Making better decisions by applying mathematical optimization to cost accounting: An advanced approach to multi-level contribution margin accounting

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ABSTRACT

The purpose of multi-level contribution margin accounting in cost accounting is to analyze the profitability of products and organizational entities with appropriate allocation of fixed costs and to provide relevant information for short-term, medium- and longer-term decisions. However, the conventional framework of multi-level contribution margin accounting does not usually incorporate a mathematical optimization method that simultaneously integrates variable and fixed costs to determine the best possible product mix within hierarchically structured organizations. This may be surprising in that operations research provides an optimization model in the form of the fixed-charge problem (FCP) that takes into account not only variable costs but also fixed costs of the activities to be planned. This paper links the two approaches by expanding the FCP to a multi-level fixed-charge problem (MLFCP), which maps the hierarchical decomposition of fixed costs in accordance with multi-level contribution margin accounting. In this way, previously hidden optimization potentials can be made visible within the framework of multi-level contribution margin accounting. Applying the linkage to a case study illustrates that the original assessment of profitability gained on the sole basis of a multi-level contribution margin calculation might turn out to be inappropriate or even inverted as soon as mathematical optimization is utilized: products, divisions, and other reference objects for fixed cost allocation, which at first glance seem to be profitable (or unprofitable) might be revealed as actually unprofitable (or profitable), when the multi-level contribution margin calculation is linked to the MLFCP. Furthermore, the proposed concept facilitates assessment of the costs of an increasing variant diversity, which also demonstrates that common rules on how to interpret a multi-level contribution margin calculation may have to be revised in some cases from the viewpoint of optimization. Finally, the impact of changes in the fixed cost structure and other parameters is tested via sensitivity analyses and stochastic optimization.

1. Introduction

In the concept of variable costing (also called direct costing), the contribution margin is calculated by deducting all variable costs from revenues. In the simplest version of variable costing, all fixed costs are then subtracted in one block, resulting in operating income. In German-speaking countries, such as Germany, Austria, and Switzerland, this approach is known as single-level contribution margin accounting. In these countries – especially in Germany – the contribution margin approach has been expanded to a more detailed version: multi-level contribution margin accounting (Berkau, 2020, p. 339), which is also called multistage contribution margin accounting (Friedl et al., 2009, p. 40) or multi-step variable costing (Taschner and Charifzadeh, 2016, p. 231). In multi-level contribution margin accounting, fixed costs are not treated as one block but split into different categories or levels based on accountability – resulting in a hierarchical structure of fixed costs. Commonly used categories may comprise (e.g., Friedl et al., 2005, p. 59; Taschner and Charifzadeh, 2016, pp. 231–232):

- product-related fixed costs
- product group-related fixed costs
- division-related fixed costs
- and, finally, company-wide fixed costs
As illustrated in the right-hand side of Figure 1, these layers of fixed costs are subtracted stepwise. The intermediate contribution margins obtained by this approach are typically indexed using Latin numbers; starting with I after deduction of variable costs, continuing with II after deduction of product-related fixed costs and so on. The abbreviations CM I, CM II and so on are also used throughout this paper. Finally, company-wide fixed costs are subtracted from the last contribution margin in order to obtain the operating income. Note that the hierarchical structure of fixed costs as depicted in Figure 1 is not the only one possible. Depending on the context of application and the purpose of accounting, companies may use different decompositions of fixed costs with different categories and structures of levels (Taschner and Charifzadeh, 2016, p. 232).

Figure 1 shows that multi-level contribution margin accounting is part of a sophisticated cost accounting system, the so-called Grenzplankostenrechnung (GPK). 1 Translated from German, it roughly means “flexible margin costing” (Friedl et al., 2005, p. 56) or “marginal costing” (Sharmar, 2003, p. 32). According to Friedl et al. (2005, p. 56), GPK has become “... arguably the most important cost accounting system for industrial firms in German-speaking countries.” Embedded in enterprise-resource-planning (ERP) systems made by SAP, GPK is widely applied in this area today. Also in other European countries, as well as in the United States, the methods of “German cost accounting” have received considerable attention (Friedl et al., 2009, p. 38). Kajüter and Schröder (2019, p. 17) state that “… there is now also an influence of GPK on US cost accounting.” Resource consumption accounting (RCA), which has been promoted in the United States, is based on principles of GPK, and combines it with elements of activity-based costing (ABC) where appropriate (e.g., Clinton and Webber, 2004, p. 22; White, 2004, p. 6; Thomson and Gurowka, 2005, p. 31; Clinton and van der Merwe, 2006, p. 17; Krumwiede and Suessmair, 2007b).

Referring to the structure of GPK as shown in Figure 1, the three subsystems cost type accounting, cost center accounting, and product cost accounting, together with revenue accounting, provide the information required for executing multi-level contribution margin accounting. In cost type accounting, cost types, such as material, labor, and depreciation, are firstly separated into variable and fixed components. Following this, direct costs (which are always variable costs) are passed straight on to product cost accounting. In contrast, variable indirect costs and fixed indirect costs are handed down to cost center accounting. Here, costs are assigned to relatively small organizational entities, where these costs arise, and allocation rates for variable indirect costs are calculated. After that, only variable indirect costs are passed on through the allocation rates to product cost accounting, where total variable costs for the various products are calculated as sum of direct costs and variable indirect costs. Since GPK assumes linear cost functions throughout, variable costs per unit are constant with respect to output (Friedl et al., 2005, p. 57). Figure 1 broadly outlines how all the pieces of information generated by the described subsystems finally flow into multi-level contribution margin accounting.

Corresponding to the hierarchical decomposition of fixed costs, multi-level contribution margin accounting facilitates analysis of the profitability of products and organizational entities, providing relevant information for short-term as well as medium- and longer-term decisions. However, the conventional framework of multi-level contribution margin accounting does not usually incorporate a mathematical optimization method that simultaneously integrates variable and fixed costs to determine the best possible production and sales program with regard to multiple production and market restrictions. This may be astonishing as the field of operations research provides a model structure that includes both variable costs as well as fixed costs of the activities that are under consideration: the so-called fixed-charge problem (FCP, also called the fixed-cost problem). This paper links the two approaches in order to show how previously hidden optimization potentials can be revealed within the framework of multi-level contribution margin accounting, thus enabling the latter to provide better information for the practitioner’s decision making. Likewise, the proposed linkage addresses scholars by formulating an optimization model and analyzing some conceptual relationships between cost accounting in the form of GPK and mathematical optimization, thus providing a basis for further development at this interface.

The remainder of this paper is organized as follows: In Section 2, I first present the basic FCP and then remodel it as a multi-level fixed-charge problem (MLFCP) that mirrors the hierarchical decomposition of fixed costs in multi-level contribution margin accounting. The model formulated here is classified against the background of the relevant existing literature. I close Section 2 with a conceptual framework for the

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1 GPK was originally developed by Wolfgang Kilger, a cost accounting researcher, and Hans-Georg Plaut, a practitioner (Friedl et al., 2005, p. 56; Friedl et al., 2009, p. 39).

2 Friedl et al. (2009) refer to the following sources: Sharmar (2003); Sharmar and Vilks (2004); White (2004); Smith (2005); Krumwiede (2005); Cheney (2005). See also Kajüter and Schröder (2019, p. 16); Krumwiede and Suessmair (2007a); Krumwiede and Suessmair (2008).
The FCP is a well-known problem in operations research. It describes situations in which the level of activities (for example, production or transportation quantities) is to be decided, taking into account not only variable costs but also fixed costs of the activities. A fixed cost (or fixed charge) is incurred if an activity is executed at any positive level and is not incurred if the activity is not executed, i.e., if its level is zero. (on the left-hand side of a constraint (3), the corresponding binary variable \( y_j \) on the right-hand side is pushed to the value one, otherwise this constraint would be violated. At the same time, the production quantity \( x_j \) is limited to the upper bound \( M_j \). If upper bounds on sales volumes apply to the products, these can be used directly as values for \( M_j \). Conditions (3) then perform a dual function as linking constraints and restrictions on sales volume (e.g., Baker, 2016, p. 234). If there are no upper bounds on sales volumes, suitable values must be determined for \( M_j \). This can be achieved by expression (6), which ensures that the values for \( M_j \) are sufficiently large, but not unnecessarily so.

\[
M_j = \min_{i=1}^{n} \frac{\text{cap}}{a_{ij}}, \quad j = 1, \ldots, n
\]  

(6)

The constraints (4) ensure that all \( x_j \) are nonnegative and, if necessary, integers. Conditions (5) state that all \( y_j \) are binary integer variables (i.e., binary variables). The FCP in the form of (1)-(5) is an integer (linear) programming problem.

Within the scope of this paper, I expand the basic FCP to a multi-level fixed-charge problem that can be related to multi-level contribution margin accounting with any number of levels. As a basis for this, the next section sketches a general structure of multi-level contribution margin accounting.

2.2. General structure of multi-level contribution margin accounting

Figure 2 outlines a generalized contribution margin calculation with any number of levels and any number of reference objects for fixed cost allocation within each level. The product-related fixed costs in row 11 are only incurred if the product in question is produced and in this case are independent of the quantity produced. The product group-related fixed costs in row 13 are only incurred if at least one product (i.e., subordinate reference object) within the product group in question (i.e., superordinate reference object) is produced; otherwise the group is inactive and the associated fixed cost is not incurred. For example, the fixed cost of product group G2 is only incurred if at least one of the products P3, P4, and P5 is produced. This relationship holds for all levels: the fixed cost of a product group is only incurred if at least one product of the group is produced. Therefore, if a product group is inactive, the fixed costs associated with it are not incurred. This relationship is formally described by constraints (6) for each product group.
superordinate reference object is only incurred if at least one of the associated subordinate reference objects is active.

