Fractal and topological sustainable methods of overcoming expected uncertainty in the radiolocation of low-contrast targets and in the processing of weak multi-dimensional signals on the background of high-intensity noise: A new direction in the statistical decision theory

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Abstract. The main purpose of this work is to interpret the main directions of radio physics, radio engineering and radio location in “fractal” language that makes new ways and generalizations on future promising radio systems. We introduce a new kind and approach of up-to-date radiolocation: fractal-scaling or scale-invariant radiolocation. The new topologic signs and methods of detecting the low-contrast objects against the high-intensity noise background are presented. It leads to basic changes in the theoretical radiolocation structure itself and also in its mathematical apparatus. The fractal radio systems conception, sampling topology, global fractal-scaling approach and the fractal paradigm underlie the scientific direction established by the author in Russia and all over the world for the first time ever.

1. Introduction

The intensive development of the modern radar technology establishes new demands to the radiolocation theory [1, 2]. Some of these demands do not refer to the theory basis and contribute to the precision improvement and the development of new calculation methods. The other ones are fundamental and related to the basis of the radiolocation theory. The last demands are the most important both in theory and in practice.

The entire current radio engineering is based on the classical theory of an integer calculation. Thus, an extensive area of mathematical analysis which is called the fractional calculation and deals with derivatives and integrals of a random (real or complex) order as well as the fractal theory has been historically “outboard” (!) [2-14]. At the moment the integer measures (integrals and derivatives with integer order), Gaussian statistics, Markov processes etc. are mainly and habitually used everywhere in the radio physics, radio electronics and processing of multidimensional signals. It is worth noting that the Markov processes theory has already reached its satiation and researches are conducted at the level of abrupt complication of synthesized algorithms. The radar systems should be considered with relation to the open dynamical systems. The improvement of classical radar detectors of signals and its mathematical support have basically reached their saturation and limit. It forces to look for fundamentally new ways of solving the problem of increasing the sensitivity or the range of coverage for various radio systems.

Nowadays it is absolutely clear that the application of scale invariance ideas – “scaling” along with the set theory, fractional measure theory, general topology, measure geometrical theory and dynamical systems theory reveals big opportunities and new prospects in the processing of multidimensional signals in the related scientific and engineering fields.
In this paper the presented alternative solutions for actual problems of modern radiolocation are based on the ideas and methods of the new scientific fundamental direction “Fractal Radio Physics and Fractal Radio Electronics: Designing of Fractal Radio Systems”. This direction was initiated by Professor A. Potapov in 1980 at the IREE RAS (Kotelnikov Institute of Radio Engineering and Electronics of the Russian Academy of Sciences, Moscow, Russia) and it is currently widely developed in his works and acknowledged in the Scientific World.

2. “Fractal” Conception in Radiolocation
In general terms a radar image (RI) can always be presented as a set of elements \( X_k \), whose values are proportional to the scattering cross-section (SCS) of a \( k \)-th element of the radar resolution [6-10]. In Figure 1a the RI of the terrain which was obtained at the wavelength \( \lambda = 8.6 \text{ mm} \) from a helicopter is shown. In Figure 1b the RI of the same terrain region which was obtained by a radar at the wavelength \( \lambda \approx 30 \text{ cm} \) is shown. Both images are two-dimensional with gray level proportional to SCS.

![Figure 1. a - d) Examples explaining the matter of fractal processing, e) fractal space signature](image)

Let us suppose that for every RI a surface (Figure 1c) with the height \( h \) which is proportional to the gray level is built. Let us suppose that we need to measure the square of the resulting surface. On the RI which corresponds to \( \lambda \approx 30 \text{ cm} \) the square will be less than for the RI on \( \lambda \approx 8.6 \text{ mm} \) since the smaller wavelength is the more terrain details can be recognized.

A probing electromagnetic wave is some kind of a “ruler” in this case. Due to that an increasingly finer structure of time-spatial signals or wave fields begins to have an effect.

If we have a RI which was obtained at even shorter waves then its square will be bigger. By decreasing the wavelength \( \lambda \) we will get increasing values of the squares. Then the question arises: what is the square of the surface which the RI was obtained from in reality? If the surface is covered with simple objects, for example a rectangular eminence (Figure 1d), and the sizes of this eminence are much higher than the wave length then the squares of the objects on the RI will be approximately equal for short and long waves. Then we would answer the mentioned question by calculating the number of resolution elements covering the object. The surface area \( S \) in this case would be equal to:

\[
S \equiv S(\lambda) = N(\lambda) \delta(\lambda),
\] (1)
where \( \delta(\lambda) \) - the square of the resolution element of the radar; \( N(\lambda) \) - the number of the resolution elements required to cover the object, \( \lambda \) - the wavelength of the radar. As it was already noted, for a simple object (Figure 1d) value \( S(\lambda) = \text{const} \).

