Monkey Optimization System with Active Membranes: A New Meta-heuristic Optimization System

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Abstract. Optimization techniques, used to get the optimal solution in search spaces, have not solved the time-consuming problem. The objective of this study is to tackle the sequential processing problem in Monkey Algorithm and simulating the natural parallel behavior of monkeys. Therefore, a P system with active membranes is constructed by providing a codification for Monkey Algorithm within the context of a cell-like P system, defining accordingly the elements of the model - membrane structure, objects, rules and the behavior of it. The proposed algorithm has modeled the natural behavior of climb process using separate membranes, rather than the original algorithm. Moreover, it introduced the membrane migration process to select the best solution and the time stamp was added as an additional stopping criterion to control the timing of the algorithm. The results indicate a substantial solution for the time consumption problem, significant representation of the natural behavior of monkeys, and considerable chance to reach the best solution in the context of meta-heuristics purpose. In addition, experiments use the commonly used benchmark functions to test the performance of the algorithm as well as the expected time of the proposed P Monkey optimization algorithm and the traditional Monkey Algorithm running on population size \(n\). The unit times are calculated based on the complexity of algorithms, where P Monkey takes a time unit to fire rule(s) over a population size \(n\); as soon as, Monkey Algorithm takes a time unit to run a step every mathematical equation over a population size \(n\).

Keywords: Membrane Computing, P systems, Monkey Algorithm, Bio-inspired Computing, Swarm Optimization.

1 Introduction

Membrane computing is a computational model to simulate and abstract the functionality and structure of biological living cells. It belongs to natural computing
science, which investigates the creation of well-designed computational systems based on natural biological systems [26, 37]. Illustrations of natural computing contain: neural computation inspired by simulating the functionality of brain, evolutionary computation formulated by the Darwinian evolution of species, swarm intelligence inspired by the behavior of groups of organisms, and life systems modeled by the functionalities of natural life such as membrane computing [11, 6, 4]. Membrane computing is a theoretical computational model, designed by Gheorghe Păun in 1998, providing distributed and parallel computation devices able to solve NP-hard problems in linear or polynomial time [18, 15, 31]. Over the past ten years, there were many attractive studies in building connectivity solutions between the potentiality and features of membrane computing and real-life applications, such as membrane algorithms for solving optimization problems [41, 19, 21]. Furthermore, a wide variety of membrane computing applications have been proposed in solving hard computational problems in an efficient way [8, 21, 22, 4]. By comparing the results of previous and the existing evolutionary algorithms in membrane computing studies, membrane computing compromises as a more competitive methodology, because of these three advantages: better convergence, stronger robustness and a better balance between exploration and exploitation [19, 36, 20].

P systems with active membranes and electrical charges were introduced in [17]. This is the first variant in Membrane Computing allowing to solve computationally hard problems in polynomial time. It consists of a hierarchical structure composed of several inner polarized membranes; surrounded by a skin membrane; regions encircled by the membranes containing objects; and evolution rules [29, 68]. Bio-Inspired swarms and biological systems have been widely used as a computational and optimization solutions for many real applications [25, 40, 9]. Their models work sequentially, while in reality, swarms’ members work together in a governed parallel system. The sequential methodologies used to implement swarms lead to time consumption problem and weaken the power of natural parallel behavior of swarms. One of those swarms models is Monkey Algorithm (MA) [42]. It represents the movement of monkeys over mountains to reach the highest mountaintop. Monkeys work in parallel to get into their purpose, but the algorithm does it sequentially.

This paper aims to enhance the computational efficiency of optimization algorithms by introducing a new algorithm based on a swarm technique called P Monkey System with Active Membranes (PMSAM). It is a new methodology to support the natural parallel behavior of monkeys’ movements. It uses the computational power of membrane computing to simulate their movements over mountains, based on a predefined mathematical model. The objective of this paper is to propose a significant solution to time-consuming problem and simulating the natural parallel behavior of monkeys.

The rest of the paper is organized as follows: Section 2 introduces P system with active membranes, MA, and its applications. In section 3 the setup of PMSAM is presented. While section 4 reports the outcome of the numerical and
empirical experiments. Finally, the main conclusions of the study are summarized and directions for future work are outlined.

2 Related Work

In this section, we provide the definitions of P system with active membranes and MA in addition to a discussion about some previous studies that are related to both of them.

2.1 P system with active membranes

P system is a research area in computer science targeting the abstraction of the structure and functionality of living cells, in addition to the cells systematized way in tissues or higher cells order structure. Membrane computing has three basic types; cell-like, tissue-like, and neural-like P systems. Cell-like P systems are working on the cell level. P system with active membranes is a class of P systems, which, together with the basic transition systems and the symport/antiport systems is one of the three central types of cell-like P systems [17].

We recall the definition of P system with active membranes that will be used in this paper see [17, 28, 29] for more details.

Definition 1. A P system with active membranes of degree $m \geq 1$ is a tuple:

$$\Pi = (O, H, \mu, W_1, ..., W_m, R, 0)$$

Where

1. $O$ is the alphabet of objects;
2. $H$ is a finite set of labels for membranes;
3. $\mu$ is a membrane structure consisting of $m$ membranes, injectively labeled with elements of $H$. Each membrane is supposed to have an "electrical polarization", one of the three possible: positive (+), negative (-), or neutral (0) and is presented as $S = \{+, -, 0\}$;
4. $W_1, ..., W_m$ represents a string of $O$ describing multisets of objects in a region $m$ in $\mu$;
5. $R$ is a finite set of developmental rules with the following types:

(a) $p^n_{s_1} h \rightarrow a^n_{h}$, for $h \in H, s_1 \in S, p^n \in O, a \in O^*$
(Object evolution rules; associated with membranes and contingent of the label and the charge of membranes);

(b) $p^{n_1 s_1}_{h_1} h_1 \rightarrow p^{n_2 s_2}_{h_2} h$, for $h \in H, s_1, s_2 \in S, p^{n_1}, p^{n_2} \in O$
(in-communication rules; an object is sent into the membrane, perhaps modified through this process; also the polarization of the membrane can be modified, but not its label);

(c) $p^{n_1 s_1}_{h_1} h_1 \rightarrow p^{n_2 s_2}_{h_2} h$, for $h \in H, s_1, s_2 \in S, p^{n_1}, p^{n_2} \in O$
(out-communication rules; an object is sent out of the membrane, perhaps modified through this process; also the polarization of the membrane can be modified, but not its label);
(d) $p^{v_1}_{h_1} s_1 h \rightarrow p^{v_2},$ for $h \in H, s_1 \in S, p^{v_1}, p^{v_2} \in O$
(Dissolution rules; in reaction with an object, a membrane can be dissolved, while the object specified in the rule can be modified);

(e) $s_1 h p^{v_1}_h \rightarrow s_2 h p^{v_2}_h$ for $h \in H, s_1, s_2, s_3 \in S, p^{v_1}, p^{v_2}, p^{v_3} \in O$
(Division rules for elementary membranes; in reaction with an object, the membrane is divided into two membranes with the same label, and possibly of different polarizations);

6. 0 is the output region. It represents the output environment.

Cell-like P systems are popular paradigm used to solve NP-problems based on their efficiency and computational power such as P system with active membranes [16, 32, 13]. Accordingly, a previous study investigated the computational efficiency of the cell-like P system with Symport/Antiport rules to solve NP-complete problems, QSAT problem, in linear time solution [30]. It used some membrane operations (division rules for elementary membranes and communication rule) in the proposed method. The authors further proved that such systems can efficiently solve this complete problem in linear time. A new automatic CNV segmentation method based on an unsupervised and parallel machine learning technique named density cell-like P systems, which injects a clustering algorithm into cell-like P system. Therefore, that study achieved a better accuracy in comparison with previous studies [34]. A new study introduced the idea of limited number of membranes to solve satisfiability problem to make P system more realistic model [12].

There are two studies introduced the idea of timed P system with active membranes to prove that the correctness of the solution does not depend on the precise timing of involved rules [28, 29]. The authors assumed that firing a rule takes a time unit according to objects interactions. Those studies proved the computational efficiency of those systems with respect to the execution times of inner operations. Also, the types of membranes keep the same initial configurations during the computation. The time unit was calculated from an external clock out of the membrane structure, and it may affect the synchronization between membrane rules and time consumption units.

