Investigation of spatial manifestation of $\alpha$ clusters in $^{16}$O via $\alpha$-transfer reactions

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I. INTRODUCTION

Cluster structure is one of the essential features of nuclei besides a mean-field feature. Theoretically, various cluster states are predicted in excited states of not only light-stable nuclei but also sd-shell or unstable nuclei (for instance, in Refs. [1–7] and references therein). A double magic nucleus, $^{16}$O, has the ground state with the dominant p-shell closed configuration, while there are many low-lying excited states that are difficult to be understood by shell-model pictures. Cluster models [4–19] assuming $^{12}$C + $\alpha$ and 4$\alpha$ structures have suggested that those excited states are cluster states. However, it is difficult for such cluster models to answer the questions whether four nucleons form an $\alpha$ cluster in dynamics of sixteen nucleons and how the formed $\alpha$ cluster is distributed in the system, since the models rely on a priori assumption of the cluster structures.

Very recently, the fully microscopic calculations of the antisymmetrized molecular dynamics (AMD) [20, 21] and the chiral nuclear effective field theory [22] have shown the formation of the cluster structures in the sixteen-nucleon system, $^{16}$O, starting from nucleon degrees of freedom. In addition, a five-body model (5BM) [23] based on the $^{12}$C + $ppnn$ orthogonality condition model (OCM) has been achieved and demonstrated $\alpha$ cluster formation at the nuclear surface. The 5BM is more sophisticated than the conventional $^{12}$C + $\alpha$-OCM [5, 6] (we refer to this model as just OCM) in a sense that it is able to describe $\alpha$ cluster dissociation in the inner region although the 5BM calculation assumes the single $0p_{1/2}$ configuration of $^{12}$C core ignoring configuration mixing and core polarization, which are taken into account in the OCM calculation. As a result of suppression of the $\alpha$-cluster formation in the inner region, the $\alpha$ distribution in the $0^+_2$ and $0^+_3$ states of $^{16}$O of the 5BM is qualitatively different from that obtained by the OCM; surface peaks of the $\alpha$ probabilities are shifted outward in both states.

The assignment regarding the structures of the $4^+$ states in $^{16}$O have been attracting much attention, lately. By the OCM calculation, the $0^+_2(6.06$ MeV), $2^+_1(6.92$ MeV), and $4^+_1(10.36$ MeV) states were concluded to be the $^{12}$C$(0^+_1)+\alpha$-cluster band [5, 6]. Likewise, the AMD has assigned the $4^+_1$ and $4^+_2(11.10$ MeV) states to the $^{12}$C + $\alpha$- and the tetrahedral 4$\alpha$-cluster states, respectively, with significant state mixing between them [20, 21]. This assignment is consistent with the observed $\alpha$-decay widths [24, 25], strong $\alpha$-transfer yields [26, 27], and weak two-nucleon transfer cross sections [28, 29]. However, latest results by Bijker and Iachello [18, 19], who have revived the algebraic approach of 4$\alpha$ system [1], have introduced different assignment. They have attributed the $0^+_2$, $3^+_1(6.13$ MeV), and $4^+_4$ states to the tetrahedral-4$\alpha$ band associated with the $T_d$ symmetry, as well
as the $4_{2}^{−}$ state to a vibration mode on it.

For verification of $α$-cluster states, $α$-transfer reactions are useful, as they are able to produce nuclei in not only a ground state but also excited states, within a consistent reaction condition. Experimental studies of $(^6\text{Li},d)$ reactions on $^{12}\text{C}$ and its inverse have been carried out since 1960s [26, 30–47]. Mostly in these studies, spectroscopic factors (SFs) have been extracted in order to identify cluster states of $^{16}\text{O}$, with an astrophysical interest as well. However, such SFs have exceeded unity due mainly to uncertainty of their reaction models, and therefore relative SFs with respect to that of the $2_{1}^{+}$ state have been regarded as an indication to verify the $α$-cluster structure, although it is difficult to argue the spatial manifestation of the cluster from the SFs. Moreover, the $4_{2}^{−}$ state has been paid attention [37, 40, 41, 48] because $α$-transfer reactions such as $^{12}\text{C}(^6\text{Li},d)^{16}\text{O}$ and $^{12}\text{C}(^7\text{Li},t)^{16}\text{O}$, anomalously yield $4_{2}^{−}$-cross sections larger than predictions evaluated from the $α$-decay width by almost two orders of magnitude.

