Source Term in Einstein’s Field Equations

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Abstract

A basic problem that confronts the standard cosmological models is the problem of Initial Singularity characterised by infinite material density, infinite temperature and infinite spacetime curvature. The inevitable existence of such a phase of the universe may be considered to be one of the major drawbacks of Einstein’s field equations. To some extent inflation models ameliorate this. In the present work we postulate that whenever matter (radiation) arises in flat spacetime, it introduces curvature and causes a repulsive interaction to develop. A modified energy momentum tensor is introduced towards this end, which invokes the temperature and entropy. This redefinition of the energy momentum distribution when applied to the early universe dynamics has the effect of producing a non-singular initial behaviour. The repulsive interaction introduces some features of an accelerating universe consistent with present observations of high redshift supernovae, etc.

1. Introduction

In this paper we postulate that whenever matter is introduced in flat spacetime, it is stimulated to respond in the following manner;

1. spacetime curvature corresponding to $G_{ij}$ (Einstein tensor) is produced and

2. spacetime responds to temperature and entropy of material distribution in such a way that it develops a peculiar kind of repulsive interaction.

Thus gravitational field equations in presence of matter could be written in the form

$$-KT_{ij} = G_{ij} + G^*_{ij}$$

$G_{ij}$ is Einstein Tensor and $G^*_{ij}$ is the tensor corresponding to repulsive interaction of spacetime.
The very presence of temperature and entropy of material distribution stimulates spacetime to develop the repulsive interaction. Thus we assume the existence of a repulsive fluid having energy tensor $S_{ij}$ such that

$$G^*{}_{ij} = -KS_{ij}$$

where $S_{ij}$ is assumed to be of the form

$$S_{ij} = f(t)\phi \theta U_i U_j - \alpha(t)g_{ij}$$

with the restriction $S^{ij}_{\;}_{;j} = 0$

$f(t)$ is called the degree of repulsive interaction to the presence of entropy and temperature of material distribution. It is dimensionless. $\phi(t)$ is the entropy density and $\theta(t)$ is temperature of the material distribution. $U_i$ is the 4-velocity vector. $C$ is the velocity of light in freespace. $\alpha(t)$ is a cosmic scalar field depending on cosmic time $t$. When $\theta = $ Zero Kelvin we assume that $\alpha(t) = 0$.

Hence the modified form of gravitational field equations take the form

$$G_{ij} = -K(T_{ij} - S_{ij}) = -KT_{ij} \text{(effective)}$$
i.e., $G_{ij} = -K[T_{ij} + (-S_{ij})]$.

$T_{ij}$ is the total energy tensor. $S_{ij}$ is the energy tensor corresponding to the repulsive interaction. $T_{ij} \text{(effective)}$ represents the effective energy tensor.

$$ \left[ T_{ij} \text{(effective)} - \frac{1}{2} g_{ij} T \text{(effective)} \right] U^i U^j \geq 0$$

implies

$$ \left[ T_{ij} - \frac{1}{2} g_{ij} T \right] U^i U^j \geq \alpha(t) + \frac{1}{2} f(t) \phi \theta C^2 \geq 0$$

Details regarding the above inequality are being carried out by us.

2. Basic Assumptions

(A1) For the sake of convenience, we restrict to study the dynamics of the universe for the case when cosmic time is non-negative. But however without loss of generality one can extend to study the nature of the universe for cosmic time $t$ less than zero. It could be analysed if one assumes the discontinuity of Hubble parameter at $t = 0$. The universe is already in the state of BEING at $t = 0$ for there is NO geodesic singularity at $t = 0$. 

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(A2) To begin with a peculiar kind of blackbody radiation field exists and only this radiation dominates throughout the radiation dominated era for $t \geq 0$, the entropy density is given by $(\frac{4}{3})a\theta^3$ throughout the radiation dominated era. Here $a$ is the radiation constant. In this model only thermal entropy is considered. During evolution the total entropy of the universe is conserved. The law $\theta R$ is constant is true for all $t \geq 0$, $\theta$ is the temperature and $R$ is the cosmic scale factor. The expression $C' = \frac{\phi R^4}{C^2} = \text{constant}$ is true for $t \geq 0$.

(A3) There exists $R(0) \neq 0$ at $t = 0$ and $R(0)$ is finite and greater than zero. $\theta(0)$ is the finite initial temperature at cosmic time $t = 0$. Thus the law $\theta R = \text{constant}$, gives $\theta Y = \theta(0)$ and $Y = \frac{R}{R(0)}$.

(A4) The form of energy tensor $T_{ij}$ is assumed to be of the form

$$T_{ij} = \left[ \rho + \frac{p}{C^2} \right] U_i U_j - \frac{p}{C^2} g_{ij}$$

for all time $t \geq 0$

In matter dominated era, this form of $T_{ij}$ still holds with $p$ becoming vanishingly small.

