A DEA-based multi-response fusion model in the context of Taguchi method

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Abstract. Taguchi methods have been widely used to optimize machining parameters of a manufacturing process so that the product performances or responses are close to the desired targets and not sensitive to external noises. For single-response problems, the optimal machining parameters can be obtained based on Taguchi signal-to-noise ratio or response surface models. For multi-response problems, a multi-objective optimization approach or a fusion model is needed before obtaining the optimal parameters; and the latter is preferred due to its simplicity. This paper proposes a fusion model to combine multiple responses into a composite response variable. The proposed model first transforms the response data obtained from Taguchi experiments into the data with smaller-the-better or larger-the-better quality characteristics, and then combines the transformed data into the composite response variable in the way similar to Data Envelopment Analysis (DEA). Once the fusion model is built, the problem reduces into a single-response problem, which can solved using existing approaches. The proposed model can be easily implemented using a spreadsheet program; and one real-world example is included to illustrate its appropriateness.

1. Introduction

Taguchi method aims to obtain an optimal combination of design parameter levels so that the performances (or responses) of a product are close to the desired targets and not sensitive to external noises [1-2]. Due to its excellent idea, Taguchi method has been widely recognized and used in all phases of product life cycle [3].

For single-response problems, the optimal design parameters can be obtained based on Taguchi signal-to-noise ratio (SNR). Some drawbacks resulting from the SNR have been identified (e.g., see [4-5]) and various improvements have been developed in two main directions: modifications of the SNR (e.g., [6]) and development of response surface models (e.g., [7-8]). As a result, there are a number of methods or models can be used to solve the single-response problem.

However, many practical optimization problems in the context of Taguchi method involve multiple response variables [9-13]. In this case, it is not easy to obtain the optimal design variables; and a multi-objective optimization approach or a performance fusion model is needed before obtaining the optimal parameters. The fusion approach is preferred due to its simplicity. Through a fusion model, multiple response variables are combined into a single composite response variable so that a multi-response problem reduces into a single-response problem.

A typical method to build a fusion model is the generalized mean model, which includes the weighted sum models (e.g., see [14-15] and the literature cited therein). Almeida et al. develop a quality loss function model to fuse the response variables into a single loss function [16]. This
approach needs to specify some cost parameters, and this may be a hard task. The idea of Data Envelopment Analysis (DEA) also has been used to build a fusion model. DEA uses a weighted sum model to combine smaller-the-better (SB) characteristic variables into an input and to combine larger-the-better (LB) characteristic variables into an output. An efficiency measure is defined as the ratio of the output and input (e.g., [17-18]). However, a problem with the DEA model is that it cannot directly include nominal-the-best (NB) characteristic variables in the model. To address this issue, and a new transformation method can be introduced to transform a NB characteristic variable into a SB characteristic variable so that the transformed variable can be viewed as an input.

The use of the weighted sum model involves data transformation and determination of the weight parameters. The weight parameters can be are mathematically determined from the experiment data. For example, Emovon et al. determine the weights under the assumption that the weight is proportional to the sample variance [19]; Jiang et al. determine the weights under the assumption that the weight of a variable (or criterion) is proportional to the coefficient of variation (cv) of its sample [20]. However, for some data transformation methods, the sample variance or cv changes after the transformation. This raises another problem with the DEA model: how the compatibility between the data transformation method and the method to determine the weight is ensured. To address this issue, and a proportional transformation is proposed to normalize the SB or LB response data. The proportional transformation has two important features. The first is that the transformed response variables have the same average and hence the transformation is called the equal-magnitude transformation. This feature ensures that the weight is not distorted by the “magnitudes” of the response variables. The second is that the cv values of a response variable before and after the transformation are the same. Th is feature ensures the above-mentioned compatibility. Based on these two transformations, a DEA-based fusion model is proposed in this paper. The proposed model can be easily implemented using a spreadsheet program; and one real-world example is included to illustrate its appropriateness.

The paper is organized as follows. The proposed model is presented in Section 2 and illustrated in Section 3. The paper is concluded in Section 4.

