The Unholey Solution to Black Hole Information Loss

Neil J. Cornish
Department of Physics, University of Toronto
Toronto, Ontario M5S 1A7, Canada

The simplest solution to the black hole information loss problem is to eliminate black holes. Modifications of Einstein gravity which accomplish this are discussed and the possibility that string theory is free of black holes is considered.

INTRODUCTION

In recent years, the application of quantum mechanics to black holes has been considered the best arena in which to find clues to theoretical physics’ holy grail – a quantum theory of gravity. The possible connection between gravity, quantum mechanics and thermodynamics embodied in the laws of black hole thermodynamics is seen as an important step on the road to a deeper understanding of quantum gravity. The seductive beauty of these results is in itself enough to make any suggestion that black holes should be eliminated a highly unpalatable thesis. However, beauty can sometimes be beguiling. We believe it can be argued that black hole thermodynamics, and the information loss problem, are merely symptoms of the unphysical causal division of spacetime engendered by black holes. From this standpoint, black holes are seen as a signal that the classical gravity theory which predicts them has broken down and a replacement should be sought.

By definition, a black hole divides a spacetime \( (M, g_{\mu\nu}) \) into two causally disconnected regions such that \( M = B + J^- (I^+) \), where \( B \) is the black hole region and \( J^- (I^+) \) is the causal past of future null infinity. This causal disconnection is the root cause of the information loss problem in spacetimes where a black hole is produced by the collapse of matter and subsequently evaporates away via the Hawking process.

The three main approaches to solving the information loss problem can be roughly classified as 1) reconciling quantum mechanics with non-unitary evolution, 2) reformulating gravity to store or return the information, 3) there is no problem. Each of these possibilities is supported by ingenious and plausible arguments, and one of the three might well be correct. However, the question of which approach is correct is rendered moot if we insist that black holes do not exist.

The motivation for eliminating black holes goes beyond achieving a quick fix to the information loss problem. Additional motivation is provided by considering the Hawking-Penrose singularity theorems and the role that trapped surfaces play in forcing singular behaviour. Moreover, black hole event horizons cause physical measurables such as redshifts to diverge. In the spirit of the cosmic censor conjecture and the chronology protection conjecture of general relativity we shall demand that the entire spacetime manifold lies in the causal past of future null infinity, i.e. \( M = J^- (I^+) \). This global causality demand also requires that the spacetime is free of singularities in order for the spacetime to be strongly asymptotically predictable, as we have sacked the cosmic censor.

Clearly, Einstein gravity is at odds with the no black hole condition and alternative theories must be sought which ensure causal connectivity. In section I we shall consider what form the modifications to Einstein’s theory must take to eliminate black holes. A concrete example which appears to satisfy the global causality demand is reviewed in section II. The possibility that string theory may be compatible with the no-black hole conjecture is considered in section III.

I. MODIFYING GRAVITY

In physical terms, the elimination of black holes comes down to ensuring that gravity never gets so strong, or spacetime so bent, that light cannot escape from regions of spacetime. This probably cannot be achieved in any theory which employs point particles, obeys the weak equivalence principle and is entirely local.

What we are seeking is essentially a redshift-limited theory in which the redshift between any two points in spacetime is finite. The difficulty is that redshift is an intrinsically non-local quantity, and any attempt to construct a theory based on non-local notions is likely to produce acausal effects far worse than the problems of causal disconnection it is trying to solve. More promising possibilities are offered by theories which violate the weak equivalence principle or employ extended structures such as strings.

In a theory which violates the weak equivalence principle local measurements can be made to determine the strength of the gravitational field. A freely falling observer would be able to tell that a high redshift surface was being approached. The gravitational field can respond to such information. One might try and formulate such a theory along the lines of the Limited Curvature Hypothesis \([1]\) by explicitly constructing a limited redshift Lagrangian. In section II we shall review a different approach in which an equivalence principle violating the-
ory, formulated on a non-Riemannian manifold, is able to eliminate black holes from static, spherically symmetric spacetimes.

