Comparison of Bus Frequency Models for Power System 
Electro-mechanical Simulations

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Abstract. With more and more frequency-related devices interconnected into power grid, accurate frequency estimation becomes important for power system electro-mechanical simulations. This paper reviewed the methods for calculation of bus frequency including difference method, difference method with low-pass filter. Trapezoidal method and damping trapezoidal method which are commonly used in numerical computation are also discussed in this paper for the calculation of bus frequency. In order to analyze advantages and disadvantages of these methods in the aspect of numerical accuracy and stability, a comparison is made on their amplitude-frequency and phase-frequency characteristics. Voltage angle samples from both ideal function and numerical simulation are provided to test the performance of these methods on estimating bus frequency.

1. Introduction
Numerical simulation is one of the most important means for power system analysis. Accurate models are critical for checking the dynamics of power systems. In recent years, there is a growing number of non-synchronous devices such as distributed generation, grid friendly appliances, energy storage devices and flexible loads which are able to participate in frequency regulation [1-4]. To properly model the impact of those devices on power system dynamics, it is necessary to calculate bus frequency when performing studies with electro-mechanical simulations.

There are several methods for calculating bus frequency presented in literatures. Average system frequency is commonly used to estimate bus frequency by computing the arithmetic average of all synchronous machines’ rotor speeds with weights of inertia [5]. Though computational efficiency is achieved for the average system frequency method, the assumption of uniform frequency is inappropriate for power system since the bus frequency at different locations is different which has already been proven by both numerical simulation and measurements [6].

Another method consists in computing the numerical derivative of bus voltage phase angle [7]. A difference quotient is approximately used as a substitute for the derivative. Due to the difficulty of calculating derivative numerically, a low-pass filter is usually added to the difference quotient to avoid spikes. It is widely implemented in some commercial tools like PSS/E and DigSILENT Power Factory. However, additional delay is introduced by the low-pass filter and the frequency calculated with the filter should be used with caution.

This paper presents a detailed comparison of bus frequency models for power system electro-mechanical simulations including difference method, difference method with low-pass filter, trapezoidal method and damping trapezoidal method. The rest of the paper is organized as follows. In
section 2, calculation formulas in time domain and transfer functions in \( z \)-domain and \( s \)-domain of methods are represented. In section 3, the abscissa scope of frequency characteristic graph is discussed. On this basis, a comparison of amplitude-frequency and phase-frequency characteristics is made to analyze the advantages and disadvantages in the aspect of numerical accuracy and stability. In section 4, voltage angle samples from both ideal function and numerical simulation are provided to test the performance of these methods on estimating bus frequency. Conclusions are made in section 5.

2. Numerical calculating methods

2.1. Definition of bus frequency

It is recognized that bus voltage phasor in power system is composited by the instantaneous values of its three-phase voltage. The dynamic bus frequency is usually defined as the speed increment variation of the voltage phasor rotating in the phase plane over time [8]. That is, given a sinusoidal voltage signal, \( v(t) = V_m \cos[\psi(t)] \), frequency is defined as shown in equation (1):

\[
f(t) = \frac{d[\psi(t) / 2\pi]}{dt}
\]

where \( V_m \) is voltage magnitude, \( \psi(t) \) is the phase angle, and \( f \) is the bus frequency.

When the cosine argument is represented as \( \psi(t) = \omega_0 t + \phi(t) = 2\pi f_0 t + \phi(t) \), the deviation of frequency from nominal is defined as shown in equation (2):

\[
\Delta f(t) = f(t) - f_0 = \frac{d[\phi(t) / 2\pi]}{dt}
\]

where \( f_0 \) is nominal frequency.

Since bus frequency is proportional to the derivative of bus voltage phase angle, the numerical methods for calculating bus frequency should approximate the derivative in power system simulations. The transfer function of the control link with input signal \( \phi(t) \) and output signal \( \Delta f(t) \) that describes the derivative relationship is shown as equation (3):

\[
G_\phi(s) = \Delta F(s) / \Phi(s) = s / 2\pi
\]

2.2. Numerical calculation methods of bus frequency

This section presents time-domain and frequency-domain expressions of the numerical calculating methods considered in this paper, including difference method, difference method with low-pass filter, trapezoidal method and damping trapezoidal method. Besides that, the advantages and disadvantages of them are analysed.

2.2.1. Difference method. Difference method is the simplest and most straightforward method of numerical differentiation that uses difference quotient as a substitute for the derivative [9]. It proved to be of good stability and not produce numerical oscillation. The calculation formula in time domain is shown in equation (4) and a subscript \( D \) is used to mark the bus frequency calculated by difference method.

\[
\Delta f_{D}(t) = \frac{\phi(t) - \phi(t - h)}{2\pi h}
\]

where \( h \) is the step size of simulation.

