Statistics of dressed modes in a thermal state

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Abstract

By a Wigner-function calculation, we evaluate the trace of a certain Gaussian operator arising in the theory of a boson system subject to both finite temperature and (weak) interaction. Thereby we rederive (and generalize) a recent result by Kocharovsky, Kocharovsky, and Scully [Phys. Rev. A 61, 053606 (2000)] in a way that is technically much simpler. One step uses a special case of the response of Wigner functions to linear transformations, and we demonstrate the general case by simple means. As an application we extract the counting statistics for each mode of the Bose gas.

Key words: Gaussian operator, Wigner function, cold Bose gas.

In a recent paper on dilute boson gases [1], Kocharovsky, Kocharovsky, and Scully faced the problem of evaluating the trace of a product of two 2-mode Gaussian operators. They solved it by exploiting the algebra of 2-mode squeezing operators in a remarkable tour de force occupying the appendix of [1]. The main objective of the present Brief Report is to put on record an alternative derivation that is simpler and more direct.

In a few words, the essence of our calculation is this: We recall (i) that the trace of a product of two operators can be obtained as a phase space integral of the product of the respective Wigner functions [2]; (ii) that the Wigner functions of Gaussian operator functions are Gaussian functions of the phase space variables; and (iii) that integrals of Gaussians are immediate.

The trace of interest is

$$T(u, v; \epsilon) = \text{tr}\{G(u, v)\rho_\epsilon\} = \langle G(u, v) \rangle_\epsilon$$

(1)
Table 1
Dictionary for switching from the present notation to the one in Eqs. (63), (68), (69) of Ref. [1].

| here    | there               | here    | there               |
|---------|---------------------|---------|---------------------|
| $T(u, u; \epsilon)$ | $\Theta_{\pm k}(u)$ | $u, v$  | $u, u$              |
| $a, a^\dagger$ | $\hat{\beta}_k, \hat{\beta}_k^\dagger$ | $\epsilon$ | $\varepsilon_k/T$  |
| $b, b^\dagger$ | $\hat{\beta}_{-k}, \hat{\beta}_{-k}^\dagger$ | $\tanh \vartheta$ | $A_k$              |
| $A, A^\dagger$ | $\hat{b}_k, \hat{b}_k^\dagger$ | $Y_+$  | $z(-A_k)$           |
| $B, B^\dagger$ | $\hat{b}_{-k}, \hat{b}_{-k}^\dagger$ | $Y_-$  | $z(A_k)$            |

with

$$G(u, v) = e^{iua^\dagger a + ivb^\dagger b}$$  \hspace{1cm} (2)$$

and

$$\rho_\epsilon = (1 - e^{-\epsilon})^2 e^{-\epsilon(A^\dagger A + B^\dagger B)} ,$$  \hspace{1cm} (3)$$

where $a^\dagger, a$ and $b^\dagger, b$ are the ladder operators of the physical excitations (two independent harmonic oscillators), and the Bogolubov transformation

$$A = a \cosh \vartheta - b^\dagger \sinh \vartheta , \quad A^\dagger = a^\dagger \cosh \vartheta - b \sinh \vartheta ,$$

$$B = b \cosh \vartheta - a^\dagger \sinh \vartheta , \quad B^\dagger = b^\dagger \cosh \vartheta - a \sinh \vartheta$$  \hspace{1cm} (4)$$

(with real $\vartheta$) relates them to the ladder operators of the “dressed modes.” The statistical operator $\rho_\epsilon$ is a thermal state (with temperature parameter $\epsilon > 0$) that is diagonal in the dressed number operators $A^\dagger A$ and $B^\dagger B$. Table 1 states how our notation is related to the one used in [1]. The factor $G(u, v)$ is the generating function for the count of physical excitations inasmuch as

$$\delta a^\dagger a, m \delta b^\dagger b, n = \int \frac{du}{(2\pi)^2} \int \frac{dv}{(2\pi)^2} e^{-imu - inv} G(u, v) ,$$  \hspace{1cm} (5)$$

the integrations covering any interval of length $2\pi$. Accordingly, upon expanding $T(u, v; \epsilon)$ in powers of $e^{iu}$ and $e^{iv}$ we get the probabilities for having certain numbers of quanta in the physical excitations.

