Scaling relations for dark matter core density and radius from Chandra X-ray cluster sample

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A large number of studies have found that the dark matter surface density, given by the product of the dark matter core radius \(r_c\) and core density \(\rho_c\), is approximately constant for a wide range of galaxy systems. However, there has been only one systematic study of this ansatz for galaxy clusters by Chan [1], who found that the surface density for clusters is not constant and \(\rho_c \sim r_c^{-1.46}\). We implement a test of this ansatz for an X-ray sample of 12 relaxed clusters from Chandra observations, studied by Vikhlinin et al [2], implementing the same procedure as in Chan [1], but also accounting for the gas and star mass. We find that \(\rho_c \propto r_c^{-1.08\pm0.055}\) with an intrinsic scatter of about 18%. Therefore, we get a much shallower slope for the relation between core density and radius as compared to previous estimates, and the dark matter surface density shows deviations from a constant value at only about 1.4\(\sigma\).

I. INTRODUCTION

The current concordance (ΛCDM) cosmological model consisting of 25% cold dark matter and 70% dark energy, agrees very well with Planck CMB and large scale structure observations [3]. However, at scales smaller than about 1 Mpc, the cold dark matter paradigm runs into a number of problems such as the core/cusp problem, missing satellite problem, too big to fail problem, satellites plane problem etc (See Refs. [4, 5] for recent reviews on this subject). At a more fundamental level, another issue with the ΛCDM model is that there is no laboratory evidence for any cold dark matter candidate, or theories beyond the Standard Model of Particle Physics, which predict such candidates [6]. Therefore, a large number of theoretical alternatives to ΛCDM model have been proposed, and a variety of observational tests devised to test these myriad alternatives.

An intriguing observational result discovered more than a decade ago is that the dark matter halo surface density is constant, for a wide variety of systems spanning over 18 orders in blue magnitude for a diverse suite of galaxies, such as spiral galaxies, low surface brightness galaxies, dwarf spheroidal satellites of Milky way [7–16] etc. See however Refs. [17–21] and references therein, which dispute these claims and argue for a mild dependence of the dark matter surface density with halo mass. These results for a constant dark matter surface density were obtained by fitting the dark matter distribution in these systems to a cored profile, either Burkert [22], pseudo-isothermal profile [7], or a simple isothermal sphere [23]. All these cored profiles can be parameterized by a central density \(\rho_c\) and core radius \(r_c\); and the halo surface density is defined as the product of \(\rho_c\) and \(r_c\). The existence of a constant dark matter surface density was found to be independent of which cored profile was used [8]. Alternately, some groups have also calculated a variant of the above dark matter halo density, which has been referred to as the dark matter column density [17, 19] \(^1\), whose value remains roughly invariant with respect to the choice of the dark matter profile. This column density is equivalent to the product of \(\rho_c\) and \(r_c\) for a Burkert profile [19], and provides a more precise value of the surface density for non-cored profiles, such as the widely used NFW profile [24]. The best-fit values for the dark matter surface density for single galaxy systems using the latest observational data is given by \(\log(\rho_c r_c) = 2.15 \pm 0.2 M_{\odot} pc^{-2}\) [16].

A large number of theoretical explanations have been proposed to explain the constancy of dark matter halo density. Within the standard ΛCDM model, some explanations include: transformation of cusps to cores due to dynamical feedback processes [25], self-similar secondary infall model [17, 19, 26], dark matter-baryon interactions [27], ultralight scalar dark matter [28], super-fluid dark matter [29], self-interacting dark matter [30–33], MOND [34], etc. This observation may be in tension with some fuzzy dark matter models [35].

It behooves us to test the same relation for galaxy clusters. Galaxy clusters are the most massive collapsed objects in the universe and are a wonderful laboratory for a wide range of topics from cosmology to galaxy evolution [36, 37]. In the last two decades a large number of new galaxy clusters have been discovered through dedicated optical, X-ray, and SZ surveys, which have provided a wealth of information on Astrophysics and Cosmology. However, tests of the constancy of dark matter surface density for galaxy clusters have been very few.

