Linear response correlation functions in strained graphene

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After deriving a general correspondence between linear response correlation functions in graphene with and without applied uniaxial strain, we study the dependence on the strain modulus and direction of selected electronic properties, such as the plasmon dispersion relation, the optical conductivity, as well as the magnetic and electric susceptibilities. Specifically, we find that the dispersion of the recently predicted transverse plasmon mode exhibits an anisotropic deviation from linearity, thus facilitating its experimental detection in strained graphene samples.

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I. INTRODUCTION

Graphene is a truly two-dimensional (2D) electronic system, based on an atomically thin carbon honeycomb lattice. Low-energy quasiparticles can be described as massless Dirac fermions, with a cone dispersion relation in reciprocal space around the so-called Dirac points $K$, $K'$, and a linearly vanishing density of states (DOS) at the Fermi level. Such a linear spectrum and reduced dimensionality yield remarkable behaviors already in the non-interacting limit of several electronic properties of graphene. These include, inter alia, the reflectivity, the optical conductivity, the plasmon dispersion relation, as well as a newly predicted transverse electromagnetic mode, which is characteristic of a 2D system with a double band structure, such as graphene. Moreover, the relevance of lossless plasmons in graphene in the infrared frequency range has been emphasized, with possible applications in nanophotonics. These properties can be extracted from the study of the appropriate correlation functions within linear response theory.

Within the Dirac approximation, rotational invariance implies that the current-current correlation function may be decomposed in a longitudinal and a transverse contribution, the former being related to the density-density correlation function via the continuity equation. Moreover, in the case of massless Dirac fermions, it has been shown that the current-current response is simply proportional to its pseudospin-pseudospin counterpart. On the other hand, the magnetic susceptibility is related to the transverse contribution. Specifically, the noninteracting Dirac model yields an orbital magnetic susceptibility $\chi_m(q \rightarrow 0) \propto \delta(\mu)$, in the long wavelength limit. This is consistent with earlier results for graphite, obtained using Wallace’s two-dimensional band structure for a graphene layer. Such a finding would predict no response to a uniform, static magnetic field, away from half-filling. This is of course partially compensated by a smearing of the $\delta$-function at finite temperatures, already in the noninteracting limit. Still at zero temperature and in the noninteracting limit, one recovers a nonzero magnetic response also away from half-filling, when the honeycomb lattice structure is considered. The effect of the interactions has been considered in Ref. 26, where it is shown that an interacting 2D Dirac electron liquid develops a magnetic response also at finite doping. Concerning the response to an external electromagnetic field, graphene is also unique among other conventional 2D electron systems, in that it has been predicted that it can sustain a transverse plasmon mode, as a consequence of its double band structure. More recently, such a transverse mode has been predicted also for bilayer graphene.

Here, we will consider the effect of strain on the various electronic properties that may be described by linear response correlation functions. Indeed, a deformation of the lattice through the application of uniaxial strain or hydrostatic pressure is expected to produce modifications also in the electronic structure of graphene. Recently, it has been proposed that nanodevices based on graphene could be engineered on the basis of the expected strain-induced modifications of the deformed graphene sheet (origami electronics). This is made possible by the exceptional mechanical properties of graphene, as is the case for other carbon compounds. For instance, despite its reduced dimensionality, graphene is characterized by a sizeable tensile strength and stiffness, with graphene sheets being capable to sustain elastic deformations as large as 20%. Larger strains would then induce a semimetal-to-semiconductor transition, with the opening of an energy gap, and it has been demonstrated that such an effect critically depends on the direction of applied strain.

The paper is organized as follows. In Sec. III we present our formalism for treating graphene under uniaxial strain, and derive our central result for a generic linear response function for a deformed graphene sheet.
This, in particular, applies to the density-density and current-current correlation functions. In Sec. III we then study the electron polarization, with emphasis on the strain dependence of the plasmon dispersion relation and the conductivity. Specifically, we find a strain-induced anisotropic enhancement of the deviations from linearity of the transverse plasmon, which should facilitate its experimental detection. In Sec. IV we derive the strain dependence of the magnetic and electric susceptibilities. Finally, we summarize our results in Sec. V.