In order to study cluster structures, $α$-cluster probability in the surface and outer regions is more important. In particular, the surface-peak position of $α$-cluster wave functions should be extracted from reaction observables instead of SFs given as integrated quantities. In Ref. [49] we have reported that $α$-transfer reactions are suitable to extract the $α$-cluster probability at the surface in the ground state of $^{20}\text{Ne}$. In addition to $α$-transfer reactions, for example, proton-induced $α$-knockout reactions [50–52] and $^{12}\text{C}+α$ inelastic scattering [53] have been applied to extract the $α$-cluster probability.

Our purpose of this paper is to determine the surface-peak position in the ground and excited states of $^{16}\text{O}$, through an analysis of the $α$-transfer reaction $^{12}\text{C}(^6\text{Li},d)^{16}\text{O}$. To this end, a phenomenological potential model (PM) is introduced. We test the $α$-cluster wave functions of the $0_{1}^{+}$ and $0_{2}^{+}$ states computed microscopically with the OCM [5, 6] and 5BM [23]. Furthermore, we discuss what we can learn from the $α$-transfer cross sections about structures of the two states, $4_{1}^{−}$ and $4_{2}^{−}$, and try to give an answer to the controversial assignments for them. To avoid complicated reaction mechanism originating from compound-nucleus formation and to be consistent with parameterization of a $^6\text{Li}$-optical potential [54] we adopt, the experimental data of the transfer reactions at 42.1 [39] and 48.2 MeV [45] are analyzed.

This article is organized as follows. Section II is dedicated to sketch our theoretical framework, namely the model setting. In Sec. III, results on the $0^{+}$ and $4^{+}$ states are shown. We discuss how the spatial manifestation is probed on transfer-cross sections. A summary is given in Sec. IV.

II. THEORETICAL FRAMEWORK

The $α$-transfer reaction $^{12}\text{C}(^6\text{Li},d)^{16}\text{O}$ is described by the finite-range distorted-wave Born approximation (DWBA) [55, 56]. The $^{12}\text{C}-α$ relative wave function $φ_{l}$ required in the DWBA framework, with the relative orbital angular momentum $l$, is taken from Refs. [5, 6] (Ref. [23]), where the OCM (5BM) was employed to obtain $φ_{l}$. Both the OCM and 5BM provided the reduced-width amplitude (RWA) as the $^{12}\text{C}-α$ wave function. Since the RWA is not able to be an input for reaction calculations directly [23], we normalize it by letting its norm be unity. The normalized RWA is suitable to discuss the spatial manifestation of the wave function and corresponding diffraction pattern of the cross section, rather than the SF and absolute value of the cross section. Moreover, because the asymptotic form of the RWA by the OCM was not reported in Refs. [5, 6], we connect it with the Whittaker function at $r = 6$ fm, where $r$ is the relative distance between $^{12}\text{C}$ and $α$.

In addition to microscopic wave functions, those simulated with the phenomenological PM [49] based on a Woods-Saxon potential are introduced in order to clarify how the wave function is probed on the transfer-cross sections. The parameters of the Woods-Saxon potential found in Tables I and III in Sec. III are chosen so that the PM-wave functions can be references to compare them with the microscopic wave functions (see Sec. III for more detail). The depth of the potential is adjusted to reproduce the $α$ separation energy of $^{16}\text{O}$. The PM-wave functions of both the $0_{1}^{+}$ and $0_{2}^{+}$ states are obtained using the experimental binding energy [57], whereas we approximate the $4_{1}^{−}$-resonance states as bound states having a binding energy of 0.01 MeV. Note that we adopt experimental $Q$-values of each $4_{1}^{−}$ state in the DWBA calculation.

