Accuracy evaluation of broadcast ephemeris for BDS-2 and BDS-3

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Abstract: The BDS-3 system was completed in July 2020 and began to provide services to users around the world. The inspection of its operation, especially the detailed evaluation of the orbit, clock error, TGD and other indicators, plays an important role in the subsequent positioning. This study conducts an investigation of the satellite broadcast ephemeris of the BDS-2 and BDS-3. The difference between the satellite orbit position calculated by the broadcast ephemeris and the position calculated by the precise ephemeris is used for analysis. First, the ephemeris form January 2020 to February 2020 are investigated. The results show that the broadcast ephemeris accuracy of the BDS-2 MEO satellite is the highest, while the GEO satellite broadcast ephemeris accuracy is the lowest. And their three-dimensional orbit difference is 3m and 7.5m, respectively. Second, the BDS-3 MEO satellite broadcast ephemeris accuracy is higher than the BDS-2, its three-dimensional orbit accuracy is about 0.39m, while its clock error is slightly smaller than the BDS-2. The result of ephemeris calculation is basically equivalent to the clock error of satellite-to-earth observation, which is related to the addition of the clock error of the inter-satellite link in the BDS-3. Finally, the clock error of the BDS-3 MEO satellite with the H clock is basically the same as that of the MEO satellite with the Rb clock.

Keywords: BDS-3, Broadcast ephemeris, SISRE, Clock error,
Satellite System (BDS-1), Regional Navigation Satellite System (BDS-2), and Global BeiDou Navigation System (BDS-3). At present, the development trend of BDS continues to improve, and BeiDou-related services and industries are developing rapidly around the world. The full resolution of the compatibility of BDS with other global navigation satellite systems will make BDS more important in global positioning, navigation and timing (PNT) (China Satellite Navigation Office, 2019). Up to July, 2020, satellites working in-orbit in BeiDou navigation satellite system (BDS) consist of 15 BDS-2 satellites and 30 BDS-3 satellites, and most of the BDS-2 satellites clocks in-orbit are in the final phase, so it is essential to evaluate the performance of the satellites clocks and broadcast ephemeris. Thanks to the new inter-satellite link payloads on BDS-3 satellites, more than 98% of ephemerides and 93% of clock parameters are uploaded within one hour (Lv et al. 2019).

Many current research mainly focuses on the signal-in-space range error (SISRE), the satellite orbits and clock offsets calculated by broadcast ephemeris are compared with the precise orbit and clock offset products. With the contribution of BDS-3, the number of global average visible satellites has increased from 5.1 to 10.7 (Zhang et al. 2019), and all the results show that the accuracy level of BDS-3 is significantly higher than that of BDS-2 both in satellite orbit and in satellite clock offset. The corresponding RMS and STD of all BDS-3 satellite clock offsets are improved by 40.34% and 52.49% than that of BDS-2, respectively. Meanwhile, the mean RMS and STD are 1.78 m and 0.40 m for BDS-2 SISRE, 1.72 m and 0.34 m for BDS-2 orbit-only SISRE, 0.50 m and 0.14 m for BDS-3 SISRE, and 0.17 m and 0.04 m for BDS-3 orbit-only SISRE (Jiao et al. 2020). And similar results can also be found in other study. Yang found that the average RMS satellite clock error is 1.12 nanoseconds and the average SISRE is 0.44 m by evaluating 8 satellites of BDS-3 (Yang et al. 2019). Meanwhile, that of BDS-2 MEO satellites would be about 1 m (Wang et al. 2019). In terms of the clock offsets, BDS-3 satellites are equipped with high-precision domestic new rubidium clocks and passive
hydrogen atomic clocks. Compared with the satellites of BDS-2, the performance of BDS-3 has great promotion (Mao et al. 2020).

This article mainly studies the accuracy of the BDS-3 broadcast ephemeris, and compares it with the BDS-2 broadcast ephemeris. We present an assessment of BDS-3 MEO satellites and BDS-2 three kinds of satellites broadcast orbit and clock offset accuracy from DOY 001-060, 2020. The precise products by the Wuhan University, which is one of the IGS MGEX analysis centers, are selected as the reference for the comparison. Finally, a standard single point positioning (SPP) test is used to evaluate the correctness of the results. Satellites positions and clock offsets derived from broadcast ephemeris are compared with precise orbit determination (POD) orbits and clock offsets. Then, the corresponding SISIRE is computed according to the SISRE definition.

