APPLICATION OF THE LIMIT-CYCLE MODEL TO STAR FORMATION HISTORIES IN SPIRAL GALAXIES: VARIATION AMONG MORPHOLOGICAL TYPES

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Received 1999 October 28; accepted 2000 April 27

ABSTRACT

We propose a limit-cycle scenario of star formation history for any morphological type of spiral galaxy. It is known observationally that an early-type spiral sample has a wider range of present star formation rate (SFR) than a late-type sample. This tendency is understood in the framework of the limit-cycle model of the interstellar medium (ISM), in which the SFR changes cyclically in accordance with the temporal variation of the mass fraction of the three ISM components. When the limit-cycle model of the ISM is applied, the amplitude of variation of the SFR is expected to change with the supernova (SN) rate. Observational evidence indicates that early-type spiral galaxies should show smaller rates of present SNe than late-type ones. Combining this evidence with the limit-cycle model of the ISM, we predict that the early-type spiral galaxies will show larger amplitudes in their SFR variation than the late types. Indeed, this prediction is consistent with the observed wider range of the SFR in the early-type sample than in the late-type sample. Thus, in the framework of the limit-cycle model of the ISM, we are able to interpret the difference in the amplitude of SFR variation among the morphological classes of spiral galaxies.

Key words: galaxies: evolution — galaxies: ISM — galaxies: spiral — ISM: evolution — stars: formation

1. INTRODUCTION

In this paper, we propose a scenario of star formation histories in giant spiral galaxies. Our standpoint is based on a very interesting observational result presented by Kennicutt, Tamblyn, & Congdon (1994, hereafter KTC). According to their sample galaxies, there is a difference in present star formation activity among morphological types of spiral galaxies. This is found in KTC (the third paragraph of their § 5.2). The early-type spiral galaxies have a 1 order-of-magnitude range in $b$, which denotes the present-to-past ratio of star formation rate (SFR), as being $b = 0.01-0.1$. On the other hand, $b = 0.5-2$ (a range of just a small factor) in the late-type spiral sample. Moreover, in the framework of the scenario proposed in this paper, we can incorporate another relevant observational result: the difference of supernova (SN) rate with galactic morphology (Cappellaro et al. 1993).

Not only KTC, but also Tomita, Tomita, & Saitô (1996), found a difference in present star formation activity among different morphological types of spiral galaxies (see also Devereux & Hameed 1997). Tomita et al. (1996) commented that this variation may indicate a short-term change in the SFR in spiral galaxies and that the duration of an episode of star formation activity is less than $10^8$ yr.

Kamaya & Takeuchi (1997, hereafter KT97) independently suggested an observational interpretation of the difference in present star formation activity among morphological types. They pointed out that the short duration of star formation proposed by Tomita et al. (1996) may indicate that the interstellar medium (ISM) in a spiral galaxy is a nonlinear open system. That is, they suggested that the duration may result from the period of a limit-cycle of mass exchange among various phases of the ISM (see Ikeuchi & Tomita 1983, hereafter IT83, for the limit-cycle behavior). If the ISM in a galaxy is regarded globally to be a nonlinear open system, the evolution of the whole ISM on a galaxy-wide scale may result in a limit-cycle star formation history. Indeed, the variance of the star formation activities of spiral galaxies can be understood as a short-term ($\lesssim 10^8$ yr) variation of their activities. Recent results by Rocha-Pinto et al. (2000) suggest that the Galactic star formation history indeed shows such a galaxy-wide variability.

KT97's discussion was based on the sample of Tomita et al. (1996). In the subsequent discussion, we reexamine KT97's proposal more quantitatively than their original considerations through a proper comparison with KTC. Moreover, we check whether their scenario is consistent with the difference of SN rate among the morphological types of spirals, since the amplitude of the limit-cycle orbit is determined by the SN rate (§ 4). Thus, we can state clearly our motivation here: to interpret the difference in star formation history among morphological types in the framework of the limit-cycle model of the ISM in spiral galaxies.

