Precision muon $g - 2$ results and light Higgs bosons in the 2HDM(II)

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Abstract

We discuss the implications of the recent evaluations of the SM contribution to $a_\mu = (g - 2)_\mu / 2$ in the light of the latest E821 measurement, which indicate $\sim 3 \sigma$ deviation. We derive the 95% CL interval, $\delta a_\mu$, to be used to constrain any additional contribution to $a_\mu$, beyond the SM ones; it has to have a positive sign. We apply the $\delta a_\mu$ to constrain the light Higgs-boson scenarios in a Two-Higgs-Doublet-Model ("Model II"). If the constraints from the new $(g - 2)_\mu$ results are combined with the other existing constraints, one can exclude a light-scalar scenario at 95% CL, while a light-pseudoscalar scenario can be realized, for a pseudoscalar mass between 25 and 70 GeV, with $\tan \beta$ in the range $25 < \tan \beta < 115$.

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1 Introduction

A precision measurement of the \( g - 2 \) for the muon at BNL is expected to test the electroweak (EW) sector of the Standard Model (SM) and at the same time to shed light on possible effects of “new physics”. After a release of the new E821 result [1], based on the \( \mu^+ \) data collected in the year 2000, a current mean of experimental results for \( (g - 2)_\mu \) is [1]:

\[
a^\text{exp}_\mu \equiv \frac{(g - 2)^{\text{exp}}}{2} = 11\,659\,203 \times 10^{-10},
\]

with an uncertainty (in parentheses) that is almost two times smaller than in the previous measurement [2], and only two times larger than the ultimate goal of the E821 experiment.

The Standard Model prediction for \( a_\mu \) consists of the QED, EW and hadronic contributions:

\[
a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{had}}.
\]

The QED contribution, which constitutes the bulk of the SM contributions, is calculated up to four loops and the \( O(\alpha^5) \) term is estimated [3]; its uncertainty is very small \( \sim 3 \times 10^{-11} \). The EW contribution, based on one- and two-loop diagrams, is also known to a similar accuracy [4–8]. The EW contribution is small \( (152 \times 10^{-11}) \) and is only about two times larger than the present experimental uncertainty (see (1)). The hadronic contribution, \( \sim 7000 \times 10^{-11} \), is the second largest contribution to \( a_\mu^{\text{SM}} \). It is known at present with an accuracy of 1%. This uncertainty is the main source of the uncertainty in the SM prediction. Predictions for the hadronic contribution [9–21] differ among themselves, however, these differences seem to be much less significant than, say, one year ago (see e.g. discussions in [11,12,17–21], and also in [23, 24]). The dominant contribution to \( a^{\text{had}}_\mu \), as well as one of the dominant errors in its value, come from the leading vacuum-polarization (vp1) term. Some preliminary results of the improved calculations of the vp1 have been presented recently [13, 14]. These are data-driven analyses using the most recent data on hadron production in \( e^+e^- \) collisions from BES, CMD-2, SND [22]. The corresponding uncertainties for the vp1, \( \sim 58 \times 10^{-11} \), are now even smaller than those obtained previously by using the data on \( \tau \) decays in addition to the then available \( e^+e^- \) data, e.g. see [10, 12]. Another important issue has been the hadronic contribution to the light-by-light (lbl) scattering, and its contribution to \( (g - 2)_\mu \). After a sign error, first pointed out in [18], was found in the earlier calculations of this contribution, a few re-evaluations [19] of this part have appeared during the last few months. All of them confirm the finding of [18], i.e. a positive sign of the lbl contribution. The central value for the lbl varies from 80 to 110 \( \times 10^{-11} \), depending on the analysis. Since this contribution can be estimated only on a purely theoretical ground, it has a sizeable uncertainty of the order of \( 40 \times 10^{-11} \) [20] (or maybe even larger, as discussed in [21]).

