Neutrino diffraction induced by many-body interaction

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Abstract

The neutrino produced in the pion decay reveals a new diffraction phenomenon due to many-body interactions in an intermediate time region when wave functions of the parent and daughters overlap. Because of diffraction, the probability to detect the neutrino involves a large finite-size correction that depends on the neutrino mass, $m_\nu$, and energy, $E_\nu$, the speed of light, $c$, and the distance $L$ between the positions of the initial pion and final neutrino, $m_\nu^2 c^4 L / (2 E_\nu \hbar)$. The correction vanishes for the charged leptons and is finite for the neutrino at a macroscopic distance, $L$, of near-detector regions in ground experiments. A new method for determining the absolute neutrino mass is proposed.
1 Neutrino interference. Interference of photons, electrons, neutrons, and other heavy elements are important for confirming quantum mechanics and other basic principles. A wave composed of many components of different kinetic energies behaves non-uniformly in space. In the above cases, they are formed by potential energy. We present a diffraction theory due to many-body interactions that provides a varying kinetic energy to the many-body system in a finite time. A neutrino produced in pion decay reveals the wave nature, and the probability for detecting the neutrino displays diffraction. Consequently the absolute value of the currently unknown neutrino mass could be deduced from the unique interference pattern of this diffraction phenomenon.

A neutrino interacts extremely weakly with matter. Being undisturbed by matter, the neutrino behaves purely as a quantum mechanical wave with a negligible one-particle potential, except in case of degenerate flavor states [1, 2]. Instead of potential energy, interaction energy carried by a weak Hamiltonian becomes finite when the wave functions of the pion and decay product overlap. Then, kinetic energy becomes different from that of the initial state and varies with time, because total energy is constant. This system shows a non-uniform spatial behavior, called neutrino diffraction, similar to the above cases of ordinary particles. Having its origin in the weak Hamiltonian, neutrino diffraction appears in vacuum and has universal properties. The diffraction pattern is easily observed without an obstacle or potential in the time region $T \leq \tau$, where $T$ is the time interval between initial and final states, and $\tau$ is the mean life-time for a large number of events.

Consider the system described by the Hamiltonian $H = H_0 + H_1$, where $H_0$ is a bi-linear form of fields, and $H_1$ is a higher-order polynomial that causes many-body interactions. Kinetic energy is defined by eigenvalue $E$ of $H_0$. The Schrödinger equation, $i\frac{\partial}{\partial t} |\psi(t)\rangle = (H_0 + H_1)|\psi(t)\rangle$ is solved using $H_1$ and an initial state $|\psi(0)\rangle$ of the kinetic energy, $E_0$, in the interaction picture by $|\tilde{\psi}(t)\rangle = i\int_0^t dt' e^{-iH_1(t')} |\tilde{\psi}(0)\rangle$, where $i\int_0^t$ denotes the time-ordered product. Hence, the wave function $|\tilde{\psi}(\infty)\rangle$ is written in the following form;

$$|\tilde{\psi}(\infty)\rangle = a(\infty)|\tilde{\psi}(0)\rangle + 2\pi \int d\beta \delta(\omega) |\tilde{\beta}\rangle \langle \tilde{\beta} |\tilde{S}|\tilde{\psi}(0)\rangle$$

(1)

with a reduced matrix $\tilde{S}$, where $H_0|\tilde{\beta}\rangle = E_\beta |\tilde{\beta}\rangle$, $\omega = E_\beta - E_0$, and $a(\infty)$ is a constant. The state $|\tilde{\beta}\rangle$ has the kinetic energy of the initial state, $E_0$. Accordingly, this state has the property of free particles. Kinetic energy is conserved in the asymptotic regions $t \rightarrow \pm \infty$, and a scattering matrix $S[\infty]$ satisfies $[S[\infty], H_0] = 0$. Now, at finite $t$, the wave function is
written as follows:

