Robust Optimization Design of Supply Chain Based on Correlated Multi-Performance Responses

Lianyan Zhu 1*, Linhan Ouyang 2 and Feng Wu 3

1Department of Education and Science, Nanjing Polytechnic Institute, Nanjing 210048, China
2College of Economics and Management, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
3College of Economics and Management, Anhui Polytechnic University, Wuhu 241005, China
*Email: zlynjust@163.com

Abstract. Aiming at the problem of supply chain robust optimization with multiple performance responses, a robust optimization method was proposed by combining principal component analysis with double response surface method based on Kriging meta-model. Firstly, through principal component analysis, the location characteristics and divergence characteristics of multiple related performance responses were transformed into the principal component comprehensive score of supply chain performance; secondly, the Kriging meta-models of location characteristics and divergence characteristics of multiple performance responses were constructed respectively, and the principal component comprehensive score model of comprehensive performance based on Kriging meta-model was constructed. Thus the optimal operating conditions by optimizing the constructed robust optimization strategy were obtained in order to realize the robust optimization of supply chain system with correlated multiple responses. Finally, a supply chain simulation case was given to illustrate the effectiveness of our proposed optimization method, and the robustness of different methods was also discussed. The comparison results showed that our proposed was more robust.

1. Introduction
Robust parameter design (RPD) is an important quality improvement methodology to reduce the variations incurred by choosing appropriate values of the control variables[1-2]. In recent years, the robust parameter design methodology has been gradually applied to the quality design of supply chain [3-5]. Shukla et al. [6] proposed a hybrid approach incorporating simulation, robust parameter design method, robust multiple non-linear regression analysis and the Psychoclonal algorithm to determine the optimal operating conditions of three-level supply chain network incurring minimum total costs. The results showed that the proposed hybrid approach improved the robustness of supply chain. Owing to the complexity of the proposed multi-product economic production quantity (EPQ) model to be tackled in real-sized problems, Kangi et al. [7] combined the robust parameter design method with genetic algorithm (GA) and invasive weed optimization (IWO) algorithm to obtain the optimal level of parameters of the developed algorithms. Considering the uncertainty involved in the supply chain, a novel robust design optimization method based on the robust parameter design methodology is proposed to assure service level rate requirements and minimize total service level costs [8]. Dynamic simulation and Taguchi method for the robust parameter design are applied to design a robust blood
supply chain system considering the total cost and the safety of patients to improve the blood supply chain efficiency [9]. The above studies only consider the supply chain system with single performance response, and the robust parameter design methodology assumes that polynomial models have the capability to objectively reflect the real process response (quality characteristic) and the process variance. With the development of economic globalization, the number of simulation input factors in the supply chain system is increasing, and there is a strong nonlinear relationship between input factors and performance response, and the degree of approximation to the real process response using polynomial models for such strong nonlinear problems is not high. In addition, the simulation cost is expensive [10-11]. Therefore, to enhance the accuracy of the response predictions, we introduced kriging model which can fit the nonlinear relationship between input and output, especially for strong nonlinear problems [12-13]. Zhu et al. [14] proposed an optimization design method based on Kriging model to solve the problem of three-level supply chain optimization, which effectively overcame the shortcomings of polynomial model and provided a new research idea for supply chain optimization. Aiming at the inventory management (s, S) model in the supply chain environment, Zhu et al. [15] combined Kriging meta-model with conditional value at risk criterion to effectively solve the problem of supply chain optimization with risk aversion.

With the deepening of the research, the research object has gradually transformed into a supply chain system with multiple performance responses. Hussain et al. [16] regarded information sharing, production delay and average sales time as controllable factors. The influence of the above controllable factors on the performance of supply chain was studied by using the method of robust parameter design. Tiwari et al. [17] combined the robust parameter design methodology with artificial immune algorithm for the five-stage supply chain design, and the optimal controllable factor level is determined. Then the theoretical guidance for the supply chain management is provided. However, the above literatures took the optimal operation factor level as the research point and ignored the correlation between multiple performance responses. In fact, if the correlation between these responses is ignored in the optimization process, it may cause a large deviation to the optimization results [18]. In view of the risk of supply interruption in the supply chain system, Shi et al. [19] suggested the response surface method to improve the robustness of the supply chain system by adjusting the controllable factor level and the satisfaction function method was used to transform multiple optimization objectives into a single optimization objective function. However, Shah et al. [20] pointed that there were some defects in satisfaction function when there was strong correlation between responses, the 'optimal' result optimized by satisfaction function would not be reliable. Principal component analysis is one of the effective methods to solve the correlation between responses, but the method ignores the robustness of controllable variable fluctuations [21]. Therefore, scholars [22-23] at home and abroad have improved PCA. Through literature analysis, the domestic studies on the correlation between multiple performances responses of the supply chain are relatively few from the perspective of multi response optimization.

