Abstract

This paper describes how to convert a machine learning problem into a series of map-reduce tasks. We study logistic regression algorithm. In logistic regression algorithm, it is assumed that samples are independent and each sample is assigned a probability. Parameters are obtained by maximizing the product of all sample probabilities. Rapid expansion of training samples brings challenges to machine learning method. Training samples are so many that they can be only stored in distributed file system and driven by map-reduce style programs. The main step of logistic regression is inference. According to map-reduce spirit, each sample makes inference through a separate map procedure. But the premise of inference is that the map procedure holds parameters for all features in the sample. In this paper, we propose Distributed Parameter Map-Reduce, in which not only samples, but also parameters are distributed in nodes of distributed filesystem. Through a series of map-reduce tasks, we assign each sample parameters for its features, make inference for the sample and update parameters of the model. The above processes are executed looply until convergence. We test the proposed algorithm in actual hadoop production environment. Experiments show that the acceleration of the algorithm is in linear relationship with the number of cluster nodes.

Keywords: Distributed Machine Learning, Logistic Regression, Map-Reduce, Hadoop, Large-scale Machine Learning

1. Introduction

Internet companies now collect a large number of user logs every day. How to explore user’s interest from these logs, so as to provide a personalized service has become the focal point of major internet companies attracting customers and increasing revenue. But exploring this treasure is not an easy task. Storing these logs requires number of machines. Analysis of these logs requires a lot of processors work in parallel. In recent years, hadoop platform is adopted by more and more companies. Hdfs [Ghemawat et al., 2003] provides a high reliable distributed filesystem. Lot of log files are divided into many small data blocks, which stored in hdfs nodes. Map-Reduce [Dean and Ghemawat, 2004] runs hdfs, it provides a simple and efficient concurrency framework calling each node’s processors to solve the same problem.

As the name suggests, map-reduce is divided into map stage and reduce stage. In map phase, it seeks subtasks are independent. Ideally, for each data block, a separate subtask is started without the need to interact with other subtasks. In typical machine learning
methods, we assume independence of samples. So it seems that a separate subtask can be started to process each sample block. However, samples share same parameter space and sample independence is only established in the condition of parameters determined. In traditional large-scale machine learning research, parameter space is in centralized storage. Subtasks on each sample block query parameters for its own sample from centralized parameter storage and then make inference independently. DistBelief [Dean et al., 2012] divides parameter space into many parameter servers, and each sample server is responsible for a part of parameter space. Samples are stored in large-scale clusters. Each cluster node read parameters from parameter servers, compute parameter gradient each stored sampler and update parameter servers using these gradients. They proposed two different parameter update strategy. Downpour SGD, each machine in the cluster interacts only with it’s sample block involved parameter servers, reading current parameter values or update the parameters. The parameter updating of each sample server is independent. Sandbluster L-BFGS, storage of parameter and sample is same with Downpour SGD, and parameter updating of different servers is controlled by a unified parameter coordinator. Parameter gradient generated by each machine in the cluster is sent to parameter servers or aggregated to parameter servers through a tree structure, but does not update the parameter server immediately. After all the machines have generate the gradients, parameter are updated uniformly. For the parameter server, there have been a lot of research. For example, YahooLDA [Ahmed et al., 2012] implemented a dedicated server with user-definable update primitives(set, get, update) and a more principled load distribution algorithm. Petumn [Ho et al., 2013], Graphlab [Low et al., 1968] devote to a more efficient parameter synchronization mechanism. In [Li et al., 2014], they develop third-generation parameter server. Their main contributions are: 1, support a variety of algorithms, such as sparse logistic regression, LDA. 2, the asynchronous communication model does not block computation. 3, the globally shared parameters are represented as vector or matrices to facilitate development of machine learning application. 4, related consistency hides synchronization cost and latency. They allow the algorithm designer to balance algorithmic convergence rate and system efficiency. 5, scalability fault tolerance and ease of use.

Reduce phase, results generated by subtasks in the map phase are aggregated. Records with same key will be aggregated one final result. Reduce process is particularly suitable for commutative, associative operations, such as addition operation. Plus a large number of items, first plus part of the items, the intermediate result are then added with other items. It does not change the final result. Therefore, we can distribute large amounts of addition operations into nodes of cluster, intermediate results generated by each node are then added to produce the final result. (Chu et al., 2014) pointed out that a large number of machine learning algorithms belong to Statistical Query Models. These algorithms can be written into a specific summation form. Therefore, they proposes multicore map-reduce framework. A map-reduce engine is responsible for splitting the samples into multiple parts. It runs a master, which transfer samples to different mappers, collect intermediate results from each mapper and activate a reducer to aggregate these intermediate results.

