Hard diffractive electroproduction, 
transverse momentum distribution 
and QCD vacuum structure

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Abstract:

We study the impact of the "intrinsic" hadron transverse momentum on the pre-asymptotic behavior of the diffractive electroproduction of longitudinally polarized ρ-meson. Surprisingly, we find the onset of the asymptotic regime in this problem to be rather low, $Q^2 \simeq 10 \text{ GeV}^2$ where power corrections due to the transverse momentum do not exceed 20 % in the amplitude. This drastically contrasts with exclusive amplitudes where the asymptotics starts at much higher $Q^2 = 50 - 100 \text{ GeV}^2$. The sources of such unexpected behavior are traced back to some general (the quark-hadron duality) as well as more silent (properties of higher dimensional vacuum condensates) features of QCD.

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1 Introduction

Over the last couple of years, following the paper [1], the diffractive electroproduction of the light vector mesons has been investigated within the framework of perturbative QCD. Since this paper a few more important results have been obtained. First of all it is a rigorous QCD-based proof [2] that an amplitude of diffractive electroproduction of vector mesons has required factorization properties. The second important step in the same direction is an introduction of a new type of functions (the so-called antisymmetric structure functions) [3] which are relevant objects for such kind of problems. Therefore, the standard perturbative QCD (pQCD) approach to this new class of problems which are currently intensively studied experimentally [4] has obtained a solid basis for the future investigation.

We should note that the diffractive electroproduction had been discussed many times before the recent paper [1], see e.g. [5] and references therein. However, all previous approaches to the problem were mainly based on some kind of quark models with inevitable to such methods specific assumptions not following from QCD. This is certainly a self-consistent approach for a heavy quark system (where the size of a hadron is of the order of $1/m_Q$ and is always small by kinematical reasons), but not for the light hadrons, where the problem of the separation of small and large distance physics cannot even be formulated in an appropriate way within the quark model. Let us remind that the main idea of the QCD-based approach is just such a separation of the large and small distance physics, the so-called Wilson Operator Product Expansion (OPE). At small distances one can use the standard perturbative expansion due to the asymptotic freedom and smallness of the coupling constant. All nontrivial, large distance physics is hidden in nonperturbative hadron wave functions (WF’s) in this approach. They cannot be found by the perturbative technique, but rather should be extracted from elsewhere. Therefore, the problem is reduced to the analysis of the bound states within OPE.

The problem of bound states in the relativistic quantum field theory with large coupling constant is, in general, an extremely difficult problem. Understanding the structure of the bound state is a very ambitious goal which assumes the solution of a whole spectrum of tightly connected problems, such as confinement, chiral symmetry breaking phenomenon, and many others which are greatly important in the low energy region. Fortunately, a great deal of information can be obtained even in absence of such a detailed knowledge. This happens in high energy processes where the only needed nonperturbative input are the so-called light-cone WF’s with a minimal number of constituents. In this case a problem becomes tractable within existing nonperturbative methods.

We have therefore a well-formulated problem of the diffractive electroproduction of vector mesons. The formulation is based on the solid background of QCD. However, calculations of the pQCD approach refer, strictly speaking, only to asymp-
totically high energies \[1\]. Therefore, as usual, the main question remains: at what energies do the asymptotic formulae of Ref.\[1\] start to work? The prime goal of the present work is to attempt to answer this question. We should note that there are two very different types of pre-asymptotic corrections to the asymptotically leading formula \[1\]. First, there exist the so-called shadowing (rescattering) corrections \[6\] which we do not discuss in this paper at all. Our concern is rather with a second kind of corrections which are related to the internal hadron structure of the vector meson. This kind of corrections was discussed for the first time in Ref.\[7\], and we will discuss the relation of the paper \[7\] to our results later on.

However, before deepening in details, we would like to recall that a very similar question about an applicability of the QCD-based approach to exclusive amplitudes has been discussed during the last fifteen years. The history of this development is very instructive and we believe it is worthwhile to mention it shortly here.

As is known, at asymptotically high energies the parametrically leading contributions to hard exclusive processes can be expressed in terms of the so-called distribution amplitude \(\phi(x)\) \[8\], which itself can be expressed as an integral \(\int d^2\mathbf{k}_1 \psi(\mathbf{k}_1^2, x)\) with nonperturbative wave function \(\psi(\mathbf{k}_1^2, x)\), see reviews \[9\],\[10\] for details. Distribution amplitudes \(\phi(x)\) depend only on longitudinal variables \(x_i\) and not on transverse \(\mathbf{k}_1^2\) ones. The same is true for inclusive reactions where structure functions depend on \(x\), but not on \(\mathbf{k}_1^2\). Thus, any dependence on \(\mathbf{k}_1^2\) gives some power corrections to the leading terms. Naively one may expect that these corrections should be small enough already in the few \(\text{GeV}^2\) region. These expectations are mainly based not on a theoretical analysis but rather on the phenomenological observation that the dimensional counting rules, proposed in early seventies (see \[11\]), agree well with the experimental data such as the pion and nucleon form factors, large angle elastic scattering cross sections and so on. This agreement can be interpreted as a strong argument that power corrections are small in the few \(\text{GeV}^2\) region.

However, in mid eighties the applicability of the approach \[8\] at experimentally accessible momentum transfers was questioned \[12\],\[13\]. In these papers it was demonstrated, that the perturbative, asymptotically leading contribution is much smaller than the nonleading ”soft” one. Similar conclusion, supporting this result, came from the different side, from the QCD sum rules \[14\],\[15\], where the direct calculation of the form factor has been presented at \(Q^2 \leq 3\text{GeV}^2\). This method has been extended later for larger \(Q^2 \leq 10\text{GeV}^2\) \[16\],\[17\] with the same qualitative result: the soft contribution is more important in this intermediate region than the leading one.

* The formal reason for that can be seen from the following arguments. At large energies the quark and antiquark are produced at small distances \(z \sim 1/Q \to 0\), where \(Q\) is typical large momentum transfer. Thus one can neglect the \(z^2\) dependence everywhere and one should concentrate on the one variable \(zQ \sim 1\) which is of the order of one. One can convince oneself that the standard Bjorken variable \(x\) is nothing but Fourier conjugated to \(zQ\).
Therefore, nowadays it is commonly accepted that the asymptotically leading contribution to the exclusive amplitudes cannot provide the experimentally observable absolute values at accessible momentum transfers. If we go along with this proposition, then the natural question arises: How can one explain the very good agreement between the experimental data and dimensional counting rules if the asymptotically leading contribution cannot explain the data for experimentally accessible energies? A possible answer was suggested recently [18] and can be formulated in the following way: very unusual properties of the transverse momentum distribution of a hadron lead to the mimicry of the dimensional counting rules by the soft mechanism at the extended range of intermediate momentum transfers. Numerically, the soft term is still more important than the asymptotically leading contribution at rather high \(Q^2 \sim 50 \div 100 GeV^2\).

Now we come back to our original problem of the hard diffractive electroproduction. Having the experience with exclusive processes in mind, one could expect a similar behavior (i.e. a very slow approach to the asymptotically leading prediction) for the diffractive electroproduction as well. The main goal of the present work is to argue that this naive expectation is wrong. The asymptotically leading formula starts to work already in the region \(Q^2 \sim 10 GeV^2\), where power corrections do not exceed the 20% level in the amplitude. This surprising result is a consequence of very special properties of the WF in the \(\vec{k}^2_\perp\) variable (as we mentioned earlier, the \(\vec{k}^2_\perp\) dependence determines corrections to the leading term). At the same time, as we argue below, these properties of WF are very sensitive to the QCD vacuum structure, more exactly to some higher dimensional vacuum condensates. As a result, we find that the pre-asymptotic behavior of the \(\rho\)-meson diffractive electroproduction amplitude essentially depends on the (non-) factorizability of particular mixed quark-gluon condensates. In our opinion, this makes diffractive processes an excellent laboratory for testing our current understanding of the QCD vacuum structure. In contrast to the standard QCD sum rules for static properties of hadrons [21] which probe most crude characteristics of the QCD vacuum like \(\langle \bar{\psi}\psi \rangle, \langle g^2 G^2 \rangle\), diffractive amplitudes are remarkably influenced by far more silent vacuum features. Moreover, in the latter case all these theoretical ideas can in principle be given a dynamical test by varying the photon virtuality \(Q^2\) and measuring a deviation from the asymptotic predictions of Ref.[1] (provided, of course, all other sources of the \(Q^2\) dependence such as shadowing corrections [6] and the \(Q^2\) evolution of the gluon distribution are properly taken into account). One can therefore say that by studying the pre-asymptotic behavior of diffractive amplitudes we actually study the fine structure of the QCD vacuum.

Our presentation is organized as follows. In Sect.2 we collect relevant definitions and discuss general constraints on the nonperturbative \(\rho\)-meson WF stemming from general principles of QCD. Sect.3 deals with the calculation of the lowest moments \(\langle \vec{k}^2_\perp \rangle\) and \(\langle \vec{k}^4_\perp \rangle\) of the \(\rho\)-meson transverse momentum distribution from the equations of motion and QCD sum rules. The main observation of this section is a presence of strong fluctuations in the transverse direction \(\langle \vec{k}^4_\perp \rangle/\langle \vec{k}^2_\perp \rangle^2 \gg 1\). We show how this
ratio is fixed by particular mixed quark-gluon condensates. In Sect. 4 we propose a model WF satisfying the general constraints of Sect. 2 as well as the lowest moments \( \langle \vec{k}_2^2 \rangle \) and \( \langle \vec{k}_4^4 \rangle \) of Sect. 3. All these results are applied to the study of pre-asymptotic effects due to the "intrinsic" transverse momentum (the Fermi motion) in the hard diffractive electroproduction in Sect. 5, where we also present our understanding of a fundamental difference between exclusive and diffractive processes. As a by-product of our study, we propose a simple reformulation of the diffractive amplitude in terms of a current Green function. Appendix A establishes a connection between the transverse momentum distribution we are dealing with and higher twist distribution amplitudes of the standard light-cone OPE approach. Methods for estimates of relevant higher dimensional vacuum condensates are described in Appendix B. Sect. 6 presents a summary of our results.

