Mitigation of microbunching instability in x-ray free electron laser linacs

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The microbunching instability seeded by small initial density modulation and driven by collective effects can cause significant electron beam quality degradation in next generation x-ray free electron lasers. A method exploiting longitudinal mixing derived from the natural transverse spread of the electron beam through a dispersive bending magnet was proposed to suppress this instability several years ago [Phys. Rev. Lett. 111, 054801 (2013)]. Instead of using bending magnets to introduce the transverse-to-longitudinal coupling, which will lead to an inconvenient deflection of the downstream beam line, in this paper, we propose a scheme to mitigate the microbunching instability by inserting a quadrupole magnet inside a bunch compressor of the accelerator. This results in transverse-to-longitudinal phase space mixing and large slice energy spread that can efficiently mitigate the growth of the microbunching instability through the major accelerator section. Finally, at the exit of the accelerator, a dogleg section is used to restore the emittance and slice energy spread before entering the undulator radiation section. Multiparticle simulations show that the transverse space charge, structure wakefield, and the coherent synchrotron radiation effects will have a relatively small impact on this scheme.

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I. INTRODUCTION

X-ray free electron lasers (FEL) provide an important tool for scientific discovery in physics, chemistry, biology, and material science. The performance of those x-ray FEL facilities critically depends on the electron beam quality out of linear accelerators, i.e., linacs. However, in the linear accelerator, due to the presence of collective effects such as the longitudinal space-charge effect, an initial small current density modulation inside an electron beam (e.g., from shot noise) will generate sufficiently large energy modulation of the beam after transporting through a section of the accelerator. The induced energy modulation will cause even larger density modulation inside the beam after passing through a magnetic bunch compressor to increase the electron beam peak current. This leads to the amplification of the initial density modulation, i.e., the microbunching instability. Typically, in a x-ray FEL linear accelerator, two stage magnetic bunch compressors or multiple stage magnetic bunch compressors are used to generate final high electron beam peak current. The microbunching instability will be dramatically amplified through the accelerator and causes significant degradation of the electron beam quality at the entrance of undulator for x-ray radiation.

The microbunching instability was extensively studied in the past [1–14]. A conventional method to control the instability is to use a laser heater to increase the electron beam uncorrelated energy spread before bunch compression to damp the instability [4,5,15]. This is typically tolerable for the operation of self-amplified spontaneous emission (SASE) FELs. However, for seeded FELs it could degrade the FEL performance due to the increase of final electron beam energy spread [16]. Recently several new methods based on transverse-to-longitudinal coupling were proposed to suppress the microbunching instability [17–21]. These schemes are attractive because of the “reversible feature,” which means the transverse emittance and slice energy spread can be recovered meanwhile effectively suppresses the microbunching instability [22]. So far, none of those methods has been applied to an entire linear accelerator system. The scheme based on two bending magnets [17] is a simple inexpensive method without using sophisticated hardware such as a rf cavity. However, the use of bending magnets changes the beam line direction, which is not convenient and difficult to apply to the existing FEL linacs.

In this paper, we propose a scheme to mitigate the microbunching instability by inserting a quadrupole magnet inside a four dipole bunch compressor chicane to allow dispersion leakage and transverse-to-longitudinal coupling
through the rest of the linac in a two-stage bunch compression accelerator. This method does not require the use of any new type of hardware in comparison to the other reversible methods and also keeps the original straight linac beam line geometry without changing direction. One effect of coupling is to smear out the already induced current or energy modulation before the bunch compressor. The other effect is to increase slice energy spread due to the presence of correlated energy chirp before the bunch compressor, which efficiently suppresses the microbunching instability in the following accelerator sections. At the end of the accelerator, a dogleg section is used to remove the dispersion and the coupling. By properly tuning the quadrupole strength inside the dogleg, one can obtain an achromatic lattice from the entrance of the bunch compressor to the exit of the dogleg. This removes the transverse-to-longitudinal coupling after the dogleg. Meanwhile, the increased emittance and slice energy spread are also restored.

