Sommerfeld enhancement of resonant dark matter annihilation

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Abstract

The dark matter annihilation cross section can be amplified by orders of magnitude if the annihilation occurs into a narrow resonance, or if the dark-matter particles experience a long-range force before annihilation (Sommerfeld effect). We show that when both enhancements are present they factorize completely, that is, all long-distance non-factorizable effects cancel at leading order in the small-velocity and narrow-width expansion. We then investigate the viability of “super-resonant” annihilation from the coaction of both mechanisms in Standard Model Higgs portal and simplified MSSM-inspired dark-matter scenarios.
1 Introduction

In many models, the dark matter (DM) annihilation cross section, which constitutes the fundamental quantity determining the relic abundance and cosmic ray signals of DM, is enhanced beyond the naive expectation derived from the mass and couplings of the DM particle. The two most common enhancement mechanisms are non-relativistic scattering before pair annihilation, the Sommerfeld effect \[1\]–\[3\], and annihilation through a narrow resonance \[4\]–\[5\]. The Sommerfeld effect is generic for DM with SU(2) electroweak charge and mass in the TeV region and exhibits resonant behaviour for specific mass values. Resonant annihilation requires the resonance mass to be close to twice the DM mass, and can significantly boost the annihilation of weakly coupled DM. Interesting scenarios of this type can be found in the parameter space of the minimal supersymmetric standard model (MSSM) and Higgs portal models.

The present work is motivated by two theoretical questions that appear when both effects, Sommerfeld and resonant enhancement, occur simultaneously. In this case the annihilation process is not only no longer a short-distance process, but it contains two long-distance phenomena possibly intertwined: a long-range force before annihilation, and a long-lived intermediate state thereafter, as sketched in Figure 1. Can one factorize the two and achieve a simple description as is the case when only one of the enhancements is present?\[1\] Furthermore, the velocity-dependent Sommerfeld factor is then to be folded with an annihilation cross section into the resonance, which varies rapidly with the three-momentum of the DM particle. Does this lead to a breakdown of the partial-wave expansion in the calculation of the Sommerfeld effect?

The answer to both questions turns out to be simple, as will be seen, but to our knowledge has not been discussed previously in the literature. We exemplify the theoretical discussion with the Standard Model (SM) Higgs portal model augmented by a light mediator in the dark sector, and a MSSM-like model with TeV scale DM annihilating through a heavy Higgs boson resonance.

\[1\] The factorization of the Sommerfeld from another long-distance effect holds for the electroweak Sudakov resummation, see for example \[6\].

![Figure 1](image-url)

Figure 1: Schematic representation of the process in question. In red, the exchange of a light mediator that produces the long-distance Sommerfeld effect, in blue the long-lived intermediate resonance. The question arises whether there are long-distance non-factorizable corrections connecting the two (black curly lines, including real emission into the final state).
2 Factorization of the Sommerfeld and resonant enhancement

The salient features of simultaneous Sommerfeld- and resonance-enhanced annihilation can be exposed by a simple model of a spin-1/2 DM field $\chi$ with gauge interactions in a representation of some gauge group $G$. The DM field couples to a resonance $H$, which may (but does not have to) transform under the same gauge symmetry, through the interaction

$$\lambda y_{abc} \bar{\chi}_a \chi_b H_c + \text{h.c.},$$

where $a, b, c$ denote gauge-group indices, $\lambda$ is the coupling strength, and $y_{abc}$ are Clebsch-Gordan coefficients such that the product is gauge-invariant. We assume that the gauge symmetry is spontaneously broken such that the gauge boson acquires a small mass $m_X$.

We are interested in a situation, where the DM particles are non-relativistic and twice the DM mass, $2m_\chi$, is close to the resonance mass $m_h$, and define

$$\delta M = m_h - 2m_\chi \ll m_h,$$

where $|\delta M| \ll m_h$. In the centre-of-mass (cms) frame of the annihilating $\chi\chi$ pair, the kinetic energies of the DM particles and resonance are small. The problem involves the high-mass, “hard” scales $m_\chi$ and $m_h$, as well as the low-energy scales $\delta M$, the width of the resonance $\Gamma_h$, and the kinetic energies.

One can integrate out systematically the hard scales to construct an effective description of the annihilation process in terms of an effective Lagrangian. The resonance dynamics is described by the scalar field version of a static field (similar to heavy quark effective theory), $h_w$, generalized to an unstable particle \[7,8\], given by

$$\mathcal{L}_{\text{HSET}} = h_w^\dagger \left( iD_0 - \delta M + \frac{i\Gamma_h}{2} \right) h_w .$$

$D_0$ is the time-component of the $G$-covariant derivative and the quantity $\delta M$ appears, since we measure the non-relativistic energy $E$ in the rest frame of the $\chi\chi$ pair, that is, from $2m_\chi$ rather than from $m_h$. In this frame $w^\mu = (1, 0)$. The Lagrangian receives small corrections suppressed by powers of $E/m_h, \Gamma_h/m_h$, which will be consistently neglected.

