Unlocking the frequency domain for high-dimensional quantum information processing

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High-dimensional photonic entanglement is a promising candidate for error-protected quantum information processing with improved capacity. Encoding high-dimensional qudits in the carrier frequency of photons combines ease of generation, universal single-photon gates via electro-optic modulation and wave shaping, and compatibility with fiber transmission for high-capacity quantum communication. Recent landmark experiments have impressively demonstrated quantum interference of a few frequency modes, but the certification of massive-dimensional frequency entanglement has remained an open challenge. Here we report a record certification of discretized frequency entanglement, combined with a novel approach for certification that is both highly efficient and nonlocally implementable, opening the possibility for utilizing this encoding in quantum communication.

I. INTRODUCTION

Entanglement is a unique and powerful quantum feature with a multitude of applications in quantum information processing. The nonlocal correlations of the entangled states may be used in quantum communications, imaging, metrology, and quantum processors. In the case of photons, polarization-entangled states have been traditionally used to demonstrate a multitude of quantum gates and quantum information protocols [1] and the principles of rapidly developing quantum networks [2, 3]. These qubit states are easy to manipulate with linear optics and can be distributed in fiber or free-space links because of their low interaction with the environment. Nevertheless, larger alphabets in quantum communication are highly pursued not only to increase the capacity of the quantum channel. High-dimensional encoding also provides a stronger tolerance to noise [4], an essential asset to overcome the transmission limits of polarization qubits. On the other hand, an increase in dimensionality can also boost the computational power of quantum computers [5]. These important building blocks opened the possibility to measure in the superposition basis at optical scales and were used to demonstrate discretized frequency entanglement with few dimensions (up to 4). Since then, approaches have been developed to perform arbitrary manipulations of single-frequency qubits [17] and the first steps have been taken towards full control of single qudits, its high-dimensional version [18] [19]. While Hong-Ou-Mandel interference has also been used to verify frequency entanglement [20] [21], the requirement of local measurements limits the utility of these methods in quantum networks.

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Entangled frequency states can also be generated in combination with entanglement in other DOFs like polarization or even the time domain, whenever the relevant time and frequency properties can be manipulated independently, i.e., because they refer to vastly different time scales. Such hyperentangled states can also be used to enlarge the dimensionality of the system, to generate cluster states [22] or to perform more advanced quantum gates [23]. Yet, all these approaches are only able to manipulate a small frequency mode set of typically extensive underlying quantum states. Certifying real high-dimensional entanglement is not trivial, particularly in the frequency domain, and its immense potential is still unexploited.

In this Letter, we show that it is not always necessary to design quantum sources with a discretized frequency space, such as those built in cavities, and therefore continuous spectra can also provide access to massive-dimensional and well-controlled Hilbert space. We show quantum interference with up to 7 modes and >98% visibility, providing tools for complete state control in a 7-mode state space. Subsequently, based on previous work to characterize time-bin qudits [24, 25], we certify genuine high-dimensional frequency entanglement without prior assumptions regarding, e.g., the purity of the quantum state. Exploiting a novel bucket detection approach that requires very few measurement settings, resembling the compressed measurements used to characterize spatial correlations [26, 27], we alleviate the harsh requirements on the number of measurements necessary to characterize the relevant correlations of the state, which grows with dimensionality. Finally, by recovering the information from high-quality quantum interference of two-dimensional (2D) subspaces, we are able to certify a minimum of 33 entangled frequency modes, the highest dimensionality of entanglement reported in time and frequency degrees of freedom.

II. SETUP

In our work, we analyze the frequency content of two-photon states generated in standard $\chi^{(2)}$ non-linear crystals: periodically poled lithium niobate (ppLN) waveguides (see Methods). They provide continuous and broadband spontaneous parametric down-converted photons (SPDC) with high efficiency and 60 nm of bandwidth. The SPDC process is temperature tuned to a degeneracy wavelength of 1548 nm to cover the entire C band with high uniformity (see inset in Fig. 1).

At this point, the frequency space of the generated photon pairs would typically be discretized, e.g., with etalon cavities to carve the spectrum [28, 29]. This approach is useful for the isolation of frequency modes and for performing sideband modulation. However, a considerable contribution of the photon spectrum is directly rejected, reducing the total throughput. For this reason, we avoid this step and employ the maximum bandwidth per frequency mode.

