Manipulation of optical solitons in Bose-Einstein condensates

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We propose a method to control the optical transparency of a Bose-Einstein condensate with working energy levels of the Λ-type. The reported effects are essentially nonlinear and are considered in the framework of an exactly solvable model describing the interaction of light with a Λ-type medium. We show how the complicated nonlinear interplay between fast and slow solitons in the Λ-type medium points to a possibility to create optical gates as well as to a possibility to store optical information.

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I. INTRODUCTION.

Recent progress in experimental techniques for the coherent control of light-matter interaction opens many opportunities for interesting practical applications. The experiments are carried out on various types of materials such as cold sodium atoms 1, 2, rubidium atom vapors 3, 4, 5, solids 5, 6, photonic crystals 7. These experiments are based on the control over the absorption properties of the medium and study slow-light and superluminal light effects. The control can be realized in the regime of electromagnetically induced transparency (EIT), by the coherent population oscillations or other induced transparency techniques. The use of each different materials brings in specific advantages important for the practical realization of the effects. For instance, the cold atoms have negligible doppler broadening and small collision rates, which increases ground-state coherence time. The experiments on rubidium vapors are carried at room temperatures and this does not require application of complicated cooling methods. The solids are obviously one of the strongest candidates for realization of long-living optical memory. Photonic crystals provide a broad range of ways to guide and manipulate the slow light.

In this paper we study the interaction of light with a gaseous active medium whose working energy levels are well approximated by the Λ-scheme. Our theoretical model is a very close prototype for a gas of sodium atoms, whose interaction with the light is approximated by the structure of levels of the Λ-type. The structure of levels is given in Fig. 1, where two hyperfine sub-levels of sodium state 32S1/2 with F = 1, F = 2 are associated with |2⟩ and |1⟩ states, correspondingly 2. The excited state |3⟩ corresponds to the hyperfine sub-level of the term 32P3/2 with F = 2. We consider the case when the atoms are cooled down to microkelvin temperature in order to suppress the Doppler shift and increase the coherence life-time for the ground levels. The atomic coherence life-time in sodium atoms at temperature of 0.9μK is of the order of 0.9 ms 2. Typically, in the experiments the pulses have length of microseconds, which is much shorter than the coherence life-time and longer than the optical relaxation time of 16.3ns. The gas cell is illuminated by two circularly polarized optical beams co-propagating in the z-direction. One beam, denoted as channel a, is a σ−-polarized field, and the other, denoted as b, is a σ+-polarized field. The corresponding fields are presented within the slow varying amplitude and phase approximation (SVEPA) as

\[ \hat{E} = \hat{E}_a e^{i(k_a z - \omega_a t)} + \hat{E}_b e^{i(k_b z - \omega_b t)} + c.c. \] (1)

Here, k_{a,b} are the wave numbers, while the vectors \( \hat{E}_a, \hat{E}_b \) describe polarizations of the fields. It is convenient to introduce two corresponding Rabi frequencies:

\[ \Omega_a = \frac{2 \mu_a E_a}{\hbar}, \quad \Omega_b = \frac{2 \mu_b E_b}{\hbar}. \] (2)

Here \( \mu_{a,b} \) are dipole moments of quantum transitions in the channels a and b.

Within the SVEPA the Hamiltonian describing the interaction of a three-level atom with the fields assumes the form

\[ H_\Lambda = H_0 + H_I, \quad H_0 = \omega_{12} |1⟩⟨1| + \omega |3⟩⟨3|, \] (3)

\[ H_I = -\frac{i}{2} (\Omega_a e^{i(k_a z - \omega_a t)} |3⟩⟨1| + \Omega_b e^{-i(k_a z - \omega_b t)} |3⟩⟨2|) + h.c. \]

Here \( \hbar = 1 \). The description can be further simplified in the interaction representation for the density matrix \( \rho \), namely,

\[ \tilde{\rho} = e^{-i H_E (t - \frac{\Delta}{2})} \rho e^{i H_E (t - \frac{\Delta}{2})}, \]

\[ H_E = \left( \frac{\Delta}{2} + \omega_{12} \right) |1⟩⟨1| + \left( \frac{\Delta}{2} |2⟩⟨2| + (\omega - \frac{\Delta}{2}) |3⟩⟨3| \right). \]
FIG. 1: The Λ-scheme for working energy levels of sodium atoms. The parameters of the scheme are the following: \( \omega_{12}/(2\pi) = 1772\text{MHz} \), \( \omega_{12}/(2\pi) = 5.1 \cdot 10^{14}\text{Hz} \) (\( \lambda = 589\text{nm} \)), and \( \Delta \) is the variable detuning from the resonance.

