One never walks alone: the effect of the perturber population on subhalo measurements in strong gravitational lenses

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ABSTRACT
Analyses of extended arcs in strong gravitational lensing images to date have constrained the properties of dark matter by measuring the parameters of one or two individual subhalos. However, since such analyses are reliant on likelihood-based methods like Markov-chain Monte Carlo or nested sampling, they require various compromises to the realism of lensing models for the sake of computational tractability, such as ignoring the numerous other subhalos and line-of-sight halos in the system, assuming a particular form for the source model and requiring the noise to have a known likelihood function. Here we show that a simulation-based inference method called truncated marginal neural ratio estimation (TMNRE) makes it possible to relax these requirements by training neural networks to directly compute marginal posteriors for subhalo parameters from lensing images. By performing a set of inference tasks on mock data, we verify the accuracy of TMNRE and show it can compute posteriors for subhalo parameters marginalized over populations of hundreds of subhalos and line-of-sight halos, as well as lens and source uncertainties. We also find the MLP Mixer network works far better for such tasks than the convolutional architectures explored in other lensing analyses. Furthermore, we show that since TMNRE learns a posterior function it enables direct statistical checks that would be extremely expensive with likelihood-based methods. Our results show that TMNRE is well-suited for analyzing complex lensing data, and that the full subhalo and line-of-sight halo population must be included when measuring the properties of individual dark matter substructures.

Key words: dark matter – gravitational lensing: strong – methods: statistical

1 INTRODUCTION
Determining the microphysical properties of the dark matter (DM) comprising about 85% of the universe’s mass is one of the key problems in physics. The distribution of DM on scales larger than dwarf galaxies is well-characterized and consistent with DM behaving as an approximately cold, collisionless, classical fluid (see e.g. Profumo (2017) for an overview). On the other hand, the distribution of DM on smaller scales is currently only roughly mapped out. At present, there is continued debate over whether the known abundance of dwarf galaxies and the density profiles of low-mass galaxies are in tension with the predictions of ΛCDM (respectively dubbed the cusp-core problem (de Blok & McGaugh 1997), reviewed in Bullock & Boylan-Kolchin (2017)). DM models which are warm instead of cold (Colín et al. 2000; Hogan & Dalcanton 2000), collisional instead of collisionless (Spergel & Steinhardt 2000), or quantum on macroscopic scales rather than classical (Hu et al. 2000) predict a diverse array of possible configurations of low-mass halos and could potentially resolve these tensions (Buckley & Peter 2018).

Unfortunately, light DM halos are difficult to probe as they are not expected to accumulate enough baryonic matter to form stars (Efstathiou 1992; Fitts et al. 2017). If DM has significant self-interactions, such halos might be detectable by searching for the self-annihilation or decay products of DM. However, even if such interactions are not present, light halos can potentially be probed through their irreducible gravitational effects. In this work we study one such probe: galaxy–galaxy strong gravitational lensing.

In galaxy–galaxy strong lenses the light from a source galaxy is dramatically distorted into a ring shape by the mass of a lens galaxy lying close to the line of sight to the source. This leads to multiple magnified and distorted images of the source, as explained by general relativity. A perturber (i.e., a subhalo or line-of-sight (LOS) halo lying somewhere between the observer and source) positioned near one of these images contributes additional, much more localized distortions. By carefully analyzing the relationship between the multiple images of the source, the distortions from a perturber can be disentangled from possible variations in the source light and its properties can be measured. From measuring the distributions of perturbers’
masses and other parameters, it is possible to infer population-level properties like the (sub)halo mass function parameters, which are dictated by the fundamental properties of dark matter.

Such analyses have been developed for two types of lensing systems: quadruply-lensed quasars (“quads”) and lenses with extended arcs. In the former, the source is a nearly point-like quasar that is lensed into four compact images. These images’ positions and flux ratios comprise the summary statistics for these systems. The presence of a perturber near one of these images would cause anomalies in the ratios of these fluxes relative to what would be predicted assuming a smooth lens mass distribution. Evidence for flux ratio anomalies due to perturber was first found in Mao & Schneider (1998); Dalal & Kochanek (2002) developed the first statistical analysis to measure perturbers’ properties from flux ratios.

Here we focus on gravitational imaging, which refers to the analysis of lenses with extended arcs (Koopmans 2006; Vegetti & Koopmans 2009a; Vegetti & Koopmans 2009b). The observation in this case consists of a whole image. On one hand, such images cover a larger area of the sky than the four point-like images in quads, potentially providing more sensitivity to detect perturbations due to perturber. On the other hand, extracting this information requires modeling the source galaxy’s light, which generally has a complex morphology. Gravitational imaging has so far yielded several detections of \( \sim 10^9 \, M_\odot \) perturbers using deep, high-resolution observations in the optical from the Hubble Space Telescope and Keck as well as in radio data from Atacama Large Millimeter/submillimeter Array (Vegetti et al. 2010a,b, 2012; Hezaveh et al. 2016b). Near-future telescopes such as the Rubin Observatory, Euclid, JWST and the Extremely Large Telescope will greatly increase the quality of future telescopes such as the Rubin Observatory, Euclid, JWST and Array (Vegetti et al. 2010a,b, 2012; Hezaveh et al. 2016b).

Gravitational imaging analyses such as the method in Vegetti & Koopmans (2009a); Hezaveh et al. (2016b) use likelihood-based inference to infer the properties of perturbers. Measurements and non-detections of individual perturbers can be converted to constraints on the (sub)halo mass function and thus dark matter’s properties. The central mathematical object in such approaches is the likelihood, a probabilistic model \( p(x|\theta) \) for the data \( x \) given some parameters \( \theta = (\eta_{\text{lens}}, \eta_{\text{src}}, \theta_{\text{sub}}, \theta_{\text{other}}) \) for the lens, source, perturber and possibly other (hyper)parameters. Statistical inference of perturber parameters \( \theta_{\text{sub}} \) such as mass and position given an observation \( x_0 \) amounts to computing marginal posteriors \( p(\theta_{\text{sub}}|x_0) \) by means of Markov-chain Monte Carlo (MCMC) or nested sampling (Skilling 2004). Likelihood-based inference tools do not directly produce marginal posteriors but instead compute the joint posterior \( p(\theta|x_0) \), which must then be marginalized over.

