FT prediction method based on adaptive model of meteorological data

Wei-jun Pan, Hengheng Zhang*, Tao Liu, Tianyi Wu,
Civil Aviation Flight University of China, Guanghan, 618307, China

*Corresponding author e-mail: 1135167071@qq.com

Abstract: Due to the various meteorological conditions encountered in the flight process, the uncertainty of FT will change. In order to obtain the FT error, the secondary surveillance radar and aviation weather forecast data are used for analysis. According to the propagation mechanism of uncertainty, an adaptive prediction model of FT uncertainty is established. The input parameters of the adaptive model include Mach number, flight distance, vector wind and temperature. Cluster analysis and linear regression analysis are used to analyze the accuracy of the model. Compared with the static time-of-flight prediction model, the dynamic time-of-flight prediction model can accurately predict the FT even if the weather conditions are very bad. The dynamic time-of-flight prediction model is applied to air traffic management to further test the accuracy of the model. The results show that the adaptive time-of-flight prediction model based on meteorological conditions can accurately predict the arrival time of a flight to a certain waypoint.

1. Introduction

With the rapid development of air transport industry, air transportation has the advantage of speed, and the carrier has more and more strict requirements for the accuracy of flight time (FT). At present, Heathrow airport has used time-based operation mode to cope with the capacity growth in terminal area and route. The operation mode based on time interval can effectively cope with the aircraft delay caused by weather conditions. Due to the change of meteorological conditions, the influence of various uncertainties in flight operation increases. By reasonably predicting the uncertainty of FT, flight safety and time efficiency can be improved. The factors that lead to the uncertainty of FT are as follows: 1. The uncertainty of departure time, the error of weather forecast data, the error of navigation and communication delay, and the control instruction issued by the controller. This paper focuses on the uncertainty of FT caused by the dynamic change of weather. The purpose of this paper is to accurately predict the arrival time of the flight and improve the efficiency of FT.

In the past, flight delay rate is an important index to evaluate the efficiency of flight transportation. Previous studies only theoretically verified that the standard deviation of FT is positively correlated with flight distance [1, 2]. According to the empirical analysis, it is confirmed that the aircraft delay also depends on the meteorological conditions of the route [3], and then the solution of the concept of "dynamic interval" is proposed. The FT prediction based on meteorological conditions can improve the control and flight efficiency.

For the prediction of FT based on meteorological conditions, the correlation models between different aircraft combinations and meteorological conditions have been established [4, 5]. In addition, the application of downwind and upwind variation in forecasting FT based on meteorological data
[6,7] is established. This method is also suitable for aircraft fuel consumption analysis[8]. At present, the relationship between the four-dimensional wind[9,10] and the flight path is analyzed. Based on the change mechanism of meteorological conditions, the prediction model of FT with meteorological changes is established. By calculating the estimated FT with the airborne computer, the Mach number and ground speed (GS) are converted, and the FT prediction model is established. Based on the data of meteorological conditions, time and weather forecast, the evaluation coefficient of the model is analyzed by cluster analysis and regression analysis[11].

In this paper, the QAR data of flights over Liuting Airport in March, June, September and December 2020 are used. The pressure altitude is more than 25000 feet, the real track angle is between 45-135 degrees, the Mach number is less than 0.02, the real track angle is less than 15 degrees, and the pressure altitude is less than 100 feet. The flight trajectory that continuously meets the above flight conditions for more than 200 kilometers is extracted.

2. Time of flight uncertainty modeling

2.1 FT error calculation

FT error refers to the time difference between the target FT and the actual FT. The GS values measured by ins and GPS are used to predict the FT. The predicted FT $t_p$ is calculated by using the initial time $GS_i$ in the extracted QAR flight data as shown in equation (1), while the actual FT $t_a$ is calculated by integrating the actual time $aGS$ recorded in equation (2), as shown in equation (3). Time of flight error $t_{err}$ is defined as

$$t_p = \frac{D}{GS_i} \quad (1)$$

$$\int aGS \, dt = D \quad (2)$$

$$t_{err} = t_a - t_p \quad (3)$$

The formula (1) (2) conforms to the following assumption: the data in SSR mode $GS$ is correct and the FT error between different aircraft is consistent. The mean, standard deviation and root mean square of FT error are 0.27s, 8.69s and 8.72s, respectively. The results show that the FT error is distributed with the mean value of almost zero. In this paper, root mean square value is used as the evaluation parameter of FT uncertainty.

