Cartesian products of the $g$-topologies are a $g$-topology

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Abstract

We show that unlike the usual topologies the $g$-topologies are closed with respect to the Cartesian products. Moreover, we bring much detailed explanations some examples of concepts related the statistical metric spaces.

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1 Introduction

As mentioned in [1] the three principal applications of statistical metrics are to macroscopic, microscopic and physiological spatial measurements. Statistical metrics are designed to provide us firstly with a method removing conceptual difficulties from microscopic physics and transferring them into the underlying geometry, secondly with a treatment of thresholds of spatial sensation eliminating the intrinsic paradoxes of the classical theory.

The notion of distance is defined in terms of functions, points and sets. Indeed, in many situations, it is appropriate to look upon the distance concept as a statistical rather than a determinate one. More precisely, instead of associating a number to the distance $d(p,q)$ with every pair of points $p$, $q$, one should associate a distribution function $F_{pq}$ and for any positive number $x$, interpret $F_{pq}(x)$ as the probability that the distance from $p$ to $q$ be less than $x$.

Using this idea, K. Menger in [1] defined a statistical metric space using the probability function in the year 1942. In 1943, shortly after the appearance of Mengers article, Wald published an article [2] in which he criticized Mengers generalized triangle inequality.

In [3] the following questions raised by Thorp in statistical metric spaces:

- What are the necessary and sufficient conditions that the $g$-topology of type $V$ to be of type $V_D$?

- What are the necessary and sufficient conditions that the $g$-topology of type $V_\alpha$ to be the $g$-topology of type $V_D$?

- What conditions are both necessary and sufficient for the $g$-topology of type $V_\alpha$ to be a topology?

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In [4] it had given partial answer to the above questions. Also, it was provided the basis for carrying out analysis in statistical metric spaces, in particular for the development of various $g$-topologies, neighborhoods defined in a statistical metric space and also the improvement of $\lambda_\Omega$-open sets in a generalized metric space. The authors of [4] had given more examples of the neighborhoods defined in a statistical metric space and the special kind of relationship between various $g$-topologies defined by Thorp in a $SM$ space. We seem that the examples have shortcomings. That is why we in the present paper completed the shortcomings and give more detail clarifies.

Further, we give positive answers to the question, expressed in [5], which asks: Is the Cartesian product of the $g$-topological spaces a $g$-topological space?

2 Preliminaries

A statistical metric space ($SM$ space) is an ordered pair $(S, F)$ where $S$ is a non-null set and $F$ is a mapping from $S \times S$ into the set of distribution functions (that is, real-valued functions of a real variable which are everywhere defined, non decreasing, left-continuous and have infimum 0 and supremum 1).

The distribution function $F(p, q)$ associated with a pair of points $p$ and $q$ in $S$ is denoted by $F_{pq}$. Moreover, $F_{pq}(x)$ represents the probability that the “distance” between $p$ and $q$ is less than $x$.

The functions $F_{pq}$ are assumed to satisfy the following:

\begin{align*}
(SM-I) & \quad F_{pq}(x) = 1 \text{ for all } x > 0 \text{ if and only if } p = q. \\
(SM-II) & \quad F_{pq}(0) = 0. \\
(SM-III) & \quad F_{pq} = F_{qp}. \\
(SM-IV) & \quad \text{If } F_{pq}(x) = 1 \text{ and } F_{qr}(y) = 1, \text{ then } F_{pr}(x + y) = 1.
\end{align*}

We often find it convenient to work with the tails of the distribution functions rather than with these distribution functions themselves. Then the tail of $F_{pq}$, denoted by $G_{pq}$, is defined by $G_{pq}(x) = 1 - F_{pq}(x)$ for all $x \in \mathbb{R}$.

