Kähler Potentials for Hilltop F-term Hybrid Inflation

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Abstract

We consider the basic supersymmetric (SUSY) models of F-term hybrid inflation (FHI). We show that a simple class of Kähler potentials ensures a resolution to the η problem and allows for inflation of hilltop type. As a consequence, observationally acceptable values for the spectral index, ns, can be achieved constraining the coefficient c_{4K} of the quartic supergravity correction to the inflationary potential. For about the central value of ns, in the case of standard FHI, the grand unification (GUT) scale turns out to be well below its SUSY value with the relevant coupling constant κ in the range (0.0006 - 0.15) and c_{4K} ≃ -(1100 - 0.05). In the case of shifted [smooth] FHI, the GUT scale can be identified with its SUSY value for c_{4K} ≃ -16 [c_{4K} ≃ -1/16].

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1 Introduction

One of the most natural and well-motivated classes of inflationary models is the class of supersymmetric (SUSY) F-term hybrid inflation (FHI) models [1]. In particular, the basic versions of FHI are the standard [2], shifted [3] and smooth [4] FHI. They are realized [2] at (or close to) the SUSY Grand Unified Theory (GUT) scale \( M_{GUT} \approx 2.86 \times 10^{16} \) GeV and can be easily linked to several extensions [5] of the Minimal Supersymmetric Standard Model (MSSM) which have a rich structure. Namely, the \( \mu \)-problem of MSSM is solved via a direct coupling of the inflaton to Higgs superfields [6] or via a Peccei-Quinn symmetry [7], baryon number conservation is an automatic consequence [6] of an R symmetry and the baryon asymmetry of the universe is generated via leptogenesis which takes place [8] through the out-of-equilibrium decays of the inflaton’s decay products.

Although quite successful, these models have at least two shortcomings: (i) the so-called η problem and (ii) the problem of the enhanced (scalar) spectral index, ns. The first problem is tied [1, 9, 10] on the expectation that supergravity (SUGRA) corrections generate a mass squared for the inflaton of the order of the Hubble parameter during FHI and so, the η criterion is generically violated, ruining thereby FHI. Inclusion of SUGRA corrections with canonical Kähler potential prevents [1, 11] the generation of such a mass term due to a mutual cancellation. However, despite its simplicity, the canonical Kähler potential can be regarded [1] as fine tuning to some extent and increases, in all cases, even more ns. This aggravates the second problem of FHI, i.e., the fact that, under the assumption that the problems of the standard big bag cosmology (SBB) are resolved exclusively by FHI, these models predict \( n_s \) just marginally consistent with the fitting of the five-year results [12] from the Wilkinson Microwave Anisotropy Probe Satellite (WMAP5) data with the standard power-law cosmological model with cold dark matter and a cosmological constant (ΛCDM). According to this, \( n_s \) at the pivot scale \( k_p = 0.002/Mpc \) is to satisfy [12] the following range of values:

\[
0.963^{-0.014} \leq n_s \leq 0.991 \quad \text{at 95\% confidence level.}
\]
One possible resolution (for other proposals, see Ref. [13, 14]) of the tension between FHI and the data is [15, 16] the utilization of a quasi-canonical [17] Kähler potential with a convenient choice of the sign of the next-to-minimal term. As a consequence, a negative mass term for the inflaton is generated. In the largest part of the parameter space the inflationary potential acquires a local maximum and minimum. Then, FHI of the hilltop type [18] can occur as the inflaton rolls from this maximum down to smaller values. This set-up provides acceptable values for both \( \eta \) and \( n_s \) but it requires [15, 16, 20] two kinds of mild tuning: (i) the relevant coefficient in the Kähler potential is to be sufficiently low (ii) the value of the inflaton field at the maximum is to be sufficiently close to the value that this field acquires when the pivot scale crosses outside the inflationary horizon.

In this paper, we propose a class of Kähler potentials which supports a new type of hilltop FHI (driven largely by the quartic rather than the quadratic SUGRA correction) without the first kind of tuning above. In particular, the coefficients of Kähler potentials are constrained to natural values (of order unity) so as the mass term of the inflaton field is identically zero. The achievement of the observationally acceptable \( n_s \)'s requires a mild tuning of the initial conditions similar to that needed in the case with quasi-canonical Kähler potential. The suggested here form of Kähler potentials has been previously proposed in Ref. [21] in order to justify the saddle point condition needed for the attainment of \( A \)-term or MSSM inflation [22]. A similar idea is also explored in Ref. [23] without, though, the \( n_s \) problem to be taken into account.

Below, we describe the proposed embedding of the basic FHI models in SUGRA (Sec. 2) and we derive the inflationary potential (Sec. 3). Then we exhibit the observational constraints imposed on our models (Sec. 4) and we end up with our numerical results (Sec. 5) and our conclusions (Sec. 6). Throughout the text, we set \( \hbar = c = k_B = 1 \). Hereafter, parameters with mass dimensions are measured in units of the reduced Planck mass \( m_P = 2.44 \cdot 10^{18} \text{ GeV} \) which is taken to be unity.

## 2 FHI in non-Minimal SUGRA

In this section we outline the salient features of our set-up (Sec. 2.1), we extract the SUSY potential (Sec. 2.2), we calculate the SUGRA corrections (Sec. 2.3) and present the proposed class of Kähler potentials (Sec. 2.4).
2.1 The General Set-up

The F-term hybrid inflation can be realized within SUGRA adopting one of the superpotentials below:

\[ W = \hat{W} + W_{\text{FHI}} \text{ with } W_{\text{FHI}} = \begin{cases} \hat{k}S \left( \hat{\Phi} \hat{\Phi} - M^2 \right) & \text{for standard FHI}, \\ \hat{k}S \left( \hat{\Phi} \hat{\Phi} - M^2 \right) - S \frac{(\hat{\Phi} \hat{\Phi})^2}{M^2} & \text{for shifted FHI}, \\ S \left( \frac{(\hat{\Phi} \hat{\Phi})^2}{M^2} - \hat{\mu}_S^2 \right) & \text{for smooth FHI}. \end{cases} \] \tag{2.1}

Here we use the hat to denote quantities (such as the part \( \hat{W} \) of \( W \)) which depend exclusively on the hidden sector superfields, \( h_m \). Also, \( \hat{\Phi} \) and \( \hat{\Phi} \) is a pair of left handed superfields belonging to non-trivial conjugate representations of a GUT gauge group \( G \) and reducing its rank by their vacuum expectation values (v.e.v.s), \( S \) is a gauge singlet left handed superfield, \( \hat{M}_S \sim 0.205 \) is an effective cutoff scale comparable with the string scale and the parameters \( \hat{k} \) and \( M, \hat{\mu}_S \sim M_{\text{GUT}} = 4.11 \times 10^{-3} \) are made positive by field redefinitions.