Due to the energy conservation of the SPDC process, the emitted photon pairs are strongly anticorrelated in frequency, and the state can be described as:

$$|j\rangle_s|j\rangle_i = \int \Pi(\Omega - j\Delta \omega, \Omega + j\Delta \omega)|\omega_0 + \Omega, \omega_0 - \Omega\rangle_s \, d\Omega$$

where $|j\rangle_{s,i}$ is the label of the $j^{th}$ frequency mode corresponding to the signal and the idler photon. $\omega_0$ is the degeneracy frequency of the SPDC process, $\Delta \omega$ is the FSR between modes and $\Pi$ is the spectral shape of each mode. We discretize the system in bins of 25 GHz bandwidth and the same FSR over the whole C-band, yielding the state:

$$|\psi_d\rangle = \sum_{j=1}^{d} \alpha_j |j\rangle_s |j\rangle_i$$
The term $\alpha_j$ refers to the phase and amplitude of the mode and is determined by the spectral characteristics of the source. For a very broad and uniform spectrum as here, $\alpha_j \approx 1$. Although after propagation, each photon pair corresponding to a mode $j$ accumulates a different phase due to material dispersion.

In a quantum key distribution (QKD) scenario, the frequency-entangled photon pairs emerging from a single optical fiber can be distributed into different paths, e.g., via wavelength- or polarization demultiplexing. In our experiment, we use only one device simultaneously for both photons, but in principle, manipulation would be just as easily possible at two separate locations. To reveal the entanglement content, we use a commercial pulse shaper to control the phase and amplitude of each of the frequency modes [16], and electro-optic modulation to achieve mode superpositions [13] (see Methods).

### III. THE SUPERPOSITION BASIS

For either quantum state characterization via full state tomography (FST), evaluation of Bell-type tests, or implementation of QKD protocols, measurements in superposition bases are fundamental to uncover quantum correlations and statistics of the state. The eigenvectors of these bases may be the superposition of some or all elements of the computational basis, here the frequency basis, with certain phases for each mode. Here, we show that with standard levels of RF signal amplification ($P_{\text{max}} = 26\text{dBm}$), it is possible to perform a full superposition of up to 7 modes with a low contribution of accidental coincidence detection events. To prove this, we performed high-dimensional Bell-type tests, also known as CGLMP [30], based on the CHSH inequality for two-photon qubits [31]. Instead of measuring quantum correlations only for fixed phase settings, we performed a phase scan for all contributing modes [32]. The measurement projector we use for each photon is:

$$|\Psi_{\text{proj}}\rangle = \frac{1}{\sqrt{d}} \sum_{j=1}^{d} (e^{ij\theta_{s,i}} |j\rangle_{s,i})$$

where $\theta_{s,i}$ is the phase applied to the signal or the idler photon, and we use $\theta_s = \theta_i$. The phase of interference depends on the sum of the signal and idler phases. By scanning their phase, we obtain the quantum interferences shown in Fig. 2 for dimensions $d = 2, 3, 5, 7$. The visibilities are 96.7%, 97.7%, 98.1% and 98.2%, respectively, without fitting or subtraction of accidental coincidences and much above the thresholds 70.7%, 77.5%, 84.6% and 88.3% to rule out hidden variable theories [32]. To perform these measurements, we have selected states centered on the $6^{th}$ mode. Electro-optic modulation shifted photons from the neighboring modes into the $6^{th}$ mode, and demultiplexing filters (DEMUX) postselected the superposition state. Our constrained RF signal only allowed a limited efficiency of the frequency modulation of photons; thus, added mode loss by the pulse shaper provided an equal contribution. Even dimensionality can also be evaluated by using the same parameters as here.

