will be the relevant harmonic function, while $V$ is the harmonic function for the D5-branes, which are dual to the D0-branes in the IIA picture. The metric is given by

$$ds^2 = U^{-1/2}V^{-1/2} \left[-(dt - A)^2 + (dy + B)^2\right] + U^{1/2}V^{1/2} \left[d\theta^2 + r^2(d\phi^2 + d\psi^2) + 2r^2 \cos \theta d\phi d\psi\right] + U^{1/2}V^{-1/2} \sum_{i=1}^{4} (dx^i)^2,$$  \hspace{1cm} (3.19)

where $C_2 = U^{-1}(dt \wedge B + dy \wedge A + 2A \wedge B + (U - 1)dt \wedge dy) + C$ and $e^{2\Phi} = U/V$.

The field strengths $dA$ and $-dB$ are Hodge dual in the four dimensions transverse to the branes, and the field strength $dC$ is dual in four dimensions to $-dV$. For our particular choice of $A$ and $V$, we find that outside of the domain-wall $B = -A$, and $C = -Q_5 \cos^2 \theta d\phi \wedge d\psi$. Inside the shell we get $B = A$, and $C = 0$.

This space is similar in some ways to a rotating black hole, but it lacks a finite size horizon. Since the existence of closed time-like curves is a T-duality invariant notion [18], it also suffers from
closed time-like curves, to see them we should consider a vector $\alpha \partial_{\psi} + \partial_{y}$. This represents a closed curve if $y$ is compact, and we may adjust $\alpha$ so $dy + B$ acting on this vector is arbitrarily small\(^5\). With this choice of $\alpha$, the norm squared of this vector is proportional to $-J^{2}/r^{4} + UVr^{2}$, as in the dual case. Assuming small $r$ this vanishes again at a radius where the angular momentum is related to the product of the D1 and D5 charges by $J^{2} = R^{2}Q_{1}Q_{5}$. Recall that if the branes form a single bound state the angular momentum satisfies $J = Q_{1}Q_{5}$, so the radius where the closed time-like curves appear is $R^{2} = Q_{1}Q_{5}$.

If we were not to include the domain-wall, it would have had closed time-like curves. With the domain-wall the interior is one of the Gödel spaces of type IIB.

There is another possibility for the exterior metric, if we drop the constant from the harmonic functions $U$ and $V$. This corresponds to taking a near-horizon limit, and after the above chain of dualities, it is the near horizon geometry of the D1-D5 system. That is $AdS_{3} \times S^{3}$ with angular momentum in two directions \([54, 55, 56]\).

3.4 Three-charge black hole

An even more interesting example is the three charge black hole itself \([29, 57]\). Unlike the above examples, it preserves only four supersymmetries, not eight, and therefore the probe analysis does not apply. Instead there would be some noncommutative gauge theory with four supercharges, but many of the same features. We therefore will still try to extrapolate our results to this case.

Those black holes carry three charges and two angular momenta, we expect the singular source to be replaced, as in the previous example by a domain-wall with the geometry $S^{3} \times \mathbb{R}^{6}$. Actually such constructions were already done in \([58, 59]\). In the first, the D1 and D5-branes were left at the origin and only a wave carrying linear and angular momentum was put on the shell. In the second paper the D-branes were wrapped on a K3 surface and the enhançon mechanism was invoked to put branes on a spherical domain-wall.

But following the example above, we expect at least the D1, the D5 and the angular momentum to sit on the shell. It seems unnatural to leave a singular source for the momentum at the origin, therefore it is very reasonable to assume that our mechanism should work for a supertube-like object carrying also momentum.

Actually this object was found recently by Bena and Kraus \([27]\). If we follow backwards the same chain of dualities as above, it would bring the BMPV to the system of D0-D4 branes and strings with two angular momenta. Those are exactly the charges carried by their generalized supertubes.

Again it is very simple to patch together two solutions inside and outside of the domain-wall.

\(^5\)It can be set to zero, but then perhaps it would not be closed, so one has to approximate the ratio of the period of $y$ and $2\pi/\alpha$ by a rational number and get a closed curve.
In type IIB the metric of the BMPV black hole (in ten dimensions) is

\[ ds^2 = f_{1}^{-1/2} f_{5}^{-1/2} \left[ -\left( dt + \frac{J}{2r^2} \sigma_{L}^{3} \right)^2 + \left( dy + \frac{J}{2r^2} \sigma_{L}^{3} \right)^2 + \frac{Q_{k}}{r^2} (dy - dt)^2 \right] + f_{1}^{1/2} f_{5}^{1/2} \left[ dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 + r^2 \cos^2 \theta d\psi^2 \right] + f_{1}^{1/2} f_{5}^{-1/2} dx_{i}^2, \]

(3.20)

\[ C_2 = f_{1}^{-1} dt \wedge dy + f_{1}^{-1} \frac{J}{2r^2} \sigma_{L}^{3} \wedge (dy - dt) - Q_{5} \cos^2 \theta d\phi \wedge d\psi, \]

\[ e^{2\Phi} = f_{5}^{-1} f_{1}. \]

