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A GPU Based Explicit Solid-Shell Finite Element Solver

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Abstract. In this work we present a co-rotational/updated Lagrangian, strain-rate based explicit finite element code which uses hexahedral solid-shell tri-linear elements, intended for simulation of the incremental sheet forming (ISF) process. This element is based heavily on the elements described in [1, 2, 3]: it is under-integrated with a single stack of stress integration points in the thickness direction passing through the elements center; it uses Assumed Natural Strain (ANS) interpolates for the thickness and transverse shear strains; it uses a single parameter Enhanced Assumed Strain (EAS) for the thickness strain; and it selectively scales the mass in the through thickness direction to increase the stable time-step. We combine these methods with a hypo-elastic constitutive model to simulate the ISF process. Initial results obtained with a GPU implementation of the element are presented.

1. Introduction
Translation based solid shell-shell elements were first introduced by [4] to overcome a number of problems associated with degenerate shell elements; e.g., the complicated update required for the rotational degrees of freedom in geometrically non-linear problems, the difficulties in connecting degenerate shell elements to solid elements, and the lack of compatibility with fully three-dimensional material laws, to list a few. The solid-shell elements, however, (re-)introduced a number of inadequacies found in standard brick elements, primarily associated with different modes of locking, which required special treatment if results comparable to those generated using degenerate shell elements are to be obtained. Since then, there has been a significant amount of work dedicated to the development of solid shell elements for both implicit and (more recently) explicit simulation of (non-linear) problems involving large deformations (See [5] for a good summary). Common to most of these elements is the use of assumed natural strain (ANS) interpolants to remove transverse shear locking and curvature thickness locking, and an enhanced assumed strain (EAS) method to compensate for Poisson thickness and volumetric locking. Most of these elements are also formulated in terms of the second Piola-Kirchhoff stress and Green-Legrange strain, and use hyper-elastic constitutive models.

In this work we present an explicit solid-shell element formulated in terms of the co-rotational rate of deformation. This element is based heavily on the elements of [1, 2, 3]: it uses ANS interpolants for the covariant thickness and transverse shear (rate-of-) strains, and a single param-
eter enhanced thickness (rate-of) strain. Unlike these elements, however, the current element accounts for geometric non-linearity by accumulating the stresses and strains in a co-rotating coordinate system using a hypoelastic constitutive model. While introducing little overhead into the strain calculation — as we now accumulate the strain via the co-rotational rate-of-deformation — the constitutive update is greatly simplified (compared to hyperelastic solid shell models) while still maintaining a formulation appropriate for large deformation problems.

After outlining the element formulation in Section 2, as a first test of the proposed element we reproduce an elastic ring-plate benchmark [6], as well as simulate a simple incremental sheet forming (ISF) problem in Section 3. As the current element formulation is explicit, it lends to natural parallelisation. In the present case simulations were performed using a CUDA (GPU) based implementation of the proposed element. Final remarks are given in Section 4.

2. Shell Formulation

2.1. Shell Geometry

We begin with a standard tri-linear hexahedral brick element which maps to the $2 \times 2 \times 2$ isoparametric cube with coordinates $(\xi, \eta, \zeta)$, centered at $(0, 0, 0)$. The direction of $\zeta$ is identified with the fiber direction of the sheet. The element has a single stack of stress integration points lying on the line $\xi = \eta = 0$. For each element we define an orthonormal (Cartesian) co-rotational coordinate system with origin at the elements center.

2.2. Element Strains

In this work we adopt the (co-rotational) rate-of-deformation as the measure of each element’s strain

$$D := \frac{1}{2} \left( \nabla v + (\nabla v)^T \right), \quad (1)$$

where $v$ is the (interpolated) element velocity. In this expression both the velocity and the gradient are defined with respect to the co-rotational system. The rate-of-deformation defined with respect to the convective coordinates $g_i = \partial x / \partial \xi_i$, where $x$ is the position defined in the co-rotational system, is calculated as

$$\tilde{D} = J^T \cdot D \cdot J, \quad (2)$$

where $J$ is the Jacobian of the transformation $x = x(\xi)$, which is written as a polynomial in $\xi$, $\eta$ and $\zeta$ as (see e.g [2])