The first such study for galaxy clusters was done by Boyarsky et al [38], who used the dark matter profiles from literature for 130 galaxy clusters and showed that the dark matter column density (S) goes as \(S \propto M_{200}^{0.21}\).

\(^1\) See Eq. 1 of Ref. [17] for the definition of dark matter column density.

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where $M_{200}$ is the density contrast at $\Delta = 200$ [39]. Hartwick [12] used the generalized NFW profile [24] fits in Ref. [40] (using strong and weak lensing data) for the Abell 611 cluster, and found that $\rho_cr_c = 2350M_{\odot}pc^{-2}$. This is about twenty times larger than the corresponding value obtained for galaxies [16]. Lin and Loeb [30] estimated $\rho_cr_c \approx 1.1 \times 10^4M_{\odot}pc^{-2}$ for the Phoenix cluster, using multi-wavelength data obtained by the SPT collaboration [41]. Using a model for self-interacting dark matter including annihilations, they also predicted the following relation between the surface density and $M_{200}$ [30] \[
abla\rho \rho = 41M_{\odot}pc^{-2} \times \left( \frac{M_{200}}{10^{10}M_{\odot}} \right)^{0.18}
\]
Del Popolo et al also predicted [19] a similar relation between the dark matter column density and $M_{200}$, within the context of a spherical infall model [26] valid for cluster scale haloes \[
\log(S) = 0.16 \log \left( \frac{M_{200}}{10^{12}M_{\odot}} \right) + 2.23
\]

The first systematic study of the correlation between $\rho_c$ and $r_c$ for an X-ray selected cluster sample, and without assuming any dark matter density model, was done by Chan [1] (C14, hereafter). C14 first considered the X-ray selected HIFLUGCS cluster catalog consisting of ASCA and ROSAT observations [42]. They considered 106 relaxed clusters from this catalog. From the hydrostatic equilibrium equation and parametric models for the gas density and temperature profiles, the total mass ($M(r)$) was obtained as a function of radius. The total density as a function of radius ($\rho(r)$) was then obtained from the total mass, assuming spherical symmetry.

One premise in C14 is that the total mass is dominated by the dark matter contribution, while the stellar and gas mass can be ignored. $\rho_c$ was obtained from extrapolating the dark matter density distribution to $r = 0$. The core radius was obtained by finding the radius $r_c$ at which $\rho(r) = \rho_c/4$. This emulates the definition of $r_c$ in the Burkert profile [22]. The core radius can also be interpreted as the length scale associated with a turnover in logarithmic slope of density to radius [30]. Therefore, the estimate of cores density and radius was done in C14 without positing any dark matter profile. We note that from weak and strong lensing observations, galaxy clusters are estimated to have cored or shallower than cuspy NFW dark matter profiles [43, 44]. However, these results have been disputed [45], and some works have also found evidence for cuspy haloes in clusters [46]. Therefore, there is no consensus on this issue [47]. Nevertheless, no assumptions about the dark matter profile was made in C14, while obtaining the dark matter core density and radius. We also note that in some models, for example the cusp to core transformation model [25] or the self-interacting dark matter with annihilations [30], the product of the core density and core radius for the cored profile is same as the product of scale density and scale radius of cuspy NFW-like profiles.

In their analysis, C14 used two different density profiles (single-$\beta$ and double-$\beta$ model) for the gas density. They also did separate fits for both the cool-core and the non cool-core clusters. Using the double-$\beta$ model, they obtained $\rho_c \propto r_c^{-1.46 \pm 0.16}$ for the HIFLUGCS sample. Results from fits with other profiles for the same sample as well as other samples can be found in C14. Therefore, their result shows that the dark matter surface density is not constant for clusters. C14 also carried out a similar analysis on the LOCUS cluster sample analyzed in Shan et al [48] and found that $\rho_c \propto r_c^{-1.64 \pm 0.10}$. Therefore, these results indicate that unlike single-galaxy systems, the dark matter surface density is not constant for galaxy clusters and is about an order of magnitude larger than for single galaxy systems.

