THE PROPOSED QUADRUPLE SYSTEM SZ HERCULIS: REVISED LITE MODEL AND ORBITAL STABILITY STUDY

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ABSTRACT
In a recent study, Lee et al. presented new photometric follow-up timing observations of the semidetached binary system SZ Herculis and proposed the existence of two hierarchical cirumbinary companions. Based on the light-travel time effect, the two low-mass M-dwarf companions are found to orbit the binary pair on moderate to high eccentric orbits. The derived periods of these two companions are close to a 2:1 mean-motion orbital resonance. We have studied the stability of the system using the osculating orbital elements as presented by Lee et al. Results indicate an orbit-colliding architecture with short-term dynamical instabilities leading to the escape of one of the proposed companions. We have examined the system’s underlying model parameter space by following a Monte Carlo approach and found an improved fit to the timing data. A study of the stability of our best-fitting orbits also indicates that the proposed system is generally unstable. If the observed anomalous timing variations of the binary period is due to additional cirumbinary companions, then the resulting system should exhibit a long-term stable or semi-stable configuration much different from the orbits suggested by Lee et al. We, therefore, suggest that based on Newtonian-dynamical considerations, the proposed quadruple system cannot exist. To uncover the true nature of the observed period variations of this system, we recommend future photometric follow-up observations that could further constrain eclipse-timing variations and/or refine light-travel time models.

Key words: binaries: close – binaries: eclipsing – stars: individual (SZ Herculis)

1 INTRODUCTION

Formation of multiple star systems is complex and is believed to occur either by interaction/capture mechanisms (van den Berk et al. 2007, and references therein) during the formation and dynamical evolution of globular star clusters, or directly from a massive primordial disk involving accretion processes or disk instabilities (Lim & Takakuwa 2006; Duchêne et al. 2007; Marzari et al. 2009). The exact formation channel is not yet fully understood. However, these mechanisms might be capable of producing various star systems characterized by different orbital architectures and/or hierarchies (Evans 1968).

Cirumbinary objects belong to a category of hierarchical star systems where one (or more) massive companion(s) orbits a pair of stars. An example of such systems is the quadruple (or quaternary) system of HD 98800 consisting of two distinct spectroscopic binaries orbiting a common center of mass (Furlan et al. 2007). Single or multiple low-mass cirumbinary companions of planetary nature have been recently discovered from ground-based observations (Lee et al. 2009; Beuermann et al. 2010; Potter et al. 2011; Qian et al. 2011) and with the Kepler space telescope (Doyle et al. 2011; Welsh et al. 2012). However, some of these multi-planet cirumbinary systems (NN Ser, HW Vir, HU Aqr, and DP Leo) have proposed orbital properties that seem to render their orbits unstable (Horne et al. 2011, 2012; Hinse et al. 2012; Wittenmyer et al. 2012; Funk et al. 2011; Goździewski et al. 2012). In addition, Parsons et al. (2010) present photometric follow-up observations of a number of eclipsing post-common-envelope binaries where they have been able to rule out previous claims for single-object cirumbinary companions (e.g., Qian et al. 2009, 2010a, 2010b).

In a recent work, Lee et al. (2012) presented new photometric observations of the Algol-type semidetached binary star system SZ Herculis (SZ Her(AB)) hereafter. Based on more than 1000 eclipse measurements (spanning more than a century), these authors were able to detect a significant change in the system orbital period manifesting itself as eclipse-timing variations (ETVs). Such a change in the binary period can be due to (1) the interaction between the two binary components through their magnetic fields, mass transfer, or tidal interactions (resulting in apsidal motion), (2) gravitational perturbations by additional massive objects (companions), and/or (3) by the light-travel time effect (LITE) also known as Romer effect (Irwin 1952, 1959).

It is important to note that timing measurement errors can be uncorrelated (white noise following a Gaussian distribution) and/or of systematical (correlated or red noise) origin. For timing measurements of pulsars, this has recently been pointed out by Coles et al. (2011) as a possible cause of errors in estimating the model parameters. In the past, neglecting the effect of red noise was responsible for the false detection of planets around pulsars (Bailes et al. 1991).

The LITE effect implies the presence of one or more massive object(s), which can result in the reflex motion of the binary barycenter about the total system’s center of mass. This reflex motion (or binary wobble) gives rise to time-varying light-travel paths resulting in differences in the periodic mid-eclipse timing. It is important to note that the LITE effect is a geometrical effect and does not involve gravitational perturbations. For instance, in a hierarchical triple stellar system (a third companion orbiting a close binary), the third object creates a single-body LITE effect, introducing a sinusoidal-like variation in the binary orbital period (Irwin 1959).

