Difficulties in probing density dependent symmetry potential with the HBT interferometry

LI QingFeng * & SHEN CaiWan
School of Science, Huzhou Teachers College,
Huzhou 313000, China

Abstract

Based on the updated UrQMD transport model, the effect of the symmetry potential energy on the two-nucleon HBT correlation is investigated with the help of the coalescence program for constructing clusters, and the CRAB analyzing program of the two-particle HBT correlation. An obvious non-linear dependence of the neutron-proton (or neutron-neutron) HBT correlation function \( C_{np,nn} \) at small relative momenta on the stiffness factor \( \gamma \) of the symmetry potential energy is found: when \( \gamma \lesssim 0.8 \), the \( C_{np,nn} \) increases rapidly with increasing \( \gamma \), while it starts to saturate if \( \gamma \gtrsim 0.8 \). It is also found that both the symmetry potential energy at low densities and the conditions of constructing clusters at the late stage of the whole process influence the two-nucleon HBT correlation with the same power.

Keywords: Density dependence of the symmetry energy, sensitive observables, HBT correlation

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* Corresponding author (e-mail: liqf@hutc.zj.cn)
In recent years, the isospin dependent equation of state (EoS) of both infinite and finite nuclear matter has been investigated in both deeper and broader range. Theoretically, a large amount of probes have been brought out to be sensitively affected by the isovector part of the EoS, i.e., the symmetry energy term. Based on different theories, the symmetry energy has been parametrized in different ways (please see [1, 2, 3] for details). Among these parametrizations, the $E_{\text{sym}} = S_0 u^\gamma$ is often used ($S_0$ being the symmetry energy at the normal density, and $u = \rho/\rho_0$ the reduced density, while $\gamma$ is the stiffness factor of the density dependence of the symmetry energy). Especially, the density dependence of the symmetry potential energy is paid much more attention since the uncertainty of the symmetry energy mainly comes from this term. Recently, with some comparisons between the results of experimental data and dynamical model IBUU, the uncertainty in the density dependence of the symmetry energy at subnormal densities is reduced into the range of $\gamma \sim 0.7 - 1.1$, depending on the probable medium modifications of the nucleon transport [4, 5]. Nevertheless, recently a quite soft symmetry energy (the corresponding $\gamma$ factor is only about 0.3) was implied experimentally at very low nuclear densities ($\rho_N$ is $0.01 - 0.05$ times normal density) [6]. Furthermore, in the most recent analyses by Tsang’s group [7], although they found that several different observables can provide consistent constraints on the density dependence of the symmetry energy, the constraints are still relatively loose.

To bring forward more sensitive probes for symmetry energy at subnormal densities is still urgent and prerequisite. However, we think that one should firstly check more carefully the difficulties of detecting the existing observables for forthcoming experiments. Therefore, the currently crucial question becomes: Can the sensitive observable suggested by theoretical physicists be experimentally taken as a sensitive candidate for detecting the symmetry energy at subnormal densities? In the previous work [8] this question has been concerned for the free neutron/proton ($n/p$) ratio as well as the $\Delta^0/\Delta^{++}$ ratio.

In this paper, we would like to continue this topic to further check the two-nucleon correlation, which is based on the Hanbury-Brown-Twiss interferometry (HBT) [9, 10, 11] technique. It was pointed out that it is a good candidate to probe the symmetry energy since it is quite unsensitive to the iso-scalar part of EoS as well as the in-medium NN cross section [12]. In their initiated investigations, the isospin effect is found to occur only in the late stage of the heavy ion collisions (HICs) where the nuclear medium becomes dilute. Therefore, the two-nucleon correlation has the advantage in probing the density dependence
of the symmetry potential at low densities. It was further claimed that the sensitivity of the
correlation function of the nucleon pair to the density dependence of the symmetry energy
is reduced after considering a (isospin-dependent) momentum dependent nuclear potential
\cite{13}. However, in their studies, only several parameter sets of the stiffness of the symmetry
energy were employed. It is believed to be enough in the studies of the density dependent
symmetry energy by adopting the conventional single-particle observables since they behave
monotonously with the increase of the stiffness factor \( \gamma \). But, is that still true in the
symmetry-energy dependence of the two-nucleon HBT correlation? Here we would like to
vary the \( \gamma \) factor with a larger range to see the variance of the correlation function.