The non-relativistic dynamics of the DM field $\chi$ is described by potential non-relativistic dark matter (PNRDM) effective theory \[2,9\] with the Lagrangian

$$\mathcal{L}_{\text{PNRDM}} = \chi_w^\dagger \left( iD_0 + \frac{\partial^2}{2m_\chi} \right) \chi_w - \int d^3r \, \mathcal{V}(r) \left[ \chi_w^\dagger \chi_w \right] (t, \mathbf{x}) \left[ \chi_w^\dagger \chi_w \right] (t, \mathbf{x} + \mathbf{r}).$$

Gauge-boson exchange generates the static Yukawa potential $\mathcal{V}(r)$

$$\mathcal{V}(r) = -\frac{\alpha_\chi}{r} e^{-m_\chi r},$$

which leads to Sommerfeld-enhanced DM annihilation. We denote the non-relativistic DM field by the symbol $\chi_w$ to distinguish it from $\chi$ above. We also assumed that the $\chi_a\chi_b$
scattering amplitude has been decomposed in irreducible group representations and that the corresponding Casimir has been absorbed into the coupling $\alpha_X$. In the following, we consider a single representation with an attractive potential and therefore drop the gauge index on the non-relativistic $\chi_w$ field.

The Sommerfeld effect is a long-distance effect related to the range $1/m_X$ of the potential. For specific values of $m_\chi, m_X$ and the gauge coupling $a$, attractive potential and therefore drop the gauge index on the non-relativistic $\chi_w$ field.

The Sommerfeld effect is a long-distance effect related to the range $1/m_X$ of the potential. For specific values of $m_\chi, m_X$ and the gauge coupling $a$, zero-energy DM bound-state appears in the spectrum. At these values the annihilation cross section is resonantly enhanced and grows as $1/v^2$ with the cms velocity $v$ of the annihilating particles. Similarly, the annihilation through a resonance is an enhanced long-distance effect relative to the point-like annihilation cross section, related to the life-time $1/\Gamma_h$ of the resonance. For small width $\Gamma_h$ and $\delta M = 0$, the resonance enhancement grows as $1/v^4$. In the following, we first consider the factorization of both effects and then discuss their interplay.

The inclusive DM annihilation cross section can conveniently be obtained from the discontinuity of the forward-scattering amplitude $T$ through the optical theorem via

$$\sigma v_{rel} = \frac{1}{m_\chi \sqrt{s}} \text{Im} T,$$

where $\sqrt{s} = 2m_\chi + E$ the cms energy of the process and $v_{rel} = 2v$. Unstable-particle effective theory \cite{7,8} provides the systematic framework to describe resonant processes in an expansion in $\Gamma_h/m_h$. The forward-scattering amplitude of two non-relativistic DM particles can be expressed as

$$i T = \sum_{m,n} \int d^4x \langle \chi(0)|iJ_m(x)iJ_n(0)|\chi(0)\rangle + \sum_k \langle \chi|iT_k(0)|\chi\rangle,$$

where the first term involves production operators $J_n$ for the long-lived resonance, whilst the second captures non-resonant local interactions through operators $T_k$. This second term is suppressed relative to the first by one power of $\Gamma_h/m_h$, and can therefore be dropped for the following leading-power analysis. For the simple model above, the production operator is

$$J(x) = \frac{C}{\sqrt{2m_h}} y_{abc} \chi_w^+ h_w^r \epsilon_{abc} \chi_w (x),$$

where $C$ denotes the hard matching coefficient and $C = \lambda$ at tree level.

We now show that at leading power in the expansion in $\Gamma_h/m_h$ and the non-relativistic velocity, the long-distance dynamics before annihilation (Sommerfeld effect) is completely factorized from the long-lived resonance, that is, there are no soft gauge boson exchanges connecting the initial state to the resonance or the SM final state. To this end, we redefine the effective fields as

$$h_w(x) \to Y_w(x_0) h_w(x), \quad \chi_w(x) \to Y_w(x_0) \chi_w(x),$$

where

$$Y_w(x) = \text{P exp} \left( i g_X \int_{-\infty}^0 ds w \cdot A^d_s(x + sw) T^d \right)$$

\footnote{The local term naturally factorizes by the standard non-relativistic analysis in the absence of a resonance, see, e.g., the discussion for the neutralino in \cite{10}.}
is a time-like Wilson line, and the space-time point \(x_0\) is \((t, 0)\). The generator \(T^d\) is taken in the representation of the field to which the Wilson line is multiplied. The Wilson line satisfies the key property
\[
Y_w^+ i w \cdot D Y_w = i w \cdot \partial .
\] (10)

