Thermal convection in the crust of the dwarf planet (1) Ceres

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ABSTRACT
Ceres is the largest body in the Main Belt, and it is characterized by a large abundance of water ice in its interior. This feature is suggested by its relatively low bulk density (2162 kg m$^{-3}$), while its partial differentiation into a rocky core and icy crust is suggested by several geological and geochemical features: minerals and salts produced by aqueous alteration, icy patches on the surface, lobate morphology interpreted as surface flows. In this work we explore how the composition can influence the characteristics of thermal convection in the crust of Ceres. Our results suggest that the onset of thermal convection is difficult and when it occurs it is short lived and this could imply that Ceres preserved deep liquid until present, as recent suggested by the work of Castillo-Rogez et al.. Moreover, cryovolcanism could be driven by diapirism (chemical convection) rather than thermal convection.

Key words: minor planets, asteroids: individual: Ceres - planets and satellites: interiors - physical data and processes: convection

1 INTRODUCTION
The in-depth study of the surface of the dwarf planet Ceres based on the observations returned by the Dawn spacecraft over the past few years, has highlighted the extensive presence of minerals produced by aqueous alteration (e.g. VIR observations) (De Sanctis et al. 2016; Combe et al. 2016; Ciarriello et al. 2017; Raponi et al. 2018; Carrozzo et al. 2018; McSween Jr. et al. 2018; Combe et al. 2019), geomorphological evidence for ground ice (Schmidt et al. 2017), and an ice table via hydrogen detection (Prettyman et al. 2017) all indicating that a large quantity of water is present both on the surface and in the interior of this planetary object. Moreover, past studies, e.g. McCord & Sotin (2005); Castillo-Rogez & McCord (2010); McCord et al. (2011) investigated the role of this huge quantity of water in the thermodynamic evolution of Ceres.

The inversion of the gravity and topography data returned by Dawn (Park et al. 2016; Fu et al. 2017; Ermakov et al. 2017) suggests a two-layer structure for Ceres’ interior: an inner rocky layer with a density of $\approx$2400 kg m$^{-3}$ and a crust (40 km thick) with a density of $\approx$1300 kg m$^{-3}$. Crustal density estimates vary among studies depending on underlying assumptions: Konopliv et al. (2018) derived the range 1200-1600 kg m$^{-3}$, whereas King et al. (2018) estimated a crustal density lower than 1300 kg m$^{-3}$, consistent with Ermakov et al. (2017). All these estimates indicate a partially differentiated structure with abundant water ice in the crust. Based on mechanical strength observations, Bland et al. (2016) imposed further constraints on crustal composition, suggesting no more than 40 vol% of a weak phase (water ice and/or porosity).

Formisano et al. (2016) suggested that, in the case of a pure water ice mantle, the primordial crust would not survive until present (only until 3 Ga after formation), since it experienced gravitational overturn. On the other hand, a 1-km thick crust with a low volume fraction of ice above a muddy mantle could be characterized by a stability time span compatible with the lifetime of the Solar System. However, even if the muddy mantle scenario could be still considered a possibility, a 1-km thick crust seems to be unlikely given the recent Dawn observations.

The abundant water ice in Ceres raises the possibility that cryovolcanism could occur in Ceres’ crust. This paper investigates this question and tests if that process could be triggered by thermal convection. Cryovolcanism has been suggested as responsible for the formation of domes across Ceres’ surface (Sori et al. 2018) and in particular for Ahuna Mons, an isolated, 4-km high mountain located at 10$^\circ$S. This outstanding feature has been interpreted as a cryovolcanic extrusive edifice (Ruesch et al. 2016). Cryovolcanism could be linked to thermal convection that is a
function of material viscosity, and thus of temperature. The viscosity can change by several orders of magnitude during the thermal/geophysical evolution of a planetary body and this affects the longevity and the shape of the convective cells. Here, we will investigate, through a 2-D finite element method (FEM) numerical modelling, how the composition of Ceres’ crust can affect the onset of thermal convection, selecting two different composition on the basis of the constraints of the mean crustal density. The aim of this work is to evaluate the feasibility of the thermal convection in the current crust (or potentially also in the past - with the same temperature boundary conditions) given the temperature at the bottom set by geochemical considerations related to the possible presence of liquid water near the top of the mantle (Castillo-Rogez et al. 2019). Moreover, we will discuss the possibility to produce features such as Ahuna Mons with the stress induced by thermal convection. The lack of numerical modelling of thermal convection in Ceres’ crust, as well as the importance of the physical consequences of this mechanism in a dwarf planet as Ceres are the main motivations behind this work. Moreover, the prospect for thermal convection, or absence thereof, in Ceres bears important implications for the until present preservation of liquid in the dwarf planet (Castillo-Rogez et al. 2019), as also proposed by a recent thermophysical model (Neumann et al. 2020).