This paper extends [Chu et al., 2014]'s work from a single multicore machine to a distributed cluster. Unlike Statistical Query Models, our study includes only logistic regression algorithm. We use gradient ascent algorithm to optimize parameters. we assume these samples are independent, making it suitable for map process. As we pointed out
earlier, different samples can only be independent in the case of the parameters determined. Each mapper must obtain the parameters contained in its sample block. In a distributed cluster, this is a key factor affecting the performance of parallel algorithms. Unlike previous works (Smola and Narayanamurthy 2010; Power and Li 2010; Feng et al. 2012), we don’t use special parameter servers to store parameter space. But like samples, parameter space is dispersed into nodes of the cluster. In parameter server mode, each mapper queries parameters from parameter server initially. But in our approach, each mapper obtain parameters passively. Because samples and parameters are dispersed in the sample cluster, a separate map-reduce process is started to assign each sample the current values of its parameters. The output of this map-reduce process is that each line is a sample, but these 'samples' are different from usual samples. The usual samples contain only features and feature counts. But these 'samples', contain not only features and feature counts, but also the current values of its parameters. So we call these 'samples' sufficient samples. Different sufficient samples are independent. For each sufficient sample block, start a mapper to compute parameter gradients generated by the sufficient sample block. We update parameter values by maximum likelihood estimation method. Therefore, an update, each parameter increment is summation of parameter gradients produced by different samples. Thereby parameter increment calculation is applicable for reduce process. After obtaining the parameter increments, start a separate map-reduce process to generate new parameter values based on old parameter values and increments. These new parameter values are assigned to each sample and new sufficient samples are produced.

Later sections in this article are organized as follows. In the second chapter, we briefly the logistic regression model, including variables, parameters, objective function and inference process. Logistic regression is applicable to the Distributed Parameter Map-Reduce Framework we propose. In third chapter, we describe Distributed Parameter Map-Reduce method in detail. Through a series of map-reduce tasks, we build parameter invert index, generate sufficient sample, compute gradient and update parameter. In this chapter, we describe each map-reduce process in detail. To get sufficient samples, we start a map-reduce process to build parameter invert index. Every line in the output is 'parameter→sample' relationship. If a parameter occurs in large amount of samples, the line will be very long and take up a lot of bytes, making sample blocks containing invert index uneven. Some parameter invert index takes several data blocks, and some data block contains lots of invert indexes. This also affects the map-reduce process generating sufficient samples. Some mapper will take a very long time to finish. In fourth chapter, we introduce high-frequency parameter sharding, which produces invert index in uniform distribution. In the fifth chapter, we use a series of map-reduce tasks to make prediction for test samples. The procedure is similar to one training iteration. Unlike training process, we don’t compute gradients for each sample, but make prediction for each sample. In the sixth chapter, we introduce the experimental design and results. In this chapter, we mainly validate the acceleration effect of Distributed Parameter Map-Reduce. Our experiments show that the acceleration proportion of Distributed Parameter Map-Reduce and the number of cluster nodes is the kind of linear relationship. Therefore, Distributed Parameter Map-Reduce is especially suitable for large-scale machine learning in distributed cluster.
2. Logistic Regression

Logistic regression is a typical regression algorithm. In the following, \( x^{(i)} \) represents a document, \( y^{(i)} \) represents the label. Assume samples are \((x^{(1)}, y^{(1)}), \ldots, (x^{(D)}, y^{(D)})\), \( y^{(i)} \in \{0, 1\} \), \( \theta \) represents a vector of all parameters, \( \theta_j \) represents parameter corresponding to feature \( j \). Each sample \( x^{(i)} \)'s probability is expressed as a sigmoid function. The probability that \( x^{(i)} \) is assigned label 1 given \( \theta^{(i)} \) (\( \theta^{(i)} \) represents parameter corresponding to \( x^{(i)} \)) is equal to

\[
p(y^{(i)} = 1 | x^{(i)}; \theta^{(i)}) = \frac{1}{1 + e^{\theta^{(i)T} x^{(i)}}} = \sigma(\theta^{(i)T} x^{(i)}) \tag{1}
\]

We use maximum likelihood estimation to solve the optimal parameters, so the objective function is

\[
L(Y, X; \theta) = \prod_{i=1}^{D} p(y^{(i)} | x^{(i)}, \theta^{(i)}) \tag{2}
\]

