On the Geometry of the
Batalin-Vilkovisky Formalism

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Abstract

An invariant definition of the operator $\Delta$ of the Batalin-Vilkovisky formalism is proposed. It is defined as the divergence of a Hamiltonian vector field with an odd Poisson bracket (antibracket). Its main properties, which follow from this definition, as well as an example of realization on Kählerian supermanifolds, are considered. The geometrical meaning of the Batalin-Vilkovisky formalism is discussed.
1 Introduction

The Batalin–Vilkovisky formalism (BV–formalism) is the most general method of quantization of gauge field theories [1], [2]. In recent years the interest in studying its geometrical nature has increased. It was stimulated by Witten’s paper [3], where the necessity of such an investigation is pointed out and particularly for the formulation of the background independent open-string field theory on the base of the BV-formalism. The realization of this program began in [4], [5].

It is known that the BV-formalism uses unusual structures – odd Poisson brackets (antibrackets) and the operator ∆. As one of the main obstacles to the construction of a background independent open string field theory, indicated by Witten [3], was the nonexistence of an invariant definition of the operator ∆, and of a naturally defined integral measure. However, such a definition of the operator ∆ was shown by one of us (O. K.) in [6] before the cited Witten’s paper appeared (see also [7]). Its realization on Kählerian supermanifolds [8] and its simplest properties [9] were considered.

In the present paper we study the operator ∆ in more detail. We propose an invariant definition of this operator and show that the condition of its nilpotency defines (in some sense) the choice of the integration measure.

In Section 2 we propose an invariant definition of the operator ∆ on supermanifolds, given by the odd symplectic structure and by the volume form as the divergence of the Hamiltonian vector field. We show that all the relations between the antibrackets and the operator ∆, which are satisfied for canonical ones in the BV-formalism, are satisfied for any generalized operator ∆ and the corresponding odd brackets. However the nilpotency condition for such an operator holds only for a certain class of the integral density.

In Section 3 we consider the realization of the operator ∆ on supermanifolds, given by the odd (and even) Kählerian structure, and show that it is nilpotent, if the integral density is the characteristic class of the basic Kählerian manifold (the function from Chern classes). Then it corresponds to the divergence operator δ = *d* of the basic manifold.

In Section 3 we discuss the geometrical nature of the Batalin-Vilkovisky formalism.

When this paper was in preparation, we received two very important papers of A.S. Schwarz [10], [11] where the geometry of the BV-formalism is analyzed in detail and in particular the same definition of the operator ∆, as in [6], [8], [9] and in the present paper is given.
2 Odd Poisson Brackets and Operator $\Delta$

The odd Poisson bracket (odd bracket, antibracket, Buttin bracket) of the functions $f$ and $g$ on the supermanifold $\mathcal{M}$ is defined by the following conditions \cite{12}, \cite{13}:

\begin{align}
\{f, ga + hb\}_1 &= \{f, g\}_1 a + \{f, h\}_1 b, \quad \text{where} \quad a, b = \text{const} \\
p(\{f, g\}_1) &= p(f) + p(g) + 1 \quad \text{(grading condition)} \\
\{f, g\}_1 &= -(-1)^{(p(f)+1)(p(g)+1)} \{g, f\}_1 \quad \text{"antisymmetricity" condition} \tag{2.1} \\
\{f, gh\}_1 &= \{f, g\}_1 h + (-1)^{(p(f)+1)p(g)} g \{f, h\}_1 \quad \text{(Leibnitz rule)} \\
(-1)^{(p(f)+1)(p(h)+1)} \{f, \{g, h\}_1\}_1 + \text{cycl. perm.}(f, g, h) &= 0 \quad \text{(Jacobi id.)}
\end{align}

