QCD thermodynamics on the lattice: approaching the continuum limit with physical quark masses

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Abstract. The $T>0$ QCD transition is an analytic cross-over. This non-trivial result is shown by means of lattice simulations using physical quark masses and finite size scaling analysis extrapolated to the continuum limit. The transition temperature ($T_c$) is determined. The discrepancy between our (Wuppertal-Budapest) results and the results of the Bielefeld-Brookhaven-Columbia-Riken Collaboration & hotQCD Collaboration is discussed. Preliminary results on the equation of state are presented.

1. Introduction
During the evolution of the universe there was a transition at $T \approx 200$ MeV. It is related to the spontaneous breaking of the chiral symmetry of QCD. The nature of the QCD transition affects our understanding of the universe’s evolution (see e.g. Ref. [1]). Extensive experimental work is currently being done with heavy ion collisions to study the QCD transition (most recently at the Relativistic Heavy Ion Collider, RHIC and at the Large Hadron Collider, LHC). Both for the cosmological transition and for RHIC/LHC, the net baryon densities are quite small, thus the baryonic chemical potentials ($\mu$) are much less than the typical hadron masses (below 50 MeV at RHIC, even smaller at LHC and negligible in the early universe). A calculation at $\mu=0$ is directly applicable for the cosmological transition and most probably also determines the nature and absolute temperature of the transition at RHIC/LHC. Thus we carry out our analysis at $\mu=0$. (For a review on the QCD transition at $\mu=0$ and $\mu>0$ see e.g. [2].)

When we analyze the nature and/or the absolute scale of the $T > 0$ QCD transition for the physically relevant case two ingredients are quite important.

First of all, one should use physical quark masses. The nature of the transition depends on the quark mass, for small or large quark masses it is a first order phase transition, whereas for intermediate quark masses it is an analytic crossover. Since the nature of the transition influences the absolute scale ($T_c$) of the transition –its value, mass dependence, uniqueness etc.– the use of physical quark masses is also essential for the determination of $T_c$, too. The absolute scale than goes into all observables, also into the equation of state.

Secondly, the nature of the $T > 0$ QCD transition is known to suffer from discretization errors [3, 4]. The three flavor theory with a large, $a \approx 0.3$ fm lattice spacing and standard action predicts a critical pseudoscalar mass of about 300 MeV. This point separates the first order and cross-over regions. If we took another discretization, with another discretization error,
the critical pseudoscalar mass turns out to be much smaller, well below the physical pion mass of 135 MeV. The only way to determine whether the physical point lies in the crossover or first-order region, is to carry out a careful continuum limit analysis.

In this review the recent results of the Wuppertal-Budapest group are discussed (for the computational setup see [5]). The order and the absolute scale of the transition \(T_c\) was determined using physical quark masses and a continuum extrapolation. The findings are compared to those of our competitor’s (Bielefeld-Brookhaven-Columbia-Riken, which later merged with some part of the MILC Collaboration and formed the hotQCD Collaboration). Preliminary results on the \(T_c\) determination for temporal extensions upto 16 and equation of state results for temporal extensions upto 12 are presented.

In this section we summarize the qualitative features of the T>0 QCD transition. We use the water-vapor transition as an illustration.

One of the most important piece of information we have is our knowledge about the nature of the transition. Though many take it for granted, it is a highly non-trivial result, that the transition is an analytic one and usually called as a cross-over [6]. In order to show this by means of lattice QCD, physical quark masses were taken, and a finite size scaling analysis was carried out for the continuum extrapolated chiral susceptibilities. (Some details of the analysis will be given in the next section.) This analytic behavior has important consequences for any \(T_c\) determination in QCD.

In order to illustrate the most important differences between a real phase transition and an analytic cross-over we recall the water-vapor phase diagram on the temperature versus pressure plane (c.f. [7], Figure 1). We study the transition by fixing the pressure to a given value and than varying the temperature. For smaller pressures (below about 22 MPa) there is a first order phase transition. The density jumps, the heat capacity is infinite, and these singular features appear simultaneously, thus exactly at the same critical temperature. At \(p \approx 22.064\) MPa pressure and \(T \approx 647.096\) K temperature there is a critical point with a second order phase transition. This phase transition is also characterized by a singular behavior. (Note, that a real singularity, a phase transition takes place only in infinite size systems. In our example we have a macroscopic amount of water with \(O(10^{23})\) molecules. From the practical point of view this is an infinitely large system).