II. MODEL

The low-energy Hamiltonian for noninteracting quasiparticles around a Dirac point, say $K$, has the well-known linear form

$$H^{(0)} = \hbar v_F \sigma \cdot q,$$

(1)

where $v_F$ is the Fermi velocity, $\sigma = (\sigma_x, \sigma_y)$, with $\sigma_x, \sigma_y$ Pauli matrices, and $q$ is the wavevector displacement from the Dirac point one is referring to, i.e. $K = K + q$. Here and below, a superscript zero denotes absence of strain. The effect of strain is then that of modifying the lattice vectors as $\delta_l = \{1 + \varepsilon \cdot \delta_l^{(0)} \}$ ($l = 1, 2, 3$), where $\delta_l^{(0)} = (1/2 \sqrt{3}, 1/2, 0)$, $\delta_2^{(0)} = (0, 1/\sqrt{3}, 1/2)$, $\delta_3^{(0)} = (1/2, 1/2, 0)$ are the relaxed (unstrained) vectors connecting two nearest-neighbor (NN) carbon sites, with $a = 1.42$ Å, the equilibrium $C-C$ distance in a graphene sheet, and $\varepsilon$ is the strain tensor,

$$\varepsilon = \frac{1}{2} [ (1 - \nu) \mathbb{I} + (1 + \nu) A(\theta)],$$

(2)

where

$$A(\theta) = \sigma_z e^{2i\theta \sigma_x} = \cos(2\theta) \sigma_z + \sin(2\theta) \sigma_x.$$ 

(3)

In Eq. (2), $\theta$ denotes the angle along which the strain is applied, with respect to the $x$ axis in the lattice coordinate system, $\varepsilon$ is the strain modulus, and $\nu$ is Poisson’s ratio. While in the hydrostatic limit $\nu = -1$ and $\varepsilon = \varepsilon$, in the case of graphene one has $\nu = 0.14$, as determined from ab initio calculations, to be compared with the known experimental value $\nu = 0.165$ for graphite. The special values $\theta = 0$ and $\theta = \pi/6$ refer to strain along the zig zag and armchair directions, respectively.

The overall effect of a moderately low applied uniaxial strain on the low-energy Hamiltonian is that of shifting the location of the Dirac point in momentum space as $K \rightarrow K_D$, and changing the shape of the Dirac cone, into a deformed one, with elliptical section. Such a picture applies for strain moduli below the value at which a gap opens in the energy spectrum, which takes place at $\varepsilon \simeq 20\%$. 

In particular, setting $K = K_D + q$, with $q$ measuring now the vector displacement from the shifted Dirac point, the Fermi velocity, defined as the slope of the Dirac cone in the direction of $q$, will now have anisotropic components $c_\parallel v_F$, $c_\perp v_F$ along the direction of applied strain and the direction orthogonal to it, respectively, with

$$c_\parallel = 1 - 2\kappa\varepsilon, \quad (4a)$$

$$c_\perp = 1 + 2\kappa v_F \varepsilon, \quad (4b)$$

where $\kappa = (a/2t)|\partial t/\partial a| - \frac{1}{2} \simeq 1.1$ is related to the logarithmic derivative of the nearest-neighbor hopping $t$ at $\varepsilon = 0$. Thus, the low-energy Hamiltonian around $K_D$ maintains a linear form even in the presence of strain, and can still be written as

$$H = \hbar v_F \sigma \cdot q',$$

(5)

where now

$$q' = R(\theta)S(\varepsilon)R(-\theta)q,$$

(6)

with $R(\theta)$ the rotation matrix in the direction of applied strain, and $S(\varepsilon) = \text{diag}(c_\parallel, c_\perp)$ the matrix describing the deformation of the Dirac cone. Explicitly, for the compound transformation matrix $R(\theta)S(\varepsilon)R(-\theta)$ mapping $q$ onto $q'$ one finds

$$R(\theta)S(\varepsilon)R(-\theta) = \mathbb{I} - 2\kappa \varepsilon.$$ 

(7)