Then the Liouville equation for the transformed operator \( \bar{\rho} \) assumes the form

\[
i\partial_t \bar{\rho} = \left[ (H_0 - H_E + \bar{H}_I), \bar{\rho} \right],
\]

where

\[
\bar{H}_I = -\frac{1}{2} (\Omega_a |3\rangle \langle 1| + \Omega_b |3\rangle \langle 2|) + \text{h.c.} = -\frac{1}{2} \begin{pmatrix}
0 & 0 & \Omega_a^* \\
0 & 0 & \Omega_b^* \\
\Omega_a & \Omega_b & 0
\end{pmatrix}. \tag{4}
\]

The dynamics of the fields is described by the Maxwell equations

\[
\begin{align*}
(\partial_t^2 - c^2 \partial_z^2) \Omega_a e^{i(k_a z - \omega_a t)} &= -\frac{2\nu_a}{\omega_a} \partial_t \bar{\rho}_{11} e^{i(k_a z - \omega_a t)}, \\
(\partial_t^2 - c^2 \partial_z^2) \Omega_b e^{i(k_b z - \omega_b t)} &= -\frac{2\nu_b}{\omega_b} \partial_t \bar{\rho}_{12} e^{i(k_b z - \omega_b t)},
\end{align*}
\]

where \( \nu_a = (n_A|\mu_a|^2 \omega_a)/\epsilon_0 \), \( \nu_b = (n_A|\mu_b|^2 \omega_b)/\epsilon_0 \), \( n_A \) is the density of atoms, and \( \epsilon_0 \) is the vacuum susceptibility. For many experimental situations it is typical that the coupling constants \( \nu_a, \nu_b \) are almost the same. Therefore we assume that \( \nu_a = \nu_b = \nu_0 \). Within the SVEPA the wave equations are reduced to the first order PDEs:

\[
\partial_\tau \Omega_a = i\nu_0 \bar{\rho}_{31}, \quad \partial_\tau \Omega_b = i\nu_0 \bar{\rho}_{32}. \tag{5}
\]

Here \( \zeta = z/c, \tau = t - z/c \).

Equations Eqs. (5) can be rewritten in a matrix form as

\[
\partial_\zeta \bar{H}_I = \frac{i\nu_0}{4} [D, \bar{\rho}], \tag{6}
\]

where

\[
D = \begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & -1
\end{pmatrix}.
\]

In the new variables the Liouville equation takes the form

\[
\partial_\tau \bar{\rho} = i \frac{\Delta}{2} D - \bar{H}_I, \bar{\rho}. \tag{7}
\]

As was indicated above, the system of equations Eqs. (5), (7) is exactly solvable in the framework of the inverse scattering (IS) method [10].

This means that the system of equations Eqs. (6), (7) constitutes a compatibility condition for a certain linear system, namely

\[
\begin{align*}
\partial_\zeta \Psi &= U(\lambda) \Psi = \frac{i}{\lambda} D \Psi - i\bar{H}_I \Psi, \tag{8} \\
\partial_\tau \Psi &= V(\lambda) \Psi = \frac{i}{\pi} \frac{\nu_0}{\lambda^2} \Psi. \tag{9}
\end{align*}
\]

Here, \( \lambda \in \mathbb{C} \) is the spectral parameter. The comparison \( \Psi_{\zeta} \) against \( \Psi_{\zeta} \), leads to the zero-curvature condition \( U(\lambda) - V(\lambda) + [U(\lambda), V(\lambda)] = 0 \), which holds identically with respect to the linearly independent terms in \( \lambda \). It is straightforward to check that the resulting conditions coincide with the nonlinear equations Eqs. (6), (7). At this point it is worth discussing the initial and boundary conditions underlying the physical problem in question. We consider a semi-infinite \( \zeta \geq 0 \) active medium with a pulse of light incident at the point \( \zeta = 0 \) (initial condition).

This means that the evolution is considered with respect to the \( \text{space} \) variable \( \zeta \), while the boundary conditions should be specified with respect to the variable \( \tau \).

In our case we use as the asymptotic boundary conditions the asymptotic values of the density matrix as \( |\tau| \to \infty \).

To solve the nonlinear dynamics as described by equations Eqs. (6), (7), the IS method considers the scattering problem for the linear system Eq. (5), while the auxiliary linear system Eq. (8) describes the evolution of the scattering data. The purpose of this work is, in particular, to study an essentially nonlinear interplay of the fields in the both channels. This goal leads to consideration for equation Eq. (8) the scattering problem of finite density type (cf. [10] and references therein), i.e. \( \Omega_{a,b} \to \Omega_{a,b}^{(0)} \) as \( |\tau| \to \infty \).

