Reducing Penguin Pollution

Bhubanjyoti Bhattacharya\textsuperscript{a}\textsuperscript{1}, Alakabha Datta\textsuperscript{b}\textsuperscript{2} and David London\textsuperscript{a}\textsuperscript{3}

\textsuperscript{a}: Physique des Particules, Université de Montréal, C.P. 6128, succ. centre-ville, Montréal, QC, Canada H3C 3J7
\textsuperscript{b}: Department of Physics and Astronomy, 108 Lewis Hall, University of Mississippi, Oxford, MS 38677-1848, USA

Abstract

The most common decay used for measuring $2\beta_s$, the phase of $B_s^0\bar{B}_s^0$ mixing, is $B_s^0 \rightarrow J/\psi \phi$. This decay is dominated by the colour-suppressed tree diagram, but there are other contributions due to gluonic and electroweak penguin diagrams. These are often referred to as “penguin pollution” (PP) because their inclusion in the amplitude leads to a theoretical error in the extraction of $2\beta_s$ from the data. In the standard model (SM), it is estimated that the PP is negligible, but there is some uncertainty as to its exact size. Now, $\phi_s^{CCs}$ (the measured value of $2\beta_s$) is small, in agreement with the SM, but still has significant experimental errors. When these are reduced, if one hopes to be able to see clear evidence of new physics (NP), it is crucial to have the theoretical error under control. In this paper, we show that, using a modification of the angular analysis currently used to measure $\phi_s^{CCs}$ in $B_s^0 \rightarrow J/\psi \phi$, one can reduce the theoretical error due to PP. Theoretical input is still required, but it is much more modest than entirely neglecting the PP. If $\phi_s^{CCs}$ differs from the SM prediction, this points to NP in the mixing. There is also enough information to test for NP in the decay. This method can be applied to all $B_s^0/\bar{B}_s^0 \rightarrow V_1V_2$ decays.

Keywords: $B$ decays, CP violation, penguin pollution, angular analysis

PACS numbers: 11.30.Hv, 12.15.Ji, 13.25.Hw, 14.40.Nd
1 Introduction

In the presence of $B_s^0 - \bar{B}_s^0$ mixing, the mass eigenstates $B_{L,H}^s$ (where $L$ (H) corresponds to “light” (“heavy”)) are admixtures of the flavour eigenstates $B_s^0$ and $\bar{B}_s^0$:

$$|B_{L,H}^s\rangle = p |B_s^0\rangle \pm q |\bar{B}_s^0\rangle ,$$  \hspace{1cm} (1)

where the complex coefficients $p$ and $q$ satisfy $|p|^2 + |q|^2 = 1$. States which are $B_s^0$ or $\bar{B}_s^0$ at $t = 0$ then evolve in time into an admixture of both states, leading to the time-dependent states $B_s^0(t)$ and $\bar{B}_s^0(t)$. If both $B_s^0$ and $\bar{B}_s^0$ can decay to the final state $f$, there is an indirect (mixing-induced) CP-violating asymmetry (CPA) between the rates $|B_s^0(t) \to f|^2$ and $|\bar{B}_s^0(t) \to f|^2$. The indirect CPA measures

$$\text{Im} \left( \frac{q A_s^f}{p \bar{A}_s^f} \right) ,$$  \hspace{1cm} (2)

where $A_s^f$ and $\bar{A}_s^f$ are the amplitudes for $B_s^0 \to f$ and $\bar{B}_s^0 \to f$, respectively. $\bar{A}_s^f$ can be obtained from $A_s^f$ by changing the signs of the weak phases.

$B_s^0 - \bar{B}_s^0$ mixing is dominated by the box diagram with an internal $t$ quark, so that $q/p = (V_{tb} V_{cs}^* / V_{tb} V_{ts}^*) = \exp(2i \arg(V_{tb} V_{ts}))$. This is phase-convention dependent. Suppose now that $A_s^f$ is dominated by a single decay amplitude with the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements $V_{cb}^* V_{cs}$. We then have $A_s^f/\bar{A}_s^f = (V_{cb} V_{cs}^* / V_{cs} V_{cs}) = \exp(2i \arg(V_{cb} V_{cs}))$, which is also phase-convention dependent. However, the product of these two quantities is

$$\frac{q A_s^f}{p \bar{A}_s^f} = \frac{V_{tb}^* V_{ts} V_{cb} V_{cs}^*}{V_{cb}^* V_{cs} V_{tb} V_{ts}^*} = e^{2i \beta_s} ,$$  \hspace{1cm} (3)

where

$$\beta_s \equiv \arg \left[ -\frac{V_{tb}^* V_{ts}}{V_{cb}^* V_{cs}} \right] .$$  \hspace{1cm} (4)

This is phase-convention independent, and hence physical. The indirect CPA measures $\sin 2\beta_s$.

In the standard model (SM), $2\beta_s$ is expected to be very small: the SM prediction is $2\beta_s = 0.03636 \pm 0.00170$ rad [1]. For this reason there is great interest in measuring this quantity. A value for $2\beta_s$ significantly different from zero would be a smoking-gun signal of new physics (NP).

The main decay mode used for measuring $2\beta_s$ is $B_s^0 \to J/\psi \phi$, which is the analogue of the “golden” mode $B_d^0 \to J/\psi K_S$ used to measure the $B_d^0 - \bar{B}_d^0$ mixing phase $2\beta$. Since $B_s^0 \to J/\psi \phi$ involves two final-state vector mesons, a time-dependent angular analysis of this decay (with $J/\psi \to \ell^+ \ell^-$ and $\phi \to K^+ K^-$) must be used to disentangle the CP + and − final states. The most precise measurement of the
indirect CPA has been performed by the LHCb Collaboration \[2\]. They find
\[\phi^{cs}_{s} = -0.001 \pm 0.101 \text{ (stat) } \pm 0.027 \text{ (syst) } \text{rad}, \] (5)
in agreement with the SM. (Still, the errors are large enough that NP cannot be excluded.)

Now, the above formalism holds for the case where \(A_{s}^{f} \) is dominated by a single
decay amplitude. However, if \(A_{s}^{f} \) contains a second decay amplitude with a differ-
ent weak phase, then the indirect CPA no longer cleanly measures \(2\beta_{s} \) – there is
a theoretical uncertainty due to the presence of the second amplitude. It is often
the case that the first and second amplitudes correspond to tree and penguin dia-
grams, respectively. The theoretical uncertainty is thus generally called “penguin
pollution.”

