DIFFUSIVE NUCLEAR BURNING OF HELIUM ON NEUTRON STARS

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ABSTRACT

Diffusive nuclear burning (DNB) of H by an underlying material capable of capturing protons can readily consume H from the surface of neutron stars (NSs) during their early cooling history. In the absence of subsequent accretion, it will be depleted from the photosphere. We now extend DNB to He, motivated by the recent observation by Ho & Heinke of a carbon atmosphere on the NS in the Cassiopeia A supernova remnant. We calculate the equilibrium structure of He on an underlying α capturing material, accounting for thermal, mass defect, and Coulomb corrections on the stratification of material with the same zeroth order μ$_c$ = A/Z. We show that Coulomb corrections dominate over thermal and mass defect corrections in the highly degenerate part of the envelope. We also show that the bulk of the He sits deep in the envelope rather than near the surface. Thus, even if the photospheric He abundance is low, the total He column could be substantially larger than the photospheric column, which may have implications for rapid surface evolution ($\approx 1$ yr timescales) of NSs. When nuclear reactions are taken into account, we find that for base temperatures $\gtrsim 1.6 \times 10^6$ K, He is readily captured onto C. As these high temperatures are present during the early stages of NS evolution, we expect that the primordial He is completely depleted from the NS surface like the case for primordial H. We also find that magnetic fields $\lesssim 10^{12}$ G do not affect our conclusions. Armed with the results of this work and our prior efforts, we expect that primordial He and H are depleted, and so any observed H or He on the surfaces of these NS must be due to subsequent accretion (with or without spallation). If this subsequent accretion can be prevented, the underlying mid-Z material would be exposed.

Key words: diffusion – nuclear reactions, nucleosynthesis, abundances – pulsars: general – stars: abundances – stars: interiors – stars: neutron

1. INTRODUCTION

The surface composition of neutron stars (NSs) remains an outstanding problem. Observations of young NSs have failed to find indisputable signatures of any particular element on their surface. Some NSs completely lack any spectral features such as RX J1856.5−3754 (Burwitz et al. 2001, 2003; Pons et al. 2002; Ho et al. 2007). However, when spectral features are seen, the degeneracies in our (currently) limited understanding of the chemistry and physics of highly magnetized material makes secure identification a challenge. Theory also fails to constrain the initial conditions, i.e., supernova fallback, spallation, subsequent accretion, that would make a convincing case for a particular composition.

Even after the initial composition is set, subsequent evolution can occur. For instance, the small mass ($\approx 10^{-17} M_\odot$) and rapid diffusion time ($\approx 1$ s) at the NS photosphere suggest that it should consist of H due to gravitational settling. However, we showed in Chang & Bildsten (2003, 2004; hereafter Papers I and II) and Chang et al. (2004; hereafter Paper III) that H is easily destroyed by diffusive nuclear burning (DNB). The central idea behind DNB, which was first proposed by Chiu & Salpeter (1964; see also Rosen 1968), is that H can diffuse to great depth where the temperature and density are sufficiently large to consume H by proton captures onto heavier elements. In Papers I and II, we showed that this process is so effective that we expect NS surfaces to be depleted of any primordial H. The observation of H on the surfaces of NS would then point to late-time or continuous accretion. This conclusion is insensitive to the strength of the magnetic field and the size of an inert He buffer that sits between the H and the underlying proton capturing elements (Paper III).

With all the H consumed, one would expect surfaces of He. However, recent observations by Ho & Heinke (2009) suggest that the NS at the center of the Cassiopeia A supernova remnant has a carbon surface with an effective temperature $T_e = 1.8 \times 10^6$ K and radius $R = 12–14$ km. This paper shows that He is also vulnerable to DNB on the surface of Cas A. All primordial H/He is consumed during its early cooling history, exposing the underlying material. If subsequent accretion does not cover this underlying material, we would expect a population of NSs with mid-Z surfaces.

We first review the physics of DNB in Section 2, outlining the major results of Papers I–III. We then discuss the physics of material in diffusive equilibrium on NSs in Section 3. We show how to calculate the electric field in degenerate material accounting for thermal, mass defect, and Coulomb corrections. Using this electric field, we then calculate the structure of an envelope composed of He and C. We show that the diffusive tail of He penetrates sufficiently deep to be readily captured by C and find that He is easily consumed in young, hot NSs for a range of α-capturing elements. In Section 5, we conclude with a discussion of the implications of our work for young NS surface compositions.

2. BASIC PHYSICS FOR HYDROGEN DEPLETION

The diffusion rates on NSs imply that H will rapidly float to the photosphere where the density is $\rho \approx 1$ g cm$^{-3}$ and the temperature of $T \approx 10^6$ K. Hydrogen at this density and temperature will not burn in a Hubble time. However,
$10^3$–$10^4$ cm below the photosphere, the higher densities ($\rho > 10^8$ g cm$^{-3}$) and temperatures ($T \sim 1$–$10 \times 10^8$ K) allow H to burn. The H that diffuses down to this depth will be readily captured, driving an H current from the surface to the burning layer. Over time, this results in the total depletion of H.

In Paper I, we studied the case where the rate of H burning is sufficiently slow that H always remain in diffusive equilibrium, and the rate limiting step is the rate at which H is consumed by nuclear reactions in the burning layer. This burning layer is defined by the competing effects of an exponential decline in H number density and a rising temperature. For cooler NSs ($T_c < 10^8$ K), DNB takes place in this nuclear-limited regime. For hotter NSs ($T_c > 1.5 \times 10^8$ K), DNB is not limited by nuclear reactions in the burning layer, but rather by the rate at which H can diffuse down to the burning layer, i.e., the diffusion-limited regime (Paper II). In this case, the burning occurs in a thin layer at which the nuclear burning time becomes comparable to the downward diffusion time, breaking the assumption of diffusive equilibrium (see Papers I and II). Below this burning layer, the hydrogen concentration profile is rapidly cut off.

