Research Article

Subcarrier Group Assignment for MC-CDMA Wireless Networks

Tallal El Shabrawy¹ and Tho Le-Ngoc²

¹ Faculty of Information Engineering and Technology, German University in Cairo (GUC), Main Entrance, Al Tagamoa Al Khames, New Cairo, Egypt
² Department of ECE, McGill University, 3480 University, Montréal, Québec, Canada H3A 2A7

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Two interference-based subcarrier group assignment strategies in dynamic resource allocation are proposed for MC-CDMA wireless systems to achieve high throughput in a multicell environment. Least interfered group assignment (LIGA) selects for each session the subcarrier group on which the user receives the minimum interference, while best channel ratio group assignment (BCRGA) chooses the subcarrier group with the largest channel response-to-interference ratio. Both analytical framework and simulation model are developed for evaluation of throughput distribution of the proposed schemes. An iterative approach is devised to handle the complex interdependency between multicell interference profiles in the throughput analysis. Illustrative results show significant throughput improvement offered by the interference-based assignment schemes for MC-CDMA multicell wireless systems. In particular, under low loading conditions, LIGA renders the best performance. However, as the load increases BCRGA tends to offer superior performance.

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1. INTRODUCTION

Multicarrier CDMA (MC-CDMA) [1] has drawn significant interest as a major candidate for next generations of high data rate mobile networks. MC-CDMA is a multicarrier transmission technique that might be viewed as a combined OFDM/CDMA communication system [2]. Accordingly, MC-CDMA benefits from advantages of both technologies. OFDM modulation [3] offers robustness against multipath fading characterizing frequency selective channels by subdividing the wideband system bandwidth into numerous narrowband subcarriers that approximately exhibit flat fading. On the other hand, CDMA [3] has the advantage of efficient multiplexing. Multiple transmissions from different users within the same cell are possible by allocating different spreading codes.

MC-CDMA systems considered in this study spread the information symbols in the frequency domain. Each chip is transmitted over one of the subcarriers after being multiplied by the information symbol. Initial proposals for MC-CDMA systems have adopted spreading over all system subcarriers. However, grouped MC-CDMA systems (e.g., [4]) have the potential of enhancing prospective system capacity. Grouped MC-CDMA subdivides system subcarriers into a set of nonoverlapping subcarrier groups. This opens an avenue for isolating the received target signal from sources of intolerably high interference, and potentially resulting in capacity improvements. It is worth mentioning that in this paper MC-CDMA and grouped MC-CDMA are used interchangeably to indicate the same multicarrier modulation system.

Wireless channels in mobile networks are known to display significant variations across active users’ subcarriers as well as among subcarriers of the same user. In order to fully accomplish MC-CDMA aspirations for high throughput, radio resource management (RRM) must play a key role in adequately adapting to channel dynamics to guarantee efficient resources utility. RRM in MC-CDMA is governed by two main functions: subcarriers group assignment and power allocation. Group assignment depicts selection of appropriate subcarriers to support information bit streams of individual users. Power allocation is the scheme by which users share the power available at the serving base station.

Research on RRM in MC-CDMA systems has been generally limited to a single-cell environment [2, 5–7]. The RRM schemes described in such references share a common subcarrier group assignment theme. It is primarily based on allocating transmissions over subcarriers with best channel
response from the perspective of corresponding users. However, in a multicell environment, the use of only channel response information in subcarrier group assignment overlooks the severe impact of intercell interference on the quality and reliability of communications. In other words, signal-to-interference/noise ratio (SINR) of a given user assigned on a subcarrier group with adequate channel response might be restricted by the amount of interference received due to the power transmitted and large-scale path-loss of intercell base stations towards the user of interest. Such constraint is not quite as evident in a single-cell scenario, due to orthogonality of codes as well as a result of interference and signal transmissions sharing a common large-scale path-loss channel. Therefore, in this paper, interference-based assignment for MC-CDMA multicell networks is proposed. While there exists some limited work on MC-CDMA for multicell networks in the literature (e.g., [8, 9]), none has addressed the importance of interference-based assignment.

In this paper, two interference-based subcarrier group assignment strategies are introduced for multicell MC-CDMA systems. In least interfered group assignment (LIGA), users are assigned to groups experiencing minimum interference. In best channel ratio group assignment (BCRGA), the user is assigned to the subcarrier group that holds the best ratio of channel response-to-received interference. Throughput analysis of interference-based assignment in multicell networks constitutes a challenging problem. In general, analytical approaches available in the literature assume independence of power allocation profiles across base stations of the network (e.g., [10, 11]). The notion of power allocation profile independence reflects the assumption that the number of users per subcarrier group as well as their power allocation does not depend on the corresponding functionalities at other base stations of the network. Unfortunately, independence of RRM across base stations may no longer be considered the case when interference-based assignment is employed. The distribution of users across subcarrier groups as well as their transmission powers within a given cell has a significant effect on how users and power are accordingly distributed elsewhere in the network. Therefore, in this paper, an iterative framework is devised for throughput analysis in MC-CDMA multicell networks. In [12], an iterative framework has been also developed to investigate SIR-based power control in a CDMA network. The approach introduced in this paper is different from [12] in that it iterates on the basis of distribution functions rather than statistical parameters, such as the mean or variance. A simulation model is also developed for throughput evaluation, which is more effective for large networks and sophisticated power allocation schemes. Throughput distribution curves obtained from both analytical and simulation models for a simple three-cell network scenario are shown to be in an excellent agreement. This indicates that the results collected from the developed simulation platform should be considered reliable. Since the main interest of this paper is throughput distribution functions, the introduced iterative platform relies only on Monte Carlo simulations using random variables characterizing the wireless channel (i.e., large-scale path-loss and small-scale fading). Accordingly, it is considered to be superior in terms of simulation time compared to extensive simulations that have to deal with details such as calls arrival/departure, mobile speeds, correlated channel behavior, and so forth. Performance results confirm that the proposed interference-based assignment schemes significantly outperform best subcarrier assignment techniques. With no constraints on the number of users per subcarrier group, LIGA supports the best throughput performance. However, since the number of users per assignment group is limited by the spreading factor of the orthogonal codes, the least interfered group might not always be feasible for assignment. Results show that with limited number of codes, LIGA maintains superiority for low-load scenarios. BCRGA offers the most superior performance at high loads.

The rest of the paper is organized as follows: In Section 2, the system model is presented, while in Section 3, interference-based assignment schemes are proposed for MC-CDMA multicell networks. Section 4 presents both analytical framework and simulation models for throughput evaluation. Results and discussions are featured in Section 5. Finally, conclusions are presented in Section 6.

