Nuclear Three-body Force Effect on a Kaon Condensate in Neutron Star Matter

W. Zuo\textsuperscript{1,2,3,4}, A. Li\textsuperscript{2}, Z. H. Li\textsuperscript{1,3}, U. Lombardo\textsuperscript{5}

\textsuperscript{1} Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, P. R. China

\textsuperscript{2} School of Physics and Technology, Lanzhou University, Lanzhou 730000, P. R. China

\textsuperscript{3} Graduate School of Chinese Academy of Sciences, Beijing 100039, P. R. China

\textsuperscript{4} Institut für Theoretische Physik der Justus-Liebig-Universität, D-35392, Giessen, Germany

\textsuperscript{5} INFN-LNS, Via Santa Sofia 44, I-95123 Catania, Italy

We explore the effects of a microscopic nuclear three-body force on the threshold baryon density for kaon condensation in chemical equilibrium neutron star matter and on the composition of the kaon condensed phase in the framework of the Brueckner-Hartree-Fock approach. Our results show that the nuclear three-body force affects strongly the high-density behavior of nuclear symmetry energy and consequently reduces considerably the critical density for kaon condensation provided that the proton strangeness content is not very large. The dependence of the threshold density on the symmetry energy becomes weaker as the proton strangeness content increases. The kaon condensed phase of neutron star matter turns out to be proton-rich instead of neutron-rich. The three-body force has an important influence on the composition of the kaon condensed phase. Inclusion of the three-body force contribution in the nuclear symmetry energy results in a significant reduction of the proton and kaon fractions in the kaon condensed phase which is more proton-rich in the case of no three-body force. Our results are compared to other theoretical predictions by adopting different models for the nuclear symmetry energy. The possible implications of our results for the neutron star structure are also briefly discussed.

Key words: Dense nuclear matter, nuclear symmetry energy, kaon condensation, nuclear three-body force, Brueckner-Hartree-Fock approach

PACS numbers: 21.65.+f, 26.60.+c, 97.60.Jd
I. INTRODUCTION

As well known that around $\rho \simeq \rho_0$ where $\rho_0 = 0.16 \text{ fm}^{-3}$ is the empirical saturation density of nuclear matter, neutron star matter in chemical equilibrium consists mainly of nucleons and leptons, whereas at high enough densities it may become more or less exotic\cite{1}. Among the possible exotic phases in dense nuclear matter, the kaon condensation is a subject of great interest in nuclear physics, hadronic physics and neutron star physics\cite{2, 3, 4}. Since been proposed by Kaplan and Nelson\cite{5}, the possibility of kaon condensation and its implications for astrophysical phenomena of neutron stars have been extensively discussed by many physicists\cite{6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16}. It has been suggested in Ref.\cite{17} that the condensation of kaons may be understood as a chiral rotation away from a $V$-spin scalar ground state and it is related to a partial restoration of the chiral symmetry explicitly broken in the vacuum. The presence of kaon condensation may have important consequences for determining the structures and evolutions of neutron stars. For example, the kaon condensation may soften substantially the equation of state (EOS) of neutron star matter and consequently lower the predicted maximum mass of neutron stars\cite{3}. The phase transition from the normal matter to the kaon condensed matter is also expected to affect the transport properties and the glitch behavior of pulsars\cite{8}.

It is shown\cite{5} that the kaon-nucleon sigma term $\Sigma_{KN}$ provides a strongly attractive interaction between kaons and nucleons. This attraction reduces the $K^-$ energy $\omega_K$ in nuclear medium and it becomes strong enough at a high enough baryon density above which the kaon condensed phase is energetically favorable. It is shown that the kaon–nucleon sigma term plays an essential role in determining the formation of such a phase. Besides the kaon-nucleon interaction, the high-density behavior of the nuclear symmetry energy is also important for determining the threshold density for kaon condensation and the composition of the kaon condensed phase. Associated to the high-density symmetry energy there is a large uncertainty due to the lack of experimental constraints\cite{18}. The nuclear symmetry energy plays its role somewhat in a different way from the kaon–nucleon interaction. It determines to a large extent the electron chemical potential in $\beta$-equilibrium neutron star matter below the critical density for kaon-condensation\cite{19, 20, 21}. The electron chemical potential $\mu_e$ serves as the energy threshold for kaon condensation since the kaon condensation becomes energetically favorable as soon as $\mu_e > \omega_K$. 
Since realistic nucleon-nucleon (NN) interactions have not been fully determined from the chiral theory up to now \cite{22}, various theoretical models of the symmetry energy have been adopted for studying the properties of kaon condensation. In the work of Ref. \cite{8}, the simple parametrizations proposed by Prakash et al. \cite{23} has been chosen for the nuclear symmetry energy. In the investigations of Ref. \cite{16}, the symmetry energy derived from the variational approach \cite{24} has been applied. It turns out that besides the dependence on $\Sigma_{KN}$, the condensation is also sensitive to the high-density behavior of the symmetry energy. Within the framework of a non-relativistic microscopic model based on realistic NN interactions, nuclear three-body forces are critical for reproducing the empirical saturation properties of nuclear matter \cite{27, 28, 29, 30, 31}. In our previous work \cite{21, 30}, the EOS of nuclear matter has been explored in the Brueckner-Hartree-Fock (BHF) approach by adopting a microscopic three-body force (TBF) from the meson-exchange current method \cite{28}. The TBF turns out to affect strongly the high-density behavior of the symmetry energy, i.e., it makes the density dependence of the symmetry energy much stiffer as compared to the result without the TBF contribution. The aim of the present work is to investigate the critical density for kaon condensation in neutron stars and the composition of the kaon condensed phase in neutron star matter by using the BHF approach for the nuclear symmetry energy. In the calculations, we adopt the chiral Lagrangian \cite{5} to extract the kaon-nucleon part of the interactions as in Refs. \cite{9, 16}. Special attention has been paid on the effects of the nuclear three-body force.

