Distributed adaptive formation control for underactuated quadrotors with guaranteed performances

Xingling Shao · Xiaohui Yue · Jun Liu

Received: 10 December 2020 / Accepted: 17 July 2021 / Published online: 7 August 2021
© The Author(s), under exclusive licence to Springer Nature B.V. 2021

Abstract This paper investigates a distributed adaptive formation control problem for underactuated quadrotors with guaranteed performances. To ensure a robust and stable formation pattern with predefined behavior bounds, by transforming the original constrained formation synchronization error dynamics into an equivalent unconstrained one, a prescribed performance mechanism is introduced in the translational loop to render the formation regulation as a prior. An adaptive consensus strategy is developed according to undirected graph theory and Lyapunov stability rules for follower quadrotors to achieve a distributed cooperative formation with prescribed tracking abilities via exchanging local information with neighbors. The presented control scheme has the following salient merits: (1) the formation synchronization errors can be guaranteed within pre-assigned bounds with desired transient behaviors despite of uncertain disturbances; (2) by using a state estimation error to update neural network (NN) parameters, rather than the tracking error that widely applied in traditional NN approximators, and with the help of MLP technique, the proposed SE-MLP observer capable of decreasing the computational complexity can achieve a fast identification of lumped disturbances without causing high-frequency oscillations even using a large adaptive gain, and the transient solutions of $L_2$ norm of the differential of neural weights are established to illustrate the mechanism of SE-MLP observer in reducing chattering behaviors. The merits of presented algorithm are confirmed by sufficient simulations.

Keywords State estimator · Prescribed performances · Neural network · Formation · Underactuated quadrotors

1 Introduction

Recently, there have been a surge of interests in formation control for multiple quadrotors, owing to their practical applications in collectively fulfilling difficult missions across civilian and military fields, such as distributed environmental monitoring with portable chemical and biological sensor arrays to detect toxic pollutants, disaster relief in an inaccessible area, aerial transportation of large objects, etc. [1–6]. However, the coordinated control problem is rather challenging, due to the strong nonlinear coupling in quadrotor modeling and various sources of
uncertainties arising from unmodeled dynamics and time-varying disturbances acting on each quadrotor dynamics [7–9]. In addition, some time-urgent tasks always entail the design of swarm control strategy equipped with a preset convergence rate to ensure a high-efficiency mission outcome. Therefore, complicated model dynamics [10, 11], uncertain circumstances, and rigid performance constraints [12, 13] pose great difficulties on guaranteeing the success of formation flight mission.

To realize a distributed cooperative control of multi-agents, consensus is a fundamental issue where each agent reaches an agreement on the interested states utilizing only local communication flow [14–17]. As described in [15–17], a consensus topic for strict-feedback multi-agent systems is investigated in a single-input single-output (SISO) form. Consensus strategies for second-order nonlinear multi-agent systems are discussed in [18–20]. Consider a formation system with a union of unmanned surface vehicles; a consistent tracking result can be found in [21]. However, the aforementioned consensus schemes only concentrate on system dynamics obeying SISO or fully actuated features, which are hard to be generalized to formation control problems of multiple quadrotors exposed to multiple-input multiple-output (MIMO) and underactuated characteristics [22]. Recently, some formation control approaches of multiple quadrotors have been reported. For example, a leader–follower control approach based on a suboptimal $H_\infty$ is developed in [23] for quadrotors suffering from external disturbances and model parameter uncertainties, contributing to an enhanced control performance. In [24], a robust formation problem is studied, and with the aid of a robust compensation strategy, a distributed controller is developed to accomplish a desired formation of networked uncertain quadrotors. By employing a finite-time stability and optimal control principle, a non-smooth cooperative formation control algorithm is discussed in [25] under a leader–follower structure. Unfortunately, most of the previous formation control methods can merely provide ultimately uniformly bounded (UUB) tracking via various anti-disturbance avenues, where the final upper bound of synchronization error primarily relies on some design scalars and unknown auxiliary terms, i.e., the preset performance indices cannot be easily met even tuning parameters tediously, inevitably leading to a severe design conservatism and further cause unpredictable tracking errors in the presence of large uncertainties. Additionally, little consideration has been focused on transient performances, while transient performances dominate an important role in ensuring an autonomous and effective execution of cooperative flight mission. For instance, in the situation of performing a formation task in an uncertain environment with randomly distributed obstacles, to avoid collision between neighbor quadrotors and communication interruption, practical conditions typically require high-performance formation control algorithms to hold certain transient and steady-state specifications [26–28]. If the preselected performances cannot be specified as a prior, the unexpected consequences, including a relatively large overshoot, a slow convergence rate, or an enlarged steady-state deviation, may deteriorate system performances and even cause the failure of missions. Thus, it is imperative to design an enhanced anti-disturbance formation control for multiple quadrotors with preselected characteristics. Unfortunately, up to now, few results have been available for the guaranteed performance formation control design of multiple quadrotors.

Neural network (NN) is an efficient self-learning tool to counteract the adverse effect of unmodeled dynamics [29–32]. During the past decades, NN-based adaptive estimation algorithms have drawn much attentions, and a great deal of research advancements have been made in [33–38]. For instance, in [33], a radial basis function (RBF) neural network is employed for multiagent systems to neutralize unknown system dynamics and external disturbances. A constrained backstepping adaptive NN control scheme is presented in [35] for MIMO aeroelastic systems to approximate the system uncertainties. A RBF neural network is developed in [36] to identify unknown dynamics existing in multi-agent systems. In [37], by integrating with a neural adaptive observer, a consensus coordination controller is introduced for nonlinear multi-agent systems considering unknown external disturbances. In [38], by synthesizing a hybrid rescheduling policy, a fuzzy NN observer is studied for spacecrafts to neutralize the uncertainties with fuzzy features. It is worthwhile stating that unknown time-varying uncertainties can be straightforwardly recovered with a favorable convergence speed and estimation accuracy by utilizing a fast learning solution, which corresponds to a high adaptive gain.
in NN [33–38]. However, a sizeable adaptive gain may deteriorate transient performances and even make control inputs suffer from high-frequency oscillations during the initial stage [34–37], which will directly lead to collisions and communication interruptions among quadrotors. Meanwhile, another crucial issue inherent in implementing the calculation of NN is the computational burden owing to the involved huge number of NN hidden nodes. It is acknowledged that a large amount of NN nodes are inevitably needed to realize an excellent system identification capability against a vast range of uncertainties [39–41]. Nevertheless, the increase in the computation load results in a remarkable controlling error, especially for the maneuver formation control scenario that requires a real time computational capability. Consequently, it is paramount to explore an improved NN approximator for uncertainty identification with less chattering transient behaviors and reduced computational complexity even using a large adaptive gain.

Based on the above observations, a distributed adaptive consensus policy comprising of a low cost approximator is proposed to achieve a rapid and precise formation maintenance for multiple quadrotors with performance requirements and uncertainties. By introducing a prescribed performance control (PPC) technique, communication interruption and every possible collision between quadrotors under aggressive maneuvers can be circumvented. To handle unknown uncertainties, SE-MLP observers are effectively designed in the translational and rotational loop, respectively, greatly alleviating the computational complexity and avoiding high-frequency oscillations even using a large adaptive gain. The specific contributions of presented control algorithm are twofold:

1. Differing from the available cooperative issues concentrating on multi-agents formulated as single integrator [42], high-order nonlinear uncertain form [43, 44], fractional-order systems [45, 46], herein a nonlinear collective control policy is devised for quadrotor robust formation suffering from nonlinearity, underactuated MIMO properties, and uncertainties. Distinguished with the existing distributed consensus solutions [23–25], where synchronization errors can only converge to an unknown and conservative performance bound, a PPC technique consisting of an exponential decaying behavior bound and an error transformation function is enforced, allowing for a satisfaction of preset time-varying formation constraint rather than a constant limitation executed by the reported output constraint approach [46], indicating that the tracking precision, the maximum overshoot, and convergence time can be prescribed as a priori despite of uncertainties. And it is worth stressing that the proposed design can be suitable for a general type of intelligent unmanned systems considering nonlinear uncertain dynamics, including flocking and swarming control of autonomous surface vehicles [21, 47], circular formation of ground vehicles [48].

2. Different from the published approximation-based methods [39–41, 49] where transient oscillations are frequently encountered due to nonzero initial tracking errors and the usage of large adaptive gain, by using a state estimation error instead of tracking error to update NN parameters, a SE-MLP observer is proposed to decouple the control loop and learning loop via appropriately selecting the bandwidth of state estimator, which enables a fast and smooth transient behavior even using a sizeable adaptive gain. And transient solutions of L2 norm of the differential of neural weights are established to illustrate the mechanism in reducing chattering behaviors. In addition, unlike the traditional NN schemes subject to explosion of learning [36, 50], herein an artificial quantity rather than each elements in NN weight vector is entailed to be updated online; thus, a competitive observation performance can be realized by virtue of only regulating one parameter in each subsystem, effectively reducing the computational complexity and ensuring an affordable neuro-adaptive cooperative control framework to robust formation configuration under unsigned network, and comparative simulations are conducted to show the merits and demerits of developed SE-MLP observer.

The rest of this paper is arranged in the following order. Preliminaries and problem formulation are illustrated in Sect. 2. Formation controller design together with stability analysis is presented in Sect. 3. Section 4 provides simulation results, and conclusions are drawn in Sect. 5.
2 Preliminaries and problem formulation

2.1 Graph theory

Given the multiple quadrotors consisting of one virtual leader and N followers, the data communication among N followers is described by an undirected graph \( \mathcal{G} = \{\mathcal{V}, \mathcal{E}, \mathcal{A}\} \), with \( \mathcal{V} = \{v_i\} \) being the set of nodes and \( v_i (i = 1, 2, \ldots, N) \) being the i-th quadrotor. \( \mathcal{E} = \{(v_i, v_j) \subseteq \mathcal{V} \times \mathcal{V}\} \) indicates the information exchange between i-th quadrotor and its neighbor, i.e., the j-th quadrotor. The adjacency matrix \( \mathcal{A} = [a_{ij}] \in \mathbb{R}^{N \times N} \) denotes topology of undirected graph, given by \( a_{ij} = a_{ji} = 1 \) if \( (v_i, v_j) \in \mathcal{E} \), and \( a_{ij} = a_{ji} = 0 \), otherwise. The set of neighbors of quadrotor \( i \) is denoted by \( N_i = \{v_j \in \mathcal{V} : (v_j, v_i) \in \mathcal{E}, j \neq i\} \). Define the graph Laplacian matrix as \( \mathcal{L} = \mathcal{D} - \mathcal{A} \), where \( \mathcal{D} = \text{diag}(d_i) \) with the weighted in-degree \( d_i = \sum_{j=1}^{N} a_{ij} \). Specify the adjacency matrix between the follower quadrotors and the virtual leader as \( \mathbf{b} = \text{diag}(b_1, \ldots, b_N) \), in which \( b_i = 1 \) indicates that i-th quadrotor has access to the leader, or else, \( b_i = 0 \). For simplicity, denote \( \mathcal{H} = \mathcal{L} + \mathbf{b} \).

Assumption 1 [20, 21, 25, 26] Consider the involved multiple nodes, the unsigned graph for N follower quadrotors, and one virtual leader preserve connected if and only if there exists at least one path from the virtual leader to each follower, demonstrating that all the eigenvalues of matrix \( \mathcal{H} \) have positive real parts.

