Dispersion of bubbles in fully developed channel flow

M J W Harleman¹,², R Delfos¹, T J C van Terwisga² and J Westerweel¹

¹ Laboratory for Aero & Hydrodynamics, 3ME Faculty, Delft University of Technology, Leeghwaterstraat 21 2628CA Delft, The Netherlands
² Section Ship Hydromechanics and Structures, Department of Marine and Transport Technology, 3ME Faculty, Delft University of Technology, Leeghwaterstraat 21 2628CA Delft, The Netherlands
E-mail: m.j.w.harleman@tudelft.nl

Abstract. Dispersion and preferential concentration of small, low Stokes number bubbles in horizontal turbulent channel flow is studied by DNS and experiments. A DNS of turbulent channel flow at \( Re_\tau = 360 \) with Lagrangian tracking of one-way coupled bubbles (\( d < \eta, St = 1.3 \times 10^{-3} \)) shows that equilibrium bubble concentration profiles can be described by a gradient diffusion hypothesis in analogy to flows with suspended sediment as studied by Rouse (1937). The conditionally averaged flow around the bubbles is measured by simultaneous PIV and bubble shadowgraphy and confirms the finding of the DNS that bubbles are preferentially concentrated in large-scale downward flowing fluid regions, which compensates for the rise velocity of the bubbles. This clustering is not an inertia effect, but results from the combination of a concentration gradient and turbulent mixing.

1. Introduction

The dispersion of bubbles in turbulently flowing fluid is commonly observed in oceanography and the oil and food processing industry. Mixing properties and the effect of the bubbles on the flow are often unknown. By now much is known about the forces on very small (\( Re_d \ll 1 \)) and on larger bubbles and spheres (Maxey & Riley (1983), Magnaudet & Eames (2000)) which enables detailed numerical simulations of bubble and particle motion in turbulence. In the absence of walls and gravity the Stokes number defined by the ratio of the bubble and fluid time scales, \( St = \tau_b/\tau_f \), defines whether clustering occurs (Squires & Eaton (1990), Mazzitelli et al. (2003)). When gravity is included, bubbles are only trapped inside vortices if the bubble trapping parameter, the ratio between the velocity difference of a vortex and the bubble rise velocity, is large i.e. \( \beta = \Delta U/\nu_{\text{rise}} \gg 1 \) (Sene et al. (1994)). In vertical ducts and boundary layers the lift force has a large effect on the bubble distribution. Bubbles rising in upflowing liquid are pushed towards the wall, while in downflowing liquid bubbles are pushed away from the wall (Giusti et al. (2005), Moursali et al. (1995)).

This work focusses on bubble dispersion in horizontal turbulent channel flow. Despite the large interest in drag reduction by air bubbles in the turbulent boundary layer under a ship, the dispersion of bubbles in the turbulent boundary layer and the prediction of the bubble concentration profiles did not get much attention. In this paper the dispersion of small bubbles in low Reynolds number turbulent channel flow is studied by DNS and by experiments. The
bubble distribution profile will be modelled based on gradient diffusion in analogy with the work on sediment flow by Rouse (1937) and van Rijn (1984).

2. DNS of turbulent channel flow with Lagrangian bubbles

The dispersion of bubbles in fully developed channel flow is studied in a direct numerical simulation. The Navier-Stokes equations are discretised by a second-order accurate finite volume code with a staggered grid (van Haarlem et al. (1998)). The calculation domain has a length of $5H$, width of $3H$ and height $H$. Periodic boundary conditions are imposed in the streamwise ($x$) and spanwise ($y$) directions and no-slip boundary conditions at the walls at $z = 0$ and $z = H$. The domain is divided into $150 \times 108 \times 180$ equidistant cells, giving a grid spacing of $\Delta x = 12$, $\Delta y = 10$ and $\Delta z = 2$ at a Reynolds number $Re = u_\tau H/\nu = 360$. The motion of 100,000 rigid (polluted) bubbles with diameter $d = 0.4$ in water ($\rho_b \ll \rho_f$) is calculated in a Lagrangian framework by solving the equation of motion

$$\frac{dU_b}{dt} = 3 \frac{Du_f}{Dt} - \frac{36\nu}{d^2}(U_b - U_f) - 2g$$

This equation of motion is based on Maxey & Riley (1983) for rigid bubbles in the Stokes flow regime with

