Abstract

It is assumed that the radial propagation of light with respect to the naive coordinate system of the observer is uniform and isotropic and that the physical rate of propagation of light is the same for all observers. In accelerated frames of reference, these assumptions lead to the findings that the measured value of \( c \) is a function of the gravitational energy per unit mass (GEPUM) of the observer, and that this is due to the physical characteristics of the standard measuring-devices being a function of their GEPUM. The consequences of these findings include observers who at rest with respect to each other assigning different values to the same physical separation, the mixed metric tensor \( g^{\mu\nu} \), describing how gravitation affects measuring-devices, and the De-Broglie wavelength being a function of an object’s GEPUM. How the measured values of various types of physical quantities are affected is described. The gravitational Doppler shifting of light is viewed differently. The correct value for the deflection of light is obtained without the use of Huyghen’s Principle. The Schwarzschild solution is re-examined: The physical size of radial coordinate \( 2m \) is 0, a traveler must perceive himself to go an infinite distance to reach the radial coordinate of \( 2m \), and gravitational self-potential energy reaches a minimal value at the radial coordinate \( 3m \). Therefore, black holes do not exist in this theory; Gravitational collapse forms hyper-massive star-like objects instead.
INTRODUCTION

The cornerstone of the theories of Relativity is the statement that the propagation of light is uniform and isotropic for all observers. The term “uniform” usually refers to Einstein’s assumption that all observers measure the same value for the speed of light \( c \) \[1,2\]. This assertion is based on two additional assumptions:

- \( c \) is physically the same for all observers, which will be referred to as Absolute Uniformity. Absolute Uniformity is the heart and soul of Relativity, and is retained in the theory proposed in this article.
- All measuring devices of identical construction are physically identical. This will be referred to as Construction Uniformity.

In Special Relativity (SR), another form of uniformity involving the propagation of light exists: The radial propagation of light with respect to the naive coordinate system of every observer is instantaneously the same throughout the universe, where the naive coordinate system is that created by the observer when it is assumed that light travels in straight lines. This will be referred to as Extended Uniformity.

Although Extended Uniformity and Construction Uniformity peacefully co-exist in SR, the same does not apply to General Relativity (GR). This is because the coordinate size of a standard measuring-rod and the rate at which a standard clock ticks both decrease as their gravitational energy per unit mass (GEPUM) decreases. When Absolute Uniformity and Construction Uniformity are assumed, it is predicted that the rate of propagation of light with respect to a naive coordinate system will be diminished as it descends into a gravitational field \[2\], thereby violating Extended Uniformity. The resulting theory, which is currently accepted, is referred to in this article as Measurement-Based GR (MGR).

On the other hand, when Extended Uniformity is used instead of Construction Uniformity, the GR effects on the coordinate features of clocks and measuring-rod produce the prediction that \( c \) increases as one’s GEPUM decreases. If Absolute Uniformity is also assumed, then it cannot be a change in the physical quantity which is the rate of propagation of light (\( \tilde{c} \)) which causes the change in \( c \). Instead, the physical attributes of the standards of length and time must now be a function of their GEPUM and not solely a function of their construction.

In this article, it is assumed that Absolute Uniformity and Extended Uniformity are the underlying principles on which the universe operates. The theory constructed on these postulates is referred to as Propagation-based GR (PGR). In this theory, physical space, time, mass, and charge are as measured with respect to the naive coordinate system of an observer with a given GEPUM. The resulting model of the universe makes a number of surprising predictions, including a finding that black holes can not exist, and that gravitational collapse halts itself before a singularity can be formed.

\[1\]This qualifier is not needed in flat space-times. However, the distorting effects of curved space-times can make the rate of lateral propagation of light appear to be different than that of its radial propagation. The resulting loss of isotropy would invalidate the resulting theory if these lateral effects were treated as being physically as perceived.
FUNDAMENTALS

A. Quantities, Values, and Effects

Given the loss of Construction Uniformity, how should standards of measurement be created and/or reconciled? In PGR, the physical standards continue to be described by their construction because:

- We are physical beings and what we experience is a physical world. The physical standards therefore should help a physical being to describe the various phenomena of the universe as perceived by itself.

- The existing tensors of GR already assume the use of measuring-devices which are of identical construction, and permit the measurements made with them in different frames of reference to be reconciled. To go with another type of standard would require the development of a new mathematical framework to support it.

The physical size of the standard measuring-devices being a function of their GEPUM means that different observers may assign different values to the same physical quantity. Not only does this principle apply to c, but to all velocities as well as the measurement of spatial and temporal distances, mass2, acceleration3, etc. As a result one must differentiate between a physical quantity and the value an arbitrary observer measures it to have. In this article, this is done symbolically by placing a star over a normal symbol to represent a quantity (such as \( \ast c \)), while using the normal symbol to represent its measured value (such as c). For example: For two observers K and K′ in an accelerated system both MGR and PGR agree that \( \ast c′ = \ast c \). However, in MGR c′ = c, while that is not true in PGR.

The quantity notation will be used when it does not matter which observer is measuring the quantities in the expression. In that case, what matters is having those quantities measured in a consistent fashion. For example, the PGR expression for determining how much faster a clock at a different potential than ones own will run in a short accelerated box is\[4 \sqrt{1 + 2g\ast z/\ast c^2} \]
where \( \ast g \) is the quantity of acceleration throughout the box, and \( \ast z \) is the quantity of distance in the direction opposite that of gravity in the box between ones own position and that of the clock being examined. Who measures \( g \), z, and c is irrelevant. Since the result is dimensionless, all of the length and time variances cancel out. Instead, what matters is that all of these quantities be measured by a given observer while that observer has a given GEPUM.

In many cases, the difference between a quantity and its value is trivial. For a single observer whose position in a stable accelerated system is not changing, or who is making all of their measurements at a given instant in time, the measured value of a quantity is a reasonable description of the quantity itself. Another example is \( E = mc^2 \): There is

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2The effect of a change in GEPUM on the standard mass has been documented by Einstein [3].

3This includes the measurement of gravity by different observers in a uniformly accelerated box.

4This may be derived from Eq. (1) (in §B).
no practical difference between it and $\dot{E} = \dot{m} c^2$. On the other hand, it is quantities that are subject to the conservation laws of physics. (The consequences of this observation will manifest themselves in §9).

In PGR, gravitation affects measured values in two ways: When two observers obtain different values for the same quantity, that is called an observer-based effect, since what has changed is the observer’s units of measure as opposed to the quantity being measured.

