Exploration of Finite 2D Square Grid by a
Metamorphic Robotic System

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Abstract. We consider exploration of finite 2D square grid by a meta-
morphic robotic system consisting of anonymous oblivious modules. The
number of possible shapes of a metamorphic robotic system grows as the
number of modules increases. The shape of the system serves as its mem-
ory and shows its functionality. We consider the effect of global compass
on the minimum number of modules necessary to explore a finite 2D
square grid. We show that if the modules agree on the directions (north,
south, east, and west), three modules are necessary and sufficient for ex-
ploration from an arbitrary initial configuration, otherwise five modules
are necessary and sufficient for restricted initial configurations.

Keywords: Metamorphic robotic system · autonomous modules · ex-
ploration.

1 Introduction

Distributed systems consisting of mobile computing entities, called robots, agents,
particles have gathered much attention in these twenty years as computational
models for mobile networks, biological systems, and chemical reactions. Each
computing entity is often assumed to have very weak capabilities, i.e., it is an-
onymous and oblivious (memory-less), and have neither any communication
capability nor any access to the global coordinate system. Most existing papers
focus on shape formation that requires the mobile computing entities to form a
specified shape. Suzuki and Yamashita investigated the pattern formation prob-
lem by anonymous autonomous mobile robots, each of which moves in continuous
2D space by sensing the positions of other robots and computing its next posi-
tion by a common (deterministic) algorithm. They pointed out that the pattern
formation problem is essentially related to the agreement problem because once
the robots agree on a global coordinate system, they can form any arbitrary
pattern [19]. Derakhshandeh et al. first presented a shape formation algorithm
for the amoebot model [5]. The system consists of programmable particles mov-
ing in the 2D triangular grid by performing extension and contraction. Each
vertex of a triangular grid is occupied by at most one particle that is equipped
with constant size memory and communication capability with other particles in the neighboring vertex. Their algorithm is based on a randomized leader election algorithm so that formation of an arbitrary shape is accomplished. Dumitrescu et al. considered the metamorphic robotic system model, that consists of autonomous modules moving in the 2D square grid [10,11]. Each module can perform local movement called rotation and slide, but the modules should keep their connectivity. They showed a canonical shape to which any shape can be transformed. Because of reversibility, this result guarantees transformation between any pair of shapes via the canonical shape. The goal of all these results is the structure of shapes. Reachability among shapes decomposes the system’s configuration space into subspaces that express the degree of global agreement and coordination, in other words, distributed computing ability of a considered system. However, when we take a closer look at existing shape formation algorithms, we find that intermediate shapes are used to guarantee the progress of distributed formation. That is, geometric configuration of the system is used as global memory though each computing entity is often memory-less or equipped with constant-size memory.

In this paper, we investigate the functionality of shapes of a distributed system consisting of mobile computing entities. We focus on the exploration problem in the metamorphic robotic system model. The problem requires the system to find a target put in one cell of a given field from an arbitrary initial configuration, where the field is a finite rectangular subspace of the 2D square grid. Clearly, as the number of modules increases, the number of possible shapes increases and the system can memorize more information with its shape. We show the minimum number of modules to accomplish the exploration problem. We also investigate the effect of the global compass that allows the modules to agree on north, south, east, and west.

Our results. In this paper, we consider the exploration problem by a metamorphic robotic system. Although shape formation [10,18] and locomotion [2,11] by the metamorphic robotic system have been discussed, to the best of our knowledge, this is the first time the exploration problem is discussed for the model. We show the effect of the global compass on the number of modules. We first show when the modules are equipped with the global compass, three modules are necessary and sufficient for the exploration problem from an arbitrary initial configuration. Then we show that when the modules lack the global compass, five modules are necessary and sufficient, but there are initial shapes from which the metamorphic robotic system cannot accomplish exploration.

