On the fractal structure of the universe: methods, results and theoretical implication

Francesco Sylos Labini

Dépt. de Physique Théorique, Université de Genève, 24, Quai E. Ansermet, CH-1211 Genève, Switzerland and INFM Sezione Roma1 P.le A. Moro, 2, I-00185 Roma, Italy.

Abstract. The fact that galaxy distribution exhibits fractal properties is well established since twenty years. Nowadays, the controversy concerns the range of the fractal regime, the value of the fractal dimension and the eventual presence of a cross-over to homogeneity. Fractal properties maybe studied with methods which do not assume homogeneity a priori as the standard statistical methods do. We show that complementary to the adoption of new methods of analysis there are important theoretical implications for the usual scenario of galaxy formation. For example, we focus on the concept of bias and we show that it needs a basic revision even if future redshift surveys will be able to identify an eventual tendency to homogenization.

1. Introduction

The assumption of homogeneity in the distribution of matter lies at the heart of the Big Bang cosmology. The nature of the evidence, if any, for this assumption has, however, been the subject of very considerable controversy [1, 2]. A central point made by Pietronero [3] has been that the standard methods of analysis of galaxy red-shift catalogues, which provide the most direct probe of the (luminous) matter distribution, actually assume homogeneity implicitly. In this report we review the main points of this controversy: we discuss the different methods of analysis of redshift surveys.
samples, and the corresponding results. The basic point we try to clarify is: what do we learn from the redshift surveys? We show that complementary to the adoption of a new method of analysis there are important theoretical implications for the usual scenario of galaxy formation. Moreover we briefly discuss the results of N-body simulations. We refer to [4] for a more detailed and complete picture.

2. Redshift surveys and the usual methods of analysis

Until the seventies galaxy distributions were only known in terms of angular catalogues. These catalogues are limited by the apparent luminosity of galaxies. Since the intrinsic luminosity can vary over a range of the order of one million, the points of the angular catalogues correspond to extremely different distances and they are a complex convolution of these. These angular distributions appeared rather smooth and, as such, they justified the usual statistical assumptions of homogeneity at large angular separations. Actually the measurement of the amplitude of angular fluctuations with respect to the (angular) average density shows that at large enough scale galaxy distribution turns out to be very smooth and uniform [5, 6].

The redshift measurements permit to locate galaxies in 3-d space. They immediately showed a clumpy distribution with large clusters and large voids, in apparent contrast with the angular data. This finally led to 3-d catalogues from which one could make volume limited samples (i.e. samples which are not biased by any observational luminosity selection effect [4]), which provide the best and most direct information for the correlation analysis. These 3-d data have been and are extensively analysed with the usual $\xi(r)$ method. By using the standard two-points correlation function it has been found [6, 1] that galaxy structures are characterized by having a very well defined "correlation length" that is found to be $r_0 \approx 5h^{-1}Mpc$. The physical interpretation of such a scale being the distance above which the density fluctuations become of the same order of the average density. At twice this distance the fluctuations have small amplitude and the linear theory holds, i.e. the distribution becomes homogeneous.

Most authors who analyse galaxy (and cluster) catalogs use these methods, and they are the instruments in terms of which the predictions of all standard type theories of structure formation are framed. That the "correlation length" is a real physical scale characterizing the clustering of galaxies is affirmed by producing evidence for the stability of this scale in different samples. In practice, however, correlation lengths are not observed to be very stable, and it is here that the concept of "luminosity bias" enters [7, 8, 9]: Galaxies of different brightness are supposed to be clustered differently, and it is this which is proposed as the physical explanation of the
observed variation, rather than the absence of real underlying homogeneity. Rather than their being one real scale characterizing the correlations of galaxies, there is then an undetermined number of such scales. The luminosity bias effect is therefore the cornerstone of the concept of bias: In the standard biasing picture the distribution of dark matter, galaxies and galaxy clusters are Gaussian but with different amplitudes, i.e. different correlation lengths.