2. Proposed fusion model

Consider a multi-response problem with \( m \) design variables \( (X_1, X_2, \ldots, X_m) \). Let \( (Y_{Si}, i = 1, 2, \ldots, n_S) \) denote the SB responses, \( (Y_{Lj}, j = 1, 2, \ldots, n_L) \) denote the LB responses, and \( (Y_{Nk}, k = 1, 2, \ldots, n_N) \) denote the NB responses; and \( x_l \) and \( y_{ij} \) denote the measurement values of \( X_l \) and \( Y_{ij} \), respectively.

Taguchi method is based on orthogonal array experiments; and the array \( L_N \) is selected by \( m \) and the number of levels of \( X_l \), where \( N \) is the number of experiments. For each experiment, there are \( M \) trials; and the total number of trials is \( N \times M \).

The process to construct the proposed fusion model includes four steps, and specific details are outlined as follows.

2.1. Step 1: transform NB responses into SB responses

For a NB response variable \( Y^* \), let \( T \) denote its target value. The following transformation makes it into a SB variable:

\[
Y = (1 - Y^*/T)^2. \tag{1}
\]

2.2. Step 2: normalize the experiment data by proportional transformation

Without loss of generality, suppose that \( n \) performance measures, \( (P_1, P_2, \ldots, P_n) \), are SB. Let \( p_i \) denote the measurement value of \( P_i \) for a trial. The overall performance of the trial can be evaluated by

\[
P = \sum_{i=1}^{n} w_i q_i, 0 \leq w_i \leq 1, \sum_{i=1}^{n} w_i = 1. \tag{2}
\]
Where \( w_i \)'s are the weight parameters, and can be determined under the assumption that the weight is proportional to \( cv \); 
\[ q_i = p_i/\mu_i, \]
where \( \mu_i \) is the sample mean of \( P_i \). Clearly, \( q_i \) is dimensionless and can be viewed as a realization of \( Q_i \), given by 
\[ Q_i = P_i/\mu_i. \]  
Equation (3) is called the proportional transformation, which meets the following relations:
\[ E(Q_i) = E(P_i)/\mu_i = 1, \quad \text{var}(Q_i) = \text{var}(P_i)/\mu_i^2, \quad \text{cv}(Q_i) = \text{cv}(P_i). \]
Equation (4) implies that the transformation is of equal-magnitude and constant-cv. These properties are particularly desired due to the above-mentioned reasons.

2.3. Step 3: aggregation of responses with the same characteristic
The proportionally transformed SB variables are aggregated into a composite SB indicator using the weighted sum model and denoted as \( Z_S \). Similarly, the proportionally transformed LB variables are aggregated into a composite LB indicator, which is denoted as \( Z_L \).

The NB variables are first transformed by Equation (1), and then the transformed NB variables are normalized by Equation (3). Finally, the normalized NB variables are aggregated into a composite NB indicator using the weighted sum model, which is denoted as \( Z_N \).

2.4. Step 4: construction of fusion model
Let \( (Q_i, 0 \leq i \leq I) \) denote the normalized output parameter values and \( (I_j, 1 \leq j \leq J) \) denote the normalized input parameter values. DEA defines the relative efficiency of a decision-making unit as [17]
\[ E = \sum_{i=1}^I u_i Q_i / \sum_{j=1}^J v_j I_j. \]  
Where \( u_i \)'s are the weights of outputs and \( v_j \)'s are the weights of inputs.

For the multi-response optimization problem considered in this paper, the input is \( Z_S + Z_N \) and the output is \( Z_L \). Thus, the composite response variable (i.e., fusion model) can be defined as
\[ Z = (Z_S + Z_N)/Z_L. \]
This definition is slightly different from Equation (5) so that \( Z \) is smaller-the-better. Once the fusion model is obtained, the optimal combination of design parameter levels can be determined using a proper Taguchi method.

3. Illustration

3.1. Problem and earlier results
The problem deals with optimizing the welding parameters of flux-cored arc welding of stainless-steel cladding process [16]. Input variables are
- wire feed rate \( (X_1) \),
- voltage \( (X_2) \),
- welding speed \( (X_3) \), and
- the distance from the contact tip to the work piece (N or \( X_4 \)).