5 This notation follows the notation as suggested by Hessman et al. (2010).
6 Sometimes referred to as LTTE or LTT in the literature.
a discussion of the light-travel time orbit in a coordinate system with origin at the center of the LITE orbit. For reasons of symmetry, when discussing properties of the LITE orbit, this choice is suitable for the geometric interpretation of the resulting light-travel time curve (or $O-C$ diagram). However, a more natural choice, especially in systems with multiple companions, would be the system’s barycenter. From Figure 1, if $z$ measures the distance of the single-binary object from the line perpendicular to the line of sight and passing through the systems’ barycenter, then the eclipsing period variation (or the observed minus computed, $O-C$, timing difference) will be

$$
\tau_i = \frac{z_i}{c} = K_{b,i} \left[ 1 - \frac{e_{b,i}^2}{1 + e_{b,i} \cos f_{b,i}} \sin(f_{b,i} + \omega_{b,i}) \right],
$$

where $K_{b,i}$ is its semiamplitude. The expression in Equation (1) is obtained by evaluating the $z$-component of the binary orbit in the $O-C$ diagram with $c$ denoting the speed of light and $i = 1, 2$ referring to the LITE binary orbit when considering either one of the two companions. In Equation (1), $a_{b,i}$ is the semimajor axis of the binary (single-object) orbit, $\epsilon_{b,i}$ is its eccentricity, $i_{b,i}$ is the orbital inclination with respect to the plane of the sky, $\omega_{b,i}$ measures the binary’s argument of pericenter, and $f_{b,i}$ denotes its true anomaly. We note that we do not include any secular timing variation due to mass transfer between the two components. As pointed out by Lee et al. (2012), this effect is minimal and we here assume that it is negligible. Furthermore, it is important to note that $i = 1, 2$ describes two separate two-body problems. For $i = 1$, we consider the binary and the inner companion whereas for $i = 2$, we consider the binary and the outer companion. To obtain the resulting period variation of the measured mid-eclipse times due to both companions, one then usually assumes the superposition principle and uses the expression $\tau = \tau_1 + \tau_2$ for the combined LITE.

The expression in Equation (1) is obtained by evaluating the $z$-component of the binary that is along the line of sight, in a coordinate system with origin at the system’s barycenter. This is different from the formulation in Irwin (1952), who employs a coordinate system with origin at the center of the LITE orbit. The result is the omission of the $e_{b,i} \sin \omega_{b,i}$ term when comparing with the corresponding expression for the light-travel timing difference presented in Irwin (1952). We find that a barycentric coordinate system is more intuitive when carrying out the subsequent dynamical analysis. Since $\tau_i$ is a quantity measuring a time difference, it follows that there should be no difference in the derived orbits when formulating the light-travel timing difference in the two coordinate systems. We have tested the latter and used the best-fitting parameters from Table 6 in Lee et al. (2012) as an initial guess for the two formulations. The results are shown in Figure 2 showing a shift along the secondary axis for the two best-fitting models. The final best-fitting orbital parameters were similar for the two models and were close to the parameters as determined by Lee et al. (2012).

Finally, it is worth mentioning some properties of the LITE orbit for clarification. The previously mentioned Kepler elements all describe the binary system’s orbit (as a single object). None of these elements are directly attributed to the (possibly unseen) companion orbit. The Keplerian elements of the companion(s) are inferred only indirectly from first principles. We refer to Murray & Dermott (2001) for the details of the properties of orbits in a barycentric coordinate system. In the following, we qualitatively outline some relationships between the two orbits.
and refer to Figure 1. First, the two semimajor axes are related to each other via the masses. Second, the orbital eccentricity, inclination, and orbital periods ($P_{1,2}$) are the same for the single-binary object and the companion orbit. Also the apsidal lines of the two orbits are antialigned (see Figure 1) giving rise to a $180^\circ$ difference between the two argument of pericenter angles. Orbital quantities related to a companion are denoted with a numeral subscript starting with the inner companion first (i.e., $e_1$ shows the eccentricity of the inner companion). We note that this convention is different in Lee et al. (2012). In their work the authors use the subscript 4 (3) to denote the inner (outer) companion.

3. STABILITY STUDY

We examined the orbital stability of the proposed three-body system (binary and two M-type stars) of SZ Her. Orbital parameters of the two proposed companions are listed in Table 1.
with formal 1σ uncertainties as reproduced from Lee et al. (2012). The masses and orbital semimajor axes are stated without formal uncertainties and were determined along with other orbital parameters, as outlined below. We show a graphical representation of the two osculating M-star orbits in Figure 3 for two values of the argument of pericenter. The figures assume the case where \( I_{b,1} = I_{b,2} = 90° \) with the combined binary pair placed at the origin, defining the dynamical center in an astrocentric system. An inspection of the derived osculating elements indicates that the pericenter distance \( e_{1,2}(1 - e_{1,2}) \) of the inner and outer companions are at 8.63 and 7.45 AU, respectively, implying an orbit-crossing geometry. Considering the large masses of the companions, such an orbital architecture is expected to be highly unstable. In the following, we will study the orbital stability of the two proposed circumbinary companions in more detail.

