Exploring Ackermann and LQR stability control of stochastic state-space model of hexacopter equipped with robotic arm

I N Ibrahim, M A Al Akkad, I V Abramov

Kalashnikov Izhevsk State Technical University, 7, Studencheskaya, Izhevsk, 426069, Russia

E-mail: ibncfe@gmail.com, aimanakkad@yandex.ru, abramov@istu.ru

Abstract. This paper discusses the control of Unmanned Aerial Vehicles (UAVs) for active interaction and manipulation of objects. The manipulator motion with an unknown payload was analyzed concerning force and moment disturbances, which influence the mass distribution, and the center of gravity (CG). Therefore, a general dynamics mathematical model of a hexacopter was formulated where a stochastic state-space model was extracted in order to build anti-disturbance controllers. Based on the compound pendulum method, the disturbances model that simulates the robotic arm with a payload was inserted into the stochastic model. This study investigates two types of controllers in order to study the stability of a hexacopter. A controller based on Ackermann's method and the other - on the linear quadratic regulator (LQR) approach - were presented. The latter constitutes a challenge for UAV control performance especially with the presence of uncertainties and disturbances.

1. Introduction

UAVs have been used recently in surveillance, rescue operations, exploration, and others. However, there are complex and dangerous operations that require performing tasks instead of humans, e.g., inspection and maintenance of gas and oil pipes, and dealing with chemical and radioactive materials, by integrating robotic arms with UAVs. Such integration can be seen in assembly tasks using 3 link manipulators [1], assemble bars structures [2], transportation tasks using 2 DOF manipulators [3], valve turning using dual 2 DOF manipulators [4], bridge inspection using 4 DOF manipulators [5], and industrial applications based on a 7 DOF manipulator combined with a helicopter [6, 7]. In those applications, the vehicle's CG and mass distribution change with the manipulator movement. A nonlinear controller like back-stepping [6] was suggested for resolving the disturbances problem. In [8], PID controllers were used to ensure flight stability by applying a feedback compensation for reactionary forces during arm movement. In [9], an adaptive neural network was designed for attitude tracking of a quadcopter with a robotic arm and an unknown payload, but the inertial moment generated by the arm movement was not considered explicitly. On the other hand, estimation techniques for unknown payload were used in [3]. Furthermore, a nonlinear model predictive controller was presented to follow 3D trajectories with the end-effector. Several researchers suggested a light-weight aerial robotic arm in order to constrain the CG like in [10]. An aerial manipulation system consisting of an ultralight-weight and low inertia dual-arm prototype, integrated with a multirotor aircraft platform with torque estimation, was presented [11]. However, the CG distribution and the aerodynamic effects by the slim design that reflects real dynamics in uncertain environment, e.g., adding noise like wind, were not detailed explicitly. Modelling and control of such system are not trivial because it is characterized as nonlinear, coupled and underactuated [12]. Getting a real model without approximation, taking into account the environment, and manipulator movements emulated as
forces and moments disturbances [13], allows sketching a stochastic model for optimum control. The equations of motion including disturbances and the stochastic state space model effectiveness were presented in the second section as a stage for studying Ackermann controller in the third section and LQR in the fourth section. Results and conclusions were discussed in the fifth section.

2. Dynamics Model and Stochastic State-Space

The earth inertial frame (E-frame) and the body-fixed frame (B-frame) are needed to describe the hexacopter motion. The E-frame uses north, east, and down (NED) coordinates as in figure 1. The (X, Y, Z) axes are directed to North, East, and Down, respectively. The mobile frame \((X_B, Y_B, Z_B)\) is fixed at the hexacopter CG. The B-frame angular position with respect to the inertial one is defined by Euler angles: roll \(\phi\), pitch \(\theta\), and yaw \(\psi\), where vector: \(\sigma = [\phi \theta \psi]^T\) and \(\theta \in [\frac{-\pi}{2}, \frac{\pi}{2}]\); \(\psi \in [-\pi, \pi]\). The aircraft’s E-frame position is \(\xi = [x \ y \ z]^T\) [12, 14]. The transformation from the B-frame to the E-frame is realized by using rotation matrix \(C_B^E\) [12, 14]. The transformation matrix for angular velocities from the B-frame to the inertial one is \(S\) [12, 14], where \(\dot{\sigma} = S \cdot \Omega\); \(\dot{\xi} = C_B^E \cdot \Omega\); \(V\); the angular velocity is \(\Omega = [p \ q \ r]^T\); and the linear velocity is \(V = [u \ v \ w]^T\) defined in the B-frame. To describe the hexacopter dynamics assumed as a rigid body with a symmetrical structure, Newton-Euler equations that govern linear and angular motion are used [3, 12]. This model’s thrust and torque are as in [12]:

\[
\begin{align*}
F & = [F_{dx} \ F_{dy} \ F_{dz}]_E^n, \\
T & = [T_{dx} \ T_{dy} \ T_{dz}]_E^n,
\end{align*}
\]

