Delta-Hole Interaction in Nuclei and the Gamow-Teller Strength in $^{90}$Nb

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Abstract

The recently obtained experimental result for the quenching of the GT sum rule in the reaction $^{90}$Zr(p,n)$^{90}$Nb is used to extract the value of the Landau-Migdal parameter $g'_{N\Delta}$, taking into account also the effect of the finite range meson exchange interactions. The extracted value is compared to the one obtained in the $\pi + \rho$ exchange model by explicitly taking into account the effects of antisymmetrization and short range correlations. Although the $\pi + \rho$ exchange model tends to give a somewhat stronger quenching than observed experimentally, the results are consistent within the experimental error bars if the quark model value for the parameter $f_\Delta/f_\pi$ is used.

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The study of the renormalization of Gamow-Teller (GT) matrix elements in nuclei has revealed valuable information on nuclear structure effects and non-nucleonic degrees of freedom like mesons and delta-isobars [1, 2]. If one considers only the low energy part of the GT strength function, as provided by the GT $\beta$-decay matrix elements or the charge exchange reactions at low excitation energies, one cannot decide a priori whether the experimentally observed quenchings originate from nuclear structure effects, like configuration mixing and core polarization, or from non-nucleonic degrees of freedom. Although theoretical calculations with realistic interactions have shown that a considerable amount of strength is shifted to the higher energy region due to the admixture of $2p - 2h$ components in even-even nuclei [3], the observed quenchings have also been attributed to the admixture of $\Delta$-hole components by assuming a large value of the Landau-Migdal parameter $g'_{N\Delta}$ characterizing the strength of the $NN \rightarrow N\Delta$ transition potential [4]. In order to

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discriminate between these two interpretations, Wakasa et al. have recently used the reaction \(^{90}\text{Zr}(p,n)^{90}\text{Nb}\) to extract the value of the GT sum rule (or more precisely the Ikeda sum rule) integrated up to about 50 MeV excitation energy. Since this value of the excitation energy is sufficiently high to include most, if not all, of the strength due to \(2p-2h\) excitations, their result \((S_- - S_+)_{\text{exp}}/S_{-0} = 0.9 \pm 0.05\) indicates that the quenching due to the \(\Delta\)-hole excitations should be about 10% or even smaller.

In order to extract information on the value of \(g'_{N\Delta}\) from this observation, Suzuki and Sakai recently performed an analysis by assuming that the relevant part of the transition potential is solely given by the delta-function type Landau-Migdal interaction. For the case of vanishing \(N\Delta \rightarrow N\Delta\) interaction, their analysis gave \(g'_{N\Delta} = 0.18\) \((g'_{N\Delta} = 0.12)\) if the quark model (Chew-Low model) value of the parameter \(f_\Delta/f_\pi\) (see eqs. (1) and (2) below) is assumed, which is considerably smaller than one would expect on the basis of the \(\pi + \rho\) exchange model. The assumption of a pure delta function potential is justified in nuclear matter, since the direct term due to the 'bare' \(\pi + \rho\) meson exchange transition potential \((V_{\pi+\rho}(q))\) vanishes for \(q = 0\). In finite nuclei, however, due to the lack of momentum conservation, \(V_{\pi+\rho}(q)\) acts attractively and enhances the GT matrix elements, which would imply a larger value of \(g'_{N\Delta}\) in order to balance this attraction. The purpose of this letter is to investigate this possibility and to comment on the question whether the \(\pi + \rho\) model is consistent with the experimentally observed quenching or not.

Following the perturbative treatment of Arima et al, the GT sum rule

\[ S_{\pm} = S_{+0} = 3(N-Z), \]  

\[ S_{-0} = 3(N-Z), \]  

\[ S_{+0} = 0. \]
including the effects of $\Delta$-hole excitations in lowest order is represented diagrammatically by fig.1.

\[
\mathcal{S}_-= \sum_{k=1}^{3} \sum_{m,i} \left[ \begin{array}{c}
  m \\
  \tau_i
\end{array} \right] + \sum_{m,i} \left[ \begin{array}{c}
  m \\
  \tau_i
\end{array} \right] + \sum_{m,i} \left[ \begin{array}{c}
  m \\
  \tau_i
\end{array} \right] + \sum_{m,i} \left[ \begin{array}{c}
  m \\
  \tau_i
\end{array} \right] + \sum_{m,i} \left[ \begin{array}{c}
  m \\
  \tau_i
\end{array} \right]
\]

Figure 1: Diagrammatical representation of the GT sum rule including the effects of $\Delta$h-states in lowest order perturbation theory.

