Two-body hadronic decay of K$^+$ in the presence of a circularly polarized laser field

M. Baouahi$^1$, I. Dahiri$^1$, M. Ouali$^1$, B. Manaut$^1$(a), R. Benbrik$^2$ and S. Taj$^1$

$^1$ Recherche Laboratory in Physics and Engineering Sciences, Team of Modern and Applied Physics, Polydisciplinary Faculty of Beni Mellal, Sultan Moulay Slimane University - Beni Mellal, Morocco

$^2$ LPFAS, Polydisciplinary Faculty of Safi, Cadi Ayyad University - Marrakech, Morocco

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Abstract – In this study, we have investigated the two-body hadronic decay of the charged kaon, K$^+ \rightarrow \pi^+ + \pi^0$, in the presence of a laser field with circular polarization. We have derived, by analytical techniques, the laser-assisted decay width and the branching ratio of the charged kaon decay via the two-body hadronic channel. We have also taken into consideration the impressive results obtained for the laser-assisted charged kaon decay via the leptonic mode in order to understand more clearly the effect of the laser field on the quantities related to the charged kaon decay such as the decay width, the branching ratio and lifetime. A precise comparison of the ratios of hadronic-to-muonic decay in the presence of the laser field is made to show that the hadronic mode becomes slightly more important by increasing the laser field intensity.

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Introduction. – The interaction between the electromagnetic field and charged particles is a very interesting research topic in the study of particle physics processes, especially after the introduction of the laser which is one of the most important technological discovery in 1960 by Maiman [1]. Today, the breakthrough progress in laser technology allows us to observe in real time the electrons and atoms that compose matter, and to deform, in an ultra-fast way, the molecular structure around the atoms in order to transform materials into magnetic materials [2]. Moreover some studies and discoveries, that have been made in 2018, have demonstrated that the energy of sub-relativistic electrons is strongly modified on the scale of a few femtoseconds by the interaction with a progressive wave created in the vacuum by the collision of two laser pulses of different frequencies [3].

Studies in the field of laser-matter interactions usually deal with non-relativistic [4–6] and relativistic atomic physics [7–9]. Today, the technological progress gives us the opportunity to introduce an external electromagnetic field in high-energy physics interactions. Indeed, several studies have been made to study the impact of the laser field on the decay width [10–14] and on the effective cross-section [9,15–17] by introducing the phenomenon of emission and/or absorption of photons [18]. For instance, in [19,20], the authors illustrated the effect of an electromagnetic field with circular polarization on the decay widths and lifetime of the charged pion and the Z boson. In [21], it is found that the circularly polarized laser field has a great impact on the cross-section of the charged Higgs pair production via electron-positron annihilation. These processes are called laser-assisted processes for they may occur also in the presence of the electromagnetic field.

There is another category of very well-known processes in laser physics, and they are induced by the laser field. We cite, here, an example in which the creation of the \{e$^+$, e$^-$\} pair is induced by the presence of an external field. In [22–24], the authors showed that the creation of electron-positron pairs is possible by a strong plane electromagnetic wave in the presence of an additional electromagnetic energy source. For all these studies, the production of ultra-short pulses is important, not only because time compression obviously implies an increase of the intensity at a given laser energy, but also because high intensities allow, in general, to control fast physical processes [25,26].

In our previous work [14], we have found impressive results on the laser-assisted charged kaon decay in the leptonic channel. In addition, we have indicated that the CPT symmetry might be preserved or broken in both the muonic and the electronic channels. Therefore, we can control the dominance of matter over antimatter or vice
versa. In this respect, we have investigated the two-body hadronic decay of the charged kaon in the presence of an electromagnetic field of circular polarization by using the same laser parameters as in [14]. Thus, the aim of this work is to present the analytical and numerical results about the laser-assisted decay of the charged kaon, which has a free lifetime $\tau = (1.2380 \pm 0.0020) \times 10^{-8}\text{s}$ [27].

The remainder of this work is organized as follows: in the next section, we will deal with the theoretical calculation of the charged kaon decay width in the absence and presence of a circularly polarized laser field. Then, we will present and discuss the obtained results. Finally, a short conclusion is given in the last section.

