Viability of nuclear $\alpha$-particle condensates

A reply to N.T. Zinner and A.S. Jensen, arXiv:nucl/th0712.1191

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In a recent paper \[1\], arXiv:nucl/th0712.1191, Zinner and Jensen (ZJ) expressed strong doubts about the concept of alpha-particle condensation in finite nuclei. In this article we give a reply which, essentially, is point by point (but not in the order).

I. DEFINITIONS

First let us define how we understand the concept of “alpha-particle condensation in nuclei”. As explained in our previous work \[2\], the word “condensation” is not to be understood in the macroscopic sense when talking about nuclei. It rather is to be seen in analogy to nuclear pairing, to nuclear deformation and rotation, etc. Nuclear physicists became used to employ those macroscopic terms for things which are in reality only in a (slowly) fluctuating state. They well know this and it is only to be understood as a semantic short cut when they talk about “nuclear superfluidity”, “nuclear deformation” and “rotation”, etc. In reality e.g. the number of Cooper pairs in a nucleus is very limited and in no way one can consider this to be a macroscopic condensate. One only can say that an antisymmetrised product of Cooper pairs is a good approximation for certain nuclear states and phenomena which reflect pairing and superfluidity and that this product state goes continuously over into the macroscopic BCS state when the number of pairs is increased indefinitely. That is there is as much link between a macroscopic alpha-particle condensate and the product state of a few alphas in a nucleus as there is link between pairing in a nucleus with half a dozen of Cooper pairs and neutron superfluidity in a neutron star! Nobody will deny that such a strong correspondence exists for the latter case. We then think that there is complete analogy between nuclear pairing, nuclear deformation, etc., and nuclear alpha-particle condensation, with the only difference that alpha-particle condensation has only recently been suggested as a new nuclear state. As is the case for pairing, this new nuclear property reflects, in a finite system, the state which it would acquire in a corresponding infinite system. For infinite nuclear matter alpha particle condensation has recently been suggested from a theoretical investigation to exist at low densities \[3,4\]. A quartet phase at low density was also found by a QMC solution of a 1D Hubbard model with four different fermions \[5\]. We, therefore, define a state of condensed \(n\alpha\)'s, if in a nuclear state the latter forms in good approximation a bosonic product state. So far in what concerns definitions.
II. \(\alpha\)-PARTICLE DENSITY MATRIX

One of the main points in the paper by ZJ is that they contest the uniqueness of the definition of our alpha-particle density matrix whose eigenvalues show that e.g. in the Hoyle state the three alpha particles form to nearly 75% a bosonic product state with the bosons all in the identical 0S state. We have recently published a longer paper on this subject on arXiv [6] and only repeat here the main conclusions. ZJ base their arguments on the fact that for self-bound systems like alpha-particles in nuclei, one necessarily has to define a density matrix corresponding to the intrinsic system where the center of mass coordinate has been removed. This question has recently been debated with respect to Bose-Einstein condensation of cold atoms [7, 8]. N.K. Wilkin et al. [7] found that a BEC which rotates with its c.o.m. in a trap potential but stays with its intrinsic state in the ground state, i.e. no internal excitations are present, exhibits a so-called fragmented condensate, that is there are several eigenvalues of the single particle density matrix which show occupancies of the order of the total number of particles. This is to be contrasted with the situation of a uniform system where ALL condensed particles sit in the lowest momentum state \(k = 0\). In a subsequent paper Pethick and Pitaevskii (PP) [8] argued that on physical grounds the situation of condensed particles should not be different in a uniform system from a Bose-system in a trap when the intrinsic system is not excited and that for that one has to work with a suitably defined density matrix of the “internal” system. Their internal density matrix is defined with “internal” coordinates \(q_i = r_i - R\) where \(R\) is the total c.o.m. coordinate. Our study in Ref. [6] shows, however, that with this so-defined internal density matrix one again obtains a fragmented condensate what is contrary to the initial claim and objective of PP. It turns out that the outcome of the study strongly depends on the definition of the internal coordinates: the coordinates chosen by PP are not orthogonal, this being the reason for the occurrence of a fragmented condensate. In choosing Jacobi coordinates which are orthogonal, we could show that bosons in a harmonic trap which form an ideal condensate in the laboratory frame, i.e. all particles in the lowest 0S orbit, remain an ideal, i.e. non-fragmented condensate, once the c.o.m. coordinate has been removed, that is internally. This, in agreement with the original objective of PP, seems to us the correct physical situation [9]. In addition we could show that the internal density matrix defined with non-orthogonal coordinates leads to a fragmented condensate even in
the macroscopic limit [6]. At this point we should mention that in previous publications on alpha-particle condensation always the internal density matrix was defined with the Jacobi coordinates [10, 11, 12, 13]. We, therefore, conclude on this point that our previous statement that the Hoyle state in $^{12}$C is to nearly 75% a product state of three alpha particles condensed into an identical 0S-orbit is unambiguous [10, 11, 12]. Similarly we recently have found in an extended investigation of $^{16}$O that the sixth $0^+$ state at 15.1 MeV also is a strong candidate to be of alpha-particle condensation nature with over 60% of the alpha-particles condensed [13]. Therefore, those states fulfill our criterion of $\alpha$-particle condensation. At the same time, this brings to fall the main argument of ZJ which, initially, anyway was based on an erroneous formula [15].