On the other hand, consider the case of an observer with two identical atomic clocks who sends one of them to another gravitational potential. After the transfer, the observer will find that the clock which was moved is no longer keeping the same time as that of the clock which he kept. This is because in the process of being transferred, the quantity of time which is a standard unit of time came to be different for the clock which was moved. This is called an object-based effect, since it is caused by the object being measured having a different GEPUM.

The physical constants of nature may be either quantities or values. Those that describe the “construction” of the universe such as $c$, Plank’s Constant ($\hbar$), etc. are constant quantities; while those that which describe the construction of ordinary matter such as the quantum of electrical charge ($e$) and the mass of the electron ($m_e$) are locally measured values or local constants. The difference is important since constant quantities are not subject to object-based effects, while the quantities described by local constants are.

B. Effect on the metric tensor $g_{\mu\nu}$

Extended Uniformity is not inconsistent with either the Equivalence Principle or general covariance. As a result, The basic tensor mathematics of GR are retained in PGR, including the Einstein Field Equations. However, the interpretation of the components of the metric tensor of spacetime $g_{\mu\nu}$ is modified in PGR.

First of all, the components of the mixed metric tensor $g^{\mu\nu}$ describe how the physical properties of the standard clock and measuring-rod are modified by a change of GEPUM. For example, $g^{\mu\nu}$ expressed in terms of the Newtonian potential $\Phi$ for $\Phi < 0$ is [4]:

\begin{align*}
g^{0\,0} & = 1 + 2\Phi/c^2, \quad (1a) \\
g^{1\,1} = g^{2\,2} = g^{3\,3} & = (1 + 2\Phi/c^2)^{-1}, \quad (1b) \\
g^{\mu\nu} & = 0, \mu \neq \nu. \quad (1c)
\end{align*}

Eq. (1a) indicates that a clock ticks $(1 + 2\Phi/c^2)^{1/2}$ as fast as one at the base potential, and Eq. (1B) indicates that spatial distances as measured in the $x$, $y$, and $z$ directions from a position at potential $\Phi$ will be $(1 + 2\Phi/c^2)^{-1/2}$ as large as those measured from the base potential.

Secondly, $g_{\mu\nu}$ continues to describe how to reconcile the views from the various frames of reference. This use is more important given that the coordinate values now have direct physical significance. Therefore, the transformation rule for physical measurements of space and time between observers $K$ and $K'$ in an accelerated system is given in PGR by:

\[ x'^\mu = a^{\alpha\beta} g^{\alpha\beta} (g^{-1})^{\beta \nu} x^\nu, \quad (2) \]
where $a^{\mu \alpha}$ is the Lorenz tensor, $g^{\alpha \beta}$ is the mixed metric tensor for observer $K'$, and $(g^{-1})^\beta_\nu$ is the inverse mixed metric tensor for observer $K$. This replaces the MGR rule of using measuring-rods at rest in the lattice to measure distance with.

Thirdly, the expression for the invariant spacetime interval $ds$ is still given by $ds^2 = g_{\mu \nu} dx^\mu dx^\nu$. The $dx^\mu$ and $dx^\nu$ continue to be locally measured values.

Finally, one's view of spacetime may be distorted by gravitational lensing and the overall curvature of spacetime. In this case, there are occasions where the field equations predict that the lateral effects of gravitation on the measurement of space and time are not the same as the radial effects. The radial values then describe the direct physical effects of gravitation, while the variance between the radial and lateral effects describe the form and magnitude of the distortions caused by the curvature of spacetime.

As was previously mentioned, $c$ is not the same for all observers in PGR. Under Extended Uniformity, $c$ is given by:

$$c = c_0 \sqrt{\sum_{\nu=1}^{3} \frac{g^{1\nu}}{g^{00}}},$$

where $c_0$ is the value of $c$ at some base potential where $g^{\mu \nu} = \delta^\mu_\nu$ by definition\(^5\), $\delta^\mu_\nu$ is the Kronecker delta, and $x^1$ is the direction of the radial propagation of light from a position at the base potential. Eq. (3) indicates that all observers will measure the same value for $c$ if $\sum_{\nu=1}^{3} g^{1\nu} = g^{00}$, but this is not normally the case. In fact, it is usual for $\sum_{\nu=1}^{3} g^{1\nu} = 1/g^{00}$, which results in Eq. (3) taking on the form:

$$c = c_0/g^{00}.$$

**GENERAL RESULTS**

**C. The observer-based effects of gravitation on measured values**

In this section, the effects of an observer’s GEPUM on the measured value of various types of quantities is studied. This is done by associating the values for quantities as measured by observers $K$ and $K'$ who are at rest with respect to each other in an accelerated system, and using standards for time, length, mass-energy, and electrical charge which are of identical construction. Each effect will be expressed in the form:

$$q' = q \Upsilon f,$$

where $q$ is a quantity as measured by observer $K$, $q'$ is that same quantity as measured by observer $K'$, $\Upsilon$ is the factor for the expansion in the measurement of length with Eqs. (1) and (2) indicating that $\Upsilon = \left[ g^{00} (g^{-1})^{00} \right]^{-1/2}$, and $f$ is an arbitrary factor. Several of these effects have been previously documented, but in a different context \([\text{1}]\).

\(^5\)This is not to be confused with the SR case, where $g^{\mu \nu} = \delta^\mu_\nu$ for all of spacetime.
A complete list of the observer-based effects is given in Table I. How the entries in this table were obtained is described below.

By definition, the measurement of lengths using measuring-rods of identical construction is modified by the relationship:

$$l' = l \Upsilon.$$  \hfill (5)

For clocks of identical construction, it is shown by Eq. (1) that the time effect is given by:

$$t' = t \Upsilon^{-1}.$$  

These fundamental observer-based effects (and the others that will be described in this section) may be combined to create other observer-based effects using the rules for powers. For example, velocity is given by $v = l/t$; Therefore $v' = l'/t' = (l/t) \Upsilon^{[1-(-1)]} = v \Upsilon^2$.

