To the question of the distribution of the workload between the teeth of chain gear sprockets

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Abstract. Approaches and methods for solving the problem of the distribution of the workload between the teeth of chain gear sprockets are considered. A review of research on this topic is given. In accordance with the premise that the elastic deformations of chain links and sprocket teeth have a decisive influence on the distribution of workload between teeth, an automated calculation technique is proposed. According to the results of the calculation, the number of simultaneously transmitting the main share of the working load of the teeth of the sprockets is limited and ranges from one to three.

1. Introduction

In mechanical engineering, chain drives are widely used (Fig. 1). They have the following advantages: compactness, high load capacity, high efficiency, small forces acting on the shafts, the ability to easily replace the chain, the ability to use in a significant range of interaxal distances.

![Figure 1. Chain drive circuit with drive roller chain.](image)

When designing chain gears, it must be borne in mind that in many cases the durability of their work is limited by wear of the teeth of the sprockets. In addition, plastic sprockets are widely used, the performance of which is primarily determined by the strength of the tooth, which transfers the
maximum load. Therefore, there is a need for a comprehensive study of the distribution of workload on the teeth of the power sector of the sprocket. The uneven distribution of the load leads to increased local tension, which reduces the strength of the teeth, and negatively affects the bearing capacity of the chain transmission. The analysis of the load distribution scheme along the teeth of the sprockets will allow, firstly, to clarify the calculation of the chain transmission; and, secondly, to give recommendations on changing the design parameters in order to obtain the most uniform distribution of the workload across the contacting teeth and, thereby, increase the reliability of the transmission, reduce its dimensions, weight and cost.

2. Related research survey
A significant contribution to the study of the loading, strength and stiffness of parts and their elements in chain transmissions was made by the staff of the Machine Parts department of the Moscow Machine-Tool Institute «Mosstankin»: V.A. Khlunov, N.I. Zeitlin, G.B. Stolbin, A.K. Kuznetsova, K.P. Zhukov. Also important and interesting results were obtained as a result of research conducted by N.V. Vorobyov, G.K. Ryabov, I.P. Glushchenko, A.A. Gotovtsev and others.

V.A. Khlunov in the article [1] analyzes the effect of the relationship between the chain pitch and the sprocket pitch on the engagement of the hinges of the sleeve-roller chains with the teeth of the sprockets. Tests are described in which sprockets with increased and reduced spacing were used at a nominal chain pitch. At the same time, there was an increased bilateral wear of the teeth of the leading and especially the driven sprockets, leading to the rapid chipping of almost all the teeth. There was also increased noise and sharp shocks, leading to cracking of the rollers and breaking the chain along the eyes of the internal link.

One of the first major publications on the study of the problems of static loading of teeths of chain gears sprockets is an article by N.I. Zeitlin and M.V. Mintz [2]. The results of theoretical and experimental studies presented in it showed that the previously assumed law of the distribution of the load on the sprocket teeth in geometric progression is not confirmed. In fact, 3-4 teeth are involved in the work.

In the work [3] N.V. Vorobyov derives calculated dependences, from which it follows that the pressure exerted by the chain hinges on the teeth of the drive sprocket decreases according to the law of decreasing geometric progression. It is also noted that the main feature of the chain operation on the driven sprocket is the fact that not a weakly stretched, but a loaded leading branch escapes from the sprocket, therefore, the operation of the chain on the driven sprocket is possible only if the chain pitch is equal to or greater than the sprocket pitch.

In the work of I.P. Glushchenko and A.A. Petrik [4], the authors noted that the pressure of the hinges on the teeth of the sprocket with increasing number decreases according to the law of decreasing geometric progression. However, it should be noted that the conclusion about the distribution of efforts on the sprocket teeth according to the law of decreasing geometric progression is considered critically by some authors, for example, G.K. Ryabov.

In articles [5, 6] G.K. Ryabov considers the force interaction between the chain and the sprocket, taking into account the elastic deformations of the chain links and the gear teeth of the sprocket, which, in the author's opinion, are a determining factor in influencing the load distribution between the teeth.

Having written down the equilibrium condition of the sprocket relative to its axis of rotation, the equilibrium conditions of the chain hinges on the sprocket teeth, and taking into account the relationship of additional deformations of the elements that are in force interaction as the sprocket rotates, the author obtains a system of equations for calculating the static component of the pressure force on the sprocket teeth, like metal ones, and plastic ones.

The author also constructed the dependence of the load values on the serial number of the tooth for some new and worn chains.
The results were confirmed by experimental studies and led to the conclusion that the number of teeth simultaneously absorbing the load in gears with a new and little worn roller chain is in the range of 3-7 depending on the number of teeth of the drive or driven sprocket. This number increases to some extent (by 1-2 teeth) as the chain wears out, varies slightly depending on the magnitude of the transmitted load and the lubrication conditions of the gear, and does not depend on the angle of girth of the chain sprockets. On the driven sprocket in relation to the leading one, the number of simultaneously loaded teeth is less by 1-2 teeth, which is explained by the fact that the chain runs onto the driven sprocket from the idle branch in a slightly tightened state, when the chain links have increased flexibility.

