Understanding disk galaxy rotation velocities without dark matter contribution – a physical process for MOND?

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Abstract

An impact model of gravity designed to emulate Newton’s law of gravitation is applied to the radial acceleration of disk galaxies. Based on this model (Wilhelm et al. 2013), the rotation velocity curves can be understood without the need to postulate any dark matter contribution. The increased acceleration in the plane of the disk is a consequence of multiple interactions of gravitons (called "quadrupoles" in the original paper) and the subsequent propagation in this plane and not in three-dimensional space. The concept provides a physical process that relates the fit parameter of the acceleration scale defined by McGaugh et al. (2016) to the mean free path length of gravitons in the disks of galaxies. It may also explain the modification of the gravitational interaction at low acceleration levels in MOND (Milgrom 1983, 1994, 2013, 2016). Three examples are discussed in some detail: The spiral galaxies NGC 7814, NGC 6503 and M 33.

Keywords

Disk galaxies, rotation curves, modified gravity

1 Introduction

Since Oort (1932) and Zwicky (1933) introduced the concept of Dark Matter (DM; dunkle Materie), because (1) of discrepancies of velocity distributions in the Milky Way Galaxy and (2) the large speed deviations within the Coma galaxy cluster that are in conflict with its stability, respectively, DM concepts have been applied, in particular, to the flat rotation curves of disk galaxies, but also to the anomalous deflection of light (cf., e.g., Rubin 1982, 1986; Ellis 2010).

We mainly consider the rotation dynamics of galaxies and propose a physical process which explains the observations without the need to introduce any DM.

Binney et al. (1987) states that there is independent ‘global’ and ‘local’ evidence for galactic DM and asks if the same mysterious material can be responsible in both cases. We will not address the global evidence, but question the local one. As far as the global aspect is concerned, hundreds of articles discuss Cold Dark Matter (CDM) models of cosmology with DM halos dominating the dynamics of the universe and—combined with the ΛCDM paradigm, where Λ is the cosmological constant—explain both the large-scale cosmic structures as well as galaxy formation and evolution over an enormous span of redshifts (e.g. Navarro et al. 1995; Moore et al. 1998; Weinberg et al. 2015; Oh et al. 2015). Nevertheless, important features related to the nature and origin of DM are still missing and the empirical knowledge of dark halos remains very sparse (Burkert 1995; Treu and Koopmans 2004; Boylan-Kolchin et al. 2011).

The interesting question: ‘Are the tensions between CDM predictions and observations on the scales of galactic cores and satellite halos telling us something about the fundamental properties of dark matter, or are they telling us something interesting about the complexities of galaxy formation?’ asked by Weinberg et al. (2015) leads to their suggestion that investigating the influence of baryons on the DM halo profiles could be a direction for future progress.
McGaugh (2005a) observed a fine balance between baryonic and dark mass in spiral galaxies that may point to new physics for DM or a modification of gravity. Fraternali et al. (2011) have also concluded that either the baryons dominate the DM or the DM is closely coupled with the luminous component. Samurovic (2016) could describe the dynamics of NGC 5128 both with a DM halo and MOND. Salucci and Turini (2017) have suggested that there is a profound interconnection between the dark and the stellar components in galaxies. McGaugh et al. (2016) have presented a correlation between the radial acceleration and the observed distribution of baryons in 153 rotationally supported galaxies and have concluded that the DM contribution is fully specified by that of the baryons (cf. Milgrom 2016). Salucci (2017) supports this correlation, but sees no reason that it challenges the DM scenario in galaxies. A brief review of the literature on spiral galaxies is required, before we will argue that the DM scenario in galaxies is not required.