**Theoretical framework.** – In this part, we start with the theoretical calculation of the laser-free two-body hadronic decay of the charged kaon, $K^+$, at the lowest order without taking into account the quarks composition of mesons. Then, we will deal with the decay process by dressing the charged mesons. The plane wave functions used to describe the neutral meson, $\pi^0$, and charged mesons that are involved in the studied decay, in the absence of the electromagnetic field, are derived from the free Klein-Gordon equation. However, in the presence of an electromagnetic field, the charged particle are described by the Volkov functions [18].

**Laser-free kaon decay.** In the absence of an external field, the transition matrix element $S_{f_i}$ which is associated with the decay of the charged kaon $K^+$ to $\{\pi^+, \pi^0\}$ is written as a product of two currents such that

$$S_{f_i} = \frac{-iG_F}{\sqrt{2}} \int d^4x J^{(K^+ \otimes \pi^+)}(x), J^{(0 \otimes \pi^0)}(x) \mu(x), \quad (1)$$

where $G_F$ represents the Fermi constant. The two hadronic currents $J$ [28,29] are defined as follows:

$$J^{(K^+ \otimes \pi^+)}(x) = \frac{1}{2\sqrt{V E_3}} e^{(K^+) \cdot O_1(\pi^+)} e^{(P_1 - P_3) \cdot x},$$

$$J^{(0 \otimes \pi^0)}(x) = \frac{1}{2\sqrt{V E_2}} \langle 0 \mid O_2(\pi^+) \rangle e^{P_2 \cdot x},$$

where $V$, $P_i = (E_i, \vec{p}_i)$ ($i = \{1, 2, 3\}$) and $\Delta$ are, respectively, the quantum volume, the 4-momentum of the hadrons (successively for $K^+$, $\pi^+$ and $\pi^0$) and the transfer momentum such that $\Delta = (P_1 - P_3)^2$. $F_{\pi^+}$ is the decay constant associated with the particle $\pi^+$. The operator $O_1$ corresponds to the transition in the vector current $[K^+, \pi^+]$, and $O_2$ corresponds to that in the weak vector-axial current. The form factors $f_{\pm}$ are given, as the case of $K_{e3}^+$ and $K_{\mu3}^+$ decays, by the following expression [27]:

$$f_\pm(\Delta) = f_\pm(0) \left[ 1 + \lambda_\pm \frac{\Delta}{m_{\pi^\pm}} \right]. \quad (4)$$

We have developed the form factors $f_\pm$ based on the semileptonic $K_{3}$. This approximation is based on the fact that the $K_{3}$ transfer momentum is conditioned by $m_\ell^2 \leq \Delta \leq (m_{K^+} - m_{\pi^\pm})^2$ which is still valid also for $\Delta = m_{\pi^\pm}^2$ as it locates in the range of those of $K_{3}$ [29].

We use the parameterization $\{\lambda_+, \lambda_0\}$ to define the factors $f_\pm$ by the function $f_0$ [27] which is given by:

$$\begin{align*}
  f_0(\Delta) &= f_+(\Delta) + \frac{\Delta}{m_{K^+} - m_{\pi^0}} f_-(\Delta), \\
  f_0(\Delta) &= f_0(0) \left[ 1 + \lambda_0 \frac{\Delta}{m_{\pi^\pm}} \right].
\end{align*} \quad (5)$$

In this parameterization, we use the universality assumption $\mu - e$ to define $\lambda_+$ and $\lambda_0$ [27]. After weighting the square of the matrix element $S_{f_i}$ by the phase space and per unit time $T$, the decay width [30] becomes as follows:

$$\begin{align*}
  \Gamma &= \frac{1}{T} \int \frac{d^4p_{\pi^0}^*}{(2\pi)^3} \int \frac{d^4p_{\pi^0}^0}{(2\pi)^3} |S_{f_i}|^2, \\
  \Gamma &= \frac{G_F^2 F_{\pi^+}^2}{32\pi m_{K^+}} \times \left[ (m_{K^+}^2 - m_{\pi^0}^2) f_+(m_{\pi^+}^2) + m_{\pi^+}^2 f_-(m_{\pi^+}^2) \right]^2, \\
  &\times \left[ (m_{K^+}^2 - m_{\pi^0}^2)^2 + m_{\pi^0}^4 \right], \\
  &\times \left( m_{K^+}^2 - m_{\pi^+}^2 + m_{\pi^0}^2 \right)^{1/2}.
\end{align*} \quad (6)$$

