We compare two most relevant loop induced muon number violating processes, \( \mu \rightarrow e \gamma \) and \( \mu \rightarrow e \) conversion in nuclei, in constraining new physics. Because of much richer structure of the latter process it may be enhanced with respect to \( \mu \rightarrow e \gamma \) by a large \( \ln(m^2_\mu/\Lambda^2) \), where \( \Lambda \) is the scale of the new physics. After model-independent considerations we present two examples constraining the \( R \)-parity violating couplings of MSSM and the off-diagonal couplings of the doubly charged Higgs bosons from the non-observation of \( \mu-e \) conversion.

1 Introduction

The precision reached in the last years in the experiments searching for \( \mu-e \) conversion in nuclei at PSI\(^1\) and the expected improvements in the sensitivity of the experiments at PSI in the next years by more than two orders of magnitude\(^2\) will make \( \mu-e \) conversion the main test of muon flavour conservation for most of the extensions of the standard model (SM). Moreover, according to the recent BNL proposal\(^4\) further improvements in the experimental sensitivity down to the level \( 10^{-16} \) are feasible. The experimental prospects in this field have been recently reviewed by A. Czarnecki\(^5\).

In any renormalizable model \( \mu \rightarrow e \gamma \) can occur only at the loop level. For a wide class of models, discussed in the present talk, this is also true for \( \mu-e \) conversion (which, in principle, can be also a tree level process). If this is the case, and given the present experimental accuracy, one often finds\(^6\) that the bounds on new physics coming from \( \mu \rightarrow e \gamma \) are stronger than the bounds found from \( \mu-e \) conversion. However, this does not need to be always the case. Indeed, it has been noticed already in the early works of Ref.\(^7\) that in some cases \( \mu-e \) conversion constrains new physics more stringently than \( \mu \rightarrow e \gamma \). More recently it has been shown rigourously\(^8\) using the effective quantum field theory in which class of models \( \mu-e \) conversion is enhanced by large logarithms compared with \( \mu \rightarrow e \gamma \). Using this enhancement new strong bounds have been derived for the doubly charged Higgs interactions\(^8\) and for the \( R \)-parity violating couplings of the minimal supersymmetric standard model (MSSM)\(^9\).

2 Effective Lagrangian description of \( \mu-e \) conversion

Assuming that the relevant physics responsible for muon-number nonconservation occurs at some scale \( \Lambda > \Lambda_F \equiv \text{Fermi scale} \), we can write the relevant
factors where the four-fermion interaction gives a large enhancement for the form factors. We compare the branching ratios of the form factors to eq. (2) (with $\Lambda = 1$ TeV in the logarithms) we find definiteness only the right-handed operators in Lagrangian (1). Substituting $\Gamma_{\text{capt}}^P$ for $\Gamma_{\text{capt}}^L$ in eq. (1), we find

$$\Gamma_{\text{capt}}^P = \frac{8\alpha^5 m^5 \langle f_{E_0} \rangle^2 Z_{eJ}^L}{\Gamma_{\text{capt}}^L} . \frac{\xi_0^2}{q^2},$$

where $\xi_0^2 = |f_{E_0}|^2 + |f_{ME}|^2$, $C_T = 1.0$, $C_P = 1.4$, $Z_{eJ}^L = 17.61$, $Z_{eJ}^R = 33.81$, $\Gamma_{\text{capt}}^L = 2.59 \times 10^6$ s$^{-1}$, $\Gamma_{\text{capt}}^P = 1.3 \times 10^7$ s$^{-1}$ and the proton nuclear form factors are $T_i^P(q) = 0.55$ and $T^P_{c_{L/R}}(q) = 0.25$. One should note that the $\mu \to e \gamma$ branching ratio, $R_\gamma = 96\pi^2 \alpha/(G_F^2 m_\mu^2)(|f_{ME}|^2 + |f_{E_1}|^2)$, depends on a different combination of the form factors.

