Simplified characteristic time method for accurate estimation of the soil hydraulic parameters from one-dimensional infiltration experiments

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Abstract
Recently, a novel approach with excellent performance based on the concept of the characteristic infiltration time, the characteristic time method (CTM), is proposed to infer soil sorptivity (S) and saturated hydraulic conductivity (Ks) from one-dimensional (1D) cumulative infiltration. The current work provides a simplified version of the CTM, called the SCTM, by eliminating the necessity of the iteration method used in CTM and providing a similar accuracy as the original method when estimating S and Ks. We used both synthetic and experimental data to evaluate SCTM in comparison with the original CTM, as well as Sharma (SH) and curve-fitting methods. In the case of synthetically simulated infiltration experiments, the predicted S and Ks values showed an excellent agreement with their theoretical values, with Nash–Sutcliffe (E) values higher than 0.9 and RMSE values of 0.11 cm h\(^{1/2}\) and 0.35 cm h\(^{-1}\), respectively. In the case of experimental data, the SCTM showed E values larger than 0.73 and RMSE values of 0.64 cm h\(^{1/2}\) and 0.35 cm h\(^{-1}\), respectively. The accuracy and the robustness of SCTM was comparable with the original CTM when applied on synthetic infiltration curves as well as on experimental data. Similar to the original CTM, the simplified approach also does not require the knowledge of the time validity, which is needed when using approaches based on Philip’s infiltration theory. The method is applicable to infiltrations with durations from 15 min to 24 h. The supplemental material presents the calculation of S and Ks using SCTM in an Excel spreadsheet.

INTRODUCTION

Water infiltration plays a fundamental role in controlling, for example, surface runoff, groundwater recharge, the soil water available for evapotranspiration, and thus crop growth. Knowledge of the soil infiltration properties is also essential in managing irrigation systems and controlling soil salinity and...
sodicity. Monitoring of infiltration rates under natural conditions is difficult if not impossible, and therefore the use of the hydrological models that correctly describe infiltration are very important.

Water infiltration into soils can either be described by using soil water balance models or by using theoretical, semiempirical, and empirical hydrological models (Corradini et al., 1997; Green & Ampt, 1911; Haverkamp et al., 1994; Parlange et al., 1982; Philip, 1957; Swartzendruber, 1987) for specific infiltration conditions. The former models are typically used for the characterization of soil sorptivity (S), a measurable physical quantity, which expresses the capacity of a porous medium to absorb or release liquids by capillarity (Philip, 1957), and saturated hydraulic conductivity (Ks), a measure of the soil’s ability to transmit water under the influence of gravity.

In general, two major approaches are used to estimate S and Ks: linearization approaches (Sharma et al., 1980; Smiles & Knight, 1976; Vandervaere et al., 2000) and inverse estimation using curve-fitting method (Bonell & Williams, 1986; Bristow & Savage, 1987; Marquardt, 1963; Vandervaere et al., 2000). Both approaches usually result in very good agreement (R² > 0.9) between measured and predicted infiltration curves (Rahmati et al., 2020). However, linearization approaches suffer from substantial arbitrariness in deciding which part of the data fully meets linearity, leading to uncertainty in estimated infiltration parameters. On the other hand, the curve fitting methods suffer from equifinality, the principle that the minimum of the objective function can be obtained by a broad set of parameter values (Beven & Freer, 2001), as well as possible nonrealistic parameter values when the optimization is unconstrained.

Rahmati et al. (2020) introduced a new procedure based on the use of the characteristic time (named characteristic time method, CTM) to predict S and Ks from one-dimensional (1D) infiltration experiments. Their method uses an iterative procedure to ensure accurate predictions of S and Ks regardless of the infiltration regime measured (i.e., the transient regime only or both transient and steady-state regimes). Contrary to previously published methods, the CTM does not require knowledge of the time validity of the applied semi-analytical solution for transient infiltration and can be applied to any infiltration duration (from a few minutes to several days). Rahmati et al. (2020) demonstrated the usefulness and strength of CTM in comparison with a suite of existing methods including classical methods of Sharma et al. (1980) (SH) and curve-fitting methods using two- (Haverkamp et al., 1994) (CF2) and three-term (Rahmati et al., 2019) (CF3) approximate expansions based on the quasi-exact implicit (QEI) formulation proposed by Haverkamp et al. (1994). Note that the QEI model addresses the case of 1D water infiltration and was extended to the case of water infiltration through discs (three-dimensional [3D] axisymmetric geometry) by Smettem et al. (1994).

Core Ideas

- A simplified version of the CTM is proposed to estimate the S and Ks from a 1D infiltration curve.
- The SCTM is nearly as accurate as the original CTM in prediction of S and Ks.
- The SCTM eliminates the necessity of the iteration method used in original CTM.