For the RI on Figure 1a and Figure 1b one can build a dependence \( S(\lambda) = f(\lambda) \) and assuming that \( \delta(\lambda) = K(\lambda) \), where \( K \) is a known function then one can build a dependence \( S(\lambda) = f(\delta) \). It happens that the measured square \( S \) is described well by the formula
\[
S(\lambda) = k\lambda^{-D}.
\]

Then just using the logarithm we can calculate the parameter \( D \). The dependence which determines a fractal signature \( D(t, f, \vec{r}) \) of a RI by itself is shown on Figure 1e. This dependence describes a space fractal cepstrum of an image (this conception was introduced by the author in 1990s).

The fractional parameter \( D \) is called the Hausdorff-Besikovitch dimension or the fractal dimension [3,5,7,8]. For the RIs of the objects with a simple geometric form (rectangles, circles, smooth curves) this dimension coincides with the topological one that is equal to 2 for the two-dimensional RIs and it is determined by the slope of straight lines (2) in binary logarithmic coordinates. However, the value of \( D \) for the majority of real coverings images and meteorological formations turns out to be higher than the topological dimension \( D_0 = 2 \) that emphasizes its complexity and random nature.

3. Textural and Fractal Measures in Radio Physics and Radiolocation

A radar along with the observation objects and radio waves propagation medium forms a space-time radar-location probing channel. During the process of radio location the useful signal from the target is a part of the general wave field which is created by all reflecting elements of the observed fragments of the target surrounding background, that is why in practice the signals from these elements form an interfering component. It is worthwhile to use the texture conceptions to create radio systems for the landscape real inhomogeneous images automatic detecting [6-8]. A texture describes spatial properties of the earth covering images regions with locally homogenous statistical characteristics [15]. Target detecting and identification occurs in the case when the target shades the background region at that changing integral parameters of the texture.

Many natural objects such as soil, flora, clouds and so on reveal fractal properties in certain scales [5-10]. Today the analysis of natural textures is undergone by significant changes due to the use of metrics taken from the fractal geometry. After the texture they introduced a conception of fractals that is based on the fractional measure theory for fundamentally different approach to solving modern radio location problems. The fractal dimension \( D \) or its signature in different regions of the surface image is a measure of texture i.e. properties of spatial correlation of radio waves scattering from the corresponding surface regions. At the very beginning the author initiated a detailed research of the texture conception during the process of the radio location of the earth coverings and objects against its background. Further on a particular attention was paid to the development of textural methods of objects detecting against the earth coverings background with low ratios of signal/background (see for example [6-10,14,16-19] and references).

4. Textural and Fractal Measures in Radio Physics and Radiolocation

The author for the first time ever has shown that the fractal processing excellently suits for solving the problem of processing the low-contrast images and detecting superweak signals in high-intensity noise when the modern radars do not practically function [2, 6-11, 14, 16, 17]. When using the fractal approach, as it was pointed out above, it is natural to focus attention on the description and also on the processing of radio physical signals (fields) exceptionally in the fractional measure space using the hypothesis of the scaling and universal distributions with “heavy tails” or stable distributions [7, 8, 20].
The author has developed a fractal classification which was personally approved by B. Mandelbrot [9, 10] in the USA in 2005. It is presented on Figure 2 where the fractal properties are described on the assumption that $D_0$ is the topological dimension of the space of embeddings.

![Fractal Classification Diagram]

**Figure 2.** The author's classification of fractal sets and signatures

The textural and fractal digital methods developed by the author and his followers (Figure 3) allow to partially overcome a prior uncertainty in the radar problems using the geometry or the sampling topology (one- or multidimensional) [6, 16]. At that topological peculiarities of the sampling become very important as opposed to the average realizations which have different behavior.

It turns out that the concepts of fractal signatures and fractal spectra are very helpful for measurements. For example, these concepts are effectively applied to solve the problems of detecting the low-contrast targets and weak signals in the presence of intense non-Gaussian (!) interference. The methods of fractal processing should take into account the scaling effect of real radio signals and electromagnetic fields. The introduction of a fractional measure and scaling invariants necessitates the predominant use of power-series probabilistic distributions. These distributions result from the feedback that amplifies the events. Note that, for the distributions with the heavy tails, the sample means are unstable and carry little information because the law of large numbers cannot be applied in this situation.
Figure 3. Textural and fractal methods for processing low-contrast images and super weak signals in high-intensity noise

Figure 4 shows the general view of distributions with the fractal dimension $D$. While fractal processing the signals in noise it is shown that at the signals in noise ratio (SNR) $q_0^2 = +10$ dB we precisely measure the statistics of a signal. With the reduction of value aside negative values (for example, $q_0^2 = -3$ dB) there is a displacement of maximum final fractal distributions aside the values fractal dimensions of the noise or cluttering.

