Resolving the effects of frequency dependent damping and quantum phase diffusion in YBa$_2$Cu$_3$O$_{7-\delta}$ Josephson junctions

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We report on the study of the phase dynamics of high critical temperature superconductor Josephson junctions. We realized YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) grain boundary (GB) biepitaxial junctions in the submicron scale, using low loss substrates, and analyzed their dissipation by comparing the transport measurements with Monte Carlo simulations. The behavior of the junctions can be fitted using a model based on two quality factors, which results in a frequency dependent damping. Moreover, our devices can be designed to have Josephson energy of the order of the Coulomb energy. In this unusual energy range, phase delocalization strongly influences the device’s dynamics, promoting the transition to a quantum phase diffusion regime. We study the signatures of such a transition by combining the outcomes of Monte Carlo simulations with the analysis of the device’s parameters, the critical current and the temperature behavior of the low voltage resistance $R_0$.

I. INTRODUCTION

A correct understanding of the phase dynamics of a Josephson circuit relies on the possibility to distinguish the contributions to dissipation coming from the junction itself from those due to the external circuit. This is especially relevant in the moderately damped regime for junctions with low critical current. High temperature superconductor (HTS) Josephson junctions (JJ) often fall in this category. Their phase dynamics is made particularly rich by the HTS unconventional superconductivity. The high value of the critical temperature ($T_c \approx 90$ K) and of the superconducting gap ($\Delta \approx 20$ meV) impose a unique energy scale to HTS JJs. Some effects generally observed in HTS junctions, as for example the values of the $I_cR_N$ parameter (with $I_c$ and $R_N$ the critical current and normal state resistance respectively) on average one order of magnitude lower than the expected value of $2\Delta$, may signify the relevance of other energy scales in these devices. One possibility is the Thouless energy associated to single nanoscale channels in a filamentary approach to transport across the GB. Despite this complexity, recent experiments demonstrate that macroscopic quantum phenomena can be observed also in HTS JJs, revealing coherence beyond expectations. Ultrasmall HTS junctions were also used to realize single electron transistors with unprecedented energy resolution, and proposed for the fabrication of ultra-sensitive superconducting quantum interference devices to use in the detection of small spin systems. These studies confirm the interest in nanoscale HTS devices and the need for a systematic and reliable study of their phase dynamics.

A detailed analysis of phase dynamics in moderately damped low temperature superconductor (LTS) JJs was performed by Kautz and Martinis in the early 90s. Here it emerges the need of a frequency dependent damping to fully account the phenomenology of the junctions, with clear indications of distinct behaviors at low and high frequency respectively. These arguments offer the possibility to disentangle the quality factor of the junction from the one of the external circuit. More recently, moderately damped JJs based on both LTS and HTS and operating in the phase diffusion regime, were investigated through the analysis of the switching current distribution (SCD) histograms. All these devices are, however, characterized by values of the Josephson energy $E_J = hI_0/2e$ (where $I_0$ is the critical current in absence of thermal fluctuations) much larger than those of the charging energy $E_C = e^2/2C$ (where $C$ is the junction capacitance). Devices characterized by $E_J \approx E_C$, on the other hand, were first studied by Iansiti et al. using Sn based junctions with nominal area of $\sim 0.1 \mu m^2$ and $I_c$ in the range 1-10nA. It was shown that this energy scale favours the access to a quantum phase diffusion regime, which is quite unexplored and whose nature is still unsettled.