The fixed costs at the various levels of the fixed cost hierarchy may exhibit differing commitment periods, i.e., cutting back the fixed cost of a reference object completely in case of its elimination or closing can require a certain time span; while cutting back product-related fixed costs may be possible rather quickly, this may take comparatively more time in case of division-related fixed costs. However, I refrain from explicitly mapping varying commitment periods in the optimization model formulated below, which is an acceptable simplification in view of the function I attribute to it. This issue is addressed in the interpretation of the results of the case study.

2.3. Formulating, discussing, and classifying the MLFCP

2.3.1. Formulating a general MLFCP

The basis of the notation shown in Table 2 are the levels \( h = 1, \ldots, k \), which correspond to the levels of the fixed cost hierarchy but do not include the level of company-wide fixed costs. For example, \( h = 1 \) can denote the level of the products, \( h = 2 \) the level of the product groups, \( h = 3 \) the level of the divisions, and so on (see rows 3 to 7 in Figure 2). Within each level, there can be any number of reference objects for fixed cost allocation. These reference objects are denoted by \( j_h = 1, \ldots, n_h \). Thus, in continuation of the above example, the index \( j_1 = 1, \ldots, n_1 \) denotes the products, the index \( j_2 = 1, \ldots, n_2 \) the product groups, the index \( j_3 = 1, \ldots, n_3 \) the divisions, etc.

Given the notation in Table 2, I formulate the MLFCP in the following form:

**Model:** Multi-level fixed-charge problem (MLFCP)

\[
\text{max } \text{CM}_{k+1} = \sum_{j_1=1}^{n_1} c_{j_1} x_{j_1} - \sum_{h=2}^{k} \sum_{j_h=1}^{n_h} f_{h,j_h} y_{h,j_h} \tag{7}
\]

Subject to

\[
\sum_{h=1}^{k} a_{i,j_h} x_{j_h} \leq \text{cap}_i \quad i = 1, \ldots, m \tag{8}
\]

\[
x_{j_h} \leq M_{j_h} y_{h,j_h} \tag{9}
\]

\[
\sum_{h=Q(h,j_h)}^{Q(h)} y_{h,j_h} \leq (Q(h,j_h)) y_{h,j_h} \tag{10}
\]

\[
x_{j_1} \geq 0 \text{ (or } x_{j_1} \in \mathbb{N}_0 \text{ if necessary)} \tag{11}
\]

\[
y_{h,j_h} \in \{0;1\} \tag{12}
\]

In the objective function (7), the fixed costs associated with the reference objects at the various levels are deducted from contribution margin \( I \) (CM I), finally resulting in CM \( I + 1 \), which is to be maximized. The reason for the index value \( k + 1 \) is that CM I (or CM 1) is obtained after deduction of variable costs, CM II (or CM 2) after deduction of the fixed costs of level 1, CM III (or CM 3) after deduction of the fixed costs of level 2, and so on. Hence, after deducting the fixed costs associated with the last level \( k \), we obtain CM \( k+1 \), (see also cell A20 in Figure 2). The objective function thus reflects the hierarchical decomposition of the fixed cost block and the gradual subtraction of the fixed costs associated with the levels. Because a \((k+1)-\)level contribution margin calculation includes \( k \) levels of decision-relevant fixed costs, it corresponds to a \( k \)-level fixed-charge problem. Finally, it should also be noted that in the double summation of the objective function (7), the index set chosen in the inner summation \((i.e., j_h = 1, \ldots, n_h)\) depends on the value of the index of the outer summation \((i.e., h)\).

The constraints (8) reflect the restrictions imposed by the production system regarding the available capacities \( \text{cap}_i \) of the resources \( i \). The constraints (9) represent linking constraints, which ensure consistent value assignment for the production quantities \( x_{j_h} \) and the binary variables \( y_{h,j_h} \) of the products \( j_h \). These logical constraints work in the same manner as in the basic FCP described in Section 2.1.

The linking constraints (10) establish logical links between the binary variables \( y_{h,j_h} \) of the various levels \( h \). \( Q(h,j_h) \) represents the set of those reference objects at the subordinate level \( h-1 \), which belong to a particular superordinate reference object defined by a given value of \( h \) and one of \( j_h \). The upper section of Figure 3 illustrates the sets \( Q(h,j_h) \) for an exemplary structure of levels \((h = 1, \ldots, 4)\) and reference objects. Since all index sets for the reference objects start with the same numbering \((i.e., j_h = 1, \ldots, n_h)\), the denotation of the members within the

| A | B | C | D | E | F | G | H | I | J | K | L | M | N | O | P | Q | R | S |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | **Multi-Level Contribution Margin Accounting with any Number of Levels and Reference Objects** | | | | | | | | | | | | | | | | | | |
| 2 |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 3 | **Regions** | **Region R1** | **Region R2** | *** | **Region R3** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 4 | **(further levels)** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 5 | **Divisions** | **Division D1** | **Division D2** | *** | **Division D3** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 6 | **Product groups** | **Group G1** | **Group G2** | **Group G3** | *** | **Group G4** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 7 | **Products** | P1 | P2 | P3 | P4 | P5 | P6 | P7 | P8 | P9 | P10 | P11 | P12 | P13 | P14 | P15 | P16 | P17 | P18 |
| 8 | **Revenues** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 9 | **Variable costs** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 10 | **Contribution Margin I** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 11 | **Product-related fixed costs** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 12 | **Contribution Margin II** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 13 | **Prod. group-rel. fixed costs** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 14 | **Contribution Margin III** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 15 | **Division-related fixed costs** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 16 | **Contribution Margin IV** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 17 | **(further levels)** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 18 | **Contribution Margin k** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 19 | **Region-related fixed costs** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 20 | **Contribution Margin k + 1** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 21 | **Company-wide fixed costs** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| 22 | **Operating income** |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |

Figure 2. A general structure of multi-level contribution margin accounting.
sets $Q(h, j_h)$ is not disjoint across the various levels $h$. This is not an issue, however, as the numbering of the reference objects within each level $h$ is uniquely defined and the variables $y_{h-1,t}$ are uniquely defined by the combination of level index (i.e., $h - 1$) and reference object index (i.e., $t$). On this basis, the constraints (10) perform the function of linking constraints in the following manner: If at least one of the binary variables $y_{h-1,t}$ on the left-hand side of a constraint (10) has the value one (i.e., at least one of the subordinate reference objects within $Q(h, j_h)$ is active), then on the right-hand side the binary variable $y_{h,t}$ of the corresponding superordinate reference object is pushed to the value one. $Q(h, j_h)$ represents the number of subordinate reference objects contained in the set $Q(h, j_h)$. We need this number as coefficient for $y_{h,t}$ in order to allow the possibility that all subordinate reference objects within $Q(h, j_h)$ are active (i.e., that all corresponding binary variables can take on the value one).

### Table 2. Notation for the multi-level fixed-charge problem (MLFCP).

| Indices and sets |  |
|------------------|---|
| $h = 1, \ldots, k$ | Levels (in the following, $h = 1$ denotes the level of products) |
| $j_h = 1, \ldots, n_h$ | Reference objects within each level $h$ (in the following, $j_1 = 1, \ldots, n_1$ denotes the products) |
| $i = 1, \ldots, m$ | Resources |

| $Q(h, j_h)$ | Set of those reference objects at the subordinate level $h - 1$, which belong to a particular superordinate reference object defined by a given value of $h$ and one of $j_h$ ($h = 2, \ldots, k; j_h = 1, \ldots, n_h$) |

| $t \in Q(h, j_h)$ | Set members of $Q(h, j_h)$ |

### Parameters

- $c_{j_1}$: Contribution margin per unit of product $j_1$ |
- $f_{j_h}$: Fixed cost associated with reference object $j_h$ at level $h$ |
- $a_{i,j_1}$: Resource requirement per unit of product $j_1$ with respect to resource $i$ |
- $c_{i}$: Available capacity of resource $i$ |
- $M_{j_1}$: Upper bound on $x_{j_1}$ (“Big M”) |

### Decision variables

- $x_{j_1}$: Production quantity of product $j_1$ |
- $y_{h,t}$: $\begin{cases} 1, & \text{if reference object } j_h \text{ at level } h \text{ is active} \\ 0, & \text{otherwise} \end{cases}$ |

### Figure 3. Exemplary structure of levels, reference objects, and linking constraints (9) and (10).
In continuation of the example in the upper section of Figure 3, the lower section illustrates the linking constraints (10) and, at the very bottom, the linking constraints (9).

The linking constraints (9) and (10) are chained together via the binary variables $y_{h,1}$, which appear on the right-hand side of (9) and on the left-hand side of (10) in case of $h = 2$ (here denoted by $y_{h-1,1}$). The linking constraints (10) are chained together via the binary variables $y_{h,1}$, so putting it all together, a consistent value assignment is guaranteed starting from the production quantities $x_j$ through the binary variables $y_{1,1}$ of the first level $h = 1$ up to and including the binary variables $y_{h,1}$ of the last level $h = k$. The chaining of the linking constraints (9) and (10) as well as the chaining within the linking constraints (10) is indicated in the lower section of Figure 3 by the double-headed arrows. It becomes clear that the structure and the chaining of the linking constraints correspond to the hierarchical decomposition of fixed costs into levels and to the related structure of superordinate and subordinate reference objects for fixed cost allocation.

A peculiarity arises if a superordinate reference object for allocating fixed costs contains only one subordinate reference object, for example, a product group contains only one product or a division contains only one product group. According to the exemplary structure of levels and reference objects in the upper section of Figure 3, product group $j_2 = 7$ only contains product $j_1 = 15$. In this case, either the product-related fixed cost $f_{1,15}$ for product $j_1 = 15$ or the group-related fixed cost $f_{2,7}$ for product group $j_2 = 7$ might be irrelevant. The easiest and most flexible way to deal with this situation is to retain both variables $y_{1,15}$ and $y_{2,7}$ as well as the corresponding linking constraint as in the lower section of Figure 3, and to set either the fixed cost parameter $f_{1,15}$ or $f_{2,7}$ to the value zero.

