Three-cluster models for light nuclei

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Abstract. A three-body cluster model involving microscopic nonlocal interactions is developed and compared with a fully microscopic cluster model. The energy-independent nonlocal interactions are obtained from a renormalization of the energy-dependent kernels of the resonating-group method. Such interactions are derived for the $\alpha\alpha$ and $\alpha n$ systems. These interactions are employed in three-body studies of the $\alpha nn$, $\alpha\alpha n$ and $3\alpha$ descriptions of the $^6\text{He}$, $^9\text{Be}$ and $^{12}\text{C}$ nuclei. A comparison with fully microscopic calculations is performed.

1. Introduction
Microscopic cluster models provide a consistent description of the spectroscopy and reactions of light nuclei [1]. In these models, the nucleons are grouped into substructures known as clusters. Some nuclei, such as $^9\text{Be}$ or the halo nucleus $^6\text{He}$, display a clear three-cluster structure. Others such as $^{12}\text{C}$ display this structure in some excited states but its presence in the ground state has raised questions. For light nuclei, microscopic cluster models, i.e. involving all nucleon coordinates, and taking the Pauli principle exactly into account, have been developed. This approach can however be very heavy. Simpler three-cluster models involving phenomenological local interactions are therefore often used. These phenomenological cluster models are however less well established and sometimes controversial. A well known example is the $^{12}\text{C}$ nucleus, where a microscopic three-$\alpha$ model provides a good description of low-lying states and of the $0^+_2$ resonance, but where non-microscopic models, based on phenomenological local $\alpha - \alpha$ interactions, fail to reproduce the ground-state properties [2, 3].

A first attempt to use energy-dependent nonlocal $\alpha \alpha$ RGM kernels has been developed in Refs. [4, 5] in a study of $^{12}\text{C}$. However, the energy dependence of the $\alpha \alpha$ nonlocal interaction raises a difficulty. This energy is well defined in a two-body system but not in a three-body system. Using some average energy provides fair results for $^{12}\text{C}$ [6] but not for other three-cluster systems [7]. Hence the energy dependence must be eliminated from the RGM equation. The principle of this elimination is known for a long time [8, 9, 10, 11], but is difficult to apply.
Recently, we have shown that this procedure leads to a consistent description of the $0^+$ states of $^{12}$C case, in semi-quantitative agreement with fully microscopic calculations [12].

The purpose of this contribution is to present an extension of the model to $^{9}$Be (as $\alpha\alpha n$) and $^{6}$He (as $\alpha nn$) in order to improve our understanding of the importance of nonlocality.

2. Renormalized RGM

We have recently developed a new approach to derive nucleus-nucleus interactions based on the Resonating Group Method (RGM) [12]. This approach combines the predictive power of fully microscopic models and the relative simplicity of non-microscopic approximations. The interaction is derived from a NN interaction, and is non-local.

The RGM wave function of a two-cluster $A+B$ system with the relative motion function $\chi$ is expressed as

$$\Psi = A\phi_A\phi_B\chi(x),$$

where $x$ is the relative coordinate between the clusters, and $\phi_A$ and $\phi_B$ are the cluster internal wave functions, assumed to be described by harmonic-oscillator orbitals with a common size parameter. In this definition, $A$ is the $A$-nucleon antisymmetrizer.

The RGM equation for $\chi$ reads [1, 8, 13]

$$(T + V + \varepsilon K)\chi = \varepsilon\chi,$$  \hspace{1cm} (2)

where the energy $\varepsilon$ is defined with respect to the $A+B$ threshold. In Eq. (2), $T$ is the intercluster kinetic energy, and the potential $V$ contains local and nonlocal terms. $K$ is related to the overlap kernel $N$ according to (see Ref. [14] for details)

$$N(r, r') = \langle \phi_A\phi_B\delta(x - r)|A\phi_A\phi_B\delta(x - r')\rangle = \delta(r - r') - K(r, r').$$  \hspace{1cm} (3)

By introducing a renormalized relative motion function

$$g = \sqrt{N}\chi,$$  \hspace{1cm} (4)

Eq. (2) can be converted into an equation involving an energy-independent potential

$$(T + V^{RGM})g = \varepsilon g.$$  \hspace{1cm} (5)

The nonlocal potential reads

$$V^{RGM} = N^{-1/2}(T + V)N^{-1/2} - T.$$  \hspace{1cm} (6)

It involves contributions from the kinetic and potential energies. The calculation of $V^{RGM}$ is detailed in Ref. [14]. It is based on an expansion of the overlap kernel on a basis of harmonic oscillator functions. For two-cluster systems, Eq. (5) is fully equivalent to the RGM equation. This property provides a strong test of the numerical calculation. Our aim here is to apply the renormalized RGM potential to three-cluster systems.
3. Application to $^6$He, $^9$Be and $^{12}$C

We compare the fully microscopic results [15] with the renormalized RGM for $^6$He, $^9$Be and $^{12}$C. The three-cluster calculations are performed within the hyperspherical method, with the Minnesota interaction. Various values of the exchange parameter $u$ are adjusted to reproduce, either the $\alpha - n$ and $\alpha - \alpha$ phase shifts, or the experimental binding energies of the three-cluster system.

