Analysis of Nonlinear Soil Behavior under Vertically Loaded Steel Plate Considering Local Shear Failure of Soil

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Abstract
In previous studies, an analytical method has been developed for design of spread foundations. A hybrid analysis model with nonlinear discrete springs was proposed (2002) and predictions were made of load-settlement behavior of circular plates in loading tests. The analysis results were in good agreement with test results for small settlement but departed gradually from the test results as the settlements increased, because soil shear failure was neglected. Then, a method was developed for taking into account local soil shear failure. A model was built up and applied in loading tests on square plates for clay and sandy soils (2003). The purpose of current study is to expand the use of this hybrid model considering local shear failure for analysis of circular and square plates for both clayey and sandy soils. Vertical loads are predicted for given settlements. Comparisons are made between test results and analyses with and without consideration of local shear failure. And the results are shown to be greatly improved after consideration of local shear failure. It shows that the proposed method can be very validly used for nonlinear behavior analysis of steel plate even considering local shear failure.

Keywords: nonlinear behavior; local shear failure; loading test

Introduction
In structural design, loading tests are sometimes performed on steel plate to evaluate bearing capacity of spread foundations. Test results often show a nonlinear relationship between plate settlement and external load. This indicates that the soil under the plate probably behaves nonlinearly under working loads. It is now necessary for designers to consider this soil nonlinearity in design of building foundations.

Analytical methods for predicting the behavior of a vertically loaded plate have been presented by some investigators, but most of them deal with ultimate resistance only [for example, K. Terzaghi & R. Peck (1967) 1], or elastic behavior only [for example, H. Takahashi (2000) 2, 3]. Only a few of the studies investigated the vertical behavior of a plate to the ultimate state [H. Yamaguchi (1977) 4, M. Georgiadis et al. (1988) 5, Architectural Institute of Japan (2001) 6 and so on].

The authors are developing an analytical method to study soil from its elasticity up to general failure through analysis of a vertically loaded steel plate. A hybrid model comprising a rigid plate, elastic soil and nonlinear discrete springs connecting one side to the plate and the other to soil has been proposed. As shown in Fig.1, the soil’s non-linearity is considered in the spring while the soil in this model is assumed as a linearly elastic body expressed by Boussinesq solution. Plate behaviors are predicted for circular plates without consideration of soil shear failure 7 and it is found that for large settlements the predictions were not in good agreement with test results. Further study was made for square plates considering local shear failure 8 and it showed that analysis considering soil shear failure greatly improved the results.

This study expands the use of the proposed model for analysis of both circular and square plates taking into account local soil shear failure. Spring stress is limited to the ultimate stress that the actual soil can bear. The elastic modulus, ultimate soil stress and nonlinear coefficient of spring are back-figured from test data of

Fig.1. Spring-Soil Model
loads and settlements. Using these parameters, external loads are estimated for given settlements. Analyses with and without consideration of local shear failure are compared with test results for clay, loam, sand and gravel.

**Analysis Model and Governing Equations**

As proposed in previous studies, the steel plate on the tested soil in loading tests as shown in Fig. 1a is analytically modeled by a hybrid spring-soil system as Fig. 1b. The system consists of a rigid plate, linearly elastic soil and nonlinear springs connected one side to the soil the other to the plate.

1. **Analysis model**

As specified in previous studies, the spring-soil system can be summarized as follows:

1) The response of soil in this model can be expressed by Boussinesq solution as:

\[
\delta = \frac{P(1-\nu^2)}{\pi E_s} \tag{1}
\]

where \( \delta \) represents the deformation of soil at the evaluation point; \( E_s \) is the soil’s elastic modulus and \( \nu \) is the Poisson’s ratio; \( P \) is the external vertical load; and \( \varsigma \) is the distance from external load’s location to the evaluation point.

2) Non-linearity of actual soil is represented by discrete springs, where the force-displacement relation can be expressed as:

\[
N = kx^{1/2} \tag{2}
\]

where \( N \) is the internal spring force; \( k \) is the spring’s nonlinear coefficient (taken as constant everywhere in the soil); and \( x^{1/2} \) is the spring deformation from its natural state. In this study, the spring stress is limited to ultimate stress of tested soil, which is back-figured from the load-settlement relationship of loading test data.

3) The elastic soil under rigid plate in this model is discretized accordingly with the nonlinear springs, as shown in Fig. 2. Reactions of soil and spring are evenly distributed on each section.

For circular plate, the under soil is divided into \( m \) sectors and each sector is divided into \( n \) sections, each of equal area. In view of axial symmetry of load and plate, the reaction along the circumference is the same and difference appears along the radial direction only.

For square plate, the soil is divided into \( n \) sections with equal grid span in \( x \)- and \( y \)-directions along its two perpendicular edges.