Let $(S, F)$ be a statistical metric space. Then the Menger inequality is,

\begin{align*}
(SM-IVm) & \quad F_{pr}(x + y) \geq T(F_{pq}(x), F_{qr}(y))
\end{align*}

holds for all points $p, q, r \in S$ and for all numbers $x, y \geq 0$ where $T$ is a 2-place function on the unit square satisfying:

\begin{align*}
(T-I) & \quad 0 \leq T(a, b) \leq 1 \text{ for all } a, b \in [0, 1]. \\
(T-II) & \quad T(c, d) \geq T(a, b) \text{ for all } a, b, c, d \in [0, 1] \text{ such that } c \geq a, d \geq b \text{ (monotonicity).} \\
(T-III) & \quad T(a, b) = T(b, a) \text{ for all } a, b \in [0, 1] \text{ (commutativity).} \\
(T-IV) & \quad T(1, 1) = 1. \\
(T-V) & \quad T(a, 1) > 0 \text{ for all } a > 0.
\end{align*}
Let \((S, \ F)\) be a statistical metric space, \(p \in S\) and \(u, v\) be positive numbers. Then

\[
N_p(u, v) = \{q \in S : F_{pq}(u) > 1 - v\} = \{q \in S : G_{pq}(u) < v\}
\]

is called the \((u, v)\)-sphere with the center \(p\).

The following example shows the existence of \((u, v)\)-sphere in a statistical metric space.

**Example 1** Consider the \(SM\) space \((S, \ F)\) where \(S\) denotes the possible outcomes of getting a tail when a coin is tossed once. Then \(S = \{0, 1\}\). Let \(F_{pq}(u)\) be the probability that the “distance” between \(p\) and \(q\) is less than \(u\) where \(u > 0\) and \(p, q \in S\). We have (A. A. Zaitov):

\[
F_{00}(u) = F_{11}(u) = 1 \quad \text{for all } u > 0; \quad \text{and}
\]

\[
F_{01}(u) = F_{10}(u) = \begin{cases} 
0, & \text{if } 0 < u \leq 1, \\
1, & \text{if } u > 1.
\end{cases}
\]

Fix \(p = 0\). Then

\[
N_0(u, v) = \begin{cases} 
\{0\}, & \text{if } 0 < u \leq 1, 0 \leq v \leq 1, \\
\{0, 1\}, & \text{if } 0 < u \leq 1, v > 1, \\
\{0, 1\}, & \text{if } u > 1, v \geq 0.
\end{cases}
\]

Fix \(p = 1\). Then

\[
N_1(u, v) = \begin{cases} 
\{1\}, & \text{if } 0 < u \leq 1, 0 \leq v \leq 1, \\
\{0, 1\}, & \text{if } 0 < u \leq 1, v > 1, \\
\{0, 1\}, & \text{if } u > 1, v \geq 0.
\end{cases}
\]

For fixed positive numbers \(u\) and \(v\), define a set

\[
U(u, v) = \{(p, q) \in S \times S : G_{pq}(u) < v\}.
\]

Let us illustrate such sets in the following example.

**Example 2** Consider the \(SM\) space \((S, \ F)\) where \(S\) denotes the possible outcomes of rolling a dice. Then \(S = \{1, 2, 3, 4, 5, 6\}\) and the distribution function \(F_{pq}(x)\) is the probability that the “distance” between \(p\) and \(q\) is less than \(u\) where \(u > 0\) and \(p, q \in S\). Consider the usual metric on \(S\) induced on the real line, i. e. \(d(p, q) = |q - p|\). We have (A. A. Zaitov):
\[ F_{pp}(u) = 1 \text{ for all } u > 0 \text{ and } p = 1, 2, 3, 4, 5, 6; \]
\[ F_{p(p+1)}(u) = F_{(p+1)p}(u) = \begin{cases} 0, & 0 < u \leq 1, \\ 1, & u > 1, \end{cases} \text{ where } p = 1, 2, 3, 4, 5; \]
\[ F_{p(p+2)}(u) = F_{(p+2)p}(u) = \begin{cases} 0, & 0 < u \leq 2, \\ 1, & u > 2, \end{cases} \text{ where } p = 1, 2, 3, 4; \]
\[ F_{p(p+3)}(u) = F_{(p+3)p}(u) = \begin{cases} 0, & 0 < u \leq 3, \\ 1, & u > 3, \end{cases} \text{ where } p = 1, 2, 3; \]
\[ F_{p(p+4)}(u) = F_{(p+4)p}(u) = \begin{cases} 0, & 0 < u \leq 4, \\ 1, & u > 4, \end{cases} \text{ where } p = 1, 2; \]
\[ F_{p(p+5)}(u) = F_{(p+5)p}(u) = \begin{cases} 0, & 0 < u \leq 5, \\ 1, & u > 5, \end{cases} \text{ where } p = 1. \]

