Collective Excitations of Dilute Bose-Fermi Superfluid Mixtures

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Using the effective action formalism, we investigate collective excitations of a dilute mixture of a Bose gas and a two component Fermi gas when both bosons and fermions have undergone superfluid transitions. We show that there is a repulsion between Bogoliubov and Anderson modes, which has important implications including disappearance of boson superfluidity. We derive an analytic expression for the long-wavelength dispersion relation of the mixture at zero temperature and give a condition for the instability. We also numerically calculate the damping rate due to boson-fermion interaction at finite temperatures and show that the two modes are stable at sufficiently low temperatures.

I. INTRODUCTION

Experimental realization of Bose-Einstein condensation (BEC) [1] of trapped atomic Bose gases and achievement of trapped quantum degenerate Fermi gas (DFG) [2] are two of the most important prospects in the field of ultracold atomic gas. Recently, $^{40}$K atoms has been cooled to one-fifth of the Fermi temperature ($0.2T_F$) [3]. One of the cardinal goals for the cooling of Fermi gas is the observation of BCS-type superfluidity [4]. Except for $^6$Li which has an enormously large negative scattering length [5], creation of effective attractive interaction for pairing is necessary, methods such as Feshbach resonance [6–8] and phonon exchange in Fermi-Bose mixtures have been proposed [9].

Sympathetic cooling of atomic Bose-Fermi gas mixtures has also risen to a thriving field. Since Bose gas can act as a coolant for fermions, severe decrease of rethermalization due to Pauli blocking can be avoided. In addition, this leads us to a fascinating system of dilute Bose-Fermi mixture to explore. Success in simultaneous trapping of Bose-Fermi isotopic mixture has been reported [10,11] and simultaneous quantum degeneracy at temperature as low as $0.25T_F$ has been observed in bosonic $^7$Li and fermionic $^6$Li mixture [12]. Density profiles in harmonic traps [13,14] and collective excitations [15,16] of Bose-Fermi mixtures has been investigated so far.

Collective excitations of superfluid Fermi gas are predicted to be detectable [17,18] and sound propagation in BEC has already been observed [19]. It would be interesting to investigate how these two sound modes act upon each other. In this paper, we study the collective excitations of dilute Bose-Fermi mixture when both boson and fermion have undergone superfluid transition. Spatial homogeneity is assumed throughout the calculation since overlapping of bosons and fermions can only be large in a gradual or box-like trap. Based on the effective action formalism, long-wavelength dispersion relation of the superfluid mixture at zero temperature is derived and damping rates due to coupling of the Bogoliubov and Anderson mode are also obtained by analytic continuation of the RPA-polarization bubbles. Instability of the superfluid modes in this mixture is also discussed.

II. FORMULATION

A. Model

We shall start from a imaginary time path integral for which the grand canonical partition function reads

$$Z = \int \mathcal{D}\phi^* \mathcal{D}\phi \mathcal{D}\psi^* \mathcal{D}\psi \exp \left\{ -\frac{1}{\hbar} (S_B[\phi^*, \phi] + S_F[\psi^*, \psi] + S_I[\phi^*, \phi, \psi^*, \psi]) \right\} ,$$

where the total action consists of terms representing the Bose gas,

$$S_B[\phi^*, \phi] = \int_0^{\hbar\beta} d\tau \int dx \left\{ \phi^*(x, \tau) \left( \frac{1}{2m_B} \frac{\partial^2}{\partial x^2} - \mu_B \right) \phi(x, \tau) + \frac{g_B}{2} |\phi(x, \tau)|^4 \right\} ,$$

the Fermi gas,

$$S_F[\psi^*, \psi] = \sum_{\alpha=1}^{N} \int_0^{\hbar\beta} d\tau \int dx \left\{ \psi^*_\alpha(x, \tau) \left( \frac{1}{2m_F} \frac{\partial^2}{\partial x^2} - \mu_\alpha \right) \psi_\alpha(x, \tau) + \frac{g_F}{2} |\psi_\alpha(x, \tau)|^2 |\psi_{-\alpha}(x, \tau)|^2 \right\} ,$$
and boson-fermion interaction

