Supersymmetry Hierarchy Problems and Anomalous Horizontal U(1) Symmetry

Kiwoon Choi†, Eung Jin Chun*, and Hyungdo Kim†

Department of Physics, Korea Advanced Institute of Science and Technology
Taejon 305-701, Korea†

Department of Physics, Chungbuk National University
Cheongju, Chungbuk 360-763, Korea*

Abstract

It is suggested that various hierarchy problems in supersymmetric standard model, i.e. the Yukawa hierarchies, the $\mu$ problem, and the suppression of dangerous baryon and/or lepton number (B/L) violating couplings, are resolved altogether in the framework of horizontal U(1) symmetry whose spontaneous breaking results in the appearance of one expansion parameter (the Cabibbo angle). Within a reasonable range of U(1) charges, there exist a few models compatible with experiments. The specific sizes of B/L violating couplings of these models are calculated and several phenomenological consequences are discussed.

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1. Introduction

In some sense, the minimal supersymmetric standard model (MSSM) suffers from more hierarchy problems than the standard model (SM). The gauge invariance under $SU(3)_c \times SU(2)_L \times U(1)_Y$ would allow the following superpotential,

$$
W_{\text{MSSM}} = \mu_0 H_1 H_2 + Y_u^{ij} H_2 Q_i U^c_j + Y_d^{ij} H_1 Q_i D^c_j + Y_e^{ij} H_1 L_i E^c_j + \mu_i L_i H_2 + \Lambda^{u}_{ijk} U^c_i D^c_j D^c_k + \Lambda^{d}_{ijk} Q^c_j D^c_k + \Lambda^{e}_{ijk} L_i L_j E^c_k + \ldots,
$$

where $Y$’s and $\Lambda$’s are Yukawa couplings, $\mu_0$ and $\mu_i$ are dimension-one parameters, $\Gamma$’s denote the coefficients of B/L violating $d = 5$ operators, and the other big letters denote the superfields of Higgses, quarks and leptons. Concerning the above superpotential, one fundamental question which applies also for the SM is why the quark and lepton masses are hierarchical, e.g. why the up quark Yukawa coupling $Y_u \simeq 10^{-5}$ is much smaller than the top quark coupling $Y_t \simeq 1$. Unlike the case of the SM, the baryon number (B) and the lepton number (L) violating Yukawa couplings ($\Lambda$’s) generate also a kind of hierarchy problem since they are required to be highly suppressed. For instance, proton stability forces $\Lambda^u$ and/or $\Lambda^d$ to be extremely small: $\Lambda^u \Lambda^d \leq 10^{-24}$ [1]. Another hierarchy problem concerns the mass parameters, $\mu_0$ and $\mu_i$. The Higgs mass parameter $\mu_0$ should be of order of the electroweak scale. The $\mu$ problem [2] consists in understanding why $\mu_0$ is so small compared to the fundamental scale of the theory, e.g. the Planck mass $M_P$: $\mu_0/M_P \simeq 10^{-16}$. The parameters $\mu_i$ are required to be further suppressed by the smallness of neutrino masses [3] unless one assumes a special form of soft supersymmetry breaking [4]–[8]. Finally even the coefficients of B/L violating $d = 5$ operators, i.e. $\Gamma$’s, are required to be suppressed to a certain degree, e.g. $\Gamma_{112}^{i} \leq 10^{-8}$ and $\Gamma_{12j}^{0} \Lambda^{d}_{ijk} \leq 10^{-8}$.

It is certainly appealing to assume that the above-mentioned hierarchies in $W_{\text{MSSM}}$ have a common origin. Recently, the pattern of quark mass matrices are studied in the framework of supergravity (SUGRA) model [9]–[15] in which nonrenormalizable couplings of quarks and leptons to a SM singlet field $\phi$ are constrained by a horizontal abelian symmetry $U(1)_X$ to generate Yukawa hierarchies [16]. The vacuum expectation value of a singlet $\phi$ which breaks $U(1)_X$ yields the expansion parameter of Yukawa couplings:

$$
\lambda = \langle \phi \rangle / M_P \simeq 0.22 \quad (\text{Cabibbo angle}).
$$
It has been noted that the $\mu$ problem can be resolved also by means of $U(1)_X$ \[12\]. In this scheme, supersymmetry breaking is assumed to occur spontaneously in a hidden sector and is transmitted to the observable sector by supergravity interactions. The size of supersymmetry breaking in the observable sector is of order $m_{3/2}$ which can be identified as the electroweak scale. Then for a certain $U(1)_X$ charge assignment \[12\], $\mu$ appears to be of order $\lambda m_{3/2}$ as a consequence of the $U(1)_X$ selection rule. As was discussed recently, the horizontal symmetry $U(1)_X$ can be useful also for suppressing the dangerous B/L violating couplings \[14, 15\].

An interesting feature of the model with $U(1)_X$ is its connection to superstring theory. In the simple model with one expansion parameter $\lambda = \langle \phi \rangle / M_P$, the observed quark mass eigenvalues requires the Green-Schwarz mechanism to cancel the anomalies \[17\]. The ratio between the anomalies would be determined by the canonical value of $\sin^2\theta_w = 3/8$ at the string scale \[18\]. In this paper, we show how a horizontal abelian gauge symmetry compatible with the observed quark masses and mixing can constrain the B/L violating operators and also the $\mu$ terms to be phenomenologically safe. We then pick out several viable models with a reasonable range of $U(1)_X$ charges and discuss their phenomenological consequences.

In the models we found, all hierarchies in the MSSM superpotential, i.e. the hierarchical fermion masses and mixings, the hierarchically small $\mu$, and finally the hierarchically small B/L violating couplings including those of nonrenormalizable terms, can be understood by the $U(1)_X$ selection rule alone. It turns out that there exists only one such model (Model 1) if the maximum magnitude of the $U(1)_X$ charges is limited to be less than 10 for the basic unit of charge normalized to one. There appear several more models (Models 2 and 3 for instance) if one relaxes the limit to 15. Although quite attractive in the sense that all hierarchies have a common origin, we feel that the models, particularly Models 2 and 3, have a flaw that the magnitudes of the required $U(1)_X$ charges are still big (although not unreasonably big) in view of the the anomalous $U(1)$ charges in various string model constructions \[19\]. This would make their appearance as a low energy limit of string theory not very plausible. In this regard, an interesting possibility is that the model contains another spontaneously broken gauge symmetry (in addition to $U(1)_X$) which would be responsible for the weak scale value of $\mu$ and/or the suppression of some B/L violating couplings \[20\]. The $U(1)_X$ charges in this context can be smaller and thus fit better for string theory.
2. Basic properties