Related works. Computational power of a distributed system consisting of mobile computing entities with very weak capabilities is currently one of the most active topics in distributed computing theory. The quest also reveals the minimum capabilities necessary to accomplish a given task. The results serve as a design guideline for robotic systems with cheap hardware and are expected to give clues to understand complex behavior of natural systems. There are a variety of indicator tasks such as gathering, shape formation, leader election, computing a function, exploration, and decomposition. Regarding the autonomous mobile
robot model, formable patterns have discussed by considering various aspects of robots, such as obliviousness [19], asynchrony [14,20], limited visibility [22], and randomness [23]. These papers showed that instead of above properties, symmetry of initial positions of robots determines formable shapes, thus obliviousness and asynchrony essentially have no effect. Randomness allows probabilistic symmetry breaking and realizes universal pattern formation. Yamauchi et al. extended these results in 2D space to 3D space by using rotation groups [21].

Weaker formation tasks also play an important role in this quest. The embedded target pattern problem, where the target pattern consists of landmarks on the plane, gave an important techniques for asynchronous pattern formation [13]. Such mobile robots can be used in surveillance. Izumi et al. proposed the set cover formation problem that requires to cover landmarks on 2D space by the minimum number of robots [15]. Real robotic sensors are equipped with different sensors and battery, thus the system consists of heterogeneous robots. Liu et al. proposed the team formation problem that requires the colored robots to form a team according to a given specification [16].

Distributed shape formation in the amoebot model is investigated for shapes consisting of triangles [5] and arbitrary shapes [8]. Di Luna et al. considered the limit of deterministic leader election and characterized formable shapes by the symmetry of an initial configuration [8]. Derakhshandeh et al. proposed the universal coating problem of a given obstacle [4].

Shape formation in the metamorphic robotic system model is investigated in distributed settings and centralized settings. Dumitrescu et al. considered distributed transformability of an initial (horizontally) convex shape to a line (also called a chain) shape with global compass [10]. Dumitrescu et al. considered locomotion of a metamorphic robotic system and showed the shape that realizes fastest locomotion [11]. While these two papers assume unlimited visibility, Chen et al. considered locomotion with limited visibility [2]. However, when the move is restricted to rotations, there are pairs of shapes that are not transformable. Michail et al. considered the complexity of deciding transformability of a pair of shapes only by rotations [18].

Michail and Spirakis proposed the network constructor model that considers finite-state agents under passive movement [17]. The communication model is based on the population protocol model [1], while the agents can construct an edge when they interact. They discussed distributed transformation of shapes in the network constructor model.

All these results consider reachability and classification of shapes. Little is known about the functionality of shapes. Das et al. investigated the formation of a sequence of patterns, which also serves as finite memory formed by oblivious robots [3]. Simulating a Turing Machine by a line shape of computing entities has been separately discussed for the metamorphic robotic system model [6,10], the network constructor model [17] and the amoebot model [8]. Di Luna et al. showed a constant number of oblivious mobile robots can simulate a robot with memory [7]. In this paper, we focus on the fact that geometric configuration of

\[1\] In [10], the authors considered a restricted Turing Machine.
mobile computing entities functions as memory and processor, and we investigate how a small number of oblivious computing entities accomplish exploration of a given field. We finally note that exploration by a single metamorphic robotic system is different from exploration of a team of ants [13], mobile agents [6], and mobile robots [9,12] because a metamorphic robotic system cannot be separated into several small fragments.

