Exact Event Rates of Lepton Flavor Violating Processes in Supersymmetric $SU(5)$ Model

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Abstract

Event rates of various lepton flavor violating processes in the minimal supersymmetric $SU(5)$ model are calculated, using exact formulas which include Yukawa vertices of lepton-slepton-Higgsino. We find subtlety in evaluating event rates due to partial cancellation between diagrams. This cancellation typically reduces the event rates significantly, and the size of the reduction strongly depends on superparticle mass spectrum.

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Lepton Flavor Violation (LFV) is an indication of physics beyond the standard model. Supersymmetric (SUSY) extension of the standard model, which has been strongly motivated as a solution of the gauge hierarchy problem, generally possesses LFV as generation mixing of slepton masses \[^1\]. In particular, interaction of superheavy particles may induce LFV in the slepton masses through renormalization effects, giving non-negligible event rates for LFV processes \[^3\]. Concrete realization of the mechanism has been considered in supersymmetric grand unified theories (GUTs) such as $SU(5)$ \[^2\] \[^3\] \[^4\] \[^4\] \[^5\] and flipped $SU(5)$ \[^7\], as well as in models with right-handed neutrinos \[^8\] \[^9\] \[^10\].

A remarkable observation has been made in Refs. \[^3\] \[^4\], which has pointed out that large top-quark Yukawa coupling yields large rates of LFV processes in SUSY GUTs only one or two orders of magnitudes below present experimental bounds. The large top Yukawa coupling gives a sizable flavor violation in slepton masses squared through the renormalization effects above the GUT scale. Importantly \[^3\] \[^4\], the amount of the off-diagonal elements in the slepton mass matrix has mild dependence on the value of the top Yukawa coupling even when it closes to its quasi-infrared fixed point.

However, this stable prediction of the LFV source in the slepton masses will not necessarily lead to a stable prediction of event rates. The purpose of this paper is to give exact event rates of $\mu \to e\gamma$, $\mu \to eee$, $\mu$-e conversion in nuclei, and $\tau \to \mu \gamma$ in the case of the minimal SUSY $SU(5)$ model, and show that this is the case. When calculating these event rates, we include Yukawa type vertices of lepton-slepton-Higgsino \[^10\], which was not taken into account in Ref. \[^4\]. As was already suggested in Ref. \[^10\], new diagrams coming from the Yukawa vertices interfere destructively to the other dominant diagrams, leading to reduction of the event rates. We find that this partial cancellation is subtle and generally has complicated dependence on superparticle mass spectrum. It takes place significantly under the universal scalar mass hypothesis because contribution from each diagram is comparable, and event rates we obtain are typically much smaller than the ones previously analyzed \[^2\] \[^4\]. Moreover, we will show that, given a LFV scalar mass matrix, variation of other parameters within a reasonable range will change the event rates by one order of magnitude or more. Without detailed knowledge on superparticle mass spectrum, it would be difficult to predict these LFV event rates.

First of all, let us introduce the model we consider. In the minimal SUSY $SU(5)$ model
both quarks and leptons are embedded in the $5^*$ and $10$ representations. The left-handed ($SU(2)_L$ doublet) leptons are in $5^*$’s, while the right-handed ($SU(2)_L$ singlet) ones are in $10$’s. The Yukawa interaction in this model is described by a superpotential

$$W = \frac{1}{4} f^{ij}_u \psi_i \psi_j H + \sqrt{2} f^{ij}_d \psi_i \Phi_j \overline{H}$$

with $f^{ij}_u$ and $f^{ij}_d$ being Yukawa couplings. Here supermultiplets $\Psi_i$ and $\Phi_i$ correspond to the $10$ and $5^*$ representations, respectively, and indices $i$ and $j$ ($i,j = 1,2,3$) stand for a generation. $H$ ($\overline{H}$) is a Higgs supermultiplet in $5$ ($5^*$) representation. In the decomposition of the standard model, $f_u$ becomes Yukawa coupling matrix of the up-type quark sector, whereas $f_d$ involves those of the down-type quark sector and of the lepton sector. The Yukawa couplings in Eq. (1) break leptonic number conservation as well as its quark counter part: one can diagonalize one of the Yukawa couplings matrices, $f_d$ for example, with respect to generation indices. When it is done, the other Yukawa couplings $f_u$ among the $10$’s cannot be diagonalized in general and thus yield flavor violation. In the model we are considering, we assume that this is the only source of LFV.

