Λ* hypernuclei with chiral dynamics

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Abstract. The properties of the strangeness $S = -1$ and the baryon number $B = 2$ system is studied from the viewpoint of Λ* hypernuclei with chiral SU(3) dynamics. The Λ* = Λ(1405) is treated as an effective baryon degree of freedom and the two-body Λ*N potential is constructed in the meson exchange picture. Based on the chiral unitary approach, the physical Λ* is described as a superposition of the two states whose coupling constants are estimated through the nonperturbative meson-baryon dynamics. The potential in the total spin $S = 0$ channel turns out to be attractive so that a bound state is generated. Taking into account the mixing of two Λ*N channels, we find the ground state of the bound Λ*N system in the energy region above the πΣN threshold.

1. Introduction

One of the most important issues in strangeness nuclear physics is to understand the mechanism of the possible bound state of the $\bar{K}$ meson in nuclei [1]. The key ingredient here is the $\Lambda^* = \Lambda(1405)$ resonance which exists slightly below the $\bar{K}N$ threshold and can be regarded as a quasi-bound $\bar{K}N$ state [2]. If the quasi-bound picture is appropriate, then the $\bar{K}N$ interaction should be strongly attractive, and $\bar{K}$ meson may form a bound state in nuclei. Experimental searches for the possible bound $\bar{K}$ system have been performed [3, 4, 5]. A broad bump structure is observed for the baryon number $B = 2$ system in the $\Lambda N$ mass spectra [3, 5], while the interpretation of the bump is still controversial [6]. In the $B = 3$ system, no narrow structure was observed [4]. From the theoretical side, many efforts are made to calculate the simplest $\bar{K}NN$ system using the few-body calculation technique [7, 8, 9, 10, 11, 12, 13], where the interactions are constrained by the present $NN$ and $\bar{K}N$ database. Although all the results show that the $\bar{K}NN$ system bounds, the binding energies are quantitatively different from each other. The main reason of the difference is the lack of the experimental information of the $\bar{K}N$ interaction below the threshold which is relevant to the study of the possible $\bar{K}$ bound state in nuclei. In this way, theoretical study is called for to understand the physical mechanism of the binding of the $\bar{K}NN$ system and the origin of the peak structures observed in experiments.

To study the $\bar{K}$-nuclei system, an interesting picture, the $\Lambda^*$ hypernuclei, has been proposed in Ref. [14] where the $\bar{K} + N^A$ system is regarded as the $\Lambda^* + N^{A-1}$ system due to the strong correlation of the $\bar{K}N$ pair. This picture may be supported by the variational calculations of the $\bar{K}NN$ system [11, 12, 13] which have revealed that the wave function of the $\bar{K}N(I = 0)$ pair in the $\bar{K}NN$ system behaves similarly to that of the $\Lambda^*$ in vacuum. Then the system has a large overlap with $\Lambda^*N$ state, which is bound by exchanging $\bar{K}$ and mixing $\Lambda^*N$ and $NN\Lambda^*$. In the present study, we take a viewpoint that the main components of the $\bar{K}NN$ (and $\bar{K}$-nuclear bound states) are the $\Lambda^*N$ ($\Lambda^*$-nucleus) state.
There are two ways to proceed. One way is to study the property of the $\Lambda^*N$ potential, by assuming the experimentally observed peak structure as the $\Lambda^*N$ bound state. This approach has been taken in Ref. [14], with the experimental results by FINUDA [3]. Alternatively, one can construct the $\Lambda^*N$ potential theoretically, and then predict the mass of the $\Lambda^*N$ bound state and study the mechanism of the binding. In this approach, we need to determine the $\Lambda^*$ coupling constants appearing in the potential.

There is a suitable model for this purpose, called chiral unitary approach [15, 16, 17, 18]. In this approach, the low energy interaction is constrained by the chiral symmetry of QCD [19, 20] through the matching with chiral perturbation theory, and the scattering amplitude satisfies the coupled-channel unitarity condition [21] which guarantees the probability conservation of the scattering process. Applied to the strangeness coupling constants appearing in the potential.

