The O(N) Nonlinear Sigma Model in the Functional Schrödinger Picture

Dae Kwan Kim* and Chul Koo Kim†

Department of Physics and Institute for Mathematical Sciences, Yonsei University, Seoul

120-749, Korea

Abstract

We present a functional Schrödinger picture formalism of the (1+1)-
dimensional

$O(N)$ nonlinear sigma model. The energy density has been calculated
to two-loop order using the wave functional of a gaussian form, and from
which the nonperturbative mass gap of the boson fields has been obtained.
The functional Schrödinger picture approach combined with the variational
technique is shown to describe the characteristics of the ground state of the
nonlinear sigma model in a transparent way.

11.10.Ef, 11.10.Lm

Typeset using REVTEX

*e-mail:dkkim@phya.yonsei.ac.kr
†e-mail:ckkim@phya.yonsei.ac.kr
I. INTRODUCTION

The ground state of the interacting quantum fields, in general, has a complicated structure to investigate, rendering ordinary perturbation theories without much success. Especially, concerning spontaneous symmetry breaking and bound states, perturbative ground states mislead to wrong results [1]. In this respect, the functional Schrödinger picture (FSP) approach with variational approximation is expected to be a useful tool to examine the nonperturbative aspects of quantum field theory.

In contrast to the usual perturbative expansion, the Schrödinger picture approach has the merit that one does not need to specify a particular Fock basis for the ground state of the Hamiltonian under consideration. Therefore, when there is no well defined Fock vacuum, this method appears to be a convenient choice [2].

In particular, the nonlinear sigma (NLS) model [3–5] has a nontrivial vacuum structure that is composed of particle-antiparticle pairs. The ground state is not easily tractable applying the usual perturbation expansion. Therefore, it appears as a natural candidate for application of the FSP approach. The aim of this paper is to analyze the NLS model in the framework of the FSP method combined with a variational approach [6–9], which is known to go beyond the perturbative scheme in some cases. We will show that the nonperturbative phenomena like the mass gap and asymptotic freedom can be described in the Schrödinger picture in a direct way.

The NLS model in lower dimensions has attracted much attention, since it has relevance to the low energy limit of QCD as well as condensed matter systems such as antiferromagnets. The NLS model in two dimensions is classically scale invariant and asymptotically free [3–5]. According to Mermin-Wagner and Coleman [10], the continuous symmetry can not be broken in 1 + 1 dimensions; the massless Goldstone bosons tend to acquire their masses. And it was shown that there is a mapping between the NLS model and the effective long wavelength action of the quantum Heisenberg antiferromagnet [11].

In section II, we briefly introduce the NLS model and its formulation in the FSP approach.
In the section III, we first calculate the energy density to two-loop order using a gaussian-type wave functional, and then derive the mass gap for the boson fields by minimizing the energy density. We will show that in the NLS model, the massive ground state is more stable than the massless one. In the last section, a brief summary and discussion of our results will be given.

II. THE NONLINEAR SIGMA MODEL IN THE FUNCTIONAL
SCHRÖDINGER PICTURE

We start with the $O(N)$-invariant Lagrangian density

$$\mathcal{L} = \frac{1}{2\lambda} \partial_{\mu} \Phi_{a} \partial^{\mu} \Phi_{a},$$

where $N$ scalar fields $\Phi_{a}, a = 1, \ldots, N$, obey the constraint,

$$\sum_{a=1}^{N} \Phi_{a} \Phi_{a} = 1.$$  

This constraint makes the theory quite complicated, since the $N$ components of the scalar field $\Phi$ are mutually dependent on each other. The coupling constant $\lambda$ is a measure of the strength of the self interaction of the $N$ scalar fields $\Phi_{a}$, and a small value of $\lambda$ corresponds to a weak interaction. The constraint in Eq.(2) means that one degree of freedom among the $N$ variables, $\Phi_{a}$, is not a real dynamical variable. Thus, we follow the standard prescription to get rid of the $N$th field $\Phi_{N}$ through the following nonlinear transformation

