Implementation of the Explicit Group Iterative Method for Solving Image Blurring Problem using Non-Linear Diffusion Equations

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Abstract. Diffusion equations have been known to solve various image processing problems. This study employs the diffusion equations as the partial difference equations (PDEs)-based image processing techniques for image blurring which also can be refer as the process of image smoothing. The solutions of diffusion equations were obtained using the iterative algorithms and thereafter applied in the image blurring processes. The images were blurred without destroying the crucial information of an image that need to be preserve such as sharp edges, lines and shapes as the diffusion occurs inside the image locations, where the images with different sizes were tested. In terms of performance comparison, the standard point Gauss-Seidel and two-point Explicit Group (2-EG) methods were considered to produce the same quality image of classical point iterative method which is Jacobi. The numerical results showed that 2-EG iterative method capable to smooth the inner region of the images faster compared to the standard point iterative method. It was shown that the 2-EG iterative method more efficient in reducing the number of iterations and computational time than the standard point iterative method.

1. Introduction
Image processing technique applications have played an important role in various fields of science and technology. These techniques have been applied widely in medical [1, 2] and agriculture [3,4] fields while the studies are more focused on generating effective algorithms to solve the problems raised. This paper looks at on an efficient numerical method via iterative method to solve the nonlinear diffusion equation for image blurring problem. The Partial Difference Equations (PDEs) based image processing using nonlinear diffusion equations was first proposed by [5]. They developed an algorithm that solved the blurring and localization problems of linear diffusion filtering. Nonlinear diffusion filtering also suitable for image blurring, smoothing, denoising and edge detection purposes. It imposed inhomogeneous process that lessen the diffusivity at those areas which have a high probability to be edges or important features of an image. Thus, finite difference method is applied to solve the diffusion equation numerically to formulize the approximation equations through implicit scheme which unconditional stable. Then, the diffusion approximation system form is solved and produced a newly blurring image. Further details are being discussed in the following section.

The successful of the nonlinear diffusion filtering in resolve vast image processing issues have persuaded the recent researcher to define the effective algorithms for better problem solving. Most recent study in image denoising and restoration using nonlinear diffusion conducted by [6] have proposed fourth order PDE denoising schemes. This schemes have provided an optimal trade-off between noise reduction, avoiding of undesired impacts and preservation of image edges. The findings shown the algorithms performs better than many second-order PDE-based algorithms in terms of
overcoming the staircasing, running faster and outperforms the You-Kaveh [7] by providing a better deblurring and speckle removal.

Other than that, the fastest method to denoise an image especially from speckle noise called Faster Oriented Speckle Reducing Anisotropic Diffusion filter (FOSRAD have been suggested by [8]. The findings indicate a look-ahead decomposition technique has efficiently reduced the execution time than the other existing methods. Besides image denoising, current researcher also intrigued to develop better algorithms for solving image segmentation and edge detection problems. The study conducted by [9] with aims to separate the infectious breast tissue has formulated the efficient algorithms of anisotropic diffusion filter that improved the edge information from breast thermograms. The result has shows the segmented region of interests (ROIs) was identified through evolving the initial level set function according to the much better edge information. Hence, the sharp region boundaries of ROIs were useful to improve the performance of segmentation algorithm.

There are plenty of studies on utilizing diffusion equation including nonlinear diffusion [10,11] or anisotropic diffusion equation [12]. However, the goal of this paper to evaluate the nonlinear diffusion equation numerically in accordance with the recent researcher that has used the iterative method to solve other image processing problem known as Poisson image blending [13,14,15,16] in faster way. Therefore, this paper examined the potential of 2-point explicit group iterative method to solve the nonlinear diffusion equation for image blurring problem. Hence, Jacobi and Gauss-Seidel iterative methods are used to make comparison with 2-point Explicit Group method. The performance of those iterative methods are examined by the number of iterations and the computational time taken.

