Why is there more matter than antimatter? 
Calculational methods for leptogenesis and electroweak baryogenesis

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Abstract

We review the production of the matter-antimatter asymmetry in the early Universe, that is baryogenesis, in out-of-equilibrium conditions induced by decays of heavy particles or the presence of phase boundaries. The most prominent examples are given by leptogenesis and electroweak baryogenesis, respectively, and for both cases, we derive the equations that govern the production of the asymmetries. We first use more intuitive arguments based on classical fluid equations in combination with quantum-field-theoretical effects of $CP$-violation. Then, in order to provide a more thorough approach that is well-suited for systematic improvements, we obtain the real-time evolution of the system of interest based on first principles, using the closed time-path approach. We thus aim to provide a simple and practicable scheme to set up phenomenological fluid equations as well as to provide a more thorough deduction of these. An important possibility for generating the $CP$-even phases that are necessary for baryogenesis in conjunction with odd phases is the coherent superposition of quantum states, i.e. mixing. These coherence effects are essential in resonant leptogenesis as well as in some scenarios of electroweak baryogenesis, and recent theoretical progress on asymmetries from out-of-equilibrium decays may also be applicable to baryogenesis at phase boundaries.
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1 Introduction

This article is mainly concerned with calculational methods for baryogenesis, i.e. the hypothetical process that produces the matter-antimatter asymmetry in the early Universe. Besides summarizing these methodical matters, we also provide some minimal context on the problem of the matter-antimatter asymmetry in this introduction. In addition, we refer to the reviews [1–3] that contain a general overview on baryogenesis.

1.1 The matter-antimatter puzzle

If matter and antimatter had, apart from charges that are precisely opposite, exactly the same properties, and we assumed the Universe to start in a big bang with symmetric initial conditions, there would be no way for Nature to develop a preference of one above the other. Nonetheless, the evidence for an asymmetry is overwhelming. No events where astrophysical objects composed of antimatter annihilate with matter objects have been observed [4]. And while it could yet be conceivable that individual stars or entire galaxies are composed of antimatter, such an hypothesis is untenable in view of the observationally established big bang scenario, according to which the baryonic components of the Universe have been in an almost homogeneous state at least from the time of big bang nucleosynthesis (BBN) to the appearance of nonlinear structures.

However, it is a generic prediction of quantum field theories (QFTs) that matter and antimatter can have different properties because certain discrete symmetries are not conserved (see Ref. [5] for a comprehensive discussion): Chiral gauge theories maximally violate parity \( P \) and charge \( C \) conjugation, and, crucially, mass terms or underlying Yukawa couplings can violate the combined charge-parity symmetry \( CP \). While these discrete symmetries are observed in quantum electrodynamics, the Standard Model (SM) of particle physics realizes these possibilities leading to their violation. The experiments that have discovered \( P \) and \( CP \) violation \([6, 7]\) are therefore among the most celebrated in physics and have been pivotal for the development of the SM. The responsible theory of electroweak symmetry breaking and the Cabibbo-Kobayashi-Maskawa (CKM) model of fermion masses have been continuously confirmed by present day experiments in conjunction with precision calculations.

Adding time-reversal \( T \), there is another important combination: \( CPT \). QFTs that are local and that observe relativistic causality predict the conservation of this symmetry. The SM is of course an example for such a theory and no violation of \( CPT \) has been observed to the present date. While ultimately, it could still turn out that the more underlying theory realized in Nature violates \( CPT \), we therefore apply the working hypothesis that this is either not the case or is not of relevance in the effective theory responsible for baryogenesis. Then, \( CPT \) conservation predicts that particles and antiparticles still have precisely opposite charges, precisely the same mass and even the same lifetime, i.e. the same inclusive decay rates. However, particular exclusive decay channels can have different decay rates provided they sum up to the same total, inclusive decay rate.

Considering thus a theory respecting \( CPT \), we further add the violation of baryon number \( B \) or, more appropriately in the context of the SM, some combination of \( B \) and lepton number \( L \). If we put particles of that theory in a box with perfect heat insulation and wait long enough, thermal equilibrium will be reached. This implies a vanishing baryon asymmetry because we assume that baryon number is violated and baryon and antibaryons have exactly the same mass. Equal numbers of particles and antiparticles must therefore be present in the equilibrium state of maximum entropy. The question of whether in the early Universe, a preexisting asymmetry can survive the washout via baryon number violating reactions and whether baryogenesis, i.e. to produce an asymmetry dynamically, is possible therefore depends on how far the particle contents of the expanding Universe deviate from thermal equilibrium.

By 1967, \( CP \) violation \([7]\) and the Cosmic Microwave Background (CMB) Radiation had been
Remarkably, the temperature of the latter turned out to be consistent with what had been predicted from BBN. This pioneering work had therefore established that reactions from particle and nuclear physics have taken place in the early Universe and that theory can make quantitative predictions about their turnout. The recent discoveries at that time may therefore have played a part in leading Sakharov in 1967 to propose that the matter-antimatter asymmetry is a question that can indeed be answered by QFT. More precisely, he has formulated the minimal requirements that successful baryogenesis poses on particle theory and its embedding in a cosmological context. These have subsequently been paraphrased in terms of the Sakharov conditions, i.e. necessary for baryogenesis are:

1. the presence of baryon-number violating interactions,
2. $C$ and $CP$ violation (the latter being necessary for asymmetries in left- and right-handed sectors not to cancel one another),
3. a deviation from thermal equilibrium.

The SM turns out to qualitatively meet Sakharov’s criteria:

1. Baryon-plus-lepton number $B + L$ is violated through the chiral anomaly and the pertaining weak sphaleron transitions at finite temperature.
2. $C$ is violated through the weak interactions and $CP$ through the CKM mechanism.
3. The expansion of the Universe leads to a deviation from thermal equilibrium and in particular, if the mass of the Higgs boson were below 70 GeV, there would be a first order phase transition, i.e. the coexistence of symmetric and broken electroweak phases in a certain temperature range. At the phase boundaries, which are the walls of broken phase bubbles expanding into the sea of symmetric phase, there would be a substantial deviation from equilibrium.

Curiously, it is only because of the values of its free parameters that the SM cannot solve the puzzle. Quantitatively, it falls short of explaining the asymmetry for the following reasons:

- The 125 GeV Higgs boson is too heavy in order to support a first order phase transition, such that an electroweak crossover with continuous evolution of the expectation value of the Higgs field has occurred instead. As a consequence, the plasma remains too close to thermal equilibrium because all of its degrees of freedom participate in gauge interactions, which are fast and therefore very effective in suppressing any deviation from equilibrium due to the expansion of the Universe.

- The first $CP$ violating and rephasing invariant quantity appears at eighth order in Yukawa couplings and involves second-generation couplings at fourth order, i.e.

$$\text{Im} \left[ \det[m_u^a, m_d^a] \right] \approx -2m_t^4m_b^2m_s^2,$$

what we have expressed in terms of the masses $m_q$ of the quarks of flavour $q$ and where

$$J = \text{Im}[V_{us}V_{cb}V_{ub}^*V_{cs}^*] \approx 3 \times 10^{-5}$$

is constructed from the elements $V_{ij}$ of the CKM matrix. At high temperatures, where non-perturbative effects that help to make $CP$ violation more accessible in the laboratory are absent, $CP$-violating effects are largely suppressed, which can be estimated through the ratio

$$J \frac{m_t^4m_b^2m_s^2}{T^{12}} \approx 3 \times 10^{-19} \text{ for } T = 100 \text{ GeV},$$

that appears too small in order to explain the observed value below.
1.2 Asymmetry observed

When it comes to the calculation of reaction networks in the early Universe, it is most convenient to use entropy-normalized number or charge densities, which are conserved unless substantial amounts of energy are injected into the approximately thermalized plasma, e.g. through the far-from-equilibrium decay of an abundant heavy species. We thus assume that entropy is conserved in a comoving volume elements, i.e. \( sa^3 = \text{const} \) where \( s \) is the entropy density and \( a \) the scale factor of the Friedmann-Robertson-Walker metric.

For this reason, the calculations for BBN are formulated in terms of entropy-normalized densities. The key cosmological parameter entering BBN is the baryon asymmetry, which controls the density of the nucleons and the light nuclei that are fused from these eventually. The currently reported best-fit value is\(^{23}\)

\[
5.8 \times 10^{-10} \leq \frac{n_B}{n_\gamma} \leq 6.6 \times 10^{-10}
\]  
(4)

at 95% confidence level, where instead of the entropy density the number density of CMB photons \( n_\gamma \) has been used for normalization. Assuming that there are three chiral relativistic neutrino species that have a temperature of \( T_\nu = (4/11)^{1/3} T_{\text{CMB}} \) (because photons are heated by electron-positron annihilation after neutrinos decouple), the entropy density today is \( s = (2\pi^2/45)(2T_{\text{CMB}}^3 + 7/8 T_\nu^3) \), where \( T_{\text{CMB}} = 2.725\text{K} \) is the present temperature of the CMB. Integrating the Bose distribution for two massless photon degrees of freedom yields \( n_\gamma = (2/\pi^2)\zeta(3)T_{\text{CMB}}^3 \), such that \( s/n_\gamma \approx 7.04 \), i.e. \( n_B/s = Y_B \approx n_B/(7.04 n_\gamma) \). The value\(^{1} \) can therefore be interpreted such that at temperatures above the phase transition of quantum chromodynamics (QCD), there has been roughly one extra quark per ten billion particle-antiparticle pairs.

An entirely complementary method of determining the BAU, which has become more precise thanks to the precision data from the WMAP\(^{24} \) and Planck\(^{25} \) probes, is to infer it from CMB anisotropies, in particular from the imprint of baryon acoustic oscillations. We quote the up-to-date value as \( \Omega_B h^2 = 0.0224 \pm 0.0001 \) with 68% confidence\(^ {25}\), where \( \Omega_B \) is the fraction of the baryon mass in terms of the critical energy density \( \rho_c \) (i.e. the energy density in a spatially flat Friedmann model) and \( H_0 = 100\text{kms}^{-1}\text{Mpc}^{-1}h \) is the Hubble rate in the present Universe. The baryon number density is then obtained dividing the baryon mass density by the mass of a nucleon \( m_{\text{nuc}} \) as \( n_B = \rho_c \Omega_B/m_{\text{nuc}} = [3/(8\pi m_{\text{nuc}})]H_0^2\rho_c \Omega_B h^2 \). Taking \( n_\gamma \) as given above then yields \( n_B/n_\gamma \approx 2.74 \times 10^{-8}\Omega_B h^2 \), such that the baryon-to-photon ratio inferred from the CMB is

\[
\frac{n_B}{n_\gamma} = 6.14 \pm 0.02 \times 10^{-10}.
\]

(5)

The agreement of these results is a key achievement of particle cosmology and impressively validates the approach of formulating and calculating networks of QFT reactions taking place during the early stages of the Universe.

1.3 Outline of this article

In the present work, we present basic aspects about calculations of the baryon asymmetry of the Universe. To illustrate the methods on rather paradigmatic examples, we choose leptogenesis for out-of-equilibrium decays and electroweak baryogenesis for \( CP \) violation on phase boundaries.

The generation of the baryon asymmetry is typically described through fluid equations. These can be derived in classical mechanics using conservation laws, i.e. energy, momentum and current conservation. Microscopically, they can also be derived from Boltzmann equations. In the given context of particle physics, reaction rates then have to be added from scattering theory, assuming that individual scattering
events are well separated in spacetime, which is not always justifiable. One may therefore prefer to combine the derivation of the reaction rates with the kinematic evolution of the system by evolving in in real time based on QFT. An approach that is based on first principles that proves efficient in specifying initial conditions, evolving the system and evaluating observables is given by the closed-time path (CTP) formalism \[26–28\]. We present and compare these different approaches in Section 2.

In Section 3, we discuss aspects of the calculation of $CP$-violating decay rates of heavy particles. In view of comparing with reaction rates obtained in the CTP framework, we explicitly evaluate these rates in Section 3.1 using the optical theorem. We also discuss how the decay asymmetries for Dirac fermions can be reduced to results for Majorana fermions, thus covering and relating both of these scenarios for out-of-equilibrium decays. In Section 3.2 we review different approaches to computing the decay rates of mixing right-handed neutrinos (RHNs), the differences of which become relevant for very mass-degenerate mixing systems.

The results for the decay asymmetry are then used in Section 4 in order to derive fluid equations governing leptogenesis in the early Universe. Important technical points are the subtraction of real intermediate states (RIS) that is necessary in order to unitarize the system such that no asymmetry can be generated or persist in equilibrium. This procedure is owed to the fact that scattering processes with intermediate RHNs are not clearly distinguishable from decay and inverse decay processes. Also the expansion of the Universe is accounted for. Individual rates appear in the fluid equations when integrated over phase space. These are closely related to expressions that we encounter in calculations using the CTP approach and are therefore presented for comparison in integral form.

In Section 5 we then derive the fluid equations from first principles of QFT using the CTP formalism. The results in integral form will be compared with those from Section 4.4. This should give some concrete insights into the relation between standard methods using Boltzmann equations with scattering rates and the CTP approach. Notably, in the CTP approach no subtraction of RIS is necessary because the correct counting is implemented by construction.

To give an example for the benefit of using the CTP method, we discuss leptogenesis in the resonant regime in Section 6 with a particular focus on the extremely mass-degenerate case where the masses of the RHNs are only separated within their decay width. It turns out that the off-diagonal correlations of the RHNs, that the decay asymmetry depends on, can be computed by solving the Schwinger-Dyson equations on the CTP, what effectively amounts to the correct resummation of the one-loop absorptive self-energy corrections. This method therefore resolves the questions brought up in Section 3.2.

In Section 7 we then briefly review how to solve the fluid equations for leptogenesis in the strong washout regime and arrive at an analytic approximation in a form that can be compared with the observed value \[5\] for the asymmetry.

Similar to resonant leptogenesis, mixing and oscillating systems may play a role in electroweak baryogenesis. In Section 8 we review the computation of $CP$-violating source terms using the CTP approach and gradient expansion. As an introduction, we first discuss force terms in classical kinetic equations. Applying the CTP approach to a fermionic system, we next discuss the semiclassical force that is present independent of mixing and then identify a resonantly enhanced force term for mixing systems. We eventually give some directions how to complete the calculation in order to arrive at a prediction for the asymmetry that is to be compared with the observed value \[5\].

This outline also sets the scope of the present article which are calculational methods for baryogenesis. Given the large number of possible scenarios for baryogenesis and variants thereof a comprehensive phenomenological survey of the field would require a far more extensive treatment as it is offered by this work. We note nonetheless that recently, a series of review articles on leptogenesis has appeared that provides some comprehensive, yet detailed, review of the state of the art \[29–34\]. The present article presents basic elements needed in order to set up some minimal fluid equations for baryogenesis, and it additionally justifies these equations in a more detailed way based on the CTP methods. It is therefore aimed to be complementary to the more detailed but but also more specific and less general
calculations presented in the typical research literature.

We further remark that while discussing leptogenesis in the strong washout regime and electroweak baryogenesis, we do not cover the many other aspects where CTP or finite-temperature techniques are relevant for baryogenesis calculations. Among these are leptogenesis involving relativistic RHNs \cite{30,35,40}, flavoured leptogenesis \cite{41,46} or thermal and other radiative corrections to leptogenesis in general \cite{47,53,53}. While noting that many of these works use methods from equilibrium field theory or the time evolution in the canonically quantized formalism, we emphasize that the CTP approach is of particular appeal because it uses the exact time evolution of the system as derived from first principles of QFT as a platform and nonetheless allows for a very efficient representation in terms of Feynman diagrams. This time evolution then has to be approximated systematically, which to some large extent is the work lying ahead.

2 Theoretical foundation of kinetic and fluid equations for baryogenesis

Predictions for the BAU are usually obtained from a solution to fluid equations describing the processes that generate the asymmetry in the early Universe. These are cast as ordinary, linear differential equations that can easily be solved numerically or even using analytic approximations (cf. Section 7 for leptogenesis in the strong washout regime). To some large extent, the fluid equations can be formulated in terms of averaged interaction rates for charge and number densities. Balancing these rates for the creation and annihilation of particles is a very efficient way of setting these up these equations, cf. the discussion in Section 4.1 on leptogenesis. This procedure can be justified on a more basic level from the Boltzmann kinetic equations, i.e. from classical statistical mechanics, combined with $S$-matrix elements from QFT. Alternatively, we can derive fluid equations based on QFT altogether, where the closed time-path approach turns out to be very suitable and powerful. In this section, we give an overview about both methods.

2.1 Boltzmann equations from classical statistical mechanics

**Kinetic equations** In many cases, reaction networks can be effectively described in terms of Boltzmann equations for distribution functions $f(x, p, t)$ of particles with mass $m$, i.e. in the form

$$\frac{df}{d\tau} = \gamma(v) \frac{df}{dt} = \gamma(v) \left( \frac{\partial f}{\partial t} + \frac{\partial f}{\partial x} \cdot v + \frac{\partial f}{\partial p} \cdot \frac{dp}{dt} \right) = C,$$

where $\tau$ is the proper time of the particles with momentum $p$, partial derivatives with respect to $x$ and $p$ are understood to be the spatial and momentum gradients, $v = d\mathbf{x}/dt = p/\gamma(v)m$ and $\gamma(v)$ is the Lorentz factor. The left hand side of this equation is referred to as the convection term. In the convection term, we can also identify a term that is relevant in presence of a force field $dp/d\tau = F$. On the right-hand side, there is the collision term $C$.

When we let $X$ label the particle that the distribution $f(p_X)$ refers to, the collision term takes the form

$$C = \frac{1}{2p^0} \int \prod \frac{d^3p_i}{(2\pi)^32p_i^0}(2\pi)^4 \delta^4(p + pA1 + \cdots - pB1 - \cdots) \times \{(1 \pm f)(1 \pm fA1) \cdots fB1 \cdots |M_{B1B2\cdots \rightarrow XA1A2\cdots}|^2$$

$$- ffA1 \cdots (1 \pm fB1) \cdots |M_{XA1A2\cdots \rightarrow B1B2\cdots}|^2\}$$
where $p^0 = \sqrt{\mathbf{p}^2 + m^2}$, $p^0_i = \sqrt{\mathbf{p}_i^2 + m_i^2}$, the index $i$ runs through the particles labeled by $B_1, B_2, \ldots$ and $A_1, A_2, \ldots$, and $\mathcal{M}$ is the invariant matrix element of the reaction indicated in the subscript. We have included here the Bose enhancement and Pauli suppression terms $\pm f_X$ such that this collision term also account for quantum statistics.

A key assumption here is that the range of the force between two particles is much shorter than their average distance. Famously, this does not apply to the Coulomb force, which is a long-range interaction, leading to divergences in the collision integral. In classical electrodynamics, long range interactions can be included through Vlasov equations from which one can obtain important plasma phenomena such as Landau damping and Debye screening. Another approximation made is that we assume that the system can be described by a set of distribution functions for the single particle species rather than one joint phase-space distribution function accounting for the trajectory of each single particle, i.e. we truncate the Bogoliubov-Born-Green-Kirkwood-Yvon (BBGKY) hierarchy at its lowest order.

**Fluid equations** In cosmological calculations one is mostly interested in the evolution of charge and number densities that are conserved under the long-range gauge interactions (e.g. baryon or lepton number or the number of dark matter particles) such that the pertaining issues not necessarily need to be dealt with. The role of the gauge interactions is then mainly to maintain kinetic equlibrium, i.e. to establish the Fermi-Dirac or Bose-Einstein distributions

$$f(p, x) = \frac{1}{e^{\frac{p_\nu - \mu}{T} \mp 1}}. \quad (8)$$

Rather than on the distribution function accounting for an infinite number of degrees of freedom at each spacetime point, the problem now depends on the temperature field $T(x)$, the field of bulk velocities $u^{\nu}_b(x)$ and the field of chemical potentials $\mu(x)$. These fields can be extracted from the distributions that appear in the kinetic equations by taking moments

$$\varrho(x) = \int \frac{d^3p}{(2\pi)^3} f(p, x), \quad (9a)$$

$$n^\mu(x) = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{m} f(p, x), \quad (9b)$$

$$\ldots,$$

where the dots indicate that these generalize to a tower of quantities. In many cases – as in the discussions of the present article – it is sufficient to consider only the number density $\varrho$ or in addition the fluid-density current $n^\mu$. A charge density $q$ and a current are obtained from taking the difference of the currents for particles and antiparticles. In kinetic equilibrium, the chemical potentials are opposite such that e.g.

$$q = \varrho - (\varrho|_{\mu \to -\mu}) = \left\{ \begin{array}{cl} \frac{T^2}{6} \mu + \cdots & \text{(massless chiral fermions)} \\ \frac{T^2}{3} \mu + \cdots & \text{(massless bosons)} \end{array} \right\}, \quad (10)$$

where it has been assumed that $\mu/T \ll 1$ and the dots indicate higher order terms in this parameter.

It is often also the case that bulk velocity can be non-relativistically approximated as $u^{\nu}_b = (1, v_b)$, i.e. $|v_b| \ll 1$, such that we can extract it through

$$\int \frac{d^3p}{(2\pi)^3} \mathbf{D} f(p) = \left\{ \begin{array}{cl} \frac{7}{90} \pi^2 v_b T^4 + \cdots & \text{(massless chiral fermions)} \\ \frac{4}{45} v_b T^4 + \cdots & \text{(massless bosons)} \end{array} \right\}. \quad (11)$$
In terms of these variables, we may therefore obtain from the Boltzmann equations a simplified network of fluid equations. A very typical form of these is (cf. Section 8.4)

\begin{align}
\dot{q} + \nabla \cdot j &= -\Gamma q, \\
j &= v_b q = -D\nabla q,
\end{align}

where \( j \) is the current density of the charge density \( q \). The coefficients \( \Gamma \), which may be referred to as decay rates, and \( D \), the diffusion constants, have to be extracted by taking the corresponding moments of the collision terms in the Boltzmann equations. While the fluid Eqs. (12) form a closed network of four equations for the four variables which are \( q \) and the components of \( v_b \), this closure is typically an idealization and relies on the approximate validity of discarding terms involving higher moments of the distribution functions.

Fluid equations in the form of Eqs. (12) are a typical starting point for setting up calculations of particle reactions in the early Universe. In the absence of quantum coherence, \( q \) and \( j \) are promoted to carry a species index and \( \Gamma \) and \( D \) will then in general be matrix valued. It is also possible to account for quantum coherence when promoting \( q \) and \( j \) to matrices in the space of particle species. In that case, additional commutator and anticommutator structures appear in the equations that will play an important part in the remainder of this review, cf. Eqs (115) and (159).

### 2.2 First principles of QFT: closed time-path approach

As an alternative to introducing distribution functions within classical kinetic theory in order to obtain results for e.g. current densities and their time evolution, we may also note that current densities can be defined as expectation values of two-point functions, i.e.

\[ j^\mu(x) = \langle [i\partial^\mu\phi^*(x)]\phi(x) - \phi^*(x)[i\partial^\mu\phi(x)] \rangle, \quad j^\mu(x) = \langle \bar{\psi}(x)\gamma^\mu\psi(x) \rangle = \text{tr}(\psi(x)\gamma^\mu\bar{\psi}(x)) \]

for scalar and fermion fields, respectively. For fermion fields, one may in addition compute e.g. an axial current or currents associated with states of definite helicity or spin, as we will get back to in more detail in Section 8 [cf. Eqs. (146) and (147)].