The quark mixing matrix $V_{\text{CKM}}$ in the Wolfenstein parameterization \[21\] is approximately given by

$$V_{\text{CKM}} \simeq \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix},$$

where all the coefficients of order 1 are omitted. The class of models under consideration assume that the Cabibbo angle originates from the spontaneous breaking of $U(1)_X$ as $\lambda = \langle \phi \rangle/M_P$. Under the additional assumption that $U(1)_X$ breaking is described entirely by the order parameter $\lambda$, the eigenvalues of up and down quark masses at the Planck scale are given by \[22\]:

$$(M^u)_{\text{diagonal}} \simeq m_t(\lambda^8, \lambda^4, 1),$$

$$(M^d)_{\text{diagonal}} \simeq m_b(\lambda^4, \lambda^2, 1).$$

As shown in ref. \[15\], two informations in eqs. (2) and (3) are enough to reconstruct the corresponding up and down quark mass matrices in our scheme. The observed up and down quark masses and mixing determine the six $U(1)_X$ charges of the MSSM superfields. Throughout this paper, we will use the small letters $q, u, d, l, e, h_1, h_2$ to denote the $U(1)_X$ charges of the corresponding MSSM superfields. The charge of $\phi$ can be any integer, say $-N$. But for the purpose of convenience we will normalize it to $-1$ which means that the charges of the MSSM superfields can be fractional numbers with $N$ in the denominator. Note that this means that the MSSM possesses an unbroken $Z_N$ parity for $N \geq 2$. The large top quark mass says that the top quark Yukawa coupling comes from the renormalizable term $H_2 Q_3 U_3^c$ in the SUGRA superpotential, and thus

$$h_2 + q_3 + u_3 = 0.$$

The bottom quark Yukawa coupling could well be obtained by the nonrenormalizable term $H_1 Q_3 D_3^c(\phi/M_P)^x$ where the positive integer $x$ is given by

$$x = h_1 + q_3 + d_3.$$

Here $x$ can be 0, 1, 2 or 3 with $\tan \beta \simeq \lambda^x m_t/m_b$. Denoting $q_{ij} \equiv q_i - q_j$ etc., the charge assignments

$$(1) \quad (q_{13}, q_{23}) = (3, 2), \quad (u_{13}, u_{23}) = (5, 2), \quad (d_{13}, d_{23}) = (1, 0)$$
(II) \((q_{13}, q_{23}) = (-3, 2), \ (u_{13}, u_{23}) = (11, 2), \ (d_{13}, d_{23}) = (7, 0)\) (4)

are known to yield the acceptable quark Yukawa matrices [14]. However, as we will see, the pattern (II) does not yield any acceptable model for the range of \(U(1)_X\) charges not exceeding 15 when the basic unit of charge is normalized to unity. Therefore here we quote the up and down quark Yukawa matrices only for the pattern (I):

\[
Y_u \simeq \begin{pmatrix}
\lambda^8 & \lambda^5 & \lambda^3 \\
\lambda^7 & \lambda^4 & \lambda^2 \\
\lambda^5 & \lambda^2 & 1
\end{pmatrix}, \quad Y_d \simeq \lambda^x \begin{pmatrix}
\lambda^4 & \lambda^3 & \lambda^2 \\
\lambda^3 & \lambda^2 & \lambda^2 \\
\lambda & 1 & 1
\end{pmatrix}.
\]

As a gauge symmetry, the horizontal symmetry \(U(1)_X\) has to be anomaly free. The mixed anomalies of \(SU(3)_c^2 - U(1)_X\), \(SU(2)_L^2 - U(1)_X\), \(U(1)_Y^2 - U(1)_X\) and \(U(1)_Y - U(1)_X^2\) are given by

\[
A_3 = \sum_i (2q_i + u_i + d_i),
\]
\[
A_2 = \sum_i (3q_i + l_i) + (h_1 + h_2),
\]
\[
A_1 = \sum_i \left( \frac{1}{3} q_i + \frac{8}{3} u_i + \frac{2}{3} d_i + l_i + 2 e_i \right) + (h_1 + h_2),
\]
\[
A'_1 = \sum_i (q_i^2 - 2u_i^2 + d_i^2 - l_i^2 + e_i^2) - (h_1^2 - h_2^2).
\]

As shown by Binetruy-Ramond [11], the observed quark masses are not compatible with the usual anomaly-free condition; \(A_3 = A_2 = A_1 = 0\). But the MSSM with \(U(1)_X\) symmetry may come from superstring theory which allows the Green-Schwarz mechanism of anomaly cancellation [17]. Furthermore, the gauge coupling unification near the Planck scale can be understood in terms of the Green-Schwarz mechanism when the anomalies satisfy the relation; \(A_3 : A_2 : A_1 = 1 : 1 : 5/3\) [18]. Therefore we assume that the horizontal symmetry \(U(1)_X\) is a gauge symmetry coming from superstring theory. In this case, the identity \(A_1 + A_2 - 8A_3/3 = 0\) implies

\[
h_1 + h_2 = \sum_i (q_{i3} + d_{i3}) - \sum_i (l_{i3} + e_{i3}).
\]

(7)

Throughout this paper, we assume that \(h_1 + h_2 = -1\) for which \(\mu_0\) appears to be of order the weak scale as a consequence of \(U(1)_X\). (See the subsequent discussion on the \(\mu\) parameters.) Then combined with the \(b-\tau\) unification condition,

\[
l_3 + e_3 = q_3 + d_3,
\]
the above relation from the anomaly cancellation provides an information on the charged lepton Yukawa couplings $Y^e$. Since $\sum_i (q_i + d_i) = 6$ from eq. (4), we have $\sum_i (l_i + e_i) = 7$, implying $\det(Y^e) = \lambda^6$. Then the observed charged lepton masses indicate that the eigenvalues of $Y^e$ are given by

$$\det(Y^e)_{\text{diagonal}} = \lambda^6(\lambda^5, \lambda^2, 1).$$

It is also useful to recall that the desired charged lepton mass matrix with the above eigenvalues follows from the charge relations \[15\]

$$(e_{13}, e_{23}) = (5 - l_{13}, 2 - l_{23}), \quad (e_{13}, e_{23}) = (9 - l_{13}, -2 - l_{23}).$$

