Heat Transfer by Steady Streaming in a Horizontal Annulus with Inner Heating at Boundary Vibration

N V Kozlov
Institute of Continuous Media Mechanics UrB RAS,
1, Ac. Koroleva St., Perm, 614013, Russia
E-mail: kozlov.n@icmm.ru

Abstract. Convection and heat transfer are studied experimentally in a horizontal annulus, the outer wall of which is formed by an elastic silicone tube, and the inner one – by a copper cylindrical heat exchanger. The working volume is filled with a viscous fluid. The temperature of the outer boundary is kept constant. At the inner boundary, a constant heat generation power is maintained. The elastic outer wall is driven in a periodic motion by two linear motors, installed symmetrically to the left and to the right of the container and performing synchronous oscillations. Thermal measurements are carried out at steady state temperature. In the absence of vibrations, free convection is observed. Vibrations lead to the generation of steady averaged flows in the form of a system of eight pairs of rolls, parallel to the cylinder axis. Vibrational convection enhances heat transfer across the annulus. The structure of steady flows and the rate of heat transfer are determined by the interaction between two convective mechanisms.

1. Introduction
One of the actual challenges for the modern industry is the intensification of heat and mass transfer in fluids. Succeeding in this suggests opportunities in product quality enhancement or in production costs’ reduction. Traditional methods, for example mechanical stirring, may have limitations in quite a number of hydrodynamic systems, which include droplets, microfluidic devices and others. An outstanding approach to fluid mixing is steady streaming [1] that is generated in dynamic viscous boundary layers, also referred to as Stokes layers, due to fluid oscillations [2].

It is known that at free oscillations of rising (falling) droplets in extraction columns, the mass transfer coefficient increases with the dimensionless oscillation frequency, see for example [3, 4]. Oscillations of a drop lead to the generation of steady flows both inside it and in the surrounding fluid [5]. This leads to the intensification of mass transfer [6–8]. For example, flow stagnation inside a droplet due to surfactant accumulation on its surface [9] can be successfully compensated by steady streaming. Another example is an original experiment [10] whose authors studied the heat transfer from a hot upper boundary of a vertical cylinder by steady streaming from an oscillating disk: the vibrational mixing produced an efficient heat transfer and a homogeneous temperature distribution in the fluid bulk. As found in theoretical studies [5, 7], steady flows in an oscillating droplet have the shape of toroidal vortices and lead to the interface renewal by tangential time-average flows. In recent experiments on fluid oscillations in cylindrical and spherical containers with elastic boundaries, similar steady flows were modeled [11, 12]. The experiments were systematically conducted in a large range of dimensionless frequencies, and the impact of the latter on the flow structure was theoretically analyzed.

The purpose of this study is to find out how the structure and velocity of convection are related to the contribution of the steady streaming to heat transfer under the condition of the simultaneous existence of free convection. A horizontal annulus filled with a viscous fluid is chosen as experimental model. Its inner boundary is rigid and hot, while the outer one is elastic and is kept at a constant temperature via the water cooling. It is known that in a horizontal annulus with a hot inner cylinder, natural thermal convection develops in the form of two symmetric rolls with a hot plume in the region.
of their contact, above the inner cylinder [13–16]. If the annulus is vibrated as a whole perpendicular to its axis, in the fluid the thermal vibrational convection develops, which alters the fluid motion and enhances the heat transfer [17, 18]. In contrast to [17, 18], in the present experiments, the annulus remains immobile as a whole, and only its outer boundary is brought into oscillations by external electrodynamic actuators. Consequently the boundary takes alternately the oblate and prolate shape, while its average boundary shape remains circular. The dimensionless frequency of vibrations is set relatively low, so as to be of interest for the application to droplet-size systems (see the argumentation in [11]). The impact of the competing convection mechanisms is analyzed.

