Hawking temperature of a Kerr–Newman–dS black hole from tunneling

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Abstract
We study the Hawking radiation of the Kerr–Newman–dS black hole on its event horizon and cosmological horizon from the tunneling approach of Parikh and Wilczek. In order to eliminate the motion on the rotating degree of freedom of a tunneling particle coming from the rotation of the Kerr–Newman–dS black hole, we choose a reference system that is co-rotating with the black hole horizon. Using such a method, we can derive the Hawking temperature of a general rotating black hole from the tunneling approach clearly. Then, to use the general result to the Kerr–Newman–dS case, we reproduce the Hawking temperatures of the Kerr–Newman–dS black hole on its event horizon and cosmological horizon from the tunneling approach.

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1. Introduction
Black hole radiation is one of the most interesting problems of black hole physics. Since its primal discovery by Hawking more than three decades ago [1], people have given many different explanations for this phenomenon [2–11]. The same result has been obtained from different derivation methods. Nearly 10 years ago, in [10], Parikh and Wilczek studied the Hawking radiation of Schwarzschild and Reissner–Nordström black holes from the quantum tunneling approach. It is a semiclassical description of the black hole radiation which has considered the self-gravitation and back-reaction effects in the process of the black hole radiation [9]. In such a description, Hawking radiation is interpreted as the tunneling effect of the radiated particles from the black hole horizon. The potential barrier that a particle tunnels is formed by the radiated particle itself due to the horizon’s contraction that comes from the self-gravitational effect of the radiation.

After the original work of Parikh and Wilczek [10], the tunneling method of black hole radiation has been developed in many aspects and has been applied for the derivation of the
Hawking radiation of many different types of black holes [12–27]. In [28–30], such a method has been generalized to the tunneling of fermions from some different types of black holes. In this paper, we study the Hawking radiation of the Kerr–Newman–de Sitter (Kerr–Newman–dS) black hole from the tunneling approach. Although recently in [29], the tunneling of the Kerr–Newman–dS black hole has been investigated in the Dirac particle tunneling picture, the tunneling of scalar particles from the Kerr–Newman–dS black hole still needs to be given. This is because scalar particles compose the main part of the spectrum of the radiation of a black hole. As a typical charged rotating black hole in de Sitter spacetime, we need to give an exact derivation of its Hawking radiation based on the tunneling of scalar particles.

For a rotating black hole, the tunneling particles will be dragged by the rotation of the black hole when they are tunneling across the horizon, thus, in the action of a tunneling particle, we need also to include the part that comes from the motion on the rotating degree of freedom, like that in [17, 18] in studying the tunnelings from Kerr and Kerr–Newman black holes. However, the calculation of the action of a tunneling particle will be rather complicated in the case of the Kerr–Newman–dS black hole if we use the method of [17, 18]. We find that the motion on the rotating degree of freedom of a tunneling particle can be eliminated in a reference system that is co-rotating with the black hole horizon, then, if we pass to the reference system that is co-rotating with the black hole horizon, we need not consider the action that comes from the motion on the rotating degree of freedom of a tunneling particle, and thus the calculation of the action of a tunneling particle can be greatly simplified. Using such a method, we can derive the Hawking temperature of a general rotating black hole from the tunneling approach clearly. Then, to use the general result to the special case of the Kerr–Newman–dS black hole, the Hawking temperature of the Kerr–Newman–dS black hole can be reproduced from the tunneling approach, which is independent from other more conventional methods for the calculation of the Hawking temperatures of black holes.

On the other hand, a Kerr–Newman–dS black hole is a black hole which has a cosmological horizon as well as the event horizon. Although in [13, 14, 21, 23], the tunnelings of a scalar particle from the cosmological horizons of the de Sitter space, Schwarzschild–de Sitter black hole and Reissner–Nordström–de Sitter black hole have been studied by several authors, it is still necessary for us to give a clear manipulation of the tunneling from the cosmological horizon of the Kerr–Newman–dS black hole. We can ascribe the de Sitter space, Schwarzschild–de Sitter black hole and Reissner–Nordström–de Sitter black hole to the special cases of the Kerr–Newman–dS black hole.

There are some differences in the calculation between the tunneling of a particle from the cosmological horizon and the event horizon. For an observer in the spacetime of the Kerr–Newman–dS black hole, because he (or she) is lying inside the cosmological horizon, the tunneling particles from the cosmological horizon which the observer has received are the ingoing particles, so their geodesics are the ingoing null geodesics, instead of the outgoing null geodesics for the radiation particles from the event horizon. Corresponding to this fact, we find that in order to obtain the correct result of the Hawking temperature on the cosmological horizon of the Kerr–Newman–dS black hole from the tunneling approach, the Painlevé time coordinate transformation in the tunneling of the cosmological horizon is different from that in the tunneling of the event horizon. To use the method of rotating coordinate transformation, we can reproduce the Hawking temperature on the cosmological horizon of the Kerr–Newman–dS black hole from the tunneling approach independent from other more conventional methods.

The content of this paper is organized as follows. In section 2, we study the Hawking radiation of the Kerr–Newman–dS black hole on its event horizon from the tunneling approach. We reproduce the Hawking temperature of the Kerr–Newman–dS black hole on its event horizon. In section 3, we study the Hawking radiation of the Kerr–Newman–dS black hole on
its cosmological horizon from the tunneling approach. We reproduce the Hawking temperature of the Kerr–Newman–dS black hole on its cosmological horizon. In section 4, we discuss some of the problems.

