Exclusive $b \to s \nu \bar{\nu}$ induced transitions in RS$_b$ model

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We study a set of exclusive $B$ and $B_s$ decay modes induced by the rare $b \to s \nu \bar{\nu}$ transition in the RS$_b$ model, an extra-dimensional extension of the standard model with warped 5D metric and extended gauge group. We emphasize the role of correlations among the observables, and their importance for detecting the predicted small deviations from the standard model expectations.

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I. INTRODUCTION

The current searches for deviations from (or for further confirmation of) the Standard Model (SM) involve observables of increasing sophistication and difficulty. This is what happens for several quark flavour observables that are able to provide us with access to large energy scales, complementing the direct searches at the CERN LHC [1]. The flavour changing neutral current (FCNC) processes, loop-induced and heavily suppressed in SM, play a prominent role, and an important case to be studied is the $b \to s \nu \bar{\nu}$ transition, which in the Standard Model proceeds through $Z^0$ penguin and box diagrams dominated by the contribution with the intermediate top quark [2].

Rare $b$ decays with neutrino pairs in the final state are experimentally challenging. Nevertheless, the advent of new high-luminosity $B$ factories opens the possibility to access these modes which, on the other hand, present remarkable features of theoretical clarity, as we discuss below. We are mainly interested in the exclusive $B \to K \nu \bar{\nu}$ and $B \to K^* \nu \bar{\nu}$ decays, the branching fractions of which were predicted in SM of $\mathcal{O}(10^{-6})$ [3, 4]. Since the results are affected by the uncertainty of the form factors parametrizing the hadronic matrix elements, particular attention has to be paid to such an issue. Using form factors from light-cone QCD sum rules together with experimental information on the $B \to K^{\pm} \gamma$ decay rate [5], new predictions were obtained in SM [6].

\[
\begin{align*}
B(B^+ \to K^+ \nu \bar{\nu}) &= (4.5 \pm 0.7) \times 10^{-6} \\
B(B \to K \nu \bar{\nu}) &= (6.8 \pm 0.6) \times 10^{-6}
\end{align*}
\]

(considering in the final state the sum over the three neutrino species), that must be compared to the present experimental upper bounds. The Belle Collaboration has established the limits, at 90% C.L. [7],

\[
\begin{align*}
B(B^+ \to K^+ \nu \bar{\nu}) &< 5.5 \times 10^{-5} \\
B(B^0 \to K_s \nu \bar{\nu}) &< 9.7 \times 10^{-5} \\
B(B^+ \to K^{*+} \nu \bar{\nu}) &< 4.0 \times 10^{-5} \\
B(B^0 \to K^{*0} \nu \bar{\nu}) &< 5.5 \times 10^{-5}
\end{align*}
\]

(2)

The bounds (at 90% C.L.) obtained by the BaBar Collaboration [8],

\[
B(B^+ \to K^+ \nu \bar{\nu}) < 1.6 \times 10^{-5}
\]

are derived combining the results of the semileptonic tag reconstruction method [9] and of the hadronic tag reconstruction method [8].

In addition to $B \to K^{(*)} \nu \bar{\nu}$, other modes are induced by the $b \to s \nu \bar{\nu}$ transition, namely $B_s \to (\phi, \eta, \eta', f_0(980)) \nu \bar{\nu}$ that we also discuss in the following. At present, the experimental upper bounds for their rates are still quite high [10, 11], however they are also expected to be sizeably reduced at the new high-luminosity $B$ facilities.

The importance of the rare $b \to s \nu \bar{\nu}$ process relies on its particular sensitivity to new interactions. In [12] the effects of scalar and tensor interactions have been discussed, with particular attention to the distortion of the $q^2$ spectra (with $q^2$ the dilepton squared momentum) with respect to SM. The role of new right-handed operators has also been discussed [13], and the possibility of non-standard $Z$ couplings to $b$ and $s$ quarks has been considered [13]. An overview of the expected results in several new physics (NP) scenarios is in Ref. [6]. In an analysis of the effects of a new neutral gauge boson $Z'$, the correlations between the branching ratios, as well as between these modes and the decay $B_s \to \mu^+ \mu^-$, have been analyzed under different assumptions for the $Z'$ couplings [14]. In extensions of SM based on additional spatial dimensions, predictions have been given for the decay rates and distributions in minimal models with a single universal extra-dimension [15]. Here, we consider the case of a single warped extra-dimension, as formalized in the Randall-Sundrum model [16], in particular in the realization with custodial protection of the $Zb_L b_L$ coupling [17, 19]. In [20] a range for the $B \to K^{(*)} \nu \bar{\nu}$ branching fractions has been predicted in this framework. Here we extend the analysis focusing on other observables, such as several differential distributions, and on various correlations, reconsidering the predictions using model parameters singled out in a study of the rare semileptonic $B \to K^{\pm} \ell^+ \ell^-$ modes [21].

In section II we describe the general form of the effective $b \to s \nu \nu$ Hamiltonian, and in sect. [11] we define several $B \to K^{(*)} \nu \bar{\nu}$ observables. Generalities of
the custodially-protected Randall-Sundrum model are described in sects. [V] and [VI] with particular attention to the parameter space bound for the model. The predictions are presented in sects. [VII] and [VIII] with a discussion of possible improvements. The conclusions are collected in the last section.

II. $b \to s\nu\bar{\nu}$ EFFECTIVE HAMILTONIAN

In SM the effective $b \to s\nu\bar{\nu}$ Hamiltonian is written as

$$H_{eff}^{SM} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2 \pi \sin^2 \theta_W} V_{tb}^* V_{ts} X(x_t) (\bar{b}s)_{V-A}(\bar{\nu}\nu)_{V-A}$$

$$\equiv C_L^{SM} O_L , \quad (4)$$

with $O_L = (\bar{b}s)_{V-A}(\bar{\nu}\nu)_{V-A} [2]$. $G_F$ and $\alpha$ are the Fermi and the fine structure constant at the $Z^0$ scale, respectively, $V_{tb}$ and $V_{ts}$ are elements of the Cabibbo-Kobayashi-Maskawa (CKM) matrix, and $\theta_W$ is the Weinberg angle. The contribution of the operator with opposite chirality $O_R = (\bar{b}s)_{V+A}(\bar{\nu}\nu)_{V-A}$ is negligible. The master function $X$ depends on the top quark mass $m_t$ and on the W mass through the ratio $x_t = m_t^2/M_W^2$.

$$X(x_t) = \eta_X X_0(x_t) , \quad (5)$$

The function $X_0$.

$$X_0(x_t) = \frac{x_t}{8} \left[ \frac{x_t + 2}{x_t - 1} + \frac{3x_t - 6}{(x_t - 1)^2} \log x_t \right] , \quad (6)$$

results from the calculation of the loop (penguin and box) diagrams at leading order (LO) in $\alpha_s$ [22], while the factor $\eta_X = 0.994$ accounts for NLO $\alpha_s$ corrections [23]. $X$ is flavour-universal and real, implying that, in SM, it is possible to relate different modes with a neutrino pair in the final state, namely $B_d \to X_s d\nu\bar{\nu}$ and $K^+ \to \pi^+ \nu\bar{\nu}$ or $K^0 \to \pi^0 \nu\bar{\nu}$. Such relations continue to hold in NP models with minimal flavour violation.

