Accretions of dark matter and dark energy onto \((n + 2)\)-dimensional Schwarzschild black hole and Morris-Thorne wormhole

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Received: 24 August 2015 / Accepted: 24 October 2015 / Published online: 5 November 2015
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Abstract In this work, we have studied accretion of the dark matter and dark energy onto \((n + 2)\)-dimensional Schwarzschild black hole and Morris-Thorne wormhole. The mass and the rate of change of mass for \((n + 2)\)-dimensional Schwarzschild black hole and Morris-Thorne wormhole have been found. We have assumed some candidates of dark energy like holographic dark energy, new agegraphic dark energy, quintessence, tachyon, DBI-essence, etc. The black hole mass and the wormhole mass have been calculated in term of redshift when dark matter and above types of dark energies accrete onto them separately. We have shown that the black hole mass increases and wormhole mass decreases for holographic dark energy, new agegraphic dark energy, quintessence, tachyon accretion and the slope of increasing/decreasing of mass sensitively depends on the dimension. But for DBI-essence accretion, the black hole mass first increases and then decreases and the wormhole mass first decreases and then increases and the slope of increasing/decreasing of mass not sensitively depends on the dimension.

Keywords Accretion · Black hole · Wormhole · Dark energy

1 Introduction

Recent observations of type Ia supernovae indicate that the expansion of the Universe is accelerating rather than slowing down (Bachall et al. 1999; Perlmutter et al. 1997, 1999; Riess et al. 1998). These results, when combined with cosmic microwave background (CMB) observations of a peak in the angular power spectrum on degree scales (de Bernardis et al. 2000; Lange et al. 2001; Balbi et al. 2000), strongly suggest that the Universe is spatially flat with \(\sim 1/3\) of the critical energy density being in nonrelativistic matter and \(\sim 2/3\) in a smooth component with large negative pressure. This acceleration is caused by some unknown matter is known as “dark energy” (DE) (Briddle et al. 2003; Spergel et al. 2003; Peebles and Ratra 1988; Caldwell et al. 1998). Recent WMAP (Bennett et al. 2003) and Chandra X-ray Observations (Allen et al. 2004) strongly indicate that our universe is undergoing an accelerating phase. The most appealing and simplest candidate for DE is the cosmological constant \(\Lambda\) which is characterized by the equation of state \(p = w \rho\) with \(w = -1\). Many other theoretical models have been proposed to explain the accelerated expansion of the universe. Another candidate of dark energy is quintessence satisfying \(-1 < w < -1/3\) (Peebles and Ratra 1988; Caldwell et al. 1998). When \(w < -1\), it is known as phantom energy (Alam et al. 2004) which has the negative kinetic energy. Recently many cosmological models have been constructed by introducing dark energies such as quintessence (Peebles and Ratra 1988; Caldwell et al. 1998), DBI-essence (Gumjudpai and Ward 2009; Martin and Yamaguchi 2008), Tachyon (Sen 2002), holographic dark energy (Li 2004; Hsu 2004; Cohen et al. 1999), new agegraphic dark energy (Wei and Cai 2007, 2008, 2009), etc.

