Modelling for Water Resources Protection

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Abstract. This paper emphasizes the importance of water resource modelling. Due to the transferability and tradability of water resources, all water users will pursue a cost-effective strategy, thus forming the behavior of rational water use and saving water in subconscious. To increase the reliability of water supply and to provide a hedge against future water price volatilities, financial derivatives are incorporated in the water market and analytical solutions for pricing the European option are provided. This research provides a solution for water management problem if water resource is fully marketized, and the solution is applicable for other resources management such as gas, oil, electricity.

1. Introduction
Water scarcity, including water stress, water shortage and water crisis will be exacerbated in the foreseeable future, due to major challenges such as population growth, economic recovery, drought, and climate change ([1]). It is also an important environmental challenge. According to the world bank, water scarcity has already had impact on 40 percent of the global population and most countries are facing pressure on water resources. It is predicted that by 2030, there is a 40 percent shortfall between demand and supply of water around the world. For example, the Figure 1 shows the rapid increase of population and decrease of per capita water supply in India ([2]), which clearly demonstrates that per capita water supply will be only one third in 2050 compared to 1955.

![Figure 1 The rapid increase of population and decrease of per capita water supply in India](image)

It is of vital importance to better manage water resource. One good way is to marketize water resources. Water markets have already been developed in some regions in the United States and Australia to some extent. To meet the creasing water demands with limited resources, a well-established water trading market for water is anticipated to emerge in the near future. Prospective studies on water resources optimization under financial forces are therefore imperative.
The marketization of water is a solution for water shortage, which has been promoted as one of the most efficient ways to reallocate water, but has also been subject to criticism due to its possible social, economic and environmental impacts. Taking the advantages and benefits of water market while limiting its possible downsides is a key focus. For example, the marketization of water encourages more sufficient market competition which accelerates the development of sewage treatment and desalination industry. The success of the marketization of water in Australia provides another sophisticated example for the whole world, especially in the situation of water shortage. The emergence and expansion of water market in Australia results in a more efficient and sustainable way for water management and allocation ([3]). In the southern Murray-Darling Basin (MDB), where over 90% water trading occurs, the trading has expanded rapidly over the past decades, reaching an annual turnover of 3 billion in 2009 – 2010 ([4]). Some studies have focused on water market option pricing problems, such as Michelsen and Young ([5]), Villinski ([6]), Hasen et al ([7]), Plummer and Schreider ([8]) et al.

The next sections will be organised in the following: Section 2 will pop up the research questions and hypothesis. Section 3 will focus on the mathematical formulation for pricing the water resource derivatives. Section 4 will provide conclusions, merits of this research as well as the applicable applications in other resources.

2. Research Questions
Our research project will focus on modelling water resources under a completely marketized water market assumption. We aim to incorporate financial derivatives in the water market to better manage the water resources, including reducing the risk and variation of water prices and water supplies. Our hypothesis is that with a completely marketized water market in the future in the world, the no-arbitrage trading provides a more reasonable market value for water and a more proper way to reallocate the water resources, which will lead to the saving and effective use of water resources. We note that the commodity trading market has proven that marketization is an efficient way to solve the shortage of resources and materials.

3. Methodology
Suppose we have a lake containing $V_0 m^3$ of water at time 0, with the total price of the whole water of $S_0$. Thus, the water price per unit at time 0 is $P_0 = \frac{S_0}{V_0}$. At time $t$, we have $P_t = \frac{S_t}{V_t}$ where $S_t$ follows geometric Brownion motion, which is

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

Due to the dynamic of water movement (see Figure 2), we let

$$dV_t = dI_t - dO_t$$

where $I_t$ is the water inflow including rain and inflows from streams, ground water, reservoir, etc. And $O_t$, the water outflow contains evaporation, leaking, and other water usage.

Figure 2

To hedge the water price fluctuations in the future, the concept of option, which is a contract giving the buyer the right, not the obligation, to buy or sell an underlying assets at a strike price prior to or on the expiration data [ref wiki], can be brought to the water market. For example, at time 0, if we predict
the future water price per unit will increase at time $T$, we would like to buy a European call option with strike price $K$ and duration $T$. If at time $T$, the water price per unit $P_T$ is higher than $K$, we can exercise the call option and purchase at price $K$, however, if $P_T$ is lower than $K$, we can discard the option. Thus, the value of this call option at time $T$, is $(P_T - K)^+$, and the option price at time 0 is

$$C = e^{-rT}E[(P_T - K)^+]$$

where $r$ is the risk free interest rate. Thus, how to obtain the value of $C$ becomes the major problem.

Recall that $dP_t = \frac{dS}{V_t}$, if we assume $V_t = V_0$ is a constant value, we can obtain the value $C$ by using Black-Shole model ([9]), which gives:

$$C = V_0 P_0 N(d_1) - Ke^{-rT}N(d_2)$$

where
- $P_0$ is the water price per unit at time 0;
- $C$ is the price of the call option as a formula of water price per unit and time;
- $K$ is the strike price;
- $T$ is the time to maturity;
- $N(d_1)$ and $N(d_2)$ are cumulative distribution of a standard normal distribution with

$$d_1 = \frac{\ln V_0 P_0}{\sigma \sqrt{T}} + \frac{r - \frac{1}{2} \sigma^2}{\sigma \sqrt{T}}$$
$$d_2 = d_1 - \sigma \sqrt{T}$$

where
- $\sigma$ is the underlying volatility;
- $r$ is the risk-free interest rate.

