Bayesian Receiver Design for Grant-Free NOMA with Message Passing Based Structured Signal Estimation

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Abstract—Grant-free non-orthogonal multiple access (NOMA) is promising to achieve low latency massive access in Internet of Things (IoT) applications. In grant-free NOMA, pilot signals are often used for user activity detection (UAD) and channel estimation (CE) prior to multiuser detection (MUD) of active users. However, the pilot overhead makes the communications inefficient for IoT devices with sporadic transmissions and short data packets, or when the channel coherence time is short. Hence, it is desirable to improve the efficiency by avoiding the use of pilot signals, which can also further achieve lower latency. This work focuses on Bayesian receiver design for grant-free low density signature orthogonal frequency division multiplexing (LDS-OFDM), where each user is allocated a unique low density spreading sequence. We propose to use the low density spreading sequences for active user detection, thereby avoiding the use of pilot signals. Firstly, the task of joint UAD, CE and MUD is formulated as a structured signal estimation problem. Then message passing based Bayesian approach is developed to solve the structured signal estimation problem. In particular, belief propagation (BP), expectation propagation (EP) and mean field (MF) message passing are used to develop efficient hybrid message passing algorithms to achieve trade-off between performance and complexity. Simulation results demonstrate the effectiveness of the proposed receiver for grant-free LDS-OFDM without the use of pilot signals.

Index Terms—Grant-free, non-orthogonal multiple access (NOMA), multiuser detection, message passing, Bayesian inference.

I. INTRODUCTION

The explosion of small and cheap machine-type devices with sensing and communication capability is paving the way towards smart home, smart city, smart health care and factory automation [1]. As one of the major application scenarios in the fifth generation (5G) wireless communications, massive machine type communications (mMTC) aims to accommodate massive connections and sporadic short-burst transmissions in Internet of Things (IoT) systems [2]–[4]. Due to the limited spectral resource, the conventional orthogonal multiple access (OMA) techniques cannot meet the demands on massive connections in massive IoT networks. Non-orthogonal multiple access (NOMA), where a resource block can be used to serve multiple users, is considered as a promising technology to support mMTC [5]–[8]. In addition, the conventional grant-based access protocols with handshaking procedure may lead to excessive overhead, long and uncertain latency, which can be unacceptable in sporadic short-burst IoT traffic as the communication becomes inefficient due to the small amount of payload data [9], [10]. Therefore, grant-free access without handshaking procedure is highly desirable, where users can transmit data at any time slot and active users have to be identified by the access point before data detection.

The major tasks for the access point in grant-free NOMA system includes user activity detection (UAD), channel estimation (CE) and multiuser detection (MUD) of active users. In existing works, UAD is coupled with CE and/or MUD, e.g., CE is followed by joint UAD and MUD [11]–[16], pilot assisted joint UAD and CE [17]–[20], and UAD, CE and MUD are performed jointly with the aid of pilot signals [21]–[25]. By exploiting that only a small fraction of the users in the network are active at a time, i.e., the distribution of active users is sparse, the problem of joint UAD and MUD or joint UAD and CE has been formulated under the compressive sensing (CS) framework [27]–[29]. In [11]–[16], with the assumption that the channel state information (CSI) is available to the receiver, various approaches such as those based on approximate message passing (AMP) [11], orthogonal matching pursuit (OMP) [12] and prior-information aided adaptive subspace pursuit (PIA-ASP) [13] were developed for joint UAD and MUD. However, in many scenarios, particularly dynamic ones such as Internet of vehicles (IoV), the CSI varies over time and has to be estimated frequently. With the assist of pilot signals, joint UAD, CE and MUD are performed jointly in [21]–[25], and joint UAD and CE are carried out in [17]–[20] to identify active users and estimate their CSI (which are then used for MUD). However, the use of pilot introduces excessive overhead, making communications inefficient for IoT devices with sporadic transmission and short data packets, or when...
the channel coherence time is short. Hence, it is desirable to improve the efficiency by avoiding the use of pilot signals, which can also further achieve lower latency.

In this work, we consider the NOMA scheme low density signature orthogonal frequency division multiplexing (LDS-OFDM) [23] and investigate the receiver design for grant-free LDS-OFDM, where pilot signals are not used. In LDS-OFDM, as each user is allocated a unique low density spreading (LDS) sequence, it is possible to identify active users based on the LDS sequences, therefore avoiding the use of pilot signals. We first show that joint UAD, CE and MUD without the use of pilot can be formulated as an interesting structured signal estimation problem, where the structures of the signals are brought by the LDS matrix of LDS-OFDM and the discreteness of transmitted signals. The structures can be fully exploited to identify active users and estimate active users’ channel gain and their symbols, so that signals are not necessary. We then consider Bayesian approaches to the structured signal estimation problem, and develop efficient message passing based Bayesian inference algorithms which run in a graph representation of the system. Specifically, belief propagation (BP) [30] and expectation propagation (EP) [31], [32] are combined for UAD and CE, and mean field (MF) based message passing [33], [34] is used at observation factors for noise power estimation and MUD. It is noted that UAD, CE and MUD are seamlessly integrated based on their message passing implementation. By introducing some auxiliary variables to break down the observation factors, we further develop another hybrid message passing algorithm, where BP and MF are merged to improve the system performance substantially. Extensive simulation results are provided to demonstrate the effectiveness of the proposed algorithms.

The rest of this paper is organized as follows. Section II describes the system model and the problem formulation. Message passing based Bayesian receiver is developed in Section III. Numerical simulation results are provided in Section IV, followed by conclusions in Section V.

**Notation-** Lowercase and uppercase letters denote scalars. Boldface lowercase and uppercase letters denote column vectors and matrices, respectively. The superscriptions (·)T and (·)H denote the transpose and conjugate transpose operations, respectively, and \( \propto \) denotes equality of functions up to a scale factor. The functions \( \mathcal{C}N(x; \bar{x}, \sigma_x^2) \) stands for a complex Gaussian distribution with mean \( \bar{x} \) and variance \( \sigma_x^2 \). As the convention, \( \langle f(x, y, z) \rangle = \frac{\int f(x, y, z) dy dz}{\int dy dz} \), where \( f(x) = \int f(x) dx \) is used to denote the marginalization operator. The expectation operator with respect to a probability density function (PDF) \( f(x) \) is expressed by \( \langle x \rangle = \frac{\int x f(x) dx}{\int f(x) dx} \), and \( \text{Var}[x] = \langle x^2 \rangle - \langle x \rangle^2 \) stands for the variance of \( x \).

