Fermionic decays of scalar leptoquarks and scalar gluons in the minimal four color symmetry model

P.Yu. Popov*, A.V. Povarov†, A.D. Smirnov‡

Division of Theoretical Physics, Department of Physics,
Yaroslavl State University, Sovietskaya 14,
150000 Yaroslavl, Russia.

Abstract

Fermionic decays of the scalar leptoquarks $S = S_1^{(+)}$, $S_1^{(-)}$, $S_m$ and of the scalar gluons $F = F_1$, $F_2$ predicted by the four color symmetry model with the Higgs mechanism of the quark-lepton mass splitting are investigated. Widths and branching ratios of these decays are calculated and analysed in dependence on coupling constants and on masses of the decaying particles. It is shown that the decays $S_1^{(+)} \rightarrow tl^+_j$, $S_1^{(-)} \rightarrow \nu_i \tilde{b}$, $S_m \rightarrow t \tilde{\nu}_j$, $F_1 \rightarrow t \tilde{b}$, $F_2 \rightarrow tl^+$ are dominant with the widths of order of a few GeV for $m_S, m_F < 1$ TeV and with the total branching ratios close to 1. In the case of $m_S < m_t$ the dominant scalar leptoquark decays are $S_1^{(+)} \rightarrow c l^+_j$, $S_1^{(-)} \rightarrow \nu_i \tilde{b}$, $S_m \rightarrow b l^+_j$, $S_m \rightarrow c \tilde{\nu}_j$ with the total branching ratios $Br(S_1^{(+)} \rightarrow c l^+) \approx Br(S_1^{(-)} \rightarrow \nu_i \tilde{b}) \approx 1$, $Br(S_m \rightarrow b l^+) \approx 0.9$ and $Br(S_m \rightarrow c \tilde{\nu}) \approx 0.1$. A search for such decays at the LHC and Tevatron may be of interest.

Keywords: Beyond the SM; four-color symmetry; Pati–Salam; leptoquarks; decay modes.

PACS number: 12.60.-i

One of the goals of the forthcoming experiments at LHC (in addition to the search for the Higgs boson and to the further studies of the Standard Model (SM)) will be the search for the possible effects of the new physics beyond the SM. There is a lot of the variants of new physics beyond the SM predicting new effects at energies of LHC (supersymmetry, left - right symmetry, two Higgs model, etc.).

One of such variants can be the variant induced by the possible four color symmetry between quarks and leptons of Pati–Salam type [1]. The immediate consequence of this symmetry is the prediction of the gauge leptoquarks which, however, occur to be relatively heavy (for example, the resulted from the unobservation of the $K_L^0 \rightarrow \mu^\pm e^\mp$ decays the most stringent lower mass limit for the vector leptoquarks is of order of $10^3$ TeV). By this reason it is usually thought that the effects of the four color symmetry at the colliders energies are too small to be directly detectable in the collider experiments of the near future.

*E-mail: popov.p@univ.uniyar.ac.ru
†E-mail: povarov@univ.uniyar.ac.ru
‡E-mail: asmirnov@univ.uniyar.ac.ru
It should be noted however that in addition to the gauge leptoquarks the four color symmetry can predict also the new scalar particles. Thus, in the case of the Higgs mechanism of splitting the masses of quarks and leptons the four color symmetry in its minimal realization on the gauge group

\[ G = SU_V(4) \times SU_L(2) \times U_R(1) \]  

(MQLS-model \[2, 3\]) needs the existence of the scalar particles belonging to the (15,2,1) - multiplet of the group G. This multiplet contains fifteen scalar \(SU_L(2)\)-doublets: two scalar leptoquark doublets \(S^{(\pm)}\), scalar gluon doublet \(F\) and a \(SU_c(3)\)-colorless scalar doublet which in admixture with the (1,2,1) - doublet forms the SM Higgs doublet \(\Phi^{(SM)}\) and an additional colorless scalar doublet \(\Phi'\). All these scalar doublets are necessary \[4\] for splitting the the masses of quarks from those of leptons by the Higgs mechanism and for generating the quark - lepton mass splittings including the so large mass splittings as the \(b - \tau\) and \(t - \nu_\tau\) ones.

