Construction of mathematical models the parachute jumper with change position acrobatic

Asmianto Asmianto, Hariyanto Hariyanto and Iis Herisman
Department of mathematics, Faculty of mathematics and natural science,
Institut Teknologi Sepuluh Nopember (ITS)
Jl. Arief Rahman Hakim, Surabaya 60111 Indonesia
e-mail: asmianto57@gmail.com

Abstract. Construction of mathematical models the movement of a parachutist during the air using newton's II law is $\Sigma F = ma$. Position parachutist after exiting the plane immediately unfurled his body so as to create a large air resistance. The presence of air drag resulting movement indirectly parachutist moves down vertically downwards but also shifting toward horizontal and form a parabolic trajectory. Parachutist speed getting down increased until eventually the parachutist reaches terminal velocity it's the position where the air drag is equal to the gravitational force (gravity) jumpers. In this paper is assumed to be parachutist with 91.6 kg mass (including equipment) jumping from a plane at an altitude of 3,000 meters and reach a height of parachutist ± 1000 meters with velocity ±57 m/s . So the parachutist have to be clever in taking account of everything, because if just a little too late can dangerous the safety of the parachutist.

1. Introduction
Skydiving is one of the most extreme sports. Beside that, the parachute is also commonly used to save the life of a pilot and distribute soldiers in remote places. Now this is a typical skydiving used is at an altitude of 4000 meters by opening the parachute at an altitude of 800 meters off the ground. Parachuting lasted less than a minute. [1]

In doing parachuting, before making the jump parachutist always pay attention to the air pressure, minimum height early jump and the minimum distance from a point that will be targeted. Because, it was very influential on the safety and speed of the parachutist. To account for this, many of the scientists who developed the numerical model of the parachute problem. The power given by the air resistance is very influential with the speed of a parachute. The altitude of the plane to start a jump should also attention, that the parachutist could land on the desired point. [2]

Mathematical model for the motion of parachute shaped based on Newton's II Law $\Sigma F = ma$. Where $m$ is the mass of the parachutist, $a$ is the acceleration, and $F$ is the sum of $F_g$ and grafitasi style of drag air $F_d$. Therefore, $F = F_g + F_d$. On air drag there is drag coefficient which are influenced by the magnitude of the Reynolds number. Reynold number depends also on the shape of the canopy used by parachutist. In this paper is the canopy in the shape of a half ball with Reynolds number $Re > 10^3$ and with a coefficient of drag $C_D = 1.33$. [4]

Based on the above problems then suggested the paper about, how the movement and position of sitting in the air when the parachutist started earlier jumped from the plane to the position of the parachutist ready to open the parachute. In this mathematical models it’s will be easier to know the various influences before making the jump. So the parachutist can jump safely and land at a position already determined.

2. Research and methodology
The establishment of mathematical model of parachutist with acrobat changes the position used newton’s II. The second law explains how the velocity of an object changes when it is subjected to an external force. The law defines a force to be equal to change in momentum (mass times velocity) per
change in time. Newton also developed the calculus of mathematics, and the "changes" expressed in the second law are most accurately defined in differential forms. (Calculus can also be used to determine the velocity and location variations experienced by an object subjected to an external force.) For an object with a constant mass \( m \), the second law states that the force \( F \) is the product of an object's mass and its acceleration \( a \). In the formation of the model there are several steps including assumption of the position of the parachutist after jumping from the plane, the parachutist during motion analysis in air and differentials the formula of speed and position with respect to time and analyze the movement of the parachutist started from motion free fall from the airplane in a position ready to open the parachute. After that, establishment of mathematical model of parachuting approached from the second law of Newton, start from parachutist free fall from the airplane, until the parachutist in a position ready to open the parachute. Finally, the result mathematical modelling are simulated and MATLAB will be used as a simulator. After the obtained model equation of parachutist, then will be created for equation of algorithms are simulated.

3. Analysis and discussion

Construction of mathematical model on the parachutist will be constructed is starting to get out of the plane in a position ready to open the parachute. In this paper assumed parachutist doing motion free fall from an airplane and instantly make her spread her down position (Figure 1) so that it created a large air resistance.

![Figure 1: Position of parachutist in the air](image1)

![Figure 2: Forces that works on parachutist during in air](image2)

By the time the parachutist began jumping from the plane then Newton's law II can be applied and in this case assumed parachutist doing motion free fall. During his time in the air doing translational...
motion of parachutist and rotation at the position until ready to open the parachute. Mathematically it
can be written as follows:

\[ \sum F = ma \]
\[ \sum F = F_g + F_d \]
\[ F_g + F_d = ma \]

\( F_g \) is the gravitational force of parachutists that the direction is always toward the center of the
earth while the \( F_d \) is the air drag (Force drag) the direction is always opposite to the movement of
the parachutist.

\( m \) is the mass of the whole, it’s mass parachutist with the mass of equipment under by parachutists
(including parachute).

