Fourth Generation Leptons and Muon $g - 2$

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We consider the contributions to $g - \mu^2$ from fourth generation neutral and charged leptons, $N$ and $E$, at the one-loop level. Diagnostically, there are two types of contributions: boson-boson-$N$, and $E$-$E$-boson in the loop diagram. In general, from the Standard Model to the Two-Higgs Doublet Models, the effect from $N$ is suppressed by off-diagonal lepton mixing element $V_{N\mu}$. With contribution from $E$, we consider flavor changing neutral couplings.

1. MOTIVATION

It was recently pointed out [1] that the existence of a 4th generation could have great implications on the baryon asymmetry of the Universe (BAU). By shifting the Jarlskog invariant product [2] for CP violation (CPV) of the 3 generation Standard Model (SM3) by one generation, i.e. from 1-2-3 to 2-3-4 quarks, one gains by more than $10^{13}$ in effective CPV, and may be sufficient for BAU! On the other hand, with renewed interest in the existence of a sequential 4th generation for CPV studies in $B$ decays (see the references in [1]), and with experimental discovery or refutation expected at the LHC in due time, we turn to the lepton sector.

The difference between the experimental value of muon $g - 2$ and the SM3 prediction has been around for some time now [3], i.e.

$$a_{\mu}^{\exp} - a_{\mu}^{SM} = 295(88) \times 10^{-11},$$

where $a_{\mu} = (g_{\mu} - 2)/2$. The difference is over $3.4\sigma$, which has aroused a lot of interest. On the other hand, we have very stringent bounds on lepton flavor violating (LFV) rare decays, such as [4]

$$B(\mu \rightarrow e\gamma) < 1.2 \times 10^{-11},$$

at 90% C.L. These limits could be improved further in the near future. However, the calculations of $a_{\mu}$ and $B(\mu \rightarrow e\gamma)$ are intimately related, coming from similar diagram, Fig. 1(a), hence giving the similar structures,

$$\epsilon_{\lambda q_e} e^{\lambda\mu}(C_L L + C_R R).$$

2. EFFECTS FROM NEUTRAL LEPTON $N$ AND CHARGED LEPTON $E$

2.1. SM + $N$

The contribution from a fourth generation lepton $N$, Fig. 1(b), has been considered before [5, 6]. We find

$$a_{\mu}^{SM}(W^+W^- N) \sim 233 \times 10^{-11} |V_{N\mu}|^2 F(x),$$

where $x = m_N^2/M_W^2$ and $V_{N\mu}$ is the lepton mixing matrix element. We depict $F(x)$ versus $x$ in Fig. 1(e). We see that $F(x)$ is a well-behaved function and bounded. From $m_N \gtrsim 90$ GeV [4], we see that $|V_{N\mu}|$ needs to be 0.7 or higher to reach within $2\sigma$ of Eq. 1. Considering the stringent constraint from Eq. 2, however, this is clearly unrealistic. We conclude that the difference of Eq. 1 cannot come from the addition of a 4th neutral lepton $N$.

2.2. 2HDM-II + $N$

Going beyond SM, we turn to 2HDM-II (which occurs for MSSM), where up and down type quarks receive masses from different Higgs doublets. We find (see Fig. 1(c))

$$a_{\mu}^{2HDM-II}(H^+H^- N) \sim -233 \times 10^{-11} |V_{N\mu}|^2 [f_H(x) + g_H(x) \cot^2 \beta + x_\mu q_H(x) \tan^2 \beta],$$

where
where $x = m_N^2/M_{H^+}^2$, and $x_\mu = m_\mu^2/M_{H^+}^2$. We plot $f_{H^+}(x)$, $g_{H^+}(x)$ and $q_{H^+}(x)$ in Fig. 1(e). Because $N$ has isospin $+1/2$, large cot $\beta$ could lead to enhancement. If we take $|V_{N\mu}\cot\beta|^2 \sim 1$ in the large cot $\beta$ limit, and if $m_N$ is large compared to $m_{H^+}$, it could generate a finite, but unfortunately negative contribution to $\Delta a_\mu$.

