Stability and Robustness Analysis of Plug-Pulling using an Aerial Manipulator

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Abstract—In this paper, an autonomous aerial manipulation task of pulling a plug out of an electric socket is conducted, where maintaining the stability and robustness is challenging due to sudden disappearance of a large interaction force. The abrupt change in the dynamical model before and after the separation of the plug can cause destabilization or mission failure. To accomplish aerial plug-pulling, we employ the concept of hybrid automata to divide the task into three operative modes, i.e., wire-pulling, stabilizing, and free-flight. Also, a strategy for trajectory generation and a design of disturbance-observer-based controllers for each operative mode are presented. Furthermore, the theory of hybrid automata is used to prove the stability and robustness during the mode transition. We validate the proposed trajectory generation and control method by an actual wire-pulling experiment with a multirotor-based aerial manipulator.

I. INTRODUCTION

Aerial manipulation has been a growing research topic which aims to utilize the maneuverability of an aerial vehicle and the versatility of a robotic manipulator. Different from physically non-interacting passive tasks such as surveillance and remote sensing, an aerial manipulator can execute active tasks involving physical interaction such as grasping [1], [2], valve turning [3], drawer opening [4], contact inspection [5], transportation [6], [7], and door opening [8].

Despite various demonstrations of aerial manipulation tasks involving contact with environments, they usually involved a relatively low level of changes in the dynamic characteristics, and they rarely dealt with the transition during the physical interaction explicitly. In fact, there is a lack of research on the stability of aerial manipulation before and after the physical interaction. When a mode switch entails a significant change in the system response, neglecting it can lead to destabilization. Therefore, it is necessary to systematically analyze dynamical modes and design a robust controller to more realistically embrace the whole operation.

As an example of aerial manipulation involving a drastic change in dynamics, this paper deals with the problem of pulling a plug from a socket using a multirotor equipped with a two degree-of-freedom (DOF) robotic arm. In this task, the large force exerted on the end-effector suddenly disappears after the plug is separated from the socket. For formal analysis of the stability, we first formulate hybrid automata [9] which enclose all the dynamic models and the operative modes that have their own control laws different from another. Then, we design disturbance-observer (DOB)-based controllers [10] for the respective operative modes and prove the stability and robustness of the formulated hybrid automata [11].

A. Related Works

There have been several works which explain aerial vehicles using the concept of hybrid automata. In [12], [13], [14], the contact task using an aerial vehicle is divided into two operative modes, i.e., docking and free-flight, and each mode are controlled by a different controller from one another. However, these works did not conduct an analysis on stability and robustness and the transition between those modes was not explicitly discussed.

Some studies addressed the stability and robustness of an aerial vehicle involved in the physical interaction using the hybrid automata theory. In [15], the stability of path-following control considering mode changes was investigated for a robust contact of a ducted-fan aerial vehicle on a vertical surface. In [16], the process of a multirotor landing on the slope was divided into several modes and the stability and robustness was proved in a similar way to [15]. However, in such settings, the effect of dynamic change can be reduced by slowly approaching the wall or landing site. Thus, there is no guarantee that such control methods can maintain the stability and robustness of the aerial manipulation involving an abrupt change such as plug-pulling.

B. Contributions

To the best of the authors’ knowledge, this is the first attempt to conduct a plug-pulling task using an aerial manipulator, which involves a significant mode change, and present a thorough analysis on the stability and robustness of the aerial manipulator using the hybrid automata theory.
We propose a trajectory generation strategy and DOB control structure for each operative mode. Especially, for the situation of pulling the wire, we derive a dynamical model of the aerial manipulator constrained to the wire and the socket. In addition, we construct a DOB structure corresponding to the model of the plug-pulling aerial manipulator and prove the stability and robustness of the proposed controller.

C. Outline

In Section II, we briefly explain the concept of hybrid automata, describe notions utilized throughout the paper and introduce the aerial plug-pulling scenario. Section III formulates hybrid automata for the aerial manipulator conducting the plug task, and the trajectory generation and controller design is described in Section IV. Section V shows the stability and robustness analysis, and Section VI presents the experimental setup and results.

II. PROBLEM SETUP

A. Preliminary: Hybrid Automata

The following elements define hybrid automata [9] with the state variable \( x \in \mathbb{R}^{n_x} \) and the control input \( u_x \in \mathbb{R}^{m_x} \).

- **Set of operative modes**, \( \mathcal{M} \), contains names of the control modes. With respect to \( \mathcal{M} \), we let \( t_{0,\mu} \) and \( t_{d,\mu} \) denote the time when the mode \( \mu \) begins and the desired time to terminate the mode \( \mu \), respectively.

- **Domain mapping**, \( \mathcal{D} : \mathcal{M} \Rightarrow \mathbb{R}^{n_x} \times \mathbb{R}^{m_x} \), means the possible region where \( x \) and \( u_x \) can evolve while maintaining a specific mode \( \mu \). It is expressed as \( \mathcal{D}(\mu) = \mathcal{D}_x(\mu) \times \mathcal{D}_{u_x}(\mu) \).

- **Flow map**, \( f : \mathcal{M} \times \mathbb{R}^{n_x} \times \mathbb{R}^{m_x} \rightarrow \mathbb{R}^{n_x} \), describes the dynamics in each operative mode \( \mu \).

- **Set of edges**, \( \mathcal{E} \subseteq \mathcal{M} \times \mathcal{M} \), means all possible pairs of operative mode changes \((\mu_1, \mu_2)\).