The latest SM predictions and the present world average of the experimental result (1) differ by \( \sim 3\sigma \) (a theoretical and experimental error combined in quadrature), if preliminary results of the evaluation of the vp1 contribution from [13, 14] are used together with the estimation for the lbl given in [20]. The significant progress in the reduction of the error on the experimental side and the stabilization of the SM prediction of \( a^{\text{had}}_\mu \) makes it essential for
the possible implications of the \((g - 2)_\mu\) results to be reanalysed. The issue of whether this is a signal of supersymmetry is already being addressed \[25\]. Here we focus on the implication of the latest \((g - 2)_\mu\) results for the light-Higgs boson scenarios in the non-supersymmetric, CP-conserving Two-Higgs-Doublet-Model (2HDM) in its version called “Model II”; this is a continuation of earlier studies \[24, 26, 27\].

In sec. 2 we collect some results based on the recent calculations of \(a^\mu_{SM}\). As a reference for the vp1 contribution we take the \(e^+e^-\) data-driven analysis done by Jegerlehner \[13\], FJ02, where the new CMD-2 results were used. The difference between the experimental data and the SM prediction, \(\Delta a_\mu = a^\mu_{exp} - a^\mu_{SM}\), can be used to derive stringent constraints on the parameters of models, which give additional contribution(s) to \(a_\mu\). We calculate the interval \(\delta a_\mu\), which can then be used to constrain any such contribution, at 95% CL. In sec. 3 we introduce the 2HDM(II), which we wish to constrain using this interval, and discuss briefly the existing constraints. In sec. 4 we derive the 95% CL limits on the parameters of this model, using \(\delta a_\mu\). Section 5 contains the combined constraints, while the conclusions and outlook are given in sec. 6.

## 2 The \(g - 2\) for the muon: the new experimental and theoretical results

Here we collect the SM contributions (and their uncertainties), which we take into account in our analysis. First we discuss the hadronic contributions (see table 1). We use the higher order contribution from \[16\] and, for the lbl scattering contribution, we take an estimate from \[20\]. The leading vacuum polarization contribution is taken from a preliminary result of an analysis by Jegerlehner, FJ02 \[13\], where the experimental input is based only on the \(e^+e^-\) data, including the latest ones from CMD-2 \[22\]. We sum all the hadronic contributions, adding in quadrature the corresponding errors. This leads us to the result for \(a^\mu_{had}\) given in the last row of table 1 (which we label by the author of the analysis of the vp1 contribution).

### Table 1

| Hadronic contribution | \([in \, 10^{-11}]\) |
|-----------------------|------------------|
| ho \[16\]             | -100 (6)         |
| lbl \[20\]            | 80 (40)          |
| vp1 \[13\]            | 6889 (58)        |
| had [FJ02]            | 6869 (71)        |

We take the QED and EW terms from \[3\] and \[7, 8\], respectively (see table 2). We then calculate the total SM prediction presented in this table, by adding the QED and EW to the full hadronic contributions, and by adding in quadrature the corresponding errors. This leads us to the SM prediction (we label it, as above, by the author of the analysis of the vp1 contribution):

\[
[FJ02] \quad a^\mu_{SM} = 116\,591\,726.7 \quad (70.9) \times \, 10^{-11}. \tag{2}
\]
Taking the new world average we calculate the quantity $\Delta a_\mu$, defined as the difference between the central values of the experimental and theoretical (SM) predictions for $a_\mu$. The error for this quantity we estimate by adding in quadrature the corresponding experimental and theoretical errors, $\sigma = \sqrt{\sigma_{\text{exp}}^2 + \sigma_{\text{SM}}^2}$.