Consequently:
for every pair of \( u, v \) such that \( 0 < u \leq 1, 0 < v \leq 1 \) we have:
\[ U(u, v) = \{(p, q) \in S \times S : d(p, q) = 0\} = \{(p, p) : p \in S\} = \Delta(S); \]
for every pair of \( u, v \) such that \( 1 < u \leq 2, 0 < v \leq 1 \):
\[ U(u, v) = \{(p, q) \in S \times S : d(p, q) \leq 1\} = \Delta(S) \cup \{(p, p + 1) : p = 1, 2, 3, 4, 5\} \cup \{(p + 1, p) : p = 1, 2, 3, 4, 5\} \overset{\text{def}}{=} \Delta_1(S); \]
for every pair of \( u, v \) such that \( 2 < u \leq 3, 0 < v \leq 1 \):
\[ U(u, v) = \{(p, q) \in S \times S : d(p, q) \leq 2\} = \Delta_1(S) \cup \{(p, p + 2) : p = 1, 2, 3, 4\} \cup \{(p + 2, p) : p = 1, 2, 3, 4\} \overset{\text{def}}{=} \Delta_2(S); \]
for every pair of \( u, v \) such that \( 3 < u \leq 4, 0 < v \leq 1 \):
\[ U(u, v) = \{(p, q) \in S \times S : d(p, q) \leq 3\} = \Delta_2(S) \cup \{(p, p + 3) : p = 1, 2, 3\} \cup \{(p + 3, p) : p = 1, 2, 3\} \overset{\text{def}}{=} \Delta_3(S); \]
for every pair of \( u, v \) such that \( 4 < u \leq 5, 0 < v \leq 1 \):
\[ U(u, v) = \{(p, q) \in S \times S : d(p, q) \leq 4\} = \Delta_3(S) \cup \{(p, p + 4) : p = 1, 2\} \cup \{(p + 4, p) : p = 1, 2\} \overset{\text{def}}{=} \Delta_4(S); \]
for every pair of \( u, v \) such that \( u > 5, 0 < v \leq 1 \):
\[ U(u, v) = \{(p, q) \in S \times S : d(p, q) \leq 5\} = \Delta_4(S) \cup \{(p, p + 5) : p = 1\} \cup \{(p + 5, p) : p = 1\} = S \times S. \]

Also, it is easy to see that \( U(u, v) = S \times S \) for every \( u, v \) such that \( u > 0, v > 1 \).
For any set \( Z \) of ordered pairs of positive numbers, i. e. \( Z \subset (0, +\infty)^2 \), let
\[
\mathcal{N}(Z) = \{N_p(u, v) : (u, v) \in Z, \ p \in S\} \quad \text{and} \quad \mathcal{U}(Z) = \{U(u, v) : (u, v) \in Z\}.
\]

A non-null collection \( \{N_p\} \) of subsets \( N_p \) of a set \( S \) associated with a point \( p \in S \) is a family of neighborhoods for \( p \) if each \( N_p \) contains \( p \). Let the family of neighborhoods be associated with each point \( p \) of a set \( S \).

\( V \) In this case the set \( S \) and the collection of neighborhoods is called the \( g \)-topological space of type \( V \) [3].

The closure of a subset \( E \) of \( S \), written \( \overline{E} \), is the set of points \( p \) such that each neighborhood of \( p \) intersects \( E \). The interior of \( E \) is the complement of the closure of the complement of \( E \). A \( g \)-topological space \( S \) is symmetric if, for every pair of points \( p \) and \( q \), \( p \) is in \( \{q\} \) iff \( q \) is in \( \{p\} \).

E. Thorp introduced the following \( g \)-topologies in a statistical metric space \((S, F)\).

- \( N_0 \) is type \( V \).
- \( N_1 \). For each point \( p \) and each neighborhood \( U_p \) of \( p \), there is a neighborhood \( W_p \) of \( p \) such that for each point \( q \) of \( W_p \), there is a neighborhood \( U_q \) of \( q \) contained in \( U_p \).
- \( N_2 \). For each point \( p \) and each pair of neighborhoods \( U_p \) and \( W_p \) of \( p \), there is a neighborhood of \( p \) contained in the intersection of \( U_p \) and \( W_p \).