At even larger pressures (\(p > 22.064\) MPa) the water-vapor transition is an analytic one (the behaviors of various observables are analytic, even in the infinite volume limit). As a consequence, in this pressure region if we change the temperature there is no jump in the density only a rapid but continuous change. The inflection point of this density-temperature function (the point with the largest, though finite, derivative) can be used to define the pseudocritical temperature (another usual name for it is "transition temperature") related to the density. Similarly, the heat capacity is always finite, but it has a pronounced peak as we increase the temperature. The position of this peak can be used to define the pseudocritical temperature related to the heat capacity. Despite the fact that there is no singularity the inflection point and peak position are well defined.

The most important message here is that the various transition temperatures (e.g. those related to the density or heat capacity) behave differently depending on whether we are in the...
Figure 1. The phase diagram of water around its critical point (CP). For pressures below the critical value ($p_c$) the transition is first order, for $p > p_c$ values there is a rapid crossover. In the crossover region the critical temperatures defined from different quantities are not necessarily equal. This can be seen for the temperature derivative of the density ($d\rho/dT$) and the specific heat ($c_p$). The bands show the experimental uncertainties (see [8]).

singular (real phase transition) or non-singular (analytic cross over) region. As it is indicated on the figure for a real phase transition these critical temperatures coincide, whereas in the non-singular region (for pressures above 22.064 MPa) the pseudocritical temperatures can differ considerably. The fast change (though no jump) in the density is at a lower temperature than the peak in the heat capacity. The transition is a broad cross-over. The pseudocritical temperatures, related to various observables, are separated, but both of them are in the broad transition temperature region. This separation does not mean that we have two transitions (one for the density and one for the heat capacity), it merely reflects the broadness of the transition.

It is easy to see that different observables can give different pseudocritical temperatures. Let us study an observable $X$, which characterizes the transition as a function of the temperature $X(T)$. For a real phase transition its singular behavior appears at the same temperature even if we multiplied it by $T$ (an infinitely high peak keeps its position). For an analytic cross-over we have a peak with a finite height and a finite width. Multiplying it by $T$ (thus constructing a new observable $T \cdot X$, which is also sensitive to the transition) shifts the peak position to larger temperature values. The value of $T_c$ is shifted. The pseudocritical temperature is well defined for any definition, but it is not unique. Though a $T_c$ related to some observable is informative a more complete description is given by the whole temperature dependence of $X(T)$.

The determination of such curves is the main goal of any study on the QCD transition (c.f. our earlier studies [7, 9]). Since the QCD transition (at vanishing chemical potential) is an analytic cross-over one wants to obtain these smooth curves for several observables. Though the characteristic points of such curves contain obviously less informations than the curves themselves, their knowledge tells a lot about the transition, too.

Before we list the observables we study in detail, it is worth to mention that the cross-
over nature of the QCD transition is related to the specific values of the quark masses we have in nature. For two or three flavor QCD with vanishing quark masses or with infinitely massive quarks one has real phase transitions. There are order parameters (in the former case the chiral susceptibility/condensate signaling the chiral phase transition; in the latter case the Polyakov line signaling the deconfinement phase transition) which show a non-analytic behavior as we change the temperature. As we pointed out earlier, the highly non-trivial result about the analytic nature of the QCD transition with physical quark masses implies, that no observable can be treated as an order parameter. All of the observables show analytic temperature dependence. There is neither a chiral nor a deconfinement phase transition. Note however, that similarly to the density or to the heat capacity in the water-vapor cross-over transition, the observables chiral susceptibility/condensate and the Polyakov line can develop a pronounced peak or show a rapid change. The peak position or the inflection points for such a cross-over are usually expected to be at different temperatures. Again, we do not say \([7, 9]\) that there are two phase transitions and one of them is at a lower temperature than the other. The separation of the pseudocritical temperature is merely a sign of the broad analytic transition \([6]\).