In order to discuss how the couplings other than $Y^{u,d,e}$ and also the $\mu$ terms are constrained by the spontaneously broken U(1)$_X$, one needs to write down the most general Kähler potential invariant under SU(3)$_c \times$ SU(2)$_L \times$ U(1)$_Y \times$ U(1)$_X$ gauge symmetry. The U(1)$_X$ distinguishes the Higgs doublet $H_1$ from the lepton doublet $L_i$. (We call $H_1$ the field having the largest $\mu$, viz $\mu_0 \geq \mu_i$.) Let us write down the Kähler potential containing only $L_i$ and $H_{1,2};$

$$K = \frac{1}{2} H_1 H_1^\dagger + \frac{1}{2} H_2 H_2^\dagger + \frac{1}{2} L_i L_i^\dagger \left[ \left( \frac{\phi}{M_P} \right)^{l_i - l_j} \theta(l_i - l_j) + \left( \frac{\phi^\dagger}{M_P} \right)^{l_j - l_i} \theta(l_j - l_i) \right]$$

$$+ L_i H_1^\dagger \left[ \left( \frac{\phi}{M_P} \right)^{l_i - h_1} \theta(l_i - h_1) + \left( \frac{\phi^\dagger}{M_P} \right)^{h_1 - l_i} \theta(h_1 - l_i) \right]$$

$$+ H_1 H_2 \left( \frac{\phi^\dagger}{M_P} \right)^{-h_1 - h_2} \theta(-h_1 - h_2) + L_i H_2 \left( \frac{\phi^\dagger}{M_P} \right)^{-l_i - h_2} \theta(-l_i - h_2) + \text{h.c.},$$

where $\theta(y) = 1$ if $y$ is a non-negative integer, and $\theta(y) = 0$ otherwise. Note that the holomorphic operators $H_1 H_2$ and $L_i H_2$ can appear in the Kähler potential as well as in the superpotential.

The “effective” MSSM superpotential \[1\] generated after the spontaneous breaking of both supersymmetry and U(1)$_X$ contains the $\mu$ terms given by

$$\mu_0 = M_P \lambda^h + m_{3/2}, \quad \mu_i = M_P \lambda^{l_i + h_2} + m_{3/2},$$

where the first terms in the right hand sides arise from the underlying SUGRA superpotential, while the second terms are the contributions from the SUGRA Kähler potential. Here $\lambda^x \equiv \lambda^x \theta(x)$ and $\tilde{\lambda} = \lambda^*$. For the desirable value of $\mu_0 \simeq m_{3/2}$, the charge $h_1 + h_2$ may happen
to be $h_1 + h_2 = 23 \sim 25$, yielding $\mu_0 = M_P \lambda^{h_1+h_2} \simeq m_{3/2}$. In our approach, however, the anomaly-free condition does not allow such a large value of $h_1 + h_2$ [see eq. (7)]. As noted by Nir [12], the acceptable fermion mass matrices are compatible with the choice $h_1 + h_2 = -1$, which may actually solve the “$\mu_0$ problem” in the context of horizontal symmetry. Therefore, in this paper we will assume

$$h_1 + h_2 = -1,$$

(12)

for which $\mu_0 \simeq \bar{\lambda} m_{3/2}$. The smallness of $\mu_i$ can be understood in the similar manner. Even though the anomaly-free condition allows a large positive value of $l_1 + h_2$, one may still assume that $l_1 + h_2$ are all negative, and thus $\mu_i \simeq m_{3/2} \bar{\lambda} |l_1 + h_2|$.

Although we assume $h_1 + h_2 = -1$ in this paper, another choice of $h_1 + h_2 = 0$ can also give rise to acceptable fermion mass matrices while satisfying the anomaly-free condition (7). However then we need an independent mechanism, e.g. other spontaneously broken gauge symmetry [20], ensuring $\mu_0$ to be of order the weak scale since the horizontal symmetry allows $\mu_0$ to be of order $M_P$.

The Yukawa couplings of other renormalizable operators appearing in the effective MSSM superpotential are given by

$$Y^d_{ij} = \lambda^{h_1+q_i+d_j} \lambda^{h_1+l_i+e_j},$$

$$\Lambda^d_{ijk} = \lambda^{l_i+q_j+d_k} + \frac{m_{3/2}}{M_P} \bar{\lambda}^{l_i-q_j-d_k},$$

$$\Lambda^e_{ijk} = \lambda^{l_i+l_j+e_k} + \frac{m_{3/2}}{M_P} \bar{\lambda}^{l_i-l_j-e_k},$$

$$\Lambda^u_{ijk} = \lambda^{u_i+d_j+d_k} + \frac{m_{3/2}}{M_P} \bar{\lambda}^{u_i-d_j-d_k}.$$  

(13)

where we ignored the the Kähler potential contributions, i.e. the parts suppressed by $m_{3/2}/M_P$, to $Y^d,e$. Note that the up and down quark Yukawa couplings $Y^{u,d}$ are already given in eq. (7).

The above $\mu$'s in eq. (1) and the Yukawa couplings in eq. (13) are given in the non-canonical basis where the Kähler metric has off-diagonal components:

$$K_{Q_iQ_j} = \lambda^{q_{ij}}, \quad K_{U_i^{\dagger}U_j} = \lambda^{u_{ij}}, \quad K_{D_i^{\dagger}D_j} = \lambda^{d_{ij}},$$

$$K_{L_i^{\dagger}L_j} = \lambda^{l_{ij}}, \quad K_{E_i^{\dagger}E_j} = \lambda^{e_{ij}}, \quad K_{H_1^{\dagger}H_1} = \lambda^{h_{1i}}.$$  

(14)

The above Kähler metric can be diagonalized as $(K)_{\text{diagonal}} = (U^\dagger K U)$ where $U$ takes the same form as the Kähler metric in the order of magnitude estimate [13, 14]. This diagonalization
would alter the original estimate of the $\mu$’s and the couplings in eqs. (11) and (13). Especially, diagonalizing away the Kähler metric components $K_{L_iH_1^\dagger}$ leads to the change

$$
\mu_i \rightarrow \lambda^{L_i-h_1}_0|\mu_0 + \sum_j \lambda^{L_j}_0|\mu_j \ ,
$$

$$
\Lambda^d_{ijk} \rightarrow \sum_{n,p} \lambda^{L_i-h_1}_0|q_{jn}|+|d_{kp}|Y^d_{np} + \sum_{m,n,p} \lambda^{L_{im}+|q_{jn}|+|d_{kp}|}_0|\Lambda^d_{mnp} .
$$

As we will see, the above change of $\Lambda^d$ is essential for constraining the charges $l_i$ from the experimental bounds on $\Lambda^d$ since it is related to $Y^d$ which is known to us as eq (5). However the change of $\mu_i$ is not so relevant for us since it does not change the size of $\mu_i$ for the models under consideration.

We have to yet consider two more redefinitions of the couplings which would alter the estimated size of the couplings. First, normally the Yukawa couplings $\Lambda^d$ are defined after the $\mu_i$ terms, i.e. $\mu_i L_i H_2$, in the superpotential are rotated away. This results in an additional contribution to $\Lambda^d$, which is given by $\delta \Lambda_{ijk} \simeq Y^d_{jk}\mu_i/\mu_0$. In some cases, e.g. the case (iii) of the section 3, this contribution becomes dominant and thus alters the order of magnitude estimate of $\Lambda^d$, while in other cases, e.g. the case (ii) of the section (3), it doesn’t.