Next we calculate the regions of $\delta a_\mu$, allowed at 95% CL, assuming Gaussian errors. This leads to an interval symmetric around $\Delta a_\mu$, quoted in the last row of table 2.

| TABLE 2 |
|----------------|
| SM contribution | $\text{[in 10}^{-11}\text{]}$ |
| QED            | 116 584 705.7 (2.9) |
| had[FJ02]      | 6 869.0 (70.7) |
| EW             | 152.0 (4.0) |
| tot            | 116 591 726.7 (70.9) |
| $\Delta a_\mu(\sigma)$ | 303.3 (106.9) |
| $\text{lim(95%)}$ | $93.8 \leq \delta a_\mu \leq 512.8$ |

The 95% CL interval $\delta a_\mu$ thus obtained is positive, and hence it leads to an allowed positive contribution (an allowed band). At the same time it also leads to the exclusion of any negative contribution to the $a_\mu$. Note that the additional positive contribution to $a_\mu$ can be up to a few times larger than the EW one.

Using results for the vp1 contribution obtained by the HMNT group [14], from their “exclusive” analysis, the values we get for $a_\mu^{\text{SM}}$, $\Delta a_\mu(\sigma)$ and $\delta a_\mu$, are numerically very close to those obtained above, using the FJ02 analysis. Keeping all the other contributions as before, but with the HMNT(ex) results for the vp1 term [14], we get, in units of $10^{-11}$,

$$\text{[HMNT(ex)] : } \Delta a_\mu(\sigma) = 297.0 (107.2) \quad 87.2 \leq \delta a_\mu \leq 507.4,$$

while their “inclusive” analysis leads to a more stringent constraint, namely

$$\text{[HMNT(in)] : } \Delta a_\mu(\sigma) = 357.2 (106.4) \quad 148.7 \leq \delta a_\mu \leq 565.7.$$

Note that the above 95% CL intervals are positive for both the HMNT results, just as in the FJ02 case. Also note that the relative differences between the upper bounds in all three analyses are small (below 10%). However, the use of HMNT “inclusive” analysis leads to a lower bound, which is relatively much higher (up to 70%) than in the FJ02 and HMNT(ex) cases.

The other recently published SM predictions [10, 12], to which the BNL paper [1] refers, were obtained from analyses of the vp1 contribution based on the older, often not very precise $e^+e^-$ data. To improve the accuracy of the estimation of the vp1 part, $\tau$ decay data were included in addition in those analyses. However, predictions obtained there for $a_\mu^{\text{SM}}$ and for $\sigma_{\text{SM}}$ are not very different from these new preliminary results used by us. It is worth mentioning that the following trend is observed (see discussion e.g. in [13]): if one uses the $\tau$-decay data in the calculation of vp1, then its estimation for the value of vp1 increases while that for the uncertainty decreases. On the other hand, the preliminary FJ02 and HMNT analyses of the
vp1 contribution, the ones used here, rely solely on the low-energy data for the $e^+e^-$ collisions. The accuracy of these analyses increases significantly with respect to the earlier analyses of the same kind thanks to an inclusion of new, high-precision measurements [22] and the use of more refined theoretical methods. Moreover, the central value for the vp1 contribution did not increase in these new analyses. Thus, the preliminary analyses by FJ02 and HMNT lead to a deviation from the experimental value for the muon $(g - 2)$ larger than the published results mentioned above, which differ by “only” 1.6 to 2.6 $\sigma$, as pointed out in [1].

Let us conclude this general part by the following comment. It becomes clear that already at present there is a need of a coherent and a comprehensive error analysis for all components contributing to the calculation of $\sigma_{\mu}^{SM}$. It will be even more necessary in a near future, when the BNL experiment will reach the planned accuracy. Since an estimate combining all the different contributions does not exist for $a_{\mu}^{SM}$, and hence for $\Delta a_{\mu}$, which is needed for estimating the effects of new physics, is not available at the moment, a simplified error analysis, as presented in this section, is unavoidable. This is sufficient for a rough estimation of new effects; however, to reach a final conclusion, better error analyses are necessary.

