Research Article

Intercarrier Interference in OFDM: A General Model for Transmissions in Mobile Environments with Imperfect Synchronization

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Intercarrier Interference (ICI) is an impairment well known to degrade performance of Orthogonal Frequency Division Multiplexing (OFDM) transmissions. It arises from carrier frequency offsets (CFOs), from the Doppler spread due to channel time-variation and, to a lesser extent, from sampling frequency offsets (SFOs). Literature reports several models of ICI due to each kind of impairment. Some studies describe ICI due to two of the three impairments, but so far no general model exists to describe the joint effect of all three impairments together. Furthermore, most available models involve some level of approximation, and the diversity of approaches makes it cumbersome to compare power levels of the different kinds of ICI. In this work, we present a general and mathematically exact model for the ICI stemming from the joint effect of the three impairments mentioned. The model allows for a vis-a-vis comparison of signal-to-ICI ratios (SIRs) caused by each impairment. Our result was validated by simulations. An analysis of ICI in IEEE-802.16e-type transmissions shows that during steady-state tracking and at speeds below 150 km/h, SIR due to CFO is typically in the range between 25 dB and 35 dB, SIR due to Doppler spread is larger than 25 dB, and ICI due to SFO is negligible.

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1. Introduction

Mathematical models of Intercarrier Interference (ICI) in Orthogonal Frequency Division Multiplexing (OFDM) and techniques for mitigating it have been reported by many authors. Studies modeling and dealing with ICI stemming individually from channel variation in time are [1–9]. Likewise, the works of [10–15] address ICI due to Carrier Frequency Offset (CFO) and those of [16, 17] ICI solely due to Sampling Frequency Offset (SFO). Work modeling ICI produced jointly by two of the three impairments is significantly less common. The joint effect of CFO and SFO has been studied in [18, 19], while [20] reports on ICI due to CFO and channel mobility. Despite the attention that the topic has received so far, there is as yet no general model in literature that describes ICI resulting from the joint effect of all three impairments.

Many of the above cited references model ICI by using discrete-time and discrete-frequency signals. Unfortunately, discrete-domain approaches are inaccurate for representing impairments that affect signals outside the time-frequency grid of discrete analysis, such as the SFO, or often restrict the time-frequency properties of the channels for which the approaches are valid. Another limitation of the models in the cited references is the difficulty of combining them in order to make a fair comparison of each kind of ICI under the same conditions.

Our contribution with this paper is the derivation of the general and mathematically exact model of ICI for OFDM transmissions subject to the joint effect of CFO, SFO, and time-varying channels with arbitrary statistics. By including continuous-domain analysis, our derivations yield a model that is general and is devoid of the limitations of purely discrete-domain approaches, and by modeling all three impairments together, we obtain a tool that allows for a direct and clear comparison of the ICI caused by each impairment.
The remainder of this work is organized as follows. Section 2 presents the development of our model of ICI with deterministic signals; Section 3 analyzes the statistical behavior of ICI; in Section 4, the statistical behavior predicted by our model is validated by simulations of IEEE-802.16e-type transmissions ("mobile WiMAX") [21]. Signal-to-interference ratio curves for a broad range of mobile speeds and CFOs are provided; finally, Section 5 sets out our conclusions.

2. Deterministic Model of ICI

In what follows, we derive a mathematical model that includes the effects of CFO, SFO, and channel mobility on OFDM transmissions.

We begin by modeling the signal of the $m$th OFDM symbol in continuous time $t$ using complex baseband notation as

$$x(t) = \sum_{v=-\frac{N_t}{2}}^{\frac{N_t}{2}} X_m(v)e^{j2\pi v(t-mT_g)\delta_t}.$$  

In this equation, $X_m(v)$ is the modulation on subcarrier $v$ of a total of $N_t$ subcarriers. The separation between subcarriers is $\delta_t = 1/N_t T$ Hertz, where $T$ is the sampling period of the transmitter. The cyclic prefix has $N_p$ samples and duration of $T_c = N_t T$ seconds. Thus, the complete OFM symbol has $N_s = N_p + N_t$ samples and duration of $T_g = N_t T$ seconds. The symbol $j$ denotes the imaginary unit $\sqrt{-1}$. Finally, $\oplus$ is the rectangular function, equal to 1 when $x$ is between $[-1/2,1/2]$ and 0 elsewhere.

In [22, 23], the general input-output relationship of time-variant linear systems is described as

$$y(t) = \int_0^\infty h(t,\tau)x(t-\tau)d\tau,$$  

where $x(t)$ and $y(t)$ are the respective input and output signals in the time domain. The function $h(t,\tau)$ is the time-variant impulse response of the system observed at instant $t$ due to an impulse at time $t - \tau$.