2.2 Monkey Algorithm

Monkey Algorithm is employed to solve global numerical optimization problems with continuous variables [42]. The algorithm entails three main processes; Climb, Watch-Jump and Somersault processes, plus the initialization of the algorithm parameters and monkey positions; and the termination process to apply stopping criteria. Climb process finds the local optimal solution in local search space. The Watch-Jump process looks for other points whose objective values exceed those of the current solution. Somersault process makes monkeys find new search spaces rapidly.

Monkey Algorithm processes are given below [42, 33, 43]:

1. Solution representation:
(a) \( n \) is the population size of monkeys;
(b) For the monkey \( i \), \( i \in \{1, \ldots, n\} \) its position is denoted as a vector \( p_i = (p_{i1}, \ldots, p_{id}) \) where \( d \) is a search space dimension.

2. Climb process:
   (a) A random vector is generated as \( \Delta p_i = (\Delta p_{i1}, \ldots, \Delta p_{id}) \), where
       \[
       \Delta p_{ij} = \begin{cases} 
       a & \text{with probability } \frac{1}{2} \\
       -a & \text{with probability } \frac{1}{2}
       \end{cases}
       \]
       The parameter \( a \) (\( a > 0 \)), called the step length of the climb process, can
       be determined by specific situations.
   (b) For each \( i, j, 1 \leq i \leq n, 1 \leq j \leq d \), calculate \( f'_{ij}(p_i) = f(p_i + \Delta p_i) - f(p_i - \Delta p_i) \)
       The vector \( f'_i(p_i) = (f'_{i1}(p_i), f'_{i2}(p_i), \ldots, f'_i(p_i)) \) is called the pseudo-
       gradient of the objective function \( f \) at the point \( p_i \);
   (c) For each \( i, 1 \leq i \leq n \), set \( y_i = p_{ij} + a \cdot \text{sign}(f'_{ij}(p_i)) \), for \( j = 1, \ldots, d \), and
       let \( y = (y_1, y_2, y_3, \ldots, y_d) \);
   (d) For each \( i, 1 \leq i \leq n \), let \( p_i \rightarrow y_i \) if \( y_i \) is feasible, otherwise \( p_i \) is kept
       unchanged;
   (e) Repeat steps a) to d) until there is a little change in the values of objective
       function in the neighborhood iterations, or the maximum allowable
       number of iterations (called the climb number, denoted by \( N_c \)) has been
       reached.

3. Watch-Jump process:
   (a) For each \( j, 1 \leq j \leq d \), randomly generate a real number \( y_j \in [p_{ij} - b, p_{ij} + b] \), where \( b \) is the eyesight of monkey. Let \( y = (y_1, \ldots, y_d) \);
   (b) For each \( i, 1 \leq i \leq n \), let \( p_i \leftarrow (y_1, \ldots, y_d) \) if \( f(y_1, \ldots, y_d) \geq f(p_i) \) and
       \( y = (y_1, \ldots, y_d) \) is feasible, otherwise repeat step a) until finding a feasible
       \( y \);
   (c) Repeat the climb process by considering \( y \) as an initial position.

4. Somersault Process:
   (a) Generate a real random number \( \alpha \) from the interval \([c, g]\) (called the
       somersault interval);
   (b) For each \( j, 1 \leq j \leq d \), set \( y_j = p_{ij} + \alpha(x_j - p_{ij}) \), where \( x_j = \frac{1}{n} \sum_{i=1}^{n} p_{ij} \)
       and the point \( x = (x_1, \ldots, x_d) \) is called a somersault pivot;
   (c) For each \( i, 1 \leq i \leq n \), let \( p_i \leftarrow y \) if \( y = (y_1, \ldots, y_n) \) is feasible. Otherwise, repeat steps a) and b) until finding a feasible solution \( y \).

5. Termination Process:
   Monkey Algorithm will terminate either after reaching a given number (called
   the cyclic number, denoted by \( N \)) of cyclic repetitions of the above steps, or
   if the optimal value hasn’t been changed.

Monkey Algorithm has many applications and variants in different research
fields [2]. It’s modified to minimize the energy loss, improve the voltage levels,
and reduce the carbon dioxide emission. The results show that the proposed
algorithm is efficient and achieving good quality solutions [5]. MA is employed and
modified to solve the optimal sensor placement problem. The results show that the modified MA is a better solution than the base one [35, 23]. The modifications include chaotic initialization, step length, and adaptive watching time. Other many studies introduced modification versions to solve real-life applications such as in [39]. All previous studies focused on enhancing randomization problem with better solutions, but the performance of MA still needs enhancements to overcome the time-consuming problem. No study had proposed a solution for MA enhancement. Two other studies have introduced an abstract idea about a combination between P system and particle swarm optimization technique [27][44]. Those studies proved the efficiency and realism of the proposed methodology using benchmark functions. The previous studies neither showed the computational power of P system from the theoretical side nor addressed P system in swarm processes deeply.

3 Membrane Monkey Algorithm - Main Definition and Theorems

Membrane Monkey Algorithm is responsible for finding the objective value and eliminating the optimal solution of monkey positions. It needs arithmetic operations (addition, subtraction, multiplication, and division), some logical operators (greater than, less than or equal), and ranking algorithm to apply rules of MA processes. We employed previous studies to remove the obstacles of applying the mathematical equations of MA with P system [42][33][43].

Definition 2. Membrane Monkey Algorithm of degree \( m \geq 1 \) is defined as a tuple

\[
\Pi = (O, H, \mu, W_1, \ldots, W_m, R_g, R_l, 0)
\]

Where,

1. \( O \) is an alphabet of elements called objects. They are corresponding to monkey positions;
2. \( H \) is a set of labels for membranes;
3. The following membrane structure \( \mu = [g]_{l=1}, [l]_{l=2}, [l]_{l=3}, \ldots, [l]_{l=m} \) represents a set of membranes. The skin membrane \([g]_{g}\) denotes the global search space, and \([l]_{l}\) refers to a local search space (local membrane). These membranes are initially labeled with elements of \( H \), where \( m \) is the number of regions (local search spaces), \( g \) refers to the label of global search space, and \( 1 \leq l \leq m \) is the label of a membrane in the (local) search space. \( \mu \) is considered a weighted representation of the natural behavior of monkey movements, through mountains in a large search space to get the highest top mountain. Mountains are local search spaces in large search space named global search space;
4. The strings \( W_1, \ldots, W_m \) represent multisets of objects placed in the corresponding regions, where \( W = S \cap \Pi_{i=1}^{p^e} \) for \( e \leq (n + k) \), \( S \) represents the signal of sending and receiving objects, and \( S \in \{-, +, 0\} \). \( e \) represents the message that shows when there is an incoming object to a membrane from another. \( n \) indicates the number of monkeys (population size),
$k \in \{n, m, d, b, \alpha, f, g, l, p, \Delta Rand, t_{\text{max}}, p_c, P_{\text{max}}\}$, is a set of algorithm parameters in which defined in Table 1, and $p^v$ is an object of the alphabet. The membrane polarization, overall system rules, will not change through firing any rule. It will be the same in both the left and right sides of the rules;  
5. $R_g$ is a finite set of developmental rules in the global search region. It is formulated at Theorem 1 and 2;  
6. For each $l$, $1 \leq l \leq m$, $R_l$ is a region inside a global region. The purpose of those regions is to apply MA processes to get an optimal solution. Each local membrane has a number of rules to perform MA processes (Climb, Watch-Jump and Somersault process). Every local region has the same processes proved at Theorem 3, 4 and 5;  
7. $0$ referred to an environment. The optimal solution and objective value are eliminated to that environment after performing all algorithm processes. This fact is implemented in the model based on some dissolution rules in the termination process.

| Table 1. Algorithm Initialization Parameters. |
|---------------------------------------------|
| **Parameter** | **Symbol** |
| Population size | $n$ |
| Number of local membranes | $m$ |
| Monkey step length | $l$ |
| Monkey eyesight | $b$ |
| Random real vector | $\Delta Rand$ |
| A random real number | $\alpha$ |
| Somersault interval values | $[f - g]$ |
| Maximum time of algorithm running | $t_{\text{max}}$ |
| Number of problem dimensions | $d$ |
| Number of climb process iterations | $P_c$ |
| Number of algorithm iterations | $N_{\text{max}}$ |

**Theorem 1.** Initialization process in MA can be performed in one membrane $[g]_g$.

*Proof.* Consider a global membrane $[g]_g$ has a set of rules $R_g$, which is employed to implement the initialization process of MA. We assume that $[g]_g$ specific responsibilities such as create monkey position objects $p^v$ and algorithm parameters, create local membranes $[l]_l$ and sending objects to them, and have a full control over the time object. In order to prove that a P system can implement the previous steps in one membrane, we only need to apply $R_g$ in this sequence:

1. Evolution rule:

$$[s_1^h p^v \rightarrow a]^h_v$$ for $h \in H, s_1 \in S, p^v \in O, a \in O^*$. The rule is responsible for evolving all objects of monkey position and algorithm parameters in the
global $[g]_g$ and local $[i]_l$ regions. Every object $p^i$, $O$ is specified for example real numbers;

2. in-Parameters rule:
\[
\gamma^i_{[s_1]^1_{[s_2]}_{[h]}} \rightarrow \gamma^i_{[s_1]^2_{[s_2]}_{[h]}} \quad \text{for} \ h \in H, h = \{h_g, h_l\}, \{s_1, s_2\} \in S, \gamma^i \in O, 1 \leq i \leq m.
\]
Using this rule, the values of Membrane Monkey parameters are passed to their objects in global $[g]_g$ and local $[i]_l$ regions. This rule is fired in $[g]_g$ as a first rule after the evolution rule, and it’s fired after the creation rule in $[i]_l$;

3. in-Timing rule:
\[
T^i_{[s_1]}_{[T]} \rightarrow T^i_{[s_2]}_{[T]} \quad \text{for} \ h \in H, h = \{h_g, h_l\}, \{s_1, s_2\} \in S, \{T, T^i\} \in O, 1 \leq l \leq m, 0 \leq t \leq t_{max}.
\]
The rule is used to increment the timestamp for firing every rule in regions. Every firing process sends a message $\epsilon$ to fire the Time-Access to increase timestamp $T^i$ by a time unit as $T^i_{new}$. The addition operation is performed on timestamp $T^i$ according to the addition rules in $[1]$ with time complexity $O$.

4. Time-Access rule:
\[
\epsilon^i_{[s_1]}_{[T]} \rightarrow \epsilon^i_{[s_2]}_{[T]}_{[T_{new}]} \quad \text{for} \ h \in H, h = \{h_g, h_l\}, \{s_1, s_2\} \in S, \{T, T^i, T_{new}\} \in O, 1 \leq l \leq m, 0 \leq t \leq t_{max}.
\]
The rule is fired to inject the consumption message object $\epsilon$ into the global and local regions and defined as $T^{i}$. The time operation $T^{i}$ is introduced to control the time parameter through algorithm processes and changes during these processes;

5. Division rule:
\[
\gamma\rightarrow [z^n, z^m]_{l} \rightarrow [z^n, z^m]_{l} \quad \text{for} \ h_g \in H, \{s_1, s_2\} \in S, \{z^n, z^m\} \in O.
\]
The rule is fired to inject the consumption message object $\epsilon$ into the global and local regions and defined as $T^{i}$. The division operation is performed according to the division rules in $[1]$. The division needs two inner membranes inside the local membrane, and the time complexity is based on $z^n$, $z^m$, and $z^m$ are considered constant parameters inside the membrane structure;

6. Positions rule:
\[
P^i_{[s_1]}_{[h_g]} \rightarrow P^i_{[s_2]}_{[h_g]} \quad \text{for} \ h_g \in H, \{s_1, s_2\} \in S, P^i_{[s]} \in O, 1 \leq i \leq n.
\]
The rule is developed to modify positions’ objects $P^i_{[s]}$ with positions’ values;

7. Membranes-Creation rule:
\[
\epsilon^i_{[s_1]}_{[h_g]} \rightarrow [z^n, z^{m+1}]_{l} \rightarrow [z^n, z^{m+2}]_{l} \rightarrow \cdots \rightarrow [z^n, z^{m+2}]_{l} \rightarrow [z^n, z^{m+2}]_{l} \quad \text{for} \ h_g, h_l \in H, \{s_1, s_2, s_{l+2}\} \in S, 1 \leq l \leq m, z^m \in O.
\]
The rule is developed to generate local membranes (local regions), according to the number of local search spaces $m$ in each iteration $N_{cre}$. The membrane creation process is done, when an incoming message object $\epsilon$ reaches $[g]_g$;

8. Distribution rule:
\[
P^i_{[h]} \rightarrow P^{i+1}_{[h]} \rightarrow P^{i+2}_{[h]} \rightarrow \cdots \rightarrow P^{i+m}_{[h]} \rightarrow P^{i+m}_{[h]} \quad \text{for} \ h_l \in H, \{s_1, s_{l+m}\} \in S, 1 \leq l \leq m, P^i_{[s]} \in O.
\]
The rule eliminates monkey objects $P^i_{[s]}$ from the global region to local regions based on a threshold object $z^{th}$. Injecting position values $P^i_{[s]}$ is considered a message $\epsilon$ to local regions to commence their processes;

9. Comparison rule:
In respect to the maximum number of iterations $N$, operations, as follows: the monkey positions and objective value need to be applied by P system operations. Let $T_{new}^{t}$.

**Proof.** Let $[g]_{g}$ has a responsibility to implement the termination process of PMSAM. As a result, the main processes of PMSAM can start.

**Theorem 2.** The global membrane $[g]_{g}$ can get the optimal solution $p_{g}$ with respect to the maximum number of iterations $N_{max}$ and running time $T_{max}$.

**Proof.** Let $[g]_{g}$ is a set of local membranes $[g]_{g}$ will be created and PMSAM Time $T$ will be controlled. Therefore, a set of rules is constructed based on dissolution, and evolution operations, as follows:

1. **in-Global rule:**
   \[ p_{g}^{s_{1}} h_{g}^{s_{1}} \rightarrow h_{g} p_{g}^{s_{2}} h_{g}^{s_{2}}, \quad \text{for } h_{g} \in H, \{s_{1}, s_{2}\} \in S, \{p_{g}, p_{g}^{s_{2}}\} \in O. \]
   When the global region receives a message $\epsilon$ at the first iteration, the rule is developed to create a global solution object $p_{g}^{s_{1}}$ in the global membrane.

2. **Comparison rule:**
   \[ p_{g}^{opt} v_{g}^{opt} h_{g}^{s_{1}} \rightarrow h_{g} p_{g}^{v_{g}} h_{g}^{s_{2}}, \quad \text{for } h_{g} \in H, \{s_{1}, s_{2}\} \in S, \{p_{g}^{opt}, p_{g}^{v_{g}}\} \in O. \]
   The rule is developed to compare incoming optimal solution object $p_{g}^{opt}$ at the current iteration with the current global solution object $p_{g}^{v_{g}}$ in a membrane. The result is the better object value $p_{g}^{v_{g}}$ from an incoming object and the current object.

3. **Timing rule:**
   \[ h_{g}^{s_{1}} T_{i}^{s_{1}} p_{g}^{opt} (T_{max}^{T_{i}^{s_{1}}} h_{g}^{s_{1}} \rightarrow P_{g}^{v_{g}}, \quad \text{for } h_{g} \in H, s_{1} \in S, \{P_{g}^{v_{g}}, p_{g}^{v_{g}}, T_{i}^{s_{1}}\} \in O, 0 \leq t \leq t_{max}, 1 \leq i \leq n. \]
   After global region receives all monkey positions from local membranes, the rule is fired to check if the timestamp $T_{i}$ exceeds the maximum time $T_{max}^{s_{1}}$, and the optimal solution and objective value objects will be eliminated to the environment. The comparison process between $T_{i}$ and $T_{max}^{s_{1}}$ is done similar to a sorting algorithm as in [10] with linear time complexity. Regarding time stopping criterion, it is the first study to put the time factor as a stopping condition.

4. **Dissolution rule:**
   \[ h_{g}^{s_{1}} P_{g}^{v_{g}} h_{g}^{s_{2}}, \quad \text{for } h_{g} \in H, s_{1} \in S, \{P_{g}^{v_{g}}, p_{g}^{v_{g}}\} \in O, 1 \leq i \leq n. \]
   If the current solution $p_{g}^{v_{g}}$ is feasible, the rule eliminates final monkey positions $P_{g}^{v_{g}}$ and the object value $p_{g}^{v_{g}}$. This occurs when all local membranes finished their work. In PMSAM, the solution feasibility can be checked by creating an object to represent the desired value in the global region;
5. Iteration-Checker rule: 
\[ [s_i P_{i1}]_{h_{i1}} \rightarrow [s_i P_{i1}]_{h_{i1}} \] \( l_i \in H, s_i \in S \), \( \{ P_{i1}^{v_i}, z_{N_{curr}} \} \), \( z_{N_{curr}} \) \( h_{i1}, 1 \leq i \leq n \). The role of this rule is to realize if the current PMSAM iteration \( z_{N_{curr}} \) exceeds the maximum number of iterations \( z_{N_{max}} \). In this case, the objective value \( P_{i1}^{v_i} \) monkey position objects \( P_{i1}^{v_i} \) will be eliminated to the environment and stop working in PMSAM. The comparison process between \( z_{N_{curr}} \) and \( z_{N_{max}} \) is done similarly to a sorting algorithm as in [10].