2.2 Theoretical basis and physical explanation

The mathematical expression of FT is as follows:

$$t = \frac{D}{GS} \quad (4)$$

Where, the GS $GS$ is the sum of the vacuum speed $TAS_v$ and the downwind along the route $W$, as follows:

$$TAS_v = GS - W \quad (5)$$

In addition, using airborne equipment measurement $TAS_m$, Mach number $M$, temperature $T$ and crosswind $W$, the actual gas constant $R = 287.1$ and adiabatic index of air $\kappa = 1.42$ are given as shown $TAS_v$ in equation (6)

Finally, the FT $t$ is as follows:

$$t = \frac{D}{\sqrt{M^2 + \kappa RT - W^2 + W_i}} \quad (6)$$

By differential analysis of the variables $W$, $W_c$, $M$, $T$, the expression of FT error is as follows:
According to the uncertainty propagation principle, it is assumed that there is no correlation between variables, and the variance of the variation of $F_T$ is as follows:

$$
\sigma^2_t = \frac{D}{G_S^2} \sigma^2_{GS} = \left( \frac{D}{G_S^2} \right)^2 \left( 1 + \left( \frac{W_c}{TAS_i} \right)^2 \right) \sigma^2_w + \left( \frac{M^2 \kappa R}{2TAS_i} \right)^2 \sigma^2_T + \left( \frac{M \kappa RT}{TAS_i} \right)^2 \sigma^2_M
$$

(8)

Assuming that the fluctuation of meteorological wind is uniform in all directions, formula (8) shows that the variance of $F_T$ is inversely proportional to the fourth power of $G_S$, directly proportional to the square of flight distance, and directly proportional to the variance of $G_S$. In the past studies, variance of $G_S$ was statically treated. However, equation (8) shows that variance of $G_S$ can be expressed as a function of $W_c, W_e, M, T$ and their variances. This shows that it is feasible to use the function input of these parameters to predict the uncertainty of $F_T$. It is concluded from the model that: (1) The wind fluctuation directly affects the $G_S$ fluctuation of the aircraft; (2) the lateral wind fluctuation also affects the $G_S$ fluctuation of $F_T$; (3) the temperature fluctuation affects the conversion between Mach number and $TAS$; (4) the fluctuation of Mach number will also lead to the fluctuation of $TAS$.

In the paper, the adaptive prediction model is the uncertainty propagation equation (8). In formula (8), since the flight distance and $G_S$ are usually determined by the pilot's operation, the adaptive uncertainty prediction is equivalent to the prediction of variance of $G_S$. Therefore, the final adaptive prediction model is formula (9)

$$
\sigma^2_{GS} = (1 + \left( \frac{W_c}{TAS_i} \right)^2) \sigma^2_w + \left( \frac{M^2 \kappa R}{2TAS_i} \right)^2 \sigma^2_T + \left( \frac{M \kappa RT}{TAS_i} \right)^2 \sigma^2_M
$$

(9)

Because numerical weather forecast basically describes the average weather behavior, but cannot accurately describe the instantaneous weather behavior that affects the $G_S$ fluctuation, it is difficult to predict the instantaneous flight state of aircraft. In this study, a coefficient of $\alpha$ is introduced to compensate for the instantaneous change of aircraft state during flight. In addition, $\sigma^2_M$ is not easy to obtain in prediction, so it is considered constant together with $\alpha$. Then $x$ is introduced to describe the explanatory variables, and the adaptive prediction model function is obtained as follows:

$$
\sigma^2_{GS} = \alpha \left( 1 + \left( \frac{W_c}{TAS_i} \right)^2 \right) \sigma^2_w + \alpha \left( \frac{M^2 \kappa R}{2TAS_i} \right)^2 \sigma^2_T + \alpha \left( \frac{M \kappa RT}{TAS_i} \right)^2 \sigma^2_M = \alpha x_1 + \alpha x_2 + \alpha x_3
$$

(10)

2.3 Clustering analysis and regression modeling method

The accurate definition of some constant parameters in formula (10) requires the preparation of a sufficient number of $F_T$ uncertainty data sets, as well as flight and meteorological conditions.