A central result of the present work is that a similar correspondence holds between a generic linear response function $\chi(q, \omega)$ under applied strain, with respect to its unstrained limit, $\chi^{(0)}(q, \omega)$. This follows from the fact that any linear response function $\chi(q, \omega)$ of a noninteracting electron system can be expressed as an integral over the first Brillouin zone (1BZ) of a suitable matrix operator over pseudospins, which is itself a function of $q$. Such an operator then admits a unique expression in terms of the Pauli matrices $\sigma_x, \sigma_y, \sigma_z$, and the identity matrix $\mathbb{I} \equiv \sigma_0$. The simplest cases are then given by the density operator and the density current operator, which, in reciprocal space read,

$$\rho^{(0)}(q) = \sum_k \Psi^\dagger_{k-q} \mathbb{I} \Psi_k, \quad (8a)$$

$$J_i^{(0)}(q) = -e\nu_F \sum_k \Psi_{k-q}^\dagger \sigma_i \Psi_k, \quad i = x, y, \quad (8b)$$

respectively, where $\Psi_{k}^\dagger = (\psi_{kA}, \psi_{kB})$, and $\psi_{qA}$ destroys a quasiparticle with momentum $q$ and pseudospin $\alpha = A, B$, and summations run over the 1BZ. While the density operator does not change under applied strain, for the generic component of the density current operator one has

$$J_{ij} = [\mathbb{I} - 2\kappa \varepsilon]_{ij} J_i^{(0)}.$$ 

(9)

Here and below a summation will be understood over repeated indices ($j = x, y$).

Defining now eigenvalues and eigenvectors in pseudospin space of the Hamiltonian with and without applied strain, Eqs. (1) and (2), as $H^{(0)}|q', \lambda^{(0)} = \epsilon^{(0)} q', \lambda^{(0)} = \epsilon^{(0)}$.
and the effect of strain on their dispersion relation has

Under applied strain are mapped onto \( E_{\lambda q}^{(0)} \) and \( |q', \lambda(0) \rangle = E_{\lambda q}(q', \lambda) \), respectively, with \( \lambda \) a pseudospin index, it follows that both \( E_{\lambda q} \) and \( |q', \lambda(0) \rangle \)

where the curly brackets in the last term denote a matrix anticommutator.

III. POLARIZATION

A. Charge response: Plasmons and conductivity

We now specifically turn to consider the density-density correlation function with linear response theory, i.e. the electron polarization \( \Pi_{\mu \nu}(q, \omega) \). Plasmon modes are then recovered as poles of the polarization, and the effect of strain on their dispersion relation has been studied in Refs. [10][11]. In particular, by including local field effects, as a consequence of the two-band character of the band structure of graphene, we found two main plasmon branches, the lower branch being characterized by the standard square-root dependence on \( q \), at long wavelengths, as expected for a two-dimensional system.

In given limits, the asymptotic form of the noninteracting polarization in the absence of strain, say \( \Pi_{\mu \nu}^{(0)}(q, \omega) \) is known explicitly. For instance, in the long wavelength limit \((q \to 0)\), one finds:

\[
\Pi_{\mu \nu}^{(0)}(q \to 0, \omega) = \frac{g_{\mu \nu} q^2}{8 \pi \hbar \omega} \left[ \frac{2 \mu}{|\hbar \omega|} + \frac{1}{2} \log \frac{2 \mu - \hbar \omega}{2 \mu + \hbar \omega} \right] - i \frac{\pi}{2} \theta(\hbar \omega - 2 \mu),
\]

where \( g_{\mu \nu} = g_{\mu \nu} = 2 \) take into account for spin and valley degeneracies, respectively. In other words, \( \Pi_{\mu \nu}^{(0)}(q \to 0, \omega) = Z(\omega) q^2 \), at a given \( \omega \), with the complex factor \( Z(\omega) \) implicitly defined by Eq. (12).

