Minimalistic optic flow sensors applied to indoor and outdoor visual guidance and odometry on a car-like robot

To cite this article: Stefano Maffica et al 2016 Bioinspir. Biomim. 11 066007

View the article online for updates and enhancements.

You may also like

- Experimental Verification of a Vehicle Localization based on Moving Horizon Estimation Integrating LRS and Odometry
  Kuniyuki Sakaeta, Kenichiro Nonaka and Kazuma Sekiguchi

- Fish-inspired robots: design, sensing, actuation, and autonomy—a review of research
  Aditi Raj and Atul Thakur

- An indoor positioning framework based on panoramic visual odometry for visually impaired people
  Weijian Hu, Kaiwei Wang, Hao Chen et al.
Bioinspiration & Biomimetics

PAPER

Minimalistic optic flow sensors applied to indoor and outdoor visual guidance and odometry on a car-like robot

Stefano Mafraica, Alain Servel and Franck Ruffier

1 PSA Peugeot Citroën, 78140 Vélizy-Villacoublay, France
2 Aix-Marseille Univ., CNRS, ISM, Inst. Movement Sci., Marseille, France
3 Author to whom any correspondence should be addressed.

E-mail: stefano.mafraica@mpsa.com, alain.servel@mpsa.com and franck.ruffier@univ-amu.fr

Keywords: optic flow, bio-inspired sensor, visual odometry, car-like robot, extended Kalman filter, adaptive pixels

Supplementary material for this article is available online

Abstract

Here we present a novel bio-inspired optic flow (OF) sensor and its application to visual guidance and odometry on a low-cost car-like robot called BioCarBot. The minimalistic OF sensor was robust to high-dynamic-range lighting conditions and to various visual patterns encountered thanks to its M²APIX auto-adaptive pixels and the new cross-correlation OF algorithm implemented. The low-cost car-like robot estimated its velocity and steering angle, and therefore its position and orientation, via an extended Kalman filter (EKF) using only two downward-facing OF sensors and the Ackerman steering model. Indoor and outdoor experiments were carried out in which the robot was driven in the closed-loop mode based on the velocity and steering angle estimates. The experimental results obtained show that our novel OF sensor can deliver high-frequency measurements (>300 Hz) in a wide OF range (1.5–15 rad s⁻¹) and in a 7-decade high-dynamic light level range. The OF resolution was constant and could be adjusted as required (up to 0.05 rad s⁻¹), and the OF precision obtained was relatively high (standard deviation of 0.17 rad s⁻¹ with an average OF of 4.5 rad s⁻¹, under the most demanding lighting conditions). An EKF-based algorithm gave the robot’s position and orientation with a relatively high accuracy (maximum errors outdoors at a very low light level: 0.95 m and 0.58 rad over about 32 m and 8π rad) despite the low-resolution control systems of the steering servo and the DC motor, as well as a simplified model identification and calibration. Finally, the minimalistic OF-based odometry results were compared to those obtained using measurements based on an inertial measurement unit (IMU) and a motor’s speed sensor.

1. Introduction

Optic flow (OF) has been studied by many authors during the last few decades [1–4]; various OF sensors [5–7] and algorithms [8–10] have been developed and used for robotic applications, such as autonomous navigation [11, 12], speed control [13, 14], simultaneous localization and mapping (SLAM) [15, 16], and visual odometry (VO) [17–20].

Although standard cameras have been widely used in this context, they have often proved to be unsuitable for OF computation purposes, especially outdoors, because of their low dynamic range, their low frame rate and the high computational cost of the image processing. In addition, in camera-based VO and SLAM applications, small motion is usually assumed to occur between frames, which is often not the case, and a ‘scale factor’ is also required to convert the visual information (e.g. the OF) into metric measurements, which is often provided by other sensors [20, 21]. These issues can be overcome by using high-dynamic-range, high-frame-rate cameras and stereo-vision systems, but these solutions require even larger computational resources and are too expensive for most robotic and automotive applications.

Ethological findings on insects have shown that complex navigation tasks, such as terrain following [22, 23] (see [24] for a review), speed control [25] and odometry [26] are performed by many flying insects, such as flies and bees, on the basis of OF cues, although...
their compound eyes have a very poor spatial resolution in comparison with our modern high-resolution cameras. In addition, it has been behaviorally shown that nocturnal insects also use these OF cues for flight control in dim light conditions [27].

Custom-made bio-inspired visual sensors have therefore been developed using technologies of various kinds [28–31] in order to deal with the issues encountered with standard cameras. However, designing effective low-cost OF sensors is a real challenge, and additional information given by other sensors or based on assumptions about the environment is still required to convert the OF to distance and/or velocity values. In addition, OF alone is generally thought to be unsuitable for VO purposes when working in complex environments where the sweeping angular speed between the sensor and the objects that induce the OF is subjected to sharp, large variations, especially in the presence of occluded objects [20, 21].

Visual odometry for mobile robots and automobiles has been recently developed using downward-facing standard cameras [32–36] and optical mice [37–39], as well as custom-made OF sensors [40, 41], since the visual patterns and light conditions encountered in this way are relatively uniform, and the distance between the sensors and the ground usually changes slightly. In most cases, the authors used the car-like non-holonomic constraint to estimate the vehicle’s position and orientation using simple methods involving low computational costs.

However, solutions based on standard cameras [33, 36] still fail to cope with high-dynamic-range lighting conditions, as well as being impeded by the low frame rate and the high computational cost of the image processing: only a narrow range of low velocity measurements can often be obtained using this approach.

Solutions based on optical mice [37, 39] are certainly very cheap and deliver high-frequency measurements, but their main disadvantage is that they have to operate very near the ground to be able to work properly, and are therefore unsuitable for use in environments with an uneven terrain. In addition, these sensors are usually highly sensitive to the lighting conditions, and like standard cameras, deliver measurements in a rather narrow velocity range.

Solutions based on custom laser or LED-lighted OF sensors [40, 41], have been developed in order to reduce the sensitivity to height and improve the performance while traveling over terrains of various kinds, but no tests were run by these authors under various lighting conditions and under the robot’s normal driving conditions.

Physiological studies on vertebrates have shown that the retina obeys a process of adaptation whereby each photoreceptor’s response is normalized by a representative value of the average local luminosity, in line with the Michaelis–Menten equation [42, 43]. In light of these and subsequent findings, many efforts have been made to mimic the outer plexiform layer circuitry in silicon retinas [44, 45], or to implement the model in software for image processing [46, 47]. More recently, we presented a novel auto-adaptive pixel called M²APix, which stands for Michaelis–Menten auto-adaptive pixel, that auto-adapts in a 7-decade range and responds appropriately to both small and large contrasts [48].

Several versions of local motion sensors (LMSs) have been developed at our laboratory [5, 13, 49, 50]: local 1D OF measurements have been obtained using very few pixels and applying methods based on findings previously obtained on the fly’s visual system [51].

In this paper, we present:

- a novel bio-inspired OF sensor giving measurements which are robust to high-dynamic-range lighting conditions and to the various visual patterns encountered;
- an application to visual guidance and odometry, in which these minimalist OF sensors were mounted on a low-cost car-like robot called BioCarBot (see figure 1) and tested both indoors and outdoors in a 7-decade light-level range.

An extended Kalman filter (EKF) was used to estimate the robot’s velocity and steering angle using only the OF measurements delivered by two downward-facing sensors, as presented in [52] in the case of a single indoor environment. In the present paper, the tests were extended to various indoor (see online supplementary video 1) and outdoor (see online supplementary video 2) testing conditions, including various light levels, ground textures, trajectories and vibrations, and the robot’s absolute position and orientation were estimated in real time using the EKF estimates and the Ackerman steering model. These position and orientation estimates were finally compared with those obtained by applying the same model to the measurements from an inertial measurement unit (IMU) and a motor’s speed sensor.

The results presented here show that our novel OF sensors were robust to changes in light in a 7-decade range (from about $10^{-10}$ A to $10^{-3}$ A of the photodiode current), including sharp changes of up to 2 decades occurring within 0.5 s. Although low-cost, low-resolution servos and motors and a simplified model identification and calibration method were used, the robot was able to estimate its velocity and steering angle accurately. The robot’s position and orientation were estimated both indoors and outdoors while it was traveling through unstructured environments (on ground consisting of asphalt, gravel, sand, and leaves, including shadows and holes), both during the day and at night. In addition, the visual odometry method was robust to vibrations liable to change the sensors’ local height by up to 6% (i.e. ±10 mm over 175 mm).

In section 2, we will introduce the principles underlying a 2-pixel LMS as well as the method and the hardware used to construct our novel OF sensor.
In section 3, we will present the BioCarBot robot, the simplified model used to estimate the robot’s velocity and steering angle, and the control scheme implemented on the robot. In section 4, we will present and discuss the results obtained in the indoor and outdoor experiments we performed. Some conclusions will be reached in the last section.

