Quantum three-body calculation of the nonresonant triple-$\alpha$ reaction rate at low temperatures

Kazuyuki Ogata

(in collaboration with M. Kan and M. Kamimura)

Department of Physics, Kyushu University
The triple $\alpha$ reaction

\[ \alpha (^4\text{He}) \rightarrow ^8\text{Be} (0^+) \quad (\tau \sim 10^{-17} \text{ sec}) \]

\[ ^8\text{Be} (0^+) + \alpha \rightarrow ^{12}\text{C} (0^+) \]

\[ ^{12}\text{C} (0^+) + \gamma \rightarrow ^{12}\text{C} (2^+) \]

Fred Hoyle

I think, therefore it is!

Question:
How is this picture of the triple alpha reaction accurate?
The resonant and nonresonant $3\alpha$ process

- $T > \text{a few } 10^8 \text{ K: resonant capture is dominant.}$
  - $T < 10^8 \text{ K: nonresonant capture is important.}$
Results of Nomoto’s method

— K. Nomoto et al., Astrophys. J. 149, 239 (1985).

nonresonant contribution!

“nonresonant” contribution

resonant rate

total rate
Nomoto’s method for “nonresonant” capture

\[ \phi(r) \]

\[ \varepsilon_1 \]

\[ \chi(R) \]

\[ R_1 \]

\[ \alpha + \alpha + \alpha \]

\[ 12C \ (0^+ \ (379.5 \text{ keV}) \]

\[ 8Be + \alpha \]

\[ E_\text{res}^{\text{res}} = 287.5 \text{ keV} \]

\[ E_\text{res}^{\text{res}} = 92.04 \text{ keV} \]

\[ \varepsilon_2 \]

\[ \varepsilon_1 \]

\[ \phi^* (r) \left[ T_R + V_{\alpha-\alpha} (R_1) + V_{\alpha-\alpha} (R_2) + h_{\alpha\alpha} - E \right] \phi(r) \chi(R) dr = 0 \]

\[ \left[ T_R + V_{\alpha\alpha-\alpha} (R) + \varepsilon_1 - (\varepsilon_1 + \varepsilon_2) \right] \chi(R) = 0 \]

Accurate only if \( \alpha\alpha-\alpha \) interaction is independent of the \( \alpha\alpha \) states!
The nonresonant $3\alpha$ reaction: now and past

- **Preceding studies**
  - Pioneering study on nonresonant capture by Nomoto (*Nomoto’s method*)
    - K. Nomoto, Astrophys. J. 253, 798 (1982);
    - K. Nomoto et al., Astrophys. J. 149, 239 (1985).
  - Potential model by Langanke
    - K. Langanke et al., Z. Phys. A 324, 147 (1986).

  Still based on the **resonance picture** with an “energy shift” of the Hoyle state as a correction

- **This work**

  Accurate description of the three-body reaction treating the **resonant and nonresonant** processes on the same footing.

  c.f. M. Kamimura and Y. Fukushima, *Proceedings of the INS International Symposium on Nuclear Direct Reaction Mechanism*, Shikanoshima, Fukuoka, Japan, 1978, p. 409.
  P. Descouvemont and D. Baye, Phys. Rev. C 36, 54 (1987).
\[ \Psi_{i,E}^{3\alpha}(\mathbf{r}, \mathbf{R}) = \phi_i(\mathbf{r}) \chi_{i,E}(\mathbf{R}) \]

\[ \left[ T_R + V^{i}_{\alpha_1\alpha_2 - \alpha_3}(\mathbf{R}) + \varepsilon_2 \right] \chi_{i,E}(\mathbf{R}) = 0, \]

\[ V^{i}_{\alpha_1\alpha_2 - \alpha_3}(\mathbf{R}) = \left\langle \phi_i(\mathbf{r}) \left| V^{N+C}_{\alpha_1\alpha_3}(\mathbf{R}_1) + V^{N+C}_{\alpha_2\alpha_3}(\mathbf{R}_2) \right| \phi_i(\mathbf{r}) \right\rangle_r. \]
Constraints on $V^N_{\alpha\alpha}$

$V^N_{\alpha\alpha}$: 2-range Gaussian (with repulsive part simulating the Orthogonal Condition Model; OCM)

1. $^8$Be resonance properties

   $\varepsilon_{1\text{res}} = 92.0$ keV, $\Gamma = 4.8$ eV
   exp. $92.04\pm0.05 \quad 5.57\pm0.25$

