Fast high-resolution imaging method for wide-band rotating targets

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Abstract: To reduce the computation burden of inverse synthetic aperture radar on rotating targets imaging, a fast distributed simultaneous multiple orthogonal matching pursuit method is proposed in this study. In the proposed method, the distributed matching sparse representation model of the rotating targets is firstly established. Also then the total number of iterations is reduced by using multi-atom recognition strategy. Simultaneously, the computational complexity of each iteration is also reduced through QR decomposition. High-resolution image of the rotating targets can thus be obtained efficiently. Theoretical analysis and simulation results verify the effectiveness of the proposed method.

1 Introduction

In inverse synthetic aperture radar (ISAR) applications, the micro-motion effect caused by the rotation parts of targets will seriously deteriorate the imaging quality [1, 2]. The traditional idea is to take the echo generated by the rotating parts as interference and remove it as noise. However, this processing would lose important information of the targets. To achieve high-resolution imaging of target with rotating parts, many methods have been studied in the past decades. The processing of these traditional methods can be divided into two steps, i.e. separation and imaging. The first step is how to separate the echo generated by rigid body from the echo generated by rotating parts. The typical methods include short-time Fourier transform, Wigner-Ville distribution [3], complex empirical mode decomposition method [4], complex local mean decomposition method [5] and so on. The second step is how to image rotating parts, such as single-range Doppler interferometry [6], Hough transform (HT) and extended Hough transform [7]. However, when the rotating parts contain time-varying Doppler information, the methods described above cannot effectively implement imaging.

In this paper, a fast distributed simultaneous multiple orthogonal matching pursuit (FDSMOMP) method is proposed based on the distributed compressed sensing (DCS) algorithm, in which the total number of iterations and computation per iteration are reduced. Being different from the traditional imaging ideas, the FDSMOMP searches the positions of the scattering points directly; thus avoiding the estimation of the time-varying Doppler.

2 FDSMOMP algorithm

A schematic diagram of the relative relationship between the targets’ rotating parts and radar is shown in Fig. 1. Assume that the rotating parts contain $K$ strong scattering points, and the initial coordinate position of the $k$th point in the target rectangular coordinate system $X - Y$ is $(x_k, y_k)$. Suppose the angular velocity of rotating parts $\omega$ has been estimated. The complex envelope of the transmitted linear frequency modulation (LFM) signal can be expressed as $s(t, t_0) = \text{rect}(t/\tau_p) \exp(j\pi f_0 t/\tau_p)$ ($\tau_p$ is the time width, $\mu$ is the frequency modulation rate). The baseband frequency echo of the rotating parts can be expressed as [8]

$$s_k(t, t_0) = \sum_{k=1}^{K} \sigma_k \exp\left(- \frac{2R_k(t_0)}{c}\right) \exp\left(-j4\pi R_k(t_0)/\lambda\right) + e$$  \hspace{1cm} (1)

where $t$ and $t_0$ are fast time and slow time, respectively. $R_k(t_0) = R_k + \nu_k \sin(\omega t_0 + \varphi_k)$ denotes the instantaneous slant distance from the $k$th scattering point to radar, $(\tau_0, \varphi)$ is the initial polar co-ordinate position of the $k$th scattering point. $R_k$ is the slant distance between radar and the rotating centre, $c$ is the speed of light, $\lambda$ is the wavelength, $\sigma_k$ is the scattering intensity of the $k$th scattering point, and $e$ is the additive noise. Using the conventional pulse compression technique to deal with the target echo, we can obtain the range profile of target, which can be expressed in 2D time domain as

$$s_{pc}(t, t_0) = \sum_{k=1}^{K} \sigma_k \tau_p \sin\left[B(t - \frac{2R_k(t_0)}{c})\right] \cdot \exp\left(-j4\pi R_k(t_0)/\lambda\right) + e'$$  \hspace{1cm} (2)

where $B = \mu \tau_p$ is the signal bandwidth, $e'$ denotes the result of pulse compression processing of $e$. Apply the Fourier transform to (2) along fast time $t$, we get

$$S_{pc}(f, t_0) = \sum_{k=1}^{K} \sigma_k \exp\left(-j4\pi \left(f + f_0\right) \frac{\nu_k \sin(\omega t_0) + \nu_k \cos(\omega t_0)}{c}\right)$$

$$+ \text{FT}(e')$$  \hspace{1cm} (3)

where $\sigma_k = (\tau_0/B) \nu_k \text{rect}(f/B) \exp(-j4\pi(f + f_0)R_k/c)$, $f$ denotes the fast time frequency, and $\text{FT}(\cdot)$ denotes Fourier transform operator.

