Initial state effects in the Color Glass Condensate

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Abstract. The Color Glass Condensate provides a systematic power counting of initial state effects in high energy QCD. We briefly discuss in this framework quark and gluon production in high energy collisions.

1. Introduction

The Color Glass Condensate (CGC) is an effective theory of the hadronic wavefunction at high energies. For a review and references, see Ref. [1]. Key features, as summarized in the McLerran-Venugopalan model (MV-model) [2], are as follows. The dynamical degrees of freedom are small $x$ partons with occupation numbers of $O(\alpha_s^{-1})$ best described by the classical gauge fields $A^a_\mu$. The large $x$ partons are static light-cone sources of color charge represented by the two dimensional color charge density $\rho_a(x_\perp)$. The classical field of the nucleus obeys the Yang-Mills equations $[D_\mu,F^{\mu\nu}]_a = \delta^{\nu+}\delta(x^-)\rho_a(x_\perp)$. The sources $\rho_a$ are described by the gauge invariant weight function $W_{x_0}[\rho_a]$ where $x_0$ separates “fields” and “sources”. As $x_0$ decreases, fields are transformed into sources – the weight functional obeys a renormalization group equation (the JIMWLK equation) describing its evolution with $x_0$. The evolution of the system is characterized by the saturation scale $Q_s(x) \gg \Lambda_{\text{QCD}}$. One determines $a \text{ posteriori}$ that $\alpha_s(Q_s) \ll 1$. Observables $O[A^\mu]$ are computed in the classical field for fixed $\rho_a$ and then averaged over $W_{x_0}[\rho_a]$ to obtain the gauge invariant expectation value: $\langle O \rangle = \int [D\rho_a] O[A^\mu(\rho_a)] W_{x_0}[\rho_a]$.

In the CGC framework, the ratio of the density $\rho$ of hard parton color charges to the transverse momentum scale $k_\perp$ of interest ($\rho/k^2_\perp$) enables a systematic power counting of initial and final state effects in the scattering of both dilute and dense partonic systems [1]. These will now be discussed.

2. Gluon and Quark production in the dilute/pp regime ($\rho_{p1}/k^2_\perp, \rho_{p2}/k^2_\perp \ll 1$)

Inclusive gluon production in the CGC is computed by solving the Yang-Mills equations $[D_\mu,F^{\mu\nu}]_a = J^\nu_a$, where $J^\nu_a = \delta(x^-)\delta^{\nu+}\rho_{p1} + \delta(x^+)\delta^{\nu-}\rho_{p2}$, with initial conditions given by the Yang-Mills fields of the two nuclei before the collision. To lowest order in $\rho_{p1}/k^2_\perp$ and $\rho_{p2}/k^2_\perp$, one can compute inclusive gluon production analytically. This was first done in
the $A^r = 0$ gauge \[3\] and subsequently in the Lorenz gauge $\partial_\mu A^\mu = 0$ \[4\]. The result at this order is $k_\perp$-factorizable into the product of the unintegrated gluon distributions in the two projectiles.\[‡\] The comparison of this result to the collinear pQCD $gg \to gg$ process and the $k_\perp$-factorized $gg \to g$ was performed in ref. \[9\]. This result for gluon production is substantially modified, as we shall discuss later, by high parton density effects in the projectiles.

The result for inclusive quark pair production can be expressed in $k_\perp$-factorized form as \[6\]

$$\frac{d\sigma_1}{dy_pdy_qd^2p_\perp d^2q_\perp} = \frac{1}{(2\pi)^6d_A} \int \frac{d^2k_1\perp d^2k_2\perp}{(2\pi)^2} \frac{\delta(k_1\perp + k_2\perp - p_\perp - q_\perp)}{k_1^2 k_2^2} \times \varphi_1(k_1\perp) \varphi_2(k_2\perp) \frac{\text{Tr} \left( |m_{ab}^+(k_1, k_2; q, p)|^2 \right)}{k_1^2 k_2^2}, \tag{1}$$

