Invited Comment

Unified description of structure and reactions: implementing the nuclear field theory program

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Abstract
The modern theory of the atomic nucleus results from the merging of the liquid drop model of Niels Bohr and Fritz Kalckar, and of the shell model of Marie Goeppert Meyer and Hans Jensen. The first model contributed the concepts of collective excitations. The second, those of independent-particle motion. The unification of these apparently contradictory views in terms of the particle-vibration and particle-rotation couplings carried out by Aage Bohr and Ben Mottelson has allowed for an ever more complete, accurate and detailed description of nuclear structure. Nuclear field theory (NFT), developed by the Copenhagen–Buenos Aires collaboration, provided a powerful quantal embodiment of this unification. Reactions are not only at the basis of quantum mechanics (statistical interpretation, Max Born), but also the specific tools to probe the atomic nucleus. It is then natural that NFT is being extended to deal with processes which involve the continuum in an intrinsic fashion, so as to be able to treat them on an equal footing with those associated with bound states (structure). As a result, spectroscopic studies of transfer to continuum states could eventually make use of the NFT rules, properly extended to take care of recoil effects. In the present contribution we review the implementation of the NFT program of structure and reactions, setting special emphasis on open problems and outstanding predictions.

Keywords: nuclear field theory, renormalization, collective motion, single-particle motion, transfer reactions, nuclear superfluidity

(Some figures may appear in colour only in the online journal)

1. Foreword

In what follows, one of the authors (RAB) elaborates on the background which is at the basis of the present article, written to commemorate the 40 year anniversary of the 1975 Nobel prize...
in Physics (see also RA Broglia, 40 years of the Nobel Prize in Physics: then and now, invited lecture, Niels Bohr Archive, 14th December 2015, arXiv:1603.09271v1 [physics.hist-ph]).

**Background for subject and title**

In the morning of 4th October 1965, I (RAB) sat in a rather crowded auditorium A of the Niels Bohr Institute to attend the first of a series of lectures on nuclear reactions which were to be delivered by Ben Mottelson. In the following spring term, the Monday lectures were expected to deal with the subject of nuclear structure and the lecturer to be Aage Bohr, as it duly happened. After Ben’s lecture, an experimental group meeting took place in which experimentalists, as it was the praxis, showed their spectra, likely not yet completely analyzed, while theoreticians attempted at finding confirmation of their predictions in connection with specific peaks of the spectra.

In the afternoon I would continue with the calculation of pairing vibrations I was carrying out in collaboration with Daniel Bès, as well as discuss with Claus Riedel on how to use this information to work out two-nucleon transfer differential cross sections for lead isotopes, quantities newly measured at the Aldermaston facility by Ole Hansen and co-workers. Within this context it did not seem surprising to me, nor to the rest of the attendees of Ben’s lecture as far as I recall, that reactions and structure went hand in hand, to the extent that practitioners aimed at checking theory with experiment. Given this background, reinforced through the years by my association with Aage Winther and Daniel Bès, aside from that with Aage Bohr and Ben Mottelson, it is only natural that I view structure and reactions as the two inseparable faces of the same medal.

The pages where I wrote down the notes of Ben’s and Aage’s lectures have somewhat yellowed in the intervening years. On the other hand their content, in particular concerning the deep interweaving of structure and reactions, has not lost a single drop of its depth and simplicity, the mathematics used being elemental, the physical concepts employed the right ones. This is even more so concerning nuclear physics today, where the study of halo nuclei has blurred the distinction between bound and continuum states and forced practitioners to develop inverse mechanisms to break gauge invariance in nuclei leading to superfluidity and superconductivity. Let us use as reference for such comparison condensed matter physics. Tunneling experiments, both of single electrons as well as of Cooper pairs, involving superconducting metals, were swiftly analyzed employing the same many-body techniques used to work out BCS. Such an approach was instrumental to allow for a quantitative description of quasiparticle and of pair motion in metals. Within this context one can posit that our lagging behind is due to at least two facts. One, that many nuclear structure

9 Another one, this time regarding style, is also called for (within this context see also, R.A. Broglia 40 Years of the Nobel Prize in Physics: then and now, arXiv:1603.09271v1 [physics.hist-ph]). Subordinated clauses play very different roles in many languages, in particular in Latin ones, than in English. Such a role is deeply rooted in the way people having one or the other as mother language, think and express themselves. This is in keeping with the fact that a language reflects, not only a way to communicate, but a conception of the whole of life and of the circumstances in which we live (‘am me and my circumstance’ José Ortega y Gasset, Meditations on Quixote, 1914). Within this context, the final text of the article reproduced here likely adheres more correctly to English use than the original one made out of long sentences, with many subordinate clauses. Weather it also conveys better the facts and the physics than before, is an open question (RAB; within this context and with all due provisos see Niels Bohr, The structure of the atom, Nobel Lecture 11 December, 1922, in which, from the starting, sentences running for seven lines and containing a large number of subordinate clauses are a rule).

10 Namely, bare \(v_{\text{bare}}(p)\) and induced \(v_{\text{ind}}(p)\) pairing interactions. Because part of the time a physical nucleon is a bare one, and part of the time it is a bare nucleon carrying along vibrational modes, the interaction between clothed nucleons contains a short-range component \(v_{\text{bare}}(p)\), arising from meson exchange (range \(a \approx 1 \text{ fm}\), and long-range component \(v_{\text{ind}}(p)\) resulting from the exchange of collective modes between nucleons moving in time reversal states around the Fermi energy. The wavelength of surface vibrations of multipolarity \(L\) is of the order of \(\lambda = 2\pi R/L\), where \(R\) is the nuclear radius. In keeping with the fact that \(L\) is, for collective modes less or approximately equal to 5, the range of \(v_{\text{ind}}(p)\) is of the order of the nuclear dimensions. Furthermore, because of the important role of spatial quantization in nuclei, \(v_{\text{ind}}(p)\) is expected to display an important \(\omega\)-dependence. Within this context it is of notice the richness of modes which can be used, together with the bare nuclear interaction, to bind Cooper pairs (density, spin, etc (p-h) collective vibrations, as well as monopole and multipole pairing vibrational modes), let alone the fact that one can study the binding of single Cooper pairs in actual nuclear systems, essentially as in the original model.

11 The BCS papers were published in 1957 [3, 4]. Josephson’s paper [5, 6] and Anderson’s interpretation [7] as the specific probe of gauge phase coherence appeared in 1962 (the same year of Giaever’s paper [8], see also [9]) and in 1964 respectively. The use of the Cooper pair tunneling results by Scalapino, to provide a quantitative account of the electron–phonon coupling phenomena within a 10% error, is of 1968 [10]. Within this context see also the contribution of McMillan and Rowell to [11] (see also [12]). The translation of BCS to atomic nuclei carried out by Bohr, Mottelson and Pines is dated 1958 [13]. The specificity of two-nucleon transfer differential cross sections to quantitatively probe pairing correlations in nuclei was promptly recognized [14–16] (see also [17]), while its implementation as a quantitative tool which can be used throughout within the 10% error level is only of recent date (see [18], in particular figure 10, and references therein).

12 The fact that in Coulomb excitation the kinematic factors associated with the coupling of the relative motion and the intrinsic degrees of freedom can be analytically extracted may induce practitioners to think that one can directly compare the results of structure models. This is of course not true. Conversely, one cannot compare the lifetimes obtained from tunneling probabilities and barrier attacking frequencies (reaction) with experiment, without weighting them with the associated formation probabilities of the outgoing particles (structure) (see e.g. chapter 7 of [1] and references therein).

Let us conclude these remarks with a methodologic consideration. It concerns the worrisome delay, compared to other fields of physics, with which we are understanding the nuclear embodiment of a variety of universal renormalization, many-body phenomena. In particular: (a) physical, clothed, single-particle motion; (b) mechanisms to break gauge invariance in nuclei leading to superfluidity and superconductivity. Let us use as reference for such comparison condensed matter physics. Tunneling experiments, both of single electrons as well as of Cooper pairs, involving superconducting metals, were swiftly analyzed employing the same many-body techniques used to work out BCS. Such an approach was instrumental to allow for a quantitative description of quasiparticle and of pair motion in metals. Within this context one can posit that our lagging behind is due to at least two facts. One, that many nuclear structure
practitioners do not deem reactions relevant, let alone those who consider them boring. Another, that reaction experts often combine state of the art reaction theories and their software implementation with less than same level nuclear structure inputs. Arguably, a third, indirect reason for our delay is connected with the single-minded search for a Hamiltonian, instead than for a physical understanding of the many-body nuclear phenomena.

Within this context one can posit that the physics at the basis of the achievements rewarded by the 1975 Nobel prize, resulted in the paradigm of broken symmetry restoration as a tool to determine the nuclear collective variables (CV, elementary modes of excitations) and of their couplings: restoration of the violation of the translational invariance by the mean field and by scattering states (single-particle motion); restoration of rotational invariance in a variety of spaces, in particular in 3D- and in gauge-spaces, leading to surface vibrations and to quadrupole rotations, as well as to pairing vibrations and rotations, with associated emergent properties of generalized rigidity in these spaces, resulting from the coupling to the single-particle degrees of freedom, properties which can be directly probed by experiment.

2. Introduction

The year 1975 was important for nuclear physics. The second volume of the monograph ‘Nuclear Structure’ written by Aage Bohr and Ben Mottelson was published [19], and the authors co-shared the Nobel prize in physics [20, 21]. In hindsight, it was important also because of the unexpected contents of vol II as compared to those originally planned [22], reflecting the fact that a three volume project had become a two volume one12, thus lying a heavy responsibility squarely on the shoulders of the younger collaborators of Aage and Ben.

In particular regarding the implementation of the nuclear field theory (NFT) program. This theory, tailored after Feynman’s version of QED [23, 24] and based on the concept of elementary modes of excitation and of their interweaving through the particle-vibration coupling (PVC) mechanism [19, 25–31] (within this context see footnote number 14 of [21]) was, at the time, essentially developed conceptually, mainly as the result of the Copenhagen–Buenos Aires13 collaboration [32–44] (within this context see also [45–48]). On the other hand, its actual workings and its power and limitations had still to be tested and the associated protocols for doing so, worked out.

This fact became particularly poignant during the ‘Enrico Fermi’ International School of Physics on ‘Elementary Modes of Excitation in Nuclei’ which took place in July 1976 at Varenna (Como Lake), under the direction of Aage Bohr and of one of the authors (RAB) [49]. Although a number of applications of NFT were discussed at the school, it was clear that there were ample zones of nuclear structure, let alone nuclear reactions, which had been barely touched upon like: (1) renormalization and damping of collective modes, including giant resonances (GR) and rotational motion; (2) the clothing of single-particle motion to make them physical particles, (3) the role of retardation and state dependent effects, in particular in nuclear pairing correlations, (4) the calculation of two-nucleon transfer absolute differential cross sections. In the present paper we report on some of the latest developments which testify to the fact that the validity of the implementation of the NFT program has received strong experimental confirmation regarding important predictions for light halo nuclei and has, arguably, reached an important milestone14 (one would be tempted to say, ‘been recently completed’, to the extent that a scientific endeavour can ever be considered completed).

In the following we shall see that subjects (1) (regarding GR) and (3) surprisingly, have become strongly connected within the scenario of exotic halo nuclei15, in particular in the description of $^{11}$Li (section 3). In the meantime subjects (1)–(4) are found to be strongly linked in the case of the description of the structure of superfluid nuclei, the corresponding results manifesting a deep physical unity which can be represented at profit, in terms of a well funnelled nuclear structure landscape (section 4). Finally in section 5 a number of open problems are discussed16.

In keeping with the fact that a central issue touched upon both in connection with exotic halo nuclei and with superfluid nuclei is pairing, it is not surprising that subject (4) [18] plays an important role in the examples discussed below.

3. The exotic, halo nucleus $^{11}$Li

The weak, but finite stability of light halo dripline exotic nuclei like $^{11}$Li, is associated with the $s_{1/2}$ and $p_{1/2}$ levels at threshold.

14 This could be closer to becoming ‘true’ if also the optical potential needed to describe direct reactions, in particular one- and two-nucleon transfer processes, had become incorporated among the standard quantities calculated within the NFT framework.

15 High-lying giant resonances, the elastic response of the atomic nucleus to rapidly varying external fields and controlled by $\hbar\omega_{BC}=(41/A^{1/3})$ MeV, give rise to a variety of low-energy, $\omega$–independent effects, like polarization charges, polarization contributions to effective two-particle interactions (see e.g. [119] p 432 and 515 respectively) and to two-nucleon transfer amplitudes (see figure 1 of [50]). Low-lying modes, the plastic response of nuclei to time varying external probes, lead to retarded, $\omega$–dependent effects, which play an essential role both in the clothing of single-particle motion and in the induced interactions arising from the exchange of these modes between pairs of nucleons (see [19], last lines of section 6.3f, p 432), a subject intimately related to the melting of points (1) and (3) above, in the case of the halo nucleus $^{11}$Li.

16 Among the subjects we do not discuss are the extension of NFT to finite temperature based on Matsubara’s formalism (see chapter 9 of [51] and references therein; within this context one is reminded of the fact that at room temperature ($\approx 25\text{ meV}$) the atomic nucleus is in the ground state and thus at zero temperature in keeping with the fact that the first excited state of any nucleus is to be found at least at tens of keV), the connections between inhomogeneous damping and motional narrowing both regarding GR and rotations in hot and warm systems respectively (see [51–54] and references therein), the applications of NFT methods to describe specific aspects of atomic clusters [55–58]. Neither the systematic treatment of over completeness and non-orthogonality of the basis states nor the breakdown of symmetries discussed in [59] (in connection with reactions, see [60] (adjoint basis)),Within the above context we refer to the contribution of Daniel Bézu to this topical issue.
They are thus unavailable for the short range bare NN-pairing interaction [61–63]. This is the so called pairing anti-halo effect. It requires a mechanism of Cooper binding mediated by the exchange of long wavelength collective modes. Because of the presence of the 'continuum' low-energy s → p transition, it is the natural scenario for the appearance of extremely low-lying collective dipole modes, that is of the giant dipole pygmy resonance (GDPR). This is in keeping with the fact that the neutron halo displays a very large radius, as compared with that of the core nucleons [17] and thus, a small overlap with it (within this context, see appendix B and table B1). This phenomenon has a threefold consequence: (i) to screen the bare NN−1S0 short range pairing interaction, (ii) to screen the (repulsive) symmetry interaction, and (consequently) (iii) to bring down at low energies a consistent fraction of the Thomas–Reiche–Kuhn (TRK) energy weighted sum rule associated with the GDR. The two last effects allow for the presence of a dipole mode at very low energies [18].

Exchanged between the s, p orbitals heavily dressed by mainly quadrupole core vibrations, resulting into parity inversion (10Li) (see figure 1(I)), it provides essentially all of the glue needed to bind the neutron halo Cooper pair to the core by ≈ 400 keV, and thus the weak stability of the halo field needed to sustain the pygmy resonance. In fact, the contribution of the bare pairing interaction is, in the case of 11Li, subcritical (≈ 100 keV) (see figure 1(II)) [67]. Before elaborating on this

\[
\begin{align*}
|s_{1/2}^2(0)\rangle, r_1 = 5 \text{ fm} \\
|\tilde{\phi}\rangle = |0\rangle + 0.7 \left( p_{1/2}, s_{1/2}\right)_1^- \otimes 1^-; 0\rangle + 0.1 \left( s_{1/2}, d_{5/2}\right)_2^+ \otimes 2^+; 0\rangle
\end{align*}
\]

Figure 1. Parity inversion and Cooper pair binding in the N = 6 closed shell isotope 9Li. (I): schematic representation of the phonon clothing of single-particle motion in 10Li through the particle-vibration coupling (PVC). (II) of the induced pairing interaction in 11Li, again in terms of phonons and the PVC. The first process is mainly associated with quadrupole vibrations of the core. The second, with the exchange of the GDPR between the halo neutrons of 11Li (after [67]). In the boxed inset, a schematic representation of the bare NNinteraction is given (four point vertex entering rule II of NFT, see appendix H).

