Pragmatic approach to the little hierarchy problem
- the case for Dark Matter and neutrino physics -

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We show that the addition of real scalars (gauge singlets) to the Standard Model can ameliorate the little hierarchy problem and provide realistic Dark Matter candidates. To this end, the coupling of the new scalars to the standard Higgs boson must be relatively strong and their mass should be in the 1–3 TeV range, while the lowest cutoff of the (unspecified) UV completion must be \( \gtrsim 5 \) TeV, depending on the Higgs boson mass and the number of singlets present. The existence of the singlets also leads to realistic and surprisingly reach neutrino physics. The resulting light neutrino mass spectrum and mixing angles are consistent with the constraints from the neutrino oscillations.

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Introduction
The goal of this project is to provide the most economic extension of the Standard Model (SM) for which the little hierarchy problem is ameliorated while retaining all the successes of the SM. We focus here on leading corrections to the SM, so we will consider only those extensions that interact with the SM through renormalizable interactions (below we will comment on the effects of higher-dimensional interactions). Since we concentrate on taming the quadratic divergence of the Higgs boson mass, it is natural to consider extensions of the scalar sector: when adding a new field \( \varphi \), the gauge-invariant coupling \( |\varphi|^2 H^\dagger H \) (where \( H \) denotes the SM scalar doublet) will generate additional radiative corrections to the Higgs boson mass that can serve to soften the little hierarchy problem. In this paper we will consider a class of modest extensions by adding several real scalar fields which are neutral under the SM gauge group. The extension we consider, although renormalizable, will still be understood to constitute an effective low-energy theory valid up to energies \( \sim 5 – 10 \) TeV; we shall not discuss the UV completion of this model.

The little hierarchy problem
Within the SM the quadratically divergent 1-loop correction to the Higgs boson (\( h \)) mass is given by

\[
\delta^{(SM)} m_h^2 = \left[3m_t^2/2 - (6m_W^2 + 3m_Z^2)/8 - 3m_h^2/8\right] \Lambda^2/(\pi^2 v^2)
\]  

(1)

where \( \Lambda \) is a UV cutoff (we use a cutoff regularization) and \( v \approx 246 \) GeV denotes the vacuum expectation value of the scalar doublet (SM logarithmic corrections are small since we assume \( v \ll \Lambda \lesssim 10 \) TeV); the SM is treated here as an effective theory valid below the physical cutoff \( \Lambda \), the scale at which new physics becomes manifest.

Since precision measurements (mainly from the oblique \( T_{\text{obli}} \) parameter \([10]\)) require a light Higgs boson, \( m_h \sim 120 – 170 \) GeV, the correction (1) exceeds the mass itself even for small values of \( \Lambda \), e.g. for \( m_h = 130 \) GeV we obtain \( \delta^{(SM)} m_h^2 \simeq m_h^2 \) already for \( \Lambda \approx 580 \) GeV. On the other hand constraints on the scale of new physics that emerge from analysis of operators of dim 6 require \( \Lambda \gtrsim \) few TeV. This difficulty is known as the little hierarchy problem.

There are two ways to solve this problem: one adds new particles whose effects either \( i \) generate radiative corrections that partially cancel (1), as is done in supersymmetric theories (for which \( \delta m_h^2 \ll m_h^2 \) up to the GUT scale); or \( ii \) increase the allowed value of \( m_h \) by canceling the contributions to \( T_{\text{obli}} \) from a heavy Higgs (see e.g. [3]).

Here we follow the first strategy, but with a modest goal: we construct a simple modification of the SM within which \( \delta m_h^2 \) (the total correction to the SM Higgs boson mass squared) is suppressed only up to \( \Lambda \lesssim 3 – 10 \) TeV. Since (1) is dominated by the fermionic (top) terms, the most economic way of achieving this is by introducing new scalars \( \varphi_i \) whose 1-loop contributions balances the ones derived from the SM. In order not to spoil the SM predictions we assume that \( \varphi_i \) are singlets under the SM gauge group. It is then easy to see that the oblique parameters will remain unchanged if \( \langle \varphi_i \rangle = 0 \) (which we assume hereafter), so that the SM prediction of a light Higgs is preserved. An extension of the SM by an extra scalar singlet was also discussed in [2], there however (classical) conformal symmetry was adopted to cope with the hierarchy problem.

