Heavy Quark Effective Field Theory at $O(1/m_Q^2)$. II.
QCD Corrections to the Currents

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Abstract

We present a calculation of the renormalized heavy-light and heavy-heavy currents in HQET at order $O(1/m_Q^2)$.

1 Introduction

In [1] we have presented a new calculation of the $O(\alpha)$ renormalized Lagrangian of Heavy Quark Effective Field Theory (HQET) at $O(1/m_Q^2)$. This result has been confirmed using different calculational strategies in [2] (see

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also [3]). In this note we proceed with the results of a calculation of the renormalized heavy-light and heavy-heavy currents in the same order. The goal is a systematic calculation of the matrix element

\[ \langle D(v')|\bar{c}\gamma_\mu(1-\gamma_5)b|B(v) \rangle \]  

up to and including \( O(1/m_Q^2) \), including leading order QCD radiative corrections.

A full calculation of the matrix element (I) of the heavy-heavy current at non-zero recoil is technically feasible, but still of limited phenomenological value. The appearance of the second independent velocity vector \( v' \) leads to a proliferation of operators whose matrix element would have to be parametrized. This results in a substantial decrease in predictive power. Instead we will follow [4] and concentrate on the zero recoil case \( v = v' \), which should be considered as the lowest order of an expansion of the matrix element (I) at the symmetry point \( \omega = vv' = 1 \). In this case Luke’s celebrated theorem [5] guarantees that there are no \( O(1/m_Q) \) corrections to the current, but there are non-trivial contributions at \( O(1/m_Q^2) \) [II, 4].

This note is organized as follows: in section 2 we define our operator basis for the effective lagrangian, heavy-light and heavy-heavy currents up to and including \( O(1/m_Q^2) \). Our result for the anomalous dimensions will be presented in section 3 followed by a discussion of various consistency checks in section 4. In section 5 the numerical solution of the renormalization group equation (RGE) for the currents at \( O(1/m_Q^2) \) is presented and is discussed with respect to phenomenological applications.

## 2 Operator basis

### 2.1 Heavy-light currents

We start with the current up to \( O(1/m_Q) \) and the operators in the effective lagrangian needed for the construction of time-ordered products in \( O(1/m_Q^2) \). The kinetic and chromomagnetic operators in the effective lagrangian of dimensions 5

\[ O^{(1)c/b}_1 = \bar{h}^{c/b}_v(iD)^2h^{c/b}_v, \quad \quad \quad O^{(1)c/b}_2 = g\bar{h}^{c/b}_v\sigma^{\mu\nu}G_{\mu\nu}h^{c/b}_v \]  

and 6

\[ O^{(2)c/b}_1 = \bar{h}^{c/b}_v i\sigma^{\mu\nu}D_\mu(i\gamma_D)D_\nu h^{c/b}_v, \quad \quad \quad O^{(2)c/b}_2 = \bar{h}^{c/b}_v i\sigma^{\mu\nu}iD_\mu(i\gamma_D)D_\nu h^{c/b}_v \]
are labeled by a flavor index for future use. We choose the operator basis for the currents of dimensions 3 and 4 as

\[ O_{HL,1}^{(0)} = \bar{c}_L v^\mu h_v, \quad O_{HL,2}^{(0)} = \bar{c}_L \gamma^\mu h_v \]  
\tag{3a}

and \( O_{HL,i}^{(1)} \):

\[ -\bar{c}_L i \bar{D}^\mu v^\mu h_v, \quad \bar{c}_L i \bar{D}^\mu h_v, \quad -\bar{c}_L (ivD) v^\mu h_v, \quad -\bar{c}_L i \bar{D} v^\mu h_v, \]
\[ \bar{c}_L i \bar{D} v^\mu h_v, \quad -\bar{c}_L (ivD) \gamma^\mu h_v, \quad -\bar{c}_L i \bar{D} \alpha i \sigma ^{\alpha \mu } h_v, \quad \bar{c}_L i \bar{D} \alpha i \sigma ^{\alpha \mu } h_v \]  
\tag{3b}