The scattering problem for equation Eq. (8), reformulated as a matrix Riemann-Hilbert problem, is then posed on the two-sheet Riemann surface for the spectral parameter \( \lambda \). The purpose of this work is to discuss certain experimentally relevant solutions of the \( \Lambda \)-system, while the technicalities of the IS method are very well reflected in the existing literature anyway. For an account of results for the \( \Lambda \)-system accessible through the IS method see, for example, references [11, 12, 13, 14]. We only wish to indicate that in our analysis we use elements of the approach based on the Riemann-Hilbert matrix problem, along with algebraic techniques of the theory of solitons.

We use Darboux-Bäcklund transformations, in the spirit of [15, 16, 17, 18], up to certain modifications, however (cf. Appendix). The paper is organized as follows. In the next section we introduce the notions of slow and fast solitons and describe a nonlinear mechanism of formation of the transparency gate for the slow soliton. Section III
describes the transparency gate for the fast soliton. The section VI is devoted to conclusions. The details of the Darboux-Bäcklund transformation for the $\Lambda$-system are given in the Appendix.

II. THE TRANSPARENCY GATE FOR THE SLOW SOLITON

In this section we introduce a concept of slow and fast solitons in the $\Lambda$-medium and explain how the nonlinear interplay between the solitons leads to a possibility to control transparency of the medium. We discuss first the mechanism of transparency control for the slow soliton. We explain how the fast soliton propagating in the $a$ channel hops to the $b$ channel where the slow soliton is propagating. The fast soliton then "knocks down" the slow soliton, thus stopping the propagation of the latter, and then disappears itself due to the strong relaxation in the system.

As was indicated above, in this work we consider exact solutions of the Maxwell-Bloch system Eqs. (3, 4) existing on a non-vanishing finite background. In our considerations the background field plays the same role as the controlling field in the conventional linear theory of EIT. The background field enters the exact solutions as a parameter in a substantially nonlinear fashion. Because of reasons of experimental relevance we specify the background field as

$$\Omega_{a}^{(0)} = \cos(\eta)\Omega, \quad \Omega_{b}^{(0)} = \sin(\eta)\Omega, \quad \Omega = \Omega_{0}e^{ik\zeta}. \quad (10)$$

Therefore $\Omega_{0}^{2}$ specifies the total intensity of background fields existing in the both channels, $\eta$ defines the relative intensities, while $k \ll k_{a,b}$ is introduced in order to take into account small spatial variations of the phase. The intensity of the background field $\Omega_{0}$ is an experimentally adjustable parameter. We show in the paper that this parameter provides the control over the transparency of the optical gates and determines the speed of the slow soliton in the system.

For simplicity, we first assume $k = 0$. The atoms are initially unexcited and are prepared in the so-called dark-state

$$|\psi_{d}\rangle = \cos(\eta)|2\rangle - \sin(\eta)|1\rangle \quad (11)$$

Hereafter we assume $\eta = 0$, and therefore the dark state is simply $|2\rangle$. Taking this state of the atomic subsystem as the initial state we find the following solutions describing the formation of the transparency gate (cf appendix for details):

$$\tilde{\Omega}_{a} = \Omega_{0} - \frac{2(a_{3}^{2}\Omega_{0}e^{-\sqrt{\Delta^{2}+\epsilon_{0}^{2}}+\sqrt{\Delta^{2}+\epsilon_{0}^{2}}+\sqrt{\Delta^{2}+\epsilon_{0}^{2}}}}{a_{3}^{2}e^{-\sqrt{\Delta^{2}+\epsilon_{0}^{2}}+a_{3}^{2}e^{-\sqrt{\Delta^{2}+\epsilon_{0}^{2}}+2a_{3}^{2}e^{-\sqrt{\Delta^{2}+\epsilon_{0}^{2}}}}+a_{3}^{2}e^{-\sqrt{\Delta^{2}+\epsilon_{0}^{2}}+2a_{3}^{2}e^{-\sqrt{\Delta^{2}+\epsilon_{0}^{2}}}}+a_{3}^{2}e^{-\sqrt{\Delta^{2}+\epsilon_{0}^{2}}+2a_{3}^{2}e^{-\sqrt{\Delta^{2}+\epsilon_{0}^{2}}}}+a_{3}^{2}e^{-\sqrt{\Delta^{2}+\epsilon_{0}^{2}}+2a_{3}^{2}e^{-\sqrt{\Delta^{2}+\epsilon_{0}^{2}}}}}, \quad (12)$$

The solution Eq. (12) describes the nonlinear interaction of the slow and fast solitons. As is explained in the appendix, the solution is parameterized by the constants defining the position and phase of the solitons. Without loss of generality we assume $a_{1}$ and $a_{3}$ to be real constants, while $a_{2} = 1$. We show below that $a_{1}$ determines the position of the slow soliton whereas $a_{3}$ determines the position of the fast signal. In practice these constants are defined by the initial condition, which specifies the actual pulse of light entering the medium at the point $\zeta = 0$.