Given a certain state (that in general can be a statistically mixed quantum state), we aim to evaluate expectation values of the above type. In equilibrium field theory, the state is given by a canonical or grand canonical ensemble and is time-translation invariant. In general however, the system evolves with time, and we specify the state through some initial condition at one time and evaluate the expectation values at another. We therefore need the time evolution, and a particularly powerful method of computing it results from combining the Schwinger-Keldysh CTP method \cite{26, 27} with Schwinger-Dyson equations that are derived from a two-particle irreducible (2PI) effective action \cite{28, 54, 56}.

In order to discuss this approach, we begin with a real scalar field \( \phi \) for simplicity. For this, we...
define the in-in generating functional:

\[ Z[J_+, J_-] = \int \mathcal{D}\phi(\tau) \mathcal{D}\phi^- \mathcal{D}\phi^+ \langle \phi^- | \phi(\tau) \rangle \langle \phi(\tau) | \phi^+ \rangle \langle \phi^- | \varrho | \phi^+ \rangle \] (14)

\[ = \int \mathcal{D}\phi^- \mathcal{D}\phi^+ e^{i \int d^4 x (\mathcal{L}(\phi^+) - \mathcal{L}(\phi^-) + J_+ \phi^+ - J_- \phi^-) \langle \phi^- | \varrho | \phi^+ \rangle}. \]

In the first step (cf. Figure 1), we integrate over a set of in states \( |\phi^\pm_{\text{in}}\rangle \) that can be weighted statistically by a density matrix \( \varrho \). Then, we evolve \( |\phi^+_\text{in}\rangle \) specified at the time \( \tau_0 \) to some finite time \( \tau \), where we insert a complete set of states \( |\phi(\tau)\rangle \langle \phi(\tau)| \), and eventually we evolve back to \( \langle \phi^-_{\text{in}} | \varrho | \phi^+_{\text{in}} \rangle \). For both steps of this time evolution, we introduce Lagrangian terms \( J_\pm(x) \phi^\pm(x) \) for variational purposes. When writing down the corresponding path-integral representation in the second step, we note that this procedure generates two branches of integration that we denote by \(+\) and \(-\). The factor of minus one from the backward time integration along the \(-\) branch is then attributed to the integrand, such that both branches appear as integrands in an integral over \( d^4x \) where each of the Lagrangian terms pertaining to the minus branch attains a factor of minus one.

Taking variational derivatives, we arrive at path-ordered Green functions:

\[ \begin{align*}
\iota\Delta^{ab}(x, y) &= -\frac{\delta^2}{\delta J_a(x) \delta J_b(y)} \log Z[J_+, J_-] \Big|_{J_\pm=0} = \iota \langle C[\phi^a(x) \phi^b(y)] \rangle,
\end{align*} \]

(15)

where the subscript \( C \) indicates that operators are arranged from the right to the left according to the sequence in which their time arguments appear on the closed time-path shown in Figure 1, and higher correlation functions are obtained in an analogous way. Observables such as the currents \( (13) \) can then be calculated straightforwardly from these Green functions.

From the path integral representation in Eq. (14), we infer that the standard Feynman rules hold, with the generalization that there are \(+\) and \(-\) vertices. These vertices are connected through the Green functions \( \iota\Delta^{ab} \) with the matching CTP superscripts, and vertices of the \(-\) type receive an extra factor of minus one because of the negative sign in front of the \(-\) Lagrangian in the exponent in Eq. (14).

It is further customary to introduce specific superscripts for the various types of two-point functions on the CTP that are more or less connected to their mathematical and physical significance. For a two-point function \( G^{ab} \), we write

\[ G^>(x, y) = G^{--}(x, y), \quad G^<(x, y) = G^{+-}(x, y), \quad G^T(x, y) = G^{++}(x, y), \quad G^\bar{T}(x, y) = G^{--}(x, y), \]

(16)

where \( G^T \) (\( G^\bar{T} \)) is the time (anti-time) ordered two-point function and \( G^{<>} \) are Wightman functions. Of practical importance are the causal, retarded and advanced two-point functions

\[ G^a = G^T - G^c = G^> - G^\bar{T}, \quad G^a = G^T - G^\bar{a} = G^< - G^\bar{T} \]

(17)

as well as the spectral function

\[ G^A = \frac{1}{2} (G^> - G^<) = \frac{1}{2i} (G^a - G^r) \]

(18)

and the Hermitian function

\[ G^H = \frac{1}{2} (G^a + G^r). \]

(19)

These relations also imply the identity

\[ G^T + G^\bar{T} = G^> + G^<. \]

(20)
To progress further towards models of phenomenological interest, we now proceed with complex scalar fields $\phi$ and four-component spinors $\psi$, i.e.

$$i\Delta(x,y) = \langle C[\phi(x)\phi^*(y)] \rangle, \quad iS(x,y) = \langle C[\psi(x)\bar{\psi}(y)] \rangle. \quad (21)$$

Note that the spinor indices are not contracted here, i.e. that $S$ is endowed with $4 \times 4$ structure in spinor space. We have suppressed here the CTP indices as well as possible flavour indices.

Throughout this article, we are concerned with complex fermion masses or mass matrices, that are in general Nonhermitian. For a complex mass, we use the notation of a lower case $m$ that is decomposed into its real and imaginary part as

$$m^R = \text{Re}[m], \quad m^I = \text{Im}[m]. \quad (22)$$

We may also add subscripts indexing the particle species to $m$ and $m^R, I$. It is further useful to define the term pertaining to this mass that appears in spinor products,

$$\hat{m} = m^R + i\gamma^5 m^I, \quad \hat{m}^* = m^R - i\gamma^5 m^I. \quad (23)$$

When dealing with mass matrices, we use capital letters (aside from the exception of masses for RHNs, where customarily capital letters are used for the mass terms). A matrix $M$ of general form can be decomposed into Hermitian and Antihermitian parts as

$$M^H = \frac{1}{2} (M + M^\dagger), \quad M^A = \frac{1}{2i} (M - M^\dagger), \quad (24)$$

which appear in spinor products as

$$\hat{M} = M^H + i\gamma^5 M^A. \quad (25)$$

General expressions for mass matrices can be reduced to the case of a single fermion species by taking $M^H \rightarrow m^R$ and $M^A \rightarrow m^I$.

In order to derive kinetic equations, it turns out useful to note the Hermiticity properties of the Wightman functions

$$[i\Delta^{<,>}(x,y)]^\dagger = i\Delta(y,x), \quad (26a)$$

$$[i\gamma^0 S^{<,>}(x,y)]^\dagger = i\gamma^0 S(y,x). \quad (26b)$$

Here, we have explicitly replaced the spinor matrix $A$ defined in Eq. (A1) with $\gamma^0$, as it is appropriate for all common representations of Dirac matrices such as the Weyl and the Dirac representations.

Green functions on the CTP can be expanded using straightforward perturbation theory. However, a more powerful computation method arises from using Schwinger-Dyson equations and deriving the latter from the two-particle-irreducible (2PI) effective action. The main advantage is that the Schwinger-Dyson equations readily express the time evolution of the system in an integro-differential form similar to kinetic equations. The 2PI effective action can be expressed as

$$\Gamma[\Delta, S] = B + i\text{tr}[\Delta^{(0)-1}\Delta] - i\text{tr}[S^{(0)-1}S] + i\text{tr}\log\Delta^{-1} - i\text{tr}\log S^{-1} + \Gamma_2[\Delta, S] \quad (27)$$

where $B$ is the classical action, $\Delta^{(0)-1}$ the Klein-Gordon, $S^{(0)-1}$ the Dirac operator,

$$\Gamma_2[\Delta, S] \equiv -i \times \text{the sum of 2PI vacuum graphs}, \quad (28)$$

and the relative signs between the terms concerning fermions and bosons arise from the contributions of the quadratic fluctuations about the classical field configuration in the path integral.
Defining the self energies

\[
\Pi^{ab}(x, y) = i a b \frac{\delta \Gamma_2[\Delta, S]}{\delta \Delta^{ba}(y, x)},
\]

(29a)

\[
\Sigma^{ab}(x, y) = -i a b \frac{\delta \Gamma_2[\Delta, S]}{\delta S^{ba}(y, x)}.
\]

(29b)

and taking functional derivatives, we obtain

\[
\frac{\delta \Gamma[\Delta, S]}{\delta \Delta(y, x)} = 0 \iff i \Delta^{(0)-1}(x, y) - i \Delta^{-1}(x, y) - i \Pi(x, y) = 0,
\]

(30a)

\[
\frac{\delta \Gamma[\Delta, S]}{\delta S(y, x)} = 0 \iff -i S^{(0)-1}(x, y) + i S^{-1}(x, y) + i \Sigma(x, y) = 0.
\]

(30b)

The convolution from the right with the full propagators yields the Schwinger-Dyson equations on the CTP

\[
[-\partial^2 - M^2] i \Delta^{ab}(x, y) = a \delta_{ab} i \delta^4(x - y) + \sum_c c \int d^4 z \Pi^{ac}(x, z) i \Delta^{cb}(z, y),
\]

(31a)

\[
[i \phi - M^H - i \gamma^5 M^A] i S^{ab}(x, y) = a \delta_{ab} i \delta^4(x - y) + \sum_c c \int d^4 z \Sigma^{ac}(x, z) i S^{cb}(z, y).
\]

(31b)

To this end, we have presented the expressions for scalar fields and Dirac fermions, in order to highlight agreements and differences in their treatment. As the examples in this article are mainly concerned with the evolution of fermions, we proceed with these and leave the scalar particles aside for now. The theory for the latter, along with fermions, is developed in detail in Refs. [54, 55].

Since for the Green functions on the CTP, there are two linearly independent combinations, the same holds true for the Schwinger-Dyson-equations as well. It proves useful to set up equations for the retarded or advanced Green functions

\[
[i \phi - M^H - i \gamma^5 M^A] S^{r,a}(x, y) - \int d^4 z \Sigma^{r,a}(x, z) S^{r,a}(z, y) = \delta^4(x - y),
\]

(32)

and for the Wightman functions

\[
[i \phi - M^H - i \gamma^5 M^A] S^{<,>}(x, y) - \int d^4 z \Sigma^{<,>}(x, z) S^{<,>}(z, y) + \Sigma^{<,>}(x, z) S^H(z, y) = \frac{1}{2} \int d^4 z \left[ \Sigma^{<,>}(x, z) S^{<}(z, x) - \Sigma^{<}(x, z) S^{<,>}(z, x) \right],
\]

(33)

where this latter equation is called the Kadanoff-Baym equation.

To proceed with deriving kinetic equations that are suitable for a reduction to the Boltzmann or fluid form, we perform a Wigner transformation of the two-point functions,

\[
G(k, x) = \int d^4 r e^{i k \cdot r} G \left( x + \frac{r}{2}, x - \frac{r}{2} \right),
\]

(34)

where \( A \) stands here either for a propagator or a self energy. When applying this transformation to the Schwinger-Dyson equations, we encounter convolutions that transform into \([57, 59]\)

\[
\int d^4 r e^{i k \cdot r} \int d^4 z G \left( x + \frac{r}{2}, z \right) F \left( z, x - \frac{r}{2} \right) = e^{-i \omega} \{ G(k, x) \} \{ F(k, x) \},
\]

(35)
where

\[
\frac{1}{2} \left( \frac{\partial G(k, x)}{\partial x^\mu} \frac{\partial F(k, x)}{\partial k_\mu} - \frac{\partial G(k, x)}{\partial k^\mu} \frac{\partial F(k, x)}{\partial x_\mu} \right).
\]

(36)

In Wigner space, the Kadanoff-Baym equation thus reads,

\[
\left[ \hat{k} + \frac{i}{2} \slashed{\partial} - M^H e^{-i \frac{1}{2} \gamma^5 \slashed{\partial} k} - i \gamma^5 M^A e^{-i \frac{1}{2} \gamma^5 \slashed{\partial} k} \right] S^{<,>} - e^{-i \sigma^i} \{ \Sigma^{<,>} \} \{ S^{<,>} \} = \frac{1}{2} e^{-i \sigma^i} \{ \Sigma^{<,>} \} \{ S^{<,>} \} ,
\]

(37)

and, similarly, the retarded and advanced propagators obey

\[
\left[ \hat{k} + \frac{i}{2} \slashed{\partial} - M^H e^{-i \frac{1}{2} \gamma^5 \slashed{\partial} k} - i \gamma^5 M^A e^{-i \frac{1}{2} \gamma^5 \slashed{\partial} k} \right] S^{r,a} - e^{-i \sigma^i} \{ \Sigma^{r,a} \} \{ S^{r,a} \} = 1 ,
\]

(38)

where we recall that we have suppressed the spinor and flavour indices, such that the right-hand side is to be understood as a an identity operator in tensor space. The arrows over the partial derivatives are indicating onto which side these are acting, and the subscript \( k \) indicates a partial derivative with respect to the four-momentum \( k^\mu \).

It turns out that the right-hand side of the Kadanoff-Baym equation (37) can be physically interpreted as the QFT analogue of the collision term in the classical Boltzmann equations. It should therefore vanish in thermal equilibrium which can be readily seen from the Kubo-Martin-Schwinger (KMS) relation [60, 61]: Any two-point function \( G \) (that here may be a propagator or a self energy) in equilibrium at a temperature \( T \) satisfies

\[
G^{>}(k, x) = \pm e^{k^0/T} G^{<}(k, x) ,
\]

(39)

where the plus sign holds for bosons, the minus sign for fermions. Since the collision term vanishes in equilibrium, this automatically implies that no matter-antimatter asymmetry can then be generated in compliance with Sakharov’s conditions.

The Kadanoff-Baym equations can the be decomposed into kinetic and constraint equations, where the former can be further reduced to Boltzmann and then to fluid equations, while the latter yield the necessary input on the spectral properties. These matters are best illustrated on concrete examples, as we present in Section 5 for leptogenesis in the strong washout regime and in Section 8 for baryogenesis at phase boundaries with the most important example of electroweak baryogenesis. We therefore outline the remaining steps in formulating kinetic equations by pointing to the relevant results on the subsequent sections.

At tree level, when neglecting self energies, the solutions to Eqs. (37) and (38) are shown in Appendix B. While depending on the particular problem, these solutions may or may not be a suitable starting point for a perturbative expansion, their form is useful in order to understand the physical meaning of the particular terms that appear in the Schwinger-Dyson equations on the CTP. To see already to this end how to proceed toward fluid equations, we note that the Kadanoff-Baym equations can be decomposed in kinetic and so-called constraint equations. Kinetic equations for the Wightman functions of nonrelativistic fermions are given by Eq. (95a) which are then reduced to kinetic equations (97a) for the distribution functions. The general (fully relativistic) theory for fermionic Wightman functions turns out to be more involved and is discussed for spatially varying mixing mass matrices in Section 8 on electroweak baryogenesis.
2.3 Comparing remarks concerning Boltzmann and the closed time-path methods

When substituting $S$-matrix elements into the Boltzmann equations, we implicitly make use of the full machinery of scattering theory: amplitudes are computed in time-ordered perturbation theory and related to matrix elements using the Lehmann-Symanzik-Zimmermann (LSZ) reduction formula. This constitutes a conceptual detour because for baryogenesis, we are interested in the time-evolution of observables such as current densities that can be expressed as expectation values of operators. Moreover, in a finite density system, it is often not possible to identify free asymptotic states as scattering theory requires. As we discuss in this article, in the context of baryogenesis, this leads to problems concerning the counting of real intermediate states or the treatment of quasi-degenerate mixing states.

The method of directly computing the time evolution of the correlation functions of interest, as it is done in the CTP approach, is therefore more direct. The evolution of the system is expressed entirely in terms of correlation functions. These can also be used to specify initial conditions as well as to derive the observables of interest. In this functional approach, there is no reference to operators nor do we have to rely on the computation of amplitudes that need to be related to observables via the LSZ machinery. The price to pay for this simplification is to give up Lorentz symmetry because it is explicitly broken by the finite density background that implies a preferred plasma frame, such that we have to give up the simple form in which the Green functions behave under Wick rotations that is enjoyed in vacuum field theory. A combination of both methods (and in addition with methods of equilibrium field theory) may therefore often prove as the most efficient means of achieving the particular calculational goal.

3 $CP$ violation

The violation of $CP$ symmetry is a hallmark of quantum physics that requires the interference of amplitudes with $CP$-even and $CP$-odd phases. For mesons of the SM, where $CP$-violation has actually been observed, obtaining the $CP$-even phases requires experimental or numerical input. In contrast, in many scenarios of baryogenesis, the $CP$-even phase emerges in a comparably simple manner and can be computed in terms of on-shell cuts of loop amplitudes. We review the basics of such calculations as well as the interplay with the $CP$-odd phases in Section 3.1. We first discuss the calculation of the decay asymmetry in a model with heavy Dirac fermions and then show how this can be reduced to the case of Majorana neutrinos that is relevant for leptogenesis $[62]$. It should therefore be clear that baryogenesis from out-of-equilibrium decays does not necessarily rely on Majorana fermions. For the calculation of the asymmetry produced in the early Universe, we nonetheless focus on leptogenesis as the most plausible scenario.

For the most time since its inception, there has been some debate on wave-function or mixing contributions to the $CP$ asymmetry in leptogenesis, a point that is most relevant in the resonant regime, where there is a pronounced mass degeneracy for the RHN states. We review some approaches concerning this matter in Section 3.2 while in Section 6 we show that the decay asymmetry in the limit of strong mass degeneracy can strongly depend on the dynamical circumstances, i.e. on the oscillation time and washout strength of the system of mixing RHNs in the early Universe.

3.1 Decay asymmetries, odd and even phases under $CP$ conjugation

Some salient features of $CP$-violating decays can be explained on a model given by the Lagrangian

$$\begin{align*}
\mathcal{L} &= \bar{F} \left( i\bar{\phi} - m_F^R - i\gamma^5 m_F^L \right) F + \bar{G} \left( i\bar{\phi} - m_G^R - i\gamma^5 m_G^L \right) G + \phi^* \partial^2 \phi + \bar{f}i\bar{\phi}f \\
&- \sum_{X = F, G} \left( y_X^* \bar{f} \phi P_R X + \bar{y}_X \bar{f} \phi P_L X + \text{h.c.} \right),
\end{align*}$$

(40)
where $F, G$ are Dirac spinors, $f$ a left-chiral fermion, $\phi$ a massless complex scalar field and h.c. denotes Hermitian conjugation. The left and right chiral projectors are given by

$$P_{L,R} = \frac{1 \mp \gamma^5}{2}.$$  

(41)

We have written the Yukawa interactions in a form such that it is manifest that $F$ and $G$ can decay into both, $f$ and $f^C$ but note that alternatively, we could express the Yukawa interaction terms as

$$\overline{f^C} \phi^* P_L X = \overline{X^C} \phi^* P_L f.$$  

(42)

From the transformation properties stated in Appendix A, in particular Eqs. (A6a) and (A6b), we see that $CP$ conjugation takes the effect $m_X \rightarrow m_X^*$, $y_X \rightarrow y_X^*$. We therefore refer to the arguments of these complex parameters as $CP$-odd phases. In general, there are also extra arbitrary phases (i.e. $\alpha^{CP}$ in Appendix A) from the definition of the action of $CP$ conjugation on the fields, that we choose to zero for convenience in the present discussion. However, due to the freedom of field redefinitions, individual terms in the Lagrangian that violate $CP$ do in general (and typically) not lead to $CP$ violation. Rather, physical $CP$ indiscretion relies on rephasing invariants, i.e. combinations that are $CP$ odd and that are invariant under field redefinitions by complex phases. We will shortly come back to this point on the present example.

For definiteness, we focus on the decays of the massive fermion $F$. Working in the rest frame of the decaying particle $F$, where $p^\mu = (|m_F|, 0)$, up to one loop order, the decay rate into fermions and antiscalars can be written as

$$\Gamma_{F \rightarrow f\phi^*} = \frac{1}{2|m_F|} \int \frac{d^3q}{(2\pi)^3 2|q|} \frac{d^3q'}{(2\pi)^3 2|q'|} (2\pi)^4 \delta^4(p - q - q') \sum_{\text{pol}} |i\mathcal{M}_{F \rightarrow f\phi^*}^{LO} + i\mathcal{M}_{F \rightarrow f\phi^*}^{\text{vert}} + i\mathcal{M}_{F \rightarrow f\phi^*}^{\text{wv}}|^2,$$

(43)

where $q$ and $q'$ are the momenta of $f$ and $\phi^*$, and the $CP$-conjugate rate can be obtained accordingly. Here $i\mathcal{M}$ are the invariant matrix elements, LO indicates the tree level, leading order contribution and vert and wv the one-loop vertex and wave-function type corrections. Under the sum sign, “pol” indicates the polarization sum over the particles $f$ and the bar above the average over an unpolarized sample of $F$. The one-loop terms decompose into absorptive and dispersive contributions,

$$i\mathcal{M}_{F \rightarrow f\phi^*}^{\text{vert}} + i\mathcal{M}_{F \rightarrow f\phi^*}^{\text{wv}} = i\mathcal{M}_{F \rightarrow f\phi^*}^{\text{abs}} + i\mathcal{M}_{F \rightarrow f\phi^*}^{\text{dis}},$$

(44)

and correspondingly for the additional decays implied by the Lagrangian (40). The absorptive contributions can be isolated by the use of the optical theorem, or, equivalently by extracting the discontinuous part of the loop integrals, which gives rise to an extra $CP$-even phase of $i = e^{i\pi/2}$. (In general, it is not necessary to attribute the $CP$ even phase to purely absorptive contributions to the amplitude, i.e. we could add absorptive and dispersive contributions at one loop together such that the phase in general is different from $\pi/2$.) For the $CP$-conjugate rates, this implies the respective relations

$$i\mathcal{M}_{F \rightarrow f\phi^*}^{LO} = (i\mathcal{M}_{F^{CP} \rightarrow f^{CP}\phi}^{LO})^*,$$

(45a)

$$i\mathcal{M}_{F \rightarrow f\phi^*}^{\text{abs}} = - (i\mathcal{M}_{F^{CP} \rightarrow f^{CP}\phi}^{\text{abs}})^*,$$

(45b)

$$i\mathcal{M}_{F \rightarrow f\phi^*}^{\text{dis}} = (i\mathcal{M}_{F^{CP} \rightarrow f^{CP}\phi}^{\text{dis}})^*.$$  

(45c)

We note that if we were not setting the explicit phases in the definition of $CP$ conjugation to zero, there would be additional overall phases multiplying each of the three amplitudes on the right-hand side. These are spurious phases that turn out to be immaterial when it comes to physical observables, i.e. the decay rate.
In our setup, we can therefore explicitly attribute the complex conjugation in Eq. (45b) to a $CP$-odd, the factor of minus one to the square of a $CP$-even phase. As a consequence,

$$|iM_{F \to f\phi^*}^{\text{LO}} + iM_{F \to f\phi^*}^{\text{abs}}|^2 \neq |iM_{F \to f\phi^*}^{\text{CP}} + iM_{F \to f\phi^*}^{\text{abs}}|^2,$$

and therefore the the decay rates of particles and antiparticles as per Eq. (43) are different.