- **Guard mapping**, \( \mathcal{G} : \mathcal{E} \Rightarrow \mathbb{R}^{n_x} \times \mathbb{R}^{m_x} \), describes the conditions where the transition from \( \mu_1 \) to \( \mu_2 \) occurs. It is represented as \( \mathcal{G}(\{\mu_1, \mu_2\}) \).

- **Reset Map**, \( \mathcal{R} : \mathcal{E} \times \mathbb{R}^{n_x} \times \mathbb{R}^{m_x} \rightarrow \mathbb{R}^{n_x} \), means the jump of the state variable \( x \). It is expressed as \( \mathcal{R}(\{\mu_1, \mu_2\}, (x, u_x)) \).

B. Notations

In this work, we use \( 0_{ij}, I_j \) and \( e_3 \) to denote the \( i \times j \) zero matrix, the \( i \times i \) identity matrix and \([0 \ 0 \ 1]^\top\). Also, we define \( a_{i,j}, [a]_j, \dim(a), A_{i,j}, A_{i,j,k,l} \) and \( B_{\sigma}(a) \) as the \( i \)-th element of a column vector \( a \), the \( \text{so}(3) \) operator representing the cross product \([a]b = a \times b\), the dimension of \( a \), the \((i,j)\)-th element of a matrix \( A \), the block matrix of \( A \) containing from \((i,k)\)-th to \((j,l)\)-th elements and the set \( \{c \in \mathbb{R}^{\dim(a)} | ||c-a|| \leq \sigma\} \) where \( \sigma \) is a constant positive number. The Kronecker product is expressed as \( \otimes \).

As in Fig. 2, we denote the frame of the inertial coordinate, multirotor, 1st, 2nd servo motor and the end-effector by \( I \), \( B \), \( \{i\} \), \( \{j\} \), \( \{E\} \) respectively.

To express the state of an aerial manipulator, we define the generalized coordinate \( q \) as \([p_{IB} \quad \eta^\top \quad \gamma^\top]^\top\) consisting of the position of the multirotor \( p_{IB} \), the body frame of the end-effector, and \( \eta \) and \( \gamma \), the Euler angles. Specifically, we let \( \chi, r, x_q \) and \( x_r \) denote \([p_{IB} \quad \eta^\top \quad \gamma^\top]^\top, [\eta^\top \quad \gamma^\top]^\top, [q^\top \quad r^\top]^\top \) and \([q^\top \quad r^\top]^\top \). To represent inputs, we use \( u_f \) and \( u \) to denote \([T \quad \tau \quad \tau_b]^\top \) and \([T \quad \tau_b \quad \tau_r]^\top \) where \( T, \tau_b \) and \( \tau_r \) mean a total thrust, moments with respect to \( i_B, j_B \) and \( k_B \) and torque inputs exerted on the servo motors. We let \( Q = \mathbb{R}^{3 \times 3} \) denote a matrix which satisfies \( \omega_{1B}^\top = Q \tilde{\eta} \) and the scalar \( g \) the gravitational acceleration. Moreover, we set \( a_{d, \tilde{a}}, \dot{\tilde{a}} \) and \( \tilde{a} \) as a desired trajectory, an estimate of \( a \) and the nominal value of \( A \).

We use \( m_B, m_1 \) and \( m_2 \) to denote mass of the multirotor, the 1st and the 2nd servo motor while diagonal matrices \( J_0, J_1, J_2 \) in \( \mathbb{R}^{3 \times 3} \) are the moments of inertia of the corresponding components. Additionally, \( J_E \in \mathbb{R}^{3 \times 3} \) means the moment of inertia of the end-effector.

C. Scenario

As in Fig. 2, the aerial manipulator tries to unplug in \(-\dot{i}_1\) direction from the socket installed on the wall aligned with \( j_1-k_1 \) plane. After the plug is separated from the socket, the vehicle quickly stabilizes its attitude in a short time and maintains the hovering state.

III. HYBRID AUTOMATA OF AERIAL PLUG-PULLING

We construct the elements of the hybrid automata listed in Section II-A for the aerial manipulator pulling the plug.

A. Set of Operative Modes, \( \mathcal{M} = \{WP, ST, FF\} \)

In WP (wire-pulling) mode, the aerial manipulator tries to unplug by pulling the wire in \(-\dot{i}_1\) direction. In ST (stabilizing) mode, the vehicle quickly stabilizes its attitude immediately after the separation of the plug. In FF (free-flight) mode, the aerial manipulator returns to the original location and keeps the hovering state.

B. Domain Mappings, \( \mathcal{D}(WP) \), \( \mathcal{D}(ST) \) and \( \mathcal{D}(FF) \)

- \( \mathcal{D}(WP) \triangleq \{(x_r, u_f) \in \mathbb{R}^{10} \times \mathbb{R}^6 | F_{E,1} < F_{TH}\} \) where \( F_E \) is the interaction force acting on the end-effector due to the friction between the plug and the socket and \( F_{TH} \) is the force limit up to which the plug can resist from separating.

- \( \mathcal{D}(FF \text{ or } ST) \triangleq \mathcal{V}_F - \mathcal{D}(WP) \) where \( \mathcal{V}_F \) means an \( \mathbb{R}^{16} \times \mathbb{R}^6 \) space representing the flight envelope, i.e.,
all possible regions of the state and inputs for the flight experiment.

C. Flow Maps, \( f(WP, x_r, u_f) \) and \( f(ST \) or FF, \( x_\eta, u_f) \)

1) WP mode: The derivation of \( f(WP, x_r, u_f) \) is based on [1], but since the position of the end-effector is fixed, we newly derive it in the form of the Euler-Lagrange equation with \( r = [\eta^T \gamma]^T \) which fully describes the dynamics of the wire-pulling aerial manipulator.

