How directional mobility affects biodiversity in rock-paper-scissors models

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This work deals with a system of three distinct species that changes in time under the presence of mobility, reproduction and predation, as in the popular rock-paper-scissors game. The novelty of the current study is the modification of the mobility rule to the case of directional mobility in which the species move following the direction associated to a larger (averaged) number density of preys in the surrounding neighborhood. Directional mobility can be used to simulate eyes that see or a nose that smells, and we show how it may contribute to limit biodiversity.

I. INTRODUCTION

It is a known fact in the study of population dynamics that mobility affects biodiversity. As pointed out by several authors [1–8], mobility may contribute to end biodiversity, or to the extinction of some species in systems of several species that interact cyclically. In particular, in the recent work [8] the authors review several aspects of the cyclic dominance in evolutionary games, including pattern formation and the impact of mobility that motivates the current study.

Mobility plays a crucial role to promote or destroy coexistence of species in rock-paper-scissors games [1]. The movement of predators can be motivated by the geographic distribution of preys, leading to directional dispersal of individuals on the grid. For example, in Refs. [9–11], the authors have studied particles moving in the average direction of the particles in their neighborhood. Furthermore, individuals can move following the fluid in which the species are dispersed [12, 13].

In recent works [14–17], we have studied dynamical, geometrical and topological properties of competing networks that depend crucially on the mobility, reproduction, predation, and competition interactions, assuming a standard mobility. In the current work, we focus on how directional mobility modifies the dynamics of the system, with a particular focus on its impact on the coexistence of the species. The motivation is to make mobility more realistic, introducing an extra feature, which works like an eye to see or a nose to smell the direction to move.

To make the investigation easier to follow, we consider the system described by three distinct species, A, B, and C, and in Sec. [18] we describe how the stochastic rules are implemented in our spatial system defined in a square lattice, and we also explain how the directional mobility is modeled. In Sec. [19] we deal with the time evolution of the system, firstly reproducing the typical spiral patterns that appear in the standard situation and then studying the modifications associated to the introduction of directional mobility. We then investigate temporal features of the system and the related spatial behavior in the square lattice used to implement the stochastic simulations. In Sec. [20] we investigate the impact of directional mobility on biodiversity. We end the work in Sec. [21] with our comments and conclusions.

II. THE MODEL

In this work we implement stochastic network simulations in a system composed of three species that evolve in a square lattice having $N$ sites, obeying periodic boundary conditions. The initial state is formed by a lattice where each site contains a single species or is empty. Species and empty sites are initially distributed randomly, in such a way that, in the initial state, there are $N/4$ sites associated to each one of the three species and the empty sites.

At each time step an individual is randomly chosen on the grid. This individual can interact with one of its eight immediate neighbours (we are using the Moore vicinity).
An interaction can be summarized as follows: select the individual, pick up the neighbor, choose the rule and implement it in the lattice. The unit of time $\Delta t = 1$ is defined as the time necessary for $N$ interactions to occur - one generation time. All simulations are done for 15000 generations, with the first 5000 generations discarded, allowing that all the investigation are implemented after the pattern formation.

The stochastic model is characterized by mobility, predation, and reproduction. In the current work, these interactions happen with probabilities $m = 0.60$, $r = 0.20$, and $p = 0.20$, respectively (probabilities are same for all species).

Reproduction only happens if the neighbor grid point is empty. Predation requires that the neighbor is a prey, with the prey being substituted by an empty site. Predation follows the rock-paper-scissors game, as illustrated in Fig. 1.

Directional mobility is implemented as follows: when mobility is sorted, a region of radius $r_c$ around the chosen individual is delimited. Inside this region one vectorially identifies the preys of chosen individual. The direction of maximum likelihood of finding a prey (denoted by $\vec{r}$) is the direction the individual will move. For this, the individual switches position with its neighbor, in the direction dictated by $\vec{r}$. This is illustrated in Fig. 2 for the case $r_c = 4$, with $a = 2$.

Figure 2: As an illustration of the directional mobility, the left panel identifies the preys of the central individual inside the euclidean circle of radius $r_c$. The total distance is shown in the middle panel, and the predator moves in this direction, as it appears in the right panel.

We calculate the vector $\vec{r}$ as

$$\vec{r} = \sum_{i=1}^{k} d(r_i) \vec{r}_i,$$

where $\vec{r}_i$ is the distance from the predator to the prey, and $k$ stands for the number of preys inside the circle of radius $r_c$. Moreover, in this paper we choose the function $d(r_i)$ to be defined by

$$d(r_i) = \exp \left( \frac{-(r_i - 1)^2}{a} \right),$$

where $a$ is a parameter that controls the maximum reach of the directional mobility. In our simulations, we assume $a = 2^m$, where $m = 0, 1, \ldots, 6$, and cutoff radius $r_c = r_c(a)$ so that $d(r \geq r_c) = 0$. Table I shows the cutoff radius and the respective values of $r_c$, used in our simulations. In the case of $r \leq r_c$, we consider $d(r) = 0$ for $d(r) < 1 \times 10^{-2}$.

