Thermal action of pulse radiation on the carbon conic shells loaded with internal pressure

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Abstract. The new mechanism of constructions destruction as a result of thermal action of radiations and particles fluxes was considered. It is supposed that interruption of construction operability comes owing to non-stationary processes of deformation and destruction. Non-stationary deformation and destruction take place as a result of action of quasi-stationary working loadings at saltatory change of construction rigidity. Jump of rigidity arises at volume and impact heating of material by means of action of radiations and particles fluxes.

A numerical method for predicting of the thermal and tension-strain states of thin-walled orthotropic constructions having variable thickness under the action of energy fluxes of different physical nature are considered. Physical and chemical processes in result of heating of composite construction materials were taken into account. These processes are the binder pyrolysis, chemical reactions of carbon and air flow components, and the ablation of carbon residue. Non-stationary processes occurring in the shell has been numerically simulated. Model of layer-by-layer destruction is used in this simulation.

1. Introduction

Forecasting of consequences of interaction of radiations and particles fluxes (RPF) having various physical nature with composite structural elements of flight vehicles (VF) is of practical interest [1, 2, 3, 4]. In many cases thermal effect of RPF is had prevailing because on condition of sufficient remoteness of RPF source from VF mechanical action is not realized any more.

Vehicles can be irradiated in flight by RPF having spectrums which are characteristic for optical radiation and up to γ-quanta [4]. The main danger of thermal action to the thin-walled VF constructions consists in warming up materials and reduction of its thickness as a result of ablation in all this interval of quanta energy. Calculation of parameters of this danger (non-stationary temperature profile and distribution of thickness of the carried-away material) can be made by means of common physical and mathematical model of thermal action of RPF on multilayered composite targets [5, 6, 7] for all practically realized durations of irradiation and area densities of RPF energy. Such model was considered in quasione-dimensional statement [6] earlier. In the present work the model is generalized on a three-dimensional case.

Fast heating causes change of deformation and strength properties of materials and also change of thickness of the thin-walled construction. Then changes of these properties and
thickness lead to development of non-stationary processes of deformation and destruction under the action of quasi-stationary flight loadings. Such mechanism of destruction can take place for thin-walled constructions of FV which are in flight. In particular nozzles of the working rocket engines of the top steps [8] are in severe heat-strength conditions. Thermal RPF action appears the largest in comparison with other s factors of powerful explosion (for example action of a shock wave) for the top steps. Orthotropic shell of rotation is made from a coal plastic can serve as nozzle model. This shell is jammed in the place of fastening and is free in the output section of a nozzle.

Now there is a representative set of numerical models of deformation of multilayered shells [9, 10, 11, 12, 13]. Unevenly heated shells having a variable thickness are among of such models. However problem calculation of non-stationary destruction for composite multilayered shells in non-stationary temperature fields are still under development. In present paper the numerical method [13] is used for a research of non-stationary deformation of composite multilayered shells. Calculations are made taking into account layer-by-layer destruction of thin-walled constructions at change of their stress-strain state.

Let’s note that thermal and deformation parts of a problem can be considered independently. It is proved by the fact that change of a temperature profile can be neglected during process of non-stationary deformation and destruction of a shell (it is supposed that characteristic time of redistribution of heat is much more than the period of construction oscillation).

2. Temperature fields and thermal ablation

Calculation of temperature fields and distributions of ablation thickness in composite FV constructions is required for justification of their working capacity (it includes strength to flight loadings). Flight internal and external heat fluxes are axisymmetric in many settlement cases and the corresponding problem about a heat-mass transfer for such constructions is two-dimensional. However the regime of one-sided action is realized at irradiation of FV constructions by RPF. Calculation of temperature fields and thermal ablation non-stationary deformation and destruction of a shell (it is supposed that characteristic time of redistribution of heat is much more than the period of construction oscillation).