$$Re_d = \frac{|U_b - U_f|}{\nu} \ll 1$$

so that the drag coefficient is $C_d = 24/Re_b$ and the added mass coefficient is 0.5. Since the focus is on slow bubble dispersion the history force is not included and Faxen corrections are neglected since bubbles are smaller than the Kolmogorov lengthscale $\eta = (\nu^3/\epsilon)^{1/4}$. Note that lift forces do not play a role in the Stokes flow regime. The forces in the equation of motion are calculated at the bubble centre. Therefore $2^{nd}$ order quadratic interpolation is used to obtain the fluid velocities at the bubble location from the nearest 27 nodes of Eulerian grid.

Bubble fluid interaction and bubble collisions are not considered (one-way coupling), but it is essential to consider the bubble-wall collisions. When fully elastic bouncing at the wall is implemented, no equilibrium bubble concentration profile is obtained, but instead all bubbles are trapped at a distance of $a$ from the wall. Therefore elastic bouncing is implemented at $10\delta_\nu$ from the wall so that the bubbles remain in the buffer region where fluctuations in the fluid velocity are large.

The simulation parameters are summarised in table 1. They are chosen to be comparable to the experimental condition described later in this paper. The Stokes number is defined as $St = \tau_b/\tau_f$ were the bubble time scale is defined as $d^2/36\nu$ and the fluid time scale is defined as the Kolmogorov time scale $\tau_\eta = (\nu/\epsilon)^{1/2}$. The Kolmogorov scales $\eta$ and $\tau_\eta$ are based on the bulk dissipation $\epsilon_{bulk} = 2U_{\text{bulk}}u_\tau^2/H$.

The simulation is started without bubbles and run for about 20 dimensionless times scales ($t^* = tu_\tau/H$) to obtain fully developed turbulent channel flow. Next, the 100,000 bubbles are randomly distributed in the fluid domain and the simulation is continued for 6.5 time scales until the bubble distribution reaches an equilibrium. In this time the bubble are advected over 100 channel heights. Finally bubble statistics are obtained in the next 8$t^*$ at intervals of 0.05$t^*$.

The mean velocity and Reynolds stress profiles are plotted in figure 1 and compared with the DNS data by Moser et al. (1999). The agreement is quite good, but the peaks in the Reynolds stresses are all slightly lower than in the DNS of Moser, confirming that the grid resolution is a bit coarse. Nevertheless the channel flow is considered sufficiently resolved to study bubble dispersion and clustering in wall-bounded turbulence.
Table 1. Overview of the simulation parameters

| $Re_\tau$ | $gH/\mu^2$ | $d^+$ | $d/\eta$ | $Re_d$ | $St$ | Rouse |
|-----------|-------------|------|----------|--------|------|-------|
| 360       | $11.25 \times 10^3$ | 0.40 | 0.22     | 1.11   | $1.3 \times 10^{-3}$ | 0.68  |

Figure 1. Overview of DNS velocity and velocity fluctuation profiles at $Re_\tau = 360$. The left graph shows the velocity profile in viscous scaling for the present DNS (solid, blue) and the one by Moser et al. (1999) (dashed, red). Viscous and logarithmic profiles ($\kappa = 0.37, B = 4.5$) are indicated. The vertical dotted lines indicate the edges of the log layer; $30\delta_\nu < z < 0.15H$ as defined by Pope (2000). As it was also indicated by Moser, the log-layer, if even present, is very thin at the current low Reynolds number. The graph on the right shows the velocity fluctuation profiles in the top half of the channel.

