Relativistic nuclear matter equation of state in fractional exclusion statistics approach

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We apply the fractional exclusion statistics (FES) to the quantum hadrodynamics model (QHM). The FES typically describes in an effective way the remanent interaction between the constituent particles. We determine the phenomenological parameters of the QHM with FES from the conditions at the saturation point of the symmetric nuclear matter, independent of the exclusion statistics parameter, $\alpha$. We calculate the relevant physical quantities of the system, e.g. the effective mass of the nucleons and its dependence on the baryon density, temperature, and $\alpha$. We also calculate the isothermal curves (pressure vs. baryonic density) for different temperatures and $\alpha$s. We observe that in the region of phase transition the isotherms do not depend much on the exclusion statistics parameter, but a strong dependence on $\alpha$ is observed at higher densities or at higher temperatures.

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I. INTRODUCTION

The relativistic mean-field model (RMF) is a general framework for the relativistic nuclear many-body problem based on hadronic degrees of freedom. The RMF model has successfully been used to describe many nuclear phenomena. In the models of nuclear
matter studied so far, the statistics of particles involved was not a subject of discussions. The baryons are fermions and mesons are bosons. However, in the hot and dense nuclear medium the statistics of quasi-baryons may change. Therefore, the main purpose of this paper is to investigate the influence of the statistics of particles on the thermodynamic properties of the isospin symmetric nuclear matter. For this reason, we apply the concept of the fractional exclusion statistics to the RMF of Refs. [3, 4]. In FES, the statistics of the ideal particles is deformed by the additional parameter $\alpha$, and so it is possible to fit more of the known properties of nuclear matter.

The fractional exclusion statistics, introduced by Haldane in Ref. [6] has received very much attention since its discovery and has been applied to many models of interacting systems (see for example Refs. [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28]). Several authors have also discussed the microscopic reason for the manifestation of FES [17, 18, 19, 20, 21, 22, 23, 29, 30, 31, 32, 33]. Iguchi and Sutherland [33] showed that liquids of particles in three dimensions, interacting through long-range forces exhibit the nature of quantum liquids with FES, the characteristics of the FES being determined by the interaction. Murthy and Shankar [20] analysed a system of fermions in the Calogero-Sutherland model and showed that by redistributing in a certain way the interaction energy in the system among the single-particle states, one obtains a gas which is described by FES.

Although the concept received so much attention and has been applied in general to many types of systems, in Ref. [34] it was proven that it has basic inconsistencies. These inconsistencies have been corrected in the same paper, by introducing mutual exclusion statistics parameters which are proportional to the dimension of the subspace on which they act. In a related paper [28] it was also shown that the FES is not an exceptional statistics, manifesting only in certain exotic systems, but it is generally present in interacting particle systems. Concretely, a non-ideal system of interacting particles may be described as an “ideal” FES system, in which the exclusion statistics parameters are determined by the interaction Hamiltonian.

The structure of the article is the following. In Section II we briefly describe basic ingredients of the relativistic mean-field model in fractional exclusion statistics approach and the methodology to evaluate the model parameters. The thermodynamic results are discussed in Section II C. The main conclusions are summarized in the final Section III.
Throughout the paper we use the natural units system, \( \hbar = c = k_B = 1 \).

II. FRACTIONAL EXCLUSION STATISTICS APPLIED TO QUANTUM HADRODYNAMICS

A. The relativistic mean-field model

As already mentioned, RMF is a relativistic quantum theory model for the nuclear many-body system. The motivation to introduce the RMF was the observation of large Lorentz scalar and four-vector components in the nucleon-nucleon interaction.

To introduce the notations and write the basic equations, we follow the treatment from [3]. In the model we have the barionic field, \( \psi \), that describes the nucleons, \( p \) and \( n \), the neutral scalar meson field, \( \phi \), for the meson \( \sigma \), and the neutral vector meson field, \( V_\mu \).

We assume that the nuclear scalar meson couples to the scalar density of barions through \( g_s \bar{\psi} \psi \phi \) and that the neutral vector meson couples to the conserved barion current through \( g_v \bar{\psi} \gamma_\mu \psi V_\mu \).

