On the existence and stability for noninstantaneous impulsive fractional integrodifferential equation

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In this paper, by means of Banach fixed point theorem, we investigate the existence and Ulam–Hyers–Rassias stability of the noninstantaneous impulsive integrodifferential equation by means of $\psi$-Hilfer fractional derivative. In this sense, some examples are presented, in order to consolidate the results obtained.

KEYWORDS
Banach fixed point, existence, noninstantaneous impulsive integrodifferential equation, Ulam–Hyers–Rassias stability, $\psi$-Hilfer fractional derivative

1 | INTRODUCTION

The study of impulsive differential and integrodifferential equations has been the subject of several papers, in particular, equations involving fractional differential and fractional integrodifferential operators. However, there are still few works on the existence, uniqueness, and stability of solutions of fractional differential equations, in particular equations involving the Hilfer fractional derivative. The many works available usually employ the Caputo fractional derivative and the Riemann–Liouville fractional derivative. With the extension and emergence of new fractional derivatives of an even number $N$ of variables, one possible way to carry new studies of existence, uniqueness, and Ulam–Hyers and Ulam–Hyers–Rassias stabilities, is to use more general fractional derivatives.

The first ideas about Ulam–Hyers stability began in 1941 with Ulam and Hyers, from a response by Hyers to a problem proposed by Ulam, stating the result that is now known as the Ulam–Hyers stability theorem. In studying stability problems, we are trying to answer the following question: when can we say that the solutions of an inequality are close to an exact solution of the corresponding equation? Until recently, this type of question had been raised only for equations involving integer-order differential operators.

With the expansion of fractional calculus and the increasing number of researchers working with fractional derivatives and integrals, the studies about the existence, uniqueness, attractivity, and stability of solutions of fractional differential equations have also grown in number.

We can highlight here the investigation by Wang et al on a new class of switched systems of differential equations with nonlocal and impulsive initial conditions, using the Caputo derivative, by means of fixed point methods. On the other hand, Yang et al, using the Caputo fractional derivative, extended some results on the Hausdorff orbital depen-
dence of solutions of nonlinear differential non-instantaneous equations. We must also cite the work by Wang et al. in which the authors investigated the generalized Ulam–Hyers–Rassias and Ulam–Hyers–Rassias stabilities of a new class of noninstantaneous fractional differential equation solutions.

Recently, Sousa and Oliveira introduced the so-called $\psi$-Hilfer fractional derivative, of major importance to fractional calculus as it contains, as particular cases, a wide class of other fractional derivatives, which consequently, will have the same properties of the more general fractional derivative. In this sense, there are already some works on the existence, uniqueness, and Ulam–Hyers type stability for some classes of equations involving the $\psi$-Hilfer fractional derivative.22-25

In this new work, we present some results on the existence, uniqueness, and stability of the solutions for a noninstantaneous impulsive fractional integrodifferential equation in order to consolidate a previous work11 and to strengthen the theory of stability within the unified mathematical analysis fractional calculus.26

Consider the fractional integrodifferential equation

$$
\begin{aligned}
H_D^{\alpha, \beta}_{\eta, \gamma} x(t) &= f \left( t, x(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{W}_\psi^{\alpha}(t, s, x(s)) \, ds \right), \quad t \in (s_i, t_{i+1}), \ i = 0, 1, \ldots, m \\
x(t) &= g_i \left( t, x(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{M}_\psi^{\alpha}(t, s, x(s)) \, ds \right), \quad t \in (t_i, t_{i+1}), i = 1, \ldots, m
\end{aligned}
$$

(1)

where $H_D^{\alpha, \beta}_{\eta, \gamma}(\cdot)$ is the $\psi$-Hilfer fractional derivative, whose parameters satisfy $0 < \alpha \leq 1, 0 \leq \beta \leq 1$.11

To simplify the notation, we introduce $\mathcal{W}_\psi^{\alpha}(t, s, x(s)) := N_\psi^{\alpha}(t, s) K(s, x(s))$ and $\mathcal{M}_\psi^{\alpha}(t, s, x(s)) := N_\psi^{\alpha}(t, s) \mathcal{E}(s, x(s))$, where $N_\psi^{\alpha}(t, s) = \psi(t) - \psi(s)^{\gamma - 1}$ and $0 = t_0 = s_0 < t_1 \leq s_1 \leq t_2 < \cdots < t_m \leq s_m < t_{m+1} = T$ are fixed numbers. Also, $f : [0, T] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous; $K, \mathcal{E} : [0, T] \times \mathbb{R} \to \mathbb{R}$ are continuous, and $g_i : [t_i, s_i] \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ is continuous for all $i = 1, 2, \ldots, m$, which is called not instantaneous impulse.

When we investigate the existence, uniqueness and Ulam–Hyers type stability of the solution of a particular integrodifferential equation, we expect that the results we will obtain will provide contributions to the area. On the other hand, as it is well known, when we consider noninteger order derivatives, in some applications, we usually obtain a better description of the phenomena under study. Then, the problem of choosing the fractional operator arises, since the definitions are numerous. Thus, the main motivation for this work is to present a new class of noninstantaneous impulsive fractional integrodifferential equations, by means of the $\psi$-Hilfer fractional derivative and to investigate the existence and Ulam–Hyers–Rassias type stability of the solutions of Equation (1), making use of Banach’s fixed point theorem. In this sense, this paper provides new results, which are valid for all possible particular cases of the $\psi$-Hilfer fractional derivative, which is an advantage of this fractional operator, as the properties of the more general operator are preserved in the particular cases.

