Analytic Electroweak Dyon

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We present analytic monopole and dyon solutions whose energy is fixed by the electroweak scale. Our result shows that genuine electroweak monopole and dyon could exist whose mass scale is much smaller than the grand unification scale.

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Ever since Dirac [1] has introduced the concept of the magnetic monopole, the monopoles have remained a fascinating subject in theoretical physics. The Abelian monopole has been generalized to the point-like non-Abelian monopole by Wu and Yang [2, 3], and to the finite energy soliton by 't Hooft and Polyakov [4, 5].

In the interesting case of electroweak theory of Weinberg and Salam, however, it has generally been believed that there exists no topological monopole of physical interest. The basis for this “non-existence theorem” is, of course, that with the spontaneous symmetry breaking the quotient space $SU(2) \times U(1)/U(1)_{em}$ allows no non-trivial second homotopy. This belief, however, is unfounded. Indeed, recently Cho and Maison [6, 7] have established that Weinberg-Salam model has exactly the same topological structure as Georgi-Glashow model, and demonstrated the existence of a new type of monopole and dyon solutions in the standard electroweak theory. This was based on the observation that Weinberg-Salam model, with the hypercharge $U(1)$, could be viewed as a gauged $CP^1$ model in which the (normalized) Higgs doublet, $\phi$, allows no finite energy soliton by 't Hooft and Polyakov [4, 5].

To understand the physical content of the ansatz we now perform the following gauge transformation on (2)

$$\xi \rightarrow i \begin{pmatrix} \cos(\theta/2) & \sin(\theta/2) e^{-i\varphi} \\ -\sin(\theta/2) e^{i\varphi} & \cos(\theta/2) \end{pmatrix} \xi = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and let $\tilde{A}_{\mu} = (A_{\mu}^{1}, A_{\mu}^{2}, A_{\mu}^{3})$ in this unitary gauge. Now, introducing the electromagnetic potential $A_{\mu}^{(em)}$ and the neutral Z-boson $Z_{\mu}$ with the Weinberg angle $\theta_{w}$

$$\begin{pmatrix} A_{\mu}^{(em)} \\ Z_{\mu} \end{pmatrix} = \begin{pmatrix} \cos \theta_{w} & \sin \theta_{w} \\ -\sin \theta_{w} & \cos \theta_{w} \end{pmatrix} \begin{pmatrix} B_{\mu} \\ A_{\mu} \end{pmatrix}$$

$$= \frac{1}{\sqrt{g^2 + g'^2}} \begin{pmatrix} g & g' \\ -g' & g \end{pmatrix} \begin{pmatrix} B_{\mu} \\ A_{\mu} \end{pmatrix},$$

where $(t, r, \theta, \varphi)$ are the spherical coordinates. Notice that the apparent string singularity along the negative z-axis in $\xi$ and $B_{\mu}$ is a pure gauge artifact which can easily be removed with a hypercharge $U(1)$ gauge transformation. So the above ansatz describes a most general spherically symmetric ansatz:

$$\phi = \frac{1}{\sqrt{2}} \rho(r) \xi(\theta, \varphi),$$

$$\xi = i \begin{pmatrix} \sin(\theta/2) e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix}, \quad \tilde{n} = \xi^\dagger \tilde{\tau} \xi = -\hat{r},$$

$$\tilde{A}_{\mu} = \frac{1}{g} A(r) \partial_{\mu} t \hat{n} + \frac{1}{g} (f(r) - 1) \hat{n} \times \partial_{\mu} \tilde{n},$$

$$B_{\mu} = -\frac{1}{g'} B(r) \partial_{\mu} t - \frac{1}{g'} (1 - \cos \theta) \partial_{\mu} \varphi,$$

with $A_{\mu}^{1}$, $A_{\mu}^{2}$, $A_{\mu}^{3}$ the gauge potentials of SU(2) and $B_{\mu}$ the magnetic field. Notice that $A_{\mu}^{1}$ and $B_{\mu}$ are pure gauge fields, while $A_{\mu}^{2}$ and $A_{\mu}^{3}$ are non-gauge fields.

The purpose of this Letter is to show that this is indeed possible, and to present finite energy electroweak monopole and dyon solutions which are analytic everywhere, including the origin.

