Study on vibration characteristics of plastic helical gear assembly with prestress

Wu Baogui, Du Zaiyou, Huang Ruchen

College of Mechanical and Electronic Engineering in China University of Petroleum, Qingdao 266580, China

E-mail: buoguiwu@163.com

Abstract. In order to obtain the vibration characteristics of helical gear transmission system in actual working conditions, two kinds of CAE software are combined to analyse the helical gear vibration with prestress. CAD software interface is used to import the model into ANSYS, and then multi-point coupling constraint is proposed to obtain the first six natural frequencies and main modes of the plastic and steel gears with speed and load. The results suggest that the design of plastic gears meets the requirement of bending strength, and prestress causes the natural frequency of the plastic gears to increase faster than that of the steel gears. The dynamics simulation of ADAMS multi-body system shows that the meshing frequency of plastic gear is obviously lower than the natural frequency, so the system will not resonate. Research shows that plastic helical gears can replace steel gears for low-speed gear pumps, which provides a research method for miniaturization, lightweight and precision designing of low-speed gear pumps.

1. Introduction
Modern gear transmission system is developing towards the direction of light weight, precision, high strength and low noise. The new type of plastic helical gear with light weight, high strength, low noise and corrosion resistance has been widely used. They have gradually replaced the traditional steel gear in some products and have a broad application prospect. At present, the performance analysis of plastic helical gear is mainly based on the finite element method. Many scholars in the industry have conducted relevant studies on plastic gear. Zheng Zheng [1] carried out the structural statics analysis of a single plastic spur gear, and obtained the most vulnerable part of failure in transmission process of plastic gear. Wang Xiang [2] analyzed the mechanical properties of a single composite straight gear. An Juan [3] compared the modes of a single plastic spur gear and a steel gear without considering prestress, and obtained the vibration pattern of each order of two kinds of gear. Mr. Samadhan and M. Lande [4] conducted static analysis of a single plastic spur gear and verified the feasibility of finite element simulation with theoretical calculation results. In summary, the existing finite element analysis of plastic gear is mostly static analysis of a single straight gear or modal analysis without prestress. Modal analysis of plastic helical gear systems in operating conditions with load and speed is rare. Gear mesh belongs to the typical nonlinear contact problem, and stress in transmitting process can produce great influence on the natural frequency of the driving system [5, 6]. Therefore, the modal analysis of plastic helical gear assembly under the action of prestress is of great significance to improve the stability and reliability of transmission system and reduce the working noise.

2. Assembly modelling of helical gear system
The transmitting characteristics of a pair of standard helical gears with small modulus in low speed external gear pump transmission system are studied. Study purpose is to reduce the vibration impact of gear pairs and the oil-trapping pressure of low speed gear pump. SolidWorks is used to create 3D model of the gear pair. After that, the model is imported into ANSYS for subsequent analysis. Relevant structural parameters of the meshing gear pair are given in table 1.

| Table 1. Some structural parameters of gears. |
|---------------------------------------------|
| Module/mm | Driving gear | Driven gear |
| Nominal pressure angle/(°) | 2 | 2 |
| Helix angle/(°) | 20 | 20 |
| Number of teeth | 12 | 12 |
| Face width/mm | 20 | 25 |
| Face width/mm | 15 | 15 |

3. Static prestress analysis of helical gear structure

3.1. Finite element modelling

The copolymerized formaldehyde (POM) was selected as the plastic gear material [7], and the steel gear material is 45 # steel. The property parameters of the two types of gear materials are given in table 2.

| Table 2. Material and property parameters of gears |
|-----------------------------------------------|
| Material | Plastic gear | Steel gear |
| Density/(kg/m³) | POM | 1410 | 7850 |
| Poisson ratio | 0.4 | 0.3 |
| Elastic modulus/MPa | 2.3×10³ | 2.1×10⁵ |
| Yield strength/MPa | 78 | 520 |

Considered the solution accuracy and operating time, ANSYS adopts the intelligent grid division of volume scanning and controlling the number of nodes to obtain a finite element model with high grid quality. The finite element model is shown in figure 1. After inspection, a total of 90,336 nodes and 52,508 grid elements are generated.