2 General constraints on the nonperturbative wave function \( \Psi(\xi, k_\perp^2) \).

The aim of this section is to provide necessary definitions and establish some essentially model independent constraints on the non-perturbative WF \( \Psi(\xi, k_\perp^2) \) which follow from the use of such general methods as dispersion relations, duality and large order perturbation theory. For definiteness, we will consider the longitudinally polarized charged \( \rho \)-meson.

Let us start with the definition of the non-perturbative WF \( \Psi(\xi, b^2) \) in the so-called impact parameter representation as an infinite series of local gauge invariant operators (a low normalization point \( \mu^2 \sim 1 \text{ GeV}^2 \) is implied and \( b^2 \equiv -z^2 \))

\[
\sum_n \frac{\gamma^n}{n!} \langle 0 | \bar{d}(0) \gamma_\mu (iz_\nu \leftrightarrow \nabla_\nu) \hat{u}(0) | \rho(q) \rangle = f_\rho m_\rho \varepsilon_\mu^{(\lambda)} (zq, b^2) \\
= f_\rho m_\rho \varepsilon_\mu^{(\lambda)} \int_{-1}^{1} d\xi e^{-iqz} \Psi(\xi, b^2) \tag{1}
\]

Here \( \nabla_\nu = \hat{\nabla}_\nu - \hat{\nabla}_\nu - igA_\nu \) and \( \varepsilon_\mu^{(\lambda)} \) stands for the polarization vector.

In the asymptotic limit of hard exclusive processes [8, 9] the quark and antiquark are produced at small distances \( z \sim 1/Q \to 0 \) where \( Q \) is a typical large momentum transfer. In this case the only remaining dependence on the variable \( zq \sim 1 \) is given by the leading twist distribution amplitude (DA) \( \phi(\xi) \):

\[
\langle 0 | \bar{d}(0) \gamma_\mu (iz_\nu \leftrightarrow \nabla_\nu) \hat{u}(0) | \rho(q) \rangle = f_\rho m_\rho \varepsilon_\mu^{(\lambda)} (zq) \phi(\xi) = f_\rho m_\rho \varepsilon_\mu^{(\lambda)} (zq) \int_{-1}^{1} d\xi \xi^n \phi(\xi) \\
\xi = 2u - 1, \quad z^2 = 0 \tag{2}
\]

In the infinite momentum frame (IMF) \( q_z \to \infty \) the DA \( \phi(\xi) \) describes the distribution of the total longitudinal momentum \( q_z \) between the quark and antiquark.
carrying the momenta \( u q_z \) and \( (1 - u)q_z \), respectively. In what follows we will use the both variables \( \xi \) and \( u \), \( \bar{u} \equiv 1 - u \) interchangeously.

A portion of model independent information on the DA \( \phi(\xi) \) can be obtained using the dispersion relations and quark-hadron duality. To this end, we study the asymptotic behavior of the correlation function

\[
i \int dx e^{iqx} \langle 0|T J^\parallel_n(x), J_0(0)|0\rangle = (zq)^{n+2} I_n(q^2), \quad J^\parallel_n = \bar{d} \gamma_\mu z_\mu(i \not\nabla_{\mu} z_\mu)^n u \tag{3}
\]

(here the label \( \parallel \) stands for the longitudinal direction which is selected by the projector \( z_\mu : z^2 = 0, z_\mu q_\mu \neq 0 \).) At large \( q^2 \to -\infty \) the exact answer for the correlation function (3) is given by the perturbative one-loop diagram

\[
\frac{1}{\pi} \int_0^\infty ds \frac{\text{Im} I_n^{\text{pert}}(s)}{s - q^2}, \quad \text{Im} I_n^{\text{pert}}(s) = \frac{3}{4\pi(n+1)(n+3)}. \tag{4}
\]

On the other hand, in the IMF the correlation function (3) can be parametrized in terms of intermediate hadron states, the lowest one being the \( \rho \)-meson. Therefore the following relation takes place

\[
\frac{1}{\pi} \int_0^{S^\parallel_n} ds \text{Im} I(s)_n^{\text{pert}} = \frac{1}{\pi} \int_0^\infty ds \text{Im} I(s)_n^\rho, \tag{5}
\]

The only assumption made here is that the \( \rho \)-meson gives a non-zero contribution in (5), i.e. the duality interval \( S^\parallel_n \) (the subscript \( \parallel \) specifies the longitudinal direction) does not vanish for arbitrary \( n \). No further assumptions on highly excited states (like those imposed in the QCD sum rules approach) are needed. In this case Eq. (5) yields

\[
f_\rho^2 \langle \xi^n \rangle (n \to \infty) \to \frac{3S_n^{\infty}(\parallel)}{4\pi^2 n^2} \tag{6}
\]

(we have used the fact that for the longitudinally polarized \( \rho \)-meson in the IMF \( \varepsilon^{(\lambda)}_\mu = q_\mu/m_\rho + O(1/q_z) \).) This formula unambiguously fixes the end-point behavior of the DA (10):

\[
\langle \xi^n \rangle = \int_{-1}^{1} d\xi \xi^n \phi(\xi) \sim 1/n^2, \quad \phi(\xi \to \pm 1) \to (1 - \xi^2). \tag{7}
\]

We want to emphasize that the constraint (7) is of very general origin and follows directly from QCD. No numerical approximations were involved in the above derivation. Pre-asymptotic as \( q^2 \to -\infty \) perturbative and non-perturbative corrections are only able to change the duality interval in Eq. (5) (which is an irrelevant issue, anyhow) but not the parametric \( 1/n^2 \) behavior.

The same analysis can be done for the transverse distribution. It moments are defined analogously to Eq.(2) through gauge invariant matrix elements

\[
\langle 0|\bar{d} \gamma_\mu (i \not\nabla_{\nu} t_{\nu})^{2n} u |\rho(q)\rangle = f_\rho m_\rho \varepsilon_\mu^{(\lambda)} (-t^2)^n (2n - 1)!! (k_\perp^{2n})^n, \tag{8}
\]
where transverse vector $t_\mu = (0, \vec{t}, \bar{0})$ is perpendicular to the hadron momentum $q_\mu = (q_0, 0_\perp, q_z)$. The factor $(\frac{2(n-1)!!}{(2n)!!})$ is introduced to (8) to take into account the integration over $\phi$ angle in the transverse plane: $\int d\phi (\cos \phi)^{2n}/\int d\phi = (\frac{2(n-1)!!}{(2n)!!})$.

By analogy with a non-gauge theory we call $\langle \vec{k}_2 \perp \rangle$ in this equation the mean value of the quark perpendicular momentum, though it does not have a two-particle interpretation. The usefulness of this object and its relation to the higher Fock components will be discussed at the end of this section. Here we note that Eq. (8) is the only possible way to define $\langle \vec{k}_2 \perp \rangle$ in a manner consistent with gauge invariance and operator product expansion.

To find the behavior $\langle \vec{k}_2 \perp \rangle$ at large $n$ we can repeat the previous duality arguments with the following result $\dag$:

$$f^2_\rho \langle \vec{k}_2 \perp \rangle (\frac{2(n-1)!!}{(2n)!!}) \sim n! \Rightarrow f^2_\rho \langle \vec{k}_2 \perp \rangle \sim n!$$

This behavior (for the case of pion) has been obtained in Ref.[19] by the study of large order perturbative series for a proper correlation function. Dispersion relations and duality arguments transform this information into Eq.(9). It is important to stress that any nonperturbative wave function should respect Eq.(9) in spite of the fact that apparently we calculate only the perturbative part (see the comment after Eq.(7)). The duality turn this perturbative information into exact properties of the nonperturbative WF.

The most essential feature of Eq. (9) is its finiteness for arbitrary $n$. This means that higher moments

$$\langle \vec{k}_2 \perp \rangle = \int d\vec{k}_2 \perp d\xi \vec{k}_2 \perp \Psi(\vec{k}_2 \perp, \xi)$$

do exist for any $n$. In this formula we introduced the nonperturbative $\Psi(\xi, k_2 \perp)$ normalized to one $\‡$. Its moments are determined by the local matrix elements (8). The relation to the longitudinal distribution amplitude $\phi(\xi)$ looks as follows:

$$\int d\vec{k}_2 \perp \Psi(\xi, k_2 \perp) = \phi(\xi) , \int_{-1}^{1} d\xi \phi(\xi) = 1$$

The existence of the arbitrary high moments $\langle \vec{k}_2 \perp \rangle$ means that the nonperturbative $\Psi(\xi, k_2 \perp)$ falls off at large transverse momentum $\vec{k}_2 \perp$ faster than any power function. The relation (9) fixes the asymptotic behavior of $\Psi(\xi, k_2 \perp)$ at large $\vec{k}_2 \perp$. Thus, we arrive at the following constraint:

$$\langle \vec{k}_2 \perp \rangle = \int d\vec{k}_2 \perp d\xi \vec{k}_2 \perp \Psi(\vec{k}_2 \perp, \xi) \sim n! , \quad n \to \infty$$

$\dag$ Here and in what follows we ignore any mild (nonfactorial) $n$-dependence.