For \(B_{s}^{0} \to J/\psi \phi \), the decay amplitude can be written
\[A_{s}^{J/\psi \phi} = \lambda_{c}^{(s)} C' + \left( \lambda_{t}^{(s)} P'_{t} + \lambda_{c}^{(s)} P'_{c} + \lambda_{u}^{(s)} P'_{u} \right) + \frac{2}{3} \lambda_{t}^{(s)} P'_{EW}, \] (6)
where \(\lambda_{q}^{(q')} \equiv V_{q'q}^{*} V_{qq'} \). Here, \(C'\), \(P'\) and \(P'_{EW}\) are the colour-suppressed tree, gluonic
penguin and electroweak penguin diagrams, respectively. (As this is a \(b \to \bar{s}\) transition,
the diagrams are written with primes.) The index ‘\(i\)’ of \(P'_{i} \) indicates the flavour
of the quark in the loop. The unitarity of the CKM matrix \((\lambda_{u}^{(s)} + \lambda_{c}^{(s)} + \lambda_{t}^{(s)} = 0) \)
can be used to eliminate the \(t\)-quark contribution:
\[A_{s}^{J/\psi \phi} = \lambda_{c}^{(s)} (C' + P'_{c} - \frac{2}{3} P'_{EW}) + \lambda_{u}^{(s)} (P'_{u} - \frac{2}{3} P'_{EW}) \equiv e^{i \arg(V_{ub}^{*} V_{us})} [A_{1} + e^{i \gamma} A_{2}], \] (7)
where \(P'_{ct} \equiv P'_{c} - P'_{t} \), \(P'_{ut} \equiv P'_{u} - P'_{t} \). In the second line, \(A_{1}\) and \(A_{2}\) are combinations of
diagrams, and include the magnitudes of the corresponding CKM matrix elements.
We have explicitly written the weak-phase dependence of the terms in the ampli-
tude\[^{5}\]. (The phase information in the CKM matrix is conventionally parametrized
in terms of the unitarity triangle, in which the interior (CP-violating) angles are
known as \(\alpha, \beta\) and \(\gamma^{3}\).) \(A_{1}\) and \(A_{2}\) contain strong phases.

We now have
\[\frac{q \tilde{A}_{s}^{f}}{p A_{s}^{f}} = e^{2i\beta_{s}} A_{1} + e^{-i\gamma} A_{2} \frac{A_{1} + e^{i\gamma} A_{2}}{A_{1} + e^{i\gamma} A_{2}} . \] (8)
\[^{4}\] \(\beta_{s}\) is defined in Eq. (4). For the measured \(B_{s}^{0}-\bar{B}_{s}^{0}\) mixing phase, it is common to use the
symbol \(\phi^{cs}_{s}\) (or occasionally just \(\phi_{s}\)), which is equal to \(-2\beta_{s}\) in the SM.
\[^{5}\] The weak phase in the coefficient of \(A_{2}\) is \(\arg(V_{ub}^{*} V_{us}/V_{cb}^{*} V_{cs})\), and so should technically be
called \(\gamma_{s}\). However, it differs very little from \(\gamma \equiv \arg(V_{ub}^{*} V_{ud}/V_{cb}^{*} V_{cd})\), and so we refer to it by this
name.
The point here is that the $A_2$ term represents the penguin pollution (PP). There are several ingredients in estimating its size. First, $|V_{us}^* V_{ub}| / |V_{cs}^* V_{cs}| \approx 0.02$. Second, in Ref. [4, 5], it is estimated that $|P'_{ct}/C'| \sim |P'_{us}/C'| = O(1)$. However, here the penguin diagrams are both OZI-suppressed, so these ratios are much smaller. Third, it is expected that $|P'_{EW}/C'| = O(\tilde{\lambda})$, where $\tilde{\lambda} \sim 0.2$ [4, 5]. Putting everything together, it is estimated theoretically that $|A_2/A_1| = O(10^{-3})$ [6, 7, 8]. Thus, to a good approximation, the $A_2$ term, i.e. the PP, is negligible. In this case, the indirect CPA in $B^0_s \to J/\psi \phi$ cleanly measures $2\beta_s$.

Indeed, in the determination of $\phi_s^{c\bar{s}s}$ [Eq. (3)], the LHCb Collaboration neglected the PP [2] ($A^f_s = A^f_s$ was assumed). However, the result reintroduces the question of the size of the PP. To be specific, we now know that $\phi_s^{c\bar{s}s}$ is small. Suppose a future measurement finds $\phi_s^{c\bar{s}s} = 0.1 \pm 0.01$ rad (error is statistical only). This disagrees with the SM prediction and therefore suggests NP. But perhaps the size of the PP has been underestimated, so that there is really a nonzero theoretical error associated with this measurement. It is possible that, when this error is taken into account, the discrepancy with the SM disappears, so that there is no signal of NP. This situation is not at all improbable – the theoretical prediction of $O(10^{-3})$ for the size of the PP includes estimates of the hadronic matrix elements. But these are notoriously difficult to determine with certainty. It is not impossible that the true size of the PP is larger than its theoretical estimate, and that this introduces a theoretical error which is important when the measured value of $\phi_s^{c\bar{s}s}$ is small.

This issue was raised several years ago in Ref. [9]. There it was suggested that the PP term can be measured in the time-dependent angular distribution of the flavour-specific $\bar{b} \to \bar{d}$ decay $B^0_s \to J/\psi K^{*0}(\to \pi^+ K^-)$, and then related to $B^0_s \to J/\psi \phi$ using flavour SU(3) symmetry. The difficulty here is that the value of SU(3) breaking is unknown, and this is problematic given that we need a precise value of the size of the PP.

The purpose of this paper is to point out that, in fact, using the present time-dependent angular analysis, the theoretical error due to PP associated with $\phi_s^{c\bar{s}s}$ can be reduced. In this method, one retains the PP term. One piece of theoretical input is still needed to extract $\phi_s^{c\bar{s}s}$, but the theoretical error is quite a bit smaller than when the PP term is neglected from the beginning. In addition, if it is concluded that NP is present, there is enough measured information to determine whether the NP is in the mixing and/or the decay.

In Sec. 2, we describe the method for extracting $\phi_s^{c\bar{s}s}$ from the full angular analysis. Here we follow the procedure as described in Ref. [10]. The theoretical error, and how it can be reduced, are discussed in Sec. 3. In Sec. 4, we show how the PP or NP parameters can be extracted, and discuss the application of the method to other $B^0_s/\bar{B}^0_s \to V_1 V_2$ decays. We conclude in Sec. 5.
Angular Analysis

Given that $B_s^0 \to J/\psi \phi$ has two final-state vector mesons, its amplitude can be separated into 3 helicities: the polarizations of the vector mesons are either longitudinal ($A_0$), or transverse to their directions of motion and parallel ($A_\parallel$) or perpendicular ($A_\perp$) to one another. In addition, since the $\phi$ is detected through its decay to $K^+K^-$, one must also allow for the possibility that the observed $K^+K^-$ pair has relative angular momentum $l = 0$ ($S$-wave) \cite{11}. That is, the angular distribution of the decay $B_s^0 \to J/\psi(\to \ell^+\ell^-)\phi(\to K^+K^-)$ involves the 4 helicities 0, $\parallel$, $\perp$, $S$.

In going from the decay to the CP-conjugate decay, $A_h \to \eta_h \tilde{A}_h$ ($h = 0, \parallel, \perp, S$), in which the $\tilde{A}_h$ are equal to the $A_h$, but with weak phases of opposite sign, and $\eta_0 = \eta_\parallel = +1$, $\eta_\perp = \eta_S = -1$.

