Dark states of dressed Bose-Einstein condensates

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We combine the ideas of dressed Bose-Einstein condensates, where an intracavity optical field allows one to design coupled, multicomponent condensates, and of dark states of quantum systems, to generate full quantum entanglement between two matter waves and two optical waves. While the matter waves are macroscopically populated, the two optical modes share a single photon. As such, this system offers a way to influence the behaviour of a macroscopic quantum system via a microscopic “knob”.

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I. INTRODUCTION

The recent experimental realization of multicomponent Bose-Einstein condensates has opened up new directions of research, including the study of the miscibility and stability of quantum fluids, the nonlinear dynamics of component separation, the generation of spatial patterns, such as e.g. the antisymmetric collective mode of two-component condensates, the generation of ferromagnetic and antiferromagnetic states, etc. In particular, the interaction of condensates with light leads to fascinating effects including the extreme slowing down of the speed of light, matter-wave four-wave mixing, and the superradiant scattering of light and atoms.

So far, multicomponent condensates have been realized by several different methods: the first one, achieved in $^{87}$Rb, relies on a fortuitous coincidence of the scattering lengths of two Zeeman sublevels; the second one, in $^{23}$Na, uses optical dipole traps to achieve the trapping of the three magnetic sublevels of the $F = 1$ hyperfine ground state. More recently, multicomponent condensates have also been achieved in nonlinear atom optics experiments where the distinction between components is via their center-of-mass motion rather than their internal state.

In a recent paper, we suggested yet another approach to the creation of multicomponent condensates, relying neither on distinct electronic nor on center-of-mass levels, but rather on the dressing of a scalar condensate by the photons of a high-$Q$ optical resonator.

The goal of the present paper is to further examine this idea in light of the recent work using dark states to achieve coherent quantum dynamics in condensates and in particular to reduce the speed of light. In that case, the dark states consist of a coherent superposition of two hyperfine ground states $|F = 1, M_F = -1\rangle$ and $|F = 2, M_F = -2\rangle$ of Sodium, but the center-of-mass motion of the atoms and the optical fields are treated classically. In contrast, the present study treats both the optical fields and the atomic center-of-mass motion quantum-mechanically. Section II introduces our model and discusses the dark states of a single isolated atom. This result is extended to the case of dressed condensates in section III. Section IV considers the effect of ground-state collisions on these states and evaluates their resulting lifetime. Finally, section V is a summary and conclusion.

II. SINGLE-ATOM DARK STATES

The experimental situation we have in mind consists of an atomic sample trapped inside a high-$Q$ optical ring resonator, where the atoms can interact with two counterpropagating light fields. In addition, they are subject to spontaneous emission resulting from their interaction with the continuum of modes of the electromagnetic field. This system is described by the Hamiltonian

$$H = H_A + H_C + H_R + H_{AC} + H_{AR}$$

where the atomic Hamiltonian

$$H_A = \frac{p^2}{2m} + \hbar \omega_0 |e\rangle \langle e|,$$

contains both the kinetic energy of the atom of momentum $p$ and mass $m$ and its internal energy $\hbar \omega_0$. Note that the form of the atomic Hamiltonian takes the ground states energy to be equal to zero without loss of generality.

The present section deals with a single atom, which can be taken to be either a two-level atom or a three-level atom with degenerate ground states $|g_+\rangle$ and $|g_-\rangle$. However, in section III, which deals with condensates, it will be necessary to consider a true $\Lambda$-type three-level system. Hence, we consider this latter situation from the very beginning. We note however at the onset that the use of a three-level system does not imply that we need to consider three different electronic levels; the full description of the atom requires to specify both its internal and its center-of-mass state, hence it is possible e.g. to construct a three-level system consisting of an excited electronic state and a single ground electronic state, but with two different momenta corresponding, for instance, to the atomic wave function propagating in opposite directions. For now, however, we assume for simplicity...
three electronic levels, since the extension to other situations is straightforward. We will return to this point following Eq. (11).