**Laser-assisted kaon decay.** An electromagnetic field with a circular polarization is described by the classical 4-vector potential $A^\mu(\phi) = a_1^\mu \cos(\phi) + a_2^\mu \sin(\phi)$, where $\phi = k.x$ is its phase. The 4-vector $A^\mu$ verifies the transversality condition $k.A = 0$ (Lorentz gauge), where $k = (\omega, \vec{k})$ is the 4-vector wave. The multi-polarizations $a_1^\mu = |a| \delta(0, 1, 0, 0)$ and $a_2^\mu = |a| \delta(0, 0, 1, 0)$ are equal in magnitude such that $a_1^2 = a_2^2 = \frac{|a|^2}{6} = \frac{-\epsilon}{\omega^2}$, and they verify the orthogonality relation $a_1 a_2 = 0$. The quantity $\omega$ represents the amplitude of the electric field of the laser while $\omega$ is its frequency. By adding the formalism of the laser field to describe the decay $K^+ \rightarrow \pi^+ + \pi^0$, the Klein-Gordon equation in the presence of an electromagnetic field is given by

$$[(i\partial - eA)^2 - m_\pi^2] \psi_\mu(x) = 0, \quad (8)$$

where $m_\pi$ may be either the mass $m_{K^+}$ of the charged kaon or $m_{\pi^+}$ of the charged pion. Equation (8) admits as solution the following Volkov functions:

$$\begin{align*}
  \psi_{K^+}(x) &= \frac{1}{\sqrt{2Q_1^V}} e^{iS(q_1, x)}, \\
  \psi_{\pi^+}(x) &= \frac{1}{\sqrt{2Q_2^V}} e^{iS(q_2, x)},
\end{align*} \quad (9)$$

where

$$\begin{align*}
  S(q_1, x) &= q_1.x - e \frac{a_1}{k} P_i \sin \phi + e \frac{a_2}{k} P_i \cos \phi, \\
  \psi_{\pi^+}(x) &= \frac{1}{\sqrt{2Q_2^V}} e^{iS(q_2, x)},
\end{align*} \quad (10)$$
$q_i = (Q_j, q'_j)$ ($j = \{1, 2\}$) is the four-momentum of the charged particle in the presence of the laser field where $Q_j$ denotes its effective energy. It is related to its corresponding free momentum by: $q_i = P_j - [(e^2 a^2)/(2k P_j)]k$, such that $q_j^2 = P_j^2 - e^2 a^2 = m_P^2 - e^2 a^2 = m_P^2$, where $m_P$ is the effective mass of the charged particle. In the presence of the electromagnetic field, the matrix element $S'_{fi}$ in the first Born approximation is given by the following expression:

$$S'_{fi} = \frac{G_F}{\sqrt{2}} \int d^4x \frac{F_{\pi^+}}{\sqrt{8Q_1Q_2E_3V^3}} [f_+(\Delta')|q_1 + P_3|\mu q'_2^\mu \delta f_-(\Delta')|q_1 - P_3|\mu q'_2^\mu] e^{-iS(q_1,x) + S(q_2,x) + P_3} x, \quad (11)$$

where $\Delta'$ is the new 4-momentum transfer. To expand the element $S'_{fi}$, we will perform some transformations in order to find an element expandable into a series of ordinary Bessel functions, with argument $z$ and phase $\phi_0$ such that

$$z = \sqrt{\alpha_1^2 + \alpha_2^2} \quad \text{and} \quad \phi_0 = \arctan \left( \frac{\alpha_2}{\alpha_1} \right). \quad (12)$$

where

$$\alpha_1 = e \left( \frac{a_1 \cdot P_1}{k \cdot P_1} - \frac{a_1 \cdot P_2}{k \cdot P_2} \right) \quad \text{and}$$

$$\alpha_2 = e \left( \frac{a_2 \cdot P_1}{k \cdot P_1} - \frac{a_2 \cdot P_2}{k \cdot P_2} \right). \quad (13)$$