We also assume that the baryon density is high, so that we can use mean field theory (MFT) and replace the meson field operators by their expectation values, which are the classical fields \( \langle \phi \rangle \equiv \phi_0 \) and \( \langle V_\mu \rangle \equiv \delta_\mu 0 V_0 \). For a static, uniform system, the quantities \( \phi_0 \) and \( V_0 \) are constants, independent of \( x_\mu \). With these simplifications (see Refs [3, 4] for details), we have \( \phi_0 = \langle \bar{\psi} \psi \rangle g_s / m_s^2 \equiv \rho_s g_s / m_s^2 \) and \( V_0 = \langle \bar{\psi} \psi \rangle g_v / m_v^2 \equiv \rho_B g_v / m_v^2 \), with \( m_s \) and \( m_v \) the masses of the scalar and vector mesons, respectively, and the mean field Lagrangian is written as

\[
\mathcal{L}_{\text{MFT}} = \bar{\psi} \left[ i \gamma_\mu \partial^\mu - g_v \gamma^0 V_0 - (M - g_s \phi_0) \right] \psi - \frac{1}{2} m_s^2 \phi_0^2 - \frac{1}{2} m_v^2 V_0^2. \tag{1}
\]

Applying the Lagrange equations to \( \mathcal{L}_{\text{MFT}} \) one obtains the equation for the baryonic Dirac field,

\[
[i \gamma_\mu \partial^\mu - g_v \gamma^0 V_0 - (M - g_s \phi_0)] \psi = 0, \tag{2}
\]

which, when solving for plane-waves, gives the single-particle spectrum

\[
\epsilon(k) \equiv \sqrt{k^2 + M^2} + g_v V_0 \\
\equiv E^*(k) + g_v V_0, \tag{3}
\]
and
\[ \bar{\epsilon}(k) = E^*(k) - g_v V_0 \]  
with \( M^* \) being the effective mass,
\[ M^* = M - g_s \phi_0. \]  
The baryonic number,
\[ B \equiv \int_V d^3x B^0 = \int_V d^3x \psi^\dagger \psi \equiv V \rho_B, \]  
is a constant of motion, so it is conserved. (Throughout the paper, we shall denote the volume of the hadronic system by \( V \)–without any index.)

From the energy-momentum tensor calculated with (1) one can derive the expressions for the pressure and energy density,
\[ p = \frac{1}{3} \psi^\dagger (-i\alpha \cdot \nabla) \psi + \frac{1}{2} m_v^2 V_0^2 - \frac{1}{2} m_s^2 \phi_0^2, \]  
\[ \mathcal{E} = \psi^\dagger [-i\alpha \cdot \nabla + \beta M^* + g_v V_0] - \frac{1}{2} m_v^2 V_0^2 + \frac{1}{2} m_s^2 \phi_0^2. \]  
The total Hamiltonian of the system is
\[ H = \int_V d^3x T_{00} = \int_V d^3x \mathcal{E}. \]  
In the second quantization, the baryon number and the effective Hamiltonian operators are
\[ \hat{B} = \sum_{k,\lambda} (A^\dagger_{k\lambda} A_{k\lambda} - B^\dagger_{k\lambda} B_{k\lambda}), \]  
\[ \hat{H}_{\text{MFT}} = \sum_{k,\lambda} \sqrt{k^2 + M^*} (A^\dagger_{k\lambda} A_{k\lambda} + B^\dagger_{k\lambda} B_{k\lambda}) + g_v V_0 \hat{B} - V \left( \frac{1}{2} m_v^2 V_0^2 - \frac{1}{2} m_s^2 \phi_0^2 \right), \]  
where \( A^\dagger_{k\lambda} \) and \( A_{k\lambda} \) are the creation and annihilation operators for the baryon state of momentum \( k \) and species \( \lambda \) (\( \lambda = n \) or \( p \), of spin up or spin down), whereas \( B^\dagger_{k\lambda} \) and \( B_{k\lambda} \) are the corresponding antiparticle operators.

