Compressed and Bending Concrete Elements with Confinement Reinforcement Meshes

A L Krishan¹, V I Rimshin² and E A Troshkina³

¹Department of Building Design and Constructions, Nosov Magnitogorsk State Technical University, 11 Uritsky, Magnitogorsk 455000, Russia
²Research Institute of Building Physics of Russian Academy of Architecture and Construction Sciences, 21 Lokomotivny pr., Moscow 127238, Russia
³Department of Construction Engineering, Nosov Magnitogorsk State Technical University, 11 Uritsky, Magnitogorsk 455000, Russia

E-mail: kris_al@mail.ru

Abstract. Practical application limits and scope related to confinement reinforcement of load-bearing concrete structures are continuously expanding. However, strength and deformability of compressed and bending elements with confinement reinforcement are calculated based on empirical equations. The purpose of this article is to propose new equations for determining strength and ultimate deformation of volume-compressed concrete that would reflect the key mechanisms of their force resistance and ensure higher accuracy of calculations when compared to the currently used ones. The comparison with previously reported empirical data demonstrated that such equations were obtained. An important advantage of these equations is that they take account of all the main factors influencing mechanical properties of volume-compressed concrete and are universally applicable.

1. Introduction
Confinement reinforcement by means of meshes is well known as one of the methods applied to improve the performance efficiency of reinforced concrete elements. Such reinforcement — arranged perpendicular to the compressive force — creates favorable conditions for volumetric compression of concrete due to the limitation of its transverse deformations. As a result, strength and ultimate deformability of the concrete are increased [1-10], which should be used to the maximum possible extent when designing reinforced concrete elements.

2. Relevance
Over the last years, the application scope related to confinement reinforcement of load-bearing concrete structures is expanding. For example, reinforcement meshes come into use to reinforce compression areas of bending elements [11-16]. The main reason for this is aiming for the improvement of progressive collapse resistance of buildings. In case of emergencies, a significant improvement in ultimate deformability of concrete structures with confinement reinforcement allows avoiding brittle fracture and provides the possibility to redistribute forces towards the adjacent less loaded elements.
The above circumstances imply the necessity to develop more accurate methods of calculating the strength of compressed and bending concrete elements with confinement reinforcement meshes. The advanced methods, which adequately evaluate the main features of structural force resistance to the actual loads, will contribute to more effective use of confinement reinforcement. Eventually, these methods will make it possible to objectively take account of the existing reserve of structural load-bearing capacity and to apply more cost-effective design solutions.

3. Statement of the problem

The most important components of advanced calculation methods are the formulas for calculating strength and ultimate deformations of compressed concrete with confinement reinforcement. Most researchers use empirical equations for such purposes. For example, the volume-compressed concrete strength \( f_{cc} \) is in most cases determined using the following formula developed back in 1928 based on experiments conducted by American researchers F. Richart, A. Brandtzaeg and R. Brown:

\[
f_{cc} = f_i + k \sigma_{cr},
\]

where \( f_i \) is the strength of uniaxially compressed concrete;
\( \sigma_{cr} \) is the lateral pressure applied to the concrete core;
\( k \) is the lateral pressure coefficient that is generally taken as equal to 4.1 or 4.0.

According to the design and construction specifications of the Russian Federation – SP 63.13330.2012 “Concrete and reinforced concrete structures. General provisions” - this equation for confinement reinforcement meshes is made more precise based on empirical data and transformed as follows:

\[
f_{cc} = f_i + K \mu_y f_y,
\]

where \( f_y \) is the design tensile strength of confinement reinforcement;
\( \mu_y \) is the confinement reinforcement coefficient;
\( K \) is the performance coefficient of confinement reinforcement determined according to the following formula:

\[
K \equiv \left(0.23 + \frac{\mu_y f_y}{f_i + 10}\right)^{-1}.
\]

Volume-compressed concrete deformations at the vertex of the deformation curve are also calculated using empirical formulas. Most researchers use the following equation:

\[
\varepsilon_{cc1} = \varepsilon_{c1}\left(1 + k_1 \frac{\sigma_{cr}}{f_i}\right),
\]

where \( \varepsilon_{c1} \) is the deformation of uniaxially compressed concrete under maximum stress;
\( k_1 \) is the empirical coefficient taken in the 17.5-20.5 range by different researchers [2,17].

