Evolution of the primordial axial charge across cosmic times

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We investigate collisional decay of the axial charge in an electron-photon plasma at temperatures 10 MeV–100 GeV. We demonstrate that the decay rate of the axial charge is first order in the fine-structure constant $\Gamma_{\text{flip}} \propto \alpha m_e^2/T$ and thus orders of magnitude greater than the naive estimate which has been in use for decades. This counter-intuitive result arises through infrared divergences regularized at high temperature by environmental effects. The decay of axial charge plays an important role in the problems of leptogenesis and cosmic magnetogenesis.

The origin of cosmic magnetic fields remains a subject of intense debate, see [1–4] for reviews. One of the leading hypotheses is that these fields originated in the hot and homogeneous early Universe. If this hypothesis is correct, the requirements that the magnetic fields (i) were generated before and (ii) survived till the beginning of the structure formation epoch (when the process of their amplification started) impose tight constraints on the possible history of the Universe, likely implying the existence of new physics [4, 5]. This potential for serving as a bridge between the observational data and the properties of the early Universe makes both primordial magnetogenesis and magnetohydrodynamics (MHD) of ultrarelativistic plasmas research topics of fundamental importance.

It has been argued that due to the weakness of non-conservation of the axial charge current in an ultrarelativistic plasma, the proper description of the evolution of primordial cosmic magnetic fields requires an extension of MHD called chiral magnetohydrodynamics [6–8], see also [9, 10]. In chiral MHD the system of Maxwell and Navier-Stokes equations is supplemented with an extra degree of freedom – the axial chemical potential. Such an extension materially affects the predictions of the theory. In particular, chiral MHD admits for the transfer of magnetic energy from short- to long-wavelength modes of helical magnetic fields, partially compensating Ohmic dissipation in the early Universe and thus increasing their chance to survive till today [6–8, 10–12]. It is worth noting that chiral MHD has drawn a lot of recent interest not only because of its importance for the description of primordial magnetic fields, but also due to its relevance to the theory of neutron stars and quark-gluon plasmas (see, e.g., Refs. [5, 11, 13]).

The chiral MHD description is only appropriate inasmuch as the axial current can be treated as conserved on microscopic time scales such as the momentum and energy relaxation rates. This requires the typical kinetic energy of an electron in the plasma to significantly exceed the electron mass $m_e$, so one can meaningfully assign chirality to each particle. In such a high-temperature regime, $T \gg m_e$, the axial charge decays through rare chirality-flipping processes, which are still possible due to the non-conservation of chirality introduced by a perturbatively small mass term. Surprisingly, the chirality flipping rate resulting from such processes has never been rigorously calculated [9]. The previous body of work relied on the naive estimate of the chirality flip rate

$$\Gamma_{\text{flip}} \propto \left( \frac{m_e}{T} \right)^2 \alpha^2 T$$

as being second-order in the small parameter responsible for chirality non-conservation, $m_e/T$, and first order in the electron scattering rate $\Gamma_{\text{scat}} \propto \alpha^2 T$ (see, e.g., [7, 36–38]) where $\alpha = e^2/(4\pi)$ is the fine structure constant. This estimate is based on the simple rationale that for an ultrarelativistic particle in a definite helicity state, which up to a correction on the order of $m_e/T$ is the same as a definite chirality state, the helicity can only be flipped by a sideways scattering process having the rate $\Gamma_{\text{scat}}$.

The aim of the present work is to show that contrary to the naive expectation, Eq. (1), the actual chirality flipping rate in an ultrarelativistic plasma is first order in $\alpha$, see Eq. (1). We focus, in particular, on the analysis of infrared singularities in the matrix elements of chirality-flipping Compton scattering and show how they effectively lead to the cancellation of one power of $\alpha$. We also briefly discuss other scattering channels which contribute to chirality flipping in the same order of perturbation theory and give the leading-order asymptotic expression for the chirality flipping rate as resulting from all such contributions. We perform their in-depth analysis from the first principles in our companion paper [39]. We note that although it is natural for kinetic coefficients associated with electron-photon scattering to be second order in the fine structure constant, there exists another known
exception from this rule — the axial charge diffusion coefficient $[41]$

\begin{align*}
\text{FIG. 1. The } t \text{-channel Compton scattering (a) and electron-}
\text{positron annihilation (b) with the chirality flip in the intermediate state contributing to the chirality equilibration rate.}
\end{align*}

Although naively they are of the second order in $\alpha$, their amplitudes contain infrared singularities. Regularization of these singularities leads to the result which is of the first order in $\alpha$.