2 Preliminary

We consider the rectangular metamorphic robotic system introduced in [2,10,11,18]. Consider a 2-dimensional (2D) square grid where each square cell \( c_{i,j} \) is labeled by the underlying \( x-y \) coordinate system. We consider finite subspace of width \( w \) and height \( h \) and call it the field. Without loss of generality, we assume that \( c_{0,0} \) is the southwesternmost cell and \( c_{w-1,h-1} \) is the northeasternmost cell. See Fig. 1. Each cell \( c_{i,j} \) has eight adjacent cells (E)ast \( c_{i+1,j} \), (N)orth\( (E)ast \) \( c_{i+1,j+1} \), (N)orth \( c_{i,j+1} \), (N)orth\( (W)est \) \( c_{i-1,j+1} \), (W)est \( c_{i-1,j} \), (S)outh\( (W)est \) \( c_{i,j-1} \), (S)outh \( c_{i,j-1} \), and (S)outh\( (E)ast \) \( c_{i+1,j-1} \). The four cells N, S, E, and W are said to be side-adjacent to \( c_{i,j} \). An infinite sequence of cells with the same \( x \) coordinate is called a column and those with the same \( y \) coordinate is called a row. The field is surrounded by walls, the \((-1)\)th column (the west wall), the \(w\)th column (the east wall), the \((-1)\)th row (the south wall), and the \(h\)th row (the north wall). These cell labels are used just for description and there is no way to distinguish the cells.

A metamorphic robotic system \( R \) consists of \( n \) anonymous modules, each of which occupies a distinct cell in the grid at discrete time steps \( t = 0, 1, 2, \ldots \). The configuration \( C_t \) of \( R \) at \( t \) is the set of cells occupied by the modules at \( t \). An execution is an evolution of configurations \( C_0, C_1, C_2, \ldots \).

The evolution is generated by movement of modules. Let \( M_t \) be the set of modules that move at time \( t \). We call the modules in \( B_t = C_t \setminus M_t \) a backbone, that does not move at time \( t \). There are two types of moves, rotation and sliding, guided by backbone modules. See Figure 2. A rotation of a moving module \( m \) side-adjacent to a backbone module \( b \) is a rotation around \( b \) by an angle of \( 2/\pi \) either clockwise or counter-clockwise. A 1-sliding of a moving module \( m \) is a sliding to a side-adjacent cell. In this case, there should be two backbone modules one is \( b_1 \) that is side-adjacent to \( m \) and the other is \( b_2 \) that is side-adjacent to \( b_1 \) and the target cell of \( m \). A \( k \)-sliding \(( k \geq 2, 3, \ldots \) ) is defined in the same way, but it requires \((k+1)\) backbone modules along the track. In rotation and sliding, the cells that \( m \) just passes are required to contain no module.

The connectivity of configuration \( C_t \) is represented by a connectivity graph \( G_t = \langle V_t, E_t \rangle \). Each vertex \( V_t \) corresponds to a module and \( E_t \) contains an edge \((c, c')\) if cells \( c \) and \( c' \) are occupied by modules and \( c \) is side-adjacent to \( c' \). When \( G_t \) is connected, we say \( C_t \) is connected. Then any execution \( C_0, C_1, C_2, \ldots \) must satisfy the following three conditions:

\[ \text{The original metamorphic robotic system model in [2,10,11,18] allows rotation and 1-sliding. We extended the original model by allowing } k \text{-sliding for } k \geq 2. \]
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1. Connectivity: For any \( t = 0, 1, 2, \ldots, C_t \) is connected. Thus, we assume that any initial configuration is connected.
2. Single backbone: For any \( t = 0, 1, 2, \ldots, B_t \) is connected.
3. No interference: For any \( t = 0, 1, 2, \ldots \), the trajectories of two moving modules \( m \) and \( m' \) never overlap.

The modules are uniform, i.e., they are anonymous and execute a common distributed algorithm. At each time step, each module observes the modules in its neighborhood and decides its movement. Thus they are synchronous. A cell \( c_{i,j}' \) is a \( k \)-neighborhood of cell \( c_{i,j} \) if \( |i' - i| \leq k \) and \( |j' - j| \leq k \). A distributed algorithm of neighborhood size \( k \) is a total function that maps a \( (2k+1) \times (2k+1) \) square grid to one cell. Thus the modules are oblivious. We assume that \( k \) is constant regarding \( w \) and \( h \), and a module can observe whether each cell in its \( k \)-neighborhood is occupied by a module or a target and whether the cell is a part of the walls or not. When the modules are equipped with the global compass, they share common north, south, east, and west directions. When the modules are not equipped with the global compass, they do not know directions and their observations can be inconsistent. However, we assume that the modules agree on the clockwise direction, i.e., they share the common handedness.