B. Thermodynamics

We shall consider in this paper the FES in its simplest form, which is the diagonal form. We introduce the spin-isospin degeneracy factor, \( \gamma \), which takes the value 2 for neutron
matter (two spin projections and one isospin projection) and 4 for nuclear matter (two spin and two isospin projections). The net baryon number $B$ is a conserved quantity, so the chemical potential, $\mu$, is associated to it. In the grand canonical ensemble the partition function is written as

$$Z = Tr\left(e^{-\frac{\hat{H}_{MFT} - \mu \hat{B}}{T}}\right).$$

(12)

Plugging Eqs. (10) and (11) into (12) and maximizing the partition function, we obtain in the standard way the thermodynamic quantities,

$$\rho_B = \frac{\gamma}{(2\pi)^3} \int_{R^3} [n_k(T) - \bar{n}_k(T)]d^3k,$$

(13)

$$\mathcal{E} = \frac{g^2}{2m^2_v} \rho_B^2 + \frac{m^2_s}{2g^2_s}(M - M^*)^2 + \gamma \frac{(2\pi)^3}{3} \int_{R^3} d^3k E^*(k) [n_{ks}(T) + \bar{n}_k(T)],$$

(14)

$$p = \frac{g^2}{2m^2_v} \rho_B^2 - \frac{m^2_s}{2g^2_s}(M - M^*)^2 + \gamma \frac{(2\pi)^3}{3} \int_{R^3} d^3k k^2 \frac{k^2}{E^*(k)} [n_k(T) + \bar{n}_k(T)],$$

(15)

where $n_k(T)$ and $\bar{n}_k(T)$ are the baryon and antibaryon mean occupation numbers of the single particle level $k$. We assume that all the baryons have the same mass, and therefore their spectra and single-particle level populations are identical.

If the baryons are ideal fermions, then $n_k(T)$ and $\bar{n}_k(T)$ are simply the Fermi functions,

$$n_k(T) = \frac{1}{e^{(E^*(k)-\nu)/T} + 1}, \quad \bar{n}_k(T) = \frac{1}{e^{(E^*(k)+\nu)/T} + 1},$$

(16)

where $\nu \equiv \mu - g_v V_0 = \mu - g_v \rho_B / m^2_v$.

In general, in a FES system the mean occupation numbers are not given by Eq. (16), but by the formula [29, 30, 34]

$$n_{k,\alpha} = [w(\xi) + \alpha]^{-1},$$

(17)

where $w$ is a solution of the equation

$$w^\alpha(\xi)[1 + w(\xi)]^{1-\alpha} = \xi \equiv e^{(\epsilon - \mu)/T}$$

(18)

(note that we added the subscript $\alpha$ to specify the statistics). In this case the formalism remains the same, except that we work with the more general levels population (17), instead of (16).

The equilibrium values of the fields $\phi_0$ and $V_0$, and therefore of $M^*$, are determined by maximizing the grandpartition function, $Z$, with respect to $\phi$, which gives [3, 4]

$$M - M^* - \frac{g^2}{m^2_s (2\pi)^3} \int_{R^3} d^3k \frac{M^*[n_{k,\alpha}(T) + \bar{n}_{k,\alpha}(T)]}{\sqrt{k^2 + M^2}} = 0,$$

(19)
that has to be solved for $M^*$. 

The phenomenological constants of the model, expressed as $C_s^2 \equiv g_s^2 M^2 / m_s^2$ and $C_v^2 \equiv g_v^2 M^2 / m_v^2$ are determined from the conditions at the saturation point: at $T = 0$ and $\rho_B = \rho_0$, the binding energy, $\mathcal{E}_b \equiv [\mathcal{E}(\rho_B) - \rho_B M] / \rho_B$ attains its minimum (i.e. equilibrium) value, $\mathcal{E}_{b0}$. For the nuclear matter we take $\mathcal{E}_{b0} \approx -16$ MeV, $\rho_0 = 0.16$ fm$^{-3}$.