In the article [7] G.K. Ryabov and L.P. Sergeantova give a theoretical justification and determine the values of the minimum allowable angles of girth of the stars from the condition of ensuring the reliability of chain engagement. The authors note that the theory of the distribution of the transmitted load along the teeth of the sprockets according to the law of geometric progression is in good agreement with the experiment at small angles of rotation of the sprockets with a small number of teeth. However, with an increase in the angle of rotation of the sprockets, in accordance with theoretical and experimental data, it is revealed that, due to the elastic deformations of the loaded engagement elements, not all teeth in engagement are simultaneously involved in the load transfer, but a completely limited number of teeth. The values of the pressure forces on the teeth are found from the solution of a system of equations that take into account the influence of elastic deformations of the engagement elements.

It should be noted that in the scientific and technical literature of recent years, the authors have not identified publications that refute or clarify the conclusions presented in the considered works.

3. A mathematical model of idealized chain gearing
Automation of calculations of machine parts is one of the main means of increasing design efficiency in mechanical engineering, therefore, at the engineering and technical departments of Moscow State Technological University "STANKIN" increasing the level of automation of calculations is one of the priority tasks [8-12]. Research on this topic was carried out using an automated system (AS) developed by the authors, the conceptual foundations and software of which are discussed in articles [13, 14].

One of the basic provisions for creating an AS is the following premise: on the distribution of the workload between the contact elements in gears with a multi-pair power contact, which include a chain gear, the elastic deformations of the contact elements and the foundations of the links of the mechanism have a decisive influence [13, 14]. For chain transmission, the contact elements are the teeth of the sprockets and chain links, and the basis is the chain itself and the rims of the sprockets.

When developing a mathematical model of multi-pair force interaction in an idealized chain mesh, the following prerequisites and assumptions are made.

- The materials of the chain, teeth and rim of the sprocket are assumed to be elastic and obeying Hooke's law.
- The compliance coefficients of all the teeth and the bases of the links along the entire length of the connection are the same.
- The workload is transmitted due to elastic compression deformations of the tooth flanks.
- Based on the principle of Saint-Venant, we accept that the workload is distributed over the teeth in the form of concentrated forces applied at the mid-height of the tooth (Figure 2).
- The influence of thermal deformations, surface roughness of the contacting parts, mounting clearances on the load distribution along the teeth is not taken into account.
- We consider the tooth profile to be straight-sided, in connection with which the influence of radial forces and transverse deformations is not taken into account.
- The discrete solution method is applied, that is, the isolated operation of each pair of teeth is considered.
Let us draw up the equation of compatibility of deformations of the 1st and 2nd teeth of links A (leading) and B (driven) and sections of the bases of links [13-15]. Since the bases of the links A and B undergo multidirectional deformation - stretching and compression, the resulting deformation of the sections of the bases of the links in the first step section will be (Figure 3):

$$\delta_{p_{1-2}} = (\delta_{A_{1-2}} + \delta_{B_{1-2}}).$$

Designations in figures 2 and 3:
- A – transmission lead;
- B – transmission slave;
- $F$ – moving vector of transmitted force (work load);
- $F_i$ – the main load vector for the $i$-th tooth in fractions of force $F$;
- $t_{i-1,i}$ – tooth pitch on the $(i-1, i)$ section of the chain base (sprockets);
- $\Delta_{l}$ – longitudinal elastic deformation (compression) of the $i$-tooth, mm;
- $\delta_{l-1,i}$ – longitudinal elastic deformation (tension or compression) of the base of the link at the pitch $(i-1, i)$, mm;
• $C_A$, $C_B$ – tooth compliance factors of the leading (A) and driven (B) links, respectively, mm/N;

• $\lambda_A$, $\lambda_B$ – the compliance coefficients of the bases of the leading (A) and driven (B) links, respectively, in the area of one step, mm/N.

Using the notation introduced, we write the equation for strain compatibility:

$$(\Delta_{A_1} + \Delta_{B_1}) - (\Delta_{A_2} + \Delta_{B_2}) = (\delta_{A_{12}} + \delta_{B_{12}}),$$

or:

$$\Delta_{A_2} + \Delta_{B_2} = (\Delta_{A_1} + \Delta_{B_1}) - (\delta_{A_{12}} + \delta_{B_{12}}).$$

The physical meaning of this equation is as follows: the algebraic difference in the total longitudinal deformations of the first and second teeth of links A and B compensates for the total longitudinal deformation of the sections of the bases of links A and B in the step (1-2), which is the reason for the uneven load distribution between teeth [13-15].

