Four component magnetized dusty plasma containing non-thermal electrons

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Abstract. Multicomponent plasmas are of great attraction for research in dusty plasmas. In the present research paper, dusty plasma consisting of non-thermal electrons, Maxwellian ions, negatively and positively charged warm adiabatic dust particles, is considered. The Korteweg-de Vries (KdV) equation which describes the basic features of the electrostatic solitary structures is derived by use of reductive perturbation method and solved for solitary wave solution. The effect of externally applied magnetic field and non-thermal electrons is found to modify the properties of the dust acoustic solitary potential significantly. The implications of these results for some space and astrophysical dusty plasma systems especially in planetary ring and cometary tails, are briefly mentioned.

1. Introduction
Since the discovery of the dust acoustic wave (DAW) [1], there has been a great deal of interest in investigating numerous collective processes in dusty plasmas. Specifically, attention has been focused on waves and instabilities as well as coherent nonlinear wave structures in weakly coupled dusty plasmas. The history, occurrence and characterization of dusty plasma in space and laboratory environments are well described and documented in recent monographs [2,3]. In a weakly coupled plasma, the presence of charged dust grains can modify the propagation of existing plasma wave spectra through the quasi-neutrality condition when the dust grains are stationary, whereas the dust dynamics provides the possibility of new wave modes such as dust acoustic wave[1], dust Alfven wave [4] and dust-Whistler wave[5,6] etc. Dust grains are usually negatively charged as they collect electrons from the background plasma [2,3,7,8]. The presence of positively charged dust particles has also been observed in different regions of space such as Cometary tails [9,10,11,12,13], Jupiter's magnetosphere[14], the earth's polar mesosphere [15] and in the Martian atmosphere[16]. There are different mechanisms by which dust grains become positively charged [17,18]. These are (i) photoemission in the presence of a flux of ultra-violet (UV) photons, (ii) thermionic emission induced by the radiative heating, and (iii) secondary emission of electrons from the surface of the dust grains. Wang et. al. [19] have studied the properties of positively and negatively charged dust particles. Rao et. al. [1] are the first to report in theory the existence of dust-acoustic (DA) wave and verified experimentally by Barken et. al. [20]. On the other hand, dusty plasmas containing grains of...
opposite polarity have been investigated theoretically\cite{21,22,23,24,25,26} and experimentally\cite{9,10,14,22,27}. F. Sayed and Mamun \cite{25} studied the basic properties of small amplitude solitary wave structures in cold four-component dusty plasma containing both positive and negative dust particles. On the other hand, Shukla \textit{et. al.} \cite{26} have reported a purely growing instability in positive-negative dusty plasma.

Observations of space plasmas and particle in cell simulation have confirmed that the particle distributions play a crucial role in characterizing the physics of the wave structures. To explain the observation of solitary wave structures with density depression, the role of non-thermal electron distribution on characterization of solitary wave/ solitons have been reported \cite{28,29}. Most of the investigations in dusty plasma containing opposite polarity dust particles have been studied with Maxwellian electron and ion distributions \cite{21,25,30,31}. The non-thermal distributions associated with the particle flows resulting from the force fields present in the space and astrophysical plasmas, have abundance of super thermal particles. Since the electron and ion distributions play important role for the formation of nonlinear structures, it is interesting to study the coherent nonlinear wave structures with non-Maxwellian distribution of electrons/ions. In the present research work, we consider a magnetized dusty plasma system consisting of positively and negatively charged adiabatic dust particles, Maxwellian ions and nonthermally distributed electrons. It is noticed that the presence of opposite polarity dust components not only significantly modify the basic properties of the solitary potential structures, but also leads to existence of the positive and negative solitary potential structures in dusty plasma.

The paper is organized as follows. The basic equations governing the dynamics of dusty plasma system under consideration are given in section 2. Using reductive perturbation method the KdV equation describing the properties of the small amplitude solitary potential structures is derived in section 3. Section 4 is devoted to discussion of the present investigation.

