Research on low cycle fatigue life prediction considering average strain

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Abstract

To address the difficult problems in the study of the effect of average strain on fatigue life under low-cycle fatigue loads, the effect of average strain on the low-cycle fatigue life of materials under different strain cycle ratios was discussed based on the framework of damage mechanics and its irreversible thermodynamics. By introducing the Ramberg-Osgood cyclic constitutive equation, a new low-cycle fatigue life prediction method based on the intrinsic damage dissipation theory considering average strain was proposed, which revealed the correlation between low-cycle fatigue strain life, material properties, and average strain. Through the analysis of the low-cycle fatigue test data of five different metal materials, the model parameters of the corresponding materials were obtained. The calculation results indicate that the proposed life prediction method is in good agreement with the test, and a reasonable characterization of the low-cycle fatigue life under the influence of average strain is realized. Comparing calculations with three typical low-cycle fatigue life prediction models, the new method is within two times the error band, and the prediction effect is significantly better than the existing models, which is more suitable for low-cycle fatigue life prediction. The low-cycle fatigue life prediction of different cyclic strain ratios based on the critical region intrinsic damage dissipation power method provides a new idea for the research of low-cycle fatigue life prediction of metallic materials.

1. Introduction

Fatigue is a gradual and irreversible permanent plastic cumulative damage process caused by cyclic stress and strain loading of materials or components during service [1]. Under the action of fatigue load, fatigue crack initiation, propagation, and rapid fracture have led to the failure of structural parts. As one of the most serious failure forms of materials, metal components are often subjected to cyclic loads and cause fatigue failure. It affects the safe and stable operation of the engineering structure and causes major safety accidents. Fatigue life prediction is an effective means of avoiding major accidents or disasters to the maximum extent. Therefore, to improve the service reliability and safety of structural parts, in-depth research on the low-cycle fatigue life prediction of materials [2, 3] is of great significance and a guiding role.

China is moving from a ‘manufacturing power’ to a ‘manufacturing power’, and the country has a major strategic need for equipment safety. In recent years, fatigue plastic deformation has had a significant impact on the fatigue life of engineering materials and mechanical components, which is related to the service safety, service life, and economic benefits of components. Fatigue deformation behavior has received extensive attention in anti-fatigue design [4–6]. At present, the fatigue life failure criterion uses the point-domain method focusing on the stress-strain field at the dangerous point to construct the damage cumulative failure model or the stress-strain field failure criterion. The main methods are the stress invariant method, meso-integral method, critical damage surface method, energy method, and damage mechanics method. Fatigue damage is a...
complex evolution process that is generated from a specific area of the damaged material and coupled with local crack initiation and propagation. Therefore, to explore the fatigue life prediction method based on the critical region stress and strain field to evaluate the service performance of the component has more definite physical and mechanical meanings.

The fatigue process is a complex energy dissipation process with energy storage and heat dissipation. Under the action of fatigue load, large-scale plastic deformation is formed owing to the irreversible distortion of the material microstructure, which is the main cause of component fatigue damage. Hasegawa and Rauch et al. [7, 8] elaborated on the physical mechanism of energy storage and release of fatigue damage evolution from the perspective of the dislocation evolution of deformed metals. Tanaka et al. [9] established a single crystal dislocation plugging model to quantitatively characterize the energy dissipated by cyclic plastic deformation, however, this model is not suitable for low-cycle fatigue with a high plastic strain amplitude. Based on the thermodynamic energy model, ZHU [10] considered the combination of thermodynamics and heat dissipation to establish a thermodynamic dissipation model, which reveals the evolution behavior of cyclic heat dissipation with high accuracy. The damage mechanics method has attracted much attention because of its definite physical and mechanical significance in deriving the damage evolution equation and considering the evolution of micro-damage [11].

Fatigue failure under cyclic loading is a process of crack initiation and propagation, and it is also the result of damage accumulation or material performance degradation [12]. Materials in the modern engineering field are affected by more severe environments, and low-cycle fatigue is the main form of environmental conditions. At the same time, mechanical components or structures often bear asymmetric fatigue loads during service, so the effect of average strain is considered to improve the prediction accuracy of low-cycle fatigue life. Yokobori [13] first proposed the equivalent strain theory, and its main equivalent strain models were the maximum shear strain model, maximum principal strain model, and von Mises equivalent strain model. Since the equivalent strain model does not consider the interaction with the stress and the factors of the loading path, it lacks a clear physical meaning between the fatigue life and fatigue life. Morrow [14] proposed an energy method based on the irreversible damage evolution and failure phenomenon caused by the accumulation of plastic work. Ostergren [15] used plastic work to predict the fatigue life. Garud [16] proposed a power-function life prediction model. Lee et al. [17] introduced damage mechanics and used strain energy as the fatigue damage parameter to predict the low-cycle fatigue life, and the accuracy of the fitting results was higher. Owing to the lack of physical meaning in characterizing fatigue failure by plastic work, the complicated constitutive model, and the difficulty of obtaining parameters, the energy method has been greatly restricted in practical engineering applications. Brown and Miller [18] considered the size and direction evolution behavior of crack initiation and growth and proposed the critical surface method, which has clear mechanical and physical meanings and is widely used in engineering. Considering the effect of shear force, a critical surface model with strain as the damage parameter is established, and its prediction accuracy is higher than that of traditional methods [19–24]. Owing to the complexity of fatigue problems, no universally practical fatigue life models for various materials and loads have been developed.