First we express position and angular velocity as

\[
\begin{align*}
p_{IB} &= p_{IE} - R_{IBPBE}, \quad p_{II} = p_{IE} + R_{IBPBE}, \\
p_{I2} &= p_{IE} + R_{IBPBE} - R_{IBPBE}, \\
\omega_{IB}^R &= \dot{\theta}, \quad \omega_{II}^R = R_{IBPBE}^R \omega_{IB}^R, \\
\omega_{I2}^R &= R_{IB}^R \omega_{IB}^R.
\end{align*}
\]

with the kinematic constraint \( \dot{p}_{IE} = 0 \). By substituting \( \dot{q} \) into the derivation process presented in [1], an Euler-Lagrange equation of the wire-pulling aerial manipulator model can be derived. Then, the obtained equation of motion can be analyzed in the form of flow map as follows.

\[
\dot{v} = f(WP, x_r, u_f) = M_{\eta}^{-1}(-C_{\eta} - K_{\eta} + J_{ur}^T u_f + \tau_{\eta, r})
\]

where \( M_{\eta} \in \mathbb{R}^{5 \times 5}, \ C_{\eta} \in \mathbb{R}^{5}, \ K_{\eta} \in \mathbb{R}^{5}, \)

\[
J_{ur} \triangleq \frac{\partial \ddot{p}_{IB}^R \omega_{IB}^R \gamma^T}{\partial \gamma} \in \mathbb{R}^{6 \times 5},
\]

and \( \tau_{\eta, r} \) means the external disturbance applied to the wire-pulling aerial manipulator system. Thus, the variable \( x_r \) evolves in correspondence with \( \ddot{q} \).

2) ST and FF modes: The Euler-Lagrange equation for the multirotor equipped with the 2 DOF robotic arm is derived in [1]. Then, the flow map \( f(ST \) or FF, \( x_\eta, u_q) \) can be derived as follows.

\[
\dot{q} = f(ST \) or FF, \( x_\eta, u_q) = M_{\eta}^{-1}(-C_\eta - K_\eta + J_{uq}^T u_f + \tau_{\eta, q})
\]

where \( M_q \in \mathbb{R}^{8 \times 8}, \ C_q \in \mathbb{R}^{8}, \ K_q \in \mathbb{R}^{8}, \ J_{uq} \in \mathbb{R}^{6 \times 8}, \) and \( \tau_{\eta, q} \) means the external disturbance applied to the system in free flight.

D. Set of Edges, \( E = \{(WP, ST), (ST, FF)\} \)

There would be only two possible edges in our scenario because the transition from ST or FF mode to WP mode does not occur unless the plug is attached to the socket again. Also, a change from FF to ST mode is impossible because the ST mode is primarily designed as an intermediate stage between the WP and FF modes.

E. Guard Mappings, \( G\{(WP, ST)\} \) and \( G\{(ST, FF)\} \)

A transition from WP mode to ST mode occurs when the \( \dot{t} \) element of \( F_E \) exceeds \( F_{T_2} \). Therefore, the guard map \( G\{(WP, ST)\} \) is defined as \( \{(x_r, u_f) \in D(WP) \mid F_{E,1} = F_{T_2}\} \). Also, since \( (\phi_d, \theta_d) \) in the ST mode are user-defined values while they are computed from the user-defined values of \( (x_d, \eta_d) \) in the FF mode, an undesirable abrupt change in \( (\phi_d, \theta_d) \) would provoke a failure in attitude control. Therefore, the guard map \( G\{(ST, FF)\} \) is defined as \( \{(x_\eta, u_f) \in D(ST) \mid ||\eta|| < \eta, t_\eta,ST \leq t\} \) where \( \eta \) is defined as the threshold of \( \eta \) for the mode change.

F. Reset maps, \( R\{(WP, ST), (x_r, u_f)\} \) and \( R\{(ST, FF), (x_\eta, u_f)\} \)

If the operative mode changes from WP to ST, there would be jumps in \( x_\eta \) due to the sudden disappearance of the force exerted on the end-effector. However, since we cannot know the exact magnitude of the jumps in \( x_\eta \), we denote by \( x_\eta \) which satisfies \( \dot{p}_{IE}(x_\eta) \neq 0 \). Then, the reset map \( R\{(WP, ST), (x_\eta, u_f)\} \) is expressed as \( \{x_\eta \in D_{x_\eta}(ST) \mid \dot{p}_{IE}(x_\eta) \neq 0\} \) where \( x_\eta \in D_{x_\eta}(WP) \). On the other hand, the change from ST to FF does not entail any jump in \( x_\eta \) because they evolve under the same dynamics. Therefore, the reset map \( R\{(ST, FF), (x_\eta, u_f)\} \) can be derived as \( \{x_\eta \in D_{x_\eta}(FF) \mid x_\eta = x_\eta \ where \ x_\eta \in D_{x_\eta}(ST)\} \).

IV. Trajectory Generation and Controller Design

A. Trajectory Generation

It is assumed that \( \gamma \) and \( \dot{\gamma} \) exactly follow \( \gamma_d \) and \( \dot{\gamma}_d \) respectively and the desired values that are not defined at each mode are set to be the same as the current values.

1) WP mode: In this mode, the aerial manipulator tries to tilt its body with respect to \( -y_p \) in order to exercise a pulling force to the socket. Therefore, \( \eta_d(t) \) is given as below,

\[
\eta_d(t) = \begin{cases} 
0 & \text{if } t_\eta,WP < t < t_{d,WP} \\
\theta_m(t_{d,WP} - t_{0,WP}) & \text{if } t_{0,WP} \leq t < t_{d,WP} \\
0 & \text{if } t_{d,WP} \leq t
\end{cases}
\]

where \( \theta_m \) means the maximum absolute value of the pitch angle. It prevents a sudden transition to ST mode by gradually tilting the vehicle’s body.