### 2.3.2. Discussing the relationship between fixed costs and the quantities of resources

The fixed costs $f_{h,1}$ partly depend on the quantities of the resources for production, i.e., on the right-hand sides $c_{a,i}$ of some of the resource constraints (8). This may particularly hold, for instance, for fixed costs associated with machines. The above formulation of the MLFCP does not explicitly consider such relationships, which is an acceptable simplification in the present context for the following reason: If a fixed cost related to the quantity of a resource is categorized as clearly attributable to a certain reference object for fixed cost allocation (e.g., a product group), this implies that the relevant resource is exclusively used by this reference object (and, as the case may be, by its subordinate reference objects). Hence, the resource requirements of products not (directly or indirectly) associated with this reference object must be zero with respect to this resource. This argumentation holds for these products not associated with the eliminated reference object must be zero with respect to this resource. This argumentation holds for these products not associated with the eliminated reference object for a certain reference object for

### 2.3.3. Classification of the model

To the best of the author's knowledge, the following sources in particular can be discussed for a classification of the MLFCP formulated in Section 2.3.1:

At first glance, one may be tempted to classify it as a “multi-level fixed-charge problem” in the sense of Jones and Soland (1969) but this would be essentially only terminological. In contrast to this paper, Jones and Soland concentrate on step costs in such a way that different fixed charges are incurred at different activity levels, resulting in “...a finite number of jump discontinuities...” (Jones and Soland, 1969, p. 67) in the objective function. Furthermore, they consider a vendor selection problem with different set-up costs and scaled unit costs, i.e., the unit price decreases stepwise when a larger quantity is purchased (Jones and Soland, 1969, p. 74–75).

Warszawski and Peer (1973) present a “multi-level fixed-charge model” for optimizing the location of facilities on a building site, taking into consideration that “...on most building sites work is carried out in stages.” (Warszawski and Peer, 1973, p. 36). Erenguc and Benson (1986) and Benson and Erenguc (1988) formulate an “interactive fixed charge linear programming problem”. They consider groups of activities with fixed costs and focus primarily on economies of scale where several activities are executed simultaneously. Possible reference may also be made to the “endogenous fixed charge problem” in Ross (2016, p. 1 and pp. 9–11) and Ross et al. (2018, pp. 267–268). Here, harvest and road reconstruction scheduling decisions are integrated, making “...the revenue and cost structure of the problem endogenous.” (Ross et al., 2018, p. 260). Shavarani (2019) proposes a multi-level facility location problem to determine the best network design for post-disaster humanitarian relief distribution. However, none of these studies address the structure of hierarchically decomposed fixed costs in multi-level contribution margin accounting as described in this paper.

Letchford and Soulé (2020, p. 240) consider “... (not necessarily disjoint) sets of variables...”, where, “... for each set, a fixed charge is incurred if and only if at least one of the variables in the set takes a positive value”. In the context of the present paper, such a set may be related, for instance, to a product group, whose fixed cost is only incurred if at least one of the products within the group is produced. The case of nested sets described by Letchford and Soulé (2020, p. 241) can span a hierarchical structure. However, their paper also does not address multi-level contribution margin accounting as considered in my approach.

Certain references can be made to the “cascading fixed charge structure” of Dykstra and Riggs (1977, pp. 272–273) and Arthur and Dykstra (1980, p. 493) as well as to the related models in Bont (2012, p. 4 and pp. 71–72), Bont et al. (2014) and Bont et al. (2015), all of which deal with the field of timber harvesting. Although the area of application in the sources mentioned is quite different from that in this article, one may see a certain analogy in an abstract view that decisions about activities at downstream levels cause fixed costs at upstream levels. The fixed costs are thus caused by interlinked or “cascading” decisions. Compared to the “cascading fixed charge structure” of Dykstra and Riggs (1977, pp. 272–273), the model presented in Section 2.3.1 contains both a limitation and an extension. A limitation insofar as the assignment between objects or activities of different levels is variable for Dykstra and Riggs and therefore the subject of the decision, while my model assumes a predefined assignment in this respect. For example, in my model the assignment of products as subordinate objects to the product groups as superordinate objects is predefined. As a consequence, however, this requires an extension insofar, as an adequate formal representation of the given hierarchical structure of levels and reference objects has to be incorporated into the model; my above formulation serves this requirement. Furthermore, my notation accommodates any number of levels, while the Dykstra and Riggs model is formulated for a specifically predefined number of levels.

Possible reference may also be made to the comprehensive model of Ruvwwe-Glosenkamp (2014, pp. 105–107), which relates detailed planning of activities and capacities in hospitals to multi-level contribution margin accounting. Again, this model is formulated for a specifically predefined number of levels and configured for the considered area of application. While my model is less complex, my approach aims at providing a conceptual framework for the linkage with cost accounting, within which some general relationships are pointed out.
Finally, because of the hierarchical decomposition of the fixed cost block and the interlinked decisions at the various levels, one may be tempted to classify the MLFCP as a hierarchical planning system as framed in hierarchical production planning (Hax and Candea, 1984, pp. 393–464; Schneeweiss, 2003, pp. 159–203). However, the core elements of a hierarchical planning system involve splitting the overall planning problem into sub-problems and linking these by appropriate coordination mechanisms. By contrast, my above formulation integrates the decision variables of all levels into a monolithic model.

2.4. A framework for the linkage between multi-level contribution margin accounting and MLFCP

Figure 4 provides a conceptual overview of my framework for linking multi-level contribution margin accounting with the MLFCP and highlights important interfaces between the two approaches (“linking framework”). The right part of the figure roughly sketches the subsystems of the cost accounting system GPK as outlined in the Introduction and Figure 1 (the subsystem cost type accounting is not explicitly shown in Figure 4). Within these subsystems, the module at the bottom right of Figure 4 symbolizes the general structure of multi-level contribution margin accounting as displayed in Figure 2. The MLFCP as formulated in Section 2.3.1 is indicated at the middle left of Figure 4. These two modules are illustrated for a hierarchical structure including products, product groups, and divisions as an example. On this basis, the structure of the linking constraints sketched in the MLFCP module corresponds to the hierarchical structure of reference objects indicated in the multi-level contribution margin accounting module. Finally, the framework is completed by resource-related and market-related data in the left of Figure 4.

One of the constitutive features of this framework is that the resource requirements per product unit \((a_{ij})\) simultaneously flow into the resource constraints (8), and, via cost type and product cost accounting, into the objective function (7) of the MLFCP. Regarding the latter relationship, I integrate a detailed resource-based product cost calculation into the framework. In GPK, data on resource consumption are also considered in cost center accounting for planning the variable indirect costs and allocation bases of the cost centers. This eventually results in the

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**Figure 4. Overview of the framework for the linkage and data flows.**

| Resource-Related Data | Market-Related Data |
|-----------------------|---------------------|
| Resource requirements per product unit | Selling prices |
| Available quantities of resources | Upper bounds on sales volumes |
| Decision variables | Lower bounds on sales volumes (if applicable) |
| Objective function | |
| LHS ≤ RHS Resource constraints | |
| LHS ≤ RHS Linking constraints products & upper bounds | |
| LHS ≤ RHS Linking constraints product groups | |
| LHS ≤ RHS Linking constraints divisions | |
| LHS ≤ RHS Lower bounds sales volumes (if applicable) | |

| Multi-Level Fixed-Charge Problem (MLFCP) | Cost Accounting (GPK) |
|-----------------------------------------|-----------------------|
| Decision variables | Variable indirect costs |
| Objective function | + Allocation bases |
| LHS ≤ RHS Resource constraints | = Allocation rates |
| LHS ≤ RHS Linking constraints products & upper bounds | |
| LHS ≤ RHS Linking constraints product groups | |
| LHS ≤ RHS Linking constraints divisions | |
| LHS ≤ RHS Lower bounds sales volumes (if applicable) | Fixed indirect costs |

| Product Cost Accounting | Multi-Level Contribution Margin Accounting |
|-------------------------|-------------------------------------------|
| Direct costs | Divisions D1 D2 |
| + Variable indirect costs | Groups G1 … G5 |
| = Variable costs | Products P1 … P12 |
| | CM I |
| | … |
| | CM IV |

1) The subsystem cost type accounting is not explicitly shown here; for the complete structure of GPK, see Fig. 1.
2) Via cost type accounting.
3) In my implementation, this interface is captured by assuming given values for the allocation rates.
4) Fixed costs partly depend on the quantities of the resources. However, I assume given values for the fixed costs. For a discussion of the relationship between fixed costs and the quantities of resources in the MLFCP, see Section 2.3.2.
5) As far as attributable to reference objects for fixed cost allocation.
6) Optimal values of decision variables are finally fed into multi-level contribution margin accounting. This enables restructuring of the components of the objective function according to the levels and reference objects in multi-level contribution margin accounting.
7) Lower bounds on sales volumes are not considered in the model formulation in Section 2.3.1. They are introduced in Section 3.2.

LHS: left-hand side; RHS: right-hand side
the determination of allocation rates for charging the variable indirect costs of (final) cost centers to products. In cost center accounting, a detailed planning procedure is applied, including diverse cost relationships and allocation of costs between cost centers (Friedl et al., 2005, pp. 57–58; Sharman, 2003, pp. 32–33). However, it is not appropriate to include this level of complexity into the optimization model; in view of the linearity of all cost functions, a more suitable approach is to capture this interface to cost accounting by assuming given allocation rates for the cost centers. Furthermore, I also premise given values for the fixed costs provided by cost center accounting.