Now consider $h(t,\tau)$ to represent the baseband-equivalent impulse response of a time-varying wireless channel, and substitute (1) for $x(t)$ in (2) to represent an OFDM transmission through that channel. At the receiver, the arriving (passband) signal becomes corrupted by Additive White Gaussian Noise (AWGN), is then downconverted to baseband with a CFO of $\Delta f$ Hertz, and sampled with an SFO of $\Delta T$ seconds. Following the steps outlined in Appendix A for including these impairments, we obtain the following sampled received signal for the $m$th OFDM symbol:

$$y_s(t) = (T + \Delta T) \sum_{p=0}^{N_t-1} \sum_{v=-\frac{N_t}{2}}^{\frac{N_t}{2}} X_m(v)e^{j2\pi f_0(T + \Delta T)} \cdot \delta\left(t - p(T + \Delta T) - mN_g(T + \Delta T)\right)$$

$$\cdot \int_0^\infty h(t,\tau)e^{j2\pi \delta_t(t-\tau)}d\tau$$

$$\cdot \left(\left(1/2\right)N_s + mN_g\right)(T + \Delta T)$$

$$+ n_s(t).$$ 

Above, $\delta(\cdot)$ is the impulse function and $n_s(t)$ is AWGN sampled at instants $t = p(T + \Delta T) + mN_g(T + \Delta T)$, with $p = 0,\ldots,N_t - 1$. Equation (3) is a continuous-time signal, but its value is 0 at every instant except when $t = p(T + \Delta T) + mN_g(T + \Delta T)$, with $p = 0,\ldots,N_t - 1$.

To recover the symbols in an actual OFDM system implementation, the Fast Fourier Transform (FFT) of the $N_t$ samples is calculated. That operation is equivalent to calculating the continuous Fourier transform of (3) with respect to $t$, but with the origin fixed at time $mN_g(T + \Delta T)$ (i.e., taking the transform of $y(t + mN_g(T + \Delta T))$). It is to be noted that we assume that the cumulative drift of the FFT window due to SFO has not yet reached the previous or following OFDM symbol. Intersymbol interference is therefore not considered in our model. Upon following the algebraic steps detailed in Appendix B, we obtain

$$Y_s(f) = \left(\sum_{v=-\frac{N_t}{2}}^{\frac{N_t}{2}} X_m(v) e^{-j2\pi f (1/2)N_s + mN_s(T + \Delta T)}\right)$$

$$\cdot \sin\left[\pi(f - \Delta f - \nu_0)(T + \Delta T)N_s\right]$$

$$\cdot \sin\left[\pi(f - \Delta f + \nu_0)(T + \Delta T)N_s\right]$$

$$\cdot e^{-j2\pi \delta_t e^{-j2\pi \delta_t f mN_g(T + \Delta T)}}$$

$$+ N_s(t).$$ 

In this equation the function $s^\cap(f,\tau)$ is the Doppler-variant impulse response [22], time-limited to the duration of the $m$th OFDM symbol. Thus, $s^\cap(f,\tau)$ is given by the Fourier Transform:

$$s^\cap(f,\tau) = \mathcal{F}\left\{h(t,\tau) \cap \left(\left(1/2\right)N_s + mN_s\right)(T + \Delta T)\right\}$$

$$= N_s(T + \Delta T)\sin\left(\pi f N_s(T + \Delta T)\right)$$

$$\cdot e^{-j2\pi f((1/2)N_s + mN_s)(T + \Delta T)}s(f,\tau),$$ 

$$\cdot \left(1/2\right)N_s + mN_s\right)(T + \Delta T)$$

$$+ n_s(t).$$ 

In this equation the function $s^\cap(f,\tau)$ is the Doppler-variant impulse response [22], time-limited to the duration of the $m$th OFDM symbol. Thus, $s^\cap(f,\tau)$ is given by the Fourier Transform:
where \( s(f, \tau) \) is the Doppler-variant impulse response defined as the Fourier transform in \( t \) of \( h(t, \tau) \) [22]. For a fixed delay \( \tau \), and if the channel is static, then \( s(f, \tau) \) is a frequency domain impulse. As channel mobility increases, so does the frequency spread of \( s(f, \tau) \). The symbol \( \delta \) denotes continuous convolution in the frequency domain.

Expression (4) gives an exact description of the continuous spectrum of an OFDM signal received over a time-domain.

For a fixed delay \( \tau \), with \(-N_s/2 \leq l \leq N_s/2-1\) and \( \delta_r = 1/N_s(T + \Delta T) \) being equal to the separation of the subcarrier frequencies used by the receiver. Imposing these conditions on (4) and considering an arbitrary subcarrier \( k \), we obtain (Appendix C) the discrete output of the system:

\[
Y(k) = \beta(k,k)\overline{h}(k)X_m(k) + I(k) + W(k) + Q(k) + N(k),
\]

where \( \beta(\cdot, \cdot) \) is a phase and magnitude distortion given by

\[
\beta(d, \nu) = \frac{1}{N_s} e^{j2\pi(d-\nu)\delta_m T_s} e^{-j2\pi[(d-\nu)/N_s-(\nu/N_s)(\Delta T/T)-\Delta fT(1+\Delta T/T)](mN_s+(N_s-1)/2)} \sin[\pi[(d-\nu)/N_s-(\nu/N_s)(\Delta T/T)-\Delta fT(1+\Delta T/T)]N_s] \sin[\pi[(d-\nu)/N_s-(\nu/N_s)(\Delta T/T)-\Delta fT(1+\Delta T/T)]],
\]

