The process of coevolutionary competitive exclusion: speciation, multifractality and power-laws in correlations

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\textit{New Journal of Physics} \textbf{10} (2008) 023006 (10pp)
Received 30 September 2007
Published 6 February 2008
Online at http://www.njp.org/
doi:10.1088/1367-2630/10/2/023006

\textbf{Abstract.} Competitive exclusion, a key principle of ecology, can be generalized to understand many other complex systems. Individuals under surviving pressure tend to be different from others, and correlations among them change correspondingly to the updating of their states. We show with numerical simulation that these aptitudes can contribute to group formation or speciation in social fields. Moreover, they can lead to power-law topological correlations of complex networks. By coupling updating states of nodes with variation of connections in a network, structural properties with power-laws and functions like multifractality, spontaneous ranking and evolutionary branching of node states can emerge simultaneously from the present self-organized model of coevolutionary processes.
The process of competitive exclusion [1] occurs in some real systems—evolutionary branching of species in ecosystems, citations in scientific research and designation of consumer goods are examples among many others. It is actually a fundamental ingredient governing the main property of dynamical behaviors of systems which are often described with complex networks [2]–[4] nowadays. However, the contribution of competitive exclusion to the interactional structure of networks and to their functional features has not been widely realized up to now.

In modeling a system, individuals are usually represented as nodes and correlations among them are represented as edges of a graph. The scale-free property [3], characterized by a power-law degree distribution, has attracted extensive attention since it reflects a general feature of diverse systems such as the Internet, citation networks, protein–protein interaction, and so on [2]. In most previous models, dynamics of networks and dynamics on networks are separated [4]–[6]. The interplay between the formation of topological structure and functions that emerge from the network is usually neglected, which is reasonable when the structure is independent of the dynamical states of nodes, or when these two sides vary at rather different speeds. However, in many practical phenomena like academia and art creation [7], financial transactions [8], global climate fluctuation [9] and synaptic plasticity of neuron networks in the brain [10, 11], both the structure and functions emerge from an identical process, and time-dependent variations of both individual states and local connections of nodes feedback with each other. Therefore, novel models with coevolution mechanisms [12] underlying them appeared to be appropriate. Unfortunately, rarely can one produce both scale-free structure and collective dynamics of nodes simultaneously. On the other side, new nodes are often assumed to know the global information of the whole growing network, which is usually impossible for huge-size systems. In this sense it is necessary to set-up models based on local interactions to see if structure and functions at a system level will emerge from self-organized dynamics [13].

As is well known, competitive exclusion plays a key role in the formation of species. There is strong competition among species occupying the same or nearest loci. Surviving pressure forces them to drift their traits away from the local average level, and gradually induces evolutionary branching of species. Sympatric speciation [14] in an ecosystem is a recent focus of naturalists. It refers to the origin of two or more species from a single local population. The seceder model [15] based on a simple rule of local third-order collision succeeded in mimicking such a process and capturing its similarity to group formation in society. A network version [16] of it has been reported, giving rise to a possible mechanism of community structure and clustering properties in social networks.

In this paper, the principle of competitive exclusion is generalized outside the realm of ecology, the seceder model is modified to describe temporally updated states of nodes and corresponding variation of connections among them together. We show that the generic nature of members in diverse systems, i.e. to be different from others under the pressure of competition, and coevolution between updating node states and varying connection among nodes, can lead to simultaneous emergences of the evolutionary branching of individual traits, spontaneous ranking and multifractality of node states and, power-law topological structure of correlations in a system. In this way we are able to understand scale-free phenomena and other characteristics in various fields with a novel common mechanism. Self-organized coevolution models of scale-free networks with both structural and functional properties integrated like the present one are still few to the best of our knowledge.
Figure 1. Three iteration rules. (a) A new node links to old ones in the primitive complete graph arbitrarily ($m = 2$, $m_0 = 4$). (b) Every node $i$ picks up its neighbor $J_{sed}(i)$ randomly, and let it update the state value based on the state of its neighbor $J_{max}(i)$. (c) A young node ($I$) cuts off the edge with the node whose relative ratio of state value is less than threshold $h$, meanwhile, add an edge to the node $J_{sed}(j)$ which is the seceder of its neighbor $j$ if it has a higher ratio ($w(J_{sed}(j))/w(I) > h$) to $I$s.