Because low-cycle fatigue is a common failure mechanism in many engineering projects, it is very important to accurately predict the low-cycle fatigue life of components. Researchers have done a lot of work on low-cycle fatigue performance, but there are still many challenges. Aiming at the difficulties in the current low-cycle fatigue life prediction research, around the theoretical modeling of low-cycle fatigue life prediction considering average strain, this paper proposes a low-cycle fatigue life prediction method based on critical domain intrinsic damage dissipation. Based on continuum damage mechanics and its irreversible thermodynamics framework, a damage evolution model with elastic modulus as the damage variable was established. Taking the maximum damage dissipation work in the equivalent critical region as the equivalent life condition, a strain-controlled low-cycle fatigue life prediction model is proposed. Through the verification and analysis of the data from the uniaxial tension-compression low-cycle fatigue test of five metal materials, the accuracy and reliability of the new model were compared and demonstrated.

2. Energy dissipation model based on continuous damage mechanism

The nature of material damage evolution is the process of internal defect expansion. The evolution of defects changes the properties of the materials in the damaged area. Due to the complexity of the tissue structure, it is impossible to directly measure the damaged area and the non-destructive area calculation of the damaged micro-element body to characterize the damage variable, and it is difficult to measure the evolution characteristics of the elastic modulus of the material in the damaged area. To measure the damage degree of the component material under fatigue action, the damage variable $D$ is introduced based on the Lemaitre strain equivalence hypothesis. which is
\[ D = \frac{E_0 - E}{E_0} \]  

where \( E \) is the effective elastic modulus, \( E_0 \) is initial elastic modulus.

The low-cycle fatigue damage evolution process takes the strain amplitude as the control variable. The critical damage variable \( D_f \) is defined as the function of the mean strain \( \varepsilon_m \) and strain ratio \( R_c \), expressed as follows:

\[ D_f = f(\varepsilon_m, R_c) \]  

where \( \varepsilon_m \) is the average strain; \( R_c \) is the cyclic strain ratio.

Fatigue damage is irreversible. As a continuous medium, the damaged micro-element body simultaneously satisfies the law of conservation of mass, law of conservation of momentum, and law of conservation of energy under the action of fatigue loads [25], which are obtained as follows:

\[
\begin{aligned}
\rho + \rho \dot{\varepsilon}_{ij} &= 0 \\
\sigma_{ij} + \rho \dot{\varepsilon}_{ij} &= \rho \dot{\varepsilon}_{ijkl} \\
\sigma_{ij} \dot{\varepsilon}_{ij} - \dot{q}_{ij} &= \rho \dot{\varepsilon}
\end{aligned}
\]  

where \( \rho, \dot{\varepsilon}_{ij} \) are the element density and partial differential of the heat flux, respectively. \( \sigma_{ij} \) is the stress tensor, \( \varepsilon_{ij} \) is the velocity of the element in different directions, \( \dot{\varepsilon} \) is the rate of change of the internal energy of the damaged medium.

The physical processes include elastic deformation, plastic deformation, and damage evolution. The evolution of each process is accompanied by a change in energy, and its evolution process needs to satisfy both the first law of thermodynamics and the second law of thermodynamics in terms of thermodynamic self-consistency. By introducing Helmholtz free energy, which is commonly used in engineering thermodynamics, the energy evolution of multiple physical processes under fatigue loading is correlated, and a thermodynamic self-consistent equation of fatigue damage evolution based on energy conservation is established as follows:

\[
\left( \sigma_{ij} - \rho \frac{\partial g}{\partial \varepsilon_{ij}} \right) \dot{\varepsilon}_{ij}^{\text{e}} + \sigma_{ij} \dot{\varepsilon}_{ijkl}^{\text{mp}} - \rho \left( s + \frac{\partial g}{\partial T} \right) \dot{T} - \rho \frac{\partial g}{\partial D} \dot{D} - \rho T \dot{s} - \dot{q}_{ij} = 0
\]  

where \( g \) is the potential free enthalpy deduced from the Helmholtz free energy, which is a function of the elastic strain, damage variable and temperature, and \( \rho \) is the mass density.