Inserting the above field redefinitions into the Lagrangians \([3, 4]\) removes the gauge field from the covariant derivative due to the identity \([10]\) and also leaves the Yukawa potential interaction invariant \([11]\). Thus the Lagrangians take the same form as before except for \(D_0 \to \partial_0\). The field redefinition puts the gauge interactions into the production vertex,
\[
y_{abc} (\chi^+ w a \chi_w b_w h_{wc})(x) \to y_{abc} \left[ Y_{w, a d} \ Y_{w, b b} Y_{w, c c'} \right] (x_0) \left( \chi^+ w a \chi_w b_w h_{wc'} \right)(x) .
\] (11)

However, since \(Y_w(x_0)\) is simply a gauge transformation with gauge parameter \(\alpha^d(x_0) = g_X \int_0^\infty \! ds \ w \cdot A^d_c(x_0 + sw)\), it follows from the gauge invariance of the production vertex that the right-hand side is in fact equal to the left. This demonstrates that gauge interactions are completely absent in the low-energy effective theory and hence all non-factorizable effects (curly lines in Figure [1]) vanish.

We emphasize the importance of the fact that the gauge interactions are decoupled from \(\chi_w\) and \(h_w\) by Wilson lines with the same vector \(w^\mu\). The production of a resonance in the annihilation of two heavy particles is very different in this respect from the production in the high-energy collision of massless particles, where non-factorizable soft effects do exist. The difference is that for such a process the decoupling from the Lagrangian leads to Wilson lines \(Y_{n+}, Y_{n-}\) and \(Y_w\) in different directions, where the first two are related to the light-like directions of the colliding particles, and the production operator is not simply a gauge-transformation of itself after decoupling.

The factorization of the forward-scattering amplitude \([9]\) proceeds as follows. The absence of any long-distance interaction between the DM and the resonance field allows us to split the matrix elements into the two factors
\[
\int d^4x \braket{\chi \bar{\chi}}{i J^T(x) i J(0) } \chi \bar{\chi} \]
\[
= i^2 \frac{C^2}{2\hbar} y_{abc}^+ y_{a'b'c'} \int d^4x \braket{\chi \bar{\chi}}{ i J^T(x) \left[ \chi^+ a b \chi_w (x) \chi^+ a' b' \chi_w (x) \right] (0) } \chi \bar{\chi} \langle 0 | T \left\{ h_{a a'}^c (x) h_{b b'}^c (0) \right\} | 0 \rangle .
\]

The matrix element of non-relativistic fields can now be evaluated as in the standard treatment of the Sommerfeld effect \([9]\), resulting in
\[
\braket{\chi \bar{\chi}}{ i J^T(x) \left[ \chi^+ a b \chi_w (x) \chi^+ a' b' \chi_w (x) \right] (0) } \chi \bar{\chi} = e^{i E t} \braket{\chi \bar{\chi}}{ \chi^+ a b \chi_w (0) } \chi \bar{\chi} \langle 0 | \chi^+ a b \chi_w (0) \rangle \]
\[
= e^{i E t} \left[ \psi_E^{a b} (0) \right]^* \psi_E^{a' b'} (0) \times \text{Born} \equiv e^{i E t} S_{SF}(v) \times \text{Born} ,
\] (13)

neglecting corrections of order \(v^2\). In the second line we identified the Sommerfeld factor \(S_{SF}(v)\), which depends on \(v\) through \(E = m_\chi v^2\) in terms of the wave-function at the
origin of the DM two-particle scattering state in the Yukawa potential. Substituting into \( (13) \), leaves the resonance propagator

\[
Z_d 4 \chi e^{iEt} \langle 0 | T\{h^{+}_w(x)h_c^r(0)\} | 0 \rangle = \frac{i \delta^{cc'} E - \delta M + i \Gamma_h}{E - \delta M + i \Gamma_h} . \tag{14}
\]

Putting everything together and taking the imaginary part, the total DM annihilation cross section through the resonance can be written as

\[
\sigma_{v_{\text{rel}}} = \frac{C^2 Y}{4m^2_{\chi}} S_{\text{SF}}(v) R(v) . \tag{15}
\]

Here \( Y \) is a schematic notation for the projection of \( y^{\frac{1}{2}}_{abc} y_{a'c'} \) on the irreducible “colour” channel of the \( \chi \chi \) state with respect to the gauge symmetry \( G \). The value of the Sommerfeld factor is also channel-dependent. We further introduced the dimensionless quantity

\[
R(v) = \frac{m_{\chi} \Gamma_h}{(m_{\chi} v^2 - \delta M)^2 + \frac{\Gamma_h^2}{4}}, \tag{16}
\]

which quantifies the resonant enhancement normalized to a point-like interaction. The annihilation cross section now contains two separate enhancements and the factorization of the two long-distance effects is manifest in \( (15) \). If both enhancements overlap in some velocity region, one obtains “super-resonant” annihilation cross sections. A few simple scenarios that can exhibit such behaviour will be considered below.