Our main idea can be summarized as follows. We consider $2 \leftrightarrow 2$ chirality flipping processes, starting from massless QED limit and treating both the electron mass and the electron-photon coupling as perturbations (see Fig. 1). As is well known such processes have a non-integrable infrared singularity at small momentum transfer $[41, 42]$. This signals the need for the resummation of the leading infrared divergence in all orders of the perturbation theory series. Such a resummation should generally result in an answer $\Gamma_{\text{flip}} \propto \alpha^2 T m_e^2 / q_{\text{in}}^2$, where $q_{\text{in}}$ is the infrared regulator scale associated with either the effective mass or the lifetime of the quasiparticle associated with the electron propagator. In a hot plasma a natural infrared scale arises from the thermal self-energy corrections to the dispersion relations of (quasi)particles. In $[43]$, Sec. A we use hard thermal loops (HTL) resummation to show that such corrections are of the order $q_{\text{in}} \sim \sqrt{\alpha} T$, which results in $\Gamma_{\text{flip}} \propto \alpha m_e^2 / T$. We note that such an approach is not valid in the regime where the self-energy corrections are less than the electron mass. Therefore the validity range of our analysis is $T \geq m_e / \sqrt{\alpha} \sim 10$ MeV.

Next we describe our calculations in some detail. Particle chiralities are well-defined for free massless particles. Therefore we start from massless QED and treat mass as a perturbation. In plasma this means that we consider each chirality obeying its own Fermi-Dirac distribution

\begin{align*}
f_{L,R}(k) &= \frac{1}{\exp[\pm \mu_k / T] + 1} = n_F(\epsilon_k \pm \mu_5), \quad (2)
\end{align*}

with chemical potentials $\pm \mu_5$ for right- and left-chiral particles. [For the corresponding antiparticles the chemical potentials should be taken with the opposite sign, $f_{L,R}(k) = n_F(\epsilon_k + \mu_5)$]. The left-right chirality imbalance is then characterized by the density of axial charge

\begin{align*}
q_5 &= \int \frac{d^3 k}{(2\pi)^3} \left[ f_R - f_R - f_L + f_L \right] = \frac{T^2 \mu_5}{3} \quad (3)
\end{align*}

where in the last equality we assumed that $\mu_5 \ll T$.

The electron mass $m_e$ breaks the axial symmetry and thus the axial charge relaxes to zero: $q_5 = -\Gamma_{\text{flip}} q_5 [44]$. Assuming that the chirality relaxation is the slowest equilibration process in the plasma (we give a posterior justification of the assumption of the slowness of the chirality relaxation) the thermodynamic state $[2]$ with slowly varying $\mu_5 \neq 0$ can still be defined. We can then use Boltzmann’s kinetic theory to compute $\Gamma_{\text{flip}}$ as an asymptotic series in $m_e / T \ll 1 [15]$.

We now proceed to the calculation of the chirality relaxation rate due to the $2 \leftrightarrow 2$ processes of Fig. 1 within the framework of Boltzmann’s kinetic theory. The rate of change of the axial charge due to the $2 \leftrightarrow 2$ scattering processes is given by

\begin{align*}
\dot{q}_5 &= -\int \frac{d^3 k}{(2\pi)^3} \left[ C_R - C_R - C_L + C_L \right] \quad (4)
\end{align*}

where

\begin{align*}
C_a(k) = \sum_{\{bcd\}} \int \frac{d^3 k'}{(2\pi)^3} \frac{d^3 p}{(2\pi)^3} \frac{d^3 p'}{(2\pi)^3} \left| \mathcal{M}_{ab}^{cd}(kp \rightarrow k'p') \right|^2 \frac{\delta^4(k + p - k' - p')}{16\epsilon_k \epsilon_{k'} \epsilon_p \epsilon_{p'}} \times \left[ f_a(k) f_a(p) (1 \pm f_a(k')) (1 \pm f_d(p')) - (1 \pm f_a(k)) (1 \pm f_a(p)) f_c(k') f_d(p') \right], \quad (5)
\end{align*}

is the Boltzmann’s collision integral. In Eq. 5, $k = (k^0, \mathbf{k})$ is the 4-momentum, with $k^0 = \epsilon_k = |\mathbf{k}|$ (the hard particles with $k \gg T$ can be treated effectively as massless). The delta-function takes into account the energy-momentum conservation in scattering. The subscripts $a, b, c, d$ run through the set of particle species $R, L, \bar{R}, \bar{L}, \gamma$; $f_a(k)$ is the distribution function for the particle of type $a$ and in the expression $\pm f_a(k)$ the sign depends on the statistics of the particle $a$ (plus for a boson and minus for a fermion). The amplitudes $\mathcal{M}_{ab}^{cd}$ are found by applying Feynman’s rules to the diagrams shown in Fig. 1.
Expanding the thermal Fermi-Dirac distribution functions in the collision integral on the right-hand side of Eq. (11) to the linear order in \( \mu_5 \) and using equation (3) we find that the chirality imbalance decays exponentially with the relaxation rate given by