In Papers I and II, we demonstrated that DNB can easily consume all the H on NS surfaces early in their cooling history. We also studied the effect of magnetic fields on the rate of H burning and found that for fields $< 10^12$ G, this basic evolution is unaffected. In Paper III, we extended our study of DNB to magnetar surfaces. There, we showed that the high temperatures associated with magnetars, i.e., soft gamma repeaters and anomalous X-ray pulsars, allow H to be captured onto elements as heavy as Fe. In addition, we showed that the effect of an inert He buffer between the H and C does little to damp the effectiveness of DNB. Our calculation of the structure of an anomalous X-ray pulsars, allow H to be captured onto elements of the ion, $A_{i}/Z_{i}$, the differentiating force between trace and background ion species $i$, Equation (1) becomes

$$\frac{\partial P_i}{\partial r} = -n_i m_p g \left( A_i - Z_i \frac{A_i}{Z_i} \right).$$

For $A_{j}/Z_{j} = A_{i}/Z_{i}$, the differentiating force between trace and background ion species is zero for the zeroth-order electric field of Equation (3). Note this is not the case of H on C in Papers I, II, and III, which have very different $\mu_e = A/Z$ and so the zeroth-order electric field imposes a differentiating force. Here ions with the same $A_{i}/Z_{i}$ experience no differentiating force (ignoring mass defect corrections) in a strongly degenerate plasma. Hence, higher order corrections to the electric field due to thermal effects and Coulomb interactions determine the differentiating forces between ion species and the equilibrium compositional structure.

### 3.1. Thermal, Mass Defect, and Coulomb Corrections

We now give a qualitative discussion of the thermal and Coulomb corrections to the electric field in degenerate plasmas. We first consider corrections to the zeroth-order electric field due to purely thermal effects because this physics is simpler and illustrative. Using Equations (1) and (2) and the condition of charge neutrality, the electric field is (Paper I)

$$eE = \frac{A_{i}}{Z_{i}} m_p g \left( 1 - \frac{n_1 k_B T}{\gamma P_e} \right),$$

where $z = r - r_{\text{ref}}$ is the height measured from a reference point, $r_{\text{ref}}$. Note that we have ignored the mass of the electron in the derivation of Equation (5) (see Paper I for a detailed derivation). As expected, we reproduce the zeroth-order electric field (Equation 3) from Equation (5) for a degenerate plasma.

Now consider a two-component plasma where $n_1 \gg n_2$ (1 is the background and 2 is the trace) with the same $\mu_e$, i.e., $A_{1}/Z_{1} = A_{2}/Z_{2}$. For a degenerate plasma ($P_e \gg n_2 k_B T$), we expand Equation (5) in the ratio of the thermal pressure to the degenerate pressure, $n_1 k_B T / P_e$, to find

$$eE = \frac{A_{i}}{Z_{i}} m_p g \left( -\frac{n_1 k_B T}{\gamma P_e} \right),$$

where $\gamma = \partial \ln P_e / \partial \ln n_e$, demonstrating that the thermal correction yields a differentiating force between ions with the same $A_i/Z_i$. We should note that Hameury et al. (1983) derived similar results.

Plugging Equation (6) into hydrostatic balance for each ion, we find

$$\frac{\partial \ln P_i}{\partial r} = -\frac{\rho g}{\gamma P_e},$$

$$\frac{\partial \ln P_2}{\partial r} = \frac{Z_2 \rho g}{Z_1 \gamma P_e}.$$

In Papers I, II, and III, the electric field in Equations (1) and (2) results from the condition of charge neutrality $n_e = Z_i n_i$. For an ideal, non-degenerate plasma ($P_e = Z P_i$), this electric field is $eE = A_i m_p g/(Z_i + 1)$. For a strongly degenerate plasma ($P_e \gg P_i$), the electric field becomes

$$eE = \frac{A_{i}}{Z_{i}} m_p g.$$
where we presume that the ions obey the ideal gas equation of state, \( P_i = n_i k_B T \). Subtracting these equations from each other and using hydrostatic balance \( \partial P / \partial r = -\rho g \), where \( P \approx P_e \), we find a simple power law between the concentration, \( n_2/n_1 \), and pressure, \( P \),

\[
\frac{\partial \ln n_2/n_1}{\partial \ln P} = \gamma^{-1} \left( \frac{Z_2}{Z_1} - 1 \right). \tag{8}
\]

Equation (8) demonstrates that thermal corrections to the electric field yield a differentiating force which separates lighter ions from heavier ions with the same \( A_i/Z_i \) in a degenerate plasma.

Our discussion of purely thermal effects above, while illustrative ignores mass defect corrections and Coulomb corrections. Mass defect corrections arise from the fact that \( A_i/Z_i \) is only \( \approx A_i/Z_i \) for different \( i \) and \( j \) due the slight variation in atomic mass. While this is usually ignored, as we have done in Papers I, II, and III, the anonymous referee has reminded us that mass defect corrections are important in the degenerate plasmas at this level of approximation that we are interested in (see Blaes et al. 1992). Fortunately this is an easy fix as our derivation of Equation (5) is independent of any assumption regarding \( A \) and \( Z \) of the ions so including mass defect corrections merely requires including the appropriate atomic masses of the ions. However, we will show below that Coulomb corrections ultimately dominate over both thermal and mass defect corrections.

The alert reader may worry that these mass defect corrections may change the results of Papers I, II, and III. However, he or she may rest assured that these corrections are orders of magnitude smaller than the order unity corrections of \( H \) on \( C \) which we discussed in Papers I, II, and III. The only possible exception may be in regard to our discussion of the \( He \) buffer in Paper III.

However as we will show below, Coulomb effects continue to dominate, which argues that our results on the \( He \) buffer in Paper III remain valid.

Figure 1 plots the \( He \) (dashed lines) and \( C \) (solid lines) as a function of column depth, \( y = \int p dz \), for an equilibrium atmosphere with temperature, \( T = 10^8 \) K. Note that \( y = P/g \) in a plane–parallel, constant gravity atmosphere. In thin lines, we show the \( He \) and \( C \) abundance accounting for only thermal corrections to the electric field (thin lines marked by thermal) and accounting for both thermal and Coulomb corrections (thin lines marked by Coulomb; see Section 3.2 for a discussion on how these Coulomb corrections are calculated). In thick lines, we also include the mass defect corrections to the masses of \( He \) and \( C \).

