Gyrokinetic investigation of magnetic islands in tokamaks

M. Siccinio, E. Poli, W. A. Hornsby*, A. G. Peeters**
Max Planck Institut für Plasmaphysik, Boltzmannstrasse 2, 85748 Garching bei München, Germany
* Centre for Fusion, Space and Astrophysics, Physics Department, University of Warwick, CV4 7AL Coventry, United Kingdom
** University of Bayreuth, 95440 Bayreuth, Germany
E-mail: mattia.siccinio@ipp.mpg.de

Abstract. A powerful new numerical tool to carry out investigations into magnetic islands by means of the gyrokinetic equations is described in the present paper. An island structure, with imposed width and rotation, has been implemented in the flux-tube code GKW. Numerical simulations retaining finite Larmor radius effects (FLRs), toroidicity, kinetic electrons and self-consistent electrostatic potential are presented, concerning, among others, the incomplete density profile flattening for small magnetic islands, the dependency of the island potential on the island rotation frequency and the emission of drift waves connected to the island rotation. Turbulent fluctuations are not fully resolved by filtering out short wavelengths.

The tearing mode is a non-ideal magnetohydrodynamic instability leading to a degradation of the confinement capability of a tokamak, and acting at times as a trigger for disruptive events. Therefore, a precise comprehension of its dynamics represents a primary objective in the present fusion research. In this paper, a numerical investigation has been worked out by means of the flux-tube gyrokinetic code GKW (GyroKinetics@Warwick [1]), which allows, in principle, to account for most of the physical effects of possible interest, e.g. finite Larmor radius effects, collisions, toroidicity and self-consistent calculation of electromagnetic disturbances. In this preliminary work, however, we restrict ourself to some particular aspects of this otherwise extremely broad topic.

1. Implementation of a magnetic island structure
The code GKW is a gyrokinetic flux-tube code, which employs Hamada coordinates [2] and exploits the periodicity constraint of the flux-tube approximation to solve the equations in the Fourier space on the plane perpendicular to the magnetic field lines (such codes are sometimes referred to as spectral codes). The gyrokinetic equation is solved for both ions and electrons, and the perturbed electric and magnetic fields can be calculated self-consistently from the particles response by means of Poisson and Ampère equations, respectively. In the simulations presented in this paper, however, the Ampère’s law has been switched off, as, for the moment, we are not interested in the island evolution (the island is completely described by a perturbation on the parallel vector potential). Subsequently, an island of fixed width and angular rotation frequency is imposed at every timestep, entering the gyrokinetic equation for both species. It is worth in
any case to stress that the problem of the self-consistent island evolution is in principle treatable with GKW.

The coordinate system of the code is constituted by a coordinate along the magnetic field lines (indicated by $s$, varying from -1/2 to 1/2 in the poloidal plane and such that $s = 0$ corresponds to the outer midplane of the tokamak), a poloidal flux $\psi$, defined to be zero at the centre of the flux-tube domain and playing the role of a radial-like coordinate, and a helical coordinate $\zeta = q s - \gamma$, where $q$ is the safety factor (which is in general a function of $\psi$) and $\gamma$ is a toroidal angle normalized to $2\pi$. The flux-tube approximation constrains every perturbed quantity in the code to be periodic in the directions $\nabla \zeta$ and $\nabla \psi$, calculated at the centre of the computational domain. In general, the perturbed vector potential $A_\parallel$ associated to the magnetic island is identified by a mode of the form

$$A_\parallel = C \exp \left[ 2\pi i (m s - n \gamma) - i \omega t \right],$$

where $C$ is the amplitude, $m, n$ are the poloidal and toroidal mode number, respectively, and $\omega$ is the island rotation frequency. Supposing the island to be located at the centre of the computational box (where by consequence one has to fix $q = m/n$), and expanding the safety factor as $q \approx m/n + (\partial q / \partial \psi) \psi$, Eq.(1) can be rewritten as

$$A_\parallel = C \exp \left[ 2\pi i n \left( \zeta - \frac{\partial q}{\partial \psi} \psi s \right) - i \omega t \right].$$

It is evident that this mode is not in general periodic in $\psi$ on the computational domain. Thus, an approximation of the mode is required. The solution we adopted consists in projecting Eq.(2) on the radial Fourier harmonics, adding moreover an artificial Gaussian smoothing, depending on $s$, in order to avoid the appearance of abrupt “jumps” at the boundary of the computational domain. The interested reader is referred to [3] for further details. The amplitude of the mode $C$, as well as the island rotation frequency $\omega$ and the magnetic shear $\partial q / \partial \psi$ are given as input values ($C$ is calculated from the island width, which is the actual input data).

