Nonlinear $k_{\perp}$-factorization for Quark-Gluon Dijet Production off Nuclei

N.N. Nikolaev,1,2, W. Schäfer,1, B.G. Zakharov,2, and V.R. Zoller3

1Institut für Kernphysik, Forschungszentrum Jülich, D-52425 Jülich, Germany
2L.D. Landau Institute for Theoretical Physics, 142432 Chernogolovka, Russia
3Institute of Theoretical and Experimental Physics, 117259 Moscow, Russia

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Abstract

The breaking of conventional linear $k_{\perp}$-factorization for hard processes in a nuclear environment is by now well established. Here we report a detailed derivation of the nonlinear $k_{\perp}$-factorization relations for the production of quark-gluon dijets. This process is of direct relevance to dijets in the proton hemisphere of proton-nucleus collisions at energies of the Relativistic Heavy Ion Collider (RHIC). The major technical problem is a consistent description of the non-Abelian intranuclear evolution of multiparton systems of color dipoles. Following the technique developed in our early work [N.N. Nikolaev, W. Schäfer, B.G. Zakharov and V.R. Zoller, J. Exp. Theor. Phys. 97 (2003) 441], we reduce the description of the intranuclear evolution of the $qg\bar{q}$ state to the system of three coupled-channel equations in the space of color singlet 4-parton states $|3\bar{3}\rangle$, $|6\bar{6}\rangle$ and $|15\bar{15}\rangle$ (and their large-$N_c$ generalizations). At large number of colors $N_c$, the eigenstate $(|6\bar{6}\rangle - |15\bar{15}\rangle)/\sqrt{2}$ decouples from the initial state $|3\bar{3}\rangle$. The resulting nuclear distortions of the dijet spectrum exhibit much similarity to those found earlier for forward dijets in Deep Inelastic Scattering (DIS). Still there are certain distinctions regarding the contribution from color-triplet $qg$ final states and from coherent diffraction excitation of dijets. To the large-$N_c$ approximation, we identify four universality classes of nonlinear $k_{\perp}$-factorization for hard dijet production.

*Electronic address: N.N. Nikolaev@fz-juelich.de
†Electronic address: Wo.Schaefer@fz-juelich.de
‡Electronic address: B.Zakharov@fz-juelich.de
§Electronic address: zoller@heron.itep.ru
I. INTRODUCTION

According to the conventional perturbative QCD (pQCD) factorization theorems the hard scattering cross sections are linear functionals (convolutions) of the appropriate parton densities in the projectile and target \[1\]. An implicit assumption behind these theorems is that the parton densities in the beam and target are low and the relevant partial wave amplitudes are small, so that the unitarity constraints can be ignored. In the case of hard processes in a nuclear environment, the properly defined partial wave amplitudes become proportional to the nuclear thickness and, for a sufficiently heavy nucleus, overshoot the \(s\)-channel unitarity bound. The unitarization makes the nuclear partial waves a highly nonlinear functional of the free nucleon amplitudes. Alternatively, in the pQCD language, the unitarity constraints bring in a new dimensional scale into the problem - the so-called saturation scale. Important implication of the nonlinear unitarity relation between the free-nucleon and nuclear partial waves is that the properly defined density of gluons in a nucleus becomes a nonlinear functional of the gluon density in a free nucleon; the first discussions of the fusion of partons in deep inelastic scattering (DIS) off a nucleus go back to 1975 \[2\].

The emergence of a new large scale and the ensuing nonlinearity call for a revision of the pQCD factorization for hard processes in a nuclear environment. A consistent analysis of forward hard dijet production in DIS off nuclei revealed a striking breaking of linear \(k_\perp\)-factorization \[3, 4\] confirmed later on in the related analysis of single-jet spectra in hadron-nucleus collisions \[5, 6\]. Namely, following the pQCD treatment of diffractive dijet production \[7, 8\], one can define the collective nuclear unintegrated gluon density such that it satisfies the \(s\)-channel unitarity constraints and such that the familiar linear \(k_\perp\)-factorization (see e.g. the recent reviews \[9\]) would hold for the nuclear structure function \(F_{2A}(x, Q^2)\) and forward single-quark spectrum in DIS off nuclei because of their special Abelian features. However, the dijet spectra in DIS and single-jet spectra in hadron-nucleus collisions prove to be a highly nonlinear functionals of the collective nuclear gluon density. Furthermore, the pattern of nonlinearity for single-jet spectra was shown to depend strongly on the relevant partonic subprocess \[6\]. Our conclusions on the breaking of linear \(k_\perp\)-factorization for hard scattering off nuclei were recently
taken over by other authors [10, 11, 12].

In this communication we extend the analysis [4, 6, 13] of the excitation of heavy flavor and leading quark dijets in DIS, $\gamma^* g_N \rightarrow Q\bar{Q}$, where $g_N$ stands for the gluon exchanged with the nucleon, to the excitation of quark-gluon dijets (pQCD Bremsstrahlung tagged by a scattered quark) in the pQCD subprocess $q^* g_N \rightarrow qg$ off free nucleons and its generalization to heavy nuclear targets. In the latter case multiple gluon exchanges between the involved partons and a nucleus are enhanced by a large nuclear radius. The issues are (i) to which extent such multiple gluon exchanges can be described in terms of the unintegrated collective nuclear gluon density and (ii) whether the nuclear factorization for quark-gluon dijets in $qA$ collisions is similar to that for the quark-antiquark dijets in DIS, i.e., in $\gamma^* A$ collisions. To a certain extent, our answer is in the affirmative - the nonlinear $k_{\perp}$-factorization properties for two processes exhibit much similarity. Still, the two cases differ substantially. For instance, the production of coherent diffractive dijets makes about 50% of the total cross section in DIS but becomes marginal in $qA$ collisions. Furthermore, the contributions from quark-gluon dijets in different color multiplets have a very distinct nonlinear $k_{\perp}$-factorization properties. Also the effects of the initial state interaction change substantially from the color-singlet $\gamma^*$ in DIS to the color-triplet quark in $qA$ collisions. On the other hand, the unifying aspect is a treatment of the excitation of final-state color dipoles in the higher color multiplets - color-octet in DIS and sextet and 15-plet in $qA$ collisions.

The starting point of our analysis is the master formula (14) for the inclusive dijet spectrum. It is derived based on the technique developed in [4, 6, 14, 15] and allows to calculate the dijet spectrum in terms of the $S$-matrices for interaction with the target nucleon or nucleus of the color-singlet n-parton states, $n = 2, 3, 4$. Within this technique, one deals with infrared-safe quantities despite the fact that the incident parton - the quark $q^*$ - is carrying a net color charge. The calculation of the two-parton and three-parton $S$-matrices is the single-channel problem with the known solution [15, 18, 20]. The stumbling block is the calculation of the 4-body $S$-matrix. In the case of the quark-gluon dijets it describes the non-Abelian intranuclear evolution of the color-singlet $qg\bar{q}g$ system of dipoles. It can be reduced to a $3 \times 3$ coupled-channel problem. In our earlier work [4] we published a full solution of the related two-channel problem for the $q\bar{q}q\bar{q}$ system which
emerges in the description of quark-antiquark dijets in DIS. Here we report the solution for the $qg\bar{q}g$ system of diopes which has some new features compared to the $q\bar{q}q\bar{q}$ state in DIS\(^1\). We go to fine details of this derivation - specifically, the diagonalization of the coupled-channel $S$-matrix and to the formulation of explicit nonlinear $k_{\perp}$-factorization formulas for the dijet spectrum - for several reasons. First, the production of quark-gluon dijets without the soft gluon approximation has not been treated before. Second, regarding the color properties of the incident and final states, it is a process of sufficient generality to set a basis for the description of other pQCD processes. In conjunction with our earlier results, it allows to formulate four universality classes of nonlinear $k_{\perp}$-factorization. Third, recently the formal representation for the dijet cross section similar to our master formula has been discussed by several groups\([10,11,12]\), but these works stopped short of the diagonalization of their counterpart of our 4-body $S$-matrix. Correspondingly, they do not contain explicit nonlinear $k_{\perp}$-factorization formulas.

A very rich pattern of the process-dependent nonlinear $k_{\perp}$-factorization emerges from the studies presented here and reported in \([4,6,13,16]\). For instance, it becomes increasingly clear that a heavy nucleus cannot be described in terms of a universal collective glue, rather the nuclear glue must be described by the density matrix in the color space. Furthermore, the collective glue defined for the slice of a nucleus rather than the whole nucleus is an integral part of the description of excitation of color dipoles in higher color representations. The linear $k_{\perp}$-factorization for the single-quark jets in DIS found in our earlier study\([4]\) is an exception due to the Abelian incident parton - the photon.

From the point of view of practical applications, the discussed quark-gluon dijets are of direct relevance to the large (pseudo)rapidity region of proton-proton and proton-nucleus collisions at RHIC (for the discussion of the possible upgrade of detectors at RHIC II for the improved coverage of the proton fragmentation region see\([17]\)). Our treatment is applicable when the beam and final state partons interact coherently over the whole longitudinal extension of the nucleus, which at RHIC amounts to the proton

\(^1\) A brief discussion of the main results has been reported elsewhere\([16]\).
fragmentation region of

\[ x = \frac{(Q^*)^2 + M^2_\perp}{2mE_q^*} \lesssim x_A = \frac{1}{2R_A m_p} \approx 0.1 A^{-1/3}, \tag{1} \]

where \( R_A \) is the radius of the target nucleus of mass number \( A \), \((Q^*)^2\) and \( E_q^*\) are the virtuality and energy of the beam quark \( q^* \) in the target rest frame, \( M_\perp \) is the transverse mass of the dijet and \( m_p \) is the proton mass (\([2,18]\), for the related color dipole phenomenology of the experimental data on nuclear shadowing see \([19]\)).

The presentation of the main material is organized as follows. The master formula for the dijet spectrum is presented in Sec. II. The two-body density matrix - the Fourier transform of which gives the dijet spectrum - contains the \( S \)-matrices for the interaction of two-, three- and four-parton color-singlet systems of dipoles with the target. Based on the technique developed in \([15]\), in Sec. III we report single-channel \( S \)-matrices in terms of the quark-antiquark and quark-antiquark-gluon color-dipole cross sections \([18,20]\). Sec. IV contains all the technicalities of the derivation of the coupled-channel \( S \)-matrix for the \( qg\bar{q}g \) state: the decomposition into color multiplets; projection onto the final states; the color-flow diagram technique for the calculation of the \( 3 \times 3 \) matrix of color-dipole cross sections; the derivation of the relevant Casimir operators; the explicit diagonalization of the \( S \)-matrix at large number of colors \( N_c \) and the Sylvester expansion. The quark-gluon dijet spectrum for the free-nucleon target is derived in Sec. V. Here we also comment on a direct relationship between the dijet and single-jet spectra for the free-nucleon reactions described by the single-gluon exchange in the \( t \)-channel. The principal result of this study - the nonlinear \( k_\perp \)-factorization for the dijet spectrum produced off nuclear targets - is reported in Section VI. Here we compare the pattern of nonlinear \( k_\perp \)-factorization for quark-gluon dijets in \( qA \) collisions to that for the quark-antiquark dijets in DIS and \( gA \) collisions and identify four universality classes depending on the color representation of the incident parton and final-state dijet. In Section VII we apply our results to the nuclear broadening of the dijet acoplanarity distribution. In Sec. VIII we comment on a limiting case when the quark-gluon dijets merge to one jet. Such monojets can be identified with the fragmentation of the quark jet formed by the quasielastically scattered incident quark. The separation into the dijet and monojet final states changes with the mass number of the target nucleus and the centrality of the collisions. We also comment
on the possible nuclear modification of the fragmentation function. In the Conclusions section we summarize our main results.

II. THE MASTER FORMULA FOR QUARK-GLUON DIJET PRODUCTION OFF FREE NUCLEONS AND NUCLEI

A. Kinematics and nuclear coherency

To the lowest order in pQCD the underlying subprocess for quark-gluon dijet production in the proton fragmentation region of proton-nucleus collisions is a collision of the quark $q^*$ from the proton with the gluon $g_N$ from the target,

$$q^* g_N \rightarrow qg.$$  

It is a pQCD Bremsstrahlung tagged by a scattered quark. We don’t restrict ourselves to soft gluons. In the case of a nuclear target one has to deal with multiple gluon exchanges which are enhanced by a large thickness of the target nucleus.

From the laboratory, i.e., the nucleus rest frame, standpoint it can be viewed as an excitation of the perturbative $|qg\rangle$ Fock state of the physical projectile $|q^*\rangle$ by one-gluon exchange with the target nucleon or multiple gluon exchanges with the target nucleus. Here the collective nuclear effects develop if the coherency over the thickness of the nucleus holds for the $qg$ Fock states, i.e., if the coherence length is larger than the
diameter of the nucleus,

\[ l_c = \frac{2E_{q^*}}{M^2_{\perp}} = \frac{1}{x m_N} > 2 R_A, \]  

(2)

where

\[ M^2_{\perp} = \frac{p^2_{q}}{z_q} + \frac{p^2_{g}}{z_g} \]  

(3)

is the transverse mass squared of the \( qg \) state, \( p_{q,g} \) and \( z_{q,g} \) are the transverse momenta and fractions of the the incident quarks momentum carried by the quark and gluon, respectively \( (z_q + z_g = 1) \). In the antilaboratory (Breit) frame, the partons with the momentum \( x p_N \) have the longitudinal localization of the order of their Compton wavelength \( \lambda = 1/x p_N \), where \( p_N \) is the momentum per nucleon. The coherency over the thickness of the nucleus in the target rest frame is equivalent to the spatial overlap of parton fields of different nucleons at the same impact parameter in the Lorentz-contracted ultrasllativistic nucleus. In the overlap regime one would think of the fusion of partons form different nucleons and collective nuclear parton densities [2]. The overlap takes place if \( \lambda \) exceeds the Lorentz-contracted thickness of the ultrarelativistic nucleus,

\[ \lambda = \frac{1}{x p_N} > 2 R_A \cdot \frac{m_N}{p_N}, \]  

(4)

which is identical to the condition (2).

