Proximity effect, quasiparticle transport, and local magnetic moment in ferromagnet-\textit{d}-wave superconductor junctions

Jian-Xin Zhu\textsuperscript{1} and C. S. Ting\textsuperscript{1,2}

\textsuperscript{1}Texas Center for Superconductivity and Department of Physics, University of Houston, Houston, Texas 77204
\textsuperscript{2}National Center for Theoretical Sciences, P.O.Box 2-131, Hsinchu, Taiwan 300, R.O. China

The proximity effect, quasiparticle transport, and local magnetic moment in ferromagnet–\textit{d}-wave superconductor junctions with \{110\}-oriented interface are studied by solving self-consistently the Bogoliubov-de Gennes equations within an extended Hubbard model. It is found that the proximity induced order parameter oscillates in the ferromagnetic region. The modulation period is shortened with the increased exchange field while the oscillation amplitude is depressed by the interfacial scattering. With the determined superconducting energy gap, a transfer matrix method is proposed to compute the subgap conductance within a scattering approach. Many novel features including the zero-bias conductance dip and splitting are exhibited with appropriate values of the exchange field and interfacial scattering strength. The conductance spectrum can be influenced seriously by the spin-flip interfacial scattering. In addition, a sizable local magnetic moment near the \{110\}-oriented surface of the \textit{d}-wave superconductor is discussed.

PACS numbers: 74.20.Mn, 74.80.Fp, 74.50.+r

I. INTRODUCTION

The electronic transport in ferromagnetic-superconducting hybrid structures is currently a very active area of research due to their interesting physical properties and potential device applications. A fundamental transport process through the interface between the normal conducting and superconducting materials is the Andreev reflection (AR). An electron incident with energy below the superconducting energy gap cannot enter the superconductor, it is instead reflected at the interface as a hole by transferring a Cooper pair into the superconductor. The earlier spin polarization experiments involving ferromagnet and superconductor were performed on tunnel junctions where the AR is unimportant due to the strong interface barrier. Recently, the effect of spin polarization on the AR has been investigated in ferromagnet–conventional superconductor contacts experimentally where the AR plays an important role. In an earlier theoretical work, this effect was studied in the zero-bias limit. Several recent spin injection experiments have been done with \textit{d}-wave superconductors. Common to both ferromagnet–conventional high-\textit{T}_c superconductors, the subgap conductance at a given bias is suppressed due to the suppression of AR by the spin splitting of energy bands in the ferromagnet. In particular, a zero-bias conductance dip was observed in the ferromagnet–high-\textit{T}_c superconductor contacts. It has been widely accepted that the high-\textit{T}_c superconductors have a \textit{d}-wave pairing symmetry. The above interesting observation may indicate the importance to take into account the unconventional pairing symmetry of the cuprate superconductors. It has been shown that, due to the formation of midgap states at the \{110\}-oriented interface the conductance spectrum of \textit{d}-wave superconductor junctions differs dramatically from that of conventional \textit{s}-wave superconductor junctions. Thus the difference should also exist between ferromagnet–\textit{d}-wave and \textit{s}-wave superconductor junctions. In recent theoretical works, the novel features of AR have been exhibited in the subgap conductance of ferromagnet–\textit{d}-wave superconductor junctions. More recently, the effect of spin injection into \textit{s}- and \textit{d}-wave superconductors has also been studied with an emphasis on the interplay between boundary and bulk spin transport processes.

In parallel, there also has been much interest in the interplay of superconductivity and ferromagnetism in these combined structures. In the case of ferromagnet–superconductor multilayers, the transition temperature of the \textit{(s-wave)} superconductor changes nonmonotonically with the thickness or the exchange field strength of the ferromagnetic layers. In superconductor–ferromagnet–superconductor junctions, the exchange field in the ferromagnetic layer leads to oscillations of the Josephson critical current. More recently, the influence of the exchange field on the Josephson current in superconductor–ferromagnet–superconductor junctions with unconventional pairing symmetry has also been studied. In ferromagnet–superconductor–ferromagnet multilayers, the appearance of the superconducting energy gap causes a reduction of the indirect magnetic coupling which exists in the normal state. For the case of ferromagnet–superconductor junctions, the superconducting proximity effect will also change in the presence of an exchange field. The previous works with an emphasis on the transport through the ferromagnetic–superconductor junctions were based on a simplified continuum model and did not calculate the order parameter (i.e., pairing amplitude) self-consistently so that the proximity effect cannot be included.

The purpose of this work is to present a unified and rigorous treatment of the proximity effect, transport and magnetic properties in a ferromagnet–\textit{d}-wave superconductor junctions. Within the framework of an ex-
tended Hubbard model, we solve self-consistently the 
Bogoliubov-de Gennes (BdG) equations to obtain the 
spatial variation of the order parameter and supercon- 
ducting energy gap. With the obtained energy gap, a 
transfer matrix method is then proposed to calculate the 
subgap conductance within the scattering approach. The 
self-consistent calculation also allows us to study the local 
magnetic moment in the superconducting region due to 
the presence of the exchange field or Zeeman coupling. 
The main procedure in the present work is parallel to 
the study of transport properties in the normal-metal– 
anisotropic superconductor junctions using the quasiclas- 
sical theory, where the pair potential first obtained from 
the quasiclassical formalism is substituted into the An-
dreev equation to calculate the reflection and transmis-
sion coefficients.

The paper is organized as follows: In Sec. II, the BdG 
equations for the ferromagnet–superconductor junctions 
are derived within the extended Hubbard model. In 
Sec. III, the order parameter and pair potential are de-
termined self-consistently. The subgap differential 
current and the local magnetic moment are presented in 
Secs. IV and V, respectively. Finally, conclusions are 
given in Sec. VI.

II. THE BOGOLIUBOV-DE GENNES 
EQUATIONS FOR THE 
FERROMAGNET–SUPERCONDUCTOR 
JUNCTIONS

We use a single-band extended Hubbard model to de-
scribe the ferromagnet–superconductor junctions. The 
geometry is shown in Fig. 1 for (a) a ferromagnet–s-
wave superconductor junction with a \{100\}-oriented 
interface and (b) a ferromagnet–\(d_{x^2-y^2}\)-wave supercon-
ductor junction with a \{110\}-oriented interface. The signifi-
cant difference between s-wave and d-wave superconduc-
tors can be exhibited most clearly in these two struc-
tures. For such a crystalline orientation of the \(d_{x^2-y^2}\)-wave superconductor, a \(d_{xy}\)-wave superconductor 
junction is formed. For the junction involving the s-wave 
superconductor, the qualitative features are independent 
of the interface orientation. Hereafter, we will call the 
ferromagnet–s-wave superconductor junction with the 
\{100\}-oriented interface the FS junction while the 
ferromagnet–\(d_{x^2-y^2}\)-wave superconductor junction with 
the \{110\}-oriented interface the FD\(_{xy}\) junction. In the 
interface geometry, both the ferromagnet and the super-
conductor are treated as semi-infinite. Here we choose 
the interface to be at the 0-th layer. We further assume 
that the transition temperature of the superconductor is 
much smaller than the Curie temperature of the ferro-
magnet so that fluctuation effects on the magnetism are 
negligible.

