Spin transport and spin torque in a magnetic nanowire with a non-collinear magnetic order

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Abstract. A theory is developed for the treatment of magneto transport properties and spin dynamics in magnetic nanowire that contains a domain wall (DW), i.e. a region of non-collinearity. The method is applicable when the wavelength of spin up and down electrons are larger than the DW width in which case the problem is solvable exactly as a spin-resolved scattering from a strongly localized spin-dependent effective potential created by DW. We present calculations for the current-induced spin density, the distribution of the spin currents, and the spin torque acting locally on the magnetic moments within DW. We also discuss an extension based on the weak gradient (adiabatic) perturbation arising after a local transformation to a uniform magnetization.

1. Introduction
Transport properties of magnetic nanowires with domain walls (DWs) are attracting much attention \cite{1, 2, 3, 4} due to a number of observed phenomena such as the huge positive magnetoresistance (MR) \cite{5, 6}, or in some cases the small negative MR \cite{7}, and the current-induced motion of the domain walls. In stripe-shaped wires, the formation of magnetic vortices and other topological objects have been observed \cite{4}. The theoretical explanation and in particular the quantitative theory of these properties is hampered with some difficulties which provoked ongoing discussions \cite{8, 9, 10, 11}. The main problem is related to the spin torque mechanism of the dynamics of the magnetic system \cite{12, 13}.

Here we concentrate on the current-induced spin torque and its relation to the current-induced spin density and spin-current density of electrons. We consider a one-dimensional electron system in a limiting regime corresponding to a sharp domain wall, i.e. when the DW width $L$ is much smaller than the wavelength of electrons at the Fermi surface. Strictly speaking, this model is justified only in some specific cases like, for example, in the case of magnetic semiconductors with

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a small carrier density (sharp DW) or a thin ferromagnetic wire with the DW in a constriction. We discuss more realistic cases in the conclusions.

2. Model and the calculation of spin torque

The Hamiltonian of 1D electron gas in a magnetization profile \( \mathbf{M}(z) \) has the following form

\[
H = -\frac{1}{2m} \frac{d^2}{dx^2} - g \mathbf{\sigma} \cdot \mathbf{M}(x),
\]

where \( g \) is the coupling constant and we take \( \mathbf{M}(x) = [M_0 \cos \varphi(x), M_0 \sin \varphi(x), 0] \). For \( x \) far from the DW, the vector \( \mathbf{M} \) is directed along \( x \) for \( x < 0 \), and in the opposite direction for \( x > 0 \) and the function \( \varphi(x) \) changes from 0 to \( \pi \) in a small region \( 2L \) near \( x = 0 \). The domain wall lies in the \( x - y \) plane.

In the case of a sharp DW it is convenient to use the scattering states. For the case corresponding to the up and down spin polarized waves incoming to the DW from the left, we get

\[
\psi_{k\uparrow,\downarrow}(x) = \left[ (e^{ik\uparrow,\downarrow x} + r_{\uparrow,\downarrow} e^{-ik\uparrow,\downarrow x}) |\uparrow, \downarrow \rangle + r^f_{\uparrow,\downarrow} e^{-ik\uparrow,\downarrow x} |\downarrow, \uparrow \rangle \right] \theta(-x)
+ \left[ t_{\uparrow,\downarrow} e^{ik\uparrow,\downarrow x} |\uparrow, \downarrow \rangle + t^f_{\uparrow,\downarrow} e^{ik\uparrow,\downarrow x} |\downarrow, \uparrow \rangle \right] \theta(x),
\]

where \( k_{\uparrow,\downarrow} \) are the momenta of the spin up and down electrons with the same energy in the incoming waves, \( r_{\uparrow,\downarrow}, t_{\uparrow,\downarrow} \) are the reflection and transmission amplitudes, and we take the point \( x = 0 \) at the center of DW. Analogously, we can present the scattering states corresponding to the waves incoming from the right side of the wire. The transmission and reflection amplitudes can be easily calculated for the sharp DW [14].

We calculate the spin density from

\[
\mathbf{S}_{\uparrow,\downarrow}(x) = \psi_{k\uparrow,\downarrow}^\dagger(x) \mathbf{\sigma} \psi_{k\uparrow,\downarrow}(x),
\]

and the spin current density profiles as

\[
\mathbf{j}^s_{\uparrow,\downarrow}(x) = \psi_{k\uparrow,\downarrow}^\dagger(z) v_{\uparrow,\downarrow} \mathbf{\sigma} \psi_{k\uparrow,\downarrow}(x)
\]

and relate them to the separate scattering state. Here \( v_{\uparrow,\downarrow} \) is the velocity of electrons in the spin up and down waves.

Using Eqs. (2)-(4) we find that some components of \( \mathbf{S}_{\uparrow,\downarrow}(x) \) and \( \mathbf{j}^s_{\uparrow,\downarrow}(x) \) are nonzero far from the DW and contain strongly oscillating parts. Besides, the one-wave-induced spin current (4) has a jump at the domain wall. In the equilibrium, the total electron spin density and spin current density distributions result from the sum over all occupied electron states, which leads to the cancellation in the total spin current jump at the DW but the oscillating factors remain. It corresponds to the electron-induced exchange interaction acting on the magnetic moments.