$\‡$ To avoid possible misunderstanding, we stress that our definitions are very different from those made in the light-cone perturbation theory $\‡$ where $\langle \vec{k}_2 \rangle_{BS} = \int_{-1}^{1} dx \int d^2\vec{k}_2 \perp \langle |\Psi_{BS}(\xi, \vec{k}_2 \perp)|^2 \rangle$ (here $BS$ stands for bound state). Our approach does not suppose at all the existence of a bound state equation in QCD, but rather relies entirely on the logic of OPE.
We can now repeat our duality arguments again for an arbitrary number of transverse derivatives and large \((n \to \infty)\) number of longitudinal derivatives. The result reads \([20]\):

\[
\int d\vec{k}^2 \Psi(\vec{k}_\perp^2, \xi \to \pm 1) \sim (1 - \xi^2)^{k+1}
\]  \(\text{(13)}\)

The constraint \((13)\) is extremely important and implies that the \(\vec{k}_\perp^2\) dependence of \(\Psi(\vec{k}_\perp^2, \xi)\) comes \textbf{exclusively in the combination} \(\vec{k}_\perp^2/(1 - \xi^2)\) at \(\xi \to \pm 1\). This means that the standard assumption on factorizability \(\Psi(\vec{k}_\perp^2, \xi) = \psi(\vec{k}_\perp^2)\phi(\xi)\) is at variance with very general properties of the theory such as duality and dispersion relations. The only form of \(\Psi(\xi, k^2_\perp)\) satisfying all the constraints \((7), (12)\) and \((13)\) is the Gaussian with a very particular argument:

\[
\Psi(\vec{k}_\perp^2 \to \infty, \xi) \sim \exp\left(-\frac{\vec{k}_\perp^2}{\Lambda^2(1 - \xi^2)}\right)
\]

\(\text{(14)}\)

(here \(\Lambda^2\) is a mass scale which can be fixed by calculating the moments \(\langle \vec{k}_\perp^2 \rangle, \langle \vec{k}_\perp^4 \rangle\) etc.) Strictly speaking, so far we have only established the validity of Eq.\((14)\) in a vicinity of the end-point region \(\xi \to \pm 1\). In Appendix A we discuss in what sense Eq.\((14)\) can be approximately valid in the whole range of the \(\xi\) variable.

Here a few comments are in order. Our result \((14)\) essentially supports the old quark model-inspired idea on the SU(6) invariance of hadron WF’s. Indeed, the very same analysis carries without any changes for pseudoscalar \(\pi\) - and \(K\) -mesons (non-zero s-quark mass effects are subleading). A possible violation of SU(6) is expected via a variation of the parameter \(\Lambda^2\) for different hadrons. Moreover, the ansatz \((14)\) coincides with the well known harmonic oscillator WF, except for the mass term. One should stress that there is no room for such term in QCD. Its inclusion violates the duality constraint \((7)\) since in this case we would have

\[
\langle \xi^n \rangle \sim \int_{-1}^{1} d\xi \xi^n \exp\left(-\frac{m^2}{\Lambda^2(1 - \xi^2)}\right) \sim \exp(-\sqrt{n}) , \ n \to \infty
\]

\(\text{(15)}\)

instead of the \(1/n^2\) behavior \((7)\). In other words, a true nonperturbative WF must respect the asymptotic freedom which is incompatible with the quark model -type mass term in the WF \([1]\). This difference leads to interesting phenomenological consequences, see Ref.\([18]\) for more detail. Furthermore, we would like to clarify the correspondence between our results \((14)\) and those obtained in Ref.\([23]\) on similar grounds. There, it was also claimed that \(\vec{k}_\perp^2\) and \(\xi\) enter the WF only in the combination \(\vec{k}_\perp^2/(1 - \xi^2)\) but a different WF was suggested, \(\Psi(\xi, k^2_\perp) \sim \Theta(s_0 - \vec{k}_\perp^2/(1 - \xi^2))\).

\(\text{§}\)To set this more accurately, one can say that a possible (scale-dependent) mass term in \(\Psi(\xi, k^2_\perp)\) must renormalize to zero at a normalization point \(\mu^2 \sim a \ few \ GeV^2\) where the duality arguments apply. This conclusion is not at variance with popular models for the QCD vacuum such as e.g. the instanton vacuum (see \([22]\) for review).
Our approach differs from that of Ref. [23] in that we apply the duality arguments for currents with derivatives and in addition use the large order perturbation theory analysis. As a result, we end up with the Gaussian instead of the $\Theta$-function.

Our final comment concerns with the meaning of Eq.(5). As has been explained above, $\langle \vec{k}^2_\perp \rangle$ is not literally an interquark transverse momentum. The notion of interquark transverse distance is senseless in a gauge theory because quark transverse degrees of freedom are undistinguishable from longitudinal degrees of freedom of higher Fock states at any normalization point due to equations of motion, see Ref.[24] and Appendix A. Thus an attempt to separate them would be at variance with exact equations of motion. Any theoretically consistent calculation of higher twist effects must include these two effects simultaneously at each given power of $Q^2$. However, the two contributions may well differ numerically. In fact, in all known examples where higher twist effects for exclusive amplitudes were calculated [10, 17] a contribution coming from the $\bar{\psi}G\psi$ Fock state turns out smaller than a two-particle contribution due to the transverse motion by typically a factor of 2-3, and comes with an opposite sign. It is probably worth noting that in the light-cone QCD sum rule approach to the pion form factor at intermediate $Q^2$ a two-particle DA of twist 4 yields the same $1/Q^4$ behavior as the leading twist DA, while a contribution due to $\bar{\psi}G\psi$ DA is down by the extra power of $Q^2$ [17]. Thus, one can expect that a calculation based on the use of the light-cone WF $\Psi(\xi, k^2_\perp)$ alone can serve as a reasonable estimate of higher twist effects. It is just the line of reasoning taken in this paper. In no sense we claim that a WF like (14) exhausts the $\rho$-meson properties, nor we pretend to give a complete evaluation of higher twist effects in diffractive electroproduction (see Sect.5).

3 Lowest moments $\langle \vec{k}^2_\perp \rangle$, $\langle \vec{k}^4_\perp \rangle$ and vacuum condensates

The general constraints of the previous section are insufficient for building up a realistic nonperturbative WF for the $\rho$-meson. To fix the latter, we follow the same logic as in the analysis of the distribution amplitudes and calculate the lowests $\langle \vec{k}^2_\perp \rangle$ and $\langle \vec{k}^4_\perp \rangle$ moments of $\Psi(\xi, k^2_\perp)$. The physical meaning of the second moment $\langle \vec{k}^2_\perp \rangle$ is clear: this quantity serves as a common scale for power corrections in physical amplitudes, cf. the next section. The fourth moment $\langle \vec{k}^4_\perp \rangle$ produces a next-to-leading power correction and signals on how strongly $\Psi(\xi, k^2_\perp)$ fluctuates in the $\vec{k}^2_\perp$ plane. In this section we will calculate the lowest moments $\langle \vec{k}^2_\perp \rangle$ and $\langle \vec{k}^4_\perp \rangle$ by a combined use of the equations of motion and QCD sum rules technique. With the knowledge of $\langle \vec{k}^2_\perp \rangle$ and $\langle \vec{k}^4_\perp \rangle$ we then construct a model WF $\Psi(\xi, k^2_\perp)$ which will be used to estimate higher twist effects in diffractive electroproduction in Sect.5.
We start with a general Lorentz decomposition of the matrix element

\[
\frac{1}{2} \sum_{\text{perm}} \langle 0 | \bar{\psi} \gamma_\mu i \nabla_\alpha \bar{\psi} \gamma_\beta u | \rho(q) \rangle = \varepsilon_\mu^{(\lambda)} q_\alpha q_\beta f_\rho m_\rho \langle u^2 \rangle
\]

\[
- \frac{1}{2} \varepsilon_\mu^{(\lambda)} \delta_\alpha \beta f_\rho m_\rho \langle \vec{k}_1^2 \rangle + (\varepsilon_\alpha^{(\lambda)} \delta_\mu \beta + \varepsilon_\beta^{(\lambda)} \delta_\mu \alpha) m_\rho^2 A_1
\]

\[
+ (\varepsilon_\alpha^{(\lambda)} q_\beta q_\mu + \varepsilon_\beta^{(\lambda)} q_\alpha q_\mu - 2 \varepsilon_\mu^{(\lambda)} q_\alpha q_\beta) A_2
\]

(here \( \text{perm} \) stands for the symmetrization in respect to \( \alpha, \beta \)). We have defined the last term such that to reproduce the usual definition of the first term as the second moment of the leading twist DA \( \phi(u) \) when all Lorentz indices are contracted with the light-like vector \( z_\mu \). The second term gives rise to the definition (8) after contraction with the transverse vector \( t_\mu \). With the definition (16) at hand, our strategy is to relate the unknown parameters \( \langle \vec{k}_1^2 \rangle, A_1, A_2 \) to some matrix elements of gauge invariant operators. At the second step these matrix elements are estimated using the QCD sum rules.

The first relation between the coefficients \( A_1, A_2 \) is obtained by multiplying Eq.(16) by \( \delta_\alpha \mu \):

\[
\frac{f_\rho}{2m_\rho} \langle \vec{k}_1^2 \rangle - 5A_1 - A_2 = 0
\]

Another relation can be derived when one multiplies Eq.(16) by \( q_\mu \). This yields the following constraint

\[
(\varepsilon_\alpha^{(\lambda)} q_\beta + \varepsilon_\beta^{(\lambda)} q_\alpha) m_\rho^2 (A_1 + A_2) = -\langle 0 | \bar{\psi} \gamma_\mu (\bar{\psi} \nabla_\alpha + gG_{\mu \alpha} \nabla_\alpha) u | \rho(q) \rangle
\]

\[
\equiv - (\varepsilon_\alpha^{(\lambda)} q_\beta + \varepsilon_\beta^{(\lambda)} q_\alpha) m_\rho^2 \eta
\]

The matrix element (18) parametrized by the number \( \eta \) can be estimated by studying the following correlation function

\[
i \int d\mathbf{x} \ e^{i q \mathbf{x}} \langle 0 | T \{ \bar{u} \gamma_\rho d(x) \bar{\psi} \gamma_\mu (gG_{\mu \alpha} \nabla_\alpha + gG_{\mu \beta} \nabla_\alpha) u(0) \} | 0 \rangle
\]

The matrix element of interest appears as the lowest intermediate hadron state contribution to the imaginary part of (19). On the other hand, the correlation function (19) can be calculated for \( q^2 \rightarrow -\infty \) in QCD. The perturbative contribution can be easily read off a very similar sum rule in Ref. [24] and numerically turns out negligible. The first power correction \( \sim \langle g^2 G^2 \rangle \) in (19) vanishes, and we end up with the following simple estimate

\[
\eta \approx 8\pi \alpha_s \frac{\langle \bar{\psi}\psi \rangle^2}{f_\rho m_\rho^3} \simeq 1.8 \times 10^{-3} \text{GeV}^2
\]

