Optimal logistics planning for modular construction using two-stage stochastic programming

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\section*{1. Introduction}

Over recent years the practice of modular construction, using the design for manufacturing and assembly (DfMA) method, has progressively entered the construction industry replacing the traditional stick-built approach to construction. The benefits of modular construction come from manufacturing building components in a factory environment, where higher efficiencies and quality can be achieved, the need for space to store materials or equipment on site is reduced, and the assembly process is significantly shortened \cite{4-6}.

The structure of the supply chain for traditional stick-built construction projects is usually composed of multiple raw material suppliers that directly deliver to construction sites, with seldom use of consolidation centres for temporary storage \cite{1}. In this paradigm, raw building materials are dispatched to sites on demand, in response to orders placed by the site. Therefore, the configuration of the overall supply chain is straightforward as all the information for decision making such as quantity and timing of materials can be obtained from the construction schedule \cite{2}. In the event of construction schedule deviations, which make the site material demands diminish, deliveries can be temporarily held in supplier warehouses or stored in consolidation centres for temporary storage \cite{11}, thus introducing in the supply chain another layer for buffering.

Due to the dependence on complex and unique production processes, and the production of tailor-made and project-specific products, modular construction supply chains are often distinguished from the logistics arrangements encountered in other sectors and are often studied separately \cite{6}. For instance, commonly adopted assumptions such as the ability to procure products at a higher price from other manufacturers, when production fails to meet the demand, may not apply to modular construction. More so, the total quantity of modular products produced in the factory normally matches the demand from the construction sites, and consequently the inventory will reach zero when a construction site is completed.

On the contrary, building materials in modular construction projects are rarely sent directly to sites. Instead, most materials are initially transported to the manufacturing facility where they are transformed into modular products and components \cite{4,5}. The modular components leaving a factory are typically large and cumbersome, necessitating extra caution when they are carried across public roads and assembled on site \cite{10}. Furthermore, many construction sites are located in urban settings with limited storage space, which makes warehouses essential for temporary storage \cite{11}, thus introducing in the supply chain another layer for buffering.

The construction sector is currently undergoing a shift from stick-built construction to modular building systems that take advantage of modern prefabrication techniques. Long established in-situ construction practices are thus being replaced by processes imported from the manufacturing sector, where component fabrication takes place within a factory environment. As a result of this transformation, current construction supply chains, which have focused on the delivery of raw materials to sites, are no longer apt and need to make way to new, strengthened, and time-critical logistics systems. The aim of this study is to establish a mathematical model for the optimisation of logistics processes in modular construction covering three tiers of operation: manufacturing, storage and assembly. Previous studies have indicated that construction site delays constitute the largest cause of schedule deviations. Using the model outlined in this paper we seek to determine how factory manufacturing and inventory management should react to variations in the demand on construction sites. A two-stage stochastic programming model is developed to capture all possible demand variations on site. The model is evaluated using a case study from the residential construction sector. The application shows that the model is effective and can serve as decision support to optimise modular construction logistics.

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project ends. This phenomenon is in contrast to the supply chains of ordinary retail products, where inventory needs to be preserved as safety stock. Furthermore, unlike ordinary retail products, additional assembly processes and costs are required after the products are delivered to the site.

Satisfying the demands on modular construction sites is often challenging as various delay factors can introduce uncertainty. The construction industry regards delays in construction schedules to be inevitable [7] and directly linked to additional costs [8]. Their causes can generally be traced to factors internal to construction sites such as human errors and equipment failures [9] as much as external factors such as extreme weather conditions. Previous studies have focused on the mitigation of construction delays in stick-built projects through supply chain interventions [3]. However, the adoption of modular construction practices introduces an additional layer of complexity, given the interplay of manufacturing, transportation and assembly (MtA) processes.

The optimal configuration for a three-tier modular construction supply chain including a manufacturing factory, a storage facility, and construction sites has not been studied before. Previous research on the supply chains of stick-built construction as well as general merchandise supply chain including a manufacturing factory, a storage facility, and construction sites has been conducted by Gündüz et al. [9] and MirHassani et al. [30]. However, the differences between these supply chains are significant, and it is not possible to extend the findings from these studies to the modular construction context.

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This research aims at addressing this knowledge gap by establishing a mathematical model to optimise logistics processes in modular construction. The model proposed in this study, accounting for stochastic demands on site, is capable of identifying the optimal factory production plan and transportation scheme, and revealing the change of inventory in the factory, warehouse and site for multiple time horizons. The outputs of the model can be of great value to managers responsible of either issuing plans to be executed in the future or making changes to existing plans to account for recently revealed data.

The rest of the paper is structured as follows. Section 2 reviews work on supply chain integration, the origins of demands uncertainty in modular construction and stochastic supply chain network design. Section 3 introduces the methodology including assumptions and model formulation and objective function. Section 4 covers model implementation, case study and discussion. Section 5 draws the conclusions.

2. Background

In this section, we provide an overview of current research in supply chain integration under demand fluctuation in both the manufacturing and construction industry. Methods for the design of optimal supply chain configurations under uncertain demands are also reviewed.

2.1. Supply chain integration

Supply chains are commonly modelled as dynamic systems involving multiple interacting parties. Careful coordination is considered essential across the organisational boundaries to create a seamless and value-added process for fulfilling all customer needs [12].

Chandra and Fisher [13] pointed out that in supply chain design, the activities in different echelons should be considered simultaneously to achieve high overall efficiency. Coelho and Laporte [14] suggested that decisions in production planning, shipment scheduling and inventory management should be modelled in a single problem statement. In this context, a model and its optimal solution can serve as the foundation for tactical decision making in complex supply chain design [15].

Supply chains integrating multiple stakeholders have been previously explored and modelled. Lei et al. [16], for example, developed a model which considered production, inventory, and distribution synchronously. In their study, the most appropriate operation schedule needed to satisfy constraints including customer demand, vehicle capacity, and production limits.

A recent review by Díaz-Madroñero et al. [19], summarising research trends in supply chain integration studies, found that a commonly adopted requirement is the absolute satisfaction of customer demands. However, in a supply chain, customers, distributors, and suppliers typically have different and occasionally conflicting objectives. For instance, customers are predominantly concerned with their demand being fulfilled on time, while distributors aim at dispatching all the goods within minimum vehicle run, and suppliers focus on producing products at the lowest cost with demand fulfilment as a secondary goal. To address them concurrently stakeholders’ objectives should be jointly considered and linked together. In this context, a network-based analysis is, therefore, well-placed to determine the freight transport configuration that maximises the profit of every stakeholder [17]. Elimam and Dodin [18] combined the production and distribution chains into one integrated supply chain (ISC), which includes all the stakeholders over multiple echelons. The ISC is represented as a project network with different types of activities, thus capturing all the flow of information and goods within the supply chain.

In the studies reviewed above, mixed integer linear programming (MILP) is the most commonly used modelling framework for supply chain design problems, with solutions typically obtained by a mix of exact algorithms, meta-heuristics and decomposition techniques [19,56,61].

2.2. Origins of demand uncertainty in modular construction

It is common practice in modern construction that the specific material demand and delivery plans are prepared during the design stage. When construction starts, there are many factors that will ultimately lead to deviations from the originally planned schedules. Gündüz et al. [9] listed 83 distinct factors causing delays in building projects with over 90% traceable to activities within construction sites. When delays occur, the actual progress of the project lags behind the original schedules. Material demands will, therefore, decrease, and project duration will have to be extended, incurring additional costs [8]. Since delays in construction schedules are almost inevitable [7], and changes in the demand often have a severe impact on the upstream logistics, their effect must be carefully taken into consideration [20,62].