In this paper, the key point is how to obtain the optimal operating conditions of the supply chain by using Kriging meta-model and multi-response optimization method in view of the correlation between the multiple performances for the supply chain. Therefore, the principal component analysis and Kriging meta-model are first briefly introduced in this paper. Secondly, the supply chain quality with multiple performance responses is given, and a robust optimization strategy based on principal component analysis and Kriging meta-model is proposed. Finally, a supply chain simulation case is used to verify the effectiveness of our proposed method.

2. Theoretical Basics

2.1. Principal Component Analysis

Principal component analysis (PCA) was first proposed by Pearson, which is used to solve the correlation between variables. However, there are two shortcomings of PCA. One is that when there are multiple principal components with eigenvalues greater than 1, the problem is still multi responses,
the other is that the proposed PCA leads to partial information loss. So Liao [24] proposed a weighted principal component analysis method to solve the multi response optimization problem where the variance contribution rate of each principal component was taken as the weight coefficient of the principal component, the weighted sum of multiple principal components was used as the final multi response optimization index. Assuming that \( y_j (j = 1, 2, L, p) \) represents the performance response variable of supply chain system, according to the PCA method, the following transformation is given as equation (1).

\[
z_i = t_{i1}y_1 + t_{i2}y_2 + L + t_{ip}y_p = t_i^T y
\]  

(1)

Where, \( t_i = (t_{i1}, t_{i2}, L, t_{ip})^T \), \( y = (y_1, y_2, L, y_p)^T \), \( t_i^2 + t_{i2}^2 + L + t_{ip}^2 = 1, i = 1, 2, L, q \).

The ratio of the explained variance of principal components to the total variance is called the variance contribution rate of each principal component, which reflects the information content of each principal component.

\[
\omega_i = \lambda_i \sum_{i=1}^{q} \lambda_i
\]  

(2)

Among them, \( \omega_i \) is the variance contribution rate of the \( i \)th principal component, \( \lambda_i \) is the \( i \)th eigenvalue of the covariance matrix of supply chain system performance response \( y = (y_1, y_2, L, y_p)^T \). The weighted sum of all principal components of variance contribution rate is the comprehensive score of principal components which is given as follows.

\[
MS = \sum_{i=1}^{q} (\omega_i z_i) = \omega^T z
\]  

(3)

where \( \omega = (\omega_1, \omega_2, L, \omega_q)^T \), \( z = (z_1, z_2, L, z_q)^T \). Combing equation (1) and equation (2), then equation (3) can be converted into equation (4).

\[
MS = \omega^T z = t_\omega^T t_\omega = (t_\omega t_\omega)^T y
\]  

(4)

The weighted principal component analysis can transform the supply chain optimization problem with multiple performance responses into a single performance response optimization problem, explain all the variance of the initial multiple performance responses of the supply chain, and effectively avoid the loss of the original variable information.

2.2. Kriging Meta-model

Kriging meta-model is a statistical method based on stochastic process. According to the minimum variance criterion of prediction model, it can estimate the value of regionalized variables in a limited area without bias. This method can not only give predicted values at observed points that exactly equal to the observations, but also give the prediction accuracy of unobserved points. \( X = [x_1, x_2, L, x_n]^T \) \((X \in R^{m\times n})\) is an \( m \times n \) design matrix, \( Y = [y_1, y_2, L, y_n]^T \) \((Y \in R^{m\times n})\) represent corresponding response variables. Kriging meta-model could be given as equation (5),

\[
Y(x) = \sum_{j=1}^{n} \beta_j f_j(x) + z(x)
\]  

(5)

Equation (5) comprises two parts, \( f(x) = [f_1(x), f_2(x), L, f_p(x)]^T \) is the first part which is the linear combination (basis function) of regression function, corresponding design matrix is \( F = [f(x_1), f(x_2), L, f(x_n)]^T \), \( \beta_j \) is estimated parameters. The first part provides the global approximation of polynomial fitting in the design space, which reflects the change process of process mean. The second part \( z(x) \) is a stochastic process which represents the deviation between the response variable and the assumed linear regression model, commonly assumed to be Gaussian
process with zero mean and covariance, the covariance between design sample points can be expressed as equation (6),

$$\text{Cov}(Z(x), Z(w)) = \sigma^2 R(\theta, x, w)$$ (6)

where $\sigma^2$ is the process variance which could be obtained by maximum likelihood method, $R(\theta, x, w)$ is spatial correlation function, $x$ and $w$ are the design points. In generally, Gaussian correlation function is taken as following,

$$R(\theta, x, w) = \prod_{j=1}^{k} \exp(-\theta \|x_j - w_j\|^2)$$ (7)

