Control parameters in turbulence, Self Organized Criticality and ecosystems.

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Abstract. From the starting point of the well known Reynolds number of fluid turbulence we propose a control parameter $R$ for a wider class of systems including avalanche models that show Self Organized Criticality (SOC) and ecosystems. $R$ is related to the driving and dissipation rates and from similarity analysis we obtain a relationship $R \sim N^{\beta_N}$ where $N$ is the number of degrees of freedom. The value of the exponent $\beta_N$ is determined by detailed phenomenology but its sign follows from our similarity analysis. For SOC, $R = h/\epsilon$ and we show that $\beta_N < 0$ hence we show independent of the details that the transition to SOC is when $R \to 0$, in contrast to fluid turbulence, formalizing the relationship between turbulence (since $\beta_N > 0$, $R \to \infty$) and SOC ($R = h/\epsilon \to 0$). A corollary is that SOC phenomenology, that is, power law scaling of avalanches, can persist for finite $R$ with unchanged exponent if the system supports a sufficiently large range of lengthscales; necessary for SOC to be a candidate for physical systems. We propose a conceptual model ecosystem where $R$ is an observable parameter which depends on the rate of throughput of biomass or energy; we show this has $\beta_N > 0$, so that increasing $R$ increases the abundance of species, pointing to a critical value for species ‘explosion’.

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1. Introduction

A central idea in physics is that complex, often intractable, behavior can be quantified by a few measurable control parameters. In fluid turbulence a single control parameter, the Reynolds number $R_E$ quantifies the transition from ordered (laminar) to disordered (turbulent) flow. This parameter is from dimensional arguments a function of macroscopic system variables, but is also expressible as a function of the number of energy carrying modes or degrees of freedom (d.o.f.)\[1\]. Although the detailed dynamics are different, many of the macroscopic features of idealized, finite (large) Reynolds number turbulence are also characteristic of other strongly correlated, out of equilibrium systems. These systems can all be driven into a disordered state with defining characteristics: they have many degrees of freedom (d.o.f.); are driven and dissipating, are out of equilibrium but on average in a steady state, and show anomalous scaling over a large dynamic range. Loosely speaking, a ‘class’ of such systems will show an order- disorder transition, captured by varying a single control parameter. This includes avalanche models exhibiting Self Organized Criticality (SOC)[2, 3, 4, 5, 6, 7]. It was originally argued[2, 21] that these systems self organize to an SOC state which in the above sense is their state of maximal disorder. Subsequent analysis has established a consensus\[11, 12, 13, 6, 5\] that SOC is a limiting behavior in the driving rate $h$ and the dissipation rate $\epsilon$, such that $h/\epsilon \to 0$ with $h, \epsilon \to 0$, (and $h \leq \epsilon$, that is, steady state). This is exemplified by the constructive definition (in[4]) as “slowly driven interaction dominated thresholded” (SDIDT) systems.

In this paper we give a prescription for obtaining a control parameter for these systems, in analogy to the Reynolds number in fluid turbulence. For avalanche models exhibiting SOC, we identify d.o.f. with realizable avalanche sizes and we show that the relevant control parameter $R_A$ is $h/\epsilon$. It follows that the SDIDT limit is reached by taking $R_A$ to zero; we show this maximizes the number of d.o.f. in the opposite sense to fluid turbulence. This result clarifies the much debated relationship between turbulence and SOC\[14, 15, 16, 17, 18, 19\]. An important corollary is that SOC phenomenology can quite generally persist under conditions of finite drive in a sufficiently large bandwidth system. As our result flows from dimensional analysis it is quite generic. An important example that we give here is as a parametrization of ecosystem models for species abundance\[8, 9, 10\]. For ecosystems we show that as the control parameter increases so does the abundance of species, or d.o.f. This points to the possibility of a critical value at which the onset of diversification of species occurs.

