Comparison between the residue-vector map and existing quality maps for phase unwrapping

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Abstract. Many robust phase unwrapping algorithms rely on having good quality or weighting maps. In essence, we present a general comparison between the residue-vector map and existing quality maps for phase unwrapping to evaluate the importance of the newly established residue-vector technique in the phase unwrapping context. Such quality maps used for comparison are phase derivative variance, maximum phase gradient, pseudo-correlation, weighted phase derivative variance and second difference. Flynn’s minimum discontinuity phase unwrapping algorithm was used on the same wrapped phase but with different quality maps for result evaluation. It was found that the residue-vector map was the most robust of all.

1. Introduction

Phase unwrapping problem has been approached by many researchers for the last two decades and many algorithms were developed trying to solve this problem. Phase unwrapping is the problem of retrieving continuous phase from wrapped phase maps wrapped into the interval $[-\pi, \pi]$. The lack of a good quality map for phase unwrapping has diminished the chance of having a phase unwrapping algorithm that could solve any phase unwrapping problem. A new quality map is proposed which relies on information provided by what we will term the residue-vector.

The residue-vector can be used as a weighting factor to optimize the results of algorithms employing local, global and hybrid phase unwrapping methods. It can optimize the results of the phase unwrapping to a degree never achieved by existing weighting quality map methods such as: phase derivative variance, maximum phase gradient, pseudo-correlation, correlation, weighted phase derivative variance and second-difference. This residue-vector map created by residue-vector extraction is more problem-specific to the phase unwrapping problem. It combines both the concept of the quality maps and the knowledge of the residues and their branch-cuts based on the fact that residues are the cause of the problem of the phase unwrapping problem.

In this paper, a brief description of the residue-vector and the method used for the residue-vector extraction is presented. Then, a general comparison is made between the residue-vector map and other existing quality maps by assessing the results obtained from Flynn’s minimum discontinuity algorithm using different quality maps.
2. Residue-Vector

Residues in a wrapped phase map generate a vector which can be viewed by calculating and displaying the first derivative in x and y directions (i.e., dx and dy). This residue-vector can be seen in the dx and dy gradient phase maps in Figs. 1(c) and (d), respectively extracted form the wrapped phase map in Fig. 1(a) of the simulated spiral object shown in Fig. 1(e) and its residue distribution is shown in Fig. 1(b). Figs. 1(c) and (d) show the behaviour of the residue-vector in the x and y directions. In Fig. 1(f), an enlarged single residue-vector is shown to illustrate how the residue-vector appears in the gradient phase map. Usually residue-vector orientation follows edges, shadows, areas with under-sampling and phase noise.

A residue-vector is a vector generated by a residue in the phase map that has a certain orientation pointing out to the balancing residue of opposite polarity.

![Residue-Vector Image]

Figure 1. (a) A 257×257 simulated spiral wrapped phase map (computer simulated object from Ghiglia and Pritt)(b) its corresponding residue map, a wrapped phase gradient in the (c) x direction, (d) y direction sense; (e) original 3D surface of the spiral and (f) a magnified version of the residue-vector of a single residue.

In essence, residues have vector fields that are very directional in nature. This directional vector field can only point to the balancing opposite polarity residue. These vector fields are very visible in the case of noise and under-sampling. With the increase of phase noise, the residue vector grows smaller but it stays strong at the residue. At very high noise, residues will get closer until they become dipoles of one sample apart. The behaviour of the residue-vector in the wrapped phase map under different conditions can give useful information on how to solve the branch-cut problem.
If the residue-vector is not disrupted with other dipoles along the same edge or discontinuity, the dipoles would share the constant residue-vector between them as in the example in Figs. 2(a) and (b) and illustrated in Fig. 2(f).

However, once the residue-vector is disrupted by a residue or a dipole lying between them and having the same residue vector direction, it will not have the necessary power to overcome the vector of the residue or the dipole lying between it and the balancing residue. This case is called zero-vector or null or neutral-vector and illustrated in Fig. 2(g). This zero-vector makes it difficult to extract the original continuous vector of the first dipoles. This case can be seen in Figs. 2(e), (h) and (i).

**Figure 2.** Residue-vector between dipole residues taken from Fig. 1(d) shows a constant vector charge shared between the residues of the dipole (a) and (b) where charge is varying with the nature of discontinuity whether descending or ascending, (c) original 3D simulated object of a quarter pyramid with a square hole in the middle, (d) its wrapped phase map, (e) the overlapped wrapped phase gradient of dx and dy, (f) a schematic showing a constant vector charge shared between the residues of the dipole, (g) a schematic showing how the zero-vector is created, (h) residue-vector orientation of the four upper right corner residues in Fig. 2(e), (i) a schematic of the residue-vector orientation of the four upper right corner residues.
3. Residue-Vector Map

In the previous section, we have defined and explained briefly the characteristics of the residue-vector. We will now move on to consider how this residue-vector can be used as a criterion for the placement of branch cuts. A neutral unwrapped phase map is defined by having all residues in the phase map neutralized by branch-cuts. This does not necessarily indicate that this set of branch-cuts will result in the correct phase estimate in the unwrapped phase. This leads to a new concept which specifies that for a set of branch-cuts in a phase map to achieve the minimum error between the estimated gradient of the unwrapped phase solution and the gradient of the wrapped phase map; branch-cuts should follow the maximum number of residue-vector pixels separating each pair of opposite polarity residues in a branch-cut and the minimum number of pixels branch-cut in the phase map. In other words, the Flynn minimum discontinuity and the minimum cost flow algorithms attempt to identify the lines of discontinuity representing the branch-cuts between residues; however, they are trying to identify the residue-vectors that result in such discontinuities. Moreover, due to the mentioned algorithms complete reliance on the weights provided; they try to approximate the position of the residue-vector to branch-cut in the phase map. In essence, they do not have problem-specific knowledge of what is causing the discontinuity they are trying to identify. We can rectify this shortcoming by using the residue-vector to orientate the dipole branch-cuts. This method is a general form of branch-cut which could form a cut or a tree of cuts, which could make it very suitable to nearly all types and variants of branch-cut phase unwrapping algorithms. The residue-vector Branch-cut method using dipole strategy can be summarized as following:

- Calculate the phase gradient in the x and y directions.
- Identify positive and negative residues in the wrapped phase map.
- Start from a random residue in the phase map.
  - Identify the residue-vector by locating consecutive pixels with high and low gradient values in the x and y phase gradient maps, respectively.
  - Follow the residue-vector on both sides of the residue until a balancing residue of opposite polarity is found or until the residue-vector reaches a zero-vector. If a zero-vector is found:
    - Identify the nearest residue-vector of another residue and include pixels of the highest and lowest gradient located between the two vectors.
    - Iteratively locate the nearest residue-vector of another residue closer to the last residue-vector until a balancing residue of opposite polarity of the starting residue is found.
  - Branch-cut all the included pixels of this pair of residues.
- Repeat the same procedure until all the residues are balanced by branch-cuts created by this method.

The residue-vector map can then be inserted as weights in different phase unwrapping algorithms such as Flynn and minimum cost flow or it may be optimized by minimizing the number of branch-cut pixels ensuring the maximum number of residue-vector pixels included and all residues are balanced. The successful branch cut placement should obey the residue-vector orientation in the gradient phase map to balance two dipole residues.

To demonstrate this fact, an example that shows the incorrect and correct branch-cut placement is displayed in Figs. 3(a), (b) and (c). Fig. 3(a) shows the dipole residue-vector with the residues positions and their polarity indicated by arrows. It can be seen from the figure that the residue-vector follows a curved path. Hence, it is incorrect to place a straight branch-cut as in Fig. 3(b) that only takes in consideration the distance of the branch-cut between the dipole residues and does not recognizes the residue-vector. On the other hand, Fig. 3(c) shows the correct branch-cut placement that completely follows the residue-vector lying between the two dipole residues with the minimum number of branch-cut pixels.

Residue-vector gives all the information necessary to know how to place a branch-cut in order to balance the discontinuity in the residue. The information provided by the residue vector includes; direction, pixels to be branch-cut, and destination to an opposite polarity pixel or a border pixel.
Figure 3. Branch-cut placement methods used to connect two residues; (a) original dy gradient map taken from Fig. 1(d), (b) incorrect branch-cut placement using straight line cuts and (c) correct branch-cut placement obeying the residue-vector rule.

Figure 4. Unwrapped phase map produced by Flynn’s algorithm\(^1\) with zero-weights provided by the mask of the (a) minimum phase variance quality map, (b) residue-vector map, (c) maximum phase gradient quality map, (d) pseudo-correlation quality map, (e) weighted phase variance quality map and (f) second difference quality map.
3. Results and Discussion

A simulated wrapped phase map with residues was used to verify the residue-vector concept. The wrapped phase map and its corresponding residue map are shown in Figs. 1(a) and (b) respectively.

Moreover, by examining the unwrapped results of the spiral by using the masks of the quality maps mentioned previously, it is clear from Fig. 4(a) that Flynn’s algorithm with phase variance quality map cannot unwrap the spiral edges properly because of the random discontinuities or branch-cuts generated by the method to overlap the residue-vector even though succeed in unwrapping the spiral. On the other hand, Flynn’s algorithm with residue-vector map, as seen in Fig. 4(b), produces successful unwrapped result with high precision in unwrapping at the spiral edges. This is because in this case it is able to solve the problem without random discontinuities or branch-cuts.

In the case of the unwrapped result by the Flynn’s algorithm using the mask of the minimum gradient quality map shown in Fig. 4(c), the algorithm fails in unwrapping the spiral successfully because the algorithm did not branch-cut zero-vector pixels located in the wrapped phase map. Moreover, the unwrapped results by the Flynn’s algorithm using the mask of both pseudo-correlation and weighted phase variance quality maps, respectively are shown in Figs. 4(d) and (e). The algorithm completely fails in unwrapping the spiral due to the same reasons as the results produced by the mask of the maximum gradient quality map.

On the other hand, the unwrapped result produced by Flynn’s algorithm using the mask of the second difference quality map is shown in Fig. 4(f). The result is partially successful except for the local failure due to a $2\pi$ discontinuity introduce by a mis-placed branch-cut from the spiral arm to the border. But generally, this quality map produces a result similar to that of the phase derivative variance quality map.

4. Conclusion

In conclusion, residue-vector extends the phase unwrapping residue theory into a new prospective and lay over the basis of future successful phase unwrapping technique. The cause of failure of many unwrapping algorithms was presented in the examples introduced in this paper. Especially, the presence of the zero-vector in the phase map which will stand against robust phase unwrapping algorithms. Moreover, it was shown that existing quality maps not always produce successful results even the best of them, this is because they are not problem-specific to phase unwrapping not like the residue-vector map that can always produce successful unwrapped results no matter the application.

From the results presented, it can concluded that the residue-vector not just give information on how to balance a set of residues; it also takes in consideration object edges and shapes. Thus, residue-vectors never violate object shape but follow object edges. In essence, a quality map based on the residue–vector information will always preserve object shape and in the mean time it balances residues effect in the phase map.

Reference

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