2 Spiral galaxies

2.1 Flat rotation curves

From the flat or slightly increasing rotation curves, Rubin (1980) has concluded that the DM is clumped about galaxies in a mostly spherical halo and is more extended than the luminous matter. Binney et al. (1987) expected the halo to be flattened in the same way although not to the same extent as the disk. Recent observations indicate for the disk light, a flattening ratio of 7.3 (Kregel et al. 2002), and for seven dwarf galaxies seen edge-on an average stellar disk scale length to height ratio of $\approx 9$ (Peters et al. 2017). The flat rotation curves seem to require that the mass interior to $R$, $M(R)$ is proportional to $R$. The observations – described, for instance, by Salucci (2008) – that the rotation curves of spiral galaxies do not show a Keplerian fall-off and, therefore, do not match the gravitational effects of the stellar plus gaseous matter are supported by many other studies (cf. Corbelli and Salucci 2000; McGaugh 2005a; Greisen et al. 2009; Corbelli et al. 2014; McGaugh et al. 2016; Kam et al. 2017). The observation that the circular velocity at large radii $R$ around a finite galaxy becomes independent of $R$ – resulting in asymptotically flat velocity curves – led Milgrom (1983, 1994) to propose a modification of the gravitational interaction at low acceleration levels, called MOrification of the Newtonian Dynamics (MOND). Recent discussions and a review indicate that MOND might describe the dynamics of spiral galaxies without DM (e.g., Girardi et al. 2000a; Famaey and McGaugh 2012; Angus et al. 2012; Kroupa et al. 2012).

2.2 Core-cusp problem

de Blok (2010) has provided a clear exposition of the problem: ‘This paper gives an overview of the attempts to determine the distribution of dark matter in low surface brightness disk and gas-rich dwarf galaxies, both through observations and computer simulations. Observations seem to indicate an approximately constant dark matter density in the inner parts of galaxies, while cosmological computer simulations indicate a steep power-law-like behaviour. This difference has become known as the “core/cusp problem”, and remains one of the unsolved problems in small-scale cosmology.’

The core-cusp problem therefore is one of the main subjects of many papers on dwarf galaxies (e.g., Navarro and Steinmetz 2000; Klypin et al. 2001; de Blok et al. 2001; Treu and Koopmans 2004; Gentile et al. 2007; Hague and Wilkinson 2014; De Geyter et al. 2014; Oh et al. 2015; Eby et al. 2016; Kam et al. 2017). Even warm DM does not do better than cold DM in solving the small-scale inconsistencies. López Fune et al. (2017) find that the Navarro-Frenk-White (NFW, Navarro et al. 1995, 1996) DM profile provides a better fit to the rotation curve data than the cored Burkert one (Burkert 1995). Bottema and Pestana (2015) conclude that the central regions of galaxies definitively have cores and not cusps. The core radii of the resultant halos are typically one to a few kiloparsec, apparently independent of galaxy mass.

2.3 Satellite galaxies

Further discrepancies between observations and numerical simulations of $\Lambda$CDM and CDM models of cosmology are (1) the “missing satellites problem”, i.e., the number of satellite galaxies of the Milky Way and other large galaxies is far fewer than the number predicted in simulations, and (2) that the observed number of dwarf galaxies as a function of rotation velocity is smaller than predicted by the standard model – the so-called “too big to fail” problem (Klypin et al. 1999; Moore 2001; Kravtsov 2010; Boylan-Kolchin et al. 2011; Tollerud et al. 2011; Kroupa et al. 2012; Strigari et al. 2012; Bottema and Pestana 2015; Eby et al. 2016; Oh et al. 2015; Schneider et al. 2017). Moreover, the dwarf galaxies are not isotropically distributed around the Andromeda Galaxy and the Milky Way, but show ‘a highly flattened distribution around each of the galaxies’ (Henkel et al. 2017).
2.4 New ideas

In view of the many discrepancies between the standard cosmological model and the observational data obtained, in particular, from dwarf galaxies, many new ideas have been discussed in the literature, and Wilczek (2001) writes: ‘We can identify deep questions that seem to call for ideas outside our present grasp’.

Weinberg et al. (2015) consider as alternatives that the small-scale conflicts could be evidence of more complex DM physics itself or that baryonic effects can account for some of the discrepancies; and a Whitepaper describes the new science opportunities and experimental possibilities for the exploration of the nature of DM by small projects (Battaglieri et al. 2017).