2. New implementation of the visual motion sensor

2.1. Principles underlying a 2-pixel local motion sensor

A defocused lens placed in front of two photoreceptors determines the interreceptor angle \( \Delta \varphi \) between the two photoreceptors’ output signals. In the Reichardt–Hassenstein model [53], the interreceptor angle \( \Delta \varphi \) and the time lag \( \tau \) are used to compute the optic flow \( \omega \):

\[
\omega(t) = \frac{\Delta \varphi}{\tau(t)}
\]

In figure 1, we show the different environments where the experiments were performed. In figure 2, we present the block diagram of the novel OF algorithm that determines the time lag \( \tau_k \) between several \( \tau_i \) in a time window \( w \), giving the maximum cross-correlation between the delayed and non-delayed signals. The OF sensor’s output does not result directly from the correlation, as in the Reichardt–Hassenstein model [53], but from the ratio between the interreceptor angle \( \Delta \varphi \) and the time lag \( \tau_k \), i.e., \( \omega(t) = \frac{\Delta \varphi}{\tau_k} \).
two photoreceptors’ axes and gives them a Gaussian angular sensitivity with an acceptance angle $\Delta \rho$ (figure 2(a)), on similar lines to what occurs in many insects’ eyes. A visual contrast moving in front of the LMS will induce a time lag $\tau$ between the photoreceptors’ output signals (figure 2(b)). After measuring this time lag, the optic flow can be computed as follows:

$$\omega(t) = \pm \frac{\Delta \varphi}{\tau(t)}$$  \hspace{1cm} (1)

where the sign depends on the orientation of the sensor’s axis and on which of the two output signals is delayed.

The acceptance angle $\Delta \rho$, namely the full width at half-height of the Gaussian angular sensitivity, determines the cut-off frequency of the low-pass spatial filter as follows: $f_c \approx \frac{1}{2 \Delta \rho}$ [54]. On the one hand, achieving a tight $\Delta \rho$ makes it possible for the photoreceptors to respond to higher spatial-frequency contrasts, but on the other hand, when $\Delta \rho < \Delta \varphi$, it is difficult to estimate the time lag $\tau$ because the two output signals are rarely correlated in a short time window. Therefore, as occurs in some diurnal insects [55], we adjusted the distance between the plane of the lens and that of the photoreceptors in order to obtain $\Delta \rho = \Delta \varphi$.

2.2. New cross-correlation method for computing the OF

To compute the 1D OF $\omega$, the time lag $\tau$ between two neighboring pixels’ output signals was estimated using a cross-correlation method inspired by the Reichardt–Hassenstein correlator model [53]. In the method presented here, the OF sensor’s output did not result directly from the correlation, as in the Reichardt–Hassenstein model, but from the ratio between the interreceptor angle $\Delta \varphi$ and the time lag $\tau$ giving the maximum cross-correlation between the delayed and non-delayed signals, i.e. $\omega(t) = \frac{\Delta \varphi}{\tau_{\text{corr}}}$, on similar lines to the method proposed in [56].

First, the two pixels’ output signals ($V_{\text{Ph}_1}$, $V_{\text{Ph}_2}$) were sampled and band-pass filtered ($V'_{\text{Ph}_1}$, $V'_{\text{Ph}_2}$) at $f_l = 3 \text{ Hz}$ and $f_h = 30 \text{ Hz}$, and the following pseudo-algorithm was then applied (figures 2(c)–(d)):

(i) delay one of the two filtered signals (e.g. $V'_{\text{Ph}_1}(t)$) by the time $\tau_i$,

(ii) compute the Pearson correlation coefficients between the delayed (e.g. $V'_{\text{Ph}_1}(t - \tau_i)$) and non-delayed signals (e.g. $V'_{\text{Ph}_2}(t)$) in a fixed time window $w_{\text{corrPh}}$;

(iii) repeat steps (i) and (ii) for every $\tau_i$ ($i = 1, \ldots, n$) in a fixed time window $w_i$;

(iv) set $\tau$ at a value equal to the time lag $\tau_k$ giving the maximum cross-correlation coefficient, as long as this maximum is greater than the fixed value $\rho_{\text{thr}}$;

(v) compute the OF $\omega$ using equation (1) (or set $\omega$ at NaN if the maximum cross-correlation coefficient is less than $\rho_{\text{thr}}$).

The threshold value of the cross-correlation coefficients ($\rho_{\text{thr}}$) directly reflects the reliability and the robustness of the OF measurements: the higher $\rho_{\text{thr}}$ is, the more reliable and robust the measurements will be, but in the presence of noise, the lower the refresh rate will be.

The time window of the signals ($w_{\text{corrPh}}$) determines the bandwidth of the OF measurements, whereas the number of samples ($n$) in the time window determines the reliability of the correlation coefficients: the smaller $w_{\text{corrPh}}$ is, the larger the bandwidth will be, and the higher $n$ is, the higher the reliability will be.

In order to obtain a constant resolution $\Delta \omega$, the time lags were chosen as follows:

$$\tau_i = \frac{\Delta \varphi}{\left| \omega_i \right|}.$$  \hspace{1cm} (2)
where \( \omega^k \) are the desired OF measurements, which are linearly separated by the resolution \( \Delta \omega^k \) required. The signals can then be delayed by the time \( \tau \) elapsing between two sampling steps, by linearly interpolating the signals sampled.

### 2.3. Hardware and software implementation

In this study, we used the auto-adaptive silicon retina presented in [48] soldered onto a tiny printed circuit board (PCB) on which an optical lens casing was mounted (figure 3).

The M²APix pixel, which stands for Michaelis-Menten auto-adaptive pixel, can auto-adapt in a 7-decade range and responds appropriately to small contrasts, such as \( \pm 2\% \), as well as large changes in light, such as \( \pm 3\) decades [48]. In the chip used here, the analog low-pass filter had a cut-off frequency of 100 Hz (instead of 300 Hz used in [48]), giving a minimum sampling frequency of 200 Hz in order to prevent the occurrence of aliasing.

The optical lens used here was taken from a Raspberry-Pi camera (focal length: 2 mm), while the lens casing was custom made using a 3D printer to precisely adjust the distance between the plane of the lens and that of the pixels during the calibration phase. The interreceptor angle \( \Delta \varphi \) and the acceptance angle \( \Delta \rho \) were measured at \( \Delta \varphi \approx \Delta \rho \approx 3.6^\circ \) using the method presented in [5], giving a cut-off frequency of the low-pass spatial filter \( F_{\rho} \approx \frac{1}{\Delta \rho} \approx 0.2826^\circ \text{s}^{-1} \) [54].

The OF algorithm presented in section 2.2 was implemented by setting:

- the threshold on the cross-correlation coefficients \( \varphi_{\text{thr}} = 0.99 \);
- the number of pixel signal samples \( n = 70 \), giving a signal time window for the cross-correlation computation \( n_{\text{corrPh}} = 0.21 \text{s} \);
- the number of time lags \( m = 30 \), because a larger number would cause saturation of the CPU load since the implementation of the algorithm was not optimized.

Depending on the velocity range required, the time lag window \( \tau_c \) ranged from 6.3 ms–63 ms in order to obtain OF measurements \( \omega^i \) ranging from 1–15 rad s\(^{-1}\) and a resolution \( \Delta \omega^i \) ranging from 0.05–0.5 rad s\(^{-1}\). In the experiments presented here, the OF range was set prior to each test at the smallest range comprising all the reachable OF values, given the robot’s velocity and steering angle commands and the sensors’ height, in order to obtain the highest OF resolution, given the computational constraint on the total number of possible time lags at each time step (\( m = 30 \)).

We note that having a very high threshold on the cross-correlation coefficients (\( \varphi_{\text{thr}} = 0.99 \)) does not imply that the two neighboring signals have to be identical but rather that they have to be linearly dependent, i.e. there might be a gain and an offset between them. Thus, the output of our OF algorithm, i.e. the OF measurements, nearly does not depend on the color, intensity and spatial frequency of visual patterns encountered, i.e. on the amplitude and shape of the pixels’ temporal signals produced, as long as these signals fall in the bandwidth of the band-pass temporal filter (see section 2.2). In other words, we can say that the pattern-based noise is very low as long as the visual patterns are not completely uniform and have spatial frequency components lower than \( F_c \).

The OF algorithm was then applied to every pair of adjacent pixels in each of the two 6-pixel rows (see figure 2(b) in [48]), giving ten local 1D OF measurements within a field of view of about 18°. The visual motion sensor (VMS) was therefore composed of ten 2-pixel LMSs and the median value \( \omega^i \) of the ten OF measurements \( \omega^i \) was used as the actual output of the VMS to robustly filter out possible outliers.

### 3. BioCarBot: a bio-inspired visually-guided car-like robot

#### 3.1. The car-like robot

Figure 1 shows the images of the BioCarBot robot, one of the two VMSs used and the indoor and outdoor testing environments equipped with a Vicon motion-capture system.

The present odometry method was tested using a car-like robot based on the 2WD Racecar Kit provided by Minds-I Robotics, which was chosen despite the low resolution of the servo and motor control because of its modularity and low price (US$275). The car-like robot was composed of a 1/10-scale car body (419 × 203 × 114 mm), one Hitec HS-311 standard servo coupled to a steering hub, one 5000-rpm DC motor connected to a 300-A electronic speed controller (ESC), one 7.2-Volt 3000-mAh Ni-Cd rechargeable battery, one mechanical slip differential and four 90 mm-diameter crawler wheels.