In (3b) the label \( i \) runs from left to right. Because of the momentum transfer at the weak current, we have to distinguish derivatives \( \bar{D} = \partial D + igT^a A^a \) acting to the left from derivatives \( D = \partial D - igT^a A^a \) acting to the right in the operator basis. It turns out that not all operators that contain two of these derivatives are independent. In fact there are arrow identities (AI), that we have to take into account:

\[ -i \bar{D}_\alpha i \bar{D}_\beta + i \bar{D}_\beta i \bar{D}_\alpha - i \bar{D}_\alpha i \bar{D} + i \bar{D} i \bar{D}_\alpha = 0 \]  
\tag{4a}

\[ i \bar{D}_\alpha i \bar{D}_\beta - i \bar{D}_\beta i \bar{D}_\alpha - i \bar{D} i \bar{D}_\alpha + i \bar{D} \beta i \bar{D}_\alpha = 0 \]  
\tag{4b}

\[ i \bar{D}_\alpha i \bar{D}_\beta + i \bar{D}_\beta i \bar{D}_\alpha - i \bar{D} i \bar{D}_\alpha - i \bar{D} \beta i \bar{D}_\alpha = 0 \]  
\tag{4c}

\[ \bar{c}_L \left[ i \bar{D}_\alpha \bar{D}_\beta + i \bar{D}_\beta \bar{D}_\alpha + i \bar{D}_\alpha \bar{D}_\beta + i \bar{D}_\beta \bar{D}_\alpha \right] h_v = i \partial_\alpha i \partial_\beta \bar{c}_L h_v \]  
\tag{4d}

The identities (4) are operator identities and we stress that their derivation \cite{7} is independent from the applicability of partial integration. The identity (4d) results in additional contributions to the anomalous dimensions from the divergent parts of the two point function with dimension 5 operator insertions. The AI and the equation of motion (EOM) for the heavy quark \((ivD)h_v = 0\) reduce the full set of 56 local operators to 26 operators \( O_{HL,i}^{(2)} \), which we choose as

\begin{align*}
(i\bar{D}_\alpha )^2 v^\mu, & \quad i\bar{D}_\alpha i \bar{D} v^\mu, & \quad -i \bar{D}_\alpha i \bar{D} v^\mu, & \quad i \bar{D} i \bar{D} v^\mu, \\
-i \bar{D}_\alpha i \bar{D} v^\mu, & \quad i \bar{D}_\alpha i \bar{D} v^\mu, & \quad i \bar{D}_\alpha i \bar{D} v^\mu, & \quad \bar{D}_\alpha i \bar{D} \beta \sigma ^{\alpha \beta } v^\mu, \\
i \bar{D}_\beta \sigma ^{\alpha \beta } v^\mu, & \quad -i \bar{D}_\beta \sigma ^{\alpha \beta } v^\mu, & \quad \bar{D}_\beta \sigma ^{\alpha \beta } v^\mu, & \quad i \bar{D} \beta \sigma ^{\alpha \beta } v^\mu, \\
(i\bar{D}_\alpha )^2 \gamma^\mu, & \quad i\bar{D}_\alpha \bar{D} \gamma^\mu, & \quad -i \bar{D} i \bar{D} \gamma^\mu, & \quad i \bar{D} \gamma^\mu, \\
i \bar{D}_\beta \gamma^\mu, & \quad -i \bar{D}_\beta \gamma^\mu, & \quad \bar{D}_\beta \gamma^\mu, & \quad i \bar{D} \gamma^\mu, \\
i \bar{D}_\beta \gamma^\mu, & \quad -i \bar{D}_\beta \gamma^\mu, & \quad \bar{D}_\beta \gamma^\mu, & \quad i \bar{D} \gamma^\mu, \\
i \bar{D}_\beta \gamma^\mu, & \quad -i \bar{D}_\beta \gamma^\mu, & \quad \bar{D}_\beta \gamma^\mu, & \quad i \bar{D} \gamma^\mu, \\
-i \bar{D}_\alpha i \bar{D} \beta v^\gamma i \epsilon ^{\alpha \beta \gamma}, & \quad i \bar{D}_\alpha i \bar{D} \beta v^\gamma i \epsilon ^{\alpha \beta \gamma}, \\
i \bar{D}_\alpha i \bar{D} \beta v^\gamma i \epsilon ^{\alpha \beta \gamma}. \\
\end{align*}
\tag{5}
where the $\bar{c}_L$ on the left and the $h_v$ on the right have been suppressed.
Since $m_c \neq 0$, the full basis contains 18 operators proportional to $m_c$ as well as $m_c^2$, which can be eliminated using the EOM for the light quark