Let’s assume that the directions of axes of an orthotropy for a composite material coincide with the directions of axes of the coordinate system connected with a shell of rotation and temperature of the solid residue equals to temperature of gaseous products of thermal destruction. Then the equation of transfer of thermal energy in each layer of a shell is wrote in the form (the index of a layer is lowered)

\[
\tilde{\rho}_s \tilde{c}_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial z} \left( \tilde{\lambda}_s \frac{\partial T}{\partial z} \right) + \frac{\partial}{\partial s} \left( \tilde{\lambda}_s \frac{\partial T}{\partial s} \right) + \frac{\partial}{\partial \varphi} \left( \tilde{\lambda}_s \frac{\partial T}{\partial \varphi} \right) + F,
\]

\[
F = \frac{\sin \alpha}{R} \tilde{\lambda}_s \frac{\partial T}{\partial s} - \tilde{c}_g \tilde{G}_{gz} \frac{\partial T}{\partial z} + \frac{d \tilde{Q}_\Sigma}{dt},
\]

\[
\tilde{c}_g \tilde{G}_{gz} = a_s a_{\varphi} c g_{gz}, \quad d \tilde{Q}_\Sigma/dt = a_s a_{\varphi} dQ_{\Sigma}/dt, \quad a_s = 1 - z \alpha/s, \quad a_{\varphi} = 1 + (z/R) \cos \alpha,
\]

where s, \varphi are longitudinal and angular coordinates of a curvilinear coordinates system connected with a shell surface, z is coordinate on a normal to a shell surface, T is temperature of a solid phase and gaseous products, \alpha = \alpha(s) is the current corner between generating line and an axis of a shell of rotation (for a conic shell we have \alpha = const is half corner of a cone); R = R(s) is distance of generating line points from an axis, \rho_s, c_s are density and specific heat capacity of a composite, \lambda_{s, \varphi} are total coefficients of heat conductivity for molecular and radiation (in a pores of a solid phase) transfers of heat in the corresponding directions, \tilde{c}_g, \tilde{G}_{gz} are specific heat and a z-component of a mass flux of gaseous products (components \tilde{G}_{gs}, \tilde{G}_{gz} are assumed by small); dQ_{\Sigma}/dt is total power of energy release per unit of volume at RPF absorption and the physical and chemical transformations (PCT).
The equation of heat transfer is complemented with relations for speeds of the motion of PCT boundaries, initial and boundary conditions. These conditions are formulated in rather common form [6] that allow to consider inputs and outputs of heat to construction surfaces. Heat contributions include losses from PCT and also convective and radiation supplies of energy from the high-temperature gases passing along an internal surface of a shell.

The solution of a problem is complicated by the fact that position and a form of boundaries of shell layers change as a result of PCT. New spatial variable is used for transforming of settlement region to area having motionless boundaries for each layer of a shell. This variable \( \tilde{z} = (z - z_l(t, s, \varphi))/\delta((t, s, \varphi) \) changes within each layer from zero to unit (\( \delta = \delta(t, s, \varphi) = z_r(t, s, \varphi) - z_l(t, s, \varphi); z_l = z_l(t, s, \varphi), z_r = z_r(t, s, \varphi) \) are boundary surfaces of layers).

The equation (1) for heat transfer is wrote in new variables

\[
\dot{\rho}_s c_s \frac{\partial T}{\partial t} = \frac{\partial}{\partial \tilde{z}} \left( \lambda_\ast \frac{\partial T}{\partial \tilde{z}} \right) + \frac{\partial}{\partial s} \left( \tilde{\lambda}_s \frac{\partial T}{\partial s} \right) + \frac{\partial}{\partial \tilde{\varphi}} \left( \tilde{\lambda}_\varphi \frac{\partial T}{\partial \tilde{\varphi}} \right) + \tilde{F},
\]

\[
\tilde{F} = C_{zs} \frac{\partial^2 T}{\partial \tilde{z} \partial \tilde{z}} + C_{z\varphi} \frac{\partial^2 T}{\partial \tilde{z} \partial \tilde{\varphi}} + C_z \frac{\partial T}{\partial \tilde{z}} + C_s \frac{\partial T}{\partial s} + C_{\varphi} \frac{\partial T}{\partial \tilde{\varphi}} + \frac{d\dot{Q}_\Sigma}{dt},
\]

