6D black string as a model of the AdS/CFT correspondence

Y. S. Myung, N.J. Kim and H.W. Lee
Department of Physics, Inje University, Kimhae 621-749, Korea

Abstract

We discuss the entropy for the extremal BTZ black hole and the extremal EBTZ black hole. The EBTZ black hole means the BTZ black hole embedded in a five-dimensional (5D) black hole. The six-dimensional (6D) black string with traveling waves is introduced as a concrete model for realizing the AdS/CFT correspondence. The traveling waves carry the momentum distribution $p(u)$ which plays an important role in counting the entropy and establishing the correspondence. It turns out that the EBTZ black hole is consistent with the AdS/CFT correspondence.
I. INTRODUCTION

Recently the AdS/CFT correspondence has attracted much interest. This is based on the duality relation between the string theory (bulk theory) on AdS\(_{d+1}\) and a conformal field theory (CFT) as its d-dimensional boundary theory \([1–3]\). The s-wave calculation of greybody factor (dynamic property) for the BTZ black hole on AdS\(_3\) is in agreement with that of a 5D black hole with three charges \(Q_1, Q_5, N_K\) \([4]\). This is obvious because in the low energy limit a 5D black hole reduces to the BTZ black hole \([4]\). Furthermore the greybody factor for the BTZ black hole agrees with the CFT\(_2\) calculation on the boundary of AdS\(_3\) \([6]\). This proves the AdS\(_3\)/CFT\(_2\) correspondence for the greybody factor. In the calculation of the greybody factor we need to study the propagation of test fields in the BTZ black hole background \([7]\). For counting the entropy we investigate the static configuration of the black hole itself \([8,9]\). A 5D black hole was originally constructed by the bound states of \(Q_1\) D1-branes and \(Q_5\) D5-branes with some momentum \(P(= 2\pi N_K/L = N_K/R_1)\) along an internal circle (\(S^1\)) with the radius \(R_1\). In order to see a closer relation between the microscopic states and the classical spacetime, it is convenient to rewrite a 5D black hole to include \(S^1\) as a space. It comes out a six-dimensional (6D) black string. Even though the extension of the spacetime, the entropy for a 5D black hole is the same as in a 6D black string. In this work a 6D black string with traveling waves is introduced as a concrete model for realizing the AdS/CFT correspondence. The traveling waves carry the momentum distribution. This plays an important role in counting the entropy and establishing the AdS/CFT correspondence.

Nowadays it seems to be a discrepancy for counting the entropy (static property) of the BTZ black hole in relation to the AdS/CFT correspondence. First we assume that the BTZ black hole is not embedded in string theory. In this sense this corresponds to an exact AdS\(_3\) background. And thus this is described as a solution to the pure three-dimensional (3D) gravity (\(S_{3D} = \frac{1}{16\pi G_3} \int d^3x \sqrt{-g}(R - 2\Lambda)\)) with the negative cosmological constant (\(\Lambda = -1/\ell^2\)).
Here $M = (\rho_+^2 + \rho_-^2)/8G_3 \ell^2$ and $J = \rho_+ \rho_- / 4G_3 \ell$ correspond to the mass and angular momentum of the BTZ black hole. $\rho_+ (\rho_-)$ are the outer(inner) horizon. This can be described by a \text{SL}(2, \mathbb{R})_L \times \text{SL}(2, \mathbb{R})_R$ Chern-Simons theory. Since it is a topological field theory, the physical degrees of freedom reside only on the boundary. In this direction an important question arises: what fields provide the relevant degrees of freedom on the boundary at infinity ($\rho = \infty$)? A single Liouville field at infinity may be a candidate. Unfortunately this has a central charge ($c = 1$ \cite{10, 11}). But we need $c = 3\ell / 2G_3$ \cite{12, 13} for AdS$_3$/CFT$_2$ correspondence at $\rho = \infty$. That is, in order to recover the Bekenstein-Hawking entropy of the extremal BTZ black hole with $\rho_- = \rho_+ = \rho_0$