II. SYSTEM MODEL AND PROBLEM FORMULATION

As show in Fig. 1, we assume an uplink LDS-OFDM system with \( N \) subcarriers and \( U \) users, where the number of active users denoted by \( K \) can be much smaller than \( U \). The input bit stream \( b_k \) of user \( k \) is coded and mapped to a symbol sequence \( x_k \in \mathbb{C}^{L \times 1} \), where \( L \) is the length of the sequence. Then, each symbol in \( x_k \) is spread onto \( N \) OFDM subcarriers using a unique low-density spreading sequence \( s_k \) of length \( N \), where \( s_k \) is a column of the LDS matrix \( S = [s_1, s_2, \cdots, s_U] \). For simplicity, we consider the LDS matrix \( S \) with a regular structure [21], where \( S \) has the same number of non-zero elements in each column denoted by \( d_c \), and also the same number of non-zero elements in each row denoted by \( d_r \), i.e., each users occupies \( d_r \) subcarriers, and each subcarrier accommodates \( d_c \) users. It is noted that the extension of this work to the case of irregular LDS structure is straightforward. As only \( K \) out of \( U \) users are active, we use \( z_k \in U \triangleq \{1, 2, \cdots, U\} \) to denote the identity of active user \( k \). Hence, we can define an active user LDS matrix \( S_a = [s_{z_1}, s_{z_2}, \cdots, s_{z_K}] \) with size \( N \times K \), i.e., \( S_a \) is a sub-matrix of \( S \). We assume that the channel is approximately static within a data block. The received signal on the \( n \)-th subcarrier is the superposition of the signals of \( K \) active users, i.e.,

\[
y_n = \sum_{k=1}^{K} g_{n,k} s_{z_n,k} x_k + w_n, n = 1, 2, \cdots, N, \tag{1}
\]

where \( y_n \in \mathbb{C}^{L \times 1} \), \( g_{n,k} \) is the channel gain of active user \( k \) on the \( n \)-th subcarrier, \( s_{n,z_n} \) is the \( n \)-th component of the spreading sequence \( s_{z_n} \), which is non-zero if active user \( k \) transmits signal on the \( n \)-th subcarrier and 0 otherwise, and the noise vector on the \( n \)-th subcarrier \( w_n \sim \mathcal{C}N(0, \sigma_w^2 I_L) \). In a matrix form, the received signals over \( N \) subcarriers can be expressed as

\[
Y = HX + W, \tag{2}
\]

where \( Y = [y_1, y_2, \cdots, y_N]^T \) is an \( N \times L \) matrix,

\[
H = G \circ S_a \tag{3}
\]

(\( \circ \) represents the Hadamard product) is an \( N \times K \) equivalent channel matrix, whose \((n,k)\)-th element \( h_{n,k} = g_{n,k}s_{n,z_k} \), \( X = [x_1, x_2, \cdots, x_N]^T \) is the transmitted symbol matrix of size \( K \times L \), and \( W \subseteq \mathbb{C}^{N \times L} \) is the noise matrix.

Our objective is to estimate \( H \) and \( X \) simultaneously based on \( Y \). This is possible thanks to the structures of \( H \) (brought by the active user LDS matrix \( S_a \)) and \( X \) (brought by the discreteness of transmitted signals), i.e., the columns of \( S_a \)
in (3) are randomly drawn from those of the LDS matrix \( S \), and the entries of \( X \) are discrete values (drawn from a constellation set), which are mapped from coded bits. These structures can be exploited to recover \( H \) and \( X \). However, when the constellation is symmetric, the solution may not be unique due to the phase ambiguity. The problem can be overcome by using rotationally-invariant coded modulation schemes, e.g., the rotationally-invariant trellis coded (RI-TCM) encoder in [35], which combines the operations of coding and modulation.

### III. Bayesian receiver design with message passing based structured signal estimation

To jointly estimate \( H \) and \( X \) in (2) from the received signal \( Y \) by exploiting the structures of \( H \) and \( X \), graphic model and message passing based Bayesian inference is investigated in this section. In addition, the computational complexity is analysed.

#### A. Factor graph representation

We note that \( z_k \in U \triangleq \{1, 2, \cdots, U\} \), which indicates that the active user \( k \) employs the \( z_k \)-th spreading sequence \( s_{zk} \). We assume the a priori PDF of \( z_k \) is \( p(z_k) = \frac{1}{U} \delta(z_k - u) \). As shown in [3], the equivalent channel matrix \( H \) is the Hadamard product of the channel gain matrix \( G \) and the active user LDS matrix \( S_a \), which are independent of each other. Hence, we have

\[
p(H, G, z) = p(H|G, z)p(G)p(z) = \prod_{k=1}^{K} \left( \prod_{n=1}^{N} p(h_{n,k}|g_{n,k}, z_k) p(g_{n,k}) \right),
\]

where \( z = \{z_1, z_2, \cdots, z_K\} \), \( p(h_{n,k}|g_{n,k}, z_k) \) represents a hard constraint, i.e., \( \delta(h_{n,k} - g_{n,k} s_{n,z_k}) \), and \( p(g_{n,k}) = \mathcal{CN}(g_{n,k}; 0, 1) \). We assume that the noise precision \( \Lambda = 1/\sigma_w^2 \) is unknown, and it has a prior \( p(\lambda) \propto 1/\lambda \). Given \( Y \), the joint a posteriori PDF of \( X, H, G, z \) and \( \lambda \) can be expressed as

\[
p(X, H, G, z, \lambda|Y) \propto \prod_{k=1}^{K} p(x_{k,1}) \prod_{k=1}^{K} p(y_{n,i}|x_{k,1}) \prod_{k=1}^{K} p(y_{n,i}|\lambda, h_{n,k}, x_{k,1}, z_k) \prod_{k=1}^{K} p(z_k) \prod_{n=1}^{N} p(h_{n,k}|g_{n,k}, z_k) p(g_{n,k}) p(\lambda),
\]

where \( p(x_{k,1}) = \sum_{x \in X} \frac{1}{\lambda} \delta(x_{k,1} - q) \) and \( p(y_{n,i}|\lambda, h_{n,k}, x_{k,1}, z_k) = \mathcal{CN}(y_{n,i}; \sum_{k=1}^{K} h_{n,k} s_{n,z_k}, 1/\lambda) \). To facilitate the factor graph representation of the factorization in [5], we introduce the notations in Table I showing the correspondence between the factor labels and the underlying PDFs they represent. The factor graph representation of [5] is shown in Fig. 2.

We divide the factor graph in Fig. 2 into two parts labeled by Part (i) and Part (ii). As we can see Part (i) represents the structure of \( H \), where message passing rules are derived by combining BP and EP, this part functions as active user detector and realizes channel estimation in conjunction with Part (ii). Part (ii) functions as multiuser detector and noise estimator, where the structure of \( X \) is included. In this part, two message passing algorithms are developed based on the pure MF and hybrid BP-MF, respectively, which can achieve different trade-off between computational complexity and performance.

In the following, we detail the forward (from left to right) and backward (from right to left) message computations at each node of Part (i) and Part (ii), and some approximations are introduced to reduce the computational complexity. We use \( I_{A \rightarrow B}(x) \) to denote a message passed from a variable (function node) \( A \) to a function node (variable node) \( B \), which is a function of \( x \). The notations \( m \) and \( v \) are used to denote the mean and variance of a Gaussian message specified by their subscripts. The arrows over \( m \) and \( v \) represent the directions of Gaussian message passing. Note that, if a forward message computation requires backward messages, we use the messages in previous iteration by default.

#### B. Message Passing in Part (i)

With the output forward message from Part (ii), BP and EP based message passing is used to realize the estimation of \( \{z_k\} \) and \( \{h_{n,k} \forall n,k\} \), where the function nodes \( \{f_{h_{n,k}}\} \) are handled by the BP rule, and some messages are approximated to be Gaussian with EP to reduce the computational complexity.