3. Equilibrium bubble distributions from a continuum approach

In order to find an expression for the bubble distributions in a water channel we use an analogous procedure as used by Rouse (1937) for the description of sediment profiles in open water channels. Conservation of bubble mass is given by

$$ \frac{\partial c_b}{\partial t} + \frac{\partial c_b U_b}{\partial x} + \frac{\partial c_b V_b}{\partial y} + \frac{\partial c_b W_b}{\partial z} = 0 $$ (3)

We consider fully developed flow in a wide (2D) horizontal channel with streamwise direction $x$, spanwise direction $y$ and wall-normal direction $z$. After a Reynolds decomposition the conservation equation for fully developed channel flow reduces to

$$ \frac{\partial}{\partial z} \left[ c_b W_{\text{rise}} + c_b' w_b' \right] = 0 $$ (4)

Here we assumed that the mean rise velocity of the bubbles relative to the fluid, $\bar{w_b}$ equals the bubble rise velocity in stagnant water

$$ W_{\text{rise}} = \frac{gd^2}{18\nu} $$ (5)

Rouse argued that the turbulent eddies that mix momentum also mix the sediment mass. In analogy we assume that the term $c_b' w_b'$ can be modelled as the $\bar{u} w'$ term in the turbulent
momentum balance by the product of an eddy diffusivity and a concentration gradient

$$
\overline{c'_{b}w'} = K_{b} \frac{\partial \overline{c_{b}}}{\partial z}
$$

(6)

The bubble diffusion coefficient $K_{b}$ is related to the diffusion of fluid momentum or turbulent viscosity $\nu_{T}$ by

$$
K_{b} = \alpha \nu_{T}
$$

(7)

In the simplest case that the bubbles can be assumed to be passive tracers $\alpha$ is a constant of order 1, but in the case that bubbles and flow interact, $\alpha$ might be a function of the wall-normal position $z$ and for example the bubble Stokes number. The turbulent viscosity is known to grow linearly with $z$ in the logarithmic layer, while it becomes constant in the channel centre. Following van Rijn (1984) we approximate $\nu_{T}$ by a parabolic expression for $z < 0.25H$ and by a constant value for $z > 0.25H$

$$
\begin{align*}
    \frac{z}{H} \leq 0.25 : & \quad \nu_{T} \simeq \frac{z}{H} \left(1 - \frac{2z}{H}\right) \kappa u_{r} \\
    \frac{z}{H} > 0.25 : & \quad \nu_{T} \simeq \frac{H}{8} \kappa u_{r}
\end{align*}
$$

(8)

with $z$ the distance to the wall. The foregoing equations can be inserted in the conservation equation 4 and when $\alpha$ does not depend on $z$ this expression can be integrated analytically to obtain the following expression for $c(z)$

$$
\begin{align*}
    \frac{z}{H} \leq 0.25 : & \quad \frac{c_{b}}{c_{a}} = \left[ \frac{z(H - 2z_{a})}{z_{a}(H - 2z_{a})} \right]^{-P/\alpha} \\
    \frac{z}{H} > 0.25 : & \quad \frac{c_{b}}{c_{a}} = \left[ \frac{H - 2z_{a}}{2z} \right]^{-P/\alpha} e^{-8P(z/H - 0.25)}
\end{align*}
$$

(9)

with $c_{a}$ the concentration at reference height $z_{a}$ and $P$ the Rouse parameter ($P = W_{\text{rise}}/\kappa u_{r}$). The approximate expressions for the turbulent viscosity and the bubble concentration profiles are compared with the results of the numerical simulation in figure 2. The profiles of $\nu_{T}$ and $K_{b}$ are calculated from

$$
\begin{align*}
    \nu_{T} & = \overline{w'w'} \left( \frac{\partial U}{\partial z} \right)^{-1} \\
    K_{b} & = \overline{c_{b}} W_{\text{rise}} \left( \frac{\partial \overline{c_{b}}}{\partial z} \right)^{-1}
\end{align*}
$$

(10)

The approximate expression of equation 8 for $\nu_{T}$ captures the trends in the top half of the $\nu_{T}$ profile. The bubble diffusion coefficient is not constant over the bulk of the channel, but its magnitude is clearly of the same order as $\nu_{T}$. As a result the concentration profile can well be described by the theoretical profile of equation 9. In fact the constant of proportionality $\alpha$, that follows from the fit, has a value of 1.0. Therefore the bubble concentration profile can be predicted reasonably well from the bubble Rouse number $P = W_{\text{rise}}/\kappa u_{r}$.