The computational expense of sampling from the joint posterior imposes restrictions on the realism of lensing models that can be analyzed. One such restriction common to most analyses is to assume no more than two perturbers are present in each image. Allowing for \( n \) perturbers would cause the joint posterior to become highly multimodal, with approximately \( n! \) modes due exact invariance of the observation under relabeling of perturbers. Transdimensional MCMC methods provide an inroad into this problem by inferring the probabilities of different possible populations of perturbers (Brewer et al. 2016; Daylan et al. 2018), albeit at substantial computational cost. Another approach is to circumvent measuring individual perturbers by instead engineering summary statistics such as the power spectrum of the residuals between the image and best-fit reconstruction excluding substructure and relating them to the (sub)halo mass function parameters (Hezaveh et al. 2016a; Bayer et al. 2018; Diaz Rivero et al. 2018; Çağan Şengül et al. 2020). It is unknown how much information such approaches discard, and more generally unknown how large an impact ignoring all but one perturber has on measurements.

Likelihood-based analyses also typically assume a particular form of the noise and source model so that the source uncertainties can be excluded from the sampling and marginalized over analytically (Hezaveh et al. 2016b; Vegetti & Koopmans 2009a; Vegetti et al. 2010b,a, 2012). This makes it difficult to explore more complex source models described by e.g. generative machine learning methods or noise artifacts like cosmic ray streaks that cannot be described by an analytic likelihood.

An additional difficulty with likelihood-based analyses is that each run of MCMC or nested sampling produces posterior samples for just a single observation. Directly exploring the systematics, biases and other statistical properties of a particular lensing model is thus extremely time-consuming, necessitating rerunning posterior sampling many times for different input observations. This also makes analyses such as mapping perturber measurement sensitivity costly.

In this work, we demonstrate that a simulation-based inference (SBI) (Cranmer et al. 2020) method called truncated marginal neural ratio estimation (TMNRE) (Miller et al. 2021, 2020) can circumvent these inference challenges to measure the properties of individual perturbers. SBI refers to a class of statistical inference methods that use the output of a stochastic simulator that need not have a known likelihood. Neural ratio estimation (NRE) (Hermans et al. 2019) in particular trains a neural network to map from observations directly to marginal posteriors for a specified subset of model parameters (e.g. the position and mass of a perturber). This bypasses the requirement of likelihood-based inference to sample the joint posterior. In contrast to methods like approximate Bayesian computation (ABC), this also removes the need to engineer summary statistics (He et al. 2020) as they are in effect learned directly from the training data. Since NRE learns a marginal posterior function, it is straightforward to check the statistical properties of the inference results for different observations. TMNRE further extends NRE by focusing training data generation in the regions of parameter space most relevant for analyzing a particular observation over a sequence of inference rounds. This substantially reduces the number of simulations required to train the inference network as well as the required network complexity. Several other works have applied machine learning and SBI to substructure lensing. We recently demonstrated that TMNRE can measure the cutoff in the warm DM subhalo mass function (SHMF) directly from images by combining multiple observations generated with a simple simulator (Anau Montel et al. 2022). In Zhang et al. (2022) a likelihood-ratio estimation technique similar to TMNRE was employed to measure density profile parameters of subhalos from images. Wagner-Carena et al. (2022) recently applied neural posterior estimation to measure the SHMF normalization in mock lensing images using real galaxy images as sources. Brehmer et al. (2019) utilized a “likelihood-based” SBI method requiring the simulator’s score\(^2\) to measure the slope and normalization of a SHMF in simple mock images. In Ostdiel et al. (2022a,b) image segmentation was used to classify whether each pixel in an image contained a subhalo

\(^1\) For example, the hyperparameters could include the pixel size for pixelated sources or strength of source regularization.

\(^2\) The score is the derivative of the log-likelihood for a given observation with respect to the model’s parameters.

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in a given mass bin. Classifiers were also used in Alexander et al. (2020) to distinguish between different DM models based on their lensing signatures.

The present work on measuring individual perturbers complements these efforts in several ways. First, it offers a path towards cross-checking current substructure measurements under different modeling assumptions. Second, inference based on perturbers provides a level of interpretability beyond measuring SHMF parameters directly from images, and moreover the opportunity to test different DM models through measuring the properties of individual subhalos. Third, measuring the heaviest subhalos in an observation enables modeling them explicitly in lensing simulations, which could reduce the training data requirements and improve inference accuracy for direct SHM measurements.

This paper is organized as follows. In section 2 we explain our lensing model, which uses an analytic source and main lens in conjunction with well-motivated perturber models. In section 3 we review TMNRE. Our analysis begins in section 4.1, where we show that TMNRE is capable of recovering posteriors for a subhalo’s mass and position in the limit where they are analytically-calculable. In the other analyses in section 4 we gradually complexify our inference tasks, first accounting for the fact that the source and lens parameters are unknown and later by incorporating a population of light perturbers to marginalize over. This work will help form the basis for TMNRE-based measurement of light DM halos in existing and future lensing data.

2 MODELING STRONG LENS OBSERVATIONS

Here we summarize the source, main lens, perturbers and instrument models we use to simulate mock images of gravitational lenses. We implement our lensing model in PyTorch (Paszke et al. 2019) so that we can leverage GPUs to rapidly generate large numbers of observations.

Before delving into modeling details we briefly summarize the key points of the physics of gravitational lensing, referring the reader to e.g. Meneghetti (2016) for a more detailed overview. We assume that mass densities are low enough to treat the gravitational field of the matter in the image plane in the Newtonian approximation of general relativity (GR). In this case the metric is fully characterized by the lens’ gravitational potential ψ. We also adopt the thin lens approximation, which assumes all the mass lies in a single image plane and all the source light is emitted from a source plane. We use ξ and x as two-dimensional angular coordinates in the image and source planes respectively and use z to indicate distances along the orthogonal dimension. Since the image plane covers a small angular patch of the sky and the lensing deflections are small in the Newtonian limit, the coordinate system can be treated as Cartesian.

In this setting, the lens’ matter distribution can be described by its surface density

$$\Sigma(\xi) = \int dz \rho(\xi, z),$$

where ρ is the lens’ three-dimensional mass density and z is its redshift. The source-plane coordinate to which a light ray through the image plane traces back is given by the lens equation

$$x = \xi - \alpha(\xi).$$

Here α is the deflection field of the lens, which can be computed through the integral

$$\alpha(\xi) = \frac{4G}{c^2} \frac{D_{LS}}{D_L D_S} \int d^2(x') \frac{|\xi - \xi'|}{|\xi - \xi'|^2} \Sigma(\xi').$$

This expression involves the (angular diameter) distances $D_{LS}$ (from the lens to the source), $D_L$ (from the observer to the lens) and $D_S$ (from the observer to the source).3 Since lensing merely alters the trajectories of photons rather than creating or destroying them, the surface brightness $B(\xi)$ in the image plane is equal to the surface brightness at the point to which it traces back in the source plane:

$$B(\xi) = B(x(\xi)).$$

Our lens model thus requires specifying the form of the deflection fields of lens components and the surface brightness of the source.

