The Solving Program of Multi-Variable Coupling Problem in Engineering State Diagnosis System Based on Laplace Transformation

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Abstract. Multi-variables coupling problems bring disadvantageous effect to macro-analyzing of large-scale engineering project implement states. In order to solve the problem, multi-variables coupling mechanism of engineering project is analyzed and described, and the universal equation which includes multi-variables coupling results is set up. Then, two key parameters to solve multi-variables coupling problems are found through decomposition analyzing the universal equation. On the basis of which, the two key parameters are obtained with Laplace transformation equation which contains system multi-variables coupling result. At last, real engineering experiments prove that the Laplace solution program not only is availability, but also has higher checking precision.

Introduction

In recent years, the engineering project implement state diagnosis system has been widely used in large-scale engineering projects in order to realize management modernization. It can be known from the application situations that it has a good effect in the micro-problems diagnosis, but the results to analyze macro-problems often appear large deviations comparing with engineering practice because of multi-variables coupling effects in the integration process of several micro-diagnosis results, which brings much difficulty to macro-analyzing and decision-making of large-scale engineering project [1].

For solving the problem, many researchers studied in different ways and put forward many methods such as the grey fuzzy combination method, state decomposition analyzing method, entropy decomposition methods, close-loop control method and so on [2-3]. Overall, these proposed methods can be classified into two categories, one is named multi-variates linear combination method, and the other is named hierarchical decomposition method. It can be known from the simulation or application results of these methods that because these methods can not reflect the polymorphic multi-variables coupling essence in the solving process of multi-variates comprehensive value, they still cannot meet the actual needs of project management. Therefore, understanding and grasping the multi-variables coupling mechanism of the large-scale engineering project is a necessary precondition for solving this problem.

Analysis of System Multi-variables Coupling Mechanism

The large engineering project is a complex nonlinear open system, which needs to exchange resources continually with inside and outside the system during the project implement process, and is also affected by uncertain various factors from the internal and external system. Therefore, according to the system science and control theory, the system state and its characteristic is the comprehensive effect of internal and external factors, which mainly depends on the internal factors in general [4], when the internal factors change, the system state will change accordingly, so the internal factors become system endogenous variables. When the system is disturbed by the external factors, the system state will also change correspondingly, the external factors are thought as system exogenous variables. Therefore, in the implement process of engineering project, the system state would appear randomness and diversity characteristics when the system emerges many variables, and there are
random combinations of multi-variables. Assuming $\dot{x}(t)$ is a set of relative multi-variables coupling behaviors and results during any period, the system state equation can be expressed as $\dot{x}(t) = Ax(t)$ when the system does not exist external interference$^6$. Here, $A$ is a parameter matrix of endogenous variables $x(t)$. If the dimension $x(t)$ is $N$, the system variables can be expressed as a function of time change$^6$, namely, $x(t) = a_0 + a_1 t + a_2 t^2 + \ldots + a_n t^{n-1}$, where $a$ is a $N$ dimension column vector. If put $x(t)$ into the system state equation, then,

$$\dot{x}(t) = A(a_0 + a_1 t + a_2 t^2 + \ldots + a_n t^{n-1}) = a_0 + 2a_1 t + 3a_2 t^2 + \ldots + na_n t^{n-1}$$

(1)

$$a_i = \frac{1}{(n)!} A^n a_0 \qquad \dot{x}(t) = a_0 + a_1 t + a_2 t^2 + \ldots + a_n t^{n-1} = \left( I + At + \frac{1}{2!} A^2 t^2 + \ldots + \frac{1}{n-1!} A^{n-1} t^{n-1} \right) a_0$$

(2)

At this time, mark the sum of infinite series in parentheses as matrix exponential function $e^{At}$, then, $x(t) = e^{At} a_0$, when $t = 0$, $e^{At} = 1$, $x(0) = a_0$. This shows that the system at this time lies in its original state. When engineering system appears multi-variables and produce coupling effects with the development of project, the system state at any time point $\tau$ can be expressed as follow:

$$x(t) - x(t_0) = \int_0^t A(\tau) x(\tau) d\tau$$

(3)