where $\varphi_1$ and $\varphi_2$ are the unintegrated gluon distributions in the projectile and target respectively (with the gluon distribution defined as $xG(x, Q^2) = \int_0^2 d(k_+^2) \varphi(x, k_\perp)$). The matrix element $\text{Tr} |m_{ab}^+(k_1, k_2; q, p)|^2$ is identical to the result derived in the $k_\perp$-factorization approach, which has been applied extensively to study heavy quark production at collider energies \[7\]. In the limit $k_1\perp, k_2\perp \to 0$, $\text{Tr} |m_{ab}^+(k_1, k_2; q, p)|^2/k_1^2 k_2^2$ is well defined – after integration over the azimuthal angles in eq. \[8\] one obtains the usual matrix element $|M|^2_{gg\to qq}$, recovering the lowest order pQCD collinear factorization result.

### 3. Gluon and Quark production in the semi-dense/pA regime

$(\rho_p/k_\perp^2 \ll 1, \rho_A/k_\perp^2 \sim 1)$

Here one solves the Yang-Mills equations $[D_\mu, F^{\mu\nu}] = J^\nu$ (with $J^\nu = \delta^{\nu+}\delta(x^-) \rho_p(x_\perp) + \delta^{\nu-}\delta(x^+) \rho_A(x_\perp)$) to determine the gauge field produced at lowest order in the proton source density and to all orders in the nuclear source density. The computations are performed in Lorenz/covariant gauge $\partial_\mu A^\mu = 0$. Gluon production, in this framework, was first computed by Kovchegov and Mueller \[8\]. In ref. \[9\], the gluon field produced in pA collisions was computed explicitly. One obtains

$$A^\mu(q) = A_p^\mu(q) + \frac{ig}{q^2 + iq^2} \int \frac{d^2k_1\perp}{(2\pi)^2} \left\{ C_U^{\mu}(q, k_1\perp) [U(k_2\perp) - (2\pi)^2 \delta(k_2\perp)] \right. \left. + C_V^{\mu}(q) [V(k_2\perp) - (2\pi)^2 \delta(k_2\perp)] \right\} \frac{\rho_p(k_1\perp)}{k_1^2 k_2}, \tag{2}$$

with $k_2 \equiv q - k_1$ and $U$ & $V$ Wilson lines containing all orders in the nuclear source density $\rho_A$. The coefficient functions $C_U^{\mu}$ and $C_V^{\mu}$ are simply related to the well known Lipatov effective vertex $C_L^{\mu}$ through the relation $C_L^{\mu} = C_U^{\mu} + \frac{1}{2} C_V^{\mu}$.

The path-ordered exponential $U$ is a color matrix arising from the rotation of the color charge density of the proton source due to multiple scattering off the nucleus. The

\[‡\] This quantity is not the usual unintegrated distribution but is closely related. See Ref. \[11\] for a discussion.
path-ordered exponential $V$ (differing from $U$ by a factor $1/2$ in the argument of the exponential) arises from the propagation of the produced gluon through the nucleus. Interestingly, the $V$’s do not appear in the final result for gluon production. This is because for gluons produced on-shell one finds remarkably that $C_U \cdot C_V = C_V^2 = 0$ and $C_U^2 = C_L^2 = 4k_{1\perp}^2k_{2\perp}/q_{\perp}^2$. Thus only bi-linears of the Wilson line $U$ survive in the squared amplitude that gives the gluon production cross-section. The result is $k_{\perp}$-factorizable, except that now one replaces $\varphi_2$ with the unintegrated nuclear gluon distribution $\varphi_A \propto \langle U^\dagger U \rangle$. This distribution contains powers of the usual unintegrated gluon distribution ($\varphi_2$ in eq. 1) at large transverse momentum.

Our result in the Lorenz gauge is exactly equivalent to that of Dumitru & McLerran in the $A^\tau = 0$ gauge [10]. The Cronin effect in proton-nucleus collisions has been studied by us in Ref. [9] and several other authors previously and since and will not be discussed further here.

