The Current Flows in Pulsar Magnetospheres

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Abstract The global structure of the current flows in pulsar magnetospheres is investigated, with rough calculations of the elements in the magnetospheric circuit. It is emphasized that the potential of critical field lines is the same as that of interstellar medium, and that the pulsars whose rotation axes and magnetic dipole axes are parallel should be positively charged, in order to close the pulsar’s current flows. The statistical relation between the radio luminosity and pulsar’s electric charge (or the spindown power) may hint that the millisecond pulsars could be low-mass bare strange stars.

Key words: pulsars: general — stars: neutron — dense matter

1 INTRODUCTION

Since Hewish et al. (1968) discovered the first radio pulsar, more and more magnetospheric models for pulsars have been proposed to explain their observed phenomena. The vacuum inner gap model, on the one hand, suggested first by Ruderman & Sutherland (1975, hereafter RS75) depends on enough binding energy of charged particles on the pulsar surface. The space charge-limited flow model (e.g., Arons & Scharlemann 1979; Harding & Muslimov 1998), on the other hand, is without any binding energy. An outer gap near light cylinder has also been proposed (e.g., Cheng, Ho, & Ruderman 1986), the existence of which may also reflect strong binding of particles on pulsar surface (Xu 2003a). However, as the observational data in radio, optical, X-ray, and γ-ray bands accumulates, there are still a great number of puzzles to be solved (e.g., Melrose 2004). Nevertheless, it is obvious that the models depend on the nature of pulsar surfaces, and one may conclude on the interior structure of pulsars (e.g., either normal neutron stars or bare strange stars, see, e.g., Xu 2003b), which is almost impossible via calculations in supranuclear physics, via investigating pulsar emission models.

The study of pulsar magnetosphere is essential for us to understand various radiative processes, and thus observed emission in different bands. Goldreich and Julian (1969) argued that a pulsar must have a magnetosphere with charge-separated plasma and demonstrated that a steady current would appear if charges can flow freely along the magnetic field lines from the
pulsar surface. Sturrock (1971) pointed that such a steady flow is impossible due to pair formation, and suggested that a simple electric circuit of the pulsar magnetosphere would be a discharge tube connected with an electromotive source. RS75 proposed that sparking process takes place in a charge-depletion gap just above the pulsar surface. The sparking points drift due to $E \times B$, which can naturally explain drifting subpulse phenomena observed in radio band. Based on the assumption that the magnetosphere has a global current loop which starts from the star, runs through the outer gap, the wind and the inner gap, and returns to the star, Shibata (1991) proposed a circuit including an electromotive source connected in series with two accelerators (the inner and outer gaps) and the wind. Providing a fully general relativistic description, Kim et al. (2005) studied the pulsar magnetosphere and found that the direction of poloidal current in neutron star magnetosphere is the same as that in black hole.

In this work, assuming that the critical magnetic-field lines are at the same electric potential\(^1\) as the interstellar medium (ISM, Goldreich & Julian 1969), we propose a circuit model for pulsar magnetosphere. The corresponding relation of the elements between magnetosphere and circuit is as follows. (1) The total inner radiation of the pulsar corresponds to an electromotive power parallel connecting with a capacitor. Note that the resistance of power is negligible due to perfect conductivity of the star. (2) The inner gaps which include the inner core gap (ICG) and the inner annular gap (IAG) (Qiao et al. 2004a, 2004b) correspond to a parallel connection of a resistor and a capacitor. When a spark takes place in gap, it can be represented by a resistor; if there is no spark, the voltage on the gap is so high that it can be described by a capacitor. (3) The outer gap and the pair-plasma wind correspond to a series connection about an inductor and a parallel connection of a resistor and a capacitor. Considering a parallel pulsar whose rotation axis is parallel to the magnetic axis, we find that the pulsar should be positively charged on the surface for the necessarian of a close circuit. The total electric field along field lines, $E_\parallel$, in the magnetosphere is then composed of two components: that due to charge-departure from the Goldreich-Julian density, and that due to the charges of pulsar.