$$m_c \bar{c}_R = -\bar{c}_L i \tilde{D}. \quad (6)$$

In [1] the operator basis could be divided naturally in four classes: vanishing by the equations of motion (EOM) or not, local or time-ordered product. This time we have to consider another class of unphysical operators, that are related to the other operators by contraction identities (CI). These identities arise in time-ordered products with $(ivD)$-operators acting on internal heavy quark propagators [8]. They are independent from the external states and can be illustrated graphically as follows:

$$\text{T} \left[ \ldots A(\overrightarrow{ivD})h_v][h_vB\ldots] \right] \propto \text{T} \left[ \ldots AB\ldots \right]$$

Even though their derivation proceeds in terms of unrenormalized operators, the CI hold true under renormalization in the $MS$-scheme, which we have used. The full basis contains 56 time-ordered products of lower order contributions to the current with operators from the effective lagrangian of the $b$-quark. After application of the CI and the EOM for the heavy quark, we are left with 26 time-ordered products. They are classified as follows:

- double-insertions from the effective lagrangian in $O(1/2m_b)$:

$$\mathcal{T}^{(011)bb}_{HL,ijk} = -(1 - \frac{1}{2} \delta_{jk}) \text{T} \left[ \mathcal{O}^{(0)}_{HL,i}; \mathcal{O}^{(1)b}_j, \mathcal{O}^{(1)b}_k \right] \quad i, j, k = 1, 2 \quad (7)$$

The prefactor accounts for insertions of identical operators. To avoid double-counting the additional constraint $j \leq k$ has to be imposed.

- single-insertions from the effective lagrangian in $O(1/(2m_b)^2)$:

$$\mathcal{T}^{(02)b}_{HL,ij} = \text{T} \left[ \mathcal{O}^{(0)}_{HL,i}; \mathcal{O}^{(2)b}_j \right] \quad i, j = 1, 2 \quad (8)$$

- mixed insertions of dimension 4 currents and the effective lagrangian in $O(1/2m_b)$:

$$\mathcal{T}^{(11)b}_{HL,ij} = \text{T} \left[ \mathcal{O}^{(1)}_{HL,i}; \mathcal{O}^{(1)b}_j \right] \quad \begin{cases} i = 1, \ldots, 8 \\ j = 1, 2 \end{cases} \quad (9)$$
2.2 Heavy-heavy currents at zero recoil

The construction of the operator basis contributing to the heavy-heavy current proceeds in analogy to the heavy-light current. Since the $c$-Quark is now static, the tensor structures of flavor changing operators are projected from the left by $P_v^+$ and we have to consider additional time-ordered products with operators in the effective lagrangian of the static $c$-Quark. To lowest order only two operators contribute:

\[
\bar{h}_v^c P_5^+ v^\mu h_b^b \\
\bar{h}_v^c P_5^+ \gamma^\mu h_b^b
\]  

(10)

At order $\mathcal{O}(1/m_Q)$, all operators sandwiched between physical states vanish by Luke's theorem and may be discarded. At order $\mathcal{O}(1/m_Q^2)$ the full basis contains 14 local and 100 nonlocal operators. After application of CI and EOM for the heavy quarks, we are left with 46 operators.

The action of spin and flavor symmetry in the matrix elements of time-ordered products reveals 20 additional relations. Using flavor symmetry, for example, the matrix elements of $T_{HH,112}^{(011)c\bar{b}}$ and $T_{HH,121}^{(011)c\bar{b}}$ between mesonic states $|\mathcal{D}\rangle$ and $|\mathcal{B}\rangle$ are described by the same reduced matrix element multiplied by Clebsch-Gordan coefficients

\[
\text{Tr} \left\{ \sigma_{\alpha\beta} \mathcal{M}_c P_5^+ v^\mu P_v^+ \sigma^{\alpha\beta} \mathcal{M}_b \right\}, \quad \text{Tr} \left\{ \sigma_{\alpha\beta} \mathcal{M}_c \sigma^{\alpha\beta} P_v^+ P_5^+ v^\mu \mathcal{M}_b \right\}
\]  

(11)

with $P_5^+ v^\mu$, $\sigma^{\alpha\beta}$ and $\mathcal{M}_c/b$ representing the Dirac-structure of the current, the chromomagnetic operator and the external states respectively. Since the former is sandwiched between projectors $P_v^+$, it conserves spin symmetry and may be moved around freely inside the traces. Consequently the traces are equal and so are the matrix elements. Other relations among matrix elements can be established analogously.