Under these assumptions, the Hamiltonian defined on 
two-dimensional square lattice is given by

\[
\mathcal{H} = -t \sum_{{\langle ij \rangle \sigma}} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{{i\sigma}} U_i n_{i\sigma} + \sum_{{i\sigma}} h_{i\sigma} n_{i\sigma} - \mu \sum_{{i\sigma}} n_{i\sigma} - \sum_{{i}} V_0(i) n_{i\uparrow} n_{i\downarrow} - \frac{1}{2} \sum_{{\langle ij \rangle \sigma\sigma'}} V_1(i) n_{i\sigma} n_{j\sigma'} . \tag{2.1}
\]

Here \(i\) and \(j\) are site indices and the angle bracket im-
plies that the hopping and interactions are only consid-
ered up to nearest-neighbor sites, \(c_{i\sigma}^\dagger \) (\(c_{i\sigma}\)) are creation (annihilation) operators of an electron with spin \(\sigma\) on 
site \(i\), \(n_{i\sigma} = c_{i\sigma}^\dagger c_{i\sigma}\) is the electron number operator on 
site \(i\), \(t\) the hopping integral, and \(\mu\) the chemical poten-
tial. The interfacial scattering potential is modeled by 
\(U_i = U_0 \delta_{n0}\), where \(n\) is the layer index along the direc-
tion perpendicular to the interface plane. The conduction 
electrons in the ferromagnet interact with an exchange 
field, \(h_{i\sigma} = -h_0 \sigma_i \Theta(-n)\), where \(\Theta\) is the Hasevise step 
function and \(\sigma_i (= \pm 1)\) is the eigenvalue of the \(z\) 
component of the Pauli matrix. A real space representa-
tion of the exchange interaction is used since the present 
system is inhomogeneous. The quantities \(V_0(i)\) and \(V_1(i)\) 
are on-site and nearest-neighbor interaction strength, re-
spectively. They are taken to be \(V_0\) and \(V_1\) in the super-
conductor and identically zero in the ferromagnet. Pos-
tive values of \(V_0\) and \(V_1\) mean attractive interactions 
and negative values mean repulsive interactions. When 
\(V_0 < 0\) and \(V_1 > 0\), the d-wave pairing state is favorable. 
Here we also would like to point out that by taking the 
same chemical potential in both the ferromagnet and the 
superconductor, we have ignored the effect of the Fermi 
wavevector mismatch between two materials. Very 
recently, this effect on the conductance spectrum has 
been well studied within the simple continuum model \[14\].

Within the mean-field approximation, the effective 
Hamiltonian Eq. (2.2) can be written as

\[
\mathcal{H}_{\text{ef}} = -t \sum_{{\langle ij \rangle \sigma}} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{{i\sigma}} U_i n_{i\sigma} + \sum_{{i\sigma}} h_{i\sigma} n_{i\sigma} - \mu \sum_{{i\sigma}} n_{i\sigma} + \sum_{{i}} [\Delta_0(i) c_{i\uparrow}^\dagger c_{i\downarrow} + \Delta_0(i) c_{i\downarrow}^\dagger c_{i\uparrow} + \Delta_0(i) c_{i\uparrow}^\dagger c_{i\downarrow}^\dagger + \Delta_0(i) c_{i\downarrow}^\dagger c_{i\uparrow}^\dagger] \tag{2.2}
\]

\[
\Delta_0(i) = V_0(i) \langle c_{i\uparrow}^\dagger c_{i\downarrow} \rangle , \tag{2.3}
\]

\[
\Delta_0(i) = V_1(i, i + \delta) \langle c_{i\uparrow}^\dagger c_{i+\delta\downarrow} \rangle , \tag{2.4}
\]

are the on-site and nearest-neighbor pair potentials, re-
spectively. The effective Hamiltonian Eq. (2.2) can be 
diagonalized as \(\mathcal{H}_{\text{ef}} = E_g + \sum_{\nu \nu'} E_{\nu \nu'} \gamma_{\nu \nu'}^\dagger \gamma_{\nu \nu'}\) by performing 
the Bogoliubov transformation,

\[
c_{i\sigma} = \sum_{\nu} [u_{i\sigma \nu}^\nu \gamma_{\nu}^\dagger - \sigma v_{i\sigma \nu}^\nu \gamma_{\nu}^\dagger] . \tag{2.5}
\]
Here $\gamma_0^\dagger$ ($\gamma_0$) are creation (annihilation) operators of fermionic quasiparticles. $E_{\nu}$ are the quasiparticle eigenvalues. The quasiparticle wavefunction amplitudes $(u_{\nu \sigma}, v_{\nu \sigma})$ satisfy the lattice BdG equations:

\[
\sum_i \left( H_{ij} + h_{ij} \delta_{ij} \right) \left( u_{\nu i}^{j \dagger} \right) = E_{\nu} \left( u_{\nu i}^{j} \right),
\]

\[
\sum_i \left( H_{ij} - h_{ij} \delta_{ij} \right) \left( u_{\nu i}^{j \dagger} \right) = E_{\nu} \left( v_{\nu i}^{j} \right),
\]

where

\[
H_{ij} = -t \delta_{i+\delta j} + (U_i - \mu) \delta_{ij},
\]

\[
\Delta_{ij} = \Delta_0(1) \delta_{ij} + \Delta_1 \delta_{i+\delta j},
\]

with $\delta = \pm \hat{x}_a, \pm \hat{x}_b$ the unit vectors along the crystalline $x_a$ and $x_b$ axis. The energy gaps for on-site and nearest-neighbor pairing are determined self-consistently

\[
\Delta_0(0) = V_0(0) F_0(0) = \frac{V_0(0)}{2} \sum_{\nu} \left[ u_{\nu i}^{j \dagger} u_{\nu i}^{j} + u_{\nu i}^{j} v_{\nu i}^{j} \right] \tanh(E_{\nu}/2T),
\]

\[
\Delta_1(0) = V_1(0) F_1(0) = \frac{V_1(0)}{2} \sum_{\nu} \left[ u_{\nu i}^{j \dagger} u_{\nu i+\delta}^{j} + u_{\nu i+\delta}^{j} v_{\nu i}^{j} \right] \tanh(E_{\nu}/2T),
\]

where the Boltzmann constant $k_B = 1$ has been taken, and $F_0(0)$ and $F_1(0)$ are the on-site and nearest-neighbor bond order parameter. Note that the $4 \times 4$ BdG equations are decoupled into two sets of $2 \times 2$ equations since the spin-flip effect is not considered in Eq. (2.1). Note that the eigenstates of the BdG equations exist in pairs: If $(u_{\nu}, v_{\nu})$ is the solution of Eq. (2.6a) with the eigenvalue $E$, then $(-v_{\nu}^*, u_{\nu}^*)$ is the solution of Eq. (2.6b) with the eigenvalue $-E$.

