Application of the generalized Kirchhoff’s law to calculation of photoluminescence spectra of one-dimensional photonic crystals

M. M. Voronov
Ioffe Institute, St. Petersburg 194021, Russia
(Dated: December 29, 2017)

The approach based on the generalized Kirchhoff’s law for calculating photoluminescence (PL) spectra of one-dimensional (1D) multi-layered structures, in particular, 1D photonic crystals has been developed. It is valid in the local thermodynamic equilibrium approximation and leads to simple and explicit expression for the photoluminescence intensity. In the framework of the present theory the analytical expression for the spontaneous emission intensity enhancement factor (IEF) for a 1D photonic crystal has been derived. It takes a particularly simple form in the case of a sufficiently large number of the layers and is well suitable for analysis; in particular, it explains the difference in emission intensity at frequencies near different edges of photonic band-gaps (PBGs), where the intensity is relatively high, and specificity of suppression of the emission in a given frequency range. Also, the developed approach is discussed in connection with the standard method using the Fermi’s golden rule and the concept of the local density of states (LDOS).

I. INTRODUCTION

The theory of light emission from microstructures has been described in many books and reviews, see, e.g., [1 and 2]. The principle difference in the calculation methods is related to the electromagnetic field quantization, which in most experiments on luminescence is not apparent. This situation corresponds to the so-called weak-coupling regime where the atom-cavity coupling constant, $g$, is much less than the cavity decay rate, $\kappa$, and the dipole decay rate, $\gamma$, which are due to two energy-loss mechanisms. The opposite case where the dipole-cavity interaction dominates over the dissipative processes and zero-point energy must be taken into account is the strong-coupling regime [3]. It corresponds to the condition $g \gtrsim \kappa, \gamma$ and is realized in high quality-factor microcavities and photonic crystals within the photonic band-gap region making spontaneous emission a reversible process and in the case of $g \gg \kappa, \gamma$ (the quantum-coherent coupling regime) leading to the vacuum Rabi oscillations [4]. The weak-coupling regime is characterized by an exponential decay of the light intensity, but with a different decay rate compared to that in free space (the Purcell effect [3]), which is perfectly realized, for instance, for a 1D photonic crystal with a moderate number of periods. This is just the situation considered in the present paper.

In this paper we present a theory of photoluminescence of one-dimensional layered structures on the basis of the generalized form of Kirchhoff’s law. It determines the spectral dependence of the photoluminescence intensity when the lifetime of excited states ($\tau$) of emitting atoms, which we will term “emitting centers” (ECs) or simply “emitters”, is long enough compared to the thermalization time ($\tau_{th}$) of electrons participating in optical transitions and short compared to the energy redistribution time for the rest of the system, so that the energy distribution stays the same while the spectrum is being measured. In this case, which is typical of steady-state luminescence experiments, it is possible to use the concept of local thermodynamic equilibrium (LTE) and introduce a local temperature to describe the quasi-equilibrium distribution [5]. (It is supposed that the local temperature doesn’t change during the measurement time.) As the condition of quasi-thermal equilibrium is sufficient for the generalized Kirchhoff’s law to hold true, it characterizes not only general thermodynamic properties of the light-emitting systems but also the specificity of the emitting material. A similar approach is used to quantitatively describe luminescence spectra of very different systems including astrophysical objects [6–9] and is sometimes mentioned in the literature as the generalized Planck’s law [10 and 11]. Also, there exists a well-known relation between the absorption and luminescence spectra called the van Roosbroeck–Shockley relation [12] (and also the Kennard-Stepanov relation [13 and 14]).

As is known, in the semi-classical theory of radiation the atom is treated as a quantum mechanical system while the electromagnetic field is described classically. However, to get the true value of the Einstein A coefficient for spontaneous emission one has to take into account both the radiation reaction field and vacuum field fluctuations [4]. Since the type of emitters in our theory plays no important role, this theory can be considered equally as classical and as semi-classical and, consequently, the question about the origin of spontaneous emission does not arise here. The emitting centers are efficient sources of spontaneous emission and, as is discussed in the paper, the Langevin-like approach in which the sources terms are contained in Maxwell’s equations [15] is equivalent to the generalized Kirchhoff’s law under study.

The analytical expressions presented in this paper can be used in calculations of the PL spectra for multi-layered structures (which in many experimental situations can be considered as quasi-one-dimensional), in particular, for a 1D photonic crystal composed of the layers uniformly doped with the emitting centers. The numerical calculation procedure using these expressions is much less cum-
II. THE GENERALIZED KIRCHHOFF’S LAW FOR 1D LAYERED STRUCTURE

As a model system, we consider a 1D layered structure consisting of alternating plane layers, A and B, with real values of the refractive indices $n_a$ and $n_b$, respectively. The active (A) layers contain a large number of emitting centers (e.g., complexes of impurity atoms or ions with the nearest neighbour ions and also defects of the crystaline structure), differing, in general, in their rates of radiative (and non-radiative) transitions; we assume for simplicity that the concentration of emitting centers in each A layer is the same and is not too large so that the effects of their interaction with one another can be neglected (the approximation of independent emission sources). In the present theory, the specific nature of such emitters is of no importance from the point of view of the general approach and is taken into account separately. It is partly due to the fact that the underlying Kirchhoff’s law of thermal radiation is independent of the equilibrium-producing material and it is valid if the energy distribution of the excited states of the emitting centers and, as a consequence, the shape of the resulting PL emission spectrum (in a homogeneous bulk material), is independent of frequency of the exciting laser beam; it is typical for thermalized photoluminescence, which is the case considered in this study.

We start with the equation for the photoluminescence intensity for a one-layer structure [13] and [18]:

$$I(\omega) \propto \omega N_{ph}(\omega) A'(\omega).$$

(1)

Here $N_{ph}(\omega)$ is the photon distribution function (which is approximately $\exp(-h\omega/k_B T)$ at not too high temperatures, $k_B T \ll h\omega$) and $A'(\omega)$ is the absorption coefficient:

$$A'(\omega) = 1 - |r(\omega)|^2 - |t(\omega)|^2,$$

where $r(\omega)$ and $t(\omega)$ are the amplitude reflection and transmission coefficients of light at normal incidence; the prime on the function $A(\omega)$ denotes that absorption is very small, i.e. $A'(\omega) = \lim_{n'' \to 0} A(\omega)$, where $n''$ is the imaginary part of the refractive index associated with the absorption of light by ECs. Hereafter we consider a layer-by-layer structure, so we define $A_m'(\omega)$ as the contribution to the absorption by the $m$-th A layer. As follows from the excitation conditions for luminescence we should disregard the reabsorption and re-emission effects and thus to keep only the first-order term in $n''_{EC}(\omega)$ in the Maclaurin series for $A_m'(\omega)$, where $n''_{EC}(\omega)$ is proportional to concentration of ECs, emitting at the frequency $\omega$. It follows then that $A_m'(\omega)$ can be represented by

$$A_m'(\omega) = \phi_m(\omega)n''_{EC}(\omega), \quad \phi_m(\omega) = dA_m'(\omega)/dn'' .$$

(2)

Then the intensity of photoluminescence from the $m$-th A layer can be written in the form:

$$I_m(\omega) \propto \omega f_T(\omega) \phi_m(\omega) F_m(\omega_0).$$

(3)

This expression is obtained from Eqs. (1) and (2) by replacing $N_{ph}(\omega)n''_{EC}(\omega)$ with a new distribution function, $f_T(\omega)$, which depends on the local temperature $T$ of the system.

