Formation of a Kerr black hole from two stringy NUT objects

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Abstract

In this paper we show that an isolated Kerr black hole can be interpreted as arising from a pair of identical counter-rotating NUT objects placed on the symmetry axis at an appropriate distance from each other.

PACS numbers: 04.20.Jb, 04.70.Bw, 97.60.Lf
As has been recently shown [1], the well–known NUT solution [2] represents a stringy object with two counter–rotating semi–infinite massive singularities attached to the poles of a central non–rotating mass. This solution is the simplest possible one among the equatorially antisymmetric spacetimes the notion of which has been introduced in [3]. Although the usual NUT object is endowed with some unphysical properties, e.g., the presence of the regions with negative mass and closed time–like curves, in the present paper we will demonstrate that configurations of several NUT constituents can give rise to physically significant models without the pathologies of a single NUT spacetime. Concretely, we shall consider a simple but convincing example: emergence of a Kerr black hole from two interacting NUT objects. The consideration will be carried out within the framework of an exact solution describing a non–linear superposition of two NUT sources.

We remind that the stationary axisymmetric vacuum problem reduces to finding the complex function \( \mathcal{E} \) satisfying the Ernst equation [4]

\[
(\mathcal{E} + \bar{\mathcal{E}})(\mathcal{E}_{,\rho,\rho} + \rho^{-1}\mathcal{E}_{,\rho} + \mathcal{E}_{,z,z}) = 2(\mathcal{E}^2 + \bar{\mathcal{E}}^2),
\]

(1)

where \( \rho \) and \( z \) are the Weyl–Papapetrou cylindrical coordinates, a comma denotes partial differentiation with respect to the coordinate that follows it, and a bar over a symbol means complex conjugation. Using Sibgatullin’s method [5], the potential \( \mathcal{E} \) satisfying (1) can be constructed from its value on the upper part of the symmetry axis. For \( z > \sqrt{m^2 + \nu^2} \), the Ernst complex potential of a single NUT solution has the form [1]

\[
e(z) \equiv \mathcal{E}(\rho = 0, z) = \frac{z - m - i\nu}{z + m + i\nu},
\]

(2)

where \( m \) is the total mass of the stringy NUT object, while \( \nu \) represents the average angular momentum per unit length of the semi–infinite NUT singularity. The non–linear superposition of two NUT solutions with equal masses and opposite angular momenta is formally defined by the axis data

\[
e(z) = \frac{z - k - m - i\nu}{z - k + m + i\nu} \cdot \frac{z + k - m + i\nu}{z + k + m - i\nu},
\]

(3)

the real parameter \( k \) representing a displacement of each NUT constituent along the \( z \)-axis. We point out that whereas the axis data (2) define the equatorially antisymmetric solution because of the characteristic relation \( e(z)e(-z) = 1 \) fulfilled in this case [3], the axis data (3) satisfy the relation \( e(z)e(-z) = 1 \) and hence define already the equatorially symmetric spacetime [6, 7].
Below we give the final form of the Ernst potential $\mathcal{E}$ constructed from the data (3) with the aid of Sibgatullin’s method, and the form of all the corresponding metric functions $f$, $\gamma$ and $\omega$ from the Papapetrou stationary axisymmetric line element

$$
\begin{align*}
\mathcal{E} &= \frac{A - B}{A + B}, \\
f &= \frac{A\bar{A} - B\bar{B}}{(A + B)(A + \bar{B})}, \\
\omega &= -\frac{4\text{Im}[G(\bar{A} + \bar{B})]}{AA - BB}, \\
A &= [(m^2 + \nu^2)(k^2 - m^2)(k^2 - m^2 - \nu^2) - 2m^2k^2\nu^2](R_+ - R_-)(r_+ - r_-) \\
&+ 2\alpha_+\alpha_- (m^2 + \nu^2)(m^2 - k^2)(R_+R_- + r_+r_-) \\
&- \alpha_+\alpha_-(2m^4 + (m^2 + \nu^2)(k^2 - m^2))(R_+ + R_-)(r_+ + r_-) \\
&- 2imkd[(\alpha_+ - \alpha_-)(R_+r_+ - R_-r_-) - (\alpha_+ + \alpha_-)(R_+r_- - R_-r_+)], \\
B &= 4d\{m\alpha_+\alpha_- [(m^2 - d)(R_+ + R_-) - (m^2 + d)(r_+ + r_-)] \\
&+ ik\nu[\alpha_- (m^2 - d)(R_+ - R_-) - \alpha_+(m^2 + d)(r_+ - r_-)]\}, \\
G &= d[d^2 + m^2(m^2 + 2ik\nu)][(\alpha_+ - \alpha_-)(R_+r_+ - R_-r_-) + (\alpha_+ + \alpha_-)(R_+r_- - R_-r_+)] \\
&- 2m^2d^2[(\alpha_+ + \alpha_-)(R_+r_- - R_-r_+) + (\alpha_+ - \alpha_-)(R_+r_+ - R_-r_-)] \\
&- m\alpha_+\alpha_- (d^2 + m^4)(R_+ + R_-)(r_+ + r_-) + m[kd^2(k + 4i\nu) - (2k^2 - m^2)(m^2 + \nu^2)^2] \\
&+ k^2\nu^4](R_+ - R_-)(r_+ - r_-) - 2m\alpha_+\alpha_- (k^2 - m^2)(m^2 + \nu^2)(R_+R_- + r_+r_-) \\
&- 2dz\{\alpha_- (m^2 - d)[m\alpha_+(R_+ + R_-) + ik\nu(R_+ - R_-)] \\
&- \alpha_+(m^2 + d)[m\alpha_-(r_+ + r_-) + ik\nu(r_+ - r_-)]\} \\
&+ 2d\alpha_+\alpha_- (2m^2 + ik\nu)[m^2(R_+ + R_- - r_+ - r_-) - d(R_+ + R_- + r_+ + r_-)] \\
&+ 2md[d^2 - m^4 - ik\nu(2m^2 + ik\nu)][\alpha_- (R_- - R_+) + \alpha_+(r_+ - r_-)] \\
&- 2md^2(m^2 - k^2 + \nu^2 - 2ik\nu)[\alpha_- (R_- - R_+) + \alpha_+(r_+ - r_-)],
\end{align*}
$$