According to a review by Ikeuchi (1988), he and his collaborators indicated that we might understand the dynamical evolution of the ISM on a galaxy-wide scale if we could describe galaxies as nonlinear open systems (see also Nozakura & Ikeuchi 1984, 1988). Here we stress that one type of model, the limit-cycle model, indicates a periodic star formation history (KT97). A more elaborate model has also been proposed by Tainaka, Fukazawa, & Mineshige (1993). To understand the behavior of star formation activity in spiral galaxies, we focus on this interesting hypothetical behavior of the ISM. Adopting the limit-cycle model, KT97 insisted that the amplitude of the cyclic part of the SFR, $\Psi^c$, should be larger than the SFR of the quiescent era, $\Psi^q$. Moreover, adopting a Schmidt (1959) law of index 2, they predicted that the amplitude of the $\Psi^c$'s, defined as max ($\Psi^c$)/min ($\Psi^c$), should be $\sim 50$. Here max ($\Psi^c$) and min ($\Psi^c$) indicate the maximum and minimum values of $\Psi^c$.
respectively. However, since \( \max(\Psi')/\min(\Psi') \) depends on the characteristic parameters for the limit-cycle model (IT83; or § 2 in this paper), we reexamine the amplitude for various parameters in this paper.

Throughout this paper we discuss the difference in the amplitude of cyclic star formation among the three morphological types of spiral galaxies (Sa, Sb, and Sc), along with the limit-cycle model. In the next section we review the limit-cycle model of the ISM. In § 3, we estimate the amplitude of the cyclic SFR and interpret the difference in the amplitude via the limit-cycle model. In § 4, a consistent scenario for the time variation of SFR is proposed. In the final section we summarize our considerations and present some discussion.

2. CYCLIC STAR FORMATION HISTORY

Since our discussions in this paper are based mainly on KTC, we first summarize KTC and also review KT97. Although KT97 discussed Tomita et al. (1996), their argument on the duration and the behavior of star formation activity is not altered even if we base it on KTC.

2.1. KTC’s Sample and KT97’s Interpretation

Treating a data set of Hα equivalent widths of galactic disks with various morphologies, KTC have shown that the star formation activities present a wide spread for each morphological type (KTC’s Fig. 6). They derived a \( b \)-parameter that indicates the ratio of the present SFR to the past averaged SFR.

Based on KT97, we reinterpret Figure 6 in KTC: KTC’s wide dispersion of the star formation activity is interpreted as evidence for a periodic star formation history on the scale of a giant galaxy. If galaxies have the same morphological type and the same age, such a large scatter as that in Figure 6 of KTC should not appear for near-constant or monotonically declining SFRs. However, if cyclic star formation occurs in any spiral galaxy, we can easily understand why such a large scatter emerges. If the period of the cyclic star formation is several times \( 10^7 \) yr, the dispersions in KTC’s Figure 6 are consistent with the hypothesis of KT97, if we see \( b = \Psi'/\Psi \), where \( \Psi' \) and \( \Psi \) are the cyclic and past averaged SFRs, respectively.

Indeed, such a periodic star formation history was proposed by Ikeuchi (1988) as cited by KT97. His discussion is based on the limit-cycle behavior of the fractional mass of each phase of the ISM shown by IT83. If the fractional component of the cold gas, where stars are formed, changes cyclically on a short timescale (\( \sim 10^7-10^8 \) yr), the SFR also varies cyclically. Thus, we review the formulation by IT83 in the next subsection.

2.2. Limit-Cycle Model of the ISM

We review the limit-cycle model for the ISM proposed by IT83 (see also Scalo & Struck-Marcell 1986). The model has been used to interpret Tomita et al. (1996) (KT97). The limit-cycle behavior emerges if we treat the ISM as a nonlinear open system. As long as the ISM is a nonlinear open system, it spontaneously presents a dissipative structure (Nozakura & Ikeuchi 1984).

