Inapproximability of actions and Kazhdan’s property (T)

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Abstract. We construct p.m.p. group actions that are not local-global limits of sequences of finite graphs. Moreover, they do not weakly contain any sequence of finite labeled graphs. Our methods are based on the study of almost automorphisms of sofic approximations: We show that the set of $\varepsilon$-automorphisms of a sufficiently good sofic approximation of a Kazhdan group by expanders form a group in a natural way.

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1. Introduction

A sequence of finite labeled graphs is locally convergent if for every $r$ the isomorphism class of a rooted $r$-ball centered at a vertex chosen uniformly at random converges in distribution. A finitely generated group is called sofic if any of its labeled Cayley graphs admits a sofic approximation, this is, a local approximation by finite labeled graphs. Sofic groups were introduced by Gromov [9], see also Weiss [18]. Quite a number of classical conjectures about groups and group rings not known in general are known to hold for the class of sofic groups: Gottschalk’s Conjecture [9], Kaplansky’s Direct Finiteness Conjecture [5, 7], Connes’ Embedding Conjecture [6], and the Kervaire-Laudenbach Conjecture [14] and its generalizations [11, 13].
For more on sofic groups see [14, 16]. It is a major open problem if every group is sofic, though it is widely believed that non-sofic groups exist. In general, we do not know much about sofic approximations. Schramm proved that the sofic approximation of an amenable group is hyperfinite [15]. On the other end of the spectrum, the first author proved Bowen’s conjecture that the sofic approximation of a Kazhdan’s Property (T) group is essentially a vertex disjoint union of expander graphs [12]. We will build on this work in order to understand almost automorphisms of sofic approximations of Kazhdan groups: This allows us to prove inapproximability results about the direct product of a Kazhdan group and another group which is not LEF. Every finitely presented LEF group is residually finite. The first main result of this paper is the following.

**Theorem 1.1.** Let $\Gamma$ be a countable Kazhdan group and $\Delta$ a finitely generated group. Let $S_\Gamma$ and $S_\Delta$ be finite generating sets of $\Gamma$ and $\Delta$. Consider a sofic approximation of $\Gamma \times \Delta$ with respect to the generating set $S_\Gamma \cup S_\Delta$. If the edges with labels in $S_\Gamma$ induce an expander sequence, then $\Delta$ is LEF. In particular, if $\Delta$ is finitely presented then it is residually finite.

**Remark 1.2.** Note that for every sofic approximation of $\Gamma$ the edges with labels in $S_\Gamma$ induce a graph that is essentially a vertex disjoint union of expander graphs. The theorem requires somewhat more, i.e., that it is (essentially) an expander graph.

As a consequence of Theorem 1.1, we show that certain group actions are not approximable by finite labeled graphs in the local-global sense [10], a notion of convergence induced by the colored neighborhood metric of Bollobás and Riordan [2]. Moreover, these actions do not weakly contain any ultra-product of a finite sequence of graphs, i.e., this half of the local-global convergence already fails.

**Theorem 1.3.** Let $\Gamma$ be a countable Kazhdan group and $\Delta$ a finitely generated group, which is not LEF. Consider an almost free, probability measure preserving action of $\Gamma \times \Delta$ on a probability measure space such that the restriction of the action to $\Gamma$ is ergodic. Then this action does not weakly contain any sequence of finite graphs. In particular, it is not a local-global limit of finite graphs.

The simplest example of such an action is the Bernoulli shift of $\Gamma \times \Delta$, this satisfies the conditions of the theorem.

**Proof of Theorem 1.3.** Consider a sofic approximation of $\Gamma \times \Delta$. We may assume that its edges are labeled by $S_\Gamma \cup S_\Delta$. Restricting labels to $S_\Gamma$ and using the main result of [12], we obtain a vertex disjoint union of expander graphs after making irrelevant changes to the labels, since it is a sofic approximation of $\Gamma$. Theorem 1.1 implies that it can not be an expander graph sequence. On the other hand, the action of $\Gamma$ is strongly ergodic, since it is ergodic and $\Gamma$ is a Kazhdan group. However, if
the graph sequence was weakly contained by a strongly ergodic action there would be only one large component. This is a contradiction.