Mass-energy raises a problem. A certain amount of matter as could be used as our standard for either mass or energy. It is known that as a given quantity of matter is loses GEPUM, its mass-energy as measured by an observer with a given GEPUM is decreased by its loss of gravitational energy \cite{2}. Given a standard mass with a mass-energy of $\mathcal{M}$ when it is at rest at the base potential, this decrease occurs in such a way that its mass-energy when it is at rest at another potential ($\mathcal{M}'$) is given by \cite{5}:

$$\mathcal{M}' = \mathcal{M} \sqrt{g_{00}} \equiv \mathcal{M} \Upsilon^{-1}. \hfill (6)$$

If the rest energy in the standard mass $E$ is taken to be a fundamental unit, then Eq. (6) produces a fundamental observer-based effect of:

$$E' = E \Upsilon. \hfill (7)$$

On the other hand, rest mass of the standard mass $m$ could be taken to be a fundamental unit. In this case, Eq. (6) produces a fundamental observer-based effect of:

$$m' = m \Upsilon. \hfill (8)$$

Eqs. (7) and (8) cannot both be assumed. This is due to the mass-energy relationship $E = mc^2$. Since $c$ varies by $c' = c \Upsilon^2$, if the rest energy in a standard mass with the observer’s GEPUM is treated as a fundamental unit, then as the observer’s GEPUM changes the measured rest mass of the standard mass $M$ is modified by the relationship:

$$M' = M \Upsilon^{-4}. \hfill (9)$$

On the other hand, if the rest mass of standard mass is used as a fundamental unit, then the measured rest energy associated with it when it has the observer’s GEPUM $\mathcal{E}$ is modified as the observer’s GEPUM changes by the relationship:

$$\mathcal{E}' = \mathcal{E} \Upsilon^4. \hfill (10)$$

Because of this conflict Table I contains two columns of observer-based effects: one for the mass-fundamental case and another for the energy-fundamental case.
Eqs. (9) and (10) are combinations of an observer-based effect and an object-based one. For example: if the observer-based effect for energy in Eq. (7) is taken to be fundamental, then for the measured value $m'$ of an absolute quantity of mass the observer-based relationship

$$m' = E'/c^2 = (E/c)\Upsilon^{(1-2\pm2)} = m\Upsilon^{-3}$$  \hspace{1cm} (11)$$

will be obtained. The object-based effect on mass-energy given in Eq. (6) is also in effect since both the observer and the object have been transitioned to the other potential. It is the product of the effects in Eqs. (11) and (6) that produces the effect in Eq. (9).

What happens to an electrical charge as its GEPUM changes? If all electrons are of identical construction and the electrical force is mediated by photons, then the absolute electrical flux per unit time for the electron decreases as its GEPUM decreases due to time dilation. Therefore, the standard electrical charge $C$ is subject to an object-based effect of:

$$C' = C\Upsilon^{-1},$$  \hspace{1cm} (12a)$$

and electrical charge itself $\mathcal{C}$ is subject to a fundamental observer-based effect of:

$$\mathcal{C}' = C\Upsilon.$$  \hspace{1cm} (12b)$$

Constant quantities such as the electrical permeability of the vacuum $\epsilon_0$ and the magnetic permittivity of the vacuum $\mu_0$ can not be subject to object-based effects, but like $c$, $\epsilon_0$ and $\mu_0$ may be subject to an observer-based effect. Suppose that observer $K$ is measuring $\epsilon_0$ by noting the force $\hat{F}$ that two charges of strength $\hat{C}_1$ and $\hat{C}_2$ with a stable GEPUM and constant separation of $\hat{r}$ exert on each other. As observer $K$’s GEPUM changes, Eq. (3) calls for the measured distance between the charges $r$ to change while Eq. (12a) calls for the measured values of the charges ($\hat{C}_1$ and $\hat{C}_2$) to change in a similar manner and the energy-fundamental effects listed in Table I indicate that they are measured to exert the same force $F$ on each other. Putting these considerations together and using the relationship $\epsilon_0 = C_1C_2/Fr^2$, an energy-fundamental observer-based effect of:

$$\epsilon'_0 = \epsilon_0$$

is obtained. For magnetism, since $c$ is subject to an observer-based effect of $c' = c\Upsilon^2$ and $c^2 = 1/\epsilon_0\mu_0$, $\mu_0$ is subject to an energy-fundamental observer-based effect of:

$$\mu'_0 = \mu_0\Upsilon^{-4}.$$  

On the other hand, the use of mass fundamental standards produces the opposite result: $\mu'_0 = \mu_0$ and $\epsilon'_0 = \epsilon_0\Upsilon^{-4}$. This is due to the mass-fundamental observer-based effect on force being $F' = F\Upsilon^4$ while the measurement of force is uniform in the energy-fundamental case ($F' = F$).
D. The gravitational Doppler shifting of light

The gravitational Doppler shifting of light provides a way of illustrating the observer-based effects and the differences between MGR and PGR. Under both theories, the rate at which an atom with a lower GEPUM is observed to vibrate is less than that of an atom with the observer’s GEPUM, and that explains the change in frequency. However, the two theories make different predictions as to how the wavelength of the light behaves.

Suppose that a photon emitted in a given atomic transition by an atom with the GEPUM of a base observer (observer A). Observer A will measure a frequency of \( \nu \) for the photon and a corresponding wavelength of \( \lambda = c/\nu \). The atom is now moved to another location which is at a gravitational potential of \( \Phi \) with respect to that of observer A and kept at rest there. From Eq. (1), \( \Upsilon = (1 + 2\Phi/\ell^2)^{-1/2} \) for an observer at the new potential (observer B).

The same atomic transition now produces a photon which is measured by observer B to have a frequency \( \nu' = \nu \). In MGR observer B will also observe the photon’s wavelength \( \lambda' = \lambda \) because Construction Uniformity demands that observer B’s value for the speed of light \( c' \) be \( c \). At the time the photon is emitted, its wavelength in observer A’s frame of reference is now given by two considerations: Because of time dilation, the photon’s frequency is \( \nu'' = \nu \Upsilon^{-1} \), while the coordinate speed of light with respect to observer A at the position of observer B is \( c'' = c \Upsilon^{-2} \); resulting in a coordinate wavelength of \( \lambda'' = c''/\nu'' = \lambda \Upsilon^{-1} \). When the photon finally reaches observer A, the coordinate speed of light will be \( c''' = c \) while the frequency of the photon will be \( \nu''' = \nu'' \), resulting in the wavelength finally becoming

\[
\lambda''' = c'''/\nu''' = c/\nu'' = \lambda \Upsilon. \tag{13}
\]

Under PGR, the situation changes: To the observer B, the photon’s frequency is still \( \nu'' = \nu \), but because \( c' = c \Upsilon^2 \) a wavelength of \( \lambda' = \lambda \Upsilon^2 \) is observed. Due to Extended Uniformity, the light propagates at constant speed with respect to all observers, and because of that its wavelength can not change. At first glance, this is a problem, since it is expected to have the wavelength given in Eq. (13) when it is reaches observer A. However, observer A measures lengths to be \( \Upsilon^{-1} \) as long as those measured by observer B. Therefore, in the frame of reference of observer A, \( \lambda'' = \lambda''' = \lambda \Upsilon \).