2. Similarity analysis and Reynolds number

The systems that we have in mind all have strongly coupled d.o.f. that transport some quantity from the driving to the dissipation scale. Provided that this ‘dynamical quantity’ is governed by a conservation law to ensure steady state (not necessarily equilibrium) on the average we insist that its precise nature, and the microscopic
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Table 1. \( \Pi \) theorem applied to homogeneous turbulence.

| Variable | dimension | description          |
|----------|-----------|----------------------|
| \( L_0 \) | \( L \)   | driving length scale  |
| \( \eta \) | \( L \)   | dissipation length scale |
| \( U \) | \( LT^{-1} \) | bulk (driving) flow speed |
| \( \nu \) | \( L^2T^{-1} \) | viscosity |

details of how it is transported are not relevant to the macroscopic ensemble average behavior. We seek a control parameter expressible in terms of the number of d.o.f. of the system that parameterizes the transition from ordered (few d.o.f.) to disordered (many d.o.f.) behavior. We identify the control parameters of the system in terms of known macroscopic variables by formal dimensional analysis (similarity analysis or Buckingham \( \Pi \) theorem, see e.g. \cite{20}). Any system’s behavior is captured by a general function \( F \) which only depends on the relevant variables \( Q_{1..V} \) that describe the system. Since \( F \) is dimensionless it must be a function of the possible dimensionless groupings \( \Pi_{1..M}(Q_{1..V}) \) which can be formed from the \( Q_{1..V} \). The (unknown) function \( F(\Pi_1, \Pi_2, \ldots, \Pi_M) \) is universal, describing all systems that depend on the \( Q_{1..V} \) through the \( \Pi_{1..M}(Q_{1..V}) \) and the relationships between them. If one then has additional information about the system, such as a conserved quantity, the \( \Pi_{1..M}(Q_{1..V}) \) can be related to each other to make \( F \) explicit. Thus this method can lead to information about the solution of a class of systems where the governing equations are unavailable or intractable, often the case for complex systems where there are a large number (\( N \) here) of strongly coupled d.o.f.. If the \( V \) variables are expressed in \( W \) dimensions (i.e. mass, length, time) then there are \( M = V - W \) dimensionless groupings.

Here, since we have that the precise nature of the transported dynamical quantity is irrelevant, the only relevant dimensions are length and time so that \( W = 2 \). We next insist that there is a single control parameter (\( R \), in the case of turbulence, the Reynolds number \( R_E \)) which may be expressed as a function of the number of active degrees of freedom \( N \). This means that the system’s behaviour is captured by some \( F(\Pi_1, \Pi_2) \); where \( R = \Pi_1 \) and \( \Pi_2 = f(N) \) and the \( \Pi_1 \) and \( \Pi_2 \) are related via some conservation property. Hence \( M = 2 \) so that \( V = 4 \); there are four relevant variables to consider.

It is useful to fix ideas in terms of a relatively well understood example of the above, namely turbulence. Our aim here is to obtain a control parameter \( R \) by analogy to \( R_E \) via dimensional analysis; for a detailed discussion of the universal scaling properties of Kolmogorov (K-41) turbulence and their origin in the Navier Stokes equations see for example \cite{1}. As above, for K-41 we have four relevant macroscopic variables (given in Table 1) and two dimensionless groups:

\[
\Pi_1 = \frac{UL_0}{\nu} = R_E, \quad \Pi_2 = \frac{L_0}{\eta} = f(N)
\]

(1)

\( \Pi_1 \) is the Reynolds number of the flow, and the ratio of lengthscales \( \Pi_2 \) is directly
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related to the number of d.o.f. \( N \) available and we now relate \( R_E \) to \( f(N) \) (or \( \Pi_1 \) to \( \Pi_2 \)). For incompressible fluid turbulence, our dynamical quantity is the time rate of energy transfer per unit mass \( \varepsilon_l \) through length scale \( l \). Conservation and steady state imply that in an ensemble averaged sense this is balanced by the rate at which energy is transferred to the fluid \( \varepsilon_{inj} \sim U^3/L_0 \) which in turn is balanced by the dissipation of energy within the fluid \( \varepsilon_{diss} \) so that \( \varepsilon_{inj} \sim \varepsilon_l \sim \varepsilon_{diss} \). Dimensional arguments (e.g. \( \Pi \)) lead to \( \varepsilon_{diss} \sim \nu^3/\eta^4 \). Conservation, that is \( \varepsilon_{inj} \sim \varepsilon_{diss} \) then gives the well known result [1] which relates \( \Pi_1 \) and \( \Pi_2 \):

\[
R_E = \frac{UL_0}{\nu} \sim \left( \frac{L_0}{\eta} \right)^{\frac{4}{3}}
\]  