This paper is organized as follows: in section 2, we present some spaces that we need for the development of the paper. We also present a version of the concept of Ulam–Hyers–Rassias type stability for the $\psi$-Hilfer fractional derivative. In section 3, the main results of the paper are presented: the investigation on the existence and Ulam–Hyers–Rassias stability of the fractional integrodifferential equation. In section 4, we present two examples in order to illustrate the results. Concluding remarks close the paper.

## 2 PRELIMINARIES

Let $J = (0, T), J' = (0, T)$ and $C(J, \mathbb{R})$ be the space of all continuous functions from $J$ into $\mathbb{R}$ and $n$-times absolutely continuous set, given by

$$AC^n(J, \mathbb{R}) = \left\{ x : J \to \mathbb{R}, x^{(n-1)} \in AC(J, \mathbb{R}) \right\}.$$ 

The weighted space of functions on $J'$ is

$$C_{\gamma, \psi}(J, \mathbb{R}) = \left\{ x : J' \to \mathbb{R}, (\psi(t) - \psi(a))^{\gamma} x(t) \in C(J, \mathbb{R}) \right\}, \quad 0 \leq \gamma < 1.$$ 

The weighted space of functions on $J'$ is

$$C_{\gamma, \psi}^{n}(J, \mathbb{R}) = \left\{ x : J' \to \mathbb{R}, x(t) \in C^{n-1}(J, \mathbb{R}), x^{(n)}(t) \in C_{\gamma, \psi}(J, \mathbb{R}) \right\}, \quad 0 \leq \gamma < 1.$$ 

The weighted space is

$$C_{\gamma, \psi}^{a, \beta}(J, \mathbb{R}) = \left\{ x \in C_{\gamma, \psi}(J, \mathbb{R}), H_D^{a, \beta}_{\eta, \gamma} x \in C_{\gamma, \psi}(J, \mathbb{R}) \right\}, \quad \gamma = a + \beta(1 - a).$$
We also recall the piecewise weighted function space

\[ \mathcal{PC}_{\alpha, \psi}(J, \mathbb{R}) = \left\{ (\psi(t) - \psi(a))^{-\alpha} x(t) \in C_{\alpha, \psi}((t_k, t_{k+1}], \mathbb{R}), \ k \in [0, m]\right\} \]

and there exist \( x(t^-_k) \) and \( x(t^+_k) \), \( k = 1, \ldots, m \) with \( x(t^-_k) = x(t^+_k) \).

Meanwhile, we set

\[ \mathcal{PC}^1_{\alpha, \psi}(J, \mathbb{R}) = \left\{ x \in \mathcal{PC}_{\alpha, \psi}(J, \mathbb{R}), \ x' \in \mathcal{PC}_{\alpha, \psi}(J, \mathbb{R}) \right\} \]

and

\[ \mathcal{PC}^{n, \beta}_{\alpha, \psi}(J, \mathbb{R}) = \left\{ x \in \mathcal{PC}_{\alpha, \psi}(J, \mathbb{R}), \ H^\alpha_0\mathcal{D}^\beta_{\alpha, \psi} x \in \mathcal{PC}_{\alpha, \psi}(J, \mathbb{R}) \right\}. \]

The function \( x \in \mathcal{PC}_{\alpha, \psi}(J, \mathbb{R}) \) is called a classical solution of the fractional impulsive Cauchy problem

\[
\begin{align*}
H^\alpha_0\mathcal{D}^\beta_{\alpha, \psi} x(t) &= f(t, x(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{W}^\alpha_{\psi}(t, s, x(s)) ds), \quad t \in (s_i, t_{i+1}], i = 0, 1, \ldots, m \\
x(t) &= g_i(t, x(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{M}^\alpha_{\psi}(t, s, x(s)) ds), \quad t \in (t_i, s_i], i = 1, \ldots, m \\
H^\alpha_0\mathcal{D}^\beta_{\alpha, \psi} x(0) &= x_0,
\end{align*}
\]

if \( x \) satisfies \( H^\alpha_0\mathcal{D}^\beta_{\alpha, \psi} x(0) = x_0 \) and

\[
x(t) = g_i \left( t, x(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{M}^\alpha_{\psi}(t, s, x(s)) ds \right), \quad t \in (t_i, s_i], \ i = 0, 1, \ldots, m
\]

and

\[
\begin{align*}
x(t) &= \Psi^\alpha_{\psi}(t, 0) x_0 + \frac{1}{\Gamma(\alpha)} \int_0^t N^\alpha_{\psi}(t, s) f \left( s, x(s), \frac{1}{\Gamma(\alpha)} \int_0^s \mathcal{W}^\alpha_{\psi}(s, \tau, x(\tau)) d\tau \right) ds, \quad t \in [0, t_1] \\
x(t) &= \frac{1}{\Gamma(\alpha)} \int_0^t N^\alpha_{\psi}(t, s) f \left( s, x(s), \frac{1}{\Gamma(\alpha)} \int_0^s \mathcal{W}^\alpha_{\psi}(s, \tau, x(\tau)) d\tau \right) ds + g_i \left( s_i, x(s_i), \frac{1}{\Gamma(\alpha)} \int_0^{s_i} \mathcal{M}^\alpha_{\psi}(s_i, t, x(s)) ds \right), \quad t \in (s_i, t_{i+1}], i = 0, 1, \ldots, m,
\end{align*}
\]

with \( \Psi^\alpha_{\psi}(t, 0) = \frac{(\psi(t) - \psi(0))^{\alpha-1}}{\Gamma(\alpha)} \) and \( N^\alpha_{\psi}(t, s) = \psi^\alpha (t) (\psi(t) - \psi(s))^{\alpha-1} \).

