EOS OF DENSE MATTER
AND
FAST ROTATION OF NEUTRON STARS

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Recent observations of XTE J1739-285 suggest that it contains a neutron star rotating at 1122 Hz.1 Such rotational frequency would be the first for which the effects of rotation are significant. We study the consequences of very fast rotating neutron stars for the potentially observable quantities as stellar mass and pulsar period.

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1. Introduction

Neutron stars with their very strong gravity can be very fast rotators. Theoretical studies show that they could rotate at sub-millisecond periods, i.e., at frequency $f = 1/\text{period} > 1000$ Hz.2,3 The first millisecond pulsar B1937+21, rotating at $f = 641$ Hz,4 remained the most rapid one during 24 years after its detection. In January 2006, discovery of a more rapid pulsar J1748-2446ad rotating at $f = 716$ Hz was announced.5 However, such sub-kHz frequencies are still too low to significantly affect the structure of neutron stars with $M > 1M_\odot$.6 Actually, they belong to a slow rotation regime, because their $f$ is significantly smaller than the mass shedding (Keplerian) frequency $f_K$. Effects of rotation on neutron star structure are then $\propto (f/f_K)^2 \ll 1$. Rapid rotation regime for $M > 1M_\odot$ requires submillisecond pulsars with supra-kHz frequencies $f > 1000$ Hz.

Very recently Kaaret et al.1 reported a discovery of oscillation frequency $f = 1122$ Hz in an X-ray burst from the X-ray transient, XTE J1739-285. According to Kaaret et al.1 “this oscillation frequency suggests that XTE J1739-285 contains the fastest rotating neutron star yet found”. If confirmed, this would be the first detection of a sub-millisecond pulsar (discovery of a 0.5 ms pulsar in SN1987A remnant announced in January 1989 was withdrawn one year later).
Rotation at $f > 1000$ Hz is sensitive to the stellar mass and to the equation of state (EOS). Hydrostatic, stationary configurations of neutron stars rotating at given rotation frequency $f$ form a one-parameter family, labeled by the central density. This family - a curve in the mass - equatorial radius plane - is limited by two instabilities. On the high central density side, it is instability with respect to axi-symmetric perturbations, making the star collapse into a Kerr black hole. The low central density boundary results from the mass shedding from the equator. In the present paper we show how rotation at $f > 1000$ Hz is sensitive to the EOS, and what constraints on the EOS of neutron stars result from future observations of stably rotating sub-millisecond pulsars.

2. Method

We studied the properties of fast rotating neutron stars for a broad set of the models of dense matter. The set of equations of state (EOSs) considered in the paper is presented in Fig. 1.

![Fig. 1. The equations of state used in the paper](image)

Out of ten EOSs of neutron star matter, two were chosen to represent a soft (BPAL12) and stiff (GN3) extreme case. These two extreme EOSs should not be considered as “realistic”, but they are used just to “bound” the neutron star models from the soft and the stiff side.

We consider four EOSs based on realistic models involving only nucleons (FPS, BBB2, DH, APR), and four EOSs softened at high density either by the appearance
of hyperons (GNH3, BGN1H1), or a phase transition (GMGSp, GMGSm). The softening of the matter in latter case is clearly visible in Fig. 1 at pressure $P \sim 10^{35} \text{dyn/cm}^2$.

EOSs GMGSp and GMGSm describe nucleon matter with a first order phase transition due to kaon condensation. In both cases the hadronic Lagrangian is the same. However, to get GMGSp we assumed that the phase transition takes place between two pure phases and is accompanied by a density jump. Assuming on the contrary that the transition occurs via a mixed state of two phases, we get EOS GMGSm. This last situation prevails when the surface tension between the two phases is below a certain critical value.

The stationary configurations of rigidly rotating neutron stars have been computed in the framework of general relativity by solving the Einstein equations for stationary axi-symmetric spacetime. The numerical computations have been performed using the Lorene/Codes/Rotstar/rotstar code from the LORENE library (http://www.lorene.obspm.fr). One-parameter families of stationary 2-D configurations were calculated for ten EOSs of neutron-star matter, presented in Fig. 1.

Stability with respect to the mass-shedding from the equator implies that at a given gravitational mass $M$ the circumferential equatorial radius $R_{\text{eq}}$ should be smaller than $R_{\text{max}}$ which corresponds to the mass shedding (Keplerian) limit. The value of $R_{\text{max}}$ results from the condition that the frequency of a test particle at circular equatorial orbit of radius $R_{\text{eq}}$ just above the equator of the actual rotating star is equal to the rotational frequency of the star. This condition sets the bound on our rotating configurations from the right side on $M(R)$ plane (the highest radius and the lowest central density).