Furthermore, since spin symmetry allows us to rewrite all mixed time-ordered products in terms of EOM-operators, they are removed from the basis applying the CI.

The basis is reduced by these relations to 6 local operators

\[
(iD)^2 v^\mu, \quad (iD)^2 \gamma^\mu, \quad i\bar{D}iD^\mu, \quad iD^\mu i\bar{D}, \quad iD_\alpha iD_\beta i\sigma^{\alpha\beta} v^\mu, \quad iD_\alpha iD_\beta v_\gamma i\epsilon^{\alpha\beta\gamma\mu}
\]  

(12)

taken between static fields $\bar{h}_v^c P_5^+$ and $h_b^b$ and 20 time-ordered products

\[
T_{HH,ijk}^{(011)c\bar{b}} |j|<k, \quad T_{HH,122}^{(011)c\bar{b}}, \quad T_{HH,122}^{(011)c\bar{b}} |j|<k \quad \text{for} \ i=1, \quad T_{HH,ij}^{(02)c}, \quad T_{HH,122}^{(02)c}.
\]  

(13)

where all indices run from 1 to 2.
3 Anomalous dimensions

As in [1], we have calculated the anomalous dimensions in the background field gauge with gauge parameter \( \xi \). For the physical operator bases presented here, all \( \xi \)-dependence has to drop out, which provides a powerful cross-check. In the case of heavy-light currents, the anomalous dimensions in leading and subleading order are well known [9, 10, 11] and are reproduced by our calculations. Our result in \( \mathcal{O}(1/m_b^2) \) can be written in block form, separating local operators from time-ordered products:

\[
\hat{\gamma}_{HL}^{(2)} = \begin{pmatrix}
\hat{O}_{HL}^{(2)} & \hat{T}_{HL}^{(11)b} & \hat{T}_{HL}^{(02)b} & \hat{T}_{HL}^{(011)bb}
\end{pmatrix}
\]

(14)

Weinberg’s theorem guarantees that local operators need no non-local counterterms. This accounts for the vanishing of seven submatrices in (14). There are, however, nonlocal contributions \( \hat{\gamma}_{n,1}^{(011)} \) and \( \hat{\gamma}_{n,3}^{(011)} \) that arise from the renormalization of the lagrangian or the lower order current. This situation is depicted diagramatically in figure 1.

Our result for the heavy-heavy currents at the same order is of similar structure:

\[
\hat{\gamma}_{HH}^{(2)} = \begin{pmatrix}
\hat{O}_{HH}^{(2)} & \hat{T}_{HH}^{(02)} & \hat{T}_{HH}^{(011)}
\end{pmatrix}
\]

(15)

The entries for all submatrices \( \hat{\gamma} \) are available from the authors on request. A
Figure 1: Nonlocal contributions to the renormalization of \( \tilde{T}^{(011)bb}_{HL} \). The operators of the effective lagrangian in \( \mathcal{O}(1/m_b) \) and the current in \( \mathcal{O}(1/m_b^0) \) are represented by open squares and triangles respectively, whereas filled symbols represent the corresponding operators of the next dimension.

A typical example is given by the anomalous dimensions of the local operators:

\[
\tilde{\gamma}^{(2)}_{HH,d} = \frac{C_A}{4} \begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 1 & 0 \\
0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & -2 & 0 \\
0 & 0 & 0 & 0 & -2
\end{pmatrix}.
\]

The observation, that the kinetic and chromomagnetic operators (the upper left and the lower right \( 2 \times 2 \)-block) renormalize as their counterparts in the effective lagrangian in \( \mathcal{O}(1/m_Q) \), is easily explained by the fact that both operator bases coincide at zero recoil up to parity violating contributions. The anomalous dimensions have been calculated with the help of FORM [12].

