In order to explain the empirical integer multiple rule for the stable mesons and baryons presented in the preceding paper we assume that the particles are held together in a cubic nuclear lattice. This is a novel approach to the particles, based on the fact that the range of the weak nuclear force is only a thousandth of the diameter of the nucleon, and that the crystals are the best-known macroscopic bodies held together by a microscopic force. We investigate the standing waves in a cubic nuclear lattice. From the frequency distribution of the waves follows that the masses of the γ-branch particles are integer multiples of $m(\pi^0)$. We show that each particle has automatically an antiparticle. Assuming that the energy of the oscillations is determined by Planck’s formula for the energy of a linear oscillator, it turns out that the $\pi^0$ meson and the other members of the γ-branch are like cubic black bodies filled with plane, standing electromagnetic waves. Our standing wave model explains the integer multiple rule of the masses of the neutral mesons and baryons of the γ-branch and uses nothing else but photons. Our results justify the cubic lattice assumption.

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1 Introduction

The masses of the so-called stable elementary particles are the best-known and most characteristic property of the particles, but have not yet been explained. In a preceding article [1] we have shown that it follows from the well-known decays of the elementary particles that the spectrum of the particles splits into a γ-branch and a neutrino branch. From the masses of the particles follows that the masses of the γ-branch particles are, within 3%, integer multiples of the $\pi^0$ meson and that the masses of the $\nu$-branch are integer multiples of the $\pi^0$ mesons times a factor 0.86 ± 0.02. For the masses and decays see Table I and II of [1]. The integer multiple rule suggests that the particles are the solution of a wave equation. In a previous article [2], we have tried to explain the masses of the elementary particles by monochromatic eigenfrequencies of standing waves in an elastic cube. These eigenfrequencies depend on the value of Young’s modulus of the material of the cube. In a subsequent paper [3], we have explained the value of Young’s modulus of the material of the elementary particles with the help of Born’s classical model of cubic crystals, assuming that the crystal is held together by the weak nuclear force acting between the lattice points. The explanation of the particles by monochromatic eigenfrequencies does not seem to be tenable because a monochromatic frequency does not accommodate the multitude of frequencies created in a high energy collision of $10^{-23}$ sec duration. We will now study whether the so-called stable elementary particles of the γ-branch cannot be described by the frequency spectrum of standing waves in a lattice, which can accommodate automatically the Fourier frequency spectrum of an extreme short-time collision.

The investigation of the consequences of lattices for particle theory was initiated by Wilson [4] who studied a fermion lattice. This study has developed over time into lattice QCD. The results of such endeavors have culminated in the paper of Weingarten [5] and his colleagues which required elaborate year long numerical calculations. They determined the masses of seven particles ($K^*, p, \phi, \Delta, \Sigma, \Xi, \Omega$), with an uncertainty of up to ±8%, agreeing with the observed particles within a few percent, up to 6%. Our theory covers all particles of the γ-branch, namely the $\pi^0$, $\eta$, $\eta'$, $\Lambda$, $\Sigma^0$, $\Xi^0$, $\Omega^-$, $\Lambda_c^+$, $\Sigma_c^0$, $\Xi_c^0$, and $\Omega_c^0$ particles and agrees with the measured masses within at most 3.3%. The masses of the $\nu$-branch can be explained separately by standing waves in a neutrino lattice.
2 The frequency spectrum of the oscillations of a cubic lattice

It will be necessary to outline the most elementary aspects of the theory of lattice oscillations since we will, in the following, investigate whether the masses of the elementary particles can be explained with the help of the frequency spectra of standing waves in a cubic lattice. The classic paper describing lattice oscillations is from Born and v. Karman [6], henceforth referred to as B&K. They looked at first at the oscillations of a one-dimensional chain of points with mass $m$, separated by a constant distance $a$. B&K assume that the forces exerted on each point of the chain originate only from the two neighboring points. These forces are opposed to and proportional to the displacements, as with elastic springs. The equation of motion is in this case

$$m\ddot{u}_n = \alpha (u_{n+1} - u_n) - \alpha (u_{n} - u_{n-1}).$$

The displacements of the mass points from their equilibrium position are given by $u_n$. The dots signify, as usual, differentiation with respect to time, $\alpha$ is a constant characterizing the force between the lattice points, and $n$ is an integer number.

