CHAOTIC DYNAMICS AND COHERENT STRUCTURE IN ELECTRON BEAM WITH VIRTUAL CATHODE IN THE DIODE WITH LOCAL NEUTRALIZATION

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Abstract — The paper consider a complex dynamics of electron beam with virtual cathode and local neutralization of the beam charge density near anode. Different types of nonlinear behaviour, including deterministic chaos, were treated. It is shown that chaotic dynamics arises as results of spatiotemporal structures interaction.

Introduction and model

In the present paper we consider a electrostatic short-circuited diode model. The immobile ion background with concentration \( n_p \) locates near left boundary (anode plasma). The electron beam with overcritical current injected into diode with nonpurhbitated velocity \( v_0 \) and charge density \( \rho_0 \). In this case virtual cathode (VC) forms in the beam as a result of the electrostatic instability \( \mathbb{1} \), and some beam part is reflected from VC to injected boundary. VC oscillates in the diode region. The electron beam with VC demonstrates wide diversity of nonlinear phenomenons, including chaotic behaviour \( \mathbb{2} \), synchronization \( \mathbb{3} \) and other. Investigation of complex dynamics attracts many researchers, that is, such behaviour is the characteristic property of the beam with VC. That counts is study the structure formation, since it is well known, that chaotic dynamics in distributed systems is connected with pattern formation (see \( \mathbb{4} \) and references therein).

The such simplest model of device with VC as a planar diode with overcritical current describes a different nonlinear phenomenons in the real vircator systems. Our model with local neutralisation is a simple model of vircator with injected plasma \( \mathbb{5} \).

The behaviour of system is determined by the dimensionless parameter related current

\[ \alpha = \omega_p L / v_0, \]

where \( \omega_p \) is the beam plasma frequency, \( L \) is the distance between diode planes and neutralisation parameter

\[ n = n_p / n_0. \]

Hence \( n_0 = \rho_0 / e \) and value of plasma region length \( x_p \) is constant \( (x_p = 0.25L) \).

The effect of neutralisation degree of anode plasma on VC dynamics was investigated with the aid of particle-in-cell simulation. The macroparticles in the simulation obey the non-relativistic equations of motion

\[
\begin{align*}
\frac{dx}{dt} &= v, \\
\frac{dv}{dt} &= -(q/m)\partial\phi/\partial x,
\end{align*}
\]

where \( x \) is the position of particles, \( v \) is the velocity of the particles, \( q \) is the charge and \( m \) is the mass of the macroparticles. The code integrates the equation of motion forward in time using a leapfrog scheme. The potential \( \phi \) is computed by the Poisson’s equation in one dimension

\[
\frac{\partial^2 \phi(x)}{\partial x^2} = -\alpha^2 (\rho(x) - \rho_p(x)).
\]

Hence \( \rho(x) \) is the spatial distribution of beam charge density and \( \rho_p(x) \) is the distribution of immobile ion background. In our case

\[
\rho_p(x) = \begin{cases} 
  n \cdot e, & x \leq x_p, \\
  0, & x > x_p.
\end{cases}
\]

System dynamics

The tentative analysis of nonlinear dynamics were effec-tuated from observation of time series of electric field oscillation in the injection plane. Power spectra and projections of attractors were reconstructed from time series. Based on this analysis, domains for distinct behaviour were isolated in parameter plane \((\alpha, n)\) (see Fig. 1).

The VC oscillation (VCO) for small value of neutralisation is regular (domain marked \( A \) in the parameter plane; Fig. 2(a)). Analysis of physical processes shows that only one electron bunch (VC) is arisen in the system. This bunch is marked on the spatiotemporal diagram (Fig.3(a)). Besides, metastable particles, which
exist in the interaction space during of more than one period of VCO, is observed in the beam. However, charge density of the metastable bunch is small, and it is little affected by VCO. The weakly chaotic VCO arises as neutralisation and current increase (domain $B$ for $n < 2.0$; Fig. 2(b)). In this case metastable bunch density grows. The further increasing of $n$ leads to formation of profound metastable bunch in the beam (Fig. 3(b)). A buildup of space charge density in VC region entails the regime with large base frequency in the VCO spectrum (compare Fig. 2(a,b) and Fig. 2(d), that obtain for the same value of $\alpha = 2.125\pi$). This behaviour of system take place for $n > 2.25 \div 2.5$ ($B$; Fig. 2(d)). A change-over from the weak chaos for small values of $n$ to the weak chaos for large $n$ derives through two domains of strongly chaotic VCO. In the first regime (domain $C$ in Fig. 1) phase portrait is homogeneous, there are not sharp peaks in the power spectrum. The second regime ($D$) may be treated as intermittence (Fig. 2(c)).