As Figure 4 indicates, the optimal solution obtained by the MLFCP is finally fed into multi-level contribution margin accounting. Through this type of linkage, the results from the optimization model can be presented and examined in more detail in the structural framework of multi-level contribution margin accounting: the total contribution margin as a final result of the MLCP (i.e., the objective function value) can be retraced step by step in multi-level contribution margin accounting by restructuring the components of the objective function according to levels and reference objects. In this way, the profit contributions of the various products, product groups, and divisions can be shown separately.

On the basis of the framework with its modules shown in Figure 4 and the subsequent case study, the resource- and market-related data, the data from cost center accounting in the form of allocation rates and fixed costs, the calculation structure of product cost accounting, the MLFCP, and the (originally non-optimized) multi-level contribution margin calculation were all implemented in Excel and formulaically linked together, resulting in an integrated tool for analysis, planning, and interpretation. The MLFCP as one module within this tool was implemented and solved using the Solver4 included in Excel as well as OpenSolver5.

Furthermore, the Frontline Systems Inc. Analytic Solver6 was used throughout this paper. The complete implementation and a comprehensive overview of the related data flows between the various worksheets in three versions (deterministic, stochastic, and robust version) are available for download at Mendeley Data; this material provides a detailed spreadsheet-based concretion of the conceptual framework as shown in Figure 4.

3. Application, results, and discussion

3.1. Case study for linking multi-level contribution margin accounting and MLFCP

The case study introduced below represents an application of the MLFCP with concretion of the indices (i.e., objects of consideration), parameters and decision variables as defined and discussed in Section 2.3. The exemplary specification of the cost parameters and contribution margins is based on the theoretical background of the cost accounting system GPK as outlined in the Introduction, Section 2.4 and subsequent Section 3.1.1.1. The multi-level contribution margin calculations considered below represent concretions of the general structure of multi-level contribution margin accounting as described in Section 2.2 and shown in Figure 2. After presenting a non-optimized initial solution, the multi-level contribution margin calculation is being optimized by means of the MLFCP within the linking framework as outlined in Section 2.4 and illustrated in Figure 4.

Against this background, the purpose of the case study is to illustrate the benefit of using the MLFCP in the sense of providing better information for decision making and to derive general conclusions. The case study considers two datasets in order to demonstrate a variety of effects of the optimization approach.

3.1.1. Case study with data set 1

Greg Greedy and Garry Grabby are the founders of the Greedy&Grabby Corporation. They produce twelve products, which are grouped into five product groups and two divisions. Figure 5 depicts the hierarchical structure of levels and reference objects. This structure represents an excerpt of the exemplary structure in Figure 3. Greg and Garry want to analyze the profitability of the products, product groups, and divisions. Having both completed a master’s degree in Munich a couple of years ago, they remember the concept of GPK and especially multi-level contribution margin accounting.

3.1.1.1. Calculating the variable costs in product cost accounting

Greg and Garry first calculate the variable costs of the products. In this connection, the resources (i = 1, …, m) required for producing the products have to be considered. In the present case, these resources comprise the raw materials A and B (necessary for all products), material C (necessary for the products in group G1), material D (necessary for the products in group G2 & G3), material E (necessary for the products in group G4 & G5), and material F (necessary for all products). Furthermore, all products in divisions D1 and D2 each pass through a multi-stage production process including machining, pre-assembly, assembly, and finishing. The production process in each division requires human resources as well as equipment.

The variable costs of the products are determined in product cost accounting according to the principles of GPK as outlined in the Introduction (see also Figure 1). As shown in Figure 4, this subsystem is an integral part of the linking framework. On this basis, the variable costs in the case study include direct costs (direct material and direct labor costs) as well as variable indirect costs (variable indirect material, manufacturing, and sales costs). The calculation of the direct costs is based on the resource requirements per product unit (a_{ij}) and the related costs per unit of resource consumption. Variable indirect costs are assigned to the products by means of allocation rates in the form of percentages, machine hour rates and other rates, which are determined in cost center accounting (Friedl et al., 2005, p. 58; Smith, 2005, pp. 37–38; for details on the concept of cost centers, see Sharman, 2003, pp. 32–33). The complete detailed product cost calculation for the Greedy&Grabby Corporation as well as all resource- and market-related data addressed here and in the subsequent sections are available for download at Mendeley Data.

The set of resources considered when calculating the variable costs and the set included in the constraints of an optimization model may differ to some extent. An optimization model is aimed at providing relevant information for making best possible decisions with respect to current and future effects important to the decision-maker. Therefore, only decision-relevant costs should be considered in the decision framework. Decision-relevant costs are incurred as a direct result of the decision to be made, arise now or in the future, and differ among the alternatives available to the decision-maker in the considered decision situation. In contrast, sunk costs are irreversible costs, which were determined by past decisions and cannot be changed through current or future decisions. (For the concept of sunk costs and relevant costs for decision making, see, e.g., Lal and Srivastava, 2009, pp. 38–39; Atkinson et al., 2012, pp. 98–100; Hilton and Platt, 2014, pp. 53–54 and 592–593; Datar et al., 2018, pp. 446–449). Sunk costs should be excluded from the optimization model because they necessarily do not depend on the decision variables, i.e., on the decisions to be made. For instance, if a stock of raw material has already been...

4 For further information, see https://support.microsoft.com and https://www.solver.com/excel-solver-online-help (accessed 17 October 2020).
5 For further information, see Mason (2012), and https://opensolver.org (accessed 17 October 2020).
6 For further information, see https://www.solver.com (accessed 17 October 2020).
7 The case study, all persons, and the company in this paper are fictitious and therefore have no connection to any real persons and companies. Any similarities with real persons and companies are purely coincidental and not intended.
purchased and cannot be economically exploited otherwise than in the in-house production process, this is a sunk cost not to be included in the variable costs when deciding on the production quantities – even if this raw material is a scarce resource and in this case definitely has to be included as a constraint in the optimization model. This applies for material A, whose cost per unit of resource consumption is recorded as zero when calculating the variable costs, but is included in the constraints. On the other hand, some resources not subject to limited availability may represent decision-relevant costs and have to be taken into account when determining the variable costs – but there is no need to include them in the constraints. In the case study at hand, this holds true for material F.

As already mentioned, GPK assumes linear cost functions throughout (Sharman and Vikas, 2004, pp. 30–31; Friedl et al., 2005, p. 57). As a consequence, the variable costs per unit – including direct costs and variable indirect costs – are constant with regard to output. Therefore, once the relevant variable costs per unit for each product have been calculated, the variable costs of any planned sales program can be determined through multiplying the variable costs per unit by the considered sales volumes.

### Table 3. Considering resources in the variable costs and constraints.

| Resources     | Material A                  | Material F                  |
|---------------|-----------------------------|-----------------------------|
| Include in variable costs? | no                           | yes                         |
| (sunk cost)    | (decision-relevant cost)     |                             |
| Include in constraints? | yes                          | no                          |
| (subject to limited availability) | (not subject to limited availability) |                             |

### 3.1.1.3. Assessment of the initial multi-level contribution margin calculation

When determining a production and sales program, the limited availability of the resources required for production – i.e., materials, human resources and equipment as mentioned in Section 3.1.1.1 – must be taken into account. Furthermore, the Greedy&Grabby Corporation must consider upper bounds on sales volumes due to maximum demand, which serve to specify the parameters $M_j$. In the present case, a production program that fully utilized all upper bounds on sales volumes would violate all the resource restrictions imposed by the available quantities ($cap_i$) of the resources. The initial production and sales program on which the multi-level contribution margin calculation in Figure 6 is based, on the other hand, is feasible with regard to all production and market restrictions. The available quantities of raw material A (necessary for all products) and material D (necessary for product groups G2 & G3) are completely exhausted. The average resource utilization amounts to 91.2%. Note, however, that due to the fully exhausted stock of raw material A, the quantity of none of the products can be increased without simultaneously reducing the quantity of another product.

In the (non-optimized) multi-level contribution margin calculation shown in Figure 6, the contribution margins I, II, III, and IV each vary to some degree within the levels of products, product groups, and divisions – nevertheless, all contribution margins are in a clearly positive order of magnitude. No urgent need for action regarding the profitability of the products, product groups, and divisions is discernible, because such a need for action would only be indicated at the point when negative contribution margins occur. A deeper insight into profitability can be gained by consulting the contribution margin ratios, i.e., the ratios of contribution margins to the corresponding revenues. Exhibiting relatively low standard deviations (SD), the contribution margin ratios are reasonably well-balanced within the levels. All in all, at first glance everything would seem to be in order with this initial multi-level contribution margin calculation.

On the sole basis of the contribution margin calculation shown in Figure 6, however, it is not possible to judge directly whether the underlying production program is optimal with regard to the scarce capacities and market restrictions. Furthermore, the question arises, whether the described assessment of profitability is actually appropriate. Answering these questions requires applying mathematical optimization based on the MLFCP as formulated in Section 2.3.1 and embedded into the linking framework as shown in Figure 4.

### 3.1.1.4. Comparison of the optimized and non-optimized solutions

While the upper section of Figure 7 shows the non-optimized multi-level contribution margin calculation (identical to Figure 6, repeated here for...
convenient comparison), the multi-level contribution margin calculation in the lower section of Figure 7 was gained by optimization through solving the MLFCP. Referring to the lower section of Figure 7, the optimal production and sales quantities are displayed in row 30. Due to respective formulas, the product-, product group-, and division-related fixed costs in the cells B9:M9, B11:M11, and B13:M13 will only be taken into account if the corresponding binary variables (\(y_h\); \(j_h\)) in the optimization model have the value one. Through this type of implementation, I achieve the above-mentioned restructuring of the components of the objective function according to the levels and reference objects in multi-level contribution margin accounting (see Section 2.4).