Although CTM is attractive, providing accurate and simultaneous estimates of S and Ks, the approach suffers from remarkable complexity due to the fact that an iterative procedure is needed to obtain the parameter values and requires programming in, for example, Python, MATLAB, Scilab, and/or VBA. This might hamper a widespread interest and reduce applicability of the CTM. In this paper, we therefore simplified the CTM to eliminate the iterative procedure, making it more practical and simpler with an only slight reduction in the model accuracy. The objective of this paper is to present STCM, a simplified version of the CTM, which is compared with CTM as well the SH, CF2, and CF3 methods. Finally, a simple calculation of the S and Ks in Excel worksheet is presented as supplemental material.

2 MATERIALS AND METHODS

2.1 Theoretical background

Following Rahmati et al. (2020), the contribution of capillary- and gravity-driven components to the cumulative infiltration are temporally variable and is illustrated in Figure 1. The contribution of the capillary-driven component shows a maximum at the start of the infiltration process (t close to 0), whereas the contribution of the gravity-driven component is 0.

![Figure 1](https://via.placeholder.com/150)

**Figure 1** Temporal variations of capillary- and gravity-driven components’ contributions to cumulative infiltration for the case of an ideal soil (adapted from Rahmati et al., 2020)
By advancing in time, the contribution of the capillary-driven component decreases, leading to symmetrical increase in contribution of the gravity-driven component. The time that both components provide an equal contribution to the cumulative infiltration was defined by Philip (1957) as the gravity time ($t_{\text{grav}}$), and this concept was used by Rahmati et al. (2020) to develop the CTM. They provided the following solutions to predict $S$ and $K_s$ by applying a characteristic time ($t_{\text{char}}$), which falls between 0 and $t_{\text{grav}}$:

$$S = (1 - \omega) \frac{I_{\text{char}}}{\sqrt{t_{\text{char}}}}$$  \hspace{1cm} (1)

$$K_s = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$  \hspace{1cm} (2)

where $a$, $b$, and $c$ are defined as below:

$$a = \frac{1}{9(1-\omega)}(\beta^2 - \beta + 1) \frac{t_{\text{char}}^2}{t_{\text{char}}}$$

$$b = \frac{2-\beta}{3} I_{\text{char}}$$

$$c = -\omega I_{\text{char}}$$  \hspace{1cm} (3)

where $\beta$ (dimensionless) is an integral infiltration constant related to the QEI model (Haverkamp et al., 1994), and $t_{\text{char}}$ is linked to $t_{\text{grav}}$ as (Rahmati et al., 2020):

$$t_{\text{char}} = \kappa t_{\text{grav}}, \text{ where } 0 < \kappa \leq 1$$  \hspace{1cm} (4)

The values of $1 - \omega$ and $\omega$ define the contribution of the capillary-driven (sorptivity) and gravity-driven components, respectively, to the cumulative infiltration $I_{\text{char}}$ obtained at time $t_{\text{char}}$:

$$1 - \omega = \frac{S\sqrt{t_{\text{char}}}}{I_{\text{char}}}$$  \hspace{1cm} (5)

$$\omega = \frac{c_2 t_{\text{char}} + c_3 t_{\text{char}}^{3/2}}{I_{\text{char}}}$$  \hspace{1cm} (6)

where

$$c_2 = \frac{2-\beta}{3} K_s$$

$$c_3 = (\beta^2 - \beta + 1) \frac{K_s^2}{9S}$$  \hspace{1cm} (7)

To obtain the above solutions, Rahmati et al. (2020) considered the three first components of the approximate expansion of the QEI model for the bulk infiltration by demonstrating that the other terms of the expansion are negligible. Hereafter, the capillary- and gravity-driven components, respectively, refer to the first and the sum of the remaining terms of three-term approximate expansion of the QEI model (Haverkamp et al., 1994).

In CTM proposed by Rahmati et al. (2020), an iterative procedure is used to determine $\omega$, as well as $t_{\text{char}} - I_{\text{char}}$ set from a measured infiltration curve. To do this, they equalized the $t_{\text{char}} - I_{\text{char}}$ set to different data points of the measured infiltration curve and examined the contribution of the capillary-driven component ($W_i$) in the vicinity of $t = 0$ where $W_i$ is defined as below (Rahmati et al., 2020):

$$W_i(t) = S \sqrt{t \over I(t)}$$  \hspace{1cm} (8)

In the case of $\lim_{t \to 0} W_i(t) = 1$, the selected datapoint for $t_{\text{char}} - I_{\text{char}}$ is correct and $S$ and $K_s$ can be identified. Otherwise [i.e., $\lim_{t \to 0} W_i(t) \neq 1$], the next data point is considered for $t_{\text{char}} - I_{\text{char}}$, the procedure is run, and the $\lim_{t \to 0} W_i(t)$ is checked again. The procedure starts with an initial value of $\omega = 1/2$, corresponding to $t_{\text{char}} = t_{\text{grav}}$. If no correct set of $t_{\text{char}} - I_{\text{char}}$ set is found when all datapoints of the infiltration curve are checked, the maximum experimental time is expected to be lower than $t_{\text{grav}}$. In that case, $\omega$ is decreased by small increment (e.g., 0.001), and the above procedure is repeated and $\omega$ is decreased until a correct set of $t_{\text{char}} - I_{\text{char}}$ can be found.