It should be remembered that empirical equations are worked out for specific conditions of conducted research and always have application limits. Therefore, it is preferred to use more universal formulas with the expanding scope of application related to concrete elements with confinement reinforcement meshes.

The purpose of this article is to propose new equations for determining strength and ultimate shortening deformation of volume-compressed concrete that would reflect the key mechanisms of their force resistance and ensure higher accuracy of calculations when compared to the currently used ones.
4. Theoretical part
The authors of this article developed a formula for determining the strength of volume-compressed concrete by way of phenomenological approach [18]. With regard to concrete elements with confinement reinforcement meshes, this formula can be written as follows:

\[ f_{cc} = f_c \left\{ \frac{1 - \rho_c}{2} + \left[ \frac{1 - \rho_c}{2} \right]^2 + \beta_c \rho_c \right\}^{1/2}, \]  

(5)

where \( \beta_c \) is the material coefficient (\( \beta_c = 9 \) for heavy-weight concrete and \( \beta_c = 7 \) for fine grained concrete);

\( \rho_c \) is the relative lateral pressure applied to the concrete core, which is determined using the following formula:

\[ \rho_c = 0.375 \mu_c \frac{f_c}{f_c}. \]  

(6)

Reported experimental data for more than 300 compressed concrete specimens with confinement reinforcement meshes were processed in the article [19] to verify the formula (5). As a result, it was demonstrated that this equation provided the best coincidence of empirical and theoretical values.

The authors of this article also analyzed known equations for determining deformations of volume-compressed concrete \( \varepsilon_{cc1} \). Based on the analysis findings, they recommended the following empirical formula, which was proposed in the article [20] following the results of statistical analysis of empirical data:

\[ \varepsilon_{cc1} = \varepsilon_{cc1}^0 \varepsilon_{cc1}, \]  

(7)

where the power exponent \( n \) was calculated using the following formula:

\[ n = \left( 2.9224 - 0.00408 f_c \right) \left( 0.9 \rho_c \right)^{0.3124 + 0.0022 f_c}. \]  

(8)

The following formula is proposed based on the comparison of elastic and plastic deformation components of uniaxially and volume-compressed concrete, carried out in the article [21]:

\[ \varepsilon_{cc1} = \varepsilon_{cc1} \left( \frac{f_{cc}}{f_c} \right)^m + \frac{f_{cc}}{E_c} - \frac{f_c}{E_c} \left( \frac{f_{cc}}{f_c} \right)^m, \]  

(9)

where \( E_c \) is the initial modulus of concrete elasticity and the value of power exponent \( m \) to a first approximation can be taken as equal to 2.5.

Table 1 provides a comparison between experimental values of deformations \( \varepsilon_{cc1}^{exp} \) and the calculated values determined from the formulas (7) and (9). The values \( f_{cc}^{(5)} \) shown in the table are calculated using the formula (5) whereas the deformations \( \varepsilon_{cc1} \) and the initial moduli of concrete elasticity \( E_c \) are obtained based on the following formulas:

\[ \varepsilon_{cc1} = 0.0012 + 0.00016 \sqrt{f_c}; \]  

(10)

\[ E_c = 56000 - 122000 \sqrt{f_c}. \]  

(11)
Table 1. Comparison between calculated values of concrete deformations at the vertex of the deformation curve and experimental data.