All the calculations performed in this paper are collisionless, and in addition the number of poloidal harmonics considered has been kept low in order to cut off turbulent effects, which are left for future work.

2. Preliminary results

In this section, numerical simulations confirming some well-known physical effects connected to the presence of a magnetic island in a tokamak plasma are presented. This is helpful both to validate our implementation and to give a further support to the existing theories. Two aspects of the island dynamics are discussed in this section: the island electrostatic potential and the nonlinear density flattening in the island region.

A rotating magnetic island can be thought of as a time-varying magnetic perturbation, and therefore it possesses an associated electrostatic potential, according to Faraday’s law. In the existing literature (see for example [4]), an analytical expression for such potential is derived under the assumption of electrons immediately shorting out every parallel electric field, i.e.

$$E_\parallel = 0 \Rightarrow \nabla_\parallel \phi + \frac{1}{c} \frac{\partial A_\parallel}{\partial t} = 0.$$  

Recalling Eq.(2), it is clear that $\partial A_\parallel / \partial t = -\omega \partial A_\parallel / \partial \xi$, where $\xi = m \theta - n \varphi - \omega t$ is a helical coordinate (with $\theta$ and $\varphi$ being the poloidal and toroidal angle, respectively). After some straightforward algebra, one obtains

$$\phi = \frac{\omega B}{k_0 c} \left[ \psi - h(\xi) \right],$$

where
Figure 1. Qualitative behaviour of the island electrostatic potential through the island O-point according to Eq.(4) for a positive rotation frequency $\omega$. Vertical dashed lines denote the location of the island separatrix.

Figure 2. Self-consistent electrostatic potential as a function of the radial coordinate through the island O-point for various values of the island rotation frequency $\omega$ calculated with GKW. The potential is normalized to $q_i\phi/T\rho_*$, where $q_i$ and $T$ are the ion charge and temperature, respectively, while $\rho_* = \rho_L/R$ is the ratio between the thermal ion Larmor radius and the tokamak major radius. For the presented simulations, $\rho_* = 2 \times 10^{-3}$. The radial coordinate is normalized to $\rho_L$, while $\omega$ is normalized to $v_{th}/R$, where $v_{th}$ is the ion thermal velocity.

where $B$ is the magnetic field, $x$ is a radial coordinate defined to be zero on the rational surface, $k_0 = m/r$, $\Omega = 2x^2/w^2 - \cos \xi$ (with $w$ indicating the island half-width) and $h(\Omega)$ a function, playing the role of an integration constant (as $\nabla \parallel \Omega = 0$), which has to be determined from boundary conditions. Fig.1 shows a qualitative plot of Eq.(4) for a positive value of $\omega$, under the assumption of vanishing electric field at some distance from the island.

This behaviour of the potential has been confirmed by GKW simulations. As presented in Fig.2, the potential is found to scale linearly with the frequency, presenting in addition a behaviour which closely resembles the analytical results. It is important to mention that Fig.2 has been derived in absence of equilibrium pressure gradients. In fact, gradients allow some particular finite orbit effect to take place, as discussed in the following sections.

The density profile inside a magnetic island is known to flatten, as the appearing perturbed
component of the magnetic field leads to a fast parallel radial transport. Indeed, such flattening is typically the first check of a proper implementation of the magnetic island. For sufficiently large islands, we see that the flattening is indeed present in our simulations, except for a relatively narrow region around the separatrix where finite banana-width effects prevent the pressure profile to completely flatten (see next section). It is known moreover (see [5]), that in the separatrix region a complete flattening of the pressure profile is inhibited, because of the competition between perpendicular and parallel transport. However, in these simulations, where no collisions are present and the turbulence is not fully resolved, we expect this latter mechanism to be of minor importance. Fig.3 highlights what discussed.

![Density Profile](image)

**Figure 3.** Ion and electron density profiles for a non-rotating island of width $w = 12\rho_L$. The equilibrium density gradient has been fixed to $R/L_n = 1$.

### 3. Incomplete flattening for small islands

Once the fundamental aspects of the island dynamics have been benchmarked, some more recent result of the literature can be investigated. As a matter of fact, GKW is able to take into account a large number of physical mechanisms, allowing to model a wide variety of phenomena with a very high degree of self-consistency.