In Newtonian theory, the problem of accretion of matter onto the compact object was first formulated by Bondi (1952). Michel (1972) has formulated the accretion process of steady-state spherical symmetric flow of matter into or out of a condensed object. The accretion of phantom dark energy onto a static Schwarzschild black hole...
was first formulated by Babichev et al. (2004, 2005) using Michell’s process and established that static Schwarzschild black hole mass will gradually decrease to zero near the big rip singularity. Recently, Jamil (2009) has investigated accretion of phantom like modified variable Chaplygin gas onto Schwarzschild black hole. Also the accretion of dark energy onto the more general Kerr-Newman black hole was studied by Ji´menez Madrid and González-Díaz (2008) and Bhadra and Debnath (2012). Nayak and Jamil (2012) have studied the vacuum energy evolution of primordial black hole in Einstein’s gravity. Analytical solutions of accreting black holes immersed in a ΛCDM model have been found by Lima et al. (2010). Sharif and Abbas (2012) have studied the phantom energy accretion by stringy charged black hole. Phantom energy accretion onto black holes in Cyclic Universe have analyzed by Sun (2008). Kim and Kang (2012) have analyzed the dark energy and the matter accretion onto a black hole in expanding universe. Rodrigues and Bernardiniz (2012) have studied the accretion of non-minimally coupled generalized Chaplygin gas into black holes. Phantom energy accretion onto a black hole in Horava Lifshitz gravity has been discussed by Abbas (2014) and also discussed (Abbas Chin 2013) the thermodynamics of phantom energy accreting onto a black hole in Einstein-Power-Maxwell gravity. Now there is a lot of interest of the investigation of dark energy accretion onto static wormhole (González-Díaz 2004a, 2004b; Faraoni and Israel 2005). González-Díaz (2006a) has discussed the phantom energy accretion onto wormhole. Madrid and Martín-Moruno (2010) and Martín-Moruno (2008) have analyzed a general formalism for the accretion of dark energy onto astronomical objects, black holes and wormholes. Subsequently, the in accelerating universe, the dark energy accretion onto wormhole has been discussed in González-Díaz (2006b), González-Díaz and Martín-Moruno (2007).

Recently, there has been a growing interest to study the accretion of higher dimensional black hole (BH). Interest in the BTZ black hole has recently heightened with the discovery that the thermodynamics of higher dimensional black holes (Kim et al. 1997). Also, non-static charged BTZ like black holes in (n + 1)-dimensions have been considered by Ghosh (2011). John et al. (2013) examined the steady-state spherically symmetric accretion of relativistic fluids (like polytropic equation of state) onto a higher dimensional Schwarzschild black hole. Also charged BTZ-like black holes in higher dimensions have been studied by Hendi (2011). By the motivations of above works, we shall assume the accretions of dark matter and dark energy onto (n + 2)-dimensional Schwarzschild black hole and Morris-Thorne wormhole. The nature of masses of black hole and wormhole will be investigated during various types of dark energies like holographic dark energy, new agegraphic dark energy, quintessence, tachyon, DBI-essence, etc. Finally we present the conclusions of the whole work.

2 Accretions of dark matter and energy onto Schwarzschild black hole and Morris-Thorne wormhole

Let us consider (n + 2)-dimensional spherically symmetrical accretion of the dark energy onto the black hole. We consider a Schwarzschild black hole (static) of mass $M$ which is gravitationally isolated (in geometrical units, $8\pi G = 1 = c$) (Kanti and Winstanley 2014; John et al. 2013) described by the line element

$$ds^2 = -\left(1 - \frac{\mu}{r^{n-1}}\right)dt^2 + \left(1 - \frac{\mu}{r^{n-1}}\right)^{-1}dr^2 + r^2d\Omega_n^2$$

(1)

Here, $r$ being the radial coordinate and $\mu = \frac{8\pi M \Gamma(\frac{n+1}{2})}{n \pi r^{n+1}}$, where, $M$ is the mass of the Schwarzschild black hole. Energy momentum-tensor for the DE, considering in the form of perfect fluid having the EoS $p = p(\rho)$, is

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$$

(2)

where $\rho$, $p$ are the density and pressure of the dark energy respectively and $u^\mu = \frac{dx^\mu}{ds}$ is the fluid (n + 2)-velocity satisfying $u^\mu u_\mu = -1$. We assume that the in-falling dark energy fluid does not disturb the spherical symmetry of the black hole.

The rate of change of mass $\dot{M}$ of the Schwarzschild black hole is computed by integrating the flux of the dark energy over the $n$-dimensional volume of the black hole and given by (John et al. 2013)

$$\dot{M} = -\frac{2\pi \frac{n+1}{2}}{\Gamma(\frac{n+1}{2})} r^n T_{01}$$

(3)

where, $A$ is a positive constant given in Babichev et al. (2004), which can be written as

$$\dot{M} = \frac{2\pi \frac{n+1}{2}}{\Gamma(\frac{n+1}{2})} AM^n (\rho + p)$$

(4)

For quintessence model, $\rho + p > 0$, so $\dot{M} > 0$, i.e., $M$ increases. But for phantom model, $\rho + p < 0$, so $\dot{M} < 0$, i.e., $M$ decreases as Universe expands.

