Moving Horizon Estimation for Vehicle Robots using Partial Marker Information of Motion Capture System

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Abstract. The measurement using a motion capture camera is fluctuated by white noise and outliers. In addition, markers to be measured are frequently hidden from cameras by occlusion, then the position and heading angle of a vehicle cannot be uniquely determined because of failure to detect sufficient number of markers. Thus, robust estimation method is required which suppresses the influence of the white noise, the outlier and the occlusion. In this study, we introduce Moving Horizon Estimation (MHE) using partial marker information of motion capture system. It optimizes the objective function using both the marker information in the evaluation range and the constraints on the robot dynamics. By virtue of introduction of constraints, even if the cameras fail to measure the actual state of the robot, the estimated value is determined by MHE. It is the difference from our previous research which assumed that sufficient number of markers are available. In this paper, we estimate the position of the vehicle robot by MHE using the information of the measured markers on the robot, even if several markers are hidden. We will prove the effectiveness of the proposed method by comparing MHE with EKF.

1. Introduction
In recent years, the visual feedback control has been intensively studied [1]. Especially, the static cameras set around the robot realize high accuracy measurement of position without any contact. However, the occlusion occurs when the robot is hidden by obstacles from cameras or enters in blind space [2] [3]. They cause the lack of measurement which strongly deteriorates the performance of visual feedback system. In addition, the measurement is contaminated with steady measurement noise.

In this study, we use a motion capture system which measures several markers on the robots with fast and/or complex motion at high sampling rate. Multiple cameras are utilized for measuring objects in various fields. For example, it is used for visual feedback control of autonomous mobile objects [4][5], while misrecognition of markers causes significant decrease in measurement accuracy on the motion capture. It is necessary for state estimation to suppress the influence of white noise and fill in gaps of missing data in order to resolve these issues.

In order to estimate state with high accuracy, the state estimation based on the model of the robot is effective. State observer, Particle Filter [6] and Extended Kalman Filter (EKF) [7] [8]...
Hiromachi et al. applied an EKF with variable covariance for localization using laser range sensors [10]. Kimura et al. proposed a state estimation method based on Moving Horizon Estimation (MHE) under intermittent lacking of measurement [11]. For vision sensors, many studies have been conducted where EKF is generally used method for interpolating data [12] [13]. Our previous research showed the effectiveness of state estimation based on MHE under outlier or occlusion on motion capture system, where it utilizes the measurement of gravity center of a group of markers called Rigid Body through tracking software [14]. It was shown that MHE achieved comparable accuracy with EKF under white noise of measurement, while MHE suppressed the influence of outlier or occlusion by imposing constraints. On the other hand, it is impossible to measure the Rigid Body when cameras fail to capture the necessary number of markers. However, part of marker’s positions are sometimes possible to be measured as Fig. 1. Hence, we can obtain reliable estimated value using partially measured markers.

Our purpose is that we estimate position and orientation of the robot directly using measured markers. In this paper, the position and heading angle of the vehicle robot are estimated by MHE using motion capture system. EKF can estimate state within steady white noise using data on infinite time, though it is difficult to suppress the influence of outlier. On the other hand, state is estimated by MHE for optimizing performance index which has evaluation range from current time to finite past time. The performance index evaluates the error of estimation, output calculation and initial estimation. In addition, MHE can impose constraints on the variation of estimated values. Thus, MHE is expected to estimate state which suppresses the influence of these issues.

In our previous research, even if there are other markers measured, we did not get the state of the Rigid Body when it is occluded. In this study, we incorporate all available data of marker into MHE so that they serve to improve the estimation of vehicle state. The measured value is the markers position on the robot in contrast with the previous study. It is necessary to match markers data to interrelate the markers on the robots between each sampling time, but the challenge is to suppress the misrecognition. By virtue of MHE, the estimated state is constrained to a possible range which works against the decrease of detected markers and avoids misrecognition of markers.