The following are various \( g \)-topologies in a statistical metric space \((S, F)\) defined by E. Thorp.

- \( V_D \). If the conditions \( N_0 \) and \( N_2 \) are satisfied, then the collection of neighborhoods on \( S \) is called the \( g \)-topology of type \( V_D \).
- \( V_\alpha \). The collection of neighborhoods on \( S \) is called the \( g \)-topology of type \( V_\alpha \) if the conditions \( N_0 \) and \( N_1 \) are satisfied.

\( Top \). A \( g \)-topology is a topology if the conditions \( N_0, N_1 \) and \( N_2 \) are satisfied.

Let \( S \) be a set and \((P, <)\) be a partially ordered set with least element 0. A generalized \( \acute{e} \)cart (\( g \)-\( \acute{e} \)cart for short) is a mapping
\[
G : S \times S \to P.
\]

If a \( g \)-\( \acute{e} \)cart \( G \) satisfies \( G(p, p) = 0 \) and the set \( S \) consists of more than one point, the \( g \)-\( \acute{e} \)cart \( g \)-topology for \( S \) is the \( g \)-topology determined from \( G \), and its partially ordered range set \( P \), as follows:

For each \( f > 0 \) in \( P \) and each \( p \in S \), the \( f \)-sphere for \( p \) is a set of the form
\[
N_p(f) = \{q \in S : G(p, q) < f\}.
\]
Then for each \( p \in S \), the collection of \( f \)-spheres
\[
\mathcal{N}_p(P) = \{N_p(f) : f > 0, p \in P\}
\]
is a family of neighborhoods for \( p \).

The \( g \)-\( \acute{e} \)cart associated with a statistical metric space \((S, F)\) is the mapping \( G \) defined by \( G(p, q) = G_{pq} \).
**Example 3** Let $S = \mathbb{N}$ and $P = \mathbb{N} \cup \{0\}$ be a partially ordered set with the relation $<$ where $\mathbb{N}$ denote the set of all positive integers. Let $A = \{1, 2, 3\}$ be a subset of $S$. Define

$$G(p, q) = \begin{cases} 
1, & \text{if } p \notin A, q \in S, \\
1, & \text{if } p \in A, q \notin S, \\
0, & \text{if } p \notin A, q \notin S.
\end{cases}$$

and for $p \in A, q \in A$ define $G(p, q)$ as follows:

$$G(1, 1) = 0, \quad G(1, 2) = 2, \quad G(1, 3) = 3, \quad G(2, 1) = 4, \quad G(2, 2) = 0, \quad G(2, 3) = 6, \quad G(3, 1) = 1, \quad G(3, 2) = 2, \quad G(3, 3) = 0.$$

The $f$-sphere for each $p \in S$ has the following form (A. A. Zaitov):

**Case** $p = 1$:

$$N_1(f) = \begin{cases} 
\emptyset, & \text{if } f = 0, \\
\{1\}, & \text{if } f = 1, \\
S \setminus \{2, 3\}, & \text{if } f = 2, \\
S \setminus \{3\}, & \text{if } f = 3, \\
S, & \text{if } f \geq 4.
\end{cases}$$

**Case** $p = 2$:

$$N_2(f) = \begin{cases} 
\emptyset, & \text{if } f = 0, \\
\{2\}, & \text{if } f = 1, \\
S \setminus \{1, 3\}, & \text{if } 2 \leq f \leq 4, \\
S \setminus \{3\}, & \text{if } 5 \leq f \leq 6, \\
S, & \text{if } f \geq 7.
\end{cases}$$

**Case** $p = 3$:

$$N_3(f) = \begin{cases} 
\emptyset, & \text{if } f = 0, \\
\{3\}, & \text{if } f = 1, \\
S \setminus \{2\}, & \text{if } f = 2, \\
S, & \text{if } f \geq 3.
\end{cases}$$

**Case** of arbitrary $p \in S \setminus A$:

$$N_p(f) = \begin{cases} 
\emptyset, & \text{if } f = 0, \\
S \setminus A, & \text{if } f = 1, \\
S, & \text{if } f \geq 2.
\end{cases}$$

