Single-atom as a macroscopic entanglement source

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We discuss the generation of a macroscopic entangled state in a single atom cavity-QED system. The three-level atom in a cascade configuration interacts dispersively with two classical coherent fields inside a doubly resonant cavity. We show that a macroscopic entangled state between these two cavity modes can be generated under large detuning conditions. The entanglement persists even under the presence of cavity losses.

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INTRODUCTION

Quantum entanglement lies at the heart of quantum computing and quantum information science. Cavity quantum electrodynamics (QED) provides an important testing ground for these ideas. For example, cavity QED can be used to not only store quantum information but also act as a source of entanglement [1-8]. The generation of entanglement in cavity QED has been studied by many authors including the generation of entangled coherent states using a single trapped photon [9, 10]. More recently, generation of macroscopic entangled states is still for atomic cloud. On the other hand, the photon number can be very large [10, 11]. The scheme lead to two-mode entanglement even when the average emission laser (CEL) [9], it was shown that a CEL can act with the two cavity modes with detunings δ with δ = |ω1 − (Ea − Eb)| = |ω2 − (Eb − Ec)|. The two atomic transitions (namely, |a⟩ ↔ |b⟩ and |b⟩ ↔ |c⟩) are also driven by two classical fields with the same detunings as their corresponding quantized field modes and Ω1 and Ω2 are the Rabi frequencies of the two classical fields. The dipole forbidden atomic transition between |a⟩ and |c⟩ are resonantly driven by another classical field of Rabi frequency Ω.

The Hamiltonian of our system under the dipole and rotating wave approximation and in the interaction picture is given by

\[ H_I = g_1 (\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) \sigma_{bc} + (\hat{a}_1^\dagger \hat{a}_2^\dagger + \hat{a}_2 \hat{a}_1) \sigma_{ab} \]

\[ + g_2 (\hat{a}_2^\dagger \hat{a}_2 + \hat{a}_2 \hat{a}_2^\dagger) \sigma_{ab} + (\hat{a}_1^\dagger \hat{a}_2^\dagger + \hat{a}_2 \hat{a}_1) \sigma_{ba} \]

\[ + \Omega (\hat{a}_1^\dagger \sigma_{ac} + \hat{a}_2 \sigma_{bc}) - \hat{\delta} (\hat{\sigma}_{aa} + \hat{\sigma}_{cc}) \]

where \( \hat{\sigma}_{ij} = |i\rangle\langle j| \) (i, j = a, b, c) are the atomic operators. \( \hat{a}_1(\hat{a}_1^\dagger) \) and \( \hat{a}_2(\hat{a}_2^\dagger) \) are the creation (annihilation) operators of the two cavity modes and \( g_1 \) and \( g_2 \) are the atom-field coupling constants and in general they are different.

The Heisenberg equations of motion for the atomic operators \( \sigma_{bc} \) and \( \sigma_{ba} \) are given by

\[ i \frac{d\sigma_{bc}}{dt} = -g_1 \hat{a}_1^\dagger \sigma_{cc} - g_2 \hat{a}_2 \sigma_{bb} + \Omega \sigma_{ba} - \hat{\delta} \sigma_{bc} \]

\[ i \frac{d\sigma_{ba}}{dt} = -g_1 \hat{a}_1 \sigma_{bb} + g_2 \hat{a}_2^\dagger \sigma_{cc} + \Omega \sigma_{bc} + \hat{\delta} \sigma_{ba} \]

(System Description and Calculations)
The approximate effective Hamiltonian for this case reduces to

\[ \hat{H}_b = \eta_1 \hat{a}_1^\dagger \hat{a}_1 + \eta_2 \hat{a}_2^\dagger \hat{a}_2 + \frac{1}{2} (\eta_1 + \eta_2) + \xi (\hat{a}_1 \hat{a}_2 + \hat{a}_1^\dagger \hat{a}_2^\dagger) \]  

(5)

where

\[ \xi = \frac{2g_1 g_2 \Omega}{\delta^2 - \Omega^2}, \]

\[ \eta_1 = \frac{2g_1^2 \delta}{\delta^2 - \Omega^2}, \]

\[ \eta_2 = \frac{2g_2^2 \delta}{\delta^2 - \Omega^2}. \]

(6)