This paper is arranged as follows. The model is introduced in §2. The total charges of pulsar are estimated in §3. In §4, we discuss about the charges of low-mass strange stars and show evidence for low-mass millisecond pulsars by observational data. Conclusions are presented in §5.

2 THE MODEL

As shown in Fig.1, the foot points of line $a$, $b$, and rotation axis on the star surface are assigned as $A$, $B$ and $P$. Point $P$ is also the magnetic pole of star. We assume that the potential of the critical field line $b$ equals to that of ISM at infinity, $\phi_B = 0$; otherwise, a close electric current in the two regions (i.e., Region I: with boundary lines labelled “a”, and Region II: that between lines “a” and “b”) of open field lines is impossible\(^2\). Then for a parallel pulsar, the potential $\phi_I < 0$ (within Region I) and $\phi_{II} > 0$ (within Region II) in the regime of $\phi_B = 0$. Therefore, the negatively charged particles should flow out along the open magnetic field lines within Region I from star, but positively charged particles flow out through Region II.

The inner gaps (including ICG and IAG in this paper) and an outer gap may work in a magnetosphere\(^3\). There is no current in the circuit until a spark takes place in the inner gaps, so

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\(^1\) We choose the potential of the interstellar medium to be zero, $\phi_{ISM} = 0$, in this paper.

\(^2\) For parallel pulsars, electric current flows outward in the two regions if one sets the potential of the polar line to equal to that of ISM, $\phi_P = 0$, since all the potentials of open-field lines, except the polar line, are greater than zero. Also one can see that the current flows inward in those two regions if one chooses the potential of the last open-field lines to be zero, $\phi_A = 0$. Current flows can not be closed in both these cases.

\(^3\) The inner annular gap and the outer gap might not exist simultaneously.
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**Fig. 1** Sketch of the magnetic field line, electric field line, and magnetospheric charge distribution for a parallel pulsar. Line $a$ is the last open magnetic field line. Line $b$ is the critical field line. Region I: with a boundary of lines labelled “b”, and Region II: between lines “a” and “b”. If the potential of line $b$ equals to that of interstellar medium, then the potential within Region II is positive but negative within Region I.

these gaps could be simulated by parallel connected a resistor and a capacitor, which connect with other parts in series (as shown in Fig.2).

In Fig.2, the power of star could be equivalently modelled by voltage $V$ and capacitance $C_{\text{star}}$. The star is magnetized and possesses an interior electric field, $E$, which satisfies

$$E + \frac{\Omega \times r}{c} \times B = 0,$$

where $\Omega$ is the angular velocity of star rotating around the dipole rotation axis, which relates with rotating period of star $P$ by $P = 2\pi/\Omega$. The magnetosphere of a rotating isolated-pulsar is thus generally concluded to be powered by an electric source with certain potential drop between the polar angle $\theta_B$ and $\theta$ of

$$\phi = \frac{R^2 \Omega B}{2c}(\sin^2 \theta - \sin^2 \theta_B) \approx 3 \times 10^{16} R_6^2 B_{12} P^{-1}(\sin^2 \theta - \sin^2 \theta_B) \text{ volts},$$

if the pulsar is assumed to be magnetized homogenously, where $R_6 = R/(10^6 \text{ cm}), B_{12} = B/(10^{12} \text{ G})$. According to the equation of dipolar field line, $r = r_4 \sin^2 \theta$ ($r_4$ is the maximum polar radius), and the polar angle of null surface $\theta_n = \cos^{-1}(\pm 1/\sqrt{3})$, one can obtain $\sin^2 \theta_B = (2/3)^{3/2} R/R_L$, where the radius of light cylinder $R_L \equiv c/\Omega = cP/(2\pi)$.

The capacitance of the star with a radius $R$ is, in cgs units,

$$C_{\text{star}} = 9 \times 10^{-21} R \text{ farads}.$$
**Fig. 2** An equivalent circuit of pulsar magnetosphere. An electromotive source connected with other parts in series corresponding to the ICG, IAG, the outer gap and the pair-plasma wind. The current is determined by the power voltage \( V \). “\( R \)” stands for plasma resistance effects if a spark happens, mainly the losses of relativistic particles. “\( C \)” represents the gap capacitance effects if there is a potential drop on the gap. “\( L \)” describes the electromagnetic characteristic of flow.

