Comparing D-branes to Black-branes*

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Abstract We calculate the potential between two different stationary D-branes and the velocity dependent potential between two different moving D-branes. We identify configurations with some unbroken supersymmetry, using a zero force condition. The potentials are compared with an eleven dimensional calculation of the scattering of a zero black-brane from the 0, 2, 4 and 6 black-brane of type IIA supergravity. The agreement of these calculations provide further evidence for the D-brane description of black-branes, and for the eleven dimensional origin of type IIA string theory.

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1 Introduction

It has been recently realized that string dualities can be understood in a simple way by assuming the existence of higher dimensional theories \[1, 2, 3\]. In particular an eleven dimensional “M”-theory is conjectured to give, when compactified on different manifolds, various string theories in ten dimensions \[1, 4, 5\]. The relationship of the M-theory to type IIA string theory is particularly simple. The M-theory on \(R^{10} \times S^1\) is conjectured to be equivalent to type IIA string theory with a particular string coupling \((g_s)\), which is determined by the radius of \(S^1\) (which we denote by \(R_{11}\)), \(g_s \sim R_{11}^{3/2}\). The low energy limit of the M-theory should be 11D supergravity \[4\].

The Kaluza-Klein mode of the 11 dimensional theory is a non perturbative state in type IIA string theory. Other non-perturbative relationships between string theories require extended objects with particular properties \[1, 4, 5\]. These object have been conjectured to be the D-branes \[8\] (for a recent review see \[9\]). The type IIA and type IIB supergravity theories have black-brane solutions. These are extended objects with horizons and singularities. Some of these solutions carry RR charges and in the extremal limit preserve half of the space-time supersymmetries. These were thought to be the extended objects needed for string dualities before the D-brane idea came. Now they are believed to be the long range space-time description of the D-branes. Thus, the description of the D-branes in terms of boundary conditions of open strings might give us a powerful tool to analyze the behavior of the black-branes. In fact, the D-brane description of black-branes (if correct) may be used to solve some long standing problems with black holes. It has been used to calculate the entropy of black holes in various dimensions \[10\], and to try and resolve the black hole paradox. Thus it has become important to test the equivalence between the D-branes and the black-branes. It has been shown \[11, 12\], that string scattering off D-branes agrees with the results (at large distances) of scattering particles off the black-branes, showing that

\[1\] It has been known for a long time that the type IIA supergravity can be obtained by a dimensional reduction of eleven dimensional supergravity on \(R^{10} \times S^1\).
they carry the same charges.

The purpose of this letter is to give further evidence that the long range space-
time description of the D-branes are the black-branes, and to show that the long
range interaction between the D-branes follows from eleven dimensional physics (see
also [13]). In this letter we compute the potential between two different stationary
branes, and between two different moving branes using the result from [8, 14]. We
then compute the potential between the black-branes of type IIA supergravity and
a moving zero black-brane. This calculation is done using an eleven dimensional
interpretation of these solutions [15].

The D-brane calculation is valid for small string coupling which is equivalent from
the point of view of 11D to a small radius of the eleventh direction. In general an
eleven dimensional computation might be expected to be valid only at strong coupling.
However we are going to use the eleven dimensional interpretation only to determine
the form of the interaction between the ten dimensional fields of the black-branes.
When the M-theory is compactified on $R^{10} \times S^1$ with a small $R_{11}$ we know that the
only massless particles are the ones from the reduction of the 11D supergravity down
to ten dimensions, so no new long range physics is involved. Thus if we interpret
the black-branes as some solutions of the low energy supergravity in 11D, and derive
the interaction between them from 11D physics, as long as we are looking at the
long range interaction and at processes that do not excite 11D modes, they can be
compared with a D-brane calculation. Comparing both calculations we find that they
agree at large distances. The agreement of the two calculations gives further evidence
to the consistency of the eleven dimensional picture.