To eliminate the necessity of the iterative procedure, we rely on the time independency of the prediction of $S$ with the iterative procedure. In fact, since the CTM predictions of $S$ are independent of the infiltration duration (Rahmati et al., 2020), one can simply choose the latest available data point to define the $t_{\text{char}} - I_{\text{char}}$ set and then predict $S$ by changing $\omega$ to achieve $\lim_{t \to 0} W_i(t) = 1$. By doing so, the necessity for iterative procedure is eliminated.

The correctness of the procedure outlined above can be shown mathematically by setting $t_{\text{char}} = t_{\text{end}}$ and $I_{\text{char}} = I_{\text{end}}$ in Equation 1:

$$S = (1 - \omega) \frac{I_{\text{end}}}{\sqrt{t_{\text{end}}}}$$  \hspace{1cm} (9)

where a correct estimate of $\omega$ is needed for accurate estimation of $S$ from the above equation.

As mentioned above, the contribution of the capillary-driven component ($W_i$) on infiltration at any time, $t$, can be calculated from Equation 8. Substituting $S$ in Equation 8 using Equation 9, we obtain

$$W_i(t) = (1 - \omega) \frac{I_{\text{end}}}{\sqrt{t_{\text{end}}}} \sqrt{t \over I(t)}$$  \hspace{1cm} (10)

From the above equation, we can estimate $\omega$ by knowing that in the case of correct estimate of $\omega$, $W_i$ will approach unity in the vicinity of the $t = 0$, where
\[
\lim_{t \to 0} W_1(t) = 1 \Rightarrow \lim_{t \to 0} (1 - \omega) \frac{I_{\text{end}} \sqrt{t}}{t_{\text{end}} I(t)} = 1 \quad (11)
\]

In the case \( t = 0 \), no solution for the ratio \( \sqrt{t}/I(t) \) can be found. This can be solved by using either a \( t \) value very close to 0 (e.g., \( t = 1 \) s), or the first nonzero value of \( t \) in the infiltration curve. If we assume that all measured and simulated infiltration curves start at time \( t_1 = 0 \) at the first datapoint, then the \( t_2 - I_2 \) set can be the best choice to determine the limit in Equation 11. We can then simply rewrite the above equation as

\[
(1 - \omega) \frac{I_{\text{end}} \sqrt{t_2}}{t_{\text{end}} I_2} = 1 \quad (12)
\]

The rearrangement of the above equation gives a correct estimate of the \( \omega \):

\[
\omega = 1 - \frac{I_2}{I_{\text{end}}} \sqrt{\frac{t_{\text{end}}}{t_2}} \quad (13)
\]

By replacing \( \omega \) in Equation 9 using Equation 13, we can simplify Equation 9 to redefine the final solution for calculating \( S \) as

\[
S = \frac{I_2}{\sqrt{t_2}} \quad (14)
\]

The same procedure is applied to determine \( K_s \). In a similar manner, we calculated the contribution of the gravity-driven component, \( W_2(t) \), at different points in time, as below:

\[
W_2(t) = \frac{c_s t + c_s t^{3/2}}{I} \quad (15)
\]

where \( c_2 \) and \( c_3 \) are defined by Equation 7 considering the estimates of \( S \) defined by Equation 14, and \( \beta \) can be set equal to 0.6, as suggested by Haverkamp et al. (1994), or a soil-dependent \( \beta \) can be used as suggested by Lassabatere et al. (2009). However, independent studies (Latorre et al., 2015, 2018; Rahmati et al., 2019, 2020) have shown that the choice of the value of \( \beta \) had only a slight effect on estimating \( S \) and \( K_s \), suggesting that we can assume a constant \( \beta \) value of 0.6.

According to Equation 15, we can estimate \( K_s \) using the fact that \( W_2(t) \) will approach \( \omega \) at \( t = t_{\text{end}} \). Note that the value of \( \omega \) was computed from the cumulative infiltration value at the latest datapoint in measured infiltration curve with \( t_{\text{char}} = t_{\text{end}} \) and the knowledge of sorptivity. This can be written as below:

\[
\lim_{t \to t_{\text{end}}} \frac{c_s t + c_s t^{3/2}}{I} = \omega \quad (16)
\]

By replacing \( \omega \) from Equation 13 and using the \( t_{\text{end}} - I_{\text{end}} \) set, we can rewrite the above equation as below:

\[
\frac{c_2 t_{\text{end}} + c_3 t_{\text{end}}^{3/2}}{I_{\text{end}}} = 1 - \frac{I_2}{I_{\text{end}}} \sqrt{\frac{t_{\text{end}}}{t_2}} \quad (17)
\]

