de Sitter Thermodynamics: A glimpse into non equilibrium

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In this article is shown that the thermodynamical evolution of a Schwarzschild de Sitter space is the evaporation of its black hole. The result is extended in higher dimensions to Lovelock theories of gravity with a single positive cosmological constant.

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I. INTRODUCTION

The study of the thermodynamics of black holes has been so far only the windows at hand into the realms of quantum gravity. One of the most remarkable results obtained was to prove that the boundary conditions define the ensemble in which the black hole is described. Unfortunately, there are not general boundary conditions that one can identify with any particular ensemble, and this must be done case by case. Furthermore, there can be more than a single set of boundary conditions that yield any ensemble. For instance in [2] was shown that the boundary conditions which define the canonical ensemble with null and negative cosmological constants are not related at all.

In [2] the case of positive cosmological constant was excluded and the present article aims to amend that in part. The simplest case with positive cosmological constant is the de Sitter space which is the maximally symmetric manifold with positive curvature. It has a horizon which is an observer depending feature. This is associated with the fact its Euclidean version is a sphere. Moreover its finite volume has led to conjecture that it could have a finite number of quantum states.

However, since maximally symmetric spaces usually are stable, if not plainly ground states, the presence of a horizon in de Sitter space may be considered in conflict with the idea that a horizon emits Hawking radiation in an underlaying decaying process. The same discussion for negative cosmological constants is solved by the presence, or not, of Killing spinors because of their connection with the definition of a BPS state [4]. For a positive cosmological constant such a connection can not be established, although Killing spinors indeed exist, because the de Sitter group does not have a supersymmetric extension (see for instance [5]). Therefore, in order to understand the role of the de Sitter as a ground state can be useful to study the thermodynamics of black holes with positive cosmological constant. Even studying the simplest case, the Schwarzschild-dS solution, gives a lot of useful information. There are several other geometries with positive cosmological constant whose thermodynamics can be relevant, for a discussion see [6], unfortunately most of them present naked singularities.

From the start the thermodynamics of black holes with positive cosmological constant presents some novelties. One usually deals with a single horizon where somehow to fix a single temperature. For a positive cosmological constant, in even for Schwarzschild-dS solution, the space where the observers inhabit is located between two horizons, and at both a temperature can be defined [7, 8].

In principle the presence of those two horizons with their own temperature defines a non-equilibrium system, which should evolve. One very interesting feature of the Schwarzschild-dS black hole geometry is that its black hole horizon can be understood as made of degrees of freedom borrowed from the cosmological horizon [9]. This is in complete agreement with the fact even if one adds cosmological horizon and the black hole horizon entropies still the result is smaller than the entropy of de Sitter space, defined as usual as proportional to the respective areas. Remarkably, with only this in mind one can predict, using the usual rule that systems evolve into larger entropy configurations, that Schwarzschild-dS space should evolve into de Sitter space. In this paper that evolution is discussed on some general grounds. The final result of this article is that the quasi statical thermodynamical evolution determines the complete evaporation of the black hole of the Schwarzschild-dS solution, leaving behind, in principle, a de Sitter space.

In this article the positive cosmological constant will be considered fixed, though it is known that even the cosmological constant can evolve [10].

Thermodynamics reviewed

Before to proceed to the next sections is worth to recall some notions of black hole thermodynamics.

Thermodynamics has two fundamental laws which are satisfied by every known physical system, therefore one should expect that they be satisfied by black holes as well. Above all stands the conservation of energy, known as the first law of thermodynamics,

\[ dE = dQ + \sum_i \mu_i dJ^i, \]

where \( dQ \) stands for the differential of heat, \( J^i \) are some extensive charges, as angular momenta, and \( \mu_i \) their associated extensive potentials (For gravity see for instance [11]). This law can be even used to recover the gravitational equations for black holes, see for instance [12].
The other fundamental law is the so called second law of thermodynamics
\[ \sum_a dS_a \geq 0, \tag{2} \]
which states that in the evolution of a composed system the total change of entropy is always positive or null.

The suitability of the other two laws of thermodynamics in black hole physics is not so clear. The zero law, which states that two systems in contact must reach thermal equilibrium, needs at least that the heat capacities be positive. This can fail in gravity (for instance it fails in Newtonian gravity). Finally, the third law of thermodynamics also represents an open question, since to step in Newtonian gravity. Finally, the third law of thermodynamics is not so clear. The zero law, which states that in the evolution of a composed system...