17 Low-energy electric dipole strength is customarily related to a neutron skin [64]. Within this context, one can hardly think of a better example than 11Li, in which case the core (9Li) radius is ≈ 0.3 ± 0.03 fm, while the halo extends to define a radius for 11Li of 4.58 ± 0.17 fm. These quantities were calculated making use of the experimental value of the mass mean square radius 2.32 ± 0.02 fm (9Li) and \( r^2 \approx 3.55 \pm 0.1 \text{ fm} (11Li) \) with the help of the relation \( R = \sqrt{3}/4 \times (r^2)^{1/2} \). It is of notice that the interplay between isoscalar and isovector modes in the presence of neutron excess, is a subject with a long tradition (see e.g. [19, 65] and references therein), and that the search to the answers to questions posed in connection with recent work on exotic nuclei can be facilitated by results to be found in the above mentioned references. It is of notice that the GDPR plays a central role in determining the value of the dipole effective charge, opposing the contribution of the GDR, in a similar way in which the giant quadrupole resonance (GQR) opposes the contribution of the isovector GQR (IGQR) to the quadrupole effective charge.

18 This is intimately related to the fact that in 9Li, there is a (continuum) single-particle dipole transition of very low energy (≈ 0.5–1 MeV) between the s_{1/2} and p_{1/2} unbound states lying essentially at threshold. This is a very subtle extension of the statement that single-particle motion is the most collective of all nuclear motions [66], emerging from the same properties of the nuclear interaction (both bare and induced) as collective motion does, and in turn at the basis of the detailed properties of each collective mode, acting as scaffolds and filters of the variety of embodiments. In fact, one has to add the characterisation of ‘physical’ to ‘single-particle motion’ (i.e. clothed) to englobe in the above statement also the present situation. In other words, while the bare s_{1/2} and p_{1/2} orbitals could never lead to a low-lying GDR, the corresponding clothed, physical states do so. Consequently, ‘physical, clothed single-particle motion, is one of the most collective nuclear motions’, is, arguably, the right statement.
point, let us discuss in more detail the results displayed in figure 1. They provide a textbook example of the workings of NFT, and of the associated rules and interactions (see appendix H), but also of the practical difficulties found in their implementation. In particular, those arising from the lack of a single effective interaction which can generate with comparable accuracy HF, RPA and HFB, QRPA solutions.

In the calculations the odd $p_{3/2}(\pi)$ proton state of the $^9$Li, $^{10}$Li, $^{11}$Li isotopes is considered as a spectator. It is of course properly taken care of in terms of angular momentum coupling to determine spin (and parity) of the states whose properties are compared with the observation. An example is provided by the absolute differential cross section $^1H(^{11}$Li,$^9$Li $(1/2^+; 2.69 \text{ MeV})^3$H which testifies to the population of the $1/2^-$ member of the multiplet $(p_{3/2}(\pi) \otimes 2^+ (^{4}$He))$_{1/2^-}$, first excited state of $^9$Li. On the other hand, $p_{3/2}(\pi)$ does not play an explicit role in the dynamical processes associated with the neutron degrees of freedom we are studying: one- and two-neutrons outside the $N=6$ closed shell in $^{10}$Li, and $^{11}$Li respectively. Within this context, it does not escape our attention the work done concerning the possible role the NFT, and of the associated rules and interactions in order to generate the different diagrams of perturbation theory. Because a significant part of the original interaction has already been included in generating the vibrational modes, the rules of NFT for evaluating these diagrams imply a number of restrictions as compared to standard perturbation theory in terms of only the nucleonic degrees of freedom and the bare NN-interaction (Feynman–Goldstone diagrams) [34, 37]. Namely, the one mentioned above and having to do with proper allowed initial and final states (rule I) and that associated with rule III restricting internal lines to participate in processes in which two fermions are created and subsequently annihilated without having participated in subsequent interactions.

In figure 1(I), the wavy lines correspond to vibrations of $^9$Li, considering the odd proton as a spectator. Particular important among these vibrations is the role played by the low-lying quadrupole vibration.

In figure 1(II), the boxed particle–particle interaction represents the bare pairing interaction (NN-Argonne $v_{14}$ [70]), while the upper two-particle interaction diagram describes the induced pairing interaction, resulting from the exchange of collective vibrations between nucleons moving in time reversal states close to the Fermi energy. In particular, of the dipole pygmy resonance of $^{11}$Li. This resonant state carries about 8% of the TRK sum rule with centroid 0.5 MeV $\leq \hbar \omega \leq 1 \text{ MeV}$ [71, 72]. It is of notice that within the framework of the present discussion, the uncertainty in the centroid energy quoted above does not affect the results in a major way.

According to rules I and II of NFT, the two-nucleon interaction diagrams shown in figure 1(II) are not proper diagrams which can be employed in initial or final states, but the interactions to be used in the calculation of the binding of the neutron halo pair addition mode to the core of $^9$Li leading to $^{11}$Li (gs) (correlation generating pair addition modes to be calculated in RPA). This can be seen from figure 12, in connection with the processes taking place between the two neutrons moving around the closed shell system $^9$Li, and leading to $^{11}$Li (double arrowed line labeled 0.7$^\text{MeV}$) up to the time denoted $t_7$. Note also in these diagrams, the lack of bubbles, as required by rule III of NFT (after [67]).

Halo and pygmy on top of it are, within this picture, two aspects of the same physics. Namely that associated with the coexistence of two ground states, the normal core- and the halo-based states$^{19}$. In a very real sense, the monopole halo Cooper pair addition mode of $^9$Li, i.e. $^1$Li(gs), and the pygmy resonance of $^{11}$Li, i.e. $^1$Li(gs)$_{1/2^-}$(0.7 MeV), are two states which can only exist in mutual symbiosis. In a nutshell, the pygmy resonance is the quantal reaction the nucleus has at disposition to stabilize dripline species by pulling back into the system barely unbound neutrons which essentially do not feel a centrifugal barrier, generating in the process the halo ground state (see figure A5).

Let us elaborate on this point. In nuclei lying along the stability valley (e.g. $^{120}$Sn), one pays a high energetic prize to

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In other words, of the realization of a low-density nuclear system in which neutron skin effects overwhelm in connection with particular states, the role of the ‘normal’ core.
separate protons from neutrons even in the case of the low-lying GDPR [64], while nucleons outside closed shells can quadrupole polarize the core with ease (see section 4). This is why the induced pairing interaction receives important contributions from low multipole surface modes with the exclusion of \( \lambda = 1 \). However, in the case of halo nuclei like \(^{11}\text{Li}\), the above arguments do not apply. Better, they are still operative, but now set upside down. In fact the large diffusivity of the halo makes it difficult to quadrupole distort it. At the same time, the ground state of the system is poised to acquire a permanent dipole moment. Thus the associated ZPF become quite large. Consequently, the most important intermediate boson being exchanged between the halo neutrons and contributing to a large extent in binding the halo Cooper pair to the \(^{\text{Li}}\text{Li}\) core, is a dipole, soft E1, giant pygmy resonance, mode \([71, 72]\). Thus the halo pair addition mode of \(^{11}\text{Li}(gs)>\) can be viewed as a van der Waals Cooper pair (dipole pygmy bootstrap mechanism to violate gauge invariance in nuclei, see appendix A, figures A1(d) and (e)).

Haloo pair addition modes can be used, in principle, as building blocks of the nuclear spectrum, as one does with standard pairing vibrational modes around closed shell nuclei (e.g. \(^{208}\text{Pb}\)). Within this context, it is an open question whether, the first excited \((0^+)^{+}\) (halo) state (2.25 MeV) of \(^{13}\text{Be}\) is the analogue of the halo pair addition mode of \(^{\text{Li}}\text{Li}\), that is \(^{11}\text{Li}(gs)>\), and the observed \(1^-\) state \([73-75]\) at 0.460 MeV on top of it (that is at 2.71 MeV above the ground state) is a member of the associated pygmy resonance, analogue state of the GDPR state of \(^{11}\text{Li}\) observed at low energy (\(\lesssim 1\) MeV).

Summing up, halo Cooper pair or better halo pair addition vibrations and pygmy dipole resonances (soft E1-excitation, vortex-like pair addition mode, appendix A) are two novel (symbiotic) plastic modes of nuclear excitation. Experimental studies of these modes, in particular of pygmy resonances based on excited states are within reach of experimental ingenuity and techniques\(^{20}\). They are expected to shed light on a basic issue which has been with us since nuclear BCS: the microscopic mechanism, aside from the bare NN-pairing force, to break gauge invariance. Thus, the variety of origins of nuclear pairing.

Furthermore, they are likely to extend the probing of the validity and limitations of the\(^{21}\) Axel–Brink hypothesis \([77, 78]\) (within this context see [19], sections 6-6(b) and references therein). This phenomenon can be instrumental in modulating the transition between warm and hot (equilibrated) excited nuclei, let alone provide a microscopic probe to study a new form of inhomogeneous damping. Namely radial isotropic distortion. The importance of this mechanism, which has partially entered the literature under the name of neutron skin, is particularly important in \(^{11}\text{Li}\). In this case, radial inhomogeneous damping is able to bring down by tens of MeV a consistent fraction (approximately 8\%) of the TRK sum-rule. As expected, changes in density can affect very strongly nuclear structure properties.

Let us conclude this section with some technical remarks, within the context of [67]. Among other things, in this paper, the dipole response function of \(^{11}\text{Li}\), calculated by diagonalizing a separable dipole–dipole interaction in QRPA, is discussed. The single-particle states were determined with the help of a Saxon–Woods potential with the parameterization of [22] (see equations (2.181) and (2.182)), also taking into account the unusually large value of \((N - Z)/A \approx 0.45\) associated with the symmetry term. This potential was placed in a spherical box of radius equal to 30 fm, so as to be able to take care of the (discretized) continuum states. By properly adjusting the value of the dipole coupling constant (close to its selfconsistent value \([19]\), a root was forced to have zero frequency (\((\approx 10\) keV). In this way one ensures that the resulting dipole strength function consists solely of intrinsic nuclear excitations, without contribution from the center of mass (C.M.) oscillations.

Within this context, calculations of the B(E1) transition probabilities were carried out, making use of the QRPA wavefunctions, in two different ways: (1) ascribing to protons and neutrons effective charges \(+Ne/A\) and \(-Ze/A\) respectively (intrinsic system of reference); (2) with the real charges, namely \(+e\) and \(0\) respectively (laboratory system). The fact that both calculations lead to the same result, testifies to the fact that the QRPA solutions were free from C.M. contributions (see appendix G).

Another question is, to which extent this statement still holds when the GDPR is employed as intermediate boson which, exchanged between the pair of neutrons mainly occupying the configurations \(s_{1/2}^2(0)\) and \(p_{1/2}^2(0)\) at threshold, stabilizes the halo system and binds it to the core \(^{\text{Li}}\text{Li}\). Or when the particle-(GDPR) vibration coupling vertex is screened through processes arising from the Pauli principle (see e.g. [1], appendix F).

Let us assume to carry out a selfconsistent, iterative calculation, in which the ground state and the associated occupation probabilities are calculated at each step. But also the polarization potential \(U_p\) to be added to the starting, global mean field \([80-82]\). \(U_p\) is the generalization of the polarization contribution to the optical potential referred to in connection with the process displayed in figure 2 (figure 12). It is, among other things, intimately related to the contribution of collective modes ZPF, to the nuclear density as well as to the total nuclear binding energy \([83-85]\). Using the resulting modified energies and effective \(U, V\) factors, a new iteration in the QRPA calculation of the GDPR is carried out. The process is continued until convergence is achieved. Following such a NFT protocol,
Figure 2. NFT representation of the reactions (a) $^{11}$Li(p, t)$^9$Li(gs), (b) $^{11}$Li(p, t)$^9$Li(1/2$^-$), (c) $^3$H($^{11}$Li, $^9$Li(gs))$^1$H and (d) $^3$H($^{11}$Li, $^9$Li(1/2$^-$))$^1$H. Time is assumed to run upwards. A single arrowed line represents a fermion (proton (p) or neutron (n)). A double arrowed line two correlated nucleons. In the present case two correlated (halo) neutrons (halo-neutron pair addition mode $|0\rangle_n$). A heavy arrowed line represents the core system $^9$Li(gs). A standard pointed arrow refers to structure, while 'round' arrows refer to reaction. A wavy line represents (particle-hole) collective vibrations, like the low-lying quadrupole mode of $^9$Li, or the (more involved) dipole pygmy resonant state which, together with the bare pairing interaction (horizontal dotted line) binds the neutron halo Cooper pair to the core. A short horizontal arrow labels the proton–neutron interaction $v_{np}$ responsible for the single-particle transfer processes, represented by an horizontal dashed line. A dashed open square indicates the particle-recoil coupling vertex (for more details see caption to figure 12). The jagged line represents the recoil normal mode (see appendix F, discussion connected with figure F1) resulting from the mismatch between the relative center of mass coordinates associated with the mass partitions $^{11}$Li+p, $^{10}$Li+d (virtual) and $^9$Li+t. It is explicitly drawn as discussed in the text and in appendix F as a mnemonic connected with the particle-recoil coupling vertex. The detector array is represented by a crossed squared box.
arguably, there would be no contribution from the C.M. motion in the dipole response. In the actual implementation, such contributions may arise because of numerical approximations.

3.1. NFT of structure and reactions: the case of the $^1H$ ($^{11}Li,^{9}Li$)$^{3}H$ two-particle transfer process

The standard setup for direct nuclear reactions involving stable species contemplates a beam of light particles aimed at a (fixed) target of a somewhat heavy nucleus, like e.g. $^{120}Sn$ ($p, t$)$^{118}Sn$, where the proton is the projectile and $^{120}Sn$ the target nucleus. The standard set up was maintained with the introduction of (long lived) light projectiles and/or heavy (target) nuclei, like e.g. in the case of $^{208}Pb$ ($t, p$)$^{210}Pb$ (unstable projectile, $t_{1/2} = 12.32$ y), $^{210}Pb$ ($p, t$)$^{208}Pb$ (unstable target, $t_{1/2} = 22.2$ y). Experiments of the first type could be carried out only at selected laboratories, like Harwell (Aldermaston) and LANL (Los Alamos).

The precise meaning of the standard set up became somewhat blurred with the advent of heavy ion accelerators, in which case both target and projectile were heavy nuclei (see e.g. [60] and references therein). Nonetheless, the incoming beam was, as a rule, made out of species lighter than that used to make the target. The situation got reversed in connection with the study of exotic nuclei [86], that is the study of species which, like $^{11}Li$ have very short lifetimes ($t_{1/2} = 9.75$ ms). The probing of pairing phenomenon through two-nucleon transfer processes is then only possible in terms of inverse kinematic, in which an ephemeral $^{11}Li$ beam is aimed at a proton (hydrogen) gas target, that is, $^{1H}(^{11}Li,^{9}Li)^{3}H$.

In figures 2(a) and (b) a fictitious standard set up to study the two-nucleon pick-up reaction from a gedanken $^{11}Li$ target is displayed. The detector is assumed to provide, in both cases, information which allows to reconstruct the kinematics of the process (energy, momentum, mass partition). Of course the standard arrangement cannot be operative due to the extremely short lifetime of $^{11}Li$, the set up used to carry out the experiment [87] being that schematically shown in figures 2(c) and (d) in terms of the initial and final asymptotic states (inverse kinematics). Let us shortly concentrate on the detection of the process populating the first excited state $^{9}Li$ ($^2S_1/2$; 2.69 MeV).

Concerning the present formulation of NFT it may look that, while one can calculate structure processes up to any order of perturbation theory in the PVC vertices (see figure 12, processes in the range $t_1 - t_2$), concerning reaction channels one is not able to do better than second order in $v_{np}$ (see figure 2). One does so, in the structure case, by diagonalizing the PVC processes is then only possible in terms of inverse kinematic, in which an ephemeral $^{11}Li$ beam is aimed at a proton (hydrogen) gas target, that is, $^{1H}(^{11}Li,^{9}Li)^{3}H$.

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Hamiltonian [19, 38], taking into account also the effect of four point vertices, and by an orthogonalization procedure in terms of overlaps (dual basis) in the reaction case [see 60, 88], see also [18, 89]. Now, this picture, as explained in more detail in section 5.2, is misleading. Within the framework of direct reaction theory in general, and of two-nucleon transfer reactions in particular, a complete description of the reaction process is obtained by considering: (a) simultaneous transfer, which is linear in the neutron-proton interaction $\mathcal{V}_{\text{npg}}$, (b) successive transfer, which is bilinear in $\mathcal{V}_{\text{npg}}$ correctly corrected, (c) non-orthogonality, again linear in $\mathcal{V}_{\text{npg}}$, and (d) Pauli principle operating between the active nucleons (figure 11). In connection with point (d), in particular, and with Pauli principle in direct reactions in general, we refer to the so-called Perey-Buck term (0.4Ee, see section 4.1) in the real part of the optical potential (see also section 5.2).