### IV. HUGE DIMENSIONALITY CERTIFICATION

While these CGLMP tests provide a (partially) device-independent certification of entanglement and demonstrate the reliability of our devices, they do not easily test for actual entanglement dimensionality, i.e. the dimension of entanglement needed to reproduce the correlations. Generically, to demonstrate even higher levels of entanglement, one would need to project distant spectral modes into superposition states, which may be limited or physically impossible to perform. In our large frequency space, this would imply unreachable RF power levels for electro-optic modulation, but similar practical limitations are to be expected for any type of encoding. On the other hand, common entanglement certification techniques, such as FST, are expensive procedures that require measurements in at least $(d+1)^2$ bases for bipar-
tite systems, an equivalent of $d^2(d+1)^2$ single-outcome measurement settings. The obvious consequence is that the number of total measurements increases rapidly with dimension. Adapting the techniques developed for the time-bin domain, we show that it is sufficient to characterize the quantum coherence of a few 2D subspaces to certify a high dimensionality of entanglement in the frequency domain.

The certification process is structured into two main blocks: the measurement of some elements of the density matrix $\rho$ and the posterior lower bound of the remaining unknowns. Measurements on the computational basis, that is, the frequency basis, can be performed with standard filters and are, in fact, directly related to the diagonal elements of the density matrix $\langle j,k | \rho | j,k \rangle$. Only this characterization step would usually require $d^2 = 10404$ filter settings, which can take arbitrary long times. Here, we propose making good use of the frequency parallelization and bucket detection of all uncorrelated frequencies with a single and broadband filter setting $\sum_{k \neq j} \langle j,k | \rho | j,k \rangle$ (i.e. 101 modes $\times$ 25 GHz), while still measuring the correlated set $\langle j,j | \rho | j,j \rangle$ with narrowband filter settings (25 GHz). This method reduces the high number of measurements required to only $2d$. The results of the maximum frequency correlation are shown in Fig. 3a. The background noise measured for uncorrelated frequencies originates from accidental coincidence detection events due to the high number of single counts and imperfect filters. The detected coincidence to accidental ratio (CAR) averaged throughout the space amounts to 1.4 $\times$ 10$^3$.

Outside of the diagonal of $\rho$ we find two types of elements: those close to zero due to energy conservation, as observed from the computational basis measurements and upper limited by the accidentals in our system, and those non-zero elements $\langle j,j | \rho | k,k \rangle$ that indicate the coherence of the entangled state. We now measure some of the coherence elements to which we have access with our measurement system. They can be estimated with the mode amplitude from the computational basis and upper limited by the accidentals in our system, and as observed from the computational basis measurements.

The fact that these high-contrast interference is preserved over the whole investigated space indicates that the quantum state at hand is very close to a maximally entangled state. However, in potentially adversarial scenarios, such as QKD, we do not want to make any assumption on the distributed state. Thus, we proceed with a rigorous analysis to demonstrate entanglement. Indeed, these strong coherences allow us to finally certify a large amount of entanglement without further measurements. Similarly to the methods proposed for entanglement certification in the time domain [24, 25], we lower bound the remaining unknown elements $\langle j,j | \rho | j+i,j+i \rangle$ by using the fact that the density matrix must be positive semidefinite to represent a valid quantum state. Thus, every element, submatrix, and therefore subdeterminant of $\rho$ must be positive or equal to zero at the very least. They can be lower-bounded iteratively by solving $3 \times 3$ subdeterminants, composed of measured parameters and one unknown, and we keep the largest bound extracted from all combinations of submatrices. Notice that this type of bound causes a rather fast loss of information, yet it is sufficient to certify high-dimensional entanglement. To visualize this loss, we compare in Fig. 3b: the measured quantities for $\langle j,j | \rho | j+2,j+2 \rangle$ and the calculated bound if we would not use those measurements. The resulting submatrix $\langle j,j | \rho | k,k \rangle$ is shown in Fig. 4.
FIG. 4. (a) Lower bound of the density submatrix $\langle j,j|\rho|k,k \rangle$ and (b) minimum certified dimensionality of our system, according to the measured data. Notice that for low space dimension up to $d = 11$, we can certify the maximum dimensionality with very few measurement settings.