Locally, the odd bracket can be written as:

\begin{equation}
\{f, g\}_1 = \frac{\partial R f}{\partial x^A} \Omega^{AB} \frac{\partial L g}{\partial x^B} \tag{2.2}
\end{equation}

where $\Omega^{AB}$ satisfies to conditions

\begin{align}
p(\Omega^{AB}) &= p(A) + p(B) + 1 \quad \text{(grading condition)} \\
\Omega^{AB} &= -(-1)^{(p(A)+1)(p(B)+1)} \Omega^{BA} \quad \text{"antisymmetricity" condition} \\
(-1)^{(p(A)+1)(p(C)+1)} \frac{\partial R \Omega^{AB}}{\partial x^D} \Omega^{DC} + \text{cycl. perm.}(A, B, C) &= 0 \quad \text{(Jacobi id.)}
\end{align}

where $x^A$ are the local coordinates of $\mathcal{M}$, $p_A \equiv p(x^A)$. $\frac{\partial R}{\partial x^A}$ and $\frac{\partial L}{\partial x^A}$ denote corresponding right and left derivatives. They are connected with each other by:

$$
\frac{\partial R f}{\partial x^A} = (-1)^{(p(f)+1)p_A} \frac{\partial L f}{\partial x^A}.
$$

If $\mathcal{M}$ has an equal number of even and odd coordinates, the odd bracket can be nondegenerate. Then one can associate with it the odd symplectic structure

$$
\Omega = dx^A \Omega^{AB} dx^B \tag{2.3}
$$

where $\Omega_{AB} \Omega^{BC} = \delta^C_A$. This form is closed because of the Jacobi identities (2.1). Locally, one can reduce (2.3) to the canonical form \cite{13}:

$$
\Omega^{\text{can}} = \sum_{i=1}^N dx^i \wedge d\theta_i \tag{2.4}
$$

where $(x^i, \theta_i)$ are some local coordinates (Darboux coordinates) ($p(x^i) = 0, p(\theta_i) = 1$).

The corresponding odd bracket takes the form

$$
\{f, g\}_1 = \sum_{i=1}^N \left( \frac{\partial R f}{\partial x^i} \frac{\partial L g}{\partial \theta_i} + \frac{\partial R f}{\partial \theta_i} \frac{\partial L g}{\partial x^i} \right) \tag{2.5}
$$
Transformations preserving odd symplectic structures (or odd brackets) have (locally) the hamiltonian form:
\[ \mathcal{L}_V \Omega = 0 \quad \text{iff} \quad V = \{\cdot, H\}_1 \equiv D_H \quad (2.6) \]
where \( H \) is an arbitrary function (Hamiltonian) on \( \mathcal{M} \), \( \mathcal{L}_V \) denotes the Lie derivative along the vector field \( V \).

It is well known that any supermanifold can be associated with some vector bundle \([12]\). The odd symplectic structure can be globally defined on the supermanifolds which are associated with the cotangent bundles of manifolds.

Let \( T^*\mathcal{M} \) be the cotangent bundle of the manifold \( \mathcal{M} \). \( x^i \) are local coordinates on \( \mathcal{M} \) and \( (x^i, v_i) \) are the corresponding local coordinates on \( T^*\mathcal{M} \). From map to map
\[ x^i \rightarrow \tilde{x}^i = \tilde{x}^i(x), \quad v_i \rightarrow \tilde{v}_i = \sum_{i=1}^{N} \frac{\partial x^j}{\partial \tilde{x}^i} v_j. \quad (2.7) \]
Considering for every map the superalgebra generated by \( (x^i, \theta_i) \), where the \( x^i \) are even and the \( \theta_i \) are odd coordinates, transforming from map to map like \( (x^i, v_i) \) in ref eq:trans) \( (v \leftrightarrow \theta) \), we come to the supermanifold \( \mathcal{M} \) which is associated with \( T^*\mathcal{M} \) in the coordinates \( (x^i, \theta_i) \).

Obviously, on this supermanifold, in the coordinates \( (x^i, \theta_i) \), one can globally define the canonical odd symplectic structure \([24] [13]\).

