Research Article

Computation of Irregularity Indices of Certain Computer Networks

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A graph is said to be a regular graph if all its vertices have the same degree; otherwise, it is irregular. In general, irregularity indices are used for computational analysis of nonregular graph topological composition. The creation of irregular indices is based on the conversion of a structural graph into a total count describing the irregularity of the molecular design on the map. It is important to be notified how unusual a molecular structure is in various situations and problems in structural science and chemistry. In this paper, we will compute irregularity indices of certain networks.

1. Introduction

In mathematics, graph theory can be used to describe different types of graphs that are computational structures. It is also used to model sensible item connections [1, 2]. An irregularity index is a statistical value connected with a graph that defines a graph’s irregularity. The theory of networks is a part of computer science and network engineering graph theory.

A topological invariant \( \text{TOP}(G) \) is referred to as a graph \( G \) irregularity index if the topological invariant \( \text{TOP}(G) \geq 0 \) and \( \text{TOP}(G) = 0 \) if \( G \) is a regular graph. However, if all its vertices have the same degree, it is said that the graph is regular. Topological invariant \( \text{TOP}(G) \) is a numeric value of a molecular structure of a chemical compound. Nonetheless, the creation of irregular indices is based on the conversion of a structural graph into a total count describing the irregularity of the molecular design on the map [3, 4]. Many networks including silicate, chain silicate, oxide, hexagonal, and honeycomb networks are identical to networks of atomic or chemical structure. There are very important unusual characteristics in such networks.

In this paper, we are concerned with simple connected graphs symbolized by \( G(V, E) \), where \( V(G) \) and \( E(G) \) represent the set of vertices and edges of \( G \), respectively. The degree of a vertex \( u \) of a graph \( G \) is the count of first neighbors of \( u \). And \( uv \) represents an edge for \( G \), connecting vertices \( u \) and \( v \) [5, 6].

Graph theory was established in 1736 when Leonard Euler presented “Solutio problematic as situspertinentis geometries” (the solution of a problem related to place theory).
Wiener is the pioneer of topological indices; he discovered the first topological index and found out the boiling point of a compound (paraffin, a member of the alkane family) in 1947. It was named as path number, but latterly, it was renamed as the Wiener index [7].

\[ W(G) = \frac{1}{2} \sum_{u,v \in V(G)} (d_u + d_v). \]  

(1)

Wiener also invented the Wiener polarity index. Milan Randić invented the first and oldest degree-based index named as the Randic index in 1975 [8]:

\[ R_{-1/2}(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u d_v}}. \]  

(2)

The graph invariant denoted by \( M_1(G) \) is called the first Zagreb index, which is equal to the sum of square of the degrees of the vertices of a graph; it was introduced by Trinjastic and Gutman in 1972 [9]. \( M_1(G) \) is linked with sum of quantities in the field of chemical graph theory. \( M_1(G) \) is known as the Gutman index; \( M_1(G) \) is bounded and attains lower and upper bound [10]:

\[ M_1(G) = \sum_{uv \in E(G)} (d_u + d_v). \]  

(3)

The second Zagreb index is a graph invariant denoted by \( M_2(G) \) which is defined as the aggregate of the product of degrees of connected pairs of vertices of the molecular compound, and it was introduced by Trinjastic and Gutman in 1972.

\[ M_2(G) = \sum_{uv \in E(G)} (d_u \cdot d_v). \]  

(4)

Bo-Zhou and Ivan Gutman presented the upper bound for these Zagreb indices w.r.t the min-max degree [11, 12].

Nevertheless, the emergence of new topological descriptors with extremely detecting capacity retains a concern without deformation for the scientific world [13, 14]. Therefore, there is a great willingness to change novel graph invariants with enormous detecting power combined with insignificant degeneration. In this paper, we compute irregularity indices for certain networks.

2. Irregularity Indices

All these selected irregularity indices belong to the family of degree-based topological indices. Tamas Reti et al. selected these irregularity indices as a molecular descriptor in the QSPR study to predict physicochemical properties of octane isomers [15]. The selected irregularity indices for certain networks are represented by

\[ \begin{align*}
VAR(G) &= \frac{M_1(G)}{n} - \left( \frac{2m}{n} \right)^2, \\
AL(G) &= \sum_{uv \in E} |d_u - d_v|, \\
IR1(G) &= F(G) - \frac{2m}{n} M_1(G), \\
IR2(G) &= \sqrt{\frac{M_2(G)}{m}} - \frac{2m}{n}, \\
IRF(G) &= F(G) - 2M_2(G), \\
IRFW(G) &= \frac{IRF(G)}{M_2(G)}, \\
IRA(G) &= \sum_{uv \in E} \left( d_u^{-1/2} - d_v^{-1/2} \right)^2, \\
IRB(G) &= \sum_{uv \in E} \left( d_u^{1/2} - d_v^{1/2} \right)^2, \\
IRC(G) &= \sum_{uv \in E} \sqrt{d_u d_v} - \frac{2m}{n}, \\
IRD1(G) &= \sum_{uv \in E} \ln \left( 1 + |d_u - d_v| \right), \\
IRGA(G) &= \sum_{uv \in E} \ln \frac{d_u + d_v}{(2 \sqrt{d_u d_v})}.
\end{align*} \]

(5)

3. Main Results and Discussion

In this section of the paper, we will discuss about the silicate network, chain silicate network, oxide network, hexagonal network, and honeycomb network briefly and will compute the irregularity indices for these networks.
3.1. Silicate Network. Silicates are the most popular, significant, and most complex mineral class. Tetrahedron \((\text{SiO}_4)\) is the basic chemical unit of silicates. By fusing metal oxides or metal carbonates with sand, silicates are produced. The fact is that all silicates contain tetrahedron of \(\text{SiO}_4\). For chemistry, the corner vertices of tetrahedron \(\text{SiO}_4\) reflect oxygen ions, and the silicon ion is represented by the middle vertex. We name the corner vertices as oxygen nodes in graph theory and the middle vertex as the silicon node. Although the tetrahedron is arranged linearly, chain silicates are produced (refer Figure 1).

