**On the conjectured gravity-related collapse rate \( E_\Delta/\hbar \) of massive quantum superpositions**

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(Dated: 18 February 2022)

Roger Penrose and the author share the proposal that the spatial superposition \(|x_1⟩ + |x_2⟩\) of a massive object collapses into its localized components \(|x_1⟩\) or \(|x_2⟩\) with the characteristic time \(\hbar/E_\Delta\) where \(E_\Delta\) is the gravitational self-energy excess of the superposition versus the localized states. Underlying arguments of such radical departure from standard quantum mechanics and different derivations of the rate equation are briefly recapitulated and discussed.

I. THE COLLABORATE RATE

Microobjects, from elementary particles to giant molecules, can exist in superpositions of different locations. As to more massive objects, however, the violation of standard quantum mechanics has been conjectured from certain purely theoretical speculations. The proposal concerned here has been surviving three and a half decades in a status of pure speculation. The overlap between Penrose’s and my results is the claim that it can not be. Independently of the details of how \(\gamma_1\) and \(\gamma_2\) represent the two geometries, they have their own Killing vectors to define stationarity but they have no single common Killing vector to define stationarity of the superposition. The equivalence principle of general relativity (general covariance, in other terms) “forbids a meaningful precise labelling of individual points in a space-time. [...] there is generally no precise meaningful pointwise identification between different space-times” — says Penrose\textsuperscript{2} and adds: “all that we can expect will be some kind of approximate pointwise identification”.

The “measure of this degree of approximation” is obtained by Penrose in the Newtonian non-relativistic limit of general relativity. The time coordinate for the two geometries \(\gamma_1, \gamma_2\) can now be taken the common \(t\), the Killing vector becomes equivalent to “\(\partial/\partial t\)” while it remains ambiguous because the point-wise identification of the spatial coordinates \(x\) remains ambiguous. Penrose argues that this ambiguity corresponds to the ambiguity of free falls determined by the ambiguity of local accelerations \(g = -\nabla \Phi\) where \(\Phi\) is Newton’s potential. If so, then the plausible measure of the ambiguity (uncertainty) will be proportional to the volume integral of the squared difference of local accelerations:

\[
\Delta \propto \int |g_1(x) - g_2(x)|^2 \, d^3x. \tag{4}
\]

One expresses \(g_1\) by the mass distribution \(\rho_1\) in state \(|x_1⟩\):

\[
g_1 = -\nabla \Phi(x) = -G\nabla \int \frac{\rho_1(x')}{|x - x'|} \, d^3x', \tag{5}
\]

and similarly for \(g_2\). The ambiguity, or “uncertainty” \(\Delta\),

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divided by $G$, takes this form:

$$E_\Delta = \text{const.} \times G \int \frac{(\rho_1(x) - \rho_2(x))(\rho_1(x') - \rho_2(x'))}{|x - x'|} d^3 x d^3 x'$$

$$= \text{const.} \times (U(x_1 - x_2) - U(0)) ,$$

where $U(x_1 - x_2)$ is the Newton interaction potential between $\rho_1$ and $\rho_2$.

The bottom line of the derivation is that the energy $E_\Delta$ should be considered the energy ambiguity of the superposition and, as for usual unstable quantum states, $E_\Delta$ leads to decay at mean lifetime $\tau$ defined in [2].

Later, Penrose supports the role of acceleration $g$ in the uncertainty measure [4] by an alternative reasoning. Consider a mass $M$ in free fall and compare its wavefunctions in the Earth system and in the free-falling system, respectively. The former has a phase factor $\exp(-i M g t^2/\hbar)$. A key part of the new arguments invokes relativistic quantum field theory where the two wavefunctions would belong to different vacua, i.e., to non-equivalent Hilbert spaces. Although alternative vacua are irrelevant in the given non-relativistic situation, as noticed by Penrose, his train of thought may still hit the target, despite the overlooked triviality of the phase factor. The strange-looking phase comes simply from the nonzero time-dependent kinetic energy in the Earth based frame: $M g t^2/6$ is the integral of $M g t^2/2$.