For the coordinates \( (x^i, \theta_i) \) on \( \mathcal{M} \), one can admit a more general class of transformations:
\[ x^i \rightarrow \tilde{x}^i(x, \theta) \quad \theta_i \rightarrow \tilde{\theta}_i(x, \theta) \]
which do not correspond to \( (2.7) \). In particular, if \( \theta_i \rightarrow \tilde{\theta}_i = \omega^{ij} \theta_j \), where \( \omega_{ij} \) is the matrix of some nondegenerate Poisson bracket on \( \mathcal{M} \), then the supermanifold \( \mathcal{M} \) in the coordinates \( (x^i, \tilde{\theta}^i) \) is associated with the tangent bundle \( T\mathcal{M} \) of \( \mathcal{M} \), e.g. \( \tilde{\theta}^i \) transform under \( (2.7) \) as \( dx^i \).

On the supermanifolds which can be associated in some coordinates with the tangent or cotangent bundle the superstructures are evidently reduced to standard geometrical objects.

Concerning the integration, the properties of the odd brackets strongly differ from the properties of odd brackets \([5], [7]\), such as:

\[ \begin{align*}
- &\quad \text{the odd bracket hasn’t an invariant volume form and invariant integral densities}; \\
- &\quad \text{it has semidensities, which depend on higher order derivatives}.
\end{align*} \]

The first of this properties plays an essential role in the Batalin–Vilkovisky quantization formalism. Using this property, we can construct on \( \mathcal{M} \) an invariant generalization of the important object of the BV-formalism – the operator \( \Delta \).
Let the supermanifold $\mathcal{M}$ be provided with the odd symplectic structure (2.3) and the volume form

$$dv = \rho(x, \theta)d^Nx d^N\theta.$$  

(2.8)

Here $\rho(x, \theta)$ is some integral density. Under coordinate transformation $\tilde{x}^A = \tilde{x}^A(x)$, it transforms as:

$$\tilde{\rho}(\tilde{x}) = \rho(x(\tilde{x})) \text{Ber} \frac{\partial^R x^A}{\partial \tilde{x}^B}.$$  

(2.9)

On this supermanifold one can invariantly define a second order odd differential operator, which we call the ”generalized operator $\Delta$”, and which is invariant under the transformations preserving the symplectic structure and the volume form [6]. Its action on a function $f(x, \theta)$ is the divergence of the Hamiltonian vector field $D_f$ with the volume form $dv$:

$$\Delta f = \frac{1}{2} \text{div}_\rho D_f \equiv \frac{1}{2} \mathcal{L}_{D_f} dv.$$  

(2.10)

where $\mathcal{L}_{D_f}$ is the Lie derivative along $D_f$ [12], [14]. In coordinate form:

$$\Delta f = \frac{1}{2 \rho} \frac{\partial^R}{\partial x^A} \left( \rho \{ x^A, f \} \right)_1.$$  

(2.11)

It has no analog within even symplectic structures. The oddness of the Poisson bracket (2.2) forces the nontrivial grading of $\Delta$, and the “antisymmetricity” condition (2.1) forces its dependence on second derivatives.

If the Poisson bracket in (2.10) is canonical, and $\rho = \text{constant}$, the generalized operator $\Delta$ takes the canonical form

$$\Delta_{\text{can}} = \frac{\partial^R}{\partial x^i} \frac{\partial^L}{\partial \theta_i}.$$  

(2.12)

used in the BV-formalism.

From the Leibnitz rule (2.1) and the definition (2.10) follows

$$(-1)^{p(g)} \{ f, g \}_1 = \Delta(fg) - f \Delta g - (-1)^{p(g)}(\Delta f)g$$  

(2.13)

From the Jacobi identity (2.1) and the definition (2.10) follows

$$\Delta \{ f, g \}_1 = \{ f, \Delta g \}_1 + (-1)^{p(g)+1} \{ \Delta f, g \}_1$$  

(2.14)

