Initial state radiation correction and its effect to data-taking scheme for $\sigma^B(e^+e^- \rightarrow ZH)$ measurement

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The measurement of Born cross section of $e^+e^- \rightarrow ZH$ process is one of the major goals of the future Circular Electron Positron Collider, which may reach a precision of 0.5%. Such unprecedented precision must be guaranteed by both theoretical and experimental sides, such as the calculations of high order corrections, the knowledge of the $\sigma^B(e^+e^- \rightarrow ZH)$ line shape. The uncertainty of radiative correction factor due to lack of knowledge of the $\sigma^B(e^+e^- \rightarrow ZH)$ line shape is evaluated in this work. Therefore, the dedicated data-taking schemes are proposed in order to precisely calculate the ISR correction factor.

Keywords: Higgs-strahlung; cross section; radiative correction; data-taking scheme.

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1. Introduction

The historic observation of the Higgs boson in 2012 at the Large Hadron Collider (LHC)\textsuperscript{1,2} declared the discovery of the last missing piece of the most fundamental building blocks in the Standard Model (SM). Although the SM has been remarkably successful in describing experimental phenomena, a precision Higgs physics program will be critically important given that the SM does not predict the parameters in the Higgs potential, nor does it involves particle candidates for dark matters. In particular, potential observable deviations of the Higgs couplings from SM expectations would indicate new physics. Therefore, the Higgs discovery marks the beginning of a new era of theoretical and experimental explorations.

China has been investigating the feasibility of a high energy Circular Electron Positron Collider (CEPC) as a Higgs factory since 2013\textsuperscript{3,4}. The CEPC will collide electrons and positrons at the center of mass energy of 240 GeV with an instantaneous luminosity of $3 \times 10^{34}$ cm$^{-2}$s$^{-1}$. With a clean environment, CPEC will provide a clearer picture of the nature of Higgs and reveal many of the most pro-
found mysteries intimately connected with the Higgs particle. The three leading Higgs production processes at a 240 GeV CEPC are: Higgs-strahlung ($e^+ e^- \rightarrow ZH$) and vector boson fusions ($e^+ e^- \rightarrow \nu \bar{\nu}H$ and $e^+ e^- \rightarrow e^+ e^- H$). The CEPC is designed to collect $5.6 \text{ ab}^{-1}$ of integrated luminosity with two detectors in seven years, producing about $1.1 \times 10^6$ Higgs events.

One of the advantages at a $e^+ e^-$ collider like CEPC is that the center of mass energy is precisely defined and many absolute measurements could be performed for Higgs boson. In a $ZH$ event, where the $Z$ boson decays to a pair of visible fermions ($Z \rightarrow e^+ e^-$, $\mu^+ \mu^-$, or $q \bar{q}$), the Higgs boson can be identified with the kinematics of these fermion pairs independent its decays. It is claimed that the CEPC is able to measure the observed cross section ($\sigma_{\text{obs}}$) of $e^+ e^- \rightarrow ZH$ at 240 GeV to a precision of 0.5% by combining all the three channels of $Z$ boson decays.$^4$

The Born cross section ($\sigma^B$), which is directly applicable to the theoretical analysis or independently comparing with results from other experiments, can then be obtained by applying corrections for initial state radiation (ISR) and other high order corrections. Unfortunately, the ISR correction depends on not only the theoretical calculations but also the line shape of the $e^+ e^- \rightarrow ZH$ cross section, which needs to be constrained by experimental data. In this paper, the dedicated data-taking schemes for the radiative correction to the cross section of $e^+ e^- \rightarrow ZH$ at 240 GeV for CEPC is investigated. The data-taking schemes are optimized to collect data samples economically and effectively in order to achieve a significant better precision of the ISR correction factor and to satisfy requirements experimentally and theoretically. Problems of determination of the center of mass energies and their integrated luminosities that need to be accumulated are carefully studied.

This paper is organized as follows: the ISR effect and theoretical formulas for the radiative correction are described in Sec. 2, followed by the procedure to calculate the radiative correction factor in Sec. 3, the data-taking schemes are suggested in Sec. 4, and finally summary and discussion about the results are presented.

2. ISR effect

The ISR effect is an issue that cannot be avoided at $e^+ e^-$ colliders. One of the incoming particles ($e^+$ or $e^-$) emits photon(s) before the interaction with the other, which reduces the beam energy prior to the momentum transfer. The ISR effect can be described with the structure function approach,$^5$ which yields an accuracy of 0.1% due to the uncertainty of the radiative function $F(x, s)$. Therefore, the experimental $\sigma_{\text{obs}}$ of $e^+ e^-$ colliders can be mathematically factorized as the integral of the Born cross section with the high order correction factors and $F(x, s)$,

$$\sigma_{\text{obs}}(s) = \int_0^{1-s_m/s} \frac{\sigma^B(s(1-x))}{|1 - \Pi(s(1-x))|^2} F(x, s) \, dx,$$

where $\sigma^B(s)$ is the Born cross section at the center of mass energy $\sqrt{s}$ of the colliding beam, and $\sqrt{s_m}$ in the upper limit of the integral is the production threshold of the
specific reaction, and $1/[1-\Pi(s)]^2$ represents all the high order corrections and must be taken into account in such a precision measurement. Since the high order factors are independent of experiments and not issues to concern in this paper, it is dropped hereafter.