2. **Equations for analysis**

If the displacement of the rigid plate under external force \( F \) is \( S \), letting \( y_i = x_{i\nu}^{1/2}, (i=1, \ldots, n) \), the governing equations for this spring-soil system can be expressed as:

\[
\begin{bmatrix}
  a_{11} & a_{12} & \cdots & a_{1n} \\
  a_{21} & a_{22} & \cdots & a_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_n
\end{bmatrix}
\begin{bmatrix}
k \end{bmatrix}
= \begin{bmatrix}
S - y_1^2 \\
S - y_2^2 \\
\vdots \\
S - y_n^2 \\
F_i
\end{bmatrix} \tag{3}
\]

where \( i (i=1, \ldots, n) \) is the number of the evaluation section. \( F = F/m \) for circular plate and \( F = F \) for square plate. The partial matrix of Eq.(3) is called linear characteristics matrix of soil and can be expressed by Boussinesq solution as:

\[
A = \begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
 a_{21} & a_{22} & \cdots & a_{2n} \\
 \vdots & \vdots & \ddots & \vdots \\
 a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}
\]

For circular plate, \( a_{ij} = \sum_{\lambda=1}^{m} \frac{1-\nu^2}{\pi E_s \zeta_{i\lambda}} \) \tag{4-1}

For square plate, \( a_{ij} = \frac{1-\nu^2}{\pi E_s \zeta_{ij}} \) \tag{4-2}

\( (a_{ij}; i=1, \ldots, n, j=1, \ldots, n) \)

In Eq.(4-1), \( \lambda = 1, \ldots, m \) denotes the number of circumference vector; \( \zeta_{i\lambda} \) is the distance from the external load on section \( j \) of sector \( \lambda \) to the evaluation point \( i \).

The internal spring force is equivalent to the soil reaction of corresponding section, and the soil reaction of any section is limited because of soil shear failure.
Therefore, it is considered in this study that the spring force should not exceed the ultimate soil resistance. For instance, if the spring force $N_j (j=1, \cdots, n)$ at section $j$ based on elastic calculation reaches the ultimate soil resistance $N_0 (j)$, it is assumed that shear failure occurred in soil of this section. The internal spring force remains constant and equal to the ultimate value, and Eq.(1) is defined by:

$$N = N_0 (j)$$  \hspace{1cm} (5)

And the governing equation should be changed accordingly with Eq.(5) substituted into Eq.(3).

**Analysis Procedure**

In using the above hybrid model for estimation of plate behavior in loading tests, the elastic modulus, ultimate resistance of soil and nonlinear coefficient of spring should be determined first.

1. **Determination of parameters**

   **Elastic modulus**

   Konder et al.\cite{Konder10} have proposed that the nonlinear stress-strain curve of soil may be approximated by hyperbolic equation with a high degree of accuracy. This is used in this study to simulate the load-settlement curve of loading test:

   $$F = \frac{S}{(aS + b)}$$  \hspace{1cm} (6)

   in which $F$ is the working load on the plate; $S$ is the plate settlement; and $a$ and $b$ are constants and, as shown in Fig.3, $a$ is the reciprocal of the asymptotic load of the hyperbolic function and $b$ is the reciprocal of the slope of the initial tangential line of the hyperbola. The soil’s elastic modulus is approximated from the initial tangential line together with the Boussinesq equation as follows:

   $$E_s = \frac{1 - v^2}{Bb}$$  \hspace{1cm} (7)

   in which $B$ is the width of plate; $I_s$ denotes the settlement coefficient. $I_s=1.00$ for circular plate and $I_s=0.88$ for square plate.

   At the same time, the $F$-$S$ relationships from the vertical loading plate test can also be simulated by two intersecting lines on logarithmic coordinates. The solid curve shown in the small window in Fig.3 indicates the hyperbolic relationship of data included in the first line, while the dotted one indicates that of all the test data. This indicates that the $E_s$-value back-figured using the first section of the logarithmic $F$-$S$ relationship is closer to the real elastic modulus of the tested site soil. This is used to evaluate the elastic modulus of soils in this study.

   **Ultimate resistance and stress distribution**

   According to J.M. Duncan et al.\cite{Duncan11} who have presented similar research to Konder et al., the ultimate resistance of soil is expected to be about 0.90 times the asymptotic value of the hyperbolic relation curve. In this study, the ultimate resistance $F_u$ of soil is calculated by:

   $$F_u = 0.90 * F_{ asymptotic}$$  \hspace{1cm} (8)

   in which $F_{ asymptotic}$ is the asymptotic load value, that is $1/a$.

   On the basis of the data of vertically loaded model pile tests, Takano\cite{Takano12} proposed the parabolic shape as the distribution of soil stresses acting on the pile tip at the general failure state in sand. From this result and the knowledge of soil mechanics, the distribution of soil stresses under a rigid plate at ultimate state is assumed to be in the shape as shown in Fig.4, that is, uniform for cohesion part and parabolic for friction part.