$$S_{\text{BTZ}}^{\text{BH}} = \frac{2\pi \rho_0}{4G_3},$$

(2)

one needs both $c = 3\ell / 2G_3$ and Cardy’s formula

$$d(c, N) = \exp \left( \frac{2\pi \sqrt{\frac{c}{6}N}}{N} \right)$$

(3)

with the level number $N \gg 1$. Here $N = \rho_0^2 / 4G_3 \ell$ is given by an eigenvalue of the angular momentum operator $J = L_0$. At this stage we ask : is this BTZ black hole dual to the CFT of the D1-D5 bound states (D-string theory)? Because it is not embedded in string theory, its boundary CFT with $c = 1$ has nothing to do with the D-string. The Liouville theory is simply a boundary description of the pure 3D gravity sector. Martinec showed that the 3D gravity is a pure gauge theory and thus it examines only its macroscopic properties (thermodynamics) by a set of Noether charges \cite{11}. On the other hand, the gauge theory of branes(dual CFT$_2$) is a tool to investigate its microscopic features. Hence the 3D gravity appears as a collective field theory of the microscopic dual CFT$_2$. On the other hand, string theory provides many more dynamical fields ($c = 6Q_1 Q_5$) and one certainly does not expect the Liouville sector to accommodate all of the states of the system \cite{12}. Hence it is not appropriate for discussing the AdS/CFT correspondence for the entropy within the pure
3D gravity. In this paper, we wish to describe how the AdS/CFT correspondence for the entropy can be realized explicitly.

II. THE EBTZ BLACK HOLE

Now we consider the BTZ black hole which resides in the near-horizon geometry of a 5D black hole. Hereafter we call this the embedded BTZ(EBTZ) black hole. At low energies (α′ → 0) the physics of an extremal 5D black hole (6D black string) is governed by the extremal EBTZ black hole. In this case we have an embedded AdS3.Explicitly a 5D black hole (M5) has the geometry: AdS3 in the near-horizon (the throat region) but with asymptotically flat space. This corresponds to the geometry which interpolates between AdS3 and flat space. This black hole has the outer horizon at r = r0 and the inner horizon at r = 0. And thus the extremal limit means r = r0 → r = 0. In this limit the coordinate relation between a 5D black hole and the EBTZ black hole is given by

\[ \tilde{\rho}^2 = \rho^2 + \tilde{\rho}_0^2, \]

\[ \tilde{\phi} = z/R_1, \tilde{t} = t\tilde{Q}_1\tilde{Q}_5/R_1^2. \]

The extremal EBTZ black hole has its horizon at \( \tilde{\rho} = \tilde{\rho}_0 \). If one starts with \( M_5 \times S^1 \times T^4 \) in the ten-dimensional (10D) type IIB superstring theory, we have \( AdS_3 \times S^3 \times T^4 \) in the throat region \( r_0 \leq r \leq R, R^2 = \sqrt{Q_1Q_5} \) but find Minkowski space after passing through the remote boundary \( r = R \). Hereafter we choose a convention with \( \alpha' = 1 \). In the extremal limit the relations between \( \tilde{Q}_i \) and \( Q_1, Q_5, N_K \) are given by

\[ v\tilde{Q}_1 = g_sQ_1, \tilde{Q}_5 = g_sQ_5, \text{and } R_1^2v\tilde{Q}_K = g_s^2N_K. \]

The 3D Newton’s constant \( (G_3) \) is replaced by

\[ \tilde{G}_3 = \frac{g_s^2}{4vR_1R_2^2}. \] (4)

where \( g_s \) is the string coupling constant and \( v = V/(2\pi)^4 \) with the volume of \( T^4(V) \). Notice that \( \tilde{G}_3 \) contains all information of the type IIB string theory, contrary to \( G_3 \). The geometry of the extremal EBTZ black hole is described by

\[ d\tilde{s}_{EBTZ}^2 = -\left(\frac{\tilde{\rho}^2 - \tilde{\rho}_0^2}{\rho^2\ell^2}\right)^2d\tilde{t}^2 + \tilde{\rho}^2 \left(d\tilde{\phi} - \frac{\tilde{J}}{2\tilde{\rho}^2}d\tilde{t}\right)^2 + \frac{\tilde{\rho}^2\ell^2}{(\rho^2 - \tilde{\rho}_0^2)^2}d\tilde{\rho}^2. \] (5)