We express the deformations in terms of force factors and ductility, and the tensile force in the cross section at the step $t_{1,2}$ of the link A and the compressive force in the section of the link B are found in Figure 2, a, using the section method. Similarly, the compressive force in the cross section of the link A and the tensile force in the cross section of the link B will be found in Fig. 2, b:

$$\delta_{A_{a2}} = \lambda_A (F - F_1); \quad \delta_{B_{a2}} = \lambda_B (F - F_1);$$

$$\Delta_{A_i} = C_A F_i; \quad \Delta_{B_i} = C_B F_i;$$

$$\Delta_{A_{12}} = C_A F_{12}; \quad \Delta_{B_{12}} = C_B F_{12};$$

We substitute expressions (2) and (3) into equation (1):

$$C_A F_{12} + C_B F_{12} = (C_A F_i + C_B F_i) - [\lambda_A (F - F_i) + \lambda_B (F - F_i)].$$

Similarly, we make the equation of compatibility of deformations for any pair of teeth $(i-1,i)$, wherein:

$$\Delta_{A_{i}} + \Delta_{B_{i}} = (\Delta_{A_{i-1}} + \Delta_{B_{i-1}}) - (\delta_{A_{i-1,i}} + \delta_{B_{i-1,i}}).$$

Expressing strain through force factors and compliance, we obtain:

$$\delta_{A_{i-1,i}} = \lambda_A (F - F_i - F_{i-1} - \cdots - F_{i-1});$$

$$\delta_{B_{i-1,i}} = \lambda_B (F - F_i - F_{i-1} - \cdots - F_{i-1});$$

$$\Delta_{A_{i}} = C_A F_i; \quad \Delta_{B_{i}} = C_B F_i;$$

$$\Delta_{A_{i-1}} = C_A F_{i-1}; \quad \Delta_{B_{i-1}} = C_B F_{i-1};$$

We substitute expressions (6) and (7) into equation (5):

$$C_A F_i + C_B F_i = (C_A F_{i-1} + C_B F_{i-1}) - [\lambda_A (F - F_i - F_{i-1} - \cdots - F_{i-1}) + \lambda_B (F - F_i - F_{i-1} - \cdots - F_{i-1})].$$

We transform equation (8) and divide all its parts by the sum $(C_A + C_B)$:

$$F_i = F_{i-1} - \left(\frac{(F - F_i - \cdots - F_{i-1}) \lambda_A + \lambda_B}{C_A + C_B}\right).$$

Having written Eq. (9) for each pair of teeth for $3 \leq i \leq n$, we obtain a system of (n-1) linear algebraic equations. The missing equation is the equilibrium condition:

$$F_1 + F_2 + \cdots + F_i + \cdots + F_n = F.$$
\[
\begin{align*}
F_1 + F_2 + \ldots + F_n &= F \\
F_2 &= F_1 - \left( F - F_1 \right) \frac{\lambda_A + \lambda_B}{C_A + C_B} \\
F_3 &= F_2 - \left( F - F_2 \right) \frac{\lambda_A + \lambda_B}{C_A + C_B} \\
&\quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \\
F_n &= F_{n-1} - \left( F - F_{n-1} - \ldots - F_2 - \ldots - F_1 \right) \frac{\lambda_A + \lambda_B}{C_A + C_B}.
\end{align*}
\]

System of equations (11) contains \( n \) unknowns: \( F_1, F_2, \ldots, F_n \), and constant values: \( F, \lambda_A, \lambda_B, C_A, C_B \).

The system of equations (11) is part of the mathematical support of the AS, with the help of which an automated calculation was performed. The purpose of the calculation was to determine the distribution scheme of the workload between the teeth of the power sector of the drive sprocket. The values of the compliance of the teeth of the sprocket in contact with the tooth of the drive sprocket and the chain are tentatively taken in accordance with the data of experimental and theoretical studies given in [1-7,16].

The following numerical parameters are accepted for calculation: circumferential force \( F = 3800 \) N; compliance: tooth sprocket \( \lambda_s = 0.0000004762 \) mm / N, chain \( \lambda_c = 0.00005 \) mm / N, chain link \( \lambda_{cl} = 0.0000114 \) mm / N. The number of teeth of the sprocket power sector is \( n = 12 \).

Figure 4 shows the calculation results for the roller chain 2PR-19.05-64 executed according to state standard of the Russian Federation number 13568-97.

**Figure 4.** The diagram of the distribution of workload between the teeth.

## 4. Conclusions
Based on the analysis of the calculation results, we can conclude that they correlate with the statements given in [5-7] that in the power gear sector of the sprocket, no more than 3-7 teeth are loaded, and the first tooth accounts for up to 60-80% of the transmitted load. According to [2], no more than 3-4 teeth of a leading sprocket are loaded. It should be noted, however, that the features of the design and operation of chain transmissions (chain tension from its mass, gear ratio fluctuations and dynamic loads), as well as the accepted assumptions of the mathematical model, allow the results
to be considered to reflect only the general picture of the distribution of the workload between the teeth of the sprocket power sector.

It is also necessary to distinguish between the operation of the chain on the drive, follower, guide or tension sprockets. The chain works on the leading and driven chain, perceiving a certain force (on the driving chain, the force acts on the chain in the direction of movement of the latter, on the driven chain - in the opposite direction), and from the side of the tension or guide sprocket (if we neglect friction in the bearings), the chain does not perceive force at all [3].

Based on the results of the analysis, the following general conclusions can be drawn about the effect of compliance coefficients on the distribution scheme of the workload between the teeth of the sprockets:

- an increase in the compliance of the teeth of the sprockets reduces the uneven distribution of the load;
- increasing the compliance of the chain raises the uneven distribution of the load.

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