Externally-driven transmission and collisions of domain walls in ferromagnetic wires

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Analytical multi-domain solutions to the dynamical (Landau-Lifshitz-Gilbert) equation of a one-dimensional ferromagnet including an external magnetic field and spin-polarized electric current are found using the Hirota bilinearization method. A standard approach to solve the Landau-Lifshitz equation (without the Gilbert term) is modified in order to treat the dissipative dynamics. I establish the relations between the spin interaction parameters (the constants of exchange, anisotropy, dissipation, external-field intensity, and electric-current intensity) and the domain-wall parameters (width and velocity) and compare them to the results of the Walker approximation and micromagnetic simulations. The domain-wall motion driven by a longitudinal external field is analyzed with especial relevance to the field-induced collision of two domain walls. I determine the result of such a collision (which is found to be the elastic one) on the domain-wall parameters below and above the Walker breakdown (in weak- and strong-field regimes). Single-domain-wall dynamics in the presence of an external transverse field is studied with relevance to the challenge of increasing the domain-wall velocity below the breakdown.

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I. INTRODUCTION

Description of the magnetic-field- and electric-current-induced motions of domain walls (DWs) in nanowires has become a hot topic because of novel methods of storing and switching the (magnetically encoded) binary information. These proposals offer a progress in the miniaturization of memory and logic elements, utilizing crucial advantages of magnetic information encoding, (when a bit is identified with a single magnetic domain). Such information is insensitive to the voltage fluctuations while its maintenance does not cost any energy, which enables the data processing with the production of a small heat amount. Currently investigated random-access memories are built of metallic nanowires, formed into a parallel-column structure, which store magnetic domains separated by DWs. Such a three-dimensional (3D) magnetic system has the potential of storing more information than devices based on 2D systems, like hard-disk drives or electronic memories, in a given volume. Also, an interesting concept of logical operation via transmitting magnetic DWs through nanowires of specific geometries is being developed. I mention that ferroelectric nanosystems offer similar capabilities while their basic properties are studied with the same dynamical (Landau-Lifshitz-Gilbert) equation even though the effects of electroelastic coupling are strong.

In order to write and switch information, one can move the DWs via the application of an external magnetic field (parallel to the easy-axis) or via the application of a voltage which induces the spin-polarized electric current through the DW. The directions of the field-driven motion are different for the tail-to-tail and head-to-head DWs; thus the magnetic field induces the DW collisions, while the direction of the voltage-driven motion uniquely corresponds to the current direction. Field-driven motion and current-driven (below the Walker breakdown) motion are possible due to the magnetic dissipation and its description demands inclusion of the Gilbert term into the Landau-Lifshitz (LL) equation. However, existing many-domain analytical solutions to the LL equation do not include the dissipation, while approximate solutions using the Walker ansatz describe a single DW only. Since the parameters of the DW solutions to the Landau-Lifshitz-Gilbert (LLG) equation determine accessible values of technological characteristics (e.g. the minimal domain length, the bit-switching time, etc.), it is of interest to know the analytic DW solution to the LLG equation. Knowledge of the many-domain solution is of importance for preventing unwanted DW collisions which can result in an instability of the record and it enables verification of DW-collision simulations regardless of the internal structure or the geometry of the simulated system.

In the present paper, I perform an analytical study of the dynamics of multi-domain systems including the dissipation. The dynamical LLG system is bilinearized following the Hirota method of solving nonlinear equations and it is extended, via doubling the number of freedom degrees and the number of equations, into a time-reversal invariant form. The field solving the extended system contains proper and virtual (unphysical) dynamical variables and the physical components of the solution are shown to satisfy the primary LLG system in a relevant time regime. In particular, aiming to analytically describe the field-induced collision, I establish asymptotic three-domain magnetization profiles (relevant in the time limits $t \to \pm \infty$). With connection to the phenomenon of...
the Walker breakdown, (a cusp in the dependence of the DW velocity on the external field and in its dependence on the current intensity), I modify the LLG model in a way to make it applicable below the breakdown. I reduce it into a model of plane rotators. In this weak-field regime, the spin alignment in the DW area is saturated to a plane while, above the breakdown (the strong-field regime), the spins rotate about the magnetic-field axis (the easy axis). These considerations supplement micromagnetic simulations of the DW collisions in terms of the studied DW-parameter regimes [10]. Within the present method, I verify the Walker-ansatz predictions on the current-driven DW motion above and below the breakdown, (including adiabatic and non-adiabatic parts of the spin-transfer torque) [11]-[12], thus, showing the applicability of the present formalism to the field-driven motion of multi-domain systems with an additional voltage applied. With relevance to the challenge of controlling the maximum DW velocity below the Walker breakdown, we analyze the longitudinal-field-driven motion and current-driven motion of the DWs in the presence of an additional perpendicular (with respect to the easy axis) field.

A complementary study of the DW collision of magnetic DWs in the subcritical regime is performed in a separate paper [16].

In Sec. II, I extend the dissipative equations of motion of the ferromagnet in such a way as to make equations applicable to the unlimited range of time \( t \in (-\infty, \infty) \). Section III is devoted to the analysis of its single- and double-DW solutions in the presence of a magnetic field and an electric current. I study the field-induced collision in detail. The plane-rotator approach to the DW dynamics is described in Sec. IV. In Sec. V, consequences of the application of an external field perpendicular to the easy axis for the DW statics and dynamics are considered. Conclusions are given in Sec. VI.

II. DYNAMICAL EQUATIONS

The dynamics of the magnetization vector \( \mathbf{m} \) (\( |\mathbf{m}| = M \)) in the 1D ferromagnet is described with the LLG equation

\[
\frac{\partial \mathbf{m}}{\partial t} = \frac{J}{M} \mathbf{m} \times \frac{\partial^2 \mathbf{m}}{\partial x^2} + \gamma \mathbf{m} \times \mathbf{H} + \frac{\beta_1}{M} (\mathbf{m} \cdot \hat{x}) \mathbf{m} \times \hat{i} - \frac{\beta_2}{M} (\mathbf{m} \cdot \hat{j}) \mathbf{m} \times \hat{j} - \frac{\delta}{M} \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial x} - \frac{\alpha}{M} \mathbf{m} \times \frac{\partial \mathbf{m}}{\partial t}. \tag{1}
\]

The first term of the right-hand side (RHS) of (1) relates to the exchange spin interaction while the second term depends on the external magnetic field \( \mathbf{H} \); thus, \( \gamma \) denotes the gigamagnetic factor (up to its sign). The constant \( \beta_{1(2)} \) determines the strength of the easy axis (plane) anisotropy and \( \hat{i} \equiv (1,0,0), \hat{j} \equiv (0,1,0) \). Note that the long axis of the magnetic nanowire is an easy axis for the majority of real systems; however, another choice of the anisotropy axes does not influence the magnetization dynamics. The constant \( \delta \) is proportional to the intensity of the electric current through the wire and \( \delta \) changes its sign under time-reversal (the inversion of the electron flow) [17],[18]. The non-adiabatic part of the current-induced torque (which depends on \( \beta \)) is of dissipative origin; thus, \( \beta \to 0 \) with decreasing Gilbert damping constant \( \alpha \to 0 \) (one takes \( \alpha, \beta \ll 1 \)). Its inclusion is necessary if one describes an observed monotonous variation of (1) into (2) of the current-induced torque (which depends on \( \alpha, \beta \)).

\[
m_+ = \frac{2M}{f^* + g^* + f} f^*, \quad m_z = M \frac{f^* - g^*}{f^* + g^* + f}, \tag{2}
\]

where \((*)^*\) denotes the complex conjugate (c.c.)]

