Cooperative Pilot Spoofing in MU-MIMO Systems

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Abstract—In this letter, we consider downlink transmission of a multiuser multiple-input multiple-output (MU-MIMO) system with zero-forcing (ZF) precoders in the presence of multiple attackers. We propose a cooperative pilot spoofing attack (CPSA), where the attackers collaboratively impair the channel estimations in the uplink channel training phase, aiming at deteriorating the downlink throughput of the whole cell. We first evaluate the impacts of CPSA on the channel estimation and the downlink ZF precoding design, and then we derive an analytical expression for the achievable downlink sum-rate. Furthermore, we investigate the optimal attack strategy to minimize the achievable downlink sum-rate. We show that the optimization problem under consideration is a convex one so the global optimum could be obtained conveniently. Numerical results show that the CPSA attack results in a severe performance deterioration with the increase in the attacking power and the number of attackers.

Index Terms—Physical layer security, pilot spoofing, achievable downlink sum-rate, convex optimization.

I. INTRODUCTION

MU-MIMO is the most promising manner of exploiting the spatial degrees of freedom provided by multiple-antennas at the base station (BS) [1]. To fully exploit benefits of MU-MIMO, accurate channel state information (CSI) is a prerequisite. In practice, the CSI needs to be estimated. In a time-division duplex (TDD) system, the BS estimates the CSI based on the uplink pilot signals due to the reciprocity of the uplink and downlink channels [2].

However, this specific pilot transmission mechanism is vulnerable to malicious interference from active attackers. In particular, the malicious attacker can attack the uplink pilot transmission by sending the same pilot signals as legitimate users, which is also known as pilot spoofing attack (PSA) [3], [4]. PSA may lead to incorrect channel estimations and consequently reduce the wireless throughput of legitimate users in the whole cell significantly.

Recently, PSA has attracted a lot of research interest [5]–[8]. In [5], the authors studied the impact of a PSA launched by a single-antenna attacker in a single user scenario, where analysis showed that this attack could drastically weaken the strength of the received signal at the legitimate user. Extreme cases were considered where the number of transmit antennas and the attackers power were very large. In [6], the authors investigated a PSA launched by a multi-antenna attacker in a multi-cell multiuser massive MIMO system, and they found that the attacker could conduct a best possible PSA by maximizing the total average estimation error variance of the desired users channel, because the leakage of the desired signal would increase when the channel estimation error increased. In [7], the authors studied a combined PSA in a single-cell massive MIMO system, and the downlink transmission rates in the presence of the attack was derived by exploiting the channel hardening effect. In [8], the authors investigated the design of a PSA carried out by multiple single-antenna attackers in a single user scenario. They constructed an optimization problem from the point of view of the attackers, which aimed to maximize the signal-to-noise ratio (SNR) and information leakage to a target adversary.

However, all these aforementioned works are limited either within a single attacker [5]–[7] or focusing the impact on a single specific user [5], [8]. In fact, on one hand, the PSA may effect all users in the whole cell, which may deteriorate the cell performance severely. On the other hand, since the user access protocol is publicly known, multiple attackers can synchronize to the BS and launch collaborative attack to improve their PSA capabilities. So far, a study on a general PSA scenario with multiple users and multiple attackers is still absent, so the ultimate impact of PSA to a MU-MIMO in a cell for multiple users has not been clearly exposed yet. Although the analysis presented in [7] is in this line but it assumes the channel hardening property so it does not hold for moderately large number of transmit antennas (dozens of antennas), which is a more practical scenario.

In this letter, we consider a MU-MIMO system under the PSA launched by multiple cooperative attackers, and investigate how multiple attackers can cooperatively perform the PSA to deteriorate the cell performance. Especially, 1) we first evaluate the impacts of CPSA on the channel estimation and the downlink ZF precoding design, and then we derive an analytical expression for the achievable downlink sum-rate. 2) Furthermore, we investigate the optimal attack strategy, which aims at minimizing the achievable downlink sum-rate. We show that this problem under consideration is a convex optimization problem so the global optimum could be obtained conveniently. 3) Our results show that the CPSA results in a severe performance deterioration for the whole cell. Several cooperative attackers could drive the sum-rate down to only 30% of the normal throughput without attack.