Because of their Higgs origin the coupling constants of these scalar doublets with the fermions are proportional to the ratios \(m_f/\eta\) of the fermion masses \(m_f\) to the SM VEV \(\eta\). The effects of these scalar leptoquarks in the processes with the ordinary \(u, d, s, b\) - quarks are small because of the smallness of the corresponding coupling constants \(m_u/\eta \sim m_d/\eta \sim 10^{-5}, m_s/\eta \sim 10^{-3}\) (but these effects can be significant in c-, b- and, especially, in top-physics \(m_c/\eta \sim m_b/\eta \sim 10^{-2}, m_t/\eta \sim 0.7\)). As a result the scalar leptoquark doublets \(S^{(\pm)}\) and the scalar gluon doublet \(F\) can be relatively light, with masses below 1 TeV, without any contradictions with the \(K_L^0 \rightarrow \mu^+\nu^-\) data or with the radiative correction limits \[5, 6\]. \(SU_c(3)\)-colored, the scalar leptoquarks \(S^{(\pm)}\) and the scalar gluons \(F\) can be in pairs produced in \(pp\)-collisions via gluon-gluon fusion and, in part, via quark-antiquark annihilation. In the case of the masses below 1 TeV the corresponding total cross section for the scalar leptoquark pairs \[7, 8\] (and, seemingly, for the scalar gluon ones) is known to be sufficient for their effective production at LHC. For the detection of these particles it is necessary to know their dominant decay modes and the corresponding widths and branching ratios.

In this paper we calculate and discuss the widths of the fermionic decays of the scalar leptoquark and gluon doublets predicted by four color symmetry with the Higgs mechanism of splitting the masses of quarks and leptons in frame of MQLS-model \[2, 3\]. We evaluate and analyze the widths of the dominant decay modes in dependence on the masses of the decaying particles.

In MQLS model the basic left \((L)\) and right \((R)\) quarks \(Q_{i_0a}^{L,R}\) and leptons \(l_{i_0a}^{L,R}\) form the fundamental quartets of \(SU_V(4)\) color group and can be written, in general, as superpositions

\[ Q_{i_0a}^{L,R} = \sum_j (A_{Q_{i_0a}}^{L,R})_{ij} Q_{jaa}^{L,R}, \quad l_{i_0a}^{L,R} = \sum_j (A_{l_{i_0a}}^{L,R})_{ij} l_{jaa}^{L,R} \]  

of the quark and lepton mass eigenstates \(Q_{i_0a}^{L,R}, l_{i_0a}^{L,R}\), where \(i, j = 1, 2, 3\) are the generation indexes, \(a = 1, 2\) and \(\alpha = 1, 2, 3\) are the \(SU_L(2)\) and \(SU_c(3)\) indexes, \(Q_{i_1} \equiv u_i = (u, c, t)\), \(Q_{i_2} \equiv d_i = (d, s, b)\) are the up and down quarks, \(l_{j_1} \equiv \nu_j\) are the mass eigenstates of neutrinos and \(l_{j_2} \equiv l_j = (e^-, \mu^-, \tau^-)\) are the charged leptons. The unitary matrices \(A_{Q_{i_0a}}^{L,R}\) and \(A_{l_{i_0a}}^{L,R}\) describe the fermion mixing and diagonalize the mass matrices of quarks and leptons.

The Higgs mechanism of the quark-lepton mass splitting needs, in general, two scalar multiplets \(\Phi^{(2)}\) and \(\Phi^{(3)}\) (with VEV \(\eta_2\) and \(\eta_3\)) transforming according to the representations \((1.2.1)\) and \((15.2.1)\) of the group \(\mathbb{U}\). The multiplet \((15.2.1)\) contains fifteen \(SU_L(2)\)
doublets:

\[
(15.2.1) : \begin{pmatrix} S_{1a}^{(1+)} \\ S_{2a}^{(1+)} \end{pmatrix}, \begin{pmatrix} S_{1a}^{(-)} \\ S_{2a}^{(-)} \end{pmatrix}, \begin{pmatrix} F_{1k} \\ F_{2k} \end{pmatrix}, \begin{pmatrix} \Phi_{115}^{(3)} \\ \Phi_{115}^{(3)} \end{pmatrix},
\]

where \(S_{a\alpha}^{(\pm)}\) and \(F_{ak}\) (k=1,2...8) are the scalar leptoquark and scalar gluons doublets. The scalar doublet \(\Phi_{115}^{(3)}\) is mixed with the \((1,2,1)\) doublet \(\Phi_{1}^{(2)}\) and gives the SM Higgs doublet \(\Phi_{1}^{(SM)}\) (with SM VEV \(\eta = \sqrt{\eta_{L}^{2} + \eta_{H}^{2}}\)) and an additional \(\Phi'\) doublet. The scalar doublets \(\Phi\) have the electric charges

\[
Q_{em} : \begin{pmatrix} 5/3 \\ 2/3 \end{pmatrix}, \begin{pmatrix} 1/3 \\ -2/3 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 0 \end{pmatrix}
\]

respectively.