The most general scalar potential consistent with \( Z_2^{(i)} \) independent symmetries \( \varphi_i \to -\varphi_i \) (imposed in order to
however that these conditions are derived from the behavior of the theory at energies

In the following numerical computations we assume for simplicity that

In the limit at large field strengths requires \( \lambda_H \phi_x > 6A_x^2 \) at tree level. The high energy unitarity behavior (known \( [3] \) for \( N_\phi = 1 \)) implies \( \lambda_H \leq 4\pi/3 \) (the SM requirement) and \( \lambda_x \leq 8\pi, \lambda_x < 4\pi \). Note however that these conditions are derived from the behavior of the theory at energies \( E \gg m \), where we don’t pretend our model to be valid, so that neither the stability limit nor the unitarity constraints are applicable within our pragmatic strategy \(^1\) that aims at a modest increase of \( \Lambda \) to the \( 3 \rightarrow 10 \text{ TeV} \) range.

The presence of \( \phi_1 \) generates additional radiative corrections \(^2\) to \( m_h^2 \). However different ways of imposing the cutoff \( \Lambda \) (cutoff regularization, higher-derivative regulators, Pauli-Villars regulators, etc.) yield different expressions for the extra corrections; for large \( \Lambda \), the coefficients of the \( \Lambda^2 \) and \( m^2 \ln(\Lambda^2/m^2) \) terms are universal, but not the sub-leading, terms are not. Since the sub-leading contributions are small for \( m \ll \Lambda \) (this is the range interesting for us) the differences between various regularization schemes are not relevant. Here we decided to adopt the simple UV cutoff regularization. Then the extra contribution to \( m_h^2 \) read

Adopting the parameterization \(|\delta m_h^2| = |\delta^{(SM)} m_h^2 + \delta^{(\phi)} m_h^2| = D_1 m_h^2 \) \(^3\), we can determine the value of \( \lambda_x \) needed to suppress \( \delta m_h^2 \) to a desired level \((D_1)\) as a function of \( m \), for any choice of \( m_h \) and \( \Lambda \); examples are plotted in fig.1 for \( N_\phi = 6 \). It should be noted that (in contrast to SUSY \(^3\)) the logarithmic terms in \(|\delta|\) can be relevant in canceling large contributions to \( \delta m_h^2 \). It is important to note that the required value of \( \lambda_x \) is smaller for larger \( m_h \), and can also be reduced increasing the number of singlets \( N_\phi \). When \( m \ll \Lambda \), the \( \lambda_x \) needed for the amelioration of the hierarchy problem is insensitive to \( m, D_1 \) or \( \Lambda \); as illustrated in fig.1 analytically we find

\[
\lambda_x = N_\phi^{-1} \left\{ 4.8 - 3(m_h/v)^2 + 2D_1[2\pi/(\Lambda/\text{TeV})]^2 \right\} \left[ 1 - m^2/\Lambda^2 \ln \left( m^2/\Lambda^2 \right) \right] + O \left( m^4/\Lambda^4 \right). \tag{4}
\]

\(^1\) These conclusions remain even if one includes higher-dimensional operators since such terms are subdominant unless the energies and/or field strengths are of order \( \Lambda \) – were the model is not valid; such operators can also generate spurious minima, but these have scale \( \sim \Lambda \) and are not within the range of validity of the model. It is also fair to note that for \( N_\phi = 1 \) the stability limit for \( m_h > 115 \text{ GeV} \) implies \( \lambda_x > 12v(m_h/v^2)^2>0.5 \lambda_x \). Then using \( \lambda_x \leq 8\pi \) we find \( \lambda_x \leq 0.68 \); this does not allow for a significant cancellation of the SM contributions \(^4\) and the little hierarchy problem remains. Increasing \( N_\phi \) suppresses \( \lambda_x \) and relaxes the unitarity constraints.