Introducing the complex dynamical parameters \( m_{\pm} = m_y \pm im_z \), I represent the magnetization components using a pair of complex functions \( g(x,t), f(x,t) \). This way I reduce the number of independent degrees of freedom. The relation between the primary and secondary dynamical variables

\[
f \left[ -\frac{\partial}{\partial x} + JD_x^2 + \delta(\beta - i)D_x + \alpha D_t \right] f^* \cdot g + Jg^2D_z^2g \cdot g - \left( \gamma H_x + \beta_1 + \frac{\beta_2}{2} \right) |f|^2 g - \frac{\beta_2}{2} f^* g^* = 0, \quad g^* \left[ -\frac{\partial}{\partial x} - JD_x^2 + \delta(\beta - i)D_x + \alpha D_t \right] f^* \cdot g - JfD_z^2f^* \cdot f^* + \left( -\gamma H_x + \beta_1 + \frac{\beta_2}{2} \right) |g|^2 f^* + \frac{\beta_2}{2} g^2 f = 0, \tag{3}
\]

where \( D_t, D_x \) denote Hirota operators of differentiation which are defined by

\[
D_t^nD_x^mb(x,t) \cdot c(x,t) \equiv (\partial/\partial x - \partial/\partial x')^m \times (\partial/\partial x - \partial/\partial x')^n b(x,t) c(x',t') |_{x=x',t=t'}. \]
The inclusion of the dissipation into the LLG equation is connected to breaking the symmetry with relevance to the time reversal. Therefore, neither \( 1 \) nor \( 3 \) can describe the magnetization evolution on the whole time axis. In particular, the application of an external magnetic field or current to the DW system (or creation of a domain in the presence of an external field) initiates a nonequilibrium process of the DW motion. Such a motion cannot be present in the distant past \( t \to -\infty \) since a nonzero value of the dissipative function [relevant to the Gilbert term in \( 1 \)] would indicate unlimited growth of the energy with \( t \to -\infty \). Thus, solitary-wave solutions to \( 1 \) are relevant only in the limit of large positive values of time \( 11, 13 \]. This fact makes impossible an exact analysis of the DW collisions using \( 3 \) and motivates extension of the dynamical system within a formalism applicable to the whole length of the time axis.

We write modified equations of motion using a similar trick to the one proposed by Bateman with application to the Lagrangian description of the damped harmonic oscillator \( 20 \). It is connected to the concept by Lakshmanan and Nakamura of removing the dissipative term from the evolution equation of ferromagnets via multiplying the time variable by a complex constant \( 21 \). However, it demands an improvement in the spirit of Bateman’s idea \( 22 \). The concept is to extend the dynamical system doubling the number of freedom degrees and adding the equations which differ from the original ones by the sign of the dissipation constants. Resulting extended system is symmetric with relevance to the time-arrow reversal, however, its solution consists of physical and virtual fields. Let us mention that different quantum dissipative formalisms (non-equilibrium Green functions, thermo-field dynamics, rigged Hilbert space) are based on the Bateman’s trick \( 23 \).

We extend the system of secondary dynamical equations \( 3 \) since it describes unconstrained dynamics unlike the primary LLG equation \( 1 \). We replace \( g, g^*, f, f^* \) in \( 3 \) with novel fields of the corresponding set \( g_1, g_2, f_2, f_2^* \) and of the set of their c.c.. For \( \alpha, \beta = 0 \), in the absence of dissipation, \( g_1 = g_2 = g, f_1 = f_2 = f \). For the case \( H_0 = H_z = 0 \), the secondary dynamical equations transform into

\[
\begin{align*}
    f_2 \left[-iD_t + JD_x^2 + \delta(\beta - i)D_x + \alpha D_t\right] f_1^* &\cdot g_1 + Jg_2^2D_x^2g_1 \cdot g_1 - \left(\gamma H_x + \beta_1 + \frac{\beta_2}{2}\right) f_2f_1^*g_1 - \frac{\beta_2}{2} f_2^*f_1^*g_1^* = 0, \\
g_2^* \left[-iD_t - JD_x^2 + \delta(\beta - i)D_x - \alpha D_t\right] f_1 \cdot g_1 - Jf_2D_x^2f_1 \cdot f_1 + \left(-\gamma H_x + \beta_1 + \frac{\beta_2}{2}\right) g_2^2g_1f_1^* + \frac{\beta_2}{2} g_2g_1f_1 = 0.  
\end{align*}
\]

Writing \( 4 \), we have replaced the last terms on the lhs of \( 3 \) in a way to be linear in \( g_2^*, f_2^* \), which ensures that they vanish (diverge) with time in the presence of \( H_x \neq 0 \) with similar damping (exploding) rates as all other terms of these equations, (in particular, their damping does not modify the anisotropy). The additional equations of the dynamical system differ from \( 4 \) by the sign of the dissipation constants \( \alpha, \beta \)

\[
\begin{align*}
    f_1 \left[-iD_t + JD_x^2 + \delta(-\beta - i)D_x - \alpha D_t\right] f_2^* \cdot g_2 + Jg_1^2D_x^2g_2 \cdot g_2 - \left(\gamma H_x + \beta_1 + \frac{\beta_2}{2}\right) f_1f_2^*g_2 - \frac{\beta_2}{2} f_1^*f_2^*g_2^* = 0, \\
g_1^* \left[-iD_t - JD_x^2 + \delta(-\beta - i)D_x + \alpha D_t\right] f_2 \cdot g_2 - Jf_1D_x^2f_2 \cdot f_2 + \left(-\gamma H_x + \beta_1 + \frac{\beta_2}{2}\right) g_1f_2^*f_2^* + \frac{\beta_2}{2} g_1g_2f_2 = 0.  
\end{align*}
\]

Though the previous dynamical variables \( g(f), g^*(f^*) \) were mutually independent, they had to be c.c. to each other in order that the system of equations \( 3 \) and their c.c. was closed. In the system of eight equations: \( 3, 4, 5, 6 \) and their c.c., \( g_1(f_1) \) is not a c.c. to \( g_2(f_2) \), while comparing \( 4 \) and \( 3 \), one sees that \( g_2(x,t) (f_2(x,t)) \) can be obtained from \( g_1(x,t) (f_1(x,t)) \) via changing the sign of its parameters \( \alpha, \beta \).