2. SYSTEM MODEL

Consider an MC-CDMA cellular network where the system bandwidth, $W$, is subdivided into $N_C$ subcarriers. Bandwidth of subcarriers is selected such that they each approximately exhibit flat fading channel characteristics (i.e., $W/N_C \leq B_C$, where $B_C$ is the coherence bandwidth). Assume for grouped MC-CDMA that each $G$ subcarriers constitute a group over which individual streams will be spread. As a result of subcarrier grouping, system bandwidth could be described in terms of a set of subcarrier groups, $C = \{C^{(1)}, C^{(2)}, \ldots, C^{(j)}, \ldots, C^{(N_G)}\}$, where $N_G = N_C/G$ is the number of subcarrier groups. Subcarriers belonging to the same group are selected such that they are $G$ subcarriers apart to guarantee independence between corresponding fading channels. For notational convenience throughout the paper, $C^{(j)}$ is used to signify the $j$th subcarrier group.

A multicell network constituted of $B = \{1, 2, \ldots, N_B\}$ base stations is considered. Subcarrier grouping defined by the set $C$ is presumed to be the same across all base stations. Each base station, $b \in B$, is effectively simultaneously supporting $K$ active data users. Each base station $b$ operates under the constraint that it has at its disposal a maximum amount of power $P_{\text{MAX}}$ to share among active sessions.

Radio resource management at each base station must conduct two primal functions for all served users—group assignment and power allocation. It is assumed for the analysis presented in this paper that each user may be assigned to only one group. LIGA and BCRGA proposed in this study are group assignment strategies and will be discussed in Section 3. Fair power allocation (FPA) and water-filling [13] (WFPA) are considered as the most popular power allocation schemes. FPA evenly distributes power across users to guarantee minimal outage probability, while WFPA allocates power with the objective of maximizing total cell-throughput [14]. Power dedicated for each user assigned to a certain group is assumed to be uniformly distributed over...
subcarriers of such group. In other words, for a user \( k \) of interest allocated with \( P_k^{(b,j)} \) by base station \( b \) over \( C^{(j)} \), the power share per subcarrier of \( C^{(j)} \) will be evenly distributed as \( \frac{P_k^{(b,j)}}{G} \). This is considered as a legitimate assumption since MC-CDMA can benefit from diversity of fading channels of subcarriers belonging to the same group. Moreover, varying power across subcarriers disrupts orthogonality of users’ signals even before being transmitted over the wireless channel (especially if channel responses are unknown).\(^1\)

Next consider the mobile user \( k \) of interest served by base station \( b \) that offers the best path-loss for such user. If it is assumed that the user has been assigned to \( C^{(j)} \), then the total signal power measured in the downlink direction at the receiver input of user \( k \) excluding thermal noise is

\[
P_k^{(b,j)} = A \sum_{c=1}^{G} \left( \frac{P_k^{(b,j)}}{G} H_{kb}^{(b,j)} S_{kb}^{(b,j)} \right) + I_k^{(b,j)},
\]

where \( A \) is the average antenna gain for the transmitted signal relative to all interferers; \( H_{xy}^{(b,j)} \) signifies the large-scale path-loss between the mobile user \( x \) and base station \( y \) given that user \( x \) is being served by base station \( z \) and assigned to \( C^{(j)} \); \( S_{xy}^{(b,j)} \) depicts the small-scale fading power for the \( c \)th subcarrier belonging to \( C^{(j)} \) defined between the mobile user \( x \) and base station \( y \) given that user \( x \) is being served by base station \( z \); \( I_k^{(b,j)} \) is the total interference measured by user \( k \) served by base station \( b \) over \( C^{(j)} \), expressed as

\[
I_k^{(b,j)} = E_k^{(b,j)} + \sum_{b' \neq b} \sum_{j=1}^{G} \sum_{c=1}^{G} \left( \frac{P_k^{(b',j)}}{G} H_{kb'}^{(b,j)} S_{kb'}^{(b,j)} \right),
\]

where \( E_k^{(b,j)} \) depicts the intracell interference from users assigned within base station \( b \) to the same group as user \( k \); \( P_k^{(b,j)} \) is the total power transmitted by base station \( y \) over \( C^{(j)} \). In a multicell environment, it is commonly assumed for intracell interference to be negligible when compared to intercell interference [14]. Furthermore, for a large number of interferers, small-scale fading variations over interfering paths are presumed to inflict minimal effects on SINR performance. Under such circumstances, the average large-scale interference is employed to represent the interference component in the SINR equation [15]. Therefore, the equation for interference affecting user \( k \) of interest reduces to

\[
I_k^{(b,j)} \approx \sum_{b' \neq b} \sum_{j=1}^{G} \sum_{c=1}^{G} \left( \frac{P_k^{(b',j)}}{G} H_{kb'}^{(b,j)} S_{kb'}^{(b,j)} \right).
\]

\(^1\) In CDMA, two codes (with the same chip levels, e.g., either 1 or -1) are orthogonal if their cross-correlation is zero. When the chip levels of two orthogonal CDMA codes are independently scaled by different power coefficients, their cross-correlation computation can result a nonzero value, that is, they are no longer orthogonal.

where \( \Omega_k^{(b,i)} = GA H_{kb}^{(b,j)} S_{kb}^{(b,j)} \) is the normalized signal-to-interference ratio (SIR) corresponding to a unit of power allocated to user \( k \) and \( \xi_{kb}^{(b,j)} = \frac{1}{G} \sum_{c=1}^{G} S_{kb}^{(b,j)} \) is the corresponding effective small-scale fading power between the mobile user \( k \) and base station \( b \) given that user \( k \) is being served by base station \( b \) over \( C^{(j)} \). Evidently, \( \xi_{kb}^{(b,j)} \) assumes that maximal ratio combining (MRC) is used at the receiver. Thermal noise is assumed to be negligible as compared to intercell interference in a multicell network. Such an assumption is convenient for the benefit of simplifying the analytical model. The impact of such approximation on throughput distribution results is discussed in Section 5 where thermal noise is incorporated in the developed simulation platform.

The achieved throughput in bits/s/Hz for user \( k \) served by base station \( b \) over \( C^{(j)} \), \( R_k^{(b,j)} \) is

\[
R_k^{(b,j)} = \log_2 \left( 1 + \alpha P_k^{(b,j)} \Omega_k^{(b,j)} \right),
\]

where \( \alpha (0 \leq \alpha \leq 1) \) represents the performance gap\(^2\) of the coding/modulation scheme in use, including both its theoretical performance and degradation due to practical implementation, with respect to the Shannon limit [14]. In practice, an adaptive coding/modulation strategy is used, where a set of coding/modulation schemes is predetermined and a particular scheme is dynamically selected from this set based on the instantaneous SINR level. Accordingly, achievable throughputs \( K_k^{(b,i)} \) are potentially drawn from a finite set \( \text{MCR} = \{ \text{MCR}_m \}, m \in \{1, 2, \ldots, N_{\text{MCR}}\} \). Each index \( m \) characterizes the achievable modulation/coding rate (MCR) for the corresponding modulation configuration measured in the number of information bits per symbol. However, for convenience of analysis in this paper, a continuous throughput distribution is assumed. Given that \( R_k^{(b,j)} \) depicts the number of users assigned on \( C^{(j)} \) by base station \( b \), the total cell-throughput is

\[
R^{(b,j)} = \sum_{j=1}^{G} \sum_{k=1}^{K} R_k^{(b,j)}.
\]

### 3. GROUP ASSIGNMENT STRATEGIES

In this section, group assignment strategies considered within this study are discussed. A brief review of simple schemes consistent with approaches adopted in single-cell analysis is presented, that is, random group assignment (RGA) [17] and best subcarrier group assignment (BSGA) [2, 5–7]. This will be followed by introducing the proposed interference-based strategies, that is, LIGA and BCRGA.