and \( \overline{h}(k) \) in (6) is the time domain average of the channel in carrier \( k \) during the transmission of the \( m \)-th OFDM symbol, given by

\[
\overline{h}(k) = \frac{1}{N_s(T + \Delta T)} \int_{0}^{\infty} s(0, \tau) e^{-j2\pi(k\delta_r \tau)} d\tau
\]

\[
= \int_{0}^{\infty} e^{-j2\pi(k\delta_r \tau)} \int_{mN_s}^{(m+1)N_s} h(t, \tau) dt d\tau.
\]

Finally \( I(k) \), \( W(k) \), and \( Q(k) \) in (6) represent various forms of intercarrier interference (ICI). Concretely, \( I(k) \) is ICI due solely to mobility, \( W(k) \) is ICI caused exclusively by imperfect synchronization, and \( Q(k) \) is an ICI that is nonzero only when both impairments are present. Their expressions are

\[
I(k) = \frac{1}{N_s(T + \Delta T)} \sum_{d=-N_s/2}^{N_s/2-1} X_m(d)\beta(d,d) \cdot \int_{0}^{\infty} s(0, \tau) e^{-j2\pi(k\delta_r \tau)} d\tau,
\]

\[
W(k) = \sum_{\nu=0}^{N_s/2} X_m(\nu)\beta(k,\nu)\overline{h}(\nu),
\]

\[
Q(k) = \frac{1}{N_s(T + \Delta T)} \sum_{d=-N_s/2}^{N_s/2-1} \sum_{\nu=\nu}^{N_s/2-1} X_m(\nu)\beta(d,\nu) \cdot \int_{0}^{\infty} s(0, \tau) e^{-j2\pi(k\delta_r \tau)} d\tau.
\]

Results (6) through (11) are deterministic and mathematically exact. In (9) (ICI due to mobility) we observe that the interference in subcarrier \( k \) is the sum of signals from all the other subcarriers, respectively weighted by the integral of \( s(0, \tau) \). The value of the integral depends on mobility and on the separation between the interfering subcarriers \( d \) and the desired subcarrier \( k \). This value is relevant only when subcarrier \( d \) is in the neighborhood of \( k \). The size of the neighborhood grows with the mobile's speed, but in any current-day OFDM systems designed for mobility (e.g., DVB-T/H [24], “mobile WiMAX” [21]), the neighborhood is mainly comprised by subcarriers \( k+1 \) and \( k-1 \). It is to be noted that (9) equals zero if the channel is static, regardless of synchronization. If synchronization is perfect (i.e., \( \Delta T = 0 \) and \( \Delta f = 0 \)), then (9) is similar to the description found by many authors [2, 6, 9, 25] for representing interference based on Doppler-variant impulse responses. However, they all use discrete-domain approaches, different from the one employed here, thus capturing the effect of ICI less accurately.

Term (10) (ICI due to imperfect synchronization) has been described by several authors for static channel conditions, either considering CFO and SFO jointly [18, 19], CFO alone [10, 12], or SFO alone [16, 17]. This term equals zero if and only if there are no frequency nor sampling offsets, regardless of mobility.

Finally, \( Q(k) \) of (11) is a new finding and clarifies a frequent misconception that ICI due to mobility and imperfect synchronization is two separate additive terms. There is an ICI enhancement when both impairments are jointly present (we will show in Section 4, however, that this ICI is negligible in practice).

3. Statistical Analysis of ICI

In this section we analyze the statistical properties of ICI on the basis of the expressions derived in the previous section. The analysis that follows assumes a WSSUS (wide-sense
stationary with uncorrelated scattering) channel [22, 23], meaning that

$$E[s(f_1, \tau_1)s^*(f_2, \tau_2)] = \delta(f_2 - f_1)\delta(\tau_2 - \tau_1)S_n(f_1, \tau_1), \tag{12}$$

where $S_n(f, \tau)$ is the scattering function [22, 23]. It is also assumed that the transmitted data symbols are not correlated either in frequency or in time and have an average energy of $E_s$.

The relationship between the Doppler power spectral density and the scattering function is [23]

$$S(f) = \int_0^\infty S_n(f, \tau)d\tau, \tag{13}$$

where $S(f)$ is the power spectral density of a pure tone received under conditions of mobility. Perhaps the most widely used model for this density is the one due to Clarke [26] (often referred to as Jakes’ Doppler spectrum):

$$S(f) = \frac{1}{\pi f_d \sqrt{1 - (f/f_d)^2}}, \quad \text{if } |f| \leq f_d, \tag{14}$$

$$= 0, \quad \text{if } |f| > f_d,$$

where $f_d$ is the maximum Doppler spread as given by $f_d = v/\lambda$, with $v$ denoting the mobile’s speed and $\lambda$ the carrier wavelength.