\[ S_I[\phi^*, \phi, \psi^*, \psi] = \sum_{\alpha} \int_0^{\hbar \beta} dt \int dx \ g_{\alpha}(x, \tau)|\phi(x, \tau)|^2, \quad (4) \]

where \( \phi(x, \tau) \) are complex fields and \( \psi_{\alpha}(x, \tau) \) are Grassman fields describing Bose component and the two hyperfine Fermi component \( \alpha = \{ \uparrow, \downarrow \} \) respectively. \( g_{\alpha} = \frac{2\pi \hbar^2 \alpha_0}{m_{\alpha}} \) is the coupling constant, for which \( a_{\uparrow}, a_{\downarrow} \) are the scattering length of boson-boson, fermion-fermion and boson-fermion interaction respectively. Note that only interaction between two different hyperfine states is considered since Pauli principle prohibits s-wave scattering between two identical fermions. \( m_x = \frac{m_{\uparrow} m_{\downarrow}}{m_{\uparrow} + m_{\downarrow}} \) is the reduced mass and \( \mu_x \) is the chemical potential of the corresponding component.

In order to introduce an auxiliary field corresponding to BCS pairing field, we perform a Stratonovich-Hubbard transformation to the fermion-fermion interaction,

\[ \exp \left\{ -\frac{g_F}{\hbar} \int_0^{\hbar \beta} dt \int dx |\psi_{\uparrow}(x, \tau)|^2 |\psi_{\downarrow}(x, \tau)|^2 \right\} \]

\[ = \int D\Delta^* D\Delta \exp \left\{ \frac{1}{\hbar} \int_0^{\hbar \beta} dt \int dx \left[ \frac{|\Delta(x, \tau)|^2}{g_F} + \Delta^*(x, \tau) \psi_{\uparrow}(x, \tau) \psi_{\downarrow}(x, \tau) + \psi_{\uparrow}^*(x, \tau) \psi_{\downarrow}^*(x, \tau) \Delta(x, \tau) \right] \right\}. \quad (5) \]

Integrating over the fermionic field, we obtain an effective action,

\[ S_{\text{eff}}[\Delta^*, \Delta, \phi^*, \phi] = \int_0^{\hbar \beta} dt \int dx \left[ -\frac{|\Delta(x, \tau)|^2}{g_F} \right] + S_B[\phi^*, \phi] - \hbar \text{Tr}[-\nabla^{-1}], \quad (6) \]

for which the inverse of Green’s function in the third term can be expanded perturbatively,

\[ -\nabla^{-1}(x, \tau; x', \tau') = \]

\[ \frac{1}{\hbar} \left( \frac{\hbar}{\partial \tau} - \frac{\hbar^2 \nabla}{2m_F} - \mu_\uparrow + g_\uparrow |\phi|^2}{\Delta^*} = \frac{\hbar}{\partial \tau} + \frac{\hbar^2 \nabla}{2m_F} - \mu_\uparrow - g_\uparrow |\phi|^2 \right) \]

\[ \times \delta(x - x') \delta(\tau - \tau'). \quad (7) \]

B. Perturbation Expansion

Since the trace of (7) is invariant under unitary transformation, we can perform a rotation in Nambu space, \( U(\theta) = e^{-\frac{i}{2} \sigma_3 \theta(x, \tau)} \), which corresponds to a gauge transformation. Separating the pairing field into phase and amplitude, i.e. \( \Delta(x, \tau) = \Delta_0(x, \tau) e^{i\psi_0(x, \tau)} \), and choosing \( \theta = \theta_F \) leads to a new inverse of Green’s function \( \nabla^{-1} \) with a real energy gap.

