| Title       | Genuine multipartite entanglement without multipartite correlations |
|-------------|---------------------------------------------------------------------|
| Author(s)   | Schwemmer, Christian; Knips, Lukas; Tran, Minh Cong; de Rosier, Anna; Laskowski, Wiesaw; Paterek, Tomasz; Weinfurter, Harald |
| Citation    | Schwemmer, C., Knips, L., Tran, M. C., de Rosier, A., Laskowski, W., Paterek, T., et al. (2015). Genuine multipartite entanglement without multipartite correlations. Physical review letters, 114(18), 180501-. |
| Date        | 2015                                                               |
| URL         | http://hdl.handle.net/10220/25996                                  |
| Rights      | © 2015 American Physical Society. This paper was published in Physical Review Letters and is made available as an electronic reprint (preprint) with permission of American Physical Society. The paper can be found at the following official DOI: [http://dx.doi.org/10.1103/PhysRevLett.114.180501]. One print or electronic copy may be made for personal use only. Systematic or multiple reproduction, distribution to multiple locations via electronic or other means, duplication of any material in this paper for a fee or for commercial purposes, or modification of the content of the paper is prohibited and is subject to penalties under law. |
Genuine Multipartite Entanglement without Multipartite Correlations

Christian Schwemmer, Lukas Knips, Minh Cong Tran, Anna de Rosier, Wiesław Laskowski, Tomasz Paterek, and Harald Weinfurter

Max-Planck-Institut für Quantenoptik, Hans-Kopfermann-Straße 1, 85748 Garching, Germany
Department für Physik, Ludwig-Maximilians-Universität, 80797 München, Germany
School of Physical and Mathematical Sciences, Nanyang Technological University, Singapore, 637371 Singapore
Institute of Theoretical Physics and Astrophysics, University of Gdańsk, PL-80-952 Gdańsk, Poland
Center for Quantum Technologies, National University of Singapore, Singapore, 117543 Singapore
MajuLab, CNRS-UNS-NUS-NTU International Joint Research Unit, Singapore, UMI 3654 Singapore

Nonclassical correlations between measurement results make entanglement the essence of quantum physics and the main resource for quantum information applications. Surprisingly, there are n-particle states which do not exhibit n-particle correlations at all but still are genuinely n-particle entangled. We introduce a general construction principle for such states, implement them in a multiphoton experiment and analyze their properties in detail. Remarkably, even without multipartite correlations, these states do violate Bell inequalities showing that there is no classical, i.e., local realistic model describing their properties.

Correlations between measurement results are the most prominent feature of entanglement. They made Einstein, Podolski, and Rosen [1] question the completeness of quantum mechanics and are nowadays the main ingredient for the many applications of quantum information like entanglement based quantum key distribution [2] or quantum teleportation [3].

Correlations enable us, e.g., when observing two maximally entangled qubits, to use a measurement result observed on the first system to infer exactly the measurement result on the second system. In this scenario, the two particle correlations are formally given by the expectation value of the product of the measurement results obtained by the two observers. Note, the single particle correlation, i.e., the expectation value of the results for one or the other particle are zero in this case. Consequently, we cannot predict anything about the individual results. When studying the entanglement between n particles, a natural extension is to consider n-particle correlations, i.e., the expectation value of the product of n measurement results. Such correlation functions are frequently used in classical statistics and signal analysis [4], moreover, in quantum information, almost all standard tools for analyzing multiparticle systems like multiparty entanglement witnesses [5,6] and Bell inequalities [7,8] are based on the n-particle correlation functions.

Recently, Kaszlikowski et al. [9] pointed at a particular quantum state with vanishing multiparty correlations which, however, is genuinely multipartite entangled. This discovery, of course, prompted vivid discussions on a viable definition of classical and quantum correlations [10,11]. Still, the question remains what makes up such states with no full n-particle correlations and how non-classical they can be, i.e., whether they are not only entangled but whether they also violate a Bell inequality.

