The analysis of the implementation inquiry based learning to improve student mathematical proving skills in solving dominating metric dimension number

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Abstract. Inquiry based learning has been promoted as a student-centered approach that can strengthen the relationship between teaching and research. Inquiry can be defined as seeking for truth, information or knowledge / understanding and is used in all facets and phases of life. In this study, researchers tried to apply inquiry-based learning with mathematical proving skills. The classes used are the control class and the experimental class. This study uses a mixed method namely quantitative and qualitative methods. This study aims to develop inquiry-based learning tools and produce learning products in the form of student worksheets (LKM), learning outcomes tests (THB), and monographs find out significant differences between control class and experimental class. This research uses 4D development methods (define, design, development, and disseminate). To find out the effect of inquiry based learning. Rshiny program is used. The overall result of product validation is 50 \% with a valid category. The effectiveness of learning outcomes is 0.304 with the medium category. The practicality of the teacher’s response results was 75.55 with a very practical category, and the student response result was 83.46 with a very practical category. With the testing criteria accept $H_0$ if the significance value or probability value $> 0.05$ then $H_0$ is accepted and $H_1$ if the significance value or probability value $< 0.05$ then $H_0$ is rejected. The results showed that there was an impact of the development of inquiry-based learning tools on mathematical proving skills of students to prove the problem of dominating metric dimension numbers.

1. Introduction
Education is one of the important factors in the progress of Indonesian society, especially in the era of globalization. Education is taught not only to elementary schools and secondary schools but also in the world of college’s teaching and learning activities, especially mathematics. In the world of mathematics education students especially discrete mathematics material. The purpose of learning activities is not only to be smart but to be able to reason, communicate, represent, solve problems, and behave well. This must be owned by students after participating in learning. Problems faced by students are not easy to overcome. Mathematics learning has a goal about the abilities students must have. This ability is better known as mathematical ability. Mathematical ability is the ability to handle problems, both in mathematics and in real life. Mathematical ability consists of: mathematical reasoning, mathematical communication, mathematical problem solving, concept understanding, mathematical understanding, creative thinking, and critical thinking. In line with this opinion, NCTM suggests a standard mathematical learning process, namely: Solving mathematical problems,
reasoning and mathematical proofs (mathematical reasoning and proof), mathematical communication (mathematical communication), mathematical connections and mathematical representations (mathematical representations).

One of the learning models that leads to mathematical proving skills is Demand-Based Learning. Inquiry-Based Learning (IBL) is a teaching technique in which the teacher engages students in the learning process through the use of questions, problem-solving activities, and critical thinking. The learning model of teaching is to help the student to be independent in finding ideas from concepts provided by the instructor. This method is student-centered with the teacher plays an important role, namely as a designer-maker of learning. From the explanation above, the researcher wants to develop an Inquiry-Based Learning model to mathematically prove students' level of thinking. According to [5], the phase of IBL in learning include three phase and describes in the table:

| Phase                | Activity                                                                 |
|----------------------|--------------------------------------------------------------------------|
| 1. Exploration       | Students are free to find and manipulate subject matter                  |
| 2. Introduction of   | Students under the guidance of the teacher, organize the data that has   |
| Concept              | been collected and look for patterns that emerge.                        |
| 3. Application       | Students are given problems that they must solve by using information    |
| Concept              | obtained through discovery and reading references.                       |

Table 2. The indicators of student mathematical prove skills

| Aspect                          | Indicator                                                      |
|---------------------------------|---------------------------------------------------------------|
| 1. Initial Attack               | Read and understand a question given                          |
| 2. Recognition the pattern      | Take several approaches that can be used and formulate and try to solve |
| 3. Generalization               | Resolve problems and reflections on what has been done and give reasons |
| 4. Develop a lemma or theorem   | Prove lemma or theorem                                        |
| 5. Analysis deductively based on the previous lemma or theorem | Analyze deductive reasoning based on lemma or theorems         |
| 6. Concluding the proof         | Student can prove mathematical skills in solving dominating metric dimension number |

Various approaches and methods have been developed, including [21] suggesting the concept of generic evidence as a way to improve students' understanding of the evidence of a statement. Generic evidence is provided at the sample level that explains concepts in general by looking at specific examples. There are several reasons why evidence teaching needs to be given, namely: 1) evidence is an integral part of mathematics, 2) for verification and discovery of facts, 3) for the development of students' logical and critical thinking skills, and 4) accelerating and increasing students' mathematical understanding [12]. Meanwhile, by dominating metric dimention we mean the problem in the following definition.

According to Haynes and Henning in [4], the D set from a simple graph point G is called the dominating set if each point \( v \in V(G) - D \) is adjacent to several points \( v \in D \). The smallest cardinality of a dominating set is called a domination number denoted by \( \gamma (G) \). According to [16], the upper bound of a domination number is the number of points in a graph.

