Agreement Threshold on Axelrod’s model of Cultural Dissemination

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Shared opinions are an important feature in the formation of social groups. In this paper, we use the Axelrod model of cultural dissemination to represent opinion-based groups. In the Axelrod model, each agent has a set of features which each holds one of a set of nominally related traits. Survey data, for example, has a similar structure, where each participant answers each of a set of items with responses from a fixed list.

We present an alternative method of displaying the Axelrod model by representing it as a bipartite graph, i.e., participants and their responses as separate nodes. This allows us to see which feature-trait combinations are selected in the final state. This visualisation is particularly useful when representing survey data as it illustrates the co-evolution of cultures and opinion-based groups in Axelrod’s model of cultural diffusion.

We also present a modification to the Axelrod model. A standard finding of the Axelrod model with many features is for all agents to fully agree in one cluster. We introduce an agreement threshold and allow nodes to interact only with those neighbours who are within this threshold (i.e., those with similar opinions) rather than those with any opinion. This method reliably yields a large number of clusters for small agreement thresholds and, importantly, does not limit to single cluster when the number of features grows large. This potentially provides a method for modelling opinion-based groups where as opinions are added, the number of clusters increase.

INTRODUCTION

To understand the societal structure of opinions, or attitudes, it is necessary to model the emergence of the groups that bind them. Much research demonstrates that people evaluate and understand their environment with reference to relevant social groups [1–3]. Importantly, shared opinions and beliefs are a defining feature of social groups [4, 5] and group identity can be fostered by coordinating attitudes. In particular, opinion-based groups are social structures in which people are connected by the opinions they share; and clusters of opinions become interlinked signifiers of group identity when they are jointly held by the members of a group [6]. We propose that these features enable us to model opinion-based groups using data from standard attitude surveys.

In this paper we demonstrate that an adapted version of Axelrod’s model of cultural dissemination [7] can be used to model opinion-based groups. Axelrod modelled people as each holding one of several available traits for each of several features. The individual’s culture is defined as their combination of traits; and multiple individuals are said to share the same culture if they are spatial neighbours who share the exact combination of traits.

The social structure of opinion-based groups has similar properties to Axelrod’s model. Specifically, agents in Axelrod’s model are linked according to the values they hold (i.e. the traits) on a given number of features. Opinion-based groups (or “cultures”) are formed by people holding a particular selection of attitudes. In principle, conceptualising Axelrod’s cultural features as attitudes will allow us to model the emergence of opinion-based groups.

Usually, the final state is presented in terms of whether convergence is reached, or if there is no consensus then in terms of the number of clusters. As we are interested in opinions, we initially present an alternate visualisation of the final state allowing one to observe the opinions held by agents in the final state. Projections of this can be made to view the clusters of agents agreeing in the final state.

The tendency toward consensus in the standard Axelrod model poses a problem for modelling the emergence of opinion-based groups with attitudinal surveys. Specifically, when the number of features $F$ is greater than the number of traits $q$, usually there will be consensus with only one cluster emerging [8]. Given that attitudinal surveys naturally assess more attitudes than response-options, an Axelrod simulation of such data would converge towards
uniformity. However, in real social systems survey data frequently reveals stable systems of cultural diversity [9]. Our adaptation of the Axelrod model is intended to account for diversity in the structure of opinion-based groups.

According to social judgment theory [10], the amount of attitude change caused by an interaction should depend on the perceived distance between the communication and the respondent’s original view. Specifically, at any given time there is a range of positions each person is open to holding, known as the "latitude of acceptance". Importantly, this range is anchored by one’s current opinion. Thus, if two individuals report attitude positions that are far apart in an ordinal scale (i.e. they disagree strongly), one is unlikely to be influenced by the other as a communicator because the position communicated will be outside the latitude of acceptance and within the latitude of rejection. However, when people hold positions that are closer together, assimilation can occur. Assimilation occurs when a person shifts their attitude closer to that of the communicator. In summary, the theory suggest that social judgments mediate attitude change [11], and this is supported by empirical evidence [12–14].

In this paper, we firstly use bipartite network (a network with two types of nodes) visualisations to identify opinion-based group structure in the standard Axelrod model. Secondly, we discuss how the non-nominal structure of opinions changes Axelrod’s agreement mechanism. Axelrod’s original interaction mechanism was based on the principle of homophily – “the likelihood that a given cultural feature will spread from one individual to another depends on how many other features they may already have in common.” If the nominal traits available to each agent are ordered then there is another dimension for homophily. A small modification to the interaction rules allows us model the types of survey-based data evident in the field of opinion-based groups.