We next contract Eq.(16) with \( \delta_\alpha \beta \). Then the matrix element in the LHS becomes

\[
\langle 0 | \bar{\psi} \gamma_\mu (i \nabla_\alpha)^2 u | \rho(q) \rangle = -\langle 0 | \bar{g} \tilde{G}_{\mu \nu} \gamma_\nu \gamma_5 u | \rho(q) \rangle \equiv -\varepsilon_\mu^{(\lambda)} m_\rho^3 \lambda
\]
The number \( \lambda = 24 \pm 3 \text{ MeV} \) was calculated a long time ago [25] by the QCD sum rules method. Using (17), (18), (20) we finally obtain

\[
\langle \bar{k}_4^2 \rangle = \frac{2m_\rho}{3f_\rho} [m_\rho \lambda + f_\rho m_\rho \langle u^2 \rangle + 3n] \approx (420 \text{ MeV})^2 \tag{22}
\]

Note that the last term in Eq.(22) is rather small in comparison with the other two terms (for the second term we used here the asymptotic DA \( \phi(u) = 6u(1 - u) \) with \( \langle u^2 \rangle = \int duu^2 \phi(u) = 0.3 \). This fact is quite understandable as the matrix element in Eq.(18) is proportional to first moments of three-particle \( qGq \) DA’s while their normalizations are of the order of \( m_\rho \lambda \). Therefore, an uncertainty in Eq.(20) does not play an important role in our estimate (22). We will use this observation in the calculation of \( \langle \bar{k}_4^2 \rangle \). Numerically \( \langle \bar{k}_4^2 \rangle \) for the \( \rho \)-meson (22) is somewhat larger than the analogous parameter for the pion \( \langle \bar{k}_4^2 \rangle_\pi \approx (330 \text{ MeV})^2 \) [10] (see also Eq.(A.3) in Appendix A).

The same type of analysis can be done for calculation of the fourth moment \( \langle \bar{k}_4^2 \rangle \) though the algebra in this case becomes somewhat more complicated. We write

\[
\frac{1}{4!} \sum_{\text{perm}} \langle 0| \bar{d} \gamma_\mu i \nabla_\alpha i \nabla_\beta i \nabla_\lambda i \nabla_\xi u |\rho(q)\rangle = \varepsilon^{(4)}_\mu [q_\alpha q_\beta q_\lambda q_\xi f_\rho m_\rho \langle u^4 \rangle + (\delta_\alpha_\beta \delta_\lambda_\xi + \text{perm}) \frac{1}{8} f_\rho m_\rho \langle \bar{k}_4^2 \rangle + (\delta_\alpha_\lambda \delta_\beta_\xi \delta_\rho_\mu + \text{perm}) C_1]
\]

\[
\begin{align*}
+ (q_\mu \varepsilon_\alpha^{(3)} q_\beta q_\lambda q_\xi + \text{perm} - 4 \varepsilon_\mu^{(3)} q_\alpha q_\beta q_\lambda q_\xi) C_2 + q_\mu (\varepsilon^{(3)}_\alpha q_\beta \delta_\lambda_\xi + \text{perm}) C_3 \\
+ (\delta_\mu_\alpha \varepsilon^{(3)}_\beta q_\lambda q_\xi + \text{perm}) C_4 + (\delta_\mu_\alpha \delta_\beta_\lambda \varepsilon^{(3)}_\xi + \text{perm}) m_\rho^2 C_5
\end{align*}
\]

(23)

Proceeding analogously to the previous case and neglecting numerically small contributions due to second moments of three-particle DA’s, we can obtain a system of linear equations on the coefficients \( C_i \). The solution reads

\[
\begin{align*}
C_1 &= -f_\rho \left( \frac{1}{5} m_\rho^3 \langle u^4 \rangle + \frac{1}{24} \frac{\langle \bar{k}_4^2 \rangle}{m_\rho^2} \right) \\
C_2 &= -\frac{3}{20} f_\rho m_\rho^3 \langle u^4 \rangle \\
C_3 &= \frac{1}{48} \frac{f_\rho}{m_\rho} \langle \bar{k}_4^2 \rangle \\
C_4 &= \frac{1}{20} f_\rho m_\rho^3 \langle u^4 \rangle \\
C_5 &= -\frac{1}{48} \frac{f_\rho}{m_\rho} \langle \bar{k}_4^2 \rangle
\end{align*}
\]

(24)

We finally contract Eq.(23) with \( \delta_\alpha_\beta \delta_\lambda_\xi \) and make use of (24). Then the following equation holds

\[
\frac{1}{4} \langle 0| \bar{d} \gamma_\mu (g \sigma G)^2 u |\rho(q)\rangle + \frac{1}{2} \langle 0| \bar{d} \gamma_\mu g^2 G_\alpha_\beta G_\rho_\sigma u |\rho(q)\rangle = \varepsilon^{(4)}_\mu m_\rho^2 \left[ -\frac{3}{5} f_\rho m_\rho^3 \langle u^4 \rangle + 2 \frac{f_\rho}{m_\rho} \langle \bar{k}_4^2 \rangle \right]
\]

(25)
We have thus reduced the calculation of the fourth moment of the matrix elements of operators containing only the gluon tensor $G_{\mu\nu}$ and not the gluon potential $A_\mu$ itself as in the original problem [3]. To estimate these matrix elements, it proves convenient [20] to consider the following non-diagonal correlation functions:

$$T_1(p) = i \int dx e^{ipx} \langle 0|\bar{u}\sigma_{\alpha\beta}d(x) \bar{d}\gamma_\mu g^2 G_{\lambda\xi}^2 u(0)|0\rangle$$

$$T_2(p) = i \int dx e^{ipx} \langle 0|\bar{u}\sigma_{\alpha\beta}d(x) \bar{d}\gamma_\mu u(0)|0\rangle$$

Perturbative contributions vanish in the chiral limit in the both correlation functions (26) due to an odd number of gamma matrices. Thus, a leading as $p^2 \to -\infty$ behavior comes from condensate terms in the OPE for $T_1, T_2$. Dividing $T_1$ by $T_2$ we get rid of the $\rho$-meson residue $\sim \langle 0|\bar{u}\sigma_{\alpha\beta}d|\rho(q)\rangle$ in (26). Then for the reduced matrix elements $\lambda_1, \lambda_2$ defined by

$$\langle 0|\bar{d}\gamma_\mu g^2 G_{\alpha\beta}^2 u|\rho(q)\rangle = \varepsilon_\mu^{(\lambda)} m_\rho \lambda_1, \quad \langle 0|\bar{d}\gamma_\mu (g\sigma G)^2 u|\rho(q)\rangle = \varepsilon_\mu^{(\lambda)} m_\rho \lambda_2$$

we obtain after a simple algebra the following relations for the matrix elements of interest in terms of mixed vacuum condensates of the dimension 7:

$$\lambda_1 \simeq \frac{f_\rho}{m_\rho} \frac{\langle 0|\bar{u}(gG)^2 u|0\rangle}{\langle 0|\bar{u}u|0\rangle}$$

$$\lambda_2 \simeq \frac{f_\rho}{m_\rho} \frac{\langle 0|\bar{u}(g\sigma G)^2 u|0\rangle}{\langle 0|\bar{u}u|0\rangle}$$

Here the second of Eqs.(28) is obtained analogously to (26). Using Eq.(29) we finally get

$$\langle \vec{k}_4^4 \rangle \simeq \frac{3}{10} m_\rho^4 \langle u^4 \rangle + \frac{1}{4} \frac{\langle 0|\bar{u}(gG)^2 u|0\rangle}{\langle 0|\bar{u}u|0\rangle} + \frac{1}{8} \frac{\langle 0|\bar{u}(g\sigma G)^2 u|0\rangle}{\langle 0|\bar{u}u|0\rangle}$$

This is the main result of this section: we have explicitly expressed $\langle \vec{k}_4^4 \rangle$ in terms of the vacuum expectation values (VEV’s) of the dimension 7 operators. Naively, one could estimate these condensates by factorizing them into the products of the quark $\langle \bar{q}q \rangle$ and gluon $\langle g^2 G^2 \rangle$ condensates. This procedure, based on the factorization hypothesis, does not work in the given case: there are essential deviations from the factorization prediction in VEV’s (28). The non-factorizability of mixed quark-gluon matrix elements of such type has been studied in [26, 20] by two independent methods with full agreement in estimates between them. The first one [20] was based on the analysis of heavy-light quark systems which allows to obtain restrictions on the VEV’s like (28), while the second method has related the vacuum condensates of the form (28) to some pion matrix elements known from PCAC. As our results essentially depend on the magnitude of the condensates (23), we shortly review the method of Ref. [20] in Appendix B. Here we only formulate the result of this analysis. A measure of non-factorizability introduced by the correction factors $K_1$, $K_2$ in the matrix elements

$$\langle 0|\bar{u}(gG)^2 u|0\rangle = \frac{1}{6} K_1 \langle 0|g^2 G^2 |0\rangle \langle 0|\bar{u}u|0\rangle$$
\[ \langle 0 | \bar{u}(g\sigma G)^2 u | 0 \rangle = \frac{1}{3} K_2 \langle 0 | g^2 G^2 | 0 \rangle \langle 0 | \bar{u}u | 0 \rangle \]  

(30)

\( K_1 = K_2 = 1 \) in the factorization limit is approximately the same \( K_1 = K_2 = K \approx 3 \) for the both mixed operators appearing in Eq. (28). A possible uncertainty of this estimate does not exceed 30 % \cite{20}. Using Eqs. (29) and (30) we arrive at the following numerical estimate

\[ \langle \vec{k}_{\perp}^4 \rangle \simeq \frac{1}{8} \left[ \frac{K}{3} \langle g^2 G^2 \rangle + \frac{K}{3} \langle g^2 G^2 \rangle + \frac{12}{5} m^4 \langle u^4 \rangle \right] \simeq 0.14 \text{ GeV}^4 \]  

(31)

This large value of \( \langle \vec{k}_{\perp}^4 \rangle \) seems rather surprising as it implies strong fluctuations of the nonperturbative WF \( \Psi(\xi, k_{\perp}^2) \). To measure the magnitude of these fluctuations it is convenient to introduce a dimensionless parameter

\[ R \equiv \frac{\langle \vec{k}_{\perp}^4 \rangle}{\langle k_{\perp}^2 \rangle^2} \simeq 4.2 \text{ if } \langle \vec{k}_{\perp}^4 \rangle \simeq (420 \text{ MeV})^2 \]  

(32)

The phenomenon of strong fluctuations in \( \Psi(\xi, k_{\perp}^2) \) has been found previously by analogous methods for the pion wave function in Ref. \cite{20} with approximately the same number for the ratio (32). In terms of the QCD vacuum structure these fluctuations are due to the numerical enhancement of the high dimensional quark gluon mixed condensates or, what is the same, the large magnitude of the parameter \( K \) in Eq. (30).