The embedded electronics included one Nanowii board (Flyduino) featuring an ATmega32u4 16-MHz CPU microcontroller (Atmel) and a MPU-6050 IMU comprising a 3-axis gyroscope and a 3-axis accelerometer (InvenSense) and one Overo IronSTORM computer-on-module (COM) (Gumstix) featuring a 1-GHz CPU DM3730 processor (Texas Instruments) comprising an ARM Cortex-A8 architecture and a C64x digital signal processor.

Thanks to the modularity of the robot’s structure, two identical VMSs (figure 3(b)) were attached to the robot’s frame on both sides of its body, aligned with the rear wheel axis (figure 1(a)). To facilitate the sensors’ installation, we used the same testing board as that which was used in the study presented in [48] to
connect the VMSs to the Nanowii board (figure 1(b)). An OSRAM BPX65 photodiode connected to an analog amplifier circuit was also included on the testing board next to the VMS in order to measure the effective light levels of the scene.

To obtain the ground-truth values, the 3D robot’s pose ([\(X \ Y \ Z \ \alpha \ \beta \ \gamma\)]^T) was measured by means of a Vicon motion-capture system thanks to the infrared markers attached to the robot’s frame (figure 1(a)). Indoor experiments were performed in the flying arena at our laboratory [57], whereas four individual Vicon cameras, each mounted on a tripod, were used outdoors (figures 1(c)–(d)).

### 3.2. Car-like robot modeling

As the robot’s velocity was relatively low and the robot did not have any suspension system, we focused here on the 2D kinematic model for a car-like robot moving on a flat surface. Figure 4 shows the kinematic diagram of the BioCarBot with the two VMSs installed on both sides, as depicted in figure 1(a).

Let us take the inertial frame \(I\) having the \(x\)- and \(y\)-axes lying on the local ground plane, with the robot’s body frame \(B\) placed in the middle of the rear wheels’ axis. Two VMSs were placed at \(x_i = [x_i \ y_i \ z_i]^T\) and \(x_r = [x_r \ y_r \ z_r]^T\) with respect to \(B\) \((x_i = x_r = 0 \ \text{mm}, \ y_i = -y_r = 140 \ \text{mm}, \ z_i = z_r = 125 \ \text{mm})\), respectively, facing downwards at a height of \(h_i, h_r\), respectively, from the ground \((h_i = h_r = h = 175 \ \text{mm})\) (figure 1(a), 4(a)). As the sensors’ frames \((l), (r)\) were taken to be parallel to the body frame \((B)\), we can consider all the position and velocity vectors projected onto \((B)\).

The ground OF can be measured using the method presented in section 2.2 thanks to the no-skidding assumption, which guarantees that \(V_z \gg V_r\) with any velocity vector \(V = [V_x, V_y, 0]^T\) located near the line passing through the two rear wheel/ground contact points. Therefore, the OF measured between the \(i - 1\)-th and \(i\)-th pixels of each VMS can be written as follows:

\[
\omega_i = -\frac{V_z \sin^2 \phi_i}{h},
\]

where \(V_i = [V_x, V_y, 0]^T\) is the velocity vector of the vector \(x_i = [x_i, y_i, z_i]^T (z_i \approx -h)\) giving the position of the intersection point \(P_i\) between the \(i\)-th pixel axis and the ground plane with respect to the sensor’s frame, i.e. \((l)\) or \((r)\), and \(\phi_i\) is the angle between \(x_i\) and the \(x\)-axis of the sensor’s frame (figure 4(b)).

Since the sensors’ frames \((l), (r)\) were taken to have their \(x\)-axis aligned with the rows of pixels of each VMS (i.e. \(y_i \approx 0\)) and to be parallel to \(B\) (figures 4(a), (b)), the position vectors of the intersection points \(P_i\) with respect to \(B\) can be written as \(x_i^l = x_l + x_i = [x_i, y_i, z_i - h]^T\) and \(x_i^r = x_r + x_i = [x_r, y_r, z_i - h]^T\), for the left and right sides, respectively, and their corresponding velocities are given as follows:

\[
\begin{aligned}
V_i^l &= V_l = V - y_i \Omega \\
V_i^r &= V_r = V - y_r \Omega
\end{aligned}
\]

where \(V, \Omega\) are the robot’s longitudinal and angular velocity, respectively (figure 4(a)).

By combining equations (3) and (4), we obtain a set of redundant linear equations that relate the local OF measurements \(\omega_i\) to the robot’s longitudinal and angular velocity \(V, \Omega\), which can be used for odometry purposes with any wheeled robot that satisfies the no-skidding assumption, regardless of how the

![Figure 4. (a) Kinematic diagram of the robot moving on a flat surface. (b) Kinematic diagram of one OF sensor.](image-url)
robot is actuated. However, if we are interested in controlling the robot in a closed loop, the dynamic equations relating $V$, $\Omega$ to the specific control parameters should be included into equation (4) for better tracking performances.

Since we are dealing here with a car-like robot which is controlled by a DC motor, giving the longitudinal velocity $V$, and a steering servo, giving the steering angle $\phi$, the angular velocity $\Omega$ can be computed as follows, according to the Ackermann steering geometry \cite{2016bomafrica}: (figure 4(a)):

$$\Omega = \tan \theta \frac{V}{L}, \quad (5)$$

where $L (= 255 \text{ mm})$ is the distance between the rear and front wheel axes (figure 1(a)).

By substituting the median value of the OF measurements obtained with each VMS $\omega_{m1}^l$, $\omega_{m1}^r$ into (3) and combining equations (3), (4) and (5), the following equation relating the output $\zeta = [\omega_{m1}^l \omega_{m1}^r]^T$ to the state $\xi = [V \phi]^T$ is obtained:

$$\zeta = \begin{bmatrix} \omega_{m1}^l \\ \omega_{m1}^r \end{bmatrix} \approx \begin{bmatrix} \frac{(L - y_{\text{in}} \tan \phi \sin \phi_{x1})}{h_L} \\ \frac{(L - y_{\text{in}} \tan \phi \sin \phi_{x1})}{h_L} \end{bmatrix} \hat{V} = h(\xi), \quad (6)$$

where $\phi_{x1}$ and $\phi_{x2}$ are the orientation of the pixel’s axis corresponding to the median OF values $\omega_{m1}^l$ and $\omega_{m1}^r$, respectively.

Lastly, the dynamics of $V$ and $\phi$, which mostly depend on the dynamics of the DC motor and the steering servo, respectively, were identified in the form of two independent first-order systems using the ground-truth measurements:

$$\dot{\xi} \approx A\xi + Bu = f (\xi, u). \quad (7)$$

where $A = \text{diag}(a_1, a_2)$, $B = \text{diag}(b_1, b_2)$. The values of $A$ and $B$ were identified using the slower time constants as follows: $a_1 = -b_1 = -2.15$, $a_2 = -b_2 = -4.87$.

It is worth noting that the identified model given in equation (7) does not take into account the non-linearities which characterize the conversion from the actual control inputs, i.e. the values delivered to the ESC and the servo controller, to the model input $u$, such as, for instance, the dependency of the DC motor’s speed to the battery charge level and the backlash in the steering system’s geometry. Such a simplified model, therefore, would not be accurate enough to be used directly for odometry purposes; however we want to show here that it is sufficient to obtain good odometry results when applied to an EKF using the measurements delivered by our novel OF sensors.

3.3. EKF and control system

In order to obtain a robust continuous estimation of the robot’s longitudinal velocity and steering angle $(V, \phi)$, an EKF based on the discrete approximation of the model presented in (6) and (7) was implemented, taking the median values of the 10 local OF measurements $(\omega_{m1}^l, \omega_{m1}^r)$ to be actual measurements.

The first-order discrete approximation for the model presented in (7) was taken to be as follows:

$$\begin{align*}
\xi_k &= \hat{f} (\xi_{k-1}, u_{k-1}, w_{k-1}) \\
&= [f (\xi_{k-1}, u_{k-1}) + w_{k-1}] + \Delta t \cdot \xi_{k-1}, \\
\zeta_k &= \hat{h} (\xi_k, u_k) = h(\xi_k) + \nu_k
\end{align*} \quad (8)$$

where the index $k$ denotes the $k$-th sampling period (i.e., $t = k \Delta t$); $w, \nu$ denotes the model and the measurement noise, respectively, and they are assumed to be independent white noises and to have normal probability distributions, i.e., $p(w) \sim N(0, Q)$ and $p(\nu) \sim N(0, R)$, where $Q = \text{diag}(\sigma_w^2, \sigma_v^2)$ and $R = \text{diag}(\sigma_v^2, \sigma_v^2)$ are covariance matrices. The assumption that $Q$ and $R$ had normal and uncorrelated distributions was adopted on the basis of what was observed statistically during several experimental tests with trajectories of various kinds (see section 4). The elements of $R$ and $Q$ were set at about $0.01^2$ and $0.02^2$, respectively, for the indoor tests and about $0.1\Delta \omega^2$ and $0.02^2$, respectively, for the outdoor tests.

Because of the backlash and other uncertainties, we did not have a very clear picture of the motor and
steering control inputs $u$ to the system. The real system inputs were therefore assumed to be equal to the outputs from the controller, i.e. $u_{k-1} = \dot{x}_{k-1}$ (see equation (9)).

