Research on the differential settlements of mat foundations

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Abstract. Mat foundation is a very commonly used foundation type. There are many advantages in adopting mat foundations including the effects of loading compensation, full utilization of underground space, lowering foundation settlements and increasing the factor of safety of bearing capacity for foundations etc. In addition, because of the high stiffness of the mat foundation and foundation girders the differential settlements could be effectively decreased, which would in turn increase the safety of structures. The differential settlements of the foundation will produce extra stresses in the structure. At the worst it will make the structure members failure. There are many factors which would affect the differential settlements of the foundation, which include the uneven distribution of structure column loading, the subgrade reaction of foundation soils (or so called soil spring coefficient), the combined stiffness of the foundation plate and foundation girders, the shape and size of the mat plate etc. The main objective of this research is to use the finite element method to simulate the interactions between the mat foundation and the underneath soils. It will focus on the systematic research on all possible factors affecting the differential settlements of mat foundations. The influence of each factor on the differential settlements will be studied and identified. The influence charts for the differential settlements for each individual factor will be proposed. Finally a reliable method using these influence charts to predict the differential settlements of mat foundations will be developed, which will be verified by practical examples.

1. Introduction

Mat foundation is a very commonly used foundation type. Mat foundations would carry both the loading from the upper structures and also the pressures from the soils underneath. Therefore this is a typical soil-structure interaction problem. Researches about the mat foundations are still in progress [1-3]. In the design of mat foundations the foundation plate would carry the moments and shears from soil reactions as well as the extra moments and shears resulting from the possible differential settlements of the foundation. Too much differential settlements will at the worst make the structures failure. Therefore allowable differential settlement for the foundations is specified in many building codes. The study of differential settlements of mat foundations is therefore very important. There are several factors that will affect the differential settlements of mat foundations. Those include the distribution of column loading; subgrade reaction coefficient of soils; the thickness of foundation plate; stiffness of foundation girders and the shape and width of the foundation plates etc. [1]. Usually the foundations resting on the clay soils suffer more obvious differential settlements. Therefore this research will focus on the study of differential settlements of mat foundations on the clay soils.

There are several methods simulating the differential settlements of mat foundations [4]. In the first simplified method as shown in figure 1, the soil springs are only imposed directly under the
foundation columns, and then the soil reaction pressures are acting on the foundation girders together with the column loads to calculate the differential settlements. In the second simplified method as shown in figure 2, the soil springs are uniformly imposed directly under the foundation girders. In this case only the column loads are necessary to be imposed to calculate the differential settlements.

Neither of the two above-mentioned methods takes into consideration the influence of the mat plates on the differential settlements. In order to correctly predict the differential settlements, a finite element method is adopted to simulate the soil-structure interactions as shown in figure 3 in this research. A commercial finite element program “SAFE” is used to simulate the interaction behavior. In this simulation, plate element is used to simulate the mat plate while the beam element is used to simulate the foundation girders. On the other hand, the soil springs are uniformly imposed directly under the foundation mat plate to correctly simulate the soil reaction pressures. The soil springs must be assumed not to take any tensile forces to match the special characteristic of foundation soils [2,3]. However only flat mat foundation as shown in figure 4 is considered in this preliminary research, the influence of the foundation girders on the differential settlements could be therefore ignored.
2. Theoretical background

2.1. Vertical coefficient of subgrade reaction
As far as the simulation of the soil behavior, the concept of vertical coefficient of subgrade reaction is adopted to simulate the foundation soils as soil springs. The factors that influence the coefficient of subgrade reaction include soil type; foundation pressure; foundation size; embedment depth of foundation and thickness of compressible soil layer [2,3,5]. In this research, the consolidation theory is used to calculate the coefficient of subgrade reaction. According to the consolidation theory, the consolidation settlement \( \Delta H_c \) could be calculated as:

\[
\Delta H_c = \frac{C_c}{1 + e_0} H_0 \log \left( \frac{\sigma_0 + \Delta \sigma'}{\sigma_0} \right)
\]

where \( e_0, C_c \) and \( H_0 \) are respectively the initial void ratio; compression index and thickness of compressible soil layer, whereas \( \sigma_0' \) and \( \Delta \sigma' \) are the initial effective stress and effective stress increment of the soils due to foundation loads.