Writing Equation 17 in terms of \( K_s \) leads to:

\[
a' K_s^2 + b' K_s + c' = 0 \quad (18)
\]

where

\[
\begin{align*}
a' &= \frac{1}{4} (\beta^2 - \beta + 1) t_{\text{end}}^2 \frac{I_{\text{end}}}{I_{\text{end}}} \\
b' &= \frac{2 - \beta}{3} t_{\text{end}} \\
c' &= I_2 \sqrt{\frac{I_{\text{end}}}{t_2}} - I_{\text{end}}
\end{align*}
\]

Finally, solving for \( K_s \) in Equation 18 gives

\[
K_s = -b' + \sqrt{b'^2 - 4a'c'} \\
2a'
\]

### 2.2 Test data

To test the proposed procedure, we used synthetic infiltration curves simulated using HYDRUS-1D (Šimůnek et al., 2008, 2016) for 12 USDA soil textural classes provided as supplemental material by Rahmati et al. (2020), as well as experimental data (648 infiltration data) selected from Soil Water Infiltration Global (SWIG) database (Rahmati, Weihermüller, Vanderborght, et al., 2018; Rahmati, Weihermuller, & Vereecken, 2018).

#### 2.2.1 Synthetic data

To synthetize test data using HYDRUS-1D (Šimůnek et al., 2008, 2016), we used the average soil hydraulic parameters (Table 1) of the van Genuchten (VG) model (van Genuchten, 1980) for all examined soils provided by HYDRUS-1D soil catalog (Carsel & Parrish, 1988). The parameters and conditions applied during the simulation process are summarized in Table 2.

Contrary to Rahmati et al. (2020), who simulated the infiltration process up to 240 h, in this study we limited the simulations to the duration of 24 h, as infiltration measurements rarely last more than 24 h. We also assumed that the water level in the system can be measured with a time interval of 1 min, in line with most experimental devices and protocols (Angulo-Jaramillo et al., 2000, 2016; Pakparvar et al., 2018; Rezaei et al., 2016, 2020). We account for this by rounding up the simulated infiltration time data expressed in minutes to...
Average values of soil hydraulic parameters of van Genuchten (VG) model (van Genuchten, 1988) used as synthetic data, as well as the sorptivity ($S$) data being obtained from the horizontal infiltration simulation (Rahmati et al., 2020). Note that the shape parameter $l$ is fixed at 1/2 for the hydraulic conductivity.

| Soil              | Parameter | $u_0$ | $u_s$ | $u_i$ | $\alpha$ | $n$ | $m$ | $K_s$ | $S$ | $\beta$ |
|-------------------|-----------|-------|-------|-------|-----------|-----|-----|-------|-----|---------|
|                   | cm$^3$ cm$^{-3}$ | cm$^{-1}$ | cm$^{-1}$ | cm h$^{-1}$ | cm h$^{-1/2}$ |
| Clay              | 0.068     | 0.380 | 0.271 | 0.008 | 1.09      | 0.083 | 0.20 | 1.02  | 1.92 |
| Clay loam         | 0.095     | 0.410 | 0.150 | 0.019 | 1.31      | 0.237 | 0.26 | 1.46  | 1.58 |
| Loam              | 0.078     | 0.430 | 0.088 | 0.036 | 1.56      | 0.359 | 1.04 | 2.20  | 1.27 |
| Loamy sand        | 0.057     | 0.410 | 0.057 | 0.124 | 2.28      | 0.561 | 14.6 | 6.22  | 0.80 |
| Sand              | 0.045     | 0.430 | 0.045 | 0.145 | 2.68      | 0.627 | 29.7 | 9.23  | 0.60 |
| Sandy clay        | 0.100     | 0.380 | 0.170 | 0.027 | 1.23      | 0.187 | 0.12 | 0.79  | 1.70 |
| Sandy clay loam   | 0.100     | 0.390 | 0.111 | 0.059 | 1.48      | 0.324 | 1.31 | 1.61  | 1.36 |
| Sandy loam        | 0.065     | 0.410 | 0.066 | 0.075 | 1.89      | 0.471 | 4.42 | 3.84  | 0.99 |
| Silt              | 0.034     | 0.460 | 0.090 | 0.016 | 1.37      | 0.270 | 0.25 | 1.35  | 1.50 |
| Silt loam         | 0.067     | 0.450 | 0.104 | 0.020 | 1.41      | 0.291 | 0.45 | 1.66  | 1.44 |
| Silt clay         | 0.070     | 0.360 | 0.266 | 0.005 | 1.09      | 0.083 | 0.02 | 0.35  | 1.92 |
| Silty clay loam   | 0.089     | 0.430 | 0.197 | 0.010 | 1.23      | 0.187 | 0.07 | 0.53  | 1.70 |

Note: $u_0$, $u_s$, and $u_i$ are saturated, residual, and initial water contents; $\alpha$, $n$, and $m$ are parameters of the van Genuchten (1980) model; $K_s$ is saturated hydraulic conductivity; $S$ is soil sorptivity; $\beta$ is an infiltration constant defined by Haverkamp et al. (1994).