$$J = J_0 + \xi_i J_{\xi_i} + \xi_j \xi_i J_{\xi_i \xi_j}. \quad (3)$$

With $D$ and $\tilde{D}$ in Voigt notation

$$\{D\} = \begin{bmatrix} D_{xx} & D_{xy} & 2D_{xz} & 2D_{yz} & 2D_{xy} \end{bmatrix}^T, \quad (4)$$

$$\{\tilde{D}\} = \begin{bmatrix} D_{11} & D_{22} & D_{33} & 2D_{12} & 2D_{23} & 2D_{13} \end{bmatrix}^T \quad (5)$$

the transformation between the Cartesian and covariant rate-of-deformation can be written

$$\{D\} = T \{\tilde{D}\}, \quad (6)$$

where the transformation matrix $T$ is a function of $J^{-1}$, and is given in [2]. Presently, good results have been obtained by considering only the constant part of $T$, and thus all other components of $T$ are ignored. We hence only need to calculate the constant part of the inverse
Jacobian. Note, however, that we still require the components of (3) for the calculation of $\tilde{D}$.

As with previous elements, to improve the element for the simulation of thin structures, where transverse shear and curvature thickness locking causes poor element performance, we adopt the assumed natural strain (ANS) interpolants for the (co-variant) thickness and transverse shear strains (see e.g. [3]). Additionally, we adopt a single parameter enhanced assumed strain (EAS) method for the thickness strain to avoid Poisson thickness and volumetric locking. The enhanced strain takes the form

$$\{\tilde{D}_e\} = \begin{bmatrix} 0 & 0 & \zeta & 0 & 0 & 0 \end{bmatrix}^T W_e,$$

which is added to (5) before the constitutive update. The enhancement parameter $W_e$ is calculated using the explicit update procedure of [3]. Currently, this update is performed every time-step.

2.3. Constitutive model
Element stresses are accumulated using a hypoelastic constitutive model. Results presented in Section 3.2 were generated using a von-Mises yield criterion with yield stress $\sigma_{\text{yield}}$, with an implicit return mapping algorithm. A Hollomon power-law of the form

$$\sigma_{\text{yield}} = \alpha \mathcal{E}_\text{eff}^\beta,$$

where $\mathcal{E}_\text{eff}$ is the effective plastic strain, was used for the isotropic (work) hardening. No kinematic hardening was considered. See [7] for details.

2.4. Contact
For contact we use a node-to-surface penalty stiffness model [8].

2.5. Selective mass scaling
To increase the stable time-step, we selectively scale the mass in the through thickness direction [9]. This solid-shell element differs from that of [3] in that the mass scale factor is chosen such that when the smallest element dimension is in the thickness direction, the maximum stable time-step is modified to match the stable time-step corresponding to the in-plane element dimensions only. Additionally, each element’s thickness mass scaling factor is replaced by the maximum thickness scaling factor over the entire sheet.

3. Results
In this section we present a first set of results generated using the proposed solid-shell element. For reference, we also generate results using the above element without the ANS strain interpolants, with (1) and (7) evaluated directly in the (Cartesian) co-rotational system. Below we name this the modified brick element.

3.1. Non-dimensional Elastic Cantilever Ring-Plate
In this section we reproduce an elastic ring-plate benchmark [6]. For this benchmark, all values given are dimensionless and care should be taken to apply the correct scaling when dimensional values are used. The geometry of this problem is described by a circular annulus of inner radius 6.0, outer radius 10.0 and thickness 0.03. The annulus is cut along a single radial line [which corresponds to the discontinuity in the sheet shown in Figure 1(b)]. On one of the straight edges of this cut the sheet is clamped. On the other (initially straight) edge of the cut a vertical line load (perpendicular to the plane of the unloaded plate) of $F = \lambda \cdot 100$ per unit length is
distributed evenly across the section area. This force does not change direction as the sheet deforms. The loading parameter $\lambda$ is varied from 0 to 2.0. The material parameters are Elastic modulus $E = 2.1 \times 10^{10}$ and Poisson ratio $\nu = 0.0$. The benchmark values are the displacements of the end points of the loaded edge. We take these values on the mid-surface of the shell. Again, we note that the values for this benchmark are dimensionless and that any choice of consistent units should not impact the results.