The mass, angular momentum, and the area of the horizon take the forms
with \( \tilde{\ell} = R^2/R_1, \tilde{\rho}_0^2 = \tilde{Q}_K = g_s^2 N_K / R_1^2 v \). Hence the \( \text{Bekenstein-Hawking} \) entropy of the EBTZ black hole leads to

\[
S_{\text{EBTZ}}^{\text{BH}} = \frac{\tilde{A}_3}{4 G_3} = 2 \pi \sqrt{Q_1 Q_5 N_K} = \frac{A_5}{4 G_5} = S_{\text{BH}}^{5D}
\]

which is exactly the same form as in the 5D black hole. Therefore all information is encoded in AdS\(_3\).

Suppose the weakly coupled D-brane description in which BPS states are described by right movers with \( N_K \) momentum in (1+1) dimensions. Then the degeneracy of D-brane system is given by the degeneracy of CFT with a central charge \( \tilde{c} = 3 \tilde{\ell}/2G_3 = 6Q_1 Q_5 \) at level \( N_K \). Plugging this into (3) leads to the Bekenstein-Hawking entropy (7). However, this is done actually in the weak-coupling limit of \( g_s Q_1, g_s Q_5, g_s^2 N_K \ll 1 \). This is precisely the opposite regime where the classical supergravity solution is good. On the supergravity side a 5D black hole appears in the strong coupling limit with \( g_s Q_1, g_s Q_5, g_s^2 N_K \gg 1 \). Due to supersymmetry, one can extrapolate results in the D-brane phase to the black hole phase.

### III. 6D BLACK STRING

#### A. 6D black string without momentum modes

We start with a 6D black string with \( \tilde{Q}_1 = \tilde{Q}_5, p = 0 \) as [16]

\[
ds_{6D}^2 = -\left(1 + \frac{\tilde{Q}_1}{r^2}\right)^{-1} dudv + \left(1 + \frac{\tilde{Q}_1}{r^2}\right) \left(dr^2 + r^2 d\Omega_3^2\right),
\]

where \( u = t - z \) and \( v = t + z \). \( z \) is on \( S^1 \). It turns out to be \( \text{AdS}_3 \times S^3 \) in the near-horizon

\[
ds_{6D}^2 = -\frac{r^2}{R^2} dudv + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_3^2
\]

\[
= \frac{R^2}{y^2} \left[-dudv + dy^2\right] + R^2 d\Omega_3^2, \quad y = R^2 / r.
\]

Here \( R \) is the effective radius of an embedded \( \text{AdS}_3 \). This amounts to the zero temperature limit of the D-string model with \( Q_1 \) and \( Q_5 \). Although this is a simple model for the
calculation of the greybody factor, it is not good for the entropy counting. The EBTZ black hole is a rotating black hole with the angular momentum $\tilde{J}$. This contains the information for the momentum modes. The important thing is how we introduce the total momentum number $N(= N_K)$ into (8).

**B. 6D black string with $p = \text{constant}$**

Recently it is shown that the chiral orbifold of AdS$_3$/Z$_N$ describes the extremal EBTZ black hole of (5) [14,17]. Here the chiral orbifold of AdS$_3$/Z$_N$ means a lens space of AdS$_3$ [18]. Also (AdS$_3$/Z$_N$) $\times$ S$^3$ corresponds to the near-horizon geometry of the extremal 6D black string with momentum modes $p = \tilde{p}^2_0 = g_s^2 N_K / R_l^2 v \equiv cN, c = 2\pi\kappa^2 / L^2$ [4],

$$ds^2_{\text{AdS}_3/Z_N} = \frac{r^2}{R^2} \left[ - du dv + \frac{cN}{r^2} du^2 \right] + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_3^2. \tag{10}$$