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and the equations $T^a = de^a + \omega^a{}_b e^b = 0$ which define a torsion free condition. When the torsion free condition is replaced in Eqs. (3) they becomes the standard Einstein equations with a positive cosmological constant.

As usually the variation of the action also yields a boundary term. This term is given by

$$\Theta(e^a, \delta \omega^{ab}) = (\delta \omega^{ab} e^c \ldots e^d) \varepsilon_{abc \ldots c_d}, \quad (7)$$

and represents the first step to fix the boundary conditions.

For a null or negative cosmological constant $\Theta$ usually diverges, however in this case, since the space is bounded, this becomes a light like vector. In Schwarschild-dS $\xi = \partial_t$. The key relation to obtain the temperature of those horizons [16]. To analyze the evolution of these charges one can use the approach developed in [19] in terms of the so called presymplectic form

$$\Xi(\phi, \delta_1 \phi, \delta_2 \phi) = \int \delta_1 \Theta(\phi, \delta_2 \phi) - \delta_2 \Theta(\phi, \delta_1 \phi), \quad (11)$$

where $\delta_1$ and $\delta_2$ correspond to functional variations of the fields. The presymplectic form defines the structure of the space of configurations, $\mathcal{F}$. One must stress that if either $\delta_1$ or $\delta_2$ are symmetries then $\Xi$ vanishes [19].

Obviously the space of classical solutions, denoted $\mathcal{F}$, is a subspace of $\mathcal{F}$. Precisely, the evolution of the Noether charges in $\mathcal{F}$ defines the thermodynamics [20]. To study their evolution is necessary to introduce a variation along the parameter of the solutions denoted $\delta$. This yields

$$\delta \delta \cdot J_X = \delta \Theta(-\mathcal{L}_X \phi, \phi) + I_X \Theta(\delta \phi, \phi), \quad (12)$$

One can notice that the left hand side, upon integration, is the presymplectic form $\Xi(\phi, \delta \phi, -\mathcal{L}_X \phi)$, therefore

$$\Xi(\phi, \delta \phi, -\mathcal{L}_X \phi) = \int \delta \delta \cdot J_X + dI_X \Theta(\delta \phi, \phi), \quad (13)$$

which must vanish since $\delta \chi = -\mathcal{L}_X \chi$ is a symmetry. The thermodynamical relations arise from this expression evaluated on $\xi$. The right side of Eq. (13) for $\xi$ is given by,

$$\Xi(\phi, \delta \phi, -\mathcal{L}_X \phi) = 0 = \kappa_1 \delta \int_{\partial \Sigma} I_\xi(\omega^{ab}) e^c \ldots e^d \varepsilon_{abc \ldots c_d},$$

where the surface is $\partial \Sigma = \partial \Sigma_+ \oplus \partial \Sigma_{++}$. Therefore, the expression turns out to be

$$\Xi(\phi, \delta \phi, -\mathcal{L}_X \phi) = \kappa_1 \delta \int_{\partial \Sigma} I_\xi(\omega^{ab}) e^c \ldots e^d \varepsilon_{abc \ldots c_d}.$$
\[ \kappa_1 \delta \int_{\partial \Sigma^+} I_\xi(\omega^{ab}) e^{c_3} \ldots e^{c_d} \varepsilon_{abc_3 \ldots c_d} = \kappa_1 \delta \int_{\partial \Sigma^+} I_\xi(\omega^{ab}) e^{c_3} \ldots e^{c_d} \varepsilon_{abc_3 \ldots c_d}, \]

which, since \( \omega^{ab} \) is fixed by the boundary conditions, yields the relation between the fluxes of heat at both horizons,

\[ T_{++} \delta S_{++} = T_+ \delta S_+. \quad (14) \]

Here it has been identified \( \delta S = 4\pi \kappa_1 \delta (A) \), where \( A \) stands for the area of any of the horizons. The usual \( S = A/4 \) is obtained using the standard units (see the discussion in [13]).

One can notice, by using Eq.(10), that \( T_{++} < 0 \). This only due to the orientation of the radial normal vectors, which were defined parallel for both horizon and not inward. Therefore, in Eq.(13) is accounted with a positive sign the emissions from the black horizon but accounted with a negative sign the emissions of the cosmological horizon. Equivalently, one can recall the Euclidean language and notice that the time coordinate, which here is an angle, has been taken globally anticlockwise, but it should be taken clockwise at the cosmological horizon to preserve an inward orientation [7]. Therefore, in thermodynamical terms the correct temperature of the cosmological horizon is given by \( T_{++} = -T_+ \).