2.1 Source

The brightness profile of our mock sources is parametrized by the widely-used Sérsic profile:

$$f(x) = I_e \exp\left(-k_n \left(\frac{r}{r_e}\right)^{1/n} - 1\right),$$

where $r_e$ is the half-light radius and $k_n$ is a normalization constant related to the index $n$. For $n > 0.36$ (Ciotti & Bertin 1999)

$$k_n \approx 2n - \frac{1}{3} \frac{4}{405n^2} \frac{46}{25515n^2} + \frac{131}{1148175n^2} - \frac{2194697}{30690717750n^4}. (6)$$

For typical galaxies $1/2 < n < 10$. The radial parameter $R(x)$ is the length of the elliptic coordinate vector

$$R_x = \left( q^{1/2} 0 \cos \varphi \sin \varphi \right) \left( 0 q^{-1/2} -\sin \varphi \cos \varphi \right) \left( x - x_0 \right),$$

which depends on the source’s position angle $\varphi$, axis ratio $q$ and position $(x_0, y_0)$. We fix the source’s redshift to $z_{src} = 2$.

Our source model therefore has seven parameters, $\eta_{src} = (x_{src}, y_{src}, \varphi_{src}, q_{src}, n, r_e, I_e)$.

2.2 Main lens

We adopt the singular power law ellipsoid (SPLE) model for the main lens galaxy, which is capable of modeling the gravitational potentials of strong lenses to near the percent level (StiYu et al. 2009). The SPL deflection field can be expressed in closed-form as a complex field $\alpha = \alpha_x + i \alpha_y$ (Tessore & Metcalf 2015; O’Riordan et al. 2020):

$$\alpha_{SPL}(\xi) = \theta_E \frac{2q_{lens}^{1/2}}{1 + q_{lens}} \left( \frac{\theta_E}{R} \right)^{\gamma - 2} e^{i \phi} \begin{pmatrix} 1 \\ 2F_1 \left( \frac{1}{2}, \frac{1}{2}; 1; \frac{\gamma - 2}{\gamma} \right) \left( \frac{1 - q_{lens}^{1/2}}{1 + q_{lens}} \right)^{2 \phi} \end{pmatrix}. (8)$$

Here $(R, \phi)$ are elliptical coordinates, related to the Cartesian coordinates $\xi$ through a transformation parametrized by the lens’ orientation $\varphi_{lens}$, axis ratio $q_{lens}$ and position $(x_{lens}, y_{lens})$:

$$\begin{pmatrix} R_x \\ R_y \end{pmatrix} = \begin{pmatrix} q_{lens}^{1/2} & 0 \\ 0 & q_{lens}^{-1/2} \end{pmatrix} \begin{pmatrix} \cos \varphi_{lens} & \sin \varphi_{lens} \\ -\sin \varphi_{lens} & \cos \varphi_{lens} \end{pmatrix} \begin{pmatrix} \xi_x - x_{lens} \\ \xi_y - y_{lens} \end{pmatrix}. (9)$$

$$\tan \phi = \frac{R_y}{R_x}. (10)$$

Since the hypergeometric function $2F_1$ is not implemented in PyTorch, we instead pre-tabulate its value as a function of $\phi$, $q_{lens}$ and $\gamma$ and interpolate, as described in Chianese et al. (2020).

3 We compute these with astropy (Astropy Collaboration et al. 2013, 2018) using the flat cosmology from Planck (Planck Collaboration 2020).
The slope γ has a complicated degeneracy with the size of the source (Schneider & Sluse 2013, 2014). Roughly, larger γ values cause the spatial scale of the source to increase (Nightingale & Dye 2015, sec. 3.3). For simplicity we fix γ = 2.1. We also assume the lens galaxy’s light has been perfectly subtracted, and fix its redshift to \( z_{\text{lens}} = 0.5 \).

To account for the weak lensing due to large-scale structure located along the line of sight to the source, we also include an external shear component, which is constant across the image plane:

\[
\alpha^{\text{shear}}(\xi) = \frac{\gamma_1}{\gamma_2} - \frac{\gamma_1}{\gamma_2} \xi, \tag{11}
\]

Our main lens model thus has seven parameters: the SPLE parameters \((x_{\text{lens}}, y_{\text{lens}}, \varphi_{\text{lens}}, \delta_{\text{lens}}, Q_E)\) and the external shear parameters \((\gamma_1, \gamma_2)\), which we denote collectively with \( n_{\text{lens}} \).

### 2.3 Perturbers

#### 2.3.1 Density profiles

We model the deflection field of subhalos using a truncated Navarro-Frenk-White (NFW) profile (Baltz et al. 2009) to account for tidal stripping by the main lens:

\[
\rho_{\text{NFW}}(r) = \frac{\rho_s}{r/r_s (1 + r/r_s)^2} \left( 1 + r^2/r_s^2 \right)^{-1}.	ag{12}
\]

Here \( r \) is the distance from the center of the subhalo, \( \rho_s \) is the density normalization, \( r_s \) is the scale radius and \( r_t \) is the truncation radius. The deflection field for this density profile is given in Baltz et al. (2009, appendix A), and differs from that of an NFW profile for \( r \geq r_t \). While the value of \( \tau \equiv r_t/r_s \) depends on the full history of the subhalo, it typically falls between 4 and 10 (Gilman et al. 2020); we fix \( \tau = 6 \) for simplicity. For simplicity, we model LOS halos using exactly the same profile even though they typically have not undergone tidal stripping.