The presence of a single operator in the Hamiltonian [4] makes the $b \to s\nu\bar{\nu}$ processes easier to study in SM with respect to other rare decays described by a richer effective Hamiltonian, for instance those induced by the $b \to s\ell^+\ell^-$ transition. Moreover, long-distance effects threatening, e.g., the modes with charged leptons in the final state due to hadron resonance contributions, are absent in modes into neutrino pairs.

In general NP extensions the new operator with opposite chirality $O_R$ can arise and the value of $C_L^{SM}$ can be modified. The effective $b \to s\nu\bar{\nu}$ Hamiltonian is given by

$$H_{eff} = C_L O_L + C_R O_R , \quad (7)$$

with $C_{L,R}$ specific of the NP model. Notice that we only consider massless left-handed neutrinos.

For the inclusive $B \to X_s d\nu\bar{\nu}$ mode, the heavy quark mass expansion allows to express the decay rate as a sum of terms proportional to inverse powers of the $b$ quark mass. The $O(1/m_b^2)$ corrections are tiny, and the same happens for the $q^2$ spectrum except for a small portion of the phase-space close to the kinematical endpoint [24]. For the exclusive modes, in SM a source of uncertainty is in the hadronic form factors describing the matrix element of the operator $O_3$, between the $B$ meson and $K$ or $K^*$. This problem can be circumvented in $K \to \pi\nu\bar{\nu}$ modes, exploiting information on the corresponding semileptonic modes (with one charged final lepton), and invoking isospin symmetry. On the other hand, the uncertainty represented by the renormalization scale in the QCD corrections is reduced by the account of NLO terms through the $\eta_X$ factor [2]. Another difference with respect to the analogous Kaon decay modes is that in $B$ decays the top quark contribution dominates, while in the Kaon case, namely the charged $K^+ \to \pi^+ \nu\bar{\nu}$ decay, also the CKM enhanced intermediate charm contribution has to be considered. This makes the role of the $\alpha_s$ correction more important in the latter channel since $\alpha_s(m_c) > \alpha_s(m_t)$.

In the study of NP effects it is useful to introduce two parameters [1].

$$\epsilon^2 = \frac{|C_L|^2 + |C_R|^2}{|C_L^{SM}|^2} , \quad \eta = - \frac{\text{Re}(C_L C_R^*)}{|C_L|^2 + |C_R|^2} \quad (8)$$

which probe deviations from SM where $(\epsilon, \eta)_{}^{SM} = (1, 0)$. In particular, $\eta$ is sensitive to the right-handed operator in the effective Hamiltonian, while $\epsilon$ mainly measures the deviation from SM in the coefficient $C_L$.

III. $B \to K\nu\bar{\nu}$ AND $B \to K^*\nu\bar{\nu}$

The analysis of the exclusive $B \to K^{(*)}\nu\bar{\nu}$ modes requires the hadronic matrix elements. The $B \to K$ matrix element can be parametrized in terms of two form factors,

$$<K(p')|\bar{s}\gamma_\mu|b(p)> \equiv (p + p')_\mu F_1(q^2) + \frac{m_B^2 - m_K^2}{q^2} \eta_\mu (F_2(q^2) - F_1(q^2)) , \quad (9)$$

with $q = p' - p$ and $F_1(0) = F_0(0)$. Only $F_1$ is relevant for decays to massless leptons. Two dimensionless quantities can be defined, the normalized neutrino pair invariant mass $s_B = q^2/m_B^2$, and the ratio $m_K/m_B$. In SM the decay distribution in $s_B$ reads:

$$\frac{d\Gamma^{SM}}{ds_B} = 3\frac{|C_L^{SM}|^2}{96\pi^3} m_B^5 \lambda^{3/2}(1, s_B, m_K^2)|F_1(s_B)|^2 , \quad (10)$$

with $C_L^{SM}$ in [4] and $\lambda(x, y, z)$ the triangular function. In the NP case this expression is generalized to

$$\frac{d\Gamma}{ds_B} = 3 \frac{|C_L + C_R|^2}{96\pi^3} m_B^5 \lambda^{3/2}(1, s_B, m_K^2)|F_1(s_B)|^2 . \quad (11)$$

In both Eqs. (10) and (11) the factor 3 accounts for the sum over the three final neutrino flavours. Modulo a
factor of two, the distributions coincide with the distributions in \(E_{\text{miss}}\), the (missing) energy of the neutrino pair, since \(s_B = 2x - 1 + \tilde{m}_K^2\), with \(x = E_{\text{miss}}/m_B\), and
\[
\frac{d\Gamma}{ds_B} = \frac{1}{2} \frac{d\Gamma}{dx} . \tag{12}
\]

For the \(B \to K^\ast\) matrix elements, we adopt the usual parametrization in terms of form factors
\[
< K^\ast(p', \epsilon)|\bar{s}\gamma_\mu(1 - \gamma_5)b|B(p) > = \epsilon_{\mu\nu\alpha\beta} \epsilon^* \nu^p p'^3 \frac{2V(q^2)}{m_B + m_{K^\ast}}.
\]

\[
\mathcal{A}_0(s_B) = - \frac{N(s_B)(C_L - C_R)}{m_{K^\ast}\sqrt{s_B}} \left[ (1 - \tilde{m}_{K^\ast} - s_B)(1 + \tilde{m}_{K^\ast})A_1(s_B) - \lambda(1, \tilde{m}_{K^\ast}, s_B) \frac{A_2(s_B)}{(1 + \tilde{m}_{K^\ast})} \right]
\]
\[
\mathcal{A}_\perp(s_B) = 2\sqrt{2}N(s_B)\lambda^{1/2}(1, \tilde{m}_{K^\ast}, s_B)(C_L + C_R) \frac{V(s_B)}{(1 + \tilde{m}_{K^\ast})}
\]
\[
\mathcal{A}_\parallel(s_B) = -2\sqrt{2}N(s_B)(C_L - C_R)(1 + \tilde{m}_{K^\ast})A_1(s_B)
\]

with \(\tilde{m}_{K^\ast} = m_{K^\ast}/m_B\) and the function \(N(s_B)\) defined as
\[
N(s_B) = \left[ \frac{m^3_{3s_B} \lambda^{1/2}(1, \tilde{m}_{K^\ast}, s_B)}{3 \cdot 2^7 \pi^3} \right]^{1/2}.
\]