The density transformation rule (2.9) implies for the generalized operator $\Delta$ the following transformation rule under canonical transformations:

$$\Delta' f = \Delta f + \frac{1}{2} \{ \log \mathcal{J}, f \}_1,$$  

(2.15)

where $\mathcal{J}$ is the Jacobian of the canonical transformation of the odd bracket, $\Delta'$ is the generalized operator $\Delta$ in the new coordinates. For example, let us demonstrate the derivation of (2.14):

$$\Delta \{ f, g \}_1 dv = \mathcal{L}_{D_{\{ f, g \}_1}} dv =$$

$$= \left( \mathcal{L}_{D_f} \mathcal{L}_{D_g} - (-1)^{(p(f)+1)(p(g)+1)} \mathcal{L}_{D_g} \mathcal{L}_{D_f} \right) dv =$$

$$= \mathcal{L}_{D_f} \Delta g dv - (-1)^{(p(f)+1)(p(g)+1)} \mathcal{L}_{D_g} \Delta f dv =$$

$$= \{ f, \Delta g \}_1 + (-1)^{p(g)+1} \{ \Delta f, g \}_1 dv$$
Let us write the following useful expressions, too:

\[ \Delta f(g) = f'(g) \Delta g + \frac{1}{2} f''(g) \{g, g\}_1, \]  

(2.16)

where \( f(g) \) is an even complete function, and \( g \) is an even function on \( \mathcal{M} \).

The properties (2.13) - (2.16) are satisfied for any \( \rho \) and in the same manner as the relations between canonical Poisson brackets (2.5) and (2.12) in the BV-formalism [2], [3]. This can be derived in the same way as (2.14).

However (2.12) satisfies the nilpotency condition

\[ \Delta^2 = 0 \]  

(2.17)

which is very important in the BV-formalism.

The latter condition is violated for arbitrary \( \rho(x, \theta) \). Indeed, if we have two densities \( \rho \) and \( \bar{\rho} \), and \( \bar{\rho} = \lambda \rho \), \( p(\lambda) = 0 \), then the corresponding operators \( \Delta \) are related by

\[ \Delta_{\bar{\rho}} f = \Delta_{\rho} f + \frac{1}{2} \{\log \lambda, f\}_1 \]  

(2.18)

It is easy to see that

\[ \Delta_{\bar{\rho}}^2 f = \Delta_{\rho}^2 f + \{\Gamma_\lambda, f\}_1, \]  

(2.19)

where

\[ \Gamma_\lambda = \lambda^{-\frac{1}{2}} \Delta_{\rho} \lambda^{\frac{1}{2}}, \quad p(\Gamma_\lambda) = 1 \]  

(2.20)

If for some \( \Delta_{\rho} \) the nilpotency condition (2.17) is satisfied, then it is also satisfied for \( \Delta_{\bar{\rho}} \) (2.18) if \( \Gamma_\lambda = \text{odd constant} = 0 \).

For example, if the symplectic structure is canonical, (2.17) holds if \( \rho(x, \theta) \) satisfies the equation

\[ \Delta_{\text{can}} \sqrt{\rho} = 0. \]  

(2.21)

But this is the master equation of the BV-formalism for the action \( S = -\frac{i}{2} \log \rho \).

The geometrical meaning of the nilpotency condition (and correspondingly of the master equation) will be illustrated on a simple example in the next Section.

## 3 Example: The operator \( \Delta \) on Kählerian Supermanifolds

As we saw in the previous section, in contrary to the case of an even symplectic structure, on supermanifolds with an odd symplectic stricture there arises a nontrivial differential geometry.

It is sufficient to show the correspondence between the generalized operator \( \Delta \) and geometrical objects on basic manifolds in the case of a Kählerian basic manifold, because
on Kählerian manifolds the symplectic structure corresponds to a Riemannian one, and a Riemannian structure has a rich differential geometry. Moreover, in this case there also exists on $\mathcal{M}$ an even Kählerian structure, and, using it, we can construct a natural integral density [8], [9].

