Magnetic Vortices in High Temperature Superconductors

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It is suggested that modes, observed in recent neutron scattering experiments by Lake et al., on La$_2-x$Sr$_x$CuO$_4$ in strong magnetic fields ($\approx 7$ T), are due to the existence of antiferromagnetic moments associated with the cores of vortices generated by the field. These moments form one-dimensional chains along the c-axis (the vortex axis), which at finite temperatures are disordered. At temperatures higher than 10 K the correlation length gets shorter than the lattice parameter, resulting in no scattering from coherent spin-waves above that temperature. The bandwidth of the spin-waves is estimated to be $\approx 4$ meV in accordance with the observations.

The superconducting and the antiferromagnetic phases are close in the copper oxide supersuperconductors. They are close in the overall phasediagram, where at low temperatures the antiferromagnetic (AF) order is replaced by a superconducting (SC) order upon doping and in certain inhomogenous phases, such as the striped phases, the two order parameters will be literally close geometrically. In the original observations of stripes in cuprates by Tranquada et al. it was found that the stripes are in fact insulating regions with antiferromagnetic order in a sea of superconducting material. An alternative way of suppressing the superconductivity is to apply an external magnetic field which will generate vortices, who’s cores are non-superconducting. It has been suggested by several, that there is a possibility, that the cores are in fact small islands of insulating antiferromagnets.

So far we have only indirect evidence, that such a possibility is realized. E.g. Scanning Tunneling Microscope (STM) measurements indicates an insulating behavior of vortex cores in the Bismuth based cuprates. Recent inelastic neutron scattering experiments on optimally doped La$_2-x$Sr$_x$CuO$_4$ in an applied magnetic field has revealed modes with energies around 3.5 meV below 10 K. In this paper it will be argued that such results are to be expected if in fact the vortex cores support antiferromagnetically ordered spins.

In the experiments by Lake et al. on optimally doped La$_2-x$Sr$_x$CuO$_4$ below $T_c$ it is found that without a field the spin excitations have a gap $\Delta = 6.7$ meV, for all measured momentum transfers. A simple BCS model for d-wave superconductivity leads one to expect that the gap in the spin-excitations should vanish for momentum transfers close to the difference between two nodes in the one particle spectrum. This is not found experimentally, from which one concludes, that in strongly correlated systems the spin excitations are not necessarily associated with the charge excitations. We shall have nothing further to say about this general problem. When a field is applied, however, the situation changes. Then new excitations are found below a temperature $T_0 = 10$ K. These excitations have energies in the gap associated with the field free superconductor. In fact the results can be summarized by saying that there is a single mode with energy 3.5 meV and width 3.8 meV, although this is not the physical picture emerging from the considerations in this paper. It is natural to associate the excitations with the vortices generated by the magnetic field, and Lake et al. also establishes that the intensity of the magnetic field scales with the strength of the external field, consistent with such an assignment.

Using the generalized Ginzburg-Landau model, which includes both the AF order parameter and the SC order-parameter, and which has an approximate SO(5) symmetry, Arovas et al. has suggested the that vortex cores can be insulating antiferromagnets. The scenario is not only possible in an SO(5) model, and more recently it has also been proposed in models with spin charge separation. In this paper we shall make a minimal assumption, that in the vortex core the AF order parameter is non-zero in the core, and described by an envelope function $\phi(r)$, who’s total weight $\int \phi(r)^2 r dr/a^2 = N$, where $N$ then can be interpreted as the total number of spins in the vortex core. The excitation of such an AF vortex core has been studied in the work by Hallundbæk et al..