With the message

\[
I_{g_{n,k} \rightarrow f_{h_{n,k}}}(g_{n,k}) \propto \mathcal{CN}(g_{n,k}; \bar{m}_{g_{n,k}}, \bar{v}_{g_{n,k}})
\]

and

\[
I_{h_{n,k} \rightarrow f_{h_{n,k}}}(h_{n,k}) \propto \mathcal{CN}(g_{n,k}; \bar{m}_{h_{n,k}}, \bar{v}_{h_{n,k}})
\]

Table I: The factors involved in the factorization in [5]

| Factor | Distribution | Functional Form |
|--------|--------------|-----------------|
| \( f_{h}(\lambda) \) | \( p(\lambda) \) | \( 1/\lambda \) |
| \( f_{x_{k,1}}(z_k) \) | \( p(x_{k,1}) \) | \( \sum_{q \in X} \frac{1}{\lambda} \delta(x_{k,1} - q) \) |
| \( f_{y_{n,i}}(\lambda, h_{n,k}, x_{k,1}, \forall k) \) | \( p(y_{n,i}|\lambda, h_{n,k}, x_{k,1}, \forall k) \) | \( \mathcal{CN}(y_{n,i}; \sum_{k=1}^{K} h_{n,k} x_{k,1}, 1/\lambda) \) |
| \( f_{h_{n,k}}(h_{n,k}, z_k) \) | \( p(h_{n,k}|z_k) \) | \( \delta(h_{n,k} - g_{n,k} s_{n,z_k}) \) |
| \( f_{g_{n,k}}(g_{n,k}) \) | \( p(g_{n,k}) \) | \( \mathcal{CN}(g_{n,k}; 0, 1) \) |
| \( f_{z_k}(z_k) \) | \( p(z_k) \) | \( \frac{1}{U} \delta(z_k - u) \) |
where \( \beta_s \) employs the LDS sequence given by

\[
I_{\beta_s} = (1 - s_n z_n) \mathcal{CN}(0; m_{h_{n,k}}, v_{h_{n,k}}) + s_n z_n \mathcal{CN}(\overline{m}_{h_{n,k}}, \overline{m}_{g_{n,k}}, \overline{v}_{h_{n,k}} + \overline{v}_{g_{n,k}}).
\]

Then, the belief of \( z_k \) can be updated as

\[
b(z_k) \propto \prod_{n=1}^{N} I_{f_{h_{n,k}} \rightarrow z_k}(z_k) I_{z_k \rightarrow f_{h_{n,k}}}(z_k) \sum_{u \in \mathcal{U}} \beta_n^u \delta(z_k - u),
\]

where \( \beta_n^u \) is calculated by (61) at the bottom of the page. Note that \( b(z_k) \) is used to determine the identity of active user \( k \), i.e., \( z_k = \text{arg max}_z b(z_k) \) indicates that active user \( k \) employs the LDS sequence \( s_{z_k} \), so active user \( k \) is identified.

The backward message \( I_{f_{h_{n,k}} \rightarrow z_k}(z_k) \) can be expressed as

\[
I_{z_k \rightarrow f_{h_{n,k}}}(z_k) = \prod_{n' \neq n} I_{h_{n',k} \rightarrow z_k}(z_k) I_{f_{z_k} \rightarrow f_{h_{n,k}}}(z_k) \sum_{u \in \mathcal{U}} \gamma_n^u \delta(z_k - u),
\]

where \( \gamma_n^u \) is calculated by (10) at the bottom of the page and it can be approximated as \( \beta_n^u \) to reduce the computational complexity. Thus, the backward message \( I_{f_{h_{n,k}} \rightarrow z_k}(z_k) \) is given by

\[
I_{f_{h_{n,k}} \rightarrow h_{n,k}}(h_{n,k}) = \{ f_{h_{n,k}}(h_{n,k}, g_{n,k}, z_k) \}_{h_{n,k} \rightarrow f_{h_{n,k}}(h_{n,k})} f_{g_{n,k} \rightarrow f_{h_{n,k}}}(z_k) \sum_{u \in \mathcal{U}} \beta_n^u \left[ (1 - s_{n,u}) \delta(h_{n,k}) + s_{n,u} \mathcal{CN}(h_{n,k}; \overline{m}_{g_{n,k}}, \overline{v}_{g_{n,k}}) \right],
\]

which is not Gaussian, so that the belief \( b(h_{n,k}) \propto I_{f_{h_{n,k}} \rightarrow h_{n,k}}(h_{n,k}) I_{h_{n,k} \rightarrow f_{h_{n,k}}}(h_{n,k}) \) is not Gaussian either. It is difficult to calculate the backward message \( I_{h_{n,k} \rightarrow f_{h_{n,k}}}(h_{n,k}) \) in (39). We propose to use the EP method to overcome this problem. We first approximate \( b(h_{n,k}) \) to be Gaussian, i.e.,

\[
b(h_{n,k}) \approx b^G(h_{n,k}) = \text{Proj}_G \left\{ I_{f_{h_{n,k}} \rightarrow h_{n,k}}(h_{n,k}) I_{h_{n,k} \rightarrow f_{h_{n,k}}}(h_{n,k}) \right\}
\]

\[
\propto \mathcal{CN}(h_{n,k}; \overline{h}_{n,k}, v_{h_{n,k}}),
\]

where \( \text{Proj}_G \{ \} \) denotes the operation of Gaussian approximation, and the mean and the variance of \( h_{n,k} \) can be calculated by moment matching, i.e.,

\[
\hat{h}_{n,k} = \{ h_{n,k} \}_{b(h_{n,k})}
\]

\[
= \frac{1}{O_{h_{n,k}}} \sum_{u \in \mathcal{U}} \beta_n^u s_{n,u} \mathcal{CN}(\overline{m}_{h_{n,k}}, \overline{m}_{g_{n,k}}, \overline{v}_{h_{n,k}} + \overline{v}_{g_{n,k}}),
\]

where \( O_{h_{n,k}} \) is a normalization coefficient

\[
O_{h_{n,k}} = \sum_{u \in \mathcal{U}} \beta_n^u (1 - s_{n,u}) \mathcal{CN}(0; m_{h_{n,k}}, v_{h_{n,k}}) + s_{n,u} \mathcal{CN}(h_{n,k}; m_{n,k}, v_{n,k})
\]

and

\[
v_{n,k} = \left( \frac{1}{v_{h_{n,k}}} + \frac{1}{v_{g_{n,k}}} \right)^{-1}, \quad m_{n,k} = \left( \frac{\overline{m}_{h_{n,k}}}{v_{h_{n,k}}} + \frac{\overline{m}_{g_{n,k}}}{v_{g_{n,k}}} \right) v_{n,k}.
\]

Then, the backward message \( I_{f_{h_{n,k}} \rightarrow h_{n,k}}(h_{n,k}) \) can be expressed as

\[
I_{f_{h_{n,k}} \rightarrow h_{n,k}}(h_{n,k}) = \frac{b^G(h_{n,k})}{I_{h_{n,k} \rightarrow f_{h_{n,k}}}(h_{n,k})} \propto \mathcal{CN}(h_{n,k}; \overline{m}_{h_{n,k}}, \overline{v}_{h_{n,k}}).
\]

where

\[
\overline{v}_{n,k} = \left( \frac{1}{v_{h_{n,k}}} - \frac{1}{v_{n,k}} \right)^{-1}, \quad \overline{m}_{n,k} = \left( \frac{\overline{h}_{n,k}}{v_{n,k}} - \frac{\overline{m}_{h_{n,k}}}{v_{h_{n,k}}} \right) v_{n,k}.
\]

As shown in Fig. 2, the outgoing messages \( \{ I_{h_{n,k} \rightarrow f_{h_{n,k}}}(h_{n,k}) \}_{\forall n,k} \) are input to Part (ii).
C. Message Passing in Part (ii)

With the incoming messages \( \{I_{h_{n,k}}} \rightarrow h_{n,k} (h_{n,k}, \forall n,k) \), the message passing in Part (ii) realizes the estimation of signal \( X \) and noise precision \( \lambda \). The key is to deal with the observation nodes \( \{f_{n,l}, \forall n,l\} \), which can be tackled with pure MF or BP-MF to achieve trade-off between complexity and performance. These two methods are elaborated in the following subsections.

