A formal model of Algorand smart contracts

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Abstract. We develop a formal model of Algorand stateless smart contracts (stateless ASC1). We exploit our model to prove fundamental properties of the Algorand blockchain, and to establish the security of some archetypal smart contracts. While doing this, we highlight various design patterns supported by Algorand. We perform experiments to validate the coherence of our formal model w.r.t. the actual implementation.

1 Introduction

Smart contracts are agreements between two or more parties that are automatically enforced without trusted intermediaries. Blockchain technologies reinvented the idea of smart contracts, providing trustless environments where they are incarnated as computer programs. However, writing secure smart contracts is difficult, as witnessed by the multitude of attacks on smart contracts platforms (notably, Ethereum) — and since smart contracts control assets, their bugs may directly lead to financial losses.

Algorand \cite{21} is a late-generation blockchain that features a set of interesting features, including high-scalability and a no-forking consensus protocol based on Proof-of-Stake \cite{9}. Its smart contract layer (ASC1) aims to mitigate smart contract risks, and adopts a non-Turing-complete programming model, natively supporting atomic sets of transactions and user-defined assets. These features make it an intriguing smart contract platform to study.

The official specification and documentation of ASC1 consists of English prose and a set of templates to assist programmers in designing their contracts \cite{1,5}. This conforms to standard industry practices, but there are two drawbacks:

1. Algorand lacks a mathematical model of contracts and transactions suitable for formal reasoning on their behaviour, and for the verification of their properties. Such a model is needed to develop techniques and tools to ensure that contracts are correct and secure;
2. furthermore, even preliminary informal reasoning on non-trivial smart contracts can be challenging, as it may require, in some corner cases, to resort to experiments, or direct inspection of the platform source code.

Given these drawbacks, we aim at developing a formal model that:
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|  |  |
|---|---|
| Users (key pairs) | Transactions in last 1000 rounds |
| \( a, b, \ldots \in X \) | \( T_{E} \subseteq T \) |
| Addresses | Asset manager |
| \( x, y, \ldots \in X \) | \( f_{\text{mngr}} \in A \rightarrow X \) |
| Assets | Lease map |
| \( \tau, \tau', \ldots \in A \) | \( f_{\text{mngr}} \in (X \times N) \rightarrow N \) |
| Values | Freeze map |
| \( v, v', w, \ldots \in Z \) | \( f_{\text{lsr}} \in X \rightarrow 2^{A} \) |
| Balances | Blockchain states |
| \( \sigma, \sigma' \in A \rightarrow N \) | \( \Gamma, \Gamma', \ldots \) |
| Accounts | Valid balance |
| \( \tau, \tau', \ldots \in T \) | \( \sigma \) |
| Transactions | Valid time constraint |
| \( v, v', \ldots \in X \) | \( f_{\text{frz}} \in X \rightarrow 2^{A} \) |
| Scripts | Authorized transaction in group |
| \( r, r', \ldots \in N \) | \( W \mid T_{i}^{W} \) |
| Rounds | Script evaluation |

Table 1: Summary of notation.

o1. is high-level enough to simplify the design of Algorand smart contracts and enable formal reasoning about their security properties;
o2. expresses Algorand contracts in a simple declarative language, similar to PyTeal (the official Python binding for Algorand smart contracts) [7];
o3. provides a basis for the automatic verification of Algorand smart contracts.

Contributions. This paper presents:

– a formal model of stateless ASC1 providing a solid theoretical foundation to Algorand smart contracts (§2). Such a model formalises both Algorand accounts and transactions (§2.1–§2.4, §2.6), and smart contracts (§2.5);
– a validation of our model through experiments [6] on the Algorand platform;
– the formalisation and proof of some fundamental properties of the Algorand state machine: no double spending, determinism, value preservation (§2.7);
– an analysis of Algorand contract design patterns (§3.2), based on several non-trivial contracts (covering both standard use cases, and novel ones). Quite surprisingly, we show that stateless contracts are expressive enough to encode arbitrary finite state machines;
– the proof of relevant security properties of smart contracts in our model;
– a prototype tool that compiles smart contracts (written in our formal declarative language) into executable TEAL code (§4).

Our model is faithful to the actual ASC1 implementation. Still, for the sake of clarity, we introduce minor high-level abstractions over low-level details. For instance, since TEAL code has the purpose of accepting or rejecting transactions, we model it using expressions that evaluate to true or false. We discuss the differences between our model and the actual Algorand platform in (§5).

2 The Algorand state machine

This section introduces our formal model of the Algorand blockchain, including its smart contracts (stateless ASC1). We present the model incrementally: we first define the basic transactions that generate and transfer assets (§2.1–§2.3), and then add atomic groups of transactions (§2.4), smart contracts (§2.5), and authorizations (§2.6). We discuss the main differences between our model and Algorand in §5.
2.1 Accounts and transactions

We use $a, b, \ldots$ to denote public/private key pairs $(k^p_a, k^s_a)$. Users interact with Algorand through pseudonymous identities, obtained as a function of their public keys. Hereafter, we freely use $a$ to refer to the public or the private key of $a$, or to the user associated with them, relying on the context to resolve the ambiguity.

The purpose of Algorand is to allow users to exchange assets $\tau, \tau', \ldots$. Besides the Algorand native cryptocurrency $\text{Algo}$, users can create custom assets.

We adopt the following notational convention:

- lowercase letters for single entities (e.g., a user $a$);
- uppercase letters for sets of entities (e.g., a set of users $A$);
- calligraphic uppercase letters for sequences of entities (e.g., list of users $A$).

Given a sequence $L$, we write $|L|$ for its length, $\text{set}(L)$ for the set of its elements, and $L.i$ for its $i^{th}$ element ($i \in 1..|L|$); $\varepsilon$ denotes the empty sequence. We write:

- $\{x \mapsto v\}$ for the function mapping $x$ to $v$, and having domain equal to $\{x\}$;
- $f \{x \mapsto v\}$ for the function mapping $x$ to $v$, and $y$ to $f(y)$ if $y \neq x$;
- $f \{x \mapsto \perp\}$ for the function undefined at $x$, and mapping $y$ to $f(y)$ if $y \neq x$.

Accounts. An account is a deposit of one or more crypto-assets. We model accounts as terms $x[\sigma]$, where $x$ is an address uniquely identifying the account, and $\sigma$ is a balance, i.e., a finite map from assets to non-negative integers. In the concrete Algorand, an address is a 58-characters word; for mathematical elegance, in our model we represent an address as either:

- a single user $a$. Performing transactions on $a[\sigma]$ requires $a$’s authorization;
- a pair $(A, n)$, where $A$ is a sequence of users, and $1 \leq n \leq |A|$, are multisig (multi-signature) addresses. Performing transactions on $(A, n)[\sigma]$ requires that at least $n$ users out of those in $A$ grant their authorization;$^5$
- a script$^6$ $e$. Performing transactions on $e[\sigma]$ requires $e$ to evaluate to true.

Each balance is required to own $\text{Algo}$, have at least 100000 micro-$\text{Algo}$s for each owned asset, and cannot control more than 1000 assets. Formally, we say that $\sigma$ is a valid balance (in symbols, $\models \sigma$) when:\$^7$

$$\text{Algo} \in \text{dom}(\sigma) \land \sigma(\text{Algo}) \geq 100000 \cdot |\text{dom}(\sigma)| \land |\text{dom}(\sigma)| \leq 1001$$

Transactions. Accounts can append various kinds of transactions to the blockchain, in order to, e.g., alter their balance or set their usage policies. We model transactions as records with the structure in Fig. 1. Each transaction has a type, which determines which of the other fields are relevant.$^8$ The field

$^5$ W.l.o.g., we consider a single-user address $a$ equivalent to $(A, n)$ with $A = \langle a \rangle$, $n = 1$.

$^6$ We formalize scripts (i.e., smart contracts) later on, in §2.5.

$^7$ Since the codomain of $\sigma$ is $\mathbb{N}$, the balance entry $\sigma(\text{Algo})$ represents micro-$\text{Algo}$s.

$^8$ In Algorand, the actual behaviour of a transaction may depend on both its type and other conditions, e.g., which optional fields are set. For instance, pay transactions may also close accounts if the CloseRemainderTo field is set. For the sake of clarity, in our model we prefer to use a richer set of types; see §5 for other differences.
open $\text{rcv, val}$ $\text{snd}$ creates account $\text{rcv}$, transferring $\text{val}$ Algo

pay $\text{rcv, val, asst}$ $\text{snd}$ transfers $\text{val}$ units of $\text{asst}$ to $\text{rcv}$

close $\text{rcv, asst}$ $\text{snd}$ gives $\text{asst}$ to $\text{rcv}$ and removes it (if Algo, closes $\text{snd}$)

gen $\text{rcv, val}$ $\text{snd}$ mints $\text{val}$ units of a new asset, managed by $\text{rcv}$

optin $\text{asst}$ $\text{snd}$ opts in to receive units of asset $\text{asst}$

burn $\text{asst}$ as $\text{asst}$ is removed from $\text{snd}$ (if it is the creator and sole owner)

gvk $\text{rcv, val}$ as $\text{asst}$’s manager transfers $\text{val}$ units of $\text{asst}$ to $\text{rcv}$

frz $\text{asst}$ as $\text{asst}$’s manager freezes $\text{snd}$’s use of asset $\text{asst}$

unfrz $\text{asst}$ as $\text{asst}$’s manager unfreezes $\text{snd}$’s use of asset $\text{asst}$

delegate $\text{asst, rvk}$ as $\text{asst}$’s manager delegates $\text{asst}$ to new manager $\text{rcv}$

Fig. 1: Transaction types (fields type, snd, fv, lv, lx are common to all types.)

snd usually refers to the subject of the transaction (e.g., the sender in an assets transfer), while rcv refers to the receiver in an assets transfer. The fields asst and val refer, respectively, to the affected asset, and to its amount. The fields fv (“first valid”), lv (“last valid”) and lx (“lease”) are used to impose time constraints.