2) ST mode: This mode is proposed for compensating the overshoot invoked by the transition of the dynamical model and avoiding an abrupt change in \( \phi_d(t) \) and \( \theta_d(t) \). In order to simultaneously minimize the overshoot and make \( \phi \) and \( \theta \) close to zero, the time interval \( [t_{0,ST}, t_{d,ST}] \) needs to be reasonably small. Therefore, \( p_{z,ST}(t) \) and \( \eta_d(t) \) are set as

\[
p_{z,ST}(t) = p_{z}(t_{0,ST})
\]

\[
\eta_d(t) = \begin{cases} 
c_2 t^2 + c_1 t + c_0 & \text{if } t_{0,ST} \leq t < t_{d,ST} \\
0 & \text{if } t_{d,ST} \leq t
\end{cases}
\]

where coefficients \( c_0, c_1 \) and \( c_2 \) satisfy the conditions \( \eta_d(t_{0,ST}) = \eta(t_{0,ST}), \eta_d(t_{d,ST}) = \eta(t_{d,ST}) \) and \( \eta_d(t_{d,ST}) = 0 \).

3) FF mode: In FF mode, \( p_{IB} \) and \( \psi \) are set to fly back to the original position as follows:

\[
p_{IB}(t) = p_{IB}(t_{0,WP}), \ psi(t) = \psi(t_{0,WP})
\]

B. Nominal Model for Each Mode

Servo motors are usually controlled by the given desired position, not torque. Therefore, the equations of motion that eliminate the term \( \tau_r \) are derived for each model.
the solution of (9) adequately follows

\[ \eta = \eta^1 + \eta^2 + \Delta_\eta \]

Fig. 3: The DOB structure of WP mode for compensating the model
discrepancy \( \Delta_\eta \) between (8) and (9).

1) WP mode: In (2), \( J_{ur} \) is computed as

\[
J_{ur}^T = \begin{bmatrix} J_{r, \gamma}^T & 0_{23} \end{bmatrix}, \quad J_{r, \gamma} \in \mathbb{R}^{1 \times 3}, \quad J_{T_r} \in \mathbb{R}^{1 \times 2}.
\]

(7)

Therefore, the model with respect to \( \eta \) is obtained with

\[
\dot{\eta} = F_\eta + G_\eta \tau_b
\]

(8)

where

\[
F_\eta = M_\eta^{-1}\{-C_{\eta} - K_{\eta} - M_{\eta}\gamma \hat{\gamma} - J_{T_{r, \gamma}}^T T - \tau_{e, r}\}
\]

\[
G_\eta = M_\eta^{-1} Q^T
\]

with block matrices \( M_\eta = M_{r, 1, 1} M_{\gamma, 1, 1} \), \( C_{\eta} = C_{r, 1, 3} \), and \( G_{\eta} = G_{r, 1, 1} \). Then based on this, the nominal model for the WP mode can be obtained as follows.

\[
\dot{\eta} = \hat{F}_\eta + \hat{G}_\eta \tau_b
\]

(9)

where

\[
\hat{F}_\eta = \hat{M}_\eta^{-1}\{-C_{\eta} - \tilde{K}_{\eta} - \hat{M}_{\eta}\gamma \hat{\gamma} - \hat{J}_{T_{r, \gamma}}^T T\}
\]

\[
\hat{G}_\eta = \hat{J}_b^{-1} Q^T
\]

(10)

with the nominal input \( u_0 = [T \; \tau_{b0}]^T \). The total thrust \( T \) is calculated in the DOB controller introduced in [17].

2) ST and FF mode: The nominal model for ST and FF mode is derived in [17] as below.

\[
\ddot{q}_u = \ddot{G}_u \Phi_0, \quad \ddot{q}_f = \ddot{F}_f + \ddot{G}_f \dot{u}_0
\]

(11)

where \( q_u \) and \( q_f \) mean the center of mass of \([p_x \; p_y]^T\) and \([q_x \; q_y]^T\) respectively. The other notations are defined in [17].

C. Controller Design

1) WP mode: If we design the nominal input \( \tau_{b0} \) to make the solution of (9) adequately follows \( \eta_0 \), the compensation of model discrepancy, \( \Delta_\eta = (F_\eta - \hat{F}_\eta) + (G_\eta \tau_b - \hat{G}_\eta \tau_{b0}) \), is conducted by the DOB structure presented in [10]. The overall diagram is shown in Fig. 3 and the detailed DOB control law is formulated as below:

\[
\dot{q}_i^n = A_{\eta,i} q^n_i + B_{\eta,i} \eta_i, \quad \dot{p}_i^n = A_{\eta,i} p^n_i + B_{\eta,i} \tau_{b2,i}
\]

\[
u_i^n = p^n_{i,1} - \sum_{j=1}^{3} A_{\eta,i,j} (q^n_{j,2} - F_{\eta,j}), \quad \tau_{b1} = \Delta_\eta \hat{G}_\eta \tau_{b0}
\]

\[
u_{b2} = \Delta_\eta \hat{G}_\eta \tau_{b0} + \Pi_{\eta}(u^n), \quad \tau_b = \tau_{b0} + (\Delta_\eta \hat{G}_\eta)^{-1} \Pi_{\eta}(u^n)
\]