Robertson Walker metric is used. Signature of a spacetime is -2. For comoving co-ordinate system $U_i = (1, 0, 0, 0)$. The effective tensor $T_{ij}$ (effective) causes $G_{ij}$

$$G_{ij} = -KT_{ij}(\text{effective}) = -K(T_{ij} - S_{ij})$$

(A5) There exist relations $\rho = 3\frac{p}{C^2}$ and $\rho_e = 3\frac{p_e}{C^2}$ which are true for all $t \geq 0$ in radiation dominated era, $\rho$ is total material density. $p = \text{total pressure}$. $\rho_e$ is the effective density causing gravitation and $p_e$ is the effective pressure of material distribution.

$$\rho_e = \rho - \frac{f(t)\phi \theta}{C^2} \geq 0, \quad \frac{p_e}{C^2} = \frac{p}{C^2} - \alpha(t) \geq 0$$
In matter dominated era, \( p \approx 0, p_e \approx 0 \) and \( \alpha \approx 0 \)

(A6) The Hubble parameter at \( t = 0 \) is given by

\[
H(0) = \left( \frac{8}{9} \pi G \rho(0) \right)^{1/2}
\]

Here \( \rho(0) \) is the total density at \( t = 0 \) and it is given by \( \rho(0) = \frac{a \theta(0)^4}{C^2} \)

\( G \) is Newtonian gravitational constant ‘a’ is the radiation constant.

(A7) There exists a temperature \( \theta_T \) at which transition from radiation domination to matter domination takes place. At this temperature, \( \rho, p_e, Y, Y \dot{Y} \) and \( f(t) \) are assumed to continuous. Dot shows differentiation with respect to time.

(A8) This model becomes identical to existing Cosmological model of the early universe when \( Y = 1.6824079 \ldots \) It is the least value of \( Y \) at which our model becomes identical to the traditional cosmological model of early universe. We assume that it occurs at cosmic time \( t = 5.39 \times 10^{-44} \) second, the Planck time.

(A9) To analyse the behaviour for \( t < 0 \), we assume discontinuity of the Hubble parameter \( H(t) \), at \( t = 0 \). However \( H(0)t \) is always Non negative quantity in the solution given by equation (15) of this MS. So for negative \( t \), \( H(0) \) has to be negative. So we can give the interpretation of the contracting phase of the universe for \( t < 0 \) (indicating a cosmological blue shift). Thus there is no geodesic singularity at \( t = 0 \). This can then be interpreted (i.e. the discontinuity in \( H(0) \)) as a ‘bouncing’ (or reflection) of the universe at \( t = 0 \). \( t > 0 \), then corresponds to the expanding phase. Here we put

\[
\lim_{t \to \Theta} H(t) = -H(0) \quad \text{and} \quad \lim_{t \to \Theta} \dot{H}(t) = +H(0)
\]

3. Repulsive Interaction of Spacetime and Sij term in the field equations

Einstein field equation \( \Box \) in its classical form
connects the dialectical categories content and form. $G_{ij}$, the Einstein tensor represents the form explicitly manifested as the curvature of spacetime and $T_{ij}$ represents the material content causing the manifestation of the form. The content and form are connected by the coupling constant $K$, the relativistic gravitational constant.

Repulsive gravity can occur in unusual situation and in recent years has been invoked in connection with inflation. In general relativity both pressure and energy density contribute to the gravitational field and inflation invokes a negative pressure as implied by a positive cosmological constant (induced by a change in vacuum energy) to drive the initial expansion of the Universe (as opposed to the usual tendency to collapse). Gravitational binding energy is negative and thus reduces the effective energy of the system and can make it vanish under certain circumstances. We invoke thermal effects in gravitational field which can reduce the system energy. In this context, we note the work of references [9], [10] where it is shown that thermal finite temperature corrections to the energy momentum tensor are of the form

$$\theta^{\mu\nu} = T^{\mu\nu} - \left(\frac{2}{3}\right)\alpha\pi\left(\frac{T^2}{E^2}\right)\delta_0^\mu\delta_0^\nu T^{00}$$

At finite temperature, the Minkowski vacuum is replaced by a thermal bath and note that it has the effect of ‘diluting’ $T_{\mu\nu}$ and indeed the expression for acceleration of a particle (mass $m$) has a term of $\sim \frac{2}{3}\left(\frac{T^2}{m^2}\right)$ of opposite sign to gravitational acceleration, suggesting thermal induced repulsion. This is the result of the detailed finite temperature calculation [11].

Here we assume that the response or reaction of spacetime consists of two parts

1. Producing curvature of spacetime or Einstein tensor $G_{ij}$

2. Producing a repulsive response of spacetime (repulsive interaction) induced by the presence of entropy and temperature of material distribution.

Now gravitational field equation assumes the form
\[ G_{ij} + G^*_{ij} = -KT_{ij}, \quad (K = \frac{8\pi G}{C^2}) \]

Einstein identified \( G^*_{ij} = \wedge g_{ij} \) Here we assume \( G^*_{ij} \) to be of the form

\[ G^*_{ij} = -KS_{ij} \]

\[ S_{ij} = \frac{f(t)\phi\theta}{C^2}U_iU_j - \alpha(t)g_{ij} \]

If we put \( f(t) = 0 \) and \( \alpha(t) = \) a constant, we get back Einstein’s field equations with \( \wedge \) term.