The state of \( R \) in \( C_t \) is the local shape of \( R \). We often denote a state of \( R \) with \( S^n \). If the modules are equipped with global compass, the state of \( R \) contains global directions, otherwise it does not because the modules cannot distinguish rotations on their state.

The exploration problem requires that the metamorphic robotic system to find the target put in one cell in a given field without any apriori information (i.e., the size of the field and the target cell.) We say that the metamorphic robotic system finds the target from a given initial configuration \( C_0 \), if some module reaches the cell with the target and the metamorphic robotic system stops in any execution from \( C_0 \). We say that the metamorphic robotic system accomplishes exploration if for any given field and a target, it can find the target in any execution from any initial configuration.
When the modules are equipped with global compass, the execution is uniquely determined by $C_0$ because the modules are synchronous. On the other hand, when the modules have no access to the global compass, there exist multiple executions from $C_0$ depending on the local compass of each module. For example, if the left endpoint module in Fig. 3 performs a rotation, the right endpoint module may also perform a rotation, when they have symmetric local compasses. More precisely, because of symmetry, the two modules cannot distinguish themselves. Another example is shown in Fig. 4. In this case, the only possible moves are rotations, but the four modules cannot move because if one of them moves, then others may also move. Then, the backbone requirement is not satisfied. Consequently, without global compass, exploration is essentially impossible from an arbitrary initial configuration.

3 Exploration with global compass

In this section, we consider the metamorphic robotic system $R$ consisting of modules with global compass. We show the following theorem.

Theorem 1. Three modules are necessary and sufficient for a metamorphic robotic system with global compass to accomplish exploration.

We show the necessity with impossibility for less than three modules and the sufficiency with an exploration algorithm for three modules.

3.1 Impossibility for less than three modules with global compass

We show the following lemma for a metamorphic robotic system with global compass.

Lemma 1. The metamorphic robotic system consisting of less than three modules with global compass cannot accomplish exploration.

Proof. A metamorphic robotic system consisting of one module cannot perform any move and exploration is impossible. We consider a metamorphic robotic system consisting of two modules, each of which can observe its $k(>0)$ neighborhood where $k$ is a constant compared to the size of the field. When we ignore directions, we just have one unique connected state. The metamorphic robotic system consumes the same state after two moves and Fig. 5 shows possible movements from the state after two steps. Note that the other (vertical) state appears as an intermediate state.

Hence, when the field is large enough so that the modules around the center of the field cannot see the wall in their constant neighborhood, the metamorphic robotic system keeps on moving to one of the eight directions. Of course, when it reaches a corner or a wall, it can perform other movements. However, in the

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3 If we allow random walk of a single module, of course, it can explore the target with probability 1.
same way, when the metamorphic robotics system is on the wall but cannot see any corner, it keeps on moving to one direction along the wall or just make a turn. If it turns on a wall (not on a corner), its track is a fixed cycle between two walls. Thus, possible tracks starting from the center of the field consists of a move to one direction until it reaches a corner or a wall, a move along the wall, and turns on the corners. Fig. 5 shows an example. In any combination of movements, depending on the initial configuration, there always exists a cell that the metamorphic robotic system cannot visit. When the target is put in such a cell, exploration cannot be done.

\[ \square \]

\[ \text{Fig. 6. Example of a track of two modules.} \]

3.2 Exploration algorithm for three modules with global compass

To show the matching sufficiency, we present an exploration algorithm. Our basic method is to make $R$ visit all cells in a given field, i.e., $R$ moves to the south with sweeping each row. However, since the initial configuration is arbitrary, when it
Fig. 7. Tracks of $R$ in the proposed algorithm. Each track starts from the black circle.