The electric flow process in a magnetosphere can be equivalently mimicked by an antenna cable with capacitance and inductance. They could be equivalently described by the inductance \( L_{\text{flow}} \) and \( C_{\text{flow}} \), the values of which can be estimated as

\[
C_{\text{flow}} \simeq 2\pi \varepsilon_0 l (\ln \frac{r_2}{r_1})^{-1} \tag{4}
\]

and

\[
L_{\text{flow}} \simeq \frac{\mu_0 l}{2\pi} \ln \frac{r_2}{r_1} \tag{5}
\]

respectively; where \( l, r_1, \) and \( r_2 \) are the length, inner radius, and outer radius of the cable, respectively. The passage length of the electric current \( l \sim \sqrt{3/2}r_L = 5.8 \times 10^7 P \) m. Near the light cylinder, the ratio \( r_2/r_1 \sim (r_L + r_L/\sqrt{2})/r_L = 1.7; \) but on the stellar surface, the ratio \( r_2/r_1 = (3/2)^{3/4} = 1.4. \) Both them are not dependent on \( P, B, \) and other parameters, and we choose thus simply \( r_2/r_1 = 1.5 \) in this paper. One therefore comes to

\[
C_{\text{flow}} \sim 8.0 \times 10^{-3} P \text{ farads,} \tag{6}
\]

and

\[
L_{\text{flow}} \sim 4.7P \text{ henries.} \tag{7}
\]

In case enough binding energy of charged particles on the stellar surface, an RS-type (RS75) vacuum gap should exist near polar cap, which can be the equivalent of a capacitor of two parallel slabs, with

\[
C_{\text{RS}} = \pi \varepsilon_0 r_p^2 h = 5.8 \times 10^{-8} P^3 h^{-1} \text{ farads,} \tag{8}
\]

where \( r_p = R \sin \theta_p = 1.45 \times 10^4 P^{3/2} P^{1/2} \) cm is the radius of polar cap, the gap height \( h \) is a model dependent parameter, \( h_3 = h/(10^3 \text{ cm}) \).
For the curvature-radiation-induced and the resonant inverse-Compton-scattering-induced cascade models, the gap heights $h$ could be (e.g., Zhang, Harding, & Muslimov 2000)

$$h_{cr} = 54\rho_6^{2/7} B_{12}^{-4/7} P^{3/7} \text{ m},$$  

and

$$h_{ics} = 279\rho_6^{4/7} B_{12}^{-11/7} P^{1/7} \text{ m},$$  

respectively, where $\rho$ is the radius of curvature of field line, $\rho_6 = \rho/(10^6 \text{ cm}).$

If the field lines which cross the light cylinder can only not co-rotate, the active region of the star is then the polar cap, from polar angle 0 to $\theta_p \simeq \sqrt{2 \pi R/(c \dot{P})} = 1.45 \times 10^{-2} B_6^{-1/2} P^{-1} \text{ m}$. In this case, the potential difference of the electrical source is therefore

$$V_{\text{cap}} = -6.58 \times 10^{12} B_6^3 B_{12}^{-2} \text{ volts}.$$  

Since the dominate source of rotation energy dissipation is through $R_{\text{wind}}$, we can estimate $R_{\text{wind}}$ as

$$R_{\text{wind}} \simeq \frac{V_{\text{cap}}^2}{\dot{E}_{\text{rot}}} = 11 M_1^{-1} R_6^4 B_{12}^2 \dot{P}^{-1} P^{-1} \text{ ohms},$$  

where the rotation loss rate $\dot{E}_{\text{rot}} = -4 \pi^2 I \dot{P}/P^3$, $I \sim 10^{45} M_1 M_\odot$ g cm$^2$ is the moment of inertia for a neutron star with mass $M_1$ M$\odot$ and radius $R_6 \sim 10$ km, the period derivative $\dot{P}/P = |\dot{P}|/10^{-15}$. Compared with $R_{\text{wind}}$, the stellar resistance $R_{\text{star}}$ is negligible due to the perfect conductivity of the star.