(12)

where \( q^n_i = [q^n_{i,1} \; q^n_{i,2} \; q^n_{i,3}]^T \), \( u^n = [u^n_{i,1} \; u^n_{i,2} \; u^n_{i,3}]^T \), and

\[
A_i^n = \begin{bmatrix} 0 & -a^n_{i,1} / c^n_\eta & -a^n_{i,2} / c^n_\eta \\ -a^n_{i,1} / c^n_\eta & 0 & -a^n_{i,3} / c^n_\eta \\ -a^n_{i,2} / c^n_\eta & -a^n_{i,3} / c^n_\eta & 0 \end{bmatrix}, \quad B_i^n = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
\]

(13)

\[
\Lambda_\eta = \hat{J}^{(1/2)}_b Q \in \mathbb{R}^{3 \times 3}
\]

with the positive constants \( a^n_{i,0}, \; a^n_{i,1} \) and the small positive constant \( \epsilon_\eta \). In this DOB structure, we use a saturation function \( \Pi_{\eta} \) defined with the conditions below.

- \( \Pi_{\eta} : \mathbb{R}^3 \rightarrow \mathbb{R}^3 \) is a globally bounded \( C_1 \) function.
- \( \Pi_{\eta}(u^n) = u^n \) for \( u^n \in S_{\Pi_{\eta}} \) where \( S_{\Pi_{\eta}} \defeq \{ u^n \in \mathbb{R}^{3 \times 1} \mid u^n = \Lambda_\eta \hat{G}_\eta \gamma^{-1} (F_{\eta} - \hat{F}_\eta + (\hat{G}_\eta - G_\eta) \tau_{b0} - \Delta_\eta) \} \).
- \( \|\partial \Pi_{\eta}(u^n) / \partial u^n\| \leq 1 \) for \( u^n \in \mathbb{R}^{3 \times 1} \). From the conditions above, the quasi-steady state range of \( u^n \) satisfies the equation \( u^n = p^n_{i,1} - \sum_{j=1}^{3} A_{\eta,i,j} (q^n_{j,2} - F_{\eta,j}) \) while avoiding the saturation [10].

3) FF mode: We will utilize the same controller presented in [17] for the FF mode.

V. STABILITY AND ROBUSTNESS ANALYSIS

In this section, an analysis of the stability and robustness will be presented. During the analysis, the term \( \text{maneuver} \) which means \([x_q \; u_f]^T\) in a particular mode will be used. \( v_0 \) and \( v_0^N \) mean the solution from the actual flow map and the nominal flow map. Additionally, \( \text{trv}_\mu \) is defined as a set including all values of \( x_q \) in the given time interval.

A. Preliminary Definitions for the Analysis on Hybrid Automata

Definition 1 (\( \sigma \)-robust \( \mu_1 \)-single maneuver in \([t_0, \mu_1, t_1] \)). For \( \sigma > 0, \mu_1 \in M \) and \( 0 < t_1 \), a maneuver \( v_{\mu_1}^N \in [t_0, \mu_1, t_1] \mu_1 \) satisfies

\[
\text{tr} \left( v_{\mu_1}^N \cap \left( \bigcup_{(\mu_1, \mu')} \mathcal{G}(\{\mu_1, \mu'\}) + B_\sigma \right) \right) = \emptyset.
\]

(13)

Here, \([t_0, \mu_1, t_1] \mu_1 \) means the time interval where \( v_{\mu_1}^N \) evolves in the mode \( \mu \) within \([t_0, t_1] \).

Definition 2 (\( \sigma \)-robust \( \mu_1 \rightarrow \mu_2 \) approach maneuver in \([t_0, \mu_1, t_1] \)). For \( \sigma > 0, \mu_1, \mu_2 \in M \) and \( t_0, \mu_1 \leq t_1 \), a maneuver \( v_{\mu_1}^N \in [t_0, \mu_1, T \mu_1] \) satisfies

\[
\text{tr} \left( v_{\mu_1}^N \cap \left( \bigcup_{(\mu_1, \mu')} \mathcal{G}(\{\mu_1, \mu'\}) + B_\sigma \right) \right) = \emptyset
\]

and \( B_\sigma (v_{\mu_1}^N) \subseteq \mathcal{G}(\{\mu_1, \mu_2\}) \).

Let \( S^{\mu_1} \) and \( S^{\mu_1} \rightarrow \mu_2 \) be compact sets defined as \( S^{\mu_1} \defeq \{\text{trv}_{\mu_1}^N + B_\sigma \} \cap \mathcal{G}(\{\mu_1, \mu_2\}) \) and

\[
S^{\mu_1} \rightarrow \mu_2 \defeq \{X_q^+ \in D_{\gamma}(\mu_2) \mid x_q^+ \in \mathcal{R}(\{\mu_1, \mu_2\}, (x_q, u_f)) \text{ for some } (x_q, u_f) \in S^{\mu_1} \}
\]

(14)

Then, for any \( x_q^+ \in S^{\mu_1} \rightarrow \mu_2 \) there exists \( u_f^+ \) such that

\[
(x_q^+, u_f^+) \notin \bigcup_{\{\mu_2, \mu_2'\} \in \mathcal{E}} \mathcal{G}(\{\mu_2, \mu_2'\}) + B_\sigma.
\]

(15)
Definition 3 \((\sigma, \delta_\sigma)\)-robust \(S^{i_1 \rightarrow i_2}\) coverage set, \(C^{i_1 \rightarrow i_2}\). A set of \(N_{\delta_\sigma}\) elements \(x_{q,1}, \ldots, x_{q,N_{\delta_\sigma}} \in D_{x_q}(\mu_2)\) which satisfies
\[
S^{i_1 \rightarrow i_2} \subset \bigcup_{j=0}^{N_{\delta_\sigma}} B_{\delta_\sigma}(x_{q,j}).
\]