In a theory which employs extended objects such as strings, there is an essential non-locality built into the physics which allows redshifts to be felt. However, since the non-locality is expected to be confined to Plankian scales, it may seem impossible for string theory to have any impact on macroscopic horizons. This is not the case [7]. Consider a fully stringy geometry near a high redshift surface. As quantum fluctuations on this geometry propagate towards the surface of high redshift they become amplified, leading to a large back reaction in the underlying geometry. In this picture, a would-be event horizon must be described by non-perturbative string theory. In section III we shall develop arguments which suggest that string theory might be free of black hole regions.

II. NON-RIEMANNIAN GRAVITY

By formulating gravity on a hypercomplex non-Riemannian manifold, the gravitational theory is endowed with additional degrees of freedom. In particular, the metric is no longer symmetric. Coordinates can be chosen such that the symmetric part of the metric is locally Minkowskian, however the skew components of the metric cannot be chosen to vanish.

Recently it was shown that one such theory was free of black holes and curvature singularities for static, spherically symmetric spacetimes. The coordinates have been chosen so that circles of radius \( r \) have circumference \( 2\pi r \). The hypercomplex skew field \( g_{[\mu\nu]} \) looks superficially like the Kalb-Ramond axion of low energy string theory, but it has significantly different dynamics. Indeed, for large \( r \) we find that \( g_{[\theta\phi]} = Q_{\text{top}} \sin \theta \) which corresponds to a purely topological contribution with topological charge \( Q_{\text{top}} \) in four dimensional string effective theories.

The constant \( Q_{\text{top}} \) is found to equal \( sM^2/3 \) where \( M \) is the ADM mass and \( s \) is a dimensionless constant. Since the skew contribution to the gravitational field does not vanish in a freely falling frame, it enables local measurements to be made of the redshift. If we define \( \nu = -2 \ln(z + 1) \), where \( z \) is the redshift between \( r \) and spatial infinity, we find that the invariant \( F^2 = g_{[\mu\nu]}g^{[\mu\nu]} = f^2/(r^4 + F^2) \) can be written as

\[
F = \frac{\sinh(aw)\sinh(bv) + s(1 - \cosh(aw)\cos(bv))\sqrt{1 + s^2}}{\cosh(aw) - \cos(bv)} \tag{2.2}
\]

where

\[
a = \sqrt{\frac{1 + s^2 + 1}{2}}, \quad b = \sqrt{\frac{1 + s^2 - 1}{2}}. \tag{2.3}
\]

The above equation provides an implicit expression for the redshift in terms of a locally measurable quantity, \( F \). When \( F \ll 1 \) we can recast the theory as Einstein gravity non-minimally and non-polynomially coupled to a skew field. From this standpoint we can study the linearised equation of motion for \( f \) in a Schwarzshild background. Near \( r = 2M \) we find that

\[
f'' \approx \frac{1}{M(r - 2M)} (-Mf' + 2f). \tag{2.4}
\]

Outside of \( r = 2M \), \( f \) can be very small and slowly decreasing \((f, f' \ll 0)\). Near \( r = 2M \), \( f'' \) becomes very large which implies \(-f' \) becomes large also. This means that \( f \) must be rising precipitously as \( r = 2M \) is approached from outside. This is confirmed by the exact solution which is displayed graphically in Fig.1. We see that the skew field responds to the increasing redshift by becoming very large. In turn, the back-reaction of the skew field on the symmetric metric ensures that the redshift, \( z_{12} = \gamma_1 - \gamma_2 \), remains finite everywhere. Importantly, the two non-vanishing curvature invariants, \( R_{\mu\nu\lambda\kappa}R^{\mu\nu\lambda\kappa} \) and \( R_{\mu\nu\lambda\kappa}R^{\rho\delta\kappa\lambda}R^\rho_{\mu\nu} \), are everywhere finite also. The region below \( r = 2M \) is characterised by having very large, but finite redshifts relative to spatial infinity. In this respect the solution describes a grey, rather than a black hole. In a practical sense it is difficult to transmit information from below \( r = 2M \), but there is no barrier in principle.

