Pion- and photo-induced transition amplitudes to $\Lambda K$, $\Sigma K$, and $N\eta$

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Received: May 1, 2014/ Revised version:

Abstract. Pion and photo-induced inelastic reactions off protons are studied in a multichannel partial-wave analysis. Properties of nucleon and $\Delta$ resonances are derived and compared to previous analyses. Amplitudes are shown for transitions to $N\eta$, $\Lambda K$, and $\Sigma K$ final states.

PACS: 11.80.Et, 11.80.Gw, 13.30.-a, 13.30.Ce, 13.30.Eg, 13.60.Le 14.20.Gk

1 Introduction

The spectrum of baryon resonances, their masses, widths, and photocouplings [1] provide a wealth of information to test theories modeling QCD [2,3,4,5,6,7,8] or, eventually, calculations of these quantities on the lattice [10,11,12,13,14]. The Particle Data Group [15] summarizes regularly the experimental information coming from a variety of different experiments. Some of the information is already highly condensed: information on the excitation spectrum of the nucleon stems from experiments on $\pi N$ elastic scattering, including experiments on $\pi^- p \rightarrow n\pi^+$ charge exchange, and from experiments in which the target nucleon is polarized, and from experiments in which the recoil polarization of the scattered nucleon is measured in a secondary reaction. All this information is exploited to determine energy-independent partial wave amplitudes. In a second step, these amplitudes are analyzed to extract resonant contributions [16,17,18,19]. Partial decay widths stem from dedicated experiments studying, e.g. pion induced hyperon production or photoproduction of the known resonances.

Unfortunately, the experimental information is not really sufficient to determine energy-independent partial wave amplitudes from $\pi N$ elastic scattering. Data on the nucleon recoil polarization do not cover the full energy and angular range. They help to differentiate between different partial wave solutions but they cannot be used to construct the amplitudes from the data. Without this information, only the absolute values of the spin-flip and spin non-flip amplitudes can be determined but not their relative phases. The recoil polarization is accessible easily in case of production of hyperons which reveal their polarization by their asymmetric $N\pi$ decays. Unfortunately, pion-induced hyperon production experiments with polarized targets have been carried only at a few momenta. Hence again, the data do not suffice to reconstruct the amplitudes in single energy bins. Even if the amplitudes are enforced to obey some continuity criteria from energy bin to energy bin, the problem does not converge to a unique solution. Dispersion relations – for fixed four-momentum transfer, for fixed cms-angles, and for fixed energy – can be used to relate the real and imaginary part of the scattering amplitude and can thus, in principle, be exploited to overcome the lack of experimental data. Further input can be derived from a comparison of the total and differential cross section for $\pi^- p$ and the Coulomb interference and from $pp \rightarrow 2\pi$ exploiting backward dispersion relations. But still, it is questionable how reliable the individual partial waves can be separated.

The lack of data has led to the unpleasant situation that about half of all nucleon and $\Delta$ resonances reported in the Karlsruhe Helsinki analysis (KH80) [16,17] and in the Carnegie-Mellon analysis [18] were not confirmed in a later analysis at GWU [19] using larger and more precise data sets, in particular also spin rotation parameters in the elastic pion-proton scattering from ITEP [20,21,22]. Predictions by [19] of the backward asymmetry [23] in the elastic pion-proton scattering favor the GWU solution over the classical solutions of KH80 and CBM. So, even in the case of $\pi N$ elastic scattering, the number of contributing resonances is controversial, and concrete predictions using the amplitudes of the KH80 and CBM analyses show significant discrepancies when compared to data.