\[
\Gamma_{\text{flip}} = \frac{3\pi}{T^3} \int \frac{d^3k}{(2\pi)^3} \frac{d^3p}{(2\pi)^3} \frac{d^3q}{(2\pi)^3} \left\{ n_F(k)n_F(p)[1 + n_B(k')][1 + n_B(p')][|M_{\text{annih}}|^2 + \right.
\]
\[\left. + n_F(k)n_B(p)[1 + n_B(k')][1 - n_F(p')][|M_{\text{Compt}}|^2] \right\} \delta(\epsilon_k + \epsilon_p - \epsilon_{k'} - \epsilon_{p'}),
\]

where \( k' = k - q, p' = p + q, \) and \( n_B(p) = 1/\exp(\epsilon_p/T) - 1 \) is the Bose-Einstein distribution function.

Since we treat mass \( m_e \) as a perturbation, we expand the matrix element of the Compton process as a perturbative series in \( m_e \) and keep only the leading term

\[
|M^{(1)}|^2 = \frac{8m_e^2e^4\epsilon_k\epsilon_p(1 - \cos \theta_{kp})}{(q^2)^2},
\]

This matrix element contains a non-integrable singularity at \( q = 0 \) which needs to be regularised by the environmental effects. To that end, we perform a partial resummation of the perturbative expansion in \( \alpha \) to take into account the thermal self-energy corrections to the dispersion relations of quasiparticles:

\[
|M^{(1),\text{therm}}|^2 = \frac{8m_e^2e^4\epsilon_k\epsilon_p(1 - \cos \theta_{kp})}{(|q - \Sigma|^2)^2},
\]

where \( \Sigma^\mu \) is the 4-vector coming from the self-energy correction to the propagator of the intermediate particle, see Eq. (A.3) in [43]. It helps to regularize the infrared divergence at \( q \sim \sqrt{\alpha T} \) scale.

Using the explicit expressions for the electron self-energy in the HTL approximation (see [43], Sec. B), we find that the chirality flipping rate

\[
\Gamma_{\text{flip}} = C \times \frac{\alpha m_e^2}{T},
\]

where the constant \( C \approx 0.24 \) (see [43], Sec. A).

Next we briefly discuss other processes that contribute to the chirality flipping rate in the same order of perturbation theory as the Compton process. One such processes is shown in Fig. 2. Its contribution to the chirality flipping rate can be estimated in a way similar to the \( \leftrightarrow 2 \) case [see Eq. (6)]

\[
\Gamma_{\text{flip}}^{1\leftrightarrow 2} \propto \frac{1}{T^3} \int d^3k d^3p d^3q n_F(\epsilon_k)[1 + n_B(\epsilon_p)][1 - n_F(\epsilon_q)]
\]
\[\times \frac{|M_{k\rightarrow pq}|^2}{\epsilon_k\epsilon_q\epsilon_p} \delta^{(3)}(k - q - p)\delta(\epsilon_k - \epsilon_q - \epsilon_p),
\]

where the matrix element reads as

\[
|M_{k\rightarrow pq}|^2 = 2e^2m_e^2 \frac{k \cdot p}{k^2}.
\]

In vacuum, \( \epsilon_k = |k| \) and the process is only allowed for strictly collinear momenta of participating particles. Due to this kinematical constraint the process has an extremely unstable phase volume which even an infinitesimal deformation of the dispersion curves of the particles wipes out completely. At the same time, the singularity of the matrix element (11) at \( k = 0 \) leads to a non-integrable divergence inside the available phase volume resulting in an uncertainty of 0/0 type. The resolution of this uncertainty is subtle and lies outside the scope of the present work. In [43], Sec. C we estimate the parametric dependence of the chirality flipping rate due to \( 1 \leftrightarrow 2 \) processes.

For details, we refer the interested reader to our companion paper [39] where we calculate all leading-order contributions to chirality flipping rate within the framework of linear response theory. The full leading order result for the chirality flipping rate has the form given in Eq. (11) with the coefficient \( C \) which is a logarithmically-varying function of \( \alpha \). For \( \alpha = 1/137 \) we find

\[
C \approx 1.17
\]

Thus, we find that the actual chirality flipping rate [43] is three orders of magnitude as high as the previously used naive estimate \( \Gamma_{\text{flip}}^{\text{naive}} \) (see, e.g., [21]).