Since the chiral susceptibility/condensate and the Polyakov loop are not order parameters, they are just used to signal the cross-over. In principle any other quantity showing rapid changes or developing a peak in the transition region can be studied. The temperature dependences of these observables can be compared with the predictions of other lattice results or model calculations.

The reason for calculating the temperature dependence of these many observables is obvious. The more observables we study the broader picture we have on the QCD transition. To be more specific, the chiral susceptibility/condensate and the Polyakov loop are remnants of the real phase transition order parameters (for other mass regions of the phase plane). The strange quark number susceptibility is a particularly attractive quantity from the theoretical point of view. It is related to a conserved current, thus no renormalization ambiguities appear, which makes direct comparisons particularly easy.

The various observables (listed in the previous paragraph) lead to different transition temperatures, they are typically between 150 and 170 MeV, thus well within the breadth of the transition. Let us emphasize again, the difference between the \(T_c\) values does not mean that one of the phase transition happens at a lower temperature than the other, quite the contrary: no phase transition happens at all. Our new results confirm our earlier findings and their interpretation by all means: the transition temperatures scatter within the broad temperature interval, characteristic to the cross-over.

### 3. Lattice formulation

Thermodynamical quantities can be obtained from the partition function which can be given by a Euclidean path-integral: \(Z = \int dU d\bar{\Psi} d\Psi \exp \left(-S_E(U, \bar{\Psi}, \Psi)\right)\), where \(U\) and \(\bar{\Psi}, \Psi\) are the gauge and fermionic fields and \(S_E\) is the Euclidean action. The lattice regularization of this action is not unique. There are several possibilities to use improved actions which have the same continuum limit as the straightforward unimproved ones. The advantage of improved actions is that the discretization errors are reduced and therefore a reliable continuum extrapolation is possible already from larger lattice spacings. On the other hand, calculations with improved actions are usually more expensive than with the unimproved one.

Our choice for \(S_E\) is to use a tree level Symanzik improved gauge action and a stout improved staggered fermion action \([5]\). This choice is motivated by at least two reasons. It is computationally relatively cheap (comparable to the unimproved actions) and the discretization effects coming from \(T = 0\) and \(T > 0\) simulations are balanced.

In order to carry out \(T > 0\) simulations we have to fix the parameters of the action, the gauge coupling and the quark masses. This is usually done at zero temperature, where the results of
lattice computations can be compared to experiments. In order to fix three parameters, three quantities are needed. We chose to use the pion and kaon masses \((m_\pi, m_K)\) and the leptonic decay constant of the kaon \((f_K)\). These can all be determined from \(T = 0\) lattice simulations. Lattice calculations can only yield dimensionless quantities, i.e. \(am_\pi, am_K, af_K\), where \(a\) is the lattice spacing. For any value of the coupling, the mass parameters can be tuned to obtain the correct ratios for \(m_\pi/f_K\) and \(m_K/f_K\). These \(m_{ud}(\beta)\) and \(m_s(\beta)\) functions are called the line of constant physics (LCP). All finite \(T\) simulations have to be carried out along the LCP.

A very important step of any lattice analysis is the continuum extrapolation. Several simulations have to be carried out at different lattice spacings and the results have to be extrapolated to the continuum. Since \(T = 1/(N_t a)\), where \(N_t\) is the number of lattice sites in the temporal direction, if we are interested in a given temperature range (around the transition) then a decreasing lattice spacing corresponds to increasing \(N_t\).

4. **The order of the transition**

In this section the nature of the QCD transition is discussed. The details of the calculations can be found in [6]. In order to determine the nature of the transition one should apply finite size scaling techniques for the chiral susceptibility \(\chi = (T/V) \cdot (\partial^2 \log Z/\partial m_{ud}^2)\). This quantity shows a pronounced peak as a function of the temperature. For a first order phase transition, such as in the pure gauge theory, the peak of the analogous Polyakov susceptibility gets more and more singular as we increase the volume \((V)\). The width scales with \(1/V\) the height scales with volume. A second order transition shows a similar singular behavior with critical indices. For an analytic transition (what we call a cross-over) the peak width and height saturates to a constant value.