Therefore we rewrite Eqs. (13)-(15), and (19) at $T = 0$:

$$\rho_B = \frac{\gamma}{(2\pi)^3 \alpha} \int_{k < k_F} d^3k = \frac{\gamma}{6\pi^2 \alpha} k_F^3,$$

(21)

$$\mathcal{E} = \frac{g_v^2}{2m_v^2} \rho_B^2 + \frac{m_s^2}{2g_s^2} (M - M^*)^2 + \frac{\gamma}{(2\pi)^3 \alpha} \int_{k < k_F} d^3k E^*(k),$$

(22)

$$p = \frac{g_v^2}{2m_v^2} \rho_B^2 - \frac{m_s^2}{2g_s^2} (M - M^*)^2 + \frac{\gamma}{3(2\pi)^3 \alpha} \int_{k < k_F} d^3k \frac{k^2}{E^*(k)},$$

(23)

$$0 = M - M^* - \frac{g_s^2}{m_s^2} \frac{\gamma}{(2\pi)^3 \alpha} \int_{k < k_F} d^3k \frac{M^*}{E^*(k)},$$

(24)

where $k_F$ is the Fermi wavevector. If we minimize $\mathcal{E}_b$ for the usual Fermi statistics, $\alpha = 1$, and with the expression for $\mathcal{E}$ given by (22), we obtain $C_s^2 \approx 330$ and $C_v^2 \approx 249$. If we keep $C_s$ and $C_v$ fixed at these values and plot $\mathcal{E}_b$ for the nuclear matter, as a function of $\rho_B$, for different $\alpha$s (see Fig. I a), we observe that the binding energy decreases and its minimum moves towards higher baryonic densities; so the characteristics of the nuclear matter change significantly with the exclusion statistics. This situation have been analysed in Ref. [35].

Since the exclusion statistics is a microscopic characteristic of matter, which is employed (typically) to describe the eventual remanent interaction in the MFT in an effective, statistical manner [28], in this paper we assume fixed the experimentally observable quantities, $\mathcal{E}_{b0}$ and $\rho_0$, from which we determine the microscopic parameters $C_s$ and $C_v$ as functions of $\alpha$.

Therefore we determine the parameters $C_s$ and $C_v$, so that the minimum of the function $\mathcal{E}_B(\rho_B)$, at $T = 0$, does not change with $\alpha$. The condition of minimum for $\mathcal{E}_B$ reads

$$\frac{d}{d\rho_B} \left( \frac{\mathcal{E} - \rho_B M}{\rho_B} \right)_{\rho_0} = \frac{1}{\rho_0} \left. \frac{d\mathcal{E}}{d\rho_B} \right|_{\rho_0} - \frac{\mathcal{E}(\rho_0)}{\rho_0^2} = 0,$$

(25)

and calculating the total derivative of $\mathcal{E}$ with respect to $\rho_B$ from Eq. (22), we obtain an equation for $C_v$,

$$\frac{C_v^2}{M^2} = \frac{\mathcal{E}_{b0} + M - E^*(k_F)}{\rho_0},$$

(26)
where \( k_{F0} \equiv [(2\pi)^3 \alpha \rho_0 / \gamma]^{1/3} \). Using Eq. (26) we eliminate \( C_v \) from the expression of \( \mathcal{E} \) (22),

\[
\mathcal{E} = \rho_B \frac{\mathcal{E}_{b0} + M - E^*(k_{F0})}{2} + \frac{M^2(M - M^*)^2}{2C_s} + \frac{\gamma}{16\pi^2\alpha} \left[ k_FE^*(k_{F0})(2k_F^2 + M^*^2) - M^*^4 \ln \frac{k_{F0} + E^*(k_{F0})}{M^*} \right],
\]

(27)

where \( k_F \) is given by Eq. (21) as a function of \( \rho_B \) and we evaluated analytically the integral over \( k \). From the effective mass equation (24), we get \( C_s \) as a function of \( M^* \),

\[
C_s = \frac{4\pi^2\alpha}{\gamma} \cdot \frac{M^2(M - M^*)}{M^*} \left[ k_F \sqrt{k_F^2 + M^*^2} - M^*^2 \ln \frac{k_F + \sqrt{k_F^2 + M^*^2}}{M^*} \right]^{-1}
\]

