ABSTRACT

Strong gravitational lensing has traditionally been one of the few phenomena said to oppose a large cosmological constant; many analyses of lens statistics have given upper limits on $\Omega_\Lambda$ that are marginally inconsistent with the concordance cosmology. Those conclusions were based on models in which the predicted number counts of galaxies at moderate redshifts ($z \sim 0.5$–1) increased significantly with $\Omega_\Lambda$. I argue that the models should now be calibrated by counts of distant galaxies. When this is done, lens statistics lose most of their sensitivity to the cosmological constant.

Subject headings: cosmological parameters — galaxies: evolution — gravitational lensing

1. INTRODUCTION

Popular opinion seems to have settled on a “concordance” cosmology dominated by dark energy. The cosmic microwave background indicates a flat geometry (e.g., de Bernardis et al. 2000; Hanany et al. 2000; Pryke et al. 2002), cluster mass-to-light ratios indicate a low matter density (e.g., Carlberg, Yee, & Ellingson 1997; Bahcall et al. 2000), and Type Ia supernovae indicate cosmic acceleration (e.g., Riess et al. 1998; Perlmutter et al. 1999). The popular cosmology has matter content $\Omega_m \approx 0.30$–0.35 and dark energy $\Omega_\Lambda \approx 0.65$–0.70 such that $\Omega_m + \Omega_\Lambda = 1$. For simplicity, I assume that the dark energy corresponds to a cosmological constant $\Lambda$, although my analysis could easily be extended to quintessence models (e.g., Waga & Miceli 1999).

One phenomenon that has traditionally stood out from the concordance model is strong gravitational lensing. The statistics of strong lenses are sensitive to the cosmological parameters via the cosmological volume element (e.g., Turner 1990; Fuks, Futamase, & Kasai 1990), and analyses of the data have yielded upper limits on the dark energy at the level of $\Omega_\Lambda < 0.66$ at 95% confidence (e.g., Kochanek 1996; Falco, Kochanek, & Muñoz 1998). There are small systematic uncertainties in the upper limit due to assumptions about the lens sample and the amount of dust extinction in lens galaxies (e.g., Falco et al. 1998; Helbig et al. 1999; Waga & Miceli 1999; Corrigan, Quashnock, & Miller 1999). Larger systematic effects arise from uncertainties in the local luminosity function of galaxies. Chiba & Yoshii (1999) argue that by adopting luminosity functions from different surveys they can relax the upper limit on $\Omega_\Lambda$ and even find models that favor values in the range $\Omega_\Lambda \sim 0.5$–0.8. Kochanek et al. (1998) respond by acknowledging the systematic uncertainties but defending their choice of luminosity function as the one that is most consistent with the observed luminosities of lens galaxies. This controversy will soon be resolved by new measurements of the luminosity function from the Sloan Digital Sky Survey (SDSS) and Two-Degree Field surveys that appear to eliminate most of the traditional problems (Blanton et al. 2001; Cross et al. 2001).

While debating the details, the previous studies agreed on the idea that raising $\Omega_\Lambda$ dramatically increases the expected number of lenses; they agreed on the trend and contested only the zero point. In this Letter I question the trend itself, based on number counts of distant galaxies. I argue that the lens statistics models used to obtain upper limits on $\Omega_\Lambda$ are inconsistent with galaxy counts at $z \sim 0.5$—not with any particular value of the counts but with the general idea that they can be measured and used as constraints on the models. I modify the models to be calibrated by galaxy counts and consider how the new models depend on $\Omega_\Lambda$. For simplicity, I consider only flat cosmologies.

2. FUNDAMENTALS

The optical depth for lensing of sources at redshift $z_s$ is

$$\tau(z_s) = \int_0^{z_s} dz_D D(z_D) \frac{dM}{dM} A(M, z_m, z_s),$$

(1)

where $dn/dM$ is the mass function of deflectors, $A(M, z_m, z_s)$ is the cross section for lensing by a deflector of mass $M$ at redshift $z_m$ and $D(z)$ is the comoving distance. When analyzing real lens data, the optical depth must be modified to account for magnification bias, or the fact that lensing magnification effectively changes a survey’s flux limit (e.g., Turner, Ostriker, & Gott 1984). Magnification bias is largely insensitive to cosmological parameters, so for our purposes the simple optical depth is adequate.