Crucially, the difference in the rates is due to the interference of amplitudes that have both, different $CP$-odd and $CP$-even phases. The latter arise due to intermediate on-shell states that may appear in the amplitude. Since physical $CP$ violation thus relies on interference it is a hallmark phenomenon of quantum physics.

In view of the comparison with the results derived in the CTP framework, it is interesting interesting to explicitly quote those crucial interference terms that [62–65] are proportional to both, the $CP$-even and $CP$-odd factors.

The leading contribution to the decay rate [43] is proportional to

$$\sum_{\text{pol}} |iM_{F \to f\phi^*}^{\text{LO}}|^2 = \sum_{\text{pol}} |iM_{F \to f\phi^*}^{\text{CP}}|^2 = \frac{|y_F|^2}{2} |m_F|^2.$$

In the following, we refer to the momenta of the leptons and Higgs bosons that appear as decay products by $q$ and $q'$, when they appear in internal cuts by $k$ and $k'$. Accounting for the extra factor of $1/(2|m_F|)$ in Eq. (43) and integrating over the phase space $\{d^2q, d^2q'\}$, what gives a factor of $1/(8\pi)$, the decay rate at leading order is

$$\Gamma_{f\phi^*}^{\text{LO}} = \Gamma_{f\phi^*}^{\text{CP}} = \frac{|y_F|^2 |m_F|}{32\pi}.$$

Using the optical theorem as illustrated in Figure 2, the vertex contribution is obtained as

$$\sum_{\text{pol}} (iM_{F \to f\phi^*}^{\text{LO}})^* iM_{F \to f\phi^*}^{\text{abs,vert}} = -\frac{1}{2} y_F \bar{y}_F \bar{y}_G y_G \int \frac{d^3kd^3k'}{(2\pi)^6 2k^0 2k'^0} (2\pi)^4 \delta^4(p - k - k') \times \text{tr} \left[ (\bar{\phi} + \hat{m}_F) P_L \bar{q} P_R (q + k' + \hat{m}_G) P_R \bar{k} P_L \right] \frac{(q - k')^2 - |m_G|^2}{(q - k')^2 - |m_G|^2}$$

$$= -\frac{1}{2} y_F \bar{y}_F \bar{y}_G y_G \frac{m_F m_G^*}{32\pi} \int_{-1}^1 d\cos \vartheta \frac{1}{\frac{1}{2}|m_F|^2 (1 - \cos \vartheta) + |m_G|^2}.$$

Note the appropriate prefactors due to the averaging over the spin states of the decaying particle and due to the application of the optical theorem. Further, $q$ is the momentum of the outgoing lepton, $k$ of the lepton running in the loop, and $\vartheta$ is the angle between these. This expression is represented by the right-hand side of the first equation in Figure 2. In particular, the integration over $d^3kd^3k'$ corresponds to the phase-space integral $d\varOmega$ in the graphical expression.

Similarly, the wave-function contribution is obtained as

$$\sum_{\text{pol}} (iM_{F \to f\phi^*}^{\text{LO}})^* iM_{F \to f\phi^*}^{\text{abs,wv}}$$

$$= -\frac{1}{2} y_F \bar{y}_F \bar{y}_G y_G \int \frac{d^3kd^3k'}{(2\pi)^6 2k^0 2k'^0} (2\pi)^4 \delta^4(p - k - k') \text{tr} \left[ (\bar{\phi} + \hat{m}_F) P_L \bar{q} P_R (\bar{\phi} + \hat{m}_G) P_R \bar{k} P_L \right] \frac{1}{|m_F|^2 - |m_G|^2}$$

$$= -\frac{1}{2} y_F \bar{y}_F \bar{y}_G y_G \frac{m_F m_G^*}{|M_F|^2 - |M_G|^2} 4q_\mu \frac{p^\mu}{32\pi} = -2i y_F \bar{y}_F \bar{y}_G y_G \frac{|m_F|^2}{64\pi} \frac{m_F m_G^*}{|M_F|^2 - |M_G|^2}.$$
which corresponds to the right-hand side of the second equation in Figure 2. In the second step, we have isolated the loop function, that again is essentially the result of a phase space integral,

$$\hat{\Sigma}^\mu = p^\mu/(32\pi),$$

which we will use to compare with corresponding quantities appearing in the CTP approach. Note that by virtue of the optical theorem, the computation of the $CP$-violating effects has been reduced to deriving tree-level amplitudes and their interference terms and eventually to performing a phase-space integral. The limit $|m_F|^2 - |m_G|^2 \to 0$ is referred to as the resonant regime. Clearly, as the mass differences approaches zero, the validity of the result (50) will break down. We discuss this further in Section 3.2 as well as Section 6, where we present the resolution relevant for scenarios of out-of-equilibrium decay in the early Universe.

Using this result and carrying out the integration in Eq. (49), we define

$$I_{\text{vert}}(m_F, m_G) = -\frac{1}{16\pi} \left[ 1 - \left( 1 + \frac{|m_G|^2}{|m_F|^2} \right) \log \left( 1 + \frac{|m_F|^2}{|m_G|^2} \right) \right],$$

$$I_{\text{wv}}(m_F, m_G) = -\frac{1}{32\pi} \frac{|m_F|^2}{|m_F|^2 - |m_G|^2}. $$

(52a)  

(52b)
In terms of these, the crucial interference terms are given by

$$\sum_{\text{pol}} (i M_{F \to f \phi^*}^{\text{LO}})^* i M_{F \to f \phi^*}^{\text{abs}} = i (\mathcal{T}^{\text{vert}} + \mathcal{T}^{\text{wv}}) y_F y^*_G y_G m_F m_G^*. \quad (53)$$

It is next convenient to parametrize the decay rate as

$$\Gamma_{F \to f \phi^*} = \Gamma_{F \to f \phi^*}^{\text{LO}} (1 + \varepsilon). \quad (54)$$

The effect of $CP$ conjugation is on the interference term (53) is the complex conjugation of the $CP$-odd phase, i.e. the conjugation of all explicit masses and couplings, while the $CP$-even phase remains unaffected. It therefore follows that

$$\Gamma_{F CP \to f CP \phi} = \Gamma_{F \to f \phi^*}^{\text{LO}} (1 - \varepsilon). \quad (55)$$

At the present level of accuracy, the parameter $\varepsilon$ can therefore be identified with the decay asymmetry

$$\varepsilon = \frac{\Gamma_{F \to f \phi^*} - \Gamma_{F CP \to f CP \phi}}{\Gamma_{F \to f \phi^*} + \Gamma_{F CP \to f CP \phi}} = \frac{\sum_{\text{pol}} \left((i M_{F \to f \phi^*}^{\text{LO}})^* i M_{F \to f \phi^*}^{\text{abs}} - (i M_{F CP \to f CP \phi}^{\text{LO}})^* i M_{F CP \to f CP \phi}^{\text{abs}} \right) + \text{c.c.}}{2 \sum_{\text{pol}} |i M_{F \to f \phi^*}^{\text{LO}}|^2} = 4 \text{Im}[y^*_F y_F y^*_G y_G m_F^* m_G^*] \left(\mathcal{T}^{\text{vert}} + \mathcal{T}^{\text{wv}}\right), \quad (56)$$

where c.c. stands for complex conjugation.

We can now check explicitly that this result is invariant under field redefinitions through rephasing $F \to e^{i \varphi^F \gamma^5} F$, $G \to e^{i \varphi^G \gamma^5} G$, $\phi \to e^{i \varphi^\phi} \phi$, $f \to e^{-i \varphi^f} f$, (57) upon which the terms in the Lagrangian (40) transform as

$$m_{F,G} \to m_{F,G} e^{-2i \varphi^F}, \quad y_{F,G} \to y_{F,G} e^{i \varphi^F + \varphi^G + i \varphi^f}, \quad y^*_{F,G} \to y^*_{F,G} e^{-i \varphi^F - i \varphi^G - i \varphi^f}. \quad (58)$$

Indeed, this leaves the decay asymmetry (56) unaffected. The combination $\text{arg}[y_F^* y_F y_G^* y_G^* m_F^* m_G^*]$ is therefore a physical $CP$-odd phase that leads in conjunction with interference effects involving a $CP$-even, absorptive phase to $CP$-violating decays.

A viable scenario of baryogenesis can emerge when attributing baryon number e.g. to the field $f$. As the Universe expands and cools, equal populations of heavy particles $F$ and $F_{CP}$ can then decay and leave behind a baryon asymmetry. We discuss such a scenario on the simpler example of leptogenesis, that is of great phenomenological relevance, however.

To obtain the decay asymmetry for leptogenesis from the above results we introduce the spinor fields $N_i = (1/\sqrt{2}) (\tilde{F} + F^C)$, $N'_i = (i/\sqrt{2}) (\tilde{F} - F^C)$, as well as $N_2 = (1/\sqrt{2}) (G + G^C)$ $N'_2 = (i/\sqrt{2}) (G - G^C)$. By construction, these are Majorana spinors, i.e. $N'_i = N_{1,i}$ and $N''_i = N'_{1,i}$. Imposing further $\tilde{y}'_X = y_X$ on the Yukawa couplings, $N'_1, 2$ decouple such that we are effectively left with the Lagrangian

$$\mathcal{L} = \sum_{i=1,2} \left[ \frac{1}{2} \tilde{N}_i \left( i \tilde{\phi} - M_i e^{i \alpha_i \gamma^5} \right) N_i - Y_i^* \tilde{f} \phi P_R N_i - Y_i \tilde{N}_i \phi^* P_L f \right] \quad (59)$$

where $M_{1,2} = m_{F,G}$, $\alpha_{1,2} = \text{arg}(m_{F,G})$, $Y_{1,2} = \sqrt{2} y_{F,G}$. Following widely used notation, we denote here the masses of the RHNs with a capital letter (not being entirely consequent about the notation declared above, where we have reserved this for mass matrices). When dealing with mixing and oscillations of
RHNs in Sections 3.2 and 5 it is convenient to take $M_{1,2}$ as entries of a mass matrix for these particles that will be defined in due course.

Further, we identify $f$ with the lepton weak isodoublet of the SM, and the scalar field is related to the Higgs doublet $H$ as $\phi = (\epsilon H)^\dagger$ (where $\epsilon$ is the totally antisymmetric rank-two tensor and we suppress the notation of all SU(2) contractions), what leads to an extra factor of $g_w = 2$ in the wave-function contributions relative to the vertex ones. From the above results, we then obtain the decay asymmetry

$$
\varepsilon = \frac{\Gamma_{N\to f\phi^*} - \Gamma_{N\to f^{CP}\phi}}{\Gamma_{N\to f\phi^*} + \Gamma_{N\to f^{CP}\phi}} = \frac{2\text{Im}[Y_1 Y_2^* M_1^* M_2]}{|Y_1|^2 M_1^2} \left( I_{\text{vert}}(M_1, M_2) + g_w I_{\text{sv}}(M_1, M_2) \right). \tag{60}
$$

### 3.2 Variants of computing $CP$ violation from mixing and oscillations

So far, we have assumed an external state for the RHN that is purely $N_1$ without an admixture of $N_2$. However, the lighter mass eigenstate, for which we have been computing the decay asymmetry contains an admixture of $N_2$ that is generated by the Yukawa couplings. This admixture is present even when attributing the renormalized dispersive loop corrections to the mass terms resulting a redefined diagonal mass matrix because the absorptive effects cannot be removed this way. In the previous section, we have apparently accounted for this mixing through the absorptive part of the wave-function correction for the RHN. Another point of view one can take calculationally is to attribute the mixing to the external RHN states of the $S$ matrix. One may do so by generalizing the LSZ reduction formula to account for the mixing induced by absorptive corrections [66, 67].

To work this out in more detail, in Ref. [66] the one-loop improved Dirac operator

$$
S^{-1} = \begin{pmatrix}
\phi - M_1 + \Sigma_{N11} & -\Sigma_{N12} \\
-\Sigma_{N21} & \phi - M_2 - \Sigma_{N22}
\end{pmatrix}
$$

is inverted as

$$
S_{11} = \left( \phi - M_1 - \Sigma_{N11} - \Sigma_{N12} \frac{1}{\phi - M_2 - \Sigma_{N22}} \Sigma_{21} \right)^{-1}, \tag{62a}
$$

$$
S_{12} = S_{11} \Sigma_{N12} \frac{1}{\phi - M_2 - \Sigma_{N22}} = \frac{1}{\phi - M_1 - \Sigma_{N11}} \Sigma_{N12} S_{22}, \tag{62b}
$$

where the remaining entries follow from replacing $1 \leftrightarrow 2$. The loop functions are obtained from the one defined in Eq. [51] as

$$
\Sigma = \gamma_{\mu} \hat{\Sigma}^\mu, \quad \Sigma_{Nij}^A = g_w Y_i^* Y_j^* P_R + g_w Y_i Y_j^* P_L, \quad \Sigma_{Nij} = \Sigma_{Nij}^\text{disp} - i\Sigma_{Nij}^A. \tag{63}
$$

We identify here the absorptive part of $\Sigma_N$ with the spectral self energy $\Sigma_{Nij}^A$, where the sign is dictated by imposing the time-ordered boundary conditions on the resummed propagator. The factor $g_w = 2$ serves also as a reminder that the two fundamental SU(2)$_L$ degrees of freedom are understood to run in the loop.

Next, consider the three-point Green function pertaining to the decay of the lighter RHN state to $\ell$ and $\phi^*$. The leg for the RHN is then given by the resummed external propagator $S$. In order to arrive at an amplitude, it is amputated by multiplying with the diagonal components $S_{11}^{-1}$. As a result, the squared amplitude reads

$$
\sum_{\text{pol}} (i\mathcal{M}_{N_1 \to \ell \phi^*})^* i\mathcal{M}_{N_1 \to \ell \phi^*} = \frac{1}{2} \text{tr} \left[ (S_{11}^{-1})^\dagger (Y_1 S_{11} + Y_2 S_{21})^\dagger \phi (Y_1 S_{11} + Y_2 S_{21}) S_{11}^{-1} P_R (\phi + M_1) \right]$

$$
\subset -i \frac{g_w}{64\pi} Y_1^2 Y_2^* \frac{M_1 M_2^2}{|M_1|^2 - |M_2|^2 + 2\phi \Sigma_{N22}^A} + \text{c.c.}, \tag{64}
$$

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where we can immediately verify the agreement with Eq. (50). In addition, there appears an extra antihermitian term in the denominator which we recognize as the decay width of the off-shell particle $N_2$: $2\rho\Sigma^A_{N22} = \frac{1}{4} \frac{i}{2} \text{tr}[\rho \Sigma^A_{N22}] = \frac{1}{4} \rho \Gamma_{22}$.

Yet another perspective arises from describing the time evolution through a Hamiltonian \[ 68, 69 \]. For non-relativistic RHNs, the kinetic energy can be neglected and the different helicity states evolve in the same way such that one can use the effective Hamiltonian

$$H = \begin{pmatrix} M_1 - \frac{i}{2} \Gamma_{11} & -\frac{i}{2} \Gamma_{12} \\ -\frac{i}{2} \Gamma_{12} & M_2 - \frac{i}{2} \Gamma_{22} \end{pmatrix},$$

(65)

where

$$\Gamma_{ij} = \frac{1}{2p^0} \text{tr}[\rho \Sigma^A_{ij}] = \frac{g_w}{32\pi} \left( Y_i Y_j^* + Y_i^* Y_j \right),$$

(66)

such that $\Gamma_{ii}$ is the total decay rate of $N_i$. This system has two eigenstates of mass and lifetime. If the lifetimes of the two states are different enough, only one of these is relevant at late times. Also, if the mass difference and hence the oscillation time is much shorter than the lifetime, one will produce states that perform fast oscillations about the eigenstates, such that the phases pertaining to these oscillations average out and can be neglected \[ 70 \]. Under these circumstances, only the relative phase that appears when constructing these eigenstates from $N_{1,2}$ is of relevance for $CP$-violation from mixing.

Provided $|\Gamma_{ij}| \ll |M_1 - M_2|$ (An exact expression can be easily found but its form is less compact and illuminating.), these eigenstates are

$$v_1 \approx \begin{pmatrix} 1 \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} \delta v_1, \quad v_2 \approx \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} \delta v_2,$$

(67)

where

$$\delta v_1 = \frac{\frac{i}{2} \Gamma_{12}^*}{M_1 - M_2 + \frac{1}{2} \Gamma_{11} - \frac{1}{2} \Gamma_{22}}, \quad \delta v_2 = -\frac{\frac{i}{2} \Gamma_{12}}{M_1 - M_2 + \frac{1}{2} \Gamma_{11} - \frac{1}{2} \Gamma_{22}},$$

(68)

and, for simplicity, we take here $M_{1,2}$ to be real. Assuming in addition that we are close to the resonance, $|M_1 - M_2| \ll M = (M_1 + M_2)/2$, we can express the admixture to $N_1$ as

$$\delta v_1 \approx \frac{i \tilde{M} \Gamma_{12}^*}{M_1^2 - M_2^2 + iM(\Gamma_{11} - \Gamma_{22})}. \quad (69)$$

Thus, we can once more compute the wave-function contribution to the decay asymmetry when taking for the RHN $N_1$ a mixing state according to $v_1$ in Eq. (67), with the result

$$\sum_{\text{pol}} (i \mathcal{M}_{N_1 \to \ell \phi^*})^* i \mathcal{M}_{N_1 \to \ell \phi^*} \left[ \frac{1}{2} \text{tr} \left[ \frac{i}{\hbar} P_R(\rho + M_1) \right] \left( Y_1 + Y_2 + \frac{i}{M_1} \frac{g_w}{32\pi} \tilde{M}^2 \right) \right] \left( Y_1^* - Y_2^* \frac{\frac{i}{2} \Gamma_{12}}{M_1^2 - M_2^2 + iM(\Gamma_{11} - \Gamma_{22})} \right) \approx -\frac{ig_w Y_1^2 Y_2^*}{64\pi} \frac{M^2}{|M_1|^2 - |M_2|^2 + iM(\Gamma_{11} - \Gamma_{22})} + \text{c.c.} \quad (70)$$

As stated above, we see that the $CP$ phase is generated by the interference of the tree-level contributions with the admixture (69). Again, when $|M_1 - M_2| \gg |\Gamma_{11} - \Gamma_{22}|$ we recognize agreement with the result (50). However, now the corrections due to the finite width of the RHNs are different from what is stated in Eq. (64).
Regarding the denominator term involving the RHN width, the expression (70) agrees with what is found in Ref. [71]. However, one needs to be aware of the fact that the admixtures only build up on the oscillation time-scale that is for non-relativistic neutrinos given by $|M_1 - M_2|^{-1}$. Given that the lifetime of the decaying RHN is $\Gamma_{11}^{-1}$, as noted in Ref. [72], the results (69) and (70) cannot be used for leptogenesis calculations in the interesting regime where $|M_1 - M_2|$ is smaller or even much smaller than $\Gamma_{11}$.

The apparent resolution to the discrepancies in the contributions from the finite width of the RHNs to the denominator terms is found when appreciating that the amount of mixing depends on the full real-time dynamics, i.e. on the way the system deviates from equilibrium as well as on the background evolution and initial conditions. In particular, it is necessary to solve systematically and in general settings for the correlation between $N_1$ and $N_2$, that yields the crucial $CP$-even phase due to quantum mechanical interference [70, 73]. While the result based on the Hamiltonian evolution in this section assumes RHN states that decay in vacuum, in Ref. [72] an approach of a vanishing initial distribution of RHNs toward equilibrium in a finite temperature background is considered. Neither setup realistically models the dynamics in the expanding Universe. In Section 6, we work out this dynamics in the context of the strong-washout regime of leptogenesis in the early Universe, using CTP methods [74–76]. Again, the result differs from Eqs. (64) and (70) in the way the finite-width of the RHNs affects the denominator of the term describing the resonant enhancement.

We finally note that when carefully specifying the initial conditions as well as decomposing into helicity eigenstates, the method based on the Hamiltonian evolution can well be applied to cosmological calculations, as it has been carried out for neutrino oscillations [77] or for leptogenesis from oscillations of RHNs with GeV-scale masses [35–38]. We nonetheless pursue here the CTP approach because it leads to a straightforward expansion in terms of Feynman diagrams as explained in Section 2.2 and because we do not need to specify a basis of quantum states for the interacting system because the Schwinger-Dyson equations are formulated in terms of Green functions.

4 Baryogenesis from out-of-equilibrium decays and inverse decays – Classical fluid equations with QFT cross sections

In order to explain the calculation for baryogenesis based on classical kinetic theory and, in contrast, based on first principles of QFT, we pick leptogenesis as the simplest and phenomenologically most relevant scenario from out-of-equilibrium decays. In addition, we choose a parametrically simple situation where $M_1 \ll M_2$ and $\Gamma_{11} \gg H_{T_{\text{scale}}} = (4\pi^3 g_*/45)^{1/2}M_1^2/m_{\text{Pl}}$, where $g_*$ is the effective number of relativistic degrees of freedom, $m_{\text{Pl}} = 1.221 \times 10^{19}$ GeV is the Planck mass and $H$ is the Hubble rate. Due to the first relation, we may assume that during the times relevant for leptogenesis, the abundance of $N_2$ is strongly Maxwell suppressed and can be neglected. The second condition characterizes the strong-washout regime. It is of great importance because any preexisting asymmetry at higher temperatures will be erased by the lepton-number violating interactions involving $N_1$ as the Universe cools.

4.1 Setting up the fluid equations

Under the assumptions stated above, we may readily proceed to formulate kinetic or fluid equations for leptogenesis. Given that the RHNs are nonrelativistic and that there is kinetic equilibrium for the charged particles, it is sufficient to only track charge and number densities rather than the distribution functions of the species involved. This is because for $\ell$ and $\phi$, kinetic equilibrium maintains the Fermi-Dirac and Bose-Einstein form of the distributions whereas for the nonrelativistic RHNs, the details of the distribution function are irrelevant in order to obtain leading order accurate results. It is therefore
the most commonly used and quickest approach to sidestep the setup of kinetic equations and to directly go for fluid equations in the first place. After having resolved some important matter regarding real intermediate states that occur in matrix elements necessary for CP violation in Section 4.2, we return in Section 4.4 to the systematic derivation of the fluid equations from Boltzmann kinetic equations along the general lines discussed in Section 2.1.