The ISR correction factor is defined to extract the Born cross section from the observed one

$$1 + \delta = \frac{\sigma^{\text{obs}}(s)}{\sigma^B_{\text{gen}}(s)},$$

where it should be noted that the $\sigma^{\text{obs}}(s)$ and $\sigma^B_{\text{gen}}(s)$ are usually calculated with some dedicated generator(s) with some experiment-dependent kinematic cuts and measured $\sigma^B$ line shape from the threshold up to $\sqrt{s}$ as inputs. Then the Born cross section at $\sqrt{s}$ can be determined by

$$\sigma^B(s) = \frac{\sigma^{\text{obs}}(s)}{1 + \delta}.$$

It is clear that the ISR correction factor depends on not only theoretical calculations, but also experiment measurements. Furthermore its uncertainty directly contributes to the $\sigma^B$. For the sake of convenience, $\Delta_{\text{ISR}}$ is used to represent relative uncertainty of the ISR correction factor $[\Delta(1 + \delta)/(1 + \delta)]$ throughout the paper.

3. Calculation of ISR correction factor

3.1. Model independent measurement of $\sigma(ZH)$

In the Higgs-strahlung process, the $e^+e^-$ annihilate into a virtual Z boson and become a real Z by emitting a Higgs boson, with the Z boson mainly decaying to a pair of fermions afterward. The center of mass energy is precisely controllable at an $e^+e^-$ collider like the CEPC. The Higgs boson can be identified with the recoil mass of the fermion pair system with the following formula $m^2_{\text{recoil}} = (\sqrt{s} - E_{f\bar{f}})^2 - \mu^2_{f\bar{f}} = s - 2E_{f\bar{f}}\sqrt{s} + m^2_{f\bar{f}}$, where $E_{f\bar{f}}, \mu_{f\bar{f}}$ and $m_{f\bar{f}}$ are the energy, momentum and invariant mass of the fermion pair system. The $ZH$ event yield can be extracted independently of the Higgs decays with the $m_{\text{recoil}}$ spectrum.

Events with $Z$ decaying to $e^+e^-, \mu^+\mu^-$, and $q\bar{q}$ are three ideal ways to identify the $e^+e^- \rightarrow ZH$ recoil mass spectrum and cover a majority of 76.6% of the $Z$ decay modes. The observed cross section is calculated using

$$\sigma^{\text{obs}}(e^+e^- \rightarrow ZH) = \frac{N^{\text{obs}}}{L_{\text{int}}\epsilon_{f\bar{f}}B_{f\bar{f}}},$$

where $N^{\text{obs}}$ is the total number of $ZH$ events observed, $L_{\text{int}}$ the integrated luminosity accumulated at a certain $\sqrt{s}$, $B_{f\bar{f}}$ is the branching fractions of $Z$ decaying to $e^+e^-, \mu^+\mu^-$, or $q\bar{q}$. The efficiency $\epsilon_{f\bar{f}}$ of event selection for the reaction is obtained by a full detector simulation and digitalization procedures. The three $Z$ decay modes are combined to form the final $\sigma^{\text{obs}}(e^+e^- \rightarrow ZH)$ in order to improve the precision.
3.2. Method to extract ISR correction factor

The ISR effect impacts on not only the production rate of \( ZH \) process but the shape of the recoil mass spectrum, which is used to determine the signal yield. Therefore, a full knowledge of ISR correction is essential for both the determinations of the \( \sigma^B(e^+e^- \rightarrow ZH) \) and the Higgs boson mass. The expressions in Eq. (1)-(3) manifest mathematically that constraining the line shape of the Born cross section from production threshold to 240 GeV is needed to get a precise measurement of the ISR correction factor at 240 GeV. The only feasible way is to collect a series of scan data samples between the threshold and 240 GeV to constrain the line shape of \( \sigma^B(e^+e^- \rightarrow ZH) \).