Figure 4. Empirical fractal distributions with heavy tails for images observed in the presence of an intense Gaussian noise: (1 and 3) scene A, (2 and 4) scene B, (1 and 2) $q_0^2 = -10$ dB, and (3 and 4) $q_0^2 = -20$ dB
Thus, always in a vicinity of value fractal dimensions of a useful component there is “a heavy tail” fractal distribution, reaching the stable size, about 20 %. The given tendency is kept at much smaller values, equal SNR $q_0^2 = -10$ dB and $q_0^2 = -20$ dB, as shown in Figure 4.

Figure 5 shows the selected results of the low-contrast objects fractal nonparametric filtering. Aircraft images were masked by an additive Gaussian noise. In this case, the SNR was -3 dB. It is seen in the figures that all the desired information is hidden in the noise. The optimum mode of filtering the necessary contours or objects is chosen by the operator using the spatial distribution of fractal dimensions $D$ of a scene. This distribution is determined automatically and is shown in the right panel of the computer display [8-11, 13, 21].

![Figure 5](image)

**Figure 5.** a) Source image, b) source image and noise $q_0^2 = -3$ dB, c) results of fractal filtration image

High sensitivity of estimating the non-integral dimension functionals to the presence of a continuous contour in images suggests a large potential of the objects contours fractal filtering in strong interference (Figure 6). The observation was made using a ground-based telescope, the distance between the telescope and objects being about 800 km. These data are presented in the book [11]. None of the modern methods of digital processing can provide comparable objects resolution!

![Figure 6](image)

**Figure 6.** a) The initial image of space complex at the time of joining “Shuttle” – “Peace”, b–d) the results of the fractal processing (targets clustering) for various values of the threshold $D$ of topological fractal nonparametric detector

This concept can be widely applied to solving the modern problems of radar, correlation-extremely navigation, artificial intelligence and dynamic systems. The algorithms developed by the authors for calculating the fractal signatures are efficient over an extremely wide range of physical sizes of the
characteristic image details and provide the detection estimation for the scaling effects, including even those masked by the noise.

5. Innovative Fractal-Scaling Technologies: Creation, Development and Application of Fractal Methods for Radiolocation Tasks and Developing the Foundations of Fractal Element Base

A critical distinction between the author's proposed fractal and scaling methods and classical ones is due to fundamentally different (fractional) approach to the main components of a physical signal. It allowed us to come to the new level of informational structure of the real non-Markov signals and fields. Thus, this is a fundamentally new radio engineering. For 35 years of scientific researches the global fractal and scaling method designed by the author has justified itself in many applications. We labeled all of this briefly and expressively – “The Fractal Paradigm” [9, 22-27].

The fractal geometry is a huge and genius merit of mathematician B. Mandelbrot [5]. But its radio physical/radio engineering implementation is a merit of the Russian (now it is international) scientific school of fractal methods and fractional operators under the supervision of Professor A.A. Potapov (V.A. Kotelnikov IREE RAS). In the current situation all the attempts to belittle their meaning and to reckon only with the classical knowledge endure the intellectual fiasco. A triad of specified problems in the general “Fractal Analysis and Synthesis” therefore creates the basis for “Fractal Radio Systems” (2005) – Figure 7, “Global Fractal–Scaling Method” (2006) and “Fractal Paradigm” (2011) [2,6-14,16-19,21-28].

Figure 7. Author’s conception of fractal radio systems and devices

The work fulfilled in Kotelnikov IREE of RAS by the author and his apprentices is based on the theoretical and experimental results in the scheduled introduction of the fractals, fractional integration-differentiation and the scaling effects in radio physics, radio engineering, and some contiguous scientific directions (Figure 8). We have published a sufficiently large number of works for each direction using the data from Figure 8 in Russia and abroad.
6. Fractal-Scaling or Scale-Invariant Radiolocation and Fractal MIMO-Radars as a New Kind of Radio System

For further instantiation of the problems regarding the weak radar signals detection, we believe the initial information to come from a variety of radio systems in the form of a one-dimensional signal and/or a radar image (RI) - Figure 9.

The simplified scheme of primary radio systems and the investigation of radar image and one-dimensional signal in millimeter wave band (MWB) were introduced by the author much earlier.

Currently the fractal radar, the MIMO-radar and the fractal MIMO radar as well as unmanned aerial vehicles (UAVs) are added to the scheme in Figure 9. The concept of fractal radar is presented in [7-11, 13, 14, 26-29], the concept of fractal MIMO-radar is considered in [7-11, 13, 26-29].