We briefly discuss the validity of the partial-wave expansion, which is equivalent to the expansion in the relative velocity of the annihilating DM particles, or \( E/m_{\chi} \). The cross section is a function of \( m_{\chi}, E \) and \( \delta M, \Gamma_h \). In the presence of a resonance, however, the cross section cannot be expanded in \( E \) and one may ask whether the expansion of the Sommerfeld effect in partial waves is justified. Factorization solves this problem as well, since it separates the problem into hard-matching coefficients and the factorized low-energy matrix elements. The computation of the Sommerfeld effect is insensitive to the existence of the resonance. In the non-relativistic theory, the partial-wave expansion can then be constructed by extending the matching of the interaction \( (11) \) to higher orders in \( v^2 \), in terms of the series

\[
\lambda y_{abc} \bar{\chi}_a \chi_b H_c = \sum_n \frac{C_n y_{abc}}{m^{2n}_{\chi}} \chi^{\dagger}_{wa} \left( -\frac{i}{2} \partial \right)^{2n} \chi_{wab} h_{wc} . \tag{17}
\]

In the above discussion we already anticipated that the term \( n = 0 \) gives the leading contribution in \( E/m_{\chi} \).

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3The DM particles can also annihilate into gauge bosons \( XX \). This is formally part of the non-resonant cross section, but not necessarily small, since this annihilation rate is controlled by the gauge coupling, which is independent of the DM coupling \( \lambda \) to the resonance. See Sec. 3 below.

4The total width is written as \( \Gamma_h = \tilde{\Gamma}_h + \Gamma_{h \rightarrow XX} \), and in the numerator of \( R(v) \) only the partial width \( \tilde{\Gamma}_h \) contributes to \( \sigma_{v_{\text{rel}}} \) in the Boltzmann equation.
It is well-known that the Sommerfeld factor for the Yukawa potential exhibits resonances near specific DM mass values, which violate partial-wave unitarity due to the presence of a zero-energy bound-state \[12, 13\]. Unitarity is self-consistently restored by including the local potential \(\delta(3)(r)\) \[13\] into the PNRDM Lagrangian (4) which is generated by the DM forward \(\chi\chi \rightarrow \chi\chi\) scattering amplitude, whose imaginary part is related to the total annihilation cross section. One may wonder what are the consequences of (15) for unitarity violation. We first note that unitarity is not violated by the intermediate-resonance enhancement \(R(v)\) in the absence of the Sommerfeld factor, \(S_{\text{SF}}(v) = 1\). To see this, we remark that the approximation of the imaginary part of the \(H\) self-energy by the on-shell width is not justified when there is a decay final-state close to threshold \[14\], as is the case here for \(h \rightarrow \chi\chi\). In this case, one must use \(\Gamma(p^2) \propto \sqrt{p^2 - 4m_{\chi}^2}\), where \(p^2 = (2m_{\chi} + E)^2\) is the off-shell momentum squared of the resonance. We can therefore write \(\Gamma_h\) as
\[
\Gamma_h = \frac{m_{\chi}}{8\pi} \left( \lambda_{\text{eff}}^2 + C^2 Y v_{\text{rel}} \right) \equiv \hat{\Gamma}_h + \Gamma_{h \rightarrow \chi\chi},
\]
where \(\lambda_{\text{eff}}^2\) parameterizes the coupling to all final states in \(h\) decay, which are not close to threshold. Then from (15)
\[
\sigma_{v_{\text{rel}}} \left|_{\text{no SF}} \right. \leq \frac{C^2 Y}{4m_{\chi}^2} \frac{2m_{\chi} \hat{\Gamma}_h}{\Gamma_h^2} = \frac{4\pi}{m_{\chi}^2 v_{\text{rel}}} \frac{\lambda_{\text{eff}}^2 \times C^2 Y v_{\text{rel}}}{(\lambda_{\text{eff}}^2 + C^2 Y v_{\text{rel}})^2} \leq \frac{\pi}{m_{\chi}^2 v_{\text{rel}}},
\]
which remains a factor of 4 below the unitarity bound, hence (S-wave) partial unitarity is guaranteed\(^5\).

Nevertheless, when \(R(v) \gg 1\) the unitarity violation from \(S_{\text{SF}}(v)\) near certain resonant DM mass values is enhanced and operative in a wider DM mass window around the resonant value. Yet, the basic mechanism of unitarity restoration remains the same as in the standard case, because the intermediate resonance that causes the enhancement \(R(v)\) also contributes to the imaginary part of the \(\chi\chi\) forward scattering amplitude. In contrast to the standard situation, however, the forward-scattering amplitude does not match to a local potential in the non-relativistic effective theory, since resonant scattering is a long-distance process. Formally integrating out the resonance, results in the temporally non-local operator
\[
-\text{const.} \times \int_0^\infty ds \, e^{-i(\delta M - i\Gamma_h/2)s} \left[ \chi_w^\dagger \chi_w \right] (t + s, \mathbf{x}) \left[ \chi_w^\dagger \chi_w \right] (t, \mathbf{x}).
\]

The PNRDM field equation then no longer takes the form of a Schrödinger equation, and the solution for the unitarized annihilation cross section becomes complicated.