The exponential term in eq. (11) takes the following form:

$$-S(q_1,x) + S(q_2,x) + P_3 \cdot x = -(q_1 - q_2 - P_3) \cdot x + z \sin(\phi - \phi_0). \quad (14)$$

The decay matrix element can be developed using the following ordinary Bessel transformation:

$$e^{iz\sin(\phi - \phi_0)} = \sum_{n=-\infty}^{n=+\infty} J_n(z)e^{-in\phi}e^{in\phi}$$

$$= \sum_{n=-\infty}^{n=+\infty} B_n(z)e^{in\phi}. \quad (15)$$

After some substitutions using eqs. (14) and (15), the matrix element $S'_{fi}$ becomes:

$$S'_{fi} = \sum_{n=-\infty}^{n=+\infty} G_F F_{\pi^+} (2\pi)^4 \delta^4(q_2 + P_3 - q_1 + nk)$$

$$\times [f_+(\Delta')|q_1 + P_3|\mu q'_2^\mu] + f_-(\Delta')|q_1 - P_3|\mu q'_2^\mu] B_n(z) = \sum_{n=-\infty}^{n=+\infty} S'^{n}_{fi}. \quad (16)$$

Following the same steps as in the absence of the laser field, the decay width takes the following form:

$$\Gamma = \sum_{n=-\infty}^{n=+\infty} G_F^2 F_{\pi^+}^2 \int \frac{d^3q_2}{64\pi^2 Q_1} \frac{d^3P_3}{E_3} \delta^4(q_2 + P_3 - q_1 + nk)$$

$$\times |M'_{fi}|^2,$$

$$= \sum_{n=-\infty}^{n=+\infty} G_F^2 F_{\pi^+}^2 \int \frac{d|q_2|F(|q_2|)\delta(G(|q_2|)) |M'_{fi}|^2}{64\pi^2 Q_1}. \quad (17)$$

where $M'_{fi}$ represents the laser-assisted decay amplitude, and it is given by

$$|M'_{fi}|^2 = |B n(z)|^2 \left[ f_+(\Delta') + f_-(\Delta') \right] + q_2 P_3 \left( f_+(\Delta') - f_-(\Delta') \right)^2, \quad (19)$$

where $\Delta' = m_{K^+}^2 + m_{e^2}^2 - 2q_1 P_3$ is the effective 4-momentum transfer. $n$ is interpreted as the number of exchanged photons such that $n \geq 0$ corresponds to emission of photons and $n < 0$ corresponds to absorption [31]. Since the charged kaon may decay via fifty decay modes [27], and since our purpose is to study the effect of the laser field on both the hadronic channel $(\pi^+, \pi^0)$ and the leptonic channel, we define the total decay width of the charged kaon $\Gamma^T$ as follows:

$$\Gamma^T = \Gamma + \Gamma_{\text{Lept}} + \Gamma_{\text{O.C.}}, \quad (20)$$

where $\Gamma$ is the two-body hadronic decay width given by eq. (17), $\Gamma_{\text{Lept}}$ represents the laser-assisted leptonic decay width of the positive kaon, and $\Gamma_{\text{O.C.}} = 8.93365 \times 10^{-9} \text{eV}$ is the sum of decay widths which correspond to other channels. The branching ratio $BR$ and the lifetime $\tau$ are defined as

$$BR = \frac{\Gamma}{\Gamma^T}, \quad \tau = \frac{1}{\Gamma^T}. \quad (21)$$

**Results and discussion.** In this section, we will discuss the different obtained results about the action of a monochromatic electromagnetic field with circular polarization on the decay. A spherical geometry is chosen such that the angle $\varphi$ associated with the produced particle $\pi^+$ be zero, and the wave vector $k$ of the laser field propagates along the direction of the $z$-axis. We have calculated the parameters $f_+(\Delta)$ and $f_-(\Delta)$ in the absence of the laser field by using the value of $f_+(0) = 0.982 \pm 0.008$ [27] in the parameterization $(\lambda_1, \lambda_0)$. The constant $(G_F \times F_{\pi^+})^2$ is obtained by normalizing the free decay width, expressed in eq. (6), by its experimental value $1.09897 \times 10^{-8} \text{eV}$ [27]. As a first step, we will present the effect of the electromagnetic field on the number of emissions and absorptions of photons $n$ in order to introduce the notion of cut-off.