Definition 4 \((\sigma, \delta_\sigma)\)-robust \(\mu_1 \rightarrow \mu_2\) transition maneuver. For \(\sigma > 0, \delta_\sigma > 0, \mu_1, \mu_2 \in \mathcal{M}\) and \(t_0, \mu_1 < t_1\), \(\bar{v}_N^{\mu_1} \) which is a union of a \(\sigma\)-robust \(\mu_1 \rightarrow \mu_2\) approach maneuver before the switching time \(T \in (t_0, \mu_1, t_1)\) and of a set of \(N_{\delta_\sigma}\) \(\sigma\)-robust single maneuvers after transition with the property \(\bar{v}_{\mu_2,j} \in C^{\mu_2 \rightarrow i_2}, \forall j \in \{1, \ldots, N_{\delta_\sigma}\}\).

B. Robustness of the Nominal Maneuver

With the assumption that \(x_d\) of the nominal maneuver \(\bar{v}_N^{\mu}\) adequately follows \(x_{d,\text{d}}\) defined through (4)–(6), we analyze the characteristics of the nominal maneuver.

1) \((\sigma, \delta_\sigma)\)-robust \(WP \rightarrow ST\) transition maneuver: With the assumption that the aerial manipulator is in a quasi-equilibrium state while perching on the wall, the pulling force \(F_{E,1}\) is equal to \(-T_i \sin \theta\) [4]. Thus, \(F_{E,1}\) increases with the gradually decreasing trajectory of \(\theta\) as generated in (4).

As a result, \(\bar{v}_N^{WP}\) does not reach \(G\{(WP, ST)\}\) + \(B_{\delta_\sigma}\) before \(F_{E,1}\) closely approaches \(F_{TH}\). Moreover, as depicted in Fig. 4a, there is no possibility of a transition from \(WP\) to \(FF\) mode because \(G\{(WP, FF)\}\) is defined as \(\emptyset\). Therefore, the maneuver \(\bar{v}_N^{WP}\) is proved to be a \(\sigma\)-robust \(WP\)-single maneuver in \(t_0, WP, t_1\).

\(\bar{v}_N^{WP}\) can also become a \(\sigma\)-robust \(WP \rightarrow ST\) approach maneuver when there exists a time instant that a change from \(WP\) to \(ST\) occurs in a finite time. Thanks to the relation that \(F_{E,1}\) equals to \(-T_i \sin \theta\), we can easily find a sufficient condition \(0 < F_{TH} < T_i \sin \theta_{max}\) where \(T_i\) means the maximum value of \(T\). The inequality above infers that \(F_{E,1}\) can reach \(F_{TH}\) with the given trajectory of \(\theta\). Additionally, since the direct change from \(WP\) to \(FF\) mode never occurs as mentioned above, the claim that \(\bar{v}_N^{WP}\) is a robust approach maneuver is proved.

After the transition from \(WP\) to \(ST\) mode, the reset map of \(\bar{v}_N^{WP}\) is uncertain. However, we can avoid the situation where the value of \(R\{(WP, ST)\}, (x_s(t_0, ST), u_f(t_0, ST))\) becomes an element of \(D\{(FF, ST)\}\) from \(ST\) to \(FF\) does not occur before the time reaches the value of \(t_{d,ST}\) by the condition \(t_{d,ST} < t\) in \(G\{(ST, FF)\}\). Therefore, a maneuver \(\bar{v}_N^{WP}\) turns out to be a \(\sigma\)-robust \(ST\)-single maneuver in \(t_0, WP, t_0, ST\) after the mode transition. From the analyses above, the union of \(\bar{v}_N^{WP}\) and \(\bar{v}_N^{\mu}\) is proved to be a \(\sigma\)-robust \(WP \rightarrow ST\) transition maneuver.

2) \((\sigma, \delta_\sigma)\)-robust \(ST \rightarrow FF\) transition maneuver: As proved in the previous section, the maneuver \(\bar{v}_N^{ST}\) is a \(\sigma\)-robust \(ST\)-single maneuver in \(t_0, ST, t_0, FF\). Also, there must be a change from \(ST\) to \(FF\) mode in a finite time since \(\eta_d\) is defined in \(ST\) mode reaches \(\eta_{d}\) when \(t\) reaches \(t_{d,ST}\). Also, since the transition from \(ST\) to \(WP\) mode is impossible, \(\bar{v}_N^{ST}\) also turns out to be a \(\sigma\)-robust \(ST \rightarrow FF\) approach maneuver in \(t_0, ST, t_0, FF\).

Accordingly, the property that \((FF, ST)\) and \((FF, WP)\) are not the elements of \(E\) also guarantees that a maneuver
For a given $\sigma > 0$, there exists $\epsilon_0^\ast$ such that, for each $0 < \epsilon_1 < \epsilon_0^\ast$, the solution of \( \dot{y}^N + \dot{y}^N(t) \tau_{60}(t)^T \), and that of \( \dot{y}^N(t) \tau_{60}(t)^T \), initiated at \( y(0)^T \tau_{60}(0)^T \), \( \dot{y}^N(0)^T \tau_{60}(0)^T \) \( \in S_{\eta} \times S_{\eta}^\ast \), satisfies
\[
||y(t)^T \dot{y}(t) \tau_{60}(t)^T \| \leq \sigma, \quad \forall t \geq 0
\]
if we set the initial conditions for both dynamic models identical, \( \dot{y}^N(0)^T \dot{y}(0) \tau_{60}(0)^T \) \( = (y(0)^T \dot{y}(0) \tau_{60}(0)^T) \). For a given \( \alpha \), the closed-loop system of the actual model describing wire-pulling mode can be rearranged in the standard singular perturbation form as follows.