We do have Einstein’s field equations with \( \wedge \) - term

\[ -KT_{ij} = G_{ij} + \wedge g_{ij} \]

i.e., matter \( \leftrightarrow \) (Attractive Interaction) + (Repulsive Interaction)

It shows that the concept of repulsive interaction was already there in the mind of Einstein. In this paper even though \( S_{ij} \) is a particular form, it is a bit more general than just \( \wedge g_{ij} \) term. And the result it generates even in perfect fluid approximation does not violate any known physical laws and hence may have some physical relevance in cosmology. Thus the existence of \( S_{ij} \) is physically plausible. We have the restriction \( S_{ij} = 0 \) and effective pressure, effective density and \( \alpha(t) \) we introduce are Non-Negative quantities. It does not lead to violation of energy condition.

Except horizon problem and dark matter problem, some fundamental basic issues are addressed well in a very natural way without imposing any other extra additional suppositions other than the introduction of the repulsive term \( S_{ij} \), the simplest form of which was already introduced by Einstein.

In this paper we hypothesize that introduction of matter is flat spacetime (in the early universe), apart from causing a curvature (given by Einstein tensor) would also give rise to another effect, i.e., spacetime acquires a black body like temperature due to the curvature and becomes filled with radiation with a corresponding entropy density and a negative pressure leading to repulsive action making the spacetime to expand. We choose a particular form for the stress tensor of this radiation field. (It could be thought to be a time depended generalisation of modified varying cosmical constant term). Such an additional term in the field equations can be justified in several physical context, e.g., in the framework of quantum gravity and
super gravity. The modified field equations are found to have solutions which can ameliorate several problems in conventional cosmology such as Initial Singularity, the Flatness problem etc. The model also smoothly joins onto the radiation era and to present matter dominated phase of the universe and have some definite predictions at the present epoch to differentiate it from conventional cosmology.

4. Justification for the repulsive interaction

Introduction of matter of density $\rho$ in flat spacetime causes it to curve. This is a response of spacetime in the sense of elastic medium following Satcharov; the gravitational constant being constant of elasticity.

Curvature scalar $\tilde{R} = K \rho$ (we use $\sim$ to distinguish from scale factor $R$), $K = \frac{8\pi G}{C^2} \cdot \frac{1}{K} = \text{Elastic constant (very high), so it needs a high } \rho \text{ to effectively curve spacetime as effective elastic constant } \frac{1}{K}$ is very high.

Any curved spacetime with average curvature $\tilde{R}$, would by Quantum Gravity effects, acquire a black body temperature given by

$$\theta = \frac{hc}{k_B} \sqrt{\tilde{R}}, \quad \sqrt{\tilde{R}} \sim 10^{33} \text{cm}^{-1}$$

If $\theta \sim 10^{32} K$, $\tilde{R} \sim \frac{C^3}{hG} = 10^{66} \text{cm}^{-2}$

$R$ changes as spacetime expands. So $\theta$ changes $\frac{\theta}{\sqrt{\tilde{R}}} = \text{constant i.e., } \theta R$ is a constant for adiabatic expansion. So $S_{ij}$ for radiation field could be expressed as

$$S_{ij} = f(t)\phi \frac{\theta}{C^2} U_i U_j - \alpha(t)g_{ij}$$

This radiation field could consist of ultra relativistic vector particles (of spin 1) the so called gravi photons of supergravity theories. It is postulated to be a mediating boson particle in addition to graviton and is expected to mediate a repulsive force field. Being massive vector field its velocity could
be less than \( C \), so it behaves like a fluid with the above form of \( S_{ij} \). Lorentz invariance would constrain \( S_{ij} \) to have its natural form as above.

Thus we have the modified form of Einstein field equations.

\[
G_{ij} = -K(T_{ij} - S_{ij}) = -KT_{ij} \text{ (effective)} \tag{1}
\]

This would reduce to Einstein’s field equations when \( \theta = 0 \) Kelvin. We assume that \( \alpha = 0 \) when \( \theta = 0 \) Kelvin. -\( S_{ij} \) corresponds to repulsive interaction. Its existence does not violate the principle of equivalence. We imposed the restriction

\[ S_{ij} = 0 \]

Already we have

\[ G_{ij} = 0 \]

and

\[ T_{ij} = 0 \]

It is simple approach to remove the Initial Singularity. Now if we calculate the quantity

\[ U_i[T_{ij} - S_{ij}]_{,j} = 0 \]

for an elemental volume being proportional to \( R^3 \) in Robertson Walker spacetime, we get

\[
d(\rho_e R^3) + p_e (\frac{R^3}{C^2}) = -d(\alpha R^3) \tag{2}
\]

where \( \rho_e \) is the effective density

\[
\rho_e = \rho - \left[ f(t)\phi t^2 C^2 \right] \geq 0 \tag{3}
\]

\[
\frac{p_e}{C^2} = \frac{p}{C^2} - \alpha(t) \geq 0 \tag{4}
\]
\( p_e \) is the effective pressure of material distribution. 
\( \rho_e = 0 \) at \( t = 0 \) implies

\[
\rho(0) = \frac{f(0)\phi(0)\theta(0)}{C^2}
\]

\[
\rho(0) = \frac{a\theta(0)^4}{C^2}
\]

and \( \phi(0) = \frac{4a\theta(0)^3}{3} \)

implies \( f(0) = \frac{3}{4} \)

\( p_e(0) = 0 \) at \( t = 0 \) implies

\[
\alpha(0) = \frac{p(0)}{C^2} = \frac{\rho(0)}{3}
\]