6. Restarting rule: 
\[ [s_i P_{i1}]_{h_{i1}} \rightarrow [s_i P_{i1}]_{h_{i1}} \] \( l_i \in H, s_i \in S \), \( \{ P_{i1}^{v_i}, z_{N_{curr}} \} \), \( z_{N_{curr}} \) \( h_{i1}, 1 \leq i \leq n \). The rule is used to restart the process from the beginning, in case of the solution is not feasible or the stopping criteria have not been met. The value of current iteration object \( N_{curr} \) incremented by one to be \( N_{new} = N_{curr+1} \). It also sends a starter message \( \epsilon \) to Membranes-Creation rule to start creating local membranes and passing monkey position objects \( P_{i1}^{v_i} \).

Formally, from the above rules, \([g]_g\) can control PMSAM runtime and the statement of the theorem holds.

**Theorem 3.** Climb process in PMSAM can be emulated in a homogeneous P system by \( m \) membranes where \( m \) is the number of local search spaces.

**Proof.** We prove the theorem by finding the local optimal solution in every local membrane \([i]_i\), which is the purpose of Climb process. Local membranes start to fire climb process rules by generating a random vector according to step length of monkey climbing. After that, they calculate the pseudo-gradient of a selected objective function and finally generate new positions vector to update monkey positions. The set of rules to be fired in \([i]_i\) are described as follows:

1. Random-Injection rule: 
\[ \Delta R_{[s_i]}_{h_{i1}} \rightarrow [s_i \Delta Rand_{i1}^{u} \] \( h_{i1}, 1 \leq i \leq n \), \( 1 \leq l \leq m \), \( 1 \leq u \leq d \)

The rule passes a random vector \( \Delta Rand_{i1}^{u} \) in every local region for all monkeys. The random vector generation is based on the step length of monkey climb \( l \) and the dimensionality of the problem.

2. Addition rule: 
\[ [s_i P_{i1}]_{h_{i1}} \rightarrow [s_i P_{i1}]_{h_{i1}} \] \( l_i \in H, s_i \in S \), \( \{ P_{i1}^{v_i}, \Delta Rand_{i1}^{u} \} \), \( \Delta Rand_{i1}^{u} \) \( h_{i1}, 1 \leq i \leq n \), \( 1 \leq l \leq m \).

The rule is employed to do the first step in calculating the objective function value. When it’s fired, an addition process occurs between monkey position value \( P_{i1}^{v_i} \) and a random value \( \Delta Rand_{i1}^{u} \) to get a randomized value \( \Delta P_{i1}^{v_i} \) for each monkey position. The addition operation is performed according to the addition rules in [1] with time complexity (1).
3. Subtraction rule:
\[
[s_i, P_i^{v_i} \Delta Rand_i^{h_i}] \rightarrow [s_i, P_i^{v_i} \Delta Rand_i^{h_i} \Delta P_i^{v_i}], \text{ for } h_i \in H, \{s_1, s_2\} \in S, \{P_i^{v_i}, \Delta Rand_i^{h_i}, \Delta P_i^{v_i}\} \in O, 1 \leq i \leq n, 1 \leq l \leq m
\]
It is employed to do the second step in calculating an objective function value. When it’s fired, a subtraction process occurs between a monkey position value \(P_i^{v_i}\) and a random value \(\Delta Rand_i^{h_i}\) through problem dimensionality \(d\) to get a randomized value \(\Delta P_i^{v_i}\) for each monkey position. The subtraction operation is performed according to the subtraction rules in \([1]\) with time complexity \((1)\).

4. Objective rule:
\[
[s_i, f' \langle \Delta P_i^{v_i} \rangle] \rightarrow [s_i, f' \langle \Delta P_i^{v_i} \rangle], \text{ for } h_i \in H, \{s_1, s_2\} \in S, \{\Delta P_i^{v_i}, \Delta P_i^{v_i}, \Delta P_i^{v_i}\} \in O, 1 \leq i \leq n, 1 \leq l \leq m.
\]
The rule is used to apply an objective function on the result of previous two rules, and it is considered the pseudo-gradient of the objective function. When the rule is fired, a subtraction process occurs between \(f' \langle \Delta P_i^{v_i} \rangle\) and \(f' \langle \Delta P_i^{v_i} \rangle\) values after applying objective function \(f'\) rules on them. The purpose of firing this rule is to get a new value \(\Delta P_i^{v_i}\) for each monkey position. To fire this rule, the objective function needs to be applied on \(\Delta P_i^{v_i}\) and \(\Delta P_i^{v_i}\), and represented in P system rules.

5. Division rule:
\[
[s_i, \Delta P_i^{v_i} \Delta Rand_i^{h_i} \Delta Rand_i^{h_i}] \rightarrow [s_i, \Delta P_i^{v_i} \Delta Rand_i^{h_i} \Delta Rand_i^{h_i}], \text{ for } h_i \in H, \{s_1, s_2\} \in S, \{\Delta P_i^{v_i}, \Delta Rand_i^{h_i}, \Delta Rand_i^{h_i}\} \in O, 1 \leq i \leq n, 1 \leq l \leq m, 1 \leq u \leq d.
\]
The rule is developed to calculate the pseudo-gradient of the objective function for each monkey position. A new value \(P_i^{v_i}\) for each monkey position is calculated by dividing \(\Delta P_i^{v_i}\) by \(\Delta Rand_i^{h_i}\) multiplied by 2. The multiplication is performed according to the multiplication rule in \([32]\) with time complexity \(O(xlogx)\), where \(x\) is the number of bits of the two numbers.

6. Sign rule:
\[
[s_i, P_i^{v_i} sign] \rightarrow [s_i, P_i^{v_i} sign], \text{ for } h_i \in H, \{s_1, s_2\} \in S, \{P_i^{v_i}, P_i^{v_i}, sign\} \in O, 1 \leq i \leq n, 1 \leq l \leq m.
\]
The rule is used to calculate an updated value \(P_i^{v_i}\) of monkey position based on the pseudo-gradient of an objective function value \(P_i^{v_i}\). The sign function is represented mathematically as:
\[
sign(x) = \begin{cases} 
1 & \text{if } x > 0 \\
0 & \text{if } x = 0 \\
-1 & \text{if } x < 0 
\end{cases}
\]
The sign function is broken down into a number of P system rules as follows:
\[
[s_i, P_i^{v_i} sign] \rightarrow [s_i, P_i^{v_i} sign], \text{ for } h_i \in H, \{s_1, s_2\} \in S, \{P_i^{v_i}, P_i^{v_i}, sign\} \in O
\]
\[
P_i^{v_i} sign = 1, P_i^{v_i} > 0, 1 \leq i \leq n, 1 \leq l \leq m
\]
11. Optimal-Solution rule:

(b) \[ [s_1]^{P_i^u}_{h_i} \rightarrow [s_2]^{P_i^u}_{h_i} \text{, for } h_i \in H, \{s_1, s_2\} \in S, \{P_i^u, P_i^v, P_i, P_i^u, P_i^v, P_i, P_i^u, P_i^v\} \in O, \]

\[ P_i^v \text{ sign} = 0, P_i^v = 0, 1 \leq i \leq n, 1 \leq l \leq m \]

(c) \[ [s_1]^{P_i^v}_{h_i} \rightarrow [s_2]^{P_i^v}_{h_i} \text{ sign}_{h_i}, \text{ for } h_i \in H, \{s_1, s_2\} \in S, \{P_i^v, P_i^v, P_i, P_i^u, P_i^v, P_i, P_i^u, P_i^v\} \in O, \]

\[ P_i^v \text{ sign} = -1, P_i^v < 0, 1 \leq i \leq n, 1 \leq l \leq m. \text{ After firing sign rules, the sign-calculation rule is fired.} \]

7. Updating-Position rule:

\[ [s_1]^{P_i^v}_{h_i} P_i^v \text{, for } h_i \in H, \{s_1, s_2\} \in S, \{P_i^v, P_i^u, P_i^v, y_i\} \in O, 1 \leq i \leq n, 1 \leq l \leq m. \text{ The algorithm eliminates the first element as the best solution and current value } y_i. \]