Above the GUT scale $M_G \approx 2 \times 10^{16}$ GeV, soft SUSY breaking terms for scalar fields are given by

$$- \mathcal{L}_{\text{SUSYbreaking}} = (m_{10}^2)^i_j \psi_i \psi_j + (m_5^2)^i_j \phi_i \phi_j + m_h^2 h^\dagger h + m_{\overline{H}}^2 \overline{H} \overline{H}$$

$$+ \left( \frac{1}{2} A_{u(i)}^j \psi_i \psi_j h + \sqrt{2} A_{d(i)}^j \psi_i \phi_j \overline{H} + \text{h.c.} \right),$$

where we have used lowercases $\psi_i$, $\phi_i$, $h$, and $\overline{H}$ to denote scalar components of the supermultiplets $\Psi_i$, $\Phi_i$, $H$, and $\overline{H}$, respectively. Soft masses for right-handed sleptons are identified with $m_{10}^2$ and those for left-handed sleptons are $m_5^2$. At the (reduced) Planck scale, $M \approx 2 \times 10^{18}$ GeV, it is assumed that the matrices $m_{10}^2$ and $m_5^2$ are diagonal and $A_u$ and $A_d$ are proportional to the Yukawa couplings $f_u$ and $f_d$, respectively, in order to avoid too large flavor changing neutral currents. When the energy scale goes down below the Planck scale, they will acquire flavor-violating components through renormalization effects which involve the flavor-violating Yukawa couplings. The magnitude of the LFV in the soft masses can be evaluated by using renormalization group equations above the GUT scale. At one-loop level, some of the equations are written as

$$16\pi^2 Q \frac{d}{dQ} (m_{10}^2)^j_i = -\frac{144}{5} g_5^2 M_5^2 \delta^j_i + \left\{ 2 f_d f_\dagger_d + 3 f_u f_\dagger_u, m_{10}^2 \right\}^j_i$$
the Kobayashi-Maskawa (KM) matrix at the GUT scale. In this basis, soft masses for the Yukawa matrix for leptons are diagonalized. Such a basis can be obtained by using Yukawa coupling. Below the GUT scale, it is more convenient to use the basis in which $A$ and $m$ in

\[ A \]

are diagonalized. Dropping the Yukawa couplings and $A$-parameters except for $f_u^{33} (\equiv f_t)$ and $A_u^{33}$ (i.e. those for top quark) in the renormalization group equations, one can solve them analytically. It turns out that the universality for the flavor indices no longer holds in $m_{10}^2$, $A_u$, and $A_d$, while it holds in $m_5^2$. In particular, the matrix $m_{10}^2$ at the GUT scale is generally written as

\[
\begin{pmatrix}
  m^2 & 0 & 0 \\
  0 & m^2 & 0 \\
  0 & 0 & m^2 - I
\end{pmatrix},
\]

In these equations, $g_5$ is the gauge coupling constant of the unified gauge group, $M_5$ is the corresponding gaugino mass and $Q$ stands for the renormalization point (energy scale).

Solutions of the one-loop renormalization group equations above the GUT scale were discussed already [4]. Following Ref. [4], let us take the basis where the Yukawa matrix $f_u$ is diagonalized. Dropping the Yukawa couplings and $A$-parameters except for $f_u^{33} (\equiv f_t)$ and $A_u^{33}$ (i.e. those for top quark) in the renormalization group equations, one can solve them analytically. It turns out that the universality for the flavor indices no longer holds in $m_{10}^2$, $A_u$, and $A_d$, while it holds in $m_5^2$. In particular, the matrix $m_{10}^2$ at the GUT scale is generally written as

\[
\begin{pmatrix}
  m^2 & 0 & 0 \\
  0 & m^2 & 0 \\
  0 & 0 & m^2 - I
\end{pmatrix},
\]

where $m^2$ is a generation blind contribution and $I$ stands for a contribution from the top Yukawa coupling. Below the GUT scale, it is more convenient to use the basis in which the Yukawa matrix for leptons are diagonalized. Such a basis can be obtained by using the Kobayashi-Maskawa (KM) matrix at the GUT scale. In this basis, soft masses for