\( U_\text{i}T_\text{j}^{\text{i}_j} = 0 \) yields

\[
d(\rho R^3) + \frac{p(dR^3)}{C^2} = 0
\]

since \( \rho = \frac{3p}{C^2} \) eqn (4) gives

\[
\rho R^4 = \rho(0)R^4(0) = C^* = \text{constant}
\]

\( U_\text{i}S_\text{j}^{\text{i}_j} = 0 \) gives

\[
\alpha(t) = C'\left[f(t)R^{-4} + 3 \int f(t)\frac{R^{-4}dR}{R}\right] + \text{constant}
\]

Where \( C' = \frac{\phi\theta R^4}{C^2} = \text{constant for } t \geq 0. \)

Now eqn (3) and eqn (4) along with the expressions \( \rho = \frac{3p}{C^2} \) and \( \rho_e = \frac{3p_e}{C^2} \)
give

\[
\alpha = \frac{1}{3}C'\left(\frac{f(t)}{R^4}\right)
\]

From eqn (8) and eqn (9) we shall have
\[ f(t) = \frac{f(0)}{\sqrt{Y}}, Y = \frac{R}{R(0)} \quad (1 \leq Y < \infty) \]

From eqn (5)

\[ f(0) = \frac{3}{4} \text{ hence } f(t) = \frac{3}{4\sqrt{Y}} \quad (10) \]

From eqn (9) and eqn (10) we obtain

\[ \alpha(t) = \frac{1}{3} \rho(0) Y^{-\frac{9}{2}} \]

From eqn (3)

\[ \rho e R^4 = \rho R^4 - \frac{f(t)\phi\theta R^4}{C^2} \]
\[ = C^*(1 - \frac{f(t)C'}{C^*}) \]
\[ \rho R^4 = C^* = \rho(0)R(0)^4 \text{ and } f(0) = \frac{C^*}{C^*} \text{, } f(t) = \frac{f(0)}{\sqrt{Y}} \]

Therefore

\[ \rho e R^4 = C^*[1 - (\frac{1}{\sqrt{Y}})] \]

i.e., \[ \rho e Y^4 = \rho(0)[1 - (\frac{1}{\sqrt{Y}})] \quad (11) \]

From eqn (5)

\[ \rho Y^4 = \rho(0) \]

Therefore \[ \rho e = \rho[1 - (\frac{1}{\sqrt{Y}})] \]

Y becomes very large implies

\[ \rho e \sim \rho \]
For a comoving co-ordinate system

\[ U^0 = 1, U_0 = 1, g_{00} = +1 \]
\[ U^1 = U^2 = U^3 = 0 \]

Now \[ T^i_j - S^i_j = (\rho_e + \alpha)U^i U_j - p_C^{-2}(\delta^i_j - U^i U_j) \]
we have \[ T^i_i - S^i_i = \alpha \neq 0 \]

since \( \rho_e = 3 \frac{p_e}{C^2} \) in radiation dominated era

Now the field equation (1) in Robertson Walker spacetime yield

\[
\left(\frac{8\pi G}{C^2}\right)(\rho_e + \alpha) = 3\left(\frac{k}{R^2} + \frac{\dot{R}^2}{C^2 R^2}\right) \\
\left(\frac{8\pi G}{C^2}\right)\left(\frac{p_e}{C^2}\right) = -\left(\frac{k}{R^2} + \frac{\dot{R}^2}{C^2 R^2}\right) - \left(\frac{2\ddot{R}}{C^2 R}\right)
\]

(12)

(13)

Here \( k \) is the curvature index. Dot shows differentiation with respect to time. Now substituting for \( \rho_e, \alpha \) and \( R \) in terms of \( Y \) variable and making use of equation

\[ \rho_e = 3 \frac{p_e}{C^2} \]

Eqn (12) and eqn (13) yield after integration

\[
\left[ \frac{dY}{dt} \right]^2 = \left(\frac{8\pi G}{3}\right)\rho(0)(Y^2 - \left(\frac{2}{3}\right)Y^{-(5/2)})
\]

(14)

The constant of integration vanishes since we assume that

\[ H(0) = \left(\frac{8\pi G \rho(0)}{9}\right)^{1/2} \]

is the Hubble parameter at \( t = 0 \). Thus Hubble parameter in radiation dominated era

\[ H(t) = \pm H(0)(3Y^{-4} - 2Y^{-9/2})^{1/2} \]

(15)

\( H(t) \) is taken to be negative when \( t < 0 \) showing blue shift. \( H(t) \) is taken to be positive when \( t > 0 \) showing red shift.
From eqn (11) it is clear that $\rho_e$ increases for $1 \leq Y < \frac{81}{64}$ and it decreases for $\frac{81}{64} < Y < \infty$, becomes maximum at $Y = \frac{81}{64}$.