The initial estimate of the state $\hat{x}_0$ was set to zero, while the initial estimate of the error covariance matrix $P_0$ was set at the identity matrix. When there were no measurements available on at least one of the robot’s sides, i.e. no $\omega_m$ or $\omega_m$, the Kalman gain $K_0$ was set at zero, so that it was still possible to have an estimation of $\dot{x}_k$ based on the ‘a priori’ prediction $\hat{x}_k$.

In that case, a timeout was set at 0.5 s, after which the EKF was reinitialized and the robot was stopped. Such an event happened only in the few cases where the robot drove for a while on a non-textured area of the floor (e.g. the white or black areas in figure 1(b)), which never happened in the tests presented here.

The robot’s longitudinal velocity and steering angle ($\xi = [V, \phi]^T$) were controlled in the closed-loop mode using their estimates ($\hat{\xi} = [\hat{V}, \hat{\phi}]^T$) and the values required ($\xi^e = [V^e, \phi^e]^T$) via a proportional and integral (PI) controller (figure 5):

$$\bar{\xi} = K_P (\xi^e - \hat{\xi}) + K_I \int (\xi^e - \hat{\xi}) \text{,}$$

(9)

where $K_P = \text{diag}(0.5, 0.4)$ and $K_I = \text{diag}(2.5, 2)$.

We note that the EKF and the PI controller were applied on the state $\xi = [V, \phi]^T$ instead of $\xi = [V, \Omega]^T$ in order to obtain a more robust estimate of $\phi$, and therefore better closed-loop trajectories. Also, a feedforward term could be added to the PI controller to increase the tracking performances/accuracy, although it was not included here since no significant improvements were obtained.

4. Experimental measurements and visual odometry results

Experiments were carried out both indoors (see online supplementary video 1) and outdoors (see online supplementary video 2) using various floor patterns and trajectories to test the performances of the OF sensors as well as the method of estimation and control presented here.

Figure 1 shows two examples of test environments, indoors (figure 1(c)) and outdoors (figure 1(d)) at the flying arena at our laboratory. In the tests presented here, the car-like robot followed paths forming a circle, a square and a figure of eight. The floors used in the indoor tests showed patterns of various colors and contrasts, from small and large high-frequency contrasts to large low-frequency contrasts. The ground used in the outdoor tests, which consisted mainly of asphalt, included holes, gravel and a steel rail.

The M$^2$APix output signals were sampled at a frequency of 333 Hz ($\Delta t = 3$ ms) by the on-chip ADC (see [48] for details), acquired by the Nanowii board via SPI communication and transmitted to the computer-on-module (COM) via serial communication (see section 3 for details). The OF algorithm presented in section 2.2, as well as the estimation and control scheme shown in figure 5, were run at the same rate on the COM. The Linux-based program running on the COM was entirely generated in the Matlab/Simulink environment using the RT-MaG toolbox [57], a custom-made toolbox for real-time applications developed at our laboratory. The ground host-PC program conveys the control set points $V^*, \phi^*$ to the robot’s COM and receives data from the robot’s COM and the Vicon system using the RT-MaG toolbox. Estimates of the absolute robot’s position $\hat{X}$ and orientation $\hat{\theta}$ were lastly computed in real time by integrating the equations provided in the Ackermann model [58] using the EKF estimates $\hat{V}$, $\hat{\phi}$ as inputs to the model.

To compare the minimalistic odometry results obtained when using the OF measurements with those obtained when using inertial measurements, the estimates of the absolute robot’s position and orientation $\hat{\xi}_\text{IMU}$, $\hat{\theta}_\text{IMU}$ were also computed offline using the measurements acquired from the MPU-6050 IMU ($a_{\text{IMU}}$, $\Omega_{\text{IMU}}$) and the measurements of the DC motor’s speed $\Omega_{\text{DC}}$ acquired by a hall-effect sensor attached to the motor’s shaft. Several methods have been proposed through the years to perform odometry using inertial measurements based on specific dynamic models of the IMU used, in particular to handle the varying bias of the gyroscopes [59, 60], but these methods require an accurate identification and calibration of the models used. Here, in order to have a meaningful comparison with our minimalistic OF-based method, estimates of the robot’s velocity and steering angle $\hat{V}_\text{IMU}$, $\hat{\phi}_\text{IMU}$ were obtained by applying an EKF to the same dynamical model, e.g. equation (7), while using the following output model:

$$\zeta_{\text{IMU}} = \begin{bmatrix} a_v & \Omega_{\text{DC}} \\ \Omega_{\text{DC}} & \Omega \end{bmatrix} \begin{bmatrix} a_v V + b_1 u_1 + g \sin \beta_{\text{IMU}} \\ \frac{\tan \beta_{\text{IMU}}}{L} V - \frac{b_2}{k_s} V \end{bmatrix} = h_{\text{IMU}}(\zeta),$$

(10)

where $g (= 9.81 \text{ m/s}^2)$ is the gravity acceleration, $r (= 14 \text{ mm})$ is the radius of the robot’s wheels, and $k_s (= 3.4)$ is the transmission gear ratio. The angle $\beta_{\text{IMU}}$ gives the rotation about the $y$-axis between the body frame ($B$) and the inertial frame ($I$) and was computed by integrating the following equations:

\[ \text{Note that in the first equation in (10) there is no contribution of } \Omega_{\text{IMU}} \text{ because the body frame } (B) \text{ was taken to be coincident to the IMU frame.} \]

\[ \text{Note that we substituted } \gamma_{\text{IMU}} \text{ with } \Omega \text{ in the third equation in (10) because in the experiments presented here we always had } \Omega_{\text{DC}} = \gamma_{\text{IMU}}. \text{ Also note that the angle } \beta_{\text{IMU}} \text{ was considered as a parameter although it depends on } \Omega, \text{ and therefore on the state } \xi, \text{ since its value varied within a small range and no significant improvement was obtained using a more complex output model.} \]
\[ \Theta_{\text{IMU}} = \begin{bmatrix} \dot{\alpha}_{\text{IMU}} \\ \dot{\beta}_{\text{IMU}} \\ \dot{\gamma}_{\text{IMU}} \end{bmatrix} = \begin{bmatrix} 1 & \sin \alpha_{\text{IMU}} \tan \beta_{\text{IMU}} & \cos \alpha_{\text{IMU}} \tan \beta_{\text{IMU}} \\ 0 & \cos \alpha_{\text{IMU}} & -\sin \alpha_{\text{IMU}} \\ 0 & \sin \alpha_{\text{IMU}} / \cos \beta_{\text{IMU}} & \cos \alpha_{\text{IMU}} / \cos \beta_{\text{IMU}} \end{bmatrix} \Omega_{\text{IMU}} \]

The initial condition of the integrator on \( \Theta \) was taken as \( \Theta_0 = [\tan(\hat{\alpha}_{0z}, \hat{\beta}_{0x}) \hat{\gamma}(\pi/2)]^T \), where \( \hat{\alpha}_{0z}, \hat{\beta}_{0x}, \hat{\gamma}_{0z} \) are the mean values of the accelerometer’s measurements along the three axes delivered within 1 s before starting each test while the robot was not moving. The covariance matrix of the model uncertainties was the same in both the OF-based and IMU-based EKF, i.e. \( Q_{\text{IMU}} = Q \), whereas the covariance matrix of the measurement noise was identified as \( R_{\text{IMU}} = \text{diag}(0.263^2, 0.047^2, 0.001^2) \).

The ground-truth values were computed from the robot’s pose measurements after being low-pass filtered at \( f_c = 100 \text{ Hz} \) \([X' Y' Z' \alpha' \beta' \gamma']^T\) as follows (see section 3 for details):

\[
\cdot V_{\text{truth}} = \sqrt{X'^2 + Y'^2}; \\
\cdot \phi_{\text{truth}} = \arctan (l_{\text{phan}}/V_{\text{phan}}); \\
\cdot h^I_{\text{truth}} = h_0 + Z'' + y'_1 \sin(\alpha'') \cos(\beta'') + z_2 \cos(\alpha'') \cos(\beta'') \text{(same for } h^I_{\text{truth} \text{ using } y_1, z_2}); \\
\cdot \omega^I_{\text{truth}} = \frac{(y'_1 - V_{\text{phan}}) \sin(\phi')}{h^I_{\text{phan}}} \text{ (same for } \omega^I_{\text{truth} \text{ using } y_1, h^I_{\text{phan}}});
\]

where \( h_0 \) is the height of the body frame \( (B) \) with respect to the local ground plane, i.e. \( h_0 = h - z_2 = 50 \text{ mm} \), and \( Z'', \alpha'', \beta'' \) are the high-pass filtered values of \( Z', \alpha', \beta' \) \((f_c = 1 \text{ Hz})\), which were adopted in order to cut off the low-frequency components due to the changes in the ground’s slope and height.