**Chirality flip across cosmic times.** Our result enables us to compute the electron-mass induced the chirality flipping rate in the early Universe at temperatures
The corresponding estimate for the reaction rate is given by
\[ \Gamma_{\text{flip}} = \frac{\alpha}{\pi} G_F T^5 \left( \frac{m_e}{3T} \right)^2. \]

Unlike the QED case, there is no zero mass singularities because of the massive intermediate vector bosons. There is also the contribution to the chirality flipping rate due to the Higgs (inverse) decay (h ↔ e_L e_L^R)
\[ \Gamma_{\text{flip,H}} = \frac{3\sqrt{2}}{\pi^3} G_F T m_e^2 \left( \frac{\pi m_H}{2T} \right)^{5/2} e^{-m_H/T}, \]
where \( m_H \) is the Higgs boson mass.

These results are summarized in Fig. 3 that demonstrates that at temperatures \( T \lesssim 80 \text{ TeV} \) \( \Gamma_{\text{flip}} \) always exceeds the Hubble expansion rate; that the slowest \( \Gamma_{\text{flip}}(T) \) occurs at \( T \approx 100 \text{ GeV} \) and that below 100 GeV the ratio \( \Gamma_{\text{flip}}(T)/H(T) \gg 1 \).

**Conclusion and outlook.** We have shown that the chirality flipping processes for electrons in QED plasma with \( T \gg m_e \) occur much faster than one would naively expect: it is proportional to the fine structure constant \( \alpha \), rather than \( \propto \alpha^2 \) (the latter dependence holds, for example, for chirality-preserving scatterings). We used Boltzmann’s collision integral to evaluate the contribution of the leading-order 2 ↔ 2 scattering processes (Fig. 3). As \( m_e/T \rightarrow 0 \), the matrix elements for these processes exhibit the infrared singularity. In order to obtain a meaningful result one has to proceed beyond tree-level analysis and invoke a partial resummation of the perturbation theory series. In plasma such a resummation results in the singularity being regularized not by the mass \( m_e \) but by the thermal mass of the electron \( m_{\text{th}} = \frac{4}{3} T \). Our result in particular means that the chirality flipping rate is \( O(10^3) \) higher than was previously believed.

Chiral anomaly provides a coupling between the magnetic field and the axial current of electrons via the chiral magnetic effect \( \kappa = 3 \). Such a coupling has, in particular, been shown to lead to a special form of “inverse cascade” (transfer of the magnetic energy from smaller to larger scales) even in the absence of turbulence. The inverse cascade is a remarkable example of macroscopic manifestation of a microscopic quantum effect. This mechanism was, in particular, shown to increase the resilience of macroscopic magnetic fields against dissipative processes. Chirality flipping suppresses the chiral magnetic effect therefore it may switch off the inverse cascade before it completes the redistribution of energy between the electromagnetic modes. The present study shows that the accurate description of time scales associated with such counteracting mechanisms in a plasma requires a good microscopic understanding of the underlying quantum processes. Chirality flipping is not the only such mechanism. Recent microscopic simulations hint that the anomaly-induced rate of redistribution of energy between the electromagnetic modes may significantly exceed its classical estimate presumably due to quantum effects arising at short length scales. These findings call for further revision of the MHD of axially-charged plasmas based on a first-principles approach along the lines of the present study.

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[44] Apart from chirality flips, conservation of the axial charge is also violated due to the chiral anomaly [53, 54]. The effect of the anomaly can be rigorously distinguished from chirality flipping collisions due to the existence of a global charge whose conservation is respected by the former however is violated by the latter [49, 55, 56]. Here we focus on the contribution of genuine chirality flipping processes only.

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SUPPLEMENTAL MATERIAL

A. COLLISION INTEGRAL INVOLVING 2 ↔ 2 PROCESSES

In this section we calculate the collision integrals corresponding to the processes of the Compton scattering and electron-positron annihilation with the flip of chirality. Let us consider these processes separately.

1. The Compton scattering. The matrix element of the t-channel process \( e_L(k) + \gamma(p) \rightarrow \gamma(k') + e_R(p') \) shown in Fig. 1(a) reads as

\[
\mathcal{M}_C = \frac{1}{i} (-ie)^2 \bar{u}_\nu(p') P_R \gamma^\nu i \hat{S}(q) \gamma^\mu P_R u_\mu(k) \varepsilon^*_\nu(k', \lambda') \varepsilon_\nu(p, \lambda),
\]

where \( P_R = (1 + \gamma_5)/2 \) is the right chiral projector and the propagator has the form

\[
\hat{S}(q) = [S_0(q) - \gamma_\mu \Sigma^\mu(q)]^{-1} = \frac{q - \Sigma + m_e 1}{(q - \Sigma)^2 - m_e^2}.
\]

