Metering error quantification under voltage and current waveform distortion

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Abstract. With integration of more and more renewable energies and distortion loads into power grid, the voltage and current waveform distortion results in metering error in the smart meters. Because of the negative effects on the metering accuracy and fairness, it is an important subject to study energy metering combined error. In this paper, after the comparing between metering theoretical value and real recorded value under different meter modes for linear and nonlinear loads, a quantification method of metering mode error is proposed under waveform distortion. Based on the metering and time-division multiplier principles, a quantification method of metering accuracy error is proposed also. Analyzing the mode error and accuracy error, a comprehensive error analysis method is presented which is suitable for new energy and nonlinear loads. The proposed method has been proved by simulation.

1 Introduction
The electrical energy metering result is the main evidence of economic accounting in power grid, and electric energy meter as the core part of electric energy measurement system, its accuracy and rationality is related to the economic benefits of power supply and demand[1,2]. With the development of power electronics technology, more and more nonlinear load or time-varying load were connected to power grid and produced a large number of harmonic. Therefore the harmonic pollution of power system has steadily increased, that make the electric energy meter has error[3-7]. In order to ensure the accuracy and fairness of the electric energy metering, researched on the mode error and accuracy error, and presented a comprehensive error analysis method that is more reasonable in harmonic environment, has important theoretical value and engineering significance.

At present, a lot of researches have been carried out on metering error in harmonic environment. Based on the metering mode, mainly of qualitative analysis[8-11], and not accurately quantify the error value. Considering accuracy error, they can be divided into two main directions, one is accuracy error base on the meter measurement principle[12-14] and the other is the improvement scheme[15]. In Ref. [12], researched on the working principle of the various types of time-division multiplier, and analyzed accuracy error, but only for the qualitative analysis. The works[13-14] derived the theoretical error expression of time-division multiplier under the condition of harmonic. In[15], a harmonic response rate mode is established, and the electrical energy is corrected by empirical parameters, that can reduce error. It can be seen that most of the researches are based on the principle of physical metering, and analyzed and calculated the accuracy error. However the influence of metering mode is not considered.

This paper comprehensive consider the metering mode and accuracy, analyzed the error in harmonic environment. In part II, compared between metering theoretical value and real recorded value under different metering modes, and derived the metering mode error expression. In part III, analyzed the accuracy error base on the working principle of the time-division multiplier. In part IV, the feasibility and rationality of the proposed method are verified by simulation of simple nonlinear circuit and
multiplier mode.

2 Comprehensive error analysis

Combined error of the electrical energy meter in the condition of harmonic can be represented as (1)

\[ \gamma = P_{\text{meter}} - P \]  \hspace{1cm} (1)

Where \( P \) is the metering theoretical value, \( P_{\text{meter}} \) is the real recorded value. \( \gamma \) is the combined metering error of the meter. This paper main discussed error caused by the metering mode and accuracy, and respectively defined as \( \alpha \) and \( \beta \), then \( \gamma = \alpha + \beta \).

2.1 Full-wave metering mode

Full wave metering mode, which uses a wide range of frequency response of electronic energy meter to measure the fundamental and harmonic. In theory, it can be included the fundamental power and harmonic power that consumed or issued by the user in the calculation of electricity charges. Most of the existing meter is full wave metering mode. In this mode, the measurement results of the electric energy meter can be represented as (2)

\[ P_{\text{meter}} = P_{m1} + P_{mh} \]

\[ = P_{r1} + P_{rh} + \beta \]  \hspace{1cm} (2)

\[ = P_{r1} + \beta + P_{rh} + \beta_h \]

Where \( P_{m1} \) and \( P_{mh} \) is the fundamental power and harmonic power of meter real recorded, respectively. \( P_{r1} \) and \( P_{rh} \) is the actual fundamental power and actual harmonic power on metering point, respectively. \( \beta \) is the error caused by meters accuracy, composed of fundamental measurement errors (\( \beta_r \)) and harmonic measurement errors(\( \beta_h \)).

