Abstract

The ability for machine learning to exacerbate bias has led to many algorithms centered on fairness. For example, fair clustering algorithms typically focus on balanced representation of protected attributes within clusters. Here, we develop a fair clustering variant where the input data is a hypergraph with multiple edge types, representing information about past experiences of groups of individuals. Our method is based on diversity of experience, instead of protected attributes, with a goal of forming groups that have both experience and diversity with respect to participation in edge types. We model this goal with a regularized edge-based clustering objective, design an efficient 2-approximation algorithm for optimizing the NP-hard objective, and provide bounds on hyperparameters to avoid trivial solutions. We demonstrate a potential application of this framework in online review platforms, where the goal is to curate sets of user reviews for a product type. In this context, “experience” corresponds to users familiar with the type of product, and “diversity” to users that have reviewed related products.

1 Fair Clustering and Group Formation

Left to make decisions based solely on training data, machine learning algorithms may make decisions that are biased or unfair towards a subset of a population [8, 14, 15]. There are now a variety of algorithmic fairness techniques to combat this issue [16, 23, 28], which broadly seek to address two overarching questions: what constitutes fairness (for a given task), and how (if at all) can an algorithm efficiently guarantee fairness? For the broad task of clustering, fairness is typically formulated in terms of protected attributes on data points — a cluster is “fair” if it exhibits a proper balance between nodes from different protected classes. The goal is then to design an algorithm that optimizes a classical clustering objective while also adhering to constraints ensuring a balance on the protected attributes. This approach has been applied to a variety of clustering objectives and types of data (e.g., point cloud or graph data) [2, 4, 3, 9, 12, 13, 17, 25].

Although balancing protected attributes in clusters is broadly applicable, this accounts for only one notion of cluster fairness that can be meaningfully encouraged in practice. In this paper we explore a new notion of fairness in clustering that focuses on forming groups that are diverse and experienced in terms of past group interactions. As a motivating example, consider a fair team formation task in which the goal is to assign a task to a group of people who (1) already have some level of experience working together on the given task, and (2) are diverse in terms of their previous work experience. As another example, a recommender system may want to display a diverse yet cohesive set of reviews for a certain class of products. In other words, the set of displayed reviews should work well together in providing an accurate overview of the product category. There are alternative fair group formation algorithms [6, 11, 24, 27, 31, 32], but these do not focus on diversity of experience, which has long been of interest in sociology, psychology, and business management [18, 20, 21, 26].

Here, we formalize diverse and experienced group formation as a clustering problem on edge-labeled hypergraphs. In this context, a hyperedge represents a set of objects (such as a group of individuals) that have participated in a group interaction or experience. The hyperedge label encodes the type or category of interaction (e.g., a type of team project). The output is then a labeled clustering of nodes, where cluster labels are chosen from the set of hyperedge labels. The goal is to form clusters whose nodes are balanced in terms of experience and diversity. By experience we mean that a cluster with label ℓ should contain nodes that have previously participated in hyperedges of label ℓ. By diversity, we mean that clusters should also include nodes that have participated in other hyperedge types.

Our mathematical framework for diverse and experienced clustering builds on an existing objective for clustering edge-labeled hypergraphs [5]. This objective encourages cluster formation in such a way that hyperedges of a certain label tend to be contained in a cluster with the same label, similar to (chromatic) correlation clustering [7, 10]. We add a regularizer to this objective that encourages clusters to contain nodes that have participated in many
different hyperedge types. This diversity-encouraging regularization is governed by a tunable parameter \( \beta \geq 0 \), where \( \beta = 0 \) corresponds to the original edge-labeled clustering objective. Although the resulting objective is NP-hard in general, we design a linear programming algorithm that guarantees a 2-approximation for any choice of \( \beta \). We show that certain values of this hyperparameter reduce to extremal solutions with closed-form solutions where just diversity or just experience is encouraged. In order to guide a meaningful hyperparameter selection, we show how to bound the region in which non-extremal solutions occur, leveraging linear programming sensitivity techniques. Finally, we develop and analyze a dynamic version of our objective in which groups iteratively evolve over time based on previous solutions to the regularized objective. We demonstrate the utility of our framework by applying it to team formation of users posting answers on Stack Overflow, and the task of aggregating a diverse set of reviews for categories of establishments and products on review sites (e.g., Yelp and Amazon).

