Non Linear Gauge Fixing for FeynArts

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Abstract. We review the non-linear gauge-fixing for the Standard Model, proposed by F. Boudjema and E. Chopin, and present our implementation of this non-linear gauge-fixing to the Standard Model and to the minimal supersymmetric Standard Model in FeynArts.

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1 Introduction

Due to the increased accuracy in the experimental area, also the theoretical calculations should become more accurate, also in extensions of the SM \cite{1}. The calculation in the perturbative approach requires to sum over all amplitudes contributing to the measured process. There are already tools developed, that are capable of automatically generating and calculating Feynman diagrams at one-loop level in the Standard Model (SM) and the minimal supersymmetric Standard Model (MSSM) (like \cite{2}, \cite{3}, \cite{4} and others), but together with the development of such tools there is also a need for ways to check the validity of the computed results, in order to use them with any reliability. One powerful tool to perform such checks arises from the procedure of gauge-fixing the theory, as the result has to be independent of the introduced gauge-fixing parameters. Naturally, the more gauge-fixing parameters we have available, the more stringent the test becomes. The non-linear gauge-fixing of the Standard Model was presented in \cite{5}, and later fully implemented in GRACE \cite{4}. In this work we want to implement the same class of non-linear gauge-fixing in the FeynArts/FormCalc package \cite{2}. The advantage of this package is that it is open-source and freely distributable.

In Sect. 2 we introduce the non-linear gauge-fixing in the SM and we discuss the extension to the MSSM in Sect. 3. We describe the implementation in FeynArts and conclude with an outlook and acknowledgements in Sect. 4.

2 Non-linear gauge-fixing in the SM

Gauge-fixing is necessary in any gauge theory, like the SM or the MSSM. Though all fields participate in the gauge transformations, fermions cannot mix with the gauge bosons or the scalar bosons. Therefore we can ignore the fermions and their gauge transformations in the discussion of gauge-fixing.

The whole theoretical issue about gauge-fixing a spontaneously broken Yang-Mills gauge theory can be found in text books like \cite{6}, \cite{7}, or others. The remaining problem is to get the conventions right. In this overview we sketch only the conventions we use to obtain our FeynArts models files. We write the Higgs doublet in terms of the higgs $h$ and the Goldstone bosons $\chi$ as

$$
\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} -i\phi_1 - \phi_2 \\ i\phi_3 + \phi_4 \end{pmatrix} = \left( \frac{1}{\sqrt{2}}(i\chi^0 + h) \right).
$$

The gauge group $U(1)$ with the coupling $g'$ and the boson $B_\mu$ and the gauge group $SU(2)$ with the coupling $g$ and the bosons $W_\mu^\pm$ are mixed to

$$
A_\mu = s_W W_\mu^3 + c_W B_\mu, \\
Z_\mu = c_W W_\mu^3 - s_W B_\mu, \\
W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2),
$$

with the electric charge and Weinberg angle given by

$$
e = \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad s_W = \frac{e}{g}, \quad c_W = \frac{e}{g'}.
$$

The gauge transformations on these bosons are

$$
\delta \phi_j = -\varphi^a T^a_{jk} \phi_k, \\
\delta W_\mu^a = \frac{1}{g_a} \partial_\mu \varphi^a + f^{abc} W_\mu^b \varphi^c,
$$

where the $U(1)$ gauge group gets the index 4: $g_4 = g'$ and $f^{abc}$ is the structure constant of $SU(2)$ for $a, b, c < 4$ and zero otherwise. $[T^a, T^b] = f^{abc} T^c$ are real.