Here $\kappa^2 = 2\pi g_s^2 / v$ is related to the 6D Newton’s constant. This corresponds to a homogeneous (translationally invariant) black string. (10) contains the geometry of the EBTZ black hole in (5). The difference is that $N$-term is explicitly shown here in terms of $r$-coordinate. However, the horizon geometry ($\tilde{g}^{rr} = 0$) is not changed under the coordinate transformation $r \leftrightarrow \tilde{r}$. Consequently, its entropy counting goes exactly to the same way as in the EBTZ black hole. This guarantees that the entropy of the EBTZ black hole is invariant under this orbifolding of AdS$_3$ $\rightarrow$ AdS$_3$/Z$_N$. At this stage let us introduce the AdS/CFT correspondence. This means that the 10D type IIB bulk theory deep in the throat is dual to the 2D CFT of D-string(world sheet theory) on its remote boundary. Further, this implies that the semiclassical limit of spacetime physics (a homogeneous black string) is related to the large $(Q_1, Q_5, N)$ limit of the CFT(D-string). Concerning the entropy, the relevant quantity is the total momentum number $N$. Furthermore $Q_1$ and $Q_5$ are fixed numbers. Considering $N$ as a variable, one can understand clearly how this correspondence is realized.
C. 6D black string with traveling waves $p(u)$

We wish to describe how the AdS/CFT correspondence can be realized in this picture. First we extend a black string to include traveling waves. The extremal 6D black string solution (8) has full (1+1)-dimensional Poincaré invariance, including a null translational symmetry. Thanks to this null symmetry, one can add traveling waves with the momentum distribution $p(u)$ to any solution as

$$ds_{6D}^2 = \left(1 + \frac{R^2}{r^2}\right)^{-1} \left(-dudv + \frac{p(u)}{r^2}du^2\right) + \left(1 + \frac{R^2}{r^2}\right) \left(dr^2 + r^2d\Omega^2_3\right).$$  \hspace{1cm} (11)

Although $p(u)$ looks like a pure gauge degree of freedom, it contains an important physical information (momentum distribution). Furthermore $p(u)$ carries this information far from the horizon. The mechanism is as follows. In the low energy limit the 6D black string without $p(u)$ becomes AdS$_3 \times$S$^3$. That is, (8) leads to (9). Hence a naive counting of the graviton ($D(D-3)/2$) implies that in (2+1)-dimensional spacetime we have no propagating graviton. This counting is suitable for the black holes and non-extremal black strings. However, our model is the extremal black string with the null Killing symmetry. In this case the graviton mode ($p(u)$) may be physically propagating. Actually this graviton becomes a propagating mode by the transmutation of the degree of freedom with the other field (for example, dilaton) in the extremal black string spacetime [19]. This is similar to the Higgs mechanism for gauge fields in the Minkowski spacetime. Therefore this graviton is not traveling along the horizon but becomes purely outgoing in the near-horizon. This is longitudinal wave moving along the string. It is important to note that this wave propagates indefinitely without radiating to infinity or falling into the horizon.

Further we review briefly how these traveling waves can be generated. This is found from the technique of Garfinkle-Vachaspati [20]. Starting from a known static solution, this produces a new solution representing waves traveling on the old black string background. However, it requires that the background metric ($\bar{g}_{MN}$) possess a null, orthogonal Killing vector ($k^M = (\partial/\partial v)^M$). As an example, if ($\bar{g}_{MN}, \bar{H}_{MNP}$) is also an exact solution to the
6D Einstein-Maxwell equations, then \((\bar{g}_{MN} + h_{MN}, \bar{H}_{MNP})\) is also an exact solution to this system \([19]\). Here \(h_{MN} = hk_{M}k_{N}\) with \(h \bar{g}_{vu} = p(u)/r^2\). This method was originally proposed for the Yang-Mills-Higgs system coupled to gravity and then applied to the low energy limit of string theory \([21]\).