   For both circular and square plates, the ultimate stress $Q_u$ can be expressed in two parts as

   $$Q_u = Q_c + Q_f$$

   where $Q_c = cN_c (N_c = 5.70)$ denotes the cohesion part and $Q_f = Q_{max} [1 - (r/r_0)^2]$ denotes the friction part.

   For circular plate, in view of axial symmetry of load and plate, the stresses are evenly distributed along each
circle as indicated in Fig.5a. If \( r \) is the radius of a circle on which the evaluation point located, the ultimate stress of friction part can be expressed as:

\[
Q_y (r) = Q_{y_{\text{max}}} \left[ 1 - \left( \frac{2r}{B} \right)^2 \right] \quad (10-1)
\]

For square plate, we assume that the stresses are evenly distributed along each concentric square, as indicated in Fig.5b. If \( x \) and \( y \) are the distances from the evaluation point to the loaded point in the \( x \)- and \( y \)-directions, respectively, the ultimate stress of friction part can be expressed by:

\[
Q_y (x, y) = Q_{y_{\text{max}}} \left[ 1 - \left( \frac{2x}{B} \right)^2 \right] \quad (x \geq y)
\]

\[
Q_y (x, y) = Q_{y_{\text{max}}} \left[ 1 - \left( \frac{2y}{B} \right)^2 \right] \quad (x \leq y) \quad (10-2)
\]

\( Q_{y_{\text{max}}} \) for both square plate and circular plate is determined by equating the ultimate resistance to the integration of ultimate stress. The equilibrium can be expressed as:

\[
F_b = cN_eA + \int Q_y dA \quad (11)
\]

where \( A \) is the area of plate, \( A = \pi B^2 / 4 \) for circular plate and \( A = B^2 \) for square plate. It can be derived that

\[
Q_{y_{\text{max}}} = 2F_b/A \quad (12)
\]

At any loading stage, the spring force of a certain section should not exceed the ultimate resistance defined by soil shear failure of that section. The spring force \( N_j \) of an arbitrary failed section \( j \) can be expressed as \( N_j = Q_{y_{\text{max}}} dA \), in which \( dA \) indicates the area of soil section \( j \) and \( Q_y \) is expressed by Eqs.(9) and (10) according to \( x \) and \( y \) values or radius \( r \).

**Nonlinear coefficient of spring**

After the elastic modulus and ultimate stress are determined, the proposed model is used to back-figure nonlinear coefficient of spring from load-settlement test data. A program using Newton-Raphson’s iterative method is built up to solve Eq.(3) with \( F \) and \( S \) known and \( k \), \( y_1 \), \( y_2 \), ..., \( y_n \) to be solved. Iterative calculation steps are made to ensure \( N_j = Q_{y_{\text{max}}} dA \) while considering local soil shear failure. The program flow chart is as shown in Fig.6, with initial assumption made for \( k \) and \( \{ x_{sp} (i) \} \), \( (i = 1, ..., n) \).

As in previous studies, \( k \)-values solved from Eq.(3) are different for each pair of \( F \)-\( S \) test data even for the same test. And the average of these \( k \)-values for different loading steps is taken as the nonlinear coefficient of spring for the tested soil.

2. Estimation of external loads

After all the parameters of the analysis model are determined, it can be used to estimate external loads under measured plate settlements. Program is developed to solve Eq.(3) with \( S \) and \( k \) known while \( y_1 \), \( y_2 \), ..., \( y_n \) and \( F \) are to be solved. The flow chart is same as in Fig.6, but initial assumptions are made for \( F \) and \( \{ x_{sp} (i) \} \), \( (i = 1, ..., n) \).

**Analysis of Test Results**

Test results of 6 sites from Obayashigumi, Ltd. and Tokyo Soil Research, Ltd., performed on clayey soil including loam and sandy soil including gravel are analyzed. There are 3 for circular plate and 3 for square plate. In this section, estimation of elastic modulus \( E_s \), ultimate soil resistance \( F_b \) and nonlinear coefficient of spring \( k \) are presented first. Then, predictions of vertical
load from plastic analyses considering local shear failure are compared with elastic analyses without considering local shear failure and test results.

1. Parameters for analysis model

Tests were performed on circular steel plates of Ø30cm×2.5cm and square steel plates of 30cm×30cm×2.5cm. Outlines of loading tests are shown in Table 1, in which \( q_u \) is unconfined compressive strength of soil. The Poisson’s ratio is taken to be 0.50 for clayey soil and 0.30 for sandy soil in this study. Soil’s elastic modulus, ultimate resistance and non-linear coefficient of spring back-figured from test result data using the method mentioned above are also listed.

