Day-Ahead and Intra-Day Planning of Integrated BESS-PV Systems providing Frequency Regulation

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Abstract—The paper proposes an optimal management strategy for a system composed by a battery and a photovoltaic power plant. This integrated system is called to deliver the photovoltaic power and to simultaneously provide droop-based primary frequency regulation to the main grid. The battery state-of-energy is controlled by power offset signals, which are determined using photovoltaic energy generation forecasts and predictions of the energy required to operate frequency regulation. A two level control architecture is developed. A day-ahead planning algorithm schedules the energy profile which is traded at the day-ahead market and defines the primary control reserve that the integrated system is able to provide in the considered day. During the day operations, a second level algorithm corrects the dispatched plan using updated information, in order to guarantee a continuous and reliable service. Both control algorithms take into account the uncertainties of the photovoltaic generation and of the frequency dynamics using stochastic optimization.

Index Terms—Battery energy storage systems, primary frequency regulation, primary control reserve, predictive control, photovoltaic systems.

I. INTRODUCTION

THE instantaneous balance between generated and consumed active power is one of the basic principles of the AC power systems operation. Any variation from such a condition causes a frequency event, namely, the deviation of the system frequency from its nominal value. The progressive displacement of conventional generation in favour of production from Renewable Energy Sources (RES) will cause the reduction of the frequency control capability of power systems. Therefore, it is necessary to involve new resources in grid ancillary services in order to ensure robustness, resiliency and efficiency of future power systems [1]–[3].

The power equilibrium in real-time can be controlled only if the production system is able to change its generation level [4]. The coupling of RES with Battery Energy Storage Systems (BESSs) is therefore investigated in order to meet the grid flexibility requirements with the aleatory characteristics of such generation systems [5]–[7]. Assessments on the capital costs of batteries have shown that, with the market condition of last years, a multifunctional storage deployment is necessary to overcome the investment costs for energy storage systems [8].

Many literature papers propose methods for allowing batteries to provide services such as energy management, peak shaving, and frequency and voltage regulation [9]–[23]. Several control strategies to perform Primary Frequency Regulation (PFR) are proposed in literature [24]–[29]. Moreover, specific markets around the world are now under development in order to integrate BESS into grid services, such as in the United States PJM interconnect and ISO New England [27], [28], in the Europe National Grid (GB) [29] and in the International Grid Control Cooperation (IGCC) which involves German, Belgian, Dutch, French, Swiss and Austrian Primary Control Reserve (PCR) markets [30].

In this work an integrated BESS-Photovoltaic system (PV) is considered. A wide literature shows how to properly manage this Integrated System (IS) to perform multiple services such as contingency management, peak shaving, demand response, etc. [31]–[33]. However, in many cases droop-based PFR is not considered. Papers combining multiple services with PFR usually assume a non-traditional provision of PFR, such as the one defined by the PJM market [27]. In this specific case, the signal provided to the regulating units is divided in two contributions, a slow one (RegA) and a fast one (RegD). The one provided to BESS and RES is RegD, which is designed to be zero-mean, in order to keep the BESS State-Of-Charge (SOC) approximately at the same level, during a given time period [31], [32], [34]. Nevertheless, most markets do not adopt this control strategy, but use the row frequency as regulating signal, which is not guaranteed to be zero-mean within a given time period. In this case, more sophisticated techniques, such as the ones in [35] and [9] should be used.

In particular, in [35] and [9] PFR is coupled with the dispatch of the active power demand of a distribution feeder. Moreover, such as other works previously cited, these two works are focused on the usage of batteries in transmission and distribution level. Differently, the present paper is focused on the generation level: the IS is operated as a power plant which simultaneously participates to the energy market, delivering to the grid the available PV generation, and provide droop-based PFR. The main contribution of this work is therefore the integration of these two services with a common formulation. Moreover, the problem is defined in order to match the current grid codes and markets requirements (see Section II-B for details).

The IS architecture is depicted in Fig. 1. The objective is to define an energy dispatch plan using the storage flexibility, to maximize the economic gain and provide a continuous and reliable PFR service. A two level strategy [36] is adopted.
IS rated power is indicated with $P_{n}^{b} = P_{n}^{pv} + P_{n}^{b}$. The BESS energy capacity is indicated with $E_{n} [\text{kWh}]$.

The IS has the objective of exporting the PV generation and provide PFR. Therefore, $P^{t}$ assumes the form

$$P^{t} = P^{m} - \alpha \Delta f,$$  \hspace{1cm} (2)

where $\alpha \ [\text{kW/Hz}]$ is the droop coefficient, $\Delta f [\text{Hz}]$ is the frequency deviation from the nominal value $f_{n}$ and $P^{m} [\text{kW}]$ is the IS market power, i.e., the power traded at the energy market. The duration of the energy market session, also called dispatch sampling time, will be indicated with $\tau$ [s].

It is assumed that the IS always operates as a generator, and therefore $P^{m} \geq 0$. A minimal droop coefficient $\alpha_{\text{min}}$ is established. It is therefore required that

$$\alpha \geq \alpha_{\text{min}}.$$  \hspace{1cm} (3)

The value of $\alpha_{\text{min}}$, can be defined, for example, according to [38], where a generator with rated power $P_{n}$ participating to PFR has to ensure a maximum statism $b_{p}^{\text{max}} [\%]$, that corresponds to $\alpha_{\text{min}}$ by the relation

$$b_{p}^{\text{max}} = \frac{100}{\alpha_{\text{min}}} \frac{P_{n}}{f_{n}}.$$  \hspace{1cm} (4)

PFR is effectively operated only by BESS. Therefore, to obtain (2), it results that the battery power exchange is

$$P^{b} = P^{m} - P^{pv} - \alpha \Delta f.$$  \hspace{1cm} (5)

The IS is controlled by a IS Management System (ISMS) that receives measurements and sends control set-points from/to the PV inverter and the Battery Management System (BMS), which controls the BESS. In particular, the ISMS receives the measurements of the current PV power generation $P^{PV}$ and of the battery State-of-Energy, indicated with $S$ [p.u.].