Under the time-arrow inversion, the system of the novel equations transforms into itself if one accompanies this operation by the transform of the novel dynamical variables \( g_{1(2)} \to f_{2(1)}, f_{1(2)} \to -g_{2(1)} \). The equations \( 4 \) and their c.c., which determine the magnetization dynamics for large positive values of time (in particular, for \( t \to \infty \)), contain the differentials of the functions \( g_1, g_1^*, f_1, f_1^* \). Therefore, the magnetization vector should be expressed with these functions in the relevant time regime. Writing the magnetization in the form

\[
m_+ = \frac{2M}{f_1^*/g_1 + g_1^*/f_1}, \quad m_x = M f_1^*/g_1 - g_1^*/f_1 (f_1^*/g_1 + g_1^*/f_1)
\]

ensures that their components satisfy \( |\textbf{m}| = M, m_x = m_+^* \), and they reproduce \( 2 \) for \( \alpha = \beta = 0 \). In the regime of large negative values of time, in particular, for \( t \to -\infty \), we can analyze the evolution of the magnetization with the inversed time arrow. It is described with the reversed magnetization vector

\[
\hat{m}_+ = -\frac{2M}{f_2^*/g_2 + g_2^*/f_2}, \quad \hat{m}_x = -M f_2^*/g_2 - g_2^*/f_2 (f_2^*/g_2 + g_2^*/f_2).
\]
III. DOMAIN-WALL MOTION

Let us analyze the multi-domain solutions to (4)-(5) in the absence of any external magnetic field, $H = 0$. We search for the solutions which describe a single DW and two DWs in the forms $f_1^t = 1$, $g_1 = w_1 e^{i k_1 x - l_1 t}$, and $f_1 = 1 + v^* e^{i k_x x - l_1 t} e^{i k_y x - l_2 t}$, $g_1 = w_1 e^{i k_1 x - l_1 t} + w_2 e^{i k_2 x - l_2 t}$, respectively, where $k_j = \text{Re}(k_j)$, and sign$(k_1) = -\text{sign}(k_2)$. Inserting these ansätze into (4)-(6), one finds

$$\begin{align*}
l_j &= \frac{1}{1 + i \alpha} \left\{-\sqrt{(Jk_j - \beta_1)} \left[J k_j^2 - (\beta_1 + \beta_2)\right] + \delta(1 + i \beta)k_j \right\}, \\
k_j &= \sqrt{\beta_1 + \beta_2(w_j + w^* j)^2/(4|w_j|^2)}.
\end{align*}$$

The single-wall (two-domain) solutions with $|k_1| \in (\sqrt{\beta_1}/J, \sqrt{\beta_1 + \beta_2}/J)$ describe moving solitary waves (topological solitons), \[7, 24\]. One sees the correspondence between the wall width and the spin deviation from the easy plane since the dependence of $k_j$ on $w_j$. When $\delta = 0$, static two-domain solutions represent the Bloch DW, $(|k_1| = \sqrt{\beta_1}/J, w_1 = -w_1^*)$, or Neel DW, $(|k_1| = \sqrt{\beta_1 + \beta_2}/J, w_1 = w_1^*)$, respectively (the nomenclature of \[24\] which differs from Neel and Bloch wall definition of e.g. \[26\]). These two solutions correspond to the ones found in \[25, 27\] within the XY model. The DW profiles can be described with the functions

$$\begin{align*}
m_+(x, t) &= M \frac{w_1 e^{-ilmt}}{|w_1|} \sech[k_1 x - \text{Re} l_1 t + \log|w_1|], \\
m_-(x, t) &= -M \tanh[k_1 x - \text{Re} l_1 t + \log|w_1|],
\end{align*}$$

$(k_1 > 0$ relates to the head-to-head structure while $k_1 < 0$ to the tail-to-tail structure). At the time points of the discrete set $t = \pi n / |l_1|$ and $n = 0, \pm 1, \pm 2, \ldots$, one finds $g_1 = g_2 = g$, $f_1 = f_2 = f$ and (4) coincide with (3). Therefore, (4) is a solitary-wave solution to (1). (in particular, it coincides with the one representing a Bloch DW or a Neel DW for $\delta \neq 0$, \[13\].) Throughout the paper, we focus our attention on the externally-driven dynamics of the Bloch and Neel DWs, since these initially-static structures are the most important with relevance to the magnetic data storage.

We establish that static double-wall (three-domain) solutions to the LLG equation cannot be written with the above Hirota expansion when $k_1 = -k_2$. In this case the coefficient $v$:

$$v = -\frac{\beta_2 J k_1^2 w_1^* w_2^*}{(J k_1^2 - \beta_1)(J k_2^2 - \beta_1 - \beta_2)}$$

diverges with $|k_1| \to \sqrt{\beta_1}/J$ or $|k_1| \to \sqrt{(\beta_1 + \beta_2)/J}$. Analogously to the XY model, the Hirota expansion is inapplicable to static three-domain configurations of the Bloch walls or Neel walls, while there exists static solution to (4)-(5) which describes a pair of different-type (Neel and Bloch) walls \[28\]. In particular, for $k_1 = \sqrt{\beta_1}/J$, $k_2 = -\sqrt{(\beta_1 + \beta_2)/J}$, and $w_1 = -w_1^*$, one finds

$$v = -\frac{\beta_2 w_1^* w_2^*}{2\beta_1 + \beta_2 - 2\sqrt{\beta_1(\beta_1 + \beta_2)}}$$

Let us emphasize that we have not excluded the coexistence of a pair of Neel walls or Bloch walls in a magnetic wire. However, the overlap of both the topological solitons induces their interaction which leads to an instability of their parameters and, unlike for non-topological solitons, is not a temporal one \[29\].