We assume that the atom interacts predominantly with two cavity modes only, further using the dipole approximation and selection rules such that each of the ground to excited state transitions is driven by just one of these modes. We then have

$$H_C = \sum_{\mu=\pm} \hbar \omega_{\mu} \hat{a}_{\mu}^\dagger \hat{a}_{\mu},$$

with $[\hat{a}_{\mu}, \hat{a}_{\mu}^\dagger] = \delta_{\mu,\mu'}$, and the atom-cavity field interaction takes the form

$$H_{AC} = \sum_{\mu=\pm} \hbar R_{\mu} e^{i k_{\mu} \cdot \mathbf{r}} \langle e | g_{\mu} | \hat{a}_{\mu} + H.c. \rangle (4)$$

Here, $\mathbf{r}$ is the center-of-mass location of the atom, with $[x_i, p_j] = i \hbar \delta_{ij}$ and $R_{\mu}$ is the strength of the dipole transition between $|g_{\mu}\rangle$ and $|e\rangle$. Finally, the atom is also dipole-coupled to the continuum of electromagnetic modes, described by the Hamiltonian

$$H_R = \sum_i \hbar \omega_i \hat{a}_{i}^\dagger \hat{a}_i,$$

with $[\hat{a}_i, \hat{a}_i^\dagger] = \delta_{ij}$, and which is responsible for spontaneous emission with

$$H_{AR} = \sum_{\mu,i} \hbar R_{\mu} e^{i k_{\mu} \cdot \mathbf{r}} \langle e | g_{\mu} | \hat{a}_i + H.c. \rangle (6)$$

However, we will not need to consider this part of the interaction explicitly in the present paper.

We consider the situation where the atom is initially so cold that the momentum width of its center-of-mass wave function is much narrower than the photon momentum $\hbar k_0$, where $k_0 = \omega_0 / c$. This is achieved in practice by cooling the atom to sub-recoil temperatures. In this case, the center-of-mass wave function can be treated as an excellent approximation as a discrete sum of plane waves separated by integer numbers of photon recoil momenta $\hbar k_{\mu}$. We further assume that the cavity initially contains only one quantum, which may be in either one of its modes, and that the atom is initially in the corresponding one of the two states of the ground electronic manifold, which interacts with the cavity photon. The coherent dynamics described by the interaction Hamiltonian (4) preserves the number of excitations in the system, but the quantum initially in the electromagnetic field can be transferred back and forth between the atom and the cavity modes. In contrast, spontaneous emission irreversibly couples the one-quantum manifold to the zero-quantum state, with the atom in its ground state manifold and the cavity in a vacuum. Hence, for the initial condition at hand we need only consider the one- and zero-quantum manifolds of states of the atom-cavity system.

Consider first the situation in the absence of spontaneous emission: Because of the electric dipole selection rules, transitions between the state $|g_{\mu}\rangle$ and $|e\rangle$ can be achieved only by absorbing or emitting a quantum from or into the “$\mu$”-mode:

$$| (g_{\mu}, Q - k_\mu); 1_\pm; 0_+ \rangle \leftrightarrow | (e, Q); 0_-, 0_+ \rangle, \quad (7)$$
$$| (g_{\mu}, Q - k_\mu); 0_\pm; 1_+ \rangle \leftrightarrow | (e, Q); 0_-, 0_+ \rangle. \quad (8)$$

Here, $Q$ is the momentum of the atom in its excited state, and the notation $|(g_{\mu}, Q)\rangle$, for example, means “atom in electronic ground state $|g_{\mu}\rangle$ with momentum $Q$”. Note that the photon recoil associated with the dipole interaction (4) has been explicitly taken into account in these relations, which show that the dipole interaction between the atom and the cavity modes only couples states within closed manifolds $\mathcal{F}_Q = \{|\psi_-(Q)\rangle, \psi_+(Q)\rangle, \psi_e(Q)\rangle\}$, with

$$|\psi_-(Q)\rangle = |(g_{\mu}, Q - k_\mu); 1_-; 0_+\rangle,$$
$$|\psi_+(Q)\rangle = |(g_{\mu}, Q - k_\mu); 0_-; 1_+\rangle,$$
$$|\psi_e(Q)\rangle = |(e, Q); 0_-; 0_+\rangle. \quad (9)$$

Within one such manifold, the general state of the system is of the form

$$|\psi(Q, t)\rangle = c_-(t)|\psi_-(Q, t)\rangle + c_+(t)|\psi_+(Q, t)\rangle + c_e(t)|\psi_e(Q, t)\rangle, \quad (10)$$

where the probability amplitudes satisfy the equations of motion

$$i \hbar \frac{d c_\mu}{dt} = \left[ \frac{\hbar^2 \mu - k_\mu}{2m} + \hbar \omega_\mu \right] c_\mu + \hbar R_{\mu}^* c_e,$$
$$i \hbar \frac{d c_e}{dt} = \frac{\hbar^2 Q^2}{2m} c_e + \sum_{\mu=\pm} \hbar R_{\mu} c_\mu. \quad (11)$$