\[
\lambda_\ast = \tilde{\lambda}_s + \left( \frac{\partial \xi}{\partial \tilde{z}} \right)^2 \tilde{\lambda}_s + \left( \frac{\partial \xi}{\partial \tilde{\varphi}} \right)^2 \tilde{\lambda}_\varphi, \quad C_{zs} = -2 \frac{\partial \xi}{\partial \tilde{z}} \tilde{\lambda}_s, \quad C_{z\varphi} = -2 \frac{\partial \xi}{\partial \tilde{\varphi}} \tilde{\lambda}_\varphi,
\]

\[
\xi = \xi(t, z, s, \varphi) = (1 - \tilde{z})z_l(t, s, \varphi) + \tilde{z}z_r(t, s, \varphi), \quad C_s = \frac{\sin \alpha}{R} \tilde{\lambda}_s - 2 \frac{\partial \xi}{\partial \tilde{z}} \tilde{\lambda}_s, \quad C_{\varphi} = -2 \frac{\partial \xi}{\partial \tilde{\varphi}} \tilde{\lambda}_\varphi.
\]

The problem formulated in new variables is solved by finite-difference method by means of the implicit integration scheme using splitting on spatial variables. Deviation from the the law of energy conservation is checked for all construction with the purpose of additional control of stability and accuracy of calculations during numerical integration.

3. Physical and chemical transformations

Temperature increase of a coal plastic at RPF action is followed by pyrolysis of binder and the corresponding losses of energy (\( \dot{Q}_{pyr} = 1...3 \text{ kJ/g} \)) in all volume of heated material. The expiring gas products of pyrolysis redistribute and carry away a part of energy from a construction. Change of density of material takes place owing to pyrolysis that leads to formation of the carbon rest finally. Velocity of pyrolysis is described by the simplest Arrhenius-type equation [14, 15, 16]

\[
\frac{\partial \rho_s}{\partial t} = -K \rho_{s0} \left( \frac{\rho_s}{\rho_{s0}} - k_C \right)^\nu exp \left( \frac{E_{pyr}}{R_g T} \right),
\]

where \( k_C \) is a mass fraction of the carbon rest (for coal plastic having phenolic binder that depend on mass contents and coke number of binder), \( K, \nu, E_{pyr} \) are constants of pyrolysis kinetics (\( K = (3...5) \times 10^6 \text{ 1/s}, \nu = 1, E_{pyr} = 70...80 \text{ kJ/mol} \) that depend on a type of phenolic binder), \( R_g \) is a universal gas constant. The equation of pyrolysis kinetics (2) is solved numerically by means of the implicit scheme and together with the equation of transfer for thermal energy (1).

The most power-intensive process is ablation of the carbon rest. Mass velocity of ablation for the carbon rest (this velocity is included into boundary conditions for the equation (1))
calculates taking into account formation of molecular $C_2$ and $C_3$ compounds [5, 14, 15] 
($C \to i = 1, C_2 \to i = 2, C_3 \to i = 3$)

\[ G_{abliC} = \sum_{i=1}^{3} \alpha_i (P_i^s(T) - P_i) \sqrt{2RgT/\mu_i} \]

\[ P_i = C_i \frac{\mu}{\mu_i}, \quad P_i^s(T) = K_i \exp \left( -\frac{E_i}{RgT} \right), \]

where $G_{abliC}$ is mass velocity of ablation, $\alpha_i$ are accommodation coefficients ($\alpha_1 = 0.24, \alpha_2 = 0.5, \alpha_3 = 0.023$), $\mu, \mu_i$ are molecular masses; $P_i^s(T), P_i, P$ are pressures of saturated steams, molecular compounds and mix of gases located at a surface, $C_i$ are concentrations, $K_i, E_i$ are constants for pressures of saturated steams.

