Achieving Efficient Distributed Machine Learning Using a Novel Non-Linear Class of Aggregation Functions

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Abstract—Distributed machine learning (DML) over time-varying networks can be an enabler for emerging decentralized ML applications such as autonomous driving and drone fleetin. However, the commonly used weighted arithmetic mean model aggregation function in existing DML systems can result in high model loss, low model accuracy, and slow convergence speed over time-varying networks. To address this issue, in this paper, we propose a novel non-linear class of model aggregation functions to achieve efficient DML over time-varying networks. Instead of taking a linear aggregation of neighboring models as most existing studies do, our mechanism uses a nonlinear aggregation, a weighted power-$p$ mean (WPM) where $p$ is a positive integer, as the aggregation function of local models from neighbors. The subsequent optimizing steps are taken using mirror descent, driving in real-time scenario [2]. By having devices perform training tasks locally and then aggregating the local models, DML let devices collaboratively build a global model, and preserves the users’ privacy for edge AI [3].

To avoid the scalability bottleneck and single point of failure issues brought by using a parameter server [4], recent DML systems propose to use decentralized architectures (e.g., All-Reduce [5], Ring-Reduce [6] and Gossip [7]), where devices share their local models with neighbors, and aggregate received local models. For example, Xiao et al. propose a linear iteration to yield distributed averaging consensus over a network based on the Laplacian matrix of the associated graph [8]. Decentralized Stochastic Gradient Descent (DSGD) methods propose to trade the exactness of the averaging provided by All-Reduce [7], [9], [10] for better scalability. Heged et al. propose a gossip-based server architecture to solve decentralized learning in mobile computing [7].

Despite substantial efforts from existing literature, they ignore the topology dynamicity, a fundamental challenge for applying DML in edge computing. Specifically, edge devices usually exchange local models through wireless communication, which is highly dynamic. As a result, two devices may be able to exchange local models in one iteration, but lose the connection in the next iteration. Moreover, doing so drastically slows down the convergence speed because the communication is bottlenecked by the limited wide-area or mobile edge network bandwidth. Specifically, when the training process iterates over time-varying networks [11], [12], this often leads to a severe drop in the training and test performance (i.e., generalization gap), even after hyper-parameter fine-tuning. Although some papers have proposed a linear class of aggregation functions for DML over time-varying networks and proven their convergence [10], [12]–[14], the convergence speed still can not match the requirements of emerging applications.

To address this challenge, we propose a novel non-linear model aggregation mechanism to achieve efficient DML over time-varying networks. We make a key observation that existing DML systems exclusively use the linear weighted-mean of neighboring local models as their aggregation functions, with different ways of assigning the appropriate weights for averaging local models. In contrast, instead of fine-tuning the weighting of local models during aggregation or designing delicate and complex architecture, our mechanism takes a weighted power-$p$ mean (WPM) of local models from neigh-
When the variance is lower, DML more efficiently simulates aggregation under a given resource budget. Huang et al. propose a distributed deep learning-based offloading algorithm for MEC networks. Ghosh et al. [19] propose a framework of iterative federated clustering learning. Li et al. [21] propose a novel momentum-based method to mitigate this decentralized training difficulty [22]. However, they assume a fixed topology and are not suitable for edge computing under network dynamics.

**DML with Topology Dynamicity.** Kovalev et al. propose the ADOM method for decentralized optimization over time-varying networks with projected Nesterov gradient descent [12]. Koloskova et al. introduce a framework covering local SGD updates and synchronous and pairwise gossip updates on adaptive network topology [23]. Eshraghi et al. propose a DABMD algorithm for decentralized learning by varying mini-batch sizes across time-varying topology [14]. Pu et al. consider new distributed gradient-based methods with an estimate of the optimal decision variable and an estimate of the gradient for the average of the agents’ objective functions [24]. Nedic et al. [10], [25] tackle the DML with topology dynamicity from the consensus perspective. They propose a model aggregation method for agents in a network with a time-varying topology to collaboratively solve a convex objective function, with a convergence guarantee. However, our preliminary results show that their methods suffer from lower accuracy, higher loss and slower convergence speed under time-varying networks compared to under a fixed topology. Additionally, their methods use linear averages for aggregation, which is a special case of our framework.

### III. DESIGN

**A. Problem Formulation**

We consider the scenario of DML on a total of $m$ devices over time-varying networks, without the existence of a centralized parameter server. Such time-varying networks are ubiquitous in many complex systems and practical applications (e.g., sensor networks, next generation federated learning systems), where the communication pattern among pairs of mobile devices will be influenced by their physical proximity, which naturally changes over time [12], [26], [27]. The connection between any two devices $i$ and $j$ is dynamic but symmetric. In other words, in the same epoch, if $i$ can send information to $j$, $j$ must be able to send information to $i$ as well; if $i$ cannot send information to $j$, then neither can $j$ to $i$. Each device $i$ has its own dataset and loss function $F_i$. Denote the model parameters as a vector $w \in \mathbb{R}^n$. Devices aim to collaboratively minimize the following objective function:

$$
\text{minimize } f(w) = \sum_{i=1}^{m} F_i(w), \text{ subject to } w \in \mathbb{R}^n,
$$

without sharing any data among devices.