We carried out a finite size scaling analysis with the continuum extrapolated height of the renormalized susceptibility. The renormalization of the chiral susceptibility can be done by taking the second derivative of the free energy density \((f)\) with respect to the renormalized mass \((m_r)\). The logarithm of the partition function contains quartic divergences. These can be removed by subtracting the free energy at \(T = 0\): \(f/T^4 = -N_t^3 \log Z(N_t, N_t)/N_t/N_t^3 - \log Z(N_0, N_0)/N_0/N_0\). This quantity has a correct continuum limit. The subtraction term is obtained at \(T=0\), for which simulations are carried out on lattices with \(N_0\) spatial and temporal extensions (otherwise at the same parameters of the action). The bare light quark mass \((m_{ud})\) is related to \(m_r\) by the mass renormalization constant \(m_r = Z_{m_r} m_{ud}\). Note that \(Z_m\) falls out of the combination \(m_r^2 \partial^2/\partial m_{ud}^2 = m_{ud}^2 \partial^2/\partial m_{ud}^2\). Thus, \(m_{ud}^2 [\chi(N_s, N_t) - \chi(N_0, N_0)]\) also has a continuum limit (for its maximum values for different \(N_t\), and in the continuum limit we use the shorthand notation \(m^2 \Delta \chi\)).

In order to carry out the finite volume scaling in the continuum limit we took three different physical volumes. For these volumes we calculated the dimensionless combination \(T^4/m^2 \Delta \chi\) at 4 different lattice spacings: 0.3 fm was always off, otherwise the continuum extrapolations could be carried out. The volume dependence of the continuum extrapolated inverse susceptibilities is shown on Figure 2.

Our result is consistent with an approximately constant behaviour, despite the fact that we had a factor of 5 difference in the volume. The chance probabilities, that statistical fluctuations changed the dominant behaviour of the volume dependence are negligible. As a conclusion we can say that the staggered QCD transition at \(\mu = 0\) is a cross-over.

5. **The transition temperature**

An analytic cross-over, like the QCD transition has no unique \(T_c\), different observables and/or definitions may lead to different temperatures.

In ref. [7] we considered three quantities to locate the transition point: the chiral susceptibility, the strange quark number susceptibility and the Polyakov-loop. The obtained temperatures were
Continuum extrapolated susceptibilities $T^4/(m^2\Delta\chi)$ as a function of $1/(T^3cV)$. For true phase transitions the infinite volume extrapolation should be consistent with zero, whereas for an analytic crossover the infinite volume extrapolation gives a non-vanishing value. The continuum-extrapolated susceptibilities show no phase-transition-like volume dependence, though the volume changes by a factor of five. The $V\to\infty$ extrapolated value is $22(2)$ which is $11\sigma$ away from zero. For illustration, we fit the expected asymptotic behaviour for first-order and O(4) (second order) phase transitions shown by dotted and dashed lines, which results in chance probabilities of $10^{-19}$ ($7\times10^{-13}$), respectively. The agreement to the experimental values shows that using any of the hadron masses to set the lattice spacing would give the same results.

A possible reason for the discrepancy could be the uncertainty coming from the scale setting. Different quantities can be used to set the lattice spacing and the results should not depend on this choice. Figure 3 shows the masses of the $\Omega$ baryon, $\phi(1020)$ meson and $K^*(892)$ meson as well as the ratio of the quark masses and $f_K/f_\pi$ obtained from our $T = 0$ simulations [9]. The agreement to the experimental values shows that using any of the hadron masses to set the lattice spacing would give the same results.

Since then both collaborations improved on their data. We have used physical quark masses also for the $T = 0$ simulations. We used physical quark masses and four lattice spacings corresponding to $N_t = 4, 6, 8$ and 10. These results, especially the one coming from the chiral susceptibility, are in contradiction with those of ref. [10]. They obtained $T_c = 192(7)(4)$ MeV from both the chiral susceptibility and Polyakov loop susceptibility. They used $N_t = 4$ and 6 lattices with a p4 improved action.