The total number of vertices and edges in \(SL_n\) is \(15n^2 + 3n\) and \(36n^2\), respectively.

**Theorem 1.** The irregularity indices for the silicate network \((SL_n)\) for \(n > 1\) are

- \(VAR(SL_n) = \frac{54n^2 + 16n - 22}{(5n + 1)^2}\)
- \(AL(SL_n) = 3n(36n + 6)\)
- \(IR1(SL_n) = \frac{9072n^3 - 3690n^2 + 486n}{5n + 1}\)
- \(IR2(SL_n) = \sqrt{\frac{(324n - 90)}{12n}} - \frac{24n}{5n + 1}\)
- \(IRF(SL_n) = 3n(54n + 18)\)
- \(IRFW(SL_n) = \frac{54n + 18}{324n - 90}\)
- \(IRA(SL_n) = 0.1716n(3n + 1),\)
- \(IRB(SL_n) = 3.0883n(3n + 1),\)
- \(IRC(SL_n) = \frac{(30.7279n - 3.7279)}{6n} - \frac{24n}{5n + 1},\)
- \(IRDI(SL_n) = 27n^2 + 9n,\)
- \(IRL(SL_n) = 0.6931(18n^2 + 6n),\)
- \(IRLU(SL_n) = 6n(3n + 1),\)
- \(IRLF(SL_n) = 4.2426n(3n + 1),\)
- \(IRLA(SL_n) = 4n(3n + 1),\)
- \(IRD1(SL_n) = 8.3178n(3n + 1),\)
- \(IRGA(SL_n) = 0.3533n(3n + 1).\)

**Proof.** By using the edge partition based on degrees of end vertices of each edge of the silicate network \((SL_n)\) given in Table 1, we compute the irregularity indices of the silicate network \((SL_n)\), and the computations are given as follows:
\[
VAF(SL_n) = \frac{M_1(SL_n)}{n} - \left( \frac{2m}{n} \right)^2 = \frac{3n(12n^2 - 22)}{3n(5n + 1)} - \left( \frac{2(36n^2)}{3n(5n + 1)} \right)^2 = \frac{54n^4 + 16n - 22}{(5n + 1)^2},
\]
\[
AL(SL_n) = \sum_{uv \in E} |d_u - d_v| = [3 - 3](6n) + [3 - 6](18^2 + 6n) + [6 - 6](18n^2 - 12n) = 3n(36n + 6),
\]
\[
IR1(SL_n) = F(SL_n) - \frac{2m}{n} M_1(SL_n) = \left( 2106n^2 - 486n \right) - \frac{2(36n^2)(378n^2 - 66n)}{3n(5n + 1)} = \frac{9072n^4 - 3690n^2 + 486n}{5n + 1},
\]
\[
IR2(SL_n) = \sqrt{M_2(SL_n)} - \frac{2m}{n} = \sqrt{\frac{3n(324n - 90)}{36n^2} - \frac{2(36n^2)}{3n(5n + 1)}} = \sqrt{\frac{(324n - 90)}{12n} - \frac{24n}{5n + 1}}.
\]
\[
IRF(SL_n) = F(SL_n) - 2M_2(SL_n) = \left( 2106n^2 - 486n \right) - 2\left( 972n^2 - 270n \right) = 3n(54n + 18),
\]
\[
IRFW(SL_n) = \frac{IRF(SL_n)}{M_2(SL_n)} = \frac{3n(54n + 18)}{3n(324n - 90)} = \frac{54n + 18}{324n - 90}
\]
\[
IRA(SL_n) = \sum_{uv \in E} \left( d_u^{1/2} - d_v^{1/2} \right)^2 = \left( \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \right)^2 (6n) + \left( \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}} \right)^2 (18n^2 + 6n) + \left( \frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}} \right)^2 (18^2 - 12n) = 0.1716n(3n + 1),
\]
\[
IRB(SL_n) = \sum_{uv \in E} \left( d_u^{1/2} - d_v^{1/2} \right)^2 = (\sqrt{3} - \sqrt{3})^2 (6n) + (\sqrt{3} - \sqrt{6})^2 (18n^2 + 6n) + (\sqrt{6} - \sqrt{6})^2 (18n^2 - 6n) = 3.0883n(3n + 1),
\]
\[
IRC(SL_n) = \sum_{uv \in E} \sqrt{d_u d_v} - \frac{2m}{n} = \sqrt{\frac{6}{3}(6n)} + (18n^2 + 6n) \sqrt{18} + (18n^2 - 12n) \sqrt{36}
\]
\[
- \frac{72n^2}{3n(5n + 1)} - \frac{(30.7279n - 3.7279)}{6n} = \frac{24n}{5n + 1}
\]
\[
IRDIF(SL_n) = \sum_{uv \in E} \left| \frac{d_u}{d_v} - \frac{d_v}{d_u} \right| = (6n) \left| \frac{3}{3} - \frac{3}{3} \right| + (18n^2 + 6n) \left| \frac{5}{6} - \frac{6}{6} \right| + (18n^2 - 12n) \left| \frac{6}{6} - \frac{6}{6} \right| = 27n^2 + 9n,
\]
\[
IRL(SL_n) = \sum_{uv \in E} |\ln d_u - \ln d_v| = (6n) |\ln (3) - \ln (3)| + (18n^2 + 6n) |\ln (3) - \ln (6)|
\]
\[
+ (18n^2 - 12n) |\ln (6) - \ln (6)| = 0.6931(18n^2 + 6n),
\]
\[
IRLU(SL_n) = \sum_{uv \in E} \left| \min (d_u, d_v) \right| = (6n) \left| \frac{3 - 3}{3} \right| + (6n(3n + 1)) \left| \frac{3 - 6}{3} \right| + (18n^2 - 12n) \left| \frac{6 - 6}{6} \right| = 6n(3n + 1).
\]
Specific values of irregularity indices of $SL_n$ for different values of parameters are given in Table 2.