To understand the structure of the slow soliton one can put $a_{3} = 0$. This choice corresponds to taking the fast soliton to $-\infty$ in the variable $\tau$. Indeed, this specification removes from the overall solution Eq. (12) the fast pulse component and thus singles out the slow soliton part. The slow soliton solution assumes then the following form:

$$\tilde{\Omega}_{a} = \Omega_{0}\tanh(\phi_{s}) \quad (13)$$

$$\tilde{\Omega}_{b} = -i\frac{\frac{\epsilon_{0}}{\epsilon_{0}^{2}-\epsilon_{0}^{2}}}{\cosh(\phi_{s})} \sqrt{2\epsilon_{0}(\epsilon_{0} - \epsilon_{0}^{2} - \Omega_{0}^{2})}, \quad (14)$$

where

$$\phi_{s} = \zeta - \frac{\epsilon_{0}\nu_{0}}{2(\Delta^{2} + \epsilon_{0}^{2})} - \frac{\tau}{2} \left( \epsilon_{0} - \sqrt{\epsilon_{0}^{2} - \Omega_{0}^{2}} \right) + \ln|a_{1}|. \quad (15)$$

is the phase of the slow soliton. From the expression above and in the simplifying approximation $\frac{\nu_{0}}{2(\Delta^{2} + \epsilon_{0}^{2})} >> 1$ the group velocity of the slow soliton can be easily derived:

$$v_{g} \approx c\frac{\Omega_{0}^{2}(\Delta^{2} + \epsilon_{0}^{2})}{2\epsilon_{0}^{3}\nu_{0}}. \quad (16)$$

The pure state of the atomic subsystem corresponding
to the slow soliton solution Eq. (14) reads
\[ |\psi\rangle = \frac{\Delta}{\sqrt{\Delta^2 + \varepsilon_0^2}} + i \frac{\varepsilon_0}{\Omega_0} |\Omega_0^1\rangle |2\rangle + \Omega_0 \frac{i \sqrt{\varepsilon_0 + \sqrt{\varepsilon_0^2 - \Omega_0^2}}}{2 \sqrt{\Delta^2 + \varepsilon_0^2 \sqrt{\varepsilon_0 - \sqrt{\varepsilon_0^2 - \Omega_0^2}}}} |\Omega_0^b\rangle |3\rangle. \] (16)

Notice that the population of the upper level \( |3\rangle \) is proportional to the intensity of the background field. The speed of the slow soliton is also proportional to \( \Omega_0^2 \). This means that the slower the soliton, the smaller the population of the level \( |3\rangle \) and, therefore, the dynamics of the nonlinear system as a whole is less affected by the relaxation process.

To understand the structure of the fast soliton one can choose \( a_1 = 0 \). We then arrive at an expression describing a signal moving on the constant background with the speed of light (fast soliton):
\[ \Omega_a = \Omega_0 \left( 1 - 2 \frac{\cosh(\phi_f)}{\cosh(\phi_f) + \frac{\varepsilon_0}{\Omega_0}} \right), \quad \Omega_b = 0, \] (17)
where the phase of the fast soliton is
\[ \phi_f = \tau \sqrt{\varepsilon_0^2 - \Omega_0^2} + \ln |\alpha_3|. \]

We wish to emphasize once again that the atomic subsystem is prepared in the dark state \( |1\rangle \), which is not affected by a signal propagating in the channel \( a \). This explains why the fast soliton propagates with the velocity of light. The fast signal propagating in channel \( a \) can be observed as a localized peak in the field intensity \( I_a \). Below we analyze how the fast soliton collides with the slow soliton, hops into channel \( b \), and then slightly slows down.

