Study on minimum rollable thickness in asymmetrical rolling

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Received: 11 March 2021 / Accepted: 9 November 2021 / Published online: 3 December 2021
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Abstract

A new model for the asymmetrical rolling is proposed to calculate the minimum rollable thickness simply and fast by the slab method. The calculation formulas of the rolling pressure, the rolling force, the critical roll speed ratio, and the critical front tension under different deformation zone configurations are proposed, and the deformation zone configuration-rolling parameter relationship diagram is given and analyzed. The results show that the minimum rollable thickness can be reached when the rolling parameters keep the deformation zone configuration as cross-shear zone + backward-slip zone (C + B) or all cross-shear zone (AC). The calculation formulas of the minimum rollable thickness and the required rolling parameters for different deformation zone configurations are proposed, respectively. The calculated value agrees well with the experimental result.

Keywords Minimum rollable thickness · Asymmetrical rolling · Deformation zone configuration · Critical reduction rate

Abbreviations Minimum rollable thickness · Asymmetrical rolling · Deformation zone configuration · Critical reduction rate

1 Introduction

Micromanufacturing technology has attracted more and more attention with the development of the product miniaturization [1]. The ultrathin metal strip rolling is one of the hot spots in the field of micromanufacturing [2]. According to the traditional rolling theory, there is a minimum rollable thickness under the certain rolling conditions due to the elastic deformation of the rolls and the strip. Stone [3, 4] proposed the classic formula for calculating the minimum rollable thickness as the following:
The multi-roll mills that are commonly used to produce the ultrathin strip are designed based on this theory. Fleck and Johnson [5, 6] considered that the plastic deformation occurs near the entrance and exit of the deformation zone during the ultrathin strip rolling. In the middle of the deformation zone, the strip does not reduce or slip relative to the rolls. Sutcliffe and Rayner [7] verified this theory through thin strip rolling experiments. Xiao et al. [8] proposed two minimum rollable thickness models based on the Fleck theory [5, 6]. The difference of the two models is whether to consider the restriction of the rolling force. When the restriction of the rolling force is not considered, the theoretical minimum rollable thickness calculated by their model is approximately 22% that of the Stone model. Wu et al. [9] proposed a minimum rolled thickness model with considering the allowable rolling pressure and production efficiency. Their theory had been used in a 1220 five-rack cold tandem mills of China [10]. Tateno et al. [11] considered that the minimum rollable thickness affected by the elastic deformation of the work rolls and the edge cracks of the ultrathin strip. Zhang [12] used the slab method and incremental analysis to study the deformation mechanism of the cold rolling ultrathin strip. Hwang and Kan [13] proposed a mathematical model to design the roll rape for foil rolling of a four-high mill. The above studies are about the minimum rollable thickness of symmetrical rolling. The production of ultrathin strip by symmetrical rolling requires relatively high equipment, and needs to perform multiple annealing processes. Using the asymmetrical rolling can improve these problems.

In the asymmetrical rolling, the roll speed, the roll radius, and the lubrication conditions on the upper and lower side of the strip could be different. Asymmetrical rolling has a prominent advantage over the symmetrical rolling in the thinning capacity, which breaks through the classical Stone minimum rolling thickness theory. Tang et al. [14, 15] proposed a model of the permissible minimum thickness in the asymmetrical rolling, and supposed that the midpoint of the deformation zone is the midpoint of the cross-shear zone and the length of the forward-slip and backward-slip zone is equal. Liu et al. [16] proposed a model for calculating the minimum rollable thickness in the asymmetrical rolling with the identical roll radius based on the Tselikov equation and the modified Hitchcock equation. Tzou and Huang [17] considered that the minimum rollable thickness in the asymmetrical rolling occurs under the all cross-shear zone configuration of the deformation zone. However, the value of DIH proposed in their study is smaller than the experimental results obtained by other researchers. Feng et al. [18] had carried out systematic analyses and experiments of the single-roll-driven asymmetrical ultrathin strip rolling, and proposed a minimum rollable thickness model. Wang et al. [19, 20] studied the influences of three asymmetrical conditions on the distribution of the rolling pressure. Wang et al. [21] proposed the relationship diagram between the deformation zone configuration and the rolling parameters; according to this diagram, the rolling parameters required for different deformation zone configurations can be determined. Sun et al. [22–24] analyzed the effects of the rolling parameters on the deformation zone configuration and the proportion of each zone in the deformation zone. Ji and Park [25] used the rigid-viscoplastic finite element method to analyze the effects of the three asymmetrical rolling conditions on the plastic deformation.

In this paper, the influences of the roll speed ratio, the back and front tensions, and the critical reduction rate on the deformation zone configuration and the proportion of cross-shear zone are studied by the slab method. According to the relationship between the rolling parameters and the deformation zone configurations, the required rolling conditions for the minimum rollable thickness in the asymmetrical rolling are analyzed. Then, a new minimum rollable thickness model for the asymmetrical rolling is proposed and verified by experiments.

