Macroeconomic aspects of maintenance optimization of critical key assets

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Abstract. The main goal of maintenance is prevention, timely detection and elimination of failures and damage. From the point of view of critical infrastructures (CIs), the main purpose of their maintenance is to increase the safety of CIs and / or to ensure life safety. CIs should be optimal in terms of their purpose, cost, as a source of income and profit at all stages of their life cycle, and also acceptable in terms of possible loss of human lives or injuries. The paper considers the assessment of necessary optimal investments in maintenance (time interval between subsequent maintenance) for ensuring life safety, the so-called life saving costs. The problem of limiting the threat to human life is as follows: how much money is society ready and able to spend to reduce the likelihood of a premature death. A quantitative criterion of risk acceptance is used, in which the marginal life saving cost (MLSC) is compared with social willingness to pay. This value is determined using the life quality index.

1. Introduction
Optimization can be carried out either by a private or social decision maker (DM). A private DM means a company that builds and/or manages an object (and possibly creates risks for society). Social DMs are government agencies or organizations that manage risks on behalf of society. The costs and benefits of making a decision, as well as preferences, which form the basis for comparing different alternatives, may vary depending on the interests of the DM.

The paper uses a quantitative criterion of risk acceptance [1], in which the marginal life saving cost is compared with social willingness to pay (SWTP). This value is determined using the life quality index (LQI) of population, which was obtained on the basis of socio-economic assumptions using the GDP per capita and the average life expectancy (ALE). The LQI criterion makes it possible to determine a threshold that separates effective investments from inefficient investments in terms of saving lives. Life saving costs can be increased only up to the effectiveness threshold, in our case, SWTP.

Investing in safety measures (in our case, during the time interval between subsequent maintenances) leads to both monetary expenditures and the benefits of a safe life. Cost reductions achieved through investments in security, such as reducing the cost of eliminating the consequences of a failure, are considered to be the benefit of a safety measure. The benefits of improved life safety are determined by a marginal change in the expected number of deaths per year. Costs and benefits can accumulate at different moments of time, so they have to be discounted.

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2. Fundamentals of the theory of discounting and social utility of consumption

The discounted or present value (PV) is an estimate of the value, that is, the current monetary equivalent of the future flow of payments based on the different value of money received at different points in time, the so-called concept of the time value of money.

As a tool to compare the benefits received by the society and the costs incurred, the market discount rate is not applicable. In world economic practice, such an instrument is the social (public) discount rate. The social discount rate is an alternative opportunity for society to use resources between two time periods, a kind of “price” at which society is ready to abandon today’s consumption for tomorrow, which reflects the social norm of time preference.

In this paper, we use the method of assessing the social rate of time preferences (SRTP), which is based on identifying society’s preferences in terms of consumption. The social rate of time preferences shows the readiness of the society to abandon consumption at the present time in order to implement the project and receive benefits from its results in the future.

Utility is the degree of satisfaction of the needs of individuals, which they receive when consuming goods or services or from conducting any activity. There are two forms of utility: total and marginal. Total utility is the utility obtained as a result of consumption of all units of the good. Marginal utility is the utility that is obtained from the use of another additional unit of the good.

3. The utility function of (whole) life

An utilitarian approach is adopted, according to which social welfare (social utility) is the sum of welfare (sum of utilities) of individual members of society, and marginal utility decreases with increasing consumption (the first Gossen’s law). This is the law of diminishing marginal utility - with the growth of consumption of one good (with a constant consumption of all other goods), total utility increases, but the growth rate slows down.

The enjoyment of life or its usefulness in the economic sense is due to the continuous influx of resources available for consumption throughout the whole life. Therefore, the income necessary for consumption, and the time to enjoy it, are two determining factors in the life quality.

According to [2,3], life quality in the economic sense can be measured using the utility function of life expectancy at age $x$ [2,3]:

$$U(x,D) = \int_{x}^{D} u(c_\tau) d\tau,$$

where $c_\tau > 0$ is the rate (intensity) of consumption at the age of $\tau$ ($$/\text{year}); u(c_\tau)$ is the utility of this consumption from age $x$ to the moment of death $D$.

The function $U(x,D)$ can be interpreted as total consumption over the remaining lifetime.

People usually appreciate the opportunity of consumption in the future less than in the present. This can be accounted for by discounting the utility function:

$$U(x,D) = \int_{x}^{D} u(c_\tau) \exp\left[ -\int_{x}^{\tau} \gamma(\theta) d\theta \right] d\tau,$$

where $\gamma(t) > 0$ is the discount rate of utility consumption at age $t$, reflecting the time preferences of the individual. As noted above, the social discount rate is used as an estimate of $\gamma(t)$.