Besides the symmetry potential energy, the freeze-out conditions of clusters should also
heavily influence the final two-nucleon correlation function. In the transport model cal-
culations, when the transport is stopped at a certain time, such as 150 fm/c or so, one
cluster constructing program is then used. Generally speaking, two methods (i.e., BUU-like
and QMD-like) for constructing clusters exist in the transport model calculations \cite{8}. In
our UrQMD calculations below, the coalescence model, which is often used for analyzing
QMD-like model outputs, is employed, in which the nucleons with relative momenta smaller
than \( P_0 \) and relative distances smaller than \( R_0 \) are considered to belong to one cluster. It
is comparable to the BUU-like coordinate density cut \( \rho_c \) when we only adopt the relative
distance \( R_0 \) but not the relative momentum \( P_0 \) in the QMD-like analysis. The freeze-out
condition certainly influences the two-nucleon correlation function since it reflects the spa-
tial (and momentum) separation of two nucleons. It is also interesting to see if the small
uncertainty in the multiplicity of the observed particle changes the HBT correlation function
accordingly.

In this paper, we use the recently updated UrQMD transport model \cite{14, 15} for studies of
intermediate energy HICs to investigate the effects of the density dependent symmetry po-
tential energy and the freeze-out conditions on the neutron-proton and the neutron-neutron
HBT correlations. The proton-proton correlation will not be shown in this paper since it has
similar isospin-dependent effect to the neutron-proton and neutron-neutron ones, although
the largest effect is shown at the relative momentum of pair \( q \sim 20 \) MeV/c but not at
\( q \lesssim 10 \) MeV/c. As a default, \( P_0 \) and \( R_0 \) used in the coalescence model are set to be 0.3
GeV/c and 3.5 fm, respectively (except otherwise stated). The central collisions (< 11%
of total cross section \( \sigma_T \)) of neutron-rich intermediate-mass and heavy systems \( ^{52}\text{Ca}+^{48}\text{Ca} \)
and $^{197}\text{Au}+^{197}\text{Au}$ are adopted in the UrQMD calculations. As stated above, the correlation function of the nucleon pair from HICs with the super-intermediate-mass system at the intermediate energy is claimed to be unsensitive to the isospin-scalar part of EoS and the NN cross section $^{[12,16]}$. It is also reported that by using the imaging method the shape of the source of the two-proton pairs from the HICs with medium sized system at beam energies around 100A MeV is sensitive to the medium modification of the NN cross section $^{[17]}$. However, the corresponding HBT correlation function is seen to be less and less affected by the medium modification with the increase of the beam energy. In this work, we focus our investigation on the correlation functions at beam energies $100A \sim 1000A$ MeV. And, we randomly adopt several parameter sets of EoS used in previous works $^{[18]}$, for checking. Those are, S-EoS, SM-EoS, and HM-EoS, without considering the medium modifications on NN cross sections.

To analyze the two-nucleon HBT interferometry, the Correlation After Burner (CRAB v3.0$^\beta$) program is employed $^{[9,10,19]}$. The correlation function is expressed as $C(P,q) = B/A$, where $B = \int d^4x_1 d^4x_2 g(x_1, P/2) g(x_2, P/2) |\phi(q,r)|^2$ is the inclusive two-particle emission probability in the same event, $A = \int d^4x_1 g(x_1, P/2) \int d^4x_2 g(x_2, P/2)$ is the product of two single-particle emission probability in different events. The squared relative two-particle wave function $|\phi|^2$ serves as a weight, while $g(x, P/2)$ is the probability for emitting a particle with momentum $P/2$ from the space-time point $x = (r,t)$. The $P = p_1 + p_2$ and the $q = (p_1 - p_2)/2$ are the total and relative momenta of the particle pair, respectively.