The procedure described here is used to demonstrate the dependence of the ISR correction factor on the line shape of \( e^+e^- \rightarrow ZH \) process. First, The Born cross section is assumed to be SM-like and the radiative function, \( F(x, s) \) is used to calculate the observed cross section. Then the MC signals of \( e^+e^- \rightarrow ZH \) process are generated at 216, 220, and 240 GeV with luminosities of 0.2, 0.2, and 5.6 ab\(^{-1}\), respectively. Next, same analysis method is repeated to get observed cross sections and evaluate their statistical uncertainties. And next, the observed cross section of all energy points are fitted using Eq. (1) and \( \sigma_{\text{Fit}}^{\text{obs}} \) and \( \sigma_{\text{Fit}}^B \) are obtained simultaneously as shown in Fig. 1. Finally, the ISR correction factor is calculated using Eq. (2): \( (1 + \delta) = \sigma_{\text{Fit}}^{\text{obs}}/\sigma_{\text{Fit}}^B \) at 240 GeV, where \( \sigma_{\text{Fit}}^{\text{obs}} \) and \( \sigma_{\text{Fit}}^B \) are the best knowledge on the observed and Born cross sections and will be implemented into generators.

After repeat the above procedure 10,000 times, the distribution of the ISR correction factor is found to satisfy a Gaussian distribution as expected. The fitted mean and standard deviation are taken as the central value and uncertainty of the ISR correction factor, respectively. Fig. 2 illustrates the fit results, and the relative

![Fig. 1. The fit to the MC samples generated at 216, 220, and 240 GeV with luminosities of 0.2, 0.2, and 5.6 ab\(^{-1}\), respectively. The blue dashed line refers to the \( \sigma^B(e^+e^- \rightarrow ZH) \) in the MC generator, the black foursquare markers with errors refer to the observed cross sections calculated with MC samples, the blue and red lines are \( \sigma^B \) and \( \sigma^{\text{obs}} \) with the fitted parameters.](image-url)
4. Optimization of Data-taking scheme

In Eq. (1), the integral kernel $\sigma^B(s)$ is Born cross section, which should be determined by experiments. The reason is that Higgs properties are still not yet solidly determined due to the limited experimental precision. On one hand, the present experimental results are sufficient to discriminate between distinct hypotheses in Higgs boson spin analysis. But on the other hand, the determination of the $CP$ properties is in general much more difficult, since in principle the observed state could consist of any admixture of $CP$-even and $CP$-odd components. If physics is the SM, i.e., a single resonance with spin-0 and $CP$-even, the Born cross section of the Higgs-strahlung process is expected to reach its maximum at 250 GeV approximately, and then decreases with increasing center of mass energy. From the experimental point of view, the center of mass energies of the data samples determine the uncertainty of the fitted line shape of the cross section. Besides, the allocation of integrated luminosity of the various energy points could also make differences on the statistical uncertainty of the ISR correction factor and of the $\sigma^B$. In conclusion, dedicated scan data samples between the threshold and 240 GeV are needed to constrain the line shape of $e^+e^\to ZH$ process.

For a measurement of 0.5% statistical uncertainty, 0.25% is a reasonable requirement for the systematic uncertainties. In order to achieve a relative precision of 0.25% for the ISR correction factor, an economical and effective way of collecting data samples should be proposed for the CEPC. The effects of energies and allocation of integrated luminosity are investigated systematically in this section.

![Fig. 2. The fit result of the distribution of calculated $(1+\delta)$ using simulated samples at 216, 220, and 240 GeV with luminosities of 0.2, 0.2, and 5.6 $ab^{-1}$, respectively. The blue histogram is the distribution of 10,000 samplings. The red line is the fit result with a Gaussian function. The fitted mean value and standard deviation are 0.855 and $2.03 \times 10^{-3}$, respectively.](image-url)
4.1. Determination of the energies

The impact of energies is studied by changing the combination of center of mass energies of MC samples. As a Higgs factory, CEPC is designed to accumulate a total of 5.6 ab$^{-1}$ integrated luminosity running at 240 GeV. The line shape of the Born cross section based on MC sample at this single energy point of 240 GeV is shown in the left of Fig. 3. The Born cross sections are constrained to the blue band with a bad performance in the low energy region which indicates that more data are in need below 230 GeV. Our study shows that besides the established 5.6 ab$^{-1}$ data-taking plan at 240 GeV, at least two more data samples at lower energy region are necessary to form a stable fit. With the luminosities fixed to 0.2 ab$^{-1}$, MC samples are generated with center of mass energies varying from 215 to 239 GeV with a step of one GeV. Then fits are performed with the data sample at 240 GeV together with MC samples randomly picked at two other lower energy points. As illustrated in the right of Fig. 3, the Born cross sections are constrained to a much narrower band in the lower energy region.