In summary, the most popular point of view, is that galaxy distribution exhibits indeed fractal properties with dimension $D \approx 1.3$ at small scale (i.e. up to $\sim 10 \div 20h^{-1}\text{Mpc}$). The main result of this approach is that a characteristic length is derived $r_0 \approx 5h^{-1}\text{Mpc}$ which should mark the tendency towards homogenization. This is in apparent agreement with the structureless angular data but it is puzzling with respect to the structures of the 3-d data. From this perspective it seems that the presence or absence of structures in the data is irrelevant for the determination of $r_0$.

Puzzled by these results we decided to reconsider the question of correlations from a broader and critical perspective [10, 4]. This allowed us to test homogeneity, instead of assuming it as in the $\xi(r)$ analysis, and to produce a totally unbiased description of the correlation properties using the methods of modern Statistical Physics. In the next section we discuss the methods and we show the main results. Very recently a wide debate on this subject is in progress [11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22].

3. Correlation properties of redshift samples

An alternative analysis has been proposed using statistical methods which do not make the assumption of homogeneity, and are appropriate for characterizing the properties of regular as well as irregular distributions [2, 3, 4]. Essentially this analysis makes use of a very simple statistic, the conditional average density, defined as

$$\Gamma(r) = \left\langle \frac{1}{S(r)} \frac{dN(<r)}{dr} \right\rangle$$

where $dN(<r)$ is the number of points in a shell of radius $dr$ at distance $r$ from an occupied point and $S(r)dr$ is the volume of the shell. Note that in Eq.1 there is an average over all the occupied points contained in the sample. If the distribution is scale invariant with fractal dimension $D$ we have

$$\Gamma(r) \sim r^{-\gamma}$$

2 See the web page http://www.phys.uniroma1.it/DOCS/PIL/pil.html where all these materials have been collected.
where $\gamma = 3 - D$. Clearly the conditional average density is an appropriate statistical tool with which to study fractal versus homogeneous properties in a given distribution, and in particular the approach (if any) to homogeneity. The problems of the standard analysis can easily be seen from the fact that, for the case of a fractal distribution the standard “correlation function” $\xi(r)$ in a spherical sample of radius $R_s$, is given by

$$\xi(r) = \frac{3 - \gamma}{3} \left( \frac{r}{R_s} \right)^{-\gamma} - 1.$$  \hfill (3)

Hence for a fractal structure the “correlation length” $r_0$ (defined by $\xi(r_0) = 1$), is not a scale characterizing any intrinsic property of the distribution, but just a scale related to the size of the sample. If, on the other hand, the distribution is fractal up to some scale $\lambda_0$ and homogeneous beyond this scale, it is simple to show that (if $\lambda_0 < R_s$, i.e. the crossover to homogeneity is well inside the sample size)

$$r_0 = \lambda_0 \cdot 2^{-\frac{\gamma}{3}}.$$  \hfill (4)

The correlation length does in this case have a real physical meaning (when measured in samples larger than $\lambda_0$), being related in a simple way to the scale characterizing homogeneity. In the case $D = 2$ we have $r_0 = \lambda_0/2$.

Finally it should be noticed that $\xi(r)$ is power law only for

$$\left( \frac{3 - \gamma}{3} \right) \left( \frac{r}{R_s} \right)^{-\gamma} \gg 1$$  \hfill (5)

hence for $r \ll r_0$: for larger distances there is a clear deviation from the power law behavior due to the definition of $\xi(r)$. This deviation, however, is just due to the size of the observational sample and does not correspond to any real change of the correlation properties. It is clear that if one estimates the $\xi(r)$ exponent at distances $r \lesssim r_0$, one systematically obtains a higher value of the correlation exponent due to the break of $\xi(r)$ in the log-log plot. In this respect it is useful to compute the log derivative of eq.\ref{eq:3} with respect to log($r$):

$$\gamma' = \frac{d(\log(\xi(r)))}{d\log(r)} = -\frac{2\gamma r_0^\gamma r^{-\gamma}}{2r_0^\gamma r^{-\gamma} - 1}$$  \hfill (6)

where $r_0$ is defined by $\xi(r_0) = 1$. The tangent to $\xi(r)$ at $r = r_0$ has a slope $\gamma' = -2\gamma$. It is clear that, even if the distribution has fractal properties, it is very difficult to recover the correct slope from the study of the $\xi(r)$ function. The $\xi(r)$ is intrinsically problematic to this end.