Qualitatively, the both descriptions of collective nuclear effects are equivalent to each other. Quantitatively, the laboratory frame approach takes advantage of the well developed multiple-scattering theory of interactions of color dipoles with nuclei [4, 18, 20, 21]. From the practical point of view, the coherency condition \( x < x_A \) restricts collective effects in hard processes at RHIC to the proton fragmentation region. The target frame rapidity structure of the considered \( q^* \rightarrow qg \) excitation is shown in Fig. 1. The (pseudo)rapidities of the final state partons must satisfy \( \eta_{q,g} > \eta_A = \log 1/x_A \). The rapidity separation of the quark and gluon hard jets,

\[ \Delta \eta_{qg} = \log \frac{1 - z_g}{z_g}, \]  

(5)

is considered to be finite. Both jets are supposed to be separated by a large rapidity from other jets at mid-rapidity or in the target nucleus hemisphere; the gaps between all jets, beam spectators and target debris are filled by soft hadrons from an underlying event.
B. Master formula for excitation of quark-gluon dijets

In the nucleus rest frame, relativistic partons $q^*, q$ and $g$, propagate along straight-line, fixed-impact-parameter, trajectories. To the lowest order in pQCD the Fock state expansion for the physical state $|q^*\rangle_{\text{phys}}$ reads

$$|q^*\rangle_{\text{phys}} = |q^*\rangle_0 + \Psi(z_g, r)|qg\rangle_0,$$

where $\Psi(z_g, r)$ is the probability amplitude to find the $qg$ system with the separation $r$ in the two-dimensional impact parameter space, the subscript "0" refers to bare partons. The perturbative coupling of the $q^* \to qg$ transition is reabsorbed into the lightcone wave function $\Psi(z_g, r)$. We also omitted a wave function renormalization factor, which is of no relevance for the inelastic excitation to the perturbative order discussed here. The explicit expression for $\Psi(z_g, r)$ in terms of the quark-splitting function will be presented below. The wave function depends on the virtuality of the incident $q^*$, which equals $(Q^*)^2 = (p^*)^2$, where $p^*$ is the transverse momentum of $q^*$ in the incident proton (Fig. 1).

For the sake of simplicity we take the collision axis along the momentum of the incident quark $q^*$, the transformation between the transverse momenta in the $q^*$-target and $p$-target reference frames is trivial [6].

If $b$ is the impact parameter of the projectile $q^*$, then

$$b_q = b - z_g r, \quad b_g = b + z_g r.$$  

By the conservation of impact parameters, the action of the $S$-matrix on $|a\rangle_{\text{phys}}$ takes a simple form

$$S|q^*\rangle_{\text{phys}} = S_q(b)|q^*\rangle_0 + S_q(b_q)S_g(b_g)\Psi(z, r)|qg\rangle_0$$

$$= S_q(b)|q^*\rangle_{\text{phys}} + [S_q(b_q)S_g(b_g) - S_q(b)]\Psi(z_g, r)|qg\rangle.$$  

In the last line we explicitly decomposed the final state into the (quasi)elastically scattered $|q^*\rangle_{\text{phys}}$ and the excited state $|qg\rangle_0$. The two terms in the latter describe a scattering on the target of the $qg$ system formed way in front of the target and the transition $q^* \to qg$ after the interaction of the state $|q^*\rangle_0$ with the target, as illustrated in Fig. 2. The contribution from transitions $q^* \to qg$ inside the target nucleus vanishes in the
FIG. 2: Typical contribution to the excitation amplitude for $q^*A \rightarrow qgX$, with multiple color excitations of the nucleus. The amplitude receives contributions from processes that involve interactions with the nucleus after and before the virtual decay which interfere destructively.

In high-energy limit of $x \lesssim x_A$ \(^2\). We recall, that the $s$-channel helicity of all partons is conserved.

The probability amplitude for the two-jet spectrum is given by the Fourier transform

$$
\int d^2b_q d^2b_g \exp[-i(p_q b_q + p_g b_g)] [S_q(b_q) S_g(b_g) - S_q(b)] \Psi(z_g, r)
$$

(9)

The differential cross section is proportional to the modulus squared of (9),

$$
\int d^2b'_q d^2b'_g \exp[i(p_q b'_q + p_g b'_g)] [S^\dagger_q(b'_q) S^\dagger_g(b'_g) - S^\dagger_q(b')] \Psi^*(z_g, r')
\times \int d^2b_q d^2b_g \exp[-i(p_q b_q + p_g b_g)] [S_q(b_q) S_g(b_g) - S_q(b)] \Psi(z_b, r).
$$

(10)

The crucial point is that the hermitian conjugate $S^\dagger$ can be viewed as the $S$-matrix for an antiparton \(^4\). Consequently, the four terms in the product

$$
[S_q(b'_q) S_g(b'_g) - S_q(b')] [S_q(b_q) S_g(b_g) - S_q(b)]
$$

admit a simple interpretation:

$$
S_{q'q}^{(2)}(b', b) = S^\dagger_q(b') S_q(b)
$$

(11)

\(^2\) In terms of the lightcone approach to the QCD Landau-Pomeranchuk-Migdal effect, this corresponds to the thin-target limit \(^{22}\).
can be viewed as an $S$-matrix for elastic scattering on a target of the $\bar{q}^*q^*$ state in which the antiparton $\bar{q}^*$ propagates at the impact parameter $b'$. The averaging over the color states of the beam parton $q^*$ amounts to the dipole $q^*\bar{q}^*$ being in the color singlet state. Similarly,

$$
\begin{align*}
S_{q'qg}^{(3)}(b', b_q, b_g) &= S_q^t(b') S_q(b_q) S_g(b_g), \\
S_{q'g'q}^{(3)}(b, b'_q, b'_g) &= S_{q'}^t(b'_q) S_{g'}^t(b'_g) S_q(b) \\
S_{q'g'g}^{(4)}(b'_q, b'_g, b_q, b_g) &= S_{q'}^t(b'_q) S_{g'}^t(b'_g) S_g(b_g) S_q(b_q).
\end{align*}
$$

(12)
describe elastic scattering on a target of the overall color-singlet $\bar{q}gg$ and $\bar{q}gqq$ states, respectively. This is shown schematically in Fig. 3. Here we suppressed the matrix elements of $S^{(n)}$ over the target nucleon, for details of the derivation based on the closure relation, see [4]. Specifically, in the calculation of the inclusive cross sections one averages over the color states of the beam parton $q^*$, sums over color states $X$ of final state partons $q, g$, takes the matrix products of $S^t$ and $S$ with respect to the relevant color indices entering $S^{(n)}$ and sums over all nuclear final states applying the closure relation. The technicalities of the derivation of $S^{(n)}$ will be presented below, here we cite the master formula for the dijet cross section, which is the Fourier transform of the two-body density matrix:

$$
\frac{d\sigma(q^* \rightarrow gg)}{dz d^2p_q d^2p_g} = \frac{1}{(2\pi)^4} \int d^2b_q d^2b_g d^2b'_q d^2b'_g \exp[-ip_q(b_q - b'_q) - ip_g(b_g - b'_g)] \Psi(z_g, b_q - b_g) \Psi^*(z_g, b'_q - b'_g) \\
\times \sum_X \left\{ |X\rangle \left\{ S_{q'g'g}^{(4)}(b'_q, b'_g, b_q, b_g) + S_{q'qg}^{(2)}(b', b) - S_{q'qg}^{(3)}(b, b'_q, b'_g) - S_{q'g'g}^{(3)}(b', b'_q, b'_g) \right\} |in\rangle \right\}.
$$

(13)

Hereafter, we describe the final state dijet in terms of the gluon jet momentum, $p \equiv p_g$, $z \equiv z_g$, and the decorrelation (acoplanarity) momentum $\Delta = p_q + p_g$. We also introduce

$$
s = b_q - b'_q,
$$

(14)
in terms of which $b_g - b'_g = s + r - r'$ and

$$
\exp[-ip_q(b_q - b'_q) - ip_g(b_g - b'_g)] = \exp[-is - ipr + ipr'],
$$

(15)
so that the dipole parameter $s$ is conjugate to the acoplanarity momentum $\Delta$.  

III. CALCULATION OF THE 2-PARTON AND 3-PARTON S-MATRICES

A. The quark-nucleon S-matrix and the $k_\perp$-factorization representations for the color dipole cross section

In order to set the formalism, we start with the S-matrix representation for the cross section of interaction of the triplet-antitriplet color dipole $q\bar{q}$ with the free-nucleon target \[4\]. To the two-gluon exchange approximation, the S-matrices of the quark-nucleon and antiquark-nucleon interaction equal, respectively,

\[
S(b_q) = 1 + iT^a V_a \chi(b_q) - \frac{1}{2} T^a T^a \chi^2(b_q), \\
S^\dagger(b_{\bar{q}}) = 1 - iT^a V_a \chi(b_{\bar{q}}) - \frac{1}{2} T^a T^a \chi^2(b_{\bar{q}}),
\]

were $T^a V_a \chi(b)$ is the eikonal for the quark-nucleon gluon exchange. The vertex $V_a$ for excitation of the nucleon $g^a N \rightarrow N^*_a$ into color octet state is so normalized that after application of closure over the final state excitations $N^*$ the vertex $g^a g^b NN$ equals $\langle N|V_a^\dagger V_b|N \rangle = \delta_{ab}$. The second order terms in (16) do already use this normalization. The S-matrix of the $(q\bar{q})$-nucleon interaction equals

\[
S^{(2)}_{q\bar{q}}(b_q, b_{\bar{q}}) = \frac{\langle N|\text{Tr}[S(b_q)S^\dagger(b_{\bar{q}})]|N \rangle}{\langle N|\mathbb{I}|N \rangle\text{Tr}\mathbb{I}}.
\]

A graphical rule for the calculation of the color traces entering (17) is shown in Fig. 4. Such color flow diagrams will extensively be used in the subsequent calculation of $S^{(4)}$. 

FIG. 3: The S-matrix structure of the two-body density matrix for excitation $q^* \rightarrow qg$. 

FIG. 4: A graphical rule for the calculation of the color traces entering (17) is shown in Fig. 4. Such color flow diagrams will extensively be used in the subsequent calculation of $S^{(4)}$. 

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\[ S_{q\bar{q}}^{(2)}(a-\bar{a}) = \begin{array}{c} S_q(a) \\ \bullet \\ \downarrow \end{array} / \begin{array}{c} S_{\bar{q}}(\bar{a}) \\ \bullet \\ \downarrow \end{array} \]

FIG. 4: The color-flow diagram for the S-matrix for the interaction of the color-single \( q\bar{q} \) dipole with the nucleon; \( a \) and \( \bar{a} \) are the impact parameters of the quark and antiquark, respectively.

The corresponding profile function is \( \Gamma_2(\vec{b}_q, \vec{b}_{\bar{q}}) = 1 - S_{q\bar{q}}^{(2)}(\vec{b}_q, \vec{b}_{\bar{q}}) \). The dipole cross section for interaction of the color-singlet \( q\bar{q} \) dipole \( r = \vec{b}_q - \vec{b}_{\bar{q}} \) with the free nucleon is obtained upon the integration over the overall impact parameter

\[
\sigma(r) = 2 \int d^2\vec{b}_q \Gamma_2(\vec{b}_q, \vec{b}_q - r) = C_F \int d^2\vec{b}_q [\chi(\vec{b}_q) - \chi(\vec{b}_q - r)]^2, \tag{18}
\]

where \( C_F = (N_c^2 - 1)/2N_c \) is the quark Casimir operator. It sums a contribution from the four Feynman diagrams of Fig. 5 and is related to the gluon density in the target by the \( k\perp \)-factorization formula [20, 23]

\[
\sigma(x, r) = \int d^2\kappa f(x, \kappa)[1 - \exp(i\kappa r)], \tag{19}
\]

where

\[
f(x, \kappa) = \frac{4\pi\alpha_S(r)}{N_c} \cdot \frac{1}{\kappa^4} \cdot \mathcal{F}(x, \kappa^2) \tag{20}
\]

and

\[
\mathcal{F}(x, \kappa^2) = \frac{\partial G(x, \kappa^2)}{\partial \log \kappa^2} \tag{21}
\]

is the unintegrated gluon density in the target nucleon. Hereafter, unless it may cause a confusion, we suppress the variable \( x \) in the gluon densities and dipole cross sections. The leading \( \log^2 \) evolution of the dipole cross section is governed by the color-dipole BFKL evolution [20, 24], the same evolution for the unintegrated gluon density is governed by the familiar momentum-space BFKL equation [25].
FIG. 5: The four Feynman diagrams for the quark-antiquark dipole-nucleon interaction by the two-gluon pomeron exchange in the $t$-channel.

The $S$-matrix for coherent interaction of the color dipole with the nuclear target is given by the Glauber-Gribov formula \[26, 27\]

$$S[b, \sigma(r)] = \exp\left[-\frac{1}{2} \sigma(r) T(b)\right],$$

(22)

where

$$T(b) = \int_{-\infty}^{\infty} dr_z n_A(b, r_z)$$

(23)

is the optical thickness of the nucleus. The nuclear density $n_A(b, r_z)$ is normalized according to $\int d^3r n_A(b, r_z) = \int d^3b T(b) = A$, where $A$ is the nuclear mass number.