3 2HDM and existing constraints

3.1 The model

The non-supersymmetric, CP-conserving 2HDM (“Model II”) [28] based on two doublets of complex, scalar-fields $\phi_1, \phi_2$. This is a simple extension of the SM, in which only the Higgs sector is enlarged. To avoid possible large effects due to the flavour-changing neutral currents (FCNC), the 2HDM potential can be chosen in a $Z_2$-symmetric form, i.e. without ($\phi_1, \phi_2$) mixing. In the general case, the potential can have terms, characterized by a mass parameter $\mu$, which break the $Z_2$ symmetry softly; see e.g. [30, 31].

The 2HDM has five Higgs particles: two neutral Higgs scalars $h$ and $H$, one neutral pseudoscalar $A$, and a pair of charged Higgses $H^\pm$. Their masses are free parameters of the model. Other parameters are: the angle $\alpha$, which describes the mixing in the neutral Higgs-scalar sector; $\tan \beta$ the ratio of two vacuum expectation values of scalar doublets, $\tan \beta = v_2/v_1$; and the parameter $\mu$. Small values of $\mu$ seem to be more natural from the point of view of the FCNC effects [30]. It is worth noticing that for such a case, a non-decoupling of the heavy Higgs sector can be realized [30, 31].

In the 2HDM one can choose the Yukawa couplings in a few different ways. Here we consider the Model (II) implementation, where one doublet of fundamental scalar fields couples to the $u$-type quarks, and the other to the $d$-type quarks and charged leptons. This way FCNC processes are avoided at the tree level [29, 28]. This Higgs sector is identical to the one in the MSSM; however, in the 2HDM (II) considered by us, there are no tree-level relations between parameters as in the MSSM case. Therefore even for very heavy supersymmetric particles, the 2HDM (II) and MSSM have very different phenomenology.
To be more specific, let us consider the ratios of the direct coupling constants of the Higgs boson $h$ or $H$ to the massive gauge bosons $V = W$ or $Z$, as well as to the fermions (i.e., Yukawa couplings) for the $u$-type quarks and $d$-type quarks and the charged leptons, to the corresponding couplings for the SM. They are determined in terms of angles $\alpha$ and $\beta$ \cite{28,30}. For $\chi^h_V \equiv g^h_V/(g^h_{SM})$ (and similarly for $H$) we have, in a form suitable for a simultaneous discussion of $h$ and $H$:

\[
\begin{align*}
\chi^h_V &= \sin(\beta - \alpha), & \chi^H_V &= \cos(\beta - \alpha), & \chi^A_V &= 0, \\
\chi^h_u &= \chi^h_V + \cot \beta \chi^H_V, & \chi^H_u &= \chi^H_V - \cot \beta \chi^h_V, & \chi^A_u &= -i \gamma_5 \cot \beta, \\
\chi^h_d &= \chi^h_V - \tan \beta \chi^H_V, & \chi^H_d &= \chi^H_V + \tan \beta \chi^h_V, & \chi^A_d &= -i \gamma_5 \tan \beta.
\end{align*}
\] (5)

Here we have $(\chi^h_V)^2 + (\chi^V_H)^2 = 1$. Observe a pattern relation between these couplings (for $h$ or $H$): $(\chi_u - \chi_V)(\chi_V - \chi_d) = 1 - \chi^2_V$, or $(\chi_u + \chi_V)\chi_V = 1 + \chi_u \chi_d$, found in \cite{30}.

For $\chi^h_V = 1$ all couplings of $h$ have the SM values, the couplings of $H$ to gauge bosons are equal to zero, while one of the couplings of $H$ to fermions may differ considerably from the corresponding SM one, for a small or large $\tan \beta$. If $\chi^h_V = 1$ then the $H$-boson has SM couplings, while $h$ has very different properties: $\chi^h_V = 0$ and the Yukawa coupling $\chi^h_d$ can be large, for large values of $\tan \beta$. This is a case that may correspond to a light-scalar scenario discussed below.