Let us start with the Lagrangian of the standard Weinberg-Salam model,

$$\mathcal{L} = -|D_{\mu} \phi|^{2} - \frac{\lambda}{2} (\phi^{\dagger} \phi - \mu^{2})^{2} - \frac{1}{4} F_{\mu \nu}^{2} - \frac{1}{4} G_{\mu \nu}^{2},$$

$$D_{\mu} \phi = (\partial_{\mu} - ig \tilde{\tau} \cdot \tilde{A}_{\mu} - ig' B_{\mu}) \phi, \quad \mu = 1, 2,$$

where $\phi$ is the Higgs doublet, $F_{\mu \nu}$ and $G_{\mu \nu}$ are the gauge field strengths of $SU(2)$ and $U(1)$ with the potentials $\tilde{A}_{\mu}$ and $B_{\mu}$. Now we choose the following static spherically symmetric ansatz

$$\phi = \frac{1}{\sqrt{2}} \rho(r) \xi(\theta, \varphi),$$

$$\xi = i \begin{pmatrix} \sin(\theta/2) e^{-i\varphi} \\ -\cos(\theta/2) \end{pmatrix}, \quad \hat{n} = \xi^\dagger \tilde{\tau} \xi = -\hat{r},$$

$$\tilde{A}_{\mu} = \frac{1}{g} A(r) \partial_{\mu} t \hat{n} + \frac{1}{g} (f(r) - 1) \hat{n} \times \partial_{\mu} \tilde{n},$$

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we can express the ansatz (2) by
\[ \rho = \rho(r) \]
\[ W_\mu = \frac{1}{\sqrt{2}} (A_\mu^1 + iA_\mu^2) \]
\[ = \frac{i}{g} f(r) e^{i\varphi} (\partial_\mu \theta + i \sin \theta \partial_\mu \varphi), \]
\[ A_{\mu}^{(em)} = -e \left( \frac{A(r)}{g} + \frac{B(r)}{g^2} \right) \partial_\mu t - \frac{1}{e} (1 - \cos \theta) \partial_\mu \varphi, \]
\[ Z_\mu = \frac{e}{gg'} (B(r) - A(r)) \partial_\mu t, \tag{5} \]
where \( \rho \) and \( W_\mu \) are Higgs boson and W-boson, and \( e \) is the electric charge
\[ e = \frac{gg'}{\sqrt{g^2 + g'^2}} = g \sin \theta_w. \tag{6} \]
This clearly shows that the ansatz is for the electromagnetic monopole and dyon.

With the spherically symmetric ansatz and with a proper boundary condition one can obtain the Cho-Maison dyon solution shown in Fig.1, which has the magnetic charge \( 4\pi/e \) \[ \mathbb{B} \]. The regular part of the solution looks very much like the Julia-Zee dyon, except that it has a non-trivial Z-boson dressing. Of course the magnetic singularity at the origin makes the energy of the Cho-Maison solutions infinite. A simple way to make the energy finite is to introduce the gravitational interaction \[ \Box \]. But the gravitational interaction is not likely remove the singularity at the origin.

To construct the analytic monopole and dyon solutions, notice that non-Abelian gauge theory in general is nothing but a special type of an Abelian gauge theory which has a well-defined set of charged vector fields as its source. This tells that the finite energy non-Abelian monopoles are really nothing but the Abelian monopoles whose singularity is regularized by the charged vector fields \[ \Box \]. From this perspective one can try to make the energy of the above solutions finite by introducing additional interactions and/or charged vector fields.

It is rather straightforward to obtain a finite energy dyon solution by introducing additional hypercharged vector fields. This can be done by enlarging the hypercharge \( U(1) \) to another \( SU(2) \) and extending the gauge group to \( SU(2) \times SU(2) \) \[ \mathbb{B} \]. But a more economic way to obtain a finite energy electroweak dyon is utilizing the already existing W-boson. In this case we could try to regularize the magnetic singularity of the Cho-Maison solutions with a judicious choice of an extra electromagnetic interaction of W-boson with the monopole.