In the background field gauge, the divergent part of the three-point function has to be calculated to derive the anomalous dimensions. The latter receive additional contributions from the two-point function only in the case of heavy-light currents as a result of the identity [11].
4 Consistency checks

In section 2 we have used several identities to reduce our operatorbasis. However, we have to verify that this reduction is compatible with the RGE. Suppose that a linear combination

\[ O_\kappa = \sum_i \kappa_i O_i \]  

(17)

of operators does not contribute to the matrix element under consideration

\[ \langle D(v) | O_\kappa | B(v) \rangle = 0 \]  

(18)

In general, this relation will not be stable under renormalization since

\[ O_{\delta\kappa} = \sum_i (\delta\kappa)_i O_i = \sum_i \kappa_i \frac{d}{d\ln \mu} O_i = \sum_{i,j} \kappa_i \gamma_{ij} O_j \]  

(19)

and

\[ \langle D(v) | O_{\delta\kappa} | B(v) \rangle \neq 0 \]  

(20)

unless \( \gamma_{ij} \) satisfies further conditions.

For a consistent reduction of the operatorbasis we have to start with the anomalous dimensions of the full operatorbasis. Whenever an operator identity [17], e.g. CI or EOM, is applied to reduce the full operatorbasis, we have verified its compatibility with the RGE. Also, the relations between matrix elements of heavy-heavy currents have been shown stable under renormalization.

In the background field gauge, only the anomalous dimensions of operators that are manifestly invariant under gauge transformations of the background field have to be calculated. The Ward identities of the underlying BRST-symmetry are particularly simple and provide a powerful cross-check of our results [11]. From the Ward identities for the background field and the gauge parameter dependence of the effective action follows, that the counterterms proportional to \( C_A \) have to be transversal and should not depend on \( \xi \). Our results pass these consistency checks.

Additional checks are provided by the construction of relations among operators. For example, the differentiation of the lowest order currents

\[ i\partial_\nu \bar{c}_L \Gamma^\mu h_\nu = \bar{c}_L i \bar{D}_\nu \Gamma^\mu h_\nu + \bar{c}_L \Gamma^\mu i \bar{D}_\nu h_\nu \]  

(21)
leaves their anomalous dimensions unaffected and relates them to subleading operators. The operators on the right hand side must mix under renormalization in such a way, that their sum can be expressed by the total derivative of the current on the left hand side, which renormalizes multiplicatively. Our result is consistent with all relations of this type, including generalizations to local operators of higher dimension and time-ordered products.

5 Solving the renormalization group equations

5.1 Matching

As in any effective field theory calculation, we sum the radiative corrections by matching the operators at the thresholds and by running their coefficients between thresholds. In the case at hand, we have two thresholds $\mu = m_b$ and $\mu = m_c$. It is convenient to express the matching conditions as matrices $M^b$

$$\langle c | \bar{c} \gamma_\mu (1 - \gamma_5) b | b \rangle_{\mu = m_b} = \sum_i M^b_i \langle c | \mathcal{O}_{HL,i} | b(v) \rangle_{\mu = m_b}$$

and $M^c$

$$\langle c | \mathcal{O}_{HL,i} | b(v) \rangle_{\mu = m_c} = \sum_j M^c_{ij} \langle c(v) | \mathcal{O}_{HH,j} | b(v) \rangle_{\mu = m_c} .$$

Since there is only one QCD-operator, $M^b$ reduces to a vector. The $\mathcal{O}_{HL/HH,j}$ contain all physical local operators and time-ordered products up to and including dimension 5, that contribute to heavy-light and heavy-heavy currents respectively. Consequently the matching matrices receive contributions suppressed by inverse powers of the c- and b-quark masses up to and including $\mathcal{O}(1/m^{2}_{c/b})$. In addition, the matching of HL-operators with a covariant derivative acting on $\bar{c}$ induces a nontrivial $m_c/m_b$-dependence in $M^c$. For example, we have to lowest order

$$-\bar{c} i \gamma_\mu D_\mu c \rightarrow -\bar{c}_i e^{im_c v x} i \gamma_\mu D_\mu c = -\bar{c}_i e^{im_c v x} i D_\mu c^{e^{im_c v x}} + m_c \bar{c}_i e^{im_c v x} .$$

5.2 RG-Running

For the lagrangian at $\mathcal{O}(1/m^2_{Q})$, only five operators do not vanish by the EOM $\square$ and an analytical solution of the RGE is straightforward. In con-


contrast, the operator bases for the heavy-light and heavy-heavy currents contain 52 and 26 operators respectively and an analytical solution of the RGE is impractical, to say the least.