8. Checker rule:

\[ [h_i y_i P_i]_{h_i} \rightarrow [h_i^2 P_i^v y_i]_{h_i}, \text{ for } h_i \in H, \{s_1, s_2\} \in S, \{y_i, P_i^v, P_i^u, y_i\} \in O, 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq u \leq d. \text{ The rule is developed to compare the updated value } y_i \text{ and current value } P_i^v \text{ of monkey position. It keeps the current value if the updated one is not feasible. The comparison process is done similar to a sorting algorithm as in [10], with linear time complexity.} \]

9. Comparison rule:

\[ [h_i y_i P_i]_{h_i} \rightarrow [h_i^2 P_i^v y_i]_{h_i}, \text{ for } h_i \in H, \{s_1, s_2\} \in S, \{y_i, P_i^v, y_i\} \in O, 1 \leq i \leq n, 1 \leq l \leq m, 1 \leq u \leq d. \text{ The rule is developed to compare updated position value } y_i \text{ and the current value } P_i^v \text{ to update current value of monkey positions with the updated value and get a better value } y_i \text{ in process iterations.} \]

10. Repeat steps from the rules in climb process until reaching the maximum number of repeating times of climb process, or get the best local optimal solution.

11. Optimal-Solution rule:

\[ [h_i]^{P_i^u}_{h_i} \text{, for } h_i \in H, \{s_1, s_2\} \in S, \{P_i^v, P_i^v, P_i^v, P_i^v, P_i^v, P_i^v, P_i^v, P_i^v\} \in O, 1 \leq i \leq n, 1 \leq j \leq m, 1 \leq u \leq d. \text{ The rule is developed to get the objective value from the optimal solution for monkey positions. Every local membrane starts to retrieve the local optimal solution } P_i^v \text{ and the objective value } P_i^v \text{ from feasible solution } P_i^v \text{ by this rule. The objective value } P_i^v \text{ needs to be ranked as a best solution from monkey positions. The ranking process is similar to the ranking sorting algorithm in [10] with linear time complexity. There is a little change in ranking sorting algorithm rules; the algorithm eliminates the first element as the best solution and stops working.} \]

12. Time-Elimination rule:

\[ [h_i]^{P_i^v}_{h_i} \rightarrow [h_i]^{T_i^v}_{h_i}, \text{ for } h_i \in H, \{s_1, s_2\} \in S, \{T_i^v, T_i^v\} \in O, 0 \leq t \leq t_{max}, 1 \leq i \leq m. \text{ The rule is employed to eliminate the time value } T_i^v \text{ from each local membrane to the global membrane as } T_i^v. \]
The local optimal solution is obtained by the application of previous rules in \(|i|\), so the theorem holds.

**Theorem 4.** The Watch-Jump process can be developed in a homogeneous P system, which gives the optimal solution \(y_i\) by migrating \(m\) local membranes in one membrane.

**Proof.** Let us recall the functionality of Watch-Jump process, which allow monkeys to decide whether there are other surrounding points higher than the current one, search for new values exceeding the current solutions and get going with a regeneration rule to create new search domains. To prove the working of above steps in P system, the dissolution operation is applied over \(m\) membranes, whereas membranes are merged with each other as follows:

1. Migration rule:
   \[
   [s_1, P_{i_{\text{opt}}}^v h_1^1, P_{i_{\text{opt}}}^v h_1^2, \ldots, P_{i_{\text{opt}}}^v h_m^1, P_{i_{\text{opt}}}^v h_m^2] \rightarrow [s_2, P_{i_{\text{opt}}}^v h_1^1, P_{i_{\text{opt}}}^v h_1^2, \ldots, P_{i_{\text{opt}}}^v h_m^1, P_{i_{\text{opt}}}^v h_m^2], \quad \text{for } \{h_i, h_q\} \in H, \{s_1, s_2, s_3\} \in S, \{P_{i_{\text{opt}}}^v, P_{i_{\text{opt}}}^v, y_i\} \in O, 1 \leq i \leq n, 1 \leq l \leq m, 1 \leq q \leq j/2.
   \]
   The rule is developed to add monkey positions objects \(P_{i_{\text{opt}}}^v\) from a membrane to another membrane that has a higher optimal solution \(P_{i_{\text{opt}}}^v\), where \(j = m\) at the first iteration. Migration rule gives a monkey a chance to search for a better position in new search spaces and is fired after finishing climb process. The migration step makes monkeys search for new optimal value and this step avoids the occurring of local minimum.

2. Updating-Position rule:
   \[
   [s_1, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1, \ldots, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1, \ldots, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1] \rightarrow [s_2, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1, \ldots, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1, \ldots, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1], \quad \text{for } h_q \in H, \{s_1, s_2\} \in S, \{P_{i_{\text{opt}}}^v, y_i\} \in O, 1 \leq i \leq n, 1 \leq q \leq j/2, 1 \leq u \leq d, l = q_{\text{max}}. \]
   The rule is fired to calculate an updated value of monkey position based on a random number, and the eyesight parameter, generating a real random number \(P_{i_{\text{opt}}}^v (\pm) z^b\) based on eyesight \(z^b\) of monkeys. Random number generation is done by subtracting or adding the eyesight \(z^b\) to the current position to get a new value of monkey position \(y_i\).

3. Comparison rule:
   \[
   [s_1, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1, \ldots, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1, \ldots, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1] \rightarrow [s_2, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1, \ldots, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1, \ldots, P_{i_{\text{opt}}}^v h_{i_{\text{opt}}}^1], \quad \text{for } h_q \in H, \{s_1, s_2\} \in S, \{P_{i_{\text{opt}}}^v, y_i\} \in O, 1 \leq i \leq n, 1 \leq q \leq j/2, 1 \leq u \leq d, l = q_{\text{max}}. \]
   The rule is fired to compare between the current value \(P_{i_{\text{opt}}}^v\) and updated value \(y_i\) after applying the objective function on both. If an updated value is larger than the current value, the current value is updated.

4. Repeat firing Watch-Jump process rules until \(q=1\).
5. Call and fire climb process rules. The last step in Watch-Jump is to call and fire climb process as in Theorem 3 until an optimal solution is satisfactory.

After applying Watch-Jump process rules, the theorem holds and then moving to Somersault process.

**Theorem 5.** A feasible solution \(y_i\) is obtained by a set of organized rules in P system; implementing Somersault process.
Proof. Consider a local membrane \([i]\) has the responsibility of Somersault process; employed to get a new optimal solution and update monkey positions according to somersault pivot. Let us prove this assumption using a membrane that has the following rules:

1. Division rule:
   \[
   [\lambda_q (\{z^n\})]_{h_q} \rightarrow [\lambda_q z^n]_{h_q}, \text{ for } h_q \in H, \{s_1, s_2\} \in S, \{z^n, z^n\} \in O, q = 1.
   \]
   The rule is developed to get the value \(z^n\) of dividing one by the number of monkeys \(z^n\). The output of this rule is used to get the somersault pivot.

2. Summation rule:
   \[
   [\lambda_q (P_{iu}^{nu})]_{h_q} \rightarrow [\lambda_q P_{iu}^{nu}]_{h_q}, \text{ for } h_q \in H, \{s_1, s_2\} \in S, \{P_{iu}^{nu}, P_{iu}^{nu}\} \in O, q = 1, 1 \leq i \leq n, 1 \leq u \leq d.
   \]
   The rule is employed to get the value \(z^n\) of summing all monkey position values \(P_{iu}^{nu}\), the output of this rule is used to get the somersault pivot.

3. Pivot rule:
   \[
   [\lambda_q z^n P_{iu}^{nu}]_{h_q} \rightarrow [\lambda_q P_{iu}^{nu}]_{h_q}, \text{ for } h_q \in H, \{s_1, s_2\} \in S, \{z^n, P_{iu}^{nu}, P_{iu}^{nu}\} \in O, q = 1, 1 \leq i \leq n, 1 \leq u \leq d.
   \]
   The rule is developed to get the value \(z^n\) of the somersault pivot. When the local membrane receives a message \(\epsilon\) about the existence of two objects \(z^n\) and \(P_{iu}^{nu}\), the rule is fired to calculate somersault pivot \(P_{iu}^{nu}\).