For linear load, in the presence of harmonic in the utility side, the load absorb fundamental and harmonic power from the system. However, linear load should be only metered fundamental power in theory, then

\[ P = P_{1} = P_{r1} \]  \hspace{1cm} (3)

Combined the formula (1)-(3), in this metering mode, the combined metering error of linear load can be represented as

\[ \gamma_{\text{liner}} = \alpha + \beta = P_{rh} + \beta \]  \hspace{1cm} (4)

For nonlinear load, in the presence of harmonic in the utility side, the load absorb fundamental power, absorb harmonic power that caused by another nonlinear load, and inject harmonic power into system at the same time. In this condition, actual harmonic power on metering point can be represented as

\[ P_{rh} = P_{sh} - P_{lh} \]  \hspace{1cm} (5)

Where \( P_{sh} \) is the harmonic power that be absorbed by nonlinear load which we considered. \( P_{lh} \) is the harmonic power that be injected by nonlinear load which we considered. Then, defined harmonic power of the nonlinear load should be metered in theory as \( P_h \), the value be related to harmonic contribution study, do not give discussion in this paper. Introduced coefficient \( M_h \) which satisfy:

\[ P_h = M_h P_{rh} \]  \hspace{1cm} (6)

In summary, when the utility side exist harmonics, nonlinear load should be metered power quantity can be represented as

\[ P = P_{1} + P_h = P_{r1} + M_h P_{rh} \]  \hspace{1cm} (7)

Combined the formula (1), (2) and (7), in this metering mode, the combined metering error of nonlinear load can be represented as

\[ \gamma_{\text{non-liner}} = \alpha + \beta = P_{rh} - M_h P_{sh} + \beta \]  \hspace{1cm} (8)

2.2 Fundamental metering mode
The fundamental metering mode, namely sample the voltage and current data, then through filtering only gain fundamental data. In this mode, only the fundamental power that consumed by the user will be calculated in electricity charges. In this condition, the real recorded value of meter can be represented as

\[ P_{\text{meter}} = P_{\text{ml}} = P_{\text{th}} + \beta_i \]  

(9)

For linear load, combined the formula (1), (3) and (9), the combined metering error can be represented as

\[ \gamma_{\text{liner}} = \beta_i \]  

(10)

For nonlinear load, combined the formula (1), (7) and (9), the combined metering error can be represented as

\[ \gamma_{\text{non-liner}} = \alpha + \beta = M_h P_{\text{th}} + \beta_i \]  

(11)

Analyzed the formula (4), (8), (10) and (11), the combined metering error of electric energy meter is related to the accuracy and the harmonic power on metering point. Next part analyzed the accuracy error of the electric energy meter (\( \beta \)).

3 Accuracy error principle

3.1 Working principle

Electric energy meter can be divided into two types of induction meter and electronic meter. Induction meter mainly is composed of the driving components, rotating components, braking components, bearings, gauges and auxiliary components. It is operated by the force between a fixed alternating magnetic field and an electric current which is induced of the movable part of the conductor. Electronic meter consists of current converter, voltage converter, multiplier, voltage frequency converter, counter and so on. Electronic meter is widely used because of its good linearity, stability and high precision. The working principle of the electronic meter is shown in Figure 1. The multiplier is the main component of the electronic meter, also is the main source of metering error. At present, time-division multiplier is the most widely used, so this section mainly discusses the error of time-division multiplier.

The pulse width modulation circuit, as the main component of the time-division multiplier, main types are: triangle wave comparison, dual level comparison, the beat square wave control voltage integration, etc. This paper analyzed metering error based on triangular wave compared type of time-division multiplier, the working principle is shown in Figure 2. Where X and Y are the input signal, T is the high frequency triangular wave modulation signal.