3. Robust Optimization of Supply Chain based on Principal Component Analysis and Kriging Meta-Model

The text of your paper should be formatted as follows: As most of the current researches often ignore the correlation between multiple performance responses when determining the optimal operation parameters to maximize supply chain performance, and polynomial model is mostly used to fit the relationship between input factors and response output of supply chain system when using the idea of robust parameter design to solve the quality design of supply chain. In fact, due to the complexity of the supply chain system, there is a strong nonlinear relationship between the input and output of the system, polynomial model is difficult to reflect this kind of functional relationship, and the optimization results based on the polynomial model are not reliable, while Kriging meta-model can effectively overcome the above defects. Therefore, the idea of robust parameter design is combined with Kriging meta-model to deal with the influence of noise factors in the supply chain; at the same time, considering the correlation between multiple system performance responses, a robust optimization method based on principal component analysis and Kriging meta-model is proposed in this paper.

3.1. Definition of the Location and Divergence Characteristics of performance responses [23]

$x = (x_1, x_2, L, x_m)^T$ is an $m \times n$ input factors (or controllable factors) matrix in the supply chain system, $y = (y_1, y_2, L, y_n)^T$ is an $n \times 1$ output performance responses matrix, the experiment is repeated $r$ times, $T_p$ is the target value of the $p$ performance response, $\hat{\mu}_{pi}$ and $\hat{\sigma}_{pi}$ are respectively mean and standard deviation, $\hat{\mu}_{pi}$ and $\hat{\sigma}_{pi}$ are respectively deviations and variances as following,

$$\hat{\mu}_{pi} = (\hat{\mu}_{pi} - T_p)^2$$
$$\hat{\sigma}_{pi} = (\hat{\sigma}_{pi})^2$$ (8)

For Large and small value quality characteristics, if the performance response itself has a target value, equation (8) is still satisfied, if the target value does not exist, it is expressed by the following formula,

$$\hat{\mu}_{pi} = (\hat{\mu}_{pi})^2$$, small target
$$\hat{\mu}_{pi} = \left(\frac{1}{\hat{\mu}_{pi}}\right)^2$$, large target (10)

The above test data was standardized according to the following formula,

$$\hat{y}_{pi} = \frac{\hat{y}_{pi} - \hat{\mu}_{pi}}{\hat{\sigma}_{pi}}$$ (11)
\[
\hat{p}^\sigma = \frac{\hat{p}^\mu - \hat{p}^\sigma}{\hat{p}^\sigma}
\]

Where \(\hat{p}^\mu\) and \(\hat{p}^\sigma\) represent the mean and standard deviation of the \(r\) deviations of the \(p\)-th response, respectively. \(\hat{p}^\mu\) and \(\hat{p}^\sigma\) indicate the mean and the standard deviation of the \(r\) variances of the \(p\)-th response, respectively. \(\hat{X}^\mu\) and \(\hat{X}^\sigma\) represent the position characteristic and divergence characteristic of the \(p\)-th response respectively. According to the definition, the location and the divergence of each performance response are as small as possible.

### 3.2. Analysis of the Location Characteristics and Divergence Characteristics of Each Performance Response with PCA

Using principal component analysis method, the eigenvalue, eigenvector of \(\hat{X}^\mu\) and \(\hat{X}^\sigma\) are calculated, then the principal component of the \(p\)-th position characteristic is expressed as equation (13),

\[
z_p^\mu = a_{p1}\hat{X}_1^\mu + a_{p2}\hat{X}_2^\mu + K \cdot a_{pm}\hat{X}_n^\mu
\]

Where \((a_{p1}, a_{p2}, \ldots, a_{pm})^T\) represent the eigenvectors of the \(p\)-th response, and \(a_{p1}^2 + a_{p2}^2 + K = a_{pm}^2 = 1\). Similarly, the \(l\)-th principal component is expressed as following,

\[
z_l^\sigma = b_{l1}\hat{Y}_1^\sigma + b_{l2}\hat{Y}_2^\sigma + K \cdot b_{ln}\hat{Y}_n^\sigma
\]

Where \((b_{l1}, b_{l2}, \ldots, b_{ln})^T\) represent eigenvectors of the \(l\)-th response, and \(b_{l1}^2 + b_{l2}^2 + K = b_{ln}^2 = 1\).