#### 3.1. Group assignment without interference consideration

**RGA**

In RGA, active user \( k \) is assigned to subcarrier group \( C^{(j)} \) in a random manner, that is, \( j = \text{rand}(1 : N_C) \), where “rand”

\(^2\) In illustrative numerical results, we assume \( \alpha = -1.5/\ln(5\text{BER}_k^{(b,j)}) \) for M-QAM (as suggested in [16]) and the threshold bit-error rate \( \text{BER}_k^{(b,j)} = 10^{-7} \) (for reliable communications) in both analysis and simulation.
is a random generator of integers from one to the number of groups \(N_G\) of the system.

**BSGA**

In BSGA, user \(k\) is assigned to subcarrier group \(C^{(j)}\) if it offers the best small-scale fading channel response, that is, 
\[
C^{(j)} = \max_{C^0} \min \{ \hat{b} \} / H_k^{(j)}(b).
\]
Note that in RGA and BSGA, the selection of \(C^{(j)}\) for user \(k\) is independent of assignments across the network as well as within the same cell.

### 3.2. Interference-based group assignment

**LIGA**

In a multicell environment, intercell interference signifies significant contribution on attained throughput performance. Subcarriers with the best small-scale fading channels no longer have the potential to support the highest transmission rates since they might coincide with intolerable interference power generated from other cells of the network. Consequently, it is provisioned in this paper that LIGA has the potential to outperform BSGA that has been popular for single-cell scenarios. In LIGA, active user \(k\) is assigned to subcarrier group \(C^{(j)}\) such that \(C^{(j)} = \min \{ \hat{b} \} / H_k^{(j)}(\hat{b})\).

**BCRGA**

BCRGA is considered as a composite group assignment scheme that is based on LIGA and BSGA. The notion “channel ratio” in BCRGA indicates that the metric used for group selection is based on the ratio of small-scale fading channel-to-interference ratio received on a particular group. Accordingly, \(C^{(j)}\) in BCRGA is selected such that it supports the best channel ratio, that is, \(C^{(j)} = \max_{C^0} \{ \hat{b} \} / H_k^{(j)}(\hat{b})\).

### 4. THROUGHPUT ANALYSIS

Throughput depicts a measure of achievable transmission rates for information streams conveyed over the wireless medium with high reliability. In the dynamic environment of cellular networks, attained throughputs for mobile users and correspondingly aggregate cell-throughput constitute random variables that are heavily dependent on instantaneous (large-scale and small-scale) channel states as well as interference profiles from intercell base stations. As a result, RRM performance might be best characterized in terms of the throughput distribution. In the following subsections, we proceed to present the analytical approach proposed for throughput analysis in a multicell MC-CDMA network using interference-based assignment. The difficulty of such type of analysis is inherent in the interdependency of users’ distribution and interference profiles across cells of the network.

#### 4.1. Analytical model

From (3), the probability density function \(f^{(k)}(r)\) characterizing attainable throughput for an arbitrary user \(k\) assigned by base station \(b\) over group \(C^{(j)}\) could be calculated from the conditional distribution \(f^{(k)}(r)\) (using transformation of random variables) and averaging over the distribution of \(f^{(k)}(r)\), that is,
\[
f^{(k)}(r) = \int_{0}^{\infty} f^{(k)}(r) f^{(k)}(r) \, dp,
\]
\[
f^{(k)}(r) = \frac{f^{(k)}(r)}{|\partial r / \partial \omega|} \left| \omega = (2^{-1}) \alpha_p \right|
\]
\[
|\partial r / \partial \omega| = \frac{1}{ln2} \left( \frac{1}{\alpha_p} + \omega \right)^{-1}.
\]

Consequently, the probability density function of the aggregate cell-throughput, \(f^{(k)}(r)\), could be evaluated by applying the convolution of \(K\) identically independently distributed random variables characterized each by \(f^{(k)}(r)\). For FPA, all \(K\) users served within a given cell \(b\) are allocated an equal share of available power. Therefore, \(f^{(k)}(r)\) resolves to
\[
f^{(k)}(r) = \frac{f^{(k)}(r)}{|\partial r / \partial \omega|} \left| \omega = K(2^{1-1}) \alpha_p \right|
\]
\[
|\partial r / \partial \omega| = \frac{1}{ln2} \left( \frac{1}{\alpha_p} + \omega \right)^{-1}.
\]

From (5), it is clear that deriving \(f^{(k)}(r)\) is the critical step in computing user- and cell-throughput performances, \(f^{(k)}(r)\) and \(f^{(k)}(r)\), respectively. In the following, it is desirable to evaluate instead the probability density function \(f^{(k)}(r)\) for the random variable \(\Omega^{(k)}\) depicting the transformation of normalized SIR \(\Omega^{(k)}\) in the dB-scale. Given \(f^{(k)}(r)\) \(f^{(k)}(r)\) \(f^{(k)}(r)\) \(f^{(k)}(r)\) \(f^{(k)}(r)\) easily deduced by employing the relation \(f^{(k)}(r) = 10 \log_{10}(\Omega^{(k)}\).

Conducting the analysis in the dB-domain is convenient for numerical computations associated with the large-variance of lognormal random variables that appear quite frequently in different stages of the analysis. Furthermore, it renders a simple expression for \(\Omega^{(k)}\) in terms of the summation of independent random variables where \(\hat{H}^{(k)}\) \(\hat{S}^{(k)}\), and \(\hat{I}^{(k)}\) are random variables corresponding to signal large-scale path-loss, effective small-scale channel (in conjunction with MRC) and intercell interference, respectively, defined in the dB-scale such that
\[
\hat{H}^{(k)} = 10 \log_{10}(H^{(k)}\),
\]
\[
\hat{S}^{(k)} = 10 \log_{10}(S^{(k)}\),
\]
\[
\hat{I}^{(k)} = 10 \log_{10}(I^{(k)}).
\]
For LIGA, \(^{3}\) \(f_{k}^{(b,0)}(\omega)\) is expressed by the convolution

\[
f_{k}^{(b,0)}(\omega) \propto f_{k}^{(b,0)}(h) \ast f_{k}^{(b,0)}(s) \ast f_{k}^{(b,0)}(i). \tag{7}\)

The distribution functions, \(f_{k}^{(b,0)}(h), f_{k}^{(b,0)}(s),\) are independent of RRM; \(\hat{I}_{k}^{(b,0)}\) follows a normal distribution (since large-scale path-loss is usually log-normal distributed) and \(\hat{\sigma}_{k}^{(b,0)}\) is related by (6) to \(\sigma_{kb}\) that follows a gamma distribution (since \(\sigma_{kb}\) is expressed as the sum of \(G\) exponential random variables for Rayleigh fading channels). On the other hand, computation of the probability density function \(f_{k}^{(b,0)}(i)\) is not necessarily straightforward since \(\hat{I}_{k}^{(b,0)}\) heavily relies on RRM operation across the entire network.