Zou et al. [21] proposed a technique for predicting the demand uncertainties on the construction site by using a probabilistic analysis on historical data. Generally speaking, the demand variation of construction material can often be conceived in a stochastic form [22–24], which is established by combing various reasons for causing schedule deviations with their severity toward the original demand. Illustrating these demand uncertainties in a mathematical model and identifying the optimal solution, which can effectively mitigate the extra cost incurred are problems that have to be addressed when building a logistic system for modular construction.

2.3. Stochastic supply chain network design

Various methods have been proposed to deal with uncertainties in supply chain design, such as stochastic programming, fuzzy set theory, robust optimisation and dynamic stochastic programming [25]. Among them, the two-stage stochastic programming has been commonly adopted, because it can capture different types of uncertainties, which may occur in the supply chain [26]. Moreover, it can output the optimal supply chain configuration in terms of production planning, inventory management, capacity planning and routing [27]. Ierapetritou and Pistikopoulos [28] sought the optimal production lot quantities when customers’ demands are uncertain. The problem was modelled by two-stage stochastic programming with an objective function for minimising the total operational cost and the expected penalty cost incurred by the unmet customer demand.

Tsikakis et al. [29] and MrHassani et al. [30] implemented a two-stage stochastic programming model to find out the optimal network
design and tactical planning in a multiple echelon supply chain with uncertain demand. Two-stage stochastic programming was also proposed to optimize the design of a multi-product, multi-period and multi-site supply chain under demand uncertainty [31–33]. A supply chain with short and long-term disruptions owing to external factors was also modelled by two-stage stochastic programming to pursue its best configuration and resilience strategies [34,35].

The uncertainties and disruptions in the supply chain are usually illustrated in scenarios, which represent situations that may happen in the future. Nevertheless, to truly reflect these uncertainties in the real world, the number of scenarios often grows exponentially. Methods such as sample average approximation and Benders decomposition are sometimes employed to solve two-stage stochastic programming with vast numbers of scenarios [36–38].

Discrete-Event Simulation (DES) has been used in several studies focusing on logistics planning for offsite construction. Bu Hamdan et al. [57] used DES to determine optimal inventory management strategies for modular construction components, whose dimensions and assembly sequences were extracted from building information models (BIM).

Mohsen et al. [58] used DES to predict the construction duration of projects making use of modular construction methods. The study presents a software tool called Simphony. NET that allows to determine project execution times under various sources of onsite assembly uncertainties.

Han et al. [59] mention that DES is often employed to model factory-based production processes for modular construction components. However, given that the outputs of DES can often be misrepresented, they suggest to use 3D visualisation to improve the communication of DES results to site engineers.

Taghaddos et al. [60] developed a simulation-based protocol for resource scheduling in large-scale construction projects. Their analysis suggests that the proposed technique can be successfully deployed for activity scheduling and resource levelling in modular construction projects.

It is worth mentioning that while DES models are extensively used in practice, it is often difficult to replicate outputs particularly when proprietary software, extensions and models are used. Furthermore, such models are particularly sensitive to changes in logic and practical arrangements, which often necessitate major modifications to the underlying models.

### 2.4. Two-stage stochastic programming and need for a new integrated logistics model

In line with previous research, this study adopts a two-stage stochastic programming approach for the simultaneous consideration of the various stochastic elements affecting supply chain configurations. In contrast to some of the simulation-based techniques used by the industry, our proposed model is based on a mathematical model that can be implemented using a range of modelling environments.

Since modular construction has both characteristics of construction and manufacturing, decisions on the most favourable production scheme can be made in the first stage, while considering the expected transportation, inventory, assembling and the penalty costs that may occur under all demand uncertainties in the second stage. However, the current practice adopted in the modular construction field in general is to learn from practice and borrow experience from traditional construction. This trial and error approach often is problematic. It is, therefore, deemed essential to proceed with the development of a tailored integrated logistics model that specifically targets modular construction.

### 3. Methodology

This section introduces the assumptions used in the development of the mathematical model, its structure and implementation. A case study with an application of the technique to a problem instance from an industrial setting is discussed in the following section.

A complete list of indices, parameters and variables is provided in the Appendix A. As discussed in the introduction, the key distinguishing trait of projects using DfMA is that most of the construction components are produced in an off-site manufacturing facility (hereafter referred to as factory for simplicity). The factory can manufacture all types of modular products \( j \in J \) (where \( J \) is the set of modular products) for supporting multiple ongoing construction sites \( i \in I \) (where \( I \) is the set of construction sites). The factory has a fixed daily overhead (\( MF \)) which exists regardless of its production quantity. The factory fix production cost (\( FPC \)) is the product of the fixed overhead and the overall factory manufacturing duration for the whole project, which encompasses the period of time for preparing the initial inventory, which can last for \( IID \) days, and the period after construction starts, which runs for \( CMD \) days.

The days are counted by indices \( p (p \in P, \text{where} \ P = 1, 2 \ldots IID) \) during the preparation phase, and \( t \) (where \( t = 1, 2 \ldots CMD \)) in the construction phase.

For the production in the factory, this research assumes that every kind of modular product has a basal unitary manufacturing cost (\( MV_j \)), a maximum daily manufacturing rate (\( MRM_j \)), which cannot be exceeded, and an adjustment factor (\( ADJ_j \)), which reflects the rise of \( MV_j \) as the daily production rate approaches \( MRM_j \) due to the deviation from the most economical marginal variable production cost. The variable production cost (\( VPC \)) is composed of two parts. The first part regards the initial inventory, which is obtained by summing up the daily quantity of each kind of modular products \( m_{pj} \) (\( B \) denotes “before construction starts”) produced in the initial inventory preparation period and then multiplying by \( MV_j \). It should be noted that \( MV_j \) needs to be adjusted for the resource utilisation ratio:\( ADJ_j(m_{pj}/MRM_j) \). Thus \( VPC \) for each kind of modular product during the initial inventory preparation stage is denoted: \( m_{pj} \cdot MV_j / [1 + ADJ_j(m_{pj}/MRM_j)] \). The second term is for the production after construction starts, and is calculated using a method similar to the first term, except that \( m_{pj} \) is replaced by the daily production quantity in this time period, \( m_{pj} \).

For the construction stage, although assembly schedule for modular products is fixed at the end of the planning phase, this research anticipates that delays may occur during the assembly process, yielding uncertainties in daily demand for modular products. The case study presented later in this paper focuses on the site demand variation aroused from the following disruptions: inclement weather, workers’ productivity fluctuations, late transportation and crane’s operation reliability issues. They are all assumed to be independent to one another and characterised by their occurrence probability. By considering all these disruptions simultaneously, a set of possible demand scenarios can be generated. In this study, \( D_{stij} \) represents the demand for modular product \( j \) on day \( t \) (\( t \in T \) where \( T = 1, 2 \ldots LWD \)) for site \( i \) in scenario \( s \) (\( s \in S \), where \( S \) is the set of demand scenarios). It is assumed that each demand scenario has a probability \( SFi \), of happening and each scenario has a working duration (\( DD_s \)) for reaching the assembling target. This duration may be different in each scenario owing to different level of disruptions. The longest working duration among all scenarios is denoted as \( LWD \).

The site fix cost (\( SFC_i \)) in each scenario is the sum of the product of the working duration (\( DD_s \)) and the site overhead (\( SF_i \)) for each site. The total assembly cost (\( AC_j \)) is determined by summing up \( D_{stij} \cdot AC_j \) for each day, each site and each product, where \( AC_j \) is the cost for assembling one unit of modular product \( j \). The extra operational cost on site (hereafter referred to penalty cost, \( PY_j \)) is incurred in a scenario when the daily demands on site are not met, then the site is forced to operate beyond the originally planned working period to assemble the late delivered modules, provoking an extra site fix cost.

