The paramagnetic photon. Absence of perpendicular component and decay in large fields

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Previous results from the authors concerning the arising a tiny photon anomalous paramagnetic moment $\mu'_{\gamma}$ due to its interaction with a magnetized virtual electron-positron background are complemented and discussed. It is argued that such magnetic moment it cannot be a linear function of the angular momentum and that there is no room for the existence of an hypothetical perpendicular component, as recently claimed in the literature. It is discussed that in the region beyond the first threshold, where photons may decay in electron-positron pairs, the photon magnetic moment cannot be defined independently of the magnetic moment of the created pairs. It is shown that for magnetic fields large enough, the vacuum becomes unstable and decays also in electron-positron pairs.

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INTRODUCTION

We have shown in [1] that in analogy to the electron anomalous magnetic moment $\mu'_{\gamma} = \alpha \mu_B / 2\pi$ (being $\mu_B = e/2m$ the Bohr magneton) shown by Schwinger [2] as due to their interaction with the virtual photon background through its self-energy, a similar effect exists for photons. Due to the magnetic properties of the photon self-energy, a photon anomalous magnetic moment $\mu_{\gamma} > 0$ arises, which is paramagnetic in the region of transparency (which is the region of momentum space where the photon self-energy, and in consequence, its frequency $\omega$, is real). The photon magnetic moment vanishes only when its momentum $k$ is parallel to the magnetic field $B$.

The photon magnetic properties are due to the dependence of $\omega$ on $B$ expressed by the photon dispersion equation dependence on the self-energy tensor $\Pi_{\mu\nu}(x, x' | A^{\mu,\nu})$. But let us recall some details about the electron-positron quantum mechanics in a magnetic field. We assume some magnetic field defined by the field invariants $F = 2B^2 > 0$, $G = 0$. In a given coordinate system, constant uniform magnetic field $B$, taken along $x_3$, produces a symmetry breaking of the space symmetry. For electrons and positrons ($e^\pm$) physical quantities are invariant only under rotations around $x_3$ or displacements along it [3]. This means that the conserved quantities, i.e., those commuting with the Hamiltonian operator, are all parallel to $B$, as angular momentum and spin components $J_3, L_3, s_3$ and the linear momentum $p_3$. By using units $\hbar = c = 1$, the energy eigenvalues for $e^\pm$ are $E_{n,p_3} = \sqrt{p_3^2 + m^2 + eB(2n + 1 + s_3)}$ where $s_3 = \pm 1$ are spin eigenvalues along $x_3$ and $n = 0, 1, 2, \ldots$ are the Landau quantum numbers. In other words, in presence of $B$, the transverse squared energy $E_{n,p_3}^2 - p_3^2$ is quantized by integer multiples of $eB$. For the ground state $n = 0$, $s = -1$, the integer is zero. Quantum states degeneracy with regard spin is expressed by a term $\alpha_n = 2 - \delta_{n0}$, whereas degeneracy with regard to orbit center coordinates leads to a factor $eB$. The quantity $1/eB$ characterizes the spread of the $e^\pm$ spinor wavefunctions in the plane orthogonal to $B$. The magnetic moment operator $\hat{M}$ is defined as the quantum average of $\hat{M} = -\partial H / \partial B$, where $H$ is the Dirac Hamiltonian in the magnetic field $B$, and it is not a constant of motion. But its time-dependent terms vanish after quantum averaging and it leads to $\hat{M} = -\partial \hat{E}_{n,p_3} / \partial B$. Obviously, it must be understood as the modulus of a vector parallel to $B$. There is no a linear relation between $\hat{M}$ and $J_3$ as it exists in non-relativistic quantum mechanics. There is also no room for conjecture the arising of a magnetic moment component in a direction different from $B$.

Due to the explicit symmetry breaking, the four momentum operator acting on the vacuum state does not have a vanishing four-vector eigenvalue, $p_{\mu} | 0, B \rangle = \neq 0$. The components $p_{1,2}$ does not commute with the Hamiltonian operator $H$, and are not conserved. The $e^\pm$ vacuum quantum energy density is given by $\Omega_{EH} = -eB \sum_{n=0}^{\infty} \alpha_n \int dp_3 E_{n,p_3}$. After removing divergences it gives the well-known Euler-Heisenberg expression $\Omega_{EH} = \frac{\alpha}{2\pi} \int_0^{\infty} e^{-B_x x / B} \left[ \frac{\coth x}{2} - 1 \right] \frac{dx}{B_x}$, which is an even function of $B$ and $B_x$, where $B_x = m^2 / e \simeq 4.4 \times 10^{13} \text{G}$ is the Schwinger critical field. The magnetized vacuum is paramagnetic $\mathcal{M}_V = -\partial \Omega_{EH} / \partial B > 0$ and is an odd function of $B$. For $B \ll B_x$ it is $\mathcal{M}_V = \frac{\alpha}{2\pi} B_x^2 / B_x^2$, which is obviously be understood as the modulus of a vector parallel to $B$.