The analytic power law (Equation (8) for \( \gamma = 4/3 \) (dotted line marked by \( y^{-1/2} \)) matches that given by a full numerical calculation for \( He \) for purely thermal corrections (thin dashed line marked thermal). Including mass defect corrections results in a significant change (thick dashed line marked thermal). The mass defect correction drives the abundance of \( He \) away from both the analytic power law and numerical calculation for purely thermal corrections (dotted line marked by \( y^{-1/2} \) and thin dashed line marked thermal, respectively). In fact, the abundance of \( He \) increases rather than decreasing at large column. To understand this, we note that the mass defect correction for \( He \) on \( C \) is

\[
\frac{\Delta F_{\text{defect}}}{m_p g} = (A_{He} - 2Z_{He}) \approx 0.003, \tag{9}
\]

where \( A_{He} \) is the corrected mass number of \( He \). Thermal corrections, using Equation (6), are

\[
\frac{\Delta F_{\text{Th}}}{m_p g} = Z_{He} \frac{A_c n c k_B T}{Z_C} \frac{\gamma P_e}{\gamma P_e} \approx 0.002, \tag{10}
\]

when the plasma is highly degenerate. The mass defect correction is fixed relative to \( m_p g \) for increasing column because it depends only on the mass numbers of \( He \) and \( C \). On the other hand, the thermal correction scales like \( n c k_B T / P_e \), which decreases for increasing column. Moreover, the nucleons in \( He \) are heavier than the nucleons in \( C \), so the larger mass defect correction opposes the thermal correction, reversing the \( He \) abundance trend at large depth.

Coulomb effects, however, overwhelm both the thermal and the mass defect corrections, which we also show in Figure 1 in the lines marked Coulomb. A quick order of magnitude estimate of the size of this Coulomb correction is (see the discussion in Section 3.2 especially around Equation (33) for details)

\[
\frac{\Delta F_{\text{Col}}}{m_p g} = \frac{2 Z_{He} Z_{C}^{2/3} e^2 / a_e}{E_F} \approx 0.006 \tag{11}
\]

for \( y = 10^{10} \) g cm\(^{-3} \) and \( g = 2.43 \times 10^{14} \) cm s\(^{-2} \) and assuming that \( n_C \gg n_He \). Clearly Coulomb corrections dominate over both the thermal and mass defect corrections as the effects of these forces on the number density of ions are exponential. However, we have included all these effects in subsequent calculations in the remainder of this paper.

Figure 1 also shows that the fraction of the \( He \) trace declines against the heavier \( C \) background, but the rate at which this fraction declines strongly depends on whether the corrections

\[\text{http://www.nist.gov/physlab/data/comp.cfm}\]

5 We have used the atomic database compiled by National Institute of Standards and Technology available at http://www.nist.gov/physlab/data/comp.cfm.
to the electric field include thermal effects, mass defect effects, or Coulomb effects. The dot-dashed line is for an abundance that varies inversely with column, i.e., $n_i/n_{\text{tot}} \propto y^{-1}$. For He abundances that decline shallower than this, the total He column, $\gamma_{\text{He}}$, rises substantially with increasing depth. For He abundances that decline steeper than $y^{-1}$, $\gamma_{\text{He}}$ is capped and does not increase substantially with decreasing $y$. If we only include thermal corrections to the electric field, the He abundance follows the power law (Equation (8)), which is shallower than $y^{-1}$. Hence, $\gamma_{\text{He}}$ continually increases. Hence, we find that the bulk of the He sits at a depth of $y_{\text{cut}}$, however, the He abundance is shallower than this and so the He column continuously increases. Hence, we find that the bulk of the He sits at a depth of $y_{\text{cut}}$, which we show explicitly below. This will have important implications on the rate of He DNB as we show in Section 4.

3.2. Coulomb Effects via a Chemical Equilibrium Approach

Now we turn our attention to the effect of Coulomb physics on the electric field. While it would be tempting to follow the derivation leading to Equation (5), but substitute Coulomb correction terms for the ion pressure, this gives erroneous results in the general case. Instead, we have calculated these corrections starting from the condition of chemical equilibrium. Chabrier & Potekhin (1998) have provided fitted forms for $f_i$ (their Equation (27)) which we use below.

Equilibrium requires $\partial \mu/\partial r = Z_e e - A_i m_p g$ for each species. Ignoring temperature gradients, we find

$$\frac{\partial \ln n_i}{\partial r} = -\frac{u_i}{3} \frac{\partial \ln n_e}{\partial r} + \frac{Z_e e E - A_i m_p g}{k_B T},$$

(20)

$$\frac{\partial \mu^{(e)}}{\partial \ln n_e} \frac{\partial \ln n_e}{\partial r} + \frac{1}{3} k_B T \sum_i \frac{n_i u_i}{n_e}$$

$$\times \left[ \frac{\partial \ln n_i}{\partial r} + \frac{\partial \ln Z_{i,e}}{\partial r} \right] = - e E,$$

(21)

where $h_i \equiv \partial \ln u_i/\partial \ln \Gamma$. For a single ion species, $n_e = Z_i n_i$ and therefore $d \ln n_i = d \ln n_e$. So, Equations (20) and (21) simplify to give

$$k_B T \left( 1 - \frac{u_i}{3} \right) \frac{\partial \ln n_i}{\partial r} = Z_i e E - A_i m_p g,$$

(22)

which recovers the equation of hydrostatic balance for each species (in the isothermal limit) and demonstrates the relationship between chemical equilibrium and hydrostatic balance.

