Pion production off the nucleon

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We have studied charged current neutrino/antineutrino induced weak pion production from nucleon. For the present study, contributions from $\Delta(1232)$-resonant term, non-resonant background terms as well as contribution from higher resonances viz. $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, $S_{11}(1650)$ and $P_{13}(1720)$ are taken. To write the hadronic current for the non-resonant background terms, a microscopic approach based on SU(2) non-linear sigma model has been used. The vector form factors for the resonances are obtained from the helicity amplitudes provided by MAID. Axial coupling in the case of $\Delta(1232)$ resonance is obtained by fitting the ANL and BNL $\nu$-deuteron reanalyzed scattering data. The results of the cross sections are presented and discussed for all the possible channels of single pion production induced by charged current interaction.

KEYWORDS: Pion production, Deuteron effects, Higher resonances, Charged current.

1. Introduction

Experimenters are using neutrino/antineutrino beam of few GeV energy in the study of neutrino/antineutrino nucleus scattering to determine some of the oscillation parameters like $\Delta m_{32}^2$, $\theta_{32}$, CP violating phase $\delta$, etc. They are also important because of the interest in understanding hadronic structure in weak sector where besides vector current they also get contribution from axial vector current. In the neutrino/antineutrino energy region of 1GeV, the dominant contribution to the charged lepton events come from quasielastic scattering and single pion production processes. The available experimental results of single pion production and their comparison with various theoretical calculations have necessitated the need to re-examine the basic reaction mechanism for the production of single pion from free nucleon target. In various theoretical calculations there is lack of consensus in the modeling of basic reaction mechanism of $\nu(\bar{\nu})$ induced pion production from free nucleon, specially concerning the contribution of background terms as well as the contribution of higher resonances in addition to the dominant $\Delta(1232)$ resonance. The tension between the experimental results from old bubble chamber experiments, ANL [1] and BNL [2], which were performed using deuterium/hydrogen targets, necessitated the need of re-performing the experiments with high precision using deuterium targets. Recently, Wilkinson et al. [3] have reanalyzed the old ANL [1] and BNL [2] data and found the differences in these two results to be within 12% at $E_{\nu} = 1GeV$ and 8% at $E_{\nu} = 2GeV$. A theoretical understanding of these data may be of great interest in the understanding of hadronic physics.

In this work, we present a study of single pion production induced by neutrinos/antineutrinos off nucleon. Besides the dominant $\Delta(1232)$-term, we have considered non-resonant background terms and also taken the contribution of higher resonances viz. $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$, $S_{11}(1650)$ and $P_{13}(1720)$. Presently there is no consensus as to how the non-resonant background terms should be added to the dominant $\Delta(1232)$ contribution. Some authors have performed calculations by coherently summing the contributions of hadronic current from the background terms and the $\Delta(1232)$-resonant term, while some have added them incoherently. The understanding of the role of back-
ground terms is specially important in determining the $N - \Delta$ transition form factors in $\nu_\mu p \to \mu^- p \pi^+$ and $\bar{\nu}_\mu n \to \mu^+ n \pi^-$ channels which are dominated by $\Delta(1232)$-excitation and receive no contribution from the nearby higher resonance which are $I = \frac{1}{2}$ resonances.

In section-2, we present the formalism in brief and discuss the results in section-3.

2. Formalism

The cross section for the single pion production process $\nu_l(\bar{\nu}_l) + N \to l^-(l^+) + N' + \pi^0$; $N,N' = p,n; i = \pm,0$, may be written as,

$$d\sigma = \frac{(2\pi)^4}{4ME} \delta^4(k + p - k' - p' - k_\pi) \frac{d\hat{p}'}{2\pi^2 E'_p} \frac{d\hat{k}_\pi}{2\pi^2 E_\pi} \frac{d\hat{k}}{2\pi^2 E_l} \Sigma |M|^2 \tag{1}$$

