Dynamics of the free jets from nozzles of complex geometries

Paolo D’Addio, Paolo Orlandi

Dipartimento di Ingegneria Meccanica e Aerospaziale, “Sapienza” Università di Roma, Italy

Abstract

The dynamics of the coherent structures in jets generated by nozzles of different shapes is analyzed through DNS at $Re = 565$ by considering circular, square, fractal and star-like nozzles. The jets generated from orifices with corners undergone a rotation proportional to the corner angular width: $\theta_{\text{rotation}} = \theta_{\text{corner}} / 2$. The velocity at which this rotation occurs is also affected by the angle of the corners, being faster for fractal and star-like nozzles which have small $\theta_{\text{corner}}$. Therefore it has been found that the velocity of the rotation is associated with enhanced spreading and entraining characteristics. The jet evolution and its rotation are dictated by the vorticity field and, in particular, by the positive and negative $\omega_x$ layers generated at each corner. The comparison between the fractal and the star-like jets at this $Re$, suggested that the effect of the smaller scales generated by the fractal nozzle does not play a role in the development of the jet, that evolves as the star-like one.

Keywords: vortex dynamics, non-circular jet

1. Introduction

Several researches on non-circular jets has been performed in the past, both experimentally (e.g. [1], [2], [3], [4]) and through numerical simulations (e.g. [5], [6]). It has been found that these jets are capable to entrain ambient fluid more effectively than comparable circular jets and, at the same time, also the mixing properties are enhanced (Gutmark and Grinstein (1999) [1]). Such characteristics of non-circular jets may have a large influence on the combustion process (Williams (1985) [7]), with important implications also in the transport, handling and storage of fuels, and may also find application in engine ignition systems (Murase et al. (1996) [8]).

Previous experimental studies have investigated the increased entrainment of non-circular jets, considering both elliptic jets (Ho and Gutmark (1987) [9], Husain and...
Hussain (1983)[10] and Hussain and Husain (1989)[3] among the others) and jets originated by nozzles with corners (Gutmark et al. (1989)[11] and Quinn (1994)[12]). The outcome of these studies is that the entrainment capability of these jets is associated either with the non-uniform curvature of the jet’s initial perimeter, or with the instabilities produced by the initial perimeter’s sharp corners through the asymmetric distribution of mean flow field. Both phenomena are expected to cause a greater degree of three-dimensionality in the coherent structures or motions (Schadow (1988)[13]), therefore causing faster entrainment. The control of the jet evolution is thus strongly dependent on understanding the dynamics of the vortical structures, because the spreading is affected by the formation, interaction, merging, and breakdown of these structures.

The starting jet generated by a circular orifice shows a leading vortex ring followed by a slender jet stem, and the associated flow dynamics has been studied for constant density configurations by Gharib et al. (1998)[14] and many other in the past. They observed that, as the boundary layer separates at the orifice, the vortex sheet rolls up to form a toroidal vortex that travels downstream, entraining the outer fluid. As a result of the roll-up, a mixed core of reactants and combustion products should form at the jet head, providing a precursor ignition kernel where chemical reactions are enabled by the high temperature. Successively, Husain and Hussain (1983)[10] and Ho and Gutmark (1987)[9] found that the entrainment of jets can be enhanced by using non-circular nozzles and that the jets started by these nozzles are characterized by the fact that undergo a rotation. The reason for the enhanced mixing and entrainment properties has then been attributed to this characteristic rotation of the jet cross section, that occurs during its development in the stream-wise direction. In the previous laboratory studies using elliptic nozzles by Ho and Gutmark (1987)[9], Husain and Hussain (1983)[10] and Hussain and Husain (1989)[3], or using nozzles with corners (e.g. rectangular, triangular, star-like) by Gutmark et al. 1989)[11] and Quinn (1992)[15], it has been observed that as the jet spreads, its cross-section can regularly evolve through shapes similar to those of the jet nozzle but with axes successively rotated at angles characteristic of the jet geometry. This main underlying ‘axis-switching’ mechanism is responsible for the increased entrainment properties of non-circular jets, and results from self-induced deformation of vortex rings with non-uniform azimuthal curvature and interaction between azimuthal and stream-wise vorticity. This axis-switching phenomena found by experiments have also been demonstrated in several numerical simulations, such as those of Husain and Hussain (1983)[10] for elliptic jets, and Grinstein (1995)[6] for rectangular jets and Miller et al. (1995)[5] for several jet geometries.