4. Preferential concentration as a result of a concentration gradient

The bubble and fluid velocities are averaged over 160 time steps and shown in figure 3. The average wall-normal velocity for both the fluid and the bubbles are zero, which confirms that an equilibrium concentration profile is reached. In addition the plot shows the fluid
Figure 2. Bubble distribution and dispersion coefficient as a function of the wall normal position. Left the PDF of the bubble distribution (black, solid) and a fitted Rouse profile (red, dashed). Right the turbulent viscosity of the fluid (solid, red), the bubble dispersion coefficient (solid, black) and the Rouse approximation for the turbulent viscosity from equation 8 (dashed, blue).

Figure 3. Time averaged profiles of the streamwise (left) and wall-normal (right) fluid velocities (black, solid), bubble velocities (red, dashed) and the fluid velocities at the bubble locations (blue, solid).

velocity conditioned at the location of the bubble. This velocity is downwards and equals the (negative) rise velocity for rigid bubbles in still water, $W_{\text{rise}}$. Consequently one can observe that $W_b - W_{f,b} = W_{\text{rise}}$ i.e. on average the velocity difference between the bubble and the surrounding fluid equals the rise velocity of bubbles in still water. This is not surprising since there is no bubble-bubble and bubble-fluid interaction. From the streamwise velocity profiles it can be observed that $U_b$ equals $U_{f,b}$ so there is no velocity difference between the bubbles and the surrounding fluid in the streamwise direction. On the other hand, the velocity of fluid regions around the bubbles do differ from the mean (Eulerian) fluid velocity. Bubbles are located in fluid regions that are moving downward, so in the top of the channel this implies that these fluid regions have a lower streamwise velocity, while in the bottom half of the channel these fluid regions have a higher streamwise velocity.
Figure 4. Visualisation of bubble positions in the slice $348 < z^+ < 350$ for $Re_\tau = 360$, $a^+ = 0.2$, $Rouse = 0.68$ and $St = 1.3 \times 10^{-3}$.

To summarise, it is observed that a steady bubble concentration profile is obtained if the buoyant flux of rising bubbles is counteracted by a preferential location of the bubbles in downward moving fluid. This preferential location or particle clustering is clearly seen in a visualisation of bubbles located in the slice $348 < z^+ < 350$ as shown in figure 4. The bubbles are preferentially located in the streamwise elongated regions of low speed fluid that are ejected away from the wall i.e. the ejection or $Q_2$ events. This clustering is not related to inertia effects since $St = 1.3 \times 10^{-3} \ll 1$, but is a result from the gradient in the bubble concentration profile and the turbulent fluctuations as indicated in figure 5.

Figure 5. Schematic drawing of preferential bubble concentration as a result of turbulence and a concentration gradient.

5. Experiments with electrolysis bubbles in fully developed channel flow

A fully developed channel flow is created in the open water channel of the laboratory for Aero & Hydrodynamics in order to study bubble dispersion at low Reynolds numbers. Transition to turbulence is stimulated by a metal screen and velocity profiles are measured at several downstream locations. Fully developed turbulence has developed about 40 channel heights downstream of the trip and bubbles are generated by electrolysis at $47H$ by circa 80, equally spaced platinum wires. With the aid of a splitter cube, bubble shadowgraphy and PIV
measurements are performed simultaneously at $75H$ (figure 6).

Shadowgraphy is performed and optimised to detect bubbles in a size range of 30 to 100 $\mu$m (figure 7). Basically, the bubbles move in between a light source and a camera and the shadows of the bubbles are recorded. The bubble diameter can be obtained by using a threshold on the light intensity distribution. A second threshold for the intensity gradient makes sure that only in-focus bubbles are detected and prevents the common bias that large bubbles are more likely to be detected than small ones (Bongiovanni et al. (1997)). The image processing for bubble detection and validation requires that bubble image diameters consist of at least 5 pixels, so a high magnification ($M \sim 1$) is used for accurate detection of bubbles as small as 30 $\mu$m. A high intensity Xenon flashlight generates 15 $\mu$s pulses of incoherent light to prevent motion blur and contrast loss by camera ‘smear’ (PCO (2011)).