\[
IRL(SL_n) = \sum_{u\neq v \in E} \frac{|d_u - d_v|}{\sqrt{d_u d_v}} = (6n) \left(\frac{3 - 3}{\sqrt{9}} + (18n^2 + 6n)\frac{6 - 3}{\sqrt{18}} + (18n^2 - 12n)\frac{6 - 6}{\sqrt{36}}\right) = 4.2426n(3n + 1),
\]

\[
IRL (SL_n) = 2 \sum_{u \neq v \in E} \frac{|d_u - d_v|}{d_u + d_v} = 2 \left(\frac{3 - 3}{6} + 2(18n^2 + 6n)\frac{6 - 3}{9} + 2(18n^2 - 12n)\frac{6 - 6}{12}\right) = 4n(3n + 1),
\]

\[
IR D1 (SL_n) = \sum_{u \neq v \in E} \ln \left(1 + |d_u - d_v|\right) = \ln \left(1 + |3 - 3| \right)(6n) + \ln \left(1 + |3 - 6|\right)(18n^2 + 6n)
\]

\[+ \ln \left(1 + |6 - 6|\right)(18n^2 - 12n) = 8.3178n(3n + 1),\]

\[
IRGA (SL_n) = \sum_{u \neq v \in E} \ln \left(\frac{d_u + d_v}{\sqrt{d_u d_v}}\right) = (6n)\ln \left(1 + \frac{3}{2}\right)(18n^2 + 6n)\ln \left(\frac{3 + 6}{2\sqrt{18}} + (18n^2 - 12n)\ln \left(\frac{6 + 6}{2\sqrt{36}}\right)\right) = 0.3533n(3n + 1).
\]

3.2. Chain Silicate Network. Chain silicate network is obtained when tetrahedra are organized in a sequence. An $n$-dimensional chain silicate network is represented by $CS_n$, and it is generated by sequentially organizing $n$ tetrahedra. An $n$-dimensional chain silicate network is shown in Figure 2.

The edge partition of the chain silicate network is given in Table 3.

The total number of vertices and edges in $CS_n$ is $3n + 1$ and $6n$, respectively.

**Theorem 2.** The irregularity indices for the chain silicate network ($CS_n$) for $n > 1$ are

\[
VAR(CS_n) = \frac{18n^2 - 18}{(3n + 1)^2},
\]

\[
AL(CS_n) = 12n - 6,
\]

\[
IR1(CS_n) = \frac{162(n^2 - 1)}{3n + 1},
\]

\[
IR2(CS_n) = \sqrt{\frac{9(13n - 8)}{6n}} \frac{12n}{3n + 1},
\]

\[
IRF(CS_n) = 36n - 18,
\]

\[
IRFW(CS_n) = \frac{4n - 2}{13n - 8},
\]

\[
IRA(CS_n) = 0.02857(4n - 2),
\]

\[
IRB(CS_n) = 0.5147(4n - 2),
\]

\[
IRC(CS_n) = \frac{3(n + 4) + 3\sqrt{2}(4n - 2) + 6(n - 2)}{6n} - \frac{12n}{3n + 1},
\]

\[
IR D1 (CS_n) = 6n - 3,
\]

\[
IRL(CS_n) = 0.6931(4n - 2),
\]

\[
IRLIU(CS_n) = 4n - 2,
\]

\[
IRLF(CS_n) = \sqrt{2}(2n - 1),
\]

\[
IRLA(CS_n) = \frac{4}{3}(2n - 1),
\]

\[
IR D1 (CS_n) = (4n - 2)\ln 4,
\]

\[
IRGA(CS_n) = (2n - 1)\ln \frac{3}{\sqrt{2}}.
\]

**Proof.** By using the edge partition based on degrees of end vertices of each edge of the chain silicate network ($CS_n$) given in Table 3, the computations for irregularities indices are given as follows:
VAR(CS_n) = \frac{M_1(CS_n)}{n} - \frac{(2m^2)}{n} = \frac{18(3n-1)}{3n+1} - \frac{(2(6n))}{3n+1}^2 = \frac{18n^2 - 18}{(3n+1)^2}.

AL(CS_n) = \sum_{uv \in E} |d_u - d_v| = |3-3|(n+4) + |3-6|(4n-2) + |6-6|(n-2) = 12n - 6,

IR1(CS_n) = F(CS_n) - \frac{2m}{n} M_1(CS_n) = (270n - 162) - \frac{(6n)(2)(54n-18)}{3n+1} = \frac{162n^2 - 162}{3n+1},

IR2(CS_n) = \sqrt{\frac{M_1(CS_n)}{m}} - \frac{2m}{n} = \sqrt{\frac{9(13n - 8)}{6n}} - \frac{2(6n)}{3n+1} = \sqrt{\frac{9(13n - 8)}{6n}} - \frac{12n}{3n+1}.

IRF(CS_n) = F(CS_n) - 2M_2(CS_n) = (270n - 162) - 18(13n - 8) = 36n - 18,

IRFW(CS_n) = \frac{IRF(CS_n)}{M_2(CS_n)} = \frac{18(2n - 1)}{9(13n - 8)} = \frac{4n - 2}{13n - 8}.