2. **Governing equations**

We consider fully ionized collisionless dusty plasma consisting of warm adiabatic positively and negatively charged dust particles, Maxwellian ions, and nonthermally distributed electrons in an external static magnetic field $B = B_0 \hat{z}$. The behavior of nonlinear wave structure in this plasma system is described by the following set of fluid equations:

\begin{align*}
\frac{\partial n_j}{\partial t} + \nabla \cdot (n_j u_j) &= 0 \\
\frac{\partial u_j}{\partial t} + (u_j, \nabla) u_j - \nabla \phi + \frac{\sigma_1}{n_1} \nabla p_1 + \omega_{1z} (u_1 \times B) &= 0 \\
\frac{\partial u_2}{\partial t} + (u_2, \nabla) u_2 + \alpha \beta \nabla \phi + \frac{\sigma_2}{n_2} \nabla p_2 + \omega_{2z} (u_2 \times B) &= 0
\end{align*}

The subscript $j=1(2)$ corresponds to negatively (positively) charged dust particles. $n_i$ ($n_2$) is the negative (positive) dust number density normalized by its equilibrium value $n_{i0}$ ($n_{20}$). $u_j$ ($u_2$) is the negative (positive) dust fluid speed normalized by its speed $c_i = (z_j k_0 T_i/m_j)^{1/2}$. $\phi$ is wave potential normalized by $(k_i T_i/e)$, $\alpha = z_2/z_1$, $\beta = m_2/m_1$, $\mu = n_0/z_1 n_{10}$, $\mu_2 = n_0/z_2 n_{20}$, $\sigma = T_i/T_e$, $\sigma_1 = 1$ and $\sigma_2 = \beta \mu T_i/T_e$, $n_{00}$ and $n_{00}$ are the equilibrium densities of electrons and ions respectively. $z_1$ ($z_2$) is the number of electrons (protons) residing on a negative (positive) dust particle. $m_j$ ($m_2$) is the mass of the negative (positive) dust particle. $T_i$ ($T_e$) is the temperature of ions.
(electrons), $k_b$ is the Boltzmann constant and $e$ is the electronic charge. The time variable $t$ is normalized by inverse of plasma frequency, $\omega_p^{-1} = (m_1/4\pi n_1 e^2)^{1/2}$ and space variable $\nabla$ is normalized by Debye length $\lambda_D = (z_1 k_b T_{i1}/4\pi n_1 e^2)^{1/2}$ respectively. $\omega_{c1}$ and $\omega_{c2}$ are the negative and positive charged dust cyclotron frequencies normalized to plasma frequency.

The un-normalized Poisson’s equation is given by

$$\nabla^2 \phi = 4\pi e n_e n_e + 4\pi e n_i - 4\pi e n_i$$ (4a)

Where the first term on the right hand side of the above equation is the total contribution due to negatively charged dust particles, second term is the contribution of positively charged dust particles, third term describes the nonthermally distributed electrons and last term represents the ion contribution due to Maxwellian distribution. We disturb the densities of various terms on the right hand side of the above equation into equilibrium values $+ \text{ perturbed quantities}$ and used the charge neutrality of the background plasma to obtain the poisson’s equation in normalized form as follow:

$$\nabla^2 \phi = n_z - (1 + \mu_e - \mu_i) n_z + \mu_e (1 - \beta \sigma \phi + \beta \sigma^2 \phi^2) \exp(\sigma \phi) - \mu_i \exp(-\phi)$$ (4)

$$p_j = C' n_j^\gamma$$ (5)

$C'$ in the above equation is a constant, $\gamma$ is the ratio of the specific heats at constant pressure $c_p$ and constant volume $c_v$.

The nonthermal parameter is defined by:

$$\beta = \frac{4\delta}{1 + 3\delta}$$ (6)

We have followed the model of Cairns et. al. [28,29], non-thermal distribution for the electrons is taken as

$$f(v) = \frac{1}{\sqrt{2\pi (1 + 3\delta)}} (1 + \delta v^4) e^{-v^2/2}$$

The parameter $\delta$ in the distribution function $f(v)$ defines the shape of the distribution and expresses the deviation from the Maxwellian state. This form of the distribution is convenient for the description of various observed particle distributions. For example, when $\delta = 0$, we get Maxwellian distribution and when $\delta \to 1$, it tends to look as two counter streaming beams with cold core distribution [32].

It is worth mentioning that $\gamma = 1 + 2/F$. Here F represents the degrees of freedom [33] and for our model $\gamma = 5/3$.