This paper is organized as follows. In Sect. 2 we review briefly the theoretical models adopted in our calculations, including the self-consistent BHF approach, the microscopic TBF, and the chiral model for kaon-nucleon interaction. Our numerical results are presented and discussed in Sect. 3. In Sect. 4 a summary of the present work is given.

## II. THEORETICAL MODELS

In the kaon condensed phase of neutron star matter, the energy density consists of three parts of contributions, i.e.,

$$\varepsilon = \varepsilon_{NN} + \varepsilon_{lep} + \varepsilon_{KN}$$

(1)

where $\varepsilon_{NN}$ is the nuclear part, $\varepsilon_{lep}$ denotes the contribution of leptons, and $\varepsilon_{KN}$ is the contribution from the kaon-nucleon interaction.
Our calculations of the nuclear part of the EOS are based on the BHF approach for asymmetric nuclear matter\cite{20}. The starting point of the BHF approach is the interaction $G$ matrix which satisfies the following Bethe-Goldstone (BG) equation\cite{20}

$$G(\rho, \beta; \omega) = v_{NN} + v_{NN} \sum_{k_1, k_2} \frac{|k_1 k_2)}{\omega - \epsilon(k_1) - \epsilon(k_2) + i\eta} G(\rho, \beta; \omega),$$

(2)

where $\omega$ is the starting energy and $Q(k_1, k_2) = [1 - n(k_1)][1 - n(k_2)]$ is the Pauli operator which prevents the two intermediate nucleons from being scattered into the states below the Fermi sea. The isospin asymmetry parameter is defined as $\beta = (\rho_n - \rho_p)/\rho_B$, $\rho_n$, $\rho_p$, and $\rho_B$ being the neutron, proton and total baryon number densities, respectively. The single particle (s.p.) energy is given by $\epsilon(k) = \hbar^2 k^2/(2m) + U(k)$, where the s.p. potential is calculated from the real part of the on-shell $G$ matrix, i.e.

$$U(k) = \sum_{k'} n(k')\text{Re}\langle k k' | G(\epsilon(k) + \epsilon(k')) | k k' \rangle_A .$$

(3)

In the present calculations, we adopt the continuous choice for $U(k)$ since it has been proved to provide a much faster convergency of the hole-line expansion for the energy per nucleon in nuclear matter up to high densities than the gap choice\cite{25}. In addition, in the continuous choice, the s.p. potential describes physically the nuclear mean field felt by a nucleon in nuclear medium. The realistic nucleon-nucleon (NN) interaction $v_{NN}$ is the Argonne $V_{18}$ ($AV_{18}$) two-body force\cite{26} supplemented with the contribution of the TBF.

The microscopic TBF adopted in the present calculations is constructed from the meson-exchange current approach\cite{28}. In this TBF model, four important mesons $\pi$, $\rho$, $\sigma$ and $\omega$ are considered\cite{29}. The meson masses in the TBF have been fixed at their physical values except for the virtual $\sigma$-meson mass which has been fixed at 540MeV according to Ref.\cite{28}. This value has been checked to satisfactorily reproduce the $AV_{18}$ interaction from the one-boson-exchange potential (OBEP) model\cite{30}. The other parameters of the TBF, i.e., the coupling constants and the form factors, have been determined from the OBEP model to meet the self-consistent requirement with the adopted $AV_{18}$ two-body force. The values of the parameters are given in Ref.\cite{30}. For a more detailed description of the model we refer to Refs.\cite{28,30}. We want to stress that the most important component of this TBF is essentially that introduced automatically in the Walecka relativistic mean field theory\cite{33} over that in the non relativistic theories.
In order to include the TBF contribution into the BG equation, we follow the standard scheme\cite{28, 32, 34} to reduce the TBF to an effective two-body interaction which is expressed in $r$ space as