In general, the technology of MIMO systems implies that each wireless device involved in the data exchange has several spatially distributed receiving and transmitting antennas. The basic idea of fractal MIMO-radars is using fractal antennas and fractal detectors [9, 13, 18, 19, 23-29]. The capability of fractal antennas to work on several frequencies simultaneously or to radiate broadband sounding signal provides a sharp increase in the number of degrees of freedom that defines many of the important advantages of this type of radiolocation and significantly expands adaptation possibilities.
To represent these specifics in [9, 13, 26-29] a new term “fractal-frequency MIMO systems (FF MIMO)” was introduced, which reflects their physical properties much better. The MIMO technologies related to the spatial multi-channel systems provide great opportunities for the application of the author’s global fractal-scaling method for signal processing, various algorithms and technologies of fractal detectors at all stages of the synthesis of information MIMO systems.

The idea of a fractal radar station (Figure 9 and Figure 10) is based on the concept of fractal radio systems developed by the author - Figure 7.

7. Postulates of Fractal Radar
Fractal radar (Figure 10) defined in [7-11, 13, 14, 16-19, 26-29] is based on four main postulates:

1) Intelligent signal processing is based on the theory of fractional measure, scaling effects and fractional operator’s theory;
2) Hausdorff dimension or fractal dimension \( D \) of a signal or a radar image is directly connected with the topological dimension;
3) Robust non-Gaussian probability distributions of the fractal dimension of the processed signal;
4) “Maximum topology with a minimum of energy” for the received signal. It allows using the advantages of fractal scaling information processing more effectively.
Figure 10. The points of application of fractals, scaling and fractional operators in a classical radar for proceeding to a fractal radar

The key point of fractal approach is to focus on describing and processing of radar signal (fields) exclusively in the space of fractional measure with the use of the scaling hypothesis and distributions with heavy-tailed or stable distributions (non-Gaussian). The methods of fractal-scaling processing of signals, wave fields and images are in a broad sense based on the pieces of information which is not usually taken into account and irretrievably lost if the classical methods of processing are applied. This work is concerned with the main area of radio physics – radiolocation and it aims to ascertain what is done and has to be done in this field on the basis of the fractal theory. The investigations carried out showed the correctness of the path chosen by the author (since 1980) to improve the radiolocation technique.

It is necessary to think about the processing of the input signals with a low threshold at high levels of false alarm and then changing to a low level of false alarms. Moreover, the false alarm probability is never measured in the real time. In principle, we need new metric and new parameters of radar detection.

8. Postulates of Cognitive Radar
Cognitive radar defined in [30] is based on 3 main postulates:

1) intelligent signal processing, which builds on learning through interactions of the radar with the surrounding environment;

2) feedback from the receiver to the transmitter, which is a facilitator of intelligence;

3) preservation of the information content of radar returns, which is realized by the Bayesian approach to the target detection through tracking.

We focus on future possibilities of radars with the special emphasis on the fractal, chaos and synergistic ideas.

9. New Features and Topological Methods for the Detection of Low-Contrast Targets
All the currently existing methods and features of detecting the low-contrast targets against the background of high-intensity reflections from the sea, ground and meteorological formations are presented on Figure 11. The correlations between various signs and methods are also shown.

**Figure 11.** Topological features and methods for detecting the low-contrast targets against the background of high-intense noise

The introduction of the textural features ensemble concept to the US in 1973 [15] made it possible for the author in the 1980-ies to be the first to calculate complete ensembles of 28 textural features and to conduct their detailed synchronic analysis for the real (optical aerial photography (OAP) and radar images within the MMW range at a wavelength of 8.6 mm) as well as for the synthesized textures based on the autoregressive models depending on the season [6].
For a long time the works on the study of radar images of the land cover at MMW using textural information have largely been carried out only in Russia and are of interest so far (especially now) [9,13,14]. After calculating the ensembles of textural features based on the optical and radar images, in 1985-1986 the author proposed the methods and algorithms for the detection of low-contrast targets against the background of high-intense noise. Those included the method of the direct use of textural features (1985), the dispersion method based on $f$-statistics (1985) and the detection method using linearly simulated patterns, i.e. textures (1986) [6].

The created methods of detection are quite valid at low signals. To the author’s knowledge, no textural methods for detecting the low-contrast target have been proposed abroad. Moreover, an important advantage of the textural methods of processing is the possibility to neutralize the speckles at coherent images of the Earth’s surface obtained by SAR.

The methods of deterministic chaos are widespread: they are shown in the right column of Figure 11. It should be only noted that the algorithms of radar detection of the low-contrast targets against the background of woodlands for the radar at a wavelength of 2.2 mm were tested by the experimenters in 2001. It was the first time when a strange attractor was reconstructed. It controlled the radar scattering of millimeter radio waves. Its dynamic and geometric characteristics were measured; $D$ fractal dimensions, depending on the value of $m$ embedding dimension were calculated. The most accurate estimate of $D$ can be obtained at the breakpoint of $D(m)$ convex curve, at that paying no attention to the reduction in scale ratio above and below.