reaches the southernmost (0th) row, it moves to the northernmost ($((w - 1)st$) row along the east wall or the west wall so that it explores unvisited cells. Fig. 7 shows possible “tracks” of $R$. Depending on the number of rows and an initial configuration, $R$ moves along one of these tracks. We address the progress of the exploration with the reference point of $R$ in each time step defined by its spine and frontier that will be defined later. Each track in Fig. 7 shows the track of reference points. We note that the reference point does not refer to some specific module, but different modules serve as a reference point as the evolution of configurations.

The proposed algorithm consists of the following basic moves.

– Move to the east and move to the west.
– Turn on the east wall and turn on the west wall.
– Turn on the southwest corner and turn on the southeast corner.
– Move to the north wall along the east wall and that along the west wall.
– Turn on the northeast corner and turn on the northwest corner.

We assume that each module can observe the cells in its 2-neighborhood. When one of the modules of $R$ reaches a cell with the target, $R$ stops there forever. More precisely, because of the sufficient visibility, each module can detect the target and never perform any movement thereafter.

For the simplicity of presentation, we first show all possible states of $R$ in Fig. 8.

Move to the east and move to the west. Fig. 9 and Fig. 10 show the moves to the east and to the west. By repeating these moves, $R$ moves to one direction. Each module can observe the state of $R$ and the two sets of configuration used in the two moves are distinct, the modules can consistently agree on to which direction $R$ is moving.

In the first state of the unit move to the east, the spine is the $i$th row and the frontier is the $j$th column in Fig. 9. At the end of a unit move, the frontier reaches $(j + 1)th$ column. During the movement, the modules do not care whether $(i + 1)st$ row, $(i - 2)nd$ row and $(j - 3)rd$ column are walls or not.

In the first state of the unit move to the west, the spine is the $i$th row and the frontier is the $j$th column in Fig. 10. The modules do not care whether $(i - 2)nd$ row, $(i + 1)st$ row and $(j + 1)st$ column are walls or not.
Fig. 8. States of $R$ consisting of three modules

Fig. 9. Move to the east ($S_3^1 \rightarrow S_3^2 \rightarrow S_3^3 \rightarrow S_3^1$)

Fig. 10. Move to the west ($S_3^4 \rightarrow S_3^5 \rightarrow S_3^6 \rightarrow S_3^4$)

Fig. 11. Turn on the east wall ($S_3^1 \rightarrow S_3^2 \rightarrow S_3^6 \rightarrow S_3^1$)

Fig. 12. Turn on the west wall ($S_3^1 \rightarrow S_3^3 \rightarrow S_3^6 \rightarrow S_3^1$)
Fig. 13. Turn on the southeast corner ($S_2^3 \rightarrow S_3^3 \rightarrow S_4^3$)

Fig. 14. Turn on the southwest corner ($S_4^3 \rightarrow S_3^3 \rightarrow S_2^3 \rightarrow S_4^3$)

Fig. 15. Move to the northeast corner ($S_4^3 \rightarrow S_5^3 \rightarrow S_3^3 \rightarrow S_4^3$)

Fig. 16. Move to the northwest corner ($S_4^3 \rightarrow S_6^3 \rightarrow S_2^3 \rightarrow S_4^3$)
Fig. 17. Turn on the north-east corner \((S_4^3 \rightarrow S_5^3 \rightarrow S_6^3 \rightarrow S_1^4)\)

Fig. 18. Turn on the north-west corner \((S_4^3 \rightarrow S_6^3 \rightarrow S_2^3 \rightarrow S_3^3 \rightarrow S_1^3)\)

Fig. 19. Exceptions. A gray column is either a wall or a non-wall cells.
Turns on the east wall and the west wall. Fig. 11 and Fig. 12 show the turns on the east wall and on the west wall. On the east wall (west wall, respectively), R changes its spine and starts a new move to west (east, respectively).