IRA(CS_n) = \sum_{uv \in E} (d_u^{1/2} - d_v^{1/2})^2 = \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}}\right)^2 (n+4) + \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}\right)^2 (4n-1)

+ \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}}\right)^2 (n-2) = 0.02857 (4n - 2),

IRB(CS_n) = \sum_{uv \in E} (d_u^{1/2} - d_v^{1/2})^2 = (\sqrt{3} - \sqrt{3})^2 (n+4) + (\sqrt{3} - \sqrt{6})^2 (4n-2) (\sqrt{6} - \sqrt{6})^2 (n-2) = 0.5147 (4n-2),

IRC(CS_n) = \sum_{uv \in E} \sqrt{d_u d_v} - \frac{2m}{n} = \frac{\sqrt{9} (n+4) + (4n-2) \sqrt{18} + (n-2) \sqrt{36}}{6n} - \frac{12n}{3n+1}

= \frac{3(n+4) + 3\sqrt{2} (4n-2) + 6(n-2)}{6n} - \frac{12n}{3n+1}.

IRD1(CS_n) = \sum_{uv \in E} \left(\frac{d_u - d_v}{\min(d_u, d_v)}\right) = (n+4) \left[\frac{3 - 3}{3} + (4n - 2) \frac{3 - 6}{3} + (n - 2) \frac{6 - 6}{6}\right] = 6n - 3,

IRL(CS_n) = \sum_{uv \in E} |\ln d_u - \ln d_v| = (n+4) |\ln(3) - \ln(3)| + (4n - 2) |\ln(3) - \ln(6)| + (n - 2) |\ln(6) - \ln(6)| = 0.6931 (4n - 2),

IRL(CS_n) = \sum_{uv \in E} \frac{|d_u - d_v|}{\min(d_u, d_v)} = (n+4) \left[\frac{3 - 3}{3} + (4n - 2) \frac{3 - 6}{3} + (n - 2) \frac{6 - 6}{6}\right] = 4n - 2,

IRLF(CS_n) = \sum_{uv \in E} \frac{|d_u - d_v|}{\sqrt{d_u d_v}} = (n+4) \left[\frac{3 - 3}{\sqrt{9}} + (4n - 2) \frac{6 - 3}{\sqrt{18}} + (n - 2) \frac{6 - 6}{\sqrt{36}}\right] = \sqrt{2} (2n - 1),

IRL(CS_n) = 2 \sum_{uv \in E} \frac{|d_u - d_v|}{d_u + d_v} = 2(n+4) \left[\frac{3 - 3}{6} + 2(4n - 2) \frac{6 - 3}{9} + 2(n - 2) \frac{6 - 6}{12}\right] = (2n - 1) \frac{4}{3},

IRD1(CS_n) = \sum_{uv \in E} \ln \{1 + |d_u - d_v|\} = \ln\{1 + |3 - 3|\} (n+4) + \ln\{1 + |3 - 6|\} (4n-2) + \ln\{1 + |6 - 6|\} (n-2) = (4n - 2) \ln 4,

IRGA(CS_n) = \sum_{uv \in E} \frac{d_u + d_v}{(2) \sqrt{d_u d_v}} = (n+4) \ln \frac{3 + 3}{(2) \sqrt{9}} + (4n - 2) \ln \frac{3 + 6}{(2) \sqrt{18}} + (n - 2) \ln \frac{6 + 6}{(2) \sqrt{36}} = (2n - 1) \ln \frac{3}{\sqrt{2}}.

(10)
Specific values of irregularity indices for different values of involved parameters for the chain silicon network are given in Table 4.

### 3.3. Oxide Network

Oxide networks play a crucial role in the analysis of silicate networks. If we detach silicone vertices out of a silicatenetwork, we get an oxide network \( \text{OX}_n \) that is referred to as an \( n \)-dimensional oxide network as shown in Figure 3.

There are two types of edge partition in the oxide network \( \text{OX}_n \) centered on degrees of end vertices. Table 5 shows the edge partition for \( \text{OX}_n \).

The total number of vertices and edges in \( \text{OX}_n \) is \( 9n^2 + 3n \) and \( 18n^2 \), respectively.

#### Theorem 3

The irregularity indices of oxide networks \( (\text{OX}_n) \) for \( n > 1 \) are

\[
\begin{align*}
\text{VAR}(\text{OX}_n) &= \frac{72n^2(3n - 1)}{9n^2 + 3n^2}, \\
\text{AL}(\text{OX}_n) &= 72n, \\
\text{IR1}(\text{OX}_n) &= \frac{144n(3n - 1)}{3n + 1}, \\
\text{IR2}(\text{OX}_n) &= \sqrt{\frac{16(3n - 1)}{3n} - \frac{12n}{3n + 1}}, \\
\text{IRF}(\text{OX}_n) &= 48n, \\
\text{IRFW}(\text{OX}_n) &= \frac{7}{24(3n - 1)}, \\
\text{IRA}(\text{OX}_n) &= 0.5136n, \\
\text{IRB}(\text{OX}_n) &= 4.1177n, \\
\text{IRC}(\text{OX}_n) &= 2\left(2\sqrt{2} + 6n - 4\right) - \frac{12n}{3n + 1}, \\
\text{IRD1}(\text{OX}_n) &= 18n, \\
\text{IRL}(\text{OX}_n) &= 8.3177n, \\
\text{IRLU}(\text{OX}_n) &= 12n, \\
\text{IRLF}(\text{OX}_n) &= \frac{12n}{\sqrt{2}}, \\
\text{IRLA}(\text{OX}_n) &= 8n, \\
\text{IR D1}(\text{OX}_n) &= 13.1833n, \\
\text{IRGA}(\text{OX}_n) &= 0.7068n.
\end{align*}
\]