This model also includes inventory management considerations. In this research, manufactured items for building up the initial inventory before the start of the construction are only stored in the warehouse, given that the storage spaces at the factory are usually occupied by
other projects during this period of time, and no inventory protection is available on site. Under this circumstance, the quantity of initial inventory in the factory \( (n_{W1}^{F}) \) and sites \( (n_{W1}^{S}) \) are zero before the commencement of construction, and the initial inventory in the warehouse \( (n_{W1}^{W}) \) increases daily. Therefore, the product holding cost in the initial inventory preparation phase \( (ICTP) \) is the sum of the daily cost for storing the initial inventory of each product, which is expressed as \( n_{W1}^{W} \cdot VOL_{t} \cdot WIC \), where \( VOL_{t} \) is the volume of modular product and \( WIC \) is the daily unitary storage cost in the warehouse. The inventories of each product at factory, warehouse and sites on each day after construction starts are demand scenario specific, and denoted as \( n_{W1}^{F} \), \( n_{W1}^{W} \) and \( n_{W1}^{S} \), respectively. The daily inventory cost of each product in each scenario in this period is composed by the inventory costs within the factory \( (n_{W1}^{F} \cdot VOL_{t} \cdot FCAP) \), warehouse \( (n_{W1}^{W} \cdot VOL_{t} \cdot WCAP) \) and site \( (n_{W1}^{S} \cdot VOL_{t} \cdot SIC) \), where \( FCAP \) and \( SIC \) are the daily unitary storage costs in the factory and site, respectively. The summation of the daily storage cost of each product in each place gives the total inventory cost in each scenario \( (IC_{i}) \). It is worth noting that, since most modular products are tailor-made for a project with exact quantity, \( n_{W1}^{F} \), \( n_{W1}^{W} \) and \( n_{W1}^{S} \) are expected to be exhausted by the end of the project. Moreover, assumptions have been made that the total volumes of inventory cannot exceed their respective storage capacities in the factory \( (FCAP) \), warehouse \( (WCAP) \) and site \( (SIC) \) at all time.

For the transportation process, all modular products are transported from the warehouse to construction sites according to the daily demand for each product at each location on a just-in-time basis. The number of truck needed daily for transporting each product is calculated by dividing the daily transportation quantity from the warehouse to the site \( (t_{W1}^{S}) \) by the quantity of product that can be loaded onto a single truck \( (NL_{S1}^{W}) \) running in this sector. The transportation cost from the warehouse to the site in each scenario \( (TCWS) \) then is determined as the sum of the product of the truck numbers, the distance in between \( (DWS) \) and the unitary transportation cost \( (CWS) \) of each day, each site and each kind of modular product. The transportation costs for moving products from the factory to the warehouse in the initial inventory preparation phase \( (TCFW) \) and the construction phase \( (TCW) \) are calculated in the same manner using the corresponding variables and parameters: the daily transportation quantities of each kind of product in the preparation phase \( (t_{W1}^{F}) \) and construction phase \( (t_{W1}^{S}) \), the truck loading quantity \( (NL_{S1}^{W}) \), the moving distance \( (DFW) \) and the unitary transportation cost \( (CFW) \).

### 3.1. Objective function

The objective of the model presented in this paper is to minimise the total cost \( TC \) required during the time period to build up the initial inventory and complete construction projects. \( TC \) as defined by Eq. (1) is, therefore, the sum of the following costs terms:

- The fixed and variable manufacturing costs \( (FPC \) and \( VPC) \).
- The inventory cost for holding inventory in a warehouse in the initial inventory preparation phase \( (ICTP) \).
- The inventory cost for holding products in a factory, warehouse and sites within each demand scenario in the construction phase \( (IC_{i}) \).
- The transportation costs for moving products from the factory to the warehouse and from the warehouse to all sites within each demand scenario in the construction phase \( (TCFW, \) and \( TCWS) \).
- The transportation costs for moving products from the factory to the warehouse in the initial inventory preparation phase \( (TCFW) \).
- The total site fix cost and assembly cost within each demand scenario \( (SFC_{i}, \) and \( AC_{i}) \).
- Anticipated penalty costs within each demand scenario \( (PY_{i}) \).

Minimise:

\[
TC = FPC + VPC + TCFW + IC + \sum_{j \in S} SP_{j} \cdot (IC_{j} + TCFW_{j} + TCWS_{j}) + SFC_{i} + AC_{i} + PY_{i} \tag{1}
\]

\[
FPC = MFP \cdot CMD + MFP \cdot IID \tag{1.1}
\]

\[
VPC = \sum_{i=1}^{\infty} \sum_{j \in J} m_{ij} \cdot MV_{ij} \cdot \left( 1 + AD_{ij} \cdot \left( \frac{m_{ij}}{MRM_{ij}} \right) \right) + \sum_{j \in J} \sum_{i=1}^{\infty} m_{ij} \cdot MV_{ij} \cdot \left( 1 + AD_{ij} \cdot \left( \frac{m_{ij}}{MRM_{ij}} \right) \right) \tag{1.2}
\]

\[
IC_{i} = \sum_{t \in T} \sum_{j \in J} n_{ij}^{W} \cdot Vol_{ij} \cdot FCAP + \sum_{t \in T} \sum_{j \in J} n_{ij}^{W} \cdot Vol_{ij} \cdot WCAP + \sum_{t \in T} \sum_{j \in J} \max(n_{ij}^{S}, 0) \cdot Vol_{ij} \cdot SIC \tag{1.3}
\]

\[
TCFW_{i} = \sum_{t \in T} \sum_{j \in J} l_{ij}^{F} \cdot DFW \cdot CFW \tag{1.5}
\]

\[
TCWS_{i} = \sum_{t \in T} \sum_{j \in J} l_{ij}^{W} \cdot DWS \cdot CWS \tag{1.6}
\]

\[
TCFW = \sum_{t \in T} \sum_{j \in J} l_{ij}^{F} \cdot DFW \cdot CFW \tag{1.7}
\]

\[
SFC_{i} = \sum_{t \in T} SF_{i} \cdot DD_{i} \tag{1.8}
\]

\[
AC_{i} = \sum_{t \in T} \sum_{j \in J} D_{ij} \cdot AC_{j} \tag{1.9}
\]

\[
PY_{i} = \sum_{t \in T} \max(CMD - DD_{i}, 0) \cdot SF_{i} \tag{1.10}
\]

Eqs. (1.1) and (1.2) calculate the fixed and variable production costs for manufacturing all types of products \( (FPC \) and \( VPC) \), respectively, in the period encompassing the initial inventory preparation and construction phases. Note that the variable production cost is a function of the relative daily manufacturing rate.

Eq. (1.3) calculates the inventory cost for holding the initial inventory at the warehouse \( (IC_{i}) \) before construction starts. Note that the initial inventory is only allowed to be stored in the warehouse. Eq. (1.4) calculates the total inventory cost after construction starts for each scenarios \( (IC_{i}) \). It includes the inventory costs in factory, warehouse and construction sites. It should be noted that in the last term of Eq. (1.4) only non-negative quantities of inventory are included in the calculation of the inventory cost on construction sites in each scenario. In some scenario the demands are not met for a certain time period, and the calculated inventories on site become negative on these days. These negative numbers stand for the unfilled demands, but do not represent the real quantities of inventory, which should be equal to 0 under these circumstances.

Eqs. (1.5) and (1.6) calculate the transportation cost for moving products from the factory to the warehouse and from the warehouse to the construction site for all scenarios \( (TCFW, \) and \( TCWS) \), respectively. Eq. (1.7) calculates the transportation cost for moving the initial inventory from the factory to the warehouse \( (TCFW) \).

Eq. (1.8) calculates the site fix cost in each scenario \( (SFC_{i}) \). Eq. (1.9) calculates the assembly cost for assembling all types of products on all sites in each scenario \( (AC_{i}) \). Eq. (1.10) calculates the penalty cost for each scenario. The penalty is incurred in a scenario when the daily demands on site are not met, then the site is forced to extend its working period giving rise to an extra site fix cost.