3 This calculation assumes a simple matching of a fractal onto a pure homogeneous distribution. For any particular model with fluctuations away from perfect homogeneity, the numerical factor will differ slightly depending on how precisely we define the scale $\lambda_0$.  

4
Table 1. The volume limited catalogues are characterized by the following parameters: - $R_d(h^{-1}Mpc)$ is the depth of the catalogue - $\Omega$ is the solid angle - $R_s(h^{-1}Mpc)$ is the radius of the largest sphere that can be contained in the catalogue volume. This gives the limit of statistical validity of the sample. - $r_0(h^{-1}Mpc)$ is the length at which $\xi(r) \equiv 1$. - $\lambda_0$ is the eventual real crossover to a homogeneous distribution that is actually never observed. The value of $r_0$ is the one obtained in the deepest VL sample. (Distances are expressed in $h^{-1}Mpc$).

| Sample      | $\Omega$ (sr) | $R_d$ | $R_s$ | $r_0$     | $D$     | $\lambda_0$ |
|-------------|---------------|-------|-------|-----------|---------|--------------|
| CfA1        | 1.83          | 80    | 20    | 6         | 1.8 ± 0.2| > 80         |
| CfA2South   | 1.23          | 130   | 50    | 15        | 2.0 ± 0.1| > 120       |
| PP          | 0.9           | 130   | 30    | 10        | 2.0 ± 0.1| > 130       |
| SSRS1       | 1.75          | 120   | 35    | 12        | 2.0 ± 0.1| > 120       |
| SSRS2       | 1.13          | 150   | 45    | 15        | 2.0 ± 0.1| > 130       |
| Stromlo-APM | 1.3           | 100   | 35    | 12        | 2.2 ± 0.1| > 150       |
| LEDA        | ∼5            | 300   | 150   | 45        | 2.1 ± 0.2| > 150       |
| LCRS        | 0.12          | 500   | 18    | 6         | 1.8 ± 0.2| > 500       |
| IRAS2Jy     | ∼5            | 60    | 20    | 5         | 2.0 ± 0.1| > 50        |
| IRAS1.2Jy   | ∼5            | 80    | 30    | 8         | 2.0 ± 0.1| > 50        |
| ESP         | 0.006         | 700   | 8     | 3         | 2.0 ± 0.2| > 700       |

The direct analysis of all available galaxy catalogues reported in [4], using the conditional average density $\Gamma(r)$. In Table we report the characteristics of the various catalogs we have analyzed by using the methods previously illustrated. We show in Fig.1 the results of the conditional density determinations in various redshift surveys [4]. All the available data are consistent with each other and show fractal correlations with dimension $D = 2.0 ± 0.2$ up to the deepest scale probed up to now by the available redshift surveys, i.e. $\sim 150h^{-1}Mpc$. A similar result has been obtained by the analysis of galaxy cluster catalogs [4]. If we consider also the determination of the radial density, which is a much weaker test because it does not involve an average [4, 16], we find a rather good continuation of the fractal behavior up to $\sim 1000h^{-1}Mpc$.

It is interesting to compare the analysis of Fig.1 with the usual one, made by the function $\xi(r)$, for the same galaxy catalogs. This is reported in Fig.2. From this point of view, the various data the various data appear to be in strong disagreement with each other. This is due to the fact that the usual analysis looks at the data from the perspective of analyticity and large scale homogeneity (within each sample). These properties have never
Figure 1. Full correlation analysis for the various available redshift surveys in the range of distance $\sim 1 \div 200h^{-1}Mpc$. A reference line with slope $-1$ is also shown, which corresponds to fractal dimension $D = 2$. 
been tested and they are not present in the real galaxy distribution so the result is rather confusing (Fig.2). Once the same data are analyzed with a broader perspective the situation becomes clear (Fig.2) and the data of different catalogs result in agreement with each other. It is important to remark that analyses like those of Fig.2 have had a profound influence in the field in various ways: first the different catalogs appear in conflict with each other. This has generated the concept of not fair samples and a strong mutual criticism about the validity of the data between different authors. In the other cases the discrepancy observed in Fig.2 have been considered real physical problems for which various technical approaches have been proposed. These problems are, for example, the galaxy-cluster mismatch, luminosity segregation, the richness-clustering relation and the linear non-linear evolution of the perturbations corresponding to the "small" or "large" amplitudes of fluctuations. All this problematic situation is not real and it arises only from a statistical analysis based on inappropriate and too restrictive assumptions that do not find any correspondence in the physical reality. It is also important to note that, even if the galaxy distribution would eventually became homogeneous at larger scales, the use of the above statistical concepts is anyhow inappropriate for the range of scales in which the system shows fractal correlations as those shown in Fig.1.