Another possibility is the change of couplings in the course of going to the mass eigenstates in order to make a contact with experiments. The quark and lepton Yukawa matrices $Y^I$ ($I = u, d, e$) can be diagonalized by biunitary transformations

$$
(Y^I)_{\text{diagonal}} = U_I Y^I V_I^\dagger .
$$

For us, the unitary matrix $U_I$ can be decomposed into three rotations described by the small angles $S^l_{12}, S^l_{13}$ and $S^l_{23}$:

$$
U_I \simeq \left( \begin{array}{ccc}
1 & -S^l_{12} & 0 \\
S^l_{12} & 1 & 0 \\
0 & 0 & 1
\end{array} \right) \left( \begin{array}{ccc}
1 & 0 & -S^l_{13} \\
0 & 1 & 0 \\
0 & 0 & 1
\end{array} \right) \left( \begin{array}{ccc}
1 & 0 & 0 \\
0 & 1 & -S^l_{23} \\
0 & S^l_{23} & 1
\end{array} \right) ,
$$

also similarly for $V_I$ with $S^{t}_1, S^{t}_1$ and $S^{t}_{23} \ [23]$. The general expressions for $S^l_{ij}$ and $S^{t}_{ij}$ are calculated in refs. [13, 14]. For the acceptable quark Yukawa matrices given by the charge assignments (I) and (II) of eq. (4), it is easy to find that

$$
S^l_{12} \simeq Y^l_{12}/Y^l_{22} , \quad S^l_{13} \simeq Y^l_{13} , \quad S^l_{23} \simeq Y^l_{23} ,
$$

$$
S^{t}_{12} \simeq Y^{t}_{21}/Y^{t}_{22} , \quad S^{t}_{13} \simeq Y^{t}_{31} , \quad S^{t}_{23} \simeq Y^{t}_{23} ,
$$

(18)
where $I = u, d$. In fact, we find also that the above expressions of the rotation angles are applicable also for the lepton Yukawa matrices satisfying all the phenomenological constraints. Therefore, the expressions of eq. (18) are valid for all $I = u, d, e$ in our scheme. For the biunitary transformations defined by the angles of eq. (18), the diagonalization of the quark and lepton mass matrices gives the same effect on the order of magnitudes of the couplings as the diagonalization of the Kähler metric. As a consequence, the mass diagonalization does not change further the order of magnitudes of the couplings once the effects of the Kähler metric diagonalization are taken into account as eq. (15).

Combining all the $U(1)_X$ charge relations discussed so far with the anomaly-free conditions

$$A_3 = A_2 = 3A_1/5, \quad A'_1 = 0,$$

we are left with four independent charges, for instance $l_i$ and $x$. In section 4, we vary $l_i$ and $x$ under the condition that $x = 0, 1, 2, \text{or } 3$ to find some reasonable charge assignments which fulfill the bounds on B/L violating couplings which will be discussed in section 3.

3. Constraints on B/L violating terms

Let us first discuss in detail the L violating terms. The existence of $\mu_i$ or $\Lambda_{d,e}$ plays an important role of generating significant neutrino masses when their values are not too small [3]. Therefore, it is interesting to see whether some phenomenologically observable neutrino masses and mixing [24] can arise naturally in our scheme. As seen from eqs. (13) and (15), $\mu_i$ and $\Lambda_{ijk}$ are closely related to $\mu_0$ and $Y^d_{jk}$ by the value of $l_i - h_1$. There are then the following three possibilities.

(i) $l_i - h_1$ is a fractional number with $N \geq 2$ in the denominator. In this case, the $U(1)_X$ charges of the operators $L_iQ_jD_k^c$ and $L_iH_2$ are fractional also, and thus neither $\Lambda_{ijk}$ nor $\mu_i$ are allowed. In order to see this, let $y_{ijk}^d$, $y_{ij}^d$, and $y_i$ denote the $U(1)_X$ charges of $L_iQ_jD_k^c$, $H_1Q_iD_j^c$, and $L_iH_2$ respectively. Then

$$y_{ijk}^d = l_i + q_j + d_k = (l_i - h_1) + y_{jk}^d,$$

$$y_i = l_i + h_2 = (l_i - h_1) - 1.$$  \hfill (19)

Since $y_{jk}^d = (q_{j3} + d_{k3}) + x$ are all integers [see eq. (14)], obviously $y_{ijk}^d$ and $y_i$ are all fractional for a fractional value of $l_i - h_1$. 

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Table 1: Bounds on the $\Lambda^d$ and $\Lambda^u$ from various experiments. $\tilde{m}$ stands for typical squark or slepton mass.

| couplings | upper bound | experiment |
|-----------|-------------|------------|
| $\Lambda^d_{i12} \Lambda^d_{i21}$ | $10^{-10} (\tilde{m}/\text{TeV})^2 \sim \lambda^{15}$ | $\epsilon$ [25] |
| $\Lambda^d_{i13} \Lambda^d_{i31}$ | $3.6 \times 10^{-7} (\tilde{m}/\text{TeV})^2 \sim \lambda^{10}$ | $\delta m_B$ [26] |
| $\Lambda^u_{i13} \Lambda^u_{i23}$ for $i = 2, 3$ | $3 \times 10^{-4} (\tilde{m}/\text{TeV}) \sim \lambda^{5}$ | $\epsilon$ [25] |
| $\Lambda^d_{j12} \Lambda^e_{ijk}$ for $j = 2, 3, k = 1, 2$ | $8 \times 10^{-7} (\tilde{m}/\text{TeV})^2 \sim \lambda^0$ | $K_L \to e\bar{e}, \mu\bar{\mu}$ [26] |
| $\Lambda^u_{11k} \Lambda^d_{ijk}$ for $j = 1, 2$ | $10^{-24} (\tilde{m}/\text{TeV})^2 \sim \lambda^{37}$ | proton decay |
| $\Gamma_{i12}$ | $10^{-8} (\tilde{m}/\text{TeV}) \sim \lambda^{11}$ | proton decay [27] |