The 4-vector \( \Sigma^\mu \) equals

\[
\Sigma^\mu = \left( \Sigma^0, \frac{q}{q} \left[ -\frac{m_{th}^2}{2q} + \frac{q_0}{q} \Sigma^0 \right] \right),
\]

where \( m_{th} = eT/2 \) is the electron thermal mass and the function \( \Sigma^0 \) is given by Eq. \( \text{(B.7)} \) in Sec. \( \text{B} \). Because of the chiral projectors, only the massive term in the numerator contributes to the matrix element. In the leading order in \( m_e \) we get

\[
\mathcal{M}^{(1)}_C = -m_e e^2 \bar{u}_{\nu}(p') \gamma^\nu \gamma^\mu P_R u_\mu(k) (q - \Sigma)^2 \varepsilon^*_\nu(k', \lambda') \varepsilon_\nu(p, \lambda).
\]

Taking the squared modulus of this matrix element and summing over all possible spins and polarizations (we sum over the spin projections because we included the chiral projectors directly in the matrix element), we get

\[
|\mathcal{M}^{(1)}_C|^2 = \frac{8m_e^2 e^4 (k \cdot p')}{(q - \Sigma)^2}.
\]

2. The annihilation process. Let us now consider the process of annihilation \( e_L(k) + \bar{e}_R(p) \rightarrow \gamma(k') + \gamma(p') \) shown in Fig. 1(b). It is worth noting that the incoming positron is the antiparticle to the right electron, i.e., it is left, so that the chirality is not conserved in this reaction. Following the same steps as in the case of Compton scattering, we obtain the matrix element

\[
\mathcal{M}^{(1)}_A = -m_e e^2 \bar{u}_{\nu}(p) \gamma^\nu \gamma^\mu P_R u_\mu(k) (q - \Sigma)^2 \varepsilon^*_\nu(k', \lambda') \varepsilon_\nu(p', \lambda)
\]

and its squared modulus

\[
|\mathcal{M}^{(1)}_A|^2 = \frac{8m_e^2 e^4 (k \cdot p)}{(q - \Sigma)^2}, \quad q = k - k'.
\]

This matrix element coincides with that of the Compton process \( \text{(A.4)} \) up to the terms \( \mathcal{O}(q) \) in the numerator. It is important to note that there is also the u-channel of annihilation when the outcome photons are interchanged. However, the matrix element is exactly the same with \( q = k - p' \) instead of \( q = k - k' \). Changing the variables \( k' \leftrightarrow p' \) in the collision integral we can see that the result is simply twice the result of t-channel. There is, however, the factor \( 1/2 \) in front of the collision integral which takes into account the indistinguishability of the outcome photons. In our calculation, we omit both, the u-channel and the factor of \( 1/2 \).

Taking into account the identity

\[
[1 - n_F(p)]n_B(p) = n_F(p)[1 + n_B(p)] = \frac{1}{2 \sinh p/T}, \quad (A.8)
\]

we conclude that the Compton scattering and the annihilation process make equal contributions to the collision integral.
3. Calculation of the chirality flipping rate. Substituting the expressions (A.5) and (A.7) for to the Compton scattering and annihilation processes into the expression for chirality flipping rate (6), we arrive at the following expression

$$\Gamma_{\text{flip}}^{2+2} = \frac{3m^2\epsilon^4 T}{128\pi} \int_0^\infty q \, dq \int_0^\pi d\cos\theta_{kq} \frac{1 - \cos^2 \theta_{kq}}{|(q^0 - \Sigma(0))^2 - (q - \Sigma)^2|^2} \bigg|_{q^0 = q \cos \theta_{kq}}. \quad (A.9)$$

Let us carefully consider its denominator

$$\psi(q, \cos\theta_{kq}) = \frac{(q^0 - \Sigma(0))^2 - (q - \Sigma)^2}{q^2} \bigg|_{q^0 = q \cos \theta_{kq}} = \left[ \cos\theta_{kq} - \frac{m_{th}^2}{4q^2} \left( \ln \frac{1 + \cos \theta_{kq}}{1 - \cos \theta_{kq}} - i\pi \right) \right]^2 - \left[ 1 + \frac{m_{th}^2}{2q^2} - \cos\theta_{kq} \frac{m_{th}^2}{4q^2} \left( \ln \frac{1 + \cos \theta_{kq}}{1 - \cos \theta_{kq}} - i\pi \right) \right]^2. \quad (A.10)$$

It is easy to see that it depends only on $q^2/m_{th}^2$ and satisfies

$$\psi(q, -\cos\theta_{kq}) = \psi^*(q, \cos\theta_{kq}), \quad (A.11)$$

so that $|\psi|^2$ is invariant under the reflection $\theta_{kq} \rightarrow \pi - \theta_{kq}$. Introducing the new integration variables

$$\xi = q^2/m_{th}^2, \quad y = \cos\theta_{kq}, \quad (A.12)$$

we get the expression for the chirality flipping rate in the form

$$\Gamma_{\text{flip}}^{2+2} = \frac{m^2}{T} \alpha \times \frac{3}{8} \int_0^\infty \xi \int_0^1 dy \frac{1 - y^2}{\xi^2|\psi(m_{th}\sqrt{\xi}, y)|^2}. \quad (A.13)$$