To consider Coulomb physics, we include an additional term to the free energy:

$$F = F_{i,d,\text{deg}}^{(e)}(N_e, V, T) + F_{i,d,\text{nondeg}}^{(i)}(N_i, V, T) + F_{\text{ex}}(N_e, N_i, V, T),$$

(17)

where $F_{\text{ex}}(N_e, N_i, V, T)$ is the excess free energy due to Coulomb interactions. Chabrier & Potekhin (1998) give an excellent discussion of the Coulomb physics of electron–ion plasmas and we adopt their formalism and notation. Following Chabrier & Potekhin (1998), we rewrite the excess free energy $F_{\text{ex}} = N_e f_e k_B T + N_i (f_i + f_a) k_B T$ in terms of dimensionless functions, $f_e(\Gamma_e), f_i(\Gamma_i)$, and $f_a(\Gamma_a)$, which denote the electron–electron, ion–ion, and screening interactions, respectively. We define $\Gamma_e = e^2 / a_i k_B T$ as the electron coupling parameter, and $\alpha_e = (4 \pi n_i / 3)^{1/3}$ as the mean electron spacing. The ion coupling parameter $\Gamma_i = Z_i^2 e^2 / a_i k_B T$ is related to $\Gamma_e$ via $\Gamma_i = Z_i^3 / \Gamma_e$, where $a_i = (4 \pi n_i / 3)^{1/3}$.

Typically the ion–ion term ($f_a$) dominates and for the purposes of simplifying the discussion, while capturing the relevant physics, we consider just this term. The chemical potentials then become

$$\mu^{(e)} = \frac{\partial F}{\partial N_e} \bigg|_{N_i, V, T} = \mu^{(e)}_{id} + \frac{k_B T}{3} \sum_i \frac{N_i}{N_e} u_i,$$

(18)

$$\mu^{(i)} = \frac{\partial F}{\partial N_i} \bigg|_{N_e, N_{\text{tot}}, V} = \mu^{(i)}_{id} + k_B T f_i,$$

(19)

where we have defined $f_i = f_i(e, \Gamma_i)$ and $u_i = \partial f_i(e, \Gamma_i) / \partial \ln \Gamma_i$.
\[
\left( \frac{\partial \mu_{id}^{(e)}}{\partial \ln n_e} \right)_T + \frac{1}{9} k_B T \frac{u_i h_i}{Z_i} \frac{\partial \ln n_e}{\partial r} = -eE. \tag{23}
\]

Including Coulomb physics introduces corrections to both the ion equation (22) and the electron equation (23). The additional correction to the electron equation would have been missed had we started from the ion hydrostatic equation (1).\(^7\) The total pressure is
\[
P = -\left( \frac{\partial F}{\partial V} \right)_T. \tag{24}
\]

Therefore, the non-ideal, Coulomb part is
\[
P_{\text{Coul}} = nkT u_i/3, \tag{25}
\]

which agrees with the sum of Equations (22) and (23).

This is further complicated in the general case of multiple species where the relation \(d \ln n_i = d \ln n_e\) does not hold. For instance, consider a trace ion species against a background \(Z_i n_i \ll n_e\). Equation (20) becomes
\[
\frac{\partial n_i}{\partial r} = -n_i \left( A_i m_{pg} - Z_i eE + \frac{1}{3} k_B T u_i \frac{\partial \ln n_e}{\partial r} \right), \tag{26}
\]

where \(n_{id} = n_i k_B T\) is the ideal gas equation of state. Here a simple substitution of a corrected ion pressure, \(P_i = n_i k_B T (1 + u_i/3)\), would have given erroneous results. Distinct terms must be included for both the ion and electron momentum equations. The Coulomb interaction must not be regarded as a pressure for each ion species, but rather as a force that is sensitive to the electron gradient which is set by the background.

This discussion of these two simple cases (single ion species and trace) highlights the problems with calculating the electric field starting from the equations of hydrostatic balance for ions and electrons (Equations (1) and (2)) when Coulomb physics plays a role. Fortunately, our approach from the viewpoint of chemical equilibrium avoids these problems.

We now solve the general case by first plugging the ion equation into the Coulomb term of the electron equation. Collecting terms, the result can be written in the intuitive form
\[
\frac{\partial \ln n_e}{\partial r} = Z_e^* eE - A_e^* m_{pg}, \tag{27}
\]

where
\[
Z_e^* = -1 - \frac{1}{3} \sum_i n_i u_i Z_i n_i, \tag{28}
\]

\[
A_e^* = -\frac{1}{3} \sum_i n_i u_i A_i n_i. \tag{29}
\]

This result can now be used to simplify the ion equation:
\[
\frac{\partial \ln n_i}{\partial r} = \frac{Z_i^* eE - A_i^* m_{pg}}{k_B T}, \tag{30}
\]

where
\[
Z_i^* = Z_i - \frac{k_B T}{E_e^*} \frac{u_i Z_i^*}{3}, \tag{31}
\]

\[
A_i^* = A_i - \frac{k_B T}{E_e^*} \frac{u_i A_i^*}{3}. \tag{32}
\]

In this form, it is as if the charge and mass of each species (ions and electrons) are shifted due to Coulomb interactions. Using charge neutrality, we solve for the electric field and find
\[
\frac{eE}{m_{pg}} = \frac{A (\gamma - 1) E_F + k_B T u_i h_i/9Z}{Z (\gamma - 1) E_F + k_B T u_i h_i/9Z} + k_B T (1 + u/3) \approx \frac{A Z}{Z} \left( 1 + \frac{3}{10(\gamma - 1)} \frac{Z^{2/3} e^2}{Z} \right), \tag{33}
\]

where \(P_{\text{id}}^{(e)} \propto n_i E_F \propto n_e^\gamma\) is the degenerate electron pressure. In analogy with the thermal correction to the electric field, the correction to the electric field due to Coulomb physics scales as \(n_i E_F / n_e E_F\), where \(E_F\) is the Coulomb energy of the ions.