(ii) $l_i - h_1$ is a negative integer or zero. In this case, $\mu_i/\mu_0 \simeq \lambda^{|l_i-h_1|} \leq 1$. Let us first consider the pattern (I). If $y^d_{i11}$ is a non-negative integer and $l_i - h_1$ is not zero, the SUGRA superpotential would give $\Lambda^d_{i11} \geq \lambda^{x+3}$ since $0 \leq y^d_{i11} \leq x+3$ from eq. (4). Diagonalization of the Kähler metric then gives rise to $\Lambda^d_{ijk} \rightarrow \lambda^{|q_{i1}|+|d_{i1}|} \Lambda^d_{i11}$, which leads to $\Lambda^d_{i12} \simeq \Lambda^d_{i21} \geq \lambda^{x+4}$. Obviously this is inconsistent with the first experimental bound in Table 1 for $0 \leq x \leq 3$. The same is trivially true when $l_i - h_1 = 0$ for which $\Lambda^d_{i12} \simeq \Lambda^d_{i21} \simeq \lambda^{x+3}$. We thus conclude that for the pattern (I), $y^d_{i11}$ (and thus all charges $y^d_{ijk}$) should be negative, and thus there is no contribution to $\Lambda^d_{ijk}$ from the SUGRA superpotential. Since $y^d_{i11} = l_i - h_1 + x + 4$ from eq. (4), a negative integer value of $y^d_{i11}$ implies

$$|l_i - h_1| \geq x + 5.$$  \hfill (20)

In this case, another contribution to $\Lambda^d_{ijk}$ from the diagonalization of the Kähler metric, i.e. $\lambda^{|l_i-h_1|} Y^d_{jk}$, can satisfy the bounds in the Table 1. More explicitly, in this case, we have

$$\mu_i/\mu_0 \simeq \Lambda^d_{ijk}/Y^d_{jk} \simeq \lambda^{|l_i-h_1|} \leq \lambda^{5+x}.$$  \hfill (21)

In the case of generic soft terms, the neutrino mass of order $\mu_i^2/\mu_0$ is generated at tree-level [4]–[8]. Therefore in the scheme under consideration, we have $m_\nu \simeq \lambda^{2|l_i-h_1|} \mu_0 \leq 30 \lambda^{2x} \text{ keV}$. If we assume the universality of soft-terms which may be necessary to suppress the flavor changing neutral currents, there will be a loop-suppression factor of order $10^{-5} \sim 10^{-6}$ [4], so that $m_\nu \leq 0.1 \lambda^{2x} \text{ eV}$.
For the pattern (II), more possibilities are allowed. For \(|l_i - h_1| \geq 10 + x\), we get \(\mu_i/\mu_0\), \(\Lambda_{ijk}^d/Y_{jk}^d \simeq \lambda^{l_i-h_1}\) as in the case of the pattern (I). The resultant tree-level neutrino masses are \(m_\nu \leq 7 \times 10^{-3} \lambda^{2x}\) eV. In addition to this, the cases with \(l_i - h_1 = 0\) (only for \(x = 2\)) and \(|l_i - h_1| = 5 + x\) also fulfill the first and second bounds on \(\Lambda^d\) in Table 1. The case with \(l_i - h_1 = 0\) would be disfavored since it leads to a too large neutrino mass when soft terms are generic. Independently of this point, it turns out that the cases with the pattern (II) can not be compatible with the proton stability bound for the range of \(U(1)_X\) charges not exceeding 15.

(iii) \(l_i - h_1\) is a positive integer large enough to make \(\mu_i \simeq \lambda^{l_i-h_1-1} M_P \leq \mu_0\). For this, we would need at least \(l_i - h_1 \geq 25\) if we take \(\mu_0/M_P \simeq \lambda^{24}\). (Throughout this paper, we will set \(m_{3/2}/M_P \simeq \lambda^{23}\) and thus \(\mu_0/M_P \simeq \lambda m_{3/2}/M_P \simeq \lambda^{24}\).) Later, we will see whether this case can be realized for \(l_i\) and \(h_1\) whose magnitudes are allowed to be as large as 15. Before rotating away \(\mu_i L_i H_2\) from the effective superpotential, \(\Lambda_{ijk}^d\) appears to be extremely small since \(y_{ijk}^d \geq x + 25\) for \(l_i - h_1 \geq 25\). However after rotating away the \(\mu_i\) terms, we have \(\Lambda_{ijk}^d \simeq Y_{jk}^d \mu_i/\mu_0 \simeq \lambda^{l_i-h_1-25} Y_{jk}^d\). Then the first experimental bound of Table 1 demands for the pattern (I) to satisfy \(l_i-h_1-25 \geq 5 - x\). In the same way, for the pattern (II), we need \(l_i-h_1-25 \geq 2 - x\). In summary, we find for the case of \(l_i-h_1 \geq 25\)

\[
\frac{\mu_i}{\mu_0} \simeq \frac{\Lambda_{ijk}^d}{Y_{jk}^d} \simeq \lambda^{l_i-h_1-25} \leq \begin{cases} \lambda^{5-x} & \text{(I)} \\ \lambda^{2-x} & \text{(II)} \end{cases}
\]

This case would allow larger \(\mu_i\) (or \(\Lambda_{ijk}^d\)) than the case (ii) and thus larger neutrino masses. However again we do not find any example of this class for the range of \(U(1)_X\) charges not exceeding 15.

Similarly to the L violating couplings \(\Lambda_{ijk}^d\), the B violating couplings \(\Lambda_{ijk}^u\) are determined also by one parameter, \(b_0 \equiv u_3 + 2d_3\). For the pattern (I) and (II), the \(U(1)_X\) charges \(y_{ijk}^u\) of \(U_i^c D_j^c D_k^c\) are given by

\[
\begin{align*}
\text{(I)} & \quad b_0 + \begin{pmatrix} 6 & 6 & 5 \\ 3 & 3 & 2 \\ 1 & 1 & 0 \end{pmatrix}, \\
\text{(II)} & \quad b_0 + \begin{pmatrix} 18 & 18 & 11 \\ 9 & 9 & 2 \\ 7 & 7 & 0 \end{pmatrix}
\end{align*}
\]

where the low is for \((u_1, u_2, u_3)\) and the column is for \((d_1 d_2, d_1 d_3, d_2 d_3)\). Obviously, if \(b_0\) is fractional, \(y_{ijk}^u\) are all fractional and thus \(\Lambda_{ijk}^u\) are all vanishing due to the unbroken \(Z_N\). For
the case that $b_0$ is an integer, $K$-$\bar{K}$ mixing, i.e. the third bound in Table 1, sets a constraint on $b_0$ but only for the pattern (I) as

$$b_0 \leq -4 \quad \text{or} \quad b_0 \geq 2. \quad (24)$$

Here we do not consider the bound on $\Lambda^u_{ijk}$ coming from the $n$-$\bar{n}$ oscillation [28] or double nucleon decay [25] since it can be as large as order one for $\tilde{m} \simeq 1$ TeV and for a generous value of the hadronic scale [29].