2. Nonlinear diffusion equation

Application of diffusion equation on image processing has becoming a significant study in smoothing or blurring an image [17, 18] and also filtering the degraded image [19] as well detecting the edge of an image [20]. The non-linear diffusion equation which is categorized as Partial Differential Equation (PDEs) based image processing has been introduced by Perona and Malik in [5]. The findings has improved linear heat equation filtering which tend to blurring the entire image without maintaining the shape of the significant features as the rate of diffusion constant throughout the image domain [21]. Thus, nonlinear diffusion equation can be described as:

$$I_t = \text{div}(c(x,y,t)\nabla I(x,y));$$  \hspace{1cm} (1)

where $\text{div}$ refer as the divergence operator and $\nabla I$ is gradient magnitude operator respect to the spatial of $x$ and $y$. Meanwhile, the diffusion rate is controlled through diffusion coefficient, $c(x,y,t) = g(\|\nabla I(x,y,t)\|)$ at any location of an image domain $(x,y)$. Then, the diffusion coefficient $g(.)$ in this study is given in the equation (2) which is suggested by [5]:

$$g(\nabla I) = \frac{1}{1+\left(\frac{\|\nabla I\|}{K}\right)^2}$$  \hspace{1cm} (2)

The diffusion coefficient served as edge stopping function where diffusion enhance in the interior of homogenous region and zero at the boundaries. The blurring effect will reduce at any places with high possibilities to be the boundaries which monitored by using the local gradient magnitude function, $\|\nabla I\|$ where $c(x,y,t) = g(\|\nabla I\|)$. In the image processing literature, Gauss-Seidel has been used for computing the solutions of nonlinear diffusion equation [22]. It shown that the Gauss-Seidel faster than Jacobi in terms of number of iterations and computational time. Recently, studies in [14, 15, 16] have employed block iterative methods for solving image blending problems. Other than that, block iterative methods also widely been studied by [23, 24,25] in solving others numerical problems.

3. Point iterative method

The utilization of equation (1) to the image blurring problems frequently brings about the large linear system with sparse coefficient matrix. In this way, numerical solution of (1) using iterative method is often used efficiently solve such linear system of the two-dimensional implicit scheme where its large and sparse. The standard five-point finite difference formula to approximate equation (1) is given as:
(1 + \lambda g_N + \lambda g_S + \lambda g_E + \lambda g_W)I_{i,j,k+1} - \lambda g_N I_{i,j+1,k+1} - \lambda g_S I_{i,j,k+1} - \lambda g_E I_{i+1,j,k+1} - \lambda g_W I_{i-1,j,k+1} \equiv I_{i,j,k}^{*}.

(3)

where \lambda = \frac{At}{h^2} as h = \Delta x = \Delta y. Let \alpha = 1 + \lambda g_N + \lambda g_S + \lambda g_E + \lambda g_W, \lambda g_N = G_N, \lambda g_S = G_S, \lambda g_E = G_E and \lambda g_W = G_W, then we have:

\begin{align*}
\alpha I_{i,j,k+1} - G_N I_{i,j+1,k+1} - G_S I_{i,j,k+1} - G_E I_{i+1,j,k+1} - G_W I_{i-1,j,k+1} \equiv I_{i,j,k}.
\end{align*}

(4)

The approximate equation (4) will result in systems of algebraic equations that can be stated as:

\begin{align*}
\sum_{j=1}^{m} a_{ij} I_j = b_i; \quad i = 1,2,3,\ldots,m.
\end{align*}

(5)

Hence, the point Gauss-Seidel method for system (5) can be shown as:

\begin{align*}
I^{(n)}_i = \frac{1}{a_{ii}} \left( b_i - \sum_{j=1}^{i-1} a_{ij} I_j^{(n)} - \sum_{j=i+1}^{m} a_{ij} I_j^{(n-1)} \right)
\end{align*}

(6)

where i = 1,2,3\ldots m and n = 1,2,3\ldots.

By applying the method of Gauss-Seidel from equation (4) into equation (6), the corresponding iteration scheme can be shown as below:

\begin{align*}
I_{i,j}^{k+1} = \frac{1}{\alpha} \left( I_{i,j}^{k} + G_N I_{i,j+1}^{k} + G_S I_{i,j,k}^{k+1} + G_E I_{i+1,j}^{k} + G_W I_{i-1,j}^{k+1} \right)
\end{align*}

(7)

for k = 1,2,3,\ldots,n.