The magnetic field $B$, is defined as such for a class of reference frames moving parallel to $B$. For frames moving in other directions, for instance, perpendicular to $B$, the magnetic field changes to $B'$ and an electric field $E$ arises, such that $B'^2 - E^2 = B^2$. We do not intend to define an anomalous electric moment in that case, since it would be of pure kinematical origin (it can be eliminated by a...
Lorentz boost) and for that reason, it is not interesting.

Most of this paper is based on a talk presented at the International Workshop on Astronomy and Relativistic Astrophysics 2009 (IWARA09), which was based on [1], as well as some new results. We have enlarged the manuscript in some details, and discussed a new reference related to the content of [1].

THE PHOTON MAGNETIC MOMENT FROM SHABAD’S DISPERSION EQUATIONS

In a similar way as the definition of the electron-positron magnetic moment, for an observable photon we define the quantity \( \mu_\gamma = -\partial \omega / \partial B \), which is to be understood also as the modulus of a vector along \( B \). It can be obtained analytically from the dispersion equations for the photon in a magnetic field and it is generated by the photon self-energy tensor dependence on \( B \). There is no any reason to impose a priori any linear relation between \( \mu_\gamma \), and classical photon spin, (or either electron-positron angular moment operators) as has been made recently in the literature (See [5]. We will return later to this reference).

The diagonalization of the photon self-energy tensor leads to the equations [6]

\[
\Pi_{\mu\nu} a^{(i)}_\mu = \kappa_i a^{(i)}_\mu,
\]

having three non vanishing eigenvalues and three eigenvectors for \( i = 1, 2, 3 \), corresponding to three photon propagation modes. One additional eigenvector is the photon four momentum vector \( k_\mu \) whose eigenvalue is \( \kappa_4 = 0 \). The first three eigenvectors

\[
\begin{align*}
a^{(1)}_\mu &= k^2 F^\mu\lambda k^\lambda - k_\mu (k^2 k), \\
a^{(2)}_\mu &= F^\mu\lambda k^\lambda, \\
a^{(3)}_\mu &= F_\mu\lambda k^\lambda,
\end{align*}
\]

satisfy the four dimensional transversality condition \( a^{(1,2,3)}_\mu k_\mu = 0 \). Here \( k_\mu \) is the photon four-momentum, \( F^\mu\nu = \partial_\mu A_\nu - \partial_\nu A_\mu \) and \( F^{\mu\nu*} = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma} \) are the external electromagnetic field tensor and its dual pseudotensor, respectively.

In reference frames which are at rest or moving parallel to \( B \) we define \( n_\perp = k_\perp / k_\perp \) and \( n_\parallel = k_\parallel / k_\parallel \) as the transverse and parallel unit vectors respectively.

By \( a^{(i)}_\mu(x) \) as the electromagnetic four vector describing these eigenmodes, its electric and magnetic fields \( e^{(i)}_\mu = -\partial_\nu a^{(i)}_\nu - \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} a^{(i)}_\rho \), \( h^{(i)}_\mu = \nabla \times a^{(i)}_\mu \) are obtained in [3].

It is easy to see that the mode \( i = 3 \) is a transverse plane polarized wave whose electric unit vector is \( e^{(3)}_\mu = (n_\perp \times n_\parallel) \) orthogonal to the plane \( (B, k) \). For \( k \perp B \), \( a^{(1)}_\mu \) is longitudinal, polarized along \( e^{(1)}_\mu = n_\perp \) and it is a non physical mode, whereas \( a^{(2)}_\mu \) is transverse, since \( e^{(2)}_\mu = n_\parallel \). Thus, in that case modes \( a^{(2,3)}_\mu \) are superposition of waves of opposite circular polarizations.