**Derivations for \(f_{k}^{(b,0)}(i)\)**

The main burden in (7) is the evaluation of the probability density function \(f_{k}^{(b,0)}(i)\) characterizing the intercell interference affecting user \(k\) over designated group \(C^{(j)}\). In fact, the difficulty of throughput analysis for interference-based assignment schemes in general is mainly due to lack of information on the distribution function \(f_{k}^{(b,0)}(i)\). This might be explained as follows: as shown in (1), \(I_{k}^{(b,0)}\) is a linear combination of aggregate group power \(P^{(b,0)}\) for all intercell base stations \(b'\). The distribution of \(P^{(b,0)}\) for any given subcarrier group \(C^{(j)}\) is heavily dependent on RRM decisions at intercell base stations reflected in terms of \(K^{(b,0)}\) (the number of users assigned to \(C^{(j)}\) at base station \(b'\)). In turn, \(K^{(b,0)}\) essentially requires knowledge of perceived interference profiles \(I_{k}^{(b,0)}\) for all \(k'\), for all \(b'\), for all \(C^{(j)}\) where user \(k'\) is served at an intercell base station \(b'\). In other words, the interference perceived by user \(k'\) over all groups (not only \(C^{(j)}\)) affect the expected outcome of \(K^{(b,0)}\). It is also to be noted that \(I_{k}^{(b,0)}\) for all \(k'\), for all \(b'\), for all \(C^{(j)}\) are not readily available and similar to \(I_{k}^{(b,0)}\) are derivable from power allocation profiles. As a result, it is evident that RRM decisions and interference profiles are coupled across base stations which reinforces the explanation of the inherent difficulties of deriving \(f_{k}^{(b,0)}(i)\).

It is worth mentioning that throughout this derivation, the use of superscript \((j)\) or \(C^{(j)}\) is strictly reserved to signify the group nominated for assignment.

Consider a network defined by the set of base stations \(B = \{1, 2, \ldots, N_{B}\}\). Let \(b \in B\) denote the serving base station for the user \(k\) of interest and \(b' \in B'\), where \(B' = B - \{b\}\) depicts the set of \((N_{B} - 1)\) base stations constituting sources of intercell interference. Let us focus on a particular assignment group \(C^{(j)}\) selected from \(C = \{C^{(1)}, C^{(2)}, \ldots, C^{(N_{G})}\}\). Furthermore, define \(C^{(j)} \in C'\) where \(C' = C - \{C^{(j)}\}\) represents the set of other subcarrier groups. LIGA will assign session \(k\) over \(C^{(j)}\) only if \(\hat{\sigma}_{k}^{(b,0)} < \hat{\sigma}_{k}^{(b,0)}\) for all \(C^{(j)} \in C'\). If \(K^{(b,0)}\) depicts the number of sessions assigned over \(C^{(j)}\) at base station \(b'\), and given FPA, then

\[
p^{(b',j)} = K^{(b',j)} \left( \frac{P_{\text{MAX}}}{K} \right), \tag{8}\]

where \(P_{\text{MAX}}\) is the available base station power and \(K\) is the number of active sessions per cell. Next, define \(Q^{(b',j)}(n) = \Pr\{K^{(b',j)} = n\}\) as the probability of assigning \(n\) users over \(C^{(j)}\) by base station \(b'\). It can be anticipated that \(f_{k}^{(b,0)}(i)\) might be fully characterized by \(\hat{I}_{k}^{(b',0)}(h)\) for all \(b' \in B'\) and \(Q^{(b',j)}(n)\) for all \(b' \in B'\), as well as \(Q^{(b',j)}(n)\) for all \(b' \in B'\), for all \(C^{(j)} \in C'\). In other words, \(f_{k}^{(b,0)}(i)\) is dependent on the user distribution profile across all \(N_{G}\) groups (rather than only the candidate group \(C^{(j)}\)). The reason \(Q^{(b',j)}(n)\) contributes to the density function \(f_{k}^{(b,0)}(i)\) might be best described as a direct consequence of the criterion employed for group selection \((C^{(j)} = \min C^{(j)}\)). This indicates that the random variable \(\hat{\sigma}_{k}^{(b,0)}\) (displaying minimum interference across groups) is correlated with remaining interference powers \(\hat{\sigma}_{k}^{(b,0)}\) for all \(C^{(j)} \in C'\). This point will be reflected in the analysis to follow.

For illustrative purposes, we will now focus on throughput analysis in a three-cell network shown in Figure 1. System bandwidth is assumed to be subdivided into two groups, \(C = \{C^{(1)}, C^{(2)}\}\). Let \(b = 1\) denote the base station of interest and \(b' = 1, 2, 3\). Since only two groups of assignment are available, then \(Q^{(b,1)}(n) = Q^{(b,2)}(K - n)\). For LIGA with FPA, user \(k\) will be assigned on \(C^{(1)}\) if \(I_{k}^{(1,1)} < I_{k}^{(1,2)}\) or alternatively

\[
K^{(2,1)}H_{k}^{(1)} + K^{(3,1)}H_{k}^{(3)} < K^{(2,2)}H_{k}^{(2)} + K^{(3,2)}H_{k}^{(3)} \nonumber\]
\[
\Rightarrow (2K^{(2,1)} - K)H_{k}^{(1)} < (K - 2K^{(3,1)})H_{k}^{(3)} \tag{9},
\]

where each base station is supporting \(K\) active sessions. It is important to note that the superscript \((j)\) has been removed from \(H_{k}^{(j)}\) as the assignment group has not been determined at this point. Let us define \(\Theta = \{\theta\}, \theta = (n^{(2,1)}, n^{(3,1)})\) as the set of all assignment profiles at intercell base stations.