The angular distribution is given in terms of the vector $\vec{\Omega} = (\theta, \psi, \varphi)$ in the transversity basis \cite{12} \cite{13} \cite{14} \cite{15} \cite{16} \cite{17}. This basis is defined as follows: in the $J/\psi$ rest frame, one has a righthanded coordinate system in which the $x$ axis is parallel to $\vec{p}_\phi$ and the $z$ axis is parallel to $\vec{p}_{K^-} \times \vec{p}_{K^+}$. In this frame, $\theta$ and $\varphi$ are the azimuthal and polar angles, respectively, of the $\mu^+$. The angle $\psi$ is the angle between $\vec{p}_{K^-}$ and $\vec{p}_{J/\psi}$ in the rest frame of the $\phi$. We have

$$\frac{d^4\Gamma(B_s^0 \to J/\psi \phi)}{dt d\vec{\Omega}} \propto \sum_{k=1}^{10} h_k(t) f_k(\vec{\Omega}). \quad (9)$$

The explicit expressions for the $f_k(\vec{\Omega})$ are given in Ref. \cite{10}.

The $h_k(t)$ are determined as follows. The time dependence of the helicity amplitudes $A_h$ ($h = 0, \parallel, \perp, S$) is given by \cite{18}

$$A_h(t) = g_+(t) A_h(t=0) + \eta_h \frac{q}{p} g_-(t) \tilde{A}_h(t=0). \quad (10)$$

$g_\pm(t)$ contain the information about the time evolution of $B_s^0(t)$ and $\bar{B}_s^0(t)$. We use the following established relations involving $g_\pm(t)$:

$$|g_\pm(t)|^2 = \frac{1}{2} e^{-\Gamma_s t} \left( \cosh (\Delta \Gamma_s / 2) t \pm \cos \Delta m_s t \right),$$

$$g_+^*(t) g_-(t) = \frac{1}{2} e^{-\Gamma_t} \left( - \sinh (\Delta \Gamma_s / 2) t + i \sin \Delta m_s t \right). \quad (11)$$

The $h_k(t)$ are given by

$$h_1(t) = |A_0(t)|^2, \quad h_2(t) = |A_\parallel(t)|^2, \quad h_3(t) = |A_\perp(t)|^2, \quad h_7(t) = |A_S(t)|^2,$$

$$h_4(t) = \text{Im} \left( A_\perp(t) A_\parallel^*(t) \right), \quad h_6(t) = \text{Im} \left( A_\parallel(t) A_\perp^*(t) \right),$$

$$h_5(t) = \text{Re} \left( A_0(t) A_\parallel^*(t) \right), \quad h_9(t) = \text{Im} \left( A_\parallel(t) A_\perp^*(t) \right),$$
\[
    h_s(t) = \text{Re} \left( A_\parallel(t)A_\parallel^*(t) \right),
    \quad h_{10}(t) = \text{Re} \left( A_0(t)A_\parallel^*(t) \right),
\]
and can be written as
\[
    h_k(t) = \frac{1}{2} e^{-\Gamma_s t} \left[ c_k \cos \Delta m_s t + d_k \sin \Delta m_s t + a_k \cosh (\Delta \Gamma_s/2) t + b_k \sinh (\Delta \Gamma_s/2) t \right].
\]

By measuring the time-dependent angular distribution and fitting to the four time-dependent functions, \( \Gamma_s \) and \( \Delta \Gamma_s \) can be determined, as well as the coefficients \( a_k-d_k \).

There are also the functions \( \tilde{h}_k(t) \) in the angular distribution of the CP-conjugate decay \( \bar{B}_s^0 \to J/\psi \phi \). They can be obtained from the \( h_k(t) \) by making the following changes: \( A_h \leftrightarrow \eta_h \bar{A}_h \) and \( \phi^{cCS} \to -\phi^{cCS} \). Each of the \( \tilde{h}_k \) of the \( \tilde{h}_k \) is the same as the \( a_k-d_k \) of the \( h_k \), up to a possible sign. Both \( h_k \) and \( \tilde{h}_k \) should be included in the fit.

Also, the \( h_k(t) \) contain both CP-conserving and CP-violating terms. However, when one calculates the sum or difference of \( h_k(t) \) and \( \tilde{h}_k(t) \), certain terms cancel, so that these sums and differences are purely CP-conserving or CP-violating. In particular, the untagged sums \( h_k(t) + \tilde{h}_k(t) \) are CP-violating for \( k = 4,6,8,10 \). These correspond to triple-product terms. Similarly, the differences \( h_k(t) - \tilde{h}_k(t) \) \((k = 1,2,3,5,7,9)\) are CP-violating. These can only be constructed through tagged decays, and contain the direct and indirect CPA’s.

Now, the ultimate goal is to measure \( \phi^{cCS}_s \). To this end, it is necessary to express the \( a_k-d_k \) in terms of all the theoretical unknowns in order to determine what exactly can be extracted. The \( b_k \) and \( d_k \) involve terms of the form
\[
    \text{Re} \left( \frac{q}{p} A_h^* \bar{A}_{h'} \right), \quad \text{Im} \left( \frac{q}{p} A_h^* \bar{A}_{h'} \right).
\]

The helicity amplitudes can be multiplied by an arbitrary phase, so that the form of the individual terms is uncertain. In order to fix the phases, in what follows we adopt the convention that the dominant SM decay amplitude has no weak phase. That is, the phase term \( e^{-2i \arg(V_{ud}^* V_{cs})} \) is factored out and combined with the phase of \( q/p \) to make \( 2\beta_s \), as in Eq. (8). Thus, any terms of the form \( \text{Re}(q/p) \) or \( \text{Im}(q/p) \) depend only on the mixing phase \( \beta_s \).

The \( a_k-d_k \) are expressed in terms of the unknown parameters. Suppose first that only the dominant SM amplitude is retained (the “1-amplitude method”) – this is \( A_1 \) of Eq. (7). In this case we have \( \bar{A}_h = A_h (h = 0, \parallel, \perp, S) \), so that there are 8 unknown parameters – the magnitudes of the \( A_h \) (4), the relative strong phases (3), and \( \phi_s^{cCS} \). (In their analysis, LHCb fix the normalization \( |A_0|^2 + |A_\parallel|^2 + |A_\perp|^2 = 1 \).) The coefficients \( a_k-d_k \) are given in Table 1 in which the strong-phase differences are
$\delta_{ij} \equiv \arg(A_i) - \arg(A_j)$. It is clear from this Table that all unknown parameters can be determined from the measurements of $a_k$-$d_k$. In addition, there is a great deal of redundancy, which allows for a reasonably precise determination of these unknowns. Indeed, this is the method used by the LHCb Collaboration [2, 10]. However, as noted above, there is a potential theoretical error associated with the neglect of the PP.