In each order $1/m_Q$, the solution of the RGE is expressed in terms of evolution matrices

$$R^{(n)}_{\text{HL/HH}}(\mu, m_{c/b}) = \exp \left( -\frac{1}{2\beta^{(1)}} \ln \left( \frac{\alpha(\mu)}{\alpha(m_{c/b})} \right) \cdot \hat{\gamma}^{(n)}_{\text{HL/HH}} \right).$$  \hspace{1cm} (25)

In $\beta^{(1)} = (33 - 2n_f)/12$, the number of active flavors takes the values $n_f = 4$ and $n_f = 3$ for heavy-light and heavy-heavy currents respectively.

Introducing their direct sum

$$R_{\text{HL}} = R^{(0)}_{\text{HL}} \oplus R^{(1)}_{\text{HL}} \oplus R^{(2)}_{\text{HL}},$$

$$R_{\text{HH}} = R^{(0)}_{\text{HH}} \oplus R^{(1)}_{\text{HH}} \oplus R^{(2)}_{\text{HH}},$$ \hspace{1cm} (26)

(27)

the combined solution for the Wilson coefficients at the hadronic scale can be written in compact form:

$$C_{\text{HH}}(\Lambda_{\text{hadr}}) = R_{\text{HH}}(\Lambda_{\text{hadr}}, m_c) \cdot M^c \cdot R_{\text{HL}}(m_c, m_b) \cdot M^b.$$ \hspace{1cm} (28)

The lowest order currents are conserved at zero recoil, therefore $R^{(0)}_{\text{HH}} = 1$ in (27). Furthermore, we can set $R^{(1)}_{\text{HH}} = 0$ because Luke’s theorem renders the corresponding contribution irrelevant.

For practical purposes, it is most convenient to sum the power series of the exponential in (25) directly. This series converges fast and any desired accuracy can be obtained. This procedure is used in the numerical results in table 1 where a factor of $1/(2m_c)^2$ must be attached to the higher order coefficients. We also present the tree-level values of the coefficients to expose the effect of the radiative corrections.

We stress that the matrices $\hat{\gamma}^{(2)}_{\text{HL/HH}}$ are defective, i.e. do not have a complete set of eigenvalues. Therefore the analysis of [13] is not applicable and we cannot derive general properties of the RG evolution from the spectrum of the matrices.

5.3 Results

Let us finally discuss phenomenological applications of our result.
From the coefficient $C_{H/\bar{H},2}^{(0)}$ of the lowest order symmetry breaking operator in the top row of table 1, one obtains a value of 1.180 for the short-distance coefficient $\eta_A$ of the axial vector current which is larger than 1. This contradicts the established result in [10] which is smaller than 1. This apparent contradiction is resolved if one takes into account the complementarity of the calculational approaches.

Our calculation uses effective field theory to resum corrections $\alpha_s \ln m_b / m_c$ from the running between $m_b$ and $m_c$, while [10] integrates out the b- and c-quarks simultaneously. The latter approach resums the powers of $z = m_c / m_b$. After applying radiative corrections, [10] includes $\alpha_s^n \ln^{n-1} z$, $\alpha_s^n \ln^n z \cdot z$ and $\alpha_s \ln z \cdot z^2$, while we have concentrated on $\alpha_s^n \ln^n z$, $\alpha_s^n \ln^n z \cdot z$ and $\alpha_s^n \ln^n z \cdot z^2$. Therefore our result for $\eta_A$ should not be taken face value. Instead, our terms $\alpha_s^n \ln^n z \cdot z^2$ with $n > 1$ are corrections to the result of [10]. In practice this means that the term $\alpha_s \ln z \cdot z^2$ should be subtracted from our result and the remaining correction should be added to the result of [10]. This correction is 0.002 and the final result for $\eta_A$ remains below 1.

Up to $O(1/m_{c/b})$ no form factors are needed to parameterize the matrix elements in the effective theory. The lowest order is normalized and Luke’s theorem states the absence of contributions at $O(1/m_{c/b})$.

In $O(1/m_{c/b}^2)$ the constraints of spin and flavor symmetry are less powerful. An expansion of the matrixelements in terms of 12 nonperturbative formfactors has been performed in [4] on tree-level.