The kinetics of mass losses for the carbon rest owing to heterogeneous chemical reactions of the carbon rest with five-component air mix is determined by the relations of papers [14, 17].

4. Deformation and destruction of a shell

The equations for the motion of a shell of rotation are wrote in the form [18]

\[ \frac{\partial R M_s}{\partial s} + \frac{\partial N_{s\varphi}}{\partial \varphi} - \gamma N_\varphi + k_\varphi R Q_s = m_{sh} R \frac{\partial^2 u}{\partial t^2}, \]

\[ \frac{\partial N_{s\varphi}}{\partial \varphi} + \gamma N_s + k_\varphi R Q_\varphi = m_{sh} R \frac{\partial^2 v}{\partial t^2}, \]

\[ \frac{\partial Q_{s\varphi}}{\partial s} + \frac{\partial N_{s\varphi}}{\partial \varphi} - R(\gamma N_s + k_\varphi N_\varphi) + p(t, \varphi) = m_{sh} R \frac{\partial^2 w}{\partial t^2}, \]

\[ Q_s = \frac{\partial R M_s}{\partial \varphi} + \frac{\partial M_{s\varphi}}{\partial s} + \gamma \frac{M_\varphi}{\nu} - N_\varphi \theta_s - N_{s\varphi} \theta_\varphi, \quad Q_\varphi = \frac{\partial M_{s\varphi}}{\partial \varphi} + \frac{\partial R M_{s\varphi}}{\partial s} + \gamma \frac{M_{s\varphi}}{\nu} - N_\varphi \theta_s - N_{s\varphi} \theta_\varphi, \]

\[ \gamma = \frac{dR}{ds}, \quad k_s = \frac{d\theta}{ds}, \quad k_\varphi = \frac{\cos \theta}{R}, \quad \theta = \arctg \left( \frac{d^2 X/ds^2}{d^2 R/ds^2} \right), \quad \theta_s = k_s u - \frac{\partial w}{\partial s}, \quad \theta_\varphi = k_\varphi v - \frac{\partial w}{\partial \varphi}. \]

where $u, v, w$ are displacements, $N_s, N_\varphi, N_{s\varphi}, N_{s\varphi}$ are intensities of the forces, $M_s, M_\varphi, M_{s\varphi}, M_{s\varphi}$ are intensities of the moments, $Q_s, Q_{varphi}$ are intensities of shear forces, $X = X(s), R = R(s)$ is parametrical description for a form of a shell of rotation, $p(t, \varphi)$ is pressure. Stresses located in a layer of an orthotropic shell at $z$ distance from the surface are wrote in the form of [19] (it is supposed that axes of an orthotropy coincide with the directions of axes of a curvilinear coordinate system)

\[ \sigma_s = \frac{E}{1 - \nu^2 - \vartheta^2} \left[ (1 + \vartheta)(\varepsilon_s + \kappa_s z) + \nu(\varepsilon_\varphi + \kappa_\varphi z) \right], \]

\[ \sigma_\varphi = \frac{E}{1 - \nu^2 - \vartheta^2} \left[ (1 - \vartheta)(\varepsilon_\varphi + \kappa_\varphi z) + \nu(\varepsilon_s + \kappa_s z) \right], \quad \sigma_{s\varphi} = \sigma_{\varphi s} = \frac{E}{1 + \psi} \left[ \omega + \tau z \right], \quad (4) \]

where Jung’s module $E$, Poisson’s coefficient $\nu$ and correction coefficients $\vartheta, \psi$ are determined by the known characteristics $E_s, E_\varphi, \nu_{s\varphi}, \nu_{\varphi}, G_{s\varphi}$ ($E_s \nu_{s\varphi} = E_\varphi \nu_{\varphi}$) for ortotropy material

\[ E = \frac{2E_s E_\varphi}{E_s + E_\varphi}, \quad \nu = \frac{E_s \nu_{s\varphi} + E_\varphi \nu_{\varphi}}{E}, \quad \vartheta = \frac{E_s - E_\varphi}{E_s + E_\varphi}, \quad \psi = \frac{E}{2G_{s\varphi}} - 1. \]