The solution shown in the lower section of Figure 7 based on the optimization calculation indicates elimination of the products P1, P5, & P7, and of the entire product group G4, as well as adjusting the production quantities (i.e., sales volumes) of the remaining products. Thereby, the total contribution margin IV can be considerably increased in comparison to the non-optimized contribution margin calculation in the upper section of Figure 7 ($228,874 vs. $128,728 as soon as the respective fixed costs have been cut back completely). This optimization potential is not discernible if the contribution margin calculation according to the upper section of Figure 7 is considered in isolation, because here all reported contribution margins I, II, III and IV are clearly positive and the related contribution margin ratios indicate a reasonably well-balanced pattern of profitability.

Even more remarkable are the following observations in Figure 7: though product group G4 exhibits only a below-average production volume among the five product groups, it achieves by far the highest contribution margin III ($48,938) and simultaneously clearly the highest contribution margin ratio III (22.6%) in the non-optimized contribution margin calculation – nevertheless, product group G4 is proposed for complete elimination as a result of the optimization. By contrast, product group G5, which has the lowest contribution margin III ($18,880) and simultaneously the lowest contribution margin ratio III (17.1%) in the non-optimized calculation, remains in the production program after optimization. (albeit again with significantly changed quantities of the associated products P11 and P12). Product group G5 then turns out to have by far the highest contribution margin IV ($107,956) as well as clearly the highest contribution margin ratio IV (35.4%).

The reason for the above described effects is that in the non-optimized contribution margin calculation, neither the contribution margin nor the contribution margin ratio as determined above considers the resource consumption of a product with respect to the bottlenecks in the production process. By contrast, the MLFCP

- weighs the contribution margins of the products against their requirements with respect to scarce resources,
- determines the production quantities accordingly, and
- simultaneously integrates the interdependent and fundamental decisions as to whether to keep or eliminate products, product groups, and divisions at the various levels of the fixed cost hierarchy (keep-or-drop decisions).

However, the elimination of entire product groups and, as the case may be, of divisions represents only the “last resort”, which should by no means automatically follow from such an optimization calculation and should be carefully weighed up against strategic market considerations, also taking into account possible interdependencies within the product
portfolio (for the latter point, see, e.g., Renn, 2016, p. 39). While determining the production quantities is a somewhat short-term decision, closing a division may require additional analysis with a medium-to-longer-term foundation. Moreover, one should bear in mind that the time spans needed to cut back the fixed costs may differ across the various levels of the fixed cost hierarchy. For these reasons, I prefer to interpret the proposed elimination of entire product groups, divisions, and other complex reference objects as a warning signal that these reference objects appear to lack profitability and therefore require further analysis and, if necessary, action. For such reference objects, before possibly eliminating

Figure 7. Comparison of multi-level contribution margin accounting with and without optimization (data set 1).
them, one should first examine whether an improvement in profitability can be achieved, for example, by reducing the resource requirements of the associated products and/or by reducing fixed costs at the respective levels in the medium term.

Finally, applying the MLFCP will often lead to a change in the calculated degree of operating leverage (for the concept of operating leverage, see, e.g., Eldenburg and Wolcott, 2011, pp. 104–105; McNair-Connolly and Merchant, 2017, pp. 118–119; Drury, 2018, pp. 184–185; Drury, 2019, pp. 63–65). As mentioned above, the MLFCP optimizes the production (i.e., sales) quantities, which in many cases will result in a change of the total variable costs and contribution margins, and at the same time it can considerably influence the total fixed costs by integrating keep-or-drop decisions regarding products, product groups, and other reference objects for fixed cost allocation. In the non-optimized contribution margin calculation shown in the upper section of Figure 7, the degree of operating leverage is 1.64 (total contribution margin IV divided by operating income = $128,728/$78,728), while the corresponding figure for the optimized contribution margin calculation according to the lower section of Figure 7 is 1.28 ($228,874/$178,874). The ratio of total fixed costs to total variable costs has decreased in the optimized version, which is due to lower fixed costs as a consequence of the proposed elimination of several reference objects. Note, however, that the higher degree of operating leverage in the non-optimized calculation refers to a far lower absolute level of operating income.

3.1.2. Case study with data set 2

While the first data set started out with a multi-level contribution margin calculation where at first glance “everything seems to be in order”, I now proceed with a modified data set that indicates a serious lack of profitability for some reference objects and exhibits a negative operating income in the non-optimized calculation. In this way, I can demonstrate by example further effects of applying mathematical optimization.

3.1.2.1. Variation of the case study. The Greedy&Grabby Corporation is facing heavy competition. Therefore, Greg and Garry have to revise most of their original estimates of the selling prices downwards, except for the products P1 and P3. Furthermore, Hugo Hoggish, head of cost center accounting at the Greedy&Grabby Corporation, has revised the fixed cost calculation of product group G5 and division D2. All other data remain unchanged. The upper section of Figure 8 shows the non-optimized contribution margin calculation, taking into account the new data but retaining the original (non-optimized) sales volumes as displayed in Figure 6. The clearly negative contribution margins III of product group G4 and IV of division D2 indicate a serious lack of profitability of this product group and division. Garry suggests closing the entire division D2 because then the operating income would increase by $19,737, resulting in a positive figure. Based on the assumption, that the fixed cost of each reference object in question can be cut back within certain periods of time, this is true. However, is it really a good idea to close the entire division D2?

3.1.2.2. Comparison of the optimized and non-optimized solutions. In addressing the question, the lower section of Figure 8 shows the optimal solution obtained by solving the MLFCP. In contrast to a non-optimized multi-level contribution margin calculation, negative contribution margins cannot occur in the optimized version (unless lower bounds on sales volumes are imposed; this will be dealt with in Section 3.2). The optimal solution indicates retaining division D2 and adjusting the production quantities, whereby the contribution margin IV of division D2 and the total contribution margin IV can be considerably increased in comparison to the non-optimized contribution margin calculation in the upper section of Figure 8 ($50,970 vs. −$19,737 and $154,045 vs. $33,123, respectively). This optimization potential is not discernible if the contribution margin calculation according to the upper section of Figure 8 is considered in isolation. It should also be noted that in the optimal solution, the contribution margin ratio IV is 18.9% for division D2 and 17.5% for division D1. This means that division D2, which was originally deemed seriously unprofitable, turns out to be even more profitable – in relation to the revenues – than division D1.

3.1.3. Main finding from the exemplary analysis

The analysis carried out on the basis of data set 1 and data set 2 has illustrated that the proposed concept facilitates exploitation of previously hidden optimization potentials. The original assessment of profitability gained on the sole basis of a multi-level contribution margin calculation might turn out to be inappropriate or even inverted as soon as mathematical optimization is applied:

Products, product groups, divisions, and other reference objects for fixed cost allocation, which at first glance seem to be profitable (or unprofitable) on the sole basis of a multi-level contribution margin calculation may be revealed as actually unprofitable (or profitable), when the calculation is linked to the MLFCP.

3.2. Lower bounds on sales volumes and the costs of variant diversity

I now continue with data set 1 as a basis and discuss the effects of introducing lower bounds on sales volumes.

3.2.1. Continuation of the case study on the basis of data set 1

Greg and Garry are discussing the Greedy&Grabby Corporation strategy. Greg strongly supports the optimal solution shown in the lower section of Figure 7. Garry counters that in this solution only six out of twelve products are produced and sold, and product group G4 is not even offered. He believes that all market segments should be served, i.e., all products in all groups should be offered. Greg points out, however, that variant diversity often entails high costs. How can Greg and Garry visualize the costs of the variant diversity in the present model framework?

3.2.2.1. Introducing lower bounds on sales volumes. The MLFCP formulated in Section 2.3.1 allows the elimination of products and product groups as well as the closure of divisions and other reference objects within the scope of optimization, in which the above-mentioned signal function is considered with regard to lack of profitability. However, actually eliminating products (i.e., specific variants) within a product group or eliminating entire product groups leads to a lower degree of variant diversity of the sales program. If one wants to know what the optimal production and sales program looks like while retaining all products (or at least a desired part of products), one can include corresponding lower bounds on sales volumes in the model. They can easily be stated in the following form:

\[ x_{ji} \geq LB_{ji} \quad j = 1, \ldots, n_j \]

The lower bounds on sales volumes stated in (13) can be interpreted as a strategy of increasing variant diversity, as they force all products to be produced and offered (provided that all \( LB_{ji} > 0 \)). Given the now always positive quantities \( x_{ji} \), the corresponding binary variables \( y_{n1,j} \) are continuously pushed to the value one by the linking constraints (9), whereby the binary variables \( y_{n2,j} \) of all higher levels are sufficiently also set to the value one according to the linking constraints (10). Therefore, the fixed costs of all reference objects (i.e., product-related, product group-related, division-related fixed costs, etc.) within this model are no longer relevant for decision making, so that one could

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8 In the related Excel implementation, these constraints have been reformulated as equivalent less-than-or-equal-to constraints. This corresponds with the illustration of these constraints in the MLFCP module in Figure 4.
ignore them and remove all binary variables and linking constraints from the model (in doing so, simple upper bounds on sales volumes would then have to be formulated instead of the constraints (9), if applicable). However, all binary variables, fixed cost parameters and linking constraints are retained for the analysis for two reasons: first, the complete model (7)–(13) represents a flexible model framework in which the right-hand sides of the lower bounds (13) can be set either to values greater than zero or to the value zero, depending on the respective purpose of the analysis; in the latter case the fixed costs of the various reference objects are certainly relevant for decision making and the corresponding binary

Figure 8. Comparison of multi-level contribution margin accounting with and without optimization (data set 2).
variables as well as the linking constraints are required, because \( x_j = 0 \) is permitted and as a consequence, products as well as product groups, divisions, etc. may be eliminated. Secondly, the fixed costs of the various reference objects are also relevant for decision making, when solutions are compared which are based on optimization either with or without such lower bounds. This comparison is made in the next section.