2. Definitions

Throughout, a graph is denoted by $G = (V(G), E(G))$, where $V(G)$ is the set of vertices and $E(G)$ the set of undirected edges. We work with sequences $\{G_n\}_{n=1}^\infty$ of finite, undirected, $d$-regular graphs. The edge boundary of $S \subseteq V(G)$ is defined to be $\partial(S) = E(S, V(G) \setminus S)$. The Cheeger constant of the graph $G$ is

$$h(G) = \min_{S \subseteq V(G), |S| \leq |V(G)|/2} \frac{|\partial S|/|S|}{|\partial S|/|S|}.$$ 

We say that a sequence $\{G_n\}_{n=1}^\infty$ of finite, undirected, $d$-regular graphs is an expander sequence if there exists an $h > 0$ such that $h(G_n) \geq h$ holds for $n \in \mathbb{N}$.

Let $\Gamma$ denote a group generated by the finite symmetric set $S = S^{-1} \subseteq \Gamma$. We will consider $|S|$-regular graphs with an edge-labeling, where every edge will be labeled by an element of $S$ in a directed way, and the label of $(x, y)$ will be the inverse of the label of $(y, x)$ for any pair of vertices. We say that an $|S|$-regular graph $G$ is regularly $S$-labeled if for every vertex $x$ and $s \in S$ there exists a unique vertex $y$ such that $(x, y)$ is an edge labeled by $s$, and we write $y = sx$. We say that a sequence $\{G_n\}_{n=1}^\infty$ is a sofic approximation of $\Gamma$ if it is a sequence of regularly $S$-labeled graphs and for every $s_1, \ldots, s_k \in S$ and $\varepsilon > 0$ if $s_1 \ldots s_k = 1$ then $x = s_1 \ldots s_k x$ holds for all but at most an $\varepsilon$-proportion of the vertices if $n$ is large enough, while if $s_1 \ldots s_k \neq 1$ then $x \neq s_1 \ldots s_k x$ holds for all but at most an $\varepsilon$-proportion of the vertices if $n$ is large enough.

We say that a group is residually finite if it can be embedded into the direct product of finite groups. Residually finite groups are sofic. We say that a group $\Gamma$ is LEF (Locally Embeddable into Finite groups) if for any finite subset $F \subseteq \Gamma$ there exists a finite group $H$ and a mapping $\varphi : F \to H$ such that $\varphi(x)\varphi(y) = \varphi(xy)$ holds if $x, y \in F$. It is easy to see that a group is LEF if it is a subgroup of an ultraproduct of finite groups. Every finitely presented LEF group is residually finite.

We will study probability measure preserving (p.m.p. for short) almost free actions of groups on probability measure spaces. A p.m.p. action of a group $\Gamma$ generated by a finite set $S$ on a probability measure space $X$ is ergodic if for any measurable $A \subseteq X$, $\nu(A) \neq 0, 1$ there exists $s \in S$ such that $\nu(sA \setminus A) > 0$. It is strongly ergodic if there exists an $h > 0$ such that for any measurable $A \subseteq X$, $\nu(A) \leq 1/2$ there exists $s \in S$ such that $\nu(sA \setminus A) > h\nu(A)$. By a result of Connes and Weiss, a group has Kazhdan’s Property (T) if every ergodic action of the group is strongly ergodic. However, we will use a different definition of Kazhdan’s property (T) using the terminology of [12]. We say that the finitely generated group $\Gamma$ has Kazhdan Property (T) if there is a finite set of generators $S$ and a Kazhdan constant $\kappa > 0$,
such that for every Hilbert space $\mathcal{H}$ and $\pi: \Gamma \to U(\mathcal{H})$ a unitary representation of $\Gamma$, either $\pi$ has a non-zero invariant vector or for $A = \sum_{s \in S} \pi(S)/|S|$ the inequality $\|A\xi\| \leq (1 - \kappa)\|\xi\|$ holds for every $\xi \in \mathcal{H}$. See the book of Bekka, de la Harpe and Valette [1] for more on Property (T).