**B. Mechanism Overview**

After presenting the problem formulation, we propose our mechanism to solve it efficiently under dynamic topologies. Algorithm 1 describes the overall training process of our mechanism on each device.

Specifically, in each iteration $t$, each device $i$ uses its local dataset to compute its local gradient $\delta_i(t)$. It then checks its wireless connections with other devices, and sends to each connected device $j$ the following information: (1) $w_i(t)$, the
Algorithm 1 The training procedure of WPM mechanism for each device.

1: Input: the dataset $D_i$, the initial model parameters $w_i(0)$, and the learning rate $\eta_i(t)$ for each device $i$, $t = 0, \ldots, T$.

2: Output: The final model parameters of all devices after $T$ iterations $w_i(T)$.

3: for $t = 0$ to $T$ do
4:     for $i = 1$ to $m$ do
5:         Compute the local gradient $d_i(t)$.
6:         Check wireless connections to other devices and count the number of connected devices as $N_i(t)$.
7:         Send $w_i(t)$ and $e_{i,j}(t) = 1/(N_i(t) + 1)$ to all connected devices $j$.
8:         Assign $\alpha_{i,j}(t)$ for all devices $j$, $j = 1, \ldots, m$.
9:         Compute $w_i(t+1)$ using Equation (2).
10:     end for
11: end for
12: Return $w_i(T)$.

The local model parameter of $i$ at the beginning of iteration $t$; and (2) $e_{i,j}(t) = \frac{1}{N_i(t)+1}$ for all devices $j$ connected to $i$, where $N_i(t)$ is the number of devices $i$ connected to iteration $t$. After all information is exchanged, for each connected device $j$, device $i$ assigns an aggregation weight $\alpha_{i,j}(t) = \min\{e_{i,j}(t), e_{j,i}(t)\}$ for the local model $w_j(t)$. It then assigns an aggregation weight $\alpha_{i,i}(t) = 1 - \sum_{j \neq i} \alpha_{i,j}(t)$ for its own local model, and an aggregation weight $\alpha_{i,k}(t) = 0$ for all devices it does not have a connection to in iteration $t$. Denote the learning rate for device $i$ at iteration $t$ as $\eta_i(t)$. The local model for device $i$ for iteration $t+1$ is then updated as the weighted power-$p$ mean of the local models $w_j(t)$ of all devices $j = 1, \ldots, m$ with weights $\alpha_{i,j}(t)$:

$$w_i(t+1) = \left( \sum_{j=1}^{m} \alpha_{i,j}(t) w_j(t)^p \right)^{1/p} - \eta d_i(t),$$

where $p$ is an integer. For vectors $v$, we define $v^C$ to be the element wise application of $x \rightarrow \text{sgn}(x) \cdot |x|^C$. Alternatively, let $v^C = \nabla h(v)$ where $h(v) = \frac{1}{p+1} \|v\|^p$. In the case that the gradient step is zero, this makes $w_i(t+1)$ the WPM of its neighbor’s estimates at time $t$, which is different from the usual linear aggregation.

Note that when $p = 1$, the WPM aggregation function in our mechanism reduces to the typical weighted linear mean aggregation function commonly used by classical distributed machine learning systems:

$$w_i(t+1) = \sum_{j=1}^{m} \alpha_{i,j}(t) w_j(t) - \eta d_i(t),$$

such as [7, 9, 10, 13].

IV. CONVERGENCE ANALYSIS

Firstly, we present several assumptions needed to prove convergence.

Assumption 1. The network $G = (V, E_i)$ and the weight matrix $P(t)$ satisfy the following [28]:

- $P(t)$ is doubly stochastic for all $t \geq 1$, that is $\sum_{j=1}^{m} [P(t)]_{ij} = 1$ and $\sum_{i=1}^{m} [P(t)]_{ij} = 1$ for all $i$, $j \in V$.
- There exists a scale $\zeta > 0$, such that $|P(t)|_{ij} \geq \zeta$ for all $i$ and $t \geq 1$, and $|P(t)|_{ij} \geq \zeta$ if $(i, j) \in E_t$.
- There exists an integer $B \geq 1$ such that the graph $(V, E_kB+1 \cup \cdots \cup E_{(k+1)B})$ is strongly connected for all $k \geq 0$.

This assumption is very common in the literature [10, 25, 29], and is a basic requirement that guarantees a necessary level of communication necessary to optimise based on the loss functions at each device.