We can now compare the lower-bounded density matrix with a target state $|\Phi\rangle$ by computing the fidelity $F(\rho, \Phi) = \text{Tr}(\sqrt{\sqrt{\Phi} \rho \sqrt{\Phi}})^2$. There exists an upper bound for the fidelity of any state of Schmidt rank $k \leq d$ \cite{33, 34}. Fidelities with the maximally entangled state above the threshold $B_k(\Phi) = k/d$ indicate a dimensionality of at least $k+1$. We thus choose a maximally entangled state as the target state $|\Phi\rangle = 1/\sqrt{d} \sum_{j=1}^{d} |j,j\rangle$ and iteratively calculate the fidelity for $d = 2$ to $d = 102$ and compare it with the threshold values for varying Schmidt rank. The final certification is plotted in Fig. 4b, where we find at least 33 entangled modes in a space of 101 to 102. It is worth emphasizing that with only these very few measurement settings, $2d$ on the computational basis and $\sim 6d$ on 2D subspaces, we are able to certify 11 dimensions in a space of 11 modes. Higher visibility would directly increase the amount of certified entanglement per number of modes, and further measurements that could fill more elements of the density matrix would also improve certification.

V. DISCUSSION AND OUTLOOK

To date, although high-dimensional frequency entanglement has been widely accepted to exist in photon pairs generated spontaneously from nonlinear processes, it has never been completely certified beyond a few dimensions. In our work, we have shown methods on how to characterize these quantum states that unquestionably possess a huge dimensionality. Similarly as in the time domain, the limits on the dimensionality for continuously pumped processes depend on the resolution of our devices, here the spectral filters.

In this work, we have shown full superpositions for $d = 2, 3, 5, 7$ and high interference visibilities, the highest values in the literature (only available up to $d = 4$). These subspaces are easily exploitable with few fiber-integrated optical components. We have also certified 11-dimensional entanglement in a subspace of 11 modes with only $\sim 8d$ measurement settings, and at least 33-dimensional entanglement in our frequency space of 102 frequency modes.

Entanglement in the frequency domain can also be used in combination with entanglement in time and path DOFs, leading to huge state spaces that can encode vast amounts of information. We hope that these results will motivate further photonic technology development, in particular low-loss electro-optic modulation and wave-shaping technologies that would inevitably render frequency-coded quantum information encoding a choice for near-term photonic quantum information processing with massive bandwidth.

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\[1\] P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, Linear optical quantum computing with photonic qubits, Reviews of Modern
[2] J.-W. Pan, Z.-B. Chen, C.-Y. Lu, H. Weinfurter, A. Zeilinger, and M. Żukowski, Multiphoton entanglement and interferometry, Reviews of Modern Physics 84, 777 (2012).

[4] S. Ecker, F. Bouchard, L. Bula, F. Brandt, O. Kohout, F. Steinlechner, R. Fickler, M. Malik, Y. Guryanova, R. Ursin, and M. Huber, Overcoming noise in entanglement distribution, Physical Review X 9, 041042 (2019).

[5] B. P. Lanyon, M. Barbieri, M. P. Almeida, T. Jennewein, T. C. Ralph, K. J. Resch, G. J. Pryde, J. L. O’Brien, A. Gilchrist, and A. G. White, Simplifying quantum logic using higher-dimensional Hilbert spaces, Nature Physics 5, 134 (2009).

[6] C. Vigilier, S. Paesani, Y. Ding, J. C. Adcock, J. Wang, S. Morley-Short, D. Bacco, L. K. Oxenløwe, M. G. Thompson, J. G. Rarity, and A. Laing, Error-protected qubits in a silicon photonic chip, Nature Physics 17, 1137 (2021).

[7] A. Mair, A. Vaziri, G. Weihs, and A. Zeilinger, Entanglement of the orbital angular momentum states of photons, Nature 412, 313 (2001).

[8] M. Krenn, M. Huber, R. Fickler, R. Lapkiewicz, S. Ramelow, and A. Zeilinger, Generation and confirmation of a (100 × 100)-dimensional entangled quantum system, Proceedings of the National Academy of Sciences 111, 6243 (2014).

[9] N. K. Fentaine, R. Ryf, H. Chen, D. T. Neilson, K. Kim, and J. Carpenter, Laguerre-Gaussian mode sorter, Nature Communications 10, 1865 (2019).

[10] J. D. Franson, Bell inequality for position and time, Physical Review Letters 62, 2205 (1989).