Within this context, but not only, it is to be noted that in all the reactions calculations discussed in this contribution, global optical potentials to describe elastic scattering in the different channels have been used, with the exception of the analysis of the $^{11}\text{Li}(p, \, ^{1}T)$ $^{11}\text{Li}$ reaction [90] in which case the empirical potential of [87] was employed. This is also in keeping with the non-standard values of the parameters needed to describe the $(^{11}\text{Li}, \, p)$ elastic channel [91]. Within this context it is of notice that we consider the calculation of the $^{11}\text{Li}(p, \, p^{'})$ optical potential among the open problems (see section 5.2).

3.2. Comparison with the data

In figure 3 the absolute differential cross sections associated with the processes $^1\text{H}(^{11}\text{Li}, ^3\text{Li(gs)})^1\text{H}$ and $^1\text{H}(^{11}\text{Li}, ^3\text{Li}(1/2^-))^1\text{H}$, calculated making use of the software Cooper and of NFT wavefunctions[22] displayed in figure 1(II), or better of the associated two-nucleon transfer spectroscopic amplitudes [67, 90], are compared with the experimental findings [87].

The population of the first excited state of $^3\text{Li}$ provides evidence for the presence of a component of the type $(s_{1/2} \otimes d_{3/2})^2 \otimes 2^+$; $0^+ \otimes \otimes |p_{3/2}(\pi)>$ in the $^{11}\text{Li(gs)}$ wavefunction (see also figure 2). The absolute value of the ground state transition depends directly on the $(s_{1/2}, \, p_{1/2}) \otimes \otimes \Delta \Pi \otimes 0^+$; $0^+ \otimes \otimes |p_{3/2}(\pi)>$ component of the $^{11}\text{Li(gs)}$ wavefunction, through normalization (see figure 1(II)b)). This result underscores the importance of having at one’s disposal a reliable description of the two-nucleon transfer process (reaction) populating these states, so as to be able to accurately calculate the associated absolute differential cross section and thus test the structure of the wavefunctions describing the states connected by the process. In other words, of having at one’s disposal a reaction theory of Cooper pair transfer, and associated software implementation, so that discrepancies between predictions and experiment can be associated solely with the structure input.

Summing up, within the general scenario of the foreword, one can posit that a representative example of the reaction face of the medal, consists in the possibility to calculate, at the 10\% level uncertainty, absolute two-nucleon transfer differential cross sections. This in turn has shaped the (structure) reversed face, which provides direct evidence of the central role the induced pairing interaction plays throughout the mass table. In the case of $^{11}\text{Li}$, accounting for close to 85\% of Cooper pair binding. For about 50\% in open shell nuclei lying along the stability valley (see next section).

4. The chain of superfluid $^{118,119,120,121,122}\text{Sn}$-isotopes lying along the stability valley

An essentially ‘complete’ description of the low-energy structure of the superfluid nucleus $^{120}\text{Sn}$ and of its odd- and even-A neighbours $^{118,119,120,121,122}\text{Sn}$ is provided by the observations carried out with the help of Coulomb excitation and subsequent $\gamma-$ decay and of one- and two-particle transfer reactions. These reactions and processes constitute specific probes of particle-hole-like vibrations, quasiparticle and pairing degrees of freedom respectively. Also of their mutual couplings. The corresponding experimental findings have been used to stringently test the predictions of a similarly ‘complete’ description of $^{118,119,120,121,122}\text{Sn}$ carried out in terms of elementary modes of excitation which, through their interweaving within the framework of NFT, melt together into effective fields [92], each displaying properties reflecting those of all the others, their individuality resulting from the actual relative importance of each one [93–95].

Independent particle and collective vibrations constitute the basic states of the structure calculations. These are implemented in terms of a SLy4 effective interaction [96] and a $\nu_{14}(S_{0}) (\equiv \nu_{p}^{\text{bare}})$ Argonne pairing potential [70]. In keeping with rule IV of NFT, HFB provides an embodiment of the quasiparticle spectrum while QRPA a realization of density ($J^{\pi} = 2^{+}$, $3^{-}$, $4^{+}$, $5^{-}$) and spin ($2^{+}$, $3^{+}$, $4^{+}$, $5^{+}$) modes (rule II, PVC vertex). Taking into account renormalization processes (self-energy, vertex corrections, phonon renormalization and phonon exchange) in terms of the PVC mechanism (figure 4), the dressed particles (rule II) as well as the induced pairing interaction $\nu_{p}^{\text{ind}}$ were calculated (see [80, 81, 97–103]). Adding $\nu_{p}^{\text{ind}}$ to the bare interaction $\nu_{p}^{\text{bare}}$, the total pairing interaction $\nu_{p}^{\text{eff}}$ was determined. With these elements, the Nambu-Gor’kov (NG) equation (see appendix E) was solved self-consistently (rule I–III) using Green’s function techniques [104–108], thus obtaining the parameters characterizing the renormalized quasiparticle states (figure 5).

It is to be noted that in carrying out the above calculations use has been made of empirically renormalized collective modes[23] (see section 4.1). These modes are determined as

22 It is of notice that these wave functions and the associated spectroscopic predictions [67] had to wait short of a decade to be tested and found to be correct [87, 90].

23 In a similar way in which it has been stated that in describing a many-body system you may choose the degrees of freedom you prefer, although if you choose the wrong ones you will be sorry, one may state that elementary modes of excitation plus renormalization (in some cases empirical renormalization, see section 4.1), provides an economic picture of structure and reactions which converges to the physical observation, in many cases, already in lowest order of perturbation. To the extent of attributing the ancient Greek meaning of ‘find’ and ‘discover’ to the word heuristic (προφητεύω) and of ‘serving to discover’ of the Oxford dictionary, one may ascribe the connotation of heuristic to the above mentioned protocol (within this context see [109]).
the QRPA solutions of a separable multipole-multipole interaction with empirical single-particle levels, adjusting the strength around the selfconsistent value to obtain the desired properties (energy and above all, B(E2)-values ($\beta_2$-deformation parameters)). In this way one obtains physical reliable results (see section 4.2) and also avoids difficulties associated with the zero-range character (ultraviolet divergencies), finite size instabilities and spurious self interactions of most Skyrme forces [110–112].

The corresponding results provided directly, or were used to work out the following structural quantities associated with $^{120}$Sn and with neighbouring Sn-nuclei [93–95]: (I) the state

![Diagram](image_url)
dependent pairing gap, (2) the quasiparticle spectrum, (3) the \( (\hbar 1/2 \otimes 2^+) \) centroid and multiplet splitting, (4) the B(E2)-transition strengths associated with the \( \gamma \)-decay of \(^{119}\text{Sn}\) following Coulomb excitation, (5) the absolute differential cross section associated with the reactions \(^{122}\text{Sn}(p, t)^{120}\text{Sn}\) (gs) and \(^{120}\text{Sn}(p, t)^{118}\text{Sn}(gs)\), and (6) the \(^{120}\text{Sn}(d, p)^{121}\text{Sn}\) and \(^{120}\text{Sn}(p, d)^{119}\text{Sn}\) absolute differential cross sections with which the \(^{120}\text{Sn}\) valence orbitals are populated, paying special attention to the breaking of the \( d_{5/2} \) strength. The relative root mean square standard deviations between theory and experiment are shown in table 1. These results provide important evidence to the fact that, choosing as basis states the elementary modes of nuclear excitations, and calculating their couplings following NFT rules, leads to a well funneled landscape of structure and reactions. Namely, a global minimum in the multidimensional space defined by (1)-(6), of the difference between theory and experiment as a function of the \( k\)-mass, the pairing strength and the collectivity of the vibrational modes (see figure 6 and [93]). In section 4.2 we elaborate on a particular example of this overall accuracy of

\[ Z_\omega = \left( \frac{m_\omega}{m} \right)^{-1} \]

\[ m^* = \frac{m_\omega m_k}{m} \]

\[ \psi_{\text{eff}} = \psi_{p}^{\text{bare}} + \psi_{p}^{\text{ind}} \]

\[ (B \bar{C} \bar{S}(A + 2)|P^1|B \bar{C} \bar{S}(A)) \]

\[ (B \bar{C} \bar{S}(A + 1)|a^1|B \bar{C} \bar{S}(A)) \]

\[ E_{\lambda \tau}; \ B(E\lambda) ; \ B(M\lambda) \]

\[ (E_{qp}(j) \otimes \lambda)_j \]
Table 1. Root mean square deviation $\sigma$ between the experimental data and the theoretical values expressed in keV for the pairing gap, quasiparticle energies, multiplet splitting, centroid and width of the $5/2^+$ low-lying single-particle strength distribution.

| Observables | SLY4 | $d_{2\varepsilon}$ shifted | Opt. levels |
|-------------|------|--------------------------|-------------|
| $\Delta$    | 10 (0.7%) | 10 (0.7%) | 50 (3.5%) |
| $E_{qp}$    | 190 (19%) | 160 (16%) | 45 (4.5%) |
| Mult. splitt. | 50 (7%) | 70 (10%) | 59 (8.4%) |
| $d_{2\varepsilon}$ strength (centr.) | 200 (20%) | 40 (4%) | 40 (4%) |
| $d_{2\varepsilon}$ strength (width) | 160 (20%) | 75 (9.3%) | 8 (1%) |
| B(E2)       | 1.4 (14%) | 1.34 (13%) | 1.43 (14%) |
| $\sigma_{2\varepsilon}(p, t)$ | 0.6 (3%) | 0.6 (3%) | 0.6 (3%) |

In single-particle units $B_{2\varepsilon}$ for the $\gamma$-decay (B(E2) transition probabilities) and in mb for $\sigma_{2\varepsilon}(p, t)$. In brackets the ratio $\sigma_{2\varepsilon} = \sigma/L$ between $\sigma$ and the experimental range $L$ of the corresponding quantities: 1.4 MeV ($\Delta$), 1 MeV ($E_{qp}$), 700 keV (mult. splitting), 1 MeV ($d_{2\varepsilon}$ centroid), 809 keV ($d_{2\varepsilon}$ width), 10 $B_{2\varepsilon}$ (B(E2)), 2250 mb ($\sigma_{2\varepsilon}(p, t)$), is given (for details see [95]).

NFT predictions for open-shell nuclei. That is, on the clothing and breaking of the $d_{2\varepsilon}$ valence orbital. Theoretically, this happens as a consequence of the coupling to vibrational states. The fact that the resulting physical (clothed) $d_{2\varepsilon}$ valence single-particle state is part of the time a vibrational mode and a particle in the same or in a different valence orbital state, reflects itself on the breaking of the strength— and thus on the value of the associated $^{120}$Sn(p, d)$^{119}$Sn (5/2)$^+$ absolute differential cross sections.

We conclude by quoting one of the important results of the work which is at the basis of this section. The values of the pairing gap $\Delta = \Delta^{\text{bare}} + \Delta^{\text{ind}}$, obtained from the solution of the NFT+NG calculations, and resulting from the contributions of $F^{\text{bare}}$ and $F^{\text{ind}}$ are about equal, density modes leading to attractive contributions which are partially cancelled by spin modes (within this context see also [113, 114]).

4.1. Empirical renormalization

The collectivity of low-lying particle-hole (two-quasiparticle (2qp)) vibrations like e.g. the lowest $2^+$ state of the Sn-isotopes ($\langle h^2 \omega \rangle \approx 1$ MeV) is specifically measured by the B(E2) transition probability. This quantity is proportional to the density of states which in turn is proportional to the effective mass $m^*$ of nucleons moving in levels close to the Fermi energy. Within an energy interval of approximately $\pm 5$ MeV around $\varepsilon_F$, experimental evidence testifies to the fact that $m^* = m$, as well as that the single-particle content of these physical levels is smaller than 1, and consistent with $Z^* \approx 0.7$ (see figure 5). This quantity being equal to $(m_\pi/m)^{-1}$ leads to $m_\pi = 1.4 m$, where $m_\pi$ is the so-called $\omega$-effective mass associated with the clothing of single-particle states through the coupling to vibrations $(m_\pi/m) = (1 - \partial \Delta E(\omega)/\partial \omega)$, $\Delta E(\omega)$ being the real part of the self-energy. In other words, the $\omega$-mass reflects retardation effects, and single-particle content of the physical [92] fermions (see [115–118] and references therein).

Nucleon elastic scattering experiments at energies of tens of MeV can be accurately described in terms of an optical potential in which the strength $V$ of the real (Saxon–Woods) potential $V(r)$ is written as $V = V_0 + 0.4E$, where $V_0 \approx -45$ MeV and $E = \frac{1}{2} \kappa - \varepsilon_F(\kappa = h^2 k^2/2m)$, the term 0.4E being known as the Perey–Buck potential [19, 22]. It is possible to obtain essentially the same results by solving the elastic scattering single-particle Schrödinger equation making use of an energy independent potential of strength $V \approx 1.4V_0$ and of an effective mass 0.7$m$, the $^{24}$ so called $k$-mass

$^{24}$ What in nuclear matter is called the $k$-mass and is a well defined quantity, in finite systems like the atomic nucleus, in which linear momentum is not a conserved quantity, is introduced to provide a measure of the non-locality of the mean field, and is defined for each state as the expectation value of the quantity inside the parenthesis, calculated making use of the corresponding single-particle wavefunction (see e.g. [119], in which case $m$ is referred to as the non-locality effective mass).
particular cases, may overwhelm the effect discussed above concerning the states calculated in QRPA with SLy4 is too weak. In fact, to reproduce collectivity, namely the strength of low-lying two-quasiparticle vibrational states, one needs to allow the quasiparticles participating in the vibration to excite other quasiparticles, and the excitation energy of the resulting particle-dressed modes coincides with the experimental ones. The empirical renormalization we are talking about in the present paper, involves one more step, namely to consider within this context the techniques developed in [116] can become important, see also [118], chapter 11, section on 'clothed skeletons'). Summing up, in this approach the fermions and the vibrational states used in the intermediate states are supposed to be fully dressed, resulting in what is known as a self-consistent perturbation theory. The empirical renormalization we are talking about in the present paper, involves one more step, namely to consider that fully dressed modes coincide with the experimental ones. If using the experiment as input one recovers, among other observables, the experiment as output (see figure 7), one can conclude that one has a sensible physical model for the bare quantities.

Before concluding this section, let us return to the question of the $k$-mass. The Pauli principle [124] leads, among other things, to the exchange (Fock) potential in nuclei $U_\text{F}(\mathbf{r}, \mathbf{r}') = -\sum_{\mathbf{k} \leq \mathbf{q}} \phi^*_\mathbf{k}(\mathbf{r}) \phi^*_\mathbf{q}(\mathbf{r}') \phi^*_\mathbf{k}(\mathbf{r})$, and thus to the $k$-mass (all of it in the case in which velocity independent forces are used to determine the mean field, a consistent fraction of it otherwise). Thus, an essential nuclear structure element arises from a symmetry-like condition (see also [125, 126]) without any possibility of fine tuning. Be as it may, a way out is that few of the energies of the bare single-particle valence orbitals are slightly modified empirically, and thus to be considered among the physical parameters (e.g. $\epsilon_{d_{12}/2}$, see section 4.2) to be adjusted to account for the set of experimental findings which provide a complete characterization of the low-energy structure of atomic nuclei. Because single-particle motion can be considered the most collective of all nuclear motions [66], adjusting simultaneously $k_2$, $k_3$ and $\epsilon_{d_{12}/2}$ has the following consequence. To force the self-consistent relations between single-particle density fluctuations, mean field ($U(r) = \int d^3r' \rho(r') \mathbf{v}((\mathbf{r} - \mathbf{r}')$, $U_\text{F}(\mathbf{r}, \mathbf{r}')$) and its fluctuations ($\delta U = \int d^3r' \delta\rho(r')$), to be physically (empirically) fulfilled.

