Non-Gaussianity after BICEP2

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(Dated: September 19, 2014)

We analyze primordial non-Gaussianity in single field inflationary models when the tensor/scalar ratio is large. Our results show that detectable levels of non-Gaussianity $f_{NL} \sim 50$ are still possible in the simplest class of models described by the effective theory of inflation. However, the shape is very tightly constrained, making a sharp prediction that could be confirmed or falsified by a future detection of non-Gaussianity.

Current cosmological data provides strong support for the simplest models of inflation. A single scalar field with a quadratic potential and canonical kinetic term $L = -(\partial \phi)^2/2 - m^2 \phi^2/2$ predicts a tilt $n_s = 1$ and tensor/scalar ratio $r$ that lie within about one sigma of the current central values reported by Planck and BICEP2 [5] [6]. It also predicts extremely Gaussian perturbations, with undetectably small non-Gaussian deviations $f_{NL} \sim \epsilon$ [2].

Planck improved the constraints on non-Gaussianity in the CMB temperature spectrum [5] (Fig. 2). In addition the recent BICEP2 results indicate that the tensor/scalar ratio $r \sim 0.2$. The level of inflationary model building, such a large value of $r$ at least naively poses difficulties for detectable non-Gaussianity. This is because the simplest way to achieve large $f_{NL}$ is an inflationary model with a non-canonical kinetic term such that the speed of sound of perturbations during inflation $c_s \ll 1$. This produces non-Gaussianity peaked on equilateral triangles, with $f_{NL} \sim 1/c_s^2$ [9] [10]. But small $c_s$ also enhances the amplitude of scalar perturbations so that $r \propto c_s$, making it difficult to obtain both large $r$ and large $f_{NL}$ in this way.

In this note we will argue that in these “next-to-minimal” models of inflation – namely, single-field models described by the effective theory of inflation [10] up to cubic order in perturbations - it is still possible to achieve relatively large (and detectable) levels of primordial non-Gaussianity even with $r \sim 0.2$. This is because there are two independent parameters that determine $f_{NL}$: the speed of sound $c_s$, and another parameter $c_p$ (that appears in the coefficient of $\pi^3$).

As mentioned above $c_s$ is fairly tightly constrained if $r \sim 0.2$. But $c_p$ is not, and, while $c_s \ll 1$ generates $c_p \gg 1$ via quantum loop corrections, the converse is not true. In other words, an effective theory with $c_s \lesssim 1$ but $c_p \gg 1$ is technically natural. Furthermore it can arise in a regime where higher derivative terms and modifications of Einstein gravity are small.

The main result we wish to report in this note is that not only a detectable $f_{NL}$ is possible in this “next-to-minimal” scenario, but its shape is very tightly constrained (Fig. 2). This means – assuming that the tensor/scalar ratio $r$ is in fact large – that a detection of non-Gaussianity with a shape that does not lie in the narrow region plotted in Fig. 2 would have dramatic implications for the theory of inflation and fundamental physics in general.

\section*{Effective Theory of Inflation}

The effective theory of inflation is based on the fundamental organizing principle of effective field theory: one should write an action containing all terms compatible with the underlying symmetries, starting from the terms most relevant at low energies. In the case of inflation the symmetries are those of de Sitter spacetime, with time-translation invariance weakly broken by slow-roll corrections. The appropriate metric to describe background plus fluctuations can be written in ADM form:

$$ds^2 = -N^2 dt^2 + h_{ij}(dx^i + N_i dt)(dx^j + N_j dt), \quad (1)$$

where the background is quasi de Sitter: $N = 1, N_i = 0$, $h_{ij} = a(t)^2 \delta_{ij} = e^{2Ht} \delta_{ij}$, with $H$ a slowly varying function of $t$. The effective action for perturbations of single field inflation, in unitary gauge and to cubic order in perturbations, is