This Hamiltonian can be rewritten as

\[ \hat{H}_b = (\eta_1 + \eta_2) \hat{K}_0 + \xi (\hat{K}_- + \hat{K}_+) + \frac{1}{2} (\eta_1 - \eta_2) \hat{N}_0. \]  

(7)

where

\[ \hat{K}_0 = \frac{1}{2} (\hat{a}_1^\dagger \hat{a}_1 + \hat{a}_2^\dagger \hat{a}_2 + 1), \]

\[ \hat{K}_- = \hat{a}_1 \hat{a}_2, \]

\[ \hat{K}_+ = \hat{a}_1^\dagger \hat{a}_2^\dagger, \]

\[ \hat{N}_0 = \hat{a}_1^\dagger \hat{a}_1 - \hat{a}_2^\dagger \hat{a}_2. \]

These operators can be verified to obey the \( SU(1, 1) \) commutation relations \( [\hat{K}_-, \hat{K}_+] = 2 \hat{K}_0, [\hat{K}_0, \hat{K}_\pm] = \pm \hat{K}_\pm, \) and \( [\hat{N}_0, \hat{K}_0] = [\hat{N}_0, \hat{K}_\pm] = 0. \) We can therefore use the \( SU(1, 1) \) Lie-algebra to expand the unitary evolution \( e^{-i\hat{H}_b t} \) as

\[ \hat{U} = e^{(A_+ \hat{K}_+) e^{-\frac{\eta_1 - \eta_2}{\phi} \hat{N}_0} e^{(A_- \hat{K}_-)} \]  

(8)

where

\[ A_0 = a_0^2, \]

\[ A_+ = A_- = -\frac{i \xi t}{\phi} a_0 \sinh \phi \]

(9)

with

\[ a_0 = \frac{1}{\cosh \phi + it \frac{(\eta_1 + \eta_2)}{2 \phi} \sinh \phi} \]

\[ \phi^2 = \frac{[-(\eta_1 + \eta_2)^2 + \xi^2]}{2}. \]

(10)

We now consider the case when the two-mode field is initially prepared in a vacuum state \( |0, 0\rangle \). The time evolution of the field state can be obtained as

\[ |\Psi_f(t)\rangle = \exp(A_+ \hat{a}_1^\dagger \hat{a}_2) \exp(\alpha_1 \hat{a}_1^\dagger) \exp(\alpha_2 \hat{a}_2^\dagger) |0, 0\rangle \]  

(11)

with

\[ \alpha_1 = \frac{\Omega_2}{g_2} A_+ + \frac{\Omega_1}{g_1} [a_0 e^{-\frac{\phi}{2} (\eta_1 - \eta_2)} - 1], \]

\[ \alpha_2 = \frac{\Omega_1}{g_1} A_+ + \frac{\Omega_2}{g_2} [a_0 e^{-\frac{\phi}{2} (\eta_1 - \eta_2)} - 1]. \]

(12)

The \( SU(1, 1) \) Lie-algebra yields

\[ e^{A_+ \hat{a}_1^\dagger \hat{a}_2} = e^{(\theta_1 \hat{a}_1 \hat{a}_2 - \theta_2 \hat{a}_1^\dagger \hat{a}_2^\dagger)} e^{A_+ \hat{a}_1 \hat{a}_2} e^{\theta_1 \hat{a}_1 \hat{a}_2 + \theta_2 \hat{a}_1^\dagger \hat{a}_2^\dagger + 1}. \]
Let $\theta = re^{i\epsilon}$, $g = \ln \cosh r$ where $r$ and $\epsilon$ are determined by the relation
$$A_+ = -e^{i\epsilon} \tanh r. \quad (13)$$
The squeezed parameter $r$ and $\epsilon$ are
$$r = \tanh^{-1} |A_+|, \quad (14)$$
$$\cos \epsilon = -\frac{\text{Re}(A_+)}{|A_+|}, \quad \sin \epsilon = -\frac{\text{Im}(A_+)}{|A_+|}.$$

The state of the system can then be written as
$$|\Psi_f(t)\rangle = e^{(\theta^* a_1 a_2 - \theta a_1^+ a_2^+)}|\alpha_1 \cos r, \alpha_2 \cosh r\rangle \quad (15)$$
$$= S(\theta)D(\alpha_1 \cosh r)D(\alpha_2 \cosh r)|0, 0\rangle. \quad (19)$$