(1) MF based message passing

As the MF rule will be used to handle the observation nodes, the incoming messages to the observation nodes are the beliefs of the relevant variables. To calculate the backward message \( I_{f_{n,l}} \rightarrow x_{k,l} (x_{k,l}) \), the beliefs \( \{b^{g}(x_{k,l}), \forall k' \neq k\}, b(\lambda) \) and \( \{b(h_{n,k}), \forall n,k\} \), which are given respectively in (24), (29) and (35), are required. So we have

\[
I_{f_{n,l}} \rightarrow x_{k,l} (x_{k,l}) = \exp \left\{ \int \ln \left[ f_{n,l} (\lambda, h_{n,k}, x_{k,l}, \forall k) \right] \prod_{k' \neq k} b^{g}(x_{k',l}) \right\}
\]

\[
\propto C_{N} \left( x_{k,l}; \hat{m}_{x_{k,l}}, \hat{v}_{x_{k,l}} \right),
\]

where

\[
\hat{m}_{x_{k,l}} = \frac{\hat{h}_{n,k} (y_{n,l} - \sum_{k' \neq k} \hat{h}_{n,k} \hat{x}_{k',l})}{\hat{h}_{n,k} + v_{n,k}}, \quad \hat{v}_{x_{k,l}} = \frac{1}{\lambda} \left( |\hat{h}_{n,k}|^2 + v_{n,k} \right).
\]

Then, the belief of \( x_{k,l} \) can be updated by

\[
b(x_{k,l}) \propto f_{x_{k,l}} (x_{k,l}) \prod_{n=1}^{N} I_{f_{n,l}} \rightarrow x_{k,l} (x_{k,l}) \]

\[
\Delta = \sum_{q \in \mathcal{X}} \beta_{k,l}^{g} \delta(x_{k,l} - q),
\]

where

\[
\beta_{k,l}^{g} = \frac{C_{N} \left( q'; \hat{m}_{x_{k,l}}, \hat{v}_{x_{k,l}} \right)}{C_{N} \left( q'; \hat{m}_{x_{k,l}}, \hat{v}_{x_{k,l}} \right)},
\]

with

\[
\hat{v}_{x_{k,l}} = \left( \sum_{n=1}^{N} \frac{1}{\hat{v}_{x_{k,l}}} \right)^{-1}, \quad \hat{m}_{x_{k,l}} = \left( \sum_{n=1}^{N} \frac{\hat{m}_{x_{k,l}}}{\hat{v}_{x_{k,l}}} \right) \hat{v}_{x_{k,l}}.
\]

Note that \( \{b(x_{k,l}), \forall k, l\} \) in the last iteration are used for soft demodulation and decoding. As \( b(x_{k,l}) \) is no longer Gaussian, it can be approximated to be Gaussian using moment matching, i.e.,

\[
b(x_{k,l}) \approx b^{g}(x_{k,l})
\]

\[
= \text{Proj}_{G} \left\{ f_{x_{k,l}} (x_{k,l}) \prod_{n=1}^{N} I_{f_{n,l}} \rightarrow x_{k,l} (x_{k,l}) \right\}
\]

\[
\Delta \propto C_{N} \left( x_{k,l}; \hat{x}_{k,l}, \hat{v}_{x_{k,l}} \right),
\]

Algorithm 1 MF based MUD joint with UAD and CE

\begin{itemize}
  \item **Input:** \( X, p(\lambda), \{p(z_{i})\}, \{p(g_{k})\}, \{p(x_{k,l})\} \).
  \item **Initialize:** \( \lambda: \{\hat{m}_{h_{n,k}}, \hat{v}_{h_{n,k}}\}; \forall k, l, \hat{x}_{k,l} = q, q \in \mathcal{X} \).
  \item For \( i = 1: N_{O_{l}} \) (Outer iteration)
    \item For \( j = 1: N_{l} \) (Inner iteration)
      \item For \( k, l \) update \( \hat{h}_{n,k} \) and \( v_{n,k} \) by (33) and (34).
      \item For \( k, l \) update \( \hat{m}_{x_{k,l}} \) and \( v_{x_{k,l}} \) by (30).
      \item For \( k, l \) update \( \hat{h}_{n,k} \) and \( v_{h_{n,k}} \) by (35).
      \item For \( k, l \) update \( \hat{h}_{n,k} \) and \( v_{h_{n,k}} \) by (36).
    \item End
  \item End
  \item For \( k, l \) update \( \hat{h}_{n,k} \) and \( v_{h_{n,k}} \) by (37). \( \Delta \)
  \item End
  \item End
  \item Output: \( b(z_{k}) = \sum_{u \in \mathcal{U}} \beta_{n}^{g} \delta(z_{k} - u), \forall k; \)
    \( \hat{h}_{n,k}, \forall n, k; b(x_{k,l}) = \sum_{q \in \mathcal{X}} \beta_{k,l}^{g} \delta(x_{k,l} - q), \forall k, l. \)
\end{itemize}

where

\[
\hat{x}_{k,l} = \left( x_{k,l} \right)_{b(x_{k,l})} = \sum_{q \in \mathcal{X}} q \beta_{n}^{g},
\]

\[
\hat{v}_{x_{k,l}} = \text{Var} \left[ x_{k,l} \right]_{b(x_{k,l})} = \sum_{q \in \mathcal{X}} |q|^{2} \beta_{n}^{g} - |\hat{x}_{k,l}|^{2},
\]

With the updated \( \{b^{g}(x_{k,l}), \forall k, l\} \) and \( \{b(h_{n,k}), \forall n, k\} \) in (35), the forward message \( I_{f_{n,l}} \rightarrow \lambda(\lambda) \) can be calculated by

\[
I_{f_{n,l}} \rightarrow \lambda (\lambda) = \exp \left\{ \int \ln \left[ f_{n,l} (\lambda, h_{n,k}, x_{k,l}, \forall k) \right] \prod_{n=1}^{N} b(h_{n,k}) \right\}
\]

\[
\propto \lambda \exp \left\{ -\lambda A_{y_{n,l}} \right\},
\]

where

\[
A_{y_{n,l}} = \sum_{k=1}^{K} \left[ \hat{h}_{n,k}^{2} v_{x_{k,l}} + |\hat{x}_{k,l}|^{2} v_{h_{n,k}} + v_{x_{k,l}} v_{h_{n,k}} \right] + y_{n,l} - \sum_{k=1}^{K} \hat{h}_{n,k} \hat{x}_{k,l}^{2}
\]

Then, the belief of \( \lambda \) can be updated by

\[
b(\lambda) \propto \prod_{l=1}^{L} \prod_{n=1}^{N} I_{f_{n,l}} \rightarrow \lambda (\lambda) f_{\lambda}(\lambda)
\]

\[
\propto \lambda^{N L - 1} \exp \left\{ -\lambda \sum_{l=1}^{L} \sum_{n=1}^{N} A_{y_{n,l}} \right\}.
\]