2.2 Quadrotor model

Define the inertial frame \( O_x X_y Z_e \) and body frame \( o_b x_b y_b z_b \), as shown in Fig. 1, where \( F_{i,n}(n = 1, \ldots, 4) \) represents the rotor thrusts of quadrotors. According to [31, 32], the kinematics and kinetics of i-th quadrotor conform to the following dynamics:

\[
\begin{align*}
\dot{X}_{i,p} &= X_{i,v} \\
\dot{X}_{i,v} &= f_{i,v}(X_{i,v}) + F_{i,v} + \Delta_{i,v} \\
\dot{X}_{i,o} &= X_{i,o} \\
\dot{X}_{i,o} &= f_{i,o}(X_{i,o}) + U_{i,o} + \Delta_{i,o}
\end{align*}
\]

where \( X_{i,p} = [X_{i,p1}, X_{i,p2}, X_{i,p3}]^T \) and \( X_{i,v} = [X_{i,v1}, X_{i,v2}, X_{i,v3}]^T \) stand for the translational motions including position and linear velocity vectors expressed in \( O_x X_y Z_e \), respectively. \( X_{i,\Omega} = [X_{i,\Omega1}, X_{i,\Omega2}, X_{i,\Omega3}]^T \) and \( X_{i,\theta} = [X_{i,\theta1}, X_{i,\theta2}, X_{i,\theta3}]^T \) represent rotational angles and angular velocity vector. \( F_{i,n}(u_{i,n} - G_i)/m_i \) denotes the equivalent control action, with \( m_i \) representing the i-th quadrotor mass. \( G_i = [0, 0, m_i g]^T \) with \( g \) being the gravity acceleration. \( \mathbf{1} = [\cos(X_{i,\Omega1}) \sin(X_{i,\Omega2}) \cos(X_{i,\Omega3}) + \sin(X_{i,\Omega1}) \sin(X_{i,\Omega2}) \cos(X_{i,\Omega3}) \cos(X_{i,\Omega1}) - \cos(X_{i,\Omega1}) \cos(X_{i,\Omega2}) \cos(X_{i,\Omega3})]^T \) indicates the interaction matrix between translation and rotational motions. \( u_{i,1} \) and \( U_{i,\theta} = [u_{i,2}, u_{i,3}, u_{i,4}]^T \) express the actual lifting force and torque vector, which are produced by a linear combination of rotor thrusts as \( u_{i,1} = F_{i,1} + F_{i,2} + F_{i,3} + F_{i,4}, u_{i,2} = J_{i,\theta} (l_{i} F_{i,1} - l_{i} F_{i,3}), u_{i,3} = J_{i,\theta} (l_{i} F_{i,2} - l_{i} F_{i,4}), u_{i,4} = J_{i,\phi} (c_{i} F_{i,1} + c_{i} F_{i,2} - c_{i} F_{i,3} + c_{i} F_{i,4}) \), where \( c_{i} \) denotes force-to-moment factor and \( l_{i} \) represents the distance between each rotor of quadrotor and the center of mass. Obviously, the quadrotor is underactuated, since the number of actual control inputs is less than that of flight states. Notations \( f_{i,v}(X_{i,v}) = -\Pi_{i,v} X_{i,v}/m_i \) and \( f_{i,o}(X_{i,o}) = -\Pi_{i,o} X_{i,o}/J_{i,\theta} \) are model perturbations induced by inaccurate aerodynamic damping matrices \( \Pi_{i,1}, \Pi_{i,2} \). \( J_i = \text{diag}(J_{i,\theta}, J_{i,\theta}, J_{i,\phi}) \) is the inertia moment matrix. \( \Delta_{i,v} = [\Delta_{i,v1}, \Delta_{i,v2}, \Delta_{i,v3}]^T \) and \( \Delta_{i,o} = [\Delta_{i,o1}, \Delta_{i,o2}, \Delta_{i,o3}]^T \) stand for unknown bounded environmental disturbances.

2.3 Radial basis function NN

For any given scalar continuous function \( f(\cdot) \), one can use a NN with a sufficient degree of accuracy as follows:

\[
f(\mathbf{x}_p) = \mathbf{w}^T h(\mathbf{x}_p) + \varepsilon(\mathbf{x}_p), |\varepsilon(\mathbf{x}_p)| \leq \varepsilon
\]

where \( \varepsilon(\mathbf{x}_p) \) stands for the unavoidable approximation.
error with its upper bound being $\bar{e}$. $\mathbf{w}^* = [w_1^*, w_2^*, \ldots, w_L^*]^T \in \mathbb{R}^L$ denotes the ideal weight with $L$ being the hidden node number. $\mathbf{x}_p = [x_1, x_2, \ldots, x_p]^T$ is the neural input with $p$ being the input number. $\mathbf{h}(\mathbf{x}_p) = [h_1(\mathbf{x}_p), h_2(\mathbf{x}_p), \ldots, h_L(\mathbf{x}_p)]^T$ represents the commonly used Gaussian basis function. Referring to [34], each component $h_j$ is chosen as

$$h_j(\mathbf{x}_p) = \exp\left(-\frac{\|\mathbf{x}_p - \mathbf{d}\|^2}{2\phi_j^2}\right), \quad j = 1, 2, \ldots, L$$

with $\mathbf{d} = [\delta_1, \delta_2, \ldots, \delta_p]^T$ and $\phi_j$ representing the center vector and width, respectively.

**Remark 1** The exponential function $\exp(\cdot) > 0$ is a monotonously increasing function and $-\|\mathbf{x}_p - \mathbf{d}\|^2 / 2\phi_j^2 \leq 0$. Obviously, $0 < h_j(\mathbf{x}_p) \leq h_j(0) = 1$. Evidently, the Gaussian basis function is limited within $\|\mathbf{h}(\mathbf{x}_p)\| \leq \bar{h}$, where $\bar{h}$ denotes the upper bound.

**Remark 2** It is well known that when traditional NNs are designed to identify unknown functions with a high precision, explosion of learning parameters unavoidably appears due to the involved numerous hidden units; especially, as argument vector dimension of the observed function increases, the number of parameters to be regulated will drastically grow, leading to a heavy computational complexity, which is not appropriate for the considered quadrotor formation control system that can only tolerate a slight time delay during the computational process.

**Control Objectives** The purpose of this paper is to design a distributed neural adaptive formation consensus for follower quadrotors with guaranteed performances, such that

(i) The follower individual can be steered to tend to the predefined trajectory of virtual leader, i.e., $\mathbf{X}_d = [X_1 d, X_2 d, X_3 d]^T$, and the anticipated formation configuration is described as

$$\lim_{t \to \infty} \left(\frac{1}{N} \sum_{i=1}^{N} \mathbf{X}_i p\right) = \mathbf{X}_d, \quad \lim_{t \to \infty} (\mathbf{X}_{i,p} - \mathbf{X}_{j,p}) = \mathbf{q}_{ij}$$

where $\mathbf{q}_{ij} = [q_{i,1}, q_{i,2}, q_{i,3}]^T$ is the expected pattern expressed in the inertial frame among networked quadrotors and $q_{i,1}, q_{i,2}, q_{i,3}$ are the expected distance deviation between the $i$-th individual and formation geometry center. $\mathbf{q}_{ij} = \mathbf{q}_i - \mathbf{q}_j$ denotes the anticipated relative position discrepancy between $i$-th and $j$-th individual.

(ii) The formation synchronization deviation $e_{i,p,k} (k = 1, 2, 3)$ obeys the following preselected condition:

$$-\sigma_{i,p,k} \rho_{i,p,k}(t) < e_{i,p,k}(t) < \sigma_{i,p,k} \rho_{i,p,k}(t)$$

where $0 < \sigma_{i,p,k} \leq 1$, $0 < \sigma_{i,p,k} \leq 1$ are design scalars, $\rho_{i,p,k}(t)$ represents the prescribed performance function, and its definition is given in Sect. 3.

**Remark 3** The control objectives imply that not only the desired formation pattern can be maintained through application of the distributed consensus protocol, but also the pre-given time-varying trajectory determined by virtual leader can be tracked accurately for followers. Equation (4) implies that the location of each individual can synchronize to the leader with a designed deviation. Besides, it should be pointed out that since only a portion of follower individuals can have access to the data of virtual leader, the developed strategy is distributed essentially; the effectiveness relies on the network topology to be designed and consensus parameters, irrespective of the number of agents. In addition, note that the anticipated relative position discrepancy $\mathbf{q}_{ij}$ can be specified according to requirements of different tasks, for example, when $\mathbf{q}_{ij}$ is designated as a time-related signal, the proposed technique can be readily expanded to a time-varying formation pattern case.

### 3 Formation consensus design and stability analysis

Here, a distributed neural adaptive consensus protocol with guaranteed performances for multiple quadrotors is proposed to acquire the previous formation control objectives. The control block diagram is given in Fig. 2.

**3.1 SE-MLP observer**

In the section, to facilitate the subsequent demonstration and calculation, and referring to the design principle of MLP [41], take the translational dynamics of $i$-th quadrotor into consideration and the lumped
disturbances can be approximated by the following MLP:

\[ \chi_{i,v} = f_{i,v}(\mathbf{X}_{i,v}) + \Lambda_{i,v} = \frac{1}{2} \mathbf{W}_{i,v}^T \| \mathbf{h}_{i,v}(\mathbf{X}_{i,v}) \|^2 + e_{i,v} \]  \hspace{1cm} (6)

where \( \mathbf{W}_{i,v} = [W_{i,v1}, W_{i,v2}, W_{i,v3}]^T \) is the ideal weight vector and it corresponds to the artificial quantity \( \| \mathbf{w}_{i,v} \|^2 e_{i,v} \) formulated in the traditional RBF neural network, where \( e_{i,v} \) denotes velocity tracking error. Besides, it should be noted that there exists an unknown positive constant \( \mathcal{W} \) such that weight vector \( \mathbf{W}_{i,v} \) conforms to \( \| \mathbf{W}_{i,v} \| \leq \mathcal{W} \). \( \mathbf{h}_{i,v}(\mathbf{X}_{i,v}) \in \mathbb{R}^L \) is the Gaussian basis function with \( L \) being the number of hidden nodes in NN. \( \mathbf{X}_{i,v} = [X_{i,v1}, X_{i,v2}]^T \), and \( e_{i,v} = [e_{i,v1}, e_{i,v2}, e_{i,v3}]^T \) represents the approximation error. Note that \( \chi_{i,v} \) is an unknown and continuous function, indicating that the precise information of \( \chi_{i,v} \) is not obtained.

**Remark 4** For classical NN approximators [39, 50], the number of learning parameters to be regulated is mainly determined by the dimension of nodes in hidden layer; thus, \( \mathbf{h}_{i,v}(\mathbf{X}_{i,v}) \in \mathbb{R}^L \) inevitably causes a substantial time-consuming computational process, especially when a high accuracy identification result is required. Here, only one adaptive learning parameter is needed to be updated online for each subsystem, regardless of the specific number of hidden nodes \( L \), which can eliminate the issue of learning explosion inherent in the existing adaptive NN algorithms [39, 40].

To approximate the unknown disturbance \( \chi_{i,v} \), the following SE-MLP observer is designed as

\[
\begin{align*}
\dot{\hat{\chi}}_{i,v} &= f_{i,v}(\hat{\mathbf{X}}_{i,v}) + \Lambda_{i,v} = \frac{1}{2} \mathbf{W}_{i,v}^T \| \mathbf{h}_{i,v}(\hat{\mathbf{X}}_{i,v}) \|^2 + e_{i,v} \\
\dot{\hat{\mathbf{X}}}_{i,v} &= \mathbf{W}_{i,v}^T \mathbf{h}_{i,v}(\hat{\mathbf{X}}_{i,v}) - \sigma_{i,v} \tilde{\mathbf{W}}_{i,v}
\end{align*}
\]  \hspace{1cm} (7)

where \( \hat{\mathbf{X}}_{i,v} = \mathbf{X}_{i,v} - \hat{\mathbf{X}}_{i,v} \) is the estimation error vector, \( \mathbf{h}_{i,v} = diag(\eta_{i,v1}, \eta_{i,v2}, \eta_{i,v3}) \) is the positive estimator bandwidth, \( \gamma_{i,v} > 0 \) represents the adaptive gain, and \( \sigma_{i,v} = diag(\sigma_{i,v1}, \sigma_{i,v2}, \sigma_{i,v3}) \) is the modification parameter matrix with \( \sigma_{i,v1}, \sigma_{i,v2}, \sigma_{i,v3} \) being positive constants. \( \tilde{\mathbf{W}}_{i,v} \) is an estimate of ideal weight vector \( \mathbf{W}_{i,v} \).