For the simple case present, there are only three densities that we need to track: \( n_{N_1} \), i.e. the number density of \( N_1 \) (counting both helicity degrees of freedom), and the charge densities of leptons and Higgs bosons \( q_\ell \) and \( q_\phi \), respectively. We define the charge densities such that they only count the contribution from one component of the weak isodoublet. Due to gauge invariance, the charge densities for the different components in a multiplet are equal. When normalized to the entropy density

\[
s = \frac{2\pi^2}{45} g_* T^3, \tag{71}
\]

these quantities are referred to as yields, \( Y_X = n_X / s \), \( Y_{\Delta X} = q_X / s = (n_X - n_{X^CP}) / s \), where \( n_X \) is the number density of the particle \( X \) and \( q_X \) is the charge density when summed over particles and antiparticles. Further, since we are working in the limit of non-relativistic RHNs where \( M_1 \gg T \), we approximate the averaged decay rate

\[
\gamma = \Gamma_{N_1 \to \ell\phi^*, \ell^CP\phi} \int \frac{d^3p}{(2\pi)^3} \frac{M_1}{\sqrt{p^2 + M_1^2}} e^{-\sqrt{p^2 + M_1^2}/T} \int \frac{d^3p}{(2\pi)^3} e^{-\sqrt{p^2 + M_1^2}/T} = \frac{K_1(z)}{K_2(z)} \Gamma_{N_1 \to \ell\phi^*, \ell^CP\phi}, \tag{72}
\]

where \( z = M_1 / T \) and \( \Gamma_{N \to \ell\phi^*, \ell^CP\phi} \equiv \Gamma_{11} \), as defined in Eq. \([66]\), is the total vacuum decay rate of a singlet neutrino \( N_1 \) into particles and antiparticles. The factor \( M_1 / \sqrt{p^2 + M_1^2} \) accounts for time dilation and can, of course, be verified when evaluating the decay rates for \( p \neq 0 \). Note that

\[
\frac{K_1(z)}{K_2(z)} = 1 - \frac{3}{2z} + \frac{15}{8z^2} + \cdots \tag{73}
\]

such that we explicitly see that this factor accounts for relativistic corrections. Further, we have approximated the distribution of RHNs by Maxwell statistics as it is appropriate for \( M_1 \gg T \). Being sterile particles, the RHNs are not maintained in kinetic equilibrium by gauge interactions. Deviations of their distribution from the equilibrium form should however only have a subdominant impact on the final result in the present nonrelativistic limit.

The fluid equations can now easily be written down by balancing the number densities of the individual particles with their rate of change in the particular reactions. For the simple model of leptogenesis under consideration, we thus obtain

\[
\frac{dY_{N_1}}{dt} = -\gamma (Y_{N_1} - Y_{N_1}^{\text{eq}}), \tag{74a}
\]

\[
\frac{dY_{\Delta t}}{dt} = Y_{N_1} \left( \frac{1 + \varepsilon \gamma}{2g_w} - Y_{N_1} \frac{1 - \varepsilon \gamma}{2g_w} + Y_{\ell^CP} \frac{1 + \varepsilon \gamma Y_{N_1}^{\text{eq}}}{2g_w} - Y_{\ell^CP} \frac{1 - \varepsilon \gamma Y_{N_1}^{\text{eq}}}{g_w} \right) - 2\gamma_{\ell\phi^* \to \ell^CP\phi} + 2\gamma_{\ell^CP\phi \to \ell\phi^*} + 2\gamma_{\ell\phi^* \to \ell^CP\phi}^{\text{RIS}} - 2\gamma_{\ell^CP\phi \to \ell\phi^*}^{\text{RIS}}, \tag{74b}
\]

where

\[
Y_{N_1}^{\text{eq}} = \frac{45}{2\pi^4 g_*} z^2 K_2(z) \approx \frac{45z^3}{2^3\pi^3 g_*} e^{-z} \tag{75}
\]

is the value that \( Y_{N_1} \) takes in thermal equilibrium (for Maxwell statistics) and the approximation holds for \( z \gg 1 \).
For the decay asymmetry $\varepsilon$, we can use the result (60) in the given nonrelativistic, nonresonant case. In the denominators, explicit factors of two are present because the individual decay rates into $\ell\phi^*$ and $\ell\phi^{CP}$ are equal in the absence of $CP$ violation, while $\gamma$ accounts for the total decay rate into both of these final states. Explicit factors of $1/g_w$ arise because the decay rate $\gamma$ accounts for both weak isodoublet final states, while $q_\ell$ and $q_\phi$ only account for an individual component.

The right hand side of Eq. (74b) can be derived from the collision term (7), where the integrations lead to terms involving the averaged decay rates $\gamma$ in Eq. (72) multiplied with the charge or number densities (i.e. the yields when normalized to entropy density). We show in more detail how these terms arise in the derivation from the Boltzmann kinetic equations in Section 4.4 and from first principles of QFT in Section 5. We next discuss some important features of the individual collision terms, i.e. why they take their particular form and the reactions that they describe.

The first two of the collision terms in Eq. (74b) account for decays of $N$ into $\ell$ and $\ell\phi^{CP}$ and the pertaining decay asymmetry, and the third and fourth term describe inverse decays. In Eq. (60), we have obtained the asymmetry $\varepsilon$ for the processes $N_1 \rightarrow \ell\phi^*, \ell\phi^{CP}$, and together with the averaged decay rate (72), this leads directly to a contribution to the rate of change in lepton asymmetry. As for the third and fourth term, the asymmetries for the inverse processes follow directly from the $CPT$ theorem. For $\varepsilon = 0$, $Y_{N_1} = Y_{N_1}^{eq}$, $Y_{\ell} = Y_{\ell}^{CP} = Y_{\ell}^{eq}$, among the four leading collision terms the decay and inverse decay terms cancel pairwise in equilibrium, which explains the rescaling of the inverse decay terms by $Y_{N_1}^{eq}/Y_{\ell}^{eq}$. (Again, this argument based on balancing the reactions can be verified explicitly by evaluation of the collision term.) The fifth to the eighth term account for two-by-two lepton-number violating scattering processes which we discuss in the following subsection.

### 4.2 Real intermediate states, $CP$ violation and deviation from equilibrium

Now, when taking $\varepsilon > 0$ for definiteness, we immediately observe from the first four of the collision terms in Eq. (74b) that leptons would be preferred over antileptons even in equilibrium. However, this cannot be the full picture because it would imply that an asymmetry can be present even in thermal equilibrium, in contradiction with Sakharov’s nonequilibrium condition based on the $CPT$ theorem along the reasoning in Section 1.1.

A way of resolving this matter appears when adding the last four collisional terms describing two-by-two scatterings [78], where we have extracted explicit factors of two because these processes change lepton number by two units. The rates $\gamma_{\ell\phi^* \rightarrow \ell\phi^{CP}}$ and $\gamma_{\ell\phi^{CP} \rightarrow \ell\phi^*}$ denote the full two-by-two rates, and one can easily see that these are $CP$ even because each Feynman diagram has a counterpart with complex conjugated Yukawa couplings, such that the $CP$-odd phases cancel. Since we assume that $M_1 \ll M_2$, we can concentrate on contributions mediated by the exchange of $N_1$. Provided $N_1$ is off shell, at tree level, the two-by-two rates are $CP$ even and of order $Y^4$. (We omit absolute values and indices on the Yukawa couplings $Y$ in these power counting arguments.) They are therefore subdominant when compared with the $CP$-even one-to-two rates that are of order $Y^2$. (Notably though, because one-to-two rates are Boltzmann suppressed when $M_1 \gg T$, two-by-two rates are phenomenologically relevant for the washout of the produced asymmetries only when the Yukawa couplings are large compared to the values generically predicted by the type-I seesaw mechanism in conjunction with the observed light neutrino masses [79]. Scenarios where the two-by-two rates are of quantitative importance for washout are studied e.g. in Refs. [12, 80].) However, when $N_1$ is exchanged in the $s$ channel, the tree-level two-by-two rates lead to contributions of order $Y^2$ from the portion of the phase-space integral where the internal $N_1$ propagator is on shell, i.e. where there is a so-called real intermediate state (RIS). Now, the production of on-shell $N_1$ is already accounted for by the explicit one-to-two processes, such that these portions need to be subtracted from the two-by-two rates.
These RIS contributions to the two-by-two rates can be written in the following suggestive way:

\[
\gamma_{\ell \ell^* \rightarrow \ell \ell}^\text{RIS} = \frac{Y_\ell}{Y_\ell} \times \frac{1 - \varepsilon}{2} \frac{\gamma_{N}^\text{eq}}{g_w} \times \frac{1 - \varepsilon}{2} \approx \frac{Y_\ell \gamma_{N}^\text{eq} (1 - 2\varepsilon)}{4 g_w} , \tag{76a}
\]

\[
\gamma_{\ell \ell^* \rightarrow \ell \ell}^\text{RIS} = \frac{Y_{\ell \ell}^\text{CP}}{Y_\ell} \times \frac{1 + \varepsilon}{2} \frac{\gamma_{N}^\text{eq}}{g_w} \times \frac{1 + \varepsilon}{2} \approx \frac{Y_{\ell \ell}^\text{CP} \gamma_{N}^\text{eq} (1 + 2\varepsilon)}{4 g_w} . \tag{76b}
\]

In each of these equations, the first factor involving \(\varepsilon\) corresponds to the \(CP\)-violating inverse decay rate of \(N_1\), while the second factor involving \(\varepsilon\) is the branching ratio of the \(CP\)-violating decays. Substituting these equations into Eq. (74b), a number of cancellations leads to

\[
\frac{Y_{\Delta \ell}}{dt} = \varepsilon \gamma \frac{g_w}{g_w} \left( Y_{N_1} - Y_{N_1}^\text{eq} \right) - 2\gamma_{\ell \ell^* \rightarrow \ell \ell}^\text{RIS} + 2\gamma_{\ell \ell^* \rightarrow \ell \ell^*} . \tag{77}
\]

Notice that, according to our above remark, the two-by-two rates are \(CP\) conserving such that all remaining effects of \(CP\) violation are proportional to the deviation of the RHN \(N_1\) from equilibrium, in agreement with Sakharov’s conditions. Further, we now refer to the first term on the right-hand side as a source term and the second one as a washout term. The washout processes are \(CP\) conserving.

Next, we recall that two-by-two washout processes contain still those mediated by RIS. In a wide range of parameter space, where washout mediated by off-shell RHNs may be neglected, it is a good approximation to only account for the RIS contributions. In that case, the fluid equation for the lepton asymmetry simplifies to

\[
\frac{dY_{\Delta \ell}}{dt} = \varepsilon \gamma \frac{g_w}{g_w} \left( Y_{N_1} - Y_{N_1}^\text{eq} \right) - \frac{Y_{\Delta \ell}}{2} \frac{\gamma_{N_1}^\text{eq}}{Y_\ell} g_w , \tag{78}
\]

where now the last two terms describe the washout, and where we note that this equation is of the form advertised in Eq. (12a).

To conclude this discussion, the first four collision terms in Eq. (74b) by themselves fail to comply with the \(CPT\) theorem (that relies on unitarity of the \(S\) matrix) and consequently also with Sakharov’s non-equilibrium condition. Operating with \(S\)-matrix elements for unstable states, here for \(N_1\), leads apparently to a nonunitary time-evolution, which is fixed through the subtraction of certain parts of the RIS contributions. Whether or not the procedure presented here corresponds to a satisfactory solution of the problem, it has motivated studies of leptogenesis in the \(CPT\) framework \([45, 81-89]\), where instead of using \(S\)-matrix elements, one operates with the real-time evolution of quantum mechanical correlation functions in first place. We review these matters in Section 5.

4.3 Final adjustments and expansion of the Universe

There are two more simple, yet substantial adjustments that we are going to apply to the fluid equations: First, we change the implicit assumption of Maxwell statistics for the relativistic leptons to the more accurate Fermi-Dirac distribution and second, we take account of the additional bias that the charge density in Higgs bosons implies for the washout term. Most importantly, we account in addition for the expansion of the Universe, as this creates the crucial nonequilibrium conditions for baryogenesis in first place.

When the active leptons \(\ell\) are energetic enough to produce a RHN \(N_1\), they will typically reside in the Maxwell tail of the distribution. Further, gauge interactions rapidly reestablish the Fermi-Dirac form when charges are lost from the distribution due to washout or when they are produced from the source term. Compared to the average decay rate of the RHNs into leptons, in the relaxation rate for the lepton charge, there thus occurs an extra factor \(\frac{1}{2} \left[ \exp(\mu T) - \exp(-\mu T) \right] \approx \mu T / T\). We replace this
chemical potential according to relation (10) (i.e. $\frac{1}{2}Y_{\Delta f}/Y^\text{eq}_f = \mu_f/T \rightarrow 6q_{\ell}/T^3$), such that the washout term becomes

$$\frac{d}{dt} q_{\ell} = -\frac{1}{g_w} \gamma s Y^\text{eq}_{N1} \frac{6}{T^2} q_{\ell} = W q_{\ell}. \quad (79)$$

Besides the lepton charge density $q_{\ell}$, also the charge density of Higgs bosons $q_{\phi}$ leads to a bias of the washout term, and noting the relation (10) between charge and chemical potentials for fermions and bosons, this amounts to a multiplication of the washout rate by a factor of 3/2. In the SM, the charges in the Higgs bosons and active leptons are further redistributed by the Yukawa interactions and by strong and weak sphaleron processes to the remaining SM particles [90, 91]. However, due to the large number of degrees of freedom, this additional correction is comparably small, and we neglect it here for simplicity. It is technically easy though to incorporate such fully equilibrated spectator effects such that these should be nonetheless included in phenomenological studies. Partially equilibrated spectator fields may lead to an effective protection of the asymmetry from washout, which is of importance in some regions of the parameter space discussed in Ref. [92].

Crucially for baryogenesis, we yet need to account for the expansion of the Universe that creates the necessary out-of-equilibrium conditions. At the present stage, where the derivations to this end have been made in Minkowski background, this is most easily implemented when describing the Friedmann-Robertson-Walker Universe through conformal coordinates and the pertaining metric tensor

$$g_{\mu\nu} = a^2(\eta) \text{diag}(1, -1, -1, -1), \quad (80)$$

where $\eta$ is conformal time and $a(\eta)$ the scale factor.

In standard scenarios for leptogenesis, the Universe is dominated by relativistic radiation at the time when the asymmetry is generated. The expansion during radiation domination is given by

$$a(\eta) = a_R \eta. \quad (81)$$

When we define a comoving temperature $T_{\text{com}} = aT$, where $T$ is the physical temperature, we note that the choice $a_R = T_{\text{com}}$ is particularly convenient, as this implies that $T = 1/\eta$. Relating the Hubble rate to the energy density of the plasma through the Friedmann equation, one obtains

$$H^2 = \left(\frac{1}{a^2} \frac{d}{d\eta} a\right)^2 = \frac{T^4}{a^2} = \frac{8\pi^2 g_{*} T^4}{3 m_{\text{Pl}}^2} \Leftrightarrow a_R = \frac{m_{\text{Pl}}}{2} \sqrt{\frac{45}{\pi^3 g_{*}}}, \quad (82)$$

where $m_{\text{Pl}} = 1.22 \times 10^{19}\text{GeV}$ is the Planck mass.

Given the metric tensor [80], we can view the kinetic equations for the charge and number densities (i.e. not the entropy-normalized versions) derived to this end as expressed in terms of comoving momenta, i.e. $k \rightarrow k_{\text{com}} = a(\eta) k_{\text{ph}}$, and the comoving temperature, i.e. $T \rightarrow T_{\text{com}}$ provided we replace the masses $M_i \rightarrow a(\eta) M_i$ (up to potential effects from the coupling of the scalar fields to the background curvature that are negligible in the present context) and the derivatives $d/dt \rightarrow d/d\eta$. When integrating over $d^3k_{\text{com}}$, we then obtain equations for the comoving number densities. In the absence of collisions, these are conserved as the Universe expands and directly proportional to the entropy-normalized yields. Next, in order to have the fluid equations temporally depend on an order-one parameter, we define $z = M_1/T = \eta M_1$, which implies that

$$\frac{d}{d\eta} = M_1 \frac{d}{dz}, \quad a(\eta) = z T_{\text{com}}/M_1. \quad (83)$$

Carrying out these rescalings, the fluid equations derived in Minkowski space are recast to a form suitable for the radiation-dominated Universe as

$$\frac{dn_i}{dt} = -\Gamma_{ij} (\{M_k\}, T) n_j \quad \rightarrow \quad \frac{dY_i}{dz} = -\frac{1}{M_1} \Gamma_{ij} (\{M_k/M_1 \times z T_{\text{com}}\}, T_{\text{com}}) Y_j. \quad (84)$$
Figure 3: Diagrammatic representation of the individual terms appearing in the fluid equations for leptogenesis. Bold solid lines without arrows stand for RHNs, with arrow for leptons and bold dashed lines with arrows stand for Higgs bosons. The on shell cuts are indicated by thin orange lines. A gap at the cut through the RHN or lepton lines indicates that the pertaining integral originates from the zeroth moment of the associated kinetic equation. Alternatively, these diagrams also appear in the collision terms of the CTP approach discussed in Section 5. In the limit $M_1 \ll M_2$, the leading contributions to $S_{\text{vert}}$ and $S_{\text{wv}}$ are those where the cut goes through an $N_1$ propagator whereas $N_2$ remains off shell.

In summary, the apparently simplest scenario of leptogenesis is described by the following set of coupled differential equations:

\begin{align}
\frac{dY_{N_1}}{dz} &= -\gamma (Y_{N_1} - Y_{N_1}^{eq}) , \\
\frac{dY_{\Delta \ell}}{dz} &= \varepsilon \frac{\bar{\gamma}}{g_w} (Y_{N_1} - Y_{N_1}^{eq}) - \bar{W} \frac{3}{2} Y_{\Delta \ell} ,
\end{align}

where, applying the replacement rule (84) to the expressions (72) and (79),

\begin{align}
\gamma &= \frac{K_1(z)}{K_2(z)} |Y_1|^2 \frac{M_1}{8\pi} \rightarrow \bar{\gamma} = \frac{K_1(z)}{K_2(z)} |Y_1|^2 \frac{z \bar{T}_{\text{com}}}{8\pi M_1} , \\
W &= 3|Y_1|^2 \frac{M_1^3}{8\pi^3 T^2} K_1 \left( \frac{M_1}{T} \right) \rightarrow \bar{W} = 3|Y_1|^2 \frac{z^2 \bar{T}_{\text{com}}}{8\pi^3 M_1} K_1(z) = \frac{3|Y_1|^2 z^2 \bar{T}_{\text{com}}}{2\pi^\frac{5}{2} M_1} e^{-z} \times (1 + O(1/z)) . \tag{86b}
\end{align}

Numerically, these can be solved easily. An approximate analytic solution, that yields some insight into the general mechanics of baryogenesis from out-of-equilibrium decays is also available and will be reviewed in Section 7.

### 4.4 Integral expressions for the rates in the fluid equations

To this end, we have inferred the fluid equations for leptogenesis by balancing the rates of change in the charge and number densities of leptons $\ell$ and RHNs $N_i$. Nonetheless, as advertised in Section 2.1, these can also be derived from the Boltzmann equations by integration over three-momentum, i.e. the fluid equations (85) can be expressed as

\begin{align}
\frac{dq_\ell}{dt} &= \frac{d}{dt} \int \frac{d^3 q}{(2\pi)^3} \left( f_\ell(q) - \bar{f}_\ell(q) \right) = S - W , & \frac{dn_{N_1}}{dt} &= \frac{d}{dt} \int \frac{d^3 p}{(2\pi)^3} \bar{f}_{N_1}(p) = -D ,
\end{align}

where the decay term $D$, the washout term $W$ and the source term $S$ can be expressed as momentum integrals over the collision term $\mathcal{C}$. The bar over the function $\bar{f}_X$ indicates that this is the distribution of the antiparticle of $X$. In the integral for the RHNs, there is an explicit factor of two accounting for
the two helicity states. For simplicity, we do not include here the expansion of the Universe and the normalization to entropy that can be reintroduced as explained in Section 4.3. These integral expressions will also be useful for comparison with the corresponding results derived in the CTP approach in Section 5. In order to condense the notation of phase-space and four-momentum integrals, we use the shorthand expressions

\[ \int = \int \frac{d^4p_1}{(2\pi)^4} \cdots \frac{d^4p_n}{(2\pi)^4}, \]  
\[ \int = \int \frac{d^3p_1}{(2\pi)^32\sqrt{p_1^2 + m_1^2}} \cdots \frac{d^3p_n}{(2\pi)^32\sqrt{p_n^2 + m_n^2}}, \]  
\[ \delta_p = (2\pi)^4\delta^4(p), \]  

where \( m_i \) is the mass of the particle with momentum \( p_i \).

We proceed by neglecting quantum-statistical factors (i.e. by replacing \((1 \pm \phi) \to 1\)) in the collision term, as it is appropriate in the strong-washout regime where the RHNs are nonrelativistic. The decay term can then be expressed as

\[ D = \gamma (n_{N_1} - n_{N_1}^{eq}) = 2g_w \int \delta_{p-q-q'} \sum_{\text{pol}} |i\mathcal{M}_{N_1 \to \ell\phi}^{LO}|^2 \delta f_{N_1} = g_w \int \delta_{p-q-q'} 4p \cdot q \delta f_{N_1}(p), \]  

where the explicit factor of two accounts for the decay channels into particles and antiparticles. The distribution \( \delta f_{N_1} = f_{N_1} - f_{N_1}^{eq} \) is the deviation from the equilibrium distribution \( f_{N_1}^{eq} \) for \( N_1 \). Note also that there appears the a polarization sum rather than the average over the RHN polarizations because we account here for both of their helicity states, i.e. “pol” under the sum now refers to the polarizations of \( N_1 \) as well as \( \ell \). The integral form of the washout rate is

\[ \mathcal{W} = WY_{\Delta\ell} = \int \delta_{p-q-q'} \sum_{\text{pol}} |i\mathcal{M}_{N_1 \to \ell\phi}^{LO}|^2 f_{\phi}(q') \left[ f_{\ell}(q) - \bar{f}_{\ell}(q) \right] \]  
\[ = \int \delta_{p-q-q'} \text{tr}[\gamma_\ell f_{\phi}(q')] \left[ f_{\ell}(q) - \bar{f}_{\ell}(q) \right], \]  

and for the \( CP \)-violating source, we write

\[ S = S^{vert} + S^{wv} = \frac{\varepsilon \gamma}{g_w} (n_{N_1} - n_{N_1}^{eq}) \]  
\[ = \int \sum_{\text{pol}} \left[ (i\mathcal{M}_{N_1 \to \ell\phi})^* i\mathcal{M}_{N_1 \to \ell\phi} - (i\mathcal{M}_{N_1 \to \ell CP\phi})^* i\mathcal{M}_{N_1 \to \ell CP\phi} + \text{c.c.} \right] \delta_{p-q-q'} \delta f_{N_1} \]  
\[ = - (Y_1^2Y_2^* - Y_1^*Y_2^2) \int \delta_{p-k-k'} \delta_{p-q-q'} \left\{ \frac{i \text{tr} \left[ \left( \phi + \tilde{M}_1^* \right) P_l \gamma P_R \left( \bar{q} - k' + \tilde{M}_2^* \right) P_R \bar{k} P_L \right]}{(q - k')^2 - |M_2|^2} \right. \]  
\[ + g_w \frac{i \text{tr} \left[ \left( \phi + \tilde{M}_1^* \right) P_l \gamma P_R \left( \phi + \tilde{M}_2^* \right) P_R \bar{k} P_L \right]}{|M_1|^2 - |M_2|^2} \left\} \delta f_{N_1}, \]  

where we have substituted Eqs. (49) and (50) for the interference terms. [Note the appropriate rescaling of the Yukawa coupling and that this result (91) accounts for the sum of both polarization states of \( N_1 \), whereas Eq. (49) does so for the average.]
One can check that these integral expressions agree with the definitions of $\gamma$ in Eq. (72), $W$ in Eq. (86b) and $\varepsilon$ in Eq. (60) when further relating the distribution functions to the densities as in Eq. (89).

The rates appearing in the fluid equations can also be represented diagrammatically as shown in Figure 3. Note that the diagrams for the $CP$-violating rates are obtained from those in Figure 2 by sewing together the lines appearing in the phase-space integrals and eventually, sewing the line associated with the momentum space integral of the particle described by the particular fluid equation. The resulting diagrams take the form of contributions to an effective action, i.e. of “vacuum graphs”. Within the CTP approach discussed in the following Section 5 we see that such an interpretation is indeed meaningful.