We will define the notion of weak containment in the spirit of Kechris [3] for sequences of (labeled) graphs. Given an integer $r$ and a graph $G$ we will consider the following probability distribution on isomorphism classes of rooted graphs: Pick a vertex uniformly at random and consider the isomorphism class of the rooted $r$-ball centered and rooted at $x$. Given a finite set of colors we can extend this to vertex-colored graphs and we call this the colored $r$-neighborhood statistics of the graph following Bollobás and Riordan [2]. An almost free p.m.p. action of a group $\Gamma$ generated by a finite set $S$ on a probability measure space $X$ weakly contains a finite sequence of $S$-regularly labeled graphs $\{G_n\}_{n=1}^{\infty}$ if for every $k, r$ and $\varepsilon > 0$ there exists $N$, such that if $n \geq N$ then for any $k$-coloring of $G_n$ there exists a measurable coloring of $X$ that is $\varepsilon$-close to the colored $r$-neighborhood statistics of the finite colored graph $G_n$.

3. Improving almost automorphism

We introduce the notion of $\varepsilon$-automorphism for some $\varepsilon > 0$, this is a key notion that we will use in the sequel.

**Definition 3.1.** Let $(V, E)$ be a finite edge-labeled graph $(V, E)$ and $\varepsilon > 0$. A mapping $c: V \to V$ is called a $\varepsilon$-almost automorphism, if there are at most $\varepsilon|V|$ edges, whose image under $c$ is not an edge with the same label.

Note that automorphisms are exactly the $0$-almost automorphisms. It is tempting to study the set of $\varepsilon$-automorphisms of a finite graph appearing in a sofic approximation of a group $\Gamma$. However, the product of two $\varepsilon$-automorphisms is in general only a $2\varepsilon$-automorphism and that is a problem that renders this idea rather useless unless there is some mechanism that allows us to improve the resulting $2\varepsilon$-automorphisms. This is exactly what the main theorem of this section achieves under additional assumptions on $\Gamma$.

Let $G$ denote a finite, simple, regularly $S$-labeled graph and let $b: V(G) \to V(G)$ be a map. The graph of $b$ will be denoted by $B = \{(x, b(x)) : x \in V(G)\} \subseteq V(G) \times V(G)$. Note that $G \times G$ is also a regularly $S$-labeled graph.

**Lemma 3.2.** Then, $b$ is an $\varepsilon$-automorphism if and only if $|\partial B| \leq 2\varepsilon|V(G)|$.

**Proof.** For every $s$-labeled edge $(x, y) \in E(G)$ the followings are equivalent:

1. $(b(x), b(y))$ is not an $s$-labeled edge (maybe not even an edge) of $V(G)$.
2. $s(x, b(x)) \notin B$
(3) $s^{-1}(y, b(y)) \notin B$

This gives a one-to-one correspondence between the bad edges of the bijection $b$ and pairs of edges on the boundary of $\partial B$. Here we use that $G$ is simple, hence $x \neq y$. The lemma follows. \hfill \Box

We will use the structural results of the first author about the sofic approximation of Kazhdan groups. The main result of [12] is that every sofic approximation of a countable Kazhdan group is essentially a disjoint union of expander graphs. However, we will need a specific proposition that will be relevant in the study of the expansion of small sets, in particular the graph of an almost automorphism.