After appears $M$ multi-variables coupling response, the system state changes into following form.

$$x(t) = \left[ I + \int_0^t A(\tau) d\tau + \int_0^t A(\tau_1) \int_0^{\tau_1} A(\tau_2) d\tau_2 d\tau_1 + \ldots + \int_0^t A(\tau_1) \int_0^{\tau_2} A(\tau_3) \int_0^{\tau_3} A(\tau_4) \int_0^{\tau_4} A(\tau_5) \ldots \right] x(t_0)$$

(4)

It can be seen from the formula four that even if only appearing the coupling effect produced by system endogenous variables, $e^{At}$ has changed very complex. In order to express clearly the relationships of multi-variables coupling effect, $e^{At}$ can be described by relationship matrix and marked it as $\Phi(t, t_0)$.

$$\Phi(t, t_0) = \begin{bmatrix}
\phi_1(t, t_0) & \phi_2(t, t_0) & \ldots & \phi_n(t, t_0) \\
\phi_1(t, t_0) & \phi_2(t, t_0) & \ldots & \phi_n(t, t_0) \\
\vdots & \vdots & \ddots & \vdots \\
\phi_1(t, t_0) & \phi_2(t, t_0) & \ldots & \phi_n(t, t_0)
\end{bmatrix}$$

(5)

However, the disturbing from external factors always exists in engineering practice. When the system exogenous variable $u(t)$ interferes, the system state equation also changes into a new form$^7$, namely, $\dot{x}(t) = Ax(t) + Bu(t)$, $B$ is a parameter matrix of $u(t)$. If transfer the equation form, then,

$$\dot{x}(t) = Ax(t) + Bu(t) \qquad \left[ \dot{x}(t) - Ax(t) \right] e^{At} = e^{At} Bu(t) = \frac{d}{dt} e^{At} Bu(t)$$

$$x(t) = A e^{At} x(t_0) + \int_0^t e^{A(t-\tau)} Bu(\tau) d\tau$$

Replace $e^{At}$ with $\Phi(t, t_0)$ and derivation from both sides, then,
\[
\frac{d}{dt} x(t) = \frac{\partial}{\partial t} \Phi(t,t_0) A x(t_0) + \frac{\partial}{\partial t} \int_{t_0}^{t} \Phi(t,\tau) B(\tau) u(\tau) d\tau \\
= A(t) \Phi(t,t_0) x(t_0) + \left[ \Phi(t,\tau) B(\tau) u(\tau) \right]_{\tau=t_0}^{\tau=t} + \int_{t_0}^{t} \frac{\partial}{\partial t} A(t) \Phi(t,\tau) B(\tau) u(\tau) d\tau + B(t) u(t)
\]

The analysis result shows that the multi-variables coupling result in the process of system state change includes three parts, the first part is the coupling result of polymorphic system endogenous multi-variables, the second part is the coupling result of polymorphic system exogenous multi-variables, and the third part is the coupling result of polymorphic system endogenous and exogenous multi-variables. Therefore, if the system multi-variables coupling problem wants to be solved, the best ideal is using the generalized equation which contains system inside and outside multi-variables and can effectively express the multi-variables coupling results to seek the solving approach based on the coupling mechanism.

**Coupling Analysis of System Multi-variables Based on Engineering Practice**

It can be seen from the analysis result of system multi-variables coupling mechanism that there are many variables and their corresponding parameters in the multi-variables coupling equation. For endogenous variables, \( x(t) \) is stable relatively in engineering practice. Otherwise, the engineering project cannot be controlled, while the case is contradiction with the actual engineering situation. So, the endogenous variables and their parameters can be controlled and measured. But for exogenous variables \( x(t) \), they are often unstable and bring different influence to engineering project.