2. Research Methods
This research is a type of mix method research that is a multimethod approach. Multimethod is a combination of qualitative and quantitative research methods. The method used is sequential
exploratory. Design is a combination of research having the collection and analysis of qualitative data in the first stage while the second stage is followed by the collection of data and quantitative data analysis to make conclusions of the results of the research in the first stage. This study aims to develop inquiry-based learning tools and produce learning tool products in the form of student worksheets (L.KM), learning outcomes tests (THB) and monographs and find out significant differences between the control class and the experimental class.

The population of this study is Jember University mathematics education students who take discrete mathematics courses. The research sample consisted of two classes consisting of a control class and an experimental class. Both classes are the same teacher but with different treatments. For example in the experimental class will use learning tools that have been developed, namely learning tools based on inquiry based learning while the control class will use conventional learning. Based on the research design, both groups showed effective results after solving the problems given.

| Class     | Pre-Test | Treatment | Post Test |
|-----------|----------|-----------|-----------|
| Experimental | $R_1$    | $X$       | $R_2$     |
| Control    | $R_3$    | $-$       | $R_4$     |

Explanation:
$R_1, R_3$: Pre-test
$R_2, R_4$: Post-test
$X$: treatment in the experimental class in the form of inquiry based learning

2.1. Population
The population of this study is Jember University mathematics education department in the FKIP University of Jember students who take discrete mathematics courses. The consisting of 28 students of experimental class and 33 students of the control class.

2.2. Instrument
Instruments used in this research were pre-test, post-test, observation and interview. The instrument used in collecting thinking skills data proves mathematically students through pre-test and post-test in the form of essay questions. The maximum number of scores that a student will get if he answers all the questions correctly is 100. The observation instrument used a Cahyani scale encompassing into five categories, namely very active (score 4), active (score 3,4), less active (2,4), and inactive (score 1).

2.3. Data Collection and Data Analysis
We gave pre-test and post test to both experimental and control class. We also apply observation and interview with the subject research. Quantitative data analysis is divided into two namely initial data analysis and final data analysis. Initial data analysis (taken from the results of the initial mathematical representation ability in order to find out the average similarity of the experimental and control classes) uses the normality test, homogeneity test, and the two average similarity test. While the final data analysis (conducted after learning using inquiry based learning) uses completeness test and average difference test using SPSS.
2.4. Task
Assignments given to students are pre-test, post-test and student worksheets. The test is given to the control class and the experimental class while the student worksheet is only given to the experimental class. Students are asked to look for resolving domination number and their functions. Assignments given to students are as follows:

**Figure 1.** Flow Chart of the Mixed Method Model

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First, determine the graph that we will look for as the dominating set. Secondly give labels on vertices and edge. Then determine the cardinality of vertices and edge. After that we look for the dominating set and then determine the dominating metric dimension number. Where for the dominating set we look for as minimum as possible and the dominating set can dominate other vertices.

\[ V = \{a_i, 1 \leq i \leq m \} \cup \{b_i, 1 \leq i \leq m \} \cup \{C\} \]

\[ E = \{a_i b_i, 1 \leq i \leq m\} \cup \{b_i b_{i+1}, 1 \leq i \leq m - 1\} \cup \{b_m b_1\} \cup \{b_i C, 1 \leq i \leq m\} \]

\[ |V| = 2m \]

\[ |E| = 3m - 1 \]

\[ D = \{b_1, b_2, b_3, b_4\} \]

\[ \text{Dom}_{Dim}(G) = \left\lceil \frac{m}{3} \right\rceil \]

3. Research finding

The study was conducted in the experimental class and the control class using qualitative methods to determine students' mathematical thinking skills. The purpose of the validity and reliability test is to determine the accuracy of the measurement instruments. The following are the results of the validity and reliability test.

| Table 4. The test result of the validity question |
|-----------------------------------------------|
| **Exercise** | **Exercise 1** | **Exercise 2** | **Exercise 3** | **Exercise 4** | **Exercise 5** | **TOTAL** |
|----------------|-------------|-------------|-------------|-------------|-------------|----------|
| Exercise 1     | Pearson Correlation | .429* | .262 | .205 | .282 | .724** |
| Sig. (2-tailed) | .013 | .141 | .253 | .111 | .000 |
| N               | 33 | 33 | 33 | 33 | 33 |
| Exercise 2     | Pearson Correlation | .429* | .440* | -.113 | .382* | .672** |
| Sig. (2-tailed) | .013 | .010 | .531 | .028 | .000 |
| N               | 33 | 33 | 33 | 33 | 33 |
| Exercise 3     | Pearson Correlation | .262 | .440* | .149 | .094 | .576** |
| Sig. (2-tailed) | .141 | .010 | .407 | .604 | .000 |
| N               | 33 | 33 | 33 | 33 | 33 |
| Exercise 4     | Pearson Correlation | .205 | -.113 | .149 | 1 | .091 | .455** |
| Sig. (2-tailed) | .253 | .531 | .407 | .614 | .008 |
| N               | 33 | 33 | 33 | 33 | 33 |
Based on Table 4. We can see that the value $r_{count}$ of number 1 is 0.724, number 2 is 0.672, number 3 is 0.576, number 4 is 0.455, and number 5 is 0.637. All of the items give the value of $r_{count} > r_{table}$ with N = 33 thus all items are valid.