The external GT operator is given in the space of $N$ and $\Delta$ states by

\[
\mathcal{O}_{k-} = \frac{1}{2} \left( \sigma_k \tau_- + \frac{f_\Delta}{f_\pi} S_k T_- + \frac{f_\Delta}{f_\pi} (S^\dagger)_k (T^\dagger)_- \right),
\]

where the transition spin $S$ (transition isospin $T$) transforms a nucleon into a delta with the reduced matrix elements given by $\langle \frac{3}{2} \mid S \mid \frac{1}{2} \rangle = \langle \frac{3}{2} \mid T \mid \frac{1}{2} \rangle = 2$.

\[\footnote{We will use the quark model value $f_\Delta/f_\pi = \sqrt{72/25}$ as in ref. [8], and also $f_\Delta/f_\pi = 2$ [9], which is based on the Chew-Low model [10]. For the transition $^{90}\text{Zr} \rightarrow ^{90}\text{Nb}$, the relevant proton-particle neutron-hole states $(m, i)$ in fig. 1.}

4 We define the spherical $\pm$ components by $A_k = A_1 \pm i A_2$. Useful relations for the transition spin are $(S^\dagger)_i S_j = \frac{2}{3} \delta_{ij} - \frac{1}{3} \epsilon_{ijk} \sigma_k$ and similar for the transition isospin.
are \((g_{9/2}, g_{9/2})\) and \((g_{7/2}, g_{9/2})\). The first diagram alone gives the single particle value \(S_{-0} = 3(N - Z) = 30\). The diagrams of fig.1 refer to a calculation in which the effects of the exchange terms and short range correlations (s.r.c.) are explicitly taken into account, that is, in the \(\pi + \rho\) exchange model the transition potential is given by \(V_{\pi + \rho}(r)f(r)\), where \(f(r)\) is a s.r.c. function \(^1\) and the 'bare' transition potential is (in momentum space)

\[
V_{\pi + \rho}(q) = -\frac{f_\Delta}{f_\pi} \left( \left( \frac{f_\pi}{m_\pi} \right)^2 \frac{\sigma_1 \cdot q}{q^2 + m_\pi^2} S_2 \cdot q \right) + \left( \frac{f_\rho}{m_\rho} \right)^2 \frac{\sigma_1 \times q \cdot (S_2 \times q)}{q^2 + m_\rho^2} \right) \tau_1 \cdot T_2 \\
+ (1 \leftrightarrow 2) + h.c.
\]

(2)

Here we use the coupling constants \(f_\pi^2 = 4\pi \cdot 0.08 = 1.01\), and \(f_\rho^2 = 4\pi \cdot 4.86 = 61.04\) \(^{11}\). We use harmonic oscillator single particle wave functions with \(\hbar \omega = 9.23\) MeV (\(b=2.12\) fm) \(^{12}\). Note that in a calculation of this type, which includes explicitly the exchange terms and s.r.c., \(g'_{N\Delta}\) does not appear explicitly. We will refer to such a kind of calculation as 'type 1'.

The resulting quenching factors \(S_{-}/S_{-0}\) obtained in this type 1 calculation are shown in the second and fourth columns of Table 1. Here '\(\pi\) only' refers to a calculation taking only the pion exchange potential (\(V_\pi\)) supplemented by the s.r.c. function, and '\(\pi + \rho\)' includes also the effects of \(\rho\) meson exchange. Comparing these results to the experimental value of ref. \(^5\), we see that, for the case of the quark model value \(\frac{f_\Delta}{f_\pi} = \sqrt{\frac{72}{25}}\), pion exchange alone leads to the observed quenching factor, but the quenching obtained by including also the \(\rho\) meson exchange is somewhat too strong, although it is still within the error bars of the experimental value. The quenching obtained with \(\frac{f_\Delta}{f_\pi} = 2\), however, seems to be too strong compared to the experimental value.