It is useful to gain some qualitative understanding of the velocity dependence of \(R(v)\), given \(\delta M\) and \(\Gamma_h/m_h \approx \Gamma_h/(2m_{\chi})\)\(^6\). We first note that for \(\delta M < 0\) the resonance

\(^5\)The factor of four arises from the initial-state spin-average and the fact that only one of the four possible initial spin states couples to the scalar resonance.

\(^6\)For simplicity, we set \(\Gamma_h\) equal to \(\hat{\Gamma}_h\) for this discussion.
Figure 2: Velocity dependence of the resonant enhancement $R(v)$ for $\delta M \leq 0$ (left) and $\delta M > 0$ (right) for $\Gamma_h/m_\chi = 0.001$ and various values of $\delta \hat{M} = \delta M/m_\chi$.

is lighter than $2m_\chi$. In this case, the annihilation occurs always beyond the resonance peak of the propagator and the resonance enhancement is monotonically decreasing as $v$ grows (Figure 2, left). When the velocity decreases, $R$ first increases as $1/v^4$ with maximal slope for $v^2 \approx 0.88 \Gamma_h/(2m_\chi)$ (as long as $|\delta M| \lesssim \Gamma_h$), and then saturates at the maximal value

$$R(0) = \frac{m_\chi \Gamma_h}{\delta M^2 + \frac{\Gamma_h^2}{4}} < R_{\text{max}} = \frac{2m_\chi}{\Gamma_h}.$$  \hfill (21)

Interestingly, this behaviour is similar to the saturation of the Sommerfeld effect in the Yukawa potential. In the latter case the saturation value depends on the strength of the potential and the value of $m_\chi/m_X$.

For positive $\delta M$, the DM particles must have finite velocity $v_{\text{peak}}^2 = \delta M/m_\chi$ to produce the resonance exactly on its peak. At this velocity, the maximally possible enhancement $R_{\text{max}}$ is attained independent of the value of $\delta M$ (Figure 2, right). For smaller velocity, $R(v)$ drops to the same $R(0)$ as for the corresponding negative $\delta M$. The annihilation cross section can now be dramatically increased in a narrow velocity interval, which is not possible for Sommerfeld enhancement. However, when $\delta M \ll \Gamma_h$, the peak becomes increasingly one-sided, since it is regularized by the finite width. A pronounced resonance peak can appear in the velocity spectrum only for $v_{\text{peak}}^2 \gtrsim \Gamma_h/(2m_\chi)$.

The velocity peak can boost freeze-out relative to late time annihilation, which occurs at smaller $v$, or vice versa \cite{4}, or bias indirect DM detection from astrophysical objects with markedly different velocity distribution, if the resonance is sufficiently narrow.

3 Resonant dark matter scenarios

DM annihilation through a resonance is by itself a long-known mechanism to boost the annihilation rate \cite{15,17}, but to our knowledge the coaction of resonant and Sommerfeld effect has not been considered up to now, even leaving aside radiative effects. In the following, we discuss two potentially interesting scenarios, focusing on the question
whether super-resonant behaviour is possible.

### 3.1 Higgs portal with dark sector mediator

We first study the SM Higgs portal model with a complex scalar DM particle $S$ of mass $m_S$, augmented by a dark-sector $U(1)_X$ symmetry, which is spontaneously broken to give the dark gauge boson a small mass $m_X \ll m_S$. The relevant terms in the Lagrangian are

$$
\mathcal{L} \supset (D_\mu S) \dagger (D^\mu S) - m_S^2 |S|^2 - \frac{1}{4} X_{\mu\nu}X^{\mu\nu} - \frac{\lambda_S}{4} |S|^4 + \lambda_{HS}(H^\dagger H)|S|^2 + \mathcal{L}_{\text{mix}}
$$

where $H$ is the SM Higgs doublet. The mass and width of the Higgs particle $h$ are given by $m_h = 125$ GeV and $\Gamma_{h,SM} = 4.1$ MeV, respectively, and since near resonance the mass of the DM scalar satisfies $m_S \approx m_h/2$, the only adjustable parameter related to the resonance is $\delta M = m_b - 2m_S$.

To be cosmologically viable, the light dark-sector gauge boson must decay into SM model particles. After electroweak symmetry breaking, $X$ mixes with the SM gauge bosons through the terms

$$
\mathcal{L}_{\text{mix}} = \frac{\epsilon_\gamma}{2} X_{\mu\nu}F^{\mu\nu} + \epsilon_Z m_Z^2 X_\mu Z^\mu
$$

with $\epsilon_\gamma, \epsilon_Z \ll 1$. The resulting decay width of $X$ into photons is given by

$$
\Gamma_{X \rightarrow \gamma} = \alpha_{em} m_X \epsilon_\gamma^2 / 3.
$$

Choosing $\epsilon_\gamma, \epsilon_Z = 10^{-9}$ ensures dark gauge boson decay before nucleosynthesis. Additionally, constraints from supernova cooling require $m_X \gtrsim 100$ MeV.