The differential distributions in \(s_B\) for a longitudinally or transversely polarized \(K^\ast\) (with helicity \(h = +1\) or \(h = -1\)) can be written in terms of these amplitudes. Exploiting the definitions \(\tag{8}\), one finds for the sum over the three neutrino flavours:
\[
\frac{d\Gamma_L}{ds_B} = 3m_B^2A_0^2 = \left( \frac{d\Gamma_L}{ds_B} \right)_{SM} \epsilon^2 (1 + 2\eta)
\]
\[
\frac{d\Gamma^\pm}{ds_B} = \frac{3}{2}m_B^2|A_\perp \mp A_\parallel|^2
\]
\[
\frac{d\Gamma_T}{ds_B} = \left( \frac{d\Gamma_T}{ds_B} \right)_{SM} \epsilon^2 (1 + 2\eta f_T(s_B))
\]
\[
\frac{d\Gamma}{ds_B} = 3m_B^2 \left( A_0^2 + A_\perp^2 + A_\parallel^2 \right) = \left( \frac{d\Gamma}{ds_B} \right)_{SM} \epsilon^2 (1 + 2\eta f(s_B)) \tag{16}
\]

with
\[
f_T(s_B) = \frac{(1 + \tilde{m}_{K^\ast})^4[A_1(s_B)]^2 - \lambda[V(s_B)]^2}{(1 + \tilde{m}_{K^\ast})^4[A_1(s_B)]^2 + \lambda[V(s_B)]^2}
\]
\[
f(s_B) = \frac{[(1 + \tilde{m}_{K^\ast})^2(1 - s_B - \tilde{m}_K^2)A_1(s_B) - \lambda A_2(s_B)]^2 + 8\tilde{m}_K^2 s_B [(1 + \tilde{m}_{K^\ast})^4[A_1(s_B)]^2 - \lambda[V(s_B)]^2]}{[(1 + \tilde{m}_{K^\ast})^2(1 - s_B - \tilde{m}_K^2)A_1(s_B) - \lambda A_2(s_B)]^2 + 8\tilde{m}_K^2 s_B [(1 + \tilde{m}_{K^\ast})^4[A_1(s_B)]^2 + \lambda[V(s_B)]^2]} . \tag{17}
\]

In Eq. \(\tag{17}\) we use the notation \(\lambda = \lambda(1, \tilde{m}_{K^\ast}, s_B)\); the factor \(3\) in Eqs. \(\tag{16}\) accounts for the sum over the neutrino species. Also in this case, the distributions in \(s_B\) can be converted in neutrino missing energy distributions using Eq. \(\tag{12}\).

Starting from the above defined quantities, several observables can be constructed. The polarization fractions \(F_{L,T}\) can be considered \(\tag{6}\),
\[
\frac{dF_{L,T}}{ds_B} = \frac{d\Gamma_{L,T}/ds_B}{d\Gamma/ds_B} \tag{18}
\]
in which several hadronic and parametric uncertainties are reduced or even canceled (namely the overall quantities, like the CKM elements in SM). The integrated polarization fractions can be obtained, integrating separately the numerator and the denominator in Eq. (18):

\[
F_{L,T} = \frac{1}{\Gamma} \int_0^{1-\alpha_k^*} ds_B \frac{dF_{L,T}}{ds_B} .
\]  

(19)

Another observable is the ratio of branching fractions involving \( K \) and the transversely polarized \( K^* \) [3],

\[
R_{K/K^*} = \frac{B(B \to K \nu\bar{\nu})}{B(B \to K^{*+} \nu\bar{\nu})} + B(B \to K_{h=+1}^{*-} \nu\bar{\nu}) ,
\]  

(20)

which is sensitive to \( \eta \).

In [4] the transverse asymmetry has been proposed

\[
A_T = \frac{B(B \to K^{*+} \nu\bar{\nu}) - B(B \to K_{h=+1}^{*-} \nu\bar{\nu})}{B(B \to K^{*+} \nu\bar{\nu}) + B(B \to K_{h=+1}^{*-} \nu\bar{\nu})} ,
\]  

(21)

for which a reduced hadronic uncertainty is expected. However, its measurement would require the determination of the lepton pair polarization [23], therefore we consider it only for a theoretical analysis.

The observables can probe NP effects, as the ones envisaged in warped five-dimensional extensions of the standard model.

IV. RANDALL-SUNDRUM MODEL WITH CUSTODIAL PROTECTION

The motivation of the Randall-Sundrum (RS) model is the possibility of addressing, among others, the hierarchy and the flavour problems invoking the same geometrical mechanism [10]. For a description of the model, in particular for the flavour phenomenology, we refer to [25]. Here we briefly illustrate the main features of the custodially-protected RS\(_5\) model, adopting the same notations of our analysis of \( B \to K^* \ell^+ \ell^- \) in this framework [21], with the parameter space defined there.

The RS\(_5\) model is a new physics scenario in which the spacetime is supposed to be five-dimensional with coordinates \((x, y)\), \(x\) being the ordinary 4D Minkowski coordinates, and metric

\[
ds^2 = e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2 ,
\]  

\[
\eta_{\mu\nu} = \text{diag}(+1, -1, -1, -1) .
\]  

(22)

The (fifth) coordinate \( y \) varies in the range \( 0 \leq y \leq L \); \( y = 0 \) is identified with the so-called UV brane, \( y = L \) with the IR brane. To address the hierarchy problem, the parameter \( k \) in the metric (22) is chosen \( k \simeq O(M_{Planck}) \); specifically, \( k \) is set to \( k = 10^{19} \text{ GeV} \). We adopt the variant of the model based on the gauge group

\[
SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_X \times P_{L,R} \]  

(23)

which, together with the metric, defines the Randall-Sundrum model with custodial protection RS\(_5\) [17][19]. Indeed, the action of the discrete \( Z_2 \) \( P_{L,R} \) symmetry, implying a mirror action of the two \( SU(2)_{L,R} \) groups, guarantees the custodial protection avoiding large \( Z \) couplings to left-handed fermions, experimentally not allowed.

Appropriate boundary conditions (BC) on the UV brane permit to break the gauge group \([25]\) to the SM gauge group, which further undergoes a spontaneous symmetry breaking through a Higgs mechanism, as in SM. Among the various SM fields, the Higgs one is chosen to be localized close to the IR brane, while all the other fields can propagate in the bulk. Here we consider a Higgs field completely localized at \( y = L \).

The existence of a compact fifth dimension leads to a tower of Kaluza-Klein (KK) excitations for all particles. As customary in extra-dimensional models, particles having a SM correspondent can be distinguished from those without SM partners by the choice of their field boundary conditions, so that only for some choices a zero mode in the KK mode expansion exists. Two choices for BC are adopted: Neumann BC on both branes (+), or Dirichlet BC on the UV brane and Neumann BC on the IR one (+). The zero modes exist only for fields with (+) BC, and are identified with the SM particles. The KK decomposition has the general form

\[
F(x, y) = \frac{1}{\sqrt{L}} \sum_k f^{(k)}(x) f^{(k)}(y) .
\]  

(24)

For each field \( F(x, y) \) the functions \( f^{(k)}(y) \) are referred to as the 5D field profiles, and \( f^{(k)}(x) \) are the effective 4D fields. The 5D profiles are obtained from the 5D Lagrangian densities for the various fields, solving the resulting 5D equations of motion. This can be performed before the EWSB takes place [20]. Afterwards, the ratio \( v/M_{KK} \) of the Higgs vacuum expectation value (vev) \( v \) and the mass of the lowest KK mode \( M_{KK} \) is treated as a perturbation. The effective 4D Lagrangian is derived integrating over \( y \), and the Feynman rules follow after the neglect of terms of \( O(v^2/M^2_{KK}) \), or higher. The mixing occurring between SM fermions and higher KK fermion modes is neglected, being \( O(v^2/M^2_{KK}) \). In the case of gauge bosons, modes up to the first KK excitation (1-mode) are taken into account [20].