Bogoliubov approximation is then applied to the Bose field and the pairing field is also separated into its average and fluctuation. i.e.

\[ \phi(x, \tau) = \sqrt{n_B} + \phi'(x, \tau), \quad (8) \]

\[ \Delta_0(x, \tau) = \Delta_0 + \delta(x, \tau). \quad (9) \]

Assuming quantum fluctuation to be small, the third term in (6) can be expanded,

\[ -\hbar \text{Tr}[-\nabla^{-1}] = -\hbar \text{Tr}[-\nabla^{-1}] + \hbar \sum_{j=1}^{\infty} \frac{1}{j} \text{Tr}[(\nabla_0 \Sigma)^j], \quad (10) \]

where the fluctuation part reads,

\[ \Sigma(x, \tau) = -\frac{1}{\hbar} \left( \begin{array}{cc} K + L + M \delta(x, \tau) & -K + L + M \delta(x, \tau) \\ -K + L + M \delta(x, \tau) & K - L - M \end{array} \right) \]

\[ = -\frac{1}{\hbar} (K\sigma_3 + (L + M)\sigma_0 + \delta(x, \tau)\sigma_1), \quad (11) \]

with,

\[ K = i\gamma(x, \tau) + \frac{m_F v_0^2(x, \tau)}{2} + g_A \sqrt{n_B}(\phi(x, \tau) + \phi^*(x, \tau)) + \phi'(x, \tau)\phi^*(x, \tau)), \quad (12) \]

\[ L = \frac{i\hbar}{2} \nabla \cdot \psi(x, \tau) + \psi(x, \tau) \cdot \nabla, \quad (13) \]

\[ M = g_D [\sqrt{n_B}(\phi(x, \tau) + \phi^*(x, \tau)) + \phi'(x, \tau)\phi^*(x, \tau)), \quad (14) \]

in which \( \psi(x, \tau) = \frac{\hbar}{2m_F} \nabla \theta(x, \tau) \) and \( \gamma(x, \tau) = \frac{\hbar}{2m_F} \theta(x, \tau) \) are the superfluid velocity and chemical potential for Cooper pairs respectively. The average of boson-fermion interaction, \( g_A = \frac{g_{\uparrow} + g_{\downarrow}}{2} \) and the deviation of interaction strength from average \( g_D = \frac{g_{\uparrow} - g_{\downarrow}}{2} \) has also been introduced in order to rewrite (11) in Pauli matrices.

Since we are dealing with a homogeneous system, it is convenient to perform a Fourier transformation. The unperturbed Green’s function reads,

\[ \tilde{G}_0(k, \omega_n) = \frac{\hbar}{D(k, \omega_n)} \left( \begin{array}{c} i\hbar \omega_n + \epsilon_j(k) \\ \Delta_0 \end{array} \right) \]

\[ = \left( \begin{array}{c} G_j(k, \omega_n) \\ \bar{F}(k, \omega_n) \end{array} \right) \tilde{G}_j(k, \omega_n), \quad (15) \]

with,

\[ D(k, \omega_n) = (i\hbar \omega_n - \epsilon_j(k))(i\hbar \omega_n + \epsilon_j(k)) - |\Delta_0|^2, \quad (16) \]

where \( \epsilon_\alpha(k) = \frac{\hbar^2 k^2}{2m_F} - \mu_\alpha + g_\alpha n_B \) and \( \omega_n = \frac{2\pi n}{\hbar^2} \) is the even Matsubara frequency.

Expanding (10) to the second order of fluctuation around its minimum leads to the dispersion relation of collective modes. The requirements of the linear terms of fluctuations to be zero (\( \delta S^{(1)} = 0 \)) are the well known Hugenholz-Pines relation

\[ \mu_B = g_B n_B + g_1 n_+ + g_1 n_+, \quad (17) \]
with \( G_\alpha(x, \tau; x, \tau) = n_\alpha \) for the Bose field and BCS gap equation
\[
\mathcal{F}(x, \tau; x, \tau) = \frac{\Delta_0}{g_F}
\] (18)
for the pairing field.