It is useful at this point to return to our earlier discussion of the two- versus three-level atomic system: It is now quite apparent that since the ground-state manifold is an electro-translational state, characterized not just by its electronic state, but also by its center-of-mass quantum numbers, it could in fact perfectly well correspond to an atom in the same electronic state, but with different and distinguishable states of motion. Of course, the dipole selection rules used to express the interaction Hamiltonian (4) are no longer justified in that case. However, that same form of interaction, where each field mode interacts with just one of the atomic transitions in the $\Lambda$-system, can still be achieved, taking into account the different momenta of the two electro-translational ground states instead of the dipole selection rules. For instance, in the case of counterpropagating cavity modes one can detune the cavity from the transition frequency of the atom at rest, so that the Doppler shift associated with its motion brings just one of the states into resonance with one of the field modes, while the other is shifted further away from resonance, very much like in Doppler cooling geometries [23]. This requires of course that the
natural linewidth of the atomic transition be less than a recoil energy, a condition that can be achieved by using a long-lived upper electronic state.

We note that the use of an electro-translational ground-state manifold involving just one electronic state is quite interesting in that it allows one to create a multicomponent condensate simply by dressing a scalar condensate with a cavity field as discussed in Ref. [21].

In the following we continue to label the electronic ground state(s) with the symbol $|g_\mu\rangle$, keeping in mind that the single electronic ground state situation can then readily be obtained by setting $|g_+\rangle = |g_-\rangle$. None of the remaining algebra is changed then.

So far, we have ignored spontaneous emission. Its effect is to induce transitions between the one-quantum and the zero-quantum manifold of states, while imposing a random photon recoil $\hbar \mathbf{q}$ on the atom. Clearly, the realization of a dark state requires that this effect be eliminated. This can be achieved provided that the probability amplitude $c_\mu(Q)$ remains equal to zero for all times. With Eqs. (11), this implies

$$i\hbar \frac{dc_\mu}{dt} = \left[ \frac{\hbar^2 |Q - k_\mu|^2}{2m} + \hbar \omega_\mu \right] c_\mu, \quad (12)$$

and

$$\sum_{\mu=\pm} \hbar R_\mu c_\mu = 0, \quad (13)$$

Equations (12) are readily solved to give

$$c_\mu(t) = c_\mu(0)e^{-i\Omega_\mu t}, \quad (14)$$

with

$$\Omega_\mu = \frac{\hbar |Q - k_\mu|^2}{2m} + \omega_\mu. \quad (15)$$

However, the additional equation (13) requires that the probability amplitudes $c_-$ and $c_+$ have a constant phase relation, so that

$$\Omega_+ = \Omega_- \equiv \Omega, \quad (16)$$

which is nothing but a statement of energy-momentum conservation in the Raman-like transitions between the two ground electronic states. This condition may be re-expressed as

$$\left[ \frac{\hbar k_+}{2m} + u_+ \right] k_+ = \left[ \frac{\hbar k_-}{2m} + u_- \right] k_- \quad (17)$$

where

$$u_\mu = c + \frac{\mathbf{Q} \cdot \mathbf{k}_\mu}{\mu k_\mu} \quad (18)$$

and the second term on the right-hand side is the component of the atomic recoil velocity $\hbar \mathbf{k}_\mu$ along $\mathbf{Q}$. Typical recoil velocities are of the order of a few cm/sec, hence $u_\mu \simeq c$, and Eq. (17) implies that

$$k_+ \simeq k_- \quad (19)$$

the equality being exact for $Q = 0$, in which case the corrections due to the recoil momentum cancel and

$$k_+ = k_- \equiv k. \quad (20)$$

In that case, the dark state $|\psi_{\text{dark}}\rangle$ of the system is simply

$$|\psi_{\text{dark}}(Q = 0, t)\rangle = \frac{1}{\sqrt{|R_+|^2 + |R_-|^2}} \times [R_-|\psi_+(Q = 0)\rangle - R_+|\psi_-(Q = 0)\rangle] e^{-i\Omega t}, \quad (21)$$

where $\Omega = \hbar k^2/2m + c k$.