By invoking (1), the total disturbances (6), and observer (7), the estimation error dynamics can be formulated in the subsequent form:

\[
\begin{align*}
\dot{\hat{\mathbf{X}}}_{i,v} &= -\frac{1}{2} \mathbf{W}_{i,v}^T \| \mathbf{h}_{i,v}(\hat{\mathbf{X}}_{i,v}) \|^2 - \eta_{i,v} \dot{\hat{\mathbf{X}}}_{i,v} + e_{i,v} \\
\dot{\tilde{\mathbf{W}}}_{i,v} &= -\frac{1}{2} \gamma_{i,v} \| \mathbf{h}_{i,v}(\hat{\mathbf{X}}_{i,v}) \|^2 \hat{\mathbf{X}}_{i,v} - \sigma_{i,v} \tilde{\mathbf{W}}_{i,v}
\end{align*}
\]  \hspace{1cm} (8)

where \( \tilde{\mathbf{W}}_{i,v} = \mathbf{W}_{i,v} - \tilde{\mathbf{W}}_{i,v} \).

Next, a rigorous mathematical derivation and theoretical analysis will be conducted to show the improved transient performances of presented SE-MLP observer by using series of algebraic operations. It is noteworthy that the truncated \( L_2 \) norm of the differential of NN weights can reflect their frequency.
features, and generally, a larger $L_2$ norm of the differential of NN weight corresponds to more oscillations included in system response. Here, $\|\hat{W}_{i,v}\|_{L_2,t^*} = \int_0^{t^*} \|\hat{W}_{i,v}\| \, dt$ indicates the truncated $L_2$ norm of $\hat{W}_{i,v}$. Thus, Theorem 1 is established.

**Theorem 1** Take the estimation error dynamics (8) into consideration, and then, $L_2$ norm of the differential of weight vector $\hat{W}_{i,v}$ during the time interval $[0, t^*]$ satisfies

$$
\|\hat{W}_{i,v}\|_{L_2,t^*} \leq \frac{\gamma_{i,v}^2 \beta_T \sqrt{2(2\lambda_{\min}(\eta_{i,v}) - 1)}}{\sqrt{\gamma_{i,v}^2 + 2\sqrt{\gamma_{i,v}^2 + 2\pi p_2 t^*}}} + \frac{\gamma_{i,v}(\lambda_{\max}(\sigma_{i,v}))}{\sqrt{2}}
$$  

with $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and maximum eigenvalues of a matrix, respectively.

**Proof** Construct the following Lyapunov function candidate:

$$
V = \frac{1}{2} \hat{X}_{i,v}^T \hat{X}_{i,v} + \frac{1}{2\gamma_{i,v}} \hat{W}_{i,v}^T \hat{W}_{i,v}
$$  

(10)

Differencing $V$ with respect to time yields

$$
\dot{V} = \hat{X}_{i,v}^T \left[ \frac{1}{2} \dot{\hat{W}}_{i,v}^T \dot{h}_{i,v}(\hat{X}_{i,v}) \right] - \frac{1}{2} \hat{W}_{i,v}^T (h_{i,v}(\hat{X}_{i,v}) - \sigma_{i,v})^T \dot{X}_{i,v} - \sigma_{i,v} \dot{W}_{i,v}
$$  

(11)

Resorting to Young’s inequality produces

$$
\hat{X}_{i,v}^T \dot{e}_{i,v} \leq \frac{1}{2} \|\hat{X}_{i,v}\|^2 + \frac{1}{2} \|\dot{e}_{i,v}\|^2, \|\hat{W}_{i,v}\| \leq \frac{1}{2} \|\dot{W}_{i,v}\|^2 - \frac{1}{2} \|\hat{W}_{i,v}\|^2
$$  

(12)

Substituting (12) into (11), $V$ further satisfies

$$
\dot{V} \leq - \left( \lambda_{\min}(\eta_{i,v}) - \frac{1}{2} \right) \|\dot{\hat{X}}_{i,v}\|^2 - \left( \frac{1}{2} \lambda_{\min}(\sigma_{i,v}) \right) \|\dot{W}_{i,v}\|^2 + \bar{e}_T
$$  

(13)

with $\bar{e}_T = \frac{1}{2} \|\dot{e}_{i,v}\|^2 + \lambda_{\max}(\sigma_{i,v}) \|\dot{W}_{i,v}\|^2$ being a positive constant.

Define $\mu_T = \min_{i=1,...,N} \left\{ 2\lambda_{\min}(\eta_{i,v}) - 1, \gamma_{i,v} \lambda_{\min}(\sigma_{i,v}) \right\}$, and to assure the convergence of $V$, let $\mu_T > 0$, i.e., $2\lambda_{\min}(\eta_{i,v}) - 1 > 0$, $\gamma_{i,v} \lambda_{\min}(\sigma_{i,v}) > 0$. Thus, (13) can be rewritten as

$$
\dot{V} \leq - \mu_T V + \bar{e}_T
$$  

(14)

Solving the inequality (14) results in

$$
V \leq \frac{\bar{e}_T}{\mu_T} (1 - e^{-\mu_T t}) + V(0)e^{-\mu_T t}
$$  

(15)

Combining the Lyapunov function candidate (10) and utilizing the inequality $\sqrt{a + b} \leq \sqrt{a} + \sqrt{b}$, $a \geq 0$, $b \geq 0$, it further generates the following bound:

$$
\|\hat{X}_{i,v}\| \leq \sqrt{\gamma_{i,v} \frac{2\beta_T}{\mu_T} + 2V(0)}
$$  

$$
\|\hat{W}_{i,v}\| \leq \frac{\gamma_{i,v} \frac{2\beta_T}{\mu_T} + 2V(0)}{\sqrt{\gamma_{i,v}}}
$$  

(16)

To obtain the transient bound of estimation error dynamics $\hat{X}_{i,v}$, by recollecting (13), we can deduce the following inequality:

$$
(\lambda_{\min}(\eta_{i,v}) - \frac{1}{2}) \|\hat{X}_{i,v}\|^2 \leq \bar{V} + \bar{e}_T
$$  

(17)

Afterwards, through integrating (17) over $[0, t^*]$, we get

$$
\|\hat{X}_{i,v}\|_{L_2,t^*} \leq \frac{1}{\lambda_{\min}(\eta_{i,v}) - 1/2 + \frac{\bar{e}_T t^*}{\lambda_{\min}(\eta_{i,v}) - 1/2}}
$$  

(18)

Following (18), one has

$$
\|\hat{X}_{i,v}\|_{L_2,t^*} \leq \frac{1}{\sqrt{2\lambda_{\min}(\eta_{i,v}) - 1}} \left( \|\hat{X}_{i,v}(0)\| + \|\hat{W}_{i,v}(0)\| + \sqrt{2\beta_T t^*} \right)
$$  

(19)
To make a quantitative evaluation for the transient performance, one concern is to derive the upper bound of \( \| \dot{\mathbf{W}}_{i,v} \| \), and aiming at the purpose, it follows from the time derivative of \( \mathbf{W}_{i,v} \) in (7) that

\[
\| \dot{\mathbf{W}}_{i,v} \| \leq \frac{1}{2} c_i v \left[ \| \mathbf{h}_{i,v}(\mathbf{X}_{i,v}) \|^2 + \lambda_{\text{max}}(\mathbf{a}_{i,v}) \| \dot{\mathbf{W}}_{i,v} \| \right]
\]  

(20)

Due to the continuous and smooth property of the Gaussian basis function, \( \| \mathbf{h}_{i,v}(\mathbf{X}_{i,v}) \|^2 \leq \mathbf{h}_T^2 \) can be ensured, where \( \mathbf{h}_T \) is a positive constant. In addition, recalling the results of error dynamics in (16), define the upper bound of \( \| \dot{\mathbf{W}}_{i,v} \| \) as \( \mathbf{W}_2 = \sqrt{\mathbf{h}_T^2 (2\mathbf{I}/\mu_T + \| \dot{\mathbf{X}}_{i,v}(0) \|) + \| \mathbf{W}_{i,v}(0) \|} \) for ease of presentation. Then, we can obtain

\[
\| \dot{\mathbf{W}}_{i,v} \| \leq \frac{1}{2} c_i v \left[ \mathbf{h}_T^2 \| \dot{\mathbf{X}}_{i,v} \| + \lambda_{\text{max}}(\mathbf{a}_{i,v}) (\mathbf{W}_1 + \mathbf{W}_2) \right]
\]  

(21)

Squaring both sides of (21) gives

\[
\| \dot{\mathbf{W}}_{i,v} \|^2 \leq \left\{ \frac{1}{2} c_i v \left[ \mathbf{h}_T^2 \| \dot{\mathbf{X}}_{i,v} \| + \lambda_{\text{max}}(\mathbf{a}_{i,v}) (\mathbf{W}_1 + \mathbf{W}_2) \right] \right\}^2
\leq \frac{1}{2} c_i v \left[ \mathbf{h}_T^2 \| \dot{\mathbf{X}}_{i,v} \|^2 + (\lambda_{\text{max}}(\mathbf{a}_{i,v}))^2 (\mathbf{W}_1 + \mathbf{W}_2)^2 \right]
\]  

(22)

Similarly, via using integration of (22) over \([0,t^*]\), we have

\[
\| \dot{\mathbf{W}}_{i,v} \|^2 \leq \frac{1}{2} c_i v \left[ \mathbf{h}_T^2 \| \dot{\mathbf{X}}_{i,v} \|^2 + \left( \lambda_{\text{max}}(\mathbf{a}_{i,v}) \right)^2 (\mathbf{W}_1 + \mathbf{W}_2)^2 \right] t^*
\]  

(23)

which further produces

\[
\| \dot{\mathbf{W}}_{i,v} \|_{L_2,t^*} \leq \frac{1}{2} c_i v \left[ \mathbf{h}_T^2 \| \dot{\mathbf{X}}_{i,v} \|_{L_2,t^*} + \left( \lambda_{\text{max}}(\mathbf{a}_{i,v}) \right)^2 \right] (\mathbf{W}_1 + \mathbf{W}_2) \sqrt{t^*}
\]  

(24)

Noticing the bound of \( \dot{\mathbf{X}}_{i,v} \) in (19), the truncated \( L_2 \) norm of \( \mathbf{W}_{i,v} \) can be rewritten as

\[
\| \dot{\mathbf{W}}_{i,v} \|_{L_2,t^*} \leq \frac{\gamma_{i,v} \mathbf{h}_T^2}{2(\lambda_{\text{min}}(\mathbf{a}_{i,v}) - 1)} \left( \| \mathbf{X}_{i,v}(0) \| + \| \dot{\mathbf{X}}_{i,v}(0) \| \sqrt{T_{i,v}} + \sqrt{2 \mathbf{h}_T^2} \right) + \frac{\gamma_{i,v} (\lambda_{\text{max}}(\mathbf{a}_{i,v}))}{\sqrt{t^*}} \left( \| \mathbf{W}_1 + \mathbf{W}_2 \| \sqrt{T_{i,v}} + \| \mathbf{W}_{i,v}(0) \| \right) \sqrt{t^*}
\]  

(25)

The proof of Theorem 1 is completed.

To facilitate comparison, the following lemma is given to show the truncated \( L_2 \) norm of differential of weight \( \mathbf{Q}_{i,v} \in \mathbb{R}^{l \times 3} \) used in traditional RBF neural networks.

**Lemma 1** [53] Consider the classical radial basis function NNs given in Sect. 2.3, where the adaptive law of the weight is described as \( \dot{\mathbf{Q}}_{i,v} = \gamma_{i,v} [\mathbf{h}_{i,v}(\mathbf{X}_{i,v})]^T - \mathbf{a}_{i,v} \mathbf{Q}_{i,v} \). Then, \( L_2 \) norm of the differential of the weight \( \mathbf{Q}_{i,v} \) during the time interval \([0,t^*]\) satisfies

\[
\| \dot{\mathbf{Q}}_{i,v} \|_{L_2,t^*} \leq \frac{\sqrt{2 \mathbf{h}_T^2}}{\gamma_{i,v}} \left( \| \mathbf{e}_{i,v}(0) \| + \| \mathbf{Q}_{i,v}(0) \| \sqrt{T_{i,v}} + \sqrt{2 \mathbf{h}_T^2} \right) + \frac{\sqrt{2 \mathbf{h}_T^2}}{\gamma_{i,v}} (\lambda_{\text{max}}(\mathbf{a}_{i,v})) \left( \mathbf{W}_1 + \mathbf{W}_2 + \| \mathbf{e}_{i,v}(0) \| \sqrt{T_{i,v}} + \| \mathbf{Q}_{i,v}(0) \| \right) \sqrt{t^*}
\]  

(26)

where \( \mathbf{k}_{i,v} = \text{diag}(k_{i,v1}, k_{i,v2}, k_{i,v3}) \) denotes the positive control gain matrix in the velocity loop.