In this work we study the phase dynamics of submicron HTS JJs in the moderately damped regime, using the tools developed for LTS JJs. We have realized YBCO junctions with lateral size down to 600nm on (La$_{0.7}$Sr$_{0.3}$)$_2$AlO$_2$(Al$_{0.65}$Ta$_{0.35}$)O$_3$ (LSAT) substrates. The reduction of the junctions’ size allows one to minimize the influence of the GB microstructure on the transport properties of the devices, while the use of LSAT substrate reduces the parasitic capacitance present in the more common SrTiO$_3$ (STO) based junctions. Using Monte Carlo simulations, we extract the frequency dependent damping of these devices and show that, for a particular range of parameters, the quantum phase diffusion regime can be attained.
II. EXPERIMENTAL

The junctions studied in the present work were realized following the design reported in Ref. [20, 22] and [23]. A CeO$_2$ thin film is deposited using RF magnetron sputtering on a (110) oriented LSAT substrate and patterned using photolithography and ion-beam etching (IBE). A 200nm YBCO film is then deposited using inverted cylindrical magnetron sputtering, obtaining (001) growth on the CeO$_2$ seed layer and (103) growth on the LSAT substrate, and subsequently covered with a protective gold layer (100nm thick). The definition of the sub-micron bridges is carried out using an electron beam lithography technique adapted to HTS requirements [24]. The electron beam pattern is transferred to a 80-nm-thick Ti layer which serves as a hard mask. The YBCO not covered by the Ti mask is removed using IBE, keeping the sample at low temperature (-140°C) in order to minimize oxygen loss. After this, the Ti mask is removed by chemical etch in a highly diluted (1:20) HF solution. Finally, the protective gold layer is removed using a last step of low-energy IBE. In the panels (a) and (c) of Fig. 1 scanning electron microscope images of 600nm wide devices (before the gold removal) are shown. The high quality of the YBCO film can be inferred from the systematic presence of elongated grains with typical size of 1μm in the (103) part and by the absence of impurities and outgrowths in the (001) part [25].

The devices were measured down to 0.25K using a four contacts technique. The measurement environment was magnetically shielded and the lines were filtered using RC filters and two stages of copper powder filters [26]. Current vs. voltage ($I - V$) characteristics of two typical devices, 1W and 6W, are shown in panels (b) and (d) of Fig. 1 respectively. The $I - V$ s are modulated by the magnetic field $H$ (panels (e) and (f)), leading to a Fraunhofer-like $I_c(H)$ pattern for junction 1W [3, 27]. Taking into account focusing effects, the $I_c(H)$ pattern periodicity in field points to an effective width of ≈500nm for device 1W (Figure 1e) and of ≈600nm for device 6W (Figure 1f). These values are very close to the nominal dimensions of the devices. The critical current density $J_c$ is 65 A/cm$^2$ for device 1W and 5 A/cm$^2$ for device 6W. The low $J_c$ values of these devices are a consequence of oxygen depletion, occurring especially in the GB region. [29] This is a quite general feature of HTS JJs [8] and is expected to be of particular relevance when decreasing the size of the junction, as in this case. We have found that the devices realized using LSAT as a substrate are characterized by higher values of the normal state resistance and are more affected by aging when compared with the ones fabricated on STO substrates. These micro-structural factors could in this case mask the influence of the d-wave order parameter in determining the magnitude of the $J_c$ as a function of the junction misorientation [22]. Grains elongated in the current direction in the device 1W (Fig. 1(a)), for instance, might be less exposed to oxygen desorption compared to grains leaning against the walls of the channel in device 6W (Fig. 1(c)), explaining the different values of $I_c$ measured for these two devices.

The reduced values of $J_c$, on the other hand, offers the possibility to have access to JJ dynamical regimes which have been poorly explored. The Josephson energy $E_J$ is ≈270 μeV (corresponding to 3K) for device 1W and 70μeV (corresponding to 0.8K) for device 6W. These energies were calculated using the $I_0$ values obtained through comparison to numerical results, as described in Section IV. They are one or two orders of magnitudes smaller than those measured for junctions where macroscopic quantum behavior has been demonstrated [5], and five orders of magnitude smaller than those observed in most HTS Josephson devices [5, 8]. More importantly, for device 6W, $E_J$ is comparable with the charging
energy \( E_c \), as will be described in Section IV, placing this device in an uncommon and interesting energy range.