Here, we generalize, highlight, and experimentally test such remarkable quantum states. We introduce a simple principle how to construct states without n-particle correlations for odd n and show that there are infinitely many such states which are genuinely n-particle entangled. We implement three and five qubit no-correlation states in a multiphoton experiment and demonstrate that these states do not exhibit n-particle correlations. Yet, due to the existence of correlations between a smaller number of particles, we observe genuine n-particle entanglement. Using our recently developed method to design n-particle Bell inequalities from lower order correlation functions only [12,13], we show that these states, despite not having full correlations, can violate Bell inequalities.

\[ T_{j_1 \ldots j_n} = \langle r_1 \ldots r_n \rangle = \text{Tr}(\rho \sigma_{j_1} \otimes \ldots \otimes \sigma_{j_n}), \]

where \( r_k \) is the outcome of the local measurement of the \( k \)th observer, parametrized by the Pauli operator \( \sigma_{j_k} \) with \( j_k \in \{x, y, z\} \). Evidently, besides the n-particle correlations, for an n-particle state, one can also define \( l < n \) fold correlations \( T_{\mu_1 \ldots \mu_l} = \text{Tr}(\rho \sigma_{\mu_1} \otimes \ldots \otimes \sigma_{\mu_l}) \) with \( \mu_i \in \{0, x, y, z\} \) and \(|\{\mu_i = 0\}| = n - l \). Nonvanishing l-fold correlations indicate that we can infer (with higher probability of success than pure guessing) an \( l \)th measurement result from the product of the other \((l - 1)\) results [see Supplemental Material [14]]. Only in the two particle scenario can we directly use the result from one measurement to infer the other result. For an n-qubit no-correlation state, the vanishing n-particle correlations do not imply vanishing correlations between a smaller number of observers, thus not necessarily destroying predictability. We will
see also in the experimentally implemented example that the various individual results still enable some possibility for inference, which is then largely due to bipartite correlations.

Constructing no-correlation states.—For any state $|\psi\rangle$ with an odd number $n$ of qubits, we can construct an “antistate” $|\bar{\psi}\rangle$, i.e., the state whose $n$-partite correlations are inverted with respect to the initial one. By evenly mixing these states

$$\rho_{\psi}^{nc} = \frac{1}{2} |\psi\rangle \langle \psi| + \frac{1}{2} |\bar{\psi}\rangle \langle \bar{\psi}|,$$

we obtain a state $\rho_{\psi}^{nc}$ without $n$-partite correlations.

The antistate $|\bar{\psi}\rangle$ of a state $|\psi\rangle$ described in the computational basis by

$$|\psi\rangle = \sum_{k_1,\ldots,k_n=0} \alpha_{k_1,\ldots,k_n} |k_1\ldots k_n\rangle,$$

with normalized coefficients $\alpha_{k_1,\ldots,k_n} \in \mathbb{C}$, is given by

$$|\bar{\psi}\rangle = \sum_{k_1,\ldots,k_n=0} (-1)^{k_1+\ldots+k_n} \alpha_{-k_1,\ldots,-k_n}^{\!*} |k_1\ldots k_n\rangle,$$

where the asterisk denotes complex conjugation. This state has inverted correlations with respect to those in $|\psi\rangle$ for every odd number of observers, whereas all the correlation function values for an even number of observers remain unchanged.

$|\bar{\psi}\rangle$ is mathematically obtained from $|\psi\rangle$ by applying local universal-not gates [24]. These gates introduce a minus sign to all local Pauli operators. Therefore, for odd $n$, the correlations of $|\bar{\psi}\rangle$ have opposite sign to those of $|\psi\rangle$. Representing the universal-not gate by $K = \sigma_x \sigma_y \sigma_z$, where $K$ is the complex conjugation operating in the computational basis, i.e., $K(a|0\rangle + b|1\rangle) = a^*|0\rangle + b^*|1\rangle$, indeed, we obtain $N\sigma_x N^\dagger = -\sigma_x$, $N\sigma_y N^\dagger = -\sigma_y$, and $N\sigma_z N^\dagger = -\sigma_z$. Applying $N$ to all the $n$ subsystems, we find the anticipated result $N \otimes \cdots \otimes N |\psi\rangle = |\bar{\psi}\rangle$.