The paper is structured as follow: in Section II we present the governing equations used in this paper, with the initial configurations adopted and the viscosity calculation; in Section III we report simulation results, and finally, in Section IV, we report the conclusions. An Appendix with additional information is provided.

2 2-D MATHEMATICAL MODEL

2.1 Governing Equations

We applied a 2-D finite element method (FEM) model in order to solve the Navier-Stokes equations coupled with the heating equation, by using the software COMSOL Multiphysics. We solve the following system of equations:

\[
\rho \frac{\partial \tilde{u}}{\partial t} + \tilde{u} \cdot \nabla \tilde{u} = -\nabla p + \tilde{W} + \rho \tilde{g},
\]

\[
\rho (\nabla \cdot \tilde{u}) = 0
\]

\[
\tilde{W} = \eta(T) \left( \tilde{v} u + (\tilde{v} u)^T \right),
\]

where \(t\) is the time, \(\rho\) is the density, \(\tilde{u}\) the convective velocity, \(\eta(T)\) the dynamic viscosity, \(T\) the temperature, \(p\) the pressure, \(g\) the gravity acceleration and \(\tilde{W}\) is the stress.

Some approximations are used: I) we consider the density as a function of temperature only in the buoyancy term (Boussinesq approximation); II) incompressible fluid (i.e. \(\rho (\nabla \cdot u) = 0\)); III) no sink or source of new materials are included. However, we can neglect the momentum term due to the large value of the dynamic viscosity which reflects in a near infinity Prandtl number (see Eq.(A8) in the Appendix).

Navier-Stokes equations are coupled with the heating equation:

\[
\rho c_p \left[ \frac{\partial T}{\partial t} + \tilde{u} \cdot \nabla T \right] + K \nabla \cdot \tilde{V} T = 0,
\]

where \(c_p\) is the specific heat and \(K\) the thermal conductivity. We model the crust as a mixture of water ice, clathrate hydrate, rock (antigorite) and salt (hydrohalite) as in Castillo-Rogez et al. (2019), studying the thermal evolution as a function of the volumetric percentages of such materials and temperature gradient across the shell. The thermophysical parameters adopted in this study are reported in the Appendix. Our simulations solve the dimensionless version of the above equations. See the Appendix for the non-dimensionalization procedure.

2.2 Initial Configurations /Assumptions and Boundary Conditions

A 2-D Cartesian geometry is adopted (see the Appendix), taking advantage of the results obtained by Vangelov & Jarvis (1994), who showed that plane layer numerical models can be scaled to spherical geometry with no substantial errors. Moreover, the assumption of a 2-D Cartesian geometry is valid since the thickness of the crust is small compared to the radius of the body. We investigate two different configurations characterized by a particular composition, whose components are chosen according to the previously discussed observation-based studies. In Tab.1 we report the volumetric percentage of the materials we assume in our simulations, the mean density, the mean thermal conductivity, the mean dynamic viscosity and finally the initial Rayleigh number. As discussed in Castillo-Rogez et al. (2019), because the porosity is not well constrained, we neglect its contribution in these simulations. The simulations start with an initial temperature profile consisting in a perturbation of the linear profile, between the temperature at the base of the crust (275 K) and at the surface (170 K), expressed as a single Fourier mode (see, e.g. Solomatov & Barr (2006)). The initial temperature chosen at the base of the crust is suggested by the ocean composition evolution model from Castillo-Rogez et al. (2018); Castillo-Rogez et al. (2019), while the surface temperature corresponds to the radiation equilibrium temperature of Ceres (at equator) at its average distance from the Sun. The value of 275 K can be considered as an extreme upper bound, since in Castillo-Rogez et al. (2019) a temperature of 250 K is compatible with a narrow set of evolutionary scenarios. While the surface temperature remains fixed during the simulation, we imposed an heat flux at the bottom of the crust, whose estimation is given by Castillo-Rogez et al. (2019). This heat flux decreases from an initial value of about 16 mW m\(^{-2}\) (at the formation time - see Fig.1 of Castillo-Rogez et al. (2019)) to a current value of about 3 mW m\(^{-2}\). Thus, the bottom temperature is fixed initially and changes during the time evolution. Free slip conditions on the sides of our integration domain are imposed: this condition requires that the normal component of fluid velocity tends to zero at the wall-fluid interface while the tangential component is unrestricted.

