Scattering and resonances on \( p \)-shell nuclei

L Canton\(^1\), P R Fraser\(^2\), J P Svenne\(^3\), K Amos\(^4\), S Karataglidis\(^5\) and D van der Knijff\(^4\)

\(^1\) Istituto Nazionale di Fisica Nucleare, Sezione di Padova, I-35131, Italy
\(^2\) Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México, 04510 México D.F., Mexico
\(^3\) Department of Physics and Astronomy, University of Manitoba and Winnipeg Institute for Theoretical Physics, Winnipeg, Manitoba, Canada R3T 2N2
\(^4\) School of Physics, University of Melbourne, Victoria 3010, Australia
\(^5\) Department of Physics, University of Johannesburg, P.O. Box 524, Auckland Park, 2006, South Africa

E-mail: luciano.canton@pd.infn.it

Abstract. We review recent results on low energy nucleon scattering from some light-mass nuclei and particularly consider resonances seen or expected in cross sections. We present results found using the multichannel algebraic scattering (MCAS) method which was developed to analyze such cross sections. The method gives both bound and resonant spectra of the compound nuclei. In 2006, using this method, we predicted nontrivial narrow resonances due to non-valence \( p \)-shell effects in the nucleus \(^{15}\)F, a nucleus that lies beyond the proton drip-line. Subsequently, in 2009, those predictions were verified experimentally. Research on other such mirror systems is underway. In particular, the structure of \(^{17}\)C has been used to define an appropriate MCAS nuclear interaction for the \( n + ^{16}\)C system. Adding a Coulomb field to that interaction, an MCAS evaluation has been made for the mirror system \( p + ^{16}\)Ne, to specify the low excitation spectrum of the yet unobserved, particle-unstable, \(^{17}\)Na.

We also investigated how the scattering cross-section changes when the spectra of the colliding nuclei have low-excitation particle-emitting resonances. As a test case, we considered \(^8\)Be, being particle-unstable, and analyzed neutron scattering cross sections and the spectra of the compound \(^9\)Be system. If the \(^8\)Be excited states have fixed-energy decay widths, we find that the bound states of the compound system are affected in an unphysical way. The shapes of the target resonances must vary from the usual Lorentzian in an energy-dependent way. Resonance width functions must go smoothly to zero at the elastic threshold. Ways of achieving this condition are being explored.

1. Introduction

The advent of radioactive ion beam (RIB) physics stimulates the consideration of new theoretical challenges involving weakly-bound systems. Radioactive nuclei, especially those close to the drip lines, can have quite low particle emission thresholds and consequently have low-lying resonance states in their spectra. Low-energy scattering of a RIB is a coupled-channel problem that involves such low-lying resonant states of the scattered ion. To address this, a Multi-Channel Algebraic Scattering (MCAS) formalism \([1]\) is used, in which momentum space solutions of coupled Lippmann-Schwinger equations are found. The method also defines all negative-energy bound states. A finite-rank separable representation using an “optimal” set of sturmian functions \([2]\) serves to construct an input matrix of nucleon-nucleus interactions. The MCAS
method has the ability to locate all compound system resonance centroids and widths, regardless of how narrow. For full details see Ref. [1]. Also, by use of orthogonalizing pseudo-potentials (OPP) in generating Sturmians, it ensures the Pauli principle is not violated [3], even if the collective model formulation of nucleon-nucleus interactions is used. Otherwise, some compound nucleus wave functions contain spurious components.

In section 2, we briefly review the MCAS method [1, 3], and in section 3 we illustrate the fundamentals of the resonance identification. Section 4 is devoted to the discussion of results for $^{15}\text{F}$, the mirror of $^{15}\text{C}$, and section 5 for the preliminary results on $^{17}\text{C}$, in view of future investigations on the mirror system $^{17}\text{Na}$. Section 6 reviews the extension of the MCAS approach to unstable target states [4]. In section 7, we display and discuss results for neutron scattering from $^{8}\text{Be}$, with and without the excited target states having widths different from zero. In section 8 we look briefly at the implications of letting the target-state widths have energy dependence, to ensure that these resonances do not extend to negative energies, where they would produce unphysical behaviour for bound, sub-threshold, states. Finally, section 9 gives our brief concluding remarks.