$$\Phi_{a} = \frac{\phi_{a}}{(1 + \phi^{2}/4)}, \Phi_{N} = \frac{(1 - \phi^{2}/4)}{(1 + \phi^{2}/4)},$$

where $\phi^{2} \equiv \sum_{a=1}^{N-1} \phi_{a}^{2}$. Substituting these expressions into the Lagrangian, we find the equivalent Lagrangian involving the $N-1$ fields $\phi_{a}$,

$$\mathcal{L} = \frac{1}{2\lambda} \frac{\partial_{\mu} \phi_{a} \partial^{\mu} \phi_{a}}{(1 + \phi^{2}/4)^{2}}.$$  

3
Here and in the following, the sum over the repeated indices are implied; otherwise, a
comment will be given explicitly.

The NLS model in terms of $\phi_a$’s has no mass parameter, so these fields are classically
massless Goldstone bosons. However, the Goldstone bosons originate from the breakdown
of the continuous $O(N)$ symmetry, which can not occur in this case [10]. In the below, it
will be seen that the Goldstone bosons become massive through the quantum mechanical
self-interaction.

The canonical quantization procedure of the classical system require the conjugate mo-
mentum of the field $\phi_a$, which becomes

$$\pi_a = \frac{\partial L}{\partial \dot{\phi}_a} = \frac{1}{\lambda (1 + \phi^2/4)^2}. \quad (5)$$

Thus, the Hamiltonian is written in terms of the canonical variables, $\pi_a$ and $\phi_a$ as

$$H = \pi_a \dot{\phi}_a - L(\phi_a, \dot{\phi}_a) = \frac{1}{2} \lambda (1 + \phi^2/4)^2 \pi^2 + \frac{1}{2\lambda (1 + \phi^2/4)^2} \dot{\phi}^2 \quad (6)$$

where we used the conventions $\phi^2 \equiv \sum_{a=1}^{N-1} \phi^2_a$, $\pi^2 \equiv \sum_{a=1}^{N-1} \pi^2_a$ and $\phi'^2 \equiv \sum_{a=1}^{N-1} (d\phi_a/dx)^2$.

In the quantum theory, the dynamical field variable $\phi_a(x)$ and $\pi_a(x)$ become field oper-
ators. These $\pi_a$’s satisfy the canonical equal-time commutation relations with the $\phi_a$’s such that

$$[\phi_a(x), \pi_b(y)]_{x_0=y_0} = i\delta_{ab}\delta(x-y). \quad (7)$$

In the FSP representation of its quantum theory, the scalar field operator $\phi_a(x)$ and its
conjugate momentum $\pi_a(x)$ are realised as [3,8]

$$\phi_a(x) \rightarrow \phi_a(x),$$

$$\pi_a(x) \rightarrow -i\hbar \frac{\delta}{\delta \phi_a(x)}. \quad (8)$$
Now, we employ a Gaussian-type wave functional \[8\] with two variational function parameters \(G_{ab}(x, y)\) and \(\hat{\phi}_a(x)\),

\[
\Psi[\phi] = \frac{1}{\operatorname{det}(2\pi\hbar G)^{1/4}} \exp \left[ -\int_{x,y} (\phi_a(x) - \hat{\phi}_a(x)) \frac{G_{ab}^{-1}(x, y)}{4\hbar} (\phi_b(y) - \hat{\phi}_b(y)) \right],
\]

where repeated indices over \(a\) or \(b\) mean sums; otherwise, an explicit comment will be given in the below. This wave functional may be seen to obey that

\[
\langle \Psi | \Psi \rangle = 1,
\]

\[
\langle \Psi | \phi_a(x) | \Psi \rangle = \hat{\phi}_a(x),
\]

\[
\langle \Psi | \pi_a(x) | \Psi \rangle = 0.
\]