First of all, we should note that the galaxy disk is treated as one zone. The interstellar medium is assumed to consist of three components, each with its temperature \( T \) and density \( n \) (McKee & Ostriker 1977); the hot rarefied gas (\( T \sim 10^6 \) K, \( n \sim 10^{-3} \) cm\(^{-3}\)), the warm gas (\( T \sim 10^4 \) K, \( n \sim 10^{-1} \) cm\(^{-3}\)), and the cold clouds (\( T \sim 10^2 \) K, \( n \sim 10^{-6} \) cm\(^{-3}\)). The fractional masses of the three components are denoted by \( X_h , X_w , X_c \), respectively. A trivial relation is

\[
X_h + X_w + X_c = 1 .
\]

The following three processes are considered in IT83 (see also Habe, Ikeuchi, & Tanaka 1981): (1) the sweeping of the warm gas into the cold component at the rate of \( a_s X_w \) (\( a_s \sim 5 \times 10^{-8} \) yr\(^{-1}\)); (2) the evaporation of cold clouds embedded in the hot gas at the rate of \( b_s X_c X_h^2 \) (\( b_s \sim 10^{-7} \) to \( 10^{-8} \) yr\(^{-1}\)); (3) the radiative cooling of the hot gas by mixing with the ambient warm gas at the rate of \( c_s X_w X_h \) (\( c_s \sim 10^{-6} \) to \( 10^{-7} \) yr\(^{-1}\)). Writing down the rate equations and using equation (1), IT83 obtained

\[
\frac{dX_c}{dt} = -BX_c X_h^2 + A(1 - X_c - X_h) ,
\]

\[
\frac{dX_h}{dt} = -X_d (1 - X_c - X_h) + BX_c X_h^2 ,
\]

where \( \tau \equiv c_s t, A \equiv a_s/c_s, \) and \( B \equiv b_s/c_s.\)

The solutions of equations (2) and (3) are classified into the following three types (IT83):

1. \( A > 1 \).—All the orbits in the \((X_c , X_h)\)-plane reduce to the node (0, 1) (node type);
2. \( A < 1 \) and \( B > B_c \).—All the orbits reduce to a stable focus \([[1 - A]/(AB + 1), A]]\) (focus type);
3. \( A < 1 \) and \( B < B_c \).—All the orbits converge on a limit-cycle orbit (limit-cycle type);

where \( B_c \equiv (1 - 2A)/A^2 \). Obviously, case 3 is important if we wish to predict a cyclic star formation history. According to the summary of the limit-cycle model by Ikeuchi (1988), the period of a cycle is several times \( 10^7 \) yr, as depicted in his Figure 4. Since this period is much smaller than the characteristic timescales in galaxy evolution, such as the gas consumption timescale (\( > 1 \) Gyr; KTC’s \( t_p \)), the cyclic change of SFR will produce a scatter in the observed star formation activity in spiral galaxies even if their ages are similar.

3. OSCILLATORY MODEL OF THE SFR

3.1. Model Description

Following KT97, we use a simple description to test our discussion. First, we define the present quiescent component of the SFR as \( \Psi \) and the oscillatory component of the SFR as \( \Psi' \). Then the total SFR is denoted as

\[
\Psi = \Psi + \Psi' .
\]

Our definitions are adequate when the period of oscillation of the SFR is much smaller than the cosmic age (see e.g., Sandage 1986). According to Schmidt (1959), the SFR in a galaxy is expressed approximately as \( \Psi \propto n^p \) (\( 1 < p < 2 \)), where \( n \) is the mean gas density of the galaxy. If we interpret \( n \) as the gas density of a cold cloud, which can contribute to the star formation activity, we expect the oscillatory part of the SFR to be

\[
\Psi' \propto X_c^{1.5} ,
\]

where we have assumed that \( p = 1.5 \) (Kennicutt 1998). Using this relation and equations (2) and (3), the variation of the star formation activity (i.e., \( \Psi' \) as a function of \( t \)) is calculated. For example, according to Figure 6 in Ikeuchi
application to KTC

