The Solution to galaxy rotation curve problem-----Dark matter and MOND

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Abstract. In 1932, Dutch astronomer Jan Hendrik Oort examined the Doppler shifts in the spectra of stars in the Milky Way Galaxy and first found that the stars moved faster than expected. Then people began to think and argue why that happen. Some people thought maybe the Newton’s law was only useful in the solar system. And someone thought there may be some matters that cannot be seen. Now there are two different hypotheses to explain this situation, Modified Newtonian dynamics (MOND) and Dark Matter. This paper will introduce the origin of these two hypotheses and the evidence supporting these hypotheses. The paper will also compare and contrast three different models to show the existence of dark matter.

1. Introduction
The galaxy rotation curve is a graph of the orbital velocity of visible stars or gas in that galaxy vs. their radial distance from that galaxy’s centre. It is by studying the galaxy rotation curves that the galaxy orbital velocity varies greatly from the predicted velocity that the following two hypotheses appear. Astronomers use the 21 centimeter line which refers to the electromagnetic spectrum lines produced by neutral hydrogen atoms due to energy level changes. The frequency is about 1420.4 MHZ. Then the orbital speeds of cloud of gas is obtained by observing the red shift of the spectral line caused by the Doppler effect. The Doppler effect is that the object radiation wavelength varies due to the relative motion of the wave source and the observer. Before moving the wave source, the wave is compressed, the shorter wavelength, and the frequency becomes higher (blue shift); when moving the wave source, the opposite effect occurs. Long wavelength, lower frequency (red shift)[1].

In 1933, astronomer Fritz Zwicky was studying Coma cluster of galaxies, noting that the amount of matter needed to cause the orbital velocity of the galaxy does not reflect the amount of visually detectable matter. Since then, more observations with similar features have been made, which scientists have used to determine that the gravitational potential needed to explain the observations means what Zwicky calls "missing mass" —— what we now often call "dark matter."[2]. After nearly a century of investigation, astrologers have learned two things about dark matter, which must be non-baryon, that is, by new versus normal matter only by gravity, the effects observed in the stellar system when only the internal accelerated gravity is below a fixed value

\[ a_\text{th} = 1.2 \times 10^{-8} \text{cm/s}^2 \]

We will propose a proof of the existence of dark matter through the fitting of actual data and different models.

An alternative to dark matter, which directly incorporates the radial acceleration relation, is the modified Newtonian dynamics (MOND). As early as 1983, after realizing that when the gravitational acceleration was lower than a fixed value, the star system would observe a mass difference, M. Milgrom proposed a modification to Newtonian dynamics (MOND) as a substitute for non-baryon dark matter. Milgrom's proposal states that gravity does not follow the prediction of Newtonian dynamics for acceleration smaller than \( a_\text{th} = 1.2 \times 10^{-8} \text{cm/s}^2 \), as derived from galaxy rotation curves fit. Below
this acceleration, the behavior of gravity changes become asymptotically \( a = \sqrt{a_0 a_0} \) where \( a_0 \) is the usual Newtonian acceleration. The transition from Newtonian to MOND regime occurs for accelerations that remain undetected within the solar system where the strong field of the sun is dominating all dynamical processes. To be more precise, it is seen that even the acceleration by Mercury on Pluto is above \( a_0 \), so that the there is no way in the solar system to test the validity of Newtonian dynamics down to the acceleration regimes typical of galaxies. As a consequence, Astrologists have no direct proof of its validity below \( a_0 \), and its applicability in the realm of galaxies is not guaranteed. The basic premise of MOND is that although Newton's laws have been extensively tested in high acceleration environments (in the solar system and on the earth), they have not yet been verified on extremely low acceleration objects, such as stars outside galaxies. This leads to Milgrom Suppose a new law of effective gravitation (sometimes called "Milgrom's law"), which relates the true acceleration of an object to the acceleration predicted by Newtonian mechanics. This law is the cornerstone of MOND. It was chosen to reproduce Newtonian results at high accelerations, but it will cause different behaviors at low accelerations. This article will explain the two mainstream hypotheses that solve the problem of galaxy rotation and point out the current problems of each hypothesis and the evidence to prove these hypotheses. Solving the problem of galaxy rotation curve can help astronomers understand the galaxy evolution and then get clues to the origin of the universe[4].