It follows from (4) that parameters $\nu, \vartheta, \psi$ have to satisfy to inequalities $\nu^2 + \vartheta^2 < 1, \psi > -1$. The parameters of deformations entering in relations (4) are determined by fields of displacements [18]

\[ \varepsilon_s = \frac{\partial u}{\partial s} + k_s w + \frac{1}{2} \vartheta \varepsilon_\varphi, \quad \varepsilon_\varphi = \frac{\partial v}{\partial \varphi} + k_\varphi w + \frac{1}{2} \vartheta \varepsilon_s, \quad \omega = \frac{\partial u}{r \partial \varphi} + R \frac{\partial}{\partial s} \left( \frac{v}{R} \right) + \theta_s \theta_\varphi, \]

\[ \varepsilon_s = \frac{\partial u}{\partial s} + k_s w + \frac{1}{2} \vartheta \varepsilon_\varphi, \quad \varepsilon_\varphi = \frac{\partial v}{\partial \varphi} + k_\varphi w + \frac{1}{2} \vartheta \varepsilon_s, \quad \omega = \frac{\partial u}{r \partial \varphi} + R \frac{\partial}{\partial s} \left( \frac{v}{R} \right) + \theta_s \theta_\varphi, \]
\[ \kappa_s = \frac{\partial \theta_s}{\partial s}, \quad \kappa_{\varphi} = \frac{\partial \theta_s}{R \partial \varphi} + \frac{\gamma_s}{R}, \quad \tau = \frac{\partial \theta_s}{R \partial \varphi} + k_{s} \frac{\partial v}{\partial s} - \frac{\gamma}{R}, \]

Intensity of forces and the moments are determined by integration on thickness of a shell (we use (4))

\[
\begin{align*}
N_s &= \int \sigma_s(1 + k_{s}z)dz = B_{\varphi}^{+}\varepsilon_s + B_{\psi}^{+}\varepsilon_{\varphi} + S_{\varphi}^{+}\kappa_s + S_{\psi}^{+}\kappa_{\varphi}, \\
N_{\varphi} &= \int \sigma_{\varphi}(1 + k_{s}z)dz = B_{\psi}^{+}\varepsilon_{\varphi} + B_{\varphi}^{-}\varepsilon_{\varphi} + +S_{\varphi}^{-}\kappa_s + S_{\psi}^{-}\kappa_{\varphi}, \\
M_s &= \int \sigma_s(1 + k_{s}z)zdz = S_{\varphi}^{+}\varepsilon_s + S_{\psi}^{+}\varepsilon_{\varphi} + D_{\varphi}^{+}\kappa_s + D_{\psi}^{+}\kappa_{\varphi}, \\
M_{\varphi} &= \int \sigma_{\varphi}(1 + k_{s}z)zdz = S_{\psi}^{+}\varepsilon_{\varphi} + S_{\varphi}^{-}\varepsilon_{\varphi} + +D_{\psi}^{+}\kappa_s + D_{\varphi}^{-}\kappa_{\varphi}, \\
N_{s\varphi} &= \int \sigma_{s\varphi}(1 + k_{s}z)dz = B_{\psi}^{+}\omega + S_{\psi}^{+}\tau, \quad M_{s\varphi} = \int \sigma_{s\varphi}(1 + k_{s}z)zdz = S_{\psi}^{+}\omega + D_{\psi}^{+}\tau \quad (s \leftrightarrow \varphi, 1 \leftrightarrow 2).
\end{align*}
\]