The Yukawa coupling $\chi_d$, relevant to a Higgs boson coupling to a muon, plays a basic role in the calculation of the 2HDM contribution to $\alpha_\mu$. It is equal (up to a factor $-i \gamma_5$) to $\tan \beta$ for a pseudoscalar and $H^+$. If in addition $\chi^h_V = \sin(\beta - \alpha) = 0$, then the same holds for a scalar $h$; more precisely then $|\chi^h_d| = \tan \beta$ (see eq. (7)). In the calculation of the two-loop contribution to $\alpha_\mu$, the coupling of $H^+$ to a scalar $h$ is involved as well, and it is given by

\[
\chi^h_{H^+} = \left(1 - \frac{M^2_h}{2M^2_{H^+}}\right) \chi^h_V + \frac{M^2_h - \mu^2}{2M^2_{H^+}} (\chi^h_d + \chi^h_A),
\] (8)

with the same normalization as that for an elementary charged scalar particle in the SM. For $\chi^h_V = 0$ one gets $\chi^h_{H^+} = (\chi^h_d - 1/\chi^h_d)(M^2_h - \mu^2)/(2M^2_{H^+})$. We see that this coupling depends on the parameter $\mu$. In this paper we consider only the case with $\mu = 0$. A more general case will be studied elsewhere.

### 3.2 Existing constraints

Many searches for a light Higgs particle in the 2HDM (II) were performed at various energies and machines; the most systematic studies were performed at LEP. All existing LEP data, see for instance \cite{27,32,33,34,35}, allows for the existence of one light neutral Higgs boson, $h$ or $A$, with a mass even below 20 GeV. According to the results presented in the left panel of Fig. 1, the other Higgs particle ($A$ or $h$, respectively) should be heavy enough to avoid the exclusion region in the $(M_h, M_A)$ plane, given roughly by $M_h + M_A \geq 90$ GeV.

This is in contrast to the case of the SM Higgs boson, which should be heavier than 114.4 GeV (95\% CL). Also the MSSM Higgs particles should be heavier than $\sim 90$ GeV \cite{32}. An
analysis of the Bjorken process leads to an upper limit on the coupling of $h$ to the gauge boson $\chi_V^h$. This limit, obtained at 95% CL, is presented in the right panel of Fig. 1. We see that this coupling is much smaller than 1 for $M_h \lesssim 50$ GeV. The Yukawa couplings $\chi_d$ of a very light scalar or of a very light pseudoscalar, with mass below 10 GeV, are constrained in the form of upper limits by the low-energy data [36, 37], whereas LEP experiments [35] do that for masses $\gtrsim 4$ GeV (see Figs. 4, 5). It is only the analysis of the decay $Z \rightarrow h/A \gamma$ at LEP [27] that gives both the upper and lower limits for $|\chi_d|$. Note that $|\chi_d|$ is equal to $\tan \beta$ for $A$ and, if $\chi_V^h = 0$, also for $h$.

The constraints from the $\Upsilon \rightarrow h(A)\gamma$ process, mentioned above, have been measured by a few groups [38]. We present their results in Fig. 4 (lines denoted by K, N and L). Unfortunately, the corresponding predictions have large experimental and theoretical uncertainties, the latter ones both being due to the QCD and relativistic corrections. Nevertheless, as we will see below, the constraints coming from this process, even when accounting for some additional uncertainties, play an important role in closing a low mass window for the scalar $h$.

Finally, note that in the 2HDM there is an important lower limit on the mass of $H^+$, coming from the NLO analysis of the $b \rightarrow s \gamma$ data, given by $M_{H^+} \geq 500$ GeV at 95% CL [39].