2. Hawking radiation on the black hole horizon

The metric of the Kerr–Newman–dS black hole written in the Boyer–Lindquist coordinates is given by

\[ ds^2 = -\frac{\Delta_r}{\Sigma} \left( dt - a \frac{\sin^2 \theta}{\Xi} \, d\phi \right)^2 + \frac{1}{\Sigma} \left( \frac{dr^2}{\Delta_r} + \frac{\Delta_y \sin^2 \theta}{\Xi} \left( r^2 + a^2 \right) \, d\phi \right)^2 + \frac{1}{\Delta_r} \, d\Omega^2, \]

(1)

where

\[ \Sigma = r^2 + a^2 \cos^2 \theta, \quad \Xi = 1 + \frac{1}{3} \Lambda a^2, \]
\[ \Delta_y = 1 + \frac{1}{3} \Lambda a^2 \cos^2 \theta, \quad \Delta_r = (r^2 + a^2)(1 - \frac{1}{3} \Lambda r^2) - 2Mr + Q^2. \]

(2)

where \( \Lambda \) is the cosmological constant, \( \Lambda > 0 \). The metric (1) represents the spacetime of a Kerr–Newman black hole embedded in a background de Sitter universe. The horizons of the metric are determined by the equation

\[ \Delta_r = 0 \]
\[ = \frac{1}{3} \Lambda \left( r^4 - \left( \frac{3}{\Lambda} - a^2 \right) r^2 + \frac{6M}{\Lambda} r - \frac{3}{\Lambda} (a^2 + Q^2) \right) \]
\[ = -\frac{1}{3} \Lambda (r - r_+)(r - r_-)(r - r_+)(r - r_-) = 0. \]

(3)

Under certain conditions for the parameters \( M, Q, a \) and \( \Lambda \) (for example, see [31]), equation \( \Delta_r = 0 \) has four roots represented by \( r_+, r_+, r_- \) and \( r_- \), where \( r_+, r_+ \) and \( r_- \) are positive and \( r_- \) is negative. The negative root \( r_- \) has no physical meaning. The largest positive root \( r_+ \) represents the cosmological horizon \( r_c \), the smallest positive root \( r_- \) represents the inner black hole horizon and the other positive root \( r_+ \) represents the black hole event horizon \( r_h \). In this section, we first study the Hawking radiation of the Kerr–Newman–dS black hole on its event horizon. There are two different methods appeared in the literature for the calculation of the classical action of a tunneling particle, the null-geodesic method of Parikh and Wilczek [10] and the Hamilton–Jacobi equation method of Angheben et al [19]. In this paper, for convenience, we use the null-geodesic method to calculate the action of a tunneling particle.

For an observer located at infinity, the radiation of a black hole is spherically symmetric, no matter whether the black hole is rotating or not. Thus, to lowest order, one can treat the radiation of a black hole as s-waves. The tunneling rate of a radiated particle can be calculated through the WKB approximation. For a tunneling particle in terms of an s-wave, the tunneling rate is obtained as [10]

\[ \Gamma = \Gamma_0 \exp(-2\text{Im} \mathcal{I}), \]

(4)

where \( \mathcal{I} \) is the classical action of the tunneling particle and \( \Gamma_0 \) is a normalization factor. To consider a spherically symmetric black hole first, in four-dimensional spacetime, the metric is given by

\[ ds^2 = -A(r) \, dr^2 + B(r) \, dr^2 + r^2 \, d\Omega^2. \]

(5)
As obtained in [10], the imaginary part of the action of a tunneling particle is expressed as
\[ \text{Im } I = \text{Im } \int_{r_h(M)}^{r_h(M-E)} p_r \, dr \]
where \( r_h \) is the radius of the event horizon, \( M \) is the total mass of the black hole before the particle is emitted, \( M - E \) is the total mass of the black hole after the particle is emitted and \( E \) is the energy of the tunneling particle. Here, a pair of virtual particles is created in the background fields near the horizon of the black hole through quantum field effects. The positive energy particle tunnels across the horizon and propagates to infinity, the negative energy particle is trapped by the horizon. To make use of the Hamilton’s equation
\[ \dot{r} = \frac{dH}{dp_r} = \frac{d(M - \omega)}{\dot{r}} \]
formally, we can write
\[ dp_r = \frac{d(M - \omega)}{\dot{r}} \cdot \dot{r} \]
To substitute (8) into (6), we obtain
\[ \text{Im } I = \text{Im } \int_{r_h(M)}^{r_h(M-E)} \int_0^E \frac{d(M - \omega)}{\dot{r}} \, dr \, d\omega. \]
In (9), \( M \) is conservative because it is also the total energy of the black hole spacetime including radiation. Usually, the Hawking temperature of a black hole is very small, zero-mass particles possess the main part of the whole radiation. For a tunneling particle of zero-mass in terms of an s-wave, it moves in a radial null geodesic. Then, to transform the metric (5) to the Painlevé form in order to eliminate the coordinate singularity on the horizon, \( \dot{r} \) can be evaluated from \( ds^2 = 0 \) [10].

For the tunneling from a rotating black hole, we cannot use (9) directly to calculate the action of a tunneling particle. This is because, for a rotating black hole, when a particle is tunneling through the horizon, it will be dragged by the rotation of the black hole. Therefore, the tunneling particle will also take part in the motion on the \( \phi \) degree of freedom. This means that in formula (6), we need also to consider the contribution to the action that comes from the motion on the \( \phi \) degree of freedom of a tunneling particle, as we can see in [17, 18]. Meanwhile, in the null geodesic equation of a tunneling particle, we cannot set \( d\phi = 0 \), thus \( \dot{r} \) cannot be obtained from \( ds^2 = 0 \) conveniently. However, the motion on the \( \phi \) degree of freedom of a tunneling particle can be eliminated in a reference system co-rotating with the black hole horizon. First, the metric (1) can be cast in the form
\[ ds^2 = -g_{rr}(r, \theta) \, dr^2 + g_{\theta\theta}(r, \theta) \, d\theta^2 + g_{\phi\phi}(r, \theta) \, d\phi^2 - 2g_{r\phi}(r, \theta) \, dt \, d\phi \]
generally. We make a rotating coordinate transformation
\[ \phi' = \phi - \Omega_h t \quad \text{or} \quad \phi = \phi' + \Omega_h t \]
to the metric (10), where \( \Omega_h \) is the angular velocity of the event horizon of a rotating black hole which is a constant and is defined by
\[ \Omega_h = \left. \frac{\partial \phi}{\partial \phi'} \right|_{r=r_h} \]
Thus, the metric (10) is transformed to the form
\[ ds^2 = -G_{rr}(r, \theta) \, dr^2 + g_{rr}(r, \theta) \, dr^2 + g_{\theta\theta}(r, \theta) \, d\theta^2 + g_{\phi\phi}(r, \theta) \, d\phi^2 - 2G_{r\phi}(r, \theta) \, dt \, d\phi', \]
\begin{equation}
G_{tt} = g_{tt} + 2g_{t\phi} \Omega_0 - g_{\phi\phi} \Omega_0^2,
\end{equation}
\begin{equation}
g'_{t\phi} = g_{t\phi} - \Omega_0 g_{\phi\phi}.
\end{equation}

