Multi-parton interactions and rapidity gap survival probability
in jet–gap–jet processes

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Abstract

We discuss an application of dynamical multi-parton interaction model, tuned to measurements of underlying event topology, for a description of destroying rapidity gaps in the jet–gap–jet processes at the LHC. We concentrate on the dynamical origin of the mechanism of destroying the rapidity gap. The cross section for jet–gap–jet is calculated within LL BFKL approximation. We discuss the topology of final states without and with the MPI effects. We discuss some examples of selected kinematical situations (fixed jet rapidities and transverse momenta) as distributions averaged over the dynamics of the jet–gap–jet scattering. The color-singlet ladder exchange amplitude for the partonic subprocess is implemented into the PYTHIA 8 generator, which is then used for hadronisation and for the simulation of the MPI effects. Several differential distributions are shown and discussed. We present the ratio of cross section calculated with and without MPI effects as a function of rapidity gap in between the jets.

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I. INTRODUCTION

Diffraction, i.e. strong interaction involving the exchange of the vacuum quantum numbers (the pomeron)\(^1\) is a very broad field of research. In the recent years the understanding of diffraction and its connection to the microscopic picture of strong interactions has been greatly improved thanks to the studies of hard diffraction, i.e. diffraction involving a hard scale, like high \(p_T\) jets.

The jet–gap–jet process is an example of the diffractive jet production, in which the pomeron is exchanged between the produced jets. Contrary to other types of diffractive jet processes (e.g. single diffractive jets), the absolute value of the four momentum carried by the pomeron is large. This provides a unique possibility to apply perturbative calculation methods to fully describe the diffractive exchange. The jet-gap-jet processes were measured at Tevatron [2] and recently at the LHC [3].

One important ingredient in the calculations of the hard diffractive cross section is the rapidity gap survival probability. In many calculations, see e.g. [4, 5], the gap survival factor is assumed to be constant with respect to the kinematics of the event, and depending only on the centre-of-mass energy. Recently, more detailed analyses were performed, in which kinematic dependence was taken into account. The calculations were done for exclusive processes (see e.g. [6] and references therein) and single diffraction [7]. Recently the gap survival in single diffractive processes was calculated dynamically by including MPIs [8].

In the present paper we study this topic for a different class of processes – the jet-gap–jet production. What is/are the process(es) responsible for destroying rapidity gap obtained in the pQCD calculation of colour-singlet exchange? In the present study we explore the role of multi parton interactions that are the main mechanism responsible for understanding of underlying event topology, see e.g. [9–12]. In particular, we wish to address the problem how much the dynamical calculation changes the somewhat academic BFKL result.

\(^1\) The spin structure of the pomeron is a matter of current discussions [1].
II. PARTICLE PRODUCTION IN JET EVENTS

The difference in the underlying mechanism of the non-diffractive jet and jet–gap–jet production, the details of which are discussed in Section IV, affect not only the cross section and angular distribution of jets, but also the distributions of particles produces in the events. This difference originate from a different flow of the colour charges in the events, which affect the hadron formation process. These effects can be studied using Monte Carlo event generators that simulate the hadronization process. The following results were obtained with PYTHIA 8.

![Rapidity distributions](image)

FIG. 1: Rapidity distributions of particles produced in non-diffractive jet (black) and jet–gap–jet (red, curve with a dip at \( y = 0 \)) events, for the selected kinematical situation.

First we wish to illustrate the situation for a selected kinematical situation. Fig. 1 presents the rapidity distribution of particles produced in a \( pp \) interaction at \( \sqrt{s} = 7 \text{ TeV} \) for events obtained with the \( gg \rightarrow gg \) hard subprocess, where the gluons are scattered with fixed transverse momentum \( p_T = 50 \text{ GeV} \) at rapidities of \( y = \pm 3 \). Two cases are studied: nondiffractive jets, when colour charges are exchanged between the scattered gluons, and jet–gap–jet production, in which interacting gluons keep their colours. One can clearly see that the rapidity density of produced particles is highest around rapidities of scattered gluons, which reflects the jet structure of the events. In this region one does not see a large difference between the two cases (colour structures). On the other hand, in the region between the jets the difference is quite dramatic. When no colour charge is transferred between the gluons, the density of produced particles is reduced by two orders of magnitude. The particles produced at rapidities outside the jet system originate
from the hadronization of the proton remnants and from the fact that there is an colour transfer between the remnants and the scattered gluons.