Here, \(\hat{\phi}_a(x)\) is the expectation value of the field operator \(\phi_a(x)\), and the expectation value of the momentum operator has been chosen to be zero. From the above trial wave functional, we readily obtain the following results, which are needed for calculation of the expectation value of the Hamiltonian,

\[
\langle \phi_a(x) \phi_b(y) \rangle = \hat{\phi}_a(x) \hat{\phi}_b(y) + \hbar G_{ab}(x, y),
\]

\[
\langle \pi_a(x) \pi_b(y) \rangle = \frac{\hbar}{4} G_{ab}^{-1}(x, y).
\]

### III. LARGE \(N\) CALCULATIONS

In order to obtain the expectation value of the Hamiltonian with the gaussian trial wave functional \(\langle \Psi | H | \Psi \rangle\), it is necessary to handle the field operator in \((1 + \phi^2/4)^{-2}\) properly. For that purpose, we expand it in terms of \(\phi^2/4\), which involves an infinite number of terms of even powers of \(\phi_a(x)\) fields.

Thus, we have to evaluate terms as follows,

\[
\langle \Psi | \phi^2(x) \cdots \phi^2(x) \pi^2(x) | \Psi \rangle \text{ and } \langle \Psi | \phi^2(x) \cdots \phi^2(x) \phi'^2(x) | \Psi \rangle,
\]

which cannot be calculated in closed forms, except for some special limiting cases. Due to this inability, we require an approximation scheme. Here, we will resort to the large-\(N\)
calculations \[13\]. The following example will show why the large-$N$ approximation is useful in the present problem;

\[
\langle \Psi | \phi^2(x) \phi^2(x) \pi^2(x) | \Psi \rangle = \left[ (\hat{\phi}_a \hat{\phi}_a + \hbar G_{aa})(\hat{\phi}_b \hat{\phi}_b + \hbar G_{bb}) \frac{\hbar}{4} G^{-1}_{cc} \right] \\
+ \hbar^2 \left[ \hat{\phi}_a G_{ab} \hat{\phi}_b G_{cc}^{-1} - \frac{3}{4} \hat{\phi}_a \hat{\phi}_a \delta_{bb} + \frac{\hbar}{2} G_{ab} G_{ba} G_{cc}^{-1} - \hbar G_{aa} \delta_{bb} \right] \\
- 2\hbar^2 \left[ \hat{\phi}_a \hat{\phi}_a + \hbar G_{aa} \right],
\]

(13)

where the three terms in the brackets on the right-hand side are of the order of $N^3$, $N^2$, and $N$ respectively. Thus in the large $N$ limit, only the leading terms of $N^3$ order dominate.

\[
\langle \Psi | \phi^2(x) | \Psi \rangle \overset{\text{Large } N}{\rightarrow} \langle \Psi | \phi^2(x) | \Psi \rangle \langle \Psi | \phi^2(x) | \Psi \rangle \langle \Psi | \pi^2(x) | \Psi \rangle.
\]

(14)

Therefore it is clear that in the large $N$ limit, the expectation values of the composite operators become \[13\]

\[
\langle \Psi | \phi^2(x) \cdots \phi^2(x) | \Psi \rangle = \langle \Psi | \phi^2(x) | \Psi \rangle \cdots \langle \Psi | \phi^2(x) | \Psi \rangle \langle \Psi | \pi^2(x) | \Psi \rangle.
\]

(15)

To construct an $1/N$ expansion \[13\] in a systematic way, we define a new parameter $g$ such that

\[
g \equiv \lambda N,
\]

(16)

where $g$ is fixed to be finite. Thus, we are allowed to write the Hamiltonian expectation value in the following form

\[
\langle \Psi | H | \Psi \rangle = \frac{1}{2N} \left[ 1 + \frac{\langle \phi^2 \rangle}{4} \right]^2 \langle \pi^2 \rangle + \frac{1}{2} \frac{N}{g} \left[ 1 + \frac{\langle \phi^2 \rangle}{4} \right]^{-2} \langle \phi^2 \rangle,
\]