This exercise indicates that the light was already Doppler shifted when it was emitted.

E. The deflection of light

The deflection of light in a gravitational field has been verified, but Einstein’s explanation for this deflection, Huyghen’s Principle \(^6\), can not be used under Extended Uniformity.
In PGR, the deflection of light is solely a consequence of the constantly changing state of motion which defines being accelerated. The PGR prediction for the magnitude of the bending of light is given below. This prediction produces the same result as that obtained using tensor mathematics [7], thereby validating that tensor mathematics may be used in PGR. However, this method is not usable in MGR because its treatment of physical distance is improper when Construction Uniformity is being assumed [8].

To compute the magnitude of the bending, suppose that a photon enters an accelerated box at time $t^* = 0$, that the box is of width $w^*$, and that the acceleration of gravity in the $-z$ direction in the box is $g^*$. The angle of the deflection of light $\theta$ is given by the relationship $\theta = \frac{c_z}{c} \cdot \gamma$, where $c_z$ is the downward velocity of light obtained as it propagates across the box, and $c_z \ll c$. Therefore, in terms of time, $\theta$ can be calculated using:

$$\theta = \frac{1}{c} \int_{0}^{t^*_\gamma} g^* dt^*, \quad (14)$$

where $t^*_\gamma = \frac{w^*}{c}.$

Since $c$ is a constant speed, Eq. (14) can be represented as a space-based integral. However, in making this conversion, one must consider that while $\int g^* dt^* \rightarrow \tilde{v}$, at the same time $\int g^* dx^* \rightarrow \tilde{v}^2/2$. Eq. (14) is also generalized for all directions of propagation by replacing $g^*$ with $\partial \Phi / \partial z^*$. The resultant equation is:

$$\theta = 2 \frac{c}{c^2} \int_{0}^{\tilde{x}^*} \frac{\partial \Phi}{\partial z^*} d\tilde{x}^*, \quad (15)$$

where $\tilde{x}^*$ is the initial direction of propagation of the light and $\tilde{z}$ is a direction perpendicular to $\tilde{x}$ for which the deflection is being measured.

**F. Quantum Mechanics and the Bohr Radius**

Combining Quantum Mechanics (QM) and PGR results in changes being made to QM. The basic reason for this involves the De Broglie wavelength and the way it is affected by an object’s GEPUM.

Take an object at the position of observer $K$ which he measures to have a rest mass of $m$ and a velocity of $v$. This object has a momentum of $p$ and a De Broglie wavelength $\lambda$ of:

$$\lambda = \frac{h}{p}, \quad (16)$$

where $h$ is Plank’s Constant. The object is then transferred to the position of observer $K'$ and is made to move with that same velocity of $v$ as measured by observer $K'$. The object’s rest mass as measured by observer $K$ will now be given by the object-oriented relationship $m' = m \Upsilon^{-1}$, and its velocity as measured by observer $K$ will then be $v' = v \Upsilon^{-2}$. Therefore, the object’s momentum as measured by observer $K$ is:

$$p' = p \Upsilon^{-3}. \quad (17)$$

Since $\lambda$ is a length, it is expected to be governed by the object-oriented relationship
\[ \lambda' = \lambda \Upsilon^{-1}. \] (18)

However, since \( h \) is a constant quantity, Eqs. (16) and (17) predict that \( \lambda' = \lambda \Upsilon^3 \), which does not agree with Eq. (18). To get the De Broglie wavelength to come out right, it is postulated that it is affected by an object’s GEPUM in such a way that

\[ \lambda = h \Upsilon^{-4}/p. \] (19)

To demonstrate that Eq. (19) holds in PGR, the Bohr radius of the atom \( a_0 \) is examined. The equation for the Bohr radius is:

\[ a_0 = 4\pi \epsilon_0 \hbar^2/m_e e^2, \] (20)

where \( m_e \) is the mass of the electron and \( e \) is the charge of the electron. The constant quantities \( \epsilon_0 \) and \( h \) do not change for observer \( K \) when the atom is moved to the position of observer \( K' \), while Eq. (8) indicates that the local constant \( m_e \) is subject to an object-based effect of \( \Upsilon^{-1} \), and Eq. (12a) indicates that the local constant \( e \) is subject to an identical object-based effect. Because of this, Eq. (20) calls for an object-based effect of \( a_0' = a_0 \Upsilon^3 \) instead of the desired \( a_0' = a_0 \Upsilon^{-1} \). It is no coincidence that this is the same problem that we had with the De Broglie wavelength. Eq. (20) is the radius at which the wavelength of the electron is exactly adequate to wrap around a proton once, and therefore it implicitly assumes that Eq. (16) holds. Because of this, PGR calls for Eq. (20) to be rewritten as:

\[ a_0 = 4\pi \epsilon_0 h^2 \Upsilon^{-4}/m_e e^2. \] (21)

The changes which produced Eq. (21) affect the binding energy of the atom \( \mathcal{E} \), which is given by \( \mathcal{E} = -e^2/\epsilon_0 a_0 \) for a Hydrogen atom in its ground state. The object-based effects \( e' = e \Upsilon^{-1} \) and \( a_0' = a_0 \Upsilon^{-1} \) predict that \( \mathcal{E}' = \mathcal{E} \Upsilon^{-1} \). This is consistent with the finding in §D that the photons emitted by atoms with lower GEPUMs were already of a lower energy when they were emitted: The atom itself has a lower binding energy when it has a lower GEPUM.