As the final step, we show that

$$\frac{1 - y^2}{\xi^2|\psi(m_{th}\sqrt{\xi}, y)|^2} = \frac{1 - y^2}{\xi^2 \left[ y - \frac{1}{4\pi} \left( \ln \frac{1+y}{1-y} - i\pi \right) \right]^2 - \left[ 1 + \frac{1}{4\pi} - \frac{1}{4\pi} \left( \ln \frac{1+y}{1-y} - i\pi \right) \right]^2} = \frac{\xi^2/(1 - y^2)}{\left( \xi + \frac{1}{4\pi} \ln \frac{1+y}{1-y} + \frac{1}{2(1+y)} \right)^2 + \frac{\pi^2}{16}} \left[ \left( \xi - \frac{1}{4\pi} \ln \frac{1+y}{1-y} + \frac{1}{2(1+y)} \right)^2 + \frac{\pi^2}{16} \right]. \quad (A.14)$$

and we end up with Eq. (9) where the constant $C$ equals to

$$C = \frac{3}{8} \int_0^1 dy \frac{\xi^2 d\xi}{1 - y^2} \int_0^\infty \left[ \left( \xi + \frac{1}{4\pi} \ln \frac{1+y}{1-y} + \frac{1}{2(1+y)} \right)^2 + \frac{\pi^2}{16} \right] \left[ \left( \xi - \frac{1}{4\pi} \ln \frac{1+y}{1-y} + \frac{1}{2(1+y)} \right)^2 + \frac{\pi^2}{16} \right] \approx 0.24. \quad (A.15)$$

B. FERMION SELF-ENERGY AND FULL PROPAGATOR IN HTL APPROXIMATION

The leading order fermion self-energy is given by the one-loop diagram shown in Fig. B.1. The corresponding analytical expression in the Matsubara formalism is given by:

$$\Sigma(i\omega_m, k) = \epsilon^2 T \sum_{\rho} \int \frac{d^3Q}{(2\pi)^3} \frac{\gamma^\mu S_0(i\omega_{m-k}, k - Q)\gamma^\nu}{(i\Omega_{\rho})^2 - Q^2}. \quad (B.1)$$
where $S_0$ is the free electron propagator, $\omega_m = (2m + 1)\pi T$ and $\Omega_p = 2\pi p T$ are the fermionic and bosonic Matsubara frequencies, respectively. The self-energy has the block-diagonal structure in the chiral basis, i.e. does not mix the left and right components as consequence of the fact that the EM interaction respects the chiral symmetry.

The leading contribution to the self-energy is captured by the HTL approximation when the external momentum is considered to be much less than the loop momentum and the former is neglected everywhere except the denominator. The final result has the following form \[57\]:

\[
\Sigma_{R,L}(i\omega_m, k) = \frac{m_{th}^2}{2} \int \frac{d\Omega_v}{4\pi} \frac{1 \pm \sigma \cdot v}{i\omega_m \pm \mu_5 - k \cdot v},
\]  \tag{B.2}

where the integration is performed over all possible directions of the unit vector $v$ and

\[
m_{th}^2 = \frac{e^2 T^2}{4} \tag{B.3}
\]

is the electron asymptotic thermal mass (meaning that, for hard momenta, the energy dispersion of the electron quasiparticle takes the usual form $\epsilon = \sqrt{k^2 + m_{th}^2}$). One of the advantages of the HTL self-energy is its gauge invariance with respect to the Lorentz covariant gauges. Spatial isotropy leads to the simple matrix structure of the self-energy

\[
\Sigma_{R,L}(i\omega_m, k) = \Sigma^0_{R,L}(i\omega_m, k) + \frac{\sigma \cdot k}{k} \Sigma^1_{R,L}(i\omega_m, k),
\]  \tag{B.4}

where

\[
\Sigma^0_{R,L}(i\omega_m, k) = \frac{1}{2} \text{tr} \Sigma_{R,L}(i\omega_m, k) = \frac{m_{th}^2}{2} \int \frac{d\Omega_v}{4\pi} \frac{1}{i\omega_m \pm \mu_5 - k \cdot v},
\]  \tag{B.5}

\[
\Sigma^1_{R,L}(i\omega_m, k) = \mp \frac{1}{2} \text{tr} \left[ (\sigma \cdot k) \Sigma_{R,L}(i\omega_m, k) \right] = -\frac{m_{th}^2}{2k} + \frac{i\omega_m \mp \mu_5}{k} \Sigma^0_{R,L}(i\omega_m, k). \tag{B.6}
\]