In general case the scalar leptoquarks \(S_{2a}^{(+)}\) and \(S_{2a}^{(-)}\) with electric charge 2/3 are mixed and can be written as superpositions

\[
S_{2a}^{(+)} = \sum_{m=0}^{3} c_{m}^{(+)} S_{m}, \quad S_{2a}^{(-)} = \sum_{m=0}^{3} c_{m}^{(-)} S_{m}
\]

of three physical scalar leptoquarks \(S_{1}, S_{2}, S_{3}\) with electric charge 2/3 and a small admixture of the Goldstone mode \(S_{0}\). Here \(c_{m}^{(\pm)}\), \(m = 0, 1, 2, 3\) are the elements of the unitary scalar leptoquark mixing matrix, \(|c_{0}^{(\pm)}|^{2} = \frac{1}{3} g_{4}^{2} \eta_{3}^{2} / m_{V}^{2} \ll 1\), \(g_{4}\) is the \(SU_{V}(4)\) gauge coupling constant, \(\eta_{3}\) is the VEV of the \((15,2,1)\)-multiplet and \(m_{V}\) is the vector leptoquark mass.

The interactions of the scalar leptoquarks doublets \(S_{a\alpha}^{(\pm)}\) with quarks and leptons can be written in the model independent form as

\[
L_{S_{1}^{(+)},u_{i}l_{j}} = \bar{u}_{ia} \left[ (h_{L}^{+})_{ij} P_{L} + (h_{R}^{+})_{ij} P_{R} \right] l_{j} S_{1a}^{(+)} + \text{h.c.,}
\]

\[
L_{S_{1}^{(-)},v_{i}d_{j}} = \bar{v}_{i} \left[ (h_{L}^{(-)})_{ij} P_{L} + (h_{R}^{(-)})_{ij} P_{R} \right] d_{j} S_{1a}^{(-)} + \text{h.c.,}
\]

\[
L_{S_{m},u_{i}l_{j}} = \bar{u}_{ia} \left[ (h_{1m})_{ij} P_{L} + (h_{1m})_{ij} P_{R} \right] l_{j} S_{ma} + \text{h.c.}
\]

\[
L_{S_{m},d_{i}l_{j}} = \bar{d}_{ia} \left[ (h_{2m})_{ij} P_{L} + (h_{2m})_{ij} P_{R} \right] l_{j} S_{ma} + \text{h.c.}
\]

where \(P_{L,R} = (1 \pm \gamma_{5}) / 2\) are the left and right projection operators, \((h_{L,R})_{ij}^{L,R}\) are the coupling constants, \(i, j\) are the generation indexes. Generally the interactions \(\Phi\) induce the scalar leptoquark decays

\[
S_{1}^{(+)} \rightarrow u_{i}l_{j}^{+}, \quad S_{1}^{(-)} \rightarrow v_{i}d_{j}, \quad S_{m} \rightarrow u_{i}v_{j}, \quad S_{m} \rightarrow d_{i}l_{j}^{+}.
\]

As a result of the Higgs mechanism of the quark lepton mass splitting the general form of Yukawa interaction in the MQLS model [2–4] gives for the coupling constants of \(\Phi\) the expressions

\[
(h_{L}^{+})_{ij} = \sqrt{3/2} \frac{1}{\eta \sin \beta} \left[ m_{ui}(K_{1}^{L}C_{ij})_{ij} - (K_{1}^{R})_{ik}m_{vi}(C_{l})_{kj} \right],
\]

\[
(h_{L}^{R})_{ij} = -\sqrt{3/2} \frac{1}{\eta \sin \beta} \left[ (C_{Q})_{ik}m_{dk}(K_{2}^{R})_{kj} - m_{lj}(C_{Q}K_{2}^{L})_{ij} \right],
\]
\begin{align}
(h^L_{-})_{ij} &= \sqrt{3/2} \frac{1}{\eta \sin \beta} \left[ (K_{1}^{+})_{ik} m_{uk} (C_{Q})_{kj} - m_{v_{l}} (K_{1}^{+} C_{Q})_{ij} \right], \\
(h^R_{-})_{ij} &= -\sqrt{3/2} \frac{1}{\eta \sin \beta} \left[ (C_{K}^{+})_{ij} m_{d_{ij}} - (C_{i})_{ik} m_{uk} (K_{2}^{+})_{kj} \right], \\
(h^L_{1m})_{ij} &= -\sqrt{3/2} \frac{1}{\eta \sin \beta} \left[ m_{u_{j}} (K_{1}^{L,R})_{ij} - (K_{1}^{R,L})_{ij} m_{v_{j}} \right] c_{m}^{(\pm)}, \\
(h^R_{2m})_{ij} &= -\sqrt{3/2} \frac{1}{\eta \sin \beta} \left[ m_{d_{j}} (K_{2}^{L,R})_{ij} - (K_{2}^{R,L})_{ij} m_{v_{j}} \right] c_{m}^{(\pm)},
\end{align}

where \( m_{u_{j}}, m_{d_{j}}, m_{e_{j}}, m_{\nu_{j}} \) are the masses of quarks, of charge leptons and of neutrinos, \( \beta \) is \( \Phi_{a}^{(2)} - \Phi_{15}^{(3)} \) mixing angle in MQLS model, \( t g \beta = \eta_{3}/\eta_{2} \), \( C_{Q} = (A_{Q_{1}})^{+} A_{Q_{2}} \) is the CKM-matrix, \( C_{i} = (A_{i_{1}})^{+} A_{i_{2}} \) is the analogous matrix in the lepton sector and \( K_{a}^{L,R} = (A_{Q_{a}})^{+} A_{L,R}^{L,R} \) are the mixing matrices specific for the model with the four color quark-lepton symmetry.