2 Potential between D-branes

In this section we compute the static potential between two different D-branes [16, 17].
The vanishing of the potential between two different D-branes is a signature of the
existence of some unbroken supersymmetries in the D-branes configuration.
A D-brane of dimension $p$ is defined by the boundary condition of open strings, Neumann boundary conditions on $p + 1$ coordinates,

$$n^a \partial_a X^\mu = 0 \quad \mu = 0, \ldots, p,$$

and Dirichlet boundary conditions on $9 - p$ coordinates,

$$X^\mu = 0 \quad \mu = p + 1, \ldots, 9.$$

We define the first D-brane to be of dimension $p$ and the second of dimension $l$, where we take $p \geq l$. The branes coordinate are either parallel or orthogonal and we label the number of orthogonal coordinates as $a$ (of course $l \geq a$, $p + a \leq 9$).

We follow [8] and compute the effective potential of a stretched open superstring between the two branes. Now the string coordinate can have three different types of boundary conditions on its two ends (the boundary condition are enforced at $\sigma = 0$ and at $\sigma = \pi$): NN, DD and ND, where N (D) stands for Neumann (Dirichlet) boundary conditions. The branes are separated by a distance $b$, which is taken to be in one of the DD directions. The number($\sharp$) of string coordinates of each type is,

1. $\sharp DD = 8 - p - a$
2. $\sharp NN = l - a$
3. $\sharp ND = p - l + 2a$

Where we have taken into account that the ghost contribution will cancel two of the coordinates (NN or DD).

In order to calculate the potential between the D-branes we calculate the one loop vacuum amplitude of open superstring in the presence of the D-branes [8]. The one loop vacuum amplitude takes the form,

$$A = C \int_0^\infty \frac{dt}{(2\pi)^{(2NN+1)}} \sum_i \int_0^\infty \frac{dt}{t} e^{-2\pi \alpha' t(k^2 + M_i^2)}$$

(3)

where $\frac{1}{2\pi \alpha'}$ is the string tension, the sum runs over all the string states,

$$M_i^2 = \frac{b^2}{4\pi^2 \alpha'^2} + \frac{1}{\alpha'} \sum(\text{oscillators})$$

(4)
and $C$ is the space time volume of an $(l-a)$ brane. Integrating over $k$ and doing the oscillator sum gives

$$A = C \int \frac{dt}{t} e^{-(\frac{k^2}{2\pi \alpha'})}(8\pi^2 \alpha' t)^{-\frac{\sharp NN + 1}{2}} B \times J.$$  \hspace{1cm} (5)

$B$ and $J$ are the contribution from the bosonic and fermionic oscillators respectively.

For the bosonic oscillators the NN and DD boundary condition gives as usual integer modes, while the ND boundary condition gives the bosonic modes a spectrum of half integers. For the fermionic oscillators in the NS (Neveu-Schwarz) sector the NN and DD are as usual half integer modes, and ND boundary conditions gives integer modes. In the R (Ramond) sector NN and DD are integer modes while ND gives half integer modes. The degeneracy of the ground states becomes $2\sharp (NN + DD)/2$ in the R sector, and $2\sharp ND/2$ in the NS sector. If $\sharp ND \neq 0$ there are fermionic zero modes in the NS$(-1)^F$ sector, and thus this sector does not contribute \(^2 \) (like the R$(-1)^F$). In terms of $q = e^{-\pi t}$, we define

$$f_1(q) = q^{1/12} \prod_{n=1} (1 - q^{2n}). \hspace{1cm} (6)$$

$$f_2(q) = \sqrt{2} q^{1/12} \prod_{n=1} (1 + q^{2n}). \hspace{1cm} (7)$$

$$f_3(q) = q^{-1/24} \prod_{n=1} (1 + q^{2n-1}). \hspace{1cm} (8)$$

$$f_4(q) = q^{-1/24} \prod_{n=1} (1 - q^{2n-1}). \hspace{1cm} (9)$$

Then the bosonic and fermionic contributions are,

$$B = f_1^{-\sharp (NN + DD)}(q)f_4^{-\sharp ND}(q), \hspace{1cm} (10)$$

$$J = \frac{1}{2}\{ -f_2^{\sharp (NN + DD)}(q)f_3^{\sharp ND}(q) + f_3^{\sharp (NN + DD)}(q)f_2^{\sharp ND}(q) \}.$$ \hspace{1cm} (11)