Assumption 2. Functions $f_{i,t}$ for $1 \leq i \leq m$ and $t \geq 0$ are convex. Additionally, they are $G_i -$ Lipschitz [28]. That is

$$\|f_{i,t}(x) - f_{i,t}(y)\| \leq G_i \|x - y\|.$$  (4)

Additionally, this implies that the gradient of $f_{i,t}$ is always

$$\|\nabla f_{i,t}(x)\| \leq G_i.$$ (5)

Let $w : \mathbb{R}^n \rightarrow \mathbb{R}$ be a function that is strictly convex, continuously differentiable, and defined on $\mathbb{R}^n$. Then, the Bregman divergence [30] is defined as

$$D_w(x, y) = w(x) - w(y) - \langle \nabla w(x), x-y \rangle.$$ (5)

Since $w$ is convex, $D_w(x, y) \geq 0$, and the Bregman divergence also functions as a distance measuring function. In the process described in this paper, we take Mirror Descent steps in the special case where $w(x) = \frac{1}{2\eta} \|x\|^p_{C+1}$ is the function used to create the distance measuring Bregman divergence function.

Mirror descent has its origins in classical convex optimization [31], while follow the regularised leader goes back to the work of [32]. The mirror descent algorithm (MDA) was introduced by Nemirovsky and Yudin for solving convex optimization problems [33]. This method exhibits an efficiency estimate that is mildly dependent in the decision variables dimension, and thus suitable for solving very large scale optimization problems.

Our algorithm is a natural extension of Mirror Descent to DML [30, 33]. Mirror Descent also strives to solve min$_x f(x)$ for convex $f : \mathbb{R}^n \rightarrow \mathbb{R}$. It generates a sequence $\{x_t\}_t$ of estimates where after calculating $x_t$, it generates the estimator function $f^{(t)}(x) = f(x_t) + \eta \nabla f_{i,t}(x)(x - x_t) + D_w(x_t, x)$, and lets $x_{t+1} = \arg \min f^{(t)}(x)$. This satisfies $\nabla w(x_{t+1}) = \nabla w(x_t) - \eta \nabla f(x_t)$, which is used in our algorithm in Equation (8).

Mirror Descent also serves as a generalization of classical gradient descent. The case when $w(x) = \frac{1}{2} \|x\|^2$ and $D_w(x, y) = \|x-y\|^2$ gives $x_{t+1} = \arg \min f^{(t)}(x) = x_t - \eta \nabla f(x_t)$.

In our algorithm, each step begins by taking a WPM as described in Equation (7). Next, $y_{i,t+1}$ is a component wise WPM of $z_{i,t}$ since $\nabla w(x)$ raises each element to the power of $p$. However, if one were to then take standard gradient steps instead of the specific Mirror Descent step in Equation (8) it would not converge to the optimal value $x^*$. Instead,
we discovered that it was necessary to use Mirror Descent where the map mapping function satisfied $\nabla w(x) = xp$ with termwise exponentiation.

The usage of the WPM is motivated by the idea that as $p$ increases, the consensus distance decreases.

The WPM model is the following

$$\mathbf{w}_i^{(t)} = \left( \sum_{j=1}^{m} \alpha_{ij} \cdot (\mathbf{w}_j^{(t)})^p \right)^{\frac{1}{p}}.$$  \hspace{1cm} (5)

$$\mathbf{w}_i^{(t+1)} = \left( \left( \mathbf{w}_i^{(t)} \right)^p - \eta \nabla f_i(\mathbf{w}_i^{(t)}) \right)^{\frac{1}{p}}.$$  \hspace{1cm} (6)

Firstly, the $\mathbf{w}_i^{(t)}$ is calculated by taking a WPM of the models at the neighbors of node $i$. Then, $\mathbf{w}_i^{(t+1)}$ is formed by taking a Mirror Descent step from $\mathbf{w}_i^{(t)}$. Additionally, in the experiments, a constant $\eta_0$ was chosen and for each value of $p$, $\eta$ was set as $\eta_0 + \varphi/p$.

We rewrite this with $y_{i,t} = \mathbf{w}_i^{(t)}$ and $z_{i,t} = \mathbf{w}_i^{(t+1)}$ for all $i, t$. Then, the process becomes

$$\nabla w(y_{i,t+1}) = \sum_{j=1}^{m} P(t)ij \nabla w(z_{i,t}).$$ \hspace{1cm} (7)

$$\nabla w(z_{i,t}) = \nabla w(y_{i,t}) - \eta \nabla f_i(y_{i,t}),$$ \hspace{1cm} (8)

where $w(x) = \|x\|_{p+1}^{-1}$. Note that we may write a general formula for $\nabla w(y_{i,t+1})$. Define the matrix $P(t,s) = P(t)P(t-1) \cdots P(s+1)P(s)$. Then,

$$\nabla w(y_{i,t+1}) = \sum_{k=1}^{m} P(t,s)_{ik} \nabla w(y_{k,s})$$ \hspace{1cm} (9)

$$- \sum_{r=s}^{t} \sum_{k=1}^{m} P(t,r)_{ik} \eta \nabla f_{k,r}(y_{k,r}).$$

Note that $\nabla w(y_{i,t}) - \nabla w(y_{i,t})$. Then, by applying the Triangle Inequality and Equation (10),

$$\nabla w(y_{i,t+1}) - \nabla w(y_{i,t}) \leq \sum_{k=1}^{m} \nabla w(y_{k,s}) + \sum_{r=s}^{t} \sum_{k=1}^{m} \nabla f_{k,r}(y_{k,r})$$ \hspace{1cm} (11)

$$\leq \sum_{k=1}^{m} \nabla w(y_{k,s}) + \sum_{r=s}^{t} \sum_{k=1}^{m} \nabla f_{k,r}(y_{k,r})$$ \hspace{1cm} (12)

