Strength Analysis and Reliability Evaluation for Speed Reducers

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Abstract. This paper studies the structural stresses of differential drive (DD) and harmonic drive (HD) for design improvement of reducers. The designed principles of the two reducers are reported for function comparison. The critical components of the reducers are constructed for performing motion simulation and stress analysis. DD is designed based on differential displacement of the decelerated gear ring as well as HD on a flexible spline. Finite element method (FEM) is used to analyze the structural stresses including the dynamic properties of the reducers. The stresses including kinematic properties of the two reducers are compared to observe the properties of the designs. The analyzed results are applied to identify the allowable loads of the reducers in use. The reliabilities of the reducers in different loads are further calculated according to the variation of stress. The studied results are useful on engineering analysis and reliability evaluation for designing a speed reducer with high ratios.

1. Introduction

Reducers are extensively applied for speed reduction in many types of machinery such as machine tools, transmission mechanisms, and robots, etc. Their functional properties always are emphasized for satisfying the performance needs, such as speed-reduction ratio, transmitted torque, and efficiency, etc. An often used reducer for high reduction ratio and small volume is harmonic drive (HD). The HD is designed using a Flexible spline which possesses the properties of small backlash, high transmission accuracy, and large reduction ratio so that it is extensively used in robotic devices [1]. The existed problems of the HD are their rigidity due to the flexible design of the decelerated gear rings [2]. The other one frequently applied is trochoidal gear reducer which has the advantages of high torsional rigidity, great shock-resistant ability and loading capacity. [3]. A new design of reducers for high rigidity, large reduction ratios is to make use differential displacements of decelerated gear rings, named as differential drive (DD) [4]. This DD is designed primarily consisting of one off-center cam and several gear rings. Different types of reducers have been proposed in the past. However, many structural problems such as motion property, induced stress, and reliability, etc., are still unknown so that the further studies are needed to avoid failures.

1.1 Stress analysis

The failures of gear-based reducers frequently occur at the decelerated gears, for example, gear tooth breakage, pitting or wearing of tooth surfaces. Gear tooth breakage is caused by internal stress
rupturing which is a direct result of the accumulated residual stress or root bending stress exceeding the strength of materials [5]. To preliminarily investigating the strength of a reducer, the working stresses must be observed. An often used approach in analyzing structural stresses and dynamic properties is by finite element method (FEM). The ANSYS software is a powerful tool of FEM, which has been extensively applied in many engineering fields. The geometric models created by CAD are imported into ANSYS for meshing and computing while performing statistic or dynamic analyses. It is sometimes difficult to obtain an optimal solution by FEM because the definitions may slightly violate the constraints, although the efficiency can be improved by replacing the search algorithms. Recently, the use of FEM to perform engineering stress analysis and support product design has been extensively studied. For example, Tsai et al. [6] made use of the limited experimental data and simulated stress obtained by FEM to evaluate the fatigue life of dental implants. A typical example reported by Tsai [7] is to integrate probabilistic theories with CAD/CAE to perform reliability optimization design for robotic arms.

1.2. Brief contents
This paper reports the design concepts of a high-ratio speed-reducer. Geometric models of the reducer are designed using parametric methods so that the related components can be rapidly created by changing the designed variables. The decelerated gear rings play a critical role in determining the ratios of speed reduction and the strength of the reducers. The structural stresses of the decelerated gear ring within the DD and HD are studied to formulate the relationship between the working loads and the tooth sizes so that the allowable loads can be decided accordingly. The strength and reliability of the two reducers are compared to assess the advantages of the designs. The FEM is used to evaluate the maximum stresses and deformations of the reducer under various loadings. The studied results are useful for stress analysis and reliability design in developing a reducers owning high reduction ratios.

2. Differential Drive (DD)
The working principles of the reducer design are based on a differential displacement of the sliding gear ring related to the fixed gear ring. The movements are similar to the sliding meter to the fixed meter in the traditional calipers. The critical components of the DD have four, the fixed ring, sliding ring, medium ring and off-center cam. The isotropic and plain perspectives of the DD are illustrated in Fig.1.