Solving (4)-(5) in the presence of a longitudinal magnetic field $H_x \neq 0$, we apply the ansätze

$$\begin{align*}
f_1^t &= (1 + v^* e^{i k_x x - l_1 t} e^{i k_y x - l_2 t}) e^{\gamma H_x t/(-2i + 2\alpha)}, \\
g_1 &= (w_1 e^{i k_x x - l_1 t} + w_2 e^{i k_2 x - l_2 t}) e^{-\gamma H_x t/(-2i + 2\alpha)}
\end{align*}$$

at the discrete time points $t = t_n \equiv 4\pi n (1 + \alpha^2)/(|\gamma H_x|)$, where $n = 0, \pm 1, \pm 2, \ldots$, (let $\delta = 0$ for simplicity). Behind these time points, in the presence of the longitudinal field, the last terms on the lhs of (4)-(5) change faster (they oscillate with the three-times higher frequency) than the other ones. Therefore, taking the above ansätze, we apply an approach similar to the ‘rotating wave approximation’ in the quantum optics. Since this ansatz describes the spin structure rotation about the $x$-axis, it is applicable when the external field exceeds Walker-breakdown critical value, $|H_x| > H_W$. From the single-wall solution, (the case of $w_2 = 0$ or $w_1 = 0$), for $k_1 = \sqrt{\beta_1}/J$, $k_2 = -\sqrt{(\beta_1 + \beta_2)/J}$, we establish that applying the magnetic field in the easy-axis direction drives the DW motion with the velocity

$$c_{1(2)} = \gamma |H_x| \alpha/[|k_{1(2)}|(1 + \alpha^2)].$$

Correspondingly, for $H_x = 0$, applying the electric current through the initially static wall drives it to move with the velocity

$$c = \frac{\delta(1 + \alpha\beta)}{1 + \alpha^2}$$

which is independent of the DW width.

The essential difference between both kinds of the driven motion emerges from the analysis of the three-domain solutions. Under the external field, the two consecutive DWs move in the opposite directions. The walls which are closing up to each other collide and eventually they can annihilate or wander off each other. The application of the electric current along the magnetic wire drives both the DWs to move in the same direction with the same velocity. Analyzing long-time limits of the magnetization vector in different regions of the coordinate $x$, we establish the consequences of the field-induced collision of the complex of a Bloch DW interacting with a Neel DW. We use the ansatz (12) and assume $\delta = 0$. Let $\eta_j \equiv k_j(x - x_0) - \gamma H_x \alpha t/(1 + \alpha^2)$, $\tilde{\eta}_j \equiv k_j(x - x_0) + \gamma H_x \alpha t/(1 + \alpha^2)$. For $H_x > 0$, at $t = t_n$ (within
the above ‘rotating wave approximation’), we find the distant-future limit of the magnetization \( |(1)\)

\[
m_+ \approx \begin{cases} \frac{m_+(1)}{m_+(2)} \eta_2 & \eta_1 \approx 0 \\ \frac{m_+(2)}{m_+(1)} \eta_1 & \eta_2 \approx 0 \end{cases} = \lim_{t \to \infty} m_+,
\]

\[
m_x \approx \begin{cases} \frac{m_x(1)}{m_x(2)} \eta_2 & \eta_1 \approx 0 \\ \frac{m_x(2)}{m_x(1)} \eta_1 & \eta_2 \approx 0 \end{cases} = \lim_{t \to \infty} m_x, \tag{15}\]

where

\[
m_+(j) = 2M \frac{v/w_j^* \sin \theta_k e^{-\gamma H_j t/(1+\alpha^2)}}{1+|v|^2/w_j^2 e^{2\eta_k}},
\]

\[
m_x(j) = -M \frac{1-|v|^2/w_j^2 e^{2\eta_k}}{1+|v|^2/w_j^2 e^{2\eta_k}}, \tag{16}\]

and \( j \neq k \). Identifying the parameters \( x_0 \) with the DW-center positions, we introduce the restriction on \( w_j \), \( |v|/|w_j| = 1 \). We notice that \( m_+(1), m_x(1) \) as well as \( m_+(2), m_x(2) \) are the Walker single-DW solutions to the primary LLG equation which describe the motion of well separated DWs \([9, 30]\). Thus, our three-domain profiles of the fields \( m \) tend to satisfy \( \eta \) in the limit \( t \to \infty \) according to the requirement formulated in the previous section.

In the distant-past limit, we describe the magnetization evolution with the reversed time arrow. Following \( 7 \),

\[
\tilde{m}_+ \approx \begin{cases} \frac{\tilde{m}_+(1)}{\tilde{m}_+(2)} \tilde{\eta}_1 & \tilde{\eta}_2 \approx 0 \\ \frac{\tilde{m}_+(2)}{\tilde{m}_+(1)} \tilde{\eta}_1 & \tilde{\eta}_2 \approx 0 \end{cases} = \lim_{t \to -\infty} \tilde{m}_+,
\]

\[
\tilde{m}_x \approx \begin{cases} \frac{\tilde{m}_x(1)}{\tilde{m}_x(2)} \tilde{\eta}_2 & \tilde{\eta}_1 \approx 0 \\ \frac{\tilde{m}_x(2)}{\tilde{m}_x(1)} \tilde{\eta}_2 & \tilde{\eta}_1 \approx 0 \end{cases} = \lim_{t \to -\infty} \tilde{m}_x, \tag{17}\]

where

\[
\tilde{m}_+(j) = -2M \frac{v/w_j^* e^{\gamma H_j t/(1+\alpha^2)}}{1+|v|^2/w_j^2 e^{2\eta_k}},
\]

\[
\tilde{m}_x(j) = M \frac{1-|v|^2/w_j^2 e^{2\eta_k}}{1+|v|^2/w_j^2 e^{2\eta_k}}, \tag{18}\]

and \( j \neq k \). In order to consider the collision of the pair of DWs which are infinitely distant from each other at the beginning of their evolution, we determine the magnetization dynamics in the limit \( t \to -\infty \). For this aim, one has to invert the propagation direction of the kinks of \( \tilde{m} \) and to reverse the arrow’s head of the field vector \( \tilde{m} \). Utilizing the properties \( \tilde{m}_+(j)(x+x_0k,0) = \tilde{m}_+(j)(-x+x_0k,0) \), \( \tilde{m}_x(j)(x+x_0k,0) = -\tilde{m}_x(j)(-x+x_0k,0) \), we arrive at

\[
m_+(x,t) = \begin{cases} -\tilde{m}_+(1)(-x+2x_{01},t) & \eta_1 \gg \eta_2 \approx 0 \\ -\tilde{m}_+(2)(-x+2x_{02},t) & \eta_2 \gg \eta_1 \approx 0 \end{cases},
\]

\[
m_x(x,t) = \begin{cases} -\tilde{m}_x(1)(-x+2x_{01},t) & \eta_1 \gg \eta_2 \approx 0 \\ -\tilde{m}_x(2)(-x+2x_{02},t) & \eta_2 \gg \eta_1 \approx 0 \end{cases}, \tag{19}\]

The applicability of the above procedure to study the asymptotic evolution of a single DW is easy to verify since any single-DW solution satisfies

\[
m_+(x,t) = -\tilde{m}_+(x-2x_{01},t),
\]

\[
m_x(x,t) = \tilde{m}_x(x-2x_{01},t). \tag{20}\]

Typically, one should consider the formulas \([15, 19]\) with relevance to the case \( \beta_1 \gg \beta_2 \), thus \( k_1 \approx -k_2 \), which corresponds to commonly studied crystalline magnetic nanowires, e.g. for Fe, FePt, \( \beta_2/\beta_1 \sim 10^{-1} \). For noncrystalline (permalloy) nanowires Fe1−xNix deposited on a crystalline substrate, the easy-axis anisotropy constant determined from uniform-resonance measurements was found to be, unexpectedly, as big as in the crystalline nanowires \([32]\). Therefore, even when neglect structural effects in real systems which lead to the saturation of the spin alignment in the DW area to the easy plain or hard plain (the Walker breakdown) which suppresses their spontaneous motion, the spectrum of spontaneously propagating DWs in nanowires would be very narrow and their velocities would be very small.