On integrating eqn (14), we shall have

$$
\Delta \left[ \left( \frac{1}{2} \right) Y^{7/4} + \left( \frac{7}{18} \right) Y^{5/4} + \left( \frac{35}{108} \right) (Y^{3/4} + Y^{1/4}) \right] + \left( \frac{35}{162} \right) \ln |Y^{1/4} + \Delta| = \left[ \sqrt{3} \right] H(0) t + \mu
$$

(16)

$\mu$ is the constant of integration and $\Delta = \left( \sqrt{Y} - \frac{2}{3} \right)^{1/2}$

At $t = 0, Y = 1$ so that $\mu = 0.985872474...$

when $Y$ becomes appreciable $\frac{1}{2} Y^2$ dominates LHS of eqn (16) and we shall have

$$
Y = (2\sqrt{3} H(0))^{1/2} \sqrt{t}
$$

(17)

Equation (14) collapses to the above equation when $Y = 1.6924079148.....$

According to our assumption this occurs when $t = 5.3903687 \times 10^{-44}$ s. Hence we obtain

$$
\theta(0) = 1.107 \times 10^{32} K
$$

From equation (17),

$$
R(t) = R(0)(2\sqrt{3} H(0) t)^{1/2}
$$

(18)

From equation (11) we get

$$
\rho_e = \frac{(\rho(0) H(0) t)^{-2}}{12}
$$

$$
= \frac{3}{32\pi G} \frac{1}{t^2}
$$

(19)

(20)

From $\theta Y = \theta(0)$, we get

$$
\theta \approx \theta(0)(2\sqrt{3} H(0))^{-1/2} \frac{1}{\sqrt{t}}
$$

(21)
Substituting the value of $H(0)$ we get

$$\theta \approx \left[\frac{3C^2}{32\pi G\alpha}\right]^{1/4} \sqrt{\frac{1}{t}}$$

(22)

i.e., $\theta \approx 1.52 \times 10^{10} \sqrt{\frac{1}{t}} \text{K}$

(23)

Expressions (18), (19) and (21) are similar to those expressions in the existing cosmological theory of early universe.

The deceleration parameter in radiation dominated era

$$q(t) = \frac{\sqrt{Y} - \frac{5}{6}}{\sqrt{Y} - \frac{2}{3}}$$

$$q(0) = \frac{1}{2}$$

and $q(t)$ tends to 1 as $Y$ tends to $\infty$.

Density parameter in the radiation dominated era is found to be

$$\Omega(t) = \frac{\rho_e}{\rho^*}, \rho^* = \frac{3H^2}{8\pi G}$$

$H$ is the Hubble parameter in the radiation dominated era, we get

$$\Omega(t) = \frac{1 - \left(\frac{1}{\sqrt{Y}}\right)}{1 - \left(\frac{2}{3\sqrt{Y}}\right)}$$

The flatness problem in the traditional standard Cosmological picture may be reviewed in the light of this model. In this model the curvature index $k$ vanishes identically in radiation dominated era (See Appendix III).
5. Matter dominated era

For matter dominated era we have \( p \approx 0, p_e \approx 0 \) yielding \( \alpha \approx 0 \)

Thus equations (12) and (13) read

\[
\frac{8\pi G}{C^2} \rho_e = 3\left( \frac{k}{R^2} + \frac{\dot{R}^2}{C^2 R^2} \right) \tag{24}
\]

and

\[
0 = -3 \left[ \frac{k}{R^2} + \frac{\dot{R}^2}{C^2 R^2} \right] - \left[ \frac{6 \ddot{R}}{C^2 R} \right] \tag{25}
\]

On adding (24) and (25)

\[
\frac{8\pi G}{C^2} \rho_e = -\left( \frac{6\ddot{R}}{C^2 R} \right) = -\left( \frac{6\dot{Y}}{C^2 Y} \right) \tag{26}
\]

But \( \rho_e Y^3 = \rho Y^3 - \frac{f(t)\phi Y^3}{C^2} \) \tag{27}

From (2) and (6) we shall have

\[
d(\rho_e Y^3) = 0 \text{ and } d(\rho Y^3) = 0 \text{ for } p_e = 0, p = 0
\]

and \( \alpha = 0 \) in the matter dominated era.

Therefore

\[
\rho_e Y^3 = B_1 = \text{constant and } \rho Y^3 = B_2 = \text{constant}
\]

Thus eqn (27) suggests

\[
\left( \frac{f(t)\phi Y^4}{C^2} \right) \left( \frac{Y^4}{Y} \right) = B_3 = \text{constant}
\]

\[
f(t) = \frac{B_3 Y}{A_3}
\]

where \( A_3 = \frac{\phi \theta Y^4}{C^2} = \frac{4}{3} \left( \frac{a\theta^4(0)}{C^2} \right) \)
Then eqn (27) takes the form $B_1 = B_2 - B_3$. Continuity of $\rho_e, \rho, Y, \dot{Y}$ and $f(t)$ at $\theta_T$ help us to determine the constants $B_1, B_2$ and $B_3$. Thus

$$B_3 = \left[ \frac{\theta_T}{\theta(0)} \right]^{3/2} \rho(0), B_2 = \frac{\theta_T}{\theta(0)} \rho(0)$$

$$B_1 = \rho(0) \frac{\theta_T}{\theta(0)} \left[ 1 - \sqrt[3]{\theta_T} \theta(0) \right]$$

we have $\rho_e = B_1 Y^{-3}$

$$\rho = B_2 Y^{-3}$$

and

$$f(t) = \frac{3}{4} \left[ \frac{\theta_T}{\theta(0)} \right]^{3/2} Y$$

since $\theta_T \ll \theta(0)$, $B_1 \sim B_2$ showing that $\rho_e \sim \rho$

Making use of equation (26) and equation (28) one obtains after integration

$$H^2(t) = H^2(0) \left\{ 3 \left[ 1 - \sqrt[3]{\theta_T} \theta(0) \right] Y^{-3} + \left[ \frac{\theta_T}{\theta(0)} \right]^{3/2} Y^{-2} \right\}$$