2 Mathematical model

In the asymmetrical rolling conditions, the roll radius asymmetry condition affects the length of the deformation zone, and the friction asymmetry condition affects the friction stress. But the roll speed asymmetrical condition makes the cross-shear zone appear in the deformation zone which changes the stress state of the deformation zone [19]. This is the essential difference between the asymmetrical rolling and the synchronous rolling. In the symmetrical rolling, the thinning ability of the mill can also be improved by reducing the roll radius and improving the lubrication conditions. Obviously, roll speed asymmetrical condition is the main reason why the asymmetrical rolling can break through the classical minimum rollable theory. In this study, only the roll speed asymmetrical condition will be considered. The strip uses the annealed 430 stainless steel; the plane deformation resistance of the strip is

\[ K = \frac{2}{\sqrt{3}} \left( 205.0 + 509.6\bar{\varepsilon}^{0.312} \right) \]  

\[ \bar{\varepsilon} = 0.4(H_0 - H)/H_0 + 0.6(H_0 - h)/H_0, \]  

in which \( H_0 \) is the initial thickness of the strip.
2.1 Derivation of rolling pressure and rolling force

Figure 1 shows four possible deformation zone configurations during the asymmetrical rolling, namely, F + C + B (forward-slip zone + cross-shear zone + backward-slip zone), C + B (cross-shear zone + backward-slip zone), AC (all cross-shear zone), and F + C (forward-slip zone + cross-shear zone), where C + B, AC, and F + C are all critical states.

As shown in Fig. 1e, the horizontal and vertical static equilibrium equations in the deformation zone can be obtained, respectively:

\[
\sigma_x dh_x + h_x d\sigma_x - \left(p_1 + p_2\right) \tan \alpha dx + \tau_\mu dx = 0 \quad (3)
\]

\[
p_x = p_1 + p_1 \mu \tan \alpha = p_2 + p_2 \mu \tan \alpha - \frac{dh_x}{dx} \quad (4)
\]

In different regions of the deformation zone, the friction stress \(\tau_\mu\) on the upper and lower sides of the strip and the average shear stress \(\bar{\tau}\) on the vertical section can be expressed as

\[
\begin{align*}
B & : \quad \tau_\mu = \mu (p_1 + p_2) \quad \bar{\tau} = 0 \quad h_2 < h_x \leq H \\
C & : \quad \tau_\mu = \mu (p_1 - p_2) \quad \bar{\tau} = \mu p_x \quad h_1 \leq h_x \leq h_2 \\
F & : \quad \tau_\mu = -\mu (p_1 + p_2) \quad \bar{\tau} = 0 \quad h \leq h_x < h_1
\end{align*}
\quad (5)
\]

Substituting Eqs. (4) and (5), and the yield criterion \(p_x - \sigma_x = K\) into Eq. (3), the following differential equations for different regions of the deformation zone can be obtained:

\[
\begin{align*}
B & : \quad \frac{1}{\delta_1} \ln \left(\delta_1 p_x - K\right) = \frac{1}{h_3} + C_1 \quad h_2 \leq h_x \leq H \\
C & : \quad \frac{1}{\delta_2} \ln \left(\delta_2 p_x - K\right) = \frac{1}{h_3} + C_2 \quad h_1 \leq h_x \leq h_2 \\
F & : \quad \frac{1}{\delta_3} \ln \left(\delta_3 p_x + K\right) = \ln h_x + C_3 \quad h \leq h_x \leq h_1
\end{align*}
\quad (6)
\]

where \(C_1\), \(C_2\), and \(C_3\) are the integral constants; \(\tan \alpha = \frac{\Delta h}{2l}\);

\[
l = \sqrt{R \Delta h + \left(8R \frac{1-x^2}{x_0}\right)^2 + 8R \frac{1-x^2}{x_0}} \approx \frac{2\mu l}{\Delta h}.
\]

\[
\delta_2 = \left(\frac{1+tan^2\alpha}{1+\mu \tan \alpha} - \frac{1+\tan^2\alpha + 2\mu \tan \alpha}{1-\mu \tan \alpha}\right) \frac{\mu l}{\Delta h} \approx -2\mu^2; \quad \text{and} \quad \delta_3 = \frac{1+tan^2\alpha}{1-\mu \tan \alpha} \frac{2\mu l}{\Delta h} \approx 2\mu l.
\]

The boundary conditions on the entrance of the deformation zone are \(h_x = H\) and \(p_x = K - \sigma_x\), the boundary conditions on the exit of the deformation zone are \(h_x = h\) and \(p_x = K - \sigma_x\), and the positional relationship between the neutral points is \(h_2 \approx i \cdot h_1\). According to Fig. 1a, substituting these boundary conditions into Eq. (6), the rolling pressure in different regions of the deformation zone under the F + C + B configuration can be obtained; the subscripts B, C, and F indicate the backward-slip zone, the cross-shear zone, and the forward-slip zone, respectively.

\[
\begin{align*}
p_B &= \frac{K}{\delta_1} \left[\left(\delta_1 \xi_1 - 1\right) \left(\frac{H}{h_3}\right)^{\frac{1}{\delta_1}} + 1\right] \quad h_2 \leq h_x \leq H \\
p_C &= \frac{K}{\delta_2} \left[\left(\frac{h_1}{h_3}\right)^{\frac{1}{\delta_2}} + 1\right] \quad h_1 \leq h_x \leq h_2 \\
p_F &= \frac{K}{\delta_3} \left[\left(\delta_3 \xi_2 + 1\right) \left(\frac{h_1}{h_3}\right)^{\frac{1}{\delta_3}} - 1\right] \quad h \leq h_x \leq h_1
\end{align*}
\quad (7)
\]

where \(\xi_1 = 1 - \frac{\gamma}{K}\), \(\xi_2 = 1 - \frac{\gamma}{K}\), and \(W = \frac{\delta_3}{\delta_1} \left[\left(\delta_3 \xi_2 + 1\right) \left(\frac{h_1}{h_3}\right)^{\frac{1}{\delta_3}} - 1\right] - 1\).