\[
\dot{\eta} = F_\eta + G_\eta (\tau_{60} + (\Lambda_\eta \tilde{G}_\eta)^{-1}) \Pi (u^\eta) + \Delta_\eta
\]

where
\[
\Delta_\eta = \begin{bmatrix} \epsilon_{\eta,1} & \epsilon_{\eta,2} \end{bmatrix}, \quad \epsilon_{\eta,1} = \epsilon_{\eta,1} - \epsilon_{\eta,2}, \quad \epsilon_{\eta,2} = \epsilon_{\eta,2} - \epsilon_{\eta,1}
\]

and
\[
\alpha = \frac{1}{\epsilon_{\eta,1} + \epsilon_{\eta,2}} = \frac{1}{\epsilon_{\eta,1} - \epsilon_{\eta,2}}
\]

and
\[
A_{\eta,1} = \begin{bmatrix} -a_{\eta,1} & 1 \end{bmatrix}, \quad A_{\eta,2} = \begin{bmatrix} 0 & 1 \end{bmatrix} - \begin{bmatrix} a_{\eta,0} & -a_{\eta,1} \end{bmatrix}
\]

The next step is to show that the only solution for \( u^\eta \) is \( u^\eta + \Delta_\eta \). Thus, we declare a function \( \Gamma_\eta(\delta) \) which replaces \( u^\eta \) in \( u^\eta + \Delta_\eta \) as below.

\[
\Gamma_\eta(\delta) \equiv u^\eta + \delta - \Lambda_\eta (\tilde{F}_\eta - F_\eta) + (\tilde{G}_\eta - G_\eta) \tau_{60} - \Delta_\eta
\]

where
\[
\delta = (\Lambda_\eta \tilde{G}_\eta \Lambda^{-1}) \Pi (u^\eta + \delta)
\]

Lemma 2. With the assumption that \( \eta \in U_\eta \), \( \Gamma_\eta(\delta) \) belongs to the sector \( [1 - \kappa, 1 + \kappa] \) with \( 0 < \kappa < 1 \). Then accordingly, \( \delta = 0 \) is the unique solution of \( \Gamma_\eta(\delta) = 0 \).

To prove that the fast dynamic system in Lemma 1 is exponentially stable, we derive 1st order differential equations for \( \xi^\eta \equiv \xi_1^\eta - \xi_2^\eta + \xi_3^\eta + \xi_4^\eta \) as

\[
\dot{\xi}_1^\eta = A_{\xi,1} \xi_1^\eta + B_{1,0} \xi_2^\eta + B_{1,1} \xi_3^\eta
\]

where
\[
B_{1,0} = \begin{bmatrix} 1 & 0 \\ 0 & -a_{\eta,0} \end{bmatrix}, \quad A_{\xi,1} = \begin{bmatrix} 0 & 1 \end{bmatrix} + \begin{bmatrix} -a_{\eta,1} & 1 \end{bmatrix}
\]

Remark 3. Since \( \Pi (u^\eta) \in S_{\eta} \), \( \Pi (u^\eta) \equiv u^\eta \) equals \( u^\eta \). Therefore, an explicit expression for \( u^\eta \) is obtained as follows.

\[
u^\eta = \Lambda_\eta \tilde{G}_\eta \Lambda^{-1}(\tilde{F}_\eta - F_\eta) + (\tilde{G}_\eta - G_\eta) \tau_{60} - \Delta_\eta
\]
above. Therefore, $A_{\eta}G_{\eta}G_{\eta}^{-1}A_{\eta}^{-1}$ is an invertible matrix. Accordingly, since $G_{\eta}$ is not singular, $\|G_{\eta}^{-1}\|$ is bounded. Moreover, since $\|G_{\eta}^{-1}\|\leq\|G_{\eta}^{-1}\|\|G_{\eta}\|\|G_{\eta}^{-1}\|$ holds and $(\hat{G}_{\eta}^{-1}, \hat{G}_{\eta}^{-1})$ are bounded in $\mathcal{U}$, based on the relationship $G_{\eta} = J_{\eta}Q^{T}, \|G_{\eta}\|$ is also bounded.

Remark 4. Since the initial condition in Theorem 1 is $[\hat{\eta}^{N}(0)^{T} \hat{\eta}^{N}(0)^{T} \tau_{60}(0)^{T}]^{T} = [\eta(0)^{T} \eta(0)^{T} \tau_{60}(0)^{T}]^{T}$ and $\eta$ is bounded due to Remark 2 and the definition of $\Pi_{\eta}$, there exists $T_{1} > 0$ such that $[\eta(t)^{T} \tau_{60}(t)^{T}]^{T}$ remains in $\mathcal{U}$ for $t \in [0, T_{1})$. Also, there exists $T_{2} > 0$ such that $[\hat{\eta}^{N}(0)^{T} \hat{\eta}^{N}(0)^{T} \tau_{60}(0)^{T}]^{T} \leq \sigma/2$ for $t \in [0, T_{2}]$.