\[ |\tilde{\psi}(t)\rangle = a(t)|\tilde{\psi}^{(0)}(t)\rangle + \int d\beta \frac{e^{i\omega t}}{\omega} |\tilde{\beta}\rangle \langle \tilde{\beta}|S|\tilde{\psi}^{(0)}\rangle, \tag{2} \]

and it is a superposition of the states of the kinetic energy, \(E_0\) and \(E_\beta \geq 0\), with a time dependent weight. The interaction energy, \(\langle \tilde{\psi}(t)|\tilde{H}_1|\tilde{\psi}(t)\rangle\), does not vanish in the region of finite \(\langle \tilde{\psi}^{(0)}(t)|\tilde{H}_1|\tilde{\beta}\rangle\). Total energy satisfies the following condition: \(\langle \tilde{\psi}(t)|\tilde{H}|\tilde{\psi}(t)\rangle = E_0\).

At finite \(t\), \(|\tilde{\psi}(t)\rangle\) has a varying kinetic energy that free particles do not possess. Consequently, \(|\tilde{\psi}(t)\rangle\) retains its wave nature, and the probability to detect a particle in the final state becomes dependent on a time interval, which we call finite-size correction. To observe this correction, an S-matrix \(S[T]\) defined according to the boundary condition at the time interval \(T\) is used. Because wave packets localize around center positions and satisfy the asymptotic boundary conditions [3, 4] of scattering experiments, \(S[T]\) is defined using wave packets. \(S[T]\) is constructed with Møller operators at finite \(T\), \(\Omega^{\pm}(T)\), as \(S[T] = \Omega^\dagger_-(T)\Omega^+_+(T).\) The term \(\Omega^{\pm}(T)\) is expressed in the form \(\Omega^{\pm}(T) = \lim_{t \to \mp T/2} e^{iHt} e^{-iH_0 t}.\) From this expression, \(S[T]\) satisfies the following equation:

\[ [S[T], H_0] = i \left\{ \frac{\partial}{\partial T} \Omega^+_T(T) \right\} \Omega^+_+(T) - i \Omega^+_T(T) \frac{\partial}{\partial T} \Omega^+_+(T). \tag{3} \]

Thus, kinetic energy is not conserved at finite \(T\). A matrix element of \(S[T]\) between eigenstates \(|\alpha\rangle\) and \(|\beta\rangle\) of eigenvalues \(E_\alpha\) and \(E_\beta\) respectively, \(\langle \beta|S[T]|\alpha\rangle\), has the components of \(E_\beta = E_\alpha\) and \(E_\beta \neq E_\alpha\). At \(T \to \infty\), only the former terms remain, and the latter terms at finite \(T\) give finite-size correction.

A neutrino wave packet [5–7] expresses a nucleon wave function in a nucleus with which the neutrino interacts and is well-localized [8–15]. The mass-squared differences, \(\delta m^{2}_\nu\), are extremely small [16–18], thus, we study a situation in which the mass-squared average, \(\bar{m}^2_\nu\), satisfies, \(\bar{m}^2_\nu \gg \delta m^{2}_\nu\), and presents the one flavor case first. Extensions to general cases are straightforward.

2 Position-dependent probability.

Now, we find the finite-size correction of the probability to detect the neutrino in the pion decay with \(S[T]\). \(H_0\) is the free Hamiltonian of the pion, charged lepton, and neutrino; and \(H_1 = g \int d\vec{x} \partial_\mu \varphi (V - A)_\mu^{\text{lepton}}\), where \(\varphi(x)\), \(V^\mu(x)\), and \(A^\mu(x)\) are pion field, lepton’s vector, and axial-vector currents respectively. The term \(|\tilde{\psi}^{(0)}\rangle\) is a one-pion state, and \(|\beta\rangle\) is a two-particle state composed of a charged lepton and neutrino. For a pion of momentum...
$p_\pi$ prepared at $T_\pi$, the amplitude for a neutrino of $p_\nu$ to be detected at $(T_\nu, \vec{X}_\nu)$ and a muon of $p_\mu$ to be un-detected in the lowest order of $H_1$, $T = \int d^4 x \langle \mu, \nu | H_1(x) | \pi \rangle$, is written in terms of Dirac spinors as follows:

$$T = \int d^4 x d\vec{k}_\nu N(0|\varphi_{\pi}(x)|\pi) \bar{u}(\vec{p}_\mu)(1 - \gamma_5)\nu(\vec{k}_\nu)$$

$$\times e^{i p_\mu \cdot x + i k_\nu \cdot (x - X_\nu) - \frac{q_\mu}{\pi}(\vec{k}_\nu - \vec{p}_\mu)^2},$$

where $N = ig m_\mu (\sigma_\mu/\pi)^{\frac{3}{2}} (m_\mu m_\nu / E_\mu E_\nu)^{\frac{1}{2}}$, and the four-dimensional coordinate, $x$, has the components $(t, \vec{x})$, and $t$ is integrated over the region $T_\pi \leq t$. In this study, a Gaussian form is assumed for simplicity. Finite-size correction has a universal property that is common to general wave packets. The size of the wave packet, $\sigma_\nu$, is estimated later. The amplitude $T$ satisfies the boundary condition at finite $T = T_\nu - T_\pi$.

By integrating $\vec{k}_\nu$, we obtain the Gaussian function of $\vec{x} - \vec{x}_0$, which vanishes at large $|\vec{x} - \vec{x}_0|$, where $\vec{x}_0$ is the center coordinate to be expressed later, and satisfies the asymptotic boundary condition. We express an integration of $|T|^2$ over $\vec{p}_\mu$ with a correlation function of coordinates. After spin summations, we have the following expressions:

$$P = \int \frac{d\vec{p}_\mu}{(2\pi)^3} \sum_{\text{spin}} |T|^2 = \frac{C}{E_\nu} \int d^4 x_1 d^4 x_2 e^{-\frac{x_1}{2\pi} \sum (\vec{x}_i - \vec{x}_i^0)^2}$$

$$\times \Delta_{\pi,\mu}^{\nu}(\delta x) e^{i\phi(\delta x)} e^{-\frac{t_1 + t_2}{\tau}},$$

$$\Delta_{\pi,\mu}^{\nu}(\delta x) = \frac{1}{(2\pi)^3} \int \frac{d\vec{p}_\mu}{E(\vec{p}_\mu)} (p_\mu p_\nu) e^{-i(p_\pi - p_\nu) \cdot \delta x},$$

where $\tau$ is pion’s life-time, $C = g^2 m_\mu^2 (4\pi/\sigma_\nu)^{\frac{3}{2}} V^{-1}$, $V$ is a normalization volume for the initial pion, $\vec{x}_i^0 = \vec{X}_\nu + \vec{v}_\nu(t_i - T_\nu)$, $\delta x = x_1 - x_2$, and $\phi(\delta x) = p_\nu \cdot \delta x$. In Eq. (5), muon momentum is integrated in the entire positive energy region so that Eq. (5) can agree with the original probability.

3 Light-cone singularity.

By using the new variable $q = p_\mu - p_\pi$ that is a conjugate to $\delta x$, we write $\Delta_{\pi,\mu}^{\nu}(\delta x)$ as the sum of integrals over regions $0 \leq q^0$, and $-p_\pi^0 \leq q^0 \leq 0$. Kinetic energy is conserved in the latter integral and not conserved in the former. Thus, they correspond to the asymptotic value and finite-size correction. The former integral is expressed as, $[p_\pi \cdot p_\nu - ip_\nu \cdot (\frac{q}{\partial \delta x})] \bar{I}_1$, and the four-dimensional integral is given as follows:

$$\bar{I}_1 = \int d^4 q \frac{\theta(q^0)}{4\pi^4} \text{Im} \left[ \frac{1}{q^2 + 2p_\pi \cdot q + m^2 - i\epsilon} \right] e^{i q \cdot \delta x},$$
and \( \tilde{m}^2 = m_{\pi}^2 - m_\mu^2 \). By expanding the denominator with \( p_\pi \cdot q \), we have an expression