\( W_{\text{FHI}} \) in Eq. (2.1) for standard FHI is the most general renormalizable superpotential consistent with a continuous R symmetry [2] under which

\[ S \to e^{i\gamma} S, \hat{\Phi} \to e^{i\gamma} \hat{\Phi}, \bar{\Phi} \to e^{-i\gamma} \bar{\Phi}, W \to e^{i\gamma} W. \] \tag{2.2}

Including in this superpotential the leading non-renormalizable term, one obtains \( W_{\text{FHI}} \) of shifted [3] FHI in Eq. (2.1). Finally, \( W_{\text{FHI}} \) of smooth [4] FHI can be produced if we impose an extra \( Z_2 \) symmetry under which \( \Phi \to -\Phi \) and, therefore, only even powers of the combination \( \hat{\Phi} \bar{\Phi} \) can be allowed.

To keep our analysis as general as possible, we do not adopt any particular form for \( \hat{W} \) (for some proposals see Ref. [24–26]). Note that our construction remains intact even if we set \( \hat{W} = 0 \) as it was supposed in Ref. [23]. This is due to the fact that \( \hat{W} \) is expected to be much smaller than the inflationary energy density (see Sec. 2.3). For \( \hat{W} \neq 0 \), though, we need to assume [21] that \( h_m \)’s are stabilized before the onset of FHI by some mechanism not consistently taken into account here [27]. As a consequence, we neglect the dependence of \( \hat{W}, \hat{k}, \hat{\mu}_S \) and \( \hat{M}_S \) on \( h_m \) and so, these quantities are treated [21] as constants. We further assume that the D-terms due to \( h_m \)’s vanish (contrary to the strategy followed in Ref. [23]).

The SUGRA scalar potential (without the D-terms) is given (see, e.g., Ref. [9, 24]) by

\[ V_{\text{SUGRA}} = e^K \left( K^{MN} F_M F_N^* - 3|W|^2 \right) \text{ where } F_M = W_M + K_M W \] \tag{2.3}

is the SUGRA generalization of the F-terms, the subscript \( M \) \[ M \] denotes derivation with respect to (w.r.t) the complex scalar field \( \phi_M \left[ \phi_M^* \right] \) which corresponds to the chiral superfield \( \phi_M \) with \( \phi_M = h_m, S, \Phi, \bar{\Phi} \) and the matrix \( K^{MN} \) is the inverse of the Kähler metric \( K_{MN} \). In this paper we consider a quite generic form of Kähler potentials, which respect the R symmetry of Eq. (2.2). Namely we take

\[ K = \hat{K} + \hat{Z}|S|^2 + \frac{1}{4}k_4 \hat{Z}^2 |S|^4 + \frac{1}{6}k_6 \hat{Z}^3 |S|^6 + |\Phi|^2 + |\bar{\Phi}|^2, \] \tag{2.4}

where \( k_4 \) and \( k_6 \) are positive or negative constants of order unity and the functions \( \hat{K} \) and \( \hat{Z} \) are to be determined. The non-vanishing entries of \( K^{MN} \) are

\[ K^{\tilde{m}\tilde{n}} \simeq \hat{K}^{\tilde{m}\tilde{n}} - \hat{K}^{\tilde{m}\tilde{n}} |S|^2 \text{ with } \hat{K}^{\tilde{m}\tilde{n}} = \hat{K}^{\tilde{m}\tilde{n}} \left( Z_{\tilde{m}\tilde{n}} - \hat{Z}_{\tilde{m}} \hat{Z}_{\tilde{n}} / \hat{Z} \right) \], \tag{2.5a}

\[ K^{mS^*} \simeq \left( \hat{K}^{m\tilde{n}} \hat{Z}_{m\tilde{n}S^*} |S|^2 - \hat{Z}^{m\tilde{n}S^*} \right) / \hat{Z}, \tag{2.5b} \]

\[ K^{S\tilde{n}} \simeq \left( \hat{K}^{m\tilde{n}} \hat{Z}_{m\tilde{n}S} |S|^2 - \hat{Z}^{m\tilde{n}S} \right) / \hat{Z}, \tag{2.5c} \]

\[ K^{SS^*} \simeq 1 / \hat{Z} + \left( \hat{Z}^m \hat{Z}_{m\tilde{n}} / \hat{Z}^2 - k_4 \right) |S|^2 + \left[ (k_4^2 - 3k_6) \hat{Z} - \hat{K}^{m\tilde{n}} \hat{Z}_{m\tilde{n}} / \hat{Z} \right] |S|^4, \tag{2.5d} \]

\[ K^{\Phi\Phi^*} = 1 \text{ and } K^{\Phi\tilde{\Phi}^*} = 1, \tag{2.5e} \]
where the indices $m$ and $n$ are raised and lowered with $\hat{K}^{mn}$ and we keep only the terms necessary in order to extract a reliable expansion of $V_{\text{SUGRA}}$ up to the order $|S|^4$ (see Sec. 2.3).

### 2.2 The SUSY Potential

The SUSY potential includes $[9, 24]$ F- and D-term contributions. Note that, as a consequence of our assumptions about the nature of $\hat{\Phi}$ and $\Phi$ and the structure of $K$ in Eq. (2.4), the D-term contribution vanishes for $|\hat{\Phi}| = |\Phi|$. Expanding $V_{\text{SUGRA}}$ in Eq. (2.11) for $|S| \ll 1$ and $W \ll 1$ and neglecting soft SUSY breaking terms (see, e.g., Ref. [24]), we can extract the F-term contribution to the SUSY potential, which can be written as

$$V_F \simeq \begin{cases} 
\kappa^2 M^4 \left( (\Phi^2 - 1)^2 + 25 \Phi^2 \right) & \text{for standard FHI,} \\
\kappa^2 M^4 \left( (\Phi^2 - 1 - \xi \Phi^4)^2 + 25 \Phi^2 (1 - 2 \xi \Phi^2)^2 \right) & \text{for shifted FHI,} \\
\mu_5^{-2} \left( (\Phi^4 - 1)^2 + 8 S^2 \Phi^6 \right) & \text{for smooth FHI,}
\end{cases}$$