Another effect from the merger of PGR and QM involves the fine structure factor \( \alpha \), which is given by \( \alpha = e^2/\epsilon_0 hc \). When observing the fine-structure factor of an atom with another GEPUM \( \alpha' \), PGR’s object-based effects indicate that the quantity which is \( e \) for that atom will be different \( [e' = e \Upsilon^{-1} \) from Eq. (12a)] while \( \epsilon_0, h, \) and \( c \) are constant quantities, resulting in an object-based effect on the fine structure factor of:

\[ \alpha' = \alpha \Upsilon^{-2}. \] (22)

For an observer with the same GEPUM as this atom, PGR’s observer-based effects (see Table I) indicate that the local value of the product \( \epsilon_0 h \) is uniform while \( c \) is modified \( (c' = c \Upsilon^2) \) and \( e \) is a local constant. This once again produces \( \alpha' = \alpha \Upsilon^{-2} \). For an observer with an

\textsuperscript{7} This is usually called the fine-structure constant, but as this paragraph shows, it is not expected to be the same for all atoms in PGR.
arbitrary GEPUM measuring the fine-structure factor $\alpha''$ of an atom at the base potential, the observer-based effect on charge [$e' = e\Upsilon$ from Eq. (12b)] kicks in, and combining the square of this effect with the effect in Eq. (22) produces a result of $\alpha'' = \alpha$. It may be possible to astronomically confirm the existence of the object-based effect on $\alpha$ as noted in Appendix A2.

MASSIVE OBJECTS

G. The Schwarzschild solution

The Schwarzschild solution is an exact external solution to the Einstein field equations for a spherically symmetric, non-rotating, massive object (which is referred to as a Schwarzschild object). To derive it, one starts with an equation for space-time intervals around a Schwarzschild object [9] which, in PGR terms, is:

$$ds^2 = A(\tilde{r})d\tilde{t}^2 - B(\tilde{r})(\tilde{r}^2 d\theta^2 + \tilde{r}^2 \sin^2 \theta d\phi^2) - C(\tilde{r})d\tilde{r}^2,$$

where $s$ is a spatial-temporal interval, $\tilde{r}$ is the quantity of distance from the center of the massive object (CMO), $\tilde{t}$ is the temporal quantity, $\theta$ and $\phi$ are spherical surface coordinates, and $A(\tilde{r})$, $B(\tilde{r})$, and $C(\tilde{r})$ are arbitrary functions.

To simplify the math for the Schwarzschild equation, a transformation is done on the radial coordinate by defining a new radial coordinate $r'$ which is:

$$r' \equiv \tilde{r}\sqrt{B(\tilde{r})},$$

which turns Eq. (23) into:

$$ds^2 = A'(r')dt^2 - B'(r')dr'^2 - r'^2 d\theta^2 - r'^2 \sin^2 \theta d\phi^2,$$

where $t$ is the value of $\tilde{t}$ as measured from $r'$.

What is the physical interpretation of $r'$ in PGR? To find the answer, first consider that $\sqrt{B(\tilde{r})}$ is the factor by which a measuring-rod oriented perpendicular the line running between itself and the CMO will be shortened with respect to that of some base observer. Since such shortenings are based on GEPUM, the same shortening holds for a ruler oriented parallel to the line running between itself and the CMO. Then take the case of a base observer located at an arbitrary $\tilde{r}$ from the CMO, who measures their own distance from the CMO to be $r_0$, and who is at rest with respect to the massive object. Since $B(\tilde{r}) \equiv 1$ for the base observer by definition, $r' = r_0$ at the position of the base observer. Since the location of the base observer is arbitrary, it may be conjectured that $r'$ is the distance that an observer at any $\tilde{r}$ from the CMO and at rest with respect to it will measure for itself. This conjecture is confirmed by considering the case of the $r'$ for a position that the base observer measures to be at distance $r_1$ from the CMO. Since $\sqrt{B(r_1)}$ is the factor by which distances as measured from $r_1$ differ from distances as measured at $r_0$, $r_1'$ is the distance that an observer at $\tilde{r}_1$ from the CMO and at rest with respect to the massive object will measure as being their own distance from the CMO.

Further steps (which are not discussed here) result in $A'(r')$ and $B'(r')$ being:
\[ A'(r') = 1 - C/r', \]  
\[ B'(r') = 1/(1 - C/r'), \]  
where \( C \) is a constant of integration. To be the same for all observers, \( C \) must be a quantity as measured from some base potential. This requires that the base potential of the Schwarzschild equation be determined. As stated above, at the base potential \( g_{\mu\nu} = \eta_{\mu\nu} \). In Eqs. (24) and (25), this can only be true for extremely large values of \( r' \). Therefore, the base potential is infinitely distant from the massive object.

When the value of \( C \) is determined, it is found that \( C = 2m \), where \( m = MG/c^2 \), \( M \) is the mass of the gravitating object as measured by a distant observer, \( G \) is the gravitational constant as measured by a distant observer, and \( c \) is as measured by a distant observer.

In MGR, it is usual to remove the prime from \( r' \). This is not done in PGR because \( r' \) and \( r \) are physically distinct in this theory. \( r \) is radial distance as measured by some base observer (in this case a very distant observer), while \( r' \) is the self-measured radial distance for the local observer. How then are \( r \) and \( r' \) related? Since the base observer’s measuring-rod is \( \sqrt{B'(r')} \) a big as that of the local observer,

\[ r = r'/\sqrt{B'(r')} = \sqrt{r'(r' - 2m)}. \]  
The inverse equation for Eq. (26) is:

\[ r' = m + \sqrt{r^2 + m^2}. \]  

Eqs. (26) and (27) show that a radial coordinate of \( r' = 2m \) corresponds to \( r = 0 \). This indicates that a black hole can not be embedded in the naive coordinate system of an outside observer. This holds true in MGR [10] as well as PGR, but since naive coordinate systems measures physical reality in PGR, the inability to embed it in the coordinate system means that the black hole can not exist. In addition:

- \( \Upsilon \equiv B'(r') \). Therefore at \( r' = 2m \), \( T = \infty \). This indicates that the Schwarzschild radius is measured locally with a measuring-rod which is physically of zero length.
- At the surface of a point object, a Newtonian potential of \( \Phi = -c^2/2 \) exists. This is consistent with SR since it means that no free-falling object can be made to go faster than \( c \) with respect to any other frame of reference.
- In Newtonian physics, an infinite amount of energy is needed to escape from the “surface” of a point mass. Similarly, in GR an infinite amount of energy is needed to escape from \( r' = 2m \) [11]. In PGR, these two views of gravitational collapse become one and the same.