Subject to:
Table 1
The input parameters of the model.

| Data names                                      | Values                                      |
|------------------------------------------------|---------------------------------------------|
| Number of the construction site                | 1 site                                      |
| Type of modular bathroom pod                   | Pod A; Pod B                                |
| Maximum number of Pod can be assembled         | 20 Pod A/day; 18 Pod B/day                  |
| Assembly cost of modular products              | £250/Pod A; £200/Pod B                      |
| Construction site fix cost (overhead)          | £1600/day                                   |
| Maximum storage capacity on site               | 40 m³                                        |
| Maximum storage capacity at warehouse          | 1600 m³                                     |
| Maximum storage capacity at factory            | 60 m³                                        |
| Inventory cost on site                         | £1/m³/day                                    |
| Inventory cost at warehouse                    | £2/m³/day                                    |
| Inventory cost at factory                      | £1/m³/day                                    |
| Volume of the modular products                 | 10 m³/Pod A; 5 m³/Pod B                      |
| Distance between factory and warehouse         | 30 km                                        |
| Distance between warehouse and site            | 8 km                                         |
| Transportation cost from factory to warehouse  | £1.73/truck/km                               |
| Transportation cost from warehouse to site      | £1.73/truck/km                               |
| Pods can be loaded on a truck from factory to warehouse | 2 Pod A/truck; 4 Pod B/truck |
| Pods can be loaded on a truck from warehouse to site | 1 Pod A/truck; 2 Pod B/truck |
| Manufacturing fix cost at factory (overhead)   | £2200/day                                    |
| Maximum manufacturing rate                     | 10 Pod A/day; 10 Pod B/day                   |
| Variable manufacturing cost                    | £800/Pod A; £600/Pod B                       |
| Adjustment factor for variable manufacturing cost | 0.5 for Pod A; 0.6 for Pod B              |

\[0 \leq m^B_j \leq MRM_j \quad \forall j \in J\]\hspace{1cm}(2.1)

\[0 \leq m^F_j \leq MRM_j \quad \forall j \in J\]\hspace{1cm}(2.2)

\[n^{F}_j = n^{RF}_j + m_j - t^{F}_j \quad \forall s \in S, t = 1, p = IID, j \in J\]\hspace{1cm}(2.3)

\[n^{W}_j = n^{BW}_j + m_j - t^{W}_j \quad \forall s \in S, t = 1, p = IID, \quad j \in J\]\hspace{1cm}(2.4)

\[n_j^S = n^{BS}_j + t^{W}_j - D_{ij} \quad \forall s \in S, t = 1, p = IID, \quad j \in J\]\hspace{1cm}(2.5)

\[n^{W}_{p} = n^{BW}_{p} + t^{W}_{p} \quad \forall p = 1, j \in J\]\hspace{1cm}(2.6)

\[n^{FW}_{p} = n^{FW}_{p} + t^{RF}_{p} \quad \forall p = 2 \quad IID, \quad j \in J\]\hspace{1cm}(2.7)

\[n^{F}_{p}, n^{W}_{p} \geq 0 \quad \forall s \in S, t \in T, p \in P, \quad i \in I, j \in J\]\hspace{1cm}(2.8)

\[\sum_{j \in J} t^{F}_{j} = \sum_{i \in I} m_{ij} \quad \forall s \in S, j \in J\]\hspace{1cm}(2.9)

\[\sum_{j \in J} t^{W}_{j} = m_{ij} \quad \forall p = 1, j \in J\]\hspace{1cm}(2.10)

\[\sum_{j \in J} t^{RF}_{j} = n^{F}_{j} \quad \forall p \in P, j \in J\]\hspace{1cm}(2.11)

\[\sum_{j \in J} t^{W}_{j} = m_{ij} \quad \forall s \in S, j \in J\]\hspace{1cm}(2.12)

\[\sum_{j \in J} t^{RF}_{j} = 0 \quad \forall p \in P, j \in J\]\hspace{1cm}(2.13)

\[\sum_{j \in J} t^{W}_{j} = 0 \quad \forall s \in S, t = \text{LWD}, i \in I, j \in J\]\hspace{1cm}(2.14)

\[\sum_{j \in J} n^{F}_{j} \cdot \text{Vol}_{j} \quad \text{and} \quad \sum_{j \in J} n^{W}_{j} \cdot \text{Vol}_{j} \leq \text{WCAP}\]\hspace{1cm}(2.15)

Constraints (2.1) and (2.2) state that the daily manufacturing rate of each type of product is within a pre-defined range during the period for the initial inventory preparation and the period after the construction starts, respectively. The balance of inventory for each kind of product for the whole period of the construction phase in each scenario is bound by constraints (2.3)–(2.10). Constraints (2.3) and (2.4) represent the balance of inventory in the factory, constraints (2.5) and (2.6) are for the warehouse, while constraints (2.7) and (2.8) handle the construction site. Constraints (2.9) and (2.10) stand for the balance of inventory in the warehouse during the period for preparing the initial inventory.

The first component of constraint (2.11) assures the non-negativity of inventory at the factory after construction starts in each scenario, while the second and third components add the same restriction in the warehouse before and after the commencement of construction. Constraint (2.12) makes sure that the total production of each kind of modular products follows the total demand for that product. This is valid for every scenario.

Constraint (2.13) states that no initial inventory is stored in the factory and construction sites. This is because the initial inventory is only stocked in the warehouse. Constraint (2.14) makes sure that the demand for each kind of products on each site is met at the end of the construction project in each scenario. Constraints (2.15) ensures that the sum of the volume of all the stored products is smaller than the warehouse capacity at all times in each scenario. Constraints (2.16) and (2.17) impose the same restrictions on the inventories at the factory and construction sites, respectively.

The first and second components of constraint (2.18) state that the
quantities of each kind of products transported from the factory to the warehouse should be non-negative all the time, while the third component adds the same bound on the quantity transported from the warehouse to each construction site in the construction phase. Constraint (2.19) ensures that all the products produced in the factory after construction starts will be shipped to the warehouse. Constraint (2.20) ensures that all the above products plus the initial inventory will be transported to the construction sites.

4. Case study: bathroom pods for residential construction

This section describes the implementation of the model and its application to a bathroom pods case study. The data, obtained both from a major UK construction company and publicly available sources, concern modular bathroom pods for a large residential development designed and built using the DfMA method. Minor modifications to the data were carried out to reproduce a more diverse set of testing conditions. The key input parameters used in the mathematical model were collected from project documents and interviews with subject matter experts, see Table 1. The data regarding costs were gathered from academic publications [3,5] and websites1,2.

The problem dealt with in the case study involves a single manufacturing facility, a warehouse for temporary storage and the assembly of two distinct types of bathroom pods. Thus, the logistic system investigated can be interpreted as a three-tier construction supply chain. Given the complexity of the service ducts and components used in their design, bathroom pods are well-suited for off-site production. Offsite manufacturing using modular construction and the DfMA method enables to achieve a higher quality of the end product on site, significant time and cost savings and less construction materials wasted compared to the traditional stick-built method.

Bathroom pods were chosen due to their prevalence in the construction industry. In large-scale developments, many identical units to the traditional stick-built method. Time and cost savings and less construction materials wasted compared to achieve a higher quality of the end product on site, significantly.

4.1. Model implementation

The model was implemented using IBM ILOG CPLEX Studio (version 12.6). The mathematical relationships introduced in Section 3 were captured using the OPL mathematical modelling language. The programme flow for the stochastic aspects of the model was implemented using the ILOG Script language, a JavaScript derivative used for logic control and data manipulation. The CPLEX Optimizer was used to solve problem instances on a relatively modest workstation (Intel Core i7-4790, 8G RAM). The computation times for solving the model with problem instances on a relatively modest workstation (Intel Core i7-4790, 8G RAM). The computation times for solving the model with different parameter configurations are presented in Table 2. It should be emphasised that our implementation uses the default branch and cut algorithm that is provided by the CPLEX engine. As such computational time requirements are heavily dependent upon the number of scenario that are considered and the overall duration of the project. This could limit the applicability of the model to large problem instances. However, it can be addressed by considering individual parts of the projects, or approximate solution techniques.

