Tackling Unit Commitment and Load Dispatch Problems Considering All Constraints with Evolutionary Computation

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Abstract—Unit commitment and load dispatch problems are important and complex problems in power system operations that have been traditionally solved separately. In this paper, both problems are solved together without approximations or simplifications. In fact, the problem solved has a massive amount of grid-connected photovoltaic units, four pump-storage hydro plants as energy storage units and ten thermal power plants, each with its own set of operation requirements that need to be satisfied. To face such a complex constrained optimization problem an adaptive repair method is proposed. By including a given repair method itself as a parameter to be optimized, the proposed adaptive repair method avoid any bias in repair choices. Moreover, this results in a repair method that adapt to the problem and will improve together with the solution during optimization. Experiments are conducted revealing that the proposed method is capable of surpassing exact method solutions on a simplified version of the problem with approximations as well as solve the otherwise intractable complete problem without simplifications. Moreover, since the proposed approach can be applied to other problems in general and it may not be obvious how to choose the constraint handling for a certain constraint, a guideline is provided explaining the reasoning behind. Thus, this paper open further possibilities to deal with the ever changing types of generation units and other similarly complex operation/schedule optimization problems with many difficult constraints.

Index Terms—Supply and demand balancing operations, Mixed integer programming, Surplus energy, Unit Commitment, Load Dispatch, Differential Evolution, Constrained Optimization, Electric Energy Dispatch, Evolutionary Computation.

I. INTRODUCTION

Unit commitment (UC) and load dispatch (LD) are very important problems in power system operations in order to make an appropriate power generation schedule over a specified time period that meets electrical power demand with minimum costs satisfying other relevant constraints [1]. Unit commitment determines which generation units should be run and which should be stopped in each time slot in the period. Load dispatch divides the total amount of power to be generated over individual running units [1]. UC and LD are essentially a single problem that finds the optimal power generation plan. However, due to differences in the nature of the problems, solving them as a single problem is difficult. Traditionally, they were solved separately, sequentially or in a hierarchical manner with UC at the higher level and LD at the lower level. Due to this, a UC-LD problem is sometimes simply referred to as a UC problem [2]. UC and LD were solved with different solution techniques, respectively. These ‘separate’ solution processes require some extent of approximation or simplification in the problem formulation. This is why efforts are being made to develop better solution techniques although there already exist a number of solution techniques. Especially, much attention are being payed, recently, to evolutionary computation and swarm intelligence techniques [3], [4]. Recently, the nature of the UC and LD problems is changing due to introduction of non-traditional types of generation units, e.g. wind turbines and photovoltaic, and large-scale storage units, which makes it hard for the existing solution techniques to obtain good solutions. The difficulty arises from the complexity of problems, which comes from a larger number of variables and more constraints of various types than the problems used to have.

In this paper a solution technique is proposed which can deal with these more complicated and thus more difficult UC-LC problems without any approximations or simplifications to the standard formulation of the problems (Figure 1). The proposed solution technique utilizes the powerful searching capability of differential evolution [5] to deal with a large number of variables and treats various types of constraints with a set of newly developed constraint handling techniques, repair-penalty and adaptive repair techniques, combined with existing but effective ones. These constraint handling techniques are not specific to the UC-LD problems but can be applied to various constrained optimization problems.

The proposed solution technique is validated on an example UC-LD problem. The target power system includes massive amount of grid-connected photovoltaic and four pump-storage hydro plants as energy storage units in addition to ten thermal power plants. One of the important roles of the pump-storage hydro plants is to store surplus energy generated by the massive photovoltaic when demand is low and to utilize the energy economically. To enhance economical benefit, we have to consider weekly periodicity of the power demand. This forces us to take into account a longer horizon of one week than the one-day horizon that is considered in typical UC and/or LD problems, which results in a huge optimization problem with thousands of variables to be determined. Including pump-storage hydro plants in UC-LD problems also increase complexity of the problems in terms of constraints. A pump-storage hydro plant stores energy by pumping up
This paper solves for the first time the combined problem of load dispatch and unit commitment without approximations or simplifications, taking into consideration realistic operation requirements and constraints of a massive amount of grid-connected photovoltaic units, four pump-storage hydro plants as energy storage units and ten thermal power plants.

Fig. 1. This paper solves for the first time the combined problem of load dispatch and unit commitment without approximations or simplifications, taking into consideration realistic operation requirements and constraints of a massive amount of grid-connected photovoltaic units, four pump-storage hydro plants as energy storage units and ten thermal power plants.

II. RELATED WORK

The use of optimization techniques in electric power systems has been massively investigated [2]. Regarding UC optimization problems and their solutions, reviews can be found in [4], [3], [6], [7].

Over decades, use of evolutionary algorithms in power systems field has been attracting much attention [1]. For UC problems, an early work by Shebl et al. [8] solved the problem with the genetic algorithm where constraints were handled by penalty methods. However, the minimum-down-time constraint, one of the most tricky constraints, was not considered. Handling the constraints in UC-LD problems is not easy. For example, in [9], evolutionary algorithms were applied to the problems. However, only relatively simple constraints were taken into account and handling the constraints by the repair method may cause some bias on the optimization and give a negative impact on the quality of the search. The situation is similar when the LD problems for pumped storage hydro plants considered in [10]. They used the classical differential evolution and an adaptive differential evolution to optimize the unit commitment. There were few constraints taken into account. In [11], the valve-point effect is taken into account and the authors used an improved DE for achieving state-of-the-art results. The LD problem considered has, however, few constraints and no hydro plants or photovoltaic units. Another approach using DE to target a LD problem considering storage hydro plants is presented in [12]. In this recent work, an adaptive DE is proposed to solve the problem. The modeling considered for hydro plants are similar to ours in complexity, however, the thermal plants have only a single constraint associated and there is no photovoltaic units and their associated spinning reserve. A hybrid approach mixing simulated annealing and ant colony optimization into a new metaheuristic method was applied to the short-term energy resource management problem where intensive use of electric vehicles is present [13]. In this case, the constraints were all inequalities. To satisfy them, penalties were added to the objective function.

