On the rôle of mass growth in dusty plasma kinetics

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It is shown that absorption of ions by the dust grain may reduce the effective translational temperature of the dust component below the ambient gas temperature.

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I. INTRODUCTION

The kinetic theory of dusty plasmas, which takes into account specific processes of charging, has been considered on phenomenological basis and used for different applications in [1,2]. The form of charging collision integrals considered in these papers has been recently rigorously justified in [3], where also the stationary solution of kinetic equation for charge and velocity distributions of grains were established. As it has been established in [3] the process of absorption of the small particles by grains in dusty plasmas can lead to inequality of the grain temperature and the temperatures of the light components (even for the case of equal temperatures of electrons and ions).

At the same time the preliminary results of MD simulations for the kinetic energy of dust particles [5] demonstrated that mass transfer from light components to dust can be essential for the large time scale. The problem of mass transfer is also very actual for the conditions of the experiments with growth of grains [6] and formation of new materials [7] in dusty plasmas, as well as for many other applications. Therefore the appropriate kinetic theory should be developed, in which a new kinetic variable — the mass of grains — must be introduced [8].

In this paper we will consider kinetics of the dust particles with variable mass, which increases due to absorption of the ambient plasma and determine the nonstationary distribution function for the grains. It is shown that asymptotically the effective temperature of the dust component is lower than the stationary temperature of the gas. It means the mass growth leads to the cooling of the dust component.

II. KINETIC EQUATIONS

Since our main purpose here is to demonstrate the importance of the mass growth, we simplify the problem ignoring the grain charge. In other words, we treat a plasma as a neutral gas. We adopt here that in process of an elementary collision a grain absorbs every atom hitting its surface. An atom transfers its momentum to a grain and, respectively, the mass of a grain changes. Therefore generally the distribution function of the dust component depends both on grain momenta, \( \mathbf{P} \), and masses, \( M \).

It should noted that the assumption of the complete absorption seems justified under the conditions of the experiments aimed at the plasma synthesis of fine grains [1][5]. Otherwise, an ion hitting the grain surface rather leaves it as a neutral atom carrying away some momentum. The latter may be ignored if the surface temperature of a grain is, by some means, below the ion temperature.

The appropriate kinetic equation describing the process may be written as

\[
\frac{df_d(\mathbf{P}, M, t)}{dt} = I_d(\mathbf{P}, M, t) =
\int d\mathbf{p} f_n(\mathbf{p}) \{w(\mathbf{p}, \mathbf{P} - \mathbf{p}, M - m)f_d(\mathbf{P} - \mathbf{p}, M - m) - w(\mathbf{p}, \mathbf{P}, M)f_d(\mathbf{P}, M)\}, \tag{1}
\]

where \( f_n(\mathbf{p}) \) is the distribution function of neutral atoms of the mass \( m \). The probability of absorption is given by
\[ w(p, P, M) = \sigma(M) \left( \frac{P}{M} - \frac{p}{m} \right), \]  

(2)

where the cross-section, \( \sigma(M) \), generally is mass dependent. For example, assuming the permanent specific gravity of the grain material results in \( \sigma(M) \propto M^{2/3} \). The distribution function in Eq. (2) is normalized to the average density:

\[ n_d = \int dP dM f_d(P, M). \]

The evolution of the neutral gas distribution is governed by

\[ \frac{df_n(p)}{dt} = - \int dP dM w(p, P, M) f_d(P, M) f_n(p). \]  

(3)

Evidently, the set of kinetic equations (1,3) provides the conservation of the net number of dust grains and the total momentum. The total energy is no longer a conserving quantity. The physical reason for this is fairly obvious: a part of kinetic energy of a colliding atom is transferred to the kinetic energy of a dust grain, while the remainder is spend for the heating of the grain surface. The latter part of the energy balance is out of our consideration.

The collision term in Eq. (1) is greatly simplified by expanding it in powers of a small \( \epsilon = m/M \) ratio. Straightforward expansion of Eq. (1) results in

\[ \frac{1}{M} \frac{\partial}{\partial P} \left[ -\beta_i g(P, M) + \lambda_{ij} P_j g(P, M) + \frac{\partial}{\partial P} \left( \pi_{ij} g(P, M) \right) \right] - \frac{\partial}{\partial M} \left( j g(P, M) \right), \]  

(4)

where \( g(P, M) = \sigma(M) f_d(P, M) \). The kinetic coefficients introduced in Eq. (4) are expressed in terms of the gas distribution:

\[ j = \int dP p f_n(p), \]  

(5)

\[ \beta_i = \int dP \frac{p_i}{m} f_n(p), \]  

(6)

\[ \lambda_{ij} = \frac{1}{M} \int dP \frac{p_i p_j}{p} f_n(p), \]  

(7)

\[ \pi_{ij} = \frac{1}{2m} \int dP pp_i p_j f_n(p). \]  

(8)

The first term in Eq. (4) arises due to the possible anisotropy of the ambient gas distribution. Formally, this term is proportional to \( \epsilon^{1/2} \). The remaining terms describing diffusion in the phase space and mass growth are of the order \( \epsilon \).