Thus, the fermion self-energy in HTL approximation is determined by the single scalar function $\Sigma^0_{R,L} = \Sigma^0(i\omega_m \pm \mu_5, k)$. Let us consider it as a function of the complex variable $k^0$ in a complex plane with the branch cut along the real axis from $k^0 = -k$ to $k^0 = k$. Then, the integration can be performed explicitly and we obtain:

\[
\Sigma^0(k^0, k) = \frac{m_{th}^2}{2} \int \frac{d\Omega_v}{4\pi} \frac{1}{k^0 - k \cdot v} = \frac{m_{th}^2}{4k} \ln \frac{k^0 + k}{k^0 - k}. \tag{B.7}
\]

Substituting the HTL self-energy \[B.4\], we obtain the full propagator in the following form:

\[
S_{R,L}(i\omega_m, k) = \frac{i\omega_m \pm \mu_5 - \Sigma^0_{R,L} + (\sigma \cdot k) \left[ 1 + \frac{m_{th}^2}{2k^2} - \frac{i\omega_m \pm \mu_5 \Sigma^0_{R,L}}{k^2 (1 + \frac{m_{th}^2}{2k^2} - \frac{i\omega_m \pm \mu_5 \Sigma^0_{R,L}}{k^2})^2} \right]}{(i\omega_m \pm \mu_5 - \Sigma^0_{R,L})^2 - k^2 \left[ 1 + \frac{m_{th}^2}{2k^2} - \frac{i\omega_m \pm \mu_5 \Sigma^0_{R,L}}{k^2} \right]^2}, \tag{B.8}
\]

It can be represented as a decomposition into the components with positive and negative helicity

\[
S_{R,L}(i\omega_m, k) = \sum_{\lambda = \pm} \frac{1}{\Lambda_{\lambda}(i\omega_m \pm \mu_5, k)} \frac{1 \pm \lambda \sigma \cdot k}{2}, \tag{B.9}
\]
where \( \hat{k} = k/k \),

\[
\Delta_\lambda(k^0, k) = k^0 - \Sigma^0(k^0, k) - \lambda k \left( 1 + \frac{m^2_{\text{th}}}{2k^2} \frac{k^0}{k^2} \Sigma^0(k^0, k) \right)
\]  

is the denominator whose zeros determine the quasiparticle spectrum, and the function \( \Sigma^0 \) is given by Eq. (B.7). The quasiparticle dispersion relations can be found from the requirement \( \Delta_{\pm}(k^0, k) = 0 \). This gives

\[
\epsilon_{\pm}(k) = \pm k \frac{A_\pm(k) - 1}{A_\pm(k) + 1},
\]

where \( A_+(k) = W_{-1}(z) \), \( A_-(k) = W_0(z) \), \( z = -\exp(-4k^2/m^2_{\text{th}} - 1) \), and \( W_{-1} \) and \( W_0 \) are the upper and lower branches of the Lambert W-function. Since \( W_{-1}(z) \leq -1 \) for \( z \in [-e^{-1}; 0) \) and \( W_0(z) \in [-1; 0) \) for \( z \in [-e^{-1}; 0) \), the values of \( \epsilon_{\pm} \) are always positive. They are shown in Fig. B.2.

\[\text{FIG. B.2. Energy dispersion for the electron quasiparticles: normal branch } \epsilon_+(k) \text{ (blue solid line) and plasmino branch } \epsilon_-(k) \text{ (red dashed line). The black dotted line shows the free massless electron dispersion in vacuum.}\]

It is possible to find the asymptotic expressions for the dispersion relations for small and large momenta [57]:

\[
\epsilon_{\pm}(k \ll m_{\text{th}}) \approx \frac{m_{\text{th}}}{\sqrt{2}} \pm \frac{k}{3},
\]

\[
\epsilon_{\pm}(k \gg m_{\text{th}}) \approx k + \frac{m^2_{\text{th}}}{2k} \approx \sqrt{k^2 + m^2_{\text{th}}} \quad \text{for } \epsilon_+(k \gg m_{\text{th}}) \approx k \left[ 1 + 2 \exp \left( -4 \frac{k^2}{m^2_{\text{th}}} - 1 \right) \right].
\]

\[\text{C. CHIRALITY FLIPPING RATE FROM } 1 \leftrightarrow 2 \text{ PROCESSES}\]