The Bekenstein-Hawking entropy for a 5D black hole and the EBTZ black hole can also be reproduced by this picture \([9]\), provided only that the distribution function \(p(u)\) does not vary too rapidly: \(p^{3/2} \gg r_0^2 |\dot{p}|\), where the dot means \(d/du\). The geometry of the near-horizon for the 10D spacetime is

\[
d\tilde{s}^2_{10D} = \frac{r^2}{R^2} \left[-du^2 + \frac{p(u)}{r^2} \, du^2 + \frac{R^4}{r^4} \, dr^2\right] + R^2 d\Omega^2_3 + dy_i^2
\]

with \(y^i\) on the \(T^4\). We note that this is neither \(AdS_3 \times S^3 \times T^4\) nor \((AdS_3/Z_N) \times S^3 \times T^4\). In order to rewrite this as \(AdS_3 \times S^3 \times T^4\), we must introduce a function \(\sigma\), periodic in \(u\) which is related to \(p(u)\) by \(\sigma^2 + \dot{\sigma} = p/r_0^4\). Then the near-horizon geometry can be described by \(AdS_3 \times S^3 \times T^4\) and the horizon area is given by \(A_{10} = 2\pi^2 r_0^2 V \int_0^L \sigma(u) du\). If \(p^{3/2} \gg r_0^2 |\dot{p}|\), one has \(\sigma = \sqrt{p/r_0^4}\). Its Bekenstein-Hawking entropy is

\[
S_{BH}^{10D} = \frac{A_{10}}{4G_{10}} = \sqrt{2\pi Q_1 Q_5} \int_0^L \sqrt{\frac{p(u)}{\kappa^2}} du.
\]

If one fixes the total momentum, the horizon area is maximized by the uniform distribution of \(p = \text{constant}\). Then one finds a homogeneous (translationally invariant) black string. In this case we have \(p/\kappa^2 = P/L = 2\pi N/L^2\) and thus \((13)\) reduces to the familiar form of \(S_{BH}^{5D} = 2\pi \sqrt{Q_1 Q_5 N} = S_{BH}^{EBTZ}\). On the other hand, fixing the momentum distribution \(p(u)\), one finds the form of \((13)\).

We wish to count BPS states. All of string fields should have the same momentum distribution as in the black string: \(p^{3/2} \gg r_0^2 |\dot{p}|\). Quantum mechanically, however, one cannot fix the momentum distribution \(p(u)\) of string fields exactly in D-string theory. This is so because \(p(u)\) and \(p(u')\) do not commute and thus its Fourier modes satisfy the Virasoro algebra. Instead one introduces a mesoscopic length scale \(\ell_m \ll L\). One divides the spacetime into \(X = L/\ell_m\) intervals \(\Delta_a(a \in \{1, \cdots, X\})\). Then the total momentum in the
interval \( \Delta_a \) is \( P_a = \kappa^{-2} \int_{\Delta_a} p(u)du \) and it is assumed to be constant. If one could view states on \( S^1 \) as consisting of a collection of \( X \)-independent systems of length \( \ell_m \), the number of states for \( \tilde{c} = 6Q_1Q_5 \) string fields with total momentum \( P_a = 2\pi N_a / \ell_m \) on the \( a \)-th interval would be \( e^{S_a} \) with \( S_a = 2\pi \sqrt{\tilde{c} N_a / 6} = \sqrt{2\pi Q_1Q_5 P_a / \ell_m} \). Here \( N_a \gg Q_1Q_5 \), because of \( p/|\dot{p}| \gg \ell_m \gg \sqrt{Q_1Q_5 \kappa^2 / p} \). The entropy of \( X \)-independent systems is additive so the total entropy with \( P_1, \cdots, P_X \) is given by

\[
S_{CFT}^p = 2\pi \sqrt{Q_1Q_5} \sum_{a=1}^X \sqrt{N_a} = \sqrt{2\pi Q_1Q_5} \int_0^L \sqrt{\frac{P}{\kappa^2}} du
\]  

which leads to (13).