Equation (32) is the central result of this section and we now briefly discuss its essential physics. When \(A_1/|Z_1| \neq A_2/|Z_2|\), i.e., dissimilar \(\mu_e\), the ion scale height is determined by \(k_B T / m_{pg}\), which is much smaller than the electron (pressure) scale height \(l_e = E_F / m_{pg}\) in the strongly degenerate part of the atmosphere. When \(A_1/|Z_1| \approx A_2/|Z_2|\), i.e., similar \(\mu_e\), then the zeroth-order field produces no differentiating force (modular thermal/mass defect corrections). Moreover, the thermal correction to the electric field produces an ion scale height \(\approx l_e\) as shown earlier. Mass defect corrections produce ion scale heights that are similarly large. While this results in a decreasing fraction for the lighter ion going deeper into the envelope, the column of this lighter ion increases as discussed earlier (and increase really fast if we include mass defect corrections). Including Coulomb physics causes the ion scale height to become \(l_e / \Gamma_i\), much smaller than \(l_e\) for large \(\Gamma\), but is still larger than the ion scale height for dissimilar \(A/Z\) by \(E_C / E_F\). Both concentration and number density for the lighter ion decrease with increasing depth.

We now utilize the discussion above to estimate \(\gamma_{\text{Coul}}\). Plugging Equation (33) into hydrostatic balance (Equation (1)) for a trace, 2, and a background, 1, we find for relativistic degenerate electrons (\(\gamma = 4/3\)) that
\[
\frac{\partial \ln n_1}{\partial r} = \frac{m_{pg}}{E_F} \Gamma_e Z_1^{5/3}, \tag{34}
\]

\[
\frac{\partial \ln n_2}{\partial r} = \frac{m_{pg}}{E_F} \Gamma_e Z_2^{5/3}. \tag{35}
\]

Subtracting Equation (34) from Equation (35) and noting that \(\partial \ln P / \partial r = \partial \ln \gamma / \partial r = -m_p g / P = -4 A m_{pg} / Z E_F\), we find
\[
\frac{\partial \ln n_2 / n_1}{\partial \ln \gamma} = \frac{1}{4} \frac{Z_1}{A} \Gamma_e \left( Z_2^{5/3} - Z_1^{5/3} \right). \tag{36}
\]

The factor of 4 comes from noting that \(P \approx P_e\), i.e., relativistic degenerate electrons dominate the pressure, and \(P_e = n_e E_F / 4\).
Plugging in values for an He/C envelope appropriate for Figure 1 with base temperature \( T_b = 1 \times 10^8 \) K (see Section 4 for a discussion of the base temperature), we find that \( d \ln (\rho z) / d \ln y = -1 \) at \( \rho = \rho_{\text{cut}} \approx 5 \times 10^6 \text{ g cm}^{-3} \) for relativistic degenerate electrons, which gives \( y_{\text{cut}} \approx 2 \times 10^9 \text{ g cm}^{-2} \).

The results of this section show that stratification occurs even for elements with the same \( \mu_e \). This has important implications in high gravity environments such as white dwarfs and NSs beyond the problem of He DNB.

4. DIFFUSIVE NUCLEAR BURNING OF HELIUM

Combining the results of Section 3, the thermal profile of the envelope, rates of He burning, and rates of diffusion, we now calculate the rate of nuclear burning of He in NS envelopes. For the thermal profile, we solve the heat diffusion equation for a constant flux envelope in radiative equilibrium:

\[
\frac{\partial T}{\partial r} = -\frac{3 \kappa \rho}{16 T^4} T^4,
\]

(37)

where \( \kappa \) is the opacity. Starting from the photosphere (with optical depth, \( \tau = 2/3 \), we integrate the plane–parallel envelope inward assuming constant flux to a depth of \( y = y_b \equiv 10^{14} \text{ g cm}^{-2} \), which we call the base of the envelope. As in Paper II, we use the tabulated conductivities of Potekhin et al. (1999) and the radiative opacities and the equation of state of Potekhin & Yakovlev (2001), which is applicable to all magnetic field strengths (see Paper II for more details).

Potekhin et al. (1997) performed a similar calculation down to a density of \( \rho = \rho_b \equiv 10^{16} \text{ g cm}^{-3} \), which they referred to as the base of the envelope. These two definitions are roughly equivalent to each other, namely that \( \rho(\rho_b) \approx 10^{10} \text{ g cm}^{-3} \). We define the temperature at \( y_b \) as the base temperature, \( T_b \). For an NS in thermal equilibrium, the base temperature and the core temperature, \( T_c \), are equal to each other as the temperature profile is nearly isothermal due to the large conductivities. For the most part, \( T_b \) and \( T_c \) can be used interchangeably, except for the very early cooling history of the NS (age \( \lesssim 100 \) yr), when the NS is not in thermal equilibrium and \( T_b \neq T_c \). This point will be important in our discussion of Figure 5.

For the diffusion coefficient of the trace He, we use the results of Paper II (also see Brown et al. 2002), where we found

\[
D \approx 10^{-3} \frac{A_{10}^{0.7} T_6^{1.3}}{Z_1^{0.2} Z_2^{0.3} \rho_5^{-0.6}} \text{ cm}^2 \text{ s}^{-1},
\]

(38)

where \( T_6 = T / 10^6 \) K and \( \rho_5 = \rho / 10^5 \text{ g cm}^{-3} \). As discussed in Paper II, Equation (38) is reasonably accurate in the liquid regime of the background C, 1 < \( \Gamma < \Gamma_M \), where \( \Gamma_M = 175 \) defines the melting point of the crystalline phase (Potekhin & Chabrier 2000). For \( \Gamma > \Gamma_M \), the material is a crystalline and we assume that there is no diffusion, though we never reach this point in our parameter regime.

We now discuss the nuclear processes that consume He. For our nuclear reaction rates, we have utilized both the NACRE\(^9\) (Angulo et al. 1999) and REACLIB\(^10\) compilation of nuclear reaction rates and experimental values (see http://nuastro.org). We have not included electron screening in our calculations, though the effect of screening would be to increase the rate of burning. For triple-\( \alpha \) reactions, we use the fit in Nomoto et al. (1985), but have found this rate to be typically very small for the parameters of interest because of the reduced number density of He in the burning region (note the reduced number fraction of He in Figure 1). Hence, we no longer discuss triple-\( \alpha \) reactions.

The local rate of He burning is

\[
y_{\text{He}} \equiv \frac{y_{\text{He}}}{\tau_{\text{He,col}}} = \int_0^z \frac{4 n_{\text{He}} n_{\rho}}{\tau_{\text{He}}(n_{\text{He}}, n_{\alpha-\text{cap}}, T)} \text{ d}z,
\]

(39)

where \( z \) is the depth and \( n_{\alpha-\text{cap}} \) is the density of an He-capturing substrate.