Proof: By using the edge partition based on degrees of end vertices of each edge of the oxide network \( (\text{OX}_n) \) given in Table 5, we have the following computations for irregularities of the oxide network \( (\text{OX}_n) \):
Specific values of irregularity indices for different values of involved parameters for the oxide network are given in Table 6.

\[ IRL(OX_n) = \sum_{u \in E} |\ln d_u - \ln d_v| = 12n|\ln 2 - \ln 4| + 18n^2|\ln 4 - \ln 4| = 8.3177n, \]

\[ IRLU(OX_n) = \sum_{u \in E} \frac{|d_u - d_v|}{\min(d_u, d_v)} = 12n \frac{|2 - 4|}{2} + 18n^2 \frac{|4 - 4|}{4} = 12n, \]

\[ IRLF(OX_n) = \sum_{u \in E} \frac{|d_u - d_v|}{\sqrt{d_u d_v}} = 12n \frac{|4 - 2|}{\sqrt{8}} + 18n \frac{|4 - 4|}{\sqrt{16}} = \frac{12n}{\sqrt{2}}, \]

\[ IRLA(OX_n) = 2 \sum_{u \in E} \frac{|d_u - d_v|}{d_u + d_v} = 2(12n) \frac{|4 - 2|}{6} + 2(18n^2) \frac{|4 - 4|}{6} = 8n, \]

\[ IRD1(OX_n) = \sum_{u \in E} \ln\left[1 + |d_u - d_v|\right] = \ln[1 + |4 - 2|](12n) + \ln[1 + |4 - 4|](18n^2) = 13.1833n, \]

\[ IRGA(OX_n) = \sum_{u \in E} \ln\left(\frac{d_u + d_v}{2\sqrt{d_u d_v}}\right) = (12n)\ln\frac{4 + 2}{2\sqrt{8}} + (18n^2)\ln\frac{4 + 4}{2\sqrt{16}} = 0.7068n. \]
3.4. Hexagonal Network. It is well known that there are three normal plane tilings with the same regular polygon type as triangle, hexagon-shaped, and square structures. Triangular tiling is used in the design of hexagonal networks. An $n$-dimensional hexagonal network is commonly referred to as $HX_n$, where $n$ is each side’s number of hexagonal vertices. A hexagonal network $HX_n$ is shown in Figure 4.

Table 5 shows the edge partition of the hexagonal network ($HX_n$). The total number of vertices and edges in hexagonal networks ($HX_n$) is $3n^2 − 3n + 1$ and $9n^2 − 15n + 6$, respectively.
Theorem 4. The irregularity indices of the hexagonal network \((HX_n)\) for \(n > 1\) are