Given a statistical metric space $(S, F)$, for each pair of points $p$ and $r$ in $S$, number $u > 0$, the $r$-sphere with center $p$, $N_p(r; u)$ is defined to be the sphere

$$N_p(r; u) = N(G_{pr}(u)) = \{ q : G_{pq}(u) < G_{pr}(u) \}.$$
The $R\cdot g$-topology for $(S, F)$ is the structure whose family of neighborhoods at each point $p$ is the collection

$$\mathcal{N}_p(r) = \{ N_p(r; u) : r \in S, u > 0 \}.$$ 

**Example 4** Consider the $SM$ space $(S, F)$ where $S = \mathbb{N}$ and the distribution function

$$F_{pq}(x) = \begin{cases} \frac{x}{d(p, q)}, & \text{if } 0 < x < d(p, q), \; d(p, q) \neq 0, \\ 0, & \text{if } x = d(p, q), \\ 1, & \text{if } x \geq d(p, q). \end{cases}$$

where $d(p, q) = |q - p|$, $p, q \in S$.

Fix $p = 1$ and $r = 2$ from $S$. Let $x = \frac{1}{4}$. Then $G_{pr}(x) = G_{12}(\frac{1}{4}) = 1 - F_{12}(\frac{1}{4}) = 0.75$. We have (A. A. Zaitov): $F_{11}(\frac{1}{4}) = 1 > 0.25$ and $F_{1q}(\frac{1}{4}) = \frac{1}{4}(q - 1) \leq 0.25$ for every $q \geq 2$. That is why

$$N_1\left(2; \frac{1}{4}\right) = N\left(G_{12}\left(\frac{1}{4}\right)\right) = \{q \in S : G_{1q}\left(\frac{1}{4}\right) < 0.75\} = \{q \in S : F_{1q}(\frac{1}{4}) > 0.25\} = \{1\}.$$ 

Note that $N(G_{pr}) = \emptyset$ if $p = r$. Really, for every $u > 0$ and $p \in S$ one has $F_{pp}(u) = 1$, consequently, $G_{pp}(u) = 0$. Since $0 \leq G_{pq}(u) \leq 1$ for all $u > 0$, $p, q \in S$, there exists no $q \in S$ such that $G_{pq}(u) < G_{pp}(u)$. Hence $N(G_{pp}) = \emptyset$, $p \in S$.

**Remark 1** In a $SM$ space $(S, F)$, we use the following notations:

(a) Let $\tau$ denote the $g$-topology of type $V$.

(b) Let $\tau_D$ denote the $g$-topology of type $V_D$.

(c) Let $\tau_\alpha$ denote the $g$-topology of type $V_\alpha$.

(d) Let $\tau_e$ denote the $g$-écart $g$-topology.

(e) Let $\tau_R$ denote the $R\cdot g$-topology.

(f) Each element in $\mathcal{N}(X)$ is called a $\tau$-neighborhood.

(g) Each element in $\mathcal{N}_p(P)$ is called a $\tau_e$-neighborhood.

(h) Each element in $\mathcal{N}_p(r)$ is called a $\tau_R$-neighborhood.

## 3 Main part

Let $(S^1, F^1)$ and $(S^2, F^2)$ be statistical metric spaces, $(p^1, p^2)$, $(q^1, q^2)$, $(r^1, r^2) \in S^1 \times S^2$. Put

$$F_{(p^1, p^2)(q^1, q^2)}^{12}(x) = F_{p^1, q^1}^1(x) \cdot F_{p^2, q^2}^2(x). \tag{1}$$

Obviously, conditions $(SM-I)$ - $(SM-III)$ are true. Let us show $(SM-IV)$ is also satisfied. Assume that $F_{(p^1, p^2)(q^1, q^2)}^{12}(x) = 1$ and $F_{(q^1, q^2)(r^1, r^2)}^{12}(y) = 1$. These equalities mean that the
“distance” between \((p^1, p^2)\) and \((q^1, q^2)\) less than \(x\), and the “distance” between \((q^1, q^2)\)
and \((r^1, r^2)\) less than \(y\). Then clearly, that the “distance” between \((p^1, p^2)\) and \((r^1, r^2)\)
less than \(x + y\). Consequently, the probability that the “distance” between \((p^1, p^2)\) and
\((r^1, r^2)\) is less than \(x + y\), is 1, i. e. \(F_{(p^1, p^2)(r^1, q^2)}(x + y) = 1\).