C. RPA Polarization Bubbles

Expanding (10) to the second order gives rise to various kinds of RPA Bubbles:
\[
f_0(k, \omega_n) = \frac{1}{\hbar \beta V} \sum_{p,m} \mathcal{F}(k, \omega_n) \mathcal{F}(p + k, \omega_m + \omega_n), \tag{19}
\]
g_0(k, \omega_n) = \frac{1}{\hbar \beta V} \sum_{p,m} G_\uparrow(k, \omega_n) G_\downarrow(p + k, \omega_m + \omega_n), \tag{20}
h_0(k, \omega_n) = \frac{1}{\hbar \beta V} \sum_{p,m} G_\downarrow(k, \omega_n) G_\uparrow(p + k, \omega_m + \omega_n), \tag{21}
k_0^+(k, \omega_n) = \frac{1}{\hbar \beta V} \sum_{p,m} G_\uparrow(k, \omega_n) F(p + k, \omega_m + \omega_n), \tag{22}
\]
Note that inverting \( G \) and \( \mathcal{F} \) at (22) gives \( h_0^-(k, \omega_n) \).

The remaining bubbles such as:
\[
g_1(k, \omega_n) = \frac{1}{\hbar \beta V} \sum_{p,m} (p + \hbar \beta V) G_\uparrow(k, \omega_n) G_\downarrow(p + k, \omega_m + \omega_n), \tag{23}
\]
g_2(k, \omega_n) = \frac{1}{\hbar \beta V} \sum_{p,m} (p + \hbar \beta V)^2 G_\downarrow(k, \omega_n) G_\uparrow(p + k, \omega_m + \omega_n), \tag{24}
\]
can be obtained as a combination of (19)-(22) by considering small rotation \( U(\chi) = e^{ \chi \sigma_3 \chi(x)} \) of phase, since gauge invariance gives rise to a Ward identity \cite{22, 23}.

The real part of the bubbles can be calculated in a purely analytical manner \cite{22, 23} at absolute zero, the result of integration is independent of the difference \( g_\uparrow - g_\downarrow \) and \( \mu_\uparrow - \mu_\downarrow \),

\[
\operatorname{Re} f_0(k, \omega) = \frac{N(0)}{2} \left[ 1 - \frac{\hbar^2}{6 \Delta_0^2} \left( -\omega^2 + \frac{v_F^2 k^2}{3} \right) + \ldots \right], \tag{25}
\]
\[
\operatorname{Re} g_0(k, \omega) = -\frac{N(0)}{2} \left[ 1 - \frac{\hbar^2}{6 \Delta_0^2} \left( -\omega^2 - \frac{v_F^2 k^2}{3} \right) + \ldots \right], \tag{26}
\]
\[
\operatorname{Re} h_0(k, \omega) = \frac{1}{g_F} + \frac{N(0) \hbar \omega}{2} \left[ 1 - \frac{\hbar^2}{36 \Delta_0^2} \left( -\omega^2 + \frac{v_F^2 k^2}{3} \right) + \ldots \right], \tag{27}
\]
\[
\operatorname{Re} k_0^+(k, \omega) = \pm \frac{N(0) \hbar \omega}{4 \Delta_0} \left[ 1 - \frac{\hbar^2}{6 \Delta_0^2} \left( -\omega^2 + \frac{v_F^2 k^2}{3} \right) + \ldots \right]. \tag{28}
\]

In order to obtain the imaginary part, we have to perform an analytic continuation carefully by representing the Matsubara sum in the form of a contour integral \cite{23}.