We used the variable-step, Bulirsch-Stoer (BS2) \( N \)-body algorithm as implemented in the orbit integration package MERCURY\(^7\) (Chambers & Migliorini 1997; Chambers 1999). The initial time step was set to 0.01 days. During the integrations the maximum relative energy error was a few times \( 10^{-13} \)

In our calculations, we combined the masses of the two binary components and treated them as one single object. We used this simplifying assumption in order to be consistent with the orbital parameters (and minimum masses) as derived from the LITE formalism (Irwin 1952). We integrated the orbits of binary and companion bodies in a “binarycentric” system with the combined binary mass at rest (this system is also referred to as a non-inertial astrocentric frame). This requires a transformation of orbital elements from the coordinate system defining the LITE orbit (barycentric frame) to the binarycentric frame. Throughout this work, we consider the two companions to be on the same plane. That is, \( I_1 = I_2 \). This consideration may be justified noting that circumbinary objects may form in the same accretion disk where the binary system was formed, and as a result, orbits are more or less aligned with each other. In such a scenario, the inclination of the orbit of the system with respect to the plane of the sky is likely to be close to 90°. However, we remind the reader that no observational data exist for SZ Her that might allow the determination of \( I_1 \).

### 3.1. Initial Conditions from LITE Orbit

In general, the inclination of the LITE orbit relative to the plane of the sky is not known. Only in the case of \( I_{1,2} \sim 90° \) where the companion is also eclipsing one (or both) of the binary components information about this angle can be obtained from photometric measurements (e.g., Kepler-16b, Doyle et al. 2011 and Kepler-34b, Kepler-35b, Welsh et al. 2012). However, in the case of SZ Her, such a detection seems difficult due to the long LITE periods involved. Possibly third/fourth-light might be detectable from spectral data. The lack of constraints in orbital inclination implies that only information about the minimum mass \( (m_{1,2} \sin I_{1,2}) \) and minimum semimajor axis \( (a_{1,2}) \) can be obtained.

Information on the minimum mass from each individual LITE orbit is extracted somewhat similarly to the case of a single-lined spectroscopic binary. The minimum mass of the two circumbinary companions was determined from a Newton–Raphson iteration using the individual mass functions

\[
f(m_i) = \frac{4\pi^2(a_{b,i} \sin I_i)^3}{GP_i^2} = \frac{(m_i \sin I_i)^3}{(M_\text{bp} + m_i)^2}
\]

for each LITE orbit and the combined mass of the binary pair \( M_\text{bp} \approx 2.32 \, M_\odot \) (Lee et al. 2012). In agreement with Lee et al. (2012), the masses of the two M-dwarfs were found to be 0.19 \( M_\odot \) for the inner and 0.22 \( M_\odot \) for the outer companion (see Table 1).

In calculating the values of the masses, we chose not to consider the formal uncertainties in the companion masses since as presented by Lee et al. (2012), these (formal) errors are on a 0.1% level. From numerical experiments, these minute changes in mass have little impact on the overall dynamical evolution of the system.

The minimum semimajor axes of the LITE orbits are adopted from Lee et al. (2012) and are stated in the barycentric frame. In order to determine the semimajor axes of the two companions in the astrocentric frame, we used Kepler’s third law, since the minimum mass and orbital period along with the total mass of the binary system are known parameters. We found the minimum semimajor axis for the inner companion to be

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**Table 1**

| Parameter | SZ Her(AB)C | SZ Her(AB)D | Unit |
|-----------|-------------|-------------|------|
| Minimum semimajor axis, \( a_{1,2} \sin I_{1,2} \) | 16.6 | 26.6 | AU |
| Eccentricity, \( e_{1,2} \) | 0.48 ± 0.17 | 0.72 ± 0.09 | … |
| Argument of pericenter, \( \omega_{1,2} \) | 105 ± 10 | 268.6 ± 7.5 | deg |
| Orbital period, \( P_{1,2} \) | 42.5 ± 1.1 | 85.8 ± 1.0 | year |
| Minimum mass, \( m_{1,2} \sin I_{1,2} \) | 0.188 | 0.222 | \( M_\odot \) |

**Notes.** Note that we have accounted for the 180° difference in the argument of pericenter angle. Parameters with formal 1σ uncertainties are from Lee et al. (2012). All other parameters are obtained as outlined in the text.