Let us consider (n + 2)-dimensional spherically symmetrical accretion of the dark energy onto the wormhole. We
consider a non-static spherically symmetric Morris-Thorne wormhole metric (Morris and Thorne 1988) given by

$$ds^2 = -e^{\Phi(r)}dt^2 + \frac{dr^2}{1 - K(r)} + r^2d\Omega_n^2$$

(5)

where the functions $K(r)$ and $\Phi(r)$ are the shape function and redshift function respectively of radial co-ordinate $r$. If $K(r_0) = r_0$, the radius $r_0$ is called wormhole throat radius. So we want to consider the outward region such that $r_0 \leq r < \infty$. Here we have assumed the redshift function $\Phi(r) = 0$. The rate of change of mass $M$ of the wormhole is given by (González-Díaz 2006b)

$$\dot{M} = -\frac{2\pi n+1}{\Gamma(\frac{n+1}{2})} BM^n (\rho + p)$$

(6)

where $B$ is positive constant. For quintessence model, $\rho + p > 0$, so $\dot{M} < 0$, i.e., $M$ decreases. But for phantom model, $\rho + p < 0$, so $\dot{M} > 0$, i.e., $M$ increases as Universe expands.

We consider the background spacetime is spatially flat represented by the homogeneous and isotropic $(n + 2)$-dimensional FRW model of the universe which is given by

$$ds^2 = -dt^2 + a^2(t)[dr^2 + r^2d\Omega_n^2]$$

(7)

where $a(t)$ is the scale factor. The Einstein’s field equations are given by ($8\pi G = c = 1$)

$$n(n+1)H^2 = 2\rho$$

(8)

and

$$n\dot{H} = - (\rho + p)$$

(9)

Conservation equation is

$$\dot{\rho} + (n + 1)H(\rho + p) = 0$$

(10)

Now assume that the universe is filled with dark matter and dark energy, so $\rho = \rho_m + \rho_D$ and $p = p_m + p_D$. Here, $\rho_m$ and $p_m$ are respectively energy density and pressure of dark matter. Also, $\rho_D$ and $p_D$ are respectively energy density and pressure of dark energy. Now, assume that the dark matter and dark energy are separately conserved. So,

$$\dot{\rho}_m + (n + 1)H(\rho_m + p_m) = 0$$

(11)

and

$$\dot{\rho}_D + (n + 1)H(\rho_D + p_D) = 0$$

(12)

Now assume that dark matter obeys the equation of state $p_m = w_m\rho_m$ and using the redshift relation $1 + z = \frac{1}{a}$ (assume, at present, $a_0 = 1$), we get the solution as

$$\rho_m = \rho_{m0}(1 + z)^{(1+n)(1+w_m)}$$

(13)

where, $\rho_{m0}$ is the present value of the energy density of dark matter.

Using Eqs. (8) and (10), Eq. (4) integrates to yield the mass of the Schwarzschild black hole as

$$M = \frac{M_0}{[1 + \frac{4(n-1)n\frac{n+1}{2}AM^{n-1}}{\Gamma(\frac{n+1}{2})}\sqrt{\frac{\rho}{2(n+1)}(\sqrt{\rho} - \sqrt{\rho_0})}]^{\frac{1}{n-1}}}$$

(14)

where, $\rho_0 (= \rho_{m0} + \rho_{D0})$ is the present value of the density and $M_0$ is the present value of the Schwarzschild black hole mass. Similar happens for wormhole mass. Using Eq. (10), Eq. (6) integrates to yield the mass of the Morris-Thorne wormhole as in the form:

$$M = \frac{M_0}{[1 - \frac{4(n-1)n\frac{n+1}{2}BM^{n-1}}{\Gamma(\frac{n+1}{2})}\sqrt{\frac{\rho}{2(n+1)}(\sqrt{\rho} - \sqrt{\rho_0})}]^{\frac{1}{n-1}}}$$

(15)

where, $M_0$ is the present value of the Morris-Thorne wormhole mass. In the following subsections, we shall assume various types of dark energies like holographic dark energy, new agegraphic dark energy, quintessence, tachyon, DBI-essence, etc.