The most accurate fit results of the energy combinations with the lowest energy of MC sample varies from 215 to 224 GeV are listed in Table 1 and the full-scale fit results can be found in Fig. 4. As an example, the red points refers to fit results with lowest energy MC sample generated at 216 GeV, the horizontal axis is the center of mass energy of MC sample at the middle energy point. Fit results with different lowest energy MC sample are shown in the same figure. The common feature is that $\Delta_{ISR}$ decrease as a function of the center of mass energy of the MC sample at middle energy point, then increase above a certain point after three to four GeV interval approximately. The energy combination is the most critical factor for the accuracy of $1+\delta$. For example, with the same luminosity combination of 0.2, 0.2, and 5.6 ab$^{-1}$, the uncertainty can differ from 0.24% to 14%. Other luminosity allocations are studied, besides the fit is also applied with more than three MC samples, we find
Table 1. Center of mass energies of three MC samples versus the uncertainty of the ISR factor at 240 GeV. The luminosities for the three samples (energies from low to high) are fixed to 0.2, 0.2, and 5.6 ab$^{-1}$, respectively.

| $\sqrt{s}$ (GeV) | $\Delta_{\text{ISR}}$ (%) |
|------------------|-------------------------|
| 215              | 220 240                 | 0.239 |
| 216              | 220 240                 | 0.237 |
| 217              | 221 240                 | 0.238 |
| 218              | 222 240                 | 0.263 |
| 219              | 223 240                 | 0.288 |
| 220              | 225 240                 | 0.336 |
| 221              | 225 240                 | 0.366 |
| 222              | 227 240                 | 0.407 |
| 223              | 227 240                 | 0.458 |
| 224              | 228 240                 | 0.499 |

that three MC samples at 216, 220, and 240 GeV gives a best accuracy on average in case of the same integrated luminosity.

4.2. Allocation of integrated luminosity

The energies of three MC samples are fixed to 216, 220, and 240 GeV, which provides the best performance on average. With the luminosity for the MC sample at 240 GeV fixed to 5.6 ab$^{-1}$, we change the luminosities of the other two MC samples for the purpose of investigating the effect of statistics. A significant improvement is that the relative uncertainty $\Delta_{\text{ISR}}$ are all below 0.3% with the energies fixed at 216, 220, and 240 GeV as shown in Fig. 5. The luminosity of the MC sample at lowest energy is fixed at a certain value, at first the uncertainty decrease rapidly as the luminosity of the MC sample at the middle energy increases. But the slope gradually

Fig. 4. Fit results with three MC samples. Different colors denote fit results with MC samples at different lowest energies (in units of GeV) as shown in the legend. The horizontal axis is the center of mass energy for MC sample at middle energy point. The center of mass energy for the MC sample at the highest energy is fixed to 240 GeV.
reduce until finally reaching a plateau region where $\Delta_{\text{ISR}}$ has a little reduction with the increase of the luminosity of the MC sample at the middle energy point. A comparison between results with different colors indicates that with a higher statistic for the MC sample at lowest energy, the slope of $\Delta_{\text{ISR}}$ is bigger and gives a better averaged precision in the plateau regions. However, in contrast with the significant improvement due to a higher statistic of the MC sample at middle energy point, the increase of the luminosity of the lowest energy MC sample does not make significant changes.

5. Summary and discussion

In summary, the procedure of the calculation of ISR correction factor of the Higgs-strahlung process is investigated to match the statistical precision of the future CEPC. The effects of energies and statistics of data samples are studied systematically and the uncertainty of the ISR correction factor is evaluated accordingly. Based on the SM assumption, a economical and effective data-taking scheme is proposed.

The study suggests an economical and effective proposal of collecting data samples. With the established 5.6 $\text{ab}^{-1}$ data-taking plan at 240 GeV as well as data samples at two other energy points, the uncertainty for the ISR correction factor of $\sigma_{\text{obs}}(e^+e^- \rightarrow ZH)$ at 240 GeV can be suppressed significantly. Selecting of energy points is the most critical factor for the precision, and it is found that three data samples at 216, 220, and 240 GeV give the best accuracy on average when fixing the total integrated luminosity. To meet the minimum requirement to measure the Born cross section of Higgs-strahlung process at the CEPC, $\Delta_{\text{ISR}} < 0.25\%$, at least 0.15 $\text{ab}^{-1}$ at 220 GeV are supposed to be collected beside 0.05 $\text{ab}^{-1}$ at 216 GeV. The study also shows that higher priority should be given to 220 GeV if the precision

![Fig. 5. The fit results based on three MC samples generated at 216, 220, and 240 GeV. Different markers refer to MC samples at 216 GeV of different luminosities as shown in the legend. The horizontal ordinate is the luminosity of MC sample at 220 GeV and luminosity for the MC sample at 240 GeV is fixed to 5.6 $\text{ab}^{-1}$.](image)
need to be improved further. It should be noted that such scan data samples for the ISR correction are also useful to determine the Higgs boson spin and $CP$.

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