In the case of applied strain, but still in the noninteracting limit, this is then readily modified through the linearized Eq. (11), yielding

\[
\Pi_{\mu \nu}(q \to 0, \omega) = [1 - 2 \kappa (1 + \nu) \varepsilon \cos(2 \theta - 2 \varphi)] Z(\omega) q^2,
\]

where \( \varphi = q(q \cdot \mu, \omega) \). Within the random phase approximation (RPA), the interacting polarization reads

\[
\Pi_{\mu \nu}(q, \omega) = \Pi_{\mu \nu}^{(0)}(q, \omega) / (1 - V(q) \Pi_{\mu \nu}^{(0)}(q, \omega), \text{ where } V(q) = e^2 / (2 \epsilon_c q) \text{ is the (bare) Coulombic electron-electron interaction, and } \epsilon_c \text{ is the dielectric constant of the medium.}
\]

Solving for the plasmon dispersion relation, Re \( \Pi_{\mu \nu}^{-1}(q, \omega) = 0 \), at low energies one finds

\[
\hbar \omega_{pl} = \frac{e^2}{2 \pi \epsilon_c} \mu [1 - \kappa (1 + \nu) \varepsilon \cos(2 \theta - 2 \varphi)] \sqrt{q},
\]

One thus finds that the prefactor \( \tilde{\omega}_1(\varphi) \) in the \( \sqrt{q} \)-dependence is maximum [resp., minimum] for \( \varphi - \theta = \pi/2 \) [\( \varphi - \theta = 0 \)], i.e., wavevector orthogonal [parallel] to the direction of applied strain. Correspondingly, one also finds for the imaginary part of the retarded polarizability along the low-energy plasmon branch

\[
\text{Im} \tilde{\omega}_1(\varphi) = -\frac{1}{2} \sqrt{\frac{2 \pi \epsilon_c}{e^2}} \mu [1 - \kappa (1 + \nu) \varepsilon \cos(2 \theta - 2 \varphi)] \times (qa)^{3/2} \delta(h \omega - h \omega_{pl}(q)).
\]

Therefore, one recovers a dependence of the plasmon spectral weight on the angle of applied strain, similar to that shown by \( \tilde{\omega}_1(\varphi) \) in Eq. (13).

Another quantity of interest which is related to the density-density correlation function is the optical conductivity, which can be obtained as

\[
\sigma_{\varphi \varphi}(\omega) = \frac{ie^2}{\omega} \lim_{q \to 0} \frac{\omega^2}{q^2} \Pi_{\mu \nu}(q, \omega).
\]
Making use of Eq. (11) and (12) one therefore finds the optical conductivity in the presence of applied strain as

\[ \sigma_{\varphi \varphi}(\omega) = \sigma_0 [1 - 2\kappa(1 + \nu)\varepsilon \cos(2\theta - 2\varphi)] \times \left( \Theta(h\omega - 2\mu) + \frac{4}{\pi} \frac{\mu}{h\omega} \frac{i}{\pi} \log \left| \frac{2\mu - h\omega}{2\mu + h\omega} \right| \right), \quad (17) \]

where \( \sigma_0 = \pi\varepsilon^2/2h \) is proportional to the quantum of conductivity. In the hydrostatic limit, \( \nu = -1 \), \( \sigma_{\varphi \varphi} \) does not depend on strain, as may be expected, as the unstrained relation does not contain the Fermi velocity.