**Table 5.** The test result of the reliability question

| Cronbach's Alpha | N of Items |
|------------------|------------|
| .578             | 5          |

Based on Table 5, it can be seen that the overall reliability value is 0.578 and $r_{table}$ of a significance level 5% with $dK = N - 1 = 32$, $r_{table} = 0.2869$. Therefore $r_{count} > r_{table}$. It concludes that the instrument items are reliable.

**Figure 3.** The graphic of students mathematical proving skills of control classes based on their pre-test result

We will show the graphic of students mathematical thinking skills of both control and experimental classes based on their pre-test result as follows.
Based on the results of the pre-test analysis between the two classes, it can be seen classes have the same variance. The results showed the mathematically thinking skills of the control class can very active is 7%, active is 11%, less active is 15%, and inactive is 17% while for the experimental class can very active is 6%, active is 12%, less active is 15%, and inactive is 17%.

Table 6. the analysis of the homogeneity of pre-test

| Levene Statistic | df1  | df2  | Sig.  |
|------------------|------|------|-------|
| 7.365            | 1    | 59   | .009  |

Table 6 shows a discussion of homogeneity test from pre-test. The value (sign.) in the analysis of homogeneity variances table test is 0.009. The obtained significance value is less than 0.05, it implies the data shows homogeneous variances.

Table 7. the result of mean scores of pre-test between control and experimental classes

| Group             | N  | Mean   | Std. Deviation | Std. Error Mean |
|-------------------|----|--------|----------------|-----------------|
| Pre Test          |    |        |                |                 |
| Experimental Class| 28 | 57.7857| 6.46562        | 1.22189         |
| Control Class     | 33 | 64.9394| 8.56205        | 1.49046         |
Table 8. The comparison of pre-test score of experimental class and control class score using independent sample t-test

| Group         | F    | Sig. | T    | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower           | Upper           |
|---------------|------|------|------|----|-----------------|-----------------|-----------------------|------------------|------------------|
| Equal variances assumed | 2.093 | .153 | -3.628 | 59 | .001 | -7.15368 | 1.97176 | -11.09917 | -3.20819 |
| Pre Test      | -3.712 | .000 | 58.272 | 58.272 | .000 | -7.15368 | 1.92730 | -11.01121 | -3.29615 |

Table 8 also shows that the result of t-test independent samples test. The research was continued by conducting learning used conventional learning model followed by post-test. The research was done to 33 students in the control class to know their mathematically thinking level after the learning. 33 subjects were tested by using post-test, in the control class it was found that 22% students were on the category of level 1, 16% students were in the category of level 2, 10% students were in the category of level 3, and 2% students were in the category of level 4. Now, the analyze the result on post-test by using the inferential statistic.

Table 9. The analysis of the normality test of both class for the post-test

| Group         | Kolmogorov-Smirnovα | Shapiro-Wilk |
|---------------|---------------------|--------------|
| POST TEST     | Statistic df Sig.   | Statistic df Sig. |
| Experimental Class | .135 28 .200' .956 | 28 .280 |
| Control Class  | .123 33 .200' .957 | 33 .216 |

The results of the normality test of each group, it was obtained the significance value are the control is 0.200, the experimental is 0.200. The significance value of the two classes is greater than the value of α (0.05), meaning that two classes of research samples are normally distributed.

Table 10. The result of mean scores of post-test between control and experimental class

| Group         | N   | Mean    | Std. Deviation | Std. Error Mean |
|---------------|-----|---------|----------------|-----------------|
| POST TEST     |     |         |                |                 |
| Experimental Class | 28 | 90.7143 | 2.65075 | .50094 |
| Control Class | 33 | 86.8485 | 4.40256 | .76639 |

Table 10 present the post-test result of control class it mean 86.8485 and Std. Deviation 4.40256 while the experimental class it mean 90.7143 and Std Deviation 2.65075.
Table 11. The comparison of post test score of experimental class and control class score using independent sample t-test

| Levene’s Test for Equality of Variances | t-test for Equality of Means |
|----------------------------------------|-------------------------------|
| F          | Sig. | T     | df | Sig. (2-tailed) | Mean Difference | Std. Error Difference | Lower  | Upper  |
| Equal variances assumed                 | 5.886 | .018  | 4.061 | 59  | .000  | 3.86580  | .95199 | 1.96087 | 5.77073 |
| Equal variances not assumed             | 4.222 | 53.591 | .000  | 3.86580  | .91558 | 2.02984  | 5.70176 |

Table 11 shows that the result of t-test indicates sign. (2-tailed) of independent sample t-test of post test is 0.000 (p =< 0.05) thus it is significant.