This is equal to minimizing the following cost function

\[
J(\theta) = -\sum_{i=1}^{D} \{y^{(i)} \log(h_{\theta^{(i)}}(x^{(i)})) + (1 - y^{(i)}) \log(1 - h_{\theta^{(i)}}(x^{(i)}))\} \tag{3}
\]

The optimal solution of \( J(\theta) \) can not be solved directly. We use optimization method (Nocedal and Wright, 2006) such as gradient descent to approach the optimal solution. The most important is to calculate the gradients:

\[
\nabla_j J(\theta) = \sum_{i=1}^{D} x_j^{(i)} (h_{\theta^{(i)}}(x^{(i)}) - y^{(i)}) \tag{4}
\]

In \( \nabla_j J(\theta) \), the key is calculation of \( h_{\theta^{(i)}} \). We call such probability calculation the inference of the sample. It is noteworthy that these probabilities are calculated independently given parameters \( \theta^{(i)} \) for sample \( x^{(i)} \).

If we use gradient descent method to approximate the optimal solution. In each iteration, the parameters are updated using the following formula

\[
\theta_j = \theta_j - \alpha \cdot \nabla_j J(\theta) \tag{5}
\]

\( \alpha \) is learning rate, which is a positive constant. In each iteration, increment for parameter \( \theta_j \) is \( \nabla_j J(\theta) \) multiplied by a constant. We substitute \( \nabla_j J(\theta) \) into formula [5]

\[
\theta_j = \theta_j + \alpha \sum_{i=1}^{D} x_j^{(i)} (y_j^{(i)} - h_{\theta^{(i)}}) \tag{6}
\]

As can be seen from the above equation, updating for parameter \( \theta_j \) is summation form. In the above discussion, we pointed out that parameter updating of this form applies to reduce process. So in mappers, \( x_j^{(i)} (y_j^{(i)} - h_{\theta^{(i)}}) \) is calculated for each sample. The output is aggregated through reduce process into final increment for parameter \( \theta_j \).
3. Distributed Parameter Map-Reduce

A machine learning task can be converted into a series of map-reduce tasks. In this chapter, we describe each map-reduce task in detail. For ease of discussion, we first determine parameter granularity. In logistic regression, one feature corresponds to a parameter, which reflects the feature’s weight reallocation between two labels. In the following discussion, we store parameter space by feature. Each row represents a key-value, with feature as key and parameter corresponding to the feature as value.

Before discussing each map-reduce task, we first introduce the overall structure of Distributed Parameter Map-Reduce. This helps to better understand the role of each map-reduce task. Algorithm 1 describes the structure of our algorithm.

The entrance of Algorithm 1 is \text{dpmr}(\text{trainInput, paraValueOutput}). Enter the training corpus and output parameters that minimize cost function. Input, output and intermediate results are stored in the form of hdfs files. There are mainly six map-reduce tasks: \text{initParameters}, \text{invertDocuments}, \text{distributeParameters}, \text{restoreDocuments}, \text{computeGradients}, \text{updateParameters}. First, extract all features from the corpus and initialize parameter for each feature. The output is in the form of 'feature-¿parameter'. \text{InvertDocuments} inverts each sample by feature, and build index with the form of 'feature \rightarrow \text{sample}'. After the completion of \text{initParameters} and \text{invertDocuments}, for each feature, we can get both the corresponding parameter value and all samples in which the feature appears. \text{DistributeParameters} combines the two. For each sample, we can get both the feature and current parameter value corresponding to the feature. \text{ComputeGradients} makes inference for each 'sufficient sample' independently and compute parameter gradient for each feature. For any feature, there is both current value and gradient of its parameter. Optimization methods can be called to update current value of the parameter. New parameter values replace the current and the copy method update parameters to the latest values. The copy method is hdfs’s own function, such as copy method of FileUtilClass. In addition, the algorithm must have the termination condition, determining whether the parameter value is already optimal. This can be done by adding one step between step 5 and step 6 in Algorithm 1 to calculate the current value of the objective functions. This
Algorithm 2 initParameters(trainInput, paraValueOutput)

mapper(longId, \(S^{(i)}\))

1: for each token in \(S^{(i)}\) do
2: \(f = \text{token} \rightarrow \text{f}\)
3: emit(f, 1)
4: end for

reducer(f, iterator)

1: static{
2: para = 0
3: type = 'p'
4: paraStr = concat(' ', type, para)
5: }
6: emit(f, paraStr)

The key step is to start a separate map-reduce task like computeGradients to complete. Record history values of the objective function to determine whether the current iteration can be terminated. Later in this section, we describe six map-reduce processes in detail.