The coefficient of subgrade reaction \( k_v \) is defined as:

\[
k_v = \frac{q}{\Delta H_c}
\]

where \( q \) is the foundation pressure. As far as the foundation pressure; foundation size; embedment depth and thickness of compressible soil layer are known, then the effective stress increment due to foundation loads could be calculated. Therefore the consolidation settlements and then vertical coefficient of subgrade reaction could be obtained. From the consolidation theory it could be seen that under the constant foundation pressure the consolidation settlement increases as the size of foundation or the thickness of compressible soil layer increases, which would therefore make the coefficient of subgrade reaction decrease. On the other hand as the embedment depth of foundation increases, a smaller consolidation settlement and higher coefficient of subgrade reaction would be obtained because the net soil pressure increment decreases due to compensation effects of soil excavation. The above-mentioned conclusions regarding the coefficient of subgrade reaction are in accordance with the results of relative researches [2,3,5]. From the consolidation theory the relationship between the foundation pressure and consolidation settlement is nonlinear. Therefore it could be further concluded that the coefficient of subgrade reaction is not constant and depends on the magnitude of foundation pressures. Based on the conclusions discussed above, it is reasonable to use the consolidation theory to predict the coefficient of subgrade reaction of foundation soils. Later a practical numerical example will be chosen to show how to calculate the coefficient of subgrade reaction based on the consolidation theory.

2.2. The interaction behavior between the mat foundation and the soils underneath
The governing equation for the interaction behavior between the mat foundation and the soils
underneath could be written as: [6]

\[
\frac{\partial^4 w}{\partial x^4} + \frac{2\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4} = q + \frac{p}{D}D(\partial x \partial y)
\]  

(3)

where \( w \) is the deflection (settlement) of the mat foundation and \( q \) is the soil reaction pressure under the foundation. Therefore,

\[
q = -k_c w
\]  

(4)

\( p \) is the column load and \( D \) is expressed as:

\[
D = \frac{E_c t^3}{12(1-\mu_c^2)}
\]  

(5)

where \( E_c \); \( \mu_c \) and \( t \) are respectively the Young’s modulus and Poisson’s ratio of concrete and thickness of the mat plates.

In this research, a commercial finite element program “SAFE” is used to simulate the interaction behavior between the mat foundation and the soils underneath [7]. In the simulation, the thick plate element is adopted to simulate the mat plate, because mat plate is usually pretty thick. The thick plate element has planar flexural plate stiffness in two directions within the plate as well as the translational stiffness in the direction normal to the plate. In addition it could take the effects of shear deformation into consideration [8]. Therefore it is appropriate to use thick plate element to simulate the mat plate. In the “SAFE” program, the beam element can be chosen to model the foundation girders, while the linear spring can be taken to model the soil under the mat foundation. Besides the soil springs are not allowed to take any tensile forces. Furthermore the point load can be chosen to model the column loads and the uniformly distributed load can be used to model the loads imposed on mat plates. Therefore the finite element approach using the “SAFE” program could model the interaction behavior between the mat plate and soils appropriately.

3. Numerical Simulation Of The Mat Foundation

A systematic approach will be taken to understand the factors that influence the differential settlement of the mat foundation in the following. As mentioned above, there are several factors that will affect the differential settlements of mat foundations. Those include the distribution of column loading; subgrade reaction coefficient of soils; the thickness of foundation plate; stiffness of foundation girders and the shape and width of the foundation plates etc. In this research, the distribution of column loading is taken as column spacing of 8 meters both in vertical and horizontal directions. Each column would take a load, which could be calculated by the floor area shared by the column multiplied by a uniformly distributed load of 20 ton/m² acting on the floors.

As far as the thickness of mat plate is concerned, 50; 70; 90; 110 and 130 cm would be used in the simulation. The vertical coefficients of subgrade reaction would be considered as 30; 50; 100 and 200 ton/m³. A square mat plate of 16m x 16m (L=B) is considered first in this simulation. The relationship between the ratio of differential settlement to the average settlement and the thickness of the mat plates under different coefficients of subgrade reaction is shown in figure 5. In this figure the average settlement of mat foundation could be calculated by the uniformly distributed load of 20 ton/m² over the coefficient of subgrade reaction. It could be seen in figure 5 that the differential settlement decreases as the thickness of mat plate increases. Furthermore the ratio of differential settlement to the average settlement of mat plate increases as the coefficient of subgrade reaction increases. This is ascribed to decrease of average settlement of mat plate as the coefficient of subgrade reaction increases. Actually the differential settlement of mat plate does decrease as coefficient of subgrade reaction increases.
In order to understand the influence of the shape of the mat plate on the differential settlement, three other shapes of mat plates with the sizes of 32m x 16m (L=2B); 48m x 16m (L=3B) and 64m x 16m (L=4B) are analyzed and the simulation results are shown in figures 6, 7 and 8. The results shown in figures 6-8 are similar to those in figure 5, but it could be seen that the differential settlement of mat plate increases as the shape of the mat plate becomes longer. This result is in accordance with previous research [1]. The above results could illustrate the influence of foundation shapes on the differential settlements.