According to \([15, 19]\), two initially closing up DWs have to diverge after the collision. If one of the colliding DWs that was initially, for \( t \to -\infty \), described with the field parameter \( \tilde{m}_+(j), \tilde{m}_x(j) \), at the same time, for \( t \to \infty \), described with the field parameter \( m_+(j), m_x(j) \). Therefore, reflecting DWs exchange their parameters \( x_0 \to x_{02}, w_1 \to w_2, k_1 \to -k_2 \). It is connected to exchanging the directions of the spin orientation in the yz-plane in the wall areas, (after the collision the Néel wall changes into the Bloch wall and vice versa as shown in Fig. 1). Our prediction corresponds to the result of the collision analysis performed for spontaneously propagating DWs (in absence of external field, electric current, and dissipation). According to findings of \([24, 32]\), the DWs reflect during the collision in a way that one can say they pass through each other without changing their widths and velocities, however, with changing their character from the head-to-head one into the tail-to-tail one and vice versa.

Up till now, we have considered systems of infinite domains whose energy cannot be defined. However, the smaller a domain is the bigger percentage of the Zeeman part of its energy is lost per time unit due to the DW motion. The condition of the domain-energy minimization determines the direction of this motion. The domains aligned perpendicularly to the external field grow while the domains aligned antiparallelly to the field diminish. Any DW reflection induces a motion which contradicts this rule. Such a motion has to be decelerated and, eventually, it has to be suppressed when the decrease of the DW interaction energy equals the increase of the Zeeman energy. The outcome of a many-collision process in a finite-size system is the appearance of a 1D magnetic-bubble structure similar to widely known 2D bubble structures \([34]\). The bubble size and concentration depend on the magnetic field intensity. Each bubble is ended with a
Neel DW at one of its sides and with a Bloch DW at the other side. Some analogy with a complex boundary of hard (quasi-2D) magnetic bubbles can be noticed since such a border contains alternating Neel and Bloch points in its structure [35]. Let us mention, that interesting concepts of storing and transforming the binary information have had been developed several decades ago with relevance to 2D magnetic bubble systems, though, they have been abandoned because of technological problems of the time [36].

Numerical analyses of the field-induced DW collision (micromagnetic studies) have been performed with relevance to flattened nanowires (quasi 1D nanostripes) below and just above the Walker breakdown using the dissipative LL equation [10]. The systems below the threshold correspond to a plane-rotator model studied in section 4, while the systems above the threshold are qualitatively described with the present model. The mentioned simulations focus on the collisions of similar-type (Neel or Bloch) DWs neglecting the anisotropy. They have predicted mutual annihilation or reflection of the walls depending on (parallel or anti-parallel) spin alignments in both the DW centers. This result is partially supported by a perturbation analysis of Bloch-wall interactions within the XY model which has shown such DWs to repel or attract each other depending on their chiralities [29]. The method of present study cannot be applied to these collisions since one is unable to determine neither the double-Neel nor double-Bloch wall analytical solutions to the dynamical equations. However, except in the case of a periodically distributed DWs, multi-Bloch or multi-Neel structures are unstable because of unbalanced DW interactions, thus, they seem to be less suitable for the information-storing purposes than the Neel-Bloch DW structures. To the best of the author knowledge, the field-induced collision of the Neel DW with the Bloch DW has not been simulated.

IV. PLANE-ROTATOR MODEL

In order to describe the DW dynamics below the Walker breakdown, we consider a system of plane rotators. Let us reduce the primary (LLG) dynamical system to its single component. Saturating the magnetization dynamics to the easy plain ($m_y = 0$), we neglect the spin rotation about the $x$-axis and $z$-axis since the relevant torque components are equal to zero. For $\mathbf{H} = (H_x, 0, 0)$, inserting

$$m_x = M \frac{1-a_1^2}{1+a_1^2}, \quad m_y = 0, \quad m_z = 2M \frac{a_1}{1+a_1^2} \tag{21}$$

(where $a_1$ takes real values) into the $y$-component of (1), one arrives at a nonlinear diffusion equation

$$\left( -\alpha \frac{\partial a_1}{\partial t} - \gamma H_x a_1 - \delta \beta \frac{\partial a_1}{\partial x} + J \frac{\partial^2 a_1}{\partial x^2} \right) (1 + a_1^2)$$

$$-2Ja_1 \left( \frac{\partial a_1}{\partial x} \right)^2 - \beta_1 a_1 (1 - a_1^2) = 0. \tag{22}$$

We use another ansatz describing the dynamics constrained to the $xy$-plain (a hard plain)

$$m_x = M \frac{1-a_2^2}{1+a_2^2}, \quad m_y = 2M a_2 \frac{a_1}{1+a_2^2}, \quad m_z = 0. \tag{23}$$

Then, we insert it into the $z$-component of (1) and we arrive at

$$\left( -\alpha \frac{\partial a_2}{\partial t} - \gamma H_x a_2 - \delta \beta \frac{\partial a_2}{\partial x} + J \frac{\partial^2 a_2}{\partial x^2} \right) (1 + a_1^2)$$

$$-2Ja_2 \left( \frac{\partial a_2}{\partial x} \right)^2 - (\beta_1 + \beta_2) a_2 (1 - a_2^2) = 0. \tag{24}$$

which differs from (22) by a constant at the anisotropy term. With relevance to the case $\delta = 0$, one finds the two-domain solution

$$a_{1(2)} = \frac{\alpha e^{k_{1(2)}x} - \gamma H_x t/\alpha}{k_{1(2)}},$$

$$|k_1| = \sqrt{\frac{\beta_1}{J}}, \quad |k_2| = \sqrt{\frac{\beta_1 + \beta_2}{J}}, \tag{25}$$

which correspond to the Bloch DW (to the Neel DW). When $H_x \neq 0$, the DW propagates with the velocity

$$c_{1(2)} = \frac{\gamma |H_x|}{|k_{1(2)}|/\alpha}. \tag{26}$$

The applicability of the plane-rotator model is limited by the Walker-breakdown condition. The magnetic field $|H_x|$ cannot exceed a critical value $H_W$ (see Fig. 2a) which corresponds to the spin deviation from the basic magnetization plane (a canting) at the center of the DW about a limit angle equal or smaller than $\pi/4$. We estimate an upper limit of the Walker critical field considering the $x$-component of the LLG equation at the DW center