3 Baryons and Dark Matter

McGaugh (2012) see Equation 9) has derived for many galaxies the Baryonic Tully-Fisher Relation (BTFR), which implies that the baryonic mass $M_{\text{bar}}$ of a spiral galaxy is related to the flat rotational velocity $v_{\text{f}}$,

$$M_{\text{bar}}(v_{\text{f}}) = A v_{\text{f}}^4,$$

(1)

where $A = (9.4 \pm 1.2) \times 10^{19}$ kg m$^{-4}$ s$^4$ provides a good fit. Any DM contribution in rotationally supported galaxies would be specified by the baryon contribution according to Equations (4), (1) and (3) of McGaugh et al. (2010) as follows:

$$g_{\text{obs}} = \frac{g_{\text{bar}}}{1 - e^{-\sqrt{g_{\text{bar}}/g_{f}}}}$$

(2)

with one fit parameter $g_{f}$, where $g_{\text{obs}}$ is calculated from the observed rotation velocities by

$$g_{\text{obs}} = \frac{v^2(R)}{R},$$

(3)

and $g_{\text{bar}}$ from the gravitational potential $\Phi_{\text{bar}}$ of the sum of the observed baryonic components:

$$g_{\text{bar}} = \left| \frac{\partial \Phi_{\text{bar}}}{\partial R} \right|.$$  

(4)

The authors note in this context: ‘There is no guarantee that $g_{\text{obs}}$ should correlate with $g_{\text{bar}}$ when dark matter dominates. Nevertheless, the radial acceleration relation persists for all galaxies of all types.’

They find for the best fit parameter the acceleration $g_{f} = (1.20 \pm 0.02 \pm 0.24) \times 10^{-10}$ m s$^{-2}$ with random and systematic uncertainties, respectively. This acceleration $g_{f}$ agrees with $a_0 \approx 1.2 \times 10^{-10}$ m s$^{-2}$

given by Kroupa et al. (2012), below which the mass-discrepancy-acceleration correlation between $v_{\text{obs}}$ and $v_{\text{bar}}$ deviates from the Newtonian value. For accelerations $a < a_0$, MOND would be applicable (Milgrom 1983). With this assumption galactic DM is not required for a description of the dynamics of galaxies (cf. Famaey and McGaugh 2012).

As we will show in Sect. 5 the flat rotational velocity curves of spiral galaxies can also be deduced without DM contributions with the help of another model of gravitational interactions based on a modified impact concept proposed for massive bodies (Wilhelm et al. 2013). The difficulties of the old impact theory (cf. Fatio de Duillier 1690; Bopp 1929) have been considered in detail as alternatives to the Special Theory of Relativity (STR, Einstein 1905) and could be removed. The basic idea is that impacting gravitons – originally called quadrupoles – with no mass and a speed of light $c_0$ are absorbed by massive particles and re-emitted with reduced energy $T_G^−$ according to

$$T_G^− = T_G (1 - Y),$$

(5)

where $T_G$ is the (very small) energy of a graviton in the background flux and $Y$ ($0 < Y \ll 1$) is defined as the reduction parameter. The corresponding momentum equation is

$$p_G = -p_G (1 - Y),$$

(6)

with $|p_G| = T_G/c_0$. The equation implies that the diminished graviton is re-emitted in the anti-parallel direction relative to the incoming one. This geometry had to be assumed in a study defining the mass equivalent of the potential energy in a gravitationally bound two-body system. An omni-directional emission, as assumed originally, led to conflicts with energy and momentum conservation (Wilhelm and Dwivedi 2015).

Newton’s law of gravitation could be explained with this model. Moreover, a physical process causing the secular perihelion advances of the inner planets and the Asteroid Icarus could be defined by assuming multiple interactions within the Sun (Wilhelm and Dwivedi 2014). The multiple interaction process is obviously coupled with the presence of large mass conglomerates and, therefore, galaxies might be the places to look for this process. Despite the high mass values of spiral galaxies, they are characterized by low acceleration levels in the outer regions due to their large dimensions. A concentration of comparable amounts of matter would lead to a more spherical geometry, which would not support the amplification process proposed in Sect. 5.
### Table 1: Baryonic mass \( M_{\text{bar}} \) and flat rotation velocity \( v_f \) for the spiral galaxies NGC 7814, NGC 6503 and M 33.