In addition to original simulated curves, similarly to Rahmati et al. (2020), we also added random noises (considering maximum error values of 5, 10, and 20%) to the original synthetic data to provide an additional dataset with more realistic features for performance evaluation. To get more “realistic” time series data assimilable with the measurement process, we defined the error for the infiltration rate and then propagated it to the cumulative infiltration, as suggested by Rahmati et al. (2020). We first obtained the infiltration rate ($i$) by differentiating the cumulative infiltration ($I$) with respect to time, and then the random noise was added on the infiltration rates as below (Rahmati et al., 2020):

$$I_{\text{noised}}(j) = I_{\text{noised}}(j - 1) + i_{\text{noised}}(j) \times [\lambda(j) - \lambda(j - 1)]$$

(23)

where $j (= 1, 2, \ldots, n)$ refers to points in infiltration curves.

True or known values of $K_s$ for synthetic data are taken from HYDRUS-1D soil catalog (Carsel & Parrish, 1988), whereas we inferred the true or known values of $S$ by numerically integrating Boltzmann variable, $\lambda(\theta)$, simulated by HYDRUS-1D for infiltration without gravity (Philip, 1957):

$$S = \int_{\theta_i}^{\theta_s} \lambda(\theta) d\theta$$

(24)

where $\theta_i$ [L$^3$ L$^{-3}$] and $\theta_s$ [L$^3$ L$^{-3}$] are the initial and saturated volumetric water contents, respectively, and $\lambda(\theta)$ [L T$^{-1/2}$] is defined as

$$\lambda(\theta) = Z(\theta, t) t^{-1/2}$$

(25)

where the characteristic function $Z(\theta, t)$ quantifies the depth at which the volumetric water content equates to $\theta$ at time $t$. As such, the values of $K_s$ and $S$ can be considered as utterly known.

2.2.2 Experimental data

As experimental data, the 1D infiltration data available in the SWIG database (Rahmati, Weihermüller, Vanderborght, et al., 2018; Rahmati, Weihermüller, & Vereecken, 2018) were used to verify the proposed methodology. The selected 1D
infiltration data from SWIG database were obtained from infiltration experiments using a zero water potential imposed at the surface. Overall, we selected 648 infiltration curves for final analysis.

In the case of experimental data, we inferred the benchmark $S$ and $K_s$ values by fitting experimental cumulative infiltrations to the QEI model of Haverkamp et al. (1994) using the online website developed by Latorre et al. (2015): http://swi.csic.es/infiltration-map/. The website was initially developed to estimate hydraulic properties from disc infiltrometer 3D infiltration curves (Latorre et al., 2015), making use of the extension of the QEI model to 3D proposed by Smettem et al. (1994). It also includes the fit of the QEI model on 1D experimental infiltration curves. In this study, we considered these values of $S$ and $K_s$ as the benchmark for the following reasons. Firstly, the SWIG database does not provide $S$ values for the selected soils, as that parameter is not measured in field campaigns. Secondly, even for $K_s$ that may be measurable, it is not only reported for a limited number of selected soils, but even the measured $K_s$ values are not representative of the real soil behaviors under infiltration measurement conditions. Several investigations have already shown that $K_s$ values measured in small cylinders significantly differ from those obtained from infiltration measurements because sampling procedures do not ensure the representativeness of samples and may ignore processes like preferential flow that may drive infiltration on the field. Such inconsistency has already confirmed by the analysis of SWIG database (Rahmati, Weihermüller, Vanderborght, et al., 2018).

### 2.3 Model comparisons

We compared SCTM with the original CTM (Rahmati et al., 2020), the Sharma et al. (1980) method (SH), and a non-linear curve-fitting method (Bonell & Williams, 1986; Bris-tow & Savage, 1987; Marquardt, 1963; Vandervaere et al., 2000) with two- (CF2, Equation 26) and three-term (CF3, Equation 27) equations, regarding their ability to estimate $K_s$ and $S$.

\[
I(t) = S \sqrt{t} + \frac{2 - \beta}{3} K_s t \tag{26}
\]

\[
I(t) = S \sqrt{t} + \frac{2 - \beta}{3} K_s t + \frac{1}{9} (\beta^2 - \beta + 1) \frac{K_s^2}{S} t^{3/2} \tag{27}
\]

where $\beta = 0.6$.

In the case of SH method, we estimated $S$ by linear fitting of $I$ vs. $\sqrt{t}$ for data points shorter than 30 min, as suggested by Rahmati et al. (2020). In this approach, the slope provides $S$ when the intercept is set equal to 0. For $K_s$ predictions, we set it to be $K_s = \lim_{t \to t_{end}} (\Delta I / \Delta t)$. In the case of CF2 and CF3, the “lsqnonlin” function is used for the fit and the optimization of $S$ and $K_s$.