Neglecting for a moment the interaction between spins in different copper-oxide layers and spin anisotropies, the spins in the core has a zero-energy, Goldstone mode, which has excitation energy zero, corresponding to a rigid rotation of all the spins in the core. This mode is model independent. Since the group of spins has a finite size, the magnons will have a discrete spectrum, and the gap to the lowest excitation is approximately $Ja/\xi$, where $J$ is the exchange interaction in the plane ($\approx 130$ meV), $a$ is the lattice parameter, and $\xi$ is the coherence length, i.e. the size of the vortex core, hence the energy is much larger than the few meV we are considering in this paper, and we can take the spins to form a rigid group described by one single vector $\vec{n}$. Different models have different other excitations of the core. E.g. will the SO(5) model have a resonance close to 40 meV, which describes a rotation of the AF order parameter into the SC plane. The details are not important — here we only need to
know that the energy is so high, that such modes can be neglected in the present context. There are two final interactions, which have low energies: the Zeeman coupling to the magnetic field associated with the vortex, and the interplane exchange coupling \( J' \). Knowing the London length one can easily estimate the Zeeman energy, and this is \( 1 \mu \text{eV} \), which is way to small to be of relevance [3]. This leaves us with \( J' \). It is known to be \( J' = 3.7 \cdot 10^{-5} J \), which also seems small, but it is an interesting feature of spin waves in a system with different exchange constants in different spatial directions, that modes moving along the direction with \( J' \) (the c-direction in our case) have energies \( \omega_k = \sqrt{J J'}(1 - \cos kx) \), i.e. a typical energy \( \sqrt{J J'} \approx 2 \text{meV} \), which is the energy scale of the experiment.

So, we are suggesting, that what is seen in the experiment is spin waves traveling up and down the vortex core column. From general principles we know that a 1-dimensional antiferromagnet does not order at any finite temperature, so we have to analyze a thermally disordered antiferromagnet. This is most easily done using the field theory representation for an antiferromagnet, namely the non-linear sigma model. We start with the model in the 2-dimensional copper-oxide planes. The (imaginary time) action is given by

\[
S_{\text{plane}} = \sum_i \int_0^\beta d\tau \int d^2 x \left( \frac{1}{2} \left( \frac{\partial \vec{m}_i}{\partial \tau} \right)^2 + aJ s^2 (\nabla \vec{m}_i)^2 + L_{sc} \right),
\]

where \( s \) is the size of the spins, \( a \) is the in-plane lattice constant and \( i \) is a plane index. The last term describes the coupling to the SC orderparameter. The precise nature of that is not specified, except, that we assume, that in the core of the vortex, the AF order parameter \( \vec{m}_i(\vec{r}) \) is a constant vector \( \vec{n}_i \); i.e. we make the ansatz \( \vec{m}_i(\vec{r}) = \vec{n}_i \phi(\vec{r}) \), where \( \phi(\vec{r}) \) is 1 in the center of the core, and decays to zero outside the core. The actual expectation value of the spin-operator is then \( \langle \vec{S}_i(\vec{r}_j) \rangle = \vec{n}_i \phi(\vec{R}_j)(-1)^j \), where the factor \((-1)^j\) denotes the AF staggering, +1 on one sublattice and -1 on the other sublattice.

Let us now introduce the inter plane coupling:

\[
L_{ip} = J' \sum_i \int d^2 x \vec{S}_i(\vec{r}) \cdot \vec{S}_{i+1}(\vec{r}),
\]

which can be rewritten in terms of the in-plane AF order parameter (and making the continuum approximation):

\[
L_{ip} = \frac{J' s^2 c N}{4} \int dz \left( \frac{\partial \vec{n}_i}{\partial z} \right)^2,
\]

resulting finally in the effective action

\[
S_{\text{eff}} = \frac{1}{2} \int_0^\beta d\tau \int dz \left( \chi \left( \frac{\partial \vec{n}_i}{\partial \tau} \right)^2 + \rho \left( \frac{\partial \vec{n}_i}{\partial z} \right)^2 \right),
\]

with \( \chi = N/(4aJ) \) and \( \rho = J's^2cN/4 \).