Let us explain the meaning of the functions in Eq. (3). The function $f_T(\omega)$ is determined by the nature of ECs and the kind of the light-emitting material and thus is related to the bulk emission spectrum (in the absence of a quasi-standing wave resulting from the light reflection from two interfaces). The function $\phi_m(\omega)$ is determined exclusively in terms of parameters of the structure (in particular, it is dependent on the total number of layers) and is responsible for modification of the spectrum compared to that from a bulk material. The function $F_m(\omega_0)$ in Eq. (3) gives the relative intensity of the light absorbed in the $m$-th A layer at the excitation frequency $\omega_0$. The expression for the function $F_m(\omega)$ and its connection to $\phi_m(\omega)$ for a 1D photonic crystal will be given below.

The photoluminescence intensity $I_N(\omega)$ from the structure containing $N$ active (A) layers is the sum of contributions: $I_N(\omega) = \sum_{m=1}^{N} I_m(\omega)$ . In the case of a small value of the imaginary part of refractive index of the constituent materials at the frequency $\omega$, one can set $A'_N(\omega) = \sum_m A'_m(\omega)$. Hence $\Phi_N(\omega) = \sum_m \phi_m(\omega)$ and, consequently, by neglecting the difference in values of $F_m(\omega_0)$ for different A layers, one gets

$$I_N(\omega) = h\omega f_T(\omega) \Phi_N(\omega),$$

(4)

which as well as Eq. (3) we call the generalized Kirchhoff’s law for a one-dimensional layered structure. The equality sign in Eq. (4) means only that the distribution function $f_T(\omega)$ is appropriately normalized. In essence, Eqs. (3) and (4) represent a modified form (suitable for a layered solid-state structure) of the Kirchhoff’s law in the theory of radiation transfer, which expresses the equality...
between the directional spectral emissivity and absorptivity for non-polarized radiation \[3\]. It is valid in the stationary case at LTE condition even when the local temperature changes in space, \(T = T(z)\) \[4\]. (In our study, for simplicity and without loss of generality, we assume that \(f_T(\omega)\) is independent of \(z\).) Thus, the generalized Kirchhoff’s law is applicable to nearly transparent media and is not suitable, for instance, for microcavities with metallic walls (though the weak-coupling regime is fulfilled).

It is worth mentioning that if the emitting centers are contained not only in the A layers but also in the B layers, the calculation of emission intensity should be made by taking into account the contributions from both types of layers on the basis of the following expression:

\[ A'(\omega) = n''_a \Phi_N^{(a)}(\omega) + n''_b \Phi_N^{(b)}(\omega), \]  

where \(N\) and \(N'\) are the numbers of the A and B layers, respectively, and

\[ \Phi_N^{(a)}(\omega) = \left( \frac{\partial A(\omega)}{\partial n''_a} \right)_{n''_a=0}, \quad \Phi_N^{(b)}(\omega) = \left( \frac{\partial A(\omega)}{\partial n''_b} \right)_{n''_b=0}. \]

The function \(\Phi_N^{(a)}(\omega)\) should be calculated in the limit \(n''_a \to 0\) and it may be written in analogy with Eq. (2) as a sum of \(\Phi_m^{(a)}(\omega)\), where the sum is over all the A layers in the structure. The function \(\Phi_N^{(b)}(\omega)\) is defined in an analogous way. With the above taken into consideration, Eqs. (3) and (4) can easily be generalized to the case of two types of active layers, A and B. In the following sections, for simplicity, we confine ourselves to the simplest case when only the A layers are luminescent (so that \(n''_b = 0\)) and for the sake of shortness, instead of \(\Phi_N^{(a)}(\omega)\), we will use the notation \(\Phi_N(\omega)\), as earlier. As shown below, the function \(\Phi_N(\omega)\) is related to the spatial distribution of the energy density and, as a consequence, to the emission intensity outside the structure, therefore we will call it the photoluminescence (PL) spectral function.

### III. A GENERAL CALCULATION OF PHOTOLUMINESCENCE INTENSITY FOR 1D PHOTONIC CRYSTAL

Now we will describe the calculation procedure of the photoluminescence intensity from a 1D photonic crystal. We shall consider the case of normal incidence of light on the structure from a medium with a refractive index \(n_b\). It is useful to give the expressions for the amplitude reflection and transmission coefficients for the structure with an arbitrary number \(N\) of the A layers \[5\]:

\[ r_N = r_1 \sin NQd/Z_N, \quad t_N = t_1 \sin Qd/Z_N, \]

where \(r_1\) and \(t_1\) are the reflection and transmission coefficients for a single A layer. The other notations used are:

\[ \varphi_a = \omega n_a a/c, \quad \varphi_b = \omega n_b b/c, \quad r = (n_a - n_b)/(n_a + n_b), \]

\[ t = 2n_b/(n_a + n_b), \quad \xi = 1 - r^2 e^{2i\varphi_a} \]

\[ Z_N(\omega) = \sin NQd - t_1 \sin (N - 1)Qd, \]

where \(a\) and \(b\) are the thicknesses of the A and B layers, respectively, and \(Q\) is the magnitude of the wave vector, which satisfies the equation

\[ \cos Qd = \cos \varphi_a \cos \varphi_b - \frac{1}{2} \left( \frac{n_a}{n_b} + \frac{n_b}{n_a} \right) \sin \varphi_a \sin \varphi_b. \]

It is convenient to introduce \(\eta_m = t_m/(1 - r_m r_{N-m})\). By considering the energy flux density through the \(m\)-th A layer with the imaginary part of the refractive index \(n''\), one can show that the function \(F_m(\omega_m)\), which in the limit of \(n'' \to 0\) turns to \(F''(\omega_m) = \phi_m(\omega) n''\), can be written in the form

\[ F_m(\omega) = |\eta_m-1|^2(1 - |r_{N-m+1}|^2) - |\eta_m|^2(1 - |r_{N-m}|^2). \]

This expression can be used for approximate calculation of the intensity of the pump, \(F_m(\omega_m)\), at an arbitrary value of \(n''(\omega_0)\). (The prime on the function \(F_m(\omega)\) implies that \(n'' \to 0\).)

Let us introduce the ratio \(\gamma_m(\omega) \equiv I_m(\omega)/I_{1(\omega)}^{(0)}(\omega)\), where \(I_{1(\omega)}^{(0)}(\omega)\) is the intensity from a single A layer of the thickness \(a\), if the surrounding medium has the same refractive index as the A layer, \(n_b = n_a\). Making use of Eq. (2) at \(m=1\), we find that in the absence of dielectric contrast the function \(\Phi_1(\omega) = \phi_1(\omega) = \Phi_{1(\omega)}^{(0)} = 2\omega a/c\), \(\eta_1 = 1\). From Eq. (3) without taking into account \(F_m(\omega)\) one gets

\[ \gamma_m(\omega) = c\phi_m(\omega)/(2a\omega) \]

and the analogous quantity for the whole structure

\[ \Gamma_N(\omega) \equiv I_N(\omega)/(NI_{1(\omega)}^{(0)}(\omega)) = c\Phi_N(\omega)/(2Na\omega), \]

which we call the spontaneous emission intensity enhancement factors. Evidently, the functions \(\gamma_m(\omega)\) and \(\Gamma_N(\omega)\) give the relative change in emission intensity for the \(m\)-th A layer and \(N\)-period structure, respectively. Hence, together with Eq. (4), one obtains the expression

\[ I_N(\omega) \propto \omega^2 f_T(\omega) a N \Gamma_N(\omega). \]