Among the particles without a SM counterpart, new gauge bosons are predicted to exist, due to the enlarged gauge group. The gauge bosons of \( SU(2)_L \) and \( SU(2)_R \) are denoted by \( W_{L,R}^\alpha \) and \( W_{L,R}^{a,\mu} \) \((a = 1, 2, 3)\), respectively; the gauge choices \( W_{L,R}^5 = 0 \) and \( \partial_\mu W_{L,R}^{a,\mu} = 0 \) are adopted, as for all the other gauge bosons. The equality \( g_L = g_R = g \) for the \( SU(2)_{L,R} \) gauge couplings is a consequence of the \( P_{L,R} \) symmetry .

The eight gauge fields corresponding to \( SU(3)_c \), remain identified with the gluons as in SM, while a new gauge field \( X_\mu \), from the \( U(1)_X \), has coupling \( g_X \). All the 5D couplings are dimensionful, and are connected to their 4D counterparts by the relation \( g^{5D} = g^{4D}/\sqrt{L} \).
A mixing occurs among the various gauge fields. Charged gauge bosons are defined as in SM:

\[ W_{\pm}^{L(R)}(\rho) = \frac{W_{1}^{L(R)\rho} \mp iW_{2}^{L(R)\rho}}{\sqrt{2}}. \]  

(25)

On the other hand, \( W_{3}^{L} \) and \( X \) mix through an angle \( \phi \). The resulting fields are \( Z_{X} \) and \( B \); the latter mixes with \( W_{3}^{L} \) with an angle \( \psi \), providing the \( Z \) and \( A \) fields as in SM.

In summary, the gauge boson content of the model, together with the BC, is: eight gluons \( G_{\mu} \) with BC \((++)\), four charged bosons \( W_{L}^{\pm}(++) \) and \( W_{R}^{\pm}(-) \), three neutral bosons \( A(++) \), \( Z(++) \) and \( Z_{X}(-) \). For each of these vector fields, the KK expansion is

\[ V_{\mu}(x,y) = \frac{1}{\sqrt{L}} \sum_{n=0} f_{V}^{(n)}(x) f_{\mu}^{(n)}(y) . \]  

(26)

The profiles of the zero-modes are flat, \( f_{V}^{(0)}(y) = 1 \). As for the 1-modes, for gauge bosons having a zero-mode they are denoted by \( g(y) \) and their mass is denoted as \( M_{++} \); for gauge bosons without a zero-mode, they are indicated by \( \hat{g}(y) \), with mass \( M_{-} \). We refer to the Appendix of \[21\] for the expressions of these quantities and for the notation. The solution of the equation of motion provides \( M_{++} \approx 2.45 f \) and \( M_{-} \approx 2.40 f \), where \( f \) is the dimensionful parameter \( f = k e^{-k L} \). We set this parameter to \( f = 1 \) TeV, coherently with other studies \[27\].

Before the EWSB the zero modes of the gauge bosons (if present) are massless, while higher KK excitations are massive. Since the two groups \( SU(3) \) (for QCD) and \( U(1)_{em} \) remain unbroken, the zero modes of gluons and photon are massless as in SM, but their KK excitations are massive.

Mixing also occurs among zero modes and higher KK modes of gauge fields. Neglecting modes with KK number larger than 1, the mixing involves the charged bosons \( W_{L}^{\pm(0)} \), \( W_{L}^{\pm(1)} \) and \( W_{R}^{\pm(1)} \), with the result

\[
\begin{pmatrix}
W_{\pm}^L \\
W_{R}^L \\
W_{L}^R \\
W_{R}^R \\
\end{pmatrix}
= G_{W}
\begin{pmatrix}
W_{1}^{L(0)} \\
W_{2}^{L(0)} \\
W_{1}^{L(1)} \\
W_{2}^{L(1)} \\
\end{pmatrix},
\]

(27)

and the neutral bosons \( Z^{(0)} \), \( Z^{(1)} \) and \( Z_{X}^{(1)} \) according to the pattern:

\[
\begin{pmatrix}
Z \\
Z_{H} \\
Z' \\
\end{pmatrix}
= G_{Z}
\begin{pmatrix}
Z^{(0)} \\
Z^{(1)} \\
Z_{X}^{(1)} \\
\end{pmatrix}.
\]

(28)

The expressions of the matrices \( G_{W} \) and \( G_{Z} \) and the masses of the mass eigenstates can be found in Ref. \[26\].

Moving to the Higgs sector, the Higgs field \( H(x,y) \) transforms as a bidoublet under \( SU(2)_{L} \times SU(2)_{R} \) and as a singlet under \( U(1)_{X} \). It contains two charged and two neutral components:

\[ H(x,y) = \left( \begin{array}{c}
\frac{\pi^{+}}{\sqrt{2}} - \frac{h^{0}_{-} - i\pi^{-} y}{\sqrt{2}} \\
\frac{\pi^{-}}{\sqrt{2}} + \frac{h^{0}_{+} + i\pi^{+} y}{\sqrt{2}} 
\end{array} \right). \]  

(29)

Its KK decomposition reads

\[ H(x,y) = \frac{1}{\sqrt{L}} \sum_{k} H^{(k)}(x) h^{(k)}(y). \]  

(30)

The localization on the IR brane leads to the profile

\[ h(y) = h^{(0)}(y) \simeq e^{k y} \delta(y - L). \]  

(31)

Furthermore, one chooses that only the neutral field \( h^{0} \) has a non-vanishing vacuum expectation value \( v = 246.22 \) GeV, as in SM.

The most involved sector is the fermion one. We refer to \[29\] for the description of the fermion representations. Here, we mention that, considering three generations of quarks and leptons, SM left-handed doublets are collected in a bidoublet of \( SU(2)_{L} \times SU(2)_{R} \), together with two new fermions. Right-handed up-type quarks are singlets, while no corresponding fields exist in the case of leptons, for left-handed neutrinos. Right-handed down-type quarks, as well as charged leptons are in multiplets, transforming as \((3,1) \oplus (1,3)\) under \( SU(2)_{L} \times SU(2)_{R} \), and additional new fermions are also present in such multiplets. The electric charge is related to the third component of the \( SU(2)_{L} \) and \( SU(2)_{R} \) isospins and to the charge \( Q_{X} \) through the equation \( Q = T^{3}_{L} + T^{3}_{R} + Q_{X} \).