| $\lambda$ | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 | 1.2 | 1.4 | 1.6 | 1.8 | 2.0 |
|-----------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| pt.1      | 1.7895 | 3.3823 | 4.7427 | 5.9047 | 6.9039 | 7.7698 | 8.5266 | 9.1945 | 9.7895 | 10.324 |
| pt.2      | 1.3071 | 2.4659 | 3.4547 | 4.3019 | 5.0359 | 5.6788 | 6.2478 | 6.7569 | 7.2168 | 7.6358 |

Table 1. Tip displacements for the non-dimensional elastic ring-plate benchmark.

Shown in Figure 1 are the results obtained with the current solid shell implementation. These results compare well with those of [6, 10, 11, 12] (also shown). For reference, the displacements are tabulated in Table 1. Here we label the end point of the loaded edge lying on the inner radius pt.2, and the end point of the loaded edge lying on the outer radius pt.1. Note that these results were generated in double precision, and could not be reproduced in single precision.

![Load-displacement curves](a) Load-displacement curves  ![Initial configuration (wire-frame) and final-configuration (shaded contour plot) for maximum load factor.](b) Initial configuration (wire-frame) and final-configuration (shaded contour plot)

**Figure 1.** Results for the non-dimensional cantilever ring-plate benchmark. In (b) the colour scale indicates vertical displacement and varies from $-1.5$ to 11.

### 3.2. ISF part

The target geometry and tool-path for the simulated ISF part are shown in Figure 2. This part was formed on a $250mm \times 250mm \times 0.063'' Al2024-O$ sheet. The tool had a $20mm$ diameter spherical head, with the tool-path comprised of $z$-levels traversed in an alternating clockwise/counter-clockwise direction. The vertical step-down between contours was $2mm$, and the tool-speed was $4000mm/minute$. The toolpath was truncated at a $z$-level $5\%$ above the part minimum. The material parameters used are given in Table 2, with yield stress and isotropic hardening data taken from [13]. For the yield stress, we used the value obtained at $0^\circ$ to the rolling direction. The process was simulated using on a mesh constructed from a grid of $63 \times 63$ uniformly spaced nodes.

![Reference Elements](b) Initial configuration (wire-frame) and final-configuration (shaded contour plot) for maximum load factor.

| Parameter | Reference Elements |
|-----------|--------------------|
| Buechter 1992 (49x7x1 nodes) | |
| Sze 2004 (81x11x1 nodes) | |
| Valente 2003 (41x7x1 nodes) | |
| Hokkanen 2018 (S7 - 49x7x1 ctrl pts) | |
Figure 2. Sample part geometry and tool-path. Note the geometry is left-right and top-bottom symmetric, and the outer curve is $C_1$ continuous.

Figure 3. Vertical displacement (mm) profiles for the simulated ISF part.

The numerically generated mid-plane geometries produced by the two solvers are shown in Figures 3 and 4, with the former shaded to show vertical displacement, and the latter shaded to show the predicted thickness.

Table 2. Material parameters for the ISF simulation. Note that the density $\rho$ corresponds to a 100× mass scaling.

|     | $E$   | $\nu$ | $\rho$ (density) | $\sigma_{\text{yield}}$ | $\alpha$ | $\beta$ |
|-----|-------|-------|------------------|-----------------|----------|---------|
|     | 71700 MPa | 0.33  | $2.81 \times 10^{-7}$ tonnes/mm$^3$ | 261 MPa | 334.590  | 0.157   |

4. Conclusions

The proposed element is shown to both reproduce a standard benchmark problem and give a physically plausible prediction for the shape and thickness distribution of an incrementally formed part. We note, however, that the practical advantage of the current element over the much simplified element given above is not clear from the examples considered, and a more thorough benchmarking is required.
Figure 4. Thickness (mm) profiles for the simulated ISF part.

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