Gradient of $p_j$ is obtained from equation (5) and is given by

$$\nabla p_j = \frac{5}{3} \nabla n_j^{2/3}$$ (7)

3. Derivation of Korteweg-de Vries (KdV) equation

In the small amplitude approximation we study ion acoustic solitary waves or dust acoustic solitary waves by using reductive perturbation method. This method is basically a weakly nonlinear theory with small but finite amplitude, which leads to scaling of the independent variables through the stretching coordinates [34].

$$\xi = \varepsilon \tau/2 (l_x x + l_y y + l_z z - V_0 t)$$ and $$\tau = \varepsilon^{3/2} t$$
Where \( \varepsilon \) is a small parameter measuring the weakness of dispersion and \( V_0 \) is the solitary wave velocity. \( l_x, l_y \) and \( l_z \) are the direction cosines of the wave vector \( k \) along the x, y and z axis respectively. In reductive perturbation method, we further expand the perturbed quantities about their equilibrium values in the powers of \( \varepsilon \) as follow:

\[
\begin{align*}
n_j &= 1 + \varepsilon n_1^j + \varepsilon^2 n_2^j + \varepsilon^3 n_3^j + \ldots \\
u_{j1} &= 0 + \varepsilon^3 u_1^j + \varepsilon^2 u_2^j + \varepsilon u_3^j + \ldots \\
u_{j2} &= 0 + \varepsilon^3 u_1^j + \varepsilon^2 u_2^j + \varepsilon u_3^j + \ldots \\
u_{j3} &= 0 + \varepsilon^3 u_1^j + \varepsilon^2 u_2^j + \varepsilon u_3^j + \ldots \\
f &= 0 + \varepsilon^3 f^1 + \varepsilon^2 f^2 + \varepsilon f^3 + \ldots
\end{align*}
\]

Using stretched coordinates along with equation (8) in equations (1)-(4) yields the following equations to the lowest order in \( \varepsilon \) as:

\[
\begin{align*}
n_1^1 - (1 + \mu_e - \mu_i)n_2^1 + \mu_e (\sigma - \beta) \phi^1 + \mu_i \phi^1 &= 0 \\
n_2^1 &= -\frac{\mu_e (\sigma - \beta) + \mu_i}{(1 + \alpha \beta_i (1 + \mu_e - \mu_i))} \\
n_3^1 &= -\frac{\mu_e (\sigma - \beta) + \mu_i \alpha \beta_i \phi^1}{(1 + \alpha \beta_i (1 + \mu_e - \mu_i))} \\
u_{1z}^1 &= \frac{l_z \phi^1}{3M - V_0} \\
u_{2z}^1 &= -\frac{l_z \alpha \beta_i \phi^1}{3M - V_0} \\
V_0 &= M l_z \\
M &= \left(\frac{\frac{5}{3} + (1 + \alpha \beta_i (1 + \mu_e - \mu_i))}{(\mu_e (\sigma - \beta) + \mu_i)}\right)^{\frac{1}{2}}
\end{align*}
\]

Equation (14) represents the dispersion relation for a given dusty plasma system.

Similarly the x and y components of the perturbed velocity from the momentum balance equation are given as follows:

\[
\begin{align*}
u_{1x}^1 &= \frac{-l_x}{\omega_{ei}^1} + \frac{5l_y}{3\omega_{ei}^1 \left(\frac{5}{3} - M^2\right)} \frac{\partial \phi^1}{\partial \xi} \\

\end{align*}
\]
Eliminating $u_{1x}', u_{2x}', u_{1y}', u_{2y}'$ from equations (16)-(19) we obtain the next higher order $x$ and $y$ components of perturbed velocity from the momentum balance equation and Poisson’s equation as follows:

\[ u_{2x}^1 = \left[ \frac{\alpha \beta \upsilon_{c1}}{\omega_{c2}} - \frac{5 \alpha \beta \sigma \upsilon_{c2}}{3 \omega_{c2} \left( \frac{5}{3} - M^2 \right)} \right] \frac{\partial \phi^1}{\partial \xi} \tag{17} \]

\[ u_{1y}^1 = \left[ \frac{l_x}{\omega_{c1}} - \frac{5 l_x}{3 \omega_{c1} \left( \frac{5}{3} - M^2 \right)} \right] \frac{\partial \phi^1}{\partial \xi} \tag{18} \]

\[ u_{2y}^1 = \left[ -\frac{\alpha \beta \upsilon_{c1}}{\omega_{c2}} + \frac{5 \alpha \beta \sigma \upsilon_{c2}}{3 \omega_{c2} \left( \frac{5}{3} - M^2 \right)} \right] \frac{\partial \phi^1}{\partial \xi} \tag{19} \]

