Site and lattice resonances in metallic hole arrays

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A powerful analytical approach is followed to study light transmission through subwavelength holes drilled in thick perfect-conductor films, showing that full transmission (100%) is attainable in arrays of arbitrarily narrow holes as compared to the film thickness. The interplay between resonances localized in individual holes and lattice resonances originating in the array periodicity reveals new mechanisms of transmission enhancement and suppression. In particular, localized resonances obtained by filling the holes with high-index-of-refraction material are examined and experimentally observed through large enhancement in the transmission of individual holes.

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Light scattering from subwavelength holes drilled in metals has been the subject of long-standing interest motivated by phenomena such as extraordinary light transmission [2, 3, 4] that challenges the severe (a/λ) 4 cut-off predicted by Bethe [1] for the transmission cross section of single holes of small radius a compared to the wavelength λ. In particular, 100% transmission was predicted to be attainable using lossless metals [3] and confirmed experimentally for quasi-perfect conductors in the microwave and THz domains [4]. Quite different from perfect-conductors, real metals are capable of sustaining surface plasmons that were readily recognized to medicate the interaction among arrayed holes at visible and near-infrared frequencies [2, 3, 6]. Furthermore, the strong correlation of the transmission enhancement with the lattice periodicity in both of these metallic regimes has prompted rigorous descriptions of the effect in terms of dynamical diffraction [7, 8, 9], which connects directly to Wood's anomalies [8, 10]. Transmission resonances in individual holes [11, 12, 13] offer an additional handle to achieve extraordinary effects. These resonances can be triggered by decorating a hole with a grating around it [11] (similar to some directional antennas designs [14]), by filling the hole with high-index-of-refraction material [12], or by changing its shape to induce strong polarization [13, 15]. The combination of lattice resonances [16] in hole arrays and site resonances at specific hole positions can be anticipated to yield interesting properties in line with recent studies of light reflection on metal surfaces patterned with nanocavities that support localized modes [12, 13].

In this Letter, we offer a systematic study to the phenomenology associated to light transmission through subwavelength hole arrays in perfect-conductor films, which permits us to establish the existence of full transmission resonances for arbitrarily small holes in thick metallic screens. Furthermore, individual-hole resonances are obtained by filling the holes with high-index-of-refraction materials. This gives rise to enhanced subwavelength-light transmission, which is demonstrated both theoretically and experimentally. Finally, the complex scenario that is presented when transmission resonances of individual holes are combined with resonances originating in the array periodicity is elucidated within our analysis.

In his pioneering development, Bethe [1] showed that the scattered far-field from a hole drilled in an infinitely-thin perfect-conductor screen can be assimilated to that of a magnetic dipole parallel to the screen plus an electric dipole perpendicular to it. Subsequent studies supplemented this result with higher-order multipole corrections [15], and eventually, with rigorous solutions for arbitrary hole radius and film thickness [12, 20]. Small holes can be still represented by induced dipoles in thick screens, as illustrated in Fig. 1 This allows defining electric (E) and magnetic (M) polarizabilities both on the same side as the applied field (αE, with ν = E, M) and on the opposite side (αM). Flux conservation under arbitrary illumination leads to an exact optical-theorem type of relationship between these polarizabilities:

\[ \text{Im} \{g^\pm\nu\} = \text{Im} \{1/\alpha_\nu \pm \alpha'_\nu\} = -2k^3/3, \quad (1) \]

where \( k = 2\pi/\lambda \) is the momentum of light in free space. The remaining real parts of \( g^\pm\nu = (\alpha_\nu \pm \alpha'_\nu)^{-1} \) are obtained numerically [12, 20] and represented in Fig. 1b-c for empty holes. Note that \( \text{Re} \{g^\pm\nu\} \) diverges for zero thickness, in which case \( \text{Re} \{\alpha_E\} = -\text{Re} \{\alpha'_E\} \), and that \( |\text{Re} \{\alpha'_E\}| \gg |\text{Re} \{\alpha'_M\}| \) in the thick-film limit, dominated by transmission via the lowest-frequency TE guided mode, that does not create electric polarization.

When a subwavelength hole is filled with dielectric material of sufficiently high permittivity \( \epsilon \), hole-cavity resonances can exist thanks to the reduction of the wavelength by a factor of \( \sqrt{\epsilon} \). These resonances give rise to enhanced transmission [12], as shown in Fig. 2a by rigorous numerical solution of Maxwell’s equations (curves)
We present experimental evidence of this hole-resonance behavior in Fig. 2, which compares the transmission of microwaves through subwavelength holes filled with teflon and air. A 5-fold enhancement in the transmission is observed.

