Hydrodynamics of the Polyakov line in SU($N_c$) Yang–Mills

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A B S T R A C T

We discuss a hydrodynamical description of the eigenvalues of the Polyakov line at large but finite $N_c$ for Yang–Mills theory in even and odd space-time dimensions. The hydro-static solutions for the eigenvalue densities are shown to interpolate between a uniform distribution in the confined phase and a localized distribution in the de-confined phase. The resulting critical temperatures are in overall agreement with those measured on the lattice over a broad range of $N_c$, and are consistent with the string model results at $N_c = ∞$. The stochastic relaxation of the eigenvalues of the Polyakov line out of equilibrium is captured by a hydrodynamical instanton. An estimate of the probability of formation of a $Z(N_c)$ bubble using a piece-wise sound wave is suggested.

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1. Introduction

Lattice simulations of Yang–Mills theory in even and odd dimensions show that the confined phase is center symmetric [1,2]. At high temperature Yang–Mills theory is in a deconfined phase with broken center symmetry. The transition from a center symmetric to a center broken phase is non-perturbative and is the topic of intense numerical and effective model calculations [3] (and the references therein). Of particular interest are the semi-classical descriptions and matrix models.

In the semi-classical approximations, the confinement–deconfinement transition is understood as the breaking of instantons into a dense plasma of dyons in the confined phase and their re-assembly into instanton molecules in the deconfined phase [4,5]. This mechanism is similar to the Berezinskii–Kosterlitz–Thouless transition in lower dimensions [6], and to the transition from insulators to superconductors in topological materials [7]. In matrix models, the Yang–Mills theory is simplified to the eigenvalues of the Polyakov line and an effective potential is used with parameters fitted to the bulk pressure to study such a transition [8,9], in the spirit of the strong coupling transition in the Gross–Witten model [10].

Matrix models for the Polyakov line share much in common with unitary matrix models in the general context of random matrix theory [11]. The canonical example is Dyson circular unitary ensemble and its analysis in terms of orthogonal polynomials or a one-component Coulomb plasma. The Dyson circular unitary ensemble relates to the one-dimensional Calogero–Sutherland model [12] which is an effective model for quantum Luttinger liquids.

A useful analysis of one-dimensional interacting electron systems relies on hydrodynamics which does not require an exact solution of the many-body problem. The method treats the system in the continuum limit as a fluid, and allows for the understanding of both small amplitude collective phenomena (phonons) as well as large amplitude effects (solitons, shockons) [13,14]. A reduction of the many-body Hamiltonian onto the hydrodynamical collective degrees of freedom makes use of the collective quantization method developed in the context of quantum field theory [15] and extended to problems in condensed matter physics [16].

In this letter we develop a hydrodynamical description of the gauge invariant eigenvalues of the Polyakov line for an SU($N_c$) Yang–Mills theory at large but finite $N_c$. We will use it to derive the following new results: 1/ a hydrostatic solution for the eigenvalue density that interpolates between a confining (uniform) and de-confining (localized) phase; 2/ explicit critical temperatures for the Yang–Mills transitions in 1 + 2 and 1 + 3 dimensions; 3/ a hydrodynamical instanton for the density distribution that captures the stochastic relaxation of the eigenvalues of the Polyakov line; 4/ an estimate of the fugacity or probability to form a $Z(N_c)$ bubble using a piece-wise sound wave.

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2. Polyakov line in 1 + 2 dimensions

The matrix model partition function for the eigenvalues of the Polyakov line for SU(Nc) in 1 + 2 dimensions was discussed in [8]. If we denote by diag(e^{θi}, . . . , e^{θNc}) with ∑θi = 0 the gauge invariant eigenvalues of the Polyakov line, then [8]

\[ Z[α, β] = \prod_{i=1}^{Nc} \int dθi \prod_{i<j} |z_{ij}|^{1/2} e^{-αz_{ij}} \sum_{i=1}^{Nc} |V(z_{ij})| \]  

(1)

with z_{ij} = z_i - z_j and z_i = e^{θi}. The perturbative potential V(z_{ij}) is center symmetric and quadratic in leading order or V(|z_{ij}|) ≈ |z_{ij}|^2, with α(T) = T^2 V_2/2π and V_2 the spatial 2-volume [8]. The mass expansion of the one-loop determinant gives β(T) = m_0^2 V_2/2π [8]. The Debye mass is self-consistently defined as m_0^2 = N_c g^2 T (ln(T/m_0) + C)/2π [17] to tame all infra-red divergences, with C ≈ 1.3 from lattice simulations [18,19].