We have computed the form factors at one loop level starting from the Lagrangian (1) in Ref. 8. There are new loop contributions only to the form factors $f_{E_0}$ and $f_{ME}$ but not to $f_{E_1}$ and $f_{ME}$. Those contributions always contain a term which is proportional to $\ln(q^2/\Lambda^2)$ or $\ln(m^2_\mu/\Lambda^2)$. This term which is completely independent of the details of the model that originate the four-fermion interaction gives a large enhancement for the form factors $f_{E_0}$ and $f_{ME}$ while the enhancement is absent in the form factors $f_{E_1}$ and $f_{ME}$. To compare the branching ratios of $\mu \to e \gamma$ and $\mu \to e \gamma$ we consider for definiteness only the right-handed operators in Lagrangian (1). Substituting the form factors to eq. (2) (with $\Lambda = 1$ TeV in the logarithms) we find

$$R_{\mu e} = 1.2 (3.5) \times 10^{-5} \text{ TeV}^4 \left( \frac{\alpha_{ee}^{RR}}{\Lambda^2} \right)^2.$$
\begin{align*}
R = 4.6 \cdot 10^{-11} & \quad 4.3 \cdot 10^{-12} & \quad 5.0 \cdot 10^{-13} & \quad 3.0 \cdot 10^{-14} \\
\text{log-enhanced } \mu-e & \quad 32 & \quad 44 & \quad 101 & \quad 158 \\
\text{non-enhanced } \mu-e & \quad 7 & \quad 9 & \quad 20 & \quad 32 \\
\mu \to e \gamma & \quad 23 & \quad 41 & \quad 70 & \quad 141 \\
\end{align*}

Table 1: Values of \( \Lambda \), in TeV, probed in \( \mu-e \) conversion and \( \mu \to e \gamma \) for different upper bounds on the branching ratios. The upper bound \( 4.6 \cdot 10^{-11} \) is the present bound for \( \mu-e \) conversion on Pb and it is very close to the present \( \mu \to e \gamma \) bound \( (4.9 \cdot 10^{-11}) \). \( 4.3 \cdot 10^{-12} \) is the present bound for \( \mu-e \) conversion on Ti. \( 5 \cdot 10^{-13} \) and \( 3 \cdot 10^{-14} \) are the expected bounds in the next year for \( \mu-e \) conversion on Au and Ti, respectively.

where the first number corresponds to \( \mu-e \) conversion in Ti and the second in Pb and Au. In the effective Lagrangian framework \( \mu \to e \gamma \) does not get contributions from loops and assuming naturally \( \alpha^\sigma_R \equiv \alpha^{RR} \) we obtain \( R_\gamma = 1.2 \cdot 10^{-5} \text{TeV}^4 (\alpha^{RR}_{k e kr} / \Lambda^2)^2 \). Thus the rates of \( \mu-e \) conversion and \( \mu \to e \gamma \) in our models are comparable in magnitude.

The present experimental upper limits on the branching ratios of the processes are \( R_{\mu e}(\exp) \lesssim 4.3 \cdot 10^{-12} \), \( R_{\mu e}^{Pb}(\exp) \lesssim 4.6 \cdot 10^{-11} \) and \( R_{\gamma}(\exp) \lesssim 4.9 \cdot 10^{-11} \). SINDRUM II experiment at PSI taking presently data on gold will reach the sensitivity \( R^{Au, expected}_{\mu e} \lesssim 5 \cdot 10^{-13} \) and starting next year the final run on Ti it should reach \( R^{Ti, expected}_{\mu e} \lesssim 3 \cdot 10^{-14} \). To show which scales of new physics \( \Lambda \) can be probed in \( \mu-e \) conversion and \( \mu \to e \gamma \) experiments we have presented the values of \( \Lambda \) in TeV-s in Table 1 for different experimental upper bounds on the branching ratios of the processes. All the couplings \( \alpha \) are taken to be equal to unity. We have considered both classes of models with and without logarithmic enhancement of \( \mu-e \) conversion. If the experimental limits for \( \mu-e \) conversion and \( \mu \to e \gamma \) are equal then \( \mu-e \) conversion enhanced by large logarithms has better sensitivity to \( \Lambda \) than \( \mu \to e \gamma \), especially in the case of Pb and Au experiments.