UC-LD problems for thermal and pumped storage hydro plants were treated in several studies. Gollmer et. al [14] applied the primal and dual method to the problems where no constraints were ignored. Here, however, only linear or piece-wise linear objective functions and constraints were assumed in order to utilize well-established optimization techniques. In more recent study [15], a full set of constraints were considered. However, some of them were relaxed to make use of quadratic programming possible. In summary, handling all the constraints present UC-LD problems is not easy even if evolutionary algorithms are used, and to the best knowledge...
III. CONSTRAINED OPTIMIZATION PROBLEMS

Constrained optimization problems are a type of optimization problems where constraints are present. Constraints modify the search space making otherwise simple problems extremely complex to solve. The general formulation of constrained optimization problems is as follows:

\[
\begin{align*}
\text{minimize} & \quad f(x), \ x \in \mathbb{R}^n \\
\text{subject to} & \quad g_i(x) \leq 0, \quad i = 1, \ldots, m \\
& \quad h_i(x) = 0, \quad i = 1, \ldots, p,
\end{align*}
\]

where \( f(x) \) is the objective function, also called fitness in evolutionary computation, and \( g_i(x) \) and \( h_i(x) \) are the two types of constraints. They are called respectively inequalities and equalities. \( m \) and \( p \) define respectively the numbers of inequalities and equalities of the problem. Thus, for one candidate solution \( x \) to be feasible it must satisfy all equalities and inequalities.

Constrained optimization problems are harder than unconstrained ones because the feasible search space itself is not always explicitly given or known. Constraints can also depend on a number of variables, making almost impossible the prediction of their feasibility. To deal with these constraint related problems many techniques have been developed, they are discussed in the next Section.

IV. CONSTRAINT-HANDLING TECHNIQUES

Here related constraint-handling techniques used together with evolutionary computation will be described briefly. An extensive review is out of the scope of this article.

A. Penalty Approaches

Penalty techniques worsen the fitness of infeasible solutions by adding a penalty function to the fitness. Penalty functions should be able to make infeasible solutions unlikely to win when competing with feasible solutions. Therefore, infeasible solutions would coexist with feasible ones but would be discarded in the long run.

There are many types of penalty functions, it is possible to make them depend on the number of generations, i.e., allowing infeasible solutions to violate constraints almost free of penalty in the beginning of evolution while increasing the penalty along the evolution. It is also possible to modify the penalties using external cooling schemes [16], adaptive heuristics [17], another evolutionary algorithm [18], parent and children population [19], among others.

There are also methods that go to the extreme of discarding any infeasible solutions. This type of approach is called death penalty [20].

B. Repair Approaches

Constraint-handling approaches that aim to make infeasible solutions feasible can be divided into two groups [21], [22].

- Variable-based methods - Methods that test and modify each variable independently to keep the candidate solution inside the feasible region.
- Vector-based methods - Methods that when the solution lies in an infeasible region, modify the solution as a vector along a given direction to bring it into feasible space. In this manner, all variables are modified including the ones that were already inside their feasible region.

C. Decoder Approaches

Decoder approaches map a solution from the original search space to another space where solutions are guaranteed to be feasible [23]–[25]. Some methods define operators and solution representations to force solutions to be feasible [26], [27].

D. Multi-objective Approaches

Constraints by themselves can be seen as other objectives to be optimized. These objectives can be easily constructed from constraints by defining a function that measures the constraint violation. In this view, a constrained single objective problem becomes a multi-objective one and can be handled with multi-objective algorithms such as NSGAII [27], GDE3 [28] or SAN [29]. This approach was used by [30]–[33].

V. DIFFERENTIAL EVOLUTION

Differential Evolution (DE) [5] is a global optimization algorithm within the subfield of evolutionary computation. Similar to other evolutionary algorithms, DE is capable of optimizing functions without knowing the functions to be optimized. The objective functions themselves are neither required to be continuous nor to be differentiable. DE is a fairly simple algorithm based on three procedures (mutation, crossover and selection). The algorithm is described succinctly in Table I and the explanation of each of its three procedures is done in the subsections below.

| TABLE I |
|--------|
| DIFFERENTIAL EVOLUTION ALGORITHM |
| 1) Initialize population with randomly sampled individuals (candidate solutions) |
| 2) For each generation until the maximum number of generations do: |
| a) For each individual in the population do: |
| i) Mutation |
| ii) Crossover |
| iii) Selection |
| 3) Return best candidate solution |
A. Mutation

For each candidate solution (vector) in the g-th iteration the mutation is applied by the following equation:

\[ v_{i,g+1} = x_{i,g} + F(x_{r2,g} - x_{r3,g}) \]  

(2)

where \( r1 \), \( r2 \) and \( r3 \) are indices of randomly selected individuals of the population, which must differ from the individual \( i \). \( F \) is a parameter which should meet the condition \( F \in [0, 2] \). The generated vector \( v_{i,g+1} \) is called mutation vector.