To show that this is indeed possible notice that in the unitary gauge \[ \mathbb{B} \] where \( \hat{a} \) assumes the trivial configuration \((0, 0, -1)\), the Lagrangian \[ \mathbb{B} \] can be written as
\[ -\frac{1}{2} (D_{\mu}^{(em)} W_{\nu} - D_{\nu}^{(em)} W_{\mu}) + ie g' (Z_{\mu} W_{\nu} - Z_{\nu} W_{\mu}) \]
\[ - \frac{1}{4} F_{\mu \nu}^{(em)} \]
\[ - \frac{1}{8} \rho^2 W^* \mu W_{\mu} \]
\[ = \frac{1}{4} g^2 (W_{\mu} W_{\nu} - W_{\nu} W_{\mu})^2 \]
\[ - \frac{1}{8} \lambda \rho^2 W^* \mu W_{\mu}, \tag{7} \]
where \( Z_{\mu \nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu \) and \( D_{\mu}^{(em)} = \partial_\mu + igA_{\mu}^{(em)} \).

We now introduce an extra interaction \( \mathcal{L}' \)
\[ \mathcal{L}' = ig F_{\mu \nu}^{(em)} W_{\mu} W_{\nu} + \beta \frac{g^2}{4} (W_{\nu} W_{\mu} - W_{\mu} W_{\nu})^2. \tag{8} \]
With this additional interaction the energy of the dyon is given by \( E = E_0 + E_1 \), where
\[ E_0 = \frac{2\pi}{g^2} \int_0^\infty \frac{dr}{r^2} \{ \frac{g^2}{g^2} + 1 - 2(1 + \alpha) f^2 + (1 + \beta) f^4 \}, \]
\[ E_1 = \frac{4\pi}{g^2} \int_0^r \frac{dr}{r^2} \{ \frac{g^2}{2} (r \rho)^2 + \frac{1}{2} (r A)^2 + \frac{g^2}{2 g^2} (r B)^2 \}
\[ + \frac{g^2}{4} f^2 \rho^2 + f^2 A^2 + \frac{g^2}{8} (B - A)^2 \rho^2 \]
\[ + \frac{A g^2}{4 \lambda} (\rho^2 - \frac{2 \mu^2}{\lambda})^2 \}. \tag{9} \]
Clearly \( E_1 \) could be made finite with a proper boundary condition, but notice that when \( \alpha = \beta = 0 \), \( E_0 \) becomes infinite. This is the reason why the Cho-Maison dyon has infinite energy. To make \( E_0 \) finite we need to remove both \( 1/r^2 \) and \( 1/r \) singularities in \( E_0 \). This requires
\[ 1 + \frac{g^2}{g^2} - 2(1 + \alpha) f^2(0) + (1 + \beta) f^4(0) = 0, \]
\[ (1 + \alpha) f(0) - (1 + \beta) f^3(0) = 0. \tag{10} \]
Thus we must have
\[ \frac{(1 + \alpha)^2}{1 + \beta} = 1 + \frac{g^2}{g^2} = \frac{1}{\sin^2 \theta_w}, \]
\[ f(0) = \frac{1}{\sqrt{(1 + \alpha) \sin^2 \theta_w}}. \tag{11} \]
With this we have the following equations of motion
\[ \dot{f} - \frac{1 + \alpha}{r^2} \left( \frac{f^2}{f^2(0)} - 1 \right) f = \left( \frac{g^2}{4} \rho^2 - A^2 \right) f, \]
\[ \dot{\rho} + \frac{2}{r^2} \dot{\rho} - \frac{f^2}{2 r^2} \rho = - \frac{1}{4} (B - A) \rho + \frac{\lambda}{2} (\rho^2 - \frac{2 \mu^2}{\lambda}) \rho, \]
\[ \dot{A} + \frac{2}{r^2} \dot{A} - \frac{f^2}{2 r^2} A = \frac{g^2}{4} (A - B) \rho^2, \tag{12} \]
\[ \dot{B} + \frac{2}{r} \dot{B} = \frac{g^2}{4} (B - A) \rho^2, \]
which can be integrated with the boundary condition
\[ f(0) = 1/\sqrt{(1 + \alpha) \sin^2 \theta_w}, \rho(0) = 0, \]}
\[ A(0) = 0, \quad B(0) = b_0, \]
\[ f(\infty) = 0, \quad \rho(\infty) = \rho_0 = \sqrt{2\mu^2/\lambda}, \]
\[ A(\infty) = B(\infty) = A_0. \]  
(13)

But notice that, although obviously sufficient for a finite energy solution, the condition (13) in general does not guarantee the analyticity of the gauge potential at the origin. This must be clear from the fact that the condition (13) does not remove the singularity in \( B_\mu \).