We also note that in the source terms $S$, depending on whether we attribute the cut to $N_1$ or to $\ell$ and $\phi$, we either obtain an interference between a tree-level and a one-loop, one-two-two amplitude or an interference between two tree-level, two-by-two scatterings, one mediated by and on-shell $N_1$ and one by an off-shell $N_2$. This ambiguity is related to the correct counting of the reaction rates that has been implemented in this section by subtracting the RIS. The CTP approach presented in the following section automatically takes care of the right counting.

### 5 Baryogenesis from out-of-equilibrium decays and inverse decays based on first principles in the closed time-path approach

When reviewing the standard approach to leptogenesis based on classical kinetic theory and QFT cross sections and decay rates, we have encountered two somewhat unsatisfactory arguments:

- In order to meet the requirement that no $CP$ asymmetry may be present or be generated in thermodynamic equilibrium, certain contributions from RHNs propagating as RIS have to be subtracted from the two-by-two scattering rates. While this procedure leads to correct results, the argument faces the complications due to unstable particles as external states in scattering theory. It appears convoluted and calls for a simpler and more direct treatment. The discussion in Section 2.3 suggests that the CTP approach offers such a method.

- The results from the wave-function or mixing contribution to leptogenesis are, as discussed in Section 3.2, inconclusive in the resonant limit where $|M_{11}^2 - M_{22}^2| \gg M_1 \Gamma_{ij}$ does not hold.

Both of these issues have to do with the appearance of long-lived states in the reactions that drive the kinetic equations. In the first case, it is the RHN $N_1$ whose lifetime by definition is of the same order as the time between decays and inverse decays, in the second case, $N_2$ can be produced from $N_1$ without external radiation within the bounds of energy uncertainty. Therefore, in both situations, a region where the particles evolve as free in and out states outside of a region in spacetime where interactions may not be neglected, cannot be clearly identified. The presence of regions of freely propagating states is however a prerequisite for defining the matrix elements that are substituted into the Boltzmann equations. In the case of leptogenesis, the one-to-two rate cannot be clearly separated from the two-to-two rates involving RIS (cf. Section 4.2) or external states may not be clearly identifiable because they correspond to unstable, mixing particles (cf. Section 3.2). In the Boltzmann approach, it therefore proves complicated to construct kinetic equations that respect unitarity and the $CPT$ theorem.

Due to these problems associated with the formulation of matrix elements, the computation of the real-time evolution of the quantum mechanical correlation functions is a more straightforward approach (save for many practitioners being more familiar with scattering theory). In fact, all observables of
interest, in particular the lepton asymmetry, can be calculated from the correlation functions. The time evolution of these is found by solving a closed system of Schwinger-Dyson equations for the causal (i.e. retarded and advanced) propagators and the Wightman functions, as reviewed in Section 2.2.

Self-consistent solutions to these equations can be obtained based on systematic approximations (e.g. perturbative expansions or resummed variants of these, numerical methods or combinations of both). The correct account of finite-width effects in Wigner space is explained in Ref. [93] but these can be safely ignored if the quasi-particles can be approximated to occupy a sharp mass shell. Further, in general kinematic regimes, the spinor structure leads to technical complications that can be dealt with using the methods developed in Refs. [54, 55, 94, 95] that we review in the context of electroweak baryogenesis in Section 8. Compared to that, in the present context, which is leptogenesis in the strong washout regime, the non-relativistic approximation for the RHNs leads to a considerably simplified treatment of the fermion fields.

5.1 Kinetic equations for leptogenesis in the strong-washout regime

We start writing down the loop contributions (28) to the 2PI effective action

\begin{equation}
\Gamma_2 = -i - i + \cdots .
\end{equation}

(92)

The bold lines that appear here represent full propagators, and when these are solid, they stand for fermions, and when dashed, for Higgs bosons. Fermion lines without arrow are for RHNs and with arrow for active leptons. Taking functional derivatives with respect to the propagators of active leptons and RHNs according to Eq. (29b) yields the self energies that appear in the Schwinger-Dyson equations (31b). For the present case, these take the diagrammatic form

\begin{equation}
\delta + \cdots ,
\end{equation}

(93a)

\begin{equation}
\delta + \cdots .
\end{equation}

(93b)

Thin lines stand for tree propagators and the inverse of these amounts to Dirac operators. The term \( \delta \) stands for the first term on the RHS of Eq. (31b). As described in Section 2.2, the Schwinger-Dyson equations then lead to Kadanoff-Baym equations (37) in Wigner space.

For the present problem of leptogenesis in the strong washout regime, we apply the following simplifications:

- We discard the terms involving \( \Sigma^H \) that amount to a correction in the dispersion relation, e.g. a thermal mass. For the RHN, it is of order \( YT \) and can be neglected compared to the masses \( M_{1,2} \gg T \). Note that this term may nonetheless be of importance in scenarios where the asymmetry is
generated when the RHNs are relativistic and when at the same time their mass splitting is very small. For the active leptons, the correction to the dispersion relation of order $gT$ is small compared to the average energy of order $M_1$, where $g$ schematically stands for the $U(1)_Y$ and $SU(2)_L$ gauge couplings.

- We further drop the term involving $S^H$ that constitutes an inhomogeneous contribution to the differential equation for $S^{<,>}$. It gives rise to the equilibrium contribution to the Green’s function in a form that also resolves the finite width [93]. In many cases of phenomenological interest (such as the present one), it is sufficient to approximate spectral distributions of finite width by Dirac-$\delta$ functions. Even when dropping this term, the equilibrium form of the RHN distribution follows from the KMS relation when imposing that the collision term should vanish, such that the solution is independent of time.

- When we furthermore neglect gradient effects that are in the present case controlled by the parameter $H/M_1$ (the temporal rate of change given by the Hubble rate divided by the typical momentum scale given by the mass of the decaying RHN), i.e. we truncate $\exp(-i\partial/\kappa)$ at zeroth order.

- We assume spatial isotropy, such that $\partial_iS^{<,>}(x) = 0$ for $i = 1, 2, 3$.

We are thus left with

$$\left[-i\kappa \gamma^0 + \partial_t + iM \gamma^0 \right] i\gamma^0 S^{<,>} = -\frac{1}{2} \left( i\Sigma^{>\gamma^0} i\gamma^0 S^{<} - i\Sigma^{<\gamma^0} i\gamma^0 S^{>} \right),$$  \hspace{1cm} (94)

where we have inserted factors of $i\gamma^0$ such that we can readily take the Hermitian and Antihermitian parts that are

$$\frac{d}{dt} \gamma^0 iS^{<,>} - \frac{i}{2} [M \gamma^0, i\gamma^0 S^{<,>}] = -\frac{1}{2} \left\{ i\Sigma^{>\gamma^0}, i\gamma^0 S^{<} \right\} + \frac{1}{2} \left\{ i\Sigma^{<\gamma^0}, i\gamma^0 S^{>} \right\},$$  \hspace{1cm} (95a)

$$\frac{1}{2} \{\kappa - M, S^{<,>}\} = -\frac{1}{2} \left[ i\Sigma^{>\gamma^0}, i\gamma^0 S^{<} \right] + \frac{1}{2} \left[ i\Sigma^{<\gamma^0}, i\gamma^0 S^{>} \right].$$  \hspace{1cm} (95b)

We refer to the Antihermitian part (95b) as the constraint equation. Neglecting the loop terms on the right-hand side, the solutions are given in terms of the tree-level propagators (B8). In addition, for a system with several flavours, the solutions support flavour-off-diagonal correlations. For now, we will leave these correlations aside and perform the calculation in a perturbative expansion based on the tree-level propagators. In Section 6, we show that this is equivalent to solving Eqs. (95) also for the off-diagonal correlations in the RHNs through most of the parameter space and that this also resolves the question of how to correctly treat the degenerate regime where $|M_i - M_j| \gg \Gamma_{ii}$ is not satisfied.

The Hermitian part (95a) is called kinetic equation. To appreciate this, we note that we can extract the distribution functions as

$$\text{tr} \int \frac{dq^0}{2\pi} i\gamma^0 S^{<,>}_{\ell}(q) = \left[ \bar{f}_{\ell}(q) - f_{\ell}(q) \right],$$  \hspace{1cm} (96a)

$$\text{tr} \int \frac{dp^0}{2\pi} \text{sign}(p^0) S^{<,>}_{Ni}(p) = -4f_{Ni}(p),$$  \hspace{1cm} (96b)

where we have made use of the tree-level solutions (B8) and where the first equation corresponds to the zero-component of the fermionic current, cf. Eq. (13). Taking the trace of Eq. (95a) and integration
over the zero component of the four momentum then leads to the following form of the kinetic equations for RHNs and active leptons:

\[
\frac{d}{dt} f_{Ni}(p) = C_N(p) = \frac{1}{4} \int \frac{dp^0}{2\pi} \operatorname{sign}(p^0) \text{tr} \left[ i\Sigma_{Ni}^{\text{LO}}(p)iS^<_{Ni}(p) - i\Sigma_{Ni}^{\text{LO}}(p)iS^>_{Ni}(p) \right], \tag{97a}
\]

\[
\frac{d}{dt} \left( f_\ell(q) - \bar{f}_\ell(q) \right) = C_\ell(q) = \int \frac{dq}{2\pi} \text{tr} \left[ i\Sigma_{\ell}^{\text{LO}}(q)iS^<_{\ell}(q) - i\Sigma_{\ell}^{\text{LO}}(q)iS^>_{\ell}(q) \right]. \tag{97b}
\]

Consequently, we obtain equations for the number and charge densities when using Eq. (87), and we calculate the particular collision terms in the following subsection.

### 5.2 Decays, inverse decays and washout

In order to derive the washout rate for the lepton asymmetry, we note that the leading order self energy on the CTP is

\[
i\Sigma_{\ell}^{\text{LOab}}(q) = P_R Y_i^* Y_j \int \delta_{p-q-q'} P_R iS_{Ni}^{ab}(p) P_i i\Delta_{\phi}^{ba}(q'),
\]

which corresponds to the amputated one-loop diagram in Eq. (93a). Assuming that \( f_\phi = \bar{f}_\phi \), we obtain

\[
\mathcal{W} = \sum_{pqq'} \left[ i\Sigma_{\ell}^{\text{LO}}(q)iS^<_{\ell}(q) - i\Sigma_{\ell}^{\text{LO}}(q)iS^>_{\ell}(q) \right] - |Y_i|^2 \int \delta_{p-q-q'} \left[ P_R \delta_{pp'} \left[ f_{N1}(p) + f_\phi(q') \right] \times \left[ f_\ell(q) - \bar{f}_\ell(q) \right] \right], \tag{99}
\]

which is proportional to the lepton asymmetry. This agrees with the result (90), up to an extra term involving \( f_{Ni}(p) \). It can also be reproduced in the Boltzmann approach when accounting for the full quantum statistics. However, it is negligible in the strong-washout regime, where the RHN distribution is exponentially suppressed compared to the one of Higgs bosons. When including spectator effects, one should also expand in terms of the charge asymmetry in Higgs bosons in addition to the one in leptons.

Next, for the rate of decays and inverse decays that drives the RHNs toward thermal equilibrium, we use the leading-order self-energy of the RHNs,

\[
i\Sigma_{Ni}^{ab}(p) = g_w \int \delta_{p-q-q'} \left\{ Y_i Y_j P_R iS_{Ni}^{ab}(q) \left[ i\Delta_{\phi}^{ab}(q') \right]^* + Y_i Y_j P_R iS_{Ni}^{ab}(q) i\Delta_{\phi}^{ab}(q') \right\}, \tag{100}
\]

i.e. the amputated one-loop diagrams in Eq. (93b).

Using the approximations \( f_\ell = \bar{f}_\ell \) and \( f_\phi = \bar{f}_\phi \), this leads to the decay term

\[
\mathcal{D} = -\frac{1}{2} \int \text{tr} \operatorname{sign}(p^0) \left[ i\Sigma_{Nii}^{\text{LO}}(p)iS^<_{Nii}(p) - i\Sigma_{Nii}^{\text{LO}}(p)iS^>_{Nii}(p) \right] = -g_w |Y_i|^2 \int \text{tr} \operatorname{sign}(p^0) \hat{\Sigma}_N^A i\delta S_{Nii} = g_w |Y_i|^2 \int \text{tr} \left[ k_{pp'} \left( 1 - f_\ell(q) + f_\phi(q') \right) \delta f_{Ni}(p) \right], \tag{101}
\]

which is in agreement with Eq. (89), again up to quantum statistical corrections. For comparison with the results in Section 8, we also define here

\[
\hat{\Sigma}_N^A(p) = \gamma_\mu \hat{\Sigma}_N^{A\mu}(p) = \frac{1}{2} \int \delta_{p-q-q'} \left( 1 - f_\ell(q) + f_\phi(q') \right), \tag{102}
\]
such that we can decompose
\[ \Sigma_{Nij}^A = g_w \left( Y_i^1 Y_j^* P_L + Y_i^* Y_j^1 P_R \right) \gamma_\mu \hat{\Sigma}^{A\mu}. \]  

(103)

Note that in the strong washout regime, we can use the zero-temperature approximation because the distribution functions are Maxwell suppressed, such that \( \hat{\Sigma}^{A\mu}(p) = p^\mu / (32\pi) \), what agrees with the corresponding quantity \(^\text{[51]}\) appearing for decays in the vacuum.

When the RHNs are relativistic, one should also account for the asymmetry in their different helicity states that is generated from decays and inverse decays when \( f_\ell \neq \bar{f}_\ell \) or \( f_\phi \neq \bar{f}_\phi \). This is e.g. of relevance for leptogenesis from oscillations of RHNs with mass below the electroweak scale \(^\text{[30]-[35]-[40]}\). The effect from the helicity asymmetry is however only material when a sizable fraction of the RHNs are produced or destroyed associated with the radiation of an extra gauge boson, i.e. in two-by-two scatterings rather than in one-two-two decay and inverse decay processes. The latter dominate however in the strong-washout regime with nonrelativistic RHNs, where it is therefore a good approximation to neglect the helicity asymmetries as we will do throughout the present discussion.

5.3 \textit{CP-violating source}

As for the \textit{CP}-violating source, we note that the two-loop, vertex-type self energy for the active lepton on the CTP, given by the amputated two-loop diagram in Eq. \((93a)\), is

\[ i\Sigma_\ell^{\text{vert}}(q) = -\sum_{cd} cd Y_i^1 Y_2^{*2} \int_{q'kk'} P_R i S_{N2}^{cd}(k-q') P_R i S_{\ell}^{CPRdc}(k) P_L i S_{N1}^{ch}(p) P_L i \Delta^{ca}(-k') i \Delta^{bd}(q') \]

\[ + 1 \leftrightarrow 2. \]  

(104)

For simplicity, we now set \( M_1 \) and \( M_2 \) to be real what can be achieved by rephasings of the fields \( N_{1,2} \).

The leading order washout terms that are proportional to \( q_\ell \) are accounted for by the contribution from \( \Sigma_\ell^{\text{LO}} \) already. We can thus substitute equilibrium Fermi-Dirac or Bose-Einstein distributions into the expressions for the fermion and scalar boson propagators. Then, the KMS relation \(^\text{[39]}\) implies that the collision term on the right-hand side of Eq. \((97b)\) can only depend on the remaining deviation from equilibrium \( \delta f_{N1} = f_{N1} - f_{N1}^{\text{eq}} \). Carrying out the sum over the CTP indices and

- expanding in to linear order in \( \delta f_{N1} \),
- accounting only for the off-shell contributions from \( N_2 \) according to the assumption that \( M_2 \gg T \),

we arrive after some laborious rearrangements \(^\text{[87]-[96]}\) that are shown in Appendix \( \square \) at

\[ S^{\text{vert}} = \int \text{tr} \left[ i\Sigma_\ell^{\text{vert}}(q) iS_{\ell}^{<}(q) - i\Sigma_\ell^{\text{vert}}(q) iS_{\ell}^{>}(q) \right] \]

\[ = - (Y_1^2 Y_2^{*2} - Y_1^{*2} Y_2) \int_{q'kk'p} \delta_{p-q'-q} \delta_{p-k-k'} \text{tr} \left[ P_R \frac{i(k'-q' + M_2)}{(k-q')^2 - M_2^2} P_R \frac{kP_L(\phi + M_1)P_L \phi'}{2} \right] \]

\[ \times (1 - f_\ell(k) + f_\phi(k')) (1 - f_\ell(q) + f_\phi(q')) \delta f_{N1}(p). \]  

(105)

In the limit \( M_1 \gg T \), where the equilibrium distribution functions for active leptons and Higgs bosons may be neglected, this result agrees with Eq. \((91)\).
Now for the wave-function contribution, we observe that there is no corresponding explicit two-loop diagram in Eq. (93a). However, we note that we can approximately solve Eq. (93b) as

\[
\text{<} = \text{<} + \text{<} + \text{<} + \text{<} + \cdots \tag{106}
\]

Substituting this approximation to the RHN propagator into the one-loop diagram in Eq. (93a) then yields a two-loop diagram. The wave function contribution can then be calculated analogously to the vertex \[87\]. In order to compare with the results of Section 6, we deviate slightly from that route and first consider separately the correction to the propagator to the RHN, which is given by

\[
iS_{N_{ij}}^{wv} = -cd iS_{N_i}^{ac} i\Sigma_{N_{ij}}^{cd} iS_{N_j}^{cb}, \tag{107}\]

what is represented diagrammatically by the loop diagrams on the right-hand side of Eq. (106). This correction then enters \[\Sigma_{<,}>\] that appears in the collision term of the kinetic equation for the active leptons \[97b\]. We specifically write down the term

\[
i\delta S_{N_i}^{<,>} = i\delta S_{N_i} \left( i\Sigma_{N_{ij}}^{<} - i\Sigma_{N_{ij}}^{T} \right) iS_{N_j}^{T} - iS_{N_i}^{T} i\Sigma_{N_{ij}}^{<} iS_{N_j}^{T} - iS_{N_i}^{T} i\Sigma_{N_{ij}}^{T} iS_{N_j}^{<}, \tag{108}\]

where \[iS_{N_{ij}}^{<}\] follows from replacing \[<,>\], \[T \leftrightarrow \bar{T}\]. Next, from the KMS relation, we know that any nonvanishing contributions to the collision term of active leptons at this order must be proportional to deviations from equilibrium. We therefore expand in \[\delta S_{N_i}^{<,>} = \delta S_{N_i}^{<} - \delta S_{N_i}^{eq<,>}\], where \[\delta S_{N_i}^{<} = \delta S_{N_i}^{>}\].

In this expression, we have dropped terms that contain products of on-shell \(\delta\)-functions pertaining to \[N_i\] and \[N_j\] with \(i \neq j\) that cannot be simultaneously satisfied. The next simplification is to drop the dispersive part from \[\Sigma_{N_i}^{T}\], such that only the absorptive, cut part is left. This is in accordance with our approximation of neglecting corrections to the dispersion relations, i.e. dropping the term involving \(\Sigma_{H}^{T}\) in Eq. (37). Furthermore, since the dispersive parts satisfy \(\Sigma_{N_i}^{T,\text{disp}} = -\Sigma_{N_i}^{T,\text{disp}}\), it can be seen that these lead to contributions to the source term for the asymmetry that cancel in total. We therefore arrive at

\[
i\delta S_{N_{ij}}^{<,>} \to i\delta S_{N_i} i\Sigma_{N_{ij}}^{A} iS_{N_j}^{T} - iS_{N_i}^{T} i\Sigma_{N_{ij}}^{A} i\delta S_{N_j}. \tag{110}\]

Note that the equality of the \[<,>\] propagators can be verified using the relation \[20\].

Substituting this result into

\[
i\mathcal{P}_{\ell}^{wvab}(q) = \sum_{ij} P_{R} Y_{i} Y_{j} \int_{pq} \delta_{p-q-q'} P_{R} iS_{N_{ij}}^{wvab}(p) P_{L} i\Delta_{\phi}^{ba}(q'), \tag{111}\]

33
we find for the wave-function contribution to the CP-violating source

\[ S^{\text{wv}} = \int q \left[ i\Sigma_{\ell}^{\text{wv}\epsilon}(q)iS_{\ell}^{\epsilon}(q) - i\Sigma_{\ell}^{\text{wv}<}(q)iS_{\ell}^{\epsilon}(q) \right] \]

\[ = \int pqq' \delta(p - q - q') \sum_{ij} Y_i^* Y_j tr \left[ P_R i\delta S_{Nij}(p) i\Delta_{\phi}^{<}(p') - i\Delta_{\phi}^{>}(p) \right] \]

\[ = -2 \int p Y_i^* Y_j \left[ P_R i\delta S_{Nij}(p) i\Delta_{\phi}^{A}(p) \right] \]

\[ = -8 g_w \int p M_1 M_2 \left[ Y_1^* Y_2^2 - Y_2^* Y_1^2 \right] \frac{\dot{\Sigma}_A(p) + \dot{\Sigma}_A^a(p)}{M_2 - M_1} \delta f_{N1}(p) + 1 \leftrightarrow 2. \quad (112) \]

When substituting Eq. (102) for \( \dot{\Sigma}^A \) and noting that the present results accounts for the non-equilibrium sources from both RHNs as well as that it includes quantum-statistical factors, we once more note agreement with Eq. (91).

5.4 Diagrammatic interpretation

We note eventually that the expressions derived in the CTP approach justify the diagrammatic representation in Figure 3. The diagrams obtained from amputating the lines with a gap correspond to the self-energies \( \dot{\Sigma} \), and the line with a gap to the propagator that is attached to these in the collision term of the Kadanoff-Baym equations. This external line is then closed by taking the spinor trace as well as the momentum integral. The diagrams with closed lines can therefore be identified with contributions to the 2PI effective action as in Eqs. (27), (28) and (92), and the self energies \( \dot{\Sigma}_{\text{LO}}^A, \dot{\Sigma}_{\text{vert}}^A \) and \( \dot{\Sigma}_{Nij} \) follow from functional differentiation as in Eq. (29b). In contrast, \( \dot{\Sigma}_{\text{wvab}}^A \), Eq. (111), cannot be directly obtained from the 2PI effective action because the vacuum graph that would give rise to this contribution by functional differentiation is not 2PI. Rather, when recalling that the diagrams in the 2PI effective action are in terms of full propagators, \( \dot{\Sigma}_{\text{wvab}}^A \) arises from the leading insertion of \( \dot{\Sigma}_{Nij} \) into the RHN propagator as indicated in Eq. (106). On the other hand, we may expect that we resolve the issues encountered in Section 3.2 concerning the mass-degenerate limit when accurately resumming these insertions. This resummation is readily accomplished by the Schwinger-Dyson equations (93b), and we will make use of this fact in the following section on resonant leptogenesis.

6 Resonant effects in out-of-equilibrium decay scenarios

In Section 5, we have derived the CP-violating source term in the CTP approach based on first principles of QFT. In view of the problems in telling apart one-to-two and two-to-two matrix elements in the Boltzmann approach and thus to comply with the CPT theorem, the CTP approach appears to be more systematic. Here, we show how the kinetic equations derived in the CTP framework also lead to a resolution of the questions pertaining to the resonant limit \( M_1 \to M_2 \) of leptogenesis. In particular, as discussed in Section 3.2, a satisfactory treatment should address the calculation of the asymmetry in the parametric regime where the mass splitting is comparable to or smaller than the decay width of the RHNs.