3 Models with doubly charged Higgses

As the first example let us consider an extension of the SM by adding just a doubly charged scalar singlet \( \kappa^{++} \). However, the limits we derive apply with a good accuracy also for the interactions of triplet scalars appearing in the models with enlarged Higgs sectors as well as in the left-right symmetric models. This is because the doubly charged component of the triplet gives the dominant contribution both to \( \mu-e \) conversion and \( \mu \to e \gamma \).

\( \kappa^{++} \) coupling to right-handed leptons is described by

\[ \mathcal{L}_\kappa = h_{i j} \bar{e}_{i R} e_{j R} \kappa^{++} + \text{h.c.} \] \( (4) \)
From this interaction we obtain easily the four-fermion interaction
\[ \frac{1}{m^2} \kappa h^*_k h^*_j \left( \overline{c}^{e_i R} c^{e_j R} \right) \left( \overline{c}^{l_i R} c^{l_j R} \right). \]
This interaction is of the type $L^{RR}$ and one can immediately identify $\alpha_{ik,ij} = h^*_k h^*_j$ and $\Lambda = m_\kappa$. By using the MS renormalization scheme and by choosing the renormalization scale $\mu = \Lambda = m_\kappa$, we obtain the following coefficients $\alpha_{ij}^{RR} = 20/9 h^*_k h^*_j$ and $\alpha_{ij}^{RR} = 2/3 h^*_k h^*_j$. Therefore the full amplitude for $\mu-e$ conversion is dominated by the running from the scale of new physics $\Lambda = m_\kappa$ to relevant scale of the process $m_\mu$. This conclusion is independent on the model as long as the effective four-fermion interactions exist. Substituting these results to eq. (3) and using the present experimental limit for $T_i$ we obtain from $\mu-e$ conversion for $m_\kappa = 1$ TeV
\[ h_{e\mu} h^*_e \lesssim \frac{6 \cdot 10^{-4}}{\sqrt{B}}, \quad h_{\tau\mu} h^*_e \lesssim \frac{9 \cdot 10^{-4}}{\sqrt{B}}, \]
while $\mu \rightarrow e\gamma$ gives $h_{k\mu} h^*_e \lesssim 3 \cdot 10^{-3}$. Here we have introduced a factor $B = R_{\mu e}^{\text{present}} / R_{\mu e}^{\text{future}}$ which takes into account the expected experimental improvements. The bounds (5) are new limits on the off-diagonal doubly charged scalar interactions (note that tree level $\mu \rightarrow 3e$ probes only $h_{e\mu} h^*_e$). The upper bounds (5) are going to be improved soon by an order of magnitude, $\sqrt{B^{TT}} \approx 12$, with the expected $\mu-e$ conversion data.

4 MSSM without $R$-parity

Within the MSSM particle content the gauge invariance and supersymmetry allow for the following $R$-violating superpotential
\[ W_R = \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}^c_k + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}^c_k + \lambda''_{ijk} \hat{U}^c_i \hat{D}_j \hat{D}^c_k - \mu_1 \hat{L}_i \hat{H}_2, \]
where $\lambda_{ijk} = -\lambda_{jik}$ and $\lambda'_{ijk} = -\lambda''_{kij}$. The $\lambda$, $\lambda'$ and $\mu$ terms violate the lepton number, whereas the $\lambda''$ terms violate the baryon number by one unit. The last bilinear term in Eq. (6) gives rise to interesting physics which has been studied elsewhere.