$$\left. \frac{\partial m_x}{\partial t} \right|_{x=x_{01(2)}} \approx \left[ \beta_2 M m_y m_z - \delta \frac{\partial m_z}{\partial x} \right]_{x=x_{01(2)}}. \tag{27}$$
where \( x_{01(2)} = -\log(w)/k_{1(2)} \). Let \( \varphi \) denotes the angle of the spin deviation (a canting) at DW center. Inserting (21) and transforming \( m_x m_z \rightarrow m_1^2 \sin(\varphi) \cos(\varphi) \) in (27) or inserting (23) and transforming \( m_x m_y \rightarrow m_2^2 \sin(\varphi) \cos(\varphi) \) in (27), one arrives at

\[
\frac{\partial a_{1(2)}}{\partial t} = \frac{\beta_2}{2} \sin(2\varphi) a_{1(2)} - \delta \frac{\partial a_{1(2)}}{\partial x}
\]

and finally, assuming \( |\varphi| \leq \pi/4 \), at

\[
|H_x| \leq H_W \leq \max H_W \approx \frac{\alpha \beta_2}{2\gamma} + \frac{\alpha \delta}{\gamma} |k_2|.
\]

This expression corresponds to the one given in [31, 38]. However, we notice that, for typical nanowires whose width-to-thickness ratio is bigger than 20 (double-atomic or triple-atomic layers of a submicrometer width), measuring \( H_W \), one has estimated the canting angle to take a value of a few degrees at most, [14, 30, 32].

For \( H = 0 \), \( \delta \neq 0 \), the two-domain solution to (22)-(24) takes the form

\[
a_{1(2)} = a e^{k_{1(2)} x - \delta \beta t/\alpha},
\]

\[
|k_1| = \sqrt{\frac{\beta_1}{J}}, \quad |k_2| = \sqrt{\frac{\beta_1 + \beta_2}{J}}.
\]

It is seen that only the non-adiabatic part of the spin-transfer torque contributes to (22)-(24) since the current-dependent term is proportional to \( \beta \). From (28), the current-induced Walker breakdown corresponds to the critical current intensity

\[
\delta_W \leq \beta_2/[2|k_1|(1 + \beta/\alpha)]
\]

if \( H_x = 0 \), (see Fig. 2b). It has been observed that \( H_W, \delta_W \) decrease with decreasing the nanowire width-to-thickness ratio, [32, 40], because this ratio determines the strength of the easy-plane anisotropy while \( H_W, \delta_W \rightarrow 0 \) with \( \beta_2 \rightarrow 0 \), [14]. Notice that analytical calculations using 2D XY model, experimental observations, and simulations of the spin ordering in nanowires show this ordering to vary along the cross-section width of the nanowire in the DW area, thus, revealing a complex topological structure [10, 42]. Therefore, our plane-rotator description is valid only for a qualitative analysis of the DW dynamics in the nanowires.

Neither finding nonstationary double-Bloch nor double-Neel solutions in the form of the Hirota expansion (including its second order) does not manage. In particular, inserting

\[
a_{1(2)} = \frac{w_1 e^{k_{1(2)} x} + w_2 e^{k_{1(2)} x}}{1 + v_{1(2)} e^{k_{1(2)} x + k_{1(2)} x}} e^{-\gamma H_x t/\alpha},
\]

into (22)-(24), for \( k_1 = -k_1' = \pm \sqrt{\beta_1'/J}, k_2 = -k_2' = \pm \sqrt{(\beta_1 + \beta_2)/J} \), leads to the divergence of \( v_{1(2)} \) as it follows from the collision of Bloch and Neel walls below the Walker breakdown. I propose to apply an effective 1D model assuming the magnetization precession to be overdamped, thus, taking the lhs of (11) to be equal to zero. Inclusion of the constraint \( |\mathbf{m}| = M \) leads to the modified (by neglecting the first terms on the lhs) system (11)-(13). Solving it, we predict the field-induced DW collision below the Walker breakdown to result in their reflection similar to the one described in section 3. The reflection is accompanied by the change of the Bloch wall into the Neel wall and vice versa. Let us emphasize that there is no spontaneous DW motion below the Walker breakdown when neglect magnetostatic effects [43].

The technological challenge of increasing the DW speed is especially important below the Walker breakdown, where the driving-field is relatively weak. Referring to this purpose, we mention an attempt utilizing an increase of the nanowire width-to-thickness ratio [43], thus, an increase of the damping constant \( \alpha \). This approach fails since, according to simulations of [39], the maximum of the field-induced DW velocity is insensitive to \( \alpha \) below the breakdown. It is because \( c_{1(2)} \propto \alpha^{-1} |H_x| \), while \( |H_x| \leq H_W \propto \alpha \). On the other hand, since \( \beta \) grows with \( \alpha \) (the non-adiabatic part of the spin-transfer torque is of a dissipative origin), the velocity of the current-induced DW motion

\[
c = \delta \beta/\alpha
\]

can be insensitive to the increase of the nanowire width-to-thickness ratio as well. One has attributed some reported DW-velocity increase due to the nanostripe-edge roughness to a decrease of its effective cross-section width. Another attempt utilized an increase of \( H_W \) due to the increase of the anisotropy constant \( \beta_2 \). It has been done via the nanowire deposition on a specific crystalline substrate [45]. However, the most efficient method of influencing the maximum DW velocity below the Walker breakdown is the application of the transverse magnetic field, which is considered in the next section [40].
V. DOMAIN WALL IN PERPENDICULAR TO EASY AXIS FIELD

Let us define \( H_\pm \equiv H_y \pm iH_x \). For \( H_x = 0 \), we search for a two-domain solution to (1) using a different ‘multilinearization’ than used in the previous sections

\[
 f_2 = -i(D + J D_x^2 + \delta(\beta - i)D_x + \alpha D) \cdot g_1 + \gamma H_+ \cdot g_1 + \frac{\gamma H_+}{2} f_1^* (f_1 f_2 + g_2^* g_1) - \left( \beta_1 + \frac{\beta_2}{2} \right) f_2 f_1^* g_1 - \frac{\beta_2}{2} f_1^* g_2^* = 0, \\
g_2^* = -i(D + J D_x^2 + \delta(\beta - i)D_x + \alpha D) \cdot f_1 + \gamma H_- \cdot f_1 + \frac{\gamma H_-}{2} g_1 f_1^* + \frac{\beta_2}{2} g_2^* f_2 = 0, \\
f_2 g_1 D_x^2 f_1^* \cdot f_1 - f_1^* g_2^* D_x^2 g_1 \cdot g_1 = 0. 
\] (34)

Let us focus our attention on the case \( \alpha, \beta, \delta = 0 \) for simplicity. Then one has \( f_1 = f_2 = f, g_1 = g_2 = g \), while in the general case the relations (16), (17) apply. We analyze two cases of the external-field direction; the one parallel to the easy plane \( H_+ = iH_z \), and the one perpendicular to the easy plane \( H_+ = H_y \).

