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Two-dimensional modelling of vehicle movement using the MATLAB Environment

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Abstract. Current vehicles are equipped with many sensors used to record traffic parameters. In addition, more and more drivers use video recorders that record the speed and location of the vehicle. The data can be used in modelling and reconstruction of the process of the incidents. The aim of the study is to assess the possibility of using the MATLAB environment for the reconstruction of road accidents by modelling the vehicle movement in a two-dimensional space. In further works, the models will be supplemented with an algorithm enabling calculations in a three-dimensional space. The assumptions necessary to create the 2D models were formulated. Then the numerical models in the MATLAB environment was applied. Several simulations were performed by testing the models’ operation. The conducted analysis is to prove the legitimacy of using the model in the reconstruction of simple road accidents and predicting the behavior of the vehicle on the basis of given input data.

1. Introduction

The current lifestyle of people is conducive to the development of the automotive industry through growing demand. In developed countries, owning a vehicle has become a standard today. The greater the number of vehicles on the road is not conducive to safety. The increasing density of vehicles on a road means that it is simply more difficult to drive. As a result of this phenomenon, the risk of possible collisions is also intensified. In addition, modern cars equipped with active safety systems, such as ABS or ESP, limit the number of marks on the scene of accidents or collisions. It is sometimes difficult to determine the causes on the basis of the visible remnants of the incident and to the police and on the place of the traffic incidents. Therefore, they turn to appropriate specialists to reconstruct the sequences of events. There are many ways to reproduce the sequences of events. The selection of appropriate tools belongs to specialists dealing with the given case. In the literature, it is possible to find information on reconstructing in analytical form by mathematically calculating maneuvers, such as braking, turning the vehicle on the road curve or collision mechanics [1].

Other tools are simulation programs that allow for complex reconstruction, constituting a causal sequence of events, starting from pre-crash movement, through modelling of the collision itself up to the superfluous movement. One of the most well-known and comprehensive programs is PC-Crash [2].

This study focuses on the assessment of the possibility of using a two-dimensional mathematical vehicle model to reconstruct simple road collisions in the Matlab environment. This will be the basis for
the modelling of pre-crash vehicle traffic in a later, more comprehensive development of a package that allows the simulation of the entire cause and effect chain in the reconstruction of road accidents.

1.1. Modelling problem
In order to perform reconstruction under computer simulation conditions it is necessary to have a model. A model, is a simplified version of reality that can be used for specific behaviors. Preparing the model involves simplifying the real object. However, the level of simplifications must be such as to reflect the features of the real object as close as possible. On the other hand, it must be high enough to allow its effective use. Various features of a real object can be modeled. If the researcher's area of interest is to use the function of a real object then a functional model will be created, if it is necessary to model physical interactions, then a physical model will be created. However, the use of a mathematical model allows the use of calculating machines to solve complex problems. Mathematical models of real objects are most often presented in the form of differential equations. In the further part of the study, the term "model" will refer to the mathematical model.

1.2. Used software
There are many environments that can be used to model vehicle dynamics. One of the most inductive ones are the multibody dynamics programs. They allow the modeling of the vehicle as a rigid body joined together by kinematics joints. Then such a model is converted by the program into a mathematical model, with the use of which it is possible to carry out simulations, eg Adams Car [3]. Another example is the VI Grade Motorsport program. The main application of this program is the simulation of the dynamics of racing vehicles [4]. In addition to the above, there are also programs designed strictly for the reconstruction of road accidents. More details on this topic can be found in Przechowski L, Unarski J, Wach W, Wicher J 2014 [1].

This study has been limited to the use of matlab software as a powerful programming environment with a range of applications in science and industry. A lot of information in this area is provided by Kiusalaas J 2006 [5]. The authors decided to use this tool due to its unlimited possibilities, allowing for the implementation and comparison any mathematical models of vehicles or the nature and extent of contact between the tire and the surface. All models used can be expanded or simplified depending on the will of the user, which determines the mathematical code of the model itself. Due to the high popularity of the matlab environment, it is used by many specialists in the industry, which help and inspiration can be used.

2. Modelling
In order to simulate vehicle movement in space, it is necessary to determine two models. The first of these is the mathematical model of the dynamics of the vehicle itself. On the other hand, the second is the model of cooperation between the rubber wheel and the road surface, hereinafter referred to as the tire model. In this study, the vehicle modelling process was focused on modelling a passenger car.

2.1. Modelling of the vehicle in two-dimensional space
Modelling a vehicle in two-dimensional space is widely discussed in the industry literature. The flat model with four wheels (of which two have the possibility of rotation) and rigid suspension is used as the most intuitive model. Differential equations and a detailed description are given in Jazar N R 2008 and Radionova L V and Chernyshev A D 2015 [6,7].