The light quark chiral condensate $\langle \bar{\psi}\psi \rangle$ is minus one times the first derivative of the free energy density with respect to the light quark mass. It is ultraviolet divergent, a possible way of removing divergences was proposed in [12]. If one assumes that the additive divergences of the free energy density depend on the quark masses only through the combination $m_{ud}^2 + m_s^2$, then one can get rid of the additive divergences in $\langle \bar{\psi}\psi \rangle$ by using the strange quark condensate.
Figure 3. Left panel: masses of $\Omega$ baryon, $\phi(1020)$ meson and $K^*(892)$ meson in MeV on our four finest lattices as a function of the lattice spacing squared. Right panel: quark mass ratio and $f_K/f_\pi$ for all five ensembles. See text for a detailed explanation.

$\langle \bar{s}s \rangle$:

$$\Delta_{l,s} = \langle \bar{\psi}\psi \rangle - \frac{2m_{ud}}{m_s} \langle \bar{s}s \rangle. \quad (1)$$

The remaining multiplicative divergences can be removed by dividing with the same quantity at zero temperature:

$$\Delta_{l,s} \rightarrow \frac{\Delta_{l,s}(T)}{\Delta_{l,s}(T = 0)}. \quad (2)$$

On Figure 4(left) we plot this quantity as a function of the temperature. There is no significant lattice spacing or volume dependence for lattices of $N_t = 8, 10$ and 12 and for aspect ratios 3-4. For comparison we take the $N_t = 8$ data of the ‘hotQCD’ collaboration from [13]. We can see a huge disagreement between the curves in the transition regime. The shift between the curves of the different groups is in the order of 35 MeV.

The strange quark number susceptibility ($\chi_s$) is defined as minus one times the derivative of the free energy density with respect to the square of the strange quark chemical potential. It is conveniently normalized by $T^2$, by which it will asymptotically reach one as the temperature is increased to infinity (Stefan-Boltzmann limit).

Our results on $N_t = 8, 10$ and 12 are shown in Figure 4(right). We also have an additional point on a very fine lattice ($N_t = 16$) at a high temperature. The comparison with the results of the ‘hotQCD’ collaboration (see Reference [14]) brings us to a similar conclusion as for the chiral condensate. Around the transition point there is an approximately 20 MeV shift between the results of the two groups. For larger than $\sim 230$ MeV temperatures our finer lattices are in good agreement with the ‘hotQCD’ results.
Figure 4. Left panel: renormalized chiral condensate as a function of the temperature. For comparison the 'HotQCD' results are also shown (see text). Right panel: strange quark number susceptibility.

Figure 5. Preliminary results for the strange susceptibility on lattices upto $N_t=16$ and a comparison with the hadron resonance gas model (HRG). In our (Wuppertal-Budapest) analysis a stout improved action is used. A good agreement is found with the hadron resonance gas model using physical hadron masses. The results of the hotQCD Collaboration (asqtad and p4 actions) can be described only by using a non-physical distorted spectrum, characteristic to their choice of action and lattice spacings [15, 16].

6. Hadron resonance gas model and equation of state
In this section some preliminary results are presented. For the transition temperature determination even finer lattices are used, the temporal extension of the lattice is increased to 16. Earlier findings are confirmed. The results are compared with the predictions of the hadron resonance gas model (c.f. the transition curve for the strange quark number susceptibility on Figure 5). The physical hadron spectrum is applied for the HRG model. A very good agreement is found upto 150-170 MeV, above which the QCD transition takes place and free hadrons are not the dominant degrees of freedom any more. Note, that results of the hotQCD Collaboration can
Figure 6. Preliminary results for the pressure as a function of the temperature with our (Wuppertal-Budapest) stout improved action. As it can be seen the hotQCD collaboration (asqtad and p4 actions) has a larger characteristic temperature, and a steeper increase than the Wuppertal-Budapest result. The difference can not be explained by a 5 MeV shift to the left expected by the cut-off dependence between \( N_t = 6 \) and 8 [11]. The hotQCD result can only be described by using a hadron resonance gas model (HRG) with a non-physical distorted spectrum, characteristic to their choice of action and lattice spacings [15, 16]. The Wuppertal-Budapest result is in complete agreement with the hadron resonance gas model with physical input.

not be reproduced by a physical spectrum. Their results has a relatively large taste symmetry violation and a mass spectrum, which is distorted to larger masses. Incorporating this non-physical distorted spectrum [15, 16] into the hadron resonance gas model one can successfully reproduce the result of the hotQCD Collaboration.

