Design of double negativity elastic metamaterial

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A metamaterial model that possesses simultaneously negative effective mass density and negative effective Young’s modulus is proposed in this study. Dispersion curves and dynamic responses of the model are investigated. In the double negative frequency region, it is demonstrated that the phase velocity is negative. In addition, it was found that the band gap region of the metamaterial can be predicted accurately by taking parts of single-unit cell to analyze the steady-state response. The design is also fabricated by a 3D printer.

Keywords: acoustic metamaterial; double negativity; local resonance; negative mass; negative modulus

1. Introduction

Materials with periodic structures are well known for their use in filtering waves. However, since its underlying principle for blocking waves is to apply impedance variation, the wavelength of the gap must be the same order as the lattice constant of the periodic structure. In the field of mechanics, blocking low-frequency waves is generally more desired than blocking those with high frequencies, thus rendering the periodic structure impractical due to its large lattice constant. To overcome the limitation of low-frequency wave mitigation in periodic structures [1], Liu et al. first applied local resonance to fabricate an acoustic metamaterial with negative effective mass [2]. Using lead balls coated with silicone rubber, they found that the metamaterial could filter waves with wavelengths two orders larger than its lattice constant. Since then, acoustic metamaterials have become a hot topic among researchers [3–5]. In the area of elastic metamaterials, Huang and Sun [6] connected resonators using rigid and massless trusses to create a negative effective Young’s modulus (NYM) metamaterial.

By combing negative effective mass and negative effective modulus, one can obtain ‘double negativity metamaterials’. Although a number of double negativity (DN) acoustic metamaterials have been investigated [7–15], there are few reports in literature on DN elastic metamaterials. Among the elastic metamaterials that have been published, Wang theoretically investigated the mechanism of negative mass and/or negative modulus of lumped mass models [16]. Jin et al. proposed a broadband DN metamaterial using active control [17]. Pope proposed the designs of DN elastic metamaterials using electrical–mechanical circuit analogies [18]. Soon after, Pope and Laalej performed the first active DN elastic metamaterial

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experiment [19]. Using composite materials to make elastic metamaterials, Wu et al. [20] and Lai et al. [21] numerically validated the DN. Liu et al. [22] proposed the idea of chiral metamaterials and validated it through theoretical analysis and numerical simulation. Expanding upon that concept, Zhu et al. [23] created a single-phase solid based on the lumped mass model. The present work is based on the lumped mass model proposed in [24], and the metamaterial was fabricated as a single-phase solid using a 3D printer.

2. Metamaterial models and design process
The metamaterial design in this study is based on the lumped mass model [24]. Since the model is still in the theoretical stage, this study realizes it using an Eden 350 3D printer (Stratasys Ltd., Eden Prairie, MN, USA).

Figure 1 shows the design process for determining the dimensions of NYM resonator. To change the lumped mass model into a single-phase solid, we used beam bending to replace the vertical spring. After performing parametric studies on the length and thickness of the horizontal beam, we found that a beam with a length of 30 mm and width of 1 mm can provide resonance at the frequencies of interest. It is noted that due to geometric complexity it is impractical to use theoretical analysis to calculate local resonance. Instead, we numerically estimate the local resonances by the finite element analysis. Since the inclined frame is only used for transmitting force, the width of the frame is designed as 1.5 mm to make the mode of frame rotation/stretching occur at higher frequencies. For the mass of the NYM resonators, we found that the arrow shape gives the best spatial usage.

After determining the geometry, we proceeded to adjust the material properties in each part. Furthermore, since the thickness is converted into effective material properties in the simulations, the range of effective material properties is set to be $E = 5–20$ GPa,

![Figure 1](image_url)
\( \rho = 1300 - 5200 \text{ kg/m}^3 \) (\( E = 5 \text{ GPa} \) and \( \rho = 1300 \text{ kg/m}^3 \) are the material properties of the high-temperature photopolymer ink that was used for the 3D printing). Figure 1b and Table 1 show the finalized parts of the unit cell of the metamaterial that produces NYM. The characteristic length of the unit cell, \( L \), is 40 mm.

In the design of the negative effective mass density (NMD) resonator, as shown in Figure 2, we use the same procedure to investigate the length of the vertical beam, which serves as the horizontal spring of the lumped mass model. The shape of the NMD resonator is also determined according to the efficiency of spatial usage. Figure 2b and Table 2 show the finalized parts of the unit that produces NMD. The vertical beams supporting two triangular-shaped masses serve as two horizontal springs. The top and

| Marked area | Mass density (kg/m\(^3\)) | Young’s modulus (GPa) | Poisson’s ratio |
|-------------|---------------------------|-----------------------|---------------|
| Blue        | 1300                      | 5.0000                | 0.49          |
| Purple      | 2350                      | 9.0385                | 0.49          |
| Green       | 1742                      | 6.7000                | 0.49          |