3.1 initParameters

Algorithm 2 describes the process of initParameters. InitParameters extracts all the features from training corpus, and initialize the parameter of each feature. Training data format is that each line stores one sample \(S^{(i)}\), and each sample stores its features in feature hash form. \(S^{(i)}\) contains label part and feature part. Label part stores annotation information of \(S^{(i)}\). Label space is generally much smaller with respect to feature space. We create dictionary for the label space. In \(S^{(i)}\), the label is an integer that represents the label index in the dictionary. In logistic regression, there are two different categories. In each sample \(S^{(i)}\), we use an integer of 0 or 1 to indicate the actual category \(S^{(i)}\) belongs.

The feature part is a series of tokens. Each token corresponds to a feature and has the form of 'f : count'. Because feature space is generally large and easy to change, we don’t build dictionary for feature space. \(f\) is a string that uniquely identifies the feature and count represents the number of times \(f\) appears in \(S^{(i)}\).

The input of mapper is sample \(S^{(i)}\) and longId automatically generated by hadoop. The mapper only processes the feature part of \(S^{(i)}\). For each token, output a record with \(f\) as key, 1 or null as value.

The input of reducer is feature \(f\) and all the values corresponding to key \(f\). These values are aggregated into a iterator. In initParameters, our goal is extracting feature \(f\), not focus on values in iterator. For feature \(f\), we must intialize the corresponding parameter. Every reduce task that hadoop started make the same initialization for all features. In static code of reduce class, the parameter value is initialized to 0. For subsequent processing, type field is added to indicate the output of initParameters is parameter information. In output of reducer, key is \(f\) and value is paraStr. paraStr contains both the type field and the initialization parameter information.

3.2 invertDocuments

The key step of Distributed Parameter Map-Reduce is to build 'index' of 'feature→sample' form. Algorithm 3 describes the detail process of building 'index'. This 'index' is not same with usual index. It is only plain composed of lines of 'key→value' pairs and stored in hdfs. With this index, for one feature, the corresponding parameter and the sample list which it
Algorithm 3 invertDocuments(trainInput, docInvertOutput)

mapper(longId, S(i))
1: docId = generateRandomId(S(i))
2: label = S(i) → label
3: type = 'i'
4: num = 1
5: for each token in S(i) do
6:   f = token → f
7:   count = token → count
8:   units = concat('!', docId, count, label)
9: value = concat(' ', type, num, units)
10: emit(f, value)
11: end for
combiner(reducer)(f, iterator)
1: sum = 0
2: docUnits =''
3: type = 'i'
4: for each value in iterator do
5:   sum += value → num
6:   units = value → units
7:   docUnits = concat(' ', docUnits, units)
8: end for
9: value = concat(' ', type, sum, docUnits)
10: emit(f, value)

Algorithm 4 distributeParamters(paraValueOutput, docInvertOutput, paraDistributeOutput)

mapper(longId, paraInfo)
1: f = paraInfo → f
2: value = paraInfo → value
3: emit(f, value)
reducer(f, iterator)
1: para =''
2: docUnits =''
3: for each value in iterator do
4:   type = value → type
5: if type == 'p' then
6:   docUnits = value → docUnits
7: end if
8: if type == 'p' then
9:   para = value → para
10: end if
11: end for
12: for each unit in docUnits do
13:   docId = unit → docId
14:   count = unit → count
15:   label = unit → label
16:   value = concat('!', label, f, count, para)
17: emit(docId, value)
18: end for

appears can be sent to the same reducer. In the reducer, sample get features it contains and current parameter values corresponding to its features.

The input of Algorithm 3 is also training corpus. In order to build inverted index, we first assign each sample a random Id. The granularity of inverted index is feature f, so the key of output is f. Value is composed of units. Each unit corresponding to one sample. In output of mapper, each value has only one unit. Reducer gathers together all the values corresponding to feature f. Every unit is composed of f, count, label. So we can recover the sample later. In addition, type field is added to represent that the output record is sample inverted index record. Reducer process and combiner process is same. Some feature f is more common, so its sample list is very long. This will affect the parallel performance of the algorithm. The count statistics is to facilitate the subsequent sharding of feature f with long sample list. The key f is subdivided into a series of sub-keys. Each sub-key corresponds
Algorithm 5 restoreDocuments(paraDistributeOutput, docRestoreOutput)

mapper(longId, docPartInfo)
1: docId = docPartInfo → docId
2: value = docPartInfo → value
3: emit(docId, value)

combiner(reducer)(f, iterator)
1: values ="
2: docUnits ="
3: type =’i’
4: for each value in iterator do
5: values=concat(’,values,value)
6: end for
7: emit(docId, values)

to part of sample list for feature $f$. We currently do not make in depth description of the sharding process. It will discussed in detail in the next section.

