Actions for dynamical fermion simulations: are we ready to go?

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A critical review, playing devil’s advocate, on present dynamical fermion simulations is given.

1. Introduction

No doubt, the lattice approach to quantum field theory has established itself as an integral part of high energy theoretical physics and has produced many results that are relevant to understand or even construct quantum field theories, as well as for phenomenology helping to interpret experimentally obtained data. Still, we are by far not at the end of the story. Serious dynamical fermion simulations with realistic number of quarks, i.e. two light and one heavy flavour, have just started and reveal already a number of unexpected problems. It is the objective of this presentation to address these problems and discuss their relevance and possible consequences. Nevertheless, we have made a lot of progress and we have understood much better to keep under control systematic errors such as discretization and finite size effects, the chiral limit and non-perturbative renormalization. However, still more patience has to be invested in our lattice computations to tell our colleagues from experiment trustworthy numbers. New computer architectures are essential and will help tremendously to step forward. Of course, we are all hoping for a miraculous invention of a new algorithm that would solve all our problems at once, although reality tells us that we will proceed in only small steps.

2. Conceptual questions

Certainly, for a non-lattice person (and even for a lattice practitioner) the plethora of actions that are used in todays lattice simulations is, to say it moderately, confusing. The initial Wilson fermion action [1] got many friends such as Symanzik improved actions [2,3,4] employing various gauge actions such as the Wilson [1], various forms of Symanzik improved gauge actions [5] and renormalization group inspired actions [6]. A recent development concerning Wilson fermions is the formulation of twisted mass QCD [7]. Of course, there is the approach of staggered fermions [8] which comes again in improved forms [9] and with different gauge actions. Newer developments are chiral invariant lattice fermions such as domain wall [10], overlap [11] and perfect actions [12]. If exact chiral symmetry is not insisted on, but only chirally improved fermions are considered, designer actions such as FLIC [13], Hypercube [14] or truncated fixed point fermions [12] enter the stage.

For me, the criterion for choosing an action should be guided by the demand that

- the results obtained with these actions agree among themselves in the continuum limit and thus provide valuable cross checks
- and if using a particular action will not waste computer time.

There are a number of advantages and disadvantages of the above mentioned actions, concerning field theoretical conceptual aspects, lattice artefacts, chiral and renormalization properties and simulation cost that will be addressed below. Let me start by discussing locality properties of the actions used.

2.1. Rigorous actions

The list of the above actions can be ordered by the level of rigor they obey in constructing a
lattice quantum field theory with the emphasis on their locality properties. The initial, standard Wilson actions (including the twisted mass formulation without clover term) have been shown to be reflection positive [15], they have a positive definite transfer matrix [16] and thus obey the reconstruction theorem (see first two refs. in [15]) guaranteeing thus the reconstruction of the Minkowski Green’s functions. Naive staggered fermions are on the same level [17] with the peculiarity that the transfer matrix has to be constructed over two lattice spacings.

2.2. Non-rigorous but local actions

The next class of actions are the ones that are ultra-local, i.e. they have an interaction range of only a few lattice points, but for which reflection positivity cannot be proven. To this class belong the Symanzik improved and the designer actions mentioned above. However, there cannot be a serious doubt that these actions are theoretically sound as the improvement terms added are irrelevant and vanish in the continuum limit. We might speak of “physical positivity”.

2.3. Exponentially localized actions

The chirally invariant lattice actions even give up ultra-locality [18,19] and only show an exponential localization [20], as shown for domain wall [21], overlap [20] and perfect fermions [22]. Again, we have no reason to doubt that these actions define a local quantum field theory and describe QCD in the continuum limit.