(2)

The 4/3 exponent that arises for K-41 is modified if we consider other turbulent flows with different phenomenologies, for example anisotropy, and intermittency. Nevertheless, for any turbulent flow we can anticipate that the relationship between \( L_0/\eta \) and the number of degrees of freedom \( N \) will be of the form:

\[
N \sim \left( \frac{L_0}{\eta} \right)^{\alpha}
\]  

(3)

with \( \alpha > 0 \); the crucial point is that for turbulence, \( N \) always grows with \( L_0/\eta \). The only property of turbulence with which we are concerned here is that

\[
R_E \sim \left( \frac{L_0}{\eta} \right)^{\beta} \sim N^{\beta_N}
\]  

(4)

and that in particular, for turbulence \( \beta_N = \beta \alpha > 0 \). This identifies the Reynolds number as the control parameter for a process (turbulence) which simply grows more active modes or d.o.f. as we increase \( R_E \), taking the system from order (few d.o.f. or laminar flow) to disorder (many coupled d.o.f.).

We will now see that more generally, similarity analysis is sufficient to obtain the relationship between the control parameter \( R \) and the number of degrees of freedom \( N \) of the form:

\[
R \sim N^{\beta_N}
\]  

(5)

The value of the exponent \( \beta_N \) will depend on the details of these systems but crucially we will see that the sign of \( \beta_N \) is obtained from similarity analysis. This is sufficient to establish if, as in the case of turbulence, increasing \( R \) increases the disorder or complexity of the system.

3. Control parameter for avalanching systems

We now envisage a generic avalanche model in a system of size \( L_0 \) where the height of sand is specified on a grid, with nodes at spacing \( \delta l \). Sand is added to individual nodes, that is, on length scale \( \delta l \) at an average time rate \( \varepsilon_{inj} = h \) per node. There is some process, here avalanches, which then transports this dynamical quantity (the sand) through structures on intermediate length scales \( \delta l < l < L_0 \). Sand is then lost to
the system (dissipated) at a time rate $\epsilon$ over the system size $L_0$. The relevant variables for the avalanching system are given in Table 2. The two dimensionless groups are:

$$\Pi_1 = \frac{h}{\epsilon} = R_A, \quad \Pi_2 = \frac{L_0}{\delta l} = f(N)$$

The second parameter, $\Pi_2 = f(N)$ is related to the number of d.o.f. of the system. The control parameter $\Pi_1 = h/\epsilon$ is analogous to the Reynolds number above in that, as we will now show, it relates the ratio of the driving to the dissipation rates to the number of active, or energy containing degrees of freedom $N$ in the system.

In Euclidean dimension $D$ there are $(L_0/\delta l)^D$ nodes so that conservation of the flux of sand (in an ensemble averaged sense), gives $h(L_0/\delta l)^D \sim \epsilon$ which simply states that the rate at which sand is added to the system must on average balance the rate at which sand leaves. On intermediate length scales $\delta l < l < L_0$, sand is transported via avalanches. There must be some detail of the internal evolution of the pile that maximizes the number of length scales $l$ on which avalanches occur. For avalanche models this is the property that transport can only occur locally if some local critical gradient is exceeded; as a consequence the pile evolves through many metastable states. If these length scales represent d.o.f. then the number $N$ of d.o.f. available will be bounded by $L_0$ and $\delta l$ so that:

$$N \sim (L_0/\delta l)^\alpha$$

with $D \geq \alpha \geq 0$ for $D > 1$ ($\alpha$ may be fractional). We then have:

$$R_A = \frac{h}{\epsilon} \sim \left(\frac{\delta l}{L_0}\right)^D \sim N^{-\alpha D}$$

This is in contrast to fluid turbulence since the number of d.o.f. decreases with increasing drive, that is, increasing $R_A = h/\epsilon$. Thus we recover the SDIDT limit for SOC, namely $R_A \to 0$, but now explicitly identify this limit with maximizing the number of d.o.f. available, that is, the disorder of the system. Our result from dimensional analysis is to obtain $R_A \sim N^{\beta N}$ and to show quite generally that $\beta_N < 0$. Following the above discussion of turbulence, we can go further and make the analogy $R_A \equiv R_E$, that is, the system’s ’effective Reynolds number’ increases with the energy/sand taken up by the system, i.e. with $h$.