The width of these resonances is dictated by coupling of the cavity modes to the continuum of light states outside the film. The resonances are of Fabry-Perot origin, but the transmission line shapes are actually determined from the noted coupling to the two continua outside the film, as described by Fano (the vanishing side the film. The resonances are of Fabry-Perot origin with teflon and air. A 5-fold enhancement in the transmission of microwaves through subwavelength holes filled with teflon and air. A 5-fold enhancement in the transmission of microwaves through subwavelength holes filled with teflon and air.

The coupling strength drops rapidly for large $\epsilon$ due to small transmission through the dielectric-air interface, as predicted by Fresnel's equations. The larger $\epsilon$, the narrower the resonance, and the higher the transmission maxima. Incidentally, the normalized transmission cross-section obtained from our effective dipole model ($16k^4|a_M|^2/3a^2$, symbols in Fig. 2a for $\epsilon = 50$) compares remarkably well with the exact result (curve) for small $a/\lambda$.

Periodic arrays of sufficiently small and spaced holes can also be described by perpendicular electric dipoles $p$ and $p'$ and parallel magnetic dipoles $m$ and $m'$, where primed (unprimed) quantities are defined on the entry (exit) side of the film as determined by the incoming light. This is an extension of previous considerations for thin screens relying on Babinet’s principle. We consider first a unit-electric-field $p$-polarized plane wave incident on a hole array with parallel momentum $k_z$ along the $x$ axis, so that the external (incident plus reflected) field in the absence of the holes has parallel magnetic field $H_y^{\text{ext}} = 2$ along the $y$ direction and perpendicular electric field $E_z^{\text{ext}} = -2k_j/k$ along $z$. One can write the following set of multiple-scattering equations for the self-consistent dipoles, that respond both to the external field and to the field scattered by the other holes:

\[ p = \alpha_E(E_y^{\text{ext}} + G_x p - H m) + \alpha_p(G_y p' - H m') \]
\[ p' = \alpha_E'(E_y^{\text{ext}} + G_x p - H m) + \alpha_p(G_y p' - H m') \]
\[ m = \alpha_M(H_y^{\text{ext}} + G_y m - H p) + \alpha_m'(G_y m' - H p') \]
\[ m' = \alpha_M'(H_y^{\text{ext}} + G_y m - H p) + \alpha_m(G_y m' + H p'), \]

where $G$ and $H$ describe the induced fields produced at a given hole by the other holes. Noticing that the dipoles depend on hole positions $R = (x, y)$ only via phase factors $\exp(ik_j x)$, one finds

\[ G_j = \sum_{R \neq 0} e^{-ik_j x} (k^2 + \partial_j \partial_j) \frac{\exp(ik_R)}{R} \]
\[ H = -i k \sum_{R \neq 0} e^{-ik_j x} \partial_j \frac{\exp(ik_R)}{R} \]

The solution of the above equations can be written

\[ m \pm m' = 2[ (g_M^+ - G_x) k_j/k + H]/\Delta \]

with

\[ \Delta = (g_E^+ - G_x)(g_M^+ - G_y) - H^2, \]

from where the zeroth-order transmission of the holey film can be evaluated as obtained from the far field set up by an infinite 2D array of dipoles:

\[ T = \frac{2\pi k^2}{A k_z} (m' - p' k_j/k)^2. \]

Here, $A$ is the lattice unit-cell area and $k_z = \sqrt{k^2 - k_j^2}$.

Similarly, the transmittance of $s$-polarized light reduces to $T = |2\pi k m'/A|^2$, with magnetic dipoles parallel to $k_j$ and no electric dipoles whatsoever ($E_z = 0$). More precisely,

\[ m \pm m' = \frac{2k_z/k}{g_M - G_x}, \]

from where one obtains

\[ T = \left(\frac{2\pi k_j}{A}\right)^2 \frac{1}{g_M - G_x} - \frac{1}{g_M - G_x} \right)^2 \]
\[ = \left| \frac{1}{1 + \frac{2\pi}{2\pi k_z} \text{Re}(g_M - G_x)} - \frac{1}{1 + \frac{2\pi}{2\pi k_z} \text{Re}(g_M - G_x)} \right|^2. \]

The last identity in Eq. 4 is derived from Eq. 1 and from the exact relation $\text{Im}(G_x) = 2\pi k_j/A - 2k^2/3$ for propagating light ($k_j < k$).

The performance of the hole array is dominated by divergences in the lattice sums when the diffraction orders $(m, n)$ go grazing. More precisely, for a square lattice of spacing $d$ and for $k_j$ along $x$, the sums $G_j$ go to $+\infty$ as

\[ G_j \propto \frac{1}{\sqrt{(k_j + 2\pi m/d)^2 + (2\pi n/d)^2 - k^2}} \]

In particular, $G_y$ and $G_x$ (p polarization) diverge on the lowest-frequency side of all grazing diffraction orders, as illustrated in Fig. 3 whereas $G_x$ (s polarization) diverges only for $n \neq 0$ (non-straight curves). This entails different peak structure patterns for $s$- and p-polarized light (see Fig. 4).