According to fig. 1, we observe that the variation of the laser field parameters induces a variation of the partial decay width, $d\Gamma^n/d\theta$, as a function of the number of
photon transferred between the laser field and the decaying system. We start with fig. 1(A), where the dressing of the charged particles has been made by a Nd : YAG laser ($\hbar \omega = 1.17 \text{eV}$) with a strength $\varepsilon_0 = 2 \times 10^5 \text{V/cm}$. In this case, the number of possible photons to be exchanged is $n = 3$. However, when the strength of this laser (Nd : YAG laser) is increased to reach $10^7 \text{V/cm}$ as it is shown in fig. 1(B), the possible number of photons $n$ that can be exchanged may be more than 100 photons. Outside of this range of photons numbers, we can see that the value of $d\Gamma_n/d\theta$ becomes zero. This means that the integral of $d\Gamma_n/d\theta$ over the angle $\theta$, formed by the direction of the outgoing particle $\pi^+$ with the $z$-axis (for a value of $\theta$ equal to $90^\circ$), becomes a constant for any number of photons $n$ greater than the possible number of photons that can be exchanged. The maximum number of photons that can be exchanged represents the “cut-off” for the laser field. Moreover, for a number of photons which is higher than the cut-off, the obtained constant value of $\Gamma$ represents also the value of the laser-free decay width. The symmetric aspect of $d\Gamma_n/d\theta$ is due to the Bessel function properties. To illustrate more clearly these results, we have shown in fig. 1(C) and (D) the effect of the number of transferred photons $n$ on the differential decay width $d\Gamma/d\theta$ for a well-defined laser field strength and by varying the angle $\theta$ from $1^\circ$ to $179^\circ$. The results presented in these figures are in full agreement with those presented in fig. 1(A) and (B). Indeed, the differential decay width in the presence of the laser field coincides with that in the absence of the laser field (red curve in fig. 1(C) for $\theta = 90^\circ$) when we sum over $n$ from $-4$ to 4 or from $-20$ to 20. In addition, we observe that in the case of a summation over $n$ from $-20$ to 20 in both fig. 1(C) and (D), the laser-assisted differential decay width (green curve) is equal to its corresponding laser-free differential decay width for all values of $\theta$ between $1^\circ$ and $179^\circ$. This result indicates that $|n| = 20$ exceeds the cut-off which corresponds to both ($\varepsilon_0 = 2 \times 10^5 \text{V/cm}; \hbar \omega = 1.17 \text{eV}$) and ($\varepsilon_0 = 10^6 \text{V/cm}; \hbar \omega = 2 \text{eV}$). In the following section, we will focus on the impact of the laser field on both the branching ratio and the lifetime, which are expressed in eq. (21).

Figure 2(A) represents the variation of the branching ratio as a function of the laser field strength for different frequencies $\hbar \omega = \{0.117 \text{eV}, 1.17 \text{eV} \text{and} 2 \text{eV}\}$ and by summing over the exchanged photons number from $-10$ to 10. It is well known that in the absence of the laser field the branching ratio is experimentally equal to $20.67 \pm 0.08 \times 10^{-2}$ [27]. According to fig. 2(A), we observe that, for low intensities, the laser field does not affect

Fig. 1: (A) and (B) (top): differential partial decay width of the charged kaon, $\Gamma_n(K^+ \rightarrow \pi^+ + \pi^0)$ (17), as a function of the photons number $n$ for $\hbar \omega = 1.17 \text{eV}$ and for different values of $\varepsilon_0$. The spherical coordinates are chosen such that $\theta = 90^\circ$ and $\varphi = 0^\circ$. (C) and (D) (bottom): dependence of the laser-assisted differential decay width on the angle $\theta$ for different numbers of exchanged photons $n$ and for different values of $\hbar \omega$ and $\varepsilon_0$. 