An important feature of the solution is that Einstein gravity can only be recovered as a limit to this theory when the redshift is small. In the limit \( s \to 0 \) the theory is identical to Einstein’s theory outside of \( r = 2M \), but no matter how small the parameter \( s \) is taken to be, Einstein gravity cannot be recovered at or below \( r = 2M \).
In the small limit the solution has the following interesting features:
1) The maximum redshift between any two points in the manifold is given by
\[ z_{\text{max}} \approx \exp \left( \frac{\pi}{2|s|} \right) \quad (2.5) \]

2) The maximum curvature occurs at \( r = 0 \) and is given by
\[ R_{\mu\nu\kappa\lambda}R^{\mu\nu\kappa\lambda} \approx \frac{1}{M^4} \exp \left( \frac{2\pi + 4|s| \ln s + 4s - 9|s| \ln 2}{|s|} \right) \approx \left( \frac{z_{\text{max}}}{M} \right)^4 ; \quad (2.6) \]

3) The proper surface area at \( r = 2M \) and \( r = 0 \) approaches \( V_S = 16\pi M^2 \) as \( s \to 0 \):
\[ V_{2M} \approx V_S \left( 1 + \frac{3}{16} s^2 (\ln |s|)^2 \right) , \quad (2.7) \]
\[ V_0 \approx V_S \left( 1 - \frac{|s|\pi}{8} \right) ; \quad (2.8) \]

4) The proper distance between \( r = 0 \) and \( r = 2M \) shrinks to zero as \( s \) tends to zero since
\[ \int_0^{2M} \sqrt{\alpha} \, dr \approx \frac{4M}{z_{2M}} , \quad (2.9) \]
where \( z_{2M} \) is the redshift between \( r = 2M \) and \( r = \infty \).

When \( s = 0 \) the redshifts \( z_{2M} \) and \( z_{\text{max}} \) become infinite and the proper distance between \( r = 0 \) and \( r = 2M \) goes to zero. The curvature at the horizon diverges when \( s = 0 \), giving rise to a light-like singularity at \( r = 2M \). Unlike Einstein’s theory, this redshift dependent theory considers horizons on the same footing as curvature singularities - for any non-zero value of \( s \) it gets rid of both at the same time.

Similar results continue to hold for the analog of electrically charged black holes, and there are promising signs that rotation will not alter our conclusions. In summary, the theory appears to be both free of black holes and asymptotically predictable. A general proof for arbitrary symmetry remains to be found.

### III. STRING GRAVITY

It is commonly hoped that string theory will temper curvature singularities and provide a consistent theory of quantum gravity. While that might seem a lot to ask, we want even more - we ask that string theory also banishes black holes.

The standard picture of string theory, or at least string effective theory, holds that the string corrections to general relativity should be important when curvatures approach the string scale \( \alpha' \). From this standpoint it would seem nonsensical to suggest that string theory should offer any assistance in removing horizons, as curvatures can be arbitrarily small at a horizon. We do not believe the possibility is that easily dismissed.

In order to be consistent with the no-black hole condition and asymptotic predictability, a theory must be free of singularities. We will assume that string theory lives up to its promise and provides us with non-singular solutions. We hope to motivate the possibility that string theory also removes horizons by appealing to three main arguments. These arguments are based on 1) string effects in black hole backgrounds; 2) the nature of lowest order string black hole solutions and duality transformations; and 3) the unitarity of full string theory.

The first suggestion that string theory should have something to say about horizons is provided by studies into the behaviour of strings in fixed black hole backgrounds. These studies indicate that strings are not only important \cite{13,14} in the description of physics near a high redshift surface, but that if string perturbation theory breaks down in such regions \cite{13,15,16}. The fixed background perspective in these studies requires that a complementarity principle is invoked to reconcile the non-perturbative stringy effects seen by static observers and the total absence of stringy effects seen by free fall observers. However, if we consider the fact that the background should also be described by string theory we arrive at a somewhat different picture. In many respects the background geometry shares the viewpoint of a static observer. The high redshift surface suggested by the lowest order solution serves to excite the string modes, requiring a higher order, non-perturbative description. Via this mechanism, which is in many respects similar to the field amplification seen in \cite{2,4}, it may be possible for the full solution to mollify the infinite redshifts suggested by the lowest order solution.

