Partonic Pole Matrix Elements for Fragmentation

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A model-independent analysis of collinear three-parton correlation functions for fragmentation is performed. By investigating their support properties it is shown, in particular, that the so-called partonic pole matrix elements vanish. This sheds new light on the understanding of transverse single spin asymmetries (SSAs) for which effects of up to 40% were observed in hard hadronic reactions. Moreover, it gives additional strong evidence for the universality of transverse-momentum-dependent fragmentation functions.

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I. Introduction and definitions. — QCD factorization theorems separate the cross section for a large number of hard processes into a perturbatively calculable part as well as nonperturbative correlation functions [1]. These correlation functions are parton distributions (PDFs), fragmentation functions (FFs), etc. [2, 3]. So far the correlation functions are parton distributions (PDFs), fragmentation functions (FFs), etc. [2, 3]. So far the correlation functions are parton distributions (PDFs), fragmentation functions (FFs), etc. [2, 3].

Nonzero PPMEs for fragmentation would contribute to a large number of transverse SSAs. By using simple models for such three-parton matrix elements pioneering phenomenological studies on the processes \( p^i p \rightarrow \pi X \), \( pp \rightarrow \Lambda^i X \), \( e^i p \rightarrow e \pi X \), and \( e^i p \rightarrow e \Lambda^i X \) were carried out in [14, 13]. Other work discussed PPMEs for fragmentation in connection with \( p^i p \rightarrow \pi \pi X \), \( p^i p \rightarrow \Lambda \pi X \) [10, 11], as well as \( pp \rightarrow (\Lambda^i \text{jet}) X \) with a Λ hyperon detected in a jet [18]. Note that this already long list of semi-inclusive reactions, which might be related to PPMEs for fragmentation, is not yet complete.

Despite the potential importance of PPMEs for fragmentation it was, however, so far not even clear whether they exist. In fact, recent spectator model calculations imply (show) that these objects vanish [19, 20, 21]. In this Letter it is proven for the first time in a fully model-independent way that PPMEs for fragmentation vanish, which constitutes the main outcome of our work. To obtain this result we analyze the support properties of collinear twist-3 three-parton fragmentation correlators.

Now in order to be more specific we consider essentially two kinds of correlators in fragmentation: the quark-gluon-quark \((qgq)\) correlator as well as the gluon-gluon-gluon \((ggg)\) correlator. The \((qgq)\) correlator (see also Fig. 1) is defined by

\[ \Delta_F \left( \frac{1}{v_X}, \frac{1}{v_T} \right) = \sum_X \int \frac{d\xi^+}{2\pi} \frac{d\eta^+}{2\pi} \, e^{i \frac{\xi}{2} P^+ - i \frac{1}{2} (\frac{1}{v_T})^P - \eta^+} \times \frac{1}{3} \text{Tr} \left[ \langle 0 | g t_a F^{-\dagger} \psi \sigma^{+} \psi (\xi^+) | P, X \rangle \right. \]

where \( v^\pm = (v^0 \pm v^3) / \sqrt{2} \) and \( \sigma_T = (v^1, v^2) \) are the light-cone components of a generic 4-vector \( v = (v^0, v^1, v^2, v^3) \) and \( P^- \) in Eq. (1) is the (large) minus momentum of the hadron in the final state. In Fig. 1 the minus momenta of the three partons are given. The correlator in Eq. (1) is a matrix in Dirac space, and the trace is acting in color space. We make use of the light-cone gauge \( A^- = 0 \), in which Wilson lines between the field operators reduce to
Eqs. (1) and (2) vanish if the longitudinal (minus) momenta have to satisfy \( p^− = 0 \) for all particles in the intermediate states, i.e., the momenta of the one we are using. In the first case they are \( \sum p_i^− = 0 \), and in the second case \( \sum p_i^− = \sum q_j^− \geq 0 \). This means, however, that the \( \delta \) functions in Eq. (3) only give a nonzero contribution if \( \frac{1}{2} \geq \frac{1}{3} \) and \( \frac{1}{2} \geq \frac{1}{3} \). The case \( \frac{1}{2} = \frac{1}{3} \) is now of particular interest because it corresponds to a PPME. From Eq. (3) one finds that here the second \( \delta \) function only contributes if \( q_j^− = 0 \) for all \( j \), which leads to \( q_j^+ = -q_j^− \). Combined with the on-shell condition \( q_j^2 = m_j^2 \geq 0 \) it immediately follows that all particles in the intermediate state \( |Y⟩ \) have to be massless and move only in the plus direction, i.e., \( q_j^− = q_j^+ = 0 \) for all \( j \), in order to obtain a nonvanishing contribution to the \( qgq \) correlator. In this case, however, the matrix element

\[
M^{−i}(q_j) = \langle 0 | g t_a F^{−i}_a (0^+) | Y⟩
\]

vanishes. This matrix element is an antisymmetric Lorentz tensor, which can be expressed in terms of the Lorentz vectors it depends on—the momenta \( q_j \) and the polarization vectors \( \epsilon(q_j) \) of the massless particles in the intermediate state—multiplied by some scalar functions. One possible decomposition of the matrix element in Eq. (5) with general Lorentz indices is

\[
M^{\mu \nu}(q_j) = \sum_{m,n} [q^\mu_m q^\nu_n A_{mn}(q_j) + q^\mu_m \epsilon^\nu(q_n) B_{mn}(q_j)] + \epsilon^\mu(q_m) \epsilon^\nu(q_n) C_{mn}(q_j) - \{\mu \leftrightarrow \nu\}. \tag{6}
\]
However, for massless, on-shell particles moving in the plus direction the minus components of the polarization vectors $e(q_j)$ have to vanish because they correspond to some combination of the unphysical timelike and longitudinal polarization components. Therefore, none of the terms in Eq. (1) can contribute to the matrix element $M^{-1}$ in Eq. (3) as we have $q_j^\mu = q_j^\nu = e^\nu(q_j) = 0$ for all $j$. This means that finally the $qgq$ correlator in Eq. (3) vanishes if all particles in the intermediate state vanish according to Eqs. (11)–(13). We repeat that the PPMEs for fragmentation, therefore, cannot be responsible for single spin asymmetries as they all vanish according to Eqs. (11)–(13).