2.4. Non-local actions (?)

Every time a lattice Dirac operator is constructed, where the action ranges over all points on the lattice, the question of locality has to be asked anew. An example for such an operator are improved staggered fermions for exact, say, two flavour simulations [23]. It is not clear, how such a staggered fermion operator can be defined and also in this contribution no solution will be provided. The suggestion that is used in exact dynamical simulations, is to take the square root of the staggered operator [24], implemented by some polynomial approximation, which clearly defines an action that spreads over all lattice points. It should hence be tested, whether this operator is at least exponentially localized (see my plenary talk [24] for such a test on a Wilson operator).

There is a clear danger that through the square root operation a non-locality is introduced.

For a fixed set of parameters, i.e. fixed pion mass and fixed coupling, it should be expected that the localization of the operator is exponential, with couplings decaying like $e^{-r/r_{\text{local}}}$ with $r_{\text{local}}$ the localization range. In the continuum limit, staying at a fixed value of $r_0 \cdot m_\pi$, in principle two outcomes are possible. In scenario I, $r_{\text{local}} \cdot m_\pi = \text{constant}$ for $a \to 0$. In this case we would obtain a continuum field theory with a localization range of the order of the Compton wavelength of the pion. This would be clearly unacceptable. In order to save such a theory at least to some extent, it would be necessary that the localization range is much smaller than the Compton wavelength of the heaviest physical particle of the target continuum field theory.

In scenario II, $r_{\text{local}} \cdot m_\pi \to 0$ for $a \to 0$. In this case we would get a local continuum quantum field theory. One may suggest a “wishlist”, i.e. to perform a check of the localization properties of the square root staggered fermion operator.

Such a work is in progress [25]. We assume first, by lack of an alternative, that the two flavour staggered operator $M_{\text{staggered}}$ is defined by taking the square root of the four flavor staggered operator. Second, we think of an exact simulation which will employ this square root operator. In particular, following the arguments for deriving a strict locality bound of ref. [20], we expect the localization range to satisfy the bound $r_{\text{local}} \cdot m_{\text{quark}} = \text{constant}$. This behaviour follows directly from considering admissible gauge field configurations, the knowledge of the maximum eigenvalue of the staggered operator and the fact that the smallest eigenvalue is determined by the square of the bare quark mass. It remains to be

There might be additional shortcomings of such a definition of a two flavour theory such as the violation of the optical theorem. However, it is not clear (at least not to me), how such a violation can be detected in practical simulations.
seen, whether this theoretical bound is satisfied in practical simulations or whether it grossly overestimates the localization range as it was found in the overlap case \[20\].

2.5. Perturbation theory

For most of the actions just discussed, despite their sometimes complicated nature, perturbation theory has been worked out. There are famous seminal papers for staggered fermions, see the first ref. in \[17\] and \[26\], as well as for Wilson fermions \[27\]. These works build an important cornerstone to understand many aspects of lattice field theory such as renormalization. For more information on perturbation theory I refer to the recent review \[28\] and the talk by H. Trotter at this conference \[29\].

The only aspect of perturbation theory I would like to mention here is the Reisz power counting theorem \[30\]. This theorem guarantees that lattice integrals that appear in lattice perturbation theory exist in the continuum limit. The theorem is satisfied for Wilson fermions. However, staggered fermions do actually not obey one of the conditions of the theorem (see \[31\]). This does certainly not imply that there is a problem for staggered fermions in perturbation theory. However, for a consistent and well founded perturbative discussion of staggered fermion it would be very helpful to construct a “Reisz theorem” for staggered fermions.

Such a theorem for staggered fermions might also be very useful to analyze and underpin improved staggered actions, possibly leading to a disentanglement of the tastes in perturbation theory.

3. Algorithmic questions

Algorithms are the backbone of simulations in lattice field theory and testing and improving the algorithms cannot be appreciated enough. A review of existing algorithms for dynamical fermion simulations can be found in \[32\]. Promising new ideas can be found in \[33\], \[34\] and \[35\], see also \[36\].