In our paper, we first introduced the method of calculating satellite coordinates using broadcast ephemeris. It is worth noting that the calculation methods of MEO satellites, IGSO satellites and GEO satellites are slightly different. In addition, there are some considerations, such as the reference frame difference, antenna phase center correction, satellite clock error correction. After the satellite orbit error is obtained, the SISRE model is used to evaluate the broadcast ephemeris orbital accuracy and satellite clock error of BDS-2 and BDS-3 satellites, and the differences between them are compared.

**Methodology**

The evaluation of the broadcast of BDS-2 and BDS-3 satellites involves orbit errors and clock errors. Precise orbit and clock products obtained from WUM are used as true values to assess the performance of the broadcast ephemeris. The space segment of BDS is a hybrid constellation composed of MEO satellites, IGSO satellites and GEO satellites. However, due to the extremely small orbital inclination angle of GEO satellites, if the calculation is performed according to the orbital element method of
MEO satellites and IGSO satellites, the normal equations are prone to singular matrices and calculation failures. Therefore, the calculation method for GEO satellites to calculate satellite positions from broadcast ephemeris is different from MEO satellites and IGSO satellites.

And the reference of the broadcast orbit and clock, as well as the precise clock, is the antenna phase center (APC), while the reference of the precise orbit is the center of mass (CoM) of the satellite. Therefore, for the orbit comparison, the differences between CoM and APC need to be carefully corrected. For the clock comparison, the time group delay (TGD) caused by different signal or signal combinations also needs taking into account.

**Satellite position calculation method**

**BDS MEO and IGSO satellite coordinates (WGS84) calculation method**

The BDS satellite ephemeris provides 16 ephemeris parameters, including 1 reference moment, 6 Kepler orbit parameters at corresponding reference moments, and 9 orbital perturbation correction parameters. The ephemeris update period is 1h. The meaning of each parameter is as follows:

**Table 1** 16-parameter calculation model broadcast ephemeris parameters

| Parameter | Parameter meaning |
|-----------|-------------------|
| $t_{oe}$ | Reference Time Ephemeris |
| $\sqrt{a}$ | Square Root of the Semi-Major Axis |
| $e$ | Eccentricity |
| $i_0$ | Inclination Angel at Reference Time |
| $\Omega_0$ | Longitude of Ascending Node of Orbit Plane at Weekly Epoch |
| $\omega$ | Argument of Perigee |
| $M_0$ | Mean Anomaly at Reference Time |
| $\Delta n$ | Mean Motion Difference From Computed Value |
| IDOT | Rate of Inclination Angle |
| $\dot{\Omega}$ | Rate of Right Ascension |
According to the ephemeris parameters to calculate the satellite position at any time \( t \), the calculation steps are as follows:

1. Calculate the mean motion \( n_0 \) of the satellite at the reference time \( t_0 \)
   \[
   n_0 = \frac{\mu}{\sqrt{A^3}} \quad (1-1)
   \]
   In the formula: \( \mu \) is the earth's gravitational constant in the BDCS coordinate system, \( \mu = 3.986004418 \times 10^{14} \text{m}^3/\text{s}^2 \), \( \sqrt{A} \) is the square root of the semi-major axis given in the navigation message.

2. Using the difference \( \Delta n \) between the satellite's average moving speed given in the navigation message and the calculated value, calculated the corrected mean motion:
   \[
   n = n_0 + \Delta n \quad (1-2)
   \]

3. Calculate the satellite mean anomaly \( M_k \) at the moment of observation
   \[
   M_k = M_0 + n(t - t_{oe}) \quad (1-3)
   \]
   Where, \( t_{oe} \) is the ephemeris reference time given in the navigation message; \( M_0 \) is the mean anomaly of the reference time \( t_{oe} \) in the navigation message; \( t \) is BDT at time of signal transmission; \( t - t_{oe} \) is the total time difference, which must be considered. The start or end of the week transformation, that is: if \( t - t_{oe} \) is greater
than 302400, subtract 604800 from $t - t_{oe}$; if $t - t_{oe}$ is less than -302400, add 604800 to $t - t_{oe}$.