**III. COLLAPSE FROM CONJECTURED GEOMETRIC AMBIGUITY**

My approach goes like this. Curvature of space-time geometry is sourced by the energy-momentum of matter which is quantized obviously. Hence quantum uncertainties of matter’s behavior should impose uncertainties of geometry as well. This unsharpness of geometry is thus unavoidable and depends on $\hbar$, but its details depend on the model that couples quantized matter and quantized (or perhaps classical) geometry. Independently of the model, we might nonetheless estimate the scale of uncertainties transferred from matter to gravity. The concept is this. The uncertainty of the geometry coincides with the optimum testability of geometry, using quantized material instruments. In particular, considering a network of quantized free falling test bodies to measure the geometry, one expects that there is a finite optimum of measurement precision.

The measure of this precision and the rate [2] is obtained in the Newtonian non-relativistic limit of general relativity. Let us analyse how precisely the free fall of a single test mass $M$ encodes the local acceleration $g = -\nabla \Phi$. Let the standing initial wave packet of $M$ have a certain size $\sim r$ and volume $V \sim r^3$. Under free fall, $r$ is approximately retained over a period $T \sim M r^2/\hbar$. Hence, the test mass encodes the average acceleration field $\bar{g}$ over the volume $V$ and time $T$. The acquired momentum $M \bar{g} T$, part of the total one, has an uncertainty $\hbar/r$. Hence $\bar{g}$ is encoded at the precision

$$\delta \bar{g} \sim \frac{\hbar}{M r T} .$$

To improve precision, one can not increase $M$ boundlessly because $M$’s Newton potential contributes to $\bar{g}$ and imposes a further uncertainty

$$\delta \bar{g} \sim \frac{GM}{r^2} ,$$

because of $M$’s position uncertainty $r$. The optimum value of the test mass $M$ is reached when the above two uncertainties coincide. Then the optimum precision of the measurement reads:

$$\delta \bar{g} \sim \frac{\hbar G}{VT} .$$

The factor $1/\sqrt{VT}$ suggests that the uncertainties of $g$ at different locations and different times are independent. One can determine the corresponding structure and scale of uncertainties $\delta \Phi$ of the Newton potential. They remain independent at different times but become correlated at different locations. One can inspect that [5] is satisfied if we choose the following correlation:

$$\langle \delta \Phi(x,t) \delta \Phi(x',t') \rangle = \text{const.} \times \frac{\hbar G}{|x - x'|} \delta(t - t') .$$

Thus we have estimated the due uncertainty of the Newton potential (i.e.: of the space-time geometry in the Newtonian limit). It means an uncertainty that is present even in empty space. It yields the instability and the decay of the massive superposition [3] because it dephases the two components. The time evolution of $|x_1\rangle$ contains a phase factor

$$\exp \left( -\frac{i}{\hbar} \int_0^t \delta \Phi(x,t') \rho_1(x) d^3 x d t' \right) \equiv e^{-i \chi_1(t)}$$

and $|x_2\rangle$ contains $e^{-i \chi_2}$ with $\rho_2$ in place of $\rho_1$. One forms the expectation value of the squared difference of the two phases. Using the correlation [10] yields

$$\langle (\chi_1(t) - \chi_2(t))^2 \rangle = \text{const.} \times \frac{E_\Delta t}{\hbar} ,$$

where $E_\Delta$ happens to be the expression [6]. Therefore the decay (i.e.: dephasing) rate of the superposition [3] coincides with [2].

**IV. DISCUSSION**

Starting point for both of us, as shown in the previous two sections, was the the inapplicability of standard quantization in general relativity. But each of us could implement his concept in the Newtonian limit.
only. While Penrose kept, correctly, interpreting the non-relativistic proposal in the context of general relativity, I was happy to recognize that the Newtonian limit is rich and self-contained, though one should not forget its roots and embedding in general relativity.

Our original derivations, outlined in Secs. II and III are bearing conceptual and even technical similarities as well as important disparities. The “uncertainties” responsible for the decay of massive superpositions was thought coming from the ambiguous Killing vectors (Sec II) or from the limited testability of the geometry (Sec III). Are the two concepts compatible, complementary, or hopelessly contradictory? The answer needs further studies beyond the scope of the present work. I deliberate on two related things.