Thus, the noise precision can be calculated as

\[
\hat{\lambda} = \langle \lambda \rangle_{b(\lambda)} = \frac{N L}{\sum_{l=1}^{L} \sum_{n=1}^{N} A_{y_{n,l}}}.
\]
$I_{f_{y_{n,i}} \rightarrow h_{n,k}} (h_{n,k})$ can be calculated by

$$I_{f_{y_{n,i}} \rightarrow h_{n,k}} (h_{n,k}) = \exp \left\{ \ln \left[ \int f_{y_{n,i}} (\lambda, h_{n,k}, x_{k,i}, \forall k) \right] b(\lambda) \prod_{n' \neq n} b(h_{n',k}) \prod_{k=1}^{K} b^{2} (x_{k,i}) d\lambda dx_{k} / h_{n,k} \right\} \propto \mathcal{CN} \left( h_{n,k}; m_{h_{n,k}}, v_{h_{n,k}} \right),$$

where

$$\bar{m}_{h_{n,k}} = \frac{z_{k,i}^{H} (y_{n,i} - \sum_{n' \neq n} \hat{b}_{n',k} x_{k,i})}{|z_{k,i}|^{2} + v_{k,i}}, \quad \bar{v}_{h_{n,k}} = \frac{1}{\lambda |z_{k,i}|^{2} + v_{k,i}}.$$

Thus, the forward message $I_{h_{n,k} \rightarrow f_{h_{n,k}} (h_{n,k})}$, which is used in Part (i), is given by

$$I_{h_{n,k} \rightarrow f_{h_{n,k}} (h_{n,k})} = \prod_{n=1}^{N} I_{h_{n,k} \rightarrow f_{h_{n,k}} (h_{n,k})} \propto \mathcal{CN} \left( h_{n,k}; m_{h_{n,k}}, v_{h_{n,k}} \right).$$

Then, with $I_{f_{h_{n,k}} \rightarrow h_{n,k}} \propto \mathcal{CN}(h_{n,k}; m_{h_{n,k}}, v_{h_{n,k}})$ from Part (i) in (17) and the updated message $I_{h_{n,k} \rightarrow f_{h_{n,k}} (h_{n,k})}$, the belief of $h_{n,k}$ is updated by

$$b(h_{n,k}) \propto I_{f_{h_{n,k}} \rightarrow h_{n,k}} (h_{n,k}) I_{h_{n,k} \rightarrow f_{h_{n,k}} (h_{n,k})} \propto \mathcal{CN} \left( h_{n,k}; \hat{m}_{h_{n,k}}, \hat{v}_{h_{n,k}} \right),$$

where

$$\hat{v}_{h_{n,k}} = \left( \frac{1}{\bar{v}_{h_{n,k}}} + \frac{1}{v_{h_{n,k}}} \right)^{-1}, \quad \hat{m}_{h_{n,k}} = \left( \frac{m_{h_{n,k}}}{\bar{v}_{h_{n,k}}} + \frac{m_{h_{n,k}}}{v_{h_{n,k}}} \right) \bar{v}_{h_{n,k}}.$$  

The algorithm of message passing based MUD joint with UAD and CE, described in Section III.B and Section III.C(1), is summarized in Algorithm 1.

(2) BP and MF based message passing

We note that the observation factors $\{f_{y_{n,i}}, v_{n,i}\}$ are functions of a number of variables in the form of multiplication and multi-signal summation, where MF is not effective to deal with. To overcome the drawback of MF, we decompose each observation factor into some sub-factors, which enables the use of both BP and MF to tackle different sub-factors, leading to considerable performance improvement. Let’s define $\psi_{n,k,i} = h_{n,k} x_{k,i}$ and $\phi_{n,i} = \sum_{k=1}^{K} \psi_{n,k,i}$, and these hard constrains can be represented by factors $f_{\psi_{n,k,i}}(\psi_{n,k,i}, h_{n,k}, x_{k,i}) = \delta(\psi_{n,k,i} - h_{n,k} x_{k,i})$ and $f_{\phi_{n,i}}(\phi_{n,i}, \psi_{n,k,i}, \forall k) = \delta(\phi_{n,i} - \sum_{k=1}^{K} \psi_{n,k,i})$, respectively. In addition, define $f_{\phi_{n,i}}(\phi_{n,i}, \lambda) = \mathcal{CN}(\phi_{n,i} ; \phi_{n,i}, \lambda^{-1})$. Then the original observation factor $f_{y_{n,i}}(\lambda, h_{n,k}, x_{k,i}, \forall k)$ can be expressed as

$$f_{y_{n,i}}(\lambda, h_{n,k}, x_{k,i}, \forall k) = \int f(\lambda, \phi_{n,i}, \psi_{n,k,i}, h_{n,k}, x_{k,i}, \forall k) d\phi_{n,i} d\psi_{n,k,i}.$$  

The factor graph representation of (38) is shown in the dashed box in Fig. 3 where MF is performed at the function node $f_{\phi_{n,i}}$ for noise precision estimation, and BP is performed at the function nodes $f_{\psi_{n,k,i}}$ and $\{f_{\psi_{n,k,i}}, \forall k\}$ for multi-signal detection. Next, we detail the message computations in the factor graph shown in Fig. 3.

With $I_{f_{\psi_{n,k,i}} \rightarrow h_{n,k}} \propto \mathcal{CN}(h_{n,k}; m_{h_{n,k}}, v_{h_{n,k}})$ given in (58) and the belief $b(h_{n,k})$, the backward message $I_{h_{n,k} \rightarrow f_{\psi_{n,k,i}} (h_{n,k})}$ can be expressed as

$$I_{h_{n,k} \rightarrow f_{\psi_{n,k,i}} (h_{n,k})} = \frac{b(h_{n,k})}{I_{f_{\psi_{n,k,i}} \rightarrow h_{n,k}} (h_{n,k})} \propto \mathcal{CN} \left( h_{n,k}; \bar{m}_{h_{n,k}}, \bar{v}_{h_{n,k}} \right),$$

where

$$\bar{v}_{h_{n,k}} = \left( \frac{1}{\bar{v}_{h_{n,k}}} + \frac{1}{v_{h_{n,k}}} \right)^{-1}, \quad \bar{m}_{h_{n,k}} = \left( \frac{\hat{m}_{h_{n,k}}}{\bar{v}_{h_{n,k}}} + \frac{\bar{m}_{h_{n,k}}}{v_{h_{n,k}}} \right) \bar{v}_{h_{n,k}}.$$  

Then, with $I_{\psi_{n,k,i} \rightarrow f_{\psi_{n,k,i}} (\psi_{n,k,i})} \propto \mathcal{CN}(\psi_{n,k,i}; \bar{m}_{\psi_{n,k,i}}, \bar{v}_{\psi_{n,k,i}})$ given in (56), the backward message $I_{f_{\psi_{n,k,i}} \rightarrow x_{k,i} (x_{k,i})}$ is given by

$$I_{f_{\psi_{n,k,i}} \rightarrow x_{k,i} (x_{k,i})} = \left\{ f_{\psi_{n,k,i}}(\psi_{n,k,i}, h_{n,k}, x_{k,i}) \right\} I_{h_{n,k} \rightarrow f_{\psi_{n,k,i}} (h_{n,k})} I_{\psi_{n,k,i} \rightarrow f_{\psi_{n,k,i}} (\psi_{n,k,i})}.$$  

Thus, the belief of $x_{k,i}$ is updated by

$$b(x_{k,i}) \propto f(x_{k,i}) \prod_{n=1}^{N} I_{f_{\psi_{n,k,i}} \rightarrow x_{k,i} (x_{k,i})} \propto \sum_{q \in X} \beta_{q}^{2} \delta(x_{k,i} - q),$$
where
\[ \beta_{k,l}^{n} = \frac{\prod_{n_{l}=1}^{N} \mathcal{CN} \left( \tilde{m}_{\psi_{n,k,l}} q \tilde{m}_{h_n,k} + q^{2} \tilde{m}_{h_n,k} \right)}{\sum_{q_{l} \in X} \prod_{n_{l}=1}^{N} \mathcal{CN} \left( \tilde{m}_{\psi_{n,k,l}} q \tilde{m}_{h_n,k} + q^{2} \tilde{m}_{h_n,k} \right)} \cdot \]  
(43)

Note that \( \{ b(x_{k,l}), \forall k,l \} \) in the last iteration are used for soft demodulation and decoding.