The function \(\Gamma_N(\omega)\), as well as \(\Phi_N(\omega)\), is expressed only in terms of the parameters of the photonic crystal and does not depend on the emission characteristics of the sources (the emitting centers). It is responsible for modification of the emission spectrum due to the dielectric environment of the emitting centers (because of numerous reflections of the light when it is propagating in the structure), while the function \(f_T(\omega)\) is determined by the population of states of the centers and is proportional to their concentration. The condition that the function \(f_T(\omega)\) is a separate problem; however, as follows from the above analysis, when the distribution of nonequilibrium carriers between the states can be described by means of quasi-Fermi levels
for electrons and holes, the function $f_T(\omega)$ contains the absorption coefficient $\alpha(\omega)$ as one of the multipliers. In a bulk sample (a dielectrically homogeneous medium), as $\Gamma_N(\omega) = 1$, the PL intensity is proportional to $\omega^2 f_T(\omega)$.

Notice that to get the maximum value of the intensity of photoluminescence, $I_N(\omega)$, the frequency corresponding to the highest peak of the function $\Gamma_N(\omega)$ should coincide with the frequency of the emission spectrum maximum of the bulk material.

Using the generalized Kirchhoff’s law, one can make a more exact calculation of the photoluminescence intensity, by taking into account the difference in the light absorption in different regions of the $m$-th active layer. We now note that the function $\phi_m(\omega)$ corresponds to the time-average power of the (monochromatic) electromagnetic field absorbed per unit volume in the $m$-th active layer, $P = (1/2\pi)\omega|e''|E(\omega, r)^2$, (see [2]) integrated over the thickness of the layer; in our case $e'' = 2n_an''$. For the structure under consideration, which is translationally invariant in $x$ and $y$, one can represent $E(\omega, r) = e^{i\omega t}E(\omega, z)$, where $\rho$ and $\mathbf{q}$ are the in-plane vectors. In the case when a plane electromagnetic wave of the frequency $\omega$ is normally incident on the structure ($q = 0$), one can write

$$\phi_m(\omega) = C\omega \int_{-a/2}^{a/2} |\mathcal{E}(m)(\omega, z)|^2 dz,$$  
(11)

where the function $\mathcal{E}(m)(\omega, z)$ describes the distribution of the electric field along the $m$-th active layer and $C$ is a coefficient. Applying the generalized Kirchhoff’s law to an infinitesimally thin layer and taking into account the last relation, after integrating over the thickness of the $m$-th A layer, we arrive at the following relationship:

$$I_m(\omega) \sim \omega^2 f_T(\omega) \int dz |\mathcal{E}(m)(\omega_0, z)|^2 |\mathcal{E}(m)(\omega, z)|^2.$$

(12)

Note that if the local temperature depends on $z$ alone, so that the temperature distribution $T = T(z)$ is smooth and constant in time, the function $f_T(\omega, z)$ should be inserted into the integrand. As $f_T(\omega, z)$ is proportional to the concentration of ECs, it will also be included under the integral if the concentration changes with $z$.

The electric field $\mathcal{E}(m)(\omega, z)$ can be written as

$$\mathcal{E}(m)(\omega, z) = \mathcal{F}_1(m)(\omega)e^{ik_az} + \mathcal{F}_2(m)(\omega)e^{-ik_az},$$

(13)

where the $z$-coordinate is measured from the center of the $m$-th A layer, $k_a = \omega n_a/c$ and the functions $\mathcal{F}_1(m)$ and $\mathcal{F}_2(m)$ are

$$\mathcal{F}_1(m) = e^{i(\varphi_0 + \varphi_m)/(\eta_{m-1} + \eta_{m}e^{i\varphi_0}/\xi)},$$

$$\mathcal{F}_2(m) = e^{i(\varphi_0 + \varphi_m)/(\eta_{m-1}e^{i\varphi_0} - \eta_{m}e^{i\varphi_0}/\xi)}.$$

The function $|\mathcal{E}(m)(\omega_0, z)|^2$ is responsible for excitation of photoluminescence by incident light with a frequency $\omega_0$.

The simplest way to get the coefficient of proportionality $C$ is to compare the function $\Phi_1(\omega)$ with Eq. (11) at $m = 1$; hence $C = (2/c)(1 - r^2)$, where $c$ is the speed of light in vacuum. The calculation of $\phi_m(\omega)$ with using Eqs. (11) and (13) leads to

$$\phi_m(\omega) = \frac{2(1 - r^2)\omega a}{c|\xi|^2} [g_1(|\eta_{m-1}|^2 + |\eta_{m}e^{i\varphi_0}|^2) + 2g_2Re(\eta_{m}e^{i\varphi_0}r_{N-m})],$$

(14)

$$g_1 = 1 + r^2 + r\sin 2\varphi_a \varphi_a, \quad g_2 = (1 + r^2)\sin \varphi_a + 2r \cos \varphi_a.$$

One can show that the calculation procedure described above is equivalent to the calculation of the light power generated by independent sources. (It is in accordance with the theory presented above, where the reabsorption and re-emission processes were ignored.) For this purpose one should express the intensity of radiation transmitted through the photonic crystal in terms of the amplitude of a plane electromagnetic wave coming from the plane $z = const$ (where $z$-axis is defined inside the $m$-th active layer with the origin in the layer center) and calculate the reflection and transmission coefficients taking into account all reflections and all re-reflections of electromagnetic waves from various interfaces of the layers of the structure. The resulting expression will have the same form as Eq. (12) in which $\mathcal{E}(m)(\omega_0, z) = \text{const}$ and with $\mathcal{E}(m)(\omega, z)$ proportional to Eq. (13). In a more rigorous consideration, one has to resort to a Langevin-like approach, where the wave equation (for the electric field $E(\omega, r)$ given above) must be solved taking into account the polarization sources, which are due to the incoherent nature of spontaneous emission and can be described by random functions of the coordinates and time. This method is demonstrated in [15] on the example of a multiple-quantum-well structure, in which case the random term is ascribed to the exciton polarization.

Also, this method was used for calculations of steady-state photoluminescence spectra of Fibonacci photonic quasicrystal containing organic dye molecules [21].

In practice to calculate the PL intensity $I_N(\omega)$ of the structure with $N$ active layers, one uses the photoluminescence spectrum $I_1(0)(\omega)$ of a single active layer (of the thickness $a_0$). As follows from Eq. (12), the photoluminescence intensity under excitation at frequency $\omega_0$ can be estimated as

$$I_N(\omega) = I_1(0)(\omega) \sum_{m=1}^{N} \int_{-a/2}^{a/2} J(m)(\omega_0, \omega, z)dz \times \left( \int_{-a/2}^{a/2} J(m)(\omega_0, \omega, z)dz \right)^{-1},$$

(15)

where $J(m)(\omega_0, \omega, z) = |\mathcal{E}(m)(\omega_0, z)|^2 |\mathcal{E}(m)(\omega, z)|^2$. The equality sign in Eq. (15) implies that the power of excitation radiation in the case of $N$-period structure is exactly equal to that for the one-layer structure, otherwise it should be replaced by the sign of proportionality. If a 1D photonic crystal is terminated from one side with a plane interface between the material $B$ and a medium with the refractive index $n_0$, so that the distance between
two interfaces (of the three materials) is equal \( b' \). Eq. (15) should be multiplied by the factor \( |\tau/(1 - g r N e^{i \varphi_0})|^2 \), where \( \varphi = (n_b - n_0)/(n_b + n_0) \), \( \tau = 2n_b/(n_b + n_0) \), \( \varphi_b = (2b' - b)\omega n_b/c \), and the electric field \( E^{(m)}(\omega, z) \) should be calculated taking this medium into account.