The \( I - V \) curves shown in Fig. 1 are highly hysteretic, with a difference between the critical \((I_c)\) and the retrapping \((I_r)\) current up to 70% at the lowest temperature (panel d). The presence and the nature of hysteresis in the \( I - V \)s of HTS junctions have been a matter of debate.\[2\] It is indeed difficult in these devices to disentangle the intrinsic capacitive effects in the GB barrier from extrinsic ones, deriving from the external circuit, also due to the high dielectric constant (above 10000 at low temperatures\[30\]) of the STO substrates on which the junctions are commonly fabricated.\[3, 5, 8, 31\] In the present work, we have used LSAT as a substrate, with a temperature independent dielectric constant \( \epsilon_r \), of 23\[32\]. As a consequence, the influence of the external circuit is greatly reduced.\[20\]

Remarkably, this neat hysteresis coexist with a slope at low voltage. The low voltage slope is a hallmark of phase diffusion effects\[13\] and is visible in Fig. 1(b) (device 1W) for temperatures greater than 2K, and in Fig. 1(d) (device 6W) in the whole temperature range, down to the 0.25K. The two phenomena, hysteresis and phase diffusion, can separately be understood in the framework of the washboard potential model for Josephson junctions.\[27\] On the other hand, their coexistence in the same \( I - V \) is unusual\[13, 17, 33\] and requires a finer analysis of the devices properties and dynamics, which we will address in the following section.

III. THE "TILTED WASHBOARD" POTENTIAL MODEL FOR THE STUDY OF JJ'S PHASE DYNAMICS

The behavior of a Josephson junction can be described, in the most general approach, by an Hamiltonian \( \mathcal{H} \), which is a function of the phase difference \( \varphi \) between the superconductive electrodes:

\[
\mathcal{H} = -4E_c \frac{\partial^2}{\partial \varphi^2} - E_J \cos \varphi
\]  

where \( E_c \) and \( E_J \) are the aforementioned charging and Josephson energies respectively.\[13\] \( E_c \) is commonly much smaller than \( E_J \), both in the HTS and in the LTS case, therefore the \( E_c \) term in equation (1) is usually disregarded. In this condition, the dynamics of the junction phase can be modelled as the motion of a fictitious particle of mass \( m = C(\Phi_0/2\pi)^2 \) in the "washboard" potential \( U(\varphi) = -E_J[\cos \varphi + (1/I_0)\varphi] \), sketched in Figure 2. This dynamics is well understood, both in the classical and in the quantum regime.\[27, 34\] For \( I < I_0 \) the potential \( U \) has local minima where the phase particle is trapped and oscillates at the plasma frequency \( \omega_0 = \sqrt{2\pi I_0/C\Phi_0} \). An increase of \( I \) has the effect of tilting the potential and decreasing the barrier between two neighbouring minima. Eventually, for \( I = I_0 \) the phase will escape from the well and a voltage will appear at the junction's edges. Decreasing the bias current, the potential tilt will be reduced and for \( I = I_r \) the particle will be retrapped in a well, returning to the zero voltage state.

In the case of underdamped junctions, with quality factor \( Q_0 = \omega_0 RC > 1 \), we find \( I_r < I_0 \), therefore an hysteresis is present in the \( I - V \) characteristic. In the case of overdamped junctions (\( Q_0 < 1 \)), only one stationary state, the one at rest at a potential minimum with zero voltage across the junction, is stable for \( I < I_0 \) and the \( I - V \) characteristics show no hysteresis.\[27\] This picture is strictly valid only at zero temperature. At finite temperature, thermal noise activates the phase over the energy barrier, favoring a slip from the potential well for \( I = I_c < I_0 \) (red line in Figure 2). In underdamped junctions, a single phase slip event is enough for the junction to switch to the running state. In overdamped junctions, on the other hand, after thermal slippage, the phase can be recaptured in the next well (blue line in Fig. 2). This prevents the access to the running state and leads to the appearance of a non zero voltage, manifesting as a "rounding" in the \( I - V \) curve at low currents. This regime is called phase diffusion.\[13, 27\]