Quark production can now be computed with the gauge field in eq. 2 [11]. The field is decomposed into the sum of ‘regular’ terms and ‘singular’ terms; the latter containing a factor $\delta(x^+)$.

Our result for quark pair production, unlike gluon production, is not $k_{\perp}$-factorizable. It can however still be written in $k_{\perp}$-factorized form as a product of the unintegrated gluon distribution in the proton times a sum of terms with three unintegrated distributions, $\varphi_A^{g,g}$, $\varphi_A^{g,q}$ and $\varphi_A^{q,q}$. These are respectively proportional to 2-point, 3-point and 4-point correlators of the Wilson lines we discussed previously. For instance, the distribution $\varphi_A^{q,q}$ can be interpreted as the probability of having a $q\bar{q}$ pair in the amplitude and a gluon in the complex conjugate amplitude. For large transverse momenta or large mass pairs, the 3-point and 4-point distributions collapse to the unintegrated gluon distribution, and we recover the result for pair production (eq. 1) in the dilute/pp limit.

Single quark distributions are straightforwardly obtained. Here, the 4-point correlator in $\varphi_A^{q,q}$ collapses to a 2-point correlator $\varphi_A^{q,q}$ corresponding to a quark (or anti-quark) in the amplitude and complex conjugate amplitude.

For Gaussian sources, as in the MV-model, these 2,3 and 4-point functions can be computed exactly as discussed in ref. [11]. Single quark distributions in the MV-model were recently computed by Tuchin [12].
4. Gluon and Quark production in the dense/AA regime

\( \rho_{A1}/k_{\perp}^2 , \rho_{A2}/k_{\perp}^2 \sim 1 \)

This case is the relevant one for particle production in heavy ion collisions. It involves solving the Yang-Mills equations to all orders in the sources of both nuclei. This problem has not been solved analytically thus far – \( k_{\perp} \)-factorization breaks down completely here, even for gluon production.

The problem has however been solved numerically \[13\]. Non-perturbative formulae are derived relating (for collisions of identical nuclei) the saturation scale \( Q_s \) in the nuclear wave-function to the energy and number distributions of gluons produced immediately after the collision (on a time scale \( \sim 1/Q_s \)). The gluon distribution is infra-red finite and is fit by a massive Bose-Einstein distribution with a “temperature” \( T \sim 0.47 Q_s \) and \( m \sim 0.04 Q_s \) for \( k_{\perp} \leq 1.5 Q_s \). For \( k_{\perp} > 1.5 Q_s \), it is fit by the tree level perturbative form \( Q_s^4/4k^4 \ln(4\pi k/Q_s) \).

The classical field description is valid as long as the occupation number \( f \) is greater than unity. As the system evolves, it becomes dilute and the classical description breaks down. Baier, Mueller, Schiff and Son \[14\] estimated this time to be \( \mathcal{O}(\alpha_s^{-3/2} Q_s^{-1}) \)

In their “bottom-up” scenario, they suggest that inelastic \( 2 \to 3 \) processes, though parametrically suppressed, are actually more efficient in driving the system from the classical stage towards thermalization. The analysis is valid for very small couplings and suggests that thermalization may take several fermis to achieve. In this light, the early thermalization required in some RHIC phenomenology appears puzzling. Arnold, Lenaghan and Moore have suggested that collective instabilities might drive the system faster towards equilibrium \[15\].

An interesting possibility is suggested by the following. As the classical field expands, one identifies a scale \( \Lambda(\tau_0) \) at an early time \( \tau_0 \) which separates high momentum particles from low momentum classical fields. The high momentum particles scatter off the fields while the classical fields interact with each other and with the particles. With time, at an appropriate \( \tau_1 \), one can define a new (“coarse graining”) scale \( \Lambda(\tau_1) \) at which one re-defines field and particle modes. We have developed an algorithm for a scalar field theory which implements this dynamical coarse graining while ensuring energy-momentum conservation \[16\]. While promising, much work remains to extend this formalism to gauge theories.

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