2. Experimental setup and techniques
The experimental model consisted of the following principal parts: an elastic silicone tube (1) (figure 1a,b), a copper cylindrical heat exchanger (2), and transparent flanges (3), that all together formed a hermetically closed annulus. The flanges of the container were integrated with the front and rear walls of the cooling jacket (4). The annulus was filled with a working fluid – an aqueous solution of glycerol with concentration 0.90. Its kinematic viscosity varied in the range \( \nu = (73.0–190) \) cSt depending on temperature. Other properties of the fluid were the thermal diffusivity \( \chi = (9.7–9.9) \times 10^{-2} \) \( \text{mm}^2/\text{s} \), the thermal conductivity \( \lambda = 3.09 \times 10^{-4} \) \( \text{W/(mmK)} \), the thermal expansion coefficient \( \beta = 4.70 \times 10^{-4} \) \( \text{K}^{-1} \). The radius of the copper cylinder was \( R_2 = 7.5 \text{ mm} \), its length \( l_0 = 194 \text{ mm} \). The interior dimensions of the silicone cylinder were \( R_1 = 25.0 \text{ mm} \) and \( L = 200.0 \text{ mm} \), the thickness of its wall 2.0 mm. The gap between the boundaries made \( h = R_2 - R_1 = 17.5 \text{ mm} \). On the silicone boundary, two rectangular activators (5) were installed symmetrically. Each activator consisted of two aluminum plates that squeezed the silicone wall in a sandwich-like configuration, their dimensions being the following: length \( l = 170.0 \text{ mm} \), height \( d = 20.0 \text{ mm} \), and thickness 1.2 mm.

The temperature of the outer boundary was maintained constant, \( T_2 = (20.0 \pm 0.1)^{\circ} \), by means of pumping water from a thermostatic bath trough the cooling jacket. The copper cylinder was heated up to the temperature \( T_1 \) by a nichrome wire with resistance \( 2.4 \pm 0.1 \) Ohm installed in a narrow shaft inside it and powered by a DC supply. The heating power \( Q \) was calculated with the values of current \( I \) and voltage \( U \) measured from the indicators of the power supply, \( Q = IU \). The temperature \( T_2 \) was controlled by two thermistors (6) of Pt100 type installed in the front and the rear sides of the jacket. The temperature difference between the inner and outer boundaries of the annulus, \( \Theta = T_1 - T_2 \), was measured directly by a differential thermocouple of type T. Its hot junction was put in a narrow groove in the heat exchanger, while the cold junction was immersed in the cooling water. The thermometers’ data was automatically collected by a multichannel measuring device Termodat 22K5. Temperature measurement error did not exceed \( 0.1^{\circ} \).

The activators were mechanically connected with the rods of electromagnetic actuators Dunkermotoren STA1112 (7) (figure 1b). Each actuator was controlled by an amplifier Copley Accelnet ACJ-055-18-S and fed by a DC power supply MASTECH HY5020E. The position of the actuators was defined by the harmonic signal from a generator GSPF-052. The actuators performed synchronous periodic motion and induced vibrations of the actuators. As a result, the elastic boundary was periodically contracted and extended along the horizontal axis. The vibration frequency was \( f = 3.00 \) Hz, and the amplitude varied in the range \( b = (0–11) \text{ mm} \). The oscillations of the opposite activators were symmetric, and the oscillation amplitude of the whole boundary was equal to the span of one activator. In experiments, \( b \) was found as an average of the values obtained for the left and the right actuators, the discrepancy between them did not exceed 0.1 mm.
The flow structure was visualized by tracers. For this, the working fluid was seeded with Rilsan light-scattering powder (particle size 40 μm and density 1030 gr/mm³). A cross-section of the annulus was illuminated with a light-sheet from a solid state continuous laser (figure 1). The time-average flow structure and velocity were studied using PIV (Particle Image Velocimetry). The images were taken all in the same oscillation phase with the timestep equal to a multiple of the oscillation period. This way, the time-average velocity was calculated directly. The test experiments demonstrated that the flow structure reproduced itself in different cross-sections along the cylinder axis, except the regions immediately near the flanges. Hence, within the considered range of experimental parameters, the steady flows under the study may be considered effectively two-dimensional. All flow studies were done at \( \Theta = \text{const} \).