For \( k \parallel B \), the mode \( a^{(2)}_\mu \) becomes pure electric and longitudinal with \( e^{(2)}_\mu = n_\parallel \) (and also non physical), whereas \( a^{(1)}_\mu \) is transverse \( e^{(1)}_\mu = n_\perp \), as it is \( e^{(3)}_\mu \). In that case \( \kappa^{(1)} = \kappa^{(3)} \), and the circular polarization unit vectors \( (e^{(1)}_\mu \pm ie^{(3)}_\mu)/\sqrt{2} \) are common eigenvectors of \( \Pi_{ij} \) and of the rotation generator matrix \( A^{ij} \). The quasi-photon moving parallel to \( B \) moves with the speed of light \( c \).

The photon magnetic moment is identified as a 3-d vector which contains the contribution from electron-positron pairs whose spin components are oriented along \( B \). This is in correspondence with the fact that after solving the dispersion equations the resulting entity embodies properties of both free photons plus electron-positron pairs, leading to a composite particle which we name quasi-photons. For them, the phase and group velocities perpendicular to \( B \) are smaller than unity, although, due to gauge invariance, quasi-photons are massless. The \( e^\pm \) virtual pairs contributing to the photon self-energy, if considered as two-particle states, must have total spin \( S = 1 \), with projections \( S_i = 1, -1, 0 \). For the quasi-photon propagating along \( k(\perp B) \), its observable projection is only \( S_3 = 0 \).

We would like to stress here that the problem of a photon interacting with the electron-positron field in a magnetic field implicitly assumes a Lagrangian describing the interaction of the electron-positron fields in presence of an external constant magnetic field (Furry representation).

From the Lagrangian a total Hamiltonian can be written in principle as \( H_T = H_{e,B} + H_{int} + H_{ph} \), where \( H_{e,B}, H_{int}, H_{ph} \) are respectively the Hamiltonian operator for the electrons in the external field, the interaction Hamiltonian of bounded electrons and positrons with the photon, and the free photon Hamiltonian (some gauge condition is assumed to be imposed). Obviously, only components \( P_3, J_3 \) of the total momentum \( P \) and angular momentum \( J \) commute with \( H_T \) and are conserved along with the total energy. This is consequence of basic principles in quantum theory.

The dispersion equations, obtained as the zeros of the photon inverse Green function \( D^{-1}_{\mu\nu} = 0 \), after diagonalizing the polarization operator \( \Pi_{\mu\nu}(z_1, z_2, B) \), are

\[
k^2 = \kappa_i(z_2, z_1, B) \quad i = 1, 2, 3.
\]

After solving the dispersion equations for \( z_2 \) in terms of \( z_2 \) we get

\[
\omega^{(i)} = |k| + m^{(i)} (z_2, B)
\]

The refraction index \( n^{(i)} \) can be defined as

\[
n^{(i)} = \frac{|k|}{\omega} = (1 + \frac{m^{(i)}(z_2, B)}{\omega^2})^{-1/2}
\]

being different for each eigenmode, leading to the phenomenon of birefringence. The propagation of light in magnetized vacuum is thus similar to that in an
anisotropic medium. Gauge invariance implies that \( \kappa_{(i)}(0,0,B) = 0 \) and \( \Omega^{(2)}(0,B) = 0 \). Thus, for parallel propagation \( n^{(i)} = 1 \). By differentiating \( \Omega^{(i)} \) with regard to \( B \) we get the relation \( \mu^{(i)} = -\frac{1}{2\omega} \frac{d\Omega^{(i)}}{dB} \), and in consequence \( \Omega^{(2)} = -2 \int_{0}^{B} \omega \mu^{(i)}(B')dB' + f(z_2) \). The function \( f(z_2) \) is zero if the series expansion of \( \kappa_{(i)} \) in powers of \( B \) is taken as linear in \( z_1, z_2 \) (see below). From \( \Omega^{(3)} \), we have the approximate dispersion equation

\[
\omega = |k| - \int_{0}^{B} \mu^{(i)}(z_2, B', |k|)dB'.
\]

(6)

For nonparallel propagation the fact that \( n^{(i)} > 1 \) is a consequence of photon paramagnetism.