**Figure 2.** Design of the NMD resonator based on the lumped mass model: (a) NMD lumped mass model; (b) NMD submodel.

| Marked area | Mass density (kg/m\(^3\)) | Young’s modulus (GPa) | Poisson’s ratio |
|-------------|---------------------------|-----------------------|---------------|
| Blue        | 1300                      | 5.0000                | 0.49          |
| Purple      | 2350                      | 9.0385                | 0.49          |
| Green       | 1742                      | 6.7000                | 0.49          |
| Orange      | 4000                      | 15.3846               | 0.49          |
| Red         | 3354                      | 12.9000               | 0.49          |
bottom horizontal beams are restrained from vertical motion. The characteristic length of the unit cell, \( L \), is also 40 mm.

The DN model is the same as the NMD model except that the horizontal beams are free to move vertically. The material parameters for the DN model are also listed in Table 2. In order to have a DN frequency region, the dimensions for the NYM and NMD resonators are designed to have overlapping band gaps.

### 3. Dispersion curves for the DN model

Dispersion curves show the frequencies at which harmonic waves can propagate without amplitude attenuation. Frequencies between two dispersion curves locate the stopping band.

Considering 1D longitudinal harmonic wave propagation, the displacement can be expressed as

\[
u(x,t) = Ae^{iq(x-ct)} = Ae^{iqx}e^{-i\omega t}
\]

where \( c \) is the phase velocity and \( q \) is the wave number. By selecting two locations as nodes (zero displacement) in simulation, Equation (1) can be treated as a free vibration problem of a finite length body with fixed end conditions. As demonstrated in Figure 3, if the harmonic wave denoted by the blue line is the longest wavelength propagating within the infinite number of metamaterial unit cells, it can be modeled by setting the harmonic waves with fixed end condition at the left and right edges of either 400 unit cell, 200 unit cell or 100 unit cell metamaterial strip. In this study, we found that the 100 unit cell strip is long enough to model the dispersion curves of the infinite metamaterial chain.

To model the infinite number of unit cells metamaterial by the 100 unit cells, we constrained the horizontal movements of the left and right boundaries of the 100 unit cell metamaterial strip, so that these two boundaries became nodal points. The result of the free vibration analysis yielded the natural frequencies and mode shapes, from which the corresponding wave numbers were obtained.

We used ABAQUS commercial software (finite element software by Abaqus Inc., Providence, RI, USA) for finite element analysis. Since beam bending is used as the

![Figure 3](image-url)

Figure 3. (a) Infinite long metamaterial chain with a harmonic wave (blue line); (b) 400 unit cell metamaterial strip modeling; (c) 200 unit cell metamaterial strip modeling; (d) 100 unit cell metamaterial strip modeling.
resonator’s stiffness, accuracy for the beam bending mode estimation is crucial. For element-type selection, the incompatible mode element (CPS4I) was used owing to its efficiency for bending calculation. Four CPS4I elements are placed along the thickness of the beam to achieve element number convergence.

Figure 4 shows the dispersion curve of the DN metamaterial. Each point on the dispersion curve represents an eigenmode (calculated by ABAQUS) of the 100 unit cell long metamaterial strip.

As shown in Figure 4, the frequencies in the DN region are 668–703 Hz. The DN region is recognized by negative phase velocity \([24]\). It is noted that the narrow passing band, 579.28–579.37 Hz, is owing to the rotational mode of the horizontal beam about the joint.

4. Wave propagation in the DN model

The negative phase velocity is one of the characteristics of DN metamaterials. This phenomenon can be observed from wave propagation. Hence, we also investigated the dynamic behavior of the DN metamaterial to demonstrate the negative phase velocity.

Since a large number of finite elements are used for beam bending calculation in each unit cell, accumulated numerical error is an issue for ABAQUS Dynamic Explicit packet. To deal with this numerical problem, Dynamic Implicit version is used instead. A long metamaterial strip (200 unit cells) was built to delay wave reflection, and a sinusoidal displacement with a frequency of 685 Hz was applied at the first unit cell (metamaterial strip with wave speed of 67.7966 m/s and wavelength of 340.4255 mm at 685 Hz). As before, the vertical displacements of the upper and lower boundaries of the unit cells are constrained because of the plane wave assumption. The negative phase velocity is observed by tracking the responses of three proximal unit cells.
Since the initial condition is quiescent before the application of the sinusoidal motion at the frequency of 685 Hz, the steady-state frequency is reached after a period of time. As shown in Figure 5, the negative phase velocity is observed after 0.186 s. It is noted that the wave amplitude has no decay since 685 Hz is within the DN frequency range and there is no damping assumption in the simulation.

5. Band gap region determination by single-unit cell

To ensure that the negative phase velocity of the DN model is caused by overlapping NYM and NMD band gaps, we present a procedure for band gap prediction using single-unit cells (NYM submodel and NMD submodel).