Nearly all arcsecond-scale lenses are produced by detectable galaxies, so many analyses of lens statistics model the deflector population with observed galaxy populations. The luminosity function of galaxies is often parameterized as a Schechter function with the form

$$\frac{dn}{dL} = n_\ast \left( L/L_\ast \right)^{-\alpha} e^{-L/L_\ast},$$

(2)

where $L_\ast$ is a characteristic luminosity, $n_\ast$ is a characteristic comoving number density, and $\alpha$ is the power-law slope at the faint end. Luminosity must be converted to mass for the lensing analyses, and this is usually done with the empirical Faber-Jackson relation $L/L_\ast = (a/\sigma)^{y}$, where $\sigma$ is the velocity dispersion.

Luminosity naturally changes with redshift due to passive evolution: stellar populations fade with time as stars die out. The number density may change with time as a result of mergers. The number evolution can be parameterized in various...
ways, but Lin et al. (1999) propose the form

$$n_s(z) = n_s(0)10^{0.4Pz},$$

so that the number evolution parameter $P$ can be combined with another parameter representing luminosity evolution expressed in observers’ units of magnitudes. I adopt this form since the exact parameterization is not very important for my analysis. Other than luminosity and number evolution, we may assume for simplicity that $\alpha$, $\gamma$, and $\sigma$ are all independent of redshift (see Lin et al. 1999).

Lens galaxies are usually approximated as isothermal spheres, which is consistent with lens data, galaxy dynamics, and X-ray elliptical galaxies (e.g., Fabbiano 1989; Rix et al. 1997; Treu & Koopmans 2002). The lensing optical depth can then be written as (e.g., Kochanek 1993)

$$\tau(z_s) = \tau_0 \int_0^{z_s} \frac{dz}{dz} \left( \frac{D_o D_s}{D_{ls}} \right)^2 n_s(z),$$

$$\tau_0 = 16\pi^3 r_h^3 n_s(0) \left( \frac{\sigma}{c} \right)^4 \Gamma \left( 1 + \frac{\alpha + 4}{\gamma} \right),$$

where $D_o$, $D_s$, and $D_{ls}$ are comoving distances between the observer, lens, and source, written here in units of the Hubble distance $r_h = c/H_0$.

3. RESULTS

If there is no number evolution ($P = 0$), then equation (4) yields the standard result for the lensing optical depth (e.g., Kochanek 1993),

$$\tau(z_s) = \frac{\tau_0}{30} D_{ls}^3,$$

where the optical depth is proportional to the cosmological volume $D_{ls}^3$ and hence very sensitive to $\Omega_\Lambda$, as shown in Figure 1. If we allow for some number evolution but assume that $n_s(z)$ is identical for all cosmologies (as done by Jain et al. 2000), the optical depth curves shift slightly but retain the same qualitative behavior. In other words, if we assume the same number density (whether constant or not) for all cosmologies, then we recover the standard result that the number of lenses increases significantly as $\Omega_\Lambda$ increases, with little sensitivity to the actual amount of number evolution. The increase is large enough that previous analyses of lens statistics were able to obtain interesting upper limits on $\Omega_\Lambda$ even from small lens samples.

Although the focus of the models is the lensing optical depth, a corollary prediction is the number of galaxies per unit redshift per unit area on the sky,

$$\frac{dN}{dz} = n_s(z)D(z)^2 \frac{dD}{dz} \Gamma(1 + \alpha).$$

Figure 2 shows the predicted number counts for the models from Figure 1. If $n_s(z)$ is the same in all cosmologies, then number counts of galaxies at moderate or high redshifts are very sensitive to $\Omega_\Lambda$. For example, at $z = 0.5$, models with $\Omega_\Lambda = 0$ and 0.7 differ by a factor of 2. This is the basis of the classic idea to use galaxy number counts to constrain the cosmology (see Sandage 1998; Driver 2002). Lens statistics boil down in some sense to galaxy counts, where the objects being counted are lens galaxies (mostly massive ellipticals at $z \approx 0.3$–1).