### 3.2.1.2. Comparison of the solutions with and without lower bounds on sales volumes.

I use data set 1 and add, for illustration and convenience, identical lower bounds \( LB_j = 100 \) on the sales volumes for all products. Figure 9 shows the results of the optimization calculation, again displayed in the format of multi-level contribution margin accounting.

Both in the multi-level contribution margin calculation without optimization according to the upper section of Figure 7 and in the case of optimization with lower bounds according to Figure 9, all products are basically produced; however, the optimization leads to differing quantities and to a significantly higher total contribution margin IV ($152,420 vs. $128,728). This optimization potential again remains unrecognized if the multi-level contribution margin calculation according to the upper section of Figure 7 is considered in isolation.

It is notable that in the non-optimized contribution margin calculation according to the upper section of Figure 7, all contribution margins across the levels are in a clearly positive order of magnitude – which signals that “everything seems to be in order” – whereas in the optimized contribution margin calculation according to Figure 9, several negative contribution margins occur – which would normally indicate that “something is not in order” (see the contribution margins II of products P1, P3, and P5). Hence, applying the MLFCP demonstrates that common rules on how to interpret a multi-level contribution margin calculation might not be appropriate in all cases.

As already mentioned above, the lower bounds on sales volumes can be interpreted as a strategy of variant diversity, as they force all products to be produced and offered. However, the optimization with these lower bounds as shown in Figure 9 leads to a significantly lower total contribution margin IV than the optimization without lower bounds as shown in the lower section of Figure 7 ($152,420 vs. $228,874); in the latter solution, as we have seen, six of a total of twelve products (and as a result, one of a total of five product groups) are proposed for elimination. The comparison of the two solutions thus shows the costs of the desired variant diversity in the sense of a loss in contribution margin. This loss must be weighed against the hoped-for benefit of the variant diversity not shown in the model. In this sense, the optimization calculation can also serve to put the increasing variant diversity, which can be observed in many cases, to the test.

### 3.2.1.3. A compromise proposal.

Greg and Garry continue discussing the Greedy & Grabby Corporation strategy regarding the desirable degree of variant diversity. Greg suggests the following compromise:

- a) At least eight out of the twelve products should be produced;
- b) out of each product group, at least one product should be produced;
- c) if a product is produced, the minimum quantity should be 100 units.

What effect on the total contribution margin IV would this compromise have?

Greg’s suggestion can be expressed formulaically as follows:

\[ \text{Sales volume (units)} \times \text{Selling price per unit ($)} - \text{Variable cost per unit ($)} = \text{Contribution margin ($)} \]

\[ \text{Contribution margin (units)} = \text{Sales volume (units)} \times \frac{\text{Selling price per unit ($)} - \text{Variable cost per unit ($)}}{\text{Quantity (units)}} \]

\[ \text{Total contribution margin IV} = \sum \text{Contribution margin (units)} \]

As already mentioned above, the lower bounds on sales volumes can be interpreted as a strategy of variant diversity, as they force all products to be produced and offered. However, the optimization with these lower bounds as shown in Figure 9 leads to a significantly lower total contribution margin IV than the optimization without lower bounds as shown in the lower section of Figure 7 ($152,420 vs. $228,874); in the latter solution, as we have seen, six of a total of twelve products (and as a result, one of a total of five product groups) are proposed for elimination. The comparison of the two solutions thus shows the costs of the desired variant diversity in the sense of a loss in contribution margin. This loss must be weighed against the hoped-for benefit of the variant diversity not shown in the model. In this sense, the optimization calculation can also serve to put the increasing variant diversity, which can be observed in many cases, to the test.
a) \[ \sum_{j_1=1}^{n_1} y_{1,j_1} \geq p \quad j_1 = 1, \ldots, n_1 \] (14)

b) \[ \sum_{(Q,j_2) \in A} y_{1,j_2} \geq 1 \quad j_2 = 1, \ldots, n_2 \] (15)

c) \[ x_{j_1} \geq LB_{j_1} \quad j_1 = 1, \ldots, n_1 \] (16)

For Greg's compromise proposal, \( p \) in (14) is specified by the value eight and \( LB_{j_1} \) in (16) by the value 100. If the binary variable \( y_{1,j_1} \) on the right-hand side of a constraint (16) has the value one (i.e., the product is produced), the corresponding production quantity \( x_{j_1} \) on the left-hand side must at least amount to \( LB_{j_1} \), otherwise this constraint would be violated. Note that the constraints (13) are omitted (or their right-hand sides are set to zero), if the constraints (14)-(16) apply. Figure 10 shows the results of the optimization calculation, once again displayed in the format of multi-level contribution margin accounting.

According to Figure 10, the total contribution margin IV now amounts to $192,859 which lies between the figures for optimization without any lower bounds on sales volumes on the one side ($228,874) and optimization with lower bounds for all products on the other side ($152,420). Again, negative contribution margins occur in the optimal solution, one of them in a significant order of magnitude (see the contribution margin III of product group G4).

### 3.2.2. Main findings from the exemplary analysis

#### 3.2.2.1. The costs of variant diversity

Table 4 summarizes the analysis carried out in this section. Optimization without any lower bounds on sales volumes is pragmatically characterized as a low degree of variant diversity, optimization with lower bounds on sales volumes of all products as a high degree, and Greg's compromise proposal as a medium degree of variant diversity. With an increasing degree of variant diversity, the total contribution margin IV decreases considerably. As mentioned in Section 3.2.1.2, this loss in contribution margin can be interpreted as costs of the desired degree of variant diversity.

#### 3.2.2.2. Interpreting a multi-level contribution margin calculation

Finally, introducing lower bounds on sales volumes has also demonstrated that common rules on how to interpret a multi-level contribution margin calculation with respect to the occurrence of positive and negative contribution margins might not be appropriate in all cases from the viewpoint of optimization. While a non-optimized and therefore considerably suboptimal contribution margin calculation may seem appealing due to positive contribution margins throughout, the

| Degree of variant diversity | Number offered | Contribution Margin IV | Loss (cumulative) |
|-----------------------------|----------------|------------------------|-------------------|
| Products                    | Product groups |                       |                   |
| Low                         | 6              | 4                      | 228,874           | 0                 |
| Medium                      | 8              | 5                      | 192,859           | 36,015            |
| High                        | 12             | 5                      | 152,420           | 76,454            |

Figure 10. Multi-level contribution margin accounting with optimization (data set 1 complemented by Greg's compromise proposal).
Table 5. Allowable ranges for changes in the fixed costs of single reference objects guaranteeing stability of all decisions and impact on CM IV.

| Percentage change in the fixed cost of... | Allowable ranges guaranteeing stability of all decisions\(^\dagger\) | Impact of percentage changes in the fixed costs on CM IV |
|-----------------------------------------|---------------------------------------------------------------|-------------------------------------------------------|
|                                        | +10% | +20% | +30% |
| **Products**                            |      |      |      |
| P1 [-43%; ∞\%]                          | 0.0% | 0.0% | 0.0% |
| P2 [-100%; 279\%]                       | -0.6%| -0.6%| -0.9%|
| P3 [-100%; 82\%]                        | -0.5%| -0.7%| -1.0%|
| P4 [-100%; 98\%]                        | 0.0% | 0.0% | 0.0% |
| P5 [-75%; ∞\%]                          | 0.0% | 0.0% | 0.0% |
| P6 [-100%; 101\%]                       | -0.7%| -0.7%| -1.1%|
| P7 [-22%; ∞\%]                          | 0.0% | 0.0% | 0.0% |
| P8 [-80%; ∞\%]                          | 0.0% | 0.0% | 0.0% |
| P9 [-82%; ∞\%]                          | 0.0% | 0.0% | 0.0% |
| P10 [-64%; ∞\%]                         | 0.0% | 0.0% | 0.0% |
| P11 [-100%; 100\%]                      | -0.3%| -0.7%| -1.0%|
| P12 [-100%; 89\%]                       | -0.4%| -0.8%| -1.2%|
| **Product groups**                      |      |      |      |
| G1 [-100%; 236\%]                       | -0.6%| -1.2%| -1.8%|
| G2 [-100%; 52\%]                        | -0.6%| -1.2%| -1.8%|
| G3 [-100%; 82\%]                        | -0.5%| -0.9%| -1.4%|
| G4 [-42%; ∞\%]                          | 0.0% | 0.0% | 0.0% |
| G5 [-100%; 53\%]                        | -0.7%| -1.3%| -2.0%|
| **Divisions**                           |      |      |      |
| D1 [-100%; 600\%]                       | -1.0%| -1.9%| -2.9%|
| D2 [-100%; 574\%]                       | -0.7%| -1.4%| -2.1%|

\(^\dagger\) Increments of one percentage point were used for the optimization runs. Reductions exceeding -100\% are not considered as this would imply negative fixed costs.

optimized version may contain (single) negative contribution margins in a significant order of magnitude, which are accepted for the sake of the overall optimum.

4. Sensitivity analysis and stochastic optimization

This section continues with the case study on the basis of data set 1 as introduced in Section 3.1.1 and refrains from lower bounds on sales volumes. Hence, the optimal solution shown in the lower section of Figure 7 serves as standard for comparison for the following considerations.