2. Hoyle resonance properties (for $i = 86$)

   $\varepsilon_{2\text{res}} = 287.5$ keV, $\Gamma = 4.0$ eV
   exp. $287.5 \quad 8.5\pm1.0$

Achieved by reducing $V^N_{\alpha\alpha}$ by only 1.5% in

$$V^{ij}_{\alpha_1\alpha_2-\alpha_3}(R) = \langle \phi_i(r) | V^{N+C}_{\alpha_1\alpha_3}(R_1) + V^{N+C}_{\alpha_2\alpha_3}(R_2) | \phi_j(r) \rangle_r.$$
Reaction rate of the $3\alpha$ reaction

- **E2 transition from $3\alpha$ scattering state**

\[
(\sigma v)_{k_{i_0},E} = \frac{2 (2\pi)^7}{75\hbar} \left( \frac{\hbar \omega}{\hbar c} \right)^5 \sum_M \left| \left\langle \Psi_M^{2+} | O_{M}^{3\alpha} \right| \Psi_{i,E}^{3\alpha} \right|^2
\]

- **Reaction rate**

\[
\langle \alpha\alpha\alpha \rangle(T) = 3N_A^2 \frac{4}{\pi (k_B T)^3} \int \left\{ \sum_{i_0=1}^{122} w_{i_0} (\sigma v)_{k_{i_0},E} \right\} \exp \left( -\frac{E}{k_B T} \right) dE
\]

\[
w_{i_0} = \frac{2\hat{\epsilon}_{12,i_0}}{\hat{k}_{i_0}} \sqrt{\hat{\epsilon}_{12,i_0}(E - \hat{\epsilon}_{12,i_0})}
\]

- **Correction with effective charge $\delta e$ to reproduce $\Gamma_\gamma$**

- We include $\delta e = 0.77 e$ so that the B(E2) value obtained by the normalized $0^+_2$ W.Fn. and the $2^+_1$ W.Fn. reproduces the exp. value of $13.4 e^2 \text{ fm}^4$. 

W.Fn obtained by Gaussian Expansion Method (GEM) with rearrangement.
Resonant and nonresonant Coulomb potentials are completely different. Nomoto’s method neglects this difference and is a crude approximation.
The reaction rate

— K.O., M. Kan, and M. Kamimura, Prog. Theor. Phys. 122 (2009) 1055; arXiv:0905.0007 [astro-ph.SR].

We have normalized our results to the rate of NACRE at $10^9$ K. Normalization factor is 1.5 that indicates the uncertainty of our calculation.

C. Angulo et al., Nucl. Phys. A656 (1999), 3.
Implication of the new reaction rate

A. Dotter and B. Paxton, arXiv:0905.2397 [astro-ph.SR].

Evolutionary implications of the new triple-α nuclear reaction rate for low mass stars

Result:
The OKK rate has severe consequences for the late stages of stellar evolution in low mass stars. Most notable is the shortening-or disappearance-of the red giant phase.

Conclusions:
The OKK triple-α reaction rate is incompatible with observations of extended red giant branches and He burning stars in old stellar systems.
Effects of a new triple-alpha reaction rate on the helium ignition of accreting white dwarfs

M. Saruwatari, M. Hashimoto, R. Nakamura (Kyushu University)
S. Fujimoto (Kumamoto National College of Technology), K. Arai (Kumamoto University)

- The helium ignitions occur in the low density by two orders of magnitude if the OKK rate is adopted.

- Nuclear flashes are triggered for all cases of A-F in the helium layers.
Summary

- The triple-$\alpha$ reaction rate is reevaluated.
  - The resonant and nonresonant processes are described on the same footing.
  - The $\alpha_1$-$\alpha_2$ nonresonant states below the resonance are essentially important.
  - The $(\alpha_1$-$\alpha_2$)$\alpha_3$ Coulomb barrier in the nonresonant capture process is much lower than that in the resonant process.
  - We obtain a markedly larger reaction rate than NACRE below $10^8$ K.
  - Nomoto’s method (used in many studies including NACRE) is shown to be a very crude approximation to the present three-body calculation.

Future plan

- How can we resolve the inconsistency of a stellar evolution calculation with our new rate and observation?
- Systematic studies of ternary processes: $\alpha(\alpha n, \gamma)^9$Be, $n(p\alpha, ^6$Li) etc.