There is an interesting prediction in this approach. It has been found that the physical $\Lambda^*$ is described by two resonance poles [22]. Due to the different coupling strength to the $K\bar{N}$ and $\pi\Sigma$ states, the resonance shape in the mass spectrum depends on the production reactions [22]. The nature of the two poles is further studied in Ref. [23]. It is found that in the limit of the vanishing $K\bar{N}-\pi\Sigma$ coupling, the strongly attractive $K\bar{N}$ interaction generates a bound state below threshold, and the moderately attractive $\pi\Sigma$ interaction develops a resonance above the threshold. The mixing between two channels eventually locates two resonance poles in the $K\bar{N}-\pi\Sigma$ amplitude. Thus, based on the chiral unitary approach, the $\Lambda^*N$ system should be treated as a two-component coupled-channel framework.

In this study, we investigate the $\Lambda^*N$ two-body system by constructing the $\Lambda^*N$ potential based on the chiral unitary approach [24]. According to the model prediction, we prepare two $\Lambda^*$ states, $\Lambda^*_1$ (the higher energy state, main component of which is the $K\bar{N}$ bound state) and $\Lambda^*_2$ (the lower energy state which is mainly the $\pi\Sigma$ resonance). We construct the diagonal $\Lambda^*_iN$ potentials, as well as the transition potential between the $\Lambda^*_1N$ and the $\Lambda^*_2N$ states, in the meson exchange picture. The coupling constants related to the $\Lambda^*_i$ are estimated by the microscopic meson-baryon dynamics. In Section 2, we construct the $\Lambda^*N$ potential. The two-body $\Lambda^*N$ bound state is studied in Section 3. The last section is devoted to a summary.

2. $\Lambda^*N$ potential

Our study is based on the nonrelativistic potential model with the two-component $\Lambda^*N$ wave function $\psi_{\Lambda^*N} = (\psi_1, \psi_2)$. With the potential in channel $i$ being $V_i$ and the transition potential being $V_t$, the wave function satisfy the coupled-channel Schrödinger equation

$$H\psi_{\Lambda^*N} = E\psi_{\Lambda^*N}, \quad H = \begin{pmatrix} T_1 + V_1 & V_t \\ V_t & T_2 + V_2 \end{pmatrix},$$

where $T_i$ is the kinetic energy. Since we are interested in the ground state of the $\Lambda^*N$ system, we take the $s$-wave component into account. Then the potential is expressed by the central force and the spin-spin interaction.

We construct the potentials $V_i$ in the meson-exchange picture, by extending the potential of the Jülich model [25]. Since the isospin of the $\Lambda^*$ is zero, only isoscalar particles can be exchanged in the direct processes. In addition, $K$ exchange is possible as the exchange process.
Figure 1. Microscopic description of the $\Lambda^*\Lambda^* X$ vertices ($X = \sigma, \omega$). The $\Lambda^*$ and the exchanged meson $X$ are represented by the double lines and the dashed lines. In the right hand side, the dotted lines and the solid lines denote the intermediate meson and baryon ($\pi \Sigma$ or $\bar{K}N$), respectively.

Figure 2. $\Lambda^*_1 N$ potentials in the $S = 0$ channel (left) and in the $S = 1$ channel (right). The dash-dotted, dotted, thin solid, and thick solid lines denote the $\sigma$ exchange, the $\omega$ exchange, the $\bar{K}$ exchange, and the total potential.

So the potential is written as

$$V(r) = V_{\sigma}(r) + V_{\omega}(r) + V_{\bar{K}}(r).$$

There are three types of the coupling constants: 1) $NN\omega$ and $NN\sigma$ couplings, 2) $N\Lambda^*\bar{K}$ coupling and 3) $\Lambda^*\Lambda^*\omega$ and $\Lambda^*\Lambda^*\sigma$ couplings. The nucleon coupling constants 1) are taken from the model A version of the Jülich potential [25]. The $N\Lambda^*\bar{K}$ coupling is extracted from the residue of the pole of the $\Lambda^*$ in the chiral unitary approach. The $\Lambda^*$ couplings in 3) are estimated by the one-loop diagrams shown in Fig. 1, in which the $\Lambda^*$ is expressed by the meson-baryon intermediate state and the exchanged meson is attached to each of these constituents. All the coupling constants in these diagrams are given either by the Jülich potential or by the $\Lambda^*$ residues, except for the $\sigma$ coupling to the intermediate mesons. The $\sigma\pi\pi$ coupling is determined by the pole position of the $\sigma$ meson [26], while we assume that the $\sigma$ coupling to the kaons is zero, as it is found to be small in the dynamical model analyses [27, 28].

