A multi-resolution based method to precise identify the 
natural frequencies of beams with application in damage 
detection

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Abstract. This paper presents an enhanced method that accurately identifies natural
frequencies of structures, via an important improvement of spectral lines density. Spectral lines
density increases due to the superposition of many spectra, each spectrum being accomplished
by subtracting certain number of samples from the original signal and using the power spectral
density analysis. The newly achieved overlapped-spectrum shows important additions to
spectral line number and makes more accurately identification of frequencies in the peak
position of relevant amplitudes. In this way, modal analysis becomes more reliable and
efficient. Method application assumes small frequency changes and the advantage of
establishing more precisely the position and dimension of damage. The analysis of all results
explicitly indicates an important assessment in the evaluation of frequency. Considering
relevant aspects, it can be appreciated that the approached method offers a new development
way in the vibration-based damage detection, allowing de possibility to perform more accurate
evaluation of frequencies and introducing new features into the domain.

1. Introduction
In time, whatever physical structure, under the application of a large variety of strength types, gets
damage. By detecting damages in their very early stage, the keeping of structure integrity and
functionality can be ensured via an adequate maintenance. The impact of early damage detection is
significant for big and complex structures, causing money and work-time saving and also decreasing
the possibility to occur accidents. There is a great area of methods that can perform non-destructive
tests as regard the damage detection. Dynamic methods offer an import advantage they do not require
access to the damaged place [1].

Within damage detection domain, the vibration-based methods are very popular and also widely
spread. Herein natural frequency shifts are determined and afterwards attentively analyzed [2]-[3].
These methods can be commonly divided into two categories those having limitations at the damage
detection or designed to predict detection, location and quantification of damage [4]. Into the second
category, all methods based on the natural frequency shifts are defined as model-based, [5] and [6],
and use finite element models. The readability of frequency shifts determines the accuracy of damage
detection. Also, a great precision in frequency reading and clear measurements lead to better results in
detection. Small frequency changes caused by early structural damage or even small damage cannot be
distinguished from those of the healthy structure, and so that the mathematical model is under the
impossibility to work well. Thus, mostly it is not possible to make a difference between improperly
models and fine changes caused by the occurrence of small or early damage [7] and [8]. And these are not all the issues regarding the frequency shifts, the temperature action or/and structural loads application induce more other interferences.

Also, other methods available as well, besides a comprehensive classification and description are presented in [3]. Each method fits a particular application, due to its specific advantage. Until now, at least one method does not meet all the requirements supposed by the high variety of analyzed structures and the diversity of imposed conditions.

In this paper, a new method to enhance the frequency evaluation is presented. It is based on an extensive study regarding the transformation of acquired time-domain signal in a frequency spectrum. The achieved spectrum has to provide many possibilities to find a better fit for a certain frequency. Herein, an overlapped-spectrum has been accomplished via an iterative algorithm, performing a modulation from the right-side of acquired signal window. A step-by-step windowing of time signal is applied to an algorithmic structure implemented in LabView. Each step of iteration consists in subtracting from the signal right-side of a certain number of time samples. Also, after subtracting the samples, an analysis applying Power Spectral Density (PSD) feature of LabView is made. The amplitude values of each frequency are stored and at the end of last iteration step, they are displayed all in one single spectrum, achieving in this way a much denser frequency overlapped-spectrum.

2. Description of frequency identification method

Method consists of three steps: signal acquisition, reference analysis and algorithm setup. Considering the spectrogram and Wigner-Vile distribution, as could be seen in figure 1, it is observed that certain frequencies are damping very fast, providing in this way a very short time length [9]-[10]. As known, time length gives the resolution of the frequency spectrum after Fourier analysis is performed. Thus, spectrum resulted by applying ordinary Fourier Transform provides poor density and in case of small damage, determining fine frequency shifts, this is impossible to distinguish. So that, even if a faster acquisition device is taken to achieve much more samples in a short time interval, any enhancement of the frequency spectrum can be accomplished.

![Figure 1. Wigner-Vile distribution (a) and power spectrum (b) of vibration signal](image)

To significantly enhance the spectrum density, the following argument can be assumed: by decreasing the length of signal, from the side of low amplitude in the time history, the number of spectral lines is decreasing as well, but also their position changes. From this last point of view, cropping the signal length become important and can be iteratively applied to achieve a multi-spectrum. If an iterative step-by-step signal cropping is imagined, which is performed by an algorithm, and also the obtained spectral lines are afterwards displayed in one single graphical spectrum, we finally can achieve a much denser frequency spectrum.
Algorithm to analysis and iteratively crop the signal is implemented in LabView under a logical loop structure. Loop considers a start point and a stop one, within these points performing a number of iteration. From the beginning have to be established some functional parameters, as initial length of time signal, entire cropping portion of signal and number of samples cropped at the iteration step.