Iterative calculation of the satellite near corner E at the moment of observation:

According to the eccentricity $e$ given in the navigation message and the calculated mean anomaly $M_k$, the Kepler’s equation $E_k = M_k + e \sin E_k$ is used to calculate in an iterative manner.

Solution method: First give the initial value of $E$: $E_0 = M$, and substitute the above formula to solve the first iteration value. Stop the iteration when $|E_{k+1} - E_k| < 10^{-12}$

Calculate the true anomaly $\nu$ at the moment of observation:

$$\sin \nu_k = \frac{\sqrt{1-e^2} \sin E_k}{1-e \cos E_k}$$
$$\cos \nu_k = \frac{\cos E_k-e}{1-e \cos E_k}$$

So the calculation formula for the true anomaly angle $\nu_k$ is:

$$\nu_k = \tan^{-1}\frac{\sqrt{1-e^2} \sin E_k}{\cos E_k-e}$$

Calculate the argument of latitude $\Phi_k$:

$$\Phi_k = \omega + \nu_k$$

In the above formula: $\omega$ is the angular distance of perigee given in the navigation message.

According to the perturbation parameters $C_{uc}$, $C_{us}$, $C_{rc}$, $C_{rs}$, $C_{ic}$, $C_{is}$ given in the ephemeris, calculate the Argument of Latitude correction $\delta u_k$, the Radius Correction $\delta r_k$, and the Inclination Correction $\delta i_k$.

$$\begin{align*}
\delta u_k &= C_{uc} \cos 2\Phi_k + C_{us} \sin 2\Phi_k \\
\delta r_k &= C_{rc} \cos 2\Phi_k + C_{rs} \sin 2\Phi_k \\
\delta i_k &= C_{ic} \cos 2\Phi_k + C_{is} \sin 2\Phi_k
\end{align*}$$

Calculate the Corrected Argument of Latitude $u_k$, Corrected Radius $r_k$ and Corrected inclination $i_k$:
\[
\begin{align*}
\{ u_k &= \Phi_k + \delta u_k \\
    r_k &= a(1 - e \cos E_k) + \delta r_k \\
    i_k &= i_0 + IDOT(t - t_{oe}) + \delta i_k
\end{align*}
\] (1-8)

In the above formula: \( a \) is the long radius of the satellite orbit, \( a = (\sqrt{A})^3 \). \( i_0 \) and IDOT are respectively the square root of the semi-major axis and the orbital inclination at the reference moment given by the broadcast ephemeris parameters and the rate of change of orbital inclination.

Calculate the coordinates of the satellite in the Cartesian coordinate system of the orbital plane:

In the orbital plane Cartesian coordinate system (the coordinate origin is at the center of the earth), the \( z_0 \) axis is perpendicular to the orbital plane, the \( x_0 \) axis points to the ascending node, and \( y_0 \) is perpendicular to the \( x_0 \) axis in the orbit plane, forming a right-handed system. The satellite's planar Cartesian coordinates are:

\[
\begin{align*}
    x_0 &= r \cos u_k \\
    y_0 &= r \sin u_k
\end{align*}
\] (1-9)

Calculate the longitude \( L \) of the ascending node at the time of observation:

\[
L = \Omega_0 + (\dot{\Omega} - \omega_e)(t - t_{oe}) - \omega_e t_{oe}
\] (1-10)

In the above formula: \( \dot{\Omega} \) and \( \Omega_0 \) are the rate of change of ascending node right ascension given by the broadcast ephemeris parameters and the ascending node right ascension calculated according to the reference moment; \( \omega_e \) is the earth rotation rate in the CGCS2000 coordinate system \( \omega_e = 7.2921150 \times 10^{-5} rad/s \).

Calculate the coordinates of the satellite in the CGCS2000 coordinate system:

First rotate the coordinate system as follows:

1) Rotate the angle \( \omega_s \) clockwise around the \( z_0 \) axis to make the \( x_0 \) axis change from perigee to ascending node;

2) Rotate the \( x_0 \) axis clockwise by the angle \( i_k \) to make the \( z_0 \) axis coincide with the sky axis;
3) Rotate the angle $\Omega$ clockwise around the $z_0$ axis so that the $x_0$ axis coincides with the X axis of the celestial coordinate system, thereby obtaining the coordinates of the satellite in the celestial rectangular coordinate system.