Algorand groups transactions into rounds $r = 1, 2, \ldots$. To establish when a transaction $t$ is valid, we must consider both the current round $r$, and a lease map $f_{lx}$ binding pairs (address, lease identifier) to rounds: this is used to enforce mutual exclusion between two or more transactions (see e.g. the periodic payment contract in §3). Formally, we define the temporal validity of a transaction $t$ by the predicate $f_{lx}, r \models t$, which holds whenever:

$$
t_{.fv} \leq r \leq t_{.lv} \quad \text{and} \quad t_{.lv} - t_{.fv} \leq 1000 \quad \text{and} \quad \left( t_{.lx} = 0 \quad \text{or} \quad (t_{.snd}, t_{.lx}) \notin \text{dom}(f_{lx}) \quad \text{or} \quad r > f_{lx}(t_{.snd}, t_{.lx}) \right)
$$

First, the current round must lie between $t_{.fv}$ and $t_{.lv}$, whose distance cannot exceed 1000 rounds. Second, $t$ must have a null lease identifier, or the identifier has not been seen before (i.e., $f_{lx}(t_{.snd}, t_{.lx})$ is undefined), or the lease has expired (i.e., $r > f_{lx}(t_{.snd}, t_{.lx})$). When performed, a transaction with non-null lease identifier acquires the lease on $(t_{.snd}, t_{.lx})$, which is set to $t_{.lv}$.

2.2 Blockchain states

We model the evolution of the Algorand blockchain as a labelled transition system. A blockchain state $\Gamma$ has the form:

$$\begin{align*}
&x_1[\sigma_1] \mid \cdots \mid x_n[\sigma_n] \mid r \mid T_K \mid f_{mngr} \mid f_{lx} \mid f_{frz}
\end{align*}
$$

where all addresses $x_i$ are distinct, $|$ is commutative and associative, and:

- $r$ is the current round;
- $T_K$ is the set of transactions in the last 1000 rounds, used to avoid duplicates;
- $f_{mngr}$ is a map from assets to addresses of their assets managers;
- $f_{lx}$ is the lease map (from pairs (address, integer) to integers), used to ensure mutual exclusion between transactions;
- $f_{frz}$ is a map from addresses to sets of assets, used to freeze assets.
We define the initial state $I_0$ as $\sigma_0 = [\{\text{Algo} \rightarrow t_0\}] | 0 | \emptyset \mid f_{mogr} \mid f_{tx} \mid f_{frz}$, where $\text{dom}(f_{mogr}) = \text{dom}(f_{tx}) = \text{dom}(f_{frz}) = \emptyset$, $\sigma_0$ is the initial user address, and $r_0 = 10^{10}$ (which is the total supply of 10 billions Algos.)

We now formalize the ASC1 state machine, by defining how it evolves by single transactions (§2.3), and then including atomic groups of transactions (§2.4), smart contracts (§2.5), and the authorization of transactions (§2.6).

### 2.3 Executing single transactions

We write $I \xrightarrow{t} I'$ to mean: if the transaction $t$ is performed in blockchain state $I$, then the blockchain evolves to state $I'$.

We specify the transition relation $\xrightarrow{t}$ through a set of inference rules (see Fig. 4 in the Appendix for the full definition); each rule describes the effect of a transaction $t$ in the state $I$ of eq. (1). We now illustrate all cases, depending on the transaction type ($t$.type).

When $\tau \in \text{dom}(\sigma)$, we use the shorthand $\sigma + v: \tau$ to update balance $\sigma$ by adding $v$ units to token $\tau$; similarly, we write $\sigma - v: \tau$ to decrease $\tau$ by $v$ units:

$$\sigma + v: \tau \equiv \sigma \{\tau \mapsto \sigma(\tau) + v\} \quad \sigma - v: \tau \equiv \sigma \{\tau \mapsto \sigma(\tau) - v\}$$

**Open.** Let $t$.snd $= x_i$ for some $i \in 1..n$, let $t$.rcv $= y \notin \{x_1, \ldots, x_n\}$ (i.e., the sender account $x$ is already in the state, while the receiver $y$ is not), and let $t$.val $= v$. The rule has the following preconditions:

- c1. $t$ has not been observed in the last 1000 rounds ($t \notin T_K$);
- c2. the time interval of the transaction, and its lease, are respected ($f_{tx}, r \models t$);
- c3. the updated balance of $x_i$ is valid ($\models \sigma_i - v : \text{Algo}$);
- c4. the balance of the new account at address $y$ is valid ($\models \{\text{Algo} \rightarrow v\}$).

If these conditions are satisfied, the new state $I'$ is the following:

$$x_i[\sigma_i - v : \text{Algo}] \mid y[\{\text{Algo} \rightarrow v\}] \mid \cdots \mid r \mid T_K \cup \{t\} \mid f_{mogr} \mid \text{upd}(f_{tx}, t, r) \mid f_{frz}$$

In the new state, the Algo balance of $x_i$ is decreased by $v$ units, and a new account at $y$ is created, containing exactly the $v$ units taken from $x_i$. The balances of the other accounts are unchanged. The updated lease mapping is:

$$\text{upd}(f_{tx}, t, r) = \begin{cases} f_{tx}(\{t$.snd, $t$.lx$\}) \mapsto t$.lv & \text{if } t$.lx \neq 0 \\ f_{tx} & \text{otherwise} \end{cases}$$

Note that all transaction types check conditions c1 and c2 above; further, all transactions check that updated account balances are valid (as in c3 and c4.)

**Pay.** Let $t$.snd $= x_i$, $t$.rcv $= x_j$, $t$.val $= v$, and $t$.asst $= \tau$. Besides the common checks, performing $t$ requires that $x_j$ has “opted in” $\tau$ (formally, $\tau \in \text{dom}(\sigma_j)$), and $\tau$ must not be frozen in accounts $x_i$ and $x_j$ (formally, $\tau \notin f_{frz}(x_i) \cup f_{frz}(x_j)$).

If $x_i \neq x_j$, then in the new state the balance of $\tau$ in $x_i$ is decreased by $v$ units, and that of $\tau$ in $x_j$ is symmetrically increased by $v$ units:

9 Note that $I \xrightarrow{t} I'$ does not imply that transaction $t$ can be performed in $I$; in fact, $t$ might require an authorization. We specify the required conditions in §2.6.
all accounts but \( x_i \) and \( x_j \) are unchanged. Otherwise, if \( x_i = x_j \), then the balance of \( x_i \) is unchanged, and the other parts of the state are as above.

**Close.** Let \( t.\text{snd} = x_i \), \( t.\text{rcv} = x_j \neq x_i \), and \( t.\text{asst} = \tau \). Performing \( t \) has two possible outcomes, depending on whether \( \tau \) is \( \text{Algo} \) or a user-defined asset. If \( \tau = \text{Algo} \), we must check that \( \sigma_i \) contains only \( \text{Algos} \). If so, the new state is:

\[
x_{j}[\sigma_{j} + \sigma_{i}(\text{Algo})] \mid \cdots \mid r \mid T_K \cup \{t\} \mid f_{\text{mngr}} \mid \text{upd}(f_{\text{tx}}, t, r) \mid f_{\text{frz}}
\]

where the new state no longer contains the account \( x_i \), and all the \( \text{Algos} \) in \( x_i \) are transferred to \( x_j \). Instead, if \( \tau \neq \text{Algo} \), performing \( t \) requires to check only that \( x_i \) actually contains \( \tau \), and that \( x_i \) has “opted in” \( \tau \). Further, \( \tau \) must not be frozen for addresses \( x_i \) and \( x_j \), i.e. \( \tau \notin f_{\text{frz}}(x_i) \cup f_{\text{frz}}(x_j) \). The new state is:

\[
x_{i}[\sigma_{i}(\tau \mapsto \perp)] \mid x_{j}[\sigma_{j} + \sigma_{i}(\tau)] \mid \cdots \mid r \mid T_K \cup \{t\} \mid f_{\text{mngr}} \mid \text{upd}(f_{\text{tx}}, t, r) \mid f_{\text{frz}}
\]

where \( \tau \) is removed from \( x_i \), and all the units of \( \tau \) in \( x_i \) are transferred to \( x_j \).

**Gen.** Let \( t.\text{snd} = x_i \), \( t.\text{rcv} = x_j \), and \( t.\text{val} = v \). Performing \( t \) requires that \( x_i \) has enough \( \text{Algos} \) to own another asset, i.e. \( \models \sigma_i(\tau \mapsto v) \), where \( \tau \) is the (fresh) identifier of the new asset. In the new state, the balance of \( x_i \) is extended with \( \{\tau \mapsto v\} \), and \( f_{\text{mngr}} \) is updated, making \( x_j \) the manager of \( \tau \). The new state is:

\[
x_{i}[\sigma_{i}(\tau \mapsto v)] \mid \cdots \mid r \mid T_K \cup \{t\} \mid f_{\text{mngr}}(\tau \mapsto x_j) \mid \text{upd}(f_{\text{tx}}, t, r) \mid f_{\text{frz}}
\]

where all accounts but \( x_i \) are already storing the asset \( \tau \), and that \( \tau \) is not frozen for \( x_i \) and \( x_j \).

In the resulting state, the token \( \tau \) no longer exists:

\[
x_{i}[\sigma_{i}(\tau \mapsto 0)] \mid \cdots \mid r \mid T_K \cup \{t\} \mid f_{\text{mngr}} \mid \text{upd}(f_{\text{tx}}, t, r) \mid f_{\text{frz}}
\]

Otherwise, if \( x_i \)’s balance has already an entry for \( \tau \), then \( \sigma_i \) is unchanged.

**Burn.** Let \( t.\text{snd} = x_i \) and \( t.\text{asst} = \tau \). Performing \( t \) requires that \( x_i \) is the creator of \( \tau \), and that \( x_i \) stores all the units of \( \tau \) (i.e., there are no units of \( \tau \) in other accounts). In the resulting state, the token \( \tau \) no longer exists:

\[
x_{i}[\sigma_{i}(\tau \mapsto \perp)] \mid \cdots \mid r \mid T_K \cup \{t\} \mid f_{\text{mngr}} \mid \text{upd}(f_{\text{tx}}, t, r) \mid f_{\text{frz}}
\]

Note that this transaction requires an authorization by the asset manager of \( \tau \), which is recorded in \( f_{\text{mngr}} \) (we address this topic in §2.6.)