2.3 Viscosity

The viscosity is the crucial parameter of this kind of simulations, since it affects the Rayleigh number and consequently the onset or not of the thermal convection as well as its evolution. There are several approaches to calculate
the average viscosity of multi-phase aggregates: volume fraction weighted arithmetic mean, harmonic mean, geometric mean, logarithmic mean. A good review regarding the way to calculate the bulk viscosity is reported in Neumann et al. (2020).

The general law adopted is a temperature-dependent viscosity, largely used in literature (e.g. Thomas et al. 1987; Sterenborg & Crowley 2013; Shoji & Kurita 2014; Rubin et al. 2014; Formisano et al. 2016):

$$ \eta = \eta_0 \exp \left[ A \left( \frac{T_{\text{melt}}}{T} - 1 \right) \right], $$

where $\eta_0$ is a reference viscosity at zero pressure melting point, $A$ is a constant which depends on the activation energy and $T_{\text{melt}}$ is the melting temperature. This law is similar to that used in Solomatov & Barr (2006), in which the "pre-exponential factor" is represented by $bT^{-n}$, with $b$ constant linked to the original Arrhenius viscosity (e.g. Solomatov & Moresi (2000)) and $\tau$ the second invariant of the stress tensor.

We start the discussion of the M2 case (see Tab.1), whose composition is dominated by clathrates and ices being salt and rock negligible so it is not a bad approximation to consider M2 as a two components mixture. Following Grindrod et al. (2008), the viscosity of this mixture can be calculated as:

$$ \eta_{\text{mix}} = \eta_{\text{cla}}^x \eta_{\text{cla}}^{1-x} + \eta_{\text{ice}}^x \eta_{\text{ice}}^{1-x}, $$

where $\eta_{\text{cla}}$ is the clathrate viscosity, $\eta_{\text{ice}}$ the ice viscosity, $x_{\text{cla}}$ the volumetric percentage of clathrate and $x_{\text{ice}}$ the volumetric percentage of ice. However, if we calculate the viscosity of the mixture as a log average as in Formisano et al. (2016):

$$ \eta_{\text{mix}} = \frac{\eta_{\text{cla}}^{\phi_{\text{cla}}} \eta_{\text{ice}}^{\phi_{\text{ice}}}}{f(x_{\text{rock}})}, $$

the result will be not very different from that obtained with the Eq.6.

The viscosity of clathrate is considered 20 times that of water ice (Durham et al. 2003), as done in Grindrod et al. (2008). In practice, the viscosity contrast between clathrate and ice is a function of the host species and varies with temperature (Durham et al. 2003, 2010). At a temperature of 250 K, the viscosity of methane clathrate may be two orders of magnitude greater than ice, whereas the viscosity of carbon dioxide clathrate is only about 5 times greater. Since mixed clathrates are expected to form in Ceres (Castillo-Rogez et al. 2018), the contrast of 20 assumed here is a reasonable assumption. The activation energy for ice used in our work is 56 kJ/mol, which corresponds to a constant $A = 25$ (Thomas et al. 1987; Friedson & Stevenson 1983) and similar to the activation energy (60 kJ/mol) used in Grindrod et al. (2008). For clathrate, the activation energy adopted ranges from 1.5 to 2.5 (Nagel et al. 2004) and can be fixed at 2.0 which is the median value of that range (Shoji & Kurita 2014). The Eq.10 is not valid if $x_{\text{rock}}$ is near $\phi_{\text{CPL}}$, but the error is not large as pointed by Rudman (1992).

This procedure leads to results very similar to those obtained using the approach of Friedson & Stevenson (1983) and Freeman (2006), i.e. to multiply the ice viscosity for a function ("relative viscosity") which depends on the rock volume percentage.

We would like to note that if we calculate the bulk viscosity in M1 case using the volume fraction weighted through an arithmetic mean, assuming $\eta_{\text{salt}} = 10^{12}$ Pa s (van Keken et al. 1993) and $\eta_{\text{rock}} = 10^{19}$ Pa s (Freeman 2006) we would obtain a viscosity very close to the rock viscosity: in this case the thermal convection will be not possible.

We carried out a simulation with $\beta = 1.5$ and $\mu_{\text{ice}} = 10^{12}$ Pa s.