A. Frequency dependent damping model

A more complete description of the Josephson phase dynamics can be achieved by incorporating the effects of the circuit the junction is embedded into. The effects of the external environment are taken into account through an additional quality factor \( Q_e \). In the case of HTS-based junctions, this external circuit is intrinsic and partly hidden, because it is embedded in the GB and, in the case of off-axis biepitaxial junctions, in the (103) oriented electrode.\[8, 31\] The study of
its contributions, as encoded in the damping of HTS devices, therefore becomes more challenging. 

The effects of the embedding circuit become particularly interesting when $Q_1 < Q_0$. At the plasma frequency $\omega_0$ (typically in the GHz range), the smaller quality factor $Q_1$ dominates the behavior of the whole system. The voltage state involving steady motion of the phase is instead dominated by the higher quality factor $Q_0$. Therefore, the system will exhibit a frequency dependent damping, which explains the coexistence of hysteresis and phase diffusion, as seen in our devices (Figure 1b and 1d).

When $E_c$ is comparable with $E_J$, the $E_c$ term in equation (1) cannot be disregarded. Its presence leads to phase delocalization effects. The value of the ratio $x = E_c/E_J$ is a measure of how strongly the charging energy acts in delocalizing the phase, being related to the width $\delta \varphi$ of the phase wave function $\psi(\varphi)$: $\delta \varphi = (x)^{1/4}$. For $x << 1$, $\psi(\varphi)$ is a narrowly peaked function, the phase is localized and can be treated as a semi-classical quantity. For values of $x$ greater than 1/4, on the other hand, the phase variable is sufficiently delocalized that quantum fluctuations cannot be neglected and quantum uncertainty, especially at low temperatures, has to be taken into account. Phase delocalization leads to an increase in the probability for the phase to escape from the potential well, both in the thermal and in the quantum regime. Multiple escape and retrapping result in a finite resistance $R_0$ at low voltage; in the quantum regime, the value of $R_0$ saturates due to freezing out of the thermal fluctuations.
in the literature. In Figure 3(b) we also show the fits to the SCD histograms (full lines). These were realized using the following parameters: $Q_1=0.56$, $Q_0=2$, $I_0=130$ nA. The switching behavior of a JJ is a high frequency phenomenon. Indeed, the study of the switching behavior of JJs in the moderately damped regime is usually performed using a single-$Q$ model to fit the experimental SCD histograms. Such procedure works well when the condition $E_J >> k_B T$ is satisfied and the quality factor is larger than one. In our case, $Q_1=0.56$, therefore, in order to preserve the underdamped dynamics of the phase after the escape process, a second quality factor $Q_0$ with a slightly increased value with respect to $Q_1$, had to be included in the model. We point out that experimental reports showing the occurrence of phase diffusion effects both in the $I–V$ curves and in the SCD histograms are extremely rare. This combined analysis has previously been carried out, to our knowledge, only in Ref. where, contrary to what happens in our work, the main contribution to the damping of the devices comes from the external impedance, and the junction intrinsic resistance plays no significant role. In our case, the reduced value of $E_J$ makes phase diffusion effects become evident not only in the behavior of the SCD histograms, but also in the shape of the $I–V$ characteristics, thereby offering two independent routes for the study of phase diffusion. An estimation of the high frequency dissipation $Q_1$ for our device, for instance, is both an output of the K-M model and a necessity for numerically reproducing the experimental SCD histograms. Finally, we point out that, in previous experiments on off-axis biepitaxial junctions realized on LSAT substrates, the $Q$ factor obtained via the simulation of SCD histograms was $1.3\pm0.05$. This value is consistent with $Q_1=0.6\pm0.1$ found in the present work, taking into account that here $I_c$ is one order of magnitude smaller and that high frequency dissipation is larger for devices with reduced $I_c$. The analysis of the behavior of junction 1W reveals that the phase dynamics of YBCO submicron JJs characterized by low values of $E_J$ is compatible with that expected, in the K-M approach, in the phase diffusion regime. A further reduction of $E_J$, making it comparable to $E_c$, induces a different behavior, as we will demonstrate for device 6W. In Fig. 4 we compare the experimental $I–V$ curves of device 6W (left panel) measured at T=0.25K, and 1.45K compared with Monte-Carlo simulations (full lines, right panel) made using $Q_1=0.6$ and $Q_0=12$ and $I_0=35$ nA. In the inset the temperature dependence of the measured low voltage resistance $R_0$ is shown.