Figure 2 illustrates the propagation and collision of the slow and fast solitons according to equation Eq. (12). The figure for \( I_a \) shows the intensities of the signals in channel \( a \). The slow soliton corresponds to a groove in the background field \( \Omega_0 \). It is clearly seen that after the act of collision the slow soliton ceases propagating in channel \( a \), while some trace of the fast soliton still can be noticed in that channel. The figure for \( I_a \) is complemented by the figure for the intensity \( I_b \) of the field in channel \( b \). We see that before the collision only the slow soliton exists in channel \( b \), while after the collision the slow soliton disappears and a fast intensive signal appears, whose velocity is slightly below the speed of light. The process described above can be summarized as if the fast soliton knocks down the slow soliton. The notion of a transparency gate requires the existence of two distinctly different regimes, which are transparent (open gate), and opaque (closed gate). In the absence of the fast soliton the gate is open for the slow soliton. When the fast soliton is present the slow soliton is destroyed, while the fast intensive signal created after the collision in channel \( b \) is attenuated due to strong relaxation in the atomic subsystem. The gate thus closes up in the course of the dynamics due to the relaxation process. To further explain this process we provide the Fig. 2 plots for populations of the levels \( |1\rangle \) and \( |3\rangle \).

Notice that before the collision the population of the upper atom level \( |3\rangle \) is negligible and is approximately given by the formula for the slow soliton solution Eq. (16) (see the lower right plot of P3). The populations of the lower levels \( |1, 2\rangle \) are determined by the slow soliton (see the lower left plot of P1). Indeed, the fast signal existing in channel \( a \) does not interact with the atoms because at the onset of the dynamics their state coincides with the dark state \( |2\rangle \). Figure 2 shows that after the collision the atoms of the active medium are highly excited and therefore the level \( |3\rangle \) is strongly populated. This leads to the fast attenuation of the speedy intensive signal in channel \( b \) due to the relaxation. The optical gate closes up.

To this point we have described a mechanism of controlling the transparency of the medium for a particular type of slowly moving signals. We now discuss a possibility to read information stored in the atomic subsystem. Let us assume that the background field vanishes, i.e. \( \Omega_0 = 0 \). As was explained above, the speed of the slow soliton then vanishes as well. However, the information about polarization of the slow signal is stored in the atomic subsystem. This effect can be interpreted in terms of the concept of a polariton, which is a collective excitation of the overall atom-field system. The notion of a polariton for the \( \Lambda \)-system has been used before. In the linear case the dark-state polariton was discussed in [19]. In the strongly nonlinear regime, which is the case for the present work, a similar interpretation is possible. Indeed, the field component of a slow soliton solution
can be interpreted as the light contribution into the slow polariton. When the controlling field \( \Omega_0 \) vanishes this contribution vanishes as well, along with the speed of the polariton. The latter then contains only excitations in the atomic subsystem. The general solution Eq. (12) is then reduced to the form:

\[
\tilde{\Omega}_a = \frac{-4ic_1c_3c_0 \exp\left[ c_0 \tau - \frac{i\phi_{\alpha 0} \tau}{2(\Delta^2 + \epsilon^2)} \right]}{2c_2c_3 \cosh(\phi_{\alpha 0}) + i\tau^2 \exp\left[ 2c_0 \tau - \frac{i\phi_{\alpha 0} \tau}{2(\Delta^2 + \epsilon^2)} \right]},
\]

\[
\tilde{\Omega}_b = \frac{c_2e^{-i\phi_{\alpha 0}} \tilde{\Omega}_a}{2\Delta^2 + \epsilon^2},
\]

where \( \phi_{\alpha 0} = \frac{\phi_{\alpha 0}}{2(\Delta^2 + \epsilon^2)} + \ln(c_2/c_3) \) is the phase of the slow soliton for the vanishing background \( \Omega_0 \). The form of the fields resembles a superposition of fast and slow solitons in Eq. (12), with the vanishing velocity of the slow soliton. It is impossible to single out the contribution of the slow pulse by choosing \( c_3 = 0 \), because the amplitude of the slow signal approaches zero as the background field vanishes (see Eq. (14)). The atomic state describing the stored information reads

\[
|\psi\rangle = \sqrt{\frac{A + i\epsilon}{A - i\epsilon}} |2\rangle + \frac{\alpha e^{-i\phi_{\alpha 0}}}{2\sqrt{A^2 + \epsilon^2}} \left( \frac{\epsilon}{c_1} e^{-\epsilon_0 \tau} |1\rangle + \frac{\epsilon}{c_1} e^{-\epsilon_0 \tau - i\phi_{\alpha 0} \tau} |2\rangle + |3\rangle \right).
\]

For \( c_1 = 0 \) the fields vanish, while the atomic state reduces to a form corresponding to a stopped polariton described by Eq. (10) with \( \Omega_0 = 0 \). In other words, when the slow soliton is completely stopped its information is stored in the spin polarization of the atoms. As long as the upper state \( |3\rangle \) is not populated, the state of the atomic subsystem is not sensitive to the destructive influence of the optical relaxation processes.