B. Crossover

During the crossover, a trial vector \( u_{i,g+1} \) is created from a combination of the mutation vector \( v_{i,g+1} \) and the original vector \( x_{i,g} \) as follows:

\[ u_{i,j,g+1} = \begin{cases} x_{i,j,g} & \text{if } \text{rand}(\cdot) > CR \text{ and } j \not= rnd_i; \\ v_{i,j,g+1} & \text{if } \text{rand}(\cdot) \leq CR \text{ or } j = rnd_i, \end{cases} \]

(3)

where \( \text{rand}(\cdot) \in [0, 1] \) is a uniformly distributed random number, \( CR \in [0, 1] \) is a parameter, \( j \) is the vector component index and \( rnd_i \) is a randomly chosen vector index.

C. Selection

Both \( u_{i,g+1} \) and \( v_{i,g} \) are evaluated and the vector with better fitness function is kept, forming the next generation vector \( x_{i,g+1} \).

VI. Problem Description

A. Overview

The target problem is to build an optimal one-week generation schedule in the emerging type of UC-LC problem settings where thermal power plants, photovoltaic (PV) generation units and pumped storage hydro plants as large-scale energy storage exist.

The purpose here is twofold: the first one is to minimize operation costs of the power plants while the second being to avoid wasting the PV unit output power. Notice that the operating costs considered here include the fuel costs and the start-up costs of the thermal plants.

When a huge number of PV units are connected to the power system, it can happen that generated output exceeds electricity demand. To avoid wasting this excessive power, the power (energy) must be stored. Here pumped storage hydro plants are used. The stored energy must be used to generate power appropriately so that the generation costs of the thermal plants are minimized.

B. Symbols

The following symbols are used:

- \( t \): time interval (an hour), (all the suffixes \( t \) below indicate values at time \( t \)),
- \( T \): set of time intervals \( \{0, 1, \ldots, T_f\} \),
- \( i \): index of a thermal plant, (all the suffixes \( i \) below indicate values associated to thermal plant \( i \)),
- \( NG \): set of indices \( i \),
- \( u_{i,t} \): \((\in \{0, 1\})\) thermal plant status, 0: halted, 1: working,
- \( g_{i,t} \): consecutive time length of halted status of a thermal plant up to time \( t \),
- \( G_{i}\max \): thermal plant generating power,
- \( G_{i}\min \): minimum output of a thermal plant,
- \( \Delta G_{i}\up \): maximum upward ramp rate of a thermal plant,
- \( \Delta G_{i}\down \): maximum downward ramp rate of a thermal plant,
- \( MDT_{i} \): minimum down time (minimum required time for a halted plant to restart) of a thermal plant,
- \( SC_{i} \): start up cost of a thermal plant,
- \( A_{i}, B_{i} \) and \( C_{i} \): coefficients in fuel cost function of a thermal plant,
- \( j \): index of a pumped storage plant, (all the suffixes \( j \) below indicate values associated to pumped storage plant \( j \)),
- \( NHG \): set of indices \( j \),
- \( h_{g_{j,t}} \): pumped storage plant generating power, positive value: generated power, negative value: consumed power for pumping-up,
- \( hv_{j,t} \): water level of the reservoir for a pumped storage plant,
- \( HG_{j}\max \): maximum generating output of a pumped storage plant,
- \( HG_{j}\min \): minimum generating output of a pumped storage plant,
- \( HP_{j}\max \): \((\geq 0)\) maximum consumed power for pumping up,
- \( HP_{j}\min \): \((\geq 0)\) minimum consumed power for pumping up,
- \( \Delta H_{j}\up \): maximum ramp rate of a pumped storage plant in the generator mode,
- \( \Delta H_{j}\down \): maximum ramp rate of a pumped storage plant in the pump mode,
- \( HV_{j}\max \): maximum water level of the reservoir for a pumped storage plant,
- \( HV_{j}\min \): minimum water level of the reservoir for a pump storage plant,
- \( D_{t} \): predicted total demand,
- \( pv_{t} \): predicted total output from all PV units,
- \( u \): vector that consists of \( u_{i,t}, i \in NG, t \in T \),
- \( g \): vector that consists of \( g_{i,t}, i \in NG, t \in T \),
- \( h \): vector that consists of \( h_{g_{j,t}}, j \in NHG, t \in T \).

Some other symbols will also be used. They will be defined.
where they first appear.

C. Objective function

The objective function of the problem is defined as follows:

$$F(u, g, h) = \sum_{i \in T} \sum_{t \in \mathbb{N}} \left[ (A_i + B_i g_{i,t} + C_i g_{i,t}^2) u_{i,t} + SC_i u_{i,t} (1 - u_{i,t-1}) \right]$$

which is to be minimized. Here, the factor \( A_i + B_i g_{i,t} + C_i g_{i,t}^2 \) represents the fuel cost of thermal plant \( i \), which is a quadratic function of its output, \( g_{i,t} \), and is considered only when the plant is working (\( u_{i,t} = 1 \)). We include the start-up cost \( SC_i \) in the objective function when \( u_{i,t} = 1 \) and \( u_{i,t-1} = 0 \), which means that the plant was halted at the previous time \( t - 1 \) but now at time \( t \) is working. Note that variable \( h \) does not appear in the function explicitly. It affects the other variables \( u \) and \( g \) through the supply demand balance constraint.

