Study of Strangeness Condensation by Expanding About the Fixed Point of the Harada-Yamawaki Vector Manifestation

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Building on, and extending, the result of a higher-order in-medium chiral perturbation theory combined with renormalization group arguments and a variety of observations of the vector manifestation of Harada-Yamawaki hidden local symmetry theory, we obtain a surprisingly simple description of kaon condensation by fluctuating around the “vector manifestation (VM)” fixed point identified to be the chiral restoration point. Our development establishes that strangeness condensation takes place at \( \sim 3n_0 \) where \( n_0 \) is nuclear matter density. This result depends only on the renormalization-group (RG) behavior of the vector interactions, other effects involved in fluctuating about the bare vacuum in so many previous calculations being “irrelevant” in the RG about the fixed point. Our results have major effects on the collapse of neutron stars into black holes.

Introduction— The calculation of strangeness condensation in neutron stars has got into great complexity because the necessary interactions, especially those of strange hadrons, are only partially known. The calculation within a chiral formulation of Thorsson, Prakash, and Lattimer [1] gave the condensation density

\[
\rho_c \approx 3.1n_0, \quad \text{where } n_0 \text{ is nuclear matter density.}
\]

This calculation was later modified by other authors by introduction of higher order chiral perturbation expansion, strange hyperon condensates, the role of \( \Lambda(1405) \) important for threshold \( K^+p \) processes, etc., in which extrapolations from low densities to densities \( \sim 3n_0 \) had to be made.

In this note, based on qualitative understanding of the condensation process extracted from chiral perturbation theory combined with renormalization group analysis, we propose to study s-wave kaon condensation starting from the vector manifestation (VM) fixed point discovered by Harada and Yamawaki in the hidden local symmetry approach to effective field theory of QCD [2, 3].

A surprise in the next-to-next chiral perturbation calculation of the kaon condensation involving in-medium two-loop graphs in [4] was that certain two-body correlations via effective four-fermi operators were found to be essential for driving the condensation but played a negligible role in locating the critical density. For instance, attractive four-fermi interactions involving \( \Lambda(1405) \) which figures crucially in threshold \( K^-p \) interactions (see [5] for a recent review with references), are necessary for the condensation but the critical point is highly insensitive to the strength of the \( K^-N\Lambda(1405) \) coupling constant: Varying the coupling constant by two orders of magnitude changed the critical density by only a few \%.

This surprising result can be understood in terms of renormalization group flow arguments [1]. In [6], generic boson condensation in dense matter with conditions commensurate with kaon condensation in neutron star matter (i.e., chiral symmetry broken by the s-quark mass and the presence of electron chemical potential \( \mu_e \)) was studied with a toy model using the renormalization group technique developed by Shankar [7] for Landau Fermi liquid theory. The model consisted of a “relevant” mass term (quadratic in boson field) and an “irrelevant” boson-baryon four-point interaction term. (From here on, we put under quotation mark the terminology in the RG sense.) It was shown there (corresponding to the case of Fig.2(c) in [1]) that while condensation was driven by a “relevant” term as is the standard case for phase change in field theory, the direction of the flow as well as the fate of the state crucially depended upon whether the leading “irrelevant” interaction is attractive or repulsive: Condensation is inevitable if attractive and does not take place if repulsive. Thus, the attractive sign of the “irrelevant” interaction was indispensable for the condensation, a feature that is unusual in condensed matter physics where marginal terms figure initially. However once one is near the critical point, the “irrelevant” attractive interaction term that determined the flow direction plays a minor role.

In terms of the chiral perturbation calculation of [4], what we have here is a consequence of an intricate interplay between the chiral limit at which the kaon is a Goldstone particle and the heavy-quark limit at which the kaon is a “heavy meson.” In the two limits, condensation does not take place. Kaons condense because the kaon is neither heavy nor light [8].