IV. THE PHOTOLUMINESCENCE SPECTRAL FUNCTION AND INTENSITY ENHANCEMENT FACTOR OF 1D PHOTONIC CRYSTAL

As is seen from Eq. (4) the main features of the luminescence spectra of 1D photonic crystal can be established from an analysis of the PL spectral function \( \Phi_N(\omega) \) or, as discussed above, the emission intensity enhancement factor \( \Gamma_N(\omega) \), see Eq. (10). Taking the sum over \( m \) from 1 to \( N \) on both sides of Eq. (2) and then calculating the derivative of \( \Lambda_N(\omega) \) one gets the following expression for the function \( \Phi_N(\omega) \):

\[
\Phi_N(\omega) = \frac{C_1 N + C_2 \sin NQd}{(1 - r^2)^2 \sin^2 Qd + (2r \sin \varphi_a)^2 \sin^2 NQd},
\]

(16)

where the functions \( C_1 \) and \( C_2 \) are given by

\[
C_1 = \frac{(1/n_b - n_b/n_a^2)}{\sin \varphi_a \sin \varphi_b} + 2 \sin \varphi_a \cos \varphi_b + 2 \sin \varphi_b \cos \varphi_a \omega a/c
\]

\[+ (1 - r^2)(2 \sin \varphi_a \varphi_b + \sin(\varphi_a + \varphi_b)), \]

\[C_2 = B_0 \sin NQd + B_1 \sin (N - 1)Qd + B_2 \sin (N - 2)Qd - C_1 \sin NQd + \cos NQd \cot Qd, \]

\[B_0 = 4 \sqrt{T} \sin \varphi_a \sin \varphi_b + r^2 (1 - r^2)(\omega a/c), \]

\[B_1 = -2(1 - r^2)(3 \sin \varphi_a \varphi_b - \cos \varphi_a \cos \varphi_b)(\omega a/c) - 8 \sqrt{T} \sin \varphi_a \cos \varphi_b \sin \varphi_b/(n_a + n_b), \]

\[B_2 = -2(1 - r^2)^2(\omega a/c). \]

It follows from Eq. (16) that in the absence of dielectric contrast of the constituent materials (A and B) \( \Phi_N^{(0)}(\omega) = 2N \omega a/c = N\Phi_1^{(0)}(\omega) \). In the long wavelength limit, at \( \omega \to 0 \), Eq. (16) converts to \( 2N \omega a/c \).

In the most interesting case, when \( N \gg 1 \), Eq. (16) is greatly simplified and takes the form

\[
\Phi_N(\omega) \approx \frac{C_1 (N - \sin NQd \cos NQd \cot Qd)}{(1 - r^2)^2 \sin^2 Qd + (2r \sin \varphi_a)^2 \sin^2 NQd}.
\]

(17)

This expression is a good approximation in the frequency region where the values of \( C_1(\omega) \) are not too small (see the case \( C_1 = 0 \) below). A further simplification of the expression for \( \Phi_N(\omega) \) can be made by setting in Eq. (16) \( C_2 = 0 \), which is valid for frequencies not too close to a PBG edge, where the condition \( |\cot Qd \sin 2NQd| \ll 2N \) is satisfied; consequently, the numerator of Eq. (17) is equal to \( C_1 N \). At the Brillouin zone center (\( Q = 0 \)) and at the Brillouin zone edge (\( Q = \pi/d \)) the function \( \Phi_N(\omega) \) becomes \( \Phi_N^{(BZ)} = 3C_1 N^3 + 3(B_0 \pm B_1 + B_2 - C_1)^2 N^2 + (C_1 + 3B_1 - 6B_2)N \)

\[3\{(2N \sin \varphi_a)^2 + (1 - r^2)^2\}

where the upper sign refers to \( Q = 0 \) and the lower sign refers to \( Q = \pi/d \). The expression for \( \Phi_N^{(BZ)} \) shows that for a sufficiently large number \( N \) of the layers (\( N \gg 1 \)), approximately, \( \Phi_N^{(BZ)} \propto N \).

As an example, Fig. 1 shows the IEF (curve 1) as a function of the wavelength \( \lambda \) for a distributed Bragg reflector (DBR) with \( N = 10 \) active (A) layers, the transmission spectrum (curve 2) calculated in the absence of absorption and also the function \( C_1(\lambda) \) (curve 3). As seen from the figure, curve 1 is essentially asymmetric and has a set of peaks in two band regions, with the highest peaks being near the PBG edges. In the region of large wavelengths there are several spectral peaks, which with increasing the wavelength become lower and wider; an analogous situation occurs in the region on the opposite.

![Graph showing the intensity enhancement factor, \( \Gamma_N(\lambda) \), (curve 1) and transmission coefficient, \( T(\lambda) \), (curve 2) that has a correlation with the IEF in the position of the spectral peaks. The graph of the function \( \Gamma_N(\lambda) \) is plotted using Eqs. (10) and (16) and coincides with that plotted by using Eqs. (10) and (17) except for a small range around \( \lambda_c = 0.5 \mu m \), where Eq. (17) is not valid. The numbers \( j = 9, 8, ... \) in the figure indicate the corresponding spectral peaks. The curve 3 shows the function \( C_1(\omega) \), which is equal to zero at \( \lambda_c = 0.5 \mu m \) (at which the second PBG collapses). The vertical arrows indicate the edges of the first PBG. The inclined arrow shows a distinguished peak at the wavelength of \( \lambda_c = 0.5 \mu m \), see also the inset of the figure. Calculated for a DBR with \( N = 10 \) periods, the thicknesses \( a = \lambda/(4n_a) = 125 \) nm, \( b = \lambda/(4n_b) = 250 \) nm and refractive indices \( n_a = 2 \), \( n_b = 1 \) of the layers A and B, respectively; the tuning wavelength \( \lambda = 1000 \) nm.]}
side of the PBG. Such a picture is typical of 1D photonic crystal; it is also observed when the emitting centers are uniformly distributed in the B layers or in both types of layers. (Calculations are made using Eq. (5)).