(a) isotropic view                     (b) plain view

Figure 1. Assembly drawing of the DD

The fixed ring is designed to guide the moving of the medium ring as a planetary gear. The medium ring is designed to receive the power inputted from the off-center cam. The medium ring would generate planetary motion and push the sliding ring to move while the off-center cam is rotating. The fixed and sliding rings are designed with different teeth so that a slight difference of position of the
two paired teeth can be generated. The paired teeth are forced by the teeth of the medium ring to align with each other. The differential displacement generated by the fixed and sliding rings would result in a large speed reduction of the sliding ring. The ratios of speed-reduction are dependent upon the number of tooth of the fixed and sliding rings.

The ratio of speed reduction can be evaluated by

\[ s = \frac{z_s}{z_s - z_c}, \]  

(1)

where \( z_s \), \( z_c \) stand for the number of teeth on the sliding and fixed rings, respectively. The off-center cam is designed according to the sizes of the paired gear rings. The off-center distance is expressed as

\[ e = \frac{D_s - D_c}{2}, \]  

(2)

where \( D_s \), \( D_c \) are the pitch diameters of the sliding and the fixed gear rings, respectively.

Possible failure modes of the DD will occur at the medium ring because it is the weakest one of the structure. They are likely to appear depending on the pitch-line velocity and transmitted torque. The failures occur at the positions of engagement of the gear teeth include both tooth breakage and tooth surface pitting. Breakage of the tooth is caused by the root bending stress exceeding the strength of the materials. Pitting of tooth surface is incurred by repeated surface contact stress as well as inadequate surface strength. Increasing the transverse contact ratio as well as enlarging the overlap between the tooth contact-pairs are effective approaches to reducing noise and vibration.

The horizontal force can be obtained by taking the force equivalence [4] as follows:

\[ F_2 = \frac{T_1}{(r_2 + e)}. \]  

(3)

The moment acting on the intermediary ring can be obtained by equivalent equivalence on acting and reacting forces as

\[ T_2 = \frac{r_2}{(r_2 + e)} T_1. \]  

(4)

The structural stress on the gear rings can be further evaluated according to the results of loading analysis, based on the theory of gear design. Aiming to the paired transmission of the inner gears, the induced stress including the Root Bending Stress (RBS) and Surface Contact Stress (SCS) can be evaluated according to the transmitted torque \( T_2 \). Generally, the RBS is commonly evaluated based on the Lewis equation. The gear tooth is modeled as taking the full load at its tip as with a simple cantilever beam. The SCS is commonly predicted based on Hertz theory in which the contact points are simulated as two contacting cylinders. In this paper, the structural stresses are obtained by FEM.

3. Harmonic drive (HD)

Harmonic drive is extensively applied in industrial robots. It is designed with toothed mechanism which is mainly composed by three elements as shown in Fig. 2 [8]:
The designed properties of the HD are introduced in following:
- The FS is designed slightly smaller in diameter than the CS and the teeth of the FS usually are two fewer than the CS. The WG is designed with elliptical shape so that the FS is fitted as an ellipse and the teeth can engage the CS at two regions at the opposite ends of the major axis of the ellipse FS.
- The WG enforces the FS for gear tooth engagement at the major axis of the ellipse and is acting as an input rotator of power. The WG clockwise movement for each 180°, the FS would move counterclockwise by one tooth relative to the CS (fixed).
- The FS would move counterclockwise by two teeth from its previous position relative to the CS for each complete clockwise rotation of the WG.
- The FS is deflected as elastically deformation while the WG is elliptically across the major axis. The teeth of the FS are engaging simultaneously with the ring gears of the CS at two zones of either end of the major elliptical axis. On the other hand, there is no engagement arisen when the elliptically deflected FS across the minor axis. The meshing zones of the FS rotate with the generator when the WG rotates. A difference of two teeth of the tooth number of the FS and the CS results in a relative movement of the two wheels when the FS is driven by the WG. The FS moves relative to the CS having an angle equivalent to two teeth after a complete rotation of the WG.