The calculation of the correlation integral $C(r)$ was conducted using the F. Takens’ theorem on the sampling out of 50000 counts which corresponds to the angle of incidence of an electromagnetic wave $\theta = 50^\circ$. Based on the found maximum Lyapunov exponent $\lambda_1 > 0.6$ bit/s it has been shown that when measuring the current conditions with an accuracy of up to 1 bit we lose all predictive power over time during 1.7 seconds. Therefore, the prediction interval of the echoed signal intensity is by about 8 times greater than the classical correlation time $\tau$ ($\tau \approx 210$ ms at a wind speed of 3 m/sec). The prediction interval provides an opportunity to estimate roughly the amplitude of further samples in the sample collection and, as noted by the author, it can be used in the radar practice. The calculations of Hurst exponent $H$ showed that in two out of three cases the scattering process of millimeter waves by woodlands corresponds to the persistent process with $H > 0.5$, i.e. to the process with the maximal rank.

The obtained results along with the family of fractal distributions underlie the new dynamical model of signals scattered by the plant coverings. The proposed model of electromagnetic waves scattering by the earth coverings has a fundamental difference from the existing classical models. It has a finite number of degrees of freedom, describes processes of the non-Gaussian scattering and introduces the interval of prediction of the received radar signal intensity and its fractal features into consideration. We particularly note that taking into account the fractality of the earth coverings allows to describe the earth coverings scattering indicatrixes more precisely than the classical models which are used now.

10. Fractal Scaling Topological Detectors of Low-Contrast Targets

Currently great interest is evinced in various fractal and scaling methods (Figure 11). Those fractal investigations started almost simultaneously in Russia, the USA and China in the 1980s. [7-11, 13, 14, 16-19, 26-29, 31-36].

Fractal dimension $D$ or its signature $D(t, f, \tilde{r})$ in different regions of the surface image is at the same time a texture measure that is the properties of the space correlation of the radio waves scattering by the corresponding surface regions. At the same time the texture determines lacunarity (Figure 11) which uses the statistics of the second order for fractal images [7, 8]. The lacunarity is small for a dense texture and large when the texture is coarse-grained. Lacunarity $\Lambda$ (by Mandelbrot [5]) is defined by the formula

$$\Lambda = \langle (M/<M>) - 1 \rangle^2. \quad (3)$$

Here $M$ is the “mass” of the fractal formation, $<M>$ is the expected “mass”, brackets $\langle \ldots \rangle$ mean the data ensemble averaging. The consideration of lacunarity as a sign of objects detection was conducted
by the author in 1997. The introduction of the fractional measure and the scaling invariants requires working with exponential probabilistic distributions.

The main principles of the fractal detector were discovered and proposed by the author as early as in 1980s and getting out (also for the first time ever - see Figure 7 and Figure 10) to the functioning prototype of the fractal nonparametric detector of radar signals (FNDRS) was done in 2003-2005. It was demonstrated in the USA on the ISTC project with Central Design Bureau “Almaz” and IREE RAS in 2005 and it earned highly positive feedback from many specialists [8-10, 31, 32]. The high stability of our proposed original algorithms of fractal-and-scaling detection was shown.

Some initial versions of radar fractal detectors generalized structures are presented on Fig. 12. A schematic view of the probable detector is given on Figure 12a. A set of textural or fractal signs $\xi$ is determined on the received radio signal or the image. Then in a threshold device the decision on signal presence $H_1$ or its absence $H_0$ is generated at the threshold value $\Pi$ and a certain level of probability of a false alarm $F$. The values of fractal dimension $D$, Hurst exponents $0 \leq H \leq 1$ for multidimensional surfaces, Holder exponents, lacunarity values and so on can be used as signs $\xi$. The Hurst exponent equals

$$H = 3 - D$$

for the RI and

$$H = 2 - D$$

for the one-dimensional signal.

A structural enlarged pattern of a radar signals fractal detector is shown on Figure 12b. It includes a contours filter and a calculator of fractal cepstrums. The further specification of the structural pattern of FNDRS is given on Figure 12c. The input signal (RI, one-dimensional sampling) comes to the input transducer. Its designation is preliminary preparing of the sampling under analysis. This preparation can include either a forced noise contamination (in case of low resolution of an analogous-and-digital transducer of a radar) or for example a contrast compression - in case of the sampling with the high dynamic range.

Figure 12. a) Initial and b, c) detailed structures of the first fractal detectors

A detector on the basis of the Hurst exponent (Figure 13) works using one or several search frequencies of the radar [29]. The Hurst exponent $H$ reflects irregularity of the fractal object – (4) and (5). The less exponent $H$ is the more irregular the fractal object becomes. So, the Hurst exponent gets higher when an object appears.
Figure 13. Fractal detector based on the Hurst exponent

On Figure 14 there is a scheme of fractal detector with the autoregressive estimation of the power spectrum of the interference from the Earth surface [29]. The autoregressive model represents a linear model of prediction which estimates the power spectrum of the interference from the surface and forms its autocorrelation matrix. The autoregressive equation describes a relation between the current and preceding counts of the sampled stochastic process. Earlier, in the 1980s, we were resolving the problem of autoregression on the basis of the canonical system of Yule-Walker equations with the transform of brightness histograms [6]. Thus, in the detector on Figure 14 the real fractal properties of the power spectrum on the basis of the autoregressive spectral estimation which are applied for detecting the low-contrast objects are used.