II. SYSTEM MODEL AND PROBLEM DESCRIPTION

A. System Model

We consider a single-cell multiuser TDD communication system, where an $M$-antenna BS serves $K$ single-antenna users using orthogonal pilot sequences for channel training, i.e., $p_k \in \mathbb{C}^{T_p \times 1}$ is the pilot sequence of the $k$th user satisfying $p_k^H p_k = 1$ with the length $T_p$. In this letter, we consider a

1 In practice, the attackers can be connected to each other via low-cost low-capacity wireless links, so they can share their CSIs for collaboration.
the attack signal is a combination of all users pilot signals. In particular, the attack signal can be expressed as

\[ \mathbf{y}_B = \sum_{n=1}^{N} \sqrt{\tau_p P_{U_k}} \mathbf{h}_{B,k} \mathbf{p}_n + \mathbf{z}_B + \mathbf{u}, \]

where \( \mathbf{z}_B \) is the equivalent noise vector with distribution \( \mathbb{CN}(\mathbf{0}, \sigma^2 I_M) \). Without awareing the CPSA attack, the BS will calculate the MMSE estimation of the legitimate channel \( \mathbf{h}_{B,k} \) via the standard process as

\[ \hat{\mathbf{h}}_{B,k} = \Psi_{B,k} \Omega^{-1}_{B,k} \mathbf{y}_B, \]

where \( \Psi_{B,k} \triangleq \mathbb{E}\{ \mathbf{h}_{B,k} \mathbf{y}_B^H \} \), and \( \Omega_{B,k}^{-1} \triangleq \mathbb{E}\{ \mathbf{y}_B \mathbf{y}_B^H \}^{-1} \) are the covariance matrices, which can be derived as

\[ \Psi_{B,k} = \beta_{B,k} \mathbf{I}_M, \]

\[ \Omega_{B,k} = \beta_{B,k} \mathbf{I}_M + \sum_{n=1}^{N} \theta_{n,k} \mathbf{P}_{A,n} \beta_{A,n} \mathbf{I}_M \]

\[ + (\sigma^2 / \tau_p P_{U_k}) \mathbf{I}_M. \]

The estimated channel vector \( \hat{\mathbf{h}}_{B,k} \) is distributed as \( \mathbb{CN}(\mathbf{0}, \hat{\mathbf{R}}_{B,k}) \) with \( \hat{\mathbf{R}}_{B,k} \) can be written as

\[ \hat{\mathbf{R}}_{B,k} = \Psi_{B,k} \Omega_{B,k}^{-1} \Psi_{B,k} = \lambda_{B,k} \mathbf{I}_M, \]

where

\[ \lambda_{B,k} = \frac{\tau_p P_{U_k} \beta_{B,k}^2}{\tau_p P_{U_k} \beta_{B,k} + \tau_p \sum_{n=1}^{N} \theta_{n,k} \mathbf{P}_{A,n} \beta_{A,n} + \sigma^2}. \]

The uncorrelated channel estimation error \( \tilde{\mathbf{h}}_{B,k} \) satisfies \( \tilde{\mathbf{h}}_{B,k} = \tilde{\mathbf{h}}_{B,k} + \mathbf{h}_{B,k} \) can be derived by invoking the orthogonality property of MMSE estimation as

\[ \tilde{\mathbf{h}}_{B,k} \sim \mathbb{CN}(\mathbf{0}, \eta_{B,k} \mathbf{I}_M), \]

where \( \eta_{B,k} \triangleq \beta_{B,k} - \lambda_{B,k} \).

**Remark 1:** Note that the attackers can also transmit Gaussian random interference to degrade the accuracy of the channel estimation. However, as shown in our previous works [8], [10], transmitting random interference can not offer any advantage over the proposed pilot spoofing signals, which will be shown later in numerical results.