The distorted wave in the initial (final) channel is calculated with the optical potential including the Coulomb interaction forming a uniformly charged sphere potential by Ref. [54] (Ref. [58]). We disregard the intrinsic spin of nuclei in the computation of the distorted waves. The no-recoil limit [49] is applied to the Hamiltonian associated with the distorted wave in the final channel. To obtain the $^6\text{Li}$ wave function, we employ the $α-d$ model [49, 59] with the two-range Gaussian interaction [60].

Here we mention the role of the excitation of the projectile $^6\text{Li}$ into continuum states. As discussed in Refs. [49, 59, 61], in $(^6\text{Li},d)$ reactions, the effect of the continuum excitation of $^6\text{Li}$ is able to be effectively taken into account by a $^6\text{Li}$-optical potential that accounts for elastic scattering of $^6\text{Li}$ appropriately. The optical potential [54] we adopt was parameterized through comparison of experimental data of $^6\text{Li}$-elastic scattering on $^{12}\text{C}$ at several incident energies. Therefore, it is expected that using this optical potential ensures effective prescription for the continuum excitation of $^6\text{Li}$.

III. RESULTS AND DISCUSSION

A. $0^{+}$ states

Figure 1 represents the $^{12}\text{C}-α$ relative wave function of the $0_{1}^{+}$ state of $^{16}\text{O}$ ($l = 0$) as a function of $r$. Each line in the figure is normalized so that the norm is unity. The thick-solid and thick-dashed lines are taken from the Refs. [5, 6] for the OCM and Ref. [23]) for 5BM, respectively, but now they are normalized. Compared to the wave function of the OCM, that of the 5BM has a surface peak spreading outward by $\sim 1.5$ fm and small amplitude in the inner region, $r \approx 3$ fm. In addition to the microscopic wave functions, the PM-wave functions are
whereas the OCM gives the smooth diffraction pattern with

total differential are given by

\[ \gamma_1 \gamma_2 \]

FIG. 1. The \(^{12}\text{C}-\alpha\) relative wave functions of the ground state of \(^{16}\text{O}\) with \(l = 0\) calculated by the OCM (thick-solid line), 5BM (thick-dashed line), PM-OCM (thin-solid line), PM-5BM (thin-dashed line), and PM-mid (dash-dotted line). The norm of each wave function is unity.

obtained by employing the parameters in Table I, where the radius and diffuseness parameters of the Woods-Saxon potential are given by \(121/3r_0\) and \(a_0\), respectively. The wave function of the PM-OCM (PM-5BM) expressed by the thin-solid line (thin-dashed line) is prepared to make its peak position at \(r \sim 2.5\) fm (\(r \sim 4\) fm) consistent with that of the OCM (5BM). Since it is difficult, only from the PM-OCM and PM-5BM, to clarify how the diffraction pattern of the transfer-cross section is sensitive to the surface-peak position of the wave function, we also adopt the PM-mid. The PM-mid wave function, plotted by the dash-dotted line, has the surface-peak position at the middle of those by the OCM and 5BM.

In Fig. 2, we compare the theoretical cross sections of \(^{12}\text{C}(^6\text{Li}, d)^{16}\text{O}(^3\text{He})\) as a function of the deuteron emitting angle \(\theta\) in the center-of-mass frame with the experimental data at the two incident energies, \(\varepsilon_1 = 42.1\) MeV [39] and \(\varepsilon_2 = 48.2\) MeV [45]. The thick solid, thick dashed, thin solid, thin dashed, and dash-dotted lines are the results obtained using \(\phi_1\) of the OCM, 5BM, PM-OCM, PM-5BM, and PM-mid, respectively. The calculated results are normalized with the normalization factor \(N_0\) listed in Table II by the \(\chi^2\) fit to the experimental data. In comparison of the lines with the measured data, we focus on the forward-angle region, namely, the first and second peaks and the first dip between the two peaks of the cross section. Then we extract the information of the \(\alpha\)-wave function from the position \(\theta\) of the peaks and dip, as well as the ratio of the first peak to the second peak.