| Physical quantity of galaxy | NGC 7814 | NGC 6503 | M 33 |
|-----------------------------|----------|----------|------|
| Mass, \( M_{\text{bar}} \)/ kg | \( 1.0 \times 10^{11} \) Corbelli and Salucci (2000) | \( 2.1 \times 10^{11} \) McGaugh (2005b) | \( \approx 1.8 \times 10^{10} \) Corbelli et al. (2014) |
| Velocity, \( v_f \)/( km s\(^{-1}\)) | \( \approx 210 \) McGaugh et al. (2016) (their Figure 2) | 115 Greisen et al. (2009) (their Figure 11) | 116 Corbelli et al. (2014) (their Figure 15; MOND) |
| Mass, \( M_{\text{bar}}(v_f) \)/ kg | \( 1.8 \times 10^{11} \) | \( 1.6 \times 10^{10} \) | \( 1.7 \times 10^{10} \) |
| Best fit in Figs. 1 to 3: \( M_{\text{bar}}^{bf} \)/ kg | \( 1.0 \times 10^{11} \) | \( 1.9 \times 10^{10} \) | \( 1.8 \times 10^{10} \) |

### 4 Details on the disk galaxies NGC 7814, NGC 6503 and M 33

We will use three galaxies with different characteristics as examples. The most relevant physical parameters of the galaxies for this study are compiled in Table 1. The mass data in the literature appear to be rather uncertain. Newer values have been selected (when available). The calculations of the mass \( M_{\text{bar}}(v_f) \) have been performed with Eq. (41) according to the BTF relation McGaugh (2012). The best fit values of \( M_{\text{bar}}^{bf} \) in the bottom row have been used in Figs. 1 to 3. Some additional details are given in the following subsections.

#### 4.1 NGC 7814

In the inner part, NGC 7814 is a bulge dominated spiral galaxy (cf., e.g., Fraternali et al. 2011; Angus et al. 2012). The stellar and gas disks are seen edge on. Its mass-to-luminosity ratio is very high. A substantial DM contribution, closely coupled to the luminous component, seems, therefore, necessary to explain the rotation curve in Table 3 of Fraternali et al. (2011). This rotation curve agrees with that of McGaugh et al. (2016). The galactic radius is not well delimited and values between \( R_{\text{gal}} = (2.8 \text{ to } 5.6) \times 10^{20} \text{ m} \) can be found in the literature.

#### 4.2 NGC 6503

The late-type spiral galaxy NGC 6503 is disk dominated and exhibits a regular kinematical structure except for a remarkable drop of the stellar velocity values in the central region (Bottema and Gerritsen 1997; Greisen et al. 2009; McGaugh et al. 2016). The optical radius is \( R_{\text{gal}} \approx 1.7 \times 10^{20} \text{ m} \) (König and Binnewies 2017).

#### 4.3 M 33

The spiral galaxy M 33 has a rather complex structure (cf., e.g., Seigar 2011). The total gas mass is of the same order as the stellar disk mass according to Corbelli (2003). The optical radius is \( R_{\text{gal}} = 2.84 \times 10^{20} \text{ m} \) (King 2015; Giraud 2000b) finds that the results obtained for the extended curve of M 33 are not straightforward. At small radii, there is a wide range of possible models with or without a disk dark component. The uncertainties related to the baryonic and DM contributions in the central region are also highlighted by other authors (cf. Corbelli and Walterbos 2007; Kam et al. 2015, 2017).

#### 5 Multiple interactions of gravitons in galactic disks

The large baryonic masses in galaxies will cause multiple interactions of gravitons with matter if their propagation direction is within the disk. For each interaction the energy loss of the gravitons is assumed to be \( Y T_G \) (for detail see Sect. 2.3 of Wilhelm et al. 2013). The important point is that the multiple interactions occur only in the galactic plane and not for inclined directions. An interaction model is formulated in Table 2 indicating that an amplification factor of approximately two can be achieved by six successive interactions. The mean free path within the plane of the disk be \( \lambda_0 \), then \( z \) of \( z_0 \) gravitons will not interact a second time within a distance \( x \) according to the equation:

\[
z = z_0 \exp \left( -\frac{x}{\lambda_0} \right)
\]  

(Westphal 1956, p.160). For \( x = \lambda_0 = R_{\text{gal}} \), we find with \( z_0 = 100 \) that \( z_0 \exp(-1) = 100 \times 0.37 = 37 \) gravitons leave the interaction region without a second reduction.
whereas $1 - a$ will be reflected with a loss of $2 \times Y$. This process will be repeated several times and has been formulated until a reduction of $6 \times Y$ is reached. The left column gives the factors $f_n$ relative to a process with only one interaction and $\lambda_0 = \infty$. As can be seen, an amplification occurs for four or more interactions. The process works, of course, along each diameter of the disk and leads to a two-dimensional distribution of reduced gravitons.