To finish, we use Claim 1 and plug $s = 0$ in to get the stated bound.

$$\|y_{i,t} - \eta y_t\|_p \leq \sqrt{2} \left[ \sum_{k=1}^{m} \nabla w(y_{k,s}) + \sum_{r=s}^{t} \sum_{k=1}^{m} \nabla f_{k,r}(y_{k,r}) \right].$$ \hspace{1cm} (13)

Theorem 1. For all nodes $i$ and finishing time $T$, the regret satisfies

$$\text{Reg}_i(T) := \frac{1}{T} \sum_{t=1}^{T} \sum_{j=1}^{m} f(y_{i,t}) - f_i(x^*)$$ \hspace{1cm} (14)

$$\leq \frac{m}{T} \eta D_w(x^*, \eta y_t) + 2 \eta G_i \|w(y_{i,t})\|_1 + \frac{m \eta G_i}{\eta}.$$ \hspace{1cm} (15)

Proof. Define $\nabla w(y_{i,t}) = \frac{1}{m} \sum_{i=1}^{m} \nabla w(y_{i,t})$. Thanks to Lemma 1, it is guaranteed that $y_t$ is close to $y_{i,t}$. This allows us to show the convergence to optimality of the comparatively simpler $y_{i,t}$.

$$\nabla w(y_{i,t+1}) = \nabla w(y_{i,t}) - \eta \frac{1}{m} \sum_{i=1}^{m} \nabla f_{i,t}(y_{i,t}).$$ \hspace{1cm} (16)
Then, by the convexity of \( f_{i,t} \) and the definition of \( y_t \)

\[
\frac{1}{m} m \sum_{i=1}^{m} f_{i,t}(y_{t,i}) - f_{i,t}(x) \leq \frac{1}{m} m \sum_{i=1}^{m} \nabla f_{i,t}(y_{t,i})^T (y_{t,i} - x)
\]

\[
= \frac{1}{m} m \sum_{i=1}^{m} \nabla f_{i,t}(y_{t,i})^T ((y_t - x) + (y_{t,i} - y_t))
\]

\[
= \frac{1}{\eta} (\nabla w(y_t) - \nabla w(y_{t+1})^T (y_t - x)
\]

\[
+ \frac{1}{m} m \sum_{i=1}^{m} \nabla f_{i,t}(y_{t,i})^T (y_{t,i} - y_t)
\]

\[
\leq \frac{1}{\eta} (D_w(x, y_t) + D_w(y_t, y_{t+1}) - D_w(x, y_{t+1})
\]

\[
+ \frac{1}{m} G_t \sum_{i=1}^{m} \|y_{t,i} - y_t\|.
\]

(16)

Additionally, note that by Lipschitz,

\[
\frac{1}{m} m \sum_{i=1}^{m} f_{i,t}(y_t) - f_{i,t}(x)
\]

\[
\leq \frac{1}{m} m \sum_{i=1}^{m} (f_{i,t}(y_{t,i}) - f_{i,t}(x)) + G_t \|y_{t,i} - y_t\|.
\]

(17)

Thus, we are left to bound \( D_w(y_t, y_{t+1}) \).

\[
D_w(y_t, y_{t+1}) = w(y_t) - w(y_{t+1}) - \nabla w(y_{t+1})^T (y_t - y_{t+1})
\]

\[
\leq \nabla w(y_{t+1})^T (y_t - y_{t+1}) - \frac{\sigma_p}{p+1} \|y_t - y_{t+1}\|^{p+1}
\]

\[
= \nabla w(y_{t+1})^T (y_t - y_{t+1}) - \frac{\sigma_p}{p+1} \|y_t - y_{t+1}\|^{p+1}
\]

\[
= (\eta \sum_{i=1}^{m} \nabla f_{i,t}(y_{t,i}))^T (y_t - y_{t+1}) - \frac{\sigma_p}{p+1} \|y_t - y_{t+1}\|^{p+1}
\]

\[
\leq \eta G_t \|y_t - y_{t+1}\| - \frac{\sigma_p}{p+1} \|y_t - y_{t+1}\|^{p+1}
\]

\[
\leq \eta G_t \frac{p}{p+1} \left( \frac{\eta G_t}{\sigma_p} \right).
\]

(18)