In this paper, the SOE dynamics is modelled by the following discrete-time system:

$$S_{k+1} = S_{k} - \frac{\tau}{3000} \frac{P^{b}}{E_{n}}.$$  \hspace{1cm} (6)

Notice that (6) describes the dynamics of a BESS with unitary efficiency. It will be shown that such an assumption in the control algorithm design do not affect the overall results. The same approximation has been done and verified in [35], [39].

The ISMS has the mission of maximizing the economic gain coming from the energy delivery and the provision of the PFR service. It uses forecasts of the PV generation and of the energy required to provide PFR. Based on this information, each day, the ISMS trades the energy delivery profile and the day PFR droop coefficient $\alpha$ for the day-ahead. During the operation the battery SOE must be kept within the security interval $[S_{\text{min}}, S_{\text{max}}]$. The violation of the SOE security interval is called failure. When a failure occurs, the provision of PFR is suspended. The percentage time during which the SOE security interval is violated is defined failure rate, indicated with $\lambda$.

Using stochastic modelling, the a priori definition of maximal failure rate is, for all $k$,

$$\lambda_{\text{max}} = 1 - \mathbb{P} (S_{k}^{\text{min}} \leq S_{k} \leq S_{k}^{\text{max}}),$$  \hspace{1cm} (7)

It is worth remarking that the problem formulation is general, there are no hypotheses on the type of battery or its performance or the ratings of the resources. Moreover, there are neither hypothesis on the coupling between the BESS and the PV plants, that could be in principle in AC, DC or even the results of an aggregation of several BESSs and PVs.

The performances of the designed method are tested by simulations in MATLAB/Simulink, the test environment adopted has been validated by on field experiments as detailed in [35].

The rest of the paper is organized as follows. Section II describes the system configuration and provides the problem formulation. Section III and Section IV introduced the DAP and HAP algorithms, respectively. Simulation results are described in Section V. Finally, conclusions are reported in Section VI.

**Notation.** $\mathbb{E}(z)$ is the expectation of the random variable $z$; $P(A)$ is the probability of event $A$; $x \sim \mathcal{N}(\bar{z}, \sigma^{2})$ indicates that $z$ is a Normally distributed random variable with mean $\bar{z}$ and variance $\sigma^{2}$; erf$^{-1}(\cdot)$ is the inverse Gauss error function; $k = a : b$, denotes the sequence $k = a, a + 1, \ldots, b$.

**II. PROBLEM FORMULATION**

The system configuration is presented in Fig. 1. The IS is composed by a BESS and a PV plant. The power $P^{t} [\text{kW}]$ is exported at the Grid Coupling Point (GCP). As indicated, $P^{t} > 0$ means that the IS is exporting power. With the same convention, the BESS exports or import power $P^{b} [\text{kW}]$ and the PV plant generates power $P^{pv} [\text{kW}]$. From the figure, it clearly follows that

$$P^{t} = P^{b} + P^{pv},$$  \hspace{1cm} (1)

The PV generation and the BESS power exchange are limited by the rated powers $P_{n}^{pv}$ and $P_{n}^{b}$, respectively.
A. Primary frequency regulation from BESS

Assume to have a BESS with capacity $\hat{E}_n$, which performs PFR with a droop coefficient $\alpha$, and divide the time into windows of length $T$ [h]. The energy required to provide PFR in the generic $i$-th time window $[iT, (i+1)T]$ is:

$$E^f_i = -\alpha \cdot \int_{iT}^{(i+1)T} f(t)dt = -\alpha W^f_i,$$

(8)

where $W^f_i$ [Hz h] is defined as the integral over the current time interval of the frequency deviation. The analysis detailed in [35] demonstrates that a time series $\{W^f_i\}$ obtained from a large database of frequency measurements [40] and a given value of $T$ (e.g. $T \in [1, 2, \ldots, 24]$h) can be modeled with an autoregressive (AR) process of order $p$ [41]. This implies that:

$$W^f_{i+1} = \hat{W}^f_{i+1} + \epsilon_{i+1},$$

(9)

$$\hat{W}^f_{i+1} = W^f_i \phi_1 + \cdots + W^f_{i-p+1} \phi_p,$$

(10)

where $\{W^f_1, \ldots, W^f_{i-p-1}\}$ are the measured value of the integral of the frequency deviation in the last $p$ periods, $\{\phi_1, \ldots, \phi_p\}$ are the AR coefficients defined by the analysis of the frequency database, $\hat{W}^f_i$ is the prediction $W^f_i$ for the upcoming period, and $\epsilon_i$ is a zero-mean Gaussian random variable with standard deviation $\sigma^w_i$. The dependence on $T$ of this standard deviation is explicitly indicated with the subscript $T$, because, in the following, different values of $T$ will be used. It is worth remarking that $\sigma^w_i$ increases with $T$.