$$\langle \vec{r}_1\vec{r}_2|V^{\tau_1\tau_2}_3|\vec{r}_1\vec{r}_2'\rangle = \frac{1}{4} \text{Tr} \sum_n \int d\vec{r}_3 d\vec{r}_3' \phi_n^*(\tau_3 r_3)(1 - \eta_{\tau_1,\tau_3}(r_3))(1 - \eta_{\tau_2,\tau_3}(r_3))$$

where the trace is taken with respect to the spin and isospin of the third nucleon and the indices $\tau_1$, $\tau_2$ and $\tau_3$ denote the isospin $z$-components. The function $\eta_{\tau,\tau'}(r)$ is the defect function. Since the defect function is directly determined by the solution of the BG equation\cite{28, 32}, it must be calculated self-consistently with the $G$ matrix and the s.p. potential $U(k)$\cite{30} at each density and isospin asymmetry.

By solving self-consistently the coupled Eqs.\cite{2}, \cite{3}, and \cite{4} we can obtain the reaction $G$ matrix. From the $G$ matrix, we calculate the nuclear contribution $\varepsilon_{NN}$ to the total energy density $\varepsilon$ for any given baryon density and isospin asymmetry. The $\varepsilon_{NN}$ of asymmetric nuclear matter can be separated into two part, i.e., the isoscalar part and the isovector part. The stiffness of the isoscalar part plays an important role in predicting the structure of a neutron star while the isovector part is crucial for the chemical properties of neutron star matter. Microscopic investigations\cite{20, 30} show that for a fixed baryon density and isospin asymmetry, the isovector part of $\varepsilon_{NN}$ is completely determined by the symmetry energy $S(u)$ and can be expressed as $\rho_0 u S(u) \beta^2$, where $u \equiv \rho_B / \rho_0$ is the dimensionless baryon density and $\rho_0 = 0.16$ fm$^{-3}$ the empirical saturation density of nuclear matter.

We evaluate the kaon-nucleon sector in the energy density of the kaon-condensed phase of matter in terms of the effective chiral Lagrange density which was suggested by Kaplan and Nelson\cite{5} and extensively investigated afterwards\cite{7}. In the Kaplan-Nelson model, the $SU(3) \times SU(3)$ chiral Lagrange density is expressed as

$$L_\chi = \frac{F^2}{4} \text{Tr} \partial_\mu U \partial^\mu U^\dagger + \text{Tr} B(i\gamma^\mu D_\mu - m_B)B$$

$$+ F \text{Tr} \bar{B} \gamma^\mu \gamma_5 \{A_\mu, B\} + D \text{Tr} \bar{B} \gamma^\mu \gamma_5 \{A_\mu, B\}$$

$$+ c \text{Tr} \mathcal{M} (U + U^\dagger) + a_1 \text{Tr} \bar{B} (\xi \mathcal{M} \xi + \xi^\dagger \mathcal{M} \xi) B$$

$$+ a_2 \text{Tr} \bar{B} B (\xi \mathcal{M} \xi + \xi^\dagger \mathcal{M} \xi) + a_3 \text{Tr} \bar{B} B \text{Tr} (\xi \mathcal{M} \xi + \xi^\dagger \mathcal{M} \xi).$$

where we adopt the same notations and symbols as in Ref.\cite{5}. The above Lagrange includes an octet of pseudoscalar masons $\mathcal{M}$ and an octet of baryons $B$. The first four terms conserve
the chiral symmetry. $f = 93\text{MeV}$ is the pion decay constant. The constants $D$ and $F$ are given by $D = 0.81$ and $F = 0.44$. The coefficients $a_1 m_s, a_2 m_s, a_3 m_s$, and $c$ determine the strength of the chiral symmetry breaking in the last four terms. We adopt $a_1 m_s = -67\text{MeV}$ and $a_2 m_s = 134\text{MeV}$ obtained from the baryon mass splittings [6]. The constant $c$ is related to the bare kaon mass by the Gell-Mann-Oakes-Renner relation $m_K^2 = 2c m_s/f^2$. The parameter $a_3 m_s$ had remained largely uncertain for many years due to our poor knowledge of the strangeness content of the proton and the kaon-nucleon sigma term $\Sigma_{KN}$ [2, 5, 6, 35]. Fortunately, the large ambiguity in this parameter has been settled recently by Dong, Lagaè and Liu [36] with small error based on the lattice calculations. In the present calculations we adopt the value of $a_3 m_s$ in the range $-310\text{ MeV} < a_3 m_s < -134\text{ MeV}$ as done in Refs. [9, 16] in order to investigate the sensitivity of the TBF effect to the varying of the proton strangeness content. Our adopted central value $a_3 m_s = -222\text{MeV}$ is very close to the value extracted from the lattice gauge calculations in Ref. [36] which is $-231\text{ MeV}$ with error of less than 4%. $a_3 m_s$ measures the proton strangeness content and plays an essential role in determining the threshold density for kaon condensation [9] since it provides the dominant attraction in the kaon-nucleon interaction via the sigma term $\Sigma_{KN} = -(a_1 + 2a_2 + 4a_3)m_s/2$. The above specified range of $a_3 m_s$ from $-310\text{ MeV}$ to $-134\text{ MeV}$ corresponds to a reasonable range $0.2 > \langle \bar{s}s \rangle_p / \langle \bar{u}u + \bar{d}d + \bar{s}s \rangle_p > 0$ of the proton strangeness content [35]. Following exactly the standard prescript in Ref. [9], we can get the kaon-nucleon energy density of the kaon condensed matter, i.e.