The model simultaneously considered the demand variations on site incurred by four major delay factors: weather, transportation of modular products, labour productivity and crane status. As an illustration, the case study was carried out in two instances (i.e. immediate future execution and near future execution), differing with respect to data quality and anticipated levels of parameter stochasticity. A flow chart that outlines the various stages followed by the algorithm is provided in Fig. 1.

4.2. Quantifying the disruptions in the pods assembly process

Data regarding the four major delay factors was collected from weather forecast and historical records, written reports obtained from the collaborating construction company and a face to face interview with the case study project manager. Using these data we quantify the severity of schedule deviations caused by the various delay factors. Furthermore, the quantified delay factors were combined with the mathematical formulation obtained from previous studies to establish different levels of disrupted pod assembly pattern on a single day. Then based on these patterns, all the possible site demand scenarios that can reach the assembly target quantity within a finite time horizon were identified. Furthermore, their probability of happening was also revealed.

4.2.1. Weather-related disruptions

Previous studies [9,20,39,40] have identified inclement weather as one of the most commonly recognised sources of disruption in construction sites. This is because weather conditions can significantly influence the performance of outdoor construction activities, and modular construction is no exemption. While manufacturing may take place in controlled factory environments, the assembly stage still relies on cranes, which remain particularly sensitive to strong winds and heavy rains. Earlier studies have developed numerical frameworks [41–44] that illustrate the relationship between weather condition and the assembly rate of modular products. Building on [41–44], this research has modelled the daily module assembly rate as a function (Eq. (3)) of wind speed and rainfall using a semi-trapezoidal relationship [41] (see also the graphical presentation of the function in Fig. 2):

$$
\Delta IF(x) = \begin{cases} 
1 & 0 \leq x < a \\
1 - \left(\frac{x - a}{b - a}\right)^{3} & a \leq x < b \\
0 & x \geq b
\end{cases}
$$

(3)

where $\Delta IF$ is the influencing factor rectified by a weather condition $x$; $a$ is the turning point, i.e. the maximum wind speed or rainfall that bears no influence on assembly rates; and $b$ is the construction cancellation point, i.e. minimum wind speed or rainfall that causes the construction to halt. For wind, the turning point is set at 36 km/h and the cancellation point at 50 km/h [41]. For rain, the turning point is set at 5 mm/h and the cancellation point at 10 mm/h [43]. The disrupted assembly rate equals to the original assembly rate $\times \Delta IF$. The possible daily assembly rates of pods and their corresponding probabilities of happening have to be derived from the weather forecast historical records. Detailed derivations are presented in Sections 4.3.1 and 4.4.1 when discussing the two instances of the case study.

4.2.2. Transport-related disruptions

Beyond weather, delays in the delivery of construction components is another commonly identified cause of disruption for original assembly timelines. According to the interview with the project manager of the case study, professional logistics contractors with adequate experience to deal with ordinary traffic deviations (e.g. traffic congestion in an urban area) were hired to ship the pods from the factory to the

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1 http://www.sanika.it/en
2 https://www.offsitesolutions.com/
site. In line with the viewpoint of Laporte et al. [45], who pointed out that every traveller possesses a preferred arrival time, and thus chooses a departure time that allows to arrive at exactly the preferred time, the logistics contractors always dispatched the trucks earlier to minimise the impact of traffic congestion. Even so, certain scenarios exist where an early departure from the trip origin would not help alleviate delays. These include traffic accidents, malfunctions or breakdowns of trucks, extreme road weather conditions, absenteeism or sickness of the drivers [46,47].

To account for this fact, this study assumes that the assembly rate on a given day is halved when any of the above incidents occur. This is based on the site event logs, which show that the daily delivery quantity reduced on average to around half of the planned quantity whenever there were occasions of serious delays in transportation. The total assembly quantity on that day then was forced to be cut down by half. This situation did not happen very often, so a small probability is assigned to these serious late deliveries as shown in Table 3.

### 4.2.3. Labour productivity and crane disruptions

In the construction industry productivity is measured by the daily output carried out by workers and equipment on different kinds of tasks (e.g. cubic meter of concrete, square meter of formwork and kilogramme of rebar). In a modular construction project, productivity is expressed as the daily assembly quantity of modules in the site.

Despite the introduction of automated construction practices [5,6], manual operations are still present in modular construction. Upon arrival to the site, modules are handled by workers who have to inspect, unwrap and facilitate crane lifts. Modules then have to be lined up before laying down, which is followed by unhooking, screwing and welding to secure them in position. Construction workers’ efficiency would, therefore, play a prominent role in determining daily assembly rates. Previous surveys mentioned fatigue, alcohol, disloyalty, misunderstanding among workers, overtime work, late payment, absenteeism or negative working attitude as factors that may affect performance [48,49].

In this case study, the site manager informed us that all the workers and equipment operators received professional skills and site safety training for 2.5 h per week. Despite this, according to the daily site event log, their outputs always fluctuated. The discount rates of the workers’ productivity and the corresponding probabilities of happening, as given in Table 4, were derived from the data in the site event log, which records a series of daily assembly rate on site, and validated by the site manager. It should be noted that there are multiple factors that can affect worker’s performance. Therefore, these figures may vary widely over times, and between places and cases.

Similarly, for cranes, we consider the possibility of failure due to mechanical malfunctions or breakdowns. By consulting the site event logs, which record when and how long the cranes were down (e.g. 5 h in 267 working hours), and carrying out interviews with practitioners and construction site managers, we established a 50% productivity reduction for the days when such incidents occurred, with a probability of happening around 2% (see Table 4).

The quantified delay factors introduced in Sections 4.2.1 through to 4.2.3 are now applied to generate the possible demand scenarios on site for the following two instances in the case study: a construction project to be executed in the immediate future; and a construction project to be executed in the near future.

### Table 3
Disrupted assembly rates by late delivery.

| Delivery Status     | Affected Assembly rate-A | Affected Assembly rate-B | Probabilities |
|---------------------|--------------------------|--------------------------|---------------|
| Normal & minor      | 20                       | 18                       | 98.00%        |
| Late                | 10                       | 9                        | 2.00%         |

### Table 4
Discount on the assembly rate by labour productivity and crane status.

| Delay factor | Status      | Discount rate | Probabilities |
|--------------|-------------|---------------|---------------|
| Labour       | Normal      | 1             | 85.00%        |
|              | Medium low  | 0.8           | 10.00%        |
|              | Low         | 0.5           | 5.00%         |
| Crane        | Normal      | 1             | 98.00%        |
|              | Down        | 0.5           | 2.00%         |

### Table 5
A weekly weather forecast data.

| Date     | Sep-28 | Sep-29 | Sep-30 | Oct-01 | Oct-02 |
|----------|--------|--------|--------|--------|--------|
| Precipitation (%) | 8   | 56    | 54    | 72    | 25     |
| Wind speed (km/h)  | 22   | 24    | 16    | 11    | 17     |
known, the daily assembly rate under the influence of various intensity intervals are also provided. 5 mm/h and 10 mm/h obtained using Eq.(4). Rainfall probabilities for an assembly can be calculated using Eq.(3). We established three levels of influence of precipitation (PoP) on a certain day, the probabilities for the special gamma distribution, known as the exponential distribution, provides a reasonable approximation for the frequency distribution of rainfall amounts [50,51]. An estimator is defined using the exponential distribution probability density function to approximate the unconditional probability of exceeding (uPoE) a selected rainfall amount (x) on a certain day:

\[ uPoE(x) = PoP \times e^{-(x/\mu)} \]  

(4)

where \( \mu \) is the predicted average rainfall amount on that day. Since this value may not be always provided by weather forecasts, we used historical data for daily precipitation in the region for a period spanning 85 years (1931–2015). These were used to calculate the average precipitation amount (excluding dry days from the dataset). Predictions obtained with this methodology are provided in Table 6, which also illustrates unconditional probabilities for rainfall amount to exceed 5 mm/h and 10 mm/h obtained using Eq. (4). Rainfall probabilities for various intensity intervals are also provided.

