Signature transition in Einstein-Cartan cosmology

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Abstract

In the context of Einstein-Cartan theory of gravity, we consider a Friedmann-Lemaitre-Robertson-Walker (FLRW) cosmological model with Weyssenhoff perfect fluid. We focus attention on those classical solutions that admit a degenerate metric in which the scale factor has smooth behavior in the transition from a Euclidean to a Lorentzian domain. It is shown that the spin-spin contact interaction enables one to obtain such a signature changing solutions due to the Riemann-Cartan ($U_4$) structure of space-time.

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1 Introduction

The choice of a matter field which is coupled with the Einstein equations through its energy-momentum tensor has always a direct effect on the study of the cosmological models. Traditionally, a perfect fluid is usually used as the matter source. However, one can not deny the constantly increasing role of the scalar fields in more recent cosmological models as the matter source \cite{1}. This of course is expected since it is somewhat easier to work with scalar field. It is also possible to imagine a Universe filled with a classical spin fluid or even a massless or massive spinor fields as the matter source. Such cosmological models have rarely been studied in the literature and, when they were, it was more often than not in the form of general formalisms \cite{2}. In general then, it would be fair to say that cosmologies with spinor fields as the matter source are the least studied scenarios. In 1923 Élie Cartan introduced the relation between the intrinsic angular momentum of matter and the space-time torsion in the framework of a generalization of general relativity (GR) \cite{3}, nowadays known as Einstein-Cartan (EC) theory \cite{4}. Indeed, there are two different methods to introduce the classical spin in GR. In the first approach, spin is considered as a dynamical quantity without changing the Riemannian structure of the space-time geometry \cite{5}. The second method, which as we mentioned above was proposed by Cartan, is based on the generalization of space-time structure by assuming the metric and the non-symmetric affine connection as independent quantities. Since the first attempts of Cartan to bring spin into the curved space-time, many efforts have been made in this area and the corresponding results have been followed and developed by a number of works, see for instance \cite{6}, \cite{7} and \cite{8}. The importance of the Cartan theory becomes more clear, if one tries to incorporate the spinor field into the torsion-free general theory of relativity. In this context one should apply the Cartan theory which possesses torsion as well as curvature \cite{9}. In EC theory, torsion is not a dynamical quantity, instead it can be expressed completely in terms of the spin sources \cite{5}. Consequently, in order to study the effects of torsion in $U_4$ geometry (it is usual to denote the Riemann-Cartan space-time as $U_4$ to distinguish it from the Riemannian space-time) one may

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consider the matter fields with intrinsic angular momentum. To do this, one of the usual ways is to consider a fluid with intrinsic spin density known as the Weyssenhoff exotic perfect fluid \[10\]. As in the case of other alternative theories of gravity, it is important to seek the cosmological solutions in the EC theory of gravity, i.e., in a theory in which the spin properties of matter and their influence on the geometrical structure of space-time are considered. This is done by some authors \[11\], who have investigated the effects of torsion and spinning matter in a cosmological setting and its possible role to remove the singularities, inflationary scenarios, explain the late time accelerated expansion of the Universe and so on.

An interesting topic related to classical and quantum cosmology is that of signature change which has attracted attention since the early 1980s. Traditionally, a feature in GR is that one usually chooses a Lorentzian signature for the space-time metric before attempting to solve the Einstein’s field equations. However, the reason for doing so is not pre-determined and it is well known that the field equations do not demand this property, that is, if one ignores this requirement one may find solutions to the field equations which, when parameterized suitably, can either have Euclidean or Lorentzian signature. The notion of signature transition first appeared in the works of Hartle and Hawking \[12\] where they argued that in quantum cosmology amplitudes for gravity should be written as the sum of all compact Riemannian geometries whose boundaries are located at the signature changing hypersurface. Since then this subject has been studied at the classical and quantum cosmology level by other authors, see for example \[13\]. In what follows by a signature changing space-time we mean a manifold which contains both Euclidean and Lorentzian region. As it is shown in \[14\], in classical GR, a signature changing metric should be either degenerate or discontinuous, though Einstein’s equations implicitly assume that the metric is non-degenerate and at least continuous.

In this letter, we consider a smooth signature changing type of flat FLRW space-time in the framework of EC gravity with exotic Weyssenhof perfect fluid. For the case of a spatially flat Universe, field equations are then solved exactly for the scale factor as dynamical variable, giving rise to cosmological solutions with a degenerate metric, describing a continuous signature transition from a Euclidean domain to a Lorentzian space-time.