In the case of \( H_+ = iH_z \), we apply the ansatz

\[
 f_1^* = f_2 = g_1 + s_1 e^{k_1 x - l_1 t}, \\
g_1 = g_2^* = 1 \cdot s_1 + g_1 e^{k_1 x - l_1 t}, 
\] (35)

where \( k_1, q_1, s_1 \) denote real constants, (the parameter \( l_1 \) can take complex values when \( \alpha \neq 0 \)). The solution in the form (35) describes the wall between two domains whose spins are deviated from the easy axis onto the external-field direction about an angle which grows with \( |H_+| \). Inserting this ansatz into (34), one finds

\[
 s_1 = \frac{\beta_1 - \sqrt{\beta_1^2 - \gamma^2 H_y^2}}{\gamma H_z} g_1. 
\] (36)

Considering the solutions which are static in the absence of the electric current, \( l_1 = 0 \) for \( \delta = 0 \), one arrives at

\[
 |k_1| = \sqrt{\frac{\beta_1^2 - \gamma^2 H_y^2}{\beta_1 J} g_1. 
\] (37)

In the case of \( H_+ = H_y \), the ansatz relating to the deviation of the domain magnetization from the easy axis onto the external-field direction takes the form

\[
 f_1^* = f_2 = q_2 + s_2 e^{k_2 x - l_2 t}, \\
g_1 = g_2^* = s_2 + q_2 e^{k_2 x - l_2 t}. 
\] (38)

with real \( k_2, q_2, s_2 \). From (38), we find

\[
 s_2 = \frac{\beta_1 + \beta_2 - \sqrt{(\beta_1 + \beta_2)^2 - \gamma^2 H_y^2}}{\gamma H_y} q_2. 
\] (39)

The static solutions correspond to

\[
 |k_2| = \sqrt{\frac{(\beta_1 + \beta_2)^2 - \gamma^2 H_y^2}{(\beta_1 + \beta_2) J^2}}. 
\] (40)

The transverse external field does not drive the DW motion even in the presence of the magnetic dissipation (\( \alpha \neq 0 \)). When the current through the wire and the magnetic dissipation are applied, under the transverse magnetic field, the DW moves with the velocity \( c \) given by (14), which is independent of the value of this field. Then the solution to (34) satisfies the bilinearized LLG system (Eqs. (3) with additional \( H_\perp \)-dependent terms) at the time points of the discrete set \( t = \pi n / \Im l_1 \), where \( n = 0, 1, 2, \ldots \), since \( f_1 = f_2 = f, g_1 = g_2 = g \) at these points. Including an additional to \( H_\perp \) longitudinal component of the magnetic field \( H_x \) drives the DW motion. For the realistic case \( H_{W} < |H_y| < |H_+| \ll |\beta_1 / \gamma|, \) neglecting small contributions to the \( H_\perp \)-dependent part of the torque, one finds the velocity of such a DW propagation (13) or (26) above and below the Walker breakdown, respectively, with \( |k_{I(2)}| \) given by (37), (40). This velocity nonlinearly increases with \( |H_x| \), (47). Searching for \( c_{I(2)} \), in the case \( |H_x| < H_{W} \), additionally, I have taken the lhs of (11) equal to zero as discussed in section 4. The manipulation of \( c_{I(2)} \) via the application of the transverse magnetic field is potentially useful for speeding up the processing with a magnetically-encoded information. We also notice that the transverse-field dependence of \( |k_{I(2)}| \) enables influencing the magnitude of the critical current of the Walker breakdown \( \delta_W \), following (31).

VI. CONCLUSIONS

We have analytically studied the DW dynamics in the presence of the external magnetic field and the electric current along the magnetic wire within the LLG approach. It has demanded overcoming the difficulty arising from breaking the time-reversal symmetry by inclusion of the magnetic dissipation. We have removed this asymmetry of the dynamical system by introducing additional (virtual) dynamical variables, which is a similar trick to the Lagrangian approach to the damped harmonic oscillator. Determining a connection of the additional dynamical variables to the evolution of the magnetization vector in specific ranges of time, we have analyzed the dynamics of a single DW and of a pair of DWs.

The magnetic-field-induced velocities of the DWs, the formulas (13) and (26), and the current-induced velocities (14) and (38) are found to correspond to the ones of the Walker approach above and below the breakdown, respectively. According to (28), static three-domain solutions to 1D LLG equation describe pairs of Neel and Bloch walls. For the purposes of the qualitative dynamics analysis of a number of DWs below the Walker breakdown, especially of the Neel-Bloch pairs, we have proposed a dynamical equation which differs from the LLG
one by neglecting the lhs in [11]. Below and above the breakdown, the neighboring Neel and Bloch walls move in the presence of the longitudinal external field in the opposite directions. Their collision results in the DW reflection accompanied by the reorientation of the Neel wall into the Bloch wall and vice versa. In other words, the DWs pass through each other without changing their widths and velocities, however, the head-to-head DW structure changes into the tail-to-tail one and vice versa.

Our method is useful for the analysis of two-domain systems under the transverse (with respect to the easy axis) external field, which enables a verification of numerical and experimental results [12-13]. A reorientation of the magnetic domains due to the transverse field induces a widening of the DW area up to the infinity when approach with the field intensity to the coercivity value. The consequence of the transverse-field application is an increase of the critical current (a shift of the current-driven Walker breakdown).