The rigidity characteristics entered in (5) for an orthotropic shell are calculated by integration on thickness in each point of a construction (calculation is necessary in each point as the shell has the variable thickness and uneven spatial distribution of temperature, \(i = 1, 2, j = 0, 1, 2\))

\[
\begin{align*}
B_{\varphi}^{+} &= I_{0}^{+}, \quad S_{\varphi}^{+} = I_{1}^{+}, \quad D_{\varphi}^{+} = I_{2}^{+}, \quad I_{j}^{+} = \int \frac{E(1 \pm \vartheta)}{1 - \nu^{2} - \vartheta^{2}}(1 + k_{s}z)z^{j}dz, \quad (k_{+} = k_{s}, k_{-} = k_{s},), \\
B_{\varphi}^{-} &= J_{0}^{+}, \quad S_{\varphi}^{-} = J_{1}^{+}, \quad D_{\varphi}^{-} = J_{2}^{+}, \quad J_{ij} = \int \frac{E\vartheta}{1 - \nu^{2} - \vartheta^{2}}(1 + k_{s}z)z^{j}dz, \\
B_{\psi}^{+} &= K_{0}^{+}, \quad S_{\psi}^{+} = K_{1}^{+}, \quad D_{\psi}^{+} = K_{2}^{+}, \quad K_{ij} = \int \frac{E}{2(1 + \psi)}(1 + k_{s}z)z^{j}dz.
\end{align*}
\]

Let’s note that intensities of forces and the moments (5) with rigidities (6) identically satisfy to the equation of the moments equilibrium for axis \(z\): \(N_{s\varphi} - N_{s\varphi} + k_{s}M_{s\varphi} - k_{\varphi}M_{s\varphi} = 0\).

The given relations are correct at elastic behavior of heated materials and when destructions are absent. Otherwise intensities of forces and moments are determined by numerical integration on relations (5) and calculation of stresses by means of model of layer-by-layer destruction \([4, 6]\).

Initial conditions for the system of the equations (3) are wrote in form

\[
u|_{t=0} = v_{0}(s), \quad w|_{t=0} = w_{0}(s), \quad u|_{t=0} = u_{0}(s), \quad \frac{\partial u}{\partial t}|_{t=0} = \frac{\partial v}{\partial t}|_{t=0} = \frac{\partial w}{\partial t}|_{t=0} = 0, \tag{7}
\]

where \(u_{0}(s), v_{0}(s), w_{0}(s)\) are displacements caused by action of flight loadings. These displacements formed from quasistationary and axisymmetric loadings are defined by means of the considered model of deformation and the relaxation method.

It is supposed that RPF action has the symmetry plane which passes through axis of a rotation shell. Then boundary conditions are

\[
\varphi = \pm \frac{\pi}{2}; \quad v = \frac{\partial u}{\partial \varphi} = \frac{\partial w}{\partial \varphi} = \frac{\partial^{2}w}{\partial \varphi^{2}}, \quad s = 0: \quad u = v = w = \frac{\partial w}{\partial s} + k_{s}u = 0, \tag{8}
\]

\[
s = L: \quad N_{s} = M_{s} = N_{s\varphi} + k_{s}M_{s\varphi} = Q_{s} + \frac{\partial M_{s\varphi}}{\partial \varphi} = 0. \tag{9}
\]

The implicit finite-difference scheme \([4, 13]\) is used for numerical integration of the equations (3) of the shell motion with initial (7) and boundary (8), (9) conditions.
5. Conclusions

(i) The new mechanism of destruction of thin-walled constructions at a pulse supply of energy of RPF which are volume absorbed is offered. The essence of the mechanism consists in formation of non-stationary processes of deformation and destruction in a shell under the action of quasistationary flight loadings when rigidity of a construction changes practically instantly (time of energy supply is much less period of construction oscillation) as a result of pulse heating of materials.

(ii) The three-dimensional physical and mathematical model of thermal action of convective and radiation flight fluxes of heat together with RPF action is developed. Physical and chemical processes (there are pyrolysis of binder, chemical reactions with components of the running air flow and ablation of the carbon rest) taking place when heating composite materials of a construction are accepted in account. The algorithm of numerical realization of this multidimensional model of thermal action of fluxes on multilayered shell of variable thickness is offered.

(iii) The mechanical and mathematical model of deformation and layer-by-layer destruction of unevenly heated composite thin-walled construction of variable thickness is developed. The implicit finite-difference scheme for numerical integration of the motion equations for a shell was used.

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