2. Vehicle dynamics and state equation, output equation

2.1. Vehicle dynamics and state equation

Figure 2 depicts a single track model of a front steering vehicle. The position and the heading angle are center point between rear wheels. Then, the position is $x$ and $y$, the heading angle is $\theta$, the steering angle is $\delta$, the velocity is $V$ and the wheelbase is $L$. The kinematic model of the vehicle is

$$\frac{dx}{dt} = V \cos \theta, \quad \frac{dy}{dt} = V \sin \theta, \quad \frac{d\theta}{dt} = \frac{V}{L} \tan \delta.$$  (1)
or, \( \dot{x}(t) = f_c(x(t), u(t)) \), where \( x = [x, y, \theta]^T \) is the state, \( u = [V, \delta]^T \) is the input and \( f_c \) is defined by (2):

\[
f_c(x, u) = \begin{bmatrix} V \cos \theta & V \sin \theta & \frac{V}{L} \tan \delta \end{bmatrix}^T.
\]

2.2. Output equation

We consider a system whose output is (3) comprised of position of three markers:

\[
y = h(x) = \begin{bmatrix} M_1^T & M_2^T & M_3^T \end{bmatrix}^T,
\]

where \( M_1, M_2 \) and \( M_3 \) are position vectors of each marker. Then, \( M_i(i = 1, 2, 3) \) is

\[
M_i = \begin{bmatrix} m_{ix} \\ m_{iy} \end{bmatrix} = \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} m_{ix} \\ m_{iy} \end{bmatrix} + \begin{bmatrix} x \\ y \end{bmatrix},
\]

where \( M_{ix} \) is position of \( x \) axis and \( M_{iy} \) is position of \( y \) axis. The relation between state and output is depicted in Fig. 3.

3. Moving Horizon Estimation [15] [16]

3.1. State and output equation on discrete-time system

The state and the output equation are transformed into a discrete time system with a sampling cycle \( \Delta t \) as follows:

\[
x[k + 1] = f(x[k], u[k]) + Gv[k],
\]

\[
y[k] = h(x[k]) + w[k].
\]

\( v[k] \) is system noise and \( w[k] \) is measurement noise. They are white noise of mean value 0. \( f_c \) is the continuous-time system as follows:

\[
f_c(x[k], u[k]) = x[k] + f_c(x[k], u[k]) \Delta t.
\]

3.2. Extended linearization for nonlinear model

To apply EKF or MHE, we apply extended linearization for \( f \) and \( h \) as follows:

\[
f(x[k], u[k]) \approx f(x[k-1], u[k]) + A(x[k] - x[k-1], u[k]),
\]

\[
h(x[k]) \approx h(x[k-1]) + C(x[k] - x[k-1]),
\]
where the matrices $A$ and $C$ are

$$A = \frac{\partial f}{\partial x}(x[k-1], u[k]), \quad (10)$$

$$C = \frac{\partial h}{\partial x}(x[k-1]). \quad (11)$$

### 3.3. Moving Horizon Estimation

Moving Horizon Estimation (MHE) is a method for estimation which utilizes the data from finite time past through current time to optimize the performance index within Estimation Window which shifts with time as it flows (Fig. 4). State of $x_0$ through $x_T$ are estimated from a certain time $k = 0$ to current time $k = T$ in Estimation Window. Then, estimated value at $T$ is the state at current time. Input $u$ and output $y$ are known value at Estimation Window. Hereafter, suffix $p$ is the estimated state of last sampling time. The performance index is (12).

$$J = \sum_{k=0}^{T-1} \{(\hat{x}[k+1] - f(\hat{x}[k], u[k]))^T S_Q(\hat{x}[k+1] - f(\hat{x}[k], u[k])) + (y[k] - h(\hat{x}[k]))^T S_R[k](y[k] - h(\hat{x}[k])) + (\hat{x}[0] - x_p[1])^T S_P(\hat{x}[0] - x_p[1])\} \quad (12)$$

$S_Q, S_R$ and $S_P$ are weight matrices. In this study, we use non-liner model to estimate $\hat{x}$. However, it is not easy to solve non-liner optimization problem on real time. Thus, it is necessary to extend linearization of the non-liner model. The performance index $J$ for extended linearized system is equation as follows:

$$J = \sum_{k=0}^{T-1} (\hat{x}Q^T S_Q e_Q) + \sum_{k=0}^{T} (\hat{x}R^T S_R[k] e_R) + e_P^T S_P e_P, \quad (13)$$

where $e_Q, e_R$ and $e_P$ are:

$$e_Q = \hat{x}[k+1] - f(\hat{x}_p[k+1], u[k]) + A(\hat{x}[k] - \hat{x}_p[k+1]), \quad (14)$$

$$e_R = y[k] - h(\hat{x}[k]) \quad (15)$$

$$e_P = \hat{x}[0] - \hat{x}_p[1]. \quad (16)$$

$e_Q$ (Fig. 5) represents the error of the dynamics. $e_R$ (Fig. 6) represents the error of the output calculation. $\hat{x}_p[T + 1]$ is computed by the model using $\hat{x}_p[T]$ because the $\hat{x}_p[T + 1]$ is not estimated at $k = T$. $e_P$ (Fig. 7) represents the error of the initial estimation.