**Revoke.** Let \( t.\text{snd} = x_i \) and \( t.\text{rcv} = x_j \). Performing \( t \) requires that both \( x_i \) and \( x_j \) are already storing the asset \( \tau \), and that \( \tau \) is not frozen for \( x_i \) and \( x_j \). In the new state, the balance of \( x_i \) is decreased by \( v = t.\text{val} \) units of the asset \( \tau = t.\text{asst} \), and the balance of \( x_j \) is increased by the same amount:

\[
x_{i}[\sigma_{i} - v: \tau] \mid x_{j}[\sigma_{j} + v: \tau] \mid \cdots \mid r \mid T_K \cup \{t\} \mid f_{\text{mngr}} \mid \text{upd}(f_{\text{tx}}, t, r) \mid f_{\text{frz}}
\]

**Freeze and unfreeze.** A \( \text{frz} \) transaction \( t \) with \( t.\text{snd} = x_i \) and \( t.\text{asst} = \tau \) updates the mapping \( f_{\text{frz}} \) into \( f'_{\text{frz}} \), such that \( f'_{\text{frz}}(x_i) = f_{\text{frz}}(x_i) \cup \{\tau\} \), whenever the asset \( \tau \) is owned by \( x_i \). This effectively prevents any transfers of the asset \( \tau \) to/from the account \( x_i \). The dual transaction \( \text{unfrz} \) updates the mapping \( f_{\text{frz}} \) into \( f''_{\text{frz}} \) such that \( f''_{\text{frz}}(x_i) = f_{\text{frz}}(x_i) \setminus \{\tau\} \).
e ::= v  \quad \text{constant}  \\
| e \circ e \quad \text{arithmetic (} \circ \in \{ +, -, \leq, =, \geq, \ast, /, \% \text{, and, or} \})  \\
| \text{not } e \quad \text{negation}  \\
| \text{txlen} \quad \text{number of transactions in the atomic group}  \\
| \text{txpos} \quad \text{index of current transaction in the atomic group}  \\
| \text{txid}(n) \quad \text{identifier of } n\text{-th transaction in the atomic group}  \\
| \text{tx}(n).f \quad \text{value of field } f \text{ of } n\text{-th transaction in the atomic group}  \\
| \text{arglen} \quad \text{number of arguments of the current transaction}  \\
| \text{arg}(n) \quad n\text{-th argument of the current transaction}  \\
| \text{H}(e) \quad \text{hash}  \\
| \text{versig}(e, e, e) \quad \text{signature verification}  \\

t_{x} = t_{x} = \text{tx}(\text{txpos}).f \quad \text{txid} := \text{txid}(\text{txpos})  \\
| \text{if } e_{0} \text{ then } e_{1} \text{ else } e_{2} ::= (e_{0} \text{ and } e_{1}) \text{ or } ((\text{not } e_{0}) \text{ and } e_{2})  \\

\text{Syntactic sugar: } \text{false} ::= 1 = 0 \quad \text{true} ::= 1 = 1  \\
Fig. 2: Smart contract scripts (inspired by PyTeal [7]).

Delegate. A delegate transaction \( t \) with \( t.\text{snd} = x_{i}, t.\text{rcv} = x_{j} \) and \( t.\text{asst} = \tau \) updates the manager of \( \tau \), provided that \( f_{\text{mngr}}(\tau) = x_{i} \). In the updated mapping \( f_{\text{mngr}}(\tau) = x_{j} \), the manager of \( \tau \) is \( x_{j} \).

Initiating a new round. We model the advancement to the next round of the blockchain as a state transition \( \Gamma \xrightarrow{\cdot} \Gamma' \). In the new state \( \Gamma' \), the round is increased, and the set \( T_{K} \) of the transactions in the last 1000 rounds is updated as \( T'_{K} = \{ t \in T_{K} \mid t.\text{lv} > r \} \). The other components of the state are unchanged.

2.4 Executing atomic groups of transactions

Atomic transfers allow state transitions to atomically perform sequences of transactions. To atomically perform a sequence \( \mathcal{J} = t_{1} \cdots t_{n} \) from a state \( \Gamma \), we must check that all the transactions \( t_{i} \) can be performed in sequence, i.e. the following precondition must hold (for some \( \Gamma_{1}, \ldots, \Gamma_{n} \)):

\[ \Gamma \xrightarrow{t_{1}} \Gamma_{1} \rightarrow \Gamma \rightarrow \Gamma_{n} \]

If so, the state \( \Gamma \) can take a single-step transition labelled \( \mathcal{J} \). Denoting the new transition relation with \( \rightarrow \), we write the atomic execution of \( \mathcal{J} \) in \( \Gamma \) as follows:

\[ \Gamma \xrightarrow{\mathcal{J}} \Gamma_{n} \]

2.5 Executing smart contracts

In Algorand, custom authorization policies can be defined with a smart contract language called TEAL [8]. TEAL is a bytecode-based stack language, with an official programming interface for Python (called PyTeal): in our formal model, we take inspiration from the latter to abstract TEAL bytecode scripts as terms,
with the syntax in Fig. 2. Besides standard arithmetic-logical operators, TEAL includes operators to count and index all transactions in the current atomic group, and to access their id and fields. When firing transaction involving scripts, users can specify a sequence of arguments; accordingly, the script language includes operators to know the number of arguments, and access them. Further, scripts include cryptographic operators to compute hashes and verify signatures.

The script evaluation function $\llbracket e \rrbracket_{\mathcal{T}, i}$ (Fig. 3) evaluates $e$ using 3 parameters: a sequence of arguments $W$, a sequence of transactions $\mathcal{T}$ forming an atomic group, and the index $i < |\mathcal{T}|$ of the transaction containing $e$. The script $\text{tx}(n).f$ evaluates to the field $f$ of the $n$th transaction in group $\mathcal{T}$. The size of $\mathcal{T}$ is given by $\text{txlen}$, while $\text{txpos}$ returns the index $i$ of the transaction containing the script being evaluated. The script $\text{arg}(n)$ returns the $n$th argument in $W$, whose length is given by $\text{arglen}$. The script $H(e)$ applies a public hash function $H$ to the evaluation of $e$. The script $\text{versig}(e_1, e_2, e_3)$ verifies a signature $e_2$ on the message obtained by concatenating the enclosing script and $e_1$, using public key $e_3$. All operators in Fig. 3 are strict: they fail if the evaluation of any operand fails.

2.6 Authorizing transactions, and user-blockchain interaction

As noted in footnote 9, the mere existence of a step $\Gamma \xrightarrow{t} \Gamma'$ does not imply that $t$ can actually be issued. For this to be possible, users must provide a sequence $W$ of witnesses, satisfying the authorization predicate associated with $t$; such a predicate is uniquely determined by the authorizer address of $t$, written $\text{auth}(t, f_{\text{mngr}})$. For transaction types open, close, pay, gen, optin the authorizer address is $t.\text{snd}$; for burn, rek, frz and unfrz on an asset $\tau$ it is the asset manager $f_{\text{mngr}}(\tau)$. Intuitively, if $\text{auth}(t, f_{\text{mngr}}) = x$, then $W$ authorizes $t$ iff:

1. if $x$ is a multisig address $(A, n)$, then $W$ contains at least $n$ signatures of $t$, made by the users in $A$; (if $x$ is a single-user address $a$: see footnote 5)
2. if $x$ is a script $e$, then $e$ evaluates to true under the arguments $W$.

We now formalize the intuition above. Since the evaluation of scripts depends on a whole group of transactions $\mathcal{T}$, and on the index $i$ of the current transaction within $\mathcal{T}$, we define the authorization predicate as $W \models \mathcal{T}, i$ (read: “$W$ authorizes the $i$th transaction in $\mathcal{T}$“). Let $\text{sig}_A(m)$ stand for the set of signatures containing $\text{sig}_a(m)$ for all $a \in A$; then, $W \models \mathcal{T}, i$ holds whenever:
A formal model of Algorand smart contracts

1. if \( \text{auth}(T, i, f_{mngr}) = (A, n) \), then \( |\text{set}(W) \cap \text{sig}_{\text{set}(A)}(T, i)| \geq n \)

2. if \( \text{auth}(T, i, f_{mngr}) = e \), then \( \llbracket \text{wT}, i = \text{true} \rrbracket \)

Given a sequence of sequences of witnesses \( W = W_0 \cdots W_{n-1} \) with \( n = |T| \), the group authorization predicate \( W \models T \) holds iff \( W_i \models T, i \) for all \( i \in 0 \cdots n-1 \).

User-blockchain interaction. We model the interaction of users with the blockchain as a transition system. Its states are pairs \( (\Gamma, K) \), where \( \Gamma \) is a blockchain state, while \( K \) is the set of authorization bitstrings currently known by users. The transition relation \( \ell = \Rightarrow \) (with \( \ell \in \{w, \checkmark : T\} \)) is given by the rules:

\[
\begin{align*}
\Gamma, K \xrightarrow{w} \Gamma' & \quad \Gamma, K \xrightarrow{\checkmark} \Gamma' & \quad \Gamma, K \xrightarrow{W : T} \Gamma', K
\end{align*}
\]

With the first two rules, users can broadcast a witness \( w \), or advance to the next round. The last rule gathers from \( K \) a sequence of witnesses \( W \), and lets the blockchain perform an atomic group of transactions \( T \) if authorized by \( W \).

2.7 Fundamental properties of ASC1

We now exploit our formal model to establish some fundamental properties of ASC1. Theorem 1 states that the same transaction \( t \) cannot be issued more than once, i.e., there is no double-spending. In the statement, we use \( \rightarrow \Rightarrow \rightarrow^* \) to denote an arbitrarily long series of steps including a group of transactions \( T \).

Theorem 1 (No double-spending). Let \( \Gamma_0 \rightarrow^* \rightarrow T \rightarrow^* \Gamma' \). Then, no transaction occurs more than once in \( \Gamma' \).

Define the value of an asset \( \tau \) in a state \( \Gamma = x_1[\sigma_1] \cdots x_n[\sigma_n] | r | \cdots \) as the sum of the balances of \( \tau \) in all accounts in \( \Gamma \):

\[
\text{val}_\tau(\Gamma) = \sum_{i=1}^n \text{val}_\tau(\sigma_i)
\]

where \( \text{val}_\tau(\sigma) = \begin{cases} 
\sigma(\tau) & \text{if } \tau \in \text{dom}(\sigma) \\
0 & \text{otherwise}
\end{cases} \)

Theorem 2 states that, once an asset is minted, its value remains constant, until the asset is eventually burnt. In particular, since Algos cannot be burnt (nor minted, unlike in Bitcoin and Ethereum), their amount remains constant.