Turns on the south corners and moves to the north wall. By repeating the above four moves, R eventually reaches the south wall. Then it turns to move along the east wall or the west wall to go to the north wall. Fig. 13 and Fig. 14 show these turns. Fig. 15 and Fig. 16 show the moves to the north wall. When R moves along the east wall (the west wall, respectively), its spine is the \((w - 1)\)th column (the 0th column, respectively) and its frontier is the northernmost module. By repeating these moves, the reference point of R moves north.

Turns on the north corners. By repeating the moves in Fig. 17 (Fig. 18, respectively), R eventually reaches the north wall. Then it turns to start moves to the west (the east, respectively). Fig. 19 and Fig. 20 show these turns.

We finally add some exceptional moves. When R moves in the center of the field, all states appear in the above moves and any move can be executed. However, when R is on the wall or in the corners, moves for some states are not defined or impossible. Fig. 19 shows additional moves to avoid deadlocks in these states.

The reference point of R visits all cells in each row except the southernmost row and the northern most row. The cells of the southernmost row are visited by the modules under the spine when R moves along the 1st row. The cells of the northernmost row are visited by the modules over the spine when R moves along the \((h - 2)\)nd row. Thus we have Theorem 1.

4 Exploration without global compass

In this section, we consider the metamorphic robotic system R consisting of modules without global compass. We show the following theorem.

**Theorem 2.** Five modules are necessary and sufficient for a metamorphic robotic system without global compass to accomplish exploration from an allowed initial configuration.

We show the necessity with impossibility for less than five modules and the sufficiency with an exploration algorithm for five modules.

4.1 Impossibility for less than five modules without global compass

We show the following lemma for a metamorphic robotic system without global compass.

**Lemma 2.** The metamorphic robotic system consisting of less than four modules without global compass cannot accomplish exploration.
Proof. Depending on the number of modules, we have the following four cases. We assume that each module can observe its $k$ neighborhood where $k$ is a constant for the size of the field.

**Case A: One module.** The single module cannot move because there is no backbone.

**Case B: Two modules.** There is a single initial connected state for the two modules, and in the worst case, the two modules move symmetrically because they are anonymous and they do not share a compass. Possible moves from the initial states are rotations, but if one module performs a rotation, the other module performs a symmetric rotation. It is impossible to keep a backbone, and the two modules cannot move.

**Case C: Three modules.** There are two possible states, i.e., the line state ($S^3_1$ and $S^3_2$ in Fig. 8) and the “L” state ($S^3_3$, $S^3_4$, and $S^3_5$ in Fig. 8).

In the line state, the possible movements are rotations of endpoint modules. However, since when one endpoint module performs a rotation, the other endpoint module also performs a rotation because of symmetry. Hence, the new state is also a line, and the two endpoint modules repeats the same rotation. The center module does not move, and the entire metamorphic robotic system does not move forward to any direction. Thus, the line state should be avoided during any movement.

In the L state, the possible movements are rotations of the two endpoint modules of “L.” For the simplicity of the notation, we consider possible movements from $S^3_3$. We call the left (upper) endpoint module the *left module* and the right (lower) endpoint module the *right module*. Since the modules share the handedness, they can agree on the left module and the right module. We start with rotations.
Case C(i): The left module performs a rotation. The possible rotation is the clockwise rotation, and the resulting state is the (vertical) line state. From the line state, $R$ cannot move forward to any direction.

Case C(ii): The right module performs a rotation. The possible rotation is the counter-clockwise rotation, and the resulting state is the (horizontal) line state. From the line state, $R$ cannot move forward to any direction.

Case C(iii): Two endpoint modules perform rotations. When the both endpoint modules perform rotations, the resulting configuration is the L shape. Again, the two modules perform rotations, and the resulting state is the initial state. $R$ cannot move forward to any direction.

