INTERACTING SINGLETONS

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**ABSTRACT.** There is a chance that singleton fields, that in the context of strings and membranes have been regarded as topological gauge fields that can interact only at the boundary of anti-De Sitter space, at spatial infinity, may have a more physical manifestation as constituents of massless fields in space time. The composite character of massless fields is expressed by field-current identities that relate ordinary massless field operators to singleton currents and stress-energy tensors. Naive versions of such identities do not make sense, but when the singletons are described in terms of dipole structures, then such constructions are at least formally possible. The new proposal includes and generalizes an early composite version of QED, and includes quantum gravity, super gravity and models of QCD. Unitarity of such theories is conjectural.
1. Introduction.

Recent developments in supergravity and string/membrane theory [1] point to a form of duality between massless fields on anti-De Sitter space (“the bulk”) and conformal field theory on the boundary. The boundary values of bulk massless fields have all the quantum numbers of composite operators of the boundary conformal field theory [2], and it is tempting to identify these two classes of objects with each other. The fields of the boundary conformal field theory are the boundary values of singleton fields in the bulk; therefore, the next step would be to identify the massless fields in the bulk with local bilinears in the singleton fields.

This was already attempted long ago, not in five dimensions, where the recent activity is taken place, but in ordinary, 4-dimensional space time. The result was a genuine composite version of QED [3]. It was also suggested, but merely suggested, that this idea may be applicable to gravity and to the strong interactions, that in some sense quarks are singletons [4]. That would, at least, account for their not being observed.

The recent developments [1] point to a concept of field-current identity that could, with some luck, be applied to construct a composite field model of massless fields in general, including photons, gravitons and perhaps gluons. Let $J$ be a conserved current; for example, the usual vector current of a conventional scalar, complex field. Let $\phi$ be a complex, spinless singleton field, and introduce the interaction

$$g^2 \int d^4x J^\mu (\bar{\phi} \partial_\mu \phi).$$

(1.1)

This is not manifestly gauge invariant.

A singleton gauge transformation is a shift of $\phi$ by a field $\lambda$ that is perfectly general except that it falls off, at spatial infinity, faster than the physical modes of $\phi$. The strong association between gauge invariance and unitarity suggests that any acceptable interaction must be insensitive to gauge fields and that, consequently, only the boundary values of the singleton field at infinity can participate [5].

Here we want to suggest that there are ways to circumvent this difficulty. It was shown in [4,6] that the interaction (5.1) can be made gauge invariant in quantum theory by an alternative form of field quantization. The effect of modifying the free field commutation relations is that the classically trivial interaction $g \int d^4x J^\mu \partial_\mu \phi$ (here $\phi$ is real) gives rise
to an effective interaction of the form
\[ g^2 \int d^4x J_\mu : (\varphi \partial_\mu \varphi) : \] (1.2)
where the colons stand for a kind of normal ordered product. In fact, this normal ordered product could be identified with the electromagnetic potential (quantum field operator).

Our new proposal is different. We suggest, in effect, that the lack of gauge invariance of (1.1) may be less destructive of unitarity than it appears at first sight. If this proves to be the case, then it may become possible to extend the field-current identity to the bulk; that is, it would then be feasible to interpret the quantity
\[ A_\mu = \bar{\varphi} \partial_\mu \varphi \] (1.3)
as the electromagnetic potential, even classically.

There is some evidence that suggests that the lack of gauge invariance of (1.1) may be of a benign sort. If both factors, \( \bar{\varphi} \) and \( \varphi \), are on shell, then the contribution of gauge modes to the field (1.3) is a gradient! This was shown in [6]. In this paper we do not present a definitive analysis of the problem of unitarity. We hope to be able to do so in the future. The last section makes some suggestions.

Let us be more precise concerning the composite operators that seem to be related to the boundary values of massless fields: they are precisely the conserved currents of the boundary conformal field theory. We expect this to be true in the bulk as well, and in fact, Eq.(1.3) identifies the massless potential with the vector current of a spinless field, usually conserved. Similarly, the gravitational potential can be expected to be related to the energy-momentum tensor of the singleton field.

But this naive identification is not possible in quantum field theory, if the current is conserved. For it is well known that the quantum field operator, the potential, is not divergenceless. In fact it is divergenceless only when projected on the physical subspace defined by the Lorentz condition. So what is needed is a current that is conserved on this physical subspace only.