(1) can be regarded as the normalization of the squared and real many-body wave-function ψ_0[z_i] which is the zero-mode solution to the Shrödinger equation H_0ψ_0 = 0 with the self-adjoint squared Hamiltonian

\[ H_0 = \sum_{i=1}^{Nc} (−\partial_i + a_i) (\partial_i + a_i) \]  

(2)

with ∂_i = ∂/∂θ_i and the pure gauge potential a_i = δ S. Here S[z] = −lnψ_0[z_i] is half the energy in the defining partition function in (1). In (2) the mass parameter is 1/2.

3. Hydrodynamics

We can use the collective coordinate method in [15] to re-write (2) in terms of the density of eigenvalues as a collective variable ρ(θ) = ∑_{i=1}^{Nc} δ(θ - θ_i). For that, we re-define H_0 → H through a similarity transformation to re-absorb the diverging 2-body part induced by the Vandermond contribution Δ = ∏_{i,j} |z_{ij}|^{θ(i,j)}, i.e. Ψ = ψ_0/√Δ and √Δ H = H_0√Δ. Now H is of the general form discussed in [15] and is amenable after some algebra to

\[ H = \int dθ (\partial_θ \rho \partial_θ \pi + ρ u[ρ]) \]  

(3)

with the potential-like contribution

\[ u[ρ] = \left( A(θ) - \frac{π β(T) ρ_H}{2} + \frac{1}{2} \partial_θ \ln ρ \right)^2 \]  

(4)

Here

\[ A(θ) = \frac{1}{2} \alpha(T) \int dθ' ρ(θ') \partial_θ V \left( 2 \sin \left( θ - \frac{θ'}{2} \right) \right) \]  

and ρ_H is the periodic Hilbert transform of ρ

\[ [ρ]_H = ρ_H = \frac{1}{2} \int ρ(θ') \cotan \left( \frac{θ - θ'}{2} \right) \]  

(5)

As conjugate pairs, π(θ) and ρ(θ) satisfy the equal-time commutation rule [π(θ), ρ(θ')] = −i(δ(θ - θ') - 1/2π). We identify the collective fluid velocity with v = ∂_θ π and re-write (3) in the more familiar hydrodynamical form

\[ H \approx \int dθ ρ(θ) \left( v^2 + u[ρ] \right) \approx \int dθ ρ(θ) |v + iA|^2 \]  

(6)

modulo ultra-local terms. The Heisenberg equation for ρ yields the current conservation law ∂_θ ρ = −2∂_θ (ρv), and the Heisenberg equation for v gives the Euler equation

\[ \dot{v}_H = i[H, v] = -\partial_θ \left( v^2 + A^2 - 2A_θ A_θ ln ρ + \pi β[A_θ V_H - 2A[A_θ]|_H^2 \right) \]  

(7)

with the sine-transform [A_θ] = f sinh(θ - θ') A_θ(θ') ρ(θ'). Note that all the relations hold for large but finite N_c.

4. Hydro-static solution

The static hydrodynamical density follows from the minimum of (6) with v(θ) = 0.

\[ β(T) π ρ_H(θ) - ρ_H ln ρ_H(θ) = 2A(θ) \]  

(9)

To solve (9), we insert the leading quadratic contribution A(θ) = 2α(T)sin²(θ/2) in (9)

\[ ρ_H - a_0 ρ_H = bc_1 ρ sin(θ) \]  

(10)

with a ≡ 1/π β(T), b ≡ 2α(T)/β(T) and c_1 the first moment of the density or π c_1 = ∫₀^2π ρ(θ)cosθdθ. Let ρ_0 = N_c/2π be the uniform eigenvalue density and ρ_1 = ρ - ρ_0 its deviation. Consider the Cauchy transform

\[ G(z) = \frac{1}{π i} \int_C \frac{ρ_1(η)}{η - z} dη \]  

(11)

with η = e^{iθ}. The contour C is counter-clockwise along the unit circle. G(z) is a holomorphic function in the complex z-plane. Let G^+ and G^- be its realization inside and outside C respectively, so that

\[ G^- / G^+ = ±1 + \rho_1(θ) + iρ_H(θ) \]  

