Quantum fractional resonances in superconducting circuits with an embedded Josephson junction

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Abstract. We present a quantum electrodynamic treatment of the generation of fractional resonances in a planar waveguide with an embedded superconducting Josephson oscillator. We analyze the dynamics of the Josephson oscillator coupled with the electromagnetic pulse which is propagating along the waveguide. The calculations are carried out entirely in the Heisenberg picture. It is shown that the quantum Josephson oscillator excited by coherent pulse field at the pump frequency, can realize frequency down-conversion and emitting sub-harmonic multiples of the fundamental (fractional harmonics). The influence of dissipation on the phenomenon of resonance capture is discussed.

1. Introduction

Superconducting quantum circuits are a leading candidate for the engineering of quantum states in quantum logic devices [1–5]. Recently, there has been an increased activity in investigation of superconducting electrodynamics of waveguides and cavities with embedded artificial atoms - Josephson junction (Josephson oscillators) [6]. A Josephson oscillator is a macroscopic superconducting loop, interrupted by a weak link and may be threaded by a magnetic flux of electromagnetic wave propagating in a superconducting coplanar waveguide. Such an oscillator is characterized by energy $\hbar \omega_J$ that is much larger than the thermal energy at cryogenic temperatures and the system must be described by quantum mechanics. As it turned out, in the microwave range of frequencies can be achieved the strong coupling of the field with the artificial atoms, which opens up new possibilities for the control of quantum states. The other side of the interest due to the possibility of generating and analysis of Fock states up to a few photons [7].

In the present work we consider a coplanar waveguide (or resonator) with embedded Josephson junctions. The waveguides of chosen architecture may be used both as electromagnetic pumping and control of an inserted Josephson junction. The Josephson junction is supposed to be coupled directly with electromagnetic waves in the line with arbitrarily large strength. For a weak driving pulse a single Josephson junction operates as a linear oscillator and demonstrated the linear response for an excitation. At the same time, a strong driving pulses can cause the transition of Josephson oscillator in nonlinear regime of excitation. In this case in the first approximation only the Kerr type nonlinearity can be taken in account. It is shown that even in this case in the spectrum of excitation along with main resonance (1:1) also the fractional resonance (1:3) may appear. It means that the Josephson oscillator is trapped into nonlinear fractional resonance for long time and it can reemit microwave photons caused by a pulse of
coherent photons. In conclusion we will discuss the application of correlated microwave photons for distribution of entanglement in quantum communication networks.

2. The model of the system and the basic equations

In this paper we will be interested in the coplanar waveguide, which consists of a superconducting wire evaporated on an insulating substrate and having superconducting ground planes adjacent to it on the same surface as shown in Fig. 1a. The planar electrical circuits can be constructed by using modern lithographic techniques and the problem of spatial mode is largely eliminated. It is well known that the current can flow through the junction: \( I_s = I_0 \sin \varphi \) - where \( \varphi \) is the phase difference on the junction (the first Josephson relation) [6]. Since the wavelength of the microwave field is much larger than the size of the weak link, its effect is equivalent to the local potential jump (see. Fig. 1b), due to the Josephson effect [6]: \( V_J(t) = \frac{\hbar}{2e} \frac{\partial \varphi(t)}{\partial t} \) (the second Josephson relation). It means that the embedded junction may be considered as a lumped element. We can thus assume the open boundary conditions for which the current produced by a current generator on the end of the waveguide.

We introduce a charge distributed over the waveguide line from a current point \( x \) up to \( \infty \): \( Q(x,t) \). The changing in the time and the space charge produces the current \( I(x,t) \) and potential drop \( V(x,t) \), which may be defined by equations: \( I(x,t) = \frac{\partial Q(x,t)}{\partial t} \) and \( V(x,t) = -\frac{\partial Q(x,t)}{C_l \partial x} \), where \( C_l \) is the capacitance per unit length. It means that the coordinate of the charge waves is taken to be \( Q(x,t) \) and the conjugate momentum is \( \Phi(x,t) = L_l \frac{\partial Q(x,t)}{\partial t} \), where \( L_l = \frac{L_1}{2} \) and \( L_1 \) is the inductance per unit length. The classical Hamiltonian, describing the electromagnetic field Josephson junction and interaction between them, has form:

\[
H = \int_{-\infty}^{\infty} dx \left\{ \frac{\Phi(x,t)^2}{2L_l} + \frac{1}{2C_l} \left( \frac{\partial Q(x,t)}{\partial x} \right)^2 \right\} + \frac{1}{2J} \left( p + \frac{\hbar}{2e} Q(0,t) \right)^2 + J \omega_J^2 (1 - \cos \varphi), \tag{1}
\]

where \( p \) is the conjugate momentum to the coordinate of the oscillator \( \varphi \); \( J = C \left( \frac{\hbar}{2e} \right)^2 \), \( C \) is the junction capacitance.