The low Reynolds number flow is fully resolved by PIV since the 32x32 pixel interrogation windows with 50% overlap give a data spacing of 126 $\mu$m or $\delta z^+ = 1.4$ at $Re_\tau \sim 500$. Fluorescent tracer particles are used to minimise possible biases from reflections of laser light on the bubbles. The bubble shadows are recorded 40 $\mu$s before the first laser pulse of the PIV measurements so no optical filtering or image post processing is required to distinguish PIV tracer particles from bubbles. Even at the highest velocity used (0.3 m/s) bubbles move only one pixel during this time interval. Frames from the PIV and shadowgraphy cameras are not perfectly overlapping, but have slightly different magnification and are rotated and transposed relative to each other. This is corrected for by calculating a displacement vector field from a correlation of two calibration images. From this vector field a linear transformation matrix is derived that maps the bubble locations on the PIV images.

**Figure 6.** Schematic overview of a horizontal channel flow setup in an open water channel and the setup used for simultaneous PIV and shadowgraphy measurements.
The concentration profiles of the electrolysis bubbles are measured $28H$ downstream of the electrolysis wires and plotted in figure 7 for $Re_T = 520$. Since bubbles with different sizes are generated and recorded simultaneously it is possible to obtain profiles for different bubble size regimes. The smallest size range ($30 < D_{\text{bubble}} < 50 \mu m$) is dispersion dominated ($0.1 < \text{Rouse} < 0.3$) and these bubbles are homogeneously spread over the channel height. Bubbles of $100 \mu m$ on the other hand experience a much larger buoyancy force and have a Rouse number of 1.3. As in the DNS the concentration in the central part of the channel can be described by an exponentially decaying Rouse profile (equation 9). Quantitatively the profiles do not agree since the fit on the experimental profile gives $\alpha = K_b/\nu_T = 3.3$ instead of 1. In the DNS the initialy homogeneous bubble concentration profile becomes fully developed only after the bubbles are convected about $100H$. The experimental profiles are measured $28H$ downstream of the electrolysis wires and can therefore not be expected to be in equilibrium. More measurements further downstream of bubble generation and with better optical access over the whole channel are scheduled to further verify if a Rouse approach is suitable to describe bubble concentration profiles in turbulent channel flow.

The simultaneous PIV and shadowgraphy measurements enable us to determine the streamwise and wall-normal velocity of the fluid surrounding the bubbles ($U_{f,b}, W_{f,b}$). For reasons explained later we focus on large scale fluid motion, so $U_{f,b}$ and $W_{f,b}$ are calculated as the mean of a 9x9 velocity vector area surrounding the bubble, with the central 5x5 vectors excluded. These velocities are calculated at several distances from the top wall and are plotted as symbols together with the average fluid velocity $U_f$ and $W_f$ (solid, black lines) in figure 8. The measurements consist of 4 series recorded at different camera heights. Therefore the profiles of the fluid velocity consist of four profiles which are not perfectly overlapping and are not exactly zero as expected. These errors are so small however, that they can already be caused by a camera rotation with respect to the light sheet of 0.1 degree. The plots in figure 8 confirm the finding of the DNS that $W_{f,b} < W_f$, so the bubbles are preferentially located in downward flowing fluid regions. Consistently, we observe again that $U_{f,b} < U_f$ in the top half of the channel and that $U_{f,b} > U_f$ in the bottom half of the channel. In order to check if the fluid around the bubbles on average moves downward with $W_{\text{rise}}$ so that the bubble concentration profile is in...
Figure 8. Streamwise and wall-normal velocity profiles conditioned on the positions of bubbles. The two top graphs correspond to bubbles between 40 and 60 µm while the bottom graphs correspond to 70 to 90 µm bubbles. The solid black lines give the average fluid velocity, $U_f(z)$, while the dashed line in the wall-normal velocity graphs indicates $W_f - W_{\text{rise}}$. The symbols indicate the velocity at the location of the bubbles; $U_{f,b}(z)$ and $W_{f,b}(z)$. Errorbars indicate statistical uncertainty in the bubble conditioned statistics.

Equilibrium, $W_f - W_{\text{rise}}$ is plotted by dashed lines for the minimum and maximum bubble size. The area in between these lines is made gray. Especially for the $40 < d < 60 \mu m$ size bubbles the symbols of $W_{f,b}$ are in the gray area, indicating that the bubbles are located in fluid regions with are moving downward with $W_{\text{rise}}$. For the larger bubble size range ($70 < d < 90 \mu m$), this is only true in the bottom half of the channel while in the top half $W_{f,b}$ is smaller and bubbles are effectively still rising.