Gauge fixing with the Faddeev Popov procedure gives the gauge-fixing part $\mathcal{L}_gf^\alpha$ and the ghost part $\mathcal{L}_gh$
of the Lagrangian in terms of the gauge-fixing functions
\[ F^A = \partial^\mu A^\mu, \]
\[ F^Z = \partial^\mu Z^\mu - \xi_Z m_Z \chi^0 - \tilde{e} \frac{e\xi_Z}{2s_Wc_W}(h\chi^0), \]
\[ F^+ = \partial^\mu W^\mu + \xi_W m_W \chi^+ - \tilde{e} \frac{e\xi_Z}{2s_Wc_W}(\delta h - i\kappa\chi^0)\chi^+, \]
and \( F^- = (F^+)^* \), expressed in the physical fields as
\[ \mathcal{L}_{gl} = \frac{1}{2}(F^A)^2 - \frac{1}{2}(F^Z)^2 - \frac{1}{\xi_W} F^+ F^-, \]
\[ \mathcal{L}_{gh} = \tilde{e}^2 \left( \frac{\delta F^a}{\delta \phi^b} \bigg|_{\alpha=0} \right) c^b, \]
where the ghost and the gauge parameters \( \varphi^a \) have to be transformed into a physical basis. For the exact procedure see [3].

Obviously there are additional vertices coming from the non-linear part of the gauge-fixing functions. These vertices are proportional to the additional gauge parameters \( \tilde{\alpha}, \tilde{\beta}, \tilde{\delta}, \tilde{\varepsilon}, \) and \( \tilde{\kappa} \).

3 Non-linear gauge-fixing in the MSSM

In the MSSM the gauge bosons eqs. (22) [4] are the same, but we have two higgs doublets instead of eq. (1):
\[ H_1 = \left( \frac{1}{\sqrt{2}}[(O_{1h} + is_\beta O_{2h})H_h - ic_\beta G^0] \right), \]
\[ H_2 = \left( \frac{1}{\sqrt{2}}[(O_{2h} + ic_\beta O_{3h})H_h + is_\beta G^0] \right), \]
where \( H_h \) are the three neutral higgs bosons and
\[ \frac{s_\beta}{c_\beta} = \frac{v_2}{v_1} = \tan \beta \]
is the ratio of the vacuum expectation values of the higgs fields. In the normal MSSM without induced CP violation in the Higgs sector \( O_{3h} = \delta_3 h, H_3 = A_0, \) and \( O_{2h} \) describes the normal mixing of the two neutral CP even higgs fields \( h^0 \) and \( H^0 \).

The definition of the parts of the Lagrangian eqs. [11] and eq. [12] stay the same, but the gauge-fixing functions eq. [9] and eq. [10] are changed to account for the extended Higgs sector:
\[ Z^Z = \partial^\mu Z^\mu + \xi_Z m_Z G^0 + \frac{e\xi_Z}{2s_Wc_W}(\tilde{e}\chi h_h G^0), \]
\[ Z^+ = \partial^\mu W^\mu + \xi_W m_W G^+ + \frac{e\xi_Z}{2s_Wc_W}(\tilde{e}\chi h_h G^0 + i\tilde{e} h_h G^+ + i\tilde{e} h_h G^+), \]
Writing the gauge transformations eq. [16] and eq. [17] in terms of the physical fields
\[ \delta A_{\mu} = -\partial_{\mu} \varphi^A + i\epsilon(W_{\mu}^+ \varphi^- - W_{\mu}^+ \varphi^-), \]
\[ \delta W_{\mu}^\pm = -\partial_{\mu} \varphi^\pm + i\epsilon(A_{\mu} + \tilde{\alpha} W_{\mu}^\pm) \varphi^\pm, \]
\[ \delta Z_{\mu} = -\partial_{\mu} \varphi^Z + i\epsilon(A_{\mu} + \tilde{\alpha} W_{\mu}^\pm) \varphi^Z, \]
\[ \delta H_h = -\frac{1}{s_Wc_W} O_{3h}^c (\kappa G^0 + iG^+ - iG^- \varphi^+) - \frac{1}{s_Wc_W} (s_\beta O_{1h} - c_\beta O_{2h})(H^+ \varphi^- - H^- \varphi^+) + \frac{1}{s_Wc_W} O_{3h} (H^+ \varphi^- + H^- \varphi^+), \]
\[ \delta G^0 = -i(m_W + \frac{s_Wc_W}{c_W} O_{3h}^c H_h^0) \varphi^Z + \frac{1}{s_Wc_W} (G^+ \varphi^- + G^- \varphi^+), \]
\[ \delta G^\pm = \pm i(m_W + \frac{s_Wc_W}{c_W} O_{3h}^c H_h^0 \pm iG^0) \varphi^\pm + \frac{1}{s_Wc_W} (G^+ \varphi^- + G^- \varphi^+), \]
where \( O_{3h}^c := (c_\beta O_{1h} + s_\beta O_{2h}) \) and the gauge parameters \( \varphi^a \) are taken in physical directions.