3.1. Other length scales

The usual analysis finds that rms fluctuations of the observed galaxy density field are very large on small scales, of order of unity within spheres of $8h^{-1}\text{Mpc}$ dropping as a power law as a function of scale, becoming few percent at several tens $h^{-1}\text{Mpc}$. In this perspective it therefore makes sense to reference to the density field of galaxies to its mean. Let $\rho(r)$ be the observed galaxy density field; the density fluctuation field is defined as

$$\delta(r) = \frac{\rho(r) - \langle \rho \rangle}{\langle \rho \rangle}$$

(7)

This quantity can be measured in redshift samples. The problem in this case is the same one which enters in the definition of $r_0$: one is comparing the amplitude of fluctuations to the mean density. As an example, one can consider a portion of a fractal structure of size $R_s$ and study the behavior of $\delta N/N$. The average density is just given by Eq.2 while the overdensity $\delta N$, as a function of the size $r$ ($r \leq R_s$) of a given in structure is:

$$\delta N = \frac{N(r)}{V(r)} - <n> = \frac{3}{4\pi}B(r^{-(3-D)} - R_s^{-(3-D)})$$

(8)

We have therefore

$$\frac{\delta N}{N} = \left( \frac{r}{R_s} \right)^{-(3-D)} - 1$$

(9)
Figure 2. Traditional analyses based on the function $\xi(r)$ of the same galaxy catalogs of the previous figure. The usual analysis is based on the a priori untested assumptions of analyticity and homogeneity. These properties are not present in the real galaxy distribution and the results appear therefore rather confusing. This lead to the impression that galaxy catalogs are not good enough and to a variety of theoretical problems like the galaxy-cluster mismatch, luminosity segregation, linear and non-linear evolution, etc. This situation changes completely and becomes quite clear if one adopts the more general conceptual framework that is at the basis the previous figure.
Clearly for structures that approach the size of the sample, the value of \( \frac{\delta N}{N} \) becomes very small and eventually becomes zero at \( r = R_s \).

Another typical length scale which is usually defined in the study of redshift samples is the scale at which the power spectrum (hereafter PS), of the *density fluctuations* has a turnover: \( dP(k(\lambda_f))/dk = 0 \). Essentially all the currently elaborated models of galaxy formation (e.g. [6]) *assume large scale homogeneity* and predict that the galaxy PS, which is the *PS of the density contrast*, decreases both toward small scales and toward large scales, with a turnaround somewhere in the middle, at a scale \( \lambda_f \) that can be taken as separating “small” from “large” scales. Because of the homogeneity assumption, the PS amplitude should be independent on the survey scale, any residual variation being attributed to luminosity bias (or to the fact that the survey scale has not yet reached the homogeneity scale). However, the crucial clue to this picture, the firm determination of the scale \( \lambda_f \), is still missing, although some surveys do indeed produce a turnaround scale around \( 100 \, h^{-1} \) Mpc. Recently, the CfA2 survey analyzed by [8] (PVGH), showed a \( n = -2 \) slope up to \( \sim 30 \, h^{-1} \) Mpc, a milder \( n \approx -1 \) slope up to \( 200 \, h^{-1} \) Mpc, and some tentative indication of flattening on even larger scales. PVGH also find that deeper subsamples have higher power amplitude, i.e. that the amplitude scales with the sample depth.