The dark gauge boson is still abundant during freeze-out, and the process $SS^* \leftrightarrow XX$ must be considered. For $m_S \gg m_X$, the annihilation rate is

$$
\sigma v_{\text{rel}}(SS^* \rightarrow XX) = \frac{4 \alpha_X^2}{m_S^2} \times S_{\text{SF}}(v),
$$

where $S_{\text{SF}}(v)$ is the Sommerfeld factor generated by the dark $U(1)$ force. Since $m_S \approx 62.5$ GeV is fixed, this implies that the dark-sector gauge coupling $g_X = \sqrt{4 \pi \alpha_X}$ cannot be too large, since then the relic density would be under-abundant compared to the observed $\Omega h^2 = 0.120$.

The model is subject to the same constraints as the ordinary Higgs portal with a real singlet scalar (and no dark mediator), which we do not repeat here. Suffice it to say that the resonant regime is in the best-fit region. It remains to show, however, that additional processes induced by the dark gauge boson do not lead to further constraints.

DM loops generate an effective interaction with the SM Higgs boson, given by

$$
\mathcal{L}_{\text{EFT}} \supset \sqrt{2}\lambda_{XXH} v h X_{\mu\nu}X^{\mu\nu} \quad \text{with} \quad \lambda_{XXH} = -\frac{\alpha_X \lambda_{HS}}{48 \pi m_S^2},
$$

8
where \( v \approx 246 \text{ GeV} \) denotes the SM Higgs vacuum expectation value. This results in a contribution to the invisible Higgs width,

\[
\Gamma(h \rightarrow XX) = \frac{\lambda_{HS}^2 \alpha_X^2 m_h^2 v^2}{4608 \pi^3 m_S^4}.
\] (27)

If \( m_h > 2m_S \), the channel

\[
\Gamma(h \rightarrow SS^*) = \frac{\lambda_{HS}^2 v^2}{8 \pi m_h} \sqrt{1 - \frac{4m_S^2}{m_h^2}}
\] (28)

contributes directly to the Higgs width, but is phase-space suppressed near resonance. However, this decay now receives a final-state Sommerfeld enhancement from \( X \)-exchange.

We checked that for scenarios of potential interest, the values of the coupling \( \lambda_{HS} \) are such that the invisible Higgs width remains far below the SM width.

The annihilation cross sections relevant for setting the relic abundance of \( S \) and for indirect detection signals are (25) and the resonant annihilation of two DM particles via the SM Higgs boson

\[
\sigma v_{\text{rel}}(SS^* \rightarrow h \rightarrow \text{SM SM}) = \frac{\lambda_{HS}^2 v^2}{4m_S^3} \frac{\frac{1}{2} \Gamma_{h,\text{SM}}}{(E - \delta M)^2 + \frac{\Gamma_h^2}{4}} \times S_{\text{SF}}(v).
\] (29)

Here \( E = m_S v^2 \) is the kinetic energy of the two-particle \( SS^* \) state and \( \Gamma_h = \Gamma_{h,\text{SM}} + \Gamma_{h,\text{inv}} \). the total Higgs decay width including the invisible one. The resonance enhancement is determined by \( \Gamma_h \) and \( \delta M \), while Sommerfeld enhancement requires

\[
\frac{\pi \alpha_X m_S}{m_X} \geq 1.
\] (30)

In the present model, the combination of parameters on the left-hand side cannot be made arbitrarily large, since \( i) \alpha_X \) is constrained to values below \( 10^{-3} \) by the correct relic abundance requirement, which limits \( \sigma v_{\text{rel}}(SS^* \rightarrow XX) \) given in (25), and \( ii) \) \( m_X \geq 100 \text{ MeV} \). Thus \( \pi \alpha_X m_S/m_X \sim 1 \) can be achieved, if \( m_X \) and \( \alpha_X \) are pushed towards their limits, but not much larger values. However, the first Sommerfeld resonance in the Yukawa potential which appears at \( \alpha_X \approx \frac{\pi^2}{6} \times m_X/m_S \gtrsim 2.5 \cdot 10^{-3} \), cannot be reached. Hence, only a moderate \( \mathcal{O}(1) \) Sommerfeld enhancement is to be expected, and it is therefore not possible to obtain truly super-resonant enhancements in the SM Higgs-portal model with a dark-sector gauge boson.