(2.6)

where $\xi = M^2 / \kappa M_\phi^2$ with $[3] \ 1/7.2 < \xi < 1/4$. In order to recover the properly normalized energy density during FHI (see below), we absorb in Eq. (2.6) some normalization pre-factors emerging from $V_{\text{SUGRA}}$, defining the quantities $\kappa = e^{K/2} \hat{Z}^{-1/2} \kappa$ and $\mu_5 = e^{K/4} \hat{Z}^{-1/4} \mu_5$. We then define $M_S = e^{-K/4} \hat{Z}^{1/4} M_\phi$ so as $\kappa M_\phi^2 = \kappa M_S^2$ and $\mu_5 M_\phi = \mu_5 M_S$. Also, we use $[3, 4]$ the dimensionless quantities:

$$\begin{cases} 
\Phi = |\Phi| / M \text{ and } S = \hat{Z}^{1/2} |S| / M & \text{for standard or shifted FHI,} \\
\Phi = |\Phi| / \sqrt{\mu_5 M_\phi} \text{ and } S = \hat{Z}^{1/2} |S| / \sqrt{\mu_5 M_S} & \text{for smooth FHI.}
\end{cases}$$

(2.7)

Recall that the scalar components of the superfields are denoted by the same symbols as the corresponding superfields.

The potential in Eq. (2.6) reveals that $W_{\text{FHI}}$ in Eq. (2.1) plays a twofold crucial role:

- It leads to the spontaneous breaking of $G$. Indeed, the vanishing of $V_F$ gives the v.e.vs of the fields in the SUSY vacuum. Namely,

$$\langle S \rangle = 0 \text{ and } |\langle \Phi \rangle| = |\langle \Phi \rangle| = v_G = \begin{cases} 
M & \text{for standard FHI,} \\
M \sqrt{1-\sqrt{1-\xi}} & \text{for shifted FHI,} \\
\sqrt{\mu_5 M_S} & \text{for smooth FHI}
\end{cases}$$

(2.8)

(in the case where $\Phi$, $\hat{\Phi}$, $\Phi$ are not Standard Model (SM) singlets, $\langle \hat{\Phi} \rangle$, $\langle \Phi \rangle$ stand for the v.e.vs of their SM singlet directions).

- It gives rise to FHI. This is due to the fact that, for large enough values of $|S|$, there exist valleys of local minima of the classical potential with constant (or almost constant in the case of smooth FHI) values of $V_F$. In particular, we can observe that $V_F$ takes the following constant value

$$V_{\text{HIO}} = \begin{cases} 
\kappa^2 M^4 & \text{along the direction(s): } \Phi = \begin{cases} 
0 & \text{for standard FHI,} \\
0 \text{ or } 1/\sqrt{2\xi} & \text{for shifted FHI,} \\
0 \text{ or } 1/\sqrt{6S} & \text{for smooth FHI,}
\end{cases}
\end{cases}$$

(2.9)

with $M_\xi = M \sqrt{1/4\xi - 1}$. It can be shown [20] that the flat direction $\Phi = 0$ corresponds to a minimum of $V_F$, for $|S| \gg M$, in the cases of standard and shifted FHI, and to a maximum of $V_F$ in the case of smooth FHI. As a consequence, topological defects such as strings [15, 28, 29], monopoles, or domain walls may be produced [4] via the Kibble mechanism [30] during the spontaneous breaking of $G$ at the end of standard FHI, since this type of FHI can be realized only for $\Phi = 0$. On the contrary, this can be avoided in the other two cases, since the form of $W_{\text{FHI}}$ allows for non-trivial inflationary valleys of minima with $\Phi \neq 0$, along which $G$ is spontaneously broken.
2.3 SUGRA Corrections

The consequences that SUGRA has on the models of FHI can be investigated by restricting ourselves to the inflationary trajectory $\Phi = \tilde{\Phi} \simeq 0$ (possible corrections due to the non-vanishing $\Phi$ and $\tilde{\Phi}$ in the cases of shifted and smooth FHI are [31] negligible). Therefore, $W$ in Eq. (2.1) takes the form

$$W = \tilde{W} + I, \quad \text{where} \quad I = -\tilde{V}_0^{1/2}S \quad \text{with} \quad \tilde{V}_0 = e^{-\hat{K}}\tilde{Z}V_{\text{HI0}}. \quad (2.10)$$

Given the superpotential above, the scalar potential in Eq. (2.3) can be written as

$$V_{\text{SUGRA}} = |\tilde{W}|^2 V_{\tilde{W}} + \tilde{W}^* W_{\tilde{W}} + \tilde{W}^* V_{\tilde{W}}^* + \tilde{V}_0 V_I, \quad \text{where} \quad (2.11)$$

$$V_{\tilde{W}} = e^K \left(K^{\tilde{M}\tilde{N}}K_MK_N - 3\right), \quad (2.12a)$$

$$V_{\tilde{W}I} = e^K \left(K^{\tilde{M}\tilde{N}}K_MK_N + K^{MS^*}K_M/S^* - 3\right), \quad (2.12b)$$

$$V_I = e^K \left[K^{SS^*} + SK^{MS^*}K_M + S^*K^{SN^*}K_N + |S|^2 \left(K^{MN}K_MK_N - 3\right)\right]. \quad (2.12c)$$

Using the Kähler potential in Eq. (2.4) we can obtain an expansion of $V_{\text{SUGRA}}$ in powers of $|S|$. To this end, we first expand in powers of $|S|$ the involved in Eqs. (2.12a) – (2.12c) expressions:

$$K^{MN}K_MK_N \simeq \hat{K}^m\hat{K}_m + |S|^2 \left(\hat{Z} - \hat{K}^{m\bar{n}}\hat{K}_m\hat{K}_{\bar{n}}\right) + O(|S|^4), \quad (2.13a)$$

$$K^{MS^*}K_M \simeq (1 - \hat{Z}^m\hat{K}_m)S^* + S^*|S|^2 \left(\hat{K}^{m\bar{n}}\hat{K}_m\hat{Z}_{\bar{n}} - k_4\hat{Z}/2\right) / \hat{Z}, \quad (2.13b)$$

$$K^{SN^*}K_N \simeq (1 - \hat{K}^{m\bar{n}}\hat{Z}_m)S + |S|^2 \left(\hat{K}^{m\bar{n}}\hat{Z}_m\hat{K}_{\bar{n}} - k_4\hat{Z}/2\right) / \hat{Z}. \quad (2.13c)$$