Figure 14. Fractal detector with autoregressive estimation of the interference spectrum and the Hurst exponent

11. New Directions in the Theory of Statistical Solutions and Testing Statistical Hypotheses
The fast development of the fractal theory in radar and radio physics led to establishing a new theoretical direction in modern radar. It can be described as «Statistical theory of fractal radio-location». This direction includes (at least at the initial stage) the following fundamental questions [1-5, 7-13, 20, 37]:

1). The theory of the integer and fractional measure.
2). Caratheodory construction in the measure theory.
3). Hausdorff measure and Hausdorff-Besicovitch dimension.
4). The theory of topological spaces.
5). The dimension theory.
6). The line from the mathematician’s point of view.
7). Non-differentiable functions and sets.
8). Fundamentals of the theory of probability.
9). Stable probability distributions.
10). The theory of fractional calculus.
11). The classical Brownian motion.
14). Anomalous diffusion.
15). The main criteria for statistical decision theory in radar.
16). Wave propagation in the fractal random media.
17). Wave scattering generalized Brownian surface.
18). Wave scattering surface on the basis of non-differentiable functions.
19). Difractals.
20). Cluster analysis.
21). Theory and circuitry of fractal detectors.
22). Fractal-scaling or scale-invariant radar.
23). The multi-radar.
24). MIMO radar.
12. Generalized Brownian motion.
13. Fractal sets.

This list of studied questions, of course, is supposed to be expanded and refined in the future. The author has been dealing with it for nearly 40 years of his scientific career.

12. Fractal Processing of Medical Images

It is known that X-ray and fluorographic images are very important for express diagnostics of such diseases as numerous fractures and initial stages of malignant tumors. One of the methods is X-ray mammography making possible to diagnose at the earliest stage of the disease. To solve the problem of isolating suspicious areas on X-ray images, we chose a series of images with accumulations of calcinates. The information about the possible size of such formations helps to identify them against the background of other elements of the image. In fractal processing, the scales $K$ and $M$ varied from 2 to 10. Correspondingly, different slices were obtained for estimating $D$ – Figure 15. The suspicious regions are in Figure 15c.

Figure 15. Images obtained for different values of the fractal dimension estimate $D$ ($D_1 > D_2 > D_3$): a) initial X-ray image, b) $D = D_1$, c) $D = D_2$, d) $D = D_3$

Figure 16 shows the results of the processing another X-ray image. The accumulations of calcinates in Figure 16a are marked by the squares.

Figure 16. An example of selecting suspicious areas in the image: a) the initial image, b) the result of the fractal processing

Figure 17 shows the final results of solving the problems of image clustering based on the value of the fractal dimension $D$ and the tasks of extracting the contours of the image.
Figure 17. a) An example of solving the problem of clustering an X-ray image, b) from the value of estimating the fractal dimension $D$, and c) image contours extraction

The results of the fractal processing show a reliable improvement in the quality of the processed medical images. The proposed methods are verified by the authors on various real medical images. It should be noted that the algorithms used are characterized by modest computational requirements and high stability.

13. Conclusions
The author created, developed and applied the fractal-scaling methods for radiolocation problems and forming the foundations of fractal element base [6-14, 16-19, 21-29, 31-36, 38, 39]. For the first time ever approaches to the development of a fractal radar and a fractal MIMO-radar were considered. The author emphasizes that the synthesis of topological (fractal, textural, chaotic, etc.) detectors makes for a fresh look at the problem of detecting the super weak actual signals. As a result of that, the author’s discovery in the 1980-ies takes the meaning of the generalized detection. Thus, pure energy and pure topological detectors are not contrary to each other and they do not duplicate but complement one another.

Due to the topological detectors it is possible to see the process of energy detection in a new light and to find some essential faults in it. Consequently, topological detection becomes not less, if not more, valuable for the theory and practice than energy detection. The theory of topological detection is formulated in [7-11, 13, 14, 16-19, 26-29, 31-36, 38, 39]. It is especially necessary for the purpose of reexamining the former theory and in that way producing new results that are not available to the traditional concepts of radiolocation.

Thus, the topological detection opens the door to a radically new field of statistical decision theory and provides an opportunity to correct the ideas in this field and even to create new ones, which is of great theoretical and practical importance. The sufficiently detailed reasoning reported here should contribute to a better understanding of proposed by the author fundamentally new interpretation of the problem of radar (and other kinds of) detection. The proposed theory has much in common with the cognitive radar.