### III. DRESSED CONDENSATE DARK STATES

With the single-atom results in mind, we now turn to the situation of a condensate dressed by the cavity field modes. We assume that the atoms forming the condensate have the same $\Lambda$-like internal structure as before, with two degenerate ground states $|g_\mu\rangle$, $\mu = \pm$, which are both coupled to a single excited state $|e\rangle$. The condensate is then described by a three-component Schrödinger field

$$\hat{\psi}(r) = \begin{pmatrix} \hat{\psi}_-(r) \\ \hat{\psi}_+(r) \\ \hat{\psi}_e(r) \end{pmatrix} \quad (22)$$

with $[\hat{\psi}_i(r), \hat{\psi}_j^\dagger(r')] = \delta_{ij} \delta(\mathbf{r} - \mathbf{r}')$ and $i,j = \pm,e$. Because of the central role photon recoil plays in this problem, it is convenient to expand these components in terms of plane waves as

$$\hat{\psi}_i(r) = \frac{1}{\sqrt{V}} \sum_q e^{i\mathbf{q} \cdot \mathbf{r}} \hat{c}_{i,q} \quad (23)$$

with

$$[\hat{c}_{i,q}, \hat{c}_{j,q'}^\dagger] = \delta_{ij} \delta(\mathbf{q} - \mathbf{q}') \quad (24)$$

in terms of which the second-quantized version $H_0$ of the Hamiltonian $H_0 = H_A + H_C + H_{AC}$ is

$$H_0 = \sum_{i,q} \frac{\hbar^2 q^2}{2m} \hat{c}_{i,q}^\dagger \hat{c}_{i,q} + \sum_{\mu,q} \hbar \omega_\mu \hat{c}_{\mu,e,q}^\dagger \hat{c}_{\mu,e,q} + \hbar R_\mu \hat{a}_\mu \hat{a}_\mu^\dagger \hat{c}_{\mu,e,q} - \mathbf{k}_\mu + H.c. \quad (25)$$

where $i = \pm, e$ and $\mu = \pm$.

We consider a condensate consisting of $N$ atoms, and assume as before that there is at most one quantum of excitation in the condensate-cavity system. It is
not difficult to generalize the single-atom discussion to this case. Instead of the family of closed manifolds \( F_{Q} \) used for the single-atom case we now need to consider the extended set of manifolds \( F_{Q}(N_{-}, N_{+}) \). Indeed, it is easily shown that the manifold \( F_{Q}(N_{-}, N_{+}) = \{ |\psi_{-}(Q, N_{-}, N_{+})\rangle, |\psi_{+}(Q, N_{-}, N_{+})\rangle, |\psi_{e}(Q, N_{-}, N_{+})\rangle \} \) forms a closed set of states for the condensate-cavity evolution we get from the atom-cavity field interaction described in \( H_{a} \), where

\[
|\psi_{-}(Q, N_{-}, N_{+})\rangle = \\
|\psi_{+}(Q, N_{-}, N_{+})\rangle = \\
|\psi_{e}(Q, N_{-}, N_{+})\rangle =
\]

and \( N_{+} + N_{-} = N + 1 \).

In the following we restrict our discussion to the case \( Q = 0, k_{-} = -k_{+} \equiv k, k = \omega/c \). The extension to more general situations is straightforward. For notational clarity we also temporarily drop the variables \( Q \) as well as \( N_{-} \) and \( N_{+} \) from the definitions of the states, and express the state of the system as in the single-atom case as

\[
|\psi(t)\rangle = c_{-}(t)|\bar{\psi}_{-}\rangle + c_{e}(t)|\bar{\psi}_{e}\rangle + c_{+}(t)|\bar{\psi}_{+}\rangle.
\]

The equations of motion for the probability amplitudes \( c_{i}(t) \) are now

\[
\begin{align*}
\frac{i\hbar}{m} & \frac{dc_{\mu}}{dt} = \left[ N \frac{\hbar^2 k^2}{2m} + \hbar \omega_{\mu} \right] c_{\mu} + \hbar R_{\mu}^{*} \sqrt{N_{\mu}} c_{e}, \\
\frac{i\hbar}{m} & \frac{dc_{e}}{dt} = (N - 1) \frac{\hbar^2 k^2}{2m} c_{e} + \sum_{\mu = \pm} \hbar R_{\mu} \sqrt{N_{\mu}} c_{\mu},
\end{align*}
\]

where \( \mu = \pm \).