When both $l_i - h_1$ and $b_0$ are integers, the sum of them is constrained by proton stability. For an integer $b_0$, $\Lambda^u_{11k} \simeq \lambda y^u_{11k}$ for $y^u_{11k} \geq 0$ and $\Lambda^u_{11k} \simeq \lambda^{-y^u_{11k}} m_{3/2}/M_P \simeq \lambda^{25-y^u_{11k}}$ for $y^u_{11k} < 0$, where $y^u_{11k} = b_0 + 6$ and $b_0 + 18$ for (I) and (II) respectively. Also $\Lambda^d_{12k} \simeq \lambda^{x+2+n_i}$ where the non-negative integer $n_i = h_1 - l_i$ or $l_i - h_1 - 25$ from eqs. (21) or (22). Then the proton stability condition reads $n_i + y^u_{11k} \geq 35 - x$ for $y^u_{11k} \geq 0$ and $n_i - y^u_{11k} + 25 \geq 35 - x$ for $y^u_{11k} < 0$.

In addition to this, one also has to consider the nonrenormalizable terms in the effective superpotential (1). For instance, $\Gamma^l_{ijk} \simeq \lambda y^l_{ijk}$ should be suppressed appropriately as in Table 1 when $y^l_{ijk} \equiv q_i + q_j + q_k + l_i$ is a nonnegative integer. Rewriting $y^l_{ijk} = (l_i - h_1) - b_0 + 2x + 1 + (q_i3 + q_j3 + q_k3)$, one can see that the operators $Q_iQ_jQ_kL_l$ are allowed even when both $l_i - h_1$ and $b_0$ are fractional numbers as long as their sum is an integer. Combined with $\Lambda^d_{ijl}$, the next $d = 5$ coupling $\Gamma^0_{ijk}$ can also induce a too fast proton decay unless $\Gamma^0_{12j}A^d_{ijk} \leq 10^{-8} \simeq \lambda^{11}$ for $k = 1, 2$. It turns out that this bound is simply satisfied as $\Gamma^0_{ijk} \leq \lambda^{11}$ in all the models which pass the proton stability conditions in Table 1. About the third $d = 5$ coupling $\Gamma^l_{ijk}$, proton stability bound depends upon the unknown mixing in the gluino couplings as $\Gamma^l_{ijk}(K_{RR})_{1j} \leq 10^{-9} \simeq \lambda^{12}$ for $j = 2, 3$ and $k, l = 1, 2$ [27]. As we do not have any information on $K_{RR}$ in our approach, this bound will not be taken into account. Combined with $\Lambda^u_{ijk}$, the coefficients of other higher dimensional operators like $[QUcE^cH_1]_F$, $[QUcT]_D$ and $[UDcEC^c]_D$ are restricted also by the proton stability. However in our scheme, typically $\Lambda^u_{ijk}$ comes from the Kähler term. As a result, $\Lambda^u \leq m_{3/2}/M_P$ and thus those bounds are trivially satisfied.

4. Models

Let us now find some models, i.e. some U(1)$_X$ charge assignments, satisfying all the bounds on B/L violating couplings in Table 1. As seen from many superstring models [19], we expect the
Table 2: Model 1. U(1)$_X$ charges of the MSSM fields in the range of maximum charge 10. Here $N = 1$, $x = 3$.

| $i$ | $q_i$ | $u_i$ | $d_i$ | $l_i$ | $e_i$ | $h_1$ | $h_2$ | $\Lambda_{11k, 2l}^u \Lambda_{32k}^d$ | $\Gamma_{112}^{1/2}$ | $\Gamma_{132}^{1/3}$ |
|-----|------|------|------|------|------|-------|-------|-------------------------------|----------------|----------------|
| 1   | 7    | 9    | -7   | -8   | 9    | 7     | -8    | $\lambda^{45}$               | $\lambda^{12}$ | $\lambda^{11}$ |
| 2   | 6    | 6    | -8   | -8   | 6    | 7     | -8    |                               |                 |                 |
| 3   | 4    | 4    | -8   | -4   | 0    | 0     | 0     |                               |                 |                 |

U(1) charges are not too large. In the former investigations [14, 15], some examples satisfying the phenomenological bounds on B/L violations are worked out, however all of them have ridiculously large U(1) charges. If the maximum charge is limited to be less than 10 (for the smallest charge normalized to the unity), we find only one acceptable charge assignment (Model 1 of Table 2) with the pattern (I) and $N = 1$. In the last three columns of the Table, we provide the predicted size of the couplings which are relevant for the proton decay. One can see that, in Model 1, proton can decay with a rate not far below the current experimental limit. This is essentially due to the nonrenormalizable couplings $\Gamma_{112}^{1/2}$ since the renormalizable couplings $\Lambda_{ij}^u \Lambda_{ij}^d$ are far below the current limit. In fact, it is a generic feature of our scheme that the nonrenormalizable couplings are somewhat close to the current experimental limits. In Model 1, we have $b_0 = u_3 + 2d_3 = -12$ and thus all the charges of $U_i^c D_j^c D_k^c$ are negative [see eq. (23)]. As a result, $\Lambda_{ij}^u$ arise only from the SUGRA Kähler potential and thus are suppressed by the extremely small factor $m_{3/2}/M_P \approx \lambda^{23}$. However the couplings $\Lambda_{ij}^{d,e}$ can be induced through rotating away the off-diagonal Kähler metric components $K_{L_i H_1}^*$, yielding $\Lambda_{ij}^{d,e} \approx \lambda^{l_i-h_1} Y_{jk}^{d,e}$. In summary, the charged lepton Yukawa couplings in Model 1 are found to be

$$Y^e \simeq \lambda^3 \begin{pmatrix} \lambda^5 & \lambda^2 & \lambda^4 \\ \lambda^5 & \lambda^2 & \lambda^4 \\ \lambda^6 & 1 & 1 \end{pmatrix},$$

and the magnitudes of the nonzero B/L violating couplings in Model 1 are given by

$$\Lambda_{ij}^u \simeq \lambda^{29} \sim \lambda^{35}, \quad \Lambda_{ij, 2j, 3j}^{d,e} \approx \lambda^{15, 15, 11} Y_{jk}^{d,e}, \quad \mu_{1, 2, 3} \simeq \lambda^{15, 15, 11} \mu_0,$$

$$\Gamma_{ij}^{1/2} \simeq \lambda^6 \sim \lambda^{16}, \quad \Gamma_{ij}^0 \simeq \lambda^{21} \sim \lambda^{27}, \quad \Gamma_{ij}^{1/3} \simeq \lambda^2 \sim \lambda^{17}.$$  

(26)
Table 3: Model 2. $U(1)_X$ charges of the MSSM fields in the range of maximum charge 15. Here $N = 2$ and $x = 0$.