### 3.4. Constraints for state estimation

MHE can include constrains to protect the estimated state against the influence of the outlier and occlusion. In this study, the imposed constrains are the relation of range for marker selection and estimated value as follows:

$$||D^{-1}(\hat{x}[T] - \hat{x}_p[T + 1])||_\infty \leq 1.0. \quad (17)$$

The matrix $D$ regulates the size of constrains range.

### 3.5. Weight matrices of MHE

The estimation suffers from significant error when the measurement fails or includes outlier. While the markers are lost, the elements related with them in $S_R[k]$ is set to be 0. It is possible for state estimation on MHE to suppress the influence of occlusion.
4. Measurement systems

4.1. Measurement system

The relation between the cameras, Host PC and the robot is depicted in Fig. 8. The vehicle robot is Robo Car made by ZMP where its scale is one-tenth of actual vehicle. The image processing software tracks the markers set on the robot through the static cameras to measure marker position. Figure 9 depicts the markers set on the robot. The gravity point of markers is center point between rear wheels.

4.2. Static camera and Tracking software

We use the static camera, Prime 17W manufactured by OptiTrack, whose specification is explained in [14]. We obtain the measurement through the tracking software called Motive:Tracker measures 6 DOF of robots position and orientation using markers. This software constructs Rigid Body which is comprised of multiple markers assuming a kinematic geometry. We can obtain the measurement of position x, y and z on each markers. The detected marker includes markers outside Rigid Body. In order to create a Rigid Body, it is necessary to detect 3 – 7 pieces of markers. If sufficient number of marker is not detected, the state of the object cannot be obtained, even though there are one or two markers are measured.

4.3. Estimation based on direct use of markers.

In this study, we obtain the position and heading angle of the vehicle robot by MHE. We use detected marker information and incorporate it into MHE to integrate model based estimation. It is noted that even if the number of detected markers are small, we can reconstruct the vehicle state estimation which is improved by the detected marker information.

4.4. Selection of measurement

To deal with markers for estimation, it is important to relate each markers with the ones detected at the previous sampling time. The process is outlined below and depicted in Fig. 10 and 11.
Figure 8. Experimental system

Figure 9. Three markers are set on the vehicle robot.

Figure 10. The image of marker selection. Data is the measurement, est.i is estimation value and sel.i is selection marker (i = 1, 2, 3).

(i) The output at current time is predicted from $x_p[k + 1]$ and output equation. The possible range of each marker position is set as a prediction window.

(ii) The detected markers inside the prediction window are compared with the predicted output.

(iii) The marker closest to the center of the prediction window is selected as an estimated marker. If the marker is not found inside the window, the measurement at current time holds on data at last time.

5. Moving Horizon Estimation with marker data

5.1. Conditions of the experiment

The robot runs steady circle which is sometimes occluded from three cameras. Figure 12 depicts the trajectories of detected markers, where the target velocity of the vehicle is set to be 300 mm/s and its steering angle is 15 deg. The constraints for the estimation is described as transition within ±2.5 cm for $x$ and $y$, and ±0.09 rad for $\theta$; then the range $D$ in condition (17) is given by

$$ D = \text{diag}(0.025, 0.025, 0.09) $$

The result on MHE is compared with the one on EKF. For EKF, the data during occlusion is replaced by the predicted state computed by the dynamics.
5.2. The estimation result and consideration

5.2.1. The estimation result

Measured markers within Rigid Body are depicted in Fig. 13 through 15. Some detected markers are lost; they cannot be identified as markers composing the Rigid Body. Since the markers are not properly detected, the marker coordinates are kept constant while they are lost from sight. Fig. 16, 18 and 20 depict the measured positions of each marker together with the ones estimated by EKF. Three markers are not properly tracked by EKF for most time. It clearly shows that it is difficult for selecting markers to consider the actual measurement. On the other hand, they are properly tracked by MHE for most time as depicted in Fig. 17, 19 and 21. The measurement using MHE (Fig. 17, 19 and 21) is much more suited than EKF for the actual measurement. As explained in section 4.4, this marker selection is conducted based on the estimated value at last sampling time. The estimation result on EKF is shown in Fig. 22, 24, 26 and 28. The estimated result on MHE is depicted in Fig. 23, 25, 27 and 29. In Fig. 22 through 25, the blue line is estimated value and the green line is the gravity center of the measured three markers. Figure 26 and 27 depict the trajectories of estimated state together with measured markers. In Fig. 26, the estimated state should go through the points between the markers, but it deviates from the center of markers. On the other hand, the estimated state of MHE successfully go through the markers; the influence of occlusion is suppressed. Hence, we proved that the state can be estimated by MHE which utilizes previous value within Estimation Window and suppresses the influence of occlusion and outlier.