Substituting Eqs. (2.13a) – (2.13c) into Eq. (2.11) and taking into account that

$$e^K \simeq e^{\hat{K}} \left(1 + \hat{Z}|S|^2 + (1 + k_4/2)\hat{Z}^2|S|^4/2\right), \quad (2.14)$$

we end up with the following expansion:

$$V_{\text{SUGRA}} \simeq V_0 + V_1|S| + V_2|S|^2 + V_4|S|^4, \quad (2.15)$$

$$V_0 \simeq e^{\hat{K}}\hat{Z}^{-1}\tilde{V}_0, \quad (2.16a)$$

$$V_1 \simeq 2e^{\hat{K}}\tilde{V}_0^{1/2}|\tilde{W}| \left(\hat{K}^m\hat{K}_m - \hat{Z}^m\hat{K}_m/\hat{Z} - 2\right) \cos \theta, \quad (2.16b)$$

$$V_2 \simeq e^{\hat{K}}\tilde{V}_0 \left[\hat{K}^m\hat{K}_m - (\hat{K}^m\hat{Z}_m + \hat{Z}^m\hat{K}_m)/\hat{Z} + \hat{Z}^m\hat{Z}_m/\hat{Z}^2 - k_4\right], \quad (2.16c)$$

$$V_4 \simeq e^{\hat{K}}\tilde{V}_0 \left[Z^{-1}\left(\hat{K}^m\hat{K}_m + \hat{K}_m\hat{K}_{\bar{n}} - \hat{Z}_m\hat{Z}_{\bar{n}}/\hat{Z} - \hat{Z}\hat{K}_m\hat{K}_{\bar{n}} + \hat{Z}^m\hat{Z}_m\right) - \hat{Z}\left(\hat{K}^m\hat{K}_m + \hat{K}_m\hat{Z}_m\right) + \left(\hat{K}^m\hat{K}_m + 1/2 - 7/4k_4 + k_4^2 - 3/2k_6\right)\hat{Z}^2\right], \quad (2.16d)$$

where the phase $\theta$ in $V_1$ reads $\theta = \arg \left(\hat{K}^m\hat{K}_m - \hat{Z}^m\hat{K}_m/\hat{Z} - 2\right) + \arg(\tilde{W}) - \arg(\tilde{V}_0^{1/2}) - \arg(S)$.

In the right hand side (r.h.s) of Eqs. (2.16a), (2.16c) and (2.16d) we neglect terms proportional to $|\tilde{W}|^2$ which are certainly subdominant compared with those which are proportional to $\tilde{V}_0$. From the terms proportional to $|\tilde{W}|V_0^{1/2}$ we present, just for completeness, the term $V_1$ which expresses the most important contribution [25, 31] to the inflationary potential from the soft SUSY breaking terms. For natural values of $\tilde{W}$ and $e^{\hat{K}}$ this term starts [28, 31] playing an important role in the case of standard FHI for $k \lesssim 5 \cdot 10^{-4}$ whereas it has [31] no significant effect in the cases of shifted and smooth FHI. For simplicity, we neglect it, in the following. Note, finally, that the well-known results in the context of minimal [11] [quasi-minimal [16, 17, 20]] SUGRA can be recovered from Eqs. (2.16c) and (2.16d) by setting $\hat{K} = 0$, $\hat{Z} = 1$ and $k_4 = k_6 = 0$ [$\hat{K} = 0$ and $\hat{Z} = 1$].
2.4 Imposed Conditions

From Eq. (2.15) we infer that a resolution to the \( \eta \) problem of FHI requires \( V_\eta = 0 \) - needless to say that there is no contribution to \( \eta \) from the neglected \( V_1 \)-term in Eq. (2.16b). Considering a well motivated, by several superstring and D-brane models [32], form for \( \hat{K} \) and \( \hat{Z} \), we can impose constraints on their parameters and on \( k_4 \) and \( k_6 \) so as the requirement above is fulfilled identically. In particular, inspired from Ref. [21, 23], we seek the following ansatz for \( \hat{K} \) and \( \hat{Z} \):

\[
\hat{K} = \sum_{m=1}^{M} \beta_m \ln(h_m + h_m^*) \quad \text{and} \quad \hat{Z} = k_Z \prod_{m=1}^{M} (h_m + h_m^*)^{\alpha_m}, \quad \text{with} \quad \beta = \sum_{m=1}^{M} \beta_m < 0. \tag{2.17}
\]

The latter restriction is demanded so as the exponential of \( V_{\text{SUGRA}} \) in Eq. (2.3) is well defined for \( h_m \sim 1 \). We further assume that \( \beta_m \)'s have to be integers and \( \alpha_m \)'s have to be rational numbers. Although negative integers as \( \beta_m \)'s are more frequently encountered, positive \( \beta_m \)'s are also allowed [33]. Since \( M \) measures the number of hidden sector fields, we restrict ourselves to its lowest possible values. Inserting Eq. (2.17) into Eqs. (2.16a), (2.16b) and (2.16d), Eq. (2.15) takes the form

\[
V_{\text{SUGRA}} \simeq V_{\text{HI0}} \left( 1 - \frac{1}{2} c_{2K} \sigma^2 + \frac{1}{4} c_{4K} \sigma^4 \right), \quad \text{where} \quad \sigma = \sqrt{2} \hat{Z}^{1/2} |S| \tag{2.18}
\]

is the canonically (up to the order \( |S|^2 \)) normalized inflaton field and the coefficients \( c_{2K} \) and \( c_{4K} \) read

\[
c_{2K} = -\frac{V_\eta}{e^K V_0} = k_4 + \sum_{m=1}^{M} \frac{(\alpha_m - \beta_m)^2}{\beta_m} \tag{2.19a}
\]

\[
c_{4K} = \frac{V_\eta}{e^K V_0 Z} = k_4^2 - \frac{7}{4} k_4^2 - \frac{3}{2} k_6 + \frac{1}{2} + \sum_{m=1}^{M} \frac{(\alpha_m - \beta_m)^3}{\beta_m^2}. \tag{2.19b}
\]

Consequently, FHI can be deliberated from the \( \eta \) problem if the following condition is valid:

\[
c_{2K} = 0. \tag{2.20}
\]

On the other hand, the data on \( n_s \) favors hilltop FHI which can be attained [18] for \( c_{4K} < 0 \). However, \( c_{4K} > 0 \) is still marginally allowed. In Table 1 we list solutions to Eq. (2.20) for the simplest case with \( M = 1 \) and \( k_6 = 0 \) with \( c_{4K} \geq 0 \). Solutions to Eq. (2.20) with the observationally favored \( c_{4K} < 0 \) can be also achieved with a variety of ways. Note, initially, that \( k_6 > 0 \) is beneficial for this purpose, since it decreases \( c_{4K} \), without disturbing the satisfaction of Eq. (2.20). A first set of solutions can be taken for \( \alpha_m = 0 \). In this case (which resembles the cases studied in Ref. [23]) setting, e.g., \( k_4 = -\beta_m = 1 \), we get \( c_{4K} = 3/4, 0, -3, -6, -9 \) for \( k_6 = 0, 1/2, 5/2, 9/2, 13/2 \).

