Fuzzy cognitive maps and induced fuzzy cognitive maps approach to traffic flow

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Abstract. This paper presents a method to analyze the traffic flow pattern at a crowded junction in Chennai, one of the metropolitan cities in India, using waiting time in the signal in different time intervals with the help of Fuzzy Cognitive Map and Induced Fuzzy Cognitive Map.

Keywords: Fuzzy Cognitive Maps, Induced Fuzzy Cognitive Maps, Traffic flow

1. Introduction

Traffic congestion has been one of major issues that most metropolises are facing. It is believed that identification of congestion is the first step for selecting appropriate alleviation measures. Congestion impacts the movement of people. Different approaches can be used to minimize the traffic problem such as improved public transport, communication mechanisms expanded, proper handling of signal time. The first two approaches require many resources as money, labour, area etc. They seem very difficult to achieve in many places due to lack of resources. Control of traffic light signal timings is one of the least expensive and most effective means of reducing vehicular congestion in metropolitan road networks. The problem is examined by Fuzzy Cognitive Maps (FCMs) and Induced Fuzzy Cognitive Maps (IFCMs).

2. Basic concepts and preliminaries

Fuzzy Cognitive Maps describe and examine human being imagining or doings on particular areas by making examples. Lofti A. Zadeh in 1965 introduced the notion of fuzziness [1]. In 1976 Axelrod [2] presented cognitive maps. In 1986 Kosko [3] proposed FCMs based on the cognitive maps structure. Fuzzy cognitive maps for practical applications is discussed in [4]. Ever since, it is widely used in different areas [5-15]. In another applications, Ritha et al.[16] used IFCMs in medical for predictors of interest in cosmetic surgery.

An FCM is a directed graph with concepts such as policies, events as nodes and causalities as edges. It is a way to denote the causal relationship between concepts. If a concept $C_j$ is in direct proportion with another concept $C_i$, then the directed edge is assigned a weight 1. If a concept $C_j$ is in inverse proportion with another concept $C_i$, then the directed edge is assigned a weight -1. If there exists no relation between two concepts the value 0 is given. The connection or adjacency matrix of the FCM is $M = (e_{ij})$, $e_{ij}$ being the edge weight.

The instantaneous behavior of each node is given as an input vector $A = (a_1, a_2, \cdots, a_n)$ where $a_i \in \{0,1\}$; 0 represents OFF and 1 represents ON position. The hidden pattern is the equilibrium state of the FCM. If the equilibrium state is a unique state vector, then is called fixed point. The dynamical system goes round and round when the causality flows through the edges like a cycle starting with concept $C_i$ and ending at $C_i$ when $C_i$ is switched ON.
Suppose $A = (a_1, a_2, \cdots, a_n)$ is a vector which is passed into a dynamical system $X$. Then $A \ast E = (a'_1, a'_2, \cdots, a'_n)$ after thresholding and updating the vector suppose we get $(b_1, b_2, \cdots, b_n)$. We denote this $(a'_1, a'_2, \cdots, a'_n) \leftrightarrow (b_1, b_2, \cdots, b_n)$. Thus the symbol $\leftrightarrow$ means the resultant vector has been threshold and updated. $(b_1, b_2, \cdots, b_n)$ is the fixed point of the dynamical system $X$.

### 3. Estimation of signal waiting time for different time intervals using FCMs

Signal co-ordination significantly improves the performance of network operations. Using traffic counts, the cycle times of the signals were calculated and offset optimization has been done. The road user cost (travel time, fuel consumption etc) can be considerably reduced by signal synchronization. Using FCMs we will estimate when the signal time is high and less.

In this work, we consider Velachery-Vijayanagar junction, the three road junction in Chennai. At this junction, Velachery Vijaynagar three roads meet at the signal, one from Medavakkam to Vijaynagar, the second from Taramani to Vijaynagar and the third from Velachery By pass to Vijaynagar. Since this area is also connected by fly-over train, daily huge no of vehicles passing through the signal to reach the railway junction. In other areas in Chennai, the waiting time in the signal is less compared to the week days, but in Velachery signal, the waiting time is high because Velachery is occupied by many malls. This junction is depicted in Figure 1. The different types of vehicles passing in all the three directions is counted for a period of time and classified for different time intervals morning 5 to 7, 7 to 9, 9 to 11, afternoon 11 to 1, 1 to 3, evening 3 to 5, 5 to 7, 7 to 9.

![Figure 1: Velachery Vijayanagar signal](image)

The pseudo code for the method is

* Collect the concepts (nodes) for the given problem.
* Obtain the connection square matrix $M$.
* Set the concept $C_i$ ($i=1, 2, 3, \ldots, n$) in ON-State
* Multiply $C_i$ ($i=1, 2, 3, \ldots, n$) with $M$ and threshold value is calculated by assigning 1 to the first state and for the values > 0 to get $C_2$.
* Multiply $C_2$ with $M$ and repeat the procedure to get the fixed point.
* Continue the above process for the remaining state vector and find the hidden pattern.