## 2 Forming Clusters Based on Diversity and Experience

We first introduce notation for edge-labeled clustering. After, we analyze an approach that seems natural for clustering based on experience and diversity, but leads only to trivial solutions. We then show how a regularized version of a previous hypergraph clustering framework leads to a more meaningful objective, which will be the focus of the remainder of the paper.

**Notation.** Let \( G = (V, E, C, \ell) \) be a hypergraph with labeled edges, where \( V \) is the set of nodes, \( E \) is the set of (hyper)edges, \( L \) is a set of edge labels, and \( \ell: E \to L \) maps edges to labels. Let \( k = |L| \) be the number of labels, \( L = \{1, \ldots, k\} \) for simplicity, \( E_c \subseteq E \) the edges with label \( c \), and \( r \) the maximum hyperedge size. Following graph-theoretic terminology, we often refer to elements in \( L \) as “colors”; in data, \( L \) represents categories or types. For any node \( v \in V \), let \( d_{c}^{v} \) be the number of hyperedges of color \( c \) in which node \( v \) appears. We refer to \( d_{c}^{v} \) as the color degree of \( v \) for color \( c \).

Given this input, an algorithm will output a color for each node. Equivalently, this is a clustering \( C \), where each node is assigned to exactly one cluster, and there is exactly one cluster for each color in \( L \). We use \( C(i) \) to denote the nodes assigned to color \( i \in L \). We wish to find a clustering that promotes both diversity (clusters have nodes from a range of colored hyperedges), and experience (a cluster with color \( c \) contains nodes that have experience participating in hyperedges of color \( c \)).

### 2.1 A flawed but illustrative first approach

We start with an illustrative (but ultimately flawed) clustering objective whose characteristics will be useful in the rest of the paper. For this, we first define *diversity and experience scores* for a color \( i \), denoted \( D(i) \) and \( E(i) \), as follows:

\[
D(i) = \sum_{v \in C(i), c \neq i} d_{c}^{v}, \quad E(i) = \sum_{v \in C(i)} d_{c}^{v}.
\]

In words, \( D(i) \) measures how much nodes in cluster \( i \) have participated in hyperedges that are *not* color \( i \), and \( E(i) \) measures how much nodes in cluster \( i \) have participated in hyperedges of color \( i \). A seemingly natural but ultimately naive objective for balancing experience and diversity is:

\[
\max_{C} \sum_{i \in L} [E(i) + \beta D(i)].
\]

The regularization parameter \( \beta \) determines the relative importance of the diversity and experience scores. It turns out that the optimal solutions to this objective are overly-simplistic, with a phase transition at \( \beta = 1 \). We define two simple types of clusterings as follows:

- **Majority vote:** Node \( v \) is placed in cluster \( C(i) \) where \( i \in \arg\max_{c \in L} d_{c}^{v} \), i.e., node \( v \) is placed in a cluster for which it has the most experience.
- **Minority vote:** Node \( v \) is placed in cluster \( C(i) \) where \( i \in \arg\min_{c \in L} d_{c}^{v} \), i.e., node \( v \) is placed in a cluster for which it has the least experience.

**Theorem 1** A majority vote clustering optimizes (2) for all \( \beta > 1 \), and a minority vote clustering optimizes the same objective for all \( \beta < 1 \). Both are optimal when \( \beta = 1 \).

**Proof** For node \( i \), assume without loss of generality that the colors \( 1, 2, \ldots, k \) are ordered so that \( d_{1}^{i} \geq \cdots \geq d_{k}^{i} \). Clustering \( i \) with cluster 1 gives a contribution of \( d_{1}^{i} + \beta \sum_{j=2}^{k} d_{j}^{i} \) to the objective, while clustering it with color \( c \neq i \) gives contribution \( d_{c}^{i} + \beta \sum_{j \neq i} d_{j}^{i} \). Because \( d_{1}^{i} \geq \cdots \geq d_{k}^{i} \), the first contribution is greater than or equal to the second if and only if \( \beta \leq 1 \). Hence, majority vote is optimal when \( \beta \geq 1 \). A similar argument proves optimality for minority vote when \( \beta < 1 \). \( \square \)
Objective (2) is easy to analyze, but has optimal points that do not provide a balance between diversity and experience. This occurs because a clustering will maximize the total diversity $\sum_{c \in L} D(c)$ if and only if it minimizes the total experience $\sum_{c \in L} E(c)$, as these terms sum to a constant:

**Observation 2** $\sum_{c \in L}[E(c) + D(c)]$ is a constant independent of $\mathcal{C}$.

### 2.2 Fairness-regularized categorical edge clustering

We now turn to a more sophisticated approach: a regularized version of the categorical edge clustering objective [5]. For a clustering $\mathcal{C}$, the objective accumulates a penalty of 1 for each hyperedge of color $c$ that is not completely contained in the cluster $C(c)$. More formally, the objective is:

$$\min_{\mathcal{C}} \sum_{c \in L} \sum_{e \in E_c} x_e,$$

where $x_e$ is an indicator variable equal to 1 if hyperedge $e \in E_c$ is not contained in cluster $C(c)$, but is zero otherwise. This all-or-nothing penalty encourages entire hyperedges to be contained inside clusters of the corresponding color. For our context, this objective can be interpreted as promoting group experience in cluster formation: if a group of people have participated together in task $c$, this is an indication they could work well together on task $c$ in the future. However, we want to avoid the scenario where groups of people endlessly work on the same type of task without the benefitting from the perspective of others with different experiences. Therefore, we regularize objective (3) with a penalty term $\beta \sum_{c \in L} E(c)$. Since $\sum_{c \in L}[E(c) + D(c)]$ is a constant (Observation 2), this regularization encourages higher diversity scores $D(c)$ for each cluster $C(c)$. The resulting objective, which we focus on in the remainder of the paper, can be stated as an integer linear program. We refer to this as fairness-regularized categorical edge clustering (FRCEC):

$$\begin{align*}
\min & \sum_{c \in L} \sum_{e \in E_c} x_e + \beta \sum_{v \in V} \sum_{c \in L} d_v^c (1 - x_v^c) \\
\text{s.t.} & \quad \text{for all } v \in V: \sum_{c=1}^k x_v^c = k - 1, \quad \text{for all } c \in L, e \in E_c: x_v^c \leq x_e \quad \text{for all } e \in E \\
& \quad \text{for all } c \in L, v \in V, e \in E: x_v^c, x_e \in \{0, 1\}. 
\end{align*}$$

The binary variable $x_v^c$ equals 1 if node $v$ is not assigned label $c$, and is 0 otherwise. The first constraint guarantees every node is assigned to exactly 1 color, while the second constraint guarantees that if a single node $v \in e$ is not assigned to the cluster of color $c$, then $x_v^c = 1$ [5].

**A polynomial-time 2-approximation algorithm.** Optimizing the case of $\beta = 0$ is NP-hard [5], so FRCEC is also NP-hard. Although the optimal solution to (4) may vary with $\beta$, we develop a simple algorithm based on solving an LP relaxation of the ILP that rounds to a 2-approximation for every value of $\beta$. Our LP relaxation of the ILP in (4) replaces the binary constraints $x_v^c, x_e \in \{0, 1\}$ with linear constraints $x_v^c, x_e \in [0, 1]$. The LP can be solved in polynomial time, and the objective score is a lower bound on the optimal solution score to the NP-hard ILP. The values of $x_v^c$ can then be rounded into integer solutions to produce a clustering that is within a bounded factor of the LP lower bound, and therefore within a bounded factor of optimality. Our algorithm is simply stated:

**Algorithm 1**

1. Solve the LP relaxation of the ILP in (4).
2. For each node $v \in V$, assign $v$ to any $c \in \text{argmin}_j x_j^c$.

**Theorem 3** For any $\beta \geq 0$, Algorithm 1 returns a 2-approximation for objective (4).

**Proof** Let the relaxed solution be $\{x_v^c, x_e^c\}_{c \in E, v \in V, e \in L}$ and the corresponding rounded solution $\{x_v^c, x_e^c\}_{c \in E, v \in V, e \in L}$. Let $y_v^c = 1 - x_v^c$ and $y_e^c = 1 - x_e^c$. Our objective evaluated at the relaxed and rounded solutions respectively are

$$S = \sum_c x_v^c + \beta \sum_{v \in V} \sum_{c \in L} d_v^c y_v^c \quad \text{and} \quad S^* = \sum_c x_v^c + \beta \sum_{v \in V} \sum_{c \in L} d_v^c y_v^c.$$

We will show that $S \leq 2S^*$ by comparing the first and second terms of $S$ and $S^*$ respectively. The first constraint in (4) ensures that $x_v^c < 1/2$ for at most a single color $c$. Thus, for every edge $e$ with $x_e = 1$, $x_e^c \geq 1/2$ for some $v \in e$. In turn, $x_v^c \geq 1/2$, so $x_v^c \leq 2x_v^c$. If $x_e = 0$, then $x_e \leq 2x_v^c$ holds trivially. Thus, $\sum_v x_v^c \leq 2 \sum_e x_e^c$.