Within the CTP framework, rather than inserting the spectral self energy into the propagator of the RHN to first or any to finite order, the solution to the Schwinger-Dyson equations in an appropriate approximation automatically leads to the all-order resummation of the effects from the finite lifetime
of the RHN. Compared to the approaches presented Section 3.2 that are inconclusive to this end, the Schwinger-Dyson equation on the CTP has two features that are crucial in order to lead to correct predictions throughout the parametric range of the seesaw mechanism, including the resonant regime:

- The state of the RHNs and their flavour correlations is solved for consistently. This is important in the extremely degenerate regime where the correlations turn out to be of the same magnitude as the deviations of the flavour-diagonal distributions from equilibrium.

- Off-diagonal correlations take a finite time to build up. In the strong washout regime, under circumstances to be specified in more detail below, this typically is not a concern. This time-dependence is however captured by the Schwinger-Dyson equations such that these are applicable also to the weak washout regime or to scenarios of leptogenesis from the oscillations of relativistic RHNs with masses below the electroweak scale [39].

Now, in general, the self-consistent solutions to the nonequilibrium portion of the Wightman functions of the RHNs \( i\partial S^<_{N} \) exhibit a more complicated spinor structure than the tree-level, equilibrium solutions [8]. This is because the \( CP \)-violating effects, besides creating charge asymmetries, also lead to asymmetries in axial charges and densities. The latter are of material importance in models of flavoured leptogenesis (i.e. when all or an important part of the asymmetry in active leptons is purely flavoured in first place) when the RHNs are relativistic. This combination of circumstances is characteristic for leptogenesis from oscillations of relativistic RHNs [35–38].

In order to capture the details of these chirality and helicity effects in the RHN sector, a decomposition of the spinor fields following the methods developed for electroweak baryogenesis in Refs. [54, 55] must be applied, which we review in Section 8 of the present work. For leptogenesis, this procedure has been carried out and is reported in detail in Refs. [39, 70]. Here, for simplicity, we restrict to the nonrelativistic regime, where the axial and pseudoscalar components of \( i\partial S^<_{N} \) are small compared to the vectorial and scalar ones. We can hence approximate that \( \text{tr} \left[ P_L R i \partial S^<_{N} (p) \right] = 2 M \delta f_N(p) \) and \( \text{tr} \left[ V P_L R i \partial S^<_{N} (p) \right] = -2 p \cdot V \delta f_N(p) \) as can be seen from Eqs. (B8), where \( M = (M_1 + M_2)/2 \) and we assume \( |M_1 - M_2| \ll M \). With these approximations, integrating Eq. (95a) over \( dp^0 \) and taking the trace as in Eq. (96b) leads to

\[
\frac{d}{dt} f_N(p) + \frac{i}{2p^0} \left[ M^2, \delta f_N \right] = -g_w \frac{p \cdot \Sigma^A}{p^0} \left\{ \text{Re}[Y^* Y^t], \delta f_N \right\}, \tag{113}
\]

where \( f_N \) and \( \delta f_N \) now take values of matrices in the flavour space of RHNs, \( Y \) is understood as a column vector and the superscript \( t \) stands for transposition. We note that through taking the trace, we have added together the different polarization states for the right-handed neutrinos. In the nonrelativistic regime, differences in the decay asymmetry for the two polarization states in the finite temperature medium remain small such that it is justified not to track the helicity asymmetry. Corrections to this treatment that become relevant toward the relativistic regime are covered by the derivations of Refs. [39, 70]. We can view Eq. (113) as a reduced version (through taking a trace and applying the nonrelativistic approximation) of the Schwinger-Dyson equation (93a) that carries out the necessary resummations (i.e. the one-loop insertions into the RHN propagator accounting for the finite width) for the mass-degenerate regime.

To account for the expansion of the Universe, we apply the rule (84) to Eq. (113), which yields

\[
\tilde{M} \frac{d}{dz} \delta f_N + \frac{a_R \tilde{z}}{2p^0} i [M^2, \delta f_N] + \tilde{M} \frac{d}{dz} f_N^{\text{eq}} = -a_R \tilde{z} g_w \left\{ \text{Re}[Y^* Y^t] \frac{p \cdot \Sigma^A}{p^0}, \delta f_N \right\}, \tag{114}
\]

where \( f_N^{\text{eq}} \) is the equilibrium Fermi-Dirac distribution of the RHNs. We have explicitly decomposed the temporal derivative acting on \( f_N = f_N^{\text{eq}} + \delta f_N \), such that the term \( d/dz f_N^{\text{eq}} \) mediates the deviation from
equilibrium that drives leptogenesis. Note that Eq. (114) is structurally similar to the equation that has been studied in Ref. [97] for a system of mixing scalar particles.

Now, since $\delta f_N$ takes the values of Hermitian matrices, it has four degrees of freedom, for which we can write down coupled first-order differential equations [73–75]. The matrix associated with the homogeneous part of these differential equations has four real eigenvalues, which depend on $M_1^2 - M_2^2$ as well as $\text{Re}[Y_i^* Y^i] p \cdot \Sigma^A (p)$. Provided these eigenvalues are large compared to $(\bar{\Gamma}/2)|M|$, we can neglect the first term in Eq. (114). We note that for two RHNs, these eigenvalues can be worked out analytically [75], but simple sufficient criteria are that $|Y|^2 / (8 \pi) \bar{M} \gg H$ or $|M_1^2 - M_2^2| / M \gg H$, implying that the characteristic scales of relaxation toward equilibrium or flavour oscillations are faster than the Hubble rate that sets the scale for the temporal derivatives. As a consequence, the system can then be solved algebraically, as we discuss below. Before we carry out this task, we note that one may choose initial conditions where $d/dt f_{Nij} \sim (M_1^2 - M_2^2) / M f_{Nij}$, i.e. where the derivative is dominated by flavour oscillations. However, such initial conditions are not realized in the strong washout scenario because when $|M_1^2 - M_2^2| / (M_1 + M_2) \gg H$, the RHNs drop continuously drop out-of-equilibrium over many oscillation times such that no coherent oscillations will occur [75]. Even if one chooses initial conditions leading to coherent oscillations, these will average about the solution to the algebraic system such that they can be neglected in calculations of the resulting lepton asymmetry [70]. Nonetheless, we note that outside the strong washout regime, the time-dependence of the RHN flavour correlations have to be accounted for by keeping the time-derivative in Eq. (114).

Dropping the derivative in Eq. (114) as per the discussion above, we obtain the algebraic solution [75]

$$\delta f_{Nij} = \frac{M}{2D} ([YY^*]_{ij} + [YY^*]_{ij}) (|YY^*|_{ii} + |YY^*|_{jj})$$

$$\times [\bar{M}^2 \bar{\Gamma} (|YY^*|_{ii} + |YY^*|_{jj}) - i(M_1^2 - M_2^2)] \times \frac{M^2}{a_R z} \frac{d}{dz} f_{N}^q,$$

where

$$D = |YY^*|_{11} |YY^*|_{22} (M_1^2 - M_2^2)^2$$

$$+ \bar{M}^4 \bar{\Gamma}^2 ([YY^*]_{11} + [YY^*]_{22})^2 ([YY^*]_{11} |YY^*|_{22} - \text{Re}([YY^*]_{12})^2),$$

and $\bar{\Gamma} = 1/(8 \pi)$. Substituting this into the source term, we obtain

$$S^v = -2 \int \sum_{ij} Y^*_i Y_j p \text{tr } P_{Ri} \delta S_{Nij} (p) P_{Lj} \Sigma^A (p),$$

which generalizes the results [91] and [112] and now also applies in the extremely degenerate regime.

We also note that in Refs. [76, 98] it is argued that besides the $CP$-violating source calculated in this section, the source according to Eq. (64) is a separate and distinct contribution. Obtaining the source term from the 2PI effective action, we find however that the source terms (112) and (117) are the same contribution and agree within the range of applicability of Eq. (112), i.e. when $|M_1 - M_2| \gg |\Gamma_{ij}|$. Certainly, further investigations into this matter, as have been initiated in Ref. [99], would therefore be of interest.

In order to verify that the above results are indeed recovered from this more general expression, we note that outside of the extremely degenerate regime the diagonal components of the matrix of the nonequilibrium RHN neutrino distributions dominate the off-diagonal ones. These are then related to the rate at which the RHNs drop out of equilibrium as

$$\frac{1}{|Y_i|^2} \frac{M^2}{a_R z} \frac{d}{dz} f_{N}^q = 2 \frac{p \cdot \Sigma^A}{p^0} \delta f_{Ni}.$$
Substituting this into Eq. (115), we obtain

$$\delta f_{Nij} = -\frac{i}{2} \frac{Y_i^* Y_j^* + Y_j^* Y_i^*}{M_i^2 - M_j^2} 2g_w \frac{p \cdot \hat{\Sigma}^A}{p^0} \tilde{M} (\delta f_{N1}(p) - \delta f_{Nj}(p)),$$

which, when used in Eq. (117) leads to

$$S^{ww} = i \left[ Y_1^* Y_2^* - Y_1^2 Y_2^* \right] \int \frac{\tilde{M}^2}{M_i^2 - M_2^2} 8\Sigma^{A0} \hat{\Sigma}^{A0} (\delta f_{N1}(p) - \delta f_{N2}(p)),$$

in agreement with the results (91) and (112).

Noting that in the strong washout regime, the evolution of the lepton asymmetry can be expressed as in Eq. (123) below and comparing with Eq. (115) and (117), we find for the decay asymmetry [75]

$$\varepsilon = \frac{i \tilde{M}^2}{2D} (M_i^2 - M_2^2) \tilde{M} (|Y_1|^2 + |Y_2|^2) (Y_1^* Y_2^* - Y_1^2 Y_2^*).$$

7 Analytic approximation to the strong washout solution

Baryogenesis calculations often rely on numerical solutions to the fluid equations for the evolution of the asymmetry. Notably, the solutions to Eqs. (85) with the rates (86) for leptogenesis have relatively simple analytic approximations, and the derivation of these offers some insights into the mechanism [100].

Provided $(Y_{N1} - Y_{N1}^{\text{eq}}) \ll Y_{N1}^{\text{eq}}$ throughout the times we are interested in, we can readily write down the approximate solution to Eq. (85a) as

$$(Y_{N1} - Y_{N1}^{\text{eq}}) = -\frac{1}{\bar{\gamma}} \frac{d}{dz} Y_{N1}^{\text{eq}}.$$ (122)

Our parametrization is chosen such that a derivative with respect to $z$ can be counted as order one in the radiation-dominated Universe, while the parameter $\bar{\gamma}$ is given in Eq. (86a). Above inequality is therefore satisfied if $\bar{\gamma}(z = 1) \gg 1$, (what as a more physical relation implies $|\gamma|_{T=M_1} \gg H$, i.e. the reaction rate is much faster than the Hubble rate when $z \sim 1$). The time where $z \sim 1$ is of relevance because this is where a Fermi-Dirac distribution for fermions of mass $M_1$ in comoving momentum that is subject to redshift deviates from the equilibrium form and because the lepton-number violating decay and inverse decay rates acquire a factor of exponential Maxwell suppression $\sim \exp(-M_1/T)$, such that these processes freeze out soon after.

Substituting the approximation (122) into Eq. (85b), we next obtain

$$\frac{dY_{\Delta \ell}}{dz} = -\varepsilon \frac{1}{g_w} \frac{d}{dz} Y_{N1}^{\text{eq}} - \bar{W}(\frac{3}{2}) Y_{\Delta \ell},$$

where $W(\frac{3}{2}) = \frac{3}{2} \bar{W}$, which has the formal solution

$$Y_{\Delta \ell}(z) = -\int_0^z dz' \frac{\bar{\gamma}}{g_w} \frac{d}{dz'} Y_{N1}^{\text{eq}}(z') \exp \left\{-\int_{z'}^z dz'' \bar{W}(\frac{3}{2})(z'') \right\}.$$ (124)

This integral can be approximately evaluated using Laplace’s method, where besides the explicit exponential in the above integrand, there is also an exponential factor contained within $Y_{N1}^{\text{eq}}$, cf. Eq. (75). The full exponent is extremal for

$$\bar{W}(\frac{3}{2})(z_f) = 1,$$ (125)
where we attach a subscript \( f \) to the solution for \( z \) because around this point in time, lepton-number violating interactions freeze out. It is as useful as customary to define the washout strength

\[
K = \left. \frac{\Gamma_{N_1 \to e \phi^* eCP \phi}}{H} \right|_{T=M_1} = \frac{|Y_1|^2 M_1}{8\pi H} = \frac{|Y_1|^2 a_R}{8\pi M_1},
\]

implying that for \( K \gg 1 \), the lepton-number violating interactions are close to equilibrium before they freeze out, what corresponds to strong washout. We can then express the Laplace approximation to Eq. (124) as

\[
Y_{\Delta \ell} = \varepsilon \frac{15\sqrt{3}}{g_u g_s \sqrt{K}} \pi^{-\frac{9}{4}} 2^{-\frac{3}{2}} z_f^{\frac{1}{4}} \exp \left\{ -\frac{z_f}{2} - \int_{z_f}^{\infty} dz' \frac{\bar{W}_{(\frac{3}{2})} (z')}{z_f} \right\}.
\]

When substituting back the solution to Eq. (125) [The integral in the exponent of Eq. (127) evaluates to one for \( z_f \gg 1 \)], we obtain

\[
Y_{\Delta \ell} = \varepsilon \frac{5\sqrt{2}}{g_u g_s K \pi^2 z_f}.
\]

While larger \( K \) increase \( z_f \), this only occurs logarithmically slow. Apart from this small dependence, most significant about this result is its behaviour \( \sim 1/K \), implying that the outcome of leptogenesis in the strong washout regime has a simple dependence on two parameters: the decay asymmetry \( \varepsilon \) and the washout strength \( K \). A more accurate analytic approximation is derived in Ref. [79].

We note that eventually, sphaleron processes convert baryon-minus-lepton number \( B - L \) to baryon number \( B \) as [101, 102]

\[
Y_B = \frac{4}{77} \frac{T^2}{869 T^2 + 333 |\sqrt{2} \phi|^2} Y_{B-L},
\]

where \( \langle |\phi| \rangle \) is the vacuum expectation value of the Higgs field through the electroweak crossover, and for simplicity, we assume here that the asymmetry is unflavoured, i.e. the lepton number is the same in all generations of SM leptons. The generalized expression for the more realistic case of flavoured asymmetries can also be found in Ref. [102]. Taking from Ref. [103] the temperature of 131 GeV for sphaleron freeze out and \( \langle |\sqrt{2} \phi| \rangle \sim 170 \) GeV at that temperature, one finds \( Y_B \approx 0.343 Y_{B-L} \), to be compared with the relation \( Y_B \approx 28/79 Y_{B-L} \approx 0.354 Y_{B-L} \) in the symmetric phase where \( \langle \phi \rangle = 0 \). Note also that because of our counting of the weak isodoublet charges, \( Y_{B-L} = 2 Y_{\Delta \ell} \). Above result for \( Y_B \) can be compared with the observed value ([4]) when switching from entropy to photon normalization.

8 Baryogenesis at phase boundaries

For baryogenesis from out-of-equilibrium decays or inverse decays, the deviation from equilibrium that is necessary for creating an asymmetry is controlled by the expansion of the Universe. Another possibility for cosmology to create non-equilibrium environments are phase boundaries. These may be present e.g. in the form of domain walls or bubble walls in first-order phase transitions (i.e. where bubbles containing the true ground state nucleate and expand into the phase of the false ground state). Across such boundaries, particle masses may change, leading to a deflection of their trajectories already at the classical level. In combination with quantum interference, this may lead to \( CP \)-violating currents. In this section, we discuss the computation of such currents using CTP methods. A detailed review on these matters is provided by Ref. [105].
Before discussing these technicalities, we very briefly explain the basic picture of electroweak baryogenesis that is covered in more detail in the review articles [105–108]. The possibility of baryon number generation during the electroweak phase transition was first proposed in Ref. [15]. Early work on the presently favoured scenario where axial asymmetries generated in the wall of the first order phase transition diffuse ahead of it, where they get converted into baryons by sphaleron processes is reported in Ref. [109], while the remaining papers on electroweak baryogenesis referred to in this section further develop this mechanism in numerous details. According to these works, electroweak baryogenesis proceeds as follows:

- Electroweak symmetry breaking occurs through a first order phase transition. This does not happen in the SM, where it would require the mass of the Higgs boson to be below 70 GeV [16, 17]. One therefore has to resort to extensions of the SM, that can be probed by collider experiments at the electroweak scale.

- In the bubble wall, the gradients of the scalar field expectation values in the Higgs sector generate CP-violating currents as we discuss in Section 8.3.

- The particle flows thus generated undergo rescatterings such that their motion effectively is diffusive. Of particular importance is the diffusion ahead of the bubble wall into the symmetric phase, where baryon-number violating processes are occurring. Moreover, the charges get transferred into other particles, most importantly the left-handed fermions of the SM, by scatterings mediated e.g. by Yukawa couplings and strong sphaleron (thermal QCD instantons) processes. The left-handed charge is then transferred by anomalous processes, so-called weak sphalerons, into baryon-plus-lepton number. We review these matters briefly in Section 8.4.

- Eventually, the bubble containing the broken electroweak phase captures the baryon-plus-lepton charge that is present in its wake. In the broken phase, sphaleron processes freeze out (provided the phase transition is strong enough) such that the captured baryon charge can possibly explain the matter-antimatter asymmetry.

### 8.1 Classical forces

The action of a classical point particle with spacetime-dependent mass \( m(x) \) is given by

\[
S = \int_{\tau_A}^{\tau_B} d\tau \left[ -m(x)c^2 \right],
\]  

(130)

where \( \tau \in [\tau_A, \tau_B] \) is proper time and where we have reintroduced the speed of light \( c \) for the time being. Imposing stationarity under variations with respect to \( \delta x^\mu(\tau) \) (and minding that \( d\tau^2 = dx_\mu dx^\mu/c^2 \)) leads to the equation

\[
\frac{dp^\mu}{d\tau} = c^2 \frac{dm}{dx^\mu},
\]  

(131)

where \( p^\mu = m \frac{dx^\mu}{d\tau} \). We can therefore identify \( c^2 \frac{dm}{dx^\mu} \) with a classical four-force. Alternatively, we may also derive this results by

\[
p^\mu \frac{d}{d\tau} p_\mu = \frac{1}{2} \frac{d}{d\tau} p^2 = \frac{1}{2} \frac{d}{d\tau} m^2 c^2 = mc^2 \frac{dm}{dx^\mu} \frac{dx^\mu}{d\tau} = c^2 p^\mu \frac{dm}{dx^\mu}.
\]  

(132)
At the level of kinetic theory, we recover the classical force when considering a distribution function $g(x, p)$ and applying the Liouville theorem, 
\[
\frac{d}{d\tau}g(x, p) = \frac{1}{m}p\mu \frac{\partial g(x, p)}{\partial x^\mu} + \frac{dp^\mu}{d\tau} \frac{\partial g(x, p)}{\partial p^\mu} = 0 \, ,
\]
(133)
where $p^\mu = m dx^\mu / d\tau$. Substituting Eq. (131), we obtain the driving force that the mass gradients apply to the distributions. The latter are often of the quasi-particle form, i.e. they approximately fulfill the on-shell relation 
\[
g(x, p) = 2\pi \delta(p^2 - m^2) f(x, p) \, .
\]
(134)
Kinetic equations in the standard form are then obtained when taking the zeroth moment of Eq. (133) equation by integration over $2dp^0$, such that 
\[
\frac{1}{p^0} \left( u^\mu \frac{\partial f(x, p)}{\partial x^\mu} + \frac{dp^\mu}{d\tau} \frac{\partial f(x, p)}{\partial p^\mu} \right) = 0 \, ,
\]
(135)
where $u^\mu = dx^\mu / d\tau$. The term in brackets is the same as the left-hand side of the Boltzmann equation (6).

### 8.2 Transport equations on the CTP

We next aim to work out how the force term emerges in a calculation based on the CTP approach. For simplicity, we first consider scalar particles. Proceeding in parallel to the fermionic case discussed in Section 2.2, we take the $>\equiv -+$ and $<\equiv +-$ components of the Schwinger-Dyson equation (31a) for scalar fields and obtain [54, 55] (setting $c = 1$ again) 
\[
\left[ p^2 - \frac{1}{4} \partial^2 + ip \cdot \partial - m^2 e^{-\frac{i}{2}\phi} \right] \Delta^{<, >} - \Pi^H e^{-\frac{i}{2}\phi} \Delta^{<, >} - \Pi^{<, >} e^{-\frac{i}{2}\phi} \Delta^H = \frac{1}{2} \left( \Pi^{>} e^{-\frac{i}{2}\phi} \Delta^{<} - \Pi^{<} e^{-\frac{i}{2}\phi} \Delta^{>} \right) \, .
\]
(136)
The Hermitian part of this yields the kinetic equation 
\[
p^\mu \partial_\mu i\Delta^{<, >} + (m^2 + \Pi^H) \sin \phi \Delta^{<, >} + \Pi^{<, >} \sin \phi \Delta^H = \frac{1}{2} \left( i\Pi^{>} \cos \phi \Delta^{<} - i\Pi^{<} \cos \phi \Delta^{>} \right)
\]
(137)
and the Antihermitian part the constraint equation 
\[
\left[ p^2 - \frac{1}{4} \partial^2 - (m^2 + \Pi^H) \cos \phi \right] \Delta^{<, >} - \Pi^{<, >} \cos \phi \Delta^H = -\frac{i}{2} \left( i\Pi^{>} \sin \phi \Delta^{<} - i\Pi^{<} \sin \phi \Delta^{>} \right) \, .
\]
(138)
Next, for the similar reasons as given for the fermions in Section 5, we neglect the terms involving $\Pi^H$ and $\Delta^H$ in the kinetic equation and expand it to first order in gradients, which yields 
\[
p^\mu \partial_\mu i\Delta^{<, >} + \frac{1}{2} \partial m^2 \frac{\partial i\Delta^{<, >}}{\partial x^\mu} = \frac{1}{2} \left( i\Pi^{>} i\Delta^{<} - i\Pi^{<} i\Delta^{>} \right) \, .
\]
(139)
Comparing the left-hand side of this CTP result with Eq. (133) from classical kinetic theory, we can identify $\partial m / \partial x^\mu$ with the four-force $dp^\mu / d\tau$.

The constraint equation (138), when truncated at zeroth order in gradients, is consistent with the tree-level solutions (B7). Substituting these into the kinetic equations and taking the zeroth moment therefore leads to 
\[
\frac{1}{2p^0} \left( p^\mu \partial_\mu f(x, p) + m \frac{\partial m}{\partial x^\mu} \frac{\partial f(x, p)}{\partial p_\mu} \right) = -\frac{1}{2} \int_0^\infty \frac{dp^0}{2\pi} \left( i\Pi^{>} i\Delta^{<} - i\Pi^{<} i\Delta^{>} \right) \, ,
\]
(140)
where the left-hand side agrees up to a prefactor with the result (135) from classical kinetic theory.
8.3 Quantum transport of fermions

For the time-dependent case of leptogenesis, we have simplified in Section 6 the spinor structure making use of non-relativistic approximations for the RHNs. Here, no simplifying assumptions in addition to the symmetry of the problem parallel to the boundary are taken. This reduces spinorial the problem to eight real degrees of freedom (from in general sixteen complex ones for a four-by-four matrix in spinor space).