It is simple to show [4] that both features, bending and scaling, are a manifestation of the finiteness of the survey volume, and that they cannot be interpreted as the convergence to homogeneity, nor to a PS flattening. The systematic effect of the survey finite size is in fact to suppress power at large scale, mimicking a real flattening. In fact we have shown that even a fractal distribution of matter, which never reaches homogeneity, shows a sharp flattening and then a turnaround. In particular, it is possible to show that in a spherical sample of radius \( R_s \), which contains a portion of a fractal structure with dimension \( D = 2 \), the PS turnover scale is given by

\[
\lambda_f \approx 1.45 R_s
\]  
(10)

and hence it is another quantity related to the sample size rather than being an intrinsic characteristic scale of galaxy distribution.

4. What do we learn from galaxy catalogs?

As we have already mentioned the usual concept of bias arises from the interpretation of the results of the \( \xi(r) \) analysis. The concept of bias fixes the relative distribution of galaxies of different types, clusters and dark matter. In general [23] one assumes that there exists a direct proportionality between the density fluctuations of galaxies \( \delta_g \) and dark matter \( \delta_{DM} \)

\[
\delta_g = b \delta_{DM}
\]  
(11)
and the same concepts apply to galaxies of different masses and galaxy clusters. Under this assumption, the biasing parameter $b$ is independent of location. One case use the two-point correlation function $\xi_{gg}(r)$ and the mass autocorrelation function $\xi_{pp}(r)$ to define the bias factor

$$
\ell = \left( \frac{\xi_{gg}(r)}{\xi_{pp}(r)} \right)^{1/2}.
$$

Let us see in more detail the origin of the concept of bias as given in eq.12. It is a well known observational fact that galaxies of different morphological types have different clustering properties. For example, the most luminous elliptical galaxies usually reside in the clusters cores, at local density maxima, and are not present in low density fields, so that these objects seem to be the product of dense environments. There are various other morphological facts of this type [4] which support the fact that brighter (more massive) galaxies are more clustered than for example spirals (less massive). The different of clustering properties has been interpreted, through the $\xi(r)$ analysis, as a different amplitude of correlation for different galaxy types. In particular while for the general galaxy field the correlation length is $r_0 \approx 5h^{-1}Mpc$, for the brighter galaxies ($L > L^*$) it has been found [8, 9] that $r_0 \approx 16h^{-1}Mpc$. This trend seems to be confirmed also by the cluster (more massive than galaxies) distribution for which $r_0 \approx 25h^{-1}Mpc$ [24].

On the contrary from our interpretation it follows a number of important implications in this respect. As a crossover to homogeneity has not been found, all the length scales found by the $\xi(r)$ analysis are artifact of an inconsistent data analysis. The "correlation lengths" $r_0 = 5, 16, 25, ...h^{-1}Mpc$ are not real physical characteristic scales, but just fractions of sample sizes. Brighter objects allow one to investigate a larger volume of space. Hence, for example the sample size of cluster catalogs is usually larger than the one of galaxy samples. This simple observation explains why one obtains different correlations lengths, and in general why the correlation length seems to increase as a function of the luminosity of objects. To this qualitative observation, one may add a detailed study of the available galaxy and cluster samples. This has been done in a detailed way by our group and we refer to [4] for an exhaustive explanations of this fact.

Therefore, contrary to the usual interpretation, we have shown that the segregation of giant galaxies in clusters arises as a consequence of self-similarity of matter distribution, and that in this case the only relevant parameter is the exponent of the correlation function, while the amplitude is a spurious quantity that has no direct physical meaning and depends explicitly on the sample size. Let us explain in more detail this important point.
In a well-known review on the galaxy luminosity function (LF) Binggeli et al.\cite{25} state that "as the distribution of galaxies is known to be inhomogeneous on all scales up to a least $100\,h^{-1}\text{Mpc}$, a rich cluster of galaxies is like a Matterhorn in a grand Alpine landscape of mountain ridges and valleys of length up to 100 Km". This point of view can be seen in the light of the concept of multifractality of the mass distribution. The main observational aspects of galaxy luminosity and space distributions are strongly related and can be naturally linked and explained as a multifractal (MF) distribution. The concept of MF is appropriate to discuss physical systems with local properties of self-similarity, in which the scaling properties are defined by a continuous distribution of exponents. Roughly speaking one can visualize this property as having different scaling properties for different regions of the system (see \cite{10,26} for a more detailed discussion). The fundamental point which is dismissed in the usual picture, is that the whole matter distribution, i.e. weighing each point by its mass, is self-similar. This situation requires the generalization of the simple fractal scaling to a MF distribution in which a continuous set of exponents is necessary to describe the spatial scaling of peaks of different weight (mass or luminosity). In this respect the mass and space distributions become naturally entangled with each other.