Then, we introduce the concept of Ulam–Hyers–Rassias stability for Equation (1), motivated by the concepts of stability.\(^{27-29}\)

Let \( \delta \geq 0 \) and \( \varphi \in \mathcal{PC}_{\alpha, \psi}(J, \mathbb{R}^+) \) be a nondecreasing function. Consider

\[
\left| H^\alpha_0\mathcal{D}^\beta_{\alpha, \psi} y(t) - f \left( t, y(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{W}^\alpha_{\psi}(t, s, y(s)) ds \right) \right| \leq \varphi(t), \ t \in (s_i, t_{i+1}], i = 0, 1, \ldots, m, \tag{3}
\]

and

\[
\left| y(t) - g_i \left( t, y(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{M}^\alpha_{\psi}(t, s, y(s)) ds \right) \right| \leq \varepsilon, \ t \in (t_i, s_i], i = 1, \ldots, m. \tag{4}
\]
Definition 1. The Equation 1 is generalized Ulam–Hyers–Rassias stable with respect to \((\varphi, \delta)\) if there exists \(C_{f, g, \varphi} > 0\) such that for each solution \(y \in PC_{\gamma, \varphi}^1(J, \mathbb{R})\) of the Equation 1 we have

\[
|y(t) - x(t)| \leq C_{f, g, \varphi}(\varphi(t) + \delta), \quad t \in J.
\] (5)

Obviously, if \(y \in PC_{\gamma, \varphi}^1(J, \mathbb{R})\) satisfies the inequality Equation (3) then \(y\) is a solution the following fractional integral inequality

\[
\left| y(t) - g_i \left( t, y(t), \frac{1}{\Gamma(a)} \int_0^t M_{\varphi}^a(t, s, y(s))ds \right) \right| \leq \delta, \quad t \in (t_i, s_i], \quad i = 0, 1, \ldots, m
\] (6)

\[
\left| y(t) - \Psi^\gamma(t, 0)x_0 - \int_0^t N_{\varphi}^a(t, s)f \left( s, y(s), \frac{1}{\Gamma(a)} \int_0^s \mathcal{W}_{\varphi}^a(s, \tau, y(\tau))d\tau \right)ds \right| \leq I_{0+}^a \varphi
\] (7)

\[
y(t) - g_i \left( s_i, y(s_i), \frac{1}{\Gamma(a)} \int_{s_i}^s M_{\varphi}^a(t, s, y(s))ds \right) - \frac{1}{\Gamma(a)} \int_{s_i}^t N_{\varphi}^a(t, s)f \left( s, y(s), \frac{1}{\Gamma(a)} \int_0^s \mathcal{W}_{\varphi}^a(s, \tau, y(\tau))d\tau \right)ds \leq \delta + t_{s_i}^a \varphi(t)
\] (8)

For a nonempty set \(X\), a function \(d : X \times X \rightarrow (0, \infty)\) is called a metric on \(X\) if and only if, \(d\) satisfies:

1. \(d(x, y) = 0\) if and only if \(x = y\);
2. \(d(x, y) = d(y, x)\) for all \(x, y \in X\);
3. \(d(x, z) \leq d(x, y) + d(y, z)\) for all \(x, y, z \in X\).

As the Banach fixed point theorem is important in the study of existence, uniqueness, and in the stability of solution of fractional differential equations, it is here after recalled.

Theorem 1. Let \((X, d)\) be a generalized complete metric space. Assume that \(T : X \rightarrow X\) is a strictly contractive with constant \(L < 1\). If there exists a nonnegative integer \(k\) such that \(d(T^k x, T^k x) < \infty\) for any \(x \in X\), then the followings are true:

1. The sequence \(\{T^n x\}\) converges to a fixed point \(x^*\) of \(T\);
2. \(x^*\) is the unique fixed point of \(T\) in \(X^* = \{y \in X|d(T^n x, y) < \infty\}\);
3. If \(y \in X^*\), then \(d(y, x^*) \leq \frac{1}{1-L} d(Ty, y)\).

3 | MAIN RESULTS

In this section, we investigate the existence and Ulam–Hyers–Rassias stability of the solution of Equation 1 by using Banach’s fixed point theorem.

For the development of this paper, we first introduce some hypotheses

\(H_1\). \(f \in C(J \times \mathbb{R} \times \mathbb{R}, \mathbb{R})\)

\(H_2\). There exists a positive constant \(L_f\) such that

\[
|f(t, u_1, v_1) - f(t, u_2, v_2)| \leq L_f(|u_1 - u_2| + |v_1 - v_2|)
\]

for each \(t \in J\) and for all \(u_1, u_2, v_1, v_2 \in \mathbb{R}\).

\(H_3\). \(g_i \in C([t_i, s_i] \times \mathbb{R} \times \mathbb{R}, \mathbb{R})\) and \(L_{g_i} > 0\) constants, \(i = 1, 2, \ldots, m\) such that

\[
|g_i(t, u_1, v_1) - g_i(t, u_2, v_2)| \leq L_{g_i}(|u_1 - u_2| + |v_1 - v_2|)
\]

for each \(t \in (t_i, s_i]\) and for all \(u_1, u_2, v_1, v_2 \in \mathbb{R}\).
\[ k \in C([t_{i}, s_{j}] \times \mathbb{R} \times \mathbb{R}) \text{ and } \overline{K} > 0 \text{ constant} \], such that

\[ |k(t, u_{1}) - k(t, u_{2})| \leq \overline{K}|u_{1} - u_{2}| \]

for each \( t \in J \) and for all \( u_{1}, u_{2} \in \mathbb{R} \).

\[ \ell \in C(J \times \mathbb{R}, \mathbb{R}) \text{ and } L > 0 \text{ constant}, \text{ such that} \]

\[ |\ell(t, u_{1}) - \ell(t, u_{2})| \leq L|u_{1} - u_{2}| \]

for each \( t \in J \) and for all \( u_{1}, u_{2} \in \mathbb{R} \).