Using the same procedure we obtain next higher order quantities given as follows:

\[ u_{1x}^2 = -\frac{V_0 \upsilon_{c1} \alpha \beta}{\omega_{c1}^2} \left[ 1 - \frac{1}{(1 - \frac{3M^2}{5})} \right] \frac{\partial^2 \phi^1}{\partial \xi^2} \tag{20} \]

\[ u_{2x}^2 = \frac{V_0 \upsilon_{c1} \alpha \beta}{\omega_{c2}^2} \left[ 1 - \frac{\sigma_2}{(1 - \frac{3M^2}{5})} \right] \frac{\partial^2 \phi^1}{\partial \xi^2} \tag{21} \]

\[ u_{1y}^2 = \frac{V_0 \upsilon_{c1} \alpha \beta}{\omega_{c1}^2} \left[ -1 + \frac{1}{(1 - \frac{3M^2}{5})} \right] \frac{\partial^2 \phi^1}{\partial \xi^2} \tag{22} \]

\[ u_{2y}^2 = \frac{V_0 \upsilon_{c1} \alpha \beta}{\omega_{c2}^2} \left[ 1 - \frac{\sigma_2}{(1 - \frac{3M^2}{5})} \right] \frac{\partial^2 \phi^1}{\partial \xi^2} \tag{23} \]

\[ (l_x'^2 + l_y'^2 + l_z'^2) \frac{\partial^2 \phi^1}{\partial \xi^2} = n_1^2 - (1 + \mu_v - \mu_i) n_2^2 + (\mu_v (\sigma - \beta) + \mu_i) \phi^2 + [\mu_v ((1 - \sigma) \beta + \frac{\sigma^2}{2}) - \frac{\mu_i}{2}] (\phi^1)^2 \tag{24} \]
\begin{equation}
-V_0 \frac{\partial n_1^2}{\partial \xi} + \frac{\partial n_2^1}{\partial \tau} + l_x \frac{\partial u_1^2}{\partial \xi} + l_x \frac{\partial u_2^1}{\partial \xi} + l_x \frac{\partial (u_2^2 + n_1^1 u_1^1)}{\partial \xi} = 0 \tag{26}
\end{equation}

\begin{equation}
-V_0 \frac{\partial u_1^2}{\partial \xi} + \frac{\partial u_1^3}{\partial \tau} + l_x u_1^1 \frac{\partial u_1^1}{\partial \xi} + l_x \frac{\partial \phi^2}{\partial \xi} = \frac{5l_x}{3} \frac{\partial n_2^2}{\partial \xi} + \frac{5l_x}{9} \frac{\partial n_1^1}{\partial \xi} \tag{27}
\end{equation}

\begin{equation}
-V_0 \frac{\partial u_3^2}{\partial \xi} + \frac{\partial u_3^1}{\partial \tau} + l_x u_2^1 \frac{\partial u_2^1}{\partial \xi} + l_x \frac{\partial \phi^2}{\partial \xi} = -l_x \alpha \beta_1 \frac{\partial \phi^2}{\partial \xi} - \frac{5l_x}{3} \frac{\partial n_2^2}{\partial \xi} + \frac{5l_x}{9} \frac{\partial n_1^1}{\partial \xi} \tag{28}
\end{equation}

Equations (9)-(28) can be used to eliminate $\frac{\partial n_1^2}{\partial \xi}, \frac{\partial n_2^1}{\partial \xi}, \frac{\partial u_1^3}{\partial \xi}, \frac{\partial u_2^2}{\partial \xi}, \frac{\partial \phi^2}{\partial \xi}$ and following straightforward but long algebraic manipulations, we finally obtain the following KdV equation:

\begin{equation}
\frac{\partial \phi}{\partial \tau} + a \phi \frac{\partial \phi}{\partial \xi} + b \frac{\partial^3 \phi}{\partial \xi^3} = 0 \tag{29}
\end{equation}