Similarly, since $x_v^c = 1$ ($y_v^c = 0$) if and only if $x_e^c \geq 1/2$ ($y_v^c \leq 1/2$), and $x_v^c = 0$ otherwise, it follows that $y_v^c \leq 2y_v^c$. Thus, $\sum_v y_v^c \leq 2 \sum_v y_v^c$. $\square$

### 2.3 Extremal LP and ILP solutions at large enough values of $\beta$

In general, Objective (4) provides a meaningful way to balance group experience (the first term) and diversity (the regularization, via Observation 2). However, when $\beta \to \infty$, the objective corresponds to simply minimizing experience, (i.e., maximizing diversity), which is solved via the aforementioned minority vote assignment. We formally
show that the optimal integral solution (4), as well as the relaxed LP solution under certain conditions, transitions from standard behavior to extremal behavior (specifically, the minority vote assignment) when \( \beta \) becomes larger than the maximum degree in the hypergraph. In Section 3 we show how to computationally bound these transition points. We first consider a simple bound on \( \beta \) above which minority vote is optimal. Let \( d_{\text{max}} \) be the largest number of edges any node participates in.

**Theorem 4** For every \( \beta > d_{\text{max}} \), a minority vote assignment optimizes (4).

**Proof** Let binary variables \( \{x_v, x_v^c\} \) encode a clustering for (4) that is not a minority vote solution. This means that there exists at least one node \( v \) such that \( x_v^c = 0 \) for some color \( c \notin \text{argmin}_v d_v^c \). If we move node \( v \) from cluster \( c \) to some cluster \( m \in \text{argmin}_v d_v^m \), then the regularization term in the objective would decrease (i.e., improve) by \( \beta(d_v^c - d_v^m) \geq \beta > d_{\text{max}} \), since degrees are integer-valued and \( d_v^c > d_v^m \). Meanwhile, the first term in the objective would increase (i.e., become worse) by at most \( \sum_{c \in \mathbb{E}} x_c = d_{\text{max}} < \beta \). Therefore, an arbitrary clustering is not a minority vote solution cannot be optimal when \( \beta > d_{\text{max}} \).

A slight variant of this result also holds for the LP relaxation. For a node \( v \in V \), let \( M_v \subset L \) be the set of minority vote clusters for \( v \), i.e., \( M_v = \text{argmin}_c d_v^c \) (treating argmin as a set). The next theorem says that for \( \beta > d_{\text{max}} \), the LP places all “weight” for \( v \) on its minority vote clusters. We consider this to be a relaxed minority vote LP solution, and Algorithm 1 will round the LP relaxation to a minority vote clustering.

**Theorem 5** For every \( \beta > d_{\text{max}} \), an optimal solution to the LP relaxation of (4) will satisfy \( \sum_{c \in \mathbb{E}_v} (1 - x_v^c) = 1 \) for every \( v \in V \). Consequently, the rounded solution from Algorithm 1 is a minority vote clustering.

**Proof** Let \( \{x_v, x_v^c\} \) encode an arbitrary solution to the LP relaxation of (4), and assume specifically that it is not a minority vote solution. For every \( v \in V \) and \( c \in L \), let \( y_v^c = 1 - x_v^c \). The \( y_v^c \) indicate the “weight” of \( v \) placed on cluster \( c \), with \( \sum_{c \in L} y_v^c = 1 \). Since \( \{x_v, x_v^c\} \) is not a minority vote solution, there exists some \( v \in V \) and \( j \notin \mathbb{M}_v \) such that \( y_v^j = \varepsilon > 0 \).