4.2. The $^{120}$Sn$(\rho, \phi)_{119}$Sn$(5/2^+)$ reaction

Within the framework of NFT and of NG (NFT+NG) equations (see appendix E) we want to ask the following question: is it possible to find an orbital which belongs to the valence states but for which pairing effects are weak so as to be able to study the effects of the PVC at the level of Hartree–Fock mean field? In other words, to find an orbital a few MeV away from the Fermi energy, but still belonging to the group of valence orbitals and carrying a sizeable single-particle strength, so as to be able to test the $m_k, m_\omega$ dependence of the results without (much of) the quasiparticle dressing? To be able to give a positive answer to the above question, two conditions have to be fulfilled by the orbital parameters: (a) $\epsilon_\rho$ be sufficiently away from $\epsilon_\rho$, so that $u_{\rho_\rho} < 1$ and $v_\rho^2$ close to 1; (b) be sufficiently close to $\epsilon_\rho$ so that the single-particle doorway damping mechanism, i.e. the coupling to three quasiparticle doorway states containing a collective vibration and responsible for the single-particle damping width $\Gamma \approx 0.5[\epsilon_\rho - \epsilon_\rho][122, 123]$, see also [51] p 74, has not become fully operative. The fulfillment of

\[(m_{\text{q}}/m = (1 + m/(\hbar^2k))dV/dk)^{-1}\]
these two apparently contradictory requirements is trying to achieve, and depends delicately on the unperturbed single-particle energy spectrum.

This is one of the reasons why, arguably, the end point of a NFT+NG study of a ‘complete’ set of experimental data ‘fully’ characterizing the structure of a nucleus, is to carry out one more iteration, in which the only parameters to be varied are the single-particle HF energies of the valence orbitals\textsuperscript{26}. This is also at the basis of why, again arguably, in studying nuclear structure with theoretical tools, one has to deal with nuclear zones, where all bare valence orbital energies are rather homogeneous and their eventual clothing, transferable (e.g. those associated with a group of spherical superfluid nuclei like the Sn-isotopes and, likely, separated from the rest by phase transition regions). Within this context is that in figure 2.30 \textsuperscript{22}, where the single-particle levels throughout the mass table are displayed with continuity as a function of $A$, similar to the way one draws, in a completely different context, a regular crystal as a function of the spatial coordinate (displaying no dislocation), can be, in spite of its attractive simplicity, misleading. In fact, according to the above parlance, a plot like that shown in figure 2.30 \textsuperscript{22}, and more recent ones worked out with the help of density functional theory, should look more like a fractal than like a regular crystal. Or like magnetic domains, separated by domain walls.

Let us now return to the discussion of the $d_{5/2}$ strength function in $^{120}$Sn. The breaking and concentration of the strength of the single-particle levels lying close to the Fermi energy (valence $d_{5/2}$, $g_{7/2}$, $s_{1/2}$, $d_{3/2}$ and $h_{11/2}$ orbitals) depends, to a large extent, on few, selected, on-the-energy shell renormalization processes. It is then not surprising that the $d$ states, due to their ability to couple to $s_{1/2} \otimes 2^+$, $d_{3/2} \otimes 2^+$ and $g_{7/2} \otimes 2^+$ doorway states, may display a particularly large fragmentation ($^{120}$Sn(p, d)$^{119}$Sn and $^{120}$Sn(d, p)$^{121}$Sn data). Thus, within the framework of a microscopic theoretical description, such states are the ones more sensitive to the associated unperturbed HF single-particles energies. This is particularly so for the $d_{5/2}$ valence orbital. Being the valence orbital lying furthest away from $\epsilon_F$, it has a large likelyhood to be surrounded by $3qp$ states containing a collective mode and lying at an energy similar to that of the unperturbed state. In the case of the $d_{5/2}$ orbital, aside from the doorway states mentioned above, one has to add the $h_{11/2} \otimes 3^+$ states. These are the reasons why one expects that the $d_{5/2}$ orbital is more prone than the other valence states to experience accidental degeneracy.

In keeping with the above discussion, to provide an overall account of the experimental findings, it has been found necessary to shift the (SLy4) bare energy of the $d_{5/2}$ orbital $\epsilon_{d_{5/2}}$ by 600 keV towards the Fermi energy. After the full NFT+NG calculation has been repeated, one obtains a pole at low energy carrying most of the $5/2^+$ strength as experimentally observed. The resulting single-particle spectroscopic amplitudes were then used, together with global optical potentials and DWBA, to calculate the absolute cross section of the different fragments of the $d_{5/2}$ valence orbital. The results for the levels predicted at energies below 2 MeV, are shown in figure 8. With the $\epsilon_{d_{5/2}}$ shift, theory provides an overall account of the experimental findings (see third column of table 1). In other words, let us renormalize empirically and on equal footing bare single-particle and collective motion of open-shell nuclei in terms of self-energy and vertex corrections. Let us also renormalize particle-hole and pairing interactions through PVC. The results lead to a detailed, quantitative account of the data. They furthermore constrain the possible values of the $k$-mass, of the $^{1}S_{0}$ bare NN interaction, and of the PVC strength within a rather narrow window. This is the natural scenario of a well funnelled nuclear structure landscape (see figure 6).

Summing up, and as indicated by the relative root mean square deviation between theory and experiment displayed in table 1, implementing NFT in terms of empirically renormalized collective modes, and allowing for a moderate variation in the bare (HF) density of levels, theory becomes accurate, in average, at the 10% level. This can be achieved with just three parameters. The strengths $k_2$ and $k_3$ of the quadrupole and octupole separable multipole–multipole interactions (empirical renormalization) and the small relative shift $\delta \epsilon_{d_{5/2}}/\epsilon_{d_{5/2}} = 0.17$ of the energy of the $d_{5/2}$ valence orbital. (For more details see \textsuperscript{93–95}). It is of notice that $k_2$ and $k_3$ are in fact not free parameters, being constrained by the experimental value of $\hbar \omega_\lambda$ and of $\beta_3 (\lambda = 2, 3)$.

4.3. Technical details: bubble subtraction

In the dressing of the single-particles through the coupling with phonons, one has to remember that, in second order perturbation theory, this procedure implies an independent summation over the intermediate single-particle states for each of the two equivalent fermion lines, i.e. that of the external particle, and that of the particle-hole excitation. The second-order term is then taken into account twice and thus to be subtracted once. In other words, and according to NFT, whenever there are

\textsuperscript{26} Within this context, see last column of table 1, which provide results of an implementation of this protocol in the case of tin isotopes.
fermion lines and a boson line which appear and disappear at the same vertices, one must include another diagram in which the phonon line is replaced by a particle-hole pair, and which is evaluated with an additional minus sign. Within this context,

Figure 9. (a) Self energy associated with the coupling of pair addition modes (pairing vibration (pv) of any multipolarity and parity). The admixture between particles and holes is apparent. (b) Vertex correction associated with a multipole pair addition mode (pairing vibration (pv)) in a self-energy process \((\Sigma_{pv})\) induced by the coupling of the single particle to a particle-hole (ph) collective mode. The presence of a hole state in the \(\Gamma_{ph}\) process instead of a particle state as in the case of \(\Gamma_{ph}\) (see inset) can lead to important effects, e.g. cancellations. This is in keeping with the fact that a hole state has the same absolute value of e.g. the quadrupole moment of a particle state, but opposite sign, a consequence of the fact that closed shell systems have zero quadrupole moment [148]. (c) The pairing renormalization processes of the type shown in (a) have important consequences on the absolute value of the two-nucleon transfer process \((N_{v}-1)\) (t, p)/(N_{v}+1) (e.g. \(N_{v} = 126\)), in keeping with the fact that the coupling of the external (t, p) field with the pair addition mode leads to an effective two-nucleon spectroscopic amplitude which is strongly renormalized [50]. For a recent experimental study in the quest to observe the giant pairing vibration (GPV) see [149]. (d) Effective two-nucleon transfer amplitude. (e) Quadrupole excitation of a single-particle state renormalized by a quadrupole (p-h)-like vibration, processes which lead to (f): effective charge. It is of notice that, as a rule, the \(\omega\)-dependent contributions to the effective (t, p) or E2 values have to be calculated explicitly. Only in the case of high lying modes, like e.g. the GPV and GQR (both isoscalar and isovector), the \(\omega\)-independent effective two-nucleon transfer amplitudes and \(e_{\text{eff}}\) can provide an accurate estimate of the renormalization processes [19].


Figure 10. Self-energy diagrams associated with the \(e_{y/2}\) neutron single-particle state of \(^{209}\text{Pb}\). The wavy curve in (a) and (b) represents the octupole vibration of \(^{208}\text{Pb}\), the most collective of all low-lying modes of \(^{208}\text{Pb}\) \((\hbar\omega_{t} = 2.62 \text{ MeV}, B(E3) = 32 \text{ B}_{\gamma}[19])\). The doubled arrowed curves in (c) represent the quadrupole and hexadecapole pair addition and pair subtraction modes of \(^{208}\text{Pb}\), while in (d) also the monopole one. Single arrowed lines pointing upwards (downwards) stand for single-particle (-hole) states.

5. Open problems

In what follows we take up one example from structure and one from reactions, namely: (1) The quantitative role multipole pairing vibrations [29–31, 39, 85, 132–141] play in clothing elementary modes of excitation, and the systematic and detailed description of the properties of non-conventional modes of vertex-like nature (\(\Gamma\) Cooper pairs); 2) the implementation in terms of NFT diagrams of a protocol which will eventually allow for the calculation of the optical potential [142–146] employing the elements worked out to describe structure.

5.1. Multipole pairing vibrations

At the basis of renormalization process one finds the coupling of single-particles and vibrations. Aside from angular momentum, parity and eventually spin and isospin quantum numbers, these bosonic modes are characterized by the transfer quantum number \(\alpha\) [14]. Particle-hole vibrations have \(\alpha = 0\), while pairing\(^{27}\) vibrations carry \(\alpha = +2\) (pair creation modes) and \(\alpha = -2\) (pair removal modes). As a rule, and with few exceptions [29, 30, 41, 128, 147], renormalization is thought to be associated with the clothing of particles with \(\alpha = 0\) vibrations.

Around closed shell nuclei, monopole, but also multipole, pairing vibrations, are very collective. Even more than

\(^{27}\) The situation is, of course, more subtle in the case of superfluid nuclei, in keeping with the associated spontaneous breaking of gauge symmetry. In the discussion above we restrict ourselves to situations around closed shell nuclei. Concerning rotations, we refer to the contribution of Daniel Bès to this topical issue.
In figure 10 graphs associated with the clothing of the \( g_{9/2} \) orbital of \(^{209}\text{Pb}\) are shown. In figures 10(a) and (b) the lowest order self energy contributions arising from the coupling to the octupole vibration considering only the valence orbitals are given. In figures 10(c) and (d) those associated to the coupling to monopole, quadrupole and hexadecapole pairing vibrations. In [147] it was found that the single-particle gap of the closed shell system \(^{208}\text{Pb}\) decreases, from the bare value \((\mu_b \approx 0.7m)\) by 1.25 MeV due to the coupling to particle-hole modes. Including the pairing vibrational modes this value becomes 1.10 MeV. Their effect seems to be small. This also seems to be in line with the result of [128], which finds that the contribution of the pairing vibrational modes to the imaginary part of the average nuclear self energy is small. Note however, that in reference [147], only the valence orbitals were considered. Consequently, most of the contributions arise from graphs of the type shown in figure 10(d) which lead, as a rule, to a contribution smaller than that associated with e.g. the processes shown in figure 10(c), which are, in the case of the \( g_{9/2} \) valence orbital, the only two LO diagrams allowed. It is of notice the presence of an intermediate hole state in keeping with \( \alpha = \pm 2 \) nature of the vibration. This is at variance with graph 10(a) in which the phonon carries transfer quantum number \( \alpha = 0 \). Returning to [147], within a major shell, the monopole pairing vibration gives no contribution to processes of the type shown in figure 10(c), but only to CO diagram (10(d)).

It is to be noted, however, that the situation may be more subtle than just indicated concerning the relative contribution of surface and pairing vibrations to the self energy of valence single-particle states around \(^{208}\text{Pb}\). This is in keeping with the fact that, as seen from figures D1(d) and (e), the associated ground state correlations interfere with each other.

Now, among the properties of a physical (dressed) elementary mode of excitation, energy is not the most qualifying property. Such a role is played by the response of the nucleus to specific external fields: one- and two-particle transfer reactions for single-particle and pair vibrational modes respectively. Inelastic scattering for surface modes. Within this context one can mention as examples \(^{207}\text{Pb}(t, p)^{209}\text{Pb}, \, ^{210}\text{Pb}(p,d)^{209}\text{Pb}\), as well as \(^{209}\text{Bi}(d, d')^{209}\text{Bi^*}, \, ^{210}\text{Po}(t, \alpha)^{209}\text{Bi} \) and \(^{208}\text{Pb}(\text{He}, d)^{209}\text{Pb}\). In these cases, the coupling to pairing vibrations is important in the two-nucleon transfer process (effective spectroscopic amplitudes) and in connection with single-particle content in the case of one-particle transfer. Also, indirectly through normalization in the case of inelastic scattering (see [19, 29, 30, 42, 132] and references therein).

The clothing of single-particle motion by pairing vibrations in rapidly rotating nuclei has important effects in the dealignment phenomenon, in particular above the critical Mottelson–Valatin frequency \((\omega_{\text{cr}})\). It also affects the difference between the kinematical and dynamical moments of inertia across \(\omega_{\text{cr}}\). Furthermore, it plays a role in the cross talk pattern between rotational bands and in band crossing frequencies. As a consequence, particle-pairing vibrational

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Figure 11. In keeping with standard direct reaction praxis, neither in figure 2 nor in figure 12 antisymmetrization is carried out between the impinging proton and the protons of \(^{11}\text{Li}\). At energies of few MeV per nucleon such processes are expected to contribute in a negligible way to the differential cross section. Within the present discussion \(^{11}\text{Li}(p, p')^{11}\text{Li}\) (see figure 12), an example of such processes corresponds to the exchange of a proton participating in the quadrupole vibration of the core, with the projectile, as shown in the figure. Such a process will not only be two orders higher in perturbation in the particle-vibration coupling vertex. It will be strongly reduced by the square of the overlap between a proton moving in the continuum, and a \(p_{1/2}\) proton of the \(^9\text{Li}\) core bound by about 10 MeV.

low-lying surface quadrupole modes as testified by the fact that the ratio of the static \( \alpha_0 = |\langle \text{BCS}|P^{+}\rangle|\text{BCS} > |(\beta_2)\) to the dynamic \( \alpha_{\text{dyn}} = 2\left(\frac{E_{\text{st}}(A+2) + E_{\text{st}}(A-2)}{E_{\text{st}}(A)}\right)\) = \( \frac{(\beta_2)_{\text{dyn}}}{(\beta_2)_{\text{st}}} \) is \( \alpha_0/\alpha_{\text{dyn}} \approx 0.7 \) as compared to \( (\beta_2)_{\text{st}}/(\beta_2)_{\text{dyn}} \approx 3 - 6 \). Consequently, one expects that processes as those shown in figure 9 (see also figures C3 and C4) will lead to important contributions to the \( \omega \)-dependent (effective) physical mass of the single-particles\(^{28}\) (see appendix C). Also to the (\( \omega \)-independent) two-nucleon transfer amplitudes (figure 9(d)). It is of notice that these quantities are similar to the effective

\(^{28}\) To what extent such couplings could partially alter the conclusions of the study presented in e.g. [150] is an open question worth assessing, let alone that associated with (empirical) renormalization of the \( p-h \) like modes as done in [93–95].
coupling constitutes an essential part of the overall picture developed in nuclei at high rotational frequencies (see \cite{140, 141, 151-157} and references therein; see also e.g. \cite{158} for the coupling to shape vibrations).

5.2. Optical potential: example of the $^{11}\text{Li}(p, p)^{11}\text{Li}$ reaction

The discussion carried out in this section is centered around figures 11 and 12. Nuclear field theory in its graphical implementation, allows for a correct description of nuclear structure...
and reactions. This is achieved through an orthogonalization prescription based on the renormalization of both single-particle and collective motion. This is to be done for each order of perturbation theory, in terms of mass (self-energy) and screening (vertex) sum rule conserving processes. Pauli principle is taken care of by allowing, through particle-vibration vertices linear in particles and vibrations, (quasi) bosonic modes to decay into pairs of fermionic ones and vice versa. Such non-orthogonality corrections carry naturally over to DWBA two-nucleon transfer calculations, including both simultaneous and successive transfer processes. The fact that two-nucleon transfer reactions are calculated in second order DWBA may, within the context of NFT, lead to a misunderstanding. In fact, in structure calculations, the small parameter is the inverse effective degeneracy \( \Omega \) of the space in which nucleons are allowed to correlate. Orthogonalization, Pauli principle, etc., are calculated to different orders in \( 1/\Omega \). In the case of transfer processes, in particular, Cooper pair transfer, two nucleons may be transferred simultaneously or successively. In the first case one neutron is acted upon by the \( v_{\text{np}} \) proton–neutron interaction. The other neutron follows suit because of (pairing) correlation with the first one. In the second case, \( v_{\text{np}} \) acts on one neutron at a time. Once this is done, there is no more to it. Second order in \( v_{\text{np}} \) exhausts all the possibilities. Also because it contains the amplitude describing the process in which a neutron is acted upon by \( v_{\text{np}} \) in the transfer process, while the other does so profiting of the non-orthogonality of the single-particle basis associated with target and projectile. Being this a spurious process, the corresponding amplitude has the right phase to subtract such contribution from the sum of simultaneous and successive transfer.