$$\varepsilon_{KN} = \frac{f^2}{2} m_K^2 \sin^2 \theta + 2m_K^2 f^2 \sin^2 \frac{\theta}{2} + \rho_B (2a_1 x_p + 2a_2 + 4a_3) m_s \sin^2 \frac{\theta}{2}$$  \hspace{1cm} (6)

where $\rho_B$ and $x_p \equiv \rho_p / \rho_B$ denote the baryon number density and the proton fraction, respectively. $\theta$ is the amplitude of the condensation and comes from applying the Baym theorem [37]. The lepton part of the energy density $\varepsilon_{lep}$ can be readily obtained in a standard way from the contributions of the filled Fermi seas of leptons [9, 16].

In the neutron star matter with kaons the chemical equilibrium can be reached through the following reactions

$$n \leftrightarrow p + l + \nu_l, \quad n \leftrightarrow p + K^-, \quad l \leftrightarrow K^- + \nu_l,$$  \hspace{1cm} (7)

where $l$ denotes leptons, i.e., $l = e, \mu$. One can determine the ground state by minimizing the total energy density with respect to the condensate amplitude $\theta$ keeping all densities fixed.
This minimization together with the chemical equilibrium and charge neutrality conditions leads to the following three coupled equations\cite{9, 16}

$$\cos \theta = \frac{1}{f^2 \mu^2} \left( m_K^2 f^2 + \frac{1}{2} u \rho_0 (2a_1 x + 2a_2 + 4a_3) m_s - \frac{1}{2} \mu u \rho_0 (1 + x) \right),$$  
\hspace{1cm} (8)\hspace{1cm}

and

$$\mu \equiv \mu_e = \mu_K = \mu_n - \mu_p = 4(1 - 2x) S(u) \sec^2 \frac{\theta}{2} - 2a_1 m_s \tan^2 \frac{\theta}{2},$$  
\hspace{1cm} (9)\hspace{1cm}

where $\mu$ represents the chemical potential of the negative charged particles and $u \equiv \rho_B / \rho_0$ is the baryon number density in unit of $\rho_0$. The last two equations are from the chemical equilibrium and charge neutrality conditions, respectively. The EOS and the composition of the kaon condensed phase in the chemical equilibrium neutron star matter can be obtained by solving the coupled equations (8), (9), and (10). The critical density for kaon condensation is determined as the very point above which a real solution for the coupled equations can be found.

**III. RESULTS AND DISCUSSIONS**

The density dependence of the nuclear symmetry energy $S(u)$ is depicted in Fig.1 where the bold solid line and the thin solid line are obtained from the BHF approach adopting the $AV_{18}$ plus the TBF and the pure $AV_{18}$ two-body force, respectively. In the figure the symmetry energy from other models\cite{23, 24} are also plotted for comparison. The dashed, dotted, and dot-dashed curves correspond to the following parameterizations proposed in Ref.\cite{23},

$$S(u) = \left( \frac{2}{3} - 1 \right) \frac{3}{5} E_F^{(0)} \left( u^2 - F(u) \right) + S_0 F(u)$$  
\hspace{1cm} (11)\hspace{1cm}

and

$$F_1(u) = u, \hspace{1cm} F_2(u) = \frac{2u^2}{1 + u} \hspace{0.5cm} \text{or} \hspace{0.5cm} F_3(u) = \sqrt{u}$$  
\hspace{1cm} (12)\hspace{1cm}

where $S_0 \simeq 30$ MeV and $E_F^{(0)} = (3 \pi^2 \rho_0 / 2)^{2/3} / 2m$ are the symmetry energy and the Fermi energy at the nuclear saturation density, respectively. The double-dot-dashed line is the result of Ref.\cite{24} by using the variational approach with the UV14+TNI interaction. It is seen from the figure that in both cases with and without the TBF contribution, our calculated $S(u)$ are monotonically increasing functions of the baryon density. Below and
around the empirical saturation density, the TBF effect is quite small, while at high densities the TBF contribution leads to a strong enhancement of the increasing rate of $S(u)$. This strong enhancement of the symmetry energy due to the TBF could be explained as follows. The symmetry energy (the isovector sector of the EOS) describes the energy required to increase the isospin asymmetry of the matter. A higher value of asymmetry implies a larger neutron excess in the matter. Therefore for a fixed density, at a higher isospin asymmetry the neutron Fermi momentum becomes larger and consequently some nucleons may have higher momentum than in the symmetric nuclear matter. Since the TBF is a short-range interaction and its effect is stronger for nucleons with larger momenta, its contribution to the EOS increases as the isospin asymmetry increases \[28, 30\].