In order to check if the turbulent mixing of the bubbles results from small or large scale fluid motion, the conditionally averaged flow field around the bubbles is determined for bubbles in a certain size range. For each bubble in the domain ($0.51H < z < 0.83H$) an 8.1x8.1 mm area around the bubble is selected to obtain the average flow field around a bubble. The result is shown in figure 9 where $x$ and $z$ are now relative to the bubble, but still $x$ is positive in the flow direction and $z$ is upward. It is observed that the downward moving fluid region on average has a size of circa $0.2H$ in both directions, which is of the order of the integral length scale of turbulent channel flow. It is therefore concluded that the large scale fluid motions are responsible for the turbulent mixing of the bubbles. The flow fields show good qualitative agreement with the DNS at $Re_\tau = 360$. Differences are that in the experiment the mean flow has an offset and that the velocity differences are slightly smaller than $W_{\text{rise}}$ as was already observed in figure 8.

Plot D of figure 9 shows that a small area of a few PIV interrogation windows around the bubble has a lower downward velocity relative to its surrounding. This effect might be explained by a measurement bias resulting from insufficient filtering of bubble reflections or from the signal from fluorescent particles on the bubble surface. However, since the 90 µm bubbles are imaged with an 11 pixel diameter, they occupy at maximum one tenth of an interrogations window. With only 50% overlap in the PIV vector spacing it is unlikely that these effects are spread over several interrogation windows. Therefore it is expected that this upward velocity is physical.
Figure 9. The wall-normal velocity around a bubble conditionally averaged over $24 \times 10^3$ bubbles of $40 < d < 60 \, \mu m$ (plot A) and $5 \times 10^3$ bubbles of $70 < d < 90 \, \mu m$ (plot B). The velocity taken at $Re_\tau = 520$ is normalised by $W_{rise}$ of a 50 \, \mu m or a 70 \, \mu m bubble respectively. The data is taken from a measurement series just above the channel centre ($0.51 < y < 0.83 \, H$). Plot C shows $W_{f,b}/W_{rise}$ from the DNS taken from bubbles in the same region ($d^+ = 0.1 \sim d = 56 \, \mu m$ at $Re_\tau=360$). Plot D shows a detail of plot B, with the axes normalised by a bubble diameter of 70 \, \mu m. It shows the local flow modification around the bubbles.

and results from fluid that is dragged along with the bubble. For these bubbles $Re_b < 1$ and it is known from the analytical solution of Stokes flow around rigid spheres that even at 2.5 diameters from the sphere centre the fluid velocity is still 15\% of the spheres rise velocity. A more quantitative comparison with the analytical Stokes flow field requires a higher resolution and even better statistics.

6. Conclusions and Outlook

A numerical simulation of small, low Stokes number bubbles in horizontal turbulent channel flow showed that the bubble concentration profile that results from buoyancy and turbulent dispersion can be described by a gradient diffusion hypothesis. When buoyancy and dispersion are in equilibrium the resulting concentration profile can be described by a Rouse profile which assumes that the bubble dispersion coefficient is about the same as the diffusion coefficient of fluid momentum. In this equilibrium situation bubbles are preferentially concentrated in downward flowing fluid regions, which compensates for the bubble rise velocity. This is not an inertia effect, but simply results from the fact that fluid regions that flow downward come from a region with a higher bubble concentration while upward flowing fluid contain less bubbles.

Simultaneous PIV and bubble shadowgraphy measurements enable the measurement of bubble concentration profiles, conditionally averaged fluid velocities at the bubble locations and conditionally averaged flow fields around the bubbles. This experimental data confirms that bubbles are located in large scale fluid regions that move downward with respect to the
surrounding flow with a velocity of about the bubble rise velocity in still water.

This work will be extended by a channel modification that enables optical measurements over the full channel height further downstream of bubble generation. Secondly bubbles will be created both by electrolysis and by air flow through a porous material in order to generate larger bubbles and study bubble dispersion over a larger range of $Re_r$. Finally a blue, high power LED will be used to perform simultaneous PIV measurements of the fluid in combination with bubble tracking on the shadowgraphs of the bubbles. With this extension the bubble rise velocity can be measured directly and will show if equilibrium bubble concentration are reached.

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