As follows from Eq. (16), the PL spectral function $\Phi_N(\omega)$ can be written as $P_N(\omega)/|\xi Z_N(\omega)|^2$, where $P_N(\omega) = C_1 N + C_2 \sin NQd$, therefore the peaks of the function $\Phi_N(\omega)$ and, consequently, $\Gamma_N(\omega)$, are located around the real values, $\omega_j$, of the complex eigenfrequencies $\omega_j = \omega_j^r + i\omega_j^i$, which can be found from the equation $Z_N(\omega_j) = 0$, see Eq. (6). When absorption can be neglected the transmission coefficient in the band region is $T(\omega) = |t_N(\omega)|^2 = (1 - r^2) \sin^2 Qd/|\xi Z_N(\omega)|^2$. As the denominator of the function $T(\omega)$ is the same as for $\Gamma_N(\omega)$, while the function $\sin^2 Qd$ is rather smooth, the transmission peaks are located around $\omega_j$ as well and are slightly shifted relative to the peaks of the function $\Gamma_N(\omega)$, see Fig. 1. Thus, the positions of peaks of the IEF, $\Gamma_N(\omega)$, are close to those at which $T(\omega) = 1$ and, consequently, the wave numbers corresponding to the local maxima of $\Gamma_N(\omega)$ are $Q_j d = \pi j / N + \delta_j$, $|\delta_j| \ll 1$, where $j$ are integers and the frequencies $\omega_j$ satisfy Eq. (7) with $Q = Q_j$. Disregarding the value of $N\delta_j$, which for the peaks nearest to the PBG is $|N\delta_j| \ll 1$, one gets from Eq. (17)

$$\Phi_N(\omega_j) \approx \frac{C_1(\omega_j)N}{(1 - r^2) \sin^2(\pi j / N)}.$$  

Upon moving away from a PBG edge and passing successive values of $\omega_j$, the denominator (the squared sine) increases in a monotonous way and after that monotonically decreases when approaching to another PBG. A more exact consideration shows that when moving away from the PBG edge the values of $\delta_j$ can increase and should be taken into account, but qualitatively the situation remains quite similar; since on the scale of the distance between the neighboring peaks the function $C_1(\omega)$ changes relatively slowly compared to $\sin^2 Q(\omega)d$, the peaks of the function $\Phi_N(\omega)$ and, consequently, $\Gamma_N(\omega)$ are getting lower. The highest peaks correspond to $j = 1$ (at the PBG edge $Q = 0$) and $j = N - 1$ (at the edge $Q = \pi / d$). This explains why enhancement of the PL intensity occurs predominantly in the band region in the vicinity of the band-gap edge, see, e.g., [22]. It is obvious from Eq. (18) that if $j \ll N$ or $j \gg N$ (in the limit $N \gg 1$) $\Phi_N(\omega_j) \propto N^3$, so that with increasing the number of periods, $N$, the peaks of the function $\Phi_N(\omega)$ increase and approach to the PBG edges, and their number increases as well (in accordance with Eq. (6) and the simplified formula $Q_j d \approx \pi j / N$).

Another feature of the functions $\Phi_N(\omega)$ and $\Gamma_N(\omega)$ is their asymmetry relative to the PBG center. It can be easily explained with the help of Eq. (18) and expression for the function $C_1(\omega)$. Let us denote by $\omega_{j-}$ and $\omega_{j+}$ the frequencies corresponding to the peaks of the function $\Gamma_N(\omega)$ on different sides of a PBG. Evidently, $C_1(\omega_{j-})$ and $C_1(\omega_{j+})$ can take substantially different values that lead to an asymmetry in the height of the peaks with the same number $j$. In the example considered above (see Fig. 1) the PBG edges satisfy the condition $Qd = \pi$, consequently the peaks of $\Gamma_N(\omega)$ nearest to the PBG correspond to the integers $j = 9, 8, ...$. It follows from Eq. (18) that $\Phi_N(\omega_{j+})/\Phi_N(\omega_{j-}) \approx C_1(\omega_{j+})/C_1(\omega_{j-})$, therefore the ratio $\zeta(j) = \Gamma_N(\omega_{j+})/\Gamma_N(\omega_{j-}) = \omega_{j-} C_1(\omega_{j+})/C_1(\omega_{j-})$ is the same as the ratio $\zeta(j) = \omega_{j-} C_1(\omega_{j+})/C_1(\omega_{j-})$. The calculation gives the following values: $\zeta(j = 9) = 3.075$, $\zeta(j = 9) = 3.045$ and $\zeta(j = 8) = 2.439$, $\zeta(j = 8) = 2.408$, which verifies the applicability of Eq. (18). The analysis of Eq. (17) allows one to determine approximate values of the local minima of the PL spectral function. They correspond to the wave numbers $Q_k d = \pi(2k + 1)/(2N)$, where $k = 1, ... N - 2$, hence $\Phi_N(\omega_k) \approx C_1(\omega_k) N/ (1 - r^2) \sin^2(\pi(2k + 1)/(2N)) + (2r \sin(\omega_k a / c))^2$.

This expression also explains the asymmetry of the PL spectral function and, as a consequence, of the IEF.

The value of the function $C_1(\omega)$ changes considerably on the scale of the distance between the edges of PBGs, therefore it essentially determines the value of the IEF at a given frequency, in particular for frequencies close to a PBG edge, where the values of $\Phi_N(\omega_j)$ and $\Gamma_N(\omega_j)$ can be large, see Eq. (18). Therefore, by choosing the appropriate parameters and, thus, “governing” the function $C_1(\omega)$ one can achieve either an anomalously large enhancement of the emission intensity or its moderate suppression. However, as follows from the analysis of the function $C_1(\omega)$, for frequencies in a photonic band the inequality $C_1(\omega) > 0$ is always satisfied. This means that in the case of a 1D photonic crystal (without a defect) the light emission in the regime $I_N \propto N$ at a PBG edge (because $\Phi_N(BZ) \propto N$) and $I_N \propto N^3$ at the spectral peak frequency $\omega_j$ (because $\Phi_N \propto N^3$) cannot be suppressed completely, but can only be decreased due to a relatively small value of $C_1(\omega_j)$. In accordance with Eq. (10), in these cases $\Gamma_N = const$ and $\Gamma_N \propto N^2$, where the latter corresponds to the superradiant regime, which is due to the periodicity of the structure. However, since in any physical system of the considered type the absorption of the emitted radiation takes place, as well as loss of coherence, for sufficiently large values of $N$ this quadratic dependence ceases to be valid; moreover, in the case of very large $N$ and an extremely small absorption coefficient the high quality modes (in the vicinity of the PBG edges) come into force, potentially leading to the strong-coupling regime, in which case the present theory is not applicable.

There is a special case when at the frequency $\omega$, the photonic band-gap vanishes and, as the analysis shows, $C_1(\omega) = 0$. It is worthwhile to notice that at this frequency the function $\Phi_N(\omega)$ and, consequently, $\Gamma_N(\omega)$ has a local maximum. In the case considered in Fig. 1 the corresponding peak appears at the wavelength $\lambda_c = 0.5 \mu m$. This peak is higher than the neighbouring ones, because at the wavelength $\lambda_c$ all of the functions $\phi_m(\omega)$, where
The partial IEFs, $\gamma_m(\lambda)$, for the layers $m = 2$ (dotted line), $m = 6$ (solid line) and $m = 10$ (dashed line). The vertical arrows indicate the PBG edges. The inset shows a set of functions $\gamma_m(\omega)$, where $m = 1, 2, ..., 10$, which at the wavelength $\lambda = 0.5 \mu m$ have the same local maximum value (the peak is indicated by arrow). Calculated for the same parameters as in Fig. 1.

A quantitative description of this effect can be made by using Eqs. (14) and (16). The qualitative explanation is that as the PBG is getting narrower two spectral peaks adjoining the PBG edges essentially evolve and at the frequency $\omega_c$ join to form a single peak. Moreover, since in the case of a 1D photonic crystal the topology of the band structure for propagation of electromagnetic waves is determined by the number of photonic band-gaps, one can expect that in energy spectrum at the point where the topology changes (at the frequency at which a PBG collapses) some peculiarity will arise. This, indeed, occurs and manifests itself as a local maximum of the PL spectral function $\Phi_N(\omega)$, which determines the power of the electromagnetic radiation absorbed per unit volume. In general, this phenomenon is associated with the anomalous absorption of electromagnetic radiation in 1D photonic crystals, which occurs in the vicinity of the frequency at which either the band-gap or allowed region collapses [23 and 24].