We note again that on the black string side the traveling waves play an important role in transferring the information of the horizon into the remote boundary at \( r = R \). The AdS/CFT correspondence means that the semiclassical limit of spacetime physics (a 6D black string with \( p(u) \)) is related to the large \((Q_1, Q_5, p(u)/\kappa^2)\) limit of the CFT(D-string). We propose here that a necessary condition for realizing the AdS/CFT correspondence is the presence of a 6D black string with traveling waves \( p(u) \). On the D-brane side, then we have the following picture. One finds \( 4Q_1Q_5 \) bosonic fields and an equal number of fermionic fields on the circle\((S^1)\). These degrees of freedom \((6Q_1Q_5 \) bosonic fields\) are not changed as the coupling constant moves from the weak to the strong. One increases the string coupling \( g_s \) to arrive at the near-horizon region and finally to form an event horizon at \( r = 0 \). But all of string fields should have the same momentum distribution as in the black string: \( p^{3/2} \gg r_0^2 |\dot{p}| \).

It is reasonable from the previous discussion to consider that the CFT of D-string with a central charge \( \tilde{c} = 6Q_1Q_5 \) is well-defined on the remote boundary. Also we can calculate the microscopic entropy at strong coupling. Considering (3) together with \( \tilde{c} = 6Q_1Q_5 \) and the large \( p(u)/\kappa^2 \) limit \((N_a \gg Q_1Q_3)\), one recovers \( S_{CFT} = \sqrt{2\pi Q_1Q_5} \int_0^L \sqrt{p/\kappa^2} du \). As a result of \( S_{BH}^{10D} = S_{CFT} \), the AdS/CFT correspondence for the entropy is established on \( AdS_3 \times S^3 \times T^4/CFT_2 \).
IV. DISCUSSION

We have shown that a realization of the AdS/CFT correspondence for the entropy is possible when the BTZ black hole is embedded in the string theory (explicitly, a 5D black hole). To recover $S_{\text{BH}}^{\text{BTZ}} = 2\pi \rho_0 / 4G_3$, we need both the central charge $c = 3\ell / 2G_3$ and the level number $N_K = \rho_0^2 / 4G_3 \ell$. However, the pure 3D gravity provides us with $c = 1$ (not $c = 3\ell / 2G_3$). This is too small to obtain the Bekenstein-Hawking entropy. This is so because the 3D gravity is a pure gauge theory and examines only its macroscopic properties. Thus the correspondence between an exact AdS$_3$ and CFT$_2$ is not confirmed until now. On the other hand, when the BTZ black hole is embedded in a 5D black hole, one finds $\tilde{c} = 3\tilde{\ell} / 2\tilde{G}_3 = 6Q_1Q_5$. This is exactly the sum of $4Q_1Q_5$-bosonic string fields and $4Q_1Q_5$-fermionic string fields on the D-brane side. The total momentum number $N_K$, in the extremal case, is given by $L_0 = J = N_K$ and $\bar{L}_0 = 0$, where $L_0$ and $\bar{L}_0$ are the generators of Virasoro algebra. However, this algebra is a realization of the asymptotic symmetry group of an exact AdS$_3$ at infinity ($\rho = \infty$) [12]. In general, the AdS/CFT correspondence is well-defined on the remote boundary at $\tilde{\rho} = \sqrt{\tilde{\rho}_0^2 + R^2}$. Hence we need the other mechanism which can transfer the information of the horizon ($\tilde{\rho} = \tilde{\rho}_0$ : inner boundary) to the outer boundary ($\tilde{\rho} = \sqrt{\tilde{\rho}_0^2 + R^2}$). As far as we know, this exists only when the background geometry which interpolates between AdS$_3$ and flat space has a null Killing symmetry. The extremal 6D black string solution has such a symmetry and thus one adds traveling waves with $p(u)$ to this solution. Then they transfer physical information (momentum distribution) from the horizon to the remote boundary. This is evident from the propagation of the graviton $p(u)$ in the extremal black string background. Also this comes from the fact that the non-AdS of (12) with $p(u)$ can be transformed to AdS without $p(u)$. This implies that the near-horizon geometry is completely independent of the wave profile $p(u)$. Hence we suggest that a necessary condition for realizing the AdS/CFT correspondence is the presence of a 6D black string with traveling waves $p(u)$. Here $p(u)$ seems to be a messenger which communicates the information of the bulk (black string) to the boundary (CFT). It plays an important role.
in counting the entropy, but not in the calculation of the greybody factor.