2. The MCAS formulation with stable collective excitations of the target

The MCAS method was developed to find solutions of coupled-channel, partial-wave expanded, Lippmann-Schwinger equations for the scattering of two nuclei. For each total system spin-parity ($J^P$), those equations are

$$T_{cc'}^{J''}(p, q; E) = V_{cc'}^{J''}(p, q) + \mu \sum_{c''=1}^{\text{open}} \int_0^\infty V_{cc''}^{J''}(p, x) \frac{x^2}{k_{c''}^2 - x^2 + i\epsilon} T_{c''c'}^{J''}(x, q; E) \, dx$$

$$- \sum_{c''=1}^{\text{closed}} \int_0^\infty V_{cc''}^{J''}(p, x) \frac{x^2}{h_{c''}^2 + x^2} T_{c''c'}^{J''}(x, q; E) \, dx$$

(1)

where the channels are denoted $c$ and where $\mu = \frac{2m}{\hbar^2}$, $\overline{m}$ being the reduced mass. There are two summations as the open and closed-channel components are separated, with wave numbers

$$k_c = \sqrt{\mu(E - \epsilon_c)} \quad \text{and} \quad h_c = \sqrt{\mu(\epsilon_c - E)}$$

(2)

for $E > \epsilon_c$ and $E < \epsilon_c$ respectively. $\epsilon_c$ is the energy threshold at which channel $c$ opens. In the nucleon scattering case, they coincide with the, presumed discrete, excitation energies of the nucleus. Henceforth the $J^P$ superscript is to be understood. Expansion of $V_{cc'}$ in terms of a finite number ($N$) of basis function leads to an algebraic representation of the scattering matrix [1]

$$S_{cc'} = \delta_{cc'} - i^{l_c - l_{c'} + 1} \pi \mu \sum_{n, n' = 1}^N \sqrt{k_c} \chi_{cn}(k_c) \left[ \eta - G_0 \right]_{nn'}^{-1} \chi_{n'n'}(k_{c'}) \sqrt{k_{c''}}$$

(3)

The indices $c$ and $c'$ refer now only to open channels, $l_c$ is the partial wave in channel $c$ and the Green’s function matrix is

$$[G_0]_{nn'} = \mu \left[ \sum_{c=1}^{\text{open}} \int_0^\infty \chi_{cn}(x) \frac{x^2}{k_c^2 - x^2 + i\epsilon} \chi_{n'n'}(x) \, dx \right] - \sum_{c=1}^{\text{closed}} \int_0^\infty \chi_{cn}(x) \frac{x^2}{h_c^2 + x^2} \chi_{n'n'}(x) \, dx$$

(4)

Here, $\eta$ is a column of weight values and $\tilde{\chi}$ are a convenient set of expansion functions. Details are given in Ref. [1].
3. Resonance identification

Sturmians (also known as Weinberg states) represent an alternative way to formulate the Quantum Mechanical problem.

Consider a two-body like Hamiltonian $H = H_o + V$: then the Schrödinger equation is written in the standard (time-independent) way

$$ (E - H_o)\Psi_E = V \Psi_E, $$

where $E$ is the spectral variable, and $\Psi_E$ is the eigenstate.

Sturmians, instead, are the eigensolutions of:

$$ (E - H_o)\Phi_i(E) = \frac{V}{\gamma_i(E)} \Phi_i(E), $$

where $E$ is a parameter. The eigenvalue $\gamma_i$ is the potential scale. Thus the spectrum consists of all the potential rescalings that give solution to that equation, for given energy $E$, and with well-defined boundary conditions.

The standard boundary conditions for Sturmians $\Phi_i(E)$ are:

- $E < 0$ Bound-state like; normalizable.
- $E > 0$ Purely outgoing/radiating waves; non-normalizable.

The spectrum of eigenvalues is purely discrete, and bounded absolutely. For short-range (nuclear-type) potentials, the eigenvalues can accumulate around 0 only.