Another way is to simplify the above model to the so-called bicycle model, i.e. a model with one rear wheel and one front one, with the possibility of rotation [8]. This type of model was used in this study and is shown in Figure 1. The transition between the four-wheeled model and the two-wheeled model is presented in Mastinu G, Rossa F D, Piccardi C 2012 [9].
The relevant parameters of the above model can be described by linear or non-linear equations. The nonlinear form is described by equations (1), (2), (3) [8].

\[
\dot{V} = \frac{F_{x_f} \cos(\alpha - \delta) + F_{x_r} \cos(\alpha) + F_{y_f} \sin(\alpha - \delta) + F_{y_r} \sin(\alpha)}{m} \tag{1}
\]

\[
\dot{\alpha} = \frac{-F_{y_f} \sin(\alpha - \delta) - F_{y_r} \sin(\alpha) + F_{y_f} \cos(\alpha - \delta) + F_{y_r} \cos(\alpha)}{mV} \tag{2}
\]

\[
\dot{\psi} = \frac{F_{x_f} \alpha \sin(\delta) + F_{y_f} \alpha \cos(\delta) - F_{y_r} b}{I} \tag{3}
\]

where \( \delta \) is steering angle, \( \alpha_f \) and \( \alpha_r \) are the side slip angles of the front and rear tires, \( F_{x_f} \) and \( F_{x_r} \) are the longitudinal forces of the front and rear tires, \( F_{y_f} \) and \( F_{y_r} \) are the lateral forces of the front and rear tires, \( \psi \) is the yaw angle, \( m \) is the car mass, \( I \) is the moment of inertia.

The linear form was characterized by equations (4) and (5) [8]. The adopted simplifications consist in the assumption of small sizes of angles \( \alpha \) and \( \psi \) and constant value of speed \( V \).

\[
\dot{\alpha} = \frac{F_{y_f} + F_{y_r} - mV \psi}{mV_0} \tag{4}
\]

\[
\dot{\psi} = \frac{F_{y_f} \alpha - F_{y_r} b}{I} \tag{5}
\]

2.2. The vehicle tire modelling

In the literature it is possible to find several models of tires. Their task is to reproduce the real character of the tire. From the tire model, the \( F_y \) and \( F_x \) forces of individual wheels are obtained, which are then used in the vehicle model. The most popular models are: linear, Pacejka tire or polynomial [10,11,12].

The linear model is described by equation (6) [8]

\[
F_y = K \alpha \tag{6}
\]

where \( K \) is the proportionality coefficient.

The polynomial model has been characterized by the equation (7) [8]
The Pacejka model is a quasi experimental model and requires much more coefficients and parameters. The basic function describing the model is expressed by equation (8) [8]

\[
F_y = \frac{\mu}{\mu_n} F_{yn} \alpha_e
\]  

(8)

\[
F_{yn} = D \cdot \sin \left[ C \cdot \arctan \left( B\alpha - E \left( B\alpha - \arctan(B\alpha) \right) \right) \right]
\]  

(9)

\[
\alpha_e = \frac{\mu_n}{\mu} \alpha
\]  

(10)

where \( \mu \) and \( \mu_n \) are the operational and nominal coefficient of friction, respectively, while \( F_{yn} \) is the nominal force given by equation (9) and \( \alpha_e \) to equivalent slip angle given by formula (10). Explanations of the coefficients B, C, D, E and the corresponding physical sense have been described in the Mendes A S, Meneghetti D D R, Ackerman M, Fleury A T 2016 and Pacejka H B 2006 [8,11]. The figure 2 shows a comparison of the shape of the characteristics of individual tire models.

**Figure 2.** The comparison of shapes of tire model characteristics

### 3. Simulation tests

This paper uses code fragments of the Vehicle Dynamics - Lateral: Open Source Simulation Package for MATLAB[8]. The aim of the research was to analyse the use of the above described models of tires and vehicles for simulating pre-crash movement in the reconstruction of vehicles. The research carried out consisted in simulating a situation of avoiding collision by change of driving line. The shape of the test track is presented in figure 3.

The adhesion coefficient \( \mu_n \) was assumed at the level of 0.8, while the initial speed of the vehicle was 16.7m/s ≈ 60km/h.
Figure 3. The test track model

The simulation was carried out in four versions, adopting different variants of the tire model and the vehicle model:

- 1 version - both the tire model and the model vehicle model were assumed as linear.
- 2 version - linear model of tires and non-linear vehicle,
- 3 version - model of Pacejka tires and rope model of the vehicle
- 4 version - nonlinear vehicle model and Pacejka tire model.

The entry parameters for the model were a Fiat Grande Punto passenger car. The basic technical data used in the model are presented in Table 1 [13]. The moment of inertia was adopted on the basis of the study [14].