We define the function $S^\top(f)$ as the Doppler power density of the baseband-equivalent of a time-limited carrier wave, that is, the Doppler density $S(f)$ convoluted in frequency with a sinc$^2(\cdot)$ function:

$$S^\top(f) = \text{sinc}^2(f N_s (T_1 + \Delta T)) * f S(f). \tag{15}$$

Based on the foregoing considerations, we show in Appendix D that the expected power of ICI on subcarrier $k$ is composed by three additive terms as

$$E\{[I(k) + W(k) + Q(k)]^2\} = E\{|I(k)|^2\} + E\{|W(k)|^2\} + E\{|Q(k)|^2\} \tag{16}$$

$$= S_I(k) + S_W(k) + S_Q(k),$$

where the notation used is self-evident. The three power terms are, respectively, given by

$$S_I(k) = E_s \sum_{d=\overline{N_s/2}}^{N_s/2} \beta(d, d)\beta^*(d, d)S^\top((k - d)\delta_s), \tag{17}$$

$$S_W(k) = E_s \sum_{\nu=\overline{N_s/2}}^{N_s/2} \beta(k, \nu)\beta^*(k, \nu)S^\top(0), \tag{18}$$

$$S_Q(k) = E_s \sum_{d=\overline{N_s/2}}^{N_s/2} \sum_{\nu=\overline{N_s/2}}^{N_s/2} \beta(d, \nu)\beta^*(d, \nu)S^\top((k - d)\delta_s). \tag{19}$$

Note that (16) implicitly states that the three ICI terms are statistically independent from each other.

In [27], the steps of Appendix D were also followed for determining the covariances of the ICI terms between different subcarriers. The result can be used for generating statistically accurate frequency-correlated ICI from a white Gaussian sequence. (By virtue of the central limit theorem with $N_s$ large enough it is commonly accepted that ICI has Gaussian random properties; see, e.g., [25].) The ICI thus generated might greatly simplify some simulations of imperfectly synchronized OFDM systems in mobile environments.

Finally, using (6) and (8) (and WSSUS conditions), we can calculate the expected power of the desired symbol actually received on subcarrier $k$:

$$S_X(k) = E\{\beta(k, k)X(k)\overline{H}(k)\beta^*(k, k)X(k)^*\overline{H}^*(k)\} = E_s \cdot \beta(k, k)\beta^*(k, k)S^\top(0), \tag{20}$$

where $E\{\overline{H}(k)\overline{H}^*(k)\} = S^\top(0)$ follows from (8) and (15).

Strictly speaking, the expected interference powers per subcarrier given by (17), (18), (19), and (20) change with $k$. In practice, however, if the transmission bandwidth is much larger than the Doppler spread bandwidth, these terms are practically constant over frequency. This is so because the time-limited Doppler spread function ($S^\top(f)$) takes significant values only in the neighborhood of a subcarrier $k$, thus ensuring statistical homogeneity in most subcarriers other than those at the band edges, which are exposed to less ICI because they have fewer neighboring subcarriers.

We define $\eta = S_X(k)/E_s$ as the fraction of energy kept by subcarrier $k$. If we assume perfect synchronization then by (20) we can calculate $\eta$ as a measure of degradation due only to channel mobility. Using (14) and (15), we obtain the precise value of $\eta$ as follows:

$$\eta = \int_{-\nu/\lambda}^{\nu/\lambda} \frac{\lambda \sin^2(f N_s T_1^2)}{\pi \sqrt{1 - (\lambda f/\nu)^2}} df. \tag{21}$$

An example of (21) is shown in Figure 1(a) for a IEEE-802.16e transmission with 512 subcarriers (“mobile WiMAX”) [21]. At speeds under 500 km/h, the subcarrier energy retention is over 95%.

A similar performance measure can be calculated for a case without mobility but with imperfect synchronization. Using (20) and (7) and considering $\nu/N_s(\Delta T/T) \approx 0$, we obtain

$$\eta = \frac{1}{N_s^2} \frac{\sin^2\left[\pi \Delta f T(1 + \Delta T/T)\right]}{\sin^2\left[\pi \Delta f T(1 + \Delta T/T)\right]}. \tag{22}$$

In this case, the WiMAX system requires a CFO smaller than 0.2 intercarrier spacing so that less than 10% of energy is lost as ICI (Figure 1(b)).
4. Computational Verification of Results

We now turn to the computational validation of (17), (18), (19), and (20) found in Section 3. For this, consider defining the signal-to-interference-plus-noise ratio (SINR) of a subcarrier $k$ as

$$\text{SINR}(k) = \frac{S_X(k)}{S_I(k) + S_W(k) + E_s/\Gamma}. \quad (23)$$

Above, the parameter $\Gamma$ is defined such that transmissions have an expected signal-to-noise ratio (SNR) of $\Gamma$ when there is no ICI (i.e., no mobility and perfect synchronization, hence (23) evaluates to $\Gamma$).