### 2.4 Statistical analysis and evaluation of the estimated hydraulic parameters

The accuracy of the SCTM and the other selected methods was evaluated using the RMSE and Nash and Sutcliffe (1970), $E$, criteria between measured and predicted $K_s$ and $S$ values:

\[
RMSE = \sqrt{\frac{\sum (X_m - X_p)^2}{n}} \tag{28}
\]

\[
E = 1 - \frac{\sum (X_m - X_p)^2}{\sum (X_m - \bar{X}_m)^2} \tag{29}
\]

where $X_m$ and $X_p$ are the logarithmic values of known and predicted parameters ($S$ and/or $K_s$), respectively. The values of RMSE and $E$ near 0 and unity, respectively, denote a great accuracy.
For the assessment and comparisons of the above methods, we used synthetic infiltration curves with a duration between 15 min and 10 h, corresponding to typical infiltration experiments (Angulo-Jaramillo et al., 2000; Rahmati, Weihermüller, Vanderborght, et al., 2018; Rahmati, Weihermüller, & Vereecken, 2018).

A quantile–quantile (q-q) plot was used to examine graphically if both predicted and true (known) or benchmark populations could be fitted with the same distribution (Wilk & Gnanadesikan, 1968). In other words, from a statistical point of view, a probability q-q plot is a graphical method aimed to compare two probability distributions of two different populations by plotting their quantiles against each other (Wilk & Gnanadesikan, 1968). A vector of 5, 25, 50, 75, and 95 probability levels was used to produce q-q plots.

3 RESULTS AND DISCUSSION

In this section, we use the synthetic and real data to analyze important features of the SCTM, including the dependence of the accuracy in estimating $S$ and $K_s$ on infiltration duration, the impact of experimental errors on SCTM’s accuracy in estimating $S$ and $K_s$, and its applicability to real experimental data.

3.1 Performance of SCTM as a function of experimental duration

We evaluated the performance of SCTM using error-free synthetic and experimental data in function of infiltration duration. For that purpose, the synthetic data were truncated to have different durations of 15 min to 24 h, while the entire curves of experimental data were used for performance evaluation and considered as the benchmark. As shown in Figure 2 and Table 3, the proposed method provided accurate estimates of $S$ and $K_s$ when applied to synthetic data lasting from 15 min to 24 h. The accuracy of the method in estimating $S$ is time independent, as it always uses the second data point of the cumulative infiltration curve to predict $S$. Although the method leads to a time-dependent accuracy for $K_s$ predictions, it has relatively high accuracy for all infiltration durations lasting for 15 min up to 24 h, showing $E$ values higher than 0.7 and RMSE values lower than 0.7 cm h$^{-1}$ (Figure 2). The need for longer time series stems from the fact that the method works better when the steady-state regime is included in the estimation procedure as in this case, the gravity time $T_{grav}$ is included in the infiltration dataset, and $\omega = 0.5$. Figure 3 shows linear q-q plots of the quantiles of the true and predicted $K_s$ and $S$, indicating that both samples come from the same log-normal distributions.

Applying the proposed method over experimental data, we found high accuracies in estimating $S$ and $K_s$ as shown in Table 3 and Figure 4, showing RMSE values of 0.64 cm h$^{-1}$ and 0.35 cm h$^{-1/2}$ and $E$ values of 0.81 and 0.73, in the case of $K_s$ and $S$ predictions, respectively. We also computed the order of magnitudes of difference between predicted and true or known values of $K_s$ ($\Delta K$) and $S$ ($\Delta S$), and a histogram was produced (Figure 5). As seen from Figure 5, overall, more than 86 and 94% of soils show a difference, with order of magnitude of $-1$ to $1$ in the case of $K_s$ and $S$, respectively. In a similar manner to synthetic data, Figure 6 also shows a linear q-q plot of the quantiles of the true and predicted $K_s$ and $S$ for the experimental data, indicating that both samples come from the same log-normal distributions.
3.2 Models comparison using error-free and noised synthetic infiltration curves

We compared the accuracy of the SCTM in estimating \( S \) and \( K_s \) with estimates obtained from CTM as well as SH, CF2, and CF3. In addition to the original error-free data, random noises were added to synthetic infiltration data, and predictions were repeated to evaluate the effects of error in infiltration data on predictions. Table 4 reports the average RMSE and \( E \) values obtained between known and predicted values of \( K_s \) and \( S \) for all the methods and data types.

Comparing SCTM with CTM shows that in the case of error-free data, the SCTM works even better than CTM for \( K_s \) predictions, with an average RMSE value of 0.38 \( \pm \) 0.19 cm h\(^{-1} \) vs. 0.57 \( \pm \) 0.20 cm h\(^{-1} \) and an \( E \) value of 0.89 \( \pm \) 0.11 vs. 0.77 \( \pm \) 0.16. Both methods (SCTM and CTM) have similar accuracy for \( S \) predictions, with an average RMSE value of 0.11 cm h\(^{-1/2} \) and \( E \) value of 0.94.

