The creation of a regression model of the Earth’s pole motion with a feature of dynamic prediction

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Abstract. This work is dedicated to the modern and relevant problem of predicting the Earth’s pole motion. Using regression modelling, we form a complex model, consisting of a set of optimal mathematical structures each describing the dependence of its step’s remnant on time. The comparison between the results produced in this paper with other works on the study of North pole dynamics has shown that the models obtained using adaptive regression modelling (ARM) approach allows predicting the Y-coordinate more accurately while conserving the accuracy of the X-coordinate. Our results confirm the promise of using the so called adaptive dynamic regressions developed currently for describing the Earth’s pole position’s dynamics. The ARM-approach compared to the classic methods for analyzing time series has a number of advantages: 1) an expansion of the concept of a mathematical model’s structure describing a certain dynamics could be performed; 2) the oscillations’ harmonics stable in time are isolated; 3) the accuracy of predicting changes over a certain time period increases several times, which has an important practical value.

1. Introduction

To process observations of the Earth’s pole dynamics, an approach of regression dynamic modeling (RDM) is used [1, 2]. Using the DRM software package, models describing the dynamics of terrestrial polar coordinates are built. This approach provides accurate combined models of observations that, to some extent, describe cause-and-effect as well as determinative connections and also provides predictive values of parameters. Unlike the determined models, regression ones do not remain constant in structure and values of parameters during the entire period of their use [3, 4]. After producing a prediction for one or several steps of discreteness in time, a model “updates” according to current values of coordinates. The RDM-approach is a special case of the adaptive regression modeling (ARM) [5]. All of the above applies to geodynamic systems as well [6]. The investigation of the polar region from the point of view of economic prospects has strengthen recently [7]. If photogrammetry of the Earth’s surface is based on the existing large observational content and flexibility of model projection of a geodetic network and provides reliable and accurate results [8], then inaccuracy in the Earth’s pole position does not allow referencing of the produced model to the geocentric coordinate system with the sufficient accuracy. At the same time, the necessary instruments for studying and analyzing terrestrial systems are models of various geodynamic processes. Although these models are rather diverse, one needs reliable and high-quality data for their description [9]. The status of analyzing geodynamic data is supposed to correspond to the level of theoretical modeling [10]. Such a simulation implies the expansion of latitude series of observations, that could be used at the
investigations of isostasy dynamics, determination of gravimetric parameters [11], and most importantly – for analyzing the Earth’s pole motion, into harmonic coefficients.

The method of RDM processing includes the steps as follows: 1) transformation of the uneven RDM to even ones with the method of spectral windows or by data averaging [12]; 2) fractal analysis of RDM, trend selection [13]; 3) spectral and wavelet analysis; 4) harmonic components selection, application of the Kalman filter [14]; 5) building Generalized AutoRegressive Conditional Heteroskedasticity (GARCH model); 6) Autoregressive moving-average model (ARMA model) construction [15]; 7) smoothing of residues using the martingale approximation [16]. At each step reducing the "internal" and "external" SSD, the significance of the error changes is controlled, and analysis of the RDM model quality is performed [17].

2. Modeling of RDM of the Earth pole dynamics

The dynamic series of coordinates of the North Pole of the Earth from January 1980 to October 2010 (http://www.iers.org) are investigated [18]. Where the author wishes to divide the paper into sections the formatting shown in table 2 should be used.

2.1 Model the dynamics of the X coordinate of the North Pole

At the first stage of data analysis, within the ARM-approach, the hypothesis on series stationarity is rejected with a probability of 0.95.

The centering of the original series is performed (σ = 0.1323, σΔ = 0.14).

According to the results of spectral analysis [19], there is a confirmed presence of harmonic components. To determine the carrier harmonics, the method of stepwise regression is used. 4 significant harmonics with periods of 363, 388, 433, 490 days are distinguished (σ = 0.0831, σΔ = 0.0962).

ARMA model (6,0) with σ = 0.0002 and σΔ = 0.0094 is built. The diagram for the combined model is given in figure 1, its form is demonstrated below.