In the multi-physical field of fatigue damage, the dissipative potential and state potential are coupled to describe the dynamic evolution law of the damage evolution process. Therefore, for the irreversible state of the second law of thermodynamics, the Clausius-Duhamel inequality description [26] is used to construct its potential energy evolution equation, as expressed as follows:

\[
\sigma_{ij} \dot{\varepsilon}_{ij}^{\text{e}} - \rho \frac{\partial g}{\partial \varepsilon_{ij}} - \frac{\dot{T}}{T} \dot{q}_{ij} \geq 0
\]  

where \( \dot{\varepsilon}_{ij}^{\text{e}}, \dot{\varepsilon}_{ij}^{\text{p}}, \dot{\varepsilon}_{ijkl}^{\text{mp}} \) are the elastic strain tensor, elastic strain rate tensor and plastic strain rate tensor of the damage material, respectively. \( s(\dot{s}), T(\dot{T}) \) and \( D(D) \) are the unit entropy (entropy change rate), transient temperature (temperature change rate), and damage variable (damage change rate), respectively.

In continuous damage mechanics, the fatigue damage process is approximated as a progressive isothermal process. Because the Helmholtz free energy is only related to elastic strain, combined with the theory of thermodynamics, the characteristic form of the elastic strain energy of the loaded micro-element body is normalized as follows:

\[ g = g(\varepsilon_{ij}^{\text{e}}, D) = \frac{1}{2\rho} \alpha_{ijkl} \varepsilon_{ij}^{\text{e}}(1 - D) \]  

where \( \alpha_{ijkl} \) is the flexibility tensor.

Heat generation is an irreversible process owing to the energy dissipation of material damage evolution and plastic flow. Under arbitrary state conditions, the conservation of the first law of thermodynamics can be obtained as follows:

\[ \sigma_{ij} = \rho \frac{\partial g}{\partial \varepsilon_{ij}} = \alpha_{ijkl} \varepsilon_{ij}^{\text{e}}(1 - D) \]  

When the loading rate was low, the damaged area was close to the surface of the component and the temperature was close. The influence of heat flow and temperature gradient can be ignored in the damage zone, and the damage variable in the damage zone and the Helmholtz elastic strain energy release rate are a pair of variables for each other [27]. Therefore, the generalized damage driving force \( Y \) is defined as
where $\sigma_{eq}$ is the von Mises equivalent stress and $\sigma_{eq} = \sqrt{3/2\sigma_{ij}\sigma_{ij}}$. $R_v$ is the stress factor, $R_v = 2(1 + \nu)/3 + \frac{3(1 - 2\nu)(1}{3\sigma_{eq}})^2$.

Under the damage theory system, considering the crack closure effect, the generalized damage driving force expression under isotropic damage conditions is analyzed by the theory of elastic mechanics, which is completely modelled as follows:

\[
Y = \frac{\partial g}{\partial \sigma} = \frac{1}{2}\alpha_{ijkl}c_ijkl = \frac{R_v \sigma_{eq}^2}{2E(1 - D)^2}
\]  

(8)

3. Low-cycle fatigue life prediction model considering average strain

3.1. Our proposal

The essence of fatigue damage evolution is the continuous accumulation of plastic strain under cyclic fatigue loading. The plastic strain of the material must be considered to improve the life prediction accuracy of the fatigue damage. Therefore, the plastic power-law model was used to describe the cyclic plastic constitutive relationship. In damage mechanics, the relationship between plastic strain and equivalent stress is defined according to the Ramberg-Osgood relationship [29].

\[
\Delta \varepsilon_{ij} = \begin{cases} 
\frac{\Delta \sigma}{2E} + \left(\frac{\Delta \sigma}{K'}\right)^{1/n'} & \text{(Single axis loading)} \\
(1 + \nu)\frac{\Delta \sigma}{E} - \nu \frac{\Delta \sigma}{E} + \frac{3(\Delta \sigma)^{1/n' - 1}}{(2K')^{n' - 1}} \Delta S_{ij} & \text{(Proportional loading)} \\
(1 + \nu)\frac{\Delta \sigma}{E} - \nu \frac{\Delta \sigma}{E} + \frac{3(\Delta \sigma)^{1/n' - 1}}{(2K')^{n' - 1}} \Delta S_{ij} & \text{(non-proportional loading)}
\end{cases}
\]

where $K'$ is the plastic strain cyclic strength coefficient, $n'$ is the plastic strain cyclic hardening index, $\sigma^P$ is the plastic strain, $S_{ij}$ is stress deflection, $\Delta \sigma^P_{ijkl}$, $K'^P_{ijkl}$ are Equivalent stress and cyclic strength coefficient under non-proportional loading, respectively.