A. Exact derivation

The proposed collapse rate (2) is based on dimensional considerations hence it contains a numeric constant which is left undefined. Interestingly, a semiclassical concept and its mathematical realization by Tilloy and myself confirms the heuristic proposal and makes the constant unique.

The concept looks radically different from that in Sec. III but it is related to it intrinsically. Assume that the distribution of quantized masses is measured everywhere continuously, by hypothetic detectors which are hidden from, i.e., not part of the physical word. They are yielding the classical mass distribution \( \rho \) as the outcomes. This \( \rho \) is random, like measurement outcomes in quantum systems used to be. The postulated presence of such universal and spontaneous measurements serves the coupling of quantized matter to gravity. The classical valued \( \rho \), used in \( \nabla^2 \Phi = -4\pi G \rho \), yields the classical Newton potential \( \Phi \) which is fed back to the Schrödinger equation of the quantized masses. Now, both the continuous measurement and the stochastic potential \( \Phi \) cause decoherence in the quantized material system. Weak (imprecise) measurement causes low decoherence at the price of high stochasticity of the outcome \( \rho \) yielding high decoherence by the feedback. At optimum measurement precision the total decoherence is the lowest, irrelevant for atomic systems but relevant for massive ones! Under it, the superposition (3) will decay exactly at the rate (2) where \( E_\Delta \) is defined by (4) with the unique prefactor const. \( = 1/2 \).

B. Footprint of Planck scale uncertainty?

It would be reassuring to see that the proposed non-relativistic “uncertainty”, whether the Killing vector’s (Sec II) or the geometry’s (Sec III), is the non-relativistic limit of the corresponding Planck scale uncertainty. Penrose talks about “decay after Planck-scale difference geometry measure” and even conjectures that the decay, according to the formula (2), happens when “two space-times in superposition differ from one another by an amount of order unity [...] measured in Planck units”. To estimate space-time differences, symplectic measure in linearized gravity is mentioned cursorily. However, the whole suggestion about connections to Planck scale is missing any quantitative support, be it heuristic or approximate.

Interestingly, a certain heuristic support existed even before Ref. [12] and was already noticed in them. Unruh proposed unusual (non-canonical) commutators between components of the metric tensor \( g \) and the Einstein tensor \( G \):

\[
[g^{\mu\nu}(x), G^{\rho\sigma}(x')] = \text{const.} \times \ell_{Pl}^2 \delta^{\mu\nu} \delta^{(4)}(x, x') ,
\]

with the Planck length \( \ell_{Pl} = \sqrt{\hbar G/c^3} \) (here \( x, x' \) stand for space-time coordinates). Unruh’s motivation was a heuristic non-canonical uncertainty relation between the 00 components averaged over four-volume \( V(4) \):

\[
\delta g^{00}(x) \delta G^{00}(x') \geq \frac{\ell_{Pl}^2}{V(4)} ,
\]

now a consequence of (13). The Newtonian limit of this relativistic bound leads to the limit (9) of Sec. III. We insert \( \delta g^{00} = 2c^{-2} \delta \Phi \) and \( \delta G^{00} = (1/2)c^{-2} \delta \nabla^2 \Phi \), as well as \( V(4) = cVT \). Quite remarkably, the velocity of light \( c \) cancels:

\[
\delta \Phi \delta \nabla^2 \Phi \geq \frac{\hbar G}{VT} .
\]

With a deliberate (though justifiable) symmetrization \( \delta \Phi \delta \nabla^2 \Phi \Rightarrow \delta \nabla \Phi \delta \nabla \Phi = (\delta g)^2 \), the obtained uncertainty of the acceleration field coincides with (9).

Much later, and only recently, Ref. [14] proposed a special relativistic construction of conform metric uncertainty around the Minkowski metric, whose Newtonian limit confirms the uncertainty (10) in Sec. III. Introducing a perturbative conformal factor \( 1 + h \) (at \( |h| \ll 1 \) where the uncertainties \( h \) are proportional to \( \ell_{Pl} \), one chooses the following relativistic invariant correlation:

\[
\langle h(x)h(y) \rangle = \text{const.} \times \frac{\ell_{Pl}^2}{(2\pi)^4} \int e^{-|k(x-y)|} \frac{\theta(-k^2)}{-k^2} d^4k ,
\]

ignoring the issues of regularizing \( \theta(-k^2)/k^2 \). One writes \( h \) into the form \( h = 2\Phi/c^2 \), anticipating that \( \Phi \) plays the role of the (uncertain) Newton potential for non-relativistic matter. The limit \( c \rightarrow \infty \) does converge, and does exactly to the expression (10).