Based on this model, the following energy offset is defined:

$$\hat{E}^o_i = \left( S_i - \frac{1}{2} + \frac{\hat{W}^f_i}{E_n} \right) \hat{E}_n,$$

(11)

where $S_i$ is the battery SOE at the beginning of the $i$-th time window. In [35] it is proved that, if $\hat{E}^o_i$ is exchanged by the BESS during $i$-th time window, then the BESS can provide PFR with a maximal failure rate $\lambda_{\text{max}}$, with respect the SOE the security interval $[0, 1]$, if the droop coefficient $\alpha$ is equal or lower than the maximal value

$$\alpha_{\text{max}} = \frac{\hat{E}_n}{2 \cdot \mu \cdot \sigma^T_i},$$

(12)

where $\mu$ is $(1 - \lambda_{\text{max}}/2)$-th percentile of a zero-mean standard Gaussian random variable, which can be computed as

$$\mu = \sqrt{2 \ln 2} (1 - \lambda_{\text{max}}).$$

B. Main requirements for PFR service

The integration of RES into grid regulating scheme requires the revision of the grid codes. In continental Europe, all the Transmission System Operators (TSOs) involved in the joint market IGCC have worked together to define pre-qualification and delivery rules for the BESSs which provide PCR [30]. In the UK, Nationalgrid (NGET) has developed the enhanced frequency response service and defined specific rules for the integration of the new resources into the markets [42]. In the United States of America, PJM has created another market in which the users are remunerated for the capacity, for the availability and for the performance in providing the service [28, 29].

By analyzing the mentioned documents, it results that the PFR markets are different each others and still changing, mainly because they are new. Therefore, the control strategy designed in this paper has the objective of matching the most important rules common between those markets rules:

a) droop-based response to the frequency variations;

b) the SOE must be kept within predefined limits;

c) as requested by the market operators [30, 38, 42], a minimum PCR offer has to be ensured;

d) according to some grid operators, the failure rate has to be kept lower than a maximal value (e.g. 5% in UK [29], 40) or equal to zero [28, 30, 43, 44] in order not to pay penalties.

Finally note that the algorithm proposed in the present paper does not respect the capacity trading time line, i.e. the droop coefficient is computed daily and not weekly as in [50]. However, it is opinion of the authors that future markets deregulation will require to operate on shorter time windows in order to integrate all the new resources.

III. DAY-AHEAD PLANNING (DAP)

The DAP problem consists in the definition of the daily power delivery profile $\{P^d_k\}$ of the IS and the droop coefficient $\alpha$, computed one day before. The objective is to maximize the economic gain, given set of available data and satisfying a set of technical constraints, as detailed in the following.

A. Available data

Given the time horizon $N = 24 \cdot 3600/\tau$, the data supposed to be available at day $d - 1$ when the planning of day $d$ is computed are:

a) a PV forecast profile $\{\hat{P}^p_k\}_{k=0}^{N-1}$, with an associated confidence interval $\Delta^p_k$, such that $|\hat{P}^p_k - \hat{P}^p_k| \leq \Delta^p_k$;

b) the prediction of the frequency integral for the day-ahead $\hat{W}^f_d$ and the associated standard deviation $\sigma^w_d$, computed as described in Section II-A with $T = 24$h;

c) the energy price profile $\{c^f_k\}_{k=0}^{N-1}$;

d) the PFR price $c^f$;

e) the day initial SOE, $S_0$.
Firstly, the equivalent BESS capacity represented with the following Gaussian model:

$$P_k^p \sim \mathcal{N}(\hat{P}_k^p, (\sigma_k^p)^2), \quad \sigma_k^p = \Delta_k^p / 3$$

so that $P(|\hat{P}_k^p - P_k^p|) \leq 0.997$. From (3), (6) and definition (3) (with $T = \tau$) it follows that, for $k = 0 : N - 1$,

$$S_{k+1} = S_k - \frac{\tau(P_k^m - P_k^p)}{3600 \cdot E_n} + \frac{\alpha W_k^f}{E_n}.$$  

(14)

Figure 2 shows the basic principle of the DAP optimization. Firstly, the equivalent BESS capacity $E_n^s$ is defined as

$$E_n^s = E_n (S_{\text{max}}^n - S_{\text{min}}^n).$$

(15)

Then, each day, the quantities $S_{\text{max}}^d$ and $S_{\text{min}}^d$ are determined by the optimization, to divide $E_n^s$ in two portions $E_n^{pv}$ and $E_n^f$:

$$E_n^{pv} = E_n (S_{\text{max}}^d - S_{\text{min}}^d),$$

$$E_n^f = E_n^s - E_n^{pv}.$$  

(16)

(17)

It is obviously required that

$$S_{\text{min}}^d \leq S_{\text{min}}^d \leq S_{\text{max}}^d \leq S_{\text{max}}^d.$$  

(18)

The idea is to use the portion $E_n^{pv}$ to correct the PV prediction errors, and the portion $E_n^f$ to provide PFR, as they were two different batteries: the PV battery and the PFR battery, respectively. Two equivalent SOE trajectories $\{S_k^{pv}\}$ and $\{S_k^f\}$ are supposed to move in these two batteries. They are defined in p.u. with respect to the two capacities $E_n^{pv}$ and $E_n^f$ (right plots in Fig. 2), by the following dynamical equations (with $k = 0 : N - 1$):

$$\bar{S}_{k+1}^{pv} = \bar{S}_k^{pv} - \frac{\tau(P_k^m - P_k^{pv})}{3600 \cdot E_n^{pv}},$$

$$\bar{S}_0^{pv} = \frac{E_n (S_0 - S_{\text{min}}^d)}{E_n^{pv}},$$

$$\bar{S}_{k+1}^f = \bar{S}_k^f + \frac{\alpha W_k^f}{E_n^f},$$

$$\bar{S}_0^f = \frac{E_n (S_{\text{min}}^d - S_{\text{min}}^d)}{E_n^f},$$

(19)