The forward message \( I_{k,l} \rightarrow f_{\psi_{n,k,l}} \) can be expressed as
\[ I_{k,l} \rightarrow f_{\psi_{n,k,l}} = f_{k,l} \prod_{n_{l} \neq n} I_{f_{\psi_{n',k,l}} \rightarrow x_{k,l}} \] 
\( = \sum_{q \in X} \gamma_{k,l}^{q} \delta (x_{k,l} - q), \)  
(44)

where
\[ \gamma_{k,l}^{q} = \frac{1}{Q} \prod_{n_{l} \neq n} \mathcal{CN} \left( \tilde{m}_{\psi_{n',k,l}} q \tilde{m}_{h_n,k} + q^{2} \tilde{m}_{h_n,k} \right). \]
(45)

Then, the forward message \( I_{f_{\psi_{n,k,l}} \rightarrow \psi_{n,k,l}} \) is given by
\[ I_{f_{\psi_{n,k,l}} \rightarrow \psi_{n,k,l}} = \left( f_{\psi_{n,k,l}(\psi_{n,k,l}, h_{n,k}, x_{k,l})} \right) I_{x_{k,l} \rightarrow f_{\psi_{n,k,l}}}(x_{k,l}) \] 
\( = \sum_{q \in X} \gamma_{k,l}^{q} |q|^{2} \mathcal{CN} \left( \tilde{m}_{\psi_{n,k,l}} q \tilde{m}_{h_n,k} + q^{2} \tilde{m}_{h_n,k} \right), \)
(46)

which is not Gaussian. To reduce the complexity, it is approximated to be Gaussian by moment matching, i.e.
\[ I_{f_{\psi_{n,k,l}} \rightarrow \psi_{n,k,l}} \approx \text{Proj}_{G} \left\{ I_{f_{\psi_{n,k,l}} \rightarrow \psi_{n,k,l}} \right\} \] 
\( \triangleq \mathcal{CN} \left( \psi_{n,k,l}; \tilde{m}_{\psi_{n,k,l}}, \tilde{v}_{\psi_{n,k,l}} \right), \)
(47)

where
\[ \tilde{m}_{\psi_{n,k,l}} = \mathbb{E} \left[ I_{f_{\psi_{n,k,l}} \rightarrow \psi_{n,k,l}}(\psi_{n,k,l}) \right] \] 
\[ = \frac{\sum_{q \in X} q_{k,l} |q|^{2} \tilde{m}_{h_n,k}}{\sum_{q \in X} q_{k,l} |q|^{2}}, \]
(48)
\[ \tilde{v}_{\psi_{n,k,l}} = \text{Var} \left[ I_{f_{\psi_{n,k,l}} \rightarrow \psi_{n,k,l}}(\psi_{n,k,l}) \right] \] 
\[ = \frac{\sum_{q \in X} q_{k,l}^{2} |q|^{2} \tilde{m}_{\psi_{n,k,l}}}{\sum_{q \in X} q_{k,l}^{2} |q|^{2}} \] 
\( - |\tilde{m}_{\psi_{n,k,l}}|^{2}. \)
(49)

Thus, the forward message \( I_{\phi_{n,l} \rightarrow \phi_{n,l}} \) is given by
\[ I_{\phi_{n,l} \rightarrow \phi_{n,l}} = \left( f_{\phi_{n,l}(\phi_{n,l}, \psi_{n,l}, k), \forall k} \right) \prod_{k_{l}} I_{\phi_{n,l} \rightarrow \psi_{n,k,l}} \] 
\( \triangleq \mathcal{CN} \left( \phi_{n,l}; \tilde{m}_{\phi_{n,l}}, \tilde{v}_{\phi_{n,l}} \right). \)
(50)

With \( I_{\phi_{n,l} \rightarrow \phi_{n,l}} \) in (55), the belief of \( \phi_{n,l} \) can be updated by
\[ b(\phi_{n,l}) \propto I_{f_{\phi_{n,l}} \rightarrow \phi_{n,l}}(\phi_{n,l}) I_{f_{\phi_{n,l}} \rightarrow \phi_{n,l}}(\phi_{n,l}) \] 
\( \triangleq \mathcal{CN} \left( \phi_{n,l}; \tilde{m}_{\phi_{n,l}}, \tilde{v}_{\phi_{n,l}} \right), \)
(51)

Then, the forward message \( I_{f_{\phi_{n,l}} \rightarrow \phi_{n,l}}(\phi_{n,l}) \) can be calculated by
\[ I_{f_{\phi_{n,l}} \rightarrow \phi_{n,l}}(\phi_{n,l}) = \exp \left\{ \int \ln \left[ f_{\phi_{n,l}}(\phi_{n,l}, \lambda) \right] b(\phi_{n,l}) d\phi_{n,l} \right\} \]
\( \triangleq \lambda \exp \left\{ -\lambda |y_{n,l} - \phi_{n,l}|^{2} \right\}, \)
(52)

Thus, with the prior of noise precision \( f_{\lambda}(\lambda) \), the belief of \( \lambda \) can be updated by
\[ b(\lambda) \propto \prod_{l=1}^{L} \prod_{n=1}^{N} I_{f_{\phi_{n,l}} \rightarrow \lambda}(\lambda) f_{\lambda}(\lambda) \]
\( \triangleq \mathcal{CN} \left( \lambda; \tilde{m}_{\lambda}, \tilde{v}_{\lambda} \right), \)
(53)

and the noise precision is given by
\[ \lambda = \frac{\mathcal{CN} \left( \lambda; \tilde{m}_{\lambda}, \tilde{v}_{\lambda} \right)}{\sum_{l=1}^{L} \sum_{n=1}^{N} |y_{n,l} - \phi_{n,l}|^{2}}. \]
(54)

The backward message \( I_{\phi_{n,l} \rightarrow \lambda}(\phi_{n,l}) \) can be expressed as
\[ I_{\phi_{n,l} \rightarrow \lambda}(\phi_{n,l}) = \exp \left\{ \int \ln \left[ f_{\phi_{n,l}}(\phi_{n,l}, \lambda) \right] b(\lambda) d\lambda \right\} \]
\( \triangleq \mathcal{CN} \left( \phi_{n,l}; y_{n,l} - \hat{\phi}_{n,l}, 1 / \hat{\lambda} \right). \)
(55)

With \( \{ I_{\phi_{n,k'} \rightarrow \psi_{n,k'}}, \forall k' \neq k \} \) in (47), the backward message \( I_{\psi_{n,k,l} \rightarrow f_{\psi_{n,k,l}}}(\psi_{n,k,l}) \) is calculated as
\[ I_{\psi_{n,k,l} \rightarrow f_{\psi_{n,k,l}}}(\psi_{n,k,l}) = \left( f_{\psi_{n,k,l}(\psi_{n,k,l}, h_{n,k}, x_{k,l})} \right) I_{x_{k,l} \rightarrow f_{\psi_{n,k,l}}}(x_{k,l}) \] 
\( \triangleq \mathcal{CN} \left( \psi_{n,k,l}; \tilde{m}_{\psi_{n,k,l}}, \tilde{v}_{\psi_{n,k,l}} \right). \)
(56)