Our goal is to compare the theoretical prediction of (23) with values of SINR obtained from simulations by averaging over 300 OFDM symbols, transmitted over the same number of independent realizations of WSSUS time-varying channels with Clarke's statistics, and with receiver-side insertion of CFO and SFO. The time-variant impulse responses $h(t, \tau)$ were generated using an autoregressive model of order 100 as set out in [28], with an $\epsilon = 10^{-5}$ bias to ensure the algorithm's stability. Unit-power QPSK was used for subcarrier modulation.

The parameters used were those of an IEEE-802.16e system [21] with $N_s = 512$ subcarriers, a cyclic prefix of $N_c = 64$, bandwidth of 5 MHz, and a carrier of 3.5 GHz. Finally, the coherence time was estimated as $T_c = 0.423/fD$ [29], the average symbol energy $E_s$ was set equal to 1, and the channels' maximum delay spread was restricted to the duration of the cyclic prefix.

Note that evaluating (17), (18), (19), and (20) for (23) requires computing continuous integrals given by (8) and (15) (the latter was computed based on (14)). These were carried out with a sampling density of 100 points between subcarriers. In order to reduce the computational complexity of the resulting calculations, we used the fact that (17) to (20) are essentially invariant in $k$ (as noted) and therefore confined ourselves for evaluating the case of $k = 0$.

A first set of simulations illustrates the individual contributions of CFO, SFO, and environment mobility to the signal-to-interference ratio (SIR) in the absence of thermal noise (Figure 2). Solid curves show evaluations of (23) and markers quantify simulation results. Curve A shows the SIR due only to mobility ($S_X/S_W$) when synchronization is ideal. For cases with imperfect synchronization, note that the SIR $S_X/S_W$ is a constant with respect to the channel's coherence time. The SIR is thus shown by asymptotes B1 for the case with CFO = 0.2 parts of one intercarrier spacing and SFO = 0 ppm, and C1 for the case with CFO = 0 and SFO = 20 ppm. Correspondingly, curves B2 and C2 present the ratios $S_X/(S_W + S_Q)$. They confirm that $S_Q$ is relevant only in transmissions with extremely high mobility and has no practical relevance in current-day OFDM systems. Similarly, the contribution of SFO to SIR (curves C1 and C2) is also negligible compared to the effect of mobility and CFO, even with the rather large SFO used here.

We now focus on the simulation results for SINR with different CFOs and coherence times. Figures 3 and 4 display some of these results for $\Gamma = 7 \text{ dB}$ and $\Gamma = 12 \text{ dB}$. Observe that for high-mobility channels, where coherence times are less than approximately 3 OFDM symbols, SINR degrades dramatically, regardless of the magnitude of CFO. This implies that for the range of coherence times just indicated, CFO-induced degradation is overshadowed by the degradation due to mobility. By contrast, with greater coherence times, the ICI produced by CFO tends to dominate the SINR. A graphic representation of how SINR varies with CFO for different levels of thermal noise is presented in Figure 5. It shows that taking full advantage of OFDM performance in high SNR regimes needs tighter synchronization requirements than at lower SNRs.
The top curves of Figures 3 and 4 present a discrepancy between theoretical and simulation results. As discussed by Baddour and Beaulieu [28], the autoregressive approach for simulating a time-varying channel uses ill-conditioned equations, which makes simulating slow-varying channels with Clarke's U-shaped spectral density difficult. As a workaround, they propose a heuristic solution equivalent to adding a very small amount of white noise to the channel's fading process. The effect is also equivalent to a slight enhancement of thermal noise and reveals itself in our simulations when thermal noise dominates over ICI, as in the curves mentioned. When this simulation bias is negligible, however, our theoretical results are well matched by the simulations.

The values used above for CFO (0.2 and 0.4) are adequate for representing initial conditions of tracking loops after an acquisition stage. But during steady-state tracking, typical RMS values of the residual CFO are in the range between...
parameters are as in Figure 1. Obtained from (23) with static channel, SNR \( \Gamma = \infty \) and system parameters are as in Figure 1.

\[ 0.01 (\Gamma = 18 \text{ dB}) \text{ and } 0.04 (\Gamma = 7 \text{ dB}) \] [19]. Because of the algorithmic limitations discussed above, simulating these cases of CFO for channels with coherence times of practical interest yields inaccurate results. However, we can now predict precise SIR levels by evaluating (23). Figure 6 shows the SIR due only to CFO for a broad range of CFO values. The surprising linearity of the relationship between SIR and CFO is not at all evident from the equations. For the residual CFOs given before, SIR is in the range between 25 dB and 35 dB. In similar fashion, Figure 7 presents the SIR for a wide range of mobile speeds. For mobile speeds below 150 km/h, SIR is larger than 25 dB. Observe that Figures 6 and 7, used along with an SNR level \( \Gamma \), provide a quick way for ranking the three impairments in terms of their contribution to SINR and for determining link-level SINR values without having to resort to time-consuming simulations.