We now briefly consider the behavior of the functions $\gamma_m(\omega)$, which are the (partial) intensity enhancement factors for different emitting layers of the photonic crystal. Figure 2 shows the partial IEFs calculated by using Eqs. (9) and (14) for the layers $m = 2, 6$ and 10 (see caption to Fig. 2). As seen from the figure, the central and first nearest layers give the largest contributions to the total intensity (at different wavelengths), while the remote layers emit relatively weakly. This behavior of the functions $\gamma_m(\omega)$ can be explained from the analysis of Eq. (14). Qualitatively, the electromagnetic radiation from a remote layer experiences strong reflection from the rest of the structure, while the transmittance of the radiation coming from the first nearest layer at some wavelengths can be sufficiently high. The highest peaks of the functions $\gamma_m(\omega)$ near the PBG edges for the central layers ($m = 5$ and 6) are due to some kind of microcavity effect, see Fig. 2. In general, as shown in the previous section, the frequency dependence of the IEF is associated with the distribution of electric field modes in the photonic crystal structure.

V. DISCUSSION (CONNECTION TO THE GREEN’S FUNCTION METHOD)

Now we briefly discuss how the theory presented above is connected with the most commonly used theory of light emission from microstructures. As is known, there are two different ways to describe the emission process and calculate intensity of the emitted light, namely, classical and quantum approaches; their equivalence and difference have been discussed in detail in many papers and books, e.g., see [2, 4, and 25]. In the classical approach the electromagnetic power is due to emitting dipoles and is given by

$$\frac{dW}{dt} \propto \omega^3 |d|^2 \langle e_d \cdot Im[G(r, r, \omega)] \cdot e_d \rangle,$$  \hspace{1cm} (19) $$

where $d$ is the dipole moment oriented along the unit vector $e_d$ and located at the point $r$. $G(r, r', \omega)$ is a dyadic Green's function (or Green’s tensor) [26] of the considered system, and the angle brackets denote an average over all possible dipole orientations. In the dipole approximation the emitting centers, just as atoms, are modelled as oscillating point dipoles having electric dipole moments. In the case of uniform and isotropic distribution of dipole moments in an active (A) layer the average in Eq. (19) can be expressed through the trace of the imaginary part of the dyadic Green’s function [2]. In this connection we study non-polarized radiation and consider the source (electric current density) and response (electric or magnetic field) as scalar functions; hence, the Green’s function with which we will deal is a scalar function corresponding to the symmetry of the system. This is in accordance with the following qualitative consideration. Let us consider a layer of (noninteracting) dipoles which is perpendicular to the $z$-axis and whose thickness is much smaller than the light wavelength. An ensemble of incoherent emitters with 3D isotropic random orientation of the electric dipole moments gives the same intensity as three incoherently radiating dipoles with moments oriented along the $x, y$ and $z$ axes [27 and 28]. Since we are interested in calculating the far-field emission intensity (in the direction perpendicular to the layered structure), one can align all the dipoles in these three directions and then replace two-thirds of them with a system of two sets of incoherently radiating infinite current sheets with mutually perpendicular currents directed along the $x$- and $y$-axes.
y-axes. Such replacement is possible because an infinite plane sheet of dipoles directed in one direction and oscillating in phase radiates in the same way as an infinite plane current sheet (which produces plane waves propagating from both sides in the perpendicular direction), while a set of z-directed dipoles gives no contribution to the total intensity. In the case of a current sheet the boundary conditions at the interfaces of the layer are easily satisfied and as a result the intensity of outgoing unpolarized radiation will depend only on the z-coordinate of the current sheet in the layer. Thus, by considering that dipoles are polarized in one plane, in the case of a one-layer structure one should take 1D scalar Green’s function, calculate its imaginary part at the position of an emitting center for coincident emission and observation points, $z = z'$, and then integrate over the layer thickness.

In the quantum approach in the case of the weak-coupling regime and low quantum efficiency [12] the rate of direct radiative transitions is given by the Fermi’s golden rule [25]. As is shown in [10], in this case the emission rate is proportional to the local density of states, $\rho(\mathbf{r}, \mathbf{e}_\mu, \omega)$, at the position of the emitting center: $\kappa_r(\omega) \propto \omega^2 \rho(\mathbf{r}, \mathbf{e}_\mu, \omega)$, where $\mu$ is the magnitude of the transition dipole moment and $\mathbf{e}_\mu$ defines its orientation. The LDOS in turn is proportional to the imaginary part of the dyadic Green’s function and calculated for the direction given by the orientation $\mathbf{e}_\mu$: $\rho(\mathbf{r}, \mathbf{e}_\mu, \omega) \propto \omega \langle \mathbf{e}_\mu | \text{Im} G(\mathbf{r}, \mathbf{r}, \omega) | \mathbf{e}_\mu \rangle$. In the weak-coupling approximation the emission intensity can be calculated by the rate equation for the excited state population (the number of emitting centers in the excited state) and is given by [30] $I(\omega) \propto \frac{\kappa^2(\omega)}{\kappa_r(\omega) + \kappa_{nr}(\omega)}$, where $P$ is the rate of excitation, and $\kappa_r(\omega)$ and $\kappa_{nr}(\omega)$ are the radiative and nonradiative decay rates for the transition from an excited state to the ground state. In the case of low quantum efficiency $\kappa_r \ll \kappa_{nr}$, so that $I(\omega) \propto P \kappa_r(\omega)/\kappa_{nr}$. Unlike the nonradiative decay rate $\kappa_{nr}$, which is mainly defined by the chemical composition of the A-layers and therefore can be taken constant, the quantities $P$ and $\kappa_r$ depend on the emitter position ($P = P(\mathbf{r})$, $\kappa_r = \kappa_r(\mathbf{r}, \omega)$). In the simplest case $P$ can be considered constant (otherwise it should be taken into account by an additional factor that is the squared magnitude of the electric field at frequency $\omega$ in the integrand of Eq. (12)) and, consequently, the spontaneous emission rate enhancement factor (the Purcell factor), $\gamma(\omega) \equiv \frac{\kappa_r(\omega)}{\kappa_r(\omega)}$, is approximately equal to $I(\mathbf{r}, \omega)/I^{(0)}(\mathbf{r}, \omega)$, where $I^{(0)}(\mathbf{r}, \omega)$ and $I(\mathbf{r}, \omega)$ are the emission intensities from a two-level quantum emitter in a homogeneous medium and from the emitter in the structure under study, respectively, and $\kappa_r^{(0)}(\omega)$ and $\kappa_r(\omega)$ are the corresponding radiative decay rates. As is known, if the atoms in a waveguide are initially in the excited state they will spontaneously emit into a propagating mode (the light can be detected in the far field) and a guided mode (light does not leave the waveguide). The corresponding channels of the radiative decay contribute to the spontaneous emission rate and thus the both propagating and guided modes contribute into LDOS. In accordance with the above considerations about the replacement of the dyadic Green’s function by the 1D scalar Green’s function, the ratio of intensities $\frac{I(\mathbf{r}, \omega)}{I^{(0)}(\mathbf{r}, \omega)}$ can be determined in a way analogous to that in which the Purcell factor is determined, as $\frac{\kappa_r^{(0)}(\omega)}{\kappa_r^{(0)}(\omega)}$, where $\kappa_r(\omega)$ and $\kappa_r^{(0)}(\omega)$ are now expressed through the imaginary part of the 1D scalar Green’s function. Then, after integrating over the emitting volume of the structure, as discussed above, one can obtain the ratios $\frac{I(\omega)}{I^{(0)}(\omega)}$ for a separate $m$-th A layer ($\Gamma_m(\omega)$, see Eq. (9)) and for the whole $N$ layer structure ($\Gamma_N(\omega)$, see Eq. (10)), which were earlier termed the spontaneous emission intensity enhancement factors.