### 2.1 Holographic dark energy

In quantum field theory a short distance cut-off is related to a long distance cut-off (infra-red cut-off $L$) due to the limit set by black hole formation, the total energy in a region of size $L$ should not exceed the mass of a black hole of the same size, i.e., $L^3\rho_D \leq LM_p^2$ (where $M_p^2 = 8\pi G = 1$). If the whole universe is taking into account, then the vacuum energy related to this holographic principle is viewed as dark energy, usually called holographic dark energy. The energy density for the holographic dark energy is given by (Li 2004; Hsu 2004; Cohen et al. 1999)
\[ \rho_D = 3c^2 L^{-2} \]

where \( L \) is the IR cut-off length and \( c \) is constant. We assume \( L = H^{-1} \). So using Eqs. (8), (13) and (16), we obtain

\[ \rho_D = \frac{6c^2 \rho_m 0}{n(n+1) - 6c^2 (1 + z)(1 + w_m)} \]

From Eq. (14), we obtain the mass of the Schwarzschild black hole as

\[ M = \frac{M_0}{[1 + A M_1 M_0^{n-1} ((1 + z) (1 + w_m) - 1)]^{1/\eta}} \]

and from Eq. (15), we obtain the mass of the wormhole as

\[ M = \frac{M_0}{[1 - B M_1 M_0^{n-1} ((1 + z) (1 + w_m) - 1)]^{1/\eta}} \]

where, \( M_1 = \frac{4n(n+1) - \sqrt{\rho_m 0}}{\Gamma(\frac{n+1}{2}) \sqrt{2(n(n+1)-6c^2)}} \) with \( n(n+1) > 6c^2 \).

The black hole mass \( M \) and wormhole mass \( M \) vs. redshift \( z \) have been drawn in Figs. 1 and 2 respectively for different values of \( n = 2, 3, 4, 5 \) (i.e., \( 4D, 5D, 6D, 7D \)) when dark matter and holographic dark energy accrete onto black hole and wormhole. For dark matter, we have taken EoS parameter \( w = 0.01 \). From the figures, we see that black hole mass increases and wormhole mass decreases during whole evolution of the Universe. The slope of mass of black hole increases when \( n \) increases i.e., mass of black hole increases more sharply for increase of dimensions. Similarly, the slope of mass of wormhole decreases when \( n \) increases i.e., mass of wormhole decreases more sharply for increase of dimensions. These are the features of dark matter and dark energy accretions onto black hole and wormhole and natures of increasing/decreasing of mass completely depends on the dimensions. For more dimension, mass of the black hole increases more sharply and mass of the wormhole decreases more sharply for holographic dark energy accretion.

### 2.2 New agegraphic dark energy

The energy density of the new agegraphic dark energy is given by (Wei and Cai 2007, 2008, 2009)

\[ \rho_D = \frac{3\alpha^2}{\eta^2} \]

where \( \alpha \) is a constant and the conformal time \( \eta = \int \frac{dt}{a} \). For simplicity, we assume the power law form of the scale factor, \( a = a_i t^m \) where \( a_i \) is positive constant and \( 0 < m < 1 \). So we find \( \eta = \frac{t(1-m)}{a_i (1-m)} \). From above we obtain,

\[ \rho_D = 3\alpha^2 a_i^2 (1 - m^2) [a_i (1 + z)]^{2(1 - m)} \]

From Eq. (14), we obtain the mass of the Schwarzschild black hole as

\[ M = \frac{M_0}{[1 + A M_2 M_0^{n-1} \sqrt{3\alpha^2 a_i^2 (1 - m^2) [a_i (1 + z)]^{2(1 - m)} + \rho_m 0 (1 + z) (1 + w_m) - \sqrt{\rho_m 0})]}^{1/\eta} \]

and from Eq. (15), we obtain the mass of the wormhole as

\[ M = \frac{M_0}{[1 - B M_2 M_0^{n-1} \sqrt{3\alpha^2 a_i^2 (1 - m^2) [a_i (1 + z)]^{2(1 - m)} + \rho_m 0 (1 + z) (1 + w_m) - \sqrt{\rho_m 0})]}^{1/\eta} \]

where, \( \rho_m = 3\alpha^2 (1 - m^2) a_i^2 + \rho_m 0 \) and \( M_2 = \frac{4(n-1) \pi^{n/2} \Gamma(\frac{n+1}{2})}{\sqrt{2(n(n+1))}} \times \sqrt{\frac{n}{2(n+1)}} \).