### III. DOWNLINK SUM-RATE ANALYSIS AND OPTIMAL ATTACK STRATEGY

#### A. Downlink ZF Beamforming

Since ZF downlink beamforming is an asymptotically optimal solution for MU-MISO transmission, we consider

\[ \text{ZF precoding can achieve asymptotically optimal throughput in the downlink of MU-MISO system, which has been proved in [11], [12]}. \]
ZF beamformer for the BS, which is
\[
\mathbf{w}_k \triangleq \frac{\mathbf{a}_{B,k}}{\|\mathbf{a}_{B,k}\|},
\]
for the \(k\)th user, where \(\mathbf{a}_{B,k}\) is the \(k\)th column of \(\mathbf{H}_B (\mathbf{H}_B^H \mathbf{H}_B)^{-1}\), and \(\mathbf{H}_B \triangleq [\tilde{\mathbf{h}}_{B,1}, \ldots, \tilde{\mathbf{h}}_{B,K}]^T\) is the channel estimation matrix. Due to the CPSA, the BS uses the impaired ZF precoder for downlink data transmission. The received signal at the \(k\)th user can be written as
\[
y_k = \sqrt{P_{B,k}} \mathbf{h}_{B,k}^H \mathbf{w}_k s_k + \sum_{i=1, i \neq k}^{K} \sqrt{P_{B,i}} \mathbf{h}_{B,k}^H \mathbf{w}_i s_i + z_k,
\]
where \(P_{B,k}\) is the transmit power allocated for the \(k\)th user, and \(z_k \sim \mathcal{CN}(0, \sigma^2)\) is the additive noise at the \(k\)th user.

As discussed in [13], without the dedicate downlink channel training, the users only have statistical effective channel gain for signal demodulation, and the signal received at the \(k\)th user can be reformulated as
\[
y_k = \sqrt{P_{B,k}} \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_k\right\} s_k + \sqrt{P_{B,k}} \left(\mathbf{h}_{B,k}^H \mathbf{w}_k - \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_k\right\}\right) s_k + \sum_{i=1, i \neq k}^{K} \sqrt{P_{B,i}} \mathbf{h}_{B,k}^H \mathbf{w}_i s_i + z_k,
\]
where due to the incorrect ZF precoding caused by CPSA, inter-user interference occurs, which will greatly deteriorate the overall throughput in the cell.

**B. Achievable Downlink Sum-Rate**

The achievable downlink rate at the \(k\)th user can be given by
\[
R_k = \log \left(1 + \gamma_k\right),
\]
where
\[
\gamma_k = \frac{P_{B,k} \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_k\right\}^2}{P_{B,k} \mathbb{V}\mathbb{a}\mathbb{r}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_k\right\} + \sum_{i=1, i \neq k}^{K} P_{B,i} \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_i\right\}^2 + \sigma^2},
\]
and \(\mathbb{E}\left\{\cdot\right\}\) and \(\mathbb{V}\mathbb{a}\mathbb{r}\left\{\cdot\right\}\) are the expectation and variance operators, respectively.

To simplify the subsequent analysis, we assume in the uplink pilot transmission phase \(P_U = P_{U,k}\) and \(P_A = P_{A,n}\) for all \(k\) and \(n\). In addition, in the downlink data transmission phase, \(P_B = P_{B,k}\). Then, the achievable downlink rate at the \(k\)th user can be derived as follows.

**Theorem 1.** Under the CPSA and ZF precoding, the achievable downlink rate at the \(k\)th user is
\[
\tilde{R}_k = \log \left(1 + \frac{A_k}{B_k + C_k \nu^T \theta_k}\right),
\]
where \(\theta_k \triangleq [\theta_{1,k}, \cdots, \theta_{N,k}]^T\), \(\nu \triangleq [\beta_{A,1}, \cdots, \beta_{A,N}]^T\), \(A_k \triangleq \xi (M - K + 1)\), 
\(B_k \triangleq (M - K + 1)\), 
\(C_k \triangleq \xi (M - K + 1)\), 
\(\gamma_k \triangleq \Gamma (x + 1/2)/\Gamma (x)\).