Due to the application of large bandwidth signal, the range resolution $\delta_r$ is usually high, i.e. the value of $\delta_r$ is small. From (2) and (3), we can find that in the fast-time slow-time domain the
Input: Measurement data $Y$, sensing matrix $\Theta$, preset sparsity $k_s$, and group selection support set $s$.

Initialization: If initial residuals $R^0 = Y$ , initial results $\tilde{X} = 0$ , and initial support sets $\hat{S} = \emptyset$.

Step1: Multi atom recognition. Calculate atomic support set $P_m$ according to (8)

Step2: Projection calculation.
Update support set as $\tilde{S} = \tilde{S} \cup P_m$ , and calculate the projection value $\tilde{X}_{\tilde{S}}$ according to (10).

Step3: Residual update: $R(f_p) = y(f_p) - \Theta(f_p)^H\tilde{X}(f_p)$.
If the stop condition is satisfied, continue; Otherwise, back to step1.

Step4: Obtain the reconstruction result by (10).

Output: Reconstruction result $\tilde{X}_{\text{inner}}$.

Fig. 2 FDSMOMP algorithm

envelope and phase of complex echo both vary according to the sinusoidal, and in the fast-frequency slow time domain the phase of fast frequency spectrum also varies according to the sinusoidal. The peak curve of range imaging is changed with $R(t)$, which reflects the characteristics of the rotating parts. Obviously, it is difficult to directly use the amplitude information, as it is varying with slow time. In order to fully exploit amplitude information, a distributed matching sparse representation model corresponding to the sparse dictionary with some parameters, which is given by

$$x = R(f_p) = \Theta(f_p)x_p + \Phi n.$$ $\forall p \in \{1, 2, \ldots, P\}$

where $X = \{x_1, x_2, \ldots, x_N\} \subset \mathbb{C}^{\mu \times P}$ is the image to be reconstructed, $\|X\|_1 = \|x_i\|_1 > 0$ denotes the non-zero row number in $X_i$, $\|x_i\|_1$ denotes the 1-norm of row $i$. $I(\cdot)$ denotes the count function. The proposed model has two main characteristics. One is that the signal model has joint sparse characteristics. The other is that different fast time frequency points correspond to different sparse dictionaries.

The joint sparse characteristics of DCS theory [9] can be used to solve the problem expressed in (6). In order to reduce the computation burden, the FDSMOMP algorithm is proposed in this paper. Compared with DCS-SOMP algorithm [10], FDSMOMP algorithm has two important improvements. One is adopting multi atom recognition strategy instead of one atom recognition to reduce the total number of iterations. The other is using QR decomposition method in inverse matrix operation in each iteration. In the following, the jth iteration is used as an example to analyse the FDSMOMP algorithm.

Suppose in the $(j-1)$th iteration, the corresponding residuals of the frequency point $f_p$ is $R(f_p)^{(j-1)}$. In the jth iteration, DCS-SOMP algorithm only selects one new support set by solving the following equation:

$$P_m = \arg \max_{i \in \Omega} \sum_{p=1}^{P} |G(f_p)|_2$$

where $P_m$ is the position of the new selected support set, $G(f_p)^{(j)} = \Theta(f_p)^{(j)}R(f_p)^{(j)-1}$ and $\Omega = \{1, 2, \ldots, V\}$. If the preset degree of sparsity is $k_s$ then the DCS-SOMP algorithm needs at least $k_s$ iterations to get the final reconstruction results.

In order to reduce the total iteration numbers, FDSMOMP algorithm selects $s (> 1)$ atoms at each iteration. $\beta_i$ is defined as

$$\beta_i = \arg \max_{i \in J} \sum_{p=1}^{P} |G(f_p)|_2$$

In this way, we have $P_m = \beta_i \beta_j \ldots \beta_i$, which denotes the position of $s$ atoms, $J$ denotes the remaining atom support set after the last iteration, $\Lambda$ is the difference set operation. It can be found that the number of final iterations can be reduced to $[k_s/s]$ by using the strategy of selecting $s$ atoms instead of one atom each iteration.

After the new atom group is selected, the projection calculation would be carried out. Denote the support set as $J' = J \times s$, which is obtained after the jth iteration. The purpose of the projection calculation is to solve $X_{\Lambda'}$ in (6). As $\Theta(f_p)_{\Lambda'}$ are different for each frequency point $f_p$, so $X_{\Lambda'}$ are also different for each $f_p$ which means that each $X_{\Lambda'}$ for each $f_p$ should be solved independently. The reconstruction result corresponding $f_p$ is

$$x(f_p)_{\Lambda'} = (\Theta(f_p)^{H}_{\Lambda'}\Theta(f_p))^{-1}\Theta(f_p)^{H}_{\Lambda'}y(f_p)$$

QR decomposition is used to improve efficiency, we get

$$x(f_p)_{\Lambda'} = (R^{H}R)^{-1}R^{H}\Theta^{H}y(f_p)$$

From (9) and (10), we can find that the QR decomposition would reduce the amount of computation burden further. The detailed analysis of the reduction amount of operations by QR decomposition can be found in [11]. The flowchart of FDSMOMP algorithm is shown in Fig. 2.