In the present work, DNS simulations at low Reynolds number have been performed to study the near-field evolution of the vortical structures, and to show their role in the ‘axis-switching’ mechanism responsible for the increased entrainment properties of jet generated by nozzles of complex shapes.
2. Numerical experiments

The details of the numerical scheme to solve the Navier-Stokes equations, together with the immersed boundary method used to reproduce the interaction between the flow and the solid, are described by Orlandi and Leonardi (2006) [16]. The shape of the orifices considered are given in figure 1. The fractal orifice has been obtained with three iterations of the basic triangular geometry and all the orifices have been designed to have the same area, thus the solidity is the same in all the cases. All the orifice have the same area $A = 4$ and $D_e$ is the equivalent diameter of the jets $D_e = 2(A\pi)^{0.5} = 2.257$. At the inlet an uniform velocity profile is imposed and the orifice is located at $x_N = 2.25$. The simulations were performed at $Re_{D_e} = U_\infty D_e/\nu = 565$ ($U_\infty = 1$) in a computational domain $L_1 \times L_2 \times L_3 = 3\pi \times 2\pi \times 2\pi$ discretized with $n_1 \times n_2 \times n_3 = 385 \times 257 \times 257$ points. The orifice is located in $y-z$ and $x$ is the downstream direction. Periodicity is assumed in $y$ and $z$. In the plots the $x$-coordinate is normalized with $D_e$ starting from $x_N$: $x^* = (x - x_N)/D_e$.

3. Results

3.1. 'Axis-switching' mechanism of jets

These DNS allow to study the dynamics governing the evolution of the jets. At $Re_{D_e} = 565$, the flow is laminar at the exit from the nozzle, and therefore it is possible to see the deformation of the jet and understand the causes leading to its deformation. In figure 2 the velocity visualizations of the square, circular, fractal and star-like orifice have been reported. The flow generated by the circular orifice ($J_C$) is spread by viscous diffusion and maintains approximately the circular shape downstream of the orifice. In the other cases the shape is the same of the nozzle only in the proximity of the orifice, and it is lost in a short distance: at the end of the domain the flow is characterized by a number of corrugations, linked to the number of the large scale corners of the orifice, that are rotated with respect to their initial position. For the square jet ($J_S$) the corrugations are rotated of $45^\circ$; for the fractal jet ($J_F$) only the six corrugations, related to the largest scales, are recognizable from the contour of $U$, and these are rotated

Figure 1: Geometry of the orifices (a) $J_S$, (b) $J_C$, (c) $J_F$, (d) $J_6$ and $s$- and $l$-directions.
Figure 2: 3D Contour plot of $U = U_\infty$ for (a) $J_S$, (b) $J_C$, (c) $J_F$ and (d) $J_6$, coloured by the distance from the nozzle in the range $0 < \tilde{x} < 3.18$.

about $30^\circ$ with respect to the initial configuration, and the same rotation occurs for $J_6$.

Therefore in the cases with corners, it is possible to deduce that the rotation of the jet is proportional to the number of spikes, in particular: $\theta_{\text{rotation}} = \theta_{\text{corner}}/2$. Such rotation is clearly visible from the three-dimensional contour in figure 2. This rotation of the jets was studied experimentally and numerically by Gutmark et al. (1987) [2], Grinstein et al. (1995) [17] and Miller et al. (1995) [5]. In particular [5] considered square, rectangular and equilateral triangles orifices, observing a $45^\circ$ rotation for the first, an axis-switching ($90^\circ$ rotation) for the second, and an overturning of $180^\circ$ for the latest.

Considering only the initial and final position, the overturning of the equilateral triangle is indeed equivalent to a $30^\circ$ rotation, with $\theta_{\text{rotation}} = \theta_{\text{corner}}/2$, corroborating the present results. Therefore, if the aspect ratio of the geometry of the nozzle is equal to 1, the jet undergoes a rotation with $\theta_{\text{rotation}} = \theta_{\text{corner}}/2$. According to the results of Miller et al. (1995) [5], such law is not valid for the rectangular and the elliptic orifices.