To generate perturber populations for our third analysis task, we must choose values for their density normalizations and scale radii. Since simulation studies typically measure the halo mass \( m_{\text{sub}} \) and the concentration \( c \), it is more convenient to sample populations from distributions over these parameters. These variables can then be mapped to the parametrization above via

\[
r_s = \frac{1}{c^3} \left[ \frac{3m_{\text{sub}}}{4\pi 200 \rho_c (z_{\text{lens}})} \right]^{1/3}, \tag{13}
\]

\[
\rho_s = \rho_c (z_{\text{lens}})^{1/3} \left( \frac{c^3}{3 \log(1 + c) - c/(1 + c)} \right). \tag{14}
\]

For simplicity we fix \( c = 15 \), which is roughly the average value for perturbers in the mass range \( 10^7 M_\odot \) to \( 10^{10} M_\odot \) (Richings et al. 2021, fig. 7). We anticipate that accounting for scatter in the mass-concentration relation might actually improve our ability to measure subhalos’ parameters as higher concentrations lead to substantially stronger lensing signals (Amorisco et al. 2022).

The parameters of an individual subhalo which are not fixed are thus \( \delta_{\text{sub}} \equiv (x_{\text{sub}}, y_{\text{sub}}, m_{\text{sub}}) \), where the second and third components are the projected position of the subhalo. In the case of LOS halos, the parameter set also includes the redshift \( z_{\text{los}} \). In the next two subsections, we describe how we sample these parameters.

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4 This is defined as the mass of the halo enclosed in a sphere where the untruncated halo’s average density is 200 times the critical density.

#### 2.3.2 Generating subhalos

We sample subhalo masses using a mass function of the form from Giocoli et al. (2010):

\[
\frac{dn}{d\log m_{200}} = m_{200}^{n} \left[ \frac{M_{200}}{m_{200}} \right] \exp \left[ -\beta \left( \frac{m_{200}}{M_{200}} \right)^{3} \right], \tag{15}
\]

where \( M_{200} \) is the mass of the main lens. The free parameters in this function were fit to hydrodynamical cosmological simulations that included baryons in Despali & Veggetti (2017). In particular, we use the fits to EAGLE, which give \( \alpha = 0.85 \) (given in the text) and \( (\lambda_M, \beta) = (2.4 \times 10^{-4} M_\odot^{-1}, 300) \) (extracted from their figures).

Integrating the mass function over a given mass interval gives the expected number of subhalos in that interval distributed throughout the whole main lens.

Despali & Veggetti (2017) found the distribution of radial coordinates in hydrodynamical simulations is well-fit by an Einasto profile, but can be approximated as uniform over the lens plane. For a given lensing system we thus precompute \( \bar{n}_{\text{sub}} \), the number of subhalos expected to fall within the lens plane. Thereafter we generate the subhalo population by sampling the number of subhalos from Poisson(\( \bar{n}_{\text{sub}} \)), drawing their masses from the subhalo mass function and sampling their projected positions uniformly over the lens plane. Since the vast majority of subhalos fall outside the lens plane, we expect their lensing effect to be mostly degenerate with external shear, and thus do not simulate them. With the lens redshift we have chosen, over a \( 5'' \times 5'' \) image and integrating over the mass range \( 10^7 M_\odot \) to \( 10^{8} M_\odot \), we find \( \bar{n}_{\text{sub}} = 3.1 \).

#### 2.3.3 Generating line-of-sight halos

As described in Anau Montel et al. (2022); Çağan Şengül et al. (2020), we first compute the average number of LOS halos in the double-pyramid geometry connecting the observer, lens-plane and source. For each simulation we sample the number of LOS halos from Poisson(\( \bar{n}_{\text{los}} \)). We then sample their redshifts and projected positions uniformly over the double-pyramid region and draw their masses from the mass function in Tinker et al. (2008), with \( \Delta \) set to 200. For the lens and source redshifts we have chosen, \( \bar{n}_{\text{los}} = 265.6 \).

To avoid expensive iterative ray-tracing through the lens planes of each LOS halo, we project them as effective subhalos into the lens plane, using the relations derived in Çağan Şengül et al. (2020) to rescale their scale radii and masses. Performing the full multiplane ray-tracing would likely lead to slightly tighter posteriors for the subhalo parameters we infer in this work, as the multiplane deflection field generally has a small curl component that cannot be mimicked by single-plane lensing. As with subhalos, we ignore any LOS halos lying outside the double pyramid volume.

### 2.4 Instrumental effects

We generate mock data with comparable quality to Hubble Space Telescope observations. All images are \( 5'' \times 5'' \) with 0.05'' resolution (100 × 100 pixels). We do not include a point-spread function (PSF). To account for the fact that each pixel in the image corresponds to a finite collecting region in the sky, we generate our images at a resolution eight times higher than the target resolution and downsample.

In experiments we found that neglecting this effect can have a significant impact on inference results. Lastly, we add Gaussian pixel noise to our observations such that the brightest pixels are approximately 30 times the noise level.
3 Inference with Truncated Marginal Neural Ratio Estimation

In the inference tasks we confront in the rest of this work, our goal is to infer 2D marginals for the position and 1D marginals for the mass of a subhalo. Each posterior is to be marginalized over the other perturber parameters and potentially another set of parameters \( \eta \) for the main lens, source and perturber population. In this section we review how TMNRE solves such inference problems.

To begin with, NRE (Hermans et al. 2019) is a technique for inferring the posterior \( p(\theta|x) \) for a model with the joint distribution \( p(x, \theta) \), where \( x \) is an observation (e.g. a lensing image) and \( \theta \) is a parameter of interest (e.g. the mass of a subhalo). The idea is to train a classifier to distinguish between data and parameters drawn from two classes labeled by the binary variable \( C \):

\[
p(C = 0|x, \theta) = \frac{p(x, \theta)}{p(x, \theta) + p(x)p(\theta)} \equiv \sigma[\log r(x, \theta)] ,
\]

where we introduced the sigmoid function \( \sigma(y) \equiv 1/(1+e^{-y}) \) and the likelihood-to-evidence ratio:

\[
r(x, \theta) \equiv \frac{p(x|\theta)}{p(x)} = \frac{p(x, \theta)}{p(x)p(\theta)} = \frac{p(\theta|x)}{p(\theta)} .
\]

Therefore, by training a neural network \( \hat{r}(x, \theta) \) to estimate \( r(x, \theta) \) via this supervised classification task,\(^5\) we obtain an estimate of the posterior through \( \hat{p}_\theta(x) = \hat{r}(x, \theta)p(\theta) \). This ratio estimator can be trained by minimizing the binary cross-entropy loss

\[
\ell[\hat{r}_\phi] = -\int dx \, d\theta \left\{ p(x, \theta) \log \sigma[\log \hat{r}_\phi(x, \theta)] + p(x)p(\theta) \log \left[ 1 - \sigma[\log \hat{r}_\phi(x, \theta)] \right] \right\}
\]

with respect to the ratio estimator’s parameters \( \phi \) using stochastic gradient descent techniques. Critically, training only requires the ability to generate samples from the simulator. This makes it straightforward to apply marginal ratio estimation in scenarios where the explicit form of the likelihood cannot be written in closed form. In practice, posterior samples can be generated by resampling prior samples (with replacement) weighted by the ratio, enabling posterior sampling even when the prior cannot be expressed in closed form.