Solving the problem of maximizing social utility for consumption

$$U(x,t) = \int_{x}^{t} u(c_\tau) \exp\left[ -\int_{x}^{\tau} \gamma(\theta) d\theta \right] d\tau \rightarrow \max_{c_\tau},$$
according to the Ramsey model [4], we obtain the social discount rate \( \gamma(t) = \rho(t) + \mu \delta(t) \), where \( \delta(t) \) is the per capita consumption growth rate; the parameter \( \rho(t) \) means the subjective discounting rate of future consumption (parameter of impatience, selfishness).

Since the time of death is random, the utility of an individual's life is also a random variable. It can be shown that the average expected discounted utility of the residual life expectancy at age \( x \) (assuming surviving to this age) can be calculated by the formula [5, 6]

\[
L(x) = \frac{1}{S(x)} \int_{x}^{\infty} S(t) u(c_t) \exp \left[ -\int_{t}^{\infty} \rho(\theta) d\theta + \mu \delta(t - x) \right] dt
\]

(4)

where \( D_n \) is the maximum age to which people live in the world (or the maximum age of the cohort under consideration (in the region, industry, company); \( S(t) \) is the survival function.

In conditions of perfect competition (ideal market), optimal consumption does not depend on time \( t \) [7]. In this case, the formula (7) takes the form:

\[
L(x) = \frac{u(c)}{S(x)} \int_{x}^{\infty} S(t) \exp \left[ -\int_{t}^{\infty} \rho(\theta) d\theta + \mu \delta(t - x) \right] dt = u(c) e_x(x, \rho, \mu, \delta)
\]

(5)

where \( e_x(x, \rho, \mu, \delta) \) is the discounted ALE at age \( x \) [5,6].

4. Life quality index

The question: how much should be sacrificed from the life quality (utility of consumption) in order to get a certain increase in life expectancy at the cost of risk reduction Nathwani et al. [8] used the hypothesis: “On average, people work just so much that the marginal cost of their wealth/welfare or the income they receive for their work equals the marginal cost of the time they spent on this work” and identified a measure of life quality as

\[
L = f(g) h(t),
\]

(6)

where \( g \) is the consumption (annual income), \( t = (1 - w)e(0) \) is the leisure/rest time; \( e(0) \) is the ALE at birth; \( w \) (\( 0 < w < 1 \)) is the proportion of life expectancy spent on the (paid) work; \( f(g) \) and \( h(t) \) are two so far unknown functions of these qualities.

The value of \( L \) is called the life quality index [8]. Thus, the LQI is a product of the function \( f(g) \) measuring the life quality and the function \( h(t) \) measuring life duration.

Accepting some assumptions and conducting a set of mathematical calculations, it can be proved that the LQI is calculated by formula [5, 6, 8]:

\[
L = \frac{c^q}{q} e(0) \quad .
\]

(7)

Thus, in this case the consumption function

\[
u(c) = \frac{c^q}{q},
\]

(8)

which corresponds to an isoelastic power function at \( q = 1 - \mu \).

As a value of \( c \), Nathwani et al. [8] consider GDP per capita (taking into account the purchasing power parity, if necessary), which is an indicator of society’s productivity. Then formula (7) takes the following form
\[ L = u(g)e(0) = \frac{g^\delta}{q}e(0), \]  
(9)

where \( g \) is the GDP per capita.

If, in formula (9), the value of ALE at birth \( e(0) \) is replaced by a discounted ALE at age \( x \):

\[ L(x, g) = u(g)e_d(x, \rho, \mu, \delta) = \frac{g^\delta}{q}e_d(x, \rho, \mu, \delta) \]  
(10)

Then the obtained LQI can be interpreted as the discounted utility of the residual life expectancy.

5. Value of statistical life and willingness-to-pay
Shepard and Zeckhauser [7] determined the “value of statistical life” (VSL) at age \( x \) by dividing equation (10) by marginal utility \( du(c)/dc = u'(c) \), which translates its dimension strictly into monetary units:

\[ VSL(x) = \frac{u(c)}{u'(c)} \int_0^x \exp \left[ \int \left( \lambda(t) + \rho(t) \right) dt + \mu \delta (t-x) \right] dt = \frac{g^\delta}{q}e_d(x, \rho, \mu, \delta). \]  
(11)

It is seen that the VLS decreases with time due to a decrease of \( e_d(x, \rho, \mu, \delta) \). The monetary expression of life does not exist - “the price of life is infinite and immeasurable” [9], if we talk about an individual. Here, however, the VSL is considered, as a kind of a formal constant, as a monetary value, which is needed in order to reduce the risk of mortality by unit.