The NYM submodel is shown in Figure 1b and Table 1. The frequency-dependent effective modulus of a metamaterial can be obtained by applying symmetrical loading $F_0 \cos \omega t$ on the representative unit cell, as shown in Figure 6. The corresponding displacements at the right and left edges of the NYM unit cell are $u = +u_0 \cos \omega t$ and $u = -u_0 \cos \omega t$, respectively. The change of the unit cell dimension is $2u_0$, and the dynamic stiffness of the unit cell is given by

$$k = \frac{F_0}{2u_0} \tag{2}$$

It is noted that the stiffness of the unit cell defined by Equation (2) is proportional to the effective Young’s modulus of the metamaterial. The NYM band gap region is defined as the frequency range in which $k$ is negative. By using this procedure, the band gap region is 588.6–704.0 Hz (see Figure 7).

For comparison, we also used 100 unit cells to calculate the dispersion curves for the NYM submodel, as shown in Figure 8, from which the NYM band gap region is
588–704 Hz, which agrees with the band gap obtained from the unit cell. The narrow passing band, 576.01–576.61 Hz, is the horizontal beam rotational mode. In Figure 7, the lower bound of the NYM band gap shows out-of-phase motion on the left and right edges of unit cell, which indicates short wavelength corresponding to dispersion curve (the shortest wavelength in dispersion curve is $2L$). On the contrary, the upper bound of the NYM band gap shows in-phase motion (zero displacement) on the left and right edges of unit cell, which corresponds to the longest wavelength of optical branch in dispersion curve.
The NMD submodel is made by suppressing the bending motions of the two green horizontal beams, as shown in Figure 2b. The effective mass density of the NMD submodel can be obtained from the dynamic response of the representative unit cell loaded as shown in Figure 9. We also set uniform displacement as an unknown on the left and right edges of the NMD unit cell.

Figure 8. Dispersion curves for the NYM submodel.

Figure 9. NMD unit cell subjected to antisymmetrical loading for testing effective mass.
The effective mass of the unit cell is given by

\[ m_{\text{eff}} = -\frac{F_0}{u_0\omega^2} \]  

From the result of the finite element simulation, Figure 10 shows the displacement spectrum. The band gap region is defined as the frequency range where \( u_0 \) becomes positive, 625.3–703.7 Hz, in which \( m_{\text{eff}} \) has negative value.

Similarly, for validation, we also use the 100 unit cells of the NMD submodel to evaluate the band gap region from the dispersion curves, as shown in Figure 11. The NMD band gap region is 625–705 Hz. Again, this agrees with that obtained from the unit cell of the NMD submodel. In Figure 10, the lower bound of NMD band gap shows out-of-phase motion on the left and right edges of NMD unit cell, which indicates shortest wavelength in the acoustic branch of dispersion curve. On the contrary, the upper bound of NMD band gap shows in-phase motion on the left and right ends of unit cell, which corresponds to the longest wavelength in the optical branch of dispersion curve.

A similar numerical method of effective medium theory can be found in \([21,22]\). The only variation is the calculation of average stress and strain in the unit cell, and their unit cell predictions have not been investigated quantitatively in the studies.

6. 3D printing fabrication

The metamaterial proposed in this study can be fabricated by a 3D printer. Since 3D printing is three-dimensional with varying thicknesses, we use the following principles to convert the 2D metamaterial model with a uniform thickness to a 3D specimen.
(1) The effective Young’s modulus in a 2D model is proportional to the thickness of the specimen.
(2) Mass density is proportional to the thickness of the specimen.

Figure 12 shows the DN metamaterial specimen, which was fabricated using an Eden 350 3D printer, based on the calculations in this study. The fabrication process can be divided into three steps: preprocessing, production, and support removal.
In the preprocessing step, we built the specimen into a 3D form of various thicknesses using the CAD software, CATIA, and input the CAD file into the Eden 350. The 3D printing jet then dripped liquid photopolymer layer-by-layer to build the 3D model. We used high-temperature material RGD 525 as the photopolymer ink to produce the specimen. The liquid photopolymer was then cured by UV light during processing. It is noted that the 3D printer also dripped removable gel-like supporting materials on the parts with weaker structures to stabilize them. Finally, we used a water jet to remove the supporting materials and obtain a metamaterial specimen with a 16 µm layer resolution and 0.1 mm accuracy.

7. Conclusion
A DN elastic metamaterial has been designed. The dynamic behavior of the elastic metamaterials was studied. The negative phase velocity of the DN model is demonstrated by both the dispersion curve and wave propagation. The metamaterial proposed in this study is fabricated using a 3D printer by converting distinct material properties into different thicknesses in the individual parts of the metamaterial.

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Disclosure statement
No potential conflict of interest was reported by the authors.

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