The average refresh rate of the sensors \( f_r \) was computed by dividing the number of median values of the OF measurements obtained during each test by the time taken to run the test. The precision of the median values of the OF measurements \( \omega^I_{\text{mn}}, \omega^I_{\text{m}} \) as well as that of the estimates \( \hat{V}, \hat{\phi} \) was computed by dividing the standard deviations of the errors with respect to their ground-truth values by their average absolute value (e.g. if \( |\hat{V}| = 0.3 \text{ m/s} \) and \( \sigma_V = 0.007 \text{ m/s} \), then the precision will be \( 0.007 / 0.3 \approx 2.3\% \)). The accuracy of the estimates of the robot’s absolute position \( \hat{X} \) and orientation \( \hat{\theta} \), instead, was computed by dividing the maximum errors with respect to the ground-truth position and orientation respectively by the distance traveled and the angle covered when these maxima were reached (e.g. if \( ||X - \hat{X}|| = 0.7 \text{ m} \) after traveling 45 m then the accuracy will be \( 0.7 / 45 \approx 1.6\% \)). We preferred to use these traditional metrics for the odometry accuracy instead of using other common metrics in visual odometry based on the errors in the relative camera poses, such as that presented in [61], because such metrics give values that are well representative of the ‘noise’ in the position and orientation estimates (error between contiguous estimates), but not necessarily of the accuracy of these estimates (absolute error). To better evaluate the odometry performances during the entire path, we also provide here the plots of the error in the position and orientation with respect to time and distance.

4.1. indoors

Figure 6 shows the results obtained indoors when the robot was driven on the floor shown in figure 1(c) on a circular path, keeping a constant steering angle at a velocity ranging from 0.3–1.3 m s\(^{-1}\), under dynamically changing lighting conditions. The OF resolution \( \Delta \omega^I \) required was set at 0.3 rad s\(^{-1}\) and the average refresh rate \( f_r \) obtained was about 327 Hz.

First the incoming sunlight was made to vary by up to two decades by slowly closing the eight blinds (from 0–24 s), giving a maximum luminosity of about 5000 Lux \( (I_{\text{ph}} \approx 5 \times 10^{-5} \text{ A}) \), then the artificial lighting was made to vary both slowly and rapidly by about two decades, by varying the neon ceiling lights and then switching them off (from 24–46 s). The robot was then driven with only the Vicon cameras’ LED lights switched on (from 46–59 s), corresponding to a luminosity of about 0.3 Lux (average \( I_{\text{ph}} \) of about \( 3 \times 10^{-10} \text{ A} \)), before switching the neon lights on again. The light levels tested therefore covered a nearly 6-decade range \( (I_{\text{ph}} \text{ from about } 10^{-10} \text{ to } 5 \times 10^{-5} \text{ A}) \), as can be seen from figure 6(f).

Indoor tests at constant light levels were also carried out in the same environment and using the same trajectory to determine the highest precision and widest range achievable with our novel OF sensors. First the robot was driven at constant velocities from the minimum to the maximum values, i.e. from 0.3–1.5 m s\(^{-1}\), setting the OF resolution \( \Delta \omega^I \) required at 0.05 rad s\(^{-1}\); figure 7(A) shows the case where the robot was driven at its maximum velocity. The minimum OF resolution \( \Delta \omega^I \) used was set at 0.05 rad s\(^{-1}\) in order to obtain a measurable OF range of 1.5 rad s\(^{-1}\) without causing saturation of the CPU load. The velocity was then varied linearly from the minimum to the maximum values with accelerations varying from \( \pm 0.3 \pm 1 \text{ m/s}^2 \) (figure 7(B)). In the latter case, the height of the VMSs with respect to the ground \( h \) was exceptionally decreased to 135 mm and the OF resolution \( \Delta \omega^I \) required was increased to 0.5 rad s\(^{-1}\) in order to achieve a larger OF range (from 1.5–15 rad s\(^{-1}\)). The average refresh rate \( f_r \) obtained was about 333 Hz in all the cases tested.
Lastly, indoor tests were carried out using various trajectories and various floor textures. Figure 8 shows the results obtained indoors when the robot was driven on paths forming a figure of eight and a square, while the velocity and the steering angle were made to vary trapezoidally. The OF resolution $\Delta \omega^*$ required was set at 0.1 and 0.2 rad s$^{-1}$ and the average refresh rate $\bar{f}$ obtained was about 325 Hz in both tests.

The results presented here show that the OF measurements did not depend on either the average light level or the changes in the light, except in the case of very large, sudden changes (see figure 6(a) at about 46 s and 59 s), nor did they depend on the visual pattern and the types of trajectory used (there was no statistical evidence in the error distributions and in the refresh frequencies). In addition, although the error in the OF measurements increased slightly with the OF magnitude, our novel sensors were able to measure the OF in a very large range, from about 1.5–15 rad s$^{-1}$ (i.e. from about 85–850 $\circ$ s$^{-1}$), and with relatively high OF resolution (0.05 rad s$^{-1}$). However, both a high resolution, and therefore precision, and a wide range could not yet be provided at the same time due to computational limitations.

In all these tests, the overall error distributions were nearly Gaussian with a quasi-zero mean (figures 6(b), 7(A)(b), (B)(b), 8(A)(b), (B)(b)), giving values of the adjusted R-square goodness-of-fit statistics, computed using the Matlab Curve Fitting Toolbox, always greater than 0.99. However, it is worth noting that the error distributions were sometimes more peaky than their fitted Gaussian curves, meaning that the errors were statistically closer to their mean values than those obtained with a pure Gaussian distribution. We also note that such a difference in the height and shape of the error distributions was due to the fact that these distributions were extracted from histograms over a rather low-resolution grid of beams ($\Delta$Beam from 0.02–0.04) in order to have smooth curves.

The greatest OF precision obtained was about 1.2% when $\Delta \omega^*$ was set at 0.05 rad s$^{-1}$, whereas the lowest was about 3.6% when $\Delta \omega^*$ was set at 0.5 rad s$^{-1}$. The robot adopted the required velocity and steering angle, giving a precision of the robot’s velocity and steering angle estimates ($\hat{V}$, $\hat{\phi}$) ranging from 1%–5% and from 6%–12%, respectively, which made it possible to drive the robot relatively close to...
the reference trajectory (figures 6(g), 7(A)(f), (B)(f), 8(A)(f), (B)(f)). When the robot was driven in the open-loop mode; however, i.e. when setting \( \xi = \xi^* \) instead of applying the control scheme presented in figure 5, the position and orientation errors were much larger than those obtained in the closed-loop mode (figures 7(B)(g), 8(A)(g), (B)(g)). Depending on the type of trajectory and on whether the robot was under or over-steered with respect to the steering values required, the robot could go very astray,
sometimes making the robot drift up to going off the carpet at a very early stage (figures 7(B)(g), 8(A)(g)), or stay in a closed path although very different to that required (figures 8(B)(g)).

By using the OF-based estimates of the robot’s velocity and steering angle for odometry purposes, we obtained an accuracy of the position and orientation estimates \(\hat{X}, \hat{\theta}\) ranging from 0.3%–2.3%, after the robot had been traveling from 18–71 m and turning from 4.1\(\pi\)–8.1\(\pi\) rad, i.e. after about 2–8 laps. It is worth noting that such an odometry accuracy was about 2–9 times higher than that obtained by using the IMU-based estimates of the robot’s velocity and steering angle \((\hat{V}_{IMU}, \hat{\phi}_{IMU})\) (figures 6(e), 7(B)(e), 8(A)(e), (B)(e)), except in the case of constant high velocity where the odometry performances were comparable (figure 7(A)(e)). Such a result could be due to the fact that the OF-based estimates \((\hat{V}, \hat{\phi})\) were not very accurate at the very beginning of the test because no measurements were delivered until the OF values reached the range required, i.e. [6.75, 8.25] rad s\(^{-1}\) and [8.25, 9.75] rad s\(^{-1}\) for the left and right side, respectively, giving large errors in the robot’s position and orientation estimates (see the first part of figures 7(A)(e)).

Some quantitative data and statistics on each indoor test are given in table 1.

### 4.2. Outdoors

Figures 9, 10 show the results obtained outdoors during the day-time (at about 12:00 PM) and at night (at about 20:00 PM), respectively, when the robot was driven on the ground shown in figure 1(d) on a circular path, keeping a constant steering angle while varying the velocity from 0.3–1.3 m s\(^{-1}\). The OF resolution \(\Delta\omega^*\) required was set at 0.3 rad s\(^{-1}\) and the average refresh rate \(I^*_{SO}\) obtained was about 300 Hz in both tests.

Outdoor tests were also carried out in the same environment using various ground textures (figure 11) and various trajectories (figure 12). In the first case, four regions with different textures were added to the floor: (1) a mixture of grass, leaves and flowers, (2) dark sand and (3) light sand, and (4) black asphalt gravel (see the pictures at the bottom of figure 11). The OF resolution \(\Delta\omega^*\) required was set at 0.2 and 0.1 rad s\(^{-1}\) and the average refresh rate \(I^*_{SO}\) obtained in the two tests was about 295 and 328 Hz, respectively.

The results presented here show that our sensors responded appropriately outdoors by delivering OF measurements regardless of the luminosity in a 6-decade range \((I_{ph} \text{ ranging from about } 10^{-9} \text{ to } 10^{-3} \text{ A})\) and despite the shadows produced by the robot itself under daylight conditions (see, for instance, pictures (1) and (2) in figure 9). In addition, our sensors delivered OF measurements that did not depend significantly on either the trajectory or the ground textures (grass, leaves, flowers, dark and light sand, dark and light gravel), although sometimes fewer measurements were delivered on the left side when the robot was driving on some of the textured regions (figure 11(a)).