**Proposition 3.3** (see [12]). Let $\Gamma$ be a finitely generated Kazhdan group with finite and symmetric generating set $S$, where $1 \in S$, and Kazhdan constant $\kappa$. For every $\alpha > 0$ there exists an integer $r$ such that for any finite, $S$-edge labeled graph $G$ and $T \subseteq V(G)$, where the ball $B(t, r)$ is isomorphic to the $r$-ball in the Cayley graph of $\Gamma$ for every $t \in T$ there exists a set $U$ such that

$$|\partial U| \leq \alpha |V_U| \quad \text{and} \quad |U \Delta T| \leq \frac{10}{\kappa^2} \|\chi_T - M\chi_T\|^2 \leq \frac{5|\partial T|}{d\kappa^2}.$$

We will use this proposition in order to prove our main technical result. The Hamming distance of two permutations will be denoted by $d_H(\sigma, \tau) := |\{1 \leq i \leq n \mid \sigma(i) \neq \tau(i)\}|$.

**Theorem 3.4.** Let $\Gamma$ be a finitely generated Kazhdan group with a fixed finite and symmetric generating set $S$ and Kazhdan constant $\kappa$. There exists $C, \varepsilon_0 > 0$ depending only on $S$ and $\kappa$, such that for all $0 \leq \varepsilon \leq \varepsilon_0$ the following holds:

Let $(G_n)_{n=1}^\infty$ be a sofic approximation by a sequence of regularly $S$-labeled expander graphs and for each $n \in \mathbb{N}$ let $c_n: V(G_n) \to V(G_n)$ be an $\varepsilon$-almost automorphism. Then for every $\delta > 0$ and $n \in \mathbb{N}$ large enough, there is a $\delta$-almost automorphism $c'_n: V(G_n) \to V(G_n)$ such that $d_H(c_n, c'_n) \leq \varepsilon C |V(G_n)|$.

**Proof.** We may assume that the graphs are regularly $S$-labeled. We choose $\varepsilon_0$ and $C$ later. Consider the sofic approximation $G_n \times G_n$ of $\Gamma$ given as the product of the sofic approximations and let $F_n \subset V(G_n) \times V(G_n)$ denote the graph of $c_n$. In order to improve $F_n$ we apply Proposition 3.3 to $\alpha$ small enough chosen later in order to get an $r$ for which the conditions of the proposition hold. Let $L^*_n \subseteq V(G_n) \times V(G_n)$ denote the set of vertices whose $r$-neighborhood is isomorphic to the rooted $r$-ball in the Cayley graph of $\Gamma$. The set $L^*_n$ is a product set: $L^*_n = K^r_n \times K^r_n$, where $K^r_n$ denotes the set of vertices in $V(G_n)$ whose $r$-neighborhood is isomorphic to the rooted $r$-ball in the Cayley graph of $\Gamma$. The sequence $(G_n)_{n=1}^\infty$ is a sofic approximation, hence $|V(G_n) \setminus K^r_n| = o_n(|V(G_n)|)$. 
Since $c_n$ is a bijection $|F_n \setminus L^\ast_n| = o(|V(G_n)|)$. Set $T_n = F_n \cap L^\ast_n$ and apply Proposition 3.3: We get a set $U_n$ such that

$$|\partial U_n| \leq \alpha |U_n| = (\alpha + o(1))|V(G_n)|$$

and

$$|T_n \triangle U_n| \leq \frac{5}{d\kappa^2} |\partial T_n| \leq \frac{10\varepsilon}{d\kappa^2} |V(G_n)|.$$

Finally, we need to modify $T_n$ in order to get the graph of a $\delta$-almost automorphism. For every $x \in V(G)$ define $\pi_1(x) = |\{(x, y) : y \in V(G), (x, y) \in T_n\}|$ and $\pi_2$ similarly. Note that

$$\sum_{k=0}^\infty |E(\pi_1^{-1}([0, k]), \pi_1^{-1}([k + 1, \infty]))| \leq |\partial U_n| \leq (\alpha + o(1))|V(G_n)|.$$