The organization of the paper is as follows: after the Introduction, in Sec. II, we present the microbunching instability mitigation scheme with matrix formalism and corresponding linear optics design. In Sec. III, the analytical calculation of the microbunching instability gain is given and benchmarked with simulations. In Sec. IV, numerical simulations including space charge effects, coherent synchrotron radiation (CSR) and the accelerator transverse structure wakefield effects are presented. Finally, we draw conclusions in Sec. V.

II. MICROBUNCHING INSTABILITY MITIGATION SCHEME

We consider an x-ray FEL linear accelerator machine layout including the microbunching instability mitigation scheme as shown in Fig. 1. This accelerator includes three accelerating linac sections separated by two bunch compressor chicanes. One quadrupole is placed in the middle of bunch compressor one (BC1) to leak out the dispersion and longitudinal mixing terms $R_{s1}$ and $R_{s2}$ for instability mitigation. The energy chirp induced by Linac-2 is canceled by Linac-3. For energy chirp generated in Linac-1, it will be maintained after Linac-3, which calls for an isochronous dogleg design to avoid further bunch length compression. The final residual energy chirp is expected to be removed by a dechirper [23,24] or resistive wall wakefield of further transport system.

The longitudinal phase space smearing brought up by the coupling terms in BC1 can greatly smooth the upstream density and energy modulation, thus suppresses further instability amplification in the following transport system. Meanwhile, due to the existence of energy chirp before BC1, additional slice energy spread will also be induced by the transverse to longitudinal coupling terms, which is given as [21,25]:

$$\sigma_{\delta}(s_3) \approx C_1 h_1 R_{s1}^1 \sigma_x(s_2),$$

where $C_1$ is the compression factor from $s_1$ to $s_3$, $h_1$ and $\sigma_x(s_2)$ are the energy chirp and the horizontal rms beam size before BC1, $R_{s1}^1$ is the coupling term from BC1 section. Substituting the lattice parameters shown in Fig. 1, assuming $R_{s1}^1 = 0.05$, $\sigma_x(s_2) = 0.2$ mm, the resultant slice energy spread could be increased to about 156 keV, which is an order of magnitude larger than the nominal value ($2 \text{keV} \times 6 = 12 \text{keV}$). Finally, at the exit of the accelerator, the extra slice energy spread and the transverse emittance growth can be removed by adjusting the quadrupoles inside the dogleg, which eliminates the induced couplings and makes the entire system an achromat.

The above accelerator system can be described using a transfer matrix approach. Here the linear transfer matrices from $s_2$ to $s_7$, written as $R = R^{D}T_{r}R^{1}$, in $(x, x' \equiv dx/ds, z, \delta)$ coordinates are given in the above scheme, where $R^1$ is the transfer matrix from $s_2 \rightarrow s_3$,

$$R^1 = \begin{pmatrix}
R_{11}^1 & R_{12}^1 & 0 & R_{16}^1 \\
R_{21}^1 & R_{22}^1 & 0 & R_{26}^1 \\
R_{s1}^1 & R_{s2}^1 & 1 & R_{s6,1}^1 \\
0 & 0 & 0 & 1
\end{pmatrix},$$

$R^D$ is the transfer matrix of the dogleg section from $s_6 \rightarrow s_7$,

$$R^D = \begin{pmatrix}
R_{D11}^D & R_{D12}^D & 0 & R_{D16}^D \\
R_{D21}^D & R_{D22}^D & 0 & R_{D26}^D \\
R_{D51}^D & R_{D52}^D & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},$$

$T_r$ is the middle accelerator section from $s_3 \rightarrow s_6$

$$T_r = \begin{pmatrix}
R_{11} & R_{12} & 0 & 0 \\
R_{21} & R_{22} & 0 & 0 \\
0 & 0 & (1 + \zeta)/C_2 & R_{s6,2}\gamma_3/\gamma_5 \\
0 & 0 & 0 & (C_2\gamma_3/\gamma_7)/(1 + \zeta)
\end{pmatrix},$$

where $\zeta = (C_1 - 1)C_2(R_{s6,2}\gamma_3)/(R_{s6,1}\gamma_5)$, $\gamma_j$ is the beam relativistic factor at position $s_j$. By using the symplectic
condition of the transfer matrices \((R^T(s_{0\to1}) \cdot S \cdot R(s_{0\to1}) = S \cdot \gamma_0/\gamma_1,\) when taking acceleration effects into account) and choosing the dogleg section so that