Case C(iv): The left module performs a slide. The possible slide is the slide to south and the resulting state is again the L shape, where the other module becomes the new left module and performs a slide. Fig. 20 shows the sequence of slide moves, and after four steps, the configuration returns to the initial configuration.

Case C(v): The right module performs a slide. In the same way as Case C(v), the configuration returns to the initial configuration. Fig. 21 shows this sequence of slides.

Consequently, three modules cannot move forward to any directions.

**Case D: Four modules.** Fig. 22 show all possible states of a metamorphic robotic system consisting of four modules. Because of the lack of global compass, rotations on one state does not generate a new state. We have seven states and Fig. 22 shows the reachability of these states. An arc shows that there is a set of moves that translates the state of its tail to the state of its head.

In $S_1^1$, no module can move because of symmetry, and $S_2^4$, $S_6^4$, and $S_7^4$ forms a loop that has no outgoing arc. There are possible sequence of states consisting of these three states (e.g., $S_2^4$, $S_6^4$, $S_7^4$, $S_2^4$), but the robotic system cannot move
forward to any direction. The remaining three states are $S_4^3$, $S_4^4$, and $S_4^5$. By repeating $S_4^3$ and $S_4^4$, the robotic system can move to one direction in the same way as Fig. 28, but neither the transition from $S_4^3$ to $S_4^5$ nor $S_4^4$ to $S_4^5$ does not move the robotic system forward. Hence, the only possible way to move the robotic system is repeating $S_4^3$ and $S_4^4$. By turning the robot system upside-down, it can move both ways, but this just results in a loop in two rows. Hence, there is no way to make the four modules explore a given square grid.

We finally note that even when the modules agree on north and south, from the above discussion, the metamorphic robotic system consisting of four modules cannot accomplish exploration.

### 4.2 Exploration algorithm for five modules without global compass

When $R$ consists of five modules, there is a state from which no module can move. Additionally, there are three states from which $R$ may transit to the deadlock state. These three states also forms a cycle. More precisely, we consider the transition diagram of the four states shown in See Fig. 24, where each arc represents the fact that there are possible moves that translates its starting state to its endpoint state. In $S_5^1$, no module can move. The three states $S_5^2$, $S_5^3$, and $S_5^4$ form a cycle and from these states, $R$ cannot transit to other states except themselves and $S_5^5$. For example, in $S_5^1$ possible moves are rotations of the two
Fig. 25. States of $R$ consisting of five modules

endpoint modules. However, when one of them moves, the other also moves in the symmetric way. Then possible next states are $S_5^2$ and $S_5^3$. In the same discussion, two endpoint modules move in $S_5^2$ and $S_5^3$, and possible next states are $S_5^1$ (by 1-slides), $S_5^2$ (by 2-slides), $S_5^3$ (by 2-slides), and $S_5^4$ (by rotations). During these transitions, the module cannot move forward to any direction. Hence, from these states, $R$ cannot accomplish exploration and these states cannot be used in the exploration algorithm. We have the following lemma.

**Lemma 3.** The metamorphic robot system consisting of five modules without global compass cannot accomplish exploration from an arbitrary initial state.

To show the sufficiency of Theorem 2, we present an exploration algorithm. In the following, we consider initial configurations where the state of $R$ is not any one of the above four states. For the simplicity of presentation, we first show
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Fig. 26. Track of the metamorphic robotic system consisting of five modules.

all the other possible states of \( R \) in Fig. 25. We note that since the modules lack global compass, they cannot distinguish the rotation of a state. We assume that each module can observe the cells in its 4-neighborhood. In addition, we use 2-slides and 3-slides.