(20)

(21)

(22)

It can be proved by induction that, for $k = 0 : N$,

$$S_k = S_k^{pv} + (S_k^f - S_{\text{min}}^d),$$

(23)

where $S_k^{pv}$ and $S_k^f$ are defined as it follows (see the left plots in Fig. 2 for an example):

$$S_k^{pv} = \frac{E_n^{pv} \bar{S}_k^{pv}}{E_n} + S_{\text{min}}^d, \quad S_k^f = \frac{E_n^f \bar{S}_k^f}{E_n} + S_{\text{min}}^d.$$  

(24)

The component $S_{\text{pv}}^f$ is driven by the dispatch power $P_m^p$ and the PV power $P_{\text{pv}}^p$, whereas the component $S_k^f$ is driven by the frequency variations. Since the (local) PV production and grid frequency can be assumed to be statistically independent, also $S_{\text{pv}}^f$ and $S_k^f$ result to be independent. This implies the following result, which is proved in the appendix section.

**Proposition 1:** If, for all $k = 0 : N$,

$$P(0 \leq \bar{S}_k^{pv} \leq 1) \geq 1 - \beta,$$  

(25)

$$P(0 \leq \bar{S}_k^f \leq 1) = 1 - \lambda_{\text{max}},$$  

(26)

then

$$P(S_{\text{min}}^d \leq S_k \leq S_{\text{max}}^d) \geq 1 - \lambda_{\text{max}}.$$  

(27)

with

$$\lambda_{\text{max}} = \lambda_{\text{max}} + \beta - \lambda_{\text{max}}^f \beta.$$  

(28)

This proposition means that if (25) and (26) hold true, then $\lambda_{\text{max}}$ is the resulting maximal failure rate of the IS.

Relation (25) is considered as a chance constraint. Using the Gaussian representation (13), assuming that the PV prediction errors and the battery modelling errors are independent, and that the sampling time $\tau$ is large enough to suppose that the PV prediction errors at different time steps are mutually independent, from (19)–(20), it follows that, for $k = 0 : N$,

$$\bar{S}_k^{pv} \sim \mathcal{N}(m_k^s, (\sigma_k^s)^2),$$

(29)

where

$$m_k^s = \bar{S}_0^{pv} - \frac{\tau}{3600 \cdot E_n^{pv}} \sum_{j=0}^{k-1} (P_j^m - \hat{P}_j^{pv}),$$

$$\lambda^2 = \left(\frac{\tau}{3600 \cdot E_n^{pv}}\right)^2 \cdot \sum_{j=0}^{k-1} (\sigma_j^{pv})^2.$$  

(30)

(31)

To obtain (25), the following separated chance constraints are defined, for all $k = 0 : N$:

$$P\left(\bar{S}_k^{pv} \leq 1 - \beta 2\right) \geq 1 - \beta 2,$$  

(32)

which, using the Gaussian model (29)–(31), can be expressed with the equivalent deterministic constraints (see [37] or [14] for details):

$$m_k^s + \theta_\sigma s_k^s \leq 1,$$  

(33)

$$-m_k^s + \theta_\sigma s_k^s \leq 0,$$  

(34)

where $\theta_\sigma = \sqrt{2} \text{erf}^{-1}(1 - \beta)$.

To obtain (26), the method recalled in Section II-A is applied to the PFR battery considering a period $T = 24$ h. Recall that $S_{\text{min}}^d$ and $S_{\text{max}}^d$ are defined by the DAP optimization. Considering (22), this implies that the initial condition $\bar{S}_0^f$, at the beginning of the day, is defined by the optimization. Therefore, by (11), if

$$\bar{S}_0^f = \frac{1}{2} - \frac{\alpha W_0^f}{E_n^f},$$  

(35)
then the required energy offset \( \hat{E}_d^0 = 0 \), and therefore satisfied with \( \alpha \) given by

\[
\alpha = \frac{E_d^f}{2\mu_\sigma^w_{24}}.
\]

Using the definition of \( E_d^f \) in (17) and the relation (18) and (19) are equivalent to

\[
2\alpha \hat{W}_d^f = E_n[(S_{\text{max}} - S_{\text{min}}) - (S_{d_{\text{max}}} - S_{d_{\text{min}}})],
\]

\[
2\alpha \mu_\sigma^w_{24} = E_n[(S_{\text{max}} - S_{\text{min}}) - (S_{d_{\text{max}}} - S_{d_{\text{min}}})].
\]

C. Power constraints

As defined in Section II the BESS power is limited by the nominal value \( P^b_n \). From (5), it results that the following inequality should be always satisfied:

\[
|P^b| = |P^m - P^pv - \alpha \Delta f| \leq P^b_n.
\]

Since it is assumed that, for \( k = 0 : N - 1, \)

\[
0 \leq P^m_k \leq P^b_n
\]

and \( P^pv_k \geq 0 \) by definition, then, for the day-ahead, there are two worst cases, which are covered with the following chance constraints (with \( k = 0 : N - 1):\)

\[
P(P^m_k - P^pv_k + \alpha \Delta f_{\text{max}} \leq P^b_n) \geq 1 - \gamma,
\]

\[
P(P^m_k - P^pv_k - \alpha \Delta f_{\text{max}} \geq -P^b_n) \geq 1 - \gamma,
\]

where \( \Delta f_{\text{max}} \) is the maximal frequency variation \( \Delta f_{\text{max}} \) defined for a given failure rate \( \lambda_{\text{max}} \), with the associated deterministic constraints (see (37) or (14) for details):

\[
P^m_k - \hat{P}^pv_k + \alpha \Delta f_{\text{max}} + \theta_k \sigma^w_k \leq P^b_n,
\]

\[
P^m_k - \hat{P}^pv_k - \alpha \Delta f_{\text{max}} - \theta_k \sigma^w_k \geq -P^b_n,
\]

with \( k = 0 : N - 1, \) and \( \theta_k = \sqrt{2\sigma^w_k} (1 - 2\gamma). \)