The extension to estimating marginal posteriors is straightforward: parameters to be marginalized over must be sampled, but not presented to the ratio estimator. In more detail, consider a model with the joint distribution \( p(x, \eta, \theta) = p(x|\eta, \theta)p(\eta, \theta) \), where \( \eta \) is a set of parameters to be marginalized over (e.g. the source and main lens parameters). If \( \eta \) is not passed to the ratio estimator, the loss function becomes

\[
\ell[\hat{r}_\phi] = -\int dx \, d\theta \left\{ p(x, \theta) \log \sigma[\log \hat{r}_\phi(x, \theta)] + p(x)p(\theta) \log \left[ 1 - \sigma[\log \hat{r}_\phi(x, \theta)] \right] \right\}
\]

where we integrated over \( \eta \) to obtain the second equality, proving our statement.

The ratio estimators discussed so far are fully amortized: that is, they attempt to learn \( r(x, \theta) \) over the whole range of the prior \( p(\eta, \theta) \). In principle, it is useful to be able to analyze any possible observation with the same network. In practice, when the posterior \( p(\eta, \theta|x_0) \) for a particular observation \( x_0 \) is much narrower than the prior, training an accurate ratio estimator requires a massive amount of training data. We instead focus on the problem of targeted inference of the posterior for \( x_0 \), which substantially reduces training data requirements and reduces the complexity of the function the ratio estimator must learn to model. Such an approach is also well-suited to individually targeting the small sample of lenses relevant to DM substructure measurement that exist at present (\( O(100) \)).

Training targeted ratio estimators is achieved by replacing the prior with a truncated prior \( p_T(\eta, \theta) \), where the parameters are restricted to a region \( \Gamma \) where they are likely to have generated \( x_0 \). Since parameters from the complement of \( \Gamma \) are unlikely to have generated \( x_0 \), training a ratio estimator with data generated from the truncated prior as opposed to the full prior has little impact on the posterior learned by our ratio estimators.

Since the highest probability density region of the true posterior \( \Gamma \) is unknown, we compute an estimate \( \hat{\Gamma} \) over a sequence of inference rounds. At the beginning of each round, we sample from \( p_T(\eta, \theta) \) (or the true prior in the first round) and train a ratio estimator. We re-estimate \( \hat{\Gamma} \) by keeping only the parts of the previous truncated prior for which \( \hat{r}_\phi(x, \theta) \) exceeds a certain threshold, as described in Miller et al. (2021). This determines the truncated prior for the next round. Our final ratio estimator is obtained when \( \hat{\Gamma} \) stops changing substantially between rounds. This whole procedure is called TMNRE.

TMNRE is related to other sequential SBI methods, such as sequential neural posterior estimation (Papamakarios & Murray 2016; Greenberg et al. 2019; Lueckmann et al. 2017) and sequential neural likelihood estimation (Papamakarios et al. 2018). These two methods use the posteriors learned in each round to generate simulations for the next round rather than the truncated prior. This approach is inefficient for learning multiple marginal posteriors simultaneously, since sampling from the marginal for a particular parameter may hinder learning the marginals for other parameters.

The fact that TMNRE learns a function that can be rapidly evaluated makes it possible to perform statistical consistency checks. In this work we perform expected coverage checks (Cole et al. 2021; Hermans et al. 2021) to test the calibration of our ratio estimators for observations generated using parameters from the truncated prior. This test measures whether credible regions of different widths have achieved their nominal coverage (i.e., whether the true parameters fall within the 68% credible interval of the estimated posterior for 68% of observations). Agreement between the nominal and empirically-measured expected coverage is a necessary (but not sufficient) condition for the ratio estimator to be a correct estimate of the posterior. While typically expected coverage tests are a statement about the ratio

\footnote{For better numerical stability, we actually train the network to learn \( \log r(x, \theta) \).}
estimator’s properties averaged over the truncated prior, at increased computational cost the coverage can be checked in a frequentist manner as a function of the true parameters.

4 RESULTS

We now apply TMNRE to three different substructure lensing problems of increasing complexity. For all tasks we use the same general ratio estimator architecture. It consists of an initial compression network that maps the 100 × 100 pixel images into a feature vector. This feature vector is concatenated to \( \eta_{\text{sub}} \) and \( \eta_{\text{lens}} \) for tasks where they are also inferred. The vector is then passed to a multilayer perceptron (MLP) which outputs an estimate of the 2D and 1D marginal likelihood-to-evidence ratios for \((x_{\text{sub}}, y_{\text{sub}})\) and \(\log_{10} m_{\text{sub}}/M_\odot\) respectively (with separate MLPs used to estimate the 1D ratios for \(\eta_{\text{src}}\) and \(\eta_{\text{lens}}\)).

For each ratio estimator we begin the first training round with 10 000 training examples. We then truncate each parameter’s prior. If none of the truncated priors shrank by at least 20\%, we increase the number of training examples by a factor of 1.5 for the next inference round. A fresh network is then trained using simulations drawn from the truncated prior. Convergence of the ratio estimator is declared after five such consecutive increases in the training set size. For tasks in which we must infer the macromodel parameters we first train the macromodel ratio estimator using this procedure and use the resulting truncated priors to generate training data for the subhalo ratio estimators using the same training procedure. We use the implementation of TMNRE in swyft\(^6\) (Miller et al. 2022), which is built on PyTorch and pytorch-lightning\(^7\).

The training data for our ratio estimators differs in important ways from typical datasets studied by machine learning researchers, making the choice of a good compression network an interesting challenge. Consider, for example, the machine learning problem of classifying the content of natural images. Natural images are distinguished by a hierarchy of visual features at different scales (for example, small-scale features such as textures and edges which comprise large-scale features like the head of an animal or part of an object). A good image classifier should be translation-invariant, producing the same output regardless of the position of an image’s contents. Since the deep convolutional neural network (CNN) architecture has an inductive bias towards learning a hierarchy of features and are translation invariant, CNNs are widely used in computer vision.