Most interestingly, with the use of this sturmian representation, the coupled-channel $S$-matrix can be expressed in the form [2]:

$$ S_{cc'}(E) = \delta_{cc'} - i^{l_{cc'}-l_{cc}+1} \pi \sqrt{k_ck_{c'}} \sum_i \Gamma_{ci}(E^{(+)}; k_c) \frac{1}{1 - \gamma_i(E^{(+)})} \Gamma_{ci}(E^{(+)}; k_{c'}) $$

where $\Gamma_{ci}(E^{(+)}; k_c)$ denote the coupled-channel functions $V\Phi_i(E)$ in momentum-space representation.

It is remarkable that the following interpretation can be given to the last expression: the scattering process initiated in the asymptotic channel $c$ is “captured” into Sturmians. Subsequently the Sturmian propagates freely in the interaction region and finally decays into the outgoing channels. This structure naturally reflects the description of the scattering process in terms of compound nucleus formation, and leads to an expression which is rather similar to that obtained in $R$-matrix formalism. However, in the Sturmian approach, such structure emerges directly from the Hamiltonian, while in the $R$-matrix formalism, the resonant (compound) structure is modeled phenomenologically in terms of specific boundary conditions given at the surface of an hypothetical $R$-space sphere.

Resonant structures can be obtained in terms of the properties of Sturmian eigenvalues. A resonance occurs when the eigenvalue, initially progressing along the real axis, becomes complex before reaching the point (1,0). Such occurs for positive energies as the scattering threshold ($E = 0$) must be passed. The energy centroid of the resonance is the energy corresponding to the real part of sturmian eigenvalue matching the value 1. The width of the resonance can also be determined geometrically by the patterns of the sturmian trajectories, and relates to the imaginary part of the sturmian eigenvalue at the resonant energy. In figure 1 one observes this situation in a realistic case, namely $n+^{12}\text{C}$ elastic scattering. The eigenvalue trajectories (eigenvalues labeled “(1)” and “(2)”) produce two overlapping resonances that can be clearly seen in the experimental cross-section as well as in the theoretical calculation.
Figure 1. (Color online) $n^{-12}$C scattering and low-energy resonances in $\frac{3}{2}^-$ channel of the $n^{-12}$C system. Sturmian-eigenvalue patterns (left) and $\frac{3}{2}^-$ resonant cross-section (right). The dashed line in the left panel denotes the unit circle in the complex plane.

Table 1. The parameter values defining the nucleon + mass-14 potential.

|                  | Odd parity | Even parity |
|------------------|------------|-------------|
| $V_{\text{central}}$ (MeV) | -44.2      | -44.2       |
| $V_{ll}$ (MeV)     | 0.42       | 0.42        |
| $V_{ls}$ (MeV)     | 7.0        | 7.0         |
| $V_{ss}$ (MeV)     | 0.0        | 0.0         |
| Geometry           |            |             |
| $R_0$ (fm)         | 3.1        |             |
| $a$ (fm)           | 0.65       |             |
| $\beta_2$         | -0.5       |             |
| Coulomb            |            |             |
| $R_c$ (fm)         | 3.1        |             |
| $a_c$              | -          |             |
| $\beta_2$         | 0.0        |             |

4. The case of $^{15}$F

The low-excitation spectrum of $^{14}$O has a $0^+;1$ ground state followed by a band starting 0.545 MeV above the proton-$^{13}$N threshold and consisting of a $1^-;1$ state at 5.173 MeV, and then known proton-unstable resonances with spin-parities of $0^+;1$ at 5.920 MeV, of $3^-;1$ at 6.272 MeV, and of $2^+;1$ at 6.590 MeV [5].