The black hole mass \( M \) and wormhole mass \( M \) vs. redshift \( z \) have been drawn in Figs. 3 and 4 respectively for different values of \( n = 2, 3, 4, 5 \) (i.e., \( 4D, 5D, 6D, 7D \)) when dark matter and new agegraphic dark energy accrete onto black hole and wormhole. For dark matter, we have taken EoS parameter \( w = 0.01 \). From the figures, we see that black hole mass increases and wormhole mass decreases during
Accretions of dark matter and dark energy onto \((n+2)\)-dimensional Schwarzschild black hole.

Fig. 3 The variations of black hole mass \(M\) against redshift \(z\) respectively for new agegraphic dark energy (NADE) accretion in various dimensions \((n=2, 3, 4, 5)\)

Fig. 4 The variations of wormhole mass \(M\) against redshift \(z\) respectively for new agegraphic dark energy (NADE) accretion in various dimensions \((n=2, 3, 4, 5)\)

whole evolution of the Universe. The slope of mass of black hole increases when \(n\) increases i.e., mass of black hole increases more sharply for increase of dimensions. Similarly, The slope of mass of wormhole decreases when \(n\) increases i.e., mass of wormhole decreases more sharply for increase of dimensions. These are the features of dark matter and dark energy accretions onto black hole and wormhole and natures of increasing/decreasing of mass completely depends on the dimensions. For more dimension, mass of the black hole increases more sharply and mass of the wormhole decreases more sharply for new agegraphic dark energy accretion.

2.3 Quintessence

The energy density and pressure for quintessence scalar field are (Peebles and Ratra 1988; Caldwell et al. 1998)

\[
\rho_D = \frac{1}{2} \dot{\phi}^2 + V(\phi) \tag{24}
\]

and

\[
\rho_D = \frac{1}{2} \dot{\phi}^2 - V(\phi) \tag{25}
\]

where \(\phi\) is the quintessence scalar field and \(V(\phi)\) is the potential. If we put the above expressions in the conservation Eq. (12), we get the wave equation, which contains \(V\) and \(\dot{\phi}^2\). Now to get the solution of \(\dot{\phi}^2\) and \(V\), we need to consider \(V\) in term of either \(\phi\) or \(\dot{\phi}^2\). If we assume \(V\) is some form of \(\phi\), it is very difficult to obtain the solution of \(V\) or \(\dot{\phi}^2\). So for simplicity of the calculation, we assume the potential term \(V\) is proportional to the kinetic term \(\dot{\phi}^2\), i.e., \(V = k\dot{\phi}^2\), and we obtain

\[
\rho_D = \left(k + \frac{1}{2}\right)(C(1+z)^{1+n})^{\frac{2}{1+n}} \tag{26}
\]

From Eq. (14), we obtain the mass of the Schwarzschild black hole as

\[
M = M_0 \frac{1}{\sqrt{\left(k + \frac{1}{2}\right)\{C(1+z)^{1+n}\}^{\frac{2}{1+n}} + \rho_{m0}(1+z)^{(1+n)(1+w_m)} - \sqrt{\rho_0}}^{\frac{1}{1+n}} \tag{27}
\]

and from Eq. (15), we obtain the mass of the wormhole as

\[
M = M_0 \frac{1}{\sqrt{\left(k + \frac{1}{2}\right)\{C(1+z)^{1+n}\}^{\frac{2}{1+n}} + \rho_{m0}(1+z)^{(1+n)(1+w_m)} - \sqrt{\rho_0}}^{\frac{1}{1+n}} \tag{28}
\]

where, \(\rho_0 = (k + \frac{1}{2})C^{\frac{2}{1+n}} + \rho_{m0}\) and \(M_2 = \frac{4(n-1)^{\frac{n+1}{2}}}{\Gamma\left(\frac{n+2}{2}\right)}\times\sqrt{\frac{n}{2(n+1)}}\).