We set-up the present model through three iteration rules. (i) Network growth starts from a primitive complete graph with $m_0$ nodes. Each node $i$ on joining the network was assigned an initial state with a random real number $w(i)$ uniformly distributed in $(0, 1)$. At each time step, a new node $i'$ is added to the pre-existing network. It gives out $m$ edges ($m < m_0$) to old nodes arbitrarily (see figure 1(a)). (ii) At every step, each node $i$ counts $\bar{w}(i)$—the average of state values $w(j)(j \neq i)$ over its nearest linked neighbors, from them it picks up the one whose $w(j)$ makes the maximum distance from the average $\bar{w}(i)$, i.e. $J_{max}(i)$ corresponds to $\max |w(j) - \bar{w}(i)|$, then, a randomly selected node $j$ among the nearest neighbors of $i$ is chosen as the offspring of $J_{max}(i)$, called $J_{sed}(i)$ (see figure 1(b)). Different from original seceder model [15], it is kept at its own site and, with its state variable updated as $w(J_{sed}(i)) = w(J_{max}(i)) + \delta$, where random number $\delta \in (0, 1)$ is also uniformly distributed and with positive numerical range for wider applications. Obviously $w(i)$ here can be accounted as a time-dependent non-decreasing fitness [17]. (iii) For the newcomer node $i'$ at every step, together with its ‘young’ enough fellows (i.e. $i' - i < \Delta I$, with $\Delta I$ a given integer constant implicating aging effect [18], hereafter we call them $I$ altogether for convenience), search seceders for all $I$’s neighbors $j$. When $w(J_{sed}(j))/w(I) \geq h$, where $h$ is a given value of threshold, a new edge is added between node $J_{sed}(j)$ and $I$ (double links and self-loops are forbidden). Meanwhile, an edge linking such node $I$ and its neighbor $j$ is removed if the condition $w(j)/w(I) < h$ or $w(I)/w(j) < h$ is satisfied (see figure 1(c)). Finally, if any node $i$ becomes isolated due to edge-cutting, directly link it to its seceder $J_{sed}(i)$. The threshold description of correlation adopted here is widely used in modeling complex systems [9, 19]. In [19] Kalisky et al assign weights randomly to all edges in an Erdos–Renyi network, and then merge all nodes connected by weights less than a threshold, in this way they got a scale-free ‘supper-node network’.

Actually the iteration rules of the model are abstracted from observation to real systems. In art creation and scientific research, people have a generic tendency to create new works so that they behave differently from others. Sparks from a collision of opinions with large difference
often result in creation. As is well known, scholars are often under pressure to publish. Papers with the same or very similar viewpoint, method and results to existing ones have less chance of being published. Here we see the competition exclusion promotes prosperity of scientific research. Suppose a graduate student just starts his academic career by joining the research on a certain topic, usually he has to focus on some papers after extensive searching due to limited time, and often he extends his reading to references of them. Generally speaking, he needs to pay more attention to ones with sharp contrasts against his knowledge background \( (w(i)) \), and understand recently published literature \( (w(J_{\text{sed}}(j))) \) to inspire new ideas for his own paper. But in the reading he may be restrained within the ability of his understanding. Therefore, it is natural to assume a suitable range of threshold ratios \( (h) \) within which papers with state values \( w(J_{\text{sed}}(j)) \geq h^*w(i) \) would be read by the learner \( I \). And papers in selective reading based on one’s local sight are likely to be cited, forming increased in-degree of a citation network. On the opposite side, papers (on the node state \( w(j) \)) that have small difference from \( w(I) \) (too low ratio of \( w(j)/w(I) \)) are less cited (the link between node \( I \) and \( j \) is trimmed in figure 1(c)). Anyway, a recently updated node state \( (w(J_{\text{sed}}(i))) \) would be more attractive to a failure (an isolated node). Artists update themselves by continuous creation so that the co-occurrence network of musicians serves as another example of competitive exclusion. We know that musicians in a similar genre are competitors for performance. Managers usually do not tend to arrange for them to appear on the same stage since audiences prefer performances with diversity. It is assumed that whoever created a playlist was using a certain criterion to group artists in them. One does not normally find concerts with a mixture of heavy rock, jazz and piano sonata, therefore a range of thresholds is used to balance the homogeneity and heterogeneity. As a result of coevolution, both the citation network \([20]–[22]\) and the musician network \([23]\) display the topology of scale-free structure although most foodwebs do not \([24]\). Suppose a man faces a job crisis, he has to improve himself to overcome the problem. And he may attempt to learn from, and even team up with a successful person on the recommendation of a common friend. But whether they can sustain a close relation, depends on whether they are mutually needed and compensate in a proper measure (e.g. \( w(i) \)). In all these cases states of nodes keep varying with time and correlations among them change corresponding to such variations along an optimal gradient.