Overall, CF3 was found to be the best at estimating both \( S \) and \( K_s \) when using error-free data, with an average RMSE value of 0.28 \( \pm \) 0.10 cm h\(^{-1} \) for \( K_s \) predictions and 0.05 \( \pm \) 0.03 cm h\(^{-1/2} \) for \( S \) predictions. In the case of noised data, both CF2 and CF3 resulted in less accurate predictions of \( K_s \), showing average RMSE values higher than 1.73 cm h\(^{-1} \) and \( E \) values lower than 0, though they still outperformed (particularly CF3) in the case of \( S \) predictions. The CTM was better than other methods for \( K_s \) predictions when applied to noised data, showing average RMSE values lower than 0.59 cm h\(^{-1/2} \) and \( E \) values higher than 0.77. Although, the SCTM ranked third after CTM and SH for \( K_s \) prediction, it still has a relatively high accuracy and satisfactory predicted \( K_s \), showing an average RMSE value lower than 0.75 cm h\(^{-1} \) and \( E \) value higher than 0.61. All applied methods were accurate enough for \( S \) predictions, showing \( E \) values higher than 0.85 and RMSE values lower than 0.17 cm h\(^{-1/2} \). In other words, CTM and SCTM methods were quite competitive when dealing...
with noised data, whereas CF2 and CF3 failed to deal with data contaminated with noise. These two methods should be avoided when using experimental data to estimate $S$ and $K_s$.

### 3.3 Models comparison using experimental data

In this section, we evaluated the accuracy of the SCTM in comparison with the other methods when applied to experimental data. The results are illustrated in Figures 7 and 8 and are summarized in Table 5. The results revealed that the accuracy of the SCTM in estimating $S$ and $K_s$ is comparable with the accuracy of the CTM, as well as the classical method of SH. It showed a slightly lower accuracy compared with CTM and SH, with an RMSE value of 0.64 cm h$^{-1}$ for $K_s$ predictions and 0.35 cm h$^{-1/2}$ for $S$ predictions. Both CF2 and CF3 methods failed in accurate predictions of $K_s$, showing an RMSE value of around 2 cm h$^{-1}$ and negative $E$ values. However, CF3 placed second in the rank for $S$ predictions, showing an RMSE value of 0.27 cm h$^{-1/2}$ and $E$ value of 0.83.
FIGURE 6  Quantile–quantile plots of quantiles (5, 25, 50, 75, and 95%) of true and predicted values of (a) sorptivity ($S$ and $\hat{S}$) and (b) saturated hydraulic conductivity ($K_s$ and $\hat{K_s}$) for experimental data. Both true and predicted values are shown in logarithmic scale.

TABLE 4  Average RMSE between known and predicted saturated hydraulic conductivity, $K_s$, and sorptivity, $S$, values using the simplified (SCTM) and original (CTM) characteristic time method, Sharma method (SH), and nonlinear curve fitting method of two- (CF2) and three-term (CF3) equations over original (error-free) and noised synthetic data for all soils