\[
\begin{align*}
\text{VAR}(HX_n) &= \frac{72n^3 + 243n^2 + 15n - 30}{(3n^2 - 3n + 1)^2}, \\
\text{AL}(HX_n) &= 6(4n - 3), \\
\text{IR}_1(HX_n) &= \frac{720n^3 - 2142n^2 + 1878n - 462}{3n^2 - 3n + 1}, \\
\text{IR}_2(HX_n) &= \sqrt{\frac{108n^2 - 268n + 156}{3n^2 - 5n + 2}} - \frac{18n^2 - 30n + 12}{3n^2 - 3n + 1}, \\
\text{IRF}(HX_n) &= 6(8n - 5), \\
\text{IRFW}(HX_n) &= \frac{(2)(8n - 5)}{(108n^2 - 268n + 156)}, \\
\text{IRA}(HX_n) &= 0.1008n - 0.0418, \\
\text{IRB}(HX_n) &= 2.4240n - 0.8981, \\
\text{IRC}(HX_n) &= \frac{4\sqrt{12} + \sqrt{18}(2) + (8n - 24) + (4n - 8)\sqrt{24} + (2)(n^2 - 33n + 30)}{3n^2 - 5n + 2} - \frac{18n^2 - 30n + 12}{3n^2 - 3n + 1}, \\
\text{IRDIF}(HX_n) &= 10n - 4, \\
\text{IRL}(HX_n) &= 4.8656n + 2.1201, \\
\text{IRLU}(HX_n) &= 6n - 2, \\
\text{IRLF}(HX_n) &= 4.8990n - 20.0913, \\
\text{IRLA}(HX_n) &= \frac{168n - 76}{35}, \\
\text{IRDII}(HX_n) &= 13.1833n - 9.7311, \\
\text{IRGA}(HX_n) &= 0.2449n - 0.0126.
\end{align*}
\]
Proof. By using the edge partition based on degrees of end vertices of each edge of the hexagonal network \((HX_n)\) given in Table 7, we have the following computations for irregularities of the hexagonal network \((HX_n)\):\n
\[
\begin{align*}
\text{VAR}(HX_n) &= \frac{M_1(HX_n)}{n} - \frac{(2m_n)^2}{n} = \frac{108n^3 - 228n + 114}{3n^3 - 3n + 1} - \frac{(2(9n^2 - 15n + 6))^2}{3n^3 - 3n + 1} = \frac{72n^3 + 243n^2 + 15n - 30}{(3n^3 - 3n + 1)^2}, \\
\text{AL}(HX_n) &= \sum_{u \in E} |d_u - d_v| = |3 - 4|(12) + |3 - 6|(6) + |4 - 4|(6n - 18) + |4 - 6|(12n - 24) \\
&\quad + |6 - 6|(9n^2 - 33n + 30) = 6(4n - 3), \\
\text{IR1}(HX_n) &= F(HX_n) - \frac{2m}{n}M_1(HX_n) = \left(648n^2 - 1560n + 906\right) - \frac{(2)(9n^2 - 15n + 6)(108n^2 - 228n + 114)}{3n^3 - 3n + 1} \\
&= \frac{720n^2 - 2142n^2 + 1878n - 462}{3n^3 - 3n + 1}, \\
\text{IR2}(HX_n) &= \sqrt{\frac{M_2(HX_n)}{m}} - \frac{2m_n}{n} = \sqrt{\frac{\left(324n^2 - 804n + 468\right)}{9n^3 - 15n + 6}} - \frac{2(9n^2 - 15n + 6)}{3n^3 - 3n + 1} \\
&= \sqrt{\frac{108n^2 - 268n + 156}{3n^3 - 5n + 2}} - \frac{18n^2 - 30n + 12}{3n^3 - 3n + 1}, \\
\text{IRF}(HX_n) &= F(HX_n) - 2M_2(HX_n) = \left(648n^2 - 1560n + 906\right) - 2\left(324n^2 - 804n + 468\right) = 6(8n - 5), \\
\text{IRFW}(HX_n) &= \frac{\text{IRF}(HX_n)}{M_2(HX_n)} = \frac{(6)(8n - 5)}{324n^2 - 804n + 468} = \frac{(2)(8n - 5)}{(108n^2 - 268n + 156)}, \\
\text{IRA}(HX_n) &= \sum_{u \in E} \left(d_u^{1/2} - d_v^{1/2}\right)^2 = \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}}\right)^2 (12) + \left(\frac{1}{\sqrt{3}} - \frac{1}{\sqrt{6}}\right)^2 (6) \\
&\quad + \left(\frac{1}{\sqrt{4}} - \frac{1}{\sqrt{6}}\right)^2 (6n - 18) + \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}}\right)^2 (12n - 24) + \left(\frac{1}{\sqrt{6}} - \frac{1}{\sqrt{6}}\right)^2 (9n^2 - 33n + 30) = 0.1008n - 0.0418, \\
\text{IRB}(HX_n) &= \sum_{u \in E} \left(d_u^{1/2} - d_v^{1/2}\right)^2 = \left(\sqrt{3} - \sqrt{4}\right)^2 (12) + \left(\sqrt{3} - \sqrt{6}\right)^2 (6) + \left(\sqrt{4} - \sqrt{6}\right)^2 (12n - 24) = 2.4240n - 0.8981, \\
\text{IRC}(HX_n) &= \sum_{u \in E} \sqrt{d_u d_v} - \frac{2m}{n} = \frac{\sqrt{12}(12) + (6)\sqrt{18} + (6n - 18)\sqrt{16} + (12n - 24)\sqrt{24} + (9n^2 - 33n + 30)(6)}{9n^3 - 15n + 6} \\
&\quad - \frac{(2)(9n^2 - 15n + 6)}{3n^3 - 3n + 1} = \frac{4\sqrt{12} + \sqrt{18}(2) + (8n - 24) + (4n - 8)\sqrt{24} + (9n^2 - 33n + 30)(6)}{3n^3 - 5n + 2} - \frac{18n^2 - 30n + 12}{3n^3 - 3n + 1}, \\
\text{IR D}(HX_n) &= \sum_{u \in E} \left|d_u - d_v\right| = (12)\left|\frac{3 - 4}{3}\right| + (6)\left|\frac{3 - 6}{3}\right| + (12n - 24)\left|\frac{4 - 6}{4}\right| = 10n - 4, \\
\text{IRL}(HX_n) &= \sum_{u \in E} \|\ln d_u - \ln d_v\| = (12)\|\ln (3) - \ln (4)\| + (6)\|\ln (3) - \ln (6)\| + (12n - 24)\|\ln (4) - \ln (6)\| = 4.8656n + 2.1201, \\
\text{IRLU}(HX_n) &= \sum_{u \in E} \left|\frac{d_u - d_v}{\min(d_u, d_v)}\right| = (12)\left|\frac{3 - 4}{3}\right| + (6)\left|\frac{3 - 6}{3}\right| + (12n - 24)\left|\frac{4 - 6}{4}\right| = 6n - 2, \\
\text{IRLF}(HX_n) &= \sum_{u \in E} \left|\frac{d_u - d_v}{\sqrt{d_u d_v}}\right| = (12)\left|\frac{3 - 4}{\sqrt{12}}\right| + (6)\left|\frac{6 - 3}{\sqrt{18}}\right| + (12n - 24)\left|\frac{6 - 4}{\sqrt{24}}\right| = 4.8990n - 20.0913, \\
\text{IRLA}(HX_n) &= 2\sum_{u \in E} \left|\frac{d_u - d_v}{d_u + d_v}\right| = 2(12)\left|\frac{4 - 3}{7}\right| + 2(6)\left|\frac{6 - 3}{9}\right| + 2(12n - 24)\left|\frac{6 - 4}{10}\right| = 168n - 76 \frac{35}{10}.
\end{align*}
\]

Specific values of irregularities of the hexagonal network for different values of involved parameters are given in Table 8.
Table 8: Values of irregularity indices for $HX_n$.