Let \( \varphi \in C(J, \mathbb{R}) \) be a nondecreasing function. There exists \( C_{\varphi} > 0 \) such that

\[
P_{0}^{\alpha} \varphi(t) := \frac{1}{\Gamma(\alpha)} \int_{0}^{t} N^{\alpha}_{\varphi}(t, s) \varphi(s) ds \leq C_{\varphi} \varphi(t) \]

for each \( t \in J \).

In what follows, we will investigate the existence and generalization of Ulam–Hyers–Rassias of the solutions of Equation 1 by means of Banach fixed point theorem (Theorem 1).

**Theorem 2.** Assume that the hypotheses \( H_1, H_2, H_3, H_4, H_5, H_6 \) are satisfied and a function \( y \in PC^{1}_{\tau, \psi}(J, \mathbb{R}) \), satisfies Equation 3. Then, there exists a unique solution \( y_{0} : J \to \mathbb{R} \) such that

\[
y_{0}(t) = \Psi^{r}(t, 0)x(0) + \frac{1}{\Gamma(\alpha)} \int_{0}^{t} N^{\alpha}_{\psi}(t, s)f \left( s, y_{0}(s), \frac{1}{\Gamma(\alpha)} \int_{0}^{s} W^{\alpha}_{\psi}(s, \tau, y_{0}(\tau)) d\tau \right) ds \]

for \( t \in [0, t_{1}] \), and

\[
y_{0}(t) = g_{i} \left( t, y_{0}(t), \frac{1}{\Gamma(\alpha)} \int_{0}^{t} M^{\alpha}_{\psi}(t, s, y_{0}(s)) ds \right) \]

for \( t \in (t_{i}, s_{i}] \) and \( i = 1, \ldots, m \), and

\[
y_{0}(t) = g_{i} \left( s_{i}, y_{0}(s_{i}), \frac{1}{\Gamma(\alpha)} \int_{0}^{s_{i}} M^{\alpha}_{\psi}(t, s, y_{0}(s)) ds \right) + \frac{1}{\Gamma(\alpha)} \int_{0}^{s_{i}} N^{\alpha}_{\psi}(t, s) f \left( s, y_{0}(s), \frac{1}{\Gamma(\alpha)} \int_{0}^{s} W^{\alpha}_{\psi}(s, \tau, y_{0}(\tau)) d\tau \right) ds \]

for \( t \in (s_{i}, t_{i+1}] \), \( i = 1, \ldots, m \) and

\[
|y(t) - y_{0}(t)| \leq \frac{(1 + C_{\psi})(\varphi(t) + \delta)}{1 - \Phi}, \quad t \in J
\]

provided that

\[
\Phi := \max_{i=1, \ldots, m} \left\{ \left( LC_{\psi} + L \frac{(\psi(T) - \psi(0))^{2}}{\Gamma(\alpha + 1)} + 1 \right) \frac{\overline{K}}{} + \left( \frac{\overline{K} \psi(T) - \psi(0))^{2}}{} \frac{C_{\psi}^{2} + \overline{K}C_{\psi}^{2} + C_{\varphi}}{L_{f}} \right) \right\} < 1.
\]

**Proof.** The proof of the theorem will be carried out in three cases. First, we consider the space of piecewise weighted functions

\[ X := \{ g : J \to \mathbb{R} / g \in PC_{\tau, \psi}(J, \mathbb{R}) \} \]

and the generalized metric on \( X \), defined by

\[
d(g, h) = \inf \{ C_{1}, C_{2} \in [0, \infty] / |g(t) - h(t)| \leq (C_{1} + C_{2})(\varphi(t) + \delta), \quad t \in J \}
\]

where

\[ C_{1} \in [C \in [0, \infty] / |g(t) - h(t)| \leq \psi(\varphi(t), \tau, y_{0}(\tau)) d\tau \}] \]

\[ t \in (s_{i}, t_{i+1}], \quad i = 0, 1, \ldots, m \]
and
\[ C_2 \in \{ C \in [0, \infty) \mid |g(t) - h(t)| \leq C\delta, \ t \in (t_i, s_i], \ i = 1, 2, \ldots, m \}. \]

Note that, the \((X, d)\) is a complete generalized metric space.

Also, we introduce the following operator \(\Omega : X \to X\) given by
\[
(\Omega x)(t) = \Psi^x(t, 0) x(0) + \frac{1}{\Gamma(a)} \int_0^t N^a_\psi(t, s) f \left( s, x(s), \frac{1}{\Gamma(a)} \int_0^s W^a_\psi(s, \tau, x(\tau)) d\tau \right) ds, \text{ for } t \in [0, t_1]
\]
and
\[
(\Omega x)(t) = g_t \left( t, x(t), \frac{1}{\Gamma(a)} \int_0^t M^a_\psi(t,s) x(s) ds \right), \text{ for } t \in (t_i, s_i] \text{ and } i = 0, 1, \ldots, m;
\]
and
\[
(\Omega x)(t) = g_t \left( s_i, x(s_i), \frac{1}{\Gamma(a)} \int_0^{s_i} M^a_\psi(t,s) x(s) ds + \frac{1}{\Gamma(a)} \int_{s_i}^t N^a_\psi(t,s) f \left( s, x(s), \frac{1}{\Gamma(a)} \int_0^s W^a_\psi(s, \tau, x(\tau)) d\tau \right) ds \right)
\]
for \(t \in (s_i, t_{i+1}]\) and \(i = 0, 1, \ldots, m\), for all \(x \in X\) and \(t \in [0, T]\). Note that, the operator \(\Omega\), as defined above, is a well-defined operator according to \(H_1, H_2, H_3, H_4, H_5\).