*The renormalization effect by the bottom Yukawa coupling decreases flavor-diagonal, third generation, masses of the left- and right-handed sleptons, but does not significantly induce LFV masses. This could be easily understood in the basis that $f_d$ is diagonal. Though we will include the non-zero bottom Yukawa coupling in our numerical analysis, for the moment we ignore it for simplicity.
the right-handed sleptons exhibit LFV and their amounts are proportional to a bilinear of elements of the KM matrix \((V)\):

\[
(m_{l_R}^2)_{ij} = m_i^2 \delta_i^j - V_i^3 V_3^j I \quad (Q \simeq M_G).
\]

If universality of soft masses is imposed at the Planck scale, the quantity \(I\) is compactly expressed in the minimal SUSY SU(5) model as \([4]\)

\[
I = \rho_G [m_0^2 + \frac{1}{3}(1 - \rho_G)A_0^2 + 0.198(1 - \rho_G)A_0 M_{5G} \\
+ (0.224 - 0.029 \rho_G) M_{5G}^2].
\]

\(M_{5G}\) is a gaugino mass at the GUT scale. Precise meaning of an universal scalar mass \(m_0\) and an universal \(A\) parameter \(A_0\) at the Planck scale will be given below. In the above equation, \(\rho_G\) is defined as

\[
\rho_G = \left( \frac{f_t(M_G)}{f_t^{\text{max}}(M_G)} \right)^2
\]

where \(f_t^{\text{max}}(M_G)\), which equals 1.56 in the minimal model, is a maximal value of the top Yukawa coupling at the GUT scale under the assumption that the theory is perturbative below the Planck scale. Conversely if one starts with a large value of the top Yukawa coupling at the Planck scale, it converges to \(f_t^{\text{max}}(M_G)\) at the GUT scale and hence it exhibits a property as a (quasi) infrared fixed point. Thus \(\rho_G \leq 1\) and equality holds if the Yukawa coupling is on the fixed point. The structure of Eq. (10) is simple. It is proportional to \(\rho_G\) as a whole, and the quantity in the parenthesis is of the order of the SUSY breaking scale. Furthermore as the top Yukawa coupling gets close to the maximal value, \(I\) converges almost to a value \(m_0^2\). This simplicity is an important finding of Refs. \([3, 4]\).

For the reader’s convenience, we present in Fig. 1 dependence of the LFV source on the top quark mass. The horizontal line is the top pole mass \((m_t^{\text{pole}})\) divided by \(\sin \beta\), which is defined as \(\tan \beta = \langle h_2 \rangle / \langle h_1 \rangle\) with \(h_1, h_2\) being the Higgs boson that give masses to down-type quarks and charged leptons (up-type quarks). The vertical line represents \((m_{l_R})_e^2/m_{l_R}^2\) where \((m_{l_R})_e^2\) is a \((\mu, e)\) component of the right-handed slepton masses squared, and \(m_{l_R}\) is a physical right-handed selectron mass. These are evaluated at an electroweak scale, i.e., at the \(Z\)-boson mass. Here we choose \(M_{5G}\) and \(m_0\) such that
a Bino mass $M_1$ is 50GeV and $m_{\tilde{\tau}_R} = 300$ GeV, and $A_0 = 0$. The real line, dashed line, and dash-dotted line correspond to the case for $\tan \beta = 30, 10, 3$, respectively.