Note that in Eq. (1) the first two terms can be interpreted as the kinetic and potential energy of the field, and the third term is the Hamiltonian of Josephson junction including of

![Figure 1](image-url)
the interaction term between the field and the oscillator. It can be seen from Eq. (1) that the system under consideration is equivalent to a nonlinear pendulum interacting with a string.

To treat field and oscillator quantum mechanically we will use Heisenberg approach. The field coordinate \( Q(x, t) \) field and the conjugate momentum \( \Phi(x, t) \) are the conjugate variables, which should be replaced by operators with commutation relations: \([\hat{Q}(x, t), \hat{\Phi}(x', t)] = i\hbar(x - x')\).

The same way we introduce for the oscillator the coordinate \( \varphi(t) \) and conjugate momentum \( p(t) \): \([\hat{\varphi}(t)\hat{p}(t)] = i\hbar \). Before proceed, we shall simplify the Hamiltonian. The leading terms that we want to included is only quadratic term: \( J\omega_J^2(1 - \cos \varphi) \approx J \left( \omega_J^2 \varphi^2 - \frac{\beta}{4} \varphi(t)^4 \right) \), \( \beta = \omega_J^2/6 \).

Finally, we have the quantized Hamiltonian to study in the form:

\[
\hat{H}(t) = \int_{-\infty}^{\infty} dx \left\{ \frac{\hat{\dot{Q}}(x, t)^2}{2L^2} + \frac{1}{2\hbar L^2} \left( \frac{\partial^2 \hat{Q}(x, t)}{\partial x^2} \right)^2 \right\} + \frac{1}{2\hbar} \left( \hat{p}(t) + \frac{\hbar}{2\epsilon} \hat{Q}(0, t) \right)^2 + J \left( \omega_J^2 \hat{\varphi}(t)^2 - \frac{\beta}{4} \hat{\varphi}(t)^4 \right).
\]

The field operators may be expanded as:

\[
\hat{Q}(x, t) = \sum_k c_k \left( e^{ikx} \hat{b}_k(t) + e^{-ikx} \hat{b}_k^\dagger(t) \right),
\]

\[
\hat{\Phi}(x, t) = -i \sum_k \omega(k)L_i c_k \left( e^{ikx} \hat{b}_k(t) - e^{-ikx} \hat{b}_k^\dagger(t) \right),
\]

where \( L \) - the length of the wave guide, \( c_k = \sqrt{\frac{\hbar}{2\omega(k)L_i L}} \), \( k \) - wavevector (as usually we will use the periodic boundary conditions for states classifications); \( \omega(k) = \upsilon |k| \), \( \hat{b}_k(t) \) and \( \hat{b}_k^\dagger(t) \) are creation and annihilation bosonic operators: \([\hat{b}_k(t), \hat{b}_k^\dagger(t)] = \delta_{k,k'} \) (for modes \( k \) and \( k' \)). Notice, that the waveguide modes form a one-dimensional continuum.

The same way for the Josephson junction oscillator the phase and moment can be written in terms of creation and annihilation with

\[
\hat{\varphi}(t) = \sqrt{\frac{\hbar}{2J\omega_J}} \left( \hat{a}(t) + \hat{a}^\dagger(t) \right), \quad \hat{p}(t) = -i \sqrt{\frac{\hbar J\omega_J}{2}} \left( \hat{a}(t) - \hat{a}^\dagger(t) \right),
\]

where \( \hat{a} \) and \( \hat{a}^\dagger \) are the bosonic creation and annihilation operators, which are defined in Fock space: \(|n\rangle (\hat{a}^\dagger \hat{a}|n\rangle = n|n\rangle \).