2. Dark matter

2.1. Background

Baryonic matter is the familiar material of the universe which composed of protons, neutrons and electrons. Dark matter is different from normal material, it may be made of baryonic matter or non-baryonic matter. To combine all the elements of the universe together, dark matter have to make up about 80% of the universe. Most scientists believe that dark matter is composed of non-baryonic matter. The main candidate WIMPS (weakly interacting massive particles) is 10 to 100 times the mass of protons, but their weak interaction with "normal" matter makes them difficult to detect. Neutralions is a kind of massive hypothetical particles which is heavier and slower than neutrinos. Although Neutralions have not to be found, they are still the foremost candidate. Sterile neutrinos are another candidate. Neutrinos are particles that do not constitute regular matter. A neutrino river flows from the sun, but as they rarely interact with normal matter, they pass through Earth and the inhabitants. There are three known types of neutrinos: muon neutrino (\( \nu_\mu \)), electron neutrino (\( \nu_e \)), tau neutrino (\( \nu_\tau \)); The fourth is the sterile neutrino, considered as a dark matter candidate. Sterile neutrinos can only interact with regular matter via gravity[5].

2.2. Evidence 1 gravitational lensing

There are many evidence to show that the dark matter is existed in the universe. One evidence is the Gravitational lensing. Fusion or multiple imaging effects of gravitational field sources on electromagnetic radiation from objects located thereafter. It is named because of the similar convergence effect of a convex lens. Gravitational lensing is a phenomenon predicted by Albert Einstein's general relativity, as space-time distorts near massive objects, bending light near massive objects (light spreads along short threads of curved space). If there is a massive foreground object from the observer to the source, two images form on both sides of the source as if a lens lay between the observer and the object. This is a phenomenon called gravitational lensing. When a dark object in the Milky Way passes by a distant star (such as a Magellanic Cloud), it is briefly illuminated as a smaller-scale gravitational lensing effect. This gravitational lensing caused by a single star is sometimes called "microlensing". In 1979, astronomers observed the light from the quasar Q0597+ 56 l bending under the gravitational action of a galaxy in front of it, forming an exactly-like quasar image. This is the first gravitational lensing effect observed.

Another kind of gravitational lensing is weak gravitational lensing. It is a shape-like deformation effect produced after light emitted from distant background galaxies is slightly distorted by foreground
galaxies. It is a purely gravitational effect that can reflect perturbations of the cosmic matter density, and unlike a strong gravitational lensing or a microgravity lens, this effect does not produce a deformation as strong as an Einstein ring or arc distortion. The weak gravitational lens effect is an optical phenomenon caused by the spatial bending of the cosmic material density field described by Einstein's general relativity and the power spectrum of material disturbance. Therefore, when astronomers observe the gravitational lensing effect, they can get the mass of the galaxy cluster causing the lensing effect. People can really see is the light from all galaxies. From the amount of light observed by starologists, they can infer how much "stellar" mass (stellar mass) the galaxies in the cluster must have. Astromologists know how gravity works and will reproduce gravitational lensing on a computer. All astrologers need to do is enter a mass value and they can see how much lensing it produces. When astrologers input the observed "stellar" mass value, they find that it does not produce much lensing effect (on average, only the stellar mass, producing only 10% of the energy required for the same lensing effect as observed). This means that more quality (about nine times more) needs to be considered to produce similar lensing effects on the computer. This particle does not glow, does not interact with light but has mass, therefore, has a gravitational effect on the spacetime around it. One of the best observational arguments for dark matter are the observations of the Bullet Cluster (1E 0657-558). The Bullet Cluster has two components that have passed through each other, leaving hot x-ray gas behind due to ram pressure. By analyzing where the x-ray gas is and what weak gravitational lensing tells about the cluster's mass distribution.[6]