3 Computational complexity analysis

The most time-consuming part of the FDSMOMP algorithm is the calculation of (8) and (9). Assuming the total iteration number of

$$\tilde{X} = \arg \min \|X \|_{1, 2}.$$ $\forall p \in \{1, 2, \ldots, P\}$

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the FDSMOMP algorithm is $L$. The main computation burden of (8) is the inner product of the measurement matrix with the residual matrix, the computation cost in one fast frequency point is $O(LN_aMN)$. So the computation burden of $P$ fast frequency points is $O(LP_aMN)$. The main computation burden of (9) comes from the solution of the least squares. If QR decomposition is used, and the Modified Gram-Schmidt (MGS) is adopted for inverse matrix operation, then the computation cost is $O(\text{INM})$ in $l$th iteration. For the $P$ fast frequency points and $L$ times of iteration, the computation cost is $O(LPMN)$. The total computation cost of the proposed algorithm is

$$C_{\text{FDSMOMP}} = O(LN_aMN + L^2PMN)$$  \hspace{1cm} (11)

Comparatively, if the DCS-SOMP algorithm is used, and the algorithm is finished after $L_1$ iterations, the total computation cost is

$$C_{\text{DCS-SOMP}} = O(L_1N_aMN + L_1^2PMN)$$  \hspace{1cm} (12)

Due to the fact that $L < L_1$, the computational complexity of FDSMOMP algorithm is greatly reduced compared with DCS-SOMP algorithm.

4 Simulation and analysis

All simulation experiments are implemented in MATLAB R2008b, the main parameters of the computer are: Intel processor Corel E7500, Core 2.93 GHz, Internal memory 2 GB. The radar parameters used in the simulation are set as follows. The carrier frequency of LFM signal transmitted by radar is 10 GHz, the signal bandwidth is 1 GHz, and the azimuth accumulating time is 0.2 s. The PRF and corresponding number of pulses are given in every simulation. The scene consists of 30 cells in range direction and 30 cells in cross-range direction. The model of propeller shape component contains 27 strong scattering points as shown in Fig. 3. The backscattering intensity of scattering point is 0.4, 0.6, 0.8 and 1 from centre to edge, respectively. The spinning frequency of propeller is set as 7.5 Hz. Image entropy [8] is used to evaluate the quality of imaging results, and the simulation running time is used to evaluate the computation efficiency of different algorithms.

Simulation 1: Efficiency verification of FDSMOMP algorithm. The PRF is four times less than the un-ambitious frequency. The echo of target contains 100 range cells which correspond to 100 fast frequency points. The real sparsity of the signal is 27, and the redundant sparsity is set from 30 to 100 with step length of 10. The number of Monte Carlo is 100. Fig. 4 gives the operation efficiency of FDSMOMP algorithm when $s$ is set as 2, 4, and 8, respectively. For comparison, DCS-SOMP is also employed in this experiment.

Fig. 4 indicates that the larger the redundant sparsity is, the longer the reconstruction time. For the FDSMOMP algorithm, the larger the size of atom group selection (from 2 to 8) is, the shorter the reconstruction time. Compared with DCS-SOMP algorithm using the same redundant sparsity, FDSMOMP is much more efficient owing to reduction of the total iteration numbers and computation burden in each iteration. Simulation results verify the efficiency of the proposed algorithm.

Simulation 2: Reconstruction ability simulation of FDSMOMP algorithm under low SNR conditions. In order to verify the reconstruction ability under low SNR conditions, noise is added before the pulse compression processing. SNR is increased from −20 to 0 dB with the step length of 4 dB. The setting of other experimental conditions is the same as that of the previous experiment. Fig. 5 shows the imaging results obtained by FDSMOMP algorithm and DCS-SOMP algorithm when the SNR is −16, −8, and 0 dB, respectively. The image entropy curve and operation time curve are shown in Fig. 6.

From Figs. 5 and 6, it can be found that the imaging quality obtained by the two algorithms only has a slightly difference under the same SNR, but the imaging time of the FDSMOMP method is much shorter than that of DCS-SOMP. The simulation results show that the proposed method has the fast imaging capability under low SNR.

5 Conclusion

In order to improve the imaging efficiency of rotating parts, the FDSMOMP algorithm is proposed based on distributed matching sparse representation model and DCS theory. Both theoretical analysis and simulation results verify the feasibility and effectiveness of the proposed method. In the future, the effectiveness of the proposed method will be verified by using the real data.

6 Acknowledgments

This work is supported in part by the National Natural Science Foundation of China under grant no. 61671469.
7 References

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