The stack of linear system was developed from approximation equation (4) also can be written in the matrix form as:

\begin{align*}
AI = b
\end{align*}

(8)

where the system unmistakably has unique solution as the coefficient of main diagonal is strictly dominant [26]. Matrix A then be decomposed into three different matrices which matrices D, F and -G refer to the diagonal, strict lower triangular and strict upper triangular parts of matrix A, respectively as following:

\begin{align*}
A = D - F - G.
\end{align*}

(9)

Thus, the corresponding Gauss-Seidel iterative scheme in the matrix form can be written as:

\begin{align*}
I^{(n)}_i = (D - F)^{-1}GI^{(n-1)} + (D - F)^{-1} b;
\end{align*}

(10)

for n = 1,2,3,\ldots.

The iteration process of equation (10) continues until the convergence criterion is satisfied, which in this study the iteration stops when the overall pixel difference between the images produced by using Jacobi and standard point Gauss-Seidel iterative method is less than 1%.

Figure 1. (a) and (b) are the finite grid network of an image pixels in computer screen for standard point Gauss-Seidel and 2-point Explicit Group method respectively with n = 8.
In this study, three difference colour images domain were used as experimental images i.e a picture of building (a), automobile (b) and air force aircraft (c). All the input images have difference resolutions where examples (a), (b) and (c) with the dimension of 512x512, 1280x1280 and 1920x1920 respectively. The images as shown in Figure 2 were obtained from [28].

Figure 2. (a), (b) and (c) are the input images.

The 2-point explicit group iterative method

The 2-point Explicit Group iterative has been constructed to solve the equation (4). By applying equation (7) to block of two node points as illustrated in finite grid of Figure 1 (b), we get:

\[
I_{i,j}^{k+1} = \frac{1}{\alpha_1} \left( I_{i,j}^k + G_N I_{i+1,j}^{k+1} + G_S I_{i-1,j}^{k+1} + G_E I_{i,j+1}^k + G_W I_{i,j-1}^k \right)
\]

\[
I_{i+1,j}^{k+1} = \frac{1}{\alpha_1} \left( I_{i+1,j}^k + G_N I_{i+1,j+1}^{k+1} + G_S I_{i+1,j-1}^{k+1} + G_E I_{i+2,j}^k + G_W I_{i+1,j-1}^k \right)
\]

The equations above can be formulated into form (7) which stated as:

\[
\begin{pmatrix}
\alpha & -G_E \\
-G_W & \alpha_1
\end{pmatrix}
\begin{pmatrix}
I_{i,j}^{k+1} \\
I_{i+1,j}^{k+1}
\end{pmatrix}
= \frac{1}{\alpha \alpha_1 - G_S G_W}
\begin{pmatrix}
\alpha_1 & G_E \\
G_W & \alpha_1
\end{pmatrix}
\begin{pmatrix}
I_{i,j}^k + G_N I_{i+1,j}^{k+1} + G_S I_{i-1,j}^{k+1} + G_E I_{i,j+1}^k + G_W I_{i,j-1}^k \\
I_{i+1,j}^k + G_N I_{i+1,j+1}^{k+1} + G_S I_{i+1,j-1}^{k+1} + G_E I_{i+2,j}^k + G_W I_{i+1,j-1}^k
\end{pmatrix}
\]

(12)

The general scheme of 2-point Explicit Group iterative method can be determined by identifying the inverse matrix of the coefficient lattice in equation (12). The general scheme in the matrix form is shown as below [27]:

\[
\begin{pmatrix}
I_{i,j}^{k+1} \\
I_{i+1,j}^{k+1}
\end{pmatrix}
= \frac{1}{\alpha \alpha_1 - G_S G_W}
\begin{pmatrix}
\alpha_1 & G_E \\
G_W & \alpha_1
\end{pmatrix}
\begin{pmatrix}
I_{i,j}^k + G_N I_{i+1,j}^{k+1} + G_S I_{i-1,j}^{k+1} + G_E I_{i,j+1}^k + G_W I_{i,j-1}^k \\
I_{i+1,j}^k + G_N I_{i+1,j+1}^{k+1} + G_S I_{i+1,j-1}^{k+1} + G_E I_{i+2,j}^k + G_W I_{i+1,j-1}^k
\end{pmatrix}
\]

(13)

Based on the equation (13), the iterative scheme for 2-point Explicit Group iterative method can be define as:

\[
I_{i,j}^{k+1} = \frac{1}{\alpha \alpha_1 - G_S G_W} \left( \alpha_1 S_1 + G_E S_2 \right)
\]

\[
I_{i+1,j}^{k+1} = \frac{1}{\alpha \alpha_1 - G_S G_W} \left( G_W S_1 + \alpha S_2 \right)
\]