MODEL

Standard Axelrod model

In the social sciences, attitudinal surveys are the most common way of measuring individual opinions. We wish to model participants holding attitudes and the groups they form as a result. To do this we start with with Axelrod’s model of cultural diffusion [7]. Here, each agent takes a position on a set of features; and that position must be one of a fixed set of traits. We begin by visualising this model using a bipartite graph – a network with two types of nodes, one representing the agents and another representing their trait on a specific feature.

The original Axelrod model on a lattice (without periodic boundary conditions) works as follows: Each individual has a feature set \( F = \{ f_1, f_2, ..., f_F \} \) where \( f_k \) is each specific feature. There are a total of \( F = \dim(F) \) features and each feature \( f_k \) has \( q \) traits (initially assigned at random). At each time-step, individual \( i \) and one of its neighbours \( j \) are chosen at random. With a probability equal to the number of common features over the total number of features, person \( i \) will copy one of the traits they do not have in common with \( j \).

Fig. 1 shows the Axelrod model for 100 nodes on a lattice with \( F = 3 \) and \( q = 5 \) paused after 40,000 time steps (i.e. not final state).
FIG. 2. The bipartite representation of the Axelrod simulation in Fig. 1 after 40,000 time-steps with \( F = 3 \) features (different colours) and \( q = 5 \) traits displaying which feature-trait combinations are selected.

as seen here. A cluster with no edges indicates that those nodes all have the exact feature configuration.

A bipartite network representation of the same data is shown in fig 2. Here, the underlying feature-trait combinations are revealed, with each feature displayed using a different colour. For example, we observe that features 1 and 2 both have a sizeable majority but feature 3 is more evenly split between \( q = 1 \) and \( q = 4 \). This is particularly useful if using this network to represent survey data. If there are three questions with a five-scale response, it is clear which response is the most favoured per question.

Agreement thresholds

One key difference between survey data and the Axelrod model is that in the Axelrod model the traits for a given feature are nominally related (i.e. not ordered) but in survey data the response options are related to one another, or on a scale (e.g., a Likert-type scale) [15]. We introduce a mechanism to allow for traits to be related to each other below.

A shortcoming of using the Axelrod model to represent survey data is a tendency toward uniform convergence in the final state as the number of features increase. There have been many variations on the standard Axelrod model (some even proposed in the original paper) often attempting to promote the formation of many clusters. Some examples of variations are the introduction of noise [16], changing the topology from a lattice to a complex network [17], the effect of media represented by an external field [18], and, more recently, modelling it on a multi-layer network where interactions can only occur between nodes on specific layers [19]. Other variations treat each feature as a continuous variable which pull closer together when they agree or repel when they disagree [20] (similar to the Deffuant model [21] with more than one feature). Given that attitudes are negotiated properties of social interaction [10], it is plausible to assume that people hold a broad range of continuous positions on a given attitude. However, we are interested in how attitudes are communicated. People typically communicate attitudes verbally, which leads to a certain level of imprecision in their interpretation [22, 23]. We argue that by scaling opinions to a limited number of points on a continuous scale, surveys capture general limitations in attitude communication and interpretation. Furthermore, continuous variable models show a tendency for opinions to move towards the mid-point of the scale [24, 25] and thereby fail to capture the stubbornness of extremists [26].

In the standard Axelrod model above, if two neighbours interact, the node originally chosen copies the state of one of its neighbour’s features that they do not already agree on. We argue that purely copying an attitude of an individual with whom you interact, even if you strongly disagree with this attitude, is unrealistic. Hence, we introduce an agreement threshold, \( a \), analogous to a latitude of acceptance [10] to compute the interactions. The idea is that there exists a range of positions a person is willing to hold with this range being anchored by their current position.

Instead of interacting with someone with probability proportional to the number of common features, we now calculate the probability to interact as the number of features within this agreement threshold over the total number of features, with interaction leading to a copying of one of those traits (i.e., only traits within the agreement threshold can be copied).