The social value of statistical life (SVSL) is defined as

\[ SVSL = \frac{g^\delta}{q}E(\rho, \mu, \delta) \]  
(12)

where \( E(\rho, \mu, \delta) \) is the discounted ALE averaged over the age distribution \( h(x,n) \):

\[ E(\rho, \mu, \delta) = \int_0^\infty e_d(x, \rho, \mu, \delta) h(x,n) dx \]  
(13)

The Willingness-to-pay (WTP) measures a person’s willingness to sacrifice one desired attribute, wealth or consumption, to get another desired attribute, in this case, increase life expectancy (improve survival). Shepard/Zeckhauser [7] introduced WTP as invariant of \( L = L(x, g) \) with respect to loss (increase) in consumption with an increase (decrease) in life expectancy:

\[ WTP = dg = \frac{\partial L(x, g)}{\partial e(a)} - \frac{\partial L(x, g)}{\partial g} \frac{de(x)}{de(x)} = -\frac{g}{q} e(x). \]  
(14)

6. The criterion of social acceptability of investments in risk reduction projects
For decisions regarding investments in life safety, LQI can be interpreted as a two-digit utility function for an average member of the society. It is assumed that a decision with a (marginal) effect on \( g \) and \( e \) (for example, any investment related to saving a life) is beneficial for society if it leads to an increase in LQI. The requirement that the full differential of LQI function be equal to or greater than zero \((dL \geq 0)\) gives rise to the net benefit criterion (acceptability of investments in risk reduction projects) [1, 5, 6]:

\[ dL(x, g) \]
Inequality (15) is the criterion for the effectiveness and accessibility of specific investments in life safety. Equality in (15) shows what measures preserving human lives for society are necessary and affordable; projects with inequality “<” are not acceptable. Such projects will actually be life threatening and be in conflict with the constitutional right to life. Every time when a small increase in ALE due to some activity preserving human lives (positive \( d_e \)) is associated with more than optimal additional costs (negative \( d_g \)), it is necessary to look for another alternative for investing in projects to save lives (reduce risk). If a given positive \( d_e \) is feasible for less money than formula (15) provides, then of course this possibility should be realized.

Expression (15) remains valid for the discounted utility of residual life expectancy at age \( x \) and after averaging over the distribution of ages \( h(x,n) \), we obtain a criterion for assessing social willingness to pay (SWP)[1,9,10]:

\[
\frac{d_g}{g} + \frac{1}{q} \frac{d e(x)}{e(x)} \geq 0. \tag{16}
\]

Converting (15), express the criterion of effectiveness in terms of limit threshold for changes of GDP per capita, \( d_g \) [1,5,6]

\[-d_g \leq \frac{g}{q} \frac{d e(x)}{e(x)} \approx \frac{g}{q} C_e d \lambda. \tag{17}\]

Usually it is more convenient to estimate the marginal reduction in mortality \( d \) than the effect that the decision has on ALE. The \( d_e/e \) ratio is calculated by multiplying the change in mortality by the demographic constant \( C_e \) [5,6], assessed using the mortality (survival) tables.

Until now, the LQI threshold has been determined based on the preferences of the average citizen (per capita). The integral total values are obtained by multiplying equation (16) by the size of population \( N_p \) and the mortality rate \( \lambda \) is substituted by \( m = \lambda \cdot N_p \), the expected number of deaths per year. This allows to determine the threshold for the marginal cost of saving lives at the infrastructure project level, \( d_c \) [1]:

\[
d_c = -d_g \cdot N_p \geq SWTP = \frac{g}{q} \frac{d E(\rho,\mu,\delta)}{E(\rho,\mu,\delta)} N_p \approx \frac{g}{q} C_e dm. \tag{18}\]

where the \( d_e/e \) ratio is calculated by multiplying the change in mortality by the demographic constant \( C_e \), assessed from the mortality (survival) tables.

Acceptance criterion (18) is based on marginal cost \( d_c \) for marginal risk reduction - \( dm \), not on average values. Note that criterion (18) is based on the annual expenditures and benefits of saving a life. If these expenditures and benefits arise at different moments of time, they must be discounted (lead to a single cut of time) for adequate comparison.

7. The relationship between acceptance criterion and monetary optimization

Since costs and benefits are accrued at different moments of time, equation (18) can be rewritten by comparing the PV present (discounted) values of future costs and benefits [1]:

\[
PV(d_c) = PV\left[-d_g \cdot N_p \right] \geq PV\left(SWTP\right) = \]

\[
= PV\left(\frac{g}{q} \frac{d E(\rho,\mu,\delta)}{E(\rho,\mu,\delta)} N_p \right) \approx \frac{g}{q} C_e PV(dm). \tag{19}\]
This criterion applies to both public and private DMs, even when the latter can have financial costs much higher than the social discount rate implies. The basis for this is the Caldor-Hicks compensation principle, according to which welfare is increased if the decision is beneficial to society as a whole, regardless of whether the investments of private DMs in saving the lives of other people are compensated or not.