| Nucleus   | Microscopic $E$ (MeV) | $\sqrt{\langle r^2 \rangle}$ (fm) | Non-microscopic $E$ (MeV) | $\sqrt{\langle r^2 \rangle}$ (fm) | Exp. $E$ (MeV) |
|-----------|-----------------------|------------------------------------|----------------------------|------------------------------------|----------------|
| $^6$He ($0^+$) | $-0.09$                | 2.57                               | $-0.08$                    | 2.93                               | $-0.98$         |
| $^9$Be ($3/2^-$) | $-2.61$                | 2.35                               | $-2.16$                    | 2.41                               | $-1.57$         |
| $^9$Be ($5/2^-$) | $-0.09$                | 2.38                               | $0.2$                      | $0.87$                             |                |
| $^{12}$C ($0^+_1$) | $-11.37$               | 2.21                               | $-9.44$                    | 2.20                               | $-7.27$         |
| $^{12}$C ($0^+_2$) | $0.597$                | 0.60                               | $0.60$                     | $0.38$                             |                |
| $^{12}$C ($2^+_1$) | $-9.22$                | 2.16                               | $-7.12$                    | 2.18                               | $-2.83$         |

The energies and rms radii are given in Table 1. Here the $u$ values are chosen so as to reproduce the phase shifts as accurately as possible. For the rms radii, we use the shell-model value as internal radius of the $\alpha$ particle. Let us point out that our goal here is not to reproduce the experimental energies with a high accuracy, but to compare both approaches. The differences are in general small, and could be compensated by a slight readjustment of the nucleon-nucleon interaction. In particular, the present non-local $\alpha - \alpha$ potential provides a fair description of the $^{12}$C spectrum (see Ref. [12] for more detail). In all cases, the semi-microscopic model provides lower binding energies. This indicates that three-cluster exchange effects, missing in this approximation, should be attractive.

![Figure 1](image1.png)

Figure 1. Neutron densities of $^6$He in the microscopic (solid line) and semi-microscopic (dashed line) models. The insert represents the densities in a logarithmic scale.

Let us first discuss the results for $^6$He, which is the lightest nucleus considered here. Accordingly, three-body exchanges, missing in the semi-microscopic approximation, should play
a minor role, and differences between both models should be reduced. This is confirmed by the results presented in Table 1. Although $^6$He is hardly bound, the differences between both models are small. The Minnesota force provides virtually identical binding energies.

In Fig. 1, we present the monopole neutron density $\rho(r)$ for the $^6$He ground state. For each nucleus, the admixture parameter of the $NN$ interaction is modified in order to reproduce the experimental binding energy. The densities provide a deeper insight in the comparison between both models. They are normalized in such a way that

$$\int_0^{\infty} \rho(r)r^2dr = N/\sqrt{4\pi},$$

with $N = 4$. The microscopic densities are determined as explained in Ref. [16]. For the semi-microscopic approximation, we first determine the form factor, which is then converted to the density through a Fourier transform. In $^6$He, both approaches are very close to each other, in agreement with the rms radii of Table 1.

For $^9$Be, the ground state is overbound by about 1 MeV in the microscopic model. The semi-microscopic model provides smaller binding energies than the microscopic approach (by 0.45 MeV). This means that three-body effects, missing in the semi-microscopic approximation, are expected to be attractive. Both models underestimate the $5/2^-$ resonance energy, but the excitation energy is in all cases in fair agreement with experiment (2.42 MeV).

In Fig. 2, we present the neutron density $\rho(r)$ for the $^9$Be ground state. Again, the force parameters are taken to reproduce the experimental binding energy. Below 3 fm, the semi-microscopic model gives a rather uniform density, while the microscopic density is more peaked near the origin.

Let us now discuss $^{12}$C. Both models provide energies much better than in the local approach [3]. However the $2^+$ excitation energy is too low, due to the lack of a spin-orbit force. The charge density is shown in Figure 3. Beyond 1.5 fm, the semi-microscopic model is a fair approximation. However, the short-distance behavior is different: whereas the microscopic approach provides a maximum near 1 fm and then decreases, the semi-microscopic density steadily increases when $r$ tends to zero. This difference is probably due to three-body antisymmetrization effects, not
included in the semi-microscopic approximation. To illustrate the sensitivity with respect to the binding energy, we also present the density with the original $u$ value (dotted line), which provides an overbinding of 4 MeV (see Table 1). The general trend is not affected, but the long-range part decreases more rapidly, which leads to an increase at short distances.

![Figure 3](image_url)

**Figure 3.** Charge densities of $^{12}$C in the microscopic (solid line) and semi-microscopic (dashed line) models. The dotted line corresponds to an interaction fitted on the $\alpha-\alpha$ phase shifts. The insert represents the densities in a logarithmic scale.

4. Conclusion

We have analyzed a new semi-microscopic model, with two-body interactions based on RGM kernels. The renormalized RGM potential provides the same phase shifts as the full RGM and is nonlocal, but energy independent. This allows us to avoid the problem of defining two-body energies in three-cluster calculations. We have used this new interaction in a three-body model, and compared it with a fully microscopic approach. This is expected to evaluate the importance of antisymmetrization and three-body effects.

Our work shows that neither the microscopic model nor its approximation are able to precisely reproduce two-cluster and three-cluster properties simultaneously. In general the differences between both approaches are small, and decrease with the nucleon number (2.5 MeV for $^{12}$C, 0.5 MeV for $^{9}$Be, less than 0.1 MeV for $^{6}$He).

With the densities, the differences between both models are more apparent than with the rms matter radii. In general, differences occur at short distances or, in other words, at large momenta in the form factors. In the lightest nucleus $^{6}$He, the densities derived from the microscopic model and from its approximation are very similar to each other.

In all cases the binding energies are slightly underestimated by the semi-microscopic approach. This means that three-body exchange effects, missing in this approximation, should be attractive. The derivation of reliable three-cluster forces is a challenge for future works.

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