Expressing the quantum Hamiltonian in Eq. (2) in terms of these operators, we have

\[
\hat{H} = \sum_k \hbar \omega(k) \hat{b}_k^\dagger \hat{b}_k + \hbar \omega_J \hat{a}^\dagger \hat{a} - \frac{1}{4} \hbar \mu \left( \hat{a}^\dagger + \hat{a} \right)^4 + i\hbar g \left( \hat{a}^\dagger - \hat{a} \right) \hat{Q}(0, t) + \frac{1}{2C} \hat{Q}(0, t)^2,
\]

where: \( g = \frac{1}{2\epsilon} \sqrt{\frac{\hbar \omega_J}{2J}}, \quad \mu = \frac{\hbar \beta}{J \omega_J^2} \);

\[
\hat{Q}(0, t) = \sum_k c_k \left( \hat{b}(t) + \hat{b}^\dagger(t) \right).
\]

3. The main resonance

We consider here the propagating of a pulse generated by the current-generator at the left end of the waveguide. Let us the carried frequency \( \omega \) of the pulse is chosen close to the frequency of the Josephson junction \( \omega_J \). We will consider the case of wave packet initially in a coherent state.
They have the properties that they are eigenstates of the annihilation operator. The coherent state of a mode $k$ defined by: $b_k|\beta_k\rangle = \beta_k|\beta_k\rangle$, and for a wave packet we will have: $|\beta\rangle = \prod_k |\beta_k\rangle$.

Notice, that in coherent basis the propagating field looks like a classical wave.

The field of source is described by the charge operator $\hat{Q}_{\text{ext}}(t - x/\upsilon)$. We consider here only the case when at the initial time the field and the oscillator does not interact, and the system wave function has the form: $|\psi_{in}\rangle = |0\rangle \otimes |\{\beta\}\rangle$, where $|\{\beta\}\rangle$ is the coherent state of the field, $|0\rangle$ is the ground state of the oscillator.

The Heisenberg equations of motion for the operators in the problem are:

$$i \frac{d \hat{b}_k}{dt} = \omega(k) \hat{b}_k + \frac{c_k}{2e} J \left( \hat{\rho}(t) + \frac{\hbar}{2e} \hat{Q}(0,t) \right),$$

$$i \frac{d \hat{a}}{dt} = \omega J \hat{a} - \mu \left( \hat{a}^\dagger + \hat{a} \right)^3 + ig \hat{Q}(0,t).$$

The first equation is simply the bosonic oscillators evolution of the charge field coupled with the Josephson junction, while the second represents the nonlinear oscillator which is excited by the field propagating inside the waveguide.

We therefore begin an appropriate solution of these operators by formally integrating Eq. (7). The solution of Eq. (7) is

$$\hat{b}_k(t) = \hat{b}_k(0)e^{-i\omega(k)t} - i \frac{c_k}{2e} e^{i\omega(k)t} \int_0^t dt'e^{i\omega(k)t'} \frac{d \hat{\varphi}(t')}{dt'},$$

where we chose to begin the problem at $t = 0$ and the Heisenberg equation for the phase is used:

$$\frac{d \hat{\varphi}(t)}{dt} = \frac{1}{J} \left( \hat{\rho}(t) + \frac{\hbar}{2e} \hat{Q}(0,t) \right).$$

The solution for $\hat{b}_k(t)$ is written as a sum of two contribution, one from free evolution of the field operator and the other from radiation of the oscillator. Substituting Eq. (9) into the Eq. (6) yields

$$\hat{Q}(0,t) = \hat{Q}_{\text{ext}}(t) - \frac{\upsilon}{2} C_l \frac{\hbar}{2e} (\hat{\varphi}(t) - \hat{\varphi}(0)).$$

Now, inserting the charge operator into the right side of Eq. (8), we obtain the equation of motion for the operator $\hat{a}$ in the form:

$$i \frac{d \hat{a}(t)}{dt} = \omega J \hat{a}(t) - i \gamma \left( \hat{a}(t) + \hat{a}^\dagger(t) \right) - \mu \left( \hat{a}(t) + \hat{a}^\dagger(t) \right)^3 + ig \left( \hat{Q}_{\text{ext}}(0,t) + \frac{\upsilon}{2} C_l \frac{\hbar}{2e} \hat{\varphi}(0) \right),$$

where $\gamma = \frac{1}{4Z_l J} \left( \frac{\hbar}{2e} \right)^2$, $Z_l = \sqrt{\frac{L_0}{C_l}}$ is the impedance per unit length. Note, that the interaction of the oscillator with an semi-infinity transmission line act like a interaction with a dissipative bath even though every one of its electrical elements is non-dissipative. By combining the solution of the Eq. (12) and the equation Eq. (11) corresponding reemitting field may be obtained.