Throughout the paper we will compare the numerical results obtained by assuming directional mobility and the standard case, where individuals move randomly. In the case of standard mobility, predators are not able to figure out the best direction to move because they do not know where their preys are. Therefore, the standard random movement cannot be recovered even if one assumes
a small radius of interaction in the directional mobility case.

| Table I: |
|---------|
| $a = 1$ | $2$ | $4$ | $8$ | $16$ | $32$ | $64$ |
| $r_c = 4$ | $5$ | $6$ | $8$ | $10$ | $14$ | $19$ |

III. RESULTS

Up to here, we defined the system and showed how the rules have to be implemented to drive its time evolution and spatial behavior. From now on, we pay closer attention to some of its main features, which we describe below.

A. Pattern formation

We first consider the long time evolution of the system, with standard and directional mobility, controlled by several distinct values of $a$.

Figure 3 displays snapshots of the patterns obtained for a square lattice of size $1000^2$ after 20000 generations, for the standard mobility, and for various values of $a$. The figure shows that the increasing of $a$ changes the spiral patterns, with the formation of clusters of groups of empty sites. This happens because predators enter domains composed of preys, moving perpendicular to the interface of empty spaces separating the spiral arms - they move into the direction with higher prey density. As a consequence, islands of empty sites grow along the boundaries between the spiral arms.

Broadly speaking, the larger $r_c$ the further individuals can move. This means that the increasing of the cutoff radius causes the enlargement of the average areas occupied by the domains. If the area of the domains is the same order (or larger) of the grid size, the probability of extinction of the species increases drastically. As a result, only one species survive since the directional mobility contributes to end diversity.

The results displayed in Fig. 3 show that the inclusion of the directional behavior changes the way the species dispose themselves in space. They suggest that we investigate both the time and spatial evolution of the species. Hence, below we start focusing on some temporal features of the systems.

B. Temporal behavior

Let us now investigate some features of the system time evolution. Toward this goal, let us first consider how the abundance of a given species changes in time. As all species play similar role, we will focus on the number density of species $A$ which we denote by $\rho_A(t)$.

Figure 4 shows how the abundance of species $A$ changes over time. The data were collected starting counting the time after 10000 generations. The set of abundances of all species is shown in the ternary diagram in Fig. 5 for several values of $a$.

The results of Figs. 4 and 5 show that the fluctuation in the size of the population increases as one increases the value of $a$ (or, equivalently, $r_c$). However, no species dies out since the average size of the domains is smaller than the grid size.

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![Figure 4](image-url)

**Figure 4:** Abundance of species $A$ as a function of the time for directional mobility considering various values of $a$. The larger the interaction radius the more the total number of individuals fluctuates.

![Figure 5](image-url)

**Figure 5:** This ternary diagram illustrates the evolution of the competition network with three species. The trajectories were taken for a single realization. Although the fluctuations of the population sizes increase with $r_c$ with $a$, the species coexist because the average size of the domains is smaller than the grid size.

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The results of Figs. 4 and 5 show that the fluctuation in the size of the population increases as one increases the value of $a$ (or, equivalently, $r_c$). However, no species dies out since the average domain size (that increases with $a$) is smaller than the lattice size. For larger interaction radius, fluctuations may lead to the extinction of species.

In order to further investigate the time evolution of the species, we make a Fourier analysis of $\rho(t)$. Following closely Refs. [3, 4, 15, 19], we introduce the discrete Fourier transform to get

$$\rho(f) = \sum_{t=0}^{N_t-1} \rho(t) \cdot e^{-2\pi it f},$$

where $N_t$ is the number of generations.
where $f = n/N_G$ with $n = [0, N_G - 1]$ and $N_G = 10000$ generations. Figure 3 shows the spectral density for the abundance corresponding to species $A$, with the results depicted for an average over 100 simulations with different initial conditions. For increasing values of $a$, the amplitude at maximum frequency $f_{max}$ increases, although $f_{max}$ itself decreases.

Another related study concerns the temporal correlation length, $\tau$, which is extracted from the autocorrelation function as $C(t = \tau) = 1/2$, that is the time for the autocorrelation to decrease to half of its value at the initial time. Following closely Refs. [2] [3] [20] [21], we introduce the autocorrelation function in the form

$$C_{AA}(t') = \frac{1}{C_{AA}(0)} \sum_{t=0}^{N_G - t'} (\langle \rho_A(t) \rangle - \langle \rho_A \rangle)(\langle \rho_A(t + t') \rangle - \langle \rho_A \rangle)$$

where $\langle \rho_A \rangle$ is the average of the abundance $\rho_A(t)$, for the species $A$. We use Eq. (4) to calculate the autocorrelation displayed in Fig. 7 in the case of standard mobility and for several values of $a$. In addition, the inset in Fig. 7 shows how the correlation time varies as a function of the parameter $a$. It shows that the correlation time increases as $a$ increases.