The cyclic stress-strain curve reflects the mechanical response characteristics of low-cycle fatigue under cyclic loading. Due to the continuous changes of the material’s load-bearing capacity and possible cyclic hardening, cyclic softening and other transient behaviors under the action of low-cycle fatigue cyclic loading, the cyclic stress-strain relationship of most metals has become smaller and smaller, and the shape of the hysteresis ring is gradually increasing. The similarity between the equivalent stress-strain response curve under multiaxial load and the cyclic stress-strain response curve under uniaxial load [30]. Therefore, the transient relationship of the hysteresis loop curve under steady state is:

\[
\Delta \sigma_{eq} = K' \Delta \sigma^P
\]

(10)

Under the action of the cycles of fatigue load, the damage-driving force evolution equation is:

\[
dD = \eta(\varepsilon_{in}, \varepsilon_f) Y^n dY
\]

(11)

where $\eta(\varepsilon_{in}, \varepsilon_f)$ is the condensed material constant, $m$ is the material parameter in our proposal.

From the damage increment relationship, the damage evolution model for $N$ can be solved, which can be represented as:
\[ \frac{dD}{dN} = \int \eta(\varepsilon_m, \varepsilon_f) Y^m dY \]

\[
= \begin{cases} 
\eta(\varepsilon_m, \varepsilon_f) K^{2m+2} \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^{2m+2} \left( \frac{1}{1 - D} \right)^{2m+2} & (0 \leq R < 1) \\
\eta(\varepsilon_m, \varepsilon_f) K^{2m+2} \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^{2m+2} \left( \frac{1}{1 - D} \right)^{2m+2} & (-1 \leq R < 0)
\end{cases}
\]

(12)

Considering that the fatigue damage dissipation forms include microplastic dissipation, internal damage dissipation, and heat conduction dissipation, the increase in dissipation work caused by intrinsic damage is

\[ dQ = Y dD \]

(13)

where \( Q \) is the intrinsic damage dissipation work.

Combining equations (12) and (13), it can be obtained from the general principle of differential that the single-cycle intrinsic damage dissipation equation is expressed as

\[
\frac{dQ}{dN} = \frac{dQ}{dD} \frac{dD}{dN} = \int Y \cdot \eta(\varepsilon_m, \varepsilon_f) Y^m dY
\]

\[
= \begin{cases} 
\eta(\varepsilon_m, \varepsilon_f) K^{2m+2} \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^{2m+2} \left( \frac{1}{1 - D} \right)^{2m+2} & (0 \leq R < 1) \\
\eta(\varepsilon_m, \varepsilon_f) K^{2m+2} \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^{2m+2} \left( \frac{1}{1 - D} \right)^{2m+2} & (-1 \leq R < 0)
\end{cases}
\]

(14)

Under the action of a single cyclic load, there is only one incremental cyclic load \((-1 \leq R < 0)\), and the corresponding equation of the relationship between intrinsic damage energy dissipation and damage \( D \) is

\[
\frac{dQ}{dD} = \frac{\eta(\varepsilon_m, \varepsilon_f) K^{2m+2} \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^{2m+2} \left( \frac{1}{1 - D} \right)^{2m+2}}{(m + 2)(2E)^{m+2} \left( \frac{1}{1 - D} \right)^{2m+2}} = \frac{(m + 2)K^2 \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^2}{(m + 1)(2E)(1 - D)^2}
\]

(15)

According to the principle of integration, it can be obtained

\[
Q = \int (m + 2)K^2 \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^2 (1 - D)^2 dD = \left( \frac{m + 2)K^2 \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^2}{(m + 1)(2E)(1 - D)^2} \right) \frac{dQ}{dD}
\]

(16)

The maximum damage dissipation power at failure can be solved by combining the critical damage conditions \( D = 0 \) and \( D = 1 \), the maximum damage dissipation power at failure can be solved. Which is

\[
Q_c = \int (m + 2)K^2 \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^2 (1 - D)^2 dD = \frac{(m + 2)K^2 \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^2}{(m + 1)(2E)(1 - D)^2} f(\varepsilon_m, R_c)
\]

(17)

For the symmetric tension-compression cyclic load with \( R = -1 \), the critical region intrinsic damage dissipation work is

\[
Q_{cr-1} = \int (m + 2)K^2 \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^2 (1 - D)^2 dD
\]

\[
= \frac{(m + 2)K^2 \left( \frac{\varepsilon_{\text{max}}}{\varepsilon_{\text{max}}} \right)^2}{(m + 1)(2E)(1 - D)^2} f(0, -1)
\]

(18)

In the process of low-cycle fatigue loading, the fracture failure of the damaged component is the complete loss of the load-bearing energy capacity of the loaded section, that is, the damage degree \( D \) of the material point reaches its critical value \( D_c = f(\varepsilon_m, R_c) \). This is the theoretical failure and fracture criterion of the damaged unit.
When the stress ratio is $0 < R < 1$

$$\frac{dQ}{dD} = \frac{\eta(\varepsilon_m, \varepsilon_f) K^{2m+4}}{(p + 2)(2E)^{m+2}} \left( \frac{\varepsilon_{p_m}^{max}}{1 - D} \right)^{2m+4} - \left( \frac{\varepsilon_{p_m}^{min}}{1 - D} \right)^{2m+4}$$

$$= \frac{\eta(\varepsilon_m, \varepsilon_f) K^{2m+2}}{(p + 1)(2E)^{m+1}} \left( \frac{\varepsilon_{p_m}^{max}}{1 - D} \right)^{2m+2} - \left( \frac{\varepsilon_{p_m}^{min}}{1 - D} \right)^{2m+2}$$

$$= \frac{m + 2}{(m + 1)(2E)(1 - D)^2} K^2 \left( \frac{\varepsilon_{p_m}^{max}}{1 - D} \right)^{2m+4} - \left( \frac{\varepsilon_{p_m}^{min}}{1 - D} \right)^{2m+4}$$