Although $N$ is antiunitary, $|\bar{\psi}\rangle$ is always a proper physical state and can be obtained by some global transformation of $|\psi\rangle$. In general, $N$ can be approximated [25], but if all the coefficients $\alpha_{k_1,\ldots,k_n}$ are real, complex conjugation can be omitted and no-correlation states can be generated by local operations.

This construction principle can be generalized to mixed states using $\tilde{\rho} = N^\otimes n \rho (N^\otimes n)^\dagger$, which changes every pure state in the spectral form to the respective antistate. Evenly mixing $\rho$ and $\tilde{\rho}$ therefore produces a state with no $l$-party correlations for all odd $l$.

One may then wonder whether the principle of Eq. (2) can also be applied to construct a no-correlation state for every state with an even number of qubits. The answer is negative as shown by the following counterexample. Consider the Greenberger-Horne-Zeilinger state of an even number of qubits $|\psi\rangle = (1/\sqrt{2})(|0\ldots 0\rangle + |1\ldots 1\rangle)$. It has nonvanishing $T_{z\ldots z}$, $2^n-1$ multipartite correlations in the $xy$ plane, and also, $2^n-1$ correlations between a smaller number of subsystems, all equal to $\pm 1$. However, for a state with inverted correlations between all $n$ parties (making no assumptions about the correlations between smaller numbers of observers), the fidelity relative to the GHZ state, given by

$$\frac{\sum_{\mu_1,\ldots,\mu_n=0} T_{\mu_1,\ldots,\mu_n}^\text{GHZ} T_{\mu_1,\ldots,\mu_n}^{\text{mix}}}{\sum_{\mu_1,\ldots,\mu_n=0} T_{\mu_1,\ldots,\mu_n}^\text{GHZ}}$$

is negative because more than half of the correlations are opposite. Hence, this state is unphysical and there is no such “antistate”. In fact, so far we were unable to find an antistate to any genuinely multipartite entangled state of even $n$.

Entanglement without correlations: infinite family.—Consider a three-qubit system in the pure state

$$|\phi\rangle = \sin \alpha \cos \beta |001\rangle + \sin \alpha \sin \beta |010\rangle + \cos \beta |100\rangle,$$

where $\alpha, \beta \in (0, \pi/2)$ (which includes the state $|W\rangle$ with $\alpha = \pi/3$ and $\beta = \cos^{-1}(1/\sqrt{3})$). Together with any local unitary transformation thereof, this defines a three-dimensional subspace of genuinely tripartite entangled states within the eight-dimensional space of three qubit states. To show that all the respective no-correlation states $\rho_{\phi}^{nc}$ are genuinely entangled, we use a criterion similar to the one in [6], i.e.,

$$\max_{\rho^{\text{bi-prod}}} \langle T, T^{\text{exp}} \rangle < \langle T, T^{\text{excl}} \rangle \Rightarrow \rho^{\text{exp}} \text{ is not biseparable},$$

where maximization is over all biproduct pure states and

$$\langle U, V \rangle \equiv \sum_{\mu,\nu=0}^3 U_{\mu \nu} V_{\mu \nu}$$

denotes the inner product in the vector space of correlation tensors. Condition [Eq. (6)] can be interpreted as an entanglement witness $W = \alpha^2 - \rho_{\phi}^{nc}$, where $\alpha = L/8$ and $L = \max_{\rho^{\text{bi-prod}}} \langle T, T^{\text{bi-prod}} \rangle$ is the left-hand side of Eq. (6). In the ideal case of preparing $\rho^{\text{exp}}$ perfectly, $T^{\text{exp}} = T$, the right-hand side of our criterion equals four for all the states of the family, and thus, the expectation value of the witness is given by $\text{Tr}(W \rho_{\phi}^{nc}) = (L - 4)/8$.