## 4 Model wave function \( \Psi(u, k_{\perp}^2) \)

The results obtained so far are essentially model independent. We have fixed the form of the high-\( k_{\perp}^2 \) tail of the true nonperturbative WF \( \Psi(\xi, k_{\perp}^2) \) Eq. (14) by the use of the quark-hadron duality and dispersion relations. Furthermore, we calculated the lowest moments \( \langle \vec{k}_{\perp}^2 \rangle \) and \( \langle \vec{k}_{\perp}^4 \rangle \) using the equations of motion and QCD sum rules. Now our purpose is to build some model for the true nonperturbative WF \( \Psi(\xi, k_{\perp}^2) \) which would respect all general constraints of Sect. 2 and incorporate the effect of strong fluctuations in the transverse \( k_{\perp}^2 \) plane found in the previous Sect. 3 (see Eq. (32)). In spite of the fact that this inevitably brings some model dependence into any physical quantity calculated within such a wave function, there remains a region where the results of such a calculation become practically model independent. This happens when the answer is essentially determined only by the lowest \( \langle \vec{k}_{\perp}^2 \rangle \) and \( \langle \vec{k}_{\perp}^4 \rangle \) but not higher \( \langle \vec{k}_{\perp}^6 \rangle \), etc. moments of the WF. As we shall see in Sect. 5, this is precisely the case in the hard diffractive electroproduction for sufficiently high \( Q^2 > 10 \text{ GeV}^2 \).

The effect of strong \( k_{\perp}^2 \) fluctuations (32) cannot be taken into account by a simple Gaussian ansatz of the type (14) which would yield the factor of 15/7 in Eq. (32).
The only possible way to satisfy Eq. (32) is to have a second pick in $\Psi(\xi, k_2^\perp)$ which can have a small magnitude but reside far away from the first Gaussian one. To reconcile the existence of this second pick with the asymptotic behavior (14) it must enter as a pre-asymptotic term which would provide the constraint (32). Following Ref. [18] we suggest the following ansatz for $\Psi(\xi, k_2^\perp)$:

$$\Psi(u, k_2^\perp) = A \left\{ \exp \left[ -\frac{k_2^2}{8\beta^2 u \bar{u}} \right] + c \exp \left[ -\left( \frac{k_2^2}{8\beta^2 u \bar{u}} - l \right)^2 \right] \right\}$$ (33)

In this ansatz the parameters $c$ and $l$ determine the magnitude and position of the second pick, respectively. We have found that the best fit corresponds to the choice $\beta = 125 \text{ MeV}$, $c = 0.15$, $l = 30$. For these values of parameters $R \approx 4.1$, $\langle k_4^2 \rangle \approx 0.13$. For $u \approx 1/2$ the second pick is located at $\vec{k}_2^2 \approx 1 \text{ GeV}^2$. The existence of two characteristic scales in $\Psi(\xi, k_2^\perp)$ (0.4 GeV and 1 GeV) is rather interesting and may probably be linked to phenomenological models for the QCD vacuum like the instanton vacuum model [22] and/or the constituent quark model. If one does not insist on (or disbelieves) the effect of amplified fluctuations in the transverse $\vec{k}_2^\perp$ plane (32), then the second exponent in Eq. (33) should be omitted. It is important, however, that in order to have the same second moment $\langle \vec{k}_2^2 \rangle$ the parameter $\beta$ must be simultaneously rescaled to $\beta \approx 330 \text{ MeV}$. In this case the WF takes the following form:

$$\Psi(u, k_2^\perp) = A \exp \left[ -\frac{k_2^2}{8\beta^2 u \bar{u}} \right], \beta = 330 \text{ MeV}$$ (34)

We will use in Sect. 5 the both forms in order to reveal the importance of large $\langle \vec{k}_4^2 \rangle$ (31) and, respectively, of the non-factorizability of VEV's (30) in the pre-asymptotic behavior of the diffractive amplitude.

The model WF (33) meets all general and numerical constraints derived in previous sections of this paper. The integration of (33) over $\vec{k}_2^2$ yields the asymptotic form of the distribution amplitude $\phi(u) = 6u(1 - u)$ which is known to be a good approximation to the true non-perturbative DA of the $\rho$-meson. Furthermore, numerical coefficients we have obtained for the ansatz (33) are not very different from those found in [18] for the case of the pion WF. Our conclusion is that the transverse momentum distributions in pion and $\rho$-meson are to a large extent alike.

5 Hard diffractive electroproduction

In this section we come back to our initial aim. The results of proceeding sections expressed in condensed form by Eq. (33) will be used for the study of the pre-asymptotic effect due to the Fermi motion in diffractive electroproduction of the $\rho$-meson. Our prime goal is to get an estimate for the onset of the asymptotic regime in this problem.
The applicability of perturbative QCD (pQCD) to the asymptotic limit of the hard diffractive electroproduction of vector meson was established in Ref. [1] using the light-cone perturbation theory. The authors of [1] have proved that for the production of longitudinally polarized vector mesons by longitudinally polarized virtual photons the cross section can be consistently calculated in pQCD. There was found that at high $Q^2$ the amplitude factorizes in a product of DA’s of the vector meson and virtual photon, the light-cone gluon distribution function of a target, and a perturbatively calculable on-shell scattering amplitude of a $q\bar{q}$ pair off the gluon field of the target. After the factorization of gluons from the target the problem seems to be tractable within OPE-like methods.

It would be desirable to have an explicit realization to the idea of treating the process of interaction of the virtual photon with the gluons from the target and a sequent conversion into a vector meson as some kind of a current Green function, to which methods of the light-cone OPE could be applied. Such reformulation has proved to simplify enormously the analysis of the pion form factor and some other exclusive amplitudes. This approach is particularly convenient for the study of higher twist effects in exclusive processes, where calculations of light-cone pQCD become tremendously complicated. Here we propose such a method. Though no new result will be given, we believe it is still worthwhile to present it in this paper. The reason for this is several-fold. First, our method reproduces the results of Ref. [1] more easily and moreover extents them by taking into account an asymmetry of the gluon distribution of the target. Second, it allows to establish a connection with the nonperturbative $\rho$-meson WF defined within the OPE. Note in this respect that the question on what WF should be substituted in the amplitude is of peculiar interest and causes some controversy in the literature (see below on this). Third, the approach we suggest is ideally suited for a complete evaluation of twist 4 effects including three-particle $\bar{q}Gq$ DA’s (see discussion in Sect.2). Though we do not carry such a calculation, it can be done within this formalism provided the $\bar{q}Gq$ DA’s are known. Thus, our reformulation will be used below merely to rederive the formulæ of Ref. [1, 7] within OPE. This will enable to show that the WF which enters Eq.(41) below is in fact the soft light-cone WF (10) defined as a set of matrix elements which has been discussed in the previous sections of this paper.

Let us start with the general form of the amplitude as a matrix element of the electromagnetic current $j_\mu = e(2/3\bar{u}\gamma_\mu u - 1/3\bar{d}\gamma_\mu d)$:

$$M = \epsilon_\mu \langle N(p-r)\rho(q+r)|j_\mu(q)|N(p)\rangle,$$

where $\epsilon_\mu$ is the polarization vector of the photon (only the longitudinal polarization is considered, see [1]) and $r$ stands for the momentum transfer. We will neglect the masses of the nucleon and $\rho$-meson in comparison with the photon virtuality $Q^2 : m_\rho^2 = m_N^2 = 0$. For the momentum transfer $r$ we consider the limit $r^2 = 0$, but $r_\mu \neq 0$. We next note the following. The factorization of the gluon field of the target [1, 2] means that that these gluons act as an external field on highly virtual quarks produced by the photon with $Q^2 \rightarrow \infty$. This statement can be
given a formal meaning by introducing into the QCD Lagrangian an additional term \( \delta S = ig \int dz j^a_\mu(z) G^a_\mu \) where \( j^a_\mu = \bar{q} \gamma_\mu t^a q \) is the color current and \( G^a_\mu \) stands for the external gluon field from the target. Expanding the path integral to the second order in this external field corresponds to the two-gluon exchange:

\[
M = -g^2 \epsilon_\mu \int dz dy \langle N(p-r)|G^a_\mu(z)G^b_\nu(y)|N(p)\rangle \langle \rho(q+r)|T\{j_\mu(0)j^a_\alpha(z)j^b_\beta(y)\}|0\rangle \tag{36}
\]

One has to stress that our Eq. (36) is not another "proof" of factorization, but rather should be considered as a reformulation of results of Ref. 2 (and also 3, see Eq. (39) below) in a way which is very suitable for practical calculations. Following Ref. 3, we find it convenient to choose the light-cone gauge for the external gluons

\[
G_\mu(z) = n^\nu \int_0^\infty d\sigma G_{\mu\nu}(z + \sigma n) \quad n^\mu G_\mu = 0
\]

in which the nucleon matrix element can be parametrized in terms of the asymmetric gluon distribution function 3:

\[
\langle N(p-r)|G^a_\mu(z)G^b_\nu(y)|N(p)\rangle = \frac{\delta^{ab}}{N_c^2 - 1} \frac{\bar{u}(p-r)\hat{n}u(p)}{(-4pn)} \left( g_{\alpha\beta} - \frac{p_\alpha n_\beta + p_\beta n_\alpha}{pn} \right) \times \int_0^1 dX \frac{F_\xi^g(X)}{X(X - \xi + i\delta)} \left( e^{-iXpz + i(X - \xi)py} + e^{i(X - \xi)pz - iXpy} \right) \tag{38}
\]

Here one exploits the fact that in the limit \( r^2 = 0 \) the momentum transfer is proportional to the nucleon momentum \( r_\mu = \xi p_\mu \) with \( \xi = Q^2/(2pq) \) being the Bjorken variable. Under this circumstance the fractions of the total nucleon momentum carried by the gluon are \( xp_\mu + yr_\mu = (x + \xi y)p_\mu \equiv Xp_\mu \) and \( (x - \xi \tilde{y})p_\mu \equiv (X - \xi)p_\mu \), where \( \tilde{y} \equiv 1 - y \). It is thus convenient 3 to parametrize the gluon distribution in terms of the asymmetric gluon distribution \( F_\xi^g(X) \) which goes over to the usual gluon distribution function \( Xf_g(X) \) in the symmetric limit \( \xi \to 0 \).