Such currents (and energy-momentum tensors) are in fact characteristic of singleton field theories. The field equation for a scalar singleton field, in quantum field theory, is
\[ (\Box + u)\varphi = b, \quad (\Box + u)b = 0, \] (1.4)
where \( u \) is a constant, \( b \) is the Nakanishi-Lautrup field and \( \Box \) is the covariant d’Alembertian. In this theory the conserved current is of the form \( \bar{\varphi} \partial_{\mu} b \); it vanishes on the physical subspace defined by the physicality (Lorentz) condition, which in this theory is \( b^+ \ldots \geq 0 \). We set

\[
\hat{j}^\mu = \sqrt{-g}g^{\mu\nu}(\varphi \partial_{\nu} \varphi - (\partial_{\nu} \bar{\varphi})\varphi), \quad \varphi_{\mu} := \partial_{\mu} \varphi,
\]

where \((g_{\mu\nu})\) is the anti-De Sitter metric, and find that

\[
\partial_{\mu} \hat{j}^\mu = \bar{\varphi} b - \bar{b} \varphi.
\]

The divergence of this current, but not the current itself, vanishes on the physical subspace. A field current identification of the form (1.3), between quantum field operators,

\[
A_{\mu} \propto \hat{j}_{\mu},
\]

is therefore a possibility. We learn that the constituents of massless particles have to be described by gauge fields.

The conserved energy momentum tensor of a real, scalar singleton field is

\[
t_{\mu\nu} = \frac{1}{2} \sqrt{-g} g^{\mu\sigma} g^{\nu\tau} (\varphi_{\sigma} b_{\tau} + b_{\sigma} \varphi_{\tau} - g^{\lambda\rho} \varphi_{\lambda} b_{\rho} + u \varphi b).
\]

We define

\[
\hat{t}_{\mu\nu} := \sqrt{-g} g^{\mu\sigma} g^{\nu\tau} (\varphi_{\sigma} \varphi_{\tau} - \frac{1}{2} (g^{\lambda\rho} \varphi_{\lambda} \varphi_{\rho} - u \varphi \varphi)).
\]

It satisfies

\[
\partial_{\mu} \hat{t}_{\mu\nu} = \sqrt{-g} g^{\nu\sigma} b_{\varphi_{\sigma}},
\]

and it makes sense to suggest that

\[
h_{\mu\nu} \propto \hat{t}_{\mu\nu},
\]

with \((h_{\mu\nu})\) the gravitational potential (a perturbation of the anti-De Sitter metric).

Outline.

Section 2 deals with the scalar singleton field and shows in some detail the basis for the construction summarized above. Sections 3 and 4 introduce the spinor singleton and the super singleton. It is shown, but with less attention to the details, that the super singleton is the natural object with which to construct a version of super gravity in which all the massless particles are singleton composites. In Section 5 we make some additional remarks about the problem of unitarity of these theories.
2. The scalar singleton.

Vector potential.

The scalar singleton is a scalar field that satisfies the dipole equation \[6\]

\[(\Box + u)^2 \varphi = 0, \quad u = -\frac{5}{4} \rho, \tag{2.1}\]

where \(\rho\) is the anti-De Sitter curvature constant, henceforth set equal to 1. The second order of the Klein-Gordon operator is required in order that the propagator contain the modes of the singleton representation \(D(\frac{1}{2}, 0)\) of the anti-De Sitter group. The ordinary wave equation \((\Box + u)\varphi = 0\) is appropriate for \(D(\frac{3}{2}, 0)\). Actually, the space of solutions of the dipole equation carries the following Gupta-Bleuler triplet,

\[D(\frac{5}{2}, 0) \rightarrow D(\frac{1}{2}, 0) \rightarrow D(\frac{3}{2}, 0), \tag{2.2}\]

including physical modes (center), gauge modes (on the right) and their canonical conjugates (on the left). It is convenient to introduce a Nakanishi-Lautrup field \(b\), then the dipole equation takes the form (1.3). The complete action, including Faddeev-Popov ghost \(c\) and anti-ghost \(d\) is \[6\]

\[
\mathcal{L} = \mathcal{L}_4 + \mathcal{L}_3,
\]

\[
\mathcal{L}_4 = \int d^4 x \sqrt{-g} \left( g^{\mu\nu} \varphi_\mu b_\nu - u \varphi b + \bar{b}b - g^{\mu\nu} \bar{c}_\mu d_\nu + u \bar{c}d \right) + \text{conj.} \tag{2.3}\]

\[
\mathcal{L}_3 = \frac{1}{4} \int d^4 x \sqrt{-g} \left( g^{\mu\nu} \varphi_\mu \varphi_\nu + \frac{1}{2} \bar{\varphi} \varphi - \frac{1}{2} \varphi b \right)
\]