Here \( x_{i} \) are the four flat directions. \( f_{1} \) and \( f_{5} \) are harmonic functions

\[ f_{1} = 1 + \frac{Q_{1}}{r^2}, \quad f_{5} = 1 + \frac{Q_{5}}{r^2}. \]

(3.21)

This will be the metric outside of the domain-wall, and we place the domain-wall at the velocity of light surface, where \((R^2 + Q_{1})(R^2 + Q_{5})(R^2 + Q_{k}) = J^2\). In the over-rotating case, where \( J^2 > Q_{1}Q_{5}Q_{k} \) this happens at positive \( R^2 \), while in the more realistic under-rotating case one has to analytically continue the metric beyond the horizon at \( R = 0 \), and place the domain wall there. By putting all the charges on the domain-wall we can find what the metric would be inside. The result is

\[ ds^2 = f_{1}^{-1/2} f_{5}^{-1/2} \left[ -\left( dt + cr^2 \sigma_{L}^{3} \right)^2 + \left( dy + cr^2 \sigma_{L}^{3} \right)^2 + \frac{Q_{k}}{r^2} (dy - dt)^2 \right] + f_{1}^{1/2} f_{5}^{1/2} \left[ dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 + r^2 \cos^2 \theta d\psi^2 \right] + f_{1}^{1/2} f_{5}^{-1/2} dx_{i}^2, \]

(3.22)

\[ C_2 = f_{1}^{-1} (dt \wedge dy + cr^2 \sigma_{L}^{3} \wedge (dy - dt)), \]

\[ e^{2\Phi} = f_{5}^{-1} f_{1}. \]

But now \( f_{1} \) and \( f_{5} \) are constants. Like in the previous examples this is a metric that if analytically continued will have closed time-like curves, and has no localized sources. It is a Gödel-like metric which also carries momentum. This is an example of the mixed Gödel/pp-wave metrics found in [5] (see for example their equation (2.50)).

One can again check the junction condition across the interface, and indeed in the dual type IIA metric the stress-energy tensor includes only terms related to the fundamental strings, D0-branes and D4-branes as well as an off-diagonal component related to the angular momentum (though in the over-rotating case the shell carries too much angular momentum). We leave for future work a fuller understanding of this geometry and possible relations to the metrics found in [60, 61].

4. Discussion

In this paper we tried to generalize the observation in [25], that domain-walls made out of supertubes cutoff spaces that otherwise would have closed time-like curves. We demonstrated this for a few

[6] These metrics were studied in more detail in [62], which appeared right after the first version of this paper was posted.
metrics of the Gödel type, as well as asymptotically flat metrics with a rotating source. As one brings a supertube towards the velocity of light surface it delocalizes over the entire surface, joining the domain wall.

There has been a lot of discussion recently in the string literature on spaces with closed time-like curves, particularly of the Gödel type. In all the examples we studied we were able to find a physical domain wall that changed the metric in a way that removed the closed time-like curves. These domain walls were also very natural from the point of view of the outside metric, and by accretion of supertubes one can create only a piece of Gödel universe without closed time-like curves. Our conclusion is that the homogeneous Gödel spaces should be considered unphysical, and is simply a naive analytical continuation, ignoring the domain wall.

It is a rather pleasing fact that string theory has a mechanism that prevents the appearance of certain closed time-like curves. This was studied here from the point of view of supertubes, which are simple probes in many string backgrounds [63]. More generally, the same must apply to all the dual metrics, where the supertube is replaced by a string, a KK monopole, or other objects. It would be very interesting to understand the analogous mechanism in those settings, and to see if they can be generalized to cases that are not dual to the supertube metrics.

This construction also gives a simple explanation of over-rotating metrics. Following the same rules we will place a domain-wall at the velocity of light surface, but this domain-wall would not be able to carry all the angular momentum seen from the outside, only the critical amount. The access angular momentum will have to come from inside the domain-wall. But there are no sources inside the domain-wall, just a Gödel like space. So this extra rotation will have to come from a singular Misner string inside this piece of Gödel universe. This is clearly singular and unphysical, by a singular gauge transformation the metric can be brought back to the critically rotating case. The secret to resolving this issue is that we blew up the source into a sphere, while leaving only the access rotation behind.

It should be possible to study other metrics that have closed time-like curves, like the black holes in Gödel universe [64, 65, 66]. Again we expect a domain wall that will cure the asymptotical form of the metric. If the black hole is rotating, we may have to also add a domain wall at the inner horizon.

Those domain-walls are not the only way to eliminate closed time-like curves from those metrics. Instead of putting a domain-wall, it is possible to replace the point-like source with a localized supertube. The difference is that those solutions are not identical to the original metric, only asymptote to it very far away. Those supertube solutions were proposed as being the microstates of black-holes [52, 53, 77, 88, 23].

If this interpretation is correct, it’s not clear what the meaning of our domain-wall solution is. One possibility is that it is a superposition of different states of localized supertubes, while another is that some states were missed in the above counting.

We considered only spherical domain-walls, but it should be possible to construct less symmetric solutions, and they may be relevant for the entropy counting problem. In the case of the three charge black hole, there is a general argument [63] that localized supertubes of different shapes may
not be enough to account for the entropy of the system. Perhaps domain-walls of arbitrary shapes would be important there.

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