Note that assignment profiles within each intercell base station could be fully characterized by the number of
users on assignment group $C^{(1)}$ since the total number of users is $K$ and $C = \{C^{(1)}, C^{(2)}\}$. From (9), it is evident that for each profile $\theta = (n^{(2,1)}, n^{(3,1)})$, the joint sample $(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)})$ distributed according to $f_{\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)}}(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)}) = f_{\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)}}(\hat{h}_{k_2}^{(1)}) f_{\hat{h}_{k_3}^{(1)}}(\hat{h}_{k_3}^{(1)})$ (or alternatively $\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)}$) distributed according to $f_{\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)}}(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)}) = f_{\hat{h}_{k_2}^{(1)}}(\hat{h}_{k_2}^{(1)}) f_{\hat{h}_{k_3}^{(1)}}(\hat{h}_{k_3}^{(1)})$ directly associates user $k$ with $C^{(1)}$ or $C^{(2)}$ for assignment. Furthermore, $(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)})$ defines the interference perceived over the selected group of assignment $C^{(j)}$. Let us define a function $\Pi_{\theta}(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)})$ that maps a sample $(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)})$ given $\theta = (n^{(2,1)}, n^{(3,1)})$ to the corresponding interference $\hat{h}_{k}^{(1)}$ experienced by session $k$ on $C^{(j)}$ (whether $C^{(j)}$ is equal $C^{(1)}$ or $C^{(2)}$). $\Pi_{\theta}(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)})$ is expressed as

$$
\Pi_{\theta}(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)}) = 10 \log_{10} \left[ \min \left(10(\hat{h}_{k_2}^{(1)})^{10} n^{(2,1)}, 10(\hat{h}_{k_3}^{(1)})^{10} n^{(3,1)}\right) + 10(\hat{h}_{k_2}^{(1)})^{10} (K-n^{(2,1)}) + 10(\hat{h}_{k_3}^{(1)})^{10} (K-n^{(3,1)}) \right].$
$$

(10)

Let us define $f_{\hat{h}_{k}^{(1)}|\theta}(\hat{h}_{k}^{(1)})$ as the conditional probability density function of $\hat{h}_{k}^{(1)}$ perceived by user $k$ over the selected group $C^{(j)}$ given an assignment profile $\theta$. For all $(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)})$ pairs, the following numerical expression holds:

$$f_{\hat{h}_{k}^{(1)}|\theta}(\hat{h}_{k}^{(1)}) = f_{\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)}}(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)}) \cdot \left(\hat{h}_{k}^{(1)}\right)\left(\Delta \hat{h}_{k}^{(1)}\right).
$$

(11)

Therefore, by accounting for all possible samples $(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)})$, $f_{\hat{h}_{k}^{(1)}|\theta}(\hat{h}_{k}^{(1)})$ could be numerically calculated. Finally, $f_{\hat{h}_{k}^{(1)}|\theta}(\hat{h}_{k}^{(1)})$ is related to $f_{\hat{h}_{k}^{(1)}|\theta}(\hat{h}_{k}^{(1)})$ through

$$f_{\hat{h}_{k}^{(1)}|\theta}(\hat{h}_{k}^{(1)}) = \sum_{\theta} f_{\hat{h}_{k}^{(1)}|\theta}(\hat{h}_{k}^{(1)}) Pr[\theta] .
$$

(12)

Therefore, from (12), the intercell interference distribution (defined in the dB scale) $f_{\hat{h}_{k}^{(1)}|\theta}(\hat{h}_{k}^{(1)})$ over the group of assignment $C^{(j)}$ requires derivation of $Pr[\theta]$ for all $\theta \in \Theta$.

**Derivation of Pr[θ]**

Define $z_{k|\theta}^{(1)}$ as the probability of assigning an arbitrary user $k$ served by base station 1 to $C^{(1)}$ conditional on the intercell assignment profile $\theta$. Furthermore, define $Q^{(1)}(n | \theta)$ as the conditional probability of $n$ users in base station 1 sharing assignment over $C^{(1)}$. By closely inspecting (9), four different scenarios may be possibly identified.

(i) $K^{(2,1)} \leq K/2$ and $K^{(3,1)} \leq K/2$. Consequently,

$$z_{k|\theta}^{(1)} = \begin{cases} \frac{1}{2} & \text{for } \theta = \left(\frac{K}{2}, \frac{K}{2}\right), \\ 1 & \text{otherwise}, \end{cases}
$$

(13)

and $z_{k|\theta}^{(1)}$ is independent of samples $(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)})$. Accordingly,

$$Q^{(1)}(n | \theta) = \begin{cases} \left(\frac{K}{n}\right)^{K} & \text{for } \theta = \left(\frac{K}{2}, \frac{K}{2}\right), \\ \delta_{nK} & \text{otherwise}, \end{cases}
$$

(14)

where $\delta$ is the Kronecker-Delta function.

(ii) $K^{(2,1)} < K/2$ and $K^{(3,1)} > K/2$. Consequently,

$$z_{k|\theta}^{(1)} = Pr[\hat{h}_{k_2}^{(1)} > 10 \log_{10} \left(\frac{2K^{(3,1)} - K}{K - 2K^{(2,1)}}\right) + \hat{h}_{k_3}^{(1)}].
$$

(15)

Accordingly, $Q^{(1)}(n | \theta)$ follows a binomial distribution with parameters $(K, z_{k|\theta}^{(1)})$. Since $\hat{h}_{k_2}^{(1)}$ and $\hat{h}_{k_3}^{(1)}$ both follow normal distributions, then $z_{k|\theta}^{(1)}$ could be numerically estimated.

(iii) $K^{(2,1)} > K/2$ and $K^{(3,1)} < K/2$. Consequently,

$$z_{k|\theta}^{(1)} = Pr[\hat{h}_{k_2}^{(1)} > 10 \log_{10} \left(\frac{K - 2K^{(3,1)}}{2K^{(2,1)} - K}\right) + \hat{h}_{k_3}^{(1)}].
$$

(16)

Accordingly, $Q^{(1)}(n | \theta)$ follows a binomial distribution with parameters $(K, z_{k|\theta}^{(1)})$. Since $\hat{h}_{k_2}^{(1)}$ and $\hat{h}_{k_3}^{(1)}$ both follow normal distributions, then $z_{k|\theta}^{(1)}$ could be numerically estimated.

(iv) $K^{(2,1)} > K/2$ and $K^{(3,1)} > K/2$. Consequently, $z_{k|\theta}^{(1)} = 0$ and $z_{k|\theta}^{(1)}$ is independent of samples $(\hat{h}_{k_2}^{(1)}, \hat{h}_{k_3}^{(1)})$. Accordingly, $Q^{(1)}(n | \theta) = \delta_{n0}$, where $\delta$ is the Kronecker-Delta function.