For a clean ferromagnet-superconductor junction with a flat interface, which we are considering, there exists the translation symmetry along the $y$ direction so that the Bloch theorem can be applied to this direction. Then for the $F_D_{xy}$ junction, the eigenfunction can be written in the form

\[
\left( \begin{array}{c} u_{n,m} \\ v_{n,m} \end{array} \right) = \frac{1}{\sqrt{N_y}} \left( \begin{array}{c} u_n(k_y) \\ v_n(k_y) \end{array} \right) e^{imk_ya/\sqrt{2}},
\]

where $a$ is the lattice constant, $k_y \in [-\pi/\sqrt{2}a, \pi/\sqrt{2}a]$, $m$ and $n$ are ionic-layer indices in the $x$ and $y$ directions, and $N_y$ is the number of unit cells along the $y$ direction. The problem becomes solving the BdG equations for $(u_n(k_y), v_n(k_y))$ corresponding to the eigenvalue $E(k_y)$:

\[
\sum_{\nu} \left( H_{nn'} + h_{nn'} \delta_{nn'} \right) \left( u_{\nu n}^{n'} \right) = E_{\nu} \left( u_{\nu n}^{n'} \right),
\]

\[
\sum_{\nu} \left( H_{nn'} - h_{nn'} \delta_{nn'} \right) \left( u_{\nu n}^{n'} \right) = E_{\nu} \left( v_{\nu n}^{n'} \right),
\]

where

\[
H_{nn'} = -2t \cos(k_ya/\sqrt{2}) \delta_{n+1,n'} + (U_n - \mu) \delta_{nn'},
\]

\[
\Delta_{nn'} = \Delta_0(n) \delta_{nn'} + \left[ \Delta_a(n, n \pm 1)e^{\pm ik_ya/\sqrt{2}} + \Delta_b(n, n \pm 1)e^{\pm ik_ya/\sqrt{2}} \right] \delta_{n+1,n'},
\]

with the gap functions given by

\[
\Delta_0(n) = \frac{V_0(n)}{2N_y} \sum_{\nu, k_y} \left[ u_{n_1\nu}^{\nu \dagger} v_{n_1\nu}^{\nu} + u_{n_1\nu}^{\nu} v_{n_1\nu}^{\nu \dagger} \right] \tanh(E_{\nu}(k_y)/2T),
\]

\[
\Delta_a(n, n \pm 1) = \frac{V_1(n, n \pm 1)}{2N_y} \sum_{\nu, k_y} \left[ v_{n\nu}^{\nu \dagger} e^{\pm ik_ya/\sqrt{2}} + v_{n\nu}^{\nu} e^{\pm ik_ya/\sqrt{2}} \right] \tanh(E_{\nu}(k_y)/2T),
\]

\[
\Delta_b(n, n \pm 1) = \frac{V_2(n, n \pm 1)}{2N_y} \sum_{\nu, k_y} \left[ u_{n\nu}^{\nu \dagger} e^{\pm ik_ya/\sqrt{2}} + u_{n\nu}^{\nu} e^{\pm ik_ya/\sqrt{2}} \right] \tanh(E_{\nu}(k_y)/2T).
\]

The problem with other orientations of the flat interface can be treated similarly.

### III. Self Determination of the Order Parameter and the Pair Potentials

We solve the BdG equations self-consistently by starting with an initial gap function. After exactly diagonalizing Eq. (2.6), the obtained Bogoliubov amplitudes are substituted into Eqs. (2.13) and (2.14) to compute a new gap function. We then use it as input to repeat the
above process until the relative error in the gap function between successive iterations is less than the desired accuracy. Throughout this work, we concentrate on the zero temperature case unless specified explicitly, and take the parameters: \( \mu = 0 \) and \( V_1 = 2t \) and \( V_0 = -2t \) for d-wave superconductors while \( V_1 = 0 \) and \( V_0 = 2t \) for s-wave superconductors. This set of parameter values give the zero-temperature energy gap \( \Delta_{20} = 0.241t \) and \( \Delta_{00} = 0.376t \) for the bulk d-wave and s-wave superconductors, respectively. The corresponding coherence lengths \( \xi_d \approx 1.3a \) and \( \xi_s \approx 3.4a \). Note that as in other works, the model parameters chosen here are not intended for realistic materials.

For the d-wave superconductor, the amplitudes of d- and extended s-wave order parameters can be defined in terms of the bond order parameters [9]

\[
F_d(i) = \frac{1}{4} [F_{\hat{x}_d}(i) + F_{\hat{z}_d}(i) - F_{\hat{\gamma}_d}(i) - F_{\hat{\gamma}'_d}(i)], \quad (3.1a) \\
F_s(i) = \frac{1}{4} [F_{\hat{x}_s}(i) + F_{\hat{z}_s}(i) + F_{\hat{\gamma}_s}(i) + F_{\hat{\gamma}'_s}(i)]. \quad (3.1b)
\]

Accordingly, the d-wave and extended s-wave pair potentials are given by:

\[
\Delta_d(i) = \frac{1}{4} [\Delta_{\hat{x}_d}(i) + \Delta_{\hat{z}_d}(i) - \Delta_{\hat{\gamma}_d}(i) - \Delta_{\hat{\gamma}'_d}(i)], \quad (3.2a) \\
\Delta_s(i) = \frac{1}{4} [\Delta_{\hat{x}_s}(i) + \Delta_{\hat{z}_s}(i) + \Delta_{\hat{\gamma}_s}(i) + \Delta_{\hat{\gamma}'_s}(i)]. \quad (3.2b)
\]

In the superconducting region, the energy gap is proportional to the order parameter because of the constant pairing interaction. In the bulk state of d-wave superconductor, the extended s-wave component is zero. For the junction systems under consideration, the induced extended s-wave component near the interface is numerically found to be vanishingly small for the value of on-site repulsive interaction we have taken. For the conventional s-wave superconductor, the order parameter and the pair potential are directly on-site defined.