As in the indoor tests, the overall error distributions were consistently nearly Gaussian with a quasi-zero mean (figures 9(b), 10(b), 11(b), 12(b)), giving values of the adjusted R-square goodness-of-fit statistics, computed using the Matlab Curve Fitting Toolbox, always greater than 0.98. The same observations on the height and shape of the error distributions done for the indoor tests can also be done for the outdoor tests.

The OF precision obtained here was slightly higher than that obtained in the indoor tests, ranging from 2.6%–3.8%, and did not significantly depend on the light levels, the ground textures or the type of trajectory\(^6\). However, the average precision of the robot’s velocity and steering angle estimates \((\hat{V}, \hat{\phi})\) is similar to that obtained in the indoor tests (2.3% versus 2.4% and 9.3% versus 9.8%, respectively), showing the robustness of the method presented here to vibrations liable to cause a change in the sensors’ local height of up to \(\pm 10\) mm (figures 9(f), 10(f)), amounting to about 6% of the nominal height.

In the case of larger changes in height, such as those caused by the rail (shaded regions in figures 9(a)–(g) and 10(a)–(g)), the robot could still estimate \(V\) and \(\phi\) but with a much lower precision, and a more complex method of estimation, including information from other sensors, would therefore have to be implemented in order to achieve a higher level of robustness. In fact, the error in the position and orientation estimates \((\hat{X}, \hat{\theta})\) seems to particularly increase when the robot was driving on the rail at relatively high speed because of larger errors in the OF-based velocity and steering angle estimates (see, for instance, the shaded regions in figure 9 at about 18 s and in figure 10 at about 12 s). However, even in the presence of such large vibrations, we still obtained a good accuracy of the position and orientation estimates, i.e. from 2%–3% after the robot had been traveling about 30 m and turning about 6–8\(\pi\) rad (i.e. about 3–4 laps), which was slightly higher than that obtained by using the IMU-based estimates of the robot’s velocity and steering angle \((\hat{V}_{IMU}, \hat{\phi}_{IMU})\) (figures 9(e), 10(e), 11(e), 12(e)). In particular, the OF-based odometry gave better performances on average than the IMU-based odometry when using trajectories where the steering angle, and therefore the angular velocity, changed in sign, i.e. when driving on an eight-shaped path

---

\(^6\) The standard deviations and the corresponding precision in the first two tests (figures 9, 10) were computed without taking into account the moments when the robot was driving on the steel rail (shaded regions in figures 9(a)–(g), 10(a)–(g)) as the impact with the rail produced very large vibrations.
The image marked (a) shows the test environment at about 12:00 PM, when the light level was about 3000 Lux. The images marked (1) and (2) show two situations where the robot was driving on the steel rail, corresponding to the shaded regions in figures 9(a)–(g).

The image marked (b) shows the test environment at about 20:00 PM, when the light level was about 1 Lux. The images marked (1) and (2) show two situations where the robot was driving on the steel rail, corresponding to the shaded regions in figures 10(a)–(g).

**Figure 9.** Robust OF measurements and odometry results obtained outdoors at a very high light level (at about 12:00 PM) when the robot was driven on a circular path at a velocity ranging from 0.3–1.3 m s⁻¹. (a) $\omega_{\text{imu}}$, $\omega_{\text{ref}}$ (dots) and $\omega_{\text{ref}}$ (solid lines). (b) Distribution of $\omega_{\text{ref}} - \omega_{\text{imu}}$ (dark blue line) and $\omega_{\text{ref}} - \omega_{\text{imu}}$ (light blue line). (c) $V$, $\phi$ (dots), $V_{\text{truth}}, \phi_{\text{truth}}$ (solid lines) and $V^*, \phi^*$ (dashed lines). (d) Distribution of $V_{\text{truth}} - V$ (dark green line) and $\phi_{\text{truth}} - \phi$ (light green line). (e) $||X - \hat{X}||$, $\theta - \hat{\theta}$ (solid lines) and $||X - \hat{X}_{\text{imu}}||$, $\theta - \hat{\theta}_{\text{imu}}$ (dashed lines). (f) (i) Left (yellow line) and right (purple line) local sensor’s height $h$, $h^*$ approximately estimated using the robot’s pose measurements. (g) $X^*$ (dashed line), $\hat{X}$ (dotted line) and $X$ (solid line). The picture in the bottom-left part of the figure shows the test environment at about 12:00 PM, when the light level was about 3000 Lux. The pictures marked (1) and (2) show two situations where the robot was driving on the steel rail, corresponding to the shaded regions in figures 9(a)–(g).

**Figure 10.** Robust OF measurements and odometry results obtained outdoors at very low light levels (at about 20:00 PM) when the robot was driven on a circular path at a velocity ranging from 0.3–1.3 m s⁻¹. (a) $\omega_{\text{imu}}$, $\omega_{\text{ref}}$ (dots) and $\omega_{\text{ref}}$ (solid lines). (b) Distribution of $\omega_{\text{ref}} - \omega_{\text{imu}}$ (dark blue line) and $\omega_{\text{ref}} - \omega_{\text{imu}}$ (light blue line). (c) $V$, $\phi$ (dots), $V_{\text{truth}}, \phi_{\text{truth}}$ (solid lines) and $V^*, \phi^*$ (dashed lines). (d) Distribution of $V_{\text{truth}} - V$ (dark green line) and $\phi_{\text{truth}} - \phi$ (light green line). (e) $||X - \hat{X}||$, $\theta - \hat{\theta}$ (solid lines) and $||X - \hat{X}_{\text{imu}}||$, $\theta - \hat{\theta}_{\text{imu}}$ (dashed lines). (f) (i) $h^*$ (yellow line) and $h^*$ (purple line). (g) $X^*$ (dashed line), $\hat{X}$ (dotted line) and $X$ (solid line). The picture in the bottom-left part of the figure shows the test environment at about 20:00 PM, when the light level was about 1 Lux. The images marked (1) and (2) show two situations where the robot was driving on the steel rail, corresponding to the shaded regions in figures 10(a)–(g).
bias. All in all, we note that the accuracy of the solely IMU-based odometry strongly depends on the non-linear dynamics intrinsic to the IMU used, therefore better odometry results could be obtained by including these non-linearities in the EKF.

Some quantitative data and statistics on each outdoor tests, the errors in the IMU-based estimates were often lower than those in the OF-based estimates in the first part of the tests and then started increasing very rapidly, probably because of the gyroscope’s varying bias. All in all, we note that the accuracy of the solely IMU-based odometry is lower than those in the OF-based estimates in the tests, the errors in the IMU-based estimates were often rapidly, probably because of the gyroscope part of the tests and then started increasing very rapidly, probably because of the gyroscope’s varying bias. All in all, we note that the accuracy of the solely IMU-based odometry strongly depends on the non-linear dynamics intrinsic to the IMU used, therefore better odometry results could be obtained by including these non-linearities in the EKF.

Some quantitative data and statistics on each outdoor test are given in Table 2.
5. Conclusions

The low-cost car-like robot called BioCarBot presented in this paper is able to estimate its velocity and steering angle, and therefore its position and orientation via an EKF, using only the OF measurements delivered by two novel downward-facing VMSs. Thanks to the cross-correlation method and the auto-adaptive pixels used, these novel VMSs have the following advantages:

- the OF measurements are robust to high-dynamic-range lighting conditions (in a 7-decade range with sharp changes of up to 2 decades within 0.5 s) and to the various visual patterns encountered;
- the refresh rate of the OF measurements is relatively high (300–333 Hz) and nearly constant, and does not depend on the bandwidth of the band-pass filter;
- the resolution on the OF measurements is also relatively high (up to 0.05 rad s\(^{-1}\)) and constant, i.e. it does not depend on the OF magnitude, and can be set at whatever value is required;
- the OF measurement range is relatively wide (from 1.5–15 rad s\(^{-1}\), i.e. 85 to 850 o s\(^{-1}\)) and can also be adjusted as required;
- the sensor’s precision can be relatively high (3.8% in the worst case), depending on the OF resolution required, the OF range and the robot’s vibrations.

With these OF sensors, the robot was able to estimate its own velocity and steering angle, and therefore its position and orientation accurately, both indoors and outdoors, and the drifts liable to occur when it was driven in the open-loop mode were greatly reduced. The main features of the minimalistic visual odometry method presented here can be listed as follows:

- robustness to high-range light levels (by daylight and at night, i.e. from about 1–30000 Lux, including shadows) and various ground textures (asphalt, gravel, sand, leaves, etc);
- robustness to vibrations liable to affect change in the sensors’ local height up to 6% (i.e. about ±10 mm–175 mm);
- fairly high-accuracy position and orientation estimation (from 0.3%–3% ) after covering a distance from 30 m–75 m and turning through an angle from \(6\pi\)–16\(\pi\) rad.

We also showed that the accuracy was higher on average than that of the IMU-based odometry, especially in the indoor tests and when the distance and the rotation angle covered were relatively large.

It is worth noting that both the sampling frequency, and hence the refresh rate, and the OF resolution, and hence the precision, could be increased by either optimizing the implementation of the algorithm or increasing the computational resources. These two parameters could be also adjusted in real time to the values required depending on the robot’s reference trajectory, and therefore on the OF setpoint profile.