### 3.3. Calculation of the Principal Component Comprehensive Scores of Location and Divergence Characteristics

With the help of weighted principal component analysis, the variance contribution rate of each principal component is selected as the weight of the principal component, and the comprehensive score of the principal component of each performance response location characteristic is as following,

\[
WPC_\mu = \sum_p (\omega_p z_p^\mu)
\]

Where \(\omega_p\) is the variance contribution rate of the \(p\)-th principal component, representing the ratio of the \(p\)-th eigenvalue to the sum of all eigenvalues and \(\sum \omega_p = 1\). Similarly, the principal component comprehensive score of each dispersion performance response is given as following,

\[
WPC_\sigma = \sum_l (\tau_l z_l^\sigma)
\]

The comprehensive score of the principal component is also a linear synthesis of the original index in essence. It can be seen that \(WPC_\mu\) is a linear relationship of location characters from (15). There may be a situation where the location character is poor when optimizing the formula (15), but \(WPC_\mu\) can still obtain the optimum. Therefore, use equation (4), the principal component contribution rate is used as the weight of each location characteristic to monitor the optimization results so as to achieve the goal of balancing multiple location characteristics. The significance and optimization method of \(WPC_\sigma\) are similar to \(WPC_\mu\).
3.4. Construction of the Kriging Meta-Models of Position Characteristics and Divergence Characteristics of Each Response

According to the data obtained from 3.1, Kriging meta-models of controllable factors and location and divergence characteristics of performance responses of supply chain system are constructed respectively as following,

\[ \hat{Y}_i^\mu = \hat{f}_i(x) \quad (i = 1, 2, \ldots, n) \] (17)

\[ \hat{Y}_i^\sigma = \hat{g}_i(x) \quad (i = 1, 2, \ldots, n) \] (18)

Where \( x = (x_1, x_2, \ldots, x_m)^T \) is an \( m \times n \) controllable factor matrix, \( \hat{f}_i(x) \) and \( \hat{g}_i(x) \) are Kriging meta-models of location characteristics and divergence characteristics of each performance response.

3.5. Construction of a Robust Optimization Strategy based on Kriging Meta-Models from an Overall Perspective

According to 3.3, the principal component comprehensive scores of the location characteristics of each response are calculated and weighted by using equation (15) and equation (17). Similarly, the principal component comprehensive scores of the divergence characteristics of each response are calculated and weighted by using equation (16) and equation (18). Combined with the Kriging meta-models obtained in 3.4, the principal component comprehensive score model for multiple performance optimization of the supply chain is constructed as followings,

\[ WPC_\mu = \sum_{p=1}^{n} \sum_{i=1}^{n} a_{ip} \hat{Y}_i^\mu = \sum_{p=1}^{n} \sum_{i=1}^{n} a_{ip} \hat{f}_i(x) \] (19)

\[ WPC_\sigma = \sum_{p=1}^{n} \sum_{i=1}^{n} b_{ip} \hat{Y}_i^\sigma = \sum_{p=1}^{n} \sum_{i=1}^{n} b_{ip} \hat{g}_i(x) \] (20)

The weighted linear combination of equation (19) and equation (20) is used as the final robust optimization objective function which is given as following,

\[ \min MS = \lambda WPC_\mu + (1 - \lambda) WPC_\sigma \]

s.t. \[ 0 \leq \lambda \leq 1 \] (21)

The weight can be given according to the relative importance of position characteristics and divergence characteristics or historical experience. Since the location characteristics and divergence characteristics of each response are as small as possible, the smallest value of \( MS \) is our expected target.

4. Simulation analysis

4.1. Simulation Test

Simulation data comes from the case in literature [25]. There are three performance responses which are profit, inventory and customer demand satisfaction rate. The profit is the larger-the-better quality characteristic, inventory is the smaller-the-better quality characteristic, and demand satisfaction rate is the nominal-the-best quality characteristic. Combined with the actual situation of the case, there are four controllable factors which are wholesaler’s order point, distributor’s order point, retailer’s order quantity and manufacturer's order quantity. Two noise factors: lead time and demand are considered. 30 sample points are selected by using Latin hypercube sampling (LHS) in the experiment, and each point is repeated 10 times. The experiment output data are collected by using arena simulation. The controllable factor data are normalized, then the mean values of profit, inventory and customer satisfaction rate are obtained which are denoted as \( y_1^\mu, y_2^\mu, y_3^\mu \) respectively, and the corresponding standard deviations are denoted as \( y_1^\sigma, y_2^\sigma \) and \( y_3^\sigma \) respectively. The results are shown in table 1.
In this case, the average profit is the larger-the-better quality characteristic, the average inventory is the smaller-the-better quality characteristic and the mean customer demand satisfaction rate is the nominal-the-best quality characteristic which target value is 1. According to equation (9), equation (10), equation (11) and equation (12), the data in table 1 are standardized, thus the position characteristics and divergence characteristics of each response are obtained which are shown in table 2. Firstly, principal component analysis is conducted for the location characteristics of the three performance responses, and the results of principal component analysis are listed in table 3.