Coefficients ‘$a$’ and ‘$b$’ of nonlinear and dispersive terms appearing in equation (29) are given by following expression

\begin{equation}
a = \frac{1}{C} \left[ \frac{1}{M^2} \left( 3 - \frac{5}{9M^2} \right) \right] \frac{1}{\left( 1 - \frac{5}{9M^2} \right)^2} \frac{1 + \alpha \beta_1 (1 + \mu_x (\sigma - \beta) + \mu_x)}{1 + \alpha \beta_1 (1 + \mu_x (\sigma - \beta) + \mu_x)} \frac{(1 - \alpha^2 \beta_1^2 (1 + \mu_x (\sigma - \beta) + \mu_x) + 2 \mu_x (1 - \sigma) \beta + \frac{\sigma^2}{2} - \mu_x)}{2 \mu_x (1 + \mu_x (\sigma - \beta) + \mu_x)} \tag{30}
\end{equation}

\begin{equation}
b = - \frac{1}{C} \left[ \frac{5 \sigma_2 (1 - l_2^2) \alpha \beta_1 (1 + \mu_x (\sigma - \beta) + \mu_x)}{3 \omega_0^2 (1 + \alpha \beta_1 (1 + \mu_x (\sigma - \beta) + \mu_x))} + \frac{5(1 - l_2^2) (\mu_x (\sigma - \beta) + \mu_x)}{3 \omega_0^2 (1 + \alpha \beta_1 (1 + \mu_x (\sigma - \beta) + \mu_x))} \right] + \frac{M^2 (1 - l_2^2) (\mu_x (\sigma - \beta) + \mu_x)}{\omega_0^2 (1 + \alpha \beta_1 (1 + \mu_x (\sigma - \beta) + \mu_x))} \tag{31}
\end{equation}

Further the expression for ‘$C$’ is given by

\begin{equation}
C = \frac{-2(\mu_x (\sigma - \beta) + \mu_x)^2}{V_0 (1 + \alpha \beta_1 (1 + \mu_x (\sigma - \beta) + \mu_x))} M^2 \tag{32}
\end{equation}
On introducing new variable \( \zeta = \xi - u_0 \tau \), \( u_0 \) is a constant velocity, the soliton solution of equation (29) in the stationary frame is given as

\[
\phi^1 = \phi_m \text{Sech}^2 \left[ \left( \frac{\zeta - u_0 \tau}{w} \right) \right]
\]  \hspace{1cm} (33)

The quantities \( \phi_m \) and \( w \) representing peak amplitude and width of the soliton are given as:

\[
\phi_m = \frac{3u_0}{a} \tag{34}
\]

\[
w = \sqrt{\frac{4b}{u_0}} \tag{35}
\]

4. Discussion

From equations (33)-(35) we find that as \( u_0 \) increases, peak amplitude (width) of the solitary waves for a given plasma system increases (decreases). Solitary potential profile is positive (negative) if \( a > 0 \) \( ( < 0 \) as obvious from the equations (29), (30) and (34). Numerical computation for analyzing the nature of nonlinear coefficient ‘\( a \)’ is performed to obtain contour maps for \( a=0 \), which is shown in figure 1. In this figure \( \beta/ \) as a function of \( \mu_i \) is plotted for the chosen set of parameters such as \( \sigma=0.5, \omega_{i,1}=0.02, \omega_{i,2}=0.025, \beta=0.3 \), for three different values of \( \mu_c \). The parametric regimes for positive and negative solitary potential profiles has been traced out. Apparently the parameter regimes is divided into two regions one for compressive solitons and other for rarefactive solitons. It may be noted that from equations (30) and (34) that the coefficient ‘\( a \)’ of the nonlinearity remains positive for the positive solitary potential profile and correspondingly compressive solitons exist in the given plasma system. However introduction of magnetic field leads to the appearance of compressive and rarefactive solitons. In Figure 2 plots of peak value of amplitude of solitary waves as a function of \( \mu_i \) for parameters such as \( \sigma=0.5, \omega_{i,1}=0.02, \omega_{i,2}=0.025 \) are shown for three different values of non-thermal parameter. As obvious from figure 2, the amplitude of positive solitary potential profile is observed to decrease with the increase of non-thermal parameter. On the contrary the magnitude of the amplitude \( \phi_m \) of negative solitary potential profile representing rarefactive solitons increases with \( \beta \) (non-thermal parameter) as displayed in figure 3. For the sake of completeness, we have also plotted corresponding width as a function of \( \mu_i \) as shown in figure 4 and figure 5. Width of both positive and negative potential profiles increases with increase in \( \beta \) but decreases with increase in \( \mu_i \).