The limit for most compact stars (the lowest radius and the highest central density) is set by the onset of instability with respect to the axisymmetric oscillations defined by the condition:

$$\left( \frac{\partial M}{\partial \rho_c} \right)_J = 0 ,$$

For stable configurations we have:

$$\left( \frac{\partial M}{\partial \rho_c} \right)_J > 0 ,$$

3. Neutron stars at 1122 Hz

In this section we present the parameters of the stellar configurations rotating at frequency 1122 Hz. For details and discussion see Bejger et al. (2007)9

It is interesting that the relation between the calculated values of $M$ and $R_{\text{eq}}$ at the "mass shedding point" is extremely well approximated by the formula for the orbital frequency for a test particle orbiting at $r = R_{\text{eq}}$ in the Schwarzschild spacetime of a spherical mass $M$ (which can be replaced by a point mass $M$ at $r = 0$). We denote the orbital frequency of such a test particle by $f_{\text{orb}}(M, R_{\text{eq}})$. The formula

\begin{align*}
\left( \frac{\partial M}{\partial \rho_c} \right)_J &= 0 , \\
\left( \frac{\partial M}{\partial \rho_c} \right)_J &> 0 ,
\end{align*}
Fig. 2. Gravitational mass, $M$, vs. circumferential equatorial radius, $R_{eq}$, for neutron stars stably rotating at $f = 1122$ Hz, for ten EOSs (Fig. 1). Small-radius termination by filled circle: setting-in of instability with respect to the axi-symmetric perturbations. Dotted segments to the left of the filled circles: configurations unstable with respect to those perturbations. Large-radius termination by an open circle: the mass-shedding instability. The mass-shedding points are very well fitted by the dashed curve $R_{min} = 15.52 \left( \frac{M}{1.4 M_\odot} \right)^{1/3} \text{km}$. For further explanation see the text.

giving the locus of points satisfying $f_{\text{Schw}}(M, R_{eq}) = 1122$ Hz, represented by a dash line in Fig. 2 is

$$\frac{1}{2\pi} \left( \frac{GM}{R_{eq}^3} \right)^{1/2} = 1122 \text{ Hz} .$$

This formula for the Schwarzschild metric coincides with that obtained in Newtonian gravity for a point mass $M$. It passes through (or extremely close to) the open circles denoting the actual mass shedding (Keplerian) configurations. This is quite remarkable in view of rapid rotation and strong flattening of neutron star at the mass-shedding point. Equation (3) implies

$$R_{\max} = 15.52 \left( \frac{M}{1.4 M_\odot} \right)^{1/3} \text{km} .$$

4. Submillisecond pulsars

In this section we present results for neutron stars rotating at submillisecond periods for a broad range of frequencies (1000 - 1600 Hz)
Fig. 3. Mass vs radius relation for submillisecond neutron stars. As we see the shape of the $M(R_{eq})$ curves is more and more flat as rotational frequency increases. For $f_{rot} = 1600$ Hz the curves $M(R_{eq})$ are almost horizontal and the mass for each EOS is quite well defined and curves for different models of dense matter are typically well separated. In principle this property could be used for selecting ”true EOS” if we detect a very rapid pulsar and are able to estimate its mass.

In each panel the dotted curve corresponds to the formula (of which Eq. (3) is a special case for $f = 1122$ Hz)

$$M = \frac{4\pi^2 f^2}{G} R_{eq}^3$$

used for a given frequency (1000 Hz, 1200 Hz, 1400 Hz, 1600 Hz). As we see this
formula works perfectly in very broad range of rotational frequencies (recently this formula has been tested by Krastev et al.\textsuperscript{10} for the frequency 716 Hz.)

5. Accretion

We studied the mechanism of the spin-up of neutron star due to the accretion from the last stable orbit (the innermost stable circular orbit - ISCO). As an example we discuss this subject for DH EOS.