It is easy to see that among the coupling constants there are the coupling constants

\begin{align}
(h^L_{+})_{3j} &= \sqrt{3} \frac{m_{t}}{2 \eta \sin \beta} (K_{1}^{L} C_{i})_{3j}, \\
(h^L_{-})_{i3} &= \sqrt{3} \frac{m_{r}}{2 \eta \sin \beta} (K_{1}^{R})_{i3} (C_{Q})_{33}, \\
(h^L_{1m})_{3j} &= -\sqrt{3} \frac{m_{t}}{2 \eta \sin \beta} (K_{1}^{L,R})_{3j} c_{m}^{(\pm)}
\end{align}

which are proportional to the heaviest mass of \( t \)-quark and hence are the largest ones.

As a result among the decays the scalar leptoquark decays

\begin{align}
S_{1}^{(+)} \to t \bar{L}_{j}^{+}, \quad S_{1}^{(-)} \to \nu \bar{b}, \quad S_{m} \to t \bar{v}_{j}
\end{align}

into the quarks of the third generation and leptons occur to be the dominant ones.

The calculation for the case of \( m_{t_{j}}, m_{\nu_{j}} \ll m_{t} \) and \( m_{\nu_{j}} \ll m_{b} \) gives the following partial widths of the dominant modes

\begin{align}
\Gamma (S_{1}^{(+)} \to t \bar{L}_{j}^{+}) = \bar{\Gamma}_{S_{1}^{(+)} (m_{S_{1}^{(+)}}, m_{t})} \frac{|(K_{1}^{L} C_{i})_{3j}|^{2}}{\sin^{2} \beta}, \\
\Gamma (S_{1}^{(-)} \to \nu \bar{b}) = \bar{\Gamma}_{S_{1}^{(-)} (m_{S_{1}^{(-)}}, m_{b})} \frac{|(K_{1}^{R})_{33}|^{2}|(C_{Q})_{33}|^{2}}{\sin^{2} \beta}, \\
\Gamma (S_{m} \to t \bar{v}_{j}) = \bar{\Gamma}_{S_{m} (m_{S_{m}}, m_{t})} \frac{|(K_{1}^{L})_{3j}|^{2}|c_{m}^{(\pm)}|^{2} + |(K_{1}^{R})_{3j}|^{2}|c_{m}^{(\pm)}|^{2}}{\sin^{2} \beta}
\end{align}

where

\begin{align}
\bar{\Gamma}_{S}(m_{S}, m_{Q}) &= m_{S} \frac{3}{32 \pi} \left( \frac{m_{t}}{\eta} \right)^{2} (1 - \frac{m_{Q}^{2}}{m_{S}^{2}})^{2}
\end{align}

which we call below as the reduced widths of the scalar leptoquarks \( S = S_{1}^{(+)} , S_{1}^{(-)} , S_{m} \).
Summarizing the partial widths (9)–(11) over the generations and accounting for the unitarity of the matrices $K_{1}^{L,R}$, $C_{t}$ we obtain the next fermion mixing independent expressions for the total decay widths

$$\Gamma(S_{i}^{(+)} \rightarrow t_{j}^{+}) \equiv \sum_{j} \Gamma(S_{i}^{(+)} \rightarrow t_{j}^{+}) = \frac{1}{\sin^{2} \beta}, \tag{13}$$

$$\Gamma(S_{i}^{(-)} \rightarrow \nu \bar{b}) \equiv \sum_{i} \Gamma(S_{i}^{(-)} \rightarrow \nu \bar{b}) = \frac{|C_{Q}|_{33}^{2}}{\sin^{2} \beta}, \tag{14}$$

$$\Gamma(S_{m} \rightarrow t \bar{v}) \equiv \sum_{j} \Gamma(S_{m} \rightarrow t \bar{v}) = \frac{k_{m}}{\sin^{2} \beta}, \tag{15}$$

where

$$k_{m} = |\epsilon_{m}^{(+)}|^{2} + |\epsilon_{m}^{(-)}|^{2}. \tag{16}$$

The interaction of the scalar gluons with quarks can be written in the model independent form as

$$L_{F_{1}u_{i}d_{j}} = \bar{u}_{i\alpha} [ (h_{F_{1}}^{L})_{ij} P_{L} + (h_{F_{1}}^{R})_{ij} P_{R} ] (t_{k})_{\alpha \beta} d_{j\beta} F_{1k} + \text{h.c.},$$