Coevolution of node states and topological connection yields most structural properties of complex networks by self-organization. Numerical simulation reveals a power-law distribution of node degree: \( p(k) \sim k^{-\gamma} \), which is illustrated in figure 2(a). Without ensemble average on network configurations, it is shown that in the case of \( h = 3.0 \) the distribution is kept invariant for all values of \( m \), with the slope \( \gamma = 2.39 \). In-degree is counted by a node to its accepted edges from younger ones. The distribution also shows essentially a power-law as shown in the inset of figure 2(a). The slope of the double-logarithmic line \( p_i(k) \sim k^{-\beta} \) is around \( \beta = 2.0 \), which is in accordance with numerical results of another model \([20]\) and empirical studies \([21, 22]\). In figure 2(b), we show the variation of power exponents \( \gamma \) depending on correlation thresholds \( h \). They lie in the range of \( (2.0, 3.0) \), which fits well with real complex systems. The inset of the figure displays that the essentially power-law behavior of in-degree distributions also exists for different thresholds. The calculated Pearson coefficients \( r \) \([25]\) which describe degree–degree correlation of the network are shown in figure 3(a). They are positive reflecting a statistical feature of social networks. Moreover, they also show an asymptotic power-law decay with the size of the system, i.e. \( r(N) \sim N^{-\alpha} \), which is, to our knowledge, a specific feature first observed by the present model. It is expected to be verified by empirical data from real complex systems.
Figure 2. Power-law degree distributions of coevolving scale-free networks. (a) Degree distributions with $h = 3.0$, lines for various values of $m$ collapse on to a single one with $\gamma = 2.39$; inset: in-degree distribution with $\beta = 2.0$ for $h = 3.0$ and $m = 1$. (b) Threshold-dependent degree distributions with $h = 3.0, 4.0, 5.0$ and $6.0$, respectively. Inset: corresponding in-degree distributions. $N = 10^4$, $\Delta I = 10$ and $m_0 = 20$ for all lines.

Figure 3. (a) Asymptotic size-dependent power-law decay of Pearson coefficients: $r(N) \sim N^{-\alpha}$ for $N \gtrsim 10^3$, where $\alpha$ depends on thresholds $h$. (b) Size-dependent power-law decay of clustering coefficient, $C(N) \sim N^{-\eta}$. Averaged on 10 realizations of network configurations.

Figure 3(b) displays the size-dependent decay of clustering coefficients $[2]: C(N) \sim N^{-\eta}$. The exponents are $(1.2 - 1.4)$ corresponding to thresholds in the range of $[3.0, 8.0]$ while for random graphs we have $\eta = 1$ for comparison.

Simulations with low threshold values (e.g. $h = 2.0$ and 1.5) reveal some different behavior of the coevolution since iteration rules no longer lead to power-law degree distributions (see full and dashed lines in figure 4, randomness $a = 0.0$). However, when we allow a small portion (10% and 20%, randomness $a = 0.1$ and 0.2, respectively) of cut-off operations not to carry rule 3, scale-free properties can be retrieved promptly (see figure 4). Moreover, degree–degree
For low values (e.g. \( h = 2.0 \) and 1.5, respectively, without relaxation: \( a = 0 \)), degree distributions deviate from a power-law, but they are retrieved by introducing 10\% (\( h = 2.0, a = 0.10 \)) and 20\% (\( h = 1.5, a = 0.20 \)) relaxation, respectively, on carrying out rule (3). Other parameters are the same as those in figure 2(a).

correlations restore assortativity corresponding to it. This implies that randomness may play an essential role in the origin of scale-free behaviors since there should be more or less relaxation on deterministic rules in complex systems [26].

The updating process of node states induced by competitive exclusion coupled with topological variation leads to collective behavior of nodes, which reflects characteristics of functional aspects apart from structural ones of the network. Among them ranking [27] and multifractality [28] are prominent results of the present model.