What we learn from the above observations is that the relevant degrees of freedom for kaon condensation are not necessarily those manifest in free space where elementary kaon-nucleon interaction takes place but it would be more astute to identify the degrees of freedom relevant in the vicinity of the condensation point. In addressing this issue, we will start with our principal assumption that kaon condensation takes place in the vicinity of – but below \( n_{\chi_{SR}} \) at which the spontaneously broken chi-
r al symmetry in Goldstone mode is restored to Wigner mode. This assumption allows us to consider fluctuating around $n_{\chi SR}$ rather than around the free space $n = 0$. The appropriate formalism for this is Harada-Yamawaki hidden local symmetry theory [8]. Were it not for the presence of electron Fermi sea, the flow would wind up at the VM fixed point. However the decay of electrons into kaons [8] followed by the condensation of kaons at $n_c \lesssim n_{\chi SR}$ stops the HLS flow.

**Physics near the VM fixed point**— In applying Harada-Yamawaki (HY) hidden local symmetry (HLS) theory to dense (and hot) matter, the key ingredient is the flow of the principal parameters of the HLS Lagrangian, the hidden gauge coupling $g$, the “bare” (parametric) pion decay constant $F_\pi$ and the ratio $\alpha \equiv (F_\pi/F_\sigma)^2$ (where $F_\sigma$ is the decay constant of the would-be Goldstone scalar that gives rise to the longitudinal component of the massive gauge field) [8]. These parameters flow as the scale is varied in a variety of different directions with a variety of fixed points in the one-loop RGE. The most important point however is that the coupled RGEs flow to one unique fixed point called vector manifestation (VM) fixed point when the HLS theory is matched to QCD at a suitable matching scale $\Lambda_M$. The VM fixed point turns out to be

$$ (g, f_\pi, \bar{a}) = (0, 0, 1) $$

where $f_\pi$ is the physical pion decay constant related to $F_\pi$ with a quadratically divergent correction. HY show at one-loop order that the VM fixed point is arrived at when the quark condensate $\langle \bar{q}q \rangle$ – the order parameter of chiral symmetry – goes to zero. This has been verified by Harada and Sasaki [9] in heat bath as $T \rightarrow T_{\chi SR}$ and by Harada, Kim and Rho [10] in dense matter as $n \rightarrow n_{\chi SR}$. We shall refer to the HLS theory endowed with the vector manifestation as HLS/VM.

The most important physical implication of HLS/VM in hot/dense matter is that the vector meson mass and the gauge coupling constant go to zero as the critical point is approached in agreement with the BR scaling [11]. Another implication which has been less extensively exploited up to date is that the constant $a$ goes to 1 at the VM. Most relevant to our present work is that the HLS/VM theory simplifies the calculation enormously near the VM point because of the particularly simple limiting behavior of the constants. Now the question is: Can one start from this point, instead of from the vacuum as is commonly done, to do physics taking place not on top of but near the VM point? This issue was addressed in [12] where several non-trivial and interesting cases were given; i.e. (1) The chiral doubling of heavy-light hadrons predicted more than a decade ago using sigma model built on the (matter-free) vacuum [13] and confirmed by the BaBar and CLEO collaborations in $D$ mesons can be very simply understood in terms of HLS/VM in the light-quark sector starting from the VM fixed point [14]. Here $a$ remains equal to 1 to the order considered; (2) the $\pi^+ - \pi^0$ mass splitting can be very well reproduced in HLS theory with $a = 1$ and $g \neq 0$ [13]; (3) in the presence of baryonic matter, even when one is far away from the VM fixed point, $a$ flows precociously to 1. For instance, $a \approx 1$ in the EM form factor for the nucleon as discussed in [13]; (4) in the presence of temperature, $a$ flows to 1 and vector dominance in the EM form factor of the pion breaks down maximally at chiral restoration [17]. This vector dominance violation has an important implication in dilepton production in heavy-ion collisions [18].

**Kaon Fluctuation Around the VM Fixed Point**— A clear hint from the above discussions is that in the presence of medium, temperature and/or density, $a$ tends to go to 1 quickly whether or not one is near the VM fixed point with $g = 0$. Since kaon condensation occurs in dense medium and most likely in the vicinity of the VM fixed point, it should be much more profitable to expand around the VM fixed point with $a = 1$ than around the matter-free vacuum. The strategy is quite analogous to that used in [14] for the chiral doubler splitting of heavy-light hadrons. One starts with a bare Lagrangian fixed by Wilsonian matching to QCD at the matching scale $\Lambda_M$.