Once the probabilities for different intervals of rainfall amount are known, the daily assembly rate under the influence of precipitation can be calculated using Eq. (3). We established three levels of influence (i.e. low, medium and high) on the pod assembly process on a single day to correspond with the three different levels of impact factor outputted by the semi-trapezoidal function. For the low level of influence (\( \Delta IF = 1 \)), the probability of occurrence is the sum of the probability of rainfall between 0 and 5 mm/h and the probability of not raining on that day. The probability for medium influence equals the probability of rainfall interval between 5 and 10 mm/h. Here \( \Delta IF \) is calculated based on an x value of the average of the rainfall interval (7.5 mm/h), and the result is 0.5. High level of influence (\( \Delta IF = 0 \)) occurs with a probability of rainfall over 10 mm/h.

As shown in Table 1, the original assembly rates without external influence are 20 type-A pod and 18 type-B pod per day. Table 7 presents the disrupted daily assembly rates for both types of pods under different levels of precipitation influence along with their probabilities of happening.

Weather forecasts usually provide average wind speed estimates. Consulting these predictions, we can calculate the probability for wind speeds to exceed the turning and cancellation points by employing the Weibull distribution [52–54]. The latter can be used to determine how often winds of different speeds will be seen at a location, with a probability density function defined as follows:

\[ p(v) = \frac{k}{c} \left( \frac{v}{c} \right)^{k-1} \exp\left[ -\left( \frac{v}{c} \right)^k \right] \]  

(5)

where \( p(v) \) refers to the probability for the selected wind speed \( v \) to happen on a certain day, \( c \) is the Weibull scale parameter, with a unit equaling that of the wind speed (km/h), and \( k \) is the unit-less Weibull shape parameter. The cumulative distribution function is written as:

\[ P_c(v) = 1 - \exp\left[ -\left( \frac{v}{c} \right)^k \right] \]  

(6)

The shape factor \( k \) and scale factor \( c \) can be calculated by using the method of moment, a common technique widely used in the field of parameter estimation [54,55]. The two parameters can be evaluated by the following equations:

\[ k = \left( \frac{0.9874}{\sigma} \right)^{1.098} \]  

(7)

\[ c = \frac{U}{\Gamma\left(1 + \frac{1}{k}\right)} \]  

(8)

where \( \sigma \) is the standard deviation of the wind speed on a given day in the area of interest, \( U \) is the average wind speed on that day given by the weather forecast, and \( \Gamma \) represents the Gamma function. A large database in the UK wind speed (known as NOABL, Numerical Objective Analysis Boundary Layer, available from the Department of Energy and Climate Change) has been built up under the Wind Power and UK Wind Speed Database Programs. They found that \( \sigma / U \) has a fixed ratio of 0.62, a value that can be applied to everywhere in the UK.\(^3\)\(^4\)

### Table 6

| Date       | PoP | \( \mu \) (mm/h) | uPOE (5 mm) | uPOE (10 mm) | Probabilities for various rainfall intervals |
|------------|-----|-----------------|-------------|-------------|---------------------------------------------|
|            |     |                 |             |             | 0 < x < 5 | 5 ≤ x < 10 | x ≥ 10 |
| Sep-28     | 8%  | 2.83            | 1.36%       | 0.23%       | 6.64% | 1.13% | 0.23% |
| Sep-29     | 56% | 3.42            | 12.96%      | 3.00%       | 43.04% | 9.96% | 3.00% |
| Sep-30     | 54% | 3.74            | 14.16%      | 3.71%       | 39.84% | 10.45% | 3.71% |
| Oct-01     | 72% | 3.65            | 18.31%      | 4.66%       | 53.69% | 13.65% | 4.66% |
| Oct-02     | 25% | 3.19            | 5.21%       | 1.09%       | 19.79% | 4.13% | 1.09% |

### Table 7

| Date       | Influence Level | Affected Assembly Rate-A | Affected Assembly Rate-B | Probabilities |
|------------|-----------------|--------------------------|--------------------------|---------------|
| Sep-28     | Low             | 20                       | 18                       | 98.64%        |
|            | Medium          | 10                       | 9                        | 1.13%         |
|            | High            | 0                        | 0                        | 0.23%         |
| Sep-29     | Low             | 20                       | 18                       | 87.04%        |
|            | Medium          | 10                       | 9                        | 9.96%         |
|            | High            | 0                        | 0                        | 3.00%         |
| Sep-30     | Low             | 20                       | 18                       | 85.84%        |
|            | Medium          | 10                       | 9                        | 10.43%        |
|            | High            | 0                        | 0                        | 3.71%         |
| Oct-01     | Low             | 20                       | 18                       | 81.69%        |
|            | Medium          | 10                       | 9                        | 13.65%        |
|            | High            | 0                        | 0                        | 4.66%         |
| Oct-02     | Low             | 20                       | 18                       | 94.79%        |
|            | Medium          | 10                       | 9                        | 4.13%         |
|            | High            | 0                        | 0                        | 1.09%         |
Using Eqs. (5)–(8) and average wind speeds obtained from weather forecasts, we can calculate the probabilities for wind speed to exceed the turning and cancellation points on a given day. Table 8 provides the estimated probabilities for the period in question.

Three distinct levels of wind influence on assembly operations can be derived using Eq. (3) and the figures in Table 8. Table 9 presents the disrupted daily assembly rates for both types of pod under different levels of wind influence along with their probabilities of happening.

### 4.3.3. Creation of the production plan for the project in the immediate future

Given that there are three possible construction durations (3–5 days) on site, only two factory variables can be examined in order to establish the most favourable manufacturing plan: the number of days for preparing the initial inventory before the construction commences, and the number of days for manufacturing modules after the construction work starts. The value of the total costs outputted by the objective function (Eq. (1)) for various combinations of the days before and after the start of construction are given in Table 11. From the table it can be seen that a total of 5 days is needed to produce all the modular products. The overall lowest objective function value appears under the condition that the factory should spend 3 days to prepare the initial inventory and continue to manufacture for another 4 days after the construction commences. In addition, a broader view of the supply chain in operation can be assessed by selecting any one of the valid demand scenarios. Information about the daily variations of inventories at the factory, the warehouse and the site, as well as the most appropriate shipping sizes of modular products between the three tiers of supply chain can be obtained, as shown in Table 12.
For a modular construction project that is going to take place in the near future, i.e. from six to twelve months from present, it is necessary to establish a long-term plan that sets forth in advance the most appropriate assembly and manufacturing schedules of modular components. In the following, an illustrative case is given to demonstrate how all the possible disrupted scenarios on the construction site for a designated time frame in the near future are established under the influences from four different major delay factors on site: weather, transportation of modules, labour productivity and crane status, and how a production plan is created.

### 4.4.1. Deriving the influence of weather

Inclement weather is generally considered the most prevalent factor disrupting schedules on site. Nevertheless, no weather forecast data is applicable at a point in time which is too far from present. Thus, historical data is typically used to predict weather conditions in the future, which can then be transformed into module assembly rates on site. For this purpose, daily precipitation data in south-eastern England over the last 85 years (1931–2015) and daily average wind speed data in the London area over the last 20 years (1995–2015) have been collected from the Met Office.