5.2.2. The consideration of MHE with marker data

A more detailed explanation of the difference between result on EKF and it on MHE are given later in this section. The measurement and the estimated value using EKF or MHE are depicted in Fig. 30 through 33. These figures show the marker position, the gravity center of markers and the corresponding predicted windows of five successive samplings. In these figures, the “data m1” through “data m3” are corresponding marker set on the vehicle robot depicted in Fig. 9. Figure 30 and 32 show that the marker position or gravity center is far different from the predicted value due to the misrecognition of markers. The estimation based on EKF is inappropriate because the appropriate marker was not selected. On the other hand, for MHE, the estimated value almostly corresponds with the gravity center of markers (Fig. 31). Moreover, Fig. 33 shows that the predicted output is very near to the measured marker in predicted window. In order to confirm that EKF does not work properly, the estimated state based on MHE is replaced the previous estimation of EKF during missing data. In this case, EKF successfully tracked the appropriate markers. Figure 36 shows the result of estimation and measurement during 14.90 s through 15.50 s. The gap between the predicted output and measurement are reduced using estimated value from MHE to suppress the influence of occlusion at 14.95 s and 15.31 s. Due to the model mismatch, EKF loses effectiveness of the estimation over time.

Figure 12. The $x, y$ plane of measured points.
Though EKF can achieve high accuracy as time advances using data on infinite time, the state estimated by EKF is sometimes strongly disturbed by unexpected error of measurement and model mismatch; the prediction window may fail to capture the markers. EKF is known to be effective under white noise without model mismatch [15]. However, the model uncertainty may cause deviation of predicted windows as depicted in Fig. 32, which renders EKF hard to track the markers. MHE can obtain the optimized state considering the measurement and estimated value at each sampling time inside the prediction window. The state can be estimated close to the measurement, possibly because MHE evaluates the past few sampling data to optimize the index function which reflects the recent trend of the motion of the vehicle.

Thus, EKF can provide optimal estimation under white noise, but the model uncertainty may cause large deviation. On the other hand, state estimation based on MHE achieves high accuracy estimation comparing with the method based on EKF.

6. Conclusion
In this study, we proposed the estimation on MHE of the robot’s state directly using each marker set on the robot. In this study, the position and heading angle of the vehicle robot is estimated based on MHE through the motion capture system. We proved that an estimation method on MHE can suppress the influence of occlusion and track the moving markers. It shows effectiveness comparing the result with the estimation by EKF. Therefore, the estimated value by MHE achieves higher accuracy than EKF. In the future, we will verify that this estimation is effective to apply visual feedback control.

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Figure 13. The marker 1 of markers set on the robot.

Figure 14. The marker 2 of markers set on the robot.

Figure 15. The marker 3 of markers set on the robot.

Figure 16. The selected marker 1 is related with marker set on the robot using data of Fig. 12 (EKF).

Figure 17. The selected marker 1 is related with marker set on the robot using data of Fig. 12 (MHE).
**Figure 18.** The selected marker 2 is related with marker set on the robot using data of Fig. 12 (EKF).

**Figure 19.** The selected marker 1 is related with marker set on the robot using data of Fig. 12 (MHE).

**Figure 20.** The selected marker 3 is related with marker set on the robot using data of Fig. 12 (EKF).

**Figure 21.** The selected marker 3 is related with marker set on the robot using data of Fig. 12 (MHE).

**Figure 22.** The estimation $x$ using EKF.

**Figure 23.** The estimation $x$ using MHE.

**Figure 24.** The estimation $y$ using EKF.

**Figure 25.** The estimation $y$ using MHE.
Figure 26. $x, y$ plane of estimation by EKF.

Figure 27. $x, y$ plane of estimation by MHE.

Figure 28. The estimation $\theta$ using EKF.

Figure 29. The estimation $\theta$ using MHE.

Figure 30. The vehicle robot position with markers (EKF).

Figure 31. The vehicle robot position with markers (MHE).
**Figure 32.** The predicted markers position (EKF).

**Figure 33.** The predicted markers position (MHE).

**Figure 34.** The position is estimated by EKF using estimated value by MHE.

**Figure 35.** The output is estimated by EKF using estimated value by MHE.

**Figure 36.** The result about measurement and estimation on marker 1.