### 2 The model

In this section we start by briefly studying the EC gravity where the action is given by (we work in units where \(c = 1\) and consider the signature \((+,-,-,-)\) for the space-time metric)

\[
S = \int \sqrt{-\tilde{g}} d^4x \left[ -\frac{1}{16\pi G} \left( \tilde{R} - 2\Lambda \right) + \mathcal{L}_M \right],
\]

where \(\tilde{R}\) is the Ricci scalar constructed by the asymmetric connection \(\tilde{\Gamma}_{\alpha\beta}^\mu\) and \(\Lambda\) is the cosmological constant. By using of the metricity condition \[6\]

\[
\tilde{\nabla}_\alpha g_{\mu\nu} = 0,
\]

and also the definition of torsion

\[
T_{\alpha\beta}^\mu := \tilde{\Gamma}^\mu_{\alpha\beta} - \tilde{\Gamma}^\mu_{\beta\alpha},
\]

the connection \(\tilde{\Gamma}_{\alpha\beta}^\mu\) can be expressed as

\[
\tilde{\Gamma}_{\alpha\beta}^\mu = \Gamma_{\alpha\beta}^\mu + K_{\alpha\beta}^\mu,
\]

where \(\Gamma_{\alpha\beta}^\mu\) is the Levi-Civita connection (Christoffel symbol) and \(K_{\alpha\beta}^\mu\) is the contorsion tensor related to the torsion \(Q_{\alpha\beta}^\mu := [\tilde{\Gamma}_{\alpha\beta}]^\mu\) via

\[
K_{\alpha\beta}^\mu := \frac{1}{2} \left( Q_{\alpha\beta}^\mu - Q_{\alpha}^\mu \beta - Q_{\beta}^\mu \alpha \right).
\]
Also $L_M$ is essentially the Lagrangian density for matter field coupled to gravity. Our assumption is that instead of usual Big-Bang singularity in the early Universe, we have signature changing event. Therefore, we focus our attention on the early Universe epoch where the matter content of the model is of the form of fermionic matter, like quarks and leptons. The dynamical equations of motion can be obtained by performing the variation of the action with respect to the metric and contorsion [6], that is

\[
\begin{align*}
G^{\mu\nu} - \Lambda g^{\mu\nu} - \left( \nabla_\alpha + 2Q_{\alpha\beta}^\beta \right) (T^{\mu\alpha\nu} - T^{\nu\alpha\mu} + T^{\alpha\mu\nu}) &= 8\pi GT^{\mu\nu}, \\
T^{\mu\alpha\nu} &= 8\pi G\tau^{\mu\alpha}\tau^{\nu\beta},
\end{align*}
\]

where

\[
T^{\mu\nu\alpha} = Q^{\mu\nu\alpha} + \delta_\mu^\alpha Q^{\nu\beta\beta} - \delta_\nu^\alpha Q^{\mu\beta\beta},
\]

and $G^{\mu\nu}$ and $\nabla_\alpha$ are respectively the Einstein tensor and covariant derivative for the full nonsymmetric connection $\Gamma$. Also

\[
\begin{align*}
T^{\mu\nu} &:= 2\sqrt{-g} \frac{\delta L_M}{\delta g^{\mu\nu}}, \\
\tau^{\mu\alpha\nu} &:= \frac{1}{\sqrt{-g}} \frac{\delta L_M}{\delta \mathcal{K}_{\alpha\mu\nu}},
\end{align*}
\]

are the energy-momentum and the canonical spin-density tensors respectively. Now by using equations [6] and [7] one can obtain modified Einstein field equations

\[
G^{\mu\nu}(\Gamma) = 8\pi G(T^{\mu\nu} + \tau^{\mu\nu}),
\]

where $G^{\mu\nu}(\Gamma)$ is the usual symmetric Einstein tensor and

\[
\tau^{\mu\alpha\nu} = \frac{1}{2} S_{\mu\nu}^\alpha u_\alpha,
\]

is the correction to the space-time curvature due to the spin [10]. If the spin vanishes then equation [9] reduces to the standard Einstein field equations. We assume that $L_M$ describes a fluid of spinning particles in the early Universe minimally coupled to the metric and the torsion of the $U_4$ theory. For the spin fluid the canonical spin tensor is given by [10]

\[
\tau^{\mu\alpha\nu} = \frac{1}{2} S_{\mu\nu}^\alpha u_\alpha,
\]