The presence of new fermions will not affect our analysis, since we only take into account the zero-modes of SM quarks and leptons. The zero-mode profiles are obtained solving the equations of motion for ordinary fermions, with result denoted as \( f_{L(R)}^{(0)}(y,c) \):

\[ f(y,c) = \sqrt{\left( \frac{1 - 2c}{e^{(1 - 2c)kL} - 1} \right) e^{-cky}}. \]  

(32)

The difference between right- and left-handed fermion profiles lies in the parameter \( c \), the fermion mass in the bulk. Fields belonging to the same \( SU(2)_{L} \times SU(2)_{R} \) multiplet share the same value of \( c \), as is the case for \( u_{L} \) and \( d_{L} \), \( c_{L} \) and \( s_{L} \), \( t_{L} \) and \( b_{L} \), as well as for \( \nu_{e} \) and \( \ell_{L}^{c} \). We choose real \( c \) parameters.

Other parameters of the model enter when considering the quark mixing. As in SM, quark mass eigenstates are obtained by a rotation of flavour eigenstates. The rotation matrices of up-type left (right) and down-type left (right) quarks are denoted by \( U_{L(R)} \) and \( D_{L(R)} \), respectively. Moreover, the CKM matrix is obtained as \( V_{CKM} = U_{L}^{T} D_{L} \). At odds with SM, in which the presence of the CKM matrix affects only charged current interactions, in RS\(_{c}\) the rotation matrices also affect neutral current interactions, and this leads to the occurrence of flavour changing neutral currents at tree level mediated
by the three neutral EW gauge bosons $Z, Z', Z_H$, as well as by the first KK mode of the photon and of the gluon (however, gluons play no role in processes with leptons in the final state, and photons do not contribute to the transitions to neutrinos). The corresponding Feynman rules involve the overlap integrals of fermion and gauge boson profiles,

$$
\mathcal{R}_{f_1 f_2} = \frac{1}{L} \int_0^L dy \, e^{ik y} j_{f_1}^{(0)}(y, c_1) j_{f_2}^{(0)}(y, c_2) g(y)
$$

$$
\bar{\mathcal{R}}_{f_1 f_2} = \frac{1}{L} \int_0^L dy \, e^{ik y} j_{f_1}^{(0)}(y, c_1) j_{f_2}^{(0)}(y, c_2) \bar{g}(y),
$$

(33)

collected in two matrices $\mathcal{R}_f = \text{diag} (\mathcal{R}_{f_1 f_1}, \mathcal{R}_{f_2 f_2}, \mathcal{R}_{f_3 f_3})$ and $\bar{\mathcal{R}}_f = \text{diag} (\bar{\mathcal{R}}_{f_1 f_1}, \bar{\mathcal{R}}_{f_2 f_2}, \bar{\mathcal{R}}_{f_3 f_3})$. After the rotation to mass eigenstates, the quantities appearing in the Yukawa couplings are given by

$$
\bar{\lambda}_{Yukawa} \text{ couplings denoted by } \lambda_1, \lambda_2 \text{ and } \tilde{\lambda}_1, \tilde{\lambda}_2 \text{ for down type quarks}, \text{ respectively. The effective 4D Yukawa couplings are given by}
$$

$$
Y_{ij}^{u(d)} = \frac{1}{\sqrt{2}} \frac{1}{M^{3/2}} \int_0^L dy \, \lambda_{ij} f_R^{(0)}(y) f_L^{(0)}(y) \lambda_{ij} f_R^{(0)}(y) \lambda_{ij} f_L^{(0)}(y) h(y) .
$$

(34)

This relation produces the fermion mass and mixing hierarchy, due to the exponential dependence of the fermion profiles on the bulk mass parameters $\mathcal{M}$. The elements of the matrices $\mathcal{U}_{L(R)}$ and $\mathcal{D}_{L(R)}$ are not all independent, not only because the constraint $V_{CKM} = \mathcal{U}_{L} \mathcal{D}_{L} \mathcal{U}_{R} \mathcal{D}_{R}$ must be fulfilled, but also because the Yukawa couplings determine the quark masses. In particular, the following relations must be satisfied:

$$
m_u = \frac{v}{\sqrt{2}} \frac{\det(\lambda^u)}{\lambda_{33}^u} \frac{e^{kL}}{L} f_{uL} f_{uR}.
$$

$$
m_c = \frac{v}{\sqrt{2}} \frac{\lambda_{12}^u - \lambda_{23}^u \lambda_{33}^u}{\lambda_x} \frac{e^{kL}}{L} f_{cL} f_{cR}.
$$

$$
m_t = \frac{v}{\sqrt{2}} \frac{\lambda_{12}^u \lambda_{33}^u}{\lambda_x} \frac{e^{kL}}{L} f_{tL} f_{tR} .
$$

(35)

as well as the analogous relations for down-type quarks with the replacement $\lambda^u \rightarrow \lambda^d$ (with the notation $f_{uL/R}^{(0)} = f_{uL/R}^{(0)}(y = L, c_{uL/R})$).

In our analysis we adopt simplifying assumptions, such as considering real entries of the matrices $\lambda^{u,d}$. As a consequence, after the quark mass constraints have been imposed, there are six independent entries among the elements of the Yukawa matrices, which we choose

$$
\lambda_{12}^u, \lambda_{13}^u, \lambda_{23}^u, \lambda_{12}^d, \lambda_{13}^d, \lambda_{23}^d.
$$

(36)

Therefore, the set of input parameters in our analysis is composed by the six quantities in (36), together with the bulk mass parameters. Before describing our strategy for the numerical study, we discuss the Wilson coefficients in the effective Hamiltonian in RS model.

V. EFFECTIVE $b \rightarrow s\nu \bar{D}$ HAMILTONIAN IN RS MODEL

In SM the Wilson coefficients of the left- and right-handed operators $O_L$ and $O_R$ in the effective Hamiltonian $\alpha \bar{s} \nu \bar{D} X$ are given by

$$
C_{L} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2 \pi} \sin^2 \theta_W V_{tb} V_{ts} X(x_t)
$$

(37)

$$
C_{R} = 0 .
$$

These coefficients are modified in the RS model, in which a right-handed operator $O_{Rb}$ is present:

$$
C_{L}^{RS} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2 \pi} \sin^2 \theta_W V_{tb} V_{ts} X_{L}^{RS}
$$

(38)

$$
C_{R}^{RS} = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2 \pi} \sin^2 \theta_W V_{tb} V_{ts} X_{R}^{RS},
$$

with $X_{L}^{RS} = X(x_t) + \Delta X_L$ and

$$
\Delta X_L = \frac{1}{V_{tb} V_{ts}} \sum \Delta^L_{\nu}(X) \Delta^\nu(X)
$$

(40)

$$
X_{R}^{RS} = \frac{1}{V_{tb} V_{ts}} \sum \Delta^R_{\nu}(X) \Delta^\nu(X)
$$

(41)

The constant $g_{SM}^2$ is defined as $g_{SM}^2 = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2 \pi} \sin^2 \theta_W$. $\Delta^L_{\nu}(X)$ is the coupling of a gauge boson X to a pair of fermions $f_1 f_2$; it can be read from the Feynman rules described in the Appendix of Ref. [21].