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Fig. 2: (A) and (B) (top): branching ratio of the charged kaon decay as a function of the laser field strength $\varepsilon_0$. (A) for different values of $\hbar\omega$ and by summing over $n$ from $-10$ to 10. (B) for different values of $n$ and for $\hbar\omega = 1.17\text{eV}$. (C) and (D) (bottom): lifetime of the positive kaon as a function of the laser field strength. (C) for different known laser frequencies and by taking the number of exchanged photons as ranging from $-10 \leq n \leq 10$. (D) for different values of $n$ and for $\hbar\omega = 1.17\text{eV}$.

the branching ratio. However, the branching ratio begins to decrease progressively from the laser field strength $\varepsilon_0 = 10^3\text{V/cm}$ for $\hbar\omega = 0.117\text{eV}$. For the CO$_2$ laser ($\hbar\omega = 0.117\text{eV}$), and for $\varepsilon_0 = 10^{12}\text{V/cm}$, the branching ratio is equal to $1.686 \times 10^{-8}\text{s}$. Moreover, with this strength value, we mention that the decay process can exchange more than 100000 photons. Therefore, the interpretation of the branching ratio decrease is more significant for strengths in the order of $10^4\text{V/cm}$ and $10^5\text{V/cm}$ for the Nd:YAG laser. In fig. 2(B) and for the Nd:YAG laser, the branching ratio always decreases by increasing the laser field strength, regardless of the number of exchanged photons $n$. The cases where the branching ratio is constant corresponds to the strengths for which the process can only exchange a number of photons $|n|$ lower than that shown in the figure. That means that the number of photons exchanged exceeds the cut-off. Now, let us move to fig. 2(C) and (D) that illustrate the variation of the laser-assisted lifetime of the charged kaon. It is clear that the lifetime increases as a function of the laser field strength especially when its strength overcomes a threshold value. Then, it stagnates as the laser field strength reaches a certain value. As we have seen for the branching ratio, the lifetime of the charged kaon (fig. 2(C)) in the presence of the laser field is more significant for the laser field strengths $\varepsilon_0 = 10^4\text{V/cm}$, $\varepsilon_0 = 10^6\text{V/cm}$ and $\varepsilon_0 = 10^7\text{V/cm}$ and for the frequencies $\hbar\omega = 0.117\text{eV}$, $\hbar\omega = 1.17\text{eV}$ and $\hbar\omega = 2\text{eV}$, respectively. By comparing these results with those obtained in the case of laser-assisted leptonic decay, we notice that the branching ratio and lifetime of the laser-assisted two-body hadronic decay of the charged kaon behaves in the same manner as in the case of leptonic decay ($K^+ \rightarrow \mu^+ + \nu_\mu$). However, we remark that the lifetime is longer in the case of the hadron dressing channel [14]. In fig. 3 the ratio of the partial decay width of $K^+ \rightarrow \pi^+ + \pi^0$ to that of $K^+ \rightarrow \mu^+ + \nu_\mu$ is considered.

Figure 3(A) illustrates the ratio of $K^+ \rightarrow \pi^+ + \pi^0$ to that of $K^+ \rightarrow \mu^+ + \nu_\mu$ for different known laser frequencies and by taking $n$ as ranging from $-10$ to 10. We recall that the experimental ratio of the two decay widths is equal to $0.3252 \pm 0.0016$ [27], and the branching ratio in the muonic channel is equal to $(63.56 \pm 0.11)\%$ in the absence of the laser. We also mention that the results of the two-body muonic decay of the charged kaon in the presence of a laser field are taken from our previous work [14]. According to fig. 3(A), it is clear that, for different laser field frequencies, the studied ratio undergoes an augmentation
Fig. 3: (A) Ratio of the two-body charged kaon hadronic decay width to the muonic decay width as a function of the laser field strength, $\varepsilon_0$, for different values of $\hbar \omega$ and by summing over $n$ from $-10$ to $10$. (B) The decay width of $K^+ \to \pi^+ + \pi^0$ as a function of the laser field strength for different values of $\hbar \omega$ and by summing over $n$ from $-10$ to $10$.