using the light-cone singularity \([19]\), \( \delta(\delta x^2) \), and the less singular and normal terms that are described with Bessel functions. The latter integral, \( I_2 \), has no singularity. By adding both terms, we have the following expressions:

\[
\Delta_{\pi,\mu}(\delta x) = 2i \left\{ p_\pi \cdot p_\nu - ip_\nu \cdot \left( \frac{\partial}{\partial \delta x} \right) \delta(\delta x) \right\} \\
\times \left[ D_{\tilde{m}} \left( -i \frac{\partial}{\partial \delta x} \right) \left( \frac{\epsilon(\delta t)}{4\pi} \delta(\lambda) + f_{\text{short}} \right) + I_2 \right],
\]

where \( \lambda = (\delta x)^2 \), \( D_{\tilde{m}}(-i \partial/\partial x) = \sum_l (1/l!)(2p_\pi \cdot (-i \partial/\partial x) \partial/x)^l \), and \( f_{\text{short}} \) is expressed with Bessel functions \([19, 20]\).

**Integration of coordinates.** Next, Eq. (7) is substituted into Eq. (5), and \( \vec{x}_1 \) and \( \vec{x}_2 \) are integrated. The light-cone singularity, \( \epsilon(\delta t)/4\pi \delta(\lambda) \), leads to the slowly oscillating term, \( J_{\delta(\lambda)} \):

\[
J_{\delta(\lambda)} = C_{\delta(\lambda)} \frac{\epsilon(\delta t)}{|\delta t|} e^{i\phi(\delta t)},
\]

where \( C_{\delta(\lambda)} = (\sigma_\nu \pi)^{3/2} \sigma_\nu /2 \), and \( \phi(\delta t) = \omega_\nu \delta t = \delta t \ m_\nu^2 c^4 / (2E_\nu) \). The phase \( \phi(\delta x) \) of Eq. (5) becomes the small phase \( \phi(\delta t) \) of Eq. (8) at the light cone \( \lambda = 0 \). The next singular term becomes much smaller than that in the present parameter region, and the normal terms oscillate or decrease rapidly with \( \lambda \) and those of \( \mathcal{F} \approx 0 \) contribute to the oscillation or decreases. Hence, the spreading effect is negligible. The terms \( f_{\text{short}} \) and \( I_2 \) in Eq. (7) lead to rapidly oscillating or decreasing terms which we denote by \( \tilde{L} \).

Finally, we integrate \( t_1 \) and \( t_2 \) over the finite region, \( 0 \leq t_i \leq T \):

\[
P = N_1 \int_0^T dt_1 dt_2 \left[ \frac{\epsilon(\delta t)}{|\delta t|} e^{i\phi(\delta t)} + \tilde{L} \right] e^{-\frac{t_1+t_2}{2}},
\]

where \( N_1 = ig^2 m_\mu^2 \pi^3 \sigma_\nu (8p_\pi p_\nu / E_\nu) V^{-1} \). In most of the places, the neutrino mass is neglected compared to \( \tilde{m}^2 \), \( p_\pi \cdot p_\nu \) and \( \sigma_\nu^{-1} \), except for the slow phase \( \phi(\delta t) \). The first term in Eq. (9) oscillates slowly with \( \delta t \), and the remaining terms oscillate or decrease rapidly. They are clearly separated.

\[
i \int_0^T dt_1 dt_2 \frac{\epsilon(\delta t)}{|\delta t|} e^{i\omega_\nu \delta t} e^{-\frac{t_1+t_2}{2}} = (\tilde{g}(\omega_\nu T) - \tilde{g}_0)
\]