Integrating the rolling pressures, Eq. (7), along the contact length, the rolling force per unit width under the F + C + B configuration can be obtained:

\[
\begin{align*}
P &= \frac{IK}{\Delta h} \left[\frac{1}{\delta_1} \left(\left(\delta_1 \xi_1 - 1\right) H^{\frac{1}{h_3}} - H^{\frac{1}{\delta_1}}\right) + \left(H - ih_1\right)\right] + \\
& \quad \frac{1}{\delta_1} \left(\frac{h_1}{h_3}\right)^{\frac{1}{\delta_2}} \left(1 - \frac{1}{h_3}\right) + (i - 1)h_1 + \\
& \quad \frac{1}{\delta_3} \left[\delta_3 \xi_2 + 1\right] \left(h_1^{\frac{1}{h_3}} - h_1^{\delta_3 + 1}\right) - \left(h_1 - h\right)\right]
(8)
\]

According to Fig. 1b, where \(h_1 = h\) and \(h_2 < H\), substituting the boundary conditions into Eq. (6), the rolling pressures under the C + B configuration can be obtained:

\[\text{Fig. 1 Four possible deformation zone configurations during the asymmetrical rolling: (a) F + C + B, (b) C + B, (c) F + C, and (d) AC and (e) stress system acting on the differential vertical element in the backward-slip zone.}\]
According to Fig. 1c, the deformation zone configuration is F + C, where \( h_1 > h \) and \( h_2 = H \). The rolling pressures can be obtained:

\[
\begin{align*}
C & \quad p_C = \frac{K}{\delta_2} \left( \left( \frac{\delta_2}{h} \right)^{\delta_1} - 1 \right) \left( \frac{\delta_2}{h} \right) ^{\delta_1} + 1 \quad h \leq h_1 \leq h_2 \\
B & \quad p_B = \frac{K}{\delta_1} \left( \left( \frac{\delta_1}{h} \right)^{\delta_2} - 1 \right) \left( \frac{\delta_1}{h} \right) ^{\delta_2} + 1 \quad h_2 \leq h_x \leq H
\end{align*}
\]  

(9)

To make the deformation zone configuration change from F + C to B + C, the roll speed ratio needs to meet

\[
i \geq i_{c1} = \frac{V_1}{V_2} = \frac{h_2}{h_1} \left( 1 - \frac{h_2 - h}{2R} \right) \approx \frac{h_2}{h_1} - \frac{(1 - \varepsilon)}{\varepsilon} = \frac{\beta}{1 - \varepsilon}
\]  

(10)

where \( h_2 = \beta \cdot H \) and \( \lambda_C = \frac{\beta - 1 + \varepsilon}{\varepsilon} \).

At the neutral point \( h_\xi = h_2 \), the rolling pressure \( p_{C-h_\xi} = p_{B-h_\xi} \), \( \lambda_C \) can be solved by the following formula:

\[
\delta_1 \left( \frac{\delta_2}{h} \right) - 1 \left( \frac{1 - \varepsilon + \lambda_C \varepsilon}{\delta_2} \right) ^{\delta_1} - \delta_2 \left( \frac{\delta_1}{h} \right) - 1 \left( \frac{1 - \varepsilon + \lambda_C \varepsilon}{\delta_1} \right) ^{\delta_2} = 0
\]  

(11)

Integrating the rolling pressures, Eq. (9), along the contact length, the rolling force per unit width under the C + B configuration can be obtained:

\[
P = \frac{IK}{\varepsilon} \left\{ \frac{1}{\delta_1} \left( \frac{\delta_1 - 1 + (1 - \beta)}{\delta_1} \right) + \frac{1}{\delta_2} \left( \frac{\delta_2 - 1 + (1 - \beta)}{\delta_2} \right) \right\}
\]  

(12)

According to Fig. 1d, the deformation zone configuration is AC, where \( h_1 = h \) and \( h_2 = H \). The rolling pressures can be obtained:

\[
p_C = \frac{K}{\delta_2} \left( \left( \frac{\delta_2}{h} \right)^{\delta_1} - 1 \right) \left( \frac{\delta_2}{h} \right) ^{\delta_1} + 1
\]  

(13)

To keep the deformation zone as AC configuration, the roll speed ratio and the front tension need to meet

\[
i \geq i_{c2} = \frac{V_1}{V_2} = \frac{h_2}{h_1} - \frac{(1 - \varepsilon)}{\varepsilon} \approx \frac{h_2}{h_1} - \frac{(1 - \varepsilon)}{\varepsilon} = \frac{\sigma_{\max}}{\sigma_f}
\]  

\[
\sigma_{\max} \geq \sigma_f \geq \sigma_{fC}
\]  

(14)

where \( \sigma_{\max} \) is the maximum engineering allowable stress.

The rolling force per unit width under the AC configuration can be obtained:

\[
P = \frac{IK}{\delta_2} \left( \left( \frac{\delta_2}{h} \right)^{\delta_1} - 1 \right) \left( \frac{\delta_2 - 1 + (1 - \varepsilon)}{\delta_2 - 1} \right) \left( \frac{1 - (1 - \varepsilon) ^{\delta_2 - 1}}{1 - (1 - \varepsilon) ^{\delta_2 - 1}} \right) + 1
\]  

(15)

According to Eqs. (10), (14), and (17), the deformation zone configuration-rolling parameter relationship diagram can be obtained as Fig. 2. In the normal asymmetrical rolling process, in order to balance the production efficiency and the energy saving, the rolling technological parameters will be as close as possible to the point \( (\sigma_{fC}, i_{c2}) \) when the maximum tension \( \sigma_{\max} \geq \sigma_{fC} \) or the point \( (\sigma_{\max}, i_{c2}) \) when the maximum tension \( \sigma_{\max} < \sigma_{fC} \). When discussing the minimum rollable thickness of asymmetrical cold rolling, the above issues do not need to be considered.