Of the low energy spectrum, the $0^+_2$ and $2^+$ were selected for coupling, as in the mirror $n+^{14}$C system they generated the negative low-energy parity states in $^{15}$C [6]. In this calculation, both are taken as zero-width (ignoring their known proton-decay widths) while in a more recent calculation [7], we took into account the upper limits of the known width of the two excited states $0^+_2$ and $2^+$. (The method for including couplings to resonant levels is explained later on in this work)

The coupled-channel nuclear interaction was generated assuming the collective model with rotor character for the structure [8]. Parameter values were chosen to be those used in Ref. [6]. For completeness, they are given in table 1. In this case, the proton distribution is taken as that of a uniformly charged sphere of radius 3.1 fm.
Table 2. Low-lying levels of $^{15}$F. Only the lowest two states were known experimentally [10] at the time of the calculation [6]. The remaining three have been observed in Ref. [9].

| $J^P$ | Theory $E, (\frac{1}{2} \Gamma)$ | Experiment $E, (\frac{1}{2} \Gamma)$ |
|-------|---------------------------------|----------------------------------|
| $\frac{1}{2}^+$ | 1.31 (0.8) | 1.47 (1.00) |
| $\frac{3}{2}^+$ | 2.78 (0.3) | 2.77 (0.24) |
| $\frac{1}{2}^-$ | 5.49 (0.005) | 4.9 (<0.2) |
| $\frac{3}{2}^-$ | 6.88 (0.01) | 6.4 (<0.2) |
| $\frac{5}{2}^-$ | 7.25 (0.04) | |
| $\frac{3}{2}^+$ | 7.21 (1.2) | |
| $\frac{5}{2}^+$ | 7.75 (0.4) | 7.8 (0.4) |
| $\frac{3}{2}^+$ | 7.99 (3.6) | |

The compound system, $^{15}$F, is particle unstable and, until recently, only its ground and first excited resonance states were known. We used known properties of $^{15}$C to predict [6] new states in $^{15}$F, in particular three narrow resonances of negative parities $\frac{1}{2}^-, \frac{3}{2}^-$, and $\frac{5}{2}^-$ in the range 5-8 MeV of excitation.

Three years later, definite spin-parity assignments have been made for the lowest three experimentally known states while the fourth and fifth given in Ref. [9], are tentatively $\frac{3}{2}^-$ or $\frac{5}{2}^-$ for the 6.4 MeV state and $\frac{3}{2}^+$ or $\frac{5}{2}^+$ for the 7.8 MeV state. These resonance state centroids are compared with the spectra predicted from our MCAS calculations in figure 2. The measured states pair up quite reasonably with the calculated ones. Further comparison between the predictions and the experiments is made in table 2.

It should be noted that the parameters that produced this spectrum were fitted only to the observed $\frac{1}{2}^+$ and $\frac{5}{2}^+$ states, before the results of Ref. [9] were published.

5. The case of $^{17}$C

We have made MCAS calculations of the $n+^{16}$C system using three states in $^{16}$C; the 0+ (ground), 2+ (MeV), and 4+ (MeV) states in a bid to improve an earlier calculation of the same system [11]. We assume that couplings to other states (presumed 0$^+_2$, 2$^+_2$, 3$^+$) in the spectrum between 3 and 5 MeV excitation are not essential and that the rotational model, as defined previously [1], suffices in seeking the spectrum of $^{17}$C. The parameter values required to get the results displayed in figure 3 are listed in table 3. The potential parameter values and Pauli blocking/hindrance weights are quite similar to the set used previously [11], now with inclusion of a small hexadecapole deformation to link the 4$^+$ state to the ground in first order, and with some Pauli hindrance of the 2$s^1_2$ orbit in the connections to the 4$^+$ target state.

The spectrum that results is shown in figure 3 and labelled ‘mcas(0+2+4)’. It is compared with the spectrum found previously from the 2-state MCAS evaluation (‘mcas(0+2)’), and with the experimental one that is a combination of states listed in table 4 of Ref. [12] and in figure 5 of Ref. [13]. The latter are displayed by the dash-dot lines. Only positive parity states are known in the low excitation spectrum and (twice) their spins are given as the integers associated with the individual levels shown.

Clearly the three known subthreshold ($n+^{16}$C) states are now matched well in energy and spin-parity by the new MCAS results. The other known and uncertain spin-parity states also have matching MCAS partners in proximity of their excitation energies. Additionally, the uncertain states from Raimann et al. [13] seem to have possible matches with the first low
Figure 2. (Color online) MCAS calculation [6] of the differential cross section at \( \theta = 147^\circ \), c.m. The horizontal bars on the down arrows at the top indicate the widths (FWHM) of the experimental \(^{15}\text{F}\) spectrum [9].