(19)

Corresponding damage dissipation power is

$$Q_e = \int \frac{m + 2}{(m + 1)(2E)(1 - D)^2} K^2 \left( \frac{\varepsilon_{p_m}^{max}}{1 - D} \right)^{2m+4} - \left( \frac{\varepsilon_{p_m}^{min}}{1 - D} \right)^{2m+4} \ dD$$

$$= \frac{(m + 2)K^2}{(m + 1)(2E)} \left( \frac{\varepsilon_{p_m}^{max}}{1 - D} \right)^{2m+4} - \left( \frac{\varepsilon_{p_m}^{min}}{1 - D} \right)^{2m+4} \ f(\varepsilon_m, R_e)$$

(20)

Based on the continuous damage mechanics and the irreversible thermodynamics framework, the theory of the maximum work equal to the intrinsic damage dissipation is applicable to common metal components and has definite physical and mechanical significance.

When $R = 0, \varepsilon_m = 2\varepsilon_{s0}, \varepsilon_{min} = 0$, it can be given

$$Q_{en} = \frac{m + 2}{m + 1} \frac{2\varepsilon_{p_m}^{max} K^2}{2E} - \frac{f(\varepsilon_m, 0)}{1 - f(\varepsilon_m, 0)}$$

(21)

According to the equal life condition in which the maximum work of the intrinsic damage dissipation is equal, that is, the intrinsic damage dissipation in the equally uniformly distributed critical region has the same fatigue life [26]. Then there is

$$f(\varepsilon_m, R_e) = 1 - \left\{ 1 - \frac{2m + 3 \eta(\varepsilon_m, \varepsilon_f) K^{2m+2}}{2E} \left( \frac{\varepsilon_{p_m}^{max}}{1 - D} \right)^{2m+2} \ N_1 \right\}^{\frac{1}{n}}$$

(23)

Under different strain ratios, the mutual transformation equation of strain amplitude is

$$\varepsilon_{p_{-1a}} = \left( \frac{\varepsilon_{p_m}^{max}}{2m + 3} \right)^{\frac{1}{m+2}} (\varepsilon_p^{n-1a})^{\frac{1}{m+2}}$$

(24)

Because low-cycle fatigue is fatigue controlled by strain amplitude, and mainly plastic strain, elastic strain is small. Therefore, equation (24) can be rewritten as $\varepsilon_{-1a} = (\varepsilon_{max}^{p})^{1 + \tau}, \varepsilon_{p_m}^{max}$. Equation (24) shows that the tension-compression low-cycle fatigue strengths of different plastic strain ratios are equivalent to symmetrical cyclic strain tension-compression fatigue effects. In addition, combined with the damage accumulation theory, a uniaxial low-cycle fatigue life prediction model was constructed.

3.2. Material parameter identification

For a symmetric tension-compression cyclic strain load with $R_e = -1$, the driving force of fatigue damage has non-monotonic characteristics [28], and the damage introduced by compression strain is ignored. From formulas (12) and (23), the $\varepsilon - N$ curve equation can be derived, that is, the predicted fatigue life based on the low-cycle strain amplitude is

$$N_f = \frac{[1 - (1 - D_{-1a}^{2m+3})(2E)^{m+1}(1 + m)](\varepsilon_{p_m}^{n-1a})^{2m-2}}{(2m + 3)\eta(\varepsilon_m, \varepsilon_f)K^{2m+2}}$$

$$= k(\varepsilon_{p_{-1a}})^{-2m-2}$$

(25)

where $D_{-1a}$ is the critical value of the fatigue damage under symmetric tension and compression. To obtain the material parameter $m$, the least-squares method is introduced and coupled with the $\varepsilon - N$ curve equation. Therefore an identification model of the damaged material parameter $m$ is established.
where $t$ is the number of uniaxial symmetric tension and compression fatigue tests, $N_i$ is the median fatigue life of the test, $a_i$ is the applied strain amplitude for each group of fatigue tests.

Through the low-cycle fatigue damage evolution model under strain control and related test data analysis, the $\varepsilon$-N curve equation was used to identify the material parameters in the cyclic strain fatigue damage evolution using material theory. In summary, the technical route of the low-cycle fatigue life prediction method considering the average strain was established, as shown in figure 1.