At both the incident energies, the 5BM produces the cross section having the peak and dip positions consistent with those of the measured data at the forward-angle region, \(\theta \lesssim 30^\circ\), whereas the OCM gives the smooth diffraction pattern with the first dip at \(\theta \sim 50^\circ\) and completely fails to explain the data. It indicates that the shift of the surface peak of \(\phi_1\) arising from the \(\alpha\)-cluster breaking in the 5BM is essential to describe the ground state of \(^{16}\text{O}\). Our result supports the \(^0\text{He}\) wave function having the surface peak at \(\sim 4\) fm obtained by the 5BM.

The PM results make it clear how the surface peak of the wave function is probed on the cross section at the forward angles. Even though the PM-5BM wave function has the shape significantly different from that of the 5BM in the inner region (see Fig. 1), it reproduces the cross section almost identical with the dashed line at the forward angles, \(\theta \lesssim 50^\circ\). Furthermore, as the surface peak is populated inward by the PM-mid from that by the PM-5BM, the second peak of the cross section at \(\theta \sim 25^\circ\) decreases and its first dip is shifted backward. From the above results on the \(^0\text{He}\) states, we find that the surface peak of the wave function is crucial to describe the diffraction pattern of the cross section, and hence, not the inner region of the wave function but its surface is probed on the cross section at the forward angles. Indeed, we confirm numerically that the wave function at the inner region is absorbed by the imaginary part of the optical potentials. The surface-peak position of the \(^0\text{He}\) wave function is able to be determined by fixing eyes on the ratio of the first to second peaks and the position of the first dip of the cross section.

Now we show the results of the \(^0\text{He}\) state of \(^{16}\text{O}\). In Fig. 3 each line is the same as that in Fig. 1 but for the \(^0\text{He}\) state. The peak position of the wave function of the 5BM at \(r \sim 5\) fm, is slightly shifted outward from that of the OCM, while in

![Comparison of the cross sections of \(^{12}\text{C}(^6\text{Li}, d)^{16}\text{O}(^3\text{He})\) at 42.1 MeV (\(\varepsilon_1\)) and 48.2 MeV (\(\varepsilon_2\)) calculated using \(\phi_1\) of the OCM (thick-solid line), 5BM (dashed line), and PM (thin lines) with the experimental data [39, 45].](image)

FIG. 2. Comparison of the cross sections of \(^{12}\text{C}(^6\text{Li}, d)^{16}\text{O}(^3\text{He})\) at 42.1 MeV (\(\varepsilon_1\)) and 48.2 MeV (\(\varepsilon_2\)) calculated using \(\phi_1\) of the OCM (thick-solid line), 5BM (dashed line), and PM (thin lines) with the experimental data [39, 45].

TABLE I. The radius parameter \(r_0\) and diffuseness parameter \(a_0\) of the Woods-Saxon potential used in the PM to obtain the \(^0\text{He}\)-wave functions.

| \(r_0\) (fm) | \(a_0\) (fm) | \(0^+\text{He}\) PM-OCM | \(0^+\text{He}\) PM-5BM | \(0^+\text{He}\) PM-mid | \(0^+\text{He}\) PM-OCM | \(0^+\text{He}\) PM-5BM |
|------------|------------|----------------|--------------------|--------------------|----------------|--------------------|
| 1.000      | 0.520      | 1.875          | 1.625              | 1.250              | 1.375          | 1.625              |
| 1.125      | 0.845      | 1.625          | 1.250              | 1.250              | 1.375          | 1.625              |
| 1.250      | 0.780      | 1.625          | 1.250              | 1.250              | 1.375          | 1.625              |
| 1.375      | 0.715      | 1.625          | 1.250              | 1.250              | 1.375          | 1.625              |
| 1.625      | 0.845      | 1.625          | 1.250              | 1.250              | 1.375          | 1.625              |
the inner region, \( r \lesssim 3 \) fm, the amplitude of them are suppressed compared to those of the surface peak. The two PM-wave functions are prepared so that their surface peak coincides with that of the microscopic models.