Despite the fact that we are now considering an \( N \)-atom problem, the selection rules of the atomic system and the choice of the initial condition result in these equations having the same form as the single-particle equations (13), with two differences: First, the dipole coupling constants \( R_{\mu} \) are now replaced by

\[
R_{\mu} \rightarrow R_{\mu} \sqrt{N_{\mu}},
\]

a result of the collective action (Bose enhancement) from the \( N_{\mu} \) atoms in the \( \mu \)'s condensate component. Second, the kinetic energy term now accounts for all \( N \) atoms, the \((N-1)\)-factor in the equation for \( c_{e} \) resulting from photon recoil. With these changes, one can readily adapt the results of section II, and find that the many-particle dark state associated with a given manifold \( \{ Q = 0, N_{-}, N_{+} \} \) is

\[
|\psi_{\text{dark}}(Q = 0, N_{-}, N_{+}, t)\rangle = \frac{1}{\sqrt{N_{+}} |R_{+}|^2 + N_{-} |R_{-}|^2} \\
\times \left[ \sqrt{N_{-}} R_{+} |\psi_{+}(Q = 0, N_{-}, N_{+})\rangle - \sqrt{N_{+}} R_{-} |\psi_{-}(Q = 0, N_{-}, N_{+})\rangle \right] e^{-i\Omega_{N} t},
\]

where

\[
\Omega_{N} = N \frac{\hbar k^2}{2m} + ck.
\]

Hence, dark states of the multicomponent condensate can exist for any value of their relative populations. However, there is an important distinction between the present case and the single-atom situation, because of the effect of ground state collisions, that we have ignored so far. For ultracold atoms, they can be described in the shapeless approximation by a local two-body interaction, that is, a very broad potential in momentum space. Ground-state collisions therefore result in velocity changes in the atoms, and hence in the atoms escaping from the dark states (23). The next section evaluates the lifetime associated with these collisions.

**IV. THE EFFECT OF GROUND-STATE COLLISIONS**

In the shapeless, s-wave scattering approximation, the ground-state collisions are described by the Hamiltonian

\[
\mathcal{V} = \frac{4\pi\hbar^2}{mV} \sum_{\mu,\mu'=\pm} \sum_{\alpha,\alpha'} \hat{c}_{\mu,\alpha}^{\dagger} \hat{c}_{\mu',\alpha'} \delta_{\mu,\mu'} \delta_{\alpha,\alpha'}
\]

where \( a \) is the scattering length of the collisions, assumed to be the same for both ground states.

The \( \kappa = 0 \) contribution to \( \mathcal{V} \) corresponds to velocity-conserving collisions, which do not lead to an escape of the condensates from the dark state. Hence, it is useful to treat it separately. Reexpressing it as

\[
\mathcal{V}_{0} = \frac{4\pi\hbar^2}{mV} \sum_{\mu,\mu'=\pm} \sum_{\alpha,\alpha'} \hat{c}_{\mu,\alpha}^{\dagger} \hat{c}_{\mu',\alpha'} \delta_{\mu,\mu'} \delta_{\alpha,\alpha'}
\]

where \( \hat{N}_{g} \) is the number operator for all ground-state atoms, we observe that the states \( |\psi_{\mu}(Q, N_{-}, N_{+})\rangle \) are eigenstates of that operator, specifically,

\[
\begin{align*}
\mathcal{V}_{0} |\psi_{-}\rangle &= \frac{4\pi\hbar^2}{mV} (N_{-} - N) |\psi_{-}\rangle, \\
\mathcal{V}_{0} |\psi_{+}\rangle &= \frac{4\pi\hbar^2}{mV} (N_{+} - N) |\psi_{+}\rangle, \\
\mathcal{V}_{0} |\psi_{e}\rangle &= \frac{4\pi\hbar^2}{mV} (N_{-} - N_{+} - N + 1) |\psi_{e}\rangle.
\end{align*}
\]
Hence, its effect is merely to change the eigenenergy \( \hbar \Omega_N \) of the dark state, see Eq. (30), to
\[
\hbar \Omega_N = N \frac{\hbar^2 k^2}{2m} + \hbar c k + \frac{4 \pi \hbar^2 a}{mV} (N^2 - N). \tag{34}
\]