We now implement the procedure recommended in C14 to determine $\rho_c$ and $r_c$ for a catalog of 12 galaxy clusters, selected using pointed X-ray and archival ROSAT observations by Vikhlinin et al [2] (V06, hereafter). Detailed parametric profiles for gas density and temperature profiles have been compiled by V06. This cluster sample has been used to constrain a plethora of modified gravity theories and also to test non-standard alternatives to $\Lambda$CDM model [49–53]. We have also previously used this sample to constrain the graviton mass [54] as well as to assess the importance of relativistic corrections to hydrostatic mass estimates [55]. Our work improves upon C14 in that, we account for the baryonic mass distribution while estimating the dark matter halo properties. The outline of this manuscript is as follows. We describe the V06 cluster sample and associated models for the density and temperature profile in Sect. II. Our analysis and results for the relation between core radius and density can be found in Sect. III. We also test for dependence vs $M_{200}$ in Sect. IV. We conclude in Sect. V.

II. DETAILS OF CHANDRA X-RAY SAMPLE

V06 (See also Ref. [56]) derived density and temperature profiles for a total of 13 nearby relaxed galaxy clusters (A133, A262, A338, A478, A907, A1413, A1795, A1991, A2029, A2390, MKW4, RXJ1159+55531, USGC 2152) using measurements from the pointed or archival observations with the Chandra X-ray satellite and ROSAT respectively. The redshifts of these clusters range approximately up to $z = 0.2$. These measurements extended up to very large radii of about $r_{500}$ for some of the clusters. The typical exposure times ranged from 30-130 ksecs. The temperatures span the range between 1 and 10 keV and masses from $(0.5 - 10) \times 10^{14} M_{\odot}$. V06 provided analytical estimates for the 3-D gas density and temperature profiles used to reconstruct the masses. The accuracy of mass reconstruction, tested with simulations was estimated to be within a few percent. From this sample of 13 clusters, we skipped USGC 2152 for our analysis, as all the relevant data was not available to us. More details about this cluster sample can be found in
We now describe the three-dimensional models for the gas density and temperature projected along the line of sight proposed by V06. These models can fit the observed X-ray surface brightness and projected temperature profiles. These parametric models are then used to derive the total gravitating mass of the clusters.

A. Gas Density Model

The analytic expression used for the three-dimensional gas density distribution is a modified version of the single-$\beta$-model [57]. These modifications were introduced to account for some additional features in the observed X-ray emission, such as a cusp at the center, steepening in X-ray brightness at large radii, to have more freedom near the cluster center. This modified emission $\beta$-profile is given as,

$$n_p n_e = n_0^2 \frac{(r/r_c)^{-\alpha}}{(1 + r^2/r_c^2)^{3\beta-\alpha/2}} \frac{1}{(1 + r^\gamma/r_c^\gamma)^{\gamma/\gamma}}$$

$$+ \frac{n_0^2}{(1 + r^2/r_c^2)^{3\beta-\alpha/2}} \quad (1)$$

where $n_p$ and $n_e$ denote the number density of protons and electrons respectively. A detailed description of all the other parameters in Eq. 1 can be found in V06. We have used the same values for the best-fit parameters for the terms in Eq. 1 as in V06, where they can be found in Table 2. From the gas particle number density profile given by Eq. 1, the gas mass density can be obtained by assuming a cosmic plasma with primordial He abundance and abundances of heavier elements $Z = 0.2Z_\odot$ as,

$$\rho_g = 1.624 m_p (n_p n_e)^{1/2} \quad (2)$$

B. Temperature Profile Model

To calculate the total dynamical mass, we need the three-dimensional temperature radial profile, whereas X-ray observations can only constrain the projected two-dimensional profile. The reconstructed temperature profile in V06 consists of two different functions, one to model the central part and another model the region outside the central cooling zone. A broken power law is used to model the temperature outside the central cooling region.