As shown in Fig. 2, when the rolling parameters are fixed and the front tension \( \sigma_f < \sigma_{fC} \), only need is to increase the roll speed ratio to make it greater than \( i_{c2} \) to ensure that the deformation zone configuration remains C + B. With the increase of the roll speed ratio, the proportion of the forward-slip zone decreases until it is 0, the rolling force and the proportion of the backward-slip zone decrease, and the proportion of the cross-shear zone decreases.

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**Fig. 2** Deformation zone configuration-rolling parameter relationship diagram (\( H = 0.01\text{mm}, \varepsilon = 10\%, R = 44\text{mm}, K = 481.3\text{MPa}, f = 0.1, \sigma_b = 50\text{MPa} \))
increases, as shown in Fig. 3a. When the roll speed ratio $i \geq i_{c1}$, the above parameters no longer change with the increase of the roll speed ratio. When the front tension $\sigma_f \geq \sigma_{fc}$, with the increase of the roll speed ratio $i$, the deformation zone configuration changes from $F + C + B$ to $F + C$ and AC in turn, as shown in Fig. 2. When the roll speed ratio increases to $i_{c2}$, the backward-slip zone disappears; when the roll speed ratio continues to increase to $i_{c3}$, the forward-slip zone also disappears; and the deformation zone is all the cross-shear zone, as shown in Fig. 3b. The rolling force decreases with the increase of the roll speed ratio, until the roll speed ratio $i \geq i_{c2}$.

The rolling parameters need to exceed line $i_{c1}$ or line $i_{c2}$ to keep the deformation zone configuration as C + B or AC and maximize the thinning capacity of the rolling mill. According to Eq. (14), the critical roll speed ratio $i_{c2}$ and the critical front tension $\sigma_{fc}$ decrease with the decrease of the reduction rate $\epsilon$, as shown in Fig. 4. Therefore, it can be considered that the upper limit of the adjustable range of the roll speed ratio and the tension of the asymmetrical rolling mill are greater than $i_{c2}$ and $\sigma_{fc}$, respectively. In this paper, only the C + B and AC configuration need to be studied for the minimum rollable thickness of the asymmetrical rolling.

### 2.2 Solution of minimum rollable thickness

#### 2.2.1 Derivation of $H_{\text{min}}$

As shown in Fig. 2, with increasing the front tension to $\sigma_{fc}$, the critical roll speed ratio $i_{c1}$ increases to $i_{c2}$, and the deformation zone configuration changes from C + B to AC. Therefore, it is only necessary to study the minimum rollable thickness of the asymmetrical rolling under the C + B configuration. The AC configuration can be approached from C + B configuration through increasing the front tension.

![Graph](image)

**Fig. 4** Variations of the critical roll speed ratio $i_{c2}$ and the critical front tension $\sigma_{fc}$ with the reduction rate ($H = 0.01\text{mm}, R = 44\text{mm}, f = 0.1, \sigma_0 = 50\text{MPa}$)

The average rolling pressure under the C + B configuration can be expressed as

$$
\bar{p} = \frac{K}{\epsilon} \left\{ \frac{1}{\delta_1} \left[ \delta_1 \xi_1 - 1 \right] \left( \beta^{1-\delta_1} - 1 \right) + (1 - \beta) + \bar{p}_c \right\}
$$

(18)

where $\bar{p}_c = \frac{1}{-2\sigma_f} \left\{ \frac{2\mu l_c + 1}{2\mu l + 1} \right\} \left( 1 - \epsilon - \beta \left( \frac{1}{\delta_1} \right)^{-2\mu} \right) + (\beta - 1 + \epsilon)$.

When the reduction $\epsilon H$ is small enough, the contact length can be expressed as

$$
l = 2\bar{p} \cdot 8R \frac{1 - \nu^2}{\pi E}
$$

(19)

Substituting Eq. (18) into Eq. (19), the following formula can be obtained:

$$
H = \frac{2\mu l}{\epsilon \delta_1} \frac{16\mu DK (1 - \nu^2)}{\pi \epsilon^2 E} \left\{ \frac{1}{\delta_1^2} \left[ \delta_1 \xi_1 - 1 \right] \left( \beta^{1-\delta_1} - 1 \right) + (1 - \beta) + \bar{p}_c \right\}
$$

(20)

![Graph](image)

**Fig. 3** Variations of the proportions of each zone in the deformation zone and the rolling force with the roll speed ratio: (a) $\sigma_f = 0\text{MPa}$ and (b) $\sigma_f = 100\text{MPa}$ ($R = 44\text{mm}, K = 481.3\text{MPa}, f = 0.1, H = 0.1\text{mm}, \epsilon = 10\%, \sigma_0 = 0\text{MPa}, \sigma_{fc} = 50.7\text{MPa}$)
When the back and front tensions are zero, Eq. (20) can be simplified to
\[
H = \frac{16 \mu DK (1 - \nu^2)}{n \epsilon^2 E} \beta l - \delta_i \beta + \delta_i \frac{\rho_x}{n \epsilon^2 E} = \frac{16 \mu DK (1 - \nu^2)}{n \epsilon^2 E} f(\delta_i)
\]
(21)