The location where the rotation of the jet terminates is also affected by the shape, and this length can be determined by monitoring the stream-wise variation of the half velocity width $r_{1/2}$ measured on the smallest ($s$-direction) and largest ($l$-direction) radii defined in figure 1. The half-width $r_{1/2}$ of the jet, at the given stream-wise location, is the distance from the center-line at which the axial velocity drops to half of its center-line value $U_{cl}$. Simple geometrical considerations suggest that for $J_S$ it is possible to
find two $s$-directions that are those parallel to the $y$ and $z$ axes, and two $l$-directions that correspond to the diagonals of the square: one inclined at $45^\circ$ and the other at $-45^\circ$ with respect to the horizontal $z$-axis. For $J_F$ and $J_6$ instead it is possible to find three $l$-directions, that correspond to the largest radii: one is parallel to the vertical $y$-axis and the other two are inclined respectively at $\pm 30^\circ$ with respect to $z$. Consequently there are three $s$-directions: one is parallel to $z$ and the other two are inclined respectively at $\pm 60^\circ$ with respect to $z$.

In figure 3 the variation of the half-widths $r_{1/2}^s$ and $r_{1/2}^l$ non-dimensionalized by $D_F$ is reported for $J_S$, $J_F$ and $J_6$, while for $J_C$ we reported only $r_{1/2}^s$ because the circular jet remains circular. The quantities in figure 3 have been obtained by averaging both in time, and also exploiting the symmetry of the geometry, that allows to consider, for example, 6 long and 6 short radii for the fractal jet. Therefore, $r_{1/2}^l$ is the distance where $U = U_{cl}/2$ along the $s$-radii, and $r_{1/2}^l$ is the distance where $U = U_{cl}/2$ along the $l$-radii. Consequently, if $r_{1/2}^l$ becomes larger than $r_{1/2}^l$ the jet has inverted its long and short diameters. $r_{1/2}^s$ becomes larger than $r_{1/2}^l$ at $\tilde{x} = 0.513$ for $J_S$, $\tilde{x} = 0.260$ for $J_F$ and $\tilde{x} = 0.275$ for $J_6$, thus $J_F$ and $J_6$ switch their axes before $J_S$. $r_{1/2}^l - r_{1/2}^l$ is also a fundamental quantity because indicates the stretching of the corner portions of the jet and, being $r_{1/2}^l$ approximately equal for all the jets (see figure 3), where $r_{1/2}^l$ grows faster, the corners undergo a stronger stretching. In figure 4 the black contour line indicates the stream-wise velocity at $\tilde{x} = 1.22$, and it shows that the six corrugations of $J_F$ have become narrower and are more elongated than the four characterizing $J_S$ (the contours of $J_6$ are not shown because similar to $J_F$). After this initial region where $J_F$ is spread faster than $J_S$, figure 3 shows that, at $\tilde{x} \approx 1.5$, $r_{1/2}^s$ of $J_F$ reaches its maximum, while it continues to grow in $J_S$. At the end of the domain $r_{1/2}^l$ of $J_S$ is larger than in $J_F$ because the former maintains its coherent shape longer, and the spreading is slower; on the other hand $J_F$, as it will be shown later, produces an intricate flow pattern, where the vortex interactions make the jet to lose its coherent shape, and $r_{1/2}^l$ decreases.

To understand whether small scales corrugations play a role at small $Re$ the fractal jet has been compared with $J_6$. Figure 3 does not show large difference among these two cases, suggesting that the smallest scales generated by the fractal jet die and do not influence the evolution far from the nozzle. The simulation $J_6$ allows to attribute the initial highest spreading of $J_F$, not to the fractal corrugations, but to the angular width of the largest corners. In figure 3a it is reported also $r_{1/2}$ for $J_C$ to show that with this Reynolds number the radius of the jet remains almost unchanged along $\tilde{x}$.