D. Smoothness constraints

Two additional constraints are defined to limit the variations of \( P^m \) and \( m^s_k \) between consecutive set-points time steps, for \( k = 0 : N - 1, \)

\[
|P^m_{k+1} - P^m_k| \leq \Delta P^m_{\text{max}},
\]

\[
|m^s_{k+1} - m^s_k| \leq \Delta m^s_{\text{max}}.
\]

E. The DAP algorithm

Given a desired maximal failure rate \( \lambda_{\text{max}} \), the DAP algorithm consists in the solution of the following linear optimization problem:

\[
J^* = \max_{\{P^m_k\}, \alpha, S^m_{\text{max}}, S^m_{\text{min}}} \sum_{k=0}^{N-1} c_k^E \tau P^m_k + c_f^E \alpha
\]

subject to (3), (15), (18), (20), (30)–(31), (33)–(44), (37)–(38), (40)–(44), (45)–(46)

The result of the optimization are the actual IS base power profile \( \{P^m_k\} = \{P^m_k^*\} \) and the droop coefficient \( \alpha^d = \alpha^*, \) both defined the day before the delivery. The value of the cost function \( J^* \) is equal to the day-ahead economical gain.

IV. Hours-Ahead Planning (HAP)

The hour-ahead planning is a lower level controller which is re-computed every hour within the delivery day. The HAP routine receives from the DAP one the power delivery plan \( \{P^md_k\} \) and the droop coefficient \( \alpha^d \). The objective of HAP is to correct the plan \( \{P^md_k\} \) to guarantee the provision of PFR, keeping the droop coefficient \( \alpha^d \) and reducing the expected DAP failure rate \( \lambda_{\text{max}} \) to a lower value \( \lambda'_{\text{max}} \), always maximizing the economical income.

Figure 3 shows the HAP time scheduling. Let \( j = 0, 1, \ldots, 23 \) indicate the hours during the day, and \( n = 3600/\tau \) be the number of intra-hour power set-points defined according to the dispatch plan sampling time. Moreover, let \( N_j = N - j \cdot n \) be the number of power set-points remaining from the \( j \)-th hour to the end of the day.

At the beginning of hour \( j \), the IS power profile \( \{P^m_k\} \) with \( k = jn : N - 1 \) is re-programmed. Then, only the first \( n \) steps, corresponding to the first hour of the dispatch plan, are applied. At hour \( j + 1 \), the HAP optimization is repeated. This time scheduling can be called reducing horizon, and, similarly to the receding horizon principle adopted by Model Predictive Control (MPC), it allows the control algorithm to be more robust with respect to modelling errors. In particular, at each hour, updated, and thus more accurate, PV generation and PFR energy requirement forecasts may be available, as well as the current value of the battery SOE. These updated data are useful to suitably correct the DAP program.

Based on this idea, as shown in Fig. 3 the time from hour \( j \) to the end of the day, is divided into two phases: the First Hour (FH) \( k = jn : (j + 1)n \), and the remaining time from hour \( j + 1 \) to the end of the day \( k = (j + 1)n : N \), from now named Rest of the Day (RoD).

At hour \( j \), the available data are:

a) the DAP power profile \( \{P^md_k\}, k = j \cdot n : N - 1 \) traded at the energy market;

b) the droop coefficient \( \alpha^d \), defined for a given failure rate \( \lambda^d \), to be guaranteed during all the day;

c) the updated PV forecasts \( \{P^pv_k\} \), with the associated standard deviations \( \sigma^w_k \), \( k = j \cdot n : N - 1 \) (using the same the Gaussian model (37) adopted for DAP);

d) the prediction of the frequency integral for the first hour \( \hat{W}_h^f \) and the associated standard deviation \( \sigma^w \), computed as described in Section II-A with \( T = 1 \) h;

e) the prediction of the frequency integral for the rest of the day \( \hat{W}_h^f \) and the associated standard deviation \( \hat{\sigma}^w_{h-j} \), computed as described in Section II-A with \( T = 23 - j \) h;
g) the penalty cost profile \( \{c_k^p\} \), \( k = j \cdot n : N - 1 \) to be paid for a difference of the energy effectively exported by the IS from the energy traded at the day-ahead market; 
h) the intra-day energy price profile \( \{c_k^p\} \), \( k = j \cdot n : N - 1 \); 
i) the current battery SOE, \( S_{jn} \).

For both the time windows FH and RoD, an approach similar to DAP is adopted. In particular, the basic idea of the partition of the BESS capacity by the definition of the thresholds \( S_d^{\text{max}} \) and \( S_d^{\text{min}} \) is re-applied with the definition of different thresholds: \( S_h^{\text{max}} \), \( S_h^{\text{min}} \), for the FH, and \( S_r^{\text{max}} \), \( S_r^{\text{min}} \), for the RoD. The partition into two time windows is adopted in order to give more degrees of freedom to the optimization for the FH. Thanks to the use of short-term, and thus more accurate, predictions, the optimization over the FH will be finer. It is worth remarking that, as mentioned before, at each hour, the optimization results are applied only for the FH.