The potential drop of outer gap, where no spark happens, corresponds to a resistor, which is presumed to be combined with the wind dissipation as a total one $R_{\text{wind}}$ in Fig.2. In the magnetodipole radiation model, the filed $B$ and the spindown $\dot{P}$ is connected by (e.g., Manchester & Taylor 1977) $B_{12} = 3.2 \times 10^7 \sqrt{P \dot{P}}$, we then have

$$R_{\text{wind}} = 11 M_1^{-1} R_6^4 \text{ ohms}.$$  

For the potential drops of inner gaps (ICG and IAG), when the spark provides necessary charges to close the pulsar circuit, they should correspond to resistors ($R_{\text{ICG}}$ and $R_{\text{ICG}}$, shown in Fig.2). When there is no sparks, the gap grows and they can be described by capacitors ($C_{\text{ICG}}$ and $C_{\text{ICG}}$, shown in Fig.2).

Let’s analyze the circuit in Fig.2. Although the electric power has fixed potential supply, the current is changing due to the inner gap sparks. In this sense, the equivalent circuit description in this paper is not simply a DC circuit model. Because of the erratic sparking, the resistance $R_{\text{RS}}$ could be as the sum of many sinusoidal functions of time, $R_{\text{RS}} = \sum_{n=0}^{\infty} R_n \sin n \omega t$. The electric current between arbitrary points M and N in circuit could also be in this form, $I_{MN} = \sum_{n=0}^{\infty} I_n \sin n \omega t$.

According to the Kirchhoff’s current and voltage laws, the complex impedances of parallel connection circuits composed of $R_{\text{ICG}}$ and $C_{\text{ICG}}$, $R_{\text{ICG}}$ and $C_{\text{ICG}}$, $R_{\text{wind}}$ and $C_{\text{flow}}$ are $z_{\text{ICG}} = R_{\text{ICG}}/(1 + i \omega R_{\text{ICG}} C_{\text{ICG}})$, $z_{\text{ICG}} = R_{\text{ICG}}/(1 + i \omega R_{\text{ICG}} C_{\text{ICG}})$, $z_{\text{wind}} = R_{\text{wind}}/(1 + i \omega R_{\text{wind}} C_{\text{flow}}) + i \omega L_{\text{flow}}$, respectively, where $i = \sqrt{-1}$, $\omega$ the angular frequency of electric current modulation. Defining $z' \equiv z_{\text{ICG}} + z_{\text{ICG}} + z_{\text{wind}}$, one obtains the total complex impedance to be

$$z_{\text{total}} = \frac{z'}{1 + i \omega z' C_{\text{star}}} \text{ ohms}.$$  

Let $\omega = \omega_0$ when $|z_{\text{total}}|$ is the smallest value. In this case, the potential drop between inner vacuum gap is the highest. It is possible that there exists an oscillation with time scale $\omega_0^{-1}$.
in the circuit. We expect that some of the variations of radio intensity in different timescales could be hints of such circuit oscillations.

In case of $C_{\text{ICG}} = C_{\text{LAG}} = L_{\text{flow}} = C_{\text{flow}} = 0$, one has the total impedance $z = R_{\text{ICG}} + R_{\text{LAG}} + R_{\text{wind}}$. The physical meaning of this result is of the DC circuit model (Shibata 1991), where $E_{\text{rot}} \approx 2 \pi^2 \rho_{GJ} \cdot V_{\text{cap}}$, which results in the deduction of pulsar magnetic fields $B \approx \sqrt{c^3 1 \dot{P} \dot{P}/(\pi^2 R^6)}$ (e.g., Manchester & Taylor 1977), is usually assumed (e.g., Xu & Qiao 2001), with the Goldreich-Julian density $\rho_{GJ}$.