In the specific case of $S^{(2)}_{\bar{q}q^*}(b', b)$ the color dipole equals

$$r_{\bar{q}q} = b - b' = s + zr - zr'$$

(24)

and $S^{(2)}_{\bar{q}q^*}(b', b)$ entering Eq. (14) will be given by the Glauber-Gribov formula

$$S^{(2)}_{\bar{q}q^*}(b', b) = S[b, \sigma(s + zr - zr')] .$$

(25)

B. The $S$-matrix for the color-singlet $\bar{q}qq$ state

Here quark and gluon couple to the color triplet. The dipole cross section for the color singlet $\bar{q}qq$ state has been derived in \[20\], the $S$-matrix derivation with the quark-antiquark basis description of the gluon is found in Appendix A of ref. \[6\]. For the generic 3-body state shown in Fig. 6 it equals

$$\sigma_{3}(r_{\bar{q}q}, r_{qq}) = \frac{C_A}{2C_F} [\sigma(r_{\bar{q}q}) + \sigma(r_{qq}) - \sigma(r_{\bar{q}q}) + \sigma(r_{qq})],$$

(26)
FIG. 6: The color dipole structure of (a) the generic quark-antiquark-gluon system of dipoles and (b) of the $\bar{q}^*qg$ system which emerges in the $S$-matrix structure of the two-body density matrix for excitation $q^* \rightarrow qg$.

where $r_{gq} = r_{gq} + r_{q\bar{q}}$. The configuration of color dipoles for the case of our interest is shown in Fig. 6 (see the related derivation in [15]). For the $\bar{q}^*qg$ state the relevant dipole sizes in (26) equal

$$r_{q\bar{q}} = b_q - b' = s - zr,$$
$$r_{gq} = b_g - b_q = r,$$
$$r_{g\bar{q}} = b_g - b' = s + r - zr',$$

whereas for the $q^*\bar{q}g'$ state we have

$$r_{q\bar{q}} = b - b'_q = s + zr,$$
$$r_{gq} = b'_g - b'_q = r',$$
$$r_{g\bar{q}} = b_g - b = s + zr - r',$$

so that

$$\sigma_{q^*\bar{q}g} = \frac{C_A}{2C_F}[\sigma(r) + \sigma(s + r - zr') - \sigma(s - zr')] + \sigma(s - zr),$$
$$\sigma_{q^*\bar{q}g'} = \frac{C_A}{2C_F}[\sigma(-r') + \sigma(s - r' + zr) - \sigma(s + zr)] + \sigma(s + zr').$$

The overall color-singlet $q\bar{q}g$ state has a unique color structure and its elastic scattering on a nucleus is a single-channel problem. Consequently, the nuclear $S$-matrix is given by
the single-channel Glauber-Gribov formula \[26, 27\]

\[
S_{q'g'gq}^{(3)}(b, b'_q, b'_g) = S[b, \sigma_{q'g'q}], \\
S_{q'gq}^{(3)}(b'_q, b, b_g) = S[b, \sigma_{q'gq}].
\] (30)

IV. COUPLED-CHANNEL S-MATRIX FOR THE 4-PARTON STATE

A. The basis of color-singlet \((q\bar{q}gg')\) states

The 4-parton S-matrix describes transitions between color-singlet \((q\bar{q}gg')\) states. It is convenient to decompose the the \(|qg\rangle\) state into the \(|3\rangle, |6\rangle\) and \(|15\rangle\) states and their \(SU(N_c)\) generalizations (our reference to the triplet, sextet and 15-plet states at arbitrary \(N_c\) should not cause any confusion). Then the basis of color-singlet states \(|q\bar{q}gg')\) will consist of the \(|3\rangle, |6\rangle\) and \(|15\rangle\) systems of color dipoles and the intranuclear evolution in the elastic scattering of the 4-parton state off the nucleus is the three-channel problem. The evolution starts from the \(|3\rangle\) state what is evident from Fig. 3. Technically, once the \(3 \times 3\) matrix \(\hat{\Sigma}\) of 4-body dipole cross sections is known, the corresponding nuclear S-matrix will be given by the Glauber-Gribov formula

\[
S_{q'g'gq}^{(4)}(b'_q, b'_g, b, b_g) = S[b, \hat{\Sigma}].
\] (31)

Our immediate task is a calculation of the coupled-channel operator \(\hat{\Sigma}\).

We chose a description of the gluon in the quark-antiquark basis:

\[
g_k^i = \bar{a}^i a_k - \frac{1}{N_c} (\bar{a}a) \delta_k^i.
\] (32)

In the calculation of the S-matrices both the quark \(a\) and the antiquark \(\bar{a}\) must be considered as propagating at the same impact parameter. The generic quark-gluon state is described by a tensor

\[
v_{kl}^i = g_k^i c_l = \bar{a}^i a_k c_l - \frac{1}{N_c} (\bar{a}a) c_l \delta_k^i.
\] (33)

There is a unique color-triplet quark-gluon state (the normalization of the states will be defined at the level of the \(|33\rangle, |66\rangle\) and \(|15\overline{15}\rangle\) systems of color dipoles)

\[
t_k = (\bar{a}c) a_k - \frac{1}{N_c} (\bar{a}a) c_k
\] (34)
The sextet state is described by the traceless tensor antisymmetric in \((k, l)\),

\[
A^i_{k l} = \bar{a}^i (a_k c_l - a_l c_k) + \frac{1}{N_c - 1} [\bar{(ac)} a_l - (\bar{aa}) c_l] \delta_k^i - \frac{1}{N_c - 1} [\bar{(ac)} a_k - (\bar{aa}) c_k] \delta_l^i ,
\]

while the 15-plet is described by the traceless symmetric tensor

\[
S^i_{k l} = \bar{a}^i (a_k c_l + a_l c_k) - \frac{1}{N_c + 1} [\bar{(ac)} a_l + (\bar{aa}) c_l] \delta_k^i - \frac{1}{N_c + 1} [\bar{(ac)} a_k + (\bar{aa}) c_k] \delta_l^i .
\]

The quark, antiquark and two gluons in the color-singlet \((q\bar{q}gg')\) system of dipoles all propagate at different impact parameters. To avoid a confusion, the gluon in the complex conjugated state will be described by the tensor

\[
(g')^i_k = \bar{b}^i b_k - \frac{1}{N_c} (\bar{bb}) \delta^i_k ,
\]

and the antitriplet state is

\[
\bar{t}^k = (\bar{cb}) \bar{b}^k - \frac{1}{N_c} (\bar{bb}) \bar{c}^k .
\]

The overall color-singlet \(|33\rangle\), \(|6\bar{6}\rangle\) and \(|15\bar{15}\rangle\) states will be decomposed into six 6-body color-singlet states. The corresponding 6-body vertices (projection operators) equal

\[
V_1 = (\bar{ab})(\bar{ba})(\bar{cc}) , \quad V_2 = (\bar{ab})(\bar{bc})(\bar{ca}) , \quad V_3 = (\bar{aa})(\bar{bc})(\bar{cb}) ,
\]

\[
V_4 = (\bar{ac})(\bar{bb})(\bar{ca}) , \quad V_5 = (\bar{ac})(\bar{ba})(\bar{cb}) , \quad V_6 = (\bar{aa})(\bar{bb})(\bar{cc}) .
\]

some of which are pictorially represented in Fig. 7. For instance, the normalized color-singlet triplet-antitriplet state will be

\[
|33\rangle = \left[ -\frac{1}{N_c} V_3 - \frac{1}{N_c^2} V_4 + \frac{1}{N_c^2} V_6 \right] \frac{\sqrt{N_c}}{(N_c^2 - 1)} .
\]

Similarly, one finds

\[
|6\bar{6}\rangle = \left[ V_1 - V_2 + \frac{1}{N_c - 1} (V_3 + V_4 - V_5 - V_6) \right] \frac{1}{\sqrt{2N_c(N_c + 1)(N_c - 2)}} .
\]

\[
|15\bar{15}\rangle = \left[ V_1 + V_2 - \frac{1}{N_c + 1} (V_3 + V_4 + V_5 + V_6) \right] \frac{1}{\sqrt{2N_c(N_c - 1)(N_c + 2)}} .
\]

These states are normalized to unity, \(|33|33\rangle = |6\bar{6}|6\bar{6}\rangle = |15\bar{15}|15\bar{15}\rangle = 1\), the normalization coefficients are readily derived using the color-flow diagram technique described in...
\[ \mathbf{V}_1 = (\bar{a}b)(\bar{b}a)(cc) \quad \mathbf{V}_2 = (\bar{a}b)(\bar{b}a)(cc) \quad \mathbf{V}_5 = (ac)(\bar{b}a)(\bar{c}b) \]

FIG. 7: Examples of the 6-body vertices (projection operators) which emerge in expansions of the \( qg\bar{q}g \) states in the quark-antiquark basis.

in Sec. IV-C below. The diagonal and off-diagonal matrix elements of the 4-body cross section operator in the basis of \(|3\bar{3}\rangle, |6\bar{6}\rangle \) and \(|15\bar{15}\rangle \) of color dipole states will be decomposed in terms of the matrix elements

\[ \sigma_{ik} = \langle V_i | \sigma | V_k \rangle \]  

with the coefficients which are readily read from the expansions (40)–(42).

Note, that each of the \( \sigma_{ik} \) is a matrix element between the overall color-singlet 6-body configurations composed of the three color-singlet quark-antiquark pairs. As such all of them are infrared-safe quantities.

**B. Projection onto the final states**

In the case of the inclusive dijet spectrum with summation over all colors of final state quarks and gluons the projection onto the final state is of the form (see the discussion in [4])

\[
\sum_X \langle X | = \sum_R \sqrt{\text{dim}(R)} \langle R | R \rangle = \\
= \sqrt{N_c |3\bar{3}\rangle} + \sqrt{\frac{1}{2} N_c (N_c + 1)(N_c - 2)} |6\bar{6}\rangle + \sqrt{\frac{1}{2} N_c (N_c - 1)(N_c + 2)} |15\bar{15}\rangle ,
\]

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where \( \text{dim}(R) \) is the dimension of the corresponding representation. The averaging over the colors of the initial quark \( q^* \) amounts to taking

\[
|\text{in}\rangle = |3\bar{3}\rangle \cdot \frac{1}{\sqrt{\text{dim}(3)}}.
\] (45)

Then the calculation of the inclusive dijet cross section requires the evaluation of the combination of matrix elements

\[
\sum_X \langle X| S[b, \hat{\Sigma}]|\text{in}\rangle = \langle 3\bar{3}| S[b, \hat{\Sigma}]|3\bar{3}\rangle + \frac{\text{dim}(6)}{\text{dim}(3)} \langle 6\bar{6}| S[b, \hat{\Sigma}]|3\bar{3}\rangle + \frac{\text{dim}(15)}{\text{dim}(3)} \langle 15\bar{15}| S[b, \hat{\Sigma}]|3\bar{3}\rangle
\] (46)

Besides the inclusive spectrum one can readily consider the excitation of quark-gluon dijets in specific color representations. We reiterate that they also will be infrared-safe observables.

\textbf{C. Color-flow diagrams}

The calculation of the matrix elements (43) is greatly simplified by the technique of color-flow diagrams. Each matrix element (43) corresponds to a certain color flow diagram. Altogether there are 21 different color flow diagrams, the three selected cases are shown in Fig. \( \text{Fig. 8} \). The number of closed loops varies from three to one. In the calculation of the \( S \)-matrix elements,

\[
S_{ik} = \langle V_i| S| V_k \rangle,
\] (47)

each horizontal quark line is multiplied by the quark \( S \)-matrix \( S(b_i) \) taken at the appropriate impact parameter \( b_i \), while the antiquark line is multiplied by \( S^\dagger(b_i) \). The trace of the product of \( S \)-matrices is calculated for each closed loop.

The first application of the color-flow diagrams is to the derivation of the normalization factors in expansions (41). They are obtained by assigning the factor \( N_c \) to each and every loop.

Now we present the results for the three matrix elements shown in Fig. \( \text{Fig. 8} \). For the sake of brevity the impact parameters of quarks and antiquarks will be denoted by their
FIG. 8: Examples of color flow diagrams for the calculation of the components of the 6-body dipole cross sections. The horizontal quark lines are multiplied by the quark $S$-matrix $S(b_i)$ taken at the appropriate impact parameter, while each horizontal antiquark line is multiplied by $S^\dagger(b_i)$, the trace is taken for each closed loop.

symbols. One readily finds

$$S_{11} = \langle V_1 | S| V_1 \rangle = \text{Tr}[S(a)S^\dagger(\bar{b})] \text{Tr}[S(b)S^\dagger(\bar{a})] \text{Tr}[S(c)S^\dagger(\bar{c})]$$

$$= N_c^3 [1 - \Gamma(a - \bar{b})][1 - \Gamma(b - \bar{a})][1 - \Gamma(c - \bar{c})].$$ (48)

The multibody $S$-matrices must be evaluated up to the terms quadratic in the QCD eikonal, i.e., to the terms linear in the triplet-antitriplet color-dipole profile function $\Gamma$, and the corresponding matrix element of the dipole cross section equals

$$\sigma_{11} = \langle V_1 | \sigma_4 | V_1 \rangle = N_c^3 [\sigma(a - \bar{b}) + \sigma(b - \bar{a}) + \sigma(c - \bar{c})]$$

$$= N_c^3 [2\sigma(a - b) + \sigma(c - \bar{c})].$$ (49)

Each quark-antiquark loop gives the corresponding dipole cross section, times $N_c$ to the power equal to the number of loops. Here we took into account that the quark $a$ and antiquark $\bar{a}$, and $b$ and $\bar{b}$ as well, propagate pairwise at equal impact parameters.

The case of $\sigma_{12}$ is a bit more complicated. Here $S_{12}$ is a product of two traces:

$$S_{12} = \langle V_1 | S| V_1 \rangle = \text{Tr}[S(b)S^\dagger(\bar{a})] \text{Tr}[S(a)S^\dagger(\bar{b})S(c)S^\dagger(\bar{c})]$$

$$= N_c [1 - \Gamma(b - \bar{a})]\text{Tr}[S(a)S^\dagger(\bar{b})S(c)S^\dagger(\bar{c})].$$ (50)
FIG. 9: Examples of interaction with the target nucleon of the (a) quark-antiquark and diquark dipole in the $\bar{b}c\bar{c}a$ state

The latter trace $\text{Tr}[S(a)S^\dagger(\bar{b})S(c)S^\dagger(\bar{c})]$ was already encountered in our derivation of the 4-parton $S$-matrix for the production of dijets in DIS [4]. The corresponding color-flow diagram is shown in Fig. 9. Here one needs to sum the contributions to the $\bar{b}c\bar{c}a$ scattering amplitude from the exchange by the 2-gluon pomerons in the $t$-channel. The familiar diagram of Fig. 9a gives a contribution $-\chi(c)\chi(\bar{c})\text{Tr}(T^aT^a)$. The new case is when the two gluons are attached to the diquark $ac$ as shown in Fig. 9b. Straightforward color algebra shows that the corresponding contribution to the profile function equals $\chi(a)\chi(c)\text{Tr}(T^aT^a)$. This gives rise to a simple rule: each quark-antiquark pair, $a\bar{b}$, $a\bar{c}$, $\bar{c}b$ and $\bar{c}c$, contributes the corresponding triplet-antitriplet dipole cross section, whereas the diquark $ac$ and the anti-diquark $\bar{b}\bar{c}$ contribute the triplet-antitriplet dipole cross section taken with the negative sign. The color traces give a factor $N_c$ per each loop, one of these factors has already been put in evidence in Eq. (50). The final result is

$$\sigma_{12} = \langle V_1 | \sigma_4 | V_1 \rangle = N_c^2[\sigma(b - \bar{a}) + \sigma(a - \bar{b}) - \sigma(a - c) + \sigma(a - \bar{c}) + \sigma(c - \bar{b}) - \sigma(b - \bar{c}) + \sigma(c - \bar{c})]. \quad (51)$$