### C. Spatial behavior

We now turn attention to the spatial behavior of the system. In order to quantify this behavior we introduce the quantity

$$C(r') = \sum_{|r'|=x+y} \frac{C(\vec{r}')}{{\min}(2N - (x + y + 1), x + y + 1)} .$$

where $C(\vec{r}')$ is the spatial autocorrelation function, calculated from the Fourier transform of the spectral density

$$S(\vec{k}) = \frac{\mathcal{F}^{-1}\{S(\vec{k})\}}{C}\ .$$

The spectral density $S(\vec{k})$ is given by

$$S(\vec{k}) = \sum_{k_x, k_y} \varphi(\vec{k})\varphi(\vec{k})' ,$$

where $\varphi(\vec{k}) = \mathcal{F}\{\phi(\vec{r}) - \langle \phi \rangle\}$ and $\phi(\vec{r})$ represents the species in the position $\vec{r}$ in the lattice; here we are using 0 for the empty sites, and 1, 2, and 3 for species $A, B$, and $C$, respectively.

Figure 3 displays $C(r')$ for the directional mobility for various choices of $a$. The inset shows the characteristic length $l$ which we define as $C(r' = l) = 0.15$. The results show that the length $l$ increases as one increases the value of $a$. This fact appears clearly in Fig. 3 since there one notes the enlargement of the colored regions which identify the distinct species in the system, as $a$ increases.

### IV. MOBILITY VERSUS DIVERSITY

In the previous sections, we have studied how the systems evolve in time and modify their spatial features when one increases the parameter $a$ which controls the directional mobility. Now we will explore further the impact of directional mobility on biodiversity. To this purpose, we run a large number of numerical simulations with $100^2$ and $200^2$ sites, for $a = 1, 2, 4, 8,$ and $32$.

We introduce some auxiliary parameters $p', r'$ and $m'$ and rewrite the probabilities of predation, reproduction, and mobility as $p'(p' + r' + m')$, $r'(p' + r' + m')$ and $m'(p' + r' + m')$, respectively. By setting $p' = r' = 1$, the Mobility parameter can be written as $M = m'/2N^2$. 
that is proportional to the typical area explored by an individual, per unit time $[1]$.

Hence we study the extinction probability, that is, the probability of extinction of two species as a function of $M$. The results are shown in Fig. 9. The vertical lines show the critical mobilities, $M_c = (5.5 \pm 0.5) \times 10^{-4}$ in the case of standard mobility, and $M_c = (2.5 \pm 0.5) \times 10^{-4}$ for $a = 1$. The result for the standard mobility is in good accordance with results obtained in Refs. [1, 5, 6], whereas the result for $a = 1$ shows that the critical mobility decreases when directional mobility is assumed.

The data of Fig. 9 were taken from the average over 20000 simulations for lattices with 100$^2$ sites, and from the average over 1000 simulations for grids with 200$^2$ sites. The results displayed in Fig. 10 show that for $a = 1$ biodiversity is lost at a smaller value of the critical mobility, when compared with the standard case. This happens because the average size of the domains is larger in the case of directional movement, for same probability of mobility.

The results for the lattice with 200$^2$ sites, shown in Fig. 10 confirm those presented in Fig. 9: directional mobility jeopardizes biodiversity. Moreover, the larger the interaction radius the more the biodiversity is affected.

V. COMMENTS AND CONCLUSIONS

In this work we studied a system of three distinct species that changes in time in cyclic predator-prey interactions, following the rules of the popular rock-paper-scissors game. We consider that the movement of the individuals on the grid depends on the prey density spatial distribution. This introduces a directional mobility, that means that individuals move into the direction with larger number of preys. As a result, both the time evolution and the spatial organization of the species change significantly. The results show that directional mobility affects ecosystems by jeopardizing biodiversity. This effect is stronger for larger interaction radius of individuals because the further the individual mobility reaches, the larger are the average size of the domains. As long as the average size of the domains is smaller than the grid size, the species coexist, but the increasing of the mobility to larger and larger values contributes to end biodiversity.

The results of the work are of current interest, because directional mobility may contribute to change both the time evolution and the spatial behavior of the system. In particular, one can use directional mobility to model species whose interactions are dependent on space, adding effects due to spatial inhomogeneities in the lattice. These issues open new routes of applications in problems of current interest in several areas of research, including agriculture, ecology, and other related areas of nonlinear science. We hope to report on these and in other related issues in the near future.

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