D. Constraints

1) Supply demand balance: Total power supply and total demand must be equal at any time:

$$\sum_{i \in \mathbb{N}G} g_{i,t} u_{i,t} + \sum_{j \in \mathbb{N}HG} h_{j,t} + pv_t = D_t, \text{ for } \forall t.$$  

(5)

To distinguish clearly our decision variables \( u_{i,t} \) and \( h_{j,t} \) from exogenous variables \( hv_t \) and \( D_t \), the following equivalent expression can be helpful:

$$\sum_{i \in \mathbb{N}G} g_{i,t} u_{i,t} + \sum_{j \in \mathbb{N}HG} h_{j,t} = D_t - pv_t, \text{ for } \forall t.$$  

(6)

Here we term the right hand side, \( D_t - pv_t \), as net demand because it indicates power to be supplied from the thermal and pumped storage plants owned by the power utility company.

When PV units generate huge power in total, the right hand side of equation (6) becomes small. In this situation, it may be necessary to make \( h_{j,t} \) negative (pump mode), which means that the excessive power from the PV units is stored in the reservoirs.

2) (Extended) spinning reserve: We do not know PV unit outputs and demand for sure when we plan the plant operation. Only available are their predicted values, \( pv_t, t \in T \) and \( D_t, t \in T \), and of course they are prone to errors. To cope with the errors, we introduce the following constraints:

$$\sum_{i \in \mathbb{N}G} g_{i,t}^\text{min} u_{i,t} + \sum_{j \in \mathbb{N}HG} h_{j,t}^\text{min} \leq (1 - \alpha_t)(D_t - pv_t), \text{ for } \forall t,$$

(7)

and

$$\sum_{i \in \mathbb{N}G} g_{i,t}^\text{max} u_{i,t} + \sum_{j \in \mathbb{N}HG} h_{j,t}^\text{max} \geq (1 + \beta_t)(D_t - pv_t), \text{ for } \forall t.$$  

(8)

Here we introduce the following new symbols:

\( \alpha_t: \ (\in [0, 1]) \) the maximum possible decrease in net demand from its predicted value,

\( \beta_t: \ (\in [0, 1]) \) the maximum possible increase in net demand from its predicted value,

\( g_{i,t}^\text{min}: \) thermal plant minimum generating output at time \( t \),

\( h_{j,t}^\text{min}: \) pumped storage plant minimum generating output at time \( t \),

\( g_{i,t}^\text{max}: \) thermal plant maximum generating output at time \( t \),

\( h_{j,t}^\text{max}: \) pumped storage maximum generating output at time \( t \).

3) Thermal power plants: Each thermal plant has the maximum and minimum output constraints:

$$G_{t}^\text{min} u_{i,t} \leq g_{i,t} u_{i,t} \leq G_{t}^\text{max}, \text{ for } \forall i, \forall t.$$  

(9)

the ramp rate constraints:

$$-\Delta G_{i}^\text{down} \leq g_{i,t} - g_{i,t-1} \leq \Delta G_{i}^\text{up}, \text{ for } \forall i, \forall t.$$  

(10)

and the minimum downtime constraints:

If \( 0 < u_{i,t}^\text{off} < MDT_i \); then \( u_{i,t} = 0, \forall i, \forall t. \)  

(11)

which involves the decision variable \( u_{i,t} \) over a certain period.

4) Pumped storage power plants and their reservoirs: A pumped storage plant has the maximum and minimum output constraints when working as a generator:

$$h_{j,t} > 0 \text{ then } 0 \leq HG_{j}^\text{min} \leq h_{j,t} \leq HG_{j}^\text{max}, \text{ for } \forall j, \forall t,$$

(12)

and when working as a pump:

$$h_{j,t} < 0 \text{ then } -HF_{j}^\text{max} \leq h_{j,t} \leq -HF_{j}^\text{min} \leq 0, \text{ for } \forall j, \forall t.$$  

(13)

It also has the ramp rate constraints:

$$h_{j,t} > 0 \text{ then } h_{j,t} - h_{j,t-1} \leq \Delta H_{j}^\text{up}, \text{ for } \forall j, \forall t,$$

(14)

$$h_{j,t} < 0 \text{ then } h_{j,t} - h_{j,t-1} \geq \Delta H_{j}^\text{down}, \text{ for } \forall j, \forall t.$$  

(15)

Water level of a reservoir changes depending on operation of the pumped storage plant:

$$\varepsilon_j hv_{j,t} = \varepsilon_j hv_{j,t-1} + \eta_j (-h_{j,t}), \text{ for } \forall j, \forall t.$$  

(16)

The constraints also involve the decision variables \( hv_{j,t} \) over a certain period.

Here we use the following new symbols:

\( \varepsilon_t: \) coefficient that converts water level to power (energy),

\( \eta_j: \) efficiency coefficient of a pumped storage plant.

The reservoir has a limited capacity:

$$HV_{j}^\text{min} \leq hv_{j,t} \leq HV_{j}^\text{max}, \text{ for } \forall j, \forall t.$$  

(17)

Values of some variables at time \( t \) depend on the values at time \( t - 1 \). This implies that they also depend on the values at time \( t - 2, t - 3 \) and so on. Therefore, in theory, the problem
must be solved considering an infinitely long time period. This, however, is of course impossible. So we focus on just one week. However, focusing on one week provides us with a solution where all the water stored in the reservoir is used up at the end of the week, which is not a long-term optimal solution. To avoid this, we impose the following constraint on the initial water level, \( h_{v,j,0} \) and the final water level, \( h_{v,j,T_f} \):

\[
h_{v,j,T_f} = h_{v,j,0}, \text{ for } \forall j.
\]

(18)

**VII. Solution Representation (i.e., Chromosome Encoding)**

The solution representation used is a concatenation of two matrices, one vector and a positive integer. The details of the representation are explained below:

- **Thermal Generator Output Matrix** - It is a matrix describing the power generated at a given time by a thermal generator. The value at a given \( i, j \)-th index of the matrix represents the output of the \( i \)-th thermal plant at the \( j \)-th time step. If its value is negative, it means the generator is turned off.

