Evolutionary Game Analysis of Remanufacturing Entry Decision of Manufacturers and Distributors

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Abstract. The economic and environmental benefits of remanufacturing have been widely recognized. In the context of competition between manufacturers and distributors in a closed-loop supply chain, the dynamic evolution process of manufacturer and distributor's remanufacturing decision-making is analyzed by evolutionary game model. By analyzing the stability of the system, the optimal strategy combination is obtained. The results show that the equilibrium results of system evolution are affected by the authorization fees paid by distributors to manufacturers, the difficulty of recycling remanufactured products and the technical level. When the relationship between profit and authorization fees of remanufactured products changes, the system will evolve to different equilibrium. Finally, the correlation is verified by numerical simulation.

1. Introduction
Remanufacturing refers to the process of disassembly and recycling of products in reverse or closed-loop supply chains [1]. Remanufacturing allows enterprises to meet customers' needs for lower-priced and more sustainable products, and to extend the life cycle of products through renovation and technological upgrading to achieve the goal of saving resources [2]. For decades, the environmental performance of products has gained considerable attention in the closed-loop supply chain process (CLSC). Remanufacturing is carried out by original equipment manufacturers (OEMs) or independent remanufacturers [3]. Technology authorization is an application from the original equipment manufacturer to authorize other manufacturers to manufacture new products. Technology authorization also applies to remanufacturing. Apple subcontracted out-of-life iPhones to Foxconn for remanufacturing [4]. This case highlights the key role of technology authorization in remanufacturing.

Many literatures have studied remanufacturing in closed-loop supply chains. Gendao Li and Marc Reimann [5] proposed that when the cost of product quality improvement increases slightly, manufacturers would stop remanufacturing when introducing new products to maximize profits. Jelena and others [6] considered the application of lean improvement to remanufacturing. Dou et al. [7] proposed that manufacturers make production planning decisions according to the carbon tax law. The results show that raising the carbon tax price does not necessarily lead to the reduction of remanufacturing production level.

There are also some papers using evolutionary game theory to study. Ma Xiang and Zhang Guoxing [8] constructed the evolutionary game model of Beijing-Hebei Coalition of Fog and Haze Control from the perspective of ecological compensation and ecological claims. Guo Benhai and Fang Zhigeng [9]
constructed the exit mechanism of regional high-energy-consuming industries. The above literature focuses on the stability of decision-making from a dynamic perspective by using evolutionary game theory.

2. Evolutionary Game Model
Considering the closed-loop supply chain composed of manufacturers, distributors, and consumers, this paper establishes a two-stage model. The first stage is the manufacture and sale of new products. In the second stage, manufacturers and distributors need to decide whether to enter remanufacturing or not, and those who enter remanufacturing policies need to consider factors such as recovery price. When a retailer chooses to enter remanufacturing, he has to pay the manufacturer for the technology licensing fee. An evolutionary game model is introduced to study the equilibrium point and stability analysis of the evolution of manufacturer and distributor entering remanufacturing in the second stage.

2.1. System Description
This paper mainly studies a two-stage closed-loop supply chain consisting of manufacturers and distributors. In the first stage, the manufacturers sell new products to the distributor at wholesale price $w$, and the distributors sell new products to the consumer at retail price $p$. In the second stage, considering that remanufacturing reduces production costs, it will also sell new products to the distributor at wholesale price $w$. With factors such as recovery, transportation, and inventory costs, manufacturers and distributors must make decisions on whether to remanufacture or not in the second stage, which can be divided into the following four situations: if manufacturers and distributors choose not to enter remanufacturing, it is similar to the first stage; if manufacturers choose to enter remanufacturing, they will produce new products at the same time. If the manufacturers choose to go into remanufacturing, after selling the new product to the distributor, the product of the first stage should be retrieved for remanufacturing, and the remanufactured product should be sold directly to the consumer; if the distributors choose to enter remanufacturing, not only the first stage should be retrieved. If both parties choose to go into remanufacturing, the manufacturers and distributors will make efforts to recycle and remanufacture the discarded products, and each side will sell the remanufactured products.