Thus

$$\left( \frac{H(t)}{(\text{matter})} \right) = \pm \sqrt{\text{RHS of the above equation}} \quad (29)$$

$H(t) < 0$ for $t < 0$, $H(t) > 0$ for $t > 0$

Here constant of integration is obtained on the assumption of continuity of $H(t)$ at $\theta_T$. $(H(t)_{(\text{matter})})$ is the Hubble parameter in matter dominated era.

Now the present value of $Y = Y_N = \frac{\theta(0)}{\theta_N} = \frac{R_N}{R(0)}$

Substituting this value of $Y_N$ in expression (21) and evaluating $\theta(0)$

$$\theta(0) = \frac{\theta_T \left[ \frac{\theta_T}{\theta_N} - 3 \right]}{\left[ \frac{\theta^*}{\theta_T} - 3 \right]^2}$$

Where $\theta^* = \frac{9H_N^2 C^2}{\theta_N^3 8\pi Ga}$
\[ H_N = \text{present value of Hubble parameter}, \theta_T = 4000 \text{ K}, \text{ the transition temperature from radiation domination to matter domination and} \theta_N = 2.7 \text{ K}. \]

Since the exact value of \( H_N \) is unknown the determination of \( \theta(0) \) using the above expression is inappropriate.

Now consider the assumption (A8), it is well accepted that the classical limit up to which one can march towards \( t = 0 \) is the Planck time \( t \approx 5.34 \times 10^{-44} \text{ sec.} \) All the cosmical parameters shall have classical meaning only from \( t \approx 5.39 \times 10^{-44} \text{ sec.} \) And the variable \( Y \) becomes the classical expression

\[ Y = \left( \frac{1}{2\sqrt{3}H(0)} \right)^{1/2} \sqrt{t} \]

When \( Y \) lies in the range

\[ 1.6924079 \cdots \leq Y < \text{very large number} \]

Hence the least value of \( Y = 1.6924079 \cdots \). It occurs when \( t \approx 10^{-44} \text{ sec} \) (See Appendix II), and make use of the equation

\[ Y = \left( \frac{1}{2\sqrt{3}H(0)} \right)^{1/2} \sqrt{t} \]

i.e., \( Y = \left[ 2\sqrt{3}\left( \frac{8}{9}\pi \alpha C^{-2} \right) \right]^{1/2} \theta(0) \sqrt{t} \)

When \( Y = 1.69240 \cdots, t = 5.39036 \cdots \times 10^{-44} \text{ s.} \)

So that \( \theta(0) = 1.10677 \cdots \times 10^{32} \text{ K} \)

Deceleration parameter in matter dominated era takes the form

\[ q(t) = \frac{1}{2} \left[ 1 + \left( \frac{1}{3} \right) \eta_T^3 Y \left( 1 - \sqrt{\eta_T} \right)^{-1} \right]^{-1} \quad (30) \]

where \( \eta_T = \frac{\theta_T}{\theta(0)}, q < \frac{1}{2} \)

Substituting for \( \rho_e \) and \( H^2 \) from eqn (28) and eqn (29) in eqn (24) we obtain

\[ k = -1 \text{ and } R(0) = \frac{C}{H(0)} \left( \frac{\theta(0)}{\theta_T} \right)^{5/4} \]

\( R(0) \) being the scale factor at \( t = 0 \) (See Appendix IV).
The density parameter in the matter dominated era assumes the form

\[
\left( \frac{\Omega(t)}{(\text{matter})} \right) = \left[ 1 + \left( \frac{1}{3} \right) \eta_T^{3/2} Y (1 - \sqrt{\eta_T})^{-1} \right]^{-1}
\]

The present value of density parameter is obtained by putting

\[ Y = Y_N = \frac{\theta(0)}{\theta_N} \]

We have

\[ \Omega_N = \left[ 1 + \left( \frac{1}{3} \right) \eta_T^{3/2} Y_N (1 - \sqrt{\eta_T})^{-1} \right]^{-1} \]

which is less than 1, but very nearly equal to 1.

\[ \Omega_N = \frac{\rho_e N}{\rho_N^*} \]

\( \rho_e N \) = present value of effective density. \( \rho_N^* \) = present value of critical density.