In Figures 2 and 3, we plot the spatial variation of the order parameter for various values of exchange field in the FS junction and the FD\(_{xy}\) junction. In this case, there is no interfacial scattering potential. As can be seen, the exchange field does not influence the order parameter in the superconducting region. In the normal metal case (\( h_0 = 0 \)), the proximity induced order parameter monotonically decays into the normal metal region. Interestingly, common to both the FS and FD\(_{xy}\) junctions, the order parameter in the ferromagnetic region no longer changes monotonically, it instead oscillates around the zero value of order parameter. In addition, as the exchange field is increased, the oscillation period becomes shorter. This behavior could be understood in the following way. Take the FS junction as an example. Since the component of the wavevector parallel to the interface is conserved, we can just consider the normal component. In the superconducting region, the wavevectors (momenta) of the spin-up and spin-down electrons forming the Cooper pairs have the same amplitude (but opposite directions) \( q_x \). Upon entering into the ferromagnetic region, the pair amplitude decays. Simultaneously, the spin-up electron lowers its energy by \( h_0 \), while the spin-down electron gain the energy \( h_0 \). In order for each electron to conserve its total energy, the spin-up electron should adjust its momentum to \( q_x \), while the spin-down electron to \( q_x \). Therefore, from the expressions of the order parameter given by (2.9), we can approximately write the order parameter as \( \cos((q_x - q_y)n)\Phi(n) \), where \( \Phi(n) \) is a slow-varying envelope function. Therefore, the exchange field in the ferromagnet causes the spatial modulation of the order parameter, which now roughly varies at the scale of \( (q_x - q_y)^{-1} \). In the continuum model, the difference \( q_x - q_y \approx 2h_0/hv_Fx \), where \( v_Fx \) is the normal component of the Fermi velocity. This also explains the decrease of the modulation period with the exchange field \( h_0 \). In Figs. 4 and 5, the spatial variation of order parameter are plotted for various values of the interfacial scattering potential in the FS and FD\(_{xy}\) junctions with the exchange field fixed at \( h_0 = 0.125D \) (\( D = 8t \) is the band width). Our numerical results show that as the interfacial scattering potential becomes stronger, the oscillation amplitude of the order parameter in the ferromagnet is decreased. This is because the amplitude of the slow-varying envelope function mentioned above has been suppressed by the interfacial scattering at the interface. However, in the superconducting region, the order parameters of the FS and FD\(_{xy}\) junctions show different behavior in the presence of the interfacial scattering. As shown in Figs. 4, for the FS junction, the depression of the order parameter near the interface is decreased by the interfacial scattering. As the interface is strongly reflecting (large \( U_0 \)), the superconductor and the ferromagnet are almost decoupled, and since the opaque interface itself is not pair breaking for s-wave superconductivity, the s-wave order parameter is not depressed. In contrast to the s-wave case, in the FD\(_{xy}\) junction, the reflected quasiparticles from the \{110\}-oriented interface are subject to a sign change of the order parameter, which makes the opaque interface itself pair-breaking. Thus the d-wave order parameter is strongly depressed (see Fig. 5). Finally, since we have assumed that there is no pairing interaction in the ferromagnet, the pair potential or energy gap in this region is zero. It is the pair potential that acts as an off-diagonal scattering potential in the BdG equations.

IV. THE SUBGAP DIFFERENTIAL CONDUCTANCE

A. The case of nonmagnetic interfacial scattering

Once the BdG equations (2.1) are solved self-consistently, we can use the obtained pair potential to calculate the differential conductance. The transport properties through the normal-metal–superconductor junc-
tions can be studied within the Blonder-Tinkham-Klapwijk (BTK) scattering formalism, which expresses the differential conductance in terms of the normal and Andreev reflection coefficients. In contrast to the tunneling Hamiltonian model, which requires an opaque barrier at the interface, the BTK theory can consider the case of an arbitrary barrier strength. Also noticeably, the BTK formalism can be regarded as the earliest version of the Landauer-Büttiker formula applied to the coherent transport through a normal-metal–superconductor structure. Recently, the BTK theory has been extended to the spin-dependent transport through ferromagnet–superconductor junctions. Within the tight-binding model, the averaged differential conductance can be obtained as:

\[
G = \frac{e^2}{hN_y} \sum_{k_x, \sigma} \left[ \frac{1 + R_{h, \sigma \sigma} - R_{e, \sigma \sigma}}{1 + R_{h, \sigma \sigma} + R_{e, \sigma \sigma}} \right],
\]  
(4.1)

which shows clearly that an incoming electron of spin \( \sigma \) (\( \uparrow, \downarrow \)) is normally reflected as an electron of the same spin \( \sigma \) with probability \( R_{e, \sigma \sigma} = |r_{\sigma\sigma}|^2 \), and Andreev reflected as a hole of the opposite spin \( \sigma \) with probability \( R_{h, \sigma \sigma} = |\sin(q_0 a)/\sin(q_0 a)|^2 \). Here the summation is over all the transverse modes and over the spin indices. Without confusion, \( h \) in Eq. (4.1) is the Planck constant. In contrast to the continuum model, the factor \( \sin(q_0 a)/\sin(q_0 a) \) comes from the band structure effect. Our previous work within the continuum model concentrated on the direction-dependent subgap conductance through the ferromagnet–superconductor junctions, which can be experimentally explored with the scanning tunneling spectroscopy. For a point contact junction a summation over the transverse modes is needed.

The remaining thing is to obtain the Andreev and normal reflection coefficients, which can be calculated using the transfer matrix method. As an illustration, we give a detailed procedure for the calculation of these coefficients for the FS junction, which has a \( \{100\} \)-oriented interface. From the BdG equations (2.4), we can write the relation of wavefunctions among consecutive layers:

\[
\begin{pmatrix}
  u_{n+1} \\
  v_{n+1} \\
  u_n \\
  v_n
\end{pmatrix} = \hat{M}_n
\begin{pmatrix}
  u_n \\
  v_n \\
  u_{n-1} \\
  v_{n-1}
\end{pmatrix},
\]  
(4.2)

where the transfer matrix for \( n \)-th layer is given by

\[
\hat{M}_n = \begin{pmatrix}
  \bar{\epsilon}_n - \delta_n - \Delta_n - 1 & 0 \\
  -\Delta_n^* & \bar{\epsilon}_n + \delta_n + \Delta_n & 0 & -1 \\
  \delta_n & \bar{\epsilon}_n & 0 & -1 \\
  0 & 0 & 1 & 0
\end{pmatrix},
\]  
(4.3)

with \( \bar{\epsilon}_n = U_0 \delta_{n0} - 2t \cos k_y a - \mu \) and \( h_n = h_0 \Theta(-n) \).