For the comparison between the model prediction and the observational data, we relate b, defined in KTC (the ratio of the present SFR to the past-averaged SFR), to the model prediction. The parameter b is calculated from the equivalent width of Hα emission. According to KTC97, we can assume that b ≈ Ψ/Ψ if the large variance of b in Figure 6 of KTC originates from a short-term variation. We combine the IT83 model with the star formation history via the Schmidt law (eq. [5]). For example, when X_α = 0.1 at the minimum SFR and X_α = 0.7 at the maximum (Fig. 1 of IT83), the value of X_α changes from 0.03 to 0.59 during the cycle if p = 1.5. Accordingly, we find that the maximum SFR is about 20 times higher than the minimum SFR, since Ψ is proportional to X_α (eq. [5]). Thus, using the cyclic star formation scenario, we find the maximum of b also becomes 20 times larger than the minimum b in this numerical example.

To summarize, the large variance of b in Figure 6 of KTC is naturally derived through the limit-cycle model, if the limit-cycle model is a real evolutionary picture of the ISM. In the next section, we examine this point more precisely, in order to reproduce the variance of star formation activity for each morphological type of spiral galaxy. In the following discussions, we examine max (Ψ)/min (Ψ), where max (Ψ) and min (Ψ) are maximum and minimum values of the oscillatory SFR (the maximum and minimum are defined by the maximum and minimum star formation rates during a period of the limit-cycle, respectively), and thus max (Ψ)/min (Ψ) ≡ max (X_α)/min (X_α). In the rough estimate in the previous paragraph, max (Ψ)/min (Ψ) = 20 with p = 1.5. Here we define

\[ F_ε ≡ max (Ψ)/min (Ψ) \]  

for convenience in the subsequent sections. Using this relation and equations (2) and (3), the variation of the star formation activity (i.e., Ψ as a function of t) is calculated, and E_ε is evaluated finally.

4. SCENARIO OF LIMIT-CYCLE STAR FORMATION

To propose a scenario of the star formation history for spiral galaxies based on the limit-cycle model, let us start with the key observational result. According to KTC, the early-type sample has a larger variance of SFR than the late-type sample. As a first step, we reconstruct this observational tendency in the framework of IT83. Then, we perform several numerical analyses to examine parameters that implement the limit-cycle oscillation of the cold phase of the ISM. For the ISM in spiral galaxies, the full possible ranges of the parameters A and B (see e.g., Habe et al. 1981) are \[ A = a_*/c_0 \] of ~0.05 to ~0.5 and \[ B = b_*/c_0 \] of ~0.01 to ~0.1. In the following discussions, we focus on the parameter sets for the limit-cycle type (case 3 in §2.2).

Two of the results of differently parameterized limit-cycle behavior are displayed in Figures 1a and 1b, where the SFRs are normalized to the minimum SFR. In Figure 1a, we find an amplitude (F_ε) of about 10, which might correspond to the result of the Sa sample in KTC. Figure 1b corresponds to the amplitude of about 4 for Sc in KTC.

Clearly, the difference in the amplitude between the Sa and Sc galaxies for the KTC data sets can be reproduced via the models that yield Figures 1a and 1b. We also present F_ε for different A and B in Table 1, from which we observe that the value of F_ε is more sensitive to A than to B. Indeed, from the rough estimate, \[ δF_ε/δA ≈ -133/0.08 \sim -1600 \] for \[ B = 1.0 \] and \[ δF_ε/δB ≈ -119/1.5 \sim -80 \] for \[ A = 0.34 \].