**Remark 5** From (25) and (26), it is noteworthy that as the adaptive gain increases, the truncated \( L_2 \) norms of differential of the weights for both classical NN and presented SE-MLP gradually become larger, contributing to a fact that the transient oscillations will aggravate. But compared with the current NN approaches [39, 50], the proposed SE-MLP observer provides two extra design freedoms to alleviate oscillations with the precondition of the same adaptive gain. One is that by employing an observation error \( \dot{\mathbf{X}}_{i,v} \) instead of tracking error \( \mathbf{e}_{i,v} \) to learn the adaptive
parameters, the developed SE-MLP observer renders a time-scale separation between controlling and leaning loop with the selection of observer bandwidth \( \eta_{ik} \) satisfying \( \eta_{ik} \geq 2k_{i,ik} \), and a larger observer bandwidth will lead to a less fluctuation transient, implying that the presented SE-MLP can tolerate a relatively higher adaptive gain compared with classical NNs. Another is that by setting \( X_{i,v}(0) = \dot{X}_{i,v}(0) \), the proposed SE-MLP observer effectively removes the undesired transient learning process induced by nonzero initial tracking errors. Obviously, the above two avenues cannot be provided by utilizing the current NN methods.

### 3.2 Formation synchronization error transformation

To facilitate the formation controller design, based on graph theory, define the formation synchronization error \( e_{i, pk} \) as follows:

\[
e_{i, pk} = \sum_{j \in N_i} a_{ij}(X_{i, pk} - X_{j, pk} - q_{ij,k}) + b_i(X_{i, pk} - X^d_k - q_{ik} + \frac{1}{N} \sum_{i=1}^{N} q_{i,k})
\]

where \( N_i \) denotes a set consisting of quadrotors excluding quadrotor \( i \), while the subscript \( k \) denotes \( k \)-th component of position vector. Note that only a team of agents can have access to the leader in (27), leading to a reduced communication cost compared to centralized formation framework [54].

Differentiating \( e_{i, pk} \) along (1) gives

\[
\dot{e}_{i, pk} = \sum_{j \in N_i} a_{ij}(\dot{X}_{i, pk} - \dot{X}_{j, pk}) + b_i(\dot{X}_{i, pk} - X^d_k - q_{ik}) = (d_i + b_i)X_{i,v} - \sum_{j \in N_i} a_{ij}X_{j,v} - b_i X^d_k
\]

(28)

Note that it is difficult to establish a control law with respect to inequality (5). To tackle this problem, based on prescribed performance control principle [55–58], a formation error transformation function \( S_{i, pk}(Z_{i, pk}) \) is introduced to transform the limited formation synchronization deviation to an equivalent one free of constraints:

\[
e_{i, pk}(t) = \rho_{i, pk}(t)S_{i, pk}(Z_{i, pk})
\]

(29)

with

\[
S_{i, pk}(Z_{i, pk}) = \frac{\sigma_{i, pk} \exp(Z_{i, pk}) - \sigma_{i, pk} \exp(-Z_{i, pk})}{\exp(Z_{i, pk}) + \exp(-Z_{i, pk})}
\]

(30)

where \( Z_{i, pk} \) represents the transformed formation error, and the performance function is selected as \( \rho_{i, pk}(t) = (\rho_{i, pk}^0 - \rho_{i, pk}^\infty) e^{-\lambda_{i, pk} t} + \rho_{i, pk}^\infty \). The decreasing rate \( \lambda_{i, pk} \) is applied to regulate the decaying rate of \( e_{i, pk} \). The positive constant \( \rho_{i, pk}^\infty \) is the upper bound of allowable steady-state error, while \( \rho_{i, pk}^0 \) is tuned to maintain \(-\sigma_{i, pk} \rho_{i, pk}(0) < e_{i, pk}(0) < \sigma_{i, pk} \rho_{i, pk}(0) \).

The inverse transformation of (29) is given as

\[
Z_{i, pk} = \frac{1}{2} \ln \left( \frac{\sigma_{i, pk} + e_{i, pk}/\rho_{i, pk}}{\sigma_{i, pk} - e_{i, pk}/\rho_{i, pk}} \right)
\]

(31)

with its time derivative

\[
\dot{Z}_{i, pk} = \hat{z}_{i, pk}(\dot{e}_{i, pk} - \dot{\rho}_{i, pk} e_{i, pk} / \rho_{i, pk}) = \hat{z}_{i, pk} \left( (d_i + b_i)X_{i,v} - \sum_{j \in N_i} a_{ij}X_{j,v} - b_i X^d_k - \dot{\rho}_{i, pk} e_{i, pk} / \rho_{i, pk} \right)
\]

(32)

where \( \hat{z}_{i, pk} = \frac{1}{2\rho_{i, pk}} \left( \frac{1}{\sigma_{i, pk}/\rho_{i, pk} + \sigma_{i, pk}} - \frac{1}{e_{i, pk}/\rho_{i, pk} - \sigma_{i, pk}} \right) \).

**Lemma 2** [55, 59, 60] Considering the formation synchronization error \( e_{i, pk} \) and transformed error \( Z_{i, pk} \), the prescribed limitation imposed on \( e_{i, pk} \) will always be achieved as long as \( Z_{i, pk} \) is bounded, i.e., (5) is satisfied.

### 3.3 Neural adaptive control design

**Step 1** Assuming \( x_{i,v} \) as a virtual control input of position dynamics, to make the transformed error \( Z_{i, pk} \) tend to zero, a virtual control policy is derived:

\[
x_{i,v} = -k_{i,v} \bar{e}_{i, pk} Z_{i, pk} + \frac{1}{d_i + b_i} \sum_{j \in N_i} a_{ij}X_{j,v} + \frac{b_i}{d_i + b_i} X^d_k + \frac{1}{d_i + b_i} \dot{\rho}_{i, pk} e_{i, pk} / \rho_{i, pk}
\]

(33)

For the sake of facilitating the analysis and design, (33) can be reformulated as the following compact form:

\[
x_{i,v} = -k_{i,v} \bar{e}_{i, pk} Z_{i, pk} + \frac{1}{d_i + b_i} \sum_{j \in N_i} a_{ij}X_{j,v} + \frac{b_i}{d_i + b_i} X^d_k + \frac{1}{d_i + b_i} \dot{e}_{i, pk}
\]

(34)
where $\Gamma = \text{diag}(\dot{\rho}_{i,p1}(t)/\rho_{i,p1}(t), \dot{\rho}_{i,p2}(t)/\rho_{i,p2}(t), \dot{\rho}_{i,p3}(t)/\rho_{i,p3}(t))$, $\alpha_{i,v} = [\alpha_{i,v1}, \alpha_{i,v2}, \alpha_{i,v3}]^T$, $Z_{i,p} = [Z_{i,p1}, Z_{i,p2}, Z_{i,p3}]^T$, $k_{i,p} = \text{diag}(k_{i,p1}, k_{i,p2}, k_{i,p3})$ with $k_{i,p1}, k_{i,p2}, k_{i,p3}$ being nonnegative control gains.

To avoid analytically differentiating $\alpha_{i,v}$, inevitably causing complexity explosion problem due to the recursive analytic computation of backstepping, a dynamic surface control (DSC) principle is embedded here. Let $\alpha_{i,v}$ inject into the following filter to yield an estimate of $\alpha_{i,v}$, denoted as $\tilde{\alpha}_{i,v} = [\tilde{\alpha}_{i,v1}, \tilde{\alpha}_{i,v2}, \tilde{\alpha}_{i,v3}]^T$:

$$\tau_{i,v} + \tilde{\alpha}_{i,v} = \alpha_{i,v}, \quad \tilde{\alpha}_{i,v}(0) = \alpha_{i,v}(0)$$  \hspace{1cm} (35)

where $\tau_{i,v} = \text{diag}(\tau_{i,v1}, \tau_{i,v2}, \tau_{i,v3})$ is the positive time constant matrix. Define the first filtering error $y_{i,v} = \tilde{\alpha}_{i,v} - \alpha_{i,v}$, combining with (34), the time derivative of $y_{i,v}$ is derived as

$$\dot{y}_{i,v} = -\tau_{i,v}^{-1}y_{i,v} - \dot{\alpha}_{i,v} = -\tau_{i,v}^{-1}y_{i,v} + B_{i,v}(Z_{i,p}k, \dot{Z}_{i,p}, \dot{X}_{i,v}, \hat{\dot{X}}_{i,v}, \hat{\dot{e}}_{i,v})$$ \hspace{1cm} (36)

where $B_{i,v}(Z_{i,p}k, \dot{Z}_{i,p}, \dot{X}_{i,v}, \hat{\dot{X}}_{i,v}, \hat{\dot{e}}_{i,v})$ is a continuous function.

Step 2 In this step, $F_{i,v}$ is treated as a virtual control signal to stabilize velocity $\dot{X}_{i,v}$. The tracking error of translational velocity is defined as

$$e_{i,v} = X_{i,v} - \tilde{X}_{i,v}$$ \hspace{1cm} (37)

whose time derivative along (1) is

$$\dot{e}_{i,v} = F_{i,v} + f_{i,v}(X_{i,v}) + A_{i,v} - \dot{\tilde{X}}_{i,v}$$ \hspace{1cm} (38)

where $e_{i,v} = [e_{i,v1}, e_{i,v2}, e_{i,v3}]^T$.

Considering system (38), based on (7), the velocity subsystem control law is designed as follows:

$$F_{i,v} = -k_{i,v}e_{i,v} - \frac{1}{2}\dot{W}_{i,v}(X_{i,v}) + \dot{\tilde{X}}_{i,v}$$ \hspace{1cm} (39)

where $k_{i,v} = \text{diag}(k_{i,v1}, k_{i,v2}, k_{i,v3})$ with $k_{i,v1}, k_{i,v2}, k_{i,v3}$ being control gains.

In fact, to implement the velocity adjustment for follower quadrotors, the magnitude of $F_{i,v}$ is mainly determined by thrust force $u_{i,v}$, and the desired body attitude $(\phi_i, \theta_i, \psi_i)$ depends on its orientation, with $\phi_i, \theta_i, \psi_i$ being the anticipated roll, pitch, and yaw angles. $F_{i,v} = [F_{i,v1}, F_{i,v2}, F_{i,v3}]^T$ corresponds to the expected position control input meeting

$$\begin{align*}
F_{i,v1} &= \frac{m_i}{u_{i,v}}(\cos(\psi_i)\sin(\theta_i)\cos(\phi_i) + \sin(\psi_i)\sin(\phi_i)) \\
F_{i,v2} &= \frac{m_i}{u_{i,v}}(\sin(\psi_i)\sin(\theta_i)\cos(\phi_i) - \cos(\psi_i)\sin(\phi_i)) \\
F_{i,v3} &= \frac{m_i}{u_{i,v}}(\cos(\theta_i)\cos(\phi_i) - g_i)
\end{align*}$$  \hspace{1cm} (40)

Subsequently, the lifting force $u_{i,1}$ and the anticipated angles $\phi_i^d, \theta_i^d$ can be derived as

$$\begin{align*}
u_{i,1} &= \sqrt{F_{i,v1}^2 + F_{i,v2}^2 + (F_{i,v3} + g_i)^2} \\
\phi_i^d &= \arcsin\left(\frac{m_i}{u_{i,v}}(F_{i,v1}\sin(\psi_i) - F_{i,v2}\cos(\psi_i))\right) \\
\theta_i^d &= \arctan\left(\frac{1}{F_{i,v3} + g_i}(F_{i,v1}\cos(\psi_i) + F_{i,v2}\sin(\psi_i))\right)
\end{align*}$$ \hspace{1cm} (41)

with $\psi_i^d$ being the yaw command planned by users.

For the sake of simplicity, define $X_{i,\Omega}^d = [\phi_i^d, \theta_i^d, \psi_i^d]^T$ as the attitude command. Based on DSC method, let $X_{i,\Omega}^d$ inject into the following linear filter to acquire the estimate of $X_{i,\Omega}^d$, i.e.,

$$\tau_{i,\Omega}\dot{X}_{i,\Omega}^d + X_{i,\Omega}^d = X_{i,\Omega}^d$$ \hspace{1cm} (42)

where $\tau_{i,\Omega} = \text{diag}(\tau_{i,\Omega1}, \tau_{i,\Omega2}, \tau_{i,\Omega3})$ is a time constant matrix and $X_{i,\Omega}^d = [X_{i,\Omega1}^d, X_{i,\Omega2}^d, X_{i,\Omega3}^d]^T$ denotes filtered output vector.