especially for the laser field strengths that allow the exchange of a number $|n| = 10$ of photons (e.g., the strengths which are greater than $\varepsilon_0 = \{10^3 \text{V/cm} \text{ for the case of } \hbar \omega = 0.117 \text{eV}\}$). As a hypothesis, the increase of the ratio of the hadronic decay width to the muonic decay width may be interpreted by the fact that the laser field increases the two-body hadronic decay width of the charged kaon, and decreases its muonic decay width. Another hypothesis is that both the hadronic and muonic decay width increase or decrease, but they change with different rhythms. To check the validity of our hypotheses, we have presented different values of the ratio $\Gamma\{\pi^+, \pi^0\}/\Gamma\{\mu^+, \nu_\mu\}$ in the table associated to fig. 3(B) to illustrate the variation of the decay width of $K^+ \to \pi^+ + \pi^0$ as a function of the laser field strength. As presented in fig. 3(B), the two-body hadronic decay width of the charged kaon decreases by increasing the laser field strength. This means that our second hypothesis is the correct one. Therefore, the laser field has a strong effect of decreasing the charged kaon muonic decay width, $K^+ \to \mu^+ + \nu_\mu$, as compared to its effect on the hadronic decay width presented in fig. 3(B). We should mention that in the histogram which corresponds to $\hbar \omega = 2 \text{eV}$ and for the intensities $\varepsilon_0 = \{10^3 \text{V/cm}, 10^4 \text{V/cm}, 10^5 \text{V/cm} \text{ and } 10^6 \text{V/cm}\}$ the decay width has different values others than the ones represented in fig. 3(B). The real values of this decay width are successively $\Gamma = 1.09897 \times 10^{-8} \text{eV}$ and $\Gamma = 1.09894 \times 10^{-8} \text{eV}$ for the intensities $\varepsilon_0 = \{10^3 \text{V/cm}, 10^4 \text{V/cm}, 10^5 \text{V/cm} \text{ and } 10^6 \text{V/cm}\}$ $\hbar \omega = 1.17 \text{eV}$. This change is made to illustrate clearly the decrease of the decay width. We will now move to the last obtained result which represents the simultaneous dependence of the charged kaon decay width on both the laser field strength and the number of exchanged photons for $\theta = 90^\circ$.

Figure 4 represents some values of the charged kaon decay width for different combinations of the laser field and the number of exchanged photons. The range of laser field strengths $\varepsilon_0 = 10^5 \text{V/cm} \to \varepsilon_0 = 10^7 \text{V/cm}$ have been selected based on the results obtained in figs. 2 and 3. As fig. 4 shows, the red zone corresponds to the maximum decay width. By comparing the results obtained in fig. 1(A) and (B) with those of fig. 4(A) and (B), we can deduce that the number of photons in this zone is higher than its corresponding cut-offs. The violet zone in both fig. 4(A) and (B) corresponds to values of the charged kaon decay width which are near to zero. This zone is less probable since the required photons number representing the cut-off in this zone is very large than
that shown in these figures. Therefore, we predict that the charged kaon decay width is more significant in the region between the red and violet zones. Another important point to be mentioned, here, is that by comparing the results of fig. 4(A) with those of fig. 4(B), for the same range of laser strengths, it appears that the use of high frequencies requires high laser field strengths in order to introduce the phenomena of emission and/or absorption of photons. In contrast, for the Nd:YAG laser, low laser field strengths can induce the exchange of photons with the decaying system.

Conclusion. – In this paper and based on the interesting results found for laser-assisted leptonic decay [14], we have investigated the laser-assisted charged kaon decay in the hadronic channel. We have found that the decay width decreases by increasing the laser field strength, and the emission and/or absorption phenomena appear and increase by increasing the laser field strength. In addition, the study of mesons as particles without structure introduces the decrease of the branching ratio with an increase in the lifetime. We have also found that the ratio of the hadronic decay width to the muonic decay width increases by increasing the strength of the laser field. This result is due to the strong and rapid decrease of the muonic decay width as compared to the two-body hadronic decay width. All these results invite us to study other channels of the charged kaon decay in the presence of the electromagnetic field in order to have a complete picture about its effect on the decay.