Although Eq.(13) shows the fluxes of heat, this does not give information about the evolution yet. To address that one must compute the heat capacities of each horizon. Generically the heat capacity is given by

\[ C = \frac{\partial E}{\partial T} = \left( \frac{\partial r_H}{\partial M} \right)^{-1} \left( \frac{\partial T}{\partial r_H} \right)^{-1}. \]

However, it is direct to notice that

\[ \frac{\partial T}{\partial r_H} = -\frac{d - 3}{r_H^2} - \frac{d - 1}{r_H^2} < 0, \]

and therefore the sign of \( C \) actually depends only on \( \partial r_H/\partial M \). In [7] was shown that

\[ \frac{\partial r_{++}}{\partial M} < 0 \text{ and } \frac{\partial r_{++}}{\partial M} > 0, \]

which proves that the cosmological horizon has positive heat capacity.

One can confirm this result by rewriting the heat capacity in terms of the radii. This is given by

\[ C(r_H) = 2\pi r_H^{d-2} \left( \frac{r_H^2 - r_0^2}{r_H^2 + r_0^2} \right)^d, \]

where \( r_H \) stands for either \( r_+ \) or \( r_{++} \).

Fortunately, the fact that the heat capacity of the cosmological horizon be positive permits to foresee the evolution of the space in absence of any external source. Taking the correct signs for the temperatures one can notice that, for a given value of \( M, T_+ > T_{++} \). Therefore, during their interaction due to its positive heat capacity the cosmological horizon would increase its temperature, and so its radius. Conversely, the black hole horizon would become even hotter because of its negative heat capacity and shrink. In this way, there should be a net flux of energy from the black hole horizon into cosmological horizon. In principle this process should not stop until the complete evaporation of the black hole.

Although the complete description of the final stage of the evaporation, when the temperature of the black hole diverges, probably would be only obtained when a theory of quantum gravity truly exist, still one can expect that the final outcome of this process be the de Sitter space.

V. DECAYING PROCESS

The second law of thermodynamics allows to confirm the evolution of the Schwarzschild-dS solution. Since each horizon has its own entropy [9], in this case the second law of thermodynamics [3] implies the relation between the areas of the horizons

\[ \delta A_+ + \delta A_{++} \geq 0, \quad (15) \]

which in terms of the radii can be rewritten as

\[ r_+^{d-3} \delta r_+ + r_{++}^{d-3} \delta r_{++} \geq 0. \quad (16) \]

However, it is straightforward to prove the variation satisfy, since \( r_+ \) and \( r_{++} \) are not independent, that

\[ \delta r_+ = -F(r_+, r_{++}) \delta r_{++}, \quad (17) \]

with \( F(r_+, r_{++}) > 0 \). The exact expression of \( F(r_+, r_{++}) \) can be obtained from differentiating the relation between \( r_+ \) and \( r_{++} \) in the corresponding dimension (See Eq.(A1)). For \( d = 4 \) the relation reads

\[ \delta r_+ = -\left( \frac{2r_+ + r_{++}}{2r_+ + r_{++}} \right) \delta r_{++}, \]

which allows to rewrite Eq.(16) as

\[ \frac{(r_+^2 - r_{++}^2)}{(2r_+ + r_{++})} \delta r_{++} \geq 0 \Leftrightarrow \frac{(r_+^2 - r_{++}^2)}{(2r_+ + r_{++})} \delta r_+ \geq 0. \quad (18) \]

This result determines that the radius of the cosmological horizon must expand, or equivalently the radius of the black hole must decrease, in order to the second law of thermodynamics be satisfied. Analogously, for \( d = 5 \) the eq.(17) reads

\[ \delta r_+ = -\left( \frac{r_{++}}{r_+} \right) \delta r_{++} \]
and thus Eq. (16) in this case reads \((r_0^2 - r_r^2)\delta r_+ \geq 0\), which also implies that the radius of cosmological horizon increases.

After a straightforward, but cumbersome, computation one can prove that in higher dimensions, using relation (A1), the same result stands, and the radius of the cosmological horizon must expand due to the second law of thermodynamics. This result is extremely powerful and general since is based only on the laws of thermodynamics.

VI. OTHER THEORIES OF GRAVITY

In higher dimensions there are several possible proper theories of gravity [11], and in principle one could extend some of thermodynamical definitions above to them. For instance the thermodynamics for Gauss-Bonnet theory is discussed in [21]. Both Einstein and Gauss Bonnet theories belong to a larger family of theories called Lovelock gravities, whose thermodynamics has been also discussed in several articles.