How many measuring-rods of identical construction at rest with respect to a massive object and on a line going through its center are needed to bridge the gap between two radial coordinates \( r'_1 \) and \( r'_2 \)? This is given by:

\[ \int_{r'_1}^{r'_2} \sqrt{g^1_1} \, dx^1, \]  
where \( dx^1 \) is an increment of distance in the radial direction and \( g^1_1 \equiv B'(r') \). In MGR, Eq. (28) is converted into:
\[
\int_{r_1'}^{r_2'} \sqrt{1/(1 - 2m/r')} \, dr'.
\]

(29)

However, in the PGR viewpoint, there is a problem with Eq. (29): The shortening of a unit measuring-rod to being \([B'(r')]^{-1/2}\) units long occurs with respect to the units of a distant observer, and that observer measures distances using \(r\) instead of \(r'\). Therefore, in PGR Eq. (28) is converted into:

\[
\int_{r_1}^{r_2} \sqrt{1/(1 - 2m/r')} \, dr.
\]

(30)

To solve Eq. (30), \(1/(1 - 2m/r')\) is first modified to \(r'/((r' - 2m))\). Then using Eq. (27) to convert \(r'\) into \(r\) produces:

\[
\sqrt{r'/((r' - 2m))} = \frac{m + \sqrt{r^2 + m^2}}{r}.
\]

(31)

Substituting Eq. (31) into the integral from Eq. (30) produces:

\[
\int \frac{m + \sqrt{r^2 + m^2}}{r} \, dr = \sqrt{r^2 + m^2} + m \ln \left( \frac{r^2}{m + \sqrt{r^2 + m^2}} \right) + C,
\]

(32)

where \(C\) is a constant of integration. Eq. (26) is then used to represent Eq. (32) in term of \(r'\):

\[
\sqrt{r^2 + m^2} + m \ln \left( \frac{r^2}{m + \sqrt{r^2 + m^2}} \right) + C = r' - m + m \ln(r' - 2m) + C.
\]

Therefore:

\[
\int_{r_1}^{r_2} \sqrt{1/(1 - 2m/r')} \, dr = r'_2 - r'_1 + m \ln \left( \frac{(r'_2 - 2m)/(r'_1 - 2m)}{1/2} \right).
\]

(33)

At radial coordinates \(r' \gg 2m\), Eq. (33) reduces to \(r'_2 - r'_1\), which is normal for unaccelerated frames of reference. But as one approaches \(r' = 2m\), the term \(m \ln \left( ((r'_2 - 2m)/(r'_1 - 2m)) \right)\) comes to dominate. This term has a peculiar effect: To travel from a radial coordinate of \(r'_1 = 2m + a\) to \(r'_2 = 2m + a/2\), one must travel for a locally perceived distance of at least \(m \ln(1/2) = -0.693m\). Furthermore this applies to every decrease of the difference between one’s radial coordinate and \(2m\) by \(\frac{1}{2}\). This is called the Achilles Effect, because it is reminiscent of Zeno’s Paradox.

H. Gravitational Collapse

Because \(r' = 2m\) describes a point object in PGR, black holes can not exist, and the Achilles Effect [Eq. (33)] makes this singularity unreachable. However, these considerations do not eliminate the possibility of infinite gravitational collapse. It will now be shown that PGR does not permit that. The key concept in disallowing infinite gravitational collapse is that it is quantities which are conserved, and that locally measured values will not describe
physical quantities in a consistent manner. Instead, quantities must be measured from a base frame of reference.

Take the case of a massive object which is a hollow sphere of infinitesimal thickness. All of its mass is at the same radial coordinate \( r' \) and therefore it all has the same GEPUM. Eq. (33) indicates that after an object with an initial rest mass of \( M \) falls though a potential of \( \Phi \), its final rest mass will be \( M' = M \Upsilon^{-1} \) where, in the vicinity of a Schwarzschild object,

\[
\Upsilon \equiv B'(r') = \sqrt{r'/(r' - 2m)}.
\]

The potential energy of self-gravitation is a function of the radius of the object and the square of its mass. This results in the quantity for the potential energy of self-gravitation \( \Psi \) of the hollow sphere as measured by a distant observer being given by:

\[
\Psi = -\frac{M^2G}{r} \Upsilon^{-2} = -\frac{M^2G}{\sqrt{r'(r' - 2m)}} \frac{r' - 2m}{r'} = -M^2G\sqrt{\frac{r' - 2m}{r'^3}}. \quad (34)
\]

There are two situations where Eq. (34) goes to 0: At \( r' = \infty \) due to \( \infty \) being the denominator, and \( r' = 2m \) due to the numerator going to 0. (The case of \( r' = \infty \) represents the mass being spread out across the whole universe, while the case of \( r' = 2m \) represents the matter in the object as having lost all of its rest mass.) Therefore, there exists a radius at which gravitational self-potential energy has a minimal value. At that radius \( d\Psi/dr' = 0 \).

To solve this equation, \( \Psi^2 = -M^4G^2(r' - 2m)/r'^3 \) will be used so that is is now \( d\Psi^2/dr' = 0 \) which is being solved. Ignoring the \(-M^4G^2\) term (since it is constant), this produces:

\[
\frac{d\Psi^2}{dr'} = \frac{d}{dr'} \left( \frac{1}{r'^2} - \frac{2m}{r'^3} \right) = -\frac{2}{r'^3} + \frac{6m}{r'^4} = 0, \quad r' = 3m. \quad (35)
\]

So at \( r' = 3m \), an object’s potential energy of self-gravitation is at its minimal value. A radial coordinate of \( r' = 3m \) corresponds to a Newtonian potential of \( \Phi = -\frac{c^2}{3} \). This means that an object which is too massive to be stabilized by quantum electrostatic forces will be stabilized by gravity in such a way that its mass exists as an average Newtonian potential of \( \Phi = -\frac{c^2}{3} \).

It is fair to note that a collapsing object will retain its kinetic energy of collapse, and may therefore collapse to a Newtonian potential of \( \Phi < -\frac{c^2}{3} \). In this case, the Achilles effect comes into play to halt the collapse. As one approaches the limiting potential of \( \Phi = -\frac{c^2}{2} \), the locally measured radius of the object will become almost constant at something slightly greater than \( 2m \), but the effective depth of the object [which is obtained from Eq. (33)] will begin to increase dramatically. The result is a large increase in the effective volume at a given potential. The collapsing object will therefore begin to spread out within that effective volume. Eventually, the increase in volume will allow quantum electrostatic forces to reassert themselves, and the atoms being put into excited states will absorb the kinetic energy of collapse\[^8\]. Once those forces stop the collapse, the object will now find that it can

\[^8\]This is important since Conservation of Mass-Energy demands that the rest mass \( M \) of the object as measured by a distant observer must be the same at the end of the collapse as it was at the beginning if no energy is radiated away during the collapse.
go to a lower self-gravitational potential energy by expanding to a larger size, and it will rebound. The kinetic energy will then be in place again, but it will be sending the object’s mass outward instead of inward. The object may then oscillate, but will be losing energy as it does so. Once enough energy is lost, it will settle into a stable state with an average Newtonian potential of $\Phi = -\frac{c^2}{3}$.

