Gold-plated moments of nucleon structure functions in baryon chiral perturbation theory

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Abstract

We obtain leading- and next-to-leading order predictions of chiral perturbation theory for several prominent moments of nucleon structure functions. These free-parameter free results turn out to be in overall agreement with the available empirical information on nearly all of the considered moments, in the region of low-momentum transfer ($Q^2 < 0.3 \text{ GeV}^2$). Especially surprising is the situation for the spin polarizability $\delta_{LT}$, which thus far was not reproducible in chiral perturbation theory for proton and neutron simultaneously. This problem, known as the “$\delta_{LT}$ puzzle,” is not seen in the present calculation.
The recent advent of muonic hydrogen spectroscopy [1] is probing the limits of our understanding of the nucleon’s electromagnetic structure. The unveiled discrepancy in the charge radius value between probing the nucleon with muons [1, 2] or electrons [3, 4] is only 4%, but is of great statistical significance (5 to 8 std deviations) at the current level of precision. Interestingly enough, the accuracy of both muonic-hydrogen and electron-scattering measurements is limited by the knowledge of subleading effects of nucleon structure, entering through the two-photon exchange (TPE). The main aim of our present studies is to provide predictions for these contributions from first principles using a low-energy effective-field theory of QCD, referred to as the baryon chiral perturbation theory (BχPT), see, e.g. [5].

In this endeavor we are primarily concerned with the doubly-virtual Compton scattering (VVCS) process which carries all the nucleon structure information of the TPE. Unitarity (optical theorem) relates the imaginary part of the forward VVCS amplitude to nucleon structure functions, and then the use of dispersion relations allows one to write the low-energy expansion of VVCS in terms of moments of structure functions [6]. The low-energy expansion of VVCS can, on the other hand, be directly computed in χPT. Of course, not all of the moments enter the low-energy expansion of VVCS: either only odd or only even ones do, depending on the structure function. Here we shall present the leading-order (LO) and next-to-leading-order (NLO) BχPT predictions for the following moments:

\[ \alpha_{E1}(Q^2) + \beta_{M1}(Q^2) = \frac{8\alpha M_N}{Q^4} \int_0^{x_0} dx \, x F_1(x, Q^2), \]  
\[ \alpha_L(Q^2) = \frac{4\alpha M_N}{Q^6} \int_0^{x_0} dx \, F_L(x, Q^2), \]  
\[ \gamma_0(Q^2) = \frac{16\alpha M_N^2}{Q^6} \int_0^{x_0} dx \, x^2 g_{TT}(x, Q^2), \]  
\[ \delta_{LT}(Q^2) = \frac{16\alpha M_N^2}{Q^6} \int_0^{x_0} dx \, x^2 \left[ g_1(x, Q^2) + g_2(x, Q^2) \right], \]  
\[ \bar{d}_2(Q^2) = \int_0^{x_0} dx \, x^2 \left[ 2g_1(x, Q^2) + 3g_2(x, Q^2) \right], \]  
\[ I_A(Q^2) = \frac{2M_N^2}{Q^2} \int_0^{x_0} dx \, g_{TT}(x, Q^2), \]  
\[ \Gamma_1(Q^2) = \int_0^{x_0} dx \, g_1(x, Q^2), \]

where

\[ F_L = -2xF_1 + (1 + 4M_N^2x^2/Q^2)F_2, \]
\[ g_{TT} = g_1 - (4M_N^2x^2/Q^2)g_2, \]

and \( F_{1,2}, g_{1,2} \) are respectively the unpolarized and polarized inelastic structure functions, which depend on the photon virtuality \( Q^2 \) and the Bjorken variable \( x = Q^2/(2M_N\nu) \), with \( M_N \) the nucleon mass and \( \nu \) the photon energy; \( x_0 \) corresponds with an inelastic threshold, such as that of a pion production; \( \alpha \) is the fine-structure constant.

These gold-plated moments have already been the subject of intense experimental studies [7–13], including an ongoing experimental program at Jefferson Laboratory [14, 15], see Ref. [16] for review. The first four moments have the interpretation of generalized nucleon
polarizabilities [6]. $\tilde{d}_2$ at high $Q^2$ represents a color polarizability [17] or a color-Lorentz force [18], $I_A$ is the generalized GDH integral and $\Gamma_1$ is the Bjorken integral.

We have computed the VVCS amplitude to next-to-next-to-leading order (NNLO) in the $\chi$PT expansion scheme with pion, nucleon, and $\Delta(1232)$ degrees of freedom, where the $\Delta$-nucleon mass difference $\Delta = M_\Delta - M_N \approx 300$ MeV is an intermediate small scale, viz. the “$\delta$ expansion” [19, 20]. This allows us to obtain the LO [i.e., $O(p^3)$] and NLO [i.e., $O(p^4/\Delta)$] contributions to the moments listed above. The diagrams we needed to evaluate these two orders are shown in Figs. 1 and 2 respectively. Their detailed description can be found in Ref. [21], where they are worked out for the case of real Compton scattering, i.e. $Q^2 = 0$. The extension to VVCS done in this work is rather tedious and will be discussed elsewhere [22]. Here we only note that the extension to finite $Q^2$ for the $\Delta$-isobar contributions, arising here at NLO, follows closely Ref. [23]; in particular, the magnetic $\gamma N \Delta$ coupling $g_M$, entering the first graph of Fig. 2, acquires a dipole form factor. As in [21], there are no free parameters to fit at these orders, hence this calculation is ‘predictive’.