Summing up, physically, second order DBWA transfer fully describes Cooper pair tunneling. Although pairing correlations are weak as compared with mean field effects, they lead to Cooper pair transfer cross sections proportional to the density of levels \( N(0) \), quantity squared. That is, \( \text{d} \sigma / \text{d} \Omega \sim (\Delta/G)^2 \approx N(0)^2 \), \( \Delta = GN(0) \) being the pairing gap and \( G \) the pairing coupling constant [17]. This result testifies to the fact that Cooper pair is, essentially, successive transfer. This has been duly confirmed in systematic studies [18]. It is of notice that the same picture of Cooper pair tunneling, is at the basis of the quantitative understanding of the Josephson effect (see section D1). Also of the fact that making use of this effect, the most accurate measurements available of the ratio of fundamental constants \( h/e \) were obtained, let alone of the validity of BCS to describe superconductivity and to act as paradigm of theories of broken symmetry [5, 7, 12].

NFT is based on the concept of elementary modes of excitation. These modes, which carry a large fraction of nuclear correlations, are specially suited to be used in reaction theory. Because furthermore, NFT rules apply equally well to both bound and continuum states, it allows, in principle, for a unified description of both structure and reactions. An example of the above statements is provided by the two NFT-diagrams displayed in figure 12. They describe one-particle transfer polarization contributions to the optical potential associated with the elastic reaction \(^{11}\text{Li}(p, p)^{11}\text{Li(gs)}\). The elementary modes of excitation, appearing in these diagrams, drawn for simplicity in the language of ‘traditional’ kinematics, and not inverse kinematics as the experiment has been actually performed [87], are:

1. single-particles (structure, arrowed and solid arrowed lines; reaction, relative CM motion, curved arrows); 2. vibrations (p-h, wavy lines), 3) pairing vibrations (double arrowed lines), 4) recoil mode associated with a change in mass partition (jagged curve).

In each PVC vertex, namely those related to structure (solid, dot, (solid dot, dotted open circle) and to reaction (dashed open square), symmetries are preserved. Momentum is not conserved at structure vertices, in keeping with the fact that a nucleus is a finite system. On the other hand, momentum is conserved at the reaction vertices. This is because the relative motion between projectile and target takes place in an ‘infinite’ homogeneous, isotropic space. This is the reason why the recoil modes (jagged curves) are created by Galilean transformations. These operators smoothly join the entrance (eventually also intermediate) and exit relative motion quantal trajectories. That is, they provide the proper scaling between entrance and exit channel for each partial wave. Recoil modes propagate together with the asymptotic outgoing particle to the corresponding detector, providing a mnemonic for the fact that at transfer reaction vertices, matrix elements of the corresponding form factors which also involve recoil phases are to be used (see appendix F).

Summing up, the above NFT polarization contribution to the mean field optical potential awaits to be carried out, and constitutes one of the important challenges in the unification program of structure and reactions.

6. Conclusions

The results presented above constitute representative examples of the implementation of the NFT program. It can be concluded that this effective field theory provides the rules for designing, and calculating, the diagrams describing the variety of physical phenomena associated with nuclear structure as probed by direct reactions, and for predicting the values of the different observables.

The corresponding results reproduce, within experimental errors, data which completely characterize the spectroscopic properties of open shell nuclei lying along the stability valley as well as of exotic nuclei around novel closed shells (\( N = 6 \)). This is intimately related to the fact that NFT–Feynman diagrams lead to a graphical solution, through the interweaving of single-particle and collective modes of nuclear excitation, of the problems of over completeness (non-orthogonality) of the associated basis.
of states. Also of those problems arising from the identity of the particles appearing explicitly and those participating in the collective modes (Pauli principle). NFT thus provides the theoretical framework to obtain an accurate microscopic solution of the many-body nuclear problem. Because of the validity of these solutions concerning both bound and continuum states, the NFT rules allow for a unified treatment of both structure and reactions. The program concerning reactions is lively under way, the calculation of the polarization contribution to the optical potential constituting one of the important challenges.

Summing up, the implementation of NFT rules allows for a correct and unified description of nuclear structure and reactions. In a very real sense this (NFT diagrams shown in e.g. figures 2 and 12, and eventually those describing anelastic processes) is a nucleus. Namely, the summed information (figures 1–3 and 12 (\(^{11}\)Li); table 1 and figures 6 and 8 (\(^{120}\)Sn)), carried to the detectors by asymptotic states, of the outcome of probing the system with a complete array of processes (elastic, anelastic, one- and two-nucleon transfer processes).

This is unarguably what NFT can do, as testified by the documentation presented here or referred to. What it cannot do, is to solve problems regarding ill behaved bare forces, or the consequences of associated PVC vertices which eventually lead to divergences. Within this context, empirical renormalization has proven to be a powerful and physically consistent prescription to implement the NFT program and make connection with experimental data.

7. Hindsight

As a result of the ground breaking contributions of Bohr and Mottelson which we celebrate in the present volume, our understanding of the nuclear structure is based on independent particle and collective elementary modes of excitations and of their couplings. The NFT program uses these modes and couplings to consistently build a many-body effective field theory (see e.g. figure 7), removing spurious Pauli principle violating terms and non-orthogonality contributions (see e.g. figure 4). It is possible then to utilize the resulting many-body correlated wavefunctions for the description of the nuclear structure observables (see e.g. figure 1) and nuclear reaction cross sections (see e.g. figures 3 and 8). The program, although rather well implemented, is not yet fully operative (in particular concerning its reaction part). Not only for the intrinsic difficulties in developing such a complete description of the many-body nuclear structure and reactions phenomena in itself, but also due to inconsistencies in mean field generators leading to uncontrollable spuriousities which only now are being addressed.

To overcome such and others, external, present-day limitations to the validity of the NFT treatment of the nuclear many-body problem, we implemented an empirical renormalization procedure: tune the PVC vertex to reproduce the experimental properties of low-lying collective states with separable interactions making use of experimental single-particle levels, and treat this procedure as an ansatz which has to consistently recover itself in the RPA calculation of the collective states.

To quote just but two results and one outstanding open problem of the implementation of the NFT program:

(a) The pairing gap of spherical open-shell nuclei is made of essentially equal contributions arising from the bare NN–Λ\(_0\) and from the induced pairing interactions. That is from short and from long range interactions.

(b) Making use of NFT wavefunctions and associated spectroscopic amplitudes, one can calculate two-nucleon transfer absolute differential cross sections which provide an overall account of the observations within experimental errors.

(c) Making use of NFT wavefunctions and associated spectroscopic amplitudes, calculate the polarization contribution to the nucleon–nucleus optical potential in general, and to that describing the elastic scattering of a proton off \(^{11}\)Li in particular.

Appendix A. Neutron halo pair addition mode and GDPR: symbiotic elementary mode of nuclear excitation

Halo states like \(^{11}\)Li(gs)), in which a consistent fraction of the two weakly bound neutrons forms an extended low density misty cloud, imply the presence of a low-lying dipole state. It results from the sloshing back and forth of the neutron cloud involving also those of the core, with respect to the protons of the core. Microscopically, to form a halo, the two neutrons have to move in weakly bound or virtual single-particle states, with no or little centrifugal barrier. That is, s- and p-states strongly renormalized, and as a result, both lying essentially at threshold. Thus the presence of low-energy (s, p)\(_1\) configuration which, coupling to the giant dipole resonance can bring down a fraction of the dipole (TRK) sum rule.

Because of the small overlap existing between halo neutrons and core nucleons both the \(^{1}\)S\(_2\), NN- and the symmetry-potential become strongly screened, resulting in a subcritical value of pairing strength and in a weak repulsion to separate protons from neutrons in the dipole channel. As a result, neither the \(^{2}\)J\(_-\) \(= 0^+\) correlated neutron state (Cooper pair), nor the \(J = 1^-\) one (vortex-like) are bound (although both qualify\(^{31}\) to do so) to the core \(^{9}\)Li.

Having essentially exhausted the bare NN-interaction channels, the two neutrons can correlate their motion by exchanging vibrations of the medium in which they propagate, namely the halo and the core. Concerning the first one, these modes could hardly be the \(\lambda = 2^+, 3^+\) or \(5^+\) surface vibrations found in nuclei lying along the stability valley. This is because the diffusivity of the halo is so large that it blurs the very definition of surface. Those associated with the core (\(2^+\) see figure A1 (c), \(3^+\), \(5^+\) etc) provide some glue, but insufficient to bind any of the two dineutron states in question.

The next alternative is that of bootstrapping. Namely, that in which the two partners of the (monopole) Cooper pair exchange pairs of vortices (dipole Cooper pair), as well as one dipole Cooper pair and a quadrupole pair removal mode, while those of the vortex exchange pairs of Cooper pairs (monopole

\(^{31}\) Within this context note the detailed dependence on quantal size effects of these ‘exotic nuclei’ excitations as compared to those discussed in [160].
pairing vibrations, but also pairs of dipole pairs, as shown in figures A1 and A2. In other words, by liaising with each other, the two dineutrons contenders at the role of $^{11}$Li ground state settle the issue. As a result the Cooper pair becomes weakly bound ($S_{2n} = 389$ keV), the vortex state remaining barely unbound, by about 0.5–1 MeV [71, 72]. There is no physical reason why things could not have gone the other way, at least none that we know. Within this context we refer to $^3$He superfluidity, where condensation involve $S=1$ pairs. It is of notice that we are not considering spin degrees of freedom in the present case, at least not dynamic ones.

For practical purposes, one can describe the $I^-$ as a two quasiparticle state and calculate it within the framework of QRPA adjusting the strength of the dipole–dipole separable interaction to reproduce the experimental findings [67]. In this basis it is referred to as a GDPR. Exchanged between the two partners of the Cooper pair (figure A1(d)) leads to essentially the right value of dineutron binding to the $^9$Li core. Within this context one can view the $^{11}$Li neutron halo as a van der Waals Cooper pair (figure A1(e)). The transformation between this picture and that discussed in connection with (a) and (b) as well as with figure A2 can be obtained expressing the GDPR, QRPA wavefunction, in terms of particle creation and destruction operators (Bogoliubov–Valatin transformation) as seen from figures A1(a) and (b). A vortex–vortex stabilized Cooper pair emerges.

**Figure A1.** NFT-Feynman diagrams describing the interweaving between the neutron halo pair addition monopole and dipole modes (double arrowed lines labeled $0^+$ and $1^-$ respectively). Above, the exchange of dipole modes binding the $0^+$ pair addition mode through forwards going particle–particle $p–p$ ($h–h$) components. Below, the assumption is made that the GDPR of $^{11}$Li can be viewed as a $p–h$ (two quasiparticle), QRPA mode.

**Figure A2.** NFT-Feynman diagrams describing, (a) and (c) some of the particle-particle ($pp$), $hh$ and $ph$ processes binding the Cooper pair neutron halo and stabilizing $^{11}$Li, as well as (b) and (d) giving rise to the GDPR.

**Figure A3.** Schematic representation of levels of $^{11}$Li populated in two-nucleon transfer reactions. Indicated in keV are the two-neutron separation energies $S_{2n}$. In labelling the different states, one has not considered the quantum numbers of the $p_{32}$ odd proton.

**Figure A4.** Levels of $^{12}$Be expected to be populated in two-nucleon transfer and knockout processes. $S_{2n}$ are the two-neutron separation energies.
Which of the two pictures is more adequate to describe the dipole mediated Cooper binding is an open question, as each of them reflects important physical properties which characterize the GDPR. In any case, both indicate the symbiotic character of the halo Cooper pair addition mode and of the pygmy resonance built on top of, and almost degenerate with it. Insight into this question can be obtained by shedding light on the question of whether the velocity field dofe of these symbiotic states is more similar to that associated with irrotational or vortex-like flow\footnote{Within this context, one can mention that a consistent description of the GQR and of the GIQR is obtained assuming that the average eccentricity of neutron orbits is equal to the average eccentricity of the proton orbits\cite{65}, the scenario of neutron skin. The isoscalar quadrupole-quadrupole interaction is attractive. Furthermore, the valence orbitals of nuclei have, as a rule and aside from intruder states, homogeneous parity. These facts preclude the GQR to play the role of the GIQR. In fact, there will always be a low-lying quadrupole vibration closely connected with the aligned coupling scheme and thus with nuclear plasticity. Within this context one can nonetheless posit that the GQR, related to neutron skin, is closely associated with the aligned coupling scheme. Making a parallel, one can posit that the GDPR is closely connected with vortical motion. Arguably, support for this picture is provided by the low-lying E1 strength of $^{11}\text{Li}$. It results from the presence of $s_{1/2}$ and $p_{1/2}$ orbitals almost degenerate and at threshold, leading to a low-lying Cooper pair coupled to angular momentum $1^-$. (Dipole pair addition mode). The scenario of vortical motion.}

could be shed through electron scattering experiments, likely not an easy task when dealing with unstable nuclei. On the other hand, two-nucleon transfer reactions, specific probe of (multipole) pairing vibrational modes, contain many of the answers to the above question (figure A3). In fact, ground state correlations will play a very different role in the absolute value of the $^{9}\text{Li}(t, p)^{11}\text{Li}$ cross section, depending on which picture is correct. In the case in which it can be viewed as a vortex (pair addition dipole mode) it will lead to an increase of the two-particle transfer reaction (positive coherence). It will produce the opposite effect if the correct interpretation of the GDPR is that of a $(p–h)$-like excitation\cite{76}. Insight in the above question may also be obtained by studying the properties of a quantal vortex in a Wigner cell with parameters which approximately reproduce the halo of $^{11}\text{Li}$. Within this context, and for the solely purpose of providing an analogy, we refer to what is done in the study of vortices in the environment of neutron stars\cite{162, 163}.

A test of the soundness of the physics discussed above, concerns the question of whether the first excited, $0^+$ halo state ($E_x = 2.25\, \text{MeV}$) of $^{10}\text{Be}$ can be viewed as the $|gs(4^\text{Be})\rangle$ in a new environment. In other words, to consider the halo neutron pair addition mode a novel mode of elementary excitation: neutron halo pair addition mode of which the $|1^-(4^\text{Be})\rangle$;
2.71 MeV) is a fraction of its symbiotic GDPR partner. One can gain insight concerning this question, by eventually measuring the E1-branching ratio $|1^- (2.71\text{ MeV}) \rightarrow 0^{++} (2.25\text{ MeV})|$, and possibly finding other low-energy E1-transitions populating the $0^+$ state, as well as through two-nucleon stripping process, as well as two-nucleon pickup and knockout reactions (figure A4). A resumé of the picture discussed above is given in figure A5.

**Appendix B. The pairing vibrational spectrum of $^{10}\text{Be}$**

Calculations similar to the ones discussed in previous sections have been carried out in connection with the expected $N = 6$ shell closure in $^{10}\text{Be}$ (see e.g. [167]). In figure B1 we display the associated pairing vibrational spectrum in the harmonic approximation. Also given are the absolute two-nucleon transfer differential cross sections associated with the excitation of the one-phonon pair addition and pair subtraction modes excited in the reactions $^{12}\text{Be}(p, t)^{10}\text{Be(gs)}$ and $^{10}\text{Be}(p, t)^{6}\text{Be(gs)}$ respectively, calculated for a bombarding energy appropriate for planned studies making use of inverse kinematic techniques [168].