For the full gauge transformations we have to transform the fermion and sfermion fields, too, since they transform under the same gauge groups as the higgs fields eqs. [18]. In the interaction basis, when we look at the unbroken Lagrangian, we know that the gauge transformations are flavour-blind: they transform each generation independently in the same way. But once spontaneous symmetry breaking gives mass to gauge bosons, fermions, and sfermions the Lagrangian has to be written in terms of these mass eigenstates that no longer coincide with the interaction eigenstates. The mixing of these states is described by the CKM matrix. In order to obtain the gauge-fixing and the ghost parts of the Lagrangian we write the gauge transformation in terms of the physical fields. But these new gauge transformations are no longer flavour-blind, as the SU(2) gauge transformation mixes the up and down type quarks, which are now superpositions of the different generations. And therefore the full Lagrangian is no longer invariant under these new gauge transformations. But since the U(1) em gauge group corresponding to the electric charge is unbroken, the full Lagrangian is still invariant under the \( \varphi^A \) part of the gauge transformations.

4 Non-linear gauge-fixing in FeynArts

4.1 In the SM

From eqs. [11][11] it is easy to see, that we get non-standard interactions, that cannot be described by the generic couplings of the SM or MSSM model files, which are defined in the file Lorentz.gen of FeynArts. Most of these non-standard interactions can be found in Lorentzbgf.gen, but there the ghost-ghost vertices are missing. They are needed for the mass- and field-renormalisation constants of the ghosts. So we modify the files in order to write a model file, that FeynArts can use to implement our non linear gauge fixing.

For the SM our Lagrangian was obtained using different conventions than FeynArts. Therefore we had to...
We create the model file for the MSSM with non-linear gauge-fixing, we only had to add the ghost-ghost vertex to such a way as to be compatible with FeynArts, we only unified Lorentz file. Since we choose our conventions in http://terra.ar.fi.lt/~garfield/MSSM/ the model file can be downloaded from Mathematica/. The Lorentz file and the model file can be downloaded from the subdirectory FeynHiggs release of this summer, this feature is incorporated in the original distribution, too. When linking our model file for non-linear gauge-fixing, one has to set our mixing matrix \( \text{Ohiggs} \) to the FeynHiggs mixing matrices \( \text{Uhiggs} \) or \( \text{Zhiggs} \), which describe two different forms of mixings.

As an additional feature our model file includes the possible mixing of the two CP even with the CP odd neutral higgs bosons. With the new FeynArts and FeynHiggs release of this summer, this feature is incorporated in the original distribution, too. When linking our model file for non-linear gauge-fixing, one has to set our mixing matrix \( \text{Ohiggs} \) to the FeynHiggs mixing matrices \( \text{Uhiggs} \) or \( \text{Zhiggs} \), which describe two different forms of mixings.

We use \( \text{Lorentzbgf.gen} \) as the basis for our modified Lorentz file. Since we choose our conventions in such a way as to be compatible with FeynArts, we only had to add the ghost-ghost vertex to \( \text{Lorentzbgf.gen} \). We create the model file for the MSSM with non-linear gauge-fixing, \( \text{MSSMnlgf.mod} \), with the package ModelMaker, which is part of FeynArts. The Lorentz file and the model file can be downloaded from http://terra.ar.fi.lt/~garfield/MSSM/.

The Mathematica programs for obtaining our MSSM-Lagrangian can be downloaded from the subdirectory Mathematica/. They have no documentation, though.