Theorem 2 (Value preservation). Let \( \Gamma_0 \rightarrow^* \rightarrow \Gamma \). Then:

\[
\text{val}_\tau(\Gamma') = \begin{cases} 
\text{val}_\tau(\Gamma) & \text{if } \tau \text{ occurs in } \Gamma \text{ and it is not burnt in } \Gamma \rightarrow^* \Gamma' \\
0 & \text{otherwise}
\end{cases}
\]

Theorem 3 establishes that the transition systems \( \rightarrow \) and \( \Rightarrow \) are deterministic: crucially, this allows reconstructing the blockchain state from the transition log. Notably, by item 3 of Theorem 3, witnesses only determine whether a state transition happens or not, but they do not affect the new state. This is unlike Ethereum, where arguments of function calls in transactions may affect the state.
Theorem 3 (Determinism). For all $\lambda \in \{\checkmark, \mathcal{T}\}$ and $\ell \in \{\checkmark, w\}$:

1. if $\Gamma_0 \xrightarrow{\lambda} \Gamma'$ and $\Gamma_0 \xrightarrow{\lambda} \Gamma''$, then $\Gamma' = \Gamma''$;
2. if $(\Gamma, K) \xrightarrow{\ell} (\Gamma', K')$ and $(\Gamma, K) \xrightarrow{\ell} (\Gamma'', K'')$, then $(\Gamma', K') = (\Gamma'', K'')$;
3. if $(\Gamma, K) \xrightarrow{\ell} (\Gamma', K')$ and $(\Gamma, K) \xrightarrow{\ell} (\Gamma'', K'')$, then $\Gamma' = \Gamma''$ and $K' = K'' = K$.

3 Designing secure smart contracts in Algorand

We now exploit our formal model to design some archetypal smart contracts, and establish their security (§3.2). First, we introduce an attacker model.

3.1 Attacker model

We assume that cryptographic primitives are secure, i.e., hashes are collision resistant and signatures cannot be forged (except with negligible probability). A run $\mathcal{R}$ is a (possibly infinite) sequence of labels $\ell_1 \ell_2 \cdots$ such that $(\Gamma_0, K_0) \xrightarrow{\ell_1} (\Gamma_1, K_1) \xrightarrow{\ell_2} \cdots$, where $\Gamma_0$ is the initial state, and $K_0 = \emptyset$ is the initial (empty) knowledge; hence, as illustrated in §2.6, each label $\ell_i$ in a run $\mathcal{R}$ can be either $w$ (broadcast of a witness bitstring $w$), $\mathcal{W}:\mathcal{T}$ (atomic group of transactions $\mathcal{T}$ authorized by $\mathcal{W}$), or $\checkmark$ (advance to next round). We consider a setting where:

- each user $a$ has a strategy $\Sigma$, i.e. a PPTIME algorithm to select which label to perform among those permitted by the ASC1 transition system. A strategy takes as input a finite run $\mathcal{R}$ (the past history) and outputs a single enabled label $\ell$. Strategies are stateful: users can read and write a private unbounded tape to maintain their own state throughout the run. The initial state of $a$’s tape contains $a$’s private key, and the public keys of all users;\(^{10}\)
- an adversary $\text{Adv}$ who controls the scheduling with her stateful adversarial strategy $\Sigma_{\text{Adv}}$: a PPTIME algorithm taking as input the current run $\mathcal{R}$ and the labels output by the strategies of users (i.e., the steps that users are trying to make). The output of $\Sigma_{\text{Adv}}$ is a single label $\ell$, that is appended to the current run. We assume the adversarial strategy $\Sigma_{\text{Adv}}$ can delay users’ transactions by at most $\delta_{\text{Adv}}$ rounds, where $\delta_{\text{Adv}}$ is a given natural number.\(^ {11}\)

A set $\Sigma$ of strategies of users and $\text{Adv}$ induces a distribution of runs; we say that run $\mathcal{R}$ is conformant to $\Sigma$ if $\mathcal{R}$ is sampled from such a distribution. We assume that infinite runs contain infinitely many $\checkmark$: this non-Zeno condition ensures that neither users nor $\text{Adv}$ can perform infinitely many transactions in a round.

---

\(^{10}\) Notice that new public/private key pairs can be generated during the run, and their public parts can be communicated as labels $w$.

\(^{11}\) Without this assumption, $\text{Adv}$ could arbitrarily disrupt deadlines: e.g., $\Sigma_{\text{Adv}}$ could make $a$ always lose lottery games (like the ones below) by delaying $a$’s transactions.
3.2 Smart contracts

We now exploit our model to specify some archetypal ASC1 contracts, and reason about their security. To simplify the presentation, we assume $\delta_{Adv} = 0$, i.e., the adversary $Adv$ can start a new round (performing $\checkmark$) only if all users agree. The table below summarises our selection of smart contracts, highlighting the design patterns they implement.

| Use case / Pattern | Signed witness | Timeouts | Commit/reveal | State transfer | Atomic transfer | Time windows |
|--------------------|----------------|----------|--------------|----------------|-----------------|--------------|
| Oracle             | ✓              | ✓        | ✓            |                |                 |              |
| HTLC               | ✓              | ✓        | ✓            |                |                 |              |
| Mutual HTLC (§B.1) | ✓              | ✓        | ✓            |                |                 |              |
| $O(n^2)$-collateral lottery (§B.2) | ✓ | ✓ | ✓ | ✓ | ✓ |
| Periodic payment   | ✓              | ✓        | ✓            |                |                 | ✓            |
| Escrow (§B.3)      | ✓              | ✓        | ✓            |                |                 | ✓            |
| Two-phase authorization | ✓ | ✓ | ✓ | ✓ | ✓ |
| Limit order (§B.4) | ✓              | ✓        | ✓            |                |                 | ✓            |
| Split (§B.5)       | ✓              | ✓        | ✓            |                |                 | ✓            |

**Oracle.** We start by designing a contract which allows either $a$ or $b$ to withdraw all the Algos in the contract, depending on the outcome of a certain boolean event. Let $o$ be an oracle who certifies such an outcome, by signing the value 1 or 0. We model the contract as the following script:

$$
Oracle \triangleq tx.type = close \text{ and } tx.asst = Algo \text{ and } \begin{cases} (tx.fv > r_{max} \text{ and } tx.rcv = a) \\
(\arg(0) = 0 \text{ and versig}(\arg(0), \arg(1), o) \text{ and } tx.rcv = a) \\
(\arg(0) = 1 \text{ and versig}(\arg(0), \arg(1), o) \text{ and } tx.rcv = b) \end{cases}
$$

Once created, the contract accepts only close transactions, using two arguments as witnesses. The argument $\arg(0)$ contains the outcome, while $\arg(1)$ is $o$’s signature on $(Oracle, \arg(0))$, i.e., the concatenation between the script and the first argument. The user $b$ can collect the funds in $Oracle$ if $o$ certifies the outcome 1, while $a$ can collect the funds if the outcome is 0, or after round $r_{max}$. Theorem 4 below proves that $Oracle$ works as intended. To state it, we define $T_p$ as the set of transactions allowing a user $p$ to withdraw the contract funds:

$$
T_p \equiv \{ t \mid t.type = close, t.snd = Oracle, t.rcv = p, t.asst = Algo \}
$$

The theorem considers the following strategies for $a$, $b$, and $o$:

- $\Sigma_2$: wait for $s = \sigma_2(Oracle, 0)$; if $s$ arrives at round $r \leq r_{max}$, then immediately send a transaction $t \in T_s$ with $t.fv = r$ and witness 0 $s$; otherwise, at round $r_{max} + 1$, send a transaction $t \in T_b$ with $t.fv = r_{max} + 1$;
- $\Sigma_b$: wait for $s' = \sigma_2(Oracle, 1)$; if $s'$ arrives at round $r$, immediately send a transaction $t \in T_b$ with $t.fv = r$ and witness 1 $s'$;
- $\Sigma_o$: do one of the following: (a) send $o$’s signature on $(Oracle, 0)$ at any time, or (b) send $o$’s signature on $(Oracle, 1)$ at any time, or (c) do nothing.

All results can be easily adjusted for $\delta_{Adv} > 0$, but this would require more verbose statements to account for possible delays introduced by $Adv$.12
Theorem 4. Let $\mathcal{R}$ be a run conforming to some set of strategies $\Sigma$, such that:

1. $\Sigma_o \in \Sigma$; (ii) $\mathcal{R}$ reaches, at some round before $r_{max}$, a state $(\text{Oracle}[\sigma] \mid \cdots$; (iii) $\mathcal{R}$ reaches the round $r_{max} + 2$. Then, with overwhelming probability:

1. if $\Sigma_a \in \Sigma$ and $o$ has not sent a signature on $(\text{Oracle}, 1)$, then $\mathcal{R}$ contains a transaction in $T_a$;
2. if $\Sigma_b \in \Sigma$ and $o$ has sent a signature on $(\text{Oracle}, 1)$ at round $r \leq r_{max}$, then $\mathcal{R}$ contains a transaction in $T_b$.

Notice that in item (1) we are only assuming that $a$ and $o$ use the strategies $\Sigma_a$ and $\Sigma_o$, while $b$ and Adv can use any strategy (and possibly collude.) Similarly, in item (2) we are only assuming $b$’s and $o$’s strategies.