The limit of \(r^{1/2} \varphi(r, t, \Omega)\), as \(r\) tends to infinity, is the boundary field \(\tilde{\varphi}(t, \Omega)\), while the others fall off as \(r^{-5/2}\) and make no contribution to the boundary theory. Gauge transformations yield to the BRST transformation

\[
\delta(\varphi, b, c, d) = (c, 0, 0, b). \tag{2.4}\]

It is easy to solve the field equations, and the result is that the solution space carries the non-decomposable representation

\[
D[\varphi] := D(\frac{1}{2}, 0) \rightarrow D(\frac{1}{2}, 0) \rightarrow D(\frac{1}{2}, 0) \tag{2.5}\]
of the anti-De Sitter group. The physical modes, in $D(\frac{1}{2}, 0)$, are distinguished from the others in that they fall off slowly, as $r^{-1/2}$, at spatial infinity. The free field $b$ is identified with the invariant subspace associated with the representation on the right; the free fields $c$ and $d$ transform the same way and $b, c, d$ all fall off as $r^{-5/2}$ at infinity.

The canonical vector current is closed, and exact up to the contribution of the boundary term, which testifies to the fact that free singletons do not contribute to the charge of the bulk. It is conserved, and therefore it cannot be identified with the vector potential.

Instead, we consider the current

$$\hat{j}^\mu[\varphi] = \sqrt{-g}g^{\mu\nu}(\bar{\varphi}\varphi_\nu - \bar{\varphi}_\nu\varphi) := \sqrt{-g}g^{\mu\nu}\hat{j}^\nu, \quad (2.6)$$

and the vector potential

$$A_\mu = e\hat{j}_\mu[\varphi] = e(\bar{\varphi}\varphi_\nu - \bar{\varphi}_\nu\varphi). \quad (2.7)$$

The action of $SO(3, 2)$ on the constituent field $\varphi$ induces an action on the composite field $A$. When the field $\varphi$ is free, then so is $A$, in the sense that the induced representation is now contained in the direct product

$$D[\varphi] \otimes D[\varphi], \quad (2.8)$$

that is equivalent to a direct sum of massless representations [3]. Because the fields are multiplied locally, only a small part of the direct product is carried by the field, namely the non-decomposable representation

$$D[A] := D(3, 0) \rightarrow D(2, 1) \rightarrow D(3, 2). \quad (2.9)$$

This is precisely a Gupta-Bleuler triplet of anti-De Sitter electrodynamics [8].

It has been shown [6] that the BRST transformation of $\varphi$ induces the usual BRST transformation of $A$. Therefore the Lorentz condition, $b = 0$, of the free singleton field theory, induces the usual Lorentz condition on $A$; this tallies perfectly with the formula

$$\partial_\mu A^\mu = e(\bar{\varphi}b - \bar{b}\varphi) \quad (2.10)$$

that comes from the definitions of $A$ and $\hat{j}[\varphi]$ and the free field equations.

When free singleton modes are inserted for $\varphi$ and for $\bar{\varphi}$ in the expression for $A$, then a physical massless mode (transverse polarization) is produced. If one of the two factors
in the product is physical and the other is a gauge mode, then the field mode $A$ that results is a gauge mode; in other words, it is a gradient. Therefore, if the potential is coupled to a conserved current, then free singleton gauge modes decouple, so long as both factors in $A$ are free fields. This gives some encouragement for hoping that, under the right circumstances, such a coupling may give a unitary field theory.

Precisely, our proposal is as follows. Instead of the ordinary electromagnetic potential, couple singleton fields to any conserved current $J$ by introducing the interaction

$$\int d^4x J^\mu(x) A_\mu(x), \quad (2.11)$$

with

$$A_\mu(x) = e \hat{j}_\mu(x). \quad (2.12)$$

The physical singleton Fock space includes massless particles with all integer spins, as 2-singleton composites, but only two-singleton states with the quantum numbers of photons couple directly to the vector current $J$ of ordinary particles. One-particle singleton states are extraordinarily hard to detect, for kinematical reasons if not in principle [3].