![Figure 1. Diagram of the combined model for RDM of the North Pole X-coordinate dynamics.](image)

The final model is represented as the sum of periodic component and the ARMA (6.0) model:
\[ X = 0.1179 + 0.080861 \times \sin \left( \frac{2\pi t}{363} + 171.67 \right) + 0.024544 \times \sin \left( \frac{2\pi t}{388} + 0.75733 \right) \\
+ 0.15803 \times \sin \left( \frac{2\pi t}{433} + 72.725 \right) + 0.019003 \times \sin \left( \frac{2\pi t}{490} + 221.37 \right) \\
+ 2.3691 \times X_2(t - 1) - 1.8415 \times X_2(t - 2) + 0.43787 \times X_2(t - 3) \\
+ 0.049839 \times X_2(t - 4) + 0.061644 \times X_2(t - 5) - 0.076996 \times X_2(t - 6) + X_3(t), \] 

where \( X_2(t) \) are residue after removal of the harmonic component, \( X_3(t) \) are residue after combined model.

### 2.2 Model the dynamics of the Y coordinate of the North Pole

The tested hypothesis of series stationarity is rejected with a probability of 0.95.

The centering of the original series is performed (\( \sigma = 0.1306, \sigma_\Delta = 0.11 \)).

To determine the carrier harmonics, the method of stepwise regression is used [20]. This method selects four significant harmonics with periods of 363, 388, 433, 490 days. The DS of the models are respectively: \( \sigma = 0.044, \sigma_\Delta = 0.043 \).

ARMA (6,0) model is built with \( \sigma = 0.000135, \) and with \( \sigma_\Delta = 0.0039 \). The diagram of the complex model is given in figure 2. The final model is represented as the sum of the periodic component and the ARMA (6,0) model:

\[ Y = 0.322 + 0.072974 \times \sin \left( \frac{2\pi t}{363} + 260.65 \right) + 0.024456 \times \sin \left( \frac{2\pi t}{388} + 91.789 \right) \\
+ 0.15762 \times \sin \left( \frac{2\pi t}{433} + 162.51 \right) + 0.019638 \times \sin \left( \frac{2\pi t}{490} - 44.806 \right) \\
+ 2.4153 \times Y_2(t - 1) - 1.9991 \times Y_2(t - 2) + 0.66257 \times Y_2(t - 3) \\
- 0.12927 \times Y_2(t - 4) + 0.16416 \times Y_2(t - 5) - 0.11385 \times Y_2(t - 6) + Y_3(t) \]

### 3. Summary and conclusions

![Figure 2. Diagram of the combined model for RDM of the North Pole Y-coordinate dynamics.](image)
The deterministic mathematical model allows for a prediction of the studied characteristic value for future moments of time. Attempts to build such models have been made repeatedly, however, their predictive values turn out to be low. The development of statistical methods for time series (RDM) modeling allows us to hope for the successful application of statistical models in the geophysical systems in the description of latitude variability in time.

Unlike deterministic models, statistical (regression) ones do not remain constant in structure and parameter values for the entire period of use. After getting the forecast for a step or several steps of discreteness in the future, the model is "updated" according to the current latitude values.

The regression dynamic modeling (ARM - approach) is a special case of the adaptive regression modeling approach (ARM approach) [21]. With its application, a complex RDM model is formed, consisting of a set of optimal mathematical structures, each describing the dependence of the "remnants" of its step on time. Let us call such a model of the regression dynamic modeling, keeping in mind that time is the main argument, and the final form of ARM is formed as a result of computational adaptation to the properties of the residuals of one or another step and to violations of the conditions of application of the least squares method (LSM).

Comparison with the work of other researchers of the North Pole dynamics has shown that the proposed models in the application of ARM -approach allows for a more accurate prediction of the coordinate Y while maintaining the accuracy of the coordinate X.

The results obtained in the paper confirm the promise of using the so-called adaptive dynamic regressions, first proposed in [22] and being developed at the present time, for describing changes in latitudes. Their advantages, compared with the traditional approaches to the analysis of time series, in particular, to the analysis of the variability of geographical latitude, are: 1) expansion of the concept of the structure of the mathematical model describing the dynamics, 2) isolation of time-stable harmonics of oscillations, 3) several times increased accuracy of forecasting the changes on a certain time interval forward, which can have practical consequences.

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