The MF implies a strong correlation between spatial and mass distribution so that the number of objects with mass $M$ in the point $\vec{r}$ per unit volume $\nu(M,\vec{r})$, is a function of space and mass and it is not separable in a space density multiplied by a mass (or luminosity) function \cite{25}. This means that we cannot express the number of galaxies $\nu(M, x, y, z)$ lying in volume $dV$ at $(x,y,z)$ with mass between $M$ and $M + dM$ as the product of a space density and a luminosity function. This would assume that galaxy positions are not correlated with their luminosities, while the observations show just the opposite. Moreover we cannot define a well defined average density, independent on sample depth as for the simple fractal case. It can be shown\cite{26} that the mass function of a MF distribution, in a well defined volume, has indeed a Press-Schechter behavior.

The continuous set of exponents which describes a MF distribution can characterize completely the galaxy distribution when one considers the mass (or luminosity) of galaxies in the analysis. In this way many observational evidences are linked together and arise naturally from the self-similar properties of the distribution. Considering a MF distribution, the usual power-law space correlation properties correspond just to a single exponent of the MF spectrum: such an exponent simply describes the space distribution of the support of the MF measure. Furthermore the shape of the luminosity function (LF), i.e. the probability of finding a galaxy of a certain luminosity per unit volume, is related to the MF spectrum of exponents. We have shown that, under MF conditions, the LF is well
approximated by a power law function with an exponential tail. Such a function corresponds to the Schechter LF observed in real galaxy catalogs. In this case the shape of the LF is almost independent on the sample size, but the amplitude of the LF depends on the sample size as a power law function.

These results have important consequences from a theoretical point of view. In fact, when one deals with self-similar structures the relevant physical phenomenon which leads to the scale-invariant structures is determined by the exponent and not the amplitude of the physical quantities which characterizes such distributions.

The geometric self-similarity has deep implications for the non-analyticity of these structures. In fact, analyticity or regularity would imply that at some small scale the profile becomes smooth and one can define a unique tangent. Clearly this is impossible in a self-similar structure because at any small scale a new structure appears and the distribution is never smooth. Self-similar structures are therefore intrinsically irregular at all scales and correspondingly one has to change the theoretical framework into one which is capable of dealing with non-analytical fluctuations. This means going from differential equations to something like the Renormalization Group to study the exponents. For example the so-called "Biased theory of galaxy formation" \cite{27} is implemented considering the evolution of density fluctuations within an analytic Gaussian framework, while the non-analyticity of fractal fluctuations implies a breakdown of the central limit theorem which is the cornerstone of Gaussian processes \cite{4}.

In this scheme the space correlations and the luminosity function are then two aspects of the same phenomenon, the MF distribution of visible matter. The more complete and direct way to study such a distribution, and hence at the same time the space and the luminosity properties, is represented by the computation of the MF spectrum of exponents. This is the natural objective of theoretical investigation in order to explain the formation and the distribution of galactic structures. In fact, from a theoretical point of view one would like to identify the dynamical processes which can lead to such a MF distribution.

In this perspective, it would be extremely interesting to study the distribution of dark matter and to determine its correlation exponent. It could be that dark matter is distributed like an homogeneous fluid, having hence $D = 3$ even at small scale. In such a way one may save the usual FRW metric (which needs an homogenous density to be developed), while a substantial revision to the models of galaxy formation is required. On the contrary if dark matter is found to have the same distribution of luminous one, than a basic revision of the theory must be considered. In fact, if dark matter is essentially associated to luminous matter, then the use of FRW metric is not justified anymore. This does not necessarily imply that there
is no expansion or no Big Bang. It implies, however, that these phenomena should be described by more complex models \cite{1}.

It is worth to notice that from an observational point of view there are various arguments for the proposition that galaxies are fair tracers of the mass. For example no survey since, in 21-cm, infrared, ultraviolet or low surface brightness optical has revealed a void population. There is a straightforward interpretation: the voids are nearly empty because they contain little mass \cite{20}.