Let us focus on the distribution $Q^{(1)}(n | \theta)$ characterizing the number of users $K^{(1)}$ assigned in cell 1 to group $C^{(1)}$. This could be derived by averaging of $Q^{(1)}(n | \theta)$ over all $\theta \in \Theta$ such that

$$Q^{(1)}(n) = \sum_{\theta} Q^{(1)}(n | \theta) Pr[\theta].
$$

(17)

It is important to note that since only two assignment groups are assumed, then $Q^{(1)}(n) = Q^{(2)}(K - n)$. Furthermore, since $C^{(1)}$ has been arbitrarily chosen for investigation, then it should be expected that $Q^{(1)}(n) = Q^{(2)}(n)$ and exhibits a symmetric structure around $(K + 1)/2$. Therefore, let us drop the superscript reflecting the assignment group and define $Q^{(1)}(n)$ as the probability of assigning $n$ users on any group within base station 1. The probability of the event $\theta = (n^{(2,1)}, n^{(3,1)})$ can be expressed as $Pr[\theta = (n^{(2,1)}, n^{(3,1)})] = Q^{(2)}(n^{(2,1)}) Q^{(3)}(n^{(3,1)})$. Therefore, (17)
might be rewritten as

\[ Q^{(1)}(n) = \sum_{(n^{(2,1)},n^{(3,1)})} Q^{(1)}(n \mid (n^{(2,1)},n^{(3,1)})) \times [Q^{(2,1)}(n^{(2,1)}) \times Q^{(3,1)}(n^{(3,1)})]. \]  

(18)

In a homogeneous network, it is possible to assume that the statistical properties for the number of sessions per group eventually converge to the same distribution such that

\[ Q^{(1)}(n) = Q^{(1)}(n), \]

for all \( b' \), for all \( n = 1, 2, \ldots, K \). Subsequently, (18) becomes

\[ Q^{(1)}(n) = \sum_{(n^{(2,1)},n^{(3,1)})} Q^{(1)}(n \mid (n^{(2,1)},n^{(3,1)})) \times Q^{(1)}(n^{(2,1)}) \cdot Q^{(1)}(n^{(3,1)}). \]  

(19)

In other words, the distribution of number of users per assignment group resolves to the solution of \( K + 1 \) quadratic equations.

The key idea inspiring the discussion above is to attempt to break the dependence of the operation of power allocation and LIGA across base stations by assuming that they share identical statistical properties in a homogeneous network. In order to solve the system of equations of (19), it is possible to exploit the homogeneous nature of the considered network in devising an iterative methodology. The approach relies on the postulation that all base stations employ the same criteria and procedures for group assignment and power allocation. Consequently, stochastic processes across cells of the network tend to converge to distribution functions that share common parameters.

In view of that, for a base station of interest, it is possible to set a particular stochastic configuration for all other base stations in the network. The configurations assumed influence group assignment and power allocation decisions within the base station under investigation such that the probability measures within the corresponding cell could be evaluated. In light of the homogeneous system, it is legitimate to expect all other base stations to behave similarly such that stochastic outcomes within the cell of interest from a given iteration might be redeployed at external base stations to improve probability estimates in further iterations. Iterations are repeated until stochastic parameters characterizing cells of the network (such as mean, variance, etc.) or estimated probability density functions exhibit minimum variation from iteration to the next one.

Since in the first iteration, there is no prior information or estimates with regards to the transmission power profile across surrounding cells, an initial configuration is assumed where users are uniformly distributed over available groups. Consequently, it is assumed for the first iteration that

\[ Q^{(1)}(n^{(b',1)}) = \begin{cases} 1 & n^{(b',1)} = \frac{K}{2} \\ 0 & \text{otherwise}, \end{cases} \]  

(20)

if \( K \) is even, \( Q^{(1)}(n^{(b',1)}) \) is odd,

\[ Q^{(1)}(n^{(b',1)}) = \begin{cases} 1 & n^{(b',1)} = \frac{K + 1}{2} \\ 0 & \text{otherwise}, \end{cases} \]  

\[ Q^{(1)}(n^{(b',1)}) = \begin{cases} 1 & n^{(b',1)} = \frac{K - 1}{2} \\ 0 & \text{otherwise}. \end{cases} \]

New values for \( Q^{(1)}(n) \) are computed using (18) and inserted in the next iteration as \( Q^{(b',1)}(n) \) to attain a better estimate for the distribution of number of users per group. The process is repeated until the distribution \( Q^{(1)}(n) \) shows minimal variations over successive iterations, that is, reaches a steady state. Finally, \( \Pr[\theta = (n^{(2,1)},n^{(3,1)})] = Q^{(1)}(n^{(2,1)}) \cdot Q^{(1)}(n^{(3,1)}) \) is attained and inserted in (12) to compute \( f^{(1)}(k_1) \).

4.2. Simulation model

The analysis in the previous subsection for intercell interference distribution within a simple three-cell grouped MC-CDMA network while adopting LIGA has demonstrated that an iterative framework constitutes a suitable platform for performance evaluation. However, as the number of cells and/or assignment groups increases, the analysis becomes quite cumbersome due to the inherent difficulty in tracing correlations across groups as well as base stations. As a result, it becomes of interest to develop a simulation model that is more effective in throughput evaluation of large networks and/or more sophisticated power allocation schemes, for example, WFPA.

Let us assume a base station of interest \( b \) with registered users receiving measurable interference from only \( (N_B - 1) \) surrounding cells. The base station is simultaneously supporting \( K \) active data users. Let us assume that each iteration is comprised of \( M \) samples. For each and every sample \( m, K \) mobile terminals are randomly positioned within the cell of interest. The corresponding large-scale path-loss and small-scale fading channel random variables are sampled in accordance with their corresponding characteristic distributions.

Let us define a vector \( p^{(b')}_{2M} = [p^{(b',1)}_n, p^{(b',2)}_n, \ldots, p^{(b,N_B)}_n]^T \) as the power allocation profile in base station \( b' \) over subcarrier groups considered for the \( m \)th sample. In the first iteration, \( p^{(b')}_{2M} = P_{\text{MAX}}/N_B \) for all \( m \), for all \( b' \), for all \( C^{(j)} \in C \) is assumed. In other words, interference power from intercell base stations is evenly distributed across all assignment groups.

Given interference measurements for all \( K \) users across all groups \( C^{(j)} \in C \) within the cell of interest, group assignment, and power allocation are performed in accordance with the deployed strategies (LIGA/BRCGA and FPA/WFPA). As a result of the first iteration across all samples, a matrix \( P = [p^{(b)}_n : p^{(b)}_n : \ldots : p^{(b)}_n] \) of size \( (N_B \times M) \) could be defined that stores \( p^{(b)}_n \) measurements across all groups \( C^{(j)} \in C \) computed for all \( M \) samples within the cell of interest \( b \).

In the next iteration, the sample values for elements of \( P^{(b)}_n \) for each intercell base station \( b' \) are drawn from \( P \). Therefore, in the second iteration, for each new sample \( m \) and for all base stations \( b' \), \( p^{(b')}_{2M} \) is sampled as \( p^{(b')}_{2M} = \text{randperm}(p^{(b')}_{2M}) \), where \( \text{randperm} \) is a function that randomly permutes components of the vector \( p^{(b')} \), computed during the first iteration. In other words, for each base station \( b' \), a random column \( v \) is selected in \( P \). Following that, the corresponding stored profile \( p^{(b)}_{2M} \) is randomly permuted and deployed as the group power profile.
for the second iteration. It is to be noted that by adopting such approach, it becomes possible to conserve the total power constraint across groups to be $P_{\text{MAX}}$ (i.e., maintain correlation properties of power distribution across assignment groups). Consequently, the matrix $P$ could be updated through the second iteration. The procedure is continuously repeated until statistical parameters reach a steady state.