Hash Time Lock Contract (HTLC). A user $a$ promises that she will either reveal a secret $s_a$ by round $r_{max}$, or pay a penalty to $b$. More sophisticated contracts, e.g. gambling games, use this mechanism to let players generate random numbers in a fair way. We define the HTLC as the following contract, parameterised on the two users $a, b$ and the hash $h_a = H(s_a)$ of the secret:

$$\text{HTLC}(a, b, h_a) \triangleq \text{tx.type} = \text{close} \text{ and tx.asst = Algo and}$$

$$((\text{tx.rcv} = a \text{ and } H(\text{arg}(0)) = h_a) \text{ or } (\text{tx.rcv} = b \text{ and } \text{tx.fv} \geq r_{max}))$$

The contract accepts only close transactions with receiver $a$ or $b$. User $a$ can collect the funds in the contract only by providing the secret $s_a$ in $\text{arg}(0)$, effectively making $s_a$ public. Instead, if $a$ does not reveal $s_a$, then $b$ can collect the funds after round $r_{max}$. We state the correctness of HTLC in Theorem 5; first, let $T_p$ be the set of transactions allowing user $p$ to withdraw the contract funds:

$$T_p = \{ t \mid t\text{.type} = \text{close}, t\text{.snd = HTLC}(a, b, h_a), t\text{.rcv} = p, t\text{.asst = Algo}\}$$

We consider the following strategies for $a$ and $b$:

- $\Sigma_a$: at a round $r < r_{max}$, send a $t \in T_a$ with $t\text{.fv} = r$ and witness $s_a$;
- $\Sigma_b$: at round $r_{max}$, check whether any transaction in $T_a$ occurs in $\mathcal{R}$. If not, then immediately send a transaction $t \in T_b$ with $t\text{.fv} = r_{max}$.

Theorem 5. Let $\mathcal{R}$ be a run conforming to some set of strategies $\Sigma$, such that:

1. $\mathcal{R}$ reaches, at some round before $r_{max}$, a state $(\text{HTLC}(a, b, h_a) \mid \cdots$; (ii) $\mathcal{R}$ reaches the round $r_{max} + 1$. Then, with overwhelming probability:

1. if $\Sigma_a \in \Sigma$, then $\mathcal{R}$ contains a transaction in $T_a$;
2. if $\Sigma_b \in \Sigma$ and $\mathcal{R}$ does not contain the secret $s_a$ before round $r_{max} + 1$, then $\mathcal{R}$ contains a transaction in $T_b$.

Lotteries. Consider a gambling game where $n$ players bet 1Algo each, and the winner, chosen uniformly at random among them, can redeem $n$ Algos. A simple implementation, inspired by [11–13] for Bitcoin, requires each player to deposit

13 If $s_a$ is a sufficiently long bitstring generated uniformly at random, collision resistance of the hash function ensures that only $a$ (who knows $s_a$) can provide such an $\text{arg}(0)$.}
$n(n - 1)$ Algos as collateral in an HTLC contract.\textsuperscript{14} For $n = 2$ players $a$ and $b$, such deposits are transferred by the following transactions:

$t_{Ha} = \{\text{type: open, snd: } a, \text{rcv: HTLC}(a, b, h_a), \text{val: } 2, \ldots\}$

$t_{Hb} = \{\text{type: open, snd: } b, \text{rcv: HTLC}(b, a, h_b), \text{val: } 2, \ldots\}$

The bets are stored in the following contract, which determines the winner as a function of the secrets, and allows her to withdraw the whole pot:

$\text{Lottery} \triangleq \text{tx.type = close and tx.asst = Algo and } H(\text{arg}(0)) = h_a \text{ and } H(\text{arg}(1)) = h_b$

and if $(\text{arg}(0) + \text{arg}(1)) \text{\%} 2 = 0$ then $\text{tx.rcv} = a$ else $\text{tx.rcv} = b$

with $h_a \neq h_b$.\textsuperscript{15} Players $a$ and $b$ start the game with the atomic transactions:

$t_{La} = \{\text{type: open, snd: } a, \text{rcv: Lottery, val: } 1, \ldots\}$

$t_{Lb} = \{\text{type: pay, snd: } a, \text{rcv: Lottery, val: } 1, \text{asst: Algo, } \ldots\}$

The transaction $t_{La}$ creates the contract with $a$’s bet, and $t_{Hb}$ completes it with $b$’s bet. At this point, there are two possible outcomes:

(a) both players reveal their secret, then the winner can withdraw the pot, by performing a close action on the Lottery contract, providing as arguments the two secrets, and setting her identity in the rcv field;

(b) one of the players does not reveal the secret. Then, the other player can withdraw the collateral in the other player’s HTLC (and redeem her own).

To formalise the correctness of the lottery, consider the sets of transactions:

$T_{\text{secr}}^{a,b} = \{t | t.\text{type} = \text{close}, t.\text{snd} = \text{HTLC}(p, q, h_p), t.\text{rcv} = p, t.\text{asst} = \text{Algo}\}$

$T_{\text{out}}^{a,b} = \{t | t.\text{type} = \text{close}, t.\text{snd} = \text{HTLC}(p, q, h_p), t.\text{rcv} = q, t.\text{asst} = \text{Algo}\}$

$T_{\text{lott}}^{a,b} = \{t | t.\text{type} = \text{close}, t.\text{snd} = \text{Lottery, t.rcv} = p, t.\text{asst} = \text{Algo}\}$

and consider the following strategy $\Sigma_a$ for $a$ (the one for $b$ is analogous):

1. at some $r < r_{\text{max}}$, send a transaction $t \in T_{\text{secr}}^{a,b}$, witness $s_a$;
2. if some transaction in $T_{\text{secr}}^{a,b}$ occurs in $\mathcal{R}$ at round $r' < r_{\text{max}}$, then extract its witness $s_b$ and compute the winner; if $a$ is the winner, immediately send a transaction $t \in T_{\text{lott}}^{a,b}$ with $t.\text{fv} = r'$ and witness $s_a s_b$;
3. if at round $r_{\text{max}}$ no transaction in $T_{\text{secr}}^{a,b}$ occurs in $\mathcal{R}$, immediately send a transaction $t \in T_{\text{out}}^{a,b}$ with $t.\text{fv} = r_{\text{max}}$.

Theorem 6 below establishes that the lottery is fair, implying that the expected payoff of player $a$ following strategy $\Sigma_a$ is at least negligible; instead, if $a$ does not follow $\Sigma_a$ (e.g., by not revealing her secret), the expected payoff may be negative; analogous results hold for player $b$. This result can be generalised for $n > 2$ players, with a collateral of $n(n - 1)$ Algos. As in the HTLC, we assume that $s_a$ and $s_b$ are sufficiently long bitstrings generated uniformly at random.

Theorem 6. Let $\mathcal{R}$ be a run conforming to a set of strategies $\Sigma$, such that:

(i) $\mathcal{R}$ contains, before $r_{\text{max}}$, the label $T_{a,b}$; (ii) $\mathcal{R}$ reaches round $r_{\text{max}} + 1$. For $p \neq q \in \{a, b\}$, if $\Sigma_p \in \Sigma$, then: (1)$\mathcal{R}$ contains a transaction in $T_{\text{secr}}^{a,p}$; (2) the probability that $\mathcal{R}$ contains $T_{\text{lott}}^{a,p}$ or $T_{\text{lott}}^{b,p}$ is $\geq \frac{1}{2}$ (up-to a negligible quantity).

\textsuperscript{14} A zero-collateral lottery is presented in §B.

\textsuperscript{15} This check prevents a replay attack: if $a$ chooses $h_a = h_b$, then $b$ cannot win.
Periodic payment. We want to ensure that a can withdraw a fixed amount of \( v \) Algos at fixed time windows of \( p \) rounds. We can implement this behaviour through the following contract, which can be refilled when needed:

\[
PP(p, d, n) \triangleq \text{tx.type} = \text{pay} \quad \text{and} \quad \text{tx.val} = v \quad \text{and} \quad \text{tx.asst} = \text{Algo} \quad \text{and} \\
\quad \text{tx.rcv} = a \quad \text{and} \quad \text{tx.fv} \% p = 0 \quad \text{and} \quad \text{tx.lv} = \text{tx.fv} + d \quad \text{and} \quad \text{tx.lx} = n
\]

The contract accepts only \( \text{pay} \) transactions of \( v \) Algos to receiver \( a \). The conditions \( \text{tx.fv} \% p = 0 \) and \( \text{tx.lv} = \text{tx.fv} + d \) ensure that the contract only accepts transactions with validity interval \( [kp, kp + d] \), for \( k \in \mathbb{N} \). The condition \( \text{tx.lx} = n \) ensures that at most one such transactions is accepted for each time window.

Finite-state machines. Consider a set of users \( A \) who want to stipulate a contract whose behaviour is given by a finite-state machine with states \( q_0, \ldots, q_n \). We can implement such a contract by representing each state \( q_i \) as a script \( e_i \); the current state/script holds the assets, and each state transition \( q_i \rightarrow q_j \) is a clause in \( e_i \) which enables a \texttt{close} transaction to transfer the assets to \( e_j \). This clause requires \( \text{tx.rcv} = e_j \), except in case of loops, which cannot be encoded directly.\(^{16} \) In this case, we identify the next state as \( \text{tx.rcv} = \text{arg}(0) \), also requiring all users in \( A \) to sign \text{arg}(0) to confirm its correctness. To ensure that any user in \( A \) can trigger a state transition (by firing the corresponding transaction), their signatures must be exchanged before the contract starts. An instance of this pattern is illustrated below.