In order to solve (1) B&K set

$$u_n = Ae^{i(\omega t + n\phi)},$$

which is obviously a temporally and spatially periodic solution. $n$ is an integer, with $n \leq N$, where $N$ is the number of points in the chain. At $2n\phi = \pi$ there are nodes, where for all times $t$ the displacements are zero, as with standing waves. If a displacement is repeated after $n$ points we have $na = \lambda$, where $\lambda$ is the wavelength, and it must be $n\phi = 2\pi$ according to (2). It follows that $\lambda = 2\pi a/\phi$. Inserting (2) into (1) one obtains a continuous frequency spectrum given by

$$\nu = 2\sqrt{\frac{\alpha}{m}} \sin(\phi/2).$$

B&K point out that there is not only a continuum of frequencies, but also a maximal frequency which is reached when $\phi = \pi$, or at the minimum of the possible wavelengths $\lambda = 2a$.

B&K then discuss the three-dimensional lattice, with lattice constant $a$ and masses $m$, the monatomic case, i.e. when all lattice points are of the same mass. They reduce the complexities of the problem by considering only the 18 points nearest to any point. These are 6 points at distance $a$, and 12 points at distance $a\sqrt{2}$. B&K assume that the forces between the points are linear functions of the small displacements, that the symmetry of the lattice is maintained, and that the equations of motion transform into the equations of motion of continuum mechanics for $a \to 0$. We cannot reproduce the lengthy equations of motion of the three-dimensional lattice. In the three-dimensional case we deal with the forces caused by the 6 points at the distance $a$ which are characterized by the constant $\alpha$ in the case of central forces. There are also the forces which originate from the 12 points at distance $a\sqrt{2}$, characterized by the constant $\gamma$, which is important later on. We investigate the propagation of plane waves in a three-dimensional monatomic lattice with the ansatz

$$u_{\ell,m} = u_0 e^{i(\omega t + \ell\phi + m\phi_2)}$$

and a similar ansatz for $v_{\ell,m}$, with $\ell, m$ being integer numbers $\leq N$, where $N$ is the number of lattice points along a side of the cube. We also consider higher order solutions, with $i_1 \cdot \ell$ and $i_2 \cdot m$, where $i_1, i_2$ are integer numbers. The boundary conditions are periodic. The number of normal modes must be equal to the number of particles in the lattice. B&K arrive, in the case of two-dimensional waves, at a secular equation for the frequencies

$$\begin{vmatrix} A(\phi_1, \phi_2) - m\nu^2 & B(\phi_2, \phi_1) \\ B(\phi_1, \phi_2) & A(\phi_2, \phi_1) - m\nu^2 \end{vmatrix} = 0,$$

The formulas for $A(\phi_1, \phi_2)$ and $B(\phi_1, \phi_2)$ are given in equation (17) of B&K.

The theory of lattice oscillations has been pursued in particular by Blackman [7], a summary of his and other studies is in [8]. Comprehensive reviews of the results of linear studies of lattice dynamics have been written by Born and Huang [9], and by Maradudin et al. [10].

3 The masses of the particles of the $\gamma$-branch

We will now assume, as seems to be quite natural, that the particles of the $\gamma$-branch consist of the same particles into which they decay, i.e., of photons. We base this assumption on the fact that photons are the principal mode of decay of the $\gamma$-branch particles, the characteristic example is $\pi^0 \to \gamma\gamma$ (98.8%). The composition of the particles of the $\gamma$-branch suggested here offers a direct route from the formation of a $\gamma$-branch particle, through its lifetime, to its decay products. Particles that are made of photons are necessarily neutral, as the majority of the particles of the $\gamma$-branch are.

We also base our assumption that the particles of the $\gamma$-branch are made of photons on the circumstances of the formation of the $\gamma$-branch particles. The most simple and straightforward creation of a $\gamma$-branch particle is the reaction $\gamma + p \to \pi^0 + p + \gamma'$. A photon impinges on a proton and creates a $\pi^0$ meson. In a timespan of order of $10^{-23}$ sec the pulse of the incoming electromagnetic wave is, according to Fourier analysis, converted into a continuum of electromagnetic waves with frequencies ranging from $10^{13}$ sec$^{-1}$ to $\nu \to \infty$. The wave packet so created decays, according to experience, after $8.4 \cdot 10^{-17}$ sec, into two electromagnetic waves or $\gamma$-rays. It seems to be very unlikely that Fourier analysis does not hold for the case of an electromagnetic wave impinging on a proton. The question then arises of what happens to the electromagnetic waves in the timespan of $10^{-16}$ seconds between the creation of the wave packet and its decay into two $\gamma$-rays? We will investigate whether the electromagnetic waves cannot continue to exist for the $10^{-16}$ seconds until the wave packet decays.