1 Phase-space evaluation

Our starting point is the 1-mode thermal state,

$$\rho_\epsilon^{(1)} = (1 - e^{-\epsilon}) e^{-\epsilon a^\dagger a} ,$$  \hspace{1cm} (6)$$
and its well known Wigner function,

$$[\rho^{(1)}_\epsilon]_W = 2 \tanh(\frac{1}{2} \epsilon) e^{-2 \alpha^* \alpha \tanh(\frac{1}{2} \epsilon)}, \quad (7)$$

normalized in accordance with

$$\text{tr}\{\rho^{(1)}_\epsilon\} = \int \frac{d^2 \alpha}{\pi} [\rho^{(1)}_\epsilon]_W, \quad (8)$$

where $\alpha$ is the complex phase space variable associated with $a$, and the standard parameterization of the phase space integral — specified by $\alpha = x + iy$, $\alpha^* = x - iy$, and $d^2 \alpha = dx \, dy$ — suffices for our purposes.

First, we need the Wigner function (or Weyl transform) of $G(u, v)$ in (2), which (among other methods) can be found efficiently by analytic continuation from (6) and (7) [taking $\epsilon \to -iu$ and $\epsilon \to -iv$ in two copies of (7), discarding its prefactor]:

$$[G(u, v)]_W = e^{-\frac{1}{2} i (u + v) \sec(\frac{1}{2} u) \sec(\frac{1}{2} v)} \times e^{2i \alpha^* \alpha \tan(\frac{1}{2} u)} + 2i \beta^* \beta \tan(\frac{1}{2} v), \quad (9)$$

where $\beta$ is the phase space variable for $b$.

Second, to get the Wigner function of the statistical operator (3) we recall a fundamental property of Wigner functions, namely that linear similarity transformations of the ladder operators carry over to the Wigner function (see Sec. 4 below). For the Bogolubov transformation (4) this means that $[F(a^\dagger, a, b^\dagger, b)]_W = f(\alpha^*, \alpha, \beta^*, \beta)$ implies [3]

$$[F(A^\dagger, A, B^\dagger, B)]_W = f(\alpha^* \cosh \vartheta - \beta \sinh \vartheta, \ldots, \beta \cosh \vartheta - \alpha^* \sinh \vartheta), \quad (10)$$

and therefore we have

$$[\rho_\epsilon]_W = 4 \tanh^2(\frac{1}{2} \epsilon) \times e^{-2[(\alpha^* \alpha + \beta^* \beta) \cosh(2\vartheta) - (\alpha^* \beta^* + \alpha \beta) \sinh(2\vartheta)] \tanh(\frac{1}{2} \epsilon)} \quad (11)$$

Given (9) and (11), the trace (1) is a 4-dimensional Gaussian integral that is routinely evaluated,

$$T(u, v; \epsilon) = \int \frac{d^2 \alpha}{\pi} \int \frac{d^2 \beta}{\pi} [G(u, v)]_W [\rho_\epsilon]_W$$

$$= 4(e^\epsilon - 1)^2 \left[ (e^\epsilon - 1)^2 (1 + e^{iu})(1 + e^{iv}) + (e^\epsilon + 1)^2 (1 - e^{iu})(1 - e^{iv}) + 2(e^{2\epsilon} - 1)(1 - e^{iu} + iv) \cosh(2\vartheta) \right]^{-1}. \quad (12)$$
While this formula is the most direct and natural result of the calculation, it may appear physically inscrutable. However, it can be worked into the form

$$T(u, v; \epsilon) = \frac{2(Y_+ - 1)(Y_- - 1)}{(Y_+ - e^{iu})(Y_- - e^{iv}) + (Y_- - e^{iu})(Y_+ - e^{iv})} = T(v, u; \epsilon),$$