4. Pivot-Process rule:
   \[
   [\lambda_q P_{iu}^{nu}, P_{iu}^{nu}]_{h_q} \rightarrow [\lambda_q P_{iu}^{nu}, P_{iu}^{nu}]_{h_q}, \text{ for } h_q \in H, \{s_1, s_2\} \in S, \{P_{iu}^{nu}, P_{iu}^{nu}, P_{iu}^{nu}\} \in O, q = 1, 1 \leq i \leq n, 1 \leq u \leq d.
   \]
   The rule is developed to subtract the current position of monkey \(P_{iu}^{nu}\) from somersault pivot \(P_{iu}^{nu}\), to get a value \(P_{iu}^{nu}\) used in calculating the new position.

5. Alfa rule:
   \[
   [\alpha_q P_{iu}^{nu}, P_{iu}^{nu}]_{h_q} \rightarrow [\lambda_q P_{iu}^{nu}, P_{iu}^{nu}]_{h_q}, \text{ for } h_q \in H, \{s_1, s_2\} \in S, \{P_{iu}^{nu}, P_{iu}^{nu}, P_{iu}^{nu}\} \in O, q = 1.
   \]
   The rule is employed to multiply a new value \(P_{iu}^{nu}\) obtained from the previous rule with a real random number \(\alpha\) to get a new object \(P_{iu}^{nu}\).

6. New-Positions rule:
   \[
   [\lambda_q P_{iu}^{nu}, P_{iu}^{nu}]_{h_q} \rightarrow [\lambda_q P_{iu}^{nu}, P_{iu}^{nu}]_{h_q}, \text{ for } h_q \in H, \{s_1, s_2\} \in S, \{P_{iu}^{nu}, P_{iu}^{nu}, y_{iu}\} \in O, q = 1, 1 \leq i \leq n, 1 \leq u \leq d.
   \]
   The rule is developed to update the value of monkey position \(y_{iu}\), by the somersault pivot \(P_{iu}^{nu}\) and the real random number \(P_{iu}^{nu}\).

7. Repetition-Checker rule:
   \[
   [\lambda_q y_{iu}, P_{iu}^{nu}]_{h_q} \rightarrow [\lambda_q P_{iu}^{nu}, P_{iu}^{nu}]_{h_q}, \text{ for } h_q \in H, \{s_1, s_2\} \in S, \{y_{iu}, P_{iu}^{nu}, P_{iu}^{nu}\} \in O, q = 1, 1 \leq i \leq n, 1 \leq u \leq d.
   \]
   The rule is used to check if the optimal solution is not feasible \(y_{iu}\) to repeat somersault process.

8. Call the previous rules of somersault process in case of the optimal solution \(y_{iu}\) is not feasible.

9. Feasibility-Checking rule:
   \[
   [\lambda_q y_{iu}, P_{iu}^{nu}]_{h_q} \rightarrow [\lambda_q y_{iu}, P_{iu}^{nu}]_{h_q}, \text{ for } h_q \in H, \{s_1, s_2\} \in S, \{y_{iu}, P_{iu}^{nu}, y_{iu}\} \in O, q = 1, 1 \leq i \leq n, 1 \leq u \leq d.
   \]
   The rule is employed to make sure that the new optimal solution \(y_{iu}\) for monkey positions is feasible.
10. Solution-Elimination rule:
\[ [s^1, y_{iu}^1]_{s^1} \rightarrow P_{iu}^{y_{iu}}P_{q_{a-curr}}^{y_{iu}} : \text{for } h_q \in H, \{s_1, s_2\} \in S, \{y_{iu}, P_{iu}^{y_{iu}}, P_{q_{a-curr}}^{y_{iu}}\} \in O, q = 1, 1 \leq i \leq n, 1 \leq u \leq d. \] 
After performing somersault process, the rule is fired to eliminate the feasible solution \( y_{iu}' \) to the global membranes. After the elimination process, the optimal solution (monkey positions \( P_{iu}^{y_{iu}} \), and the objective value \( P_{q_{a-curr}}^{y_{iu}} \)) will be injected into the global membrane.

11. Time-Elimination rule:
\[ [s^1, T^t]_{s^1} \rightarrow [s^2]_{s^2}T_{out}^t : \text{for } h_q \in H, \{s_1, s_2\} \in S, \{T^t, T_{out}^t\} \in O, q = 1, 0 \leq t \leq t_{max}. \] 
The rule is fired to eliminate time value \( T^t \) as \( T_{out}^t \) from each local membrane to the global membrane, but the local membrane will be dissolved.

As a result, the global membrane can get a feasible solution for every iteration and determine the optimal solution, so the theorem holds.

The progress of PMSAM and the sequence of firing algorithm rules is shown in Algorithm 1. The algorithm starts to get the input parameters and evolve the need objects. After that, the Climb, Watch-Jump, and Somersault processes are performed based on PMSAM theorems. At every iteration, the algorithm checks the stopping criteria.

4 Experiments and Discussion

In this section, we imply two different approaches of PMSAM experiments. The first approach is a theoretical trace to prove that the proposed algorithm is halt and reaches the success case. On the other hand, there are experiments to evaluate the time complexity, convergence and optimal solution.

4.1 Numerical Experiment

A numerical example is introduced to prove the computational power and efficiency of the proposed algorithm and how it halts successfully. The following experiment starts with population initialization (monkeys, and algorithm parameters). Let us suppose that a monkey population has \( n = 20 \) monkeys and \( m = 4 \) membranes initialized as
\[ P^{vi} = \{(v_1, rand_1), (v_2, rand_2), ..., (v_{20}, rand_{20})\} \]
where \( rand \) is a random number. Also, the algorithm parameters were initialized as
\[ z^k = \{(n, 20), (m, 4), d, b, l, f, g, \alpha, p, t_{max}\} \]
Table 2 shows the workflow of PMSAM among regions (global and local regions), and examines the changes in variable objects (member of membranes \( Mem. \), monkey positions \( Mpos. \), and timestamp \( t \)) over the previous population. The progress of algorithm steps starts with the initialization process at which, the global region receives messages about incoming objects. It fires the evolution rule
Algorithm 1 Pseudo-code algorithm of PMSAM

Input: $\mu = [g]$, $\Delta R$, $p$ and $k$ \{P system input configuration.\}

Output: $p_g$ and $T$

1: $N \leftarrow 1, m \leftarrow 1$ and $T \leftarrow 0$
2: **Step 1:** evolve objects by Evolution rule
3: **Step 2:** fire in-Parameters, in-Positions and in-Timing rules
4: **Step 3:** $T \leftarrow 1$ by Time-Access rule
5: **while** $N \leq N_{\text{max}}$ and $T \leq T_{\text{max}}$ **do**
6:  **create** $m$ membranes by Division, Membranes-Creation and Time-Access rules
7:  **send** $p$ and $k$ to $m$ membranes by in-Parameters, Distribution, in-Timing and Time-Access rules
8:  **while** $i \leq p_c$ \{check maximum number of climb iteration inside local membranes in parallel\} **do**
9:  fire Evolution, Random-Injection and Time-Access rules
10:  apply Objective function and Time-Access rules
11:  update $p$ by Updating-position and Time-Access rules
12:  update $p_{\text{feasible}}$ by Comparison and Time-Access rules
13:  set $i = i + l$ by checker and Time-Access rules
14:  **end while**
15:  set $p_g$ with $p_{\text{feasible}}$ by optimal-Solution and Time-Access rules
16:  fire Time-Access and Time-Elimination rules
17:  update $T_{\text{new}}$ with $T$ by Timing and Time-Access rules
18:  **while** $l \geq 2$ \{continue firing rules until number of local membranes $= 1$\} **do**
19:  **dissolve** in parallel $[q]_l$ into $[q]_q$ by Migration and Time-Access rules
20:  inject and fire Watch-Jump rules into $[q]_q$ \{Call Theorem 4\}
21:  **end while**
22:  **while** $p$ is not feasible \{Feasibility-Checking, Repetition-Checker and Time-Access rules\} **do**
23:  inject and fire Somersault rules into $[q]_q$ \{Call Theorem 5\}
24:  **end while**
25:  **dissolve** $p$ and $T$ by Solution-Elimination, Time-Elimination and Time-Access rules
26:  fire $T_{\text{new}}$ by Global, Time-Updating and Time-Access rules
27:  set $N = N + l$ by Time-Access, Iteration-Checker, Restarting and Timing rules
28:  **end while**
29:  **return** $p_g$ by Dissolution rule
to evolve these objects (algorithm parameters and initial monkey positions). Then, the objects convey their values of the algorithm parameters (monkey position values, algorithm parameters and time object in parallel). At that time, the Division rule is fired to compute a threshold value and access time object for the first time to increase it by a one-time unit. Whenever a rule is fired throughout the algorithm, the Time-Access rule is fired to increment the time value by a unit. Membranes-Creation rule is fired to create four local regions according to the number of local membranes \( m \). After that, three communication rules are fired in parallel to send objects (time object, positions, and algorithm parameters) to local membranes according to the threshold.