Here it should be noted that 1D models can satisfactorily describe the experimental SE spectra of quasi-one-dimensional structures composed of isotropic layers [15, 22, 31, 35]. As an example, there is an approach using the concept of the electromagnetic density of modes in 1D periodic structures [26, 33], that can be exploited in calculations of SE spectra; it is discussed in comparison with the indirect method from Kirkhoff’s law in [10]. Also, for this purpose the scalar formalism based on the Green’s function method and the concept of plane scatterers was developed in [11] and the scattering matrix formalism for the electromagnetic-field quantization in [12]. However, only straightforward calculations [28, 32, 34, 42] taking into account the vectorial nature of the electromagnetic field and based on using the dyadic Green’s function can allow a detailed study of the emission from a layered structure, in particular, the near-field emission pattern, spectral energy density and LDOS. Such calculations are based on the so-called direct method, which is equivalent to the indirect method [10] used in the present study. (In this section we focus on a new aspect of this equivalence related to the replacement of the dyadic Green’s function by the scalar one, which is possible because “the LDOS of planar structures is independent of the polarization states” [16].)

Now we will establish a relationship between the Kirchhoff’s law approach and Green’s function method. For this purpose, we use Eq. (23) given in Appendix, from which one gets

$$I_m G^{(m)}(z, z) = \frac{D_0}{2 k_n z} + \frac{D_0}{2 k_n z} \cos 2 k_n z + \frac{D_2}{2 k_n z} \sin 2 k_n z,$$

$$D_0 = \frac{1 - |r_L r_R|^2}{1 - r_L r_R}, \quad D_1 = \frac{r_L + r_R}{1 - r_L r_R}, \quad D_2 = \frac{r_R - r_L}{1 - r_L r_R}.$$

These three functions are related to the functions $F^{(m)}_{1,2}$, which are proportional to the field amplitudes outside the photonic crystal, in the following way:

$$\frac{2 D_0}{1 - r^2} = |F^{(m)}_1|^2 + |F^{(m)}_2|^2 + |\gamma^{(N-m+1)}_1|^2 + |\gamma^{(N-m+1)}_2|^2,$$
Re(D_1) = (1 - r^2) Re(\mathcal{F}_1^{(m)} \mathcal{F}_2^{(m)*} + \mathcal{F}_1^{(N-m+1)} \mathcal{F}_2^{(N-m+1)*}),

D_2 = (1 - r^2) (\mathcal{F}_1^{(m)*} \mathcal{F}_2^{(m)} - \mathcal{F}_1^{(N-m+1)*} \mathcal{F}_2^{(N-m+1)}) .

Squaring the modulus of the left and right parts of Eq. (13) and multiplying by the transmission coefficient \( T_{ab} \), one gets the relationship between the total emission intensity and the imaginary part of the Green’s function:

\[ n_b |\varepsilon_L^{(m)}(\omega, z)|^2 + |\varepsilon_R^{(m)}(\omega, z)|^2 = 4n_b^2 (\omega/c) ImG^{(m)}(z, z), \]

where \( n_b |\varepsilon_L^{(m)}(\omega, z)|^2 \) and \( n_b |\varepsilon_R^{(m)}(\omega, z)|^2 \) determine the intensities of the light (emitted at point \( z \)) from the outermost layers \([47]\), therefore the many-layer structure quasicrystal (it becomes symmetric after removal of two and for nearly mirror-symmetric ones, e.g., the Fibonacci spectra (in the case of a spatially homogeneous excitation coefficient and square of the transition (dipole) moment. Its determination together with \( \alpha(\omega) \) is a problem which is to be solved individually for each type of optical transition associated with a given absorption mechanism. In practice, the transition moment is a parameter which is estimated from an experimentally measured spectrum. (In the case of forbidden lines the transition moment, for instance, corresponds to the electric-quadrupole or magnetic-dipole moment operator.)

Thus, the calculation of emission intensity based on Eq. (19) and with using the 1D scalar Green’s function given by Eq. (23) reduces to the calculation on the basis of the generalized Kirchhoff’s law, see Eq. (4). Note that the correspondence between the two approaches (for the weak-coupling regime) is related to the use of several analogous conditions which should be briefly mentioned here: i) the thermal reservoir or (Markovian) bath and the typical hierarchy of time-scales (in steady-state luminescence experiments), which is expressed by the cascading inequalities: \( \tau_{ph} \ll \omega^{-1} \ll \tau_{th} \ll \tau \ll \tau_{exc} \ll \tau_{res} \), where \( \tau_{ph} \sim l/c \) is the time of light propagation in an atomic system with a dimension \( l \) (compare to the value of \( \tau_b \sim 10^{-18} \) s in \([17]\)), the vibrational relaxation time \( \tau_{th} \sim 10^{-11} - 10^{-15} \) s, the luminescence decay time \( \tau_{exc} \sim 10^{-7} \) s, and \( \tau_{exc} \) and \( \tau_{res} \) are the luminescence excitation time and the time of change in reservoir temperature, respectively. ii) the absence of temporal correlation (the reservoir is memoryless, i.e., the coupling of a quantum emitter to the reservoir does not depend on its past, the memory function is approximated by a delta function) which is supposed in both the Langevin approach and Markovian approximation (in the Weisskopf-Wigner theory \([3]\); iii) the small dimensions of the emitting centers compared to the light wavelength, that allows one to express the PL intensity in terms of both the squared magnitude of the electric field \( E(z) \) (in Eq. (12)) and the imaginary part of the Green’s tensor (in Eq. (19), where the electric-dipole approximation is exploited); iv) a disregard of the stimulated emission (when deriving Eqs. (3) and (4)) and re-emission (which is taken into account by the first-order Maclaurin expansion of the absorption function \( A(\omega) \) in Eq. (2)), and correspondingly a disregard of reversible spontaneous emission, which is taken into account in the first-order of the perturbation theory by the Fermi’s golden rule (in the quantum analogue of Eq. (19)); v) the real dielectric function, which allows us to naturally introduce the PL spectral function as well as the local density of states.
VI. CONCLUSION

In conclusion, we have stated the generalized Kirchhoff’s law for one-dimensional layered structures, which is applicable to the calculation of the photoluminescence spectra if local thermodynamic equilibrium holds between the matter and radiation. The overlapping integrals of intensity distributions of the photonic crystal modes and pump excitation modes are expressed in terms of the amplitude reflection and transmission coefficients, which is convenient for numerical calculations and qualitative analysis. The developed approach is also convenient to obtain the expression for the spontaneous emission intensity enhancement factor whose analysis has allowed us to establish some features of the modification of the light emission in the case of 1D photonic crystals, in particular, the enhancement of emission intensity at the photonic band edges. At the same time, the necessity of calculating the integrals of the imaginary part of the Green’s function and then summing over all emitting layers, while the method based on the generalized Kirchhoff’s law allows one to avoid this problem altogether. The correspondence between these two methods in a transparent region was analytically established for a 1D photonic crystal and confirmed by numerical calculations; in essence, this approach demonstrates the effective manipulation of 1D Green’s function. Expressions obtained in this paper (Eqs. (12) and (15)) agree with the model of independent (incoherent) sources and therefore can be used for calculating photoluminescence spectra of multi-layered absorbing structures while the local thermodynamic condition is met. As a rule, quasi-one-dimensional structures allow a much simpler theoretical description of their optical properties than two- and three-dimensional systems, reducing the complexity of the problem and leading to explicit analytical expressions; such is the theory of light emission presented here. In general, the proposed approach provides the physical basis for luminescence engineering of 1D layered structures, in particular, photonic crystals and microcavities.