Application of the same technique to $\sigma_{25}$ gives

$$S_{25} = \langle V_1 | S | V_1 \rangle = \text{Tr}[S(a)S^\dagger(\bar{a})S^\dagger(\bar{c})S(b)S^\dagger(\bar{a})S(c)S^\dagger(\bar{b})] \quad (52)$$
with the cross section

\[ \sigma_{25} = \langle V_2 | \sigma_4 | V_5 \rangle = N_e [\sigma(a - \bar{c}) - \sigma(a - b) + \sigma(a - \bar{a}) - \sigma(a - c) + \sigma(b - \bar{b}) + \sigma(b - \bar{c}) - \sigma(b - \bar{c}) - \sigma(b - c) + \sigma(b - \bar{b})] = N_e \sigma(c - \bar{c}) \] (53)

Here we used the obvious properties \( \sigma(a - \bar{a}) = \sigma(b - \bar{b}) = 0 \) and cancellations due to equalities of the form \( \sigma(c - a) = \sigma(c - \bar{a}) \). For the sake of completeness, we cite all the remaining \( \sigma_{ik} \):

\[ \sigma_{13} = N_e \sigma(c - \bar{c}) , \]
\[ \sigma_{14} = N_e \sigma(c - \bar{c}) , \]
\[ \sigma_{15} = N_e^2 [2\sigma(a - b) + \sigma(c - \bar{c}) + \sigma(a - c) + \sigma(b - c) - \sigma(a - \bar{c}) - \sigma(b - c) - \sigma(a - \bar{c})] , \]
\[ \sigma_{16} = N_e^2 \sigma(c - \bar{c}) , \]
\[ \sigma_{22} = N_e^3 [\sigma(a - b) + \sigma(b - c) + \sigma(a - \bar{c})] , \]
\[ \sigma_{23} = N_e^3 [\sigma(b - c) + \sigma(b - \bar{c})] , \]
\[ \sigma_{24} = N_e^3 [\sigma(c - a) + \sigma(a - \bar{c})] , \]
\[ \sigma_{26} = N_e \sigma(c - \bar{c}) , \]
\[ \sigma_{33} = N_e^3 [\sigma(b - c) + \sigma(b - \bar{c})] , \]
\[ \sigma_{34} = N_e \sigma(c - \bar{c}) , \]
\[ \sigma_{35} = N_e^2 [\sigma(b - c) + \sigma(b - \bar{c})] , \]
\[ \sigma_{36} = N_e^2 \sigma(c - \bar{c}) , \]
\[ \sigma_{44} = N_e^3 [\sigma(a - c) + \sigma(a - \bar{c})] , \]
\[ \sigma_{45} = N_e^2 [\sigma(a - c) + \sigma(a - \bar{c})] , \]
\[ \sigma_{46} = N_e^2 \sigma(c - \bar{c}) , \]
\[ \sigma_{55} = N_e^3 [\sigma(a - b) + \sigma(a - c) + \sigma(b - \bar{c})] , \]
\[ \sigma_{56} = N_e \sigma(c - \bar{c}) , \]
\[ \sigma_{66} = N_e^3 \sigma(c - \bar{c}) . \] (54)
D. The $3 \times 3$ matrix of 4-parton dipole cross sections $\hat{\Sigma}$.

A simple algebra gives the following $3 \times 3$ matrix $\hat{\Sigma}$ of 4-body dipole cross sections (we go back to the dipole parameters defined in Sect. 2):

$$\Sigma_{11} = \langle 3\bar{3}|\sigma_4|3\bar{3}\rangle = \frac{C_A}{2C_F}\left[\sigma(s - r' + r) + \sigma(r) + \sigma(r')\right]$$
$$- \frac{1}{N_c^2 - 1}\sigma(s) - \frac{C_A}{2C_F}\cdot \frac{1}{N_c^2 - 1}\Omega,$$

(55)

where

$$\Omega = \sigma(s - r') + \sigma(s + r) - \sigma(s - r' + r) - \sigma(s).$$

(56)

Similar calculation gives

$$\Sigma_{22} = \langle 6\bar{6}|\sigma_4|6\bar{6}\rangle = \frac{3N_c + 1}{N_c + 1} \cdot \frac{1}{2}\cdot \left[\sigma(s - r' + r) + \sigma(s)\right]$$
$$+ \frac{N_c^2 + 1}{2(N_c^2 - 1)}\cdot \left[\sigma(s - r' + r) - \sigma(s)\right]$$
$$+ \frac{N_c}{N_c^2 - 1}\left[\sigma(r) + \sigma(r')\right]$$
$$- \frac{N_c}{2(N_c + 1)}\cdot \left[1 + \frac{1}{(N_c - 1)^2}\right]\Omega,$$

(57)

$$\Sigma_{33} = \langle 15\bar{15}|\sigma_4|15\bar{15}\rangle = \frac{3N_c - 1}{N_c - 1} \cdot \frac{1}{2}\cdot \left[\sigma(s - r' + r) + \sigma(s)\right]$$
$$+ \frac{N_c^2 + 1}{2(N_c^2 - 1)}\cdot \left[\sigma(s - r' + r) - \sigma(s)\right]$$
$$- \frac{N_c}{N_c^2 - 1}\left[\sigma(r) + \sigma(r')\right]$$
$$- \frac{N_c}{2(N_c - 1)}\cdot \left[1 + \frac{1}{(N_c + 1)^2}\right]\Omega.$$

(58)

All the off-diagonal matrix elements for transition between different color representations are proportional to $\Omega$:

$$\Sigma_{21} = \langle 6\bar{6}|\sigma_4|3\bar{3}\rangle = -\frac{N_c^2}{(N_c - 1)(N_c^2 - 1)}\sqrt{\frac{N_c - 2}{2(N_c + 1)}}\Omega,$$

(59)

$$\Sigma_{31} = \langle 15\bar{15}|\sigma_4|3\bar{3}\rangle = -\frac{N_c^2}{(N_c + 1)(N_c^2 - 1)}\sqrt{\frac{N_c + 2}{2(N_c - 1)}}\Omega,$$

(60)
\[ \Sigma_{32} = \langle 15 \bar{15} | \sigma_4 | 6\bar{6} \rangle = -\frac{1}{2} \frac{N_c^2}{(N_c^2 - 1)} \sqrt{\frac{N_c^2 - 4}{N_c^2 - 1}} \Omega. \]  

(61)

Note, that the off-diagonal \( \Omega \) has precisely the same color-dipole structure as the off-diagonal \( \sigma_{18} \) which describes the excitation \( q\bar{q} \) dipole from the color-singlet to color-octet state \([4]\). This off-diagonal matrix element vanishes if either \( r = 0 \) or \( r' = 0 \), when the pointlike \( |qg\rangle \) and \( |q'g'\rangle \) Fock states cannot be resolved.

E. The pointlike triplet, sextet and 15-plet dipoles and Casimir operators

In the limit of \( r = r' = 0 \), the 4-body states reduce to the pointlike triplet-antitriplet, sextet-antisextet and 15-\( \bar{15} \) dipoles.

Indeed, in this limit

\[ \Sigma_{11} = \sigma(s) \]  

(62)

as expected, while

\[ \Sigma_{22} = \frac{3N_c + 1}{N_c + 1} \sigma(s), \]
\[ \Sigma_{33} = \frac{3N_c - 1}{N_c - 1} \sigma(s). \]  

(63)

The Feynman diagrams of Fig. 5 make it obvious that for partons in the representation \( R \) the dipole cross section must be proportional to the Casimir operator \( C_R \). Consequently, the coefficients in (63) must equal the ratio \( C_R/C_F \) (a factor \( C_F \) for the triplet-antitriplet color dipole had been absorbed into the definition of \( \sigma(s) \), see Eq. (18)). The derivation of \( C_R \) by the color-flow diagram technique proceeds as follows:
We recall that the calculation of $C_F$ for the quark.

$$C_F = \frac{\text{Tr}(T^a T^a)}{\text{Tr} \mathbb{I}}, \tag{64}$$

can be represented in terms of traces of color loop diagrams as shown in Fig. 10. In order to avoid a confusion in the description of the conjugate states, it is convenient to represent the sextet $qg$ state in terms of the three different quark fields,

$$A^i_{kl} = \bar{a}^i (b_k c_l - b_l c_k) + \frac{1}{N_c - 1} [(\bar{a} c) b_l - (\bar{a} b) c_l] \delta_k^i - \frac{1}{N_c - 1} [(\bar{a} c) b_k - (\bar{a} b) c_k] \delta_l^i. \tag{65}$$

One readily finds that

$$\bar{A} A \propto (\bar{a} a)(\bar{b} b)(\bar{c} c) - (\bar{a} a)(\bar{b} c)(\bar{c} b) + \frac{1}{N_c - 1} (\bar{a} c) (\bar{b} a)(\bar{c} b) + \frac{1}{N_c - 1} (\bar{a} b) (\bar{c} a)(\bar{c} b) + \frac{1}{N_c - 1} (\bar{a} a)(\bar{b} b)(\bar{c} c) + \frac{1}{N_c - 1} (\bar{a} c)(\bar{b} b)(\bar{c} a) + \frac{1}{N_c - 1} (\bar{a} b)(\bar{c} a)(\bar{c} c). \tag{66}$$

In the quark representation the Casimir operator equals

$$(T_b + T_c - T_a)^2 = 3C_F + 2(T_b T_c) - 2(T_a T_b) - 2(T_c T_a). \tag{67}$$

The six color-flow diagrams generated by the expansion (66) are shown in Fig. 11. The straightforward calculation of the corresponding traces, putting the $T_i$ on the relevant horizontal lines in the loops gives

$$C_6 = \frac{3N_c + 1}{N_c + 1} C_F \tag{68}$$

The similar expansion for the 15-plet state reads

$$\bar{S} S \propto (\bar{a} a)(\bar{b} b)(\bar{c} c) + (\bar{a} a)(\bar{b} c)(\bar{c} b) + \frac{1}{N_c + 1} (\bar{a} c)(\bar{b} a)(\bar{c} b) + \frac{1}{N_c + 1} (\bar{a} b)(\bar{c} a)(\bar{c} b) + \frac{1}{N_c + 1} (\bar{a} c)(\bar{b} b)(\bar{c} a) + \frac{1}{N_c + 1} (\bar{a} b)(\bar{c} a)(\bar{c} c) \tag{69}$$

and

$$C_{15} = \frac{3N_c - 1}{N_c - 1} C_F. \tag{70}$$

This completes the check of the formulas (63).
FIG. 11: The color-flow diagrams for the derivation of the Casimir operator $C_F$ for sextet and 15-plet $qg$ states in the quark-antiquark representation.

F. The $N_c \to -N_c$ transformation between the sextet and 15-plet matrix elements

As a function of $N_c$, the Casimir operators and matrix elements for transitions containing the sextet and 15-plet states satisfy a curious symmetry

\begin{align*}
C_{15}(N_c) &= C_6(-N_c), \\
\Sigma_{33}(N_c) &= \sigma_{22}(-N_c), \\
\Sigma_{13}(N_c) &= -\sigma_{12}(-N_c).
\end{align*}

(71)

Evidently, the relative minus sign in the last line of (71) is a matter of convention for the basis states. We do not offer any straightforward group-theoretic explanation for this transformation (see, however, a discussion of the correspondence between the symmetric and antisymmetric representations in Cvitanovic's lectures [28]).
G. Large-$N_c$ properties of $\Sigma$

The application of the above derived $\hat{\Sigma}$ to the dijet production of the free-nucleon target is straightforward. In the case of the nuclear target one has to solve the secular equation for the eigenvalues and eigenstates of $\hat{\Sigma}$. It is a cubic equation, can be solved in radicals and the corresponding eigenfunctions are directly calculable. The further application of the Sylvester expansion \[4\] to (31) is straightforward. Unfortunately, because of the radicals the relevant Fourier transforms in (14) can only be performed numerically. Simple algebraic formulas for eigenvalues and analytic results for the dijet spectra are, however, obtained in the large-$N_c$ approximation. The higher order terms of expansion in inverse powers of $N_c$ can also be presented in an analytic form [4].

Note, that for large $N_c$

$$
\Sigma_{31} = \Sigma_{21} = \frac{1}{N_c\sqrt{2}}\Omega, \\
\Sigma_{32} = \frac{1}{2}\Omega, \\
\Sigma_{33} = \Sigma_{22} = 2\sigma(s - r' + r) + \sigma(s) - \frac{1}{2}\Omega,
$$

which shows that one must first diagonalize the matrix $\hat{\Sigma}$ in the $|6\bar{6}\rangle, |15\bar{15}\rangle$ sector. The two eigenvalues are

$$
\Sigma_{2,3} = \sigma_{22} \pm \frac{1}{2}\Omega
$$

and the corresponding eigenstates are

$$
|2\rangle = \frac{1}{\sqrt{2}}(|6\bar{6}\rangle + |15\bar{15}\rangle) = \frac{V_1}{N_c^{3/2}}, \\
|3\rangle = \frac{1}{\sqrt{2}}(|6\bar{6}\rangle - |15\bar{15}\rangle) = \frac{V_2}{N_c^{3/2}}.
$$

In the basis of the states $|1\rangle = |33\rangle$, $|2\rangle$ and $|3\rangle$ the matrix $\hat{\Sigma}$ takes the form ($\Sigma_1 = \Sigma_{11}$)

$$
\hat{\Sigma} = \begin{pmatrix}
\Sigma_1 & \frac{1}{N_c}\Omega & 0 \\
\frac{1}{N_c}\Omega & \Sigma_2 & 0 \\
0 & 0 & \Sigma_3
\end{pmatrix},
$$

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\[ \Sigma_1 = \sigma(s + r - r') + \sigma(r) + \sigma(r') , \]
\[ \Sigma_2 = 2\sigma(s + r - r') + \sigma(s) = C_2\sigma(s + r - r') + \sigma(s) . \]  

(76)

Here we show an explicit dependence on the Casimir operator for the large-\(N_c\) eigenstate
\[ C_2 + 1 = \frac{C_6}{C_F} = \frac{C_{15}}{C_F} = 3 . \]
(77)

As a matter of fact, at large \(N_c\) the quark and gluon colors in the sextet and 15-plet states become decorrelated, so that \(C_6 = C_{15} = C_A + C_F\) and
\[ C_2 = \frac{C_A}{C_F} . \]
(78)