- **Pump Generator Output Matrix** - It is a matrix representing the power generated or consumed by a pumped storage hydro plant at a given time. The value at a given \( i, j \)-th index of the matrix represents the output of the \( i \)-th pump generator plant at the \( j \)-th time step. If its value is negative, it means the generator is consuming power rather than producing it.

- **Preference Vector \((P)\)** - This vector defines the order to modify the output of thermal power plants in order to satisfy a given constraint. In this article, this preference vector is used to satisfy the supply-demand constraint.

- **Maximum Change per Step \((\text{MaxC})\)** - While the chromosome is being repaired this variable controls the maximum output change of one plant in one step.

Note that the values determined by the chromosome are pre-repair values that might be repaired later to fit constraints. The details of how a chromosome is repaired will be explained later on.

**VIII. Proposed Method: Overview**

In this paper, a global optimization algorithm is combined with several new and existing constraint handling techniques. The DE algorithm is chosen as the global optimization algorithm [5] for its simplicity and good performance. The UC-LD problem is a complex optimization problem. Therefore, it is reasonable to choose DE, which is capable of searching in its huge search space.

In addition to the DE algorithm, appropriate constraint handling mechanisms should be employed. Adding penalty functions is perhaps the simplest form of constraint handling mechanism which is enough to deal with some form of inequalities. However, penalty functions are not effective to deal with equalities because in general there are much more infeasible solutions. Nevertheless, given a candidate solution, satisfying equalities and single-variable inequalities often requires only a trivial repair mechanism. This is why we use a series of repair mechanisms to rebuild a candidate solution and consequently satisfy many equalities and single-variable inequalities (Section IX). The hardest part of designing repair mechanisms for most of the equalities comes from deciding the best order as well as repair mechanisms that are as mutually independent as possible to avoid violating previously satisfied constraints (i.e., avoid undoing the work of previous repair mechanisms).

Having said that, there are some exceptions though. For some equalities of the problem described (Section VI), satisfying them is either (a) difficult or (b) prone to induce biases by repairing mechanisms (the repaired solution satisfies the constraints but is not necessarily the best or a recommended one). To solve the cases (a) and (b) we adopt respectively a repair-penalty mechanism (i.e., a repair mechanism with penalty function) and a adaptive repair mechanism. These two approaches are briefly described below:

- **Repair-penalty mechanism** - An additional penalty function is included to repair mechanisms when constraints are left unsatisfied. This penalty function is set to zero if the repair mechanism can satisfy the constraint, only penalizing a candidate solution in which the mechanism cannot repair.

- **Adaptive repair mechanism** - Instead of setting a fixed and biased repairing, an unbiased procedure is developed.

In the next section, the details about the repair and evaluation procedures are given.

**IX. Proposed Method: Repair and Evaluation**

Before evaluating a candidate solution, it is repaired by a series of methods that try to satisfy as much as possible the constraints and/or penalized when necessary. Notice that the order in which constraints are satisfied is important as changes in this order may alter the result. In fact, modifying a candidate solution to satisfy one constraint may turn this candidate solution infeasible in previously satisfied constraints. Therefore, careful attention should be paid to how to repair a candidate solution as well as to the order in which these repairs occur. With the above in mind, below we describe the procedures used to repair single variable inequality constraints and in which order they are executed:

1) Satisfy ramp rate constraints for thermal plants (Equation [10]) - Anything exceeding the maximum ramp rate is set to its maximum value;

2) Satisfy maximum/minimum value for thermal plants (Equation [9]) - Anything exceeding the maximum or minimum is set to its respective minimum/maximum value.

3) Satisfy minimum downtime constraints of thermal plants (Equation [11]) - If a given plant violates the constraint, it is turned off for as many time steps as necessary to satisfy the constraint.

4) Satisfy maximum/minimum generation output for pumped storage plant (Equation [12]) - Anything exceeding the maximum/minimum is set to its maximum/minimum value for pumped storage plants when working as a generator (i.e., positive value).
5) Satisfy maximum/minimum power consumption for pumped storage plant (Equation [13]) - Same as above but for pumped storage plants when working as a pump (i.e., negative value).

6) Satisfy ramp rate limits for power storage hydro plant’s generation (Equation [14]) - Any increase in generation that exceeds the ramp rate is set to its maximum ramp rate.

7) Satisfy ramp rate limits for power storage hydro plant’s consumption (Equation [15]) - The same as above, but for the increase in consumption rather than generation.

8) Satisfy maximum/minimum water levels (Equation [17]) - If a given power storage plant’s consumption or generation would cause its water level to go above or below its maximum/minimum water level according to Equation [16] the consumption or generation power is set to satisfy the maximum/minimum water level.

By repairing the chromosome with the mentioned procedures, the candidate solution can now easily satisfy many constraints without inserting any kind of bias. This happens because the satisfied constraints are simple in nature, i.e., there is only one procedure for satisfying them.

However, there are still some constraints which need to be satisfied. Among the remaining constraints there are the supply demand balance constraint (Equation [6]), the constraint on the initial and final water levels (Equation [18]) and the spinning reserve constraint (Equation [7] and [8]), i.e., two equalities and one inequality.