ACKNOWLEDGMENTS

The author greatly thanks E. L. Ivchenko and A. B. Pevtsov for helpful discussions and also M. Glazov and an anonymous referee for making valuable comments on the paper.

APPENDIX

One-dimensional scalar Green’s function, $G(z, z', \omega)$, defined for the points $z$ and $z'$ of a uniform layer with the thickness $a$ (the both coordinates, $z$ and $z'$, are measured from the center of the layer in the same direction) satisfies the equation

$$\left(\frac{d^2}{dz^2} + k_0^2 \varepsilon(z)\right)G(z, z', \omega) = -\delta(z, z'),$$

where the dielectric function $\varepsilon(z) = \varepsilon_a$ for an A layer and $\varepsilon(z) = \varepsilon_b$ for a B layer. For $|z'| \leq a/2$, the Green’s function is

$$G(z, z', \omega) = \frac{i}{2k_a} |e^{ik_a|z-z'|}| +$$

$$\frac{r_L e^{ik_a(z+z')} + r_R e^{-ik_a(z+z')} + 2r_L r_R \cos k_a(z - z')}{1 - r_L r_R^*}, \quad (23)$$

where $r_L$ and $r_R$ are the reduced reflection coefficients, on the left and right sides of the layer, respectively. (Equation (23) can be obtained by the standard method for calculating the electric field taking into account multiple reflections from the left and right interfaces and using the expression for the sum of a geometric series.) In the case of a periodic structure the coefficients $r_L$ and $r_R$ related to the $m$-th A layer are given by

$$r_L = e^{i\varphi_a} \frac{r + r_{m-1} e^{i\varphi_a}}{1 + r r_{m-1} e^{i\varphi_a}}, \quad r_R = e^{i\varphi_a} \frac{r + r_{N-m} e^{i\varphi_a}}{1 + r r_{N-m} e^{i\varphi_a}}.$$
17 W. L. Vos and L. A. Woldering, *Light Localisation and Lasing: Random and Quasi-Random Photonic Structures*, Ch. 8, Cambridge University Press (2015).
18 M. M. Voronov, E. L. Ivchenko, M. V. Erementchouk, L. I. Deych, A. A. Lisiansky, J. Lumin. **125**, 112 (2007).
19 E. L. Ivchenko, *Optical spectroscopy of semiconductor nanostructures*, Alpha Science International, Harrow, UK, (2005).
20 L. D. Landau, E. M. Lifshitz, *Electrodynamics of Continuous Media*, Pergamon Press (1984).
21 V. Passias, N. V. Valappil, Z. Shi, L. Deych, A. A. Lisiansky, V. M. Menon, Opt. Express **17**, 6636 (2009).
22 A. V. Medvedev, N. A. Feoktistov, A. B. Pevtsov, V. G. Golubev, Semiconductors **39**, 1356 (2005).
23 M. M. Voronov, E. L. Ivchenko, V. A. Kosobukin, A. N. Poddubny, Phys. Solid State **49**, 1792 (2007).
24 A. P. Vinogradov, Yu. E. Lozovik, A. M. Merzlikin, A. V. Dorofeenko, I. Vitebskiy, A. Figotin, A. B. Granovsky, A. A. Lisiansky, Phys. Rev. B **80**, 235106 (2009).
25 R. Loudon, *The quantum theory of light*, Oxford University Press (2000).
26 Chen-To Tai, *Dyadic Green functions in electromagnetic theory*, IEEE Press (1994).
27 W. Lukosz, J. Opt. Soc. Am. **71**, 744 (1981).
28 L. Novotny, J. Opt. Soc. Am. A **14** 91 (1997).
29 M. Sargent III, M. O. Scully and W. E. Lamb, *Laser Physics*, Westview Press (1978).
30 A. F. Koenderink, L. Bechger, A. Lagendijk, W. L. Vos, Phys. Stat. Sol. (a) **197**, 648 (2003).
31 Levanyuk A. P., Osipov V. V., Sov. Phys. Usp. **24** 187–215 (1981).
32 M. D. Tocci, M. Scalora, M. J. Bloemer, J. P. Dowling, C. M. Bowden, Phys. Rev. A **53** 2799 (1996).
33 A. A. Dukin, N. A. Feoktistov, V. G. Golubev, A. V. Medvedev, A. B. Pevtsov, Appl. Phys. Lett., **77**, 3009 (2000).
34 K. Kuroda, T. Sawada, T. Kuroda, K. Watanabe, K. Sakoda, J. Opt. Soc. Am. B **27**, 010045 (2010).
35 M. V. Rybin, A. V. Zherdlev, N. A. Feoktistov, A. B. Pevtsov, Phys. Rev. B **95** 165118 (2017).
36 J. M. Bendickson, J. P. Dowling, M. Scalora, Phys. Rev. E, **53**, 4107 (1996).
37 A. Settini, S. Severini, N. Mattiucci, C. Sibilia, M. Centini, G. D’Aguanno, M. Bertolotti, M. Scalora, M. Bloemer, C. M. Bowden, Phys. Rev. E **68**, 026614 (2003).
38 I. S. Fogel, J. M. Bendickson, M. D. Tocci, M. J. Bloemer, M. Scalora, C. M. Bowden, J. P. Dowling, Pure Appl. Opt. **7**, 393 (1998).
39 J. P. Dowling and C. M. Bowden, Phys. Rev. A **46**, 612 (1992).
40 C. M. Cornelius, J. P. Dowling, Phys. Rev. A **59**, 4736 (1999).
41 M. Wubs, A. Lagendijk, Phys. Rev. E **65**, 046612 (2002).
42 M. A. Kaliteevski, V. A. Mazlin, K. A. Ivanov, A. R. Gubaydullin, Opt. Spectrosc. **119**, 832 (2015).
43 M. S. Tomas, Phys. Rev. A **51**, 2545 (1995).
44 S. M. Dutra and P. L. Knight, Phys. Rev. A **53**, 3587 (1996).
45 C. Creatore, L. C. Andreani, Phys. Rev. A **78**, 063825 (2008).
46 L. P. Wang, S. Basu, Z. M. Zhang, J. Heat Transfer **133**, 072701 (2011).
47 M. A. Kaliteevski, V. V. Nikolaev, R. A. Abram, S. Brand, Opt. Spectrosc., **91** 109 (2001).