Figure 4 represents comparison of the calculated cross sections with the experimental data. The legends stand for the same as those in Fig. 2 but now for the \( 0^+ \) state. The lines are normalized to the measured data by the \( \chi^2 \) fitting, which results are in \( N_0 \) given in Table II. At both the incident energies, the OCM adequately describes the measured data at the forward angles, \( \theta \lesssim 40^\circ \), where the first peak, second peak, and first dip are explained. In contrast, the 5BM does not account for the experimental diffraction pattern. In particular, it is crucial that the thick-dashed line has the first peak of the cross section at the \( \theta \approx 0^\circ \) smaller than the second peak at \( \theta \approx 25^\circ \) for \( \varepsilon_1 \), and its dip is shifted forward. It suggests that the \( 0^+ \)-wave function has the surface peak at \( \sim 4.5 \) fm predicted by the OCM, and the 5BM does not provide appropriately the \( \alpha \) probability of the \( 0^+ \) state at the surface, which is sensitively affected on the transfer-cross section.

Although, on the PM-wave functions, their amplitude in the inner region is significantly larger than that of the microscopic-wave functions, each PM produces the cross sections almost same as those by corresponding microscopic models, i.e., the transfer reactions are peripheral as in the \( 0^+ \) case. The main difference between the PM-OCM and PM-5BM on the wave function can be seen at the surface-peak position, which essentially determines the cross sections. By comparing the cross sections of the PM-OCM and PM-5BM, we see that the ratio of the first peak to the second peak becomes smaller and the first dip moves forward, with manifestation of the surface peak on the wave function, as clarified for the \( 0^+ \) state. Thus we find that the inspection of the cross section at the forward angles enables us to identify the surface-peak position of the \( \alpha \)-wave function of the \( 0^+ \) states.

To draw a conclusion from the \( 0^+ \) results, we recall features of the two microscopic models. The OCM describes \({}_{16}^\text{O}\) with the \( ^{12}\text{C} + \alpha \) configuration, where the core state \( ^{12}\text{C}(0^+) \) is calculated based on the mixing of the \( 0p_{3/2}^- \)-subshell-closed configuration and the \( 3\alpha \) configuration, involving the excitation of \( ^{12}\text{C} \) as well. The 5BM addresses a dynamical process of \( \alpha \) clusters by the four-nucleon correlation, which induces the dissociation (manifestation) of \( \alpha \) particles at the interior (exterior) of \( ^{16}\text{O} \), though \( ^{12}\text{C} \) is assumed to have the \( 0p_{3/2}^- \)-subshell-closed configuration only. Our DWBA analysis for the \( 0^+ \) state supports not the OCM-wave function but that of the 5BM. This is due to that the \( \alpha \) cluster is hard to form at the surface owing to its dissociation, and hence its probability is shifted outward. For the \( 0^+ \) state, however, the OCM-wave function rather than that of the 5BM is reasonable to account for the transfer reaction. For further clarification, it is desired to perform a calculation that addresses simultaneously the \( \alpha \) dissociation and core polarization of \( ^{12}\text{C} \).

As a summary of this subsection, we comment on the normalization factor \( N_0 \) in Table II, where two features are found; (i) some of them exceed unity and (ii) they strongly depend on the incident energy for each \( 0^+ \) state. As we argued the former point (i) in Ref. [49], the normalization factors extracted from our DWBA calculations are not necessarily same as physical SFs, because the transfer reactions we analyze in this work probe only the surface region of the \( ^{12}\text{C}-\alpha \) wave function, and thus we may need an artificial enhancement in order to increase the tail amplitude of the wave function. To clarify the origin of the latter fact (ii), in future, a systematic analysis of \((^6\text{Li}, d)\) reactions at several incident energies is desirable.