5. Comparison of N-body simulations with real data

Given the strong evidence for power-law correlations in the galaxy catalogs continuing to scales considerably larger than the characteristic scale \( r_0 = 5h^{-1} Mpc \) for galaxy clustering which is reproduced by any ‘adequate’ standard theory of structure formation, the value of a detailed comparison with the predictions of such theories is questionable. Further, the evidence in all the 3-d catalogs suggest a continuation of scale invariance in clustering to a scale where the contradiction does not require detailed investigation, and the results instead open up a completely new set of problems for this field. In some recent debate on the issue of homogeneity, however, it has been suggested that the properties of galaxy catalogs observed in the statistics used here can in fact be quite compatible with such standard theories. For example Wu \textit{et al} \cite{19} have suggested that the change in the fractal dimension \( D \) as estimated by various authors is consistent with the increase towards the asymptotic value of \( D = 3 \). In Figure 3 we reproduce a figure from this paper showing the predictions for the “fractal dimension” as a function of scale, as well as the findings of \cite{4} over the same range of scale, and the contradiction is manifest.

In any standard model of structure formation the range we are interested in is that in which the non-linear evolution of the perturbations is very important, and it is therefore necessary to go beyond the analysis on which a figure such as that of Wu \textit{et al} is based. We have performed \cite{18} a comparison with the predictions of standard theories by making use of mock catalogs compiled by Cole \textit{et al} \cite{30}, designed specifically for comparison with the Sloan Digital Sky Survey. We thus conclude that our results are in disagreement from the predictions for standard theories of structure formation. Further the striking consistency with all the galaxy 3-d catalogs, some of which probe considerably large scales, suggests the need for a fundamental revision of such theories (see for a detailed discussion \cite{1, 28, 29}). In particular the central point of the analysis is that attention should be shifted from correlation amplitudes to correlation exponents, as it is only in terms of these concepts that the structures seen in red-shift surveys are correctly characterized.
Figure 3. The fractal dimension versus distance in three Cold Dark Matter models of power spectra with shape and normalized parameters as reported in the labels (solid lines) (from Wu et al., 1998). We show also the different experimental determinations of the fractal dimension we have obtained. No agreement can be found at any scale.
6. Conclusions

From the theoretical point of view the fact that we have a situation characterized by self-similar structures implies that we should not use concept like $\xi(r)$, $r_0$, $\delta N/N$ and certain properties of the power spectrum, because they are not suitable to represent the real properties of the observed structures. To this end also the N-body simulations should be considered from a new perspective. One cannot talk about "small" or "large" amplitudes for a self-similar structure because of the lack of a reference value like the average density. The Physics should shift from "amplitudes" towards "exponent" and the methods of modern statistical Physics should be adopted. This requires the development of constructive interactions between two fields.

Possible Crossover. We cannot exclude of course, that visible matter may really become homogeneous at some large scale not yet observed. Even if this would happen the best way to identify the eventual crossover is by using the methods we have described and not the usual ones. From a theoretical point of view the range of fractal fluctuations, extending at least over three decades ($1 \div 1000h^{-1}Mpc$), should anyhow be addressed with the new theoretical concepts. Then one should study the (eventual) crossover to homogeneity as an additional problem. For the moment, however, no tendency to such a crossover is detectable from the experimental data and it may be reasonable to consider also more radical theoretical frameworks in which homogenization may simply not exist at any scale, at least for luminous matter.

Dark Matter. All our discussion refers to luminous matter. It would be nice if the new picture for the visible universe could reduce, to some extent, the importance of dark matter in the theoretical framework. At the moment however this is not clear. We have two possible situations: (i) if dark matter is essentially associated to luminous matter, then the use of FRW metric is not justified anymore. This does not necessarily imply that there is no expansion or no Big Bang. It implies, however, that these phenomena should be described by more complex models\[29]. (ii) If dark matter is homogeneous and luminous matter is fractal then, at large scale, dark matter dominate the gravity field and the FRW metric is again valid\[29]. The visible matter however remains self-similar and non analytical and it still requires the new theoretical methods mentioned before\[28].