The vanishing of gluonic pole terms in Eq. (6) can contribute to the matrix element of $\Delta$ vanishing which agrees with the well-established existence of relators on the distribution side can be found by a corresponding analysis. In that case, however, PPMEs do not vanish which agrees with the well-established existence of these objects. The key difference between both analyses is the absence of the vacuum state on the PDF side.

III. Universality of transverse-momentum-dependent fragmentation functions. — The vanishing of gluonic pole matrix elements (GPMEs) for fragmentation is intimately connected with the universality of transverse-momentum-dependent two-parton fragmentation functions (TMD FFs) [17, 22]. To illustrate the relation in more detail we consider the correlator

$$\Delta^{[R]}(\frac{1}{z}, \vec{k}_T) = \sum_X \int \frac{d^2 \vec{k}_T}{2\pi} \frac{d^2 \vec{p}_i}{(2\pi)^2} e^{i k \cdot \vec{p}_i} \frac{1}{3} \text{Tr}\left[0| W^{[R]}(0, \xi) \times \psi(\xi) | P, X \right) \langle P, X | \tilde{\psi}(0) | 0 \rangle \right]_{\xi = 0}.$$  

which defines TMD FFs when appropriate traces in Dirac space are taken [23, 24]. At leading twist there exist eight FFs. (We restrict the explicit discussion here to the quark FFs, but our general conclusions apply to gluon FFs as well.) The TMD FFs depend both on the longitudinal momentum $k^- = P^- - z$ and the transverse momentum $\vec{k}_T$ (relative to the detected hadron) of the fragmenting quark. The Wilson line $W^{[R]}$ cannot be neglected in any gauge [23, 26], and its path $U(0; \xi)$ is determined by the physical process under consideration [22, 27, 28]. More details about the precise definition of TMD correlators and, in particular, the choice of Wilson lines can be found in [29, 31, 31] and references therein.

The path dependence of the correlator in Eq. (14) implies a potential nonuniversality of TMD FFs. For instance, in semi-inclusive deep inelastic scattering (DIS) one has, a priori, past-pointing Wilson lines, while they are future pointing in $e^+ e^-$ annihilation. In Ref. [20] it was argued, however, that factorization can be established such that TMD FFs have the same Wilson lines in both processes. But the analysis in [20] made use of a spectator model and was not carried out to arbitrary order in perturbation theory. As a consequence, in the community doubts remained concerning the generality of this result.

In the following we consider $k_T$ moments of the correlator in Eq. (14). We define the zeroth moment by

$$\Delta(\frac{1}{z}) = \int d^2 \vec{k}_T \Delta^{[R]}(\frac{1}{z}, \vec{k}_T).$$

In this case the $k_T$ integration eliminates all the path dependence implying, in particular, that the three leading twist collinear FFs [22, 24], which are given by $\Delta$ in Eq. (15), are universal. For the more interesting case of the first $k_T$ moment of the correlator in Eq. (14) one finds [17, 22]

$$\Delta^{[R]}(\frac{1}{z}) = \frac{2}{\pi} \int d^2 \vec{k}_T \frac{d^2 \vec{k}_T}{(2\pi)^2} e^{i k \cdot \vec{p}_i} \frac{1}{3} \text{Tr}\left[0| W^{[R]}(0, \xi) \times \psi(\xi) | P, X \right) \langle P, X | \tilde{\psi}(0) | 0 \rangle \right]_{\xi = 0}.$$ 

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Here $\tilde{A}^i_j$ is the universal part, while the second term on the rhs in Eq. (10) contains all the potential nonuniversal behavior of the correlator. The latter is given by a GPME multiplied with the calculable and path-dependent so-called gluonic pole factor $g_F^{[d]}$. According to Eq. (11) the GPME vanishes, and, therefore, we conclude that the first moment of the TMD correlator in Eq. (11) is universal as well. This leads to universal second $k_T$ moments of four specific leading twist TMD FFs, where currently the Collins FF represents the most important one. This universality is a prerequisite for a combined analysis of transverse-momentum-dependent data from semi-inclusive DIS and $e^+ e^-$ annihilation [22, 23], and, in particular, for the first extraction of the transversity distribution of the nucleon [24].

We repeat that universality of TMD FFs was already obtained previously in low order spectator model calculations [19, 20, 34]. However, so far no model-independent proof has been given. Therefore, our result, even though the present analysis is restricted to certain $k_T$ moments of FFs, provides the first strong model-independent support for the universality of TMD FFs.

IV. Conclusions. We have proved in a model-independent way that the so-called partonic pole matrix elements for fragmentation vanish. Therefore, in contrast to conjectures in the literature, transverse single spin asymmetries in hard semi-inclusive reactions are not related to such objects. Our finding does not mean that single spin asymmetries, in general, cannot be connected to collinear fragmentation correlators. This topic requires further investigation.

The vanishing of the partonic pole matrix elements also implies that certain $k_T$ moments of transverse-momentum-dependent fragmentation functions are universal. This result confirms earlier studies, and represents the first fully general and model-independent proof of this kind. Hence, we now have additional strong evidence for the universality of transverse-momentum-dependent fragmentation functions. For the future one might hope that higher $k_T$ moments of fragmentation functions can be analyzed along the lines presented in this Letter.

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