2.2. Research hypothesis
Hypothesis 1: The market demand function of the product is $D(p) = \phi - \beta p$, where $D$ is the demand of the product and $\phi$ represents the potential market demand.
Hypothesis 2: Suppose that there is no difference between a new product and a remanufactured product for consumers. It is assumed that the retail price of new products and remanufactured products is the same.
Hypothesis 3: Suppose that the product will lose in the process of recycling, and the proportion of loss is $\rho_l$. Manufacturers and distributors reproduce the recycled product. There are untreatable recyclables in the process of construction, the proportion of which is $\rho_d$.
Hypothesis 4: The recovery of manufacturers and distributors depends on their respective efforts, assuming that the efforts of manufacturers and distributors are the same. Therefore, the proportion that cannot be recycled is the same.
Hypothesis 5: When the manufacturer authorizes the distributor to remanufacture, it is assumed that the retailer's remanufacturing capability is the same as that of the manufacturer under the technical authorization. The proportion of $\rho_d$ that cannot be processed in the remanufacturing process is the same.
Hypothesis 6: In the case that the distributor enters remanufacturing, it is assumed that the manufacturer charges an authorization fee according to the amount of recovery. The loss of the distributor in the process of recovery and remanufacturing is not considered.
2.3. Symbol Description

| Variable | Description |
|----------|-------------|
| \( w \)  | Unit new product wholesale price |
| \( p \)  | Unit retail price of new products and remanufactured products |
| \( r \)  | Unit product recycling price |
| \( \rho_l \) | Unrecoverable proportion |
| \( \rho_d \) | Proportion that cannot be processed during remanufacturing |
| \( c_m \) | Production cost of new products per unit |
| \( D(p) \) | Total market demand for products |
| \( G \)  | Demand for remanufactured products |
| \( \phi \) | Potential market demand |
| \( \beta \) | Price elasticity coefficient |
| \( f \)  | Authorization fee paid by the distributor for remanufacturing |
| \( c_s \) | Unit production cost savings in remanufactured products |

\( \pi_{NN}^{M}, \pi_{NN}^{D} \): Both parties do not enter the remanufacturing, the profit of the manufacturers and distributors

\( \pi_{NR}^{M}, \pi_{NR}^{D} \): When the manufacturer enters remanufacturing unilaterally, the profit of the manufacturers and distributors

\( \pi_{RN}^{M}, \pi_{RN}^{D} \): When the distributor enters remanufacturing unilaterally, the profit of the manufacturers and distributors

\( \pi_{RR}^{M}, \pi_{RR}^{D} \): Both parties enter remanufacturing, the profit of the manufacturers and distributors

3. Model analysis

3.1. Payment matrix

According to different manufacturers and distributors, there are different remanufacturing mode choices. Strategy N means not to enter remanufacturing in the second stage, and strategy R means entering remanufacturing in the second stage. Thus, the manufacturer and distributor strategy space can be obtained. The two-dimensional payment matrix for \{ (N, N), (N, R), (R, N), (R, R) \} is shown in Table 2:

| Manufacturer | Distributor |
|--------------|-------------|
|              | \( N \)     | \( R \)     |
| \( N \)      | \( \pi_{NN}^{M}, \pi_{NN}^{D} \) | \( \pi_{NR}^{M}, \pi_{RR}^{D} \) |
| \( R \)      | \( \pi_{RN}^{M}, \pi_{RN}^{D} \) | \( \pi_{RR}^{M}, \pi_{RR}^{D} \) |

Among them,

\[
\pi_{NN}^{M} = (w - c_m)D(p) \quad (1)
\]

\[
\pi_{NN}^{D} = (p - w)D(p) \quad (2)
\]

\[
\pi_{NR}^{M} = (w - c_m)(D(p) - G) + Gf \quad (3)
\]

\[
\pi_{NR}^{D} = (p - w)(D(p) - G) + G(1 - \rho_l)(1 - \rho_d)(p - c_m - r + c_s) - Gf \quad (4)
\]
3.2. Replication dynamic equation

Suppose that manufacturers and distributors can randomly select strategy N and strategy R in multiple games. The ratio of manufacturers selection strategy N and strategy R is x and 1-x respectively, and the ratio of distributors selection strategy N and strategy R is y and 1-y. For the manufacturer, the benefit of choosing strategy N and strategy R is

\[ e_{11} = y\pi_M^{NN} + (1 - y)\pi_M^{NR} \]  
\[ e_{12} = y\pi_M^{RN} + (1 - y)\pi_M^{RR} \]

For distributor, the benefit of choosing Strategy N and Strategy R is

\[ e_{21} = xe_{11} + (1 - x)e_{12} \]
\[ e_{22} = ye_{21} + (1 - x)e_{22} \]

The expected return from manufacturers and distributors is

\[ \bar{E}_1 = xe_{11} + (1 - x)e_{12} \]
\[ \bar{E}_2 = ye_{21} + (1 - x)e_{22} \]