Further, the calculation in above Section 8.2 for scalar fields is truncated at first order in gradients and does not account for flavour mixing in the presence of several species. Mixing particles often observe a global U(1) symmetry, such that $CP$ violation may only be present in terms of flavoured asymmetries with cancelling contributions to the total U(1) charge. For fermions, the Dirac mass terms violate chiral symmetry such that $CP$ violation can manifest itself even in single-flavour systems in terms of an axial asymmetry. We therefore consider in the following systems of mixing fermion flavours that exhibit both types of asymmetry.

Methodically, we follow here the developments on the spinor decomposition and gradient expansion in Refs. [54, 55, 94, 95, 110, 111] on electroweak baryogenesis. These have been adapted to leptogenesis in Ref. [70], thus leading to a relativistic generalization of the results in Section 6. Starting point are the Kadanoff-Baym equations for fermions in Wigner space [37] that we now aim to decompose within the background of the phase boundary into kinetic and constraint equations. It should be noted that many phenomenological papers compute the source term using the methods of Refs. [112, 113], that however do not rely on a detailed spinor decomposition. Also, the gradients are not expanded from the full spacetime dependent mass terms but are inserted perturbatively into the collision term. As a result, an apparent difference is that the sources computed in Refs. [54, 55, 94, 95, 110, 111] do not rely on a collision term, while the results from Refs. [112, 113] crucially depend on the finite width of the mixing particles.

In order to facilitate the decomposition the fermionic Kadanoff-Baym equations, we first make use of the symmetry of the problem. Without loss of generality, the bubble wall is assumed to propagate in $z$-direction, that we identify with the three-component of the position vector. (I.e., we write $z = x^3$ and $k^z = k^3$ but maintain writing $\gamma^3$ for notational appeal.) Angular momentum pointing in that direction is therefore conserved, such that for fermions, the $z$-component of the spin operator

$$S^z = \frac{1}{\tilde{k}^0} \left( \gamma^0 k^0 - \gamma^1 k^1 - \gamma^2 k^2 \right)$$

(141)

where

$$\tilde{k}^0 = \text{sign} k^0 \sqrt{(k^0)^2 - (k^1)^2 - (k^2)^2}$$

(142)

is a good quantum number. The Wightman function can hence be decomposed as

$$iS^{<,>} = \sum_{s = \pm 1} iS_s^{<,>} ,$$

(143)

where

$$iS_s^{<,>} = -P_s \left[ s \gamma^3 \gamma^5 g^{<,>} - s \gamma^3 g^{<,>} + \gamma^2 g^{<,>} - i \gamma^5 g^{<,>} \right] ,$$

(144)

with the spin projector

$$P_s = \frac{1}{2} (1 + s S^z) .$$

(145)

The spinor structures in Eq. (144) are chosen such as to commute with the spin operator.
As commented in Section 2.2, it is of interest to extract currents from the fermionic Wightman functions. These currents can readily be expressed in terms of the decomposition (144):

\[ g^+_0 + g^-_0 = -\frac{1}{2} \sum_{s=\pm} \text{tr} \left[ s \gamma^3 \gamma^5 iS^\langle s \rangle \right], \text{ charge density}, \]

\[ g^+_3 + g^-_3 = -\frac{1}{2} \sum_{s=\pm} \text{tr} \left[ s \gamma^3 iS^\langle s \rangle \right], \text{ axial charge density}, \]

\[ g^+_1 + g^-_1 = -\frac{1}{2} \sum_{s=\pm} \text{tr} \left[ iS^\langle s \rangle \right], \text{ scalar density}, \]

\[ g^+_2 + g^-_2 = -\frac{1}{2} \sum_{s=\pm} \text{tr} \left[ i\gamma^5 iS^\langle s \rangle \right], \text{ pseudoscalar density}, \]

(146)

and

\[ g^+_0 - g^-_0 = -\frac{1}{2} \sum_{s=\pm} \text{tr} \left[ s \gamma^3 \gamma^5 iS^\langle s \rangle \right], \text{ axial current density}, \]

\[ g^+_3 - g^-_3 = -\frac{1}{2} \sum_{s=\pm} \text{tr} \left[ s \gamma^3 iS^\langle s \rangle \right], \text{ current density}, \]

\[ g^+_1 - g^-_1 = -\frac{1}{2} \sum_{s=\pm} \text{tr} \left[ siS^\langle s \rangle \right], \text{ spin density}, \]

\[ g^+_2 - g^-_2 = -\frac{1}{2} \sum_{s=\pm} \text{tr} \left[ si\gamma^5 iS^\langle s \rangle \right], \text{ axial spin density}. \]

(147)

It is further useful to define

\[ \hat{k}^0 = k^0 - \frac{i}{2} \frac{k^0 \partial_z + k^\parallel \cdot \nabla}{k^0}, \quad \hat{k}^z = k^z - i \frac{\partial}{2 \partial z}. \]

(148)

Multiplication of the Kadanoff-Baym equation (37) by

\[ \frac{1}{2} \{ \mathbb{1}, s \gamma^3 \gamma^5, -is \gamma^3, -\gamma^5 \} \]

(149)

and taking the trace, one obtains

\[ 2ik^0 g_0^s - 2is \hat{k}^z g_3^s = 2iM^H e^{-i \frac{3}{2} \hat{\nabla} \cdot \hat{\nabla}} g_1^s - 2iM^A e^{-i \frac{3}{2} \hat{\nabla} \cdot \hat{\nabla}} g_2^s = 0, \]

(150a)

\[ 2ik^0 g_1^s - 2s \hat{k}^z g_2^s - 2iM^H e^{-i \frac{3}{2} \hat{\nabla} \cdot \hat{\nabla}} g_0^s + 2M^A e^{-i \frac{3}{2} \hat{\nabla} \cdot \hat{\nabla}} g_3^s = 0, \]

(150b)

\[ 2ik^0 g_2^s + 2s \hat{k}^z g_1^s - 2M^H e^{-i \frac{3}{2} \hat{\nabla} \cdot \hat{\nabla}} g_3^s + 2iM^A e^{-i \frac{3}{2} \hat{\nabla} \cdot \hat{\nabla}} g_0^s = 0, \]

(150c)

\[ 2ik^0 g_3^s - 2is \hat{k}^z g_0^s + 2M^H e^{-i \frac{3}{2} \hat{\nabla} \cdot \hat{\nabla}} g_2^s - 2M^A e^{-i \frac{3}{2} \hat{\nabla} \cdot \hat{\nabla}} g_3^s = 0. \]

(150d)

Here, we have set the collision term in Eq. (37) to zero. We will comment on this approximation in relation to other approaches pursued in phenomenological calculations toward the end of this section.

Before proceeding to decompose into kinetic and constraint equations, we change to working with quantities that have definite properties under transformations of the mass matrix \( M \). Let \( M_d = UM^\dagger \) be diagonal (such that both \( UMM^\dagger U \) and \( VM^\dagger MV^\dagger \) are diagonal as well) and define

\[ X = P_L \otimes V + P_R \otimes U = \frac{1}{2} \left[ \mathbb{1} \otimes (V + U) - \gamma^5 \otimes (V - U) \right], \]

(151a)

\[ Y = P_L \otimes U + P_R \otimes V = \frac{1}{2} \left[ \mathbb{1} \otimes (V + U) + \gamma^5 \otimes (V - U) \right]. \]

(151b)
Then, $\tilde{M}_d = X\tilde{M}Y^\dagger$ is diagonal, where we recall the definitions (24) and (25). When transforming the Dirac operator accordingly, such as to obtain a diagonal mass, the Wigner function transforms as

$$S_d = YSX^\dagger.$$  

(152)

As a consequence, the functions $g_i^s$ transform in a definite manner when arranged in chirality-blocks

$$g_{L,R}^s = g_0^s \mp g_3^s, \quad g_N = g_1^s + ig_2^s, \quad g_N^\dagger = g_1^s - ig_2^s,$$

such that these transform to the mass-diagonal basis as

$$g_R^d = Vg_RV^\dagger, \quad g_L^d = Ug_LU^\dagger, \quad g_N^d = Ug_NV^\dagger, \quad g_N^{d\dagger} = Vg_NU^\dagger.$$  

(154)

Taking the according linear combinations of Eqs. (150), the Kadanoff-Baym equations then read

$$i\left(\frac{k^0 g_R^s - s k^z g_R^s}{k^0} - M^\dagger e^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_N^s\right) = 0,$$

$$i\left(\frac{k^0 g_L^s + s k^z g_L^s}{k^0} - M e^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_N^s\right) = 0,$$

$$i\left(\frac{k^0 g_N^s}{k^0} + 2i k^z g_N^s - M e^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_R^s + ig_N e^{t\cdot \vec{\gamma}} \cdot \vec{\partial} M = 0,$$

$$i\left(\frac{k^0 \partial_\parallel + \kappa \cdot \nabla}{k^0} g_N^s\right) - 2i k^z g_N^s + M^\dagger e^{t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_R^s + ig_N e^{t\cdot \vec{\gamma}} \cdot \vec{\partial} M = 0.$$  

(155a)

(155b)

(155c)

(155d)

Taking the Hermitian part leads to the kinetic equations

$$\frac{k^0}{k^0} \partial_\parallel + \kappa \cdot \nabla g_R^s - s \frac{\partial}{\partial z} g_R^s - M^\dagger e^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_N^s + ig_N e^{t\cdot \vec{\gamma}} \cdot \vec{\partial} M = 0,$$

(156a)

$$\frac{k^0}{k^0} \partial_\parallel + \kappa \cdot \nabla g_L^s - s \frac{\partial}{\partial z} g_L^s - i Me^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_N^s + ig_N e^{t\cdot \vec{\gamma}} \cdot \vec{\partial} M = 0,$$

(156b)

$$\frac{k^0}{k^0} \partial_\parallel + \kappa \cdot \nabla g_N^s + 2i k^z g_N^s - i Me^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_R^s + ig_N e^{t\cdot \vec{\gamma}} \cdot \vec{\partial} M = 0,$$

(156c)

$$\frac{k^0}{k^0} \partial_\parallel + \kappa \cdot \nabla g_N^s - 2i k^z g_N^s + M^\dagger e^{t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_R^s + ig_N e^{t\cdot \vec{\gamma}} \cdot \vec{\partial} M = 0.$$  

(156d)

Next, we move to a frame that is comoving with the phase boundary such that we can replace $\partial_t \to 0$ and make use of the isotropy parallel to the wall implying $\kappa \cdot \nabla \to 0$. From Eqs. (156c, 156d), it then follows that

$$g_N^s = \frac{1}{2sk^z} \left(M e^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_R^s - g_L e^{t\cdot \vec{\gamma}} \cdot \vec{\partial}_k \vec{\partial} M\right),$$

(157)

which, substituted in Eqs. (156a) and (156b) leads to

$$\frac{\partial}{\partial z} g_L^s - \frac{i}{2} Me^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k \frac{1}{k^z} \left(g_R e^{t\cdot \vec{\gamma}} \cdot \vec{\partial}_k \vec{\partial} M^\dagger - M^\dagger e^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_R^s\right)$$

$$+ \frac{1}{2} \frac{1}{k^z} \left(M e^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_R^s - g_L e^{t\cdot \vec{\gamma}} \cdot \vec{\partial}_k \vec{\partial} M\right) e^{t\cdot \vec{\gamma}} \cdot \vec{\partial}_k \vec{\partial} M^\dagger = 0,$$

(158a)

$$\frac{\partial}{\partial z} g_R^s - \frac{i}{2} M^\dagger e^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k \frac{1}{k^z} \left(M e^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_R^s - g_L e^{t\cdot \vec{\gamma}} \cdot \vec{\partial}_k \vec{\partial} M\right)$$

$$+ \frac{1}{2} \frac{1}{k^z} \left(g_R e^{t\cdot \vec{\gamma}} \cdot \vec{\partial}_k \vec{\partial} M^\dagger - M^\dagger e^{-t\cdot \vec{\gamma}} \cdot \vec{\partial}_k g_R^s\right) e^{t\cdot \vec{\gamma}} \cdot \vec{\partial}_k \vec{\partial} M = 0.$$  

(158b)
These equations are symmetric under $R \leftrightarrow L$ and $M \leftrightarrow M^\dagger$. Expanding to second order in gradients (i.e. expanding the exponentiated operator up to second order), we therefore only quote the equation for $g_L$:

\[
\begin{align*}
\frac{k^z}{2} \frac{\partial}{\partial z} g_L^s + \frac{i}{2} \left[ [M M^\dagger, g_L^s] \right] \\
- \frac{1}{4} \left\{ (M M^\dagger) ', \partial_k g_L^s \right\} - \frac{1}{4k^z} \left( M' g_R^s M^\dagger + M g_R^s M^\dagger' \right) + \frac{1}{4k^z} \left( M' M^\dagger g_R^s + g_L^s M M^\dagger' \right) \\
+ \frac{i}{8} \left( M'' M^\dagger \partial_k^2 g_L^s - \partial_k^2 g_L^s M M^\dagger' \right) - \frac{i}{8} \left( m'' \partial_k^2 \frac{g_R^s}{k^z} - \partial_k^2 \frac{g_R^s}{k^z} M M^\dagger' \right) \\
- \frac{i}{16} \left\{ (M M^\dagger) '' , \partial_k^2 g_L^s \right\} + \frac{i}{8k^z} \left[ M'M^\dagger', \partial_k g_L^s \right] = 0.
\end{align*}
\] (159)

For reference in the further discussion, we label here some of the particular terms with the tags under the braces.

**Semiclassical force for a single flavour** Considering a single flavour, it turns out that there is no $CP$-violating source at first order in derivatives. At second order, the terms involving commutators in Eq. (159) vanish, and we are left with the semiclassical force. The relevant linear combination of the left and right chiral as well as spin-dependent quantities is the one that leads to the axial current

\[
\text{tr} \left[ \gamma^3 \gamma^5 i S^{<,>} \right] = 2 \left( g_0^+ - g_0^- \right) = j^5 z.
\] (160)

Taking the corresponding combination from Eq. (159) and dropping the terms that do not contribute leads to

\[
k^z \frac{\partial}{\partial z} \left[ 2g_0^+ - 2g_0^- - (|m|^2)' \partial_k^2 \left( g_0^+ - g_0^- \right) - \frac{1}{2} \left( |m|^2 \vartheta' \right)' \partial_k^2 \frac{g_0^+ - g_0^-}{k^z} \right] = 0,
\] (161)

where we have parametrized $m = |m| \exp(i \vartheta)$, such that

\[
(m'' m^* - mm'') = 2i \left( |m|^2 \vartheta' \right)'.
\] (162)

To further evaluate Eq. (161), we integrate over $dp^0$ in order arrive at a compact form of the fluid equations. We note that

\[
\int dp^0 \frac{1}{2\pi} g_0^s = \frac{1}{2} \frac{\bar{p}^0}{p^0} \left( f^s(p) - \bar{f}^s(p) \right),
\] (163a)

\[
\int dp^0 \frac{1}{2\pi} g_3^s = \frac{s p^3}{2 |p^0|} \left( f^s(p) + \bar{f}^s(p) \right).
\] (163b)

We further define

\[
h^s(p) = f^s(p) - \bar{f}^s(p)
\] (164)
appearing in Eq. (163a) as the charge distribution function for particles with a certain spin orientation perpendicular to the boundary. Taking the zeroth moment of Eq. (161) eventually leads to

\[ \frac{k^0}{s^0} \left( k^z \frac{\partial}{\partial z} h^s - \frac{1}{2} (|m|^2) ( |m|^2 \partial_k s^k) - \frac{s}{2} ( |m|^2 \partial^r) \right) \frac{2}{|k^0|} \partial_k s^k = 0, \]  

(165)

where the second term acts as a semiclassical force. This interpretation follows by comparison with e.g. Eq. (140). Therefore, in the wall frame, there are spin-dependent charge distributions present, and these are opposite for the two different spin states. In the plasma frame, this leads to an axial current density, i.e. \( k^z \frac{\partial}{\partial z} (h^+ + h^-) \to \partial_{\mu} j^{\mu} \) when replacing this term by a Lorentz-covariant form.

**Two flavours: resonant enhancement**  For two fermion flavours mixing in the background of the bubble wall, an axial current can also be generated when neglecting the semiclassical force term, which can be justified due to the possible resonant enhancement even in the presence of a moderate mass degeneracy. This axial current arises from the interplay of the mixing terms and the classical force. Computing it requires a numerical solution of Eq. (159) or an analytic approximation.

In order to obtain the latter, we recall that in Section 5 we found suitable solutions for the system of mixing sterile neutrinos in leptogenesis by neglecting the derivative terms in the kinetic equations, provided the oscillation frequency or the damping rate exceed the time-dependence of the system. In the present case of mixing in the background of a phase boundary, we may use a corresponding approximation provided that the oscillation length \( \sim 1/|m_1 - m_2| \) is small compared to the size of the boundary wall \( \ell_w \). Besides the wall width, also collision terms that are neglected in Eqs. (150) should be expected to cap the resonant enhancement, cf. Section 3 on leptogenesis.

Even when applying these simplifications, the result for the axial current generated from a general space-dependent mass matrix remains involved. In order to compare with the relevant results derived for resonant mixing scenarios that have drawn some attention in the historic context of the Minimally Supersymmetric Standard Model [111–113, 117–119], we choose a mass matrix similar to what one encounters in that setup,

\[ M = \begin{pmatrix} m_1 & e^{i \varphi} v_b \\ v_a & m_2 \end{pmatrix} \to \tilde{M} = \begin{pmatrix} m_1 & v_a P_L + e^{i \varphi} v_b P_R \\ v_a P_R + v_b e^{-i \varphi} P_L & m_2 \end{pmatrix} = \begin{pmatrix} m_1 & 0 \\ 0 & m_2 \end{pmatrix} + \delta \tilde{M}. \]  

(166)

The spatial dependence resides in \( v_{a,b} \) that smoothly change from zero in the phase of electroweak symmetry across the bubble wall to nonvanishing values in the broken phase. The masses \( m_{1,2} \) are assumed constant.

We then take Eq. (159) as well as the corresponding one obtained via \( g_L \leftrightarrow g_R \) and \( M \leftrightarrow M^\dagger \) and neglect the derivatives with respect to \( z \) acting on \( g_{LR}^s \). As a zeroth order input, we set \( g_{0,3}^{(0)_{ii}} \neq 0 \) and \( g_{0,3}^{(0)_{ij}} = 0 \) for \( i \neq j \), where the relation with \( g_{LR}^s \) is given by Eq. (153). Then, we compute the off-diagonal elements \( g_{0,3}^{(1)_{ij}} \) for \( i \neq j \) as a first perturbation from substituting these into the mixing term and substituting \( g_{0,3}^{(0)_{ij}} \) into the gradient-mixing and the classical-force terms while neglecting all other terms appearing in the kinetic equation. We eventually substitute \( g_{0,3}^{(1)_{ij}} \) into the gradient-mixing and the classical-force terms to obtain from the diagonal elements the divergence of the current

\[ k^z \frac{\partial}{\partial z} g_{011} = k^z \frac{\partial}{\partial z} (g_L + g_R)_{11} = -k^z \frac{\partial}{\partial z} (g_L + g_R)_{22} = \frac{1}{2} \frac{m_1 m_2 v_a' v_b' \sin \varphi}{m_1^2 - m_2^2} \left( \frac{1}{k^z} \frac{\partial}{\partial k^z} g_{311} - \frac{1}{2k^z^2} (g_{311} + g_{322}) \right). \]  

(167)

Upon partial integration in \( dk^z \), this can be cast into the more appealing form

\[ k^z \frac{\partial}{\partial z} g_{011} = \frac{1}{4k^z^2} \frac{m_1 m_2 v_a' v_b' \sin \varphi}{m_1^2 - m_2^2} (g_{311} - g_{322}). \]  

(168)
When comparing with the semiclassical force (165) for a single fermion flavour, we observe here two orders in derivatives as well. However, the denominator $m_1^2 - m_2^2$ leads to a resonant enhancement within the validity of the present approximation, i.e. $|m_1 - m_2|$ must no be smaller than $1/\ell_w$. We also note that the resulting total current is axial [cf. Eq. (147)] because the equilibrium form of $g_{3ii}^{s}$ is odd in the spin $s$, cf. Eq. (163b).

Since the present approximation treats the off-diagonal elements of the mass matrix as small perturbations, we also compare with an apparently related approach taken in Refs. [112, 113]. There, the Kadanoff-Baym equations are extended to include a collision term as on the right-hand side of Eq. (37). The latter is approximated by considering mass insertions as vertices. It turns out that this does not readily lead to a $CP$-violating source but only in case the propagators $iS^{<,>}$ that appear explicitly and implicitly through $\Sigma^{<,>}$ are replaced with finite-width expressions that provide extra $CP$-even phases. As a result, the source for the axial current is of the schematic form

$$\sim \frac{(v_av_b' - v_bv_a')(m_1 - m_2)(\Gamma_1 + \Gamma_2)}{[(m_1 - m_2)^2 + (\Gamma_1 + \Gamma_2)^2]^2} \sin \varphi,$$

where $\Gamma_{1,2}$ are the values for the widths of the mixing particles. In future work, it may be of interest to investigate whether the finite-width terms crucial for the source (169) can be reproduced in a systematic extension of the present approach based on the gradient expansion. To this end, we note that the source (167) leads to $CP$-violating effects without relying on a finite-width, while the result (169) vanishes in the zero-width limit. It therefore appears that both sources are distinct and that the source (169) may possibly be reproduced when including a collision term in Eqs. (150) and then solving these equations algebraically for the axial current, neglecting the derivative terms, as it has been done for the collision-free equations in this section. Further, it may then be relevant to carefully distinguish between finite-width effects that directly suppress the off-diagonal correlations and those that preserve these, as it has been investigated for correlations among active leptons in the context of flavoured leptogenesis. In that example, Yukawa interactions directly damp flavour correlations whereas gauge interactions preserve these to leading order [45]. Note however that for electroweak baryogenesis, due to electroweak symmetry breaking, typically degrees of freedom with different gauge charges are allowed to mix such that the finite-width effects should at least not be largely suppressed.

We may conclude to this end, that in approximations relying on the local insertion of spacetime-dependent mass-terms and neglecting the spatial gradients acting on the distribution functions, there is an extra source term (167) in addition to the one (169) reported previously. Phenomenologically, the source (167) is suppressed by an extra order in derivatives whereas Eq. (169) is suppressed by the finite width of the mixing particles. In addition, there are extra factors of $v_{a,b}$ in the term (169) that may lead to an important relative suppression because the asymmetry may most effectively be produced close to the symmetric phase where $v_{a,b}$ are small because of the rapid damping of the chiral asymmetries in the broken phase for large values of $v_{a,b}$.

Before drawing phenomenological conclusions, one should of course spell out that the approximations (167) and (169), besides neglecting spatial gradients on the distribution functions, rely on using the particle propagators of the symmetric phase even inside the wall. The advantage of numerical solutions is that such a far-reaching assumption is not required [111]. However, a realistic coupling to the background degrees of freedom in the SM appears challenging and has yet to be accomplished.