| $i$ | $q_i$ | $u_i$ | $d_i$ | $l_i$ | $e_i$ | $h_1$ | $h_2$ | $\Lambda^u_{11k} \Lambda^d_{32k}$ | $\Gamma^3_{112}$ | $\Gamma^{1,2}_{132}$ |
|-----|-------|-------|-------|-------|-------|-------|-------|-------------------------------|----------------|----------------|
| 1   | 11/2  | 7     | -5    | -4    | 11/2  | 7/2   | -9/2  | $\Lambda^3$                  | $\lambda^{11}$ | 0              |
| 2   | 9/2   | 4     | -6    | -7    | 11/2  | 7/2   | -9/2  | $\Gamma^0_{ijk}$              | $\lambda^{13}$ | $\lambda^{19}$ |
| 3   | 5/2   | 2     | -6    | -9/2  | 1     | 1     | 1     | $\Gamma^3_{ijk}$              | $\lambda^{13}$ | $\lambda^{19}$ |

Since $\mu_i$ are very small, $\mu_i \leq \lambda^{11} \mu_0 \simeq 6$ keV, the tree-level neutrino masses due to $\mu_i$ are negligible, $m_\nu \leq 3 \times 10^{-4}$ eV, even for generic soft supersymmetry breaking terms.

When the maximum value of the $U(1)_X$ charges is relaxed up to 15, there appear basically two types of additional models and all of them possess an unbroken $Z_2$ subgroup of $U(1)_X$. The first type of models allows $\Lambda^u_{ijk}$ to be nonvanishing since the operators $U^c_i D^c_j D^c_k$ are $Z_2$ even, while the second type does not. In Table 3, we show a representative model (Model 2) of the first type which has the following lepton Yukawa couplings

$$Y^e \simeq \begin{pmatrix} \lambda^5 & \lambda^5 & 0 \\ \lambda^2 & \lambda^2 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

and the nonvanishing B/L violating couplings:

$$\Lambda^u_{ijk} \simeq \lambda^{27} \sim \lambda^{33}, \quad \Lambda^d_{3jk} \simeq \lambda^8 Y^d_{jk}, \quad \Lambda^e_{131,132,231,232} \simeq \lambda^{13,13,10,10},$$

$$\mu_3 \simeq \lambda^8 \mu_0, \quad \Gamma^3_{ijk} \simeq \lambda^5 \sim \lambda^{11}, \quad \Gamma^0_{ijk} \simeq \lambda^{13} \sim \lambda^{19}, \quad \Gamma^3_{ijk} \simeq \lambda \sim \lambda^7.$$  

Notice that $U^c, D^c, L_{1,2}, E^c_3$ are $Z_2$ even, while the others are odd. Therefore after $U(1)_X$ breaking into $Z_2$, the operators like $L_3 Q D^c, U^c D^c D^c$, and $QQQL_3$ can be induced, leading to the proton decay. Contrary to Model 1, proton life-time is on the verge of the experimental limit due to the larger values of both $\Lambda^u_{11k} \Lambda^d_{32k}$ and $\Gamma^3_{112}$. Neutrino mass in Model 2 can be large as $m_{\nu_1} \simeq \lambda^{12} \mu_0 \simeq 3$ keV for generic soft terms. It could in fact be smaller than the cosmological bound ($m_\nu \lesssim 100$ eV) depending upon other parameters of the theory [4]–[8]. In addition to Model 2, there is another model of first type with almost same properties except that e.g. $\mu_2$ is allowed instead of $\mu_3$. 

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Table 4: Model 3. $U(1)_X$ charges of the MSSM fields in the range of maximum charge 15. Here $N = 2$ and $x = 3$.

| $i$ | $q_i$ | $u_i$ | $d_i$ | $l_i$ | $e_i$ | $h_1$ | $h_2$ | $\Lambda^u \Lambda^d$ | $\Gamma_{112}^2$ | $\Gamma_{132}^2$ |
|-----|-------|-------|-------|-------|-------|-------|-------|----------------|---------------|---------------|
| 1   | 6     | 15/2  | -7/2  | -7/2  | 7     | 0     | 0     | $\lambda^{13}$   | $\lambda^{10}$ |               |
| 2   | 5     | 9/2   | -9/2  | -4    | 9/2   | -11/2 | 0     |               |               |               |
| 3   | 3     | 5/2   | -9/2  | -7/2  | 2     | 0     | 0     | $\lambda^{13}$   | $\lambda^{10}$ |               |

The models of the second type have $Z_2$ parity which forbids both $U^c D^c D^c$ and $QQQH_1$. In particular, the representative model (Model 3) in Table 4 has the $Z_2$ parity under which $Q$, $L_2$, $E^c_{1,3}$ are even and the other are odd. In Model 3, the charged lepton Yukawa couplings are given by

$$ Y^e \simeq \lambda^3 \begin{pmatrix} \lambda^5 & 0 & 1 \\ 0 & \lambda^2 & 0 \\ \lambda^5 & 0 & 1 \end{pmatrix}.$$

(29)

and the magnitudes of the nonvanishing B/L violating couplings are

$$ \Lambda^d_{ijk,3jk} \simeq \lambda^8 Y^d_{jk}, \quad \Lambda^e_{101,122,232,133} \simeq \lambda^{0,13,13,5}, $$

$$ \mu_{1,3} \simeq \lambda^8 \mu_0, \quad \Gamma_{ijk} \simeq \lambda^7 \simeq \lambda^{13}, \quad \Gamma_{ijk}^2 \simeq \lambda^7 \simeq \lambda^{13}. $$

(30)

The neutrino mass from $\mu_{1,3} \simeq \lambda^8 \mu_0$ is $m_\nu \simeq 3$ eV. As mentioned earlier, the neutrino masses are further suppressed if soft terms satisfy certain universality conditions. We found also three more models of the second type in which $\Gamma_{112}^l$ are in the range of $\lambda^{15} \sim \lambda^{13}$ and $\Gamma_{ijk}^l$ are usually much larger than $\lambda^{10}$. Therefore, the proton decay rate in the second type models is smaller than the previous models by factor of $\lambda^{2,4}$ or less.