![FIG. 3.](image) Transport properties of device 1W. In panel (a) the experimental $I–V$ characteristics (left part, points) measured at T=0.25K, 1.0K and 2.0K are compared with Monte-Carlo simulations (right part, full lines) realized using the following parameters: $Q_1=0.6$, $Q_0=5$ and $I_0=130$ nA. Panel (b) shows the comparison between experimental (points) and simulated (black full lines) SCD histograms. The experimental SCD histograms were measured using a voltage criterion of 100 nV. The inset shows the behavior of the simulated (triangles) and experimental (dots) histograms width versus the temperature.

![FIG. 4.](image) $I–V$ characteristics of device 6W (points, left panel) measured at T=0.25K, and 1.45K compared with Monte-Carlo simulations (full lines, right panel) made using $Q_1=0.6$ and $Q_0=12$ and $I_0=35$ nA. In the inset the temperature dependence of the measured low voltage resistance $R_0$ is shown.
| device | $E_J$ (µeV) | $E_c$ (µeV) | $x$ | $Q_0$ | $Q_1$ | $I_0$ |
|-------|------------|------------|----|-------|-------|-------|
| 1W    | 270         | 45       | 0.16 | 5±0.5 | 0.6±0.1 | 130nA |
| 6W    | 70          | 47        | 0.65 | 12±0.5 | 0.6±0.1 | 35nA  |

TABLE I. Parameters of devices 1W and 6W. All the parameters refer to $T=0.25K$, except for $Q_0$, $Q_1$ and $I_0$ of device 6W, which refer to $T=1.45K$.

expected to be relevant, and promoting quantum phase diffusion.\[17, 11\]

The reduced value of $I_c$ of device 6W (a factor of 10 lower compared to device 1W) is consistent with this estimation of the fundamental energies. As discussed in the previous section, $x$ is related to the width of the phase function $\delta \varphi$ and therefore to the delocalization of the phase. For $x \approx 0.65$, $\delta \varphi$ is $\approx 0.9$. Although the phase $\varphi$ is still confined in one well of the washboard potential, the barrier height of such a well, which depends on both $E_J$ and $E_c$, is reduced, influencing the critical current.

For $x > 1/4$, the critical current $I_c$ is indeed scaled by $E_B/E_J$ where $E_B$ is the binding energy: [17]

$$E_B \approx E_J 2x[(1 + 1/8x^2)^{1/2} - 1] \tag{5}$$

leading to a temperature-independent $I_c = 2eE_B/\hbar$ which is less than the value $I_0 = 2eE_J/\hbar$ which would be observed in the absence of quantum fluctuations. Using the values of $E_J$ and $x$ to calculate $E_B$, we obtain $I_c=6.5nA$, in good agreement with the experimental value measured at 0.25K (see Fig. 4).

