We present a leading order (LO) estimate for the longitudinal-transverse spin asymmetry ($A_{LT}$) in the nucleon-nucleon polarized Drell-Yan process at RHIC and HERA-$\vec{N}$ energies in comparison with $A_{LL}$ and $A_{TT}$. $A_{LT}$ receives contribution from $g_1$, the transversity distribution $h_1$, and the twist-3 distributions $g_T$ and $h_L$. For the twist-3 contribution we use the bag model prediction evolved to a high energy scale by the large-$N_c$ evolution equation. We found that $A_{LT}$ (normalized by the asymmetry in the parton level) is much smaller than the corresponding $A_{TT}$. Twist-3 contribution given by the bag model turned out to be negligible.

The nucleon-nucleon scattering provides us with a new opportunity to probe nucleon’s internal structure. In particular, polarized Drell-Yan lepton pair production opens a window toward new types of spin dependent parton distributions – chiral-odd distributions $h_1(x, \mu^2)$ and $h_L(x, \mu^2)$ which can not be measured by the deep inelastic lepton-nucleon scatterings [1]. There are three kinds of double spin asymmetries in the nucleon-nucleon polarized Drell-Yan process: They are $A_{LL}$ (collision between the longitudinally polarized nucleons), $A_{TT}$ (collision between the transversely polarized nucleons), and $A_{LT}$ (longitudinal versus transverse). The experimental data on these asymmetries will presumably be reported by RHIC at BNL and HERA-$\vec{N}$ at DESY. By now several reports are already available for the estimate of $A_{LL}$ and $A_{TT}$ in the next-to-leading order (NLO) level [3]. In this talk we present a first estimate on $A_{LT}$ in comparison with $A_{LL}$ and $A_{TT}$ at RHIC and HERA energies in the LO QCD [4]. $A_{LT}$ is particularly interesting, since it receives the twist-3 contribution as a leading contribution (although it is proportional to $1/Q$), giving a possibility of seeing quark-gluon correlation in hard processes.

In LO QCD, the double spin asymmetries are given by

\begin{align}
A_{LL} &= \frac{\sigma(\uparrow, \uparrow) - \sigma(\downarrow, \downarrow)}{\sigma(\uparrow, \uparrow) + \sigma(\downarrow, \downarrow)} = \frac{\sum_a e_a^2 g_1^a(x_1, Q^2) g_1^\bar{a}(x_2, Q^2)}{\sum_a e_a^2 f_1^a(x_1, Q^2) f_1^{\bar{a}}(x_2, Q^2)}, \\
A_{TT} &= \frac{\sigma(\uparrow, \uparrow) - \sigma(\downarrow, \downarrow)}{\sigma(\uparrow, \uparrow) + \sigma(\downarrow, \downarrow)} = a_{TT} \frac{\sum_a e_a^2 h_1^a(x_1, Q^2) h_1^{\bar{a}}(x_2, Q^2)}{\sum_a e_a^2 f_1^a(x_1, Q^2) f_1^{\bar{a}}(x_2, Q^2)}, \\
A_{LT} &= \frac{\sigma(\uparrow, \uparrow) - \sigma(\downarrow, \downarrow)}{\sigma(\uparrow, \uparrow) + \sigma(\downarrow, \downarrow)} = a_{LT} \frac{\sum_a e_a^2 [g_1^a(x_1, Q^2) x_2 g_1^{\bar{a}}(x_2, Q^2) + x_1 h_1^a(x_1, Q^2) h_1^{\bar{a}}(x_2, Q^2)]}{\sum_a e_a^2 f_1^a(x_1, Q^2) f_1^{\bar{a}}(x_2, Q^2)},
\end{align}
where \( \sigma(S_1, S_2) \) represents the Drell-Yan cross section with the two nucleons’ spin \( S_1 \) and \( S_2 \), \( e_a \) represent the electric charge of the quark-flavor \( a \) and the summation is over all quark and anti-quark flavors: \( a = u, d, s, \bar{u}, \bar{d}, \bar{s} \), ignoring heavy quark contents (\( c, b, \cdots \)) in the nucleon. The variables \( x_1 \) and \( x_2 \) refer to the momentum fractions of the partons coming from the two nucleons “1” and “2”, respectively. In (2) and (3), \( a_{TT} \) and \( a_{LT} \) represent the asymmetries in the parton level defined as \( a_{TT} = \sin^2 \theta \cos 2 \phi / (1 + \cos^2 \theta) \) and \( a_{LT} = (M/Q)(2 \sin 2 \theta \cos \phi) / (1 + \cos^2 \theta) \), where \( \theta \) and \( \phi \) are, respectively, the polar and azimuthal angles of the virtual photon in the center of mass system with respect to the beam direction and the transverse spin. We note that \( A_{LL} \) and \( A_{TT} \) receive contribution only from the twist-2 distributions, while \( A_{LT} \) is proportional to the twist-3 distributions and hence \( a_{LT} \) is suppressed by a factor \( 1/Q \).