Where the first inequality comes from Claim 2 and the last inequality follows from \( ax - bx^{p+1} \leq a \frac{p}{p+1} \sqrt{\frac{a}{p+1}} \) for \( a, b \geq 0 \). Combining the bounds in Equations (16), (17), and (18) gives

\[
\sum_{t=0}^{T-1} \frac{1}{m} m \sum_{i=1}^{m} f_{i,t}(y_t) - f_{i,t}(x)
\]

\[
\leq \sum_{t=0}^{T-1} \frac{1}{\eta} (D_w(x, y_t) + D_w(y_t, y_{t+1}) - D_w(x, y_{t+1})
\]

\[
+ \frac{2}{m} G_t \sum_{t=0}^{T-1} \sum_{i=1}^{m} \|y_{t,i} - y_t\|
\]

\[
\leq \frac{1}{\eta} (D_w(x, y_0) - D_w(x, y_T)) + \frac{1}{\eta} T \eta 2G_t \frac{p}{p+1} \left( \frac{\eta G_t}{\sigma_p} \right)
\]

\[
+ 2G_t T^p 2^{p-1}(\eta \kappa - 1) \sum_{k=1}^{m} \|\nabla w(y_{k,s})\|_1 + m \eta G_t \frac{1}{\kappa - 1}.\]

(19)

Which easily manipulates to the final theorem result. \( \square \)

V. Evaluations

A. Experimental Setup

The experimental platform is composed of 8 Nvidia Tesla T4 GPUs, 4 Intel XEON CPUs and 256GB memory. We build the experimental environment based on Ray [35], an open-source framework that provides universal API for building high performance distributed applications.

Topology. We randomly generated fixed per-iteration-changing topology sequences to simulate the time-varying networks. Each element of a sequence is a randomly generated symmetric adjacent matrix of undirected graph topology. We use density to measure the proportion of non-zero elements in upper triangle of the adjacent matrix (i.e., the proportion of available connections in the topology). The generated topology is a fully connected topology when the density = 1, or a ring topology when the density = 0.1.

Datasets and Models. We performed our experiment on the classic MNIST [36] and CIFAR-10 [37] datasets. Both are constructed for image classification tasks. We use two typical convex models, the Logistic Regression (LR) model [38] and the support vector machines (SVM) [59] for our experiment. Specifically, we select the LR model for the MNIST, and SVM for CIFAR-10.

Baselines. We compared the WPM with \( P = 1 \) methods and SwarmSGD [40] over time-varying topologies. In order to provide a fair comparison between all three methods, we use the same dynamic topology sequences to conduct the experiment. And we made some minor adjustments for each baseline method.

• For the \( P = 1 \), we define it as a class of methods using a linear aggregation function [13], [34], [41]. All of these methods are a special case of WPM.

• For the SwarmSGD, we set the number of local SGD updates equal to 1, where the selected pair of devices perform only one single local SGD update before aggregation.
Data Partition. We consider two data distributions scenarios [42]. For the IID setting, the training dataset is equally divided into $M$ random sub-datasets, and assigned to $M$ devices. For the Non-IID setting, we define an extreme case where the data on different devices are generated from different distributions (i.e., each device hold only one label of data).

Metrics. To measure the performance of WPM, we study three metrics in the experiments.

- **Model Accuracy.** This measures the proportion between the amount of incorrect data by the model and that of all data. Assume the number of samples at each device $i$ is $\chi_i$, and the number of right measured samples is $\chi_i(r)$. The accuracy of the WPM is measured as $\frac{1}{M} \sum_{i=1}^{M} \frac{\chi_i(r)}{\chi_i}$. The average accuracy is computed as $\frac{1}{N} \sum_{r=1}^{N} \frac{\chi_{i}(r)}{\chi_i}$.  
- **Test Loss.** This metric measures the quantification difference of probability distributions between model outputs and observation results.  
- **Convergence Speed.** We record the accuracy of methods at each iteration and then relate the accuracy to the number of iterations to study how many iterations are required to achieve a specific loss or accuracy.

Hyperparameters. For all of experiments, the learning rate and batch size are fixed at 0.01 and 128. We randomly generate dynamic topology sequences with devices of 10, 15 and 20, and conduct the evaluation of total 500 iterations.

B. Experiment Results

Vertical Comparison. To study how different $P$ affects the performance of WPM over time-varying networks. We evaluate WPM with different $P \in \{3, 5, 9, 15\}$, respectively, and choose LR model on MNIST dataset. We empirically choose $P = 15$ as the max $P$ value. Due to the operation of power mean with $P$, it results in the model parameters extremely small before gradient descent, which may lead to the vanishing gradient problem. Moreover, the computation complexity exponentially increased with larger $P$. At the same time, we found the computation cost unacceptable after $P > 15$ in the experiment.

Horizontal Comparison. We use WPM ($P = 3$) and WPM ($P = 15$) compare with $P = 1$ and SwarmSGD. Results in Figure 5(b) and Figure 6(b) indicate that WPM ($P = 15$) achieves the fastest convergence speed in IID and Non-IID distributions. And also slightly improved accuracy and loss as shown in Figure 1(b) and Figure 2(b). Due to the extreme distinct Non-IID data setting, all methods perform worse than IID setting. This also caused the sharply loss increase for WPM at the beginning of the training process. However, WPM ($P = 15$) still keep the best accuracy and loss, while other baselines especially the SwarmSGD struggles significantly because its less model aggregation rounds.