### 5.1 Multiple interactions in NGC 7814

The bulge-dominated galaxy NGC 7814 conforms best to the assumptions made in Table 2. In Fig. 1 the data presented by McGaugh et al. (2010) (first example in their Figure 2) are plotted as $g_{\text{obs}}$ and $g_{\text{bar}}$, respectively, and compared with the fit of Eq. (2) as well as with the Keplerian acceleration in three dimensions, $g_{\text{M,3D}}$, with the total baryonic mass $M_{\text{bar}}$ (see Table 1) at the centre. Below an acceleration of $g_1$, the curves for $g_{\text{bar}}$ and $g_{\text{M,3D}}$ agree signaling that most of the baryonic mass is indeed inside the corresponding radial distance

$$\lambda_0 = \sqrt{\frac{G_N M_{\text{bar}}}{g_1}}. \tag{8}$$

In line with our assumptions in Table 2, we suggest that $\lambda_0$ provides an estimate of the mean free path in the plane of the disk from the centre outwards, because at this distance $g_{\text{obs}}$ is larger than $g_{\text{bar}}$ by a factor of $f_5 = 1.75$ shown as arrow [3]. Taking into account the results of Table 2, five iterations of gravitational interactions are thus required. Since the reflections can only occur within the plane of the disk, the accelerations $g_{\text{mult,2D}}$ at distances greater than $\lambda_0$ decrease with $\lambda_0/R$ in two dimensions. The agreement with $g_{\text{obs}}$ is excellent.

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Note that Eq. (4) gives a ratio $g_{\text{obs}}/g_{\text{bar}} \approx 1.6$ for $g_{\text{bar}} = g_1$. 

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### Table 2: One-dimensional model of multiple interactions within a disk galaxy. A large baryonic mass is assumed in the centre C.

| $Y$ | Left direction | Left side of disk | $C$ | Right side of disk | Right direction | Left or right sum | Factor $f$ |
|-----|----------------|-------------------|-----|-------------------|----------------|-----------------|------------|
| 1×  | $\Leftarrow a$ | $\Leftarrow 100\% = z_0$ | $z_0 = 100\% \Rightarrow$ | $a \Rightarrow$ | $z \approx 0.37$ | $a = e^{3/\lambda_0} = e^{-1} = 0.37$ | $f_1 = 0.37$ |
| 2×  | $\Leftarrow (1-a)a^2$ | $\Leftarrow (1-a)a$ | $\Rightarrow (1-a)$ | $\Rightarrow (1-a)a^2$ | $2 \times 0.85 \approx 0.17$ | $2 (1-a)a^2$ | $f_2 = 0.54$ |
| 3×  | $\Leftarrow (1-a)^2a$ | $\Leftarrow (1-a)^2a$ | $\Rightarrow (1-a)^2a \Rightarrow$ | $\Rightarrow (1-a)^2a$ | $3 \times 0.166 \approx 0.50$ | $3 (1-a)^2a(1+a^2)$ | $f_3 = 1.04$ |
| 4×  | $\Leftarrow (1-a)^3a^2$ | $\Leftarrow (1-a)^3a^2$ | $\Rightarrow (1-a)^3a^2 \Rightarrow$ | $\Rightarrow (1-a)^3a^2$ | $4 \times 0.073 \approx 0.29$ | $4 (1-a)^3a(2a+a^2)$ | $f_4 = 1.33$ |
| 5×  | $\Leftarrow (1-a)^4a$ | $\Leftarrow (1-a)^4a$ | $\Rightarrow (1-a)^4a \Rightarrow$ | $\Rightarrow (1-a)^4a$ | $5 \times 0.084 \approx 0.42$ | $5(1-a)^4a(1+3a^2+a^4)$ | $f_5 = 1.75$ |
| 6×  | $\Leftarrow (1-a)^5a^2$ | $\Leftarrow (1-a)^5a^2$ | $\Rightarrow (1-a)^5a^2 \Rightarrow$ | $\Rightarrow (1-a)^5a^2$ | $6 \times 0.049 \approx 0.29$ | $6 (1-a)^5a^2(3+4a^2+a^4)$ | $f_6 = 2.04$ |
5.2 Multiple interactions in NGC 6503