\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|c|c|}
\hline
$a_k$ & $b_k$ & $c_k$ & $d_k$ \\
\hline
$h_1$ & $2|A_0|^2$ & $-2|A_0|^2 \cos \phi_s^{\text{CS}}$ & 0 & $2|A_0|^2 \sin \phi_s^{\text{CS}}$ \\
$h_2$ & $2|A_\perp|^2$ & $-2|A_\perp|^2 \cos \phi_s^{\text{CS}}$ & 0 & $2|A_\perp|^2 \sin \phi_s^{\text{CS}}$ \\
$h_3$ & $2|A_{1\perp}|^2$ & $2|A_{1\perp}|^2 \cos \phi_s^{\text{CS}}$ & 0 & $-2|A_{1\perp}|^2 \sin \phi_s^{\text{CS}}$ \\
$h_4$ & 0 & $-2|A_{1\perp}| |A_\perp| \cos \delta_{1\perp} \sin \phi_s^{\text{CS}}$ & $2|A_{1\perp}| |A_\perp| \sin \delta_{1\perp}$ & $-2|A_{1\perp}| |A_\perp| \cos \delta_{1\perp} \cos \phi_s^{\text{CS}}$ \\
$h_5$ & $2|A_\parallel| |A_0| \cos \delta_{0\parallel}$ & $-2|A_\parallel| |A_0| \cos \delta_{0\parallel} \cos \phi_s^{\text{CS}}$ & 0 & $2|A_\parallel| |A_0| \cos \delta_{0\parallel} \sin \phi_s^{\text{CS}}$ \\
$h_6$ & 0 & $-2|A_{1\perp}| |A_0| \cos \delta_{0\perp} \sin \phi_s^{\text{CS}}$ & $2|A_{1\perp}| |A_0| \sin \delta_{0\perp}$ & $-2|A_{1\perp}| |A_0| \cos \delta_{0\perp} \cos \phi_s^{\text{CS}}$ \\
$h_7$ & $2|A_{S\perp}|^2$ & $2|A_{S\perp}|^2 \cos \phi_s^{\text{CS}}$ & 0 & $-2|A_{S\perp}|^2 \sin \phi_s^{\text{CS}}$ \\
$h_8$ & 0 & $-2|A_{S\parallel}| |A_S| \sin \delta_{S\parallel} \sin \phi_s^{\text{CS}}$ & $2|A_{S\parallel}| |A_S| \cos \delta_{S\parallel}$ & $-2|A_{S\parallel}| |A_S| \sin \delta_{S\parallel} \cos \phi_s^{\text{CS}}$ \\
$h_9$ & $2|A_{S\perp}| |A_S| \sin \delta_{S\perp}$ & $2|A_{S\perp}| |A_S| \sin \delta_{S\perp} \cos \phi_s^{\text{CS}}$ & 0 & $-2|A_{S\perp}| |A_S| \sin \delta_{S\perp} \sin \phi_s^{\text{CS}}$ \\
$h_{10}$ & 0 & $-2|A_0| |A_S| \sin \delta_{0S} \sin \phi_s^{\text{CS}}$ & $2|A_0| |A_S| \cos \delta_{0S}$ & $-2|A_0| |A_S| \sin \delta_{0S} \cos \phi_s^{\text{CS}}$ \\
\hline
\end{tabular}
\caption{Coefficients $a_k$-$d_k$ of Eq. (13) in terms of $|A_h|$ ($h = 0, \parallel, \perp, S$) and $\delta_{ij}$ [Eq. (15)] for the case where $A_h = A_h$.}
\end{table}

Suppose instead that no assumptions about the decay amplitude are made, and we allow for the possibility of more than one contribution to the amplitude. In fact, this was first considered in Ref. [19]. Here (and in Ref. [9]), the focus was specifically on the SM. The PP term was included, and the tree ($A_1$) and penguin ($A_2$) amplitudes were separated, as in Eq. (7). It was then noted that, if one takes the mixing phase as known, the angular analysis gives enough information to extract $\gamma$, given one piece of theoretical input. This is similar to one of the points made later in Sec. 4. However, the possibility of NP was not considered. Indeed, in the presence of NP, the method of Ref. [19] no longer holds. On the other hand, as we show below, one can go well beyond this analysis by not separating $A_1$ and $A_2$. By applying the angular analysis to the full amplitudes $A_h$ and $A_{\bar{h}}$, one can still extract $\phi_s^{\text{CS}}$, even if there is NP in the PP. Furthermore, we show how the theoretical error due to the PP can be reduced in this way.

The most general way to allow for the possibility of more than one contribution to the amplitude (the “2-amplitude method”) is to consider the full amplitudes $A_h$ and $A_{\bar{h}}$, and take $A_h \neq A_{\bar{h}}$. This inequality can arise due to enhanced PP within the SM and/or the presence of NP. In general, the $a_k$-$d_k$ are expressed in terms of 16 unknown parameters: the magnitudes of the $A_h$ and $A_{\bar{h}}$ (8), their relative phases (7), and $\phi_s^{\text{CS}}$. For the phase differences we define

$$\delta_{ij} \equiv \arg(A_i) - \arg(A_j)$$
angular analysis.

given in Tables 2-4. All of these quantities can be measured in the time-dependent

to choose a value for one of the $D$.

$\phi$ is not possible to separate

an accident – it is a direct consequence of the fact that one requires a convention to

where the PP has been retained, theoretical input is necessary. (Note that this is not

For the missing relative phase – a $D$ – one has to look at the $b_k$ and $d_k$ (Tables

when $\bar{A}_h \neq A_h$, the coefficients $a_k$-$d_k$ of Table 1 are modified – in particular,

All of these quantities can be measured in the time-dependent angular analysis.

|   | $(a_k + c_k)/2$ | $(a_k - c_k)/2$ |
|---|----------------|----------------|
| $h_1$ | $|A_0|^2$ | $|A_0|^2$ |
| $h_2$ | $|A_\|-|^2$ | $|A_\|-|^2$ |
| $h_3$ | $|A_\|=|^2$ | $|A_\|=|^2$ |
| $h_4$ | $|A_\|=||A_\|=|| \sin \delta_\|= | - |\bar{A}_\|=||A_\|=|| \sin \bar{\delta}_\|= |$ |
| $h_5$ | $|A_\|=||A_0|| \cos \delta_0 | |\bar{A}_\|=||A_0|| \cos \bar{\delta}_0 |$ |
| $h_6$ | $|A_\|=||A_0|| \sin \delta_0 | | - |\bar{A}_\|=||A_0|| \sin \bar{\delta}_0 |$ |
| $h_7$ | $|A_S|^2$ | $|A_S|^2$ |
| $h_8$ | $|A_\|=||A_S|| \cos \delta_S | | - |\bar{A}_\|=||A_S|| \cos \bar{\delta}_S |$ |
| $h_9$ | $|A_\|=||A_S|| \sin \delta_S | |\bar{A}_\|=||A_S|| \sin \bar{\delta}_S |$ |
| $h_{10}$ | $|A_0||A_S|| \cos \delta_0S | | - |\bar{A}_0||A_S|| \cos \bar{\delta}_0S |$ |

Table 2: Coefficients $a_k$ and $c_k$ of Eq. (13) in terms of $|A_h|$ and $|\bar{A}_h|$ ($h = 0, ||, \perp, S$), and $\delta_{ij}$ and $\bar{\delta}_{ij}$ [Eq. (15)].

$|A_h|$ and $|\bar{A}_h|$ can be obtained from $a_k$ and $c_k$ for $k = 1, 2, 3, 7$ (Table 2). The 3 independent relative phases among the $A_h$ (the $\delta_{ij}$) and the 3 independent relative phases among the $\bar{A}_h$ (the $\bar{\delta}_{ij}$) can be found from $a_k$ and $c_k$ for $k = 4-6$ and 8-10. For the missing relative phase – a $D_{ij}$ – one has to look at the $b_k$ and $d_k$ (Tables 3 and 4). However, all terms in these tables are a function of $\phi_s^{cbs} - D_{ij}$. Thus, it is not possible to separate $\phi_s^{cbs}$ and the $D_{ij}$ solely from the data. Even in this case, where the PP has been retained, theoretical input is necessary. (Note that this not an accident – it is a direct consequence of the fact that one requires a convention to fix the phases in Eq. (14).)