2.3. Evidence 2 galaxy rotation curve
Another evidence of dark matter is the galaxy rotation curve. There are three best known models in history to match the galaxy rotation curves. The first and the easiest model——Kepler Model. In this model, scientists think that all the mass is concentrated in the center and the galaxy is a rigid rotor. So the relative positions of all stars in the same galaxy remain the same, in other words, they have the same angular velocity \( \Omega \). According to the law of gravitation, the formula is obtained:

\[
v(r) = \sqrt{\frac{GM}{r}} \tag{1}
\]

\( G \) is Gravitational constant and \( M \) is Galaxy center mass. Both of them will not change. So the \( v(r) \propto \frac{1}{\sqrt{r}} \). Then Using the formula(1), the curve is obtained:

Figure 1. Kepler model predicted galactic rotation curve 1.
Figure 2. Kepler model predicted galactic rotation curve 2.

For the solar system’s planet, this curve seems right. But the real data curve is as follows:

Figure 3. Kepler model curve compare to the real data curve.

It is easy to see that the data curve is really different from the Kepler model. The Kepler model curve showed that the speed of the star decreases as the distance from the center increases, but the real data curve showed that the speed of the star increase really fast between 0 to 3 light years and then almost stabilized.

That is because in fact the mass is not concentrated in the center. In fact, the mass of galaxies is distributed in a certain proportion—the Exponential Disk model. In this model, it is assumed that the amount of mass is proportional to the amount of light. The surface brightness of galaxies have the form of:

\[ I(r) = I_0 e^{-R/R_D} \]  

(2)

And making a further reasonable assumption is that the surface mass density is proportional. And getting the form:

\[ \mu(r) = \mu_0 e^{-R/R_D} \]  

(3)

In these forms, RD is the disk “scale-length”, it is about 7000 light years in the case of the Milky way. In the Exponential Disk model, the mass still tends to be concentrated in the center of the galaxy. The surface mass density figure and the fraction of mass at r R figure:
In figure 3, it seems like most of the mass of the galaxy is concentrated to the center but in fact, that's a fallacy. Because although the density in the center is biggest, the area of the center circle is small. From figure 4, it is easy to point out that about 80% mass is concentrated between 0 to 20,000 ly. According to Freeman’s form about $v$:

$$v^2(r) = \frac{GMr^2}{2R_D} \left[ I_0(u)K_0(u) - I_1(u)K_1(u) \right]$$  \hspace{1cm} (4)

Then using the formula (4) to Exponential Disk model curve:
The blue curve is the ED model; the red curve is the real data curve; the green curve is Kepler model. This model is still not match the real data curve. And after \( r \) is bigger than 20,000 ly, the Kepler model and ED model give the similar result, that’s because in the Kepler model, we think there is almost no mass in other places except the center of galaxy and in the Exponential Disk model, we can find from figure 2 that the place more than 20,000 light years from the center contains a small percentage of mass. So the farther away from the center of the galaxy, the more similar the results of the two models.

In the third model, adding the dark matter halo to make the curve more match the real data curve. Dark matter halos are a fundamental unit of the cosmological structure. It is a hypothetical region that is decoupled from the cosmic expansion and contains gravitationally bound matter. A commonly used model for galactic dark matter halos is Isothermal Sphere model. The density function is given that:

\[
\rho(r) = \frac{\rho_0 R^2 + a^2}{r^2 + a^2}
\]  
(5)

According to the Shell Theorem could get the form:

\[
M_c(r) = 4\pi \int_0^r \rho(r')r'^2dr' = 4\pi \rho_0 (R_0^2 + a^2) \left[ r - a \tan^{-1} \frac{r}{a} \right]
\]  
(6)

Then the halo force is:

\[
F_h(r) = \frac{GM_c(r)m}{r^2}
\]  
(7)

Then using the formula (6) get the speed in this model \( v \), now the isothermal sphere model and Exponential Disk model are combined to get the new curve:

![Figure 7. Isothermal sphere model plus Exponential Disk model curve.](image)

The yellow curve is our isothermal sphere model plus Exponential Disk model curve. It shows that this new curve almost coincides with our data curve after 10,000 light years. This can be used as one of the evidences for the existence of dark matter.