With sufficiently small number of dust grains one can ignore the deviation of ambient gas from initial distribution. Assuming that \( f_n(p) \) is given by Maxwellian distribution with the temperature, \( T_n \), and particle density, \( n_n \), we get

\[ \frac{df_n(P, M, t)}{dt} = j_0 \left\{ \frac{g(P, M)}{M} + \frac{P}{3M} \frac{\partial g(P, M)}{\partial P} + \frac{2}{3} T_n \frac{1}{P^2} \frac{\partial^2 g(P, M)}{\partial P^2} \frac{P^2}{\partial P} \frac{\partial g(P, M)}{\partial P} \frac{\partial P}{\partial M} \right\}, \]  

(9)

where it is also supposed that the dust distribution is isotropic. The coefficient, \( j_0 \), in Eq. (4) is the mass flow at the grain surface, \( j_0 = n_n \sqrt{8T_n/\pi} \).

It should be noted that with the last term in the right-hand side of Eq. (4) omitted, i.e., in neglecting the process of the mass growth, there is an exact stationary solution to Eq. (4) in the form of Maxwellian function with the temperature \( T_d = 2T_n \). The same conclusion stems also from the more general approach of [3].

III. TEMPERATURE EVOLUTION

With the help of Eq. (4) we study the evolution of the temperature of the dust component. Although it is possible to obtain the general solution to Eq. (4), the corresponding expression is rather bulky (see Appendix) and little informative. To grasp the role of the mass growth one can neglect the mass dispersion of the dust component looking for the solution in the form of
Substituting this into Eq. (9) yields

\[
\frac{d\mu(t)}{dt} = j_0 \sigma(\mu(t))
\]

(11)

\[
\frac{\partial F(P, t)}{\partial t} = j_0 \sigma(\mu(t)) \left\{ \frac{F(P, t)}{\mu(t)} + \frac{P}{3\mu(t)} \frac{\partial F(P, t)}{\partial P} + 2 \frac{T_n}{P^2} \frac{\partial}{\partial P} P^2 \frac{\partial F(P, t)}{\partial P} \right\}
\]

(12)

Eq. (11) shows that the mass of all grains increases with the rate determined by the current value of the cross-section. The solution to the second equation (12) is sought in the form of the Maxwellian distribution

\[
F(P, t) = \frac{n_d}{(2\pi)^{3/2}} e^{-P^2/2\Delta}
\]

(13)

with the time-varying effective temperature, \(\Delta = T_{\text{eff}}(t)\mu(t)\). Substituting Eq. (13) to Eq. (12) we get

\[
\frac{dT_{\text{eff}}(t)}{dt} = 2 \frac{j_0}{3} \frac{2}{\sigma(\mu)(2T_n - T_{\text{eff}}(t))}.
\]

(14)

In neglecting the mass growth, as it was already mentioned, the stationary state of the dust component is characterized by the effective temperature twice as the gas temperature, \(T_{\text{eff}} = 2T_n\). However, the joint solution of Eqs. (11,14) results in

\[
T_{\text{eff}}(t) = \frac{4}{5} T_n + C\mu(t)^{-5/3},
\]

(15)

where \(C\) is an integration constant. Thus, the mass growth yields cooling of the dust component below the gas temperature, \(T_{\text{eff}} \rightarrow \frac{4}{5} T_n\).

IV. CONCLUSIONS

We have considered the kinetic equation for the ensemble of grains imposed in neutral gas. The process of gas absorption by grains leads to the time dependence of grain distribution function due to the mass growth of the dust particles. For the Maxwellian distribution of neutral gas we found the general nonstationary solution of the kinetic equation with variable mass. The average kinetic energy of grains, that is, the effective temperature of the dust component, tend to the stationary values. The process of establishing of the effective temperature can be interpreted in this case as an effective cooling.

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APPENDIX A:

It is a matter of straightforward substitution to verify that the general solution to Eq. (9) with an initial condition \(f_d(P, M, 0) = f_0(P, M)\) is given by

\[
f_d(P, M, t) = \int_0^\infty dP' P'^{2/3} \frac{\sigma(M, t)}{P' \mu(M, t)^{1/3} \sigma(M)} \frac{1}{\sqrt{\pi \Delta(M, t)}} \exp \left( - \frac{P'^2 M^{2/3} + P'^2 \mu(M, t)^{2/3}}{4 \Delta(M, t)} \right) \sinh \left( \frac{P' \mu^{1/3}(M, t)^{1/3}}{2 \Delta(M, t)} \right) f_0(P', \mu(M, t)),
\]

(A1)
where \( \mu(M,t) \) is a root of the equation

\[
\int_{\mu(M,t)}^{M} \frac{dM'}{\sigma(M')} = j_0 t
\]

and \( \Delta(M,t) = \frac{2}{5} T_n (M^{5/3} - \mu(M,t)^{5/3}) \). Evaluating the average kinetic energy, \( \langle P^2/2M \rangle \), with the help of Eq. (A1) one can verify that it tends to \( 6/5 T_n \) even for an arbitrary mass distribution.

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