The parameter levels of input variables are presented in Table 1.

| Parameter | Unit | Levels         |
|-----------|------|----------------|
|           |      | 1   2   3   4   5 |
| \( X_1 \) | m/min | 5.5 7 8.5 10 11.5 |
| \( X_2 \) | volt | 24.5 27 29.5 32 34.5 |
| \( X_3 \) | cm/m | 20 30 40 50 60 |
| \( X_4 \) | mm  | 10 15 20 25 30 |

The Table 1. Levels of input variables.
Responses are
• bead width \( (Y_1, \text{LB}) \);
• penetration \( (Y_2, \text{SB}) \);
• reinforcement \( (Y_3, \text{LB}) \);
• dilution percentage \( (Y_4, \text{SB}), \) which is equal to (penetration area of the weld)/(total area of the weld);
• percentage of productivity \( (Y_5, \text{LB}) \); and
• electric current \( (Y_6, \text{SB}), \) which will be used to calculate the energy costs of the process.

The experiment design is shown in the first five columns of Table 2; and the results of the experiments are shown in the last six columns.

**Table 2.** Experiment design and results.

| X_1 | X_2 | X_3 | X_4 | Y_1 | Y_2 | Y_3 | Y_4 | Y_5 | Y_6 |
|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| m/min | volt | cm/m | mm | mm | mm | mm | % | % | A |
| 1 | 7.0 | 27.0 | 30.0 | 15.0 | 11.2 | 1.3748 | 2.6278 | 26.44 | 89.74 | 172 |
| 2 | 10.0 | 27.0 | 30.0 | 15.0 | 13.0 | 1.6609 | 3.1158 | 25.82 | 89.71 | 214 |
| 3 | 7.0 | 32.0 | 30.0 | 15.0 | 12.7 | 1.6891 | 2.4963 | 31.49 | 89.14 | 181 |
| 4 | 10.0 | 32.0 | 30.0 | 15.0 | 15.0 | 1.9768 | 2.7782 | 31.25 | 89.47 | 233 |
| 5 | 7.0 | 27.0 | 50.0 | 15.0 | 9.2 | 1.6468 | 2.1651 | 36.22 | 91.58 | 173 |
| 6 | 10.0 | 27.0 | 50.0 | 15.0 | 10.0 | 1.9361 | 2.6666 | 33.69 | 90.70 | 205 |
| 7 | 7.0 | 32.0 | 50.0 | 15.0 | 9.7 | 1.5379 | 2.0649 | 37.12 | 87.43 | 176 |
| 8 | 10.0 | 32.0 | 50.0 | 15.0 | 11.5 | 1.9768 | 2.7782 | 31.25 | 89.47 | 233 |
| 9 | 7.0 | 27.0 | 50.0 | 15.0 | 9.2 | 1.6468 | 2.1651 | 36.22 | 91.58 | 173 |
| 10 | 10.0 | 27.0 | 50.0 | 15.0 | 10.0 | 1.9361 | 2.6666 | 33.69 | 90.70 | 205 |
| 11 | 7.0 | 32.0 | 50.0 | 15.0 | 11.3 | 1.3215 | 2.8479 | 23.71 | 90.60 | 152 |
| 12 | 10.0 | 32.0 | 50.0 | 15.0 | 13.3 | 1.1005 | 3.1793 | 21.96 | 89.81 | 179 |
| 13 | 7.0 | 27.0 | 50.0 | 15.0 | 8.0 | 1.1114 | 2.5543 | 24.96 | 94.03 | 143 |
| 14 | 10.0 | 27.0 | 50.0 | 15.0 | 8.6 | 1.2254 | 2.7967 | 23.31 | 90.17 | 177 |
| 15 | 7.0 | 32.0 | 50.0 | 15.0 | 8.5 | 1.3697 | 2.3628 | 28.77 | 93.52 | 151 |
| 16 | 10.0 | 32.0 | 50.0 | 15.0 | 10.8 | 1.6370 | 2.5983 | 30.19 | 91.74 | 183 |
| 17 | 5.5 | 29.5 | 40.0 | 20.0 | 9.1 | 1.3822 | 2.2068 | 31.56 | 92.62 | 141 |
| 18 | 11.5 | 29.5 | 40.0 | 20.0 | 12.2 | 2.1393 | 3.0557 | 30.95 | 89.52 | 213 |
| 19 | 8.5 | 24.5 | 40.0 | 20.0 | 9.4 | 1.2045 | 3.0263 | 22.84 | 90.41 | 175 |
| 20 | 8.5 | 34.5 | 40.0 | 20.0 | 11.7 | 1.8644 | 2.4578 | 35.58 | 90.04 | 188 |
| 21 | 8.5 | 29.5 | 20.0 | 20.0 | 14.9 | 0.9476 | 3.4536 | 18.58 | 90.27 | 187 |
| 22 | 8.5 | 29.5 | 60.0 | 20.0 | 8.5 | 1.4328 | 2.2498 | 35.78 | 93.08 | 172 |
| 23 | 8.5 | 29.5 | 40.0 | 10.0 | 11.7 | 2.1784 | 2.6103 | 40.44 | 88.15 | 223 |
| 24 | 8.5 | 29.5 | 40.0 | 30.0 | 9.2 | 1.2825 | 2.8912 | 24.16 | 92.05 | 152 |
| 25 | 8.5 | 29.5 | 40.0 | 20.0 | 10.8 | 1.7082 | 2.5960 | 31.05 | 93.04 | 180 |
| 26 | 8.5 | 29.5 | 40.0 | 20.0 | 10.9 | 1.7229 | 2.5923 | 31.67 | 91.91 | 181 |
| 27 | 8.5 | 29.5 | 40.0 | 20.0 | 10.7 | 1.6230 | 2.6549 | 30.88 | 92.51 | 179 |
| 28 | 8.5 | 29.5 | 40.0 | 20.0 | 10.6 | 1.8014 | 2.4950 | 32.83 | 91.98 | 176 |
| 29 | 8.5 | 29.5 | 40.0 | 20.0 | 10.6 | 1.4854 | 2.6208 | 29.99 | 92.15 | 175 |
| 30 | 8.5 | 29.5 | 40.0 | 20.0 | 10.6 | 1.4897 | 2.6119 | 31.09 | 92.40 | 172 |
| 31 | 8.5 | 29.5 | 40.0 | 20.0 | 10.6 | 1.5041 | 2.5574 | 31.02 | 92.58 | 174 |