The above expression for the conductivity, Eq. (17), can be exploited to study the strain dependence of the transverse electromagnetic mode, that has been recently predicted theoretically in graphene\textsuperscript{14}, and in a graphene bilayer\textsuperscript{22}. In a 2D electron gas, the spectrum of electromagnetic modes obeys the equations\textsuperscript{12}

\[
1 + i \frac{\sigma}{2\varepsilon_0 \omega} \zeta(q, \omega) \equiv 0, \quad (18a) \\
1 - i \frac{\sigma}{2\varepsilon_0 \xi(q, \omega)\omega} \equiv 0, \quad (18b)
\]

for the longitudinal and transverse plasmons, respectively, where \( \zeta(q, \omega) = q^2 - (\omega/c)^2 \), where \( c \) is the velocity of light in vacuum. While conventional 2D electron systems cannot sustain a transverse electromagnetic mode, it has been predicted\textsuperscript{14} that graphene can develop a transverse plasmon mode, as a consequence of a negative imaginary part in the interband contribution to its optical conductivity, Eq. (17). Its logarithmic divergence as \( h\omega/\mu \rightarrow 2 \) is in turn related to the discontinuous behavior of the interband absorption of radiation at frequencies \( h\omega > 2\mu \). Such a feature is a generic consequence of causality, and is related through a Kramers-Kröning transformation to the step-like behavior of the real part of the optical conductivity. This is in turn due to the existence of a Fermi surface, which is however expected to be smeared at finite temperature, thus implying the reduction of the logarithmic singularity into a pronounced (but finite) peak.

Indeed, making use of Eq. (17) in Eq. (18a), one consistently recovers Eq. (14) for the longitudinal plasmons. On the other hand, substituting Eq. (17) in Eq. (18b), one obtains the strain-dependence of the dispersion relation of the transverse plasmon implicitly as

\[
\frac{hc}{\alpha\mu} \zeta(q, \omega) = (1 - 2\kappa(1 + \nu)\varepsilon \cos(2\theta - 2\varphi)) \times \left[ \frac{h\omega}{2\mu} \log \left| \frac{2\mu + h\omega}{2\mu - h\omega} \right| - 2 \right], \quad (19)
\]

where \( \alpha = c^2/(4\pi\varepsilon_0 hc) \) is the fine structure constant.

Because of the small factor \( \alpha \) in the left-hand side of Eq. (19), the dispersion relation of such a transverse mode is close to the linear dispersion relation of the electromagnetic radiation itself, \( \omega - cq \ll 0 \). However, one may expect that applied strain enhances deviations from linearity (i.e., from the photon’s dispersion relation), as a consequence of a strain-induced modification of the band dispersion. Fig. 1 shows indeed deviations from linearity, \( \omega - cq \) of the transverse plasmon, for \( q \) in the allowed range, for strain modulus \( \varepsilon = 0.1 \), and strain direction ranging from \( \varphi - \theta = 0 \) (top) to \( \varphi - \theta = \pi/2 \) (bottom).

**FIG. 1:** Showing deviations from linearity of the frequency of the transverse plasmon, Eq. (19), for \( q \) in the allowed range, for strain modulus \( \varepsilon = 0.1 \), and strain direction ranging from \( \varphi - \theta = 0 \) (top) to \( \varphi - \theta = \pi/2 \) (bottom).

### B. Current response

In the case of an applied vector field (e.g., an electric field \( E_i \)), one may in general decompose the linear response function in a longitudinal and a transverse component as

\[
\chi_{ij}(q, \omega) = \frac{q_i q_j}{q^2} \chi_{||}(q, \omega) + \left( \delta_{ij} - \frac{q_i q_j}{q^2} \right) \chi_{\perp}(q, \omega), \quad (20)
\]
where \( q = |q| \), for a homogeneous system\(^{44}\). In particular, in the case of the current-current correlation function, the latter being proportional to the pseudospin-pseudospin counterpart, this can be further simplified as

\[
\Pi_{ij}^{(0)}(q, \omega) = \Pi_{+}^{(0)}(q, \omega) \delta_{ij} + \Pi_{-}^{(0)}(q, \omega) A_{ij}(\varphi),
\]

where

\[
\Pi_{\pm}^{(0)}(q, \omega) = \frac{1}{2} [\Pi_{\parallel}^{(0)}(q, \omega) \pm \Pi_{\perp}^{(0)}(q, \omega)].
\]