The different concepts considered to study the signal waiting time are given in the table-1.
The connection matrix $M$ is:

\[
\begin{array}{cccccccccccc}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\
\end{array}
\]

Case 1:
All the states are OFF state except “Total no of vehicles passing in the time 5-7 A.M”
i.e., $A_1 = (1000000000000000)$. The outcome vector is
\[
A_1^M = (0000000001000010) \\
\leftrightarrow (100000001000010) = A_2 \\
A_2^M = (210121002110020) \\
\leftrightarrow (110111001110010) = A_3 \\
A_3^M = (220232003230080) \\
\leftrightarrow (110111001110010) = A_4.
\]

Case 2:
All the states are OFF state except “Total no of vehicles passing in the time 9-11 A.M”
i.e., $A_3 = (0010000000000000)$. The outcome vector is
\[
A_3^M = (000000000000101) \\
\leftrightarrow (001000000101) = A_4.
\]
\[ A_{0}M = (0 \ 0 \ 2 \ 0 \ 0 \ 0 \ 1 \ 2 \ 0 \ 0 \ 1 \ 2 \ 0 \ 2) \]
\[ \leftrightarrow (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1) = A_{5} \]
\[ A_{0}M = (0 \ 0 \ 2 \ 0 \ 0 \ 0 \ 2 \ 0 \ 0 \ 0 \ 2 \ 3 \ 0 \ 5) \]
\[ \leftrightarrow (0 \ 0 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1) = A_{6}. \]  

Using Fuzzy Cognitive Maps from the case 1 we are getting \((1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 0)\) as a fixed point, i.e. total no of vehicles passing in the time 5-7 a.m, 7-9 a.m the traffic is very low, 11-1 p.m, 1-3 p.m the traffic is medium and 3-5 p.m there is less waiting time in signal. For two time intervals the traffic is low or medium. To overcome this and to correctly identify the signal waiting time Induced Fuzzy Cognitive Map is used. From the case 2 we are getting \((0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1)\) is a fixed point, i.e. total no of vehicles passing in the time 9-11 a.m, total no of vehicles passing in the time 5-7 p.m, total no of vehicles passing in the time 7-9 p.m, traffic is high, traffic is very high are more waiting time in the signal.

4. Estimation of signal waiting time for different time intervals using IFCMs

Induced Fuzzy Cognitive Maps (IFCMs) is altered way of Fuzzy Cognitive Maps. The algorithmic approach for IFCM is explained in detail in [6].

Identification of the concepts, incorporating expert’s opinion and formation of connection matrix is similar to FCM. We study the effect of each concept, by setting one concept at a time in ON state and the remaining in OFF state i.e., set the concept \(C_i\) \((i=1, 2, 3, \ldots, n)\) in ON-State. First consider \(C_1\). Multiply \(C_1\) with M. Similar to FCM, threshold to get the resultant vector \(C'_1\). Now each component in \(C'_1\) is taken separately and the process of multiplication and thresholding is repeated. The vector with maximum number of one’s is identified as a fixed point (say) \(B_1\). Multiply \(B_1\) with M and threshold to obtain \(B'_1\). Now each component in \(B'_1\) is taken separately and the process of multiplication and thresholding is repeated. The vector with maximum number of one’s is identified as a fixed point. If this fixed point is same as \(B_1\), the process is terminated. At this stage we conclude that \(B_1\) is the fixed point of \(C_1\). This procedure is repeated for the remaining concepts and the hidden pattern is identified. Based on the hidden patterns derived the resultant flow can be identified precisely. The relation between the concepts is plotted as a graph based on the resultant flow.

The IFCM calculations are done for the FCM model discussed in section-3 and the results of are presented in the following table. In Figure 2, the IFCM of the traffic flow problem is depicted.

Using Induced Fuzzy Cognitive Maps we proved that the waiting time for the signal is less in the time interval 5-7 a.m, 7-9 a.m, 11-1 p.m, 1-3 p.m, 3-5 p.m. When the traffic is very low, low and medium the waiting time is signal is less. In the time interval 9-11 a.m, 5-7 p.m, 7-9 p.m signal waiting time is more. When the traffic is high and very high the signal waiting time is more. These observations clearly indicate the relationship between the time intervals, traffic volume and signal waiting time, which were the concepts of FCM.

| No. | Concepts ON State | Resultant Flow |
|-----|-------------------|----------------|
| Step 1 | \((1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_1 \rightarrow C_{14} \rightarrow C_{14}\) |
| Step 2 | \((0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_2 \rightarrow C_{14} \rightarrow C_{14}\) |
| Step 3 | \((0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_3 \rightarrow C_{15} \rightarrow C_{15}\) |
| Step 4 | \((0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_4 \rightarrow C_{14} \rightarrow C_{14}\) |
| Step 5 | \((0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_5 \rightarrow C_{14} \rightarrow C_{14}\) |
| Step 6 | \((0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_6 \rightarrow C_{14} \rightarrow C_{14}\) |
| Step 7 | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_7 \rightarrow C_{15} \rightarrow C_{15}\) |
| Step 8 | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_8 \rightarrow C_{15} \rightarrow C_{15}\) |
| Step 9 | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_9 \rightarrow C_{14} \rightarrow C_{14}\) |
| Step 10 | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_{10} \rightarrow C_{14} \rightarrow C_{14}\) |
| Step 11 | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 0)\) | \(C_{11} \rightarrow C_{15} \rightarrow C_{15}\) |
| Step 12 | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0)\) | \(C_{12} \rightarrow C_{15} \rightarrow C_{15}\) |
| Step 13 | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0 \ 0)\) | \(C_{13} \rightarrow C_{15} \rightarrow C_{15}\) |
| Step 14 | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0 \ 0)\) | \(C_{14} \rightarrow C_{9} \rightarrow C_{14} \rightarrow C_{14}\) |
| Step 15 | \((0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1 \ 0)\) | \(C_{15} \rightarrow C_{13} \rightarrow C_{15} \rightarrow C_{15}\) |
5. Conclusion

The concepts of Fuzzy Cognitive Maps and Induced Cognitive Maps were applied to analyse the traffic flow problem. The results indicate precisely the relation between the chosen concepts. This approach can be applied to examine the traffic flow pattern in any crowded road junction.

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