where $S_{\mu\nu}$ is the antisymmetric spin density and $u^\alpha$ is the 4-velocity of the fluid [15]. Then the energy-momentum tensor can be decomposed into the two parts: the usual perfect fluid $T_F^{\alpha\beta}$ and an intrinsic-spin part $T_S^{\alpha\beta}$, as

\[
T^{\alpha\beta} = T_F^{\alpha\beta} + T_S^{\alpha\beta},
\]

so that we have explicitly for intrinsic-spin part

\[
T_S^{\alpha\beta} = u^{(\alpha S^\beta)}_{\mu} u^\nu u_{\mu\nu} + (u^{(\alpha S^\beta)}_{\mu} u^\nu)_{;\mu} + Q^{(\alpha S^\beta)}_{\mu\nu} - u^\nu S_{\mu}^{(\beta S^\alpha)} - u^\nu \omega_{\mu\nu} + u^{(\alpha S^\beta)}_{\mu} \omega_{\mu\nu} u^\nu,
\]

where $\omega$ is the angular velocity associated with the intrinsic spin and semicolon denotes covariant derivative with respect to Levi-Civita connection. If as usual interpretation of EC gravity we assume that $S_{\mu\nu}$ is associated with the quantum mechanical spin of microscopic particles [11], then for unpolarized spinning field we have $<S_{\mu\nu}> = 0$ and if we define

\[
\sigma^2 := \frac{1}{2} <S_{\mu\nu} S^{\mu\nu}>,
\]
we get

\[ < \tau^{\alpha\beta} > = 4\pi G \sigma^2 u^\alpha u^\beta + 2\pi G \sigma^2 g^{\alpha\beta}, \]  

(15)

and

\[
\begin{align*}
<T_F^{\alpha\beta} > &= (\rho + p)u^\alpha u^\beta - pg^{\alpha\beta}; \\
<T_S^{\alpha\beta} > &= -8\pi G \sigma^2 u^\alpha u^\beta.
\end{align*}
\]  

(16)

Consequently the simplest EC generalization of standard gravity will be

\[ G^{\alpha\beta}(\Gamma) = 8\pi G \Theta^{\alpha\beta}, \]  

(17)

where \( \Theta^{\alpha\beta} \) describes the effective macroscopic limit of matter field

\[ \Theta^{\alpha\beta} := < T^{\alpha\beta} > + < \tau^{\alpha\beta} > = (\rho + p - 4\pi G \sigma^2) u^\alpha u^\beta - (p - 2\pi G \sigma^2) g^{\alpha\beta}. \]  

(18)

In analogy with the usual GR, equations (17) and (18) show that EC field equations are equivalent to the Einstein equations coupled to a fluid with a particular equation of state as the matter source. Indeed, in a hydrodynamical description the contribution of the torsion can be carried out by a spin fluid such that

\[ \rho_{\text{tot}} = \rho - 2\pi G \sigma^2, \quad p_{\text{tot}} = p - 2\pi G \sigma^2. \]  

(19)

It is important to note that the signs of the correction terms in (19) are negative which is in agreement with the semi-classical models of spin fluid \[10\], \[11\]. This means that the effect of spin in EC theory is like a perfect fluid with negative energy density and pressure. In what follows, we will see that these negative signs are required to get the signature changing solutions. However, we would like to emphasize that our model, in some senses, is different with the model considered in \[9\] in which a Dirac field plays the role of a spin fluid with positive energy density. In such a model, although under some conditions an accelerated expansion of the universe will occur, the metric of space-time does not experience a change of signature and hence the problem of the initial singularity is still not resolved.

### 3 Signature changing cosmology

According to the Hartle-Hawking no-boundary proposal \[12\] space-time is partly Euclidean and partly Lorentzian (see figure \ref{fig:SignatureChanging}). The main motivation for this idea is the path integral formulation of quantum gravity. To have a better understanding of this idea is the path integral formulation of quantum gravity. To have a better understanding of the quantum theory it is necessary to have an understanding of the associated classical theory by constructing the classical space-time with signature changing structure. In fact, there are two main proposals for this purpose. In the first proposal, the metric of space-time is everywhere non-degenerate but fails to be continuous at the signature changing hypersurface that divides the Euclidean from the Lorentzian region. On the other hand, in the second proposition, the metric is smooth everywhere but is degenerate at the hypersurface of signature change \[16\]. Here, we are interested in using the second one. The authors of \[17\] have shown that for smooth signature changing space-time there exist coordinates such that

\[ ds^2 = tdt^2 - h_{ij}dx^i dx^j. \]  