The remaining collisions, described by \( \mathcal{V}' \), couple the atoms in the dark state of the condensate to a continuum of momentum states, which is essentially flat in the shapeless approximation. These momentum states may be thought of as a bath, and the decay of the dark state into that reservoir can be evaluated via Fermi’s Golden Rule. Hence, its effect is merely to change the eigenenergy \( \bar{\epsilon} \) of the dark state, see Eq. (30), to
\[
\hbar \Gamma = 32 \pi \frac{\hbar a^2}{mV} N_+ N_-, \tag{40}
\]
where we have used Eq. (38) to express the energy \( \epsilon \) in terms of the photon recoil momentum \( \hbar k \). For typical condensates containing of the order of \( 10^6 \) atoms at a density of \( 10^{12} \text{ cm}^{-3} \), a scattering length \( a \approx 10^{-7} \text{ cm} \) and visible light we find that \( \Gamma \approx 10^6 \text{ sec}^{-1} \). For such relatively short lifetimes, it is acceptable to neglect the effects of optical cavity loss as we have done here, provided that one uses the high-\( Q \) cavities familiar e.g. from cavity QED experiments. Since \( \Gamma \) scales as the square of the number of atoms in the condensate, longer lifetimes can easily be achieved, but in that case, cavity losses are expected to start playing an important role and to dominate the lifetime of the condensate dark states.

V. SUMMARY AND CONCLUSION

Dressing scalar condensates with electromagnetic fields allows one to design coupled, multicomponent condensates that permit to study the dynamics of coupled matter-wave fields under well controled conditions. In addition, coherent control techniques such as electromagnetically induced transparency allow one to manipulate the optical properties of condensates in dramatic ways, opening up the way to novel investigations involving e.g. the coupling of slow optical waves and acoustic waves, etc. In this paper, we have combined these two techniques to realize dark states of dressed condensates. These states exhibit full quantum mechanical entanglement between the four “modes” involved, two matter waves and two optical modes. While the matter waves are macroscopically populated, the two cavity modes share a single photon. As such, this system offers a way to influence the behaviour of a macroscopic quantum system via a microscopic “knob” and might be of use in elucidating fundamental questions in quantum measurement theory and quantum information processing. In particular, this is precisely the situation Schrödinger had in mind in his cat “paradox.”

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[1] C. J. Myatt et al., Phys. Rev. Lett. 78, 586 (1997).
[2] D. M. Stamper-Kurn et al., Phys. Rev. Lett. 80, 2027 (1998).
[3] J. Stenger et al., Nature 396, 345 (1998).
[4] E. Timmermans, Phys. Rev. Lett. 81, 5718 (1998).
[5] E. V. Goldstein and P. Meystre, Phys. Rev. A 55, 2935 (1997).
[6] H. Pu and N. P. Bigelow, Phys. Rev. Lett. 80, 1130 (1998).
[7] H. Pu and N. P. Bigelow, Phys. Rev. Lett. 80, 1134 (1998).
[8] J. Williams et al., cond-mat/9904399 (1999).
[9] T. Ohmi and K. Machida, J. Phys. Soc. Japan 67, 1882 (1998).
[10] T.-L. Ho, Phys. Rev. Lett. 81, 742 (1998).
[11] C. K. Law, H. Pu, and N. P. Bigelow, Phys. Rev. Lett. 81, 5257 (1998).
[12] L. V. Hau, S. E. Harris, Z. Dutton, and C. H. Behroozi, Nature 397, 594 (1999).
[13] E. V. Goldstein, K. Pl¨ attner, and P. Meystre, Quant. Semiclas. Optics 7, 743 (1995).
[14] E. V. Goldstein, K. Pl¨ attner, and P. Meystre, Jrnl. Res. Nat. Inst. Stand. Technol. 101, 583 (1996).
[15] L. Deng et al., Nature 398, 218 (1999).
[16] M. G. Moore and P. Meystre, Phys. Rev. A 58, 3248 (1998).
[17] M. G. Moore and P. Meystre, Phys. Rev. A 59, R1754 (1999).
[18] S. Inouye et al., unpublished (1999).
[19] E. V. Goldstein and P. Meystre, Phys. Rev. A 59, 1509 (1999).
[20] E. V. Goldstein and P. Meystre, Phys. Rev. A 59, 3896 (1999).
[21] E. V. Goldstein, E. M. Wright, and P. Meystre, Phys. Rev. A 57, 1223 (1998).
[22] P. Meystre and M. Sargent III, Elements of Quantum Optics (Springer-Verlag, Heidelberg, 1998).
[23] C. Cohen-Tannoudji, in Fundamental Systems in Quantum Optics, edited by J. Dalibard, J.-M. Raimond, and J. Zinn-Justin (North-Holland, Amsterdam, 1992), p. 1.