Table 3. The parameter values used to define the channel coupling properties of the \( n^{+16}\text{C} \) system. Energy units are MeV, length units are fm.

| \( V_0 \) | \( V_{ll} \) | \( V_{ls} \) | \( V_{ls} \) |
|--------|--------|--------|--------|
| -37.0  | -2.0   | 9.0    | 1.6    |
| \( R \) | \( a \) | \( \beta_2 \) | \( \beta_4 \) |
| 2.9    | 0.8    | 0.33   | 0.1    |

state in \(^{16}\text{C}\) (1s\(_{1/2}\), 1p\(_{3/2}\), 1p\(_{1/2}\)) OPP \( \Lambda_{i,j} \)
| 0\(^+\) (0.000) | 10\(^6\) | 2.7 | 0.0 |
| 2\(^+\) (1.766) | 10\(^6\) | 2.7 | 0.0 |
| 4\(^+\) (4.142) | 10\(^6\) | 0.0 | 2.0 |

lying state above threshold seeming to be a \( \frac{7}{2}^+ \) resonance. The MCAS spectrum has a number of aspects in common with that shown in figure 1 of Ref. [14]. Besides the three closely spaced and weakly bound sub-threshold states, there is a cluster of \( \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+ \), and \( \frac{9}{2}^+ \) states in the region around 3 MeV excitation and a second such cluster in the region of 5 MeV excitation. There is also a higher excited \( \frac{11}{2}^+ \) state found with both calculations above 6 MeV excitation, notable by being very broad (\( \Gamma = 2.46 \) MeV with MCAS). Our MCAS result shows more states, including one of spin-parity \( \frac{11}{2}^+ \) at 4.24 MeV excitation.

More specifics of the states in the \(^{17}\text{C}\) spectrum are listed in table 4. Columns 1 to 3 display values of spin-parity \( J^\pi \), excitation energy (or centroid) \( E \), and widths \( \Gamma \), of resonances.
ascertained in a study [12] of three neutron transfer cross sections for $^{12}$C scattering from $^{14}$C. The excitation energies of states shown in figure 5 of Ref. [13] are listed in column 4, while the MCAS results are displayed in columns 5, 6, and 7. The three nucleon transfer reaction widths in general do not match those from the MCAS evaluation but the two sets are quite different; the latter being single nucleon removal values. The cluster of resonant states found using MCAS in the region of 3 MeV excitation have widths that agree well with the matching ones from the MCM evaluation of Timofeyuk and Descouvement [14]. Specifically their widths quoted in Ref. [14] are $\frac{7}{2}^+$ (10$^{-12}$ MeV), $\frac{9}{2}^+$ (10$^{-6}$ MeV), and $\frac{5}{2}^+$ (0.015 MeV), while the $\frac{3}{2}^+$
resonance width of 0.265 MeV is slightly larger than the MCAS value of 0.096.

Now, it is said that $^{17}$C has a ‘peculiar’ structure connected perhaps with the neutron separation energy from the ground state being only 0.728 keV. That is typical of a halo nucleus. Indeed the coupled channel interaction we require to give the spectrum of $^{17}$C reflects that with diffuseness being large. As well, features of this interaction are just as identified by Timofeyuk and Descouvement [14] as being required in a two-body potential model, viz. “The bound $^{17}$C spectrum cannot be understood in the two-body potential model with deformation and the $2^+$ excitation of the $^{16}$C core either, if standard sets of potentials are used. An $\ell$-dependent and nonstandard spin-orbit $n+^{16}$C potential must be used for these purposes.”