3 RESULTS

In this section we report the results obtained by our numerical simulations; in particular, we show the temperature maps covering the whole thermal convection evolution and the 1-D temperature profile vs time at the center of the convective shell.

3.1 Model M1

We start the discussion with the model M1, similar to the one discussed in Castillo-Rogez et al. (2019): 30 vol.% ice, 40 vol.% clathrate, 20 vol.% salt and 10 vol.% rock. This composition is characterized by a mean density of 1400 kg
The numerical simulations presented in this work demonstrate that thermal convection is possible for a narrow set of thermophysical parameters. In particular, the onset of thermal convection requires a high temperature (> 250 K) at the base of the crust. As pointed out by Castillo-Rogez et al. (2019), such temperatures would require a crustal abundance of clathrate hydrates > 55 vol.%, consistent with the geological constraints of Pu et al. (2017) and Bland et al. (2016) but hard to reconcile with Ceres’ impacting history since clathrate hydrates, formed during the freezing of the ocean, destabilize upon impacting. In a crust with a composition similar to those explored in Castillo-Rogez et al. (2019) (30 vol.% ice, 40 vol.% clathrate, 20 vol.% salt and 10 vol.% rock) thermal convection is not very vigorous, involving only the 50% of the crust. Reducing the volumetric percentage of the clathrate and increasing that of the rock, convection is not possible. Departing from the literature physical constraints on the crust composition, and considering clathrate (50 vol.%) and ice (40 vol.%) the most predominant components (salt and rock the remaining 10 vol.%), thermal convection can take place.

In case of ice grain size in the range 0.1 - 0.1 mm, the thermal convection is very vigorous (Rayleigh number up to $10^7$) involving about the 50% of the crust. In case of grain size of 1 mm, small plumes arises at the basis of the
Figure 1. Case M1: temperature maps from 10 Myr to 450 Myr at different times. Top colour scale refers to the convective velocity (m s$^{-1}$), while bottom colour scale refers to the temperature (K). Last panel (bottom right) shows the 1-D temperature profile at the center of the cell, at given times: it is a vertical profile, where 0 represents the bottom of the crust while 1 the surface. In the x-axis we report the temperature (K).

crust so the thermal convection is very weak. However, this composition (M2) has a density less than the mean crustal density obtained by e.g. Ermakov et al. (2017); King et al. (2018); Konopliv et al. (2018).

Another supporting argument that thermal convection is unlikely in Ceres’ crust is provided by the experiments of Qi et al. (2018). These authors demonstrated that just a few percent of strong particles are able to impede grain boundary sliding and we expect at least 30 vol.% of strong particles (salt and silicates) in Ceres’ crust. It is interesting to calculate the thermal stress induced by thermal convection in order to evaluate if some of the domes observed on Ceres’ surface are the product of convection. By applying a simple formula to quantify the maximum convective stress Pappalardo & Barr (2004):

\[
p \approx 0.1 \rho g \alpha \Delta T D,
\]

where \( \rho \) is the density of the outer shell, \( D \) its thickness, \( g \) the gravity, \( \alpha \) the coefficient of thermal expansion and \( \Delta T \) the gradient across the shell, we calculate that the maximum uplift of the crust, in the most favorable case, is less than 50 m. We would like to note that if the crust were 100-km thick (as suggested by past theoretical models) the maximum uplift would be < 100 m (Formisano et al. 2018).
However, this estimation (even small) can be considered an upper limit. In fact Solomatov & Barr (2006) replaced $D$ with $\delta$ (the thickness of the boundary layer) and treated the crust as elastic, reducing in this way the amplitude of surface topography. This suggests that thermal convection can not produce sufficient driving pressures to produce domes like Ahuna Mons (4-km high) (Ruesch et al. 2016). On the other hand, diapirism Nimmo & Manga (2002); Pappalardo & Barr (2004) could produce structure like Ahuna Mons. The possibility that in Ceres a compositional density difference produced a diapir was discussed for the first time by McCord & Sotin (2005). They proposed that deep materials could reach the surface if the crust was weakened by impacts or faulting, Shoji & Kurita (2014) predicted overturn by diapirism over Ceres’ history but only near its equator while Neveu & Desch (2015) suggested a complete overturn of the crust linked to compositional diapirism from a subsurface ocean in case Ceres formed in 4-7 Myr from the Calcium - Aluminum - Inclusions (CAIs) formation. The fact that thermal convection is difficult to establish in Ceres’ crust could support the idea of preservation of liquid water until present, as suggested by Castillo-Rogez et al. (2019) and recently supported also by a numerical model (Neumann et al. 2020). However, if locally the composition was dominated by
Ceres’ crust convection