The problem with our dynamical fermion simulations is the “cost wall”. We know to some extent the scaling behaviour of the algorithms and one formula for the cost $C$ in the case of Wilson fermions according to ref. \[37\] is

$$C = A \cdot \left( \frac{m_\pi}{m_\rho} \right)^{-z_\pi} L^{z_L} a^{-z_a}. \quad (1)$$

Here the exponents are given by $z_\pi = 6$, $z_L = 5$ and $z_a = 7$; the value of the constant $A$ can be found in \[37\]. Similar formulae are found by other groups \[38\]. Although this cost formula might still be plagued by a rather large uncertainty, it provides a good measure for the costs of the algorithms employed.

![Figure 1](image-url). A comparison of the cost of dynamical fermion simulations with (inexact) staggered (dotted line) and improved Wilson fermions (full line). The dashed line is the cost for Wilson fermions if the algorithms would perform a factor of four better than found in \[37\]. The left plot is for a value of the lattice spacing $a = 0.09\text{fm}$ and the right plot for $a = 0.045\text{fm}$. In fig. 1 we show a comparison of simulations with staggered (inexact, i.e. using the R-algorithm \[40\]) and Wilson fermions. Several things are remarkable in this graph. The first is that for both kind of fermions at some value of the quark mass the costs explode. This is the famous cost wall where the simulations become essentially un-feasible. The second point is that for
staggered fermions this point is reached at much lower values of the quark mass than for Wilson fermions. For the left plot we have used measured cost data from improved staggered fermion simulations [39] and the cost formula eq. (1) for Wilson fermions. A third observation is that the functional dependence of the cost on the quark mass seems to be the same for both actions, at least, given the accuracy of the data used here. The right plot shows again a cost comparison at a lower value of the lattice spacing, exhibiting a frightening need of teraflop years. The apparent advantage of the simulation cost using improved staggered fermions makes it even more important and urgent to test the theoretical basis of this approach as discussed in section 2.

One shortcoming of the staggered fermion simulations so far is that only a non-exact algorithm is used [40]. In principle we know how to execute an exact odd flavour algorithm [41]. A comparison of the in-exact algorithm and the exact one is shown in fig. 2. A striking effect of this graph is that the extrapolation to zero step size as needed in the in-exact simulation is non-monotonic. Hence such an extrapolation has to be done with very great care using many simulation points. Since exact odd flavour algorithms are on the market, it should be urged that they are indeed used in the staggered fermion simulations.

As a conclusion, it would be very useful to use and test exact algorithms and to perform a fair cost comparison of exact algorithms including the approach to the continuum limit.

The actions discussed in section 2 are sometimes very complicated, including often fattening of links. A good news is that methods have been developed that allow for simulations of such actions either by reject/accept steps [33] or by computing the force explicitly [42] using special projection methods.

As a last topic in this algorithm section I would like to discuss a problem of principle. Random matrix theory (RMT) [43] predicts the eigenvalue distribution for the lowest non-vanishing eigenvalue $\lambda$ in topological charge sector zero to be $P(\lambda) = \frac{1}{\lambda} e^{-\lambda^2}$ with $z = \lambda \Sigma V$ and $\Sigma$ the infinite volume scalar condensate [44]. This prediction is confirmed in practical simulations [45]. Such a distribution predicts the occurrence of very small eigenvalues to be frequent, and hence close to the critical point huge fluctuations have to be expected, rendering the simulations very expensive. Of course, a non-vanishing quark mass will regulate the problem, but it can do so only to a limited amount when the physical point at realistic values of the quark mass is reached. The conclusion is therefore that we have to expect that dynamical fermion simulations are becoming even more problematic than the cost formula of eq. (1) would imply when approaching small quark masses. As a result, the need to make contact with chiral perturbation theory is most urgent.

4. Scaling in the quenched approximation

Before turning to any results of unquenched simulations, let me discuss scaling in the quenched case first. Surprising results were presented by Aoki [46] at Lattice2000. He used different actions: staggered fermions with plaquette action, Wilson and improved Wilson fermions with plaquette and renormalization group inspired gauge actions. His results for $m_N/m_V$ for the different
actions as a function of \( m_{PS}/m_V \) (i.e., the Edinburgh plot) did not agree in the continuum limit even for for heavy quark masses. The staggered results disagreed with the Wilson results, which agreed among themselves. Surprisingly, there was not much response to this striking and worrisome observation afterwards.