**Proof:** By calculating the following three terms in \(\gamma_k\) in (10), the derivation of the achievable downlink rate at the \(k\)th user is outlined.

For the numerator \(\mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_k\right\}^2\), it can be calculated by
\[
\mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_k\right\}^2 \leq \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{h}_{B,k}\right\} \mathbb{E}\left\{\mathbf{w}_k^H \mathbf{w}_k\right\} + \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_k\right\} \mathbb{E}\left\{\mathbf{w}_k^H \mathbf{h}_{B,k}\right\} + \mathbb{E}\left\{\mathbf{w}_k^H \mathbf{w}_k\right\} \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{h}_{B,k}\right\} \leq \xi (M - K + 1)\lambda_{B,k},
\]
where \(\xi (x) \triangleq \Gamma (x + 1/2)/\Gamma (x)\), step (a) is obtained by applying the MMSE channel estimation error model, step (b) holds since \(\mathbf{h}_{B,k}^H \mathbf{w}_k\) are uncorrelated, and \(\mathbf{h}_{B,k}^H \mathbf{w}_k = 1/\|\mathbf{a}_{B,k}\|\), and step (c) results from the Gamma distribution.

For the denominator \(\mathbb{E}\left\{1/\|\mathbf{a}_{B,k}\|^2\right\}\), it can be calculated by
\[
\mathbb{E}\left\{1/\|\mathbf{a}_{B,k}\|^2\right\} \geq \frac{\mathbb{E}\left\{\mathbf{a}_{B,k}^H \mathbf{a}_{B,k}\right\}}{\mathbb{E}\left\{\mathbf{a}_{B,k}^H \mathbf{a}_{B,k}\right\}} = \xi (M - K + 1)\lambda_{B,k},
\]
where \(\xi (x) \triangleq \Gamma (x + 1/2)/\Gamma (x)\), step (a) is obtained by applying the definition of variance, step (b) holds since \(\mathbf{h}_{B,k}^H \mathbf{a}_{B,k}\) are independent of each other, step (c) is obtained by applying the definition of variance, and step (d) holds since \(\mathbf{h}_{B,k}^H \mathbf{w}_k\) are uncorrelated.

The term \(\frac{\sum_{i=1, i \neq k}^{K} \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_i\right\}^2}{\mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{h}_{B,k}\right\}}\) in denominator is
\[
\sum_{i=1, i \neq k}^{K} \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_i\right\}^2 = \sum_{i=1, i \neq k}^{K} \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{h}_{B,k}\right\} \mathbb{E}\left\{\mathbf{w}_i^H \mathbf{w}_i\right\} + \sum_{i=1, i \neq k}^{K} \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{w}_i\right\} \mathbb{E}\left\{\mathbf{w}_i^H \mathbf{h}_{B,k}\right\} + \sum_{i=1, i \neq k}^{K} \mathbb{E}\left\{\mathbf{w}_i^H \mathbf{w}_i\right\} \mathbb{E}\left\{\mathbf{h}_{B,k}^H \mathbf{h}_{B,k}\right\} \leq (K - 1)\eta_{B,k},
\]
where step (a) results form the independence of \(\mathbf{h}_{B,k}^H \mathbf{w}_i\) and \(\mathbf{h}_{B,k}^H \mathbf{h}_{B,k}\), and step (b) holds since \(\mathbf{h}_{B,k}^H \mathbf{w}_i = 0\) for \(i \neq k\).

Substituting (5), (10), (13) and (14) into (10) yields the expression (11). This completes the proof.