The BCS coherence factors are given by

\[
\begin{align*}
  u_{\pm}^2 &= \frac{1}{2} \left[ 1 + \frac{\sqrt{E^2 - |\Delta(k_{\pm})|^2}}{E} \right], \\
v_{\pm}^2 &= \frac{1}{2} \left[ 1 - \frac{\sqrt{E^2 - |\Delta(k_{\pm})|^2}}{E} \right].
\end{align*}
\]  
(4.10a, 4.10b)

For clarity, we have written the energy gap explicitly depending on the wavevector \( \Delta(k_{\pm}) \equiv \Delta(\pm q_0, k_y) \)
with the \(x\)-component of the Fermi wavevector \(q_{_0}a = \cos^{-1}[-2t \cos k_y a + \mu]/2t\). For a conventional \(s\)-wave superconductor, \(\Delta(k_x) \equiv \Delta_{_0}\). Then the reflection amplitudes can be obtained by solving the linear equation \(4t\). The reflection coefficients for the \(FD\) \(xy\) junction can be calculated similarly, where \(q_{_0}a/\sqrt{2} = \cos^{-1}[-\mu/4t \cos(k_y a/\sqrt{2})]\) and \(\Delta(k_x) = -\Delta(k_-) = 4\Delta_{_0} \sin(q_{_0}a/\sqrt{2}) \sin(k_y a/\sqrt{2})\) so that the internal phase \(\phi_+ = 0\) (or \(\pi\)) while \(\phi_- = \pi\) (or 0) depending on \(\Delta(k_-)\) being positive or negative. Note that the reflection coefficients, which we are calculating here, can also be used to study the conductance spectrum for the spin current.

Before presenting the results for the conductance, we give a physical analysis of the effect of exchange field on the AR. In Fig. 5, we schematically draw the spin-split energy band in the ferromagnet and the AR process in the continuum model. As shown in the figure, the incident electrons and the Andreev reflected holes occupy different spin bands. Thus the AR is sensitive to the relative spin-dependent density of states at the Fermi energy \(E_F\). In the normal metal \((h_0 = 0)\), the energy band is spin degenerate, the AR is thus not suppressed. However, if the exchange field is sufficiently strong that there are no electrons occupying the spin-down band in the ferromagnet, the incident spin-up electrons have no spin-down electrons to drag in order to form Cooper pairs. As a consequence, the AR is completely depressed. For the numerical calculation, we take \(N_y = 625\). In Figs. 6 and 8, the subgap conductance spectrum \(G\) versus the scaled energy \(E\) is plotted for the FS junction and the \(FD\) \(xy\) junction with various values of \(h_0\) but without the interface scattering. As can be seen, for both the FS and \(FD\) \(xy\) junctions, the averaged conductance at a given energy \(E\) is suppressed due to the blocking of AR. However, because the effective energy gap for the \(d\)-wave pairing symmetry is momentum-dependent while that of \(s\)-wave pairing symmetry is a constant in the momentum space, the different conductance behaviors between the FS and \(FD\) \(xy\) junctions are exhibited. In the FS junction, before the subgap conductance is completely suppressed, a flat zero-bias maximum always shows up in the conductance spectrum. When the exchange field is sufficiently strong, the conductance is zero within the energy gap, and sharply increases to a finite value as the bias crosses the gap edge. This is because, outside the energy gap, the normal conduction process becomes important. In the \(FD\) \(xy\) junction, as the exchange field becomes strong, a zero-bias conductance maximum gives way to a zero-bias conductance dip. The strikingly similarity between the lowest curve in Fig. 6 and the experimental measurement performed on the \(La_{2/3}Ba_{1/3}MnO_3/DyBa_2Cu_3O_7\) junction demonstrates that the high degree of spin polarization in the doped lanthanum manganite compounds and the \(d\)-wave pairing symmetry of the \(Tc\) superconductors are essential to explain the observed conductance behavior. In addition, as is shown, the conductance spectrum in the ferromagnet–superconductor junction is symmetric to the zero bias, which has been observed in many experiments on spin polarized transport. A general proof of this property is given in the Appendix A.

In Figs. 9 and 10, we plot the conductance spectrum for a variety values of exchange field with the barrier strength fixed at \(U_0 = 0.2D\) in the FS junction and \(U_0 = 0.625D\) in the \(FD\) \(xy\) junction. In this case, the overall conductance spectrum is also reduced by the increase of \(h_0\). In the FS junction, a gap-like structure is exhibited in the conductance, and the peak at the gap edge is remarkably depressed by the exchange field, which is consistent with the recent experimental observations on ferromagnet–\(s\)-wave superconductor junctions where the degree of spin polarization is small and a small barrier scattering potential may still exist. In the \(FD\) \(xy\) junction, due to the existence of midgap states at the interface, a sharp zero-bias conductance peak (ZBCP) shows up. The amplitude of this conductance peak is strongly suppressed by the exchange field. Meanwhile, as the exchange field becomes much stronger, the highly suppressed ZBCP is split. Physically, the suppression of the \(d\)-wave order parameter near the interface allows the ferromagnetic effect to penetrate into the superconductor side through the tunneling of electrons, which leads to a small imbalance of the local occupation of electron with different spin direction so that a small magnetization at the \(d\)-wave superconductor side is induced. The small magnetization in turn causes the shift of the energy of the midgap states and the conductance peak is split. This splitting depends on the transparency of the interface. For a very strong barrier, the splitting is almost unobservable. The splitting of the ZBCP by the exchange interaction can also be realized by the application of a magnetic field. If an in-plane magnetic field \(B\) is applied parallel to the interface, the orbital coupling between electrons and the magnetic field can be neglected and only the Zeeman coupling \(\mp \mu_B B\) \((\mu_B = \) the Bohr magneton) is present. Unlike the ferromagnetic effect on the electronic structures in the superconducting region near the interface, which is essentially of dynamic origin, the Zeeman coupling is purely a local interaction. In this situation, the electron energy globally shifts to \( E \pm \mu_B B\) so that the energy of the midgap states shifts \(\mu_B B\). Consequently, as shown in Fig. 11, the ZBCP in the normal-metal–\(d_{x^2−y^2}\)-wave superconductor junction with \((110)\)-oriented interface (From now on we call it the ND \(xy\) junction) can be readily split. The range of splitting is just \(2\mu_B B\).