Fig. 27. Move to the east \((S_5^5 \rightarrow S_6^5 \rightarrow S_5^5)\)

Fig. 28. Move to the west \((S_7^5 \rightarrow S_8^5 \rightarrow S_9^5 \rightarrow S_10^5 \rightarrow S_7^5)\)

We adopt the same method as Section 3, i.e., \( R \) visits every cell in the field. However, the modules cannot use global compass, and even with five modules it is not easy to realize all the ten moves in Section 3. Instead, \( R \) uses one track that checks each row from west to east and from north to south. As shown Fig. 26, \( R \) rotates the track by \( \pi/2 \) at the southwest corner so that it visits all cells. It repeats the movement until it finds the target. We explain the basic case where the movement of directions coincides with the global compass. When one of the modules of \( R \) reaches a cell with the target, \( R \) stops there forever.

**Move to the east and move to the west.** Fig. 27 and Fig. 28 show the moves to the east and to the west. By repeating these moves, \( R \) moves to one direction.
Fig. 29. Turn on the east wall \((S_6^0 \rightarrow S_6^1 \rightarrow S_2^5 \rightarrow S_{11}^5 \rightarrow S_9^5 \rightarrow S_{10}^5 \rightarrow S_7^2)\)

Fig. 30. Turn on the west wall \((S_7^5 \rightarrow S_8^5 \rightarrow S_{12}^5 \rightarrow S_7^5 \rightarrow S_{13}^5 \rightarrow S_11^5 \rightarrow S_9^5 \rightarrow S_{10}^5 \rightarrow S_{14}^5 \rightarrow S_5^5)\)

Fig. 31. Turn on the east south corner \((S_7^5 \rightarrow S_8^5 \rightarrow S_{11}^5 \rightarrow S_{10}^5 \rightarrow S_9^5 \rightarrow S_{14}^5 \rightarrow S_5^5)\)
Fig. 32. Exceptions

Fig. 33. Exceptions with walls
In the move to the east (the west, respectively), its spine is the $i$th row and its frontier is the $j$th column in Fig. 27 (Fig. 28, respectively).

**Turn on the east wall and the west wall.** Fig. 29 and Fig. 30 show the turns on the east wall and on the west wall. The spine changes after a turn on the west wall, while it does not after a turn on the east wall.

**Turn on the southwest corner.** When the spine of $R$ reaches the 1st row and it comes back to the west wall, it turns its track by $\pi/2$ as shown in Fig. 31.

Note that the cells of the 0th row have been visited by the modules under the spine on the 1st row when $R$ moves from the east wall to the west wall. The final state of the turn is $S_{5}^{0}$ and $R$ moves along the 0th column by the moves in Fig. 27. Here, the spine is the 1st column, and the 0th column is visited by the modules on the spine.

We finally add some exceptional moves. Fig. 32 shows all states where no move is defined yet. To be more precise, for states $S_{1}^{0}, S_{2}^{0}, \ldots, S_{10}^{0}$, almost all states (including walls) are used in the above algorithm except $S_{5}^{0}$ and $S_{10}^{0}$ with walls. For states $S_{11}^{0}, \ldots, S_{18}^{0}$, only six states with walls are used in the above algorithm. Hence in the remaining states, $R$ changes its state to one of $S_{1}^{0}, S_{2}^{0}, \ldots, S_{10}^{0}$ through at most two steps as shown in Fig. 33.

The reference point of $R$ visits all cells in each row and its progress is shown in the algorithm. Thus we have Theorem 2.

## 5 Conclusion and future work

We proposed the exploration problem of finite square grid by a metamorphic robotic system. We showed the effect of global compass on the necessary and sufficient number of modules to accomplish exploration. Our basic method is to make the metamorphic robotic system visit all cells. In this paper, we considered rectangular field. One of the most important directions is to consider other fields, for example, a convex field in the 2D square grid, finite 3D square grid, torus, and graphs. The current model can be exploited in continuous 2D space and 3D space. Another direction is to consider speed-up with more modules or multiple metamorphic robotic systems.

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