### 4. Validation and analysis of new models

Five metal materials, namely 7050-T7451 and 2124-T851 aluminum alloy (used in aerospace and maritime applications), S550 high-strength steel (commonly used in ship and floating structures), SM45C steel (widely used in construction machinery), and AM30 magnesium alloy (automotive and aerospace applications), were selected. Based on the low-cycle fatigue test data, the model parameters of the corresponding materials and predicted life values are calculated. The low-cycle fatigue test life is compared with the predicted lifetime value of the built model to verify the correctness and applicability of the proposed life prediction model.

### 4.1. Test conditions

The experimental fatigue data of aluminum alloy 2124-T851 [31] and 7050-T7451 [32], high-strength steel S550 [33], and magnesium alloy AM30 [34, 35] were extracted from related literature. For the test material 45 steel [36], a low-cycle fatigue test was performed to obtain the corresponding fatigue life data. All experiments were performed at room temperature, using a strain-controlled loading method (equation (27)) on a closed-loop hydraulic servo experimental machine. A biaxial extensometer was used to measure and control the tensile strain on the outer surface of the test piece, and the load and strain of the test piece were recorded by a data acquisition system within a predefined time interval. During the experiment, the loading frequency was low, which can be regarded as quasi-static loading, so the effect of the loading rate was not considered. The static parameters of various materials are given in the references cited in table 1.
4.2. Material parameter identification

To obtain multiple groups of asymmetric cyclic loads, the load was applied asymmetrically by changing the average value of the tension and compression sine wave strains. The instantaneous strain values of the tension and compression loads are described by the following formula:

\[ \varepsilon_t = \varepsilon_a \sin (wt) + \varepsilon_m \]

where \( \varepsilon_x \) is the instantaneous strain value of the axial load, \( \varepsilon_a \) is the axial load strain amplitude, \( w \) is the frequency of the load.

Completed 45 steel symmetrical cyclic strain uniaxial tension-compression low-cycle fatigue test, citing documents \[37, 38\] to extract low-cycle fatigue test of two aluminum alloys 7050-T7451 and 2124-T851 and related low-cycle fatigue tests of S550 high-strength steel and AM30 magnesium alloy data. For 7050-T7451 and 2124-T851 aluminum alloys, S550 high-strength steel, AM30 magnesium alloy, multiple sets of uniaxial tension-compression experiments under different strain ratios were performed. Because the material constant identification model equation \( (23) \) depends on the \( \varepsilon-N \) curve equation of the material, it is used to identify the material model parameters of 7050-T7451 and 2124-T851 aluminum alloy, S550 high-strength steel, AM30 magnesium alloy and 45 steel. In this study, based on the von Mises equivalent criterion of the damage model and the Ramberg-Osgood relationship, the non-cyclic symmetric \( R_1 = -1 \) tension-compression loading strain is equivalent to the cyclic symmetric \( R_1 = -1 \) tension-compression loading strain. The material parameters for obtaining the material fatigue damage evolution model are listed in Table 2.

4.3. Verification and Analysis

The damage degree \( D \) of the material determines the service performance of the material and directly affects the constitutive relationship of the material fatigue damage evolution process. SM45C steel were selected for the low-cycle fatigue test, and low-cycle fatigue test data under different strain ratios were used to verify the reliability of the equivalence relationship model and the feasibility of the material identification method. The fatigue performance parameters of the SM45C steel materials were determined (see Table 2). The cyclic symmetrical low-cycle fatigue test data and the calculated life prediction values are listed Table 3.

To facilitate comparison, argumentation and analysis, the life prediction model error index \( I_{error} \) is introduced as follows:

\[ I_{error} = \frac{|N_f - N_{exp}|}{N_f} \times 100\% \]
\[ I_{\text{error}} = \frac{|N_{\text{pre}} - N_{\text{exp}}|}{N_{\text{exp}}} \times 100\% \]  

where \( N_{\text{pre}} \) and \( N_{\text{exp}} \) are the predicted low-cycle fatigue life and test the low-cycle fatigue life of the model, respectively.

At the same time, the fatigue test parameters of two aluminum alloys 7050-T7451 and 2124-T851, are cited, and the life expectancy is predicted using the model established in this study (see table 4). The comparative analysis with the test life data shows that the life prediction model established in this study can better conform to the actual life.