We will show that as long as \( \beta > d_{\text{max}} \), we could obtain a strictly better solution by moving this weight of \( \varepsilon \) from cluster \( j \) to a cluster \( \mathcal{M}_v \). Choose an arbitrary \( m \in \mathbb{M}_v \), and define a new set of variables \( \hat{y}_v^j = 0, \hat{y}_v^m = y_v^m + \varepsilon \), and \( \hat{y}_v^i = y_v^i \) for all other \( i \notin \{m, j\} \). Define \( \hat{x}_v^c = y_v^c \) for all \( c \in L \). For any \( u \in V \), \( u \neq v \), we keep variables the same: \( \hat{y}_u^c = y_u^c \) for all \( c \in L \). Set edge variables \( \hat{x}_v \) to minimize the LP objective subject to the \( \hat{y}_c \) variables, i.e., for \( c \in L \) and every \( e \in \mathbb{E}_c \), let \( \hat{x}_v = \max_{u \in e} \hat{x}_u^c \).

Our new set of variables simply takes the \( \varepsilon \) weight from cluster \( j \) and moves it to \( m \in \mathbb{M}_v \). This improves the regularization term in the objective by at least \( \beta \varepsilon \):

\[
\beta \sum_{c \in L} d_v^c(y_v^c - \hat{y}_v^c) = \beta d_v^m(y_v^m - \hat{y}_v^m) + \beta d_v^j(y_v^j - \hat{y}_v^j) = -\beta d_v^m \varepsilon + \beta d_v^j \varepsilon = \beta \varepsilon (d_v^j - d_v^m) \geq \beta \varepsilon .
\]

Next, we will show that the first part of the objective increases by at most \( \varepsilon d_{\text{max}} \). To see this, note that for \( e \in E_j \) with \( v \in e \), \( \hat{x}_v \geq 1 - \hat{y}_v^j = 1 \Rightarrow \hat{x}_v = 1 \) and \( x_v \geq 1 - y_v^j = 1 \). Therefore, for \( e \in E_j \), \( v \in e \), we know that \( \hat{x}_v - x_v \geq 1 - x_v \leq 1 - (1 - \varepsilon) = \varepsilon \). For \( e \in E_m \) with \( v \in e \) we know \( \hat{x}_v - x_v \leq 0 \), since \( \hat{x}_v = \max_{u \in e} \hat{x}_u^c (1 - \hat{y}_v^m) + x_v = \max_{u \in e} (1 - y_u^m) \), but the only difference between \( y_v^m \) and \( \hat{y}_v^m \) variables is that \( \hat{y}_v^m = y_v^m + \varepsilon \Rightarrow (1 - \hat{y}_v^m) < (1 - y_v^m) \). For all other edge sets \( E_c \) with \( c \notin \{m, j\} \), \( \hat{x}_v = x_v \). Therefore, for optimality, for every \( v \in V \), \( \sum_{c \in \mathbb{E}_v} \hat{x}_v - x_v \leq \varepsilon d_{\text{max}} \). In other words, as long as \( \beta > \varepsilon \), we can strictly improve the objective by moving around a positive weight \( y_v^j = \varepsilon \) from a non-minority vote cluster \( j \notin \mathbb{M}_v \) to some minority vote cluster \( m \in \mathbb{M}_v \). Therefore, at optimality, for every \( v \in V \), \( \sum_{c \in \mathbb{E}_v} y_v^c = 1 \).

Theorem 5 implies that if there is a unique minority vote clustering (i.e., each node has one color degree strictly lower than all others), then this clustering is optimal for both the original objective and the LP relaxation whenever \( \beta > d_{\text{max}} \). Whether or not the optimal solution to the LP relaxation is the same as the ILP one, the rounded solution still corresponds to some minority vote clustering that does not meaningfully balance diversity and experience. The bound \( \beta > d_{\text{max}} \) is loose in practice; our numerical experiments show that the transition occurs for smaller \( \beta \). In the next section, we use techniques in LP sensitivity analysis that allow us to better bound the phase transition computationally for a given labelled hypergraph. For this type of analysis, we still need the loose bound as a starting point.