Assume that $\alpha$ is small enough and $n$ is large enough so that $|T_n \triangle U_n| \leq |V(G_n)|/2$, hence $|\pi_1^{-1}(1)|, |\pi_2^{-1}(1)| \geq |V(G_n)|/2$. Let $h$ denote the Cheeger constant of the expander sequence $(G_n)_{n=1}^\infty$. Since $\pi_1^{-1}([0, k]) \geq |V(G_n)|/2$ for $k \geq 2$ and $\pi_1^{-1}(0) \leq |V(G_n)|/2$ we can conclude that

$$\pi_1^{-1}(0) + \sum_{k \geq 2} (k - 1)\pi_1^{-1}(k) \leq (\alpha + o(1))|V(G_n)|/h.$$

The same inequality holds for $\pi_2$. Hence we need to add and remove at most $(4\alpha + o(1))|V(G_n)|/h$ vertices in order to get the graph of a bijection. The boundary changed by at most $(4\alpha + o(1))d|V(G_n)|/h$. If $\alpha$ is small and $n$ is large enough this will be the graph of a $\delta$-almost automorphism $c'_n : V(G_n) \to V(G_n)$. \qed

4. The group of clusters of almost automorphisms

Consider the symmetric group $\text{Sym}(n)$ for $n \in \mathbb{N}$. Given a set of permutations $S = \{f_1, \ldots, f_d\} \subset \text{Sym}(n)$ define the $2d$-regular undirected graph $G_S$ with possible loops and multiple edges with $V(G_S) = \{1, \ldots, n\}$ and $E(G_S)$ the disjoint union of the graphs of the maps $f_i$. The following idea of using expansion originates in the work of Simon Thomas [17].

**Lemma 4.1.** Consider a set of permutations $S = \{f_1, \ldots, f_d\} \subset \text{Sym}(n)$ and let $\delta > 0$. Moreover, consider two $\delta$-automorphisms of $G_S$, $c, c' : V(G_S) \to V(G_S)$. Then, we have

$$d_H(c, c') \leq \frac{2\delta n}{h(G_S)} \quad \text{or} \quad n - d_H(c, c') \leq \frac{2\delta n}{h(G_S)}.$$

**Proof.** Set $A = \{x \in V(G_S) : c(x) = c'(x)\}$. Note that if $x \in A$ and there is an $1 \leq i \leq d$ that $f_i(x) \notin A$ holds then $c$ or $c'$ does not map $(x, f_i(x))$ to the graph of the permutation $f_i$. Hence the number of such elements $x$ in $A$ is at most $2\delta n$. On the
other hand, if \( |A| \leq n/2 \) then we can use the bound on the number of edges between \( A \) and \( A' \) to get that this number is at least \( h(G_S)|A| \). Since \( |A| = n - d_H(c,c') \) this implies the second inequality. If \( |A| > n/2 \) we get the first inequality applying the same bound to \( A' \). \( \Box \)

The previous lemma implies that having small Hamming distance defines an equivalence relation on \( \delta \)-almost automorphisms if \( \delta \) is small enough. The next lemma shows that if there is a \( \delta \)-almost automorphism close to every \( 2\delta \)-almost automorphism then we can define a group structure on these equivalence classes in a natural way.

**Lemma 4.2.** Consider a set of permutations \( S = \{f_1,\ldots,f_d\} \subset \text{Sym}(n) \) and \( \delta > 0 \). Assume that

1. for every \( 2\delta \)-almost automorphism \( c \) of \( G_S \) there exists a \( \delta \)-almost automorphism \( \alpha(c) \) such that \( d_H(c,\alpha(c)) \leq n/5 \),
2. and for any two \( \delta \)-almost automorphisms \( c,c' : V(G_S) \to V(G_S) \), we have \( d_H(c,c') \notin [n/5, 4n/5] \).