To the leading order in the \(1/N_c\) expansion, the state \(|3\rangle\) is not excited by single-gluon exchange from the initial state \(|1\rangle = |3\rangle\). This decoupling is obvious also from the projection onto the final states (71), which at large \(N_c\) reads
\[ \sum_X \langle X | = \sum_R \sqrt{\text{dim}(R)} \langle R \bar{R} | = \sqrt{N_c} \langle 1 | + (\sqrt{N_c})^3 \langle 2 | . \]
(79)

In the new basis the non-Abelian intranuclear evolution of the 4-body \(qqqg\) state becomes the two-channel problem. Expansion over the final states takes the form
\[ \sum_X \langle X | S[b, \dot{\Sigma}] | 1 \rangle = \sqrt{N_c} \langle 1 | \exp \left[ -\frac{1}{2} \dot{\Sigma}^T \right] | 1 \rangle + (\sqrt{N_c})^3 \langle 2 | \exp \left[ -\frac{1}{2} \dot{\Sigma}^T \right] | 1 \rangle . \]
(80)

To the leading order in \(N_c\), matrix element of the \(S\)-matrix in the first term equals
\[ \langle 1 | \exp \left[ -\frac{1}{2} \dot{\Sigma}^T \right] | 1 \rangle = \exp \left[ -\frac{1}{2} \Sigma_1 T \right] = \exp \left\{ -\frac{1}{2} [\sigma(s) + \sigma(r) + \sigma(r')]T \right\} . \]
(81)

Making use of the Sylvester expansion technique used in [4], for the second matrix element one finds
\[ \langle 2 | \exp \left[ -\frac{1}{2} \dot{\Sigma}^T \right] | 1 \rangle = \Omega \cdot \frac{1}{N_c} \cdot \frac{\exp \left[ -\frac{1}{2} \Sigma_1 T \right] - \exp \left[ -\frac{1}{2} \Sigma_2 T \right]}{\Sigma_2 - \Sigma_1} . \]
(82)

The integral representation of Ref. [4],
\[ \exp \left[ -\frac{1}{2} \Sigma_1 T \right] - \exp \left[ -\frac{1}{2} \Sigma_2 T \right] \]
\[ \frac{\Sigma_2 - \Sigma_1}{\Sigma_2 - \Sigma_1} = \frac{1}{2} T \int_0^1 d\beta \exp \left[ -\frac{1}{2} \beta \Sigma_1 + (1 - \beta)\Sigma_2 T \right] , \]
(83)
makes explicit a decomposition into the Initial State and Final State distortions described by the cross sections $\Sigma_1$ and $\Sigma_2$, respectively. Our final result for the sum over final states reads

$$\sum_X \langle X | S[b, \hat{\Sigma}] | 1 \rangle = \sum_X \langle X | \exp \left[ -\frac{1}{2} \Sigma T \right] | 1 \rangle =$$

$$\sqrt{N_c} \left\{ \exp \left[ -\frac{1}{2} \left[ \sigma(s + r - r') + \sigma(r) + \sigma(r') \right] T \right] +$$

$$+ \Omega \cdot T \int_0^1 d\beta \exp \left[ -\frac{1}{2} \left( \beta \Sigma_1 + (1 - \beta) \Sigma_2 \right) T \right] \right\}.$$  \hfill (84)

The systematic approach to perturbation $1/N_c$ expansion in has been developed in [4] on an example of quark-antiquark dijets in DIS. Its extension to quark-gluon dijets is straightforward, we will not dwell into that in this communication.

V. THE LINEAR $k_\perp$-FACTORIZATION FOR DIJETS FORM THE FREE NUCLEON TARGET

The $S$-matrices in the master formula depend only on the dipole parameters $s, r, r'$. In the case of the free nucleon target one can integrate over the overall impact parameter and represent the integrand of Eq. in terms of the dipole cross sections:

$$2 \int d^2b \sum_X \langle X | \left\{ S_{qg'g}^{(4)}(b', b_g, b_q, b_g) + S_{qg}^{(2)}(b', b) - S_{qg'g}^{(3)}(b, b_g, b_g') - S_{qg}^{(3)}(b', b_g, b_g) \right\} | in \rangle =$$

$$= \sigma_{q'g} + \sigma_{qg'} - \Sigma_{11} + \sqrt{\frac{\text{dim}(6)}{\text{dim}(3)}} \Sigma_{21} + \sqrt{\frac{\text{dim}(15)}{\text{dim}(3)}} \Sigma_{31}$$

$$= \frac{C_A}{C_F} \left[ \sigma(s + r - zr') + \sigma(s + zr - r') - \sigma(s + r - r') - \sigma(s + zr - zr') \right]$$

$$- \frac{1}{N_c^2} \left[ \sigma(s - zr') + \sigma(s + zr) - \sigma(s) - \sigma(s + zr - zr') \right]$$

$$+ \frac{C_A}{C_F} \Omega.$$  \hfill (85)

Now we apply the $k_\perp$-factorization representation for the free-nucleon dipole cross section. For instance, one readily finds

$$\Omega = \int d^2 \kappa f(\kappa) [1 - \exp(\im \kappa r)][1 - \exp(-\im \kappa r')] \exp(\im \kappa s).$$  \hfill (86)
The momentum space wave function of the $qg$ Fock state of the physical quark is defined by the Fourier transform

$$\Psi(z, p) = \int d^2 r \Psi(z, r) \exp(-i p r).$$  \hspace{1cm} (87)

We discuss the cross sections averaged over the helicities of the incident parton and summed over helicities of the final-state partons. Then $\Psi(z, p)$ would always enter in combinations of the form

$$|\Psi(z, p) - \Psi(z, p - \kappa)|^2 = 2 N_c \alpha_s \left( (Q^*)^2 \right) P_{gq}(z) \cdot \left( \frac{p}{p^2 + \varepsilon^2} - \frac{p - \kappa}{(p - \kappa)^2 + \varepsilon^2} \right)^2,$$  \hspace{1cm} (88)

where $P_{gq}(z)$ is the familiar splitting function,

$$P_{gq}(z) = C_F \frac{1 + (1 - z)^2}{z},$$  \hspace{1cm} (89)

and, neglecting the mass of the incident light quark,

$$\varepsilon^2 = z(1 - z)(Q^*)^2,$$  \hspace{1cm} (90)

where $(Q^*)^2 = (p^*)^2$ is the virtuality of the incident quark $q^*$, given by the square of its transverse momentum in the projectile hadron. If $\varepsilon^2$ is negligible small compared to $p^2$, then one can use the large-$p$ approximation,

$$\left( \frac{p}{p^2} - \frac{p - \kappa}{(p - \kappa)^2} \right)^2 = \frac{\kappa^2}{p^2(p - \kappa)^2},$$  \hspace{1cm} (91)

and it is worth to recall the emerging exact factorization of longitudinal and transverse momentum dependencies which is a well known feature of the high energy limit.

Then the master formula for the free-nucleon cross section takes the form

$$\frac{d\sigma_N(q^* \rightarrow qg)}{dz d^2 p_{q} d^2 p_{g}} = \frac{1}{2(2\pi)^4} \int d^2 \kappa f(\kappa)$$

$$\times \int d^2 s d^2 r d^2 r' \exp[-i \Delta s - i p r + i p r'] \exp(i \kappa s) \Psi(z, r) \Psi^*(z, r')$$

$$\times \left\{ \frac{C_A}{2C_F} \left[ 1 - \exp(i \kappa r) \right] \left[ 1 - \exp(-i \kappa r') \right] \right.$$  

$$+ \frac{C_A}{2C_F} \left[ \exp(i z \kappa r) - \exp(i \kappa r) \right] \left[ \exp(-i z \kappa r') + \exp(-i \kappa r') \right]$$  

$$- \frac{1}{N_c^2 - 1} \left[ 1 - \exp(i z \kappa r) \right] \left[ 1 - \exp(-i \kappa r') \right] \right\}.$$
\[
\frac{1}{2(2\pi)^2} f(\Delta) \left\{ \frac{C_A}{2C_F} |\Psi(z, p) - \Psi(p - \Delta)|^2 + \frac{C_A}{2C_F} |\Psi(z, p - \Delta) - \Psi(p - z\Delta)|^2 - \frac{1}{N_c^2 - 1} |\Psi(z, p) - \Psi(p - z\Delta)|^2 \right\} \tag{92}
\]

A direct comparison shows that the dijet spectrum (92) is precisely the differential form of the inclusive single gluon spectrum from the excitation \( q^* \rightarrow qg \) which was derived in [6]. The reason emphasized in [16] is that the excitation \( q^* \rightarrow qg \) proceeds via one-gluon exchange and the acoplanarity momentum is precisely the transverse momentum of the exchanged gluon. Remarkably, the color dipole structure of the integrand of the dijet cross section only differs from the one for the single-jet spectrum by the shift of arguments of all the dipole cross sections by \( s \).

The free-nucleon cross-section is a linear functional of the unintegrated gluon density. Then, with certain reservations on the region of soft \( \Delta \), the acoplanarity distribution is a direct probe of \( f(x, \Delta) \). First, on the pQCD side, the unintegrated gluon density \( f(x, \Delta) \) is well-defined only for sufficiently large momenta \( \Delta \) above the soft scale. Second, from the practical point of view, any definition of the jet momentum has an intrinsic uncertainty with whether the soft hadron belongs to the jet or to the underlying soft event, so that experimentally the acoplanarity momentum is well-defined only when it is above the soft scale.

VI. THE NONLINEAR \( k_t \)-FACTORIZATION FOR THE DIJET PRODUCTION OFF NUCLEI

A. The color-dipole representation at large \( N_c \)

The final Fourier representation for the leading term of the large-\( N_c \) expansion for the dijet cross section per unit area in the impact parameter space reads

\[
\frac{d\sigma(q^* \rightarrow qg)}{d^2b dz d^2\Delta d^2p} = \frac{1}{(2\pi)^4} \int d^2s d^2r d^2r' \times \exp[-i\Delta s - ipr + ip'r']|\Psi(z, r)\Psi^*(z, r')| \left\{ \frac{1}{2} \Omega \cdot T(b) \right\} d\beta \exp \left[ -\frac{1}{2} [\beta \Sigma_1 + (1 - \beta) \Sigma_2] T(b) \right]
\]
Recall that the first term, $\propto \Omega$, describes the excitation from the color-triplet dipole to sextet and 15-plet dipole states. Note, how the large-$N_c$ suppression of the off-diagonal matrix element $\Sigma_{12}$ is compensated for by a large number of final states in the higher representations. At large $N_c$, once the sextet and 15-plet states have been excited, their de-excitation back to the triplet state is suppressed and the further intranuclear evolution consists of the color rotations within the higher representations. The remaining four terms in (93) describe the rotations within the color triplet states.

\begin{equation}
+ \exp \left[ -\frac{1}{2} \sigma (s + r - r') + \sigma (r) + \sigma (r') T(b) \right] \\
+ \exp \left[ -\frac{1}{2} \sigma (s - z r' + z r) T(b) \right] \\
- \exp \left[ -\frac{1}{2} \sigma (r) + \sigma (s + r - z r') T(b) \right] \\
- \exp \left[ -\frac{1}{2} \sigma (r') + \sigma (s - r' + z r) T(b) \right] \right\} \quad (93)
\end{equation}

The utility of $\phi(b, x, \kappa)$ stems from the observation that the driving term of small-$x$ nuclear structure functions, the amplitude of coherent diffractive production of dijets off nuclei and the single-quark spectrum from the $\gamma^* \rightarrow q\bar{q}$ excitation off a nucleus all take the familiar $k_{\perp}$-factorization form in terms of $\phi(b, x, \kappa)$. The so defined collective nuclear glue $\phi(b, x, \kappa)$ satisfies the sum rule

\begin{equation}
\int d^2 \kappa \phi(b, x, \kappa) = 1 - \Sigma[b, \sigma_0(x)],
\end{equation}

where $\sigma_0(x)$ is the dipole cross section for large dipoles. The multiple-scattering expansion of $\phi(b, x, \kappa)$ in terms of the collective glue of $j$-overlapping nucleons in the
Lorentz-contracted nucleus and its nuclear shadowing and antishadowing properties are found in \[4, 8, 29, 30\] and need not be repeated here. We only cite the formula for the so-called saturation scale

\[ Q^2_A(b, x) \approx \frac{4\pi^2}{N_c} \alpha_s(Q^2_A) G(x, Q^2_A) T(b) \] (96)

and reiterate that at a large saturation scale \( \phi(b, x, \kappa) \) is well-defined not only for perturbative values of \( \kappa^2 \) below \( Q^2_A(b, x) \), its continuation to the soft region is also stable. To this end we recall that although \( \sigma_0(x) \) enters the multiple-scattering expansion for \( \phi(b, x, \kappa) \), the final form of \( \phi(b, x, \kappa) \) is exclusively controlled by \( Q^2_A(b, x) \) and does not depend on the auxiliary soft parameter \( \sigma_0(x) \) \[4\]. We also note, that the nuclear profile function satisfies the \( s \)-channel unitarity bound for the partial waves of the dipole-nucleus scattering, \( \Gamma[b, \sigma(x, r)] \leq 1 \), while the partial wave of the impulse approximation (IA) overshoots the \( s \)-channel unitarity bound for sufficiently heavy nucleus, \( \Gamma^{(IA)}[b, \sigma(x, r)] = \frac{1}{2} \sigma(x, r) T(b) > 1 \). As such, the unintegrated collective nuclear gluon density \( \phi(b, x, \kappa) \) defined by Eq. \(94\) unitarizes the density of partons in a Lorentz-contracted ultrarelativistic nucleus.

Still another convenient quantity is

\[ \Phi(b, x, \kappa) = S[b, \sigma_0(x)] \delta(\kappa) + \phi(b, x, \kappa) \] (97)

in terms of which

\[ \exp[-\frac{1}{2} \sigma(x, r) T(b)] = \int d^2 \kappa \Phi(b, x, \kappa) \exp(i \kappa r). \] (98)

We shall also encounter the collective glue for a slice \( 0 < \beta < 1 \) of the nucleus:

\[ \exp[-\frac{1}{2} \beta \sigma(x, r) T(b)] = \int d^2 \kappa \Phi(\beta; b, x, \kappa) \exp(i \kappa r), \] (99)

and the intranuclear attenuation-distorted wave function in the dipole and momentum representations,

\[ \Psi(\beta, x; z, r) = \Psi(z, r) \exp[-\frac{1}{2} \beta \sigma(x, r) T(b)], \]

\[ \Psi(\beta, x; z, p) = \int d^2 r \Psi(\beta; z, r) \exp(-i p r) = \int d^2 \kappa \Psi(z, p - \kappa) \Phi(\beta; b, x, \kappa). \] (100)

Hereafter, unless it may cause a confusion, we suppress the variable \( x \) in gluon densities, dipole cross sections and distorted wave functions.