The inertia matrix of the aircraft is \(J\) defined as:

\[
J = \begin{bmatrix}
J_{xx} & 0 & 0 \\
0 & J_{yy} & 0 \\
0 & 0 & J_{zz}
\end{bmatrix}; J \in R_{3 \times 3} [3, 4, 12].
\]

The hexacopter’s mass is \(m\) and \(I\) is the distance from CG to the propeller’s Centre. The thrust moment vector is \(M_T = [M_p \ M_q \ M_r]^T\), where \(M_p, M_q, M_r\) are the moments about axes \(X_B, Y_B, Z_B\) in the B-frame [3, 4, 12]. The aerodynamic moment is expressed by \(M_{AI} = K_{RI} \cdot I\); \(\Omega^2 = K_{RI} \cdot [\phi^2 \ \theta^2 \ \psi^2]^T\), where \(K_{RI}\) is a diagonal matrix related to the rotational aerodynamic friction constant by parameter \(K_r\) [12, 13]. The total disturbance moment, affecting the torque around the aircraft axes, is expressed as \(M_{DI} = [M_{dx} \ M_{dy} \ M_{dz}]^T\). Propeller gyroscopic effect \(M_{ggyro} = [-J_r \cdot \dot{\theta} \cdot \omega_r - J_\phi \cdot \dot{\phi} \cdot \omega_r]^T\) [3]. \(J_r\) is propeller’s rotational inertia \([NmS^2]\). The overall propeller speed is \(\omega_r = -\omega_1 + \omega_2 - \omega_3 + \omega_4 - \omega_5 + \omega_6\) [rad/sec]. Yaw counter moment is the differences in the propellers’ rotational acceleration defined as \(M_{counter} = [0 \ 0 \ J_r \cdot \dot{\omega}_r]^T\) [3]. The translational and rotational motions with respect to the E-frame [12] are in equation (1).

\(U = [U_x \ U_y \ U_z \ U_p \ U_q \ U_r]^T\) is the system’s control input. The derived dynamic model, characterized as nonlinear, coupled and underactuated [12, 13], is rewritten in the stochastic state-space form in equation (3), where the state-variables vector is \(X = [x \ \dot{x} \ y \ \dot{y} \ z \ \dot{z} \ \phi \ \dot{\phi} \ \theta \ \dot{\theta} \ \psi \ \dot{\psi}]^T\). A, B, C, and D specify the system, input, output, and transmission matrices respectively. \(G\) and \(H\) specify the process noise matrix related to the model state, and the outputs, respectively, and \(v\) is the measurement noise vector. \(X, u\) and \(W\) are the model state vector, the input vector, and the process noise vector respectively. The uncertainties resulted from the robotic arm appeared in all state's variables and their rates due to the coupling characteristic. The nonlinearities gave smooth-shaped curves, affecting the accuracy of control [12, 13]. Therefore, the stochastic space model was implicitly separated into two motion spaces, rotational and translational as in figure 2. After linearizing the equations and cancelling the internal coupling, the stochastic state space was reformulated. Equation (4) defines the rotational motion, while (5) and (6) define the translation along x, y, and z axes regarding the E-frame. The robotic arm movement with a payload, represented mathematically as a compound pendulum [13], and
modeled as disturbances $F_{DI}$ and $M_{DI}$, changes the aircraft’s CG with time due to the manipulator’s angles change in all directions, making the motion equations variable with time.

\[
\begin{align*}
\dot{x} &= \frac{x}{m} + \frac{F_{dx}}{m} \\
\dot{y} &= \frac{y}{m} + \frac{F_{dy}}{m} \\
\dot{z} &= \frac{z}{m} - g + \frac{F_{dz}}{m}
\end{align*}
\]

and

\[
\begin{align*}
\dot{\phi} &= u_p + b_1 \dot{\psi} + c_1 \phi^2 + \frac{M_{dp}}{J_x} \\
\dot{\theta} &= u_q + b_2 \phi \dot{\phi} + c_2 \theta^2 + \frac{M_{dq}}{J_y} \\
\dot{\psi} &= u_r + c_3 \psi^2 + \frac{M_{dq}}{J_z}
\end{align*}
\]