6. The MCAS formulation for unstable target states

The $S$-matrix equations in the MCAS methodology take the form given in Eq.(3). Traditionally, all target states are taken to have eigenvalues of zero width. Then the integrals in the (complex) Green’s functions are evaluated using the method of principal parts, where, in the limit $\epsilon \to 0$, the Green’s functions take the form given in Eq.(4). This method assumes time evolution of target states is given by

$$|x,t\rangle = e^{-iH_0t/\hbar} |x,t_0\rangle = e^{-iE_0t/\hbar} |x,t_0\rangle,$$

(8)

However, if states decay, they evolve as

$$|x,t\rangle = e^{-(\Gamma t/2\hbar)} e^{-iE_0t/\hbar} |x,t_0\rangle.$$

(9)

Thus, in the Green’s function, channel energies become complex, as do the squared channel wave numbers,

$$\hat{k}_c^2 = \mu (E - \epsilon_c + i\frac{\Gamma_c}{2}) ; \quad \hat{h}_c^2 = \mu (\epsilon_c - E - i\frac{\Gamma_c}{2}),$$

(10)

where $\frac{\Gamma_c}{2}$ is half the width of the target state associated with channel $c$. The Green’s function matrix elements are then

$$[G_0]_{nn'} = \mu \left[ \sum_{c=1}^{\text{open}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{k_c^2 - x^2 + i\mu \frac{\Gamma_c}{2}} \hat{\chi}_{cn'}(x) dx \right. \left. - \sum_{c=1}^{\text{closed}} \int_0^\infty \hat{\chi}_{cn}(x) \frac{x^2}{h_c^2 + x^2 - i\mu \frac{\Gamma_c}{2}} \hat{\chi}_{cn'}(x) dx \right].$$

(11)

When poles are moved significantly off the real momentum axis, integration along this axis is feasible.

7. The case of $^9$Be

The low excitation spectrum of $^8$Be has a $0^+$ ground state that decays into two $\alpha$-particles (width: $6 \times 10^{-6}$ MeV), a $2^+$ resonance state (centroid: 3.03 MeV; width: 1.5 MeV) and a $4^+$ resonance state (centroid: 11.35 MeV; width: $\sim 3.5$ MeV). In table 5 are given the parameters in the channel-coupling potentials, as well as the $\lambda^{OPP}$ values giving the strengths of Pauli hindrance. Two evaluations of the $n+^8$Be cross section are obtained using this spectrum and set of parameter values; one with target-state widths set to zero, and the other taking into account the widths of the excited levels. The two results will be identified by the terms ‘no-width’ and ‘width’, respectively. Both cases are calculated with the same nuclear interaction, taken from a rotor model [7]. In table 6, the resonance widths of the $^9$Be states that result from these calculations are given and they are compared with experimental values.
Table 5. The parameter values defining the $n^+{^8}\text{Be}$ interaction.

|                | Odd parity | Even parity |
|----------------|------------|-------------|
| $V_{\text{central}}$ (MeV) | -31.5      | -42.2       |
| $V_{l}$ (MeV)    | 2.0        | 0.0         |
| $V_{ls}$ (MeV)   | 12.0       | 11.0        |
| $V_{ss}$ (MeV)   | -2.0       | 0.0         |
| Geometry         | $R_0 = 2.7$ fm, $a = 0.65$ fm, $\beta_2 = 0.7$ |

Table 6. Widths of resonances in $n^+{^8}\text{Be}$ scattering for the no-width and width cases, compared to experimental values.

| $J^\pi$ | $\Gamma_{\text{exp.}}$ | $\Gamma_{\text{no-width}}$ | $\Gamma_{\text{width}}$ |
|---------|-----------------------|-----------------------------|-------------------------|
| $1^+$   | 0.217±0.001           | —                           | 1.595                   |
| $7^-$   | 1.210±0.230           | 2.08×10^{-5}                | 1.641                   |
| $1^+$   | 1.080±0.110           | 0.495                       | 1.686                   |
| $5^+$   | 0.282±0.011           | 0.187                       | 0.740                   |
| $7^+$   | 1.330±0.360           | 0.466                       | 3.109                   |
| $5^-$   | 7.8×10^{-4}           | 0.060                       | 2.772                   |
| $9^+$   | 1.330±0.090           | 0.386                       | 2.498                   |
| $3^+$   | 0.743±0.055           | 3.286                       | 5.162                   |

Taking the excited states of $^8\text{Be}$ to be resonances gives the same spectral list as when they are treated as zero-width, but the evaluated widths of the compound nuclear states significantly increase, as reflected in the cross sections. These increases bring the theoretical $^9\text{Be}$ state widths closer, often significantly, to experimental values, as evidenced by comparison of the final columns of the two tables. Only the $5^-$, also poorly recreated in centroid, has a worse match with widths applied. The resultant elastic and reaction cross sections are shown in figure 4. The line labeled no widths depicts the results obtained without considering the decay widths in the target states, and the line labeled widths does take these decay widths into consideration, as constant values multiplied by the Heaviside (step) function: $H(E) = 0$ for $E \leq 0.0$; $H(E) = 1$ for $E > 0$. (For the line labeled $A = 0.5$, see section 8.)