The twist-3 distributions \( g_T \) and \( h_L \) can be decomposed into the twist-2 contribution and the “purely twist-3” contribution:

\[
\begin{align*}
g_T(x, \mu^2) &= \int_x^1 dy \frac{g_1(y, \mu^2)}{y} + \tilde{g}_T(x, \mu^2); \\
h_L(x, \mu^2) &= 2x \int_x^1 dy \frac{h_1(y, \mu^2)}{y^2} + \tilde{h}_L(x, \mu^2).
\end{align*}
\]

The purely twist-3 pieces \( \tilde{g}_T \) and \( \tilde{h}_L \) can be written as quark-gluon-quark correlators on the lightcone. In the following we call the first terms in (4) \( g_T^{WW}(x, \mu^2) \) and \( h_L^{WW}(x, \mu^2) \) (Wandzura-Wilczek parts).

For the present estimate of \( A_{LT} \), we use the LO parametrization for \( f_1 \) by Glück-Reya-Vogt [1] and the LO parametrization (standard scenario) for \( g_1 \) by Glück-Reya-Stratmann-Vogelsang (GRSV) [6]. For \( h_1 \), \( g_T \) and \( h_L \) no experimental data is available up to now and we have to rely on some theoretical postulates. Here we assume \( h_1(x, \mu^2) = g_1(x, \mu^2) \) at a low energy scale (\( \mu^2 = 0.23 \text{ GeV}^2 \)) as has been suggested by a low energy nucleon model [6]. These assumptions also fix \( g_T^{WW} \) and \( h_L^{WW} \). For the purely twist-3 parts \( \tilde{g}_T \) and \( \tilde{h}_L \) we employ the bag model results at a low energy scale, assuming the bag scale is \( \mu^2_{bag} = 0.081 \) and \( 0.25 \text{ GeV}^2 \). In particular, we set the strangeness contributions to the purely twist-3 contributions equal to zero. By these boundary conditions for \( h_1 \), \( g_T \) and \( h_L \) at a low energy side and applying the relevant \( \mu^2 \) evolution to them, we can estimate \( A_{LT} \).

The \( \mu^2 \) evolution of the twist-3 distributions is quite complicated [7]. However, it has been proved that at large \( N_c \) their \( \mu^2 \)-dependence can be described by a simple DGLAP evolution equation similarly to the twist-2 distributions and the correction due to the finite value of \( N_c \) is of \( O(1/N_c^2) \approx 10 \% \) level [8]. Here we apply this large-\( N_c \) evolution to the bag model results [7].

The double spin asymmetries are the functions of the center-of-mass energy \( s = (P_1 + P_2)^2 \) (\( P_1 \) and \( P_2 \) are the four momenta of the two nucleons), the squared invariant mass of the lepton pair \( Q^2 = (x_1 P_1 + x_2 P_2)^2 = x_1 x_2 s \) (\( M^2 << Q^2 \)) and the Feynman’s \( x_F = 2q_2/\sqrt{s} = x_1 - x_2 \). Using these variables, momentum fractions of each quark and anti-quark in (4) can be written as \( x_1 = \left( x_F + \sqrt{x_F^2 + (4Q^2/s)} \right) / 2 \) and \( x_2 = \left( -x_F + \sqrt{x_F^2 + (4Q^2/s)} \right) / 2 \).