| $N_0$ | $f_c$ (MPa) | $\mu_s$ | $f_s$ (MPa) | $f_s^{(5)}$ (MPa) | $\varepsilon_{el} \times 10^5$ | $E_s$ (MPa) | $\varepsilon_{el}^{exp}$ $\times 10^5$ | $\varepsilon_{el}^{(7)}$ | $\varepsilon_{el}^{(9)}$ |
|-------|-------------|---------|-------------|------------------|--------------------------|------------|-----------------------------|-------------------|-------------------|
| 1     | 13.1        | 0.0402  | 575         | 30.9             | 180                      | 22290      | 1340                        | 1.04              | 1.16              |
| 2     | 23.8        | 0.0402  | 575         | 46.8             | 200                      | 30990      | 890                         | 0.86              | 1.10              |
| 3     | 30.9        | 0.0402  | 575         | 56.5             | 210                      | 34050      | 680                         | 0.75              | 0.97              |
| 4     | 13.1        | 0.0452  | 575         | 32.1             | 180                      | 22290      | 1300                        | 0.95              | 1.03              |
| 5     | 23.8        | 0.0452  | 575         | 48.4             | 200                      | 30990      | 870                         | 0.79              | 1.00              |
| 6     | 30.9        | 0.0452  | 575         | 58.3             | 210                      | 34050      | 580                         | 0.60              | 0.77              |
| 7     | 30.0        | 0.016   | 475         | 52.3             | 210                      | 33730      | 500                         | 0.91              | 0.90              |
| 8     | 30.0        | 0.016   | 475         | 52.3             | 210                      | 33730      | 600                         | 1.09              | 0.80              |
| 9     | 31.5        | 0.049   | 427         | 79.3             | 210                      | 34260      | 1300                        | 1.53              | 0.92              |
| 10    | 34.5        | 0.049   | 427         | 87.0             | 210                      | 35230      | 1400                        | 1.73              | 0.99              |
| 11    | 49.0        | 0.016   | 475         | 79.3             | 230                      | 38570      | 350                         | 0.85              | 0.63              |
| 12    | 49.2        | 0.016   | 475         | 79.3             | 230                      | 38570      | 370                         | 0.90              | 0.67              |
| 13    | 49.2        | 0.049   | 427         | 102.4            | 230                      | 38610      | 900                         | 0.99              | 0.98              |
| 14    | 49.2        | 0.049   | 427         | 102.4            | 230                      | 38610      | 950                         | 1.04              | 1.03              |
| 15    | 42.7        | 0.0205  | 447         | 59.0             | 220                      | 37330      | 430                         | 0.88              | 1.06              |
| 16    | 58.0        | 0.0202  | 445         | 75.9             | 240                      | 39980      | 440                         | 1.07              | 1.16              |
| 17    | 58.0        | 0.052   | 455         | 91.3             | 240                      | 39980      | 600                         | 0.95              | 1.13              |
| 18    | 58.0        | 0.102   | 440         | 107.0            | 240                      | 39980      | 800                         | 0.70              | 1.12              |
| 19    | 58.0        | 0.075   | 460         | 100.0            | 240                      | 39980      | 810                         | 0.95              | 1.29              |
| 20    | 22.3        | 0.054   | 400         | 43.8             | 200                      | 30160      | 800                         | 0.78              | 1.00              |
| 21    | 42.0        | 0.0186  | 400         | 56.2             | 220                      | 37180      | 430                         | 0.93              | 1.13              |
| 22    | 42.0        | 0.0186  | 400         | 56.2             | 220                      | 37180      | 450                         | 0.98              | 1.18              |
| 23    | 41.2        | 0.0186  | 400         | 55.3             | 220                      | 36990      | 430                         | 0.93              | 1.13              |
| 24    | 42.2        | 0.0419  | 400         | 66.1             | 220                      | 37220      | 550                         | 0.84              | 1.06              |
| 25    | 56.3        | 0.026   | 300         | 72.4             | 240                      | 39740      | 416                         | 1.04              | 1.13              |
| 26    | 53.4        | 0.026   | 300         | 69.2             | 240                      | 39300      | 415                         | 1.01              | 1.12              |
| 27    | 58.5        | 0.026   | 300         | 74.8             | 240                      | 40050      | 365                         | 0.93              | 1.00              |
| 28    | 46.0        | 0.0519  | 300         | 69.5             | 230                      | 38010      | 423                         | 0.90              | 1.08              |
| 29    | 57.2        | 0.0519  | 300         | 82.7             | 240                      | 39870      | 426                         | 1.03              | 0.94              |
| 30    | 58.1        | 0.0519  | 300         | 83.8             | 240                      | 39990      | 426                         | 1.05              | 0.94              |

Mean value: $\sigma = 0.97$, $\sigma = 1.02$

Statistical analysis of the above comparison demonstrated significantly higher accuracy of calculations when using the formula (9). In this case, the mean square deviation $\sigma$ equals 0.15 as opposed to 0.22 obtained when using the formula (7). Maximum and minimum deviations for individual specimens are also significantly lower (+29/-37% as opposed to +73/-40%). In addition, formula (9) is more universal since it is developed theoretically and not related to any design parameters of elements under calculation.