Therefore, as a consequence of Eq. (10), it follows that even though an unstrained system is characterized only by transverse response in the static limit\(^{43}\), it may develop a nonzero parallel response as a result of a strain-induced deformation. Making use of Eq. (11b), one finds

\[
\Pi_{ij}(q, \omega) = \Pi_{ij}^{(0)}(q, \omega) - 2\varepsilon \kappa (1 + \nu) \left[ \Pi_{-}^{(0)}(q, \omega) \cos(2\theta - 2\varphi) \delta_{ij} + \Pi_{+}^{(0)}(q, \omega) A_{ij}(\theta) + \Pi_{-}^{(0)}(q, \omega) A_{ij}(\varphi + \pi/4) \sin(2\theta - 2\varphi) \right] - \kappa[(1 - \nu) + (1 + \nu) \cos(2\theta - 2\varphi)] \varepsilon \left[ q \frac{\partial \Pi_{\parallel}^{(0)}(q, \omega)}{\partial q} \delta_{ij} + q \frac{\partial \Pi_{\perp}^{(0)}(q, \omega)}{\partial q} A_{ij}(\varphi) \right].
\]

In the static limit \( (\omega = 0) \), Eq. (23) can be further simplified, by considering the analytic result of Ref. 20, with \( \Pi_{\parallel}^{(0)}(q, 0) = 0 \), and

\[
\Pi_{\perp}^{(0)}(q, 0) = \frac{g_\ast g e^2 v_F}{16 h q} \left[ 1 - \Theta(h v_F q - 2\mu) \frac{2}{\pi} \left[ \arcsin \left( \frac{2\mu}{h v_F q} \right) - \frac{2\mu}{h v_F q} \sqrt{1 - \left( \frac{2\mu}{h v_F q} \right)^2} \right] - \Theta(2\mu - h v_F q) \right].
\]

In particular, one recovers a vanishing response, \( \Pi_{qq}(q, 0) = 0 \), with \( \Pi_{qq} \) denoting the current-current correlation function for both vector potential and response field aligned with \( q \), when \( q \) is aligned with the applied field also in the presence of strain, as expected in the static limit.

**IV. ELECTRIC AND MAGNETIC SUSCEPTIBILITIES**

A magnetic field applied in the direction perpendicular to the graphene plane can be described as \( B_{\text{ext}}(q) = B_{\text{ext}}(q) \hat{z} = iq \times A \), where \( A = i(q_y - q_x) B_{\text{ext}}/q^2 \) in reciprocal space. The linear response to such a magnetic field is then given by a current \( J \), which in turn produces a magnetization term \( \delta B \equiv \chi_m B_{\text{ext}} \).

In the case of a static, uniform magnetic field, oriented in the direction orthogonal to the graphene sheet, one is interested in the magnetic susceptibility defined as

\[
\chi_M = \lim_{\mu \to 0} \int \frac{d\varphi}{2\pi} \chi_m(q, 0).
\]

Making use of Eq. (23), one obtains

\[
\chi_M = \lim_{\mu \to 0} \left( -\frac{\mu_0}{q^2} \right) \left[ 1 - \kappa(1 - \nu) \varepsilon q \frac{\partial}{\partial q} \right] \Pi_{\perp}^{(0)}(q, 0).
\]

In the strained case, this reads

\[
\chi_M = -\mu_0 [1 - 2\kappa(1 - \nu) \varepsilon] \frac{g_\ast g e^2 v_F^2}{6\pi} \delta(\mu).
\]