**B. The effects of spin-flip interfacial scattering**

In the preceding treatment, the spin-flip interfacial scattering effects are ignored. In case of the junctions with the ferromagnet involved, this type of scattering may be important. To study the effect, we introduce a new term into the Hamiltonian.
\[
\mathcal{H}_{sp} = \sum_{i, \sigma \neq \sigma'} U_{i, \sigma \sigma'}^{sp} c_i^{\dagger} c_{i'} ,
\]
where \( U_{i, \sigma \sigma'}^{sp} = U_i \delta_{hi} \delta_{2 \sigma - \sigma'} \) is assumed to be nonzero at the interface (layer index \( n = 0 \)). In the spin-space, the spin-flip scattering term is represented by
\[
\hat{U}^{sp} = \begin{pmatrix} 0 & U_1 \\ U_1 & 0 \end{pmatrix} .
\]

The spin-nonflip term has been represented by \( U_1 \) in the total Hamiltonian, which is the diagonal elements in the spin space. In the normal state junction with the spin-flip scattering, a beam of incident electrons with spin \( \sigma \) will be reflected as electrons with the same spin and opposite spin. In the junction made up of the superconductors, one can expect that, when a beam of electrons with spin \( \sigma \) incident from the normal metal or ferromagnet, the spin-flip scattering will also lead to the normally reflected electrons with the opposite spin \( \sigma \) and Andreev reflected holes with the same spin \( \sigma \), in addition to the reflected electrons with the same spin \( \sigma \) and holes with the opposite spin \( \sigma \). By working with the coupled \( 4 \times 4 \) BdG matrix equations, we can generalize the previous transfer matrix technique to obtain the normal reflection amplitudes, \( r_{e, \sigma \sigma}, r_{e, \sigma' \sigma} \), and the Andreev reflection amplitudes, \( h_{e, \sigma \sigma}, r_{h, \sigma \sigma} \). Correspondingly, the conductance is generalized to be
\[
G = \frac{e^2}{h N_y} \sum_{k_y, \sigma} \left[ 1 + R_{h, \sigma \sigma} + R_{e, \sigma \sigma} - R_{e, \sigma' \sigma} - R_{e, \sigma' \sigma} \right] ,
\]
where \( R_{h, \sigma \sigma} = |\sin(q_{h, \sigma})/\sin(q_{e, \sigma})||h_{e, \sigma \sigma}|^2 \), \( R_{e, \sigma \sigma} = |\sin(q_{h, \sigma})/\sin(q_{e, \sigma})||h_{e, \sigma \sigma}|^2 \), \( R_{e, \sigma' \sigma} = |r_{e, \sigma \sigma}|^2 \), and \( R_{h, \sigma' \sigma} = |r_{h, \sigma \sigma}|^2 \) with \( q \)’s the wave vectors associated with different types of electrons and holes. In Figure 12, we plot the conductance spectrum and the reflection coefficients for various values of the spin-flip interfacial scattering strength \( U_1 \) in a ND_{xy} junction with the spin-flip interfacial scattering strength \( U_0 = 0.625 D \). The transverse momentum is taken to be \( k_y = \pi/3\sqrt{2}a \). In the absence of the spin-flip scattering, i.e., \( U_1 = 0 \), the coefficients \( R_{e, \sigma \sigma} \) and \( R_{h, \sigma \sigma} \) vanish, and \( R_{e, \sigma' \sigma} \) decreases while \( R_{e, \sigma \sigma} \) increases monotonically with the bias within the effective energy gap \( |\Delta| = 2\Delta_{0} \). Especially, \( R_{h, \sigma \sigma} = 1 \) and \( R_{e, \sigma \sigma} = 0 \) at \( E = 0 \), which accounts for the appearance of the ZBCP. (see Fig. 12(a)) The effects of the spin-flip scattering on the conductance spectrum depends on its strength in detail. For a relative small value of the spin-flip scattering \( (U_1 = 0.125 D) \), \( R_{h, \sigma \sigma} \) and \( R_{h, \sigma \sigma} \) decreases monotonically with the bias. \( R_{e, \sigma \sigma} \) is finite and \( R_{e, \sigma \sigma} \) is depressed at \( E = 0 \), which leads to a suppressed ZBCP. (see Fig. 12(b)) As the spin-flip scattering strength is further increased \( (U_1 = 0.25 D) \), \( R_{e, \sigma \sigma} \) varies nonmonotonically, it first increases to reach a maximum and then decreases with the bias. In addition, the zero-bias \( R_{e, \sigma \sigma} \) and \( R_{e, \sigma \sigma} \) are enhanced. Consequently, a flat conductance maximum at a finite bias shows up. \( \text{as Fig. 12(c)} \) As the spin-flip scattering strength is comparable to the spin-nonflip part, the complementary behavior in the variation between \( R_{e, \sigma \sigma} \) and \( R_{h, \sigma \sigma} \) and weak bias-dependence of \( R_{h, \sigma \sigma} \) (highly suppressed within the gap) and \( R_{h, \sigma \sigma} \) leads to an almost constant conductance spectrum. \( \text{as Fig. 12(d)} \) If the spin-flip scattering is much stronger than the spin-nonflip part \( (U_1 = 1D) \), the induction of a peak at finite bias in both \( R_{h, \sigma \sigma} \) and \( R_{h, \sigma \sigma} \) causes a finite-bias conductance peak. \( \text{as Fig. 12(e)} \) Whether such a extremely strong spin-flip scattering compared with the spin-nonflip scattering exits experimentally is unclear and we will not discuss this extreme limit further. As shown in Figure 13, the ZBCP can be completely depressed in the averaged conductance spectrum of a ND_{xy} junction \( (U_0 = 0.625 D) \) by the spin-flip scattering. Figure 14 plots the averaged conductance for various values of \( U_1 \) in a FD_{xy} junction with \( U_0 = 0.625 D \) and \( h_0 = 0.475 D \). Since the spin-flip interfacial scattering tends to spoil the pre-oriented spin direction of conduction electrons incident from the ferromagnet, it seriously influences the conductance spectrum. In particular, the splitting of the ZBCP induced by the exchange field is washed out in the presence of a strong spin-flip interfacial scattering so that the conductance spectrum becomes completely structureless. In addition, as shown in Figure 14, the suppression of the conductance at the region away from zero bias by the exchange field is reduced by a strong spin-flip interfacial scattering.

V. LOCAL MAGNETIC MOMENT

It has been predicted\(^3\) that besides the ZBCP in the quasiparticle tunneling, one of the other consequences of mid gap states is the possibility of a sizable magnetic moment at the \( \{110\} \) surface of the \( d_{x^2-y^2} \)-wave superconductor. Actually, the splitting of the ZBCP in the \( \text{FD}_{xy} \) junction with a strong exchange field or in the \( \text{ND}_{xy} \) junction with an in-plane magnetic field has supported this prediction. In a recent theoretical work\(^4\) a formal expression for the magnetic moment has been given, but a serious calculation of this quantity has not been done. In this section, we give a detailed analysis of the local magnetic moment (LMM).