It is assumed that a construction project to design and assemble two types of pods (A and B) is going to commence in the London area in January 2018 (i.e. four months in the future). The original assembly rates in the absence of any external influence are 20 type-A and 18 type-B per day. We aim to determine all the possible assembly scenarios and identify their probability of happening for one week (5 days) in January 2018.

Statistical analyses on the historical data are used to find the distribution of rainfall amount and wind speed. The probabilities and cumulative probabilities for the occurrence of different rainfall amounts and wind speeds in January are shown in Fig. 4. The probabilities of their respective values lie in the three intervals defined by their turning points, and cancelling points are given in Table 13 (columns 2 and 3). These data are then used to calculate the assembly rates under different weather conditions.

| Pattern | A | B | Sep-28 | Sep-29 | Sep-30 | Oct-01 | Oct-01 |
|---------|---|---|-------|-------|-------|-------|-------|
| I       | 20| 18| 67.76%| 56.62%| 67.02%| 66.50%| 72.82%|
| II      | 16| 14| 7.97% | 6.66% | 7.88% | 7.82% | 8.57% |
| III     | 10| 9 | 17.35%| 23.63%| 18.16%| 18.19%| 14.75%|
| IV      | 8 | 7 | 1.57% | 2.39% | 1.67% | 1.68% | 1.23% |
| V       | 5 | 4 | 1.09% | 1.59% | 1.10% | 1.10% | 0.80% |
| VI      | 4 | 3 | 0.03% | 0.05% | 0.03% | 0.03% | 0.02% |
| VII     | 2 | 2 | 0.01% | 0.02% | 0.02% | 0.02% | 0.01% |
| VIII    | 0 | 0 | 4.27% | 9.04% | 4.13% | 4.66% | 1.80% |

### Table 11

The total costs (in £) under different combinations of the duration for preparing initial inventory and that for manufacturing after the start construction works.

| Days for preparing initial inventory | 1 | 2 | 3 | 4 | 5 |
|-------------------------------------|---|---|---|---|---|
| 125,499                             | na| na| na| 127,637 | 125,936 |
| 123,660                             | na| Na| Na| 124,660 | 124,131 |
| 123,460                             | na| Na| Na| 122,930 | 123,310 |
| 123,060                             | 125,499 | 123,660 | 123,460 | 123,310 |
| 124,810                             | 123,910 | 124,110 | 124,810 | 125,895 |

**na:** the solution is not available because the duration is too short to produce the target amount of modules.
used to derive the disrupted assembly rates for both types of pods under the influence of different levels of precipitation or wind by relying on Eq. (3). They are presented in the last two columns of Table 13. Table 13 shows that each type of weather disruption can have three levels of influence, which can be applied to any day in January 2018.

4.4.2. Generation of assembly scenarios for the project in the near future

Further consideration has been given to the disruptions owing to transportation (Tables 3, 2 levels of influence), labour productivity (Tables 4, 3 levels) and crane status (Tables 4, 2 levels). After going through a process of compilation similar to that discussed in Section 4.3.2 and presented in Fig. 2, we found 8 different assembly patterns of pod A and pod B. Table 14 lists the probabilities of happening of these 8 different patterns on any day in January 2018.

4.4.3. Creation of the production plan for the project in the near future

Since the construction project will be carried out for 5 consecutive days in a week in January 2018, there will be 85 kinds of possible assembly scenarios to be expected on site. Among them, we find a probability of 99.5% that the total weekly assembly capacity can reach or exceed 30 type-A and 26 type-B, which are then set as the assembly targets. There are 25,044 scenarios that can meet these criteria. The scenarios which fail to reach the target are not considered as valid scenarios. Their probability of happening is less than 0.5%.

An adjustment then has to be made to cut down the daily assembly quantity of some scenarios which can go over the target amounts, so that in every valid scenario only the exact target number of modules are assembled within the designated time period. After the process, 2014 distinct weekly demand profiles are identified. Among these demand profiles, 97.59% of them can reach the target amount within 2–4 days, and only 2.41% of valid scenarios need to meet the target on the fifth day.

The total costs outputted by the objective function (Eq. (1)) for different production durations composed of the initial inventory preparation time and manufacturing time after the construction starts are shown in Table 15. As it can be seen, it needs at least 3 days to produce all the modules needed for a week.

The lowest cost appears under the condition that the factory should spend 3 days to prepare the initial inventory and continue to manufacture for another 2 days after the construction commences (highlighted with a framed box). This is the most beneficial way to manufacture pods when all

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### Table 12
The production schedule for best scheme recommended by the model, and information about the inventory and transportation of a chosen demand scenario.

| Date         | Period for initial inventory | Period for assembling modules |
|--------------|------------------------------|-----------------------------|
| 9/25-9/26   | A 4 6 6 6 6                  | A 6 6 6 6 6                  |
| 9/27-9/30   | B 4 6 6 6 6                  | B 6 6 6 6 6                  |
| Pod type     |                              |                             |
| Transportation quantity (Factory to warehouse) | 6 4 6 6 6 6 | 6 4 6 6 6 6 |
| Factory inventory | -- -- -- -- -- -- | 4 2 6 0 5 2 4 4 0 0 |
| Transport quantity (warehouse to site) | -- -- -- -- -- -- | 13 8 3 14 11 9 13 4 4 4 |
| Warehouse inventory | 6 4 12 9 18 15 | 7 11 8 5 5 0 0 0 0 0 |
| Demand on site | -- -- -- -- -- -- | 9 8 7 6 11 10 9 7 6 |
| Site inventory | -- -- -- -- -- -- | 4 0 0 8 0 7 3 2 0 0 |

### Table 13
The probabilities of rainfall amount and wind speed lie in the various intervals and the disrupted daily assembly rate for both types of pods under different levels of influence.

| Rainfall | Influence Level | Rainfall (mm/h) | Probabilities | ΔIF | Affected Assembly Rate-A | Affected Assembly Rate-B |
|----------|-----------------|-----------------|---------------|-----|--------------------------|--------------------------|
| Low      | 0 ≤ X < 5       | 82%             | 1             | 20  | 18                       |
| Medium   | 5 ≤ X < 10      | 12%             | 0.5           | 10  | 9                        |
| High     | X ≥ 10          | 6%              | 0             | 0   | 0                        |

| Wind speed | Influence Level | Rainfall (km/h) | Probabilities | ΔIF | Affected Assembly Rate-A | Affected Assembly Rate-B |
|------------|-----------------|-----------------|---------------|-----|--------------------------|--------------------------|
| Low        | 0 ≤ X < 36      | 83%             | 1             | 20  | 18                       |
| Medium     | 36 ≤ X < 50     | 13%             | 0.5           | 10  | 9                        |
| High       | X ≥ 50          | 4%              | 0             | 0   | 0                        |

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Fig. 4. The probabilities and cumulative probabilities for the occurrence of different (a) rainfall amounts and (b) wind speeds in January in London.
study, five major delay factors typically occurring at construction sites are employed in scenario generation, including inclement weather, namely precipitation and wind, late module delivery, low labour productivity and variations in crane failure. However, the model is flexible and can be adapted to account for any number of delay factors, which are quantifiable. In our case, it is the changes in the assembly rate of the modules on a certain day that are of interest. In addition, delay factors must be associated with a probability of occurrence, which can be derived from historical data, as shown in this study for the case of weather, or from the result of a real-world case study, similar to the examples provided for labour productivity, transportation and crane status. If uncertainty is in the form of a continuous distribution, it needs to be approximated by a discrete distribution known as probability mass function through a proper discretisation method. In this research, semi-trapezoidal functions were defined to discretise the distributions of rainfall and wind speed [41].