Now near the VM fixed point, the quark condensates are rotated out, so the sigma term which is more “irrelevant” than the Weinberg-Tomozawa term will no longer be important. Furthermore, the similarly “irrelevant” four-point interactions that intervene to drive kaon condensation when initiated from the matter-free vacuum such as those involving $\Lambda(1405)$ and $p$-wave interactions involving hyperons would have flowed to zero at the VM fixed point as mentioned above. What is left would then be the Weinberg-Tomozawa-type term – which is the least “irrelevant” from the point of view of RGE – from the exchange of the $\omega$-meson (and the $\rho$-meson in non-symmetric matter) between the kaon and the baryon.

Now to compute the Weinberg-Tomozawa term in dense medium, we need baryons in the theory. But Harada-Yamawaki HLS/VM theory has no baryons. Baryons must therefore be generated as skyrmions. Near the VM fixed point, however, baryons are not the correct fermionic degrees of freedom. We postulate that the relevant fermionic degrees of freedom at some density above that of nuclear matter - not yet precisely pinned down – in HLS/VM theory are constituent quarks or quasi-quarks [16], just as in heat bath above the “flash temperature” $T_{\text{flash}} \sim 125$ MeV indicated in lattice calculations. The flash temperature occurs at the point nucleons change into quasi-quarks [14]. Its value of 125 MeV is obtained by lattice calculations. (See Fig.1 and Eq. (3) of [20]).

We assume that the quasi-quark notion is applicable for $n \gtrsim n_0$. The density $n_0$ may be regarded as an analog to the flash temperature $T_{\text{flash}}$ in the sense that the gauge coupling $g$ starts dropping and $a \approx 1$ from that point to the critical point. This will become clearer later, where the results are not very sensitive to the choice of the change-over density which could be $\sim 2n_0$ without
changing much our scenario. In the mean field, we estimate that the ω exchange
\[ V_{K^-}(\omega) = -\frac{3}{8F^2}n \]
will give a potential
\[ V_{K^-}(\omega) \simeq -57 \text{ MeV } \frac{n}{n_0} \]
when using the parametric pion decay constant \( F_\pi \approx 90 \text{ MeV} \). Note that \( F_\pi \) should be distinguished from the physical pion decay constant \( f_\pi \), the order parameter that goes to zero whereas the former does not.

Now the Walecka vector mean field, because of the three quasiquarks in the nucleon as compared with one nonstrange anti-(quasi)quark in the \( K^- \), should be
\[ V_N(\omega) = -3V_{K^-}(\omega). \]
But at \( n = n_0 \), \( V_N(\omega) \approx 171 \text{ MeV} \) is well below the empirical value of more like \( V_N(\omega) \sim 270 \text{ MeV} \) usually used. Clearly we are missing something here. What we are missing is precisely the BR scaling of the pion decay constant.

From the analysis of deeply bound pionic atoms \( m^* \) we have learned that in medium the parametric \( F_\pi \) must be decreased
\[ F_\pi \rightarrow f_\pi^* \approx 0.8F_\pi \]
~ 20% at \( n = n_0 \) in movement towards chiral restoration. The Walecka vector mean field, obtained for \( n = n_0 \), already has this dependence empirically built into it; i.e., using \( f_\pi^* \) instead of \( F_\pi \) valid at tree order it is increased by \((1/0.8)^2\).

Our next task is to show that the critical density for strangeness condensation is close to the density at the fixed point. For this, most important for the extrapolation from fixed point to critical density \( n_c \) for kaon condensation is the renormalization group flow in the parameter \( a = (F_\pi/F_\pi^*)^2 \).

Using full (unquenched) lattice calculation results for \( SU(2) \times SU(2) \), Park et al. \( \frac{23}{23} \) have shown that the degrees of freedom giving the entropy at \( T_c \), in the finite temperature chiral restoration transition are the quark-antiquark-antiquark combination states \( \pi, \sigma \) and vector and axial vector mesons that go massless at \( T_c \). It seems reasonable to expect that these are the only degrees of freedom left also at \( n_{\chi SR} \).