(17)

where on the right side, the expectation has been taken with respect to the gaussian wave functional in Eq.(9). Using the results for $\langle \phi^2 \rangle$, $\langle \pi^2 \rangle$ in Eq.(11), this equation can be rewritten as
\[ \langle H \rangle = \frac{1}{2N} \left[ 1 + \frac{1}{4} (\hat{\phi}_a \hat{\phi}_a + \hbar G_{aa}) \right]^2 \frac{\hbar}{4} G^{-1}_{cc} \\
+ \frac{N}{2g} \left[ 1 + \frac{1}{4} (\hat{\phi}_a \hat{\phi}_a + \hbar G_{aa}) \right]^{-2} (\nabla \hat{\phi}_c \nabla \hat{\phi}_c - \nabla^2 \hbar G_{cc}). \] (18)

At this stage, we confine ourselves to the constant field configuration for the \( \hat{\phi}_a(x) \) fields, so that the square of the gradient of \( \hat{\phi}_a(x) \) in Eq.(18) vanishes. The scheme to expand \( \langle \Psi | H | \Psi \rangle \) in powers of \( \bar{\hbar} \) and discard terms higher than second order in \( \bar{\hbar}^2 \) will be adopted in the below. The \( \hbar \) expansion is equivalent to the loop expansion \([5,14]\). Thus, we are going to study the system to two-loop order. The energy density given in Eq.(18) can be expanded to second order in \( \bar{\hbar} \) to yield

\[ \langle H \rangle = \frac{1}{2N} f^2(\hat{\phi}) \left[ 1 + \frac{1}{2} \frac{\hbar G_{aa}}{f(\hat{\phi})} \right] \frac{\hbar}{4} G^{-1}_{cc} \\
- \frac{N}{2g} f^{-2}(\hat{\phi}) \left[ 1 - \frac{1}{2} \frac{\hbar G_{aa}}{f(\hat{\phi})} \right] (\nabla^2 \bar{\hbar} G_{cc}), \] (19)

where \( f(\hat{\phi}) = 1 + \hat{\phi}^2/4 \) has been defined.

We now vary this equation with respect to \( G_{bb}(x, x) \) to determine the parameters \( G_{bb}(x, x) \) that minimize the energy expectation value. Thus, one gets the following relation:

\[ \delta \langle H \rangle = \frac{g}{N} \frac{\hbar f(\hat{\phi})}{8} \left[ \frac{\hbar}{2} G^{-1}_{aa} - f(\hat{\phi}) \left[ 1 + \frac{\hbar}{2f(\hat{\phi})} G_{aa} \right] \right] \frac{1}{G_{bb}} \delta G_{bb} \\
+ \frac{N}{g} \frac{\hbar}{2f^2(\hat{\phi})} \left[ - \frac{\hbar}{2f(\hat{\phi})} G_{aa} \nabla^2 + \frac{\hbar}{2f(\hat{\phi})} \nabla^2 G_{aa} \right] \delta G_{bb} \\
= 0. \] (20)

Here and in the below, we will use the following notations. The repeated indices over the letter \( b \) does not indicate a sum; while, the repeated ones over \( a \) means a sum. This equation gives the relation that the variational parameter \( G_{bb} \) must satisfy,

\[ G_{bb}^{-2}(x, y) = \left[ - \frac{N^2}{g^2} \frac{4}{f^4(\hat{\phi})} \left( 1 - \frac{3}{2} \frac{\hbar G_{aa}(z, z)}{f(\hat{\phi})} \right) \right] \nabla_x^2 \delta(x - y), \] \( G_{aa}^{-1}(z, z) \) (21)

which has also been expanded in powers of \( \bar{\hbar} \) and terms higher than \( \bar{\hbar} \) have been discarded, so that only terms up to \( \bar{\hbar}^2 \) can be retained in Eq.(19). Since it is practically impossible to solve this equation directly, we separate the equation into two parts as follows:
\[ G_{bb}(x, y) = \frac{g}{N} \sum \frac{1}{2} \left[ 1 - (3\hbar/2f)G_{aa}(z, z) \right]^{1/2} \int \frac{dp}{2\pi \sqrt{p^2 + m^2}} \exp[ip(x - y)] \] \hspace{1cm} (22)

and

\[ m^2 = \frac{g^2}{N^2} \frac{\hbar f^3(\dot{\phi})}{8} G_{aa}^{-1}(x, x), \] \hspace{1cm} (23)

where the latter one turns out to define the mass parameter of the boson operator \( \phi(x) \), as will be seen in Eq.(26).