\[
R_{51}^D = \frac{[r_{21}(R_{52}^1 R_{11}^1 - R_{51}^1 R_{12}^1) + r_{22}(R_{52}^1 R_{12}^1 - R_{51}^1 R_{11}^1)]}{C_2/(1 + \zeta \gamma_1)},
\]

\[
R_{52}^D = \frac{[r_{11}(R_{52}^1 R_{11}^1 - R_{51}^1 R_{12}^1) - r_{12}(R_{52}^1 R_{12}^1 - R_{51}^1 R_{11}^1)]}{C_2/(1 + \zeta \gamma_1)}.
\]

\[ (5) \]

the accelerator system from the entrance of BC1 to the exit of the dogleg section can be made as an achromat with the linear transfer matrix from \(s_2\) to \(s_7\)

\[
R_{s_2-s_7} = \begin{pmatrix}
R_{11} & R_{12} & 0 & 0 \\
R_{21} & R_{22} & 0 & 0 \\
0 & 0 & (1 + \zeta)/C_2 & \frac{r_{61}}{C_6} + \frac{C_1 r_{62} \gamma_1}{\gamma_1} \\
0 & 0 & 0 & \frac{C_7 \gamma_1}{(1 + \zeta) \gamma_1}
\end{pmatrix}
\]

Following the schematic layout shown in Fig. 1, we did linear optics design based on the above first-order transfer map without including collective effects. In order to generate the coupling terms for the instability mitigation, one quadrupole is inserted at the center location of the bunch compressor chicane BC1. The normalized quadrupole strength of 0.83/m² with 0.2 m length is used as an example, which leaks out \(R_{51}^1 = 0.05, R_{52}^1 = 0.18\) m for the microbunching instability mitigation. The initial beam transverse distribution is a uniform round cross section with 0.4 mm radius and 0.3 μm normalized transverse emittance. The longitudinal electron position is uniformly distributed with a flattop current of 20 A. A Gaussian function is used for the electron energy distribution. The initial uncorrelated energy spread is 2 keV (rms) with zero energy chirp. The beam is accelerated from the initial 100 MeV at the entrance to Linac-1 to final 5 GeV with a total compression factor of 72. FODO structures are applied in each linac sections to control the beam size. The energy chirp induced in Linac-2 is canceled by Linac-3. The dogleg section, which consists of two dipole pair sandwiching three independent quadrupoles, is designed to be both isochronous and achromatic at the same time. The residual energy chirp from Linac-1 after the dogleg can be dechirped by additional beam line elements such as a dechirper. The beam and the lattice parameters are summarized in Table I.

The transverse beam size evolution without including collective effects is shown in Fig. 2. Starting from the location of BC1, the horizontal beam size increases significantly due to the leakage of dispersion. This dispersion is present through the Linac-2, the bunch compressor BC2, the

![FIG. 2. Evolution of the beam transverse rms size along the longitudinal position.](image-url)
where the kernel of the above integral equation is given as:

\[
K(\tau; s) = ik(s)R_{56}(\tau \rightarrow s)\frac{I(\tau)Z[k(\tau); \tau]}{I_0} \exp \left( -\frac{k_0^2}{2} \sigma_{s0}^2 \right) \times \exp \left( \frac{-k_0^2 \epsilon_{c0}}{2Y_0 \beta_{s0}} T \right)
\]  

(8)

with the Alfvén current \( I_A \approx 17.045 \, \text{kA} \).