Complete formulas of the event rates for the LFV processes were given in Ref. [10] where we took a mass-eigenstate basis for the sleptons. Instead of repeating them, we would like to discuss the qualitative behavior of the LFV rates using an approximate mass-insertion formula. In the approximation, the mass-eigenstate basis for the leptons is taken, and LFV masses squared as well as left-right mixing slepton masses are treated as perturbation. (This approach gives us a good approximation when the generation mixing in the chirality conserving scalar masses dominates over that in the $A$ parameters. This is checked numerically in the model we are considering.) In this discussion, we concentrate on $\mu \to e\gamma$. The process is described by effective electromagnetic-dipole type matrix element:

$$T = e\epsilon^\alpha \bar{u} \gamma_\mu i\sigma_{\alpha\beta} q^\beta (A^L_2 P_L + A^R_2 P_R) u_\mu,$$

where $P_{R/L} = (1 \pm \gamma_5)/2$, $m_\mu$ is a muon mass, $e$ the electric charge, $q$ a photon momentum, and $\epsilon^\alpha$ is the photon polarization vector. From this equation, the event rate of $\mu \to e\gamma$ is given as

$$\Gamma(\mu \to e \gamma) = e^2 \frac{m_\mu^5}{16\pi}(|A^L_2|^2 + |A^R_2|^2).$$

In the model we are discussing, $A^R_2$ is almost zero since left-handed sleptons have only tiny LFV soft-breaking masses with the non-vanishing bottom Yukawa coupling. The diagrams contributing to this matrix element have to have chirality flip of lepton and, to be proportional to $SU(2)_L \times U(1)_Y$ breaking vacuum expectation values. Such diagrams in this model are shown in Figs. 2. Fig. 2(a), in which the chirality flips on an external line, gives the following contribution to $A^L_2$:

$$A^L_2 |_a = -\frac{1}{6} \frac{\alpha_Y}{4\pi} \frac{1}{m^2_{\tilde{\tau}_R}} \frac{(m^2_{\tilde{\tau}_R})^\mu_e}{(m^2_{\tilde{\tau}_R})^\tau_e},$$

where, $\overline{m}_{\tilde{\tau}_R}$ is an averaged right-handed charged slepton mass, $(m^2_{\tilde{\tau}_R})^\mu_e$ a $(\mu, e)$ component of the right-handed charged slepton mass squared matrix, and $\alpha_Y \equiv g_Y^2/4\pi$ with $g_Y$.

\footnote{In our case, diagrams with two mass insertions like $(m^2_{\tilde{\tau}_R})^\mu_e (m^2_{\tilde{\tau}_R})^\tau_e$ could give a non-negligible contribution. However, we do not incorporate such higher terms since they do not change our argument drastically. Especially they do not change the sign of each contribution discussed here and subsequently.}
being the $U(1)_Y$ gauge coupling constant. To make our points clearer, here we adopt an expansion with respect to $M_1^2/m_{\tilde{\ell}}^2$, though we will show the exact results in the following figures. This expansion is motivated by the fact that the right-handed slepton masses become larger than the Bino mass through the renormalization effects, unless sfermion masses are negative at the Planck scale. (For example, $m_{\tilde{\ell}} > 2M_1$ in the case of the minimal SUSY $SU(5)$ model.)

In Figs. 2(b), (c) and (d), the chirality flips on an internal line. Fig. 2(b) gives the following contribution,

$$A_{2|b}^L = \frac{1}{2} \frac{\alpha_Y}{4\pi} \frac{1}{m_{\tilde{\ell}}^2} \frac{(m_{\tilde{\ell}}^2)^\mu}{m_{\tilde{\ell}}^2} \frac{M_1 (A_\mu f_\mu + \mu \tan \beta)}{m_{\tilde{\ell}}^2}. \quad (14)$$

In this equation $m_{\tilde{\ell}}$ is an averaged left-handed charged slepton mass, $A_\mu$ a SUSY-breaking trilinear coupling proportional to muon Yukawa coupling constant $f_\mu$. It is notable that $A_{2|b}^L$ grows as $\tan \beta$, and can dominate over $A_{2|a}^L$ if $3M_1 \mu \tan \beta/m_{\tilde{\ell}}^2 \approx 1$. Remember that matrix element Eq. (11) is proportional to the Yukawa coupling of the lepton, and to one of $SU(2)_L \times U(1)_Y$ breaking vacuum expectation values, $\langle h_1 \rangle$ or $\langle h_2 \rangle$. Some diagrams hitting the vacuum expectation value of $h_2$ result in the enhancement if $\tan \beta$ is large [9, 10].