(12)

We now carry the Hilbert transform on both sides of (10). Setting G(z) = G^+(z) and using 2[ρ_1ρ_H]|_H = ρ_H² - ρ_H², we have for (10)

\[ \frac{1}{2} G^2 + (ρ_0 - \frac{1}{2} bc_1 (z - z^{-1})) G + azc_2 G = bc_1 ρ_0 z + \frac{1}{2} bc_1^2 \]  

(13)

on the boundary C, thus within the circle. Here, we should require G(z = 0) = 0 to ensure that ρ_1 integrates to zero.

\[ a ≈ 1/V_2 \]  

is subleading and will be dropped. Thus (13) is algebraic in G(z). Since ρ(θ) = ρ_0 + Re G^+(z = e^{iθ}), careful considerations of the singularity structures of the quadratic solutions to (13) yield (θ is a step function)

\[ ρ(θ) = \sqrt{bc_1} (cosθ + 1)^2 - \sqrt{bc_1} (cosθ - cosθ_0) \theta(\{|θ| - |θ_0|\}) \]  

(14)

The analytic properties of G(z) fix c_1/ρ_0 = 1 + (1 - 1/b) and θ_0 at cosθ_0 = 1 - 2ρ_0/bc_1. For b < 1 the non-uniform solution with ρ_1 = 0 is absent. For b >> 1, c_1 → 2ρ_0 and

\[ ρ(θ) → \frac{N_c}{2π} \sqrt{80 - 4b^2 a^2} \]  

(15)

Therefore (14) interpolates between a uniform density distribution ρ_0 (confined phase) and a Wigner semi-circle (deconfined phase) with a transition at b = 1 or T_c = m_0. In 1 + 2 dimensions the fundamental string tension is given to a good accuracy by √g^2/2N_c = ((1 - N_c^2)/8πr)^1/2 [22]. Thus the ratio in 1 + 2 dimensions

\[ \frac{T_c}{√g^2} = \frac{C}{2π} \left( \frac{8π}{1 - 1/N_c^2} \right)^{1/2} \rightarrow \sqrt{2/π} C \]  

(16)

with C ≈ 1.3 [18,19]. In Fig. 1 we show the behavior of (16) (upper curve) versus N_c, in comparison to the numerical fit T_c/√g^2 = 0.9026 + 0.880/N_c^2 to the lattice results (lower curve) in [23]. Amazingly, (16) at large N_c is consistent with √g^2/π in the string model [20].
5. Dyson Coulomb gas

We note that (9) coincides with the saddle point equation to (1) by re-writing it using Dyson charged particle analogy on $S^1$ with the energy $2S_z = \sum_{i<j} G(z_{ij})$ and the pair interaction

$$G(z_{ij}) = -\ln|z_{ij}| + \alpha(T)V(|z_{ij}|) \equiv G(\theta_i - \theta_j)$$

(17)

At large $N_c$ the ensemble described by (1) is sufficiently dense to allow the change in the measure. Following Dyson [11] we obtain

$$Z[\alpha, \beta] \to \int D\rho \ e^{-\Gamma[\alpha, \beta; \rho]}$$

(18)

with the effective action

$$\Gamma[\alpha, \beta; \rho] = \frac{1}{2} \int \rho(\theta)G(\theta - \theta')\rho(\theta')$$

$$- \left(\frac{\beta(T)}{2} - 1\right) \int d\theta \rho(\theta) \ln \left(\frac{\rho(\theta)}{\rho_0}\right)$$

(19)

The $\beta$ contribution is the self-Coulomb subtraction and is consistent with the subtraction in the Hilbert transform. The saddle point equation $\delta\Gamma/\delta\rho = 0$ following from (18)-(19) is in agreement with the hydro-static equation (9),

$$\frac{d}{d\tau} \frac{\delta\Gamma[\alpha, \beta; \rho]}{\delta\rho(\theta)} = 2A_0 = 0$$

(20)