The final result is,
\[
\operatorname{Im} \frac{1}{\hbar \beta V} \sum_{p,m} G_\downarrow(p, \varepsilon) \mathcal{F}(p + k, \varepsilon + \omega_n)
\]
\[
= \frac{1}{(2\pi)^4} \int dp \int_{-\infty}^{\infty} d\varepsilon \operatorname{Im} G_\downarrow R(p, \varepsilon) \operatorname{Im} F_R(p + k, \varepsilon + \omega)
\]
\[
\times \left( \tanh \frac{\hbar \varepsilon}{2 k_B T} - \tanh \frac{\hbar (\varepsilon + \omega)}{2 k_B T} \right). \tag{29}
\]

In the limit of long-wavelength, i.e. \( \frac{\hbar \omega}{2 \Delta_0} \ll 1 \) and \( \frac{\hbar \omega}{\Delta_0} \ll 1 \), the imaginary parts of bubbles can be obtained. Assuming \( g_\uparrow = g_\downarrow \) and \( \mu_\uparrow = \mu_\downarrow \) for simplicity, we get

\[
\operatorname{Im} f_0(k, \omega, T) = -\frac{N(0) \pi \Delta_0^2 \omega}{2 k_B T \hbar \omega} \int_{\Delta_0}^{\infty} \frac{dE \sech^2 E}{E^2 - \Delta_0^2}, \tag{30}
\]
\[
\operatorname{Im} g_0(k, \omega, T) = -\frac{N(0) \pi \omega}{2 k_B T \hbar \omega} \int_{\Delta_0}^{\infty} \frac{dE}{E^2 - \Delta_0^2} \times \frac{2 E^2 - \Delta_0^2 \sech^2 E}{2 k_B T}, \tag{31}
\]
\[
\operatorname{Im} h_0(k, \omega, T) = \operatorname{Im} f_0(k, \omega, T), \tag{32}
\]
\[
\operatorname{Im} f_0(k, \omega, T) = \frac{\Delta_0}{\hbar \omega} \operatorname{Im} (k_0^+(k, \omega, T) - k_0^-(k, \omega, T)), \tag{33}
\]

where \( E = \sqrt{E^2 + \Delta_0^2} \) and \( c = \frac{\omega}{v_F k} < 1 \). (33) can be verified from the Ward identity of \( \sigma_0 \) rotation \cite{23}. The remaining integration is done numerically.

D. Inverse Green’s Function of Mixture

The second order of fluctuation gives us the desired dispersion relation of the mixture. In analogy to the pairing field, we separate the Bose field to it’s phase and amplitude as well,
\[
\phi'(k, \omega) = \phi_\Lambda(k, \omega) + i \phi_\varphi(k, \omega). \tag{34}
\]

Introducing a vector notation,
\[
\Psi^1 = \left( \phi_\Lambda(k, \omega), \ \phi_\varphi(k, \omega), \ \delta(k, \omega), \ \theta(k, \omega) \right), \tag{35}
\]
the second order of fluctuation can be written in a matrix form,
\[
S^{(2)} = \frac{1}{2 \pi V} \sum_k \int d\omega \Psi^1 G^{-1}(k, \omega, T) \Psi, \tag{36}
\]
where \( G^{-1} \) is the inverse of Green’s function of the mixture,
The dispersion relation of the mixture can be found by requiring determinant of (37) equals zero.

\[ G^{-1} = \begin{pmatrix}
M_{AA} & i\hbar\omega & M_{A\delta} & M_{A\theta} \\
-i\hbar\omega & \hbar^2 k^2 & 0 & 0 \\
-M_{A\delta} & 0 & M_{\delta\delta} & 0 \\
-M_{A\theta} & 0 & 0 & M_{\theta\theta}
\end{pmatrix}, \quad (37)
\]

with matrix elements:

\[
M_{AA} = \frac{\hbar^2 k^2}{2m_B} + 2g_B n_B + g_A^2 n_B(g_0(k) - f_0(k)) + g_B^2 n_B(g_0(k) + f_0(k)), \quad (38)
\]

\[
M_{A\delta} = g_D\sqrt{n_B}(k_0^+(k) + k_0^-(k)), \quad (39)
\]

\[
M_{A\theta} = -ig_A\sqrt{n_B}(k_0^+(k) - k_0^-(k)), \quad (40)
\]

\[
M_{\delta\delta} = h_0(k) + f_0(k) - \frac{1}{g_F}, \quad (41)
\]

\[
M_{\theta\theta} = \Delta_0^2(h_0(k) - f_0(k) - \frac{1}{g_F}). \quad (42)
\]

The dispersion relation of the mixture can be found by requiring determinant of (37) equals zero.