In such a co-rotating reference system, an observer located at the horizon will not see the rotation of the black hole, he (or she) will find that the angular velocity \( \Omega_0 \) of the black hole is zero. Because the tunneling of a particle takes place at the horizon, the tunneling particle will not be dragged by the rotation of the black hole to observe from such a co-rotating reference system. This makes \( d\phi' = 0 \) for a tunneling particle, i.e. a tunneling particle will have no motion on the \( \phi' \) degree of freedom. Thus, the action related with the motion on the rotating degree of freedom of a tunneling particle does not need to be considered in such a co-rotating reference system, and we can use (6) directly to calculate the action of a tunneling particle. Meanwhile, in obtaining the expression of \( \dot{r} \) from the null-geodesic equation, we can set \( d\phi' = 0 \) (in (19) in the following). Because of (12), \( g'_{t\phi} \) satisfies
\begin{equation}
g'_{t\phi}|_{r = r_h} = 0.
\end{equation}

On the other hand, from the equation \( \xi^\mu \xi_\mu|_{r = r_h} = 0 \) satisfied by the Killing vector \( \xi^\mu = \frac{\partial}{\partial t} + \Omega_0 \frac{\partial}{\partial \phi} \), we have
\begin{equation}
G_{tt}|_{r = r_h} = 0.
\end{equation}

The horizon’s radius of the metric (13) is determined by \( g_{rr}|_{r = r_h} = 1 \), which is the same equation for the horizon’s radius of the metric (10); therefore, the horizon’s radius of a rotating black hole is not changed under the coordinate transformation (11). Because of (17), the horizon’s radius for the metric (13) now is also determined by (17).

For the metric (13) of a rotating black hole, \( g_{rr} \) is singular on the horizon. In order to obtain the action of a tunneling particle, we need to eliminate such a coordinate singularity first. This can be realized through the Painlevé coordinate transformation [10]. We use \( T \) to represent the Painlevé time coordinate and make the coordinate transformation
\begin{equation}
\frac{d\sigma}{G_{tt}(r, \theta_0)} = \frac{G_{tt}(r, \theta_0)}{G_{tt}(r, \theta_0)} \frac{g_{rr}(r, \theta_0) - 1}{G_{tt}(r, \theta_0)} \frac{d\sigma}{\sqrt{g_{rr}(r, \theta_0)}} + \frac{d\phi'}{G_{tt}(r, \theta_0)} \frac{d\phi'}{d\sigma} + d\phi^2.
\end{equation}

\begin{equation}
d\sigma^2 = -G_{tt}(r, \theta_0) dT^2 + 2 \sqrt{G_{tt}(r, \theta_0)} \sqrt{g_{rr}(r, \theta_0)} - 1 dT + d\phi^2.
\end{equation}

The horizon’s radius for the metric (19) is determined by \( G_{tt}|_{r = r_h} = 0 \), thus the horizon’s radius for the metric (13) is not changed after the coordinate transformation (18). As mentioned above, for a tunneling particle in the co-rotating reference system, it satisfies \( d\phi' = 0 \). Thus we have
\begin{equation}
d\sigma^2 = -G_{tt}(r, \theta_0) dT^2 + 2 \sqrt{G_{tt}(r, \theta_0)} \sqrt{g_{rr}(r, \theta_0)} - 1 dT + d\phi^2.
\end{equation}
To suppose that the mass of the tunneling particle is zero, then its motion is determined by the null-geodesic equation \( ds^2 = 0 \). To solve this equation, we obtain

\[
\dot{r} = \sqrt{G_{tt}(r, \theta_0) \cdot g_{rr}(r, \theta_0)} \left( \pm 1 - \frac{1}{g_{rr}(r, \theta_0)} \right).
\]  

(21)

Because \( G_{tt}(r_h, \theta) = 0 \), \( g_{rr}^{-1}(r = r_h) = 0 \), \( r_h \) is a simple zero point of \( G_{tt} \) and \( g_{rr}^{-1} \). \( G_{tt} \cdot g_{rr} \) is regular at the horizon. The two solutions indicated by the plus and minus signs in (21) correspond to outgoing and ingoing radial null geodesics, respectively. For an outgoing tunneling particle, \( \dot{r} \) is positive, we have

\[
\dot{r} = \sqrt{G_{tt}(r, \theta_0) \cdot g_{rr}(r, \theta_0)} \left( 1 - \frac{1}{g_{rr}(r, \theta_0)} \right).
\]  

(22)

To substitute (22) into (9), we obtain

\[
\text{Im} \mathcal{I} = \text{Im} \int_{0}^{E} \int_{r_h(M-E)}^{r_h(M)} \frac{dr}{\sqrt{G_{tt}(r, \theta_0) \cdot g_{rr}(r, \theta_0) \left( 1 - \frac{1}{g_{rr}(r, \theta_0)} \right)}} \ d\omega.
\]  

(23)

To multiply \( 1 + \sqrt{1 - \frac{1}{g_{rr}(r, \theta_0)}} \) in the numerator and denominator of the integrand at the same time, we obtain

\[
\text{Im} \mathcal{I} = \text{Im} \int_{0}^{E} \int_{r_h(M-E)}^{r_h(M)} \frac{1 + \sqrt{1 - \frac{1}{g_{rr}(r, \theta_0)}}}{G_{tt}(r, \theta_0) \cdot g_{rr}(r, \theta_0) \left( 1 - \frac{1}{g_{rr}(r, \theta_0)} \right)} \ dr \ d\omega.
\]  

(24)