Interestingly, Eq. 4 predicts 100% transmission whenever the condition

\[ 1 + \left(\frac{A}{2\pi k_z}\right)^2 \text{Re}(g_M^+ - G_x) \text{Re}(g_M - G_x) = 0 \]

is fulfilled. Eq. 6 is a second-order algebraic equation in $\text{Re}(g_M)$ that admits positive real solutions (one or two depending on the signs of $\text{Re}(g_M)$) when

\[ \frac{A}{4\pi k_z} |g_M - g_M^+| > 1, \]
features of these plots can be classified as follows: In particular under the conditions of (ii) distribution (see Fig. 3) for which one can neglect the first term inside the squared modulus of Eq. (4) (\(Re(g_M^+ >> Re(g_M^-)\), see Fig. 1), so that 100% transmittance maxima come about near \(n \neq 0\) grazing diffraction orders (see Fig. 3) for which one can have \(Re(g_M^+) \approx Re(G_x)\).

(iii) Dispersionless regions of vanishing transmission. Eq. (4) predicts \(k_\|\)-independent vanishing transmission when \(g_M^+ = g_M^-\), which is a property of single holes. This is the case of feature B in Fig. 1, as illustrated geometrically in the right part of Fig. 3.

(iv) Strong mixing of site and lattice resonances. Avoided level crossings are particularly evident in Figs. 4g near \(k_\parallel = \pi/d\). Non-avoided crossings are also observed, as well as splitting of full transmission maxima.

(v) Film-bound states. For incident evanescent light with \(k < k_\| < 2\pi/d - k\), the lattice sums satisfy \(\text{Im}(G_y) = -2k^3/3\) and \(\text{Im}(H) = 0\). This implies that \(\Delta_\perp\) [Eq. (3)] is real and can vanish for specific combinations of \(k\) and \(k_\parallel\), leading to simultaneous infinite transmittance and reflectance (evanescent waves do not propagate energy) in what constitute film-bound resonances, as recently predicted for related metal structures [2].

In summary, a simple and powerful formalism has been used to analyze transmission through hole arrays leading to surprising results such as 100% transmission for thick perfect-conductor films perforated by arbitrarily small holes. Both theoretical and experimental evidence of single-hole resonances obtained by filling the holes with large-index-of-refraction material have been presented, resulting in enhanced transmission through isolated holes. Finally, filled-hole arrays have been shown to exhibit a colorful phenomenology, including new types of suppressed transmission and a complicated interplay between hole-site resonances and lattice resonances that is explained within the present approach.

FIG. 1: (a) The field scattered by a subwavelength hole drilled in a perfect-conductor film in response to external electric (\(E^0\)) and magnetic (\(H^0\)) fields is equivalent (at a large distance compared to the radius \(a\)) to that of effective electric (\(p\)) and magnetic (\(m\)) dipoles, which allow defining polarizabilities (\(\alpha_E\) and \(\alpha_M\), respectively) both on the same side as the external fields (\(\alpha_x\)) and on the opposite side (\(\alpha'_x\)). Only the perpendicular component of the electric field and the parallel component of the magnetic field induce dipoles. (b)-(c) Real part of the hole response functions \(g_m^\pm\) for \(\epsilon = \mu = 1\).

FIG. 2: (a) Light transmittance through a circular hole drilled in a perfect metal film and filled with dielectric material for different values of the permittivity \(\epsilon\) (see labels). The transmitted power is normalized to the incoming flux times the hole area. The ratio of the film thickness to the hole radius is 0.1. (b) Ratio of transmission for \(\epsilon = 10.2\) (teflon) and \(\epsilon = 1\) (air) under the same conditions as in (a): theory (solid curve) vs experiment (symbols).

FIG. 3: Lattice sum \(G_x\) [Eq. (2)] for a square lattice of period \(d\) as a function of parallel momentum \(k_\parallel\) and wavelength.
FIG. 4: Zeroth order light transmittance through square arrays of circular holes drilled in perfect-conductor films as a function of parallel momentum $k_\parallel$ and wavelength $\lambda$. The ratio of the hole radius to the lattice spacing is taken as $a/d = 0.2$. Different values of the film thickness $t$ and the dielectric constant inside the holes $\epsilon$ are considered, as shown by labels. Both p-polarized ($\mathbf{H}$ parallel to the film) and s-polarized ($\mathbf{E}$ parallel to the film) incident light are considered.

FIG. 5: Normal-incidence transmittance (top), lattice sums and hole response functions (middle), and hole polarizability (bottom; see Fig. 4b) under the same conditions as in Fig. 4 ($a/d = 0.2$, $t/a = 0.1$, $\epsilon = 100$).

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