On the other hand, it is obvious how to formulate the theoretical input – one has to choose a value for one of the $D_{ij}$. For example, suppose we set $D_{00} = 0$. With this input, we can now obtain $\phi_s^{cbs}$ from a fit to the data. Note that no assumptions
where the \( \Delta \) is the strong-phase difference between \( A \) things. First, the theoretical error associated with the assumption that \( \text{Eq. (15)} \), and \( A \) are made about the magnitudes and other relative phases of the \( b \) are also obtained from the fit.

The best-fit value of \( \phi_{D} \) and the \( D \) the values of the other \( |D| \) we chosen, for example, the \( D \) best-fit values of the \( |D| \) equal to 0? The answer to both questions is as follows. The best-fit values of \( \phi_{D} \) and \( \bar{D} \) are also obtained from the fit. This choice of input does raise two questions: why choose \( D_{00} \) and why set it equal to 0? The answer to both questions is as follows. The best-fit values of \( |A| \), \( |A_{h}| \), \( \delta_{ij} \) and \( \bar{\delta}_{ij} \) are all independent of the choice of input. In changing the input, \( \phi_{c\bar{s}} \) and the \( D_{ij} \) all shift by known quantities. For example, had we chosen, say, \( D_{||} = 0 \), the best-fit value of \( \phi_{c\bar{s}} \) would be shifted by \( D_{||} - D_{00} = \delta_{||0} - \delta_{00} \) [Eq. (16)], and the values of the other \( D_{ij} \) would also be changed according to Eq. (15). And had we chosen, for example, \( D_{00} = \pi/6 \), this would simply have the effect of shifting the best-fit values of the \( D_{ij} \) and \( \phi_{c\bar{s}} \) by \( \pi/6 \). Since, as we will see in the next section, the \( D_{ij} \) are all expected to be small, we choose as input \( D_{00} = 0 \) (which is the same as in the 1-amplitude method).

### 3 Theoretical Error

In the 2-amplitude method, the theoretical error on \( \phi_{c\bar{s}} \) is equal to that associated with the assumption that \( D_{00} = 0 \). In the SM we write

\[
A_{0} = A_{1,0} + e^{i\gamma}A_{2,0}
\]

where the \( A_{2,0} \) term represents the PP. Then

\[
D_{00} = \text{arg}(A_{0}^{*}A_{0}) \simeq 2 \frac{|A_{2,0}|}{|A_{1,0}|} \cos \Delta \sin \gamma ,
\]

where \( \Delta \) is the strong-phase difference between \( A_{2,0} \) and \( A_{1,0} \). This shows two things. First, the theoretical error associated with the assumption that \( D_{00} = 0 \)

| \( h \) | \( b_{k} \) |
|---|---|
| \( h_{1} \) | \( -2|A_{0}||A_{0}| \cos(\phi_{c\bar{s}}^{cs} - D_{00}) \) |
| \( h_{2} \) | \( -2|A_{||}||A_{||}| \cos(\phi_{c\bar{s}}^{cs} - D_{||}) \) |
| \( h_{3} \) | \( 2|A_{\perp}||A_{\perp}| \cos(\phi_{c\bar{s}}^{cs} - D_{\perp}) \) |
| \( h_{4} \) | \( -|A_{||}||A_{\perp}| \sin(\phi_{c\bar{s}}^{cs} - D_{||\perp}) - |A_{\perp}||A_{||}| \sin(\phi_{c\bar{s}}^{cs} - D_{\perp||}) \) |
| \( h_{5} \) | \( -|A_{||}||A_{0}| \cos(\phi_{c\bar{s}}^{cs} - D_{0||}) - |A_{0}||A_{||}| \cos(\phi_{c\bar{s}}^{cs} - D_{||0}) \) |
| \( h_{6} \) | \( -|A_{\perp}||A_{0}| \sin(\phi_{c\bar{s}}^{cs} - D_{0\perp}) - |A_{\perp}||A_{0}| \sin(\phi_{c\bar{s}}^{cs} - D_{\perp0}) \) |
| \( h_{7} \) | \( 2|A_{S}||A_{S}| \cos(\phi_{c\bar{s}}^{cs} - D_{SS}) \) |
| \( h_{8} \) | \( -|A_{||}||A_{S}| \cos(\phi_{c\bar{s}}^{cs} - D_{||S}) + |A_{||}||A_{S}| \cos(\phi_{c\bar{s}}^{cs} - D_{S||}) \) |
| \( h_{9} \) | \( -|A_{\perp}||A_{S}| \sin(\phi_{c\bar{s}}^{cs} - D_{\perp S}) + |A_{\perp}||A_{S}| \sin(\phi_{c\bar{s}}^{cs} - D_{S\perp}) \) |
| \( h_{10} \) | \( -|A_{0}||A_{S}| \cos(\phi_{c\bar{s}}^{cs} - D_{0S}) + |A_{0}||A_{S}| \cos(\phi_{c\bar{s}}^{cs} - D_{S0}) \) |

Table 3: Coefficients \( b_{k} \) of Eq. (13) in terms of \( |A_{h}| \) and \( |\bar{A}_{h}| \) (\( h = 0, ||, \perp, S \)), \( D_{ij} \) [Eq. (15)], and \( \phi_{c\bar{s}}^{cs} \).
Table 4: Coefficients $d_k$ of Eq. (13) in terms of $|A_h|$ and $|\bar{A}_h|$ ($h = 0, ||, \perp, S$), $D_{ij}$ [Eq. (15)], and $\phi_{c\bar{s}}^{c\bar{c}s}$.

| $d_k$ | $2|A_0||A_0|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{00})$ |
| $h_1$ | $2|A_{\parallel}||\bar{A}_{\parallel}|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{\parallel\parallel})$ |
| $h_2$ | $-2|A_{\perp}||A_{\perp}|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{\perp\perp})$ |
| $h_3$ | $-|A_{\perp}||A_{\parallel}|\cos(\phi_{c\bar{s}}^{c\bar{c}s} - D_{\perp\parallel}) - |A_{\perp}||A_{\parallel}|\cos(\phi_{c\bar{s}}^{c\bar{c}s} - D_{\parallel\parallel})$ |
| $h_4$ | $|A_{\parallel}||A_0|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{00}) + |A_{\parallel}||A_0|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{00})$ |
| $h_5$ | $-|A_{\perp}||A_0|\cos(\phi_{c\bar{s}}^{c\bar{c}s} - D_{00}) - |A_{\perp}||A_0|\cos(\phi_{c\bar{s}}^{c\bar{c}s} - D_{00})$ |
| $h_6$ | $-2|A_S||A_S|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{SS})$ |
| $h_7$ | $|A_{\perp}||A_S|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{0S}) - |A_{\perp}||A_S|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{0S})$ |
| $h_8$ | $-|A_{\perp}||A_S|\cos(\phi_{c\bar{s}}^{c\bar{c}s} - D_{0S}) + |A_{\perp}||A_S|\cos(\phi_{c\bar{s}}^{c\bar{c}s} - D_{0S})$ |
| $h_9$ | $|A_0||A_S|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{0S}) - |A_0||A_S|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{0S})$ |
| $h_{10}$ | $|A_0||A_S|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{0S}) - |A_0||A_S|\sin(\phi_{c\bar{s}}^{c\bar{c}s} - D_{0S})$ |

is $O(|A_{2,0}|/|A_{1,0}|)$. Second, if one wishes to calculate its exact value, one has to compute the ratio $|A_{2,0}|/|A_{1,0}|$, which involves hadronic matrix elements, and one also needs to determine the value of the strong phase $\Delta$. The upshot is that it is virtually impossible to reliably calculate the size of the theoretical error associated with $D_{00} = 0$.