We plot the $\Lambda^*_1 N$ potentials in coordinate space in Fig. 2, together with the individual contribution from each meson exchange. The right (left) panel shows the result of the spin
Figure 3. Wave functions of the ground state of the $\Lambda^*N$ system. The $\Lambda_1^*N$ and $\Lambda_2^*N$ components are shown by the solid and dashed lines.

$S = 0$ ($S = 1$) channel. The coupling constants are determined using the chiral unitary model in Refs. [29, 30] where the masses are given by $M_{\Lambda_1^*} = 1427$ MeV and $M_{\Lambda_2^*} = 1400$ MeV. In both cases, the repulsive $\omega$ exchange and attractive $\sigma$ exchange are largely cancelled each other. The $K$ exchange contributes attractively (repulsively) in $S = 0$ ($S = 1$) channel, which can be understood as in the following way. Since the $\Lambda^*$ has negative parity, the $N\Lambda^*K$ coupling is given as the scalar type vertex. This is generically attractive, but the exchange factor is also included:

$$-\frac{1 + (\vec{\sigma}_1 \cdot \vec{\sigma}_2)}{2} = \begin{cases} 1 & (S = 0) \\ -1 & (S = 1) \end{cases},$$

which is the origin of the spin dependence. The attractive contribution in $S = 0$ channel enhances the attractive pocket at the intermediate range region, while the repulsive contribution in $S = 1$ channel prevents the attractive component at the intermediate range region. This will be important for the spin of the ground state of the $\Lambda^*N$ system.

The potential for the lower energy $\Lambda_2^*N$ system shows qualitatively similar behavior to the $\Lambda_1^*N$ potential. However, the strength of the potential is smaller than the $\Lambda_1^*N$ case. The $\sigma$ and $\omega$ exchanges are proportional to the $\Lambda^*\Lambda^*\sigma$ and $\Lambda^*\Lambda^*\omega$ couplings which are evaluated by the diagrams shown in Fig. 1. The loop function is enhanced when the intermediate meson-baryon state becomes close to on-shell. This means that the $\Lambda_1^*$ state, whose mass is closer to the $\bar{K}N$ threshold, has larger coupling constants than the $\Lambda_2^*$ state. In addition, the $\Lambda_1^*\bar{K}N$ coupling is stronger than the $\Lambda_2^*\bar{K}N$ one. The strong $\bar{K}N$ coupling not only enhances the above mentioned effect further, but also strengthens the $\bar{K}$ exchange contribution. As a consequence, the total $\Lambda_1^*N$ potential should be stronger than the $\Lambda_2^*N$ potential, since all the coupling constants are enhanced.

3. $\Lambda^*N$ bound state

Here we solve the obtained $\Lambda^*N$ potential in the coupled-channel Schrödinger equation (1), and study the property of the $\Lambda^*N$ bound state. As a first step, we calculate the single-channel $\Lambda_1^*N$ system by setting $V_i = 0$. The $S = 1$ system does not generate any physical bound state. In the $S = 0$ channel, the $\Lambda_1^*N$ system develops a bound state with the binding energy of 52 MeV, while the $\Lambda_2^*N$ system has no bound state, since the interaction strength is weaker. The mass of the $\Lambda_1^*N$ bound state (2315 MeV) lies below the $\Lambda_2^*N$ threshold of 2340 MeV.
With the transition potential $V_t$ being switched on, we obtain the ground state of the $\Lambda^*N$ bound system at $M_{\Lambda^*N} = 2285$ MeV. The coupling to the $\Lambda_2^*N$ scattering states lowers the energy of the bound state, as a result of the level repulsion. Although the bound state has no width in the present model space, it should decay into the $\pi\Sigma N$ and $YN$ channels via strong interaction.