The Eq. (12) shows that the linear oscillator free evolution operator $\hat{a}(t)$ is given by $\hat{a}(t) = \hat{a}(0)e^{-i\omega_J t}$. For the chosen parameters it is supposed that term containing the frequency $\omega_J$ is dominant in right side of the Eq. (12). This means that we can neglect the rapidly oscillating terms in equations and keep only the slowly varying dynamical variables that are free from the rapid oscillations at resonant frequencies (rotating-wave approximation). In order
to do this we decompose the field operator $\hat{Q}(0,t)$ into positive and negative frequency parts writing $\hat{Q}(0,t) = \hat{Q}^{(+)}(0,t) + \hat{Q}^{(-)}(0,t)$, where

$$\hat{Q}^{(-)}(0,t) = \sum_k c_k \hat{b}_k(t), \quad \hat{Q}^{(+)}(0,t) = \sum_k c_k \hat{b}_k(t).$$  \hspace{1cm} (13)

At this point it is convenient to introduce slowly varying variables $\hat{a}_s(t) = \hat{a}(t)e^{i\omega t}$. In the terms of these new variables Eq. (12) become

$$i \frac{d\hat{a}_s(t)}{dt} = (\omega - 3\mu - \omega - i\gamma)\hat{a}_s(t) - 3\mu \left(\hat{a}_s(t)\hat{a}_s(t)\right) \hat{a}_s(t) + ig \left(e^{i\omega t}\hat{Q}_{ext}^{(+)}(0,t)\right)_{av},$$  \hspace{1cm} (14)

where $\langle \cdots \rangle_{av}$ means average per period in right side Eq. (14). Note that here and below all the non-resonant terms in the equations will be omitted.

We calculate now the backward current. As it is followed from Eq. (11) for $x \rightarrow -\infty$ we have

$$\hat{I}_R(x,t) = -\frac{\nu}{2C_I} \frac{\hbar}{2e} \frac{d\hat{\varphi}(t + x/\nu)}{dt}.$$  \hspace{1cm} (15)

The relevant quantity reemission characteristic of the oscillator that may be calculated is the ratio of the averaged reflected power to the equivalent power, that is produced by the exited current:

$$R = \frac{\langle \psi_m | \hat{P}_R(t) | \psi_m \rangle_{av}}{\langle \psi_m | \hat{P}_{ext}(t) | \psi_m \rangle_{av}}.$$  \hspace{1cm} (16)

Using Eqs. (4) and (14), the Eq. (15) may be rewritten as

$$R = 2R_0 \frac{\omega_j^2}{g^2 Q_m^2} \langle 0 | \left(\hat{a}(t)\hat{a}^\dagger(t) + \hat{a}^\dagger(t)\hat{a}(t)\right) | 0 \rangle \quad R_0 = \left(\frac{1}{22|\bar{C}\omega_j|}\right)^2.$$  \hspace{1cm} (17)

Here we are going to investigate a simple case when a coherent single mode in the state $|\beta_{k*}\rangle$ with $k_\nu = \omega/\nu$ propagating from the current source. Then we have $\langle \beta_{k*}| \hat{Q}_{ext}(x - vt) | \beta_{k*}\rangle = c_{k*} \left(\beta_{k*}e^{i(k_\nu(x-vt))} + \beta_{k*}e^{-i(k_\nu(x-vt))}\right) = Q_m \cos(k_\nu(x-vt) + \theta)$, $\beta_{k*} = |\beta_{k*}|e^{i\theta}$, $Q_{k*} = 2|\beta_{k*}|c_{k*}$ is the wave amplitude. Notice that the phase $\theta$ may be removed from wave equation by shift of time origin.