where $k(k')$ is the four momentum of the incoming(outgoing) lepton having energy $E_l(E_i)$ while $p(p')$ is the four momentum of the incoming(outgoing) nucleon and the pion momentum is $k_\pi$ having energy $E_\pi$. $|M|^2$ is the square of the matrix element given by

$$|M|^2 = \frac{G_F^2}{2} L_{\mu\nu} J^{\mu\nu}. \tag{2}$$

where $L_{\mu\nu}$ is the leptonic tensor

$$L_{\mu\nu} = j_{\mu} j_{\nu}^{\dagger} = 8 \left( k_{\pi} k_{\nu} + k_{\nu} k_{\mu} - g_{\mu\nu} k \cdot k' \pm i \epsilon_{\mu\nu\alpha\beta} k'^\alpha k^\beta \right). \tag{3}$$

the upper(lower) sign in the antisymmetric term stands for (antineutrino)neutrino induced processes.

To get the expression for hadronic tensor $J^{\mu\nu}(= j_{\mu} j_{\nu}^{\dagger})$, the hadronic current has been obtained for the Feynman diagrams shown in Fig. 1. The contributions of non-resonant background terms are obtained using a chiral invariant Lagrangian based on non-linear sigma model \[6\]. At tree level the various diagrams which may contribute to the pion-production mechanism are direct and cross nucleon pole, contact diagram, pion pole and pion in flight diagram labeled as NP, CNP, CT, PP and PF respectively. As the non-linear sigma model assumes hadrons as point like particle, therefore, to take into account the structure of hadron, the form factors are introduced at the $W^\pm N \to N$ transition vertex. The details are given in Refs. \[4, 5\].

The final form of hadronic current for non-resonant background are obtained as \[5\]

$$j^\mu|_{NP} = A^{NP} \bar{u}(\beta') \gamma^\mu \frac{\not{p} + \not{q} + M}{(p + q)^2 - M^2 + i\epsilon} \left[ V_N^{\mu}\gamma^5(q) - A_N^{\mu}(q) \right] u(\bar{p}),$$

$$j^\mu|_{CNP} = A^{CNP} \bar{u}(\beta') \left[ V_N^{\mu}(q) - A_N^{\mu}(q) \right] \frac{\not{p}' - \not{q} + M}{(p' - q)^2 - M^2 + i\epsilon} \gamma^5 u(\bar{p}),$$

$$j^\mu|_{CT} = A^{CT} \bar{u}(\beta') \gamma^\mu \left( g_{\alpha\beta} f^{\alpha\beta}_{\gamma^5}(Q^2) \gamma^\gamma - f_\rho \left( (q - k_\pi)^2 \right) \right) u(\bar{p}),$$

$$j^\mu|_{PP} = A^{PP} f_\rho \left( (q - k_\pi)^2 \right) \frac{\not{q} + M}{m_\pi^2 + Q^2} \bar{u}(\beta') \not{q} u(\bar{p}),$$

$$j^\mu|_{PF} = A^{PF} f_{\rho\rho}(Q^2) \frac{2(2m_\pi - q)^\mu}{(2m_\pi - q)^2 - m_\pi^2} 2M\bar{u}(\beta') \gamma^\mu u(\bar{p}), \tag{4}$$

where for NP and CNP currents, at the transition vertex $W^\pm N \to N$, we have introduced form factors in $V_N^{\mu}(q)$ and $A_N^{\mu}(q)$ to account for the nucleon structure, given by

$$V_N^{\mu,CC}(q) = f_1(Q^2) \gamma^\mu + f_2(Q^2) i\sigma^{\mu\nu} q_\nu \frac{q_\mu}{2M} \tag{5}$$
\[ A_N^{\mu CC}(q) = \left( f_A(Q^2) \gamma^\mu + f_P(Q^2) \frac{q^\mu}{M} \right) \gamma^5, \]  