Local membranes creation and objects assignment is the trigger of starting the Climb process in every local membrane in parallel as in Table 2. All of them start to fire their rules according to climb process rules sequence. The goal of this process is reaching the local optimal solution in every region. In Table 2, Climb process is based on applying an objective function, that needs to map its mathematical formula to P system rules. For example, the following objective function \( x^2 \) is mapped to be:

\[
[s_1 h x s_1 h] \rightarrow [s_2 h x' s_2 h], \ h \in H, \{s_1, s_2\} \in S, \{x, x'\} \in O
\]  

in Equation 1, the rule needs a time unit to be fired, so the time increasing in this step depends on the breakdown of the objective function. After that, the sign function is calculated with the inner number of rules, and monkey positions are updated. The climb process will continue to run until it reaches the maximum number of processes. At every iteration, a comparison rule is fired to update the current solution with the calculated optimal solution in that iteration. The last step in climb process is eliminating the time object in the global membrane, to update the timestamp in global and local membranes. Before updating time object in local membranes, Migration rule is fired to merge monkeys in two membranes to be in one membrane and start the Watch-Jump process. After the migration process, there are two local membranes instead of four at the first step in Watch-Jump process as in Table 3.

Watch-Jump process continues to work with updating monkey position, applying the objective function again, comparing monkey positions with updated objects, and all the previous steps are repeated once again to get a number of local membranes equal to one. Watch-Jump process calls climb process again, then, the progress of PMSAM is moved to somersault process. It starts with calculating somersault pivot by firing a number of rules to update monkey positions.
Table 2. The first part of the tracing scenario for a numerical example of Membrane Monkey Algorithm.

| Global mem. | Local mem. | Variables | Rule(s) |
|-------------|------------|-----------|---------|
| \(p_i T_{[i]}\) | \(5\) | \(P_i = P_k, \ldots, P_{20} \) | 0 | Evolution |
| \(P_{k}^{\circ} T_{[k]}\) | \(5\) | \(P_i^\circ\) | 4 | Evolution |
| \(P_{k}^{\circ} T_{[k]}\) | \(5\) | \(P_i^\circ\) | 5 | Time-Access |
| \(P_{k}^{\circ} T_{[k]}\) | \(5\) | \(P_i^\circ\) | 2 | Membranes-Creation |
| \(P_{k}^{\circ} T_{[k]}\) | \(5\) | \(P_i^\circ\) | 3 | Parameters |

| Climb Process |
|---------------|
| \(P_{k}^{\circ} T_{[k]}\) | \(\Delta Rand_k z^T P_k^\circ \) | \(5\) | \(P_i^\circ\) | 7 | Square-function |
| \(P_{k}^{\circ} T_{[k]}\) | \(\Delta Rand_k z^T P_k^\circ \) | \(5\) | \(P_i^\circ\) | 8 | Objective |
| \(P_{k}^{\circ} T_{[k]}\) | \(\Delta Rand_k z^T P_k^\circ \) | \(5\) | \(P_i^\circ\) | 9 | Sign |
| \(P_{k}^{\circ} T_{[k]}\) | \(\Delta Rand_k z^T P_k^\circ \) | \(5\) | \(P_i^\circ\) | 10 | Updating-Position |
| \(P_{k}^{\circ} T_{[k]}\) | \(\Delta Rand_k z^T P_k^\circ \) | \(5\) | \(P_i^\circ\) | 11 | Optimal-Solution |
| \(P_{k}^{\circ} T_{[k]}\) | \(\Delta Rand_k z^T P_k^\circ \) | \(5\) | \(P_i^\circ\) | 12 | Time-Access |
| \(P_{k}^{\circ} T_{[k]}\) | \(\Delta Rand_k z^T P_k^\circ \) | \(5\) | \(P_i^\circ\) | 13 | Time-Access |
| \(P_{k}^{\circ} T_{[k]}\) | \(\Delta Rand_k z^T P_k^\circ \) | \(5\) | \(P_i^\circ\) | 14 | Time-Access |

Two rules are fired, in parallel, to check the feasibility of an optimal solution. In Table 2, the algorithm finds the optimal solution from the first time, so it moved to fire time and solution elimination rules. Otherwise, the somersault process is repeated from the beginning until finding the optimal solution.

In the termination process, stopping criteria depend on three objects (maximum running time, maximum number of iterations, and feasible solution). Stopping criteria have three rules fired to eliminate the optimal solution to the environment and stop working in PMSAM. In case of the solution is not feasible and the current time object does not exceed the maximum time object, or the current iteration number does not exceed the maximum number of iteration, the
Table 3. The second part of the tracing scenario for a numerical example of Membrane Monkey Algorithm.

| Global mem. | Local mem. | Variables | Rule(s) |
|-------------|------------|-----------|---------|
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Updating-Position, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Pivot, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Migration, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Updating-Position, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Objective, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Comparison, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Time-Access |

| Global mem. | Local mem. | Variables | Rule(s) |
|-------------|------------|-----------|---------|
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Pivot, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Pivot-Process, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Alfi, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | New-Position, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Repetition-Checker, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Solution-Elimination, Time-Access |
| $[a, b, c]_N$ | $[1, 2, 3]_N$ | $P$ | Time-Elimination, Time-Access |

4.2 Empirical experiment

PMSAM is implemented using Java parallel programming features (ExecutorService, ForkandJoin framework and streams). The implementation is just a simulation for P system over the current CPUs and is not a true implementation because there is no any device can work with P system. The need of performing experiments on the proposed algorithm makes the possibility of presenting it by the current devices and environments is valid and acceptable. PMSAM is
feasible functions reported in many previous works \cite{42,3} and listed in Table 4.

Table 4. Test benchmark functions.

| Test functions | Feasible spaces | d | $f_{min}$ |
|----------------|----------------|---|---------|
| $f_1(x) = \sum_{i=1}^{d} x_i^2$ | [-100, 100] | 30 | 0 |
| $f_2(x) = \max\{|x_i|, 1 \leq i \leq d\}$ | [-100, 100] | 30 | 0 |
| $f_3(x) = \sum_{i=1}^{d} (\sum_{j=1}^{d} x_j)^2$ | [-100, 100] | 30 | 0 |
| $f_4(x) = \sum_{i=1}^{d} [x_i^2 - 10 \cos(2\pi x_i) + 10]$ | [-5, 12, 5, 12] | 30 | 0 |
| $f_5(x) = \sum_{i=1}^{d} x_i^4 + \text{random}(0, 1)$ | [-1, 28, 1, 28] | 30 | 0 |
| $f_6(x) = \sum_{i=1}^{d} x_{i} \text{sin}(\sqrt{x_{i}})$ | [-500, 500] | 30 | -418.9834 |
| $f_7(x) = \frac{1}{4000} \sum_{i=1}^{d} x_i^4 - \frac{1}{2} \sum_{i=1}^{d} \cos(\frac{\pi x_i}{6}) + 1$ | [-600, 600] | 30 | 0 |
| $f_8(x) = \sum_{i=1}^{d} (100x_i^2 - x_{i+1})^2 + (x_i - 1)^2$ | [-5, 10] | 30 | 0 |
| $f_9(x) = \sum_{i=1}^{d} x_i^2 + \sum_{i=1}^{d} |x_i|$ | [-10, 10] | 30 | 0 |
| $f_{10}(x) = \sum_{i=1}^{d} \sin(x_i) \cdot (\sin(\frac{x_i}{\pi}))^{2m} \cdot m = 10$ | [0, \pi] | 30 | -4.687 |
| $f_{11}(x) = -20 \exp(-0.2 \sqrt{\sum_{i=1}^{d} x_i^2}) - 20 \exp\left(\frac{1}{\pi} \sum_{i=1}^{d} \cos(2\pi x_i)\right) + 20 + \exp(1)$ | [-32, 32] | 30 | 0 |
| $f_{12}(x) = (\sum_{i=1}^{d} \sin^2(x_i) - \exp(-\sum_{i=1}^{d} x_i^2)) \cdot \exp(-\sum_{i=1}^{d} \sin^2(\sqrt{x_i}))$ | [-10, 10] | 30 | -1 |

PMSAM was tested by 20 runs on each benchmark function. Over the running times, the algorithm parameters were selected to validate the convergence behavior as listed in Table 5. The somersault interval and eyesight are changed across the excursion times between larger and smaller values to validate the search space increasing and decreasing. The test functions used several sets of algorithm parameters and gave the results in Figures 1, 2, and 3. From those figures, we found that the algorithm can find the feasible optimal solution for the tested functions; moreover, maximizing the number of climb iteration based on the parallelism in PMSAM is a good factor.