The first term slowly approaches \( \tilde{g}_0 \) with \( T \), where \( \tilde{g}(\omega_\nu T) \) satisfies \( \frac{\partial}{\partial T} \tilde{g}(\omega_\nu T)|_{T=0} = - \omega_\nu \), and \( \tilde{g}(\infty) = 0 \). The constant \( \tilde{g}_0 \) cancels the short-range term, \( \tilde{L} \), in Eq. (9). Here \( \tilde{g}(\omega_\nu T) \) is generated by the light-cone singularity, and its effect remains within a macroscopic distance
of order $\frac{2\hbar E_\nu}{m_\nu^2 c^2}$. We call this as the **diffraction** term. From the last term of Eq. (9), $G_0(T)$ is defined as $i \int dt_1 dt_2 \tilde{L}(\delta t) e^{-\frac{t_1 + t_2}{\tau}} = G_0(T) + \tilde{g}_0$. Because of rapid oscillation in $\delta t$, the **normal** term, $G_0(T)$, receives contributions from the microscopic $|\delta t|$ region, is proportional to $T$ in the region $T < \tau$, and approaches a constant for large $T$.

The present method of extracting the light-cone singularity is valid if the series $D_m\left(-\frac{\partial}{\partial x}\right)f_{\text{short}}$ converges. This condition is fulfilled, [20], in the region $2p_\pi \cdot p_\nu \leq \tilde{m}^2$, and the series rapidly oscillates. Outside this region, the method is not applicable, and $\Delta_{\pi,\mu}(\delta x)$ has no light-cone singularity and has only the short-range term.

**4 Total probability that depends on time interval.**

From the integration of the neutrino coordinate $\vec{X}_\nu$, the total volume emerges and cancels with $V^{-1}$. The total probability becomes

$$P = N_2 \int \frac{d\tilde{p}_\nu}{(2\pi)^3} \frac{p_\pi \cdot p_\nu}{E_\nu} \left[\tilde{g}(\omega_\nu T) + G_0(T)\right],$$

where $N_2 = 8g^2m_\mu^2\sigma_\nu$, and $L = cT$ is the length of the decay region. Because $G_0(T)$ and $\tilde{g}(\omega_\nu T)$ have their origins in the conserving and non-conserving terms of kinetic energy, respectively, $p_\pi \cdot p_\nu = \tilde{m}^2/2$ in $G_0(T)$ but not in $\tilde{g}(\omega_\nu T)$. By integrating the neutrino angle, we find that the normal term is independent of $\sigma_\nu$ [12] and agrees with the value computed via plane waves. However, $\tilde{g}(\omega_\nu T)$ is present in the kinematical region; i.e., $|\tilde{p}_\nu|(E_\pi - |\tilde{p}_\pi|) \leq p_\pi \cdot p_\nu \leq \tilde{m}^2/2$ from the convergence condition, and $\tilde{g}(\omega_\nu T)$ is integrated in this region. This is slightly different from $p_\pi \cdot p_\nu = \tilde{m}^2/2$; hence, the latter region cannot be distinguished from the former. Therefore, we add the two terms. Total probability thus obtained is presented in Fig. 11 for $m_\nu = 1$, 0.2 [eV/c^2], $E_\pi = 4, 40$ [GeV], and $E_\nu = 700$ [MeV]. The size of the nucleus of a mass number, $A$, is used for the wave packet, $\sigma_\nu = A^{\frac{2}{3}}/m_\pi^2$, and $\sigma_\nu = 6.4/m_\pi^2$ is used for the evaluation of the $^{16}$O nucleus. From Fig. 11 we can observe that the diffraction term becomes finite in $L/c \leq \tau$, where the wave functions of the initial and final states overlap. Their fractions at $L \approx 0$ vary from 0.02 for $m_\nu = 0.2$ [eV] to 0.2 for $m_\nu = 1.0$ [eV] at 4 [GeV], and they become approximately 1.0 for $m_\nu = 0.2$ [eV] and $m_\nu = 1.0$ [eV] at 40 [GeV]. They decrease rapidly with $L$ at 4 [GeV] and slowly at 40 [GeV], because the life-time is longer for the latter energy. The diffraction term slowly varies with $L$ in the high energy region, in which the life-time effect becomes negligible, and a typical length, $L_0$, for this behavior is given as

$$L_0 [m] = 2E_\nu hc/(m_\nu^2 c^4) = 400 \times E_\nu [GeV]/m_\nu^2 [eV^2/c^4].$$