(14)

where

\[
S_1 = I_{i,j}^k + G_N I_{i+1,j}^{k+1} + G_S I_{i-1,j}^{k+1} + G_E I_{i,j+1}^k + G_W I_{i,j-1}^k
\]

\[
S_2 = I_{i+1,j}^k + G_N I_{i+1,j+1}^{k+1} + G_S I_{i+1,j-1}^{k+1} + G_E I_{i+2,j}^k + G_W I_{i+1,j-1}^k
\]

(15)

It can be seen that the calculations for both points \(I_{i,j}^{k+1}\) and \(I_{i+1,j}^{k+1}\) are totally independent. As shown in Figure 1 (b), the implementation of 2-EG iterative method occurs in each group of two node points meanwhile the ungroup nodes positioned next to the boundary are computed using direct method [27].

5. Experimental examples

In this study, three difference colour images domain were used as experimental images i.e a picture of building (a), automobile (b) and air force aircraft (c). All the input images have difference resolutions where examples (a), (b) and (c) with the dimension of 512x512, 1280x1280 and 1920x1920 respectively. The images as shown in Figure 2 were obtained from [28].
6. Experimental results
In this experiment, three differences resolution of colour image were used as shown in Figure 2. For the image blurring purpose, we set the threshold value at 5 and time-step or value of $\lambda$ at 1.0. The linear system then computed using considered iterative methods, i.e. standard point Gauss-Seidel and 2-point Explicit Group (2-EG). Then, the performance of the iterative method to produce the same quality image of classical point iterative method which is Jacobi where its evaluated based on the number of iterations and computational time.

The results of image blurring techniques using the example images in the Figure 1 are shown on the Figure 3. Since this study use colour experimental images, so the iterations are running three times for each colour which corresponds to the Red, Green and Blue channels. The results presented in Table 1 are the average number of iterations $k$ and computational time $t$ of the three channels of are each image. Based on these results, the fastest method to blur the image while preserving the significant edges is the 2-point Explicit Group iterative method with error of an approximation less than 1% of image produced by the Jacobi iterative method.

![Figure 3](image)

**Figure 3.** the output images (a), (b) and (b) generated by iterative methods of (i) Jacobi (ii) Standard point Gauss-Seidel (iii) 2-point Explicit Group iterative method respectively.

From the findings in the Table 1, 2-EG used the least number of iterations and computational time to produce same blurring images. As compared to Jacobi, it has improved the blurring iterations approximately 58.00%-61.33%. Meanwhile, the number of iterations $k$ for 2-EG have been reduced approximately 38.46% to 41.66% compared to Gauss-Seidel iterative method. In parallel with the 2-
EG results on the blurring iterations, the blurring time for 2-EG iterative method also less compared Jacobi and Gauss-Seidel. 2-EG iterative method has improved the time taken for Jacobi and Gauss-Seidel by approximately 53.73%-62.22% and 33.45%-46.02% respectively. Hence, the 2-EG obtains the smallest number of iterations and the fastest computational time were achieved in this experiment.

Table 1. The \( k = \) number of iterations, \( t = \) computational time (seconds) and the error represent the percentages of the overall pixel difference between following and Jacobi iterative method to achieve the same blurring effect.

| Example | Jacobi | Gauss-Seidel | 2-EG |
|---------|--------|--------------|------|
| (a)     | 20     | 1.753        | 13   | 1.269 | 0.82% | 8    | 0.685 | 0.68% |
| (b)     | 50     | 49.509       | 36   | 32.592 | 0.93% | 21   | 18.704 | 0.85% |
| (c)     | 75     | 167.740      | 48   | 116.612 | 0.92% | 29   | 77.607 | 0.89% |

7. Conclusion
In this paper, the performance of three iterative methods i.e Jacobi, Gauss-Seidel and 2-point Explicit Group (2-EG) to produce blurring images have been conducted to evaluate the capability of the iterative methods to produce the same quality image by reducing the number of iterations and computational time. The experimental findings show that the Explicit Group is the best method as compared to the other iterative methods in producing the identical output images with least number of iterations and computational time. This because the reduction percentage has reduced and the solution domain considered is halved. For future work, other iterative method involving family of one parameters i.e 4-EGSOR [15] and EDG [14] also should be conducted to investigate the capability of the methods to reduce the number of iterations and computational time.

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