The condition for an analytic solution is given by
\[ \alpha = 0, \quad f(0) = \frac{1}{\sin \theta_w} = \frac{g}{e}. \]  
(14)

To understand this we need to review the analytic Julia-Zee dyon in Georgi-Glashow model
\[ \mathcal{L}_{GG} = -\frac{1}{2}(D_\mu \Phi^*)^2 + \frac{\lambda}{4} \left( \Phi^2 - \frac{\mu^2}{\lambda} \right)^2 - \frac{1}{4} F_{\mu\nu}^2, \]  
(15)

where \( \Phi \) is the Higgs triplet. With \( \Phi = \rho \hat{n} \) one can easily show that the Georgi-Glashow model acquires the following Abelian form in the unitary gauge
\[ \mathcal{L}_{GG} = -\frac{1}{2}(\partial_\mu \rho)^2 - \frac{\lambda}{4} \left( \rho^2 - \frac{\mu^2}{\lambda} \right)^2 - g^2 \rho^2 W^*_\mu W^\mu, \]
\[ -\frac{1}{4} F_{\mu\nu}^2 + ig F_{\mu\nu} W^*_\mu W^\nu + \frac{g^2}{4} (W^*_\mu W^\nu - W^*_\nu W^\mu)^2 \]
\[ -\frac{1}{2} |D_\mu W^\nu - D_\nu W^\mu|^2. \]  
(16)

Now, with the spherically symmetric ansatz
\[ \Phi = \rho(r) \hat{n}, \]
\[ \vec{A}_\mu = \frac{1}{g} A(r) \partial_\mu t \hat{n} + \frac{1}{g} (f(r) - 1) \hat{n} \times \partial_\mu \hat{n}, \]  
(17)

one has the following equation
\[ \ddot{f} - \frac{f^2 - 1}{r^2} f = \left( \rho^2 + A^2 \right) f, \]
\[ \ddot{\rho} + \frac{2}{r} \dot{\rho} - \frac{2 f^2}{r^2} \rho = \left( \rho^2 - \frac{\mu^2}{\lambda} \right) \rho, \]
\[ \ddot{A} + \frac{2}{r} \dot{A} - \frac{2 f^2}{r^2} A = 0. \]  
(18)

With the boundary condition
\[ f(0) = 1, \quad \rho(0) = 0, \quad A(0) = 0, \]
\[ f(\infty) = 0, \quad \rho(\infty) = \rho_0, \quad A(\infty) = A_0, \]  
(19)

one can integrate (18) and obtain the Julia-Zee dyon. Notice that the boundary condition, in particular \( f(0) = 1 \), is crucial to make the ansatz (17) analytic at the origin.

To derive the analyticity condition (14) notice that, with (14), what the extra interaction \( (8) \) does is to modify the coupling strength of the \( W \)-boson quartic self-interaction from \( g^2/4 \) to \( e^2/4 \). So, in the absence of the

\[ \begin{align*}
\text{FIG. 1: The electroweak dyon solutions. The solid line represents the finite energy dyon and dotted line represents the Cho-Maison dyon, where } Z = B - A \text{ and we have chosen } \sin^2 \theta_w = 0.2325, \lambda/g^2 = M_H^2/\lambda M_W^2 = 1/2, A(\infty) = M_W/2.
\end{align*} \]

Z-boson we have
\[ \mathcal{L} + \mathcal{L}' \rightarrow \frac{1}{4} (\partial_\mu \rho)^2 - \frac{\lambda}{8} \left( \rho^2 - 2 \frac{\mu^2}{\lambda} \right)^2 - \frac{g^2}{\lambda} \rho^2 W^*_\mu W^\mu \]
\[ -\frac{1}{4} F_{\mu\nu} (em)^2 + ie F_{\mu\nu} (em) W^*_\mu W^\nu + \frac{e^2}{4} (W^*_\mu W^\nu - W^*_\nu W^\mu)^2 \]
\[ -\frac{1}{2} |D_\mu (em) W^\nu - D_\nu (em) W^\mu|^2 \]
\[ = -\frac{1}{4} (\partial_\mu \rho)^2 - \frac{\lambda}{8} \left( \rho^2 - 2 \frac{\mu^2}{\lambda} \right)^2 - \frac{g^2}{\lambda} \rho^2 W^*_\mu W^\mu \]
\[ -\frac{1}{4} F_{\mu\nu}^2, \]  
(20)

where now \( F_{\mu\nu} \) is the “electromagnetic” \( SU(2) \) gauge field made of \( W^1, W^2, \) and \( A^\mu (em) \), with the gauge coupling constant \( e \). Furthermore, with \( Z = 0 \), the ansatz (11) is written as
\[ \rho = \rho(r), \]
\[ A_\mu = \frac{e}{g^2 + g^2} A(r) \partial_\mu t \hat{n} \]
\[ + \frac{1}{e} \left( f(r) - 1 \right) \hat{n} \times \partial_\mu \hat{n}. \]  
(21)