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C. Excitation of color-triplet quark-gluon dipoles

First, we rewrite the last four terms in the integrand of (93) in terms of the distorted wave functions. Then we make use of the Fourier representation (98), (100):

\[
\frac{d\sigma(q^* \rightarrow qg)}{d^2b dz d^2p} = \left. \frac{1}{(2\pi)^4} \int d^2s d^2r d^2r' \exp[-i\Delta s - i pr + i pr'] \right|_{\Delta d^2p} = \frac{1}{(2\pi)^4} \int d^2s d^2r d^2r' \kappa_0(b, \kappa) \exp[-i\Delta s - i pr + i pr']
\]

\[
\left\{ \begin{array}{l}
\Psi(1; z, r)\Psi^*(1; z, r') \exp \left[ -\frac{1}{2} \sigma(s + r - r')T(b) \right] \\
+ \Psi(z, r)\Psi^*(z, r') \exp \left[ -\frac{1}{2} \sigma(s - z r' + zr)T(b) \right] \\
- \Psi(1; z, r)\Psi^*(z, r') \exp \left[ -\frac{1}{2} \sigma(s + r - z r')T(b) \right] \\
- \Psi(z, r)\Psi^*(1; z, r') \exp \left[ -\frac{1}{2} \sigma(s - r' + zr)T(b) \right]
\end{array} \right.
\]

\[
= \frac{1}{(2\pi)^2} \Phi(b, \Delta) \left| \Psi(1; z, p - \Delta) - \Psi(z, p - z \Delta) \right|^2
\]

\[
= \frac{1}{(2\pi)^2} \phi(b, \Delta) \left| \Psi(1; z, p - \Delta) - \Psi(z, p - z \Delta) \right|^2
\]

\[
+ \frac{1}{(2\pi)^2} \delta(\Delta) \left| \Psi(1; z, p) - \Psi(z, p) \right|^2 S[b, \sigma_0(x)]
\]

Now recall [8] that the amplitude of the coherent diffractive excitation \( qA \rightarrow (qg)A \) is precisely proportional to

\[
\Psi(z, p) - \Psi(1; z, p) = \int d^2r \Psi(z, r) \left\{ 1 - \exp \left[ -\frac{1}{2} \sigma(r)T(b) \right] \exp[-i pr] \right\},
\]

so that the last term in (101) describes the coherent diffractive production of dijets. In the approximation of very large nucleus the diffractive dijets are produced exactly back-to-back. For finite nuclei instead of the delta-function \( \delta(\Delta) \) one finds the sharp peak of the width \( \Delta^2 \lesssim 1/R_A^2 \) which is described by the form factor of the nucleus, the details
are found in [8] and must not be repeated here. The former term describes inelastic, incoherent production of color-triplet $qg$ states.

D. The contribution from sextet and 15-plet final states

The evaluation of the contribution from the excitation of higher color representations in (93) proceeds as follows. First, we make use of the integral representation (86) for the off-diagonal cross section. Second, keeping an explicit dependence on the Casimir operators $C_6$, $C_{15}$, we have

$$
\int_0^1 d\beta \exp \left[ -\frac{1}{2} (\beta \Sigma_1 + (1 - \beta) \Sigma_2) T(b) \right] \\
= \int_0^1 d\beta \exp \left\{ -\frac{1}{2} \beta [\sigma(s + r - r') + \sigma(r) + \sigma(r')] T(b) \right\} \\
\times \exp \left\{ -\frac{1}{2} (1 - \beta) [C_2 \sigma(s + r - r') + \sigma(s)] T(b) \right\} \\
= \int_0^1 d\beta \exp \left[ -\frac{1}{2} \beta \sigma(r) T(b) \right] \exp \left[ -\frac{1}{2} \beta \sigma(r') T(b) \right] \\
\times \int d^2\kappa \Phi(\beta; b, \kappa_3) \exp[i\kappa_3(s + r - r')] \\
\times \int d^2\kappa_2 \Phi(C_2(1 - \beta); b, \kappa_2) \exp[i\kappa_2(s + r - r')] \\
\times \int d^2\kappa_1 \Phi(1 - \beta; b, \kappa_1) \exp[i\kappa_1 s] 
$$

(103)

In this decomposition we keep the dipole form of the two attenuation factors $S[b, \beta\sigma(r)]$ and $S[b, \beta\sigma(r')]$. They describe the coherent intranuclear distortion of the color-triplet quark-gluon dipole before the excitation into the sextet and 15-plet representations at the depth $\beta$ from the front face of the nucleus. The way to handle these distortion factors has already been clarified above. Note, that in contrast to the quark-antiquark dijet production in DIS off nuclei, both the ISI and FSI distortion factors depend on the dipole parameter $s$ and explicitly contribute to the acoplanarity distribution.

Combining together (86), (100) and (103) we obtain the dijet spectrum from the excitation of the sextet and 15-plet dipoles

$$
\frac{d\sigma(q^* \rightarrow qg)}{d^2bdzd\Delta d^2p}_{6+15} = \frac{1}{(2\pi)^2} T(b) \int_0^1 d\beta \\
\times \int d^2\kappa d^2\kappa_1 d^2\kappa_2 d^2\kappa_3 \delta(\kappa + \kappa_1 + \kappa_2 + \kappa_3 - \Delta)
$$

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The acoplanarity momentum $\Delta$ manifestly receives four distinct contributions which can be classified as follows. The excitation from the color-triplet to the sextet and 15-plet states by single-gluon exchange with one of the nucleons of the nucleus contributes the transverse momentum $\kappa$. The momentum $\kappa_3$ comes from the ISI of the incident quark, the FSI of the $qg$ dipole in the sextet and 15-plet representations contributes $\kappa_1$ and $\kappa_2$.

The emergence of the collective nuclear glue $\Phi(C_2(1 - \beta); b, \kappa_2)$ in the integrand of (104) is not accidental. While $(1 - \beta)$ is a thickness of the slice of the nuclear matter traversed by the sextet and 15-plet $qg$ dipoles, the factor $C_2$ derives from the Casimir operators of higher representations, see Eq. (77). That is one more illustration of our point [4, 6] that the collective gluon field of the nucleus cannot be described by a single density function, it is a density matrix in the space of color representations. In the considered large-$N_c$ approximation, $C_2 = C_A/C_F$ and $\Phi((1 - \beta)C_A/C_F; b, \kappa_2)$ is precisely the collective nuclear glue defined in terms of the color-singlet gluon-gluon dipole.

The ISI and FSI distortions can partly be combined taking the convolution [4]

$$\int d^2\kappa_3 d^2\kappa_2 \Phi(C_2(1 - \beta); b, \kappa_2) \Phi(\beta; b, \kappa_3) \delta(\kappa - \kappa_2 - \kappa_3) = \Phi(\beta + C_2(1 - \beta); b, \kappa).$$

(105)

which is also obvious from the color-dipole form in (103).

E. Nonlinear $k_\perp$-factorization for dijets: the universality classes

1. Quark-gluon vs. quark-antiquark dijets

After application of the convolution (105), the final result for the inclusive dijet spectrum takes the form

$$\frac{(2\pi)^2 d\sigma_A(q^* \rightarrow qg)}{d^2b dz d^2pd^2\Delta} = \frac{1}{2} T(b) \int_0^1 d\beta \int d^2\kappa_1 d^2\kappa f(x, \kappa) \times \Phi(1 - \beta, b, x, \Delta - \kappa_1 - \kappa) \Phi(\beta + C_2(1 - \beta), b, x, \kappa_1)$$

$$\times |\Psi(\beta; z, p - \kappa_1) - \Psi(\beta; z, p - \kappa_1 - \kappa)|^2$$
\begin{align}
+ \phi(b, x, \Delta)\left|\Psi(1; z, p - \Delta) - \Psi(z, p - z\Delta)\right|^2 \\
+ \delta(\Delta) S[b, \sigma_0(x)]\left|\Psi(1; z, p) - \Psi(z, p)\right|^2.
\end{align}

which must be compared to the large-\(N_c\) version of the free-nucleon cross section \(92\).

The free-nucleon cross-section is a linear functional of the unintegrated gluon density. The \(k_\perp\)-factorization properties of the nuclear cross section are much more complicated. At this point, it is instructive to discuss \(106\) in conjunction with the quark-antiquark dijet spectrum in DIS \(4\) and gluon-nucleus collisions \(16\). The spectrum of dijets in DIS equals

\begin{align}
\left(2\pi\right)^2 \frac{d\sigma_A(\gamma^* \rightarrow Q\bar{Q})}{d^2 p d^2 b d^2 \Delta} = \frac{1}{2} T(b) \int_0^1 d\beta \int d^2 \kappa_1 d^2 \kappa \times f(\kappa) \Phi(1 - \beta; b, \Delta - \kappa_1 - \kappa) \Phi(1 - \beta; b, \kappa_1) \\
\times \left|\Psi(\beta; z, p - \kappa_1) - \Psi(\beta; z, p - \kappa_1 - \kappa)\right|^2 \\
+ \delta(\Delta) \left|\Psi(1; z, p) - \Psi(z, p)\right|^2. \tag{107}
\end{align}

where the first term describes the excitation of the color-dipole from the lower (color-singlet) to higher (octet) representation, whereas the second term is the contribution from coherent diffractive excitation. The spectrum of the quark-antiquark dijets in \(gA\) collisions is of the form

\begin{align}
\left(2\pi\right)^2 \frac{d\sigma_A(g^* \rightarrow Q\bar{Q})}{d^2 z d^2 p d^2 b d^2 \Delta} = \int d^2 \kappa \Phi(1; b, \kappa) \Phi(1; b, \Delta - \kappa) \\
\times \left|\Psi(z, p - \kappa) - \Psi(z, p - z\Delta)\right|^2. \tag{108}
\end{align}

Now we can identify the four universality classes of the nonlinear \(k_\perp\)-factorization which differ by the pattern of transitions between the initial and final state color multiplets. They describe the leading transitions in the large-\(N_c\) approximation, the higher order excitation and regeneration processes result in still higher nonlinearity in gluon densities, the examples are found in \(4\).

2. Excitation of higher color representations from partons in the lower representations

Excitation of color-octet states in DIS, and of sextet and 15-plet states in \(qA\) interactions, belong to this universality class. The two reactions have much similarity. In
both cases the nonlinear $k_\perp$-factorization formulas contain the free-nucleon gluon density $f(x, \kappa)$, which describes the transition from the $qg$ color dipole from the lower - triplet for $qg$ and singlet for DIS - to higher - sextet and 15-plet for $qg$ and octet in DIS - color dipoles. In both cases, the number of states in higher representations is by the factor $N_c^2$ larger than in the lower representation. In $q\bar{q}$ excitation in DIS the corresponding contribution to the dijet spectrum is the fifth order functional of gluon densities. In the $qg$ case it is the sixth order functional of gluon densities, only after the application of the convolution (105) it takes the form of the fifth order functional. Two powers of the collective nuclear glue enter implicitly via the coherent ISI distortions of the wave function $\Psi(\beta; z, p)$ in the slice of the nuclear matter before excitation of color dipoles in the higher representation, two more powers of the collective nuclear glue describe the ISI and FSI broadening of the acoplanarity distribution.

The principal difference between DIS and $qA$ interactions is in the nuclear thickness dependence of the distortion factors. Namely, the factor

$$\Phi((1 - \beta), b, \Delta - \kappa_1 - \kappa)\Phi((1 - \beta), b, \kappa_1)$$

in DIS is the symmetric function of the nuclear gluon momenta $\kappa_1$ and $\kappa_2 = \Delta - \kappa_1 - \kappa$ which flow from the nucleus to the quark and antiquark (or vice versa), respectively. It describes equal, and uncorrelated, distortion of the outgoing quark and antiquark waves by pure FSI. The independence of the two distortion factors is a feature of the large $N_c$ approximation. For $qg$ dijets in $qA$ collisions the distortion factor

$$\Phi(1 - \beta, b, \kappa_2)\Phi(C_2(1 - \beta) + \beta, b, \kappa_1)$$

is an asymmetric one. The first source of the asymmetry is the non-singlet color charge of the projectile parton. The second source is that the two partons in the final state belong to different color representations. This is best seen from in the overall distortion factor in (104),

$$\Phi(\beta; b, \kappa_3)\Phi(C_2(1 - \beta); b, \kappa_2)\Phi(1 - \beta; b, \kappa_1),$$

before taking the convolution (105). The FSI distortions in the slice $(1 - \beta)$ of the nucleus are given by the two last factors, of which $\Phi(1 - \beta; b, \kappa_1)$ is a broadening due to final-state rescatterings of the quark. Because $C_2 = C_A/C_F$, see Eq. (78), the second
FSI factor, $\Phi(C_2(1-\beta); b, \kappa_2)$, corresponds to the FSI distortion of exactly the outgoing gluon wave. To the large-$N_c$ approximation the rescatterings of the quark and gluon are uncorrelated.

The coherent ISI distortion of the wave functions in DIS and $qA$ collisions is identical. However, in $qA$ collisions this coherent distortion is accompanied by an incoherent ISI distortions of the incident quark wave described by $\Phi(\beta; b, \kappa_3)$. In DIS the incoherent ISI distortions are absent because the photon is a color-singlet particle. We can anticipate that gluon-nucleus collisions with excitation of gluon-gluon dijets in higher color representations will belong to this universality class.

3. Excitation of final state diopes in exactly the same color state as the incident parton: coherent diffraction

To this universality class belong the exactly back-to-back dijets. Another experimental signature of the coherent diffraction is a retention of the target nucleus in the ground state and large rapidity gap between the hadronic debris of the diffractive dijet and the recoil nucleus. It is most important for DIS where coherent diffraction dissociation of the photon into $q\bar{q}$ dijets makes for heavy nuclei $\approx 50\%$ of the total DIS rate \[36\]. The origin of the coherent diffraction is a coherent nuclear distortion of the wave function of the $q\bar{q}$ Fock state over the whole thickness of the nucleus.