The resulting predictions for the moments of interest are shown in Table I for $Q^2 = 0$, and in Figs. 3 to 6, as function of $Q^2$. In the figures, the LO $B\chi$PT is given by the red solid curves, while the complete result, including the NLO and the uncertainty estimate (cf. Ref. [23]), is given by the blue bands. In all the plots, the black dotted curves represent the empirical evaluation using the 2007 version of the Mainz online partial-wave analysis of meson electroproduction (MAID) [25]. Some of the plots contain data points described in the legends. Other curves represent previous $\chi$PT evaluations, as will be discussed further.

The scalar polarizabilities of the proton and the neutron are shown in Fig. 3. Here the blue dashed lines denote the LO of heavy-baryon (HB) $\chi$PT. It exactly corresponds with the static-nucleon approximation of the LO $B\chi$PT. Given the large differences between
TABLE I: The NLO BχPT predictions for the forward VVCS polarizabilities (at $Q^2 = 0$) compared with the available empirical information. Where the reference is not given, the empirical number is provided by the MAID analysis \[24, 25\], with unspecified uncertainty.

| Polarizability | Proton          | Neutron         |
|----------------|-----------------|-----------------|
| $\alpha_{E1} + \beta_{M1}$ | This work: 15.12(82) | Empirical: 13.8(4) Ref. [26] |
|                | This work: 18.30(99) | Empirical: 14.40(66) Ref. [27] |
| $\alpha_L$    | 2.31(12) [MAID]  | 3.21(17) [MAID] |
| $\gamma_0$    | -0.93(5) Ref. [8] | 0.05(1) [MAID] |
| $\delta_{LT}$ | 1.35(7) [MAID]  | 2.20(12) [MAID] |

The spin polarizabilities $\gamma_0$ and $\delta_{LT}$ are shown in Fig. 4. These quantities deserve a more extensive discussion since they were traditionally hard to reproduce in $\chi$PT. In the case of $\delta_{LT}$ this problem became known as the "$\delta_{LT}$ puzzle". Obviously our complete result (blue bands) is in a reasonable agreement with the empirical information, so where is the problem?

The $\delta_{LT}$-puzzle was first observed in the HB variant of $\chi$PT \[30–32\], which invokes an additional semi-relativistic expansion, in the inverse nucleon mass. Evidently, this expansion works poorly for these quantities: compare the HB (blue dashed) curves, which only for $\delta_{LT}$ are within the scale of the figure, with the corresponding BχPT calculation (blue bands). First attempts to go beyond HB were done in the infrared-regularized (IR) version of BχPT \[34\], which has an incorrect analytic structure (unphysical branch cuts), leading to results shown by the red bands \[33\]. Having the relativistic result with unphysical analytic structure obviously did not solve the problem — the disagreement of the red bands with the data or the MAID is too large.

More recently, a first BχPT calculation has appeared \[29\], shown by the grey bands in the figure. As one can see, for $\gamma_0$ it works much better than the HB and IR counterparts. In the lower panel, it seems to resolve the $\delta_{LT}$-puzzle for the neutron, albeit at the expense of introducing it for the proton. Indeed, despite having presently no experimental data for the proton, we anticipate them to follow closely to the MAID result, shown by the black dotted line. Again, $\delta_{LT}$ would not be reproduced simultaneously for the proton and neutron.

In contrast, the present calculation (blue bands) shows no puzzle in either the proton or the neutron, and hence the question of what exactly is the difference between the two BχPT calculations is to be addressed. At the level of $\pi N$ loops they are equivalent, however the inclusion of the $\Delta$-isobar is done in different counting schemes: "$\delta$ counting" here vs. the....
FIG. 3: Scalar polarizabilities of proton and neutron. Red solid lines and blue bands represent, respectively, the LO and NLO results of this work. Blue dashed line is the LO result in the HB limit. Black dotted line represents the empirical result of MAID2007 [25]. The data points at $Q^2 = 0$ correspond with Refs [27] and [26] (purple and red point, respectively) for the proton, and [27] for the neutron. The data point in the left upper panel at $Q^2 = 0.3$ GeV$^2$ is from Ref. [28].

“small-scale expansion” in Ref. [29]. In the latter case, more graphs with $\Delta$ are included, particularly those with photons coupling to the $\Delta$ in the loops. They are the only good candidates to account for the difference between the two calculations. We have checked that our result for the $\Delta$-isobar contribution to $\delta_{LT}$ agrees with the expectation from the MAID analysis, where a separate estimate of this contribution can be obtained. The corresponding effect in Ref. [29], measured by the difference between the grey and red curves in the figure for $\delta_{LT}$ of the proton, is about an order of magnitude larger and has an opposite sign.