The property that the system generates many coupled d.o.f. is, for SOC, captured by avalanching phenomenology. This sets conditions on the microscopic details of the

| Variable | dimension | description |
|----------|-----------|-------------|
| $L_0$    | $L$       | system size |
| $\delta l$ | $L$   | grid size   |
| $\epsilon$ | $ST^{-1}$ | system average dissipation/loss rate |
| $h$      | $ST^{-1}$ | average driving rate per node |
system; specifically, there must be a separation of timescales in that the relaxation time for the avalanches must be short compared to the time taken for the drive to on average cause a cell to be come unstable so that avalanching is the dominant mode of transport. The critical gradient can be a random variable but provided it has a defined average value \( g \), we have an average number of timesteps to drive a cell unstable \( (g \delta l)/(h \delta t) \) where \( \delta t \) is the timestep. This gives two conditions for avalanching to dominate transport [25]:

\[
h \delta t \ll g \delta l, \ h \delta t \ll g \delta l \left( \frac{L_0}{\delta l} \right)^D
\]

(9)

The first condition is that avalanches only occur after many \( g \) rains of sand have been added to any given cell in the pile and is the strict SDIDT [11, 12, 13] limit. However, if the system has large bandwidth \( L_0 \gg \delta l \), one can consider an intermediate behavior \( gL_0 \gg h \delta t > g \delta l \) where the driver is large enough to swamp the smallest avalanches, but larger avalanches persist [25]. For fixed \( L_0 \) and \( \delta l \), increasing \( h \delta t \) above \( g \delta l \) successively erodes the available d.o.f since each addition of sand swamps \( h \delta t/(g \delta l) \) cells of the pile. Ultimately as \( h \) and hence \( R_A \) is increased to the point where \( h \delta t \sim g \delta l \left( \frac{L_0}{\delta l} \right)^D \) there will be a crossover to laminar flow.

We now show that this intermediate, finite \( R_A \) behavior will be ‘SOC like’, with power law avalanche statistics sharing the same exponent as at the SDIDT limit; confirming our assumption above that \( \beta_N \) is independent of the control parameter \( R_A \). To see this, consider passing through the regime of \( h \delta t \sim g \delta l \) with \( h \delta t \ll g \delta l \left( \frac{L_0}{\delta l} \right)^D \), which can be achieved by increasing both \( h \) and \( L_0 \) such that \( h \rightarrow Ah \) and \( L_0 \rightarrow L_0 A^{(1/D)} \). This is equivalent to coarse graining the pile spatially, so that provided the system has self similar spatial scaling we can anticipate obtaining the same solution (subject to a rescaling) provided \( L_0 \rightarrow L_0 A^{(1/D)} \). Under nonlocal feeding and non overlapping avalanches (\( A \) times as many grains added at well separated positions over the pile) this coarse-graining may not occur.