In the light of this finding, we would like to discuss again the content of the THSR alpha-particle condensate wave function [16]. This wave function is given by

$$\Phi_{n\alpha}(B, b) = A \\left\{ \prod_{i=1}^{n} \exp \left( -\frac{2}{B^2} X_i^2 \right) \phi_{\alpha_i} \right\}$$

with $X_i$ the coordinates of the c.o.m. motion of the $\alpha$-particles, and, e.g.

$$\phi_{\alpha_1}(r_1, r_2, r_3, r_4) = \exp \left( -\frac{1}{8b^2} \sum_{i<j=1}^{4} (r_i - r_j)^2 \right).$$

It is very important to remark, as is explained in Ref. [16], that this condensate wave function contains two limits exactly. On the one hand, for $B = b$ we have a pure HO Slater determinant because the antisymmetriser generates out of the product of simple Gaussians all higher nodal wave functions of the HO. On the other hand, for $B \gg b$ the THSR wave function tends to a pure product state of alpha-particles, i.e. a mean field wave function, since in this case the antisymmetriser can be neglected. Indeed $B$ triggers the extension of the nucleus, i.e. its average density. For alpha particles kept at their free space size (small $b$), the alpha-particles are then for large $B$-values far apart from one another and do not feel any action from the Pauli principle. The question is then whether, e.g. for the Hoyle state, the above wave function is closer to a Slater or to alpha-product state. Precisely this question is answered by the above discussed eigenvalues of the density matrix. In this respect it is important to point out that in the calculation of the afore-mentioned density matrix always the total c.o.m. motion has been split off in the wave function of Eq. (1) and that for the remaining relative c.o.m. coordinates the Jacobi ones have been used, as is clearly explained in [10, 11]. In Refs. [10, 11] it has been shown, as explained, that the alphas occupy to over
70% the 0S-orbit. Therefore, the Hoyle state is in good approximation a product of three alpha particles, that is a condensate.

III. DECAY PROPERTIES

This brings us to a further critics of ZJ where it is claimed that besides the Hoyle state in $^{12}\text{C}$, no heavier self-conjugate nuclei can show analogous alpha-particle structure. The argument is based on the fact that the alpha-particle condensate states occur near the alpha-particle disintegration threshold which rapidly grows in energy and, thus, the level density in which such a condensate state is embedded raises enormously. For example the alpha-disintegration threshold in $^{12}\text{C}$ is at 7.24 MeV and in $^{16}\text{O}$ it is already at 14.4 MeV. Under ordinary circumstances this could mean that the alpha-particle condensate state in $^{16}\text{O}$, which we suppose to be the well known $0^+$ state at 15.1 MeV, has a very short lifetime and ZJ make a Fermi gas estimate in this respect. However, on the one hand it is a fact that the supposed $^{16}\text{O}$ “Hoyle”-state at 15.1 MeV has experimentally, for such a high excitation energy, a startling long lifetime (decay width 160 keV!) and on the other hand it is easily understandable that such an exotic configuration as four alpha-particles moving almost independently within the common Coulomb barrier, has great difficulties to decay into states lower in energy which all have very different configurations! How else one could explain such a long life time of a state at 15.1 MeV excitation energy? It is precisely one of the strong indications of alpha-particle condensation that the state should be unusually long lived! It is furthermore well known that the Hoyle state cannot be explained even with the most advanced shell model calculations. Its energy comes at 2-3 times its experimental value $^{17}$. This is a clear indication that shell model configurations only couple extremely weakly to alpha condensate states. One can argue that many of the states in $^{16}\text{O}$ below 15.1 MeV are of shell model type. There are also alpha-$^{12}\text{C}$ configurations but since $^{12}\text{C}$ also has shell model configuration, it again is difficult for the four alpha condensate state to decay into. This brings to fall a further argument of ZJ.
IV. LOCALISATION