3.2. Vortex dynamics

In this paragraph the causes leading the jets to have different spreading are explained by looking at the vorticity fields which, being affected by the corners, dictate the evolution of the velocity structures previously described. The effect of corners is enlightened in figure 4 and 5 where the positive and negative contours of $\omega_x$ are respectively
Figure 3: Evolution of the jet half-width normalized with $D_e$ vs stream-wise direction $\tilde{x}$. Symbols: (■) $r_{1/2}^J$ for $J_S$; (○) $r_{1/2}^J$ for $J_F$; (▲) $r_{1/2}^J$ for $J_6$; (×) $r_{1/2}^J$ for $J_C$. Dashed lines for the $r_{1/2}^J$.

coloured in red and blue, and are overlapped to the contour of $U = U_\infty/2$. The figure clearly shows that in $J_S$, $J_F$ and $J_6$, the thin layers of $\omega_x$ produced at the corners, are responsible for the rotation and successive stretching of the jets. The formation of $\omega_x$ can not be instead observed in $J_C$. At the nozzle exit, in correspondence of every corner, a pair of thin layers of $\pm \omega_x$ vortices forms. While these are convected downstream, the positive vortices come up against the negative ones generated by another corner, thus forming a new couple. This interaction between $\omega_x$ of opposite sign stretches the spikes and spreads the jet. For the fractal $J_F$, $\pm \omega_x$ of different size are produced at the inlet but, due to the low $Re$, the small patches die and do not affect the flow; therefore at a certain distance the jet is affected only by the six largest structures and does not differ from $J_6$.

In addition to these vortical structures, at the end of the computational domain of $J_F$ (figure 5c) and $J_6$ (figure 5d), other structures with $\pm \omega_x$, that are not connected to the layers at corners, are visible. These structures are due to the vorticity dynamics that, through the processes of vortex-stretching and vortex-tilting are generated from the other vorticity components. In fact at the inlet $\omega_y$ and $\omega_z$ are also generated by the orifice forming a corrugated sheet of intense vorticity. Such structures for the three jets $J_C$, $J_S$ and $J_F$ are shown in figure 6, where iso-contours of $\Omega_\theta = \sqrt{\omega_y^2 + \omega_z^2}$ and $\Omega_x = |\omega_x|$, in the plane $yz$ are plotted at three distances $\tilde{x}$. The quantity $\Omega_\theta$ for $J_C$ shows a structure with the same shape of the vorticity $\omega_\theta$ for a circular jet, evaluated in cylindrical coordinates (Verzicco and Orlandi (1993) [18]). In the figure the region with a clustering of the contour lines are those with the highest values of $\Omega_\theta$, and represents the boundaries of the vortex sheet. The increase of the width of this region with $x^*$ indicates that the vortex
sheet of $J_C$ is spreading both towards the center-line and the surroundings. The other jets present a more complicated pattern at $x^* = 3.18$, because the jet spreading is greater than for $J_C$, and the fluid transport between the jet and its surroundings is enhanced. This fast mechanism, triggered by the $\Omega_x$ formed at the corners of the nozzle, is the cause for the intricate vorticity pattern, where also $\Omega_x$ not generated at the inlet by the orifice is present.

Figure 4 reports the evolution of $\langle \Omega_\theta \rangle$ and $\langle \Omega_x \rangle$ averaged in the plane $yz$ and in time, along the stream wise direction. The values achieved by $\langle \Omega_\theta \rangle$ are larger than $\langle \Omega_x \rangle$ for any jet, and the plot shows that $\langle \Omega_\theta \rangle \propto 1/\theta_{\text{corner}}$; in fact the highest values are achieved by $J_F$ and $J_6$ that have a smaller $\theta_{\text{corner}}$ with respect to $J_S$. The plot shows that $\langle \Omega_x \rangle$ is null along the whole domain for the circular jet. In the other cases $\langle \Omega_x \rangle$ is approximately constant, until $\tilde{x} \approx 2.25$. The subsequent growth is due to the vorticity dynamics that produces $\langle \Omega_x \rangle$ from $\langle \Omega_\theta \rangle$ by vortex stretching and vortex tilting. The high $\langle \Omega_\theta \rangle$ is characterized by a first region where this quantity decreases, followed by a second region where it grows linearly. At the end the decrease is mainly due to the viscosity. The slope and the length of the linear region depend on the geometry of the
Figure 5: 3D Contour plot of $U = U_\infty/2$ for (a) $J_S$, (b) $J_C$, (c) $J_F$ and (d) $J_6$ in yellow, overlapped with $\omega_x/\omega^{\max} = \pm 0.5$ in red and blue respectively. $0 < \tilde{r} < 3.18$. 
Figure 6: Contour plot of $\Omega_\theta = \sqrt{\omega_\gamma^2 + \omega_\xi^2}$ normalized with its maximum value in the range $[0.1 : 1]$ with 10 contour lines in red for (a) $J_S$, (b) $J_C$ and (c) $J_F$ at $\tilde{x} = 0$; (d), (e) and (f) are the same at $\tilde{x} = 1.22$; (g), (h) and (i) at $\tilde{x} = 3.18$. The plot is overlapped with the contours of $\Omega_x$ normalized with its maximum value in the range $[0.3 : 1]$ with 7 contour lines in blue.
Figure 7: (a) Evolution of \( \langle \Omega_\theta \rangle \) and \( \langle \Omega_x \rangle \) vs the stream-wise direction \( \tilde{x} \). Symbols: (□): \( \langle \Omega_x \rangle \) for \( J_S \); (■): \( \langle \Omega_\theta \rangle \) for \( J_S \); (○): \( \langle \Omega_x \rangle \) for \( J_F \); (●): \( \langle \Omega_\theta \rangle \) for \( J_F \); (◇): \( \langle \Omega_x \rangle \) for \( J_6 \); (△): \( \langle \Omega_\theta \rangle \) for \( J_6 \); (▲): \( \langle \Omega_x \rangle \) for \( J_C \); (▲): \( \langle \Omega_\theta \rangle \) for \( J_C \).