In the coherent diffractive excitation of $gg$ diopes in $qA$ collisions the $gg$ dipole must propagate in exactly the same color state as the incident quark. The nuclear suppression factor $S[b, \sigma_0(x)]$ has the meaning of

$$S[b, \sigma_0(x)] = \left( S[b, \frac{1}{2}\sigma_0(x)] \right)^2$$

and the factor $S[b, \frac{1}{2}\sigma_0(x)]$ in the diffractive amplitude corresponds to the intranuclear attenuation of the quark wave with the total cross section

$$\sigma_{qN} = \frac{1}{2}\sigma_0(x).$$

Coherent diffractive excitation of color-octet gluon-gluon dijets in gluon-nucleus collisions is expected to exhibit similar properties.

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Coherent diffractive excitation of $Q\bar{Q}$ dipoles in $gA$ collisions is allowed, but it is suppressed at large $N_c$ by the condition that the $Q\bar{Q}$ dipole must propagate in exactly the same color state as the incident gluon.

4. Incoherent excitation of final state dipoles in the same lower color representation as the incident parton

An example of this universality class is an inelastic excitation of color-triplet $qg$ states in $qA$ collisions followed by a color excitation of the target. Here both the incident parton and dijet belong to the fundamental, i.e., lower, representation of $SU(N_c)$. The intranuclear evolution of such a dipole is confined to rotations within the color-triplet state. This contribution is not suppressed at large $N_c$. The dijet cross section for this universality class looks like satisfying the linear $k_\perp$-factorization in terms of $\phi(b,x,\Delta)$. But this is not the case: one of the wave functions, $\Psi(1;z,p_g)$, is coherently distorted over the whole thickness of the nucleus, so that this contribution is a cubic functional of the collective nuclear glue.

We can anticipate that gluon-nucleus collisions with excitation of color-octet gluon-gluon dijets will belong to this universality class, although one has to account for the existence of the two, $F$-coupled and $D$-coupled, octet states.

Although superficially it looks like a subclass of this universality class, the coherent diffraction is a distinct class for the property of the exact back-to-back dijets and the rapidity gap between the dijet and the recoil nucleus in the ground state.

5. Excitation of final state dipoles in the same higher color representation as the incident parton

In the realm of QCD with gluons in the adjoint representation and quarks in the fundamental representation, this universality class consists of the quark-antiquark dijets in gluon-nucleus collisions. Only in this case the initial parton (gluon) belongs to the higher (octet) color multiplet of the final $Q\bar{Q}$ state. At large $N_c$, the intranuclear evolution of $Q\bar{Q}$ will consist of color rotations within the space of color-octet states.
The de-excitation from the color-octet to color-singlet $Q\bar{Q}$ dipoles is suppressed at large $N_c$. Consequently, the non-Abelian evolution of the $Q\bar{Q}Q'\bar{Q'}$ state becomes the single channel problem. The coherent diffraction excitation, in which the initial and final color states must be identical, is likewise suppressed. The emerging pattern of quadratic non-linearity can be related to the large-$N_c$ gluon behaving like the color-uncorrelated quark and antiquark.

The above classification exhausts reactions caused by incident photons, quarks and gluons. However, technically all the universality classes have a much broader basis. Indeed, instead of an incident gluon one can think of the projectile which is a compact lump of many partons in the highest possible color representation. For instance, compact diquarks in the proton can be viewed as sextet partons.

6. **Is an experimental separation of events belonging to different universality classes possible?**

We reiterate that for all the universality classes their separate contributions to the dijet cross section are infrared-safe quantities. Coherent diffraction has distinct signatures and the experimental separation of events from this universality class is not a problem. Production of very forward dijets in proton-nucleus collisions evidently tags quark-nucleus collisions. Production of open charm in the proton hemisphere of proton-nucleus collisions tags gluon-nucleus collisions. Incoherent processes belonging to different universality classes are characterized by distinct color charge of the hard dijet and this distinction is well defined at the parton level. Translating the cross-talk between color charges in the dijet, the spectator partons of the proton and the color-excited nucleus remnant into properties of hadronic final states can only be done within nonperturbative hadronization models. As an example we cite the impact of color reconnection effects on the flow of slow hadrons and the accuracy of the $W^\pm$ mass determination in $e^+e^-$ annihilation ([31], for the review see [32]).
F. The impulse approximation

In the impulse approximation (IA) one only has to keep the terms linear in $T(b)$. The transition to the IA is best seen in the color-dipole representation (93). Recall, that our formulas for nuclear cross section were derived in the large-$N_c$ approximation. Here the first term, the contribution from the sextet and 15-plet final states, is already linear in $T(b)$ and one must put the attenuation factors equal to unity. The remaining four exponentials must be expanded to terms linear in $T(b)$. Then one would find precisely the large-$N_c$ version of Eq. (85) times $T(b)$. The integration over impact parameters gives $\int d^2bT(b) = A$. Such a comparison does not expose the rôle of coherent diffraction and we revisit the issue in the momentum representation.

We start with the sextet and 15-plet contribution in (106). It already contains the factor $T(b)$. Consequently, one must neglect ISI distortions in the wave function, $\Psi(\beta; p) \Rightarrow \Psi(p)$, and take

$$\Phi(1 - \beta, b, x, \Delta - \kappa_1 - \kappa)\Phi(2 - \beta, b, x, \kappa_1) = \delta(\Delta - \kappa_1 - \kappa)\delta(\kappa_1).$$

This way one would recover the first term in the rhs of Eq. (92). In the contribution from the excitation of the triplet dipoles one must neglect the distortion of the wave function and take

$$\phi(b, x, \kappa) = \frac{1}{2}T(b)f(x, \kappa).$$

The second term in the rhs of Eq. (92) is recovered. Finally, according to Eq. (102) the diffractive amplitude starts with the term linear in $T(b)$. Consequently, the coherent diffractive contribution to the dijet cross spectrum starts with the terms $\propto T^2(b)$ and vanishes in the IA.

VII. NUCLEAR BROADENING OF THE ACOPLANARITY DISTRIBUTION

The nuclear broadening of the acoplanarity distribution of hard quark-gluon dijets from $qA$ collisions is somewhat different from the broadening of quark-antiquark jets in DIS and now we comment on those differences.
A. Coherent diffractive contribution

The first striking difference is in the rôle of the coherent diffractive production. It gives exactly back-to-back dijets. In the considered approximation of single-gluon exchange in the $t$-channel diffractive production off the free-nucleon target vanishes. Experimentally, at HERA energies a fraction of DIS which is diffractive does not exceed 10% [33]. In contrast to that, in DIS off heavy nuclei a fraction of coherent diffraction was shown to be as large as $\approx 50\%$ [36]. The existence of coherent diffractive mechanism in the quark-nucleus collisions is interesting by itself. From the practical point of view, it is suppressed by nuclear absorption and is marginal.

B. Excitation of the color-triplet states

Inelastic excitation of color-triplet dipoles is a specific feature of $qA$ collisions in the sense that it has no counterpart in DIS. One must compare

$$\phi(b, x, \Delta) \left| \Psi(1; z, p - \Delta) - \Psi(z, p - z\Delta) \right|^2$$

with its IA form

$$\frac{1}{2} T(b)f(\Delta)|\Psi(z, p - \Delta) - \Psi(p - z\Delta)|^2.$$  \hspace{1cm} (114)

The first striking distinction is that that for the free-nucleon target the contribution of this process vanishes at $z \to 1$, when the incident quark’s momentum is transferred entirely to the forward gluon jet. For the nuclear target this is not the case because one of the wave functions in (113) is the nuclear-distorted one. Because $p - \Delta = -p_q$, it takes the form $\phi(b, x, \Delta) \left| \Psi(1; z, p_q) - \Psi(z, p_q) \right|^2$; as a function of the quark-jet momentum, it is reminiscent of the coherent diffractive contribution, but the acoplanarity momentum distribution is given by the unintegrated nuclear gluon density $\phi(b, x, \Delta)$. Hereafter we consider the case of finite $(1 - z)$.

A comprehensive discussion of nuclear properties of the ratio

$$R_g(b, \Delta) = \frac{2\phi(b, \Delta)}{T(b)f(\Delta)}$$  \hspace{1cm} (115)
is found in \[4, 6\]. It is nuclear-shadowed, \( R_g(b, \Delta) < 1 \), for \( \Delta^2 \lesssim Q_A^2(b) \) and it exhibits antishadowing property, \( R_g(b, \Delta) > 1 \) in a broad region of \( \Delta^2 \gtrsim Q_A^2(b) \). The maximum value of \( R_g(b, \Delta) \) is reached at a value of \( \Delta^2 \) which is larger than \( Q_A^2(b) \) by a large numerical factor.

Now we turn to distortions of the wave function. We are interested in hard dijets. If the incident quark is a valence quark of the proton, its transverse momentum and virtuality have the hadronic scale and can be neglected. For hard jets

\[
\Psi(z, p) \propto \frac{p}{p^2}
\]

and, upon averaging over the azimuthal angle \( \varphi \) of the gluon momentum \( \kappa \),

\[
\langle \Psi(z, p - \kappa) \rangle_{\varphi} \propto \frac{p}{p^2} \theta(p^2 - \kappa^2).
\]

Consequently, the wave function distortion factor equals

\[
\rho_{\psi}(b, z, p) = \frac{\Psi(1; z, p)}{\Psi(z, p)} = \int d^2\kappa \Phi(b, \kappa) = 1 - \int d^2\kappa \Phi(b, \kappa).
\]

For the weakly virtual incident quark it does not depend on \( z \). For hard jets, \( p^2 \gtrsim Q_A^2(b) \), the remaining integral \((118)\) can be evaluated following the analysis of the Cronin effect in \[6\]. Namely, here we can use the leading-twist approximation,

\[
\Phi(b, \kappa) = \frac{1}{2} T(b) f(k),
\]

and the definition \((20)\) with the result

\[
\delta_{\psi} = 1 - \rho_{\psi}(b, z, p) = \int \frac{d^2\kappa \Phi(b, x, \kappa)}{\alpha_s(p^2)} \left. \frac{\partial G(x, p^2)}{\partial \log p^2} \right| \frac{1}{2} Q_A^2(b) \frac{\alpha_s(p^2)}{\alpha_s(Q_A^2) G(x, Q_A^2)} \left. \frac{\partial G(x, p^2)}{\partial \log p^2} \right| \frac{2\pi^2 T(b) \alpha_s(p^2)}{N_c p^2}.
\]

It is important that \( \delta_{\psi} \) is a manifestly positive valued quantity. It has a form similar to, but is numerically smaller than, the nuclear higher twist correction to \( \phi(b, x, \kappa) \).

In Fig. 12 we show the numerical results for the wave-function distortion factor for the gold nucleus at several values of the optical thickness \( \nu(b) = \frac{1}{2} \sigma_0(x) T(b) \). At this point one needs to pay a due attention to an explicit dependence on the QCD running coupling \( \alpha_s(r) \) on the small dipole size \( r \) in Eq. \((20)\). The discussion of its impact is
FIG. 12: The left panel shows the impact-parameter dependence of the optical thickness of the gold nucleus for several values of the gluon-jet momentum $p$. The momentum dependence of the wave-function distortion factor $\rho_\psi(b, z, p)$ for several values of the optical thickness of the nucleus is presented in the right panel.

found in [4, 8], in the evaluation of the momentum spectra this running coupling must be taken at the largest relevant hard parameter, which in our case is $p^2$. Correspondingly, in all the formulas for the dijet spectra, the dipole cross section for large dipoles, $\sigma_0(x)$, must be understood as

$$\sigma_0(x) \Rightarrow \alpha_S(p^2) \cdot \frac{4\pi^2}{N_c} \int \frac{d\kappa^2}{\kappa^4} \cdot \mathcal{F}(x, \kappa^2) = \alpha_S(p^2)\sigma_0(x, \infty).$$  \hspace{1cm} (121)

For this reason, the optical thickness of the nucleus $\nu(b)$ as a function of the impact parameter $b$, shown in the left panel of Fig. 12 depends on the hard scale - the jet momentum. The wave-function distortion factor $\rho_\psi(b, z, p)$ is shown in the right panel of Fig. 12. The hard regime [120] for $\delta_\psi$ sets in at the momenta $p \gtrsim 1$ GeV. We reiterate that the saturated cross section $\sigma_0(x, \infty)$ is only an auxiliary parameter which does not enter directly the observable cross sections - the latter only depend on the saturation scale $Q_A^2(b)$, the discussion is found in Ref. [4].
In terms of the distortion factor \( \rho_\psi(b, p) \) one readily finds
\[
R_\psi(b, p, \Delta) = \frac{|\Psi(1; z, p - \Delta) - \Psi(z, p - z\Delta)|^2}{|\Psi(z, p - \Delta) - \Psi(p - z\Delta)|^2} = \frac{|(1 - z)\Delta - \delta_\psi(p - z\Delta)|^2}{(1 - z)^2\Delta^2} = \frac{|(1 - z)\Delta + \delta_\psi(p_q - (1 - z)\Delta)|^2}{(1 - z)^2\Delta^2}
\]
(122)

The overall nuclear modification factor, the ratio of the nuclear, (113), and free-nucleon, (114), target contributions, is a product
\[
R_{A/N}^{(3)}(b, p, \Delta) = R_g(b, \Delta) R_\psi(b, p, \Delta)
\]
(123)

Here \( R_g(b, \Delta) \) does not depend on the jet momentum \( p \) except for the weak dependence through \( \alpha_S(p^2) \). Evidently, \( R_\psi(b, p, \Delta) \) is azimuthally asymmetric and favors \( \Delta \) anticollinear to the gluon momentum and collinear to the quark momentum: in the back-to-back configuration, the gluon jet tends to have the transverse momentum smaller than the quark jet. The dominant contribution to the nuclear dijet cross section comes from \( \Delta^2 \sim Q_A^2(b) \), and for hard dijets the asymmetry will be weak, of the order of \( \sqrt{\delta_\psi} \sim Q_A(b)/p \).

Alternatively, if one keeps the quark transverse momentum fixed and increases the target mass number \( A \), i.e., \( Q_A^2(b) \) and \( \delta_\psi \) thereof, the transverse momentum of the away gluon jet will decrease with \( A \). The form of the \( q \to qg \) splitting function favors production of the gluon jet at rapidities smaller than the quark jet. Then, the above correlation between the acoplanarity and quark momenta shall exhibit itself as a nuclear suppression of the away jet produced at rapidities smaller than the rapidity of the forward trigger jet. The numerical studies of this effect will be reported elsewhere.