![Finite element model of assembly](image)

3.2. Structural constraints and loads are applied

According to the normal pressure to simulate the working state of the transmission system, the driving wheel rotation speed applied is 300r/min, and the equivalent torque of the driven wheel is 15000Nꞏmm. In order to effectively simulate the loading condition of gear transmission system, a novel and efficient constraint method is proposed in this paper: MPC multi-point coupling constraint, which is to set the
reference nodes of the center of the driving gear and the driven gear respectively to establish the motion coupling constraint. The simulation of constraints in gear transmission is more accurate and effective.

3.3. Analysis of static prestress results
A complete gear meshing cycle usually goes through three stages of "double teeth meshing - single tooth meshing - double teeth meshing", so it is necessary to calculate the Von Mises equivalent stress of two meshing states of single tooth and double teeth. The equivalent stress cloud diagram of the meshing of the single tooth of the two kinds of gears is shown in figure 2 (a) and (b) respectively. The meshing stress is dominated by bending stress. The maximum meshing stress is located near the tooth root of the driving gear, and the equivalent stress value of the meshing of the single tooth of the gear is larger than that of the two teeth. The maximum equivalent stress value in the meshing area of the gear pairs of the two materials is shown in table 3.

![Von Mises stress cloud diagram of Single teeth-meshing](image)

The comparison results in table 3 show that: Under the same load condition, the stress value in the meshing area of the plastic gear is close to but lower than that of the steel gear, because the influence of tooth surface friction cannot be ignored in the meshing comparison analysis of the gears of different materials. Compared with the steel gear, the POM gear has a smoother surface, and the tooth surface friction has less influence on the meshing stress, so the meshing stress is smaller.

|               | Single teeth-meshing | Double teeth-meshing | Average value |
|---------------|----------------------|----------------------|--------------|
| Plastic gear  | 56.8                 | 52.4                 | 54.6         |
| Steel gear    | 61.2                 | 58.6                 | 59.9         |

According to the allowable bending stress calculation equation of gear:

\[
[\sigma_f] = \frac{\sigma_{f\text{lim}}Y_nY_x}{S_{F\text{min}}}
\]  

Where, \([\sigma_f]\) represents allowable bending stress of gear, MPa; \(\sigma_{f\text{lim}}\) is bending fatigue limit, MPa; \(Y_n\) is life coefficient calculated by bending fatigue strength; \(Y_x\) is size coefficient; \(S_{F\text{min}}\) is minimum safety factor for bending fatigue strength.

Appropriate parameters of working condition were substituted into Equation (1). The allowable bending stress of steel gear and plastic gear was calculated to be 504.8MPa and 72.6MPa respectively. The allowable bending stress of the two kinds of gear pairs is less than the yield limit of material, and the maximum equivalent stress at the tooth root is less than the allowable bending stress, indicating that the plastic (POM) gear also meets the bending strength requirement under the set working condition.

4. Modal analysis with prestress

4.1. Finite element modal analysis theory
The theoretical calculation formula of the natural vibration frequency of the gear is as follows:

\[ f = \frac{1}{2\pi} \sqrt{\frac{k}{m}} \quad (2) \]

Where, \( f \) represents natural vibration frequency, Hz; \( m \) is equivalent mass, kg; \( k \) is equivalent stiffness, N/m.

Gear transmission is a complex nonlinear contact process, equation (2) cannot accurately calculate the natural frequency of gear transmission. Therefore, the modal analysis of gear pair should be carried out by using finite element theory. Based on the relevant theories of elastic mechanics and finite element analysis, the free vibration equation of the gear drive system in the undamped state [8] is:

\[ ([K] - \omega^2[M]) \{\varphi_i\} = \{0\} \quad (3) \]

Where, \([K]\) represents stiffness matrix, \(\omega\) is the natural frequency of the \(i\)-th mode \((i=1, 2, 3\ldots)\), \([M]\) is mass matrix, \(\varphi_i\) is the mode vector of the \(i\)-th mode, independent of time.

Since the amplitude of each node of the gear with free vibration is not all 0, the coefficient determinant of equation (3) must be equal to zero, therefore:

\[ ([K] - \omega^2[M]) = \{0\} \quad (4) \]

According to equation (4), the natural frequency of mode I of gear pair can be obtained, as shown in equation (5):

\[ \omega_i = \sqrt{\frac{K_i}{M_i}} \quad (5) \]

4.2. Modal analysis solution

The obtained static prestress was applied to the modal structure, and Block Lanczos method was used to extract and extend the first six modes in the meshing state of single tooth and double teeth respectively. The results were analyzed and the modal modes of the plastic and steel gears were found to be the same.