The new contributions can be evaluated scanning the parameter space of the RS model. As described in [21], we require that the elements of the matrices $\lambda^{u,d}$ lie in a range assuring the perturbativity of the model up to the scale of the first three KK modes: $|\lambda_{ij}^{d,u}| \leq 3/k$. Moreover, the diagonal elements of such matrices are fixed imposing the quark mass constraints. The six remaining parameters, those in (36), must be fixed together with the bulk mass parameters for the quarks, enforcing quark mass and CKM constraints. Imposing for the quark masses the values obtained at the scale $O(M_{KK})$ through renormalization group evolution, starting from

$$
m_d = 4.9 \text{ MeV}, \ m_s = 90 \text{ MeV}, \ m_b = 4.8 \text{ GeV},
$$

(42)

the following quark mass bulk parameters have been determined [32]:

$$
c_L^{u,d} = 0.63, \ c_L^{c,s} = 0.57, \ c_L^{b,t} \in [0.40, 0.45],
$$

1 A parametrization of the matrices $\lambda^{u,d}$ with complex entries is described in [27].
c^b_R = 0.67, \quad c^b_L = 0.35, \quad c^d_L = -0.35, \quad c^d_R = 0.66, \quad c^s_L = 0.60, \quad c^s_R = 0.57.

We quote a range in the case of the left-handed doublet of the third quark generation, since further constraints are imposed, i.e. those derived in [21] using the measurements of the coupling $Zb\bar{b}$, of the $b$-quark left-right asymmetry parameter and of the forward-backward asymmetry for $b$ quarks [34].

For leptons, the bulk masses are set to $c_\ell = 0.7$ [20]. Other numerical determinations of the fermion bulk mass parameters can be found in [32, 33, 40].

In correspondence to the values fixed above, we generate the six $\lambda$ parameters in Eq. (36) imposing the CKM constraints. Specifically, we require $|V_{cb}|$ and $|V_{ub}|$ in the largest range found from their experimental determinations from inclusive and exclusive $B$ decays [11], and that $|V_{us}|$ lies within 2% of the central value reported by the Particle Data Group [10]. Hence, the selected ranges are:

$|V_{cb}| \in [0.038 - 0.043]$  
$|V_{ub}| \in [0.00294 - 0.00434]$  
$|V_{us}| \in [0.22 - 0.23]$.

The parameter space is further restricted, as in [21], imposing that the $B \to K^*\mu^+\mu^-$ and $B \to X_s\gamma$ branching fractions lie within the $2\sigma$ range of the measurements

$B(B \to K^*\mu^+\mu^-)_{exp} = (1.02 \pm 0.14 \pm 0.05) \times 10^{-6}$, \quad (43)

$B(B \to X_s\gamma)_{exp} = (3.43 \pm 0.21 \pm 0.07) \times 10^{-4}$. \quad (44)

The datum [45] is the BaBar average of the branching fractions of the four modes $B^{\pm,0} \to K^{*+,0}\mu^+\mu^-$ and $B^{+,0} \to K^{*+,0}\phi^\pm e^\mp e^\mp$, while the value in [46] is the HFAG Collaboration average for this inclusive rare radiative $B$ decay width [11]. With the selected set of points in the parameter space it is also possible to reproduce in the RS$_c$ model, using the expressions in [27], the mass difference of the neutral $B_s$ mesons $\Delta M_s$, within 20% of the central value of the experimental measurement $\Delta M_s = 17.69 \text{ ps}^{-1}$ [11].

Scanning the parameter space resulting from all the constraints, we obtain the coefficients $C^{RS}_L$ and $C^{RS}_R$ and their correlation, as shown in Fig. 1, in the RS$_c$ model (blue curve). The red dot corresponds to SM.

FIG. 1. Correlation between $C^{RS}_L$ and $C^{RS}_R$ in the RS$_c$ model.

VI. \textit{$B \to K\nu\bar{\nu}$ AND $B \to K^*\nu\bar{\nu}$ OBSERVABLES IN RS$_c$}

To compare the RS$_c$ predictions to the SM results for the exclusive $B \to K\nu\bar{\nu}$ and $B \to K^*\nu\bar{\nu}$ decay observables defined in section [11] we need the $B \to K^{(*)}$ form factors. Here we use the light-cone QCD sum rule determination [42]. Other QCD sum rule determinations, as the one in [43] from three-point correlation functions, have larger uncertainties. The one in [35], which includes QCD factorization corrections, only provides the $B \to K^*$ matrix elements, while we need the full set of $B \to K$, $B \to K^*$, as well as $B_a \to \phi$ matrix elements. Lattice QCD results are now available [44, 45], and we comment below on the differences.

In Fig. 2 we depict the differential distribution $d^3p_{BM}(B^0 \to K^0\nu\bar{\nu})$ in the whole kinematical range $0 \leq s_B \leq (1 - \frac{m_K}{m_B})^2$ in SM, including the uncertainty on the form factor $F_1(0)$ quoted in [42] and using the measured lifetime $\tau(B^0) = 1.519 \pm 0.005 \text{ ps}$ [11]. The predicted branching fraction

$B(B^0 \to K^0\nu\bar{\nu})_{SM} = (4.6 \pm 1.1) \times 10^{-6}$

has a larger uncertainty than the one in [11], due to our more conservative errors on the form factors. The modifications in RS$_c$, obtained for the central value of $F_1(0)$

FIG. 2. Correlation between the parameters $\eta$ and $\epsilon$, defined in [4], in the RS$_c$ model (blue curve). The red dot corresponds to SM.
and accounting for the uncertainty on $F_1(0)$, are also shown in Fig. 3 and produce a prediction for the branching fraction spanning a somewhat wider range,

$$B(B^0 \to K^0 \nu\bar{\nu})_{RS} \in [3.45 - 6.65] \times 10^{-6}.$$  (48)

A similar result is obtained for the charged mode. Hence, the present experimental upper bounds require an improvement by a factor of 3-4 in the case of BaBar, Eq. (3), and of about one order of magnitude in the case of Belle, Eq. (2), to become sensitive to these processes, a task within the reach of high-luminosity facilities such as Belle II.