The definition of metric \(d\) over the space \(X\) for any \(g, h \in X\), allows to find \(C_1, C_2 \in [0, \infty)\) such that
\[
|g(t) - h(t)| \leq \begin{cases} 
C_1 \varphi(t) & t \in (s_i, t_{i+1}], \ i = 0, 1, \ldots, m, \\
C_2 \delta & t \in (t_0, s_i], \ i = 1, \ldots, m.
\end{cases}
\] (15)

By means of the definition of \(\Omega\) in Equation 14, \(H_2, H_3, H_4, H_5\), and Equation 15 we have the following cases:

Case 1. \(t \in [0, t_1]\).

We have
\[
|(\Omega g)(t) - (\Omega h)(t)| = \left| \frac{1}{\Gamma(a)} \int_0^t N^a_\psi(t, s) f \left( s, g(s), \frac{1}{\Gamma(a)} \int_0^s W^a_\psi(s, \tau, g(\tau)) d\tau \right) - f \left( s, h(s), \frac{1}{\Gamma(a)} \int_0^s W^a_\psi(s, \tau, h(\tau)) d\tau \right) ds \right|
\]
so,
\[
|(\Omega g)(t) - (\Omega h)(t)| \leq \frac{1}{\Gamma(a)} \int_0^t N^a_\psi(t,s)L_f \left\{ |g(s) - h(s)| + \frac{1}{\Gamma(a)} \int_0^s N^a_\psi(s, \tau) |k(\tau, g(\tau) - k(\tau, h(\tau))| d\tau \right\} ds
\]
\[
\leq \frac{1}{\Gamma(a)} \int_0^t N^a_\psi(t,s)L_f \left\{ C_1 \varphi(s) + \frac{1}{\Gamma(a)} \int_0^s N^a_\psi(s, \tau)K g(\tau) - h(\tau))| d\tau \right\} ds
\]
\[
\leq \frac{1}{\Gamma(a)} \int_0^t N^a_\psi(t,s)L_f \left\{ C_1 \varphi(s) + C_1 K \frac{C_1 \varphi(s)}{\Gamma(a)} \int_0^s N^a_\psi(s, \tau) \varphi(\tau d\tau \right\} ds
\]
\[
\leq \frac{1}{\Gamma(a)} \int_0^t N^a_\psi(t,s)L_f \left\{ C_1 \varphi(s) + C_1 K C_1 \varphi(s) \right\} ds
\]
or,

\[
|(\Omega g)(t) - (\Omega h)(t)| \leq \frac{L_f C_1 (1 + \overline{K} C_\varphi)}{\Gamma(\alpha)} \int_0^t N_\psi^a(t, s) \varphi(s) ds \leq L_f C_1 (1 + \overline{K} C_\varphi) C_\varphi \varphi(t).
\]

Therefore, we have

\[
|(\Omega g)(t) - (\Omega h)(t)| \leq L_f C_1 (1 + \overline{K} C_\varphi) C_\varphi \varphi(t).
\]

**Case 2.** \( t \in (t_i, s_i] \).

It holds

\[
|(\Omega g)(t) - (\Omega h)(t)| = \left| g_i \left( t, g(t), \frac{1}{\Gamma(\alpha)} \int_0^t M_\psi^a(t, s, g(s)) ds \right) - g_i \left( t, h(t), \frac{1}{\Gamma(\alpha)} \int_0^t M_\psi^a(t, s, h(s)) ds \right) \right|
\]

so,

\[
|(\Omega g)(t) - (\Omega h)(t)| \leq L_i \left[ \left| g(t) - h(t) \right| + \frac{1}{\Gamma(\alpha)} \int_0^t N_\psi^a(t, s) \varphi(s) \varphi(s) - \varphi(s, h(s)) ds \right]
\]

\[
\leq L_i \left[ C_2 \delta + \frac{1}{\Gamma(\alpha)} \int_0^t N_\psi^a(t, s) L(C_1 + C_2)(\varphi(s) + \delta) ds \right]
\]

\[
= L_i \left[ C_2 \delta + L(C_1 + C_2) \left( \frac{1}{\Gamma(\alpha)} \int_0^t N_\psi^a(t, s) \varphi(s) ds + \frac{1}{\Gamma(\alpha)} \int_0^t N_\psi^a(t, s) \varphi(s) \right) \right]
\]

\[
\leq L_i \left[ C_2 \delta + L(C_1 + C_2) \left( C_\varphi \varphi(t) + \delta \frac{(\psi(T) - \psi(0))^a}{\Gamma(\alpha + 1)} \right) \right].
\]

Therefore, we have

\[
|(\Omega g)(t) - (\Omega h)(t)| \leq L_i \left[ C_2 \delta + L(C_1 + C_2) \left( C_\varphi \varphi(t) + \delta \frac{(\psi(T) - \psi(0))^a}{\Gamma(\alpha + 1)} \right) \right].
\]

**Case 3.** For \( t \in (s_i, t_{i+1}] \), we have

\[
|(\Omega g)(t) - (\Omega h)(t)| = \left| g_i \left( s_i, g(s_i), \frac{1}{\Gamma(\alpha)} \int_0^{s_i} W_\psi^a(t, s, g(s)) ds \right) \right|
\]

\[
+ \frac{1}{\Gamma(\alpha)} \int_{s_i}^t N_\psi^a(t, s) f \left( s, g(s), \frac{1}{\Gamma(\alpha)} \int_0^{s} W_\psi^a(s, \tau, g(\tau)) d\tau \right) ds \right|
\]

\[
- \left| g_i \left( s_i, h(s_i), \frac{1}{\Gamma(\alpha)} \int_0^{s_i} M_\psi^a(t, s, h(s)) ds \right) \right|
\]