4.2.1. Modal analysis pattern diagram. The first six modes of plastic gear are shown in figure 3 (a) to (f) under the meshing state of single tooth.

![The first six modes of plastic gear](image)

Figure 3. first six order mode diagram of single-tooth meshing of plastic gears

As can be seen from the modes of each order in figure 3, the first and sixth orders of the driving gear and driven gear all deform, but the deformation of the latter is more obvious. The second to the fifth stages are the deformation of the driven gear, indicating that the driving gear has better working stability than the driven gear. Therefore, the vibration deformation of the driven gear should be considered in the analysis of the vibration characteristics of the plastic gear system.
4.2.2. Natural frequency comparison of gear pairs of the two materials. A large number of experiments show that the natural frequency of gear is constantly changing, and the average stiffness can be calculated to solve the contradiction that "the stiffness matrix remains unchanged" in modal analysis. $K_1$ and $K_2$ respectively represent the stiffness of single tooth and double teeth when meshing. The average stiffness is:

$$K = \frac{K_1 + K_2}{2}$$  \hspace{1cm} (6)

The natural frequency value of single tooth meshing, double teeth meshing and average stiffness is represented by $\omega_1$, $\omega_2$, and $\omega_3$ respectively. Then, according to equation (5), we can get equation (7):

$$\omega_1 = \sqrt{\frac{K_1}{M_1}}, \omega_2 = \sqrt{\frac{K_2}{M_2}}, \omega_3 = \sqrt{\frac{\omega_1^2 + \omega_2^2}{2}}$$  \hspace{1cm} (7)

The natural frequencies of the gear pairs of the two materials in the first six orders in the three states are calculated as shown in table 4. Compare the data in table 4, meshing stiffness of single tooth is smaller than that of double teeth, the natural frequency of single tooth is lower than that of double teeth meshing. The natural frequency of each order of the steel gear pair is obviously higher than that of the plastic gear pair, and the natural frequency of each order of the steel gear pair is obviously higher than that of the plastic gear pair.

| Number of order | Plastic gear | Steel gear |
|-----------------|--------------|------------|
|                 | $\omega_1$   | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ | $\omega_6$ |
| 1               | 176.10       | 189.32     | 182.83     | 812.73      | 845.62      | 829.34      |
| 2               | 249.19       | 258.68     | 253.98     | 948.33      | 986.65      | 967.68      |
| 3               | 250.36       | 272.39     | 261.60     | 966.19      | 1020.32     | 993.62      |
| 4               | 264.76       | 298.45     | 282.11     | 1097.69     | 1162.86     | 1130.74     |
| 5               | 297.38       | 334.12     | 316.28     | 1274.82     | 1356.23     | 1316.15     |
| 6               | 395.34       | 436.69     | 416.53     | 1697.40     | 1802.36     | 1750.67     |

4.3. analysis of the impact of prestress on the mode

The natural frequency values of the first six orders of steel and plastic gear pairs in the state of non-prestress were obtained under the ANSYS simulation environment, as shown in table 5.

| Number of order | Plastic gear | Steel gear |
|-----------------|--------------|------------|
|                 | $\omega_1$   | $\omega_2$ | $\omega_3$ | $\omega_4$ | $\omega_5$ | $\omega_6$ |
| 1               | 145.62       | 180.46     | 163.97     | 767.71      | 823.14      | 795.91      |
| 2               | 205.86       | 245.54     | 226.57     | 892.43      | 960.13      | 926.90      |
| 3               | 207.98       | 254.72     | 232.53     | 906.21      | 988.24      | 948.11      |
| 4               | 223.65       | 272.38     | 249.21     | 1031.82     | 1114.26     | 1073.83     |
| 5               | 249.72       | 303.53     | 277.93     | 1200.36     | 1279.34     | 1240.48     |
| 6               | 322.43       | 400.89     | 363.78     | 1595.56     | 1681.71     | 1639.20     |

Take each order $\omega_3$ value in table 4 and 5 as the natural frequency, growth of the natural frequency of each order of two gear pairs under the effect of prestress is calculated as shown in table 6.