For the metric of a four-dimensional rotating black hole, because \( g_{rr} \) is singular on the horizon, generally, we can write \( g_{rr} \) in the form

\[
g_{rr}(r, \theta) = \frac{C(r, \theta)}{r - r_h}.
\]  

(25)

where \( C(r, \theta) \) is a function regular on the horizon. To substitute (25) into (24), we have

\[
\text{Im} \mathcal{I} = \text{Im} \int_{0}^{E} \int_{r_h(M-E)}^{r_h(M)} \frac{1 + \sqrt{1 - \frac{r - r_h}{C(r, \theta)}}}{G_{tt}(r, \theta_0) \cdot g_{rr}(r, \theta_0) \frac{r - r_h}{C(r, \theta)}} \ dr \ d\omega.
\]  

(26)

In (26), \( r_h \) is a simple pole of the integrand. To add a small imaginary part to the variable \( r \) and to let the integral path round the pole in a semicircle, the integral of \( dr \) can be evaluated which yields

\[
\text{Im} \mathcal{I} = 2\pi \int_{0}^{E} \frac{C(r_h, \theta_0)}{\sqrt{G_{tt}(r_h, \theta_0) \cdot g_{rr}(r_h, \theta_0)}} \ d\omega.
\]  

(27)

It is reasonable to suppose that the energy \( E \) of the tunneling particle is far less than the total mass \( M \) of the black hole, i.e. \( E \ll M \), thus, in (27), the integrand can be treated as a constant which yields

\[
\text{Im} \mathcal{I} = 2\pi E \frac{C(r_h, \theta_0)}{\sqrt{G_{tt}(r_h, \theta_0)}} \cdot \sqrt{g^{rr}(r_h, \theta_0)}.
\]  

(28)

Because \( G_{tt}(r_h, \theta) = 0 \), \( g^{rr}(r_h, \theta) = 0 \), near the horizon, we can expand \( G_{tt}(r, \theta_0) \) and \( g^{rr}(r, \theta_0) \) in the form

\[
G_{tt}(r, \theta_0) = G_{tt}'(r_h, \theta_0)(r - r_h) + \cdots,
\]  

(29)

\[
g^{rr}(r, \theta_0) = g^{rr}'(r_h, \theta_0)(r - r_h) + \cdots.
\]  

(30)
where in (29) and (30), \( \ldots \) are higher-order terms of \( (r - r_h) \). From (25), we have

\[
g''''(r_h, \theta_0) = \frac{1}{C(r_h, \theta_0)}.
\]

(31)

To substitute (29)–(31) into (28), we obtain

\[
\text{Im} \mathcal{I} = \frac{2\pi E}{\sqrt{G_{tt}'(r_h, \theta_0)g''''(r_h, \theta_0)}}.
\]

(32)

To substitute (32) into (4), we obtain the tunneling rate of a massless particle from the event horizon of a rotating black hole

\[
\Gamma = \Gamma_0 \exp \left( -\frac{4\pi E}{\sqrt{G_{tt}'(r_h, \theta_0)g''''(r_h, \theta_0)}} \right).
\]

(33)

Equation (33) is a Boltzmann distribution. This means that the tunneling particles from the black hole form a thermal radiation. To compare (33) with the Boltzmann distribution

\[
\Gamma = \Gamma_0 \exp(-\beta E),
\]

(34)

where \( \beta = 1/T_H \), \( T_H \) is the supposed Hawking temperature, we obtain that the Hawking temperature of the event horizon of a four-dimensional rotating black hole is given by

\[
T_H = \sqrt{G_{tt}'(r_h, \theta_0)g''''(r_h, \theta_0)}/4\pi.
\]

(35)

Equation (35) is derived from the tunneling approach. Although in (35), the expression of the Hawking temperature involves the parameter \( \theta_0 \), we can see in the following that the explicit result for the Hawking temperature of a rotating black hole does not depend on the parameter \( \theta_0 \).

Using (35), we can calculate the Hawking temperature of the Kerr–Newman–dS black hole. We first calculate the Hawking temperature at the special angle \( \theta_0 = 0 \). In metric (1), to expand \( g''''(r, \theta_0) = 0 \) near the event horizon \( r_h = r_+ \), we obtain

\[
g''''(r, \theta_0 = 0) = \frac{-\frac{1}{2} \Lambda (r_+ - r_{++})(r_+ - r_{--})(r_+ - r_-)}{r_+^2 + a^2} (r - r_+) + \ldots,
\]

(36)

where \( \ldots \) are higher-order terms of \( (r - r_+) \). \( G_{tt} \) is defined by (14), generally, we can rewrite it in the form

\[
G_{tt} = g_{tt} + g_{t\phi} \Omega_h + (g_{t\phi} - \Omega_h g_{\phi\phi}) \Omega_h = g_{tt} + g_{t\phi} \Omega_h + g_{t\phi} \Omega_h.
\]

(37)

According to (16), \( g_{t\phi}' \) is zero on the horizon, thus the last term of (37) does not need to be considered when we expand \( G_{tt}(r, \theta) \) near the horizon. The angular velocity of the Kerr–Newman–dS black hole defined by (12) is \( \Omega_h = \frac{g_{t\phi}}{r_+^2 + a^2} \). At \( \theta_0 = 0 \), \( G_{tt}(r, \theta_0 = 0) \) can be expanded as

\[
G_{tt}(r, \theta_0 = 0) = \frac{-\frac{1}{2} \Lambda (r_+ - r_{++})(r_+ - r_{--})(r_+ - r_-)}{r_+^2 + a^2} (r - r_+) + \ldots,
\]

(38)

where \( \ldots \) are higher-order terms of \( (r - r_+) \). To compare (38) and (36) with (29) and (30), we can obtain \( G_{tt}'(r_h, \theta_0 = 0) \) and \( g''''(r_h, \theta_0 = 0) \). To substitute \( G_{tt}'(r_h, \theta_0 = 0) \) and \( g''''(r_h, \theta_0 = 0) \) into (35), we obtain, for the Kerr–Newman–dS black hole,

\[
T_H = -\frac{\Lambda}{12\pi(r_+^2 + a^2)} (r_+ - r_{++})(r_+ - r_{--})(r_+ - r_-).
\]