In the experiments, neutrino’s energy is measured with uncertainty $\Delta E_\nu$, which is of the order.
FIG. 1. Detection probabilities of the neutrino for \( E_\pi = 4 \) and 40 [GeV] at distance \( L \). In (A), solid green (4 [GeV]) and dotted black line (40 [GeV]) represent normal and diffraction terms, respectively, for for \( m_\nu \) of 0.2 and 1.0 [eV] is mentioned on top of normal terms. Values are normalized to one at \( L = \infty \). In (B), fractions of diffraction terms that vary with pion’s energy and neutrino mass are shown. The horizontal axis represents distance in [m]. Neutrino energy is 700 [MeV].

\[ 0.1 \times E_\nu \] and is 100 [MeV] for energy 1 [GeV]. Diffraction components are almost constant in this energy range. For a larger energy uncertainty, the computation is easily made using Eq. (11). Hence, the diffraction component is observable if \( m_\nu \geq 0.2 \text{[eV}/c^2\text{]} \) using the near detector, but it is not observable if \( m_\nu \leq 0.1 \text{[eV}/c^2\text{]} \) using the muon neutrino. In the latter case, an electron neutrino may be used.

The process described using \( S[T] \) has the total probability same as that shown in Eq. (11). In the same experiment, the detection rate of the muon, after the neutrinos are integrated,
has the same excess value. Ordinary experiments of observing the muon, however, do not consider the neutrino and are described by a different $S[T']$, which satisfies the boundary condition for the muon, and $T' = T_\mu - T_\pi$ is the time interval for muon observation. The probability to detect a muon is computed with a free neutrino, and then it is expressed in the form of Eq. (11) with $\omega \nu \to \omega \mu = m^2_{\mu} c^4 / (2E_{\mu} \hbar)$. Because the muon is heavy, $\omega \mu T'$ becomes very large, and $\tilde{g}(\omega \mu T')$ vanishes at at macroscopic $T$. Thus, the probability of detecting the muon is not modified, and it agrees with the normal term. The light-cone singularity is formed in both cases, but the diffraction is large for the neutrino and small for the charged lepton.

The probability of detecting the muon depends on the boundary condition of the neutrino. When the neutrino is detected at $T_\nu$, the muon spectrum includes the diffraction component, but when the neutrino is not detected, the muon spectrum does not include the component. The latter condition is standard, and the former is non-standard but may be verified experimentally.

In case of three masses $m_{\nu_i}$, and a mixing matrix $U_{i,\alpha}$, the diffraction term for an $\alpha$ flavor neutrino is expressed as $\sum_i \tilde{g}(\omega_\nu T)|U_{i,\alpha}|^2$, whereas the normal term is expressed as $|\sum_i U_{i,\mu}D(i)U^\dagger_{i,\alpha}|^2$, where $i$ is the mass eigenstate, $\alpha$ is the flavor eigenstate, and $D(i)$ is the free wave of $m_{\nu_i}$. Hence, the diffraction term depends on the average mass-squared, $\bar{m}^2_\nu$, but the normal term depends on mass-squared differences, $\delta m^2_\nu$. At $L \to \infty$, the diffraction term disappears, and the normal terms become constants in the mass parameter region of the current study, $\bar{m}^2_\nu \gg \delta m^2_\nu$.

Neutrino diffraction is different from the diffraction of light passing through a hole. For the neutrino, the diffraction pattern is formed in a direction parallel to the momentum with the phase difference $\omega_\nu \delta t$ of the non-stationary wave. The size of the pattern is determined by $\omega_\nu$, which is extremely small and stable with variations in parameters. For light, the diffraction pattern is formed in a direction perpendicular to the momentum with the phase difference $\omega^{dB}_\gamma \delta t$ of the stationary wave, where $\omega^{dB}_\gamma = c|\vec{p}_\gamma|/\hbar$. The shape of the pattern is determined by $\omega^{dB}_\gamma$, which is large and varies rapidly with light’s energy. Thus, for observation, fine-tuning of initial energy is necessary in case of light but unnecessary in case of neutrino.