In order to validate correctness of the developed simulation model, Figure 2 compares results for the probability distribution of number of users per group against analytical results for the three-cell network discussed in Section 4.1, where $K = 35$. The curves display very good agreement suggesting reliability of the adopted simulation approach. Figure 3 also shows proximity of analytical and simulation curves for the distribution function of intercell interference $I_k^{(1)}$. It is to be remarked that since background noise is assumed to be negligible, intercell interference (in dB-scale) falls to $-\infty$ when $\Theta = (K_{2,i}^{(1)} = 0, K_{3,i}^{(1)} = 0)$ over $C(\theta)$. It is important to stress that the main objective of Figures 2 and 3 is to validate reliability of the iterative simulation model rather than presenting quantitative results. The assumption of negligible noise was found reasonable in simulation results for larger networks presented in the next section.

5. SIMULATION RESULTS

5.1. Simulation scenario

Consider a cellular network composed of $N_B = 19$ cells with a cell radius of 400 m as shown in Figure 4. The number of system subcarriers $N_C$ is 64. With an assumed spreading factor $G$ of 8, the number of subcarrier groups $N_G$ is $N_C/G = 8$. All base stations are assumed to use an isotropic antenna with $A = 0$ dB and a maximum power $P_{\text{MAX}}$ of 33 dBm. The Hata small-to-medium city path-loss model is used, that is, $PL(\text{dB}) = 137.744 + 35.255 \log_{10}(D_{km})$ [18], where a carrier frequency of 2 GHz, base station antenna height of 30 m, mobile antenna height of 2 m are assumed. Mobile users register with the base station $b$ that offers the best (lowest) path-loss. The remaining path-losses from other base stations are considered as interfering channels. We have tested the convergence of the proposed iterative framework by considering different number of iterations as well as various initial power allocations. In all cases, results attained have converged to the same distributions and 5 iterations were deemed as a suitable value for reliable measurements. We are interested in showing the throughput cumulative curves for individual users $F_{R_k^{(i,j)}}(r)$ as well as aggregate cell-throughput $F_{R_k^{(i,j)}}(r)$ within the central base station (i.e., cell of interest) for numerous RRM (group assignment/power allocation) configurations. Furthermore, we investigate the effects of constraints on the number codes per assignment group on the performance of proposed group assignment schemes. Code limitation per assignment group is necessary to conserve orthogonality between spreading codes employed within the same cell. It is important to reiterate that even though simulations have been used for investigating throughput capacity, the results collected using the developed platform should be presumed to be accurate (as confirmed by results in Figures 2 and 3 for the three-cell network scenario). The iterative platform proposed should be expected to display superiority in simulation time requirements as it avoids the need to simulate time-correlated system behavior.

5.2. Group assignment without code limitations

We will commence by considering an MC-CDMA system that sets no restrictions on the number of users per assignment group. Results without code limitation constraints help to demonstrate the tendency of the proposed assignment schemes, and to indicate the expected limits on their achievable throughput performance of group assignment strategies for practical scenarios (where the number of assigned
codes per group is limited by the spreading factor). Figure 5 compares the cumulative probability distribution of total cell-throughput for different assignment schemes assuming \( K = 64 \) users per cell and WFPA is used. It is evident that LIGA has the best performance. This might be counterintuitive since BCRGA attempts to minimize SINR while LIGA only addresses minimizing interference. Figure 5 also reflects that the assumption of ignoring the thermal noise results in slightly optimistic results for the throughput distribution. This is most evident in the case of LIGA. Nevertheless, more important is that incorporating thermal noise in the simulation platform maintains the trend of LIGA superiority.

In order to explain superiority of LIGA over BCRGA, Figure 6 depicts the probability distribution for the number of users per assignment group for \( K = 16 \) and \( K = 64 \) users per cell. We can notice that in LIGA there exists a higher probability of groups with no assignment (i.e., \( K(b,j) = 0 \)). Consequently, this opens an avenue for mobiles in the cell of interest to avoid sources of intolerable intercell interference from base stations where they might suffer from degraded large-scale path-loss. Therefore, even though BCRGA employs a more optimal assignment criterion from the perspective of a single-cell, the resultant distribution of \( p(b,j) \) across assignment groups overshadows such gains and accordingly hinders throughput performance.

### 5.3. Group assignment with code limitations

In this subsection, we move on to consider more practical scenarios when number of users per group is limited to the spreading factor. Figure 7 shows total throughput performance comparison of group assignment schemes for a low-load scenario (e.g., \( K = 16 \)) and WFPA is used. The curves indicate that LIGA maintains its superiority due to relative flexibility on availability of desired assignment groups for each user. However, at full load \( (K = 64) \) where the number of users per group equals the spreading factor, BCRGA tends to offer the best performance as shown in Figure 8. The figure also indicates that the performance of LIGA falls even below that of BSGA under such scenario. It is to be noted that due to code limitations, the attained performance is dependent on the sequence of users’ assignment. In the results shown, we sort users in ascending order giving priority to users with better assignment profile over their desired subcarrier group. For example, in LIGA, we prioritize users of the candidate group that offers the least interference compared to other active sessions.

As mentioned above, the performance of group assignment strategies without code limitation can be viewed as an expected limit on the achievable throughput performance under practical scenarios. This is confirmed in Figure 9.
where the throughput performance for code-limited scenarios at low load (i.e., $K = 16$) approaches that of the unlimited case since, at low load, most users are assigned to their desired groups and the effects of code constraints are negligible. However, at high loads (e.g., $K = 64$), some users are not assigned to their optimal groups and, as a result, the loss in throughput performance due to code constraints becomes more significant.

In Figure 10, we show the impact of the power allocation scheme on throughput performance. It is clear the performance drop of FPA compared to WFPA. However, by inspecting throughput curves for a single user in Figure 11, WFPA is limited by unacceptable outage probabilities. Nevertheless, both power allocation schemes display similar trends with respect to group assignment where LIGA has better performance at low load while BCRGA demonstrates superior performance at high load as shown in Figure 12.

5.4. Discussions

As discussed above, results show a large decline in performance due to code limitations. In order to restore some of the lost performance, two approaches might be considered and are issues of further studies. Firstly, the sequence of group assignment might be proven to be critical in improving throughput within the limited code environment. Secondly, pseudorandom sequence codes might be used to
help loosen the limitations on number of codes per group. However, the drawback of such an approach would be loss of orthogonality and therefore an associated increase in intercell as well as intracell interference is inevitable. One advantage of the second approach is that it has the prospect to restore superiority of the LIGA strategy in high-load scenarios. The LIGA could be considered as a favorable scheme because it does not require tracking of fast fading channel responses.