(39)
At an arbitrary value of $\theta_0$, through explicit calculation, $g^{rr}(r, \theta_0)$ and $G_{tt}(r, \theta_0)$ can be expanded as

\begin{align}
g^{rr}(r, \theta_0) &= -\frac{1}{3} \frac{\Lambda}{\Lambda_1} (r_+ - r_-) (r_+ - r_{-\pm}) (r - r_+) + \cdots, \\
G_{tt}(r, \theta_0) &= -\frac{1}{3} \frac{\Lambda}{\Lambda_1} (r_+ - r_{-\pm}) (r_+ - r_-) (r_+^2 + a^2 \cos^2 \theta_0) (r - r_+) + \cdots.
\end{align}

(40) (41)

To compare (41) and (40) with (29) and (30), we can obtain $G'_{tt}(r_h, \theta_0)$ and $g^{rr'}(r_h, \theta_0)$. To substitute $G'_{tt}(r_h, \theta_0)$ and $g^{rr'}(r_h, \theta_0)$ into (35), we obtain again

\begin{equation}
T_H = -\frac{\Lambda}{12\pi (r_+^2 + a^2)} (r_+ - r_{-\pm}) (r_+ - r_-) (r_+ - r_{-\pm}).
\end{equation}

(42)

From this example, we can see that the explicit result of the Hawking temperature given by (35) does not depend on the parameter $\theta_0$ in fact. In [32], the expression of (42) has been obtained through calculating the strength of the outgoing particle flux that generated at the event horizon of the Kerr–Newman–dS spacetime. In [31], another expression for the Hawking temperature on the event horizon of the Kerr–Newman–dS black hole has been obtained which is given by

\begin{equation}
T_H = -\frac{3 r_+^2 + (a^2 - l^2) r_+^2 + l^2 (a^2 + Q^2)}{4\pi l^2 r_+ (r_+^2 + a^2)},
\end{equation}

(43)

where $\Lambda = 3/l^2$. One can verify that these two expressions of $T_H$ for the Kerr–Newman–dS black hole are equivalent.

3. Hawking radiation on the cosmological horizon

A Kerr–Newman–dS black hole is a black hole which has a cosmological horizon as well as the event horizon. In [13, 14, 21, 23], the tunnelings of scalar particles from the cosmological horizons of the de Sitter space, Schwarzschild–de Sitter black hole and Reissner–Nordström–de Sitter black hole have been studied by several authors, but it is still necessary for us to give a clear manipulation of the tunneling from the cosmological horizon of the Kerr–Newman–dS black hole. Although recently in [29], the tunneling of a Dirac particle from the cosmological horizon of the Kerr–Newman–dS black hole has been studied, the tunneling of a scalar particle from the cosmological horizon of the Kerr–Newman–dS black hole still needs to be given.

As we know from [31, 33, 34], for the Kerr–Newman–dS black hole, when we discuss its thermodynamic property on the cosmological horizon, the black hole horizon should be regarded as a boundary of the spacetime, the total energy of the spacetime will change a sign and becomes $-M$. We still discuss the problem first with respect to the spherically symmetric metric (5). At the cosmological horizon, the tunneling rate of a scalar particle is still given by (4) [13, 14]. Supposing that a pair of virtual particles is created in the background fields near the cosmological horizon of the Kerr–Newman–dS black hole through quantum field effects, the positive energy particle tunnels inside the cosmological horizon and is observed by the observer. Similar to (6), the imaginary part of the action of a tunneling particle in terms of an s-wave is given by

\begin{equation}
\text{Im} \mathcal{I} = \text{Im} \int_{r_+(-M)}^{r_+(-M+\epsilon)} p_r \, dr = \text{Im} \int_{r_+(-M)}^{r_+(-M+\epsilon)} \int_0^{p_r} dp'_r \, dr',
\end{equation}

(44)

1 In [32], in the expression of the Hawking temperature $T_H$ of the Kerr–Newman–dS black hole, there is an additional factor $1/\Xi_1$ relative to the expression of (42). The difference comes from the fact that in [32], $g_{tt}$ of the metric of the Kerr–Newman–dS black hole has multiplied an additional factor $1/\Xi_1^2$ relative to metric (1) of this paper.
where $r_c$ is the radius of the cosmological horizon, $-M$ is the total energy of the spacetime before the particle is emitted, $-M + E$ is the total energy of the spacetime after the particle is emitted and $E$ is the energy of the tunneling particle. The Hamilton’s equation for the tunneling particle now is

$$\dot{r} = \frac{dH}{dp_r} = \frac{d(-M + \omega)}{dp_r},$$

(45)

thus, we can write

$$dp_r = \frac{d(-M + \omega)}{\dot{r}}$$

(46)

formally. To substitute (46) into (44), we obtain

$$\text{Im} \int_{r_c(-M)}^{r_c(-M+E)} \int_0^E \frac{d(-M + \omega)}{r} dr d\omega = -\text{Im} \int_0^E \int_{r_c(-M)}^{r_c(-M+E)} \frac{d(-M + \omega)}{r} d\omega dr.$$  \hspace{1cm} (47)

Here, because $-M$ is also the total energy of the spacetime including the radiation, we have supposed that it is conservative.