We illustrate this in Figures 1 and 2 with simulations of the BTW [2] sandpile in 2D, where the driving occurs randomly in time and is spatially restricted to the ‘top’ of the pile. In all cases the critical gradient (threshold for avalanching) is \( g = 4 \). Figure 1 shows two simulations of size \( L_0 = 100 \) with \( h = [4, 16] \) (\( \delta t = 1 \) in the simulations) i.e. just at, and above, the regime \( h \delta t \sim g \delta l \), but in both cases with \( h \delta t \ll g \delta l \left( \frac{L_0}{\delta l} \right)^D \). At \( h = 16 \) we see that the power law statistics of the smallest avalanches is lost but there is still scaling over a more restricted range of avalanche sizes, i.e. we have the same scaling, and same exponent, but over a reduced number of d.o.f. Rescaling the avalanche sizes of the \( h = 16 \) run \( S \rightarrow S/16 \) (that is, lengthscales \( l \rightarrow l/4 \)) recovers the behaviour of the \( h = 4 \) run except at the largest decade. To recover the full range we repeat the \( h = 16 \) run in a larger box, \( L_0 = 400 \) (that is, \( L_0 \rightarrow 4L_0 \) which is shown alongside the \( h = 4, L_0 = 100 \) run in Figure 2. Rescaling the \( L_0 = 400 \) run with \( S \rightarrow S/16 \), i.e. lengthscales \( l \rightarrow l/4 \) then reproduces the \( h = 4, L_0 = 100 \) results. This establishes a general property of avalanching systems that has been seen in several representative SOC models [27, 25, 26, 28], see also [29, 30].
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Figure 1. Avalanche size normalized distributions for two runs of the 2D BTW sandpile driven at the top corner formed by two adjacent closed boundaries, the other boundaries are open. $L_0 = 100$ and $h = 4$ (●) and $h = 16$ (×); (a-left) probability densities; (b-right) as (a) with probability density for the $h = 16$ avalanche sizes rescaled $S \rightarrow S/16$.

Figure 2. Avalanche size normalized distributions for $L_0 = 100$, $h = 4$ (●) and $L_0 = 400$, $h = 16$ (+); (a-left) probability densities; (b-right) as (a) with probability density for the $h = 16$ avalanche sizes rescaled $S \rightarrow S/16$.

The above assumes the BTW case where avalanches relax instantaneously. One can develop this idea to introduce “running sandpiles” (as extensively studied by, for example, [29, 27, 28, 30]) where redistribution is no longer instantaneous. Instead, wherever the critical gradient is exceeded locally, $f$ grains are moved in time $\delta t_f$. Thus there is a local redistribution rate per cell $h_f = f/\delta t_f$ which we can compare with the driving rate $h$. This introduces a new dimensionless parameter $(h\delta t_f)/(h_f\delta t)$ which modifies $h$ in the discussion above; in other words the SDIDT limit is approached for both $h \to 0$ and $\delta t_f/\delta t \to 0$ [11, 12]. However, consistent with studies of running sandpiles [29, 27, 28, 30], we can anticipate that avalanching phenomenology will persist for a range of finite $\delta t_f/\delta t$. 

### Table 3. Π theorem applied to a simple model for an ecosystem in a space with Euclidean dimension $D$. Interactions between species processes a quantity (biomass, here) with physical dimension $B$.

| Variable | Dimension | Description |
|----------|-----------|-------------|
| $L_0$    | $L$       | system size |
| $L_c$    | $L$       | normalization length scale |
| $M_p$    | $BT^{-1}$ | top predator rate of consumption of biomass over system |
| $M_f$    | $BT^{-1}L^{-D}$ | rate of supply of biomass/unit volume |

Depending on the details, some SOC systems may show scaling in systems where the drive is in fact highly variable. One could argue (see also [11]) that such robustness against fluctuations in the driver is necessary for SOC to provide a ‘working model’ in real physical systems where the idealized SDIDT limit may not be realized.

### 4. Control parameter for model ecosystem

Finally, we apply the above framework to simple models for ecosystems. We consider a large number of connected ‘meta- species’ (or groups of species/variations occupying a given niche [8]) with diverse sizes and rates of predation. Each meta- species is a d.o.f. in the model. We insist that the details are unimportant except that each of the $N$ meta- species, by acting as predator of one set of neighbors in the food web and prey to another set, processes some dynamical quantity, say, biomass or energy. The ecosystem then has a driving rate, or rate of supply $H$ of biomass/energy per unit volume at the ‘bottom’ of the web and a dissipation rate, or rate of consumption $P$ of biomass by the top predators. We consider a steady state on the average which includes secular change in these parameters that is slow compared to the timescale for information to propagate through the web. For a given habitat, the abundance of species (i.e. the relative number of distinct meta- species) grows with the size of the habitat. Although the details may vary, a good working approximation for the ‘species- area relationship’ [32] is a power law, so that the number of species $N$ in a habitat of size $L_0$ is given by $N \sim (L_0^2)^\gamma$. A dimensionally balanced expression in $D$ Euclidean dimensions is:

$$N \sim (L_0/L_c)^{D\gamma} \tag{10}$$

with $\gamma > 0$. The length scale $L_c$ captures details of the sampling, as well as specifics of a given habitat and terrain. The relevant system variables are shown in Table 3.