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REFERENCES

[1] MAHAN T. H., Nature, 187 (1960) 493.
[2] CAMMARATA M., ZERDANE S., BALDUCCI L. et al., Nat. Chem., 13 (2021) 10.
[3] KOZK M., ECKSTEIN T., SCHÖNENBERGER N. et al., Nat. Phys., 14 (2018) 121.
[4] LATINNE O., JOACHAIN C. J. and DÖRR M., Europhys. Lett., 26 (1994) 333.
[5] Hu S. X. and KEITEL C. H., Europhys. Lett., 47 (1999) 318.
[6] JOACHAIN C. J., Europhys. Lett., 108 (2004) 44001.
[7] ATTAOURTI Y., MANAUT B. and MAHIOUTE A., Phys. Rev. A, 69 (2004) 063407; MANAUT B., TAJ S. and EL IDRISI M., Can. J. Phys., 91 (2013) 696.
[8] ATTAOURTI Y., TAJ S. and MANAUT B., Phys. Rev. A, 71 (2005) 062705.
[9] ATTAOURTI Y., MANAUT B. and TAJ S., Phys. Rev. A, 70 (2004) 023404; MANAUT B., TAJ S. and ATTAOURTI Y., Phys. Rev. A, 71 (2005) 043401.
[10] NIKISHOV A. I. and RITUS V. I., Sov. Phys. JETP, 19 (1964) 1199.
[11] AKHMEDOV E. K., Sov. Phys.-JETP, 58 (1983) 883.
[12] REISS H. R., Phys. Rev. C, 27 (1983) 1199.
[13] BECKER W., SCHLICHER R. R. and SCULLY M. O., Nucl. Phys. A, 426 (1984) 125.
[14] BAOUAHI M. et al., Laser Phys. Lett., 18 (2021) 106001.
[15] MÜLLER C., VOITKIV A. B. and GRÜN N., Nucl. Instrum. Methods Phys. Res. B, 205 (2003) 306.
[16] TAJ S., MANAUT B. and EL IDRISI M., Afr. J. Math. Phys., 11 (2012) 27; TAJ S., MANAUT B. and HROUR E. and EL IDRISI M., Acta Phys. Pol. A, 136 (2019) 78.
[17] OUAHMOUN M., OUALI M., TAJ S. and MANAUT B., Laser Phys. Lett., 18 (2021) 076002; DAIHRI I. et al., Laser Phys. Lett., 18 (2021) 096001; OUALI M., OUAHMOUN M., MEKAOUI Y., TAJ S. and MANAUT B., Chin. J. Phys., 77 (2021) 1182; MEKAOUI Y. et al., Laser Phys. Lett., 19 (2022) 066003.
[18] VOKLOV D. M., Z. Phys., 94 (1935) 250.
[19] MOUSLHI S., JAKHA M., TAJ S., MANAUT B. and SHIER E., Phys. Rev. D, 102 (2020) 073006.
[20] JAKHA M., MOUSLHI S., TAJ S. and MANAUT B., Laser Phys. Lett., 18 (2021) 016002.
[21] OUALI M., OUAHMOUN M., TAJ S., MANAUT B. and BENBIRIK R., Phys. Lett. B, 823 (2021) 136761.
[22] SAUTER F., Z. Phys., 69 (1931) 742.
[23] SCHWINGER J., Phys. Rev., 82 (1951) 664.
[24] BREZIN E. and ITZYKSON C., Phys. Rev. D, 2 (1970) 1191.
[25] MOUROU G. and TAJIMA T., Science, 331 (2011) 41.
[26] PIAZZA A., MÜLLER C., HAYASHI T. and KEITEL C. H., Rev. Mod. Phys., 84 (2012) 1177.
[27] ŽYLA P. A. et al., Prog. Theor. Exp. Phys., 2020 (2020) 093C01.
[28] OKUN L. B., Leptons and Quarks (Elsevier, Amsterdam) 1984, p. 79.
[29] HAYASHI T. and NAKAGAWA M., Prog. Theor. Phys., 35 (1966) 515.
[30] GREINER W. and MÜLLER B., Gauge Theory of Weak Interactions, 3rd edition (Springer, Berlin) 2000, p. 211.
[31] BERESTETSKII V. B., Lifshitz E. M. and Pitaevskii L. P., Quantum Electrodynamics, 2nd edition (Oxford) 1982.