The suppression of particles production in the region between the jets will lead to large rapidity regions devoid of particles – rapidity gaps. However, the actual values of particles rapidities, and thus the size of the rapidity gap, is to some extend random and will fluctuate from event to event. This is true both for for non-diffractive jets as well as for jet–gap–jet events. On average, for the former ones one expects rather small gaps, and much bigger gaps for the latter ones. This is illustrated in Fig. 2, where the distributions of gap size are shown for jets with \( p_T = 50 \text{ GeV} \) and \( y = \pm 3 \). One can see that even though the distributions are well separated, their tails are rather long and they have a non-negligible overlap. This shows that, even neglecting other effects discussed later, these two processes cannot be fully separated experimentally (at least based solely on the rapidity gap size).

![Rapidity gap distributions](image)

**FIG. 2:** Rapidity gap distributions for non-diffractive jet (red, with maximum at \( \Delta \eta \sim 0 \)) and jet–gap–jet events (black, with maximum at \( \Delta \eta \sim 5 \)), for our selected kinematical configuration. No MPI effects are included here.

### III. MULTIPLE PARTON INTERACTIONS

An additional complication to the picture presented in the previous section comes from the fact that hadrons are complicated objects that consist of many partons. In a single hadron–hadron collision more than one parton–parton interaction can take place. This phenomenon is known as the multiple parton interactions (MPI) or the underlying
event (UE) activity and it was extensively studied at Tevatron and the LHC [9, 11, 12].

The MPIs are modeled in PYTHIA with the help of minijets calculated in collinear factorization approach with a special treatment at low transverse momenta of minijets by multiplying standard cross section by a somewhat arbitrary suppression factor [13]

\[ F_{\text{sup}}(p_t) = \frac{p_t^4}{(p_t^2 + p_t^2)^2} \theta(p_t - p_{t,\text{cut}}). \] (3.1)

Typically, MPI effects are responsible for increasing the particle production in the events, but for diffraction they have particularly important consequences. If the \( gg \to gg \) or another partonic subprocess with a colour-singlet exchange is accompanied by another independent parton–parton interaction, additional particles can be produced in the region where a gap was expected. This is presented in Fig. 3, where rapidity distributions of particles produces in jet–gap–jet events are shown for the MPI effects in PYTHIA turned off (black) and turned on (red). It is crucial that even though the particle density

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{Rapidity distributions of produced particles for jet–gap–jet events without (black) and with (red) multi parton interactions for our selected kinematical configuration.}
\end{figure}

in the region between the jets is greatly increased, it is possible to observe events with very large gap sizes. This is contrary to the case of non-diffractive jets, and it originates from the fact that it is possible to have events with no additional parton–parton interaction, in which a large gap can survive. The distributions of the gap size for events with and without MPI effects are presented in Fig. 4.

For events with MPI effects the gap distribution consists of two parts. The distributions for low gap sizes is steeply falling, similarly to the non-diffractive events. This comes from the fact that additional interactions produce particles between the jets. The
large-gap part originates from events where no additional interactions occurred. The distribution is similar to the one obtained without MPI effects, but reduced by a factor of about one order of magnitude. Fig. 4 shows also the gap size distribution plotted with MPI effects included, but only for events that do not contain any additional interactions. For very large gap sizes this distribution agrees with the one for all events. However, for medium gap sizes it is not. On the other hand, the shape of this distribution is the same as for the distribution without MPI effects, but scaled down. The difference between the red and the blue curves comes from events in which the additional interactions produce very few particles. In these events the initial rapidity gap is reduced, but not completely filled.