\(^2\) The \( \Lambda^2 \) corrections to \( m^2 \) can also be tamed within the full model with additiona fine tuning, but we will not consider them here, see \(^4\). The advantage of the present model is that this tuning allows small corrections to \( m, m_H \) while keeping them below the cutoff.

\(^3\) Note that in SUSY the corresponding logarithmic stop contributions survive and constitute a source of concern.
where the first contribution is from the annihilations into SM final states $\phi \rightarrow \text{SM SM}$, assume that the see-saw mechanism explains the smallness of three light neutrino masses, and accordingly we require $\Lambda \ll \frac{\sqrt{2} \pi}{\lambda}$. Since we consider $\lambda \sim 1$, it is pertinent to estimate the effects of higher order corrections to $m_h$ to a given level of fine-tuning as in [6]; we will return to this in a future publication [4].

Finally, the requirement that the $\phi_i$ account for the inferred DM abundance, $\Omega_{DM} h^2 = \sum_i \Omega_{\phi_i} h^2 = 0.106 \pm 0.008$ [10], can be used to fix $\langle \sigma_i v \rangle$, which translates into a relation $\lambda_i = \lambda_i(m)$ though the use of [11]. Substituting this into $|\delta m_i^2| = D_i m_i^2$, we find a relation between $m_i$ and $\Lambda$ (for a given $D_i$), which we plot in fig.2 for $N_{\phi} = 6$. It is important to stress that it is possible to find parameters $\Lambda$, $\lambda_i$ and $m_i$ such that both the hierarchy is ameliorated to the prescribed level and such that $\Omega_{\phi} h^2$ is consistent with the DM requirement (we use a 3$\sigma$ interval). It also is useful to note that the singlet mass (as required by the DM) scales with their multiplicity as $\lambda_i^5$, therefore increasing $N_{\phi}$ implies smaller scalar mass, e.g. changing $N_{\phi}$ from 1 to 6 leads to the reduction of mass by a factor $\sim 15$.

Neutrinos We now consider consequences of the existence of $\phi$ for the leptonic sector, which we assume consists of the SM fields plus three right-handed neutrino fields $\nu_R$ ($i = 1, 2, 3$) that are also gauge singlets; in this section we assume only one singlet for simplicity. The relevant Lagrangian is then

$$L_Y = -\bar{L}Y\ell R - \bar{Y}_e\nu_R - \frac{1}{2} (\nu_R)^\dagger M \nu_R - \nu_R^\dagger Y_e \nu_R + \text{H.c.}$$

where $L = (\nu_L, l_L)^T$ is a SM lepton isodoublet and $l_R$ a charged lepton isosinglet (we omit family indices); we will assume that the see-saw mechanism explains the smallness of three light neutrino masses, and accordingly we require $M \gg M_D \equiv Y_e v/\sqrt{2}$. The symmetry of the potential under $\phi \rightarrow -\phi$ can be extended to $\nu_R$ by requiring $L \rightarrow S_{LL}, l_R \rightarrow S_{lR} l_R, \nu_R \rightarrow S_{\nu_R} \nu_R$.
The allowed region in the \((m, \Lambda)\) plane for \(D_l = 0\), \(N_\nu = 6\) and \(\sum_{i=1}^{N_\nu} \Omega_\nu^{(i)} t^2 = 0.106 \pm 0.008\) at the 3\(\sigma\) level for \(m_\nu = 130, 170, 210\) GeV (as indicated above each panel).

where the unitary matrices \(S_{L,R,\nu R}\) obey
\[
S_L^T Y_L S_R = Y_l, \quad S_R^T Y_R S_{\nu R} = Y_\nu, \quad S_{\nu R}^T M S_{\nu R} = +M, \quad S_{\nu R}^T Y_\varphi S_{\nu R} = -Y_\varphi
\]
(11)