When the island width starts to be close to or below the ion banana width, an important finite orbit effect occurs. Because of their large banana width, in fact, trapped ions are able to spend part of their bounce time inside the separatrix and part outside. It has been observed [6] that for islands of the order of the banana width, ions exhibit an “adiabatic” response to the banana-averaged island potential $\phi_b$, defined as

$$
\phi_b(x) = \frac{\phi(x + \rho_b) + \phi(x - \rho_b)}{2}.
$$

(5)

Actually, the comparison between the perturbed ion density calculated with GKW and the quantity $n_b$, defined as

$$
n_b = -\sqrt{\epsilon}n_0\frac{q_i\phi_b}{T_i}
$$

(6)

(where the factor $\sqrt{\epsilon}$ appears as only trapped particles contribute), exhibits a remarkable agreement in the island region, as shown in Fig.4. On the contrary, the difference between the two curves outside the island depends on the fact that the adiabatic response, according to the existing theoretical models [7], takes place only if the scale of variation of the potential is small (or comparable) with respect to the ion orbit. This circumstance is fulfilled inside the island, but not outside.
Figure 4. Comparison of the perturbed ion density and $n_i$ for an island of width $w = 8\rho_L$ and $\omega = -0.04$. With the parameters chosen for the simulations, the inverse aspect ratio amounts to $\epsilon = 0.19$, while the ion banana width $\rho_b$ amounts to $\rho_b/\rho_L = 3.44$.

Figure 5. Ion and electron density profiles for an island of width $w = 4\rho_L$ for $\omega = 0.06$ and $\omega = -0.06$ (this means $|\omega|$ slightly larger than the diamagnetic frequency).

This effect on the perturbed density profile implies that for islands rotating in the electron diamagnetic direction (i.e. for positive $\omega$) the ion response opposes the flattening, while on the contrary the rotation in the opposite direction leads to a complete flattening, or even to a reversed pressure profile. The density profiles in a small island calculated by GKW for different frequencies are represented in Fig.5, confirming the drift-kinetic results of Ref.[6]. This phenomenon has potentially a major impact on the bootstrap current suppression inside the island region [8]. The numerical investigation of this effect, which requires a full benchmark of the collision operator, is left however for future work.

4. Emission of drift-waves

One of the most important effects which can be observed only if FLRs are retained is the emission of drift-waves by a rotating magnetic island. This phenomenon, which is supposed to have a significant impact on the background turbulence, has been first pointed out by Waelbroeck and
others [9]. In that paper, a difference between islands rotating in the electron diamagnetic
direction and those rotating in the ion diamagnetic direction has been pointed out. In fact,
the emission of drift-waves has been foreseen to take place only in the former case, as the
expression for the potential in the Fourier space exhibits a pole for $0 < \omega < \omega_{*,e}$ [9]. This is
clearly visible in the GKW numerical simulations, see Fig.6. For an island rotating in the ion-
diamagnetic direction (right) the potential is well localized in the island region, quickly going
to zero outside the island separatrix. On the contrary, for an island rotating in the electron-
diamagnetic direction, the potential is much more spread on the computational domain. The

\[ \omega = \frac{\xi}{\pi} \frac{x}{\rho L} \]

\[ k = \frac{\xi}{\pi} \frac{x}{\rho L} \]

**Figure 6.** Contour plot of the potential for an island of width $w = 8 \rho L$ and rotating in the electron- (left) and ion- (right) diamagnetic directions. With the chosen parameters, we have $\omega_{*,e} = -\omega_{*,i} = 0.015$. The O-point is identified by $\xi = 0$, and the two figures have been realized with the same number of contours.

**Figure 7.** Potential in the Fourier space (radial wavenumbers) for $k_{\xi} = k_{\xi,isl}$, for the same island and the same frequencies considered in Fig.6. $k_{isl}$ is the wavenumber corresponding to the radial box extent.

appearance of the drift-waves is moreover confirmed in Fig.7, where the “radial” Fourier spectra
of the electrostatic potentials of Fig.6 (for $k_{\xi} = k_{\xi,isl}$ corresponding to the island periodicity) are
depicted. It is clear that, for islands rotating in the electron-diamagnetic direction, the Fourier
components are much larger than in the opposite case, and this can be related to the occurrence
of drift waves.
5. Summary and Outlook
We have employed the flux-tube gyrokinetic spectral code GKW to carry out an investigation of some relevant physical phenomena caused by the presence of a magnetic island in the plasma. This code allows to take into account many relevant physical aspects (such as FLRs, toroidicity and self-consistent electrostatic potential), yielding therefore a more complete description of the island dynamics in comparison to most of the numerical investigations available in literature. This work represents the first step in the direction of achieving a self-consistent numerical description of the island dynamics retaining the large majority of physical effects potentially playing a role.

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