I. Hyperstars

If black holes can not form, then what is a gravitationally collapsed object like? First of all, it can not end up having its surface at a radial coordinate of $r' < 3m$. If the lowest self-potential energy of a stable object occurs at $r' = 3m$, then the mass of a fully gravitationally collapsed object to will be distributed around the corresponding potential of $\Phi = \frac{c^2}{3}$ and the surface of such an object will always be at $r' > 3m$.

This observation leads to the possibility that millisecond pulsars are neutron stars which are too massive to be stabilized by nuclear/quantum effects and have undergone runaway gravitational collapse. However, since the Schwarzschild radius increases as a function of mass, while volume increases with the cube of an object’s radius, the average density of a gravitationally collapsed object decreases as its mass increases. So the heaviest of the pulsars may not be any more dense than those stabilized by quantum effects, and may therefore appear to be “normal” pulsars.

For heavier objects, the average density decreases still further until at 50 million solar masses you have an object with a Schwarzschild radius of approximately 1 A.U., and a physical radius of $> 1.5$ A.U. Its average density would be comparable to that of the Sun, and it could be expected to burn hydrogen like a star. However its hydrogen burning would be occurring in a much bigger volume than is the case for a normal star. Such an object is referred to as hyper-massive star-like object, or hyperstar. Hyperstars must generate and emit tremendous amounts of energy, just as the accretion disks surrounding super-massive black holes are expected to do in MGR. It is therefore speculated that all of the existing black hole candidates are in fact hyperstars, including the quasars.

CONCLUSIONS

In this article, it has been demonstrated that it is possible to construct a general theory of relativity when Extended Uniformity is used instead of Construction Uniformity. In the process, it has been shown that if PGR is true:

- $c$ varies as a function of the observer’s GEPUM.
- The main predictions of Einstein’s MGR are preserved.
- The singularities of the Schwarzschild solution are rendered unreachable.
- Gravitational collapse produces hyperstars instead of black holes.

At the same time, there are many questions left to answer, such as:

- Can subatomic particles be treated as “raw matter” which is stabilized at a gravitational potential of $-\frac{c^2}{3}$ or as close to it QM will allow?
- Can a workable theory of quantum gravity be created using Extended Uniformity?
• What are the detailed astrophysics of hyperstars?
• What becomes of the Big Bang singularity? How is it reinterpreted in PGR, or does it even need reinterpretation?

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APPENDIX A: EXPERIMENTATION AND OBSERVATION WITH PGR

1. PGR’s Consistency with existing experimental evidence

With regards to existing experiments and observations [12], PGR and MGR produce identical results, due to the preservation of Einstein’s field equations. For example:
• The deflection of light, as measured photographically and by radio measurements of occultations of the quasar 3C279: PGR predicts the same amounts for the deflection of light, as shown in §E.
• Relativistic time dilation, measured using atomic clocks on aircraft and rockets: This effect is retained as-is in PGR.
• The red-shifting of light, measured for light coming from the Sun and certain stars, and verified using the Mossbaur effect: Both PGR and MGR predict the same values for the red-shifting of light, although the theories differ on how the red-shifted wavelength comes to be observed as detailed in §D.
• The precession of the perihelion of an orbit, as observed for the planet Mercury and of the binary pulsar PSR 1913+16: These effects are obtained directly from Einstein’s field equations, and as such are not affected by the differences between MGR and PGR.
• Radar ranging in the Solar System: Under PGR, the results are interpreted as representing an expansion of space caused by the presence of the Sun instead of a decrease in the rate of the propagation of light. Thus the light is traveling a longer distance at constant speed, and the “delay” expected remains the same.

2. Experiments that may test PGR

In spite of the inability of existing experiments to discriminate between MGR and PGR, a variation of the Michaelson-Morely experiment can. Take a laser beam, split it, send its parts down two paths of substantially different total travel length and then recombine
them, producing an interference pattern. The experiment then has its GEPUM changed by either gradually moving it to another elevation above the Earth’s surface or keeping it in place and looking for changes caused by tidal potential and possibly changes in the Earth’s GEPUM caused by interactions with the Moon and the other planets.

MGR predicts that $c$ as measured at the experiment will be unaffected by changes in the experiment’s GEPUM, which means that the interference pattern will be unaffected. PGR predicts that changes in the experiment’s GEPUM will cause $c$ as measured at the experiment to change, resulting in the relative phases of the light coming in from the two paths to change and the interference pattern to be modified as a result. The effects being search for are quite small, amounting to 1 part in $10^{13}$ for a change of 1 kilometer in the elevation of the experiment. The experiment will therefore need to be quite sensitive. Even so, it should be possible to detect the PGR effects if PGR is indeed physically correct.

Examining the propagation of light along a standard measuring-rod with another GEPUM provides another way of testing PGR. In MGR, all local observers will measure the same amount of time $t$ for a beam of light to go back and forth along a standard measuring-rod at their position in a gravitational field. As a result, if the movement of light along a standard measuring-rod with another GEPUM is observed, a back-and-forth time of $t' = t \Upsilon$ (due to time dilation) is expected in MGR. On the other hand, in PGR the rate of propagation of light with respect to our coordinate system stays constant while the size of the measuring-rod is decreased from $l$ to $l \Upsilon^{-1}$. This produces the prediction that the back-and-forth time observed for the measuring-rod with another GEPUM will be $t' = t \Upsilon^{-1}$. Therefore, timing the movement of light along a standard measuring-rod with another GEPUM will indicate whether MGR or PGR is physically correct.

Another experiment is the precise and continuous measurement of the time it takes for light to go back and forth in an evacuated tube, thereby directly measuring $c$, and detecting variations in it caused by changes in the experiment’s GEPUM.

The fine-structure factor effect given in Eq. (22) provides a means of doing an observational test of PGR by examining the fine structure factor in the emission lines from a gravitationally collapsed object such as a neutron star or quasar/hyperstar. For example, suppose that the surface of a quasar exists at a Newtonian potential of $\Phi^* = \frac{c^2}{6}$. In this case, a fine structure factor of $\alpha' = 2\alpha/3$ will be observed in the quasar’s light, where $\alpha$ is the fine structure factor for an atom at the potential of a distant observer.