A. Mechanism to Repair the Supply Demand Balance Constraint

To satisfy the supply demand balance constraint a newly developed repair method is used, increasing/decreasing the output value of power plants in sequence until the supply matches demand. In this repair mechanism the order of repairs for each power plant matters. Therefore, fixing the order gives a strong and undesirable bias. Moreover, another bias would be inserted if the repair’s maximum change per step is fixed.

To avoid these biases a new type of constraint handling mechanism called Adaptive Repair Mechanism is proposed. This new constraint handling mechanism, instead of fixing the order of repairs and MaxC, allows the repair mechanism to be controlled by both a vector of parameters called preference vector and MaxC. The values inside the preference vector set the priority of repair for each thermal plant, i.e., thermal plants with higher values in the preference vector are repaired first. The preference vector and MaxC are added to the parameters to be optimized. Consequently, every candidate solution has its own way of repairing itself. In fact, an interesting byproduct of this approach is that the way of repairing itself is optimized as well.

Aside from the repair of values to set supply equal to demand, it is also necessary to verify if ramp constraints continue to be satisfied afterwards. In case they are not satisfied, a simple repair mechanism described before is employed again to satisfy them. Actually, it is possible that even after the repair mechanism supply does not equal demand. If this happens, a huge penalty is added to the evaluation of the candidate solution. The value of this huge penalty is equal to the difference between an ideal value that could satisfy the constraint and the current value multiplied by the supplyDemandCoefficient. The detailed description of the repair mechanism can be seen in Algorithm [1]. The supplyDemandCoefficient is a value that starts as zero and is constantly increasing during optimization. Its maximum value is given by MaxSupplyDemandCoefficient.

B. Mechanism to Repair the Initial and Final Water Level Constraint

The constraint on the initial and final water levels (Equation [18]) is also an equality which makes it more suitable to be tackled with repair mechanisms. In fact, there is an easy repair mechanism which consists of starting from the last time step and going backwards in time updating the output of pumped storage hydro plants. In every update, the output of pumped storage hydro plants are modified to set the difference between initial and final water levels closer to zero. By going back in time, we guarantee that the modification will not make it surpass, in subsequent time steps, the maximum or minimum water levels.

In case the repair mechanism is not able to correct the candidate solution, a penalty (WaterLevelPenalty) is added.
for every time step and while diff ≠ 0 do

diff = supply demand difference;

for MaxAdjustments iterations and while diff ≠ 0 do

for all thermal generators do

Modify thermal generator;

Update diff to the current supply demand difference;

end /* Notice that all thermal generators are passed in the
order given by the preference vector */

end

if diff ≠ 0 then

Add a huge penalty;

end

Repair thermal ramp rate constraints (Equation 10)

end

Algorithm 1: Supply Demand Balance Constraint Repair Algorithm

to the objective. The penalty is defined as:

\[ WLP_j = |hv_{j,T_i} - hv_{j,0}|, \text{ for } \forall j, \]

\[ WaterLevelPenalty = waterCoefficient \sum_{j \in NHG} WLP_j, \]

The waterCoefficient is similar to supplyDemandCoefficient in that it starts as zero and is constantly increasing during optimization to a maximum value of MaxWaterCoefficient. For each generation of the optimization algorithm, the waterCoefficient and supplyDemandCoefficient increases their value by coefficientStep parameter.

C. Spinning Reserve’s Penalty Function

The spinning reserve constraint (Equations 7 and 8) is included in the problem in the form of a penalty function. The penalty for the spinning reserve constraint (SpinningReservePenalty) is defined as:

\[ S_{1t} = \sum_{i \in NG} g_{i,t}^{\min} u_{i,t} + \sum_{j \in NHG} h_{j,t}^{\min} - (1 - \alpha_t)(D_t - pv_t), \]

\[ S_{2t} = (1 + \beta_t)(D_t - pv_t) - \sum_{i \in NG} g_{i,t}^{\max} u_{i,t} + \sum_{j \in NHG} h_{j,t}^{\max}, \forall t, \]

\[ SpinningReservePenalty_t = \max\{S_{1t}, 0\} + \max\{S_{2t}, 0\} \]  

D. Bridging gap between pre-repair solutions and post-repair solutions

Recall that the chromosomes (candidate solution) are only the pre-repair values. They are, when necessary, repaired and/or penalized and the fitness is evaluated for the post-repair values. To bridge the gap between the pre-repair solutions expressed by chromosomes and their post-repair solutions, every one in RC function evaluations a chromosome is set to the repaired values (RC is set to 10000 in all experiments). Notice that a low value of RC would make infeasible solutions disappear rapidly from the population. This can be dangerous because the space of feasible solutions are much smaller and disconnected, therefore premature convergence is very likely to occur.

E. A Guideline for Building Repair Mechanisms Independent of the Problem

The optimization method together with its repair mechanisms can be applied to any other optimization problem. This section aims to aid the construction of repair mechanisms in general.

- Separate constraints into multivariable ones and single-variable ones.
- (a) For single-variable constraints do:
  1) Check - If the constraint’s variable depend on other unsatisfied constraints, treat this as a multivariable constraint and go to item (b) or (c) if the relationship between the constraints is respectively an equality of inequality. If the constraint’s variable does not depend on other unsatisfied constraints go to the next step:
  2) Solve - Use a greedy procedure to satisfy it (e.g., if a value exceeds the maximum set it to the maximum);
  3) Loop - Go to the next unsatisfied single-variable constraint and repeat the first step ‘Check’.
- (b) For multivariable inequalities do:
  1) Add a Penalty - Multivariable inequalities accept many solutions, i.e., probably it is not difficult to go from feasible to infeasible and vice-versa.
- (c) For multivariable equalities do:
  1) Special Solution - Search for a special solution that could solve the multivariable equality constraint without adding bias. For example, solving the multivariable equality constraint backward in time (see Section IX-B for an example). If such unbiased solution is not found go to the next step.
  2) Add Adaptive Repair Mechanism - Add a adaptive repair mechanism to avoid bias and at the same time be certain that the equality will be satisfied. The general idea of an adaptive repair mechanism is described in Section VIII and an example is given in Section IX-A.