Considering the second filtering error as $y_{i,\Omega} = X_{i,\Omega}^d - X_{i,\Omega}^d$, we can obtain

$$\dot{y}_{i,\Omega} = -\tau_{i,\Omega}^{-1}y_{i,\Omega} - X_{i,\Omega}^d = -\tau_{i,\Omega}^{-1}y_{i,\Omega} + B_{i,\Omega}(\dot{X}_{i,\Omega}^d)$$ \hspace{1cm} (43)

Step 3 In this step, to construct attitude control law for individual quadrotor to track attitude commands, the tracking error in attitude loop is defined as

$$e_{i,\Omega} = X_{i,\Omega} - \dot{X}_{i,\Omega}$$ \hspace{1cm} (44)

with its derivative along (1) being computed as

$$\dot{e}_{i,\Omega} = \dot{X}_{i,\Omega} - \dot{\dot{X}}_{i,\Omega} = X_{i,\Omega} - \ddot{X}_{i,\Omega}$$ \hspace{1cm} (45)

Thus, the virtual angular velocity law is calculated as

$$\alpha_{i,\omega} = -k_{i,\Omega}e_{i,\Omega} + \dot{\dot{X}}_{i,\Omega}$$ \hspace{1cm} (46)

where $k_{i,\Omega} = \text{diag}(k_{i,\Omega1}, k_{i,\Omega2}, k_{i,\Omega3})$ with $k_{i,\Omega1}, k_{i,\Omega2}, k_{i,\Omega3}$ denoting nonnegative values.
Following the same line with Step 1, let \( \alpha_{t,0} \) pass through the following filter:
\[
\tau_{t,0}\hat{x}_{t,0} + \bar{x}_{t,0} = \alpha_{t,0}, \quad \bar{x}_{t,0}(0) = \alpha_{t,0}(0)
\]
where \( \tau_{t,0} = \text{diag}(\tau_{t,01}, \tau_{t,02}, \tau_{t,03}) \) is a time constant to be regulated.

Define the third filtering error \( y_{t,0} = \bar{x}_{t,0} - \alpha_{t,0} \), and combining with (46), we have
\[
\begin{aligned}
\dot{y}_{t,0} &= -\tau_{t,0}^{-1}y_{t,0} - \ddot{z}_{t,0} = -\tau_{t,0}^{-1}y_{t,0} + k_i\dot{\epsilon}_{i,\Omega} - \ddot{X}_{i,\Omega} \\
&= -\tau_{t,0}^{-1}y_{t,0} + B_{i,0}(\dot{\epsilon}_{i,\Omega}, \ddot{X}_{i,\Omega})
\end{aligned}
\]
(48)

Step 4 Consider the controlling deviation of rotational level as
\[
e_{i,\Omega} = X_{i,\Omega} - \bar{x}_{i,\Omega}
\]
(49)

The eventual practical controller consisting of a neural approximator is given by
\[
\begin{aligned}
U_{i,\omega} &= -k_{i,\omega}e_{i,\omega} - \frac{1}{2}W_{i,\omega}[h_{i,\omega}(X_{i,\omega})]^2 + \dot{X}_{i,\omega} \\
\hat{X}_{i,\omega} &= U_{i,\omega} + \frac{1}{2}W_{i,\omega}[h_{i,\omega}(X_{i,\omega})]^2 + \eta_{i,\omega} \dot{X}_{i,\omega}, \quad \hat{X}_{i,\omega}(0) = X_{i,\omega}(0) \\
W_{i,\omega} &= \frac{1}{2}W_{i,\omega}[h_{i,\omega}(X_{i,\omega})]^2 \dot{X}_{i,\omega} - \sigma_{i,\omega} W_{i,\omega}
\end{aligned}
\]
(50)
where \( U_{i,\omega} \) is the rotational control vector; other notations discussed in (50) are similar with (7).

3.4 Convergence Analysis

To promote the convergence analysis, define \( \dot{W}_{i,\omega} = W_{i,\omega} - W_{i,\omega} \). For the formation deviation, state estimation error, weight updating error, and filtering error discussed aforementioned in controller design, the resulting distributed dynamics of formation deviation can be derived as
\[
\begin{aligned}
\dot{Z}_{t,p} &= (d_i + b_i)[-k_{i,p}Z_{t,p} + \xi_{i,p}(e_{i,v} + y_{i,v})] \\
\dot{e}_{i,v} &= -k_{i,v}e_{i,v} - \frac{1}{2}W_{i,v}[h_{i,v}(X_{i,v})]^2 \\
\dot{e}_{i,\Omega} &= e_{i,\Omega} + y_{i,\omega} - k_{i,\Omega}e_{i,\Omega} \\
\dot{e}_{i,\omega} &= -k_{i,\omega}e_{i,\omega} - \frac{1}{2}W_{i,\omega}[h_{i,\omega}(X_{i,\omega})]^2
\end{aligned}
\]
(51)

\[
\begin{aligned}
\dot{X}_{i,v} &= X_{i,v} - \dot{X}_{i,v} = \frac{1}{2}W_{i,v}[h_{i,v}(X_{i,v})]^2 - \eta_{i,v} \dot{X}_{i,v} + e_{i,v} \\
\dot{X}_{i,\omega} &= X_{i,\omega} - \dot{X}_{i,\omega} = \frac{1}{2}W_{i,\omega}[h_{i,\omega}(X_{i,\omega})]^2 - \eta_{i,\omega} \dot{X}_{i,\omega} + e_{i,\omega} \\
\dot{W}_{i,v} &= -\frac{1}{2}W_{i,v}[h_{i,v}(X_{i,v})]^2 \dot{X}_{i,v} - \sigma_{i,v} W_{i,v} \\
\dot{W}_{i,\omega} &= -\frac{1}{2}W_{i,\omega}[h_{i,\omega}(X_{i,\omega})]^2 \dot{X}_{i,\omega} - \sigma_{i,\omega} W_{i,\omega}
\end{aligned}
\]
(52)

**Theorem 2** Considering the multiple quadrotors (1) and overall error dynamics comprising of (51), (52), given that Assumption 1 satisfies and controllers are devised as (34), (39), (46), (50) along with adaptive laws (7), (50), all error variables are steered to convergence within a small vicinity of zero. In particular, for the initial condition of formation synchronization error fulfilling \(-\sigma_{i,\omega}(t) < e_{i,\omega}(0) < \sigma_{i,\omega}(t)\), formation synchronization errors of all quadrotors will be constrained within the prescribed bounds, i.e., \(-\sigma_{i,\omega}(t) < e_{i,\omega}(t) < \sigma_{i,\omega}(t)\).

**Proof** Consider the positive definite Lyapunov function as below:
\[
V = \frac{1}{2} \sum_{i=1}^{N} \left\{ Z_{t,p}^T Z_{t,p} + e_{i,v}^T e_{i,v} + e_{i,\Omega}^T e_{i,\Omega} + e_{i,\omega}^T e_{i,\omega} \\
+ y_{i,v}^T y_{i,v} + y_{i,\omega}^T y_{i,\omega} + y_{i,\omega}^T e_{i,\omega} + X_{i,v}^T \dot{X}_{i,v} + X_{i,\omega}^T \dot{X}_{i,\omega} \\
+ \gamma_{i,v}^{-1} W_{i,v}^T W_{i,v} + \gamma_{i,\omega}^{-1} W_{i,\omega}^T W_{i,\omega} \right\}
\]
(53)

Computing the time differential of (53) and using (36), (43), (48), (51) as well as (52) produces
\[ \dot{V} = \sum_{i=1}^{N} \left\{ \mathbf{Z}_{i,p}^T (d_i + b_i) \left( \xi_{i,p}(e_{i,v} + y_{i,v}) - k_{i,p} \mathbf{Z}_{i,p} \right) \\
+ e_{i,v}^T \left( -k_{i,v} e_{i,v} - \frac{1}{2} \mathbf{h}_{i,v}(\mathbf{X}_{i,v}) \right)^2 \right\} + e_{i,\Omega}^T (e_{i,\Omega} + \mathbf{y}_{i,\Omega} - k_{i,\Omega} e_{i,\Omega}) \]
\[ + e_{i,\Omega}^T \left( -k_{i,\Omega} e_{i,\Omega} - \frac{1}{2} \mathbf{h}_{i,\Omega}(\mathbf{X}_{i,\Omega}) \right)^2 \right\} - \tau_{i,v}^{-1} \mathbf{y}_{i,v} + \mathbf{y}_{i,v}^T \mathbf{B}_{i,v}(\cdot) - \tau_{i,\Omega}^{-1} \mathbf{y}_{i,\Omega} \right\} + e_{i,\Omega}^T \mathbf{B}_{i,\Omega}(\cdot) - \tau_{i,\Omega}^{-1} \mathbf{y}_{i,\Omega} + e_{i,\Omega}^T \mathbf{y}_{i,v} \right\} \\
+ \mathbf{X}_{i,v}^T \left( \frac{1}{2} \mathbf{h}_{i,v}(\mathbf{X}_{i,v}) \right)^2 - \eta_{i,v} \mathbf{X}_{i,v} + e_{i,v} \right\} \\
+ \mathbf{X}_{i,\Omega}^T \left( \frac{1}{2} \mathbf{h}_{i,\Omega}(\mathbf{X}_{i,\Omega}) \right)^2 - \eta_{i,\Omega} \mathbf{X}_{i,\Omega} + e_{i,\Omega} \right\} \\
- \frac{1}{2} \mathbf{X}_{i,v}^T \left( \mathbf{h}_{i,v}(\mathbf{X}_{i,v}) \right)^2 \mathbf{X}_{i,v} - \sigma_{i,v} \mathbf{W}_{i,v} \right\} \\
- \frac{1}{2} \mathbf{X}_{i,\Omega}^T \left( \mathbf{h}_{i,\Omega}(\mathbf{X}_{i,\Omega}) \right)^2 \mathbf{X}_{i,\Omega} - \sigma_{i,\Omega} \mathbf{W}_{i,\Omega} \right\} \right\} \right) \]

(54)

Define the sets \( \Omega_0 \triangleq \left\{ \left\| \mathbf{X}^d \right\|^2 + \left\| \mathbf{X}^d \right\|^2 + \left\| \mathbf{W}^d \right\|^2 \right\|^2 \leq \zeta_0 \} \) and \( \Omega_1 \triangleq \left\{ \sum_{i=1}^{N} \left( \sum_{k=1}^{3} \mathbf{Z}_{i,\Omega k}^T e_{i,v} + \mathbf{e}_{i,\Omega}^T e_{i,\Omega} + \mathbf{e}_{i,\Omega}^T e_{i,\Omega} \right) \right\} \).

(55)

with \( \zeta_{i,v}, \zeta_{i,\Omega} \) and \( \zeta_{i,\Omega} \) being nonnegative parameters.

By virtue of the aforementioned procedures, one has
Obviously,
\[ V(t) \leq \frac{\sigma}{\mu} (1 - e^{-\mu t}) + V(0)e^{-\mu t} \]  

Therefore, all error states of overall dynamics are UUB, that is, when \( t \to \infty \), one has
\[ |Z_{i,pk}| \leq \frac{2\sigma}{\mu} \| e_{i,v} \| \leq \frac{2\sigma}{\mu} \| e_{i,\Omega} \| \leq \frac{2\sigma}{\mu} \| e_{i,o} \| \]
\[ \leq \sqrt{\frac{2\sigma}{\mu} |\bar{X}_{i,v}\|} \leq \frac{2\sigma}{\mu} \| \bar{X}_{i,o} \| \leq \frac{2\sigma}{\mu} \]  

(60)

It is obvious from (59) that convergence rate of error states in the overall dynamics depends on parameter \( \mu \). Theoretically, increasing \( \mu \) will result in a faster convergence speed. Moreover, the eventual size of error relies on \( \sigma \). In addition, the formation synchronization deviation can be forced to decay to a predefined arbitrary small neighborhood around the origin by improving adaptive learning parameter, control gains, and decreasing time constants in filtering as \( t \to \infty \). Simultaneously, the converted formation synchronization deviation \( Z_{i,pk} \) converges to an arbitrary small neighborhood of zero. Furthermore, by appropriately tuning the constants \( \mu_{i, pk}, \phi_{i, pk} \), and \( \lambda_{i, pk} \) in the behavior function, and we can easily derive that the formation synchronization error \( e_{i, pk} \) can always evolve within the prescribed performance bound based on Lemma 2.