Galaxy counts at moderate redshifts ($z \sim 0.5$–1) can now be measured directly (e.g., Lin et al. 1999; Davis et al. 2001; Eisenstein et al. 2001), so we can imagine using them to test the models. Let us consider not any particular value for the counts, but just the general idea that they can be measured. If the counts at $z \sim 0.5$–1 are measured, then at most one of the curves in Figure 2 can be correct. What happens if instead we insist that all models agree with some observed galaxy counts $dN/dz$? In an idealized situation where number counts could be measured well at all redshifts, the optical depth would be

$$\tau(z_s) = 16\pi^3 \left( \frac{\sigma}{c} \right)^4 \frac{\Gamma(1 + \alpha + 4/\gamma)}{\Gamma(1 + \alpha)} \int_0^{z_s} \frac{dz}{D(z)D_o} n_s(z) dN.$$
In other words, the optical depth would simply be an integral over the observed number counts, weighted by the distance ratio \( D_L/D_d \). The direct dependence on the cosmological volume \( D^3 \) would disappear, leaving only a weak dependence on cosmology through the distance ratio.

In practice, the number counts will be measured well for some finite range of redshifts. Given counts \( dN/dz \) at redshift \( z_{\text{obs}} \) and parameterizing the evolution as in equation (3), we would infer a number evolution parameter \( P \)

\[
P = \frac{2.5}{z} \log \left[ \frac{dN/dz}{n_z(0)\Gamma(1 + \alpha)D^3 dD/dz} \right]_{z_{\text{obs}}}. \tag{9}
\]

Note that this value depends on the assumed cosmology through the volume element \( D^3 dD/dz \). Assuming two different cosmologies (“1” and “2”) would lead to two different values of \( P \) that are related by

\[
P_2 - P_1 = \frac{2.5}{z} \log \left( \frac{D^3_1 dD_1/dz}{D^3_2 dD_2/dz} \right)_{z_{\text{obs}}}. \tag{10}
\]

Thus, when models are calibrated by observed galaxy counts at moderate redshifts, the function \( n_z(z) \) cannot be the same for all cosmologies. The number density is the number counts divided by the cosmological volume element, so it necessarily depends on the assumed cosmological parameters. By using the same \( n_z(z) \) in all cosmologies, traditional models for lens statistics have violated this condition.

Let us construct new models that do not violate it. Imagine that from observed galaxy counts we infer a number evolution parameter \( P \) in a cosmology with \( \Omega_{\Lambda} = 0 \). For other values of \( \Omega_{\Lambda} \) we can use equation (10) to compute the self-consistent value of \( P \), as in Figure 3. Models with different values of \( \Omega_{\Lambda} \) now agree on the predicted number counts at \( z_{\text{obs}} \) (by construction) and are very similar over a wide range of redshifts.

\[
\text{Fig. 3.—Change in the number evolution parameter } P \text{ with } \Omega_{\Lambda} \text{ if the models are calibrated by galaxy number counts at redshift } z_{\text{obs}} = 0.5 \text{ (solid curves) or } z_{\text{obs}} = 1.0 \text{ (dotted curves); see eq. (10). Curves correspond to } P = 1 \text{ (top), } 0 \text{ (middle), and } -1 \text{ (bottom).}
\]

In other words, the new models are consistent in that they agree on an observable quantity, galaxy number counts, independent of \( \Omega_{\Lambda} \).

Figure 5 shows the lensing optical depth as a function of \( \Omega_{\Lambda} \) for these new models. The difference from the old models is striking: the dependence of the optical depth on \( \Omega_{\Lambda} \) is much smaller than before. In the old models, going from \( \Omega_{\Lambda} = 0 \) to 0.7 increased the number of lenses by a factor of 2.7–3.3 (depending on the value of \( P \)), while in the new models the increase is only 40%–43% if the calibration is at \( z_{\text{obs}} = 1.0 \) or a mere 5%–16% if \( z_{\text{obs}} = 0.5 \). Models constrained by galaxy counts were known over a range of redshifts, this expression and eq. (10) would include appropriate averages over \( z_{\text{obs}} \).