Figure 5. The distribution of student mathematical proving skills in the control class based on the post-test result

Then, based on the post-test result it showed the mathematical proving skills of the control class is very active of 15%, active of 15%, less active of 12%, and inactive of 8%. The results of both classes can be seen in Figure 3 and Figure 4.
Figure 6. the distribution of student mathematical proving skills in the experimental class based on the post-test result

4. Portrait phase

The portrait phase is taken to draw a process. The portrait phase taken to draw a process of mathematical proof of thinking skills. I have chosen three objects from the experimental and control groups as illustration. Interviews were conducted on selected subjects to find out their thoughts the process of completing dominating metric dimension.

From the picture above, there are two graphs that are the same. For working on the problems asked to look for labeling points, sides, cardinalities of points and sides, dominating set. In figure 5, the second graph has one dominating set namely $x_1$ dan $y_2$. In the first graph the dominating set is $x_1$ dan $y_1$. This ability is clearly explained by the results of the interview below.

Research: Can you understand the dominating metric dimension number?

Student 1: Yes, I’m understand dominating metric dimension number.

Research: How do you know about that?
Student 1: From LKM Research: What did you do to solve the problem?
Student 1: First, determine the graph that we will look for as the dominating set. Secondly give labels on vertices and edge. Then determine the cardinality of vertices and edge. After that we look for the dominating set and then determine the dominating metric dimension number. Where for the dominating set we look for as minimum as possible and the dominating set can dominate other vertices.
Research: Do you have difficulty in solving this problem?
Student 1: No, I don’t.

![Diagram](image)

**Figure 8.** The phase portrait student mathematical proving skills process of the high criteria

![Graph](image)

**Figure 9.** Graph of the subject 2

From the picture above, there are two graphs that are the same. For working on the problems asked to look for labeling points, sides, cardinalities of points and sides, dominating set. In figure 5, the second graph has one dominating set namely $x_1$ dan $y_2$. In the first graph the dominating set is $x_1$ dan $y_1$. This ability is clearly explained by the results of the interview below.

Research: Can you understand the dominating metric dimension number?
Student 2: Yes, I understand dominating metric dimension number.
Research: How do you know about that?
Student 2: From LKM
Research: What did you do to solve the problem?
Student 2: First, determine the graph that we will look for as the dominating set. Secondly give labels on vertices and edge. Then determine the cardinality of vertices and edge. After that we look for the dominating set and then determine the dominating metric...
dimentions number. Where for the dominating set we look for as minimum as possible and the dominating set can dominate other vertices.

Research: Do you have difficult in solve this problem?
Student 2: Yes
Research: What difficulties did you experience?
Student 2: Determine the dominating set of graph n

![Graph](image)

**Figure 10.** the phase portrait student mathematical proving skills process of the average criteria

The following is an analysis of the work of students dominating metric dimention to determine the completion process of the LKM so that students find the final results. Analysis of the results of student work is carried out to support that the information provided in the interview matches the work.

5. Discussion
This study aims to analyze the implementation of inquiry-based learning (IBL) on mathematical proving skills in solving dominating metric dimension numbers problems. The findings of this study indicate that the research conducted in this study has a significant significance on the alleged abilities of students in the experimental class. The results showed that an increase in student learning outcomes and thinking skills were seen from the post-test. Experimental class grades are significantly better than those supported by IBL learning to improve mathematical proofing thinking skills. The results obtained in the control class are as follows: 21% in the category of less speculated, 29% in the category of quite speculated, 27% in the category of speculated and 23% in the category of highly speculated. Whereas in the experimental class, 14% were in the less suspected category, 27% in the reasonably suspected category, 32% in the presumptive category, and 27% in the high presumptive category. Independent test results obtained the sig value variance. (2-tailed) 0.007 <0.05. It can be concluded that the post-test results between the control class and the experimental class have significant differences after IBL-based learning is implemented. The results of this study are in line with the theory that is converted by [3] that research data-based learning methods help students to pass a predetermined value target. This result achieved by the experimental class shows that learning objectives play an important role in solving students' problems.

6. Conclusion
Based on the results of the study, the application of IBL has a significant effect on students' mathematical thinking abilities in the experimental class. The results showed an increase in student learning and thinking skills seen from the post-test. The experimental class produces better grades.

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