Let \( e_{i, pk} = [e_{1, pk}, \ldots, e_{N, pk}]^T \), then
\[ e_{i, pk} = \mathcal{H} \left( X_{i, pk} - \bar{X}_{i,v}^q + \bar{1} \sum_{i=1}^{N} q_{i,k} / N \right) \]
where \( X_{i, pk} = [X_{1, pk} - q_{1,k}, \ldots, X_{N, pk} - q_{N,k}]^T \), \( \bar{1} = [1, \ldots, 1]^T \in \mathbb{R}^N \). Revisiting Assumption 1, \( \mathcal{H} \) is nonsingular; it yields
\[ \| X_{i, pk} - \bar{X}_{i,v}^q + \bar{1} \sum_{i=1}^{N} q_{i,k} / N \| \leq \| e_{i, pk} \| / \sigma_{\min}(\mathcal{H}). \]
where $\sigma_{\text{min}}(\mathcal{H})$ represents the minimum singular value of $\mathcal{H}$, showing that the formation controlling deviation $X_{i,pk} - q_{i,k} - X_k^q + \sum_{i=1}^{N} q_{i,k} / N$ is bounded.

The proof of Theorem 2 is completed.

Remark 6 A parameter tuning guideline for the proposed controller is summarized as follows:

1. For PPC, the transient performance of formation error can be determined by the initial value $\rho_{i,pk}^0$, and generally, a larger $\rho_{i,pk}^0$ will result in a wider range and a smoother transient. And $\rho_{i,pk}^\infty$ corresponds to the eventual boundary of controlling deviation, which should be adjusted larger than the equipped sensor resolution; $\lambda_{i,pk}$ is a decaying factor of performance boundary. A faster convergence speed can be accomplished by increasing $\lambda_{i,pk}$.

2. For the presented SE-MLP, $\mathbf{h}_{i,v}$ and $\mathbf{h}_{i,o}$ should be chosen 2 to 3 times greater than the controlling parameters in the corresponding level to allow for a decoupling between tracking and learning channels. In addition, a huge adaptive learning parameter is usually required to permit a rapid learning of high-frequency uncertainties.

4 Simulation results

In the section, all the simulation results are carried out under MATLAB /SIMULINK environment with a sampling frequency being 50 Hz. The suggested algorithms, as shown in Fig. 2, are basically formulated as algebraic equations and differential dynamics, which are prone to be programed and performed via resorting to Interpreted MATLAB Fcn or S-Functions. For the purpose of validating the effectiveness and advantages of the derived results, we provide simulations on a networked system constituted by one virtual leader and three follower quadrotors, while pseudocode for suggested formation control policy is provided in Table 1 to illustrate the proposed control scheme clearly. Model parameters and external perturbations are collected in Table 2, where model parameters are borrowed from [51], which is well recognized and extensively adopted in academia as a benchmark data to testify the efficacy of quadrotor control design. And regarding external perturbations, to validate the proposed SE-MLP in terms of enhanced adaptiveness against fast time-varying uncertainties, we choose external perturbations as a compound formulation involving sine and cosine functions with different frequencies and amplitudes, and According to Theorem 2, the control gains of formation controller are given in Table 3 via a trial-and-error manner. It is worth mentioning that for PPC, select $\rho_{i,pk}^0$, $\rho_{i,pk}^\infty$, initial value of $X_{i,p}$, and ensure initial formation synchronization error $e_{i,pk}(0)$ satisfying $-\mathcal{A}_{i,pk}\rho_{i,pk}(0) < e_{i,pk}(0) < \mathcal{A}_{i,pk}\rho_{i,pk}(0)$, then transform the constrained error dynamics $e_{i,pk}(t)$ can be constrained by $-\mathcal{A}_{i,pk}\rho_{i,pk}(t) < e_{i,pk}(t) < \mathcal{A}_{i,pk}\rho_{i,pk}(t)$. For SE-MLP observer, the detailed training procedure is stated as follows: Firstly, setting the inputs and neurons of SE-MLP and configuring the center and width of neurons (see line 5 in Table 1). Then, $\mathbf{W}_{i,v}$ and $\mathbf{W}_{i,o}$ are online updated according to adaptive laws (7) and (50). As a consequence, the learning results are calculated by $\| \mathbf{W}_{i,v}(\mathbf{h}_{i,v}(\mathbf{X}_{i,v})) \|^2, \| \mathbf{W}_{i,o}(\mathbf{h}_{i,o}(\mathbf{X}_{i,o})) \|^2$.

According to Assumption 1, to assure the coordinated control design and stability derivation, the designed communication topology should fulfill the subsequent property, i.e., there exists at least one path from the virtual leader to each follower; herein we just give a representative example shown in Fig. 3, where the weights of information exchange among quadrotors are selected as: $b_1 = 1$, $a_{12} = a_{21} = 1$, and $a_{23} = a_{32} = 1$. Note that recalling the graph theory, one can know that $b_1 = 1$ indicates that 1-th quadrotor has access to the leader, $a_{12} = a_{21} = 1$ denotes that these exists information exchange between 1-th and 2-th quadrotors, and similarly, $a_{23} = a_{32} = 1$ declares that the information of 2-th and 3-th quadrotors can be obtained from each other. The desired relative position deviation is given by $\mathbf{q}_{12} = -\mathbf{q}_{21} = \mathbf{q}_1 - \mathbf{q}_2 = ([2, 0, 0]^T - [2 \sin(-\pi/6), 2 \cos(-\pi/6), 0]^T) \ (m)$, $\mathbf{q}_{13} = -\mathbf{q}_{31} = \mathbf{q}_1 - \mathbf{q}_3 = ([2, 0, 0]^T - [2 \sin(-\pi/6), 2 \cos(5\pi/6), 0]^T) \ (m)$. $\mathbf{q}_{23} = \mathbf{q}_2 - \mathbf{q}_3 = ([2 \sin(-\pi/6), 2 \cos(-\pi/6), 0]^T - [2 \sin(-\pi/6), 2 \cos(5\pi/6), 0]^T) \ (m)$. The initial conditions of three quadrotors are chosen as: $X_{1,p}(0) = [0, -5.5, 5.5]^T (m)$, $X_{2,p}(0) = [-4.5, 0, 3.3]^T (m)$, $X_{3,p}(0) = [-4, 0, -5.5]^T (m)$, $X_{1,\Omega}(0) = [0,$
Initialization:
2. Configure the desired leader’s trajectory to be tracked, i.e., \( \mathbf{X}^d \).
3. Set initial conditions of three quadrotors \( \mathbf{X}_{i,p}(0), \mathbf{X}_{i,\Omega}(0) \). Select \( \rho_{i,pk}^0, \rho_{i,pk}^\theta \) and define the desired relative position deviation as \( q_{ij} \) and yaw reference as \( \psi_i^d \).
4. Define communication topology and expected formation configuration based on Fig. 3.
5. Set the inputs and neurons of SE-MLP observer. Configure the center and width of neurons as 0 and 20 in (7) and (50), respectively. Initialize the weight vector \( \hat{\mathbf{W}}_{i,\omega k}(0) = 0, \hat{\mathbf{W}}_{i,\omega k}(0) = 0 \).
6. Configure model parameters, external perturbations as well as controller parameters in the following Table.2 and Table.3.

Kinematic Loop for quadrotors
8. Receive position information of neighbors \( \mathbf{P}_j \). Calculate the formation synchronization error \( e_{i,pk} \) using (27). Then, compute transformed formation error \( Z_{i,pk} \) utilizing (31). Calculate \( \mathbf{a}_{i,v} \) via (34), and generate \( \hat{\mathbf{a}}_{i,v}, \hat{\mathbf{a}}_{i,v} \) based on (35).
9. Compute tracking error of linear velocity, i.e., \( e_{i,v} \) through (37).
10. Establish SE-MLP observer based on (7) to update \( \hat{\mathbf{W}}_{i,v} \).
11. Calculate \( \mathbf{F}_{i,v} \) via (39), and generate the anticipated angles \( \varphi_i^d, \theta_i^d \) by inverse transformation in (41). Calculation of \( \mathbf{X}_{i,\hat{\Omega}}, \hat{\mathbf{X}}_{i,\hat{\Omega}}^d \) is based on (42).
12. End loop

Kinetic Loop for quadrotors
14. Calculate angle tracking error \( e_{i,\Omega} \) and angular velocity reference \( \mathbf{a}_{i,\theta} \) following (44) and (46), respectively. Generate \( \hat{\mathbf{a}}_{i,\theta}, \hat{\mathbf{a}}_{i,\theta} \) utilizing (47).
15. Compute tracking error of angular velocity, i.e., \( e_{i,\theta} \) through (49).
16. Establish the resulted controller and the SE-MLP observer to update \( \hat{\mathbf{W}}_{i,\omega} \) using (50).
17. End loop

\[ \begin{align*}
\mathbf{X}_{2,\Omega}(0) &= [0, 0, 0.2]^T \text{ (rad)}, \\
\mathbf{X}_{3,\Omega}(0) &= [0, 0, 0.2]^T \text{ (rad)}. \\
\mathbf{L} &= \mathbf{D} - \mathbf{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}
\end{align*} \]
Table 2 Model parameters and external perturbations

| Sections          | Values                                                                 |
|-------------------|------------------------------------------------------------------------|
| Model parameters  | \( m_i = 2 \text{ kg } (i = 1, 2, 3), \ g_i = 9.8 \text{ m/s}^2, \ l_i = 0.4 \text{ m}, \ c_i = 0.05, \ J_i = \text{diag}\{J_{i1}, J_{i2}, J_{i3}\} = \text{diag}\{0.16, 0.16, 0.32\} \text{ kg m}^2, \ \Pi_{i1} = \text{diag}\{0.002, 0.002, 0.002\} \text{ N m}^2, \) |
| External         | \( \Lambda_{1,v} = [\sin(t) + \sin(0.5t) - \cos(0.8t), \cos(t) + \sin(0.5t) - \cos(0.8t), \sin(1.5t)]^T, \) |
| perturbations     | \( \Lambda_{2,v} = [\sin(t) + \sin(1.5t) - \cos(0.8t), \cos(t) + \sin(0.5t), \sin(0.5t)]^T, \) |
|                   | \( \Lambda_{3,v} = [\sin(1.5t) \cos(t), \sin(t) + \sin(0.5t) - \cos(0.8t), \sin(t) + \cos(0.5t)]^T, \) |
|                   | \( \Lambda_{1,o} = [1.25(\sin(t) + \sin(0.5t)), 1.25(\cos(0.5t) - \cos(0.8t)), 0.625(\sin(t) \sin(0.5t))]^T, \) |
|                   | \( \Lambda_{2,o} = [1.25(\sin(t) + \cos(0.5t)), 1.25(\cos(0.5t) - \sin(0.8t)), 0.625 \sin(0.5t) \cos(0.5t)]^T, \) |
|                   | \( \Lambda_{3,o} = [1.25(\sin(t) \sin(0.5t)), 1.25 \cos(0.8t), 0.625 \sin(0.5t) \cos(t)]^T \) |

Table 3 Control gains of the proposed scheme

| Sections          | Values                                                                 |
|-------------------|------------------------------------------------------------------------|
| PPC               | \( \rho_{i,pk} = 12, \rho_{i,pk} = 0.6, \lambda_{i,pk} = 0.4, \sigma_{i,pk} = 1, \Sigma_{i,pk} = 0.9 \) |
| SE-MLP observer   | \( \gamma_{i,\psi} = 5, \sigma_{i,\psi} = \text{diag}\{0.6, 0.6, 0.6\}, \mathbf{u}_{i,\psi} = \text{diag}\{6.5, 6.5, 6.5\}, \) |
|                   | \( \gamma_{i,\Theta} = 20, \sigma_{i,\Theta} = \text{diag}\{0.3, 0.3, 0.3\}, \mathbf{n}_{i,\Theta} = \text{diag}\{16, 16, 16\} \) |
| Controller        | \( \mathbf{k}_{i} = \text{diag}\{1.05, 1.05, 1\}, \mathbf{k}_{i,\psi} = \text{diag}\{3, 3, 3\}, \mathbf{k}_{i,\Theta} = \text{diag}\{4, 4, 4\}, \mathbf{k}_{i,\Theta} = \text{diag}\{8, 8, 8\} \) |
| DSC               | \( \tau_{i,\psi} = \tau_{i,\Theta} = \mathbf{a}_{i,\Theta} = \text{diag}\{0.01, 0.01, 0.01\} \) |

Fig. 3 Communication topology and expected formation configuration

and the leader adjutancy matrix is \( \mathbf{b} = \text{diag}\{1, 0, 0\} \). Based on the knowledge of graph theory, \( \mathcal{H} \) can be computed by \( \mathcal{H} = \mathcal{L} + \mathbf{b} \). Thus, all the eigenvalues of matrix \( \mathcal{H} \) are computed as 0.1981, 1.5550 and 3.2470 with positive real parts, which clearly illustrates the correctness of Assumption 1.