**III. RESULTS AND DISCUSSIONS**

In the limit of long-wavelength linear dispersion, the real part of the dispersion relation in absolute zero can be derived analytically,

\[
\omega^2 = \left\{ \frac{1}{2} \left( \frac{g_B n_B}{m_B} + \frac{v_F^2}{3} \right) \right. \left. \pm \sqrt{\frac{1}{4} \left( \frac{g_B n_B}{m_B} + \frac{v_F^2}{3} \right)^2 + \frac{g_A^2 N(0)v_F^2 n_B}{6m_B}} \right\} k^2, \quad (43)
\]

plus and minus sign in front of the square root corresponds to the two eigen modes of the mixture, Anderson mode and Bogoliubov mode. We can see from the last term inside the square root that there is a repulsion depending on average interaction between boson and fermion \(g_A\). If the Anderson velocity exceeds Bogoliubov velocity, this effect stiffens the Anderson mode and softens the Bogoliubov mode, and vice versa. A simple plot of \(g_A\) dependence to this effect is plotted for a superfluid mixture of fermionic \(^6\)Li (Fig.1) and bosonic \(^{87}\)Rb (Fig.2) as an example.

As the interaction becomes strong enough to satisfy the condition

\[
a_A^2 > \frac{4\pi a_B m_B^2}{k_F m_B m_F}, \quad (44)
\]

the frequency of slower mode becomes a purely imaginary number. This corresponds to a instability, however, phase separation of boson and fermion occurs before this can be observed.

Including the imaginary parts of bubbles, damping of the two modes due to boson-fermion coupling can be obtained by finding the numerical poles of (37), roots are found to be in the form of \(\omega = (c - i\gamma)k\) where the imaginary part is \(k\) dependent. A plot of \(\gamma\) in mixture of \(^6\)Li and \(^{87}\)Rb is shown in Fig.3. Note that there is no damping at absolute zero, the reason is that quasi-particle does not exist and therefore no real process of quasi-particle excitation can take place. The imaginary part is rather small even at relatively high temperature which implies that the two superfluid modes are stable. However, the effect of pair breaking and Landau damping \(\gamma\) will be important near the superfluid transition temperature of fermion \(T_c,F\), which is out of the scope of our calculation. Bose-Fermi superfluid mixture near \(T_c,F\) will be discussed elsewhere.

**FIG. 1.** Modification of Anderson mode in \(^6\)Li and \(^{87}\)Rb superfluid mixture varying the boson-fermion interaction. Densities of fermion and boson are taken to be \(4 \times 10^{12}\) cm\(^{-3}\) and \(10^{15}\) cm\(^{-3}\) respectively. \(a_0\) is the Bohr radius.

**FIG. 2.** Modification of Bogoliubov mode in \(^6\)Li and \(^{87}\)Rb superfluid mixture varying the boson-fermion interaction. Densities of fermion and boson are taken to be \(4 \times 10^{12}\) cm\(^{-3}\) and \(10^{15}\) cm\(^{-3}\) respectively. \(a_0\) is the Bohr radius.
IV. CONCLUSIONS

We have calculated the dispersion relation and damping of a dilute mixture of boson and two component fermion. Repulsion between the Anderson mode and Bogoliubov mode due to boson-fermion interaction is found. Instability of boson superfluidity is predicted in strong coupling regime, where phase separation is also predicted [9]. Damping is found to be small in low temperature region where pair breaking effects are negligible.

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