$\mu-e$ conversion probes the products of the couplings of the type $\lambda \lambda$ only at one loop level, $\lambda' \lambda'$ only at tree level and $\lambda' \lambda''$ both at tree and one loop level. The previous bounds have been collected and updated in the recent reviews. We have calculated the new loop level bounds in Ref. The $\mu-e$ conversion tree level bounds are all taken from Ref. assuming that there is no cancellations between different contributing terms. For $\lambda \lambda$-s comparison with the previously obtained bounds in Table 1 shows that in three cases out of six our new bounds are more stringent. This is because the conversion is enhanced by large logarithms $\ln(m_f^2 / m^2)$. If SINDRUM II will reach $\sqrt{B^{TT}} = 12$ then all the $\mu-e$ conversion bounds will be more stringent than the previous ones.
By far the most stringent constraint on the products $|\lambda\lambda|$ testable in $\mu-e$ conversion are very strong for most of the couplings but not for all of them. The reason is that for some combinations of the couplings, especially if the third family squarks are involved, their contribution to the $\mu-e$ conversion is strongly suppressed by small off-diagonal CKM matrix elements (as much as $\lambda_W^8$ where $\lambda_W \sim 0.2$ is the Wolfenstein parameter). With the present constraints only the bound $|\lambda'_{231}\lambda'_{131}| \lesssim 1.2 \cdot 10^{-5}$ for the relevant sfermion mass $m_f = 100$ GeV is stronger than the tree level $\mu-e$ conversion bounds. Importantly, as the result of our calculation the bounds $|\lambda'_{232}\lambda'_{132}| \lesssim 8.7 \cdot 10^{-5}$ and $|\lambda'_{233}\lambda'_{133}| \lesssim 8.7 \cdot 10^{-5}$ derived from the loop induced $\mu-e$ conversion are more stringent than the ones from the tree level $\mu-e$ conversion.

| previous bounds | $\mu-e$ conversion at loop level/$\sqrt{B}$ |
|-----------------|------------------------------------------|
| $m_f = 100$ GeV | $m_f = 100$ GeV | $m_f = 1$ TeV |
| $|\lambda_{121}\lambda_{122}|$ | $6.6 \cdot 10^{-7}$ Ref. 4 | $4.2 \cdot 10^{-6}$ | $3.2 \cdot 10^{-4}$ |
| $|\lambda_{131}\lambda_{132}|$ | $6.6 \cdot 10^{-7}$ Ref. 4 | $5.3 \cdot 10^{-6}$ | $3.9 \cdot 10^{-4}$ |
| $|\lambda_{231}\lambda_{232}|$ | $5.7 \cdot 10^{-5}$ Ref. 6 | $5.3 \cdot 10^{-6}$ | $3.9 \cdot 10^{-4}$ |
| $|\lambda_{231}\lambda_{131}|$ | $6.6 \cdot 10^{-7}$ Ref. 6 | $8.4 \cdot 10^{-6}$ | $6.4 \cdot 10^{-4}$ |
| $|\lambda_{232}\lambda_{132}|$ | $1.1 \cdot 10^{-4}$ Ref. 6 | $8.4 \cdot 10^{-6}$ | $6.4 \cdot 10^{-4}$ |
| $|\lambda_{233}\lambda_{133}|$ | $1.1 \cdot 10^{-4}$ Ref. 6 | $1.7 \cdot 10^{-5}$ | $1.0 \cdot 10^{-3}$ |

Table 2: Upper limits on the products $|\lambda\lambda|$ testable in $\mu-e$ conversion for two different scalar masses $m_f = 100$ GeV and $m_f = 1$ TeV. The previous bounds scale quadratically with the sfermion mass. The scaling factor $B$ is defined in the text and currently $B = 1$.