The average electron density for each spin direction is given by
\[
\langle n_{\sigma} \rangle = \sum_{\nu} |u_{\nu \sigma}^r|^2 f(E_\nu) + |u_{\nu \sigma}^i|^2 [1 - f(E_\nu)] ,
\]
where \( f(E) = [1 + \exp(E/T)]^{-1} \) is the Fermi distribution function. The local magnetic moment can be defined as
\[
m = -\mu_B (n_{\uparrow} - n_{\downarrow}) .
\]
For the \( \text{FD}_{xy} \) junction at temperature \( T = 0.08 \Delta_{0} \), the LMM at the distance \( x = a/\sqrt{2} \) away from the interface in the superconducting region is found to be: When
$h_0 = 0.5D$, $m(x = a/\sqrt{2}) = 0.074\mu B$ for $U_0 = 0.625D$, and $0.008\mu B$ for $U_0 = 2.5D$. Corresponding to the splitting of the ZBCP in the FD$_{xy}$ junction, the exchange-field induced local magnetization at the surface of $d$-wave superconductor is sensitive to the interfacial barrier strength. In Fig. 13, we plot the magnetic-field dependence of the LMM at $x = a/\sqrt{2}$ for different temperatures. At low fields, the LMM varies linearly with the field. The slope increases with the decreased temperature. At higher fields (about six times of the temperature) so that the width of the midgap peak in density of states is surpassed, the LMM begins to saturate. In the presence of interfacial scattering, the ZBCP is independent of the barrier strength. In addition, we also show that the spin-flip interfacial scattering can seriously influence the quasiparticle transport properties. As one of the consequences of the midgap states, a sizable local magnetic moment in the FD$_{xy}$ junction or in the ND$_{xy}$ junction in the presence of the Zeeman coupling has been found. Inspired by the observation of the zero-bias conductance dip in the ballistic ferromagnet–high-$T_c$ superconductor junctions, we believe that the other interesting behaviors predicted in this paper are also experimentally accessible.

**VI. CONCLUSIONS**

In conclusion, we have presented a unified theory for the proximity effect, quasiparticle transport, and local magnetic moment in the FD$_{xy}$ junctions by solving the Bogoliubov-de Gennes equations within an extended Hubbard model. As a comparison, the calculations are also made for the FS junctions. The energy gap appearing in the BdG equations have been determined self-consistently by using exact diagonalization technique. It is found that the proximity induced order parameter oscillates in the ferromagnetic region but is almost unchanged in the superconducting region by the exchange field. The modulation period of the proximity induced order parameter is shortened by the exchange field but the oscillation amplitude is decreased with the interfacial scattering. Once the superconducting energy gap for various interfacial scattering potentials is determined self-consistently, a transfer matrix method has been proposed to calculate the the subgap conductance within a scattering approach. We find that the subgap conductance is suppressed by the spin splitting of the energy band in the ferromagnet. For a ballistic FD$_{xy}$ junction, a conductance dip is exhibited with strong exchange fields. In the presence of interfacial scattering, the ZBCP is split by the strong exchange field. The degree of this splitting depends on the barrier strength. In contrast, the ZBCP can be split very easily by an in-plane magnetic field due to the local nature of the Zeeman coupling and the range of splitting is independent of the barrier strength. This interesting behavior directly reflects the existence of zero-energy peak at $x = (2n + 1)a/\sqrt{2}$ with $n$ being non-negative integer, and it decays into the bulk of the superconductor. This feature is special to the lattice model.

**ACKNOWLEDGMENTS**

This work was supported by the Texas Center for Superconductivity at the University of Houston, by the Robert A. Welch Foundation, and by the grant NSF-INT-9724809.

**APPENDIX A: SYMMETRY PROPERTY OF THE CONDUCTANCE SPECTRUM**

The eigenstates of the BdG equations given by Eq. (2.6) exist in pairs: Each eigenstate from Eq. (2.6a) is related to a counterpart from Eq. (2.6b) as:

$$
\begin{pmatrix}
    u_L \\
    v_L
\end{pmatrix}
- E =
\begin{pmatrix}
    -v_L^* \\
    u_L^*
\end{pmatrix}
E.
$$

This mirror image property makes the conductance spectrum is symmetric about the zero of bias. Suppose we have an incident, and outgoing waves as solutions to Eq. (2.6a) with energy $E$

$$
\begin{pmatrix}
    a_{in}e^{iq_{\uparrow}x} \\
    b_{in}e^{-iq_{\downarrow}x}
\end{pmatrix}
$$

and

$$
\begin{pmatrix}
    a_{out}e^{-iq_{\uparrow}x} \\
    b_{out}e^{iq_{\downarrow}x}
\end{pmatrix}.
$$

The amplitudes of the incoming and outgoing waves are then connected by the scattering matrix:

$$
\begin{pmatrix}
    r_{e\uparrow,e\uparrow} & r_{e\uparrow,h\downarrow} \\
    r_{h\downarrow,e\uparrow} & r_{h\downarrow,h\downarrow}
\end{pmatrix}
E
\begin{pmatrix}
    a_{in} \\
    b_{in}
\end{pmatrix} =
\begin{pmatrix}
    a_{out} \\
    b_{out}
\end{pmatrix}.
$$

From the mirror image property of the eigenfunctions, the incident and outgoing waves at energy $-E$ must be given by

$$
\begin{pmatrix}
    -b_{in}^*e^{iq_{\downarrow}x} \\
    a_{in}^*e^{-iq_{\uparrow}x}
\end{pmatrix}
$$

and

$$
\begin{pmatrix}
    -b_{out}^*e^{-iq_{\downarrow}x} \\
    a_{out}^*e^{iq_{\uparrow}x}
\end{pmatrix}.$$
These wavefunctions are related by the scattering matrix at energy $-E$:

\[
\begin{pmatrix}
  r_{e\uparrow, e\uparrow} & r_{e\uparrow, h\uparrow} \\
  r_{h\uparrow, e\downarrow} & r_{h\uparrow, h\uparrow}
\end{pmatrix}
\begin{pmatrix}
  -b_{in}^\ast \\
  a_{in}^\ast
\end{pmatrix}
= \begin{pmatrix}
  -b_{out}^\ast \\
  a_{out}^\ast
\end{pmatrix} . \tag{A3}
\]

The above equation can be rewritten as:

\[
\begin{pmatrix}
  r_{h\uparrow, h\uparrow} & -r_{h\uparrow, e\downarrow} \\
  -r_{e\downarrow, h\uparrow} & r_{e\downarrow, e\downarrow}
\end{pmatrix}
\begin{pmatrix}
  a_{in}^\ast \\
  b_{in}^\ast
\end{pmatrix}
= \begin{pmatrix}
  a_{out}^\ast \\
  b_{out}^\ast
\end{pmatrix} . \tag{A4}
\]

Comparing Eq. A4 with Eq. A2, we find the relation:

\[
\begin{align*}
  r_{e\uparrow, e\uparrow}(E) &= [r_{h\uparrow, h\uparrow}(-E)]^\ast, \\
  r_{e\uparrow, h\uparrow}(E) &= -[r_{h\uparrow, e\downarrow}(-E)]^\ast, \\
  r_{h\downarrow, e\downarrow}(E) &= -[r_{e\downarrow, h\uparrow}(-E)]^\ast, \\
  r_{h\downarrow, h\uparrow}(E) &= [r_{e\downarrow, e\downarrow}(-E)]^\ast .
\end{align*} \tag{A5}
\]

The symmetry properties of the reflection amplitudes yields the identity:

\[
G_e(E) = G_e(-E) . \tag{A6}
\]

Therefore, we have extended the symmetry properties to the magnetic case.