The metric of a four-dimensional asymptotically de Sitter rotating black hole can be cast in the form of (10). Like that in section 2, in order to eliminate the motion of $\phi$ degree of freedom of a tunneling particle coming from the rotation of the Kerr–Newman–dS black hole, we can choose a reference system which is locally static on the cosmological horizon. Thus, we make a rotating coordinate transformation

$$\phi' = \phi - \Omega_c t \quad \text{or} \quad \phi = \phi' + \Omega_c t$$

(48)

to the metric (10), where $\Omega_c$ is the angular velocity of the cosmological horizon which is a constant and is defined by

$$\Omega_c = \frac{g_{t\phi}}{g_{\phi\phi}} \bigg|_{r = r_c}.$$  \hspace{1cm} (49)

The coordinate transformation (48) makes the metric (10) be transformed to the form of (13), but now

$$G_{tt} = g_{tt} + 2g_{t\phi} \Omega_c - g_{\phi\phi} \Omega_c^2,$$

$$g'_{t\phi} = g_{t\phi} - \Omega_c g_{\phi\phi}.$$  \hspace{1cm} (50-51)

In such a co-rotating reference system, an observer located at the cosmological horizon will not see the rotation of the spacetime, the tunneling particle will not be dragged by the rotation of the spacetime to observe from such a co-rotating reference system. Therefore we have $d\phi' = 0$ for a tunneling particle, i.e. a tunneling particle has no motion on the $\phi'$ degree of freedom. Thus, the action that comes from the motion on the $\phi$ degree of freedom of a tunneling particle can be neglected and we can use (44) directly to calculate the action of a tunneling particle. Meanwhile, in obtaining the expression of $\dot{r}$ from the null-geodesic equation, we can set $d\phi' = 0$ (in (55) in the following). Because of (49), $g'_{t\phi}$ satisfies

$$g'_{t\phi} \big|_{r = r_c} = 0.$$  \hspace{1cm} (52)

On the other hand, we have the equation $\xi^\mu \xi_\mu \big|_{r = r_c} = 0$ for the Killing vector $\xi^\mu = \frac{\partial}{\partial t} + \Omega_c \frac{\partial}{\partial \phi}$ which gives

$$G_{tt} \big|_{r = r_c} = 0.$$  \hspace{1cm} (53)

The radius of the cosmological horizon of the metric (13) is determined by $g'_{tr} \big|_{r = r_c} = g_{rr}^{-1} \big|_{r = r_c} = 0$, which is the same equation for the radius of the cosmological horizon of the
metric (10); therefore, the radius of the cosmological horizon of the metric (10) is not changed under the coordinate transformation (48). Because of (53), the radius of the cosmological horizon of the metric (13) now is also determined by (53).

For the metric (13), \( g_{rr} \) is singular on the cosmological horizon. In order to obtain the action of a tunneling particle, we need to eliminate such a coordinate singularity first. This can be realized through the following Painlevé type coordinate transformation [10]:

\[
ds^2 = -G_{tt}(r, \theta_0) dT^2 - 2\sqrt{G_{tt}(r, \theta_0)} \sqrt{g_{rr}(r, \theta_0)} - 1 \, dT + dr^2 + g_{\phi\phi}(r, \theta_0) \, d\phi^2
\]

where \( T \) is the Painlevé time coordinate. A sign in the right-hand side of (54) has been changed to that of (18), we have set \( \theta \) to be a constant in order to make the coordinate transformation integrable. Such a manipulation is reasonable because for a tunneling particle in terms of an \( s \)-wave, it satisfies \( d\theta = 0; \) therefore, we can consider the tunneling of a particle at a constant angle \( \theta_0 \). At last we can verify that the physical result does not depend on the angle \( \theta_0 \).

Through such a coordinate transformation, at \( \theta = \theta_0 \), the metric (13) turns to the form

\[
ds^2 = -G_{tt}(r, \theta_0) dT^2 - 2\sqrt{G_{tt}(r, \theta_0)} \sqrt{g_{rr}(r, \theta_0)} - 1 \, dT + dr^2
\]

which is now regular on the cosmological horizon. The radius of the cosmological horizon of the metric (55) is determined by \( G_{tt}|_{r=r_c} = 0 \), because of (53), its value is not changed relative to that of the metric (13). As mentioned above, for a tunneling particle in the co-rotating reference system, it satisfies \( d\phi^r = 0 \). Therefore we have

\[
ds^2 = -G_{tt}(r, \theta_0) dT^2 - 2\sqrt{G_{tt}(r, \theta_0)} \sqrt{g_{rr}(r, \theta_0)} - 1 \, dT + dr^2. \tag{56}
\]

To solve this equation, we obtain

\[
\dot{r} = \sqrt{G_{tt}(r, \theta_0) \cdot g_{rr}(r, \theta_0)} \left( \pm 1 + \sqrt{1 - \frac{1}{g_{rr}(r, \theta_0)}} \right). \tag{57}
\]

Because \( G_{tt}|_{r=r_c} = 0, g_{rr}^{-1}|_{r=r_c} = 0, r_c \) is a simple zero point of \( G_{tt} \) and \( g_{rr}^{-1}, G_{tt} \cdot g_{rr} \) is regular at the cosmological horizon. The two solutions indicated by the plus and minus signs in (57) correspond to outgoing and ingoing radial null geodesics respectively. Because the observer is located inside the cosmological horizon, we should choose the ingoing radial null geodesic. For an ingoing particle, \( \dot{r} \) is negative, we have

\[
\dot{r} = \sqrt{G_{tt}(r, \theta_0) \cdot g_{rr}(r, \theta_0)} \left( -1 + \sqrt{1 - \frac{1}{g_{rr}(r, \theta_0)}} \right). \tag{58}
\]

To substitute (58) into (47), we obtain

\[
\text{Im}\, \mathcal{I} = \text{Im} \int_{\sqrt{E}}^{\sqrt{E+r_c(-M)}} \frac{dr}{\sqrt{G_{tt}(r, \theta_0) \cdot g_{rr}(r, \theta_0)(1 - \sqrt{1 - \frac{1}{g_{rr}(r, \theta_0)}})}} \, d\omega. \tag{59}
\]

To multiply \( 1 + \sqrt{1 - \frac{1}{g_{rr}(r, \theta_0)}} \) in the numerator and denominator of the integrand at the same time, we obtain

\[
\text{Im}\, \mathcal{I} = \text{Im} \int_{\sqrt{E}}^{\sqrt{E+r_c(-M)}} \frac{1 + \sqrt{1 - \frac{1}{g_{rr}(r, \theta_0)}}}{\sqrt{G_{tt}(r, \theta_0) \cdot g_{rr}(r, \theta_0)} \frac{1}{g_{rr}(r, \theta_0)}} \, dr \, d\omega. \tag{60}
\]
For the metric of a four-dimensional asymptotically de Sitter rotating black hole, because \( g_{rr} \) is singular on the cosmological horizon, generally, we can write \( g_{rr} \) in the form

\[
g_{rr}(r, \theta) = D(r, \theta) \frac{r}{r_c} - 1,
\]