The contribution to the chirality flipping rate from the \( 1 \leftrightarrow 2 \) process shown in Fig. 2 can be estimated by Eq. (10). Let us take into account the thermal corrections and show that they lead to the finite answer. For further convenience, let us decompose the momenta into components along the momentum \( k \) of the incoming electron and transverse to it,

\[
p = p_\parallel \hat{k} + p_\perp, \quad q = (k - p_\parallel)\hat{k} - p_\perp,
\]

where \( \hat{k} = k/k \) and in the second expression we used the momentum conservation law. Treating the longitudinal components of all momenta to be \( \sim T \) (we will see that this is true \textit{a posteriori}), we can expand the dispersion relations as follows

\[
\epsilon_k \approx k + \frac{m^2_{\text{th}}}{2k}, \quad \epsilon_q \approx k - p_\parallel + \frac{m^2_{\text{th}} + p_\perp^2}{2(k - p_\parallel)}, \quad \epsilon_p \approx p_\parallel + \frac{m^2_{\text{th}} + p_\perp^2}{2p_\parallel}.
\]  

Here \( M_{\text{th}} = eT/2 \) and \( m_{\gamma} = eT/\sqrt{6} \) are the asymptotic thermal masses of the electron and photon, respectively [57].

The HTL effective theory predicts the modification of the dispersion relations (see Sec. [B]). However, this would immediately wipe out all the available phase space for \( 1 \leftrightarrow 2 \) processes and lead to the vanishing contribution to the chirality flipping rate. Fortunately, the higher order (beyond HTL) corrections give rise also to the finite decay width of the quasiparticles [58–60] and allow for a slight violation of the energy conservation in the collision event. The electron decay width equals to \( \gamma_e \approx e^2 T/(4\pi) \log e^{-1} \), while the photon decay width is of higher order in \( e \) and thus can be neglected. At technical level, we can incorporate this finite decay width by replacing the delta function of energies in Eq. (10) by the corresponding Lorentz contour of the width \( 2\gamma_e \).

The Lorentz function works only when its argument is less or of the order its width,

\[
|\epsilon_k - \epsilon_q - \epsilon_p| \approx \frac{m_e^2}{2p_\|} + \frac{m_{\text{th}}^2 p_\|}{2(k - p_\|)} + \frac{p_\|^2 k}{2p_\|(k - p_\|)} \lesssim 2\gamma_e \sim T^2 \log e^{-1}. \tag{C.3}
\]

This immediately gives the restrictions on the longitudinal and transverse components of the momenta

\[
k > p_\| \gtrsim \frac{m_{\text{th}}^2}{\gamma_e} \sim 2\gamma_e \sim T/\log e^{-1}, \quad p_\perp \approx m_{\text{th}}. \tag{C.4}
\]

Now, let us consider the matrix element (11). The scalar products equal to

\[
k \cdot p \approx \frac{k(m_e^2 + p_\perp^2)}{2p_\|} + \frac{p_\| m_{\text{th}}^2}{2k}, \quad k^2 \approx m_{\text{th}}^2. \tag{C.5}
\]

Taking into account constraints (C.4), we obtain that

\[
|M_{k \rightarrow pq}|^2 = O(1) \times e^2 m_e^2. \tag{C.6}
\]

Then, the chirality flipping rate can be written as follows

\[
\Gamma_{1 \leftrightarrow 2 \text{flip}} \propto \frac{e^2 m_e^2}{T^3} \int_0^k dk \int_0^k dp_\| \frac{n_F(k)[1 + n_B(p_\|)][1 - n_F(k - p_\|)]}{kp_\|(k - p_\|)}
\]

\[
\times \int p_\perp dp_\perp \delta_{2\gamma_e} \left( \frac{m_e^2}{2p_\|} + \frac{m_{\text{th}}^2 p_\|}{2k(k - p_\|)} + \frac{p_\|^2 k}{2p_\|(k - p_\|)} \right)
\]

\[
\propto \frac{e^2 m_e^2}{T^3} \int_0^k dk \int_0^k dp_\| n_F(k)[1 + n_B(p_\|)][1 - n_F(k - p_\|)] \frac{4\gamma_e}{\pi} \frac{m_e^2}{p_\|} + \frac{4\gamma_e}{\pi} \frac{m_{\text{th}}^2 p_\|}{k(k - p_\|)}. \tag{C.7}
\]

Without the arctangent, the integral over \( p_\| \) would be logarithmically divergent for small momenta because of the Bose-Einstein distribution function. However, at the scale \( p_\|,\min \approx m_e^2/\gamma_e \sim T/\log e^{-1} \) the arctangent cuts this divergence. Finally, we get the estimate

\[
\Gamma_{1 \leftrightarrow 2 \text{flip}} \approx \frac{m_e^2}{T} \times \alpha \log \log \alpha^{-1}, \tag{C.8}
\]

which is again of the first order in the electromagnetic coupling constant with a slight logarithmic enhancement. Thus, we confirm that the nearly collinear \( 1 \leftrightarrow 2 \) processes also contribute to the leading order chirality flipping rate. This contribution is systematically studied in our companion paper [58].