As a result, we provide a concrete model which shows a realization of the AdS/CFT correspondence for the entropy in the near-horizon. The other realization of $\text{AdS}_3/\text{CFT}_2$ correspondence was studied by using a 3D black string in Ref. [22].

ACKNOWLEDGEMENT

This work was supported in part by the Basic Science Research Institute Program, Ministry of Education, Project NO. BSRI-98-2413 and grant from Inje University, 1998.
REFERENCES

[1] J. Maldacena, Adv. Theor. Math. Phys. 2, 231(1998), hep-th/9711200.

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, Phys. Lett. B428, 105(1998), hep-th/9802109.

[3] E. Witten, Adv. Theor. Math. Phys. 2, 253(1998), hep-th/9802150.

[4] H.W. Lee and Y.S. Myung, Phys. Rev. D58, 104013(1998), hep-th/9804095; H.W. Lee, N.J. Kim, and Y.S. Myung, hep-th/9805050.

[5] K. Skenderis, hep-th/9901050.

[6] D.B. Birmingham, I. Sachs and S. Sen, Phys. Lett. B413, 281(1997), hep-th/9707188; E. Teo, hep-th/9805014; H.W. Lee and Y.S. Myung, hep-th/9808002.

[7] H.W. Lee, N.J. Kim, and Y.S. Myung, Phys. Rev. D58, 084022(1998), hep-th/9803080; Phys. Lett. B441, 83(1998), hep-th/9803227.

[8] A. Strominger and C. Vafa, Phys. Lett. B379, 99(1996).

[9] G. Horowitz and D. Marolf, hep-th/9605224; hep-th/9606113.

[10] D. Kutasov and N. Seiberg, Nucl. Phys. B358, 600(1991); O. Coussaert, M. Henneaux, and P. van Driel, Class. Quant. Grav. 12, 2961(1995); S. Carlip, hep-th/9806026; Y.S. Myung, Phys. Rev. D59, 044028(1999), hep-th/9809059.

[11] E. Martinec, hep-th/9809021.

[12] J.D. Brown and M. Henneaux, Commun. Math. Phys. 104, 207(1986).

[13] N. Seiberg, Prog. Theor. Phys. Suppl. B102, 319(1990); A. Strominger, hep-th/9712251; M. Banados, T. Brotz, and M. Ortiz, hep-th/9802070; K. Behrnt, I. Brunner, and I. Gaida, hep-th/9806195; A. Giveon, D. Kutasov, and N. Seiberg, hep-th/9806194.

[14] J. Maldacena and A. Strominger, hep-th/9804085.
[15] Y. Satoh, hep-th/9810135.

[16] M. Taylor-Robinson, hep-th/9806132; M. Cvetič, H.Lü, C.N. Pope, and T.A. Tran, hep-th/9901002.

[17] K. Behrndt, hep-th/9809015; hep-th/9812169.

[18] M.J. Duff, H. Lü, and C.N. Pope, hep-th/9807173.

[19] H.W. Lee, Y.S. Myung, J.Y. Kim, and D.H. Park, Mod. Phys. Lett. A12, 545(1997); Mod. Phys. Lett. A13, 701(1998).

[20] D. Garfinkle and T. Vachaspati, Phys. Rev. D42, 1960(1990); D. Garfinkle, Phys. Rev. D46, 4286(1992).

[21] R. Cartas-Fuentevilla and A. Hernadez-Castillo, Phys. Rev. D58, 104019(1998).

[22] N. Kaloper, hep-th/9804062.