Each component of the ground state wave function of the $\Lambda^*N$ is shown in Fig. 3. By normalizing the wave function $\psi_{\Lambda^*N} = (\psi_1, \psi_2)^T$, we define the relative weights of the $\Lambda_1^*N$ and $\Lambda_2^*N$ components ($C_i^2$ for channel $i$) as

$$C_1^2 = \int d^3r |\psi_1|^2, \quad C_2^2 = \int d^3r |\psi_2|^2, \quad C_1^2 + C_2^2 = 1. \quad (4)$$

To obtain the information of the size, we calculate the mean squared radius as

$$\langle r^2 \rangle = \int d^3r (\psi_1 \psi_2)^* r^2 (\psi_1 \psi_2) = \int d^3r r^2 |\psi_1|^2 + \int d^3r r^2 |\psi_2|^2. \quad (5)$$

The relative weights and the root mean squared radius $\sqrt{\langle r^2 \rangle}$ are summarized in Table 1. From the value of the $C_i^2$, we find that the $\Lambda_1^*N$ component, the original bound state without mixing, dominates the wave function of the ground state, while the contribution from the $\Lambda_2^*N$ is also substantial. We observe that the size of the system is as compact as 1 fm, while the wave functions are moderately suppressed at $r < 0.5$ fm due to the short-range repulsion in the potential.

### 4. Summary and discussions

The $\Lambda^*$ hypernuclei model is applied to the $B = 2$ system with the meson-exchange $\Lambda^*N$ potential constrained by chiral SU(3) dynamics. The use of the chiral unitary approach introduces two $\Lambda^*$ states, and enables us to construct the $\Lambda^*N$ potential theoretically. For the spin $S = 0$ channel, the $\Lambda^*N$ potential shows an attractive pocket at intermediate range region, thanks to the attractive $K$ exchange contribution. Solving the Schrödinger equation for the higher energy $\Lambda_1^*N$ channel, we obtain the bound state with 52 MeV below the threshold, which is lower than the mass of the $\Lambda_2^*N$ channel where no bound state appears. With the mixing between $\Lambda_1^*N$ and $\Lambda_2^*N$ channels, the energy level of the $\Lambda_1^*N$ bound state is pushed downward to 2285 MeV. The solution is dominated by the $\Lambda_1^*N$ component with the 25% mixture of $\Lambda_2^*N$ component, and the root mean square radius is about 1 fm.

The bound state energy is higher than the $\pi\Sigma N$ threshold, so the decay width into the $\pi\Sigma N$ channel should be calculated. In addition, it is mandatory to take into account the dynamics of the $YN (= \Lambda N, \Sigma N)$ channel in order to compare the result with the experimental spectrum of the $\Lambda N$ final state. The estimation of the two-nucleon absorption width will be possible by combining the wave function obtained here with the amplitude given in the study of the non-mesonic decay of the $\Lambda^*$ [31].

There are several issues to be pursued in future. We utilize the $\Lambda^*$ properties in the chiral unitary model to construct the $\Lambda^*N$ potential. These properties, especially the pole position of

| $M_{\Lambda^*N}$ | $\sqrt{\langle r^2 \rangle}$ | $C_1^2$ | $C_2^2$ |
|------------------|-----------------|--------|--------|
| 2285 MeV         | 1.09 fm         | 0.75   | 0.25   |

Table 1. Properties of the $\Lambda^*N$ bound state: mass of the ground state of $\Lambda^*N$ system, root mean square radius, and relative weights of the $\Lambda_1^*N$ components $C_i^2$.  

the lower energy \( \Lambda^*_s \) state, is found to be sensitive to adopted models [23]. Therefore, the model dependence within the chiral unitary approach should be examined to check the robustness of the result.

The \( \Lambda^* \) hypernuclei model is applicable as far as the \( \Lambda^* \) keeps its identity in the bound system. In this sense, the solution should not be far away from the original \( \Lambda^*N \) threshold, where the effective degrees of freedom may be different. The comparison of the \( \Lambda^*-N \) bound state model with the three-body Faddeev calculation of \( KNN - \pi \Sigma N \) system with two poles for the \( \Lambda^* \) [32] will be useful to study the validity of the \( \Lambda^* \) hypernuclei picture.

Acknowledgments

We thank Dr. Hiyama for useful discussion. This work is partly supported by the Grant for Scientific Research (No. 19540275, 21840026, 22105503) from the Ministry of Education, Culture, Sports, Science and Technology (MEXT) of Japan. T.H. thanks the support from the Global Center of Excellence Program by MEXT, Japan through the Nanoscience and Quantum Physics Project of the Tokyo Institute of Technology.

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