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\(^7\) www.arm.ac.uk/~jec

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**Figure 3.** Geometry of the two orbits (in the orbital plane) corresponding to the LITE fit parameters in Table 1. Both companion orbits are relative to the binarycentric reference frame (with the binary taken to be a single object) with origin at the center of the cross-hair. The orbits were integrated for one orbital period considering massless objects to visualize the two osculating single LITE orbits. We show both the \( \omega = 0° \) (1a, 2a) orbits and the orbits for \( \omega = (285–180)° \) (1b) and \( \omega = (88.6–180)° \) (2b).
resulting in an escape of one of the companions. The upper panel in this figure shows the case where the orbital apsidal lines of the two companions are initially parallel and both companions start with the same mean longitude ($\lambda_{1,2} = 0^\circ$). The result is a quick capture of the two companions into a binary system. Although this initial condition results in a long-term stability, it does not reflect the derived Keplerian parameters of the two-LITE orbits proposed by Lee et al. (2012). The middle panel of Figure 4 shows the result of an integration where the outer circumbinary component escapes the system. In this integration, we used the fitted argument of pericenter for each LITE orbit to derive the argument of pericenter for the two companions. Finally, we considered the case where the outer M-dwarf companion is started at its apocenter $a_2(1-e_2)$ with $\lambda_2 = 180^\circ$. The remaining Kepler elements were chosen to be similar to those of the previous case. The corresponding orbits are shown in the lower panel of Figure 4. As shown here, for this set of initial conditions, the inner companion escapes from the system in less than 100 years.

In general, for the escape scenarios, because of the conservation of energy, one companion is transferred to a smaller orbit as a result of the escape of the other companion. We encountered this behavior in all our integrations where an escape took place. We then considered various orbits with semimajor axes and eccentricities within and beyond their formal 1σ uncertainties as given by Lee et al. (2012). The majority of these integrations (especially those with higher orbital eccentricities) resulted in unstable orbits. Low-eccentricity orbits showed a somewhat longer lifetime. The main mechanism resulting in the system’s instability was the escape of one of the companions as a result of a close encounter with the other.

In order to search for a stable orbital configuration that reflects the main characteristic of the two proposed one-companion LITE orbits, we considered initial conditions with different LITE orbit inclinations for the two companions on coplanar orbits. Although the orbital separations increase with decreasing inclination, mutual gravitational perturbations are still important since the masses of the two companions increase as well. Therefore, it is not obvious a priori if a stable configuration can be found. We integrated the equations of motion for 10 different values of the inclination with respect to the plane of the sky in the range of $I_{1,2} \in [1^\circ, 90^\circ]$ for each inclination, we scaled the minimum mass and semimajor axis for the two companions accordingly in order to obtain their true (assumed) mass. In each integration, we placed the outer component at $\lambda_2 = 180^\circ$. A representative subsample of our results is shown in Figure 5. In all 10 cases we found the system to be highly unstable on short timescales. To double-check our results, we repeated the numerical integrations using the RADAU algorithm (also available in the MERCURY package). The outcome confirmed our previous results.

In each integration, we found multiple close approaches that either led to ejections or collisions between the two proposed circumbinary companions. We repeated our integrations with various initial $\lambda_{1,2}$ resulting in no difference in the overall inferred orbital instability. Considering a wide range of orbital possibilities, we find that all our orbital integrations to result in highly unstable systems.

4. METHODS AND RESULTS FROM REVISED LITE MODELS

In order to find a possibly stable LITE model to the proposed quadruple system, we carried out an extensive search of the $\chi^2$
parameter space. The analysis, methodology, and technique are similar to those described in detail in Hinse et al. (2012). The only exception is that we now formulate the LITE model in the barycentric frame by omitting the \( e \sin \omega \) term in Equation (3) of Hinse et al. (2012) as outlined earlier. In the following, we briefly repeat main aspects of the underlying analysis.

We used the Levenberg–Marquardt least-squares minimization algorithm as implemented in MPFIT (Markwardt 2009) software. The goodness-of-fit statistic of each fit was evaluated from the weighted sum of squared errors, \( \chi^2 \). Here we use the reduced chi-square statistic \( \chi_r^2 \). We seeded 107,625 initial guesses within the framework of a Monte Carlo experiment.

Each guess was allowed a maximum of 500 iterations before termination. Converged solutions were accepted with initial guess and final-fitting parameters recorded. Initial guesses of the model parameters were chosen at random from either a uniform or Gaussian distribution. Lee et al. (2012), for example, provided a Lomb–Scargle period analysis on the complete timing data set. They determined two possible dominant frequencies of 3.53 \( \times 10^{-5} \) cycle day\(^{-1} \) and 7.06 \( \times 10^{-5} \) cycle day\(^{-1} \) associated with the two proposed circumbinary companions. The shorter period is relatively well determined given the long observational period in agreement with the two-LITE model found in Lee et al. (2012). However, a small number (3758 or 12\%) of initial guesses had a final reduced goodness-of-fit parameter in the interval 1.0086 \( \leq \chi_r^2 \leq 1.0186 \). This is in agreement with the two-LITE model found in Lee et al. (2012). However, a small number (3758 or 12\%) of initial guesses had a final reduced goodness-of-fit statistic in the interval 0.9886 \( \leq \chi_r^2 \leq 0.9986 \) with the best-fitting model resulting in \( \chi_r^2 = 0.9886 \). No initial guesses had a goodness-of-fit statistic in the interval 0.9886 \( \leq \chi_r^2 \leq 1.0086 \). All remaining fits had \( \chi_r^2 > 1.05 \).