In the following, the desired leader’s trajectory for follower quadrotors to track is

\[
\mathbf{x}^d = \begin{cases} 
[0, 0, 0.9(1 - e^{-0.3t})]^T, t \leq 9 \\
[10(1 - \cos(2\pi(t - 9)/23)), 5\sin(4\pi(t - 9)/23), 9(1 - e^{-0.3t})]^T, t > 9.
\end{cases}
\] (61)

4.1 Validation of the proposed algorithm

Simulation results adopting the presented method are depicted in Figs. 4, 5, 6 and 7. The formation
trajectories with maneuvering flight of follower quadrotors are presented, and it can be seen that a robust and stable triangle formation pattern is well established regardless of severe external disturbances and unknown system dynamics. The position responses and attitude responses are given in Fig. 7. It is evident that a UUB tracking result in the position level can be discerned and smooth attitude responses can also be acquired. Additionally, to analyze the effect of prescribed performance bounds on formation synchronization errors, different values of decaying rate, i.e., $\lambda_{i,pk}$; $\rho_{i,pk} = 0.4$ (Case 1), $0.05$ (Case 2), are chosen in the following, and the remaining control gains are the same. Figure 8 demonstrates that formation synchronization errors can be successfully regulated with guaranteed performances for all cases, where $e_{i,pk}(1)$ and $e_{i,pk}(2)$ are formation synchronization errors corresponding to Case 1 and Case 2, respectively. In contrast with Case 2, a smaller convergence time with a consistent steady-state deviation is obtained in Case 1 by tuning a bigger $\lambda_{i,pk}$, confirming that even with a vast range of disturbances, the cooperative formation behaviors can be preserved within predefined envelops without the need of repeatedly tuning controller parameters.

4.2 Comparative studies with RBF [39], MLP [41]

To demonstrate the advantages of proposed SE-MLP observer in improving transient behaviors, taking the roll channel of rotational loop of the first follower quadrotor as an example, we make comparisons with the existing MLP approximator [41]. Wherein according to (6), a MLP is designed as $\hat{\mathbf{W}}_{i,o}^2 \mathbf{h}_{i,o}(\mathbf{X}_{i,o})^2$ to approximate the lumped disturbances, where the weight updating law based on angular rate tracking error is established as $\dot{\mathbf{W}}_{i,o}^1 = \frac{1}{2} \mathbf{h}_{i,o}(\mathbf{X}_{i,o}) \mathbf{e}_{i,o} - \sigma_{i,o} \mathbf{W}_{i,o}$ and every element of basis function $\mathbf{h}_{i,o}(\mathbf{X}_{i,o})$ is expressed as $h_j(\mathbf{X}_{1,o}) = \exp[-\|\mathbf{X}_{1,o} - \delta\|^2 / 2\phi_j^2], j = 1, 2, \ldots, L$.

In the comparative simulation, modification parameter matrix $\sigma_{i,o}$ is set as $\text{diag}(0.3, 0.3, 0.3)$, the center vector $\delta$ is set as $[0, 0]^T$, the width $\phi_j$ is given as 70, the node number of hidden layer, i.e., $L$ is selected as 9. In addition, to make the fairness of comparisons, the values of adaptive gains are respectively adjusted for...
SE-MLP and MLP, such that nearly the same steady-state observation accuracy can be obtained.

Firstly, to validate the estimation performances against low-frequency disturbances, the external disturbance of roll channel of the first follower quadrotor is formulated as

$$D_1; x_1 = 0.2 \sin(t) + \sin(0.5t).$$

Estimation performances of low-frequency disturbances using SE-MLP and MLP are described in Fig. 9. It is demonstrated that the presented SE-MLP observer can obtain a slightly better estimation for disturbances with smooth transients.

Next, to illustrate the necessity of employing big adaptive gains to acquire a rapid approximation ability, we modify the disturbance of roll channel of first follower quadrotor as a high-frequency one, i.e.,

$$D_1; x_1 = 0.2 \sin(8t) + \sin(4t).$$

As shown in Fig. 10, as the adaptive gain increases, although the steady-state estimation ability tends to be better for MLP, the transient behaviors aggravate with significant oscillations, whereas for the proposed SE-MLP observer, a smooth and rapid identification result can be obtained using a large adaptive gain without incurring high-frequency vibrations, implying that the proposed SE-MLP observer can further relax the constraints on the maximum learning rate of MLP and permit a relatively wider range of adaptive gain, which is more suitable for solving coordinated formation consensus considering fast time-varying uncertainties.

In addition, to demonstrate the effect of observer bandwidth on improving transient estimation performances, different bandwidths of state observer are used for the developed SE-MLP, and Fig. 11 shows that when the observer bandwidth is selected at least two times larger than that of controller gain, as depicted in the right column of Fig. 11, in this sense, the time-scale separation between control and learning loops satisfies, leading to smooth and fast learning profiles in disturbance estimates and control actions. Thus, one can increase the observer bandwidth to achieve a smooth transient performance, which is consistent with the results in Theorem 1.

Finally, to quantitatively evaluate the merits and demerits of proposed SE-MLP observer, Table 4 gives observation comparisons among RBF [39], MLP [41], and SE-MLP for disturbances with different frequencies; we can see that for the involved two cases, SE-MLP observer performs the best in reducing transient chattering without influencing the steady-state
Fig. 9 Estimation performances for low-frequency disturbances using SE-MLP and MLP [41]

Fig. 10 Estimation performances for high-frequency disturbances using SE-MLP and MLP [41]
Fig. 11  Estimation performances for high-frequency disturbances using SE-MLP with different observer bandwidths

Table 4  Observation comparisons among RBF [39], MLP [41], SE-MLP for disturbances with different frequency

| Condition                | Index                          | RBF  | MLP  | SE-MLP |
|--------------------------|-------------------------------|------|------|--------|
| Low-frequency disturbances| Steady-state observation accuracy | 0.159 | 0.167 | 0.160  |
|                          | Computational burden\(^a\)     | 20.95% | 4.30% | 5.75%  |
|                          | Chattering degree\(^b\)        | 0.073 | 0.062 | 0.009  |
| High-frequency disturbances| Steady-state observation accuracy | 0.143 | 0.141 | 0.132  |
|                          | Computational burden\(^a\)     | 21.90% | 4.32% | 6.96%  |
|                          | Chattering degree\(^b\)        | 0.365 | 0.377 | 0.067  |

\(^a\)Computational burden is computed by the ratio of accumulated adaptive updating time to the total simulation time

\(^b\)Chattering degree \(L_c\) is calculated in the following form: 
\[
L_c = \sqrt{\frac{\sum_{n=1}^{N} |\hat{W}_{1,\text{rol}}(n\Delta T) - \hat{W}_{1,\text{rol}}((n - 1)\Delta T)|^2}{\sum_{n=1}^{N} |\hat{W}_{1,\text{rol}}(n)|^2}}
\] (62)

where \(\Delta T\) is the time interval and \(N\) is the number of recorded digital signals

Table 5  Transient performance comparisons using SE-MLP with different bandwidths

| Index | SE-MLP \((\gamma_{1,w} = 3450, \eta_{1,w1} = 8)\) | SE-MLP \((\gamma_{1,w} = 3450, \eta_{1,w1} = 16)\) |
|-------|-----------------------------------------------|-----------------------------------------------|
|        | Chattering degree\(^b\)                       | Chattering degree\(^b\)                       |
|        | 0.733                                         | 0.115                                         |
precision, which can be treated as the consequence of using observation errors, instead of tracking deviations to implement the weight learning, and in contrast with RBF [39] suffering from learning explosion, the computational burden of SE-MLP is obviously eased via using the artificial quantity rather than the weight vector, and it is acceptable that with the introduction of state observer, the design complexity is a bit more than that of MLP, which can be easily circumvented by the existing digital processors. Moreover, Table 5 collects transient performance comparisons using SE-MLP with different bandwidths, which further demonstrates that a large bandwidth can facilitate a smooth and fast learning, avoiding the undesired oscillations arising from strong coupling between tracking and learning loops. Consequently, the proposed SE-MLP observer can enable a rapid and smooth learning with a reduced computational complexity and chattering degree.

5 Conclusion

This paper deals with an adaptive formation consensus scheme for a group of quadrotors with prescribed performances subject to unknown uncertainties. Prescribed performance constraints acting on formation synchronization errors are explicitly considered in the controller design. By introducing the technique of MLP and constructing state estimators, the proposed SE-MLP observer not only can offer a smooth disturbance estimate even with a high adaptive learning rate, but also can avoid the issue of heavy calculation burden encountered in the available NN approximators. Finally, a distributed formation consensus approach is achieved to provide a robust formation configuration despite of unknown system perturbations. Comparative studies are illustrated to reveal the superiority and efficacy of the presented algorithm. Future direction will include an event-triggered control design for multiple quadrotors to pursue a resource efficient tracking result.

Acknowledgements This research has been supported in part by National Natural Science Foundation of China under Grant 61803348, National Nature Science Foundation of China as National Major Scientific Instruments Development Project under Grant 61927807 , State Key Laboratory of Deep Buried Target Damage under Grant DXMBJJ2019-02, Scientific and Technological Innovation Programs of Higher Education Institutions in Shanxi under Grant 2020L0266, Shanxi Province Science Foundation for Youths under Grant 201701D221123 , Youth Academic North University of China under Grant QX201803, Program for the Innovative Talents of Higher Education Institutions of Shanxi, and Shanxi “1331 Project” Key Subjects Construction (1331KSC).

Declarations

Conflict of interest The authors declare that they have no conflict of interest.