| Irregularity indices | $n = 1$ | $n = 2$ | $n = 3$ | $n = 4$ | $n = 5$ |
|----------------------|----------|----------|----------|----------|----------|
| $VAR(HX_n)$          | 30       | 31.5918  | 11.4848  | 6.2279   | 4.0633   |
| $AL(HX_n)$           | 6        | 30       | 54       | 78       | 102      |
| $IR1(HX_n)$          | $-6.0000$| 69.4286  | 280.7368 | 509.6757 | 743.9016 |
| $IR2(HX_n)$          | $\infty$| 0.1770   | 0.3896   | 0.3377   | 0.2846   |
| $IRF(HX_n)$          | 18       | 66       | 114      | 162      | 210      |
| $IRFW(HX_n)$         | $-1.5000$| 0.4231   | 0.1173   | 0.0665   | 0.0462   |
| $IRA(HX_n)$          | 0.0590   | 0.1598   | 0.2606   | 0.3614   | 0.4622   |
| $IRB(HX_n)$          | 1.5259   | 3.9499   | 6.3739   | 8.7979   | 11.2219  |
| $IRC(HX_n)$          | $\infty$| 0.1568   | 0.2887   | 0.3529   | 0.2146   |
| $IRD1(HX_n)$         | 6        | 16       | 26       | 36       | 46       |
| $IRL(HX_n)$          | 7.2455   | 7.6111   | 12.4767  | 17.3423  | 22.2079  |
| $IRLU(HX_n)$         | 4        | 10       | 16       | 22       | 28       |
| $IRLF(HX_n)$         | 4.8077   | 7.7067   | 12.6057  | 17.5047  | 22.4037  |
| $IRLA(HX_n)$         | 2.6286   | 7.4286   | 12.2286  | 17.0286  | 21.8286  |
| $IRD1(HX_n)$         | 3.4522   | 16.6355  | 29.8188  | 43.0021  | 56.1854  |
| $IRGA(HX_n)$         | 0.2323   | 0.4772   | 0.7221   | 0.9670   | 1.2119   |

3.5. Honeycomb Network. Honeycomb networks are commonly used as an illustration of benzoid hydrocarbons in chemistry in digital effects, cell phone transmitters, and image analysis. If we use dynamically hexagonal tiling in a specific pattern, honeycomb networks will be formed. An $n$-dimensional honeycomb network is referred to as $HC_n$, where $n$ is the number of hexagons between the hexagon core and the boundary. Honeycomb network $HC_n$ is built from $(HC_{n-1})$ by adding a hexagon sheet across the boundary of $HC_{n-1}$ (refer Figure 5).

Table 9 shows the edge partition of the honeycomb network $(HC_n)$. The total number of vertices and edges in the honeycomb network $(HC_n)$ is $6n^2$ and $9n^2 - 3n$.

Theorem 5. The irregularity indices of the honeycomb network $(HC_n)$ for $n > 1$ are

\[
VAR(HC_n) = \frac{n - 1}{n^2},
\]

\[
AL(HC_n) = 12(n - 1),
\]

\[
IR1(HC_n) = 30(n - 1),
\]

\[
IR2(HC_n) = \sqrt{\frac{(27n^2 - 21n + 2)}{n(3n - 1)}} - \frac{3n - 1}{n},
\]

\[
IRF(HC_n) = 12(n - 1),
\]

\[
IRFW(HC_n) = \frac{14(n - 1)}{(27n^2 - 21n + 2)},
\]

\[
IRA(HC_n) = 0.2016(n - 1),
\]

\[
IRB(HC_n) = 1.2120(n - 1),
\]

\[
IRC(HC_n) = \frac{4 + 4\sqrt{6}(n - 1) + 9n^2 - 15n + 6}{n(3n - 1)} - \frac{3n - 1}{n},
\]

\[
IRD1(HC_n) = 9.9996(n - 1),
\]

\[
IRL(HC_n) = 4.8660(n - 1),
\]

\[
IRLU(HC_n) = 6(n - 1),
\]

\[
IRLF(HC_n) = 4.8990(n - 1),
\]

\[
IRLA(HC_n) = 4.8000(n - 1),
\]

\[
IRD1(HC_n) = 8.3172(n - 1),
\]

\[
IRGA(HC_n) = 0.2448(n - 1).
\]
Proof. By using the edge partition based on degrees of end vertices of each edge of the honeycomb network \((HC_n)\) given in Table 9, we have the following computations for the irregularity indices of the honeycomb network \((HC_n)\):

\[
\text{VAR}(HC_n) = \frac{M_1(HC_n)}{n} \left( \frac{2m}{n} \right)^2 = \frac{3n(18n - 10)}{6n^2} - \left( \frac{2(9n^2 - 3n)}{6n^2} \right)^2 = \frac{36n^2(n - 1)}{6n^2} = \frac{n - 1}{n^2},
\]

\[
\text{AL}(HC_n) = \sum_{u \in E} |d_u - d_v| = |2 - 2(6)| + |2 - 3(12(n - 1))| + |3 - 3(9n^2 - 15n + 6)| = 12(n - 1),
\]

\[
\text{IR1}(HC_n) = F(HC_n) = 2M_2(HC_n) = (162n^2 - 114n) - \left( \frac{2(9n^2 - 3n)(54n^2 - 30)}{6n^2} \right) = 30(n - 1),
\]

\[
\text{IR2}(HC_n) = \sqrt{\frac{M_2(HC_n)}{m}} = \sqrt{\frac{2m}{n}} = \frac{2m}{n} = \frac{\sqrt{3(27n^2 - 21n + 2)}}{3n(3n - 1)} = \frac{2(3n)(3n - 1)}{6n^2} = \frac{(27n^2 - 21n + 2)}{n(3n - 1)} - \frac{3n - 1}{n},
\]

\[
\text{IRF}(HC_n) = F(HC_n) - 2M_2(HC_n) = (162n^2 - 114n) - 2(81n^2 - 63n + 6) = 12(n - 1),
\]

\[
\text{IRFW}(HC_n) = \frac{F(HC_n) - 2M_2(HC_n)}{M_2(HC_n)} = \frac{12(n - 1)}{81n^2 - 63n + 6} = \frac{14(n - 1)}{(27n^2 - 21n + 2)}.
\]

\[
\text{IRA}(HC_n) = \sum_{u \in E} \left( \frac{d_u^{1/2} - d_v^{1/2}}{2} \right)^2 = \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}} \right)^2 (6) + \left( \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right)^2 (12(n - 1))
\]