### 3 PULSARS CHARGED ELECTRICALLY?

If charged particles distribute as the Goldreich-Julian density, $\rho_{GJ}$, they will be in balance about electrostatic force. The equivalent “Possion” equation in comoving frame then is (e.g., Beskin, Gurevich & Istomin 1993)

\[
\nabla \cdot E = 4\pi(\rho - \rho_{GJ}).
\]

As mentioned by RS75, the electric field, $E_{GJ}$, on star surface due to the lack of charge density respective to the Goldreich-Julian density (e.g., for vacuum outside the star, $\rho = 0$) is normal to star surface. The solution of RS75 (see its appendix 1.b) for the electric field on the stellar surface is $E_a = -2 \Omega B h / c < 0$, which is equivalent to that of choosing the potential of field line $a$ to be the same one of the ISM, $\phi_A = 0$. From equation (15), one can also find generally the surface electric field $E_{GJ} < 0$ in both Regions I and II, as shown in Fig.3.

When the potential of the line $b$ is zero, as mentioned above, the vector direction of electric field $E_I$ within Region I is inward but that in Region II, $E_{II}$, is outward. How can one understand consistently this picture? Why should this be reasonable if one choose $\phi_B = 0$, rather than $\phi_A = 0$? The answer could be that there must be positive charges on the star surface which increases the potential of star. These charges provide a monopole electric field $E_{\text{mon}}$, which combined with $E_{GJ}$ to form the total electric field, as shown $E_I$ and $E_{II}$ in Fig.3.

The electricity induced by the charges on the star surface is so high that the electric field is reversed across Region II when electric field of star self and the field caused by charged particles are combined. Therefore, the current can flow out and come back in magnetosphere to close the pulsar’s generator circuit. Reversely, the increased electric field supports our assumption that the potential of line $b$ is zero.

Provided that two conditions are satisfied (1, the star has positive charges on the surface; 2, the potential of the line $b$ is zero), the potential increase of star relative to that of ISM could then be estimated to be order of the potential drop between $A$ and $B$ (from equation 2)

\[
V_{\text{star}} \sim \phi_A - \phi_B = \frac{3 \times 10^{16} R_6^2 B_{12}}{P} (\sin^2 \theta_A - \sin^2 \theta_B) \approx \frac{3 \times 10^{12} R_6^2 B_{12}}{P^2} \text{ volts. (16)}
\]

From equations (8) and (10), the total charges on stellar surface could then be

\[
Q = V_{\text{star}} \times C_{\text{star}} \approx \frac{3 \times 10^{-3} R_6^4 B_{12}}{P^2} \text{ coulombs. (17)}
\]

The collapse of an evolved massive star should form temporally a rotating magnetized compact star, and then become a black hole. The black hole could be charged too, with a quantity to be proportional to $B/P^2$. As magnetic field $B \sim \sqrt{P \dot{P}}$, one has then $Q \sim \sqrt{\dot{P}/P^3} \sim \dot{E}_{\text{rot}}^{1/2}$, where the rotation energy loss rate $\dot{E}_{\text{rot}} \sim \Omega \dot{\Omega}$. Observationally, the X-ray luminosity could be a function of the spin-down energy loss for all rotation-powered pulsars, $L_x \sim \dot{E}_{\text{rot}}$ (Becker & Trümper 1997); but the $\gamma$-ray luminosity $L_{\gamma} \sim \dot{E}_{\text{rot}}^{1/2}$ (Thompson 2003). One can also address that energetic $\gamma$-ray luminosity is proportional to the electric charge $Q$, rather than to $\dot{E}_{\text{rot}}$. 

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Fig. 3 An illustration of the total electric field $E$ as a function of polar angle $\theta$. $E_{\text{GJ}}$ is due to the charge-separation in pulsar magnetosphere, provided that the potential of line $a$ is the same of the ISM. $E_{\text{mo}}$ is induced by the charges of star. The total electric field (middle solid thick curve), $E_I$ and $E_{\text{II}}$ within the corresponding regions, could be the combination of $E_{\text{GJ}}$ (lower solid thin curve) and $E_{\text{mo}}$ (upper solid thin straight line).