This is a two-mode squeezed state which can also be generated by a parametric amplifier. The total average photon number of the two-mode field for an initial vacuum state is
$$N = 2 \sinh^2 r. \quad (20)$$

The entanglement condition still has the form of Eq.$(18)$. Next we consider the effect of the cavity losses by including the cavity damping in the equation of motion for the density operators. The equation of motion for the density operator is given by
$$\dot{\rho} = -i[\hat{a}_1^+ \hat{a}_1 \hat{a}_2^+ \hat{a}_2 + \eta_1 \hat{a}_1^+ \hat{a}_1 + \eta_2 \hat{a}_2^+ \hat{a}_2 + \Omega_1 \hat{a}_1 \hat{a}_1^+ \hat{a}_2 \hat{a}_2^+ - \Omega_2 \hat{a}_1 \hat{a}_2^+ \hat{a}_2 \hat{a}_1^+]$$
$$+ (\eta_1 \Omega_1 \frac{\Omega_2}{g_2} + \kappa \frac{\Omega_1}{g_1} \Omega_2 \frac{\Omega_2}{g_2} + \kappa \frac{\Omega_1}{g_1} \Omega_2 \frac{\Omega_2}{g_2})] \hat{\rho} - 2 \kappa \sum_{i=1,2} (\hat{a}_i \hat{a}_i^+ - \hat{a}_i^+ \hat{a}_i) \hat{\rho} + \hat{\rho} \hat{a}_i^+ \hat{a}_i.$$

The resulting equations for the expectation values of the field operators are
$$\frac{d\langle\hat{a}_1^+ \hat{a}_1\rangle}{dt} = -i[\xi(\langle\hat{a}_1^+ \hat{a}_1\rangle - \langle\hat{a}_1\hat{a}_2\rangle) + \langle\eta_1 \frac{\Omega_1}{g_1} \Omega_2 \frac{\Omega_2}{g_2} + \kappa \frac{\Omega_1}{g_1} \Omega_2 \frac{\Omega_2}{g_2} \rangle]$$
$$+ (\eta_1 \frac{\Omega_1}{g_1} \Omega_2 \frac{\Omega_2}{g_2} + \kappa \frac{\Omega_1}{g_1} \Omega_2 \frac{\Omega_2}{g_2})] \langle\hat{a}_1\rangle,$$
$$\frac{d\langle\hat{a}_1 \hat{a}_2\rangle}{dt} = -i[\xi(\langle\hat{a}_1 \hat{a}_2\rangle - \langle\hat{a}_1^+ \hat{a}_2^+\rangle + 1) + \langle\eta_1 \frac{\Omega_1}{g_1} \Omega_2 \frac{\Omega_2}{g_2} + \kappa \frac{\Omega_1}{g_1} \Omega_2 \frac{\Omega_2}{g_2} \rangle] \langle\hat{a}_2\rangle$$
$$+ (\eta_1 \frac{\Omega_1}{g_1} \Omega_2 \frac{\Omega_2}{g_2} + \kappa \frac{\Omega_1}{g_1} \Omega_2 \frac{\Omega_2}{g_2})] \langle\hat{a}_2^+\rangle,$$
$$\frac{d\langle\hat{a}_1^+ \hat{a}_2^+\rangle}{dt} = -i[\xi(\langle\hat{a}_1^+ \hat{a}_2^+\rangle - \langle\hat{a}_1^+ \hat{a}_2^+\rangle) - (\kappa + i\eta_2)] \langle\hat{a}_2^+\rangle.$$

On interchanging the subscripts 1 and 2 and taking the Hermitian conjugate, we can obtain the remaining five differential equations of $\langle\hat{a}_2^+ \hat{a}_2\rangle$, $\langle\hat{a}_1 \hat{a}_1\rangle$ etc.. These eight equations can be solved by using the standard techniques such as those based on Laplace transform method. We can then evaluate the average photon numbers and the quantity $(\langle\hat{a}_1\rangle^2 + \langle\hat{a}_2\rangle^2)$ for this system. These solutions are long and tedious and we do not reproduce them here. Instead, we present a numerical solutions for these equations in the next section.
DISCUSSION

FIG. 2: The time evolution of the total average photon number $N$ and $(\Delta \hat{u})^2 + (\Delta \hat{v})^2$. The two-mode field is entangled when $(\Delta \hat{u})^2 + (\Delta \hat{v})^2 < 2$ (Eq. (13)). The parameters are $\Omega_1 = 10, \Omega_2 = 40, g_1 = 1, g_2 = 2, \delta = 1000, \text{and } \Omega = 200$.