Once the Yukawa couplings of lepton-slepton-Higgsino are correctly included, Figs. 2(c) and (d) also contribute to $\mu \to e\gamma$ as

$$A_{2|c}^L = -\frac{\alpha_Y}{4\pi} \frac{1}{m_{\tilde{\ell}}^2} \frac{(m_{\tilde{\ell}}^2)^\mu}{m_{\tilde{\ell}}^2} \frac{M_1 \mu \tan \beta}{f_1(\mu^2/m_{\tilde{\ell}}^2)}, \quad (15)$$

$$A_{2|d}^L = \frac{\alpha_Y}{4\pi} \frac{1}{m_{\tilde{\ell}}^2} \frac{(m_{\tilde{\ell}}^2)^\mu}{m_{\tilde{\ell}}^2} \frac{\mu^2}{f_2(\mu^2/m_{\tilde{\ell}}^2)}, \quad (16)$$

with

$$f_1(x) = \frac{-8 - 11x + 4x^2 - x^3 + 2(2 + x) \log x}{2(1 - x)^4}, \quad (17)$$

$$f_2(x) = \frac{1 + 4x - 5x^2 + 2x(2 + x) \log x}{2(1 - x)^4}. \quad (18)$$

Functions $f_1(x)$ and $f_2(x)$ are monotonously decreasing functions of $x$ and positive-definite. As $x$ goes infinity, both functions vanish. Thus $A_{2|c}^L$ and $A_{2|d}^L$ are not negligible unless $|\mu|$ is much larger than $m_{\tilde{\ell}}$. As a result, $A_{2|b}^L$ and $A_{2|c}^L$ are important when
$M_1 \mu \tan \beta$ is large, otherwise $A_L^b |a$ and $A_L^d |d$ will dominate. An important point is that the relative signs between $A_L^b |b$ and $A_L^d |c$, and that between $A_L^d |a$ and $A_L^d |d$ are both opposite. This fact implies possible cancellation between the diagrams. We will soon show that this indeed occurs.

We are now at a position to present our result of numerical computation using the formulas in Ref. [10]. First, we consider the case where all scalars have a common soft mass at the reduced Planck scale (i.e., a universal scalar mass). The model is characterized by the following parameters at the scale $M$: a universal scalar mass $m_0$, a $SU(5)$ gaugino mass $M_5$, a universal trilinear scalar coupling $A_0$, a mixing mass parameter of the two Higgs scalars $B$, and a supersymmetric Higgsino mass parameter $\mu$. Then, the boundary conditions for the parameters in Eq. (2) are given by $(m^2_{i0})^i_j = (m^2_5)^j_i = m^2_0 \delta^i_j$, $m^2_h = m^2_{h_0}$, $A^i_u = f^i_u A_0$, and $A^i_d = f^i_d A_0$ at the Planck scale. Two of the five parameters (usually, $B$ and $|\mu|$) are fixed to obtain the desired values of the Z-boson mass and $\tan \beta$.

In this analysis, we fix the top Yukawa coupling at the Planck scale to be $f_t(M) = 2.4$ which corresponds to, at the GUT scale, $f_t(M_G) = 1.4$. We choose this value partly for comparison with Ref. [4]. It gives the top mass $m_t \simeq 170$ (180, 190) GeV for $\tan \beta = 2$ (3, 10).‡ Also we choose $A_0 = 0$. Dependence of the size of the generation mixing in the slepton masses squared on these parameters was discussed earlier.

Branching ratio for $\mu \rightarrow e\gamma$ is shown in Fig. 3. The horizontal line in the figure is taken to be the physical right-handed selectron mass $m_{\tilde{e}_R}$, and $M_5$ is fixed such that the Bino mass $M_1$ is 50 GeV. In our numerical analysis, we imposed theoretical and experimental constraints, i.e., negative searches for the SUSY particles at LEP1 and Tevatron [11], a muon anomalous magnetic dipole moment [12] $(-1.34 \times 10^{-8} \leq \frac{1}{2}(g - 2) \leq 2.34 \times 10^{-8})$, and tree-level stability of the scalar potential along $h_1 = h_2$ direction. We present results in the case of $\tan \beta = 3, 10, 30$ for each sign of $\mu$.