3.3 distributeParameters and restoreDocuments

Algorithm 2 initialize the parameter of the feature. Algorithm 3 build 'feature→sample’ index. The output keys of two algorithms are both feature $f$, which provides the basis for starting the map-reduce process in Algorithm 4. Parameter and sample list with same key are sent to the same reducer. Reducer determines whether the record is parameter or sample list according to the type field. From above, we know that, in sample list docUnits, each unit corresponds to one sample. For each unit, reducer will output one record. So docId and $f$ swap positions. The docId is key and $f$ is part of value. This is similar to the process of recovering sample from 'feature→sample’ index. But this is not simple recovery, in which parameter corresponding to the feature is also included. From one ‘feature→sample’ record, we can only recover one feature for the sample. Algorithm 5 gathers together all the features corresponding to same docId, resulting in the complete 'sample’ for this docId.

We call each sample in the output of Algorithm 5 sufficient sample. The sample contains not only its features ($f$ and count of $f$) and label information., but also current values of parameters corresponding to the features that is necessary to make inference for the sample. The advantage of sufficient sample is that it can be made inference independent of others sufficient samples.

3.4 computeGradients

From Section 2, we know that, in logistic regression the parameter gradient can be expressed as the summation form. Algorithm 5 output all the sufficient samples. The mapper of Algorithm 6 makes inference independently for each sample. The minimum granularity of parameter space in logistic regression is feature. Each feature corresponds to one parameter. grad, prob, empir respectively store the gradient, inference probability, experience value. In logistic regression, we make inference for each parameter and calculate its posterior probability. Gradient is equal to the expectation value minus experience value. Reducer adds up all the parameter gradient with same key, as the final parameter gradient for the feature.
Algorithm 6 computeGradients(docRestoreOutput, gradComputeOutput)

mapper(longId, Suff(i))
1: grad = 0
2: prob = 0
3: empir = 0
4: type = ’q’
5: label = Suff(i) → label
6: inf = inference(Suff(i), label)
7: emp = empirCount(Suff(i), label)
8: for each value in iterator do
9:   empir = emp
10:  prob = inf
11:  grad = func(empir, prob, label)
12:  value = concat(’’, type, grad)
13:  emit(f, value)
14:  values
15: end for

combiner(reducer)(f, iterator)
1: summation = 0
2: type = ’q’
3: grad = 0
4: for each value in iterator do
5:   grad = value → grad
6:  summation+ = grad
7: end for
8: value = concat(’’, type, summation)
9: emit(f, value)

Algorithm 7 updateParameters(paraValueOutput, gradComputeOutput, paraUpdateOutput)

mapper(longId, paraInfo)
1: f = paraInfo → f
2: value = paraInfo → value
3: emit(f, value)

reducer(f, iterator)
1: para = 0
2: grad = 0
3: for each value in iterator do
4:   type = value → type
5: if type == ’p’ then
6:   grad = value → grad
7: end if
8: if type == ’p’ then
9:   para = value → para
10: end if
11: end for
12: newPara = optimize(para, grad)
13: type = ’p’
14: value = concat(’’, type, newPara)
15: emit(f, value)

3.5 updateParameters

The final step is updating parameter. Algorithm 7 describes the process of updating parameter in detail. ParaValueOutput stores current value of the parameter. GradComputeOutput is output of Algorithm 6 and stores the parameter gradient. Use gradient descent method to update the parameter. \( \theta_j = \theta_j - \alpha \cdot \nabla J(\theta) \), where \( \alpha \) is the learning rate. According, step (12) in Algorithm 7 is actually newPara = para - \( \alpha \cdot \text{grad} \). para and grad respectively store current parameter and gradient value corresponding to feature f. newPara is new parameter corresponding to feature f.