| Data                  | Method | $\log_{10}(K_s)$ cm h$^{-1}$ | $\log_{10}(S)$ cm h$^{-1/2}$ | $E$ $\log_{10}(K_s)$ cm h$^{-1}$ | $\log_{10}(S)$ cm h$^{-1/2}$ |
|-----------------------|--------|-------------------------------|-------------------------------|----------------------------------|-------------------------------|
| Error-free data       | SCTM   | 0.376 ± 0.187                 | 0.112 ± 0.000                 | 0.889 ± 0.107                    | 0.935 ± 0.000                 |
|                       | CTM    | 0.568 ± 0.200                 | 0.107 ± 0.000                 | 0.769 ± 0.160                    | 0.941 ± 0.000                 |
|                       | SH     | 0.552 ± 0.220                 | 0.122 ± 0.015                 | 0.776 ± 0.173                    | 0.923 ± 0.016                 |
|                       | CF2    | 0.315 ± 0.099                 | 0.156 ± 0.072                 | 0.931 ± 0.045                    | 0.851 ± 0.119                 |
|                       | CF3    | 0.282 ± 0.095                 | 0.053 ± 0.026                 | 0.944 ± 0.039                    | 0.983 ± 0.014                 |
| Noised data (5%)      | SCTM   | 0.749 ± 0.209                 | 0.147 ± 0.000                 | 0.613 ± 0.221                    | 0.889 ± 0.000                 |
|                       | CTM    | 0.463 ± 0.202                 | 0.123 ± 0.003                 | 0.839 ± 0.131                    | 0.922 ± 0.003                 |
|                       | SH     | 0.585 ± 0.241                 | 0.130 ± 0.012                 | 0.746 ± 0.202                    | 0.912 ± 0.015                 |
|                       | CF2    | 1.927 ± 0.668                 | 0.159 ± 0.069                 | −1.650 ± 1.83                    | 0.847 ± 0.119                 |
|                       | CF3    | 1.958 ± 0.512                 | 0.062 ± 0.019                 | −1.624 ± 1.22                    | 0.978 ± 0.012                 |
| Noised data (10%)     | SCTM   | 0.747 ± 0.213                 | 0.147 ± 0.000                 | 0.614 ± 0.225                    | 0.889 ± 0.000                 |
|                       | CTM    | 0.462 ± 0.199                 | 0.123 ± 0.003                 | 0.840 ± 0.131                    | 0.921 ± 0.004                 |
|                       | SH     | 0.585 ± 0.243                 | 0.131 ± 0.013                 | 0.746 ± 0.204                    | 0.910 ± 0.015                 |
|                       | CF2    | 1.733 ± 0.435                 | 0.159 ± 0.066                 | −1.048 ± 0.98                    | 0.845 ± 0.113                 |
|                       | CF3    | 1.946 ± 0.448                 | 0.062 ± 0.019                 | −1.558 ± 1.04                    | 0.979 ± 0.012                 |
| Noised data (20%)     | SCTM   | 0.753 ± 0.209                 | 0.147 ± 0.000                 | 0.609 ± 0.221                    | 0.889 ± 0.000                 |
|                       | CTM    | 0.585 ± 0.118                 | 0.123 ± 0.003                 | 0.771 ± 0.094                    | 0.922 ± 0.004                 |
|                       | SH     | 0.591 ± 0.243                 | 0.130 ± 0.012                 | 0.741 ± 0.206                    | 0.913 ± 0.014                 |
|                       | CF2    | 1.751 ± 0.475                 | 0.172 ± 0.064                 | −1.109 ± 1.27                    | 0.828 ± 0.109                 |
|                       | CF3    | 2.046 ± 0.569                 | 0.064 ± 0.018                 | −1.886 ± 1.54                    | 0.977 ± 0.102                 |
FIGURE 7  Quantile–quantile plots of quantiles (0, 1, 5, 10, 25, 50, 75, 90, 95, and 100%) of logarithms of benchmark and predicted saturated hydraulic conductivity, $K_s$, values using the simplified (SCTM) and original (CTM) characteristic time method and Sharma et al. (1980) (SH) method, as well as curve fitting methods of two- (CF2) and three-term (CF3) equations over the data selected from the Soil Water Infiltration Global (SWIG) database.

FIGURE 8  Quantile–quantile plots of quantiles (0, 1, 5, 10, 25, 50, 75, 90, 95, and 100%) of logarithms of the benchmark and predicted sorptivity, $S$, values using the simplified (SCTM) and original (CTM) characteristic time method and Sharma et al. (1980) (SH) method, as well as curve fitting methods of two- (CF2) and three-term (CF3) equations over the data selected from the Soil Water Infiltration Global (SWIG) database.
The SCTM, CTM, and SH methods not only provided higher accuracy in $S$ and $K_s$ predictions compared with CF2 and CF3, but they also provided a constrained fitting of the parameters ensuring that predictions will be within the physically meaningful range. Conversely, this is not the case for CF2 and CF3.

4 | CONCLUSIONS

In this paper, we simplified the characteristic time method (CTM), which was proposed based on the concept of characteristic time ($t_{char}$) to estimate the soil sorptivity, $S$, and the saturated hydraulic conductivity, $K_s$. A novel method, coined SCTM (simplified characteristic time method), maintains all the features of the original CTM but does not require the complex iteration procedure needed to estimate $S$ and $K_s$. The SCTM uses the first point of the cumulative infiltration to estimate $S$ and the last data point to predict $K_s$, rather than iterating over all data points, as is done in CTM. The SCTM performance and accuracy was analyzed using simulated and experimental infiltration curves and compared with CTM, as well as typically used methods in the literature, including nonlinear curve fitting of two- (CF2) and three-term (CF3) equations, and the methods proposed by Sharma et al. (1980) (SH). The HYDRUS-1D-simulated synthetic infiltration curves provided as supplemental material by Rahmati et al. (2020), as well as experimental data selected from the SWIG database (Rahmati, Weihermüller, Vanderborght, et al., 2018; Rahmati, Weihermüller, & Vereecken, 2018), were used to evaluate the applied methods performances. The SCTM provided accurate predictions of $K_s$ and $S$, being comparable with the original CTM when tested against both synthetic and experimental data. The SCTM benefits from all advantages of the original CTM method, while it is much simpler than the original method and more practical. The SCTM was found to be applicable to any infiltration duration within a typical time window used for infiltration experiments (15 min to 10 h), and even up to 24 h.

AUTHOR CONTRIBUTIONS

Mehdi Rahmati: Conceptualization; Data curation; Formal analysis; Investigation; Methodology; Validation; Visualization; Writing-original draft; Writing-review & editing. Meisam Rezaei: Writing-original draft; Writing-review & editing. Renato Morbidelli: Writing-original draft; Writing-review & editing. Harry Vereecken: Project administration; Supervision; Writing-original draft; Writing-review & editing.

CONFLICT OF INTEREST

The authors declare no conflict of interest.

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