A simple argument for $\rho_{\phi}^{nc}$ being genuinely tripartite entangled can be obtained from the observation that $|\phi\rangle$ and $|\bar{\phi}\rangle$ span a two-dimensional subspace of the three qubit Hilbert space [9]. As none of the states $|\Phi\rangle = a|\phi\rangle + b|\bar{\phi}\rangle$ is a biproduct (for the proof see Supplemental Material [14]), states in their convex hull do not intersect with the subspace of biseparable states and thus all its states, including $\rho_{\phi}^{nc}$, are genuinely tripartite entangled. To evaluate the entanglement in the experiment, we calculated $L$ for all states of Eq. (5). We obtain $L_{|\phi\rangle} < 4$ in general, with $L_{|W\rangle} = 10/3$. Similar techniques were used to analyze five-qubit systems.

Quantum correlations without classical correlations.—The cumulants and correlations were initially proposed as a measure of genuinely multiparty nonclassicality in Ref. [26]. Kaszlikowski et al. [9], however, showed that such a quantification is not sufficient as the state $\rho_{\phi}^{nc}$ has
vanishing cumulants, yet contains genuinely multiparty entanglement. They suggested that the vanishing cumulants or standard correlation functions [Eq. (1)] indicate the lack of genuine multiparty “classical” correlations. This initiated a vivid discussion on a proper definition and measure of genuine multiparticle “classical” and quantum correlations. Bennett et al. proposed a set of axioms for measures of genuine multiparticle correlations [11]. They showed that the correlation function [Eq. (1)] does not fulfill all the requirements, but also still strive for computable measures that satisfy these axioms [15,27]. An information-theoretic definition of multiparticle correlations was given by Giorgi et al. [15]. Their measure combines the entropy of all sizes of subsystems. Applying their definitions to $\rho_{\text{nc}}$, we obtain genuine classical tripartite correlations of 0.813 bit and genuine quantum tripartite correlations of 0.439 bit resulting in total genuine tripartite correlations of 1.252 bit (see Supplemental Material [14] for calculations for all $\rho_{\text{nc}}$).

While this approach does assign classical correlations in the states $\rho_{\text{nc}}$, it does not fulfill all requirements of [11] either.

**Experiment.**—The three photon state $|W\rangle$ can be observed either using a multiphoton interferometer setup [28] or by suitably projecting the fourth photon of a 4-photon symmetric Dicke state [29]. The latter scheme has the advantage that it also offers the option to prepare the states $|W\rangle$ and $\rho_{\text{nc}}$, respectively. The states $|W\rangle$ and $|W\rangle$ are particular representatives of the symmetric Dicke states, which are defined as

$$|D_n^{(e)}\rangle = \left(n/e\right)^{-1/2} \sum P_i (|H\rangle \otimes |V\rangle)^e,$$

where $|H/V\rangle$ denotes horizontal (vertical) polarization and $P_i$ all distinct permutations, and with the three photon states $|W\rangle = |D_1^{(1)}\rangle$ and $|W\rangle = |D_2^{(2)}\rangle$. We observed four- and six-photon Dicke states using a pulsed collinear type II spontaneous parametric down conversion source together with a linear optical setup (see Fig. 1) [30,31]. The $|D_n^{(e)}\rangle$ states were observed upon detection of one photon in each of the four or six spatial modes, respectively. We characterized the state $|D_4^{(2)}\rangle$ by means of quantum state tomography, i.e., a polarization analysis in each mode, collecting for each setting 26 minutes of data at a rate of 70 events per minute. The fidelity of the experimental state $|D_4^{(2)}\rangle^\text{exp}$ was directly determined from the observed frequencies together with Gaussian error propagation as 0.920 ± 0.005, which due to the high number of detected events [16] is compatible with the value 0.917 ± 0.002 as obtained from a maximum likelihood (ML) reconstruction and nonparametric bootstrapping [14,20]. The high quality achieved here allowed a precise study of the respective states. The fidelities of the observed three qubit states with respect to their target states are 0.939 ± 0.011 for $|W\rangle^\text{nc,exp}$, 0.919 ± 0.010 for $|W\rangle^\text{nc,exp}$, and 0.961 ± 0.003 for $\rho_{\text{nc}}$, respectively. We observed four- and six-photon Dicke states using a pulsed collinear type II spontaneous parametric down conversion source together with a coincidence logic [30].