| \( R_e \) | \( \varepsilon_a \) | \( \varepsilon_m \) | \( N_{\text{exp}} \) | \( N_{\text{pre}} \) | \( I_{\text{error}} \) | \( R_e \) | \( \varepsilon_a \) | \( \varepsilon_m \) | \( N_{\text{exp}} \) | \( N_{\text{pre}} \) | \( I_{\text{error}} \) |
|---|---|---|---|---|---|---|---|---|---|---|---|
| -1 | 0.4% | 0 | 30448 | 19139 | 37.14% | -1 | 0.4% | 0 | 41980 | 22106 | 47.34% |
| -1 | 0.6% | 0 | 3076 | 4206 | 36.74% | -1 | 0.6% | 0 | 2870 | 4581 | 59.62% |
| -1 | 0.8% | 0 | 956 | 1435 | 50.1% | -1 | 0.8% | 0 | 1111 | 1500 | 35.01% |
| -1 | 1.0% | 0 | 609 | 624 | 2.46% | -1 | 1.0% | 0 | 454 | 631 | 38.99% |
| -1 | 1.5% | 0 | 146 | 137 | 6.16% | -1 | 1.5% | 0 | 147 | 131 | 10.88% |
| -1 | 2.0% | 0 | 58 | 47 | 18.97% | -1 | 2.0% | 0 | 60 | 43 | 28.33% |
| -0.06 | 0.4% | 0.35% | 12920 | 17399 | 34.67% | -0.06 | 0.4% | 0.75% | 16505 | 20717 | 25.52% |
| -0.06 | 0.6% | 0.53% | 2991 | 3821 | 27.75% | -0.06 | 0.6% | 1.12% | 2781 | 4296 | 54.48% |
| -0.06 | 0.8% | 0.70% | 1129 | 1305 | 15.59% | -0.06 | 0.8% | 1.50% | 888 | 1406 | 58.33% |
| -0.06 | 1.0% | 0.88% | 510 | 567 | 11.18% | -0.06 | 1.0% | 1.87% | 450 | 591 | 31.33% |
| -0.06 | 1.5% | 1.31% | 178 | 125 | 29.78% | -0.06 | 1.5% | 2.79% | 137 | 123 | 10.22% |
| -0.06 | 2.0% | 1.74% | 69 | 43 | 37.68% | -0.06 | 2.0% | 3.70% | 57 | 40 | 29.82% |
| 0.06 | 0.4% | 0.45% | 12486 | 17072 | 36.73% | 0.06 | 0.4% | 0.85% | 14169 | 20451 | 44.34% |
| 0.06 | 0.6% | 0.67% | 2842 | 3754 | 32.09% | 0.06 | 0.6% | 1.27% | 2307 | 4240 | 83.79% |
| 0.06 | 0.8% | 0.90% | 1016 | 1280 | 25.98% | 0.06 | 0.8% | 1.69% | 850 | 1388 | 63.29% |
| 0.06 | 1.0% | 1.12% | 548 | 556 | 1.46% | 0.06 | 1.0% | 2.1% | 415 | 584 | 40.72% |
| 0.06 | 1.5% | 1.67% | 148 | 122 | 17.57% | 0.06 | 1.5% | 3.14% | 132 | 121 | 8.33% |
| 0.06 | 2.0% | 2.21% | 77 | 42 | 45.45% | 0.06 | 2.0% | 4.16% | 46 | 40 | 13.04% |
| 0.5 | 0.4% | 1.19% | 10570 | 14681 | 38.89% | 0.5 | 0.4% | 1.59% | 9306 | 19172 | 106.02% |
| 0.5 | 0.6% | 1.78% | 2384 | 3413 | 43.16% | 0.5 | 0.6% | 2.37% | 2189 | 3976 | 81.64% |
| 0.5 | 0.8% | 2.37% | 941 | 1165 | 23.80% | 0.5 | 0.8% | 3.15% | 810 | 1302 | 60.74% |
| 0.5 | 1.0% | 2.95% | 489 | 506 | 3.48% | 0.5 | 1.0% | 3.92% | 380 | 548 | 44.21% |
| 0.5 | 1.5% | 4.40% | 156 | 111 | 2.88% | 0.5 | 1.5% | 5.83% | 134 | 114 | 14.93% |
| 0.5 | 2.0% | 5.81% | 72 | 38 | 47.22% | 0.5 | 2.0% | 7.69% | 51 | 37 | 27.45% |

Table 4. Calculation of experimental values and predicted values of low strain fatigue life for 7050-T7451 and 2124-T851 aluminum alloy at low cycle fatigue.

Figure 2. Predicted versus experimental fatigue lives of SM45C, Al7050, Al2124, S550 and MgAM30 samples based on the Proposed prediction model.
The equivalent relationship model in this study was used to compare and analyze the low-cycle fatigue test data of S550 high-strength steel and AM30 magnesium alloy under different cyclic strain ratios (see table 5). The prediction results of the life prediction model built in this study are in good agreement with the experimental data, but as the strain ratio increases, the life prediction error of the built model increases. The life prediction effects of the two materials show that the life prediction model built in this study can better predict the low-cycle fatigue life.