The $\langle 2p-2h \rangle$-like two-phonon pairing vibration state of $^{10}\text{Be}$ is expected, in this approximation, to lie at 4.8 MeV, equal to the sum of the energies of the pair removal $W_1(\beta = -2) = 0.5\text{ MeV}$ and of the pair addition $W_1(\beta = 2) = 4.3\text{ MeV}$ modes. In keeping with the fact that the lowest known $0^+$ excited state of $^{10}\text{Be}$ appears at about $6\text{ MeV}$ [169], we have used this excitation energy in the calculation of the $Q$-value associated with the $^{12}\text{Be}(p, t)^{10}\text{Be}$ (pv) cross section. The associated shift in energy from the harmonic value of 4.8 MeV can, arguably, be connected with anharmonicities of the $^{10}\text{Be}$ pairing vibrational spectrum, (see figures B2 and B3) [43]. Medium polarization effects (see e.g. figures B2(b) and B3(b)) may also lead to conspicuous anharmonicities in the pairing vibrational spectrum.

The two-nucleon spectroscopic amplitudes corresponding to the reaction $^{10}\text{Be}(p, t)^{6}\text{Be(gs)}$ and displayed in table B1, were obtained solving the RPA coupled equations (determinant) associated with the $^{10}\text{Be(gs)}$ pair-removal mode, making use of two pairing coupling constants, to properly deal with the difference in matrix elements (overlaps) between core–core, core–halo and halo–halo two-particle configurations (for details see [164]). In other words with a ‘selfconsistent’ treatment of the halo particle states ($\delta_k > \delta_F$), in particular of the $d_{3/2}(0)$ halo state. The absolute differential cross sections displayed in figure B1 were calculated making use of the optical parameters of [166, 170] and of COOPER [89].
The single-particle energies were deduced from experimental binding and excitation energies, and making use of the coupling constants $G_{12} = 2$ MeV and $G_{02} = G_{01} = 0.68$ MeV [164].

The calculated energy and the B(E2) transition strength of the low lying $2^+$ are 2.3 MeV and 49.6 $e^2 fm^4$ respectively. These results are to be compared with the experimental values of 3.3 MeV and 52 $e^2 fm^4$. The quantities $\varepsilon_i$ and $\varepsilon_f$ indicate the energy of the hole and of the particle states respectively for either protons ($p$) or neutrons ($n$). $E$ denotes the associated two-quasiparticle energies, while $X$ and $Y$ are the QRPA amplitudes of the mode.

The two-nucleon spectroscopic amplitudes associated with the reaction $^{12}$Be$(p,t)^{10}$Be(gs) correspond to the numerical coefficients appearing in equation (B3) below, and associated with the wavefunction describing the neutron component of the $^{12}$Be ground state (see [167])

$$|\bar{0}\rangle = |0\rangle + \alpha |p_{1/2},s_{1/2}\rangle + \beta |s_{1/2},d_{5/2}\rangle + \gamma |p_{3/2},d_{5/2}\rangle,$$

$$\alpha = 0.10, \quad \beta = 0.35, \quad \text{and} \quad \gamma = 0.33$$

and

$$|0\rangle = 0.37|s_{1/2}(0)\rangle + 0.50|p_{1/2}(0)\rangle + 0.60|d_{5/2}(0)\rangle.$$  \hspace{1cm} (B3)

The states $|1^-\rangle$, $|2^+\rangle$, $|3^-\rangle$ are the lowest states of $^{10}$Be, calculated with the help of a multipole separable interaction in RPA (see e.g. Table B2). It is of notice that a rather similar absolute differential cross section to the one displayed in figure B1 for the $^{12}$Be $(p,t)^{10}$Be(gs) reaction is obtained making use of the spectroscopic amplitudes provided by the RPA wavefunction describing the $^{10}$Be pair addition mode (see table B1), provided use of two pairing coupling constants is made. This can be seen from the results displayed in figure B4.

Figure B4. Absolute differential cross section associated with the reaction $^{12}$Be$(p,t)^{10}$Be(gs) at $E_{CM} = 7$ MeV, calculated making use of: (a) the wavefunction (B1) and (b) the RPA wavefunction describing the $^{10}$Be pair addition mode (see Table B1). Reprinted with permission from [164]. Copyright Pleiades Publishing, Ltd. 2014.

To assess the correctness of the structure description of $^{10}$Be(gs) provided by the wavefunction (B1–B3) and of second order DWBA-reaction mechanism (successive, simultaneous plus non-orthogonality) employed to calculate
Figure B5. Absolute differential cross section measured [166] in the reaction $^8\text{Be}(t, p)^{10}\text{Be}(gs)$ at 17 MeV triton bombarding energy (solid dots). The theoretical calculations (continuous solid curve) were obtained making use of the spectroscopic amplitudes associated with the wavefunction in equations (B1)-(B3), and the optical parameters of [166] and [170], taking into account successive, simultaneous and non-orthogonality processes [18, 89]. Reproduced with permission from [18] Copyright IOP Publishing 2013.

The absolute value of the $^{12}\text{Be}(p, t)^{10}\text{Be}(gs)$ differential cross section [89], we compare in figure B5 the predictions of the model for the reaction $^{12}\text{Be}(p, t)^{10}\text{Be}(gs)$ at 17 MeV triton bombarding energy with the experimental data. Theory provides an overall account of observation within experimental errors.

It is of notice that the components proportional to $\alpha$, $\beta$ and $\gamma$ of the state (B1) can lead, in a $^{12}\text{Be}(p, t)$ reaction, to the direct excitation of the $1^+$, $2^+$ and $3^-$ states of $^{10}\text{Be}$. Such results will add to the evidence obtained in the reaction $^4\text{He}(^{11}\text{Li}(gs), ^3\text{Li}(1/2^+); 2.69\text{ MeV})^4\text{He}$ [87] of phonon mediated pairing [90]. The role of these components is assessed by the fact that (wrongly) normalizing the component (B3) to 1, one obtains a value of $\sigma = 4.5\text{ mb} (4.4^\circ < \theta_{CM} < 57.4^\circ)$, a factor 2 larger than the experimental value [166] (see figure B5).

Let us now return to figure B1. The ratio of the integrated absolute cross section at $E_{CM} = 7\text{ MeV}$ in the range $10^\circ < \theta_{CM} < 50^\circ$ appropriate for planned experimental studies making use of inverse kinematic techniques [168] is

$$ R = \frac{\sigma(^{12}\text{Be}(p, t)^{10}\text{Be}(pv); 6\text{ MeV})}{\sigma(^{12}\text{Be}(p, t)^{10}\text{Be}(gs))} = 16.0\text{ mb} \approx 2.3, $$

(B4)

a result which testifies to the clear distinction between occupied and empty states taking place at $N = 6$, and thus of the bona fide nature of this magic number for halo, drip line nuclei. The ratio (B4) reflects the fact that the pairing zero point fluctuations (ZPF in gauge space) displayed by the $^{10}\text{Be}(gs)$ as embodied in the pair addition and pair removal modes, and quantified by the absolute values of the associated two-nucleon transfer cross sections, are of the same order of magnitude. This is an intrinsic property of the vibrational modes, in the same way in which e.g. the width (lifetime) of a nuclear state is an intrinsic (nuclear structure) property of such a state. An experiment displaying an energy resolution better than the intrinsic width of the states under study will provide structure information. Otherwise, eventually an upper limit. Within this scenario and in keeping with the fact that the successive transfer induced by the single-particle potential is the intrinsically (structure) dominant contribution to the absolute two-particle transfer cross section, Q-value (kinematic) effects can strongly distort the picture. In particular in the case in which single-particle transfer channels are closed at the studied bombarding energies.

Appendix C. Renormalization and pairing vibrations

Renormalization processes associated with the clothing of nucleons moving in valence orbitals around closed shell nuclei through the coupling to surface, particle-hole like vibrational modes have become fairly customary (see e.g. [81, 150] for two recent examples). The same cannot be said, with rare exceptions (e.g. [147, 171]) regarding clothing through the coupling to pairing vibrations. As mentioned in the text, this state of affairs can hardly be justified in terms of their numerical importance ($\sigma_{ab}/\sigma_{aa} \approx 0.7$ while $<\beta_2>_{ab}/<\beta_2>_{aa} \approx 3-6$), let alone the lack of experimental information (see e.g. [18, 172, 173] and references therein). Nor because pairing vibrations cannot be viewed as bona fide elementary modes of excitation [14]. They join smoothly the particle-hole like vibrations and the single-particle motion, to generate a unified description of the nuclear structure and reactions, based on the concept of $\alpha = 0$, $\pm 1, \pm 2$ elementary modes of excitation, where $\alpha$ is the transfer quantum number (see figures C1–C3).

The processes summarized in graph (e) of figure C1 lead to the real part of the mean field (both direct and exchange, i.e. Saxon–Woods potential strength $V_0 = U_0 + 0.4E$, and thus $m_t \approx 0.6m_t$, cf. equation (2.0.9) [115], cf. also equation (2.6.7)). Processes displayed in figures C2(a) and (b) give rise to the real and imaginary state-dependent contributions to the particle self-energy. That is, to the polarization part of the optical potential in the case in which the motion of the nucleon takes place in the continuum (e.g. projectile). In connection with transfer reactions that populate weakly bound or unbound (resonant/virtual state) the information carried out by the above mentioned polarization potential is particularly important. Similar considerations can be made regarding the self-energy processes implying pairing vibrations (pairing resonances in the case of the continuum). Detailed nuclear structure as probed by transfer to the continuum is finally becoming integrated with more standard nuclear structure. This is a consequence, among other things,
of the studies of halo exotic nuclei, and of the associated physics of low-density, highly extended nuclear systems.

Within this context, it is of notice the detailed treatment of a number of the points mentioned above carried out in \[171\] in connection with a paradigmatic nuclear structure study of transfer to continuum states provided by the reaction \(^9\text{Li}(d, p)^{10}\text{Li}\). In particular, the treatment of monopole pairing correlations in the continuum with the help of the NG equation, used to calculate the radial dependence of the occupation factors and the associated pairing gap (in this connection see \[174\]).

As seen from figure C4, multipole pairing modes can renormalize in an important way also the continuum states. Furthermore, because parity inversion in e.g. \(^{10}\text{Li}\) is related to a Pauli principle (Lamb shift-like) process, it can also influence this phenomenon.

Summing up, not considering single-particle renormalization processes is like ignoring the dielectric constant (function) in trying to describe the motion of electrons and photons in vacuum or in water. Similarly, considering only the effect associated with the coupling to particle-hole modes and neglecting those arising from the coupling to pairing vibrations, is like ignoring protonation of water due to acidic conditions (pH), and its overall consequences for the phenomena under study.

Appendix D. Elementary modes of nuclear excitation

In what follows we comment on specific aspects associated with the ‘parallels’ one can make between collective modes in 3D-space (essentially surface vibrations and quadrupole rotations) and in gauge space (pairing vibrations and pairing rotations).

D.1. Conserving and non-conserving approximation

Let us consider for concreteness a closed shell system and only surface and pairing vibrational modes. To become even more concrete, let us choose \(^{208}\text{Pb}\) as the closed shell system, and the low-lying octupole vibration (\(\alpha = 0, J^z = 3^-, E_v = 2.62\text{ MeV}\)), and monopole and quadrupole pair addition (\(\alpha = +2, J^z = 0^+, |\text{gs}^{(208}\text{Pb})\), \(S_{2n} = 9.1225\text{ MeV}; J^z = 2^+, |0_1^{(208}\text{Pb})\), \(E_v = 795\text{ keV}\)) and pair removal (\(\alpha = -2, J^z = 0^+, |\text{gs}^{(208}\text{Pb})\), \(S_{2n} = 14.8133\text{ MeV}; J^z = 2^+, |2_1^{(208}\text{Pb})\), \(E_v = 803\text{ keV}\)) modes. The labels \(\alpha, J\) and \(\pi\) indicate the transfer (baryon \[14\]), angular momentum
and parity quantum numbers. Making use of above basis states calculated in the RPA to lowest order of perturbation NFT of structure [31] and reactions [181] provides a quantitative account of the experimental findings (see [42] and also [172] and references therein). In spite of this, one can choose to neglect $\alpha$ and $J^\pi$ as labels of the variety of states and concentrate on $\alpha = 2 - 2 = 0$ modes and $J^\pi \otimes J^\pi = 0^+$, 2$\pi$–2$\hbar$ excitations of the closed shell system [175].

We prefer to base our discussion in terms of elementary modes of excitation (structure), which allow to directly connect, through specific probes (reactions), with experiment. Taking care of the couplings of these modes in terms of NFT rules leads to the clothing of both fermionic and bosonic degrees of freedom. The results are the physical elementary modes of excitation. Their properties provide the spectroscopic input to reaction theories, which properly corrected for non-orthogonality effects, lead to the absolute differential cross sections (Coulomb excitation, inelastic scattering, $\gamma$–decay, one-, two- etc., transfer reactions), to be directly compared with the experimental findings. To close the circle and emphasizing the property of absolute cross section values, one should be able to work out these quantities with the help of microscopically determined optical potentials. These potentials should be calculated making use of the same elements, employed in working out the spectroscopic input of the absolute cross sections.

Summing up, the protocol reads: use a basis of elementary modes of excitation and associated specific probes (reaction channels). Treat their interweaving in terms of a unified implementation of NFT of structure and reactions. The resulting physical modes and channels, together with the microscopically determined optical potential, provide a physically consistent theoretical description of nuclear measurements. It connects with experiment through the variety of absolute differential cross sections [34].

Let us now return to the main subject, namely that of the $\alpha = +2$ and $\alpha = -2$ modes in general and of those of [g$\alpha$(Pb)] in particular. Simplifying, one can say that pairing in nuclei was introduced two times. At first, in terms of the odd–even mass difference (see [176] and references therein). Then in terms of the excitation spectrum [13], closely following the BCS theory of superconductivity [3, 4]. The fact that all these works had introduced pairing because of important physical reasons, but not the specific ones, came clearly forward with the work of Josephson [5] and the ensuing arguments with Bardeen [177, 178]. Arguments which were resolved in favour of the first ones [7, 179, 180]; (see also [181]). A fact which underscores the difficulty of the task people were confronted with. That of specifically probing the structure of superconductors. That is, of systems whose internal long-range order parameter was assumed to be a phase, the gauge phase $\phi$. In such a case quantal fluctuations of the order parameter lead, in the absence of unsymmetrical external forces, to rotations in gauge space ($\omega = \phi = \lambda / \hbar$) and thus to a restoration of the original symmetry [35]. In keeping with the above parliance, it is thus not surprising that similar problems were encountered in nuclear physics regarding the description of Cooper pair transfer, and the question of whether successive transfer breaks pairing or not.

The external fields (experiment) necessary to ‘pin down’ the quantal fluctuations in gauge space, can only come from systems which themselves violate gauge symmetry. The Josephson effect predicts that the superconducting tunneling of electron Cooper pairs across a thin barrier (oxide layer) separating two superconductors leads to a DC current $J = J_1 \sin (\phi_1 - \phi_2)$ (AC current $J \sim \sin \left( \frac{2}{\pi} (V + 2\phi) \right)$ if biased). Its importance resides in the fact that it provided, for

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**Figure C3.** (a) At the basis of superconductivity one finds a class of ladder graphs which contribute to the electron–phonon vertex function $\Gamma$ and leads to a generalized Cooper pair instability (see [104], p 166). (b) and (c): to understand how the above graph enters single-particle self-energy, one needs to close one of the electron lines. (d) We redraw (c) to make connection with the nuclear case. In (e) (same as (d)) and (f) the dynamical violation of gauge invariance (mixing of particles and holes) is explicitly expressed in terms of self-energy and of vertex correction diagrams. (g) and (h) As the collectivity of the pairing mode increases, eventually the frequencies $W_a$ and $W_b$ of the pair addition and pair removal modes coincide and become equal to zero ($W_a = W_b = 0$). This situation is encountered for the value of $1/G$ where the corresponding horizontal line encounters the RPA dispersion relation parabola at the minimum (see Figure 5 of reference [15]). At this point the system undergoes a transition into the superfluid phase (critical value of $G$).
the first time, an instrument, a clamp, which can pin down the difference in gauge phase existing between two superconducting systems [7, 186]. In fact, a metallic superconductor has a rather perfect internal gauge phase order, but the zero point motion of the order parameter is large and rapid \( \dot{\phi} = \frac{\lambda}{\hbar} \). Placing two such deformed systems (rotors) in weak coupling with each other through Cooper pair transfer acting as tweezers, allows to pin down the time dependent variation of the relative gauge phase. This is testified by the oscillations of the \( 2e \) current across the layer, which reflects the behavior of the two coupled rotors. From the above narrative, it emerges that two-particle single Cooper pair transfer processes is the specific probe of pairing correlations in atomic nuclei [14]. This is true not only to measure the gauge phase coherence of superfluid nuclei (emergent generalized rigidity\(^{36}\) and associated pairing rotational bands [15, 187, 188]), but also the dynamic one, in connection with the excitation of pairing vibrational bands\(^{37}\). It is to be noted that all what has been said for deformation in gauge space can be repeated verbatim for deformation in 3D-space, for both static (quadrupole rotational bands) or dynamic (like e.g. the 2.61 MeV state of \(^{208}\)Pb). This state does not carry angular momentum and parity \( +0 \) but \( -3 \), getting it out of the \( \{ \text{gs}(^{208}\text{Pb}) \}, J^P = 0^+ \) group of states. But this does not bother anybody. This is because while one is accustomed to work with measuring instruments which themselves are not rotational invariant\(^{38}\) like, e.g., a proton beam which in the laboratory defines a privileged orientation and can thus set a 3D-deformed nucleus into rotation, one does not usually have around devices displaying gauge space coherence. In other

\(^{36}\) That is pushing one pole of the deformed body in gauge space with an external field like \( (p, t) \), the whole body reacts at once (no finite velocity propagation of information).