(61)

where \( D(r, \theta) \) is a function regular on the cosmological horizon. To substitute (61) into (60), we have

\[
\text{Im} I = \text{Im} \int_0^E \int_{r_c(-M+E)}^{r_c(-M)} \frac{1 + \sqrt{1 - \frac{r-r_c}{D(r, \theta_0)}}}{\sqrt{G_{tt}(r, \theta_0) \cdot g_{rr}(r, \theta_0)}} \frac{r}{D(r, \theta_0)} \, dr \, d\omega.
\]

(62)

In (62), \( r_c \) is a simple pole of the integrand. To add a small imaginary part to the variable \( r \) and to let the integral path round the pole in a semicircle, the integral of \( dr \) can be evaluated which yields

\[
\text{Im} I = 2\pi E \int_0^E \frac{D(r_c, \theta_0)}{\sqrt{G_{tt}(r_c, \theta_0) \cdot g_{rr}(r_c, \theta_0)}} \, d\omega.
\]

(63)

It is reasonable to suppose that the energy \( E \) of the tunneling particle is far less than the absolute value of the total energy of the spacetime, i.e. \( E \ll M \), thus, in (63), the integrand can be treated as a constant which yields

\[
\text{Im} I = 2\pi E \frac{D(r_c, \theta_0)}{\sqrt{G_{tt}(r_c, \theta_0) \cdot g_{rr}(r_c, \theta_0)}}.
\]

(64)

Because \( G_{tt}(r_c, \theta) = 0 \), \( g_{rr}(r_c, \theta) = 0 \), near the cosmological horizon, we can expand \( G_{tt}(r, \theta_0) \) and \( g_{rr}(r, \theta_0) \) in the form

\[
G_{tt}(r, \theta_0) = G_{tt}'(r_c, \theta_0)(r - r_c) + \cdots,
\]

(65)

\[
g_{rr}(r, \theta_0) = g_{rr}'(r_c, \theta_0)(r - r_c) + \cdots.
\]

(66)

where \( \cdots \) in (65) and (66) are higher-order terms of \( (r - r_c) \). From (61), we have

\[
g_{rr}'(r_c, \theta_0) = \frac{1}{D(r_c, \theta_0)}.
\]

(67)

To substitute (65)–(67) into (64), we obtain

\[
\text{Im} I = \frac{2\pi E}{\sqrt{G_{tt}'(r_c, \theta_0)g_{rr}'(r_c, \theta_0)}}.
\]

(68)

To substitute (68) into (4), we obtain the tunneling rate of a massless particle from the cosmological horizon of a rotating asymptotically de Sitter black hole

\[
\Gamma = \Gamma_0 \exp \left( -\frac{4\pi E}{\sqrt{G_{tt}'(r_c, \theta_0)g_{rr}'(r_c, \theta_0)}} \right).
\]

(69)

Equation (69) is just a Boltzmann distribution. This means that the tunneling particles from the cosmological horizon form a thermal radiation. To compare (69) with the Boltzmann distribution

\[
\Gamma = \Gamma_0 \exp(-\beta E),
\]

(70)

where \( \beta = 1/T_c \), \( T_c \) is the supposed Hawking temperature of the cosmological horizon, we obtain that the Hawking temperature of the cosmological horizon of a rotating asymptotically de Sitter black hole is given by

\[
T_c = \frac{\sqrt{G_{tt}'(r_c, \theta_0)g_{rr}'(r_c, \theta_0)}}{4\pi}.
\]

(71)
Equation (71) is derived from the tunneling approach. Although in (71), the expression of the Hawking temperature involves the parameter $\theta_0$, the explicit result for the Hawking temperature $T_c$ of a rotating asymptotically de Sitter black hole will not depend on the parameter $\theta_0$.

Using (71), we can calculate the Hawking temperature on the cosmological horizon of the Kerr–Newman–dS black hole. We first calculate the Hawking temperature at a special value of $\theta_0$ and choose $\theta_0 = 0$. In metric (1), to expand $g''''(r, \theta_0 = 0)$ near the cosmological horizon $r_c = r_{++}$, we obtain

$$g''''(r, \theta_0 = 0) = \frac{-\frac{1}{3} \Lambda (r_{++} - r_{--})(r_{++} - r_0)(r_{++} - r_--)}{r_{++}^2 + a^2} (r - r_{++}) + \ldots, \quad (72)$$

where ‘…’ are higher-order terms of $(r - r_{++})$. $G_{tt}$ is defined by (50), generally, we can rewrite it in the form

$$G_{tt} = g_{tt} + g_{t\phi} \Omega_c + (g_{t\phi} - \Omega_c g_{\phi\phi}) \Omega_c = g_{tt} + g_{t\phi} \Omega_c + g''_{t\phi} \Omega_c.$$ \quad (73)

According to (52), $g''_{t\phi}$ is zero on the cosmological horizon, thus the last term of (73) does not need to be considered when we expand $G_{tt}(r, \theta)$ near the cosmological horizon. The angular velocity of the cosmological horizon of the Kerr–Newman–dS black hole defined by (49) is $\Omega_c = \frac{a \Omega_1}{r_{++}^2 + a^2}$. At $\theta_0 = 0$, $G_{tt}(r, \theta_0 = 0)$ can be expanded as

$$G_{tt}(r, \theta_0 = 0) = \frac{-\frac{1}{3} \Lambda (r_{++} - r_{--})(r_{++} - r_0)(r_{++} - r_--)}{r_{++}^2 + a^2} (r - r_{++}) + \ldots, \quad (74)$$

where ‘…’ are higher-order terms of $(r - r_c)$. To compare (74) and (72) with (65) and (66), we can obtain $G''_{tt}(r_c, \theta_0 = 0)$ and $g''''(r_c, \theta_0 = 0)$. To substitute $G''_{tt}(r_c, \theta_0 = 0)$ and $g''''(r_c, \theta_0 = 0)$ into (71), we obtain, for the Kerr–Newman–dS black hole, the Hawking temperature on its cosmological horizon is

$$T_c = \frac{\Lambda}{12\pi (r_{++}^2 + a^2)} (r_{++} - r_{--})(r_{++} - r_0)(r_{++} - r_--). \quad (75)$$