Figure 3. Case M2B ($\mu_{i,\text{ice}} = 10^{13}$ Pa s): temperature maps from 10 Myr to 450 Myr at different times. Top colour scale refers to the convective velocity (m s$^{-1}$), while bottom colour scale refers to the temperature (K). Last panel (bottom right) shows the 1-D temperature profile at the center of the cell, at given times: it is a vertical profile, where 0 represents the bottom of the crust while 1 the surface. In the x-axis we report the temperature (K).

ice and clathrate, as also recently proposed by the work of Sori et al. (2020), thermal convection would be possible.

For the presence of water ice and/or liquid Ceres represents an interesting astrobiological target to be explored by future space missions.

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Figure 4. Case M2C ($\mu_{ice} = 10^{14}$ Pa s): temperature maps from 10 Myr to 450 Myr at different times. Top colour scale refers to the convective velocity (m s$^{-1}$), while bottom colour scale refers to the temperature (K). Last panel (bottom right) shows the 1-D temperature profile at the center of the cell, at given times: it is a vertical profile, where 0 represents the bottom of the crust while 1 the surface. In the x-axis we report the temperature (K).

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Solomatov V., Barr A., 2006, Physics of the Earth and Planetary...
We start from the Navier-Stokes equations. We divide by $A_1$ reducing Navier-Stokes equations to

\[ \frac{\partial u}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \frac{1}{\rho_0} \nabla \cdot \left[ \eta(T) \nabla \mathbf{u} + \eta(T) (\nabla \mathbf{u})^T \right] - \frac{\nabla \cdot p I}{\rho_0} - g \left[ 1 - \alpha (T - T_s) \right]. \]  

(A1)

Note that $(\nabla \mathbf{u})^T$ does not contribute to the momentum equations because $\nabla \cdot (\nabla \mathbf{u})^T = 0$ for incompressible fluid, so:

\[ \frac{\partial u}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \frac{\nabla \eta(T) \nabla \mathbf{u}}{\rho_0} + \frac{\eta(T) \nabla^2 \mathbf{u}}{\rho_0} - \frac{\nabla \cdot p I}{\rho_0} - g \left[ 1 - \alpha (T - T_s) \right]. \]  

(A2)

where $\eta(T) = \eta_0 \exp \left( \frac{1 + T}{\epsilon} \right)$ and $\epsilon = RT_b/E$. Substituting the expression for $\eta(T)$, introducing $\nu_0 = \eta_0/\rho_0$, and calling for simplicity $V(T) = \exp \left( \frac{T + T_T}{T_T} \right)$, we can rewrite the above equation as:

\[ \frac{\partial u}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = \nu_0 \nabla \cdot \nabla^2 \mathbf{u} - \frac{\nabla \cdot p I}{\rho_0} - g \left[ 1 - \alpha (T - T_s) \right] \]  

(A3)

\[ \alpha = \frac{L^2}{\nu_0} \]  

We use the following non-dimensional factors, as in Khaleque et al. (2015); Fowler et al. (2016):

\[ u = \frac{k}{L^2} u^*; \quad T = T_b T^*; \quad \nabla = \frac{1}{L} \nabla^*; \quad t = \frac{L^2}{k^*} t^*. \]  

(A4)

We can rewrite the Navier-Stokes equation as follows:

\[ \frac{k}{\nu_0} \left( \frac{\partial u^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla \mathbf{u}^* \right) + \frac{\nabla p}{\rho_0} = \frac{\eta_0 k}{L^3} \left[ \nabla^* \cdot \nabla^* \mathbf{u}^* + V(T) \nabla^* \mathbf{u}^* \right] - g + \frac{g \alpha}{T_b} \left( T_b T^* - T_b \right). \]  

(A5)

following Khaleque et al. (2015); Fowler et al. (2016), we define the pressure as:

\[ p = \rho_0 g L (1 - z^*) + \frac{\eta_0 k}{L^3} \nu^*. \]  

(A6)

and multiply all the terms for $\frac{k}{\nu_0}$, we obtain:

\[ \frac{k}{\nu_0} \left( \frac{\partial u^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla \mathbf{u}^* \right) + \frac{\nabla p}{\rho_0} = \frac{\eta_0 k}{L^3} \left[ \nabla^* \cdot \nabla^* \mathbf{u}^* + V(T) \nabla^* \mathbf{u}^* \right] + \frac{g \alpha L^3}{\nu_0} \left( T_b T^* - T_b \right). \]  