Lemma 3. For $t_{f} = \min\{T_{1}, T_{2}\}$, there exists $\epsilon_{\eta}^{n}$ such that the solutions of (24) initiated from any $\xi(0)$ and $\xi(0)$ satisfies

$$[\hat{\xi}^{N}(t)^{T} \hat{\xi}^{N}(t)^{T}]^{T} \leq \lambda_{1}e^{-\lambda_{2}(t/\epsilon_{\eta}^{n})}[\hat{\xi}^{N}(0)^{T} \hat{\xi}^{N}(0)^{T}]^{T} + \Omega(\epsilon_{\eta}^{n})$$

for $\forall t \in [0, t_{f})$ and $0 < \epsilon_{\eta}^{n} \leq \epsilon_{\eta}^{n}$ with some positive constants $\lambda_{1}$ and $\lambda_{2}$, and a class-\(K\) function $\Omega$.

Proof. The proof of Lemma 3 is presented in [17].

D. Stability and Robustness Analysis for the Entire Operation

It is shown that the solution of $\hat{\xi}_{WP}^{N}, \hat{\xi}_{SF}^{N}$, and $\hat{\xi}_{FF}^{N}$ is a $\sigma$-robust $WP \mapsto ST$ and $ST \mapsto FF$ transition maneuver. Also, with the condition $\sigma \leq \delta_{\eta}$, $\eta$ in $ST$ mode unconditionally reaches $G_{\eta}(t, ST, FF)$ in a finite time since $[\eta(0)^{T} \tau_{60}(0)^{T}]^{T} \leq \sigma/2$ for $t \in [0, T_{2})$ holds and $[\hat{\eta}^{N}(t)^{T} \hat{\eta}^{N}(t)^{T}]^{T} \leq \sigma/2$ for $t \in [0, T_{2})$ holds.

VI. EXPERIMENTAL RESULT & DISCUSSION

A. Experimental Setup

The experimental setup for this study consists of three parts: a hexacopter, a robotic arm, and a frame for the plug-socket system. We assembled the hexacopter with the off-the-shelf frame DJI F550, six KDE2315XF-967 motors with corresponding KDEXF-UAS35 35A+ electronic speed controllers and 9-inches T-Motor polymer rotors, two Turnigy LiPo batteries for power supplement, and Intel NUC for computing. It executes the proposed DOB controller, the Kalman filter for estimating the value of $\hat{\xi}$ and the navigation algorithm with OptiTrack on Robot Operating System (ROS) in Ubuntu 18.04, and a flight controller Pixhawk 4, all onboard. The robotic arm is comprised of ROBOTIS dynamixel XH430 and XM540 servo motors. We manufacture a stand for a 110 V socket and firmly attach it to the wooden plate which weighs about 11 kg as in Fig. 5.

As depicted in Fig. 1, the plug is connected to a wire held by the end-effector. Since this is different from the perching model described in Fig. 2, we modify some strategies for the trajectory generation and the controller design to maintain the end-effector’s position fixed. For this, we turn on the position controller in $\dot{j}_{I}$ and $k_{I}$ directions as in Fig. 6.

We set $\theta_{m}$ as 20 deg and the stabilizing time $t_{d, ST} - t_{0, ST}$ as 0.08 sec. Also, the threshold for the mode change from ST to FF, $\delta_{\eta}$, is set as 5 deg.

The scenario of the experiment is as follows. First, the air Manipulator takes off until the end-effector aligns with the plug attached to the socket. Then, the vehicle flies in $-\dot{j}_{I}$ direction to tighten the wire. When the wire becomes tight enough, we send a command to begin the WP mode and the vehicle gradually tilts its body. By the time when the plug is separated from the socket, we manually switch to ST mode and the vehicle automatically turns into FF mode when the Euler angles become reasonably small. Finally, the aerial manipulator returns to the original position and maintains the hovering state.
A manipulator can be included. The plug-pulling task is executed without destabilization. For the proposed DOB controller, and the overall sequence of the actual experiment. As a result, we confirm that the pitch robustness of the plug-pulling and validate them through the mode are presented. Then, we prove the overall stability and controllers based on the dynamical models of each operative modes. A white region expresses the FF mode, a magenta region expresses the WP mode and a green region expresses the ST mode. The dashed blue line describes the desired values of the states. The red line represents the measured values of the states.

### B. Experimental Results

Fig. 7 shows that the measured value of $\theta$ adequately follows $\theta_d$ until it reaches $-14.4$ deg. From this result, we can confirm that the control structure introduced in (9) is valid for the given task. Also, from the plots of $p_x$, $p_y$ and $p_z$ in Fig. 7 we can observe that the aerial manipulator recovers its original position at about 12 sec. This result shows that the hybrid automata of the aerial manipulator conducting the plug-pulling are stable and robust to the sudden change of interaction force.

On the other hand, as can be seen in the attached video, if a standard PID controller is employed for the wire-pulling, $\theta$ cannot adequately follow the desired value $\theta_d$. Accordingly, after the plug is separated from the socket, the vehicle fails to maintain its stability and crashes to the floor.

### VII. CONCLUSION

This paper presents an aerial manipulator consisting of a multrotor and a 2-DOF robotic arm pulling a plug out of a socket. To demonstrate aerial plug-pulling, the concept of hybrid automata is used to divide the mission into three operative modes of wire-pulling, stabilizing, and free-flight. The strategy for trajectory generation and design of DOB controllers based on the dynamical models of each operative mode are presented. Then, we prove the overall stability and robustness of the plug-pulling and validate them through the actual experiment. As a result, we confirm that the pitch angle can robustly track the desired pitch trajectory by our proposed DOB controller, and the overall sequence of the plug-pulling task is executed without destabilization. For future work, autonomous grasping of a plug with an aerial manipulator can be included.

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Fig. 7: History of the states of multirotor through the entire operative modes. A white region expresses the FF mode, a magenta region expresses the WP mode and a green region expresses the ST mode. The dashed blue line describes the desired values of the states. The red line represents the measured values of the states.