$$L_{F_{2}u_{i}d_{j}} = \bar{u}_{i\alpha} [ (h_{F_{1}}^{L})_{ij} P_{L} ] (t_{k})_{\alpha \beta} u_{j\beta} F_{2k} + \text{h.c.}, \tag{17}$$

$$L_{F_{2}d_{i}d_{j}} = \bar{d}_{i\alpha} [ (h_{F_{1}}^{R})_{ij} P_{R} ] (t_{k})_{\alpha \beta} d_{j\beta} F_{2k} + \text{h.c.},$$

where $(h_{F_{1}}^{L})_{ij}, (h_{F_{1}}^{R})_{ij}, (h_{F_{2}}^{R})_{ij}$ are the corresponding coupling constants and $t_{k}, k = 1, 2 \ldots 8$, are the generators of the $SU_{c}(3)$ group. The interactions (17) induce the scalar gluon decays

$$F_{1} \rightarrow u_{i} \bar{b}_{j}, \ F_{2} \rightarrow u_{i} \bar{u}_{j}, \ F_{2} \rightarrow d_{i} \bar{d}_{j}. \tag{18}$$

From the general form of Yukawa interaction in the MQLS model [2–4] the Higgs mechanism of the quark lepton mass splitting gives for the coupling constants of (17) the expressions

$$(h_{F_{1}}^{L})_{ij} = \sqrt{3} \frac{1}{\eta \sin \beta} \left[ m_{u_{i}} (C_{Q})_{ij} - (K_{1}^{R})_{ik} m_{v_{k}} (K_{1}^{L})_{kj} \right],$$

$$(h_{F_{1}}^{R})_{ij} = -\sqrt{3} \frac{1}{\eta \sin \beta} \left[ (C_{Q})_{ij} m_{d_{i}} - (C_{1} K_{2}^{L})_{ik} m_{u_{k}} (K_{1}^{R})_{kj} \right],$$

$$(h_{F_{2}}^{L})_{ij} = -\sqrt{3} \frac{1}{\eta \sin \beta} \left[ m_{u_{i}} (\delta)_{ij} - (K_{1}^{L})_{ik} m_{v_{k}} (K_{1}^{R})_{kj} \right],$$

$$(h_{F_{2}}^{R})_{ij} = -\sqrt{3} \frac{1}{\eta \sin \beta} \left[ m_{d_{i}} (\delta)_{ij} - (K_{1}^{R})_{ik} m_{u_{k}} (K_{1}^{L})_{kj} \right]. \tag{19}$$

As seen among the coupling constants (19) there are the coupling constants which are proportional to the $t$-quark mass

$$(h_{F_{1}}^{L})_{33} = \sqrt{3} \frac{m_{t}}{\eta \sin \beta} (C_{Q})_{33},$$

$$(h_{F_{2}}^{L})_{33} = -\sqrt{3} \frac{m_{t}}{\eta \sin \beta}$$

and hence they are the largest ones.
As a result among the decays \( \text{(18)} \) the decays

\[
F_1 \rightarrow t\bar{b}, \quad F_2 \rightarrow t\bar{t}
\]

are the dominant ones.

The calculation gives the following widths of the dominant modes \( \text{(20)} \)

\[
\Gamma(F_1 \rightarrow t\bar{b}) = m_{F_1} \frac{3}{32\pi} \left( \frac{m_t}{m_{F_1}} \right)^2 (1 - \frac{m_t^2}{m_{F_1}^2})^2 |(C_Q)_{33}|^2 \equiv \bar{\Gamma}_{F_1}(m_{F_1}, m_t) \frac{|(C_Q)_{33}|}{\sin^2\beta},
\]

\[
\Gamma(F_2 \rightarrow t\bar{t}) = m_{F_2} \frac{3}{32\pi} \left( \frac{m_t}{m_{F_2}} \right)^2 \left( 1 - 2 \frac{m_t^2}{m_{F_2}^2} \right) \sqrt{1 - \frac{4m_t^2}{m_{F_2}^2 \sin^2\beta}} \equiv \bar{\Gamma}_{F_2}(m_{F_2}, m_t) \frac{1}{\sin^2\beta},
\]

where the relation \( m_b << m_t \) has been taken into account.

The decay widths \( \text{(13)}, \text{(15)}, \text{(21)}, \text{(22)} \) depend on the masses of the decaying particles through the reduced widths \( \bar{\Gamma}_S(m_S, m_Q), \tilde{\bar{\Gamma}}_F(m_F, m_Q) \) and on the mixing angle \( \beta \) and on the scalar leptoquark mixing parameters \( k_m \). As mentioned above the indirect limits on the masses of the scalar leptoquarks under consideration are weak. The current data on the direct search for the leptoquarks set the lower mass limits \( \text{(9)} \)

\[
m_{LQ} > 242 \text{ GeV}, \ 202 \text{ GeV}, \ 148 \text{ GeV}
\]