In the top panels of Figures 2 and 3, we plot the rate of He burning (solid lines) as a function of column, i.e., \( dy_{\text{He}} / dy \) for effective temperatures of \( T_e = 1.25 \times 10^9 \) K and \( 2 \times 10^9 \) K with photospheric He abundances of 50%. We also plot the \( y_{\text{He}} \) (short-dashed lines) as a function of column. Plotted in the bottom panels of these two figures is the background temperature profile (dotted lines) and the composition profile (long-dashed lines) of the envelope. Figures 2 and 3 represent He DNB in the nuclear limited regime and the diffusion-limited regime, respectively (see Section 2 and Paper II for a detailed discussion of the nuclear limited and diffusion-limited regimes of DNB). In both cases, the peak of the burning is set by a combination of a rising temperature profile and a rapidly falling He concentration. The cutoff in the nuclear-limited regime (Figure 2) results from the equilibrium profile of He embedded in a C background. Hence, the He profile remains in diffusive equilibrium as it is slowly consumed. On the other hand, the sharp dropoff in the He profile in the diffusion-limited regime (Figure 3) is not a result of the equilibrium structure, but rather due to nuclear burning forcing the He profile out of diffusive equilibrium. As in the case of diffusion-limited H-DNB discussed in Paper II, the rate at which H is consumed is ultimately limited by the rate at which H diffuses down to the burning layer. Below this layer H does not exist as the burning layer captures all downwardly diffusing...
The base temperature is $T_b = 1.7 \times 10^6$ K, photospheric He abundance is 50%, and the total He column depth is $y_{\text{He}} \approx 4600 \, \text{g cm}^{-2}$. Unlike Figure 2, the rate of He burning is limited not by the rate of alpha capture onto carbon, but on the rate of He diffusion down to the burning layer. We term this regime as the diffusion-limited regime of DNB. The He abundance is not driven by the rate of alpha capture onto carbon, but on the rate of diffusion down to the burning layer, i.e., the diffusion-limited regime of DNB. The He tail which penetrates into the C is not in a diffusive equilibrium below the burning layer.

Figure 3. Same as Figure 2 but with $T_e = 2 \times 10^6$ K. The base temperature is $T_b = 1.7 \times 10^6$ K, photospheric He abundance is 50%, and the total He column depth is $y_{\text{He}} \approx 4600 \, \text{g cm}^{-2}$. Unlike Figure 2, the rate of He burning is limited not by the rate of alpha capture onto carbon, but on the rate of He diffusion down to the burning layer, i.e., the diffusion-limited regime of DNB. The He tail which penetrates into the C is not in a diffusive equilibrium below the burning layer.

protons. The rate of nuclear burning is not limited by the rate of nuclear reactions, but rather by the rate of diffusion down to this boundary. We see this same behavior for He at sufficiently high temperatures.

To calculate the case of diffusion-limited He DNB, we have generalized the argument of Paper II. The details can be found in Paper II, but for completeness, we write the additional equation to be solved below:

$$\frac{\partial^2 f_{\text{He}}}{\partial z^2} + \frac{\partial f_{\text{He}} \partial \ln n_{\text{He},0}}{\partial z} = \frac{f_{\text{He}}}{D \tau_{\text{He,m}}},$$

where $n_{\text{He},0}$ is the He number density in the absence of nuclear reactions, $f_{\text{He}} \equiv n_{\text{He}}/n_{\text{He},0}$ is the correction factor in the presence of nuclear reactions, and $n_{\text{He}}$ is the He number density. In the regime where Equation (40) becomes important, the second term on the left-hand side becomes small compared to all the other terms. In other words, the scale height of $f_{\text{He}}$ is small compared to the equilibrium scale height, $h_{\text{eq}} = (\partial \ln n_{\text{He},0}/\partial z)^{-1}$.

As we found in Paper II, Equation (40) becomes a diffusion equation in the presence of a nuclear-driven source.

We note that the small difference between the effective temperature between Figures 2 and 3 ($1.25 \times 10^6$ K versus $2 \times 10^6$ K) yields a 7 order of magnitude difference in burning rate and He column lifetime. This results from the strong temperature sensitivity of $\alpha$ capture onto C.

We now discuss how the lifetime of an He layer, $\tau_{\text{He}} = y_{\text{He}}/y_{\text{He}},$ scales with its column, $y_{\text{He}}$. In Table 1, we show the photospheric He abundance ($n_{\text{He}}/n_{\text{tot}}$ at the photosphere), He column $y_{\text{He}}$ (for He sitting on top of C), and the He lifetime, $\tau_{\text{He}}$, for an NS with $T_e = 2 \times 10^6$ K. We note two remarkable things about this table. First, the He lifetime is independent of the total column. This was not the case for H DNB (Papers I, II, and III). Second, even though the photospheric abundance of He can be low, the total column of He is high. Namely, for a photospheric He abundance of 10%, the total He column is $\approx 2500 \, \text{g cm}^{-2}$ much greater than the photospheric column of $y_{\text{He}} \approx 1 \, \text{g cm}^{-2}$.

We now demonstrate that the independence of $\tau_{\text{He}}$ and $y_{\text{He}}$ and $y_{\text{He}} \gg y_{\text{ph}}$ are due to the same physics. The reason why $y_{\text{He}} \gg y_{\text{ph}}$ is because the bulk of the He is not near the photosphere, but rather deep in the envelope. We show this in the top panels of Figures 2 and 3 where we plot $y_{\text{He}}$ (short-dashed lines) as a function of $y$. Note that the bulk of the He does not reside near the surface, but increases substantially until it asymptotes at a column of $y \approx y_{\text{cut}}$.