5. Conclusions

The suggested equations for determining strength and ultimate shortening deformation of volume-compressed concrete ensure higher accuracy of calculations when compared to the currently used ones. An important advantage of these equations is that they reflect the key force resistance mechanisms of volume-compressed concrete and are universally applicable.
References

[1] Attard M M and Samani A K 2012 A stress–strain model for uniaxial and confined concrete under compression *Eng. Struct.* 41 335–49

[2] Fattah A M 2012 Behaviour of Concrete Columns under Various Confinement Effects (Kanzas, USA: Kanzas State University) p 399

[3] Fafitis A and Shah S P 1985 Lateral reinforcement for high-strength concrete columns *ACI Materials J.* 87(12) 213–32

[4] Han L-H and an Yufeng 2014 Performance of concrete-encased CFST stub columns under axial compression *J. of Constructional Steel Research* 93 62–76

[5] Imran I and Pantazopoulou S J 1996 Experimental study of plain concrete under triaxial stress *ACI Materials J.* 93(6) 589–601

[6] Li Q and Ansari F 2000 High-strength concrete in triaxial compression by different sizes of specimens *ACI Materials J.* 97(6) 684–9

[7] Lu X and Hsu C T 2007 Stress–strain relations of high-strength concrete under triaxial compression *J. Mater. Civil Eng.* 19(3) 261–8

[8] Muguruma H, Watanabe S, Katsuta S and Tanaka S 1980 A stress-strain model of confined concrete *Proc. JCA Cement and Concrete* vol 34 (Tokyo, Japan) pp 429–32

[9] Subramanian N 2011 Design of confinement reinforcement for RC columns *The Indian Concrete Journal* 85(6) 1–9

[10] Watson S, Zahn F A and Park R 1994 Confining Reinforcement for Concrete Columns *Journal of Structural Engineering* 120(6) 1798–824

[11] Vanus D S 2011 Experimental researches of reinforced concrete beams with mesh confinement reinforcement of the compressed zone *Industrial and Civil Engineering* 5 56–7

[12] Grinev V D and Belevich S D 1993 Behaviour of concrete beams with reinforced compressed zone *Industrial and Civil Engineering* 10 12–3

[13] Hadi M and Elbasha N 2008 Displacement ductility of helically confined HSC beams *The Open Construction and Building Technology Journal* 2 270–9

[14] Rastorguev B S and Vanus D S 2009 Calculation of reinforced concrete elements with cross-mesh reinforcement *Industrial and Civil Engineering* 10 53–4

[15] Tamrazyan A G and Manaenkov I K 2016 To the calculation of bending reinforced concrete elements with confinement reinforcement of the compressed zone *Industrial and Civil Engineering* 7 41–4

[16] Yarkin R A and Strulev V M 2003 Bending of reinforced concrete beams with confinement reinforcement of the compressed zone of concrete *Bulletin of TSTU* 9(3) 486–92

[17] Mander J B, Priestley M J N and Park R 1988 Theoretical stress-strain model for confined concrete *Journal of Structural Engineering* 114(8) 1804–26

[18] Krishan A L 2014 Power resistance of compressed concrete elements with confinement reinforcement by means of meshes *Advances in Environmental Biology* 8(7) 1987–90

[19] Manaenkov I K 2018 To the improvement of the diagram of compressed concrete with confinement reinforcement *Construction and reconstruction* 2(76) 41–50

[20] Attard M M and Setunge S 1996 Stress-Strain Relationship of Confined and Unconfined Concrete *ACI Materials J.* 93(5) 432–42

[21] Krishan A L, Rimshin V I and Astafeva M A 2018 Deformability of a Volume-Compressed Concrete *IOP Conf. Series: Materials Science and Engineering* 463(2) 022063

Acknowledgments

The article is written on the basis of the results of the research conducted by the RAASN No. 7.4.11.