The goodness-of-fit statistic corresponding to our best-fitting model is somewhat smaller than the two-LITE fit presented in Lee et al. (2012). We discarded all guesses that reached a lower or upper boundary in one of the model parameters (since no formal errors are supplied for these parameters within the MPFIT algorithm). The total number of qualified guesses was then reduced to 30,700. The majority of guesses (15,659 or 50\%) had final reduced goodness-of-fit parameter in the interval 1.0086 \( \leq \chi_r^2 \leq 1.0186 \). This is in agreement with the two-LITE model found in Lee et al. (2012). However, a small number (3758 or 12\%) of initial guesses had a final reduced goodness-of-fit statistic in the interval 0.9886 \( \leq \chi_r^2 \leq 0.9986 \) with the best-fitting model resulting in \( \chi_r^2 = 0.9886 \). No initial guesses had a goodness-of-fit statistic in the interval 0.9886 \( \leq \chi_r^2 \leq 1.0086 \). All remaining fits had \( \chi_r^2 > 1.05 \).

The goodness-of-fit statistic corresponding to our best-fitting model is somewhat smaller than the two-LITE fit presented in Lee et al. (2012). We show the best-fitting model in Figure 6 with the best-fitting parameters listed in Table 2. We note that the orbital eccentricity of the inner companion increased significantly from 0.48 to 0.76 when comparing with the work in Lee et al. (2012). Orbital radii, periods, as well as the minimum masses of the two companions are almost unchanged.

We used the same timing data set as in Lee et al. (2012), but applied our model on times defined by a uniform time standard. In Lee et al. (2012), timing data for SZ Her were recorded in the UTC time frame, which is known to be non-uniform (Guinan & Ribas 2001). We therefore transformed the HJD (Heliocentric Julian Date) timing records in UTC (Coordinated Universal Time) into the terrestrial time (TT) standard (Bastian 2000). The resulting unit of TT time is HJED (Heliocentric Julian Ephemeris Date).

### 4.1. Results

The results from our Monte Carlo experiment are somewhat similar to the results presented in Hinse et al. (2012). We discarded all guesses that reached a lower or upper boundary in one of the model parameters (since no formal errors are supplied for these parameters within the MPFIT algorithm).

Figure 5. Unstable orbits from numerical integrations for various LITE orbital inclinations. Each panel shows the orbits (in the orbital plane) using initial conditions that considered a scaled semimajor axis and mass for the two companions simultaneously. The inclination between the line of sight and the plane of the sky were \( I_{1,2} = 5^\circ, 10^\circ, 30^\circ, 50^\circ, 70^\circ, 80^\circ \). In all integrations, \( \lambda_2 = 180^\circ \). The orbits of the two companion were assumed to be coplanar. The stipulated line always represents the outer binary companion. The center of the hair-cross marks the origin of the barycentric reference frame.
Figure 6. Best-fit model with $\chi^2 = 0.989$ (dash-dot-dotted) from our many-guess Monte Carlo experiment and the two-companion sinusoidal-like variations: inner (dash) and outer companions (dash-dotted). The corresponding orbital parameters for the two-LITE orbits are shown in Table 2. The root-mean-square scatter of data around the best fit is $\sim$292 s. We show the three $\pm$1σ timing error bars in the lower left corner corresponding to 0.0036, 0.0020, and 0.0013 days as adopted by Lee et al. (2012) for various observation techniques.

Table 2

| Parameter | Two-LITE | Unit |
|-----------|----------|------|
| $\chi^2$  | 0.989    |      |
| rms       | 0.00339  | days |
| $T_0$     | 2,434.98738455(31) | HJED |
| $P_0$     | 0.818095801(11) | days |
| $w_{b,i}$ | $\sin b_i$ | AU   |
| $e_{b,i}$ | $\cos b_i$ | deg  |
| $T_{b,i}$ | 2,422.649(322) | HJED |
| $P_{b,i}$ | (or $P_i$) | days |
| $K_{b,i}$ | 15286 ± 224 |        |
| $f(\mu_i)$ | 0.00864(32) | days |
| $m_{\mu_i}$ | 0.00191(65) | days |
| $m_{\mu_i}$ | 0.00138(37) | $M_\odot$ |
| $m_i$ | $\sin i$ | $M_\odot$ |
| $m_i$ | $\cos i$ | AU   |
| $e_i$   | $\sin i$ | AU   |
| $r_i$   | 182 ± 7.1 | deg  |
| $P_i$   | 15286 ± 224 | days |

Notes. Subscripts 1, 2 refer to the circumbinary companions with $i = 1$, the inner, and $i = 2$, the outer, companions. rms measures the root-mean-square scatter of the data around the best fit. Formal uncertainties obtained from the covariance matrix are valid for the last digit and shown in parenthesis. Note that the eccentricity and orbital period are shared quantities as outlined in the text. The last five lines are quantities of the two companions in the astrocentric coordinate system.