The working principle can be described as: the input signal X and triangular wave signal T through comparator, then can get the pulse signal S. The signal S is used to control the switch to turn on the input signal Y to make the pulse amplitude modulation, so that the signal amplitude is equal Y. Then use the filter to smoothing filtering to get the DC signal, and the amplitude is equal the power value.
3.2 Error analysis

When the modulation signal uses high frequency triangular wave, the input signal X can be regarded as a DC signal. In this condition, the positive and negative width of the pulse signal S will be proportional to the signal X, as shown in Figure 3.

Assuming that the slope of the triangular wave is K, then

\[ K = \frac{E_x - E_y}{\frac{1}{2}T_1} = \frac{E_x + E_y}{\frac{1}{2}T_2} \]  \hspace{1cm} (12)

And the cycle of signal S is \( T = T_1 + T_2 \), then

\[ T_1 = \frac{2(E_y - E_x)}{K} \] \hspace{1cm} (13)

\[ T_2 = \frac{2(E_y + E_x)}{K} \] \hspace{1cm} (14)

Use the signal Y to make the pulse amplitude modulation, and through low-pass filter, and the amplitude is satisfy:

\[ P = \frac{1}{T}(-T_1Y + T_2Y) \] \hspace{1cm} (15)

Combined the formula (13), (14), the output signal can be simplified to

\[ P = \frac{1}{T}(-T_1Y + T_2Y) = \frac{1}{E_y}E_xE_y \] \hspace{1cm} (16)

Therefore, the output value of the time-division multiplier is proportional to the active power input.
4 Simulation results

4.1 Metering mode

Establish a single-phase circuit with linear load and nonlinear load in Simulink, as shown in Figure 4.

![Figure 4: System simulation diagram](image)

Assuming the system exists harmonic, the amplitude of system fundamental voltage is 1 kV, and the 3rd-order, 5th-order, 7th-order harmonic voltage is the 3%, 1%, 0.5% of the fundamental voltage, respectively. System impedance $Z_u$ is composed of 5 $\Omega$ resistors and 0.1 mH inductors in series. Linear load consists of 100 $\Omega$ resistors and 1 mH inductors in series. Nonlinear load consists of 100 $\Omega$ resistors, 1 mH inductors and diode in series. The simulation step is 5 $\mu s$, and total time is 5s.

The actual fundamental and harmonic power at the point of attention can be represented as

$$P_r = \frac{1}{N} \sum_{n=1}^{N} u(n) i(n) = VI \cos \theta$$  \hspace{1cm} (17)

$$P_{rl} = \frac{1}{N} \sum_{n=1}^{N} u_l(n) i_l(n) = V_l I_l \cos \theta_l$$  \hspace{1cm} (18)

$$P_{rh} = P_r - P_{rl}$$  \hspace{1cm} (19)

The voltage and current data are directly measured by the meter, and extract the fundamental component of voltage and current by the filter. Then calculated the actual fundamental, harmonic and full wave power at the point of attention according to the formula (17)-(19), as shown in Table 1.

| Linear Load | Nonlinear Load |
|-------------|----------------|
| Fundamental Power (W) | 4327.7 | 2114.3 |
| Harmonic Power (W)     | 10.4  | -45.3  |
| Full-wave Power (W)    | 4338.1 | 2069.0 |

According to the data of Table 1, in the case that utility side exists harmonic, the harmonic power of the linear load is positive, and the measurement result in the full wave metering mode is larger than the fundamental metering mode. Harmonic power of the nonlinear load is negative, it absorb the fundamental power, inject a portion of a harmonic power at the same time, making the full wave power is less than the fundamental power. Namely under the condition of waveform distortion, the nonlinear load not only makes the harmonic pollution to the power grid, but also receives the electric charge, which is obviously unfair. Therefore, there are a certain metering mode error.

According to the formula (4), (8), (10) and (11), and the coefficient $M_h$ is taken as -0.5, the error value caused by different metering mode is shown as Table 2.

| Linear Load | Nonlinear Load |
|-------------|----------------|
| Fundamental metering mode (W) | 0 | -22.65 |
| Full wave metering mode (W)    | 10.4 | -67.95 |

According to Table 1, the metering mode error of linear load is positive if the meter works in
fundamental metering mode. The metering mode error of nonlinear load is both negative no matter what metering mode that meter works. Mainly due to the nonlinear load inject harmonic current, then harmonic power is negative, making the meter less metering electric energy power.