The density dependence of the chemical potential $\mu$ of the negative charged particles in $\beta$-equilibrium neutron star matter is given in Fig.2 for three values of $a_3m_s = -310, -222,$ and $-134$ MeV denoted by $a, b, c$, respectively. In the figure the solid and dashed curves are the results with and without including the nuclear TBF contribution. Below the critical density for kaon condensation, the matter is made up of neutrons, protons, and leptons. The chemical potential $\mu$ is determined by the density dependence of the symmetry energy and it increases gradually with density in the case that the symmetry energy is an monotonically increasing function of density. The TBF enhances the increasing rate of $\mu$. Above the critical density, $\mu$ becomes a decreasing function of density. The decreasing rate of $\mu$ depends strongly on the choice of the proton strangeness content (i.e., on the value of $a_3m_s$), but completely insensitive to the choice of different models for the symmetry energy $S(u)$ in the case that $S(u)$ is an monotonically increasing function of density as shown in Ref.\[9\]. As a consequence, the TBF contribution has almost no any effect on the chemical potential in the condensed phase.

The values of the critical density $u_c$ for different models of the symmetry energy $S(u)$, and some typical values of the proton strangeness are presented in Tab.1. The results in Ref.\[9\] (denoted with $F1, F2,$ and $F3$) and Ref.\[16\] (denoted with $UV14 + TNI$) are also given. As shown in Tab.1 the critical density $u_c$ for kaon condensation depends on both the proton strangeness content (i.e., the value of $a_3m_s$) and the high-density behavior of the symmetry energy. The critical density $u_c$ is more sensitive to the proton strangeness than to the symmetry energy. The value of $u_c$ turns out to be lowered by increasing the proton strangeness, since the attractive interaction provided by the kaon-nucleon sigma
TABLE I: Critical density $u_c$ for kaon condensation in unit of $\rho_0$.

| $a_3m_3$ [MeV] | F1 | F2 | F3 | BHF | BHF+TBF | UV14+TNI |
|----------------|----|----|----|-----|---------|---------|
| a             | -310 | 2.4 | 2.3 | 2.6 | 2.4     | 2.8     |
| b             | -222 | 3.1 | 2.9 | 3.4 | 2.9     | 4.1     |
| c             | -134 | 4.2 | 3.8 | 4.9 | 5.0     | -       |

term becomes smaller as the strangeness increases. This is in good agreement with the previous investigations by adopting different models for the symmetry energy. It is seen from the table that the predicted values of $u_c$ become less sensitive to different models for the symmetry energy as the strangeness increases. The TBF affects the critical density $u_c$ via its contribution to the symmetry energy and its effect is to reduce the critical density since it provides an additional repulsive contribution to the isospin symmetry energy. Our predicted critical density is in the range of $2.4\rho_0 - 3.8\rho_0$ if the TBF is included and $2.6\rho_0 - 5.0\rho_0$ in the case without the TBF contribution. In the case of no strangeness, inclusion of the TBF contribution in the symmetry energy reduces $u_c$ by more than 20% from 5 to 3.8. However, if the strangeness is higher, the TBF effect on $u_c$ becomes somewhat smaller. For instance, in the case of $a_3m_3 = -310$ MeV, the TBF leads to only a less than 10% reduction of $u_c$. This can be readily understood since on the one hand, for larger values of the strangeness, the kaon condensation sets in at lower densities where the TBF contribution to the symmetry energy is relatively small. On the other hand, if the strangeness is higher, the in-medium energy of $K^-$ drops down faster as a function of baryon density. As a consequence, the role played by the strangeness becomes more predominant over that by the symmetry energy for a higher value of the strangeness.