(28)

which we insert into (27) to obtain a self-consistent equation for \( M^* \):

\[
\frac{\mathcal{E}_0}{2} = -\rho_0 E^*(k_{F0}) + \gamma M^*(M - M^*) \left\{ k_{F0}E^*(k_{F0}) - M^*^2 \ln \frac{k_{F0} + E^*(k_{F0})}{M^*} \right\} + \frac{\gamma}{16\pi^2\alpha} \left[ k_{F0}E^*(k_{F0})(2k_{F0}^2 + M^*^2) - M^*^4 \ln \frac{k_{F0} + E^*(k_{F0})}{M^*} \right].
\]

(29)

With \( M^* \) calculated from the equation above, we go backwards and calculate \( C_s(\alpha) \) and \( C_v(\alpha) \) from Eqs. (28) and (26).

### C. Results

In Fig. 1 (b) we show the binding energy of the particles in the nuclear matter for \( \alpha = 1, 0.5, \) and 0.1. We see there that all the curves \( \mathcal{E}_{b,\alpha} \) have their minima in the same point and that the binding energy start to depend on \( \alpha \) roughly at \( \rho_B > \rho_0 \).

In Fig. 2 we show the dependence of \( C_s^2 \) and \( C_v^2 \) on \( \alpha \), whereas in the inset we plot the ratio between the effective and the free-particle masses, \( M^*/M \). As \( \alpha \) increases to 1, \( C_s^2(\alpha) \) and \( C_v^2(\alpha) \) decrease monotonically to their values corresponding to the Fermi statistics, \( C_s^2(1) \approx 330 \) and \( C_v^2(1) \approx 249 \).

To calculate \( C_s \) and \( C_v \) at \( \alpha = 0 \) we have to calculate first the asymptotic expression of \( M^* \), for \( \alpha \ll 1 \). We notice from Eq. (29) that if \( M^*_{\alpha=0} \) is finite, then the second and the third terms at the r.h.s. of Eq. (29) diverge. But since the l.h.s. is a constant and the first term at the r.h.s. converges to \(-\rho_0 M^*_{\alpha=0}/2\), we conclude that Eq. (29) cannot hold in such a case; therefore \( M^* \) must converge to zero as \( \alpha \to 0 \).

We can also notice by direct inspection of Eq. (29) that at small values of \( \alpha \), \( M^* \) must have an asymptotic behavior \( M^* \sim \alpha^{1/3} \), which is similar to the \( \alpha \) dependence of \( k_{F0} \). If we
denote $M^* \equiv c_M \alpha^{1/3}$ and $k_{F0} \equiv c_F \alpha^{1/3} = [(2\pi)^3 \alpha \rho_0 / \gamma]^{1/3}$ and plug them into Eq. (29), we obtain an equation for $c_M$:

$$\mathcal{E} = 2 = \frac{\gamma M c_M}{8\pi^2} \cdot \left\{ c_F \sqrt{c_F^2 + c_M^2} - c_M^2 \ln \frac{c_F + \sqrt{c_F^2 + c_M^2}}{c_M} \right\}. \tag{30}$$

Using (30) we obtain

$$C_v^2(0) = \frac{M^2 \mathcal{E}_0}{\rho_B^2} \quad \text{and} \quad C_s^2(0) = \frac{M^4}{\mathcal{E}_0}. \tag{31}$$

Note that when $\alpha$ converges to zero, $M^*/M$ converges also to zero, whereas $C_s$ and $C_v$ converge to finite values.

In Fig. 3 we plot the ratio $M^*/M$ for nuclear matter (a) and neutron matter (b) as a function of temperature. In both plots the three upper curves correspond to zero baryon density ($\rho_B = 0$), whereas the lower curves correspond to the barion density $\rho_0$. We notice that even though the values of $M^*(T)$ are different for different $\alpha$s, the general behavior is similar, i.e. for $\rho_B = 0$, $M^*$ is a monotonically decreasing function of $T$, whereas for finite baryon densities, $M^*$ first increases and then decreases with $T$.