0.57 ± 0.17, respectively. For the outer LITE orbit, the average final semimajor axis and eccentricity were 2.32 ± 0.41 AU and 0.59 ± 0.22, respectively.

4.2. Orbit Stability of Best-fit and LITE Computation

In the previous section, we obtained an improved fit to the existing timing data of the eclipsing SZ Her system. The resulting best-fit osculating orbits of the two proposed companions showed an orbit-crossing architecture. To examine the orbital stability of the best-fit quadruple system, the equations of motion were integrated in an astrocentric system using MERCURY. The binary pair was treated as a single object. We considered a large combination of different initial conditions and studied the final outcome for different values of semimajor axis, eccentricity, and orbital angular variables. In particular, we studied the dynamics of the system near the suggested 2:1 mean-motion resonance. All integrations were carried out for 10,000 years. Total system energy was conserved to within a few times $10^{-12}$. In all cases, integrations resulted in the escape of one of the two companions from the system. We show four examples of the time evolution of the orbits of the two M-dwarfs in Figure 7. It was assumed, initially, that the two companions were coplanar ($I_{1,2} = 90^\circ$) with the binary system. The upper (lower) two panels of Figure 7 show the case for $\omega_1 = 0$ ($\omega_1 = 180$ deg).

In Figure 7(c), the outer companion escapes the system within a few years. The resulting system consists of a single companion on a stable Keplerian orbit causing (at most) a single sinusoidal LITE effect.

To demonstrate this single sinusoidal variation, we computed the LITE from a direct numerical integration for two smaller values of the two-companion masses. We considered the case for which the two companions are coplanar with the binary plane ($I_{1,2} = 90^\circ$). The upper panel of Figure 8 shows an unstable system and is similar to the orbit shown in Figure 7(a). As explained earlier, one companion is ejected from the system within 50 years. The resulting LITE effect exhibits an initial variation in the binary period. Because of the conservation of the total linear momentum, the binary (single-object) and bound companion move in a direction opposite to that of the ejected companion. The result is a one-component LITE effect superimposed on a constant period change (linear trend) due to the systemic motion of the whole system toward the observer (negative $O-C$ values). The systemic velocity is obtained from the slope of the linear trend. The lower panel of Figure 8 shows the orbits of the two companions with their mass reduced by...
The time evolution of the two M-dwarf companions using initial conditions from Table 2 and marked by a star-like symbol in each panel. The mean longitude is denoted by $\lambda$. Upper two panels are for $\omega_1 = 0$ and bottom two panels for $\omega_1 = 180^\circ$. In each panel the binary pair is placed at the origin of the coordinate system.

Figure 7. Time evolution of the two M-dwarf companions using initial conditions from Table 2 and marked by a star-like symbol in each panel. The mean longitude is denoted by $\lambda$. Upper two panels are for $\omega_1 = 0$ and bottom two panels for $\omega_1 = 180^\circ$. In each panel the binary pair is placed at the origin of the coordinate system.

Figure 8. Numerical computation of the orbit (left) and the resulting LITE effect (right) for two different scenarios of companion mass. Initial conditions are the same as in Figure 7(a) with $I_{12} = 90^\circ$. Upper panel: companion masses are $m_1 = 0.23 M_\odot$ and $m_2 = 0.21 M_\odot$. Lower panel: companions with masses of $m_1 = 0.023 M_\odot$ and $m_2 = 0.021 M_\odot$. Vertical bars in the right panels represent $\pm 1\sigma$ uncertainties with $\sigma$ corresponding to 0.0013 (112 s), 0.0020 (173 s), and 0.0036 days (311 s).

a factor of 10. The initial conditions in these simulations are similar to those in the upper panel of Figure 8. Because of the smaller mass of the circumbinary companions, the system is now stable on a 200 year timescale. However, for longer integration times, the outer companion is ejected from the system as a result of strong mutual interactions with the inner one.

Within the 200 year integration the resulting LITE effect shows a quasi-periodic modulation of the binary period. The
semimajor axis $a_{\text{J}}$ of this signal is around 0.0015 days or 130 s. In order to reliably detect such an LITE signal would require $1\sigma$ timing uncertainties to be significantly smaller than 112 s (0.0013 days).