6. Hydrodynamical instanton

The fixed time zero energy solution to (7) is an instanton with imaginary velocity $v = -iA$. We have checked that this is a solution to (8) for all times. The current $J = \rho v = -i\rho A$ is conserved. Thus $\partial_t \rho + \partial_\theta \rho = 0$ or

$$\partial_t \rho + \beta(T)\partial_\theta (\pi \rho \rho_H) = \partial_\theta^2 \rho + 2\partial_\theta (\rho A(\theta))$$

(21)

for Euclidean times $\tau = i t$. For $A = 0$ and $\beta(T) = 2$, (21) agrees with the viscous Burger's equation describing large Wilson loops in $1 + 3$ dimensions [21]. Following [11] we identify $\tau$ with the stochastic (Langevin) time. (21) describes the stochastic relaxation of the eigenvalue density of the Polyakov line (out of equilibrium) to its asymptotic (in equilibrium) hydro-static solution.

7. Sound waves

The hydrodynamical action follows from standard procedure. The momentum $\pi(\theta) = (1/\partial_\theta) v$ is canonically conjugate to the density $\rho$, and the Lagrange density is $L = \pi \partial_\theta \rho - H$. Thus the action $S = \int d\tau d\theta \rho(\theta) \left( v^2 - D\rho(\theta) \right)$, which is linearized by

$$\rho \approx \rho_0(\theta) + 2\partial_\theta \varphi \quad \text{and} \quad \rho v \approx -\partial_\theta \varphi$$

(22)

Inserting (22) into $S$ yields

$$S_2 = \int d\tau d\theta \rho_0(\theta) \left( \partial_\theta \varphi^2 - \rho_0^2(\theta) W^2 \varphi^2 \right)$$

(23)

with the potential

$$W[\varphi] = 2\pi \alpha(T) \varphi \ln(\pi \rho_0(\theta))$$

(24)

For constant $\rho_0$ and large $N_c$, (23) simplifies to

$$S_2 \approx \frac{m_D^2}{2} V_2 \int d\theta \rho(\theta) \left( \partial_\theta \varphi^2 - \rho_0^2 \right)$$

(25)

after the rescaling $\nu = \pi \rho_0(\theta)$. (25) describes sound waves in the large $N_c$ space of holonomies.

8. Z($N_c$) bubble

In a de-confined phase of infinite volume, the Yang–Mills ground state settles in one of the degenerate $Z(N_c)$ vacua. In a finite volume, bubbles of different vacua may form [25]. Consider a de-confined bubble of volume $V_2$ immersed in a confined volume $V_2$. In $V_2$ all the eigenvalues are localized initially within a small $\Delta \theta$ around the origin with $\rho(\tau = 0, \theta) = N_c/\Delta \theta \equiv \rho_0$, and zero otherwise.

Using this piece-wise wave as an initial condition we solve (21) with $A = 0$ for simplicity. For large times $\tau$, the result is

$$\rho(\tau, \theta) \approx \rho_0 - \left(\frac{2}{\pi} \rho_0 \varphi \sin \left(\frac{\Delta \theta}{2}\right)\right) \cos \theta e^{-\nu_1 \tau}$$

(26)

which shows the relaxation of the piece-wise wave over a time $\tau \approx 1/\nu_1$ set by the speed of sound. Using (26) in $S$ yields the Euclidean action estimate for small $\Delta \theta$

$$S_2(V_2) \approx V_2 \left( \pi m_D \rho_0 \sin \left(\frac{\Delta \theta}{2}\right) \right)^2 \to V_2 \left( \frac{\pi}{2} \right)^2 (\rho_0^2)$$

(27)

The bubble formation probability or fugacity is $e^{-S_2(V_2)}$.

9. Polyakov line in 1 + 3 dimensions

To extend our analysis to $1 + 3$ dimensions, we approximate the Yang–Mills thermal state by a dense plasma of dyons and anti-dyons [4,5]. This semi-classical description reproduces a number of key features of the Yang–Mills phase both in the confined (center-symmetric) and de-confined (center-broken) phase. There are two key differences with the $1 + 2$ dimensional partition function in (1). First the many-body energy $2S_z = -2\ln \Psi_0[z]$ in (1) is now shifted

$$S[z] - S[z] = \gamma(T) \prod_{i=1}^{N_c} \left( \theta_{i+1} - \theta_i \right)$$