(6)

where \( f_{1,2}(Q^2) \) and \( f_{A,P}(Q^2) \) are the isovector vector and axial vector form factors for nucleons. Similarly \( f_{\rho}(Q^2) \) accounts for dominant contribution that comes from \( \rho \)-meson cloud at \( \pi \pi NN \) vertex in the case of PP diagram. Finally, CVC relates the \( f_{PF}(Q^2), f_{CT}^V(Q^2) \) with \( f_1(Q^2) \) and PCAC relates \( f_\rho(Q^2) \) with the axial part of CT diagrams. In the case of resonances, apart from the dominant \( \Delta(1232) \) resonance, we have also considered the contribution of various resonances from second resonance region viz: \( D_{13}(1520) \) and \( P_{13}(1720) \) which have \( J = \frac{3}{2}, I = \frac{1}{2} \) and \( P_{11}(1440), S_{11}(1535) \) and \( S_{11}(1650) \) which have \( J = \frac{1}{2}, I = \frac{1}{2} \). The Feynman diagrams for the resonant contributions are depicted in Fig. 1 and are labeled as \( R \) and \( CR \) corresponding to direct and cross terms. \( R/CR \) represents both spin half and three half resonances. The currents for spin \( J = \frac{3}{2} \) resonances are obtained as

\[ j_{\mu R}^{\frac{3}{2}} = i \cos \theta_c C_R \frac{k_R}{p_R^2 - M_R^2 + i M_R \Gamma_R} \bar{u}(\vec{p}) \Gamma_R \frac{\gamma^\mu}{2} \bar{u}(\vec{p})\gamma^5, \quad p_R = p + q, \]

and for spin \( J = \frac{3}{2} \) resonances are obtained as

\[ j_{\mu R}^{\frac{3}{2}} = i \cos \theta_c C_R \bar{u}(\vec{p}) \gamma_5 \frac{k_R}{p_R^2 - M_R^2 + i M_R \Gamma_R} \frac{\gamma^\mu}{2}, \quad p_R = p' - q. \]  

(7)
The weak vertex \( \nu \mu \rightarrow \nu \mu \) is generally expressed in terms of four form factors as,

\[
\langle \bar{\nu} \gamma_5 u | \gamma_\mu | \bar{\nu} \rangle = i \cos \theta_c C^R \bar{u}(\bar{p}') \Gamma^\mu \frac{\bar{p}' - \bar{q} + M_R}{\bar{p}'(p' - q)^2 - M_R^2 + i\Gamma_R M_R} \gamma_5 u(\bar{p}),
\]

where \( C^R \) is the coupling strength for \( R \rightarrow N\pi \) and \( M_R \) is the mass of the resonance. \( P_{3/2}^{3/2} \) is spin three-half projection operator and is given by

\[
P_{3/2}^{3/2}(P) = - (P + M_R) \left( g_{a\beta} - \frac{2}{3} P_a P_\beta + \frac{1}{3} P_\alpha \gamma_\beta - P_\beta \gamma_\alpha - \frac{1}{3} \gamma_\alpha \gamma_\beta \right),
\]

The weak vertex \( \Gamma^{\mu}(\nu^\mu) \) for spin \( \frac{3}{2}(\frac{1}{2}) \) resonances has V-A structure, given by

\[
\Gamma^{\mu}_{\nu^\mu} = \left[ V_{\nu^\mu} - A_{\nu^\mu} \right] \gamma_5 \hspace{1cm} \Gamma^{\mu}_{\nu^\mu} = V_{\nu^\mu} - A_{\nu^\mu}
\]

where the superscript (+) stands for positive (negative) parity state. For spin three half states the vertex \( \Gamma_{\nu^\mu} \) may be written in terms of six form factors viz:

\[
V^{\frac{3}{2}}_{\nu^\mu} = \frac{\tilde{C}^V}{M} (g_{\mu\nu} q - q_\nu \gamma_\mu) + \frac{\tilde{C}^4}{M^2} (g_{\mu\nu} q \cdot p' - q_\nu P_\mu) + \frac{\tilde{C}^5}{M^2} (g_{\mu\nu} q \cdot p - q_\nu P_\mu) + g_{\mu\nu} \tilde{C}^6
\]

\[
A^{\frac{3}{2}}_{\nu^\mu} = - \left[ \frac{\tilde{C}^4}{M} (g_{\mu\nu} q - q_\nu \gamma_\mu) + \frac{\tilde{C}^4}{M^2} (g_{\mu\nu} q \cdot p' - q_\nu P_\mu) + \frac{\tilde{C}^5}{M} g_{\mu\nu} + \frac{\tilde{C}^6}{M^2} q_\nu q_\mu \right] \gamma_5
\]

while for the case of spin half states \( \Gamma_\mu \) is generally expressed in terms of four form factors as,

\[
V^{\frac{1}{2}}_\mu = \frac{F_1(Q^2)}{2M} (Q^2 \gamma^\mu + \bar{q} q^\mu) + \frac{F_2(Q^2)}{2M} i\epsilon \mu^\alpha q_\alpha
\]

\[
A^{\frac{1}{2}}_\mu = - F_A(Q^2) \gamma^\mu \gamma^5 - \frac{F_P(Q^2)}{M} q^\mu \gamma^5,
\]

The vector form factors for the resonant states (except for the \( \Delta \)-resonance) are parameterized using helicity amplitudes from the MAID analysis. The parameterizations and various form of vector form factors used in the present calculations are given in Ref. [4]. For the axial form factors we have used the Goldberger-Trieman relation which relates the \( R \rightarrow N\pi \) coupling to the \( C^A_2(0) \) for the spin
three-half(half) resonances. To get \( R \rightarrow N\pi \) coupling strength, we have used partial decay width for the different resonant states following PDG values for the partial decay rates. The various properties of the resonances along with their couplings are tabulated in Table-I. Furthermore, assuming PCAC and pion pole dominance at the weak vertex the pseudoscalar form factors \( \tilde{C}_A(Q^2)(F_P(Q^2)) \) are related to \( \tilde{C}_A(Q^2)(F_A(Q^2)) \). We have neglected the contribution of \( \tilde{C}_{3,4} \) form factors for \( D_{13}(1520) \) and \( P_{13}(1720) \) resonances.

We have also taken deuteron effect in our calculations by following the prescription of Hernandez et al. [6] and write

\[
\left( \frac{d\sigma}{dQ^2 dW} \right)_{vd} = \int dp_d |\Psi_d(p_d)|^2 \frac{M}{E_p} \left( \frac{d\sigma}{dQ^2 dW} \right)_{\text{off shell}}.
\]  

(13)

In the above expression \( |\Psi_d|^2 = |\Psi_0|^2 + |\Psi_2|^2 \), where \( \Psi_0 \) and \( \Psi_2 \) are the deuteron wave functions for the S–state and D–state, respectively and have been taken from the works of Lacombe et al. [7].

3. Results and discussions

![Image](image_url)

**Fig. 2.** Total scattering cross section for \( \nu_p \rightarrow \mu^- p\pi^+ \) process. Data points are reconstructed/reanalyzed data of ANL and BNL experiments by Wilkinson et al. [3]. Here no invariant mass cut has been applied. In the left panel change in cross section with the variation(by 10%) of axial dipole mass \( M_A \) has been shown by taking central value as the world average value. While in the right panel the effect of variation of axial charge for \( \Delta(1232) \) resonance has been shown. The central curve has \( \tilde{C}_A(0)_\Delta = 1.1 \) and the shaded region has been obtained by varying \( \tilde{C}_A(0)_\Delta \) by 10%.