5. CONCLUSION AND DISCUSSION

In this work, we have reanalyzed the complete timing data set of SZ Her and determined an improved two-LITE fit using an extensive Monte Carlo based $\chi^2$ parameter space search. Our model used the center of mass of the one-companion system as the origin of the underlying coordinate system. Using this new approach, we find no significant change in the derived parameters when comparing our results with those presented in Lee et al. (2012).

In comparison with Lee et al. (2012) and assuming that the two-LITE model is correct, our fit with $\chi^2 = 0.989$ provides a slightly better description of the underlying timing data. The existence of the improved fit presented in this work is most likely explained by our thorough (quasi-global) $\chi^2$ parameter search. However, the majority of initial guesses resulted in a slightly larger $\chi^2$ statistic, which is consistent with the first fit found in Lee et al. (2012). Furthermore, our fit also suggests the two proposed companions to be in a near 2:1 mean-motion resonance in Lee et al. (2012). Furthermore, our fit also suggests the two companions. All numerical integrations considered in this work resulted in unstable quadruple systems (considering the three-body problem). In particular, our parameter study of the unknown orbital inclination with respect to the line of sight indicated that reducing the inclination from 90° to 1° results in an increase in the companions' semimajor axes. However, at the same time, the companions' masses also increase, which in turn introduces large gravitational perturbations between these two objects. Our inclination survey concluded that all low-inclination orbits also result in unstable systems on very short timescales. We stress at this point that our study did not consider mutually inclined orbits between the two companions. All numerical integrations considered in this study assumed the orbits of the two companions to be coplanar. However, very recently Horner et al. (2011) presented an analysis of the stability of a similar system (HU Aqr) in which the authors considered mutually inclined orbits. In their study, Horner et al. determined an increase in the lifetime of the system when considering companions on mutual inclined orbits. This suggests that the two companions around SZ Her could also be in stable orbits, if they had significant relative inclinations. We plan to explore this possibility in a future study.

If the observed timing variation is real and caused by the presence of two circumbinary objects, then the system needs to be in a stable configuration. We therefore conclude that either the two-LITE model is an inadequate description of the data, and/or the underlying data set is insufficient to draw concrete conclusions about the stability of the system. For that reason, in order to constrain any future modeling efforts, we encourage the acquisition of additional photometric observations of SZ Her.

There may also be an alternative, although unlikely, explanation for the observed period modulation in the SZ Her binary orbit as originally outlined by Horner et al. (2011). This possibility depicts a scenario for which the two companions currently undergo a dramatic dynamical evolution with a transition from a stable to an unstable configuration with one companion escaping the system shortly thereafter, as suggested in this work. The dynamical reason for this instability would remain an open question and additionally renders this scenario unlikely. If true, the observational consequence of such a scenario should make it possible within the next few decades to detect a linear trend in the measured $O-C$ diagram. This was demonstrated numerically in the top panel of Figure 8. However, due to the short orbital instability timescales found in this work, it seems unlikely that we are currently witnessing a breakup scenario in which the system enters the very end state of its dynamical evolution with one companion on the verge of escaping the binary system.

On the other hand, and as mentioned earlier, a dynamical inspection of currently known proposed circumbinary planets reveals that these systems are unstable as well (at least three out of four). This finding could be attributed to the fact that ground-based photometric observations are more sensitive to detect sub-stellar circumbinary objects introducing large-amplitude period variations superimposed on the linear ephemeris of the mid-eclipse times of the binary. In this case the quintessence would be: larger masses would introduce larger perturbations and cause larger period modulations in the $O-C$ diagram and therefore such systems would be prone to disintegrate on a short timescale due to strong mutual interactions. Another argument for the non-existence of the two components with orbits proposed in Lee et al. (2012) comes from numerical $N$-body orbit calculations. Studies of the dynamical stability of hierarchical four-body systems were carried out by Széll et al. (2004). Their work suggests that low-mass stellar objects on circumbinary orbits result in unstable hierarchical systems. Considering symmetric pairs of masses, these authors showed that for large mass ratios, the most likely event is a double binary configuration (e.g., HD98800, Furlan et al. 2007). Indeed, in one of our experiments, we confirmed such an outcome from a direct numerical integration. On the other hand, for small mass ratios (possibly comparable to planetary masses), their work suggests that the most likely outcome is circumbinary orbits. This again was demonstrated when decreasing the masses of the two companions by a factor of 10. However, a thorough stability analysis of circumbinary four-body low-mass stellar systems, considering a large range of orbital parameters and masses, would be helpful to identify stable domains of circumbinary orbits.