An example of a viable scenario that accounts for \( \Omega_{\text{DM}} h^2 = 0.120 \) uses \( m_S = 62.2 \text{ GeV} \), corresponding to \( \delta M = 0.6 \text{ GeV} \), \( m_X = 0.1 \text{ GeV} \), \( \alpha_X = 4.7 \cdot 10^{-4} \) and the DM coupling to the Higgs resonance \( \lambda_{HS} = 2 \cdot 10^{-4} \). For these parameter values the spin-independent direct detection cross section is [20]

\[
\sigma_{\text{SI}} = 2 \frac{m_N^4}{4 \pi (m_S + m_N)^2} \frac{(2 \lambda_{HS})^2 f_N^2}{m_h^4} \approx 8 \cdot 10^{-49} \text{ cm}^2.
\] (31)
Figure 3: Upper panel: Velocity dependence of the relevant DM annihilation processes with and without Sommerfeld enhancement (ratio subtended) in the SM Higgs-portal scenario. Lower panel: Thermally averaged annihilation cross section.

about two orders of magnitude below the present limits [21]. The value of $\delta M > 0$ is chosen such that a narrow resonant enhancement appears for velocity $v \approx 0.1$, which boosts the annihilation into SM particles when freeze-out happens (upper panel, Figure 3). Except for this narrow region, the annihilation cross section is dominated by the annihilation into a pair of dark gauge bosons. The impact on the thermally averaged cross section is clearly visible in the lower panel of the figure. The annihilation rate at small velocity relevant today is small and evades the constraints, which mainly arise from the Fermi-LAT, MAGIC [22] and H.E.S.S. [23] experiments’ non-observation of annihilation into the $b\bar{b}$ final state. The Sommerfeld enhancement reaches nearly a factor of two (subtended panel), but, as expected, does not cause a dramatic increase of
cosmic-ray signals.

As a general remark, we note that scenarios with $\delta M < 0$ or small positive $\delta M \sim 0.120$ are likely to produce too large annihilation cross sections at small $v$ and are therefore strongly constrained by the absence of indirect detection signals.

### 3.2 MSSM template

The large MSSM parameter space features both the Sommerfeld enhancement through the standard electroweak gauge forces and resonant annihilation. The mass of the $W$ boson and the size of the SU(2) and U(1)$_Y$ gauge couplings requires DM masses of a few TeV or larger in order to have an $\mathcal{O}(1)$ Sommerfeld effect. Resonant annihilation then requires a decoupling scenario with a SM-like Higgs boson and a heavy Higgs doublet consisting of nearly degenerate $A^0$, $H^0$ and $H^\pm$ bosons. The dominant decay channels of the heavy Higgs bosons are into gauge bosons, top quarks, and bottom quarks, depending on the value of $\tan \beta$. The $A^0$ and $H^0$ resonances may overlap, when the mass splitting is small enough. In the following simplified description, we neglect the interference effects that can occur in this situation and assume a single resonant intermediate state.

The treatment of the Sommerfeld effect in the full MSSM can be found in [9,10]. In these works the case of resonant annihilation was excluded, since theBorn cross sections were expanded in $E$. With the result of the previous section, this restriction could now be removed. Leaving a discussion of the full MSSM to future work, we consider here a grossly simplified template for resonant neutralino annihilation in the MSSM, consisting of the neutralino $\chi$, a heavy Higgs boson $A^0$ (not necessarily CP-odd) with mass close to $2m_\chi$ and a massive U(1) gauge boson $X$ with Lagrangian

$$\mathcal{L} = \bar{\chi}(i\slashed{D} - m_\chi)\chi - \frac{1}{4}X_{\mu\nu}X^{\mu\nu} + yA^0\bar{\chi}\chi - \frac{1}{2}m_A^2(A^0)^2 + \mathcal{L}_{\text{SM}}.$$  \hfill (32)

To allow for a $U(1)$ force the “neutralino” is assumed to be a Dirac fermion. The relevant DM annihilation cross sections are

$$\sigma v_{\text{rel}}(\chi\bar{\chi} \to A^0 \to \text{SM SM}) = \frac{y^2}{2m_\chi^2} \frac{\Gamma_{A,\text{SM}}}{(m_\chi v - \delta M)^2 + \frac{r^2}{4}}, \hfill (33)$$

$$\sigma v_{\text{rel}}(\chi\bar{\chi} \to XX) = \frac{\pi\alpha_X^2}{m_\chi^2}. \hfill (34)$$

For multi-TeV Higgs bosons in the MSSM, their decay width into SM particles typically results in $\Gamma_{A,\text{SM}}/m_A \sim 10^{-3} \ldots 10^{-2}$. Since the neutralino-Higgs coupling $y$ is related to the electroweak gauge couplings, the decay rate for $A^0 \to \chi\bar{\chi}$,

$$\Gamma(A^0 \to \chi\bar{\chi}) = \frac{y^2m_\chi^2}{2\pi m_A} \sqrt{1 - \frac{4m_\chi^2}{m_A^2}} \theta(m_A - 2m_\chi), \hfill (35)$$