The \( z \) transfer function of the discrete control link with discrete input signal \( \phi^*(t) \) and discrete output signal \( \Delta f^*(t) \) is shown as equation (5):

\[
G_\phi(z) = F(z) / \Phi(z) = (1 - z^{-1}) / 2\pi h
\]

The corresponding transfer function in \( s \)-domain is shown as equation (6):

\[
G_\phi(s) = \left[1 - \exp(-hs)\right] / 2\pi h
\]
2.2.2. **Difference method with low-pass filter.** The results of difference method are usually put through a discrete, low-pass filter in some commercial software tools for power system simulation, e.g., PSS/E. The low-pass filter partly diminishes the pronounced sudden changes for better convergence of the system simulation, and simultaneously introduces a time delay that is detrimental for the control of local bus frequency. In this way, this method can only be used in the case where the requirement of computational accuracy for bus frequency is quite low, and thus can not be applied to networks with modern frequency-responding devices. The calculation formula in time domain is shown in equation (7) and a subscript \( _{DF} \) is used to mark the bus frequency calculated by difference method with filter.

\[
\Delta f_{_{_{DF}}}(t) = \frac{\tau/h \times \Delta f(t-h) + \left[ \phi(t) - \phi(t-h) \right]/2\pi h}{1 + \tau/h}
\]  

(7)

where \( \tau \) is the filter time constant, which is usually set to four times of the step size in PSS/E to obtain a trade-off between accuracy and numerical efficiency.

The transfer function in \( z \)-domain and \( s \)-domain are respectively shown as equation (8) and (9):

\[
G_{_{_{DF}}}(z) = \frac{1 - z^{-1}}{2\pi h 1 + \tau/h - (\tau/h)z^{-1}}
\]

(8)

\[
G_{_{_{DF}}}(s) = \frac{1 - \exp(-hs)}{2\pi h 1 + \tau/h - (\tau/h)\exp(-hs)}
\]

(9)

2.2.3. **Trapezoidal method.** The trapezoidal method is usually used to replace the difference one to compensate for its disadvantages of low accuracy, and widely applied in the digital simulation of power system due to its simple algorithm, high precision and good adaptability to rigid equations. However, when it comes to the electromechanical transient simulation of the power system, non-prototype oscillations will be brought with the jumping of non-state variant, if there is any change on the network structure or switch operation. The calculation formula in time domain is shown in equation (10) and a subscript \( _{T} \) is used to mark the bus frequency calculated by trapezoidal method.

\[
\Delta f_{_{_{T}}}(t) = \left[ \phi(t) - \phi(t-h) \right]/\pi h - \Delta f_{_{_{DF}}}(t-h) \]

(10)

The transfer function in \( z \)-domain and \( s \)-domain are respectively shown as equation (11) and (12):

\[
G_{_{_{T}}}(z) = \frac{1}{\pi h 1 + z^{-1}}
\]

(11)

\[
G_{_{_{T}}}(s) = \frac{1 - \exp(-hs)}{\pi h 1 + \exp(-hs)}
\]

(12)

2.2.4. **Damping trapezoidal method.** An improved trapezoidal method i.e. damping trapezoidal method is proposed in some literatures to settle the numerical oscillation issues. The introduction of a damping factor \( (\alpha) \) on trapezoidal method can effectively prevent the non-prototype oscillation problem in the numerical calculation of it. The accuracy of the method is between that of difference method and trapezoidal method and varies with the magnitude of the damping. Empirical evidence suggests that \( \alpha \) is generally recommended to take 0.15. The calculation formula in time domain is shown in equation (13) and a subscript \( _{DT} \) is used to mark the bus frequency calculated by damping trapezoidal method.

\[
\Delta f_{_{_{DT}}}(t) = \frac{1}{1 + \alpha} \frac{1}{2\pi} \frac{2 \left[ \phi(t) - \phi(t-h) \right]}{h} \frac{1 - \alpha}{1 + \alpha} \Delta f_{_{_{T}}}(t-h)
\]

(13)

The transfer function in \( z \)-domain and \( s \)-domain are respectively shown as equation (14) and (15):

\[
G_{_{_{DT}}}(z) = \frac{1}{\pi h 1 + \alpha + (1 - \alpha)z^{-1}}
\]

(14)
where the frequency reaches $\omega_h$. 

3. Frequency-domain analysis

In order to intuitively illustrate the accuracy of these calculating methods including difference method, difference method with low-pass filter, trapezoidal method and damping trapezoidal method in dynamic system, we make a comparison between these numerical methods and the target function, the derivative, on the amplitude-frequency characteristic and phase-frequency characteristic via frequency-domain method. $h$ is 0.005s in the calculation.

In fact, since the frequency of input signal, that is the voltage phase angle in the actual power grid, is generally not too large, only the frequency characteristic of the low frequency part is needed to be taken into consideration for comparison. However, the processed signal in power system numerical simulation is not an absolutely smooth curve but be accompanied by some tiny glitches on account of the discreteness of data and computation error. It will produce some high frequency harmonic, the frequency of which is related to the simulation step size. The performance on these high frequency signals of the method determines the numerical stability to some extent. The highest frequency of these harmonics is $1/(2h)$ in simulation with a defined step size $h$, corresponding to the situation where the input signal fluctuates at every step size. In conclusion, the range of angular frequency need to observe is from 0 to $\pi/h$ and the low frequency part of the frequency characteristic reflects the numerical accuracy of the calculating methods while the high frequency part reflects the numerical stability in the simulation. The comparison is shown in figure 1 and figure 2.