(1), where

\[
\begin{align*}
U_p &= \frac{M_p}{J_x} = \frac{\sqrt{l^2 + \omega_1^2 - \omega_2^2}}{2J_x} \\
U_q &= \frac{M_q}{J_y} = \frac{\sqrt{l^2 + \omega_1^2 + 2\omega_2}}{2J_y} \\
U_r &= \frac{M_r}{J_z} = \frac{\sqrt{l^2 + \omega_1^2 + \omega_2^2}}{2J_z} \\
b_1 &= \frac{\omega_1}{J_x}, b_2 = \frac{\omega_1 - \omega_2}{J_y}, c_1 = \frac{\omega_1}{J_x}, c_2 = \frac{\omega_1}{J_y}, c_3 = \frac{\omega_1}{J_z}
\end{align*}
\]

\[
\begin{align*}
\dot{X} &= AX + Bu + GW \\
Y &= CX + Du + HW + v
\end{align*}
\]

(3)

\[
\begin{align*}
X_8 &= \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} X_7 + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} X_6 + \begin{pmatrix} \frac{M_p}{J_x} \\ \frac{M_q}{J_y} \\ \frac{M_r}{J_z} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} F_{dx} + \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} F_{dy}
\end{align*}
\]

(4)

\[
\begin{align*}
X_2 &= \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix} X_3 + \begin{pmatrix} 0 \\ 0 \end{pmatrix} X_4 \\
X_4 &= \begin{pmatrix} 0 & 0 & -k_1 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} X_4 + \begin{pmatrix} 0 \\ 0 \end{pmatrix} X_5 + \begin{pmatrix} 0 \\ 0 \end{pmatrix} F_{dx} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} F_{dy}
\end{align*}
\]

(5)

The pendulum motion occurring with the aircraft hovering position is fixed, i.e., the aerodynamic effects resulting from airflow through the pendulum become neglected. The computed results of disturbances and the maximum reached values affecting the CG relative to aircraft body axes $X_B$, $Y_B$, $Z_B$ are in table 1; besides, the disturbances in the inertia moments of the aircraft’s overall dynamics model were calculated mathematically and experimentally based on SolidWorks software [13]. The maximum values of the inertia moments’ matrix elements were defined in table 2. Upon the results in tables 1 and 2, the statistics noise model for the disturbances was computed based on the mean and covariance, and implemented using LabVIEW software [13].

**Figure 1.** The hexacopter structure with an attached aerial manipulator.

**Figure 2.** Translational and rotational stochastic state-space of the hexacopter.
3. Ackermann’s Controller Design

In this control method, the pole-assignment technique is investigated for controlling the hexacopter [15], by determining the desired closed-loop poles based on the transient-response, such as speed, damping ratio, and the steady-state requirements, assuming that all state variables are measurable for the feedback. If the considered system is completely state controllable, then the closed-loop system poles may be located at any desired location by means of the state feedback through a suitable state feedback gain matrix. The controller is designed such that the governing closed-loop poles have desired damping ratio $\zeta$ and undamped natural frequency $\omega_n$ as in figure 3.

Control law $u(t) = r(t) - Kx(t)$ is chosen. The $I \times n$ matrix, $K$, is called the state feedback gain matrix. Its computation is based on Ackermann’s Formula. Considering deterministic system $\dot{X} = AX + Bu$ from the previous stochastic system, $\dot{X} = (A - BK)X + ru$ is resulted. Hence, the full hexacopter system is separated into six deterministic subsystems sketched from the stochastic system as in figure 4. These subsystems have two states (order $n = 2$) such as position and velocity.

The stability and transient-response characteristics defined by the eigenvalues of matrix $\bar{A} = A - BK$, are called the regulator poles. The desired characteristic equation is defined as $S^2 + 2\zeta\omega_nS + \omega_n^2$. The desired closed-loop poles are assumed to be at $S = \mu_1, S = \mu_2$. Based on Ackermann’s formula, let us have $K = [0 \ 1][B \ AB]^{-1}\phi(A)$, where $\phi(A)$ is the characteristic equation based on Cayley-Hamilton theorem [15]. The results in figure 5 are obtained from the simulation data taken from table 3.

This method was applied to all parts of the system model in order to get an appropriate behaviour. The choice of the desired closed-loop poles or characteristic equation is a compromise between the error-vector response speed and the sensitivity to disturbances and measurement noises, i.e., if the error response speed is increased, the adverse effects of disturbances and measurement noises increase. If the system is of second order, then its dynamics can be definitely correlated to the desired closed-loop poles and zeros location of the system’s model. Therefore, in determining the state feedback gain matrix, $K$, for a given system, it is desirable to check by simulation the system’s characteristics response for several different $K$ matrices and choose the one giving the best overall system performance.