2.4. MOND

Milgrom considers the following temporary modification to Newton’s law, the gravity \( m \) exerts to the mass due to another mass \( M \) at the distance:

\[
F = \begin{cases} 
ma_N, & a_N \gg a_0, \\
(ma_N a_0)^{1/2}, & a_N \ll a_0 
\end{cases}
\]  
(8)

This is the usual gravity induced Newtonian acceleration, and it is assumed that there are some smooth interpolation functions between the two limits. If the acceleration generated by Newtonian gravity is greater than the acceleration of the object, Milgrom gravity is directly proportional to the same Newtonian force, but if the acceleration generated by Newtonian force is less than Milgrom gravity, Milgrom gravity is greater than the corresponding Newtonian force, like \( a_N = \frac{GM}{r^2}a_0 r^{-2}a_0 r^{-1} \).

Notably, under the assumption that stars in disk galaxies follow a circular orbit around the center of the galaxy, this simple modification is very successful to reproduce the rotation curve of spiral galaxies only from the visible matter distribution, that is, the absence of dark matter. Many rotation curves are fitted
in this way; The rotation curves of some galaxies do not conform to this formula, but these galaxies usually do not conform to the dynamic equilibrium visually, or have strong strips or other deviations from the circular symmetry, so they may violate the assumption that stars move in circular orbits. Milgrom calls this force law modification as MOND, to modify Newtonian dynamics. There is already an extensive literature on MOND. This paper aims to provide a brief overview of the current state of the subject and to provide entry points useful to the literature, but not a full review of the literature or science.[7][8]

A rough analysis of the law of force in [8] reveals a fundamental drawback, that it does not conserve the momentum. It is simple to construct a case with two different masses, unlike the small mass to the small mass. Shortly after Milgrom's original paper, Beckenstein and Milgrom proposed a generalization of the Poisson's equation rather than Newtonian's force law, which overcomes this difficulty:

\[
\nabla \cdot \left[ \mu (|\nabla \phi|/a_0) \nabla \phi \right] = 4\pi G \rho
\]

(9)

\(\Phi\) is the physical gravitational potential, and \(\rho\) is the baryonic mass density. \(\mu(x)\) is some smooth function with the asymptotic forms.

One of the observation evidence for Mond is tidal dwarf galaxies. Tidal dwarf galaxies form during the interaction, collision or merger of massive spiral galaxies. They can resemble “normal” dwarf galaxies in terms of mass, size, and become dwarf satellites orbiting around their massive progenitor. For some reasons, this galaxy does not contain dark matter and hence show no mass discrepancy. Three objects unambiguously identified as Tidal Dwarf Galaxies appear to have mass discrepancies in close agreement with the MOND prediction.[9]

3. Conclusion

Since 1932 scientists discovered that galaxies did not rotate faster than they were predicted by Newton's law, there has been much debate over why galaxies would rotate much faster than Newton’s law predicted. One of the most accepted theories until now is dark matter and the other is Modified Newtonian dynamics. There are many evidence to proof the existence of dark matter like Gravitational lensing and the rotation curves are perfectly consistent for some galaxies. By now, however, we still have no real observation of the dark matter itself. Modified Newtonian dynamics also have many evidence to proof it like the rotation curve of the tidal dwarf galaxies which must not contain dark matter is correctly predicted. Now the two-party doctrines are still constantly discovering new evidence to prove their own theories. Perhaps to end the debate between these two theories, Large Underground Xenon experiment (LUX) observing dark matter is the only way. Now the two-party doctrines are still constantly discovering new evidence to prove their own theories. Perhaps to end the debate between these two theories, LUX observing the dark matter is the only way.

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