\(^{37}\) Quoting Aage Bohr: ‘The gauge space is often felt as a rather abstract construction but, in the (two) particle-transfer process, it is experienced in a very real manner’ [189].

\(^{38}\) This ... (angular momentum \( L \)) is also a conserved quantity, reflecting the isotropy of space ...But states of different \( L \) interfere. Otherwise, we would have no sense of orientation. Anything we observe is not invariant under rotations ... represents a wavepacket of components with different \( L \) [20, 189].
words, objects which are wave packets of states with different number of particles, with which one can set a superfluid nucleus, or a nucleus displaying pair addition and subtraction modes, into rotation (vibration) in gauge space. It is of notice that the fingerprint of deformation of finite many body systems (FMBS) are rotation bands (in 3D-gauge-etc space).

**D.2. A-dependence of the pairing contribution to the mass formula and of the pairing gap**

Let us now shortly discuss, making use of a simplified model, the self consistent value of the (bare) nuclear pairing interaction. In particular the $A$–dependence of the pairing contribution to the mass formula, as well as to the pairing gap ([1, 19, 22, 175], and references therein). We assume, for the sake of simplicity

$$V(r_1 r_2) = -4\pi V_0 \delta (\vec{r}_1 - \vec{r}_2),$$  \hspace{0.5cm} (D1)

to be a simple representation of the nuclear pairing interaction. The relation between $V_0$ and $G_b$ (constant matrix element bare pairing force) can be written as

$$G_b \approx V_0, \quad I(j) \approx 1.2 \text{ fm}^3 \frac{V_0}{A} / \text{ferm},$$  \hspace{0.5cm} (D2)

where $I(j)$ is the delta-force radial matrix element corrected for nucleon spillout. From the self consistent relation

$$U(r) = -4\pi V_0 \int d^3r' \rho(r') \delta(\vec{r} - \vec{r}') = -4\pi V_0 \rho(r),$$  \hspace{0.5cm} (D3)

between the single-particle potential and the density one obtains

$$V_0 = -\frac{U_0}{4\pi \rho_0} \approx \frac{294}{4\pi} \text{ MeV fm}^3$$  \hspace{0.5cm} (D4)

and thus

$$G_b \approx \frac{27}{A} \text{ MeV},$$  \hspace{0.5cm} (D5)

which can be viewed as the bare pairing interaction. With the help of the single j-shell model, in which case the BCS occupation amplitudes are

$$\sqrt{\frac{N}{2\Omega}} \quad \text{and} \quad \sqrt{1 - \frac{N}{2\Omega}},$$  \hspace{0.5cm} (D6)

one obtains

$$\Delta_b = G_b \sum_{\nu > 0} U_\nu V_\nu = G_b \Omega \sqrt{\frac{N}{2\Omega} \left(1 - \frac{N}{2\Omega}\right)}$$

$$= \frac{18}{A^{1/3}} \sqrt{2N \left(1 - \frac{N}{2\Omega}\right)} \text{ MeV},$$  \hspace{0.5cm} (D7)

where use was made of $\Omega = (2/3)A^{2/3}$. In the case of $^{126}\text{Sn}$ the neutrons in open shells are $N = 26$, and $N/2\Omega = 0.78(A = 126)$. One then obtains

$$\Delta_b \approx \frac{7}{A^{1/3}} \text{ MeV} \approx \frac{7}{5} \text{ MeV} \approx 1.4 \text{ MeV};$$  \hspace{0.5cm} (D8)

in overall agreement with the experimental findings.

Now, it is well established that taking into account, in medium-mass and heavy nuclei, the coupling between single-particle motion and vibrations, half of the pairing gap arises from the bare pairing interaction and half from the induced one, resulting from the exchange of low-lying collective modes between Cooper pair partners (see [195] and references therein). Treating the induced pairing interaction within the framework of the slab model [190–193], see also [1]

$$\Delta_{\text{slab}} = \frac{9.5}{A^{0.62}} \text{ MeV},$$  \hspace{0.5cm} (D9)

close to a 2/3 $A$-dependence in keeping with the surface character of the modes.

Thus

$$\Delta = \frac{1}{2} \Delta_b + \Delta_{\text{slab}} \approx \frac{1}{2} \left(\frac{7}{A^{1/3}} + \frac{20}{A^{0.62}}\right) \text{ MeV}.\quad \text{(D10)}$$

For $A = 126$ one obtains

$$\Delta \approx \frac{1}{2} (1.4 + 1.0) \text{ MeV} \approx 1.2 \text{ MeV}.\quad \text{(D11)}$$

The situation is of course more involved, in keeping with the fact that the coupling to the variety of low-lying collective modes of the Cooper pair partners is a retarded ($\omega$–dependent) process leading to a state dependent pairing gap arising also from the presence of $\omega$-dependent spectroscopic amplitudes intimately related to $\Omega$, which can hardly be accurately parameterized in terms of the slab model (see App. E and [195]). Within this context a similar effect is expected concerning the contribution of the ZPF to the nuclear mass (binding energy) associated with the different collective modes in general and the multiple pair addition and pair subtraction modes in particular (see figure D1 [85]; see also [1], section 8.4).

**D.3. The two-nucleon transfer formfactor**

Two-nucleon transfer is the specific tool to probe pairing. Absolute two-nucleon transfer cross sections are thus the quantities to relate theory with experiment. It is of notice that such quantities do not depend on $G_a$ as

$$\frac{d\sigma}{d\Omega} \sim |\alpha|^2 = |\sum_{\nu>0} U_\nu V_\nu|^2.$$  \hspace{0.5cm} (D12)

In fact, the order parameter is different from zero also in regions in which $G = 0$, e.g in the barrier of a Josephson junction, a fact that does not prevent pair tunneling. The same of course applies to the case of pairing vibrational nuclei, where $\alpha$ is replaced by $\alpha_{\text{dyn}}$.

The field that causes a pair transfer in actual nuclei has a rather involved and subtle structure. This is due to the fact that the pair transfer process is mainly induced by the mean single-particle fields in a second order process. One may, however, introduce an effective pair field which in first order perturbation theory, causes the successive transfer of a nucleon pair.

This can be obtained from the expression of the successive transfer amplitude (see [60], equation (V.11.44), p 423), for the reaction $\alpha \equiv a(=b + 2) + A \Rightarrow \gamma \equiv f(=b + 1) +$
\( F(-A+1) = \beta \equiv b + B(-A+2) \). That is

\[
(a)_{\text{succ}} = -\sum_{n\ell} B^{a(1)}(\alpha_1; 0) B^{b(1)}(\alpha_1') 0 \left( \frac{j_1' + 1}{2j_1' + 1} \right)^{1/2} \times 2 \sum_{\mu \nu \mu'} \frac{(-1)^{\mu + \mu'}}{2\lambda + 1} D_{\mu \nu}^a(0, \pi / 2, \pi) D_{\mu' \nu'}^b(0, \pi / 2, \pi) \times |C^{(2)}(0; a_1; \ell)|^2 |C^{(b)}(0; a_1'); \ell|^2 \int_0^\infty \frac{dt}{\sqrt{t}} \rho_{a_1 a_1'}(t) c^{\dagger}(E_{a_1} - E_{a_1'} + \gamma(t)) \delta(t + \mu - \nu) + \rho_{a_1 a_1'}(t) c^{\dagger}(E_{\ell} - E_{\ell'} + \gamma(t)) \delta(t + \mu' - \nu').
\]  

(D13)

Let us now make use of a number of approximations (parabolic as well as slow phase changes) to perform summations over the single-particle distribution \( |C^{(2)}|^2 \) and \( |C^{(b)}|^2 \), i.e., over \( \gamma = f + F \). Let us furthermore introduce the expression

\[
\sum_{n\ell} B^{a(1)}(\alpha_1; 0) B^{b(1)}(\alpha_1') 0 \left( \frac{j_1' + 1}{2j_1' + 1} \right)^{1/2} \times 2 \sum_{\mu \nu \mu'} \frac{(-1)^{\mu + \mu'}}{2\lambda + 1} D_{\mu \nu}^a(0, \pi / 2, \pi) D_{\mu' \nu'}^b(0, \pi / 2, \pi) \times |C^{(2)}(0; a_1; \ell)|^2 |C^{(b)}(0; a_1'); \ell|^2 \int_0^\infty \frac{dt}{\sqrt{t}} \rho_{a_1 a_1'}(t) c^{\dagger}(E_{a_1} - E_{a_1'} + \gamma(t)) \delta(t + \mu - \nu) + \rho_{a_1 a_1'}(t) c^{\dagger}(E_{\ell} - E_{\ell'} + \gamma(t)) \delta(t + \mu' - \nu').
\]  

(D14)

which provides the matrix element of the pair density in the target (see [60], p 444). One can then write (see [60], equation (V.13.8) p 444),

\[
(a)_{\text{succ}} = \frac{1}{\hbar k_f} \int \frac{d\tau}{\sqrt{\tau}} F(r_0) e^{-\xi \tau}.
\]  

(D15)

In the above expression the collision time is given by the relation \( \tau = (2k_f)^{-1/2} e^{-\xi \tau} \) is an adiabatic cutoff factor (see [60], equation (V.10.3), p 406) and \( r_0 \) the distance of closest approach. The function (see [60], equation (V.13.8a), p 445)

\[
F(r) = (\langle \beta | \psi^0 \rangle(R_a)|\beta\rangle\langle \beta | \psi^0 \rangle(R_b)|\alpha\rangle \times \left( \frac{R_a R_b}{R_a + R_b} \right)^2 e^{-2\xi (R_a - R_b)} L(\tau),
\]

(D16)

thus acts as an effective form factor for simultaneous pair transfer, \( L(\tau) \) being a function which only depends on the collision time. One may identify \( F(r) \) with the matrix element of the effective pair interaction in the post representation (see [60], equation (V.13.10), p.445 see also [174]),

\[
F(r) = \langle \beta | V | \alpha\rangle,
\]  

with

\[
V(r) = \delta \rho^{AB}(r - R_a) \left( \frac{R_a R_b}{R_a + R_b} \right)^2 \delta \rho^{AB} R_a L(\tau).
\]  

(D18)

Comparing equation (D15) with the expression (III.19) of [60] p 108, one finds that \( F(r) \) is proportional to the square of the ion-ion potential. Having made use, in writing (D.17), of the exponential function to extrapolate the pair density in the target to the surface of the projectile, the effective pair field \( \Delta (V) \sim \int d^3r d^3r' \rho \Delta \psi^{\mu}(\text{2\,}\pi) \), see equation (V.13.3), p 443 [60] is essentially proportional to \( U_{ab} \), where \( U_{ab} \) is the mean single-particle field of the projectile.\(^{39}\) In connection with appendix F it is of notice that in equation (D13) one has

\[ \text{taken into account full recoil effects through the single-particle form factors } \tilde{f}_{\mu}^{0\ell}(r), \text{ in terms of a recoil phase } \sigma_{\beta a} \equiv \frac{\hat{k}_{\beta a}(t) \cdot (\hat{r}_a - \hat{r}_b)}. \]

**Appendix E. The NG equation and particle vibration coupling**

The basic vertex associated with the coupling of particles (or holes) to collective surface vibrations reads [19]

\[
h(ab \ell \nu) = -(-1)^{\ell - \ell} \beta a \langle a | \Delta U \partial_{\ell \nu} | b \rangle
\]

\[
(\beta a, \ell, \nu) | (\beta \ell, \ell, \nu) \rangle \left( \frac{2\ell + 1}{(2\ell + 1)(2\lambda + 1)} \right)^{1/2}.
\]  

(E1)

The single-particle states \( a, b \) have energy \( \epsilon_a, \epsilon_b \) and occupation amplitudes \( \nu_a, \nu_b \) (=1 for holes and =0 for particles). The multipolarity and the energy of the phonon are denoted by \( \lambda \) and \( \hbar \omega \).

This coupling induces the renormalization of the single particle energies and the fragmentation of the associated strength, which can be computed by solving selfconsistently the energy dependent self-energy equations

\[
\tilde{E}_{\alpha a}(n) = E_a + \sum_{\beta m, \lambda, \nu} V^{2\alpha}(b(m) \lambda \nu) + \sum_{\beta m, \lambda, \nu} W^{2\alpha}(b(m) \lambda \nu),
\]

(E2)

where

\[
V^{2\alpha}(b(m) \lambda \nu) = \hbar(ab \ell \nu) (\alpha \beta a \mu \nu) - \nu_b \tilde{\psi}_{a, \mu}(n)
\]

\[
W^{2\alpha}(b(m) \lambda \nu) = \hbar(ab \ell \nu) (\alpha \beta a \mu \nu) + \nu_b \tilde{\psi}_{a, \mu}(n).
\]  

(E3)

The \( n \)-th solution (fragment) of the equation is denoted by \( a(n) \). Its energy (referred to the Fermi energy \( \epsilon_F \)) is given by \( \tilde{E}_{\alpha a} = \langle \beta a | \tilde{E}_{\alpha a}(n) - \epsilon_F \rangle \), and the associated occupation amplitude is denoted by \( \tilde{\nu}_{a, \mu}(n) \).

In the case of a superfluid system the self-energy equation becomes the 2 \( \times \) 2 energy dependent NG eigenvalue problem

\[
\left( \begin{array}{cc} \epsilon_{\alpha a}(n) & \tilde{\Delta}_{\alpha a}(n) \\ \tilde{\Delta}_{\alpha a}(n) & (\epsilon_{\alpha a}(n) - \epsilon_F) \end{array} \right) \left( \begin{array}{c} \tilde{\nu}_{a, \mu}(n) \\ \tilde{\nu}_{\beta a, \mu}(n) \end{array} \right) = \tilde{E}_{\alpha a}(n) \left( \begin{array}{c} \tilde{\nu}_{a, \mu}(n) \\ \tilde{\nu}_{\beta a, \mu}(n) \end{array} \right).
\]  

(E4)

where the renormalized pairing gap is given by [105]

\[
\Delta_{\alpha a}(n) = \frac{Z_{\alpha a}(n)}{2} \sum_{b(m)} \frac{2j_b + 1}{2} V_{\text{eff}}(a(n)b(m)) N_b \frac{\Delta_{\beta b}(n)}{2E_{\beta b}(n)}.
\]  

(E5)

The matrix elements of the effective pairing interaction \( V_{\text{eff}}(a(n)b(m)) = V_{\text{bare}}(ab) + V_{\text{ind}}(a(n)b(m)) \) are the sum of the matrix element of the bare interaction \( V_{\text{bare}}(ab) \) and of

\[ \text{taken into account full recoil effects through the single-particle form factors } \tilde{f}_{\mu}^{0\ell}(r), \text{ in terms of a recoil phase } \sigma_{\beta a} \equiv \frac{\hat{k}_{\beta a}(t) \cdot (\hat{r}_a - \hat{r}_b)}. \]
the induced interaction:

\[ V_{\text{ind}}(a(n)b(m)) = \sum_{J} 2\hbar^2 (ab\lambda\nu) \frac{1}{(2J + 1)} \times \left[ \frac{1}{E_{\text{a}(n)} - E_{\text{b}(m)} + \hbar\omega_{\lambda\nu}} \right] \]  

(E6)

The eigenvalues of equation (E4) are the renormalized quasiparticle energies. They are related to the renormalized pairing gap and to the renormalized single-particle energies \( \tilde{\epsilon}_{\text{a}(n)} \) by a BCS-like equation:

\[ \tilde{E}_{\text{a}(n)} = \sqrt{(\tilde{\epsilon}_{\text{a}(n)} - \epsilon_{F})^2 + \Delta_{\text{a}(n)}^2}, \]  

(E7)

where \( \tilde{\epsilon}_{\text{a}(n)} - \epsilon_{F} = Z_{\text{a}(n)}(\epsilon_{\text{a}} - \epsilon_{F}) + \Sigma_{\text{a}(n)}^{\text{even}} \)  

(E8)

In turn, the spectroscopic factor \( Z_{\text{a}(n)} \) is given by

\[ Z_{\text{a}(n)} = \left( 1 - \frac{\tilde{\epsilon}_{\text{a}(n)}^{\text{odd}}}{\tilde{\epsilon}_{\text{a}(n)}^{\text{even}}} \right)^{-1}, \]  

(E9)

where \( \Sigma_{\text{a}(n)}^{\text{even}} \) and \( \Sigma_{\text{a}(n)}^{\text{odd}} \) are the even and odd parts of the self-energy \( \Sigma_{\text{a}(n)} \).