In Fig. 4 we plot the isotherms $p(\rho_B)$, for three values of $T$ ($T = 10, 15, \text{and} 100$ MeV) and $\alpha = 1, 0.5, \text{and} 0.1$. We notice that at low temperatures and baryon densities the isotherms depend very little on the exclusion statistics parameter, whereas at high temperatures the isotherms become very sensitive to $\alpha$. This is due to the fact that the phenomenological constants $C_s$ and $C_v$ are recalculated for each $\alpha$, so that the saturation point is independent of the exclusion statistics. On the contrary, if the Hamiltonian of the system would be independent of $\alpha$, i.e. we would keep $C_s$ and $C_v$ the same for all $\alpha$s, then the high temperature results would coincide (statistics has smaller and smaller influence on the thermodynamic results as the temperature increases), whereas at low temperatures the isotherms would have a strong dependence on $\alpha$.

III. CONCLUSIONS

We applied the fractional exclusion statistics [6] to the relativistic mean field model of Walecka and Serot [3, 4]. Fixing the observable (i.e. physical) parameters of the system, which are the binding energy, $\mathcal{E}_b$, and the baryonic density, $\rho_0$, we determine the model’s phenomenological mean field parameters, which are $C_s$ and $C_v$ (see Fig. 1). Therefore in
our model, $C_s$ and $C_v$ are functions of the exclusion statistics parameter, $\alpha$, as depicted in Fig. 2. Using $C_s(\alpha)$ and $C_v(\alpha)$ we calculated the effective masses of the nucleons in nuclear matter (Fig. 3 a) and neutron matter (Fig. 3 b).

Using further the effective mass in the expression 13 for pressure, we draw in Fig. 4 the isotherms for nuclear and neutron matters. We observed that at low temperatures and baryonic densities the isotherms depend very weakly on $\alpha$, whereas at high temperatures the dependence on $\alpha$ is strong (Fig. 4). This dependence is different from the model with fixed $C_s$ and $C_v$, in which case the $\alpha$ dependence of the isotherms is strong at low temperatures and weaker and weaker as the temperature increases.

The phase transition appears in systems of any FES, but depends on the exclusion statistics parameter.

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FIG. 1: The energy of the nuclear matter, at zero temperature: (a) the constants $C_s$ and $C_v$ are fixed at $C_s = 330$ and $C_v = 249$, which corresponds to the binding energy $E_{b0} = -16$ MeV and the baryon density $\rho_0 = 0.16$ fm$^{-3}$ for Fermi statistics ($\alpha = 1$); (b) the constants $C_s$ and $C_v$ are calculated for each value of $\alpha$, so that the observables $E_{b0}$ and $\rho_0$ remain the same.

Figures:
FIG. 2: The constants $C_s$ and $C_v$, as functions of the exclusion statistics parameter, $\alpha$, when the binding energy $E_{b0} = -16$ MeV and baryon density $\rho_0 = 0.16$ fm$^{-3}$ are independent of $\alpha$. In the inset is plotted the effective mass vs. $\alpha$, under the same conditions.
FIG. 3: Effective mass, $M^*$, as a function of the temperature, for $\alpha = 1, 0.5, 0.1$ and for two values of $\rho_B$: $\rho_B = 0$ (the upper three curves in each graph) and $\rho_B = \rho_0 = 0.16 \text{ fm}^{-3}$ (the lower three curves in each graph). The constants $C_s$ and $C_v$ are determined so that the binding energy and the net baryonic density at $T = 0$ are $E_{b0} = -16 \text{ MeV}$ and $\rho_B = \rho_0 = 0.16 \text{ fm}^{-3}$ for each $\alpha$. The curves in (a) and (b) correspond to nuclear ($\gamma = 4$) and neutron ($\gamma = 2$) matter respectively.
FIG. 4: Isotherms, $p(\rho_B)$, for three temperatures, $T=10$, 15, 100 MeV (the isotherms corresponding to $T = 100$ MeV are given in the insets), and $\alpha = 1$ (solid line), 0.5 (dash line), 0.1 (dot line). Up: nuclear matter, $\gamma = 4$; low: neutron matter, $\gamma = 2$. 