\[
- \frac{1}{\Gamma(\alpha)} \int_{s_i}^t N_\psi^a(t, s) f \left( s, h(s), \frac{1}{\Gamma(\alpha)} \int_0^{s} W_\psi^a(s, \tau, h(\tau)) d\tau \right) ds \right|.
\]
Therefore,

\[
(\Omega g)(t) - (\Omega h)(t) \leq L g \left\{ [g(s_i) - h(s_i)] + \frac{1}{\Gamma(\alpha)} \int_0^{s_i} N^\alpha(t, s) [\mathcal{C}(s, g(s)) - \mathcal{C}(s, h(s))] \, ds \right\} \\
+ \frac{1}{\Gamma(\alpha)} \int_0^t N^\alpha(t, s) L_f [g(s) - h(s)] \, ds \leq L g \left\{ C_2 \delta + \frac{1}{\Gamma(\alpha)} \int_0^{s_i} N^\alpha(t, s) L g(s) \, ds \right\} \\
+ \frac{1}{\Gamma(\alpha)} \int_0^t N^\alpha(t, s) L_f \left\{ C_1 \varphi(s) + \frac{1}{\Gamma(\alpha)} \int_0^s N^\alpha(s, \tau) \mathcal{K}(\tau - h(\tau)) \, d\tau \right\} \, ds \\
\leq L g \left\{ C_2 \delta + \frac{L}{\Gamma(\alpha)} \int_0^{s_i} N^\alpha(t, s) (C_1 + C_2)(\varphi(s) + \delta) \, ds \right\} \\
+ \frac{L_f}{\Gamma(\alpha)} \int_0^t N^\alpha(t, s) L_f \left\{ C_1 \varphi(s) + \frac{\mathcal{K}}{\Gamma(\alpha)} \int_0^s N^\alpha(s, \tau) (C_1 + C_2)(\varphi(\tau) + \delta) \, d\tau \right\} \, ds \\
\leq L g \left\{ C_2 \delta + L(C_1 + C_2) \left( C_{\varphi} \varphi(t) + \delta \frac{\mathcal{C}(T) - \mathcal{C}(0))^n}{\Gamma(\alpha + 1)} \right) \right\} \\
+ L_f \left\{ C_1 C_{\varphi} \varphi(t) + \mathcal{K}(C_1 + C_2) \left( C_{\varphi} C_{\varphi} \varphi(t) + \delta \frac{\mathcal{C}(T) - \mathcal{C}(0))^{2n}}{\Gamma(\alpha + 1)} \right) \right\} \\
\leq L g \left\{ C_2 \delta + L(C_1 + C_2) \left( C_{\varphi} \varphi(t) + \delta \frac{\mathcal{C}(T) - \mathcal{C}(0))^n}{\Gamma(\alpha + 1)} \right) \right\} \\
+ L_f \left\{ C_1 C_{\varphi} \varphi(t) + \mathcal{K}(C_1 + C_2) \left( C_{\varphi}^2 \varphi(t) + \delta \frac{\mathcal{C}(T) - \mathcal{C}(0))^{2n}}{\Gamma(\alpha + 1)} \right) \right\} \\
\leq \left[ \left( L C_{\varphi} + \frac{\mathcal{C}(T) - \mathcal{C}(0))^{n+1}}{\Gamma(\alpha + 1)} \right) + \frac{\mathcal{K}}{\Gamma(\alpha + 1)} \int_0^t N^\alpha(t, s) L_f \right] (C_1 + C_2)(\varphi(t) + \delta).
\]

Then, we have

\[
|(\Omega g)(t) - (\Omega h)(t)| \leq \Phi(C_1 + C_2)(\varphi(t) + \delta), \quad t \in J
\]

with

\[
\Phi := \left( L C_{\varphi} + \frac{\mathcal{C}(T) - \mathcal{C}(0))^{n+1}}{\Gamma(\alpha + 1)} \right) + \frac{\mathcal{K}}{\Gamma(\alpha + 1)} \int_0^t N^\alpha(t, s) L_f < 1.
\]

Therefore, from Equation 16, we conclude that

\[
d(\Omega g, \Omega h) \leq \Phi d(g, h)
\]

for all \( g, h \in X \), provided the condition given by Equation 11.

Now, consider \( g_0 \in X \). From the piecewise continuous property of \( g_0 \) and \( \Omega g_0 \), exists a constant \( 0 < H_1 < \infty \), so that

\[
|(\Omega g_0)(t) - g_0(t)| = \int_0^t N^\alpha(t, s) f \left( s, g_0(s), \frac{1}{\Gamma(\alpha)} \int_0^s N^\alpha(s, \tau) \mathcal{N}(\tau, x(\tau)) \, d\tau \right) \, ds - y_0(t) \leq \frac{H_1}{\Gamma(\alpha + 1)} \int_0^t N^\alpha(t, s) L_f \, ds.
\]
for \( t \in [0, t_1] \). On the other hand, also for \( 0 < \tilde{H}_2 < \infty \) and \( 0 < \tilde{H}_3 < \infty \),
\[
|\Omega_0(t) - g_0(t)| = \left| g_0(t) \left( t, \frac{1}{\Gamma(\alpha)} \int_0^t M^\alpha_{\psi}(t, s, g_0(s))ds \right) - g_0(t) \right| \leq \tilde{H}_2 (\varphi(t) + \delta)
\]
for any \( t \in (t_i, s_i] \) and \( i = 1, 2, \ldots, m \), and
\[
|\Omega_0(t) - g_0(t)| = \left| g_0 \left( s_i, \frac{1}{\Gamma(\alpha)} \int_0^{s_i} M^\alpha_{\psi}(t, s, g_0(s))ds \right) + \frac{1}{\Gamma(\alpha)} \int_{s_i}^t N^\alpha_{\psi}(t, s)\psi(s)ds \right| \leq \tilde{H}_3 (\varphi(t) + \delta)
\]
for any \( t \in (s_i, t_{i+1}] \) and \( i = 1, 2, \ldots, m \), since \( f, g, g_0, \frac{1}{\Gamma(\alpha)} \int_0^{s_i} M^\alpha_{\psi}(t, s, g_0(s))ds \) and \( \frac{1}{\Gamma(\alpha)} \int_0^{s_i} N^\alpha_{\psi}(t, s)\psi(s)ds \) are bounded on \( J \) and \( \varphi(\cdot) + \delta > 0 \).