Acknowledgments

I would like to thank L. Pietronero for continuous collaborations. I am also in debt with Y. Baryshev, R. Durrer, M. Joyce, M. Montuori, and P. Teerikorpi with whom various parts of these work have been
done. I warmly thank A. Amici, H. De Vega, H. Di Nella, J.P. Eckmann, A. Gabrielli, B.B. Mandelbrot, D. Pfenninger, N. Sanchez and F. Vernizzi for useful discussions and collaborations. Finally I thank the organizers for the invitation to this stimulating meeting. This work has been partially supported by the EEC TMR Network ”Fractal structures and self-organization” ERBFMRXCT980183 and by the Swiss NSF.

References

[1] Davis, M., 1997 p.50 in the Proc. of the Conference ”Critical Dialogues in Cosmology” N. Turok Ed. World Scientific
[2] Pietronero, L., Montuori, M. and Sylos Labini, F., 1997 p.24 in the Proc of the Conference ”Critical Dialogues in Cosmology” N. Turok Ed. World Scientific
[3] Pietronero L., 1987 Physica A, 144, 257
[4] Sylos Labini F., Montuori M., Pietronero L., 1998 Phys.Rep. 293, 66
[5] Peebles, P.E.J., 1980 ”The Large Scale Structure of The Universe” Princeton Univ.Press.
[6] Peebles, P.E.J., 1993 ”Principles of Physical Cosmology”, Princeton Univ.Press
[7] Davis M. et al., 1988 Astrophys. J. Lett. 333, L9
[8] Park C., Vogeley M. and Geller M. 1994 Astrophys. J. 431, 569
[9] Benoist C., 1996 et al Astrophys. J. 472, 452
[10] Coleman, P.H. and Pietronero, L.,1992 Phys.Rep. 231,311
[11] Guzzo L., 1997 New Astronomy 2, 517
[12] Sylos Labini F., Montuori M., Pietronero L., 1998a New astronomy submitted (astro-ph/9801151)
[13] Coles P., 1998 Nature 391, 120
[14] Scaramella et al., 1998 Astronom.Astrophys. 334,404
[15] Teerikorpi, P., et al Astron.Astrophys. 334, 395
[16] Joyce M., Sylos Labini F., Montuori M. and Pietronero L. Astron.Astrophys. 1998 Submitted (astro-ph/9805126)
[17] Cappi A., Benoist C., da Costa L.N. and Maurogordato S. Astron.Astrophys. 1998 335,779
[18] Joyce M., Sylos Labini F., Montuori M 1998 preprint
[19] Wu K.K.S., Lahav O. and Rees M., 1998 Nature submitted (astro-ph/9804062)
[20] Peebles, P.E.J., 1998 In the Proceed. of the Conference ”les Rencontres de Physique de la Vallee d Aosta” ed. M. Greco (astro-ph/9806201)
[21] Mandelbrot B., 1998 in the Proceedings of the *Erice Chalonge School, "Current Topics in Astrofundamental Physics: Primordial Cosmology*" , NATO ASI series C511,N. Sanchez and A. Zichichi Eds, Kluwer.

[22] L. Pietronero and F. Sylos Labini in the Proc. of the Euroconference 5eme “Colloque Cosmologie” on “Fundamental Problems in Classical Quantum and String Cosmology” N. Sanchez and H. De Vega Editors (In the press) (astro-ph/9807138)

[23] Strauss M.A. 1998 *Nature* in the press (astro-ph/9807199)

[24] Bachall N.A., & Soneira R.M., 1988 *Astron. Astrophys.* 270, 20

[25] Binggeli, B., Sandage, A., Tammann, G. A. 1988 *Astron. Astrophys. Ann. Rev.* 26, 509

[26] Sylos Labini F. & Pietronero L.,1996 *Astrophys.J.*, 469, 28

[27] Kaiser N., 1984 *Astrophys.J.* 284, L9

[28] Durrer R. and Sylos Labini F., 1998 *Preprint* (astro-ph/9804171)

[29] Y. Baryshev, F. Sylos Labini, M. Montuori , L. Pietronero, and P. Teerikorpi *Fractals* (1998) In the Press.

[30] Cole S., Hatton S., Weinberg D., Frenk C. 1998 *Mon.Not.R.acad.Soc* In the Press