In real life, it is necessary to combine the monetary optimization with the acceptability of social risk. It is necessary to clearly distinguish between monetary optimization and the acceptance criterion, that should be always evaluated from a social viewpoint. In this paper, optimization is performed by maximizing the objective function that represents the individual (private or social) preferences of the decision maker [1]. Acceptability in relation to life safety is ensured by the mandatory use of social acceptability criterion (equation (19)) as the boundary condition for optimization. The marginal costs of saving a life (MLSC) $d_c$ are defined as the direct investments $d_{c_S}$ needed for improving safety, assessed from a social viewpoint.

8. Illustrative example. Impact of probability of failure (POF) on a LQI acceptance criterion
Consider the example from [1] - construction of a steel bridge, that is built using its own finances and then operated by a private company. The return on investment is organized by demanding a toll fee. The problem is considered from both the societal and private perspective. From the later’s perspective, a standard cost benefit optimization analysis is performed (see [1]). The LQI criteria (Eq. (19)) is used as a boundary condition which is derived from a societal DMs perspective.

According to Figure 1, the feasible time intervals between subsequent maintenance actions from the private DM’s point of view is in the interval between 2.5 and 19.5yrs (private discount rate $\gamma_p = 5\%$, time horizon $T = 80$ yrs). The monetary optimum is reached when $p^* = 8$ yrs ($p^*$ is the time distance between adjacent maintenances).

Now analyze how the bridge POF affects the objective function. Consider the case when the parameter $\lambda$ (scale parameter of POF (Weibull) distribution) changes from 0.00005/year to 0.005/year. Figure 2 shows how the dependence of the optimal time interval $p^*$ between subsequent maintenance (according to the monetary optimum) on parameter $\lambda$. In can be seen, that the optimal $p^*$ decreases sharply for parameter values $\lambda$ from 0.00005 to 0.002 (from 17 to 5 yrs).

The societal point of view has to be used to guarantee the project acceptability. Hence, the LQI assessment has to be performed accordingly. The Societal Willingness to Pay (SWTP) is evaluated as the product of the marginal change in risk to life $d_m(t,p)/dp$ and the demographic constant $C_x$ multiplied by $g/q$. In our case, it is assumed that $g = $45 900, $q = 0.104$ and $C_x = 15.795$. The MLSC are evaluated as the marginal change in maintenance costs for a small change in the decision parameter $p$, i.e. $d_{c_S}(t,p)/dp$.

![Figure 1](image_url). Objective function for a private DM for $T = 80$ yrs.
The acceptable decisions can now be determined by comparing the MLSC and the SWTP (see Figures 3-4 for scale factor \( \lambda = 0.00005/\text{year} \) and \( 0.0005/\text{year} \)). For comparison, both the SWTP and the MLSC are evaluated as present values (PV) by discounting to \( t = 0 \). The acceptance criterion needs to be evaluated from a societal point of view; hence, a societal discount rate \( \gamma_s \) of 2\% is used. Observe, that the PV of MLSC (black line) is larger than the PV of the SWTP (dashed line) for maintenance intervals smaller than 15 yrs for scale factor \( \lambda = 0.00005 \) and 7.7 years for scale factor \( \lambda = 0.0005 \). For \( \lambda = 0.005 \) it will be 3.9 yrs. Hence, the monetary optimum found from the private DM's perspective is not an acceptable from a societal point of view, although very close to the acceptance threshold.

In order to get more practical results, this illustrative example should include the compensation costs for human fatalities and the change in construction costs for different design of the bridge – the less /more reliable it is, the less/more the initial costs.
9. Conclusion
The proposed methodology is based on the differentiation between monetary optimization and the social acceptability criterion, which is included in the decision-making problem as a boundary condition [1].

In a market economy, the option of optimization is usually provided to the individual, at least within an acceptable area as defined by the LQI criterion. On the other hand, the acceptance criterion is based on social preferences for saving lives. Hence, MLSC should be evaluated from a social viewpoint. At the design level, they can be considered as direct investments needed to improve safety.

Monetary benefits arising from the same decision (for example, reduced damage from failure) are not included in the acceptance criterion, since this leads to an unnecessarily strong restriction on the optimization conducted by the individual DM.

In monetary terms, from the society viewpoint, the obtained safety level may not always be optimal, since safety of life is the main focus in the criteria for the acceptability of decisions: it ensures the fulfillment of all effective life-saving investments.

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