By iterating the above integral equation three times, neglecting collective effects inside the two bunch compressors and the dogleg, we obtain the final modulation factor at the exit \((s_7)\) of the dogleg as:

\[
b[k(s_7); s_7] = b_0[k(s_7); s_7]
+ b_{11}[k(s_7); s_7] + b_{12}[k(s_7); s_7] + b_{13}[k(s_7); s_7]
+ b_{21}[k(s_7); s_7] + b_{22}[k(s_7); s_7] + b_{23}[k(s_7); s_7]
+ b_{31}[k(s_7); s_7],
\]  

(9)

where \( k(s) = C(s)k_0 \) and \( C(s) \) is the compression factor from \( s_1 \) to \( s \), and \( k_0 \) is the initial modulation wave number. Here \( b_0 \) denotes evolution of the modulation factor in the absence of all collective effects, and is given as:

\[
b_0[k(s_7); s_7] = b_0 \exp \left[ -k^2(s_7)R_{56}(s_{1-7})\sigma_{s0}^2 / 2 \right];
\]  

(10)

The three terms of \( b_{11}, b_{12}, b_{13} \) denote amplification of the initial microbunching due to collective effects between the linac section of \( s_{1-2}, s_{3-4}, s_{5-6} \) respectively, and are given as:

\[
b_{11}[k(s_7); s_7] = ib_0[k(s_7)R_{56}(s_{1-7})I(s_1)I(s_7) / Y_0] \times \exp \left( -\frac{k_0^2 D^2(s_{1-7})\sigma_{s0}^2}{2} \right) \times \exp \left( -\frac{k_0^2 H(s_{1-7})\epsilon_{c0}}{2Y_0 \beta_{s0}} \right),
\]  

\[
b_{12}[k(s_7); s_7] = ib_0[k(s_7)R_{56}(s_{3-4})I(s_3)I(s_7) / Y_0] \times \exp \left( -\frac{k_0^2 D^2(s_{3-4})\sigma_{s0}^2}{2} \right) \times \exp \left( -\frac{k_0^2 H(s_{3-4})\epsilon_{c0}}{2Y_0 \beta_{s0}} \right),
\]  

(11)

\[
b_{13}[k(s_7); s_7] = ib_0[k(s_7)R_{56}(s_{5-6})I(s_5)I(s_7) / Y_0] \times \exp \left( -\frac{k_0^2 D^2(s_{5-6})\sigma_{s0}^2}{2} \right) \times \exp \left( -\frac{k_0^2 H(s_{5-6})\epsilon_{c0} \beta_{s0}}{2Y_0 \beta_{s0}} \right);
\]  

(12)

\[\]
The following three terms $b_{21}$, $b_{22}$, $b_{23}$ denote the two-stage coupling collective effects between the section $s_{1-2}$ and $s_{3-4}$, $s_{1-2}$ and $s_{5-6}$, $s_{3-4}$ and $s_{5-6}$ respectively, and are given as:

\[
b_{21}[k(s_7); s_7] = -b_0 k(s_1) k(s_7) \hat{R}_{56}(s_{1-3}) \hat{R}_{56}(s_{3-7}) \times \frac{I(s_1)I(s_3)}{(\gamma_0)^2} \hat{Z}(s_{1-2}) \hat{Z}(s_{3-4})
\]
\[\times \exp \left( \frac{-k_0^2 D^2(s_{1-3-7}) \sigma_{s_0}^2}{2} \right) \times \exp \left( \frac{-k_0^2 \mathcal{H}(s_{1-3-7}) e_{x,n}}{2\gamma_0 \beta_{s_0}} \right), \tag{14}\]

\[
b_{22}[k(s_7); s_7] = -b_0 k(s_1) k(s_7) \hat{R}_{56}(s_{1-5}) \hat{R}_{56}(s_{5-7}) \times \frac{I(s_1)I(s_5)}{(\gamma_0)^2} \hat{Z}(s_{1-2}) \hat{Z}(s_{5-6})
\]
\[\times \exp \left( \frac{-k_0^2 D^2(s_{1-5-7}) \sigma_{s_0}^2}{2} \right) \times \exp \left( \frac{-k_0^2 \mathcal{H}(s_{1-5-7}) e_{x,n}}{2\gamma_0 \beta_{s_0}} \right), \tag{15}\]

\[
b_{23}[k(s_7); s_7] = -b_0 k(s_1) k(s_7) \hat{R}_{56}(s_{3-5}) \hat{R}_{56}(s_{5-7}) \times \frac{I(s_3)I(s_5)}{(\gamma_0)^2} \hat{Z}(s_{3-4}) \hat{Z}(s_{5-6})
\]
\[\times \exp \left( \frac{-k_0^2 D^2(s_{3-5-7}) \sigma_{s_0}^2}{2} \right) \times \exp \left( \frac{-k_0^2 \mathcal{H}(s_{3-5-7}) e_{x,n}}{2\gamma_0 \beta_{s_0}} \right). \tag{16}\]