When the reduction rate \( \epsilon \) is determined, the minimum rollable thickness appears at the minimum value of the function \( f(\delta_i) \). Derivative of \( f(\delta_i) \) can be obtained:
\[
f'(\delta_i) = -\frac{\delta_i \beta l - \delta_i \beta l - \delta_i \ln \beta - 2 \beta l - \delta_i + 2 \beta}{\delta_i^4}
\]
(22)

Mathematically, it can be easily obtained that \( f'(\delta_i) = 0 \) has a unique solution in the interval \((0, +\infty)\), and \( f(\delta_i) \) reaches the minimum value at this solution. Iteratively solve Eqs. (11) and (22), if the calculation results converge, the minimum rollable thickness is obtained by Eq. (21). Because of the positions of the neutral points in the deformation zone is determined by the velocity relationship between the strip and the work rolls, the reduction rate \( \epsilon \) should be greater than 0. This causes the result to fail to converge when calculating the value of the minimum rollable thickness \( H_{\min} \) by using the above formulas. If \( \epsilon \to 0^+ \) is assumed when the strip reaches the minimum rollable thickness, the proportion of each part in the deformation zone is obviously uncertain. Therefore, after confirming that the minimum rollable thickness appears in the C + B or AC configuration, the deformation zone is further assumed to be a flat plate compression process to obtain a convergence result of \( H_{\min} \).

### 2.2.2 Solution of \( H_{\min} \)

When the reduction rate reaches a critical value \( \epsilon_{\text{cr}} \), the reduction \( \epsilon_x H \) is small enough, the rolling process is regarded as an asymmetrical flat plate compression process, as shown in Fig. 5.

![Fig. 5 Schematic of the asymmetrical flat plate compression process](image)

According to Fig. 5, the rolling force model is remodeled. In the backward-slip zone, the following differential equation can be obtained from the force balance equation and the yield criterion:
\[
\text{dp}_x = -\frac{2\mu}{H} \text{dx}
\]
(23)

The boundary conditions on the entrance of the deformation zone are \( x = l \) and \( p_x = K - \sigma_b \). Substituting the boundary conditions into the differential equation, the rolling pressure in the backward-slip zone can be expressed as
\[
p_x = (K - \sigma_b) \exp \left( \frac{2\mu x}{H} \right)
\]
(24)

where \( \frac{2\mu x}{H} \)

At the neutral point of the slow roll, \( \lambda_2 = \lambda_C l \), where \( \lambda_C \in [0, 1] \), the rolling pressure \( p_x \) can be expressed as
\[
p_x = (K - \sigma_b) \exp \left[ \lambda(1 - \lambda_C) \right]
\]
(25)

The distribution of the rolling pressure in the cross-shear zone can be expressed as
\[
\text{dp}_x = C_0
\]
(26)

The boundary conditions on the exit of the deformation zone are \( x = 0 \) and \( p_x = K - \sigma_f \). Substituting Eq. (25) and the boundary conditions into Eq. (26), the rolling pressure in the cross-shear zone can be obtained:
\[
p_x = C_0 x + K - \sigma_f
\]
(27)

where \( C_0 = (K - \sigma_b) \exp \left[ \lambda_2 \right] \)

Integrating the rolling pressure along the contact length, the average rolling pressure can be obtained:
\[
\bar{p} = \frac{\lambda K}{2} \left( \xi_1 e^m + \xi_2 \right) + (1 - \lambda_C) K \xi_1 e^m - \frac{1}{m}
\]
(28)

where \( m = \delta(1 - \lambda_C) \).

Similar to Eq. (19), the thickness \( H \) can be expressed as
\[
H = \frac{16 \mu DK (1 - \nu^2)}{n \epsilon^2 E} g(\nu) = \frac{16 \mu DK (1 - \nu^2)}{n \epsilon^2 E} \left[ \frac{\lambda_C (1 - \lambda_C) \xi_1 e^m + \xi_2}{2} + (1 - \lambda_C) \frac{\xi_1 e^m - 1}{m} \right]
\]
(29)

The minimum rollable thickness can be obtained by solving the minimum value of the function \( g(\nu) \). Derivation of \( g(\nu) \) can be obtained:
\[
g'(\nu) = 1 - \lambda_C m^2 + \left( \frac{3 \lambda_C}{2} \right) e^m - 2(1 - \lambda_C)(e^m - 1) - \frac{\lambda_C \xi_2}{2 \xi_1} m
\]
(30)
When the critical reduction rate $\varepsilon_0$, the proportion of the cross-shear zone $\lambda_C$, and the back and front tensions are determined, $m$ can be obtained by the following formula:

$$j(m) = \frac{\lambda_c}{2} m^2 e^m + \left(1 - \frac{3\lambda_c}{2}\right) m e^m - 2(1 - \lambda_c) (e^m - 1) - \frac{\lambda_c \varepsilon_0}{2\xi_1} m = 0$$  

(31)

where $m \in [0, +\infty)$, $\lambda_C \in [0, 1]$, and $\frac{\delta}{\xi_1} \in \left[\frac{2-\sqrt{3}}{2}, \frac{2}{2-\sqrt{3}}\right]$.