Behavior of Parameters in the Vicinity of the Fixed Point — First we should estimate (in the chiral limit) at which density \( n_{\chi SR} \) the fixed point is reached. We do this by finding out at which density \( m^*_\rho \) goes to zero.

Although initially \( m^*_\rho \) decreases as \( \sqrt{\langle \bar{q}q \rangle} \) following the scaling of \( F_\pi^* \) the Harada and Yamawaki work shows that once it starts dropping it scales as \( \langle \bar{q}q \rangle^* \). (See the empirical verification of this in Koch and Brown \( \frac{24}{24} \) who showed that the entropy matched that in LGS if the meson masses were allowed to scale as \( \langle \bar{q}q \rangle^* \), which was referred to as “Nambu scaling.”) As noted by Brown and Rho \( \frac{16}{16} \), \( g \) does not seem to scale up to nuclear matter density \( n_0 \), but then Nambu scaling sets in. Nambu scaling is \( \sqrt{2} \) times faster than the initial scaling of \( m^*_\rho \) from \( n = 0 \) to \( \sim n_0 \), which decreases \( m^*_\rho \) by 20%. Thus, we believe in the interval \( n_0 \rightarrow 2n_0 \), \( m^*_\rho \) will decrease \( \sqrt{2} \) times 20%, or \( \sim 28\% \), and the same from \( 2n_0 \) to \( 3n_0 \), and from \( 3n_0 \) to nearly \( 4n_0 \) where \( m^*_\rho = 0 \) in the chiral limit.

Thus, the fixed point at \( n_{\chi SR} \) is at \( n \lesssim 4n_0 \).

From the Brown and Rho \( \frac{16}{16} \) argument that \( g^* \) scales as \( m^*_\rho \) for \( n > n_0 \), but up to \( n_0 \), \( g \) remains constant, whereas \( m^*_\rho \) scales, we find
\[ \frac{g^2}{m^2} = \frac{1}{a^2 F_\pi^2} \simeq \frac{1}{a^2} \left( \frac{1}{0.8 F_\pi} \right)^2 \]
and at \( n_{\chi SR} \), \( a^* = 1 \), together with \( m^*_\rho = 0 \) and \( g^* = 0 \).

Compared with the matter-free expression \( m^2 = 2F^2_\pi g^2 \), which is the KSRF relation, we see that \( \frac{m^2}{m^2} \) at fixed point \( n_{\chi SR} \) in medium density is 23% at fixed point, or 28% from 2\( n_0 \) to 3\( n_0 \), or 28% from 3\( n_0 \) to nearly 4\( n_0 \).

Thus, the mean field felt by the \( K^- \) is increased by the factor 3.1 when the scaling of both \( F_\pi^* \) and \( a^* \) are included, at the fixed point at \( n_{\chi SR} \), the final doubling coming from the scaling in \( a^* \). Note that the scalar contribution from the rotation of the sigma term is gone.

Now we calculate \( m^*_\rho \) for neutron rich matter. If we calculate for 90% neutrons and 10% protons at \( n_c = 3.1 n_0 \) without medium dependence
\[ V_{K^-} = -\frac{1}{a F_\pi^2} \left( \frac{x_n}{2} + x_p \right) n_c = -129 \text{ MeV} \]
where we have included \( \rho \) as well as \( \omega \) exchange, and \( x_{n,p} \) are the neutron and proton fractions. But now we have to incorporate the enhancement factor 3.1. Thus, taking \( n = 4n_0 \) and multiplying by 3.1 in order to take into account the medium dependence, we have
\[ V_{K^-} \approx -\frac{4}{3.1} \times 3.1 \times 129 \text{ MeV} = -516 \text{ MeV} \lesssim -m_{K^-.} \]
Thus, the vector mean field at \( n = 4n_0 \) is sufficient to bring the \( m^*_\rho \) to zero. That is, the vector mean field is strong enough to bring \( m_{\rho^-} \) to zero at \( n_{\chi SR} \), which would imply \( \mu_c = 0 \) for strangeness condensation at this density. Of course \( \mu_c \neq 0 \), so the condensation must take place at a lower density. But the fact that \( m^*_\rho \) goes to zero at \( n_{\chi SR} \) means that the \( K^- \) behaves more like a normal nonstrange meson than a Goldstone boson. This aspect may be related to the RGE finding mentioned above that kaon condensation takes place because the kaon is neither heavy nor light.