Figure shows the neutron-proton correlation function ($C_{np}$) within the relative momentum bin $q = 2.5 - 5$ MeV/c and the rapidity region $|y_{cm} < 0.5|$ ($y_{cm} = \frac{1}{2} \log(\frac{E_{cm}+p_{//}}{E_{cm}-p_{//}})$, $E_{cm}$ and $p_{//}$ are the energy and longitudinal momentum of the nucleon in the center-of-mass system) (top plot), and the free $n/p$ ratio in the rapidity region $|y_{cm} < 0.2|$ (bottom plot) as a function of the stiffness factor $\gamma$ of the symmetry potential energy. The $^{52}\text{Ca}+^{48}\text{Ca}$ central collisions ($<11\%$ of total cross section $\sigma_T$) at $E_b = 100$A MeV are adopted. It is known that the $n/p$ ratio of free nucleons is obviously rapidity dependent. But conclusions of the two-nucleon HBT correlation drawn in this paper is not changed by the small rapidity-cut difference used in both observables $^{[20]}$. The S-EoS and the SM-EoS are selected for comparison in each plot. It is seen that the iso-scalar part of the EoS only affects slightly the sensitivity of both the $C_{np}$ and the $n/p$ ratio to the stiffness of the symmetry potential energy. Thus, in the following calculations, we will randomly use various EoS. It is also known
FIG. 1: Neutron-proton correlation function \( (C_{np}) \) within the relative momentum \( q = 2.5 - 5 \) MeV/c and the rapidity region \( |y_{cm}| < 0.5 \) (top plot), and free neutron-proton ratio in the rapidity region \( |y_{cm}| < 0.2 \) (bottom plot) as a function of the stiffness factor \( (\gamma) \) of the symmetry potential energy. The S-EoS and SM-EoS are adopted for \(^{52}\text{Ca}^{48}\text{Ca} \) central collisions (< 11% of total cross section \( \sigma_T) \) at \( E_b = 100A \) MeV. The marked area illustrates the value \( C_{np} = 3.15 \pm 0.08 \). The open squares in the top plot show the \( C_{np} \) values when the symmetry potential energy is switched off at the reduced densities \( u < 0.2 \).

that the \( n/p \) value decreases monotonously with increasing \( \gamma \) factor, which is again shown in the bottom plot of this figure. It is interesting to see that, however, the \( C_{np} \) moves up rapidly with increasing \( \gamma \) when \( \gamma \) is less than \( \sim 0.8 \), and then saturates with the further increase of the \( \gamma \) value, the saturation value is \( C_{np} = 3.15 \pm 0.08 \), which is shown in a marked area. It is
known that the symmetry potential plays obvious role on the two-nucleon HBT correlation at the late stage ($t > 50 \text{ fm/c}$) of the whole process [12], where the density surrounding the emitted protons is very small. Therefore, we further calculate the $C_{np}$ with $\gamma = 0.3$ and 0.5 (open squares) but without considering the symmetry potential energy at very low densities (the reduced density $u = \rho/\rho_0 < 0.2$). It is obvious that the $C_{np}$ values move back to the marked area. We know that the stiffer the symmetry energy is, the smaller the value of the symmetry energy at subnormal density becomes. At densities $u < 0.2$ the stiff symmetry energy is too weak to influence the two-particle correlation, while it plays strong role with a very soft symmetry energy [8]. It is concluded that, firstly, the correlation function can hardly distinguish the stiffness of the symmetry energy when $\gamma > 0.8$. Secondly, the soft symmetry energy plays more obvious role on the neutron-proton correlation function at the late stage. Therefore, the neutron-proton correlation function is suitable to explore the symmetry energy in the dilute nuclear medium, especially for a very soft symmetry-energy assumption.

At beam energy around 100A MeV, it is found that the isospin effect on the correlation function is reduced in heavy-system HICs such as Sn+Sn. It is also confirmed in our calculations with the Au+Au reaction which is shown in the upper-left plot of Figure 2 even when a different stop time $250 \text{ fm/c}$ is chosen. Further, we calculate the $C_{np}$ values at higher beam energies, i.e., at $E_b = 200A, 400A, 600A, 800A, \text{ and } 1000A \text{ MeV}$, which are shown in other plots of Figure 2. It is interesting to see that with the increase of the beam energy, the effect of the density dependence of symmetry energy becomes visible. It might be understandable when considering the fact that with the increase of beam energy, more unstable light fragments emit and decay with a longer time and in the dilute medium so as to be influenced by the soft symmetry energy at the late stage.

For the neutron-neutron correlation, the phenomena shown in Figs. 1 and 2 are even more obvious. In Figure 3 we show the $\gamma$-dependence of the correlation functions of both neutron-proton and neutron-neutron pairs from central Au+Au collisions at $E_b = 1000A \text{ MeV}$. The strong non-linear dependence of the neutron-neutron correlation function on the $\gamma$ factor of the symmetry potential energy is due to the long time evolution of the neutron-rich unstable light fragments at the late stage.