C. Excitation of the sextet and 15-plet jets states

Here one must compare the contribution to the nuclear dijet spectrum (107) with its IA counterpart
\[
\frac{\left. T(b)d\sigma_N(p, \Delta) \right|_{6+15}}{dzd^2pd^2\Delta} = \frac{1}{2(2\pi)^2}T(b)f(\Delta)|\Psi(z, p) - \Psi(p - \Delta)|^2.
\]
(124)
Note, that the nuclear cross section can be cast in the from reminiscent of a triple convolution
\[
\frac{d\sigma_A(q^* \to qg)}{d^2b dz dp d\Delta} \bigg|_{6+15} = T(b) \int_0^1 d\beta \int d^2\kappa_1 d^2\kappa_2 d^2\kappa_3 \delta(\kappa + \kappa_1 + \kappa_2 + \kappa_3 - \Delta) \\
\times \Phi(1-\beta; b, \kappa_1) \Phi(C_2(1-\beta); b, \kappa_2) \Phi(\beta; b, \kappa_3) \frac{d\sigma_N(p - \kappa_2 - \kappa_3, \kappa)}{dz dp d\kappa} \bigg|_{6+15}.
\]
which suggests that at a fixed gluon-jet momentum \( p \), it will be a broader distribution of \( \Delta \) than the free-nucleon cross section (for the related discussion see [4]). This broadening is best seen for hard dijets, \( p^2 \gg \Delta^2, Q_A^2(b) \). Because the dominant contribution comes from \( \kappa_1^2 \lesssim Q_A^2(b) \), one can neglect \( \kappa_{2,3} \) compared to \( p \) in the free-nucleon cross section in the integrand of (125). Then the nuclear cross section takes the manifest convolution form
\[
\frac{d\sigma_A(q^* \to qg)}{d^2b dz dp d\Delta} \bigg|_{6+15} = T(b) \int_0^1 d\beta \int d^2\kappa \Phi(1 + C_2(1-\beta); b, \Delta - \kappa) \frac{d\sigma_N(p, \kappa)}{dz dp d\kappa} \bigg|_{6+15}.
\]
The saturation scale for the distribution \( \Phi(1 + C_2(1-\beta); b, \Delta - \kappa) \) equals
\[
Q_{A,eff}^2 \approx [1 + C_2(1-\beta)]Q_A^2(b)
\]
and the broadening of the acoplanarity distribution for the quark-gluon dijets is substantially stronger than that for the quark-antiquark dijets in DIS discussed in [4].

**VIII. THE MONOJETS FROM DIJETS: FRAGMENTATION VS. GENUINE DIJETS**

**A. Monojets from dijets in the free-nucleon reactions**

In the above discussion we implicitly assumed that the quark and gluon hard jets are separated by a large azimuthal angle and the acoplanarity momentum is small compared to the jet momenta, \( \Delta^2 \lesssim p^2, (p - \Delta)^2 \). The interesting new situation is encountered
when the quark and gluon jets start merging. Specifically, the wave function $\Psi(z, p - z\Delta)$ has a pole when $p - z\Delta = 0$, i.e., when the gluon and quark are collinear,

$$p_g = z\Delta, \quad p_q = z_q\Delta = \Delta - p = (1 - z)\Delta = z_q\Delta.$$  \hspace{1cm} (128)

In the vicinity of the pole the $qg$ production cross section has the factorized form

$$\left. \frac{d\sigma^N_N(q^* \rightarrow qg)}{dzd^2pd^2\Delta} \right|_{\text{monojet}} = \frac{1}{2(2\pi)^2} f(\Delta) |\Psi(z, p - z\Delta)|^2.$$ \hspace{1cm} (129)

Now recall that $\Psi(z, p - z\Delta)$ is precisely a probability amplitude to find the gluon with the momentum $k_\perp = p - z\Delta$ transversal with respect to the axis of the quark jet with the momentum $\Delta$, and $|\Psi(z, p - z\Delta)|^2$ of Eq. (88) is proportional to the familiar splitting function $P_{qg}(z)$, which is precisely the driving term of the quark-jet fragmentation function. Consequently, the contribution (129) must be treated as a fragmentation of the scattered quark into the quark and gluon, $q' \rightarrow qg$. The quark pole contribution will dominate if

$$k_\perp^2 \ll (p - \Delta)^2 = p_q^2.$$ \hspace{1cm} (130)

From the experimental point of view, the corresponding final state is a monojet of the transverse momentum $\Delta$. The transverse momentum of such a monojet will be compensated by an away jet produced at midrapidity or the nucleus hemisphere of $pA$ collisions.

In terms of Feynman diagrams of Fig. 2 - for the free-nucleon target one takes the single-gluon exchange, - the monojet production is a property of the diagram (c). Indeed, the cross section (129) is proportional to precisely the differential cross section of quasielastic scattering of the projectile quark off the nucleon target - the latter is evidently proportional to the unintegrated gluon density of the target proton $f(\Delta)$. The two classes of Feynman diagrams in Fig. 2 (b) and (c), are integral parts of the gauge-invariant description of the QCD Bremsstrahlung excitation of the $qg$ state. Still, the isolation of the pole contribution from the gauge-invariant combinations

$$\left| \Psi(1; z, p - \Delta) - \Psi(z, p - z\Delta) \right|^2 = \left| \Psi(1; z, p_q) - \Psi(z, p - z\Delta) \right|^2$$

in (92), and of the monojet contribution to the generic dijet cross section wouldn’t conflict gauge invariance. In order to conform to the jet-finding algorithms, the production of
the quark and gluon within the jet-defining cone must be treated as a fragmentation of the monojet; if the azimuthal angle between the quark and gluon is larger than the jet-defining angle, the two jets must be viewed as independent ones. The combination of the wave functions, which enters the excitation of the sextet and 15-plet final states, see Eq. (124), has the form

\[ |\Psi(z, p) - \Psi(z, p - \Delta)|^2 \propto \frac{(p - p_q)^2}{p^2 p_q^2} \]

and is finite for all orientations of the quark and gluon jets.

The quark-tagged pQCD gluon Bremsstrahlung considered here is already the higher order process, the lowest order pQCD process in \( qN \) interaction is the radiationless quasielastic scattering of the quark. Naive application of fragmentation \( q' \rightarrow qg \) to this lowest order process would evidently lead to a double counting, because the fragmentation is manifestly a monojet part of our dijet cross section. The integration over the gluon momentum \( k_\perp \) in the inclusive cross section would yield the familiar collinear logarithm, which must be reabsorbed into the definition of the fragmentation function at the starting scale. Simultaneously, one must include the virtual radiative correction to the radiationless quasielastic scattering of the incident quark off the target nucleon. The treatment of these virtual corrections to quasielastic scattering and elimination of double counting go beyond the scope of the present study and will be addressed elsewhere. We only want to comment that if one would insist on the description of monojets in terms of the fragmentation of the quark, then the interplay of the virtual correction to the radiationless quasielastic scattering and of the collinear logarithm in the monojet component of the dijet cross section may entail a departure of the fragmentation function from that defined in the \( e^+e^- \) annihilation.

\[ \text{B. Monojets from dijets off a nuclear target} \]

The presence of the monojet pole (128) in the nuclear dijet cross section (106) is manifest:

\[ \frac{d\sigma_A(q^* \rightarrow qg)}{d^2b dz dp d\Delta} \bigg|_{\text{monojet}} = \frac{1}{2(2\pi)^2} T(b) \phi(b, x, \Delta) |\Psi(z, p - z\Delta)|^2. \] (131)
It factorizes precisely as the free-nucleon cross section: the differential cross section of quasielastic quark-nucleus scattering, proportional to the unintegrated collective gluon density of the nucleus, times the fragmentation of the scattered quark to the gluon and quark given by $|\Psi(z, p - z\Delta)|^2$, which does not depend on the target. However, the virtual radiative correction to the radiationless quasielastic scattering of the incident quark off the target nucleus and the elimination of double counting are likely to depend on the acoplanarity momentum $\Delta$ and the shape of the collective nuclear glue $\phi(b, x, \Delta)$. Should this be the case, such a dependence could be reinterpreted as a nuclear modification of the fragmentation function; this issue will be addressed elsewhere.

As it was the case for the free-nucleon target, excitation of the sextet and 15-plet final states is free of the monojet singularities. To be more precise, the wave-function singularities in the integrand of the sextet and 15-plet contribution to (106) occur in the intermediate state, at $p - \kappa_1 - \kappa = 0$ and $p - \kappa_1 = 0$. However, they are integrated out in the observed dijet cross section. It is still instructive to look at the effect of these singularities in the monojet kinematics $\Delta^2 \gg p^2 \gtrsim Q_A^2(b)$.

Consider first the contribution from the intermediate pole at $p - \kappa_1 = 0$. The relevant $\kappa_i$ integrations are of the form

$$
\int d^2\kappa_1 d^2\kappa f(x, \kappa) \Phi(1 - \beta, b, x, \Delta - \kappa_1 - \kappa) \Phi(\beta + C_2(1 - \beta), b, x, \kappa_1) \\
\times |\Psi(\beta; z, p - \kappa_1)|^2
$$

$$
= \Phi(\beta + C_2(1 - \beta); b, x, p) \int d^2k |\Psi(\beta; z, k)|^2
\times \int d^2\kappa f(x, \kappa) \Phi(1 - \beta, b, x, \Delta - p - \kappa)
$$

(132)

For the considered hard jets

$$
\Phi(1 - \beta, b, x, \Delta - p - \kappa) = \frac{1}{2} (1 - \beta) T(b) f(\Delta - p - \kappa)
$$

(133)

and the convolution in (132) equals

$$
\int d^2\kappa f(x, \kappa) \Phi(1 - \beta, b, x, \Delta - p - \kappa) = (1 - \beta) T(b) f(\Delta - p).
$$

(134)

The resulting contribution from the intermediate pole of the wave function at $p - \kappa_1 = 0$ is proportional to

$$
T^2(b) f(\Delta - p) f(p) P_{gg}(z) = T^2(b) f(p_g) f(p_q) P_{gg}(z)
$$

(135)
and has the form of the product of the differential cross sections of independent quasielastic scattering of the quark and gluon fragments of the incident quark. It does not depend on the azimuthal angle between the quark and gluon jets at all, and has no collinear singularity. A similar situation has been found to occur in our previous study of the production of hard quark-antiquark dijets in $\pi A$ collisions. The contribution from the pole at $p - \kappa_1 - \kappa = 0$ is entirely similar.

IX. CONCLUSIONS

We presented a derivation of nuclear modifications of the quark-gluon production in quark-nucleus collisions. Our principal result is the nonlinear $k_\perp$-factorization relation (106). The derived dijet cross section can be decomposed into three major contributions. The excitation of $qg$ dijets in higher-sextet and 15-plet-color representations gives rise to the sixth order nonlinearity in gluon fields, compared to the fifth order nonlinearity for $q\bar{q}$ dijets in DIS. A part of the nonlinearity comes from the free-nucleon gluon density which emerges in all instances of excitation of higher color representations (see also the related discussion of the $1/(N_c^2 - 1)$ expansion in Ref. [4]). The matrix elements of transitions from lower to higher color representations are suppressed at large $N_c$, but this suppression is compensated for by the large number of states in higher representations. The coherent diffraction, in which the final dipole is produced in exactly the same color state as the incident quark, is not suppressed by large $N_c$, but because of the color-nonsinglet incident partons the diffractive contribution is suppressed by an overall nuclear attenuation and will only come from collisions at the diffuse edge of a nucleus. A new feature of $qA$ collisions in contrast to DIS is inelastic production of $qg$ states in the same color representation as the incident parton. Such color rotations within the same representation are not suppressed at large $N_c$. This contribution has the form which superficially looks like satisfying the linear $k_\perp$-factorization in terms of the collective nuclear gluon density. However, it contains the nuclear-distorted wave function of the $qg$ Fock state and, consequently, is a cubic functional of the collective nuclear glue.

The above three components of the dijet cross section differ by more than the degree of the nonlinearity. The coherent diffractive mechanism and the excitation of quark-gluon
dijets in the same color representation as the incident quark are explicitly calculable in terms of the collective nuclear glue of Eq. (94) which is defined for the whole nucleus. This is not the case for the excitation of quark-gluon dijets in higher color multiplets. It is proportional to the unintegrated gluon density in the free nucleon. The coherent initial state interaction, before the excitation of higher color multiplets at the depth $\beta$ of the nucleus, must be described in terms of the unintegrated collective glue (99) defined for the slice $\beta$ of the nucleus. Coherent distortions of the $qg$ wave function are complemented by incoherent broadening of the incident quark transverse momentum distribution in the same slice of the nucleus. Likewise, the final state interactions after the excitation of higher multiplets must be described in terms of the unintegrated collective glue defined for the slice $(1 - \beta)$ of the nucleus. This reinforces the point [4] that hard processes in a nuclear environment can not be described in terms of a nuclear gluon density defined for the whole nucleus, as it was advocated, for instance, within the Color Glass Condensate approach [37]. Furthermore, besides the collective nuclear glue defined for color-singlet quark-antiquark dipole, there emerges a new nuclear gluon density which depends on the Casimir operators of higher quark-gluon color representations, i.e., gluon field of the nucleus must be described by a density matrix in the space of color representations. Based on a comparison of the excitation of quark-gluon dijets in quark-nucleon collisions to the excitation of quark-antiquark dijets in DIS and gluon-nucleus collisions, we formulated four universality classes for nonlinear $k_1$-factorization.

The representation for the dijet cross section similar to our master formula (14) has been discussed recently by several authors [10, 11, 12], but these works stopped short of the solution of the coupled-channel intranuclear evolution for the for 4-parton state. Although major ingredients for the diagonalization of the four-body $S$-matrix are found in our earlier work on dijets in DIS [4], the case of the $qg$ dijets has its own tricky points. For this reason, we felt it imperative to present full technical details of this diagonalization.

The emphasis of the present communication was on the formalism, the numerical applications will be reported elsewhere. The nuclear coherency condition, $x \lesssim x_A \approx 0.1 \cdot A^{-1/3}$, restricts the applicability domain of our formalism to the forward part of the proton hemisphere of $pA$ collisions at RHIC. Although the required coherency condition
does not hold for the mid-rapidity dijets studied so far at RHIC [38], our predictions could be tested after the detectors at RHIC II will be upgraded to cover the proton fragmentation region [17].

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