Table 1. Normalized experiment data.

| Points | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $y_1^e$ | $y_2^e$ | $y_3^e$ | $y_1^a$ | $y_2^a$ | $y_3^a$ |
|--------|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| 1      | 0.354 | 0.901 | 0.755 | 0.563 | 201955| 2114.79| 0.905 | 39908.81| 340.39 | 0.066 |
| 2      | 0.742 | 0.189 | -0.695| -0.946| 202198| 1222.98| 0.886 | 31347.86| 184.58 | 0.069 |
| 3      | 0.685 | 0.415 | -0.230| -0.881| 195478| 2164.33| 0.839 | 35784.02| 184.25 | 0.101 |
| 4      | 0.859 | 0.993 | -0.841| 0.195 | 199390| 2167.08| 0.886 | 38641.17| 222.42 | 0.074 |
| 5      | 0.992 | -0.353| 0.263 | 0.006 | 202843| 1909.43| 0.906 | 39371.73| 251.46 | 0.056 |
| 6      | 0.624 | -0.082| -0.954| -0.180| 205595| 1773.84| 0.889 | 38066.16| 268.75 | 0.0624 |
| 7      | 0.445 | -0.035| 0.815 | 0.363 | 205233| 1809.66| 0.910 | 39225.38| 320.17 | 0.0608 |
| 8      | 0.089 | 0.661 | -0.313| 0.312 | 197906| 2055.36| 0.825 | 34406.11| 253.19 | 0.105 |
| 9      | -0.516| 0.706 | -0.475| 0.722 | 201500| 1812.56| 0.896 | 34817.54| 268.68 | 0.068 |
| 10     | -0.106| -0.874| 0.058 | -0.442| 198622| 1278.57| 0.887 | 31470.24| 270.47 | 0.077 |
| 11     | -0.649| -0.607| 0.327 | 0.201 | 202635| 1471.26| 0.901 | 35827.69| 295.05 | 0.059 |
| 12     | -0.882| -0.186| -0.401| -0.011| 201086| 1357.05| 0.887 | 31403.03| 278.27 | 0.069 |
| 13     | -0.884| -0.953| 0.972 | -0.284| 196634| 1665.57| 0.915 | 35012.47| 308.68 | 0.056 |
| 14     | -0.398| 0.093 | 0.089 | -0.759| 205082| 1398.53| 0.893 | 34411.84| 193.73 | 0.076 |
| 15     | -0.458| -0.264| -0.115| 0.662 | 205245| 1550.01| 0.861 | 34516.64| 170.19 | 0.092 |
| 16     | -0.206| -0.696| -0.769| -0.379| 200628| 1169.8 | 0.888 | 30885.78| 304.52 | 0.071 |
| 17     | 0.042 | -0.513| -0.007| 0.107 | 206254| 1665.15| 0.882 | 35770.48| 277.27 | 0.081 |
| 18     | 0.242 | 0.749 | -0.909| 0.468 | 202581| 2004.65| 0.887 | 38772.16| 240.55 | 0.067 |
| 19     | -0.979| 0.371 | 0.392 | -0.560 | 189767| 1764.15| 0.873 | 33737.31| 262.43 | 0.073 |
| 20     | 0.493 | 0.023 | 0.591 | -0.508| 204700| 1782.73| 0.885 | 38057.96| 321.84 | 0.081 |
| 21     | 0.147 | 0.859 | 0.527 | -0.077| 200909| 2087.76| 0.875 | 37733.44| 273.97 | 0.087 |
| 22     | 0.575 | -0.862| -0.542| -0.225| 201514| 1452.86| 0.899 | 33778.72| 248.60 | 0.063 |
| 23     | -0.147| 0.588 | -0.611| 0.923 | 203160| 1929.06| 0.898 | 38180.83| 268.98 | 0.059 |
| 24     | -0.006| 0.516 | 0.654 | 0.816 | 201567| 2143.57| 0.895 | 38421.75| 294.86 | 0.079 |
| 25     | -0.851| 0.213 | 0.912 | 0.409 | 192639| 1562.14| 0.89491| 32114.08| 237.92 | 0.065 |
| 26     | -0.547| -0.566| 0.446 | -0.823| 199149| 1396.8 | 0.87393| 35215.75| 191.96 | 0.083 |
| 27     | 0.307 | 0.282 | -0.348| -0.712| 205802| 1588.46| 0.91179| 37046.85| 216.61 | 0.056 |
| 28     | -0.303| -0.293| 0.717 | -0.619| 201187| 1543.71| 0.90034| 35939.55| 284.87 | 0.0746 |
| 29     | 0.785 | -0.752| 0.140 | 0.765 | 198769| 1648.53| 0.89837| 35414.05| 226.18 | 0.069 |
| 30     | -0.709| -0.402| -0.180| 0.945 | 202807| 1536.48| 0.85023| 33430.24| 135.90 | 0.0976 |
Table 2. Standardized data.