One therefore obtains a qualitatively similar result to the case of undeformed graphene, treated within the Dirac approximation and neglecting the electron-electron interaction\(^{20,23}\). On the other hand, applied strain causes a reduction of the magnetic response, Eq. (26). Although Eq. (27) would imply no response to a static, uniform magnetic field away from half-filling, one expects that finite-temperature effects would broaden the \( \delta \)-function, already in the noninteracting limit. A qualitatively similar smearing of the peak in the dependence on the chemical potential may also be induced by disorder\(^{25}\). Still at zero temperature and in the noninteracting limit, one recovers a nonzero magnetic response also away from half-filling, when the honeycomb lattice structure is considered\(^{25}\). The effect of the interactions has been considered in Ref. 20, where it is shown that an interacting 2D Dirac electron liquid develops a magnetic response also at finite doping.

An analogous procedure may be followed to derive the electric susceptibility \( \chi_e \), entering the relationship \( \delta E = \chi_e E_{\text{ext}} \) between the electric polarization and an external electric field. One is then interested in the static \( (\omega = 0) \) limit of the density-density polarization. In the presence of applied strain, at arbitrary \( \mu = h v_F k_F \), using Eq. (11), one explicitly finds
In particular, at zero doping ($\mu = 0$), Eq. (30) reads

$$\Pi_{\rho\rho}(q, \omega = 0) = [1 + 2\kappa(1 - \nu)\varepsilon] \left[ -\frac{g_s g_v \mu}{2\pi \hbar^2 v_F^2} + \frac{g_s g_v q}{8\pi \hbar v_F} G^+_\chi \left( \frac{2\mu}{\hbar v_F q} \right) \Theta(\hbar v_F q - 2\mu) \right]$$

$$-\kappa[(1 - \nu) + (1 + \nu)\cos(2\theta - 2\varphi)]\varepsilon \frac{g_s g_v q}{8\pi \hbar v_F} G^-_\chi \left( \frac{2\mu}{\hbar v_F q} \right) \Theta(\hbar v_F q - 2\mu), \quad (28)$$

where

$$G^\pm_\chi(x) = \pm x \sqrt{1 - x^2} - \arccos x, \quad |x| < 1. \quad (29)$$

In particular, at zero doping ($\mu = 0$, $G^+\chi(0) = -\pi/2$), one finds in general that

$$\chi_e(q, 0) = V(q)\Pi_{\rho\rho}(q, 0). \quad (30)$$

It should be emphasized that, while Eq. (30) describes the response of the system to a static electric field lying in the undoped case, Eq. (30) reads

$$\chi_e = \lim_{q \to 0} \chi_e(q, 0) = -\frac{g_s g_v e^2}{32\varepsilon_0 c \hbar v_F} \times [1 + \kappa(1 - \nu)\varepsilon - \kappa(1 + \nu)\varepsilon \cos(2\theta - 2\varphi) ], (31)$$

where $\varphi$ is the direction of the electric field on the graphene plane, thus showing that uniaxial strain introduces a modulation in the angle of applied strain. Moreover, comparison with Eq. (20) in the hydrostatic limit ($\nu = -1$) shows that strain enhances the electric response, while suppressing the magnetic one.

V. CONCLUSIONS

We have studied the dependence on applied uniaxial strain of several linear-response electronic correlation functions of graphene. After deriving a general correspondence between strained and unstrained correlation functions, we have derived the strain dependence of the plasmon dispersion relation and of the optical conductivity. Specifically, we find that the prefactor in the $\sqrt{q}$-dependence of the plasmon frequency develops an anisotropic character, with maxima occurring when the wavevector is orthogonal to the direction of applied strain. Moreover, we derive a strain-induced anisotropic enhancement of the deviations from linearity of the recently predicted transverse plasmon, which should facilitate its experimental detection in suitably strained graphene samples. Finally, we have compared and contrasted the strain dependences of the magnetic and electric susceptibilities, showing that strain enhances the response of strained graphene to an applied electric field, while suppressing the response to an magnetic field.

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