We therefore see that the 2-amplitude method requires theory input, with an associated theoretical error of $O(|A_{2,0}|/|A_{1,0}|)$. At first glance, this does not seem to be much of an improvement over the 1-amplitude method. However, let us examine the theory errors of both methods in more detail. While the 2-amplitude method requires one assumption -- $D_{00} = 0$ -- the 1-amplitude method in fact requires 8 assumptions:

$$|\bar{A}_h| = |A_h| \quad (h = 0, ||, \perp, S), \quad \tilde{\delta}_{ij} = \delta_{ij} \quad (ij = ||0, \perp 0, 0S), \quad D_{00} = 0 . \quad (19)$$

In general, there is a theoretical error associated with each assumption. This is due to the fact that, if the true values of the parameters do not obey a given assumption, the imposition of that assumption will contribute to the theoretical error in the extraction of $\phi_{c\bar{s}}^{c\bar{c}s}$. That is,

$$E(1) \leq E(2) \leq ... \leq E(8),$$

where $E(N)$ is the theoretical error on $\phi_{c\bar{s}}^{c\bar{c}s}$ in the scenario where $N$ assumptions are made. (Note that, for each of $N = 8$ (the 1-amplitude method) and $N = 1$ (the 2-amplitude method), there is only a single scenario, but there are many possibilities for $N = 2-7$.) It is true that the theoretical error associated with each assumption is only $O(|A_{2}/A_{1}|)$. However, when one takes into account the fact that multiple assumptions are involved, $E(8)$ is quite a bit larger than $E(1)$. The theoretical error...
of the 2-amplitude method could well be an order of magnitude smaller than that of the 1-amplitude method.

Note that this is not necessarily rigorously true. It is logically possible that the entire theoretical error comes from the $D_{00}$ assumption, with all other assumptions being true. In this case all the $E(N)$ of Eq. (20) are equal. However, this is exceedingly unlikely. It is far more reasonable to expect that each assumption contributes to the theoretical error. In this case, fewer assumptions lead to a smaller theoretical error, so that the 2-amplitude method is indeed an improvement over the 1-amplitude method.

In fact, this can be tested experimentally by comparing the best-fit values of $\phi_s^{cs}$ in the 1- and 2-amplitude methods – their difference is $E(8) - E(1)$. (To be conservative, in computing this difference, one should consider the full allowed ($1\sigma$) range for $\phi_s^{cs}$.) If the theoretical error were due solely to the $D_{00}$ assumption, one would have $E(8) - E(1) = 0$. If not, this proves that $E(8) \neq E(1)$. This then quantifies the improvement of the 2-amplitude method over the 1-amplitude method.

This type of analysis can be pushed further. Starting with the 1-amplitude method, we can relax an assumption by adding a new unknown parameter. For example, suppose we allow $|\bar{A}_0| \neq |A_0|$, but the other assumptions of Eq. (19) are retained. Now, when doing the fit to the data, there are 9 unknown parameters – the $|A_h|$ and $|\bar{A}_0|$ (5), the relative strong phases (3), and $\phi_s^{cs}$. In addition, the theoretical expressions in Table I are modified; the correct expressions can be found from Tables 2-4, but imposing the assumptions that have been retained. The fit will give the preferred values of $|\bar{A}_0|$ and $|A_0|$, so that we can see to what extent, if any, the assumption that $|\bar{A}_0| = |A_0|$ is violated. The fit will also give the preferred value of $\phi_s^{cs}$. This can be compared with the value found in the full 1-amplitude method, so this will give $E(8) - E(7)$, where the 7 assumptions of $E(7)$ are those that have been retained.

In fact, LHCb has performed an analogous analysis [2, 20]. They allow for $|\bar{A}_h| \neq |A_h|$, but take this difference to be helicity-independent. That is, they assume that

$$\frac{|\bar{A}_h|}{|A_h|} = \lambda \quad \forall h,$$

and add the unknown parameter $\lambda$ to the fit. This non-equality has the effect of making nonzero, but helicity-independent, the $a_k$ and $c_k$ which vanish in Table I. In the fit, the idea is to see how much $\lambda$ deviates from 1. They find that this deviation can be at most $\pm 5\%$, and leads to a deviation in $\phi_s^{cs}$ of $\pm 0.02$ rad. We therefore have $E(8) - E(7_{LHCb}) = 0.02$ rad, where here the 7 assumptions of $E(7_{LHCb})$ are those that have not been relaxed. (One way to write the LHCb condition of Eq. (21) is: $R_0 = R_\parallel$, $R_\parallel = R_\perp$, $R_\perp = RS$, $RS = \lambda$, where $R_h \equiv |\bar{A}_h|/|A_h|$. Written in this way, we see that the assumption $RS = 1$ of the 1-amplitude method has been relaxed, but 3 other assumptions remain. This is indeed a 7-assumption scenario.)
Now, above we noted that $E(1)$, the theoretical error in the 2-amplitude method, cannot be reliably calculated and is unmeasurable. However, because only one assumption is involved, this error is $O(|A_2/A_1|)$. Similarly, we expect that $E(N) - E(N-1)$ is $O(|A_2/A_1|)$ since the two scenarios differ by one assumption. One can therefore use the measured value of $E(N) - E(N-1)$ to estimate $E(1)$. To give a concrete example, if one assumes that the measured value of $E(8) - E(7)_{LHCb} = 0.02$ rad is a “typical” error, then one can infer that this is approximately $E(1)$. Of course, this is a dangerous conclusion to draw. The expression for a theoretical error [e.g. Eq. (18)] generally also depends on a function of a strong phase. If this function happens to be small for $E(7)_{LHCb}$, $E(8) - E(7)_{LHCb}$ will be atypically small. It is therefore more prudent to repeat the above analysis by relaxing a different assumption in Eq. (19), and perhaps to do so more times. When it is found that the value of $E(8) - E(7)$ associated with several different assumptions is about the same size, that value can also be assigned to $E(1)$. Of course, this prescription is clearly not rigorous – it is meant only to give a rough estimate at best.

We have argued that it is best to perform a fit with as many unknown parameters as possible. Ideally, 7 out of the 8 assumptions would be relaxed, making a fit with 15 unknown parameters. This is the full 2-amplitude method. However, if a fit with 15 unknowns is not possible, one can reduce the number of unknowns by adding assumptions and incurring a larger theoretical error in $\phi_s^{c\bar{s}c}$. The point is that one has a choice in the type of analysis that is performed. In order to maximally reduce the effect of PP, one should include as many free parameters in the fit as possible, i.e. make the fewest possible assumptions.