### 3 Bounding Hyperparameters that Yield Extremal Solutions

In order to find a meaningful balance between experience and diversity, we would like to first find the smallest value of \( \beta \), call it \( \beta^* \), for which \( \beta > \beta^* \) yields a minority vote clustering. After, we could consider the hyperparameter

\[
\text{objective} = \sum_{c \in L} d_v^c(y_v^c - \hat{y}_v^c) + \beta d_v^i(y_v^i - \hat{y}_v^i) \geq \beta d_v^i \varepsilon + \beta d_v^j \varepsilon = \beta \varepsilon (d_v^j - d_v^m) \geq \beta \varepsilon .
\]
Consider a perturbation of (8): \( \theta \) where \( \beta \)

Finding \( \beta \) so that we can re-write objective (7) with a relaxed solution to a clustering objective \([19, 30]\). We have adapted these results for our fairness-regularized objective. Our approach for computing \( \hat{\beta} \) is based on previous techniques for bounding the optimal parameter regime for a relaxed solution to a clustering objective \([19, 30]\). We have adapted these results for our fairness-regularized clustering objective. This technique can also be used to compute the largest value of \( \beta \) for which LP relaxation of our objective will coincide with the relaxed solution when \( \beta = 0 \) (i.e., the unregularized objective), though we focus on computing \( \beta \) here.

The LP relaxation of our regularized objective can be written abstractly in the following form

\[
\min_x c^T x + \beta c_d^T x \quad \text{s.t. } Ax \geq b, x \geq 0, \tag{7}
\]

where \( x \) stores variables \( \{x_e, x^*_e\} \), \( Ax \geq b \) encodes constraints given by the LP relaxation of (4), and \( c_e, c_d \) denote vectors corresponding to the experience and diversity terms in our objective, respectively. Written in this format, we see that this LP-relaxation is a parametric linear program in terms of \( \beta \). Standard results on parametric linear programming \([11]\) guarantee that any solution to (7) for a fixed value of \( \beta \) will in fact be optimal for a range of values \([\beta_\ell, \beta_u]\) containing \( \beta \). The optimal solutions to (7) as a function of \( \beta \) correspond to a piecewise linear, concave, increasing curve, where each linear piece corresponds to a range of \( \beta \) values for which the same feasible LP solution is optimal.

We begin by solving this LP for some \( \beta_0 > d_{\max} \), which is guaranteed to produce a solution vector \( x_0 \) that is at least a relaxed form of minority vote (Theorem 5) that would round to a minority vote clustering via Algorithm 1. Our goal is to find the largest value \( \hat{\beta} \) for which \( x_0 \) no longer optimally solves (7). To do so, define \( c^T = c_e^T + \beta c_d^T \)

so that we can re-write objective (7) as

\[
\min_x c^T x \quad \text{s.t. } Ax \geq b, x \geq 0. \tag{8}
\]

Finding \( \hat{\beta} \) amounts to determining how long the minority vote solution is “stable” as the optimal solution to (8). Consider a perturbation of (8):

\[
\min_x c(\theta)^T x = c^T x - \theta c_d^T x, \quad \text{s.t. } Ax \geq b, x \geq 0, \tag{9}
\]

where \( \theta = \beta_0 - \beta \) for some \( \beta < \beta_0 \), so that (9) corresponds to our clustering objective with the new parameter \( \beta \). Since \( x_0 \) is optimal for (8), it is optimal for (9) when \( \theta = 0 \). Solving the following linear program provides the range \( \theta \in [0, \theta^+] \) for which \( x_0 \) is still optimal for (9):

\[
\max_{y, \theta} \theta \quad \text{s.t. } A^T y \leq c - \theta c_d, \quad b^T y = c^T x_0 - \theta c_d^T x_0, \quad \theta \geq 0, y \geq 0. \tag{10}
\]

Let \( (y^*, \theta^*) \) be the optimal solution to (10). The constraints in this LP are designed so that \( (x_0, y^*) \) satisfy primal-dual optimality conditions for the perturbed linear program (9) and its dual, and the objective function simply indicates that we want to find the maximum value of \( \theta \) such that these conditions hold. Thus, \( \theta^* = \theta^+ \), and \( \beta = \beta_0 - \theta^+ \) will be the smallest parameter value such that \( x_0 \) is optimal for the LP relaxation of our objective.

Finally, after entering a parameter regime where \( x_0 \) is no longer optimal, the objective function strictly decreases. Again, by Theorem 5, for large enough \( \beta \), the relaxed LP solution is a (relaxed) minority vote one. Since we find the minimizer of the LP, the solution is the (relaxed) minority vote solution with the smallest objective. Thus, moving to the new parameter regime will no longer correspond to minority vote, either in the LP relaxation or in the rounding from Algorithm 1.