Almeida et al. develop a total quality loss function to fuse the response variables into a composite response variable, which is SB [16]. Based on a principal component analysis, the composite response variable is related to the welding parameters and two second order polynomial response surface models with 15 terms (including constant term) are fitted. From the fitted models yield the optimal parameter combinations, which are shown in the second and third columns of Table 3. As seen, the optimal combination obtained from Model 1 is close to the one of Experiment 21; and the optimal
combination obtained from Model 2 is close to the one of Experiment 10. Clearly, the two solutions are not quite consistent, particularly for $X_3$, which is the most important parameter, as shown later. These imply that the response surface models are somehow complex and the results are not quite reliable.

### 3.2. Fusion model

Using the fusion model proposed in the above section, we reanalyze the data. Table 4 shows the model parameters. A large cv value implies that the corresponding response variable is sensitive to the change of controllable variables and hence is a key factor. Clearly, the most important response variable is $Y_2$, the second important response variable is $Y_4$, and the least important response variable is $Y_5$.

### Table 4. Model parameters.

| Responses | $Y_1$ | $Y_2$ | $Y_3$ | $Y_4$ | $Y_5$ | $Y_6$ |
|-----------|------|------|------|------|------|------|
| Characteristic | LB | SB | LB | SB | LB | SB |
| Value of $\mu$ | 10.84 | 1.5415 | 2.6846 | 29.52 | 90.92 | 179.6 |
| Value of cv | 0.1593 | 0.2169 | 0.1303 | 0.1984 | 0.0184 | 0.1291 |
| Output weight | 0.5172 | 0.2169 | 0.1303 | 0.1984 | 0.0184 | 0.1291 |
| Input weight | 0.3984 | 0.3644 | 0.0598 | 0.0598 | 0.0598 | 0.0598 |

The values of composite response variable are computed from the resulting fusion model, and the results are shown in Figure 1. Clearly, among all the 31 experiments, Experiment 21 has the best overall performance and Experiment 10 has the second best overall performance. Their combination of parameter levels are shown in the fourth and fifth columns of Table 3. It can be noted that the combination for Experiment 21 is similar to the one of Model 1 in [16] and the combination for Experiment 10 is similar to the one of Model 2 in [16]. Therefore, Model 1 may be better than Model 2.