orifice, in fact \( a_{J_6} = 0.14 \) and \( a_{J_F} = 0.14 \), while \( a_{J_S} \) is 0.10 and the value of \( \langle \Omega_\theta \rangle \) for \( J_C \) remains approximately constant (\( a_{J_C} = 0.01 \)).

4. Conclusion

In the current work, a detailed study of the jets produced by nozzles of complex geometries at \( Re_{D_e} = 565 \) has been presented through direct simulations. The DNS results allowed to visualize the velocity structures generated by the orifices and the complex vortex dynamics that has been found to be responsible for the jet deformation.

A total of four different nozzle shapes have been analyzed, considering both circular and non-circular orifices. The different evolution of the flow pattern generated by the different shape has been analyzed by looking at the velocity field. The jets generated from nozzles with sharp corners has been found to evolve differently from circular jets, in fact in the former case it is possible to see that the initial shape of the jet is dictated by the orifice, and that downstream the jet undergoes a rotation that leads to an axis switching. The circular jet maintain instead an axisymmetric circular shape also far from the orifice. This rotation is produced at the nozzle and has been found to be initiated by the vorticity self-induction process. Given the angular width of the corners (\( \theta_{\text{corner}} \)) and the measured angle for the rotation (\( \theta_{\text{rotation}} \)), it has been possible to relate the rotation of the jet to the width of the corners: \( \theta_{\text{rotation}} = \theta_{\text{corner}} / 2 \). Considering also the data found in literature, the relationship between \( \theta_{\text{rotation}} \) and \( \theta_{\text{corner}} \) can be considered valid for nozzles with aspect ratio 1. The velocity at which the rotation occurs can be measured by the growth rates of the \( s- \) and \( l- \) diagonals and it has been shown that, while \( r_{1/2}^l \)
is similar for all the nozzles with corners, $r_{1/2}^s$ is strongly affected by the shape. The growth of $r_{1/2}^s$ is associated with the stretching of the corrugations, in fact in $J_F$ and $J_6$ $r_{1/2}^s$ grows faster, the corners undergo a faster stretching, and this process contributes to enhance the spreading.

Looking at the vorticity field, it has been possible to understand the mechanism leading to the ‘axis-switching’ of jets. The deformation of the jet is in fact linked to the vortex dynamics because patches of $\pm \omega_x$ are formed in correspondence of the corners of the orifice when the flow is initiated. These thin layers start to roll and grow, until these separate from the orifice and are convected downstream. The dynamics governing the evolution of vortices of $\pm \omega_x$ are then responsible for the jets deformation. The spikes of the jet are stretched by these vortices and consequently the structure is spread more rapidly, higher is the strength of the $\omega_x$ patches. As a result the fluid transport between the jet and its surroundings is enhanced, so as the mixing properties. The low Reynolds number assumption guarantees that the flow remains laminar in the near-field and, in such condition, the wider range of scales generated by the fractal nozzle, does not affect the flow, therefore the evolution of $J_F$ and $J_6$ does not show large differences.

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