4 Numerical Experiments

Here we present two sets of experiments on real-world data to illustrate our theory and methods. The first involves the fair clustering objective as stated above to measure the quality of the LP relaxation and our bounds on

\[
\theta < \beta^*. \quad \text{Given that the objective is NP-hard in general, computing } \beta^* \text{ exactly may not be feasible. However, we will show that, for a given edge-labeled hypergraph, we can compute exactly the minimum value } \beta \text{ for which a relaxed minority vote solution is no longer optimal for the LP-relaxation of our objective. This has several useful implications. First, when the minority vote clustering is unique, Theorem 5 says that this clustering is also optimal for the ILP for large enough } \beta. \quad \text{Even when the minority vote clustering is not unique, it may still be the case that an integral minority vote solution will still be optimal for the LP relaxation for large enough } \beta; \quad \text{indeed, we observe this in experiments with real data later in the paper. In these cases, we know that } \beta^* \leq \hat{\beta}, \text{ which allows us to rule out a wide range of parameters leading to solutions that effectively ignore the experience part of our objective. Still, even in cases where an integral minority vote solution is never optimal for the LP relaxation, computing } \hat{\beta} \text{ lets us avoid parameter regimes where Algorithm 1 does not return a minority vote clustering.}
\]
Table 1: Summary statistics of datasets. The computed $\hat{\beta}$ bounds using the tools in Section 3 are much smaller than the $d_{\text{max}}$ bound in Theorem 5.

| Dataset                  | $|V|$ | $|E|$ | $L$ | $d_{\text{max}}$ | $\hat{\beta}$ |
|--------------------------|------|------|-----|-----------------|----------------|
| music-blues-reviews      | 1106 | 694  | 7   | 127             | 0.50           |
| madison-restaurants-reviews | 565  | 601  | 9   | 59              | 0.42           |
| vegas-bars-reviews       | 1234 | 1194 | 15  | 147             | 0.50           |
| algebra-questions        | 423  | 1268 | 32  | 375             | 0.50           |
| geometry-questions       | 580  | 1193 | 25  | 260             | 0.50           |

$\hat{\beta}$, and we find that regularization costs little in the objective while substantially improving the diversity score. The second set of experiences studies what happens if we apply this clustering iteratively, where an individual’s experience changes over time based on the assignment algorithm; here, we see a clear effect of the regularization on team dynamics over time. All code and datasets are available at https://github.com/ilyaamburg/fair-clustering-for-diverse-and-experienced-groups.

Datasets. We first briefly describe the datasets that we use, which come from online user reviews sites and the MathOverflow question-and-answer site. In all cases, the nodes in the hypergraphs correspond to users on the given site. Hyperedges correspond to groups of users that post reviews or answer questions in a certain time period. Table 1 contains summary statistics.

music-blues-reviews. This dataset is derived from a crawl of Amazon product reviews that contains metadata on the reviewer identity, product category, and time of reviews [29]. We consider all reviews on products that include the tag “regional blues,” which is a tag for a subset of vinyl music. We partition the reviews into month-long segments. For each time segment, we create hyperedges of all users who posted a review for a product with a given sub-tag of the regional blues tag (e.g., Chicago Blues and Memphis Blues). The hyperedge category corresponds to the sub-tag.

madison-restaurants-reviews, vegas-bars-reviews. These two datasets are derived from reviews of establishments on Yelp [22] for restaurants in Madison, WI and bars in Las Vegas, NV. We perform the same time segmentation as the music-blues-reviews dataset, creating hyperedges of groups of users who reviewed an establishment with a particular sub-tag (e.g., Thai restaurant for Madison, or cocktail bar for Las Vegas) in a given time segment. The sub-tag is the hyperedge category.

algebra-questions, geometry-questions. These two datasets are derived from users answering questions on mathoverflow.net that contain the tag “algebra” or “geometry”. We use the same time segmentation and hyperedge construction as for the reviews dataset, and the sub-tags are given by all tags matching *algebra* or *geometry* (e.g., lie-algebras or hyperbolic-geometry).