One thinks conventionally, according to equation (15), that no acceleration (e.g., $E_{\parallel} = 0$) occurs if $\rho = \rho_{\text{GJ}}$ in pulsar magnetosphere. We note, however, that this conclusion is valid only if no solenoidal force field appears. In other words, $E_{\parallel} \neq 0$ though $\rho = \rho_{\text{GJ}}$ if one adds any solenoidal force field in the magnetosphere. A charged pulsar contributes a solenoidal electric field, which results in an acceleration near the pulsar (to damp as $1/r^2$). In the close field line region, this electric field causes a re-distribution so that $E_{\parallel} = 0$. In the open field line region, this field accelerates particles (for parallel pulsars, to accelerate negative particles in Region I, but positive particles in Region II; see Fig.1). Certainly, extra acceleration due to $\rho \neq \rho_{\text{GJ}}$ exists too. As demonstrated in Fig.3, a very large acceleration may exist near the last open field lines, which could be favorable for the high energy emission in the caustic model (Dyks & Rudak 2003). An outer gap may not be possible if particles can flow out freely either from the surface (for negligible binding energy) or from the pair-formation-front (for enough binding energy) of a charged pulsar.

4 LOW-MASS BARE STRANGE STARS

Pulsars could be bare strange stars, some of them could be of low-mass (Xu 2005). Due to the color self-confinement of quark matter, the density of low-mass bare quark star is roughly homogeneous, and its mass would be

$$M_{\text{QS}} = \frac{4}{3} \pi R^3 (4\bar{B}) = 0.9 R_6^3 \bar{B}_{60} M_{\odot},$$  \hspace{1cm} (18)

where the bag constant $\bar{B} = 60\bar{B}_{60}$ MeV fm$^{-3}$ (i.e. $1.07 \times 10^{14}$g cm$^{-3}$). For a star with pure dipole magnetic field and a uniformly magnetized sphere, the magnetic moment is

$$\mu = \frac{1}{2} BR^3.$$  \hspace{1cm} (19)
If the magnetized momentum per unit mass is a constant \( \mu_m = (10^{-4} \sim 10^{-6}) G \cdot \text{cm}^3 \cdot \text{g}^{-1} \), the magnetic moment then is (Xu 2005)

\[
\mu = \mu_m M. \tag{20}
\]

Combining equations (18)-(20), one can obtain the magnetic field strength

\[
B = 1.8 \times 10^{-18} \mu_m B_{60} M_\odot. \tag{21}
\]

Therefore, for a low-mass bare strange star, if the values of its period \( P \), radius \( R \) (or the mass \( M_{QS} \)), and the polar magnetic field \( B \) (or the parameter \( \mu_m \)) are all known, the total charges on the surface could be obtained by equation (17).

### 4.1 Evidence for low-mass millisecond pulsars?

Pulsar’s radius is assumed as a constant in above sections. In fact, the radius should be a variable in different models (of either normal neutron star or strange quark star). The radius of bare strange stars could be as small as a few kilometers (even a few meters). Could one find any observational hints about the star radius? It is suggested that normal pulsars could be bare strange stars with solar masses, whereas millisecond pulsars are of low masses (Xu 2005). Can we show evidence for low-mass millisecond pulsars in the bare strange star model for pulsars? These are investigated, based on the observational pulsar data. There are 1126 pulsars with known \( P, \dot{P} \) and radio luminosity \( L_{1400} \) (mJy kpc\(^2\)) at 1400 MHz simultaneously. The numbers of millisecond and normal radio pulsars are 35 and 1091, respectively, if the dividing line of them is \( P = 15 \) ms. Here we assign the 35 millisecond radio pulsars \( (P < 15 \) ms) as Sample I and the 1091 normal radio pulsars \( (P > 15 \) ms) as Sample II.