Derivation of function $j(m)$ can be obtained:

$$\begin{cases} 
    j'(m) = e^m(m-1)\left(1 - \frac{\lambda_c}{2}\right) + \frac{\lambda_c}{2} \left(m^2 e^m - \frac{\delta}{\xi_1}\right) \\
    j''(m) = me^m \left(1 + \frac{\lambda_c}{2}\right) + \frac{\lambda_c}{2} m^2 e^m
\end{cases}$$  

(32)

In the interval $[0, +\infty)$, $j''(m) \geq 0$, function $j'(m)$ monotonously increases. Because of $j'(0) < 0$ and $j'(m \to +\infty) > 0$, there must be $j'(m_1) = 0$, where $m_1 > 0$. When $m \in [m_1, +\infty)$, the function $j'(m) \geq 0$. Because of $j'(m_1) < j(0) = 0$ and $j(m \to +\infty) > 0$, there must be $j(m_2) = 0$, where $m_2 > m_1$. Therefore, function $g(m)$ has a minimum value in the interval $(0, +\infty)$, that is, $m_2$.

Substituting $m$ into Eq. (29), the minimum rollable thickness $H_{min}$ can be obtained. But the value of $\lambda_C$ needs to be calculated iteratively through Eqs. (11) and (29); the calculated result still does not converge. $\lambda_C$ needs to be confirmed by other means.

### 2.2.3 Solution of proportion of cross-shear zone $\lambda_C$

According to the locations of the neutral points, $h_1$ and $h_2$, the proportions of each part of the deformation zone can be obtained, when the reduction rate is determined and the back and front tensions are not considered. Figure 6a shows the variations of the proportions of each part of the deformation zone with the roll speed ratio. With the increase of the roll speed ratio, the proportions of the backward-slip zone and the forward-slip zone decrease almost equally, as shown in Fig. 6b. When the proportion of the forward-slip zone decreases to zero, the deformation zone configuration changes from $F + C + B$ to $C + B$.

Therefore, the proportion of the cross-shear zone under the $C + B$ configuration can be obtained by calculating the proportion of the forward-slip zone in the symmetrical rolling. When other rolling parameters are the same, the proportion of the cross-shear zone under the $C + B$ configuration is twice as large as that of the forward-slip zone in the symmetrical rolling and can be expressed as

$$\lambda_C = \frac{2(1 - \varepsilon_0)}{\varepsilon_0} \left\{1 + \frac{1 + (\delta_1, \delta_2 - 1)(\delta_1, \delta_2 + 1)(1 - \varepsilon_0)^{\gamma}}{\delta_1, \delta_2 + 1}\right\}^{\gamma} - 1$$  

(33)

where $\delta_j = \frac{\delta}{\varepsilon_0}$.

### 3 Results and discussion

Figure 7 shows the comparison of the proportions of the cross-shear zone under $C + B$ configuration calculated by Eqs. (11) and (33), marked as $\lambda_{C1}$ and $\lambda_{C2}$, respectively. With the decrease of the critical reduction rate $\varepsilon_0$, the error between $\lambda_{C1}$ and $\lambda_{C2}$ decreases. When the critical reduction rate $\varepsilon_0 = 10\%$, the error is 2.48%. In fact, when $\varepsilon_0 = 20\%$ the error between $\lambda_{C1}$ and $\lambda_{C2}$ does not exceed 5%. Since the minimum rollable thickness is sensitive to changes in the proportion of the cross-shear zone, the large the critical reduction rate $\varepsilon_0$ is, the larger the error of the minimum rollable thickness calculated by Eq. (33) is. At the same time, when the value of $\varepsilon_0$ is large, the value of $\varepsilon_0 H$ is not small enough and does not meet the previous assumptions. Therefore, it is appropriate to set the value range as $10\% \geq \varepsilon_0 > 0$.

**Fig. 6 (a)** Variations of proportions of each part in the deformation zone with the roll speed ratio and (b) variations of reductions of $\lambda_F$ and $\lambda_B$ with the roll speed ratio
When the back and front tensions are zero and the critical reduction rate $\varepsilon_0$ is determined, $m$ and $\lambda_C$ can be iteratively calculated by Eqs. (31) and (33) to obtain a unique solution; then the minimum rollable thickness $H_{\text{min}}$ can be obtained by Eq. (29). Therefore, $H_{\text{min}}$ can be expressed as a function of the critical reduction rate $\varepsilon_0$ as the following:

$$
\begin{align*}
H_{\text{min}} &= \left(2.1949\varepsilon_0^2 + 1.9033\varepsilon_0\right) \frac{E_{\text{max}}}{E} \quad 0 < \varepsilon_0 \leq 0.1 \\
i \geq i_1 &= 2.033\varepsilon_0^2 + 0.8361\varepsilon_0 + 1
\end{align*}
$$

(34)

The effects of the back and front tensions on the minimum rollable thickness are reflected in two aspects. At first, the average rolling pressure decreases with the increase of the tensions; it is conducive to the decrease of $H_{\text{min}}$. On the other hand, when the deformation zone configuration is C + B, the proportion of cross-shear zone $\lambda_C$ and the critical roll speed ratio $i_{c1}$ decrease with the increase of the back tension and increase with the increase of the front tension [21]. As long as the coils on both sides of the rolling mill are the same, it can ensure that $\sigma_f \geq \sigma_b$, so that the influence of the tensions is always conducive to the reduction of the minimum rollable thickness $H_{\text{min}}$. When the front tension is $\min\{\sigma_f, \sigma_{\text{max}}\} \geq \sigma_f \geq \sigma_b$, $H_{\text{min}}$ can be obtained by Eqs. (29), (31), and (33) and the following formula.