The data set used for clustering analysis consists of 3581 vectors, which are defined as follows:

$$
x_i = (I_{err}, M_i, W_{i, 1}, W_{i, 2}, T_i, \sigma^2_{w,i}, \sigma^2_{T,i})^T
$$

(11)

Where $M_{ini}, W_{i, 1}, W_{i, 2}, T_i, \sigma^2_{w,i}, \sigma^2_{T,i}$ are the initial values of the absolute values of Mach number, downwind speed, crosswind speed and temperature, and the differences between these data and the initial values obtained from the weather forecast data. The variance of weather forecast data is calculated by the following formula:
The typical number of NWP samples contained in a trajectory is 10.

The GMM (Gaussian mixture model) of expectation maximization algorithm is usually used in clustering analysis. This algorithm creates clustering, which makes the distribution of each parameter close to the Gaussian distribution.

In order to obtain the minimum number of accurate clusters, the sensitivity of BIC [14] is analyzed.

The appropriate cluster number to minimize BIC value is 30. The variance $\sigma_{GS}^2$ of each data cluster is calculated as the variance of GS fluctuation $GS$ as follows:

$$\sigma_{GS}^2 = \frac{\text{GS}_{\text{err}}}{D_{\text{ver}}} \times (14)$$

$$\sigma_{GS,a}^2 = (GS^2) (15)$$

Among them $D = 200km$, other parameters include the mean value and variance of the initial values of Mach number, tail wind speed, side wind speed and absolute value of temperature of each group of data. They are expressed as $M_c, \sigma_{M,c}^2, W_{t,c}, W_{c,c}, T_e, \sigma_{T,c}^2, \sigma_{T,c}^2$. Using these parameters, the adaptive prediction model function is described as follows:

$$\sigma_{GS,a}^2 = \alpha(1 + \frac{W_{c,c}}{TAS_{t,c}})^2 \sigma_{W,c}^2 + \alpha_2(\frac{M_c^2 \kappa R}{2TAS_{t,c}})^2 \sigma_{T,c}^2 + \alpha_3(\frac{M_c^2 \kappa RT_c}{TAS_{t,c}})^2 \sigma_{M,c}^2 (16)$$

Where

$$TAS_{t,c} = \sqrt{M_c^2 \kappa RT_c - W_{c,c}^2} (17)$$

Finally, multiple linear regression was used to determine the coefficients $\alpha_1 - \alpha_3$.

Through GMM cluster analysis, 27 clusters were obtained. The correlation between parameter $\sigma_{GS,a}^2$ and parameter $x$ is shown in Figure 1, and their related parameters are shown in Table 1.

Obviously, the correlation between $x_3$ and $\sigma_{GS,a}^2$ is not significant, because $\sigma_{M,c}^2$ is not easy to use in prediction. Therefore, the included $x_3$ terms are regarded as constants and used as intercept parameters in multiple linear regression.

$$\sigma_{GS,a}^2 = 4.02 + 2.68x_1 + 98.7x_2 (18)$$

![Figure 1: Correlation between $\sigma_{GS,a}^2$ and $x$](image)
Figure 2. Shows the regression curve between $\sigma_{GS,a}^2$ and $\sigma_{GS,e}^2$.

### Table 1. Correlation between x1-x3 and $\sigma_{GS,a}^2$

| Parameter | Correlation Coefficient | P Value |
|-----------|-------------------------|---------|
| x1        | 0.99                    | 0.0096  |
| x2        | 0.67                    | 0.0082  |
| x3        | -0.31                   | 0.23    |

### Table 2(a). Result

| Error RMS ($m^2/s^2$) | Adjusted $R^2$ | P Value |
|-----------------------|----------------|---------|
| Total                 | 0.95           | 0.94    | 0.0099 |

### Table 2(b). Parameters

| Parameter | Standard Error ($m^2/s^2$) | T Value | P Value |
|-----------|-----------------------------|---------|---------|
| Intercept | 0.31                        | 13.82   | 0.0099  |
| x1        | 0.09                        | 34.42   | 0.0099  |
| x2        | 11.9                        | 8.05    | 0.0099  |

### Table 3. Linear regression results

| Prediction using | Error RMS ($m^2/s^2$) | Adjusted $R^2$ | P Value |
|------------------|-----------------------|----------------|---------|
| $\sigma_{M,e}^2$ | 9.2                   | 0.85           | 0.0099  |
| Static Prediction| 1.75                  | 0.89           | 0.0099  |

3. Conclusion

The results show that the proposed adaptive prediction model can accurately predict the FT, while the prediction error of static prediction model is relatively large. The accuracy of the adaptive prediction model can also be proved by assuming that it is based on time interval operation mode. The adaptive prediction model can accurately predict the arrival time in severe weather conditions.

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