In order to determine the consequences of this symmetry we find it convenient to adopt the basis in which \(M\) and \(Y_l\) are real and diagonal; for simplicity we will also assume that \(M\) has no degenerate eigenvalues. Then the last two conditions in (11) imply that \(S_{\nu R}\) is real and diagonal, so its elements are \(\pm 1\). For 3 neutrino species there are then two possibilities (up to permutations of the basis vectors): we either have \(S_{\nu R} = \pm 1\), \(Y_\varphi = 0\), or, more interestingly,
\[
S_{\nu R} = \epsilon \text{ diag}(1, 1, -1); \quad Y_\varphi = \begin{pmatrix} 0 & 0 & b_1 \\ 0 & 0 & b_2 \\ b_1 & b_2 & 0 \end{pmatrix}, \quad \epsilon = \pm 1,
\]
(12)
where \(b_{1,2}\) are, in general, complex. The first conditions in (11) now requires \(S_{\nu R} = S_L\) with
\[
S_L = \text{ diag}(s_1, s_2, s_3), \quad |s_i| = 1
\]
(13)
Before discussing the explicit solutions for \(Y_\nu\), we first diagonalize (to leading order in \(M^{-1}\)) the neutrino mass matrix in terms of the light \((n)\) and heavy \((N)\) eigenstates:
\[
\mathcal{L}_m = -(\bar{n} M_n n + \bar{N} M N / 2) \quad \text{with} \quad M_n = \mu^* P_R + \mu P_L, \quad \mu = -4 M_D M^{-1} M_D^T
\]
(14)
where \(n\) and \(N\) are related to \(\nu_R\) and \(\nu_L\) through \(\nu_L = n_L + (M_D M^{-1}) N_L\) and \(\nu_R = N_R - (M^{-1} M_D^T) n_R\).

The remaining condition in (11) allows ten (up to permutations of the basis vectors) inequivalent solutions for \(Y_\nu\). Of those, assuming single massless neutrino and the absence of \(\varphi \to n, n_j\) decays, only one is acceptable; it corresponds to \(s_{1,2,3} = \epsilon\) (cf. (12)). To compare our results with the data, we use the so-called tri-bimaximal (11) lepton mixing matrix that corresponds to \(\theta_{13} = 0, \theta_{23} = \pi/4\) and \(\theta_{12} = \arcsin(1/\sqrt{3})\). One can undo the diagonalization of light neutrino mass matrix and check against the one implied by \(Y_\nu\) as a consequence of (11). We find that there are only two possible forms of \(Y_\nu\) that are consistent with (11) and independent of \(M\), and that agree with tri-bimaximal mixing:
\[
Y_\nu = \begin{pmatrix} a & b & 0 \\ -a/2 & b & 0 \\ -a/2 & b & 0 \end{pmatrix}, \quad m_1 = -3v^2 a^2 / M_1 \quad m_2 = -6v^2 b^2 / M_2 \quad \text{and} \quad Y_\nu = \begin{pmatrix} a & b & 0 \\ -a/2 & b/2 & 0 \\ a & -b/2 & 0 \end{pmatrix}, \quad m_1 = -3v^2 b^2 / M_1 \quad m_2 = -6v^2 a^2 / M_2
\]
(15)
where \(a\) and \(b\) are real (for simplicity) parameters. The resulting mass spectrum is consistent with the observed pattern of neutrino mass differences, see e.g. (13). For this solution only \(N_3\) and \(\varphi\) are odd under the \(Z_2\) symmetry hence the \(\varphi\) will be absolutely stable if \(m < M_3\).

\(^{5}\) The conditions (11) were also investigated in (12).
It is noteworthy that the presence of $Y_\phi$ also leads to an additional contribution $-(\Lambda/\pi)^2 \text{tr} Y_\phi^2$ to $\delta m^2$ (we assumed $Y_\phi$ real for simplicity) so the neutrinos can be used to ameliorate the little hierarchy problem associated with $m$ (for this however $Y_\phi$ cannot be too small) thereby “closing” the solution to the little hierarchy problem in a spirit similar to supersymmetry. This interesting scenario will be discussed elsewhere [4].

**Conclusions**  We have shown that the addition of real scalar singlets $\varphi_i$ to the SM may ameliorate the little hierarchy problem (by lifting the cutoff $\Lambda$ to multi TeV range) and also provide realistic candidates for DM. In the presence of right-handed neutrinos this scenario allows a light neutrino mass matrix texture that is consistent with experimental data while preserving all the successes of leptogenesis as an explanation for the baryon asymmetry.

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