In photoproduction of hyperons, even more experiments are needed to construct amplitudes from the data but experimental prospects are better. As shown in [24,25,26], precise measurements of eight carefully chosen observables are sufficient to construct – up to an overall phase – the four complex amplitudes describing photoproduction of a baryon with spin-parity $J^P = 1/2^+$ and a pseudoscalar meson. Measurements are then needed of the differential cross section, experiments with polarized photons yielding the beam asymmetry $\Sigma$, measurements of the hyperon polarization $P$, of the target asymmetry $T$, and double polarization experiments in which the polarization transfer from the initial photon or of the target proton to the final state hyperon is studied. This ambitious program is presently underway in several laboratories but so far, data of a “complete” experiment are not yet available.
Even though the amplitudes cannot yet be constructed from existing data, it is nevertheless possible to fit the data directly with energy-dependent amplitudes. These amplitudes contain the physics: the number and the properties of resonances. Several groups have fitted existing data on hyperon production and reported which resonances are required to achieve a good description of the data.

In a series of recent papers, we reported fits to data on pion and photo-induced hyperon production: the exploratory study was described in \[27,28,29\] and the Karlsruhe-Helsinki amplitudes are replaced by the GWU amplitudes. In \[30\], where also references to the data used can be found. Interpretations of the results were given in \[31,32\]. In this paper we compare our amplitudes for hyperon and \(\eta\) production with other PWA models. Such a comparison allows for a deeper insight into the model-dependence of the fits than a comparison of contributing resonances and their properties can provide.

As mentioned above, the number of nucleon and \(\Delta\) resonances in the Karlsruhe-Helsinki analysis \[16,17\] and in the Carnegie-Mellon analysis \[18\] is much larger than in the analysis at GWU \[19\]. In \[27\], we have tested which impact this difference has on the number of resonances observed in a multi-channel analysis which is constrained by either the Karlsruhe-Helsinki amplitudes or by the GWU amplitudes. It was found that the number of required resonances is the same and that the properties of resonances change only little when the Karlsruhe-Helsinki amplitudes are replaced by the GWU amplitudes. In order not to bias the analysis towards a multitude of resonances, we use for our fits the GWU amplitudes.

The paper is organized as follows: In the subsequent sections (2 and 3) we give a brief survey of data on hyperon and \(\eta\) production and of the fits which have been performed so far. In section 4 we present the amplitudes for pion- and photo-produced production of hyperons and of \(\eta\) mesons. Our amplitudes for \(\gamma p \to p\pi^0\) and \(n\pi^+\) have been presented elsewhere \[33\]; however, the amplitude were based on the solution BnGa2010. We show here the new amplitudes but refrain from a renewed discussion since the new fits gave only small changes in the amplitudes.

2 Hyperon production

Data on hyperon production by pion beams were taken in the 70ies and 80ies, in those glorious times when pion beams were still available for experiments at all major laboratories for particle physics. Two reactions are of particular importance:

i) production of \(\Lambda\) hyperons in \(\pi^- p \to \Lambda K^0\) gives access to nucleon resonances. The reaction was studied from the threshold region \[34\] up to 2.32 GeV in invariant mass \[35\]. The resonance parameters obtained in a partial wave analysis \[36\] of the data were found to be in good agreement with those of \(\pi N\) analyses. In the final fits, masses and width of nucleon resonances were therefore fixed to values derived from elastic \(\pi N\) scattering; the main result was thus the determination of \(\pi N \to \Lambda K^0\) matrix elements. Later measurements of the spin-rotation parameter \[37\] confirmed the main predictions of \[36\].

ii) Only isospin \(I = 3/2\) resonances can contribute to \(\pi^- p \to K^+ \Sigma^+\). The reaction was studied by Candlin et al. from threshold to 2.35 GeV centre-of-mass energy \[38\]. An energy-dependent partial wave analysis found a unique solution giving a satisfactory fit to the data. Most resonances found in the analyses of \(\pi N\) elastic scattering were confirmed except some one-star states \[39\]. Spin rotation parameters were measured later at two incident pion momenta \[40\]. The results were at most in fair agreement with predictions from their earlier PWA \[39\]. Hence their published amplitudes are certainly not fully correct.

iii) The third reaction, \(\pi^- p \to K^0\Sigma^0\), receives contributions from both isospins \[41\].

All three reactions benefit from the weak hyperon decay which reveals the polarization status of the final-state baryon, i.e. the recoil polarization.