Scalability. To analyze the performance and scalability of all methods under different scales, each method is performed with topology sequences of 10, 20, and 100 devices. The results are illustrated in Figure 5(c) and Figure 6(c). Vertically, as the number of devices increased to 20, the convergence speed of WPM is accelerated by 10.13% with IID setting, 13.74% with Non-IID setting, while keep a similar accuracy. Horizontally, when comparing with DPSGD under the scale of 20 devices, WPM converge 25.26% faster with IID setting, and significantly 57.99% faster with Non-IID setting. We present all the detailed data in Table III of Appendix.

Found Lessons. We found our proposed WPM is very sensitive to the learning rate with mirror gradient descent. We use a learning rate scaling factor to keep the corresponding learning rate at the same magnitude with model parameters. However, in our previous experiments, when we use an inappropriate learning rate scaling factor liking $\frac{(\|\nabla f_{x_i}\|)}{\|\nabla f_{x_{j}}\|}$ for mirror descent. The WPM could not work under an inadequate learning rate.
We study the important problem of DML over time-varying networks. We propose a novel non-linear mechanism that takes weighted power-$p$ means on the local models, where $p$ is a positive odd integer. To our best knowledge, it is the first time to do non-linear aggregation in DML. We rigorously prove its convergence. Experimental results show that our mechanism significantly outperforms the common linear weighted aggregation over time-varying networks, in terms of convergence speed and scalability.

VI. CONCLUSION

REFERENCES

[1] C.V.Networking, “Cisco global cloud index: Forecast and methodology, 2015–2020.” San Jose, CA, U.S.A: Cisco Public, 2016.

[2] E. Talpes, D. D. Sarma, G. Venkataaramanan, P. Bannon, B. McGee, B. Floering, A. Jalote, C. Hsiong, S. Arora, A. Gorti, and G. S. Sachdev, “Compute solution for tesla’s full self-driving computer,” IEEE Micro, vol. 40, no. 2, pp. 25–35, 2020.

[3] A. Koloskova, S. Stich, and M. Jaggi, “Decentralized stochastic optimization and gossip algorithms with compressed communication,” in International Conference on Machine Learning. PMLR, 2019, pp. 3478–3487.

[4] A. Smola and S. Narayanamurthy, “An architecture for parallel topic models,” Proceedings of the VLDB Endowment, vol. 3, no. 1-2, pp. 703–710, 2010.

[5] T. Hoeffer, W. Gropp, R. Thakur, and J. L. Traff, “Toward performance models of mpi implementations for understanding application scaling issues,” in Recent Advances in the Message Passing Interface. R. Keller, E. Gabriel, M. Resch, and J. Dongarra, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2010, pp. 21–30.

[6] J.-W. Lee, J. Oh, S. Lim, S.-Y. Yun, and J.-G. Lee, “Tomradaaggregate: Accurate and scalable federated learning via the ring-based architecture,” arXiv preprint arXiv:2012.03214, 2020.

[7] I. Hegedüs, G. Danner, and M. Jelasity, “Gossip learning as a decentralized alternative to federated learning,” in IEEE International Conference on Distributed Applications and Interoperable Systems. Springer, 2019, pp. 74–90.

[8] L. Xiao and S. Boyd, “Fast linear iterations for distributed averaging,” in 42nd IEEE International Conference on Decision and Control (IEEE Cat. No.03CH37475), vol. 5, 2003, pp. 4997–5002 Vol.5.

[9] X. Lian, C. Zhang, H. Zhang, C.-J. Hsieh, W. Zhang, and J. Liu, “Can decentralized algorithms outperform centralized algorithms? a case study for decentralized parallel stochastic gradient descent,” in Proceedings of the 31st International Conference on Neural Information Processing Systems, ser. NIPS’17. Red Hook, NY, USA: Curran Associates Inc., 2017, p. 5336–5346.

[10] A. Nedic, “Distributed gradient methods for convex machine learning problems in networks: Distributed optimization,” IEEE Signal Processing Magazine, vol. 37, no. 3, pp. 92–101, 2020.
[11] D. Kovalev, A. Koloskova, M. Jaggi, P. Richtarik, and S. Stich, “A linearly convergent algorithm for decentralized optimization: Sending less bits for free!” in International Conference on Artificial Intelligence and Statistics. PMLR, 2021, pp. 4087–4095.

[12] D. Kovalev, E. Shulgin, P. Richtarik, A. V. Rogozin, and A. Gasnikov, “Adaom: Accelerated decentralized optimization method for time-varying networks,” in Proceedings of the 38th International Conference on Machine Learning, ser. Proceedings of Machine Learning Research, M. Meila and T. Zhang, Eds., vol. 139. PMLR, 18–24 Jul 2021, pp. 5784–5793.

[13] G. Neglia, C. Xu, D. Towsley, and G. Calbi, “Decentralized gradient methods: does topology matter?” in International Conference on Artificial Intelligence and Statistics. PMLR, 2020, pp. 2348–2358.

[14] N. Eshraghi and B. Liang, “Distributed online optimization over a heterogeneous network with any-batch mirror descent,” in Proceedings of the 37th International Conference on Machine Learning, ser. Proceedings of Machine Learning Research, H. D. III and A. Singh, Eds., vol. 119. PMLR, 13–18 Jul 2020, pp. 2933–2942.