Through the analysis of the fatigue life data of five materials, the results show that the life prediction models proposed in this paper are all in the double error band (see figure 2). As the low-cycle fatigue load is close to or exceeds the fatigue strength of the material, a certain amount of plastic deformation occurs during the fatigue damage process, and is accompanied by the internal cyclic softening and hardening phenomenon, mainly plastic damage. The model in this paper is based on the continuum damage mechanics and its irreversible thermodynamics framework. It focuses on plastic strain control, fully considers the effect of average strain, and describes the influence of different strain ratios on fatigue life. Compared with other low-cycle fatigue life prediction models, the model in this paper has clear mechanical and physical meanings. The fatigue life prediction accuracy is higher and the life error is smaller. At the same time, due to low-cycle fatigue, mainly plastic strain, ignoring elastic deformation and test equipment, there is a certain error between this model and the test. But the error is within the double error band, which is in good agreement with the experimental data, as shown in figure 2.

To better compare the prediction results, combined with the low-cycle fatigue test data of the 2124-T851 aluminum alloy [38], the prediction method proposed in this paper, the modified Ohji fatigue life prediction

**Table 5. Comparison of test values and predicted values of low strain fatigue life for S550 and AZ30 low cycle fatigue.**

| Material          | Strain Ratio | εm | N0exp | N0pre | Lerror |
|-------------------|--------------|----|-------|-------|--------|
| S550 high-strength steel | −1 | 0.5% | 0 | 3701 | 3328 | 10.08% |
|                   | −1 | 0.75% | 0 | 1021 | 1203 | 17.83% |
|                   | −1 | 1% | 0 | 549 | 584 | 6.38% |
|                   | −1 | 1.25% | 0 | 376 | 334 | 11.17% |
|                   | 0.1 | 0.5% | 0.61% | 2384 | 2728 | 14.43% |
|                   | 0.1 | 0.75% | 0.92% | 891 | 986 | 10.66% |
|                   | 0.1 | 1% | 1.22% | 593 | 479 | 19.22% |
|                   | 0.1 | 1.25% | 1.53% | 388 | 274 | 29.38% |
|                   | 0.5 | 0.5% | 1.5% | 3063 | 2357 | 23.05% |
|                   | 0.5 | 0.75% | 2.25% | 856 | 852 | 0.47% |
|                   | 0.5 | 1% | 3.00% | 753 | 414 | 45.02% |
|                   | 0.5 | 1.25% | 3.75% | 394 | 236 | 40.1% |

**Figure 3.** 2124-T851 and 7050-T751 aluminum alloys prediction model error dispersion diagram.
model [39] and the Basiquin–Manson–Coffin (BMC) life are drawn. A comparison chart of the prediction model [40] and the semi-empirical unified strain-life model [41] on the life prediction effect of the 2124-T851 aluminum alloy under low-cycle fatigue loading is shown in figure 2.

It can be seen from the error dispersion diagram of the low-cycle fatigue life prediction model in figure 3 that the results of the life prediction method proposed in this paper are in good agreement with the experimental data, and the model fitting error has certain volatility, but it is completely within the two times error bands. The model proposed in this paper is the same as the modified Ohji fatigue life prediction model and is basically within two times the error.

At the same time, the established equivalent relationship life prediction model is significantly better than the Basiquin–Manson–Coffin (BMC) life prediction model and the strain-life semi-empirical unified model. The life prediction effect of the low-cycle fatigue test of 2124-T851 aluminum alloy shows that the new method of low-cycle fatigue life prediction proposed in this study is in good agreement with the low-cycle fatigue test data.

According to the fatigue test data of S550 high-strength steel, Al7050, Al2124, S550, and MgAM30 samples, the life calculation and analysis of different methods can be carried out, and similar conclusions can be obtained (see figure 4).

Figure 4. S550 high-strength steel, SM45C steel, and AM30 magnesium alloy prediction models error dispersion diagram.

Compared with the Basiquin–Manson–Coffin (BMC) life prediction model, the modified Ohji fatigue life prediction model, and the strain-life semi-empirical unified model, it is more suitable for low-cycle fatigue life prediction considering average strain.

5. Conclusions

(1) A low-cycle fatigue life prediction method that considers the average strain under different cyclic strain ratios is proposed, which reveals the correlation between the maximum strain, strain amplitude, material properties, and fatigue life. The rationalized characterization of the low-cycle fatigue life under the influence of average strain is realized.

(2) The fatigue test analysis results of five metal materials show that the proposed method for predicting the equivalent relationship between the symmetric tension-compression equivalent strain amplitude and the asymmetric tension-compression fatigue strain amplitude and maximum strain can effectively predict the low-cycle fatigue life. A comparative analysis with three typical low-cycle fatigue life prediction methods shows that the new method is significantly better than the existing low-cycle life prediction models such as Basiquin–Manson–Coffin. The newly created method was within two times the error bands.

(3) The low-cycle fatigue life prediction method based on the critical region intrinsic damage dissipation method considering different cyclic strain ratios has high prediction accuracy, which provides a new idea for the research of low-cycle fatigue life prediction of metal materials.
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Data availability statement

All data that support the findings of this study are included within the article (and any supplementary files).

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Ethical statement

Not applicable.

Conflicts of interest

The authors have declared that no competing interests exist.

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