According to the analysis method of the stability of differential equations proposed by Taylor and Jonker, the replicating dynamic equations of the manufacturer N and the distributor adopting the strategy N are respectively

\[ \frac{dx}{dt} = x(1 - x)[y(\pi_M^{NN} - \pi_M^{RN}) + (1 - y)(\pi_M^{NR} - \pi_M^{RR})] \]  
\[ \frac{dy}{dt} = y(1 - y)[x(\pi_D^{NN} - \pi_D^{RN}) + (1 - x)(\pi_D^{NR} - \pi_D^{RR})] \]

3.3. Evolutionary stability analysis

The Jacobian matrix can be used to analyze the evolutionary stability point of manufacturers and distributors's decision-making in remanufacturing. The Jacobian matrix of the system consisting of (15) and (16) is as follows.

\[ J = \begin{pmatrix} \frac{\partial F(x)}{\partial x} & \frac{\partial F(x)}{\partial y} \\ \frac{\partial F(y)}{\partial x} & \frac{\partial F(y)}{\partial y} \end{pmatrix} \]

Substituting (1)–(8) into Jacobian matrix, and assuming that \( A = (1 - \rho_l)(1 - \rho_d)(p - c_m - r + c_e) \) \( B = (w - c_m) \) \( H = (p - w) \), when J satisfies \( \det J > 0 \) and \( \text{tr} J < 0 \) at the equilibrium point, the equilibrium point is the local asymptotic stability point of the evolutionary dynamic process,
corresponding to the evolutionary stability strategy ESS. Table 4 shows that the stability of the system depends on the relationship between $A$, $B$, $H$ and $f$, and can be divided into four situations: (i) $B > A, f > A, A-f < H$; (ii) $B < A, f > A, A-f < H$; (iii) $B < A, f < A, A-f > H$; (iv) $B < A, f < A, A-f < H$. There are four other cases which do not conform to reality, so they are discarded. Table 3 and Table 4 show the evolution results of the system under case (i) - (iv).

### Table 3. Local Stability of Equilibrium Points under (i) and (ii)

| Equilibrium point | det$J$ | tr$J$ | Equilibrium | det$J$ | tr$J$ | Equilibrium |
|-------------------|--------|-------|-------------|--------|-------|-------------|
| (0,0)             | +      | +     | Instable    | +      | +     | Instable    |
| (0,1)             | -      | -     | Indefinite saddle point | + | - | ESS |
| (1,0)             | -      | -     | Indefinite saddle point | - | Indefinite saddle point |
| (1,1)             | +      | -     | ESS         | -      | Indefinite saddle point |

### Table 4. Local Stability of Equilibrium Points under (iii) and (iv)

| Equilibrium point | det$J$ | tr$J$ | Equilibrium | det$J$ | tr$J$ | Equilibrium |
|-------------------|--------|-------|-------------|--------|-------|-------------|
| (0,0)             | +      | -     | ESS         | +      | -     | ESS         |
| (0,1)             | -      | -     | Indefinite saddle point | - | Indefinite saddle point |
| (1,0)             | +      | +     | Instable    | -      | Indefinite saddle point |
| (1,1)             | -      | -     | Indefinite saddle point | + | - | ESS |

In case (i), (1,1) is an evolutionary stable point, and neither manufacturer nor distributor enters the remanufacturing process. Manufacturing is the situation with the lowest social and economic benefits.

In case (ii), (0,1) is the evolutionary stable point, the manufacturer's decision is to enter remanufacturing, and the distributor's decision is not to enter remanufacturing.

In the case (iii), where (0,0) is the evolutionary stable point, both manufacturer and distributor enter remanufacturing. It has the highest social and economic benefits. At this time, for the manufacturer, it can be found that, in order to ensure that both sides enter remanufacturing, there is a certain requirement for the profit of remanufacturing. The profit of the new product of the manufacturer unit shall be greater than that of the new product of the distributor unit after deducting the licensing fee.

### 4. Conclusion

Using evolutionary game theory, this paper studies the evolutionary process of manufacturer and distributor entering remanufacturing under competitive environment. The results show that when $B < A, f < A, A-f > H$, the system will evolve to the point where both manufacturers and distributors enter remanufacturing.

There are also some shortcomings in this paper. For example, we do not consider the differences of price between new products and remanufactured products. Subsequent research directions can consider the impact of the secondary market on the reinvention of the remanufactured product market.

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