In order to deeper understand the isospin effect on the correlation function, it is necessary to check the influence of the freeze-out conditions on the HBT correlator. Figure
FIG. 2: Neutron-proton correlation functions in central Au+Au collisions at $E_b = 100A$, 200A, 400A, 600A, 800A, and 1000A MeV. The UrQMD calculations are stopped at 150 fm/c. At $E_b = 100A$ MeV, the results at 250 fm/c are also shown. The HM-EoS and the symmetry energies with $\gamma = 0.5$ and 1.5 are adopted in calculations.

Illustrates the neutron-proton correlator as a function of the relative momentum $q$ of the nucleon pair. Central $^{52}\text{Ca} + ^{48}\text{Ca}$ collisions at $E_b = 100A$ MeV are calculated with UrQMD and sets of relative momentum ($P_0$) and relative distance ($R_0$) are chosen for the following cluster construction in the coalescence model. The final correlators by the CRAB analyzing program are shown with different lines in the figure. The multiplicities of the light clusters ($A < 5$) with different ($P_0, R_0$) sets are shown in the top-right plot. The stiffness of the symmetry potential energy $\gamma = 1.5$ is chosen in current calculations. When the values of ($P_0, R_0$) are adjusted from (0.3 GeV/c, 3.0 fm) to (0.23 GeV/c, 3.5 fm), the multiplicities of clusters keep almost unchanged, and so does the correlator. However, when the values of ($P_0, R_0$) are changed from (0.3 GeV/c, 3.0 fm) to (0.3 GeV/c, 3.5 fm), the multiplicities of free nucleons alter slightly, and so does the correlator. This vivid phenomenon reveals that the freeze-out condition influences the value of the correlator visibly and should be paid more attention. In order to constrain the density dependence of the symmetry energy by
FIG. 3: Neutron-proton (line with squares) and neutron-neutron (line with circles) correlation functions within the relative momentum $q = 2.5 - 5$ MeV/c and the rapidity region $|y_{cm} < 0.5|$ as a function of the stiffness factor ($\gamma$) of the symmetry potential energy. The HM-EoS is adopted for Au+Au central collisions at $E_b = 1000A$ MeV.

using the two-nucleon HBT correlation, the uncertainty of the multiplicity of nucleons, with whatever experimental cuts, should be largely reduced firstly.

To summarize, based on the updated UrQMD transport model, the coalescence program for constructing clusters, and the CRAB analyzing program of the two-particle HBT correlation, a more detailed analysis of the effect of the symmetry potential energy on the two-nucleon HBT correlation is investigated for HICs with super medium-sized system at intermediate energies. It is found that, for HICs with medium-sized neutron-rich system at 100A MeV, there exists a non-linear dependence of the neutron-proton (or neutron-neutron) HBT correlation function ($C_{np,nn}$) at small relative momenta $q$ on the stiffness factor $\gamma$ of the symmetry potential energy: when $\gamma \lesssim 0.8$, the $C_{np,nn}$ at small $q$ increases rapidly with increasing $\gamma$, while it starts to saturate if $\gamma \gtrsim 0.8$. This phenomenon is also seen for the HICs with the heavy system at higher beam energies. At $E_b = 1000A$ MeV, the non-linear dependence of the $C_{nn}$ is even more obvious than the neutron-proton case. With the upgrading of the LAND facility at GSI to the “newLAND”, it is expected to detect the stiffness of the symmetry energy with more neutron-related observables in the near future.
FIG. 4: Neutron-proton correlation functions from central $^{52}$Ca$+^{48}$Ca collisions at $E_b = 100A$ MeV are illustrated with different relative momenta $P_0$ and relative distances $R_0$ in the coalescence model, and in the top-right plot the multiplicities of light clusters ($A < 5$) are shown, correspondingly. The S-EoS and the symmetry potential energy with $\gamma = 1.5$ are used.

In order to understand the non-linear dependence of the HBT correlator on the $\gamma$ factor, the different freeze-out conditions, such as the stop times and the $(P_0, R_0)$ parameter sets for the coalescence model, are taken into account. It is found that both the symmetry potential energy at low densities and conditions of constructing clusters at the late stage of the whole process influence the two-nucleon HBT correlation with the same power and should be studied more carefully.

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