According to the formula (3), (7) and the data in Table 2, it can be obtained that the power should be metering of linear load and nonlinear load is 4327.7 W and 2136.95 W, respectively.

### 4.2 Accuracy error

Establish time-division multiplier in Simulink, as shown in Figure 5.

Where the frequency of the modulated triangular wave signal is 5 kHz, namely the modulation coefficient is 100. The amplitude of the modulated triangular wave signal is 2000. The voltage, current of the full wave, the fundamental wave as input voltage, input current signal as the shown in Figure 5. The simulation step is 5 us, and total time is 5s.

Obtained the full wave power ($P_m$) and fundamental power ($P_{m1}$) values by the simulation, and the former minus the latter to get the harmonic power ($P_{mh}$). Then calculated the fundamental and harmonic metering accuracy error $\beta$ according to the data in Table 2, as shown in Table 3.

| Table 3: Measurement value of linear and nonlinear load (W) |
|-----------------|---------------|---------------|
| **Power**       | Linear Load   | Nonlinear Load|
| Fundamental     | 4335.8        | 2118.1        |
| Harmonic        | 32.8          | -21.6         |
| Full-wave       | 4368.6        | 2096.5        |
| **Accuracy Error** | **Fundamental** | **Harmonic** | **Full-wave** |
| Fundamental     | 8.1           | 22.4          | 30.5          |
| Harmonic        | 3.8           | 23.7          | 27.5          |

From Table 3, it is obvious that the harmonic accuracy error is lager. Combined with the metering error principle of Part III, this is due to the harmonic signal frequency higher, when it compared with the high frequency triangular wave signal, it should not be regarded as DC signal.

### 4.3 Combined error

Calculated the combined error $\gamma$ according to the data in Table 2, Table 3, as shown in Table 4.
Table 4: Synthetic error

|                  | Fundamental Metering Mode | Full-Wave Metering Mode |
|------------------|---------------------------|-------------------------|
|                  | Linear Load   | Non-linear Load | Linear Load | Non-linear Load |
| Actual Value (W) | 8.1           | -19.65         | 37.4        | -42.85         |
| Percentage       | 0.187%        | 0.920%         | 0.864%      | 2.005%         |

In Table 4, the combined error value is obtained by considering the metering mode and the metering accuracy of the meter. In this case, the results showed that if only blindly increase the metering accuracy, such that reducing harmonic power accuracy error of nonlinear load, then the harmonic power metering value will increase, further makes the metering mode error increase, namely there are some contradictions between the metering mode error and the metering accuracy error. Therefore, when considering the error of the electric meter in the harmonic condition, should make a comprehensive analysis, and then put forward the improvement plan.

5 Conclusion

As the evidence of electricity charges, the electric energy metering results should be fair and accurate. In this paper, based on the metering principle of electric meter, considering the metering mode and metering accuracy error, the paper puts forward the comprehensive error analysis method of electric meter under the harmonic condition.

(1) The error of linear load and nonlinear load is related to metering mode of meters in presence of harmonics. In full-wave metering mode, the linear load more metering electric energy, and the nonlinear load less metering electric energy while produce the harmonic power. Quantify the produced error is related to the harmonic power at the point of attention.

(2) Analyzed the working principle of time-division multiplier. The metering accuracy error of meter is related to the frequency of the modulated signal in presence of harmonics, Due to the change of the frequency ratio can not be regarded as a DC signal and makes a large metering accuracy error.

(3) The error of the electric meter needs to consider two aspects of the metering mode and the metering accuracy in the harmonic condition. The simulation prove that if only consider accuracy error, and ignore the metering mode error, may increase the combined error of meters, reduce the accuracy and fairness of electric energy metering.

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