**EXPRESSIONS FOR THE PHOTON ANOMALOUS MAGNETIC MOMENT**

We will write now the explicit expressions for the photon magnetic moment found in \( \mathfrak{1} \). It was shown that in the regions \(-z_1, z_2 \leq 4m^2 \) and \( 0 < B \leq B_c \), the photon is paramagnetic, since \( \mu^{(2)} > 0 \). In that small frequencies and weak field limit the expressions for \( \kappa_{(i)} \) were expanded in series of even powers of \( eB \) and in all positive powers of \( z_1, z_2 \). We will take only the linear approximation in \( z_1, z_2 \). By taking the first two terms in the \( \kappa^{(0)} \) series expansion, the magnetic moments are,

\[
\mu^{(2)} = \frac{14\alpha z_2}{45\pi B_c |k|} \left( \frac{b - 52b^3}{49} \right) > 0,
\]

(7)

\[\text{and}\]

\[
\mu^{(3)} = \frac{8\alpha z_2}{45\pi B_c |k|} \left( \frac{b - 24b^3}{7} \right) > 0.
\]

(8)

**High frequencies and strong fields case**

In \( \mathfrak{1} \) was pointed out that in the case \( m^2 \lesssim -z_1 \leq 4m^2, B \lesssim B_c \), the energy gap between successive Landau energy levels for electrons and positrons is of order close to the electron rest energy. The photon self-energy diverges for values of \( -z_1 = k^2 \).

In the vicinity of the first threshold \( n = n' = 0 \) and by considering \( k_\perp \neq 0 \) and \( k_\parallel \neq 0 \), according to \( \mathfrak{2}, \mathfrak{3} \) the physical eigenvalues are described by the second and third modes, but only the second mode has a singular behavior near the threshold. The photon magnetic moment found in \( \mathfrak{1} \) is

\[
\mu^{(2)} = \frac{(1 + \frac{\gamma}{\gamma'}) \left( 4m^2 + z_1 \right) am^3e^{-\frac{z_2}{2b}}}{\omega B_c \left( \left( 4m^2 + z_1 \right)^{3/2} + bam^3e^{-\frac{z_2}{2b}} \right)} > 0,
\]

(9)

This solution is valid in the vicinity of the first threshold and agrees with the previously mentioned numerical result in its paramagnetic property, which is valid throughout the whole region of transparency (below the first pair creation threshold).

The expression \( \mathfrak{9} \) has a maximum near the threshold \( \mathfrak{4} \), \( z_2 \simeq k^2/3 \). By calling

\[
m_\gamma = \omega(k^2/3) = \sqrt{4m^2 - m^2} \left( 2\alpha b \exp(-\frac{z_2}{b}) \right)^{2/3}
\]

(10)

that maximum is \( \mu^{(2)} = \frac{e(1+2b)}{3m_b} \left( 2\alpha b \exp (-\frac{z_2}{b}) \right)^{2/3} \). Numerically for \( b \sim 1, \mu^{(2)} \approx 3\mu' (\frac{1}{3})^{1/3} \approx 12.85 \mu' \).

At this point the authors want to refer again to paper \( \mathfrak{5} \) dealing with topics close to the content of \( \mathfrak{1} \), but restricted to asymptotically large magnetic fields. In that paper it is imposed a linear dependence of the photon magnetic moment with regard to the photon spin in analogy to the non-relativistic electron, which is not true at the relativistic level, and it is claimed the existence of a non-zero photon magnetic moment component orthogonal to \( B \). This also cannot be justified a priori since the symmetry is broken along \( B \), and a direction orthogonal to it is not related to any observable. For instance, \( J_1, J_2 \) did not commute with the total Hamiltonian \( H_T \). Such alleged “perpendicular component”, is shown in the paper \( \mathfrak{6} \) to come from the scalar product of two orthogonal vectors, and in consequence it is recognized as giving a zero contribution to the \( B \) dependent part of \( \omega \). Thus, that “perpendicular component” comes from differentiating zero with regard to \( B \), which is mathematically nonsense. It is also argued to be obtainable by an independent procedure, which a simple analysis shows that it actually involves again differentiating zero with regard to \( B \). Based on these results, it is claimed some precession of the magnetic moment around \( B \) which has no basis at all (See the Appendix).

**THE ABSORPTIVE REGION**

Beyond the first threshold \( n = n' = 0 \), for frequencies such that \( -z_1 > 4m^2 \) starts the so-called region of absorption, i.e., for \( 4m^2 + z_1 < 0, \kappa_{(2)} \) becomes complex, its imaginary part leading to complex frequencies \( \omega + i\Gamma \) after solving the dispersion equations (the thresholds for absorption would be slightly lower if the pair resulting from the photon decay forms a bound state, or positronium. (See \( \mathfrak{8}, \mathfrak{9} \)). The quantity \( \Gamma \) is finite on the photon mass shell \( \mathfrak{1} \) and it accounts for the probability of photon decay in electron-positron pairs (the same occurs for higher thresholds). Thus, in this region the photon magnetic moment cannot be considered independently of the created electron-positron pairs.