5 Conclusions

In conclusion, using the effective Lagrangian description of new physics we show that in a wide class of models loop induced $\mu-e$ conversion in nuclei is enhanced by large logarithms. With the present upper limits on $\mu-e$ conversion and $\mu \rightarrow e\gamma$ branching ratios bounds on new physics (occurring at loop level) derived from these processes are more restrictive in the case of $\mu-e$ conversion. This result is confirmed by explicit calculations in the models with doubly charged Higgses and MSSM without $R$-parity. Due to the expected improvements in the sensitivity of already running $\mu-e$ conversion experiments this process will become the most stringent test of muon number conservation.
1. SINDRUM II Coll., C. Dohmen et al., Phys. Lett. B317 (1993) 631.
2. SINDRUM II Coll., W. Honecker et al., Phys. Rev. Lett. 76 (1996) 200.
3. A. van der Schaar (spokesman of SINDRUM II), PSI proposal R-87-03, 1987, and private communication.
4. M. Bachman et al., BNL Proposal P940, 1997.
5. For a review see, A. Czarnecki, hep-ph/9710425.
6. See, e.g., R. Barbieri and L. Hall, Phys. Lett. B338 (1994) 212; R. Barbieri, L. Hall and A. Strumia, Nucl. Phys. B445 (1995) 219; T.S. Kosmas and J.D. Vergados, Phys. Rep. 264 (1996) 251, and references therein.
7. W.J. Marciano and A.I. Sanda, Phys. Rev. Lett. 38 (1977) 1512; G. Altarelli et al., Nucl. Phys. B125 (1977) 285.
8. M. Raidal and A. Santamaria, FTUV/97-56, hep-ph/9710389.
9. K. Huitu, J. Maalampi, M. Raidal and A. Santamaria, FTUV/97-45, hep-ph/9712249.
10. S. Weinberg and G. Feinberg, Phys. Rev. Lett. 3 (1959) 111.
11. H.C. Chiang et al., Nucl. Phys. A559 (1993) 526.
12. R.M. Barnett et al., Review of Particle Physics, Phys. Rev. D54 (1996) 1.
13. K. Babu, Phys. Lett. B203 (1988) 132.
14. J.C. Pati, A. Salam, Phys. Rev. D10 (1974) 275; R.N. Mohapatra, J.C. Pati, Phys. Rev. D11 (1975) 566, ibid. 2558; G. Senjanovic, R.N. Mohapatra, Phys. Rev. D12 (1975) 1502; R.N. Mohapatra, G. Senjanovic, Phys. Rev. Lett. 44 (1980) 912; R.N. Mohapatra, G. Senjanovic, Phys. Rev. D23 (1981) 165.
15. S. Weinberg, Phys. Rev. D26 (1982) 287; N. Sakai and T. Yanagida, Nucl. Phys. B197 (1982) 533; C.S. Aulakh and R.N. Mohapatra, Phys. Lett. B119 (1982) 136; L.J. Hall and M. Suzuki, Nucl. Phys. B231 (1983) 419; J. Ellis et al., Phys. Lett. B150 (1985) 142; S. Dawson, Nucl. Phys. B261 (1985) 297; S. Dimopoulos and L.J. Hall, Phys. Lett. B207 (1987) 210.
16. J.C. Romao, A. Ioannisyan and J.W.F. Valle, Phys. Rev. D55 (1997) 427; M.A. Diaz, J.C. Romao and J.W.F. Valle, hep-ph/9706315 and references therein; M.A. Diaz, these proceedings.
17. G. Bhattacharyya, Nucl. Phys. Proc. Suppl. 52A (1997) 83; and IFUP-TH 43/97, hep-ph/9709395, H. Dreiner, hep-ph/9707433.
18. J.E. Kim, P. Ko and D.-G. Lee, Phys. Rev. D56 (1997) 100.
19. M. Chaichian and K. Huitu, Phys. Lett. B384 (1996) 157.
20. D. Choudhury and P. Roy, Phys. Lett. B378 (1996) 153.