Because when using BDS positioning, the position of the observation satellite and the observation station should be in a unified coordinate system, and the coordinates in the celestial coordinate system need to be converted to the earth space rectangular coordinate system. The coordinate between the two points only on the X axis. When the direction is different from the Greenwich star, so only one rotation is needed to find the position of the satellite in the instantaneous earth coordinate system.

In summary, after knowing the longitude $L$ of the ascending node and the inclination $i$ of the orbital plane, the position coordinates of the satellite in the ground-fixed coordinate system can be easily obtained through two rotations.

The coordinates of the MEO/IGSO satellite in the CGCS2000 coordinate system are

$$\begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = R_z(-L)R_x(-i_k) \begin{bmatrix} x_0 \\ y_0 \\ 0 \end{bmatrix} = \begin{bmatrix} x_0 \cos L - y_0 \sin i_k \sin L \\ x_0 \sin L + y_0 \sin i_k \cos L \\ y_0 \sin i_k \end{bmatrix}$$

(1-11)

In the formula, $L$ is the ascension of the ascending node in the earth-solid system.

**BeiDou GEO satellite coordinates (WGS84) calculation method**

Due to the small inclination of the GEO orbit, the use of GPS ephemeris parameters to fit the GEO satellite orbit may not converge due to the singularity of the matrix. Literature proposes a coordinate rotation method to solve this problem. Specific steps are as follows:

1) Transform the satellite ephemeris in the geo-fixed system to the quasi-J2000 coordinate system by rotating the GAST angle clockwise around the Z axis (the Greenwich side time corresponding to the satellite ephemeris);

2) Rotate clockwise by $n^\circ$ (counterclockwise by $n^\circ$) around the X-axis or Y-axis in the quasi-J2000 coordinate system to obtain the satellite ephemeris in the new inertial system;
Convert the new inertial system ephemeris obtained in the second step to the new ground-fixed coordinate system by rotating the GAST angle counterclockwise around the Z axis;

In the new ground-fixed coordinate system, the parameters of the broadcast ephemeris are fitted according to the MEO calculation method.

In practical applications, when users calculate the longitude of the ascending node at the instant of GEO satellite observation, the first step of rotation around the Z axis can be omitted without considering the \( \omega t_k \) term, that is, the satellite position can be obtained by two-step coordinate transformation, reducing calculations the amount.

Ascension of the ascending node in the inertial frame is

\[
L = \Omega_0 + \dot{\Omega}(t - t_{oe}) - \omega_e t_{oe} \tag{1-12}
\]

3) Solving the Greenwich side angle GAST of the instantaneous epoch will bring a lot of computation to the receiver and bring inconvenience to the design of the receiver.

\[
GAST = GAST_{toe} + \omega t_k \tag{1-13}
\]

In the above formula, \( GAST_{toe} \) represents the Greenwich side angle of the reference \( t_{oe} \) time, \( \omega \) is the angular velocity of the earth's rotation, and \( t_k = t - t_{oe} \) is the time difference from the instantaneous epoch to the reference epoch.

In order to avoid the above problems, rotate the reference plane under the inertial system that coincides with the fixed coordinate system corresponding to the reference time \( t_{oe} \), which eliminates the need to calculate the more complicated \( GAST_{toe} \) term and only calculates the \( \omega t_k \) part.

The GEO broadcast ephemeris parameters obtained by the coordinate rotation method are fitted. When calculating the GEO satellite orbit, the user only needs to calculate the satellite position according to the MEO satellite calculation method, and then perform the corresponding coordinate inverse transformation process, and the GEO satellite can be obtained. The position in the ground-fixed coordinate system, namely the BeiDou satellite orbit algorithm.
\[
\begin{align*}
X_G &= x_0 \cos L - y_0 \sin i \sin L \\
Y_G &= x_0 \sin L + y_0 \sin i \cos L \\
Z_G &= y_0 \sin i
\end{align*}
\] (1-14)

In the above formula, L is the right ascension of the ascending node in the inertial coordinate system.