Preliminary results of our Wuppertal-Budapest Collaboration for the equation of state calculation are also presented on Figure 6. Temporal extensions of 6 and 8 are used. In order to verify the correctness of the result in the continuum limit the calculations were repeated at various temperatures on \( N_t = 10 \) and 12 lattices. Since there is a large discrepancy between the absolute scales of the Wuppertal-Budapest result and that of the hotQCD Collaboration one expects a large discrepancy in the equation of state, too. Indeed, there is a remarkable difference between the two results. The message is the same as that of the previous paragraph. The Wuppertal-Budapest results can be described by the hadron resonance gas model using physical mass parameters (c.f. Figure 6). Again, the agreement is valid up to 150-170 MeV, which signals the onset of the QCD transition. In contrast to that, the hotQCD results are reproduced only by using a non-physical distorted spectrum [15, 16].

More details on these results will be presented in a forthcoming publication [17].

7. Conclusions

Recent results of the Wuppertal-Budapest group were reviewed. The order of the cosmological is found to be an analytic crossover. Due to the crossover nature of the transition different definitions may lead to different transition temperatures. The \( T_c \) values published by the different groups are given in Table 1. Our latest results are consistent with the ones from 2006. The small difference comes from the fact that the experimental value of \( f_K \) has changed slightly since 2006. The discrepancy with the 'HotQCD' results is still present and has to be resolved.
Table 1. Continuum extrapolated transition temperatures at the physical point for different observables and in different works. The first three columns give $T_c$ obtained from the chiral susceptibility using different normalizations. The other three columns give $T_c$ from the renormalized chiral condensate, renormalized Polyakov-loop and the strange quark number susceptibility.

by future work.

Preliminary results were presented for the transition temperature and for the equation of state on even finer lattices. All results are in agreement with our earlier findings. The results of the Wuppertal-Budapest collaboration can be reproduced by using a hadron resonance gas model with physical hadron spectrum, whereas the hotQCD results can only be described by using a hadron resonance gas model with a non-physical distorted spectrum, characteristic to their choice of action and lattice spacings [15, 16].

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References
[1] D. J. Schwarz, Annalen Phys. 12, 220 (2003) [arXiv:astro-ph/0303574].
[2] Z. Fodor and S. D. Katz, arXiv:0908.3341 [hep-ph].
[3] P. de Forcrand, S. Kim and O. Philipsen, PoS LAT2007, 178 (2007) [arXiv:0711.0262 [hep-lat]].
[4] G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, PoS LAT2007, 182 (2007) [arXiv:0710.0998 [hep-lat]].
[5] Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, JHEP 0601, 089 (2006) [arXiv:hep-lat/0510084].
[6] Y. Aoki, G. Endrodi, Z. Fodor, S. D. Katz and K. K. Szabo, Nature 443, 675 (2006) [arXiv:hep-lat/0611014].
[7] Y. Aoki, Z. Fodor, S. D. Katz and K. K. Szabo, Phys. Lett. B 643, 46 (2006) [arXiv:hep-lat/0609068].
[8] B. Spang, http://www.cheresources.com/iapwsif97.shtml
[9] Y. Aoki, S. Borsanyi, S. Durr, Z. Fodor, S. D. Katz, S. Krieg and K. K. Szabo, JHEP 0906, 088 (2009) [arXiv:0903.4155 [hep-lat]].
[10] M. Cheng et al., Phys. Rev. D 74, 054507 (2006) [arXiv:hep-lat/0608013].
[11] A. Bazavov et al., arXiv:0903.4379 [hep-lat].
[12] M. Cheng et al., Phys. Rev. D 77, 014511 (2008) [arXiv:0710.0354 [hep-lat]].
[13] F. Karsch [RBC Collaboration and HotQCD Collaboration], J. Phys. G 35, 140906 (2008) [arXiv:0804.4148 [hep-lat]].
[14] F. Karsch, PoS POD07, 026 (2007) [arXiv:0711.0656 [hep-lat]].
[15] P. Huovinen and P. Petreczky, Nucl. Phys. A 837, 26 (2010) [arXiv:0912.2541 [hep-ph]].
[16] P. Huovinen and P. Petreczky, arXiv:1005.0324 [hep-ph].
[17] S. Borsanyi et al., work in preparation.