So, \(H\) is defined correctly. Its tail

\[
G_{(p^1, q^1)(p^2, q^2)}^{12}(x) = 1 - F_{(p^1, q^1)(p^2, q^2)}^{12}(x) \quad \text{for all} \quad x \in \mathbb{R}.
\]

Now we check the Menger inequality. Let \(T(a, b) = ab\), \(a, b \in [0, 1]\). Clearly, it
satisfies conditions \((T-I) - (T-V)\). Suppose

\[
F_{p^1+q^1}^{1}(x + y) \geq T(F_{p^1,q^1}^{1}(x), F_{q^1+r^1}^{1}(y)) \quad \text{and} \quad F_{p^2+q^2}^{2}(x + y) \geq T(F_{p^2,q^2}^{2}(x), F_{q^2+r^2}^{2}(y)).
\]

Then

\[
F_{(p^1, p^2)(r^1, r^2)}^{12}(x + y) = F_{p^1+q^1}^{1}(x + y) \cdot F_{p^2+q^2}^{2}(x + y) \geq
\]
\[
\geq T(F_{p^1,q^1}(x), F_{q^1+r^1}(y)) \cdot T(F_{p^2,q^2}(x), F_{q^2+r^2}(y)) =
\]
\[
= F_{p^1,q^1}(x) \cdot F_{q^1+r^1}(y) \cdot F_{p^2,q^2}(x) \cdot F_{q^2+r^2}(y) =
\]
\[
= F_{(p^1, p^2)(q^1, q^2)}^{12}(x) \cdot F_{(q^1, q^2)(r^1, r^2)}^{12}(y) =
\]
\[
= T(F_{(p^1, p^2)(q^1, q^2)}^{12}(x), F_{(q^1, q^2)(r^1, r^2)}^{12}(y)),
\]
i. e.

\[
F_{(p^1, p^2)(r^1, r^2)}^{12}(x + y) \geq T(F_{(p^1, p^2)(q^1, q^2)}^{12}(x), F_{(q^1, q^2)(r^1, r^2)}^{12}(y)).
\]

Now we claim the following statements the proofs each of them consists just directly
verification.

**Theorem 1** Let \((S^1, \tau^1)\) and \((S^2, \tau^2)\) be \(g\)-topological spaces of type \(V\). Then \(\tau^1 \times \tau^2\)
is a \(g\)-topology of type \(V\) on \(S_1 \times S_2\).

**Theorem 2** Let \((S^1, \tau^1_D)\) and \((S^2, \tau^2_D)\) be \(g\)-topological spaces of type \(V_D\). Then \(\tau^1_D \times \tau^2_D\)
is a \(g\)-topology of type \(V_D\) on \(S_1 \times S_2\).

**Theorem 3** Let \((S^1, \tau^1_\alpha)\) and \((S^2, \tau^2_\alpha)\) be \(g\)-topological spaces of type \(V_\alpha\). Then \(\tau^1_\alpha \times \tau^2_\alpha\)
is a \(g\)-topology of type \(V_\alpha\) on \(S_1 \times S_2\).

**Theorem 4** Let \((S^1, \tau^1_e)\) and \((S^2, \tau^2_e)\) be \(g\)-écart \(g\)-topological spaces. Then \(\tau^1_e \times \tau^2_e\) is a
\(g\)-écart \(g\)-topology on \(S_1 \times S_2\).

**Theorem 5** Let \((S^1, \tau^1_e)\) and \((S^2, \tau^2_e)\) be \(R\)-\(g\)-topological spaces. Then \(\tau^1_e \times \tau^2_e\) is a \(R\)-\(g\)-
topology on \(S_1 \times S_2\).

**Remark 2** Note that the Cartesian product of topologies must not be a topology, i. e. usual
topology does not close under product. Unlike usual topology, \(g\)-topologies are closed with respect to product.
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