The coordinates of the GEO satellite in the CGCS2000 coordinate system are
\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = R_Z(\omega_e(t - t_{oe})) R_X(-5^\circ) \begin{bmatrix}
X_G \\
Y_G \\
Z_G
\end{bmatrix}
\] (1-15)

In the above formula,
\[
R_X(-5^\circ) = \begin{bmatrix}
1 & 0 & 0 \\
0 & \cos(-5^\circ) & \sin(-5^\circ) \\
0 & -\sin(-5^\circ) & \cos(-5^\circ)
\end{bmatrix}
\] (1-16)
\[
R_Z(\omega_e(t - t_{oe})) = \begin{bmatrix}
\cos(\omega_e(t - t_{oe})) & \sin(\omega_e(t - t_{oe})) & 0 \\
-\sin(\omega_e(t - t_{oe})) & \cos(\omega_e(t - t_{oe})) & 0 \\
0 & 0 & 1
\end{bmatrix}
\] (1-17)

**Precautions**

**Reference frame difference**

The Satellite positions derived from the BDS broadcast ephemeris are based on the BeiDou Coordinate System (BDCS), and the precise orbits are based on the International Terrestrial Reference Frame (ITRF). Compared with the broadcast track accuracy, the impact is negligible, and the impact of the difference between the two frames can be ignored in the evaluation. However, it should be mentioned that Earth’s gravitational constant, GM, and Earth’s rotation rate, \( \omega \), and the flattening of the ellipsoid adopted by the BeiDou differ from those used in GPS. For example, the value of GM for BDS is \( 3.986004418 \times 10^{14} (m^3/s^2) \) and the value for GPS is \( 3.986005 \times 10^{14} (m^3/s^2) \). The Earth rotation rate adopted for BDS is \( 7.2921150 \times 10^{-5} \), and the value for GPS is \( 7.2921151467 \times 10^{-5} \) (Qin et al. 2019). If these differences are not taken into account, it is possible to cause errors of tens of meters.

**BDS satellites broadcast ephemeris calculation**
The BDS satellite ephemeris provides 16 ephemeris parameters, including 1 reference moment, 6 Kepler orbit parameters at corresponding reference moments, and 9 orbital perturbation correction parameters. And the algorithm of the BDS MEO satellites and GEO satellites broadcast ephemeris is described in detail in the interface documents (China Satellite Navigation Office, 2017a, b; 2018; 2019a, b).

Antenna phase offset

The satellite position provided by the precision ephemeris is the center of mass (CoM) of the satellite, while the position provided by the broadcast ephemeris is the antenna phase center (APC). Starting from 7 January 2017, BDS changed the orbit reference point from the CoM to the APC, which is the same as other GNSS systems. For WUM products, the antenna phase center offset (PCO) used in broadcast ephemeris is provided by the Test and Assessment Research Center of China Satellite Navigation Office (TRAC-China Satellite Navigation Office), while PCO of BDS-2 are estimated by Wuhan University.

BDS Satellite Clock Offsets

The clock difference between the broadcast ephemeris and precise ephemeris is a systematic deviation. Considering that the indicator SISRE refers to the ranging error from the user receiver to the phase center of the satellite antenna when evaluating the accuracy of the broadcast ephemeris, there should be no systematic deviation. Therefore, we must eliminate the systematic error of satellite clock error.

The hardware delay, for example the group delay, is part of the systemic error. The group delay (time group delay, TGD) parameter of the BDS broadcast ephemeris is one of the important components for dual-frequency users to achieve positioning. The BDS broadcast clock offset is based on the B3 frequency. The precision clock difference between BDS-2 and BDS-3 is based on B1/B3 frequency. Therefore, in order to unify the time base, processing needs to be performed when evaluating the satellite broadcast ephemeris clock difference.
\[ d_t = t_{B3} - \frac{f_{12}^2 T_{GD1}}{f_1^2 - f_3^2} \]  

(1-18)

In the above formula, \( f_1 \) and \( f_3 \) are the frequencies of B1 and B3 frequency points, respectively; \( t_{B3} \) is the satellite clock difference calculated from the broadcast ephemeris parameter; \( T_{GD1} \) is the group delay parameter of the BDS broadcast ephemeris (Jiao et al. 2020).