![Figure 3. Feedback control of the stochastic state-space system using Ackermann’s method.](image)

![Figure 4. Detailed control of a hexacopter based on Ackermann’s method.](image)
The LQR controller was implemented on a stochastic state space in equations (4) to (6), where: $X = [x \dot{x} y \dot{y} z \dot{z} \phi \dot{\phi} \theta \dot{\theta} \psi \dot{\psi}]^T$ is the state vector; $U = [\phi \theta u_T M_p M_q M_r]^T$ is the control signal; $W = [M_{d\phi} M_{d\theta} M_{d\psi}]^T$ is the disturbance vector of moments, and finally the reference vector is $X_r = [x_r \dot{x}_r y_r \dot{y}_r z_r \dot{z}_r \phi_r \dot{\phi}_r \theta_r \dot{\theta}_r \psi_r \dot{\psi}_r]^T$. The LQR controller was implemented on a state vector to follow the reference vector. The stochastic state space allows adding the disturbances to the model; therefore, it is considered closer to the real aircraft model, where its information is taken based on the covariance and mean calculations of previously studied measurements of disturbances. Figure 6 illustrates the application of the LQR controller to the aircraft. The closed-loop of the LQR method has been simulated on LabVIEW with simulation results in table 4, and figure 7 shows the results of applying the LQR controller and the control signals.

Table 3. Parameters used in the simulation.

| Parameter | Value |
|-----------|-------|
| $m$ | 4 [kg] |
| $g$ | 9.806 [m/s$^2$] |
| $l$ | 0.5 [m] |
| $J_{xx}$ | 3.8e-4 [kg.m$^2$/rad] |
| $J_{yy}$ | 7.1e-4 [kg.m$^2$/rad] |
| $C_T$ | 0.01458 |
| $C_Q$ | 1.037e-3 |
| $\rho$ | 1.293 [kg/m$^3$] |

4. LQR Controller Design

The quadratic optimal control strategy is used to determine the desired closed-loop poles corresponding to a satisfactory response and energy. Obtaining a high-speed response implies a large amount of control energy, heavier motors, and consequently higher cost. This type of the controller aims to minimize quadratic cost function $J = \int_0^\infty [X^T(t)QX(t) + U^T(t)R U(t)]dt$ [14][16]. Feedback controller gain $k$, such that $U(t) = -kX(t)$, was used, where $Q$ and $R$ are weighting matrices. The task in the LQR design is to choose the appropriate weighing matrices. $Q$ and $R$ are diagonal matrices, where $Q$ limits the amplitude of the state variables while $R$ limits the amplitude of the inputs, and these coefficients treat the optimization and energy terms in the model. In other words, the main goal is to make the vehicle reach its desired position as fast as possible. The LQR controller was designed based on the stochastic state space in equations (4) to (6), where:

Figure 5. State trajectory graphs of the UAV stochastic subsystems based on Ackermann’s method.

Figure 6. A block diagram of the LQR controllers connected to the Hexacopter model.
Table 4. Stability Response of the LQR Controller.

| Time Response Parametric Data | \( \phi \) | \( \theta \) | \( \psi \) |
|-------------------------------|--------|--------|--------|
| Settling Time [s]             | 0.05   | 0.05   | 0.05   |
| Rise Time [s]                 | 0.05   | 0.05   | 0      |
| Peak Time [s]                 | 0.05   | 0.05   | 0.05   |
| Peak value                    | 1.168  | 1.168  | 1.09   |
| Overshoot                     | 1.75E-5| 1.75E-5| 1.88E-5|
| Steady-State Gain             | 1.168  | 1.168  | 1.09014|

Figure 7. Stability response of the Roll, Pitch and Yaw angles and their error signals.

5. Conclusion
A hexacopter real dynamics model was derived, which is characterized by nonlinearity, time variance, underactuation and coupling among the equations’ variables. The stochastic state-space formulation was used to write the equations in addition to decoupling and linearization procedures in order to design the controllers, allowing one to append the disturbances to the model which emulate the motion of a robotic arm attached to the aircraft. Two controllers, one based on Ackermann method and the other on the LQR approach, have been analysed. Ackermann’s method is a pole-assignment technique that determines the desired closed-loop poles based on the transient-response, such as the speed and damping ratio, and the steady-state requirements. The LQR approach is an optimization technique used to stabilize the hexacopter’s attitude, based on the stochastic model which considers the noise model, where the control signals amplitude values are very small in comparison with Ackermann’s controller, reducing the consumed power; hence it is ideal and has low power consumption.

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