$$t(r) = \frac{(r/r_1)^{-a}}{[1 + (r/r_1)^b]^{c/b}} \quad (3)$$

The temperature decline in the central region can be expressed as [58],

$$t_{cool}(r) = \frac{x + T_{min}/T_0}{x + 1}, \quad x = \left(\frac{r}{r_{cool}}\right)^{a_{cool}} \quad (4)$$

The three-dimensional temperature profile of the cluster is given:

$$T_{3D}(r) = T_0 t_{cool}(r) t(r)$$  \hspace{1cm} (5)$$

The best-fit parameters for this model can be found in Table 3 of V06.

C. Mass and Density Profile in Clusters

The total mass of the galaxy cluster can be derived through hydrostatic equilibrium equation given the temperature and gas density models as [36],

$$M(r) = -\frac{k T(r) r}{G \mu m_p} \left(\frac{d \ln \rho_g}{d \ln r} + \frac{d \ln T}{d \ln r}\right)$$  \hspace{1cm} (6)$$

where $M(r)$ is the mass within radius $r$; $T$ and $\rho_g$ denote the gas temperature and density; $\mu$ is the mean molecular weight equal to 0.5954 as in V06, and $m_p$ is the mass of the proton.

We can estimate the total dark matter mass distribution by subtracting the gas and stellar mass from the total mass, given by Eq. 6. The gas mass can be simply obtained by assuming spherical symmetry and integrating the gas density profile ($\rho_g(r)$ from Eq. 2)

$$M_{gas} = \int 4\pi r^2 \rho_g(r) dr$$  \hspace{1cm} (7)$$

To calculate the stellar mass at any radius ($M_*(r)$), we first estimated the mass at $r = r_{500}$, where $r_{500}$ is the radius at which overdensity is equal to 500. This was estimated using the empirical relation proposed in Ref. [59].

$$\frac{M_*(r = r_{500})}{10^{12} M_\odot} \approx 1.8 \left(\frac{M_{500}}{10^{14} M_\odot}\right)^{0.71}$$

From this, one can estimate the star mass at any radius by assuming an isothermal profile [49]

$$M_{star}(r) = \frac{r}{r_{500}} M_*(r = r_{500}) \quad (8)$$

Alternately, the stellar mass can also be estimated using the stellar-to-gas mass relation obtained in Chiu et al [60], as used in Ref. [61]. However, since the stellar mass contribution to the total mass is negligible, this will not make a large difference to the final result. Therefore, once we estimate the star and gas mass, we can determine the total dark matter mass distribution ($M_{DM}(r)$) at any radius ($r$) by subtracting the gas and star mass from the total mass distribution ($M(r)$) calculated in Eq. 6:

$$M_{DM}(r) = M(r) - M_{gas}(r) - M_{star}(r) \quad (9)$$

From Eq. 9, the density profile of the dark matter halo can be easily calculated by assuming spherical symmetry:

$$\rho_{DM}(r) = \frac{1}{4\pi r^2} \frac{dM_{DM}}{dr} \quad (10)$$
To obtain $\rho_c$ and $r_c$ from $\rho_{DM}(r)$, we follow the same prescription as in C14, which we now describe. To recap, $\rho_c$ is estimated from the dark matter density at the centre of the cluster. Therefore, similar to C14, we extrapolated our dark matter density profile $\rho_{DM}(r)$ (obtained from Eq. 10) to $r = 0$ in order to calculate $\rho_c$. The core radius $r_c$ was estimated by determining the radius at which the local dark matter density (defined in Eq. 10) reaches a quarter of its central value. As mentioned earlier, this is how $r_c$ is defined in the Burkert profile [9, 22, 62]. However, in this case and similar to C14, we have estimated $\rho_c$ and $r_c$ in a model-independent way without positing any dark matter profile.