Photo (and electro-)production of hyperons gives access to additional quantities like helicity amplitudes and, in electroproduction, the dependence of transition amplitudes on the (squared) momentum transfer. The latter variables are particularly sensitive to the internal structure of baryon resonances. Again, three reactions are relevant in this context: \(\gamma p \to \Lambda K^+\) and \(\gamma p \to \Sigma^0 K^+\), \[42,43,44,45,46,47,48\]. \(\gamma p \to \Sigma^+ K^0\) \[49,50\], and the corresponding reactions off neutrons \(\gamma n \to \Lambda K^0\), \(\gamma n \to \Sigma^0 K^+\), \(\gamma n \to \Sigma^+ K^0\). For electroproduction, the \(\gamma\) is replaced by a virtual photon \(\gamma^*\); we quote recent results here even though they are not the topic of this paper \[52,53\].

The conventional strategy to study the reaction dynamics and to determine the properties of contributing resonances proceeds in steps. In a first step, the \(\pi N\) elastic scattering amplitudes are determined in an energy-independent reconstruction. So far, this step was carried out by three groups only \[16,17,18\]. In a second step, a fit to the resulting \(\pi N\) elastic partial wave amplitudes is performed and masses, widths and \(\pi N\) couplings of contributing resonances are determined. Reactions like \(\pi^- p \to \Lambda K^0\) are used to derive \(\pi N \to \Lambda K^0\) transition matrix elements. In a third step, helicity amplitudes \(A_{1/2}\) and \(A_{3/2}\) are specified by including photoproduction data. Their \(Q^2\) dependence, including polarization transfer coefficients, follows from fits to electroproduction data \[54,55\]. In the latter fits, hadronic properties of resonances are mostly frozen.

The Gießen group was the first one which embarked the enterprise to analyze simultaneously many reactions in a coupled channel fit. The Giessen group starts from a chiral Lagrangian from which background terms can be constructed with a minimum number of parameters. The strong interaction parameters were frozen from coupled-channel fits to pion-induced reactions and then, photon-induced reactions were included \[56,57\]. The spin of resonances was first limited to \(3/2\); in more recent studies \[58,59\], spin \(5/2\) resonances have been included. The Giessen group fits a large number of different reactions like \(\pi N \to \pi N, KY, \eta N, \omega N\) and the corresponding photo-induced reactions. The recent measurements high precision results from photo-production experiments which include measurements of double-polarization variables like \(C_2, C_3\) and \(O_5\), \(O_2\) were not yet available at the time when the study was performed.

In many cases, photoproduction data were fitted using masses and widths of known resonances as input. In this way, the effect of coupled channels \[60\] or of resonances in the \(u\)-channel can be studied \[61\] as well as the impact of specific resonances. \[62,63,64\]: helicity amplitudes of known resonances can be determined \[65,66\], and new resonances (the so-called...
“missing resonances”) can be searched for \[67,68\]. In the high-energy region, inclusion of meson trajectories in the \( t \)-channel allows for a description of various photoproduction channels over an extended energy range \[69,70,71,72\]. Elastic \( \pi N \) scattering and the reaction \( \pi^+ p \to K^+ \Sigma^+ \) were described simultaneously in a unitary coupled-channels approach by Döring et al. \[73\]. The Bonn-Jülich approach starts with a chiral Lagrangians and takes dispersive parts of intermediate states fully into account.

Amplitudes for strangeness production are presented by Candlin et al. \[39\], by the Bonn-Jülich group \[73\], and by KAON-MAID \[74\]. The three amplitudes are compared in section \[4\].

### 3 Production of \( \eta \) mesons

Rather few data exist on the reaction \( \pi^- p \to \eta \eta \) and conflicting results have been reported \[75,76,77,78,79,80,81,82\]. A lengthier discussion on the reliability of different data sets is summarized \[83\]. In the BnGa analysis, these data are used to constrain magnitudes for \( N^+ \to N \eta \eta \) couplings. Hence we decided to use the data from \[76,78\] as main input, and those in \[80,81\] with low weight to control our solutions. The inconsistencies between the different data sets demonstrate that techniques used in these - rather old - experiments were not really adequate to identify \( \eta \) mesons and to measure their yield.