In Fig. is plotted the predicted composition of $\beta$-equilibrium neutron star matter for three different values of $a_3m_3 = -310$ MeV (the upper panel), $-222$ MeV (the middle panel), and $-134$ MeV (the lower panel) for both cases with the TBF (curves with symbols) and without the TBF (curves without symbols). In the figure the proton fraction $x_p \equiv \rho_p/\rho_B$, the kaon fraction $x_K \equiv \rho_K/\rho_B$, and the lepton fraction $x_{\text{lep}} = x_e + x_\mu \equiv (\rho_e + \rho_\mu)/\rho_B$ are given by the solid curves, dashed curves, and dotted curves, respectively. Below the critical density,
the matter is in its normal phase which is a highly neutron-rich and charge-neutral mixture of neutrons (n), protons (p), electrons (e), and muons (µ). In the normal $n, p, e, µ$ matter, the proton fraction $x_p$ increases gradually with density and the TBF contribution makes the $x_p$ rise faster. However, in the condensed phase, the matter becomes more or less proton-rich instead of neutron-rich, since a large fraction of protons is required to balance the negative charge of $K^-$ which is so abundant in the kaon condensed phase. At high enough density, the matter contains even positrons in chemical equilibrium to ensure the charge neutrality. These results are in agreement with the previous investigations[9, 16]. From Fig.3 one can see that the proton and kaon fractions in the condensed phase depend sensitively on the high-density behavior of the symmetry energy. Inclusion of the TBF contribution makes the kaon condensed neutron star matter more symmetric in protons and neutrons. As the matter goes from the normal phase to the kaon condensed phase, the redistribution of charge depends strongly on the high-density behavior of the symmetry energy. If the TBF contribution is included, the symmetry energy rises much faster as a function of density. As a consequence, the matter is driven to more symmetric in neutrons and protons in order to reduce the additional repulsive isospin energy due to the TBF contribution.

In Ref.[16] the kaon condensation has been studied by adopting the symmetry energy from the variational calculations in Ref.[24]. In contrast to the present results from the BHF approach as well the predictions from the Dirac-Brueckner method[38] and the relativistic mean field theory[39], the symmetry energy calculated in Ref.[24] decreases with density at high enough densities. As a consequence, the composition in the kaon condensed phase in Ref.[16] strongly deviates from our result due to the completely different high-density behavior of the symmetry energy. For example, in the most extreme case of the UV14+TNI interaction, the obtained neutron star becomes almost a “proton” star just above the critical density for kaon condensation as shown in Fig. 4. While in the present calculations, the neutron star matter with kaons prefers to be symmetric nuclear matter in agreement with the results reported in Ref.[8]. The discrepancy between the high-density behavior of the symmetry energy obtained from the BHF and the variational approaches is still not clear[20, 27] and deserves further investigations.

It should be stressed that in the present investigations, we do not consider the mixed phase of the normal matter and the condensed matter. It has been pointed out[8] that in the first-order phase transition to the kaon condensed phase, the conservation law(s) can only be.
fulfilled in a global rather than a local sense. This means that in the mixed phase, the local charge-neutrality, i.e. Eq. (10) becomes invalid. Since our aim is to investigate the nuclear TBF effects on the critical density and the chemical composition of the condensed phase, the present simplification is somewhat desirable for our purpose. However, for practical applications in neutron stars we should consider the problem of the mixed phase formation as done in Ref. [8] which will be discussed elsewhere.

Before summary, we discuss briefly the possible implications of our results for the maximum mass of neutron stars. In Fig. 5 we display the predicted EOS of neutron star matter for both cases with kaons (solid curves) and without kaons (dashed curves). In the figure the results by using the $AV_{18}$ plus the TBF (right panel) are compared to the ones by adopting the pure $AV_{18}$ two-body force (left panel). It is seen that in the case of no kaons, inclusion of the TBF contribution makes the EOS at high densities become much more stiffer. As a consequence, the maximum mass of neutron stars becomes much larger if the TBF contribution is included as shown in Ref. [40] where the calculated maximum mass is about 2.3 $M_\odot$ and 1.6 $M_\odot$ respectively for the two cases with and without including the TBF in the calculations for our central value of $a_3 = -222$ MeV. The latter value is close to the 1.5 $M_\odot$ found in Ref. [9] for a compression modulus of 210 MeV. In the case that kaons are allowed, the EOS of the matter in the condensed phase is softened considerably due to the attractive interaction between nucleons and kaons. Since a softer EOS implies a smaller maximum neutron star mass, the kaon condensation may lower the predicted maximum mass in both cases with and without the TBF. In our present work, however, we do not consider the role of other possible strange particles, such as $\Lambda$, $\Sigma$, $\Xi$ hyperons. The appearance of hyperons may provide an additional softening of the neutron star EOS and consequently reduce further the predicted maximum mass [41]. One would have to minimize the free energy at each density in order to choose the appropriate configuration.

**IV. SUMMARY AND CONCLUSION**

In summary, we have investigated the nuclear TBF effects on the kaon condensation in neutron star matter in the framework of the BHF approach. The TBF affects the critical density for the kaon condensation through its repulsive contribution to the symmetry energy. In both cases with and without the TBF, the calculated symmetry energy is an monotonically
increasing function of baryon density, in agreement with the results from the relativistic mean field approach and the DB approach. The TBF repulsion turns out to increase rapidly as the baryon density increases. As a consequence the high-density behavior of the symmetry energy becomes much stiffer as compared to the one obtained by adopting only the two-body nuclear force. Our results show that inclusion of the TBF contribution in the symmetry energy reduces the critical density by about 8% ~ 25% depending on the choice of the proton strangeness content. The influence of the TBF on the critical density becomes smaller if the strangeness is higher. The predicted critical density is in the range from $2.4\rho_0$ to $3.8\rho_0$ when the TBF is included.