Then, we can define a group \( \Gamma \) in the following way: The elements of \( \Gamma \) are the equivalence classes of \( \delta \)-almost automorphisms, where two almost automorphisms are equivalent if their Hamming distance is at most \( n/5 \). Given two \( \delta \)-almost automorphisms \( c,c' : V(G_S) \to V(G_S) \) representing their class define the product of the two classes as the class of \( \alpha(cc') : V(G_S) \to V(G_S) \). Then, the elements of \( \Gamma \) and the binary product are well defined and give a group structure.

**Proof.** The defined binary relation on \( \delta \)-almost automorphisms is an equivalence relation: It is clearly reflexive and symmetric. If \( a,b,c \) are \( \delta \)-almost automorphisms and \( d_H(a,b),d_H(b,c) \leq n/5 \) then by the triangle inequality \( d_H(a,c) \leq 2n/5 \), and by (2) we get \( d_H(a,c) \leq n/5 \). This proves transitivity.

Now we show that the product is well defined. Let \( a_1,a_2,b_1,b_2 \) \( \delta \)-almost automorphisms, assume that \( d_H(a_1,a_2) \leq n/5 \) and \( d_H(b_1,b_2) \leq n/5 \). Then

\[
d_H(\alpha(a_1b_1),\alpha(a_2b_2)) \leq 4n/5
\]

by (1) and the triangle inequality, hence it can be at most \( n/5 \) by (2) as required.

The class of the identity will be the identity of \( \Gamma \) and the inverse of a class will be the class of the inverses of its elements. Finally, we need to prove associativity. Given three \( \delta \)-automorphisms \( a,b,c \) we suffice to show

\[
d_H(\alpha(aa(bc)),\alpha(ab)c)) \leq 4n/5.
\]

We know that \( d_H((ab)c,\alpha(ab)c)) \leq 2n/5 \), and similarly \( d_H(a(bc),\alpha(a(bc))) \leq 2n/5 \). Since \( a(bc) = (ab)c \) this gives the required bound. The lemma follows. \( \Box \)
Proof of Theorem 1.1. Consider a sofic approximation of $\Gamma \times \Delta$ and a finite subset $F \subseteq \Delta$. Assume that the edges of the approximating graph sequence are labeled by elements of $S_\Gamma \cup S_\Delta$, where $S_\Gamma \subset \Gamma, S_\Delta \subset \Delta$ are finite subset. Assume that the sofic approximation of $\Gamma$ by the induced subgraphs containing the edges labeled by elements of $S_\Gamma$ is an essentially expander sequence.

Let $\delta > 0$ small and $n$ large enough such that the clusters of $\delta$-almost automorphisms form a group, where two $\delta$-almost automorphisms are in the same cluster if their Hamming distance is at most a fifth of the size of the vertex set. Moreover, let $n$ be large enough (and hence the ”error of the sofic approximation” small enough) such that the elements of $FF$ are all in different clusters, and for any $x,y \in F$ the cluster of $xy$ is the product of the clusters of $x$ and $y$ (in the group of clusters). We know that the group of clusters is finite. Since this holds for any finite $F \subset \Delta$ the group $\Delta$ is LEF.

Remark 4.3. It is a very interesting problem to classify or understand almost subgroups of $\text{Sym}(n)$, such as the almost centralizers of expander sofic approximations of Kazhdan groups arising from our results. In this context, it seems to be an open problem to decide whether every uniform almost homomorphism from a finite group to $\text{Sym}(n)$ is uniformly close to a homomorphism to say $\text{Sym}((1+\varepsilon)n)$. However, the analogous question for $U(n)$ (equipped with the normalized Hilbert-Schmidt distance) has been answered in [4,8], which already gives some information since $\text{Sym}(n) \subset U(n)$ in a way compatible with the metrics.

Acknowledgments

The work of the authors has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme (Consolidator Grant No. 681207, Consolidator Grant No. 617747 and Advanced Grant No. 741420).

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