(13)

where

$$Y_+ = \frac{e^\epsilon + \tanh \vartheta}{1 + e^\epsilon \tanh \vartheta}, \quad Y_- = \frac{e^\epsilon - \tanh \vartheta}{1 - e^\epsilon \tanh \vartheta}. \quad (14)$$

This matches the result, for the special case $u = v$, reported in (69) of [1] (translated with Table 1):

$$T(u, u; \epsilon) = \langle e^{iu(a^\dagger a + b^\dagger b)} \rangle_\epsilon.$$

(15)

The challenge of rederiving (15) by simpler means has thus been met; in fact, (13) is more general because $u$ and $v$ need not be the same.

Remark: One can also calculate the trace (1) by using the Q-function for $\rho_\epsilon$ and the P-function for $G(u, v)$, rather than the Wigner function for both. But that approach is more involved because the statement corresponding to (10) is more complicated.

2 Occupation probabilities

Let us now see what we can learn from this gain in flexibility. Special cases are those of $v = 0$ and $u = 0$, the generating functions for the count of excitations in either the $a$ mode or the $b$ mode,

$$T(u, 0; \epsilon) = \langle e^{iu a^\dagger a} \rangle_\epsilon = \frac{Y - 1}{Y_+ - e^{iu}},$$

$$T(0, v; \epsilon) = \langle e^{iv b^\dagger b} \rangle_\epsilon = \frac{Y - 1}{Y_- - e^{iv}},$$

(16)

where

$$Y = \frac{2Y_+ Y_- - Y_+ - Y_-}{Y_+ + Y_- - 2} = \frac{e^\epsilon + \tanh^2 \vartheta}{1 + e^\epsilon \tanh^2 \vartheta} > 1. \quad (17)$$

The symmetry in (16) is, of course, just a particular example of $T(u, v; \epsilon) = T(v, u; \epsilon)$, which we noted at (13) in passing. The geometric series that result from expanding the right-hand sides of (16) in powers of $e^{iu}$ or $e^{iv}$, respectively, tell us the individual counting statistics for each mode,

$$\langle \delta_{a^\dagger a, n} \rangle_\epsilon = \langle \delta_{b^\dagger b, n} \rangle_\epsilon = (1 - Y^{-1})Y^{-n}. \quad (18)$$
So, if there were no statistical correlations between the two modes, the probability for having a total of $N$ quanta in both modes would be given by

$$\sum_{n=0}^{N} \langle \delta a^+_a, n\rangle \epsilon \langle \delta b^+_b, N-n\rangle \epsilon = (N+1)(1-Y^{-1})^2 Y^{-N}. \quad (19)$$

But there are correlations and, indeed, the correct value of $\langle \delta a^+_a + b^+_b, N\rangle \epsilon$, obtained by expanding $T(u, u; \epsilon)$ of (15) in powers of $e^{iu}$,

$$\langle \delta a^+_a + b^+_b, N\rangle \epsilon = (1-Y^{-1})(1-Y^{-1}) \frac{Y_+^{N-1} - Y_-^N}{Y_+ - Y_-}, \quad (20)$$

differs noticeably from (19), unless we are in the limiting situation of $\vartheta = 0$ when $Y_+ = Y_- = Y$. In that limit we have

$$T(u, v; \epsilon)|_{\vartheta = 0} = \frac{e^{\epsilon} - 1}{e^{\epsilon} - e^{iu}} \epsilon \frac{e^{\epsilon} - 1}{e^{\epsilon} - e^{-iu}}, \quad (21)$$

and this factorization states the obvious: For $\vartheta = 0$ the modes do not get dressed and so they maintain their statistical independence.