(20)

For this case, Kossowski and Kriele \[18\] have shown that the in GR energy-momentum tensor of the matter field becomes bounded if and only if the signature change hypersurface (\(\Sigma\)) is totally geodesic and \(\partial_t h_{ij} = 0\) at \(\Sigma\). To proceed further, let us consider the signature changing FLRW metric as

\[ ds^2 = tdt^2 - a(t)^2 g_{ij}dx^i dx^j, \]  

(21)
where $g_{ij}$ is the metric on the constant-curvature spatial section. Inserting this signature changing line element into the \((17)\) and \((18)\) gives the field equations

\[
\begin{align*}
\frac{1}{t} \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} &= \frac{8\pi G}{3} \rho - \frac{(4\pi G)^2}{3} \sigma^2, \\
\frac{1}{t} \ddot{a} - \frac{1}{2t^2} \frac{\dot{a}}{a} &= -4\pi G (3p + \rho) + \frac{2}{3} (4\pi G)^2 \sigma^2,
\end{align*}
\]

(22)

where dot denotes the derivation with respect to $t$ and $k$ defines the curvature of the spatial section, taking the values 0, 1, $-1$ for a flat, positive-curvature or negative-curvature Universe, respectively. The combination of field equations (22) gives

\[
\frac{d}{dt} (\rho - 2\pi G \sigma^2) = -3 \frac{\dot{a}}{a} (\rho + p - 4\pi G \sigma^2),
\]

(23)

which is a generalization of the covariant energy conservation law to include the spin. Now, we consider the matter field as an unpolarized fermionic perfect fluid with equation of state $p = \gamma \rho$. Consequently, we have

\[
\sigma^2 = \frac{1}{2} < S^2 > = \frac{1}{8} \hbar^2 < n^2 >,
\]

(24)

where $n$ denotes the particle number density, and averaging procedure gives \[(19)\]

\[
\sigma^2 = \frac{\hbar^2}{8} B_\gamma \rho_0^2, \quad \gamma \rho_0^2, \quad \gamma \rho_0^2
\]

(25)

where $B_\gamma$ is a dimensional constant dependent on $\gamma$. Therefore, conservation equation (23) gives

\[
\rho = \rho_0 a^{-3(1+\gamma)},
\]

(26)

where $\rho_0$ is energy density at present time. If we define for simplicity

\[
C := \frac{4\pi G}{3} \rho_0, \quad D := \frac{(4\pi G)^2}{24} \frac{\hbar B_\gamma^2}{\gamma \rho_0^2},
\]

(27)

then the Friedmann equation (22) will be

\[
\begin{align*}
\frac{1}{t} \left( \frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} &= 2Ca^{-3(1+\gamma)} - Da^{-6}, \\
\frac{1}{t} \ddot{a} - \frac{1}{2t^2} \frac{\dot{a}}{a} &= -C(1 + 3\gamma)a^{-3(1+\gamma)} + 2Da^{-6}.
\end{align*}
\]

(28)

The avoidance of the singularity is due to the repulsive force $F := -\partial_a (-D/a^3)$ extracted from the spinning matter potential. In fact the quantum mechanical nature of spin induces the negative pressure which is important at the very early Universe and is responsible for the existence of signature
change hypersurface. From now on we will focus our attention to the special case \( k = 0 \), for which the sign of the left-hand side of the first Friedmann equation is negative for the negative values of \( t \) and positive for \( t > 0 \). Consequently, the sign of the right-hand side changes as well. Hence the right-hand side vanishes at \( t = 0 \). Now, since we have solutions both for \( t < 0 \) and \( t > 0 \), there should therefore exist signature changing hypersurface so that \( a = \left( \frac{D}{2C} \right)^{\frac{1}{3(1-\gamma)}} = a_0 \). Also, it is easy to see from Friedmann equation that the scale factor is less than \( a_0 \) for negative values of \( t \) and greater than \( a_0 \) when \( t \) is positive. Hence, this equation predicts the existence of three regions, namely, a Lorentzian domain, a signature changing hypersurface, and an Euclidean domain.