5 Summary and implications.

We presented a new mechanism for diffraction due to a many-body interaction in the


decay region where the parent and daughters overlap. The probability to detect the neutrino is given in Eq. (11), where $G_0$ is the normal term, and $\tilde{g}(\omega_\nu T)$ slowly decreases with $T$. The former agrees with the standard value obtained by an $S$-matrix of plane waves, whereas the latter is a new term that can be computed by $S[T]$ and has its origin in diffraction due to waves at finite $t$. In the many-body state consisting of the pion, neutrino, and muon, the overlap of the wave functions gives a finite-interaction energy in $t \leq \tau$. Because kinetic energy is the difference between total and interaction energies, it varies with time. Consequently, this many-body state becomes non-uniform in space and time and shows a diffraction pattern that is unique in the non-asymptotic region. This diffraction pattern is determined by the difference of the angular velocities, $\omega_\nu = \omega_\nu^E - \omega_\nu^{dB}$, where $\omega_\nu^E = E_\nu/\hbar$ and $\omega_\nu^{dB} = c|\vec{p}_\nu|/\hbar$. The term $\omega_\nu$ becomes an extremely small value equal to $m_\nu^2c^4/(2E_\nu\hbar)$ for neutrinos because of the unique features of neutrinos [16–18]. Consequently, the diffraction term becomes finite in a macroscopic spatial region $r \leq 2\pi E_\nu\hbar c/(m_\nu^2c^4)$ and affects experiments in a mass-dependent manner at near-detector regions. The area of this region is exceptionally large for neutrinos. Waves accumulating at the velocity of light form the light-cone singularity, which is peculiar in relativistic invariant systems, and exhibit neutrino diffraction.

Neutrino diffraction gives new corrections to neutrino fluxes but not to the fluxes of charged leptons; thus, it is consistent with all previous experiments involving charged leptons. The new term has various implications for existing neutrino anomalies and future experiments. One anomaly is an excess of neutrino flux at the near-detectors of ground experiments. Fluxes measured by the near detectors of K2K [21] and MiniBooNE [22] show excesses of 10%-20% in Monte Carlo estimations, whereas the excess is not clear in MINOS [23]. These excesses may be connected with the diffraction component. With additional statistics, quantitative analysis might become possible to test the diffraction term. Another anomaly is LSND [24] in which electron neutrinos in pion decays have excesses. Because diffraction occurs in the non-asymptotic region, helicity suppression does not work. An electron mode is studied with a $(V - A) \times (V - A)$ current interaction in [25], and it is found that excess in near-detector regions is attributed to the diffraction component. The controversy between LSND and others is resolved. Finally, a new method that involves consideration of the distance or energy dependence of neutrino flux may be developed for determining the absolute neutrino mass.
Thus neutrino diffraction is visible at macroscopic distances and can be confirmed with near-detectors. At much larger distances than that mentioned above, the diffraction component disappears, and only the normal component, including the neutrino flavor oscillation, remains. If masses do not satisfy $\bar{m}_\nu^2 \gg \delta m^2_\nu$ but satisfy $\bar{m}_\nu^2 \approx \delta m^2_\nu$, then the neutrino flux behaviors are more complicated.

A new quantum phenomenon of neutrinos on a macroscopic scale due to the many-body weak interaction was derived, and its physical quantity determined by the absolute neutrino mass was presented.

In this study, we used the Hamiltonian expressed by the pion field and neglected higher-order effects such as the pion mean-free-path and the unified gauge theory. The interaction of $(V - A) \times (V - A)$ does not modify the result on the muon mode but modifies the electron mode, and other higher-order effects do not give a correction. We will study these problems and other large-scale physical phenomena of low-energy neutrinos in subsequent studies.

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