We now explain why the bulk of He resides near $y_{\text{cut}}$. In Paper I, we showed that the total lifetime of H on a proton capturing substrate is dominated by the time it takes to remove a photospheric H column. Namely, we found that the H lifetime, $\tau_{\text{H}}$, scales with the H column as $\tau_{\text{H}} \propto y_{\text{H}}^{1+\delta}$, where $\delta = A_2/(Z_1 + 1)/A_1 - Z_2 - 1$, where 1 and 2 denote the background and trace ion species, respectively, i.e., $1 + \delta = -5/12 < 0$ for an H on a C envelope (paper I). As we discussed in Section 5.2 of Paper I, this scaling arises from the power-law fall off of H abundance in a nondegenerate atmosphere, which is $n_{\text{H}}/n_{\text{tot}} \propto y^\delta = y^{-17/12}$ (see Equation (31) of Paper I). For He on C, the He abundance follows $n_{\text{He}}/n_{\text{tot}} \propto y^\delta = y^{-2/3}$. The He abundance on a C substrate falls off steeper than $y^{-1}$, whereas the He abundance on a C substrate falls off shallower than $y^{-1}$. Thus, the H column is capped at the H/C boundary, the column where the number density of H and C are equal, but the He column continues to increase substantially below the H/C boundary. Indeed, one can show that the He column always increases below the H/stratosphere boundary for any substrate with $A/Z < 2$. This increasing He column carries through to the Coulomb case as discussed earlier in Section 3.2. Indeed, the He column continues to rise until it is capped at a column of $y_{\text{He}}$ as we argued in Section 3.1.

We now note that the burning layer is close to the region where He begins to get cut off, $y_{\text{burn}} \sim y_{\text{cut}}$ (see, for instance, Figures 2 and 3). If we say that they are the same, we are left with the result that the He abundance in the burning layer and the total He column are $\textit{linearly}$ related to each other. Since the rate of He DNB (via captures onto C) is linearly related to the He abundance (and hence $y_{\text{He}} \propto y_{\text{He}}$), we find that the lifetime of the He layer, $\tau_{\text{He}} = y_{\text{He}}/y_{\text{He}}$, is independent of $y_{\text{He}}$, as demonstrated in Table 1, explaining the independence of $\tau_{\text{He}}$ and $y_{\text{He}}$.

In Figure 4, we show the He lifetime, $\tau_{\text{He}}$, for a range of $\alpha$-capturing material as a function of base temperature, $T_b$ (left), effective temperature, $T_e$, and $B = 0$ (solid lines). We also plot (dotted lines) the same case but for a radial $B$ field of $B = 10^{12}$ G. Magnetic fields up to pulsar strengths do not make a significant difference. For sufficiently large base temperatures, the lifetime to deplete the atmosphere of He is a few weeks.

### Table 1

| He Photospheric Abundance | $\log_{10} y_{\text{He}}$ (g cm$^{-2}$) | $\log_{10} \tau_{\text{He}}$ (yr) |
|---------------------------|----------------------------------------|-----------------------------------|
| 0.1                       | 2.888                                 | 1.947                            |
| 0.3                       | 3.397                                 | 1.947                            |
| 0.5                       | 3.664                                 | 1.947                            |
| 0.9                       | 4.144                                 | 1.947                            |
| 0.999                     | 4.851                                 | 1.947                            |
| 0.999999                  | 5.848                                 | 1.949                            |
Figure 4. Lifetime for He sitting on top of C, Si, or Ar envelope as a function of base temperature, $T_b$ (left plot), and as a function of effective temperature, $T_e$ (right plot) for $B = 0$ (solid lines) and $B = 10^{12}$ G (dotted lines). For sufficiently high $T_b$ ($T_e$), i.e., $T_b \gtrsim 1.6 \times 10^8 \text{K}$ ($T_e \gtrsim 2 \times 10^6 \text{K}$), the lifetime of He against capture on C is very short ($\lesssim 10^2 \text{yr}$). As these $T_b$’s are reached during the early cooling history of the NS, primordial He should be burned off. At low temperatures, there is a steep power-law dependence, as would be expected for DNB in the nuclear-limited regime. At higher temperatures, the dependence on $T_b$ becomes shallower, indicative of DNB in the diffusion-limited regime.

To compare the lifetime of an He layer with the typical cooling history of an NS, we plot in Figure 5 a few representative cooling tracks (long-dashed lines) and overlay the $\tau_{\text{He}}$–$T_b$ relation shown in Figure 4. Here, we have plotted $T_b$, which during the very early history (age $\lesssim 100 \text{yr}$) of the NS $\neq T_e$. Only after the cooling wave sweeps through the star, which can be seen by the sudden drop in $T_b$ at $\sim 100 \text{yr}$, is the NS in thermal equilibrium. As we have done in Paper II, we include a standard cooling track which presumes modified URCA cooling on a 1.3 $M_\odot$ NS, another with core proton superfluidity (pSF; Potekhin et al. 2003), and one with triplet-state core neutron superfluidity (nSF) in the core with a maximum critical temperature of $8 \times 10^8 \text{K}$ (calculated by D. G. Yakovlev previously for Paper II).

The remarkable aspect of this plot is that independent of the cooling model assumed (where we have taken a few representative models that span the range from standard cooling to fast cooling), the lifetime of an He atmosphere is short compared to the cooling age (during its early history). For instance, He on a C envelope will be depleted even in the fast cooling case, if it was deposited up to 1000 yr after formation. For slowly cooling models, this can extend up to almost 1 Myr. For a much heavier material, i.e., Ar, the lifetime is still short $\sim 100 \text{yr}$ for both fast and slow cooling. Thus, primordial He should be depleted on NS atmospheres during its early cooling history.

As we close this section, we note that the lack of observed light elements on NS photosphere may not indicate that there are no light elements present. As we discussed above, in diffusive equilibrium, the bulk of the He resides near $y_{\text{cut}}$. What this implies is that while the photospheric abundance of He could be low, the total He column could be large (see Table 1). This result has implications for the subsequent accretion or production of light elements on NS surfaces. For instance, suppose a sudden accretion or spallation event produces a total He column of $\gamma_{\text{He}} \approx 10^3 \text{g cm}^{-2}$ on the NS surface, which previously consisted of C. The NS surface would initially appear to be composed of light elements. Over time, as the He diffuses downward to $y_{\text{cut}}$, the photospheric abundance of He drops and the NS surface appears more C like.