where

$$
R_\pm = \sqrt{\rho^2 + (z \pm \alpha_+)^2}, \quad r_\pm = \sqrt{\rho^2 + (z \pm \alpha_-)^2},
$$

and

$$
\alpha_\pm = \sqrt{m^2 + k^2 - \nu^2 \pm 2d}, \quad d = \sqrt{m^2k^2 + \nu^2(k^2 - m^2)}.
$$
The metric obtained is asymptotically flat and it describes a spinning body with the total mass $2m$, total angular momentum $2k\nu$ and mass–quadrupole moment $2m(k^2 - m^2 - \nu^2)$. Since the well–known Kerr solution [9] is characterized by the parameters $m$ and $a$ (total mass and angular momentum per unit mass, respectively, the mass–quadrupole moment of the source being equal to $-ma^2$), one could suppose that in the particular case $k = m$ the metric (5) reduces to the Kerr solution with the total mass $2m$ and total angular momentum per unit mass $\nu$. To see whether this supposition is true, let us put $k = m$ in (5) and (7).

Then we get

$$\alpha_{\pm} = \sqrt{2m^2 - \nu^2 \pm 2m^2}, \quad d = m^2,$$

and the expressions for $A$, $B$ and $G$ take the form

$$A = [(\alpha_+ - i\nu)R_+ + (\alpha_+ + i\nu)R_-]F,$$

$$B = 4ma_+ F,$$

$$G = m[(\alpha_+ - 2m - i\nu)R_+ + (\alpha_+ + 2m + i\nu)R_- + 2a_+(2m + i\nu - z)]F,$$

$$F = 4m^4[(\alpha_- + i\nu)r_+ + (\alpha_- - i\nu)r_-],$$

$F$ being the common factor. Cancelling out $F$ in the Ernst potential and in the metric functions, we arrive, after the introduction of spheroidal coordinates

$$x = \frac{1}{2\alpha_+}(R_+ + R_-), \quad y = \frac{1}{2\alpha_+}(R_+ - R_-),$$

at the formulas

$$\mathcal{E} = \frac{\alpha_+ x - i\nu y - 2m}{\alpha_+ x - i\nu y + 2m}, \quad f = \frac{\alpha_+ x^2 + \nu^2 y^2 - 4m^2}{(\alpha_+ x + 2m)^2 + \nu^2 y^2},$$

$$e^{2\gamma} = \frac{\alpha_+^2 x^2 + \nu^2 y^2 - 4m^2}{\alpha_+^2 (x^2 - y^2)}, \quad \omega = -\frac{4m\nu(\alpha_+ x + 2m)(1 - y^2)}{\alpha_+^2 x^2 + \nu^2 y^2 - 4m^2}. $$

Introducing now the constants

$$p = \frac{\alpha_+}{2m}, \quad q = \frac{\nu}{2m}, \quad p^2 + q^2 = 1,$$

we rewrite (11) in the form

$$\mathcal{E} = \frac{px - iqy - 1}{px - iqy + 1}, \quad f = \frac{p^2 x^2 + q^2 y^2 - 1}{(px + 1)^2 + q^2 y^2},$$

$$e^{2\gamma} = \frac{p^2 x^2 + q^2 y^2 - 1}{p^2(x^2 - y^2)}, \quad \omega = -\frac{2\alpha_+ q(px + 1)(1 - y^2)}{p(p^2 x^2 + q^2 y^2 - 1)}.$$
which is one of the standard representations of the Kerr solution [4, 10].

Therefore, we have demonstrated that a specific non–linear superposition of two NUT solutions can give rise to the Kerr spacetime. We hope that a future investigation will be able to reveal that this result is not simply a mathematical curiosity but could have a solid physical nature.

This work was partially supported by Project 45946–F from CONACyT, Mexico, by Project FIS2006–05319 from MCyT, Spain, and by Project SA010C05 from Junta de Castilla y León, Spain.

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