$$
\begin{align*}
\lambda'_{C} &= \frac{\sigma_f - \sigma_b}{\sigma_f - \sigma_b} (1 - \lambda_C) \\
i \geq i_1 &= \frac{\sigma_f - \sigma_b}{\sigma_f - \sigma_b} (i_{c2} - i_{c1}) \\
\min\{\sigma_f, \sigma_{\text{max}}\} &\geq \sigma_f \geq \sigma_b
\end{align*}
$$

(35)

Figure 8 shows the comparison of the critical roll speed ratio $i_{c1}$ calculated by Eqs. (9) and (33) ($H = 0.01\text{mm}$, $\varepsilon = 10\%$, $R = 44\text{mm}$, $K = 481.3\text{MPa}$, $f = 0.1$, $\sigma_b = 50\text{MPa}$). When the maximum engineering allowable stress $\sigma_{\text{max}} \geq \sigma_f$, the AC configuration can be obtained by adjusting the roll speed ratio and the tensions; the proportion of cross-shear zone $\lambda_C = 1$. According to Eq. (29), $H_{\text{min}}$ can be expressed as

$$
\begin{align*}
H_{\text{min}} &= 0^+ \\
\sigma_{\text{max}} &\geq \sigma_f \geq \sigma_b \& i \geq i_{c2}
\end{align*}
$$

(36)

Figure 9 shows the comparison between the contact length ignoring the plastic deformation (calculated by Eq. 19) and that considering the plastic deformation (calculated by Hitchcock formula), denoted by $l_1$ and $l_2$, respectively. With the increase of the critical reduction rate, the error between $l_1$ and $l_2$ increases. According to Eq. (34), the minimum rollable thickness increases with increasing the critical reduction rate. When the critical reduction rate
is larger, the reduction $\epsilon H$ cannot be ignored. When the critical reduction rate $\epsilon_0 = 10\%$, the error between $l_1$ and $l_2$ is 1.98%; it is reasonable to set the value range of $\epsilon_0$ as $[0, 10\%]$.

Figure 10 shows the comparison of the theoretical value with the experiment result. The experimental platform is a four-high reversing asymmetrical cold mill. The diameter of the work roll is 88mm. Lubricants use the mineral emulsifiers; the friction coefficient $\mu = 0.1$. The strip uses the annealed 430 stainless steels, and has been rolled from 0.5 to 0.01mm in 18 rolling passes without annealing. The critical reduction rate $\epsilon_0 = 10\%$; according to Eq. (34), the theoretical minimum rollable thickness $H_{\text{min}} = 5.9\mu$m, and the theoretical value is in good agreement with the experimental result. In fact, the reduction rate of the last two passes is very close to 10%; the rolled piece has the potential to continue to thin, as long as the rolling force is slightly increased. The plasticity of the strip is very poor after multiple passes, since it has not been annealed. This is a reason to limit further thinning of the rolled piece.

Figure 11a shows the variation of the exit thickness of deformation zone with the rolling pass under different roll speed ratios, when the rolling force is constant. When the symmetrical rolling is used, the strip is rolled from 0.5 to 0.046mm through 25 passes rolling and multiple annealing. After the third annealing, annealing is required for each rolling pass, but the reduction rate still decreases with the increase of the rolling pass, as shown in Fig. 11b. When the asymmetrical rolling is used, the strip can be thinned from 0.5mm to about 1$\mu$m without annealing, but the required number of rolling pass decreases with the increase of the roll speed ratio. Obviously, choosing an appropriate roll speed ratio and critical reduction rate can improve the production efficiency of the ultrathin strip and reduce the production costs.

4 Conclusion

A new minimum rollable thickness model for the asymmetrical rolling is proposed based on the slab method. The calculation formulas of rolling pressure, rolling force, required
critical roll speed ratio, and critical front tension for \( F + C + B, C + B, AC, \) and \( F + C \) deformation zone configurations are proposed, respectively. Thus, the deformation zone configuration-rolling parameter relationship diagram is obtained. Based on the analysis of this diagram, when the rolling parameters keep the deformation zone configuration as \( C + B \) or \( AC \), the rolling mill has the strongest thinning ability. The minimumrollable thickness can be obtained as the following:

1. When \( \min \{ \sigma_f, \sigma_{\max} \} > \sigma_f \geq \sigma_b \geq 0 \) and the deformation zone is \( C + B \) configuration, the minimumrollable thickness and the required critical roll speed ratio can be calculated by Eqs. (29), (31), (33), and (35). Specially, when the tension \( \sigma_f = \sigma_b = 0 \) MPa, the minimumrollable thickness and the required critical roll speed ratio can be calculated by Eq. (34);

2. When the tension \( \sigma_{\max} \geq \sigma_f \geq \sigma_b \) and the deformation zone is \( AC \) configuration, the minimumrollable thickness is close to 0 under ideal rolling conditions. The required critical roll speed ratio can be calculated by Eq. (14).

The efficiency of using asymmetrical rolling for the ultrathin strip is very significant. After 18 rolling passes, the 430 stainless steel strip is rolled from 0.5 mm to 10 \( \mu \)m without annealing. The calculated minimumrollable thickness is 5.9 \( \mu \)m when the critical reduction rate is 5.9\%. When the tension \( \sigma_f = \sigma_b = 0 \) MPa, the calculated minimumrollable thickness is 5.9 \( \mu \)m when the critical reduction rate is 10\% and agrees with the experimental result well. This model is simple and easy to use, and has high application value for the design of asymmetrical rolling mills and the formulation of the asymmetrical rolling ultrathin strip technology.

**Author contribution** Ji Wang: conceptualization, methodology, writing — original draft, formal analysis, investigation, validation, visualization.