| Number of order | Plastic gear | $\omega_3$ | Steel gear | $\omega_3$ |
|-----------------|--------------|------------|------------|------------|
|                 |              |            |            |            |
| 1               | 11.5         | 12.1       | 12.5       | 13.2       | 13.8       | 14.5       |
| 2               | 4.2          | 4.4        | 4.8        | 5.3        | 6.1        | 6.8        |

The increasing value of the plastic gear with prestress are higher than those of the steel gear. This is due to the "stress rigidization" effect, which makes the stiffness of the gear increase after being subjected to the stress generated in the transmission process. Moreover, the injection molding residual stress in plastic gear intensifies the stress rigidization effect more obviously than steel gear.
5. Simulation of meshing frequency based on ADAMS
In order to judge whether the gear transmission system will resonate, based on the natural frequency of the plastic gear assembly with prestress, the multi-body system dynamics simulation software ADAMS was used to analyze the meshing frequency of the gear transmission under the actual working conditions. According to the simulation results, the speed curve of the driven wheel and the meshing frequency domain diagram of the drive system are drawn, as shown in figure 4 and figure 5.

\[ f_0 = \frac{1}{60}n \theta \] (8)

Where, \( f_0 \) represents gear theoretical meshing frequency, Hz; \( n \) is the gear speed, r/min; \( \theta \) is number of teeth.

By substituting relevant data into Equation (8), it can be obtained that \( f_0 \) is 100Hz, which is slightly less than the simulation frequency value. Because the theoretical calculation formula does not take into account the influence of the contact friction of the gear pair. The simulated meshing frequency of 100.25Hz is the excitation frequency, which is far less than the natural frequency of the first order. Therefore, the plastic helical gear transmission system works stably and reliably without resonance.

6. Conclusion
The modal analysis and meshing frequency simulation analysis of steel and plastic gear transmission system were carried out by ANSYS and ADAMS, and the following conclusions were obtained:

(1) In same load condition, considering the influence of tooth surface friction in meshing area, the meshing stress of plastic gear is close to but slightly less than that of the steel gear, and the design of the plastic gear meets the bending strength requirements.

(2) The stress rigidization effect makes the natural frequency increase, and the growth of the natural frequency is higher than that of steel gear. The average meshing frequency of the plastic gear transmission system is obviously lower than its natural frequency, and the system will not resonate.

(3) In the low-speed gear pump and some low-speed, non-heavy-duty gear transmission occasions, the metal gear can be replaced by plastic gear, in order to achieve miniaturization and lightweight design, improve production efficiency, and reduce production costs.

References
[1] ZHENG Zheng, LEI JunXiang, LUO YuZhou. Static analysis of plastic gear structure based on ansys [J]. Manufacturing Automation, 2010(05): 163-166 (In Chinese).
[2] WANG Xiang, HUANG Wei, LI Jin. Finite element analysis of composite gear based on ansys [J]. Journal of Wuhan University (Natural Science Edition), 2012(03): 215-220 (In Chinese).
[3] AN Juan, PAN HongXia, ZHOU YunFeng. A comparative analysis of natural vibratory characteristics of metal gear and plastic gear based on ansys [J]. Journal of Mechanical Transmission, 2011(08): 75-77 (In Chinese).

[4] Mr. Samadhan M.Lande, Prof. Dr.A.D.Desai, Prof. A.B.Verma. Stress analysis of polycarbonate spur gears for sugarcane juice machine using FEA [J]. International Journal on Recent Technologies in Mechanical and Electrical Engineering (IJRMEE), 2015(10): 14-19.

[5] J.E.Mehner, L.D.Gabbay, S.D.Senturia. Computer-aided generation of nonlinear reduced-order dynamic macromodels.II.Stress-stiffened case [J]. Journal of Microelectromechanical Systems, 2000(2): 270-278.

[6] M.Kupnik, I.O.Wygant, B.T.Khuri-Yakub. Finite element analysis of stress stiffening effects in CMUTS [J]. 2008 IEEE Ultrasonics Symposium. Beijing, 2008: 487-490.

[7] OUYANG ZhiXi, SHI ZhaoYao. Design and manufacturing of plastic gears [M]. Beijing: Chemical Industry Press, 2011: 147-151 (In Chinese).

[8] XIE LongHan. Finite element analysis and simulation by ANSYS [M]. Beijing: Publishing House of Electronics Industry, 2013: 296-298 (In Chinese).