We calculate the correlation function Eq. (12) and time evolution of the oscillator in coherent states of a single oscillator $\hat{a}([\alpha]) = \alpha|[\alpha]\rangle$. The equation of motion Eq. (14) for the nonlinear oscillator may be written as:

$$i \frac{d\alpha}{dt} = (\omega_j - 3\mu - \omega - i\gamma)\alpha(t) - 3\mu \left|\alpha(t)\right|^2 \alpha(t) + ig \left(\bar{Q}_m\right)$$  \hspace{1cm} (18)

We can rewrite Eq. (17) for the correlation function by using the expression $\langle 0 \langle 0 | (\hat{a}(t)\hat{a}^\dagger(t) + \hat{a}^\dagger(t)\hat{a}(t)) | 0 \rangle = 2|\alpha(t)|^2 + 1$, $|\alpha(t)|^2 \gg 1$) Subsisting steady state solutions Eq. (18) into Eq. (17) become

$$R = R_0 \frac{\omega_j^2}{(\omega_j - \omega)^2 + \gamma^2},$$  \hspace{1cm} (19)

$\omega_j = \omega_j - 3\mu - 3\mu|\alpha|^2$, and $|\alpha|^2$ is a solution of equation $\left((\omega_j - \omega)^2 + \gamma^2\right)|\alpha|^2 = \left(\frac{g}{2}Q_m\right)^2$. Thus, in this case the nonlinear oscillator is captured into a nonlinear resonance and emits at a frequency close to the $\omega_j$. Notice, that when nonlinear term is negligibly small, the shape of line is Lorentzian (see Fig. 2).
4. Fractional resonance

We now turn to the problem of down-conversion of the wave frequency in the wave guide with an embedded lumped nonlinear element. Let us consider the case when the characteristic frequency of the coherent incoming field $\hat{Q}_{\text{ext}}(t-x/v)$ (drawing field) is near $3\omega$ and the resonant condition $\omega \sim \omega_J$ may be performed. To investigate the excitation of the nonlinear oscillator on the fractional resonance $(1:3)$ we made a canonical transformation of the field and the oscillator variables. In order to extract the fractional resonance we introduce auxiliary variables oscillating on the frequency $\omega$: $\hat{\phi}(x,t)$ is the conjugate momentum to the coordinate of the field $\hat{q}(x,t)$ (and the similar for the oscillator $\hat{P}$, $\hat{\varphi} = \hat{\varphi}_0 + \hat{\psi}$). The canonical transformation reads as:

$$
\hat{Q}(x,t) = \hat{Q}_0(x,t) + \hat{q}(x,t), \hat{\Phi}(x,t) = \hat{\Phi}_0(x,t) + \hat{\phi}(x,t), \hat{p} = \hat{p}_0 + \hat{P}, \hat{\varphi} = \hat{\varphi}_0 + \hat{\psi}.
$$

The transformed Hamiltonian takes form:

$$
\hat{H}(t) = \int_{-\infty}^{\infty} dx \left\{ \frac{\hat{\phi}^2(x,t)}{2J} + \frac{1}{2} \left( \frac{\partial \hat{q}(x,t)}{\partial x} \right)^2 \right\} + \frac{1}{2J} \left( \hat{P}(t) + \frac{\hbar}{2e}\hat{q}(x,t) \right)^2 + J \left( \frac{\omega_J^2 \hat{\psi}^2(t)}{2} - \frac{\beta}{4} \left( \hat{\psi}(t) + \hat{\varphi}_0(t) \right)^4 \right),
$$

(20)

where the auxiliary field $\hat{\varphi}_0$ is defined from equation:

$$
\frac{\partial^2 \hat{\varphi}_0}{\partial t^2} + 2\gamma \frac{\partial \hat{\varphi}_0}{\partial t} + \omega_J^2 \hat{\varphi}_0 = \frac{\hbar}{2e} \hat{I}_{\text{ext}}(0,t),
$$

(21)

In the modes representation the Hamiltonian is

$$
\hat{H} = \sum_k \hbar \omega(k) \hat{b}_k^\dagger \hat{b}_k + \hbar \omega_J \hat{a}^\dagger \hat{a} - \frac{1}{4} \hbar \mu \left( \hat{a}^\dagger + \hat{a} + \xi \hat{\varphi}_0(t) \right)^4 + i\hbar g(\hat{a}^\dagger - \hat{a})\hat{q}(0,t) + \frac{1}{2C} \hat{q}(0,t)^2,
$$

(22)

where $\xi = \sqrt{\frac{2J\omega_J}{\hbar}}$.