Figure 1 shows the three asymmetries normalized by the asymmetries in the parton level, \( \tilde{A}_{LL} = -A_{LL} \), \( \tilde{A}_{TT} = -A_{TT}/a_{TT} \), \( \tilde{A}_{LT} = -A_{LT}/a_{LT} \). They are plotted as a function of \( x_F \) for fixed values of \( Q = \sqrt{Q^2} \) (= 8, 10 GeV) and \( \sqrt{s} \) (= 50, 200 GeV), which are
within or close to the planned RHIC and HERA-$\vec{N}$ kinematics. ($50 \text{ GeV} < \sqrt{s} < 500 \text{ GeV}$ for RHIC, and $\sqrt{s} = 39.2 \text{ GeV}$ for HERA-$\vec{N}$.) $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$ are symmetric with respect to $x_F = 0$, while $\tilde{A}_{LT}$ is not symmetric as is obvious from the kinematics. In general all these asymmetries are larger for larger $Q^2/s$. $\tilde{A}_{LT}$ with only the twist-2 contributions in $g_T$ and $h_L$ are shown by solid lines. They are typically 5 to 10 times smaller than $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$. $\tilde{A}_{LT}$ with complete $g_T$ and $h_L$ is shown by the short dash-dot ($\mu_{bag}^2 = 0.25 \text{ GeV}^2$) and the dotted ($\mu_{bag}^2 = 0.081 \text{ GeV}^2$) lines. Since large $|x_F|$ corresponds to small $x_1$ or $x_2$, and the bag model prediction for the distribution function becomes unreliable in the small-$x$ region, we only plotted these lines for the region $x_1, x_2 > 0.07$. As can be seen from Fig. 1, the purely twist-3 contribution brings only tiny correction to $\tilde{A}_{LT}$. Larger value of the bag scale $\mu_{bag}^2$ would not make it appreciably larger. The smallness of $\tilde{A}_{LT}$ can be ascribed to the factors $x_1$ or $x_2$ in (3). In the kinematic range considered either $x_1$ or $x_2$ (or both) take very small values. If it were not for those factors, $\tilde{A}_{LT}$ would be comparable to $\tilde{A}_{LL}$ and $\tilde{A}_{TT}$. We remind in passing that what is measured experimentaly is $A_{LT}$ itself which receives the suppression factor $M/Q$ from $a_{LT}$.

To summarize, we presented a first estimate of the longitudinal-transverse spin asymmetry $A_{LT}$ for the polarized Drell-Yan process at RHIC and HERA-$\vec{N}$ energies in comparison with $A_{LL}$ and $A_{TT}$. $A_{LT}$ normalized by the asymmetry in the parton level turned out to be approximately five to ten times smaller than the corresponding $A_{TT}$, although the prediction on its absolute magnitude suffers from the uncertainty of the distributions, in particular, of $h_1$ as was the case for $A_{TT}$. The purely twist-3 contribution to $g_T$ and $h_L$ was modeled by the bag model, and it turned out its effect on $A_{LT}$ is negligible compared with the Wandzura-Wilczek contribution to $g_T$ and $h_L$.

REFERENCES

1. J.P. Ralston and D.E. Soper, Nucl. Phys. B152 (1979) 109; X. Artru and M. Mekhfi, Z. Phys. C45 (1990) 669; J.L. Cortes, B. Pire and J.P. Ralston, Z. Phys. C55 (1992) 409.
2. R.L. Jaffe and X. Ji, Nucl. Phys. B375 (1992) 527.
3. B. Kamal, Phys. Rev. D57 (1998) 6663; V. Barone, T. Calarco and A. Drago, Phys. Rev. D56 (1997) 527; T. Gehrmann, Nucl. Phys. B498 (1997) 245; O. Martin, A. Schäfer, M. Stratmann, and W. Vogelsang, Phys. Rev. D57 (1998) 3084; hep-ph/9902250.
4. Y. Kanazawa, Y. Koike and N. Nishiyama, Phys. Lett. B430 (1998) 195.
5. M. Glück, E. Reya and A. Vogt, Z. Phys. C67, (1995) 433.
6. M. Glück, E. Reya, M. Stratmann and W. Vogelsang, Phys. Rev. D53 (1996) 4775.
7. See, for example, Y. Koike and K. Tanaka, Phys. Rev. D51 (1995) 6125; J. Kodaira, Y. Yasui, K. Tanaka and T. Uematsu, Phys. Lett. B387 (1996) 855.
8. A. Ali, V.M. Braun and G. Hiller, Phys. Lett. B266 (1991) 117; I.I. Balitsky, V.M. Braun, Y. Koike and K. Tanaka, Phys. Rev. Lett. 77 (1996) 3078; See also Y. Koike and N. Nishiyama, Phys. Rev. D55 (1997) 3068.
9. Y. Kanazawa and Y. Koike, Phys. Lett. B403 (1997) 357; M. Stratmann, Z. Phys. C60 (1993) 763; X. Song, Phys. Rev. D54 (1996) 1955.
Figure 1. Double spin asymmetries, $\tilde{A}_{LL}$, $\tilde{A}_{TT}$, $\tilde{A}_{LT}$, for the polarized Drell-Yan using the GRSV parton distribution and the bag model at $Q = 8, 10$ GeV and $\sqrt{s} = 50, 200$ GeV. The solid line denotes $\tilde{A}_{LT}$ with only the Wandzura-Wilczek contributions in $g_T$ and $h_L$. The short dash-dot line denotes $\tilde{A}_{LT}$ with the bag scale $\mu_{bag}^2 = 0.25$ GeV$^2$, and the dotted line denotes $\tilde{A}_{LT}$ with the bag scale $\mu_{bag}^2 = 0.081$ GeV$^2$. The long dashed line corresponds to $\tilde{A}_{LL}$, and the long dash-dot line corresponds to $\tilde{A}_{TT}$. 