can be neglected due to its phase-space suppression even in the presence of the final-state Sommerfeld effect, except near a Sommerfeld-resonant $m_\chi$ mass value. Nevertheless, the numerical evaluations below include this decay channel.
The Sommerfeld effect is important in the MSSM for a variety of situations (see, for example, [24–27]). We are interested here in models that yield the observed relic abundance through a standard thermal freeze-out. In this case, Sommerfeld enhancement is particularly important for wino-dominated neutralinos, which can have mass between about 2 and 3.5 TeV [25]. Much (but not all) of this parameter space is, however, already excluded by the absence of indirect detection signals [26,27] even when assuming a cored DM profile of the Milky Way. Adding a resonant component to the already large annihilation rate into gauge bosons [34] that enhances late-time annihilation ($\delta M < 0$ or $\delta M \lesssim \Gamma_h$) worsens this tension. Bino-dominated DM, on the other hand, does not exhibit a significant Sommerfeld effect.

An interesting situation may arise for the wino- or Higgsino-dominated neutralino when $\delta M > \Gamma_h$. We consider explicitly a Higgsino-like setting in the template model in the following. In the pure Higgsino (minimal SU(2) doublet) model, the correct relic abundance is obtained for $m_\chi \approx 1.1$ TeV, at which mass the Sommerfeld enhancement is only a few percent effect on the relic density and about a factor of two for small velocities $v \approx 10^{-3}$ [2,28]. This allows for the possibility to raise the neutralino mass while avoiding over-abundance by resonant annihilation through a resonance peak in the velocity spectrum at early times. The annihilation cross section in the present universe then experiences super-resonant enhancement by a sizable Sommerfeld effect (due to the large mass) and off-peak resonant enhancement. In the full MSSM this requires a bino admixture to the dominantly Higgsino-neutralino to generate the DM coupling to the heavy Higgs boson.

We mimic this situation in our template model by choosing the effective gauge coupling $\alpha_X = \frac{1}{2} \alpha_2(m_Z)$ and $y = \sqrt{4\pi\alpha_2(m_Z)} \cdot 0.152$, where $\alpha_2(m_Z) = 1/29.792$ is the standard SU(2) gauge coupling. We further adopt the heavy Higgs width $\Gamma_{A,SM} = m_A/150$ and $m_\chi = m_W = 80.385$ GeV. The correct relic abundance $\Omega_{DM} h^2 = 0.120$ is now obtained for the significantly larger “dominantly-Higgsino” DM mass $m_\chi = 6500$ GeV and Higgs mass $m_A = 13230$ GeV. In this model $\delta M = 230$ GeV is positive and exceeds $\Gamma_{A,SM} = 88.2$ GeV, which leads to a pronounced resonance peak in the annihilation cross section at velocity $v \approx 0.2$, as shown in Figure 4. The resonant enhancement is crucial to sufficiently deplete the DM abundance during freeze-out. The Sommerfeld enhancement, on the other hand, is relatively small during this period and does not influence the freeze-out noticeably. However, it reaches $O(10^2)$ below $v \approx 10^{-3}$ and boosts the annihilation cross section in the late universe to a level that might be observable with future cosmic-ray experiments even for a cored DM profile in the Galactic Center. While the freeze-out is entirely determined by the DM annihilation through the $A^0$ resonance, the annihilation into gauge bosons dominates at small velocities and late times.

A similar scenario can be found for wino-like DM in the few TeV mass range between the first and second Sommerfeld resonance.
4 Conclusion

In this letter we investigated the factorization properties of the DM annihilation cross section when the two long-distance effects of resonant annihilation and Sommerfeld enhancement operate together. Our main theoretical result is that unlike in the production of a resonance in high-energy scattering of ultra-relativistic particles, non-factorizable long-distance effects cancel completely in heavy-particle annihilation up to corrections of higher-order in the non-relativistic and small-width expansion. As a consequence, the annihilation cross section is the product of hard amplitudes, the Sommerfeld factor and the Breit-Wigner factor.
When the resonance is slightly lighter than $2m_X$, or $\delta M$ is small compared to the resonance width, the Sommerfeld and resonant enhancement both show saturation behaviour as $v \to 0$. In such cases, it is difficult to obtain super-resonant behaviour from the coaction of both mechanisms, since satisfying indirect detection constraints usually leads to an overabundance of DM when produced through thermal freeze-out.

We specifically analyzed the SM Higgs-portal scalar DM model with a dark-sector gauge symmetry to illustrate this point. The requirement of resonance fixes the DM mass to $m_h/2$. In contrast, the mass of the DM is unconstrained in the MSSM, since the heavy Higgs mass is not fixed. Simplifying to a template model that features electroweak-size gauge couplings and a Yukawa coupling of the neutralino to a heavy MSSM Higgs boson, we find the interesting possibility of heavy Higgsino- or wino-like DM, whose relic abundance is set by resonant annihilation, while the late-time annihilation leads to signals, potentially observable in the near future, via a strong Sommerfeld effect. This motivates a closer inspection of the relevant parameter space in the full MSSM along the lines of [25–27].

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