The last term denotes the three-stage coupling amplification due to the collective effects between the region $s_{1-2}$ and $s_{3-4}$ and $s_{5-2}$, and are given as:

\[
b_{31}[k(s_7); s_7] = -ib_0 k(s_3) k(s_5) k(s_7) \hat{R}_{56}(s_{1-3}) \hat{R}_{56}(s_{3-7}) \times \frac{I(s_1)I(s_3)I(s_5)}{(\gamma_0)^3} \hat{Z}(s_{1-2}) \hat{Z}(s_{3-4}) \hat{Z}(s_{5-6})
\]
\[\times \exp \left( \frac{-k_0^2 D^2(s_{1-3-7}) \sigma_{s_0}^2}{2} \right) \times \exp \left( \frac{-k_0^2 \mathcal{H}(s_{1-3-7}) e_{x,n}}{2\gamma_0 \beta_{s_0}} \right), \tag{17}\]

where $\sigma_{s_0}$ is the initial rms relative energy spread, $\gamma_0$ is the initial electron beam relativistic factor, $I(s_j) = C(s) I_0$, $I_0$ is the initial current. The impedance term in the above equations are defined as:

\[
\hat{Z}(s_{j-k}) = \int_{s_j}^{s_{j-k}} \frac{4\pi Z[k(\tau); \tau]}{I_A Z_0} d\tau,
\]

where $Z[k(\tau); \tau]$ is the impedance per unit length of collective effects and $Z_0$ is the vacuum impedance. The exponential damping to the modulation amplification due to initial energy spread is given as:

\[
\mathcal{D}^2(s_{1-7}) = U^2(s_7, s_1),
\]
\[
\mathcal{D}^2(s_{3-7}) = U^2(s_7, s_3) + U^2(s_3, s_1),
\]
\[
\mathcal{D}^2(s_{5-7}) = U^2(s_7, s_5) + U^2(s_5, s_1),
\]
\[
\mathcal{D}^2(s_{1-3-7}) = U^2(s_7, s_3) + U^2(s_3, s_1),
\]
\[
\mathcal{D}^2(s_{1-5-7}) = U^2(s_7, s_5) + U^2(s_5, s_1),
\]
\[
\mathcal{D}^2(s_{3-5-7}) = U^2(s_7, s_3) + U^2(s_3, s_1) + U^2(s_3, s_1),
\]
\[
\mathcal{D}^2(s_{1-3-5-7}) = U^2(s_7, s_3) + U^2(s_3, s_1) + U^2(s_3, s_1), \tag{18}\]

where

\[
U(s, \tau) = C(s) \hat{R}_{56}(s) - C(\tau) \hat{R}_{56}(\tau). \tag{19}\]

The damping effects due to transverse-to-longitudinal couplings are given as:

\[
\mathcal{H}(s_{1-7}) = T(s_7, s_1),
\]
\[
\mathcal{H}(s_{3-7}) = T(s_7, s_3) + T(s_3, s_1),
\]
\[
\mathcal{H}(s_{5-7}) = T(s_7, s_5) + T(s_5, s_1),
\]
\[
\mathcal{H}(s_{1-3-7}) = T(s_7, s_3) + T(s_3, s_1),
\]
\[
\mathcal{H}(s_{1-5-7}) = T(s_7, s_5) + T(s_5, s_1),
\]
\[
\mathcal{H}(s_{3-5-7}) = T(s_7, s_3) + T(s_3, s_3) + T(s_3, s_1),
\]
\[
\mathcal{H}(s_{1-3-5-7}) = T(s_7, s_3) + T(s_3, s_3) + T(s_3, s_1), \tag{20}\]

where

\[
T(s, \tau) = [\beta_{s_0} V(s, \tau) - \alpha_{s_0} W(s, \tau)]^2 + W(s, \tau)^2, \tag{21}\]

where $\beta_{s_0}$ and $\alpha_{s_0}$ are the initial horizontal Twiss parameters, $V$ and $W$ are defined as:

\[
V(s, \tau) = C(s) \hat{R}_{51}(s) - C(\tau) \hat{R}_{51}(\tau),
\]
\[
W(s, \tau) = C(s) \hat{R}_{52}(s) - C(\tau) \hat{R}_{52}(\tau). \tag{22}\]