The additional repulsive contribution to the isospin energy due to the TBF drives the kaon condensed phase of neutron star matter to become more symmetric in neutrons and protons as compared to the results without the TBF. In the normal phase of neutron star matter the proton fraction is small and the matter is highly neutron-rich. The proton fraction gradually increases with baryon density. While in the kaon condensed phase, the matter becomes proton-rich in order to balance the negative charge of the kaon field. The composition in the kaon condensed phase depends sensitively on the high-density behavior of the symmetry energy. We find that the additional repulsion from the TBF lowers the proton and kaon fractions in the kaon condensed phase and it is so strong at high densities as to make the condensed matter almost symmetric in neutrons and protons. The EOS of neutron star matter is found to be softened considerably by the kaon-nucleon interaction in the kaon condensed phase.

V. ACKNOWLEDGMENT

The work is supported in part by the Knowledge Innovation Project of the Chinese Academy of Sciences (KJCX2-SW-N02), the Major State Basic Research Development Program of China (G2000077400), and the Major Prophase Research Project of Fundamental Research of the Ministry of Science and Technology of China (2002CCB00200), the National Natural Science Foundation of China (10235030, 10175082) and DFG, Germany.

[1] C. J. Pethick, Rev. Mod. Phys. 64, 1133 (1992).
[2] C. H. Lee, Phys. Rep. 275, 255 (1996) and references therein; C. H. Lee, G. E. Brown, D. P. Min, and M. Rho, Nucl. Phys. A 585, 401 (1995).
[3] G.Q.Li, C.-H.Lee, and G.E.Brown, Phys. Rev. Lett. 79, 5214(1997); Nucl. Phys. A 625, 372 (1997).
[4] G. E. Brown, C.-H.Lee, and R. Rapp, Nucl. Phys. A 639, 455c (1998).
[5] D. B. Kaplan and A. E. Nelson, Phys. Lett. B 175, 57 (1986).
[6] H. D. Politzer and M. B. Wise, Phys. Lett. B 273, 156 (1991).
[7] G. E. Brown, K. Kubodera, M. Rho, and V. Thorsson, Phys. Lett. B 291, 355 (1992).
[8] N. K. Glendenning, Phys. Rev. D 46, 1274(1992); N. K. Glendenning and J. Schaffner-Bielich, Phys. Rev. Lett. 81, 4564 (1998); Phys. Rev. C 60, 025803 (1999).
[9] V. Thorsson, M. Prakash, and J. M. Lattimer, Nucl. Phys. A 572, 693(1994); A 574, 851(1994), Erratum.
[10] P. J. Ellis, R. Knorren, and M. Prakash, Phys. Lett. B 349, 11 (1995).
[11] M. Yasuhira and T. Tatsumi, Nucl. Phys. A 663, 881(2000); A 670, 218(2000); T. Muto, M. Yasuhira, T. Tatsumi, and N. Iwamoto, Phys. Rev. D 67, 103002(2003).
[12] J. A. Pons, J. A. Miralles, M. Prakash, and J. M. Lattimer, Astrophys. J. 553, 382 (2001).
[13] A. Ramos, J. S. Bielich, and J. Wambach, Lett. Notes. Phys. 578, 175 (2001).
[14] J. Carlson, H. Heiselberg, and V. R. Pandharipande, Phys. Rev. C63, 017603 (2001).
[15] T. Norsen and S. Reddy, Phys. Rev. C63, 065804(2001).
[16] S. Kubis and M. Kutschera, Nucl. Phys. A 720, 189(2003).
[17] G. E. Brown, K. Kubodera, and M. Rho, Phys. Lett. B 192, 273(1987).
[18] P. Danielewicz, R. Lacey, and W. G. Lynch, Science 298, (2002)1592; B. A. Li, Phys. Rev. Lett. 88, 192701 (2002); A. E. L. Dieperink, Y. Dewulf, D. Van Neck, M. Waroquier, and V. Rodin, Phys. Rev. C 68, 064307 (2003).
[19] J. M. Lattimer, C. J. Pethick, M. Prakash, and P. Haensel, Phys. Rev. Lett. 66, 2701 (1991).
[20] I. Bombaci and U. Lombardo, Phys. Rev. C 44, 1892 (1991); W. Zuo, I. Bombaci, and U. Lombardo, Phys. Rev. C 60, 024605(1999).
[21] A. Lejeune, U. Lombardo, and W. Zuo, Phys. Lett. B477, 45 (2000).
[22] N. Kaiser, S. Fritsch, and W. Weise, Nucl. Phys. A 697, 255 (2002).
[23] M. Prakash, T. L. Ainsworth, and J. M. Lattimer, Phys. Rev. Lett. 61, 2518(1988).
[24] R. B. Wiringa, V. Fiks and A. Fabrocini, Phys. Rev. C 38, 1010(1988).
[25] H. Q. Song, M. Baldo, G. Giansiracusa, and U. Lombardo, Phys. Rev. Lett. 81, 1584(1998); M. Baldo, A. Fiasconaro, H. Q. Song, G. Giansiracusa, and U. Lombardo, Phys. Rev. C65, 017303 (2002).