Figure 5. Base temperature as a function of He lifetime/age. Overlaid are cooling tracks (long-dashed lines) for various NS models (Potekhin et al. 2003) including standard cooling (no superfluidity, modified Urca cooling represented by 1.3 $M_\odot$ noSF), proton superfluidity (pSF) and triplet-state core neutron superfluidity (nSF). Note that for sufficiently young NS or sufficiently slow cooling, $\tau_{\text{He}} < \text{age}$, which indicates that any He on the NS will be consumed by DNB.

The timescale for this mixing is the diffusion time down to $y_{\text{cut}}$ for He on C:

$$\tau_{\text{diff, cut}} \approx \frac{h_{p, \text{cut}}^2}{D} \approx 2 \rho_7^{1.3} T_8^{1.3} \text{yr},$$

where $h_{p, \text{cut}}$ is the pressure scale height at $y_{\text{cut}}$, $\rho_7 = \rho/10^7 \text{g cm}^{-3}$, and $T_8 = T/10^8 \text{K}$. This timescale is enticingly similar to the timescale of the observed spectral variations in RXJ0720.4−3125 (de Vries et al. 2004; Vink et al. 2004; Haberl et al. 2006; Hohle et al. 2009). Haberl et al. (2006) proposed that NS precession may be responsible for RXJ0720.4−3125’s
observed spectral hardening and subsequent softening. However, we speculate that slow evolution of the NS surface over a period of years may be the culprit (see also van Kerkwijk et al. 2007). Hydrogen on RX J0720.4–3125 is consumed by DNB, whereas He would diffusively mix down to a layer \(y_{\text{cut}}\). A more detailed study of this process of diffusive mixing for He is needed to make a more quantitative comparison, but such a calculation is well beyond the scope of this paper.

5. CONCLUSIONS

We have now extended our prior work on H DNB to He. We now include thermal, mass defect, and Coulomb corrections to the electric field to calculate the structure of He on C (or any other \(A/Z \approx 2\) material) in diffusive equilibrium. We show that the bulk of He in diffusive equilibrium sits at a layer \(y_{\text{cut}}\) and show that as a result of this the lifetime of an He on an NS is independent of its column. We demonstrate that young NSs undergo a phase where He is consumed via DNB (see especially Figure 5). Combined with the results of Papers I, II, and III (see Figure 8 of Paper II), we conclude that all primordial H and He are depleted on the surfaces of NS in the absence of external sources. In addition, the effects of magnetic field, as discussed in Section 4 (also see Papers II and III for H DNB), do little to change this basic result. We require the presence of H and He capturing material in NS envelopes, which the initial nuclear evolution of a cooling NS produces (Hoffman & Heyl 2009), i.e., proton and He capturing elements as light as Si. Finally, we comment on the timescale for He on C to reach diffusive equilibrium and suggest that such an evolution may explain the observed long-term spectral variation seen in RX J0720.4–3125.

Since H and He are consumed during the NS’s early history, we expect that the NS surfaces will be dominated by mid-Z elements such as C, N, and O. The evidence for mid-Z elemental compositions on NSs is sparse. Observations of 1E1207.4–5209 by Sanwal et al. (2002) and Mereghetti et al. (2002) found X-ray spectral lines at 0.7 keV and 1.4 keV with additional spectral features at 2.1 and 2.8 keV, which have been disputed (Mori et al. 2005). Subsequent modeling by Hailey & Mori (2002), Mori & Hailey (2006), and Mori & Ho (2007) suggested that its atmosphere consists of mid-Z elements such as O or Ne with a magnetic field strength of \(6 \times 10^{13}\) G. Given the age of 1E1207-52 of \(\approx 7\) kyr (Roger et al. 1988), we found that H is easily consumed (Paper II). Using the results of this paper, we suggest that He could have been consumed during the early cooling history of the NS exposing the underlying O or Ne. Subsequent accretion was O-rich and not subject to spallation or that the strong pulsar wind prevents accretion. We also note that removal via pulsar winds remains a strong possibility.

However, more recent work by Gotthelf & Halpern (2007) challenges the interpretation that these spectral lines on 1E1207-52 are due to mid-Z materials. Rather, the spin-down rate which Gotthelf & Halpern (2007) found suggests a B field strength \((B < 3 \times 10^{11}\) G), consistent with electron cyclotron lines for the higher energy (1.4 keV) line.

Currently the best evidence for mid-Z atmospheres comes from Ho & Heinke (2009). They recently argued that the surface of the NS in the center of Cassiopeia A consist of a C atmosphere. Their modeling of archival observations of this compact X-ray source showed that a \(T_{e} = 1.8 \times 10^{6}\) K carbon atmosphere NS with a low magnetic field provides both a good fit to the observed spectrum and is consistent with the theoretical expectation of the radii of NSs, i.e., \(R = 12–14\) km. Additional observations of other young NS may provide further evidence of mid-Z elements in their atmospheres.

We should mention that even though DNB should deplete the atmospheres of young NSs of any primordial H or He, subsequent accretion with or without spallation (Bildsten et al. 1992) could still lead to an H/He atmosphere in spite of DNB. As the mass of the photosphere is low and the timescales still fairly long, even a small amount of accretion will pollute the photosphere. A quick estimate of the amount of accretion needed to overwhelm DNB is

\[
M > 10^{-30} \left( \frac{y_{\text{ph}}}{1 \text{ g cm}^{-2}} \right) \left( \frac{\tau_{\text{ph}}}{100 \text{ yrs}} \right) M_{\odot} \text{ yr}^{-1}. \tag{42}
\]

A pulsar wind from the NS may suppress this accretion, but the amount of suppression has to be fairly strong as indicated by Equation (42).

Armed with the results of this work and prior efforts, we expect that primordial H and He are depleted, and so any observed H or He on the surfaces of these NS must be due to subsequent accretion (with or without spallation). If this subsequent accretion can be stopped or prevented via a pulsar wind, for instance, the underlying mid-Z material would be exposed.

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