Rather more involved is the expression for the probability of $m$ quanta in mode $a$ and $n$ quanta in mode $b$,

$$\langle \delta a^+_a, m \delta b^+_b, n\rangle \epsilon = (1-Y^{-1})(1-Y^{-1}) \frac{1}{2} (Y_+^{-1} + Y_-^{-1})^{m+n}$$

$$\sum_{k=0}^{\infty} \frac{(m+n-k)!}{k!(m-k)!(n-k)!} \left[ \frac{-4Y_+Y_-}{(Y_+ + Y_-)^2} \right]^k, \quad (22)$$

where the summation terminates when $k$ equals the smaller one of $m$ and $n$. Note that the right-hand side is invariant under the interchange $m \leftrightarrow n$, which is another manifestation of $T(u, v; \epsilon) = T(v, u; \epsilon)$. Incidentally, one could write (22) in terms of a Jacobi polynomial, but that doesn’t seem to add transparency to the result.

## 3 Bogolubov transformation of a single mode

Also worthy of study is a Bogolubov transformation mixing one mode with itself by

$$A = a \cosh \vartheta - a^\dagger \sinh \vartheta \quad (23)$$

in place of (4). In fact, the dressing of a pair of modes as in (4) can be diagonalized into two decoupled transformations of the form (23); in the Bose condensation problem this corresponds to replacing traveling waves by standing waves.
In this case we should evaluate

\[ T(u; \epsilon) = \int \frac{d^2 \alpha}{\pi} [G(u)]_W [\rho_{\epsilon}]_W , \quad (24) \]

where \([G(u)]_W\) is the \(v = 0\) version of (9) and

\[ [\rho_{\epsilon}]_W = 2 \tanh(\frac{1}{2} \epsilon) e^{-[2 \alpha^* \alpha \cosh(2 \vartheta) - (\alpha^2 + \alpha^2) \sinh(2 \vartheta)] \tanh(\frac{1}{2} \epsilon)} \quad (25) \]

is obtained by \(\alpha \rightarrow \alpha \cosh \vartheta - \alpha^* \sinh \vartheta\) in (7). [Note the subtle difference in the relative sizes of the terms in the exponent in (25) versus (11).] The Gaussian integration is more elementary than in the previous case and yields

\[ T(u; \epsilon) = 2(e^\epsilon - 1) \left[ (e^\epsilon - 1)^2 (1 + e^{i u})^2 + (e^\epsilon + 1)^2 (1 - e^{i u})^2 \right. \\
+ 2(e^{2 \epsilon} - 1)(1 - e^{2i u}) \cosh(2 \vartheta) \left. \right]^{-\frac{1}{2}} \quad (26) \]

as the analog of (12). Note that \(T(u; \epsilon)^2 = T(u, u; \epsilon)\), as it should. On the other hand, \(T(u; \epsilon)T(v; \epsilon)\) does not equal \(T(u, v; \epsilon)\), as these functions are giving the statistics for two different bases of single-quantum states, which have different dynamics. If desired, the counterpart of (22) could be obtained by expanding (26) in powers of \(e^{i u}\), which is easily done because we meet here a familiar generating function for Legendre’s polynomials.

One motivation for investigating a single mode was the hope of elucidating the structure of equations (14), which cry out to be interpreted as addition formulas for the hyperbolic tangent function with \(e^\epsilon = \tanh(\ldots)\). There is, in fact, a way of constructing the thermal state of a boson mode by means of another Bogolubov transformation [4]; one would think that (14) then expresses the composition of the two transformations. However, the appropriate thermal parameter in that construction is \(\frac{1}{2} \epsilon\) rather than \(\epsilon\). This discrepancy may be related to the fact, stressed in [1], that quanta in the interacting Bose gas are present in strongly correlated pairs. Calculations following [4] are in progress, but so far have not yielded expressions as simple as those provided by the method reported here.