More generically, taking as input parameters \( \alpha_m \)'s and \( \beta_m \)'s we can assure the fulfillment of Eq. (2.20) constraining \( k_4 \) via Eq. (2.19a). We confine ourselves to the values of \( k_4 \) in the range \( 0.1 - 10 \), which we consider as natural - note that the realization of FHI within quasi-canonical SUGRA requires [16, 19, 20] \( k_4 \) significantly lower, i.e., \( 10^{-3} \lesssim k_4 \lesssim 0.01 \). Then, for given \( k_6 \), we can extract \( c_{4K} \) through Eq. (2.19b). In Fig. 1 we display the resulting, this way, \( c_{4K} \) versus \( \alpha_1 \) for \( M = 1 \) and \( k_6 = 0 \) (gray points) or \( M = 2 \) and \( k_6 = 1 \) (black points). We present six families of points of different shapes corresponding to different values of \( \beta_1 \) (gray points) or \( \alpha_2, \beta_1 \) and \( \beta_2 \) (black points). The adopted values for these parameters are shown in the r.h.s of Fig. 1. We observe that a wide range of negative \( c_{4K} \)'s can be produced with natural values of the parameters related to the structure of Kähler potential \( (k_4, k_6, \alpha_m \text{ and } \beta_m) \). As we verify below (see Sec. 5) these \( c_{4K} \)'s assist us to achieve hilltop-type FHI consistently with the data on \( n_s \) for all possible values of \( \kappa \) or \( M_S \).
where \( \sigma \) the amount of this tuning in the initial conditions, we define the quantity:

\[
\Phi = \frac{\kappa}{2} \frac{\kappa x}{M} \frac{M^2}{Q^2} + f_c(x) / 16 \pi^2
\]

where \( V_{\text{HI}} = V_{\text{HI0}} \left( 1 + c_{\text{HI}} + \frac{1}{4} c_{4K} \sigma^4 \right) \),

where, besides the contributions originating from \( V_{\text{SUGRA}} \) in Eq. (2.18) (with \( c_{2K} = 0 \)), we include the term \( c_{\text{HI}} V_{\text{HI0}} \) which represents a correction to \( V_{\text{HI}} \) resulting from the SUSY breaking on the inflationary valley, in the cases of standard [2] and shifted [3] FHI, or from the structure of the classical potential in the case of smooth [4] FHI. \( c_{\text{HI}} \) can be written as follows:

\[
c_{\text{HI}} = \begin{cases} 
\kappa^2 N \left( 2 \ln \left( \kappa x M^2 / Q^2 \right) + f_c(x) \right) / 32 \pi^2 & \text{for standard FHI}, \\
\kappa^2 \left( 2 \ln \left( \kappa x M^2_x / Q^2 \right) + f_c(x) \right) / 16 \pi^2 & \text{for shifted FHI}, \\
-2 \mu_x^2 M_{\xi}^2 / 27 \sigma^4 & \text{for smooth FHI},
\end{cases}
\]

with \( f_c(x) = (x + 1)^2 \ln(1 + 1/x) + (x - 1)^2 \ln(1 - 1/x) \Rightarrow f_c(x) \simeq 3 \) for \( x \gg 1 \), \( x = \sigma^2 / 2 M^2 \) and \( x_\xi = \sigma^2 / M_{\xi}^2 \). Also \( N \) is the dimensionality of the representations to which \( \Phi \) and \( \bar{\Phi} \) belong and \( Q \) is a renormalization scale. Note that renormalization group effects [34] remain negligible in the available parameter space of our models.

For \( c_{4K} < 0 \), \( V_{\text{HI}} \) reaches a maximum at \( \sigma = \sigma_{\text{max}} \) which can be estimated as follows:

\[
V''_{\text{HI}}(\sigma_{\text{max}}) = 0 \Rightarrow \sigma_{\text{max}} \simeq \begin{cases} 
\left( \kappa^2 N / 8 \pi^2 \vert c_{4K} \vert \right)^{1/4} & \text{for standard FHI}, \\
\left( \kappa^2 / 4 \pi^2 \vert c_{4K} \vert \right)^{1/4} & \text{for shifted FHI}, \\
\left( 8 \mu_x^2 M_{\xi}^2 / 27 \vert c_{4K} \vert \right)^{1/8} & \text{for smooth FHI},
\end{cases}
\]

with \( V''_{\text{HI}}(\sigma_{\text{max}}) < 0 \), where the prime denotes derivation w.r.t. \( \sigma \). The system can always undergo FHI starting at \( \sigma < \sigma_{\text{max}} \). However, the lower \( n_s \) we want to obtain, the closer we must set \( \sigma_s \) to \( \sigma_{\text{max}} \), where \( \sigma_s \) is the value of \( \sigma \) when the scale \( k_s \) crosses outside the horizon of FHI. To quantify somehow the amount of this tuning in the initial conditions, we define [15] the quantity:

\[
\Delta_{m*} = (\sigma_{\text{max}} - \sigma_*) / \sigma_{\text{max}}.
\]
4 Observational Constraints

Under the assumption that (i) the curvature perturbations generated by $\sigma$ is solely responsible for the observed curvature perturbation and (ii) there is a conventional cosmological evolution (see below) after FHI, the parameters of the FHI models can be restricted imposing the following requirements:

- The number of e-foldings $N_{\text{HI}}$ that the scale $k_*$ suffers during FHI is to account for the total number of e-foldings $N_{\text{tot}}$ required for solving the horizon and flatness problems of SBB, i.e.,

$$N_{\text{HI}} = N_{\text{tot}} \Rightarrow \int_{\sigma_f}^{\sigma_*} d\sigma \frac{V_{\text{HI}}}{V_{\text{HI}}'} \simeq 64.94 + \frac{2}{3} \ln V_{\text{HI}0}^{1/4} + \frac{1}{3} \ln T_{\text{Hrh}}.$$  \hspace{1cm} (4.1)

where $\sigma_f$ is the value of $\sigma$ at the end of FHI, which can be found, in the slow-roll approximation, from the condition

$$\max\{\epsilon(\sigma_f), |\eta(\sigma_f)|\} = 1,$$

where $\epsilon \simeq \frac{1}{2} \left(\frac{V_{\text{HI}}'}{V_{\text{HI}}}\right)^2$ and $\eta \simeq \frac{V_{\text{HI}''}}{V_{\text{HI}}}$.