3. Experimental results

3.1. Steady flows. Isothermal case

Oscillations of the container boundary lead to the generation of steady time-averaged flows in the form of two-dimensional rolls elongated parallel to the cylinder axis. In the cross section, the structure consists of eight pairs of vortices, regularly spaced along the curvilinear boundary (figure 2). The large vortices are generated near the outer boundary, and the small ones – near the inner boundary. The angular coordinates of both coincide. The four vortices in the bottom of the container fall into the shadow of the copper cylinder and are only partially visible, but the regular structure of the flows indicates their presence.
At oscillations, distinctly localized regions of the boundary remain almost motionless. In figure 2a, they are indicated by points $A$, $B$, $C$, $D$. In these points, steady flows are directed perpendicular to the average position of the solid boundary, which coincides with the radial direction. This applies both to the external and the internal boundaries. Since the fluid oscillates with respect to the internal and external boundaries, a viscous boundary layer forms on each of the surfaces. In turn, the averaged vorticity is generated in each boundary layer. This explains the two-level structure of steady flows.

As a quantitative measure of the intensity of steady flows, a characteristic velocity is often chosen at some point [10–12]. In the present problem, it is convenient to take the velocity of flows directed along the radius from points $A$, $B$, $C$, $D$ towards the container center (figure 2a). Let us denote the arithmetical average value of these velocities as $v_j$. It increases with the oscillation amplitude according
to a quadratic law (figure 3a), which is typical for steady flows generated in Stokes layers [2]. Such approach to mathematical description is justified in the case of self-similar flows. It is worth noting, however, that in the case of simultaneous existence of the vibrational and the free convection, the flow structure transforms making it difficult to choose the characteristic velocity. Due to this, it is more convenient to evaluate the convection intensity by the vorticity of the averaged velocity (figure 2b). Let us introduce as a characteristic the average vorticity $\frac{1}{N}\sum_{i=1}^{N}(\text{rot } v_i)$, where $(\text{rot } v_i)$ is the maximal value of vorticity in the center of some vortex, $N = \{12 - 14\}$ is the number of measured vortices. As it is shown in figure 3b (symbols 1), for the average vorticity, the quadratic scaling law is also satisfied. At the same time, $Y$ may be used as a characteristic to compare the steady flows that have differences in the structure (compare figures 2b and 4a–d).

\[ \frac{1}{N}\sum_{i=1}^{N}(\text{rot } v_i) \]

Figure 3. Intensity of steady flows as a function of the oscillation amplitude: a) the graph for $v_j$ in the isothermal situation; b) the graph for $Y$ at various heating power.

### 3.2. Steady flows at heating
At heating, in the layer, thermal convection emerges, which has been studied in detail in the absence of vibrations, see for example [13–16]. The free convection affects the flow structure (figure 4). Now, in the annulus, two motion mechanisms are present: the free convection excited by the heating of the inner cylinder, and the forced convection – steady streaming excited by the oscillations of the outer boundary. The relative contribution of each of the mechanisms to the fluid motion can be judged by the structure of steady flows.

At relatively low vibration amplitudes, the impact of the free convection is the most pronounced: above the hot cylinder, an intensive upward flow is formed that resembles the convective plume [14], while two descending flows are formed near the outer boundary (figure 4a–c). In this case, the return radial flow in the upper half of the container is absent (compare to figure 2, points A and B). The deviation of the $Y(b)$ dependence from the quadratic law is characteristic for such convection regimes (figure 3b, points 2 and 4).

At $Ra \sim 10^4$, and lower values, it is possible to suppress the free convection by increasing the amplitude of vibrations. Here, the Rayleigh number is calculated as $Ra = g \beta \theta h^3 / (\nu \gamma)$. In this situation, the flow structure takes the form that is typical for the vibrational convection (figure 4b). Simultaneously, the average vorticity approaches to the values observed in the absence of heating (figure 3b, points 3). At $Ra \sim 10^7$, in the whole studied range of vibrational parameters the impact of
the free convection remains noticeable. This is evidenced by the flow structure (figure 4d), as well as by experimental points 4 (figure 3b), which have a discrepancy with the isothermal case.