Then, with the message \( I_{x_{k,l} \rightarrow f_{\phi_{n,l}}} \) in (44), the forward message \( I_{f_{\phi_{n,l}} \rightarrow \phi_{n,l}} \) is given by
\[ I_{f_{\phi_{n,l}} \rightarrow \phi_{n,l}} = \left( f_{\phi_{n,l}(\phi_{n,l}, \psi_{n,l}, x_{k,l})} \right) I_{x_{k,l} \rightarrow f_{\phi_{n,l}}}(x_{k,l}) I_{f_{\phi_{n,l}} \rightarrow f_{\phi_{n,l}}}(x_{k,l}) \]
\( \triangleq \mathcal{CN} \left( \phi_{n,l}; \tilde{m}_{\phi_{n,l}}, \tilde{v}_{\phi_{n,l}} \right), \)
(57)

which can be approximated to be Gaussian by
\[ I_{f_{\phi_{n,l}} \rightarrow \phi_{n,l}} \approx \text{Proj}_{G} \left\{ I_{f_{\phi_{n,k,l}} \rightarrow h_{n,k}}(h_{n,k}) \right\} \]
\( \triangleq \mathcal{CN} \left( \phi_{n,l}; \tilde{m}_{\phi_{n,l}}, \tilde{v}_{\phi_{n,l}} \right). \)
(58)
Algorithm 2 BP-MF based MUD joint with UAD and CE

Input: Y, p(λ), {p(zk)}, {p(γk)}, {p(xk,l)}, the system performance at the cost of increased computational complexity. As the message computations need to be executed for a certain number of iterations, it is desirable to use the least number of iterations in the BP-MF method. Our strategy is to design a low-complexity pre-processor for providing initial messages for the BP-MF method, so that the BP-MF method can converge rapidly while achieving good performance. The low-complexity pre-processor is elaborated in the following.

Inspired by the pure MF based method, the belief of xk,l can be approximated to be Gaussian, given in (73) with I[fψn,k,l](xk,l) given in (71), the forward message I[fψn,k,l](xk,l) is calculated by

$$I_{f_{ψn,k,l}}(xk,l) = \frac{b(xk,l)}{I_{f_{ψn,k,l}}(xk,l)} \propto \mathcal{CN}\left(\hat{x}_{k,l}; \bar{v}_{xk,l}, v_{xk,l}\right),$$

where

$$\bar{v}_{xk,l} = \left(\frac{1}{v_{xk,l}} - \frac{1}{v_{xk,l}}\right)^{-1} \approx \frac{\bar{v}_{xk,l}}{v_{xk,l}} \approx CN(\hat{x}_{k,l}; \bar{v}_{xk,l}, v_{xk,l}).$$

With I[fψn,k,l](hk,n) given in (69), b(hk,n) can be calculated in the same way to (35) and (61)-(62). Then, I[fψn,k,l](hk,n) \propto CN(hk,n; \bar{m}_{hk,n}, \bar{v}_{hk,n}) is calculated in the same way to (39). Thus, the forward message I[fψn,k,l](ψk,l) can be approximated to Gaussian be by

$$I_{f_{ψn,k,l}}(ψk,l) \approx \text{Proj}\left\{\left\{f_{ψn,k,l}(ψk,l, hk,n, xk,l)I_{f_{ψn,k,l}}(xk,l)I_{f_{ψn,k,l}}(hk,n)\right\}\right\} \propto \mathcal{CN}(\psi_{k,l}; \bar{m}_{ψk,l}, \bar{v}_{ψk,l}),$$

where

$$\bar{v}_{ψk,l} = \left(\frac{1}{v_{ψk,l}} - \frac{1}{v_{ψk,l}}\right)^{-1} \approx \frac{\bar{v}_{ψk,l}}{v_{ψk,l}} \approx CN(\hat{ψ}_{k,l}; \bar{v}_{ψk,l}, v_{ψk,l}).$$

It is worth mentioning that, compared to the pure MF based method in Section III.C.1, the BP-MF based method improve the system performance at the cost of increased computational complexity.
where the mean and variance of $\psi_{n,k,l}$ are given by

$$\tilde{m}_{n,k,l} = \tilde{m}_{h_{n,k}} \tilde{m}_{x_{k,l}},$$
$$\tilde{v}_{n,k,l} = \tilde{v}_{h_{n,k}} |\tilde{m}_{x_{k,l}}|^2 + \tilde{v}_{x_{k,l}} |\tilde{m}_{h_{n,k}}|^2 + \tilde{v}_{h_{n,k}} \tilde{v}_{x_{k,l}}.$$  (66)

With $I_{\psi_{n,k,l} \rightarrow f_{\psi_{n,k,l}}}$ $\psi_{n,k,l}) \propto CN(\psi_{n,k,l}; \tilde{m}_{\psi_{n,k,l}}, \tilde{v}_{\psi_{n,k,l}})$ in (66), the function node $f_{\psi_{n,k,l}}(\psi_{n,k,l}, h_{n,k}, x_{k,l})$ can be integrated with respect to $\psi_{n,k,l}$, leading to a new function $\hat{f}_{\psi_{n,k,l}}(h_{n,k}, x_{k,l})$,

$$\hat{f}_{\psi_{n,k,l}}(h_{n,k}, x_{k,l}) = \left( f_{\psi_{n,k,l}}(\psi_{n,k,l}, h_{n,k}, x_{k,l}) \right) I_{\psi_{n,k,l} \rightarrow f_{\psi_{n,k,l}}} \psi_{n,k,l}.$$

Then, MF is performed to calculate the forward message $I_{f_{\psi_{n,k,l}} \rightarrow h_{n,k}}(h_{n,k})$ and the backward message $I_{f_{\psi_{n,k,l}} \rightarrow x_{k,l}}(x_{k,l})$ as

$$I_{f_{\psi_{n,k,l}} \rightarrow h_{n,k}}(h_{n,k}) = \exp \left\{ \int \ln \left[ f_{\psi_{n,k,l}}(h_{n,k}, x_{k,l}) \right] b(x_{k,l}) dx_{k,l} \right\} \propto CN(\hat{m}_{h_{n,k}}, \hat{v}_{h_{n,k}})$$

with

$$\tilde{m}_{h_{n,k}} = \frac{\tilde{m}_{h_{n,k}} h_{n,k} \tilde{m}_{w_{n,k}}}{|\tilde{h}_{n,k}|^2 + |\tilde{v}_{x_{k,l}}|^2}, \quad \tilde{v}_{h_{n,k}} = \frac{\tilde{v}_{w_{n,k}} |\tilde{h}_{n,k}|^2 + \tilde{v}_{h_{n,k}} |\tilde{v}_{x_{k,l}}|^2}{|\tilde{h}_{n,k}|^2 + |\tilde{v}_{h_{n,k}}|^2},$$

and

$$I_{f_{\psi_{n,k,l}} \rightarrow x_{k,l}}(x_{k,l}) = \exp \left\{ \int \ln \left[ \hat{f}_{\psi_{n,k,l}}(h_{n,k}, x_{k,l}) \right] b(h_{n,k}) dh_{n,k} \right\} \propto CN(x_{k,l}; \tilde{m}_{x_{k,l}}, \tilde{v}_{x_{k,l}})$$

with

$$\tilde{m}_{x_{k,l}} = \frac{\tilde{h}_{n,k} \tilde{m}_{\psi_{n,k,l}}}{|\tilde{h}_{n,k}|^2 + \tilde{v}_{h_{n,k}}}, \quad \tilde{v}_{x_{k,l}} = \frac{\tilde{v}_{\psi_{n,k,l}} |\tilde{h}_{n,k}|^2 + \tilde{v}_{h_{n,k}} |\tilde{v}_{x_{k,l}}|^2}{|\tilde{h}_{n,k}|^2 + \tilde{v}_{h_{n,k}} |\tilde{v}_{x_{k,l}}|^2}.$$  (72)