The corresponding reaction with photons, \( \gamma p \to \eta p \), has been studied in a number of high-statistics experiments; a survey of early experiments can be found in \[84\]. The data are mostly consistent, with some deviations between the data from CLAS at Jlab \[85,86\] and Crystal Barrel at ELSA; we decided to use only high statistics data with direct detection of the \( \eta \) mesons \[84,87,88,89,90,91\]. Electroproduction has been studied as well; here we give reference to the latest publication only \[92\] in which earlier experiments are quoted.

In experiments on \( \eta \) production, the polarization state of the outgoing nucleon is difficult to measure (and so far, was never determined). Hence the corresponding amplitudes suffer from considerably larger uncertainties.

### 4 Amplitudes

The amplitudes presented here are derived from the BnGa multichannel analysis of a large fraction of the world data on nucleon resonances. Due to the complexity of the problem and missing information on further polarization variables, the fits due not converge to a well defined minimum. Instead, we found a number of solutions, called BnGa2011-01 and BnGa2011-02. These two classes of solutions are defined by the number of resonances with \( J^P = 3/2^+ \), one has one \( 7/2^+ \) resonance, the other one two. Within both classes of solutions, a number of different solutions were found which differ in smaller details. These provide error bands for the amplitudes.

In Figs. \[4\] - \[5\] we present our amplitudes and compare them to earlier results. The amplitudes for \( \pi^- p \to \Lambda K^0 \) in \[36\] are given only in the form of Argand diagrams without a mass scale, hence a comparison of the amplitudes is not possible. The error bands of the two BnGa-Gatchina solutions and their consistency is relatively good even though considerable differences can be seen for smaller amplitudes. These differences can be qualitative like for the \( J^P = 3/2^+ \) wave where the observed pattern seems to be shifted in energy; mass and width of the resonances in this wave optimize, however, for nearly the same pole positions; only the relative phases between consecutive resonances are different. In contrast, the \( 7/2^+ \) wave shows a significant structure in solution BnGa2011-01 which is absent in BnGa2011-02.

For the \( \pi N \to \Sigma K \) reaction, there is a much wider spread of results. For isospin 1/2, for nucleon excitations, no comparison with other work is available. The amplitudes for isospin 3/2 are shown in Fig. \[2\] upper panel. The amplitudes are from the work of Candlin et al. \[59\] and from the recent Bonn/Jülich analysis \[73\]. Candlin et al. and our amplitudes are in fair agreement; those from Bonn/Jülich show partly larger discrepancies. In particular, the Bonn/Jülich \( S_{31}(1/2^-) \) waves deviates significantly from the ones reconstructed by Candlin et al. and from our amplitude, even at the lowest energies. The \( P_{33}(3/2^+) \) partial wave seems to be not very well defined by the data, sizable differences are seen between our two solutions, the Bonn/Jülich and the solutions from the work of Candlin et al. where the imaginary part is comparatively small. The \( P_{31}(1/2^+) \) wave from Candlin et al. disagrees with the findings by Bonn/Jülich and our findings: in this wave, no resonance is supposed to contribute in the work of Candlin et al. while in the other two analyses, one \( P_{31}(1/2^+) \) resonances is used to describe the data. This work finds no contribution from the \( D_{35}(5/2^-) \) wave while Candlin et al. and Bonn/Jülich identify a small contribution. In the higher partial waves, no structural differences are seen.

To trace the differences we have reconstructed the total cross section from the sets of three partial wave amplitudes. These are compared with the data in Fig. \[5\]. It is obvious, that some intensity is missed in the amplitudes of \[23\]. Dynamical coupled-channels models based on effective chiral Lagrangians have the advantage of a much reduced number of fit parameters since they provide a microscopical description of the background \[77,78\]. In some cases, resonances can even be constructed from the iteration of background terms \[99,100,101\]. However, there is the possibility that not all background amplitudes provided by Nature are included in the calculation. It is then unclear to which minimum the fit converges.