![Figure 4](image)

**Figure 4.** Steady-flow structures at simultaneous heating and vibration: a) $Q = 6.6 \text{ W}$, $b = 5.1 \text{ mm}$, $Ra = 13.2 \times 10^3$, $Re_p \cdot \omega = 136$; b) $Q = 6.6 \text{ W}$, $b = 9.2 \text{ mm}$, $Ra = 9.12 \times 10^3$, $Re_p \cdot \omega = 383$; c) $Q = 27.2 \text{ W}$, $b = 6.2 \text{ mm}$, $Ra = 81.8 \times 10^3$, $Re_p \cdot \omega = 683$; d) $Q = 27.2 \text{ W}$, $b = 9.2 \text{ mm}$, $Ra = 64.3 \times 10^3$, $Re_p \cdot \omega = 1.22 \times 10^3$. The vector plots represent the average velocity (the scale of arrows varies between figures), and the colour maps – the average vorticity.

4. **Discussion**
As it is shown in the equations of thermal vibrational convection [19], the action of vibrations on hydrodynamic systems is described by two control parameters: the dimensionless frequency $\omega = \Omega h^2 / v$ and the pulsation Reynolds number $Re_p = b^2 \Omega / v$, where $\Omega = 2\pi f$. In experiments [11, 12], it was shown that $\omega$ determines the structure of steady flows, while $Re_p$ is responsible for the intensity of vibrational convection at a given dimensionless frequency. The analysis of experimental results carried on in works [11, 12] revealed that the intensity of steady flows is determined on the parameters plane $(\omega, V / Re_p)$, where $V = vh / v$, $v$ and $h$ are the characteristic average velocity and
container size. In the low-frequency region, at $\omega \sim 1 - 10$, the following scaling law holds:

$$\frac{V}{Re} \sim \omega. \quad (1)$$

In the present experiments, in the case of heating, due to the dependence of fluid viscosity on the temperature, the dimensionless frequency varies in the range $\omega = 30 - 80$, despite the experiments have been performed at a constant vibration frequency. In terms of the present problem, taking into account that $v \sim Yh$, we obtain $V \sim Yh^2 / \nu$. By renormalizing the expression (1) we obtain

$$Yh^2 / \nu \sim Re_\omega \sim b^2 \Omega^2 h^2 / \nu^2. \quad (2)$$

As shown in figure 5, the law (2) is found in the case of purely vibrational convection (points 1). It is also satisfied when the free convection can be neglected compared to the steady streaming (points 3). The latter is evidenced by the flow structure (figure 4b). In the situations when the vibration intensity is not sufficiently high compared to the Rayleigh number, the dependence law between $Yh^2 / \nu$ and $Re_\omega$ is different from (2) (figure 5, points 2, 4).

**Figure 5.** The steady flows’ intensity versus the vibration intensity; $\omega = 30.4$ (1), 37 – 41 (2, 3), 62 – 79 (4).

**Figure 6.** The heat transfer rate versus the vibration intensity; $Ra = (8.5 – 15.2) \cdot 10^3$ (1, 2), $(54.9 – 94.2) \cdot 10^3$ (3).

The heat transfer rate is characterized by the Nusselt number, $Nu = Q / \dot{Q}_{m}$. Here, the molecular heat transfer is calculated as $\dot{Q}_{m} = 2\pi\lambda L \Theta / \ln(R_2 / R_1)$. At relatively low values of $Re_\omega$ ($< 60$ at $Ra \sim 10^4$ and $< 200$ at $Ra \sim 10^5$), vibrations do not intensify the heat transfer (figure 6). When the vibrational convection dominates over the free convection, the dependence $Nu \sim \left(Re_\omega \omega\right)^{1/3}$ is found (points 2). It is interesting to notice that, despite the pronounced contribution of the free convection to the flow structure at $Ra \sim 10^5$, in the region $Re_\omega \omega > 10^3$ points 3 approach the 1/3 power law (compare curves I and II in figure 6). Let us find the limit between the parameters domains, which determine what type of convection dominates. For this, from the graph in figure 6 we obtain the marginal values of $\left(Re_\omega \omega\right)^{1/3}$ by tracing the points of intersection of the laws $Nu = \text{const}$ and $Nu \sim \left(Re_\omega \omega\right)^{1/3}$. Then from the experimental data we find the corresponding values of $Ra^\ast$. This procedure yields the following:
\( \left( \text{Re}_p \omega \right)^* = 110 \), if \( \text{Ra}^* = 14 \cdot 10^3 \), and \( \left( \text{Re}_p \omega \right)^* = 650 \), if \( \text{Ra}^* = 82 \cdot 10^3 \). In both cases, the ratio \( \left( \text{Re}_p \omega \right)^*/\text{Ra}^* \approx 8 \cdot 10^{-3} \) is obtained.