Thus, the belief of $x_{k,l}$ can be updated and approximated to be Gaussian by

$$b(x_{k,l}) \approx \text{Proj}_G \left\{ I_{f_{\psi_{n,k,l}} \rightarrow x_{k,l}}(x_{k,l}) \prod_{n=1}^{N} I_{f_{\psi_{n,k,l}} \rightarrow x_{k,l}}(x_{k,l}) \right\} \propto CN(x_{k,l}; \tilde{x}_{k,l}, \tilde{v}_{x_{k,l}}),$$

where $\tilde{x}_{k,l}$ and $\tilde{v}_{x_{k,l}}$ can be calculated in a similar way to (25) and (26).

The algorithm of message passing based MUD joint with UAD and CE, described in Section III.B and Section III.C.(2), is summarized in Algorithm 2.

D. Decoding and active user identification

Both Algorithm 1 and Algorithm 2 provide the beliefs of $\{x_{k,l}, \forall k, l\}$ denoted by $\{b(x_{k,l}), \forall k, l\}$, and the beliefs of $\{z_k, \forall k\}$ denoted by $\{b(z_k), \forall k\}$, and $\{h_{n,k}, \forall n, k\}$, $\{b(x_{k,l}), \forall k, l\}$ are used for soft demodulation and decoding, $\{b(z_k), \forall k\}$ are used for active user identification, i.e., $\tilde{x}_k = \arg\max x_k b(z_k)$ indicates that active user $k$ employs the LDS sequence $s_{x_k}$. As each user is allocated a unique LDS sequence, so active user $k$ is identified.

E. Complexity Analysis

Both Algorithm 1 and Algorithm 2 employ message passing in Part (i) for UAD and CE, and the complexity is the order of $O(NKU) + O(NKU)$ per iteration. The MUD part of Algorithm 1 has a complexity of $O(NKU) + O(3NKU)$ per iteration, and that of Algorithm 2 has a complexity of $O(NKU) + O(2N^2K)$ per iteration. The pre-processor used in Algorithm 2 has a complexity of $O(NKU) + O(NKU)$ per iteration.

IV. SIMULATION RESULTS

We assume an uplink LDS-OFDM system with parameters shown in Table II. The number of subcarriers $N = 128$ and the number of users $U = 256$, i.e., the overloading factor is 2. The coded modulation scheme RI-TCM in [35] based on QPSK modulation is employed. We set the number of inner iteration $N_{\text{iter}} = 5$ both in Algorithm 1 and Algorithm 2. Later, we will show both of the algorithms converge fairly fast, e.g., $N_{\text{iter}}$ is about 10 for Algorithm 2 and 40 for Algorithm 1. All the simulation results presented in this section are obtained by averaging over $10^5$ trials.

To the best of our knowledge, the problem of MUD performed jointly with UAD and CE of grant-free LDS-OFDM without the use of pilot is investigated in the paper for the first time, which is formulated as a structured signal estimation problem. Moreover, there are no existing algorithms to solve the formulated problem. So we compare the two proposed algorithms with some corresponding performance bounds. The bit error rate (BER) is used to evaluate the proposed receiver
with different algorithms. To examine the performance of user activity detection, we define active user identification error rate (AER) as

$$\text{AER} = \frac{\text{# of active users} - \text{# of active user identified successfully}}{\text{# of active users}}.$$  

The BER performance of the proposed schemes is shown in Fig. 4 for different SNRs, where two additional schemes ID-aided scheme and CSI-ID-aided scheme are also included as benchmarks. In the ID-aided scheme, we assume that the ID of the active users are perfectly known, but their CSI are not known at the receiver, i.e., the structure of $H$ is known but the values of its non-zero entries are unknown. In the CSI-ID-aided scheme, we assume that both the ID and CSI of active users are perfectly known, i.e., $H$ is perfectly known, representing the ideal benchmark, and this is served as performance lower bound. It can be seen from the Fig. 4 that, although having lower complexity, Algorithm 1 suffers from significant performance loss due to the poor efficiency in dealing with observation factors. In contrast, with combined BP and MF to handle the observation factors, Algorithm 2 can improve the performance significantly, with about a 1 dB away from the ID-aided scheme and 2 dB away from the ideal CSI-ID-aided scheme at relatively high SNR range.

Fig. 5 compares the AER performance of different algorithms with respect to SNRs. It can be seen that Algorithm 2 can outperform Algorithm 1 dramatically at relatively high SNR. The BER and AER convergence of the proposed schemes are shown in Fig. 6 and Fig. 7 respectively. We can see that the convergence rate of the receiver with Algorithm 2 is obviously quicker than that of Algorithm 1. The AER and BER of Algorithm 2 falls rapidly within the first 10 iterations and it converges in about $N_{O_{itr}} = 10$ for SNR = 6dB and about $N_{O_{itr}} = 20$ for SNR = 2dB. Here we note that, the first 5 iterations are used for Algorithm 2 to provide initial values in Algorithm 1 By comparison, Algorithm 1 converges slower, e.g., it requires about $N_{O_{itr}} = 50$ for SNR = 6dB and about $N_{O_{itr}} = 20$ for SNR = 2dB.

With different numbers of subcarriers occupied by each user, the BER and AER performance of the receiver with two algorithms are shown in Fig. 8 and Fig. 9 respectively. More subcarriers occupied by each user will lead to larger frequency diversity gain, but stronger multi-user interference. It can be observed that, Algorithm 1 with $d_c = 32$ delivers worse performance than that with $d_c = 16$. This is because Algorithm 1 has limited capability to handle the multi-user interference at the observation factors and it is overwhelmed by multi-user interference, thereby leading to poor performance when $d_c = 32$. In contrast, Algorithm 2 is able to handle the multi-user interference much more effectively. As we can see from Fig. 8 and Fig. 9 that, when SNR=2dB, Algorithm 2 with $d_c = 32$ performs considerably better than that with $d_c = 16$, i.e., Algorithm 2 enjoys the diversity gain after mitigating the multi-user interference. It is noted that, in Fig. 9, when SNR=3dB, all active users are identified correctly over $10^5$ trials.

Finally, we examine the BER and AER performance of the proposed algorithms by varying the number of active users, and the results are shown in Fig. 10 and Fig. 11 respectively, where the ID-aided scheme is also included for reference. It can be seen that, with the decrease of active user number
$K$, the multi-user interference is alleviated, which leads to better BER and AER performance for the proposed algorithms. As we can see, with the decrease of $K$, the performance of Algorithm 2 approaches the ID-aided scheme closely.

V. CONCLUSION

In this paper, we have investigated the receiver design for grant-free LDS-OFDM, where pilot signals are not used to improve the transmission efficiency in IoT applications. The receiver has been implemented by solving a formulated structured signal estimation problem, where the structures of the equivalent channel matrix $H$ and signal matrix $X$ are fully exploited. Efficient hybrid message passing algorithms have been developed to solve the structured signal estimation problem. Simulation results have verified the effectiveness of the proposed algorithms.

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