[15] J. Verbraken, M. Wolting, J. Katzy, J. Kloppenburg, T. Verbel, and J. S. Rellermeyer, “A survey on distributed machine learning,” ACM Computing Surveys (CSUR), vol. 53, no. 2, pp. 1–33, 2020.

[16] K. Donahue and J. Kleinberg, “Model-sharing games: Analyzing federated learning under voluntary participation,” vol. 35, pp. 5303–5311, May 2021. [Online]. Available: https://ojs.aaai.org/index.php/AAAI/article/view/16669

[17] J. Lee, I. Hwang, S. Shah, and M. Cho, “Flexreduce: flexible all-reduce for distributed deep learning on asymmetric network topology,” in 2020 57th ACM/IEEE Design Automation Conference (DAC). IEEE, 2020, pp. 1–6.

[18] S. Wang, T. Tsor, T. Salonidis, K. K. Leung, C. Makaya, T. He, and K. Chan, “Adaptive federated learning in resource constrained edge computing systems,” vol. 37, no. 6, 2019, pp. 1205–1221.

[19] L. Huang, X. Feng, A. Feng, Y. Huang, and L. P. Qian, “Distributed deep learning-based offloading for mobile edge computing networks,” Mobile Networks and Applications, 2018. [Online]. Available: https://doi.org/10.1007/s11036-018-1177-x

[20] A. Ghosh, J. Chung, D. Yin, and K. Ramchandran, “An efficient framework for clustered federated learning,” pp. 60–72, 06 2020.

[21] M. Li, D. G. Andersen, A. Smola, and K. Yu, “Communication efficient distributed machine learning with the parameter server,” in Proceedings of the 27th International Conference on Neural Information Processing Systems - Volume 1, ser. NIPS’14. Cambridge, MA, USA: MIT Press, 2014, p. 19–27.

[22] T. Lin, S. P. Karimireddy, S. Stich, and M. Jaggi, “Quasi-global momentum: Accelerating decentralized deep learning on heterogeneous data,” in Proceedings of the 38th International Conference on Machine Learning, ser. Proceedings of Machine Learning Research, M. Meila and T. Zhang, Eds., vol. 139, 18–24 Jul 2021, pp. 6654–6665.

[23] A. Koloskova, N. Loizou, S. Boreiri, M. Jaggi, and S. Stich, “A unified theory of decentralized SGD with changing topology and local updates,” in Proceedings of the 37th International Conference on Machine Learning, ser. Proceedings of Machine Learning Research, H. D. III and A. Singh, Eds., vol. 119. PMLR, 13–18 Jul 2020, pp. 5381–5393.

[24] S. Pu, W. Shi, J. Xu, and A. Nedić, “Push–pull gradient methods for distributed optimization in networks,” IEEE Transactions on Automatic Control, vol. 66, no. 1, pp. 1–16, 2020.

[25] A. Nedic and A. Ozdaglar, “Distributed subgradient methods for multi-agent optimization,” IEEE Transactions on Automatic Control, vol. 54, no. 1, pp. 48–61, 2009.

[26] L. A. Zadeh, “Time-varying networks, i,” Proceedings of the IRE, vol. 49, no. 10, pp. 1488–1503, 1961.

[27] M. Kolar, L. Song, A. Ahmed, and E. P. Xing, “Estimating time-varying networks,” The Annals of Applied Statistics, pp. 94–123, 2010.

[28] D. Yuan, Y. Hong, D. W. Ho, and S. Xu, “Distributed mirror descent for online composite optimization,” IEEE Transactions on Automatic Control, vol. 66, no. 2, pp. 714–729, 2020.

[29] J. Li, G. Chen, Z. Wu, and X. He, “Distributed subgradient method for multi-agent optimization with quantized communication,” Mathematical Methods in the Applied Sciences, vol. 40, no. 4, pp. 1201–1213, 2017.

[30] A. Beck and M. Teboulle, “Mirror descent and nonlinear projected subgradient methods for convex optimization,” Operations Research Letters, vol. 31, no. 3, pp. 167–175, 2003.

[31] A. Nemirovski, “Efficient methods for large-scale convex optimization problems,” Ekonomika i Matematicheskie Metody, vol. 15, no. 1, 1979.
A. Proof of Claim 1 (Lower Bound)

Proof. Substitute \(a = x - y, b = x + y\). Then, this becomes
\[
2^p \left| \frac{(a + b)^p}{2} + \frac{(a - b)^p}{2} \right| \geq 2|a|^p. \tag{20}
\]

Without loss of generality set \(a > 0\), and then it suffices to verify
\[
(a + b)^p + (a - b)^p \geq 2a^p. \tag{21}
\]
This final inequality is true because after cancellation, the only terms left on the LHS are of the form \(a^{p-2k}b^{2k}\), so the inequality is true.