The reaction \( \pi N \to \eta N \) lacks good data; polarization data are not available. In some partial waves, even the sign of coupling constants is not defined. Nevertheless, the data are useful since they constrain the \( N \eta \) decay modes of nucleon resonances. The signs of the two partial wave amplitudes \( P_{11}(1/2^+) \) and \( P_{13}(3/2^+) \) can both flip; this ambiguity leads to ambiguities in the \( \eta N \) contribution of \( N(1710)1/2^+ \) and \( N(1720)3/2^+ \). Measurements of the helicity dependence of the \( \eta \) photoproduction cross section should thus help to resolve this ambiguity.

Photoproduction multipoleos for \( \gamma p \to \Lambda K^+ \), for the two isobar contributions to \( \gamma p \to \Sigma K \), and for \( \gamma p \to p \eta \) are presented in Figs. \[45\] and compared with the amplitudes from KAON-MAID \[74,103\]. For \( \gamma p \to \Lambda K^+ \), the two Bonn-Gatchina solutions show often similar trends in the region up to 2 GeV even though the spread of results is still remarkable. This was unexpected since this reaction is, at present, the best studied.
photoproduction reaction for which even data on double polarization ($C_2, C_2, O_2, O_2$) are available, and used in the Bonn-Gatchina fits.

Here, it has to be stressed that the amplitudes are much more sensitive to the structure of an amplitude than the position of singularities in the complex energy plane. Large differences in these amplitudes may still be compatible with a reasonably good definition of the masses and widths of resonances.

The KAON-MAID amplitudes show nearly no similarity to our results at all. However, sizable discrepancies can be seen in some variables which can be calculated from their amplitudes. Fig. 3 shows an example. We emphasize that KAON-MAID did not fit those data. The comparison hence shows that the discrepancies between KAON-MAID and the Bonn-Gatchina solutions is not too worrisome. The multipoles for the two isobar contributions to $\gamma p \rightarrow \Sigma K$ show an even larger spread, and sizable differences between the two BnGa solutions, likely due to the absence of data on double polarization variables. For the reaction $\gamma p \rightarrow \eta p$, the amplitudes exhibit a large spread even in important waves. In this case, the disagreement between the BnGa and the MAID amplitudes is very unsatisfactory since basically, the same data are used. Obviously, care has to be taken when $\eta$ decay modes of resonances are to be interpreted.

Our multipoles on pion photoproduction were presented already in [53]. Those were based our 2009 solution. In Fig. 6 we give an update based on our present solutions. The over-
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**Fig. 2.** Decomposition of the $\pi N \rightarrow N\Sigma$ isospin-3/2 (top) and $\pi^- p \rightarrow n\eta$ transition amplitudes. The light (red) shaded areas give the range from a variety of different fits derived from solution BnGa2011-01, the dark (blue) shaded area from solution BnGa2011-02. The dotted curve represents the PWA from [39], dashed one from [73].