Both the “mass” and the “spring constant” scales with the size of the vortex core, \( N \), but at zero temperature this model have low energy spin wave modes with a dispersion \( i\omega_n = sJ'/\sqrt{J c} \), independent of \( N \) — and equal to the dispersion of spin waves in a 3-D model with anisotropic exchange constant. The size of the vortex core can be estimated from recent measurements of \( H_{c2} \) by Ando et al. [4]. They find \( H_{c2} \approx 62T \), which result in an estimate for \( N \approx 216 \).

At finite temperatures the system is disordered. The basic physical picture put forward in this paper, is that only propagating modes with a wavelength shorter than the correlation length will show up as peaks in inelastic neutron scattering. Below a certain temperature (10 K in the experiment) the correlation length becomes sufficiently long, that modes with wave vectors in the experimental window \( \approx 50 \% \) of the c-axis Brillouin zone contribute to the scattering. Such modes has energies in the upper half of the spin wave spectrum.

Accordingly we need to find the correlation length of the spins. We use the renormalization group equations discussed extensively by Chakravarty et al. [5]. They are for the 1-D case

\[
\frac{dg}{dl} = g^2 \frac{c}{2\pi} \coth \left( \frac{g}{2\pi} \right),
\]

\[
\frac{dt}{dl} = t + g^2 \frac{c}{2\pi} \coth \left( \frac{g}{2\pi} \right),
\]

where \( g \) is a dimensionless coupling constant:

\[
g = \frac{1}{\sqrt{N\rho}} = \frac{4}{sN} \sqrt{\frac{J a}{J' c}}
\]

and \( t \) is a dimensionless temperature:

\[
t = \frac{ck_B T}{\rho} = \frac{4k_B T}{N^2 J's^2}.
\]

The correlation length for the spin system, \( \xi_s \), is determined by the requirement that the renormalized length, \( \xi_s(l) = \xi_s e^{-l} \) is equal to the lattice parameter \( c \). This is essentially obtained when the scaling parameter, \( l \), has a value such that the renormalized temperature \( 2\pi^2 \).

The renormalization equations can be solved exactly, and the solution is

\[
t(l) = \frac{t_0 e^l}{1 - g_0 f(l)} \text{ where } f(l) = \int_0^l \frac{dt'}{2\pi} \coth \left( \frac{g_0}{2t_0} e^{-t'} \right).
\]

The correlation length is finite at all temperatures, with a limiting value

\[
\xi_s(T = 0) = \frac{c e^{2\pi / g_0}}{g_0}.
\]
For the parameters relevant to our system, the scaling starts at values

\[ g_0 \approx 5 \]  
\[ t_0 \approx 1.7 T, \]

where \( T \) is the actual temperature measured in Kelvin. These are approximate values derived from very difficult experiments (\( H_{c2} \) and \( J' \) are hard to get), so precise scale for e. g. \( t_0 \) can easily vary with a factor of 2. Choosing the temperature scale factor to be 3.4 instead of 1.7, which corresponds to the product \( NJ' \) being a factor 2 smaller than quoted above, we get the curves shown in Figure 1 for the correlation length \( \xi(T) \) as a function of temperature and for 3 values, 1, 5, 10 for \( g_0 \). We see that a strong growth of correlation length around 10 Kelvin is quite compatible with known values for the size of the vortex core and the strength of the interplane exchange coupling.

In conclusion, we have proposed an explanation of the recent experimental finding by Lake et al. of magnetic scattering from optimally doped \( \text{La}_{2-x}\text{Sr}_x\text{CuO}_4 \) in a strong magnetic field, below 10 Kelvin at an energy of approximately 3 meV. This energy scale is the natural one for spin waves in a system of antiferromagnetically ordered spins residing in the vortex cores of the superconductor. This being a 1-dimensional system no long range order exist at any finite temperature, so inelastic scattering creating reasonably longlived spin waves requires a correlation length larger than the interplane distance. We have shown that for reasonable values of the vortex core size and the interplane exchange coupling, this happens for temperatures less than 10 Kelvin. It is beyond the scope of this paper to give a detailed calculation of the actual theoretical neutron scattering spectra, but calculations of this is underway.

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