A brief review is given of attempts to understand the energies of four-quark systems calculated on a lattice in terms of nuclear-physics-inspired many-body techniques involving interquark potentials. Results are given for the next stage of this study where the wavefunctions of heavy-light mesons are also calculated on a lattice.

Over the past few years the authors have been measuring on a lattice the energies of various four-quark systems. In the original papers (see for example Refs. \cite{1,2,3}) the four quarks involved were all considered to be infinitely heavy. The resultant energies could then be reasonably well understood in terms of a many-body nuclear-physics-inspired approach involving interquark potentials – provided there was introduced a four-quark term similar to a form factor. Neglecting such a factor consistently led to an overestimate of the binding.

Later in Ref. \cite{4} a method was developed for treating on a lattice two quark systems, where one of the quarks was a light quark i.e. the case of heavy-light mesons $Q\bar{q}$. In that paper the authors concentrated on measuring the S-, P-, D- and F-wave energies.

Returning to the four-quark system, in Refs. \cite{5} the energies of the $Q^2\bar{q}^2$ system were calculated using the same techniques that proved successful in Ref. \cite{4} for the basic $Q\bar{q}$ case. In addition to the presence of light quarks, the works of Refs. \cite{4,5} had two other improvements compared with Refs. \cite{1,2,3}:

i) The gauge group used was SU(3) and not SU(2).

ii) The lattice configurations were unquenched.

In Ref. \cite{6} the earlier nuclear-physics-inspired approach in terms of interquark potentials was extended to the $Q^2\bar{q}^2$ case. This required fitting first the $Q\bar{q}$ energies of Ref. \cite{6} to extract an effective light-quark mass of about 400 MeV. The main conclusion from this work was that the $Q^2\bar{q}^2$ data could not be understood in terms of purely two-quark potentials and, as in the earlier static case of Refs. \cite{1,2,3}, a four-quark form factor was necessary.
Most of the above work has been devoted to the energies of the various quark systems – the exception being Ref. where flux-tube structures were measured. Now we are working on a lattice measurement of the radial wavefunctions of a single heavy-light meson. These wavefunctions consist of the distribution of the light quark and the colour field components around the static quark. The light quark wavefunctions of the ground state and some excited states are being measured. Such wavefunctions have not been measured before and are of relevance to various phenomenological attempts to reproduce meson-decays and scattering of mesons. These include e.g. bag models and semirelativistic Schrödinger and Blankenbecler-Sugar equations.

The actual wavefunction measurement is based on the light-quark propagators $G_{ij}$ of Ref. For a measurement of the $Q\bar{q}$ energies only one $G_{ij}$ enters in the 2-point correlation as, essentially, $C_2(t) = \sum_{ij} G_{ji} U_{ij}$ where $U_{ij}$ is the static quark propagator represented by a straight line of gauge links from point $i$ to point $j$ in a different time slice. However, for the light quark wavefunction measurement two such operators arise giving a 3-point correlation of the form $C_3(t, r) = \sum_{i jl} G_{jl} O_{l} U_{ij}$, where the site $l$ is constrained to be within $r$ spacings from the $i, j$ space coordinates. Here we use the local operators $O = \gamma_4$ and 1, which are probing respectively the light quark charge and matter distributions at a distance $r$ from the heavy quark. The latter are defined as $\langle C_3(t, r)/C_2(t) \rangle$. The result of fitting these distributions by $F^2(O)$, where $F = A \exp(-r/r_0)$, is given in Table 1. There it is seen that the charge distribution has a considerably longer range than that of the matter. Summing over the charge distribution should give the charge of the quark. With the present normalisation this should be $\approx 1$ on a lattice and, within the expected accuracy, this is indeed the case, when the sum is carried out directly on the lattice – see the column DSum. As discussed in the sum rule for $O = 1$ has a less direct interpretation.

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Table 1: Parameters for fitting the charge ($\gamma_4$) and matter (1) distributions with $F^2$, where $F = A \exp(-r/r_0)$. Dsum refers to a direct lattice estimate of the sum of $F^2$.

| Operator(O) | $r_0/a$ | $A$ | DSum |
|-------------|---------|-----|------|
| $\gamma_4$ (Charge) | 1.56(2) | 0.45(1) | 1.12(5) |
| 1 (Matter) | 1.15(5) | 0.46(2) | 0.25(5) |

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Figure 1: The correlation $\langle C_3(t,r)/C_2(t) \rangle$ as a function of $r$ in lattice units: a) Charge and b) Matter. Solid(dotted) for $t = 8(10)$.  

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