4 Linear similarity transformations

It is striking that the fundamental transformation property of Wigner functions, of which (10) is the special case we make use of, is not mentioned in any of the reviews [5–7] or textbooks [8,9], although it has been much exploited for various applications (e.g., in [10,11]). Since it deserves to be known more widely, we present the following compact derivation, which is more general than all previous ones.

As a warm-up we’ll first consider the single-mode case of Sec. 3. The Wigner function (or Weyl transform) \(F_W(\alpha^*, \alpha)\) of an operator function \(F(a^\dagger, a)\) is
given by
\[
F_W(\alpha^*, \alpha) = \int \frac{d^2\beta}{\pi} e^{\beta\alpha^* - \beta^*\alpha} \text{tr}\left\{e^{\beta^*a - \beta a^\dagger} F(a^\dagger, a)\right\}
\]
\[
= 2 \text{tr}\left\{e^{\alpha a^\dagger} - \alpha^*a S e^{\alpha^*a} - \alpha a^\dagger F(a^\dagger, a)\right\}
\]
(27)

where we encounter the unitary displacement operator \(e^{\alpha^*a - \alpha a^\dagger}\),
\[
e^{\alpha a^\dagger} - \alpha^*a a^\dagger e^{\alpha^*a} - \alpha a^\dagger = a^\dagger - \alpha^* ,
\]
\[
e^{\alpha a^\dagger} - \alpha^*a a e^{\alpha^*a} - \alpha a^\dagger = a - \alpha ,
\]
(28)

and the reflection operator
\[
S = \int \frac{d^2\beta}{2\pi} e^{\beta^*a - \beta a^\dagger},
\]
(29)

which is both unitary and Hermitean and therefore its own inverse, \(S^\dagger = S = S^{-1}\) [12]. Its name derives from the unitary transformation that \(S\) effects,
\[
S^{-1}a^\dagger S = -a^\dagger , \quad S^{-1}a S = -a .
\]
(30)

A linear similarity transformation turns \(F(a^\dagger, a)\) into \(G(a^\dagger, a)\), another operator function,
\[
G(a^\dagger, a) = V^{-1}F(a^\dagger, a)V = F(\mu a^\dagger + \nu a, \sigma a + \tau a^\dagger) ,
\]
(31)

where
\[
V^{-1}a^\dagger V = \mu a^\dagger + \nu a , \quad V a^\dagger V^{-1} = \sigma a^\dagger - \nu a ,
\]
\[
V^{-1}a V = \sigma a + \tau a^\dagger , \quad V a V^{-1} = \mu a - \tau a^\dagger.
\]
(32)

The commutation relation \([a, a^\dagger] = 1\) puts the restriction \(\mu \sigma - \nu \tau = 1\) on the numerical coefficients \(\mu, \nu, \sigma, \tau\), and otherwise they can take on any complex values permitted by \(|\mu|, |\sigma| \geq |\nu|, |\tau|\), which condition ensures that \(V^{-1}a^\dagger V\) has eigenbras and \(V^{-1}aV\) has eigenkets. It is not necessary that (32) be a unitary transformation, such as (23). In particular, the non-unitary transformation \(a^\dagger \rightarrow a^\dagger - a\), \(a \rightarrow \frac{1}{\sqrt{2}}(a + a^\dagger)\) is permissible and useful [10].

The crucial observation is now that the reflection operator is invariant under this similarity transformation,
\[
V^{-1}SV = S \quad \text{or} \quad S^{-1}VS = V .
\]
(33)

This is so because \(V\) is the exponential of a linear combination of \(a^2\), \(a^\dagger a\), and \(a^2\) (or a product of such exponentials), as illustrated by
\[
V = e^{\frac{1}{2}(\tau/\mu)a^2} - a^\dagger a e^{-\frac{1}{2}(\nu/\mu)a^2} .
\]
(34)
Clearly, the reflection (30) has no effect on $V$, which is the second statement of (33).