[11] J. Brendel, N. Gisin, W. Tittel, and H. Zbinden, Pulsed energy-time entangled twin-photon source for quantum communication, Physical Review Letters 82, 2594 (1999).

[12] D. Richart, Y. Fischer, and H. Weinfurter, Experimental implementation of higher dimensional time-energy entanglement, Applied Physics B 106, 543 (2012).

[13] L. Olijler, I. Mbojdi, E. Woodhead, J. Cussey, L. Furfaro, P. Emplit, S. Massar, K. P. Huy, and J.-M. Merolla, Implementing two-photon interference in the frequency domain with electro-optic phase modulators, New Journal of Physics 14, 043015 (2012).

[14] A. Pe’er, B. Dayan, A. A. Friesem, and Y. Silberberg, Temporal shaping of entangled photons, Physical Review Letters 94, 073601 (2005).

[15] C. Bernhardt, B. Bessire, T. Feurer, and A. Stefanov, Shaping frequency-entangled qubits, Physical Review A 88, 032322 (2013).

[16] M. Kues, C. Reimer, P. Roztocki, L. R. Cortés, S. Sciara, B. Wetzel, Y. Zhang, A. Cino, S. T. Chu, B. E. Little, D. J. Moss, L. Caspani, J. Azaña, and R. Morandotti, On-chip generation of high-dimensional entangled quantum states and their coherent control, Nature 546, 622 (2017).

[17] H.-H. Lu, E. M. Simmerman, P. Lougovski, A. M. Weiner, and J. M. Lukens, Fully arbitrary control of frequency-bin qubits, Physical Review Letters 125, 120503 (2020).

[18] H.-H. Lu, J. M. Lukens, N. A. Peters, B. P. Williams, A. M. Weiner, and P. Lougovski, Quantum interference and correlation control of frequency-bin qubits, Optica 5, 1455 (2018).

[19] H.-H. Lu, J. M. Lukens, B. P. Williams, P. Imany, N. A. Peters, A. M. Weiner, and P. Lougovski, A controlled-NOT gate for frequency-bin qubits, npj Quantum Information 5, 24 (2019).

[20] Y. Chen, S. Ecker, J. Bavaresco, T. Scheidl, L. Chen, F. Steinlechner, M. Huber, and R. Ursin, Verification of high-dimensional entanglement generated in quantum interference, Physical Review A 101, 032302 (2020).

[21] Y. Chen, S. Ecker, L. Chen, F. Steinlechner, M. Huber, and R. Ursin, Temporal distinguishability in Hong-Ou-Mandel interference for harnessing high-dimensional frequency entanglement, npj Quantum Information 7, 1 (2021).

[22] C. Reimer, S. Sciara, P. Roztocki, M. Islam, L. Romero Cortés, Y. Zhang, B. Fischer, S. Lorzanger, R. Kashyap, A. Cino, S. T. Chu, B. E. Little, D. J. Moss, L. Caspani, W. J. Munro, J. Azaña, M. Kues, and R. Morandotti, High-dimensional one-way quantum processing implemented on d-level cluster states, Nature Physics 15, 148 (2019).

[23] P. Imany, J. A. Jaramillo-Villegas, M. S. Alheshykh, J. M. Lukens, O. D. Odele, A. J. Moore, D. E. Leaird, M. Qi, and A. M. Weiner, High-dimensional optical quantum logic in large operational spaces, npj Quantum Information 5, 59 (2019).

[24] A. Martin, T. Guerreiro, A. Tiranov, S. Designolle, F. Fröwis, N. Brunner, M. Huber, and N. Gisin, Quantifying photonic high-dimensional entanglement, PHYSICAL REVIEW LETTERS , 5 (2017).

[25] A. Tiranov, S. Designolle, E. Z. Cruzeiro, J. Lavoie, N. Brunner, M. Azelins, M. Huber, and N. Gisin, Quantification of multidimensional entanglement stored in a crystal, PHYSICAL REVIEW A , 6 (2017).

[26] J. Schneeloch and G. A. Howland, Quantifying high-dimensional entanglement with Einstein-Podolsky-Rosen correlations, Physical Review A 97, 042338 (2018).