Xianghua Liu: resources, writing — review and editing, supervision.

**Availability of data and material** Data will be made available on request.

**Declarations**

**Ethical approval** Not applicable.

**Consent to participate** All authors have consented to participate in this study.

**Consent for publication** All authors have consented to the publication of this work.

**Conflict of interest** The authors declare no competing interests.

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**References**

1. Fu MW, Chan WL (2012) A review on the state-of-the-art microforming technologies. Int J Adv Manuf Technol 67(9–12):2411–2437. https://doi.org/10.1007/s00170-012-4661-7

2. Liu XH, Song M, Sun XK, Zhao QL, Yu QB (2017) Advances in research and application of foil rolling. J Mech Eng 53(10). https://doi.org/10.3901/jme.2017.10.001

3. Stone MD (1953) Rolling of thin strip. Iron Steel Eng 30(2):61

4. Stone MD (1956) Rolling of thin strip partII. Iron Steel Eng 33(12):52

5. Fleck NA, Johnson KL (1987) Towards a new theory of cold rolling thin foil.pdf. Int J Mech Sci 29:507–524. https://doi.org/10.1016/0020-7403(87)90012-9

6. Fleck NA, Johnson KL, Mear ME, Zhang LC (1992) Cold rolling of foil. Proc Inst Mech Eng 206:119–131. https://doi.org/10.1243/PIME_PROC_1992_206_064_02

7. Sutcliffe MPF, Rayner PJ (1998) Experimental measurements of load and strip profile in thin strip rolling. Int J Mech Sci 40(9):887–899. https://doi.org/10.1016/S0020-7403(97)00138-0

8. Xiao H, Ren ZK, Liu X (2017) New mechanism describing the limiting producible thickness in ultra thin strip rolling. Int J Mech Sci 133:788–793. https://doi.org/10.1016/j.ijmecsci.2017.09.046

9. Wu SM, Wei LS, Huang G (2014) Research on the minimum rolled thickness of strip steel in cold tandem rolling. Key Eng Mater 622–623:993–999. https://doi.org/10.4028/www.scientific.net/KEM.622-623.993

10. Wu SM (2007) Research of smallest permissible-rolling thickness during strip rolling. China Metallurgy 17(3):10–12. https://doi.org/10.3969/j.issn.1006-9356.2007.03.003

11. Tateno J, Hiruta T, Katsura S, Honda A, Miyata T, Kaminamaru A (2011) Experimental analysis of thickness reduction limits in ultra thin stainless steel foil rolling. ISIJ Int 51(5):788–792. https://doi.org/10.2355/tetsutohagane.98.89

12. Zhang LC (1995) On the mechanism of cold rolling thin foil. Int J Mach Tools Manufact 35(3):363–372. https://doi.org/10.1016/0890-6955(94)E0028-H

13. Hwang YM, Kan CC (2016) Roll shape design for foil rolling of a four-high mill. Int J Adv Manuf Technol 91(5–8):1587–1597. https://doi.org/10.1007/s00170-016-9843-2

14. Tang DL, Liu XH, Song M, Yu HL (2014) Experimental and theoretical study on minimum achievable foil thickness during asymmetric rolling. PLoS One 9(9):e106637. https://doi.org/10.1371/journal.pone.0106637

15. Tang DL, Liu XH, Li XY, Peng LG (2013) Permissible minimum thickness in asymmetric cold rolling. J Iron Steel Res Int 20(11):21–26. https://doi.org/10.1016/S1006-706X(13)60191-0

16. Liu X, Liu XH, Song M, Sun XK, Liu LZ (2016) Theoretical analysis of minimum metal foil thickness achievable by asymmetric rolling with fixed identical roll diameters. Trans Nonferrous Met Soc China 26(2):501–507. https://doi.org/10.1016/s1003-6326(16)64138-9

17. Tzou GY, Huang MN (2001) Study on minimum thickness for Foil forming technologies. Int J Adv Manuf Technol 67(9–12):2297–2309. https://doi.org/10.1007/s00170-018-2368-0
20. Wang J, Liu XH, Sun XK (2020) Study on asymmetrical cold rolling considered sticking friction. J Mater Res Technol 9(6):14131–14141. https://doi.org/10.1016/j.jmrt.2020.10.027
21. Wang J, Liu XH, Sun XK (2020) Study on the relationship between asymmetrical rolling deformation zone configuration and rolling parameters. Int J Mech Sci 187. https://doi.org/10.1016/j.ijmecsci.2020.105905
22. Sun XK, Liu XH, Wang J, Qi JL (2020) Analysis of asymmetrical rolling of strip considering two deformation region types. Int J Adv Manuf Technol 110(9–10):2767–2785. https://doi.org/10.1007/s00170-020-06022-1
23. Sun XK, Liu XH, Qi JL, Wang J (2020) Experimental study on deformation region types and percentages of each region in asymmetrical rolling of strip. IOP Conf Ser Mater Sci Eng 892. https://doi.org/10.1088/1757-899x/892/1/012021
24. Sun XK, Liu XH, Wang J, Qi JL (2020) Analysis of asymmetrical rolling of strip considering percentages of three regions in deformation zone. Int J Adv Manuf Technol. https://doi.org/10.1007/s00170-020-05690-3
25. Ji YH, Park JJ (2009) Development of severe plastic deformation by various asymmetric rolling processes. Mater Sci Eng A 499(1–2):14–17. https://doi.org/10.1016/j.msea.2007.11.099

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