Then the influence of the size of mat plates on the differential settlements is analyzed. The size of the mat plates is taken respectively as 24m x 24m; 32m x 32m; 40m x 40m; 48m x 48m and 56m x 56m, and the differential settlements for each size of mat plates mentioned above is respectively studied and then compared to the previous results of mat size of 16m x 16m. Let define \( B^* = 16m \) as the basic width of the mat plate. The differential settlement of mat plate of 16m x 16m is therefore taken as the basic differential settlement. The ratio of differential settlement of the mat plate with the above-mentioned five different sizes to the basic differential settlements is here defined as size modified coefficient for the differential settlement \( \alpha \) and also defined the width of mat plate to the basic width of the mat plate as mat width ratio \( B/B^* \). The relationship between the size modified coefficient \( \alpha \) and mat width ratio \( B/B^* \) for different thickness of mat plates under constant \( k_v = 30 \) ton/m\(^3\) is shown in figure 9. Similar relationships of size modified coefficient \( \alpha \) under \( k_v = 50; 100 \) and 200 ton/m\(^3\) are respectively shown in figures 10, 11 and 12. The influence of the size of mat plates on the differential settlements is therefore understood.
As discussed above, for any arbitrary mat plate (shape; size and thickness of mat plates) and coefficient of subgrade reaction, the differential settlement could be obtained by interpolation from the results in figures 5-8 once the shape and thickness of the mat plate and coefficient of subgrade reaction are given. Then the size modified coefficient \( \alpha \) could be calculated by interpolation from the results in figures 9-12 as the size of mat plate is known. Finally the actual differential settlement of mat plate is obtained by multiplying the differential settlement calculated above by the size modified coefficient \( \alpha \). Later some numerical examples will be used to illustrate the detailed procedures to get differential settlements of mat foundations.

4. Verification by numerical examples

In order to verify the above-mentioned procedures to calculate the differential settlements of mat plates, three numerical examples will be shown in the following. In the first example, \( k_v = 50 \text{ ton/m}^3 \) and the size of mat plate of \( 32 \text{m} \times 80 \text{m} \) (\( L=2.5B \)) and the thickness of mat plate of 110cm are assumed. According to \( k_v = 50 \text{ ton/m}^3 \) and \( L=2.5B \) and thickness of mat plate of 110cm, the differential settlement could be obtained as 2.56cm by interpolation from the results in figure 6 (\( L=2B \)) and figure 7 (\( L=3B \)). But the actual width of the mat plate is 32m, the mat width ratio is therefore 2.0. From the results in figure 10, the size modified coefficient \( \alpha \) could be obtained as 1.75. Therefore the actual differential settlement for this mat plate is 4.47cm. The “SAFE” finite element program also simulates the mat foundation in the first example, and the model-predicted differential settlement is 4.27cm. The deviation of the differential settlement according to the procedures suggested in this research and that by model prediction of “SAFE” program is within the error of 4.5%.

In the second example, \( k_v = 100 \text{ ton/m}^3 \) and the size of mat plate of \( 40 \text{m} \times 80 \text{m} \) (\( L=2B \)) and the thickness of mat plate of 90cm are assumed. According to \( k_v = 100 \text{ ton/m}^3 \) and \( L=2B \) and thickness of mat plate of 90cm, the differential settlement could be obtained as 1.99cm from the results in figure 6 (\( L=2B \)). But the actual width of the mat plate is 40m, the mat width ratio is therefore 2.5. From the
results in figure 11, the size modified coefficient $\alpha$ could be obtained as 1.81. Therefore the actual differential settlement for this mat plate is 3.60cm. The “SAFE” finite element program also simulates the mat foundation in the second example, and the model-predicted differential settlement is 3.20cm. The deviation of the differential settlement according to the procedures suggested in this research and that by model prediction of “SAFE” program is within the error of 11.1%.