2.1. Signal acquisition and pre-processing
In order to prove the method performance, make a right comparison of results and have a better flexibility for input signals, we provide as time signals those generated by LabView. Of course, they have been generated to meet few important requirements: short length, are mixed and have different amplitudes. Figure 2 shows an auto-generated time signal, different cropped and mixed from two signals having the following characteristics: entire time length \( t \), total number of samples \( N \), frequency \( f \) and amplitude \( A \). First signal \( S_1 \) has \( f_1 = 4.2 \) Hz and \( A_1 = 1 \) mm/s\(^2\), the second one \( S_2 \) is at \( f_2 = 23.7 \) Hz and \( A_2 = 0.5 \) mm/s\(^2\).

![Image](image.png)

Figure 2. Mixed sine signal with two variable amplitudes and cropped.

Left image of figure 2 presents the entire signal \( t = 1 \) s, middle image shows the signal cropped with a period \( T_1 = 0.0(6) \) s and the right-side one has a cropped period \( T_2 = 0.037 \). The reason of cropping is assumed by algorithm setup and will be later detailed.

2.2. The reason of applying reference analysis
Actually, reference analysis is performed to acquire an initial spectrum and establish who the frequencies of interests in a certain case are. At this time, the cropping period is chosen taking into account the lowest frequency considered as expected from experience reasons. It is very important to establish the right frequencies of interest, due to the fact that in correlation with them the algorithm must be set. In the presented case, this second step is redundant, the frequencies are well known being generated by ourselves. In real cases, the result of this test can be assumed as being similar to that shown in figure 1 b.

2.3. Explaining the algorithm implementation
Algorithm is a logic structure implemented in a virtual environment employed to ensure an explicitly work and satisfy certain requirements. This algorithm is formulated to get the signal, compute some parameters, iteratively crop and analyze the signal, and display all the results in one graphical frequency spectrum. First consideration is about the input signal, which can be auto-generated or real acquisitioned and stored as a file with a given extension. In LabView, the "Block Diagram" (BD) window ensures the placement and interconnectivity of functional blocks. Auto-generated signal can be applied by using the functional block "Simulate Signal" (SS), where the waveform, amplitude, frequency, length and number of samples were set in this case. Main part of algorithm block diagram (ABD) is the logic structure placed into the limited loop, where "Extract Portion of Signal" (EPS) and "Spectral Measurements" (SM) blocks perform signal cropping and spectral analysis. At the end of loop execution, all data achieved from the iterative analysis is sending to the "Graph" (G) block. In a different window "Front Panel" (FP), data is show as graphical spectrum. The parameters chosen to be set or visualize are also shown in FP.
Figure 3 presents the $BD$ image that is meant to comprehensively clear up all the aspects associated with the algorithm implementation.

![Figure 3. Software implementation of frequency identification algorithm.](image)

In Table 1 parameter abbreviations included in the figure 3 are presented and detailed as well. Also, for each of them the sense of consideration (as input or output) is presented, regarding the displayed values meaning that can be divided in two categories, set by hand or resulted after computing.

| Parameter | Sense | Detail |
|-----------|-------|--------|
| $A_{1,2}$ | Input | Signal amplitude |
| $F_{1,2}$ | Input | Signal frequency |
| $Ph_3$   | Input | Signal phase - not used in these tests |
| Ref.F.   | Input | Reference frequency - expected to reach by analysis |
| Ref.P.   | Output| Reference period - corresponds to Ref.F. |
| T.S.N.   | Input | Total samples number - all signal samples considered in test, equal to $N$ |
| S.I.     | Input | Sampling interval - whole signal length in time, equal to $t$ |
| S.T.     | Output| Sampling time |
| B.S.     | Input | Begin sample - first sample from the signal left-side |
| A.S.N.   | Output| All samples number - remaining samples after last iteration is done |
| Start.It.| Input | Start iteration - sample whence algorithm begins the iterative cropping |
| It.S.    | Input | Iteration step - number of samples cropped at iteration |
| Stop.It. | Output| Stop iteration - number of samples cropped at the iteration end |
| In.It.N. | Output| Initial iteration number - resulted by computing |
| Real.It.N.| Output | Real iteration number - resulted by applying It.S. to In.It.N. and rounding |
| O.S.     | Output| Original signal - graphically displayed |
| P.S.     | Output| Portioned signal - graphically displayed when the iteration ends |
| D.R.S.   | Output| Displaying of resulted data - data displayed as overlapped-spectrum |
Above parameters have not been considered only for simple signal cropping and iteration, but also to create many possibilities of data and results management from whatever section of the algorithmic structure. Therefore, considering important the way in which identification algorithm works and can be set, in order to get suitable results more details are provided in the next chapter.