Acceleration is the first derivative of speed against time, so that it can be written as follows:

\[ F_g + F_d = m \frac{dv}{dt} \]

So, by the equation of velocity with respect to time as follows:

\[ v(t) = \frac{mg - ce^{-\frac{k}{m}t}}{k} \]

To get the equation with respect to time, with the position of the integration equations of velocity with
respect to time.

So obtained

\[ x(t) = \frac{mg}{k} \left( t + \frac{m}{k} e^{-\frac{k}{m}t} \right) + c \]

By making the assumption \( x(0) = x_0 \) equation obtained a position with respect to time, parachutist

\[ x(t) = x_0 + \frac{mg}{k} t + \frac{m^2 g}{k^2} \left( e^{-\frac{k}{m}t} - 1 \right) \]

4. Simulation and analysis

In this chapter the sub carried out simulations of the equations of motion of parachutist that has
already obtained from previous calculations. From the results of the simulation will be done analysis
of the movement of the parachutist before the parachute opened in the air.

- Simulation 1

The movement of the parachutist until terminal velocity:

Parameters:

- \( x_0 = 3000 \text{ m} \)
- \( \rho = 1.2 \text{ kg/m}^3 \)
- \( t = 500 \text{ sekon} \)
- \( m = 91.6 \text{ kg} \)
- \( C_{d1} = 0.16 \)
- \( g = 9.81 \text{ m/s}^2 \)
- \( C_{d2} = 39.6 \)
Figure 3: The movement of the parachutist until terminal velocity

- Simulation 2
  Parameters:
  \[ x_0 = 1000 \text{ m} \quad \rho = 1.2 \text{ kg/m}^3 \quad t = 60 \text{ sekon} \]
  \[ m = 91.6 \text{ kg} \quad C_{d1} = 0.16 \]
  \[ g = 9.81 \text{ m/s}^2 \quad C_{d2} = 39.6 \]

Figure 4: The movement of the parachutist after the parachute opened
Simulation 3

Parameters:

\[
\begin{align*}
    x_0 &= 3000 \text{ m} \\
    m &= 91.6 \text{ kg} \\
    \rho &= 1.2 \text{ kg/m}^3 \\
    C_d &= 0.25 \\
    t &= 60 \text{ sekon} \\
    g &= 9.81 \text{ m/s}^2
\end{align*}
\]

**Figure 5**: The velocity of the parachutist before the parachute opened

From the results of the simulation of the movement of the parachutist before and after the parachute opened can be seen that the movement of the parachutist formed a parabolic trajectory. This is caused by the presence of air drag created by the body when the parachutist in the air so that the movement of the parachutist is not likely to fall free.

Analysis of the terminal velocity parachutist

Terminal velocity is speed skydiving where great value force weight objects \((W)\) of parachutist equals great value force of drag or air resistance \((F_d)\). Gravity which causes the parachutist moved vertically down one point valuation will be the same with the air drag, because the gravity constant value where as the air drag increases its value. Gravity has a constant value, because it only affected by mass and gravity. While their drag value varies according to the speed of the parachutist. At the time of parachutist jumping from a plane, the force that causes the parachutist move is gravity. Because of the gravitational acceleration of parachutist have gravity then the object will move with the speed of the faster all the time. But at the moment there are also free fall parachutist drag air is affected by the speed of the parachutist. So on one side of the gravity increased speed of objects but on the other hand with increasing speeds also adds drag. Air drag is a force of anchoring that is caused by fluid flow against the moving object. With the increase of the air drag and gravity then someday there will be equilibrium, meaning that large air drag equals huge gravity In according with the laws of Newton II, then the force acting on an object will be equal to 0. And in according with the laws of Newton, when I force acting on an object is equal to 0 then the objects tend to remain stationary or moving with constant speed. [3]
5. Conclusion

Based on the results of the analysis and discussion in the previous chapter, some things can be summed up as follows:

1. Mathematical Model of movement of a parachutist with the change of the position are as follows:

\[ v(t) = \frac{mg}{k} \left( 1 - e^{-\frac{k}{m}t} \right) \]
\[ x(t) = x_0 + \frac{mg}{k} t + \frac{m^2 g}{k^2} \left( e^{-\frac{k}{m}t} - 1 \right) \]

2. Safe time used parachutist for opening the parachute if he jumped from a plane at an altitude of 3000 meters was before in altitude of about 1000 metres ± and certainly with fixed pay attention to surroundings.

3. Large terminal velocity depends on the weight of the parachutist and the position of the body during the parachutist in the air.

References

[1] Moniuszko, J. (2010). Modelling dynamics and aerodynamics tests of a sport parachute jumper during flight in sitfly position. Polnad. Vol. 12 No. 3

[2] Ru-Yi, T. (2013). Characteristic Analysis of Flying Process of Dispensed Object with Deceleration Parachute. Vol. 3 No. 1

[3] Long, Lyle N. (1999). Velocity dependence of Aerodynamic drag. Univercity park. Washington.

[4] Meade, Douglas B. & Struthers, Allan A. (1999). Differential Equations in the New Millenium: the Parachute Problem. Dept.of Mathematic University of South Corolina, Colombia.