We analyze the equation using the following iterative method. First, we approximate the unknown function \( G_{bb}(x, y) \) by \( G_{bb}^{(0)}(x, y) \):

\[ G_{bb}^{(0)}(x, y) = \frac{g}{N} \sum \frac{1}{2} \left[ 1 - (3\hbar/2f)G_{aa}^{(0)}(x, x) \right]^{1/2} \int \frac{dp}{2\pi \sqrt{p^2 + m^2}} \exp[ip(x - y)]. \] \hspace{1cm} (24)

Second, to improve this approximation, we substitute the equation back into the coefficient of the right-hand side of Eq.(22). Thus, we have

\[ G_{bb}^{(1)}(x, y) = \frac{g}{N} \sum \frac{1}{2} \left[ 1 - (3\hbar/2f)G_{aa}^{(0)}(x, x) \right]^{1/2} \int \frac{dp}{2\pi \sqrt{p^2 + m^2}} \exp[ip(x - y)]. \] \hspace{1cm} (25)

Note that the multiplicative factor in front of the integral has a divergence involving a cutoff \( \Lambda \). This must be removed by a proper renormalization of the wave function \( \phi(x) \). Then, the wave function renormalized expression for \( G_{bb}(x, y) \) becomes

\[ G_{bb}^{(f)}(x, y) = \frac{g}{N} \sum \frac{1}{2} \left[ 1 - (3\hbar/2f)G_{aa}^{(0)}(x, x) \right]^{1/2} \int \frac{dp}{2\pi \sqrt{p^2 + m^2}} \exp[ip(x - y)]. \] \hspace{1cm} (26)

This form will be used in the subsequent discussions in calculating the energy expectation value. As a result of this renormalization, Eq. (21) is now rewritten

\[ G_{bb}^{\lambda^2}(x, y) = \left[ -\frac{N^2}{g^2} \sum \frac{4}{f^4(\dot{\phi})} \nabla_x^2 + \frac{1}{2f(\dot{\phi})} G_{aa}^{-1}(z, z) \right] \delta(x - y). \] \hspace{1cm} (27)

Now, let us evaluate the Hamiltonian expectation value using Eq.(23) and (26). The \( \nabla^2 G_{aa} \) in the Hamiltonian can be calculated multiplying Eq.(27) by \( G_{bb}(y, z) \) and integrating over the volume \( \int dy \):

\[ \nabla^2 G_{aa}(x, x) = -\frac{g^2}{N^2} \frac{f^4(\dot{\phi})}{4} G_{aa}^{-1}(x, x) \left[ 1 - \frac{\hbar}{2f(\dot{\phi})} G_{aa}(x, x) \right]. \] \hspace{1cm} (28)
The Hamiltonian density has no classical potential energy part $V(\phi)$, so we are allowed to set

$$f(\hat{\phi}) = 1. \quad (29)$$

Thus, the Hamiltonian to the second order in $\hbar$, that is, to the two-loop order, is given by

$$\langle \Psi | H | \Psi \rangle = \frac{g \hbar}{N} G^{-1}_{aa}(x, x)[1 + \frac{\hbar}{2} G_{aa}(x, x)] + \frac{g \hbar}{N} G^{-1}_{aa}(x, x)[1 - \frac{\hbar}{2} G_{aa}(x, x)]^2$$

$$= \frac{g \hbar}{N 4} G^{-1}_{aa}(x, x)[1 - \frac{\hbar}{4} G_{aa}(x, x)] \quad (30)$$