We applied this method to determine $\rho_c$ and $r_c$ for the 12 clusters in V06. Errors in the gas temperature profile at a fixed number of radii have also been provided in V06 and made available to us (A. Vikhlinin, private communication). These were used to propagate the errors in the values of $\rho_c$ and $r_c$.

III. RESULTS

The resulting values of $\rho_c$ and $r_c$ along with 1σ error bars for each of the 12 clusters estimated using the procedure outlined in Sect. II C can be found in Table I. We note that our values for $\rho_c$ and $r_c$ are of the same order of magnitude as for other galaxy cluster systems estimated in C14. They are also within the same ballpark as predicted by SIDM simulations for cluster scale haloes with $\sigma/m \approx 0.1 cm^2/g$ [63]. Our estimated dark matter surface density is about an order of magnitude larger than that found for galaxy systems [8, 16].

Figure 1 shows the log $\rho_c$ versus log $r_c$ plot, and we observe a tight scaling relation between the two. We also find that $\rho_c$ is inversely proportion to $r_c$ in agreement with C14. To determine the scaling relation between the two, we perform a linear regression ($y = mx + c$) in log-log space. Here, $y = \ln \rho_c$, and $x = \ln r_c$. Unlike C14, we also allow for an intrinsic scatter ($\sigma_{int}$) in the linear fit. This intrinsic scatter is treated as a free parameter and is added in quadrature to the observational uncertainties ($\sigma_y$ and $\sigma_x$). It can be determined along with the slope and intercept by maximizing the log-likelihood [61, 64].

The log-likelihood function ($\ln L$) can be written as,

$$-2\ln L = \sum_{i} 2\pi \sigma_i^2 + \sum_{i} \frac{[y_i - (mx_i + c)]^2}{\sigma_i^2}$$

(11)

$$\sigma_i^2 = \sigma_{y_i}^2 + m^2 \sigma_{x_i}^2 + \sigma_{int}^2$$

(12)

The maximization of the log-likelihood was done using the emcee MCMC sampler [65] with uniform priors. Our best-fit value for the scaling relation is as follows:

$$\ln \left( \frac{\rho_c}{M_{\odot} pc^{-3}} \right) = (-1.08^{+0.06}_{-0.05}) \ln \left( \frac{r_c}{kpc} \right) + (0.4^{+0.24}_{-0.25})$$

(13)

with an intrinsic scatter, $\sigma_{int} = 18.5^{+4.4}_{-4.0}$. Therefore, we obtain a much shallower slope for the core density-radius scaling relation compared to the slope obtained in C14, who found $\rho_c \sim r_c^{-1.46}$ for the HIFLUGCS sample, after assuming a double-beta profiles. Our results show deviations from a constant dark matter surface density at only about 1.3-1.4σ. A comparison of our result with the previous fits carried out in C14 can be found in Table II. As we can see all the fits done in C14 show a much steeper slope than our result.

C14 had deduced based on their estimated scaling relation that velocity dependent self-interacting cross-sections are ruled out if the observed core is produced due to dark matter self-interactions. The reason for this is that their results showed an increase of cross-section with velocity, which contradicts the predictions from some of these models [33, 66]. Using the same reasoning as in C14, since the dark matter surface density is only discrepant from a constant value by 1.4σ, our results do not rule out velocity-dependent SIDM.