**Fig. 3.** Left: Cross section for the reaction $\pi^+ p \rightarrow \Sigma^+ K^+$; Open circles [38], Black squares [93], Triangle - [94], Black circles [95], Open squares [96]. (Some symbols overlap and are difficult to recognize.) The solid curve represent this work, the dotted line is from [39], and the dashed line from [73]. Right: Re-coil Polarization $P$ for $\gamma p \rightarrow \Lambda K^+$ at 2 GeV. Data from [48] show the statistical error. The solid curve is from the BnGa fit, the dashed curve the prediction of KAON-MAID [74].
Fig. 4. Multipole decomposition of the $\gamma p \rightarrow \Lambda K^+$ (top) and $\gamma p \rightarrow \Sigma^0 K^+$ (bottom) transition amplitudes. The multipoles are given in units of $10^{-3}/M\pi$. The light (red) shaded areas give the range from a variety of different fits derived from solution BnGa2011-01, the dark (blue) shaded area from solution BnGa2011-02. The dotted line represents the fit from KAON-MAID [74].
Fig. 5. Multipole decomposition of the $\gamma p \rightarrow \Sigma^+ K^0$ (top) and $\gamma p \rightarrow p\eta$ (bottom) transition amplitudes. The multipoles are given in units of $10^{-3}/M_{\pi+}$. The light (red) shaded areas give the range from a variety of different fits derived from solution BnGa2011-01, the dark (blue) shaded area from solution BnGa2011-02. The dotted line represents the fit from KAON-MAID.
Fig. 6. Multipole decomposition of the $\gamma p \rightarrow p\pi^0$ (top) and $\gamma p \rightarrow n\pi^+$ (bottom) transition amplitudes. The multipoles are given in units of $10^{-3}/M_{\pi^+}$. The light (red) shaded areas give the range from a variety of different fits derived from solution BnGa2011-01, the dark (blue) shaded area from solution BnGa2011-02. The dashed line represents the SAID fit [102], the dotted line the MAID [103] fit.
all consistency between BnGa2009 and BnGa2011, and between BnGa2011-01 and BnGa2011-02, is good. Also, the error bands are significantly narrower. The large discrepancies between our solutions and those of MAID and SAID were, however, unexpected. In this case, new data from ELSA on the double polarization variable $G$ provide very useful information. $G$ describes the correlation between the photon polarization plane and the scattering plane for protons polarized along direction of the incoming photon. In the low-energy region ($M < 1.65$ GeV), the data are compatible with BnGa2011-01 and BnGa2011-02 predictions, while very sizable discrepancies are seen to the MAID and SAID predictions. At medium energies ($M \approx 1.65 - 1.80$ GeV), all predictions are reasonably close to the data even though improvements can be expected when the data are included in the fits.

The structure of the $M^+_1$ multipole, exciting the $J^P = 1/2^+$ partial wave, is similar in all solutions even though the quantitative agreement is not satisfying. Other multipoles compare more favorably, in particular of course the $M^+_2$ multipole. Significant differences are seen in the $E^+_2$ and $M^+_2$. Yet, these are the smallest amplitudes (and excite the $N(1675)5/2^-$ resonance).

5 Summary

We have presented amplitudes for pion and photo-induced reactions leading to final states with $\Lambda K$, $\Sigma K$, and $N\eta$ For completeness, we have added the amplitudes for $\gamma p \rightarrow p\pi^0$ and $\gamma n \rightarrow p\pi^+$. The amplitudes are the solutions of a recent multichannel analysis of the Bonn-Gatchina group [104]. The number and the properties of contributing resonances have been reported earlier.

The solutions are divided into two classes, one class in which we assume that there is one resonance with $J^P = 7/2^+$, in the other class of solutions, it is assumed that there are two of them. Within each class, there is a variety of solutions, again with different numbers of assumed resonances, different background parameterizations, or using different start values of the fit.

In most cases were the BnGa amplitudes can be compared to the amplitudes from other partial wave analyses, very severe discrepancies show up. These discrepancies are certainly due to the lack of polarization data. The gain in reliability in the definition of amplitudes can be seen when the amplitudes are used to calculate observable quantities. This can be done using our web page [103].

Significant deviations are also observed between the two classes of solutions BnGa2011-01 and BnGa2011-02 even though both solutions are compatible with the very large data base made available in the last years by intense efforts at Bonn, Mainz, Grenoble, and Jlab. But there is hope for the future: in Bonn, Mainz, and Jlab the program is ongoing, and experiments with polarized photon beams, polarized targets, and with measurements of the recoil polarization are being carried out and were partially completed and are being analyzed. Hence significant further advances can be expected.

Acknowledgements

We would like to thank the members of SFB/TR16 for continuous encouragement. We acknowledge support from the Deutsche Forschungsgemeinschaft (DFG) within the SFB/TR16 and from the Forschungszentrum Jülich within the FFZ program.

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