V. CLOSING REMARKS

For the wider community, interested but not specialized in foundations of physics, the conjectured spontaneous collapse rate \( E_\Delta /h \) is an easy and only reference. I focussed on the heuristic derivations of this aesthetic
Hypothesis of spontaneous wave function collapse is a detailed paradigm in foundations, with various models, typically unrelated to gravity and containing phenomenological parameters. We should add that the apparent parameter-independence of our gravity related rate may be illusory since the rate diverges for point-like mass distributions. A phenomenological short-length regulator might be needed. Penrose, though, avoids this problem differently.

"Ah, you are working with Penrose, aren’t you?" — I was asked a few times. No, we used to work on our own. There used to be definite parallelisms and divergences between our struggles in the field of unknowns. Roger’s interest was a gift. After many well-defined theoretical tasks that his talent famously solved, one earned his Nobel Prize 2021, he challenged the not-so-well-defined problem. How is Schrödinger’s cat bending the spacetime? We agree that the cat should collapse after a time $\sim \hbar/E_{\Delta}$. We disagree on how this happens, smoothly, suddenly, or some other way. The hard task is: gravity-related collapse dynamics that conserves energy and momentum. Worth of research. If we are on the right track at all ...

Acknowledgments. I thank Dr. Maaneli Derakhshani for useful discussions. I acknowledge support from the Foundational Questions Institute and Fetzer Franklin Fund, a donor advised fund of Silicon Valley Community Foundation (Grant No. FQXi-RFP-CPW-2008), the National Research, Development and Innovation Office for "Frontline" Research Excellence Program (Grant No. KKP133827) and research grant (Grant. No. K12435), the John Templeton Foundation (Grant 62099).

VI. AUTHOR DECLARATIONS

A. Conflict of Interest

The authors have no conflicts to disclose.

B. Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

1R. Penrose, Shadows of the Mind: A Search for the Missing Science of Consciousness (Oxford University Press, 1994).
2R. Penrose, “On gravity’s role in quantum state reduction,” General Relativity and Gravitation 28, 581–600 (1996).
3R. Penrose, “John bell, state reduction, and quanglement,” in Quantum (Un)speakables: From Bell to Quantum Information, edited by R. Bertlmann and A. Zeilinger (Springer Science & Business Media, 2002) pp. 319–331.
4R. Penrose, “On the gravitization of quantum mechanics I: Quantum state reduction,” Foundations of Physics 44, 557–575 (2014).
5L. Diósi, A quantum-stochastic gravitation model and the reduction of the wavefunction, Ph.D. thesis, Budapest (1986), in Hungarian, https://wigner.hu/diosi/prints/thesis1986.pdf.
6L. Diósi and B. Lukács, “In favor of a newtonian quantum gravity,” Annalen der Physik 499, 488–492 (1987).
7L. Diósi, “A universal master equation for the gravitational violation of quantum mechanics,” Physics Letters A 120, 377–381 (1987).
8A. Tilloy and L. Diósi, “Principle of least decoherence for newtonian semiclassical gravity,” Phys. Rev. D 96, 104045 (2017).
9W. Unruh, “Steps towards a quantum theory of gravity,” in Quantum theory of gravity. Essays in honor of the 60th birthday of Bryce S. DeWitt, edited by S. Christensen (Adam Hilger Limited, 1984) pp. 234-242.
10L. Diósi, “Planck length challenges non-relativistic quantum mechanics of large masses,” Journal of Physics: Conference Series 1275, 012007 (2019).
11A. Bassi, K. Lochan, S. Satin, T. P. Singh, and H. Ulbricht, “Models of wave-function collapse, underlying theories, and experimental tests,” Reviews of Modern Physics 85, 471 (2013).