Introducing target state widths, the resonances in $^9\text{Be}$ are suppressed but still present; their widths increasing and magnitudes decreasing so as all but the $5^+$, and arguably $5^-$, cannot be discerned from the background. The reaction cross section found in the no-width case is zero until 3.4 MeV (3.03 MeV in the centre of mass frame), the first inelastic threshold. When the target state has width different from zero the elastic cross section has flux loss from zero projectile energy upwards. There is an asymptotic behaviour as energy approaches zero. In the reaction cross section from the width calculation, there is a broad peak at 1.5 MeV, corresponding to
the $\frac{5}{2}^+$ resonance. While use of the Heaviside function ensures stability in the sub-threshold region of the compound system, it can generate ‘ghost’ levels where application of widths could move a (no-width) calculated sub-threshold state into the continuum. In such a case the action of the Heaviside function then creates two copies of the same state. An improvement of this technique is discussed in the next section.

8. Energy dependence of widths
A smooth description of the energy dependence of the decay width is needed to avoid any pathological behaviour caused by the Heaviside function. This function allows maintaining the stability of the compound system in the sub-threshold region, by setting to zero all widths for $E < 0$, but can generate “ghost” levels where the application of widths could move a sub-threshold state above $E = 0$, thus creating two copies of the same state. Also, a dramatic overestimation of the reaction cross-section occurs in the region close to threshold.

We therefore need to improve the model with a suitable parametrization of the energy-dependence of the decaying width for the excited levels, with the constraint that the width must be zero at the scattering threshold, and equal to the experimental width at the resonance centroid energy. We are presently exploring various forms of energy dependencies for the widths of these excited states ($\Gamma_c(E) = \Gamma_c \times U(E)$). Examples of such expressions are

$$U(E) = \frac{(1 + A)}{(1 + A)^2 - 1} \left\{ \frac{1 + A}{A(1 - x)^2 + 1} - \frac{1}{A(x^2) + 1} \right\} H(E) \quad (12)$$

where $x = E/E_r$, $H(E)$ is the Heaviside function defined above and $A$ is a parameter, and the Gaussian form

$$U(E) = e^{\left( \frac{E}{E_r} \right)^2} e^{-\left( \frac{E}{E_r} \right)} H(E). \quad (13)$$

In this paper we study only the first form. The second, and others, will be explored in future work. Also yet to be taken into account is causality, as needed when dealing with energy-dependent widths [15].

Figure 4. (Color online) Elastic (left panel) and reaction (right panel) cross section as a function of neutron laboratory energy with three treatments of the widths of target states (see text).
In figure 5 we give the induced energy dependence for the function $U(x = E/E_r)$ with respect to the renormalized energy $x$. The curves differ for different values of the parameter $A$. The line labeled $A = 0.5$ in figure 4 shows the effect of this energy-dependent width on elastic and reaction cross sections, respectively, when $A = 0.5$. Clearly, for this value of $A$, the asymptotic behaviour in the reaction cross section at $E = 0.0$ is suppressed, but not resolved. In principle, an energy dependence in the widths induces a corresponding energy-dependence in the positioning of the resonance centroids. This additional effect is a direct consequence of the principle of causality which demands that the real and imaginary part of any Green’s function matrix elements be related by a dispersion integral. Such an aspect has been extensively discussed in the study of the properties of the nuclear optical potential [15]. Eq. (12) is particularly convenient in this regard because the corresponding dispersion integral leads to an analytical energy-dependent shift of the resonance centroids [16].

![Figure 5.](image) (Color online) The scaling function, $U(E/E_r)$, for various values of the parameter $A$.