In the cases of standard [2] and shifted [3] FHI, the end of FHI coincides with the onset of the GUT phase transition, i.e., the slow-roll conditions are violated close to the critical point $\sigma_c = \sqrt{2}M$ [for standard [shifted] FHI, the waterfll regime commences. On the contrary, the end of smooth [4] FHI is not abrupt since the inflationary path is stable w.r.t $\Phi - \Phi$ for all $\sigma$'s and $\sigma_f$ is found from Eq. (4.2). On the other hand, the required $N_{\text{tot}}$ at $k_* = 0.002/\text{Mpc}$ can be easily derived [20] consistently with our assumption of a conventional post-inflationary evolution. In particular, we assume that FHI is followed successively by the following three epochs: (i) the decaying-inflaton dominated era which lasts at a reheat temperature $T_{\text{Hrh}}$, (ii) a radiation dominated epoch, with initial temperature $T_{\text{Hrh}}$, which terminates at the matter-radiation equality, (iii) the matter dominated era until today.

- The power spectrum $P_R$ of the curvature perturbations generated by $\sigma$ at the pivot scale $k_*$ is to be confronted with the WMAP5 data [12]:

$$P_R^{1/2} = \frac{1}{2\sqrt{3}\pi} \left| \frac{V_{\text{HI}}^{3/2}(\sigma_*)}{V_{\text{HI}}'(\sigma_*)} \right| \simeq 4.91 \cdot 10^{-5} \text{ at } k_* = 0.002/\text{Mpc}.$$  \hspace{1cm} (4.3)

Finally we can calculate the spectral index, $n_s$, and its running, $\alpha_s$, through the relations:

$$n_s = 1 - 6\epsilon_* + 2\eta_*, \quad \alpha_s = \frac{2}{3} \left(4\eta_*^2 - (n_s - 1)^2\right) - 2\xi_*.$$  \hspace{1cm} (4.4)

respectively, where $\xi \simeq V_{\text{HI}'}V_{\text{HI}''}/V_{\text{HI}}^2$ and the variables with subscript * are evaluated at $\sigma = \sigma_*$. We can obtain an approximate estimation of the expected $n_s$’s, if we calculate analytically the integral in Eq. (4.1) and solve the resulting equation w.r.t $\sigma_*$. We pose $\sigma_f = \sigma_c$ for standard and shifted FHI whereas we solve the equation $|\eta(\sigma_f)| = 1$ for smooth FHI ignoring any SUGRA correction. Taking into account that $\epsilon < \eta$ we can extract $n_s$ from Eq. (4.4). We find

$$n_s = \begin{cases} 1 - 1/N_{\text{HI}k}^* + 3\kappa^2N_{\text{HI}k}^*c_{4K}/4\pi^2 & \text{for standard FHI}, \\ 1 - 1/N_{\text{HI}k}^* + 3\kappa^2N_{\text{HI}k}^*c_{4K}/2\pi^2 & \text{for shifted FHI}, \\ 1 - 5/3N_{\text{HI}k}^* + 4c_{4K} \left(6\mu_3^2M_S^2N_{\text{HI}k}^*\right)^{1/3} & \text{for smooth FHI}. \end{cases}$$  \hspace{1cm} (4.5)

From the expressions above, we can easily infer that $c_{4K} < 0$ can diminish significantly $n_s$. To this end, in the cases of standard and shifted FHI, $|c_{4K}|$ has to be of order unity for relatively large $\kappa$’s and much larger for lower $\kappa$’s whereas, for smooth FHI, a rather low $|c_{4K}|$ is enough.
Kähler Potentials for Hilltop F-term Hybrid Inflation

5 Numerical Results

In our numerical investigation, we fix $N = 2$. This choice corresponds to the left-right symmetric gauge group $SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ for standard FHI and to the Pati-Salam gauge group $SU(4)_c \times SU(2)_L \times SU(2)_R$ for shifted [3] FHI. Note that, if $\Phi$ and $\bar{\Phi}$ are chosen to belong to $SU(2)_R$ doublets with $B-L = \pm 1$, respectively, no cosmic strings are produced [35] during this realization of standard FHI. As a consequence, we are not obliged to impose extra restrictions on the parameters (as, e.g., in Refs. [28, 29]). We also take $T_{H_{rh}} \simeq 4 \cdot 10^{-10}$ (recall that the quantities with mass dimensions are measured in units of $m_p$) as in the majority of these models [5, 8, 31] saturating conservatively the gravitino constraint [36]. This choice for $T_{H_{rh}}$ do not affect crucially our results, since $T_{H_{rh}}$ appears in Eq. (4.1) through the one third of its logarithm and so its variation upon two or three orders of magnitude has a minor influence on the value of $N_{tot}$.

The inflationary dynamics is controlled by the parameters (note that we fix $c_{2K} = 0$):

$$\sigma_* , v_G, c_{4K} \text{ and } \begin{cases} \kappa & \text{for standard and shifted (with fixed $M_S$) FHI,} \\ M_S & \text{for smooth FHI.} \end{cases}$$

In our computation, we can use as input parameters $\kappa$ or $M_S, \sigma_*$ and $c_{4K}$. We then restrict $v_G$ and $\sigma_*$ so as Eqs. (4.1) and (4.3) are fulfilled. Using Eq. (4.4) we can extract $n_s$ and $\alpha_s$ for any given $c_{4K}$ derived from Eqs. (2.19a), (2.19b) and (2.20). Turning the argument around, we can find the observationally favored $c_{4K}$’s, imposing the satisfaction of Eq. (1.1), and then we can check if these $c_{4K}$’s can be derived from Eqs. (2.19a), (2.19b) and (2.20).

Our results are presented in Fig. 2 for standard FHI and in Table 2 for shifted and smooth FHI. Let us discuss these results in the following.

5.1 Standard FHI

In Fig. 2 (a) [Fig. 2 (b)] we delineate the (lightly gray shaded) regions allowed by Eqs. (1.1), (4.1) and (4.3) in the $\kappa - c_{4K} [\kappa - v_G]$ plane for standard FHI. The conventions adopted for the various lines are also shown in the r.h.s of each graphs. In particular, the black solid [dashed] lines correspond to $n_s = 0.991$ [$n_s = 0.933$], whereas the gray solid lines have been obtained by fixing $n_s = 0.963$ – see Eq. (1.1). Below the dotted line, our initial assumption $\sigma_* < \sigma_{\text{max}}$ is violated. The various lines terminate at $\kappa = 0.15$, since for larger $\kappa$’s the two restrictions in Eqs. (4.1) and (4.3) cannot be
Note that the $\sigma_s$, $\kappa$, and $M$ potentials in Eqs. (2.4) and (2.17). Namely, for $c_{4K} > 0$, we obtain $0.0006 \lesssim \kappa \lesssim 0.15$ with $1.1 \lesssim v_G/10^{-3} \lesssim 2.5$, $-1100 \lesssim c_{4K} \lesssim -0.05$ and $0.014 \lesssim \Delta_{m_s} \lesssim 0.28$.