Our result can be used to extend this analysis to include short-distance corrections. However, Lukes’s theorem, which has been used extensively in [4], receives radiative corrections at $O(1/m_{c/b}^2)$. Therefore the analysis of [4] has to be refined to include these corrections.

6 Conclusions

We have presented a calculation of renormalized heavy-light and heavy-heavy currents at order $O(1/m_{Q}^2)$. The anomalous dimensions of the relevant operator basis of dimension 5 pass various consistency checks. We have solved the renormalization group equation numerically for the Wilson coefficients of heavy-light and heavy-heavy currents, matching both quantities at the $m_c$-threshold. We have presented the numerical values of the coefficients of the heavy-heavy current at the hadronic scale.

The two expansion parameters $\omega - 1$ and $\Lambda_{hadr} / m_Q$ are numerically of the
| \( C_{HH,1}^{(0)} \) | \( \alpha^0 \) | \( \mu = \Lambda_{hadr} \) | \( C_{HH,1}^{(0)} \) | \( \alpha^0 \) | \( \mu = \Lambda_{hadr} \) |
|-----------------|--------|-----------------|-----------------|--------|-----------------|
| \( C_{HH,2}^{(2)} \) | -0.676 | -0.534 | \( C_{HH,6}^{(2)} \) | -1.444 | -2.596 |
| \( C_{HH,7}^{(2)} \) | -1.222 | -1.702 | \( C_{HH,8}^{(2)} \) | 0.556 | 1.001 |
| \( C_{HH,13}^{(2)} \) | 2.111 | 1.825 | \( C_{HH,14}^{(2)} \) | -1.444 | -1.832 |
| \( C_{HH,11}^{(0,2),b} \) | 0.000 | 0.111 | \( C_{HH,21}^{(0,2),b} \) | -1.111 | -2.747 |
| \( C_{HH,12}^{(0,2),b} \) | 0.000 | -0.002 | \( C_{HH,22}^{(0,2),b} \) | 0.111 | 0.023 |
| \( C_{HH,211}^{(0,2),c} \) | 1.000 | 0.421 | \( C_{HH,111}^{(0,2),c} \) | 0.000 | -0.005 |
| \( C_{HH,212}^{(0,1),b} \) | 1.111 | 1.302 | \( C_{HH,112}^{(0,1),b} \) | 0.000 | -0.004 |
| \( C_{HH,222}^{(0,1),b} \) | 0.111 | 0.72 | \( C_{HH,122}^{(0,1),b} \) | 0.000 | -0.002 |
| \( C_{HH,2122}^{(0,1),c} \) | 0.111 | 0.043 | \( C_{HH,2122}^{(0,1),c} \) | 1.000 | 0.800 |
| \( C_{HH,222}^{(0,1),c} \) | 1.000 | 0.543 | \( C_{HH,111}^{(0,1),c} \) | 0.000 | -0.001 |
| \( C_{HH,211}^{(0,1),c} \) | 0.333 | 0.386 | \( C_{HH,112}^{(0,1),c} \) | 0.000 | -0.001 |
| \( C_{HH,212}^{(0,1),c} \) | 0.333 | 0.229 | \( C_{HH,221}^{(0,1),c} \) | 0.333 | 0.262 |
| \( C_{HH,122}^{(0,1),c} \) | 0.000 | 0.000 | \( C_{HH,222}^{(0,1),c} \) | 0.333 | 0.155 |

Table 1: Wilson coefficients in tree-level and one-loop approximation. A factor \( 1/(2m_c)^2 \) is to be attached to the coefficients of higher order operators. For the quark masses we use the values \( m_b = 4.5\text{GeV} \) and \( m_c = 1.5\text{GeV} \) and for the initial value of the running coupling constant we use \( \alpha_s(M_Z) = 0.120 \).
same order (for currently accessible values of \(\omega\)). Therefore phenomenological applications should consider also pieces of \(O((\omega - 1)^2)\) as well as \(O((\omega - 1)A_{\text{hadr}}/m_Q)\). While the former have been available for a long time \([14]\), the latter have been inferred by reparameterization invariance arguments, which should be used with care \([1]\). A direct calculation will be the subject of future investigations.

A more detailed discussion of the consistency checks provided by BRST and reparameterization invariance will be presented elsewhere \([7]\), together with technical details of the calculation of renormalized Lagrangian and currents.

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