The training data for our ratio estimators does not share these features. Different perturber configurations produce images with slightly different relationships between the multiple images of the source galaxy. The variations between images lie near the Einstein ring, and do not show the same rich hierarchical structure of natural images. This means that inductive biases of CNNs are not necessarily beneficial in the context of substructure lensing.

In our experiments, we used CNNs in the ratio estimators for the macromodel parameters, finding their performance to be adequate. However, we found they produced much too wide 2D marginals for the position of a subhalo. Instead, we found the MLP Mixer (Tolstikhin et al. 2021) to work well.\(^8\) Roughly, the MLP Mixer splits the image into patches, stacks the patches and passes each pixel in the stack through an MLP, acting as a dilated convolution.

\(^6\) https://github.com/undark-lab/swyft/
\(^7\) https://www.pytorchlightning.ai/
\(^8\) The MLP Mixer implementation we use can be found at https://github.com/lucidrains/mlp-mixer-pytorch.

| Parameter | True value | Initial prior | First inferred in |
|-----------|------------|---------------|-------------------|
| Subhalo   |            |               |                   |
| \(x_{\text{sub}} \ ["]\) | -1.1       | \(\mathcal{U}(-2.5, 2.5)\) | section 4.1 |
| \(y_{\text{sub}} \ ["]\) | -1.1       | \(\mathcal{U}(-2.5, 2.5)\) | section 4.1 |
| \(\log_{10} m_{\text{sub}}/M_\odot\) | 9.5        | \(\mathcal{U}(8, 10.5)\) | section 4.2 |
| \(x_{\text{lens}} \ ["]\) | 0.05       | \(\mathcal{U}(-0.2, 0.2)\) | section 4.1 |
| \(y_{\text{lens}} \ ["]\) | 0.1        | \(\mathcal{U}(-0.2, 0.2)\) | section 4.1 |
| \(\varphi_{\text{lens}} \ ["]\) | 0          | \(\mathcal{U}(0.5, 1.5)\) | section 4.2 |
| \(q_{\text{lens}}\) | 0.75       | \(\mathcal{U}(0.5, 1)\) | section 4.2 |
| \(\gamma\) | 2.1        | —             |                   |
| \(r_{\text{ein}} \ ["]\) | 1.5        | \(\mathcal{U}(1, 2)\) |                   |
| SPIE      |            |               |                   |
| \(x_{\text{src}} \ ["]\) | 0          | \(\mathcal{U}(-0.2, 0.2)\) | section 4.2 |
| \(y_{\text{src}} \ ["]\) | 0          | \(\mathcal{U}(-0.2, 0.2)\) | section 4.2 |
| \(\varphi_{\text{src}}\) | 0.75       | \(\mathcal{U}(0.5, 1.25)\) | section 4.2 |
| \(q_{\text{src}}\) | 0.5        | \(\mathcal{U}(0.2, 0.8)\) | section 4.2 |
| \(n\)     | 2.3        | \(\mathcal{U}(1.5, 3)\) |                   |
| \(r_{e} \ ["]\) | 2.0        | \(\mathcal{U}(0.5, 3)\) |                   |
| \(L_{e}\) | 0.6        | \(\mathcal{U}(0.1, 2)\) |                   |

Table 1. True subhalo and macromodel parameter values and priors used in the first TMNRE inference round in our three inference tasks. The last column references the first section in which the indicated parameter is inferred rather than being fixed to its true value. The slope of the main lens is fixed to 2.1, as explained in section 4.2. The main lens and source redshifts are set to \(z_{\text{lens}} = 0.5\) and \(z_{\text{src}} = 2\) respectively. For the analysis in section 4.3 involving a population of light perturbers, we sample the number of LOS and subhalos from Poisson distributions with means \(n_{\text{los}} = 265.6\) and \(n_{\text{sub}} = 3.1\) respectively, and restrict their masses to the range \(10^7\ M_\odot\) to \(10^9\ M_\odot\). The halo mass functions and redshift distributions are described in detail in section 2.3.

Another MLP is then applied along the channel dimension of the mixed patches, and the process is iterated.) The MLP Mixer thus directly examines the relationships between pixels in disparate parts of the image, which is exactly how the properties of subhalos are imprinted. We expect that other architectures that split the image into patches such as Vision Transformer (ViT) (Dosovitskiy et al. 2020) could work well for the compression network, though ViT is known to require large amounts of training data.

The architectures of our macromodel and subhalo compression networks are given in appendix A. While we did not perform a full hyperparameter exploration, we found the batchnorm layers to be crucial for stable training of the CNN used for the macromodel ratio estimator. Since our images are roughly one-quarter the area of the images studied in the paper introducing MLP Mixer, we use a substantially smaller model than they suggest. Using dropout in the MLP Mixer and classifier MLPs improved performance. Varying the number of hidden layers and their size in the classifiers had little impact.

We used the Adam optimizer with an initial learning rate of \(6 \times 10^{-3}\) for the macromodel ratio estimator and \(4 \times 10^{-4}\) for the subhalo ratio estimator (found through a learning rate test) and a batch size of 64. The learning rate was reduced by a factor of 0.1 whenever the validation loss plateaued for 3 epochs. Training was run for no longer than 30 epochs.

4.1 Subhalo position inference with fixed mass, source and lens

We first consider the case where the only free parameters in the lens are the position of a single \(10^9\ M_\odot\) subhalo, \(\vartheta_{\text{sub}} = (x_{\text{sub}}, y_{\text{sub}})\). The prior is taken to be uniform over the image plane (i.e., \(\mathcal{U}(-2.5, 2.5)\)).
for both coordinates). The posterior for $\theta_{\text{sub}}$ can then be computed analytically. Adopting a uniform prior over $\theta_{\text{sub}}$ covering the image plane and using the fact the posterior is much narrower, we have

$$
\log p(\theta_{\text{sub}}|x) = -\frac{1}{2} \sum_{i,j} \left( \frac{x_{ij} - f_{ij}(\theta_{\text{sub}})}{\sigma_n} \right)^2,
$$

(23)

where the sum runs over pixels and we dropped terms independent of $\theta_{\text{sub}}$.

Figure 1 shows the truncation regions for each round and compares the analytically-computed posterior with the posterior inferred using TMNRE. While the truncation regions and posterior estimates in early rounds are extremely broad compared to the analytically-computed posterior, TMNRE successfully identifies the region of the image containing the subhalo. After 10 inference rounds the truncation region stabilizes and the inferred posteriors agree well with the true ones for each coordinate. To complement this visual check we also check the coverage for samples from the final round of TMNRE in fig. 2. We find the empirical and nominal coverage to be in good agreement, with our ratio estimator very slightly underestimating the width of the posterior beyond the 95% confidence level.