The expression for \( \Omega_N \) implies that \( \rho_e N \) is less than \( \rho_N^* \) and very nearly equal to \( \rho_N^* \). That is the effective density causing gravitation is very near to the present value of critical density but less than that. We have

\[ (1 - \Omega_N) = 0.211 \times 10^{-14} \]
\[ \theta(0) = 1.10677... \times 10^{32}\text{K} \]
\[ H(0) = 1.533913... \times 10^{42}\text{s}^{-1}, R(0) = 6.974818\text{m} \]
\[ H_N = 0.61 \times 10^{-18}\text{s}^{-1}, \rho_N^* = 0.6657 \times 10^{-27}\text{kgm}^{-3} \]

6. Conclusion

Initial singularity is effectively removed at the instant \( t = 0 \), the beginning of this expanding phase of the universe. The Standard Friedmann model is described as the big bang model for the universe. It implies that the universe began in an initial singularity. If the universe is a closed one it will end in future singularity. Standard Cosmological models lead to the conclusion that the universe began from a singular state - a state with an infinite high density of matter. In our model Initial Singularity is avoided by the presence of term \( Sij \) in the field equations. There exists an upper limit of temperature \( \theta(0) \approx 10^{32} \) Kelvin at which the effective density \( \rho_e \) vanishes.
And when $Y = 1.6924...$ at $t = 5.390... \times 10^{-44}$ sec; this theory coincides with existing Cosmological theory of early universe. From the expression for density parameter in the Radiation dominated era it is clear that there is no flatness problem in this model. Curvature index vanishes identically in radiation dominated era.

Density parameter in matter dominated era shows that effective density is nearly equal to critical density but bit less than critical density. Curvature index in matter dominated era is found to be -1 showing that the 3-space is hyperbolic. Thus knowing $\theta_N = 2.7$ K, $\theta_T = 4000$ K and $\theta(0) \approx 10^{32}$ K a lot of information flows out of the theory. This is a simplified model of the universe.

In conclusion we have a model leading to $\Omega_N$ very close to 1, and an accelerating phase (due to k = -1). The latter is consistent with current observational results based on impressive data involving luminosity evolutions of high red shift supernovae which strongly suggest an accelerating phase ($q = -1$). Refs [12], [13].

The fact that $\Omega_N$ is very close to one, is supported by very recent observations of the BOOMERANG project of the cosmic microwave background as reported in the April 27, 2000 of nature [14].

Eventhough the term $S_{ij}$ is purely arbitrary and adhoc it challenges the generality of singularity theorems since $T_{ij}$ (effective) = ($T_{ij} - S_{ij}$) fits well within the conceptual structure of GTR without violating any known physical laws.
Appendix I

This is to show how we could get the number $Y = 1.6924079...$

From Equation (16) of the manuscript we have

$$\Delta \times \frac{1}{2} Y^{7/4} + \left[ \Delta \left( \frac{7}{18} Y^{5/4} + \frac{35}{108} (Y^{3/4} + Y^{1/4}) \right) \right] + \frac{38}{162} \log_e |Y^{1/4} + \Delta| - \mu = \sqrt{3} H(0)t$$

i.e., $\Delta \times \frac{1}{2} Y^{7/4} + A_1 = \sqrt{3} H(0)t$

where $A_1$ is the term within the square bracket of the above equation

$$\mu = 0.985872474...$$
$$\Delta = (\sqrt{Y} - \frac{2}{3})^{1/2}$$

Squaring the above equation and rearranging the terms we shall have

$$\frac{1}{4} Y^4 - \frac{1}{6} Y^{7/2} + [A_1^2 + A_1 \Delta Y^{7/4}] = 3H(0)^2 t^2$$

when

$$\frac{1}{4} Y^4 = 3H(0)^2 t^2$$

then

$$\frac{1}{6} Y^{7/2} = A_2$$

where

$$A_2 = A_1^2 + A_1 \Delta Y^{7/4}$$

i.e., $Y = (6A_2)^{2/7}$

Now using this equation we can find the range of value of $Y$ for which this equation is satisfied, the range being

$$1.6924079... \leq Y < \text{very very large number}.$$

Hence the least value of $Y = 1.6924079...$ It occurs when $t \sim 10^{-44}$s.
Appendix II (Some subtle features)

1. Our theory does have entirely different structure for $f(t) \neq 0$ in both radiation era and matter dominated era. The word identical may be replaced. One may say that there is one-one correspondence between our theory and accepted Big Bang theory after $10^{-44}$ sec. The correspondence is such that they appear to be identical for $Y \sim t^{1/2}$ in radiation dominated era and $Y \sim t^{2/3}$ in matter dominated era as expected by conventional cosmologists as far as the scale factor is concerned.

2. Initial Singularity is removed in a very natural way. All cosmological parameters are mathematically well behaved. There is no abnormality in any of the cosmological parameters. Explicit expressions are obtained for all of them. Existence of horizon is taken to be an initial condition. This is one of the limitations of our theory. AND THE TEMPERATURE AT THE BEGINNING OF THIS EVER EXPANDING UNIVERSE IS $10^{32}$ K. THIS COULD BE TAKEN TO BE AN UPPER LIMIT OF TEMPERATURE IN KELVIN SCALE. Some theoretical backing up to this upper limit of temperature could be found elsewhere. Knowing this temperature all other cosmological parameters are determined.

3. There is no flatness problem in this model

4. Expected behaviour of scale factor both in radiation dominated era and matter dominated era is contained in this model.

5. Curvature index vanishes identically in radiation dominated era and it becomes -1 in matter dominated era showing that the 3-space is hyperbolic in matter dominated era.

6. If we put $f(t) = 0$ and $\alpha(t)$ a constant we get back Einstein’s field equations with $\wedge$ term.
   Above all it is a very simple model of the universe with just a few physically acceptable assumptions.