Using the mass defining relation Eq.(23) and two point Green’s function Eq.(26), one can evaluate $\langle H \rangle$, $\langle H \rangle_N = \frac{2m^2}{g}[1 - \frac{\hbar}{4} G_{aa}(x, x)]$

$$= \frac{2m^2}{g} \left[1 - \frac{\hbar g}{8} \int_{-\Lambda}^{\Lambda} \frac{dp}{2\pi} \frac{1}{\sqrt{p^2 + m^2}} \right]$$

$$= \frac{2m^2}{g} \left[1 - \frac{\hbar g}{16\pi} \ln\left(\frac{4\Lambda^2}{m^2}\right) \right], \quad (31)$$

where $\Lambda$ is an ultraviolet momentum cutoff. To make the energy density finite, we may renormalize the parameter of the theory. Defining the renormalized coupling constant $g_r$,

$$\frac{1}{g} = \frac{1}{g_r} + \frac{\hbar}{16\pi} \ln\left(\frac{4\Lambda^2}{\mu^2}\right), \quad (32)$$

we obtain the finite energy density with the renormalization scale $\mu$:

$$\langle H \rangle_N = \frac{2m^2}{g_r} + \frac{hm^2}{8\pi} \ln\left(\frac{m^2}{\mu^2}\right). \quad (33)$$

Note that this is the result up to $\hbar^2$, since the mass parameter $m^2$ is of $\hbar$ order. This energy expectation value has a minimum away from the origin, since it is concave upward as $m^2$ increases.

Hence, by minimizing the energy expectation value with respect to $m^2$, one can derive the mass gap,

$$\frac{\partial \langle H \rangle / N}{\partial m^2} = \frac{2}{g_r} + \frac{h}{8\pi} \ln\left(\frac{em^2}{\mu^2}\right)$$

$$= 0. \quad (34)$$
From this relation, one obtains the dynamically generated mass gap:

$$\langle m^2 \rangle = \mu^2 \exp \left[ -1 - \frac{16\pi}{\hbar g_r} \right].$$

(35)

The NLS model has no dimensional parameters; the coupling $g$ is dimensionless in two dimensions. But, we arrived at a dimensional parameter $m^2$; this phenomena is an example of dimensional transmutation. At this value of the mass gap, the energy density to two-loop order becomes

$$\frac{\langle H \rangle}{N} = \frac{2\langle m^2 \rangle}{g_r} + \frac{\hbar \langle m^2 \rangle}{8\pi} \ln \left( \frac{\langle m^2 \rangle}{\mu^2} \right)$$

$$= -\frac{\hbar}{8\pi} \mu^2 \exp \left[ -1 - \frac{16\pi}{\hbar g_r} \right].$$

(36)

Note that the negative sign of the energy density indicates that the massive ground state is more stable than the massless one in the NLS model.

We now return to the mass defining equation, Eq.(23). Differentiating it with respect to $m^2$, one finds the relation between the coupling constant $g$ and the cutoff $\Lambda$

$$\frac{1}{g} = \frac{\hbar}{8} \int \frac{dp}{2\pi} \frac{1}{\sqrt{p^2 + m^2}}$$

$$= \frac{\hbar}{16\pi} \ln \left( \frac{4\Lambda^2}{m^2} \right).$$

(37)

This relation shows that as $\Lambda$ becomes large, the coupling constant $g$ approaches zero, thus satisfying the asymptotic freedom. This equation can be also rewritten in terms of the renormalized coupling $g_r$ and the renormalization mass scale $\mu$ in Eq.(32).

IV. CONCLUSION

In this paper, the functional Schrödinger picture approach has been applied to analyze the $O(N)$ nonlinear sigma (NLS) model. We have considered the $O(N)$ NLS model in the large $N$-limit and calculated the energy expectation value to the second order in $\hbar$ (two-loop order) systematically, using the gaussian wave functional form. The Schrödinger picture approach
combined with the variational technique produced the mass gap and the asymptotic freedom of the ground state for the $O(N)$ NLS model in a straightforward manner.