In order to calculate the long range potential one needs the expansion of the $f_i(q)$ functions as $t \to 0$.

$$f_1(q) \to \frac{1}{\sqrt{t}} e^{-\pi/(12t)}. \hspace{1cm} (12)$$

\(^2\)The case $\sharp ND = 8$ is special and will be treated later
\[ f_2(q) \to e^{\pi/(24t)}(1 - e^{-\pi/4t}). \]  
\[ f_3(q) \to e^{\pi/(24t)}(1 + e^{-\pi/4t}). \]  
\[ f_4(q) \to \sqrt{2} e^{-\pi/(12t)}. \]  

Inserting those expressions into equation (11, 10) one gets

\[
B = 2^{-\sharp ND/2} \xi^{(DD+NN)/2} \exp \left( \frac{8\pi}{12t} \right). 
\]

\[
J = \xi^{(DD + NN - ND)} \exp - \left( \frac{8\pi}{12t} \right). 
\]

Then the one loop vacuum amplitude, in the limit of large \( b \), is

\[
A = C[2 - (p - l)/2 + a] T_p T_l G_{9-p-a}(b^2), 
\]

where \( T_i = \sqrt{\pi}(4\pi^2\alpha')^{(3-i)/2} \) is the tension of an i-brane [8], and

\[
G_{9-p-a} = \frac{1}{4} \pi^{(p-9+a)/2} \Gamma((7 - p - a)/2)(b^2)^{(p-7+a)/2} 
\]

is the scalar Green function in \((9-p-a)\) dimensions.

From equation (18) the potential can be read off \((b^2 = R^2)\) to be

\[
V(R) \sim -[2 - (p - l)/2 + a] R^{-(p+a-7)}. 
\]

The absence of the sectors NS\((-1)^F\) and R\((-1)^F\) reflects the fact that different D-branes do not interact through their RR gauge fields. As one can see from equations (11, 12) the interaction between two D-branes is governed by the number of ND type coordinate, which is a T-duality invariant quantity.

From equation (11) one sees that the static potential vanishes for \( \sharp ND = 4 \). In these cases the two branes do not exert force on each other and thus these configuration preserves some supersymmetry. The only possibilities (see also [8, 18]) are the sequences \((p = 4 + l, a = 0)\), \((p = l + 2, a = 1)\) and \((p = l, a = 2)\). The space time description of some of these configuration have been constructed recently [19, 20, 21].

The case \( \sharp ND = 8 \) is slightly different. Here all the modes in the R sector are half integer and all the modes in the NS sector are integers. In this case there are no
fermionic zero modes in the $R(-1)^F$ sector and this sector contributes. One finds,

$$A \sim \int dt e^{-\frac{b^2}{2\pi^2}(8\pi^2\alpha t)^{(2DD-1)/2}} B \times J.$$  \hspace{1cm} (20)

$$B \times J = \frac{1}{2} f_{-8}^{8} \left\{ -f_{3}^{8}(q) + f_{2}^{8}(q) + f_{1}^{8}(q) \right\}$$  \hspace{1cm} (21)

In the above expression, if $\sharp DD + 1 = 0$ we should set $b^2 = 0$.

Equation (21) vanishes due to the ‘abstruse identity’, This means that a $p$ and an $l$ brane in a configuration with $p-l+2a = 8$ (with $p+a \leq 9$) do not exert force on each other. Thus this configuration preserves some supersymmetry. Some examples are the string and the nine-brane, the zero-brane and the eight-brane, the seven-brane and the D-instanton, two five-branes intersecting on a common string, two totally orthogonal four-branes, etc. (again in agreement with [9, 18]). If one of the above branes is an anti-brane then the sign of the factor $f_{-8}^{8}(q)$ in equation (21) changes, and there is no cancellation. The space time description of these configurations can be obtained using the rules in [19]. In fact the metric for the two five-branes intersecting on a common string is presented in [21] and all the rest can be obtained using T-duality transformations of [22, 23].