From the data of maximum settlements, some judgment can be made as in practical engineering. For test T-43, the settlement is about 1% of the plate width, from which it can be concluded that the soil is still in elastic state and there is no shear failure. For test 48 and 104, the settlements are about 3% and 5% of the plate width, from which it can be assumed that local shear failure has happened in the soil but the soil has not reached the ultimate state of general failure. For test 183, the settlement is already about 15% percent of the plate width, from which it is considered that the soil had undergone general failure and no further resistance could be developed.

It is considered that cohesion of sand and gravel is zero and neither friction nor cohesion of loam or sandy clay is zero. Thus, the ultimate stress of sand and gravel has a friction part only and is distributed in the form of a parabolic curve with zero at the edges and maximum at the center of the plate. However, the ultimate stress of loam and sandy clay has both a cohesion part and a friction part, with an even distribution plus a parabolic distribution as shown in Fig.4.

2. Comparisons between measured and analyzed load-settlement behavior

Figs.7 to 12 compare the predicted overall load-settlement curves with and without consideration of local shear failure in soil and the tested curves. In the figures, “Test” means test results, “Elastic” means elastic analysis without consideration of local shear failure and “shear” means plastic analysis considering local shear failure. From these figures, it is found out that the tendencies of the calculated behavior are similar to the observed ones.

Fig.7, for test T-43 performed on a square plate for clay, shows that “shear” has the same results as “elastic”, indicating that there was no shear failure during the test.

Figs.8 - 10 are for tests 48, 104 and 183, performed on circular plates on loam. From these figures, it is clear that the analysis results are in very good agreement with test results when settlement is small. When settlement becomes larger, the “Elastic” analysis gives results much larger than the test results, while “Shear” gives results in much closer agreement with test results, as expected.

Figs.11 and 12 are for test T-6 and 118 performed on square plates on sand and gravel, respectively. As shown, the analyzed results are in very good agreement with test results when settlement is small. When the settlement becomes larger, the elastic analysis gives results 1.5 times of test results for test T-6 and almost 2 times of test results for test 118. However, the plastic analysis greatly improves the results, showing predictions nearly the same as the test results.

Conclusions and Discussions

This paper has presented an analytical method for predicting nonlinear behavior of vertically loaded circular and square steel plates in loading tests up to the ultimate state of general soil failure.

The plastic analysis method takes into consideration soil shear failure with ultimate stress distribution along the plate.

Results show that, for relatively small settlements, both analyses considering and not considering local shear failure in soil show good agreement with test results; for much larger settlements, elastic analysis without consideration of shear failure shows a disagreement with the test results; while plastic analysis considering local shear failure shows results in good agreement with test data.

Table 1. Outline of Tests Applied for Analysis

| Test No. | Soil Type | Plate Type | SPT N-Value | \( q_u \) (K/N/m²) | \( E_u \) (K/N/m²) | Maximum Load (KN) | Settlement (cm) | Nonlinear Coefficient \( k \) (10²K/N/cm₁/₂) |
|----------|-----------|------------|-------------|-----------------|-----------------|----------------|----------------|-----------------|
| T-43     | Sandy Clay| Square     | 10          | 125.00          | 51498.12        | 36.00         | 0.3612         | 7.2927          |
| 48       | Loam      | Circular   | -           | 89.00           | 14506.89        | 23.100        | 0.9600         | 0.8311          |
| 104      | Loam      | Circular   | -           | 159.00          | 22714.92        | 32.000        | 1.4857         | 0.8757          |
| 183      | Loam      | Circular   | -           | 68.00           | 6500.11         | 21.600        | 4.4442         | 0.2541          |
| T-6      | Sand      | Square     | 41          | -               | 11875.63        | 25.000        | 2.8203         | 2.5848          |
| 118      | Gravel    | Square     | 56          | -               | 60543.59        | 80.000        | 3.8610         | 7.7028          |

Note: * \( q_u = 100N/8 \) (K/N/m²)
This is expected from general knowledge of geotechnics. It means that the proposed analysis method can be flexibly used for analysis of nonlinear behavior of vertically loaded circular and square steel plates for a wide range of soil types from clay and loam to sand and gravel.

In this study, the nonlinear coefficient of spring is back-figured from load-settlement results of loading test. If empirical relationships are built up between this coefficient and unconfined compressive strength or SPT N-values, this analysis method can be applied in practice even if loading tests are not performed.

This paper has presented the analysis for soil assumed as a homogeneous half-space. The same nonlinear-spring-linear-soil system can be applied for each layer in multi-layer soil case. It is expected that both nonlinear coefficient and steel plate behavior prediction will be improved if a multi-layer model is applied. This is a subject for further study in the future. However, the results of this study show that predictions made on assumption of half-space are so little different from test results that the method is already acceptable in engineering practice.

Acknowledgments
Thanks are given to Obayashigumi, Ltd. and Tokyo Soil Research, Ltd. for supplying test data.
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