![FIG. 1 (color online). Schematic of the linear optical setup used to observe symmetric Dicke states from which states with vanishing 3- and 5-partite correlations can be obtained. The photons are created by means of a cavity enhanced pulsed collinear type II spontaneous parametric down conversion source pumped at 390 nm [31]. Distributing the photons symmetrically into six modes by five beam splitters (BS) enables the observation of the state $|D_6^{(3)}\rangle$. Removing beam splitters BS$_3$ and BS$_4$ reduces the number of modes to four and thus the state $|D_4^{(2)}\rangle$ is obtained. State analysis is enabled by sets of half wave (HWP) and quarter-wave plates (QWP) together with polarizing beam splitters (PBS) in each mode. The photons are measured by fiber-coupled single photon counting modules connected to a coincidence logic [30].](image)
Although the three qubits are not tripartite correlated, the bipartite correlations shown above give rise to genuine tripartite entanglement. This can be tested for the experimental states employing Eq. (6). We observe

\[
(T, T_{W}^{nc, \text{exp}}) = 3.858 \pm 0.079 > 3.333,
\]

\[
(T, T_{D}^{nc, \text{exp}}) = 13.663 \pm 0.340 > 12.8,
\]

both above the respective biseparable bound of 10/3 = 3.333 (12.8) by more than 6.6 (2.4) standard deviations, proving that in spite of vanishing full correlations the states are genuinely tripartite (five-partite) entangled [14].

The observed five-photon state has one more remarkable property [13]. For this state, every correlation between a fixed number of observers, i.e., bipartite correlations, tripartite correlations, etc. admits description with an explicit local hidden-variable model [8]. However, some of the models are different and thus cannot be combined in a single one. Using linear programming to find joint probability distributions reproducing quantum predictions [12], we obtain an optimal Bell inequality using only two- and four-partite correlations [13]. From the observed data, we evaluate the Bell parameter to be \( B = 6.358 \pm 0.149 \) which violates the local realistic bound of 6 by 2.4 standard deviations [33]. This violation confirms the nonclassicality [14] of this no-correlation state and also offers its applicability for quantum communication complexity tasks. Contrary to previous schemes, here, the communication problem can be solved in every instance already by only a subset of the communicating parties [35].

Conclusions.---We introduced a systematic way to define and to experimentally observe mixed multipartite states with no \( n \)-partite correlations for odd \( n \), as measured by standard correlation functions. For the first time, we experimentally observed a state which allowed the violation of a Bell inequality without full correlations, thereby proving both the nonclassicality of no-correlation states as well as their applicability for quantum communication protocols. The remarkable properties of these states prompt intriguing questions. For example, what might be the dimensionality of these states or their respective subspaces, or whether we can even extend the subspace of states and antistates which give genuinely entangled no-correlation states? Moreover, can no-correlation states be used for quantum protocols beyond communication complexity, and, of course, whether these remarkable features can be cast into rigorous and easily calculable measures of genuine correlations satisfying natural postulates [11]?