One can also identify when a tachyonic instability can arise [24]. This happens when the ground state energy in the NS sector is negative which is when $\sharp(ND - NN - DD) < 0$.

3 Moving D-branes

In this section we follow [14]. We will only consider the case $a = 0$. We investigate the potential between two relatively moving different branes of dimensions $p, l$ respectively, where the $l$ brane moves with velocity $v$ with respect to the $p$ brane in one of the DD directions (taken to be $X_d$), i.e $X_d = vX_0$. As before the branes are separated in another DD coordinate by a distance $b$. As was shown in [14] the correction to the vacuum amplitude due to the moving of the branes comes from the ratio of the contribution of the $(X_0, X_d)$ coordinates, and those of the ghost. Similarly for the
ratio of the fermionic coordinates and the super-ghosts. If the branes are stationary
the ratio is 1. The ratio is different, when the branes are moving with respect to
each other, because the boundary condition for $X_0$ ($X_d$) are not NN (DD) any more.
Rather they are

$$X_d - vX_0 = \partial_\sigma (vX_d - X_0) = 0 \quad (at \quad \sigma = 0). \quad (22)$$

$$X_d = \partial_\sigma X_0 = 0 \quad (at \quad \sigma = \pi). \quad (23)$$

Similarly the boundary condition on the fermionic coordinates is also changed [26].

The one loop vacuum amplitude, $A$, takes the form

$$A = 2 \frac{C'}{4\pi} \int_0^\infty dt \frac{t}{(8\alpha'^2 t^2)} e^{-b^2 t/2}\alpha' B \times J \quad (24)$$

where $C'$ is the space volume of the $l$ brane. The bosonic and fermionic parts are ($\Theta_i$
are the usual Jacobi functions, and prime denotes the derivative with respect to the
first argument.),

$$B = f_1^{2NN}(q) f_4^{2DD}(q) \frac{\Theta'_1(0, it)}{\Theta_1(\epsilon t, it)}. \quad (25)$$

$$J = \frac{1}{2}\left\{-f_2^{2NN}(q) f_3^{2DD}(q) \frac{\Theta_2(\epsilon t, it)}{\Theta_2(0, it)} + f_3^{2NN}(q) f_2^{2DD}(q) \frac{\Theta_3(\epsilon t, it)}{\Theta_3(0, it)}\right\} \quad (26)$$

and $\tanh \pi \epsilon = v$.

We are interested in the velocity dependent long range potential between the
branes, so we evaluate equation (25,26) in the limit of small velocities and small $t$.
In order to find the correction due to the velocity one uses the differential equation
solved by the Jacobi functions,

$$\frac{\partial^2 \Theta_i(\nu, \lambda)}{\partial^2 \nu} = 4i\pi \frac{\partial \Theta_i(\nu, \lambda)}{\partial \lambda} \quad (27)$$

and the known asymptotic behavior of the theta functions at $\nu = 0$. In the limit of
small velocities and small $t$ one finds,

$$B = 2^{-\nu+1/2} \frac{1}{e^{8\pi/12t} t^{8\nu/12t} \epsilon t(1 + \epsilon^2/6)\epsilon t(1 + \epsilon^2/6)} \quad (28)$$

$$J = e^{-8\pi/12t} \{(8 - 2p + 2l) + 4(\epsilon^2)^2\}. \quad (29)$$

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The bosonic corrections are not corrections to the potential, but rather to the kinematics. Collecting all the expressions together gives

\[ A = C T_p T_l \int d\tau G_{9-p} (b^2 + \tau^2 \sinh^2(\pi \epsilon)) \]  

(30)