Two-phase authorization. We want a contract to allow user \( c \) to withdraw some funds, but only if authorized by \( a \) and \( b \). We want \( a \) to give her authorization first; if \( b \)’s authorization is not given within \( p \geq 1000 \) rounds, then anyone can fire a transaction to reset the contract to its initial state. We model this contract with two scripts: \( P1 \) represents the state where no authorization has been given yet, while \( P2 \) represents the state where \( a \)’s authorization has been given. Conceptually, the contract implements a finite-state machine, looping between two states until the contract funds are withdrawn by \( c \).

\[
P1 \triangleq \text{tx.type} = \text{close} \quad \text{and} \quad \text{tx.asst} = \text{Algo} \quad \text{and} \quad \text{versig(txid, arg(0), a)} \quad \text{and} \\
\quad \text{tx.rcv} = P2 \quad \text{and} \quad \text{tx.fv} \% (4 \ast p) = 0 \quad \text{and} \quad \text{tx.lv} = \text{tx.fv} + 1000
\]

\[
P2 \triangleq \text{tx.type} = \text{close} \quad \text{and} \quad \text{tx.asst} = \text{Algo} \quad \text{and} \\
\quad ((\text{versig(txid, arg(0), b)} \quad \text{and} \quad \text{tx.rcv} = c) \quad \text{or} \\
\quad (\text{versig(arg(0), arg(1), a)}) \quad \text{and} \quad \text{versig(arg(0), arg(2), b)}) \quad \text{and} \\
\quad \text{tx.rcv} = \text{arg}(0) \quad \text{and} \quad \text{tx.fv} \% (4 \ast p) = 2 \ast p \quad \text{and} \quad \text{tx.lv} = \text{tx.fv} + 1000)
\]

The scripts \( P1 \) and \( P2 \) use a time window with 4 frames, each lasting \( p \) rounds. Script \( P1 \) only accepts \texttt{close} transactions which transfer the balance to \( P2 \); the time constraint ensures that such transactions are sent in the first time frame. The script \( P2 \) accepts two kinds of transactions: (a) transfer the balance to \( c \), using an authorization by \( b \); (b) transfer the balance to \( P1 \), in the 4\textsuperscript{th} time frame. Note that in \( P2 \) we cannot use the (intuitively correct) condition \( \text{tx.rcv} = P1 \), as

---

\(^{16} \) This is because Algorand contracts cannot have circular references: contract accounts are referenced by script hashes, and no script can depend on its own hash.
it would introduce a circularity. Instead, we apply the state machines technique described above: we require $tx.rcv = arg(0)$, with $arg(0)$ signed by both $a$ and $b$,$^{17}$ and assume that these signatures are exchanged before the contract starts.

4 From the formal model to concrete Algorand

Our modelling approach is supported by a prototype tool, called secteal (secure TEAL), and accessible via a web interface (in anonymized form) at:

https://secteal.azurewebsites.net

The core of the tool is a compiler that translates smart contracts written as expressions, based on the script language (§2.5), into executable TEAL bytecode. In its current form, secteal supports experimentation with our model, and is provided with a series of examples from §3.2. Users can also compile their own secteal contracts, paving the way to a declarative approach to contract design and development. secteal is a first building block toward a comprehensive IDE for the design, verification, and deployment of contracts on Algorand.

5 Conclusions

This work is part of a wider research line on formal modelling of blockchain-based contracts, ranging from Bitcoin [14, 27, 32] to Ethereum [19, 23–26, 30], Cardano [20], Tezos [18], and Zilliqa [33]. These formal models are a necessary prerequisite to rigorously reason on the security of smart contracts, and they are the basis for automatic verification. Besides modelling the behaviour of transactions, in §3.1 we have proposed a model of attackers: this enables us to prove properties of smart contracts in the presence of adversaries, in the spirit of longstanding research in the cryptography area [10, 11, 15, 17, 22, 28, 29].

Differences between our model and Algorand Besides not modelling the consensus protocol, to keep the formalization simple, we chose to abstract from some aspects of ASC1, which do not appear to be relevant to the development of (the majority of) smart contracts. First, we are not modelling some transaction fields: among them, we have omitted the fee field, used to specify an amount of Algos to be paid to nodes, and the note field, used to embed arbitrary bitstrings into transactions. We associate a single manager to assets, while Algorand uses different managers for different operations (e.g., the freeze manager for $frz/unfrz$ and the clawback manager for $rvk$.) We use two different transactions types, $pay$ and $close$, to perform asset transfers and account closures: in Algorand, a single $pay$ transaction can perform both. Note that we can achieve the same effect by performing the $pay$ and $close$ transactions within the same atomic group. Our script language substantially covers all TEAL opcodes, but for a few exceptions, e.g. bitwise operations, different hash functions, jumps.

$^{17}$ We use other key pairs $a_1$ and $b_1$ to avoid confusion with the signatures on $txid$. 
Future work  In mid August 2020, Algorand has introduced stateful ASC1 contracts [4], enriching contract accounts with a persistent key-value store, accessible and modifiable through a new kind of transaction (which can use an extended set of TEAL opcodes.) To accommodate stateful contracts in our model, we would need to embed the key-value store in contract accounts, and extend the script language with key-value store updates. The rest of our model (in particular, the semantics of transactions and the attacker model) is mostly unaffected by this extension. Future work could also investigate declarative languages for stateful Algorand smart contracts, and associated verification techniques. Another research direction is the mechanization of our formal model, using a proof assistant: this would allow machine-checking the proofs developed by pencil-and-paper in §C. Similar work has been done e.g. for Bitcoin [32] and for Tezos [18].

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A The ASC1 state machine

We assume the following sets:

- $\mathbb{K}$, the set of all users;
- $\mathbb{S}$, the set of all scripts;
- $\mathbb{X} = (\mathbb{K} \times \mathbb{N}) \cup \mathbb{S}$, the set of all addresses;
- $\mathbb{A}$, the set of all assets;
- $\mathbb{T}$, the set of all transactions.

We define the partial operator $\circ$ on accounts as follows:

$$\sigma \circ v : \tau = \sigma\{\tau \mapsto \sigma(\tau) \circ v\} \text{ if } \tau \in \text{dom}(\sigma) \text{ and } \circ \in \{+, -, =, <, \ldots\}$$

We define a function $\text{auth}(t, f_{\text{mngr}})$ which gives the manager of a transaction $t$:

$$\text{auth}(t, f_{\text{mngr}}) = \begin{cases} t.\text{snd} & \text{if } t.\text{type} \in \{\text{open, close, pay, gen, optin}\} \\ f_{\text{mngr}}(t.\text{asst}) & \text{if } t.\text{type} \in \{\text{rvk, frz, unfrz, burn, delegate}\} \end{cases}$$

We define the following operators on denotations, evaluating to denotations:

$$\nu_0 \circ \perp \nu_1 \equiv \begin{cases} \nu_0 \circ \nu_1 & \text{if } \nu_0, \nu_1 \in \mathbb{Z} \\ \perp & \text{otherwise} \end{cases} \text{ for } \circ \in \{+, -, =, <, \ldots\}$$

A blockchain state $\Gamma$ is a term with the following syntax:

$$\Gamma ::= x[\sigma] \mid r \mid T_K \mid f_{\text{mngr}} \mid f_{\text{lx}} \mid f_{\text{frz}} \mid \Gamma \mid \Gamma'$$

and subject to the following conditions:

- all the terms except $x[\sigma]$ occur exactly once in a configuration;
- $r \in \mathbb{N}$ is the current round;
- $T_K \subseteq \mathbb{T}$ is the set of the transactions in the last 1000 rounds;
- $f_{\text{mngr}} : \mathbb{A} \rightarrow \mathbb{X}$ is the manager map;
- $f_{\text{lx}} : (X \times \mathbb{N}) \rightarrow \mathbb{N}$ is the lease map;
- $f_{\text{frz}} : \mathbb{X} \rightarrow 2^\mathbb{A}$ is the freeze map;
- if $x[\sigma]$ and $y[\sigma']$ occur in a configuration, then $x \neq y$.
- configurations form a commutative monoid with respect to the composition operator $|$ (with identity 0.)

We define in Fig. 4 a labelled transition relation $\rightarrow$ between blockchain states, where labels are the following:

- $\mathfrak{T}$ performs a sequence of transactions;
- $\checkmark$ advances one round.
B Additional smart contracts

B.1 Mutual HTLC

The mutual HTLC [11] is a variant of HTLC presented in §3.2: two users $a, b$ choose their own secrets, pay a deposit, and the contract ensures that either (a) both users reveal their secret and get their deposits back, or (b) whoever does not reveal the secret loses the deposit (in favour of the other user.)

Assuming that $h_a$ and $h_b$ are, respectively, the hashes of the secrets $s_a$ and $s_b$, we can implement the mutual HTLC by creating two HTLC contracts within an atomic group of transactions:

$$\{\text{snd} : a, \text{rcv} : \text{HTLC}(a, b, h_b), \ldots\} \cup \{\text{snd} : b, \text{rcv} : \text{HTLC}(b, a, h_a), \ldots\}$$

The mutual HTLC contract guarantees that once $a$ has stipulated it, then she will either learn $b$’s secret, or receive the compensation. Instead, if we create two instances of HTLC in a non-atomic way, this property is not guaranteed, since $b$ could refuse to stipulate its part of the contract.

Theorem 7 states the correctness of the mutual HTLC. For $p, q \in \{a, b\}$, let:

$$T_{p,q} = \begin{cases} \{t \mid t.type = \text{close}, t.snd = \text{HTLC}(p, q, h_p), t.rcv = p, \text{t.asst = Algo}\} \\
T'_{p,q} = \{t \mid t.type = \text{close}, t.snd = \text{HTLC}(p, q, h_p), t.rcv = q, \text{t.asst = Algo}\}
\end{cases}$$

and consider the following strategy for distinct users $p \neq q \in \{a, b\}$:

- $\Sigma_p$: at a round $r < r_{max}$, send a transaction $t \in T_{p,q}$ with $t.fv = r$ and witness $s_p$. Then, at round $r_{max}$ check whether any transaction in $T_{q,p}$ occurs in $R$: if not, immediately send a transaction $t \in T'_{q,p}$ with $t.fv = r_{max}$.

Theorem 7. Let $R$ be a run conforming to some set of strategies $\Sigma$, such that:

(i) $R$ reaches, before $r_{max}$, a state $\text{HTLC}(a, b, h_b)[\sigma] \mid \text{HTLC}(b, a, h_a)[\sigma'] \mid \cdots$; (ii) $R$ reaches $r_{max} + 1$. Let $p \neq q \in \{a, b\}$. If $\Sigma_p \in \Sigma$, then with overwhelming probability: (1) $R$ contains a transaction in $T_{p,q}$; (2) if $R$ does not contain the secret $s_q$ before round $r_{max} + 1$, then $R$ contains a transaction in $T'_{q,p}$.