Membrane Monkey Algorithm supports convergence reliability because all monkeys can discover all search spaces by membrane migration, to update their position and choose the best optimal solutions. It runs the maximum number of search spaces and gets the exact number of optimal solutions according to the number of membranes, through algorithm processes in a time unit. MA runs on one search space and gets one optimal solution through algorithm processes in a time unit. Figure 4 illustrated the running time of the proposed algorithm based on Theorem 6 and MA based on its time complexity measurements. Therefore, PMSAM is faster than MA according to their time complexity. It guarantees to visit search spaces, avoids local minima, and gets better probabilities for reaching the optimal solution.
Table 5. PMSAM Initialization Parameters.

| Parameter | Value         |
|-----------|---------------|
| n         | 50 - 10000    |
| m         | 10 - 100      |
| l         | 0.0001 - 0.000001 |
| b         | 1             |
| [f - g]   | [-1 - 10, 1 - 30] |
| d         | 30            |
| P_c       | 50 - 500      |
| N_max     | 20 - 5000     |

The previous studies in MA used sequential processes in its development, which affected the performance of the algorithm. In this study, the performance of the base algorithm has been enhanced using the true parallelism of P systems because:

1. Every membrane represents a search space, and all search spaces fire climb process rules in parallel to find local optimal solutions at the same time;
2. Watch-Jump process continues to work in parallel between merged membranes, until monkeys visit all search spaces;
3. Membrane migration ensures that all monkeys will visit all search spaces to find the optimal solution;
4. Updating monkey positions is executed in parallel;
5. In nature, monkeys are searching in parallel under deterministic conditions, which is exactly simulated by the proposed algorithm;
6. Based on results of the experiment stated in Table 6, the algorithm provides competitive results that are compared with MA [42], and Grey Wolf Optimizer (GWO) [14]. GWO is selected because it is compared to several algorithms such as particle swarm optimization, genetic algorithm, SI-based technique and a physics-based algorithm (GSA), and in addition to achieving competitive results against those algorithms [14].

In this section, we addressed the importance of P systems in the proposed algorithm from the computational power and efficiency perspectives. The previous numerical experiment introduced a proof for the efficiency of PMSAM. It was clear that the proposed algorithm depends on three variables; the number of membranes through the algorithm processes, monkey positions, and the timestamp.

**Theorem 6.** The time complexity for PMSAM is \( \frac{n}{m} \) with respect to \( N_{\text{max}} \) and \( p_g^{*} \), where \( n \) is the population size, \( m \) is number of membranes, \( N_{\text{max}} \) is maximum number of iterations and \( p_g^{*} \) is the optimal solution.

**Proof.** Local membranes can work at the same time based on the parallelism property, which means whatever the search spaces count is, it will not affect the timestamp. Timestamp will not increase if search spaces are increased (lines 58 in Algorithm 1).
Fig. 1. The results of the convergence process of $f_1(x)$, $f_2(x)$, $f_3(x)$, and $f_4(x)$ with $n = 60$ and $m = 10$.

Furthermore, PMSAM efficiency is determined by the timestamp. It is the most important variable because it represents time complexity state of the algorithm. Every timestamp, the rule firing happens many times in different regions over many objects. This means that algorithm processes are executed in parallel. From previous experiments, timestamp does not relate to the number of membranes (search spaces). Whatever the increase of the number of monkeys is, algorithm rules will fire in all objects at the same timestamp with the maximum number of monkeys (lines 920 in Algorithm 1). Based on numerical experiments, monkey positions faced a change without sequencing between monkeys every timestamp. That led to this result; timestamp will not be affected by the number
of membranes and number of monkeys, with respect to the maximum number of monkeys (lines 527 in Algorithm 1). Figure 4 shows the expected running time of PMSAM and MA over their processes. Every timestamp, MA runs on one monkey in one search space, on the contrary, PMSAM runs on $n$ monkeys and $m$ search spaces. In Figure 4, the two algorithms were traced based on the timestamp. At a time unit, one execution process is performed in the sequential mode, against $m$ execution processes in PMSAM. This means that if $m = 4$, as in the previous experiment, in the climb process, one execution is done by MA compared to 4 executions by the proposed algorithm. Therefore the theorem holds.
Fig. 3. The results of the convergence process of $f_9(x)$ to $f_{12}(x)$ with $n = 60$ and $m = 10$.

Monkey position changes across algorithm processes to find the best solution for monkey positions and the objective value. Monkeys need to discover new search spaces to find the best positions. The proposed algorithm provides a new rule in Watch-Jump process. Membrane migration does not change the monkey behavior in the real world, instead, it introduces an optimized simulation for the first step in the Watch-Jump process. It gives monkeys an advantage to avoid local minima, allowing them to discover new search spaces via deterministic rules. Furthermore, monkeys can find the best optimal solution, because they discover all search spaces in the environment before moving to somersault process.
Table 6. Results of tested benchmark functions after 20 runs.

| Func | m   | n   | PMSAM                 | MA       | GWO       |
|------|-----|-----|-----------------------|----------|-----------|
|      |     |     | Mean                  | Mean     | Mean      |
|      |     |     | Variance              | Variance | Variance  |
| $f_1(x)$ | 10  | 60  | 1.65013E-2            | 3.617E-2 | 1.729E-2  |
| $f_2(x)$ | 20  | 100 | 2.741E-4              | 4.23E-2  | 2.1864E-4 |
| $f_3(x)$ | 5   | 100 | 4.027E-3              | 1.371E-4 | 3.602E-6  |
| $f_4(x)$ | 20  | 30  | 1.5407E-2             | 4.568E-2 | 2.105     |
| $f_5(x)$ | 10  | 30  | 0.02601               | 3.602E-6 | 6201.0217 |
| $f_6(x)$ | 5   | 100 | -396.045              | 7184.83  | 1201.02E-2|
| $f_7(x)$ | 5   | 60  | 1.6010E-2             | 1.92E-3  | 4.56E-4   |
| $f_8(x)$ | 20  | 50  | 0.0127                | 0.006796 | 0.0478    |
| $f_9(x)$ | 20  | 50  | 0.2504                | 1.0594E-6| 11.5304E-8|
| $f_{10}(x)$ | 10  | 100 | 1.0071                | 1.0933   | 4.05472   |
| $f_{11}(x)$ | 10  | 30  | 1.3701E-3             | 2.3410   | 5.746E-5  |
| $f_{12}(x)$ | 20  | 100 | 0.00474               | 3.0202   | -0.5901   |

Fig. 4. Expected time of PMSAM and MA running on population size $n$. The unit times are calculated based on the complexity of algorithms, where PMSAM takes a time unit to fire rule(s) over a population size $n$; as soon as, MA takes a time unit to run a step every mathematical equation over a population size $n$.

5 Conclusions

This paper has presented an innovative method inspired by membrane computing to solve the time consuming, and sequential processing problems in MA. Its traditional method depends on a sequential mathematical model to describe
monkeys movement over mountains. In this study, a novel algorithm based on P system with active membranes was innovated to model MA processes in a distributed parallel algorithm. PMSAM is working according to the P system formulation, whereas every MA process was broken down into a number of rules to perform the process over a number of objects. It simulated the real behavior of monkeys and solved time-consuming problem based on the parallelism property of the membrane computing. The results and algorithm evaluation showed how the proposed algorithm can choose a better solution. PMSAM introduces the timestamp as a new and effective stopping criterion, to control the process of the algorithm according to the allowable time.

The contribution of this study is crystallized in three points (Time complexity, nature simulation, and a better optimal solution). Time consumption problem has been solved depending on the true parallelism in P systems. Not just in Monkey Algorithm, but also in all other nature-inspired computation algorithms. The natural behavior of monkeys is simulated to open the door for formulating new nature-inspired algorithms. The proposed design provides a way to find the best optimal solution, whereas migration process keeps on searching about the optimal solution, and avoiding local minimum. It is a start point to simulate the natural parallel behavior of swarms in their mathematical models.
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