(A7)

If we introduce the Prandtl and Rayleigh numbers, we finally obtain:

\[ \frac{1}{Pr} \left( \frac{\partial u^*}{\partial t^*} + \mathbf{u}^* \cdot \nabla \mathbf{u}^* \right) \nu^* = \left[ \nabla^* \cdot \nabla^* \mathbf{u}^* + V(T) \nabla^* \mathbf{u}^* \right] - Ra \left( 1 - T^* \right). \]  

(A8)

A2 Reducing heat equation to non-dimensional

We divide by $\rho c_p$ the heat transfer equation:

\[ \frac{\partial T}{\partial t} + u \cdot \nabla T + k \nabla^2 T = 0, \]  

(A9)

where $k = \frac{k}{\rho c_p}$ is the thermal diffusivity. We use the following non-dimensional parameters:

\[ \nabla = \frac{1}{L} \nabla^*; \quad T = T_b T^*; \quad t = \frac{L^2}{k^*} t^*. \]  

(A10)

We can rewrite the heat equation in the following way:

\[ \frac{\kappa T_b}{L^2} \frac{\partial T^*}{\partial t^*} + \frac{\kappa T_b}{L^2} \nabla^* T^* + \frac{\kappa T_b}{L^2} \nabla^2 T^* = 0. \]  

(A11)

Simplifying the common terms we obtain the dimensionless form of the heat equation:

\[ \frac{\partial T^*}{\partial t^*} + \nu^* \nabla^* T^* + \nabla^2 T^* = 0. \]  

(A12)

APPENDIX B: NUMERICAL GRID

The mesh is automatically chosen by the software COMSOL Multiphysics, based on the physical equations involved. It consists (see Fig.B1) in 10120 elements (9762 triangles and 358 quads).

APPENDIX C: MATERIAL PROPERTIES

In Tab.C1 we report the thermophysical material properties of the materials we have used for our numerical simulations.
Fixed temperature at the surface = 170 K

Figure B1. Geometry adopted for our numerical simulations. Fixed temperature (170 K) is imposed on the top while thermal insulation on the lateral sides. An heat flux is imposed in the interface mantle-crust. Free slip conditions are imposed on the lateral sides.

Table C1. Thermophysical material properties assumed in this study, in particular the density ($\rho$), the thermal conductivity ($K$), the specific heat ($c_p$) and the thermal expansion coefficient ($\alpha$). References: [1] Grindrod et al. (2008); [2] Grimm & Mcsween (1989); [3] Waite et al. (2007); [4] Schofield et al. (2014); [5] Shoji & Kurita (2014). * refers to the mean value over the range 170-275 K calculated using the temperature-dependent relations given in the specific references.

| Material          | $\rho$ [kg m$^{-3}$] | $K$ [W m$^{-1}$ K$^{-1}$] | $c_p$ [J kg$^{-1}$ K$^{-1}$] | $\alpha$ [K$^{-1}$] |
|-------------------|----------------------|---------------------------|----------------------------|---------------------|
| Ice               | 950 [1]              | 2.7 [$^{2*}$]             | 1750 [$^{2*}$]             | $10^{-4}$ [$^{5*}$] |
| Clathrate hydrate | 1000 [3]             | 0.64 [$^{3*}$]            | 1850 [$^{3*}$]             | $10^{-7}$ [$^{1*}$] |
| Antigorite (“rock”) | 2750 [1]           | 1.5 [$^{1*}$]             | 2000 [$^{3*}$]             | $10^{-7}$ [$^{1*}$] |
| Salt (hydrosalite) | 2200 [4]             | 0.60 [$^{4*}$]            | 920 [$^{4*}$]              | –                   |

APPENDIX D: RAYLEIGH NUMBER VS TIME EVOLUTION

Here we report the plot of the Rayleigh number evolution with time. The critical Rayleigh number is calculated according to Solomatov & Barr (2006) and marks the onset of the thermal convection. It depends on the geometry and physics of the problem under investigation. An appropriate definition, however, is reported in Stevenson et al. (1983): "...should more correctly be thought of as an empirical parameter chosen to be consistent with numerical and laboratory experiments". In Fig. D1 we report the Rayleigh number evolution with time for the models under investigation. The intersection with the critical value (black horizontal dashed line) gives an estimation of the timescale of the thermal convection.

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