Using the translational invariance and the fact that \( \bar{u}(p-r)\hat{n}u(p)/(-4pn) = \sqrt{1 - \xi} \), we finally present the amplitude in the form

\[
M = -\frac{g^2}{N_c^2 - 1} \left( g_{\alpha\beta} - \frac{p_\alpha n_\beta + p_\beta n_\alpha}{pn} \right) \epsilon_\mu \sqrt{1 - \xi} \int_0^1 dX \frac{F_\xi^g(X)}{X(X - \xi + i\epsilon)} \times \int dz dy e^{-iQz} \left( +e^{i(X - \xi)py} + e^{-iXpy} \right) \langle \rho(q+r)|T\{j_\mu(z)j^a_\alpha(0)j^a_\beta(y)\}|0\rangle \tag{39}
\]

We have thus reduced the amplitude of diffractive electroproduction to the three-point correlation functions of quark currents between the vacuum and the \( \rho \)-meson states. One can see that this correlation function is dominated by the light-cone and thus a leading contribution is given by a combination of non-local quark-antiquark string operators of twist 2 whose matrix elements yield the leading twist DA of the \( \rho \)-meson. Coefficients in front of these operators are due to most singular parts of quark propagators near the light-cone. Retaining only this contribution to (39), we
recover the asymptotic answer of Ref.[1] in the form presented without derivation in [3]:

$$M = \frac{4\pi\sqrt{2}e\alpha_s f_p}{N_c} \frac{1}{Q} \int_0^1 dX \frac{F_1^0(X)\sqrt{1-\xi}}{X(X-\xi+i\delta)} \int_0^1 \frac{\phi(u)}{u\bar{u}}$$

(40)

where $\phi(u)$ is the standard light-cone DA [3] (redefined to the interval $u = [0,1]$) whose asymptotic form is $\phi^{as}(u) = 6u\bar{u}$. Remind here that it is defined as $\phi(u) = \int d\vec{k}_1^2 \Psi(u,k_1^2)$ where the soft WF $\Psi(u,k_1^2)$ is introduced by Eq.(10).

Going over to higher twist corrections to the asymptotic answer (40), one has to distinguish between two classes of contributions. Corrections of the first class come from keeping less singular terms in the quark propagators in (39). Relevant terms in the light-cone expansion for the quark propagator contain an additional gluon which is to be attributed to the final meson and expressed in terms of three-particle DA’s. A corresponding calculation is technically involved, though it is feasible within the formalism presented here. The second class of corrections is due to the quark transverse degrees of freedom (the Fermi motion). Calculating only this contribution we make an educated guess on the scale of higher twist corrections in the diffractive electroproduction (see the comment at the end of Sect.2). Technically, retaining this correction amounts to the multiplication of (40) by the following correction factor [7] (following the notations of Ref.[7] we reserve the symbol $T(Q^2)$ for the correction in the cross section)

$$\sqrt{T(Q^2)} = Q^4 \int_0^1 \frac{du}{u\bar{u}} \int_0^{Q^2} d^2\vec{k}_1^2 \Psi(u,k_1^2) \left( \frac{1}{(Q^2+k_1^2/(u\bar{u}))^2} - 2 \frac{k_1^2/(u\bar{u})}{(Q^2+k_1^2/(u\bar{u}))} \right) \int_0^1 \frac{du}{u\bar{u}} \int_0^{Q^2} d^2\vec{k}_1^2 \Psi(u,k_1^2)$$

(41)

where $\Psi(u,k_1^2)$ was defined earlier in terms of the local matrix elements [11]. By definition, in the asymptotics $Q^2 \to \infty$ $\sqrt{T} = 1$. Deviations from $\sqrt{T} = 1$ determine a region of applicability of the asymptotic formula [10].

Now we are in position to discuss numerical estimates for the correction factor (41) in order to answer the main question formulated in Introduction. The authors of [4] have observed that the choice of $\Psi(u,k_1^2)$ in a factorized form $\Psi(u,k_1^2) = \phi(u)\psi(k_1^2)$ leads to a very slowly raising function $\sqrt{T(Q^2)}$ which approaches 0.8 at rather high $Q^2 \geq 40 \text{ GeV}^2$ depending on the model chosen for $\Psi(u,k_1^2)$. On the other hand, as we stated earlier, Eq.(39) unambiguously demonstrates that it is the soft nonperturbative WF [11] that has to be substituted in Eq.(41). Note in this respect that any perturbative tails like $1/k_1^2$ in the WF are by definition absent within the logic based on a use of OPE. Such contributions effectively reappear only as radiative corrections to leading $O(\alpha_s^0)$ results. It is clear that these corrections are always small unless there are special reasons which make them parametrically leading as $Q^2 \to \infty$ (this is the case e.g. for the pion form factor). Using the model WF [33] we obtain a very fast approach to the asymptotics in [41]. The correction factor $\sqrt{T(Q^2)}$ reaches the value 0.8 already at $Q^2 \simeq 10 \text{ GeV}^2$ that implies a rather low onset for the asymptotic regime in the amplitude [39].
Figure 1: The correction factor \( T(Q^2) \) in the cross section for different choices of the \( \rho \)-meson WF \( \Psi(u,k_\perp^2) \).

The technical reason for this behavior is quite clear: large values of \( \vec{k}_\perp^2/(u\bar{u}) \) are exponentially suppressed by the WF (33) and thus this term remains much smaller that \( Q^2 \) for \( Q^2 > 10 \text{ GeV}^2 \). The result of our calculation is shown for the correction \( T(Q^2) \) in the cross section on the Figure (the thick curve). To demonstrate the importance of amplified fluctuations in the \( \vec{k}_\perp^2 \)-plane (32) (and thus of the non-factorization property (30)), we also show (the thin curve) the result of the calculation when the WF (34) not including the second pick is used. One sees that the difference between the two curves disappears for \( Q^2 > 40 \text{ GeV}^2 \), while it becomes quite sizeable (20 - 30 % ) in the region \( Q^2 = 5 - 10 \text{ GeV}^2 \). One may therefore speculate that the non-factorizability of higher dimensional condensates (30) could be tested experimentally by measuring the cross section in this region. Still, one has to note that prior to any quantitative prediction, a more refined analysis is required in this region, including a complete evaluation of twist 4 corrections due to the \( (\bar{\psi}G\psi) \) Fock state. For sake of comparison with the discussion of Ref.[7] we also plot (the dashed curve) the factor \( T(Q^2) \) corresponding to the choice [7] of the WF in the factorized ”dipole” form \( \Psi(u,k_\perp^2) \sim u\bar{u}/(\vec{k}_\perp^2 + \mu^2)^2 \) for \( \mu = 300 \text{ MeV} \). Note that for such WF the second moment \( \langle \vec{k}_\perp^2 \rangle \), as defined by Eq.(11), does not exist at all, and can only be introduced by means of the bound state equation, see the footnote after Eq.(14). The choice \( \mu = 300 \text{ MeV} \) corresponds to \( \langle \vec{k}_\perp^2 \rangle_{BS} \approx (220 \text{ MeV})^2 \) (this number follows from nowhere and is taken exclusively for illustrative purposes). What matters, however, is the fact that all other options discussed in [7] for a factorized form of WF \( \Psi(u,k_\perp^2) = \phi(u)\psi(\vec{k}_\perp^2) \) ( including the above ansatz with \( \langle \vec{k}_\perp^2 \rangle_{BS} \approx (300 \text{ MeV})^2 \) and \( \langle \vec{k}_\perp^2 \rangle_{BS} \approx (600 \text{ MeV})^2 \) and some other choices) all lie under this curve. One can therefore see quite unambiguously the difference in the correction \( T(Q^2) \) for the factorized \( \Psi(u,k_\perp^2) = \phi(u)\psi(\vec{k}_\perp^2) \) and the non-factorized WF.
(33) in the whole range of $Q^2$. Furthermore, this difference is critical for the issue of the onset of asymptotic regime which is of our prime interest in this paper. While for WF (33) the correction $\sqrt{T}$ in the amplitude reaches the value 0.8 already at $Q^2 \approx 10 \text{ GeV}^2$ (or equivalently $T(10 \text{ GeV}^2) \approx 0.64$), for the "dipole" ansatz $\Psi(u, k_\perp^2) \sim u\bar{u}/(k_\perp^2 + \mu^2)^2$ with $\mu = 300 \text{ MeV}$ this happens only at $Q^2 \approx 35 \text{ GeV}^2$. For other choices of a factorized WF $\Psi(u, k_\perp^2) = \phi(u) \psi(k_\perp^2)$ discussed in [7] a corresponding value is pushed further to even higher $Q^2 > 40 \text{ GeV}^2$. On the other hand, at $Q^2 = 10 \text{ GeV}^2$ the difference between our prediction for $T(Q^2)$ and models discussed in [7] constitutes at least a factor of two.