The absorptive region is the continuation of the large frequency, large magnetic field limit to the region \(-z_1 \geq 4m^2 \) and fields \( B \geq B_c \). That region is also interesting in astrophysics, and in cosmology (stars having fields \( B \geq B_c \) and early universe). It is interesting, however,
to remark some of the new features. For instance, although larger values are expected for the photon magnetic moment than in the region of transparency, a negative peak is found for the first threshold of the third mode. This has no absolute meaning since in that region photons coexist with electron-positron pairs (the photon has some nonzero probability of decaying in pairs or either in positronium) and the magnetic moment of the created electron-positron pairs (or positronium) must be added to that of photons. This is point is not taken into account in [3], where the photon magnetic moment in supercritical magnetic fields is discussed only on real $\omega$.

The magnetized vacuum background is no longer satisfactorily described by the Euler-Heisenberg expression $\Omega_{EH}$, and radiative corrections containing the photon self-energy must be taken into account. These corrections can be written in the two equivalent forms [10]

$$\Omega_{EH} = \int_0^\infty \ln(1 + e^{\mu}) \frac{d^4k}{2k^2} \frac{\kappa_i}{k^2 - \kappa_i} \sum_{i=1}^{n'} \int_0^\infty \frac{d^3k \omega \kappa_i}{k^2 - \kappa_i}$$

(11)

The leading divergences are eliminated after integrating over $e'$. For some ranges of $k$, $\omega$ and $B$, $\kappa_i$ are divergent at the pair creation thresholds, getting some imaginary part and becoming complex, as discussed previously, and this suggests that quantum vacuum modes at these frequencies become unstable and decay for fields $B \gtrsim B_c$, in a similar way as observable photons.

Let’s consider in (11) only the second mode contribution coming from the first threshold for pair creation $n = n' = 0$:

$$\Omega_{EH2} = -\frac{1}{(2\pi)^4} \int_0^\infty \frac{d^3k \omega \kappa_2}{k^2 - \kappa_2}$$

(12)

$$= \frac{1}{(2\pi)^4} \int d^3k \omega \ln \frac{k^2 - \kappa_2}{k^2}.$$  

From (12) it is easy to obtain an expression for the imaginary part. We assume that $Re\kappa_2$ is small in the absorptive region, where the main contribution comes from $Im\kappa_2$. Thus, in the first line of (12) the quantity

$$\frac{\kappa_2}{k^2 - \kappa_2} = \frac{iIm\kappa_2}{k^2 - iIm\kappa_2}$$

$$= \frac{-Im\kappa_2^2 + ik^2Im\kappa_2}{k^4 + Im\kappa_2^2}$$

(13)

From (13)

$$Im\Omega_{EH2} = \frac{1}{(2\pi)^4} \int d^3k \omega \arctan \frac{-Im\kappa_2}{k^2}.$$  

(14)

where (see [1])

$$Im\kappa_2 = \frac{4\alpha eBm^2\Theta(-4m^2 - z_1)}{\sqrt{z_1(1 + 4m^2)}} \exp \left(\frac{-z_2}{2eB}\right),$$

and $\Theta(-4m^2 - z_1)$ is the step function. The integration over $z_1$ is extended along the half-plane $z_1 \leq -4m^2$, and for $z_2$, in the half-plane $z_2 \geq 0$. The numerical calculation of (14) is shown in fig. 1

![FIG. 1: Imaginary part of vacuum energy.](image)

By observing the figure we see that for fields $2B_\gamma < B < 12B_\gamma$, we have $Im\kappa_2 \sim 10^{22}$ in energy units. This would correspond to $\sim 10^{28}$ electron-positron pairs or photons, after the pairs decay. Magnetized vacuum becomes unstable and may decay finally in electromagnetic radiation through that mechanism, for fields $B \gtrsim B_c$. The question immediately comes about the source of matter or energy to account for the created $e^\pm$ pairs or photons. We conclude that the source of the field would contribute with a compensating amount of energy. For instance, if the field is produced by a star through the dynamo effect, the vacuum decay would be produced at the expense of a small slowing down of the rotating energy, and a consequent decrease of the magnetic field intensity. Magnetic energy would be transformed into radiation beams, propagating along $B$. Thus, the vacuum decay through the present mechanism might play a role in the $\gamma$-ray emission of strongly magnetized stars.