In moving to \( n = 3n_0 \), the change in density will produce \( m_{\rho^-} \left( \frac{4}{3} n_{\chi SR} \right) \approx m_{K^-}/4 \approx 124 \text{ MeV} \). Furthermore, the renormalization group parameter \( a \) may have
increased from the fixed point value of 1. Since \( a \) tends rapidly to 1 in the presence of baryonic matter \([12]\), we do not expect that it will be much different from 1 in the density regime we are dealing with. An upper limit may be taken to be \( a \simeq 4/3 \), the value at the matching scale \( \Lambda_{\text{QCD}} \simeq 1.1 \text{ GeV} \) which corresponds to the large \( N_c \) limit of Harada-Yamawaki’s HLS bare Lagrangian\([3]\) which is also the value obtained in holographic dual QCD that exploits AdS/QCD in string theory\([25]\). Now this increases \( m_{K^-}^* \) by that factor so that

\[
m_{K^-}^* \simeq 124 \times 4/3 \text{ MeV} \simeq 165 \text{ MeV}
\]

which should equal \( \mu_e \). It is somewhat smaller than, but not too far from, the Thorsson et al. \( \sim 220 \text{ MeV} \)\([1]\).

Special Properties of the Vector Mesons in the Hadron Free Zone— Harada and Yamawaki\([3]\) show that \( m_{\rho}^* \) goes to zero at the fixed point, at the same rate as \( g^* \), so that \( g^*/m_{\rho}^* \) goes to a constant. This was observed even earlier\([26]\) in studying the lattice calculations of the quark number susceptibility, and discussed in\([16]\). The isoscalar quark number susceptibility is the same as the isovector one. Three of the former can be added together to give the behavior of the \( \omega \), whereas two of the isovector quark susceptibilities cancel each other to make up the \( \rho \), giving

\[
g_{\omega} = 3g_{\rho}
\]

with \( g_{\rho} = \frac{1}{2}ag \). As \( T \to T_{\chi_{SR}} \)\([17]\) and \( n \to n_{\chi_{SR}} \)\([10]\), HLS/VM predicts that the interactions between hadrons go to zero; i.e., the *hadronically free region* is reached. This phenomenon was referred to as *hadronic freedom* in\([20]\). From lattice calculations and the STAR experiments\([27]\), we can reconstruct how this happens in hot matter in some detail\([10]\) and then conjecture what could happen in dense matter.

The reasoning goes as follows.

Baryonic and hyperonic interactions would not be expected to survive the hadronic freedom region. We know how this comes about in temperature\([20]\) from lattice calculations: The nucleon breaks up into three loosely bound constituent quarks at \( T \sim 125 \text{ MeV} \) and then they lose their masses, becoming noninteracting current quarks as \( T \to T_{\chi_{SR}} = 175 \text{ MeV} \). This means that the baryonic interactions go to zero as \( T \to T_{\chi_{SR}} \). In the same vein, we predict that the same phenomenon takes place as \( g^* \to 0 \) at the fixed point in finite density. However that is the region of density where kaons will condense. The matter is supported by electron pressure and flows towards the VM fixed point a la HLS/VM until it “crashes” with the kaons condensing when electrons decay into kaons. Then suddenly a neutron star will fall into a black hole in a light-crossing time and never be seen again. This scenario will be developed in more detail in a separate publication\([28]\).

**Acknowledgments**— GEB was supported in part by the US Department of Energy under Grant No. DE-FG02-88ER40388. CHL was supported by grant No. R01-2005-000-10334-0(2005) from the Basic Research Program of the Korea Science & Engineering Foundation.

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