Finally, note that after neglecting SFO the sole parameter remaining in (17), (18), (19), and (20) is the intercarrier spacing \( 1/NsT = 1/Tg \). Because all modes of operation specified by the IEEE-802.16e standard use the same intercarrier spacing [21], it follows that the curves in Figures 2 through 7 are in fact valid for any mode of mobile WiMAX transmission.

5. Conclusions

A general and mathematically exact model of the power of intercarrier interference (ICI) was derived for OFDM transmissions exposed to the joint impact of sampling frequency offset, carrier frequency offset (CFO), and channel time variation. It was shown that the ICI ensuing from these impairments has three components: one solely caused by Doppler spread, one that depends only on the synchronization offsets, and one that is nonzero only when imperfect synchronization and channel variation happen together. Similar but nevertheless approximate descriptions of the former two components are available in literature. In this paper, besides describing them without approximations, they are presented with the same power scale. This allows for a direct comparison of these two sources of ICI. The third component is a new finding. It was shown to be nonnegligible only in very-high-speed environments of no practical interest at the present.

The new model was validated by computer simulations of OFDM transmissions using IEEE 802.16e parameters (mobile WiMAX).

Signal-to-noise and signal-to-interference ratios (SIR) were used for comparing the different sources of ICI with levels of thermal noise. SIR curves for a broad range of CFOs and mobile speeds were presented. For reference, during steady-state tracking and at speeds below 150 km/h, SIR due to carrier frequency offset is typically in the range between 25 dB and 35 dB, and ICI due to Doppler spread is larger than 25 dB.

Appendices

A. Sampled Received Signal

By substituting (1) into (2) and adding the phasor \( e^{j2\pi\Delta f t} \) due to the difference of \( \Delta f \) Hertz between the receiver and transmitter carrier frequencies, we obtain

\[
y(t) = e^{j2\pi\Delta f t} \int_{0}^{\infty} \left( \int_{0}^{\infty} \left( t - \tau - (m + 1/2)T_g + T_e \right) \right)
\sum_{m=-N/2}^{N/2-1} X_m(y)e^{j2\pi\nu h(t-\tau-mT_g)} h(t, \tau) d\tau + n(t).
\]

We now sample the received signal (A.1) at a rate offset by \( \Delta T \) seconds from the transmitter rate, that is, at instants \( n \cdot (T + \Delta T) \), where \( T \) is the transmitter sampling period. If we assume that adequate filtering of the signal was conducted
prior to sampling so that the noise term \( n(t) \) is limited to the band of interest, the sampling times for the \( m \)th OFDM symbol are \( t = n(T + \Delta T) + mN_g(T + \Delta T) \), where \( n = 0, \ldots, N_r - 1 \). Thus, the sampling operation implicitly removes the cyclic prefix and extracts the OFDM symbol in the “correct” window except for a cumulative drift due to \( \Delta T \). The sampling function \( \text{shah}(\cdot) \) representing this operation is given by

\[
\text{shah}\left( \frac{t}{T + \Delta T} \right) = (T + \Delta T) \cap \left( t - \left( \frac{1/2}{N_r(T + \Delta T)} \right) \right) \\
\cdot \sum_{p=-\infty}^{\infty} \delta\left( t - p(T + \Delta T) - mN_g(T + \Delta T) \right),
\]

(A.2)

where \( \delta(\cdot) \) is the Dirac delta function. The scaling factor \( T + \Delta T \) indicates sampling by area. This formulation of the sampling function, instead of simply representing it by a sum of \( N_r \) terms, is more convenient for the derivation in Appendix B leading to (4). Applying (A.2) to (A.1) we obtain

\[
y_s(t) = \sum_{p=-\infty}^{\infty} \delta\left( t - p(T + \Delta T) - mN_g(T + \Delta T) \right) \\
\cdot (T + \Delta T) \sum_{\nu=-N_r/2}^{N_r/2-1} X_m(\nu)e^{j2\pi\nu f t} \\
\cdot \left( \frac{(1/2)N_r + mN_g(T + \Delta T)}{N_r(T + \Delta T)} \right) \\
\cdot \left( \frac{t - (m + 1/2)T_g + T_e}{T_g} \right) h(t, \nu) \\
\cdot e^{j2\pi\Delta t(t - mT_g)} \cdot \text{shah}\left( \frac{t}{T + \Delta T} \right) \cdot n(t),
\]

where \( mN_g(T + \Delta T) \leq t \leq (mN_g + N_r - 1)(T + \Delta T) \). If the cyclic prefix duration is sufficiently large that \( h(t, \nu) \equiv 0 \) for all \( \nu > T_e \) at every instant \( t \), a careful analysis of (A.3) will show that the presence of \( \cap(\cdot) \) allows \( \cap(\cdot) \) to be eliminated. This is so due to the condition on \( T_e \) and the position of the \( \cap(\cdot) \) functions when \( m \) and \( \Delta T \) are within the range of interest. Further simplification may be achieved by joining \( \cap(\cdot) \) with the series in \( p \), as given in (3).