We next turn to $I_A$ and $\bar{d}_2$ moments shown in Fig. 5. The LO result here (red solid line) is already in agreement with the experimental data where available. Going to NLO (i.e., including the $\Delta$) does not change the picture qualitatively in our $\chi$PT calculation (blue bands). The effect of the $\Delta$ is appreciably larger again for the proton in the $\chi$PT calculation of Bernard et al. [29] (grey bands). The $O(p^4)$ HB$\chi$PT result without explicit $\Delta$’s (blue dashed lines) is in disagreement with the experimental data, and in worse agreement with the empirical picture from MAID.

Note that by means of the GDH sum rule, $I_A(0) = -\kappa^2/4$, with $\kappa$ the anomalous magnetic moment of the nucleon. The $\chi$PT calculations are (at $Q^2 = 0$) fixed to this value due to renormalization, while in the MAID evaluation it comes out differently. This difference can perhaps serve as a rough uncertainty estimate of the MAID evaluation.
FIG. 4: Generalized spin polarizabilities of proton and neutron. Red solid lines and blue bands represent, respectively, the LO and NLO results of this work. Black dotted lines represent MAID2007. Grey bands are the covariant BχPT calculation of Ref. [29]. Blue dashed line is the O(p⁴) HB calculation [31]; off the scale in the upper panels. Red band is the IR calculation [33]. The data points for the proton γ₀ at finite Q² are from Ref. [7] (blue dots), and at Q² = 0 from [8] (purple square). For the neutron all the data are from Ref. [9].

The last moment in Eq. (1), Γ₁, is the first Cornwall-Norton moment of the inelastic spin structure function g₁, i.e. the inelastic part of the Bjorken integral. The isovector (p−n) combination for this moment is shown in the right panel of Fig. 6. Here the HBχPT, the previous [29] and the present BχPT calculations compare fairly well with the experimental data of Refs. [12, 13]. The MAID analysis is in worse agreement.

In the left panel of Fig. 6 we show I₁ = (2Mᴺ²/Q²)Γ₁ for the proton. Here the discrepancy of the BχPT calculations with the experimental data is most appreciable. At Q² = 0, this quantity is expressed in terms of the anomalous magnetic moment of the proton: I⁺ₚ(0) = I⁻ₐ(0) = −κ₂/4. The empirical result of MAID is not entirely consistent with this constraint, just as in the case of I₋. However it is consistent with experimental data, leaving one to wonder whether in either of them the integral I₁ is evaluated accurately.

We conclude by making the connection to the charge radius problem mentioned in the beginning. In a recent paper [5] we presented the leading-order predictions for the proton polarizability effect in the Lamb shift of muonic hydrogen. It is based on the same BχPT framework and the same VVCS amplitude as the present work. The magnitude of the effect turned out to be in agreement with models based on dispersion relations, but not with the results of HBχPT [35, 36] which indicate a substantially larger effect. Given that
FIG. 5: Generalized GDH integral and inelastic part of the $d_2$ moment. The legend is the same as in the previous figure, except for the $O(p^4)$ HB result (blue dashed line) which here is from Ref. [32], and the data points which are from Ref. [10] for $I_A$ and Ref. [11] for $\bar{d}_2$.

the longitudinal response of the nucleon is predominant in the atoms, we focus on the polarizabilities $\alpha_L$ and $\delta_{LT}$ and observe that the difference between B and HB $\chi$PT results is substantial indeed (cf., lower panels in Figs. 3 and 4). It is especially large in the scalar polarizability $\alpha_L$ which is relevant to the Lamb shift; the spin polarizability $\delta_{LT}$ may only affect the hyperfine splitting. Thanks to the available empirical information, provided by the MAID analysis, we conclude that the longitudinal response of the nucleon is largely overestimated in HB$\chi$PT.

In overall the B$\chi$PT predictions presented here are in good (within 3 std deviations) agreement with the empirical information on the gold-plated moments of nucleon structure functions. The most appreciable disagreement of the present B$\chi$PT calculation with experiment is observed in the integral $I_1$. For the first time, the spin polarizability $\delta_{LT}$ is reproduced for both the proton and the neutron within a free-parameter-free (predictive) $\chi$PT calculation, thus potentially closing the issue of the “$\delta_{LT}$ puzzle”. The latter statement relies of course on the empirical results of MAID for the proton $\delta_{LT}$. The forthcoming measurement at Jefferson Laboratory is called to provide the data for that observable, hence putting to the test the MAID and present $\chi$PT results.
FIG. 6: Left panel: $I_1 = (2M_N^2/Q^2) \Gamma_1$ for the proton. Right panel: isovector part of the Bjorken integral. Legend for the curves is as in Fig. 4. Data points for $I_1^p$ are from [7], for $\Gamma_1^{p-n}$ from [12] (squares) and [13] (dots).

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