We used the shorthand notation $\hat{R}(s) = \hat{R}(s \rightarrow s)$ here. In order to include acceleration effects, the transfer matrix terms are defined as [6]:
\[ \hat{R}_{51}(s_{j\rightarrow k}) = \hat{R}_{51}(s_{j\rightarrow k}) \sqrt{\gamma_0/\gamma_j}, \]
\[ \hat{R}_{52}(s_{j\rightarrow k}) = \hat{R}_{52}(s_{j\rightarrow k}) \sqrt{\gamma_0/\gamma_j}, \]
\[ \hat{R}_{56}(s_{j\rightarrow k}) = \hat{R}_{56}(s_{j\rightarrow k}) \gamma_0/\gamma_j, \]

(23)

where \( \gamma_j \) is the relativistic factor at location \( s_j \).

For the convenience of the following benchmark, here we listed the transfer matrix terms based on Eq. (23). The momentum compaction factors along the accelerator beam line as shown in Fig. 1 are given as:

\[ \hat{R}_{56}(s_{1\rightarrow 3}) = \frac{R_{56,1}\gamma_1}{\gamma_3}, \]
\[ \hat{R}_{56}(s_{1\rightarrow 7}) = \frac{R_{56,1}\gamma_1 + R_{56,2}C_1\gamma_1}{\gamma_5}, \]
\[ \hat{R}_{56}(s_{3\rightarrow 5}) = \frac{R_{56,2}\gamma_1}{\gamma_5}, \]
\[ \hat{R}_{56}(s_{3\rightarrow 7}) = \frac{R_{56,2}\gamma_1}{\gamma_5}, \]
\[ \hat{R}_{56}(s_{5\rightarrow 7}) = 0. \]

(24)

Applying the achromatic condition in Eq. (5), the transverse-to-longitudinal coupling terms are written as:

\[ \hat{R}_{51}(s_{1\rightarrow 3}) \equiv \hat{R}_{51}^1 = m_{11}R_{51}^1 + m_{21}R_{52}^1, \]
\[ \hat{R}_{52}(s_{1\rightarrow 3}) \equiv \hat{R}_{52}^1 = m_{12}R_{51}^1 + m_{22}R_{52}^1, \]
\[ \hat{R}_{51}(s_{1\rightarrow 5}) \equiv \hat{R}_{51}^1 \cdot \frac{1}{C_2} + (C_1 - 1)\gamma_3R_{56,2}, \]
\[ \hat{R}_{52}(s_{1\rightarrow 5}) \equiv \hat{R}_{52}^1 \cdot \frac{1}{C_2} + (C_1 - 1)\gamma_3R_{56,2}, \]
\[ \hat{R}_{51}(s_{1\rightarrow 7}) = 0, \]
\[ \hat{R}_{52}(s_{1\rightarrow 7}) = 0, \]

(25)

where \( m_{ij} \) is the transfer matrix term in \( (s_1 \rightarrow s_2) \) section. Here, we have used \( (R_{56,3} = 0, C_3 = 1) \) in the above Eqs. (24) and (25). These equations suggest that the electron beam transverse size would affect the microbunching instability gain through the transverse-to-longitudinal coupling process, which can also be seen from Eq. (1).