The conventional way [2] to read the information stored in the atoms is to increase the intensity of the background field. Our method of reading the information is different. We propose to send the fast soliton into the space domain in the active medium, where the information is stored. The polarization in the domain is then flipped by the fast signal. This is how the reading of information is realized. This way of reading optical information is advantageous because it involves fast easily detectable processes. Figure 3 illustrates the mechanism of the reading. Notice that the act of reading, based on the polarization flipping induced by the fast signal, can be realized in a very short time scale compared to typical relaxation times.

### III. The Transparency Gate for the Fast Signal

In this section we explain how to create the transparency gate for the fast signal rather than for the slow soliton as in the preceding section. Instead of decreasing the background intensity we now propose to increase it to reach the point where \( \Omega_0 = \epsilon_0 \). As we show below (cf Fig. 4 as well), the fast signal can be stopped by the slow soliton. In other words, the control of the transparency for the fast signal is realized by tuning the background intensity to the parameter \( \epsilon_0 \) entering the slow soliton solution. This effect is related to the appearance of the so-called exulon type solutions reported before [15, 24] for the SIT and NLS models.

For the constant background field with \( k = 0 \) and for \( \Omega_0 = \epsilon_0 \) the solution has a simple and transparent form, namely

\[
\tilde{\Omega}_a = \Omega_0 \frac{c_0^2 e^{i\phi_{\alpha 1}} - 2e^{-i\phi_{\alpha 1}} (c_2 + c_3 \tau \Omega_0)^2 - 3c_0^2}{c_0 e^{i\phi_{\alpha 1}} + 2e^{-i\phi_{\alpha 1}} (c_2 + c_3 \tau \Omega_0)^2 + c_0^2},
\]

\[
\tilde{\Omega}_b = -\Omega_0 \frac{4ic_1c_3c_0 \epsilon_0}{c_0 e^{i\phi_{\alpha 1}} + 2e^{-i\phi_{\alpha 1}} (c_2 + c_3 \tau \Omega_0)^2 + c_0^2}.
\]

FIG. 3: Reading the optical information by the fast soliton. The two upper plots illustrate the dynamics of the fields \( \Omega_a \) and \( \Omega_b \). The two lower plots show the populations of the levels \( |1\rangle \) and \( |3\rangle \). The standing peak on the plot for \( P_1 \) corresponds to the stored information in the form of the localized polarization. The rapidly moving localized excitation of the atoms given on the plot for \( P_3 \) represents the act of reading. The background field \( \Omega_0 = 0 \).

FIG. 4: Knocking down the fast signal. The small localized polarization flip is the slow soliton part. The intensive peak is the fast signal. The background field \( \Omega_0 = 1, \lambda_0 = i \).
and

\[ |\psi\rangle = \sqrt{\frac{\Delta + i\Omega}{\Delta - i\Omega}} \left( |2\rangle + \frac{\Omega_0}{2\sqrt{\Delta^2 + \Omega_0^2}} \right) \left( |1\rangle + \frac{\Omega_0 - \Omega_0 \tau - \frac{c_0}{c_2 + c_3 + c_3 - \epsilon_0}}{c_2 + c_3 + c_3 - \epsilon_0} |2\rangle + |3\rangle \right). \]  

Here, the phase \( \phi_{a1} \) corresponds to \( \phi \) with \( \epsilon_0 = \Omega_0 \).

To explain the mechanism underlying the functioning of an optical gate for the fast signal we analyze the solution Eq. (20) in detail. In the case \( c_3 = 0 \) we obtain the conventional slow soliton solution. If \( c_1 = 0 \) we arrive at a rational algebraic expression, which is a solution of the exulton type. In the case \( k = 0 \) and \( c_1 = 0 \) the pulse, whose algebraic form is a rational expression (exulton), moves with the speed of light without decay.

It can be readily seen that if \( k \) is not zero the exulton part of the solution vanishes in the course of the dynamics. For simplicity, and to demonstrate the typical behavior of this kind, we choose \( c_1 = 0, c_2 = 0 \). Then,

\[ \hat{\Omega}_a = \Omega_0 \left( 1 - 2e^{ik\zeta} \frac{(i\Delta(1 + \tau\Omega_0) + \Omega_0(1 - ik\zeta + \tau\Omega_0))(i\Delta(1 - \tau\Omega_0) + \Omega_0(-1 + ik\zeta + \tau\Omega_0))}{\Delta^2 + (1 + (k\zeta - \Delta\tau)^2)\Omega_0^2 + \tau^2\Omega_0^4} \right), \quad \hat{\Omega}_b = 0. \]  

The collision of the fast signal (exulton) with the slow soliton results in the disappearance of the former in channel \( a \). This means that the sol soliton controls the transparency of the medium for the signal moving through the speed of light. The reported mechanism is a realization of an optical gate for the fast signal.