Considering from engineering practice, the exogenous variables can be divided into two categories, one is moderate controllable and measurable, and the other is complete uncontrollable and unmeasurable. It has been proved by engineering practice that such complete uncontrollable and unmeasurable factors as war or earthquake rarely emerge in engineering, and bring great harmful to engineering projects when these factors once occurred. At this time, the analysis of multi-variables coupling effect has no any significance; hence, these factors can be neglected. For the moderate controllable and measurable factors like price adjustment or legal disputes, they can also be separated into two sections. The first is measurable and controllable and marked as \( x'(t) \), the second is measurable and difficult controllable and marked as \( \delta(t) \). \( x'(t) \) is similar with \( x(t) \), but the both parameters are different. In order to facilitate the analysis, replace \( x'(t) \) with \( K x(t) \). Thus, \( u(t) = K x(t) + F \delta(t) \). \( K \) is the system state feedback matrix, \( F \) is the nonsingular transformation matrix of \( \delta(t) \). If Putting it into the system state equation, then, \( \dot{x}(t) = (A + BK)x(t) + BF \delta(t) \). Meanwhile, setting the corresponding system output with the system state as the equation, that is, \( y(t) = C x(t) \). \( C \) is the system out parameter. Under the precondition, although the relative parameters \( (A, B, C) \) can be obtained according to information regularization results which are collected from information management system[^8], but because the parameters \( (K, F) \) are difficult measured which result from the randomness of external factors, the multi-variables coupling problem cannot still be solved. So, how to obtain the parameters \( (K, F) \) is the key to solve coupling problem.

However, it is can be known through above analysis that to obtain the solution of \( (K, F) \) is very difficult only by some information collected from information management system, this is the bottleneck of solving this problem at present. If standing on the macro-system view of engineering project to study the problem, it can be found that because the project object is preplanned and predefined after engineering implement plan is made in practice, the system expected assignment pole
\( \lambda_{id} \) related with system multi-variables can be captured, and the corresponding Laplace expression \( \int_i(s) \) with system state can also be obtained furtherly \(^{[9-11]}\), namely,

\[
\int_i(s) = \sum_{j=0}^{d_i} (s-\lambda_{ij}^*)^j
\]

At this time, let \( \tilde{K} \) develop with the Laplace mode as follow,

\[
\int \tilde{K}_i(s) = [sI - \tilde{a}_i + \tilde{b}_i \tilde{K}_i] = s^{d_i+1} + \tilde{k}_{id} s^{d_i} + \cdots + \tilde{k}_{i1} s + \tilde{k}_{i0}
\]

\[
\tilde{a}_i = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
\vdots & 0 & \ddots & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & \cdots & 0 & \tilde{a}_n
\end{bmatrix},
\tilde{b}_i = \begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
0
\end{bmatrix},
\tilde{b}_i = \begin{bmatrix}
\tilde{b}_1 & 0 & 0 & 0 \\
\tilde{b}_2 & 0 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & \tilde{b}_n
\end{bmatrix},
\tilde{c}_i = \begin{bmatrix}
1^{-1} \\
0 \\
0 \\
\vdots
\end{bmatrix},
\tilde{c} = \begin{bmatrix}
\tilde{c}_1 & 0 & 0 & 0 \\
0 & \tilde{c}_2 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & \tilde{c}_n
\end{bmatrix}
\]

\[
T^{-1} = \left(\tilde{V}_0 \tilde{V}_0^T\right)^{-1} \tilde{V}_0 \tilde{V}_0 = \left[\tilde{C} \tilde{C} \tilde{A} \cdots \tilde{C} \tilde{A}^{-1}\right]^{-1}
\]

Comparing \( \int_i(s) \) with \( \int \tilde{K}_i(s) \), \( \tilde{K} \) can be solved after calculate \( \tilde{K}_i \).

\[
\tilde{K} = \begin{bmatrix}
\tilde{K}_1 & 0 & 0 & 0 \\
0 & \tilde{K}_2 & 0 & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & 0 & \tilde{K}_n
\end{bmatrix}
\]

Put \( E, L, \tilde{K}, T^{-1} \) into the relation expression based on \( (A, B, C) \) and systemic object, that is, \( K = E^{-1}L + E^{-1} \tilde{K} T^{-1}, F = E^{-1}, (K, F) \) can be obtained. Hence, the system multi-variables coupling problem is also solved completely.