HD is currently used in many automotive and space industries including aviation, medicine, automatics and robotics. HD has numerous advantages in transmission compared to the classical gear chain. The main advantages are high single-stage reduction ratios including compact design, high torque capacity and high torsion stiffness, zero backlash, excellent positioning accuracy and repeatability, etc. Moreover, the drawbacks are high elasticity and nonlinear stiffness and damping. The application of harmonic drives in various industrial fields is more and wider due to the excellent performances. The disadvantages of HD is the strength problems due to the deflect design of the FS. The FS is the main component of a harmonic drive, which can generate a repeated vibration by the wave generator. With this a reason, the FS should have flexibility and good vibration characteristics.

A simplified model of the HD is constructed for analyzing the structural stresses. The geometric size and the speed-reduction ratios are designed as well as the ones of the DD for convenient in strength comparison. The tooth’s shapes are constructed based on a combination of different curve profiles which is similar to a commercial product. The geometric models including the CS, FS and WG of HD are created for structural analysis. This means that inner diameter of the FS is about 48 mm and the tooth’s numbers for the FS and the CS are 100 and 102, respectively.

4. Strength analysis
ANSYS is a tool of FEM for obtaining the simulated features of a design in real world by using mid-plane extraction of structures, boundary conditions, mesh controls, loads and perimeter welds, etc. The geometric models produced by CAD software are imported into ANSYS to analyze the static and dynamic properties. For gear transmission, the bending and contact stresses between the mated teeth
are the studied topics. Here, the induced stresses of the medium rings of the DD and the FS in the HD are the concerned problems since they are the weakest ones within the reducers. The analyzed results are further used to evaluate the strength and reliability of the reducers.

The material of the DD is designed with structural steel. To simplify the analyzed model, only the critical components, the inner-outer gear rings of the DD, are loaded into ANSYS to perform stress analysis. The augmented Lagrangian formulation method is selected as the most suitable for the analysis from among the available nonlinear methods because it is best for frictionless contact. The connection of the inner-outer gear rings are set to frictionless contact and the cam is bonded to the inner ring. A fixed support is placed at the outer gear ring as well as a frictionless support at the input hole of the off-center cam. The gear thickness is set to 10 mm and the loading is set to be 100 Nm.

![Image](image1.png)

**Figure 3.** The surface contact stress

Here, the interval of the mesh is set to 100 and the meshes in the areas of the mating are strengthened by setting a smaller meshing size. The analytical results for equivalent stress as well as the contact stress are shown in Fig. 3. The maximum value of the SCS for this example is 1840 MPa, occurring at the contacting position of the engaging teeth. The principle stresses, as well as the bending stress, are shown in Fig. 4. The maximum value of the RBS is 654 MPa, occurring at the tooth roots. The evaluated results for maximum RBS and SCS are similar to the values obtained by numerical calculations. The results imply that applying the FEM to evaluate the RBS and SCS of the reducer is feasible.

![Image](image2.png)

**Figure 4.** The root bending stress

The analyzed results show that the SCS is about 3 times of the RBS. Normally, the results of FEM analysis are closely related to the mesh settings. The convergence conditions of FEM analysis show that the maximum equivalent and principle stress will converge to one stable value representing the evaluations are reasonable.
On the other hand, the geometric models of the HD are also imported into ANSYS to perform stress analysis. The symmetrical geometries are used for simplifying the analyzed time. This is a nonlinear analysis since the FS and the WG existing an overlapping area. The CS is set to fixed support as well as the WG to frictionless support. The maximum stresses of the FS deformation caused by the WG are about 198 MPa as shown in Fig. 5. Moreover, the working stresses of the FS are obtained by setting the acting moments at the FS. The maximum principle stresses of the FS are about 1400 MPa while the loading moment is 100 Nm. The total stresses of FS would be about 1600 MPa. The analyzed results show that the working stresses of the DD are smaller than the ones of the HD at the same torques. It means that the allowable loads of the DD would be larger than the ones of the HD.