| Number | \( Y^\mu_1 \) | \( Y^\mu_2 \) | \( Y^\mu_3 \) | \( Y^\sigma_1 \) | \( Y^\sigma_2 \) | \( Y^\sigma_3 \) |
|--------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1      | -0.242         | 1.479          | 0.550          | 1.692          | 2.052          | -0.570         |
| 2      | -0.302         | -1.465         | 0.309          | -1.565         | -1.347         | -0.365         |
| 3      | 1.444          | 1.689          | 0.2066         | 0.025          | -1.352         | 2.221          |
| 4      | 0.406          | 1.701          | -0.055         | 1.160          | -0.707         | -0.037         |
| 5      | -0.018         | 0.662          | 1.315          | 1.465          | -0.135         | -1.152         |
| 6      | -1.121         | 0.168          | 0.819          | 0.925          | 0.239          | -0.786         |
| 7      | -1.035         | 0.295          | 0.943          | 1.403          | 1.497          | -0.882         |
| 8      | 0.792          | 1.234          | -2.360         | -0.491         | -0.998         | 2.616          |
| 9      | -0.129         | 0.305          | 0.368          | -0.339         | 0.237          | -0.417         |
| 10     | 0.605          | -1.328         | -0.287         | -1.524         | 0.277          | 0.185          |
| 11     | -0.410         | -0.804         | 1.034          | 0.0418         | 0.855          | -0.949         |
| 12     | -0.026         | -1.123         | 0.309          | -1.546         | 0.455          | -0.565         |
| 13     | 1.131          | -0.201         | 1.413          | -0.267         | 1.197          | -1.219         |
| 14     | -0.999         | -1.010         | -0.209         | -0.489         | 1.203          | 0.1098         |
| 15     | -1.038         | -0.568         | -1.423         | -0.451         | -1.559         | 1.415          |
| 16     | 0.090          | -1.591         | 0.159          | -1.718         | 1.091          | -0.234         |
| 17     | -1.275         | -0.202         | -0.837         | 0.019          | 0.432          | 0.486          |
| 18     | -0.397         | 0.103          | 0.486          | 1.214          | -0.358         | -0.516         |
| 19     | 3.076          | 0.134          | 0.006          | -0.736         | 0.099          | -0.993         |
| 20     | -0.909         | 0.199          | -0.602         | 0.922          | 1.542          | 0.502          |
| 21     | 0.019          | 1.367          | 0.002          | -0.938         | 0.357          | 0.983          |
| 22     | -0.133         | -0.857         | 0.785          | -0.719         | -0.194         | -0.760         |
| 23     | -0.558         | 0.736          | 1.049          | 0.972          | 0.244          | -0.962         |
| 24     | -0.146         | 1.601          | -0.408         | 1.070          | 0.851          | 0.305          |
| 25     | 2.237          | -0.531         | 0.609          | 1.305          | -0.410         | -0.618         |
| 26     | 0.468          | -1.015         | -0.767         | 0.190          | 1.231          | 0.677          |
| 27     | -1.169         | -0.449         | 1.291          | 0.516          | -0.813         | -1.135         |
| 28     | -0.051         | -0.588         | -0.108         | 0.085          | 0.610          | 0.013          |
| 29     | 0.566          | -0.257         | 0.304          | -0.116         | -0.637         | -0.361         |
| 30     | -0.452         | -0.610         | -1.827         | -0.845         | -1.995         | 1.912          |

Table 3. Principal component analysis of the location characteristics.

| Principal component | Amount | Variance contribution rate | Cumulative contribution rate | \( Y^\mu_1 \) | \( Y^\mu_2 \) | \( Y^\mu_3 \) |
|---------------------|--------|----------------------------|----------------------------|----------------|----------------|----------------|
| 1                   | 1.2779 | 42.7%                      | 42.7%                      | 0.4816         | 0.6136         | -0.6258        |
| 2                   | 0.9069 | 30.3%                      | 54.73%                     | 0.8736         | 0.3927         | 0.2873         |
| 3                   | 0.8077 | 26.99%                     | 100%                       | 0.0695         | 0.6851         | 0.7252         |

According to equation (13), the corresponding principal component function is given as following,

\[
\begin{align*}
  z^\mu_1 &= 0.4816 Y^\mu_1 + 0.6136 Y^\mu_2 - 0.6258 Y^\mu_3 \\
  z^\mu_2 &= 0.8736 Y^\mu_1 - 0.3927 Y^\mu_2 + 0.2873 Y^\mu_3 \\
  z^\mu_3 &= 0.0695 Y^\mu_1 + 0.6851 Y^\mu_2 + 0.7252 Y^\mu_3
\end{align*}
\]  

(22) (23) (24)