Our use of the ansatz (33) can, however, meet objections as apparently it yields a large contribution to the integral (41) coming from the region of small $k_\perp^2$ which is expected to be cut off on physical grounds. Here we remind that the ansatz (33) has been suggested to satisfy the general constraints of Sect.2 and the particular values of the moments (Sect.3). On the other hand, exclusive amplitudes at large $Q^2$ are generally only sensitive to the high-$k_\perp^2$ tail of the WF which dominates the moments $\langle k_\perp^2 \rangle$, $\langle k_\perp^4 \rangle$ etc. Thus one expects that these are only global characteristics like $\langle k_\perp^2 \rangle$ that matter in Eq.(41). It is easy to see that this is indeed the case as soon as the WF is defined to possess all moments, see Eq.(10). Powers of $1/(u\bar{u})$ arising when the integrand in (41) is expanded in a series over $k_\perp^2$ are harmless due to the general constraint (14) which excludes end-point singularities of the u-integration. In this case the $u-$ and $k_\perp^2-$ integrations in Eq.(41) decouple with the exponential in $Q^2$ (in practice this means $Q^2 > 5 \text{ GeV}^2$) accuracy, and the answer is therefore directly expressed in terms of moments

$$\sqrt{T(Q^2)} = 1 - 20 \frac{\langle k_\perp^2 \rangle}{Q^2} + 210 \frac{\langle k_\perp^4 \rangle}{Q^4} + \ldots = 1 - \frac{3.4 \text{ GeV}^2}{Q^2} + \frac{28.4 \text{ GeV}^4}{Q^4} + \ldots \quad (42)$$

Therefore our approach is self-consistent in the sense that any question on a large distance contribution to the integral (14) is lifted up to the definitions of the moments, as has to be expected on general grounds. Here we emphasize again that our definitions of moments are specific to OPE, see the footnote after Eq.(10). The numbers in Eq.(42) illustrate the observed fast growth of $T(Q^2)$: power corrections tend to cancel each other before they become small separately. On the other hand, it can be seen from Eq.(42) that for $Q^2 < 10 \text{ GeV}^2$ higher moments of $\Psi(\xi, k_\perp^2)$ turn out essential and thus in this region the answer for $T(Q^2)$ is to some extent model dependent.

Thus our final conclusion is that the onset of the asymptotic regime for diffractive electroproduction of the longitudinally polarized $\rho$-meson is approximately $Q^2 \approx 10 \text{ GeV}^2$ where corrections due to the quark transverse degree of freedom constitute less than 20% in the amplitude. This can be traced back to the fact that in the case at hand the power corrections are given by the matrix elements of local operators and in fact are fixed completely by the independent calculation of the moments. This is the consequence of the structure of Eq.(11) and the fact that the WF is a function
of the single variable $\vec{k}^2/(u\bar{u})$. Only global, but not local characteristics of $\Psi(u,k^2_\perp)$ in the $k^2_\perp$ plane are important. This situation can be confronted with the case of exclusive processes. In the most well studied problem of the pion form factor the asymptotic regime has been found to be pushed further to $Q^2 \gg 10 \text{ GeV}^2$ \cite{7, 18}, where the subleading ”soft” contribution is still larger than the leading asymptotic one. One reason for this \cite{17} is that a special power correction comes due to the Feynman end-point mechanism for the form factor which provides the whole answer $\sim 1/Q^4$ in absence of radiative corrections. The asymptotic answer $\alpha_s(Q^2)/Q^2$ is in fact the radiative correction to this soft term which nevertheless wins in the asymptotics $Q^2 \rightarrow \infty$. In our opinion, this is precisely where the most fundamental difference between exclusive and diffractive amplitudes lies. The academic $Q^2 \rightarrow \infty$ limit of the former is given by the hard rescattering mechanism which is $O(\alpha_s)$ and thus is superseded at all available energies by the nonperturbative soft contribution which is down by the power $1/Q^2$, but at the same time does not contain a smallness due to $\alpha_s$. On contrary, in the diffractive amplitude power corrections which are again $O(\alpha_s^0)$ (but not attached to the end-points of the u-integration) compete with the asymptotic amplitude which itself is also $O(\alpha_s^0)$. It follows from this argument that the onsets of the asymptotic regime for the exclusive and diffractive amplitudes differ parametrically by the factor $1/\alpha_s$. This conclusion is not in contradiction with the fact \cite{18} that for the pion form factor the nonleading soft contribution is larger than the asymptotic answer up to very high $Q^2 = 50-100 \text{ GeV}^2$. We are thus convinced that the analogy with the pion form factor problem does not work in the case at hand. There are no reasons to expect the onsets of the asymptotic regime to be similar in the diffractive electroproduction and exclusive processes. On contrary, they differ parametrically in $1/\alpha_s$. Moreover, the explicit calculations suggest that the asymptotic regime in the $\rho$-meson diffractive electroproduction starts already at $Q^2 \simeq 10 \text{ GeV}^2$. We stress that this conclusion refers only to the diffractive electroproduction of the longitudinally polarized $\rho$-meson, the situation with the transverse polarization or diffractive charmonium production can be quite different.

6 Summary

We hope that we have succeeded in explaining a few facts by this paper. Below we collect them together.

1. Very general principles of the theory such as the dispersion relations and quark-hadron duality allows one to fix the asymptotic form of the light-cone WF $\Psi(u,k^2_\perp)$. It is the Gaussian of the single argument $\Psi(u,k^2_\perp) \sim \exp(-\vec{k}^2_\perp/8\beta^2 u\bar{u})$, while the mass term in WF or/and the factorized WF $\Psi(u,k^2_\perp) = \phi(u)\psi(k^2_\perp)$ are absolutely forbidden by the above principles.

2. The light-cone WF $\Psi(u,k^2_\perp)$ does not have a two-particle interpretation and is related by equations of motion to higher twist distribution amplitudes. However, there exists a numerical difference between their contributions, which allows to re-
tain only the former to estimate an importance of higher twist effects for a particular process.

3. Properties of $\Psi(u,k^2_{\perp})$ are strongly influenced by the fine structure of the QCD vacuum. More precisely, the $\rho$-meson WF $\Psi(u,k^2_{\perp})$ is very sensitive to the non-factorization of quark gluon vacuum condensates of dimension 7.

4. The pre-asymptotic behavior of the $\rho$-meson diffractive electroproduction amplitude is critically depending on properties of the WF $\Psi(u,k^2_{\perp})$. When the correct WF is substituted in the amplitude, the asymptotic regime starts at rather low $Q^2 \simeq 10 \text{ GeV}^2$, in drastic contrast to exclusive processes where the onset of the asymptotic regime is pushed to much higher $Q^2 = 50 - 100 \text{ GeV}^2$.

5. This vast difference between exclusive and diffractive processes is not accidental, but rather has the clear origin: the asymptotic regime for the former is delayed in comparison to the latter parametrically by the factor $1/\alpha_s(Q^2)$.

6. In the intermediate energy region $Q^2 = 5 - 10 \text{ GeV}^2$ the diffractive amplitude is sensitive to the non-factorization of dimension 7 vacuum condensates. Thus, the fine structure of the QCD vacuum is probed there. By preliminary estimates, the non-factorization has an effect of the order of 20 - 30% in the cross section.

7. To get a quantitative description of the diffractive amplitude for intermediate energies $Q^2 = 5 - 10 \text{ GeV}^2$ a more refined calculation is required, including a careful analysis of a model dependence of predictions for such $Q^2$ and complete evaluation of twist 4 effects with account for three-particle $\bar{q}Gq$ distribution amplitudes.

8. The latter technically involved calculation can be considerably simplified using our representation of the diffractive amplitude in terms of the current Green function, Eq.(39) which re-expresses the results of [2,3] in a form convenient for practical purposes. Necessary for this calculation $\bar{q}Gq$ distribution amplitudes for $\rho$-meson will be discussed elsewhere [28].

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Appendix A

The purpose of this appendix is to establish the connection of the problem of transverse momentum distribution with the standard light-cone operator product expansion (OPE) and explain how $\langle \vec{k}_2^2 \rangle$ can be directly expressed in terms of some three-particle matrix elements. We consider this problem on the example of the pion. This choice is motivated by the fact that an analysis of higher twist distribution amplitudes in this case is considerably simplified in comparison to the $\rho$-meson due to smaller number of independent Lorentz structures. The complete set of three-particle DA’s of twist 4 beyond the asymptotic regime has been constructed in Ref. [24] using the (one-loop) conformal symmetry and the QCD sum rules technique. Here we use these results to show that the combination $b_2 u \bar{u}$ ( or $\vec{k}_1^2 / (u \bar{u})$ after the Fourier transform ) is a single argument in the pion WF to the leading conformal spin accuracy. This is a justification for our choice of the model WF (33) where the only argument $\vec{k}_2^2 / u \bar{u}$ appears.

The twist 4 three-particle DA’s of interest are introduced as follows

$$\langle 0 | \bar{d}(0) \gamma_\nu g \gamma_\mu (vx) u(x) | \pi(q) \rangle = q_\alpha (q_\mu x_\nu - q_\nu x_\mu) \frac{1}{qx} \int D(\alpha_i qx) \Phi_\parallel (\alpha_i)$$

$$+ \left[ q_\nu (\delta_{\alpha\mu} - \frac{x_\mu q_\alpha}{qx}) - q_\mu (\delta_{\alpha\nu} - \frac{x_\nu q_\alpha}{qx}) \right] \int D(\alpha_i qx) \Phi_\perp (\alpha_i)$$

Here

$$\int D(\alpha_i qx) = \int d\alpha_1 d\alpha_2 d\alpha_3 \delta(1 - \alpha_1 - \alpha_2 - \alpha_3) e^{-i qx(\alpha_1 - \alpha_2 + \alpha_3)}$$

To the leading conformal spin accuracy (i.e. in the asymptotic form) the DA’s $\Phi_\parallel (\alpha_i), \Phi_\perp (\alpha_i)$ are [24]

$$\Phi_\parallel^{as}(\alpha_i) = 10 f_\pi \delta^2 \alpha_3^2 (\alpha_1 - \alpha_2)$$

$$\Phi_\perp^{as}(\alpha_i) = 0$$

where $\delta^2$ is defined through the matrix element

$$\langle 0 | \bar{d} \gamma_\nu ig \tilde{G}_{\nu\mu} u | \pi(q) \rangle = f_\pi \delta^2 q_\mu , \delta^2 \simeq 0.2 \text{ GeV}^2$$