The majority of the literature on the $O(N)$ NLS model adopt the Lagrangian formalism to investigate its nonperturbative phenomena, where a composite auxiliary field $\sigma(x) = \sum_a \Phi_a(x)\Phi_a(x)$ is usually introduced. However, here we discuss its nonperturbative phenomena directly without resorting to the superfluous auxiliary field.

The extension of our calculations to three dimensions will be straightforward; the difference is to write the Green’s function in Eq.(26) in corresponding three dimensional form and carry out the integral in the three dimension.

Acknowledgements

We thank Prof. K. -S. Soh and J. H. Yee for helpful discussions. This work was supported in part by the Korea Ministry of Education (BSRL-96-2425), the Korea Science and Engineering Foundation through Project 95-0701-04-01-03 and through the SRC Program of SNU-CTP. Also, D. K. Kim acknowledges the Korea Research Foundation for support through the domestic postdoctorial program.
REFERENCES

[1] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, Phys. Rev. 108, 1175 (1957); Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122, 345 (1961).

[2] R. Floreanini and R. Jackiw, Phys. Rev. D 37, 2206 (1988).

[3] E. Brézin and J. Zinn-Justin, Phys. Rev. D 14, 2615 (1976); V.A. Novikov, M. A. Shifman, A. I. Vainshtein, and V. I. Zakharov, Phys. Rep. 116, 103 (1984); A. C. Davis, J. A. Gracey, A. J. Macfarlane, and M. G. Mitchard, Nucl. Phys. B 314, 439 (1989).

[4] W. A. Bardeen, B. W. Lee, and R. E. Shrock, Phys. Rev. D 14, 985 (1976); T. del R. Gaztelurrutia and A. C. Davis, Nucl. Phys. B 347, 319 (1990).

[5] J. Zinn-Justin, Quantum Field Theory and Critical Phenomena (Oxford University Press, New York, 1989).

[6] J. M. Cornwall, R. Jackiw, and E. Tomboulis, Phys. Rev. D 10, 2428 (1974); R. Jackiw and A. Kerman, Phys. Lett. 71A, 158 (1979); A. Duncan, H. Meyer-Ortmanns, and R. Roskies, Phys. Rev. D 36, 3788 (1987).

[7] S. K. Kim, J. Yang, K. S. Soh, J. H. Yee, Phys. Rev. D 40, 2647 (1989); S. K. Kim, K. S. Soh, J. H. Yee, ibid. 41, 1345 (1990); G. Amelino-Camelia and S. -Y. Pi, ibid. 50, 2356 (1994); G. Dunne, ibid. 50, 5321 (1994).

[8] S.-Y. Pi and M. Samiullah, Phys. Rev. D 36, 3128 (1987).

[9] H. S. Noh, C. K. Kim, and K. Nahm, Phys. Lett. 204A, 162 (1995); ibid. 210A, 317 (1996); J. Korean Phys. Soc. 29, 592 (1996); J. Song, S. Hyun, and C. K. Kim, ibid. 29, 821 (1996); H. S. Noh, S. K. You, and C. K. Kim, Int. J. Mod. Phys. B11, 1829 (1997).

[10] N. D. Mermin and H. Wagner, Phys. Rev. Lett. 17, 1133 (1966); S. Coleman, Comm. Math. Phys. 31, 259 (1973).

[11] F. D. M. Haldane, Phys. Lett. 93A, 464 (1983); F. D. M. Haldane, Phys. Rev. Lett. 50,
1153 (1983).

[12] P. A. M. Dirac, *Lectures on Quantum Mechanics* (Yeshiva University, New York, 1964); M. Henneaux and C. Teitelboim, *Quantization of Gauge Systems* (Princeton University Press, New Jersey, 1992).

[13] S. Coleman, *Aspects of Symmetry* (World Scientific, Cambridge University Press, Cambridge, 1985).

[14] Y. Nambu, Phys. Lett. B 26, 626 (1966).