In reality, $V_t$ is more likely to be a dynamic process, thus, we have

$$dP_t = \frac{1}{V_t} dS_t + S_t \frac{1}{V_t} d\frac{1}{V_t} dS_t$$

As $dV_t = dI_t - dO_t$, and the outflow process can be written as $dO_t = dE_t + dM_t$, where $E_t$ is the evaporation process and $M_t$ is other water outflows such as leaking and other usage. For a standardized measure height, $E_t$ follows modified Penman equation ([10]), which gives

$$E_t = 10^{-3} \frac{m_t R_t + 6.43 y_t (1 + 0.536 U_t) \delta e_t}{\lambda_t (m_t + y_t)}$$

where
- $E_t$ is potential evaporation (mm);
- $S$ is the area of the surface ($m^2$);
- $\Delta R_t$ is net radiation at the surface ($$MJ/m^2$ per time unit);
- $e_{st} = \frac{101}{760} \left( \frac{21.07}{T_{st}} \right)^{\frac{5.26}{T_{st}}} \text{kPa}$ is saturated vapor pressure of air (kPa);
- $m = \frac{d e_{st}}{d T_{st}} = \frac{709.126}{T_{st}}$ is the slope of the saturation vapor pressure curve (kPa/K);
- $y_t = \frac{0.0016286 \delta}{\lambda_t}$ is psychrometric coefficient (kPa/K);
- $\delta e_t = e_{st} - e_{at} = \frac{p_{ka}(T_{at} - T_{st})}{\lambda_t}$ is vapor pressure deficit (kPa);
- $C$ is specific heat at constant pressure;
- $\lambda_t = 2.501 - 0.002361 T_{at}$ is latent heat of vaporization (MJ/kg);
- $U_t$ is wind function (m/s).

We assume $U_t$ follows autorecorrelated Weibull distributed stochastic process ([11]), which is

$$U_t = g(x_t) = R_i^{-1} \left[ \Phi \left( \frac{x_t}{b/\sqrt{2a}} \right) \right]$$

where $d x_t = -a x_t dt + b dW_t$, and the air temperature $T_{at}$ follows Levy-based Ornstein-Uhlenbeck model ([12]) of

$$d T_{at} = dq_t + k(T_{at} - q_t) dt + \sigma_t dL_t$$
where $q_t = At + Bt + C \sin(\omega t + \phi)$ and $L_t$ is Levy noise. The surface water temperature $T_{st}$ in lakes can be converted by air temperature, $T_{at}$, through a lumped model ([13]), which gives

$$dT_{st} = \frac{1}{\delta} \{ a_1 + a_2 T_{at} - a_3 T_{st} \} dt$$

where

$$\delta = \exp \left( \frac{T_{at} - T_{st}}{a_4} \right), \quad T_{st} \geq T_{at}$$

$$\delta = 1, \quad T_{st} < T_{at}$$

and $a_1$ to $a_4$ are parameters with physical significance.

Similarity, there are formulations for $I_t$ and $M_t$, which gives an explicit solution of $dV_t$. Further research is needed for search reasonable formulations for $I_t$ and $M_t$, and the provide an analytical or numerical solution for the value of the European call option. Other financial derivatives such as European put option, American call/put option, forward/future contract can also be incorporated to hedge the water price, and this will be the future research direction.

4. Conclusion Remarks

The marketization of water has emerged and expanded in some regions, and will eventually be the best solution to the water shortage problem. The incorporation of financial derivatives in the water market increases the reliability of water supply, provides a hedge against future water price volatilities, raises current revenue for the government, industry and investors, and guarantees a minimum future price to the customers. This paper purposes one type of financial derivative, European call option to hedge water price per unit, and the analytical solution of the call option price is provided under certain circumstance, as well as points out the future research direction. This research is applicable for other financial derivatives, and can be widely applied to other resource management such as carbon emission problem, oil, electricity.

Reference

[1] Worldbank. (2019) Water. https://www.worldbank.org/en/topic/water/overview.
[2] Rajat C., Jagjit K., Harpreet K. (2019) Water Shortage Challenges and a Way Forward in India. American Water Works Association, 111(5):42-49.
[3] Australian Government National Water Commission. (2011) Water markets in australia.
[4] National Water Commission (NWC). (2011) Strengthening australia's water markets.
[5] Michelsen A., Young R. (1993) Optioning agricultural water rights for urban water supplies during drought. American Journal of Agricultural Economics, 75:1010-1020.
[6] Villinski M.. (1994) A methodology for valuing multiple-exercise options contracts for water. Centre for International Food and Agricultural Policy, University of Minnesota, Minneapolis.
[7] Hansen K., Howitt R., Williams J. (2006) Implementing options markets in California to manage water supply uncertainty. Paper presented at the American Agricultural Economics Association Annual Meeting, University of California, Davis.
[8] Plummer J., Schreider S.(2015) Predicting inter-season price jumps in the market for temporary water allocations. Journal of Hydrology, 525:676-683.
[9] Black F., Scholes M. (1973). The Pricing of Options and Corporate Liabilities. Journal of Political Economy. 81 (3): 637–654. doi:10.1086/260062.
[10] Shuttleworth, J. (2007) Putting the vap’ into evaporation http://www.hydrol-earth-syst-sci.net/11/210/2007/hess-11-210-2007.pdf.
[11] Bowden G.J., Barker P.R., Shetopatal V.O., Twidell J.W. (1983) The Weibull Distribution Function and Wind Power Statistics. Wind Engineering. Vol. 7, No. 2, pp 85-98.
[12] Benth F.E., Benth J.S. (2005) Stochastic Modelling of Temperature Variations with a View Towards Weather Derivatives. Applied Mathematical Finance 12(1):53-85.
[13] Piccolroaz S., Toffolon M., Majone B. (2013) A simple lumped model to convert air temperature into surface water temperature in lakes. Hydrology and Earth System Sciences Discussions 10(3):2697-2741.