The question arises, whether something could be wrong with the continuum limit of improved staggered fermions. In Aoki’s article, a particular form of the continuum extrapolation was taken, linear in the lattice spacing for Wilson type fermions and an \( a^2 \) dependence for the staggered results. In fig. 3 I show an alternative fitting procedure, allowing for an additional \( a^2 \)-dependence for the Wilson plaquette data\(^3\). Although this way of fitting the data might not be the final truth, it provides a possibility with the conclusion that there is nothing wrong with the continuum limit of staggered fermions. The continuum value for \( m_N/m_V \) is also consistent with results for various other improved actions as included in the plot. It would be very important, to supplement more and better data for such a plot and to

perform a precise scaling analysis for various fermion actions in the quenched approximation.

Such a test would be very re-assuring that all the calculations that are done with the many different actions lead to consistent values in the continuum limit and can hence lead to a good control over the systematic errors. For attempts for such a precise scaling tests I refer to the figure (done together with J. Zanotti) in my plenary talk [24] and the talks by A. and P. Hasenfratz at the conference [47,48].

5. Problems with dynamical fermion simulations

I now turn to somewhat surprising problems that are already encountered in practical simulations with dynamical fermions performed as of today. In the light of these problems, it may not be wise to start unthinkingly large scale and very expensive simulations with dynamical fermions but rather to concentrate on these problems for a while and try to understand and solve them.

5.1. Gauge actions

As mentioned before, improved gauge actions are not reflection positive. This leads to complex eigenvalues in the transfer matrix [5]. In fig. 4 the static potential \( V(r_0) \) is shown at a distance of the hadronic scale \( r_0 \) [49] as function of the euclidian time. The values of \( V(r_0) \) for the improved gauge actions approach their (in time) asymptotic value from below, indicating the effects of the complex eigenvalues [20]. Although for asymptotic times the value of the potential can be determined, this suggests that for these actions larger time extents would have to be used. This can have consequences in particular for determinations of glue ball masses from such actions.
Figure 4. Static potential for various gauge actions. Note that, as the effect of complex eigenvalues, in the case of improved gauge actions the potential starts from below.

From a free field analysis it also appears that different gauge actions might lead to very different lattice artefacts. In addition, the lattice spacing dependence need not to be very smooth approaching the continuum limit thus driving a reliable continuum extrapolation difficult. It might be useful therefore to perform simulations with different actions to check the continuum limit. An example, from my own experience, for such a procedure can be found in ref. where simulations using different fermion actions were performed to extract moments of parton distribution functions. We will come back to this point towards the end of this proceedings.

Another, last difficulty of improved gauge actions is that they are very inefficient to sample topological charge sectors. Although this is not too problematic for quenched simulations since there many configurations can be generated fast, it may have serious consequences for dynamical fermions when such actions are employed.

5.2. Wilson fermions

The CP-PACS collaboration started \( N_f = 3 \) simulations with improved Wilson fermions and the plaquette gauge action. By performing thermal cycles\(^4\), clear indications of a first order phase transitions were detected as can be seen in fig. where a pronounced hysteresis effect shows up. A similar effect appears also for \( N_f = 2 \) flavours if again the plaquette gauge action and non-perturbatively improved Wilson fermions are used. It might be speculated that these phase transitions are related to the phase structure of the fundamental-adjoint pure gauge action. Working in the vicinity of such a phase transition can lead, e.g., to unexpectedly large lattice artefacts that may render the continuum extrapolation difficult.