Bring equation (22), equation (23) and equation (24) into equation (15), the principal component comprehensive score model of each response position characteristic is obtained as following,

\[
WPC^\mu = 0.427 z^\mu_1 + 0.303 z^\mu_2 - 0.2699 z^\mu_3
\]

=0.4891 Y^\mu_1 + 0.3279 Y^\mu_2 + 0.0156 Y^\mu_3

(25)
Similarly, the principal component comprehensive score model of each response deviation characteristic is obtained as following.

\[ WPC_\sigma = 0.437Y_1^\sigma + 0.4576Y_2^\sigma - 0.0488Y_3^\sigma \]  
\[ (26) \]

Secondly, we use mean absolute percentage error approach [26] (MAPE, \[ MAPE= \sum_{i=1}^{n} \left| \frac{y_i - \hat{y}_i}{y_i} \right| / n \] ) to verify the effectiveness of 6 Kriging meta-models of position characteristics and divergence characteristics with the data in table 2, where \( y_i \) is the true value of performance response, \( \hat{y}_i \) is the predicted value of performance response, \( n \) is the number of sample points. The MAPE values of 6 Kriging meta-models are 1.08\%, 1.24\%, 0.78\%, 1.46\%, 1.35\% and 1.22\% respectively which are all less than the upper limit, i.e. 3\%. Compared to the results which is obtained from polynomial models in literature [23], the MAPE values of 6 polynomial models are also given as 2.15\%, 3.02\%, 3.21\%, 1.83\%, 4.13\%, 3.67\% respectively. It can be seen that the MAPE of inventory and customer fill rate exceed the upper limit, i.e. 3\%. The situation shows that 6 Kriging meta-models can well approximate the corresponding real performance responses, while 6 polynomial models have lost the capability to objectively reflect the real performance responses (quality characteristic), so the optimization results based on these Kriging meta-models are more reasonable compared to that based on polynomial models. Then 3 Kriging meta-models \( \hat{Y}_i^\mu = \hat{f}_i(x), i = 1, 2, 3 \) between the controllable factors and the position characteristics of performance responses are constructed respectively. Similarly, 3 Kriging divergence characteristics meta-models are also given such as \( \hat{Y}_i^\sigma = \hat{g}_i(x), i = 1, 2, 3 \). Thus the Kriging principal component comprehensive score model of location characteristics is obtained as following.

\[ WPC_\mu = 0.4891 \times \hat{f}_1(x) + 0.3279 \times \hat{f}_2(x) + 0.0156 \times \hat{f}_3(x) \]  
\[ (27) \]

The Kriging principal component comprehensive score model of divergence characteristics is also obtained as following.

\[ WPC_\sigma = 0.437 \times \hat{g}_1(x) + 0.4576 \times \hat{g}_2(x) - 0.0488 \times \hat{g}_3(x) \]  
\[ (28) \]

Combined equation (27) and equation (28), the optimization objective function is given as following,

\[ \min MS = \lambda WPC_\mu + (1 - \lambda)WPC_\sigma \]  
\[ s.t. \quad 0 \leq \lambda \leq 1 \]  
\[ (29) \]

As a result, the supply chain optimization problem with multiple correlation responses is transformed into a single response optimization problem considering both mean and standard deviation. The optimal parameter factor combination can be determined by optimizing equation (29).

### 4.2. Optimization Results

In the optimization step of this experiment, we used the standard optimization function \( fmincon \) in MATLAB to obtain the robust optimal solution of the problem (29). Because the initial point should be set in the function, \( fmincon \) is easily trapped into a local optimal. To obtain a global optimal, 1000 initial sample points are selected in the feasible region, which converge to the stable point or boundary point. The convergence condition is that the change ratio of response value is less than 10-6. Then our results are compared with that obtained from the method (KL) proposed by Kim and Lin [27], the traditional optimization method (DS) [28] and the method proposed in literature [23]. The optimization results of different methods are shown in table 4.
It can be seen from table 4, the optimization results are also different with the different $\lambda$. When $\lambda$ is 1, the result is only a special case of our proposed method, i.e. we only consider the principal component analysis on the mean value of the response while ignore the effect of the standard deviation of the response. At this time, the standard deviation of the response using our proposed method is larger than that obtained by other $\lambda$, especially when the standard deviation of $y_2$ reaches 253, which is much larger than that obtained by 186 when $\lambda = 0.7$ and 138 when $\lambda = 0.5$. This illustrates that different value of $\lambda$ have a great impact on the optimization results. The larger $\lambda$, the experiment results are more inclined to the optimization of the mean response, and the smaller $\lambda$, the standard deviation response has more impact on the optimization results.