The HAP optimization problem, solved at each hour \( j \), is formulated as it follows.

\[
\begin{align*}
L_j &= \max \left\{ \{P^m\}_k, \mu_h, \mu_r \right\} \\
L_j = & \sum_{k=jn}^{N-1} \left( c_k^p - c_k^p \right) (P_k^m - P_k^{\text{md}}) \delta_k^+ + c_k^p (P_k^m - P_k^{\text{md}}) \delta_k^- \\
+ & \omega_h \mu_h + \omega_r \mu_r
\end{align*}
\]

subject to:

\[
\begin{align*}
S^{\text{min}} & \leq S_h^{\text{min}} \leq S_h^{\text{max}} \leq S^{\text{max}}, \quad (48) \\
m_k^h + \theta_h \sigma_k^h & \leq S_h^{\text{max}} \quad \text{for } k = jn : j(n+1), \quad (49) \\
-m_k^h + \theta_h \sigma_k^h & \leq S_h^{\text{min}} \quad \text{for } k = jn : j(n+1), \quad (50) \\
2 \alpha^d W_{\hat{f}}^h &= E_n [(S^{\text{max}} + S^{\text{min}}) - (S_h^{\text{max}} + S_h^{\text{min}})], \quad (51) \\
2 \alpha^d \mu_h \sigma_2^w &= E_n [(S^{\text{max}} - S^{\text{min}}) - (S_h^{\text{max}} - S_h^{\text{min}})], \quad (52) \\
\mu & \leq \mu_h \leq \mu^{\text{max}}, \quad (53) \\
S^{\text{min}} & \leq S_r^{\text{min}} \leq S_r^{\text{max}} \leq S^{\text{max}}, \quad (54) \\
m_k^r + \theta_r \sigma_k^r & \leq S_r^{\text{max}} \quad \text{for } k = j(n+1) : N, \quad (55) \\
-m_k^r + \theta_r \sigma_k^r & \leq S_r^{\text{min}} \quad \text{for } k = j(n+1) : N, \quad (56) \\
2 \alpha^d W_{\hat{f}}^r &= E_n [(S^{\text{max}} + S^{\text{min}}) - (S_r^{\text{max}} + S_r^{\text{min}})], \quad (57) \\
2 \alpha^d \mu_r \sigma_2^w &= E_n [(S^{\text{max}} - S^{\text{min}}) - (S_r^{\text{max}} - S_r^{\text{min}})], \quad (58) \\
\mu & \leq \mu_r \leq \mu^{\text{max}}, \quad (59) \\
m_k^h &= S_{jn} - \frac{\tau}{3600 E_n} \sum_{i=jn}^{k-1} (P_i^m - \hat{P}_i^p) \quad (60) \\
(c_k^p)^2 &= \left( \frac{\tau}{3600 E_n} \right)^2 \sum_{i=jn}^{k-1} (\sigma_i^p)^2 \quad (61) \\
P_i^m - \hat{P}_i^p + \alpha^d f_{\text{max}} + \theta_h \sigma_i^p & \leq P_i^b \quad (62) \\
P_k^m - \hat{P}_k^p - \alpha^d f_{\text{max}} - \theta_h \sigma_k^p & \geq -P_i^b \quad (63) \\
0 & \leq P_k^m \leq P_i^b \quad (64) \\
|P_{k+1}^m - P_k^m| & \leq \Delta P_{m}^\text{max} \quad (65) \\
m_{k+1} - m_k & \leq \Delta m_{m}^\text{max} \quad (66)
\end{align*}
\]

The optimization problem results to be mixed-integer with linear constraints. Indeed, there are two binary variables: \( \delta_k^+ \) defined (through additive linear constraints not reported for clarity of presentation) to be equal to 1 when \( P_k^m \geq P_k^{\text{md}} \) and 0 otherwise, and \( \delta_k^- = 1 - \delta_k^+ \).

For each of the two time windows, starting from the definitions of the new thresholds \( S_d^{\text{max}} \) and \( S_d^{\text{min}} \), for the FH, and \( S_r^{\text{max}} \) and \( S_r^{\text{min}} \) for the RoD, the SOE constraints defined for HAP are reformulated as in (48)–(61).

Let us focus on constraints (51)–(52) and (57)–(58). They are the reformulation of the DAP constraints (37)–(38), for the FH and the RoD, respectively. In DAP, (37)–(38) have to be respected in order to assure the maximal failure rate \( \lambda_f^{\text{max}} \) due to PFR, which is related to coefficient \( \mu \) by the relation \( \mu = \sqrt{2e\pi}^{-1}(1 - \lambda_f^{\text{max}}) \) (see Section II-A).

Therefore, if (37)–(38) are satisfied with a \( \bar{\mu} \geq \mu \), the maximal failure rate \( \lambda_f^{\text{max}} \) is reduced. Indeed, by (28) \( \bar{\mu} \) results to be reduced if \( \lambda_f^{\text{max}} \) decreases. Constraints (51)–(52) for the FH and (57)–(58) for the RoD are therefore reformulated using the relevant predictions \( W_{\hat{f}}^h \) and \( W_{\hat{f}}^r \) and imposing that the droop coefficient \( \alpha \) is equal to \( \alpha^d \), computed by the DAP.