| Table 1  |
|----------|

\[ F_ε as a function of A and B \]

| A   | B = 0.5 | B = 1.0 | B = 1.5 | B = 2.0 |
|-----|--------|--------|--------|--------|
| 0.32 | 355    | 137    | 61     | 29     |
| 0.34 | 129    | 52     | 24     | 10     |
| 0.36 | 57     | 23     | 9      | 3      |
| 0.38 | 27     | 10     | 4      | ...    |
| 0.40 | 15     | 4      | ...    | ...    |

**Note:** We show the value of F_ε if B < B_ε (the condition for the limit-cycle behavior) is satisfied. See text for definitions of quantities.
Thus, the two figures are presented for different values of $A$. Here, we state an important point: the early-type spiral galaxies favor a small $A$, while the late-type ones are consistent with a large $A$. The variation of the amplitude in accordance with $A$ and $B$ is qualitatively interpreted as follows: small $A$ (or small $B$) indicates that the transition from the warm to the hot component (or the cold to the warm component) is inefficient. Thus, when $A$ (or $B$) is small, we must wait for $X_w$ (or $X_c$) to become large before the phase transition can become important, since the transition rate is described by $AX_w$ (or $BX_cX_w^2$). Thus, the amplitude and the period become large for small $A$ (or $B$). This interpretation of the relation between $A$ (or $B$) and the amplitude is qualitatively robust. This means that the scenario proposed in this paper is unchanged even if a more elaborate model such as that of Ikeuchi, Habe, & Tanaka (1984) is used.

As a next step, we examine the SFR variance via the effect of $A$. According to the definition of $A$ in § 2.2, we expect a larger rate of SNe for large $A$ (e.g., Sc) than for small $A$ (e.g., Sa). Then, the result in the previous paragraph predicts an important point, that the early-type spiral galaxies have a smaller present SN rate than the late-type spirals. This is confirmed in the following two points:

1. The higher SFR per unit optical luminosity in later-type spiral galaxies (Fig. 6 of KTC) indicates that the Type II SN rate per unit optical luminosity is higher in later types. Thus, it is natural in the context of our model that the late-type spirals have larger $A$ than the early-types.

2. The expected trend of the SN rate for early-to-late types has been found by Cappellaro et al. (1993). They examine the SN rate per blue luminosity in various types of spirals and present a summary of their results in their Table 4. We can confirm via their Table 4 that our scenario of limit-cycle SFR is consistent with the observational trend of the SN rate as the galactic morphology varies.

Moreover, the present SFR is reflected by the present rate of Type II SNe. According to Cappellaro et al., Sc types show higher Type II SN rates than Sa types. Thus, we can infer that the SFR of Sc galaxies is larger than that of Sa galaxies, which is compatible with Figure 6 of KTC.

From these pieces of evidence, we find a consistent picture of the SFR variance in the spiral sample as a function of morphology via the scenario of the limit-cycle star formation history. We note that the large gas-to-stellar mass ratio in late-type spiral galaxies is probably the reason for the large SFR, and that a larger mean SFR yields a smaller variance because of a larger SN rate.

The trend of $F_c$ with varying $B$ is also consistent with the different SN rates among morphological types. Since $B$ physically represents the efficiency of the evaporation of the cold component via conduction (one of the so-called SN feedback effects), $B$ increases with increasing SN rate. Because a large value of $B$ tends to reduce $F_c$, as can be seen in Table 1, a small $F_c$ is caused when the SN rate is large. Thus, from a similar argument to that in the previous paragraph, late-type spiral galaxies ought to have small values of $F_c$. Considering the sensitivity of $F_c$ to $A$ (§ 4), however, we insist that $A$, not $B$, is the dominant contributor to the determination of the amplitude $F_c$.

To be fair, the picture presented in this paper may not be the unique interpretation of the variation of SFR. A stochastic fluctuation may easily reproduce the observed scatter of the SFR in KTC’s sample as commented in their § 5.2.