The disk-dominated galaxy NGC 6503 also conforms quite well with the assumptions made in Table 2, although the central bulge is missing. Conceptually, this could be modelled by omitting the column “C”, i.e. the mean free path $\lambda_0$ is related to the diameter of the galaxy and not to its radius. In Fig. 2 the data presented by McGaugh et al. (2016) (second example in their Figure 2) are plotted as $g_{\text{obs}}$ and $g_{\text{bar}}$, respectively, and compared with the fit of Eq. (2) as well as with the Keplerian acceleration in three dimensions, $g_{M_{\text{bar}}}$, with the corresponding total baryonic mass $M_{\text{bar}}$ (see Table 1) at the centre. At an acceleration of $g_1$, the curves for $g_{\text{obs}}$, $g_{M_{\text{bar}}}$ and $g_{\text{bar}}$ multiplied by a factor of 1.75 agree at a radial distance that we denote by $\lambda_0/2$, taking the missing bulge into account. This distance is close to the galactic radius $R_{\text{gal}} = 1.7 \times 10^{20}$ m. Outside this radius $g_{\text{mult, 2D}}$ decreases with $(\lambda_0/2)/R$.

5.3 Multiple interactions in M 33

The galaxy M 33 has a complex structure with a very large gas contribution. Data from Corbelli and Salucci (2000) and Corbelli et al. (2014) are shown in Fig. 2 and are again compared favourably with the fit of Eq. (2). As for NGC 6503, there is no significant bulge and thus the mean free path $\lambda_0$ may again be related to the diameter of the disk. Therefore, we plot $\lambda_0/2$ in this diagram at a radial distance from the centre where $g_{\text{obs}}/g_{\text{bar}}$ is $\approx$ 1.75. Note, however, that the complex structure of M 33 casts some doubt on its position. It could well be that $\lambda_0/2$ is much larger and an amplification factor $f_5 > 2$ or higher would be applicable. Notwithstanding these difficulties the curves $g_{M_{\text{bar}}}$ and $g_{\text{bar}}$ agree outside of $R_{\text{gal}}$, where $g_{\text{obs}}$ can also be fitted very well by $g_{\text{mult, 2D}}$. 

Fig. 1 Acceleration curves for NGC 7814 versus galactic radius $R$. The data $g_{\text{obs}}$ and $g_{\text{bar}}$ as well as the fit $g_{\text{fit}}$ with $g_1 = 1.2 \times 10^{-15}$ m s$^{-2}$ are taken from McGaugh et al. (2016). Outside of $\lambda_0$, corresponding to $g_1$, the curves $g_{M_{\text{bar}}}$ with $M_{\text{bar}}$ of Table 1 (first column) and $g_{\text{obs}}$ coincide with that of $g_{\text{bar}}$, if a $1/R^2$ dependence is assumed. This distance is close to the galactic radius $R_{\text{gal}} = 1.7 \times 10^{20}$ m. Outside of $\lambda_0$, the curves coincide, if a $1/R^2$ dependence is assumed. Irrespective of this complication, a multiplication factor $f_5 = 1.75$ is probably causing this behaviour.

Fig. 2 Acceleration curves for NGC 6503 versus galactic radius $R$. The data $g_{\text{obs}}$ and $g_{\text{bar}}$ as well as the fit $g_{\text{fit}}$ with $g_1 = 1.2 \times 10^{-15}$ m s$^{-2}$ are also taken from McGaugh et al. (2016). Between $\lambda_0$ and $\lambda_*$, corresponding to $g_{\text{bar}} = 7.5 \times 10^{-11}$ m s$^{-2}$ and $\approx 10^{-11}$ m s$^{-2}$, respectively, the curve $g_{\text{bar}}$ does not coincide with that of $g_{M_{\text{bar}}}$ for $M_{\text{bar}}$ of Table 1 (second column). A substantial gas contribution between $\lambda_0/2$ and $\lambda_*$ (cf. Figure 2, McGaugh et al. 2016) is probably causing this behaviour.
6 Discussion and conclusion

The process of multiple interactions of gravitons with baryons in three spiral galaxies leads to the observed flat velocity curves without the need to invoke any DM. It thus provides a physical process for MOND in two steps: (1) Multiple interactions of gravitons with baryons and (2) a propagation confined to two dimensions. Whether it can also be applied to other galaxies remains to be studied.