| \( \varepsilon_1 \) | OCM | 5BM | PM-OCM | PM-5BM | PM-mid |
|---|---|---|---|---|---|
| \( 0^+ \) | 1.455 | 1.494 | 1.873 | 3.040 | 6.617 |
| \( \varepsilon_2 \) | 0.532 | 0.600 | 0.708 | 1.295 | 3.032 |

| \( \varepsilon_1 \) | OCM | 5BM | PM-OCM | PM-5BM |
|---|---|---|---|---|
| \( 0^+ \) | 1.499 | 1.035 | 2.593 | 0.831 |
| \( \varepsilon_2 \) | 0.617 | 0.297 | 1.430 | 0.238 |
B. \(4^+\) states

In this subsection, we report the results for the analysis of the \(^{12}\text{C}(^6\text{Li}, d)^{16}\text{O}\) reaction generating the \(4^+_1\) and \(4^+_2\) states of \(^{16}\text{O}\). First, we try to determine the surface-peak position from the angular distribution of the \(\alpha\)-transfer cross section and present that our calculation is not satisfactory to confirm it uniquely. Second, we compare surface \(\alpha\)-probability between the \(4^+_1\) and \(4^+_2\) states by means of inclusive data such as normalization factors and reduced widths.

Only for the \(4^+_1\) state, the microscopic-wave function by the OCM is available. As shown in Fig. 5(b), the OCM-wave function expressed by the solid line has a surface peak at around 4.5 fm. Using the parameters listed in Table III, we prepare three sets of \(\phi_0\) with \(l = 4\) for each \(4^+\) state by the PM, PM1a, PM1b, and PM1c (PM2a, PM2b, and PM2c) characterized by one node (two nodes). The PM2b is calculated so that its surface-peak position coincides with that of the OCM. Then, we select the Woods-Saxon parameter in order to let the PM2a (PM2c) has the surface-peak positions inside (outside) that of the PM2b. The same sets of \(r_0\) and \(a_0\) are used for the one-node wave functions, with adjusting the depth of the potential. The PM-wave functions are plotted as the thin lines in Fig. 5. The norm of each wave function is unity.

Figures 6(a) and 6(b) present the theoretical cross section of \(^{12}\text{C}(^6\text{Li}, d)^{16}\text{O}(4^+_1)\) obtained with the one-node and two-node wave functions, respectively. The lines are normalized with \(N_0\) found in Table IV to the experimental data expressed by the dots and triangles for the incident energies, \(\varepsilon_1\) [39] and \(\varepsilon_2\) [45], respectively. It is difficult to sift the reference wave functions only from the cross section at \(\varepsilon_1\), because the diffraction pattern of the measured data at \(\varepsilon_1\) is not distinct at the forward angles, and the none of the theoretical results of both the one-node and two-node cases is able to explain the experimental data. At \(\varepsilon_2\), all the calculated results except the PM1a and PM2a almost identically coincide with the measured data at forward angles, \(\theta \lesssim 30^\circ\). The PM1a at \(\varepsilon_2\) produces the dip at \(\sim 30^\circ\), while the PM2a at \(\varepsilon_2\) gives the first peak \(\theta = 0^\circ\) smaller than the second peak at \(\theta \sim 30^\circ\). In addition, the PM1a requires unphysical value of \(N_0\) such that it exceeds 10, as shown in Table IV. From these consequences, we consider that the PM1a and PM2a are not eligible for a \(4^+_1\)-wave function that accounts for the experimental cross sections. Thus, we confirm that the surface-peak position of \(4^+_1\) is outer than \(r \sim 4\) fm.