To compare row 4 and row 8 of table 4, when $\lambda$ is 0.3, it can be seen that the results obtained by our proposed method are basically the same as those obtained by the method in literature [28]. This shows that when simultaneous optimization of mean and standard deviation, our proposed method is as effective as the method mentioned in literature [28], and can get the ideal optimization results. However, the method mentioned in literature [28] only considers the performance response with the lowest satisfaction and ignores other cases which will have a certain impact on the final optimization results. Our proposed method in this paper can effectively avoid this situation. Although the optimization results obtained by our proposed method in this paper are similar to those obtained by the method proposed in literature [23], the optimization results of our proposed method in this paper are more reliable and robust (which will be illustrated in next section C), so our proposed method has more advantages compared with the method proposed in literature [23].

In addition, our proposed method adjusts the emphasis of the optimization results by changing the value of $\lambda$, and selectively optimizes the mean and standard deviation according to the actual needs of the decision-maker. Therefore the optimization results from our proposed method have some certain flexibility, moreover it can provide theoretical support and technical guidance for the decision-maker to make reasonable decisions.

### 4.3. Discussion on Robustness of Our Proposed Method

In order to discuss the robustness of our proposed method, when the weight $\lambda$ is 0.3, we take the controllable factors in row 3, row 4, row 8 and row 12 in table 4 as the central points, supposing 0.2 as the interval length, and select 1000 sample points randomly in this interval, then we optimize formula (29) using these sample points as the initial points. Thus, the mean and standard deviation of 1000 optimal solutions are obtained by using different optimization methods, and the results are shown in table 5 and table 6. Column 3, column 5 and column 7 of table 5 and table 6 give the standard deviation corresponding to 1000 optimal solutions obtained by each optimization method. It can be seen that the standard deviation corresponding to the optimal solution given by our proposed method is the smallest, which shows that our proposed method has the best robust performance. For example,
it can be seen from row 4 and row 5 of table 5 that the optimization results obtained by our proposed method are similar to those obtained by the method proposed in document [23], but the standard deviations of the three predicted mean responses are reduced by 32%, 4.9% and 33% respectively. Similarly, it can be seen from table 6 that the volatility of the three prediction standard deviation responses is reduced by 32.5%, 38.5% and 33% respectively, compared with the method proposed in document [23]. It is illustrated that the optimization results obtained by our proposed method are more reliable and robust.

Table 5. Comparison of robustness of different optimization methods for mean.

| Method      | $E(y^1_1)$ | $\sigma(y^1_1)$ | $E(y^2_1)$ | $\sigma(y^2_1)$ | $E(y^3_1)$ | $\sigma(y^3_1)$ |
|-------------|------------|-----------------|------------|-----------------|------------|----------------|
| DS method   | 201152     | 89.21           | 1469.7     | 2.53            | 0.861      | 0.23           |
| KL method   | 201691     | 53.37           | 1235.8     | 2.48            | 0.849      | 0.31           |
| Method [23] | 201752     | 43.24           | 1223.8     | 2.51            | 0.872      | 0.24           |
| our method  | 201689     | 29.36           | 1224.6     | 2.39            | 0.879      | 0.16           |

Table 6. Comparison of robustness of different optimization methods for deviation.

| Method      | $E(y^1_2)$ | $\sigma(y^1_2)$ | $E(y^2_2)$ | $\sigma(y^2_2)$ | $E(y^3_2)$ | $\sigma(y^3_2)$ |
|-------------|------------|-----------------|------------|-----------------|------------|----------------|
| DS          | 33536      | 31.36           | 252.3      | 0.54            | 0.0873     | 0.0027         |
| KL          | 20168      | 26.78           | 138.2      | 0.62            | 0.0861     | 0.0018         |
| Method [23] | 21578      | 15.73           | 138.5      | 0.26            | 0.0883     | 0.0006         |
| our method  | 20163      | 10.59           | 137.5      | 0.16            | 0.0886     | 0.0004         |

5. Conclusion
In view of the optimization problem of supply chain system with related performance responses, a robust parameter design based on principal component analysis, double response surface method and Kriging meta-model is presented in this paper. This method can not only effectively solve the correlation problem between multiple performance responses in supply chain, but also make up for the defects of traditional double response surface method. Firstly, principal component analysis is used to reduce the dimensions of multiple performance responses and transform them into irrelevant response variables; secondly, Kriging meta-model is used to describe the strong nonlinear relationship between controllable factors and performance responses in supply chain. Kriging meta-model has more advantageous over traditional response surface method. Furthermore, our method reduces the fluctuation caused by noise factors as much as possible in the process of optimization. The simulation results show the feasibility and effectiveness of the proposed method.

It should be pointed out that the proposed method still has some limitations. For example, we only consider the impact of noise factors on the system in this paper while neglect the impact caused by the volatility of controllable factors, so it is unable to effectively describe the volatility of the system. How to improve our proposed method by combining the volatility of controllable and noise factors will be the further research in the future.

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