B.2 Zero-collateral lottery

We show a variant of the two-players lottery in §3.2 which requires no collateral, similarly to [16,31]. The preconditions just require the 1Algo bets and the secrets, while the contract is the following:

\[ ZDL \triangleq \text{tx.type = close} \text{ and tx.asst = Algo and} \]
\[ (\text{tx.rcv = ZDL2 and H(arg(0)) = h_a}) \text{ or} \]
\[ (\text{tx.rcv = b and tx.fv \geq r_0}) \]

\[ ZDL2 \triangleq \text{tx.type = close} \text{ and tx.asst = Algo and H(arg(0)) = h_a and} \]
\[ (H(\text{arg(1)}) = h_b \text{ and} \]
\[ \text{if } (\text{arg(0) + arg(1)} \% 2 = 0 \text{ then tx.rcv = a else tx.rcv = b}) \text{ or} \]
\[ (\text{tx.rcv = a and tx.fv \geq r_0 + r_1}) \]

Here, $b$ must reveal first. If $b$ does not reveal his secret by the deadline $r_0$, then $a$ can redeem the 2Algos stored in the contract. Otherwise, $a$ in turn must
reveal by the deadline $r_0 + r_1$, or let $b$ redeem $2\text{Algos}$. If both $a$ and $b$ reveal, then the winner is determined as a function of their secrets. As before, the rational strategy for each player is to reveal the secret. This makes the lottery fair, even in the absence of a collateral.

B.3 Escrow

User $a$ wants to buy from seller $b$ an item that costs $v \text{Algos}$. We want to guarantee that (1) $b$ will get paid if $a$ authorizes the payment; (2) $a$ will be refunded if $b$ authorizes it; (3) if neither $a$ nor $b$ give their authorization, an escrow service $c$ will resolve the dispute, by either fully refunding $a$, or partially refunding $a$ with $v_3 \text{Algos}$ and giving the remaining $(v - v_3) \text{Algos}$ to $b$.

\[
\text{Escrow} \triangleq \text{tx.type} = \text{close} \text{ and } \text{tx.asst} = \text{Algo} \text{ and } \left( \text{versig}(\text{txid}, \text{arg}(1), a) \text{ and } (\text{tx.rcv} = b \text{ or } \text{tx.rcv} = \text{Resolve}) \right) \text{ or } \text{versig}(\text{txid}, \text{arg}(1), b) \text{ and } (\text{tx.rcv} = a \text{ or } \text{tx.rcv} = \text{Resolve})
\]

\[
\text{Resolve} \triangleq \text{tx.type} = \text{pay} \text{ and } \text{tx.asst} = \text{Algo} \text{ and } \text{versig}(\text{arg}(0), \text{arg}(1), c) \text{ and } \left( (\text{tx.rcv} = a \text{ and } \text{tx.val} = \text{arg}(0)) \text{ or } (\text{tx.rcv} = b \text{ and } \text{tx.val} = v - \text{arg}(0)) \right)
\]

B.4 Limit order

The limit order contract [2] allows a user $a$ to exchange her $\text{Algos}$ for units of a certain asset $\tau$, provided by any user. The contract imposes a lower bound $\rho_{\text{min}}$ to the exchange rate $\tau / \text{Algo}$, and guarantees its operation as long as it has enough funds, and a deadline $r_{\text{max}}$ is not reached (after then, $a$ can close it). To this purpose, the contract accepts two kinds of actions:

- an atomic group of two pay transactions: the first one transfers $v_0 \text{Algos}$ from the contract to the sender of the second one; the second transaction transfers $v_1$ units of $\tau$ to user $a$. The contract ensures that: (i) the ratio between $v_1$ and $v_0$ is greater then a given constant $\rho_{\text{min}}$; (ii) $v_0$ is greater than a given constant $v_{\text{min}}$. Note that such a group can be issued by any user owning enough units of the asset $\tau$, without requiring any interaction from $a$.

- a single transaction where $a$ closes the contract after the deadline $r_{\text{max}}$.

The following script implements this specification:

\[
\begin{align*}
\text{txlen} &= 2 \text{ and } \text{txpos} = 0 \text{ and } \\
\text{tx}(0).\text{type} &= \text{pay} \text{ and } \text{tx}(0).\text{asst} = \text{Algo} \text{ and } \text{tx}(0).\text{rcv} = \text{tx}(1).\text{snd} \text{ and } \\
\text{tx}(1).\text{type} &= \text{pay} \text{ and } \text{tx}(1).\text{asst} = \tau \text{ and } \text{tx}(1).\text{rcv} = a \text{ and } \\
\text{tx}(1).\text{val}/\text{tx}(0).\text{val} &\geq \rho_{\text{min}} \text{ and } \text{tx}(0).\text{val} \geq v_{\text{min}} \text{ and } \\
\text{or } &\left( \text{txlen} = 1 \text{ and } \text{tx.fv} > r_{\text{max}} \text{ and } \\
\text{tx.type} &= \text{close} \text{ and } \text{tx.asst} = \text{Algo} \text{ and } \text{tx}(0).\text{rcv} = a \right)
\end{align*}
\]
B.5 Split

The *split* contract [3] is created by a user $a$, who want to transfer its funds to users $b_0$ and $b_1$ in a fixed ratio $\rho$. The contract is initially funded with some Algos from $a$, and once started it accepts two kinds of actions:

- an atomic group of two *pay* transactions whose sender is the *split* contract: the first transaction transfers $v_0$ Algos to $b_0$. The contract ensures that: (i) the ratio between $v_1$ and $v_0$ is equal to a given constant $\rho$; (ii) $v_0$ is greater than a given constant $v_{\text{min}}$.
- a single transaction where $a$ closes the contract after a deadline $r_{\text{max}}$.

The following script implements this specification:

\[
\begin{align*}
\text{txlen} &= 2 \text{ and } \text{tx}(0).\text{snd} = \text{tx}(1).\text{snd} \text{ and } \\
\text{tx}(0).\text{type} &= \text{pay} \text{ and } \text{tx}(0).\text{asst} = \text{Algo} \text{ and } \text{tx}(0).\text{rcv} = b_0 \text{ and } \\
\text{tx}(1).\text{type} &= \text{pay} \text{ and } \text{tx}(1).\text{asst} = \text{Algo} \text{ and } \text{tx}(1).\text{rcv} = b_1 \text{ and } \\
\text{tx}(1).\text{val} &= \rho \times \text{tx}(0).\text{val} \text{ and } \text{tx}(0).\text{val} \geq v_{\text{min}} \text{) or } \\
\text{txlen} &= 1 \text{ and } \text{tx}.fv > r_{\text{max}} \text{ and } \\
\text{tx.type} &= \text{close} \text{ and } \text{tx.asst} = \text{Algo} \text{ and } \text{tx}(0).\text{rcv} = a
\end{align*}
\]

C Proofs

**Proof of Theorem 1**

By induction on the length of the run.

Conditions $c_1$ and $c_2$ are required at each step.

Condition $c_1$ prevents the same transaction $T$ to appear again for the next 1000 rounds ($t \not\in T_K$). Condition $c_1$ requires transaction $T$ to be valid only in a time window of at most 1000 rounds ($t.\text{lv} - t.fv \leq 1000$). Therefore, it is impossible for $t$ to appear more than once.

**Proof of Theorem 2**

By induction on the length of the run, and by cases on the rules generating each step.

Inspecting the rules, we can see that they all preserve the value for all assets, except for $[\text{Burn}]$ which completely destroys the asset, and $[\text{Gen}]$ which can create a fresh asset. The rules prevent a burnt asset to be re-created later on.

The two special cases are taken into account in the definition of $\text{val}_t(t')$, so the theorem holds.
Proof of Theorem 3

By induction on the length of the run, and by cases on the rules generating each step.

Inspecting each pair of (different) rules, we can observe that they either have distinct labels or they have conflicting side conditions, so that at most one of them can apply in each case. Further, we observe that in each rule the final state is a function of the label and of the initial state.

For part 3 of the theorem involving different labels $W:J$ and $W':J$, we further add that $(I', K) \xrightarrow{W'} (I', K)$ is possible only when $I \xrightarrow{T} I'$ holds, and the same holds for $W':J$. We conclude that the difference between $W$ and $W'$ is immaterial, since the determinism of $\Rightarrow$ is derived from the determinism of $\rightarrow$ which does not involve witnesses.

Proof of Theorem 4

For item (1), assume that $\Sigma_a, \Sigma_o \in \Sigma$. By hypothesis, $o$ has not sent a signature $s = \text{sig}_o(\text{Oracle}, 1)$. By contradiction, assume that $R$ does not contain a transaction in $T_a$. We have the following two cases:

1. $o$ has sent a signature on $(\text{Oracle}, 0)$ at round $r \leq r_{\text{max}}$. Then, since $R$ conforms to $\Sigma_a$, at round $r$, user $a$ has sent a transaction $t \in T_a$ with $t.fv = r$ and witness $0$. By construction, $t$ satisfies the script $\text{Oracle}$. Hence, to prove the thesis we show that $\text{Oracle}$ was not closed earlier in $R$. By inspecting the script $\text{Oracle}$, there are only three conditions under which a previous transaction $t'$ can close the contract:
   
   (a) $t'.fv > r_{\text{max}}$: impossible, because we would have $t' \in T_a$, contradicting the hypothesis;
   
   (b) $t'$ is validated by $o'$'s signature on $(\text{Oracle}, 1)$: impossible, since it would contradict the hypothesis;
   
   (c) $t'$ is validated by $o$'s signature on $(\text{Oracle}, 0)$: this implies that $t' \in T_a$ — contradiction.

2. $o$ has not sent a signature on $(\text{Oracle}, 0)$ at any round $r \leq r_{\text{max}}$. Then, since $R$ conforms to $\Sigma_a$, at round $r_{\text{max}} + 1$, user $a$ has sent a transaction $t \in T_a$ with $t.fv = r_{\text{max}} + 1$. By construction, $t$ satisfies the script $\text{Oracle}$. Hence, to prove the thesis we show that $\text{Oracle}$ was not closed earlier in $R$. By inspecting the script $\text{Oracle}$, there are only three conditions under which a previous transaction $t'$ can close the contract:

   (a) $t'.fv \geq r_{\text{max}}$: impossible, because we would have $t' \in T_a$, contradicting the hypothesis;
   
   (b) $t'$ is validated by $o$'s signature on $(\text{Oracle}, 1)$: impossible, since it would contradict the hypothesis;
   
   (c) $t'$ is validated by $o$'s signature on $(\text{Oracle}, 0)$: this implies that $t' \in T_a$ — contradiction.