4 Constraining 2HDM(II) by $g - 2$ for the muon data

We apply the $\delta a_\mu$, obtained in sec. 2, to constrain parameters of the 2HDM (II) (see also earlier papers [20, 11, 10]). We assume that the lightest Higgs boson, $h$ or $A$, dominates the full 2HDM (II) contribution, i.e. we have $a_{\mu}^{2HDM} \approx a_{\mu}^h$ or $a_{\mu}^A$ (a simple approach in [20]).
This approach should hold for masses below 50 GeV, according to results presented in the left panel of Fig. 1. For higher masses, which are also considered here, this is essentially equivalent to assuming a large mass gap between the lightest one, $h$ or $A$, and the remaining Higgs bosons, which lead to a light-$h$ or a light-$A$ scenario. The relevant one- and two-loop diagrams, studied in [5, 26, 40] and [41, 26], respectively, are shown in Fig. 2 for the $h$ and $A$ contributions.

Figure 2: One- and two-loop ($W^+$ and $H^+$ loops are only for a $h$-exchange) diagrams.

According to the LEP limits, discussed in sec. 3.2, we assume that $h$ does not couple to $W/Z$, and therefore we neglect the $W$-loop in the light-$h$ scenario. We include, however, an $H^+$-loop with $M_{H^+}$ equal to 400, 800 GeV (and for $\mu = 0$).

In Fig. 3 we present the contributions to $a_\mu$ obtained for an $h$ (solid lines) and for a $A$ (dashed lines), assuming Yukawa couplings $\chi_d$ equal to 1. For both $h$ and $A$, the one-loop [5, 26, 40] and two-loop [41, 26] results are shown separately. For the purpose of comparison, a one-loop $H^+$ contribution is presented. The one-loop diagram gives a positive contribution to $a_\mu$ for a scalar, whereas it is negative for a pseudoscalar, independently of the value of the Higgs-boson mass. The signs of the two-loop contributions are reversed, these diagrams contribute negatively (positively) for a $h$ ($A$) case [41]. These two-loop diagrams can give large contributions, since they allow us to avoid one small Yukawa coupling with a muon in favor of a coupling with other, potentially heavy, particles circulating in the loop [42, 41]. Indeed, the contributions of two-loop diagrams dominate over the corresponding one-loop ones when the mass of $h$ or $A$ is above a few GeV, as shown in Fig. 3. As a result, in the two-loop analysis, based on a sum of the one- and two-loop (fermionic and bosonic) contributions, a positive (negative) contribution can be ascribed to a scalar $h$ with mass below (above) 5 GeV or a pseudoscalar $A$ with mass above (below) 3 GeV.

In our calculation, $a_\mu^h$ and $a_\mu^A$ contain contributions proportional either to $\chi_d^2$, equivalently to $\tan^2 \beta$ for $A$ and for $h$ (if $\chi_d^2 = 0$), or to $\chi_d \chi_u = -1$. Assuming $a_\mu^h = \delta a_\mu$ for a light-$h$ scenario, and $a_\mu^A = \delta a_\mu$ for a light-$A$ one, and using the estimate of the interval $\delta a_\mu$ from table 2, we can derive constraints on $\tan \beta$, for $h$ and $A$. They are, as expected, in the form of allowed regions (the area between thick lines in Fig. 4) for masses below 5 GeV for $h$, shown in the left panel of Fig. 4, and for masses above 3 GeV for $A$, the right panel of Fig. 4 (see also [41]).
Figure 3: The (absolute value of) individual contributions to $a_{\mu}$ from a scalar $h$ (solid line), a pseudoscalar $A$ (dashed line) and a charged Higgs boson $H^+$ (dotted line). The one-loop contribution for $A$ and $H^+$, and the two-loop one for $h$, are negative. Two-loop diagram contributions for $A$ and $h$ (denoted “1”) are based on the down-type fermion loops only. For $h$ also, results with an additional contribution due to the $H^+$-loop are shown: line “2” (“3”) corresponds to $M_{H^+} = 800$ (400) GeV.

5 Combined 95% CL constraints

The 95% CL constraints from the $(g-2)_\mu$ are presented in Fig. 4 (area between thick lines in the left and right panels) together with current upper limits from LEP, from the Yukawa processes [34] (see ALEPH, DELPHI and OPAL results). Also the lower limits from the $Z \to h(A)\gamma$ [27] can be seen in Fig. 4. In addition, the upper 90% CL limits from the $\Upsilon$ decay (lines denoted K,N and L, with the K results rescaled by a factor 2, as discussed in [20]) and from the Tevatron [38], are presented as well.