We thank the EU-BMBF Project No. QUASAR and the EU Project No. QWAD and No. QOLAPS for supporting this work. T.P. acknowledges support by the National Research Foundation, the Ministry of Education of Singapore Grant No. RG98/13, Start-up Grant of the
Nanyang Technological University, and NCN Grant No. 2012/05/E/ST2/02352. C. S. and L. K. thank the Elite Network of Bavaria for support (Ph.D. Programs QCCC and ExQM).

tomasz.paterek@ntu.edu.sg

[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. 47, 777 (1935).
[2] A. K. Ekert, Phys. Rev. Lett. 67, 661 (1991).
[3] C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. 70, 1895 (1993).
[4] K. C. Chua, V. Chandran, U. R. Acharya, and C. M. Lim, Medical Engineering and Physics 32, 679 (2010); J. M. Mendel, Proc. IEEE 79, 278 (1991).
[5] C. Hońold, A. Grudka, M. Horodecki, P. Horodecki, and Z. Walczak, Phys. Lett. A 311, 209052 (2011).
[6] P. Badziag, Č. Brukner, W. Laskowski, T. Paterek, and M. Źukowski, Phys. Rev. Lett. 100, 140403 (2008); W. Laskowski, M. Markiewicz, T. Paterek, and M. Źukowski, Phys. Rev. A 84, 062305 (2011).
[7] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. 23, 880 (1969); N. D. Mermin, Phys. Rev. D 22, 356 (1980); H. Weinfurter and M. Źukowski, Phys. Rev. A 64, 010102 (2001); R. F. Werner and M. M. Wolf, Phys. Rev. A 64, 032112 (2001); K. Nagata, W. Laskowski, and T. Paterek, Phys. Rev. A 74, 062109 (2006); W. Laskowski, T. Paterek, M. Źukowski, and Č. Brukner, Phys. Rev. Lett. 93, 200401 (2004); M. Ardehali, Phys. Rev. A 46, 5375 (1992); A. V. Belinskii and D. N. Klyshko, Phys. Usp. 36, 653 (1993); D. Collins and N. Gisin, J. Phys. A 37, 1775 (2004); T. Vértesi, Phys. Rev. A 78, 032112 (2008).
[8] M. Źukowski and Č. Brukner, Phys. Rev. Lett. 88, 210401 (2002).
[9] D. Kaszlikowski, A. Sen De, U. Sen, V. Vedral, and A. Winter, Phys. Rev. Lett. 101, 070502 (2008).
[10] Z. Walczak, Phys. Rev. A 374, 3999 (2010); Z. Walczak, Phys. Rev. Lett. 104, 068901 (2010); D. Kaszlikowski, A. Sen De, U. Sen, V. Vedral, and A. Winter, Phys. Rev. Lett. 104, 068902 (2010); K. Modi, A. Budruch, H. Cable, T. Paterek, and V. Vedral, Rev. Mod. Phys. 84, 1655 (2012).
[11] C. H. Bennett, A. Grudka, M. Horodecki, P. Horodecki, and R. Horodecki, Phys. Rev. A 83, 012312 (2011).
[12] J. Gruca, W. Laskowski, M. Źukowski, N. Kiesel, W. Wieczorek, C. Schmid, and H. Weinfurter, Phys. Rev. A 82, 012118 (2010).
[13] W. Laskowski, M. Markiewicz, T. Paterek, and M. Wieśniak, Phys. Rev. A 86, 052105 (2012).
[14] See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.114.108051 for additional information on physical meaning of the correlation functions, details about experimentally realized states, derivations of entanglement criteria used, and correlation content of the no-correlation states, which includes Refs. [9,13,15–23].