4. Sharding

According to zip’s law, \( \text{freq}_f \propto 1/\text{rank}(f) \), \( \text{freq}(f) \) is the frequency of feature f. If feature f is stored in descending frequency, \( \text{rank}(f) \) is the position of the feature f in the sort results. This means that the frequency distribution of features is very uneven, with a
large number of features having a smaller frequency and small number of features having a greater frequency. A small amount of high-frequency features will seriously affect the performance of Distributed Parameter Map-Reduce algorithm. First, in 'feature→sample' index, if the frequency of one feature is very large, the corresponding sample list will be very large. Sample list corresponding to one feature is stored in the same line in 'feature→sample' index. If the sample list is too large, the line will be too long. In Algorithm 3, a sample is assigned a random string type docId. Suppose docId is composed of ten random letters or numbers and the sample list size is 100 million, the line will occupy at least 1G of storage space. In hdfs, block size is generally set to 64M, and the line will take up about 20 blocks. The block distribution of each line storage space in 'feature→sample' index will be very uneven. Secondly, a few features with large size sample list make mappers or reducers processing these features much slower than mappers or reducers processing large amount of low-frequency features, which affect the overall parallel effect of Distributed Parameter Map-Reduce algorithm.

4.1 sharding process

To solve this problem, we make sharding for small amount of high-frequency features. The sharding feature f is as follows: suppose complete or part count of feature f is c, feature f is divided into N sub-features, each of which is of the form 'i_N|f, 1 ≤ i ≤ N'. Sample list of feature f is assigned to N sample lists corresponding to N sub-features. In 'feature→sample' index, N sub-features substitute the parent feature f. So a long line is divided into N short lines. These N sub-features will be assigned to N mappers or reducers in the subsequent processing. Sharding process can be nested, namely sub-feature fs is then to be made sharding.

If sharding, the entrance of Distributed Parameter Map-Reduce must make appropriate adjustments, as shown in Algorithm 8. InvertDocumentSharding is similar to Algorithm 3.
but its mapper, combiner and reducer make feature sharding. Enter one feature $f$, output $N$ sub-features $f_s$. In combiner, reducer, count $c$ of feature $f$ is equal to the size of $f$’s sample list after aggregation of combiner or reducer. Sharding in mapper needs external incoming feature frequency statistics. The number of high-frequency is smaller. Make statistics in advance and pass statistics results of high-frequency into mapper. This is feasible. Of course, sharding in mapper is not necessary. This can be done only in combiner and reducer. So that no external feature frequency is needed. Sharding in mapper, combiner, reducer is performed sequentially nested.

InvertDocuments build ’parent feature $\rightarrow$ sub feature’ index for sub features after sharding and their parent features. All sub features corresponding to parent feature $f$ are stored in the same line. Suppose that sub feature $f_s$ is $i_N|f$, then its parent feature is $f$. In paraInvertOutput, feature $f$ corresponds to one line, in which key is $f$ and string representation of $f_s \in sub(f)$ is value. DistributeParametersSharding sets parameter of sub feature $f_s$ as parameter of its parent feature $f$. Therefore, all sub features corresponding to one parent feature have the same parameter. Parameter of sub feature $f_s$ in paraDistributeOutput is input into distributeParameters in place of parameter of parent feature $f$. The other processes are basically same as Algorithm 1. The only change is that computeGradients make a slight adjustment. In step (13) of Algorithm 6’s mapper, parent feature $f$ is extracted from sub feature $f_s$. The output is change from $\text{emit}(f_s, value)$ into $\text{emit}(f, value)$. Pseudocode of invertDocumentSharding, invertParameters, distributeParametersSharding respectly corresponds to Algorithm 10, Algorithm 11, Algorithm 12 in the appendix.

4.2 effective sharding

Carefully designing of sharding function in Algorithm 10 can effectively reduce network data transmission. We can start from the following two aspects. 1, try to make sharding key $f_s$ and the corresponding samples are assigned to the same reducer. So output of step (17) in Algorithm 4 is stored in local data blocks. This method is effective for high-frequency features, but has little effect on the low-frequency features. 2, in the case which the first method is not suitable, such as low-frequency features, try to make sharding key $f_s$ and the corresponding samples are allocated on machines on the same rack. This can reduce data transmission between different racks, to improve transmission efficiency. For Algorithm 4.
Table 1: Running time (minutes) of various map-reduce tasks under different conditions

| MAP-REDUCE                        | (33,25) | (100,75) | (200,150) | LOOP |
|-----------------------------------|---------|----------|-----------|------|
| invertParametersSharding          | 315     | 112      | 51        | ×    |
| invertDocuments                   | 189     | 69       | 30        | ×    |
| distributeParametersSharding      | 237     | 85       | 39        | ✓    |
| distributeParameters              | 221     | 78       | 43        | ✓    |
| restoreDocuments                  | 175     | 54       | 30        | ✓    |
| computeGradientsSharding          | 259     | 99       | 45        | ✓    |
| updateParameters                  | 161     | 51       | 27        | ✓    |
| AVERAGE OF ONE ITERATION          | 1053    | 367      | 185       |      |

Even the entire Distribute Parameter Map-Reduce algorithm, an issue which should be also noted is the inherent characteristic of hdfs. The same data block has multiple backups. Multi-backup inevitably brings more network data transmission. We should consider parallel effect of the algorithm, network data transmission, system fault tolerance and other factors to determine the number of backups. Maybe different backup strategies are adopted in different stages. For example, use single backup for Algorithm 4.