Evidently (10) and (17) of Georgi-Glashow model. In particular, the Yang-Mills part is completely identical, except that here the coupling constant is \( e \), not \( g \). This means that \( A_\mu \) becomes smooth at the origin when \( A(0) = 0 \) and \( f(0) = g/e \). Furthermore, since \( Z \) has no monopole singularity, the ansatz (11) becomes smooth everywhere when in this case. This provides the analyticity condition (14).

With (13) and (14) we can integrate (2). The results of the numerical integration for the dyon solution are shown in Fig. 1. Here we have chosen \( \sin^2 \theta_w \) to be the experimental value 0.2325. It is really remarkable that the
finite energy solutions look almost identical to the Cho-Maison solutions, even though they no longer have the singularity at the origin and analytic everywhere. The reason for this must be clear. All that we need to make the Cho-Maison solutions analytic is a simple modification of the coupling strength of $W$-boson quartic self-interaction from $g^2/4$ to $e^2/4$.

Clearly the energy of the above solutions must be of the order of the electroweak scale $M_W = \rho_0/2$. Indeed for the monopole the energy with $\lambda/g^2 = 1/2$ is given by

$$E = 1.407 \times \frac{4\pi}{e^2} M_W.$$  
(22)

This demonstrates that the finite energy solutions are really nothing but the regularized Cho-Maison solutions which have a mass of the electroweak scale.

Notice that we can even find an analytic monopole solution explicitly, if we add an extra term $\delta \mathcal{L}$ to $\mathcal{L} + \mathcal{L}'$,

$$\delta \mathcal{L} = -(\epsilon^2 - \frac{g^2}{4})r^2 W_\mu^* W_\mu.$$  
(23)

This amounts to changing the mass of $W$-boson from $g\rho_0/2$ to $e\rho_0$. With this change the electroweak Lagrangian in the absence of the $Z$-boson, becomes identical to (16) in the limit $\lambda = 0$. In this case we have the Bogomol’nyi-Prasad-Sommerfield equation for the monopole (with $Z_\mu = 0$),

$$f + e\rho f = 0,$$
$$\dot{\rho} + \frac{1}{e\rho^2} (\frac{f^2}{f(0)^2} - 1) = 0.$$  
(24)

This has the well-known analytic solution [1]

$$f = f(0) \frac{e\rho_0}{\sinh(e\rho_0)} = \frac{g\rho_0}{\sinh(e\rho_0)},$$
$$\rho = \rho_0 \coth(e\rho_0) - \frac{1}{e\rho},$$  
(25)

which has the energy $$(4\pi/e^2) M'_W, \quad (M'_W = e\rho_0).$$

But notice that, even in this case, the electroweak dyon becomes different from Prasad-Sommerfield dyon, because of the non-trivial $Z$-boson dressing.

Strictly speaking the finite energy solutions are not the solutions of Weinberg-Salam model, because their existence requires a modification of the electroweak interaction. But from the physical point of view there is no doubt that they should be interpreted as the electroweak monopole and dyon, because they are really nothing but the regularized Cho-Maison solutions. More significantly, this regularization is made possible with only a minor change of the coupling strength of $W$-boson quartic self-interaction. From this point of view one could say that, in retrospect, the existence of the finite energy electroweak dyon explains why the singular Cho-Maison dyon in Weinberg-Salam model could exist in the first place.

It has generally been assumed that the finite energy monopoles could exist only at the grand unification scale [14]. But our result suggests the existence of a totally new type of electroweak monopole and dyon whose mass is much smaller than the monopoles of the grand unification. Certainly the existence of the finite energy electroweak monopole and dyon could have important physical implications. If existed, they could be the only finite energy topological objects that one could ever hope to produce with the (future) accelerators. A more detailed discussion of our work will be published in a separate paper [13].

Acknowledgments

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