Having validated TMNRE in this simple scenario, we now turn to more complex inference tasks where the posteriors of interest cannot be derived analytically.

### 4.2 Subhalo mass and position inference

Next we aim to infer the position and mass of a single subhalo, $\theta_{\text{sub}} = (m_{\text{sub}}, x_{\text{sub}}, y_{\text{sub}})$, in a system where the source and main lens parameters are also unknown. The priors for the 17 parameters of the model are given in table 1. Due to the relatively low dimensionality, inference on this model is within the reach of likelihood-based tools such as MCMC or nested sampling. In addition, it can be implemented in a differentiable manner, making the application of methods such as Hamiltonian Monte Carlo (HMC) possible (Gu et al. 2022; Chianese et al. 2020). Running such expensive scans is beyond the scope of this paper.

The final posteriors for the subhalo parameters are shown in fig. 3. The true values of all parameters fall within the ~ 68% credible intervals of the inferred posteriors. We find the effect of the uncertain macromodel is not too strong (at least for this noise realization), with the size of the subhalo position posterior being comparable to what we found in the previous inference task. Figure 4 demonstrates that our ratio estimator has good coverage with respect to the constrained prior. In figs. 5 and 6 we display the marginal posteriors and coverage plots for all 14 source and main lens parameters, which demonstrate they are well-calibrated.

### 4.3 Subhalo mass and position inference with a population of perturbers

For our final inference task we extend the previous one by aiming to infer the position and mass of a relatively heavy target subhalo while marginalizing over a population of lighter perturbers of unknown size. The priors for the perturber population are summarized in table 1 and section 2.3. Our lensing images contain on average about 260 LOS halos and 3 subhalos in the lens plane. This means on average about 800 parameters are required to describe such images. Likelihood-based sampling of this high-dimensional, transdimensional posterior requires techniques such as reversible-jump MCMC (Daylan et al. 2018; Brewer et al. 2016). To marginalize over the perturber population with TMNRE, their parameters are sampled over during data generation but not passed to the ratio estimator.

Since the population of perturbers can contain a member with mass greater than our target subhalo, we need to make this inference task well-defined by “labeling” the subhalos of interest. We accomplish this by making the perturber population lighter than the target subhalo, with mass restricted to the range $10^7 M_\odot$ to $10^8 M_\odot$. We further assume the target subhalo has been localized to a $1.4''\times1.4''$ patch of the image around its true position.

The final-round inference results for $\theta_{\text{sub}}$ plotted in fig. 7 show that inclusion of the perturber population has a substantial effect on the posteriors. The posterior for the subhalo’s mass peaks around the true value, but has a long tail extending towards the lower boundary of the prior. This indicates we are only able to obtain an upper bound on the subhalo mass rather than a measurement, and cannot exclude the possibility its mass is the lowest value consistent with the prior. Having validated our analysis in simpler cases and checked our ratio estimator has good coverage, we conclude our marginal posteriors are in fact close to the true ones. Our results are roughly in line with the image segmentation analysis of Ostdiek et al. (2022a,b), which found subhalos of mass above roughly $10^{10.5} M_\odot$ were resolvable in similar mock observations. In addition, while the 68% credible region for the subhalo’s position contains its true position, the 95% and 99.7% credible regions cover nearly the whole prior region.

The posteriors for the source and lens parameters are shown in fig. 8. While some of the parameters’ posteriors have comparable widths to those found in the previous inference task (namely $\phi_{\text{src/lens}}$, $q_{\text{src/lens}}$, the source index, $I_r$, $\gamma_1$ and $\gamma_2$), others are measured much less precisely due to the stochastic perturber population ($z_{\text{src/lens}}$, $y_{\text{src/lens}}$, $r_e$ and $\Gamma_{\text{Ein}}$). We omit coverage plots for this analysis as they are of comparably-good quality to those in the previous subsection.

### 5 DISCUSSION AND CONCLUSIONS

Measuring the properties of individual DM halos on subgalactic scales is an important probe of the fundamental nature of DM. However, extracting their parameters from observations is difficult for a myriad of reasons, including the fact that lenses contain multiple perturbers (sub-/LOS halos). In this work we demonstrated that TMNRE enables analyses of individual perturbers’ properties in scenarios where the application of likelihood-based methods is difficult or infeasible. The key strength of TMNRE is its ability to directly learn marginal posterior functions for a set of scientifically-interesting parameters from simulated data. By truncating the range of parameters used to generate the simulations, TMNRE enables precision inference of individual observations using a targeted set of training data. This enables the previously-intractable marginalization over large perturber populations. Furthermore, the method is applicable to simulators with unknown likelihood functions and large or even variable numbers of input parameters. The resulting inference networks can be poked and prodded to confirm they are statistically well-behaved.

With three lensing simulators of varying complexity, we demonstrated the following characteristics of the method and perturber inference:

**TMNRE can recover existing results.** We verified the accuracy of TMNRE by confirming it reproduces analytically-calculable posteriors in a toy lensing scenario with known macromodel parameters and subhalo mass.
Figure 1. Validation of TMNRE through inference of the position of a subhalo, with macromodel parameters fixed to their true values and the subhalo’s mass fixed to $10^9 \, M_\odot$. The observation is shown in the left panel. The blue and dashed black contours correspond to the posterior inferred with TMNRE and computed analytically respectively, indicating the 68%, 95% and 99.7% credible regions. The red $\times$ shows the subhalo’s true position. The blue through yellow boxes in the left panel show the ranges of the truncated prior based on the 1D marginals for the subhalo’s coordinates. The zoom-in on the right encompasses the range of the final truncated prior. The distorted blue hex-bin histogram shows the magnitude of the inferred posterior.

Figure 2. Coverage plot for inference task where only the subhalo’s position is free (see fig. 1), showing our ratio estimator produces posteriors of the correct size on average. In detail, the black curve shows the empirical versus nominal coverage, estimated by computing posteriors for 10,000 observations drawn from the final truncated prior. The statistical uncertainty of this estimate is plotted in grey; its derivation is explained in detail in Cole et al. (2021). For a perfectly-calibrated ratio estimator, the black curve would lie along the diagonal green dashed line. The red dashed lines indicate the empirical and nominal coverage of the $1\sigma$ credible regions.