IV. CONCLUSIONS AND DISCUSSION

In this paper we discussed a concept of the transparency gate for the fast and slow solitons in a \( \Lambda \)-type medium. We explained how the fast soliton can knock down the slow soliton and close the gate for the latter. The opposite process of closing the gate for the fast signal, given by rational algebraic expression (exulton), is also described. We also described the process of reading optical information, written into the active medium by the slow soliton. It is worth discussing here a possibility to actually create in the \( \Lambda \)-type atomic medium the signals described above. The general physical feature underlying the mathematical property of complete integrability is a delicate balance between the workings of dispersion and nonlinearity inherent in the medium. Provided that this balance is observed and the system is completely integrable it is a general fact that virtually any sufficiently intensive localized initial condition creates solitons. The overall picture of nonlinear dynamics can be roughly described as follows. The evolving signal created by the incident pulse in the course of the dynamics breaks down to a number of solitons and a decaying tale. The latter vanishes in due course. The soliton-like signals survive (ideally, i.e., in the absence of dissipation) for infinitely long time. When the physical conditions underlying the complete integrability of the optical system are met, the general picture of the nonlinear dynamics is similar to the described above. Namely, a fairly arbitrary localized and intensive initial signal creates in the course of nonlinear dynamics a number of solitons. The number of solitons can be derived from the analysis of the corresponding zero-curvature representation. The signal Eq. (12) is a rather generic soliton-like solution of the nonlinear system Eqs. (9), (7) and therefore it is very plausible that such signal can be created. Of course an experiment accompanied by a mathematical scrutiny would be necessary to further support this claim. The purpose of this paper is merely limited to pointing to a possibility of interesting physical applications. We want to emphasize that in our considerations the distinguished role is assigned to the background field \( \Omega_0 \) that turns out to be a nonlinear analog of the conventional controlling field appearing in the linear EIT formulation. The difference between the linear and nonlinear cases lies in the fact that in the nonlinear case the control field and the soliton solution are present in the same channel in inseparable fashion of nonlinear superposition. In our future work (ArXiv: quant-ph/0411148, quant-ph/0411149) we plan to further support our results on memory reading and transparency control by an investigation of transient regimes, when the background field \( \Omega_0 \) adiabatically vanishes or adiabatically approaches the point \( \epsilon_0 \). The treatment of relaxation processes in the system will be also included. This forthcoming investigation will use the method of collective variable along with other perturbation techniques developed for the systems integrable by the inverse scattering method.

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V. APPENDIX. DARBOUX-BÄCKLUND TRANSFORMATION FOR THE $\Lambda$-SYSTEM

In this appendix we describe the Darboux-Bäcklund (DB) transformation for the $\Lambda$-system. It is plain to see that the linear system Eqs. (25) is covariant with respect to the following DB dressing transformation

$$
\mathcal{H}_1 = \mathcal{H}_1 - \frac{1}{2} [D, \sigma_1(0)], \quad \tilde{\rho} = \sigma_1(\Lambda) \tilde{\rho} \sigma_1^{-1}(\Lambda) \quad (23)
$$

$$
\tilde{\Psi} = \Psi \mathcal{L} - \sigma_1(0) \Psi, \quad \sigma_1(\Lambda) = \Psi_1 (\mathcal{L}_1 - \Delta) \Psi_1^{-1}. \quad (24)
$$

Here $\Psi$ is a matrix consisting of three linearly independent solutions of the linear system Eqs. (25) corresponding to three (not necessarily different) values of the spectral parameter $\lambda$. The matrix spectral parameter $\mathcal{L}$ in our case is defined as

$$
\mathcal{L} = \begin{pmatrix} \lambda' & 0 & 0 \\ 0 & \lambda'' & 0 \\ 0 & 0 & \lambda''' \end{pmatrix},
$$

where $\lambda', \lambda'', \lambda'''$ are certain values of the spectral parameter $\lambda$. The matrix $\Psi_1$ is a specification of $\Psi$ corresponding to the following particular value of the matrix spectral parameter:

$$
\mathcal{L}_1 = \begin{pmatrix} \lambda_0 & 0 & 0 \\ 0 & \lambda_0^* & 0 \\ 0 & 0 & \lambda_0 \end{pmatrix}.
$$

We denote the fundamental matrix of the linear system Eq. (25) for $\lambda = \lambda_0$ as $\Phi_0$.