4 The various models could be made to match over a wider range of redshifts by introducing more parameters to describe the evolution.

\[
\text{Fig. 4.—Galaxy number counts for new models calibrated by some observed counts at } z_{\text{obs}} = 0.5 \text{ (top) or } z_{\text{obs}} = 1.0 \text{ (bottom). Different line types indicate different values of } \Omega_{\Lambda} \text{. The hypothetical measured counts correspond to } Q_L, \text{ and } \Omega_{\Lambda}/H_0.\text{Note that in the limit } z_{\text{obs}} \to 0 \text{ we would recover Fig. 2.}
\]

\[
\text{Fig. 5.—Similar to Fig. 1, but with results for new models calibrated by galaxy counts at redshift } z_{\text{obs}} = 0.5 \text{ or } 1.0.
\]
counts at all redshifts as in equation (8) (not shown) lie between the $z_{\text{crit}} = 0.5$ and 1.0 curves.

The reduced density on $\Omega_L$ is a direct result of calibrating the models with observed counts of distant galaxies. This qualitative conclusion does not depend on any particular values I have assumed, but arises simply from the idea that models for lens statistics can and should be constrained to agree with measurable galaxy counts at redshifts relevant for lensing. There is some residual sensitivity to cosmology through the distance ratio $D_L/D_M$ (see eq. [8]), but it is much smaller than the volume effect and thus will be harder to detect.

The prospects for using real galaxy counts to calibrate models for lens statistics are good. The Canadian Network for Observational Cosmology 2 field galaxy redshift survey already contains $\sim 5000$ galaxies at $0.12 < z < 0.55$ (Lin et al. 1999), the SDSS Luminous Red Galaxy sample will have redshifts for $\sim 100,000$ early-type galaxies out to $z \sim 0.5$ (Eisenstein et al. 2001), and the Deep Extragalactic Evolutionary Probe redshift survey will include $\sim 60,000$ galaxies at $z > 0.7$ (Davis et al. 2001). The calibration of the models will be limited to some extent by completeness and selection effects, but with large samples it will be possible to make different cuts on the data to understand those effects (see Blanton et al. 2001). The calibration will also be limited by our understanding of the relationship between luminosity and mass—but that has always been true for models of lens statistics. The best bet is probably to turn the problem around: rather than trying to constrain $\Omega_L$, a joint study of galaxy counts and lensing could attempt to distinguish between evolution in luminosity and evolution in mass. That would be perhaps the most interesting future application of lens statistics.

4. CONCLUSIONS

The optical depth for lensing is basically proportional to the number of galaxies on the sky at redshifts $z \sim 0.3$–1. Traditionally that number was not well known, so models for lens statistics adopted a number density of galaxies and multiplied by the cosmological volume to get the number. With the assumed number density held fixed, the expected number of lenses was proportional to the cosmological volume and hence very sensitive to $\Omega_L$.

Number counts of distant galaxies can now be measured directly, and they are inconsistent with the idea that the number density is independent of $\Omega_L$. This general point holds whether there is much or little redshift evolution in the galaxy population. Constraining models for lens statistics to agree on distant galaxy counts makes them far less sensitive to $\Omega_L$. Whereas the old models saw a factor of 3 change in the number of lenses between $\Omega_L = 0$ and 0.7, in the new models the change is $\simeq 10$–40%. Using lens statistics to constrain $\Omega_L$ may still be possible, but it will be difficult.

This Letter has focused on models where the deflector population is derived from observed galaxy populations, which I refer to as phenomenology models. In an alternate class that I call theory models, the deflector population is described with a mass function from structure formation theory; the resulting models are sensitive to cosmology not only through the volume element but also through $\Omega_L$ and the growth of structure. In theory models the predicted number of lenses decreases as $\Omega_L$ increases (for flat cosmologies; Porciani & Madau 2000; Li & Ostriker 2002), which strongly disagrees with old phenomenology models but is less different from my new models. In quintessence models, phenomenology and theory models do agree that making the equation of state more negative increases the predicted number of lenses (Waga & Miceli 1999; Sarbu, Rusin, & Ma 2001). The differences between phenomenology and theory models clearly need further study. Nevertheless, I would argue that the focus of lens statistics should move away from constraining $\Omega_L$ and toward learning about the population of dark matter halos out to $z \sim 1$.

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