2.3. 2HDM-I + $N$

For 2HDM-I, where all quarks receive masses from the same Higgs doublet, we find
\[ a_{H^+}^{2\text{HDM-I}}(H^+H^-N) \sim 233 \times 10^{-11} |V_{N\mu}|^2 \tan^2 \beta [h_{H^+}(x) - x_\mu q_{H^+}(x)], \tag{6} \]
with $x = m_N^2/M_{H^+}^2$. Here we use $v_1 = v \cos \beta$ to generate all particle masses. However, in the 2HDM-I, the $t\bar{t}H^0(h^0)$ coupling relative to its SM value, $m_t/v$, is given by $\cos \alpha/\cos \beta (\sin \alpha/\cos \beta)$. Large tan $\beta$ will make the coupling strength $|g_{t\bar{t}H^0}| \gg 1$ or $|g_{t\bar{t}h^0}| \gg 1$, and becomes nonperturbative, which leads us to reject this possibility.

2.4. 2HDM-III + $E$

In the 2HDM-III, FCNCs are allowed because there exist two matrices $\eta^{(e)}$ and $\xi^{(e)}$ simultaneously for each lepton type. To regulate the FCNC in face of stringent constraints, there is the ansatz suggested by Cheng and Sher [7] for the quark sector, i.e. all $q_i q_j h^0/H^0/A^0$ couplings have the same form
\[ \Delta_{ij} \sqrt{m_i m_j} v, \tag{7} \]
where $\Delta_{ij}$ is $O(1)$. Note that CP-even Higgs $H^0$, $h^0$ give positive contributions to $a_\mu$ but negative for $A^0$. Considering the positivity of Eq. 1, we assume $A^0$ is very heavy and its effect can be neglected. For sake of illustration, we set $h^0$ to be the lightest neutral Higgs, and assume no mixing between $H^0$ and $h^0$, Fig. 1(d). Then we find
\[ \Delta a_\mu^{2\text{HDM-III}} (H^\pm e^-) \sim 233 \times 10^{-11} F_{h^0}(x), \tag{8} \]
where $x = m_E^2/M_{h^0}^2$, and $F_{h^0}(x)$ is given in Fig. 1(e). However, the LFV decay rate in Eq. 2 gives a very stringent constraint. Note that because $a_\mu$ and $B(\mu \rightarrow e\gamma)$ come from similar structure of loop diagrams, their formulas are very closely related. After some organization, we have
\[ B^{2\text{HDM-III}}(\mu \rightarrow e\gamma) \sim 1.7 \times 10^{-5} |F_{h^0}(x)|^2. \tag{9} \]
Consider the case of $\tau$ in the loop, which is the leading contribution with 3 generations. Allowing a factor of 2 uncertainty in Eq. 9, we still need $M_{h^0} > 138$ GeV in order to survive Eq. 2. The MEG experiment will soon push the bound to 530 GeV. Let us now consider 4 generations. Comparing Eq. 1 with Eq. 8, we can have $F_{h^0}(x) \sim 1$. However, Eq. 2 and Eq. 9 give $F_{h^0}(x) \sim 10^{-3}$, which requires $m_E \ll M_{h^0}$, which is unlikely. If a 4th generation is found, the Cheng-Sher ansatz does not seem applicable to the lepton sector.

2.5. MSSM + $N$ + $E$

Simply put, MSSM doubles the diagrams of SM. The corresponding loops to $W^+W^-\nu_\mu$ and $\mu\mu Z$ are chargino-chargino-$\tilde{\nu}_\mu$ and $\tilde{\mu}-\tilde{\mu}$-neutralino respectively. In the mass degeneracy limit for superparticles, $m_{\text{Higgsino}} = m_{Wino} = M_{\tilde{\nu}_\mu} = M_{\text{SUSY}}$, and with large $\tan\beta$ (to compensate the extra heaviness of $M_{\text{SUSY}}$) [8], we can get a sufficient contribution to Eq. 1, as has been elucidated in the literature [3].

3. Summary

We have discussed some models with 4th generation leptons, and calculated their impact on $a_\mu$. In the SM, 2HDM-I and II, it seems that the 4th generation is irrelevant to the $\Delta a_\mu$ puzzle because of the smallness of $|V_{N\nu_\mu}|$. However, this off-diagonal factor also protects these models from the stringent $B(\mu \rightarrow e\gamma)$ and $B(\tau \rightarrow \mu\gamma)$ constraints. For 2HDM-III, there exists a strong conflict with $B(\mu \rightarrow e\gamma)$ under the Cheng-Sher ansatz with the 4th generation. Hence, if a 4th generation is found, the Cheng-Sher ansatz cannot hold for the lepton sector. In this sense, SUSY is favored. Enhancement of $a_\mu$ (diagonal contribution) and suppression of $B(\mu \rightarrow e\gamma)$ (off-diagonal contribution) in the MSSM are both similar to the SM. Since large $\tan\beta$ suppresses the negative contribution from $H^+H^-N$, MSSM and 4th generation can coexist.

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