In this paper, a method is also used to calculate the mean value of the difference between the broadcast ephemeris and the precise ephemeris in one day for a single satellite. When comparing each ephemeris with the precise ephemeris, the difference between the clocks of different ephemeris and the mean value is used to eliminate the systematic deviation of the satellite clock.

**SISRE model**

SISRE is a common quantity used to assess the quality of broadcast ephemeris. The common SISRE expression for multi-GNSS was defined by Montenbruck et al. (2018) and is represented as

\[ SISRE = \sqrt{(\omega_1 R - T)^2 + \omega_2^2 (A^2 + C^2)} \]  

(1-19)

Where \( R \), \( A \) and \( C \) denote the orbit errors in radial, along-track and cross-track direction, while \( T \) represents clock errors. \( \omega_1 \) and \( \omega_2 \) are weight factors for the global SISRE related to a specific constellation. If we neglect clock errors \( T \), we obtain the orbit-only SISRE formulation, that is,

\[ SISRE_{orbit-only} = \sqrt{\omega_1^2 R^2 + \omega_2^2 (A^2 + C^2)} \]  

(1-20)

The detailed value of the weight factors for BDS-2 and BDS-3 satellites have been computed and presented by Montenbruck et al. (2015).

The process of broadcasting ephemeris fitting and evaluation (Fig. 1)
Fig. 1 Broadcast ephemeris accuracy assessment flowchart

Results and discussion

Table 2 BeiDou satellite navigation system satellite type

| Satellite system | Satellite type | Satellite clock type | PRN            |
|------------------|----------------|----------------------|----------------|
| GEO              |                |                      | C01~C05        |
| BDS              | IGSO           | Rb                   | C6~C10, C13, C16 |
| BDS              | MEO            |                      | C11, C12, C14  |
| BDS-3            | MEO            | Rb                   | C19~C24, C32, C33, |
| BDS-3            |                | H                    | C36, C37       |

Comparison of GEO, IGSO, MEO in BDS-2

Fig. 2, Fig. 3, Fig. 4 and Fig. 5 show the time series of the orbit error and clock error of the three types of BDS satellite broadcasting in 60 days. Fig. 2, Fig. 3 and Fig. 4 are the root mean square (RMS) orbit errors of a single GEO satellite (C04), a single IGSO
satellite (C09) and a single MEO satellite (C11) in the R, A and C directions. Fig. 5 shows the root mean square of the three-dimensional orbit error, clock error and SISRE time series of a single GEO satellite (C04), a single IGSO satellite (C09) and a single MEO satellite (C11). It can be seen that the accuracy of BDS-2’s MEO satellite broadcast ephemeris is better than that of IGSO satellites and GEO satellites, and its three-dimensional orbit error is basically around 2.6m.

For BDS-2 GEO satellites, it can be seen from Fig. 2 and Fig. 5 that the errors of GEO satellites in the R and A directions are randomly distributed and stable. The fluctuation range of GEO satellites in the R direction is within 1.8m, and the A direction is within 2.3m. The orbit error of the GEO satellite in the C direction is within 7m. The three-dimensional orbit error of GEO satellites is about 7.6m. The satellite clock error is about 0.40m.

![BDS-2 GEO satellite (C04) orbit error (RMS)](image)

**Fig. 2** BDS-2 GEO satellite (C04) orbit error (RMS)
Fig. 3 BDS-2 IGSO satellite (C09) orbit error (RMS)

Fig. 4 BDS-2 MEO satellite (C11) orbit error (RMS)
For the BDS-2 IGSO satellite, it can be seen from Fig. 3 and Fig. 5 that the errors in the R, A, and C directions of each satellite are randomly distributed, but the fluctuations are stable. The fluctuation amplitude in the R and C directions is less than 1m, and the fluctuation amplitude in the A direction is not more than 0.5m. The three-dimensional orbit error accuracy of IGSO satellite is about 2.4m. The satellite clock error is about 0.59m.

For the BDS-2 MEO satellite, it can be seen from Fig. 4 and Fig. 5 that the R, A, and C direction errors of each satellite are randomly distributed and not very stable, with a lot of fluctuations, and the values are basically concentrated around 0. The accuracy of the three-dimensional orbit error of the MEO satellite is about 2.61m. The satellite clock error is about 0.72m.