To narrow the possible theories one can requests to have a single positive cosmological constant, and so avoiding to deal with several different ground states. Within the so called Lovelock gravities is possible to define a family of theories satisfying that. The Lovelock Lagrangian is given by [22]

\[
L = \kappa \sum_{p=0}^{[(d-1)/2]} \alpha_p (R)^p (\varepsilon)^{d-2p}
\]

where \((R)^p = R^{a_1 a_2} \ldots R^{a_{2p-1} a_{2p}}, (\varepsilon)^{d-2p} = \varepsilon^{a_1 \ldots a_d}\), and \(\varepsilon = \varepsilon_{a_1 \ldots a_d}\). \([(d - 1)/2]\) stands for the integer part of \((d - 1)/2\) and

\[
\kappa_k = \frac{1}{2(d-2)!G_k \Omega_{d-2}}.
\]

By a direct translation of [13] one can determine the relation between the coefficients that yields a single cosmological constant. Provided

\[
\alpha_p = \kappa \left( \frac{-l^2 + p - k}{d - 2p} \right) \left( \frac{k}{p} \right)
\]

for \(p \leq k\) and \(\alpha_p = 0\) for \(p > k\) the action \([13]\) yields \(T^a = 0\) and the equations of motion

\[
(R - \frac{\varepsilon^2}{l^2})^k (\varepsilon)^{d-2k-1} \varepsilon = 0.
\]

This confirms the presence of a single positive cosmological constant. These theories of gravity are usually called \(k\)-gravities.

The theories above have a solution of the form of Eq. (3) with

\[
f(r)^2 = 1 - \frac{r^2}{l^2} - \left( \frac{2MG_k}{r^{d-2k-1}} \right)^{\frac{1}{k}}.
\]

As previously, to avoid naked singularities and to ensure reality \(M > 0\). Also, one can notice that when \(d - 2k - 1 = 0\) solution presents a naked singularity and thus it will not be considered either. For \(d - 2k - 1 > 0\) the function \(f(r)^2\) may have none, one or two positive solution. As previously only the case with two horizons, called respectively \(r_+\) and \(r_{++}\), will be considered. In this case the ranges of those radii are given by \(0 \leq r_+ < r_0\) and \(r_0 < r_{++} \leq l\) with

\[
r_0 = l \sqrt{\frac{d - 2k - 1}{d - 1}}.
\]

In addition \(0 \leq M < M_{\text{max}}\) where

\[
G_k M_{\text{max}} = \frac{1}{2} r_0^{d-2k-1} \left[ 1 - \frac{r_0^2}{l^2} \right]^k
\]

The definition of the temperature, since is purely geometrical, can be obtained from Eq. (8), which in this case reads,

\[
T = \frac{1}{4\pi l^2 k r_H^2} (r_0^2 - r_H^2),
\]

where \(r_H\) stands for either \(r_+\) or \(r_{++}\).

The heat capacity can also be computed in this case and it is given, in terms of the radii, by

\[
C_k (r_H) = 2\pi k r_H^{d-2k} \left[ 1 - \frac{r_H^2}{l^2} \right]^{k-1} \frac{r_H^2 - r_0^2}{r_H^2 + r_0^2}.
\]

It is direct from this expression [24] to notice that \(C_k (r_{++}) > 0\) and \(C_k (r_+) < 0\). Using the same arguments as for the Einstein theory, one can argue that the evolution of these black holes should be their complete evaporation.

The analysis using the presymplectic form is also valid for these theories. In this case this also yields the relation between the differential of heat at both horizons,

\[
T_{++}^{\delta S_{++}} = T_{++}^{\delta S_{++}} = 0,
\]

where the entropy is given by [14],

\[
S = \beta \int_{\delta S_{++}} I_k u^{ab} \tau_{ab}
\]

\[
= \kappa l^{d-2k} \sum_{p=1}^{k} \frac{p(-1)^{p-k}}{d-2p} \left( \frac{k}{p} \right) \left( \frac{r_H^2}{l^2} \right)^{d-2p}.
\]

Even though there are some negative signs in this expression one can check that this entropy is an increasing function of the radius.