Another critics of ZJ is that they say that a state of localized alpha-particles can equally well describe the Hoyle state and they cite for that the work of Chernykh et al. [18]. This again is a strong misconception. In the work of Chernykh et al. about 55 configurations are superposed. In our opinion these configurations mostly serve to delocalise the $\alpha$ particles [19].

V. THE QUANTALITY CONDITION

The next point of ZJ is the least understandable. They claim on grounds of the so-called Mottelson “quantality” condition that a mean field description of freely moving alpha particles cannot be applied. Since our wave function is a prototype of a meanfield ansatz which leads, without any free parameter, to correct results for almost all measured quantities of the Hoyle state, this statement of ZJ can only be totally fallacious. On the other hand, the Gross-Pitaevskii equation was applied to study dilute multi alpha-particle condensation in nuclei, in which we used a renormalized effective $\alpha\alpha$ potential [20] (as always for a mean field). This potential, of course, well fulfills the quantality condition. Also, using the energies of the mentioned resonances in $^8$Be and $^{12}$C* and calculating the deBroglie wave length, we do find that the latter is larger than the nuclear radius (see also [10] for same conclusion), a situation similar to the pairing concept of neutrons in heavy nuclei. The alpha-clusters are in a condensed state, a fact which probably has been observed experimentally by the emission of $^{12}$C* [21] from excited $^{52}$Fe. Again on this point ZJ are advancing erroneous statements.

VI. SIMILARITY OF $\alpha$-PARTICLE CONDENSATES WITH VARYING PARTICLE NUMBER

In Fig. 1 we show, side by side, radial parts of the single-$\alpha$ $S$ orbits (for definition, see Refs. [6, 11, 13]) of the Hoyle state ($^{12}$C) and the $0^+_6$ state in $^{16}$O [14]. We see an almost identical shape! Of course, the extension is slightly different because of the smallness of the system. The nodeless character of the wave function is very pronounced and only some oscillations with small amplitude are present in $^{12}$C, reflecting a weak influence of the Pauli principle between the $\alpha$’s! On the contrary, we show in Fig. 2 radial parts of the single-$\alpha$ $S$
orbits of the ground states in $^{12}$C [11] and $^{16}$O [13, 14]. Due to its much reduced radius the “α-like” clusters strongly overlap, producing strong amplitude oscillations which take care of antisymmetrisation between clusters. Again this example very impressively demonstrates the condensate nature of the Hoyle state and the $0^+_6$ state in $^{16}$O. This result is much in contrast with the fact that ZJ announced the similarity criterion for α-particles being very difficult to be fulfilled in finite systems with only a few bosons.
VII. CONCLUSION

We have shown that the arguments of ZJ against the existence of alpha-particle condensed states in self-conjugate nuclei are without foundation. For instance we could very clearly demonstrate that their strongest argument concerning the ambiguity of the eigenvalues of the density matrix is false [6, 15]. We also could demonstrate the similarity of condensates with different number of $\alpha$-particles, another convincing argument in favor of the condensate aspect.

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\( r \phi (r) \) is normalised as \( 4\pi \int (r \phi (r))^2 dr = 1. \)

[15] One should mention here that the original formula, Eq. (9) of ZJ [1] is wrong (v1 and v2) and consequently the conclusions thereof also. After we published the right formula, ZJ also corrected their formula (with no citation of our work!), arXiv:nucl/th0712.1191v3, but left their conclusions unchanged!

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