![Figure 1. Plot of Z.](image-url)
3.3. Optimal levels of welding parameters

Suppose that there are \( K \) experiments that correspond to Level \( j \) of \( X_i (i = 1, \ldots, 4; j = 1, \ldots, 5) \). From the values of \( Z \), we can compute the Taguchi SNR by

\[
SNR_{ij} = -4.343 \ln \left[ \frac{1}{K} \sum_{k=1}^{K} z_k^2 \right].
\]  

(7)

Where "\( z_k \)" corresponds to those values of \( Z \) from the experiments with Level \( j \) for \( X_i \). Table 5 shows the values of SNR, along with the values of \( K \). A large range of SNR implies that the response is sensitive to the corresponding input parameter and hence this input parameter is important. The last row of Table 5 shows the importance ranking. As seen, \( X_3 \) is the most important input parameter and \( X_1 \) is the least important input parameter. Since the parameter levels of \( X_3 \) associated with Models 1 and 2 of [16] are significantly different (see Table 4), the response surface models of Almeida et al. are questionable.

| Level | \( K \) | \( X_1 \) | \( X_2 \) | \( X_3 \) | \( X_4 \) |
|-------|-------|-------|-------|-------|-------|
| 1     | 1     | -0.8829 | 1.5449 | 5.2140 | -2.4137 |
| 2     | 8     | -0.2548 | 0.2509 | 1.5011 | -1.1339 |
| 3     | 13    | -0.4249 | -0.5280 | -0.5731 | -0.3946 |
| 4     | 8     | 0.0191  | -0.4616 | -1.2978 | 1.2050  |
| 5     | 1     | -0.6969 | -1.3175 | -2.0434 | 1.2147  |
| Range |       | 0.9020  | 2.8624 | 7.2575 | 3.6283  |
| Importance | 4 | 3 | 1 | 2 |

Figure 2 displays the plots of SNRs vs. levels of input parameters. It clearly shows that the optimal levels of \( X_2 \) and \( X_3 \) are Level 1. The optimal levels of \( X_1 \) and \( X_4 \) may be Level 4 and Level 5, respectively. This optimal combination is shown in the last column of Table 3.

![Figure 2. Plots of SNRs vs. levels of welding parameters.](image)

However, the optimal levels of \( X_1 \) and \( X_4 \) need to be confirmed. This is because the left-most and the right-most points in Figure 2 are less reliable since their \( K \) values are one. Therefore, we use a weighted least squared method (with \( K \) values as weights) to fit the data of \( X_1 \) and \( X_3 \) in Table 5 to the following relation:

\[
z(l) = a + bl + cl^2 + d/l, \ l > 0.
\]  

(8)
The fitted curves are shown in Figure 3. According to the fitted model, the optimal level of $X_1$ is 4.262, which corresponds to $X_1 = 10.39$. This confirms that the optimal levels of $X_1$ is really close to Level 4. For $X_4$, the SNR increases with parameter level. Therefore, the optimal level 5 is confirmed.

![Figure 3. Plots of fitted SNR curves for $X_1$ and $X_4$.](image-url)

4. Conclusions
This paper has dealt with the Taguchi experiment optimization problem with multiple responses. A multi-response fusion model has been proposed, and illustrated by a real-world example. The proposed model has the following features or advantages:

- Similar to but slightly different from DEA, the proposed fusion model includes not only SB and LB characteristics but also NB characteristics.
- The NB characteristics are transformed into SB characteristics by a binomial transformation; and an equal-magnitude and constant-cv transformation is introduced to normalize the experiment data.
- The proposed model is simple and can be easily implemented using a spreadsheet program.