The exact solution of the Friedmann equations [28] for a flat \((k = 0)\) distribution of dust \((\gamma = 0)\) reads

\[
a(t) = \left( \frac{D}{2C} \right)^{\frac{1}{3}} \left[ 1 + \frac{8C^2}{D t^3} \right]^{\frac{1}{3}},
\]

which shows a continuous transition from a finite Euclidean domain to the Lorentzian one. Another exact solution in flat case is radiation dominated \(U_4\) Universe

\[
a\sqrt{a^2 - \frac{D}{2C} + \frac{D}{2C} \ln \left[a + \sqrt{a^2 - \frac{D}{2C}}\right]} = \sqrt{2C} \frac{4}{3} t^{\frac{2}{3}}.
\]

It is clear that in the radiation case one cannot explicitly write the scale factor in terms of \( t \). To obtain solution of Friedmann equations close to the signature changing hypersurface, we can use signature changing conformal time

\[
\sqrt{\eta} d\eta = a^{3\gamma} \sqrt{\eta} d\eta,
\]

which leads to the following solution

\[
a(\eta) = \left( \frac{D}{2C} \right)^{-\frac{1}{3(1-\gamma)}} \left[ \frac{4C^2}{81(\gamma - 1)^2 D} \eta^3 + 1 \right]^{-\frac{1}{3(1-\gamma)}}.
\]

To write the scale factor in terms of signature changing time \( t \), one may expand the above solution around the signature changing hypersurface \( \Sigma \) which upon integration the relation [31], that is,

\[
\eta^3 = \left( \frac{D}{2C} \right)^{\frac{2}{3(1-\gamma)}} t^3,
\]

results the following expression for the scale factor close to the signature changing hypersurface

\[
a(t) = \left( \frac{D}{2C} \right)^{-\frac{1}{3(\gamma - 1)}} \left[ 1 + \frac{(2C)^{\frac{2}{3(1-\gamma)}}}{81(\gamma - 1)^2 D^{\frac{2}{3(1-\gamma)}}} t^3 \right]^{-\frac{1}{3(\gamma - 1)}}.
\]

Also, it is easy also to see that

\[
\frac{\partial a(t)}{\partial t} \bigg|_{t=0} = 0,
\]

\[
\frac{\partial^2 a(t)}{\partial t^2} \bigg|_{t=0} = 0,
\]

which satisfy the Kossowski and Kriele theorem mentioned above. As we have shown in figure 2 the above solution like [29] shows a continuous transition from a finite Euclidean domain to a Lorentzian one.
Figure 2: Qualitative behavior of the scale factor versus time based on relation (34) for typical numerical values of the parameters. The figures are plotted for $\gamma = -1, -1/3, 0, 1/3$ from left to right.

4 Summary

In this letter, we have shown that the EC cosmological model predicts a signature change when the singularity is approached. Moreover, the spinning matter leads to a repulsive force which results in a regular transition from Euclidean to the Lorentzian region. The above discussion shows that one of the curious features of quantum cosmology is the use of Riemannian signature spaces to explain the origin of the observable Lorentzian signature Universe. There are various interpretations of this, the simplest of which is that the signature of the Universe was initially Riemannian and then subsequently changed. It may be argued that the Lorentzian signature is an independent assumption of relativity rather than a consequence, with the theory being equally valid for Riemannian signature, and that in a quantum theory of gravity it would be unnatural to impose signature restrictions on the metric. The question arises as to whether the qualitative predictions of quantum cosmology can be obtained from purely classical relativity by relaxing the assumption of Lorentzian signature. Also in order to understand the quantum theory it is necessary to have an understanding of the associated classical theory, i.e., the theory of classical space-times with signature type change.

Finally we want to point out in connection with the EC theory of gravity and the tetrad (vierbein) formalism which is required for the coupling of spin to gravity. In view of the construction of the field equations the tetrad and the spin connection are considered as independent fields in the action of the theory. As is well known in terms of a tetrad orthonormal frame $e^\mu_a(x)$ the space-time metric at any point can take the form of the Minkowski metric. Hence, in the tetrad formalism the metric signature seems to be fixed in the signature of the Minkowski metric. Now, a question arises: Is it possible to consider the issue of signature transition in this formalism? To answer this question, note that while the space-time metric has 10 components, the tetrad field has 16 components. Indeed, renunciation of the strong equivalence principle in favor of the Galilei-Eötvös principle makes it possible to introduce in gravitational theory more field components than the ten independent components of the space-time metric. In the tetrad formalism of GR all of the 16 components $e^\mu_a(x)$ are employed to serve as gravitational field potentials and the effect of gravitation on the matter is represented in a locally Lorentz-covariant manner. Therefore, the tetrad formalism of EC theory, as respect to the fixed signature of Minkowski space-time, is not suitable to survey signature change phenomena.

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