In the third example, $k_v = 200$ ton/m$^3$ and the size of mat plate of 48m x 72m (L=1.5B) and the thickness of mat plate of 70cm are assumed. According to $k_v = 200$ ton/m$^3$ and L=1.5B and thickness of mat plate of 70cm, the differential settlement could be obtained as 2.67cm by interpolation from the results in figure 5 (L=B) and figure 6 (L=2B). But the actual width of the mat plate is 48m, the mat width ratio is therefore 3. From the results in figure 12, the size modified coefficient $\alpha$ could be obtained as 1.02. Therefore the actual differential settlement for this mat plate is 2.71cm. The “SAFE” finite element program also simulates the mat foundation in the third example, and the model-predicted differential settlement is 2.64cm. The deviation of the differential settlement according to the procedures suggested in this research and that by model prediction of “SAFE” program is within the error of 2.6%.

From three numerical examples above, it could be seen that the differential settlement evaluated according to the procedures suggested in this research is quite close to that predicted by the finite element analysis. Therefore it could be concluded that the evaluated differential settlement of mat foundation following the procedures suggested in this research is pretty accurate.

5. Application of a practical example

In the following, a practical example would be used to illustrate how to calculate the differential settlement of mat foundation as well as the coefficient of subgrade reaction as shown in the following:

\[
\begin{align*}
q &= 20 \text{ ton/m}^2 \\
\text{Water Table} &\\
10\text{m} &
\end{align*}
\]

\[
\begin{align*}
\text{Mat Foundation 24x60m Thickness 90cm} &\\
e_0 &= 1.0, C_c = 0.20 &
\end{align*}
\]

\[
\begin{align*}
10\text{m} &
\end{align*}
\]

\[
\begin{align*}
\text{Sand Unit Weight 2.0 ton/m}^3 &\\
\text{Clay Unit Weight 2.0 ton/m}^3 &
\end{align*}
\]

\[
\begin{align*}
\text{Hard Stratum} &
\end{align*}
\]

After calculation for the above example, the initial effective stress of the center of the clay stratum $\sigma_0^e$=15 ton/m$^2$ while the effective stress increment $\Delta \sigma^e$=14.5 ton/m$^2$. The thickness of the clay stratum $H_0$=10m. Substituting these values into Eq.(1), one could obtain that the average settlement of the mat foundation $\Delta H_c = 29.4$cm. Then with the use of Eq.(2), one could get the coefficient of subgrade reaction $k_v = 68.1$ ton/m$^3$. With the actual size of the mat plate of 24m x 60m (L=2.5B) and thickness of mat plate of 90cm, the differential settlement could be obtained as 2.26cm by interpolation from the results in figure 6 (L=2B) and 3.76cm by interpolation from the results in figure 7 (L=3B). Therefore the differential settlement of the mat plate with L=2.5B should be interpolated as 3.01cm. But the actual width of the mat plate is 24m, the mat width ratio is therefore 1.5. By interpolation from the results in figure 10 ($k_v=50$ ton/m$^3$) and in figure 11 ($k_v=100$ ton/m$^3$), the size modified coefficient $\alpha$ could be obtained as 0.619. Finally the actual differential settlement for this mat plate is 1.86cm.

6. Conclusions
From the discussion above, the following conclusions could be drawn as:

1. A method to predict the differential settlement of mat foundation has been proposed in this research, which is pretty accurate and easy to estimate.

2. A method to estimate the coefficient of subgrade reaction of foundation soils based on the consolidation theory has been proposed in this research.

3. The differential settlement of mat foundation predicted following the procedures suggested in this research is quite close to that obtained from the finite element analysis.

4. Finally in this research a practical example of mat foundation is used to illustrate the detailed procedures to estimate the differential settlement of the mat plate and the coefficient of subgrade reaction of foundation soils.

References

[1] ACI Committee 1987 Suggested Design Procedures for Combined Footings and Mats, ACI Committee 336 Report

[2] Bowles J E 1988 Foundation Analysis and Design 4th ed (New York: McGraw-Hill)

[3] Das B M 1997 Principles of Foundation Engineering 4th ed. (Boston, MA: PWS Publishing Co.)

[4] Chen C C, Chay Y C and Hong H K 1994 Investigation Procedures for Structure Design of High Rise Buildings Project Report of Architecture Research Institute of the Ministry of Internal Affairs MOIS830016

[5] Vesic A S 1961 Bending of beams resting on isotropic solid J. Engineering Mechanics Div. ASCE 87 35-53

[6] Timoshenko S and Woinowsky-Krieger S 1959 Theory of Plates and Shells (New York: McGraw-Hill)

[7] SAFE User’s Manual 1998 (Berkeley, USA: Computers & Structures Inc.)

[8] Ibrahimbegovic A 1993 Quadrilateral finite elements for analysis of thick and thin plates, Computer Method. Appl. Mech. Eng. 110 195-209