5. Classifying

The whole process is similar to the training process Algorithm 8. Format of testInput is same with that of trainInput. Each line has a test sample. Each row of testOutput stores a prediction result. The format is line.

```
= . < docId > < sep > < example_label > < sep > < predict_label >
```

Test corpus and model parameters (paraValueOutput) are both stored in hdfs. Dpmr_classifying uses a series of map-reduce procedures to finish prediction. The result testOutput is also stored in the form of hdfs file. The only new method is logisticTest. The procedure is similar to Algorithm 6. But it has no reduce stage, only map stage. For each line of sufficient sample $x^{(i)}$, map stage of Algorithm 6 outputs the corresponding parameter gradient values. But for each line of sufficient sample $x^{(i)}$, logisticTest outputs the probability $p(y^{(i)} = 1|\theta^{(i)}, x^{(i)})$.

6. Experiments

In order to verify the proposed Distributed Parameter Map-Reduce algorithm, we test the following data set. The dataset sample size exceeds 2T and the feature size exceeds 500G. Approximately 20 billion samples and 50 billion features. Testing focuses on the following two aspects: first test running time under different resources (number of mappers, reducers). The second is the convergence speed. Acceleration proportion of the algorithm is in linearly proportional relationship with the number of nodes (mappers, reducers). After a few iterations, the algorithm will converge. In our experiments, after two iterations, the algorithm basically reaches the level of convergence.
Figure 1: The convergence effect of Distributed Parameter Map-Reduce algorithm. Class +1, −1 are assessed separately. Compute respective scores, the average of two as the whole score. $cate_1$, $cate$ − 1 represent respectively +1, −1, $avg$ is the average of $cate_1$ and $cate$ − 1. Y-axis is score, X-axis is the number of iterations. The left is accurate rate, the middle is the recall rate, and the right is $F$ value. Accuracy and $F$ value bias Class −1 at the first iteration. At the second iteration, Accuracy and $F$ value of Class +1 reach a reasonable level. Class −1 declines slightly. When the first iteration, recalls of Class +1, −1 are almost equal. When the second iteration, Class −1 increases slightly and Class +1 decreases slightly.

6.1 acceleration proportion

In this experiment, we count running time in the following three resources: (1) 33 mappers, 25 reducers (2) 100 mappers, 75 reducers (3) 200 mappers, 150 reducers. Average running time of each map-reduce procedure of Distributed Parameter Map-Reduce algorithm and that of one iteration is shown in Table 1. Map-reduce procedure that takes less time is ignored.

InvertParameters, distributeParametersSharding, updateParameters only process parameter space, so they all take a relatively small time. InvertDocumentsSharding, distributeParameters both process sample space. Two processed are just opposite. DistributeParameters takes a slightly shorter time than invertDocumentsSharding, because the sharding method in invertDocumentsSharding is carefully designed. But currently we only implement that sharding key $f_s$ and the corresponding sample list are allocated on the same reducer. The design that they are allocated on the same rack is not really realized so far. Since invertParameters only once in the entire Distributed Parameter Map-Reduce algorithm, but distributeParameters to be repeated, carefully designed invertDocumentSharding saves the overall time cost of the algorithm. Although restoreDocuments process sample space, but the input key of mapper and the output key of reducer are both $docId$. So less network traffic, thus faster. For computeGradientSharding, calculation process takes longer time. Network data transmission is mainly parameter space, so it takes a relatively short time. But computeGradientsSharding’s speed is not slower than distributeParameters. (100,75) relative to (33,25), 3 times of the number of nodes, speedup ratio is 3.1. While (200,75) relative to (33,25), 6 times the number of nodes, speedup ratio is 6. This indicates that speedup ratio is in approximately linear relationship with the number of nodes. On
the experiment cluster, a data block has three blocks. Compression storage of data, lzo algorithm is used by default.

6.2 convergence effect

Since convergence effect and the number of nodes (mappers, reducers) have no relationship. We only make statistics of (200, 150) case. After each iteration, test procedure Algorithm [9] is called. Statistical results are shown in Figure [1].