There must be a mechanism which holds the wave packet of the newly created particle together, or else it will disperse. We
assume that the very many photons in the new particle are held together in a cubic lattice. If a particle consists of photons, alternate photons must have opposite spin, otherwise the spin of the particle could not be zero. Ordinary cubic lattices, such as the NaCl lattice, are held together by attractive forces between particles of opposite polarity. We assume that the photon lattice is held together by weak attractive forces between photons of opposite polarity. Electrodynamics does not predict the existence of such a force between two photons. However, electroweak theory says that \( e^2 \approx g^2 \), and we will now assume that there is a corresponding force in the photon lattice. It is not unprecedented that photons have been considered to be building blocks of the elementary particles. Schwinger [11] has once studied an exact one-dimensional quantum electrodynamical model in which the photon acquired a mass \( \sim e^2 \).

We will now investigate the standing waves of a cubic photon lattice. We assume that the lattice is held together by a weak force acting from one lattice point to the next. We assume that the range of this force is \( 10^{-16} \) cm, because the range of the weak nuclear force is of order of \( 10^{-16} \) cm, according to [12]. Likewise, we assume that the sidelength of the lattice is about \( 10^{-13} \) cm, which follows from the size of the nucleon as given by [13]. There are then \( 10^9 \) lattice points. Because it is the most simple case, we assume that a central force acts between lattice points of different polarity. We cannot consider spin, isospin, strangeness or charm. The frequency equation for the two-dimensional oscillations of an isotropic monatomic cubic lattice with central forces is

\[
\nu^2 = \frac{2\alpha}{4\pi^2 M} \{ 2 - \cos \phi_1 \cos \phi_2 + \sin \phi_1 \sin \phi_2 - \cos \phi_1 \}.
\]

In the isotropic case, i.e. when \( \gamma/\alpha = 0.5 \), Eq. (8) follows directly from the equation of motion for the displacements in a monatomic lattice, which are given e.g. by Blackman [7]. The minus sign in front of \( \cos \phi_1 \) means that the waves are longitudinal. All frequencies that solve (8) come with either a plus or a minus sign, which is, as we will see, important.

The frequency distribution following from (8) is shown in Fig. 2. There are some easily verifiable frequencies. For example at \( \phi_1, \phi_2 = 0,0 \) it is \( \nu = 0 \), at \( \phi_1, \phi_2 = \pi/2, \pi/2 \) it is \( \nu = v_0 \sqrt{2} \), at \( \phi_1, \phi_2 = \pi/2, -\pi/2 \) we have \( \nu = v_0 \sqrt{2} \). Furthermore at \( \phi_1, \phi_2 = \pi,0 \) we have \( \nu = v_0 \sqrt{5} \), and for all values of \( \phi_1 \) it is \( \nu = 2v_0 \) at \( \phi_2 = \pi \) and \( \phi_2 = -\pi \), with \( v_0 = \sqrt{\alpha/4\pi^2 M} \), or as we will see \( v_0 = c_s/2\pi \).

The limitation of the group velocity in the photon lattice has now to be considered. The formula for the group velocity is given by

\[
c_g = \frac{d\omega}{dk} = a\sqrt{\frac{\alpha}{M}} \frac{df(\phi_1, \phi_2)}{d\phi}.
\]

The group velocity in the photon lattice has to be equal to the velocity of light \( c_s \), throughout the entire frequency spectrum, because photons move with the velocity of light. In order to learn how this requirement affects the frequency distribution we have to know the value of \( \sqrt{\alpha/M} \) in a photon lattice. But we do not have information about what either \( \alpha \) or \( M \) might be in this case. We assume in the following that \( a\sqrt{\alpha/M} = c_s \), which means, since \( a = 10^{-16} \) cm, that \( \sqrt{\alpha/M} = 3 \cdot 10^{26} \) sec\(^{-1} \), or that the corresponding period is \( \tau = \frac{1}{3} \cdot 10^{-26} \) sec, which is the time it takes for a wave to travel with the velocity of light over one lattice distance. With \( a\sqrt{\alpha/M} = c_s \), the equation for the group velocity is

\[
c_g = c_s \cdot \frac{df}{d\phi}.
\]