To benchmark our analytical gain model, simulations using a multiparticle simulation code IMPACT [27,28] were also carried out. In the following calculation of microbunching instability gain, we included only longitudinal space-charge (LSC) effect to account for collective effects of electron beam through the accelerator since it is the dominant factor during the process of microbunching amplification [4]. The averaged free-space LSC impedance model based on round uniform electron beam is used in the analytical calculations which is given as [8]:

\[ Z_{avg}(k) = \frac{iZ_0}{\pi r_b} \frac{1 - \zeta I_0(kr/\gamma)K_1(\zeta)}{\zeta} \bigg|_{\zeta=kr_b/r}, \]

(26)

where \( r_b \) is the beam radius, \( k \) is the wave number, \( \gamma \) is the relativistic factor, and \( I_1, K_1 \) are modified Bessel functions of the first kind. The initial electron beam distribution is same as the parameters listed in Table I except that the bunch length and the charge were changed according to the initial modulation wavelength for simulations in the low frequency spectrum range. To reduce the artificial numerical noise due to a small number of macroparticles (20 millions) in the simulations, a quiet loading method based on Halton sequence samplings was used [29]. A maximum 1024 longitudinal grid points were used in the simulations, which ensures 20 grid points per modulation wavelength.

First, we consider the case when the quadrupole in BC1 is turned off. The electron beam transportation between \( (s_1 \rightarrow s_6) \) is considered, which corresponds to the conventional two-stage bunch compression linear accelerator. The accelerator lattice was tuned again to match the FODO structures in each linac section. The dipole strength in BC1 is also tuned to maintain the initial momentum compaction factor. The average beam size along the accelerator beam line is listed in Table II. Substituting the momentum compaction factors listed in Eq. (24) into Eq. (9) and setting all the transverse-to-longitudinal coupling terms listed in Eq. (25) to zeros, the microbunching gain at the exit of Linac-3 is the same as the gain at the exit of dogleg. The final microbunching gain \( |b[k(s_6); s_6]/b_0| \) with different initial current modulation wavelength is shown in Fig 5.

We also calculated the microbunching instability gain in the above case using the IMPACT code. The simulation results are also shown in Fig. 5. The discrepancy between the theory and the simulation at short wavelength could be due to the difference between the 1D LSC impedance in the analytical model and the 3D LSC model in the macroparticle simulation. These two models show larger difference toward shorter wavelength region as reported in Ref. [8].

When the quadrupole in BC1 is turned on to mitigate the microbunching instability, the final gain at the exit of the dogleg section was calculated using the above analytical model and the macroparticle simulation. The electron beam and accelerator lattice parameters listed in the previous linear optics design section are used. The final gain spectrum \( |b[k(s_f); s_f]/b_0| \) from both the analytical model calculation and from the macroparticle simulation are shown in Fig 6. Comparing the microbunching instability gain in this case with that in the case of no quadrupole in

| Parameter | Value |
|-----------|-------|
| Average \( (\sigma_x, \sigma_y) \) in Linac-1 | (0.10, 0.10) mm |
| Average \( (\sigma_x, \sigma_y) \) in Linac-2 | (0.05, 0.05) mm |
| Average \( (\sigma_x, \sigma_y) \) in Linac-3 | (0.02, 0.02) mm |

TABLE II. Average beam size (rms) in each linac section when quadrupole in BC1 is turned off.
BC1, we see that the instability is efficiently mitigated with the use of the quadrupole in BC1. The maximum instability gain in the normal two-stage bunch compression linear accelerator is reduced by more than an order of magnitude. The residual microbunching gain is mainly due to the contribution from Eq. (11). Since the \((s_2 \rightarrow s_7)\) section is an achromat but with nonzero \(R_{56}\), the energy modulations induced upstream before the BC1 can still be amplified. The instability gain from the simulation is less than that from the analytical model when the initial modulation wavelength is below 400 microns. This is because the electron beam is not a round beam but a flat beam \((\sigma_x \gg \sigma_y)\) due to the dispersion leakage after the BC1.

The average beam sizes through the accelerator are listed in Table III. An effective beam radius \(r_b = 2\sqrt{\sigma_x\sigma_y}\) was used in the analytical model.

### IV. MULTIPARTICLE SIMULATION WITH COLLECTIVE EFFECTS

The macroparticle simulation in the above benchmark includes only the longitudinal space-charge effect. The effects of transverse space-charge and wakefield and the effects of coherent synchrotron radiation (CSR) in the proposed mitigation scheme were not included. To verify the above scheme, TSC, CSR and the TESLA superconducting cavity transverse structure wakefield effects were added in the following simulations. We simulated an electron beam with an initial 0.1% current modulation at 100 \(\mu\)m wavelength following the initial beam parameters described in Table I. Figures 7 and 8 show the final longitudinal phase space distribution and the projected current profile at the exit of Linac-3 when the quadrupole in BC1 is turned off. Strong energy and current modulations can be seen after two-stage compression due to the microbunching instability.