Within our framework, a fair clustering of users from a review platform corresponds to composing groups of users for a particular category that contains both experts (with reviews in the given category) and those with diverse perspectives (having reviewed other categories). The reviews from these users could then be used to present a “group of reviews” for a particular category. A fair clustering for the question-and-answer platforms joins users with expertise in one particular math topic with those who have experiences in another topic. This serves as an approximation to how one might construct experienced and diverse teams, given historical data on individuals’ experiences.

4.1 Algorithm performance with varying regularization

Here, we examine the performance of Algorithm 1 for various regularization strengths $\beta$ and compare the results to the unregularized case (Fig. 1). We observe that including the regularization term only yields mild increases in the original objective function. This “cost of fairness” ratio is always smaller than 3 and is especially small for the questions dataset (Fig. 1, far left). Furthermore, The ratio of the original objective for regularized and unregularized ($\beta = 0$) relaxed LPs has similar properties (Fig. 1, middle left), which is not surprising given that the approximation factor of Algorithm 1 on the data is small (Fig. 1, middle right), which we obtain by solving the exact ILP. In fact, solving the relaxed LP often yields an integral solution, meaning that it is the solution to the ILP. The computed $\hat{\beta}$ bound also matches the plateau of the rounded solution (Fig. 1, far left), which we would also expect from the small approximation factors. We also examine the “edge satisfaction”, i.e., the fraction of hyperedges where all of the nodes in the hyperedge are clustered to the same color as the hyperedge [5] (Fig. 1, far right). As regularization increases, more diversity is encouraged, and the edge satisfaction decreases. Finally,
we note that the runtime of Algorithm 1 is small in practice, taking at most a couple of seconds in any problem instance.

4.2 Dynamic group formation

In this section we consider a dynamic variant of our framework in which we iteratively update the hypergraph. More specifically, given the hypergraph up to time $t$, we (i) solve our regularized objective to find a clustering $C$ (ii) create a set of hyperedges at time $t+1$ corresponding to $C$, i.e., all nodes of a given color create a hyperedge. At the next step, the experience levels of all nodes have changed. This mimics a scenario in which teams are repeatedly formed via Algorithm 1 for various tasks, where the type of the task corresponds to the color of the hyperedge. For our experiments, we only track the experiences from a window of the last $w$ time steps; in other words, the hypergraph just consists of the hyperedges appearing in the previous $w$ steps. To get an initial history for the nodes, we start with the hypergraph datasets used above. After, we run the iteration for $w$ steps to “warm start” the dynamical process, and consider this state to be the initial condition. Finally, we run the iterative procedure for $T$ times.

We can immediately analyze some limiting behavior of this process. When $\beta = 0$ (i.e., no regularization), after the first step, the clustering will create new hyperedges that increase the experience levels of each node for some color. In the next step, no node has any incentive to cluster with a different color than the previous time step, so the clustering will be the same. Thus, the dynamical process is entirely static. At the other extreme, if $\beta > d_{\text{max}}$ at every step of the dynamical process, then the optimal solution at each step is a minority vote assignment by Theorem 5. In this case, after each step, each node $v$ will increase its color degree in one color, which will change its minority vote solution in the next iteration. Assuming ties are broken at random, this leads to uniformity in the historical cluster assignments of each node as $T \to \infty$.

For each of our datasets, we ran the dynamical process for $T = 50$ time steps. We say that a node exchanges if it is clustered to different colors in consecutive time steps. Figure 3 shows the mean number of exchanges. As expected, for small enough $\beta$, nodes are always assigned the same color, resulting in no exchanges; for large enough $\beta$, nearly all nodes exchange in the minority vote regime. Figure 2 shows the clustering of nodes on a subset of the geometry-questions dataset for different regularization levels. For small $\beta$, nodes accumulate experience before exchanging. When $\beta$ is large, nodes exchange at every iteration. This corresponds to the scenario of large $\beta$ in Figure 3.

5 Discussion
We present a new framework for fair clustering that balances diversity and experience in cluster formation. Previous research on fair clustering has largely focused on balancing protected attributes; our research opens the door to new notions of fairness that are useful to optimize in practice. We cast our problem as a hypergraph clustering task, where a regularization parameter controls cluster diversity. Our results include a 2-approximation that works for any value of our parameter. As this parameter changes from zero to infinity, our problem transitions from being NP-hard to polynomial time solvable. In future work, we plan to explore how and when this transition occurs, and in particular whether we can obtain better parameter-dependent approximation guarantees.

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