For item (2), assume that $\Sigma_b, \Sigma_o \in \Sigma$. By hypothesis, $o$ has sent a signature $s'$ on $(\text{Oracle}, 1)$ at round $r \leq r_{\text{max}}$. By contradiction, assume that $R$ does not
contain a transaction in $T_b$. Then, since $\mathcal{R}$ conforms to $\Sigma_b$, at round $r$, user $b$ has sent a transaction $t \in T_b$ with $t.fv = r$ and witness $1'$. By construction, $t$ satisfies the script $Oracle$. Hence, to prove the thesis we show that $Oracle$ was not closed earlier in $\mathcal{R}$. By inspecting the script $Oracle$, there are only three conditions under which a previous transaction $t'$ can close the contract:

1. $t'.fv > r_{\text{max}}$: impossible, because $t.fv = r \leq r_{\text{max}}$;
2. $t'$ is validated by $o$’s signature on $(Oracle, 1)$: this implies that $t' \in T_b$ — contradiction.
3. $t'$ is validated by $o$’s signature on $(Oracle, 0)$: impossible, since it would contradict the honesty of $o$, which requires that the oracle never signs both 0 and 1;

Proof of Theorem 5

For item (1), assume that $\Sigma_a \in \Sigma$. By contradiction, assume that $\mathcal{R}$ does not contain a transaction in $T_a$. Since $\mathcal{R}$ conforms to $\Sigma_a$, at round $r$, user $a$ has sent a transaction $t \in T_a$ with $t.fv = r$ and witness $s_a$. By construction, $t$ satisfies the script $HTLC$. Hence, to prove the thesis we show that $HTLC$ was not closed earlier in $\mathcal{R}$. By inspecting the script $HTLC$, there are only two conditions under which a previous transaction $t'$ can close the contract:

1. $t'.fv > r_{\text{max}}$: impossible, because $t.fv = r \leq r_{\text{max}}$;
2. $t'$ is validated using a preimage of $h_a$: this implies that $t' \in T_a$ — contradiction.

For item (2), assume that $\Sigma_b \in \Sigma$, and that $s_a$ was not revealed before round $r_{\text{max}} + 1$. By contradiction, assume that $\mathcal{R}$ does not contain a transaction in $T_b$. Since $\mathcal{R}$ conforms to $\Sigma_b$, at round $r_{\text{max}}$, user $b$ has sent a transaction $t \in T_b$ with $t.fv = r_{\text{max}}$. By construction, $t$ satisfies the script $HTLC$. Hence, to prove the thesis we show that $HTLC$ was not closed earlier in $\mathcal{R}$. By inspecting the script $HTLC$, there are only two conditions under which a previous transaction $t'$ can close the contract:

1. $t'.fv > r_{\text{max}}$: this implies that $t' \in T_b$ — contradiction.
2. $t'$ is validated using a preimage of $h_a$: this implies, with overwhelming probability, that $s_a$ was used as the preimage. In such case, it was revealed and occurs in $\mathcal{R}$ at round $r_{\text{max}}$ — contradiction.

Proof of Theorem 6

The proof for the fairness of the lottery protocol when implemented on Bitcoin appeared in [11, 12]. This protocol relies on running, simultaneously, two timed commitments (for which we use $HTLC$) and a contract that transfers the bets to the winner, which can be computed after the secrets have been revealed (for which we use $Lottery$).
We already proved the security of HTLC in Theorem 5, which implies part 1 of the thesis: a honest participant \( p \) will reveal her secret in time with a transaction in \( T_{p,q}^{\text{secr}} \).

For part 2, the argument is similar to the one for the original protocol. Briefly put, if the other participant \( q \) does not to reveal \( s_q \) in time, \( p \) wins the lottery by timeout using a transaction in \( T_{q,p}^{\text{tout}} \). If instead \( s_q \) is revealed in time, since \( s_p \) was chosen in a uniformly random way, independently from \( s_q \) (which has a different hash), we have that the parity of \( s_p + s_q \) is a random bit, uniformly distributed. Hence, if \( p \) won, her strategy makes her send a transaction in \( T_{p}^{\text{lott}} \) and claim the pot.

In the argument above we neglected the case where \( q \) reveals another preimage than \( s_q \) as his secret, since that can only happen with negligible probability.

Summing up, an honest \( p \) wins with at least \( 1/2 \) probability (up-to a negligible quantity).

**Proof of Theorem 7**

The proof is analogous to the one for HTLC in Theorem 5.
\[ t.\text{type} = \text{open} \quad t.\text{snd} = x \quad t.\text{rcv} = y \quad y[\cdots] \text{ not in } \Gamma \quad f_{tx}, r \models t \quad t \notin T_K\]

\[ \models \sigma \land t.\text{val} : \text{Algo} \quad \sigma' = \{ \text{Algo} \to t.\text{val} \} \quad \models \sigma' \]

\[ x[\sigma] \mid \Gamma \mid r \mid T_K \mid f_{\text{mogr}} \mid f_{tx} \mid f_{frz} \xrightarrow{t} 1 \]

\[ x[\sigma \land t.\text{val} : \text{Algo}] \mid y'[\sigma'] \mid \Gamma \mid r \mid T_K \cup \{ t \} \mid f_{\text{mogr}} \mid \text{upd}(f_{tx}, t, r) \mid f_{frz} \]

\[ t.\text{type} = \text{close} \quad t.\text{snd} = x \quad t.\text{rcv} = y \quad t.\text{asst} = \text{Algo} \quad f_{tx}, r \models t \quad t \notin T_K \]

\[ \text{dom}(\sigma) = \{ \text{Algo} \} \quad y[\cdots] \text{ not in } x[\sigma] \mid \Gamma \land \sigma' = \{ \text{Algo} \to \sigma(\text{Algo}) \} \quad \models \sigma' \]

\[ x[\sigma] \mid \Gamma \mid r \mid T_K \mid f_{\text{mogr}} \mid f_{tx} \mid f_{frz} \xrightarrow{t} 1 \]

\[ y[\sigma'] \mid \Gamma \mid r \mid T_K \cup \{ t \} \mid f_{\text{mogr}} \mid \text{upd}(f_{tx}, t, r) \mid f_{frz} \]

\[ t.\text{type} = \text{close} \quad t.\text{snd} = x \quad t.\text{rcv} = y \quad t.\text{asst} = \tau \quad f_{tx}, r \models t \quad t \notin T_K \]

\[ \tau \notin \text{Algo} \quad \sigma(\tau) = v \quad r \in \text{dom}(\sigma') \quad \models \sigma'+v:\tau \quad \tau \notin f_{frz}(x) \cup f_{frz}(y) \]

\[ x[\sigma] \mid y[\sigma'] \mid \Gamma \mid r \mid T_K \mid f_{\text{mogr}} \mid f_{tx} \mid f_{frz} \xrightarrow{t} 1 \]

\[ y[\sigma' + \sigma(\text{Algo})] : \text{Algo} \mid \Gamma \mid r \mid T_K \cup \{ t \} \mid f_{\text{mogr}} \mid \text{upd}(f_{tx}, t, r) \mid f_{frz} \]

\[ t.\text{type} = \text{pay} \quad t.\text{snd} = x \quad t.\text{rcv} = y \quad t.\text{val} = v > 0 \quad t.\text{asst} = \tau \quad f_{tx}, r \models t \quad t \notin T_K \]

\[ \models \sigma \land v:\tau \quad \tau \in \text{dom}(\sigma') \quad \tau \notin f_{frz}(x) \cup f_{frz}(y) \]

\[ x[\sigma] \mid y[\sigma'] \mid \Gamma \mid r \mid T_K \mid f_{\text{mogr}} \mid f_{tx} \mid f_{frz} \xrightarrow{t} 1 \]

\[ x[\sigma \land v:\tau] \mid y[\sigma' + v:\tau] \mid \Gamma \mid r \mid T_K \cup \{ t \} \mid f_{\text{mogr}} \mid \text{upd}(f_{tx}, t, r) \mid f_{frz} \]

\[ t.\text{type} = \text{pay} \quad t.\text{snd} = x \quad t.\text{rcv} = y \quad t.\text{val} = 0 \quad f_{tx}, r \models t \quad t \notin T_K \]

\[ \models \sigma \land 0:\tau \quad \tau \notin \text{dom}(\sigma') \quad \tau \notin f_{frz}(x) \cup f_{frz}(y) \]

\[ x[\sigma] \mid y[\sigma'] \mid \Gamma \mid r \mid T_K \mid f_{\text{mogr}} \mid f_{tx} \mid f_{frz} \xrightarrow{t} 1 \]

\[ x[\sigma] \mid y[\sigma'] \mid \Gamma \mid r \mid T_K \cup \{ t \} \mid f_{\text{mogr}} \mid \text{upd}(f_{tx}, t, r) \mid f_{frz} \]

\[ t.\text{type} = \text{gen} \quad t.\text{snd} = x \quad t.\text{rcv} = y \quad t.\text{val} = v \quad f_{tx}, r \models t \quad t \notin T_K \]

\[ \models \sigma(\tau \to v) \quad \tau \text{ next fresh asset identifier} \]

\[ x[\sigma] \mid \Gamma \mid r \mid T_K \mid f_{\text{mogr}} \mid f_{tx} \mid f_{frz} \xrightarrow{t} 1 \]

\[ x[\sigma(\tau \to v)] \mid \Gamma \mid r \mid T_K \cup \{ t \} \mid f_{\text{mogr}} \mid \text{upd}(f_{tx}, t, r) \mid f_{frz} \]

\[ t.\text{type} = \text{optin} \quad t.\text{snd} = x \quad t.\text{asst} = \tau \quad f_{tx}, r \models t \quad t \notin T_K \]

\[ \tau \text{ occurs in } x[\sigma] \mid \Gamma \land \sigma' = \begin{cases} \sigma(\tau \to 0) & \text{if } \tau \notin \text{dom}(\sigma) \\ \sigma & \text{otherwise} \end{cases} \models \sigma' \]

\[ x[\sigma] \mid \Gamma \mid r \mid T_K \mid f_{\text{mogr}} \mid f_{tx} \mid f_{frz} \xrightarrow{t} 1 \]

\[ x[\sigma'] \mid \Gamma \mid r \mid T_K \cup \{ t \} \mid f_{\text{mogr}} \mid \text{upd}(f_{tx}, t, r) \mid f_{frz} \]
Fig. 4: The ASC1 state machine.