Class +1, -1 are assessed separately. +1, -1 ratio of the sample is roughly 3:1. Compute respective scores, the average of two as the whole score. The three targets, Precision, Recall and F value, are computed. They correspond respectively to Figure 1, 2, 3. F value is comprehensive assessment of Precision, Recall, \( f = 2pr/(p+r) \). The figures show that, whether it is Precision, Recall or F value, after two iterations, all reach fundamental convergence level. First iteration, initial parameter value is 0. After the first iteration, the algorithm does not converge. However, it makes a preliminary allocation of parameter weight, to determine whether a parameter is positive effect or negative effect for each class. Second iteration is parameter tuning refinement process. After this iteration, each parameter weight substantially reaches a reasonable value.

7. Conclusion and Future Work

In this paper, we propose Distributed Parameter Map-Reduce algorithm, implementing training and classifying for logistic regression through a series of map-reduce tasks. The input, output and intermediate results are all stored in the form of hdfs files. Through several continuous map-reduce procedures, we assign each sample involved parameters current values, make it a sufficient sample. Different sufficient samples are independently, applying to map procedure. The update of logistic regression can be expressed as summation form, applying to reduce procedure.

Distributed Parameter Map-Reduce algorithm can easily process T level samples, and more importantly, at the same time, it can also process T level feature space. Feature space is dispersed into nodes of the hdfs. All procedures are map-reduce tasks. Increased feature space will not affect the algorithm parallelism and scalability. For larger sample or feature space, spread the load on more nodes. Acceleration proportion is in linear relationship with the number of nodes (mappers and reducers) of the cluster. More nodes will linearly bear more loads or reduce algorithm's running time.

Future work focuses on the following two aspects. (1) improve sharding method. For sample space, be sure to make more data transmission local transmission. Inevitable data transmission between different machines is dispersed into different transmission inside the rack. Improve transmission concurrency, thereby improving transmission efficiency. This requires careful design and implementation of partition strategy of Algorithm [10]. (2) Distributed Parameter Map-Reduce is currently implemented on hadoop. But spark (Zaharia et al., 2012) stores its data in the form of memory blocks. Its efficiency is easily one or two orders of magnitude higher than hadoop. Distributed Parameter Map-Reduce is composed of a series of map-reduce tasks. These map-reduce tasks can be easily implemented on hadoop. Also, they can be easily implemented on spark. Therefore, Distributed Parameter Map-Reduce algorithm can be easily ported to spark.
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Appendix A.

**Algorithm 10** invertDocumentsSharding(trainInput, docInvertShardOutput)

```java
mapper(longId, S(i))
1: {extra_feature_info=getExtraInfo()}
2: docId = generateRandomId(S(i))
3: label = S(i) → label
4: type = 'i', num = 1
5: for each token in S(i) do
6:   f = token → f
7:   count = token → count
8:   units = concat(':', docId, count, label)
9:   value = concat(' ', type, num, units)
10:  for each fs in sharding(f, fcount) do
11:     emit(fs, value)
12: end for
13: emit(f, value)
14: end for
combiner(reducer)(f, iterator)
1: docUnitsList = {}
2: type = 'i', sum = 0
3: for each value in iterator do
4:   sum += value → num
5:   units = value → units
6:   docUnitsList = docUnitsList ∪ units
7: end for
8: for each fs in sharding(f, sum) do
9:   subDocUnits = docUnitsList[fs]
10:  num = subDocUnits → size
11:  value = concat(' ', type, num, subDocUnits)
12: emit(fs, value)
13: end for
```

**Algorithm 11** invetParameters(docInvertShardOutput, paraInvertOutput)

```java
mapper(longId, paraInvert)
1: fs = paraInvert → fs
2: f = fs → f
3: emit(f, fs)
combiner(reducer)(f, iterator)
1: fsList = '', type = 'e'
2: for each fs in iterator do
3:   fsList = concat(' ', type, num, units)
4:   fcount = extra_feature,fset[f]
5:   for each fs in sharding(f, fcount) do
6:     emit(fs, value)
7: end for
```

**Algorithm 12** distributeParametersSharding(paraValueOutput, paraInvertOutput, paraDistributeShardOutput)

```java
mapper(longId, paraInfo)
1: f = paraInfo → f
2: value = paraInfo → value
3: emit(f, value)
reducer(f, iterator)
1: fsList = '', para = '', otype = 'p'
2: for each value in iterator do
3:   type = value → type
4:   if type == 'e' then
5:     fsList = value → fsList
6:     end if
7:   if type == 'p' then
8:     para = value → para
9:     end if
10: end for
11: value = concat(' ', type, value, para)
12: for each fs in fsList do
13:   emit(fs, value)
```