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[1] S. S. P. Parkin, M. Hayashi, L. Thomas, Science 320, 190 (2008).
[2] M. Hayashi, et al., Science 320, 209 (2008).
[3] D. A. Allwood, et al., Science 296, 2003 (2002), D. A. Allwood, et al., ibid, 309, 1688 (2005).
[4] D. A. Allwood, G. Xiong, R. P. Cowburn, J. Appl. Phys. 101, 024308 (2007).
[5] J. F. Scott, Science 315, 954 (2007), A. Schilling, R. M. Bowman, G. Catalan, J. F. Scott, J. M. Gregg, Nano Lett. 7, 3787 (2007).
[6] A. Gruverman, et al., J. Phys.: Condens. Matter 20, 342201 (2008).
[7] M. M. Bogdan, A. S. Kovalev, JETP Lett. 31, 424 (1980).
[8] R. Hirotori, J. Phys. Soc. Japan, 51, 323 (1982).
[9] A. P. Malozemoff, J. C. Slonczewski, Phys. Rev. Lett. 29, 952 (1972), N. L. Schryer, L. R. Walker, J. Appl. Phys. 45, 5406 (1974), A. Thiaville, J. M. Garcia, J. Miltat J. Magn. Magn. Mat. 242–245, 1061 (2002).
[10] A. Kunz, Appl. Phys. Lett. 94, 132502 (2009), D. Djuhana, H.-G. Piao, S.-C. Yu, S. K. Oh, D.-H. Kim, J. Appl. Phys. 106, 103926 (2009).
[11] Z. Li, S. Zhang, Phys. Rev. Lett. 92, 207204 (2004), S. Zhang, Z. Li, Phys. Rev. Lett. 93, 127204 (2004).
[12] A. Thiaville, Y. Nakatani, J. Miltat, Y. Suzuki, Europhys. Lett. 69, 990 (2005).
[13] Y. Tserkovnyak, A. Brataas, G. E. W. Bauer, J. Magn. Magn. Mat. 320, 1282 (2008).
[14] G. S. D. Beach, M. Tsoi, J. L. Erskine, J. Magn. Magn. Mat. 320, 1272 (2008).
[15] V. V. Volkov, V. A. Bokov, Phys. Sol. State 50, 199 (2008).
[16] A. Janutka, Phys. Rev. E 83, 056608 (2011).
[17] J. C. Slonczewski, J. Magn. Magn. Mat. 159, L1 (1996).
[18] Y. B. Bazaliy, A. B. Jones, S.-C. Zhang, Phys. Rev. B 57, R3213 (1998).
[19] M. D. Stiles, W. M. Saslow, M. J. Donahue, A. Zangwill, Phys. Rev. B 75, 214423 (2007), N. Smith, ibid, 78, 216401 (2008), W. M. Saslow, J. Appl. Phys. 105, 07D315 (2009).
[20] H. Bateman, Phys. Rev. 38, 815 (1931), H. Dekker, Phys. Rep. 80, 1 (1981), D. Chrucinski, J. Jurkowski, Ann. Phys. 321, 854 (2006).
[21] M. Lakshmanan, K. Nakamura, Phys. Rev. Lett. 53, 2497 (1984), P. B. He, W. M. Liu, Phys. Rev. B 72, 064410 (2005).
[22] E. Magyari, H. Thomas, R. Weber, Phys. Rev. Lett. 56, 1756 (1986).
[23] L. V. Keldysh, Sov. Phys. JETP 20, 1018 (1965), I. Hardman, H. Umezawa, Y. Yamanaka, J. Math. Phys. 28, 2925 (1987), I. E. Antoniou, M. Gadella, E. Karpov, I. Prigogine, G. Prumo, Chaos Solit. Frac. 12, 2757 (2001).
[24] A. M. Kosevich, B. A. Ivanov, A. S. Kovalev, Phys. Rep. 194, 117 (1990), M. Svendsen, H. Fogedby, J. Phys. A 26, 1717 (1993).
[25] L. N. Bulaevskii, V. L. Ginzburg, Sov. Phys. JETP 18, 530 (1964), J. Lajzerowicz, J. J. Niez, J. de Physique Lett. 40, L165 (1979).
[26] S. Middelhoek, J. Appl. Phys. 34, 1054 (1963).
[27] S. Sarker, S. E. Trullinger, A. R. Bishop, Phys. Lett. A 59, 255 (1976).
[28] I. V. Barashenkov, S. R. Woodford, Phys. Rev. E 71, 026613 (2005).
[29] I. V. Barashenkov, S. R. Woodford, E. V. Zemlyanaya, Phys. Rev. E 75, 026604 (2007).
[30] G. S. D. Beach, C. Nistor, C. Knutson, M. Tsoi, J. L. Erskine, Nature Mat. 4, 741 (2005), G. S. D. Beach, C. Knutson, M. Tsoi, J. L. Erskine, J. Magn. Magn. Mat. 310, 2038 (2007).
[31] E. Y. Vedmedenko, et al., Phys. Rev. Lett. 92, 077207 (2004), D. Hinze, U. Nowak, R. W. Chantrell, O. N. Myrjasov, Appl. Phys. Lett. 90, 082507 (2007).
[32] H. Alouach, H. Fujikawa, G. J. Mankey, J. Vac. Sci. Technol. A 23, 1046 (2005).
[33] W. M. Liu, et al., Phys. Rev. B 65, 172416 (2002).
[34] F. H. de Leeuw, R. van der Doel, U. Enz, Rep. Prog. Phys. 43 689 (1980).
[35] A. Rosencaig, W. J. Tabor, T. J. Nelson, Phys. Rev. Lett. 29, 946 (1971).
[36] T. H. O’Dell, Rep. Prog. Phys. 49, 589 (1986).
[37] M. Hayashi, et al., Phys. Rev. Lett. 96, 197207 (2006).
[38] A. Mougin, M. Cormier, J. P. Adam, P. J. Metaxas, J. Ferre, Europhys. Lett. 78, 57007 (2007).
[39] A. Kunz, IEEE Trans. Magn. 42, 3219 (2006).
[40] S. Glathe, D. Mattheis, D. V. Berkov, Appl. Phys. Lett. 93, 025208 (2008).
[41] M. Yan, A. Kakay, S. Gliga, R. Hertel, Phys. Rev. Lett. 104, 057201 (2010).
[42] S. W. Yuan, H. N. Bertram, Phys. Rev. B 44, 12395 (1991), O. Tchernyshyov, G. W. Chern, Phys. Rev. Lett. 95, 197204 (2005), M. Klaui, et al., ibid. 95, 026601 (2005), J. Y. Lee, K. S. Lee, S. Choi, K. Y. Guslienko, S. K. Kim, Phys. Rev. B 76, 184408 (2007).

[43] D. Djuhana, et al., IEEE Trans. Magn. 46 217 (2010).

[44] Y. Nakatani, A. Thiaville, J. Miltat, Nature Mat. 2, 521 (2003), E. Martinez, L. Lopez-Diaz, L. Torres, C. Tristan, O. Alejos, Phys. Rev. B 75, 174409 (2007).

[45] J. Y. Lee, K. S. Lee, S. K. Kim, Appl. Phys. Lett. 91, 122513 (2007).

[46] A. Kunz, S. C. Reiff, J. Appl. Phys. 103, 07D903 (2008), M. T. Bryan, T. Schrefl, D. Atkinson, D. A. Allwood, ibid. 103, 073906 (2008).

[47] S. Glathe, T. Mikolajick, D. V. Berkov, R. Mattheis, Appl. Phys. Lett. 93, 162505 (2008).

[48] V. L. Sobolev, H. L. Huang, S. C. Chen, J. Magn. Magn. Mat. 147, 284 (1995), B. A. Ivanov, N. E. Kulagin, JETP 85, 516 (1997).