Then, Equation 13 implies that
\[
d(\Omega_0, g_0) < \infty.
\]

Then, there exists a continuous function \( y_0 : J \rightarrow \mathbb{R} \) such that \( \Omega^n g_0 \rightarrow y_0 \) in \( (X, d) \) as \( n \rightarrow \infty \) and \( \Omega y_0 = y_0 \) that is, \( y_0 \) satisfies Equation 9 for every \( t \in J \) by means of Banach fixed point theorem (Theorem 1).

Now, the next step, we will prove that
\[
X = \{ g \in X / d(g_0, g) < \infty \}.
\]

Let \( g \in X \). Note that \( g \) and \( g_0 \) are bounded on \( J \) and \( \min_{t \in J} (\varphi(t) + \delta) > 0 \), then there exists a constant \( 0 < C_g < \infty \) such that
\[
|g_0(t) - g(t)| \leq C_g (\varphi(t) + \delta)
\]
for any \( t \in J \). Hence, we have \( d(g_0, g) < \infty \) for all \( g \in X \) that is
\[
X = \{ g \in X / d(g_0, g) < \infty \}.
\]

Finally, we conclude the first part, ie, \( y_0 \) is the unique continuous function satisfying Equation 9.

On the other hand, by means of Equation 6 and of the hypothesis \( H_5 \), we have
\[
d(y, \Omega y) = \left| y(t) - \Psi^\alpha(t, 0)x_0 - \frac{1}{\Gamma(\alpha)} \int_0^t N^\alpha_{\psi}(t, s)f \left( s, y(s), \frac{1}{\Gamma(\alpha)} \int_0^\alpha N^\alpha_{\psi}(s, \tau, y(\tau))d\tau \right) ds \right| \leq \frac{1}{\Gamma(\alpha)} \int_0^t N^\alpha_{\psi}(t, s)\varphi(s)ds \leq C_\varphi \varphi(t) \leq 1 + C_\varphi.
\]

Multiplying both sides of Equation 17 by \( 1 - \Phi \), we have
\[
d(y, y_0) \leq \frac{d(\Omega y, y)}{1 - \Phi} \leq \frac{1 + C_\varphi}{1 - \Phi}, \quad t \in J.
\]

By this last expression, we conclude that Equation 10 is true.

\[\square\]

4 | EXAMPLES

In this section, we will present only two examples, considered as particular cases of fractional integro-differential equations, specifically one of them is the Riemann-Liouville sense (fractional) and another relative to the integer order
derivative. We recall that, our result, involving the $\psi$-Hilfer fractional derivative, is the general case because both Riemann-Liouville and integer order are recovered as particular cases.

First, we consider

$$
\begin{aligned}
H^{-\alpha,\beta,\psi}_{s,0^+} x(t) &= f \left( t, x(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{W}_{\psi}^\alpha(t, s, x(s))ds \right), \quad t \in [0, 1] \\
x(t) &= g \left( t, x(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{M}_{\psi}^\alpha(t, s, x(s))ds \right), \quad t \in (1, 2)
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
H^{-\alpha,\beta,\psi}_{s,0^+} y(t) - f \left( t, y(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{W}_{\psi}^\alpha(t, s, y(s))ds \right) &\leq \mathbb{E}_\alpha(t), \quad t \in [0, 1] \\
\left| y(t) - g \left( t, y(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{M}_{\psi}^\alpha(t, s, y(s))ds \right) \right| &\leq 1, \quad t \in (1, 2)
\end{aligned}
\tag{19}
$$

with

$$
\begin{aligned}
f \left( t, x(t), \frac{1}{\Gamma(\alpha)} \int_0^t \mathcal{W}_{\psi}^\alpha(t, s, x(s))ds \right) &= \frac{1}{5 + \psi(t)} \left[ |x(t)| + \frac{1}{\Gamma(\alpha)} \int_0^t \psi'(s)(\psi(t) - \psi(s))^{\alpha-1} \frac{|x(s)|}{10 + \psi(s)} ds \right], \quad t \in [0, 1]
\end{aligned}
$$

and

$$
\begin{aligned}
x(t) &= \frac{1}{(5 + \psi(t))(1 + |x(t)|)} \left[ |x(t)| \int_0^t \psi'(s)(\psi(t) - \psi(s))^{\alpha-1} \frac{|x(s)|}{15 + \psi(s)} ds \right], \quad t \in (1, 2)
\end{aligned}
\tag{20}
$$

where $\mathbb{E}_\alpha(\cdot) = \sum_{k=0}^\infty \frac{t^k}{\Gamma(k+1)}$ is the one-parameter Mittag-Leffler function with $0 < \alpha \leq 1$. Also, relatively to Equation 19 we simply exchange $x$ for $y$.