for the scalar leptoquarks of the first \( [10] \), of the second \( [11] \) and of the third \( [12] \) generation with assuming the branching ratios \( Br(lq) \equiv Br(LQ \rightarrow lq) \) of their quark-lepton decays to be \( Br(eq) = 1, Br(\mu q) = 1, Br(\nu b) = 1 \) respectively. For the smaller values of \( Br(lq) \) the corresponding mass limits are weaker, for example

\[
m_{LQ} > 204(205) \text{ GeV}, \ 160 \text{ GeV}
\]

for the scalar leptoquarks of the first \( [13] \) \( [14] \) and of the second \( [9,15] \) generation with \( Br(eq) = 0.5, Br(\mu q) = 0.5 \) respectively and

\[
m_{LQ} > 79(145) \text{ GeV}
\]

for the scalar leptoquarks of the first generation with \( Br(eq) = 0.0 \) \( [13] \) \( Br(eq) = 0.1 \) \( [14] \).

As seen the scalar leptoquarks can have the masses of order of 200 GeV or slightly below in dependence on the the branching ratios of their fermionic decays. It is worth noting that because of their dominant decay modes \( \text{(8)} \) the scalar leptoquarks \( S_1^{(+)}, S_1^{(-)}, S_m \) under consideration should be preferred as the third generation ones. Taking into account that the decay modes \( S_1^{(-)} \rightarrow \nu\bar{b} \) of the scalar leptoquark \( S_1^{(-)} \) are the dominant ones with the total \( Br(S_1^{(-)} \rightarrow \nu\bar{b}) \equiv \sum_i Br(S_1^{(-)} \rightarrow \nu_i\bar{b}) \approx 1 \) we have from \( \text{(23)} \) the lower mass limit \( m_{S_1^{(-)}} > 148 \text{ GeV} \) for the scalar leptoquark \( S_1^{(-)} \). As concerns the scalar gluons at the present time we have no direct experimental limits for their masses. Keeping in mind that the experimental bounds on the radiative corrections \( S, T, U \) parameters of Peskin–Takeuchi allow the scalar leptoquark and scalar gluon doublets to lie below 1 TeV slightly preferring their masses respectively of order of 400 GeV or below and of order of 800 GeV or below \( [5,6] \) for the further numerical estimations we consider the masses of these particle in sub TeV mass region.

The Figure 1 shows the reduced widths \( \bar{\Gamma}_S(m_S, m_Q), \tilde{\bar{\Gamma}}_F(m_F, m_Q) \) of the fermionic decays of the scalar leptoquarks \( S = S_1^{(+)}, S_1^{(-)}, S_m \) and of the scalar gluons \( F = F_1, F_2 \) as
the functions of the masses \( m_S, m_F \) of the decaying particles in the mass region 200–1000 GeV. Here and below we use the masses \( m_t = 174.3 \pm 5.1 \text{ GeV}, \ m_b = 4.25 \pm 0.25 \text{ GeV} \) and \( \eta = 246 \text{ GeV} \). The curves a) and b) and c) correspond to the decays of the \( S_1^{(-)} \) and \( S_1^{(+)}, S_m, F_1 \) and \( F_2 \) respectively. The total widths of these decays for the more probable mass regions are presented in Table 1.

As is seen from the the Fig.1 and from the Table 1 the fermionic widths of the scalar leptoquark and gluon doublets can be of order of a few GeV with enhancing by the factor \( 1/\sin^2\beta \). For example we obtain

\[
\begin{align*}
\Gamma(S_1^{(+)} \rightarrow tl^+) &= \Gamma(S_m \rightarrow t\bar{v}) = 0.2 - 5.8 \ (5.0 - 145.0) \text{ GeV}, \\
\Gamma(S_1^{(-)} \rightarrow \nu\bar{b}) &= 2.2 - 7.5 \ (55.0 - 187.5) \text{ GeV}, \\
\Gamma(F_1 \rightarrow \bar{t}b) &= 5.8 - 14.1 \ (145.0 - 352.5) \text{ GeV}, \\
\Gamma(F_2 \rightarrow t\bar{\nu}) &= 4.1 - 13.2 \ (102.5 - 330.0) \text{ GeV}
\end{align*}
\]

for \( m_{S^{(+)}} = 200 - 500 \text{ GeV}, m_{S^{(-)}} = 150 - 500 \text{ GeV}, m_{F_1}, m_{F_2} = 500 - 1000 \text{ GeV} \) and for \( \sin\beta = 1 \) (0.2) with using the diagonal CKM matrix element \( (C_Q)_{33} \approx 1 \) and believing \( k_m = 1 \).