5. SUMMARY AND IMPLICATIONS

5.1. Summary

In this paper, we have demonstrated that the large variety of SFRs in the spiral sample in KTC may result from the limit-cycle evolution of the ISM as suggested by KT97. We present this more quantitatively by using the numerical modeling of IT83 and explain the difference in the range of SFR between morphological types. It is known observationally that the early-type spiral sample has a wider range of present SFR than the late-type sample (KTC). In our framework of the limit-cycle scenario of star formation.
history in spiral galaxies, the early types should show a more evident time variation of the SFR than the late types to be consistent with the fact that Sc galaxies have a higher present SN rate than Sa galaxies (§ 4). Thus, the limit-cycle model by Ikeuchi and collaborators provides a consistent picture of the ISM evolution for any type of spiral galaxy.

5.2. Implications

What is the underlying physical mechanism responsible for the variation in $A$ and $B$? Since $A$ and $B$ are related to the Type II SN rate, this inquiry begs the most basic and unsolved question: what is the physical mechanism responsible for the different star formation activity among morphological types? First of all, recall that the later spirals have larger disk-to-bulge ratios than the earlier spirals. This can mean that the net volume of the disk of the later spirals is larger than that of the earlier spirals. Once we accept the larger volume of the disk of the later spiral galaxies, we expect that the Type II SN rate per galaxy is higher in the late types than in the early types because of the large disk, where ongoing star formation is generally observed. Moreover, the parameter $c_*$ is determined from the mixing rate between the warm and hot gases. Then, if the volume of the disk is effectively larger in the late types than in the early types, the late-type spirals may have smaller values of $c_*$ than the early-types because the ISM will travel a larger distance before the mixing. Since $A = a_*/c_*$ and $B = b_*/c_*$, the late type spirals tend to have larger $A$ and $B$ than the early types. Therefore, as an implication, we propose that the size of the disk of galaxies is a factor that physically produces the difference in $A$ and $B$ among the morphologies of spirals.

We expect that the difference in $A$ and $B$ is produced by an interplay between the size effect described in the previous paragraph and the SN rate, as mentioned in § 4. In fact, the question has been answered from an observational viewpoint in § 5.3 of KTC by stating “From an observational point of view, the progression in disk star formation histories with morphological type is not surprising, since one of the fundamental classification criteria is disk resolution, which should relate at least indirectly to the fraction of young stars in the disk.”

In this paper, we present a consistent picture for the variance of star formation activities in spiral galaxies by relating the differences in variance among morphological classes to the SN rate. However, since an earlier-type sample has a lower gas-to-stellar mass ratio, its mean SFR will be lower but its variance will, in any case, tend to be larger because of stochastic fluctuations.

In order to see whether the variance is caused by a purely stochastic process or not, the amplitude of a stochastic SFR should be given in a physically reasonable way. In other words, we should specify what kind of stochastic process is physically reasonable. Though our model is not stochastic, it provides a way to give an amplitude of the variable SFR. To be fair, however, a different, purely stochastic model for the variation of the SFR may provide another interpretation for the variance of SFR in KTC.

Observational study is now progressing. Recently, Rocha-Pinto et al. (2000) found that the star formation history of the Galaxy is indeed oscillatory. Based on their work, Takeuchi & Hirashita (2000) proposed that the frequency distribution function of the SFR of the Galaxy sampled every 0.4 Gyr from the formation of the Galactic disk shows a flat distribution. Comparing this distribution with the distribution of $b$ in KTC will offer us a useful element in helping to judge whether the scatter of $b$ is caused by oscillatory behavior of the SFR in any spiral galaxy.

We wish to thank the anonymous referee for invaluable comments that substantially improved the discussion of the paper. We are grateful to S. Mineshige for continuous encouragement. We also thank A. Tomita and T. T. Takeuchi for useful discussions. H. H. acknowledges the Research Fellowship of the Japan Society for the Promotion of Science for Young Scientists. We fully utilized NASA’s Astrophysics Data System Abstract Service.

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