6. CONCLUSIONS

MC-CDMA is a promising technology for supporting data traffic in next generations of wireless networks. RRM evolves as a critical component in order to cope with dynamics of the mobile environment as well as support anticipated resource requirements of future services. Studies in a single-cell suggest that RRM should be performed on the basis of deploying subcarrier groups with the best channel response. However, in a multicell environment, throughput performance over such channels might be severely hindered by intercell interference. Accordingly, in this paper, two interference-based schemes have been proposed for group assignment in multiscell MC-CDMA networks. LIGA assigns users to groups experiencing minimum interference, while BCRGA selects the subcarrier group that holds the best ratio of channel response-to-received interference. Results show that interference-based assignment renders significant performance gains compared in BSGA recommended in single-cell studies. Given no code limitation on the number of users per assignment group, LIGA exhibits the best throughput performance. However, in practice, the number of users per assignment group must be restricted to the spreading factor in order to conserve orthogonality among transmissions. This deprives some users from employing their candidate assignment groups and consequently system throughput performance declines. Under low-load scenarios, LIGA conserves its good performance, while in high-load scenarios BCRGA tends to offer best performance. Further studies are essential to devise strategies in order to reclaim some of the performance loss due to code limitations.

APPENDIX

SOME DERIVATIONS

A similar approach to that discussed in Section 4 could be used to find \( f_{\Omega_{k,b}}(r) \) and \( f_{\Omega_{k,b}}(r) \) in BCRGA. Once again deriving \( f_{\Omega_{k,b}}(r) \) is the critical step in computing user- and cell-throughput distributions. However, contrary to LIGA \( \hat{\Psi}_{k,b}^{(b,j)} \) and \( \hat{\Psi}_{k,b}^{(b,j)} \) are correlated. Therefore, \( \Omega_{k,b}^{(b,j)} \) is expressed as \( \hat{\Psi}_{k,b}^{(b,j)} = 10 \log_{10}(\hat{\Psi}_{k,b}^{(b,j)}) \) where \( \hat{\Psi}_{k,b}^{(b,j)} = 10 \log_{10}(\psi_{k,b}^{(b,j)}) = 10 \log_{10}(\psi_{k,b}^{(b,j)}) - 10 \log_{10}(\psi_{k,b}^{(b,j)}) \). Accordingly derivation of \( f_{\Omega_{k,b}^{(b,j)}}(r) \) resolves to computing \( f_{\psi_{k,b}^{(b,j)}}(r) \).

\[
\begin{align*}
\text{Derivations for } & f_{\psi_{k,b}^{(b,j)}}(r) \\
\text{For three-cell network shown in Figure 1 and assuming FPA, user } k & \text{ will be assigned on } C^{(1)} \text{ if } \psi_{k_{1}}^{(1,1)} < \psi_{k_{1}}^{(1,2)} \text{ or alternatively,} \\
K^{(2,1)}H_{k_{2}}^{(1)} + K^{(3,1)}H_{k_{3}}^{(1)} & \frac{\hat{\Omega}_{k_{1}}^{(1,1)}}{\hat{\Omega}_{k_{1}}^{(1,1)}} \\
< & \frac{(K-K^{(2,1)})H_{k_{2}}^{(1)} + (K-K^{(3,1)})H_{k_{3}}^{(1)}}{\hat{\Omega}_{k_{1}}^{(1,1)}} \\
\text{For } \Theta = \{\theta\}, \theta = (n^{(2,1)}, n^{(3,1)}) & \text{ the set of all assignment profiles at intercell base-stations, the joint sample } (H_{k_{2}}^{(1)}, \hat{H}_{k_{3}}^{(1)}, \hat{S}_{k_{1}}^{(1,1)}, \hat{S}_{k_{1}}^{(1,2)}) \text{ (or alternatively, } (H_{k_{2}}^{(1)}, \hat{H}_{k_{3}}^{(1)}, \hat{S}_{k_{1}}^{(1,1)}, \hat{S}_{k_{1}}^{(1,2)}) \text{) directly associates user } k \text{ with } C_{1}^{(1)} \text{ or } C_{2}^{(2)} \text{ for assignment.} \\
\text{Correspondingly, a function } & \Phi_{k_{1}}^{(1)}(H_{k_{2}}^{(1)}, \hat{H}_{k_{3}}^{(1)}, \hat{S}_{k_{1}}^{(1,1)}, \hat{S}_{k_{1}}^{(1,2)}) \text{ that maps a sample } (H_{k_{2}}^{(1)}, \hat{H}_{k_{3}}^{(1)}, \hat{S}_{k_{1}}^{(1,1)}, \hat{S}_{k_{1}}^{(1,2)}) \text{ given } \theta = (n^{(2,1)}, n^{(3,1)}) \text{ to the corresponding } \psi_{k_{1}}^{(1,1)} \text{ could be also defined. For all}
\end{align*}
\]
of two independently identically distributed gamma random variables, the following numerical expression holds:

\[
\hat{f}_{\psi}^{(1,1)}(\psi_{\theta}^{(1,1)}, \varphi_{\theta}^{(1,1)}, \gamma_{\theta}^{(1,1)}) = \frac{1}{\Delta_{k_1}} \left( \Delta_{h_{k_2}}^{(1)} \right) \cdot \left( \Delta_{h_{k_3}}^{(1)} \right)
\]

Therefore, \( f_{\psi}^{(1,1)}(\psi_{\theta}^{(1,1)}) \) could be numerically evaluated and \( f_{\psi}^{(1,1)}(\psi_{\theta}^{(1,1)}, \varphi_{\theta}^{(1,1)}, \gamma_{\theta}^{(1,1)}) \) might be still evaluated by numerically computing

\[
z_{k_1}^{(1,1)}(\theta) = \Pr \left[ \Lambda \left( \frac{(K_n^{(2,1)})}{n^{(2,1)}} + \frac{(K_n^{(3,1)})}{n^{(3,1)}} \right) \right]
\]

\[
= \int_0^\infty \Pr \left[ \Lambda \left( \frac{(K_n^{(2,1)})}{n^{(2,1)}} + \frac{(K_n^{(3,1)})}{n^{(3,1)}} \right) \right] f_1(\Pi) d\Pi,
\]

where \( \Pi = H_{k_3}^{(1)} / H_{k_3}^{(1)} \) follows a lognormal distribution with a probability density function \( f_1(\Pi) \). Accordingly, \( Q^{(1,1)}(n | \theta) \) follows a binomial distribution with parameters \( (K_n^{(1,1)}) \). Finally, \( \Pr[\theta] \) could be estimated iteratively by using (19) similar to the case of LIGA analysis.

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