It is important to note that other sources may exist that can create modulations in the observed binary period. This could be in the form of magnetic interaction between the two binary components, and/or mass or angular momentum transfer resulting in a secular modulation of observed timings. In their study, Lee et al. (2012) pointed out the possibility that SZ Her (being a semidetached binary pair with the less massive component filling its Roche lobe) would be currently undergoing a phase of weak mass transfer. However, these authors provide arguments that this effect is likely to be negligible. Another mechanism potentially capable of causing ETVs is the direct gravitational perturbations by a companion on the binary orbit. This possibility was not considered in Lee et al. (2012) and is left for a future study. Star spots could also introduce stellar jitter mimicking period variations as discussed in Watson & Dhillon (2004). Also, timing measurements might suffer from systematic measurement errors introducing correlated red noise possibly resulting in wrong model parameters (Coles et al. 2011). Unaccounted systematic (red) timing errors previously resulted in the false detection of planets around pulsars.
To unveil the true nature of the observed timing variation, we encourage the acquisition of future photometric/spectroscopic follow-up observations of SZ Her allowing to further constrain and refine timing models. Future models favoring a two-companion solution should be tested for orbital stability and the resulting $O-C$ variation obtained from numerical integrations compared with the inferred timing measurements.

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REFERENCES

Bailes, M., Lyne, A. G., & Shemar, S. L. 1991, Nature, 352, 311
Bastian, U. 2000, IBVS, 4822
Beuermann, K., Hessman, F. V., Dreizler, S., et al. 2010, A&A, 521, L60
Chambers, J. E. 1999, MNRAS, 304, 793
Chambers, J. E., & Migliorini, F. 1997, BAAS, 27-06-P

Coles, W., Hobbs, G., Champion, D. J., Manchester, R. N., & Verbist, J. P. W. 2011, MNRAS, 418, 561
Doley, L. R., Carter, J. A., Fabrycky, D. C., et al. 2011, Science, 333, 1602
Duchêne, G., Bontemps, S., Bouvier, J., et al. 2007, A&A, 476, 229
Evans, D. S. 1966, Q. J. R. Astron. Soc., 9, 388
Funk, B., Eggl, S., Gyergyovits, M., Schwarz, R., & Pilat-Lohinger, E. 2011, EPSC-DPS, 1725
Furlan, E., Sargent, B., Calvet, N., et al. 2007, ApJ, 664, 1176
Goździewski, K., et al. 2012, MNRAS, in press (arXiv:1205.4164)
Guinan, E. F., & Ribas, I. 2001, ApJ, 546, L43
Hessman, F. V. 2010, arXiv:1012.0707
Hinse, T. C., Lee, J. W., Haghighipour, N., et al. 2012, MNRAS, 420, 3609
Horner, J., Hinse, T. C., Marschall, J. P., et al. 2012, MNRAS, in press
Horner, J., Marschall, J. P., Wittenmyer, R., & Tinney, C. G. 2011, MNRAS, 416, L11
Irwin, J. B. 1952, ApJ, 116, 211
Irwin, J. B. 1959, AJ, 64, 149
Lee, J. W., Kim, S.-L., Kim, C.-H., et al. 2009, AJ, 137, 3181
Lee, J. W., Lee, C.-U., Kim, S.-L., Kim, H.-I., & Park, J.-H. 2012, AJ, 143, 34
Lim, J., & Takakuwa, S. 2006, ApJ, 653, 425
Markwardt, C. B. 2009, in ASP Conf. Ser. 411, Astronomical Data Analysis Software and Systems XVIII, ed. D. A. Bohlender, D. Durand, & P. Dowler (San Francisco, CA: ASP), 251
Marzari, F., Scholl, H., Thébault, P., & Baruteau, C. 2009, A&A, 508, 1493
Murray, C. D., & Dermott, S. F. 2001, Solar System Dynamics (Cambridge Univ. Press)
Parsons, S. G., Marsh, T. R., Copperwheat, C. M., et al. 2010, MNRAS, 407, 2362
Potter, S. B., Romero-Colmenero, E., Ramsay, G., et al. 2011, MNRAS, 416, 2202
Qian, S.-B., Dai, Z.-B., Liao, W.-P., et al. 2009, ApJ, 706, L96
Qian, S.-B., Liao, W.-P., Zhu, L.-Y., & Dai, Z.-B. 2010a, ApJ, 708, L66
Qian, S.-B., Liao, W.-P., Zhu, L.-Y., et al. 2010b, MNRAS, 401, L34
Qian, S.-B., Liu, L., Liao, W.-P., et al. 2011, MNRAS, 414, L16
Széll, A., Steves, B., & Érdi, B. 2004, A&A, 427, 1145
van den Berk, J., Portegies Zwart, S. F., & McMillan, S. L. W. 2007, MNRAS, 379, 111
Watson, C. A., & Dhillon, V. S. 2004, MNRAS, 351, 110
Welsh, W. F., Orosz, J. A., Carter, J. A., et al. 2012, Nature, 481, 475
Wittenmyer, R. A., Horner, J. A., Marshall, J. P., Butters, O. W., & Tinney, C. G. 2012, MNRAS, 419, 3258