B. Proof of Claim 2

Proof. \(\sigma_p = \frac{1}{2^p - 1}\) works. Holding \(b\) constant, the derivative of the LHS with respect to \(a\) is \(a^p - b^p\) and the derivative of the RHS with respect to \(a\) is \(\sigma_p(a - b)^p\). By the previous Lemma, this is true with \(\sigma_p = \frac{1}{2^p - 1}\), since the magnitude of the LHS’s derivative is larger, and for \(a < b\) both derivatives are negative, and for \(a > b\) both derivatives are positive.

C. Additional Experimental Results

Other detailed performance data were shown in Table II. The scale experiment shown in Table III. We also recorded the consensus distance in Table IV.

| Devices | i.i.d | Non-i.i.d |
|---------|-------|-----------|
|         | 500 iterations | Max | Average | 500 iterations | Max | Average |
| WPM(P=1) | 10 | 90.15% | 90.15% | 86.80% | 89.58% | 89.77% | 83.56% |
|         | 15 | 90.17% | 90.17% | 86.91% | 88.54% | 88.8% | 82.41% |
|         | 20 | 90.1% | 90.1% | 86.85% | 89.94% | 90.03% | 85.41% |
|         |     | (-0.06%) | (-0.06%) | (0.06%) | (0.4%) | (0.29%) | (2.21%) |
| WPM(P=15) | 10 | 90.92% | 90.95% | 88.82% | 89.28% | 89.3% | 86.84% |
|         | 15 | 90.93% | 90.95% | 88.71% | 88.84% | 88.91% | 86.89% |
|         |     | (0.01%) | (0%) | (-0.08%) | (-0.16%) | (-0.13%) | (-0.18%) |
|         |     | (0.01%) | (-0.02%) | (0.08%) | (0.4%) | (0.13%) | (0.68%) |

| Dataset | WPM(P=1) | WPM(P=3) | WPM(P=5) | WPM(P=9) | WPM(P=15) |
|---------|----------|----------|----------|----------|-----------|
| i.i.d   | 189      | 190      | 190      | 158      | 158       |
|         | (0%)     | (0.53%)  | (0.53%)  | (0%)     | (0%)      |
| Non-i.i.d | 345     | 332      | 322      | 213      | 127       |
|         | (3.77%)  | (3.77%)  | (5.28%)  | (3.26%)  | (3.05%)   |
|         |          |          |          |          | (62.03%) |

| Methods | i.i.d | Non-i.i.d | 10 | 15 | 20 | 10 | 15 | 20 |
|---------|-------|-----------|----|----|----|----|----|----|
| WPM(P=1) | 189   | 189       | 190 | 190 | 158 | 158 | 142 | 142 |
|         | (0%)  | (-0.53%)  | (0%) | (0%) | (0%) | (0%) | (10.15%) | (10.15%) |
| WPM(P=15)| 284   | 311       | 269 | 131 | 127 | 113 | 113 | 113 |
|         | (-9.51%) | (5.28%)  | (3.05%) | (3.05%) | (13.74%) | (13.74%) | (13.74%) | (13.74%) |
| P Value | 100   | 200   | 300   | 400   | 500   | 100   | 200   | 300   | 400   | 500   |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 3       | 0.084 | 0.074 | 0.066 | 0.058 | 0.054 | 0.179 | 0.157 | 0.140 | 0.058 | 0.054 |
| 5       | 0.311 | 0.243 | 0.201 | 0.198 | 0.199 | 0.311 | 0.243 | 0.201 | 0.123 | 0.123 |
| 9       | 0.746 | 0.601 | 0.530 | 0.519 | 0.453 | 0.225 | 0.177 | 0.155 | 0.143 | 0.139 |
| 15      | 1.250 | 0.992 | 0.969 | 0.889 | 0.877 | 0.624 | 0.433 | 0.391 | 0.282 | 0.304 |

**TABLE IV**

**Analysis of Consensus Distance with Different P**
SwarmSGD

WPM(P=1)

WPM(P=3)

WPM(P=15)

Methods

0

50

100

150

200

250

300

350

Iterations

SwarmSGD  WPM(P=1)  WPM(P=3)  WPM(P=15)
SwarmSGD

WPM(P=1)

WPM(P=3)

WPM(P=15)

methods

0

10

20

30

40

50

60

Iterations

SwarmSGD

WPM(P=1)

WPM(P=3)

WPM(P=15)
SwarmSGD

WPM(P=1)

WPM(P=15)

WPM(P=15)

Iterations

SwarmSGD  WPM(P=1)  WPM(P=3)  WPM(P=15)

methods
Accuracy vs Iterations for
- DPSGD
- WPM (P = 15)
- WPM (P = 3)
The chart shows the loss over iterations for different algorithms:

- **DPSGD**
- **WPM (P = 15)**
- **WPM (P = 3)**

The y-axis represents the loss, and the x-axis represents the number of iterations. The chart illustrates the performance of these algorithms, with DPSGD generally showing lower loss values compared to WPM (P = 15) and WPM (P = 3) as the iterations increase.
Iterations
0
200
400
600
800

Test Loss

SwarmSGD  WPM(P=3)  WPM(P=1)  WPM(P=15)

Iterations
0 100 200 300 400 500

Test Loss
0 50 100 150 200 250 300 350 400 450 500 550 600 650 700 750 800