9. Conclusions

The MCAS approach has been used with isospin mirror mass-15 systems to define the spectroscopy of the particle-unstable nucleus, $^{15}F$. The procedure involved first making an analysis of the neutron-rich $^{15}C$ system for which experimental information is known. Crucial to the description of the experimental spectrum was the concept of Pauli-hindrance of single-particle orbits coupled to the collective $0^+$ and $2^+$ excitations in the mass-14 nuclei. It leads to an appropriate description of the observed three low-lying negative-parity resonances. Then, by incorporating Coulomb interactions, the same nuclear force was used to analyze the proton-$^{14}O$ case and thus to predict the spectroscopy of $^{15}F$ up to 8 MeV excitation. Experimental information concurs with those predictions.

As a prelude to a study of the low-excitation spectrum for the exotic nucleus $^{17}Na$ treated as a $p+^{16}Ne$ system, we have used MCAS to define a nuclear interaction to give a reasonable low excitation spectrum of the mirror system, $^{17}C$, when treated as a $n+^{16}C$ coupled-channel problem. Three states, a $0^+$ (ground) a first excited $2^+$ ($\sim 1.7$ MeV), and a $4^+$ ($\sim 4$ MeV), were taken as the target states. Both sub-threshold and low excitation resonances for $^{17}C$ with
centroid energies in quite reasonable agreement with the limited set of known values were found. The $^{17}$C spectrum contains both sharp and narrow resonances. By adding Coulomb interactions for the interaction between a proton and the individual states of $^{16}$Ne, the spectrum of the nucleon emissive $^{17}$Na should also contain narrow and broad resonances. However, this system involves target states that are resonances and so is a work in progress at this stage.

An extension to the MCAS formalism that considers particle-decay widths of target nucleus eigenstates has been applied to a range of light mass nuclear targets [7], in addition to the results for $n+^{8}$Be shown in this paper. To ensure that any resonance aspect of target states does not have influence below the nucleon-nucleus threshold in our formalism, we have used a Heaviside function to cut off tails of the positive-energy resonances below zero energy of the compound system. This procedure, however, is too simplistic since it introduces non-physical singularities in cross sections as the energy tends to zero, and may also incorrectly overestimate reaction cross just above the threshold.

A better energy-dependent scaling factor is needed: one with value one at the centroid energy and approaching zero for lower and higher energies. We propose here possible means of achieving such modified Lorentzians, and show, for one example, their effect on the elastic and reaction cross sections in the scattering of neutrons from $^{8}$Be. Work is in progress to apply these and other forms to the light-nuclear systems considered in our previous work. Work is in progress, also, for preserving causality in the presence of energy-dependent widths by the use of appropriate dispersion relations.

Acknowledgments
This work is supported by the National Research Foundation, South Africa, the Natural Sciences and Engineering Research, Council, Canada, INFN, Italy and the Australian Academy of Science.

References
[1] Amos K et al. 2003 Nucl. Phys. A 728 65
[2] Rawitscher G H and Canton L 1991 Phys. Rev. C 44 60
[3] Canton L et al. 2005 Phys. Rev. Lett. 94 122503
[4] Fraser P et al. 2008 Phys. Rev. Lett. 101 242501
[5] Tilley D R et al. 2004 Nucl. Phys. A 745 155
[6] Canton L et al. 2006 Phys. Rev. Lett. 96 072502
[7] Fraser P R et al. 2011 Rev. Mex. Fis. 57 20
[8] Tamura T 1965 Rev. Mod. Phys. 37 679
[9] Mukha I et al. 2009 Phys. Rev. C 79 061301 (R)
[10] Goldberg V Z et al. 2004 Phys. Rev. C 69 031302
[11] Karataglidis S et al. 2008 Nucl. Phys. A 813 235
[12] Bohlen H G et al. 2007 Eur. Phys. J. A 31 279
[13] Raimann G et al. 1996 Phys. Rev. C 53 453
[14] Timofeyuk N K and Descouvemont P 2011 Phys. Rev. C 81 051301
[15] Mahaux C, Ngô H, and Satchler G R 1986 Nucl. Phys. A 449 354
[16] Canton L et al. 2011 Phys. Rev. C 83 047603