### 4.2 In the MSSM

For the MSSM we derive the Lagrangian using Superfields. We start from the unbroken supersymmetric Lagrangian, add the soft breaking terms and obtain the still gauge invariant full Lagrangian. After including spontaneous symmetry breaking and transforming the fields to the masseigenstates we add the gauge-fixing terms eq.(11) and the ghost terms eq.(12) that are calculated from the gauge fixing functions eqs.(8,16,17) with the gauge variation of the physical fields eqs.(18,23).

As an additional feature our model file includes the possible mixing of the two CP even with the CP odd neutral higgs bosons. With the new FeynArts and FeynHiggs release of this summer, this feature is incorporated in the original distribution, too. When linking our model file for non-linear gauge-fixing, one has to set our mixing matrix \( \text{Ohiggs} \) to the FeynHiggs mixing matrices \( \text{Uhiggs} \) or \( \text{Zhiggs} \), which describe two different forms of mixings.

We use \( \text{Lorentzbgf.gen} \) as the basis for our modified Lorentz file. Since we choose our conventions in such a way as to be compatible with FeynArts, we only had to add the ghost-ghost vertex to \( \text{Lorentzbgf.gen} \). We create the model file for the MSSM with non-linear gauge-fixing, \( \text{MSSMnlgf.mod} \), with the package ModelMaker, which is part of FeynArts. The Lorentz file and the model file can be downloaded from http://terra.ar.fi.lt/~garfield/MSSM/.

The Mathematica programs for obtaining our MSSM-Lagrangian can be downloaded from the subdirectory Mathematica/. They have no documentation, though.

### 5 Outlook and Acknowledgements

We want to do more checks of our model files. The analytic checks of the SM part could include also one-loop amplitudes, but for the MSSM the counterterms to define a fully renormalised MSSM at one loop are still missing. It is also not clear, which renormalisation scheme can be adopted, that allows general one loop calculations in the MSSM including general complex parameters. The normal on-shell scheme can not treat a decaying particle in an external line. The standard procedure [1] introduces the renormalisation condition \( \hat{\text{Re}} \), which cuts the absorptive parts in the self-energy loops when determining the mass- and field-counterterms. But when we calculate the decay-width of a decaying particle, we get a pole in our amplitude, when we do not include the width of the particle in the propagator. This can be done systematically, using the complex mass scheme [10]. Since some people are still cautious about using a complex mass, we hope to apply the non-linear gauge-fixing to investigate the gauge independence of processes calculated with the complex mass scheme.

If we can afford the time, we plan do provide a better documentation to the calculation of the Lagrangian.

In the long run we plan to calculate all counterterms in the MSSM in a suitable framework, like the complex mass scheme, and include them in our model files. The idea behind this goal is to introduce particle physics to the scientific community in Lithuania and to get students interested in this kind of work.

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### References

1. J. A. Aguilar-Saavedra et al., Eur. Phys. J. C 46 (2006) 43 [arXiv:hep-ph/0511344]
2. T. Hahn and M. Perez-Victoria, Comput. Phys. Commun. 118 (1999) 153 [arXiv:hep-ph/9807565]
3. A. Denner and T. Hahn, Nucl. Phys. B 525 (1998) 27 [arXiv:hep-ph/9711302]
4. G. Belanger, F. Boudjema, J. Fujimoto, T. Ishikawa, T. Kaneko, K. Kato and Y. Shimizu, Phys. Rept. 430 (2006) 117 [arXiv:hep-ph/0308080]
5. F. Boudjema and E. Chopin, Z. Phys. C 73 (1996) 85 [arXiv:hep-ph/9507396]
6. M. E. Peskin and D. V. Schroeder, An Introduction To Quantum Field Theory (Addison-Wesley, Reading, USA 1995) 731 ff.
7. S. Weinberg, The quantum theory of fields. Vol. 2: Modern applications, (Cambridge University Press, Cambridge, UK 1996)
8. J. Pašukonis, [arXiv:0710.0159v1 [hep-ph]]
9. A. Denner, Fortsch. Phys. 41 (1993) 307 [arXiv:0709.1075 [hep-ph]].
10. A. Denner and S. Dittmaier, Nucl. Phys. Proc. Suppl. 160 (2006) 22 [arXiv:hep-ph/0605312].