The precision of the estimates could also be improved by (i) adapting the EKF to include just a single OF measurement (i.e. on one side only) when no measurement is available on the other side, and (ii) precisely adjusting the values of the measurement and process covariance matrices of the EKF after performing a calibration phase prior to each test. In addition, the OF and inertial measurements could be combined together in one EKF by including accurately-identified dynamic models of the sensors used, which take into account their intrinsic non-linearities in order to achieve greater robustness and precision, as required for the autonomous vehicles of the future [62] and even tomorrow’s flying robots [21].

Tests involving more complex and longer trajectories as well as more challenging environments, such as forest paths, are now being considered, and we are planning to test the sensors and the estimation method presented here on a real vehicle with a view to improving the existing odometry techniques. In this way, we are willing to apply the improvements discussed above and to evaluate the results obtained also by using the metrics presented in [61].

Acknowledgments

We are most grateful to M Boyron and J Diperi for their involvement in the electronic and mechanical design of the sensors and the robot. We also thank S Viollet, A Manecy and F Colonnier for their helpful suggestions and comments during this study, S Allano and F Guillemand for their assistance, and J Blanc for correcting and improving the English manuscript. This research was supported partly by the CNRS (Life Science; Information and Engineering Science and Technology) and Aix-Marseille University. This research was also funded by a PhD grant from ANRT (Association Nationale de la Recherche et de la Technologie) as well as by PSA Peugeot Citroën via the OpenLab agreement with Aix-Marseille University and CNRS entitled ‘Automotive Motion Lab’.
Appendix

Table 1. Quantitative data and statistics on the indoor tests.

| Condition                              | $\Delta \omega^*$ | $\bar{f}_c$ | $\frac{c_{\omega}}{c_{\omega}}$, $\frac{c_{\omega}}{c_{\omega}}$ (C.P.), $\frac{c_{\omega}}{c_{\omega}}$ | $\frac{\mu}{\nu}$, $\frac{\nu}{\nu}$ ( $\nu$, $\nu$) | $\|X - X_{\text{true}}\|_2$, $\|\theta - \theta_{\text{true}}\|_2$ (d, $|\theta|$) | $\|X - X_{\text{true}}\|_2$, $\|\theta - \theta_{\text{true}}\|_2$ (d, $|\theta|$) |
|----------------------------------------|-------------------|-------------|-----------------------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|
| High-dynamic-range lighting conditions (figure 6) | 0.3 rad s$^{-1}$ | 0.327 Hz    | 3.3%, 3.2% (4.5, 5 rad s$^{-1}$) 3%, 12% (0.8 m s$^{-1}$, 0.15 rad) | 9%, 9% (4.8 m s$^{-1}$, 0.15 rad) 2%, 1.2% (45 m, 8.1 rad) | 8.4%, 10.1% (44 m, 8.1 rad) | 8.4%, 10.1% (44 m, 8.1 rad) |
| High OF resolution (figure 7(A))       | 0.05 rad s$^{-1}$ | 0.333 Hz    | 1.2%, 1.4% (7.5, 9 rad s$^{-1}$) 0.9%, 6% (1.5 m s$^{-1}$, 0.15 rad) | 2.5%, 1.5% (18 m, 4.1 rad) 2% (2.1% (22 m, 4.1 rad) | 4.3%, 4.6% (43 m, 8.6 rad) | 4.3%, 4.6% (43 m, 8.6 rad) |
| Wide OF range (figure 7(B))            | 0.5 rad s$^{-1}$  | 0.332 Hz    | 3.6%, 3.4% (7.5, 8.5 rad s$^{-1}$) 2.4%, 12% (0.8 m s$^{-1}$, 0.15 rad) | 2.2%, 2.2% (45 m, 8.6 rad) | 4.3%, 4.6% (43 m, 8.6 rad) | 4.3%, 4.6% (43 m, 8.6 rad) |
| Eight-shaped path (figure 6(A))        | 0.1 rad s$^{-1}$  | 0.326 Hz    | 2.3%, 2.1% (4, 4 rad s$^{-1}$) 1.8%, 5.3% (0.3 m s$^{-1}$, 0.3 rad) | 2.3%, 1.8% (31 m, 6.1 rad) | 6.1%, 6% (32 m, 6.1 rad) | 6.1%, 6% (32 m, 6.1 rad) |
| Square-shaped path (figure 5(B))       | 0.2 rad s$^{-1}$  | 0.324 Hz    | 3.1%, 3.3% (3.5, 3 rad s$^{-1}$) 3.3%, 11% (0.6 m s$^{-1}$, 0.2 rad) | 0.7%, 0.3% (71 m, 16.6 rad) | 1.6%, 1.7% (71 m, 16.6 rad) | 1.6%, 1.7% (71 m, 16.6 rad) |

Table 2. Quantitative data and statistics on the outdoor tests.

| Condition                              | $\Delta \omega^*$ | $\bar{f}_c$ | $\frac{c_{\omega}}{c_{\omega}}$, $\frac{c_{\omega}}{c_{\omega}}$ (C.P.), $\frac{c_{\omega}}{c_{\omega}}$ | $\frac{\mu}{\nu}$, $\frac{\nu}{\nu}$ ( $\nu$, $\nu$) | $\|X - X_{\text{true}}\|_2$, $\|\theta - \theta_{\text{true}}\|_2$ (d, $|\theta|$) | $\|X - X_{\text{true}}\|_2$, $\|\theta - \theta_{\text{true}}\|_2$ (d, $|\theta|$) |
|----------------------------------------|-------------------|-------------|-----------------------------------------------------------------|------------------------------------------------|------------------------------------------------|------------------------------------------------|
| Uneven terrains at day light (figure 9) | 0.3 rad s$^{-1}$  | 0.301 Hz    | 3.6%, 3.2% (4.5, 5 rad s$^{-1}$) 2.6%, 11% (0.8 m s$^{-1}$, 0.2 rad) | 2.6%, 2.2% (31 m, 8 rad) | 4%, 3.3% (30 m, 8 rad) | 4%, 3.3% (30 m, 8 rad) |
| Uneven terrains at night (figure 10)   | 0.3 rad s$^{-1}$  | 0.299 Hz    | 3.8%, 3.6% (4.5, 5 rad s$^{-1}$) 3.1%, 12% (0.8 m s$^{-1}$, 0.2 rad) | 3%, 2.3% (32 m, 8 rad) | 4.8%, 4.1% (30 m, 8 rad) | 4.8%, 4.1% (30 m, 8 rad) |
| Various ground textures (figure 11)     | 0.2 rad s$^{-1}$  | 0.295 Hz    | 3.7%, 3% (3, 4 rad s$^{-1}$) 2.2%, 9% (0.8 m s$^{-1}$, 0.2 rad) | 3%, 1.9% (31 m, 8 rad) | 3.3%, 3.9% (34 m, 8 rad) | 3.3%, 3.9% (34 m, 8 rad) |
| Eight-shaped path (figure 12)           | 0.1 rad s$^{-1}$  | 0.328 Hz    | 2.9%, 2.6% (3.5, 3.5 rad s$^{-1}$) 1.8%, 7.3% (0.6 m s$^{-1}$, 0.3 rad) | 1.9%, 1.6% (28 m, 6 rad) | 8.3%, 8.5% (27 m, 6 rad) | 8.3%, 8.5% (27 m, 6 rad) |
References