Using the Iwasaki action or the Symanzik gauge action the signs of the first order phase transitions seem to disappear completely. However, it is not clear, whether such effects eventually will strike back. Therefore, it would be

\(^4\)In this standard procedure to detect phase transitions, one starts at some value of \( \kappa_{\text{start}} \), increases the value of \( \kappa \) by some \( \Delta \kappa \) until a value of \( \kappa_{\text{final}} \) is reached and then runs back to \( \kappa_{\text{start}} \).
important to study and understand the $T = 0$ phase diagram of QCD, to investigate different gauge actions and to understand the nature of the phase transition.

| $N_f=0$ | $N_f=2$ |
|---------|---------|
| ![Figure 6](image) |

Figure 6. The quark mass difference of $m(\infty) - m(L)$ in quenched and in full QCD using non-perturbatively improved Wilson fermions ($L/a = \infty \equiv L/a = 16$). Note that for the unquenched case the lattice artefacts are substantially larger. As an aside, we also show results from standard Wilson fermions (W-W) where the lattice artefacts are even bigger (from [55]).

In fig. 6 we show a comparison of the lattice artefacts in the quark mass for quenched and unquenched simulations [55] using non-perturbatively improved Wilson fermions. The figure indicates that for dynamical fermions the artefacts are substantially larger, although they are still smaller than using naive Wilson fermions [56]. Whether this increase of lattice artefacts is due to the possible phase transition discussed above or originates from a completely different source, is an open question and gives additional motivation to understand the phase diagram better.

During the conference it has been pointed out [57] that in dynamical simulations using Wilson or staggered fermions the hadronic scale $r_0$ depends rather strongly on the quark mass at fixed values of $\beta$. Clearly, this effect has to be taken into account when quantities are analyzed as functions of, say, $r_0 \cdot m_{PS}$. This becomes especially important when comparisons with chiral perturbation are performed. In addition, it was shown that in the quantity $r_0(m)/r_0(m_{ref})$ with $m_{ref}$ determined by $m_{PS}/m_V = 0.45$, staggered fermions exhibit surprisingly large lattice artefacts. Clearly, it would be important to check and clarify this finding further with results from improved staggered fermion simulations.

### 5.3. Staggered fermions

For simulations with staggered fermions, naive or improved, there is a problem with the topological charge identification when coarse lattices at large values of the coupling are used. This can be seen quite evidently in studies that compare the predictions of the eigenvalue distributions predicted by RMT with the results of numerical staggered fermion simulations. Data obtained in topological charge sector 1 are fitted with the topology 0 prediction of RMT, while the curves for topology 1 fail completely [58]. These studies were mainly done with unimproved staggered fermions. For newer results with improved staggered fermions, see [57].

In [58] it was argued that this failure of staggered fermions with respect to questions related to topology has to occur when the gauge fields are not smooth enough and one is not working at small enough values of the coupling. The reason is that with such rough gauge fields, the taste breaking effects are strong and the symmetry breaking pattern is not the one of the continuum.

Indeed, in the first reference of [58] where simulations of the Schwinger model have been performed, there are indications that for larger values of $\beta$ the predictions of RMT and the numerical data get reconciled. Nevertheless, it would be very important to check explicitly that the predictions of RMT and other properties of topology are reproduced by, preferably, improved staggered fermions.
6. Results for dynamical Wilson fermions

This section is devoted to some results that have been obtained recently with dynamical Wilson fermions. Results concerning staggered fermions can be found in [60] and I will only shortly discuss staggered fermions in the context of chiral perturbation theory here.

6.1. Chiral perturbation theory

The algorithms we are using at the moment will sooner or later hit a cost wall where the exponential growth of the cost will set in with full power when the quark mass is lowered below a threshold value. In order to obtain results for physical values of the pion and \( \rho \) masses it becomes therefore mandatory to make contact to chiral perturbation theory (\( \chi PT \)) and then use \( \chi PT \) to perform the extrapolations to the “physical point”. Finding out the window where on the one hand lattice simulations can be performed and, on the other hand, \( \chi PT \) can be controlled and higher order effects can be safely neglected, is of central importance in lattice QCD and much effort will be invested to resolve this question. Fig. 1 suggests that improved staggered fermions may reach this overlap region sooner than Wilson fermions.