First steps toward a reliable numerical calculation in the mixing scenarios have been taken in Refs. [97, 120]. In both cases, a toy model of mixing scalar particles is considered. The kinetic equations that descend from the Kadanoff-Baym equations are then solved self-consistently, crucially including the collision term in a background thermal bath. The calculations in the time-dependent model [97] closely resemble the numerical solutions for resonant leptogenesis in Ref. [75], whereas the model assuming a space-dependent background [120] yields insights into electroweak baryogenesis. Since, as discussed in Section 6, analytic approximations are available for resonant leptogenesis in a substantial region of
parameter space, including also cases when the mass difference of the RHNs is smaller than their width, it would be very interesting to consider whether also for electroweak baryogenesis, more insights can be gained from a further improvement of both, numerical and analytical approximations.

8.4 CP-conserving transport effects and solving for the asymmetry

In order to complete this overview over the calculational methods for electroweak baryogenesis, we need to account for some crucial effects SM processes have on the way the axial currents are processed. First, in absence of extra collision terms, the kinetic equations just describe convective motion of the particles in a force field. In the presence of SM particles at finite temperature, the particles will however experience flavour-conserving scatterings via gauge bosons that drive the system toward kinetic equilibrium as well as flavour-converting scatterings that drive toward chemical equilibration. In the symmetric electroweak phase, left and right-chiral fermions effectively act as different species, such that we consider the Yukawa couplings of the SM as flavour-converting in the present context (different from the notion of flavour violation in the broken electroweak phase). Finally, the asymmetry in chiral fermions that has been produced inside the wall and transported ahead of it has to be converted into a baryon asymmetry, a process that is carried out by weak sphalerons.

Diffusion The flavour-conserving processes can effectively be described by a diffusion law

$$j_i = -D_i \nabla q_i ,$$

(170)

where $j_i$ is the three-current of the particle species $i$ in the plasma frame, $q_i$ is the charge density and $D_i$ a phenomenological parameter called diffusion constant.

Let us now drop the subscripts labeling the species for simplicity. When neglecting forces due to spatial gradients and restricting to stationary solutions, the kinetic equations reduce to

$$\frac{1}{|k|} k \cdot \nabla f(k) = C[f] ,$$

(171)

where the right-hand side stands for the portion of the collision term responsible for flavour-conserving processes. The leading contribution to $C$ is from two-by-two scatterings with gauge bosons. The phase-space integrals over tree-level contributions with $t$-channel exchange of gauge bosons or fermions can be logarithmically divergent, which is physically regulated by Landau damping and Debye screening and thus requires a resummation of the insertions of the pertinent self-energies [121, 122].

Now, we decompose the distribution function as $f = f^{(0)} + \delta f$, where

$$f^{(0)}(k) = \frac{1}{e^{\mu_T} - 1} \pm 1$$

(172)

as forced by local kinetic equilibrium due to fast gauge interactions. This first approximation to the solution is isotropic. The first correction then corresponds the leading anisotropic contributions and takes the form

$$\delta f(k) = h(k) k \cdot \nabla \mu .$$

(173)

In terms of this anisotropic contribution, the current is given by

$$j = \int \frac{d^3k}{(2\pi)^3} \frac{k}{|k|} \delta f(k) .$$

(174)
Since $q \propto \mu$, we thus see that determining $h(k)$ by solving Eq. (171) amounts to finding the diffusion constant. Values for the diffusion constants widely used in phenomenological studies can be found in Ref. [115]. It may presently be in order to reevaluate the estimates in that work using state-of-the art methods such as developed in Ref. [121, 122].

Taking the divergence of Eq. (170) yields $\nabla \cdot \mathbf{j}_i = -D_i \Delta q_i$. Minding that this relation holds in the plasma frame, we write down the covariant form and deduce from this the correct expression for the frame moving along with the wall in negative $z$-direction:

$$\partial_\mu j_\mu^i = v_w \partial_2 q_i - D_i \partial_2^2 q_i.$$  (175)

In general, also flavour-converting interactions contribute to the diffusion transport what should be of phenomenological relevance e.g. for the left and right-handed top quarks and the Higgs bosons. To account for this, one may promote the diffusion constants to matrices in flavour space in future work. Note that diffusion corresponds to a random walk, such that the distance a particle proceeds in a time $t$ is given by $\sqrt{Dt}$ for a diffusion constant $D$, while the wall proceeds by $v_w t$. Therefore, a particle typically remains a time $t_{\text{diff}} \sim D/v_w^2$ ahead of the wall before it is caught up by the broken electroweak phase.

**Flavour conversion**  
Flavour-converting interactions mediated by Yukawas interactions are discussed in the context of the CTP framework in Refs. [119, 123]. In that work, the thermal effects necessary to account for the conversion of massless particles in the symmetric phase are estimated through the inclusion of thermal masses. Applying techniques that have been developed more recently for leptogenesis [32, 47, 49–52], it may be worthwhile to update these calculations such that they include two-by-two scatterings involving the radiation of one gauge boson, including again the $t$-channel enhanced contributions.

At the level of fluid equations, the flavour-violating rates take the effect of forcing the system toward chemical equilibrium, i.e. they are proportional to sums of chemical potentials $\mu_i$ that are related to the charge densities $q_i$ via Eq. (10). Putting this together with the source terms that are e.g. given by the second term in Eq. (165) for the source from the semiclassical force or the right-hand side of Eq. (167) for the source from resonantly enhanced mixing, one obtains the fluid equation

$$\partial_\mu j_\mu^i = -\sum_a \Gamma^{(a)}(\mu_1 \pm \cdots \pm \mu_n) + S_{CP}^i.$$  (176)

The averaged decay rate for the reaction enumerated by $a$ is given by $\Gamma^{(a)}$, and the signs are positive for an incoming particle and negative for an incoming antiparticle (negative for an outgoing particle and positive for an outgoing antiparticle). When substituting the diffusion relation (175) for the left hand side of Eq. (176), this equation can be readily solved for the charge distributions for all relevant particle species with vanishing boundary conditions far ahead and behind the wall.

In phenomenological calculations, the the reaction rates $\Gamma^{(a)}$ also account for thermal QCD instantons, i.e. strong sphalerons as well as the damping of chiral asymmetries in the broken phase due to the vacuum expectation value of the Higgs field. The latter effect has again been computed from the insertion of vacuum expectation values of the Higgs field [113]. For the future, it would be desirable to derive this in a framework where one takes account of the locally correct dispersion relation for the quasiparticles as their mass changes through the wall.

**Baryon number violation**  
As the final step of the calculation, we sum the charge densities of all left handed SM fermions from the solution to Eq. (176) to obtain $q_{\text{left}}$ (recall that these count one isodoublet component each in the convention of the present work). Ahead of the bubble wall, also weak sphalerons
are active, turning $q_{\text{left}}$ into a baryon number density $q_B$ but at the same time wash out $q_B$. These processes are described by the equation \cite{113, 117, 118, 124}

$$-v_w \frac{d}{dz} q_B(z) + \frac{15}{4} \Gamma_{ws} q_B(z) = 3 \Gamma_{ws} 2 q_{\text{left}}(z), \quad \text{for } z < 0$$

(177)

where the weak sphaleron rate in the symmetric phase has been found to be $\Gamma_{ws} \approx 6 \kappa \omega^5 T$, where $\omega = g^2/(4/\pi)$, $g$ is the coupling constant of the SU(2)$_L$ gauge interactions and $\kappa \sim 20$ \cite{125, 127}. One eventually one finds the solution for the baryon charge density:

$$q_B(z = 0) = -3 \frac{\Gamma_{ws}}{v_w} \int_0^0 dz \ 2 q_{\text{left}}(z) e^{\frac{15}{4} \Gamma_{ws} z}.$$  

(178)

Provided the sphaleron rate inside the bubble is smaller than the Hubble rate, i.e. in the case of a so-called strong first order phase transition, the baryon-to-entropy ratio $Y_B = q_B/s$ is then conserved and may correspond to the value observed today \cite{5}. We note that in the SM, $\Gamma_{ws} \ll t_{\text{diff}}$. Therefore, only a small fraction of the chiral asymmetry typically ends up being converted into baryons.

9 Conclusions

While the idea that particle physics processes are accountable for the creation of the baryon asymmetry of the Universe \cite{5} is very compelling, it is yet unknown which mechanism precisely is at work and even in which sector of a complete theory it resides. Sakharov’s nonequilibrium condition may well be realized in a somewhat exotic manner, e.g. through the out-of-equilibrium decay of condensates relying on sometimes random or arbitrary initial conditions. Nonetheless, it is very plausible that baryogenesis is after all realized based on simple extensions of the SM, e.g. through a limited number of new particles such as RHNs or an electroweak sector featuring a first order phase transition. In that case, calculations of baryogenesis rely on statistical physics, i.e. on QFT at finite temperature in conjunction with an appropriate description of deviations from thermal equilibrium, as provided by the CTP formalism.

Even under these assumptions, there are quite a few questions pertaining to the theory of the matter-antimatter asymmetry that are not covered in the present work, most importantly those concerning the dynamics of the electroweak phase transition, i.e. whether it is of first order or a crossover as well as how large the rate for $B + L$-violating sphaleron-transitions is. These aspects come together with the matters we have been focusing on in this article, i.e. $CP$ violation in the early Universe, fluid equations and the calculational foundation of these using CTP techniques. So far, baryogenesis has thus been a challenge leading to fascinating insights into the real-time evolution of systems governed by quantum field theory and statistical physics. It is very plausible, but it remains to be seen, that these methods and results are applicable to the mechanism that is responsible for the baryon asymmetry of the Universe and that is yet to be discovered.

A Discrete symmetries

A thorough and comprehensive discussion on this topic is given in the monograph \cite{5}. Here, we quote the identities that are most useful in the present context.

For a given representation of Dirac matrices, there are nonsingular $4 \times 4$ matrices $A$ and $C$ such that

$$A \gamma_\mu = \gamma_\mu^\dagger A, \quad \gamma_\mu C = -V \gamma_\mu^\dagger,$$

(A1)

49
where $t$ denotes transposition. In the Weyl representation, $A = \gamma_0$ and $C = i\gamma_2\gamma_0$. These satisfy the useful identities

$$
A^\dag = A, \quad A\gamma_5 = -\gamma_5A, \\
C^t = -C, \quad \gamma_5C = C\gamma_5, \\
CA^*C^*A = 1, \quad A\sigma_{\mu\nu} = \sigma_{\mu\nu}^tA, \\
\sigma_{\mu\nu}C = -C\sigma_{\mu\nu},
$$

(A2)

where $\sigma_{\mu\nu} = \frac{1}{2}[\gamma_\mu, \gamma_\nu]$. For a spinor $\psi$, its conjugate is given by $\bar{\psi} = \psi^\dagger A$. The charge-conjugate spinor is

$$
\psi^C = e^{i\alpha C}C\psi^\dagger,
$$

(A3)

and with the help of one of the identities (A2), we see that $\bar{\psi}C = -\psi^tC^{-1}$. Including a parity reflection in addition, the $CP$ conjugate spinor is given by

$$
\psi^{CP} = e^{i\alpha^{CP}}\gamma^0C\psi^\dagger.
$$

(A4)

For a scalar field $\phi$, of course, $C$ and $CP$ conjugation take the same effect (up to possible arbitrary phases), namely $\phi \to \phi^{C,CP} = e^{i\alpha^{C,CP}}\phi^*$. The phases $\alpha^{C,CP}$ and $\beta^{C,CP}$ are arbitrary, i.e. physical effects of $C$ and $CP$ violation do not depend on their choice.

The action of $CP$-conjugation in terms of a unitary operator $\mathcal{CP}$ on a spinor and on a complex scalar field $\phi$ is

$$
\begin{align*}
(\mathcal{CP})\psi(\mathcal{CP})^\dagger &= \psi^{CP}, \\
(\mathcal{CP})\phi(\mathcal{CP})^\dagger &= \phi^{CP},
\end{align*}
$$

(A5a) (A5b)

such that the bilinear terms that appear in Lagrangians or correlation functions transform as (see Ref. for all relevant details as well as comprehensive tables on discrete symmetries)

$$
\begin{align*}
(\mathcal{CP})\bar{\psi}\chi(\mathcal{CP})^\dagger &= e^{i(\alpha^{CP} - \alpha^{CP})}\bar{\chi}\psi, \\
(\mathcal{CP})\bar{\psi}\gamma^5\chi(\mathcal{CP})^\dagger &= -e^{i(\alpha^{CP} - \alpha^{CP})}\bar{\chi}\gamma^5\psi, \\
(\mathcal{CP})\bar{\psi}\gamma^\mu\chi(\mathcal{CP})^\dagger &= -e^{i(\alpha^{CP} - \alpha^{CP})}\bar{\chi}\gamma^\mu\psi, \\
(\mathcal{CP})\bar{\psi}\gamma^\mu\gamma^5\chi(\mathcal{CP})^\dagger &= -e^{i(\alpha^{CP} - \alpha^{CP})}\bar{\chi}\gamma^\mu\gamma^5\psi.
\end{align*}
$$

(A6a) (A6b) (A6c) (A6d)

**B  Tree-level propagators on the CTP**

Useful basic elements of perturbative calculations on the CTP can be the tree-level propagators. For a scalar field of mass $m$, these are given by

$$
\begin{align*}
i\Delta^<(p) &= 2\pi\delta(p^2 - m^2) \left[ \partial(p_0)f(p) + \partial(-p_0)(1 + \tilde{f}(-p)) \right], \\
i\Delta^>(p) &= 2\pi\delta(p^2 - m^2) \left[ \partial(p_0)(1 + \tilde{f}(p)) + \partial(-p_0)f(-p) \right], \\
i\Delta^T(p) &= \frac{i}{p^2 - m^2 + i\varepsilon} + 2\pi\delta(p^2) \left[ \partial(p_0)f(p) + \partial(-p_0)f(-p) \right], \\
i\Delta^T(p) &= -\frac{i}{p^2 - m^2 - i\varepsilon} + 2\pi\delta(p^2 - m^2) \left[ \partial(p_0)f(p) + \partial(-p_0)f(-p) \right].
\end{align*}
$$

(B7a) (B7b) (B7c) (B7d)
and for a four-component spinor with mass $m$

\begin{align}
\text{i}S^<_\langle(p) &= -2\pi\delta(p^2 - m^2)(\dot\vartheta(p_0)f(p) - \vartheta(-p_0)(1-f(-p))) , \quad (B8a) \\
\text{i}S^>_\langle(p) &= -2\pi\delta(p^2 - m^2)(\dot\vartheta(p_0)(1-f(p)) + \vartheta(-p_0)f(-p)) , \quad (B8b) \\
\text{i}S^{T}_{\ell}(p) &= \frac{i(\dot\vartheta + m)}{p^2 - m^2 + i\varepsilon} - 2\pi\delta(p^2 - m^2)(\dot\vartheta p_0 f(p) + \vartheta(-p_0)f(-p)) , \quad (B8c) \\
\text{i}S'^{T}_{\ell}(p) &= -\frac{i(\dot\vartheta + m)}{p^2 - m^2 - i\varepsilon} - 2\pi\delta(p^2 - m^2)(\dot\vartheta p_0 f(p) + \vartheta(-p_0)f(-p)) . \quad (B8d)
\end{align}

These solutions are useful as elements in the computation of many correlation functions. Nonetheless, there are important situations where one may want to solve the Schwinger-Dyson equations such that the resulting propagators readily include effects from finite width (cf. Refs. [51, 53]), flavour mixing (cf. Section 6 and Ref. [70] on resonant leptogenesis, Ref. [45] on active-flavour effects in leptogenesis in the CTP formalism) or gradient effects (cf. Refs. [54, 55, 95, 105, 110, 111]).

C Details on the calculation of the $CP$-violating vertex contribution to leptogenesis in the CTP approach

We start from Eq. (104) and apply Eq. (16) in order to take the combination of CTP indices corresponding to the Wightman function $\Sigma_{\text{vert}}^\tau$,\n
\[ i\Sigma_{\mathbf{p}}^{-+}(q) = -Y_1^2 Y_2^* \int_{kk'qp} \delta_{p-k-k'}\delta_{p-q-q'} \left\{ P_{R_1}iS_{N_2}^{-}(k-q')P_{R_1}\Sigma_{N_1}^{CP}(p)P_{L_1}\Delta_{+}^{-}(k')i\Delta_{+}^{+}(q') - P_{R_1}iS_{N_2}^{+}(k-q')P_{R_1}\Sigma_{N_1}^{CP}(p)P_{L_1}\Delta_{+}^{+}(k')i\Delta_{-}^{+}(q') - P_{R_1}iS_{N_2}^{-}(k-q')P_{R_1}\Sigma_{N_1}^{CP}(p)P_{L_1}\Delta_{-}^{+}(k')i\Delta_{-}^{+}(q') + P_{R_1}iS_{N_2}^{+}(k-q')P_{R_1}\Sigma_{N_1}^{CP}(p)P_{L_1}\Delta_{-}^{+}(k')i\Delta_{-}^{+}(q') \right\} + 1 \leftrightarrow 2 . \]

Next, when restricting to the contributions that do not require an on shell $N_2$ (This is a good approximation in the hierarchical regime $N_1 \ll N_2$, when e.g. the temperature never is high enough to produce $N_2$), Alternatively, we can add the contribution from on-shell $N_2$ in the same way we calculate here the asymmetry generated by decays and inverse decays of $N_1$), only terms with $iS'^{T}_{N_2}$ or $iS'^{T}_{N_2}$ are left, where we can replace $iS'^{T}_{N_2} \rightarrow -iS'^{T}_{N_2}$ because $N_2$ is off shell, cf. Eqs. (B8c) and (B8d). We thus obtain

\[ i\Sigma_{\mathbf{p}}^{\text{vert}}(q) = -Y_1^2 Y_2^* \int_{kk'qp} \delta_{p-k-k'}\delta_{p-q-q'} \left\{ P_{R_1}iS_{N_2}^{T}(k-q')P_{R_1}\Sigma_{N_1}^{CP}(p)P_{L_1}\Delta_{<}^{<}(k')i\Delta_{<}^{<}(q') - P_{R_1}iS_{N_2}^{T}(k-q')P_{R_1}\Sigma_{N_1}^{CP}(p)P_{L_1}\Delta_{<}^{<}(k')i\Delta_{<}^{<}(q') - P_{R_1}iS_{N_1}^{T}(k-q')P_{R_1}\Sigma_{N_1}^{CP}(p)P_{L_1}\Delta_{<}^{<}(k')i\Delta_{<}^{<}(q') \right\} . \]
The Wightman self-energy \( \Sigma^>(q) \) follows through \(<\leftrightarrow>\) and \( T \leftrightarrow \bar{T} \). From these self energies, we build the vertex contribution to the source term \((105)\)

\[
S_{\text{vert}} = \int \text{tr} \left[ i\Sigma_{\ell}^{\text{vert}>}(q)iS_{\ell}^{<}(q) - i\Sigma_{\ell}^{\text{vert}>}(q)iS_{\ell}^{<}(q) \right]
\]

\[
= -Y_1^2Y_2^* \int_{kk'qq'p} \delta_{p-k-k'}\delta_{p-q-q'} \text{tr} \left[ P_RiS_{N2}^T(k-q')P_RiS_{\ell}^{CP>}(k)P_LiS_{N1}^T(p)P_Li\Delta^<(k')i\Delta^<(q')iS_{\ell}^{<}(q) \right. \\
- P_RiS_{N2}^T(k-q')P_RiS_{\ell}^{CP>T}(k)P_LiS_{N1}^T(p)P_Li\Delta^T(k')i\Delta^<(q')iS_{\ell}^{<}(q) \\
+ P_RiS_{N2}^T(k-q')P_RiS_{\ell}^{CP<}(k)P_LiS_{N1}^T(p)P_Li\Delta^<(k')i\Delta^<(q')iS_{\ell}^{<}(q) \\
- P_RiS_{N2}^T(k-q')P_RiS_{\ell}^{CP<T}(k)P_LiS_{N1}^T(p)P_Li\Delta^T(k')i\Delta^<(q')iS_{\ell}^{<}(q) \bigg] \\
- Y_1^2Y_2^* \int_{kk'qq'p} \delta_{p-k-k'}\delta_{p-q-q'} \text{tr} \left[ P_RiS_{N1}^>(p)P_RiS_{\ell}^{CP>T}(k)P_LiS_{N2}^T(k-q')P_Li\Delta^<(q')i\Delta^<(k')iS_{\ell}^{<}(q) \right. \\
- P_RiS_{N1}^>(p)P_RiS_{\ell}^{CP<}(k)P_LiS_{N2}^T(k-q')P_Li\Delta^<(q')i\Delta^<(k')iS_{\ell}^{<}(q) \\
+ P_RiS_{N1}^>(p)P_RiS_{\ell}^{CP<T}(k)P_LiS_{N2}^T(k-q')P_Li\Delta^<(q')i\Delta^T(k')iS_{\ell}^{<}(q) \\
- P_RiS_{N1}^>(p)P_RiS_{\ell}^{CP<}(k)P_LiS_{N2}^T(k-q')P_Li\Delta^>(q')i\Delta^<(k')iS_{\ell}^{<}(q) \bigg], \\
\text{(C11)}
\]

where, under the second integral, we have replaced \(-k' \leftrightarrow q\) and \(p \rightarrow k + q - p\).

Next, as it is shown in detail in Ref. [96], we can replace e.g.

\[
\int_{k} \delta_{p-k-k'} \left( iS_{\ell}^{CP>}(k)i\Delta^<(k') - iS_{\ell}^{CP<T}(k)i\Delta^T(k') \right) g(k) \\
\rightarrow \int_{k} \delta_{p-k-k'} \frac{1}{2} \left( iS_{\ell}^{CP>}(k)i\Delta^<(k') - iS_{\ell}^{CP<T}(k)i\Delta^T(k') \right) g(k), \\
\text{(C12)}
\]

where \(g(k)\) is an arbitrary function. The proof is straightforward and relies on substitution of the tree-level propagators \((B7), (B8)\) and considering all product terms separately. Making use of this replacement, the source term reduces to

\[
S_{\text{vert}} = -Y_1^2Y_2^* \int_{kp} \frac{1}{2} \text{tr} \left[ P_RiS_{N2}^T(k-q')P_R \left( iS_{\ell}^{CP>}(k)i\Delta^<(k') - iS_{\ell}^{CP<}(k)i\Delta^>(k') \right) \\
P_Li\Delta^<(q')iS_{\ell}^{<}(q) - \Delta^>(q')iS_{\ell}^{<}(q) \bigg] \\
+ Y_1^2Y_2^* \int_{kp} \frac{1}{2} \text{tr} \left[ P_Ri\delta S_{N1}(p)P_R \left( iS_{\ell}^{CP>}(k)i\Delta^<(k') - iS_{\ell}^{CP<}(k)i\Delta^>(k') \right) \\
P_LiS_{N2}^T(k-q')P_Li\Delta^<(q')iS_{\ell}^{<}(q) - \Delta^>(q')iS_{\ell}^{<}(q) \bigg]. \\
\text{(C13)}
\]

Substituting the tree-level propagators \((B8)\), taking the trace and carrying out the integrals over the zero-components of them momenta by making use of the on-shell \(\delta\)-functions, we obtain the result \((105)\).

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