5. Reliability evaluation
Reliability is a probability of success of failure which is formulated based on SSI theory. Reliability index can be rated based on the second-order moment method while the strength random variable \( X \) is normally distributed with a mean value of \( \mu_X \) and standard deviation of \( \sigma_X \). On other hand, the stress random variable \( Y \) is also normally distributed with mean value and standard deviation \( (\mu_Y, \sigma_Y) \), respectively. The geometric distance indicating reliability index can be rated by

\[
z = \frac{\mu_Y - \mu_X}{\sqrt{\sigma_X^2 + \sigma_Y^2}}
\]  

(5)

The relationship between reliability index, \( z \), and the reliability can be evaluated by

\[
R = 1 - \Phi(-z) = \Phi(z)
\]  

(6)

For example, reliability index \( z=3 \) represent the reliability being \( R=0.99865 \).

Focusing on mechanical design, the reliability of a design can be expressed from the strength and stress distributions. If the strength information of materials \( (\mu_X, \sigma_X) \) is known, the induced stress \( (\mu_Y, \sigma_Y) \) would be the designed variants. Considering the coefficients of variation, the standard deviation of the induced stress can be evaluated by Eq.(7) when the designed variants \( \gamma_Y \) are known

\[
\sigma_Y = \gamma_Y \mu_Y
\]  

(7)

As a result, we can derive the equation of reliability design for the reliability target being set to \( R \geq \Phi(z) \).

Based on the evaluated approach, the reliabilities of the reducers at various loads can be further evaluated by integrating the analyzed results in ANSYS and the SSI theory. The induced stresses can be regarded as normal distributions while considering the variation of the design variables. In this paper, the strength variation of the materials is set to 10 percent of the mean values. The reliability of the reducers can be calculated by above equations in cooperation with the analyzed stresses in ANSYS. The reliability degradation with respect to the loading torques can be obtained. The analyzed results
show that the reliability degradation is faster when the stress variation doesn’t be considered. The analyzed results are useful for determining the allowed loads of the reducers.

6. Conclusions
The analytical methods of structural stress are reported for performing stress and reliability evaluations. FEM is used as a tool to analyze the stresses of the reducers at different designs and loads. The maximum stresses of the DD and the HD are identified to observing the most possible failed components. The analyzed results show that the induced stresses in DD are smaller than the ones in HD at the same loads. It means that the DD have better strength compared with the HD under the same designed conditions. According to the analyzed results, the allowable loads as well as the maximum moments of the reducers can be decided accordingly. The reliabilities of the reducers under different loads are evaluated by integrating the analyzed stresses with stress-strength interference theory. The studied results may be useful in designing, structural stress analysis and reliability evaluation for developing a high-performance reducer.

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References
[1] Taghirad, H. D. and P. R. Belanger, 2009, “Modeling and parameter identification of harmonic drive systems”, Journal of Dynamic Systems, Measurement and Control, Vol.120 (4), pp. 439-444.
[2] Ghorbel, F. H., P. S. Gandhi and F. Alpeter, 2001, “On the kinematics error in harmonic drive gears”, Journal of Mechanical Design, Vol. 123(1), pp.90-97.
[3] Shuting Li, 2014, “Design and strength analysis methods of the trochoidal gear reducers”, Mechanism and Machine Theory, Vol. 81, pp.140-154.
[4] Y.T. Tsai and K. H. Lin, “Structural stress analysis and reliability evaluation for a new speed reducer”, Journal of Mechanics, Published online: 13 September 2016, pp.1-10 (DOI: http://dx.doi.org/10.1017/jmec.2016.96).
[5] Alban L. E., 1993, “Systematic analysis of gear failures”, American Society for Metals.
[6] Tsai, Y.T., K.S. Wang and J.C. Woo, 2013, “Fatigue-life and reliability evaluations of dental implants based on computer simulation and limited test data”, Proc. IMechE, Part C: J. Mechanical Engineering Science, Vol.227 (3), pp. 554-564.
[7] Tsai, Y.T., K. H. Lin and Y.Y. Hsu, 2013, “Reliability optimization design for mechanical devices based on modeling processes”, Journal of Engineering Design, Vol.24 (12), pp.849-863.
[8] General Catalogue Harmonic Drive AG, 05.2009.