Figure 6 and figure 7 displays the variation of Mach number \( V_0 \) of positive and negative solitary potential regimes. Obviously \( V_0 \) of both positive and negative potential profiles increases with increase in \( \beta \) but decreases with increase in \( \mu_i \).

Sakanaka et. al. [21] observed that the compressive dust-acoustic potential exist only when there is a significant fraction of positively charged dust grains in unmagnetized plasma. Malik and Bharuthram [35] have studied the effect of magnetic field strength, wave propagation angle, particle densities and temperature in a magnetized dusty plasma with two ion species.
Malik et. al. [36] have also observed that only rarefactive solitons can propagate in a magnetized dusty plasma having finite temperature ions, electrons with Maxwellian distribution and negatively charged dust grains. However introduction of nonthermally distributed electrons in the present investigation adds some new salient features. Particularly in this four component magnetized dusty plasma, both compressive and rarefactive dust acoustic solitons are observed depending on the sign of nonlinear coefficient ‘a’ in the KdV equation (29). It is worth mentioning that in the absence of magnetic field and positive dust component, only rarefactive solitons are obtained since ‘a’ is always negative [36,37].

Lastly since dusty plasmas are of ubiquitous nature and magnetic field is usually present in space environment, the results of the present investigation may be of useful help to identify charge coagulation, charge seperation and investigating new wave modes in a magnetized four component dusty plasma system.

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Figure 1: Showing $a=0$ ($\beta_i$ verses $\mu_i$ curves for different values of $\mu_e$)
With $\sigma=0.5$, $\sigma_1=1$, $\sigma_2=1.5$, $\omega_{c1}=0.02$, $\omega_{c2}=0.025$ and $\beta=0.3$
Figure 2: Variation of amplitude $\phi_m$ of positive solitary potential against $\mu_i$ for different $\beta$ with $\sigma = 0.5$, $\sigma_1 = 1$, $\sigma_2 = 1.5$, $\omega_{c1} = 0.02$, $\omega_{c2} = 0.05$ and $\alpha_{\beta} = 1.5$.

Figure 3: Variation of amplitude $\phi_m$ of negative solitary potential against $\mu_i$ for different $\beta$ with $\sigma = 0.5$, $\sigma_1 = 1$, $\sigma_2 = 1.5$, $\omega_{c1} = 0.02$, $\omega_{c2} = 0.05$ and $\alpha_{\beta} = 0.1$. 
Figure 4: Variation of width \( w \) of positive solitary potential against \( \mu_i \) for different \( \beta \) with \( \sigma=0.5, \sigma_1=1, \sigma_2=1.5, \omega_{c1}=0.02, \omega_{c2}=0.025 \) and \( \alpha \beta_1=1.5 \)

Figure 5: Variation of width \( w \) of negative solitary potential against \( \mu_i \) for different \( \beta \) with \( \sigma=0.5, \sigma_1=1, \sigma_2=1.5, \omega_{c1}=0.02, \omega_{c2}=0.025 \) and \( \alpha \beta_1=0.1 \)
$V_0$

$\beta=0.3$

$\beta=0.6$

$\beta=0.9$

Figure 6: Variation of Mach number $V_0$ of positive solitary potential against $\mu_i$ for different $\beta$ with $\sigma=0.5$, $\sigma_1=1$, $\sigma_2=1.5$, $\omega_{c1}=0.02$, $\omega_{c2}=0.025$ and $\alpha_{\beta}=1.5$

$V_0$

$\beta=0.3$

$\beta=0.6$

$\beta=0.9$

Figure 7: Variation of Mach number $V_0$ of negative solitary potential against $\mu_i$ for different $\beta$ with $\sigma=0.5$, $\sigma_1=1$, $\sigma_2=1.5$, $\omega_{c1}=0.02$, $\omega_{c2}=0.025$ and $\alpha_{\beta}=1.1$