$$S = \int \sqrt{-g} \left[ \frac{M_{Pl}^2}{2} R - M_{Pl}^2 (3H^2 + \dot{H}) + M_{Pl}^2 \dot{H} \frac{1}{N^2} \right. + \frac{M_{Pl}^4}{2!} \left(1 - \frac{1}{N^2}\right)^2 + \frac{M_{Pl}^4}{3!} \left(1 - \frac{1}{N^2}\right)^3 \left. - \frac{M_{Pl}^4}{2} \left(1 - \frac{1}{N^2}\right) \delta K - \frac{M_{Pl}^4}{2} \delta K^2 - \frac{M_{Pl}^4}{2} \delta K^i \delta K^i + \ldots \right]. \quad (2)$$

Here $\delta K^i_j = K^i_j - H \delta^i_j$, where $K^i_j$ is the extrinsic curvature $K^i_j = h^{ik} (\partial_j h_{kj} - \nabla_k N_j - \nabla_j N_k)$, and the $M_i, \bar{M}_i$ are constants with dimensions of mass. The ... refers to terms of higher order in derivatives or fluctuations.

The first line in (2) incorporates all single-field models with canonical kinetic terms. As mentioned above, these models produce negligible non-Gaussianity. The terms in the second line arise from non-canonical kinetic terms, and can produce large $f_{NL}$. The main result of this paper is that the Planck and BICEP2 data constrain these terms in such a way that the shape of non-Gaussianity they produce is very sharply defined (Fig. 2).
The terms in the third line come from higher derivative terms that cannot be removed by integration by parts, as in ghost condensation \([11]\), or perhaps from Lorentz-breaking modifications of gravity. These terms can produce non-Gaussianity with various different shapes, and—because they indicate exotic physics—experimental confirmation that they are important for the dynamics of inflation would be of great importance.

We introduce the Goldstone boson of broken time translations \(\pi\) by making the diffeomorphism:

\[
t \rightarrow \tilde{t} = t - \pi(t, \vec{x}) , \quad x^i \rightarrow x^i .
\]

As we are interested in non-negligible non-Gaussianities \((f_{NL} \gg 1)\), we can neglect metric perturbations.

\[
S_\pi = \int a^3 \left[ \left( -M_{Pl}^2 \dot{H} + 2 M_2^2 \right) \dot{\pi}^2 + M_{Pl}^2 \dot{H} \left( \partial \pi \right)^2 \right. \\
\left. - 2 M_2^2 \pi \left( \partial \pi \right)^2 + \left( 2 M_2^2 - \frac{4}{3} M_3^2 \right) \dot{\pi}^3 - \frac{M_3^2 H}{2} \left( \partial \pi \right)^2 \\
\left. - \frac{\dot{M}_2^2 + M_3^2 \left( \partial_2 \pi \right)^2}{2} + \frac{M_3^2 \left( \partial_2^2 \pi \right) \left( \partial_1 \pi \right)^2}{2} + \ldots \right] ,
\]

(4)

where again \(...\) indicate terms of higher order in derivatives or fluctuations.

In general, for a scalar field \(\pi\) in de Sitter space with Lagrangian

\[
S = \int a^3 N_c \left[ \pi^2 - c_s^2 \left( \partial_1 \pi \right)^2 \right] ,
\]

(5)

the speed of propagation is \(c_s\) and we get the power spectrum

\[
\Delta_\pi = \frac{1}{8 \pi^2 N_c c_s^2} ,
\]

(6)

where the dimensionless power spectrum is defined by

\[
\Delta = k^3 P(k)/(2 \pi^2) .
\]

Let us pause to briefly discuss tensors. The tensor perturbations are defined by setting

\[
\hat{h}_{ij} = a(t)^2 \gamma_{ij} \]