### Table 1: The total widths of the fermionic decays of the scalar leptoquarks and of the scalar gluons in dependence on the masses of the decaying particles.

| \( \Gamma(S_1^{(+)} \rightarrow tl^+) \) | \( \Gamma(S_m \rightarrow t\bar{v}) \) | \( \Gamma(S_1^{(-)} \rightarrow \nu\bar{b}) \) | \( \Gamma(F_1 \rightarrow \bar{t}b) \) | \( \Gamma(F_2 \rightarrow t\bar{\nu}) \) |
|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------|
| \((0.2 - 2.0 - 5.8)/\sin^2\beta \) GeV | \( 0.2 - 5.8 \) GeV | \((2.2 - 4.5 - 7.5))(C_Q)_{33}^2 /\sin^2\beta \) GeV | \((5.8 - 14.1))(C_Q)_{33}^2 /\sin^2\beta \) GeV | \((4.1 - 13.2)/\sin^2\beta \) GeV |
| \( m_{S^{(+)}} = 200 - 300 - 500 \text{ GeV} \) | \( m_S = 200 - 300 - 500 \text{ GeV} \) | \( m_{S^{(-)}} = 150 - 300 - 500 \text{ GeV} \) | \( m_{F_1} = 500 - 1000 \text{ GeV} \) | \( m_{F_2} = 500 - 1000 \text{ GeV} \) |

It should be noted that the decays [3], [18] with production of the quarks \( Q \) of the first and of second generations are very suppressed by the factor \( m_Q^2/m_t^2 \) or by the squared matrix elements of CKM matrix \( C_Q \). In the case of relatively small mass splittings \( \Delta m \) inside the scalar doublets \( \Delta m < m_W \) the weak decays

\[
S \rightarrow S'W, \ F \rightarrow F'W
\]

of the heaviest components \( S, F \) of the scalar doublets into the lightest ones \( S', F' \) are forbidden and in this case the decays [8], [20] are the dominant ones with the total branching ratios

\[
\begin{align*}
Br(S_1^{(+)} \rightarrow tl^+) &\approx Br(S_1^{(-)} \rightarrow \nu\bar{b}) \approx Br(S_m \rightarrow t\bar{v}) \approx 1, \\
Br(F_1 \rightarrow \bar{t}b) &\approx Br(F_2 \rightarrow t\bar{\nu}) \approx 1,
\end{align*}
\]

where \( Br(S_1^{(+)} \rightarrow tl^+) = \sum_j Br(S_1^{(+)} \rightarrow tl^+_j), \ Br(S_1^{(-)} \rightarrow \nu\bar{b}) = \sum_i Br(S_1^{(-)} \rightarrow \nu_i\bar{b}), \ Br(S_m \rightarrow t\bar{v}) = \sum_j Br(S_m \rightarrow t\bar{v}_j) \).

In this case the simplest way for observation of the scalar leptoquarks \( S_1^{(+)}, S_1^{(-)}, S_m \) is the search for \( t\tau^+, \ t\mu^+, \ t\tau^+ \)-pairs which can be generated by the \( S_1^{(+)} \) decays with the
total branching ratio $Br(S_1^{(+)} \to tl^+) \approx 1$ and the search for $\bar{t}b(t)$-quarks with energy missing which can be generated by the $S_1^{(-)}(S_m)$ decays with the total branching ratio $Br(S_1^{(-)} \to \nu \bar{b}) \approx 1 \ (Br(S_m \to t\nu) \approx 1)$. The search for the decays (33) at LHC may be of interest and can result or in the observation of the scalar leptoquark doublets or in setting the new limits on their masses. As mentioned above the search for $\nu b$ pairs which has been performed at Tevatron set for the scalar leptoquark $S_1^{(-)}$ the mass limit $m_{S_1^{(-)}} > 148$ GeV.

In the case of $\Delta m > m_W$ the decays (30) are also open. The analysis shows that the widths of the decays (30) can be comparable to the fermionic ones (26)−(29) and in this case the relations (31), (32) are relevant only to the lightest components of the scalar doublets.

As is seen from (23)−(25) the scalar leptoquarks with the masses below the $t$-quark mass and with the sufficiently small $Br(eq)$ and $Br(\mu q)$ are not excluded. If the scalar leptoquarks $S = S_1^{(+)}$, $S_1^{(-)}$, $S_m$ are assumed to be lighter than $t$-quark ($m_S < m_t$) then the decays with production of $t$-quark in (33) are forbidden and in this case instead of the decays (33) the dominant decays are

$$S_1^{(+)} \to c l_j^+, \quad S_1^{(-)} \to \nu \bar{b}, \quad S_m \to b l_j^+, \quad S_m \to c \nu j$$

(33)

with the total branching ratios

$$Br(S_1^{(+)} \to c l_j^+) \approx Br(S_1^{(-)} \to \nu \bar{b}) \approx 1,$$

$$Br(S_m \to b l_j^+) = m_b^2/(m_b^2 + m_c^2) \approx 0.9,$$

$$Br(S_m \to c \nu j) = m_c^2/(m_b^2 + m_c^2) \approx 0.1,$$

(34)−(36)

where $Br(S_1^{(+)} \to c l_j^+) \equiv \sum_j Br(S_1^{(+)} \to c l_j^+)$, $Br(S_1^{(-)} \to \nu \bar{b}) \equiv \sum_i Br(S_1^{(-)} \to \nu \bar{b})$, $Br(S_m \to b l_j^+) \equiv \sum_j Br(S_m \to b l_j^+)$, $Br(S_m \to c \nu j) \equiv \sum_j Br(S_m \to c \nu j)$. The search for the decays (33) at Tevatron with account of (34)−(36) is of interest and could set the mass limits for the scalar leptoquarks $S = S_1^{(+)}$, $S_m$ and the new mass limit for the scalar leptoquark $S_1^{(-)}$.