This matrix element was calculated independently and for different purposes in [10] and [27]. On the other hand, one can introduce the two-particle DA’s of twist 4 by the relation

$$\langle 0 | \bar{d}(0) \gamma_\nu \gamma_5 u(x) | \pi(q) \rangle = i f_\pi q_\mu \int_0^1 du e^{-i qx(\phi(u) + x^2 g_1(u))}$$

$$+ f_\pi \left( x_\mu - x^2 q_{\mu u} \right) \int_0^1 e^{-i qx^2} g_2(u)$$

where $\phi(u)$ is the leading twist DA ( with $\phi^{as}(u) = 6 u \bar{u}$ ) and all logs of $x^2$ are included in DA’s. The twist 4 DA’s $g_1(u), g_2(u)$ can be now expressed in terms of
the three-particle DA’s $\Phi_{\parallel}, \Phi_{\perp}$. The easiest way to do this is to use the equations of motion \[24\]

\[
\frac{\partial}{\partial x_\mu} \bar{d}(-x) \gamma_\mu \gamma_5 u(x) = i \int_{-1}^{1} dv \bar{d}(-x) x_\alpha g G_{\alpha\beta}(v x) \gamma_\beta \gamma_5 u(x)
\]

\[
\frac{\partial}{\partial x} \bar{d}(-x) \gamma_\mu \gamma_5 u(x) = i \int_{-1}^{1} dv \bar{d}(-x) x_\alpha g G_{\alpha\beta}(v x) \gamma_\beta \gamma_5 u(x)
\] (A.6)

(here $\partial_\mu$ stands for the translation operator). Taking the matrix elements and making use of the definitions (A.1), (A.5) we obtain to this accuracy

\[
g_1(u) = \frac{5}{2} \delta^2 u^2 \bar{u}^2
\]

\[
g_2(u) = \frac{10}{3} \delta^2 u \bar{u}(u - \bar{u})
\]

\[
G_2(u) = \frac{5}{3} \delta^2 u^2 \bar{u}^2
\] (A.7)

For further convenience we have introduced here the DA $G_2(u)$ defined by $g_2(u) = \frac{d}{du}G_2(u)$. Now we are in position to show that for $b^2 \equiv -x^2$ the combination $b^2 u \bar{u}$ is exactly the expansion parameter in the coordinate space. To see this, we choose the frame with $x = (x_+, x_-, x_\perp) = (0, x_-, x_\perp)$. Then

\[
\langle 0 | \bar{d}(0) \gamma_+ \gamma_5 u(0, x_-, x_\perp) | \pi(q) \rangle = i f_\pi q_+ \int_0^1 du e^{-iuq+x-} (\phi(u) + x^2(g_1 + G_2)(u)
\]

\[
= i f_\pi q_+ \int_0^1 du e^{-iuq+x-} 6u \bar{u}(1 - \frac{5}{9} \delta^2 b^2 u \bar{u} + \cdots)
\] (A.8)

It is seen that we have obtained the two first terms in the expansion of the pion WF $\Psi(\xi = 2u - 1, b^2)$ in powers of $b^2$. It is now easy to find the value of $\langle \vec{k}_{\perp}^2 \rangle_\pi$ from Eq.(A.8):

\[
\langle 0 | \bar{d}(0) \gamma_+ \gamma_5 (i \nabla_\perp)^2 u | \pi(q) \rangle = i f_\pi q_+ \frac{5}{9} \delta^2 \equiv i f_\pi q_+ \langle \vec{k}_{\perp}^2 \rangle_\pi , \langle \vec{k}_{\perp}^2 \rangle_\pi \simeq (330 \text{ MeV})^2
\] (A.9)

We have thus demonstrated two important facts:
1. The light-cone WF $\Psi(u, k_{\perp}^2)$ and higher twist DA’s containing gluons explicitly are two sides of the same coin, as they are related by exact equations of motion.
2. The combination $\vec{k}_{\perp}^2 / u \bar{u}$ is a single variable which the WF depends on:
\[
\Psi(u, k_{\perp}^2) = \Psi(\vec{k}_{\perp}^2 / u \bar{u})
\]

The same line of reasoning can be extended to the case of the $\rho$-meson. We therefore conjecture that the ansatz (14) is valid in the whole range of $\xi$ to the leading conformal spin accuracy, i.e. it corresponds to a series of asymptotic two-particle DA’s of higher twists. This approximation seems reasonable provided the WF $\Psi(\xi, k_{\perp}^2)$ is evaluated at a high normalization point $\mu^2 \sim$ a few GeV$^2$. On the other hand, it is such $\mu^2$ that enters Eq.(14) following from the duality arguments.
Appendix B

The aim of this Appendix is to present the method [20] which allows to estimate some high-dimensional condensates with a reasonable accuracy. The importance of this question (apart from the pure theoretical interest concerning a structure of the QCD vacuum) for the phenomenological purposes was emphasized in Section 3. It was observed that the large magnitude for the high dimensional condensates implies a strong momentum fluctuations in the transverse direction. Correspondingly, it has a remarkable impact on the pre-asymptotic behavior of the electroproduction.

Let us start with the formulation of the idea exploited in this method. Consider some correlation function

\[ i \int dx e^{iqx} \langle T\{J_1(x), J_2(0)\} \rangle \]

at large \(-q^2\). If the currents \(J_1, J_2\) are chosen in such a way that in the chiral limit the perturbative contribution is zero, we end up with the leading contribution in the form \(\langle O \rangle \sim q^2\) plus some nontrivial function, but with small numerical coefficient due to the loop suppression. Such a behavior, from the point of view of the dispersion relations (where each resonance \(|r\rangle\) contributes to the correlation function as \(1 - q^2 + m_r^2 r \cdot \langle 0 |J_1| \pi \rangle \langle \pi |J_2| 0 \rangle\)) implies that in the chiral limit we can keep only the \(\pi\) meson contribution and, therefore, we make an identification \(\langle O \rangle \approx \langle 0 |J_1| \pi \rangle \langle \pi |J_2| 0 \rangle\). An accuracy of this equation can be estimated by calculating corrections to the leading term which is \(\sim \langle O \rangle \sim q^2\). These corrections, which usually do not exceed \(10 \div 20\%\), are responsible for the contribution of the higher states and proportional to \(\sim \langle 0 |J_1| \pi \rangle \langle \pi |J_2| 0 \rangle\). Those contributions have the same order of magnitude as the corrections in OPE, i.e. about \(10 \div 20\%\). If it were not true, the matching of the right hand side and left hand side of the corresponding sum rules would be not possible. Therefore, if we knew the \(\pi\)-meson matrix elements \(\langle 0 |J_1| \pi \rangle \langle \pi |J_2| 0 \rangle\) independently (from PCAC, for example), we would estimate the condensate \(\langle O \rangle\) with a high enough accuracy.

We demonstrate the method by considering the following correlation function

\[ i \int dx e^{iqx} \langle T\{J_1^\mu(x), J_2(0)\} \rangle = q^\mu T(q^2), \quad J_1^\mu = \bar{d}\gamma^\mu \gamma_5 g\sigma Gu, \quad J_2 = \bar{u}\gamma_5 g\sigma Gd \]  

where \(gG \equiv \sigma_{\mu\nu} G^{a}_{\mu\nu} \lambda^a_2\) and \(\sigma_{\mu\nu} = i/2(\gamma_\mu \gamma_\nu - \gamma_\nu \gamma_\mu)\). The leading contribution is determined by the condensate of interest:

\[ T(q^2) = \frac{1}{q^2} \langle \bar{q}(g\sigma G)^2 q \rangle. \]  

At the same time, both \(\pi\) meson matrix elements which enter this correlator are known– one of them \(\langle 0 |\bar{d}\gamma^\mu \gamma_5 g\sigma Gu|\pi \rangle\) can be expressed in terms of the matrix element \(\langle A,4 \rangle\) which we already know, and another one \(\langle \pi |\bar{u}\gamma_5 g\sigma Gd|0 \rangle\) can be found from PCAC. Finally we arrive at the following equation:

\[ \langle \bar{q}g^2(\sigma G)^2 q \rangle = 4 \delta^2 \cdot \langle \bar{q}(g\sigma G)q \rangle \approx \frac{\langle \bar{q}(g\sigma G)q \rangle^2}{\langle \bar{q}q \rangle} \approx m_0^4 \langle \bar{q}q \rangle, \]  

(B.3)
where we used the following $\pi$ meson matrix element

$$\langle 0|\bar{q}g\bar{G}_\mu\gamma_\nu u|\pi(q)\rangle = if_\pi q_\mu\delta^2 \simeq if_\pi q_\mu \frac{\langle q(g\sigma G)q \rangle}{4\langle \bar{q}q \rangle}, \quad \delta^2 \simeq 0.2 \text{ GeV}^2. \quad (B.4)$$

This matrix element was calculated in Ref. [27] (in terms of $\delta^2$) and in Ref. [10] (in terms of mixed vacuum condensate). Numerically these results are in the perfect agreement to each other.

As we mentioned, an accuracy of Eq. (B.3) can be estimated by calculating corrections to the leading term (B.2). The most important loop correction can be calculated explicitly [20] and it turns out to be 30 times smaller than the main contribution (B.2); therefore, the precision of our estimate (B.3) for the condensate $\langle \bar{q}g^2(\sigma G)^2 q \rangle$ is determined mainly by the accuracy of the $\pi$ meson matrix elements which, we believe, is reasonably high. If we describe a deviation from the factorization prescription for the condensate $\langle \bar{q}g^2(\sigma G)^2 q \rangle$ by introducing the parameter $K$ as a measure of the non-factorizability ($K = 1$ if the factorization would work), we get the following estimate from Eq. (B.3) for the parameter $K$:

$$\langle \bar{q}g^2(\sigma G)^2 q \rangle = \frac{K}{3} \langle g^2 G_{\mu\nu}^2 \rangle \langle \bar{q}q \rangle, \quad K \simeq 3 \div 4. \quad (B.5)$$

which is the main result of these calculations. To be on the safe side we use $K \simeq 3$ in all our numerical estimates in the text.
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