\(^3\)In the actual calculation we use \(f(r) = \Theta(r - c)\) with \(c = 0.7\) fm.
\[ \Delta f = \sqrt{\frac{72}{25}} \]

(b) \[ \Delta f = 2 \]

\[ S_+/S_{0+} \quad \eta_{N\Delta} \quad S_+/S_{0+} \quad \eta_{N\Delta} \]

\begin{array}{cccc}
\pi \text{ only} & 0.90 & 0.25 & 0.867 & 0.25 \\
\pi + \rho & 0.86 & 0.35 & 0.805 & 0.35 \\
\end{array}

Table 1: Results for the quenching factors and corresponding values of \( g'_{N\Delta} \), referring to the two cases (a) \( \frac{\Delta f}{f_{\pi}} = \sqrt{\frac{72}{25}} \), and (b) \( \frac{\Delta f}{f_{\pi}} = 2 \). The quenching factors shown here are obtained in a calculation including exchange terms and short range correlations explicitly (‘type 1’). The corresponding values of \( g'_{N\Delta} \) are determined such that a calculation (‘type 2’) without the exchange terms and short range correlation function, but employing the \( g'_{N\Delta} \) force in addition to the bare meson exchange potentials (\( V_{\pi} \) and \( V_{\pi + \rho} \)), gives the same quenching factors as the type 1 calculation. The observed quenching factor is ref. [5] 0.9 ± 0.05.

In order to translate these results into values for \( g'_{N\Delta} \), we also performed a calculation of ‘type 2’, which considers only the direct terms in fig.1 with the bare meson exchange potentials (without s.r.c. functions), but supplemented by the contact interaction of Landau-Migdal type (in momentum space),

\[ V_{L.M.} = g'_{N\Delta} \left( \frac{f_{\pi}}{m_{\pi}} \right)^2 \frac{f_{\Delta}}{f_{\pi}} (\sigma_1 \cdot S_2 \sigma_1 \cdot T_2 + (1 \leftrightarrow 2) + h.c.) \quad (3) \]

In this ‘type 2’ calculation, the effects of the exchange terms and s.r.c. are incorporated into the interaction (3). If we determine the value of \( g'_{N\Delta} \) such as to reproduce the results of the type 1 calculation, we obtain the values shown in table 1. The resulting value \( g'_{N\Delta} = 0.35 \) shown in table 1 is very similar to the average value deduced in ref. [9] for finite nuclei. (The value obtained in nuclear matter for normal density is somewhat larger, around 0.4.)

On the other hand, we can also determine ‘empirical’ values of \( g'_{N\Delta} \) such
as to reproduce the experimental quenching factor. The results are shown in Table 2. They are obtained by assuming (i) only the Landau-Migdal type interaction (3), (ii) the interaction (3) together with $V_{\pi}$, and (iii) the interaction (3) together with $V_{\pi+\rho}$. These results, of course, refer to the 'type 2' calculation, i.e; only the direct terms of fig. 1 are considered and no s.r.c. function is used.

|                  | (a) $\frac{f_\pi}{f_n} = \sqrt{\frac{72}{25}}$ | (b) $\frac{f_\pi}{f_n} = 2$ |
|------------------|-----------------------------------------------|-------------------------------|
| $V_{L.M.}$ only  | 0.18 ± 0.09                                  | 0.13 ± 0.07                   |
| $V_{L.M.} + V_{\pi}$ | 0.26 ± 0.09                                  | 0.20 ± 0.07                   |
| $V_{L.M.} + V_{\pi+\rho}$ | 0.27 ± 0.09                                  | 0.21 ± 0.07                   |

Table 2: Values of $g'_{N\Delta}$ obtained by fitting the experimental quenching factor assuming various types of interactions (see text).

The value $g'_{N\Delta} = 0.18$ ($g'_{N\Delta} = 0.13$) for the case of the quark model (Chew-Low model) value of $\frac{f_\pi}{f_n}$ agrees with the result of ref. [8], where only the interaction (3) was used. (A small difference arises because $\frac{f_\pi}{f_n} = 2.12$ was used for the Chew-Low model value in ref. [8].) Since the 'bare' meson exchange potentials (2) vanish for $q = 0$, their direct terms give no contribution in nuclear matter due to momentum conservation, and the assumption of a pure $\delta$-function type interaction would be justified in nuclear matter. However, we see from Table 2 that the attractive contribution due to pion exchange is appreciable in finite nuclei, and in order to balance the attraction due to $V_\pi$ the value of $g'_{N\Delta}$ has to be increased. The $\rho$ meson exchange contributes only little, since it is of short range and therefore the nuclear matter picture is valid to a good approximation. From Table 2 we can conclude that, due to the presence of $V_{\pi+\rho}$, the value of $g'_{N\Delta}$ increases by about 0.1 and, for
the case of the quark model \( \left( \frac{f_\Delta}{f_\pi} = \sqrt{\frac{27}{25}} \right) \), becomes consistent with the meson exchange picture (Table 1). For the case of the Chew-Low model \( \left( \frac{f_\Delta}{f_\pi} = 2 \right) \), however, the value of \( g'_{N\Delta} \) fitted to the experimental quenching is definitely smaller than the value obtained in the meson exchange picture. This corresponds to the situation discussed above that the quenching calculated in the meson exchange picture is consistent with (stronger than) the experimental quenching for the case of the quark model (Chew-Low model) value of \( \frac{f_\Delta}{f_\pi} \).