(28)

with $\gamma(T) = 4\pi N_c f V_2$ and $f = 4\pi A^4/T g^4$ the dyon fugacity [4]. Second and more importantly $\beta(T) = 2$ and is not extensive with the spatial 3-volume $V_3$. Finally, $\alpha(T) = T^3 V_3/3$. Since $\theta_{i+1} - \theta_i \approx 1/2\pi \rho(\theta)$, then in the continuum the additional string of factors in (28) is

$$\prod_{i=1}^{N_c} \left( \theta_{i+1} - \theta_i \right) \sim e^{\frac{1}{4\pi N_c} \int d\theta \rho(\theta) \ln(1/2\pi \rho(\theta))}$$

(29)

With this in mind, a re-run of the preceding arguments yields the Hamiltonian in (3)–(4) with the shifted potential

$$A \to A + \frac{\gamma(T)}{4\pi N_c} e^{-\gamma T(\rho)} \partial_\theta \rho \ln(\rho)$$

(30)
and $N_c \sin \gamma_0 / \rho^2 = \int d\theta \rho(\theta) \ln(\rho(\theta) / N_c)$. The hydro-static equation (9) now reads

$$\beta \pi \rho_H(\theta) - 2A(\theta) = \left( 1 + \frac{\gamma(T_c)}{4\pi N_c^2} e^{-\gamma_0(\rho)} \right) \delta_0 \ln \rho(\theta)$$

(31)

The $\beta = 2$ contribution is now sub-leading and can be dropped. The corresponding solution to (31) is a localized density for $\pi \epsilon_1 = \int_0^{2\pi} d\theta \rho(\theta) \cos \theta \neq 0$, and a uniform density $\rho_0 = N_c / 2\pi$ for $\epsilon_1 = 0$. Specifically

$$\frac{\rho(\theta)}{\rho_0} = \frac{\epsilon_0 \sin \gamma_0}{I_0(\text{sinc})}$$

(32)

with $\epsilon' = \epsilon_1 / N_c$ and $\gamma' = \gamma / N_c^3$. The two parameters $\eta = 8\pi \alpha(T_c) / \gamma'$ and $x = \epsilon' \gamma_0$ are fixed by the transcendental equations

$$I_1(\frac{\pi x}{\eta \gamma_0}) = \frac{\pi x}{\eta \gamma_0} \quad \text{and} \quad I_0^2(\frac{\pi x}{\eta \gamma_0}) = \frac{2\pi^2 x}{\eta}$$

(33)

A solution exists only for $\gamma' < 2\alpha(T_c) / \pi$. Else the density is uniform. Thus the transition temperature from center symmetric (confining) to center broken (deconfining) occurs for $\alpha(T_c) / \gamma(T_c) = \pi / 2N_c^3$ or $T_1 = \frac{2}{8\pi \lambda} \epsilon_1$ with $\lambda = g^2 N_c / 8\pi^2$. For the dyon model, the fundamental string tension is given by $\sigma_1 = (N_c / \pi) \sin(\pi / N_c) \Lambda^2 / \lambda$. [4]. Thus the model independent ratio in $1 + 3$ dimensions

$$\frac{T_1}{\sqrt{\sigma_1}} = \left( \frac{3\pi}{8\pi^2 \sin^2(\pi / N_c)} \right)^{\frac{1}{2}} \rightarrow \left( \frac{3}{8\pi^2} \right)^{\frac{1}{2}}$$

(34)

(34) compares favorably to the lattice results [24] even for small $N_c$ as shown in Fig. 2. At large $N_c$, (34) is consistent with the value of $\sqrt{\sigma_1 / 2\pi}$ in the string model [20].

10. Conclusions

The hydrodynamical description of the Polyakov line captures aspects of the center dynamics in Yang–Mills theory in terms of the gauge invariant density of eigenvalues. The hydro-static equation yields solutions that interpolate between a center symmetric (confining) and a center broken (de-confining) phase. The transition temperatures normalized to the string tension compare well to the lattice results over a broad range of $N_c$, and asymptote the string model results at $N_c = \infty$. The hydrodynamical set-up supports a hydrodynamical instanton that describes the stochastic relaxation of the eigenvalues of the Polyakov line viewed as a fluid. The fluid supports sound waves that can be used to estimate the probability of formation of $Z(N_c)$ bubbles. The relaxation of a fluid of holonomies across the critical temperature may prove useful for understanding the onset of equilibration in a Yang–Mills plasma.

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