The results of the \(4^+_1\) state are shown in Fig. 7, where the legends are the same as those in Fig. 6. In Fig. 7(a), at both the incident energies, every line coincides with the experimental data within the region, \(\theta \lesssim 20^\circ\), even though the PM1a gives the first dip backward compared to the other PMs. The calculations with the two-node PM in Fig. 7(b) reasonably explain the experimental data of both the incident energies at \(\theta \lesssim 20^\circ\), except for the PM2a, which gives the cross section having the

![FIG. 5. The \(l = 4\) wave functions, which norm is unity, having (a) one node and (b) two nodes calculated by the OCM (thick-solid line) and PM (thin lines).](image1)

![FIG. 6. The angular distributed cross section of \(^{12}\text{C}(^6\text{Li}, d)^{16}\text{O}(4^+_1)\) at 42.1 MeV (\(\varepsilon_1\)) and 48.2 MeV (\(\varepsilon_2\)) calculated with (a) the one-node and (b) the two-node wave functions. In Fig. 6(a), the experimental data expressed by dots [39] (triangles [45]) are same as those in Fig. 6(b).](image2)
first peak at $\theta = 0^\circ$ smaller than second peak at $\theta \sim 30^\circ$. Due to the above results, we cannot clarify the spatial manifestation of the $\alpha$-cluster structure of the $4^+_2$ state from the transfer-cross section.

Note that, in Fig. 7, the measured cross sections at both the incident energies contain the event of the $3^+_1 (11.08 \text{ MeV})$ state, although it is expected to be small for the present case. Owing to the resolution of the experiments [39, 45], the contribution of the $3^+_1$ state being just 20 keV below the $4^+_2$ state is difficult to be separated on the cross section. However, in Ref. [41], it was revealed that the cross section of the $3^+_1$ state at the forward angles is only 15-20% of the total yield for the $(3^+_1 + 4^+_2)$-doublet, within the range of the incident energy from 20 to 34 MeV of the transfer reactions, $^{12}\text{C}(^6\text{Li}, d)^{16}\text{O}$ and $^{12}\text{C}(^7\text{Li}, t)^{16}\text{O}$. Therefore we disregard the $3^+_1$ contribution in the comparison of the calculated and measured results at forward angles.

We confirm numerically that the transfer reactions populating both the $4^+_2$ states probe only the tail part of the $\alpha$-wave function, because of absorption by the imaginary part of the optical potentials. The peripherality is also deduced from a fact that it is difficult to verify the number of nodes of the $4^+_2$ wave function. Therefore we see that the cluster component of the $4^+_2$ state is observed in the cross section of the transfer reaction only through the surface region of the wave function.

In Table IV, by comparing $N_0$ between the $4^+_1$ and $4^+_2$ states within the same incident energies and the same PM, one finds that every value of $N_0$ of the $4^+_1$ state is significantly greater than that of the $4^+_2$ state. To understand relation between the peripherality and the feature of $N_0$, we compare the experimental reduced $\alpha$-width with those extracted from $N_0$ and $\phi_l$.

In Table V we report the dimensionless reduced $\alpha$-width $\theta^2 l(a)$ at the channel radius $a = 6.0 \text{ fm}$ (see Appendix A for definitions). The experimental data listed at the rightmost column is evaluated from the measured $\alpha$-decay width [25].

![FIG. 7](https://example.com/fig7.png)

**FIG. 7.** Same as Fig. 6 but for the $4^+_1$ state.
on the coupled-channels Born approximation, as discussed in Refs. [37, 40, 41]. Such advanced reaction framework may resolve the discrepancy between the α-transfer reaction and α-decay width.

IV. SUMMARY

We have investigated the spatial manifestation of the α-cluster structure of 16O through the DWBA analysis of the α-transfer reaction 12C(6Li, d)16O. It is remarkable that we have shown how much α-cluster states spatially manifest itself from the inspection of α-transfer cross sections for not only the ground state but also excited states, without using SFs.

By testing the 12C-α relative wave functions obtained in the previous studies with the microscopic models, the OCM [5, 6] and 5BM [23], we have verified that the α-cluster structure manifests itself at the radius r ≈ 4 fm (r ≈ 4.5 fm) for the 0+ (0+) state as predicted by the 5BM (OCM). By introducing the phenomenological PM, we have clarified correspondence between the α-wave function and the transfer-cross section. It has been found that the α-transfer cross section regarding both the 0+ and 0+ states of 16O at the forward angles probes only the surface region of the wave function, and the surface peak position of the wave function is able to be determined from the first and second peaks of the cross section as well as its first dip.