4.1. Sensitivity analysis

4.1.1. Conditions and techniques for sensitivity analysis of integer programming models

The optimal solution for an optimization model depends largely on the underlying parameter values. Therefore, it is often recommended to investigate the effects of changed parameter values on the optimal solution and on the conclusions drawn within the framework of what-if analyses. These analyses can help identify critical parameters, which strongly influence the optimal solution and therefore should carefully be monitored. It should be noted, however, that the interpretation of dual values (or shadow prices), which can be consulted for sensitivity analyses in continuous linear programming models, cannot be maintained in integer programming models due to their discrete nature. Nevertheless, sensitivity analyses can be carried out assuming various scenarios for the parameter values. Data set 2 in Section 3.1.2 can be regarded as an alternative scenario to data set 1. A more systematic approach for constructing a variety of scenarios is facilitated by add-ins and software packages that automatically vary parameter values by user-defined increments within user-defined ranges. The SolverTable add-in\(^\text{10}\) invokes the Solver included in Excel for automatically repeated optimization across varying parameter values. The Frontline Systems Inc. Analytic Solver provides several sensitivity, optimization and simulation reports as well as corresponding charts. In Analytic Solver, an optimization report based on the PsiOptParam() function automatically runs multiple optimizations for varying parameter values. A sensitivity analysis based on the PsiSenParam() function does not run a new optimization but shows how a formula-dependent cell changes when parameters are varied (for both, see Frontline Systems, 2020, p. 304; more reports are available). For additional examination, I also used OpenSolver for manual initiation of specific optimization runs with respect to a limited number of different parameter values.

4.1.2. Sensitivity to changes in the fixed costs

In the present context, the fixed costs are a matter of particular interest. Taking data set 1 as a basis, I first investigated the effects of changes in the fixed cost of each single product, product group, and division (i.e., of the reference objects for fixed cost allocation) on the optimal decisions. These decisions include the keep-or-drop decisions for the respective reference object itself and all other reference objects as well as the decisions regarding the production quantities. In the sense of testing the robustness, I ask the following question: within which ranges may the fixed costs be varied without any of the mentioned decisions being changed in the optimal solution? (the objective function value, i.e., the total contribution margin IV, may change, of course). To answer this question, I considered percentage changes in the fixed cost of each single reference object using increments of one percentage point, ran multiple optimizations and tracked any changes in the decisions. The left part of Table 5 shows the results. The considerable width of the allowable ranges for percentage changes guaranteeing\(^\text{11}\) stability indicates the robustness of the optimal decisions with respect to changes in the fixed costs, making the use of the model reliable for management in the present case study.

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\(^{10}\) For further information, see Albright and Winston (2012), pp. 87–94, and https://kelley.iu.edu/albrightbooks/Free_downloads.htm (accessed 17 October 2020).

\(^{11}\) My approach does not provide a strict mathematical proof for stability across the ranges reported in Tables 5, 6, and 7. However, from a practical viewpoint the increments used seem small enough to provide fairly reliable results.
I also investigated the impact of changes in the fixed costs on the optimal objective function value. According to the right part of Table 5, an increase of 10% in the fixed cost of each single reference object leads to a decrease of total contribution margin IV (CM IV) by no more than 1%. In the case of increases of 20% and 30%, CM IV decreases by no more than 1.9% and 2.9%, respectively. The optimal decisions remain unchanged, because the increases considered lie within the ranges guaranteeing their stability.

Next, as shown in Table 6, I tested percentage changes in the fixed costs for several sets of reference objects still using increments of one percentage point. Again, the considerable width of the allowable ranges for percentage changes guaranteeing stability indicates the robustness of the optimal decisions with respect to changes in the fixed cost structure. Regarding the impact on the objective function value, a simultaneous increase in the fixed costs of all reference objects by 10% results in a decrease of CM IV by 6%. In the case of increases of 20% and 30%, CM IV decreases by 12% and 18%, respectively. Again, the optimal decisions remain unchanged.

Do the above observations regarding the robustness of the optimal decisions imply that the fixed costs are not specifically relevant for decision making in the present case? In fact, the opposite is true. For test purposes, I also applied a (potentially suboptimal) two-stage solution process: In the first stage, the problem was solved while ignoring all fixed costs, i.e., assuming that all fixed cost parameters have the value zero. Then, in the second stage the fixed costs of all actually active reference objects were simply added without a new optimization. This two-stage procedure led to quite a different solution with a significant decrease of CM IV by 6%. However, once the optimal decisions have been made and implemented, it might not be possible to revise them in the short-term. Again, this result points out the need for accurate market observation.

4.2. Stochastic optimization

4.2.1. Uncertainty in the objective function coefficients

The sensitivity of the optimal production quantities and keep-or-drop decisions to variations in the selling prices of individual products as discovered above motivates applying stochastic optimization to the problem – here meant as modeling an optimization problem that considers uncertainty in several parameters (for an overview and detailed methods of stochastic optimization, see, e.g., Marti, 2015; Kall and Mayer, 2011). Moreover, because my integrated framework as shown in Figure 4 includes a detailed product cost calculation as outlined in Section 3.1.1.1, it enables investigation of the impact of uncertainty not only at the selling side, but also at the purchasing side. One can incorporate probability distributions, for instance, for selling prices as well as for

| Area          | Percentage changes in the fixed costs of... | Allowable ranges guaranteeing stability of all decisions** |
|---------------|--------------------------------------------|----------------------------------------------------------|
| within D1 and D2 | all products                                 | [-67%; 261%]                                            |
|               | all product groups                           | [-100%; 52%]                                            |
|               | all divisions                                | [-100%; 574%]                                           |
|               | all products and product groups              | [-67%; 59%]                                             |
|               | all reference objects                        | [-67%; 59%]                                             |
| only within D1 | all products in division D1                  | [-67%; 179%]                                            |
|               | all product groups in division D1            | [-100%; 52%]                                            |
|               | all divisions, i.e. only division D1         | [-100%; 600%]                                           |
|               | all products and product groups in division D1| [-67%; 43%]                                            |
|               | all reference objects in division D1         | [-67%; 43%]                                             |
| only within D2 | all products in division D2                  | [-92%; 525%]                                            |
|               | all product groups in division D2            | [-80%; 613%]                                            |
|               | all divisions, i.e. only division D2         | [-100%; 574%]                                           |
|               | all products and product groups in division D2| [-69%; 287%]                                           |
|               | all reference objects in division D2         | [-69%; 191%]                                            |

* Identical percentage changes were applied to all reference objects within the sets.
** Increments of one percentage point were used for the optimization runs. Reductions exceeding -100% are not considered as this would imply negative fixed costs.
purchase prices of individual raw materials and parts, for labor costs and other aspects. Since no empirical data are available in this case study, I assume triangular distributions for the selling prices of all products and for the purchase prices of all materials. The triangular distribution is defined by three parameters: lower bound $a$, upper bound $b$, and mode $c$ where the density function takes its maximum. Once again based on data set 1, I specified these parameters for all products and materials as shown in Table 8, where the simplified symbol $p$ denotes either the current selling or purchase price of the individual product or material.

Table 7. Allowable ranges for changes in the selling prices guaranteeing stability and impact on CM IV.

| Percentage change in the selling price of... | Allowable ranges guaranteeing stability of the decisions regarding... | Impact of percentage changes in the selling prices on CM IV when... |
|------------------------------------------|---------------------------------------------------------------|---------------------------------------------------------------|
| production quantities                   | keep-or-drop decisions of...                                 | keeping old optimal decisions                                  |
|                                        | products groups divisions                                   | switching to new optimal decisions                            |
|                                        | -10% -20% -30%                                             | -10% -20% -30%                                               |
| P1 [-100%; 3%]                          | [-100%; 3%] [-100%; 3%]                                   | 0.0% 0.0% 0.0%                                               |
| P2 [-9%; 0%]                            | [-100%; 3%] [-100%; 3%]                                   | -6.9% -13.8% -20.6%                                          |
| P3 [-2%; 0%]                            | [-100%; 3%] [-100%; 3%]                                   | -4.2% -8.3% -12.5%                                           |
| P4 [-3%; 0%]                            | [-100%; 6%] [-100%; 6%]                                   | -7.9% -15.7% -23.6%                                          |
| P5 [-100%; 5%]                          | [-100%; 5%] [-100%; 5%]                                   | 0.0% 0.0% 0.0%                                               |
| P6 [0%; 0%]                             | [-100%; 5%] [-100%; 5%]                                   | -7.7% -15.4% -23.1%                                          |
| P7 [-100%; 4%]                          | [-100%; 4%] [-100%; 4%]                                   | 0.0% 0.0% 0.0%                                               |
| P8 [-100%; 4%]                          | [-100%; 5%] [-100%; 5%]                                   | 0.0% 0.0% 0.0%                                               |
| P9 [-100%; 4%]                          | [-100%; 4%] [-100%; 4%]                                   | 0.0% 0.0% 0.0%                                               |
| P10 [-100%; 4%]                         | [-100%; 4%] [-100%; 4%]                                   | 0.0% 0.0% 0.0%                                               |
| P11 [-3%; 0%]                           | [-100%; 4%] [-100%; 4%]                                   | -6.3% -12.7% -19.0%                                          |
| P12 [-3%; 0%]                           | [-100%; 4%] [-100%; 4%]                                   | -7.0% -14.0% -21.0%                                          |

* Increments of one percentage point were used for the optimization runs. Reductions exceeding -100% are not considered as this would imply negative selling prices.

Table 8. Specifications for the triangular distributions of selling and purchase prices.

| Skewness             | Expectations    | Parameters of triangular distributions* |
|----------------------|-----------------|---------------------------------------|
|                      | Selling prices  | Purchase prices                        |
| symmetric            | normal          | normal                                 |
| right-skewed         | pessimistic     | optimistic                              |
| left-skewed          | optimistic      | pessimistic                            |
|                      | $a$             | $b$                                    |
| symmetric            | 0.5p            | 1.4p                                   |
| right-skewed         | 0.5p            | 0.8p                                   |
| left-skewed          | 0.5p            | 1.2p                                   |

* $p$ denotes either the current selling or purchase price of the individual product or material.