[1] Koenderink J J and Doorn A J V 1987 Facts on Optic Flow Biol. Cybern. 56 247–54
[2] Beau chimpan S and Barron J L 1995 The computation of optical flow ACM Comput. Surv. 27 433–66
[3] Sun D, Roth S and Black M 2010 Secrets of optical flow estimation and their principles IEEE Conf. on Computer Vision and Pattern Recognition (CVPR) pp 2432–9
[4] Chao H, Gu Y and Napolitano M 2013 A survey of optical flow techniques for robotics navigation applications J. Intell. Robot. Syst. 73 361–72
[5] Expert F, Viollet S and Ruffier F 2011 Outdoor field performances of insect-based visual motion sensors J. Field Robot. 28 329–41
[6] Hornegger D, Meier L, Tanskanen P and Pollefeys M 2013 An open source and open hardware embedded metric optical flow cmos camera for indoor and outdoor applications IEEE Int. Conf. on Robotics and Automation (ICRA) pp 1736–41
[7] Benosman R, Clercq C, Lagorce X, Ieng S H and Bartolozzi C 2014 Event-based visual flow IEEE Trans. Neural Netw. Learn. Syst. 25 407–17
[8] Lucas B D and Kanade T 1981 An iterative image registration technique with an application to stereo vision Proc. 7th Int. Joint Conf. on Artificial Intelligence vol 2 pp 674–9
[9] Horn B K P and Schunck B G 1981 Determining optical flow Artif. Intell. 17 185–203
[10] Srinivasan M V 1994 An image-interpolation technique for the computation of optical flow and egomotion Biol. Cybern. 71 401–15
[11] Hyslop A M and Humbert J S 2010 Autonomous navigation in three-dimensional urban environments using wide-field integration of optic flow J. Guid. Control Dyn. 33 147–59
[12] Alaeddini A and Morgansen K A 2015 Bioinspired navigation for a nonholonomic mobile robot J. Aerosp. Inf. Syst. 12 688–98
[13] Expert F and Ruffier F 2015 Flying over uneven moving terrain based on optic-flow cues without any need for reference frames or accelerometers Bioinspiration Biomimetics 10 026003
[14] Garcia Carrillo I R, Fantoni I, Rondón E and Dural A 2015 Three-dimensional position and velocity regulation of a quad-rotorcraft using optical flow IEEE Trans. Aerosp. Electron. Syst. 51 358–71
[15] Blaser G and Hendehy G 2009 Using optical flow as lightweight slam alternative 8th IEEE Int. Symp. on Mixed and Augmented Reality (ISMAR) pp 175–8
[16] Alcantarilla P, Yebs J, Almazan J and Bergasa L 2012 On combining visual slam and dense scene flow to increase the robustness of localization and mapping in dynamic environments IEEE Int. Conf. on Robotics and Automation (ICRA) pp 1290–7
[17] Campbell J, Sutkhaner R and Nourbakhsht I 2004 Techniques for evaluating optical flow for visual odometry in extreme terrain Proc. IEEE Int. Conf. Intell. Robot. Syst. pp 5704–11
[18] Conkle P, Strelow D and Singh S 2004 Omnidirectional visual odometry for a planetary rover Proc. IEEE/RSJ Int. Conf. on Intell. Robot. Syst. (IROS) 4 pp 4007–12
[19] Scaramuzza D, Fraundorfer F and Siegwart R 2009 Real-time monocular visual odometry for on-road vehicles with 1-point RANSAC Proc. IEEE Int. Conf. Robot. Autom. pp 4293–9
[20] Fraundorfer F and Scaramuzza D 2012 Visual odometry: 2. matching, robustness, optimization, and applications IEEE Robot. Autom. Mag. 19 78–90
[21] Brodr A, Zufferey J C and Floreano D 2016 A method for ego-motion estimation in micro-hovering platforms flying in very cluttered environments Auton. Robots 40 789–803
[22] Williams C B 1957 Insect migration Annu. Rev. Entomology 2 163–80
[23] Snygley R B and Oliveira E G 1999 Orientation mechanisms and migration strategies within the flight boundary layer Insect Movement: Mechanisms and Consequences. Proceedings of the Royal Entomological Society’s 20th Symposium (London: CAB) pp 183–206
[24] Franceschini N, Ruffier F and Serres J 2007 A bio-inspired flying robot sheds light on insect piloting abilities Curr. Biol. 17 329–35
[25] Srinivasan M, Zhang S, Lehrer M and Collett T 1996 Honeybee navigation en route to the goal: visual flight control and odometry J. Exp. Biol. 199 237–44
[26] Srinivasan M, Zhang S and Bidwell N 1997 Visually mediated odometry for honeybees J. Exp. Biol. 200 2513–22
[27] Baire E, Kreiss E, Witsold W, Warrant E and Dacke M 2011 Nocturnal insects use optic flow for flight control Biol. Lett. 7 499–501
[28] Lichtsteiner P, Posch C and Delbrück T 2008 A 128 × 128 160 μs latency asynchronous temporal contrast vision sensor IEEE Solid-State Circuits 43 566–76
[29] Floreano D et al 2013 Miniature curved artificial compound eyes Proc. Natl Acad. Sci. USA 110 9367–72
[30] Posch C, Serrano-Gotarredona T, Linares-Barranco B and Delbrück T 2014 Retinomorphic event-based vision sensors: bioinspired cameras with spiking output IEEE Proc. 102 1470–44
[31] Posch C, Benosman R and Etienne-Cummings R 2015 Giving machines humanlike eyes IEEE Spectr. 52 44–9
[32] Chihaniyara S, Bunnun P, Seneviratne I D and Althoefer K 2008 Optical flow algorithm for velocity estimation of ground vehicles: a feasibility study Int. J. Smart Sens. Intell. Syst. 1 246–8
[33] Nourani-Vatani N, Roberts J and Srinivasan M V 2009 Practical visual odometry for car-like vehicles Proc. IEEE Int. Conf. Robot. Autom. pp 3551–7
[34] Killpack M, Deyte T, Anderson C and Kemp C C 2010 Visual odometry and control for an omnidirectional mobile robot with a downward-facing camera Proc. IEEE Int. Conf. Intell. Robot. Syst. pp 139–46
[35] Zhang J, Singh S and Kantor G 2014 Robust monocular visual odometry for a ground vehicle in undulating terrain Field and Service Robotics: Results of the 8th Int. Conf. (Berlin: Springer) pp 311–26
[36] Nagai I and Watanabe K 2013 Path tracking by a mobile robot equipped with only a downward facing camera Proc. IEEE Int. Conf. Intell. Robot. Syst. pp 6053–8
[37] Kim S and Lee S 2008 Robust velocity estimation of an omnidirectional mobile robot using a polygonal array of optical mice Int. J. Contr. Autom. Syst. 6 713–21
[38] Dille M, Grocholsky B and Singh S 2009 Outdoor downward-facing optical flow odometry with commodity sensors Proc. Int. Conf. Field Serv. Robot. pp 183–93
[39] Ross R, Devlin J and Wang S 2012 Toward refocused optical mouse sensors for outdoor optical flow odometry IEEE Sensors J. 12 1925–30
[40] Nagai I, Watanabe K, Nagatani K and Yoshida K 2010 Noncontact position estimation device with optical sensor and laser sources for mobile robots traversing slippery terrains Proc. IEEE Int. Conf. Robot. Autom. Syst. pp 3422–7
[41] Yi D H, Lee T J and Cho D I 2015 Afoical optical flow sensor for reducing vertical height sensitivity in indoor robot localization and navigation Sensors 15 11208–21
[42] Naka K I and Rushon W A H 1966 S-potentials from luminosity units in the retina of fish (Cyprinidae) J. Physiol. 185 536–55
[43] Normann R A and Perlman I 1979 The effects of background illumination on the photoreponses of red and green cones J. Physiol. 286 491–507
[44] Mead C A and Mahowald M 1988 A silicon model of early visual processing Neural Netw. 1 91–7
[45] Delbrück T and Mead C A 1994 Analog VLSI adaptive, logarithmic, wide-dynamic-range photoreceptor Proc. IEEE Int. Symp. Circuits Syst. pp 339–42
[46] Meylan L, Alleysson D and Süsstrunk S 2007 Model of retinal local adaptation for the tone mapping of color images J. Opt. Soc. Am. A 24 2807–16
[47] Ferradans S, Bertaçlio M, Provenzi E and Caselles V 2011 Analysis of visual adaptation and contrast perception for tone mapping IEEE Trans. Pattern Anal. Mach. Intell. 33 2002–12
[48] Mafrica S, Godiot S, Menouni M, Boyron M, Expert F, Juston R, Marchand N, Ruffier F and Viollet S 2015 A bio-inspired analog silicon retina with Michaelis–Menten auto-adaptive pixels sensitive to small and large changes in light Opt. Express 23 3614–35
[49] Ruffier F, Viollet S, Amic S and Franceschini N 2003 Bio-inspired optical flow circuits for the visual guidance of micro air vehicles Proc. IEEE Int. Symp. Circuits Syst. pp 846–9
[50] Sabiron G, Chavent P, Baharrijaona T, Fabiani P and Ruffier F 2013 Low-speed optic-flow sensor onboard an unmanned helicopter flying outside over fields Proc. IEEE Int. Conf. Robot. Autom. pp 1742–9
[51] Franceschini N, Riehle A and Le Nestour A 1989 Directionally selective motion detection by insect neurons Facets of Vision. (Berlin: Springer) pp 360–90
[52] Mafrica S, Servel A and Ruffier F 2016 Optic-flow based car-like robot operating in a 5-decade light level range IEEE Int. Conf. on Robotics and Automation (ICRA) pp 5568–75
[53] Reichardt W 1961 Autocorrelation, a principle for the evaluation of sensory information by the central nervous system Sensory Communication (Cambridge, MA: MIT Press) pp 303–17
[54] Snyder A W 1977 Acuity of compound eyes: physical limitations and design J. Comparative Physiol. 116 161–82
[55] Land M F 1997 Visual acuity in insects Annu. Rev. Entomology 42 147–77
[56] Albus J S and Hong T H 1990 Motion, depth, and image flow Proc. IEEE Int. Conf. on Robotics and Automation, vol 2, 1161–70
[57] Manecy A, Marchand N, Ruffier F and Viollet S 2015 X4-mag: a low-cost open-source micro-quadrotor and its linux-based controller Int. J. Micro Air Veh. 7 89–109
[58] De Luca A, Oriolo G and Samson C 1998 Feedback control of a nonholonomic car-like robot Robot Motion Planning and Control (Berlin: Springer) pp 171–253
[59] Martinelli A 2012 Vision and imu data fusion: closed-form solutions for attitude, speed, absolute scale, and bias determination IEEE Trans. Robot. 28 44–60
[60] Wu Z, Sun Z, Zhang W and Chen Q 2015 Attitude and gyro bias estimation by the rotation of an inertial measurement unit Meas. Sci. Technol. 26 125102
[61] Geiger A, Lenz P and Urtasun R 2012 Are we ready for autonomous driving? The kitti vision benchmark suite IEEE Conf. on Computer Vision and Pattern Recognition (CVPR) pp 3354–61
[62] Zhang T, Liu X, Kuhnlenz K and Buss M 2009 Visual odometry for the autonomous city explorer Proc. IEEE Int. Conf. Intell. Robot. Syst. pp 3513–8