As mentioned in Section III, the temperature dependence of the finite resistance at low voltages, $R_0$, is another indicator of the quantum phase diffusion state. Device 6W clearly shows such resistance, also at 0.25K, as marked by the black line in Figure 1d. Iansiti et al.\[17\] report that the value and the behavior of $R_0$ depends on the ratio $x$. The $R_0$ values shown in the inset of Fig. 4 are consistent with those found in Ref\[17\] resulting from numerical simulations using $x=0.65$. Moreover, $R_0$ is proportional to the tunnelling rate\[17\] $R_0 \approx \frac{1}{2\pi}\Gamma$, where $\Gamma$ can be calculated by using the Caldeira-Leggett approximation in presence of dissipation\[12\]. Using this formula with an upper bound value of $R_0 \approx 500$ Ohm, a damping $Q$ of about 1 is obtained. This value is consistent with the high frequency $Q_1$, factor inferred for this device (at high temperatures) using $I – V$ simulations. More importantly, the $R_0$ of device 6W decreases with decreasing temperature and levels off around 0.3K, as shown in the inset of Figure 4. The saturation of $R_0$ marks the entrance into the quantum regime.\[13\]

From the estimated value of the plasma frequency $\omega_0 \approx 40GHz$, we calculate a crossover temperature $T_{cr} = \hbar\omega_0/2\pi k_B$ between the classical and the quantum regime, of 120mK.\[14\] Such equation for the crossover temperature has been estimated in the regime $E_J >> E_c$. In our case, since $E_J \approx E_c$, the binding energy is modified, the phase delocalization is larger and therefore the probability for quantum tunnelling of the phase is increased.

As a result, the crossover temperature between thermal and quantum activation is pushed up. Indeed our experimental data show that quantum tunnelling of the phase influences the phase dynamics already at 0.3K.

We point out that junction 1W has similar values of $\omega_0$ and $T_{cr}$ (75GHz and 155mK respectively) but the condition $E_c << E_J$ (see the values listed in Table I) results in negligible delocalization effects, and the dynamics of the junction is classical down to 0.25K, as shown by the good agreement between the experimental data and the simulations (Fig. 3).

V. CONCLUSIONS

We have engineered YBCO grain boundary biepitaxial junctions in the submicron scale, down to 600nm, and with reduced Josephson energy $E_J$. This regime is quite rare to achieve for HTS JJs and has been, up to now, scarcely explored. The junctions behavior can be simulated using a frequency dependent damping model. The quality factors obtained by the fits indicate a moderately damped regime\[7, 20, 21\]. Classical phase diffusion, in a frequency dependent approach, describes quite well the behavior of the devices, as far as $E_c << E_J$. When $E_J \approx E_c$ delocalization starts to play an important role in the phase dynamics, the temperature at which quantum effects start to influence the phase dynamics is increased and a transition to a quantum phase diffusion regime occurs at $T \approx 0.3K$.

This work is of relevance both to define phase dynamics in HTS JJs in extreme limits and for the experimental search for quantum phase diffusion. More systematic studies will be required to obtain additional hints on the effects of microscopic factors, in particular the relation between a d-wave order parameter symmetry and dissipation.

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[1] C.C. Tsuei and J.R. Kirtley, Rev. of Mod. Phys. 72, 969 (2000)
[2] D. J. Van Harlingen Rev. Mod. Phys. 67, 515 (1995)
junction 1W, suggests an increase in effective resistance for device 6W, prevailing on the reduction of $I_c$. Iansiti et al.\cite{17} pointed out that, when charging effects become more relevant, the $I-V$ characteristic becomes more resistive in the low voltage regime.

\cite{41} Quantum phase diffusion arises when $E_c$ is comparable with $E_J$. This regime can be obtained by reducing either the $I_c$ value or the capacitance of the junction. In our case, the devices 1W and 6W are defined on the same chip and the capacitance can reasonably assumed to be constant.

\cite{42} A. O. Caldeira and A. J. Leggett, Phys. Rev. Lett. 46, 211 (1981)

\cite{43} A more quantitative analysis on the quantum phase diffusion regime for this device could be performed using SCD histograms. However, due to the extremely low values of the critical currents, such measurements are extremely difficult to perform. As a matter of fact we are not aware of SCD measurements in this range of critical currents, even for LTS JJs.

\cite{44} H. Grabert and U. Weiss, Phys. Rev. Lett 53, 1787 (1984)