### Table I: Estimated values for the core density ($\rho_c$) and the core radius ($r_c$) for the V06 cluster sample.

| Cluster | $\rho_c$ $10^{-3}M_{\odot} pc^{-3}$ | $r_c$ kpc |
|---------|---------------------------------|-----------|
| A133    | $11.68^{+0.02}_{-0.02}$       | 102.01$^{+0.08}_{-0.11}$ |
| A262    | $5.17^{+0.87}_{-0.89}$        | 136.30$^{+0.40}_{-0.49}$ |
| A383    | $9.63^{+0.62}_{-0.78}$        | 121.49$^{+3.95}_{-3.49}$ |
| A478    | $3.39^{+0.72}_{-0.84}$        | 286.14$^{+30.41}_{-35.62}$ |
| A907    | $4.15^{+0.42}_{-0.53}$        | 208.96$^{+10.66}_{-12.98}$ |
| A1413   | $6.27^{+0.49}_{-0.53}$        | 154.68$^{+6.06}_{-6.61}$ |
| A1795   | $7.15^{+0.68}_{-0.79}$        | 131.89$^{+6.32}_{-7.33}$ |
| A1991   | $111.25^{+0.83}_{-0.92}$      | 11.15$^{+0.04}_{-0.04}$ |
| A2029   | $9.39^{+0.66}_{-0.76}$        | 134.31$^{+2.72}_{-5.45}$ |
| A2390   | $5.83^{+0.22}_{-0.24}$        | 137.18$^{+2.60}_{-2.81}$ |
| RX J1159+5531 | $41.06^{+1.33}_{-1.19}$  | 34.07$^{+0.55}_{-0.49}$ |
| MKW 4   | $102.4^{+0.92}_{-0.98}$       | 10.31$^{+0.04}_{-0.04}$ |

### Table II: Summary of results for a linear regression of $\ln(\rho_c)$ versus $\ln(r_c)$ from different cluster samples. All the other results are from V06.

| Slope   | Intercept | Cluster Sample                |
|---------|-----------|-------------------------------|
| $-1.47 \pm 0.04$ | $0.75 \pm 0.08$ | ROSAT (single-β profile)   |
| $-1.46 \pm 0.16$ | $0.88 \pm 0.33$ | ROSAT (double-β profile)   |
| $-1.30 \pm 0.07$ | $0.6 \pm 0.11$  | ROSAT (cool-core clusters)  |
| $-1.50 \pm 0.24$ | $0.96 \pm 0.54$ | ROSAT (non-heat-core clusters) |
| $-1.64 \pm 0.1$  | $1.58 \pm 0.21$ | LOCUSS                      |
| $-1.08^{+0.06}_{-0.05}$ | $0.4^{+0.24}_{-0.25}$ | Chandra (this work)       |
FIG. 1: $\ln \rho_c$ versus $\ln r_c$ from V06 cluster sample [2]. The units for $\rho_c$ and $r_c$ are in $M_\odot pc^{-3}$ and kpc respectively. The black line represents the fitted line of our analysis ($\rho_c \propto r_c^{-1.08}$), and can be compared with the dashed magenta line, which has a slope equal to -1. The blue and orange lines show the slope determined by Chan [1] for ROSAT catalog and the Shan et al [48] sample respectively. A summary of all results in literature can be found in Table I.

IV. DEPENDENCE ON $M_{200}$

We now use our results for $\rho_c$ and $r_c$ to check for correlation with $M_{200}$, as suggested in some works [19, 30]. The first step in doing this is to estimate $M_{200}$ from $M_{500}$. In V06, the masses ($M_{500}$) and concentration parameters ($c_{500}$) for the overdensity level $\Delta = 500$ and its corresponding radius ($r_{500}$) have already been derived. We have determined the $M_{200}$ values using same prescription as in Ref. [67], which assumes an NFW profile

$$M_{200} = \frac{M_{500} f(c_{500})}{f(c_{200})}$$  \hspace{1cm} (14)

where $f(c)$ is a function of the concentration $c$ and the over-density ($\Delta$), and is given by [67]

$$f(c_{\Delta}) = \ln(1 + c_{\Delta}) - \frac{c_{\Delta}}{1 + c_{\Delta}}$$

The concentration at $\Delta = 200$, $c_{200}$ was obtained by solving for the following equation

$$\frac{c_{200}^3}{\ln(1 + c_{200}) - \frac{c_{200}}{1 + c_{200}}} = \frac{3\rho_s}{200\rho_{c,z}}$$  \hspace{1cm} (15)

where the NFW density scale parameter ($\rho_s$) and scale radius were fixed for each cluster and is given by