In the above expression we have set \( \pi \epsilon (1 + \frac{(\pi \epsilon)^2}{6}) \approx \sinh(\pi \epsilon) \). One can see that we should interpret \( \tau \) as the time in the frame of the moving brane. From this expression one can read off the velocity correction to the potential between an \( l \)-brane and a \( p \)-brane \( (\pi \epsilon \approx v, R^2 = b^2 + \tau^2 \sinh^2(\pi \epsilon)) \)

\[ V(R) \sim -R^{p-7} [2 - (p - l)/2 + v^2]. \]  

(31)

Equation (31) only depends on \( v^2 \). In fact equation (26) is even in \( \epsilon \), so one might wonder where are the linear velocity correction to the force, expected in the case of \( p + l = 6 \), due to the Lorentz force. However the way one finds the potential is through energy considerations. It can’t be expected to reproduce a Lorentz type force.

When \( p = l \) one has

\[ B = f_1^{-8} \frac{\Theta_1'(0, it)}{\Theta_1(\epsilon t, it)}, \]  

(32)

\[ J = \frac{1}{2} \left\{ -f_2^8 \frac{\Theta_2(\epsilon t, it)}{\Theta_2(0, it)} + f_3^8 \frac{\Theta_3(\epsilon t, it)}{\Theta_3(0, it)} \right\} \pm f_4^8 \frac{\Theta_4(\epsilon t, it)}{\Theta_4(0, it)}. \]  

(33)

Here + or − are for the case of brane-anti-brane and brane-brane scattering respectively. As before in the long range small velocity limit, one gets for the brane anti-brane scattering:

\[ B = t^4 e^{8\pi/12t} \frac{1}{\epsilon t(1 + (\pi \epsilon)^2/6)}, \]  

(34)

\[ J = 8e^{-8\pi/12t} (2 + (\pi \epsilon)^2). \]  

(35)

For the brane-brane case

\[ J = (\pi \epsilon)^4 e^{-8\pi/12t}, \]  

(36)

and \( B \) as in equation (34). So the velocity dependent potentials between a zero-brane and an anti-zero-brane, and between two zero-branes are,

\[ V(R) \sim -R^{p-7}(2 + v^2). \]  

(37)
\[ V(R) \sim -R^{p-7}v^4/8. \]  

(38)

4 Scattering of black-branes

In this section we will calculate the potential between a moving zero black-brane and the 0, 2, 4, 6 black-branes of type IIA supergravity. The zero black-brane has a very simple interpretation in eleven dimensions, as a KK mode. It is then easy to understand its coupling to the other branes from an eleven dimensional point of view.

One starts with the extremal zero black-brane solution (in the string metric) of [25]. The space-time coordinates will be labeled 1, \ldots, 11, time being the first coordinate. The relationship with the 11D metric is [1],

\[ ds_{11}^2 = e^{-2\phi/3}ds_{10}^2(string) + e^{4\phi/3}(dx_{11}^2 - A_\mu dx^\mu)^2 \]  

and we ignore all of the gauge potentials other that the one-form, as they will not enter in our discussion. In equation (39) \( \phi \) is the dilaton and \( A_\mu \) is the one-form gauge potential in the RR sector. Then by a change of coordinates \( R^2 = \sum_j x_j^2 \)

\[ R^7-p = r^7-p - r^7-p^+ \]

one arrives at the following metric for the zero black-brane \( (j = 2, \ldots, 10) \)

\[ ds_{11}^2 = -(1 - \frac{r^7}{R^7})dt^2 + 2\frac{r^7}{R^7}dtdx_{11} + (1 + \frac{r^7}{R^7})dx_{11}^2 + dx_j dx^j. \]  

(40)

For the two black-brane (where \( j = 4, \ldots, 10 \) and \( i = 2, 3 \)),

\[ ds_{11}^2 = (1 + \frac{r^5}{R^5})^{1/3}\left[(1 + \frac{r^5}{R^5})^{-1}(-dt^2 + dy_idy^i) + dx_j dx^j + dx_{11}^2\right]. \]  

(41)