Thus, during more than 35 years, almost from scratch, fundamental bases of the theory that will be applied in the following decades were formed. It is not the results and the specific solutions that are the most valuable, but the solution method and the approach to it. The created method is presented in [7-11, 13, 14, 16-19, 26-29, 31-36, 38, 39].

The author raised the foregoing problems as early as in 1980 and for more than 35 years he has been successfully working on their solution and development. Careful bibliographic studies show complete and absolute world priority of the author in all “fractal” fields of radiolocation and radio-physics (the list of the author’s works in cooperation with students has about 830 publications, including 23 monographs).
14. Acknowledgment
This work was supported in part by the project of International Science and Technology Center No. 0847.2 (2000 – 2005, USA), Russian Foundation for Basic Research (projects №№ 05-07-90349, 07-07-07005, 07-07-12054, 07-08-00637, 11-07-00203), and also was supported in part by the project “Leading Talents of Guangdong Province”, № 00201502 (2016 – 2020) in the JiNan University (China, Guangzhou).

15. References:
[1] Skolnik M 2008 Radar Handbook (New York: McGraw-Hill Co)
[2] Bunkin B V, Potapov A A, Reutov A P et al 2003 Aspects of Perspectve Radiolocation (Moscow: Radiotekhnika)
[3] Rogers C A 1970 Hausdorff Measures (London: Cambridge University Press)
[4] Oldham K B and Spanier J 1974 The Fractional Calculus (New York: Academic Press)
[5] Mandelbrot B 1983 The Fractal Geometry of Nature (San Francisco: W.H. Freeman and Co)
[6] Potapov A A 1994 Synthesis of Images of Earth Coverings in Optical and Millimeter Waves Ranges: Dissertation of Doctor of Physics and Mathematics: (01.04.03 - Radio Physics) (Moscow: IREE RAS)
[7] Potapov A A 2002 Fractals in Radiophysics and Radar (Moscow: Logos)
[8] Potapov A A 2005 Fractals in Radiophysics and Radiolocation: Sample Topology (Moscow: Universitetskaya kniga)
[9] Potapov A A. 2016 Chaos Theory, Fractals and Scaling in the Radar: A Look from 2015 in: ed C Skiadas The Foundations of Chaos Revisited: From Poincaré to Recent Advancements (Switzerland, Basel: Springer) chapter 12 pp 195–218
[10] Potapov A A 2006 Fractals and Chaos as the Basis of New Breakthrough Technologies in Modern Radio Systems in: Kronover R Fractals and Chaos in Dynamic Systems (Moscow: Technosphere) pp 374–479
[11] Potapov A A, Gulyaev Yu V, Nikitov S A, Pakhomov A A and German V A 2008 Newest Images Processing Methods ed A A Potapov (Moscow: FIZMATLIT)
[12] Podosenov S A, Potapov A A, Foukzon J and Men’kova E R 2015 Nonholonomic, Fractal and Linked Structures in Relativistic Continuous Medium, Electrodynamics, Quantum Mechanics and Cosmology: In three volumes ed A A Potapov (Moscow: LENAND)
[13] Potapov A A, Wu Hao, Foukzon J, Podosenov S A and Men’kova E R 2016 Fractals in Radar, Fields, Control and Low-Contrast Target Detection, ed. A A Potapov (China) in press
[14] Potapov A A 2009 “The Textures, Fractal, Scaling Effects and Fractional Operators as a Basis of New Methods of Information Processing and Fractal Radio Systems Designing” Proc. SPIE 7374 pp 73740E-1–73740E-14
[15] Haralick R M, Shanmugan K and Dinstein I 1973 “Textural Features for Image Classification” IEEE Trans SMC–3 no 6 pp 610–621
[16] Potapov A A 2004 “Topology of Sample” Non-Linear World (Moscow) 2 no 1 pp. 4–13
[17] Potapov A A 2007 “The Theory of Functionals of Stochastic Backscattered Fields” J. Communications Technology and Electronics 52 no 3 245–292
[18] Potapov A A and German V A 1998 “Detection of Artificial Objects with Fractal Signatures” Pattern Recognition and Image Analysis 8 no 2 pp. 226–229
[19] Potapov A A and German V A 2001 “Fractals, Fractal Target Selection and Fractal Antennas” Proc. 1st Int. Workshop on Mathematical Modeling of Physical Processes in Inhomogeneous Media (Mexico) pp 44–46
[20] Gnedenko B V and Kolmogorov A N 1954 Limit Distributions for Sums of Independent Random Variables (London: Addison–Wesley)
[21] Potapov A A and German V A 2004 “Methods of measuring the fractal dimension and fractal signatures of a multidimensional stochastic signal” J. Communications Technology and Electronics 49 no 12 pp. 1370–1391