The black hole mass \(M\) and wormhole mass \(M\) vs. redshift \(z\) have been drawn in Figs. 5 and 6 respectively for different values of \(n = 2, 3, 4, 5\) (i.e., \(4D, 5D, 6D, 7D\)) when dark matter and quintessence dark energy accrete onto black hole and wormhole. For dark matter, we have taken EoS parameter \(w = 0.01\). From the figures, we see that black hole mass increases and wormhole mass decreases during whole
The variations of black hole mass $M$ against redshift $z$ respectively for quintessence accretion in various dimensions ($n = 2, 3, 4, 5$) evolution of the Universe. The slope of mass of black hole increases when $n$ increases i.e., mass of black hole increases more sharply for increase of dimensions. Similarly, the slope of mass of wormhole decreases when $n$ increases i.e., mass of wormhole decreases more sharply for increase of dimensions. These are the features of dark matter and dark energy accretions onto black hole and wormhole and natures of increasing/decreasing of mass completely depends on the dimensions. For more dimension, mass of the black hole increases more sharply and mass of the wormhole decreases more sharply for quintessence dark energy accretion.

### 2.4 Tachyonic field

The energy density $\rho_D$ and the pressure $p_D$ of the tachyonic field are (Sen 2002)

$$\rho_D = \frac{V(\phi)}{\sqrt{1 - \dot{\phi}^2}}$$  \hspace{1cm} (29)

and

$$p_D = -\frac{V(\phi)}{\phi} (1 - \dot{\phi}^2)^{-1} + (n + 1)H = 0$$  \hspace{1cm} (30)

where $\phi$ is the tachyonic field, $V(\phi)$ is the tachyonic field potential. Put these expressions in the conservation equation (12), we get the wave equation

$$\frac{\ddot{V}}{V} + \frac{\dot{\phi}^2}{\phi} (1 - \dot{\phi}^2) + (n + 1)H = 0$$  \hspace{1cm} (31)

Now, in order to solve Eq. (31), we take a simple form of $V = (1 + \dot{\phi}^2)^{-m}$, $(m > 0)$ (Chattopadhyay et al. 2008), so that the solution of $V$ is obtained as

$$V = [1 + \{C(1 + z)^{1+n}\}^{\frac{2}{1+2m}}]^{m}$$  \hspace{1cm} (32)

So from Eq. (29), we obtain

$$\rho_D = [1 + \{C(1 + z)^{1+n}\}^{\frac{2}{1+2m}}]^{1+2m}$$  \hspace{1cm} (33)

From Eq. (14), we obtain the mass of the Schwarzschild black hole as

$$M = \frac{M_0}{\left[1 + AM_2M_0^{n-1}\{\sqrt{1 + \{C(1 + z)^{1+n}\}}^{\frac{2}{1+2m}}\}^{\frac{1+2m}{2}} + \rho_m(1 + z)^{(1+n)(1+w_m)} - \sqrt{\rho_0}\right]^{\frac{1}{n-1}}}$$  \hspace{1cm} (34)

and from Eq. (15), we obtain the mass of the wormhole as

$$M = \frac{M_0}{\left[1 - BM_2M_0^{n-1}\{\sqrt{1 + \{C(1 + z)^{1+n}\}}^{\frac{2}{1+2m}}\}^{\frac{1+2m}{2}} + \rho_m(1 + z)^{(1+n)(1+w_m)} - \sqrt{\rho_0}\right]^{\frac{1}{n-1}}}$$  \hspace{1cm} (35)

where, $\rho_0 = (1 + C \frac{2}{1+2m})^{\frac{1+2m}{2}} + \rho_m$ and $M_2 = \frac{4(n-1)n\pi^{\frac{n+1}{2}}}{\Gamma(\frac{n+1}{2})} \times \sqrt{\frac{n}{2(n+1)}}$.