Table 2: Input and output parameters consistent with Eqs. (4.1) and (4.3) for shifted (with $M_S = 0.205$) or smooth FHI, $v_G = M_{\text{GUT}}$ and selected $c_{4K}$'s indicated in Table 1 and Fig. 1.

In the cases of shifted and smooth FHI we confine ourselves to the values of the parameters which give $v_G = M_{\text{GUT}}$ and display solutions consistent with Eqs. (4.1) and (4.3) in Table 2. The selected $c_{4K}$'s are indicated in Table 1 (for $c_{4K} \geq 0$) and denoted by light gray crosses in Fig. 1 (for $c_{4K} < 0$). The entries without a value assigned for $\Delta_m$ mean that $V_{\text{HI}}$ has no distinguishable maximum.

We observe that the required (in order to obtain $v_G = M_{\text{GUT}}$) $\kappa$'s in the case of shifted FHI are rather low and so, reduction of $n_s$, $k_1$, $\beta_1$, and $\beta_2$ of different signs. On the contrary, in the case of smooth FHI, $n_s$ turns out to be quite close to its central value in Eq. (1.1) even with $c_{4K} = 0$. Therefore, in order to reach the central and the lowest value of $n_s$, in Eq. (1.1), one needs rather small $c_{4K}$'s, which can be obtained even with $M = 1$ (and only negative $\beta_m$'s) – see Fig. 1. However, the resulting $\Delta_{m_s}$'s are lower than those of shifted FHI.

### 5.2 Shifted and Smooth FHI

In the cases of shifted and smooth FHI we confine ourselves to the values of the parameters which give $v_G = M_{\text{GUT}}$ and display solutions consistent with Eqs. (4.1) and (4.3) in Table 2. The selected $c_{4K}$'s are indicated in Table 1 (for $c_{4K} \geq 0$) and denoted by light gray crosses in Fig. 1 (for $c_{4K} < 0$). The entries without a value assigned for $\Delta_{m_s}$ mean that $V_{\text{HI}}$ has no distinguishable maximum.

We observe that the required (in order to obtain $v_G = M_{\text{GUT}}$) $\kappa$'s in the case of shifted FHI are rather low and so, reduction of $n_s$ to the level dictated by Eq. (1.1) requires rather high $c_{4K}$'s. These can be derived, e.g., for $M = 2$ and $\beta_m$ of different signs. On the contrary, in the case of smooth FHI, $n_s$ turns out to be quite close to its central value in Eq. (1.1) even with $c_{4K} = 0$. Therefore, in order to reach the central and the lowest value of $n_s$, in Eq. (1.1), one needs rather small $c_{4K}$’s, which can be obtained even with $M = 1$ (and only negative $\beta_m$’s) – see Fig. 1. However, the resulting $\Delta_{m_s}$’s are lower than those of shifted FHI.
6 Conclusions

We considered the basic types of FHI in the context of a string inspired SUGRA scenario using a simple class of Kähler potentials given by Eq. (2.4) with dependence – see Eq. (2.17) – on the hidden sector fields. We imposed, essentially, two conditions so that hilltop FHI can be realized. Namely, we required the mass squared of the inflaton during FHI is zero and the parameter $c_{4K}$ involved in the quartic SUGRA correction to the inflationary potential is adequately negative so that the results on $n_s$ can be reconciled with data. We found a wide and natural set of solutions which satisfy the above requirements. Moreover the desired form of the Kähler potential is thus obtained for all hidden sector fields and not just for some carefully chosen vacua. However, our results require a proximity between the values of the inflaton field at the maximum of the potential and at the horizon crossing of the pivot scale. The amount of this tuning was measured by the quantity $\Delta_m$ defined in Eq. (3.4). In particular, for $n_s$ close to its central value, we found that (i) in the case of standard FHI, $v_G$ turns out to be well below $M_{\text{GUT}}$ with $c_{4K} \simeq -(1100 - 0.05)$ for $\kappa \simeq (0.0006 - 0.15)$ and $\Delta_m \simeq (1.4 - 28)\%$; (ii) in the case of shifted [smooth] FHI, we succeeded to obtain $v_G = M_{\text{GUT}}$ for $c_{4K} \simeq -16 [c_{4K} \simeq -1/16]$ and $\Delta_m = 26\% [\Delta_m = 17\%]$. Observationally less interesting $n_s$’s can be also achieved for $c_{4K} \geq 0$, without the presence of a maximum along the inflationary trajectory.

Trying to compare our construction with that of Ref. [21] we would like to mention that in our case (i) there is no need for cancellation of the term $V_0$ in the expansion of Eq. (2.15); (ii) higher order terms of the inflaton in the Kähler potential let intact our calculation since only terms up to the order $|S|^4$ in the inflationary potential are relevant for our analysis; (iii) the requirement of the $h_m$’s stabilization before the onset of FHI can be evaded if $\dot{W} = 0$. In the latter case, $h_m$ can represent even fields of the observable sector which do not contribute to the superpotential at all, due, e.g., to the existence of an additional symmetry (as in the case of Ref. [23]).

Throughout our investigation we concentrated on the predictions derived from the inflationary potential, assuming that we had suitable initial conditions for FHI to take place. In general, it is not clear [16, 19] how the inflaton can reach the maximum of its potential in the context of hilltop inflation. Probably an era of eternal inflation prior to FHI could be useful [18] in order the proper initial conditions to be set. On the other hand, in our regime with $c_{4K} < 0$, the potential develops just a maximum along the inflationary path and not a local maximum and minimum as in the case with quasi-canonical Kähler potential [15, 16, 19, 20]. Therefore, in our scheme, complications related to the trapping of the inflaton in that local minimum are avoided.

Let us finally note that a complete inflationary model should specify the transition to radiation domination, and also explain the origin of the observed baryon asymmetry. For FHI with canonical or quasi-canonical Kähler potential, this has been extensively studied (see, e.g., Ref. [3, 4, 8, 15, 16, 31]). Obviously our set-up preserves many of these successful features of this post-inflationary evolution which may constrain further the parameter space of our models and help us to distinguish which version of FHI is the most compelling.