For $B \to K^\ast \nu\bar{\nu}$ we separately consider the longitudinally and transversely polarized $K^\ast$, with distributions in Fig. 4. In RS$_c$ a small deviation from SM is found in the longitudinal distribution. The SM prediction, obtained including the errors on the form factors in quadrature,

$$B(B^0 \to K^{*0} \nu\bar{\nu})_{SM} = (10.0 \pm 2.7) \times 10^{-6}$$  (49)

becomes in RS$_c$ the range

$$B(B^0 \to K^{*0} \nu\bar{\nu})_{RS} \in [6.1 - 14.3] \times 10^{-6}.$$  (50)

For the charged mode the predictions are similar. Hence, the required improvement of the current upper bound to reach the expected signal is about a factor of 4 in the case of the Belle upper bounds (2), and about one order of magnitude in the case of Belle II, within the reach of new facilities. Also in the case of $K^\ast$ our result has a more conservative error than the one quoted in (1). The difference is due to the choice in (1) of exploiting additional information on the measured radiative $B \to K^\ast \gamma$ decay rate, which results in a reduction of the central value and of the error of the form factors.

Differently from the mode into the pseudoscalar $K$, the $K^\ast$ channel allows to access other observables as the polarization fractions $F_{L,T}$ in (15). Moreover, the measurements of both the $K$ and $K^\ast$ modes permit the construction of the fraction $R_{K/K^\ast}$ in (20), and to study the correlations among the various observables predicted in SM and in RS$_c$. Such correlations are important to disentangle different NP scenarios from the one we are investigating. In Fig. 5 we show the correlation between the rates of $B \to K^0 \nu\bar{\nu}$ and $B \to K^\ast \nu\bar{\nu}$, with the inclusion of the hadronic uncertainty. Although the effects of the form factor errors are at present noticeable, the SM and the RS$_c$ predictions already have a non-overlapping region, which is interesting in view of the envisaged possibility of reducing the hadronic uncertainty. In particular, the $K$ and $K^\ast$ modes are anticorrelated, hence a reduction of the $B \to K^\ast \nu\bar{\nu}$ decay rate goes in RS$_c$ with an increase of the rate of $B \to K \nu\bar{\nu}$ with respect to SM, as it is visible in Fig. 6 in the ideal case of an exact knowledge of the hadronic matrix elements.

As for the longitudinal $K^\ast$ polarization fraction, the differential distribution in Fig. 6 has a small deviation and can be below the SM; the correlation of the integrated fraction with the branching rate is depicted in Fig. 7. A precise correlation pattern hence exists in RS$_c$, among the three observables $B(B \to K^\ast \nu\bar{\nu})$, $B(B \to K^\ast \nu\bar{\nu})$ and $F_L$: the first one can be above, the other one below its SM values. We also show illustratively the correlation between the transverse asymmetry $A_T$ in (21) in $B \to K^\ast \nu\bar{\nu}$ and the branching fraction in SM and

![FIG. 3. $dS(B^0 \to K^0 \nu\bar{\nu})$ distribution in SM, including the uncertainty on the form factor $F_1(0)$ (green region), and in RS$_c$ for the central value of $F_1(0)$ (red points) and including the uncertainty of the form factor at $s_B = 0$ (blue bars).](image-url)

![FIG. 4. Distributions $dS_{B} = dS_{B} = dS_{B} = dS_{B}$](image-url)
VII. ROLE OF THE RIGHT-HANDED OPERATORS IN RS$_c$

The correlation between $\epsilon$ and $\eta$ is of particular interest, in light of general analyses where the effects of $Z'$ neutral gauge bosons are considered with no reference to the underlying NP theory [14]. In such analyses, several possible non-diagonal couplings to left- and right-handed quarks lead to models that can be distinguished by the relative weight of the couplings. As an example, a left-right symmetric scenario (LRS) corresponds to $Z'$ left- and right-handed couplings equal in size and sign; the $\epsilon - \eta$ correlation is different in the various cases.

Comparing our result in Fig. 2 with the various possibilities considered in the general analysis, Fig. 20 of [14], we infer that the RS$_c$ model looks similar to the RHS scenario, with a $Z'$ mainly coupled to right-handed quarks. Indeed, the difference $C_{RS}^{L} - C_{SM}^{L}$ and the coefficient $C_{RS}^{L}$, playing the role of the left- and right-handed quark couplings to a new gauge boson, have opposite sign, and $C_{RS}^{L} \ll C_{RS}^{S} - C_{SM}^{S}$, Fig. 1. Although in RS$_c$ there are several additional gauge bosons, the effect is similar to the case of one new boson.

The correlation of $\mathcal{B}(B \to K \nu \bar{\nu})$ and $\mathcal{B}(B \to K^* \nu \bar{\nu})$ with $\mathcal{B}(B_s \to \mu^+ \mu^-)$ provides a deeper insight. In NP models one has

$$\frac{\mathcal{B}(B_s \to \mu^+ \mu^-)}{\mathcal{B}(B_s \to \mu^+ \mu^-)_{SM}} = \frac{C_{10} - C'_{10}}{C_{10}^{SM}}$$

(51)

where $C_{10}$ and $C'_{10}$ are the Wilson coefficients of the semileptonic electroweak penguin operators with axial vector leptonic current and $V - A$ and $V + A$ structure of the quark current in the effective $b \to s\ell^+\ell^-$ Hamiltonian. In SM only $C_{10}^{SM}$ is relevant (the contribution

RS$_c$, Fig. 7, for which the two models, with the present hadronic uncertainty, have a big overlap.

The observable $R_{K/K^*}$ defined in Eq. (21) and obtained from the $K$ and $K^*$ measurements is depicted in Fig. 5 versus $F_L$. A sizable form factor uncertainty is still present, at odds with the expectation that such a variable should be quite safe; nevertheless, a region where SM and RS$_c$ results do not overlap can be observed, together with the anticorrelation with $F_L$.

An important issue concerns the hadronic error, the reliability of which cannot be asserted without the comparison among form factors obtained by independent nonperturbative methods. Recent lattice QCD determinations of the $B \to K^{(*)}$ form factors [44] [45] can be used to estimate the size of the hadronic uncertainties affecting the various observables we have considered. Using the set in [45] we have analyzed, e.g., the correlation among the $B \to K^* \nu \bar{\nu}$ decay rate and $F_L$ and $A_T$. The results reported in Fig. 7 show that the predictions already obtained are robust within the quoted errors.
of $O(10)$ is negligible). Evaluating $C_{L,R}$, $C_{10}$ and $C’_{10}$ in the RS, parameter space, the correlations in Fig. 10 are found. The rates of $B \to K \nu \bar{\nu}$ and $B_s \to \mu^+\mu^-$ are anticorrelated: in RS, a larger $B(B_s \to \mu^+\mu^-)$ than in SM implies a lower $B(B \to K \nu \bar{\nu})$. The opposite happens for $B \to K^* \nu \bar{\nu}$: $B(B \to K^* \nu \bar{\nu})$ and $B(B_s \to \mu^+\mu^-)$ are correlated, therefore finding one of them above its SM value would require an enhancement also of the other one. This again characterizes RS as an RHS scenario, as one can infer from a comparison with the general result of Fig. 21 in [14].