The black hole mass $M$ and wormhole mass $M$ vs. redshift $z$ have been drawn in Figs. 7 and 8 respectively for different values of $n = 2, 3, 4, 5$ (i.e., $4D, 5D, 6D, 7D$) when dark matter and tachyon dark energy accrete onto black hole and wormhole. For dark matter, we have taken EoS parameter $w = 0.01$. From the figures, we see that black hole mass increases and wormhole mass decreases during whole evo-
The variations of black hole mass $M$ against redshift $z$ respectively for Tachyonic field accretion in various dimensions ($n = 2, 3, 4, 5$)

The variations of wormhole mass $M$ against redshift $z$ respectively for Tachyonic field accretion in various dimensions ($n = 2, 3, 4, 5$)

2.5 DBI-essence

The energy density and pressure of the scalar field are respectively given by (Gumjudpai and Ward 2009; Martin and Yamaguchi 2008)

$$\rho_D = (\gamma - 1) T(\phi) + V(\phi), \quad (36)$$

$$p_D = \frac{\gamma - 1}{\gamma} T(\phi) - V(\phi), \quad (37)$$

where the quantity $\gamma$ is reminiscent from the usual relativistic Lorentz factor and is given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{\dot{\phi}^2}{T(\phi)}}} \quad (38)$$

where $\phi$ is the DBI scalar field and $V(\phi)$ is the corresponding potential. From energy conservation equation (12), we have the wave equation for $\phi$ as

$$\ddot{\phi} - \frac{3T'(\phi)}{2T(\phi)} \dot{\phi}^2 + T'(\phi) + \frac{(n + 1)}{\gamma^2} \frac{\dot{a}}{a} \dot{\phi}$$

$$+ \frac{1}{\gamma^3} \left[ V'(\phi) - T'(\phi) \right] = 0. \quad (39)$$

Let us assume, $\gamma = \dot{\phi}^{-2}$ (Debnath and Jamil 2011), so from (38) we have $T(\phi) = \frac{\dot{\phi}^2}{1 - \dot{\phi}^2}$. Let us also assume $V(\phi) = T(\phi)$. In this case, we have the solutions:

$$\dot{\phi}^2 = \sqrt{1 + \frac{1}{(n + 1) \log \frac{C}{a}}} \quad (40)$$

$$V(\phi) = T(\phi) = (n + 1) \log \frac{a}{C} \times \sqrt{1 + \frac{1}{(n + 1) \log \frac{C}{a}}} \quad (41)$$

where $C$ is the integration constant. Thus from Eq. (36), we obtain

$$\rho_D = (n + 1) \log \frac{1}{C(1 + z)} \quad (42)$$

From Eq. (14), we obtain the mass of the Schwarzschild black hole as

$$M = \frac{M_0}{\left[ 1 + AM_2 M_0^{n-1} \left\{ \sqrt{(n + 1) \log \frac{1}{e(1 + z)} + \rho_m(1 + z)(1+n)(1+w_m) - \sqrt{\rho_0}} \right\} \right]^{\frac{1}{n-1}}} \quad (43)$$

and from Eq. (15), we obtain the mass of the wormhole as

$$M = \frac{M_0}{\left[ 1 - BM_2 M_0^{n-1} \left\{ \sqrt{(n + 1) \log \frac{1}{e(1 + z)} + \rho_m(1 + z)(1+n)(1+w_m) - \sqrt{\rho_0}} \right\} \right]^{\frac{1}{n-1}}} \quad (44)$$
where, \( \rho_0 = (n + 1) \log \frac{1}{z} + \rho_{m0} \) and \( M_2 = \frac{4(a-1)\pi^{\frac{3+n}{2}}}{\Gamma\left(\frac{3+n}{2}\right)} \times \sqrt{\frac{n}{2\pi(z+1)}}. \)

The black hole mass \( M \) and wormhole mass \( M \) vs. redshift \( z \) have been drawn in Figs. 9 and 10 respectively for different values of \( n = 2, 3, 4, 5 \) (i.e., \( 4D, 5D, 6D, 7D \)) when dark matter and DBI-essence accrete onto black hole and wormhole. For dark matter, we have taken EoS parameter \( w = 0.01 \). From the figures, we see that black hole mass first increases up to a certain finite value and then decreases and wormhole mass decreases up to a certain finite value and then increases during whole evolution of the Universe. These are the features of dark matter and dark energy accretions onto black hole and wormhole for DBI-essence accretion, because our considered model is the phantom crossing model. The natures of increasing/decreasing of mass are nearly similar to all dimensions.