Ranking behavior of node states emerge spontaneously from coevolution. The whole range of node states is divided into 100 intervals in figure 5 to show that the values are distributed quite discontinuously. This is drastically different from the uniform initial distribution and is comparable to group formation in the original seceder model (see figure 1 of [15]). Inherited from the seceder model, two prominent traits (see figure 5) at both ends can be regarded as the result of evolutionary branching [14] with the tendency to eliminate mediate genotypes. Here, species in sympatry seem to likely drift their traits away from the local average level since the strongest competition exists between similar genotypes [29]. Anyway, scrutinizing the applicability of the co-evolutionary mechanism to sympatric speciation would be valuable.

Applied to citation networks, this means that the long term coevolution gradually eliminates the publishing chance of papers at middle level, instead, the population of quality tends to be divided and shifted approaching both ends. Beyond the seceder model [15, 16], our numerical results also give support to the assumption of the ranking model [27] of scale-free networks with self-organization mechanism. And it is also noticeable that the scale-free property as a result of coevolution can be obtained without the prerequisite of preferential attachment on the power-law function of prestige ranks of nodes.

To look for the multifractality of the entity of node states, we first put them into a series according to the time sequence of the node’s participation in the network. Once the last node joins the system, we let the coupled evolution stop, divide the series of node states into many
Figure 5. Histogram for numbers of nodes $\Delta N(L)$ with discrete ranks $L$ of node state values $w(i)$, $h = 3.0$ and $m = 18$, other parameters are the same as in figure 2(a).

equal boxes with scale $L$, and sum up the state values $w(i)$ of nodes $i$ within each box $n$ to make up $\mu_n(L)$. In the statistical sense $\mu_n(L)$ depends on the sizes of boxes in a power-law relation. The singularity strength $\alpha_n$ in the $n$th box is defined as the exponent of the power-law

$$\mu_n(L) \sim L^{\alpha_n}. \tag{1}$$

In this way the boxes can be grouped into several subsets according to the values of $\alpha_n$. The subset $\alpha$ contains the boxes with $\alpha_n$ within the window of $\alpha$. And the number density $N(\alpha, L)$ of subset $\alpha$ is a fractal with the Hausdorff dimension $f(\alpha)$, i.e.

$$N(\alpha, L) \sim L^{-f(\alpha)}. \tag{2}$$

The singularity spectrum $f(\alpha)$ completely characterizes the multifractality of the whole measure of the sequence of node states $w(i)$. We can use the properly normalized $q$th moment of $\mu_n(q, L)$ as a measure of the sequential distribution of node states:

$$\mu_n(q, L) = \mu_n^q(L) / \Sigma_n \mu_n^q(L). \tag{3}$$

Then $\alpha(q)$ and $f(q)$ are presented, respectively, in the following form:

$$\alpha(q) = \lim_{\delta \to 0} \mu(1, L) \ln \mu(1, L) / \ln \delta, \tag{4}$$

$$f(q) = \lim_{\delta \to 0} \mu(q, L) \ln \mu(q, L) / \ln \delta, \tag{5}$$

where $\delta = L/N$ denotes the ratio of box scale to the size of the system. However, the definition of Hausdorff dimension $f(\alpha)$ is valid only when the numerator and the denominator of equation (5) are kept in linear relation for different values of $\delta$. We can define $\nu(q, L) = \Sigma_n \mu_n(q, L) \ln \mu_n(q, L)$ and $x = \ln \delta$, respectively. Essential linearity can be seen for at least 4–5 center lines in figure 6 so that the singularity spectrum $f(\alpha)$ of the multifractal is shown in its inset ($N = 9728$, $L = 2^l$, where $l = 1, 2, \ldots, 6$). Our case here is not the same as that found by Song et al which is related to fractal measures on topological structures [30]. Interestingly,
Figure 6. Check the existence of multifractality of node states according to consecutive joining the network with the box-counting method. Inset: singularity spectrum \( f(\alpha) \) of multifractal of node state \( u(i) \) for the network with \( h = 3.0, m_0 = 20 \) and \( m = 18 \).
Acknowledgments

We acknowledge the partial support from the National Natural Science Foundation of China (NNSFC) under the grant nos 70471084, 10474033, 10635040 and 60676056. CPZ and BHW thank the hospitable accommodation of Bao-Wen Li at NUS, and suggestions from Bing Li. SJX and BHW acknowledge support from the National Basic Science Program of China project nos 2005CB623605, 2006CB921803 and 2006CB705500. DNS acknowledges Foundation of NCET04-0510.

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