The example analyzed in this paper does not involve the NB characteristics and more examples will be carried out to further verify its appropriateness and usefulness.

Acknowledgement
The research was supported by the National Natural Science Foundation of China (No. 71771029).

References
[1] Taguchi Genichi 1986 *Introduction to quality engineering: designing quality into products and processes* (New York: White Plains)
[2] R Jiang 2015 *Introduction to quality and reliability engineering* (Beijing: Science Press and Berlin Heidelberg: Springer-Verlag)
[3] Robinson, Timothy J, Connie M Borror and Raymond H Myers 2004 Robust parameter design: a review *Quality and Reliability Engineering International* 20 81-101
[4] Leon Ramon V, Anne C Shoemaker and Raghu N Kacker 1987 Performance measures independent of adjustment: an explanation and extension of Taguchi's signal-to-noise ratios *Technometrics* 29 253-265
[5] Box George, Søren Bisgaard and Conrad Fung 1988 An explanation and critique of Taguchi's contributions to quality engineering *Quality and reliability engineering international* 4 123-131
[6] Renyan Jiang and Xing Yao 2017 A response-based method for analyzing data from Taguchi experiments *2nd Int. Conf. on Reliability Systems Engineering (Beijing)* pp 1-6
[7] A C Shoemaker and K L Tsui 1991 Economical experimentation methods for robust design Technometrics 33 415-427
[8] Malgorzata Kowalczyk 2014 Application of Taguchi and ANOVA methods in selection of process parameters for surface roughness in precision turning of titanium advances in Manufacturing Science and Technology 38 21-35
[9] Jeyapaul R, P Shahabudeen and K Krishnaiah 2005 Quality management research by considering multi-response problems in the Taguchi method – a review The International Journal of Advanced Manufacturing Technology 26 1331-1337
[10] Chithirai Pon Selvan M and Mohana Sundara Raju N 2011 Assessment of process parameters in abrasive waterjet cutting of stainless steel International Journal of Advances in Engineering & Technology 1 34-40
[11] Jurkovic Z, Perinic M and Maricic S 2012 Application of modelling and optimization methods in abrasive water jet machining Journal of Trends in the Development of Machinery and Associated Technology 16 59-62
[12] R Senthil Kumar, S Gajendran and R Kesavan 2018 Estimation of optimal process parameters for abrasive water jet machining of marble using multi response techniques Materials Today: Proceedings 5 11208-11218
[13] Joel C and T Jeyapoovan 2020 Optimization of machinability parameters in abrasive water jet machining of AA7075 using Grey-Taguchi method Materials Today: Proceedings 37 737-741
[14] R Jiang 2013 A multivariate CBM model with a random and time-dependent failure threshold Reliability Engineering and System Safety 119 178-185
[15] Renyan Jiang 2020 Multidimensional performance degradation data fusion and residual life prediction for machine tools Global Reliability and Prognostics and Health Management Conf. (Shanghai) pp 1-8
[16] Fabricio Alves de Almeida, Ana Carolina Oliveira Santos, Anderson Paulo de Paiva, Guilherme Ferreira Gomes and José Henrique de Freitas Gomes 2020 Multivariate Taguchi loss function optimization based on principal components analysis and normal boundary intersection Engineering with Computers doi: 10.1007/s00366-020-01122-8 pp 1-17
[17] Charnes Abraham, William W Cooper and Eduardo Rhodes 1978 Measuring the efficiency of decision making units European journal of operational research 2 429-444
[18] R Jiang and C Huang 2016 Solving multi-criteria selection problems based on efficiency scores and weights derived from data Int. Conf. on Management and Operations Research (Beijing) pp 70-75
[19] R Jiang, W Wang and H Jiang 2018 Relative and absolute quality indices for evaluating overall performances of electric vehicles Int. Conf. on Reliability, Maintainability and Safety (Shanghai) pp 1-5
[20] Emovon I, Norman RA, Murphy AJ and Pazouki K 2015 An integrated multicriteria decision making methodology using compromise solution methods for prioritising risk of marine machinery systems Ocean Engineering 105 92-103