Using the expression for the differential scattering cross section given in Eq. 1 and integrating over the kinematical variables we obtain the result for total scattering cross section. To incorporate the deuteron effect we have used Eq. 13. In all the numerical calculations where \( M_A \) appears, we have taken it as the world average value, i.e. \( M_A = 1.026 \) GeV.
In Fig. 2, we have shown the results for the total scattering cross section for the charged current neutrino induced $1\pi^+$ production process on proton target i.e. for the reaction $\nu_\mu p \to \mu^- p\pi^+$. The results are presented for the total scattering cross section with $\Delta(1232)$ and non-resonant background (NRB) terms. The results presented here are obtained without using any cut on invariant mass. We have compared the results with the reanalyzed experimental data of ANL [1] and BNL [2] experiments by Wilkinson et al. [3]. Furthermore, the effect of varying $\tilde{C}_A^\Delta(0)|_\Delta$ and $M_A$ on total scattering cross section has been studied. We found that the total scattering cross section $\sigma(\nu_\mu p \to \mu^- p\pi^+)$ has minimum chi-square when $\tilde{C}_A^\Delta(0)|_\Delta = 1.0$ and $M_A = 1.026$ GeV are used in the expression of $\tilde{C}_5(Q^2)|_\Delta$. However, to see the effects of $\tilde{C}_A^\Delta(0)|_\Delta$ and $M_A$ on total scattering cross section we have shown variations of $M_A$ and $\tilde{C}_A^\Delta(0)|_\Delta$ in shaded regions. We find that the cross section changes by about $\sim 10\%$ at $E_\nu = 1$GeV if the axial dipole mass $M_A$ is varied by 10%. Similarly, at $E_\nu = 1$GeV when $\tilde{C}_A^\Delta(0)|_\Delta$ is varied by 10% the variation in the cross section is around 9%.

Fig. 3. Total scattering cross section for the charged current neutrino induced pion production processes through various channels. Legends are self explanatory.

In Fig. 3, we have presented the results of total scattering cross section for the charged current neutrino induced pion production processes in all the channels. The experimental data shown for $\pi^+p$ channel is same as in Fig. 2, while for the other channels like $\pi^0p$ and $\pi^+n$ the data are from ANL [1] and BNL [2] experiments. In the case of $\nu_\mu p \to \mu^- p\pi^+$ induced reaction, the main contribution to the total scattering cross section comes from the $\Delta(1232)$ resonance and there is no contribution from the higher resonances which are considered here. We find that due to the presence of the non-resonant background terms there is an increase in the cross section which is about 12% at $E_\nu = 1$GeV which becomes $\sim 8\%$ at $E_\nu = 2$GeV.

For $\nu_\mu n \to \mu^- n\pi^+$ as well as $\nu_\mu n \to \mu^- p\pi^0$ processes, there are contributions from the non-resonant background terms as well as from the higher resonant terms besides the $\Delta(1232)$ resonance. The net contribution to the total pion production due to the presence of the non-resonant background terms in $\nu_\mu n \to \mu^- n\pi^+$ reaction results in an increase in the cross section of about 12% at $E_\nu = 1$GeV.
which becomes 6% at $E_{\nu\mu}=2$GeV. When higher resonances are also taken into account there is a further increase in the cross section by about 40% at $E_{\nu\mu}=1$GeV which becomes 55% at $E_{\nu\mu}=2$GeV. In the case of $\nu\mu n \rightarrow \mu^- p\pi^0$ process, due to the presence of the background terms the total increase in the cross section is about 26% at $E_{\nu\mu}=1$GeV and 18% at $E_{\nu\mu}=2$GeV and due to the presence of higher resonances there is a further increase of about 35% at $E_{\nu\mu}=1$GeV and 40% at $E_{\nu\mu}=2$GeV. Thus, we find that the inclusion of higher resonant terms lead to a significant increase in the cross section for $\nu\mu n \rightarrow \mu^- n\pi^+$ and $\nu\mu n \rightarrow \mu^- p\pi^0$ processes. Furthermore, it may also be concluded from the above observations that contribution from non-resonant background terms decreases with the increase in neutrino energy, while the total scattering cross section increases when we include higher resonances in our calculations.