It can be shown that for the value of the spectral parameter $\lambda = \lambda_0^*$ the fundamental matrix is $\Phi_0 \equiv (\Phi_0^{(i)})$.

Since the subspace of solutions corresponding to $\lambda_0^*$ is two dimensional, the matrix $\Psi_1 = (\psi_1, \psi_2, \psi_3)$ is constructed as follows. The vector $\psi_1 = c_1 \Phi_0^{(1)} + c_2 \Phi_0^{(2)} + c_3 \Phi_0^{(3)}$ is a general solution of the linear problem with $\lambda = \lambda_0^*$. Here $\Phi_0^{(i)}, i = 1, 2, 3$ denotes a column in the matrix $\Phi_0$. To satisfy the structure of the operator $\sigma_1$ Eq. (24) we require that $(\bar{\Phi}_0^{(i)}, \Phi_0^{(j)}) = \delta_{ij}$. Therefore, we can easily find two appropriate orthogonal vectors $\tilde{\psi}_{1,2}$:

$$
\tilde{\psi}_1 = (c_2 + c_3 \bar{\psi}_2 + c_3 \bar{\psi}_3) \Phi_0^{(1)} - c_1 \Phi_0^{(2)} + \bar{\Phi}_0^{(3)}; \quad \tilde{\psi}_2 = c_2 \Phi_0^{(2)} - c_3 \Phi_0^{(3)}.
$$

The algorithm of finding new solutions of the nonlinear system Eqs. (22) can be recapitulated as follows. Find a solution $\Phi_0$ of the associated linear system Eqs. (25), corresponding to a certain "seed" solution of the nonlinear system Eqs. (22). Build $\tilde{\Psi}_1$, build $\sigma_1$, use then the dressing transformation Eq. (25).

In the case considered in the present work the matrix $\Phi_0$ reads

$$
\Phi_0 = \begin{pmatrix} -\tan(\eta) - \frac{\Omega_0 \cos(\eta)}{\lambda_0 + i \sqrt{-\lambda_0^2 - \Omega_0^2}} & -\frac{\Omega_0 \cos(\eta)}{\lambda_0 + i \sqrt{-\lambda_0^2 - \Omega_0^2}} & -\frac{\Omega_0 \cos(\eta)}{\lambda_0 + i \sqrt{-\lambda_0^2 - \Omega_0^2}} \\ 1 & 0 & 0 \\ -\frac{\Omega_0 \cos(\eta)}{\lambda_0 + i \sqrt{-\lambda_0^2 - \Omega_0^2}} & -\frac{\Omega_0 \cos(\eta)}{\lambda_0 + i \sqrt{-\lambda_0^2 - \Omega_0^2}} & -\frac{\Omega_0 \cos(\eta)}{\lambda_0 + i \sqrt{-\lambda_0^2 - \Omega_0^2}} \end{pmatrix} e^{\frac{i \lambda_0^{(s,\Lambda)}}{2 \Omega_0} \left( \begin{array}{ccc} \exp(\mu_1) & 0 & 0 \\ 0 & \exp(-\mu_2) & 0 \\ 0 & 0 & \exp(\mu_2) \end{array} \right)}, \quad (25)
$$

where

$$
\mu_1 = \frac{i \lambda_0^{(s,\Lambda)}}{2} \left( \tau + \frac{\kappa^2}{\lambda_0 - \Delta} \right) + i \frac{\kappa \lambda_0}{2(\lambda_0 - \Delta)} \zeta,
$$

$$
\mu_2 = \frac{\sqrt{\lambda_0^2 - \Omega_0^2}}{2} \left( \tau + \frac{\kappa^2}{\lambda_0 - \Delta} \right).
$$
The constants $c_{1,2,3}$ above define the spatial position and phase of the slow and fast solitons at a fixed moment of time. For simplicity, we assume these constants to be real.

To access the solution Eq. (12) one has to specify

$$c_1 = a_1 \Omega_0 \sqrt{\frac{\varepsilon_0}{2(\varepsilon_0^2 - \Omega_0^2)}}, \quad c_2 = a_2 \sqrt{\varepsilon_0 - \sqrt{\varepsilon_0^2 - \Omega_0^2}},$$

$$c_3 = a_3 \sqrt{\varepsilon_0 + \sqrt{\varepsilon_0^2 - \Omega_0^2}},$$

while $\lambda = i\varepsilon_0$. This concludes the construction of the Darboux-Bäcklund dressing transformation for the nonlinear $\Lambda$-model.