The second suggestion comes from considering solutions to the lowest order metric-dilaton string Lagrangian \cite{17}
\[ \mathcal{L} = \frac{1}{8\pi\alpha'} \sqrt{-g} e^\phi \left( R(g) + (\nabla \phi)^2 \right) . \quad (3.1) \]
In four dimensions the spherically symmetric solution describes a black hole with metric \cite{18}
\[ ds^2 = \left( 1 - \frac{2M}{r \cos \psi} \right)^{\cos \psi} dt^2 - \left( 1 - \frac{2M}{r \cos \psi} \right)^{\cos \psi} dr^2 \]
\[ -r^2 \left( 1 - \frac{2M}{r \cos \psi} \right)^{1 - \cos \psi} d\Omega^2 , \quad (3.2) \]
and dilaton
\[ e^\phi = \left( 1 - \frac{2M}{r \cos \psi} \right)^{\sin \psi} . \quad (3.3) \]
The constant \( M \) is the ADM mass of the black hole and \( \psi \) is a constant which can take any value. For all configurations with a non-trivial dilaton (\( \sin \psi \neq 0 \)) both the
origin at \( r = 0 \) and the horizon at \( r = 2M/\cos \psi \) suffer from infinite curvatures, rendering this lowest order solution invalid on both surfaces. An interesting feature of the solution is that it is invariant under the transformation

\[
r = r' + \frac{2M}{\cos \psi}, \quad \psi = \psi' + \pi,
\]

which interchanges the horizon and the singularity. This is reminiscent of the duality transformation which interchanges the singularity and horizon in \((1+1)\)-dimensional black hole solutions to string theory \cite{19,20}. In the \((1+1)\)-dimensional case a regular horizon gets mapped into a singularity and vice-versa. Both these classical black hole solutions to string theory suggest that if string theory is going to help remove singularities, it should also have something to say about horizons.

An alternative viewpoint might be that a singularity in the manifold associated with \( g_{\mu\nu} \) does not necessarily correspond to what constitutes a true singularity in string theory as strings “feel” a richer geometry than point particles. In other words, the stringy spacetime foam cannot be effectively modeled by a smooth manifold which is free of curvature singularities. In that event, the preceding argument fails. However, the notion of a black hole region in the manifold associated with \( g_{\mu\nu} \) would not, by the same reasoning, necessarily correspond to a black hole region as seen by strings. In this way the no-black hole condition might be satisfied in a subtler sense.

Support for the notion that strings are consistent with the no-black hole hypothesis comes from the observation that full string theory is unitary, i.e. outgoing and incoming density matrices are related by

\[
\rho_{\text{out}} = \mathcal{S} \rho_{\text{in}}, \tag{3.5}
\]

where \( \mathcal{S} \) is factorizable as the product \( SS^\dagger \). In standard point particle field theory, or light particle string field theory, the presence of black hole regions in the background spacetime destroys the factorizability of \( \mathcal{S} \). This is an alternative way of describing the black hole information loss problem.

One approach to resolving the discrepancy between the behaviour of the full theory and its approximate low energy description is to formulate self consistent string modifications of quantum mechanics which allow for apparent non-unitary evolution in the effective light particle string theory \cite{2}. While this is a valid approach, it can be viewed as being somewhat roundabout: a well behaved exact theory is replaced by a low energy approximation with black holes; the black holes in turn cause the low energy description to violate unitarity; the loss of unitarity requires quantum mechanics to be modified; the non-unitary evolution cancels out the causal disconnection so that we finally recover the consistent description we had at the outset. A simpler reconciliation is offered by the no-black hole hypothesis where we have argued that fully stringy background spacetimes are free of black holes. In this scenario both light particle string field theory and full string theory are described by unitary evolution, allowing a smooth recovery of standard quantum field theory in the point limit.

\section*{IV. CONCLUSIONS}

We have advanced the view that black holes are physical pathologies which signal the breakdown of any classical gravity theory which predicts they exist. Our viewpoint is at least consistent with observations, as there is no direct evidence that black holes exist. By exhibiting one concrete example, based on an equivalence-principle-violating modification of Einstein’s theory, we have demonstrated that the no-black hole hypothesis is not an impossible demand. By putting forward heuristic arguments, we hope to have made plausible our suggestion that string theory also banishes black holes. At the very least, we see no reason why full string theory must have black holes.

\section*{ACKNOWLEDGMENTS}

I am grateful for the support provided by a Commonwealth Scholarship. I thank Janna Levin for discussions and sharing related ideas on low energy string theory with me.

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