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References

[1] E.J. Copeland et al., Phys. Rev. D 49, 6410 (1994) [astro-ph/9401011].

[2] G.R. Dvali, Q. Shafi and R.K. Schaefer, Phys. Rev. Lett. 73, 1886 (1994) [hep-ph/9406319].

[3] R. Jeannerot et al., J. High Energy Phys. 10, 012 (2000) [hep-ph/0002151].

[4] G. Lazarides and C. Panagiotakopoulos, Phys. Rev. D 52, 559 (1995) [hep-ph/9506325];
   R. Jeannerot, S. Khalil and G. Lazarides, Phys. Lett. B 506, 344 (2001) [hep-ph/0103229].

[5] G. Lazarides, J. Phys. Conf. Ser. 53, 528 (2006) [hep-ph/0607032].

[6] G.R. Dvali, G. Lazarides and Q. Shafi, Phys. Rev. Lett. 73, 1886 (1994) [hep-ph/9406319].

[7] R. Jeannerot et al., J. High Energy Phys. 10, 012 (2000) [hep-ph/0002151].

[8] R. Jeannerot, S. Khalil and G. Lazarides, Phys. Lett. B 506, 344 (2001) [hep-ph/0103229].

[9] D.H. Lyth and A. Riotto, Phys. Rept. 314, 1 (1999) [hep-ph/9807278].

[10] E.D. Stewart, Phys. Rev. D 51, 6847 (1995) [hep-ph/9405389];
    M. Bastero-Gil and S.F. King, Nucl. Phys. B549, 391 (1999) [hep-ph/9806477];
    G. German, G.G. Ross and S. Sarkar, Phys. Lett. B 469, 46 (1999) [hep-ph/9908380].

[11] G. Lazarides, J. Phys. Conf. Ser. 53, 528 (2006) [hep-ph/0607032].

[12] E. Komatsu et al. [WMAP Collaboration], Astrophys. J. Suppl. 180, 330 (2009) [arXiv:0803.0547]
    http://lambda.gsfc.nasa.gov/product/map/dr2/parameters.cfm.

[13] R.A. Battye, B. Garbrecht and A. Moss, J. Cosmol. Astropart. Phys. 09, 007 (2006) [astro-ph/0607339].

[14] G. Lazarides and C. Pallis, Phys. Lett. B 651, 216 (2007) [hep-ph/0702260];
    G. Lazarides, Int. J. Mod. Phys. A22, 5747 (2007) [arXiv:0706.1436];
    C. Pallis, arXiv:0812.0249 (to appear in the ERE2008 Proceedings).

[15] B. Garbrecht, C. Pallis and A. Pilaftsis, J. High Energy Phys. 12, 038 (2006) [hep-ph/0605264].

[16] M. Bastero-Gil, S.F. King and Q. Shafi, Phys. Lett. B 651, 345 (2007) [hep-ph/0604198].

[17] C. Panagiotakopoulos, Phys. Lett. B 402, 257 (1997) [hep-ph/9703443].

[18] L. Boubekeur and D. Lyth, J. Cosmol. Astropart. Phys. 07, 010 (2005) [hep-ph/0502047];
    K. Kohri, C.M. Lin and D.H. Lyth, J. Cosmol. Astropart. Phys. 12, 004 (2007) [arXiv:0707.3826];
    C.M. Lin and K. Cheung, arXiv:0812.2731;
    M. Ur Rehman, Q. Shafi and J.R. Wickman, arXiv:0901.4345.

[19] M. Ur Rehman, V.N. Şenoğuz and Q. Shafi, Phys. Rev. D 75, 043522 (2007) [hep-ph/0612023].

[20] C. Pallis, “High Energy Physics Research Advances”, edited by T.P. Harrison and R.N. Gonzales (Nova Science Publishers Inc., New York, 2008) [arXiv:0710.3074].

[21] K. Enqvist, L. Mether and S. Nurmi, J. Cosmol. Astropart. Phys. 11, 014 (2007) [arXiv:0706.2355];
    S. Nurmi, J. Cosmol. Astropart. Phys. 01, 016 (2008) [arXiv:0710.1613].

[22] R. Allahverdi et al., Phys. Rev. Lett. 97, 191304 (2006) [hep-ph/0605035];
    J.C. Bueno-Sanchez et al., J. Cosmol. Astropart. Phys. 01, 015 (2007) [hep-ph/0608299].

[23] C. Panagiotakopoulos, Phys. Lett. B 459, 473 (1999) [hep-ph/9904284];
    C. Panagiotakopoulos, Phys. Rev. D 71, 063516 (2005) [hep-ph/0411143].

[24] H.P. Nilles, Phys. Rept. 110, 1 (1984).

[25] W. Buchmuller, L. Covi and D. Delepine, Phys. Lett. B 491, 183 (2000) [hep-ph/0006168].

[26] S.C. Davis and M. Postma, J. Cosmol. Astropart. Phys. 04, 022 (2008) [arXiv:0801.2116];
    S. Antusch, M. Bastero-Gil, K. Dutta, S. F. King and P. M. Kostka, arXiv:0808.2425.

[27] Z. Lalak and K. Turzynski, Phys. Lett. B 659, 669 (2008) [arXiv:0710.0613].
[28] R. Jeannerot and M. Postma, J. High Energy Phys. 05, 071 (2005) [hep-ph/0503146].
[29] J. Rocher and M. Sakellariadou, J. Cosmol. Astropart. Phys. 03, 004 (2005) [hep-ph/0406120].
[30] T.W.B. Kibble, J. Phys. A 9, 387 (1976).
[31] V.N. Şenoğuz and Q. Shafi, Phys. Rev. D 71, 043514 (2005) [hep-ph/0412102];
     V.N. Şenoğuz and Q. Shafi, hep-ph/0512170.
[32] See, e.g., L.E. Ibanez and D. Lust, Nucl. Phys. B382, 305 (1992) [hep-th/9202046];
     D. Lust, S. Reffert and S. Stieberger, Nucl. Phys. B727, 264 (2005) [hep-th/0410074].
[33] D. Lust et al., Nucl. Phys. B766, 68 (2007) [hep-th/0506090].
[34] G. Ballesteros et al., J. Cosmol. Astropart. Phys. 03, 001 (2006) [hep-ph/0601134].
[35] G. Lazarides et al., Phys. Rev. D 70, 123527 (2005) [hep-ph/0409335].
[36] M.Yu. Khlopov and A.D. Linde, Phys. Lett. B 138, 265 (1984);
     J. Ellis, J.E. Kim and D.V. Nanopoulos, ibid. 145, 181 (1984).