(7)

with \(\det \exp(\gamma) = 1, \partial_t \gamma_{ij} = 0\). The kinetic term for \(\gamma\) comes from the Einstein-Hilbert term in \([2]\), with a correction from the \(\delta K'_{ij} \delta K'_{ij}\) term:

\[
S_\gamma = \frac{M_{Pl}^2}{8} \int a^4 \left[ \left( 1 - \frac{M_2^2}{M_{Pl}^2} \right) \dot{\gamma}_{ij}^2 - \frac{\left( \partial \gamma_{ij} \right)^2}{a^2} + \ldots \right] .
\]

(8)

This shows there is a modification of the tensor speed of sound, \(c_T^2 = 1 - M_2^2/M_{Pl}^2\). This differs significantly from \(c_T = 1\) only if \(M_2^2\) is close to the Planck mass. Even in this case the resulting tensor non-Gaussianity will be only \(O(1)\) enhanced compared to its ordinary (undetectably small) value. For this reason we ignore tensors as a source for non-Gaussianity. The tensor power spectrum is given by

\[
\Delta_T = \frac{2}{\pi^2} \frac{H^2}{c_T^4 M_{Pl}^4} .
\]

(9)

**“NEXT-TO-MINIMAL” MODELS OF INFLATION**

The class of models with \(M_i = 0\) arise from theories with a non-canonical kinetic term \(P(X)\) for a scalar field \(\phi\), where \(P\) is a function and \(X \equiv (\partial \phi)^2\). With \(M_i = 0\), eq. (4) simplifies to

\[
S_\pi = \int a^3 M_{Pl}^2 \left[ \frac{\left( \dot{\pi}^2 - c_s^2 \left( \partial_1 \pi \right)^2 \right)}{a^2} \right. \\
\left. - \left( 1 - c_s^2 \right) \left( \partial_2 \pi \right)^2 \right] + \left( 1 - c_s^2 + \frac{2}{3} c_p \right) \pi^3 ,
\]

(10)

where we defined \(c_s^2 \equiv M_{Pl}^2 \dot{H}/(M_{Pl}^2 H - 2 M_2^2)\) and \(c_p = 2 M_3^2/(M_{Pl}^2 H - 2 M_2^2)\), so that \(c_p\) stays well defined in the limit \(M_2 \rightarrow 0\). For reference, the parameter \(c_3 \equiv -M_3^2/M_2^2\) is defined by \(c_p = c_3(1 - c_3^2)\). The tensor-to-scalar ratio in this model is

\[
r = 16 c_s .
\]

(11)

Since \(n_s - 1 \sim \epsilon\) and Planck measured \(n_s \sim 0.96\) \([5]\), one concludes that \(c_s\) cannot deviate substantially from \(r\):

\[
c_s \gtrsim r .
\]

(12)

The non-Gaussianity induced by the operator \(\pi (\partial \pi)^2\) is proportional to \((1 - c_s^{-2})\), and therefore cannot be very large if \(r\) is large. However, the term \(\pi^2 c_s^3 \pi^3\) can still give a sizable contribution to the non-Gaussianity if \(c_p \gg 1\).

Quantum loops in the effective theory generate \(\Delta M_3^4 \sim M_3^4/c_s^2\) \([12, 13]\). Hence, absent fine-tuning, \(c_s \ll 1\) generates \(c_p \sim c_s^{-2} \gg 1\). One might expect the converse to hold as well: that \(c_p \gg 1\) generates small \(c_s\). However this is not the case. Up to cubic order there is no diagram that generates \(\pi^3 (\partial \pi)^2\) from \(\pi^3\). Therefore, \(c_p \gg 1\) with \(c_s \sim 1\) is technically natural \([14, 15]\).

In Fig. [1] we plot the shape of the bispectrum induced by the \(\pi^3\) operator, as a function of the two independent ratios of the triangle sides. Notice that this shape has a large flattened limit (triangles with \(k_2 = k_3 = k_1/2\)).