Two optimization variables \( \mu_h \) and \( \mu_r \) are introduced for the FH and RoD time-windows. The cost function (47) is designed in order to increase their values, in order to obtain the reduction of the failure rate. With constraints (53) and (59), \( \mu_h \) and \( \mu_r \) are limited by the minimal value \( \mu \), which gives the guaranty to obtain the DAP failure rate \( \lambda_f^{\text{max}} \) and by the maximal value \( \mu_{\text{max}} = \sqrt{2e\pi}^{-1}(1 - \lambda_f^{\text{max}}) \), corresponding to the maximal reduced failure rate \( \lambda_f^{\text{max}} < \lambda_f^{\text{max}} \). The power and smoothness constraints (62)–(66) are re-written, as in DAP, for the entire interval \( k = jn : N - 1 \), with \( \alpha = \alpha^d \).

The cost function (47) considers both the economical gain, determined by the balance between penalties and intra-day energy prices, and the reduction of the DAP failure rate, which, as mentioned, corresponds to the maximization of the coefficients \( \mu_h \) and \( \mu_r \). The optimization weights \( w_h \) and \( w_r \) have a different unit from the costs \( c^p \) and \( c^f \). Therefore, they have to be suitably normalized. It is worth remarking that the minimization of the failure rate may be in contrast with the maximization of the economical income. Therefore, the sizing of the weights \( w_h \) and \( w_r \) defines the priority level between the quality of the PFR service and the economical gain.

V. SIMULATION RESULTS

A set of simulations has been performed considering real markets’ data. The Italian day-ahead (MGP) and intra-day market (MI2) results (February 2019) [45] has been selected as input of DAP and HAP problems, respectively. The penalty for the variations on the dispatched power is fixed to 0.05 \( \in \) kWh\(^{-1}\). Moreover, the frequency regulating capacity price has been selected from the the results of the International PCR markets between August 2018 and March 2019 [30].

DAP and HAP algorithms have been implemented in MATLAB/Simulink, and optimization problems have been written using the General Algebraic Modelling System (GAMS) language and solved with CPLEX. Battery is modelled with a standard equivalent circuit in which the internal resistance is a function of the SOE and of the electromotive force. Thus, a variable nonunitary battery efficiency has been implemented.
Inputs of the simulator are real PV measurements an PV forecasts registered is the low-voltage (LV) microgrid realized by the University of Genova [46]. Moreover, frequency measurements from the UK grid has been adopted in the construction of the AR models and for the simulations [40].

Simulations have been executed over a 21 days period considering the implementation only of DAP, and of both DAP and HAP. Moreover, five different cases are proposed characterized by different PV-BESS sizes, as reported in Table I. Considering devices rating, the ISMS is expected to differently balance the two services, i.e., a larger BESS will provide higher regulating capacity but can rely on smaller offsets for charge management, on the other hand, a larger PV will drive the ISMS to privilege the dispatch service.

Table I shows the parameters adopted for the IS. Among the others: the minimum droop coefficient \( \alpha_{\text{min}} \) is defined according to (3) with respect to the PV nominal power, with an equivalent maximal statism \( b^p_{\text{max}} \) fixed to 8% [38]; the maximum failure rate \( \lambda_{\text{max}} \) is fixed at 5%, according for example to the requirements of the UK market [29], [40]; the dispatch sampling time \( \tau \) is set to 15 min according to the Italian energy market [45].

Figure 4 shows a section of the simulation of the stand alone DAP controller. The top plot reports the dispatch plan \( \hat{P}_k^m \), the day ahead PV forecast \( \hat{P}_k^p \) and the battery offset program \( P_k^{md} = \{P_k^md - \hat{P}_k^p\} \). The middle plot depicts the programmed SOE trajectory and the realized ones. While the bottom plot shows the resulting profiles of the total power at the GCP \( P^t \), of the base dispatch power \( P^m \) and of the PV generation \( P^pv \).

The detailed numerical results of all the simulations in the stand alone DAP case are reported in Table I. The reported data show that the DAP is able to determine a reliable power profile, which allows the IS to perform both the services with a failure rate lower than the prescribed maximal value \( \lambda_{\text{max}} = 5\% \).

Table I. Simulation results.

| Case | \( \lambda \% \) | Total \( \text{€} \) | PCR \( \text{€} \) | Dispatch \( \text{€} \) | Penalty \( \text{€} \) |
|------|-----------------|-----------------|-----------------|-----------------|-----------------|
| A    | 0.424           | 27798           | 15756           | 12042           | 0               |
| B    | 0.530           | 35949           | 23940           | 12010           | 0               |
| C    | 0.403           | 38765           | 14420           | 24345           | 0               |
| D    | 0.234           | 41536           | 4821            | 36714           | 0               |
| E    | 0.941           | 41104           | 4274            | 36830           | 0               |

Resources sizes: A. PV 500 kW, BESS 1500 kW; B. PV 500 kW, BESS 1000 kW; C. PV 1000 kW, BESS 1000 kW; D. PV 1500 kW, BESS 500 kW; E. PV 1500 kW, BESS 320 kW.

Figure 5 shows an example of the results obtained with the DAP-HAP configuration. In particular, in the top plot the modification operated by HAP with respect to DAP can be appreciated. For example, during the night operations (from hour 20 to hour 31) the HAP commands some short power delivery in order to discharge the battery and avoid to reach the full charge condition. Also Fig. 6 makes evidence on the advantages on using the HAP procedure. Indeed, with the stand alone DAP, during the first 50 hours, the battery SOE reaches the up limit (failure), whereas this does not happen when HAP is used. It can be observed that in all the considered cases the DAP-HAP strategy allows obtaining a null failure rate, as shown in Table I. It is worth remarking that one of objective of the HAP is to reduce the expect failure rate to a value below 1%.
The bottom plot of Fig. 6 reports the droop coefficients obtained for the two configurations. They result to be comparable, even if the DAP solution allows to reach slightly higher values. As a consequence, the total economical income results to be higher. It is worth remarking that this results are not affected by some penalty that could be payed for reaching fail conditions in the HAP case.