**Example Analysis**

In order to test the effectiveness of the Laplace solving program of multi-variables coupling problem in engineering state diagnosis system, the research takes a multi-variables coupling problem which emerges in a large nuclear power project quality state analysis as an example. After understanding this engineering design plan, implement schedule and management object, according to the six determined variables (materials quality, worker technique level, implementing process effect, precision of quality test instrument, natural weather effect, special technique support degree) and related data from information management system, the corresponding multi-variables coupling relation equation is obtained as follow by MatlabR2011a based on the Laplace solving program (Please note, because the construction process of coupling equation and its related parameters includes more subjects knowledge, here no longer explicates in detail in order to highlight this study theme. The special method can be found by the references 12.)

\[
\dot{x}(t) = (A + BK)x(t) + BF \delta(t) = \begin{bmatrix}
0 & 1 & 0 & 0 \\
-1 & -2 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -1 & -2
\end{bmatrix} x(t) + \begin{bmatrix}
0 & 0 \\
1 & 0 \\
0 & 0 \\
0 & 1
\end{bmatrix} \delta(t)
\]

After replace the present multi-variables analysis mode in this project diagnosis system with above
multi-variables coupling equation model, the diagnosis system gives a comprehensive analysis result of 12 consecutive engineering quality states based on the engineering information which collected once each week from May to August in 2017(T-data in table 1). Subsequently, the researchers collected the real quality state data corresponding with above 12 analysis results by manual way(R-data in table 1).

| Collect point | T-Data | R-Data | Deviation | Absolute deviation | Deviation degree | Collect point | T-Data | R-Data | Deviation | Absolute deviation | Deviation degree |
|---------------|--------|--------|-----------|-------------------|-----------------|---------------|--------|--------|-----------|-------------------|-----------------|
| 1-May         | 0.563  | 0.507  | 0.056     | 0.056             | 0.111           | 1-Jul         | 0.474  | 0.481  | -0.007    | 0.007             | 0.015           |
| 2-May         | 0.563  | 0.624  | -0.062    | 0.062             | 0.099           | 2-Jul         | 0.523  | 0.495  | 0.028     | 0.028             | 0.057           |
| 3-May         | 0.578  | 0.632  | -0.054    | 0.054             | 0.085           | 3-Jul         | 0.623  | 0.636  | -0.012    | 0.012             | 0.019           |
| 1-Jun         | 0.563  | 0.624  | -0.062    | 0.062             | 0.099           | 1-Aug         | 0.691  | 0.759  | -0.067    | 0.067             | 0.089           |
| 2-Jun         | 0.613  | 0.565  | 0.049     | 0.049             | 0.086           | 2-Aug         | 0.725  | 0.789  | -0.064    | 0.064             | 0.081           |
| 3-Jun         | 0.593  | 0.514  | 0.079     | 0.079             | 0.154           | 3-Aug         | 0.762  | 0.835  | -0.073    | 0.073             | 0.087           |

Knowing from comparing analysis of above two group data in table 1, the maximum deviation between the analysis result of diagnosis system and real engineering state is 0.079, the minimum deviation is 0.007, the average deviation is 0.047, the degree of average deviation is 0.071. If take the present engineering quality management precision±0.1 as standard, the analysis result with Laplace solving program can meet the requirement completely.

**Conclusion**

Aiming to the multi-variables coupling problem in large-scale construction project implement state diagnosis system, the mechanism of multi-variables coupling in system state change is analyzed at first, and the state equation based on system multi-variables coupling mechanism is set up together with engineering practice. Then, through the Laplace transform, the multi-variables coupling problem is solved; the corresponding Laplace solving program is given. At last, the research obtains some following conclusions through four times of real engineering experiments.

1. The experimental results show that in the total of 74 test points, 73 test points can meet management requirement except for one point. The test result indicates that the effective degree to deal with multi-variables coupling problem with the Laplace solving program is up to 98%. Therefore, this method is feasible and effective in the solving multi-variables coupling problem.

2. In the four times of practical experiments, no matter what type of testing mode is taken as random, intermittent or continuous, the silence or no response phenomena of diagnosis system never appear from experimental beginning to end, the system output is very smoothly. This proves that the multi-variables coupling model set up by the Laplace solving program method can commendably match with the present engineering project implement state diagnosis system.

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