3. Easy frequency evaluation by adequate algorithm setup

From the beginning has to be mentioned the importance of following few steps to properly setup the identification algorithm and easy achieve clear results:

- Chose the lowest expected frequency value, decrease its value to at least 10% and use it as Ref.F.
- Perform reference analysis and make the choice regarding the frequencies of interest.
- For each of these frequencies a new algorithm setup has to be applied and also a new analysis with the identification algorithm must be taken.

Let us consider figure 4 as example for how algorithm parameters have to be managed and what for. Firstly, image from figure 4 has been accomplished by writing the following values for parameters: T.S.N. = 50000, S.I. = 1 s, Ref.F. = 3.5 and It.S. = 50. Ref.F. was deduced by assuming a lowest expected frequency in about of 4 Hz, subtracting 10% and rounding to 3.5 Hz. Afterwards, overlapped-spectrum achieved is zoomed in around of 4 Hz, so as the image offers a good vision.

![Figure 4. Overlapped-spectrum achieved at Ref.F. = 4 Hz.](image)

Overlapped-spectrum is realized from 286 graphical lines, being resulted from the same number of iteration performed by the algorithm. Image can trick us, due to its apparent complexity, and make enough difficult the evaluation of frequency because of the presence of some superposed small lobes.

To clear it up, values Ref.F. as well as Start.It. are recommended to be modified. By increasing the value of Ref.F., smaller lobes disappear and right-side of the main lobe is going to clear as well. Also, increasing the value of Start.It., left-side of the main lobe become more definite.

But in the overlapped-spectrum two values are distinguished, as well as in the auto-generated time signal provided to the input of identification algorithm. Magnifying area in around of 23 Hz, the image of the other main lobe is depicted in figure 5.
As easily can be seen, more superposed lobes are affecting the image of the main lobe at about of 23 Hz. To refine its area, values Ref.F. and Start.It. have to be managed differently than before. First modification is addressed to Ref.F., which must be significantly increased at least up to the value of expected frequency or even more, if the superposed lobes do not disappear. As for lobe's left-side, Start.It. value will be set much lower, otherwise lobe will entirely vanish.

4. Results analysis and discussion

Taking into account the previously presented technique for managing the parameters of frequency identification algorithm, we offer the values of those strictly involved in clearing lobe areas. In this way, making easy reading frequencies within the overlapped-spectrum resulted after iteration completes.

Figure 6 shows how easy is the identification of frequency after performing an adequate algorithm setup.

Figure 6 shows how easy is the identification of frequency after performing an adequate algorithm setup, frequency matching area being magnified and placed at the up-right side in the same image. Due to the clearness of the lobe area, simply identify the peak of lobe, and there must be the best frequency matching. Being in LabView, the cursor numerically shows values for frequency and amplitude in the pointed location.
The values of input parameters that suitable characterize the first frequency identification at 4.2 Hz are the following: Ref.F. = 15, Start.It. = 1300 and It.S. = 50.

For the other frequency value of 23.7 Hz that is also provided into the mixed signal, a new parameterization and analysis have been performed, and the identification was made using a graphical evolution as presented in figure 7.

Suitable values for parameters setting, ensuring a clear area of the lobe, in this case, are Ref.F. = 27, Start.It. = 1200 and It.S. = 30. By these settings, a number of 22 spectra have been accomplished, which formed the overlapped-spectrum.

Method limits can be defined as belonging to three groups: signal structure, acquisition device capability and computing power. First group refers to number of signal samples, in case of short signal length, at low-frequency value and for small frequency shifts; we have to achieve as many samples as possible. Also, signal length has to stay bigger than one period of expected lowest frequency. Second group considers the characteristics of acquisition device. As for the sensitivity, a higher number of bits for analog-to-digital conversion are requested, and for short signals faster devices have to be provided.

Third group treats computing power, regarding the number of iteration. Once frequency shift value decreases, much iteration must be performed, and greater computing power to diminish the analysing time is recommended.

In the end, it has to be specified, these frequency values actually were chosen as being nearby to those from real cases [11]-[14], in order to give significance and weighty to the results of performed tests. So that, results confirm the important improvement introduced by this method, and also highlights the great advantage in using the described new technique and algorithm, in the frequency identification field.

5. Conclusion
The paper presents an enhance method to accurately identify frequencies within a multi-resolution spectrum, here introduced as overlapped-spectrum. The method consists of a software algorithm, for iteratively cropping of the signal and generating the overlapped-spectrum, and a technique to adequate setup the algorithm, which clears the spectrum area in around of desired frequency.

Performing analyses on signals similar to real once, method confirms its effectiveness in improving the readability of frequencies, especially in the case of a short time signal.

The clarity and easiness of frequency identification, strongly suggests this method for performing in cases as rapid vibration damping or early damage occurrence.
The maximum precision in frequency reading depends only on the accuracy of the acquisition signal and the number of samples, when short time signals are analyzed.

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