[15] G. L. Giorgi, B. Bellomo, F. Galve, and R. Zambrini, Phys. Rev. Lett. 107, 190501 (2011).
[16] C. Schwemmer, L. Knips, D. Richart, H. Weinfurter, T. Moroder, M. Kleinnmann, and O. Gühne, Phys. Rev. Lett. 114, 080403 (2015).
[17] G. Tóth, J. Opt. Soc. Am. B, 24, 275 (2007).
[18] C. Schwemmer, G. Tóth, A. Niggebaum, T. Moroder, D. Gross, O. Gühne, and H. Weinfurter, Phys. Rev. Lett. 113, 040503 (2014).
[19] R. S. Bennink, Y. Liu, D. D. Earl, and W. P. Grice, Phys. Rev. A 74, 023802 (2006); P. Trojek, Ph.D. thesis, Ludwig-Maximilians-Universität München, 2007.
[20] B. Efron and R. J. Tibshirani, An Introduction to the Bootstrap (Chapman & Hall, London, 1994).
[21] N. Kiesel, Ph.D. thesis, Ludwig-Maximilians-Universität München, 2007.
[22] D. F. V. James, P. G. Kwiat, W. J. Munro, and A. G. White, Phys. Rev. A 64, 052312 (2000).
[23] J. Larsson, Phys. Rev. A 47, 424003 (2014).
[24] V. Bužek, M. Hillery, and R. F. Werner, J. Mod. Opt. 47, 211 (2000).
[25] F. de Martini, V. Bužek, F. Sciarrino, and C. Sias, Nature (London) 419, 815 (2002); J. Bang, S.-W. Lee, H. Jeong, and J. Lee, Phys. Rev. A 86, 062317 (2012).
[26] D. L. Zhou, B. Zeng, Z. Xu, and L. You, Phys. Rev. A 74, 025110 (2006).
[27] L. Zhao, X. Hu, R.-H. Yue, and H. Fan, Quantum Inf. Process. 12, 2371 (2013).
[28] M. Eibl, N. Kiesel, M. Bourennane, C. Kurtseifer, and H. Weinfurter, Phys. Rev. Lett. 92, 077901 (2004).
[29] T. Yamamoto, K. Tamaki, M. Koashi, and N. Imoto, Phys. Rev. A 66, 064301 (2002); W. Wieczorek, N. Kiesel, C. Schmid, and H. Weinfurter, Phys. Rev. A 79, 022311 (2009).
[30] N. Kiesel, C. Schmid, G. Tóth, E. Solano, and H. Weinfurter, Phys. Rev. Lett. 98, 063604 (2007); G. Tóth, W. Wieczorek, D. Gross, R. Krischek, C. Schwemmer, and H. Weinfurter, Phys. Rev. Lett. 105, 250403 (2010); R. Krischek, C. Schwemmer, W. Wieczorek, H. Weinfurter, P. Hyllus, L. Pezzé, and A. Smerzi, Phys. Rev. Lett. 107, 080504 (2011).
[31] R. Krischek, W. Wieczorek, A. Ozawa, N. Kiesel, P. Michelberger, T. Udem, and H. Weinfurter, Nat. Photonics 4, 170 (2010).
[32] W. Wieczorek, R. Krischek, N. Kiesel, P. Michelberger, G. Tóth, and H. Weinfurter, Phys. Rev. Lett. 103, 020504 (2009).
[33] Examples of Bell inequalities involving lower order correlations can be found in [34], however, none of them is violated by our state $\rho^E_{\text{no}}$.
[34] M. Wieśniak, M. Nawareg, and M. Źukowski, Phys. Rev. A 86, 042339 (2012); J. Tura, R. Augusiak, A. B. Sainz, T. Vértesi, M. Lewenstein, and A. Acín, Science 344, 1256 (2014); J. Tura, A. B. Sainz, T. Vértesi, A. Acín, M. Lewenstein, and R. Augusiak, J. Phys. A 47, 424024 (2014).
[35] Č. Brukner, M. Źukowski, J.-W. Pan, and A. Zeilinger, Phys. Rev. Lett. 92, 127901 (2004); P. Trojek, C. Schmid, M. Bourennane, Č. Brukner, M. Źukowski, and H. Weinfurter, Phys. Rev. A 72, 050305 (2005); M. Wieśniak, arXiv:1212.2388.