Now, perform the replacements

$$F(a^\dagger, a) \rightarrow V^{-1}F(a^\dagger, a)V = G(a^\dagger, a),$$

$$S \rightarrow S = V^{-1}SV$$

in (27) to establish

$$G_{W}(\alpha^*, \alpha) = 2 \text{tr} \left\{Ve^{\alpha a^\dagger} - \alpha^*aV^{-1}SVe^{\alpha^*a} - \alpha a^\dagger V^{-1}F(a^\dagger, a) \right\},$$

(36)

then note that

$$Ve^{\alpha^*a} - \alpha a^\dagger V^{-1} = e(\mu\alpha^* + \nu\alpha)a - (\sigma\alpha + \tau\alpha^*)a^\dagger,$$

(37)

and arrive at

$$G_{W}(\alpha^*, \alpha) = F_{W}(\mu\alpha^* + \nu\alpha, \sigma\alpha + \tau\alpha^*).$$

(38)

Indeed, the linear similarity transformation (31) of the operator functions carries over to their Wigner functions.

The arguments in higher dimensions are quite analogous. There are now $n$ pairs of ladder operators and $n$ pairs of phase-space variables associated with them. We write them compactly as $2n$-component rows,

$$\vec{a} = (a_1^\dagger, a_1, a_2^\dagger, a_2, \ldots, a_n^\dagger, a_n),$$

$$\vec{\alpha} = (\alpha_1^*, \alpha_1, \alpha_2^*, \alpha_2, \ldots, \alpha_n^*, \alpha_n).$$

(39)

The $n$-dimensional displacement and reflection operators are then

$$\prod_{k=1}^{n} e^{\alpha_k^*a_k} - \alpha_k a_k^\dagger = e^{\vec{\alpha}K\vec{a}^T}, \quad S = \int \frac{d^{2n}\beta}{(2\pi)^n} e^{\vec{\beta}K\vec{a}^T}$$

(40)

with the $2n \times 2n$ block matrix

$$K = \begin{pmatrix} 0 & -1 \\ 1 & 0 \\ \ddots & \ddots \\ 0 & -1 \\ 1 & 0 \end{pmatrix} = -K^{-1} = -K^T,$$

(41)

so that the analog of (27) reads

$$F_{W}(\vec{a}) = 2^n \text{tr} \left\{ e^{-\vec{a}K\vec{a}^T} S e^{\vec{a}K\vec{a}^T} F(\vec{a}) \right\}.$$
A linear similarity transformation in $n$ dimensions is of the form
\[ V^{-1} \vec{a} V = \vec{a} V, \quad V \vec{a} V^{-1} = \vec{a} V^{-1} \] (43)
where $V$ is a $2n \times 2n$ matrix restricted mainly by
\[ VKV^T = K, \] (44)
which is the analog of $\mu \sigma - \nu \tau = 1$ for the coefficients in (32). Here, too, $V$ is the exponential of a bilinear form of the ladder operators, $V = \exp(\vec{a} \vec{a}^T)$, with a (largely) arbitrary complex $2n \times 2n$ coefficient matrix $\mathcal{V}$, and so (33) is equally true in $n$ dimensions. By arguments analogous to those of (35)–(38) it then follows that the Wigner function of the transformed operator
\[ G(\vec{a}) = V^{-1} F(\vec{a}) V = F(\vec{a} V) \] (45)
is given by
\[
G_W(\vec{a}) = 2^n \text{tr} \left\{ V e^{-\vec{a} K \vec{a}^T} V^{-1} S V e^{\vec{a} K \vec{a}^T} V^{-1} F(\vec{a}) \right\} \\
= 2^n \text{tr} \left\{ e^{-\vec{a} K \vec{a}^T} S e^{\vec{a} K \vec{a}^T} F(\vec{a}) \right\} \\
= F_W(\vec{a} V). \] (46)
This closes the case.

Acknowledgements

We thank Marlan Scully for insisting that a simpler derivation of (15) was possible and worthwhile.

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