For the four black-brane (where \( j = 6, \ldots, 10 \) and \( i = 2, \ldots, 5 \)),

\[ ds_{11}^2 = (1 + \frac{r^3}{R^3})^{2/3}\left[(1 + \frac{r^3}{R^3})^{-1}(-dt^2 + dy_idy^i + dx_{11}^2) + dx_j dx^j\right]. \]  

(42)

For the six black-brane

\[ ds_{11}^2 = (1 + \frac{r^+}{R})\left[(1 + \frac{r^+}{R})^{-1}(-dt^2 + dy_idy^i) + (1 + \frac{r^+}{R})^{-2}(dx_{11} + r^+(1 - \cos \theta)d\phi)^2 + dR^2 + R^2d^2\Omega_2\right]. \]  

(43)
Let us focus on the zero brane. Equation (40) is the metric of a plane-fronted gravitational wave moving in the \( x_{11} \) direction. In fact this metric can be obtained by starting with the Schwarzschild metric in 10D and boosting it in the eleventh direction to the speed of light while keeping the combination \( r_+^7 \sinh^2 \alpha \) fixed (here \( r_+, \alpha \) are the Schwarzschild radius and the boost parameter respectively) \([27, 28]\). This shows that the zero black-brane metric is not a solution of 11D gravity, but rather requires a source which might be thought of as coming from the elementary membrane of the M-theory. The fact that zero D-branes do not exert forces on each other, is the well known fact that two parallel moving null particles do not interact. It is represented in the zero black-brane metric by the fact that \( dx_{11} = -dt \) is a geodesic of the metric (40). Of course \( dx_{11} = dt \) is not a geodesic of the metric (40), which corresponds to an interaction between a zero-brane and an anti zero-brane.

At large distances the zero black-brane does not affect the other black-brane’s metric, as its metric coefficient falls much faster than the other black-branes. From an eleven dimensional point of view the zero black-brane metric is just the metric generated by a massless scalar particle moving in the eleventh direction. The scattering of the zero black-brane from the other black-branes should then be dictated by 11D diffeomorphism invariance. To calculate the semiclassical scattering of the zero black-brane from the other black-branes all we need is to calculate the scattering of the null geodesic off the other black-branes metrics. As was explained in the introduction, we do expect to be able to compare this calculation, to a D-brane calculation, as long as we look at the long range interactions.

The potential can be basically read off the black branes metric, and one gets for the 2, 4 and 6 black-brane

\[
V_{(0-2)}(R) = -a_2 R^{-5}(1 + v^2). \quad (44)
\]
\[
V_{(0-4)}(R) = -a_4 R^{-3}v^2. \quad (45)
\]
\[
V_{(0-6)}(R) = -a_6 R^{-1}(v^2 - 1). \quad (46)
\]

In the case of the six-black-brane we have constrained the geodesic to be a line of
constant angle $\phi$, in order to suppress the Lorentz force.

Although the above reasoning does not necessarily apply when we scatter two zero black-brane we will compute the potential for these cases also. For the anti zero black-brane

$$V(R) = -a_0 2R^{-7}(2 + v^2)$$

(47)

and for the zero black-brane

$$V(R) = -a_0 R^{-7} v^4/4.$$  (48)

The absence of $v^2$ term in the potential between two identical parallel extremal black-branes was proved in [29].

Comparing the potential derived from the 11D interpretation of the black-branes (equations (44-48)) to the potential computed in the D-brane approach (equations (31,37,38)), we see that there is perfect agreement between them.

5 Discussion

We have shown that the computation of the potentials between two D-branes agrees with a computation from a space-time, black-brane approach. The interaction between the black-branes was taken to be the interaction coming from their 11D interpretation (although only a subclass of black-branes was considered). We believe this is further evidence for the existence of an underlying eleven dimensional theory.

The computation of geodesics on a given background is equivalent to the eikonal approximation, which might be expected to work for the scattering of the zero-brane from the anti zero-brane [30], but it is unclear why it reproduces the result for the zero-brane, zero-brane scattering.

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