For a photon lattice that means, since \( c_g \) must then always be equal to \( c_s \), that \( df/d\phi = 1 \). This requirement determines the form of the frequency distribution, regardless of the order of the mode of oscillation. The frequencies of the corrected spectrum must increase from the origin \( \phi_1, \phi_2 = 0,0 \) with slope 1 until a maximum is reached, from where the frequency must decrease with slope 1 to \( \nu = 0 \). The frequency distribution corrected for (8) is shown in Fig. 3. The corrected frequency distributions of higher modes are of the same type, but for the area they cover. The second mode \((i_1, i_2 = 2)\) covers 4 times the area of the basic mode, because \( 2\phi \) ranges from 0 to \( 2\pi \), (for \( \phi > 0 \)), whereas the basic mode ranges from 0 to \( \pi \). Consequently the energy \( E \) (Eq. 1) contained in all oscillations of the second mode is four times larger than the energy of the basic mode, because the energy contained in the lattice oscillations...
must be proportional to the sum of all frequencies. Adding, by superposition, to the second mode different numbers of basic modes or second modes will give exact integer multiples of the energy of the basic mode. Now we understand the integer multiple rule: within 3% of 1, which incorporate different spins, isospins, strangeness, or charm. We remember that the measured masses in Table I of [1], because the factor one takes the ratio of the energies contained in the oscillations of the different modes or for superpositions of different modes. In other words, the masses of the different particles are integer multiples of the mass of the \( \pi^0 \) meson, assuming that there is no spin, isospin, strangeness or charm. We remember that the measured masses in Table I of [1], which incorporate different spins, isospins, strangeness, and charm spell out the integer multiples rule within 3% accuracy. It is worth noting that there is no free parameter if one takes the ratio of the energies contained in the frequency distributions of the different modes, because the factor \( \sqrt{\alpha/M} \) in Eq. (8) cancels. This means, in particular, that the ratios of the frequency distributions, or the mass ratios, are independent of the mass of the photons at the lattice points, as well as of the magnitude of the force between the lattice points.

Let us summarize our findings concerning the particles of the \( \gamma \)-branch. The \( \pi^0 \) meson is the basic mode of the photon lattice oscillations. The \( \eta \) meson corresponds to the first higher mode \( (i_1, i_2 = 2) \), as is suggested by \( m(\eta) \approx 4m(\pi^0) \). \( \eta' \) is a superposition of three basic modes on the first higher mode. The \( \Lambda \) particle corresponds to the superposition of two higher modes \( (i_1, i_2 = 2) \), as is suggested by \( m(\Lambda) \approx 2m(\eta) \). This superposition apparently results in the creation of spin \( \frac{1}{2} \). The two modes would then have to be coupled. The \( \Sigma^0 \) and \( \Xi^0 \) baryons are superpositions of one or two basic modes on the \( \Lambda \) particle. The \( \Omega^- \) particle corresponds to the superposition of three coupled higher modes \( (i_1, i_2 = 2) \) as is suggested by \( m(\Omega^-) \approx 3m(\eta) \). This procedure apparently causes spin \( \frac{3}{2} \). The charmed \( \Lambda^+ \) particle seems to be the first particle incorporating a (3.3) mode. \( \Sigma^0 \) is apparently the superposition of a basic mode on \( \Lambda^+ \), as is suggested by the decay of \( \Sigma^0 \).

The easiest explanation of \( \Xi^0 \) is that it is the superposition of two coupled (3.3) modes. The superposition of two modes of the same type is, as in the case of \( \Lambda \), accompanied by spin \( \frac{1}{2} \). The \( \Omega^0 \) baryon is apparently the superposition of two basic modes on the \( \Xi^0 \) particle. All neutral particles of the \( \gamma \)-branch are thus accounted for, the agreement between the measured masses and the theoretical values is \( \approx 3\% \), see Table I of [1]. We find it interesting that all \( \gamma \)-branch particles with coupled 2 \( \cdot \) (2,2) modes, or the \( \Omega^- \) particle with the coupled 3 \( \cdot \) (2,2) mode, have strangeness. But this rule does not hold in the presence of a (3.3) mode. All \( \gamma \)-branch particles with a (3.3) mode have charm. We have also found the \( \gamma \)-branch antiparticles, which follow from the negative frequencies which solve Eq. (8). Antiparticles have always been associated with negative energies. Following Dirac’s argument for electrons and positrons, we associate the masses following from the negative frequencies with antiparticles. We emphasize that the existence of antiparticles is an automatic consequence of our theory.