References

1. Xian, B., Wang, S.Z., Yang, S.: Nonlinear adaptive control for an unmanned aerial payload transportation system: theory and experimental validation. Nonlinear Dyn. 98(3), 1745–1760 (2019)
2. Shirani, B., Najafi, M., Izadi, I.: Cooperative load transportation using multiple UAVs. Aerosp. Sci. Technol. 84(1), 158–169 (2019)
3. Ai, X.L., Yu, J.Q.: Flattness-based finite-time leader-follower formation control of multiple quadrotors with external disturbances. Aerosp. Sci. Technol. 92, 20–33 (2019)
4. Zhang, D.-F., Duan, H.-B.: Switching topology approach for UAV formation based on binary-tree network. J. Frankl. Inst. 356(2), 835–859 (2019)
5. Leahy, K., Zhou, D.J., Vasile, C.I., Oikonomopoulou, K.: Persistent surveillance for unmanned aerial vehicles subject to charging and temporal logic constraints. Auton. Robot. 40(8), 1363–1378 (2016)
6. Ghommam, J., Saad, M., Wright, S., Zhu, Q.M.: Relay maneuver based fixed-time synchronized tracking control for UAV transport system. Aerosp. Sci. Technol. 103, 105877 (2020)
7. Dong, X.W., Hua, Y.Z., Zhou, Y., Ren, Z., Zhong, Y.S.: Theory and experiment on formation-containment control of multiple multirotor unmanned aerial vehicle systems. IEEE Trans. Autom. Sci. Eng. 16(1), 229–240 (2019)
8. Yue, X.H., Shao, X.L., Li, J.: Prescribed chattering reduction control for quadrotors using aperiodic signal updating. Appl. Math. Comput. (2021). https://doi.org/10.1016/j.amc.2021.126264
9. Rekabi, F., Shirazi, F.A., Sadigh, M.J.: Distributed nonlinear H-infinity control algorithm for multi-agent quadrotor formation flying. ISA Trans. 96, 81–94 (2020)
10. Shao, X.L., Yue, X.H., Li, J.: Event-triggered robust control for quadrotors with preassigned time performance constraints. Appl. Math. Comput. (2021). https://doi.org/10.1016/j.amc.2020.125667
11. Xu, Q.Z., Wang, Z.S., Zhen, Z.Y.: Adaptive neural network finite time control for quadrotor UAV with unknown input saturation. Nonlinear Dyn. 98(3), 1973–1998 (2019)
12. Zhou, D.J., Wang, Z.J., Schwager, M.: Agile coordination and assistive collision avoidance for quadrotor swarms using virtual structures. IEEE Trans. Robot. 34(4), 916–923 (2018)
13. Arul, S.H., Manocha, D.: DCAD: Decentralized collision avoidance with dynamics constraints for agile quadrotor swarms. IEEE Robot. Autom. Lett. 5(2), 1191–1198 (2020)
14. Wu, L.B., Park, J.H., Xie, X.P., Ren, Y.W., Yang, Z.H.: Distributed adaptive neural network consensus for a class of uncertain nonaffine nonlinear multi-agent systems. Nonlinear Dyn. 106(2), 1243–1255 (2020)
15. Wang, Y., Li, Q., Xiong, Q., Ma, S.: Distributed consensus of high-order continuous-time multi-agent systems with nonconvex input constraints, switching topologies, and delays. Neurocomputing 352(7), 10–14 (2019)
16. Zhao, L., Yu, J., Lin, C.: Distributed adaptive output consensus tracking of nonlinear multi-agent systems via state observer and command filtered backstepping. Inf. Sci. 478(4), 355–374 (2019)
17. Zhang, Y.-B., Wang, D., Peng, Z.: Consensus maneuvering for a class of nonlinear multi-vehicle systems in strict-feedback form. IEEE Trans. Cybern. 49(5), 1759–1767 (2019)
18. Dong, T., Gong, Y.L.: Leader-following secure consensus for second-order multi-agent systems with nonlinear dynamics and event-triggered control strategy under DoS attack. Neurocomputing 416, 95–102 (2020)
19. Yao, D.Y., Li, H.Y., Lu, R.Q., Shi, Y.: Distributed sliding-mode backstepping control of second-order nonlinear multiagent systems: an event-triggered approach. IEEE Trans. Cybern. 50(9), 3892–3902 (2020)
20. Zhang, Z., Chen, S.M., Su, H.S.: Scaled consensus of second-order nonlinear multiagent systems with time-varying delays via aperiodically intermittent control. IEEE Trans. Cybern. 50(8), 3503–3516 (2020)
21. Peng, Z.H., Wang, D., Li, T.S., Han, M.: Output-feedback cooperative formation maneuvering of autonomous surface vehicles with connectivity preservation and collision avoidance. IEEE Trans. Cybern. 50(6), 2527–2535 (2020)
22. Liu, H., Ma, T., Lewis, F.L., Wan, Y.: Robust formation trajectory tracking control for multiple quadrotors with communication delays. IEEE Trans. Control Syst. Technol. 28(6), 2633–2640 (2020)
23. Jasim, W., Gu, D.: Robust team formation control for quadrotors. IEEE Trans. Control Syst. Technol. 26(4), 1516–1523 (2018)
24. Liu, H., Ma, T., Lewis, F., Wan, Y.: Robust formation control for multiple quadrotors with nonlinearities and disturbances. IEEE Trans. Cybern. 50(4), 1362–1371 (2020)
25. Du, H.-B., Zhu, W., Wen, G., Duan, Z., Lu, J.: Distributed formation control of multiple quadrotor aircraft based on nonsmooth consensus algorithms. IEEE Trans. Cybern. 49(1), 342–353 (2019)
26. Wang, D.-D., Zong, Q., Tian, B., Wang, F., Dou, L.: Finite-time fully distributed formation reconfiguration control for UAV helicopters. Int. J. Robust Nonlinear Control 28(18), 5943–5961 (2018)
27. Wei, C.-S., Luo, J., Dai, H., Duan, G.: Learning-based adaptive attitude control of spacecraft formation with guaranteed prescribed performance. IEEE Trans. Cybern. 49(11), 4004–4016 (2019)
28. Zhang, Q.-R., Liu, H.: UDE-based robust command filtered backstepping control for close formation flight. IEEE Trans. Ind. Electron. 65(11), 8818–8827 (2018)
29. Shao, X.L., Shi, Y.: Neural adaptive control for MEMS gyroscope with full-state constraints and quantized input. IEEE Trans. Ind. Inf. 16(10), 6444–6454 (2020)
30. Chen, Z., Huang, F.H., Sun, W.C., Gu, J., Yao, B.: RBF-neural-network-based adaptive robust control for nonlinear bilateral teleoperation manipulators with uncertainty and time delay. IEEE-ASME Trans. Mechatron. 25(2), 906–918 (2020)
31. Shahvali, M., Shojaei, K.: Distributed adaptive neural control of nonlinear multi-agent systems with unknown control directions. Nonlinear Dyn. 83(4), 2213–2228 (2016)
32. Shao, X.L., Shi, H.N., Zhang, W.D.: Fuzzy wavelet neural control with improved prescribed performance for MEMS gyroscopes subject to input quantization. Fuzzy Sets Syst. 411, 136–154 (2021)
33. Ni, J.K., Shi, P.: Adaptive neural network fixed-time leader-follower consensus for multiagent systems with constraints and disturbances. IEEE Trans. Cybern. 51(4), 1835–1848 (2021)
34. Mao, J., Karimi, H.R., Xiang, Z.: Observer-based adaptive consensus for a class of nonlinear multiagent systems. IEEE Trans. Syst. Man Cybern. Syst. 49(9), 1893–1900 (2019)
35. Gou, Y.Y., Li, H.B., Dong, X.M., Liu, Z.C.: Constrained adaptive neural network control of an MIMO aeroelastic system with input nonlinearities. Chin. J. Aeronaut. 30(2), 796–806 (2017)
36. Afaghi, A., Ghaemi, S., Ghiasi, A.R., Badamchizadeh, M.A.: Adaptive fuzzy observer-based cooperative control of unknown fractional-order multi-agent systems with uncertain dynamics. Soft. Comput. 4(50), 3737–3752 (2020)
37. Wang, F., Liu, Z., Zhang, Y., Chen, B.: Distributed adaptive coordination control for uncertain nonlinear multi-agent systems with dead-zone input. J. Franklin. Inst. 353(10), 2270–2289 (2016)
38. Li, Y.Q., Wang, R.X., Xu, M.Q.: Rescheduling of observing spacecraft using fuzzy neural network and ant colony algorithm. Chin. J. Aeronaut. 27(3), 678–687 (2014)
39. Chen, C.L.P., Wen, G.-X., Liu, Y.-J., Wang, F.-Y.: Adaptive consensus control for a class of nonlinear multiagent time-delay systems using neural networks. IEEE Trans. Neural Netw. Learn. Syst. 25(6), 1217–1226 (2014)
40. Liu, X.-M., Ge, S., Goh, C.: Neural-network-based switching formation tracking control of multiagents with uncertainties in constrained space. IEEE Trans. Neural Netw. Learn. Syst. 49(5), 1006–1015 (2019)
41. Bu, X.W., Wu, X.Y., Ma, Z., Zhang, R.: Novel adaptive neural control of flexible air-breathing hypersonic vehicles based on sliding mode differentiator. Chin. J. Aeronaut. 28(4), 1209–1216 (2015)
42. Dou, L.Y., Song, C., Wang, X.F., Liu, L., Feng, G.: Target localization and enclosing control for networked mobile agents with bearing measurements. Automatica 118, 109022 (2020)
43. Wang, Q.L., Psillakis, H.E., Sun, C.Y.: Cooperative control of multiple high-order agents with nonidentical unknown control directions under fixed and time-varying topologies. IEEE Trans. Syst. Man Cybern. Syst. 51(4), 2582–2591 (2021)
44. Shahvali, M., Askari, J.: Cooperative adaptive neural partial tracking errors constrained control for nonlinear multi-agent systems. Int. J. Adapt. Control Signal Process. 30(7), 1019–1042 (2016)
45. Shahvali, M., Naghibi-Sistani, M.-B., Modares, H.: Distributed consensus control for a network of incommensurate fractional-order systems. IEEE Control Syst. Lett. 3(2), 481–486 (2019)
46. Shahvali, M., Azarbahram, A., Naghibi-Sistani, M.-B., Askari, J.: Bipartite consensus control for fractional-order nonlinear multi-agent systems: an output constraint approach. Neurocomputing 397, 212–223 (2020)
47. Peng, Z.H., Liu, L., Wang, J.: Output-feedback flocking control of multiple autonomous surface vehicles based on data-driven adaptive extended state observers. IEEE Trans. Cybern. (2020). https://doi.org/10.1109/TCCYB.2020.3009992
48. Gu, N., Peng, Z.H., Wang, D., Zhang, F.M.: Path-guided containment maneuvering of mobile robots: theory and experiments. IEEE Trans. Ind. Electron. (2020). https://doi.org/10.1109/TIE.2020.3000120
49. Yu, Q.X., Hou, Z.S., Bu, X.H., Yu, Q.F.: RBFNN-based data-driven predictive iterative learning control for non-affine nonlinear systems. IEEE Trans. Neural Netw. Learn. Syst. 31(4), 1170–1182 (2020)
50. Gao, F., Chen, W., Li, Z., Li, J., Xu, B.: Neural network-based distributed cooperative learning control for multiagent systems via event-triggered communication. IEEE Trans. Neural Netw. Learn. Syst. 31(2), 407–419 (2020)
51. Shao, X.L., Wang, L.W., Li, J., Liu, J.: High-order ESO based output feedback dynamic surface control for quadrotors under position constraints and uncertainties. Aerosp. Sci. Technol. 89, 288–298 (2019)
52. Shao, X.L., Yue, X.H., Li, J.: Event-triggered robust control for quadrotors with preassigned time performance constraints. Appl. Math. Comput. 392, 125667 (2021)
53. Peng, Z.H., Wang, D., Wang, W., Liu, L.: Containment control of networked autonomous underwater vehicles: a predictor-based neural DSC design. ISA Trans. 59, 160–171 (2015)
54. Kocer, B.B., Tjahjowidodo, T., Seet, G.G.L.: Centralized predictive ceiling interaction control of quadrotor VTOL UAV. Aerosp. Sci. Technol. 76, 455–465 (2018)
55. Bechlioulis, C.-P., Rovithakis, G.A.: Robust adaptive control of feedback linearizable MIMO nonlinear systems with prescribed performance. IEEE Trans. Autom. Control 53(9), 2090–2099 (2008)
56. Dimanidis, I.S., Bechlioulis, C.P., Rovithakis, G.A.: Output feedback approximation-free prescribed performance tracking control for uncertain MIMO nonlinear systems. IEEE Trans. Autom. Control 65(12), 5058–5069 (2020)
57. Dai, S.L., He, S.D., Chen, X., Jin, X.: Adaptive leader-follower formation control of nonholonomic mobile robots with prescribed transient and steady-state performance. IEEE Trans. Ind. Inf. 16(6), 3662–3671 (2020)
58. Liang, H.-J., Zhang, Y., Huang, T., Ma, H.: Prescribed performance cooperative control for multiagent systems with input quantization. IEEE Trans. Cybern. 50(5), 1810–1819 (2020)
59. Wang, Y., Hu, J., Li, J., Liu, B.: Improved prescribed performance control for nonaffine pure-feedback systems with input saturation. Int. J. Robust Nonlinear Control 29(6), 1769–1788 (2019)
60. Bu, X.W.: Guaranteeing prescribed output tracking performance for air-breathing hypersonic vehicles via non-affine back-stepping control design. Nonlinear Dyn. 91(1), 525–538 (2018)

Publisher’s Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.