There are two main strategies to confront lattice results with predictions of \( \chi PT \):

- the first strategy is to extrapolate lattice data to the continuum limit first and then compare to \( \chi PT \);
- the second strategy is to take the lattice artefacts into account in \( \chi PT \) itself and compare then to lattice data.

Let me discuss the first strategy first. The big advantage of this procedure is that in the continuum chiral symmetry is restored and therefore a direct comparison to \( \chi PT \) becomes possible.\(^5\)

Clearly, the bad side of the coin is that performing the continuum limit is demanding and needs large computer resources.

This computational demand is at present “tricked out” by taking lattice data at only one value of the lattice spacing. Of course, the improved lattice actions in both the Wilson and the staggered cases substantially reduce the lattice artefacts. However, only future simulations can tell, whether the lattice data were indeed close enough to their continuum values.

In ref. [61] a recent analysis of chiral perturbation theory applied to lattice data are given. Older discussions are numerous and I have to refer to [62], the plenary discussion at Lattice 2002 and [63] for other examples (taken from the analysis of moments of parton distribution functions, my own hobby-horse).

\(^5\)Of course, using chiral invariant lattice fermions would also fulfill this requirement, but, they are, unfortunately, very expensive to simulate.
only at one value of the lattice spacing. Fig. 7 shows a comparison of the predictions from $\chi$PT in this setup and the data, which seems to be not so bad, even for quite large pion masses. However, in ref. 61 itself it is claimed: We stress again that applying the expressions to pion masses above 600 MeV is only done for illustrative purposes, for a realistic chiral extrapolation smaller pion masses are mandatory.

The reason for this becomes clear by looking at the size of corrections that can appear in a 1-loop calculation. These corrections can be very large, depending on the precise form of performing $\chi$PT. I do not have the space (and it is not the goal of this article) to discuss in detail which kind of $\chi$PT is correct, what flaws there might be and where conceptual difficulties appear. However, I believe, that finally it has to be pure $\chi$PT that has to be used, without any additional modeling or assumptions. Only then can we safely employ $\chi$PT to do for us the job of extrapolating to the physical point.

The second strategy is to take possible lattice artefacts into account in $\chi$PT itself. This idea was put to the fore by S. Sharpe 64 and was then further developed in 65 for Wilson and in 66 for staggered fermions. The advantage of this approach clearly is that chiral symmetry violating effects, taste violating effects in the case of staggered fermions and even different actions can be taken into account from the beginning. The disadvantage is that the corresponding chiral Lagrangian has twice as many low energy constants, and that the additional parameters depend on the bare coupling $g_0$ leading to a plethora of fit parameters.

This approach has been used already in practical simulations. For results with improved staggered fermions I refer to the contribution of S. Gottlieb at this conference 60. The only remark, I want to add here is that $\chi$PT provides a justification of taking the square root of the fermion determinant. This situation corresponds to a partially quenched setup in which two quarks are taken to be quenched 67. The idea can be generalized using the replica trick: take arbitrary numbers $N_u, N_d, N_s$ for the $u, d$ and $s$ quarks. Perform the computation in this general form and set in the end $N_u = N_d = N_s = 1/4 68$. In 69, although in a different context, it has been demonstrated that such a procedure is, at least to 1-loop order, completely equivalent to the super symmetric method and, although a proof to all orders is still missing, there is a good chance that the argument might work.