The process can also explain the BTF relation in Eq. (1). Equating the centripetal acceleration $g_{\text{obs}}$ in Eq. (5) with the amplified baryonic acceleration at $\lambda_0$ and taking into account its decrease with $\lambda_0/R$, we find with Eq. (5)

$$g_{\text{obs}}(R) = \frac{v^2(R)}{R} = \frac{g_1 \lambda_0}{R}.$$  

For the flat rotation velocity $v_1$ it then follows

$$v_1^4 = f_5^2 g_1 G N M_{\text{bar}},$$

where $1/(f_5^2 g_1 G N) = 4 \times 10^{19} \text{ m}^{-1} \text{ s}^4$. This value is in reasonable agreement with the coefficient $A$ in Eq. (1) considering the discrepancies between the mass values, in particular, for NGC 7814.

It should be pointed out that the multiple interactions do not increase the total reduction of graviton energy, because the total number of interactions is determined by the (baryonic) mass of the gravitational centre (cf. Wilhelm et al. 2013). A galaxy with enhanced gravitational acceleration in two dimensions defined by the galactic plane, will, therefore, have a reduced acceleration in directions inclined to this plane. This might be relevant for the “missing satellites problem” mentioned in Sect. 2.3.

The “core/cusp problem” described in Sect. 2.2 is irrelevant in the context of our discussion, because only baryonic matter is involved in the multiple interaction process without any need for DM.

Finally, an effect of gravitational lensing should be touched upon, although a detailed discussion is beyond the scope of this study. It is, however, of interest that weak lensing signals for prolate clusters can be twice that predicted under certain geometric conditions (Schneider et al. 2012).

We want to conclude with a statement by Sotiriou et al. (2017) made in the context of gravitational theories in ‘A no-progress report’: ‘[...] it is not only the mathematical formalism associated with a theory that is important, but the theory must also include a set of rules to interpret physically the mathematical laws’.

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5.4 Common aspects of the galaxies NGC 7814, NGC 6503 and M 33

Both the bulge-dominated galaxy NGC 7814 and the disk-dominated NGC 6503 could be fitted quite well with our multiple interaction process and a two-dimensional propagation of the affected gravitons. The mean free paths of gravitons, $\lambda_0$, within the disks may be quite similar near $\approx 2.4 \times 10^{20} \text{ m}$, if we take the different structures into account. Even the mean free path in M 33 might reach this value considering the uncertainties involved here. In any case, the multiple interaction process provides a good fit at distances greater $R_{\text{gal}}$. Moreover, the amplification factors of $f_n$ to 2 are appropriate at acceleration levels of $g \approx 1 \times 10^{-18} \text{ m s}^{-2}$ close to $a_0$ (Milgrom 1983, 2015) and $g_1$ (McGaugh et al. 2016).

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**Fig. 3** Acceleration curves for M 33 versus galactic radius $R$. The data for $g_{\text{obs}}$ are from Table 1 of Corbelli et al. (2014), in agreement with Figure 6 of Corbelli and Salucci (2004), where the stellar as well as gas contributions are taken from. The fit $g_{\text{fit}}$, assumed $g_1 = 1.2 \times 10^{-19} \text{ m s}^{-2}$, is calculated with Eq. (2) (Eq. 4 of McGaugh et al. 2016). A multiplication factor $f_5 = 1.75$, indicated by the arrow at $\lambda_0/2$, determines the amplified acceleration, however, the curves $g_{\text{mult}0}$ with $M_{\text{bar}}^0$ of Table 1 (third column) and $g_{\text{Rgal}}$ coincide with that of $g_{\text{bar}}$ only beyond $R_{\text{gal}} = 2.84 \times 10^{20} \text{ m}$. A significant gas contribution (cf. Figure 2, McGaugh et al. 2016) seems to be responsible for this behaviour. The two-dimensional fit $g_{\text{mult}2D}$ also agrees with $g_{\text{obs}}$ only outside the galactic radius.
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