The model was shown to be useful at recommending the most favourable production scheme after all possible demand scenarios are simultaneously considered. The scheme includes an optimal manufacturing schedule spanning from a period before the beginning of the project for initial inventory preparation to a period after the commencement of the construction. Here the most favourable scheme is the one that yields the lowest production cost and expected values of total operational cost and penalty cost for all possible demand profiles. The penalty is incurred in a scenario when the scheme cannot meet the demands on site momentarily, which is then forced to operate beyond the originally planned working period to assemble the late delivered modules, provoking an extra cost. In addition, the model can exhibit the most appropriate transportation quantities and the inventory changes for each kind of module at all echelons and for all the scenarios under the most favourable manufacturing scheme. To the best of our knowledge, a study of the supply chain, which deals with stochasticity in the demand and further considers the characteristics of both construction and manufacturing for modular construction, has never been conducted in previous construction supply chain research.

In this research, we illustrate that the model developed is able to deal with two kinds of demand situation, which address projects that take place in the immediate future and in the near future, respectively. In the first situation, our model can help a factory set up the best production plan based on the weather forecast. The model thus can be of great help to managers whose responsibility is to make decisions for production planning in a short time frame. The second situation is to deal with a demand far away from present. For prediction purpose, historical records of weather in the construction area are utilised to predict the possibility of precipitation, its amount and the wind speed in a certain time of the year, which then can be used to formulate the demand profiles and create a long-term plan using the model.

5. Conclusions

The three-tier logistics structure for modular construction projects investigated in this research is usually absent in stick-built construction projects. The most favourable responses in a manufacturing factory and construction site are also researched in this study, five major delay factors typically occurring at construction sites are employed in scenario generation, including inclement weather, namely precipitation and wind, late module delivery, low labour productivity and variations in crane failure. However, the model is flexible and can be adapted to account for any number of delay factors, which are quantifiable. In our case, it is the changes in the assembly rate of the modules on a certain day that are of interest. In addition, delay factors must be associated with a probability of occurrence, which can be derived from historical data, as shown in this study for the case of weather, or from the result of a real-world case study, similar to the examples provided for labour productivity, transportation and crane status. If uncertainty is in the form of a continuous distribution, it needs to be approximated by a discrete distribution known as probability mass function through a proper discretisation method. In this research, semi-trapezoidal functions were defined to discretise the distributions of rainfall and wind speed [41].

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In this research, we illustrate that the model developed is able to deal with two kinds of demand situation, which address projects that take place in the immediate future and in the near future, respectively. In the first situation, our model can help a factory set up the best production plan based on the weather forecast. The model thus can be of great help to managers whose responsibility is to make decisions for production planning in a short time frame. The second situation is to deal with a demand far away from present. For prediction purpose, historical records of weather in the construction area are utilised to predict the possibility of precipitation, its amount and the wind speed in a certain time of the year, which then can be used to formulate the demand profiles and create a long-term plan using the model.

5. Conclusions

The three-tier logistics structure for modular construction projects investigated in this research is usually absent in stick-built construction projects. The most favourable responses in a manufacturing factory and storage facility following demand variations at construction sites have rarely been studied. In this case, it has limited the development and effective operation of future modular construction supply chains. This is a first-of-its-kind study to develop a mathematical model to establish the optimal production, transportation and inventory schemes for the supply chain of modular construction. In addition, the optimal production durations for the initial inventory preparation and the remainder of the project (once construction commences), are also revealed. The results demonstrate how the optimal supply chain configuration is established considering multiple schedule deviation factors. The mathematical model presented in this paper can serve as the foundation for a decision support tool for practitioners involved in the design of logistics processes in modular construction.
The implementation of the model on a modular construction project assumes that practitioners are able to quantify the most prevalent schedule deviation factors. The compilation process introduced in Fig. 3 to generate all possible assembly scenarios and the flowchart of major model implementation steps presented in Fig. 1 then can be followed to determine the most efficient and effective supply chain configuration.

Future work will focus on the addition of further features to the model, such as the optimal number and location of warehouses, the potential use of outsourcing, and the inclusion of additional pre-assembly activities that take place within fabrication shops. It is also our intention to explore the impact of a larger variety of disruptions, validate the model against more real-world projects, and represent further operational aspects of modular construction that for the time being can only be modelled using simulation-based techniques.

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Appendix A

| Indices | Description |
|---------|-------------|
| i       | Construction sites, \(i \in I\), \(I\) is the set of sites |
| j       | Product types, \(j \in J\), \(J\) is the set of product types |
| s       | Demand scenario, \(s \in S\), \(S\) is the set of demand scenarios |
| t       | Working days of the construction project, \(t \in T\), \(T = \{1, 2, \ldots, LWD\}\) |
| p       | Days for preparing initial inventory, \(p \in P\), \(P = \{1, 2, \ldots, IID\}\) |

Parameters: construction sites

- \(SP_s\): The probability that scenario \(s\) occurs
- \(DD_s\): The construction duration of scenario \(s\) in days
- \(LWD\): The longest working duration among all scenarios in days
- \(D_{aj}\): Demand of product \(j\) at site \(i\) on day \(t\) in scenario \(s\)
- \(AC_j\): Assembly cost of product \(j\)
- \(SF_i\): The fixed overhead at the construction site \(i\) per day
- \(SCAP_i\): Maximum inventory capacity at construction site \(i\) in m³
- \(SIC\): Inventory cost per m³ per day at the construction site

Parameters: Warehouse

- \(VOL_j\): The volume of product \(j\) in m³
- \(WCAP\): Maximum warehouse capacity in m³
- \(WIC\): Inventory cost per m³ per day in warehouse

Parameters: Factory

- \(MRM_j\): Maximum manufacturing capacity of product \(j\) per day
- \(MF\): Fixed cost per day if factory is operating
- \(MV_j\): Basal unitary (variable) cost for manufacturing one product \(j\)
- \(ADJ_j\): Adjustment factor for the unitary manufacturing cost of product \(j\)
- \(IID\): The duration for producing initial inventory in days
- \(CMD\): The duration for factory to continue manufacturing after construction starts in days
- \(FCAP\): Maximum inventory capacity in factory in m³
- \(FIC\): Inventory cost per m³ per day in factory

Parameters: Transportation

- \(DFW\): Distance between factory and warehouse in km
- \(DWS_i\): Distance between warehouse and site \(i\) in km
- \(CFW\): Transportation cost from factory to warehouse per truck per km
- \(CWS\): Transportation cost from warehouse to all sites per truck per km
- \(NL_j^F\): Quantity of product \(j\) can be loaded onto a truck running from factory to warehouse
- \(NL_j^W\): Quantity of product \(j\) can be loaded onto a truck running from warehouse to all sites

Decision variables

- \(m_{aj}\): Manufacturing quantity of product \(j\) per day at factory on day \(t\)
- \(m_{aj}^B\): Manufacturing quantity of product \(j\) per day at factory on day \(p\)
- \(n_{aj}^F\): Quantity of initial inventory of product \(j\) in factory on day \(p\)
- \(n_{aj}^W\): Quantity of initial inventory of product \(j\) in warehouse on day \(p\)
- \(n_{aj}^{RS}\): Quantity of initial inventory of product \(j\) on site \(i\) on day \(p\)
- \(n_{aj}^{SF}\): Quantity of inventory of product \(j\) in factory on day \(t\) in scenario \(s\)
- \(n_{aj}^{SW}\): Quantity of inventory of product \(j\) in warehouse on day \(t\) in scenario \(s\)
- \(n_{aj}^{SFg}\): Quantity of inventory of product \(j\) at site \(i\) on day \(t\) in scenario \(s\)
- \(t_{aj}^F\): Transportation quantity of product \(j\) from factory to warehouse on day \(t\) in scenario \(s\)
- \(t_{aj}^W\): Transportation quantity of product \(j\) from warehouse to site \(i\) on day \(t\) in scenario \(s\)