After this excursion to staggered fermions, let me now turn back to Wilson fermions. There are two works at the moment where simulation data are confronted to the predictions of $\chi$PT when lattice artefacts are included in $\chi$PT. The first work is ref. 70 and the second is ref. 71. We show in fig. 8 a plot of $m_{PS}^2/m_q$ as a function of the quark mass at various values of $\beta$. For each value of $\beta$ two curves are shown. The curves showing a very strong curvature represent continuum $\chi$PT while the lines that fit the data represent $\chi$PT with lattice artefacts taken into account. Clearly, continuum $\chi$PT is not able to describe the simulation data while lattice $\chi$PT fits the data well. Another observation is, however, that very large lattice artefacts in $m_{PS}^2/m_q$ seem to exist. Data at various values of $\beta$ come out to be quite different.

Another comparison is shown in fig. 9 (from 71). Here a ratio of decay constants of the type, $f_{VS}/(f_{VV}f_{SS})$ is plotted with $V$ and $S$ denoting valence and sea quantities. In this double ratio, which is only one example of the many fits that can be found in ref. 71, a good agreement with the predictions of $\chi$PT is seen. However, there seem to be tremendous cancellations of cut-off effects, keeping in mind that the simulations have been done at a value of the lattice spacing of $a = 0.28$ fm.

Another, somewhat worrisome observation is that the universal $\Lambda$-parameters come out to be very different, depending on the value of $\beta$ that is used (in case of 70) or how the lattice spacing is defined (in case of 71). Clearly, we are just in the beginning of using the new approach of $\chi$PT where lattice spacing effects are taken into account and the first results, using this method, are somewhat puzzling. In particular, I would like to advocate to de-double the double ratios in order to be able to disentangle the cut-off effects and show their size, furthering thus our understand-
Figure 8. $m_{PS}^2/m_q$ as a function of the quark mass for various values of $\beta$. For each value of $\beta$ two curves are shown. The curves showing a very strong curvature represent continuum $\chi$PT while the lines that fit the data represent $\chi$PT with lattice artefacts taken into account, from [70].

7. Numerical results

In this section, I would like to discuss a few selected results from recent dynamical Wilson fermion simulations.

7.1. Finite size effects

It seems that we finally see the exponential finite size effects of hadron masses. The general form of these finite size effects are $M(L) - M = -\frac{3}{(6\pi^2 M L)^2} \int_{-\infty}^{\infty} F(iy)e^{y^2} dy$ where $F$ is the $\pi - \pi$ forward scattering amplitude in infinite volume [72]. A calculation of the integral leads to the exponential finite size correction $M(L) - M \propto L^{-3/2}$. Corrections to this formula have been computed within $\chi$PT [73]. An alternative calculation was performed and also tested in ref. [74]. At this conference there were two contributions [74,76] that showed that indeed the finite size effects are exponential and not power like of the form $M = m_{\infty} + c/L^3$ as anticipated in ref. [76]. In fig. 10 a comparison of numerical simulation data to various fits of the finite size effects are shown. It seems that indeed the exponential form of finite size corrections are preferred by the data confirming thus the exponential finite size effects in lattice simulations. A similar picture and conclusion can be found in [74]. Clearly, these first results for the finite size effects need to be confirmed in future simulations.

7.2. Fundamental parameters of QCD

The running coupling and running quark masses are basic and important parameters of QCD that can be extracted from lattice simulations. The status of these calculations were reported at this conference [77]. Although the running itself has been determined already to a quite high precision, the determination of the physical scale is still missing. In addition, it would be necessary to complement the present simulations with even larger lattices to keep the continuum limit under better control. Both of these goals will certainly be done in the near future. In the past, the Schrödinger functional, which was used for the scale dependent renormalization [78], was the domain of the ALPHA collaboration. Nowadays, there are new groups who use this renormalization scheme. In particular, a serious attempt
for $N_f = 3$ flavours of dynamical fermions has been started by the CP-PACS and JLQCD collaborations [79].

The CP-PACS and JLQCD collaborations seem not only to join their forces to compute the coupling and quark masses but also for other quantities. This is a convincing indication that dynamical fermions are a real challenge and a very difficult problem. Present simulations employ the Iwasaki gauge action of ref. [6] combined with the Symanzik improved Wilson fermion action. The reason for this choice is the occurrence of a phase transition when the standard gauge action is employed as mentioned above. The value of $c_{SW}$ is the non-perturbatively improved one [80].