Assuming the pulsar radius is a variable, from equation (17), one can obtain pulsar’s charges

\[
Q \sim R^4 \frac{B}{\dot{P}} \sim R^4 \left( \frac{\dot{P}}{P^3} \right)^{1/2} \text{ since the magnetic field strength } B \sim \sqrt{PP}.
\]

Defining \( \zeta \equiv \dot{P}/P^3 \), one comes to

\[
\log Q \sim 4 \log R + \frac{1}{2} \log \zeta. \tag{22}
\]

At the same time, the rotation energy loss rate \( \dot{E}_{\text{rot}} \simeq I \Omega \dot{\Omega} \sim R^5 \zeta \), where rotational inertia of star \( I \simeq \frac{2}{5} M R^2 \sim R^3 \) (the star is assumed to be a homogeneous rigid sphere), i.e.,

\[
\log \dot{E}_{\text{rot}} \sim 5 \log R + \log \zeta. \tag{23}
\]

The correlation between \( L_{1400} \) and \( \zeta \) and the normalized statistical distribution of \( L_{1400}/\zeta \) are shown in Fig.4. For showing the evidence for low-mass millisecond pulsars by this relation and distribution, we firstly give the best fit line by the least square method for larger Sample II. Secondly, we assign the slope of fitting line for the smaller Sample I to be the same as that of Sample II. Thirdly, we study the relation of \( L \sim R \) with two assumptions, respectively, that are the luminosity \( L = L(Q) \) (i.e., \( L \) is only a function of \( Q \)) and \( L = L(\dot{E}_{\text{rot}}) \) (i.e., \( L \) is only a function of \( \dot{E}_{\text{rot}} \)). Finally, through comparing the intercepts of two fit lines, we can obtain the ratio of radius between Sample I and Sample II.

From left panel in Fig.4, we find that the best fit line for Sample II are

\[
\log L_{1400} = 2.86 + 0.11 \log \zeta. \tag{24}
\]

Assigning the same slope for Sample I, we find that the intercept of fit line for this sample is 1.56.

[4] http://www.atnf.csiro.au/research/pulsar/psrcat/
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Fig. 4 Left: Observed luminosity $L_{1400}$ (mJy kpc$^2$) at 1400 MHz as a function of $\zeta$ for 1126 radio pulsars, where $\zeta \equiv \frac{P}{P_3}$. The solid squares represent the millisecond radio pulsars (Sample I) with $P < 15$ ms while the empty circles are the normal ones (Sample II) with period $P > 15$ ms. The solid line is the best fit result with the least square method for Sample II, and the dash line is the fit for Sample I assigned the slope to be the same as that of Sample II. Right: The normalized statistical distribution of $L_{1400}/\zeta$ for Sample I (dash step line) and Sample II (solid step line). The value of $L_{1400}/\zeta$ is only a function of pulsar radius according to Eq. (22-24).

If $L \sim Q$, from equations (22) and (24), the ratio of radii $R_{II}$ for Sample II and $R_I$ for Sample I at 1400 MHz then should be $4 \log \frac{R_{II}}{R_I} = 2.86 - 1.56 = 1.30$. Then one can obtain $R_{II} \approx 2.11 R_I$. In the same way, if $L \sim \dot{E}_{\text{rot}}$, from equations (23) and (24), we can obtain $R_{II} \approx 1.82 R_I$. From right panel in Fig.4, it is evident that the values of $L_{1400}/\zeta$, as a function of pulsar radius, distribute with two peaks; the higher one (i.e., larger radius) for normal pulsars whereas the lower one (i.e., smaller radius) for millisecond pulsars.

Summarily, the radii of Sample II are always larger than those of Sample I, which gives an evidence that the radii of millisecond pulsars are smaller and the masses are lower than those of normal radio pulsars. Especially, from the right panel in Fig.4, we can find that the normalized distributions of $L_{1400}/\zeta$ for Sample I and Sample II are almost clustered at about 13 and 15. This two-peak structure gives a crude same result and thereby supports again the evidence for low-mass millisecond pulsars. If one thinks that normal pulsars are bare strange stars with mass $\sim M_\odot$, millisecond pulsars could be bare strange stars with mass $\sim (1/2)^3 M_\odot$.