$$\rho_s = \frac{M_{500}}{4\pi r_{500}^3}$$

and $\rho_{c,z}$ is determined from

$$\rho_{c,z} = \frac{3M_{500}}{2000\pi r_{500}^3}$$

The $M_{500}$ values for the clusters A262 and MKW4 were unavailable in V06, hence, we calculated it by extrapolating the mass profiles to the corresponding $r_{500}$ values. To calculate the error in $M_{200}$, we propagated the errors in $M_{500}$ and $c_{500}$ provided in V06.
The relation between the dark matter column density, \( S = \rho \cdot r_c \) and \( M_{200} \) in logarithmic space is shown in Fig. 2. We have again done a linear regression with \( y = \ln \rho \cdot r_c \) and \( z = M_{200} \); and maximized the log-likelihood function (same as Eq. 11) using the emcee MCMC sampler with uniform priors. The best-fit parameters thus obtained are,

\[
\ln \left( \frac{\rho \cdot r_c}{M_{200} \rho c^{-2}} \right) = (-0.07^{+0.05}_{-0.06}) \ln \left( \frac{M_{200}}{M_\odot} \right) + (9.41^{+2.07}_{-1.86}) \quad (16)
\]

The intrinsic scatter for this fit is \( 17^{+4.0}_{-6.0}\% \). We now fit this data to two scaling relations predicted by two independent theoretical scenarios proposed in literature for the dark matter surface halo density. Del Popolo et al. [44] found after applying the secondary infall model for the dark matter surface halo density, Lin and Loeb deduced from numerical simulations of self-interacting dark matter with annihilations, that \( S \propto M_{200}^{0.16} \), where \( S \) is the dark matter column density. Lin and Loeb model, there is also a slight dependence of the surface density as a function of redshift (See Fig. 2 of Ref. [30]). Therefore, both these works predict a weak dependence on \( M_{200} \). We note that in the Lin and Loeb model, there is also a slight dependence of the surface density as a function of redshift (See Fig. 2 of Ref. [30]). However, since no analytic formula for variation with redshift is provided, we do not account for this. We further point out these two relations are not exhaustive and other proposed scaling relations for the dark matter surface density as a function of halo mass are discussed in Ref. [19].

However, when we compare our estimated surface density with \( M_{200} \), we find a slight decrease in dark matter core density with \( M_{200} \). Therefore, at face value our results would not be consistent with these predictions. To carry out a more definitive test, we now try to fit our data to these relations, by using the same slope (0.18 and 0.16) as predicted by these models, with only the intercept and intrinsic scatter as free parameters. We then do a model comparison with our best fits using AIC and BIC information criterion [68]. The best-fit results for these two scaling relations can be found in Table III. We find that \( \Delta \text{AIC} \) and \( \Delta \text{BIC} \) between our best-fit and that for other scaling relations is between 6-7, wherein our fit has the lowest value, indicating strong evidence for our fit as compared to the relations proposed in Refs. [19, 30].

We note that we shall obtain poorer fits for other scaling relations which predict a steeper dependence with halo mass, for eg. [38]. A comparison of our best-fit along with a fit to the theoretical relation in Lin and Loeb [30] can be found in Fig. 2. In the same figure, we also show for comparison the constant value for the surface density, obtained for single galaxy systems using the latest data [10].

| Model          | Slope | \( \sigma_{int} \) | AIC  | BIC  |
|----------------|-------|---------------------|------|------|
| Lin & Loeb [30]| 0.18  | 28%                 | 12.7 | 14.7 |
| Del Popolo et al [19] | 0.16  | 26%                 | 11.6 | 13.6 |
| This work      | \(-0.07^{+0.05}_{-0.06}\) | 17% | 5.1  | 8.0  |

TABLE III: Summary of results for a linear regression of \( \ln(\rho \cdot r_c) \) versus \( \ln(M_{200}) \) from different models and their comparison using AIC and BIC. Our best-fit (Eq. 16) has the smallest values of AIC and BIC and the difference between the other two scalings is between 6-7 indicating strong preference for our model compared to the other two.