As can be seen from this figure, there exists a region where the cancellation between the diagrams significantly reduces the event rate of $\mu \rightarrow e\gamma$. With the universal scalar mass hypothesis, an absolute value of Higgsino mass $\mu$ determined by the $SU(2)_L \times U(1)_Y$
symmetry breaking condition increases with $m_{\tilde{e}_R}$ in a range $180\text{GeV} \lesssim |\mu| \lesssim 200\text{GeV}$ for $\tan\beta = 10$ or 30, $M_1 = 50 \text{ GeV}$ and $m_{\tilde{e}_R} \lesssim 300 \text{ GeV}$ ($200\text{GeV} \lesssim |\mu| \lesssim 240\text{GeV}$ for $\tan\beta = 3$). Since the mass range for $\mu$ is comparable with other superparticle masses, we cannot ignore the diagrams which have the Yukawa type vertices. In a small $m_{\tilde{e}_R}$ region $A_2^L|_b$ tends to dominate over other contributions, and in a large $m_{\tilde{e}_R}$ region $A_2^L|_c$ does, especially if $\tan\beta$ is large. Therefore, in a middle, there is steep cancellation. If the Yukawa coupling of Higgsino were ignored, the branching ratio would exceed $10^{-11}$ in the case, for example, when $\tan\beta = 30$ and $m_{\tilde{e}_R} < 200\text{GeV}$. Under the universal scalar mass hypothesis, however, the cancellation between diagrams reduces the branching ratio to $\sim 10^{-12}$. Also, in the region of valleys $A_2^L|_a$ and $A_2^L|_d$ contribute to the amplitude with the same order as the sum of $A_2^L|_b$ and $A_2^L|_c$, and this leads to the difference of positions of valleys of each line. Moreover, if the top Yukawa coupling is smaller, the radiative symmetry breaking condition requires smaller values of $|\mu|$, and the point of cancellation tends to shift to a region with smaller right-handed selectron mass. Thus, the situation is more complicated than that previously considered in Ref. [4].

Next, we show branching ratios for $\mu \rightarrow eee$, $\mu-e$ conversion in $^{48}_{22}\text{Ti}$, and $\tau \rightarrow \mu\gamma$ in Figs.4-6. Configurations of these figures are the same as Fig. 3. These events are reduced by about one order of magnitude compared to the case where the Yukawa vertices are not taken into account, and these processes also suffer from the partial cancellation.

The behavior of the $\mu \rightarrow eee$ is similar to that of $\mu \rightarrow e\gamma$ up to normalization. This is because the penguin type contribution from Eq. (11) is enhanced by phase space integral and hence it dominates over other contributions. Steepness of the valleys is somewhat less in the case of $\mu \rightarrow eee$, because diagrams other than the penguin diagram contribute. On the other hand, the enhancement by the phase space integral does not occur for the $\mu-e$ conversion and thus behavior of this event rate is not similar to that of $\mu \rightarrow e\gamma$. Then,
the cancellation occurs at right-handed selectron mass different from the case of $\mu \to e\gamma$ and this suggests search of $\mu-e$ conversion would have a complementary role with that of $\mu \to e\gamma$. Also $\tau \to \mu\gamma$ behaves like $\mu \to e\gamma$ does and they are related with each other through the KM matrix.

So far, we have assumed the universal scalar mass at the Planck scale. It is well-known, however, the degeneracy of all soft scalar masses is not needed to avoid disastrously large flavor changing neutral currents. In the context of $SU(5)$, the mass degeneracy of the sfermions with the same quantum numbers ($5^*$ and 10) is sufficient. Thus we shall discuss the $\mu \to e\gamma$ rate as an example in a non-universal case.