![Figure 1. Amplitude-frequency characteristics of different methods](image1)

![Figure 2. Phase-frequency characteristics of different methods](image2)

The amplitude-frequency characteristics of all calculating methods are nearly consistent with that of the target function, other than the difference method with low-pass filter which is offset from the target. But for the phase-frequency characteristic, the trapezoidal method is identical to the target one while the phase-frequency characteristic of the difference method exhibits a phase lag and the phase shift comes larger with the increase of the angular frequency, which cannot be ignored, let alone the difference method with low-pass filter. As a consequence, the trapezoidal method has the highest accuracy for the input phase angle signal of the grid and yet there is a non-negligible phase lag in the difference method. The accuracy of the damping trapezoidal method is between the two above and is acceptable within the allowable range of error. As for the difference method with low-pass filter, the accuracy is too low to compare.

Unlike the low frequency band where the actual input signal is expected to be completely transferred without destruction, the fluctuation signals of high frequency band caused by the numerical calculation error are desirable eliminated. Accordingly it is the amplitude frequency characteristic which reflects the amplification and reduction of the signal amplitude that matters. As shown in figure 1, a strong amplification effect is put on the input signal in high frequency band by trapezoidal method. When the frequency reaches $1/(2h)$, the amplitude is infinite. It means that even if there is only a extremely small fluctuation with a frequency of $1/(2h)$ in the input signal, the output will form a
very serious oscillation. The damping trapezoidal method also exhibits such a characteristic of magnifying numerical fluctuations, but is weaker with respect to the trapezoidal method. Conversely, the error can be reduced by the difference method, and even reduced to near zero after adding a filter, which makes explanation for the superiority of these two methods in numerical stability.

4. Numerical illustration
To clearly illustrate the pros and cons of these calculating methods in the numerical accuracy and stability, the following two cases of simulation are conducted. One is that an arbitrary continuous phase angle is self defined, the discretization of which as the simulation input signal and the frequency calculated by equation (2) as a comparison target. It can be seen from this case the numerical accuracy of calculating methods. The other is that a phase angle signal obtained from the actual simulation case, which is not as smooth as the ideal signal but accompanied by some numerical error, is tested. This method can test the numerical stability of calculating methods.

4.1. Test with ideal signal
It is assumed that the ideal input signal of a bus voltage phase angle, as is shown in figure 3, is \( \phi(t) = 0.25\pi t^2 \), then the frequency deviation of the bus, calculated by equation 2, is \( \Delta f(t) = 0.25t \).

With this assumption, the input signal in simulation is obtained from the discretization of the ideal phase angle. The sampling period is 0.005s. Figure 4 shows the bus frequency deviation calculated by numerical methods discussed in this paper. It can be seen that the curve obtained by trapezoidal method nearly coincides that of derivative, which is the expected target. So does the damping trapezoidal method. The curve obtained by difference method exhibits a small phase lag and the difference method with filter presents much worse numerical accuracy. These features of calculating methods shown in figure 4 are consistent with the analysis in section 3.

![Figure 3. Ideal input signal of phase angle](image)

![Figure 4. Frequency deviation with ideal input signal](image)

4.2. Test with numerical simulation signal
The input signal of bus voltage phase angle used here, as is shown in figure 5, is obtained from an actual simulation case by PSS/E where a contingency of overload happened to the network. The step size of the simulation is 0.005s as well. Figure 6 shows the calculation results of bus frequency deviation in this situation through the methods discussed in this paper. It is observed that there are fast vibrations in the curve calculated by trapezoidal method all the time, which means this method obviously can not be used for bus frequency calculation, although with the highest accuracy. The oscillations in the curve obtained by damping trapezoidal method decay after a period of time but can not be eliminated. That is to say, damping trapezoidal method exhibits a better numerical stability than trapezoidal method but still has negative effects on the calculation of bus frequency in the simulation. The difference method and that with low-pass filter show a relatively good numerical stability.
5. Conclusion

The difference method is of not very high accuracy, the result calculated by which exists a phase lag with respect to the target, but has a relatively good stability. It is far from enough in accuracy for the difference method with filter commonly used in commercial tools, which apparently cannot adapt to the networks with frequency-sensitive devices nowadays. The precision of trapezoidal method is high while it introduces serious numerical oscillation, which makes it impossible to show the superiority of numerical precision. The damping trapezoidal reduces the oscillation by sacrificing the accuracy, but can not eliminate it, the remaining of which is still non-negligible and will have a negative effect on the calculation of bus frequency in the simulation.

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