Table 9 shows changes in the optimal production quantities in four out of the nine scenarios compared to the deterministic case in the lower section of Figure 7. However, an influence on the optimal keep-or-drop decisions of individual products can be observed in only one scenario and the optimal keep-or-drop decisions for the product groups and divisions remain unchanged across all possible combinations of expectations. This stability may to a certain extent relativize the picture obtained by the sensitivity analysis for selling prices in the previous section.

Again, the formulaically linked multi-level contribution margin calculation allows for restructuring the components of the objective function of the stochastic MLFCP according to levels and reference objects. Now, the expected profit contributions of the various products, product groups, and divisions can be made visible. (Not explicitly shown here, see the above-mentioned implementation available for download).

Further managerial insight can be gained by consulting the correlation between the total contribution margin IV and the selling prices on the one hand and the purchase prices on the other hand. Based on 2,500 simulation trials in Analytic Solver for the scenario with normal expectations on the selling and purchasing side, Figure 11 indicates which of the skewed distributions in the sense of scenarios; they correspond to all combinations of normal, pessimistic, and optimistic expectations. Table 9 shows these nine scenarios. For each scenario, the related – originally stochastic – optimization model is transferred by Analytic Solver into its deterministic equivalent linear program. The complete implementation (stochastic version) is available for download at Mendeley Data.

According to Table 9, the expected value of total contribution margin IV exhibits a considerable spread from $149,880 in case of pessimistic expectations on the selling and purchasing side up to $308,770 in case of optimistic expectations throughout. The lowest expected value comes with the highest coefficient of variation. A closer look reveals that the expectations on the selling side have a stronger impact on the total contribution margin IV than those on the purchasing side.

Table 9 shows changes in the triangular distributions of selling and purchase prices.
the uncertain parameters have the greatest influence on the total contribution margin IV. At the selling side, the products P6, P4, and P2 have the strongest impact. At the purchasing side, this holds for material F. This outcome can provide useful guidance for management insofar as it calls attention to the most relevant products and materials. The selling prices of products with high impact on total contribution margin IV deserve a decisive price strategy and careful monitoring. Likewise, the purchase prices of the materials with the strongest impact deserve particular attention and – if applicable – should be negotiated with adequate rigor. Finally, as already demonstrated above, the higher correlations on the selling side indicate that this side has a comparatively stronger influence than the purchasing side.

4.2.2. Uncertainty in the constraint and objective function coefficients

Finally, while keeping the symmetric triangular distributions for the selling and purchase prices, I additionally investigated the effect of uncertainty in some of the constraint coefficients. The assumption was made that due to quality issues, the processing times in the pre-assembly of division D1 are uncertain and may take up to 50% longer compared to the current processing times. From a conceptual viewpoint, it is notable in the present application that this uncertainty not only has an impact on the usage of the available capacity. In the integrated framework developed here, as shown in Figure 4, the resource requirements not only flow into the constraints, but via product cost accounting also into the objective function coefficients. As a consequence, the uncertainty in the mentioned constraint coefficients also contributes to the uncertainty in the objective function coefficients of all products made in division D1. Assuming right-skewed triangular distributions for the processing times, I set the lower bounds to 80% of the current processing times, the modes to 100% and the upper bounds to 150%. I now formulate the pre-assembly constraint for division D1 as a chance constraint that will be satisfied in most (but not necessarily all) cases across the realizations of the uncertain factors. We can demand, for example, that the chance constraint has to be satisfied 95% of the time; hence, it follows that we accept it can be violated 5% of the time (which may be resolved by doing overtime). This is one possible shape of a chance constraint, which corresponds with a Value at Risk (VaR) constraint, and is easy to interpret: the constraint will be satisfied with 95% probability. Note, however, that this approach does not consider the magnitude of the violation that might arise the other 5% of the time. (for an overview and applications of various types of chance constraints, see Frontline Systems, 2020, pp. 168–173 and 536–540; Birge and Louveaux, 2011, pp. 34, 47, 84–86, 124–134, 146–148; Kall and Mayer, 2011, pp. 88–91).

Analytic Solver automatically transforms the stochastic optimization model including the above-mentioned chance constraint into its so-called robust counterpart in the form of a deterministic linear program; solving the latter yields an approximate solution to the originally stochastic problem (Frontline Systems, 2020, p. 170; on the methods of robust optimization, see Ben-Tal et al., 2009). The complete implementation (robust version) is available for download at Mendeley Data. The chance constraints are part of a stochastic mixed-integer linear programming (SMILP) model with chance constraints (SCC) as described in (Frontline Systems, 2020, p. 170).

Table 9. Results of the stochastic optimization.

| Scenario No. | Expectations regarding… | Influence on optimal… | Expected value (CM IV) | Standard deviation (CM IV) | Coefficient of variation |
|--------------|--------------------------|------------------------|------------------------|---------------------------|-------------------------|
| 1            | normal                   | normal                 | no                     | no                        | no                      | 228,873                 | 80,040                  | 35%                      |
| 2            | normal                   | pessimistic            | no                     | no                        | no                      | 210,152                 | 80,596                  | 38%                      |
| 3            | normal                   | optimistic             | yes                    | no                        | no                      | 247,708                 | 80,711                  | 33%                      |
| 4            | pessimistic             | normal                 | no                     | no                        | no                      | 167,957                 | 82,631                  | 49%                      |
| 5            | pessimistic             | pessimistic            | yes                    | no                        | no                      | 149,880                 | 80,317                  | 54%                      |
| 6            | pessimistic             | optimistic             | no                     | no                        | no                      | 186,678                 | 83,226                  | 45%                      |
| 7            | optimistic              | normal                 | yes                    | no                        | no                      | 289,977                 | 82,890                  | 29%                      |
| 8            | optimistic              | pessimistic            | no                     | no                        | no                      | 271,069                 | 83,371                  | 31%                      |
| 9            | optimistic              | optimistic             | yes                    | no                        | no                      | 308,770                 | 83,514                  | 27%                      |

Figure 11. Product-moment correlation between total contribution margin IV and selling or purchase prices.
constraint was defined alternatively for the 99% and 95% percentile in this study. Table 10 shows the results and the actual probabilities for satisfying the chance constraint estimated by the corresponding relative frequencies across all realizations of the uncertainties. When applying the 99% percentile, the groups G3 and G4 are proposed for elimination. Accepting a lower probability for satisfying the chance constraint (95%) allows keeping group G3, resulting in a higher expected value of total contribution margin IV.

5. Conclusions

5.1. Integration of operational data, optimization techniques, and cost accounting

In general, quantitative models can play an important role in the coordination of a company’s information and planning systems (Küpper, 2007, p. 740). The MLFCP formulated in this paper shows that more information is needed to make optimal decisions than is usually considered in multi-level contribution margin accounting, in particular data on available capacities and resource requirements and, where applicable, market-related restrictions and considerations. The combination of both approaches can therefore help to better align a company’s information system to the requirements of optimal planning. Using integer programming and stochastic optimization for deriving optimal solutions and finally putting it all together, the concept proposed and implemented in this paper leads to an integration of various areas as illustrated in Figure 12. This concept may also serve as a basis for further development at the interface between cost accounting and optimization modeling.

5.2. Increasing the acceptance of operations research methods in practice

Together with the optimal values of the decision variables, the MLFCP alone initially only provides the maximum total contribution margin as an aggregated value. Once linked to multi-level contribution margin accounting, this total contribution margin can be retraced from level to level according to the hierarchical decomposition of the fixed cost block; the profit contributions of the various products, product groups, divisions, etc. can be displayed separately. This makes the use of the MLFCP much more meaningful. Displaying the results of the optimization model in the accessible analysis concept of multi-level contribution margin accounting – which is a well-known and proven format for practitioners – can increase the acceptance and persuasive power of the rather abstract mathematical approach in practice.

5.3. Gaining insight and making better decisions

If multi-level contribution margin accounting alone is considered, optimization potentials can remain unrecognized, and the assessment of profitability might be inappropriate. Once linked to the MLFCP, it may turn out that products, product groups, divisions, etc. actually show a lack of profitability despite originally being deemed clearly profitable, and are proposed for elimination or closure in the optimization calculation. Having pointed out that such a measure – particularly where complex entities like divisions are concerned – represents only the “last resort”, which should by no means automatically follow from the optimization calculation, this outcome can be understood as a warning signal indicating the need for medium- to longer-term measures to increase the profitability of the organizational entities in question. On the other hand, products, product groups, divisions, etc., which at first glance seem to be unprofitable on the sole basis of a multi-level contribution margin calculation may turn out be actually profitable, when the calculation is linked to the MLFCP. Common rules on how to interpret a multi-level contribution margin calculation should be examined carefully and may even have to be revised in some cases from the viewpoint of optimization. A non-optimized and therefore considerably suboptimal contribution margin calculation may seem appealing due to positive contribution margins throughout. However, if lower bounds on sales volumes apply, the optimized version may contain (single) negative contribution margins in a significant order of magnitude, which are accepted for the sake of the overall optimum. The proposed linkage thus enables multi-level contribution margin accounting to provide better information for decision making.

5.4. Limitations and need for supplementation by longer-term instruments

Limitations of the analysis may arise from the fact that the somewhat short-term planning of the production program contained in the MLFCP also flows into medium- to longer-term decisions. The more levels a contribution margin calculation includes, the more significant this problem might become. Due to this limitation, the benefit of the optimization carried out here is primarily seen in the above-mentioned signal function, pointing out the need for further analysis; longer-term instruments such as the methods of investment appraisal should also be used here, resulting in a reasonable mix of analytical approaches.
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Declarations

Author contribution statement

Michael Gutiérrez: Conceived and designed the experiments; performed the experiments; analyzed and interpreted the data; contributed reagents, materials, analysis tools or data; wrote the paper.

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Declaration of interests statement

The author declares no conflict of interest.

Additional information

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