[27] J. Schneeloch, C. C. Tison, M. L. Fanto, P. M. Alsing, and G. A. Howland, Quantifying entanglement in a 68-billion-dimensional quantum state space, Nature Communications 10, 2785 (2019).

[28] Z. Xie, T. Zhong, S. Shrestha, X. Xu, J. Liang, Y.-X. Gong, J. C. Bienfang, A. Restelli, J. H. Shapiro, F. N. C. Wong, and C. Wei Wong, Harnessing high-dimensional hyperentanglement through a biphoton frequency comb, Nature Photonics 9, 536 (2015).

[29] P. Imany, O. D. Odele, J. A. Jaramillo-Villegas, D. E. Leaird, and A. M. Weiner, Characterization of coherent quantum frequency combs using electro-optic phase modulation, Physical Review A 97, 013813 (2018).

[30] D. Collins, N. Gisin, N. Linden, S. Massar, and S. Popescu, Bell inequalities for arbitrarily high-dimensional systems, Physical Review Letters 88, 040404 (2002).

[31] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Proposed experiment to test local hidden-variable theories, Physical Review Letters 23, 880 (1969).

[32] R. T. Thew, A. Acín, H. Zbinden, and N. Gisin, Bell-type test of energy-time entangled qutrits, Physical Review Letters 93, 010503 (2004).
[33] R. Fickler, R. Lapkiewicz, M. Huber, M. P. Lavery, M. J. Padgett, and A. Zeilinger, Interface between path and orbital angular momentum entanglement for high-dimensional photonic quantum information, *Nature Communications* **5**, 4502 (2014).

[34] J. Bavaresco, N. Herrera Valencia, C. Klöckl, M. Pivoluska, P. Erker, N. Friis, M. Malik, and M. Huber, Measurements in two bases are sufficient for certifying high-dimensional entanglement, *Nature Physics* **14**, 1032 (2018).

[35] J. Capmany and C. R. Fernández-Pousa, Quantum model for electro-optical phase modulation, *JOSA B* **27**, A119 (2010).

[36] P. Imany, J. A. Jaramillo-Villegas, O. D. Odele, K. Han, D. E. Leaird, J. M. Lukens, P. Lougovski, M. Qi, and A. M. Weiner, 50-GHz-spaced comb of high-dimensional frequency-bin entangled photons from an on-chip silicon nitride microresonator, *Optics Express* **26**, 1825 (2018).
Appendix A: Methods

To generate our frequency-entangled state, we use a commercial second harmonic generation (SHG) module, a 40 mm type-0 ppLN waveguide (Covesion). To pump the nonlinear process, we use a standard continuous-wave telecom laser and upconvert it with a second SHG module to better align the SPDC wavelength to ITU channels. To process the frequency entanglement, we use a telecom pulse shaper (Waveshaper 16000A) to select specific frequency subspaces for the signal and the idler photons and to tune their relative phases. This device limits the operational frequency range of our source to 40 nm of the telecom C-band only. We divide this space into 102 frequency modes for the signal and the idler photon, with a standard free spectral range (FSR) of 25 GHz and the same bandwidth. This leads to a state space of $102 \times 102$ dimensions. Note that subspace selection is an actual frequency discretization procedure, and it can be employed as a resource for flexible bandwidth allocation in reconfigurable QKD networks. The photon frequencies are then modulated with an electro-optic modulator, driven by a radio-frequency sine with the same frequency as the FSR. This sideband modulation technique allows us to distribute photons into neighboring modes according to Bessel function amplitudes [35]. Choosing a low FSR and bandwidth allows slower modulation signals than in earlier work, where FSR values of 200 GHz [16] down to 50 GHz [36] were used. Lower FSR would be better suited to increase dimensionality, but would eventually be limited by the resolution of the available optical filters and the number of photons per fraction of spectral bandwidth. Lastly, due to photon scattering into distant spectral modes, postselection is necessary to measure the right superposition states. To reduce the noise that imperfect filters may introduce, we use a narrow DEMUX of 20 GHz bandwidth centered at the corresponding signal and idler frequencies prior to coincidence measurement. The colorful illustrations at the bottom of Fig. [1] depict the sideband modulation and photon scattering.