[22] Potapov A A 2012 The Fractal Method and Fractal Paradigm in Modern Natural Science (Voronezh: Publishing and Polygraphic Centre “Nauchnaya kniga”)

[23] Potapov A A 2008 “Fractal Models and Methods on the Basis of the Scaling in Fundamental and Applied Problems of the Modern Physics” Irreversible Processes in Nature and Engineering ed V S Gorelik and A N Morozov (Moscow: N. E. Bauman MSTU and P. N. Lebedev Physical Institute RAS) issue II pp 5–107

[24] Potapov A A 2013 “Fractal Paradigm and Fractal-Scaling Methods in Fundamentally New Dynamic Fractal Signal Detectors” Proc. the Eighth Int. Kharkov Symposium on Physics and Engineering of Millimeter and SubMillimeter Waves-MSMW’13 (Kharkov) pp 644–647

[25] Potapov A A 2013 “The Global Fractal Method, Fractal Paradigm and the Fractional Derivatives Method in Fundamental Radar Problems and Designing of Revolutionary Radio Signals Detectors” Zbornik radova Konferencije MIT - Matematicke i informacijske tehnologije, (Kosovska Mitrovica Prirodno-matematički fakultet Ulverziteta u Pristini Serbia) pp 539–552

[26] Potapov A A 2015 “Fractal Radar: Towards 1980 – 2015” Proc. of CHAOS 2015 Int. Conf. (Paris, Henri Poincaré Institute) pp 559–573

[27] Potapov A A 2016 “The Fractal-Scaling Radiolocation: Formation History 1980–2015” Chaotic Modeling and Simulation 3 pp 317–331

[28] Potapov A A 2016 “New Conception of Fractal Radio Device with Fractal Antennas and Fractal Detectors in the MIMO Systems” Book of abstracts 9th int. conf. (CHAOs’ 2016) on Chaotic Modeling, Simulation and Applications (London, Senate House, University of London) p 85

[29] Potapov A A 2016 “Analysis and Synthesis of Topological Radar Detectors of Low-Contrast Targets Against the Background of High Intensity Noise as a New Branch of Radiolocation and the Theory of Statistical Solutions” Eurasian Physical Technical Journal 13 no 2(26) pp 12–23

[30] Haykin S 2006 “Cognitive Radar: A Way of the Future” IEEE Signal Processing Magazine 23 pp 30–40

[31] Potapov A A 2003 “New Information Technology in Radar Detection of Low-Contrast Targets Based on Probabilistic Texture and Fractal Features J. of Communications Technology and Electronics 48 no 9 pp 1012–1029

[32] Gulyaev Yu V, Nikitov S A, Potapov A A and German V A 2006 “Concepts of Scaling and Fractal Dimension in the Design of a Fractal Detector of Radio Signals” J. Communications Technology and Electronics 51 no 8 pp 909–916

[33] Potapov A A, Il’yn E M, Chigin E P and German V A 2005 “Development and Structure of the First Etalon Dictionary of Fractal Properties of Target Classes” Electromagnetic Phenomena 5 no 2(15) pp. 107–142

[34] Potapov A A 2016 “Fractality and scaling problems in radio location and radio physics with new methods of detection of low-contrast targets against a background of high intensity noise” Proc. of XV Int. Academic Congress “Fundamental and Applied Studies in the Modern World” (Oxford, Oxford University Press) pp 314–322

[35] Potapov A A, German V A and Pahomov A A 2016 “Processing of Images Obtained From Unmanned Aerial Vehicles in the Regime of Flight over Inhomogeneous Terrain with Fractal-Scaling and Integral Method” Proc. CIE Int. Conf. on Radar “Radar 2016” (China, Guangzhou) pp 585–587

[36] Potapov A A 2016 “Strategic Directions in Synthesis of New Topological Radar Detectors of Low-Contrast Targets against the Background of High-Intensity Noise from the Ground, Sea and Precipitations” Proc. CIE Int. Conf. on Radar “Radar 2016” (China, Guangzhou) pp 692–696

[37] Lehmann E L and Romano J P 2004 Testing statistical hypotheses (New York: Springer)

[38] Potapov A A 2017 “Diffrafractals at frequency 36 GHz which are observed at radar scattering of an electromagnetic wave by a fractal surface and wave catastrophes in fractal randomly
inhomogeneous media” Proc. XIII Int. Conf. “Zababakhin Scientific Talks” Dedicated to 100th anniversary of academician E I Zababakhin (Snezhinsk) pp 137–138

[39] Potapov A A 2017 “On the fractal structure of high-altitude storm discharges in the ionosphere: elves, jets and sprites” Proc. XIII Int. Conf. “Zababakhin Scientific Talks” Dedicated to 100th anniversary of academician E I Zababakhin (Snezhinsk) pp 335–337