Since experimentally the jet–gap–jet events are distinguished by the presence of large rapidity gaps, the MPI effects lead to a reduction of the measured cross sections with respect to the cross section of the actual colour-singlet exchange. This phenomenon is often called as absorptive corrections and the corresponding probability – gap survival probability.

![Diagram](image)

FIG. 4: Rapidity gap distributions for jet–gap–jet events without MPI effects (black, the highest curve at $\Delta \eta = 3-6$) and with MPI effects (red) and with MPI effects included, but for events in which no MPIs occurred i.e. $n_{MPI} = 0$ (blue, the lowest curve).

In order to estimate its magnitude one can perform event simulation with PYTHIA and count in which fraction of events no additional parton interactions are present. For the jet–gap–jet processes it is possible to use the existing Monte Carlo generators that take into account modeling of multi parton interactions. If such a generation correctly describes the MPI effects for standard jet production, it should also provide a correct
description of the jet–gap–jet process.\textsuperscript{2}

The MPI generation in \textsc{Pythia} is based on phenomenological models that contain several arbitrary parameters. Usually the values of these parameters are tuned in order to give the best description of the Tevatron and the LHC data for observables related to MPI. Therefore, it can be expected that the results presented in the present paper should also be close to reality. A big advantage of this approach is that it allows calculations of the cross section or gap survival probability as a function of the kinematical variables of the hard (sub)process.

Fig. 5 presents the dependence of a few kinematical variables of the gap survival probability, defined as a fraction of events that do not contain any additional parton–parton interactions apart from the hard one. Since \textsc{Pythia} assumes the initial partons to be collinear with the protons, the kinematics of an event can be described by four parameters. A sensible choice is: the centre-of-mass energy $\sqrt{s}$, the invariant mass of the hard subprocess $M_{gg}$, the difference of rapidities of the scattered gluons $\Delta y$ and the rapidity of the gluon-gluon system $y_{gg}$.

The dependence of the gap survival probability as a function of the centre-of-mass energy is presented in Fig. 5a. Its value drops from about 15\% at the Tevatron energy (2 TeV) to about 5\% at the LHC nominal energy (14 TeV). Fig. 5b shows the dependence on the $M_{gg}$ for $\sqrt{s} = 7$ TeV.

Here, the gap survival increases from about 7\% for masses close to zero up to 30\% for masses close to 6 TeV. The observed behaviour can be qualitatively explained by the energy conservation: when a bigger part of the proton energy is carried by the parton participating in the hard subprocess, less energy is available for additional interactions and they become less likely.

Fig. 5c presents the dependence of the survival probability on $\Delta y_{gg}$.

Fig. 5d presents the dependence on the rapidity of the gluon-gluon system. The dependence is rather flat for central values of rapidities and rapidly grows for $|y_{gg}| > 3$. For such strongly boosted events one of the incoming partons carries a large energy and the other one very small one. In this situation the possibility of additional interactions is

\textsuperscript{2}This statement is not necessarily true for other diffractive jet processes, where the production mechanism is somewhat different and not so well understood.
FIG. 5: Kinematic dependence of gap survival probability as a function of: a) centre-of-mass energy, b) invariant mass in the hard subprocess, c) rapidity distance between the scattered gluons, d) rapidity of the digluon system.

also reduced, because additional partons from both protons are needed for extra MPI to occur.

In summary, there is a strong dependence of the gap survival probability on kinematical variables. However, not all kinematic configurations are equally probable. For example, the partonic distributions are larger at small values of Bjorken $x$, which favours small masses of the system. In addition, the dynamics of the colour-singlet exchange will also play some role.
FIG. 6: A schematic QCD diagram of color-singlet exchange for the jet–gap–jet process in a pp collision. Only gg initiated process is shown explicitly.