In conclusion we resume the results of the work. The fermionic decays of the scalar leptoquark and scalar gluon doublets predicted by the four color symmetry with the Higgs mechanism of the quark-lepton mass splitting are investigated.

The fermionic decays $S_1^{(+)} \to t l_j^+$, $S_1^{(-)} \to \nu \bar{b}$, $S_m \to t\nu j$ of the scalar leptoquarks $S = S_1^{(+)}$, $S_1^{(-)}$, $S_m$ and those $F_1 \to \bar{t}b$, $F_2 \to \bar{t}\bar{b}$ of the scalar gluons $F = F_1$, $F_2$ are shown to be in the case of relatively small mass splittings $\Delta m$ inside the scalar doublets $(\Delta m < m_W)$ the dominant ones with the total branching ratios $Br(S_1^{(+)} \to t l_j^+) \approx Br(S_1^{(-)} \to \nu \bar{b}) \approx 1$, $Br(F_1 \to \bar{t}b) \approx Br(F_2 \to \bar{t}\bar{b}) \approx 1$. The widths of these decays are found to be of order of a few GeV for the masses of the decaying particles below 1 TeV.

In the case of $m_S < m_t$ the scalar leptoquark decays $S_1^{(+)} \to c l_j^+$, $S_1^{(-)} \to \nu \bar{b}$, $S_m \to b l_j^+$, $S_m \to c \nu j$ are shown to be the dominant ones with the total branching ratios $Br(S_1^{(+)} \to c l_j^+) \approx Br(S_1^{(-)} \to \nu \bar{b}) \approx 1$, $Br(S_m \to b l^+) \approx 0.9$, $Br(S_m \to c \nu) \approx 0.1$.

The search for the considered decays at LHC and at Tevatron may be of interest.

Acknowledgments

The work was partially supported by the Russian Foundation for Basic Research under grant 04-02-16517-a.
References

[1] J. C. Pati and A. Salam, Phys. Rev. D 10, 275 (1974).

[2] A. D. Smirnov, Phys. Lett. B 346, 297 (1995).

[3] A. D. Smirnov, YaF. 58, 2252 (1995). (Physics of Atomic Nuclei 58, 2137 (1995).)

[4] A.V. Povarov and A. D. Smirnov, YaF. 64, 78 (2001). (Physics of Atomic Nuclei 64, No.1, 74 (2001).)

[5] A. D. Smirnov, Phys. Lett. B 513, 237 (2002).

[6] A.V. Povarov and A.D. Smirnov YaF. 66, No.12, 2259 (2003). (Physics of Atomic Nuclei 66, No.12, 2208 (2003).)

[7] J. Blümlein, E. Boos, A. Kryukov, Z.Phys. C76, 137 (1997); hep-ph/9610408.

[8] J. Blümlein, E. Boos, A. Kryukov, Preprint DESY 97-067; hep-ph/9811271.

[9] Particle Data Group (S. Eidelman et al.), Phys. Lett. B592, 1 (2004).

[10] C.Grosso-Pilcher et al. (The D0 and CDF Collaborations), hep-ex/9810015.

[11] F.Abe et al. (The CDF Collaboration), Phys.Rev.Lett. 78, 2906 (1997).

[12] T.Affolder et al. (The CDF Collaboration), Phys.Rev.Lett. 85, 2056 (2000).

[13] B. Abbott et al. (D0 Collaboration), Phys. Rev. Lett. 80, 2051 (1998).

[14] D. Acosta et al. (CDF Collaboration), hep-ex/0506074.

[15] F. Abe et al. (CDF Collaboration), Phys. Rev. Lett. 81, 4806 (1998).
Figure captions

Fig. 1. Reduced widths $\bar{\Gamma}_\Phi$ of the fermionic decays of the scalar leptoquarks and of the scalar gluons as the functions of the masses $m_{\Phi}$ of the decaying particles for a) $\Phi = S_1^{(-)}$, b) $\Phi = S_1^{(+)}$, $S_m$, $F_1$, c) $\Phi = F_2$. 
\[ \bar{\Gamma}_\Phi, \text{GeV} \]

\[ m_\Phi, \text{GeV} \]

Fig. 1