Let $\mathcal{M}$ be a complex supermanifold, and $z^A$ local complex coordinates on $\mathcal{M}$. A symplectic structure $\Omega^\kappa$ – here and further $\kappa = 0(1)$ if the symplectic structure is even (odd) – on $\mathcal{M}$ is called Kählerian, if in local coordinates $z^A$ it takes the following form:

$$\Omega^\kappa = i(-1)^{p(A)(p(B)+\kappa+1)}g^\kappa_{AB}dz^A \wedge d\bar{z}^B,$$

(3.1)

where

$$g^\kappa_{AB} = (-1)^{(p(A)+\kappa+1)(p(B)+\kappa+1)+\kappa}g^\kappa_{BA}, \quad p(g^\kappa_{AB}) = p_A + p_B + \kappa.$$

Then there exists a local real even (odd) function $K^\kappa(z, \bar{z})$ (Kählerian potential), such that

$$g^\kappa_{AB} = \frac{\partial L}{\partial z^A} \frac{\partial R}{\partial \bar{z}^B} K^\kappa(z, \bar{z})$$

(3.2)

To $\Omega^\kappa$ there corresponds the Poisson bracket

$$\{f, g\}_\kappa = i \left( \frac{\partial R}{\partial z^A} g^\kappa_{AB} \frac{\partial L}{\partial \bar{z}^B} - (-1)^{(p(A)+\kappa)(p(B)+\kappa)} \frac{\partial R}{\partial z^A} g^\kappa_{AB} \frac{\partial L}{\partial \bar{z}^B} \right),$$

(3.3)

where

$$g^\kappa_A B g^\kappa_{B C} = \delta^A_C, \quad g^\kappa_{AB} = (-1)^{(p(A)+\kappa)(p(B)+\kappa)} g^\kappa_{BA}.$$

Its satisfies the conditions of reality and "antisymmetry" 

$$\overline{\{f, g\}_\kappa} = \{\bar{f}, \bar{g}\}_\kappa, \quad \{f, g\}_\kappa = -(-1)^{(p(f)+\kappa)(p(g)+\kappa)} \{g, f\}_\kappa,$$

(3.4)

and the Jacobi identities :

$$(-1)^{(p(f)+\kappa)(p(h)+\kappa)} \{f, \{g, h\}_\kappa\}_\kappa + \text{cycl.perm.}(f, g, h) = 0$$

(3.5)

Let $\mathcal{M}$ be associated with the tangent bundle $TM$ of the Kählerian manifold $M$, and $z^A = (w^a, \theta^a)$ local coordinates on it, $\theta^a$ transforming from map to map like $d\theta^a$. Let

$$g_{\bar{a}b}(w, \bar{w}) = \frac{\partial^2 K(w, \bar{w})}{\partial \theta^a \partial \bar{\sigma}^b}$$

(3.6)

be a Kählerian metric on $M$, with $K$ its Kählerian potential [15]. Then the local functions

$$K_0(w, \bar{w}, \sigma, \bar{\sigma}) = K(w, \bar{w}) + ig_{\bar{a}b}(w, \bar{w})\sigma^a \bar{\sigma}^b, \quad p(K_0) = 0$$

(3.7)

$$K_1(w, \bar{w}, \sigma, \bar{\sigma}) = \epsilon \frac{\partial K(w, \bar{w})}{\partial w^a} \sigma^a + \bar{\epsilon} \frac{\partial K(w, \bar{w})}{\partial \bar{w}^a} \bar{\sigma}^a \quad p(K_1) = 1$$

(3.8)

(where $\epsilon$ is an arbitrary complex constant) correctly define an even and an odd symplectic structures on $\mathcal{M}$ (this is not the most general form of Kählerian potentials on such supermanifolds [S]).
The odd Kählerian potential (3.8) defines on $\mathcal{M}$ the following odd bracket:

$$\{f, g\}_1 = \frac{i}{\epsilon} \left( \frac{\partial^R f}{\partial \theta^a} \nabla^a g - \nabla^a f \frac{\partial^L g}{\partial \theta^a} \right) + c.c$$  \hspace{1cm} (3.9)

where

$$\nabla^a = g^{ab} \nabla_b \quad \nabla_a = \frac{\partial}{\partial w^a} - \Gamma^c_{ab} \theta^b \frac{\partial}{\partial \theta^c},$$  \hspace{1cm} (3.10)

and $\Gamma^c_{ab} = g^{dc} g_{ad,b}$ are the Christoffel symbols of the Kählerian metric on $M$.