IV. HARD COLOUR-SINGLET EXCHANGE

The calculation of the jet–gap–jet process is based on the QCD collinear factorisation, where the cross section for the hard subprocess, \( \hat{\sigma} \), is convoluted with the appropriate parton densities. An example of the diagram of the full process is presented in Fig. 6. The cross section can be written in the simple form

\[
\frac{d\sigma}{dp_T} = \int g_{e f f}(x_1, \mu_F^2)g_{e f f}(x_2, \mu_F^2) \frac{d\hat{\sigma}}{dp_T} dx_1 dx_2 ,
\]

where

\[
g_{e f f}(x_k, \mu_F^2) = g(x_k, \mu_F^2) + \frac{16}{81} \sum_f (q_f(x_k, \mu_F^2) + \bar{q}_f(x_k, \mu_F^2))
\]

and \( k = 1, 2 \). The jet–gap–jet process differs from the typical jet production in the colour structure of the subprocess. Here, the colour-singlet ladder is exchanged between partons.

To a first approximation, the colour–singlet exchange can be described in perturbative QCD as an exchange of a pair of gluons that carry opposite colour charges [14]. A better approach is to describe it as a gluon ladder, which can be performed within the BFKL framework. The first calculation of this type were done in [15]. This was followed e.g. by further studies, see e.g. [16–18].

In the present paper we shall use for illustration LL BFKL formalism used previously
e.g. in [17, 18].

\[
A(\Delta \eta, p_T^2) = \frac{16N_C\pi\alpha_s^2}{C_Fp_T^2} \sum_{p=-\infty}^{\infty} \int \frac{d\gamma}{2i\pi} \frac{[p^2 - (\gamma - 1/2)^2]}{[(\gamma - 1/2)^2 - (p - 1/2)^2][(\gamma - 1/2)^2 - (p + 1/2)^2]} \exp(\tilde{\alpha}_s K_{\text{eff}}[2p, \gamma, \tilde{\alpha}]\Delta \eta) \, ,
\]

(4.1)

where \( p_T \) is jet transverse momentum and \( \Delta \eta \) is rapidity distance between the partonic jets. The integral above runs along the imaginary axis from \( \frac{1}{2} - i\infty \) to \( \frac{1}{2} + i\infty \) and with only even conformal spins. The \( \chi_{LL} \) kernel reads

\[
\chi_{LL} = 2\psi(1) - \psi(1 - \gamma + \frac{|p|}{2}) \psi\left(\gamma + \frac{|p|}{2}\right) \, ,
\]

(4.2)

where \( \psi(\gamma) = d \log \Gamma(\gamma) / d\gamma \). In the LL BFKL approach a constant value of \( \alpha_s \) is used [17]. In Eq. (4.1) \( \tilde{\alpha} = \alpha_s N_c / \pi \).

In Fig. 7 we show the LL BFKL amplitude as a function of rapidity distance between jets for a few values of jet transverse momenta. The increase at large rapidity distance is a typical BFKL increase while the increase at low rapidity distances is “caused” by the presence of higher conformal spins. The calculation sketched above does not include gap survival factor which is a very important ingredient as discussed in the next section.

![Image of Fig. 7](image)

**Fig. 7:** Subprocess amplitude in the LL BFKL approach as a function of rapidity distance between jets for selected transverse momenta of the jets. In this calculation \( \alpha_s = 0.17 \).

The leading-logarithm approximation of BFKL may be not sufficient to provide a reasonable description of the absolute cross section normalisation and the distributions shapes, but is sufficient for presentation of the MPI effects in suppressing rapidity gap discussed in the present paper, as will be explained below.
V. REALISTIC DISTRIBUTIONS OF RAPIDITY GAP SIZE

Here we wish to discuss our results for a broad range of jet rapidities when imposing only a minimal lower cut on jet transverse momenta. We fix transverse momenta of jets to be in the interval \( 40 \text{ GeV} < p_{1t}, p_{2t} < 200 \text{ GeV} \) and impose no cuts on jet rapidities at all. The calculations presented in this section have been performed using the framework of the PYTHIA 8 Monte Carlo event generator. The BFKL leading-logarithm amplitude for the hard subprocess has been calculated following [17, 18] and implemented into PYTHIA as new \( gg \rightarrow gg, qg \rightarrow qg, qq \rightarrow qq, q\bar{q} \rightarrow q\bar{q} \) subprocesses with the appropriate colour flow.