X. EXPERIMENT 1: COMPARISON WITH ANOTHER APPROACH IN A SIMPLIFIED UC-LD PROBLEM

A. Problem Settings (Simplified Version)

This section explains the problem settings used in the first experiments in which the proposed algorithm is compared with an exact approach. Since the exact approach can not face the electrical dispatch (UC-LD) problem considering all
constraints, a simplified version of the problem described in Section VI is considered.

In the simplified version of the problem, it is assumed that the parameter $\Delta t$ and the variables $\Delta G_{i,up}$ and $\Delta G_{i,down}$ satisfy the conditions shown below. This assumption eases the difficulty that comes from the large-scale mixed-integer nature of the problem.

\[\Delta t \geq MDT_i \quad \forall i\]
\[\Delta G_{i,up} \geq G_{i,\text{max}} - G_{i,\text{min}} \quad \forall i\]
\[\Delta G_{i,down} \geq G_{i,\text{max}} - G_{i,\text{min}} \quad \forall i\]

Under these assumptions, the target problem can be partitioned into the following three stages and solved by an enumeration method followed by a quadratic programming (QP) solver.

1) First, all the feasible UC solution candidates are enumerated and their optimal output shares are calculated assuming no pumped storage hydro plants. If the sum of electricity generated by the thermal and PV units exceeds the power consumption, we can identify how much power becomes surplus at which time in this stage.

2) Afterwards, the QP solver derives the optimal pumped storage hydro operation to minimize fuel costs of the thermal units under the condition that the surplus power must be canceled. In this stage, the UC solution has been fixed, and we assume that the operating thermal units are approximated as one large-scale thermal unit ($g_t = \sum_{i \in NG} g_i u_{i,t}$). The vector $h$ influences the fuel cost of thermal units through the relationship $\Delta g_t = -h_t$.

3) Finally, the output shares of thermal units are recalculated in consideration of the determined schedule of $u$ and $h$. This procedure does not guarantee the global optimality of solutions but can provide us with stable and consistent solutions.

The numerical simulation model has 10 thermal generators each with their own set of parameters and one aggregated PV unit. The predicted values of demand and PV output are given, and their difference has to be compensated by the thermal and pump-storage hydro plants (Figure 3). Naturally, the predicted values include uncertainty and that is why the spinning reserve constraint is taken into account (Equations 7 and 8). Tables II and III describe respectively the general settings and the cost related settings of the thermal generators. To enable both methods to be compared only in terms of thermal plants’ output found and their respective cost, both methods are applied to Stage 3 assuming the results have already been obtained in Stages 1 and 2.

B. Comparison

The objective of this section is to compare the algorithm with an exact approach. The final exact solution to the simplified problem gives the minimum cost of 12049 (Figure 4).

Regarding the proposed method, it achieves a cost of 11761 surpassing the exact result. This means that the solution found satisfy all constraints and is 2.3% more efficient. To find this result the method was run 30 times. Notice that a better solution than the exact approach is possible because there are a series of approximations (e.g., using one single large-scale thermal unit) which are employed by the exact solution in order to solve the problem. In fact, every run of the proposed method achieves on average 11884 which is...
TABLE II
**THERMAL GENERATORS’ GENERAL SETTINGS**

| Generator Number | Minimum Uptime (hour) | Minimum Downtime (hour) | Maximum Output | Minimum Output |
|------------------|-----------------------|-------------------------|----------------|---------------|
| Generator 1      | 8                     | 10                      | 11             | 11            |
| Generator 2      | 6                     | 8                       | 11             | 11            |
| Generator 3      | 7                     | 10                      | 7              | 1             |
| Generator 4      | 8                     | 8                       | 11             | 3.3           |
| Generator 5      | 9                     | 5                       | 5.8            | 1             |
| Generator 6      | 8                     | 8                       | 11             | 3.3           |
| Generator 7      | 9                     | 5                       | 5.8            | 1             |
| Generator 8      | 8                     | 8                       | 7              | 1             |
| Generator 9      | 8                     | 8                       | 7              | 1             |
| Generator 10     | 6                     | 7                       | 5              | 1             |

TABLE III
**THERMAL GENERATORS’ COST RELATED PARAMETERS**

| Generator Number | Startup Cost | Cost Coefficient A | Cost Coefficient B | Cost Coefficient C |
|------------------|--------------|--------------------|--------------------|--------------------|
| Generator 1      | 1            | 0.01               | 0.5                | 0.01               |
| Generator 2      | 3            | 0.01               | 0.5                | 0.01               |
| Generator 3      | 0.8          | 1.17               | 2.4                | 0.04               |
| Generator 4      | 8            | 6.05               | 1.8                | 0.063              |
| Generator 5      | 1            | 0.01               | 4.2                | 3                  |
| Generator 6      | 8            | 6.05               | 1.9                | 0.063              |
| Generator 7      | 8            | 1.9                | 5                  | 0.038              |
| Generator 8      | 2            | 2.1                | 5                  | 0.038              |
| Generator 9      | 2            | 2.1                | 5                  | 0.038              |
| Generator 10     | 0.3          | 2                  | 5                  | 0.05               |