The perturber population matters. We demonstrated that the sensitivity with which a subhalo’s parameters are measurable can be significantly degraded when marginalizing over a population of perturbers. While the $1\sigma$ regions of our position and mass posteriors were centered on the subhalo’s true parameters, they had heavy tails extending to the boundaries of our tight, manually-fixed priors. Given our validation, statistical checks and the fact TMNRE is so far the only method capable of performing the high-dimensional marginalization required for this analysis, our results therefore suggest that the population of light perturbers should not be neglected. It is not, however, excluded that there exists an architecture for the ratio estimator capable of modeling the posterior more accurately than MLP Mixer. This is important to study in future work.

TMRNE enables statistical checks. Since the inference networks learned by TMNRE are locally amortized over a range of potential observations, we were able to test their statistical consistency. Our checks confirm that TMNRE on average produces posteriors with the correct width for the macromodel and subhalo parameters. Such tests would be extremely expensive with likelihood-based inference since they would require rerunning the sampling machinery on numerous mock observations.

While this work used simple mock lenses, TMNRE makes it possible to add realism and parameters to a simulator without significantly altering the inference procedure, or necessarily increasing the simulation budget (Cole et al. 2021). It should therefore be straightforward to incorporate various complexities we ignored in this work: a mass-concentration relation for the perturbers, the lens galaxy’s light, the (possibly uncertain) PSF, multiband observations, drizzling, and even complex noise with an unknown likelihood function. Our analysis can also in principle incorporate more complex source models based on (for example) shapelets (Birrer et al. 2015, 2017), Gaussian processes (Karchev et al. 2022) or neural networks (Chianese et al. 2020). We expect source models capable of refining fine details to improve our measurement precision since the lensing distortions from substructure scale with the gradient of the source.

Another interesting direction for further work is the use of TMNRE for model comparison. While here our ratio estimators were trained to compute the likelihood-to-evidence ratio, as pointed out in Hermans et al. (2019) it is possible to learn other ratios of densities. In particular ratio estimators can be used to learn the Bayes factor for assessing the strength of the evidence for different models. This could be used to determine whether an image contains a perturber or
The effect of perturbers on subhalo measurements

Figure 3. Marginal posteriors inferred with TMNRE for a subhalo’s 2D position (left and center) and mass (right) in a lens with unknown macromodel parameters. See the caption of fig. 1 for further details, though note we have instead used solid, dashed and dotted lines respectively to mark the 68%, 95% and 99.7% credible regions of the position posterior. The range of the $x$-axis in the right panel shows the final-round truncated prior for the subhalo’s $\log_{10} m$.

Figure 4. Coverage plots for subhalo position and mass ratio estimators learned from the observation in fig. 3. These again indicate the estimators’ credible regions are on average the correct size for observations drawn from the final-round truncated prior. See fig. 2 for an explanation of the format.

not, and to map the minimum-detectable perturber mass as a function of its position.

Overall, we believe using TMNRE to measure perturbers as described in this work in combination with measuring the (sub)halo mass function directly (Anau Montel et al. 2022) provides a promising path towards uncovering the identity of dark matter.

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This work was carried out on the Lisa Compute Cluster at SURFsara, which runs on 100% wind energy.

Software used: astropy (Astropy Collaboration et al. 2013, 2018), jupyter (Kluyver et al. 2016), matplotlib (Hunter 2007), numpy (Harris et al. 2020), PyTorch (Paszke et al. 2019), pytorch-lightning, seaborn (Waskom 2021), swyft (Miller et al. 2022) and tqdm (da Costa-Luis et al. 2022).

DATA AVAILABILITY

The data underlying this article will be shared on request to the corresponding author.

9 https://www.pytorchlightning.ai/
Figure 5. The 1D marginal posteriors for all macromodel parameters of the lensing system shown in fig. 3. The posteriors were computed using a CNN-based ratio estimator. The first seven panels correspond to the source parameters, the next five are for the main lens and the last two are for the external shear. All posteriors encompass the true parameter values (vertical red dashed lines) within the $\sim 2\sigma$ interval.

Figure 6. Coverage plots for the 1D marginal macromodel parameter posteriors of the lensing system from fig. 3, using the same format as in fig. 2. The posteriors generally have coverage, with a few being slightly conservative ($\phi_{src}$, $q_{src}$ and the source index) and the shear posteriors being slightly overconfident.
The effect of perturbers on subhalo measurements

Figure 7. Subhalo position and mass posteriors obtained with TMNRE, now marginalizing over a population of $10^7 M_\odot$ to $10^8 M_\odot$ LOS/subhalos (green dots) in addition to the unknown macromodel. See the caption of fig. 3 for details. The initial prior for the subhalo’s position is indicated by the blue box in the left panel. The range of the $x$-axis in the right panel shows the prior on the $\log_{10}$ of its mass. For both the subhalo’s mass and position, the width of the inferred posteriors prevents TMNRE from truncating the priors.

Figure 8. Macromodel 1D marginal posteriors as in fig. 6, but for the inference task where a population of $10^7 M_\odot$ to $10^8 M_\odot$ are present in the observation. This has the effect of broadening most of the posteriors.

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The convolutional compression network used in the macromodel parameter ratio estimator. The notation is taken from PyTorch: the arguments to `Conv2d` are the number of input channels, output channels, kernel size, stride and padding, respectively. The horizontal lines demarcate where the number of channels changes. The output of the network is flattened into a vector with 128 features.

| Conv2d(1, 4, 8, 2, 1, bias=False) | BatchNorm2d(4) | LeakyReLU(0.2) |
| Conv2d(4, 8, 8, 2, 1, bias=False) | BatchNorm2d(8) | LeakyReLU(0.2) |
| Conv2d(8, 16, 8, 2, 1, bias=False) | BatchNorm2d(16) | LeakyReLU(0.2) |
| Conv2d(16, 32, 8, 2, 1, bias=False) | BatchNorm2d(32) | LeakyReLU(0.2) |

APPENDIX A: COMPRESSION NETWORK ARCHITECTURES

The compressor architectures are given in table A1 and table A2. Note that we standardize the images before providing them to the networks.

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Table A2. The details of the MLP Mixer compression network in the subhalo ratio estimator. We use the implementation from https://github.com/lucidrains/mlp-mixer-pytorch, with arguments given in the table.

| Parameter     | Value |
|---------------|-------|
| image_size    | 100   |
| channels      | 1     |
| patch_size    | 10    |
| dim           | 256   |
| depth         | 4     |
| num_classes   | 32    |
| dropout       | 0.1   |