Now, above we asked the hypothetical question: suppose a future measurement finds $\phi_s^{c\bar{s}c} = 0.1 \pm 0.01$ rad (error is statistical only). This disagrees with the SM prediction ($2\beta_s = 0.03636 \pm 0.00170$ rad) and therefore suggests NP. Can we conclude that NP is indeed present when the theoretical error on $\phi_s^{c\bar{s}c}$ is taken into account? We noted that it is estimated theoretically that $|A_2/A_1| = O(10^{-3})$. Suppose it is a little larger, say 0.005. And suppose that, in the 1-amplitude method, the theoretical error is even larger, say $\approx 0.04$, because many assumptions are made. Thus, taking all errors into account, one cannot conclude that NP is present. On the other hand, in the 2-amplitude method, the theoretical error is still sufficiently small ($\approx 0.005$) that the presence of NP can be confirmed. We therefore see that there are real advantages in the search for NP to employing the 2-amplitude method.

4 Applications

If it is established that the measured value of $\phi_s^{c\bar{s}c}$ differs sufficiently from the SM prediction that one can conclude that NP is present, this NP must be in the mixing. One can also independently test whether there is NP in the decay. One uses the information about the magnitudes and relative strong phases of the $A_h$ and $\bar{A}_h$ (14 experimental results + 1 theoretical input). In order to do so, an assumption must
be made regarding the form of the decay amplitude. For example, suppose that it is assumed that NP in the decay is not present, and that the amplitude has the form of Eq. (7). Now there are 15 unknowns – the magnitudes and relative phases of $A_{1,h}$ and $A_{2,h}$ (the value of the weak phase $\gamma$ can be taken from independent measurements). The values of these unknowns can be determined from the results on $A_h$ and $\bar{A}_h$. One can then test the theoretical prediction that $|A_2/A_1| = O(10^{-3})$.

If there is a strong discrepancy with this prediction, this might suggest the presence of NP in the decay. (This is similar to the analysis of Ref. [19].)

Suppose one assumes that the SM PP term is small, but that there is NP in the decay. In this case, the decay amplitude takes the form

$$A_{s/J/\psi \phi} = A_1 + e^{i\phi_{NP}} A_{NP},$$

where $\phi_{NP}$ is the (helicity-independent) NP weak phase. But now there are 16 unknowns – the magnitudes and relative phases of $A_{1,h}$ and $A_{NP,h}$, and $\phi_{NP}$. These cannot all be determined from the 15 magnitudes and relative strong phases of $A_h$ and $\bar{A}_h$.

However, it has been argued that, to a good approximation, the NP strong phases are all negligible [21]. Briefly, the logic is as follows. Consider diagram $D$. At leading order its strong phase is zero. But diagram $D'$ can rescatter into $D'_{\text{resc}}$, which has the same Lorentz structure as $D$, and has a large strong phase. Note that $|D'_{\text{resc}}/D'| \sim 5$-10%. We now have $D_{\text{tot}} = D + D'_{\text{resc}}$, so that the diagram develops a strong phase. In the SM, the dominant strong phases of the diagrams $C'$, $P'_{ct}$ and $P'_{EW}$ are all generated via rescattering from the color-allowed diagram $T'$. Since $|T'|$ is considerably larger than $|C'|$, $|P'_{ct}|$ and $|P'_{EW}|$, the generated strong phases are sizeable. On the other hand, the strong phase of a NP operator $A_{NP}$ can only be generated by rescattering from itself: $A_{NP,tot} = A_{NP} + A_{NP,\text{resc}}$. And since $|A_{NP,\text{resc}}/A_{NP}| = 5$-10%, the NP strong phase is quite small.

In this case, the $A_{NP,h}$ helicity amplitudes all have the same phase (0), and the total number of unknown parameters is 13 – the magnitudes of $A_{1,h}$ and $A_{NP,h}$ (8), the relative strong phases (4), and $\phi_{NP}$. These can be determined from the 15 results, so that we can measure the values of the NP parameters (given the initial assumption).

Above, we have concentrated on $B^0_s \to J/\psi \phi$, but this 2-amplitude method can be applied to other $B^0_s \to V_1 V_2$ decays. For example, consider $B^0_s \to D^{*+}D_s^{-}$. Its amplitude is given by

$$A_{s/D^{*+}D_s^{-}} = \lambda_{c}^{(s)} T' + (\lambda_{t}^{(s)} P'_{ct} + \lambda_{c}^{(s)} P'_{ct} + \lambda_{u}^{(s)} P'_{ct} + 2/3 \lambda_{t}^{(s)} P'_{EW}) + \lambda_{t}^{(s)} P'_{EW}$$

$$= \lambda_{c}^{(s)} (T' + P'_{ct} - 2/3 P'_{EW}) + \lambda_{u}^{(s)} (P'_{ct} - 2/3 P'_{EW})$$

$$\equiv A_1 + e^{i\gamma} A_2.$$
Above, $T'$ and $P_{EW}^{C}$ are the colour-allowed tree and colour-suppressed electroweak penguin diagrams, respectively. As before, the $A_2$ term represents the PP, and a rough estimate is $|A_2/A_1| = O(10^{-2})$. If the PP is neglected, the indirect CPA in $B_s^0 \to D_{s}^{+}D_{s}^{-}$ cleanly measures $\phi_{cs}^{\parallel}$. However, there is a theoretical error due to its neglect.

This error can be reduced as described above. One does a time-dependent angular analysis of the decay, but allows for the possibility of more than one contribution to the amplitude, i.e. $A_h \neq A_t$. The one difference is that, in $B_s^0 \to D_{s}^{+}D_{s}^{-}$, there is no $S$ helicity. Thus, the index $h$ takes only three values: 0, $\parallel$, $\perp$. Consequently, there are only six $h_k(t)$’s in Eq. (9). But apart from this, the analysis is unchanged. $\phi_{cs}^{\parallel}$ can be measured even in the presence of PP, with a minimal theoretical error.

Another (slightly different) decay to which this method can be applied is $B_s^0 \to K^{*0}\bar{K}^{*0}$. This is a pure penguin decay, whose amplitude is given by

$$A_s^{K^{*0}\bar{K}^{*0}} = \lambda_t^{(s)} P_t' + \lambda_c^{(s)} P_c' + \lambda_u^{(s)} P_u'$$

where $P_{ut} = P_u' - P_t', P_{ct} = P_c' - P_t'$. It is expected that $|P_{ut}|$ and $|P_{ct}|$ are of similar size. Also, in contrast to $B_0^0 \to J/\psi\phi$, these penguin amplitudes are not OZI-suppressed. Now, $|\lambda_{u}^{(s)}| = O(\lambda^4)$ and $|\lambda_{c}^{(s)}| = O(\lambda^2)$, where $\lambda = 0.23$ is the sine of the Cabibbo angle. This suggests that the $\lambda_{u}^{(s)} P_{uc}$ term can perhaps be neglected.