The results reported in Table II prove the effectiveness of the control algorithms with all the different considered configurations. All cases use the same price vectors, therefore, the power ratings of the IS has a relevance on the total income. Increasing the PV power rating allows to reach higher income results to be higher. It is worth remarking that this results are not affected by some penalty that could be payed for reaching fail conditions in the HAP case.

Tests account for the efficiency. Such a model has been derived from the simulation setup presented and validated in [35], [39]. The model consists in the series of an internal voltage source and of variable resistance, the parameters obtained from measurements the original grid-scale lithium-titanate battery rated 560 kWh [47] has been scaled to match the different battery sizes simulated. The obtained results prove that such an approximation in the control design does not influence the overall performance.

VI. CONCLUSIONS

This paper presents a strategy for the optimal planning of an integrated BESS–PV system, which provides frequency regulation and generation dispatch. The control architecture is composed by two algorithms. The first one, DAP, is executed the day before the delivery and defines the power dispatch plan and a droop coefficient for the PFR, on the basis of PV forecasts and predictions of the energy required for providing PFR. The delivery day, at each hour, the second algorithm, named HAP, is executed in order to allow the IS to perform its tasks in a continuous and reliable way by using updated short-term forecasts. The two algorithms are designed to maximize the total incomes and the performance in providing PFR. They use chance-constrained optimization in order to model the forecasts errors. The control framework has been validated by simulations. Future works will consider different applications using a similar approach, also non-Gaussian representations of uncertainties and stochastic models of the energy prices.

APPENDIX A

PROOF OF PROPOSITION I

Using (24), from (25) and (26), it follows that

$$\mathbf{P}\left(0 \leq S_{k}^{pv} - S_{d}^{min} \leq \frac{E_{n}^{pv}}{E_{n}}\right) = \mathbf{P}(A) \geq 1 - \beta,$$

$$\mathbf{P}\left(0 \leq S_{k}^{f} - S_{d}^{min} \leq \frac{E_{n}^{f}}{E_{n}}\right) = \mathbf{P}(B) = 1 - \lambda_{max},$$

where, A and B indicate the two considered constraints. Since $S_{d}^{pv}$ and $S_{d}^{f}$ are independent, it results that

$$\mathbf{P}(A \cap B) = \mathbf{P}(A) \cdot \mathbf{P}(B) = (1 - \beta) \cdot (1 - \lambda_{max}) = 1 - \lambda_{max},$$

where $\lambda_{max}$ is equal to the one defined in (28). Now consider that, because of elementary set inclusion properties,

$$\mathbf{P}\left(0 \leq S_{k}^{pv} + S_{k}^{f} - S_{d}^{min} - S_{d}^{max} \leq \frac{E_{n}^{pv} + E_{n}^{f}}{E_{n}}\right) \geq \mathbf{P}(A \cap B) \geq 1 - \lambda_{max},$$

from which, taking into account (23), it follows that

$$\mathbf{P}\left(0 \leq S_{k} - S_{d}^{min} \leq \frac{E_{n}^{pv} + E_{n}^{f}}{E_{n}}\right) \geq 1 - \lambda_{max},$$

and, therefore,

$$\mathbf{P}\left(S_{d}^{min} \leq S_{k} \leq \frac{E_{n}^{pv} + E_{n}^{f}}{E_{n}} + S_{d}^{min}\right) \geq 1 - \lambda_{max}.\]

To conclude, (27) is proved by noticing that from the definitions (16) and (17) it results that

$$\frac{E_{n}^{pv} + E_{n}^{f}}{E_{n}} + S_{d}^{min} = \frac{E_{n}}{E_{n}} + S_{d}^{min} = S_{d}^{max}.$$

\begin{table}[h]
\centering
\caption{Simulation Parameters.}
\begin{tabular}{lll}
\hline
Variable & Description & Value  \\
\hline
$\tau$ & Dispatch sampling time & 15 min  \\
$\Delta P_{\text{max}}^{\text{pv}}$ & Maximal power deviation & 40% $P_{\text{pv}}^{\text{rated}}$  \\
$\Delta P_{\text{max}}^{\text{soe}}$ & Maximal SOE deviation & 10%  \\
$\gamma$ & Battery power chance-contraints coefficient & 1%  \\
$\beta$ & Battery SOE chance-contraints coefficient & 1%  \\
$\alpha_{\text{min}}$ & Minimal droop coefficient & 1%  \\
$\alpha_{\text{max}}$ & Maximal droop coefficient & inf  \\
$S_{\text{min}}^{\text{max}}$ & Maximal battery SOE & 100%  \\
$S_{\text{max}}^{\text{max}}$ & Maximal battery SOE & 0%  \\
$\Delta f_{\text{max}}$ & Maximal frequency deviation & 0.2 Hz  \\
$\mu$ & Equivalent to DAP failure rate $\lambda_{\text{max}}$ & 5%  \\
$\beta_{\text{max}}$ & Equivalent to HAP failure rate $\lambda_{\text{max}}$ & 0.3%  \\
\hline
\end{tabular}
\end{table}

Fig. 6. Simulation results for the DAP only and DAP-HAP configurations. Case D: 500 kW PV, 1500 kWBESS. Top: planned and realized SOE profiles in the stand alone DAP configuration; middle: planned and realized SOE profiles in the DAP-HAP configuration; bottom: droop coefficients obtained in the DAP only and DAP-HAP configurations.