We calculate the spin-up following the prescription given by Zdunik et al.\textsuperscript{11,12} The value of specific angular momentum per unit baryon mass of a particle orbiting the neutron star at the ISCO, \(l_{\text{IS}}\), is calculated by solving exact equations of the orbital motion of a particle in the space-time produced by a rotating neutron star, given in Appendix A of Zdunik et al.\textsuperscript{11}

Accretion of an infinitesimal amount of baryon mass \(dM_B\) onto a rotating neutron star is assumed to lead to a new quasi stationary rigidly rotating configuration of mass \(M_B + dM_B\) and angular momentum \(J + dJ\), with

\[
dJ = x_l l_{\text{IS}} \ dM_B ,
\]

where \(x_l\) denotes the fraction of the angular momentum of the matter element transferred to the star. The remaining fraction \(1 - x_l\) is assumed to be lost via radiation or other dissipative processes.

\[
\text{Fig. 4. Mass vs radius relation for accreting NS with DH EOS. The example for } f_{\text{rot}} = 1400 \text{ Hz}
\]

We present results for two choices of \(x_l\): \(x_l = 1\) and \(x_l = 0.5\) when all or half of the angular momentum of the accreting matter is transferred to the star from the ISCO. In Fig. 4 we plot the curve \(M(R)\) corresponding to the frequency of rotation.
$f_{\text{rot}} = 1400 \text{ Hz}$ for DH EOS. Point F on this curve corresponds to the onset of instability with respect to axi-symmetric oscillations (condition given by Eq. (1)). Point E is the Keplerian configuration at frequency 1400 Hz, and G - corresponds to maximum mass along the curve with fixed frequency.

The curves starting at points A,B,C and D are the track of the accreting neutron star defined by the Eq. (6) for $x_l = 1$ (solid line) and $x_l = 0.5$ (dotted line) for cases C,D and A,B respectively. To reach the configuration rotating at frequency 1400 Hz we have to start with the nonrotating neutron star between the points C and D (if $x_l = 1$) of A and B (if $x_l = 0.5$). As we see the actual frequency of rotation sets the limits on the initial mass of nonrotating star, which can be spun-up to this frequency due to the accretion. For $f_{\text{rot}} = 1400 \text{ Hz}$ these limits in the case $x_l = 1$ are: $1.7M_\odot < M_i < 1.92M_\odot$.

![Fig. 5](image_url)

Fig. 5. (Color online) The limits for the initial mass of accreting neutron star to be spun-up up to the given frequency. The labeled points correspond to the points in Fig. 3. The region to the left (red) corresponds to $x_l = 0.5$ and the central region (cyan) to $x_l = 1$. The non-shaded region (right) correspond to the allowable final masses.

In Fig 5 we plotted for DH EOS these limits for the initial mass (non-rotating) of accreting NS provided that this star could be spun-up to the given frequency. We presented results for two assumed values of the parameter $x_l$. The shaded area shows the allowed initial masses of nonrotating star to reach by the accretion a given rotational frequency. Left ”triangle” corresponds to $x_l = 0.5$, the right shaded triangle (with points C and D) to $x_l = 1$

The curves on the right represent the limits on the actual mass of rotating NS. The three curves correspond to the location of the three points E,F, G in Fig 3.
The curve with the point E (green) is defined by the Keplerian frequency of rotating star, the point F (and magenta line) correspond to the boundary resulting from the instability with respect to axi-symmetric perturbations. The point G is the mass at the maximum point of the curve with fixed frequency (1400 Hz). As the frequency increases the mass at Keplerian point increases more rapidly than that defined by the onset of instability at the maximum mass (points F and G respectively). For high frequencies the maximum mass of the stars rotating at fixed frequency is given by the value for Keplerian configuration. For DH EOS at frequency $\simeq 1500$ Hz the point G disappears and for faster rotation the curve $M(R_{eq})$ monotonically increases. The region of the masses of the stars rotating at high frequency is very narrow. For $f_{rot} > 1400$ Hz it is smaller than $0.1 M_{\odot}$ (see also discussion of Fig. 3).

6. Discussion and conclusions

The $M(R_{eq})$ curve for $f \gtrsim 1400$ Hz is flat. Therefore, for given EOS the mass of NS is quite well defined. Conversely, measured mass of a NS rotating at $f \gtrsim 1400$ Hz will allow us to unveil the actual EOS. The ”Newtonian” formula for the Keplerian frequency works surprisingly well for precise 2-D simulations and sets a firm upper limit on $R_{eq}$ for a given $f$. Finally, observation of $f \gtrsim 1200$ Hz sets stringent limits on the initial mass of the nonrotating star which was spun up to this frequency by accretion.

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