V. CONCLUSIONS

A large number of studies in the past decade have found that the dark matter surface density, given by the product of dark matter core radius \( (r_c) \) and core density \( (\rho_c) \) is constant for a wide range of galaxy systems from dwarf galaxies to giant galaxies over 18 orders in blue magnitude. This cannot be trivially predicted by the vanilla ΛCDM model, but it can be easily accommodated in various alternatives to ΛCDM or by invoking various feedback mechanisms in ΛCDM.

However, there have been very few tests of this ansatz for galaxy clusters. The first systematic study of this relation for a large X-ray selected cluster sample was done by C14 using the ROSAT sample studied in Chen et al. [42]. They considered a sample of galaxy clusters in hydrostatic equilibrium and using parametric models for gas density and temperature, obtained the total mass density profile. They assumed that this is a proxy for the total dark matter density distribution. For this sample, \( \rho_c \) was obtained by extrapolating the dark matter density distribution to the center of the cluster, whereas, \( r_c \) was obtained by determining the radius at which the core density drops by a factor of four. This emulates the definition of core radius in the Burkert cored dark matter profile [22]. Therefore, this analysis was done without positing any specific dark matter density distribution. C14 did not find a constant dark matter surface density, but found a tight scaling relation between \( \rho_c \) and \( r_c \), given by \( \rho_c \propto r_c^{-1.46\pm0.16} \).

We then carried out a similar analysis as in C14 for a Chandra X-ray sample of 12 relaxed clusters, for which detailed 3-D gas density and temperature profiles were made available by Vikhlinin et al [2]. One improvement on the analysis in C14, is that we also subtracted the gas and star mass, while non-parametrically reconstructing the dark matter density profile. Furthermore, while determining the scaling relations between the core density and radius, we also accounted for the intrinsic scatter. Our results for the dark matter core density and radius can be found in Table I. They are of the same order of magnitude as previous estimates for galaxy clusters [1], and are about an order of magnitude larger than for iso-
FIG. 2: \(\ln(\rho_c r_c)\) versus \(\ln M_{200}\) from V06 cluster sample [2]. The units for \(\rho_c r_c\) and \(M_{200}\) are in \(M_\odot pc^{-2}\) and \(M_\odot\) respectively. The black line represents the fitted line of our analysis, whereas the cyan line represents the models from Lin & Loeb [30]. We get similar fit for the scaling relation predicted by Del Popolo et al [19], which we have omitted from the plot for brevity. The red dashed line indicates the constant surface obtained from single galaxy systems of various types [16], while the orange shaded region represents 1\(\sigma\) error. Note that the mass range for these systems is much lower than for clusters. Note that in this plot the range of values 2.4-6.4 have been culled from the Y-axis in this plot, given the large difference in surface density for single galaxies and clusters.

We find that \(\rho_c \propto r_c^{-1.08+0.06^{+0.06}_{-0.05}}\). The intrinsic scatter for this fit is about 18 %. Therefore, we get only a marginal deviation from a reciprocal relation between the dark matter core density and radius in contrast to C14 who found a steeper dependence of \(\rho_c\) as a function of \(r_c\). Our estimated dark matter surface density is inconsistent with flat density core at only 1.4\(\sigma\). A comparison of our result with previous scaling relations found for galaxy clusters can be found in Table II.

We also checked for any dependence of the product of dark matter surface density with \(M_{200}\) to test some of these predictions in literature [19, 30]. We find that the dark matter surface density \((S)\) scales with \(M_{200}\) as \(S \propto M_{200}^{-0.07^{+0.55}_{-0.55}}\), which contradicts the weak logarithmic increase with \(M_{200}\) predicted in Refs.[19, 30].

Further stringent tests of this relation should soon be possible, thanks to the recent launch of the e-ROSITA satellite and the expected discovery of about 100,000 clusters [69].

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