In the $N_f = 3$ simulations, the physical extent of the lattice is about $L \approx 1.6$ fm. With such a lattice, it is not possible to perform simulations for baryonic quantities reliably since the finite size effects will be very large [81]. Hence, at the moment one concentrates on mesons. In fig. 11 the relative difference of the K and Φ meson masses as compared to the experimental data is shown. The open triangles represent the quenched approximation where for different physical input value a large variation is found. It is very reassuring that for dynamical fermions this intrinsic uncertainty of the quenched approximation seems to be eliminated completely.

Figure 10. Finite size effects of the pion, ρ and Nucleon masses compared to various fits corresponding to different analytical predictions (from [75]). The exponential form of finite size effects seem to be clearly preferred.

Figure 11. Relative difference of the K and Φ meson masses as compared to the experimental data. Open triangles represent the quenched approximation.

Similar to what has been said above for the quenched approximation, it would be very interesting to combine Wilson, staggered and other dynamical fermion action results to check the different approaches in the chiral, continuum and infinite volume limits.

7.3. Other topics

An article like the present one has to be selective and a number of topics had to be left out, unfortunately. In the first place, I could not discuss progress with dynamical domain wall [82] or even overlap [83] fermions. For domain wall fermions it seems that an appropriate gauge action has been determined where an additional adjoint piece is added which seems to solve some problems with unexpected large values of the residual mass. The simulations for producing physics have been started and I am sure that at the next conference we will see a lot of interesting results. I would also like to mention some works that discuss the study of violations of CP [84] and even...
CPT \[85,86\] without being able to go into any detail.

I could also not go into results for moments of parton distribution functions from dynamical simulations \[87\], nor could I discuss questions concerning topology in depth \[88\]. Attempts to tackle the notoriously difficult problem of the $\eta'$ meson by making use of the eigenvalue spectrum can be found in \[89\]. My personal opinion here is that this is an interesting approach, but it still has a number of open questions that should be addressed in the future.

8. Conclusion

Obviously, there are dangerous animals on the lattice \[90\]. Our present dynamical fermion simulations are still in a rather early stage but they meet already such dangerous animals: unexpected phase transitions, questions of field theoretically non-local actions, actions with unitarity violations, occurrence of large lattice artefacts, algorithms that become extremely costly when approaching the physical point, uncertainties in the continuum limit of different lattice actions, difficulties with the chiral extrapolations.

In the real world, we have learned to tame lions and tigers and in lattice field theory this will be done for sure, too. However, we have to spend work for achieving this: we have to explore and understand the phase diagram of (zero temperature) QCD, we have to study the locality properties of taking the square root, we have to explore better the phase diagrams of extended gauge actions and understand the properties of such actions. We should perform scaling tests first in the quenched approximation to demonstrate to ourselves and to the rest of the world that universality holds (with all the caveats of the notion of universality in the quenched approximation). Participate in algorithm development. It is here, where a breakthrough can happen which would solve the cost-wall problem we have with existing algorithms; it would also be very helpful to join the ILDG \[91\] initiative to combine our efforts and exchange configurations.

One recommendation could be to not put all cards on one particular action. In fact, it would be wise to first select carefully actions that do not show the flaws and problems mentioned in this contribution. Even after such identifications have been made, it may be worthwhile to perform simulations with two (good) actions to check for systematic errors.

Another recommendation could be to use actions tailored for the physics problem one is interested in. If problems with rather heavy quark masses are considered, (theoretically sound) staggered fermions or improved Wilson fermions are a good choice. Going lighter, designer actions such as FLIC, hypercube, truncated perfect actions or domain wall fermions with rather small extra dimension might enter the game. Finally, for really light quarks, chiral invariant (up to machine precision) domain wall and overlap fermions should be used.

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