For large values of $n$ (domain $E$) system demonstrates highly non-regular oscillation with noise-like spectrum and homogeneous attractors (Fig. 2(e)). In this case VC is formed out of anode plasma region. VC exists constantly and chaotic dynamics is determined by the reflection of the particles from VC. Note, VC is not moved in the space, but depth of potential barrier is oscillated in time.

Dimensions of the reconstructed attractors was estimated for different types of chaotic behaviour. Fig. 4 presents correlation dimension of attractors $D$ versus value of embedded dimension $m$ for strong chaos. Dimension is saturated for small values of $m$. Small values of embedded dimensions justify appearance of chaotic behaviour in the beam in the result of interaction between small numbers of structures.

Fig. 1: Bifurcation diagram on the parameter space $(\alpha, n)$. White area $S$ corresponds to nonuniform equilibrium

Fig. 2: Power spectra, reconstructed phase portraits and time series for different regimes

Fig. 3: Spacetime diagrams for regular (a) and chaotic (b) oscillation

Fig. 4: Dependence of attractor dimension from embedded dimension
Pattern formation

The spatiotemporal data of charge density $\rho(x,t)$ were analyzed by the Karhunen–Loeve orthogonal decomposition \[5\]. This method decomposes a data set into spatial orthogonal modes $\{\psi(x)\}_i$. This modes is the solution of the integral equation

$$\int R(x,x')\psi(x')dx' = \Lambda\psi(x),$$

where $R(x,x') = \langle \rho(x,t)\rho(x',t) \rangle_t$ is the mutual correlation function. The value of eigenvalue $\Lambda_i$ is proportional to the energy of $i$-th mode. Karhunen–Loeve method is optimal in the sense that one is optimized (from the viewpoint of energy of modes) eigensets $\{\Lambda\}_i$ and $\{\psi\}_i$. The measure of energy of modes is

$$W_i = \frac{\Lambda_i}{\sum_k \Lambda_k}.$$

The energy of several first modes for different values of $n$ is presented in Table for $\alpha = 1.35\pi$. The typical spatial distributions of modes are shown in Fig. 5. In the all regimes of VCO behaviour of beams is determined by the small number of structures, because more then 90% energy is contained in the $3 \div 4$ higher modes.

**Table:** Energy $W_i$ (in %) of Karhunen–Loeve modes

| Number of mode, $i$ | Value of neutralisation, $n$ | 0.0 | 0.35 | 0.7 | 1.05 | 1.4 |
|---------------------|-------------------------------|-----|------|-----|------|-----|
| 1                   | 67.1                          | 62.1| 60.0 | 56.1| 52.6 |
| 2                   | 17.1                          | 16.4| 20.0 | 21.8| 23.3 |
| 3                   | 5.7                           | 8.3 | 7.1  | 7.9 | 8.5  |
| 4                   | 3.1                           | 4.6 | 4.1  | 4.6 | 5.0  |

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For small neutralisation first modes demonstrates strong nonuniform charge density distribution with one peak (Fig. 5(a)), that corresponds of typical distribution of density in VC. Second mode describes are processes of destruction of VC and disposal of particles from VC to anode. Third and other modes correspond to additional bunches in the beam. Increasing of $n$ leads to growth of second modes at the expense of higher mode.

Metastable bunch is formed for values of $n \sim 2 \div 3$. In this case first and second modes together describes dynamics of VC and metastable bunch, besides cross-correlation between dynamics modes $A_1(t)$ and $A_2(t)$ is large. Hence temporal dynamics of modes is

$$A_i(t) = \int \rho(x,t)\psi_i(x)dx.$$

Spatial distribution of modes is strongly localizing in the interaction space for regime $E$ (Fig. 5(c)). Only one structure — weakly oscillating in the space virtual cathode — exists in the beam, and more than 80% energy accumulated in the first mode. Chaotic VCO is determined by the reflection of the larger part of beam from VC.

**Conclusions**

In the electron beam with VC and local neutralisation different types of nonlinear oscillation are recognized. Influence of density of anode plasma on the chaotic dynamics of VC are considered. Large neutralisation degree leads to strong chaos in the VCO. Strange attractor is most homogeneous in this case. Relationship between different type of chaotic behaviour and structure formation are shown with the help of analysis of physical processes in the diode. The typical patterns (VC and different additional structures) were recognized for regimes with small and large neutralisation. It is shown that appearance of chaotic behaviour connects with growth of charge density in the additional
structures. Investigation of chaotic behaviour in our model may be helpful for design of vircators with anode plasma grid, because changing of density of anode plasma lead to changing of radiation characteristics in these devices.

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