In Fig. [2] we plot the possible values of \(f_{NL}\) in the equilateral/orthogonal plane \([15]\) in \([10]\), with the Planck constraints superimposed \([5]\). As is evident, \(r \sim 0.2\) (and therefore a large speed of sound) puts a very sharp constraint on the possible shape of non-Gaussianity in this class of models. In the “next-to-minimal” models the region above the line in Fig. [2] can be reached only allowing negative \(c_s^2\), and as such it would signal exotic physics even if \(r\) is small.
In this section we enumerate the set of models consistent with such an observation. All of them differ sharply from the simple model considered above, and correspond to novel physics.

**OTHER POSSIBILITIES**

If a future experiment detects primordial non-Gaussianity with a shape that does not fall in the narrow region indicated in Fig. 2, what are the implications? In this section we enumerate the set of models consistent with such an observation. All of them differ sharply from the simple model considered above, and correspond to novel physics.

**Quasi-de Sitter**

If the background evolution is such that $\dot{H} \to 0$, the standard gradient evolution term for the perturbations vanishes. In this case, the additional terms in the effective action become important. To simplify the analysis, we consider two limiting cases, depending on whether $d_1$ or $d_2 + d_3$ dominates, where $d_1 \equiv 2M_1^2/M_1^3$, and $d_{2,3} \equiv M_{2,3}^2/M_1^2$. All the models in this class could potentially be tested by their prediction of near-exact scale invariance for tensor perturbations.

$d_1 \gg d_2 + d_3$: This case corresponds to a standard two-derivative kinetic term. The $\pi$ Lagrangian is

$$S_\pi = \int a^3 2M^4 \left[ \dot{\pi}^2 - \frac{d_1 H (\dot{\pi} \pi)^2}{8M^2 a^2} \right] - 2M^4 \dot{\pi}^2 (\dot{\pi} \pi)^2$$

$$+ 2M^4 \left( 1 - \frac{2}{3} c_s \right) \pi^3 + \frac{d_1 M^3 (\partial^2 \pi (\partial^3 \pi))}{4 a^4}.$$  \hspace{1cm} (13)

The last term seems to imply an additional shape of non-Gaussianity. However, using the linear equations of motion and integrating by parts, it can be rewritten in terms of the standard one-derivative cubic terms:

$$S_\pi = \int a^3 2M^4 \left[ \dot{\pi}^2 - \frac{d_1 H (\dot{\pi} \pi)^2}{8M^2 a^2} \right]$$

$$+ \frac{d_1 M^3 (\partial^2 \pi (\partial^3 \pi))}{4 a^4},$$  \hspace{1cm} (14)

where the sound speed now is $c_s^2 = \frac{d_1 H}{4M^4}$. The interesting fact is that the tensor-to-scalar ratio $r$ now does not put strong constraints on the speed of sound:

$$r = \frac{32}{\pi M_{P1}^2} \left( \frac{H}{M_{P1}} \right)^2 c_s^3 = \frac{8M^2 c_s^{3/2}}{\pi M_{P1}^2 A_s}.$$  \hspace{1cm} (15)

The requirement that the cutoff has to be $\Lambda \gg H$ gives the constraint $c_s \gg A_{\pi}^{2/11} \approx 0.03$. The resulting non-Gaussianity falls in the upper right corner of the $f_{NL}^eq - f_{NL}^P$ plane, above the line in Fig. 2.

$d_1 \ll d_2 + d_3$: This case corresponds to the ghost inflation regime [17]. The $\pi$ Lagrangian has a non-relativistic scaling for the kinetic term, which results in non-standard scaling dimensions for energy, momentum, and fields. The most relevant operators are:

$$S_\pi = \int a^3 \left[ 2M^4 \dot{\pi}^2 - \frac{(d_2 + d_3) M^2 (\partial^2 \pi)^2}{2 a^4} \right. \left. - 2M^4 \dot{\pi}^2 (\dot{\pi} \pi)^2 \right]$$

$$+ \frac{d_1 M^3 (\partial^2 \pi (\partial^3 \pi))}{4 a^4}.$$  \hspace{1cm} (16)