The eigenstates must satisfy the normalization

\[ u_{\text{a}(n)}^2 + v_{\text{a}(n)}^2 - \frac{\partial \Sigma_{\text{a}(n)}(E_{\text{a}(n)})}{\partial E_{\text{a}(n)}} v_{\text{a}(n)}^2 \]  

\[ + \frac{\partial \Sigma_{\text{a}(n)}(-E_{\text{a}(n)})}{\partial E_{\text{a}(n)}} u_{\text{a}(n)}^2 \]  

\[ - 2 \frac{\partial (\Delta_{\text{a}(n)}/Z_{\text{a}(n)})}{\partial E_{\text{a}(n)}} u_{\text{a}(n)} v_{\text{a}(n)} = 1, \]  

(E10)

where \( x = u(\nu) \) and \( y = v(\alpha) \) for particles (holes).

The single-particle strength

\[ N_{\text{a}(n)} = u_{\text{a}(n)}^2 + v_{\text{a}(n)}^2 \]  

(E11)

is thus smaller than 1 for each fragment, the strongest being the so-called quasi particle peak.

Finally the gap may be related to the occupation factors as

\[ \Delta_{\text{a}(n)} = -Z_{\text{a}(n)} \sum_{b(m)} \frac{2J + 1}{2} V_{\text{eff}}(a(n)b(m))u_{b(m)}v_{b(m)}. \]  

(E12)

where use has been made of the relation

\[ u_{b(m)}v_{b(m)} = N_{b(m)} \frac{\Delta_{b(m)}}{2E_{b(m)}}. \]  

(E13)

As a simple application of the above formalism we show the consequences that the PVC has on the pairing correlations of particles moving in a single j-shell interacting through a bare nucleon–nucleon pairing potential with constant matrix elements \( G \). For this simple model, the value of the occupation numbers \( U \) and \( V \) must be the same for all the \( 2j + 1 \) orbitals. In particular, the occupation probability for the case when the system is occupied with \( N \) particles,

\[ V = \sqrt{N/2\Omega}, \]  

(E14)

\[ U = \sqrt{1 - N/2\Omega}, \]  

(E15)

where \( \Omega = (2j + 1)/2. \) Consequently, in this case the pairing gap is given by the following relation

\[ \Delta = Z\Omega(G + v_{\text{ind}})UV = Z(G + v_{\text{ind}})\Omega/2. \]  

(E16)

The values of \( G \) and \( v_{\text{ind}} \) are about equal and close to 18/ A MeV and 19/A MeV respectively (see [195]).

For the isotopes of Sn \( {^{108}_{50}}\text{Sn}, {^{110}_{50}}\text{Sn}, {^{130}_{50}}\text{Sn} \) \( 2\Omega = 32. \) Thus, for \( {^{120}_{50}}\text{Sn} \) \( V = \sqrt{20/32} = 0.8 \) and \( U = \sqrt{1 - 20/32} = 0.61, \) leading to \( UV \approx 0.5. \) Making furthermore use of \( Z \approx 0.7 \) one obtains \( \Delta = 0.7 \times 16 \times (37/120) \times 0.5 \) MeV \( \approx 1.7 \) MeV. Taking into account that spin modes, not considered in the estimate of \( v_{\text{ind}} \) will reduce the above value by \( \approx 20\% \), \[ 95], \( \approx 0.34 \) MeV, the model prediction becomes \( \Delta \approx 1.4 \) MeV, which essentially coincides with the experimental value.

**Appendix F. NFT and reactions**

NFT was systematically developed to describe nuclear structure processes. This fact did not prevent the translation of this graphical language into expressions which embodied the transition amplitude of a variety of reaction processes. In particular, second order (in \( \nu_{\text{np}} \)) transition amplitudes associated with two nucleon transfer reactions [196].

The new feature to be considered regarding transfer processes and not encountered neither in structure, nor in inelastic or anelastic processes, is the graphical representation of recoil effects. That is, a physical phenomenon associated with two nucleon transfer reactions. In fact, nuclear structure processes, being intrinsic processes, do not affect the C.M., with a proviso. In fact, the shell model potential violates the translational invariance of the total nuclear Hamiltonian and, thus, single-particle excitations can be produced by a field proportional to the total C.M. coordinate. The translational invariance can be restored by including the effects of the collective field generated by a small displacement \( \alpha \) of the nucleus. Such a displacement, in the \( x \)-direction, gives rise to a coupling (see [19])

\[ H_{\text{coupl}} = \kappa A F, \]  

(F1)

where

\[ F = -\frac{\partial U}{\kappa \partial x}, \]  

(F2)
and

$$\kappa = \int \frac{\partial U}{\partial x} \frac{\partial \phi_0}{\partial x} d\tau = -A \frac{\partial^2 U}{\partial x^2}, \quad (F3)$$

corresponding to a normalization of $\alpha$ such that $\langle F \rangle = \alpha$.

The spectrum of normal modes generated by the field coupling (F1), namely by a Galilean transformation of amplitude $\alpha (\exp(-i k x), \alpha^2 \ll \alpha)$, contains an excitation mode with zero energy for which ZPF diverge in just the right way to restore translational invariance to leading order in $\alpha$. In fact, while

$$\lim_{\omega \to 0} \frac{\hbar^2}{2D_0 \hbar \omega_0}, \quad (F4)$$
goes to infinity, the inertia remains finite and equal to $D_0 = AM$, as expected. The additional dipole roots include, in particular, the isoscalar dipole compression modes associated with the operator $\hat{D} = \sum_{i=1}^{A} \hat{r}_i \hat{Y}_0(\hat{r}_i)$, which can be viewed as a non-isotropic compression mode (see e.g. [197] and references therein).

Naturally, the operators leading to transformations associated with the change in coordinates of relative motion (recoil effects) are Galilean operators $\exp(-i k_3 t, \alpha^2 \ll \alpha)$; see [60], section V.4 pp 308 and 309, in particular equation (8)). Their action (on e.g. the entrance channel), as that of (F1) on the shell model ground state, can be graphically represented in terms of NFT diagrams (or eventual extensions of them). In figures 2 and 12 they are drawn in terms of jagged lines. Let us elaborate on this point. When one states that the small displacement $\alpha$ of the nucleus leads to a coupling (F1) one means a coupling between the single-particle and the collective displacement of the system as a whole. When one talks about the spectrum of normal modes associated with such a coupling, one refers to the harmonic approximation (RPA). Thus, to the solutions of the dispersion relation (see [19], equation (6-244)),

$$\frac{2\kappa}{\hbar} \sum_i \frac{|F|^2 \omega_i}{\omega_i^2 - \omega_0^2} = 1, \quad (F5)$$

where the sum is over single-particle states. This dispersion relation can be represented graphically through the diagrams shown in figure F1 (see [19], figure 6.14). In particular, $\alpha$ acting on the vacuum creates the collective mode. This can also be seen by expressing $\alpha$ in second quantization, namely

$$\alpha = \sqrt{\hbar \omega_0 / 2C_0} (\Gamma^\dagger + \Gamma), \quad (F6)$$

where $\sqrt{\hbar \omega_0 / 2C_0} = \sqrt{\hbar^2 / 2D_0 \hbar \omega_0}$ is the zero-point amplitude of the collective (displacement) mode. Now, none of the above arguments lose their meaning in the case in which there is a root with $\omega_0 = 0$, also in keeping with the fact the inertia remains finite.

In figures 2 and 12 we do something similar to what is done in figure F1. The dot, which in this figure represents the PVC, is replaced by a small dashed open square, which we label ‘particle-recoil coupling vertex’. It constitutes a graphical mnemonic to count the degrees of freedom that are at play. In this case the coordinates of relative motion. Also the fact that in connection with the appearance of such vertices one has to calculate matrix elements of precise form factors which involve the recoil phases. However we do not have a simple or, better, universally agreed graphical representation of the particle-rotor coupling as we have for the PVC (see e.g. graph (c) of figure 7). This is also evident from the difficulties in trying to graphically represent such couplings starting from the vibrational (‘spherical’ or dynamically deformed) to the rotational (‘deformed’ or statically deformed) schemes (see figures C2 and C3). As a result, the representation of these couplings in both 3D- and gauge-space (see figure C2(e) and C3(h)) are, unimaginatively, equal to the mean field diagram (e) of figure C1. The only feature that changes is the label HF, N, BCS. In any case, and returning to the main issue, namely the calculation of the PVC in the case in which $\omega_0 \to 0$, an empirical way out is that of a coarse-grained-like symmetry restoration. In this case $\kappa$ is adjusted in such a way, that the lowest solution of equation (F5), although being smaller than the rest of them, remains finite (within this context we refer to [19], p 446).

Concerning the question of how to measure the recoil phases let us think for a moment of elastic scattering. In this case the process is completely determined by the elastic phase shifts $\delta_i$. These phase shifts are the difference in phase between the asymptotic form of the actual radial wave function describing the scattering process, and the radial wave function $f(kr)$ in the absence of potential. In other words, the quantities $\delta_i$ provide the change in scaling between incoming and outgoing (potential) waves, resulting in the interference between them, so that particle intensity is smaller behind the scattering region ($\theta = 0$), than in front of it. Furthermore, nuclear structure enters only through the reduced mass (aside of course $U$). Thus, measuring $\sigma(\theta)$ one can determine the values of $\delta_i$ and eventually the structure of the potential $U$. In fact, with the exception of the $l = 0$ phase shift, obtained from low energy experiments, the $\delta_i$ cannot be measured directly. They must be inferred as

40 Something which is certainly not found in [19] (pp 444–447), either in connection with the pushing model or with rotational model.

41 With no coupling the ZPF $\alpha_{i0}^{(0)}$ of the nuclear CM are small (~$A^{-3/2}$). Thus, it is possible to tune $\kappa$ so as to make the ZPF associated with the lowest root large as compared to $\alpha_{i0}^{(0)}$, but still compatible with the ansatz at the basis of RPA (BR Mottelson, private communication to PF Bortignon (1980)).
empirical quantities from the parameterization of the potential.

In the case of nucleon transfer in general, and of two-nucleon transfer in particular, the situation is similar, albeit more subtle. This is because in this case the nuclear structure input, aside from the potential \((v_{np})\) interaction, encompasses also the pair correlated wavefunctions, aside from \(Q\)-value effects. Nonetheless, a detailed measurement of the absolute differential cross sections, arguably allows for a determination of the recoil phases.

Within this context, one can posit that numerical tests of the implementation of NFT of reactions (making use of bona fide NFT structure inputs) have been carried out to the relevant order in \(v_{np}\), namely second order (see end of section 3.1).

Appendix G. Dipole operator and C.M.

The dipole operator can be written in terms of the position vectors referred to the C.M. \(\vec{r}' = \vec{r} - \vec{R} \); \(i = 1, \ldots A\) and the bare charges \((q_i, i = 1, \ldots A)\) as

\[
\vec{D} = \sum_{i=1}^{A} q_i \vec{r}_i = \sum_{i=1}^{A} q_i (\vec{r}_i - \vec{R}),
\]

that is,

\[
\vec{D} = \left( \sum_{i=1}^{A} q_i \vec{r}_i \right) - \vec{R} = \left( \sum_{i=1}^{A} q_i \vec{r}_i \right) - Ze\vec{R}.
\]

For equally massive particles we have

\[
\vec{R} = \frac{\sum_{i=1}^{A} \vec{r}_i}{A}.
\]

Substituting this expression in equation (G1) one thus obtains,

\[
\vec{D} = \sum_{i=1}^{A} q_i \vec{r}_i - Ze \frac{\sum_{i=1}^{A} \vec{r}_i}{A}.
\]

This relation, which can be expressed as

\[
\vec{D} = \sum_{i=1}^{A} q_i \left( 1 - \frac{Z}{A} \right) \vec{r}_i + \sum_{n=1}^{N} \left( \frac{0 - Z}{A} \right) \vec{r}_n,
\]

clearly suggests

\[
q_p^{\text{eff}} = e \left( 1 - \frac{Z}{A} \right) \quad \text{and} \quad q_n^{\text{eff}} = e \left( 0 - \frac{Z}{A} \right) = -e \frac{Z}{A}
\]

as the proton and neutron dipole effective charges, leading to

\[
\vec{D} = \sum_{i=1}^{A} q_i^{\text{eff}} \vec{r}_i.
\]

One has thus two equivalent forms of the dipole operator, namely

\[
\vec{D} = \left( \sum_{i=1}^{A} q_i^{\text{eff}} \right) - Ze\vec{R} = \sum_{i=1}^{A} q_i^{\text{eff}} \vec{r}_i.
\]

Acting on the ground state, the operator \(\vec{D}\) excites \(|\Gamma^-\rangle \equiv |1, m\rangle\) states, \(m\) being a discrete or continuous index which labels the full dipole response. The corresponding transition matrix elements can be written, according to equation (G8), in two equivalent ways, that is

\[
\langle 1, A | \vec{D} | \text{GS} \rangle = \langle 1^{-} | \sum_{i=1}^{A} q_i^{\text{eff}} \vec{r}_i | \text{GS} \rangle - Ze \langle 1^{-} | \vec{R} | \text{GS} \rangle
\]

\[
\langle 1, A | \vec{D} | \text{GS} \rangle = \langle 1^{-} | \sum_{i=1}^{A} q_i^{\text{eff}} \vec{r}_i | \text{GS} \rangle.
\]

If the state \(|\Gamma^-\rangle\) is decoupled from the C.M. motion, the matrix element

\[
\langle 1, A | \vec{R} | \text{GS} \rangle = 0.
\]

Consequently,

\[
\langle 1^{-} | \sum_{i=1}^{A} q_i^{\text{eff}} \vec{r}_i | \text{GS} \rangle.
\]

The above expression using either dipole effective or bare charges, must give rise to the same dipole strengths, as far as the C.M motion is not present in the excited states transition strength. Conversely, equation (G11) is a necessary (check) condition for the C.M to be decoupled from the intrinsic excitations.

Appendix H. NFT Rules

In this appendix we list, for the convenience of the reader, the NFT rules referred to throughout the paper as they appear in the first column of p 255 of [32] (Reprinted with permission).
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(I) In initial and final states, proper diagrams involve collective modes and particle modes, but not any particle configuration that can be replaced by a combination of collective modes. This restriction permits an initial state comprising the configuration \((m, 1; \tau_f = 1)\) but excludes \((m, 1; m, -1)\).

(II) The couplings are allowed to act in all orders to generate the different diagrams of perturbation lines of these diagrams.

(III) The internal lines of diagrams are, however, restricted by the exclusion of diagrams in which a particle-hole pair is created and subsequently annihilated without having participated in subsequent interactions. As an illustration of this rule, fig. 2(a) shows an excluded diagram, while fig. 2(b) is permitted.

(IV) The energies of the uncoupled particle and phonon fields are given by (5) and (6) and the contributions of all allowed diagrams are evaluated by the usual rules of perturbation theory.

Fig. 1. Diagrams representing some of the possible interactions in nuclear systems which are described in terms of fermion (arrowed lines) and phonon (wavy lines) fields. The particle-vibration coupling and the exchange of one collective phonon are displayed in (a) and (c), while (b) represents a particle-vibration scattering through the two-body bare interaction of the model.

Fig. 2. Illustration of the general diagram (a) is eliminated by rule III but (b) is allowed.

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