With a realistic description of the jet–gap–jet dynamics (BFKL) and the MPI modeling by PYTHIA it is possible to study the rapidity gap differential cross sections. In this way the kinematics-dependent rapidity gap survival probability will be properly averaged over different kinematic configurations. In addition, the effects of rapidity gap reduction (see Fig. 4) is also taken into account.

Fig. 8 presents such distributions for events with and without MPI effects. Both distributions are rapidly falling, which originates predominantly from the shape of the parton densities. In the large-gap region the distribution with the MPI effects is reduced with respect to the one without MPI. However, for small \( \Delta \eta \) the situation is opposite. This comes from the fact that the integrated cross section is in both cases the same, since it is given only by the hard partonic mechanism. The occurrence of MPI effects does not change the normalisation of the distributions, but shifts events to smaller rapidity gap sizes.

Fig. 9 presents the ratio of differential cross section for the gap distributions with and without MPI effects. This ratio can be treated as an effective gap survival factor, including all effects previously discussed and averaged over all configurations for the dynamics of the BFKL colour-singlet exchange. The occurrence of additional MPIs destroys large rapidity gaps and simultaneously increases of number of events with small rapidity gaps (see the left panels). Therefore the gap survival factor, defined in this way, depends on \( \Delta \eta \) (gap size), see the left panels of Fig. 9. It is worth considering a different definition of the survival factor, namely the ratio between the number of events in which no additional events occur. This definition is similar to the one typically used in the literature, where
it is assumed that any additional interaction destroys the rapidity gap. This assumption leads to a flat dependence of the gap survival with $\Delta\eta$, as seen in 9 (right panels). This is understandable, since here the events with additional interactions are not considered at all. In the previous case they were included, but with smaller values of $\Delta\eta$, which resulted in high ratio values in that region.

It is interesting to compare not only the shape, but also the value of the gap survival probability. This can be best done in the region of large $\Delta\eta$, where both definitions give an approximately flat behaviour. The definition that takes into account all events results in gap survival factor of about 10%. On the other hand, the definition that counts only the events with no additional MPIs give a value close to 6%. The difference is of the order of 40%, which is rather significant. It shows that latter definition may be too simplistic to provide a precise description of jet–gap–jet processes.

In addition, we also compare the situation where the jets can be created in the scattering of quarks and gluons, to the situation when only $gg \rightarrow gg$ process is included. The resulting gap survival probability in these two cases is the same within our present accuracy.

In the bottom panels of Fig. 9 we show for comparison also result obtained for the two-gluon exchange model with the regularisation parameter $m_g = 0.75$ GeV (see [14] for the cross section formula and [19] for the value of the nonperturbative parameter). The gap survival factor for the case of $n_{MPI} = 0$ is (at $\sqrt{s} = 7$ TeV) about 6-7 %, independent of modeling color–singlet exchange.

In Fig. 10 we show similar ratios but now as a function of jet transverse momentum for a few selected rapidity gap intervals. No obvious dependence on the jet transverse momentum can be observed in the left panel where we show the ratio of the distribution with MPI effect included to the corresponding distribution without MPI effects in contrast to the dependence on rapidity gap size observed in the previous figure. For smallest gaps ($0 < \Delta\eta < 1$) the ratio is with a good precision equal to 1. This seems accidental and is connected with bin size. This could be better understood by inspection of the left panels of Fig. 9 at $\Delta\eta \sim 0$. In the right panel we show the ratio with extra academic condition $n_{MPI} = 0$. Here we can observe that the result for all rapidity gap size intervals coincide within the limited Monte Carlo statistics. The ratios on the right panel are clearly smaller than those on the left panels.
FIG. 8: Rapidity gap distributions for jet–gap–jet events generated with and without MPI effects as a function of rapidity gap size. All partons combinations are included here. The left panel shows the rapidity gap distribution when MPI effects are included while the right panel shows result with extra requirement of $n_{\text{MPI}} = 0$.
FIG. 9: Ratio of rapidity gap distributions for jet–gap–jet events generated with and without MPI effects as a function of rapidity gap size. All partons are included here. For comparison result for only $gg \to gg$ is shown by the dark blue line (a,b). Results for color–singlet two–gluon exchange are shown in panels (c,d).
FIG. 10: Ratio of rapidity gap distributions for jet–gap–jet events generated with and without MPI effects (left panel) and with extra requirement \( n_{MPI} = 0 \) (right panel) as a function of jet transverse momentum for different intervals of rapidity gaps.
VI. CONCLUSIONS