### B. Spectrum of Sampled Received Signal

If we calculate the transform of \( y_s(t) \) in (3), evaluated at \( t + mN_g(T + \Delta T) \), we obtain

\[
Y_s(f) = \int_{0}^{\infty} \int_{-\infty}^{\infty} \cap(\cdot)h(t, \nu)\psi(t, \nu)e^{-j2\pi\nu f t} dt d\nu \\
\cdot e^{j2\pi f mN_g(T + \Delta T)} + N(f),
\]

(B.1)

where the function \( \psi(t, \nu) \) is defined as

\[
\psi(t, \nu) = (T + \Delta T) \sum_{\nu=-N_r/2}^{N_r/2-1} X_m(\nu) \\
\cdot \delta\left( t - p(T + \Delta T) - mN_g(T + \Delta T) \right) \\
\cdot e^{j2\pi\Delta t e^{-j2\pi\nu_0 mN_g(T + \Delta T)}}.
\]

In (B.1), \( \mathcal{F}_t[\cdot] \) is the Fourier transform in \( t \) of the product \( \cap(\cdot)h(t, \nu)\psi(t, \nu) \) for delay \( \tau \) in the time-variant impulse response. The transform is given by

\[
\mathcal{F}_t[\cap(\cdot)h(t, \nu)\psi(t, \nu)] = s^\cap(f, \nu) * \Psi(f, \nu),
\]

(B.3)

where \( s^\cap(f, \nu) \) and \( \Psi(f, \nu) \) are the Fourier transforms of \( \cap(\cdot)h(t, \nu) \) and \( \psi(t, \nu) \), respectively, given by

\[
s^\cap(f, \nu) = \int_{(N_r\pi + mN_g(T + \Delta T))}^{(N_r\pi + mN_g(T + \Delta T))} h(t, \nu) e^{-j2\pi f t} dt,
\]

(B.4)

\[
\Psi(f, \nu) = (T + \Delta T) \sum_{\nu=-N_r/2}^{N_r/2-1} X_m(\nu)e^{-j2\pi\nu_0 mN_g(T + \Delta T)} \\
\cdot e^{-j2\pi(\nu - \nu_0) mN_g(T + \Delta T)} \\
\cdot \sum_{\nu=0}^{N_r-1} e^{-j2\pi(\nu - \nu_0) p(T + \Delta T)}.
\]

(B.5)

The last step is to directly evaluate the geometric series in (B.5), following [19]. We are then left with

\[
\Psi(f, \nu) = (T + \Delta T) \sum_{\nu=-N_r/2}^{N_r/2-1} X_m(\nu)e^{-j2\pi\nu_0 mN_g(T + \Delta T)} e^{-j2\pi\nu_0 t} \\
\cdot e^{-j2\pi(\nu - \nu_0)(T + \Delta T) + (N_r - 1)/2} \\
\cdot \sin[\pi(\nu - \nu_0)(T + \Delta T) N_r] \\
\cdot \sin[\pi(\nu - \nu_0)(T + \Delta T) N_r] \\
\cdot \sin[\pi(f - \nu_0)(T + \Delta T) N_r] \\
\cdot \sin[\pi(f - \nu_0)(T + \Delta T) N_r],
\]

(B.6)

which is the expression in curly brackets in (4). Substituting (B.4) and (B.6) into (B.1) and (B.3), we obtain the desired signal model in (4).
C. Received Subcarrier Signal

We first evaluate portions of (4) at the desired discrete frequencies \( f = l \delta_r \) with \(-N_s/2 \leq l \leq N_s/2 - 1\) and \( \delta_r = 1/N_s(T + \Delta T) \) and find

\[
e^{-j2\pi \delta_r m \delta_r} e^{j2\pi l m N_s (T + \Delta T)} = e^{j2\pi l (-\gamma) \delta_r m \delta_r},
\]

\[
(f - \Delta f - \nu \delta_r)(T + \Delta T) = \frac{\nu}{N_s} \Delta f \left( \frac{1 + \Delta T}{T} \right) - \frac{\nu}{N_s} \Delta f \frac{\Delta T}{T}.
\]

(C.3)

Also, the frequency-convolution integral in (4) becomes a summation (in \( d \), below) with its differential \( df \) turning into \( \delta_r = 1/N_s(T + \Delta T) \). The term \( s^\gamma(l \delta_r, \tau) \) is evaluated at the discrete frequencies \( l \), with sample separation \( \delta_r = 1/N_s(T + \Delta T) \), and to conserve energy it must be multiplied by \( \delta_r \). Thus, sampling (4) at frequency \( l = k \) gives the following result:

\[
Y(k) = \frac{1}{N_s^2(T + \Delta T)} \sum_{d=-N_s/2}^{N_s/2-1} X_m(d) e^{-j2\pi \delta_r m \delta_r} \cdot e^{-j2\pi (d - \nu) \delta_r m \delta_r} \cdot e^{-j2\pi \nu \delta_r \tau} d \tau + N(k).
\]

(C.2)

If we now separate out \( \nu = d \) from the rest of the terms in the second summation of (C.2), we obtain

\[
Y(k) = \frac{1}{N_s(T + \Delta T)} \sum_{d=-N_s/2}^{N_s/2-1} X_m(d) \beta(d, d) \cdot \int_0^\infty s^\gamma((k - d) \delta_r, \tau) e^{-j2\pi \delta_r \tau} d \tau + N(k),
\]

(C.3)

in which \( \beta(d, \nu) \) represents the phase and magnitude effects. This function is given in (7).