TABLE IV
**HYDROPOWER PLANTS’ SETTINGS (ONLY CONSIDERED IN FULL VERSION OF THE PROBLEM)**

| Plant Number  | Maximum Output | Maximum Consumption | System Efficiency | Maximum Water Reservoir | Conversion Coefficient |
|---------------|----------------|---------------------|-------------------|-------------------------|------------------------|
| Generator 1   | 2.5            | 2.5                 | 80                | 100                     | 10                     |
| Generator 2   | 2.5            | 2.5                 | 80                | 100                     | 10                     |
| Generator 3   | 2.5            | 2.5                 | 80                | 100                     | 10                     |
| Generator 4   | 2.5            | 2.5                 | 80                | 100                     | 10                     |

still better than the exact result. Table V shows the complete statistics. The proposed method uses the same parameters for both this simplified experiment and the experiment containing all constraints explained in the next section. Parameters are shown in Table VI.

Figure 5 shows the thermal plants’ outputs for both methods. Although both methods are very distinct, they share many similarities in their solutions. For example:

- Thermal plants 1 and 2 are set to their maximum output for almost the whole period.
- Thermal plants 5, 7, 8, 9 and 10 are almost not used. They exhibit some pulse patterns.
- Thermal plants 3, 4 and 6 are switched on for long periods with some intervals.

XI. EXPERIMENT 2: ELECTRICAL DISPATCH PROBLEM CONSIDERING ALL CONSTRAINTS

In this section the same problem is tackled considering all constraints and the hydro power plants’ output is to be determined (not fixed). The following sections describe the parameters settings and discuss the achieved results.

A. Problem Settings (Full Version)

The problem settings are the same as the one mentioned in Section X-A. There are 4 hydro power plants. The parameters of all hydro power plants are shown on Table V. Regarding the algorithm, its parameters are described in Table VI.

| Variable Name                  | Value            |
|-------------------------------|------------------|
| Best Solution                 | 17071            |
| Solutions With Penalty < 0.01 | 13%              |

Variable Name               | Mean (Standard Deviation) |
|-----------------------------|---------------------------|
| Cost                        | 11927.4 (203.5)           |
| Fitness                     | −11927.4 (203.5)          |
| Total Penalty               | 0.00 (0.00)              |
| Spinning Reserve Penalty    | 0.00 (0.00)              |

Fitness and cost have the same absolute value because the penalty is zero. Mean and standard deviation of the final solutions which satisfied all constraints (13% of the runs).

B. Results

The proposed method is able to solve for the first time the complex UC-LD problem considering all constraints. The best solution found satisfies all constraints while having a thermal cost of 12650. The results of running the proposed algorithm on the problem in question are shown on Table VII. Figure 6
TABLE VI
PARAMETERS

| Parameter                              | Value  |
|----------------------------------------|--------|
| Population size                        | 2000   |
| Maximum generations                    | 80000  |
| Maximum adjustment iterations (MaxAdjustments) | 10     |
| MaxSupplyDemandCoefficient             | 1000   |
| MaxWaterCoefficient                    | 100    |
| coefficientStep                        | 0.025  |

Differential Evolution Parameters

| Parameter | Value             |
|-----------|-------------------|
| CR        | 0.8               |
| F         | random(0, 1)*     |
| Initial values | random(-10, 10)*  |

Random(a,b) means that a value is chosen by sampling from a uniform distribution with maximum value a and minimum value b.

TABLE VII
RESULTS OF THE PROPOSED METHOD IN EXPERIMENT 2. RESULTS ARE AVERAGED OVER 30 RUNS.

| Variable Name                        | Value          |
|--------------------------------------|----------------|
| Best Solution                        | 12650          |
| Solutions With Penalty < 0.01        | 36%            |
| Variable Name                        | Mean (Standard Deviation)* |
| Cost                                 | 14360.1(1283.6) |
| Fitness                              | -14360.1(1283.6) |
| Total Penalty                        | 0.00(0.00)     |
| Spinning Reserve Penalty             | 0.00(0.00)     |

Fitness and cost have the same absolute value because the penalty is zero. Mean and standard deviation of the final solutions which satisfied all constraints (36% of the runs).

Fig. 5. Thermal output comparison between the proposed method (green solid line) and the analytical approach (blue dashed line).

Fig. 6. Supply demand curve of the best solution found which has a thermal cost of 12650 while satisfying all constraints.

XII. BEYOND UC-LD PROBLEMS

The method proposed here can be easily applied to other problems of different nature. In order to do this, repair mech-
anisms need to be modified to deal with the constraints of the given problem. The modifications, however, are relatively easy and can be done by following the guidelines in Section IX-E.

XIII. CONCLUSIONS

In this paper, a method was proposed which can optimize complex electrical dispatch (UC-LD) problems considering all constraints. Generally speaking, this paper proposes a method able to tackle complex optimization problems with many constraints of varied difficulty. In fact, it proposes a new constraint handling procedure where parameters of the repair method itself are optimized. Consequently, this allows (a) the best repair method to be found for the given problem and (b) to have the bias removed. One of the greatest advantages of the proposed method is that it can be easily applied to other related problems or different settings. Guidelines were built to aid in this task.

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Fig. 7. Plot of the maximum and minimum output that the best solution is able to produce together with the desired maximum/minimum output defined by the spinning reserve constraint.
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