Unfortunately, in this case the \(1/k\) power in \(f(r)^2\) rules out the existence of an analytic relation between the variations of \(r_+\) and \(r_{++}\). One can obtained it, however, by some long numerical computations one can prove that the second law of thermodynamics also in this case determines that the respective radii of black hole horizons must decrease.
VII. CONCLUSIONS AND PROSPECTS

In this article was argued that the quasi statical evolution of the Schwarzschild de Sitter solution is the complete evaporation of its black hole. The result was obtained from the analysis of the heat capacities of the horizons, and independently confirmed by using the second law of thermodynamics. Although the analysis was not made the extension to the Kerr-dS solution seems natural, and thus one can conjecture that the evolution of any Kerr-dS solutions is also the evaporation of their black holes. Remarkably the same result stands for any other Lovelock theory of gravity with a single positive cosmological constant as well.

However, there are some fundamental question to be addressed in the future. In the picture described in this article the mass of the black hole is radiated beyond the cosmological horizon. Unfortunately this picture becomes unclear at the transition between the Schwarzschild-dS and dS spaces. The open question here is what happens with that energy radiated once the black hole disappears completely. In the de Sitter space beyond the cosmological horizon there is nothing but the de Sitter space itself, and thus, roughly speaking, the energy cannot be hidden there.

APPENDIX A: D DIMENSIONAL RELATION

The relation between \( r_+ \) and \( r_{++} \) for the Schwarzschild-dS black hole is given in d-dimensions by,

\[
(r_+ + r_{++})(r_+ + r_{++} + a_2) - r_+ r_{++} = l^2 \quad \text{(A1)}
\]

where \( a_2 \) can be obtained recursively from the relation

\[
a_{d-1} + (r_+ + r_{++})a_{d-1-1} - r_+ r_{++}a_{d-2} = 0
\]

with \( (r_+ + r_{++})a_d = r_+ r_{++}a_{d-1} \) and \( a_1 = (r_+ + r_{++}) \).

APPENDIX B: KERR-DS

The discussion has been centered on Schwarzschild-dS solution. This can be considered not general enough to be good a probe but it indeed has the structures necessary to address the general problem presented in this article. For instance, the most general four dimensional solution in vacuum with positive cosmological constant is the Kerr-de Sitter geometry. This, written Boyer-Lindquist-type coordinates, is given by the vielbein [23]

\[
e^3 = \frac{\sqrt{\Delta_\rho}}{\Xi \rho} \sin \theta (adt - (r^2 + a^2) d\varphi), \quad e^2 = \rho \frac{d\theta}{\sqrt{\Delta_\rho}},
\]

\[
e^0 = \frac{\sqrt{\Delta_\rho}}{\Xi \rho} (dt - a \sin^2 \theta d\varphi), \quad e^1 = \rho \frac{dr}{\sqrt{\Delta_r}} \quad \text{B1}
\]

with \( \Delta_r = (r^2 + a^2) \left(1 - \frac{r^2}{R} \right) - 2Mr, \Delta_\theta = 1 + \frac{a^2}{R^2} \cos^2 \theta, \Xi = 1 + \frac{a^2}{R^2} \) and \( \rho^2 = r^2 + a^2 \cos^2 \theta \).

The horizons in this case are given by the roots of \( \Delta_r = 0 \). Moreover, as for the Schwarzschild-dS solution, the region of interest is defined between the two largest positive roots, \( r_{++} \) and \( r_+ \), which define the cosmological and black hole horizons respectively. There is another internal horizon in this case [24], though. It is direct to prove that those radii are also bounded as \( r_+ < r_0 \) and \( r_0 < r_{++} < l \)

\[
r_0 = \frac{1}{6} \sqrt{\frac{6}{l^2}} \left(l^2 - a^2 + \sqrt{a^4 - 14a^2l^2 + l^4} \right).
\]

In higher dimensions the Kerr-dS solution has the same generic form of Eq. [B1] [24] with

\[
\Delta_r = (r^2 + \sum_i a_i^2) \left(1 - \frac{r^2}{l^2} \right) - 2Mr^{5-d},
\]

where \( a_i \) are the coefficients related with the angular momenta in higher dimensions. This function also defines two horizons.

These analogies with the Schwarzschild-dS solution confirm that this solution is enough general to address the general problem properly. Of course the transmission of heat in Eq. [14] should be modified by the presence of angular momenta or electric charge, nonetheless the second law of thermodynamics, which depends only on the radii, should be reducible to the form Eq. [17].

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[25] The relation (8) is the first order version of the relation \( \xi^\mu \nabla_\mu (\xi^\nu) \big|_{\partial \Sigma_H} = \kappa \xi^\nu \)
obtained in [18].