It is easy to see, that in the coordinates $(w^a, \theta^a = ig_a \bar{b})$, in which $M$ is associated with $T^* M$, the odd Poisson bracket takes the canonical form.

The generalized operator $\Delta$ corresponding to (3.9) takes the form

$$\Delta f = \left( \frac{1}{\epsilon} \nabla^a \frac{\partial^L}{\partial \theta^a} + \frac{1}{\epsilon} \nabla^a \frac{\partial^L}{\partial \theta^a} \right) f + \frac{1}{2} \{ \log \rho, f \}_1$$  \hspace{1cm} (3.11)

If $\nabla_a \rho = 0$ (or, in fact, if $\rho$ is a characteristic class of $M$) then

$$\Delta f = \frac{1}{\sqrt{\rho}} \left( \frac{1}{\epsilon} \nabla^a \frac{\partial^L}{\partial \theta^a} + \frac{1}{\epsilon} \nabla^a \frac{\partial^L}{\partial \theta^a} \right) (\sqrt{\rho} f),$$  \hspace{1cm} (3.12)

is obviously nilpotent.

The invariant density which corresponds to (3.7) is

$$\rho = det (\delta^a_b + i R^a_{bc} \theta^c \bar{d}^l)$$  \hspace{1cm} (3.13)

where $R^a_{bcd} = (\Gamma^a_{bc,d})$ is the curvature tensor on $M$. It is associated with the generating functions of the Chern classes of the underlying Kählerian manifold [15].

Obviously (3.12) corresponds to the operator of covariant divergence $\delta = * d *$ on $M$ with some effective weight.

4 Discussion

As we have seen in the previous Sections, the operator $\Delta$ has a simple geometrical nature on the supermanifolds associated with the cotangent bundles of manifolds. Obviously, the same construction holds, if we replace the basic manifold $M$ in the previous Sections by some supermanifold $\mathcal{M}_0$. Then if $x^i$ are local coordinates on $\mathcal{M}_0$ ($p(x^i) \neq 0$), then, on the supermanifold $\mathcal{M}$ which is associated with the cotangent bundle $T^* \mathcal{M}_0$, one can naturally define the odd Poisson bracket (2.3) where $\theta_i$ corresponds to coordinates of the bundle, $p(\theta_i) = p(x^i) + 1$. These coordinates are the analogs of the antifields of the BV-formalism.

But what is the reason for the introduction of antifields (and, correspondingly, for the structure of supermanifolds with an odd symplectic structure) in the BV-formalism?
In our opinion, this is connected to the peculiarity of the integration on supermanifolds. Indeed, if we have some differential form \( \omega(x^i, dx^i) \) on the supermanifold \( \mathcal{M}_0 \), its integral over \( \mathcal{M}_0 \) defined in the following way [13], [14]:

\[
\int_{\mathcal{M}_0} \omega \equiv \int_{\hat{\mathcal{M}}_0} \omega(x^i, \theta^i) D(x, \theta)[dx d\theta]
\] (4.1)

where \( p(\theta^i) = p(x^i) + 1 \), and \( \hat{\mathcal{M}}_0 \) denotes the supermanifold associated with the tangent bundle of (supermanifold) \( \mathcal{M}_0 \) (i.e. \( \theta^i \) transforms like \( dx^i \)), then \( D(x, \theta) \) is the natural density on \( \hat{\mathcal{M}}_0 \) [17].

Transiting from the description on \( \hat{\mathcal{M}}_0 \) to that on \( \mathcal{M} = T^\ast \mathcal{M}_0 \) – the supermanifold associated with the cotangent bundle of the supermanifold \( \mathcal{M}_0 \) – we saw that the integral (4.2) takes the form of the partition function of the BV-formalism.

Correspondingly, the master-equation of the BV-formalism corresponds to the close-ness of the initial differential form. This is clearly seen in the case where the basic manifold, is Kählerian (in the general case this proposition was strongly proved in [10]. Then the gauge invariance of the partition function in the BV-formalism follows from Stokes’ theorem [17], [14].

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