VIII. $B_s \to (\phi, \eta, \eta’, f_0(980)) \nu \bar{\nu}$ in RS

Several $B_s$ decay modes of great phenomenological interest are driven by the transition $b \to s \nu \bar{\nu}$. Here we focus on $B_s \to (\eta, \eta’) \nu \bar{\nu}$, on the decay $B_s \to \phi \nu \bar{\nu}$ and on $B_s \to f_0(980) \nu \bar{\nu}$ with the scalar $f_0(980)$ meson in the final state, all of them accessible at the new facilities.

The modes into $\eta$ and $\eta’$ must be considered altogether, due to the $\eta - \eta’$ mixing. Two schemes are usually adopted to describe this mixing, in either the singlet-octet (SO) or the quark-flavor (QF) basis, and both schemes involve two mixing angles [46]. We choose the quark-flavor basis, defining

$$|\eta_q\rangle = \frac{1}{\sqrt{2}} (|\bar{u}u\rangle + |\bar{d}d\rangle)$$
$$|\eta_s\rangle = |\bar{s}s\rangle,$$

in which the two mixing angles $\varphi_q$ and $\varphi_s$,

$$|\eta\rangle = \cos \varphi_q |\eta_q\rangle - \sin \varphi_q |\eta_s\rangle$$
$$|\eta’\rangle = \sin \varphi_q |\eta_q\rangle + \cos \varphi_q |\eta_s\rangle,$$

differ by OZI-violating effects. However, the difference is experimentally found to be small, ($\varphi_q - \varphi_s < 5^\circ$), therefore, within the present accuracy we can adopt an $\eta - \eta’$ mixing description in the QF basis and a single mixing angle $\varphi_q \simeq \varphi_s \simeq \varphi$. This choice is supported by a study of the radiative $\phi \to \eta \gamma$ and $\phi \to \eta’ \gamma$ transitions [17]. The KLOE Collaboration has measured the ratio

$$\frac{\Gamma(\phi \to \eta \gamma)}{\Gamma(\phi \to \eta’ \gamma)}$$

finding for the $\eta - \eta’$ mixing angle the value $\varphi = (41.5 \pm 0.3_{stat} \pm 0.7_{syst} \pm 0.6_{th})^\circ$ [15]. An improved analysis by the same collaboration, allowing a gluonium content in the $\eta’$ and making use of the measured ratio $\frac{\Gamma(\eta’ \to \gamma \gamma)}{\Gamma(\pi^0 \to \gamma \gamma)}$ confirms this determination of $\varphi$ [19].

The flavour symmetry permits to relate the $B_s \to \eta, \eta’$ form factors to the $B \to K$ ones. For a form factor $F$ one

FIG. 8. Differential longitudinal $K^*$ polarization fraction $\frac{dF_L}{ds}$ in SM including the uncertainties on $A_1(0)$, $A_2(0)$ and $V(0)$ (green region), and in RS, for the central values of $A_1(0)$, $A_2(0)$ and $V(0)$ (red points) and with the error on the form factors (blue bars).

FIG. 9. $R_{K_s}$ defined in Eq. (20), versus $F_L(B^0 \to K^{*0} \nu \bar{\nu})$. The color code is the same as in Fig. 7.
has $F_{B_s \to \eta} = -\sin \varphi F_{B \to K}$ and $F_{B_s \to \eta'} = \cos \varphi F_{B \to K}$ [50]. On the other hand, for the $B_s \to \phi \bar{\nu} \nu$ mode we use the LCSR $B_s \to \phi$ form factors in Ref. [12]. The SM predictions, obtained for $\tau(B_s) = 1.512 \pm 0.007$ ps [11],

\begin{align*}
\mathcal{B}(B_s \to \eta \bar{\nu} \nu)_{SM} & = (2.3 \pm 0.5) \times 10^{-6} \quad (54) \\
\mathcal{B}(B_s \to \eta' \bar{\nu} \nu)_{SM} & = (1.9 \pm 0.5) \times 10^{-6} \quad (55) \\
\mathcal{B}(B_s \to \phi \bar{\nu} \nu)_{SM} & = (13.2 \pm 3.3) \times 10^{-6} \quad (56)
\end{align*}

are modified in RS$_c$:

\begin{align*}
\mathcal{B}(B_s \to \eta \nu \bar{\nu})_{RS} & \in [1.7 - 3.3] \times 10^{-6} \quad (57) \\
\mathcal{B}(B_s \to \eta' \nu \bar{\nu})_{RS} & \in [1.5 - 2.8] \times 10^{-6} \quad (58) \\
\mathcal{B}(B_s \to \phi \nu \bar{\nu})_{RS} & \in [8.4 - 18.0] \times 10^{-6} \quad (59)
\end{align*}

The result is particularly relevant in the case of the $B_s \to \phi$ mode, which should be the first one accessible for the $B_s$ meson: the rate is within the reach of the new facilities, the $\phi$ can be easily identified and its decay modes allow to construct, e.g., the $F_L$ observable. The various correlation patterns are shown in Fig. [11] anticorrelation is found between the rates of $B_s \to \eta/\eta'$ with $B_s \to \phi \bar{\nu} \nu$. For $F_L$ the results are depicted in Fig. [12].

The last mode in our analysis involves the scalar $f_0(980)$ meson. The $B_s \to f_0(980)$ form factors have been determined assuming for $f_0(980)$ a dominant quark-antiquark $s\bar{s}$ structure [51]. With the updated value of $\tau(B_s)$ we find that the SM prediction is modified in RS$_c$:

\begin{align*}
\mathcal{B}(B_s \to f_0(980)\nu \bar{\nu})_{SM} & = (8.95 \pm 2.5) \times 10^{-7} \quad (60) \\
\mathcal{B}(B_s \to f_0(980)\nu \bar{\nu})_{RS} & \in [5 - 17] \times 10^{-7} \quad (61)
\end{align*}

This channel should be accessible through the $f_0(980) \to \pi \pi$ transition, providing another test of the RS$_c$ scenario.

**IX. CONCLUSIONS**

Although experimentally challenging, the FCNC exclusive $b$-hadron transitions into $\nu \bar{\nu}$ pairs are of great interest, as they can provide the evidence of possible deviations from SM through signals of remarkable theoretical significance. We have examined a set of $B$ and $B_s$ decay modes in the RS$_c$ model, with particular emphasis on the correlations among the observables that are features of the model. In the planned experimental analyses these modes can be accessible, and the predictions presented here will become testable.

FIG. 11. Correlation of $\mathcal{B}(B_s \to \phi \nu \bar{\nu})$ with $\mathcal{B}(B_s \to \eta \nu \bar{\nu})$ (top), $\mathcal{B}(B_s \to \eta' \nu \bar{\nu})$ (center) and $\mathcal{B}(B_s \to f_0(980)\nu \bar{\nu})$ (bottom). The color code is the same as in Fig. [7].

FIG. 12. $\mathcal{B}(B_s \to \phi \nu \bar{\nu})$ versus $F_L(B_s \to \phi \nu \bar{\nu})$. The color code is the same as in Fig. [7].
[51] P. Colangelo, F. De Fazio and W. Wang, Phys. Rev. D 81, 074001 (2010) [arXiv:1002.2880 [hep-ph]].