1. A. F. Andreev, Zh. Eksp. Teor. Fiz. 46, 1823 (1964) [Sov. Phys.-JETP 19, 1228 (1964)].
2. For a recent review see, R. Meservey and P. M. Tedrow, Phys. Rep. 238, 173 (1994).
3. S. K. Upadhyay, A. Palanisami, R. N. Louie, and R. A. Buhrman, Phys. Rev. Lett. 81, 3247 (1998).
4. R. J. Soulen Jr., J. M. Byers, M. S. Ososky, B. Nadgorny, T. Ambrose, S. F. Cheng, P. R. Broussard, C. T. Tanaka, J. Nowak, J. S. Moodera, A. Barry, J. M. D. Coey, Science 282, 85 (1998).
5. M. J. M. de Jong and C. W. J. Beenakker, Phys. Rev. Lett. 74, 1657 (1995).
6. V. A. Vaks’ko, V. A. Larkin, P. A. Kraus, K. R. Nikolaev, D. E. Grupp, C. A. Nordman, and A. M. Goldman, Phys. Rev. Lett. 78, 1134 (1997).
7. Z. W. Dong, R. Ramesh, T. Venkatesan, M. Johnson, Z. Y. Chen, S. P. Pai, V. Talyansky, R. P. Sharma, R. Shreekala, C. J. Lobb, and R. L. Greene, Appl. Phys. Lett. 71, 1718 (1997).
8. V. A. Vaks’ko, K. R. Nikolaev, V. A. Larkin, P. A. Kraus, and A. M. Goldman, Appl. Phys. Lett. 73, 844 (1998).
9. N.-C. Yeh, R. P. Vasquez, C. C. Fu, A. V. Samoilov, Y. Li, and K. Vakili, preprint (1999); J. Y. T. Wei, N.-C. Yeh, C. C. Fu, and R. P. Vasquez, J. Appl. Phys. 85 (April 1, 1999).
10. D. J. van Harlingen, Rev. Mod. Phys. 67, 515 (1995).
11. C. R. Hu, Phys. Rev. Lett. 72, 1526 (1994).
12. Y. Tanaka and S. Kashiwaya, Phys. Rev. Lett. 74, 3451 (1995); S. Kashiwaya, Y. Tanaka, M. Koyanagi, H. Takashima, and K. Kajimura, Phys. Rev. B 51, 1350 (1995); S. Kashiwaya, Y. Tanaka, M. Koyanagi, and K. Kajimura, ibid. 53, 2667 (1996).
38 S. Datta, P. F. Bagwell, and M. P. Anantram, Phys. Low-Dim. Struct. 3, 1 (1996).

FIG. 1. Schematic geometry of the ferromagnet–s-wave superconductor junction with a {100}-oriented interface (a) and the ferromagnet–d-wave superconductor junction with a {110}-oriented interface defined on a two-dimensional lattice. The filled circles represent the ionic positions in the ferromagnet and the filled squares represent the ionic positions in the superconductor. The interface layer is represented by empty circles. x,y are crystalline axes. The profiles of s-wave and d-wave order parameters are also shown, respectively.

FIG. 2. The spatial variation of order parameter for various values of exchange field in the FS junction without the interfacial scattering potential. Here $D = 8t$ is the band width.

FIG. 3. The spatial variation of order parameter for various values of interfacial scattering potential in the FS junction with the exchange field $h_0 = 0.125D$.

FIG. 4. The spatial variation of order parameter for various values of interfacial scattering potential in the FS junction with the exchange field $h_0 = 0.125D$.

FIG. 5. The spatial variation of order parameter for various values of interfacial scattering potential in the FD$_{xy}$ junction with $h_0 = 0.125D$.

FIG. 6. A schematic drawing of (a) the spin-split energy band in the ferromagnet within the continuum model and (b) the Andreev reflection process at the interface between the ferromagnet and superconductor: A beam of spin-up electrons incident with angle $\theta_N$ and energy within the gap are normally reflected as spin-up electrons and Andreev reflected as spin-down holes. The Andreev reflection angle $\theta_A$ is related to $\theta_N$ by the conservation of the momentum component parallel to the interface. The thick solid line represents the interfacial scattering layer. Also shown are the d-wave order parameter profile and the angle $\alpha$ of the crystalline orientation with respect to the interface. $E_F$ is the Fermi energy.

FIG. 7. The differential conductance spectrum for various values of exchange field in the FS junction without the interfacial scattering potential.

FIG. 8. The differential conductance spectrum for various values of exchange field in the FD$_{xy}$ junction without the interfacial scattering potential.

FIG. 9. The differential conductance spectrum for various values of exchange field in the FS junction with $U_0 = 0.2D$.

FIG. 10. The differential conductance spectrum for various values of exchange field in the FD$_{xy}$ junction with $U_0 = 0.625D$.

FIG. 11. The differential conductance spectrum for various values of Zeeman coupling $\mu_B$ in the ND$_{xy}$ junction with $U_0 = 0.625D$.

FIG. 12. The transverse-momentum-dependent differential conductance spectrum and the reflection coefficients in a ND$_{xy}$ junction for various values of spin-flip scattering strength $U_1 = 0$ (a), $0.125D$ (b), $0.25D$ (c), $0.625D$ (d), and $1D$ (e). The spin-nonflip scattering strength $U_0 = 0.625D$. The transverse momentum $k_y = \pi/3\sqrt{2}a$. The effective energy gap for this momentum is $|\Delta_k| = 2\Delta_{0a}$.

FIG. 13. The averaged differential conductance spectrum in a ND$_{xy}$ junction for various values of spin-flip scattering strength. $U_0 = 0.625D$.

FIG. 14. The averaged differential conductance spectrum in a FD$_{xy}$ junction for various values of spin-flip scattering strength. $U_0 = 0.625D$ and $h_0 = 0.475D$.

FIG. 15. The magnetic-field dependence of the local magnetic moment at the distance $a/\sqrt{2}$ away from the interface in superconducting region of the ND$_{xy}$ junction at different temperatures. The interfacial scattering potential $U_0 = 2.5D$.

FIG. 16. The spatial variation of the local magnetic moment into the superconducting region of the ND$_{xy}$ junction at $T = 0.08\Delta_{0a}$ and $\mu_B B = 0.4\Delta_{0a}$. The interfacial scattering potential $U_0 = 2.5D$. 

10