### 3. Evidence for the existence of hyperstars

Evidence for the existence of hyperstars comes from current observations of galaxies and quasars. In MGR, quasars are considered to be the accretion disks of super-massive galaxies

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9This experiment could not be done before the invention of the laser since incoherent light cannot produce an interference pattern with total travel paths of substantially different lengths.

10The LIGO and VIRGO evacuated tunnels may be ideal places to do the fixed version of this experiment.
which are collapsing into a back hole. This view gets some support from the finding that many quasars are accompanied by galaxies. However, quasars are known to be much more common in the early universe [13], while for more recent times one finds more in the way of active galaxies, which are believed to have quasar-like cores, prompting some researchers to consider these as being quasar remnants [14]. This creates the impression that quasars are the progenitors of galaxies instead of the other way around, which implies that they emit matter instead of absorbing it as a black hole would do. In addition, quasars and the quasar-like central objects of galaxies are known to be relatively small objects, and the central objects of some galaxies have been shown to have masses as large as 10 billion solar masses [15][16]. These observations can be interpreted as indicating that quasars and the central objects of galaxies may represent some condition whereby runaway gravitational collapse fails to form a black hole.

Another piece of evidence is the recent discovery of carbon in the early universe [17]. This may be material formed inside the hyperstar and later ejected into inter-galactic space.
REFERENCES

[1] A. Einstein, Ann. Phys. (Leipzig), 17, 547 (1905)
[2] A. Einstein, Ann. Phys. (Leipzig), 35, 898 (1911)
[3] H. Ohanian & R. Ruffini, “Gravitation and Spacetime” (W. W. Norton & Co., New York, NY), 2nd ed., 1994, Chapter 4, pp. 177-9
[4] M. G. Bowler, “Gravitation and Relativity” (Pergamon Press Ltd., Elmsford, NY), 1976
[5] R. Wald, “General Relativity” (University of Chicago Press, Chicago, Ill.), 1984, §6.2, p. 126, Eqs. 6.2.10 & 6.2.11.
[6] A. Einstein, Ann. Phys. (Leipzig), 49, 796 (1916)
[7] H. Ohanian & R. Ruffini, “Gravitation and Spacetime” 2nd ed., §4.3, p. 191.
[8] W. Rindler, Am. J. Phys. 36, 540 (1968).
[9] H. Ohanian & R. Ruffini, “Gravitation and Spacetime” 2nd ed., 1994, Chapter 7, pp. 392 – 396. This is a general reference for §7.
[10] private communications with Bill Lawson of Lawrence Livermore National Labs, 3/6/1995. The remark made was “black holes cannot be embedded in a Euclidean space”. This applies to naive coordinate systems because they are, by virtue of how they are constructed, Euclidean.
[11] H. Ohanian & R. Ruffini, “Gravitation and Spacetime” 2nd ed., §7.6, pp. 403-403 with angular momentum per unit mass ($\ell$) being 0.
[12] C. M. Will, “Theory and Experiment in Gravitational Physics” (Cambridge University Press, Cambridge, England), 1981
[13] B. J. Boyle, T. Shanks, & B. A. Peterson, Mon. Not. R. Astron. Soc. 235(3), 935 (Oct 15, 1988)
[14] F. Vagnetti & R. Spera, Astrophys. J. 436(2), 611 (Dec. 1, 1994)
[15] H. C. Ford et al, Astrophys. J. Lett. 435(1), L27 (Nov. 1, 1994)
[16] M. Miyoushi et al., Nature 373(6510), 217 (Jan. 12, 1995)
[17] T. Yamada et. al, Astrophys. J. Lett. 438(1), L5 (Jan. 1, 1995)
TABLE I. The effects changes in an observer’s GEPUM on the measured values of physical quantities (called observer-based effects), based on the general relationship \( q' = q \gamma^f \).

| Quantity            | Units\(^a\) | Observer-based Effects |
|---------------------|-------------|------------------------|
|                     |             | Mass                   | Energy                  |
|                     |             | Fundamental           | Fundamental             |
| Length              | \( d \)     | \( d' = d \gamma \)   | \( d' = d \gamma \)    |
| Time                | \( t \)     | \( t' = t \gamma^{-1} \) | \( t' = t \gamma^{-1} \) |
| Velocity            | \( d/t \)   | \( v' = v \gamma^2 \) | \( v' = v \gamma^2 \) |
| Acceleration        | \( d/t^2 \) | \( a' = a \gamma^3 \) | \( a' = a \gamma^3 \) |
| Mass                | \( m \)     | \( m' = m \gamma \)   | \( m' = m \gamma^{-3} \) |
| Force               | \( md/t^2 \) | \( F' = F \gamma^4 \) | \( F' = F \gamma^4 \) |
| Momentum            | \( md/t \)  | \( p' = p \gamma^3 \) | \( p' = p \gamma^{-1} \) |
| Energy              | \( md^2/t^2 \) | \( E' = E \gamma^5 \) | \( E' = E \gamma \) |
| Angular Momentum    | \( md^2/t \) | \( o' = o \gamma^4 \) | \( o' = o \gamma^{-4} \) |
| Gravitational Constant | \( d^4/mt^2 \) | \( G' = G \gamma^4 \) | \( G' = G \gamma^8 \) |
| Gravitational Potential | \( d^2/t^2 \) | \( \Phi' = \Phi \gamma^4 \) | \( \Phi' = \Phi \gamma^4 \) |
| GEPUM               | \( d^2/t^2 \) | \( \Phi' = \Phi \gamma^4 \) | \( \Phi' = \Phi \gamma^4 \) |
| Charge              | \( C \)     | \( C' = C \gamma \)   | \( C' = C \gamma \)    |
| Current             | \( C/t \)   | \( J = J \gamma^2 \)  | \( J = J \gamma^2 \)   |
| Electrical Permeability | \( C^2t^2/md^3 \) | \( \epsilon'_0 = \epsilon_0 \gamma^{-4} \) | \( \epsilon'_0 = \epsilon_0 \) |
| Magnetic Permittivity | \( md/C^2 \) | \( \mu'_0 = \mu_0 \)   | \( \mu'_0 = \mu_0 \gamma^{-4} \) |

\(^a\) \( d = \text{Distance}, \ t = \text{Time}, \ m = \text{Mass}, \ C = \text{Charge} \)