There may be a singleton contribution to $J$, but it has to be conserved, so that $J$ may include the canonical current $j[\varphi,b]$ (that vanishes on the physical subspace), but not $\hat{j}[\varphi]$.

**Composite gravity.**

To compose gravitons out of singletons is just as natural. However, the canonical, bulk, energy momentum tensor cannot be identified with the gravitational potential since it is conserved. The complete expression for it is, in the case of a real singleton field,

$$t_{\mu\nu}[\varphi, b] = \frac{1}{2}(\varphi_\mu b_\nu + \varphi_\nu b_\mu - g_{\mu\nu}(g^{\sigma\tau} \varphi_\sigma b_\tau - u \varphi b)) + \delta(\infty)(\tilde{\varphi}_i \tilde{\varphi}_j - \frac{1}{2} \delta_{ij} \tilde{g}^{hi} \tilde{g}^{kl} \tilde{\varphi}_k \tilde{\varphi}_l), \quad (2.13)$$

where a tilde indicates boundary values. Except for the boundary term it is BRST-exact; the physical part is thus concentrated on the boundary. The tensor $\hat{t}[\varphi]$ that is needed is

$$\hat{t}_{\mu\nu}[\varphi] = \varphi_\mu \varphi_\nu - \frac{1}{2}(g^{\sigma\tau} \varphi_\sigma \varphi_\tau - u \varphi \varphi), \quad (2.14)$$

it satisfies, by virtue of the free field equations,

$$\partial_\mu \hat{t}^{\mu\nu}[\varphi] = \varphi_\nu b. \quad (2.15)$$
This too vanishes on the physical subspace, and it makes sense to identify the tensor $\hat{t}[\varphi]$ with the gravitational potential,

$$h_{\mu\nu} = \kappa \hat{t}_{\mu\nu}[\varphi].$$  \hfill (2.16)

When both factors in the field product (2.14) are replaced by free field modes, then the action of $SO(3,2)$ on the tensor current, and on the gravitational potential, is reduced to

$$D(4,1) \to D(3,2) \to D(4,1),$$  \hfill (2.17)

which is precisely the Gupta-Bleuler triplet associated with free anti-De Sitter gravitons. We therefore propose a theory of quantum gravity in which the first order perturbation of the background anti-De Sitter metric is given by Eq.s (2.16) and (2.14). This tensor field can be coupled to any conserved energy-momentum tensor $T$, by introducing the interaction

$$\int d^4x T^{\mu\nu}(x) h_{\mu\nu}(x).$$  \hfill (2.18)

Just as was explained in connection with the vector potential, it is not possible to include $\hat{t}[\varphi]$ as a contribution to $T$; what has to be included in $T$ is the conserved energy momentum tensor $t[\varphi,b]$ given in (2.13). Of course, this interaction must be corrected by nonlinear terms, in the usual way. In addition, the metric $g$ in (2.14) and in the volume element must be replaced by the perturbed metric. The result is that (2.14) and (2.16) will give, not a closed expression for the metric in terms of the singleton field, but a nonlinear relation between both that can be solved for the metric as a power series in $\varphi$, the leading terms being $g_{\mu\nu} + \kappa \hat{t}_{\mu\nu}$.

3. Spinors.

The spinor singleton field is also governed by a dipole. The complete Lagrangian is

$$\mathcal{L} = \mathcal{L}_4 + \mathcal{L}_3,$$

$$\mathcal{L}_4 = \int d^4x \sqrt{-g} \left( \frac{i}{2} (\bar{\psi} \not{D} b - \not{D} \bar{b} \psi) + \frac{i}{2} (\bar{b} \not{D} \bar{b} \psi - \not{D} b \bar{b} \psi) - \bar{b} b + v (\bar{b} \psi + \bar{b} \psi) \right),$$  \hfill (3.1)

$$\mathcal{L}_3 = \frac{i}{2} \int d^4x \sqrt{-g} D_\mu (\bar{\psi} \gamma^\mu \not{D} \psi - \not{D} \bar{\psi} \gamma^\mu \psi + i \bar{b} \gamma^\mu \psi - i \bar{\psi} \gamma^\mu b).$$