### 4.1 Riemann-Liouville fractional derivative

In this case we consider Equation 18 and Equation 19 with $J = [0, 2]; 0 = t_0 = s_0 < t_1 = 1 < s_1 = 2$. Taking $\psi(t) = t, \alpha = 1/2$, the limit $\beta \to 0$ and the following nonlinear functions

$$
f \left( t, x(t), \frac{1}{\Gamma(1/2)} \int_0^t \mathcal{W}_{\psi}^{1/2}(t, s, x(s))ds \right) = \frac{1}{5 + t} \left[ |x(t)| + \frac{1}{\Gamma(1/2)} \int_0^t (t-s)^{-1/2} \frac{|x(s)|}{10 + s} ds \right]
$$

with $k(t, x(t)) = \frac{|x(t)|}{10 + t}, \quad k = 1/10, \quad L_f = 1/5, \quad t \in [0, 1]$ and

$$
g \left( t, x(t), \frac{1}{\Gamma(1/2)} \int_0^t \mathcal{M}_{\psi}^{1/2}(t, s, x(s))ds \right) = \frac{1}{(5 + t)(1 + |x(t)|)} \left[ |x(t)| + \frac{1}{\Gamma(1/2)} \int_0^t (t-s)^{-1/2} \frac{|x(s)|}{15 + s} ds \right]
$$

with $e(t, x(t)) = \frac{|x(t)|}{15 + t}, \quad L = 1/15, \quad L_e = 1/5, \quad t \in (1, 2)$ we get a particular case of Equation 18 and Equation 19, involving Riemann-Liouville fractional derivative.
We put \( \varphi(t) = E_n(t) \) and \( \delta = 1 \). Then, for \( C_\varphi = 1 \) we have

\[
\frac{1}{\Gamma(1/2)} \int_0^t N_1^J(t, s)E_{1/2}(s)ds \leq E_{1/2}(t).
\]

By a simple but tedious calculation, we get \( \Phi = \frac{1}{8} < 1 \) and by Theorem 2 there exists a unique solution \( y_0 : [0, 2] \to \mathbb{R} \) such that

\[
y_0(t) = \frac{t^{-1/2}}{\Gamma(1/2)}x(0) + \frac{1}{5 + s} \left[ y_0(x) + \frac{1}{(1 - + t)} \int_0^s \frac{y_0(\tau)}{10 + \tau}d\tau \right] ds, \quad t \in [0, 1],
\]

and

\[
y_0(t) = \frac{1}{(5 + t)(1 + |y_0(t)|)} \left[ |y_0(t)| + \frac{1}{5 + s} \int_0^t (t-s)^{-1/2} \frac{y_0(s)}{15 + s}ds \right], \quad t \in (1, 2).
\]

and

\[
|y(t) - y_0(t)| \leq \frac{(1 + C_\varphi)(\varphi(t) + \delta)}{1 - \Phi} = \frac{54}{19}E_{1/2}(t + 1), \quad t \in [0, 2].
\]

### 4.2 Integer derivative

As in the precedent case, consider Equation 18 and Equation 19 with \( J = [0, 2]; 0 = t_0 = s_0 < t_1 = 1 < s_1 = 2 \). Taking \( \psi(t) = t, \alpha = 1, \beta = 1/2 \) and the following nonlinear functions

\[
f \left( t, x(t), \int_0^t W_1^J(t, s, x(s))ds \right) = \frac{1}{5 + t} \left[ x(t)| + \int_0^t N_1^J(t, s) \frac{|x(s)|}{10 + s}ds \right]
\]

with \( k(t, x(t)) = \frac{|x(t)|}{10 + t}, \quad k = 1/10, \quad L_f = 1/5, \quad t \in [0, 1] \) and

\[
g_t \left( t, x(t), \int_0^t M_1^J(t, s, x(s))ds \right) = \frac{1}{(5 + t)(1 + |x(t)|)} \left[ x(t)| + \int_0^t N_1^J(t, s) \frac{|x(s)|}{15 + s}ds \right]
\]

with \( \ell(t, x(t)) = \frac{|x(t)|}{15 + t}, \quad L = 1/15, \quad L_\ell = 1/5, \quad t \in (1, 2) \), we get a particular case of Equation 18 and Equation 19, involving integer derivative. We put \( \varphi(t) = E_1(t) = e^t \) and \( \delta = 1 \). Then, for \( C_\varphi = 1 \) we have

\[
\int_0^t N_1^J(t, s)E_1(s)ds \leq E_1(t) = e^t.
\]

Also, here, after a simple and tedious calculation, we can show \( \Phi = \frac{14}{25} < 1 \) and by Theorem 2, there exists a unique solution \( y_0 : [0, 2] \to \mathbb{R} \) such that

\[
y_0(t) = x(0) + \frac{1}{5 + s} \left[ y_0(x) + \int_0^s \frac{|y_0(\tau)|}{1 - + \tau}d\tau \right] ds, \quad t \in [0, 1]
\]

and

\[
y_0(t) = \frac{1}{(5 + t)(1 + |y_0(t)|)} \left[ y_0(t)| + \int_0^t \frac{|y_0(s)|}{15 + s}ds \right], \quad t \in (1, 2)
\]
and

\[ |y(t) - y_0(t)| \leq \frac{(1 + C_\psi)(\varphi(t) + \delta)}{1 - \Phi} = \frac{50}{11}(e^\delta + 1), \ t \in [0, 2]. \]

5 CONCLUDING REMARKS

We can conclude that the main results of this paper have been successfully achieved, through Banach’s fixed point theorem. We investigated the existence and Ulam–Hyers–Rassias stability of an impulsive integrodifferential equation written with the \( \psi \)-Hilfer fractional derivative. We hope that this paper will prove to be important to mathematics, especially for researchers working with problems involving impulsive fractional differential equations.

A possible continuation of this research might be finding an answer to this question: would it be possible to obtain results similar to the ones we have presented in space \( L_{p,\alpha}([0, 1], \mathbb{R}) \) with norm \( ||(\cdot)||_{p,\alpha} \) using the \( \psi \)-Hilfer fractional derivative, Brouwer’s fixed point, and/or Schauder’s fixed point theorems.

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