7. In effect we have a time-variable $\wedge$ coupled to temperature. A time-variable $\wedge$ can also be reconciled with most of the observations [13]. A scalar field potential of the form $V(\phi) \sim \phi^{-\alpha}, \alpha > 0$, also is equivalent to a varying cosmic term [14].
Appendix III

To show that $k = 0$ in radiation dominated era

From equation (12) we have

$$3 \left[ \frac{k}{R^2} + \frac{\dot{R}}{c^2 R^2} \right] = \frac{8\pi G}{C^2} (\rho_e + \alpha)$$

$$\frac{k}{R^2} + \frac{\dot{R}}{c^2 R^2} = \frac{8\pi G}{3C^2} (\rho_e + \alpha)$$

$$k + \frac{\dot{y}^2}{c^2 Y^2} = \frac{8\pi G}{3C^2} (\rho_e + \alpha)$$

Here $\rho_e = \rho(0) Y^{-4} \left( 1 - \frac{1}{\sqrt[4]{Y}} \right)$ [From eqn (11)]

and $\alpha = \frac{1}{3} \rho(0) Y^{-\frac{3}{2}}$ [eqns: (9) & (10)]

$$\rho_e + \alpha = \rho(0) Y^{-4} \left( 1 - \frac{1}{\sqrt[4]{Y}} \right) + \frac{1}{3} \rho(0) Y^{-\frac{3}{2}}$$

$$= \rho(0) Y^{-4} - \frac{2}{3} \rho(0) Y^{-\frac{3}{2}}$$

$$\frac{k}{R^2} + \frac{\dot{y}}{c^2 Y^2} = \frac{8\pi G}{3C^2} \rho(0) Y^{-4} - \frac{2}{3} \left( \frac{8\pi G}{3C^2} \right) \rho(0) Y^{-\frac{3}{2}}$$

But $\frac{1}{c^2 Y^2} (\dot{Y}^2) = \frac{8\pi G}{3} \frac{\rho(0)}{Y^2 C^2} \left( Y^{-2} - \frac{2}{3} Y^{-\frac{5}{2}} \right)$ [from (14)]

$$= \frac{8\pi G}{3C^2} \rho(0) Y^{-4} - \left( \frac{2}{3} \right) \frac{8\pi G}{3C^2} \rho(0) Y^{-\frac{9}{2}}$$

It implies

$$\frac{k}{R^2} = 0$$

i.e. $k$ vanishes identically in radiation dominated era
Appendix IV

To show that \( k = -1 \) in matter dominated era

From equation (24) we have

\[
\frac{8\pi G}{C^2} \rho_e = 3 \left[ \frac{k}{R^2} + \frac{\circ^2}{c^2 R^2} \right] 
\]

\[
\frac{8\pi G}{C^2} B_1 Y^{-3} = 3 \left[ \frac{k}{R^2} + \frac{\circ^2}{c^2 R^2} \right] \text{ using (28)} 
\]

\[
B_1 = \rho_0 \left[ \frac{\theta_T}{\theta_0} \right] \left[ 1 - \sqrt{\frac{\theta_T}{\theta_0}} \right] 
\]

\[
\frac{8\pi G}{3C^2} \left\{ \rho_0 \left( \frac{\theta_T}{\theta_0} \right) \left[ 1 - \sqrt{\frac{\theta_T}{\theta_0}} \right] \right\} Y^{-3} = \frac{k}{R^2} + \frac{1}{C^2} \left[ \frac{\theta_T}{\theta_0} \right] \left\{ 3 \left[ 1 - \sqrt{\frac{\theta_T}{\theta_0}} \right] Y^{-3} \right. 
\]

\[
+ \left[ \frac{\theta_T}{\theta_0} \right] \frac{3}{1 - \sqrt{\frac{\theta_T}{\theta_0}}} Y^{-2} \right\} 
\]

\[
= \frac{k}{R^2} + \frac{8\pi G}{9C^2} \rho_0 \left( \frac{\theta_T}{\theta_0} \right) \left[ 1 - \sqrt{\frac{\theta_T}{\theta_0}} \right] Y^{-3} 
\]

\[
+ \frac{8\pi G}{9C^2} \rho_0 \left( \frac{\theta_T}{\theta_0} \right) \left( \frac{\theta_T}{\theta_0} \right) \frac{3}{1 - \sqrt{\frac{\theta_T}{\theta_0}}} Y^{-2} 
\]

\[
\therefore 0 = \frac{k}{R^2} + \frac{8\pi G}{9C^2} \rho_0 \left( \frac{\theta_T}{\theta_0} \right) \left( \frac{\theta_T}{\theta_0} \right) \frac{3}{1 - \sqrt{\frac{\theta_T}{\theta_0}}} Y^{-2} 
\]

\[
\therefore \frac{k}{R^2} = -\frac{8\pi G}{9C^2} \rho_0 \left( \frac{\theta_T}{\theta_0} \right) \frac{3}{1 - \sqrt{\frac{\theta_T}{\theta_0}}} Y^{-2} 
\]

It implies \( k = -1 \) and \( R_0 = \frac{C}{H_0} \left( \frac{\theta_T}{\theta_0} \right)^{\frac{3}{2}} \) for \( H_0 = \left( \frac{8}{9} \pi G \rho_0 \right)^{\frac{1}{2}} \)
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