5. Conclusion
Convective heat transfer has been studied experimentally in the horizontal annulus whose elastic outer boundary performed harmonic oscillations. Two experimental techniques have been applied independently allowing to study the flow velocity and the heat transfer simultaneously. Comparative analysis of the steady flows structure, the average vorticity and the heat transfer rate reveals a correlation between them. Namely, the vibration induced steady flows have quite a particular structure, regular with respect to the azimuthal direction, that is characterized by the scaling law \( \text{Nu} \sim \left( \text{Re}_p \omega \right)^{1/3} \). This law gives the maximal growth rate of the heat transfer rate with the increase in the vibration intensity and is valid in the situation when the steady streaming clearly dominates over the free convection. A marginal ratio exists between the vibrational parameter \( \text{Re}_p \omega \) and \( \text{Ra} \) that allows defining two regions, in which either of two convective mechanisms dominates. In the limit between two regions \( \left( \text{Re}_p \omega \right)^* \sim 0.01 \text{Ra}^* \), and this means that the condition of steady streaming dominance should be: \( \left( \text{Re}_p \omega \right) \sim 0.1 \text{Ra} \) or greater. The considered experimental model allows successfully studying the rate of heat transfer by steady streaming in the presence of free thermal gravitational convection.

6. Acknowledgements
The research was supported by the Russian Science Foundation (project 17-71-10189).

References
[1] Riley N 2001 Ann. Rev. Fluid Mech. 33 43–65
[2] Schlichting H 1968 Boundary Layer Theory (McGraw-Hill, New York (USA))
[3] Yamaguchi M, Fujimoto T and Katayama T 1975 J. Chem. Engng Japan 8(5) 361–6
[4] Kumar A, Hartland S 1999 Chem. Eng. Res. Des. 77(5) 372–84
[5] Murtsovkin V A and Muller V M 1992 J. Colloid Interf. Sci. 151 150–6
[6] Yarin A L, Brenn G, Kastner O, Rensink D and Tropea C 1999 J. Fluid Mech. 399 151–204
[7] Yarin A L 2001 J. Fluid Mech. 444 321–42
[8] Muginstein A, Fichman M and Gutfinger C 2005 Int. J. Multiph. Flow 31 263–84
[9] Wegener M, Paul N and Kraume M 2014 Int. J. Heat Mass Transf. 71 475–95
[10] Ivanova A A, Kozlov V G, Polezhaev D A, Pareau D and Stambouli M 2009 Fluid Dyn. 44 481–9
[11] Kozlov V G, Kozlov N V and Schipitsyn V D. 2017 Physical Review Fluids 2 094501
[12] Kozlov V G, Sabirov R R and Subbotin S V 2018 Fluid Dynamics 53 189–99
[13] Itoh M, Fujita T, Nishihaki N and Hirata M 1970 Int. J. Heat Mass Transf. 13 1364–8
[14] Kuehn T H and Goldstein R J 1976 J. Fluid Mech. 74(4) 695–719
[15] Kuehn T H and Goldstein R J 1976 Int. J. Heat Mass Transf. 19 1127–34
[16] Tsui Y T and Tremblay B 1984 Int. J. Heat Mass Transf. 27 103–11
[17] Ivanova A A and Kozlov V G 1985 Fluid Dynamics 20 989–92
[18] Ivanova A A and Kozlov V G 1998 Fluid Dynamics 33 324–30
[19] Gershuni G Z and Lyubimov D V 1998 Thermal Vibrational Convection (New York, USA: Wiley)