Comparison of Eqs. (5) and (20) shows that the main difference between the Hamiltonians of the main and fractional resonances is the presence of an operator $\hat{\varphi}_0$, that modulates the
nonlinear term. Doing the same way as in Sec. 3 we can find the Heisenberg equation for the operator $\hat{a}(t)$:

$$i\frac{d\hat{a}(t)}{dt} = \omega_J \hat{a} - i\gamma \left(\hat{a}^\dagger + \hat{a}\right) - \mu \left(\hat{a}^\dagger + \hat{a} + \xi \hat{\varphi}_0(t)\right)^3 + ig\hat{q}_{ext}(0, t). \quad (23)$$

The Eq. (23) should be solved simultaneously with the Eq. (21). Now we are ready to find sub-harmonics in the quantum system. Again restrict ourselves to the case when the driving field is prepared in the coherent state and has the carrier frequency $3\omega$ ($\omega \sim \omega_J$), i.e. $I_{ext}(0, t) = I_m \cos(3\omega t)$. Keeping resonant harmonics $\omega(k) \sim 3\omega$, i.e. $k_\omega \sim 3\omega/\nu$, we write the solution Eq. (21) in the form

$$\varphi_0(t) = A \cos(3\omega t) + B \sin(3\omega t), \quad (24)$$

where $A = \frac{\hbar}{2\gamma J} \frac{I_m D}{D + (6\omega_\gamma)^2}$, $B = \frac{\hbar}{2\gamma J} \frac{6\omega_\gamma}{D + (6\omega_\gamma)^2}$, $D = \omega_J^2 - (3\omega)^2$. Substituting Eq. (24) into the Eq. (23) and using slowly varying operators, we obtain:

$$i\frac{d\hat{a_s}(t)}{dt} = (\omega_J - \omega - 3\mu - i\gamma)\hat{a}_s(t) - \frac{3\mu}{2} \left(\left(\hat{a}_{s}^\dagger(t)\hat{a}_s(t) + 2\xi(\xi^2(A^2 + B^2)\right)\hat{a}_s(t)
$$

$$+ 2\xi(A + iB)\left(\hat{a}_{s}^\dagger(t)\right)^2 + ig\left(e^{i\omega t}\hat{q}_{ext}(0, t)\right)_{avg}. \quad (25)$$

![Figure 3. The phase plane Eq. (25), showing the presence of three stationary solutions (attractors) ($\mu = 0.0005$, $\gamma = 0.025\omega_J$, $\frac{\omega_{Jm}}{2\gamma J} = 1$). For instance, one of them has coordinates: $a_{s_1}^\dagger = (1.347, 0.391)$.](image-url)
The Eq. (25) may be written in the coherent basis as

\[ i \frac{d\alpha(t)}{dt} = (\omega_J - \omega - 3\mu - i\gamma)\alpha(t) \]
\[ -\frac{3\mu}{2} \left( (|\alpha(t)|^2 + 2\xi^2(A^2 + B^2)) \alpha(t) + 2\xi(A + iB)(\alpha^*(t))^2 \right) + i\frac{2g}{q_m}. \]

The investigation of the phase plane \((\text{Re}(\alpha(t)), \text{Im}(\alpha(t)))\) the nonlinear equation shows that depending on the parameters of the system in Eq. (26) possible existence of four fixed points. One point \((0,0)\) corresponds to the trivial solution, which means that no generation of fractional frequency. When the parameter are changed in the equation may appear the three stable (under certain parameters of the system) points (see. Fig. 3), which correspond to the generation of the fractional resonance. The stationary Eq. (26), determines the reflection coefficient according to

\[ R = 4R_0 \frac{\omega_J^2}{g^2 q_m^2} |\alpha|^2. \]

5. Conclusion

We have examined the interaction of a Josephson junction with the quantized electromagnetic field in a coplanar superconducting waveguide. We have used the Heisenberg picture to study system dynamics and rotating-wave approximation for the calculation of the field re-emittance by excited oscillator. We fond that it is possible to capture oscillator into main and fractional resonance. It is shown that when the Josephson oscillator is trapped to nonlinear fractional resonance it can reemit microwave photons with the fractional frequencies. This process may be used for the generation of entangled photons when the external source emit photons in the Fock states. In this case the Josephson oscillator can radiate three microwave photons for each incoming absorbed photon. Such kind of strong correlated (entangled) photons may useful for modern quantum communication networks [8].

Acknowledgments This work was supported by the Russian Science Foundation grant No. 14-07-00582.

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