Finally, by isolating the terms \( d = k \) from both sums on \( d \) in (C.3) and by using the definition of the time-domain average channel given in (8), we obtain (6), (9), (10), and (11).

D. Expected Power of ICI

For simplicity, we write \( W(k) \) and \( Q(k) \) as one term \( \varphi(k) = W(k) + Q(k) \). Then, we have

\[
E[(I(k) + \varphi(k))(I(k) + \varphi(k))^*] = E[I(k)I^*(k)] + E[I(k)\varphi^*(k)] + E[\varphi(k)I^*(k)] + E[\varphi(k)\varphi^*(k)].
\]

(D.1)

We now proceed term by term:

\[
E[I(k)I^*(k)] = \frac{1}{N_s^2(T + \Delta T)^2} \sum_{d=-N_s/2}^{N_s/2-1} \sum_{d \neq k}^{N_s/2-1} E[X_m(d_1)X_m^*(d_2)]
\]

\[
\cdot \beta(d_1, d_1) \beta^*(d_2, d_2) \cdot \int_0^\infty E[s^\gamma((k - d_1) \delta_r, \tau_1)s^\gamma((k - d_2) \delta_r, \tau_2)^*] \cdot e^{-j2\pi \delta_r (d_1 - d_2) \tau_1 \tau_2} d \tau_1 d \tau_2.
\]

(D.2)

By (12) it is readily apparent that

\[
E[s^\gamma((k - d_1) \delta_r, \tau_1)s^\gamma((k - d_2) \delta_r, \tau_2)^*] = \sin(fN_s(T + \Delta T)^2) * S_u(f, \tau) \int_{f=[k-d]} \delta_{pq} \delta_{d_1, d_1} \delta_{d_2, d_2} \delta_{\tau_2 - \tau_1},
\]

where \( \delta_{pq} \) is the Kronecker delta. Given (13) and (15) and our assumption that the data symbols are uncorrelated with average energy \( E_s \), we can simplify (D.2) to obtain

\[
E[I(k)I^*(k)] = E_s \sum_{d=-N_s/2}^{N_s/2-1} \beta(d, d) \beta^*(d, d) S_u((k - d) \delta_r).
\]

(D.4)

Similarly, for \( \varphi(k) \),

\[
E[\varphi(k)\varphi^*(k)] = \frac{1}{N_s^2(T + \Delta T)^2} \sum_{d=-N_s/2}^{N_s/2-1} \sum_{d \neq k}^{N_s/2-1} \sum_{\nu \neq d} \sum_{\nu \neq k} E[X_m(\nu_1)X_m^*(\nu_2)] \beta(d_1, \nu_1) \beta^*(d_2, \nu_2)
\]

\[
\cdot \int_0^\infty E[s^\gamma((k - d_1) \delta_r, \tau_1)s^\gamma((k - d_2) \delta_r, \tau_2)^*] \cdot e^{-j2\pi \delta_r (\nu_1 - \nu_2) \tau_1 \tau_2} d \tau_1 d \tau_2.
\]

(D.5)
Using again (D.3) and uncorrelated data symbols we get
\[
E[\phi(k)\phi^*(k)] = \sum_{d=-N/2}^{N/2-1} \sum_{\nu \neq d} S^\dagger((k - d)\delta_v)\beta(d, \nu)\beta^*(d, \nu). \tag{D.6}
\]
Equations (18) and (19) are obtained by separating the term for \(d = k\) from the rest of the sum on \(d\) in (D.6).

Finally, the cross-correlations are
\[
E[I(k)\phi^*(k)] = \frac{1}{N^2(T + \Delta T)^2} \sum_{d_1=-N/2}^{N/2-1} \sum_{d_2=-N/2}^{N/2-1} \sum_{\nu_1=-N/2}^{N/2-1} \sum_{\nu_2=-N/2}^{N/2-1} \cdot E[X_m(d_1)X_m^*(\nu_2)]\beta(d_1, d_2)\beta^*(d_2, \nu_2)
\]
\[
\cdot \int_0^{\infty} E\left[\delta^\dagger((k - d_1)\delta_v, \tau_1)\delta^\dagger((k - d_2)\delta_v, \tau_2)\right] \cdot e^{-2\delta_t(\nu_2\tau_2 - d_1\tau_1)}d\tau_1d\tau_2. \tag{D.7}
\]
In this case, assuming uncorrelated data symbols eliminates all summands in (D.7) except those for \(d_1 = d_2\) and \(d_1 = \nu_2\). Then, because the sum on \(\nu_2\) leaves out the terms \(\nu_2 = d_2\), we find that (D.7) equals zero.

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