We see that our two-loop analysis, based on the latest $(g-2)_\mu$ data and on the FJ02 estimation of $a_{\mu}^{\text{had}}$ (vp1), if combined with constraints from other experiments, allows for the existence, in the 2HDM (II), of a pseudoscalar with mass between $\sim 25$ GeV and 70 GeV, and $\tan \beta$ above $\sim 25$. The mass region for $A$ allowed by $(g-2)_\mu$ data, between $\sim 3$ and 25 GeV, is excluded by the constraints from LEP, based on OPAL and DELPHI data. On the other hand the constraints from the Tevatron close the pseudoscalar-mass window above 70 GeV. For a light scalar the combined constraints are even more severe; if the old constraints from the $\Upsilon$ decay data are taken into account, the area allowed by the latest $(g-2)_\mu$ data disappears. Note that the exclusion of a light $h$ is in agreement with the conclusion of the theoretical analysis reported in [13].
Figure 4: Current 95% CL constraints for $h$ (left) and for $A$ (right). The $(g - 2)_\mu$ data (this analysis) give the allowed area shown between thick dashed lines. Also the upper limits from LEP experiments (ALEPH, DELPHI, OPAL), Tevatron, as well as from the $\Upsilon$ decay into $h(A)\gamma$ and the $\eta$ decay (for $h$ only) are presented. The lower limits from LEP measurements on $Z \rightarrow h(A)\gamma$ are shown as well. See text for details.
6 Conclusions and outlook

Using the latest precise measurement of the \((g - 2)_\mu\) and comparing it with the improved, theoretical estimations of the SM contribution, we derive the 95% CL \(\delta a_\mu\) interval to be used to constrain an additional contribution to \(a_\mu\), beyond the SM ones. It allows one to constrain strongly the additional contribution, which arises in a CP-conserving, non-supersymmetric 2HDM (II) for a small parameter \(\mu\), studied in this paper. The additional contribution, allowed at 95% CL, has to have a positive sign, and can lead to a clear prediction for a light-scalar and a light-pseudoscalar scenario in the model considered here. These two scenarios correspond to the case when one of the Higgs bosons, \(h\) or \(A\), is very light, much lighter than the other Higgs particles of the model. It should be further noted that both of these scenarios are in agreement with existing data from other experiments. An exchange of such light particle dominates in the one- and two-loop contributions to the \(a_\mu\). Constraints from \((g - 2)_\mu\) are such that they exclude a light \(h\) with a mass above 5 GeV and a light \(A\) if its mass is below 3 GeV, as the corresponding contributions are negative in these regions.

Our two-loop analysis presented in this paper is based on the newest \((g - 2)_\mu\) data and on the (preliminary) FJ02 estimation of \(a_\mu^{\text{had}}\). Combining the constraints from the \((g - 2)_\mu\) data with those from other experiments, a pseudoscalar with mass between \(\sim 25\) GeV and \(70\) GeV and \(25 \lesssim \tan \beta \lesssim 115\) is allowed. However, a light scalar is excluded.

The main results will hold also if the \(g - 2\) constraints are based on the HMNT results for the vp1. For the HMNT(in) case the window for a pseudoscalar will be even smaller: \(35 \leq M_A \leq 70\) GeV and \(\tan \beta\) between 40 and 120. The results will not change significantly, if the uncertainty for the lbl contribution is up two times larger than in the estimation we used in the analysis. If such an error is still added in quadrature, then the lower bounds go down, with respect to results given in Fig. 4, by a factor \(\sim 1.4\). There will be no visible changes in the upper bounds.

Finally, we stress a need for a coherent and comprehensive error analysis for the SM contributions to the \(a_\mu\) (see a very recent paper [44]).

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