### 4 The mass of the \( \pi^0 \) meson

So far we have studied the ratios of the masses of the particles. We will now determine the mass of the \( \pi^0 \) meson in order to validate that the mass ratios connect with the actual masses of the particles. The energy of the \( \pi^0 \) meson is \( E(m(\pi^0)) = 134.976 \text{ MeV} = 2.1626 \cdot 10^{-4} \text{ erg} \). For the energy \( E \) of all oscillations we use the equation

\[
E = \frac{N h \nu_0}{(2\pi)^2} \int_0^{2\pi} f(\phi_1, \phi_2) \, d\phi_1 \, d\phi_2 , \tag{9}
\]

This equation originates from Born and v. Karman. \( N \) is the number of lattice points. For \( f(\phi_1, \phi_2) \) we use our Eq. (8). Using the frequency distribution shown in Fig. 2, it turns out that the numerical value of the double integral in (9) is 66.896 \text{ (radians}^2\text{)} for the corrected (1.1) state. With \( N = 10^9 \) and \( \nu_0 = 3 \cdot 10^{20} / 2\pi \) it follows from Eq. (8) that \( E(\text{corr})(1.1) = 5.36 \cdot 10^{-6} \text{ erg} \), that means 2.48 \cdot 10^{-12} \text{ times larger than } E(m(\pi^0)).

In order to eliminate this discrepancy we use, instead of the simple form \( E = h\nu \), the complete quantum mechanical energy of a linear oscillator as given by Planck,

\[
E = \frac{h\nu}{e^{\frac{h\nu}{kT}} - 1} . \tag{10}
\]
This equation was already used by BkK for the determination of the specific heat of cubic crystals or solids. Equation (10) calls into question the value of the temperature $T$ in the interior of a particle. We determine $T$ empirically with the formula for the internal energy of solids

$$u = \frac{R\Theta}{e^{\Theta/T} - 1}, \quad (11)$$

which is from Sommerfeld [14]. In this equation it is now $R = 10^9k$, where $k$ is Boltzmann’s constant, and $\Theta$ is the characteristic temperature introduced by Debye [15] for the explanation of the specific heat of solids. It is $\Theta = h\nu_m/k$, where $\nu_m$ is a maximal frequency. In the case of the oscillations making up the $\pi^0$ meson the maximal frequency is $\nu_m = \pi\nu_0$, see Figs. [11, 12, 13], therefore $\nu_m = 1.5 \cdot 10^{26}$ sec$^{-1}$, and we find that $\Theta = 7.199 \cdot 10^{15}$ K.

In order to determine $T$ we set the internal energy $u$ equal to $m(\pi^0)c^2$. It then follows from (13) that $\Theta/T = 29.16$, or $T = 2.47 \cdot 10^{14}$ K. That means that Planck’s formula (1) introduces a factor $1/(e^{\Theta/T} - 1) \approx 1/e^{29.16} = 1/4.613 \cdot 10^{-3}$ into Eq. (9). In other words, if we determine the temperature $T$ of the particle empirically through equation (13), then we arrive at a mass of the $\pi^0$ meson of $1.16 \cdot 10^{-4}$ erg. The difference between the exact $E(m(\pi^0)) = 2.16 \cdot 10^{-4}$ erg and our calculated $E(\text{corr})(1.1)$, which describes the $\pi^0$ meson, is well within the uncertainty of the number of the lattice points and the distance we have used. If we take the value of the radius of the nucleon given in [13] verbatim, $r = 0.8 \cdot 10^{-13}$ cm, and calculate the number of lattice points in a sphere of that radius, then there should be $2.14 \cdot 10^9$ lattice points in the particle. Since $E$ is directly proportional to the number of lattice points it follows that then $E(\text{corr})(1.1) = 1.15E(m(\pi^0))$. The energy in the mass of the $\pi^0$ meson and the energy in the corresponding lattice oscillations agree very well, considering the uncertainties of the parameters involved. It can be shown that the factor exp$(\Theta/T)$ remains constant for the higher modes.

To summarize. We find that the energy in the $\pi^0$ meson and the other particles of the $\gamma$-branch are correctly given by the energy of the standing waves, if the energy of the oscillations is determined by Planck’s formula for the energy of a linear oscillator. The $\pi^0$ meson is like an adiabatic, cubic black body filled with standing electromagnetic waves. We know from Bose’s work [16] that Planck’s formula applies to a photon gas as well.