To find the electric-current-induced effect on the spin density and the spin-current density, we assume that the electrons are transmitted through the DW from \( x = -\infty \) to the right, which corresponds to the current in the direction of \( -x \), and also we assume that the whole voltage drop \( \Delta \phi \) is located at the DW. For the calculation of the spin density and spin currents induced by the electric current in a magnetic nanowire, we utilize the linear response approach to weak perturbation created by the voltage drop. Then the charge and spin currents, as well as the spin density, can be presented as the weighted sums of currents for different spin-polarized scattering states at the Fermi level. Then we find that the current-induced contributions are

\[
\mathbf{S}(x) = \frac{e \Delta \phi}{2\pi} \left( \frac{\mathbf{S}_{\uparrow}(x)}{v_{\uparrow}} + \frac{\mathbf{S}_{\downarrow}(x)}{v_{\downarrow}} \right),
\]
Figure 1. The profile of current-induced spin density for different values of the spin splitting $M$.

\[ j^\alpha(x) = e\Delta\phi \left( \frac{j^\alpha_\uparrow(x)}{v^\uparrow} + \frac{j^\alpha_\downarrow(x)}{v^\downarrow} \right), \]  

where the one-wave spin- and spin current densities are calculated for the right-propagating electrons at the Fermi level. The factors $v^\uparrow, v^\downarrow$ in the denominators of (5) and (6) are related to the Fermi density of states of spin up and down electrons.

Using (2)-(6) we can calculate the current-induced spin density and the spin-current density of the 1D magnetic wire. As we see, the only problem is to calculate the scattering function but in the case of a sharp DW this problem reduces to the calculation of reflection and transmission amplitudes.

The results of our calculations for this case show that the oscillating character of both $S(x)$ and $j^\alpha(x)$. The oscillation period is determined by the inverse momentum at the Fermi level. Hence, the oscillation period of the transverse component of the spin current is much larger than the domain wall width. We recall that in 3D systems, the transverse component of the spin current decays due to an additional integration over momentum in the DW plane. In metallic ferromagnets, the decay is very fast due to the large Fermi momentum of electrons. However, there is a non-vanishing spin transfer for the transverse component in the 3D case, too.

In addition, at the DW we find $S_\uparrow(0) \neq 0$ and $S_\downarrow(0) \neq 0$, whereas $S_\parallel(0) = 0$. The functions $j^\alpha_\uparrow(x)$ and $j^\alpha_\downarrow(x)$ have some jumps at $x = 0$ but $j^\parallel_\alpha(x)$ does not have a jump (there is only a discontinuity in the derivative). As an example, the results for $S_\alpha(x)$ and $j^\alpha_\alpha(x)$ for different values of the spin splitting $M \equiv gM_0$ are shown in Figs. 1 and 2.

The jump of the spin current at $x = 0$

\[ T_{tot} = j^\uparrow(-L) - j^\downarrow(+L) \]  

is related to the total current induced spin torque acting on the DW. In the considered case of the DW in $x - y$ plane, the jumps of the spin current components $j^\alpha_\uparrow$ and $j^\alpha_\downarrow$ at $x = 0$ determine the components of the torque, pushing the DW in the of $x$ direction and also rotating the magnetic moments out of the $x - y$ plane.
Figure 2. Spin current density for different values of the spin splitting $M$.

We can find more information about the distribution of torque $T_i$ acting on a single moment $M_i$ inside the DW if we use the relation

$$ T_i = -M_0 n_i \times S(0), $$

(8)

which follows from the equation of motion of the magnetic moment, where $n_i$ is the unit vector along the moment $M_i$. Here we can neglect the variation with $x$ of the accumulated spin density $S(x)$ taking it as $S(0)$ because the variation of spin density is smooth on the length scale $L$.

Using Eqs. (2)-(5),(8) and the Büttiker-Landauer formula for the charge current $j_0$ in 1D electron system, we find that the torque acting on a single localized moment in the domain wall can be presented in the following simple form [14]

$$ T_i = \frac{j_0}{e} \left[ \eta n_i \times (n_i \times s) + \zeta n_i \times s \right]. $$

(9)

Here $s$ is the unit vector along the magnetization $M$ at $x \rightarrow -\infty$, and the coefficients $\eta$ and $\zeta$ are constant. The dependence of $\eta$ and $\zeta$ on the parameters characterizing the magnetic wire and the DW width can be calculated numerically [14].

3. Conclusions

The considered model of the magnetic nanowire is applicable to semiconducting systems with a small density of electrons. We calculated the components of accumulated spin density and spin current density determining the torque acting on a thin DW in the magnetic nanowire subject to an applied electric current. The different components induce a rotation of the magnetic moments in different directions.

In the case of metallic ferromagnets and magnetic semiconductors with a large density of electrons, the condition of sharp domain wall is not justified. In this case the problem does not reduce to the calculation of transmission and reflection amplitudes because we are interested in the variation of the electron spin density for $|x| < L$, and this variation is not small on the electron wavelength distance. This regime is called adiabatic and corresponds to a slow rotation
of the electron spin along the magnetization vector. If case of a strong adiabaticity, $k_F L \gg 1$, the reflection of electrons from the wall is exponentially small, and correspondingly, the spin torque is also very small but in the intermediate regime of $k_F L \sim 1$, it is possible to calculate the accumulated spin density profile by using a perturbation theory with a small parameter determined by the gradient of of varying magnetization [15]. For the 1D system, it leads to the same qualitative results with oscillating spin density and spin current density profiles.

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