We recall also that at present QED in unstable vacuum (see for instance [11], [12] and its references) has growing interest, and the possibility of observing vacuum decay in critical electric fields in terrestrial laboratories (see [13]) is becoming realistic thanks to the development of high power pulse lasers technology.

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APPENDIX

We are assuming throughout this paper the constancy of the external magnetic field (as also is done in [5]). If $B$ varies, we are dealing with a different problem. The “demonstration” made in the Appendix of [3] is due to a mistake. It is equivalent to the following procedure. Let us start from a rotated system of coordinates where $B_y$ and $B_z$ are the components (of the spatial part) of the magnetic field tensor $F_{ij}$. We have

$$F_{ij}k_j = ((B_z k_y - B_y k_z), -B_z k_x, B_y k_x)$$ (16)

Its square is,

$$(kF^2k) = k^2(B_y^2 + B_z^2) + (B_z k_y - B_y k_z)^2 = [B \times k]^2.$$ (17)

The contribution of $\mu_y = (\partial \omega / \partial kF^2k)(\partial kF^2k / \partial B_y)$ to the perpendicular-to-$B$ magnetic moment would be proportional to

$$\frac{\partial kF^2k}{\partial B_y} = 2k_z^2 B_y - 2(B_z k_y - B_y k_z) k_z.$$ (18)

If we assume that the limit $B_y \to 0$ can be taken, it would lead to

$$\frac{\partial kF^2k}{\partial B_y} \bigg|_{B_y=0} = -2[B] k_y k_z.$$ (19)

Since $B_z = |B|$, when $B_y = 0$, at first sight it seems that a perpendicular component to the magnetic moment arises, depending on the transversal, $k_y$, and longitudinal, $k_z$, momenta.

Let us show that this demonstration has a mistake, due to the fact that it bypasses a constraint implicit in the vector product definition, since $B \times k = B \times [k_\parallel + k_\perp] = B \times k_\perp$, and it implies $B \times (k_\parallel) = 0$.

First of all let us consider the two sides of equation (17). From the general definition we expect that $[B \times k]^2 = B^2 k_\perp^2 = (B_y^2 + B_z^2) k_\perp^2$. Thus, by comparison with the left hand side, it must be either $B_z k_y - B_y k_z = 0$ and $k_\perp = k_z$, or $B_z k_y - B_y k_z = B k'_\parallel$, must hold after some algebra, where $k'_\parallel$ is some other component of $k$ orthogonal to $B$. Let us assume that the plane $B, k$ is chosen as orthogonal to the plane $(y, z)$. This selection of the coordinate system can always be done and in it only the component $k_\parallel$ lies on the $y, z$ plane. Then

$$k_\parallel = k \sin \varphi$$ (where $\varphi$ is the polar angle of $k$ with regard to $x$-axis) has components $k_y = k_\parallel \cos \theta, k_z = k_\parallel \sin \theta$ ($\theta$ is the azimuthal angle) and in consequence in (17), (18) $B_y k_z - B_z k_y = B k \sin \varphi \cos \theta \sin \varphi = -\sin \theta \cos \varphi = 0$ identically, and $\partial(B_y k_z - B_z k_y) / \partial B_y = 0$, since derivative of zero with regard to any quantity is zero. See fig. 2

![FIG. 2: Relation between components of the vector product $B \times k$.](image)

will refer to a mechanical analogy of the “perpendicular diamagnetic component” problem: the case of a point of mass $m$ rotating uniformly around a center (as a simplified version, for instance, of a planet around the sun; we may assume the existence of a central force). With regard to some adequately chosen reference frame, its kinetic energy can be written as $E = m(\omega \times r)^2 / 2$. The angular velocity $\omega$ is taken along the 3-rd axis, since the particle rotates in the 1,2 plane. Here $\omega$ is the analog of $B$. From $E$, we get the angular momentum, (the analog of the magnetic moment)

$$L = \partial E / \partial \omega = m(e_3 \times r) \cdot (\omega \times r)$$ (20)

$$= e_3 \cdot (m r \times (\omega \times r)) = m r^2 \omega.$$
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