To study the case with non-universality, we show how the results depend on the mass spectrum of the superparticles with a fixed value of the right-handed mass matrix

$$m^2_{\tilde{e}_R} = m^2_{\tilde{e}_L} \begin{pmatrix} 1.0 & -9.8 \times 10^{-5} & -3.2 \times 10^{-3} \\ -9.8 \times 10^{-5} & 1.0 & -2.3 \times 10^{-2} \\ -3.2 \times 10^{-3} & -2.3 \times 10^{-2} & 0.25 \end{pmatrix}.$$  \hspace{1cm} (19)

(These values are realized for the universal mass case with $m_{\tilde{e}_R} = 300$ GeV, $M_1 = 50$ GeV, $A_0 = 0$, $\tan \beta = 3$ and $f_t(M) = 2.4$.) In Figs. 7 and 8, branching ratio for the process $\mu \to e\gamma$ is shown as a function of the physical left-handed selectron mass $m_{\tilde{e}_L}$ and $\mu$-parameter. Here we take $M_1 = 50$ GeV, $m_{\tilde{e}_R} = 300$ GeV, and $\tan \beta = 3$ (Fig. 7) and 30 (Fig. 8). We assume that all left-handed charged slepton have a common mass $m_{\tilde{e}_L}$ and $A(m_Z) = 0$ for simplicity. The shaded regions are excluded by the present experiments.

In general, the amplitude for the $\mu \to e\gamma$ is a complicated function of parameters at low energy scale. In addition to the right-handed slepton masses squared, it depends on the following parameters at the electroweak scale: $m_{\tilde{l}_L}$, $A$, $\mu$, $\tan \beta$, $M_1$ (and $M_2$). However, the qualitative behavior can be easily understood by using the mass insertion formula discussed above. For example, the left-handed slepton mass becomes smaller or Higgsino mass $\mu$ larger, $A^L_{2|b}$ in Eq. (14) becomes larger than the others. When the left-handed slepton mass becomes larger, $A^L_{2|a}$ and $A^L_{2|d}$ ($A^L_{2|c}$) becomes dominant for $\tan \beta = 3$ ($\tan \beta = 30$). Thus, the branching ratio becomes larger because the cancellation does not occur significantly.

We should emphasize that our low-energy approach given here should apply to a certain class of the unified models where the right-handed slepton masses are the only
source of flavor violation. The effect of this interference can be important when one computes the event rate in a given model and we expect that the rates suffer from this complicated cancellation in a large class of models. On the other hand, there is not such significant cancellation in models which left-handed sleptons have LFV masses, e.g., $SO(10)$ SUSY GUT \cite{4, 5, 6} and the minimal SUSY standard model with right-handed neutrinos \cite{8, 9, 10} since the LFV process is usually dominated by only one diagram.

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Figure 1: Top quark mass dependence of flavor-violating right handed slepton mass \( m_{\tilde{\ell}_R}^2 \) at the electroweak scale. Real line, dashed line, and dash-dotted line correspond to the case for \( \tan \beta = 30, 10, 3 \). Here we take \( M_1 = 50 \) GeV and \( m_{\tilde{\ell}_R} = 300 \) GeV, \( A_0 = 0 \). We represent the points corresponding to \( f_t(M) = 0.5, 1.0, 2.0, 2.4, 5.0 \), which is the top Yukawa coupling constant at the reduced Planck scale.

Figure 2: Feynman diagrams contributing to the process \( \mu \rightarrow e\gamma \) in the minimal SUSY SU(5) GUT. The symbols \( \tilde{\mu}_L, \tilde{\mu}_R, \tilde{e}_R, \tilde{B}^0 \), and \( \tilde{H}^0 \) represent left-handed smuon, right-handed smuon, right-handed selectron, Bino, and neutral Higgsino respectively. The blobs in the slepton line indicate the insertion of the flavor-violating right-handed slepton mass \( m_{\tilde{\ell}_R}^2 \) and left-right mixing mass which is proportional to the vacuum expectation value \( \langle h_1 \rangle \) or \( \langle h_2 \rangle \). The blobs in the Bino-Higgsino line represent the insertion of gaugino-Higgsino mass mixing, that is, \( \mu \) denotes Higgsino \( (\tilde{H}_1 - \tilde{H}_2) \) mass mixing, \( \langle h_{1,2} \rangle \) Higgsino-Bino \( (\tilde{H}^0 - \tilde{B}^0) \) mass mixing, \( M_1 \) Bino mass. And blob in the external muon line represent the chirality flip of the external muon \( \mu \).