**Fig. 5** BDS-2 satellite 3D orbit error (RMS), Clock error (RMS) and SISRE (RMS)

**Comparison between MEO of BDS-2 and MEO of BDS-3**
It can be seen from Fig. 5 and Fig. 6 that the same Rb clock is used, and the root mean square (RMS) of the orbit error of the MEO satellite of BDS-3 is smaller. The three-dimensional orbit error of the MEO satellite of BDS-2 is about 2.7m, while the three-dimensional orbit error of the MEO satellite of BDS-3 is about 0.5m.

**Fig. 6** BDS-3 MEO satellite with Rb clock (C20) orbit error (RMS)
Fig. 7 BDS-3 MEO satellite with H clock (C25) orbit error (RMS)

Comparison of MEO with H clock and MEO with Rb clock in BDS-3

It can be seen from Fig. 6, Fig. 7, and Fig. 8 that the MEO satellite with Rb clock and the MEO satellite with H clock in BDS-3 have similar three-dimensional orbit errors, and the former is slightly better than the latter. It can be seen from Figure 8 that the clock error of the MEO satellite with the H clock is not much different from that of the MEO satellite with the Rb clock, and the former is slightly better than the latter, which may be related to the higher stability of the H clock. The MEO satellite clock difference with the Rb clock is about 0.47m, and the MEO satellite clock difference with the H clock is about 0.45m.
Fig. 8 BDS-3 satellite 3D orbit error (RMS), Clock error (RMS) and SISRE (RMS)

Fig. 9 BDS satellite broadcasting 3D orbit error (RMS) histogram

Fig. 10 BDS satellite broadcasting Clock error (RMS) histogram
From Fig. 9, Fig. 10 and Fig. 11, it can be seen that the root mean square error (RMS) of the three-dimensional orbit of the BDS-3 satellite is better than that of the BDS-2 satellite, which is about 0.39m. The root mean square (RMS) difference of the clock errors between BDS-3 and BDS-2 satellites is very small. The root mean square (RMS) of SISRE of the BDS-3 satellite is generally better than that of the BDS-2 satellite, which is about 0.39m.

| Satellite type | Direction error | Direction A (m) | Direction C (m) | Direction Rb (m) | Direction H (m) |
|---------------|-----------------|-----------------|-----------------|------------------|-----------------|
| BDS-2 GEO     |                 | 1.86            | 2.21            | 1.43             | 0.10            |
| BDS-2 IGSO    |                 | 2.15            | 1.03            | 1.43             | 0.28            |
| BDS-2 MEO     |                 | 1.14            | 1.98            | 1.10             | 0.06            |
| BDS-2 ALL     |                 | 1.48            | 1.74            | 3.11             | 0.08            |
| BDS-3 MEO     |                 | 0.10            | 0.28            | 0.27             | 0.06            |
| BDS-3 ALL     |                 | 0.08            | 0.27            | 0.26             | 0.08            |

Table 3 Average results of various satellites (RMS)
Conclusion

This article uses precision ephemeris to evaluate the accuracy of BDS-2 and BDS-3 satellite broadcast ephemeris, sets up three sets of experiments, and uses standard single-point positioning results to check the veracity of the conclusions. The three sets of experiments respectively show:

1) The three-dimensional orbit errors of MEO satellites and IGSO satellites in BDS-2 are both better than those of GEO satellites. The three-dimensional orbit error of GEO satellites is about 7.6m, the error in the R direction is better than 2m, and the error in the C direction is the largest, about 7.0m. The errors of MEO satellites and IGSO satellites in all directions are basically the same, and the three-dimensional orbit error is better than 2.7m.

2) The orbit error of the BDS-3 satellite in each direction is better than 0.5m, and the three-dimensional orbit error is about 0.39m, which is obviously better than that of the BDS-2 satellite. The substantial increase in the orbital accuracy of the BDS-3 satellite may be related to the increase in interstellar links.

3) The clock difference of BDS-3 satellites equipped with H clock is better than that of satellites equipped with Rb clock, and the clock difference is about 0.45m. This is related to the type and nature of the satellite clock. The stability of the H clock is better than that of the Rb type, and the clock difference produced is smaller.

Declarations

List of abbreviations

None.

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