The cubic operators proportional to $d_2, d_3$ are suppressed by $H/M$. In this situation, the tensor-to-scalar ratio is

$$r = \frac{8M^2}{\pi M_{P1}^2 c_T} \gamma^2 (1/4) \left( \frac{M}{H} \right)^{1/2} \left( \frac{d_2 + d_3}{4} \right)^{3/4} \right)^{3/4}$$

$$\approx 282 \frac{M^2 (d_2 + d_3)^{3/5}}{M_{P1}^2 c_T}.$$  \hspace{1cm} (17)
Again, $r \sim 0.2$ does not impose strong constraints on the non-Gaussianity, which is very close to the equilateral template.

Notice that in the original work on ghost inflation [17], tensor modes are taken to be negligible because of constraints on the scale $M_2 + M_3$ from solar system tests of gravity. However, in the effective theory approach, it is perfectly possible that the Lagrangian describing the inflationary perturbations is different from the one describing modifications of gravity today, so this constraint may not play a role. For a recent discussion, see [18].

### Negative speed of sound

It is possible to have consistent models in which $\dot{H} > 0$, which violate the null energy conditions and give a blue tensor tilt. The speed of sound squared will be negative, which naively implies an instability of the theory. However, in the effective theory approach, it is possible to have consistent models in which $\dot{H} > 0$, which violates the null energy conditions and gives a blue tensor tilt. The speed of sound squared will be negative, which naively implies an instability of the theory. How-

### Higher-derivative operators

It is possible to impose a “Galilean” symmetry on the effective theory of perturbations which suppresses the single-derivative cubic interactions [19]. In this case, one has a standard kinetic term (with unit speed of sound), and a standard tensor-to-scalar ratio $r = 16c$. However, there can be sizable non-Gaussianity induced by 5-derivative cubic terms. The shape of the resulting 3-point function is peculiar, being enhanced in the flattened configurations and somewhat suppressed in the equilateral one (see [19] for details). Also, because of the different scaling of the relevant operators, the size of the 4-point function scales as $NG_4 \simeq NG_3^{8/5}$ instead of the usual $NG_3^3$.

### Modified tensor spectrum

As mentioned above, if $M_2^2/M_{Pl}^2 \approx 1$ it is possible to have $c_T \ll 1$. Generically this makes the term $(M_2^2 + M_3^2)(\partial^2 \phi)^2$ in (4) very important, which strongly affects the scalar spectrum. However if $M_2^2 + M_3^2$ is tuned to be small or zero, the effect on scalars is small. In this case since $r \sim c_s/c_T$, if $c_T \ll 1$ the tensor/scalar ratio $r$ can be large even with $c_s \ll 1$ [20], removing the constraint that allowed us to make a sharp prediction for the shape of non-Gaussianity.

### Beyond single-field

Models with multiple fields are much less well constrained. For instance, a model in which a small fraction of the perturbations are generated by a curvaton can evade isocurvature constraints, but if the fluctuations of the curvaton are highly non-Gaussian this could produce relatively large $f_{NL}$. Another class of examples are dissipative models such as [21]–[23], which produces flattened non-Gaussian. Yet another possibility is quasi-single field inflation [21], whose characteristic is a non-trivial squeezed limit, intermediate between the local and equi-

### CONCLUSIONS

While the very simplest models of inflation predict undetectably small non-Gaussianity, our analysis shows that the “next-to-minimal” models can produce detectable $f_{NL}$, but with a shape that is very tightly con-

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Expanding (4) to quartic order in fluctuations gives a term $\sim c_\pi \pi^3 (\partial \pi)^2$, which together with $\pi^3$ generates $\pi (\partial \pi)^2$, but this is suppressed both by a loop factor and the fact that it arises from a quartic term. Another way of seeing the same thing is to compute the 1-loop wave-function renormalization of $\pi$ due to the interaction $\pi^3$. This gives rise at most to $b c_\pi \sim 1$.

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