3 Discussions and concluding remarks

A proper dark-energy accretion model for black hole have been obtained by generalizing the Michel theory (Michel 1972) to the case of black hole. Such a generalization has been already performed by Babichev et al. (2004, 2005) for the case of dark-energy accretion onto Schwarzschild black hole. Astrophysically, masses of the black hole and wormhole are dynamical quantity, so the nature of the mass function is important in our black hole/wormhole model for different dark energy filled universe. Previously it has shown that the mass of black hole increases due to quintessence energy accretion and decreases due to phantom energy accretion. In González-Díaz (2006b), it was shown that for quintessence like dark energy, the mass of the wormhole decreases and phantom like dark energy, the mass of wormhole increases, which is the opposite behaviour of black hole mass.

In this work, we have studied accretion of the dark matter and dark energy onto of \((n+2)\)-dimensional Schwarzschild black hole and Morris-Thorne wormhole. The mass and the rate of change of mass for \((n+2)\)-dimensional Schwarzschild black hole and Morris-Thorne wormhole have been found. We have assumed some candidates of dark energy like holographic dark energy, new agegraphic dark energy, quintessence, tachyon, DBI-essence, etc. The black hole mass and the wormhole mass have been calculated in term of redshift when dark matter and above types of dark energies accrete onto them separately. The black hole mass \( M \) vs. redshift \( z \) have been drawn in Figs. 1, 3, 5, 7 respectively for different values of \( n = 2, 3, 4, 5, 7 \) (i.e., \( 4D, 5D, 6D, 7D \)) when dark matter and holographic dark energy, agegraphic dark energy, quintessence, tachyon accrete onto black hole. For dark matter, we have taken EoS parameter \( w = 0.01 \). From the figures, we see that black hole mass increases during whole evolution of the Universe. The slope of mass of black hole increases when \( n \) increases i.e., mass of black hole increases more sharply for increase of dimensions. On the other hand, the wormhole mass \( M \) vs. redshift \( z \) have been drawn in Figs. 2, 4, 6, 8 respectively for different values of \( n = 2, 3, 4, 5 \) (i.e., \( 4D, 5D, 6D, 7D \)) when dark matter and holographic dark energy, agegraphic dark energy, quintessence, tachyon accrete onto wormhole. From the figures, we see that wormhole mass decreases during whole evolution of the Universe. The slope of mass of wormhole decreases when \( n \) increases i.e., mass of wormhole decreases more sharply for increase of dimensions. These are the features of dark matter and dark energy accretions onto black hole and wormhole and natures of increasing/decreasing of mass completely depends on the dimensions. For more dimension, mass of the black hole increases more sharply and mass of the wormhole decreases more sharply for holographic dark energy, agegraphic dark energy, quintessence, tachyon accretion.
Lastly, the black hole mass $M$ and wormhole mass $M_4$ vs. redshift $z$ have been drawn in Figs. 9 and 10 respectively for different values of $n = 2, 3, 4, 5$ (i.e., $4D, 5D, 6D, 7D$) when dark matter and DBI-essence accrete onto black hole and wormhole. From the figures, we see that black hole mass first increases up to a certain finite value and then decreases and wormhole mass decreases up to a certain finite value and then increases during whole evolution of the Universe. These are the features of dark matter and dark energy accretions onto black hole and wormhole for DBI-essence accretion, because our considered model is the phantom crossing model. The natures of increasing/decreasing of mass are nearly similar to all dimensions. Hence, we conclude that the black hole mass increases and wormhole mass decreases for holographic dark energy, new agegraphic dark energy, quintessence, tachyon accretion and the slope of increasing/decreasing of mass sensitively depends on the dimension. But for DBI-essence accretion, the black hole mass first increases and then decreases and the wormhole mass first decreases and then increases and the slope of increasing/decreasing of mass not sensitively depends on the dimension.

Acknowledgements The author is thankful to IUCAA, Pune, India for warm hospitality where the work was carried out.

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