\[
+ \left( \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{3}} \right)^2 (9n^2 - 15n + 6) = 0.2016(n - 1),
\]

\[
\text{IRB}(HC_n) = \sum_{u \in E} \left( \frac{d_u^{1/2} - d_v^{1/2}}{2} \right)^2 = (\sqrt{2} - \sqrt{2})^2 (6) + (\sqrt{2} - \sqrt{3})^2 (12n - 12) + (\sqrt{3} - \sqrt{3})^2 (9n^2 - 15n + 6) = 1.2120(n - 1),
\]

\[
\text{IRC}(HC_n) = \sum_{u \in E} \frac{\sqrt{d_ud_v}}{m} = \frac{2m}{n} = \frac{\sqrt{4(6) + (12n - 12)\sqrt{6} + (9n^2 - 15n + 6)\sqrt{6}}}{3n(3n - 1)} = \frac{2(3n)(3n - 1)}{6n^2}
\]

\[
= \frac{4 + 4\sqrt{6}(n - 1) + 9n^2 - 15n + 6 - 3n - 1}{n},
\]

\[
\text{IRD}(HC_n) = \sum_{u \in E} \left| \frac{d_u - d_v}{d_u} \right| = (6)\left| \frac{2 - 2}{2} \right| + (12n - 12)\left| \frac{2 - 3}{3} \right| + (9n^2 - 15n + 6)\left| \frac{3 - 3}{3} \right| = 9.9996(n - 1),
\]

\[
\text{IRL}(HC_n) = \sum_{u \in E} \left| \ln d_u - \ln d_v \right| = (6)\left| \ln (2) - \ln (2) \right| + (12n - 12)\left| \ln (2) - \ln (3) \right| + (9n^2 - 15n + 6)\left| \ln (3) - \ln (3) \right| = 4.8660(n - 1),
\]

\[
\text{IRLU}(HC_n) = \sum_{u \in E} \left| \frac{d_u - d_v}{\min(d_u, d_v)} \right| = (6)\left| \frac{2 - 2}{2} \right| + (12n - 12)\left| \frac{2 - 3}{3} \right| + (9n^2 - 15n + 6)\left| \frac{3 - 3}{3} \right| = 6(n - 1),
\]

\[
\text{IRLF}(HC_n) = \sum_{u \in E} \left| \frac{d_u - d_v}{\sqrt{d_ud_v}} \right| = (6)\left| \frac{2 - 2}{\sqrt{2}} \right| + (12n - 12)\left| \frac{2 - 3}{\sqrt{6}} \right| + (9n^2 - 15n + 6)\left| \frac{3 - 3}{\sqrt{9}} \right| = 4.8990(n - 1),
\]

\[
\text{IRLA}(HC_n) = 2 \sum_{u \in E} \left| \frac{d_u - d_v}{d_u + d_v} \right| = 2(6)\left| \frac{2 - 2}{4} \right| + 2(12n - 12)\left| \frac{2 - 3}{5} \right| + 2(9n^2 - 15n + 6)\left| \frac{3 - 3}{6} \right| = 4.8000(n - 1),
\]

\[
\text{IRD}(HC_n) = \sum_{u \in E} \ln |d_u - d_v| = \ln |1 + d_u - d_v| = \ln |1 + 2| + \ln |1 + 2| + (12n - 12) + \ln |1 + 3| + (9n^2 - 15n + 6) = 8.3172(n - 1),
\]

\[
\text{IRGA}(HC_n) = \sum_{u \in E} \ln \frac{d_u + d_v}{2\sqrt{d_ud_v}} = (6)\ln \frac{2 + 2}{(2)\sqrt{(2)\sqrt{6}}} + (12n - 12)\ln \frac{2 + 3}{(2)\sqrt{6}} + (9n^2 - 15n + 6)\ln \frac{3 + 3}{(2)\sqrt{9}} = 0.2448(n - 1).
\]
Specific values for the irregularities of the honeycomb network for different values of involved parameters are given in Table 10.

### 4. Graphical Comparison

In this section, we will compare the results of irregularity indices of certain networks in graphical form. Different colors have been used to represent the behavior of irregularity indices for certain networks in the form of graphical lines. And these graphs have been generated by putting the
Figure 10: IRF of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$.

Figure 11: IRFW of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$.

Figure 12: IRA of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$.

Figure 13: IRB of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$.

Figure 14: IRC of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$.

Figure 15: IRDIF of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$. 
Figure 16: IRL of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$.

Figure 17: IRUL of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$.

Figure 18: IRLF of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$.

Figure 19: IRLA of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$.

Figure 20: IRD1 of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$.

Figure 21: IRGA of $SL_n$, $CS_n$, $OX_n$, $HX_n$, and $HC_n$. 
values of n along the X-axis in the results of irregularity indices along the Y-axis. Tables 1–5 represent such values of results which are obtained after putting values of n, and these values demonstrated in the tables will help us to generate graphical results.

In Figures 6–21, five different colors of graphical lines have been used. Red color represents irregular behavior in the silicate network (SLn), and blue color represents irregular behavior in the chain silicate network (CSn). Similarly, other colors which are green, purple, and black represent irregular behavior in the oxide network (OXn), hexagonal network (HXn), and honeycomb network (HCn), respectively. In Figures 6–21, values of n are plotted along the X-axis and behavior of all selected irregularity indices along the Y-axis.

5. Concluding Remarks

In this paper, we have computed irregularity indices of certain networks such as silicate network, chain silicate network, oxide network, hexagonal network, and honeycomb network. These results are valuable and helpful to understand deep irregular behavior of certain networks. These results are also useful for researchers to understand how these networks can be constructed with different irregular properties [16–20].

Data Availability

All the data used to support the findings of this study are included within the article..

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Authors’ Contributions

All authors contributed equally to this paper.

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