There are two dimensionless groups:

$$\Pi_1 = \frac{P}{HL_c^D} = R_B, \quad \Pi_2 = \frac{L_0}{L_c} = f(N) = N^{\frac{1}{D\gamma}} \tag{11}$$

thus we identify a control parameter $\Pi_1 = R_B$ for the simple ecosystem. To relate this to the abundance of species we require some conservation property and to insist on
steady state. One possibility is to conserve some fraction of the biomass flux propagated through the web so that for a steady state for the system as a whole, the rate of supply of biomass is balanced by the rate of removal by the top predators giving $L_0^D H \sim P$. 

In a system with losses, provided a fraction $\alpha$ of the propagated quantity is on average passed from one d.o.f. or species to the next, this expression is $A\alpha^N L_0^D H \sim P$; the factor $A$ also includes any recycling of the top predator biomass to the bottom of the web.

We can however work with any quantity which is transferred from one species to another with some conservation. If we instead consider $P$ and $H$ to refer to integrated energy consumption of the top predator population and the energy taken up by the web per unit volume (the productivity) respectively, conservation is then the original ‘energetic-equivalence rule’[31]- that the total energy flux of a population is invariant with respect to body size. The control parameter $R_B$ increases with a measure of the rate at which biomass (or energy) is utilized by the system as a whole ($P$), that is, is ultimately consumed by the top predator. Equivalently, it increases with the biomass (or energy) rate of supply to the system via the organisms at the bottom of the web, $HL_0$; these both represent the rate at which biomass/energy is processed by the ecosystem as a whole. We then have:

$$R_B \sim \left(\frac{L_0}{L_c}\right)^D \sim N^{\frac{1}{\gamma}}$$  \hspace{1cm} (12)

so we obtain that $R_B \sim N^{\beta_N}$ and quite generally that $\beta_N > 0$. The abundance of species simply increases with $R_B$ capturing the observation that diversity grows with the global flux of energy/biomass, that is, productivity times area[31] [32]. This result holds even if the species-area relationship is not a power law, it simply requires that the number of species grows with habitat size; intriguingly, a non-power law $\beta_N$ suggests a length scale dependence in the abundance of species that is intermittent.

The power law dependence implied by a power law species-area relationship suggests that the dependence of $R_B$ on $N$ is rather nonlinear. A consequence is that, if we consider slowly increasing this control parameter in a manner that does not violate our assumption of a steady state, we would expect, starting from an initial state of few species, to see a sudden ‘explosion’ in diversity at some critical value $R_L$. This will depend on the details through the non universal parameter $L_c$; but since $L_c$ can be determined through species-area abundance relationships, $R_L$ can in principle be determined.

We can consider the analogy $R_B \equiv R_E$, that is, we identify $R_B$ as the ecosystem’s ‘effective Reynolds number’ which increases with the energy/biomass taken up by the system. We have then shown that, the disorder, or complexity of the ecosystem as expressed by the abundance of species increases with effective Reynolds number in the same sense as turbulence. This analogy to turbulence may be instructive in that there is some non universal value of the Reynolds number at which a given system makes the transition to turbulence. One can speculate that dynamical systems routes to turbulence (specifically, the Ruelle-Takens or Feigenbaum scenarios, e.g. [33]) suggest a new
approach to modelling the onset of the diversity of species. Understanding ecosystems in the context of simple models for turbulence may also provide a basis for modelling the “bursty” dynamics and scaling intrinsic to some ecosystems. Our approach to a ‘generalized Reynolds number’ outlined here potentially has wider application: to living organisms and societal organizations, insofar as they can be modelled as webs of many interacting elements that process some dynamical quantity.

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