Let us finally comment on the couplings $\Gamma_{ijk}^l$. The bound on this coupling from the proton decay depends upon the mixing in the gaugino couplings: $\Gamma_{ijk}^l (K_{RR}^u)_{ij} \leq \lambda^{12}$ for $j = 2, 3$ and $k, l = 1, 2$. Models 1 and 3 can be consistent with this constraint when the flavor mixing in gaugino couplings are $(K_{RR}^u)_{1j} \simeq \lambda^{1,2}$ respectively, while in Model 2 even an arbitrary flavor mixing would not cause proton decay. If it is required, one could make the flavor mixing small in our scheme for instance by assuming the usual universality of soft terms at $M_P$.
5. Conclusion

To summarize, we suggest the relevance of an anomalous horizontal abelian symmetry for the resolution of all the hierarchy problems in the supersymmetric standard model, viz the quark and lepton mass hierarchy, the $\mu$ problem, and the highly suppressed (both renormalizable and nonrenormalizable) B/L violating interactions. This anomalous $U(1)_X$ would be a gauge symmetry as found in many superstring models endowed with the Green-Schwarz anomaly cancellation mechanism. In view of various string model constructions, the magnitudes of $U(1)_X$ charges are not likely to be so large. Observed quark masses and mixings, lepton masses and several experimental bounds on B/L violating couplings are used together with the assumption that $\mu_0 \simeq \lambda m_{3/2}$ in order to single out only a few models with reasonable charge assignments. For the most acceptable charge assignment allowing the biggest $U(1)_X$ charge to be 9, only one model with $N = 1$ (Model 1) is found. In this model, renormalizable B/L violating couplings (including $\mu_i$) are extremely suppressed, and thus not yield any observable signature. On the other hand, the coefficients $\Gamma_{112}^{1,2} \simeq \lambda^{12}$ of nonrenormalizable $d = 5$ operators $Q_i Q_j Q_k L_l$ are relatively large, so that may render proton decay observable in the near future.

Relaxing the limit of $U(1)_X$ charges to 15, we found two types of additional models with an unbroken $Z_2$ parity, i.e. models with $N = 2$. The models of the first type of allow both $L_i Q_j D^c_k$ and $U^c_i D^c_j D^c_k$ after the $U(1)_X$ breaking into $Z_2$. They predict a marginally detectable proton decay due to the renormalizable couplings $\Lambda^u_{11k} \Lambda^d_{22k} \simeq \lambda^{37}$ as well as the nonrenormalizable couplings $\Gamma_{112}^{1,2} \simeq \lambda^{11}$ which are on the verge of the current bound. In the second type of models, $U^c_i D^c_j D^c_k$ are $Z_2$ odd and thus are completely forbidden. Its representative model (Model 3) has $\Gamma_{112}^{2} \simeq \lambda^{13}$ which is away from the proton stability bound by the factor of $\lambda^2$. In our scheme, renormalizable B/L violating terms tend to be highly suppressed, while nonrenormalizable couplings are not far from the current experimental limits. In particular, the operators $U^c_i U^c_j D^c_k E^c_l$ may have coefficients larger than $\lambda^{12}$, and then (approximate) squark degeneracy has to be implemented for the proton stability. The $\mu_i$ are typically small enough to yields cosmologically safe neutrino masses even for generic forms of soft supersymmetry breaking. We however find no models with L violation patterns providing solar or atmospheric neutrinos.

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References

[1] S. Weinberg, Phys. Rev. D 26 (1982) 287; N. Sakai and T. Yanagida, Nucl. Phys. B 231 (1982) 523.

[2] J. E. Kim and H. P. Nilles, Phys. Lett. B 138 (1984) 150.

[3] L. Hall and M. Suzuki, Nucl. Phys. B 231 (1984) 419.

[4] R. Hempfling, preprint MPI-PhT/95-59, hep-ph/9511288.

[5] B. de. Carlos and P. L. White, preprint SUSX-TH/96-003, hep-ph/9602381.

[6] T. Banks, Y. Grossman, E. Nardi and Y. Nir, Phys. Rev. D 52 (1995) 5319; F. M. Borzumati, Y. Grossman, E. Nardi and Y. Nir, preprint WIS-96/21/May-PH, hep-ph/9606251.

[7] H. P. Nilles and N. Polonsky, preprint TUM-HEP-245/96, hep-ph/9606388.

[8] E. Nardi, preprint WIS-96/38/Oct-PH, hep-ph/9610540.

[9] M. Leurer, Y. Nir and N. Seiberg, Nucl. Phys. B 398 (1993) 319; Nucl. Phys. B 420 (1994) 468.

[10] L. Ibáñez and G. G. Ross, Phys. Lett. B 332 (1993) 100; E. Papageorgio, Phys. Lett. B 343 (1995) 263.

[11] P. Binetruy and P. Ramond, Phys. Lett. B 350 (1995) 49; V. Jain and R. Shrock, Phys. Lett. B 352 (1995) 83. T. Gherghetta, G. Jungman and E. Poppitz, preprint UM-TH-95-27, hep-ph/9511317.

[12] J. Nir, Phys. Lett. B 354 (1995) 107; V. Jain and R. Shrock, preprint ITP-SB-95-22, hep-ph/9507238. E. J. Chun, Phys. Lett. B 367 (1996) 226.
[13] E. Dudas, S. Pokorski and C. A. Savoy, Phys. Lett. B 356 (1995) 45.

[14] P. Binetruy, S. Lavignac and P. Ramond, Nucl. Phys. B 477 (1996) 353.

[15] E. J. Chun and A. Lukas, Phys. Lett. B 387 (1996) 99.

[16] C. D. Frogatt and H. B. Nielsen, Nucl. Phys. B 147 (1979) 277.

[17] M. Green and J. Schwarz, Phys. Lett. B 149 (1984) 117.

[18] L. Ibáñez, Phys. Lett. B 303 (1993) 55.

[19] A. Font, L. E. Ibanez, H. P. Nilles and F. Quevedo, Phys. Lett. B 210 (1988) 101; J. A. Casas, E. K. Katehou and C. Munoz, Nucl. Phys. B 317 (1989) 171; E. J. Chun, J. E. Kim and H. P. Nilles, Nucl. Phys. B 370 (1992) 105.

[20] J. C. Pati, preprint UMD-PP/97-5, hep-ph/9607416; K. Choi, E. J. Chun and H. Kim, preprint KAIST-TH 14/96, hep-ph/9610504.

[21] L. Wolfenstein, Phys. Rev. Lett. 51 (1983) 1945.

[22] P. Ramond, R. G. Roberts and G. G. Ross, Nucl. Phys. B 406 (1993) 19.

[23] L. J. Hall and A. Rasin, Phys. Lett. B 315 (1993) 164.

[24] For a review, see e.g., A. Y. Smirnov, preprint hep-ph/9311359.

[25] R. Barbieri and A. Masiero, Nucl. Phys. B 267 (1986) 679.

[26] D. Choudhury and P. Roy, preprint MPI-PTh/96-20, hep-ph/9603363.

[27] V. Ben-Hamo and Y. Nir, Phys. Lett. B 339 (1994) 77.

[28] F. Zwirner, Phys. Lett. B 132 (1983) 103.

[29] J. L. Goity and M. Sher, Phys. Lett. B 346 (1995) 69.