Table 1. Technical data of the vehicle whose model was used in the tests

| Data                                      | Value     |
|-------------------------------------------|-----------|
| Car mass m                                | 1175 kg   |
| Distance between the front axle and the center of the mass a | 1.4825m   |
| Distance between the rear axle and the center of the mass b | 1.0275m   |
| Wheelbase L                               | 2.51 m    |
| The maximum steering angle δ              | 40°       |
| Mass moment of inertia in rotational motion I | 1785 kg m² |

The tire characteristics used in the tests are shown in Figure 4.

Parameters of the Pacejka model were chosen in such a way as to obtain the tire characteristics similar to the characteristics of the actual tire in a passenger car [15].
4. Simulation tests results

On the basis of the conducted research, the results were obtained for comparisons of individual models. Figure 5 shows the centre of gravity displacement of the vehicle in XY coordinates. The curve of the linear tire model are shifted to the right in relation to the characteristics of the Pacejka tire model. This means that the centre of gravity of the vehicle with the linear model of tires has shifted to a shorter distance than that one equipped with the Pacejka tire model. In addition, for linear vehicle models, the deviation of the centre of gravity from the assumed lateral displacement is noticeable, the highest at 0.4 m for the linear model equipped with the Pacejka tire model. In comparison the shape of the obtained characteristics with the experiments presented in Prochowski L, Unarski J, Wach W, Wicher J 2014[1], it should be noted that the most similar shape has the characteristics of the non-linear vehicle model.

In addition, the above study also presents a formula (11) for determining the length of the maneuver of avoiding the obstacle[1].

\[
x_p = V_o \sqrt{\frac{y}{1.56\mu_y}} [\text{m}]
\]  

(11)

where \(V_o\) - initial speed in [m/s], \(y\) necessary lateral displacement, \(\mu_y\) coefficient of lateral friction. On the basis of the above, the length of the maneuver avoiding the obstacle in the analysed case is about 28 meters, which is very similar to the obtained simulation data (about 30m) for a non-linear vehicle model with the Pacejka tire model.
Figure 5. The car side displacement

Figure 6 shows changes of the steering angle of steered wheels $\delta$. This is the system response approximated by the step function, the theoretical course of changes in the steering angle, which generates the characteristics of Figure 5. The greater sensitivity of the linear tire model is noticeable. This means that the maximum steering angle in the linear model of tires is lower than in the Pacejka model, while it causes a faster move of the centre of gravity of the vehicle in the Y direction, faster execution of the maneuver. On the other hand, in the Pacejka tire model, the exposure to the maximum steering angle is longer, and the vehicle model reacts more slowly, which makes it less sensitive to approximation by the step function. In fact, the change in steering angle of the wheels by the driver's influence on the steering wheel deviates from the step function - it is longer, therefore the vehicle will move slightly, which makes the characteristics less as step function. Less sensitivity of the Pacejka model allows better mapping of reality despite the approximation of the step function.
Figure 6. The characteristics of changes in the steering angle of the front wheels as a function of the distance travelled.

Figure 7 shows the characteristics of the vehicle angle rotation relative to the vertical axis \( \psi \). For the Pacejka model, a gentler transition of the angle change compared to the model of linear tires is noticeable. In addition, the models equipped with the Pacejka tire model achieve lower values of the maximum angle. It should be noted that for the model of linear tires, the vehicle reaches its maximum value already at a distance of approx. 5 m from the beginning of the maneuver, which is physically difficult to achieve. The nonlinear vehicle model with the Pacejka tire model has behaviour the most close to reality.
Figure 7. The characteristics of the vehicle angle rotation relative to the vertical axis $\psi$

5. Directions for further testing
After conducting simulation tests, some suggestions for further deepening of knowledge in the discussed topic came up. The first of these is the use of a tire model approximated by complex function consisting of two linear functions. Thanks to this, it would be possible to obtain a characteristic similar to the Pacejka model, consisting of pieces of a linear function that does not require the definition of such a large number of parameters. Another issue worth considering would be the generalization of the bicycle flat model to the flat four-wheeled model. This would allow the use of different coefficients of adhesion on the left and right side of the vehicle, so that you could simulate the movement with different coefficients of adhesion under the wheels on the left and right side of the vehicle. The plans are to expand the model to a three-dimensional model and introduce the height of the centre of gravity. This would allow simulation of more complex situations and determination of the amount of contact in the planned module for the analysis of vehicle collisions at a later stage.

6. Summary
Two vehicle models were compared in two-dimensional space: linear and non-linear and two models of linear tires and Pacejka. The results of the simulation and their comparison with the experimental data included in the literature allowed to determine the most accurate combination of vehicle and tire models. These are the model of Pacejka tires and the non-linear model of the vehicle. The use of these models allows you to precisely determine the movement of the vehicle based on input data. In addition, the use of the powerful Matlab environment proves to be helpful in the conducted research, because it gives the
opportunity to further develop the model using uncomplicated methods. The analysis carried out showed the applicability of the model to the reconstruction of simple road events.

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