We have confirmed that the α-transfer reaction populating the 4+ states is peripheral. Although we have verified that the 4+ state has the surface peak at r ≈ 4 fm or outer, it is difficult to uniquely determine it. For the 4+ state, it is impossible to distinguish the α probability from the cross sections, and the puzzle between the cross section and the α-decay width has been observed. We have concluded for the 4+ state that the cluster component is not dominant but finite, and not only the 12C + α but also other configurations play a role. To extract spatial manifestation of the α-cluster structure in the 4+ states, from a theoretical point of view, calculations such as the coupled-channels Born approximation, as well as the coupled-reaction channels and compound-nucleus processes, are expected to be performed in future to describe 12C(6Li, d)16O(4+). In an experimental side, it is desirable to carry out measurement that is possible to separate the 3+ event from that of the 4+ state.

Appendix A: Calculation of reduced α-width

The reduced α-width γl, which represents the α probability at the channel radius a, is defined by

\[ \gamma_l^2(a) = \frac{\Gamma_l}{2\rho_l(a)}. \]  

(A1)

Here the Coulomb penetrability \( \rho_l \) is given by

\[ \rho_l(a) = \frac{ka}{F_l^2(ka) + G_l^2(ka)}, \]  

(A2)

with the regular and irregular Coulomb functions \( F_l \) and \( G_l \), respectively, and the 12C-α-relative wave number \( k \), which is determined uniquely from α-separation energies. Using a measured value of the α-decay width \( \Gamma_l \), we can extract experimental \( \gamma_l \). Alternatively, \( \gamma_l \) is also evaluated from \( N_0 \) and \( \phi_l \) as

\[ \gamma_l^2(a) = \frac{\hbar^2 N_0}{2\mu} |\phi_l(a)|^2, \]  

(A3)

where \( \mu \) is the reduced mass of the 12C-α system. In Table V of Sec. III B, the dimensionless reduced α-width is evaluated by

\[ \delta_l^2(a) = \frac{\gamma_l^2(a)}{\gamma_W^2(a)}. \]  

(A4)

with the Wigner single-particle limit,

\[ \gamma_W^2(a) = \frac{3\hbar^2}{2\mu a^2}. \]  

(A5)

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[1] J. A. Wheeler, Phys. Rev. 52, 1083 (1937).
[2] J. A. Wheeler, Phys. Rev. 52, 1107 (1937).
[3] D. M. Dennison, Phys. Rev. 96, 378 (1954).
[4] D. M. Brink, H. Friedrich, A. Weiguny, and C. W. Wong, Phys. Lett. B33, 143 (1970).
[5] Y. Suzuki, Prog. Theor. Phys. 55, 1751 (1976).
[6] Y. Suzuki, Prog. Theor. Phys. 56, 111 (1976).
[7] Y. Fujiwara, H. Horiuchi, K. Ikeda, M. Kamimura, K. Katô, Y. Suzuki, and E. Uegaki, Prog. Theor. Phys. Suppl. 68, 29 (1980).
[8] M. Libert-Heinemann, D. Baye, and P.H. Heenen, Nucl. Phys. A 339, 429 (1980).
[9] W. Bauhoff, H. Schultheis, and R. Schultheis, Phys. Rev. C 29, 1046 (1984).
[10] P. Descouvemont, Nucl. Phys. A 470, 309 (1987).
[11] P. Descouvemont, Phys. Rev. C 44, 306 (1991).
[12] P. Descouvemont, Phys. Rev. C 47, 210 (1993).
[13] K. Fukatsu and K. Katô, Prog. Theor. Phys. 87, 151 (1992).
[14] Y. Funaki, T. Yamada, H. Horiuchi, G. Röpke, P. Schuck, and A. Tohsaki, Phys. Rev. Lett. 101, 082502 (2008).