Accordingly, the achievable downlink sum-rate is
\[
R_{\text{sum}} = \sum_{k=1}^{K} \tilde{R}_k = \sum_{k=1}^{K} \log \left(1 + \frac{A_k}{B_k + C_k \nu^T \theta_k}\right).
\]

**Remark 2:** Note that the achievable downlink sum-rate does not depend on the small-scale fading components \(g_{B,k}\) and \(g_{A,n}\), which implies that the attackers could optimize the attack without the legitimate CSI. This makes CPSA a more practical attacking scheme.
C. Optimal Attack Strategy

The goal of the CPSA is to minimize the achievable downlink sum-rate of the target cell by allocating the attacking power. This strategy could be formulated as follows

\[
\min_{\theta_k} \sum_{k=1}^{K} \log \left( 1 + \frac{A_k}{B_k + C_k \nu^T \theta_k} \right),
\]

\[s.t. \ C1: 0 \leq \theta_{n,k} \leq 1, \ n = 1, 2, \cdots, N, \]

\[C2: \sum_{k=1}^{K} \theta_{n,k} \leq 1\]

where the constraints C1 and C2 account for the attacking power sum and individual constraints.

Fortunately, we declar that the optimization problem is a convex problem. Denote \( f(\theta_k) \triangleq B_k/A_k + (C_k/A_k) \nu^T \theta_k \). The first and second derivative of \( f(\theta_k) \) can be derived as \( \frac{d f(\theta_k)}{d \theta_k} = (C_k/A_k) \nu \) and \( \frac{d^2 f(\theta_k)}{d \theta_k d \theta_k^T} = 0_N \), respectively, where \( 0_N \) denotes a \( N \times N \) null matrix. According to the necessary and sufficient condition of convex function identification, \( f(\theta_k) \) is a convex function. Due to \( \log(1 + 1/x) \) is convex, we conclude that the composite function \( \log(1 + 1/f(\theta_k)) \) is a convex function of \( \theta_k \). Considering that the summation of convex functions is convex, C1 and C2 are convex sets, we can proof the optimization problem is convex. So, it can be efficiently solved by standard convex optimization techniques.

IV. Numerical Results

We evaluate the impact of CPSA to the achievable downlink sum-rate through numerical results. We use \( \beta = L_0 d^{-\alpha} \) to model the path loss and shadowing fading, where \( d \) is the distance between the BS and the user, \( L_0 = -45 \) dB and \( \alpha = 3.7 \) is the path loss exponent. The users and attackers are uniformly distributed in a circular cell. The inner radius is 50m, the maximum distance of the users is 400m and that of the attackers is \( D_A^{\text{max}} \). We consider communication over a 20 MHz bandwidth with noise floor of -90 dBm. We set \( P_U = 10 \) dBm, \( P_A = 10 \) dBm and \( P_B = 40 \) dBm. We set the pilot sequence length \( \tau_p = K \) symbol durations. The results are averaged over 10000 Monte-Carlo (MC) tests.

Fig. 2 depicts the achievable downlink sum-rate versus the number of antennas, where \( K = 24, D_A^{\text{max}} = 300 \). in Fig. 3 The achievable downlink sum-rate when there is no attack is taken as a benchmark. We observe that when the attackers are not so far away from the BS, the achievable downlink sum-rate has a dramatical deterioration, even there is only two attackers each with power 5 dBm. In addition, increasing attack power a little bit also has severely impact on the whole cell performance.

V. Conclusion

In this letter, we analyzed the impact of CPSA, i.e., a PSA launched by multiple cooperative attackers. This attack caused a great impact on the channel estimation in the uplink channel training phase. We have evaluated the effect of the CPSA on the achievable downlink sum-rate in a single-cell MU-MIMO system. We shown that the cooperation among attackers can significantly improve their offensive capabilities, and impose dramatic harm to the system throughput. Moreover, it should be noted that most existing pilot spoofing attack detection methods are difficult to be used directly for the CPSA in a MU-MIMO system. For example, random modulation based methods (e.g., random frequency shift [16]) will incur high computational complexity for the MU-MIMO system; it is challenging for statistic feature based methods (e.g., sparsity of
virtual channel [17]) to select the optimal detection threshold in the face of such cooperative attacks, and they need to estimate more complicated statistic features when facing the scenario of multiuser and multiple attackers. Consequently, effective detection and defense mechanisms are urgently needed, which is a critical issue for our future research.

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