When a cut of $W \leq 1.4$GeV on the center of mass energy is applied then due to the presence of the non-resonant background terms, the increase in the total scattering cross section in the energy range $E_{\nu\mu}=1$GeV for $\nu\mu p \rightarrow \mu^- p\pi^+$ process is about 10% which becomes 12% at $E_{\nu\mu}=2$GeV. For $\nu\mu n \rightarrow \mu^- n\pi^+$ reaction this increase in the cross section is about 14% at $E_{\nu\mu}=1$GeV which becomes 5% at $E_{\nu\mu}=2$GeV. When higher resonances are also taken into account there is a further increase in the cross section which is about 40% at $E_{\nu\mu}=1$GeV which becomes $\sim 55\%$ at $E_{\nu\mu}=2$GeV. While in the case of $\nu\mu n \rightarrow \mu^- p\pi^0$ due to the presence of the non-resonant background terms the total increase in cross section is about 26% at $E_{\nu\mu}=1-2$GeV. Due to the presence of other resonances there is a further increase of about 5% at $E_{\nu\mu}=1$GeV, contributions of which become 8% at $E_{\nu\mu}=2$GeV.

In Fig. 4, we have shown the results for the charged current antineutrino induced pion production processes through various channels. Legends are self explanatory.

Fig. 4. Total scattering cross section for the charged current antineutrino induced pion production processes through various channels. Legends are self explanatory.
42% at $E_{\nu_\mu}$=1GeV which becomes 20% at $E_{\bar{\nu}_\mu}$=2GeV. When higher resonances are included, the cross section further increases by ~ 2% at $E_{\nu_\mu}$=1GeV which becomes 26% at $E_{\nu_\mu}$=2GeV. In the case of $\bar{\nu}_\mu p \rightarrow \mu^- p \pi^+$ reaction, the inclusion of non-resonant background terms increases the cross section by around 16% at $E_{\nu_\mu}$=1GeV which becomes 4% at $E_{\nu_\mu}$=2GeV. When higher resonances are included the cross section further increases marginally at $E_{\nu_\mu}$=1GeV and ~ 15% at $E_{\nu_\mu}$=2GeV.

4. Conclusions

We have presented the results for charged current one pion production cross section in the energy region of $E_{\nu/\bar{\nu}} \leq 2$GeV. Our model consists of contributions from background terms due to non-resonant diagrams, $\Delta(1232)$ resonant term and the contributions from higher resonances. The $\Delta(1232)$-resonance has the dominant contribution but we also need contributions from the non-resonant background terms and the higher resonant terms to describe the experimental data for all the possible channels of single pion production induced by charged current neutrino/antineutrino induced processes. We used $\nu_\mu p \rightarrow \mu^- p \pi^+$ channel to fix the axial charge($C^5_A(0)_{\Delta}$) and axial dipole mass $M_A$, as there is no other resonance which contributes to this process. To fix these parameters, we have used reanalyzed data of ANL and BNL and the numerical values obtained from our best fit are $M_A = 1.026$GeV and $C^5_A(0)_{\Delta} = 1.0$.

When contribution of higher resonances are taken into account, we find that the major contributions to the pion production come from $P_{11}(1440)$ and $D_{13}(1520)$ resonances. The contribution due to non-resonant terms is more important for $\nu n \rightarrow \nu \pi^- n$ process and less important for $\bar{\nu} p \rightarrow \bar{\nu} p \pi^0$ process.

The present work contributes to the theoretical understanding of the role of background terms and higher resonance terms in neutrino/antineutrino induced one pion production off the nucleon. It would be interesting to apply the present formalism to study the nuclear medium effects in the neutrino/antineutrino induced pion production process from nuclear targets in the accelerator experiments being performed in the few GeV energy region.

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