At an arbitrary value of $\theta_0$, through explicit calculation, $g''''(r, \theta_0)$ and $G_{tt}(r, \theta_0)$ can be expanded as

$$g''''(r, \theta_0) = \frac{-\frac{1}{3} \Lambda (r_{++} - r_{--})(r_{++} - r_0)(r_{++} - r_--)}{r_{++}^2 + a^2 \cos^2 \theta_0} (r - r_{++}) + \ldots, \quad (76)$$

$$G_{tt}(r, \theta_0) = \frac{-\frac{1}{3} \Lambda (r_{++} - r_{--})(r_{++} - r_0)(r_{++} - r_--)}{(r_{++}^2 + a^2 \cos^2 \theta_0)} (r - r_{++}) + \ldots. \quad (77)$$

To compare (77) and (76) with (65) and (66), we can obtain $G''_{tt}(r_c, \theta_0)$ and $g''''(r_c, \theta_0)$. To substitute $G''_{tt}(r_c, \theta_0)$ and $g''''(r_c, \theta_0)$ into (71), we obtain again,

$$T_c = \frac{\Lambda}{12\pi (r_{++}^2 + a^2)} (r_{++} - r_{--})(r_{++} - r_0)(r_{++} - r_--). \quad (78)$$

From this example, we can see that the explicit result of the Hawking temperature given by (71) does not depend on the parameter $\theta_0$ in fact. In [31], another expression for the Hawking temperature on the cosmological horizon of the Kerr–Newman–dS black hole has been obtained which is given by

$$T_c = \frac{3r_{++}^2 + (a^2 - l^2)r_{++} + l^2(a^2 + Q^2)}{4\pi l^2 r_{++}(r_{++}^2 + a^2)}, \quad (79)$$

where $\Lambda = 3/l^2$. One can verify that these two expressions of $T_c$ for the Kerr–Newman–dS black hole are equivalent.
4. Discussion

In this paper, we have studied the Hawking radiation of the Kerr–Newman–dS black hole on its event horizon and cosmological horizon from the tunneling approach of Parikh and Wilczek [10]. We reproduce the Hawking temperatures on the event horizon and cosmological horizon of the Kerr–Newman–dS black hole from the tunneling approach independent from other more conventional methods for the calculation of the Hawking temperatures of black holes. For a rotating black hole, the tunneling particles will be dragged by the rotation of the black hole when they are moving across the black hole horizon, thus, in the action of a tunneling particle, we need also to include the part that comes from the motion on the rotating degree of freedom, as we can see in [17, 18] in studying the tunnelings from Kerr and Kerr–Newman black holes. However, the calculation of the action of a tunneling particle will be rather complicated in the case of the Kerr–Newman–dS black hole if we use the method of [17, 18]. But we find that the calculation can be greatly simplified in a reference system that is co-rotating with the black hole horizon, for the reason that the motion on the $\phi$ degree of freedom of a tunneling particle can be eliminated in such a reference system. Using this method, we derive the Hawking temperature of the Kerr–Newman–dS black hole on its event horizon from the tunneling approach.

On the other hand, a Kerr–Newman–dS black hole is a black hole which has a cosmological horizon as well as the event horizon. Although in [13, 14, 21, 23], the tunnelings of a scalar particle from the cosmological horizons of the de Sitter space, Schwarzschild–de Sitter black hole and Reissner–Nordström–de Sitter black hole have been studied by several authors, it is still necessary for us to give a clear manipulation of the tunneling from the cosmological horizon of the Kerr–Newman–dS black hole. There are some differences in the calculation between the tunneling of a particle from the cosmological horizon and the event horizon, as it was shown in section 3. To use the method of rotating coordinate transformation, we reproduce the Hawking temperature on the cosmological horizon of the Kerr–Newman–dS black hole from the tunneling approach. The tunnelings from the cosmological horizons of the de Sitter space, Schwarzschild–de Sitter black hole and Reissner–Nordström–de Sitter black hole can be ascribed to the special cases of the Kerr–Newman–dS black hole.

Another point that needs to be pointed out here is that there are some overlaps between the approach of this paper and the manipulation of the tunneling from the Kerr–Newman black hole in [20] using the null-geodesic method. The difference lies in that in [20] the rotating coordinate transformation for the tunneling of a rotating black hole was not proposed obviously, and it has not been used to a general rotating black hole. In this paper, we have studied the tunneling of a general four-dimensional rotating black hole to use the method of rotating coordinate transformation, and then, we applied our result to the special case of the Kerr–Newman–dS black hole. An alternative method for the calculation of the action of a tunneling particle is the Hamilton–Jacobi equation approach proposed in [19]. Such a method was applied to the tunneling of some rotating black holes in [19, 20]. For the tunneling of the Kerr–Newman–dS black hole, to use the Hamilton–Jacobi equation method of [19], we can also make a rotating coordinate transformation first to simplify the calculation. The same results of (42) and (78) will be obtained at last. However, we will not give such a derivation further in this paper.

The Hawking temperatures of (42) and (78) are obtained in the reference systems co-rotating with the event horizon and cosmological horizon. However, because the Hawking temperature of a horizon is a scalar [7], it will not change to observe from the reference system that is static relative to infinity. Thus, we can deduce that for an observer static relative to infinity, the Hawking temperatures of the Kerr–Newman–dS black hole on its event horizon
and cosmological horizon are still given by (42) and (78). The discrimination is that, for a tunneling particle, or an observer, the angular velocity of the Kerr–Newman–dS black hole is zero in the co-rotating reference system, while it is not zero in the locally static reference system. To combine the first law of thermodynamics satisfied by the Kerr–Newman–dS black hole [31], we can generalize the tunneling rates of (33) and (69) to a particle with nonzero angular momentum and nonzero charge.

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