However, for consistency, one must neglect all $O(\lambda^4)$ terms. One of these is $\text{Im}(\lambda_t^{(s)})$, so that $\lambda_t^{(s)}$ is real. But since $\beta_s$ is related to $\text{Im}(\lambda_t^{(s)})/\text{Re}(\lambda_t^{(s)})$, it too vanishes in this limit. The net effect is that, if one neglects $\lambda_{u}^{(s)} P_{uc}$ (the “penguin pollution”), along with the other $O(\lambda^4)$ terms, the indirect CPA in $B_s^0 \to K^{*0}\bar{K}^{*0}$ vanishes because $\beta_s$ must also be neglected.

That is, if one wishes to analyze the measurement of $B_s^0\bar{B}_s^0$ mixing in $B_s^0 \to K^{*0}\bar{K}^{*0}$, the PP term must be kept in the amplitude. Of course, there is now a theoretical error in this result, related to the nonzero PP term. In order to estimate this error, it has been suggested to measure the corresponding PP term in $B_d^0 \to K^{*0}\bar{K}^{*0}$ and relate it to $B_s^0 \to K^{*0}\bar{K}^{*0}$ using flavour SU(3) $^{22, 23}$. (This method also applies to $B_{d,s}^0 \to K^0\bar{K}^0$.) As always, there is an uncertainty due to the unknown value of SU(3) breaking.

Once again, one can reduce the problems through a time-dependent angular analysis of $B_s^0 \to K^{*0}\bar{K}^{*0}$. The method is identical to that in $B_s^0 \to J/\psi\phi$, but with only three helicities (0, $\parallel$, $\perp$), or six $h_k(t)$’s. Since both contributions to the amplitude [Eq. (24)] are retained, one can extract $2\beta_s$ with a small theoretical error.

As is clear from the above, the 2-amplitude method for extracting $2\beta_s$ with a minimal theoretical error in the presence of penguin pollution can be applied to all $B_s^0/\bar{B}_s^0 \to V_1V_2$ decays. (In the case of $B_s^0 \to \phi\phi$, in which each $\phi$ is detected via $\phi \to K^+K^-$, it may be necessary to add a fifth helicity. This corresponds to the case
where both $K^+K^-$ pairs are observed in $S$-wave.) It can even be used for $B_s^0/B_s^0$ decays that are governed by a $b \rightarrow d$ transition. An example is $B_s^0 \rightarrow J/\psi K^*0/\bar{B}_s^0 \rightarrow J/\psi K^{*0}$ \cite{9}. Here, in order that both $B_s^0$ and $\bar{B}_s^0$ can decay to the same final state, so as to have an indirect CPA, one must consider the flavour-nonspecific decay of the $K^*0/K^{*0}$ to $K_S\pi^0$.

5 Conclusions

To summarize: in the standard model (SM), the weak phase of $B_s^0-\bar{B}_s^0$ mixing, $2\beta_s$, is expected to be very small. For this reason, its measurement is of great interest, as a disagreement with the SM prediction ($2\beta_s \simeq 0.036$ rad) will reveal the presence of new physics (NP). The most common decay used for measuring $2\beta_s$ is $B_s^0 \rightarrow J/\psi \phi$. Since this decay involves two final-state vector mesons, a time-dependent angular analysis must be used to disentangle the CP $+$ and $-$ final states. The LHCb Collaboration has determined $\phi_s^{\text{CP}}$ (the measured value of $2\beta_s$) to be small, in agreement with the SM, but still with significant statistical and systematic errors.

In fact, there is also a theoretical error associated with this measurement. $B_s^0 \rightarrow J/\psi \phi$ is dominated by the colour-suppressed tree diagram, but there are other contributions due to gluonic and electroweak penguin diagrams. As these have a different weak phase from the tree, their inclusion in the amplitude leads to a theoretical error in the extraction of $2\beta_s$ from the data. They are therefore often referred to as “penguin pollution” (PP). In the SM, it is estimated that the ratio of the sizes of the two contributions is $O(10^{-3})$, so that the PP is negligible. Still, there is some uncertainty as to its exact size. This is important for the following reason. Suppose a future measurement finds $\phi_s^{\text{CP}}$ to be small, but in disagreement with the SM, even including the statistical and systematic errors. Unless the theoretical error is under good control, one cannot conclude that NP is present.

As $J/\psi$ and $\phi$ each have spin 1, the $B_s^0 \rightarrow J/\psi \phi$ amplitude is in fact 3 decays: the two spins are either parallel (helicity 0) or transverse (two possibilities: helicity parallel ($||$) or perpendicular ($\perp$)). In addition, since $\phi \rightarrow K^+K^-$, one must also allow for the possibility that the observed $K^+K^-$ pair has relative angular momentum $l = 0$ ($S$-wave – helicity $S$). The full $B_s^0 \rightarrow J/\psi \phi$ amplitude therefore involves 8 amplitudes $- A_h$ and $\bar{A}_h$ ($h = 0, ||, \perp, S$). When PP in the amplitude is entirely neglected (the “1-amplitude method”), one has $\bar{A}_h = A_h$. But this really corresponds to 8 theoretical assumptions – both the magnitudes and the phases of $A_h$ and $\bar{A}_h$ are taken to be equal. While the theoretical error associated with each assumption may be small, the net theoretical error due to the combination of multiple assumptions is in fact quite a bit larger.

In this paper we propose a modification of the above angular analysis in which the PP is included (the “2-amplitude method”). The amplitude is written in terms of all 8 $A_h$ and $\bar{A}_h$. We show there are enough time-dependent measurements to
extract the magnitudes of $A_h$ and $\bar{A}_h$ and 6 of the 7 relative phases. For the final relative phase, theoretical input is required. However, this is not difficult to find. For example, we can assume that the $A_0-\bar{A}_0$ relative phase is zero. With this assumption, $\phi_s^{cs\bar{s}}$ can be extracted. The key point here is that the 2-amplitude method only involves one assumption, and so the theoretical error is quite a bit smaller than that of the 1-amplitude method, which involves 8 assumptions. The 2-amplitude method is therefore more promising in the search for NP through the measurement of $B^0_s-\bar{B}^0_s$ mixing. Indeed, if $\phi_s^{cs\bar{s}}$ differs from the SM prediction, this points to NP in the mixing, and there is also enough information to test for NP in the decay.

This same method can be applied to other $B^0_s/\bar{B}^0_s \rightarrow V_1 V_2$ decays (sometimes with slight modification): e.g. $B^0_s \rightarrow D^{*+}D^{*-}$, $B^0_s \rightarrow K^{*0}\bar{K}^{*0}$, $B^0_s \rightarrow \phi\phi$, etc. It can even be used for $B^0_s/\bar{B}^0_s$ decays that are governed by a $\bar{b} \rightarrow \bar{d}$ transitions, e.g. $B^0_s \rightarrow J/\psi K^{*0}/\bar{B}^0_s \rightarrow J/\psi K^{*0}$, in which the $K^{*0}/\bar{K}^{*0}$ both decay to $K_S\pi^0$.

Acknowledgments: We are extremely grateful to Olivier Leroy for his numerous communications related to the subject of penguin pollution. We thank Max Imbeault for helpful discussions. This work was financially supported by NSERC of Canada (BB, DL), and by the National Science Foundation under Grant No. NSF PHY-1068052 (AD).

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