Figure 3: Branching ratios for the process \( \mu \rightarrow e\gamma \) as a function of the physical right-handed selectron mass, \( m_{\tilde{e}_R} \). Real line, dotted line, dash-dotted line correspond to the case for the \( \tan \beta = 30, 10, 3 \) respectively. Dashed line represents the present experimental upper bound for this process. Here we take \( M_1 = 50 \) GeV, \( f_t(M) = 2.4 \), and (a) \( \mu > 0 \) (b) \( \mu < 0 \).

Figure 4: Branching ratios for the process \( \mu \rightarrow eee \). Here we take the same parameters as Fig.3.

Figure 5: The \( \mu-e \) conversion rates in nuclei \( ^{48}_{22}\text{Ti} \). Here we take the same parameters as Fig.3.

Figure 6: Branching ratios for the process \( \tau \rightarrow \mu\gamma \). Here we take the same parameters as Fig.3.

Figure 7: Branching ratios for the process \( \mu \rightarrow e\gamma \) as a function of the physical left-handed selectron mass \( m_{\tilde{e}_L} \) and \( \mu \)-parameter for the case of non-universal scalar mass. Here we take \( M_1 = 50 \) GeV, \( m_{\tilde{\ell}_R} = 300 \) GeV, \( f_t(M) = 2.4 \), \( \tan \beta = 3 \), \( A(m_Z) = 0 \) and the typical value of right-handed slepton mass matrix as mentioned in text. For simplify we assume that all left-handed charged slepton have a common mass \( m_{\tilde{e}_L} \). The shaded regions are excluded by the present experiments.

Figure 8: Same as Fig.7 except for \( \tan \beta = 30 \).
Fig. 1
Fig. 2
$f_t(M) = 2.4 \quad \mu > 0 \quad M_1 = 50\text{GeV}$

$\beta = 30$  

$\beta = 10$  

$\beta = 3$  

$\mu > 0 \quad M = 50\text{GeV}$  

$\mu < 0 \quad M = 50\text{GeV}$

**Fig. 3**
\[
\text{Experimental bound}
\]

\[
\tan \beta = 30
\]

\[
\tan \beta = 10
\]

\[
\tan \beta = 3
\]

\[
\mu > 0 \quad M_1 = 50 \text{GeV}
\]

\[
\mu < 0 \quad M_1 = 50 \text{GeV}
\]

\[
m_{\tilde{e}_R}
\]

\[
\text{(a)}
\]

\[
m_{\tilde{e}_R}
\]

\[
\text{(b)}
\]

Fig. 4
\[ \tan \beta = 3 \tan \beta = 10 \]

\[ f_t (M) = 2.4 \quad \mu > 0 \quad M_1 = 50 \text{GeV} \]

\[ m_{\tilde{e}_R} \]

\[ R(\mu \rightarrow e; T_1) \]

\[ m_{\tilde{e}_R} \]

\[ \text{Experimental bound} \]

\[ \tan \beta = 30 \]

\[ \tan \beta = 10 \]

\[ \tan \beta = 3 \]

\[ \text{Fig. 5} \]

\[ 10^1 \quad 10^2 \quad 10^3 \quad 10^4 \quad 10^5 \quad 10^6 \quad 10^7 \quad 10^8 \quad 10^9 \]

\[ 10^{-1} \quad 10^{-2} \quad 10^{-3} \quad 10^{-4} \quad 10^{-5} \quad 10^{-6} \quad 10^{-7} \quad 10^{-8} \quad 10^{-9} \]
Fig. 6

(a) $f_t(M)=2.4, \mu > 0, M_1=50\text{GeV}$

(b) $f_t(M)=2.4, \mu < 0, M_1=50\text{GeV}$

$\frac{\text{Br}(\tau \rightarrow \mu \gamma)}{\text{GeV}}$
Fig. 7

$M_1 = 50 \text{GeV}$ $m_{\tilde{e}_R} = 300 \text{GeV}$ $\tan \beta = 3$
$M_1 = 50\text{GeV}$  $m_{\tilde{e}_R} = 300\text{GeV}$  $\tan \beta = 30$

Fig. 8