The contribution from the Faddeev-Popov ghosts was omitted, see [6]. The constant $v = \frac{i}{2} \sqrt{\rho}$. The Hamiltonian was calculated in [9], and the entire energy momentum tensor
in [10]; it is BRST exact except for a boundary term. The remarks about the canonical conserved vector and tensor currents apply here too. The vector current

\[ \hat{j}^\mu[\psi] = \sqrt{-g} \bar{\psi} \gamma^\mu \psi \]

satisfies

\[ \partial_\mu \hat{j}^\mu[\psi] = \bar{\psi}b - \bar{b}\psi; \]

The tensor current

\[ \hat{t}^{\mu\nu}[\psi] = i \sqrt{-g} g^{\mu\sigma} (\bar{\psi}_\sigma \gamma^\nu \psi - \bar{\psi} \gamma^\nu \psi_\sigma) \]

satisfies

\[ \partial_\mu \hat{t}^{\mu\nu}[\psi] = i \sqrt{-g} g^{\nu\sigma} (\bar{\psi}_\sigma b - \bar{b}_\sigma \psi - \text{conj.}) \]

The solutions of the field equations carry the triplet

\[ D(2, \frac{7}{2}) \rightarrow D(1, \frac{5}{2}) \rightarrow D(2, \frac{7}{2}) \]

When these free modes are inserted into the vector current \( \hat{j}[\psi] \) one finds that \( SO(3, 2) \) acts by the same representation \( D[A] \) as on the vector current \( \hat{j}[\varphi] \). Similarly, the tensor current transforms by the same representation as \( \hat{t}[\varphi] \). (A symmetrized form of \( \hat{t}[\psi] \) must be used.)

This makes it possible to identify the vector potential with \( \hat{j}[\varphi] \) or with \( \hat{j}[\psi] \) or with the sum of both; the same remark applies to the tensor current. For reasons having to do with the counting of states in flat limit (difficult) it is likely that the sum of both is the correct choice.

4. Supersymmetry.

The scalar and spinor singleton representations can be combined to a representation of the superalgebra \( OSp(4/1) \). A superfield formulation has been given, including a constraint-free Lagrangian formulation [11]. A scalar superfield that contains both fields has the form

\[ \Phi = \varphi + \theta \psi + \frac{1}{2} \theta \gamma^\mu \theta A_\mu + ... \]

The field equations are

\[ (\Box_b - 3)\Phi = B, \quad (\Box_b - 3)B = 0, \]
where $\square$ is a super d’Alembertian and $B$ is the Nakanishi-Lautrup superfield. The Lagrangian has the same structure as in (2.3) and (3.1), $\mathcal{L} = \mathcal{L}_4 + \mathcal{L}_3$ with

$$\mathcal{L}_4 = \int d^4x d^4\theta \sqrt{-g} \left(-\frac{i}{2}(Q^\alpha \Phi)(Q_\alpha B) + 3\Phi B + \frac{1}{2}BB\right).$$

The operators $Q_\alpha$ are the components of a supercovariant derivative. In addition to composite photons and gravitons, one can now construct neutrinos and gravitinos. The vector super current

$$\hat{j}_\alpha[\Phi] = \Phi Q_\alpha \Phi$$

satisfies

$$Q_\alpha \hat{j}_\alpha = \Phi B$$

and can be identified with the Wess-Zumino spinor super potential, in the De Sitter formulation.

In addition to the vector and tensor currents one can construct a spinorial current that may serve to construct a composite gravitino, but it is much more attractive to build the entire supergravity multiplet of fields from a super stress tensor, or rather from a tensor that is similar to the super stress tensor in the way that the vector and tensor currents $\hat{j}$ and $\hat{t}$ are patterned on the canonical current and stress energy tensors. We propose to use the tensor

$$\hat{t}_{\alpha\beta}[\Phi] = (Q_\alpha \Phi)(Q_\beta \Phi) + \frac{i}{2}(Q^\alpha \Phi)(Q_\alpha \Phi) - 3\Phi\Phi;$$

it satisfies

$$Q^\alpha \hat{t}_{\alpha\beta}[\Phi] = BQ_\beta \Phi,$$

and it is thus conserved on the physical subspace. The details of a formulation of linear anti-De Sitter supergravity, in terms of this type of superfield, remains to be worked out. Nevertheless, it is clear that this super tensor carries the right degrees of freedom to be identified as a super gravity potential.