![Figure 5](image1.png)

**FIG. 5.** Microbunching gain spectrum from the analytical model and from the numerical macroparticle simulation when the quadrupole in BC1 is turned off.

![Figure 6](image2.png)

**FIG. 6.** Microbunching gain spectrum from the analytical model and from the numerical macroparticle simulation when the quadrupole in BC1 is turned on.

![Figure 7](image3.png)

**FIG. 7.** Final longitudinal phase space with 1st order energy chirp removed when quadrupole in BC1 is turned off.

![Figure 8](image4.png)

**FIG. 8.** Final current modulation when quadrupole in BC1 is turned off.

| Parameter | Value          |
|-----------|---------------|
| Average \((\sigma_x, \sigma_y)\) in Linac-1 | (0.10, 0.10) mm |
| Average \((\sigma_x, \sigma_y)\) in Linac-2 | (0.80, 0.05) mm |
| Average \((\sigma_x, \sigma_y)\) in Linac-3 | (0.40, 0.02) mm |
When the quadrupole in BC1 is turned on, the simulation results of the longitudinal phase space and the current profile at several locations of accelerator beam line are shown in Fig. 9. The initial slice energy spread is assumed to be 2 keV, and is increased to about 160 keV (rms) after the BC1. The induced current modulation is smeared out after the passage through the BC1 as seen in Fig. 9(b). Finally at the exit of dogleg section, inside the core of the electron beam, the induced additional slice energy spread is restored to the nominal one, 144 keV, which is shown in Fig. 9(d) and the right plot of Fig. 10. Due to the additional slice energy spread after the BC1 and the corresponding smearing effects, the current profiles between the BC1 and the dogleg are smooth with little modulation, which shows the suppression of microbunching instability inside the system. A small current modulation is still visible at the exit of dogleg. For a real electron beam, the initial current modulation (∼0.001%) due to electron shot noise can be much smaller than the 0.1% modulation used in the simulation. The current modulation at the exit of dogleg would be barely noticeable. The final slice emittance is also shown in the left plot of Fig. 10. The normalized slice emittance in the horizontal direction of the beam core section increases by about 6%, while the vertical slice emittance stays about the same as the initial emittance. Up to 100 million macroparticles with $512 \times 64 \times 512$ grid points were used in the simulation to test the convergence. These results suggest that the effects of transverse space-charge and structure wakefield, and the effects of CSR through the above linear accelerator system could be relatively small. The slice emittance and energy spread outside the electron beam core increase significantly, this is mainly due to the strong space charge effects near the edge of the distribution.

V. CONCLUSIONS

In this paper, we proposed a scheme to mitigate the microbunching instability in a linear accelerator with two bunch compression stages for the x-ray FEL radiation. This scheme has the advantage of suppressing microbunching instability but without causing additional final energy spread growth, so-called reversible heater. The horizontal-to-longitudinal coupling due to the insertion of a quadrupole inside the bunch compressor BC1 in this scheme helps smear the initial microbunching and also efficiently suppress the microbunching amplification through the following accelerator sections. Inside the core of electron beam, the final slice energy spread is successfully restored to the nominal value even when space-charge, CSR and the transverse structure wakefield are taken into account in the simulations. The slice emittance in the core of the beam is also reasonably well preserved at the exit of the dogleg section.

Because of the achromatic lattice design, the current and energy modulations induced before the BC1 cannot be suppressed in this scheme. Considering the short transportation distance of Linac-1 section, the contribution to the microbunching instability gain from this section should be relatively small.

One unavoidable effect in this scheme is the beam tilt effect. The transverse-to-longitudinal correlation after the BC1 introduces chromatic effects when transporting particles through accelerator beam line elements such as a quadrupole. Careful lattice design including sextupole magnets are needed to correct these nonlinear effects and will be reported in the future study. Furthermore, we would also
like to include the machine imperfections from realistic optics in the future study.

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