[26] R. B. Wiringa, V. G. J. Stoks, and R. Schialla, Phys. Rev. C 51, 38(1995).

[27] M. Baldo, I. Bombaci, and G. F. Burgio, Astron. Astrophys. 328, 274 (1997).

[28] P. Grangé, A. Lejeune, M. Martzolf, and J.-F. Mathiot, Phys. Rev. C40, 1040(1989).

[29] R. Machleidt, Adv. Nucl. Phys. 19, 189 (1989).

[30] W. Zuo, A. Lejeune, U. Lombardo, and J. F. Mathiot, Nucl. Phys. A 706, 418(2002); Eur. Phys. J. A 14, 469(2002).

[31] C. Fuchs, Lect. Notes Phys. 641, 119 (2004).

[32] A. Lejeune, P. Grange, M. Martzolf, and J. Cugnon, Nucl. Phys. A453, 189 (1986).

[33] J. D. Walecka, Ann. Phys. (N.Y.) 83, (1974) 491; B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16, (1986) 1; Int. Journ. Mod. Phys. E 6, (1997) 515; G. E. Brown, W. Weise, G. Baym and J. Speth, Comments Nucl. Part. Phys. 17, (1987) 39.

[34] Nuclear Methods and the Nuclear Equation of State, Ed. M. Baldo (World Scientific, Singapore, 1998), Chap.1.

[35] J. F. Donoghue and C. R. Nappi, Phys. Lett. B 168, 105(1986).

[36] S. J. Dong, J.-F. Lagae, K. F. Liu, Phys. Rev. D 54, (1996) 5496.

[37] G. Baym, Phys. Rev. Lett. 30, 1340 (1973).

[38] H. Huber, F. Weber and M. K. Weigel, Phys. Rev. C57, 3484 (1998); Z. Y. Ma and L. Liu, Phys. Rev. C 66, 024321 (2002).

[39] V. Greco, M. Colonna, M. Di Toro, G. Fabbri, and F. Matera, Phys. Rev. C 64, 045203 (2001).

[40] X. R. Zhou, G. F. Burgio, U. Lombardo, H.-J. Schulze, and W. Zuo, Phys. Rev. 69, 018801 (2004).

[41] M. Baldo, G. F. Burgio, and H.-J. Schulze, Phys. Rev. C 58, 3688 (1998); C61, 055801 (2000).
FIG. 1: The density dependence of nuclear symmetry energy from different models: F1(dashed curve), F2(dotted curve), F3(dot-dashed curve), UV14+TNI(double-dot-dashed line), BHF(thin solid line), and BHF+TBF(bold solid line).

FIG. 2: The chemical potential of the negative charged particles for three different values of $a_3m_s = -310, -222, \text{ and } -134 \text{ MeV}$ denoted by a, b and c respectively. The solid curves represent the results including the TBF contribution, while the dashed curves are the results without the TBF contribution.

FIG. 3: The composition of chemical equilibrium neutron star matter for the two cases with (curves with symbols) and without (curves without symbols) the TBF contribution. The solid curves are the results for the proton fraction $x_p$, the dashed curves for kaon fraction $x_K$, and the dotted curves for lepton fraction $x_l$.

FIG. 4: The composition of neutron star matter by using different models for the symmetry energy and adopting $a_3m_s = -222 \text{ MeV}$. The curves with symbols are obtained from the BHF calculations including the TBF contribution (BHF+TBF). The bold curves without symbols correspond to the results of Ref. [16] by adopting the symmetry energy of the variational approach and the UV14+TNI interaction [24]. The thin curves are obtained by using the linear density dependent symmetry energy [9]. The solid and dashed curves are the results for the proton fraction $x_p$, and the lepton fraction $x_l$ respectively.

FIG. 5: The predicted EOS of neutron star matter for both cases with kaons (solid curves) and without kaons (dashed curves). The results by using the $AV_{18}$ plus the TBF (right panel) are compared to the ones by adopting the pure $AV_{18}$ two-body force (left panel).
$a_3 m_s = -310$ MeV

$X$

$u$

$X$

$u$

$X$

$u$