5. The problem of unitarity.

Here we present some additional considerations, going beyond the arguments given in the introduction, concerning the question of whether or not the construction proposed
in this paper may, somewhat miraculously, lead to a unitary theory of composite massless particles and fields.

We consider some of the simplest Feynman diagrams, involving a vector current \( J \) made up of ordinary fields, and a number of lines that represent singletons. If all of the latter are external, then we know that there is no problem, for when a pair of free singletons couple to a conserved current then only the physical 2-singleton modes are effective. Now let us look at a diagram that has at least one internal singleton line, and two external singleton lines extending from vertices located at \( x \) and at \( x' \). So we are dealing with the following object,

\[
\varphi(x)\varphi(x')K(x, x')
\]

and similar quantities containing a derivative of one or the other (or both) of the \( \varphi \)'s. The fields \( \varphi(x) \) and \( \varphi(x') \) are thus free. Consider an operator product expansion,

\[
\varphi(x)\varphi(x') = \sum f_n(x - x')^{(n)}\varphi(x)\partial^{(n)}\varphi(x),
\]

where \( \partial^{(n)} \) is an \( n \)’th order differential operator. The field \( \varphi(x)\partial^{(n)}\varphi(x) \) is a massless field with spin \( n \). There is nothing unphysical about massless composite states with arbitrary spin, provided that they couple in such a way that only physical states interact. But this poses very strong conditions on the factor \( K(x, x') \). Only string theory can boast of miracles of this type.

Nevertheless, one solution to this problem is already known. It was shown, in fact, that by adopting unconventional field quantization for the singleton fields, it can be arranged that the composite operator \( \hat{j}[\varphi](x) \) satisfy exactly the canonical commutation relations of a standard, massless vector potential. This merely demonstrates the existence of a solution, we do not wish to claim that it is the only solution, and in fact, we suggest that it may not be the best one. To advance the discussion, we shall explain why we think that the old solution may not be perfect.

First, there is some sense of disappointment in discovering that this old “composite electrodynamics” is precisely equivalent to ordinary electrodynamics. True, the latter does not need improvement, but the same is not true of quantum gravity. The type of softening of interactions at small distances that is expected to be the most important result of replacing elementary fields, in this case the gravitational potential, with composites, is
highly desirable in a theory that is non-renormalizable and hence internally inconsistent. Concerning quantum gravity there were two problems. First, there is little point in setting up an elaborate structure of composite gravitons if there is no gain relatively to the problematic naive version. The second problem was more technical. If we apply our old alternate quantization paradigm to a theory with just one real, scalar singleton field, then we do not get any interaction at all, simply because the tensor $\hat{t}[\varphi]$ is symmetric in the two factors. We conclude, from the discussion of Feynman diagrams, that conventional quantization is unlikely to work out, and from this paragraph that our old solution to the problem, though successful when applied to QED, is also somewhat unpromising, at least for quantum gravity.

Recall that Wigner [12] questioned conventional quantization of elementary variables on the grounds that the most important observables are second order polynomials. He therefore proposed to examine commutation relations (among the basic observables, the singletons in our case) that give reasonable commutation relations for these observables (the massless fields). This is very appropriate in our context, since isolated singletons are essentially unobservable, if not in principle, then for kinematical reasons [3]. Wigner's suggestion led to the discovery of parastatistics. (Later, it was proposed to apply parastatistics to quarks [13], but this idea was subsequently transformed into the popular concept of color [14].) And this is the only alternative to strict canonical quantization that has been attempted (and not very diligently) in quantum field theory.

Clearly, the whole difficulty with unitarity can be “solved” by postulating free field commutation relations for the composite operators. But this would be to beg the question. What is needed is to take a good look at what are the possibilities available for the quantization of singleton fields, since, like quarks, they do not need to be represented by local field operators. It is not unlikely that string theory may provide further inspiration for attacking this problem.

In this paper we have shown that massless fields may be regarded as composites, and that the constituents have to be gauge fields, hence singletons. On a certain level this works in an easy and natural manner. The problem of unitary is a serious one, and well worth investigation.
Acknowledgements.

This paper was inspired by a stimulating collaboration with Sergio Ferrara.

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