It is interesting to note that our present perturbative approach is equivalent to the RPA formulation of ref. \[8\] for the case of vanishing \( N\Delta \rightarrow N\Delta \) interaction. In order to see the connection clearly, let us derive the analytical result for the quenching factor obtained in ref. \[8\] from the diagrams of fig.1: As usual, one can approximate the energy denominators by a constant \((-1/\epsilon_\Delta\) with \( \epsilon_\Delta \approx 294 \text{ MeV} \), which allows one to perform the sum over the \( \Delta \) states by using completeness. In this way one obtains from fig.1

\[
S_- = \frac{1}{4} \sum_{k=1}^{k=3} \sum_{m_i} \langle m|\sigma_k \tau_-|i \rangle - g'_{N\Delta} \left( \frac{f_\pi f_\Delta}{m_\pi f_\pi} \right)^2 \frac{8}{9\epsilon_\Delta} \times \sum_h \langle h| \left( \delta_{kl} - \frac{i}{4} \tau_3 \epsilon_{kk'l} \sigma_{k'} \right) \right| G_{m_i}^{(l)}(r_h)|h \rangle|^2
\]

with the spin-isospin density

\[
G_{m_i}^{(l)}(r) = \langle m|\sigma_l \tau_- \delta(r - r_i)|i \rangle.
\]

By using simple angular momentum algebra one can show that the spin dependent term in eq.(4) is zero. Then the term \( \langle h|\ldots|h \rangle \) in (4) becomes \( \int d^3 r \rho_0(r) G_{m_i}^{(k)}(r) \), where \( \rho_0(r) = \sum_h \langle h|\delta(r_h - r)|h \rangle \) is the ground state nucleon density of \(^{90}\text{Zr}\). Since \( G_{m_i}^{(k)}(r) \) is strongly peaked at the nuclear surface, one can approximate this integral by replacing \( \rho_0(r) \) by its value
at the nuclear surface, i.e.; \( \rho_0(r) \rightarrow \gamma \rho_0 \), where \( \gamma \simeq 0.5 \) is the attenuation factor for nuclear surface effects, and \( \rho_0 = 0.17 \text{ fm}^{-3} \). Since the remaining integral over the spin-isospin density just gives the single-particle matrix element \( \langle m|\sigma_k \tau_-|i \rangle \), one ends up with a state independent quenching factor \( Q \equiv S_-/S_0 \) given by

\[
Q = \left( 1 - g'_{N\Delta} \left( \frac{f_{\pi} f_{\Delta}}{m_{\pi} f_{\pi}} \right)^2 \frac{8}{9 \epsilon_{\Delta} \gamma \rho_0} \right)^2,
\]

and with the quark model value \( \left( \frac{f_{\Delta}}{f_{\pi}} \right)^2 = \frac{72}{25} \), this corresponds to eq.(35) of ref. [8] for the case \( g'_{\Delta\Delta} = 0 \). The values \( g'_{N\Delta} = 0.18 \) and \( g'_{N\Delta} = 0.13 \) listed in table 2, which have been obtained without using the approximation for nuclear surface effects, correspond to \( \gamma = 0.49 \).

In conclusion, we have shown that the recently obtained experimental result for the quenching of the GT sum rule in \(^{90}\text{Nb}\) is consistent with the meson exchange picture for the \( \Delta h \) interaction in nuclei, if the quark model value for \( \frac{f_{\Delta}}{f_{\pi}} \) is assumed. In order to arrive at this conclusion, it is important to take into account also the finite range \( \pi + \rho \) exchange potentials in addition to the Landau-Migdal type interaction.

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\(^4\)We note that the term \((Z - N)/2A\) in eq. (36) of ref. [8] should actually be absent due to a cancellation between the forward and backward \( \Delta h \) contributions. However, this term is very small for the present case of \(^{90}\text{Zr}\). It is actually overwhelmed by ambiguities in the attenuation factor \( \gamma \), such that our numerical results for the case '\( V_{L,M. \, \text{only}} \)' shown in Table 2 are actually the same as in ref. [8].
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