FIG. 3: The time evolution of total average photon number $N$. Solid lines in 2a and 2b correspond to two-mode squeezed vacuum state (Eq.(19)) and the two-mode coherent-squeezed state (Eq.(16)), respectively; dotted ( $\kappa = 0.01$) and dashed lines ( $\kappa = 0.02$) are plotted from Eq. (21). In Fig. 2a, $\Omega_1 = \Omega_2 = 0$ while for Fig. 2b $\Omega_1 = 10, \Omega_2 = 40$. For all plots, $g_1 = 1, g_2 = 2$ and $\delta = 1000, \Omega = 200$.

We now discuss the entanglement properties of the amplified fields inside the doubly resonant cavity. In our plots, all of parameters are in expressed in units of $g_1$. In Fig.2, we plot the average photon number and $(\Delta \hat{u})^2 + (\Delta \hat{v})^2$ for the two-mode coherent-squeezed state from Eqs.(16) and (18), respectively. Both the quantities exhibit oscillations. This is a consequence of the terms proportional to $\eta_1$ and $\eta_2$ in the Hamiltonian (5). The period of the oscillations can however be very large as $\eta_1$ and $\eta_2$ can be small. Thus we can have entanglement for a sufficiently large interaction times.

In Figs. 3 and 4, we plot $N$ and $(\Delta \hat{u})^2 + (\Delta \hat{v})^2$ in the small time region where entanglement is present. In Fig. 3 we plot the total average photon number $N$ as a function of time under two cases: $\Omega_1 = \Omega_2 = 0$ (Fig. 2a) and $\Omega_1 = 10, \Omega_2 = 40$ (Fig. 2b). Solid lines in Figs. 2a and 2b are plotted from Eqs. (20) and (16), respectively. Dotted lines and dashed lines are plotted from Eq. (21) with the inclusion of cavity losses. Comparing the two solid lines in Fig. 2a and 2b, we note that the average photon number of two-mode coherent-squeezed state is extremely larger as compared to a two-mode squeezed vacuum state. Even with the inclusion of cavity losses (dotted lines and dashed lines), the average photon number of two-mode fields still increase dramatically for the driven system. Thus the two-mode fields still can be amplified even when cavity losses are present.

In Fig. 4, we show the time evolution of $(\Delta \hat{u})^2 + (\Delta \hat{v})^2$ in the presence of cavity losses. Notice that the entanglement exists in a lossy cavity. It is worthwhile to point out that we plot $(\Delta \hat{u})^2 + (\Delta \hat{v})^2$ and $N$ for the same set of parameters. For this set of parameters, we obtain both amplification and entanglement at the same time.

Finally, we note that the classical field $\Omega$ can not only affect entanglement between the two quantum fields but also the amplification of these fields. On the other hand, the two classical fields $\Omega_1$ and $\Omega_2$ mainly amplify the quantum field but plays no role on the entanglement criterion.

We note that Morigi et al. [15, 16] have also considered the generation of two-mode squeezing in a single atom. Their situation is however different from ours. In their work, Morigi et al consider both external and internal degrees of freedom. Under the large detuning limit, the atom’s internal degrees of freedom are eliminated and a two-mode squeezed state at certain times is obtained. At those times the atom is decorrelated from the two cavity modes. At other times, the system is in a tripartite entangled state between the cavity modes and the center-of-mass degrees of freedom of the atom. In our scheme, the entangled states are generated over a wide range of
interaction times. This is easily seen from Figs. 2 and 4. Moreover in our scheme, we can generate a two-mode coherent-squeezed state of large intensity.

**CONCLUSION**

In summary, we discussed a scheme in which a single atom in the cascade configuration inside a doubly resonant cavity can lead to amplified fields that are entangled. The resulting field, under appropriate conditions, is a two-mode coherent-squeeze state. We show that the entanglement persists even in the presence of cavity losses.

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