In the present paper we have performed detailed studies of the role of multi-parton interactions in reduction of the theoretical cross section and/or different differential distributions for the jet–gap–jet processes. The cross section and the differential distributions for jet–gap–jet processes have been calculated for illustration in the LL BFKL framework. We have also tried to use a simple two-gluon exchange model regularised by the effective gluon mass to describe the jet–gap–jet process.

The subprocess amplitudes for the color-singlet exchange (BFKL ladder or two-gluon exchange) were implemented to the PYTHIA 8 generator, which was then used to simulate multi-parton interactions and hadronisation of the generated events. The parameters of the multi-parton interactions models in PYTHIA are tuned to measurements of observables related to the underlying event. In this sense we have no freedom to modify the MPIs.

For pedagogical reasons we have first studied particle (hadron) final states for the jet–gap–jet process for fixed kinematic configurations (fixed rapidity and transverse momenta of the jets). After inclusion of MPI effects we have shown fractions of events with no extra activity (in addition to the hard jets) or no activity in some rapidity interval. Those fractions depend, but rather smoothly, on kinematic variables.

Finally we have shown similar results when imposing only a cut on jet transverse momenta and integrating over almost whole phase space (full range of jet rapidities). Again, the gap survival factor is shown as a function of the gap size and the jet transverse momenta. We have found an interesting dependence on the size of the rapidity gap and almost no dependence on jet transverse momentum. A simple explanation of the first dependence has been offered. The MPIs suppress production of events with large rapidity gaps but create events with smaller rapidity gaps. The resulting rapidity gap survival factor depends on the gap size. On the other hand, when imposing the requirement of occurring no additional MPIs, the corresponding gap survival factor is almost constant, independent of gap size. However, there is a sizeable dependence on the collision energy. The ratios obtained for colour–singlet two–gluon exchange and for the BFKL ladder are almost the same.

Summarising in one sentence, the MPI effects lead to a dependence on kinematical
variables of the so-called gap survival factor, in contrast to what is usually assumed in the literature.

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amplitude \( (1/\text{GeV}^2) \)

\( p_T \) (GeV)

\( \Delta \eta = 0.1 \)
\( \Delta \eta = 1 \)
\( \Delta \eta = 2 \)
\( \Delta \eta = 4 \)
\( \Delta \eta = 5 \)
\( \Delta \eta = 7 \)
\[ \sqrt{s_{pp}} = 7 \, \text{TeV} \]

\[ p_T = 50 \, \text{GeV} \]

\[ y_j = \pm 3 \]
\[ \sqrt{s_{pp}} = 7 \text{ TeV} \]
\[ p_T = 50 \text{ GeV} \]
\[ y_j = \pm 3 \]
\[ \frac{d\sigma_\text{(MPI)}}{d\Delta\eta} / \frac{d\sigma_\text{(noMPI)}}{d\Delta\eta} \]

\( \sqrt{s} = 7\text{TeV} \)

- Green line: two-gluon-exchange
- Red line: LL BFKL
\( \sqrt{s} = 7 \text{ TeV} \quad 1,000,000 \text{ Events} \)

Ratio: \( \frac{d\sigma(\text{MPI})}{d\Delta\eta} / \frac{d\sigma(\text{noMPI})}{d\Delta\eta} \)

\[ \begin{align*}
\eta & \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5 \quad 6 \quad 7 \\
\sigma_d (\text{MPI}) & \quad \vdots \\
\sigma_d (\text{noMPI}) & \quad \vdots
\end{align*} \]