5 CONCLUSIONS

Assuming that the magnetosphere of pulsar has a global current which starts from the star, runs through the inner core gap, the wind, the outer gap and the inner annular gap, and returns to the star, we study the circuit characteristics of four elements: the electromotive source, the inner core gap, the inner annular gap, and the outer gap and wind. It is emphasized that the potential of critical field lines equals to that of interstellar medium. We find, in this case, that the pulsar whose rotation axis and magnetic dipole axis are parallel should be positively charged.
for the pulsar’s generator circuit to be closed. The current flows out through the light cylinder and then flow to the stellar surface along the open magnetic field line.

There are five independent parameters which can describe completely the dynamics of a pulsar magnetosphere assuming a dipole field and a uniform density of the star. They are the radius $R$, the mass $M$ ($\approx 16\pi\bar{B}R^3/3$ in case of bare strange stars, with $\bar{B}$ the bag constant), the magnetic strength $B$ (or magnetic moment $\mu \sim BR^3$), the period $P$, and inclination angle $\alpha$. Typical parameters for radio pulsars are: $R \sim 10^6$ cm, $M \sim 1.4M_\odot$, $P \sim (10^{-3} - 1)$ s, $B \sim 10^8-12$ G. No solid observational evidence shows these parameters are really typical. In case that pulsars are bare strange stars (probably with low masses), some of the parameters above may not be representative, and the independent parameters could be related each other (Xu 2005). The statistics between the radio luminosity and pulsar’s electric charge (or the spindown power) may hint that millisecond pulsars could be low-mass bare strange stars (with masses of a few $0.1M_\odot$).

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References

Arons J., Scharlemenn E. T., 1979, ApJ, 231, 854
Becker W., Trümper J., 1997, A&A, 326, 682
Beskin V. S., Gurevich A. V., Istomin Y. N., 1993, Physics of the pulsar magnetosphere, Cambridge Univ. Press
Cheng K. S., Ho C., Ruderman M., 1986, ApJ, 300, 500
Dyks J., Rudak B., 2003, ApJ, 598, 1201
Goldreich P., Julian W. H., 1969, ApJ, 157, 869
Harding A. K., Muslimov A. G., 1998, ApJ, 508, 328
Hewish A., Bell S. J., Pilkington J. D., et al., 1968, Nature, 217, 709
Kim H., Lee H. M., Lee C. H., Lee H. K., 2005, MNRAS, 358, 998
Manchester R. N., Taylor J. H., 1977, QB, 843, 8 (Freeman: San Francisco)
Melrose D., 2004, in: Young Neutron Stars and Their Environments, IAU Symp. 218, eds F. Camilo & B. M. Gaensler, San Francisco, p.349
Qiao G. J., Lee K. J., Wang H. G., Xu R. X., Han J. L., 2004a, ApJ, 606, L49
Qiao, G. J., Lee, K. J., Zhang, B., Xu, R. X., Wang, H. G. 2004b, ApJ, 616, L127
Ruderman M. A., Sutherland P. G., 1975, ApJ, 196, 51
Shibata S., 1991, ApJ, 378, 239
Sturrock P. A., 1971, ApJ, 164, 529
Subbarthi R., Manuel M., José P. S. L., et al., 2004, Braz.J.Phys., 34, 310
Thompson D. J., 2003, in: Cosmic Gamma Ray Sources, eds K. S. Cheng & G. E. Romero, Kluwer
ASSL Series [astro-ph/0312272]
Xu R. X., 2003a, in: Stellar astrophysics — a tribute to Helmut A. Abt, eds. K. S. Cheng, K. C. Leung & T. P. Li, Kluwer Academic Publishers, p.73 [astro-ph/0211563]
Xu R. X., 2003b, ChJAA, 3, 33 [astro-ph/0211214]
Xu R. X., Qiao G. J., 2001, ApJ, 561, L85
Xu R. X., 2005, MNRAS, 356, 359
Zhang B., Harding A. K., Muslimov A. G., 2000, ApJ, 531, L135

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