Figure 3 displays the final state for the lattice and bipartite visualisations of this model with \( F = 6 \), \( q = 3 \) with the lowest possible agreement threshold \( a = 1 \). For standard Axelrod, this would lead to consensus, however, here we see we have differ-
FIG. 3. The lattice and bipartite visualisation for the final state of Axelrod with the lowest agreement threshold on the traits for $F = 6$ and $q = 3$. Three of the features reach consensus in this realisation.

ent regions and from the bipartite representation, half of the six features do not reach consensus. This is a simple adjustment to standard Axelrod that yields more than one cluster even where the number of features exceeds the number of traits per feature as they would in a typical survey.

Projections

Visualising Axelrod on a bipartite space helps us realise that projections can be made to observe the connections between feature-trait combinations and nodes. A 1-mode projection can be generated showing features as nodes with an edge indicating that two features are held concurrently by an agent, and therefore overall which attitudes are strongly connected.

The projection for $F = 6$, $q = 3$ is shown in Fig. 4 where the weight of an edge is the number of pairs of nodes that share that feature-trait combination. These are split in thirds from weakest to strongest for visualisation purposes. The strongest edges are between the three attitudes where consensus is reached as these are held together most commonly.

Figure 5 shows the 1-mode projection for agents where agents are nodes, linked if they share an attitude in common. The colour of the edge represents how many attitudes they share, and we can identify a number of clusters where they share all six attitudes (with white edges). Note that this figure only displays the giant component. As the edges are created by shared links, if a node has no features in common with any other node, it will not be connected and therefore not appear in any clusters.

Number of Clusters

To count the number of clusters, we count the number of components where the nodes agree on all $F$ features in the final state. (Note that for larger systems, and larger agreement thresholds, it can make sense to count clusters that differ by one instead of assuming they all agree, however, here we are just...
FIG. 5. The node-space projection (giant component). Here, two nodes are linked if they hold an attitude in common. White links represent nodes agreeing on all six attitudes, blue edges represent five common attitudes, and red represent three or four.

taking the most simplified approach initially. To display the number of clusters we plot the dispersion indicator $d$ over all $K$ clusters which is defined as:

$$d = 1/\sum_{k=1}^{K} \left( \frac{n_k}{N} \right)^2,$$

where $n_k$ is the number of nodes in cluster $k$ and $N$ is the total system size. The inverse of this indicator is a generalised number of clusters which gives the exact number of clusters when they all include the same proportion of individuals.

Figure 6 shows the dispersion indicator for $q = 5$ and $q = 7$ for $N = 100$ over all agreement thresholds. Here we see that as the number of features increases this yields a large number of clusters. The low values of $d$ here are in part due to the presence of isolated agents. As the number of features increases, there are a larger number of feature-trait combinations an agent can have. For a low agreement threshold an agent would not be able to interact with any of its neighbours. Note also that for agreement threshold $a = q - 1$, every interaction will lead to copying so this will quickly yield full consensus.

Due to interactions now occurring when agents have a similar trait instead of exactly the same as in standard Axelrod, these tend to reach their final state faster than standard Axelrod.

CONCLUSION

The Axelrod model can be easily used to view opinion-based groups from data such as survey data. However, for a large number of features (eg. responses to a survey), in the final state, all agents converge to one opinion. To compensate for this we add an agreement threshold to the model. Crucially, this threshold yields many clusters for small values of $q$.

The addition of the agreement threshold is an adjustment to standard Axelrod interaction rules that only allow those with similar features to influence each other. The clustering that emerges is useful for considering opinion-based groups in opinion dynamics models and conceptualizing survey data.

One outcome of this model is that extremists are less likely to change their position than moderates.
Due to the agreement threshold, an agent with an attitude on either extreme can only move in one direction, an agent in the middle can move in either direction so is twice as likely change their position. This helps with avoiding a problem some models have causing agents on the extremes are drawntowards a centrists attitude and thus is more likely to achieve polarised final states [28].

Further work needs to be done on computing the phase transitions for different values of $F$, $q$ and $a$. We also wish to vary $q$ per feature as there is no reason the number of traits per feature should be the same. A further extension could specify that if the agreement threshold is larger than one, the traits move towards each other rather than one agent copying the other.

Before we can seed this extended Axelrod model with real survey data, however, we need to remove the constraint of the agents interacting on a lattice. In standard Axelrod, it has been shown on random graphs and small world graphs that consensus is reached for large number of features similar to a regular lattice [29]. This needs to be tested for this extended model to compare the sensitivity or resilience of various real-world attitude network topologies to cultural diffusion. Once this is done, the model can be tested with real survey data with an attempt to model the changing nature of opinion-based groups in society.

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