5 Conclusions

Let us summarize what we have assumed and what we have learned from this study. In short, for each neutral meson and each neutral baryon of the $\gamma$-branch we have found a simple model of standing waves in a cubic lattice, the ratio of the energies of which agree within $3\%$ with the ratio of the energies of the masses of the particles. In order to arrive at this result, we have first assumed that the neutral mesons and baryons consist of the same particles into which they decay, which seems to be a quite natural assumption. For the explanation of the mesons and baryons of the $\gamma$-branch we use only photons, nothing else. We assume the existence of a weak force which holds the photon lattice of the $\gamma$-branch together. Then we apply the results of the classical study of Born and v. Karman, and subsequent studies, about lattice oscillations to the particle lattice. We determine the frequency distributions and the energy contained in plane standing waves in a cubic lattice. The $\gamma$-rays in the photon lattice must move with the velocity of light, and we impose the condition that the group velocity is equal to the velocity of light. From the frequency distributions of the standing waves in the lattice follow the ratios of the masses of the particles.

The masses of the $\gamma$-branch, the $\pi^0$, $\eta$, $\eta'$, $\Lambda$, $\Sigma^0$, $\Xi^0$, $\Omega^-$, $\Lambda^+_c$, $\Sigma^0_c$, $\Xi^0_c$, and $\Omega^0_c$ particles are found to be integer multiples of the mass of the $\pi^0$ meson, in agreement with what the data on the particle masses strongly suggest. The integer multiple rule is a consequence of the standing wave structure of the particles. It is important to note that in our theory the ratios of the masses of the $\gamma$-branch particles to the mass of the $\pi^0$ meson do not depend on the side length of the lattice, and the distance between the lattice points, neither do they depend on the strength of the force between the lattice points nor on the mass of the lattice points. The mass ratios are determined only by the spectra of the frequencies of the standing waves in the lattice. Since the equation determining the frequency of the standing waves is quadratic it follows automatically that for each positive frequency there is also a negative frequency of the same absolute value, that means that for each particle there exists also an antiparticle.

We have then determined the mass of the $\pi^0$ meson which follows from our theory. This requires consideration of the number $N^3$ of the lattice points and of the value of $\nu_0 = c/e2\pi a$. Assuming that the energy of the oscillations is determined by Planck’s formula for the energy of a linear oscillator, we arrive at a mass of the $\pi^0$ meson which differs from the experimentally determined $m(\pi^0)$ by $15\%$, which is well within the uncertainties of $N^3$ and $a$. The $\pi^0$ meson is like a cubic black body filled with plane, standing electromagnetic waves, whose wavelengths are integer multiples of the lattice constant $a$. A rather conservative explanation of the $\pi^0$ meson, and the $\gamma$-branch particles. It is worth noting that in the $\gamma$-branch of our model there is a continuous line leading from the creation of a particle out of photons or electromagnetic waves through the lifetime of the particle as standing electromagnetic waves to the decay products which are electromagnetic waves as well.

The concept of a nuclear cubic lattice provides more than just the masses of the particles of the $\gamma$-branch. The masses of the $\nu$-branch follow from the frequencies in a diatomic neutrino lattice made of electron and muon neutrinos, with $m(\nu_e) \sim 5$ meV/$c^2$ and $m(\nu_\mu) \sim 50$ meV/$c^2$. Furthermore, as discussed in [3], the theory of cubic lattices permits the determination of the potential of the force that holds the lattice together. Born and Landé [17] have shown that the potential must have an attractive part over longer distances and a repulsive part over shorter distances, otherwise the lattice would not be stable. Following the steps of Born and Landé we have found in...
equation (11) of [3] that the attractive and repulsive terms of the potential in a nuclear lattice differ at the lattice distance by only $10^{-12}$, if we replace the electric interaction constant $e^2$ in a crystal by $g^2$ from the weak nuclear force. Following a paper of Born and Stern [18] we have discussed in [3] also the force which acts, (in vacuum), between two cubic lattices. The attractive forces between two cubic lattices are the sum of all unsaturated weak forces at the sides of the lattices. Since there are about $10^6$ lattice points on a side of the nuclear cubic lattice considered here, the attractive force of a side of the lattice for one side of another lattice is $10^6$ times as large as the weak force acting between two lattice points. The empirical relation between the strength of the strong and weak forces is given by the ratio of the coupling constants, which is $\alpha_s/\alpha_w \approx 10^6$.

Our standing wave model does not only account for the masses of the mesons and baryons and the antiparticles of the $\gamma$-branch, but also provides access to an explanation of the weak force which holds the nuclear lattice together, and the strong force which emanates from the surface of the particles. The strong and the weak forces are unified in this model.

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