NMSSM in TeV-scale mirage mediation

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We study the next-to-minimal supersymmetric standard model with the TeV scale mirage mediation. The 125 GeV Higgs boson mass is realized with $O(10)$% tuning for 1.5 TeV gluino and 1TeV stop masses. This talk is based on Refs.[1, 2].

I. INTRODUCTION

Supersymmetric extension of the standard model is one of interesting candidates for the physics beyond the standard model. The LHC Run I did not find any superpartners, but put lower bounds for superpartner masses, e.g. about 1.5 TeV for the gluino mass and 700 GeV for the stop mass.

Within the framework of the minimal supersymmetric standard model (MSSM), the $m_Z$-boson mass, $m_Z$, is obtained as

$$m_Z^2 \simeq -2m_{H_u}^2 + \frac{2}{\tan^2 \beta} m_{H_d}^2 - 2\mu^2.$$  

Here, $m_{H_u}^2$ and $m_{H_d}^2$ are the soft supersymmetry (SUSY) breaking scalar mass squared of the up-sector and down-sector Higgs fields. On the other hand, $\mu$ is the supersymmetric Higgs and higgsino mass and the corresponding superpotential term is written by

$$W_{\text{MSSM-Higgs}} = \mu H_u H_d,$$

where $H_u$ and $H_d$ are up-sector and down-sector Higgs superfields. The gluino mass $M_3$ is dominant in radiative corrections on $m_{H_u}^2$ for many models, e.g. the constrained MSSM, and then we obtain $m_{H_u}^2 \sim -M_3^2$. Thus, if the gluino mass as well as the stop mass is of $O(1)$TeV, we need fine-tuning among $m_{H_u}^2$, $m_{H_d}^2$ and $\mu^2$ to derive the correct value of $m_Z$. Indeed, the stop mass is required to be of $O(1)$TeV or more in order to realize the 125 GeV Higgs mass. Furthermore, in the MSSM, $\mu$ is the supersymmetric mass and there is no reason why $\mu$ is of the same order as soft SUSY breaking masses. That is the so-called $\mu$-problem [3].

The $\mu$-problem can be solved by extending the MSSM to the next-to-minimal supersymmetric standard model (NMSSM), where we add the singlet chiral multiplet $S$ [4] (see for review e.g. [5]). Then we can write the following superpotential terms including $S$,

$$W_{\text{NMSSM-Higgs}} = \lambda S H_u H_d + \frac{\kappa}{3} S^3.$$  

We forbid the above $\mu$-term as well as the supersymmetric mass term of $S$ by assuming the $Z_3$ symmetry. Thus, there is no supersymmetric mass terms. By analyzing the scalar potential, we can determine the vacuum expectation value (VEV) of $S$, which depends only on soft SUSY breaking parameters. Then, we can obtain the effective $\mu$-term, $\mu = \lambda <S>$, which is of the same order as other soft SUSY breaking parameters. Even in the NMSSM, we face the fine-tuning problem when $m_{H_u}^2 \sim -M_3^2$ and the gluino mass as well as the stop mass is of $O(1)$TeV.

The concrete behavior of radiative corrections on $m_{H_u}^2$ as well as other soft masses depends on explicit spectrum of superpartners, that is, the mediation mechanism of SUSY breaking. The mirage mediation is one of interesting mediation mechanisms [6-8], and it is a mixture between anomaly mediation and modulus mediation. In the mirage mediation, the radiative corrections and anomaly mediation cancel each other at a certain energy scale, where the SUSY spectrum appears as the pure modulus mediation. Such an energy scale is called the mirage scale. The TeV scale mirage mediation sets this energy scale around TeV scale. Then, $m_{H_u}^2$ has no large radiative corrections due to $M_3$. Indeed, it was pointed out that the TeV scale mirage mediation can ameliorate the fine tuning problem in the MSSM [9-11]. (Non-universal gaugino masses with a certain ratio may be useful to ameliorate fine-tuning [12].)

In this talk, we apply the TeV scale mirage mediation to the NMSSM in order to improve the fine-tuning problem with deriving the 125 GeV Higgs mass.
II. TEV SCALE MIRAGE MEDIATION

Here we give a brief review on the mirage mediation. The mirage mediation is the mixture of the modulus mediation and the anomaly mediation with a certain ratio, which would be determined by the modulus stabilization mechanism and SUSY breaking mechanism. In the mirage mediation, the gaugino masses are written by

\[ M_a(M_{\text{GUT}}) = M_0 + \frac{m_{3/2}}{8\pi^2} b_a g_a^2, \]

where \( g_a \) and \( b_a \) are the gauge couplings and their \( \beta \) function coefficients, and \( m_{3/2} \) denotes the gravitino mass. We assume that the initial conditions of our SUSY breaking parameters are input at the GUT scale, \( M_{\text{GUT}} = 2 \times 10^{16} \text{ GeV} \). The first term, \( M_0 \), in the right hand side denotes the gaugino mass due to the pure modulus mediation, while the second term corresponds to the anomaly mediation contribution. In addition, we can write the soft scalar masses \( m_i \) of matter fields \( \phi^i \) and the so-called \( A \)-terms of \( \phi^i \phi^j \phi^k \) corresponding to the Yukawa couplings \( y_{ijk} \) as

\[ A_{ijk}(M_{\text{GUT}}) = a_{ijk} M_0 - (\gamma_i + \gamma_j + \gamma_k) \frac{m_{3/2}}{8\pi^2}, \]

\[ m_i^2(M_{\text{GUT}}) = c_i M_0^2 - \gamma_i \frac{m_{3/2}}{8\pi^2} - \frac{m_{3/2}}{8\pi^2} M_0 \theta_i, \]

where

\[ \gamma_i = 2 \sum_a g_a^2 C_a^0(\phi^i) - \frac{1}{2} \sum_j |y_{ijjk}|^2, \quad \theta_i = 4 \sum_a g_a^2 C_a^0(\phi^i) - \sum_j a_{ijk} |y_{ijjk}|^2, \quad \gamma_i = 8\pi^2 \frac{d\gamma_i}{d\ln \mu_R}. \]

Here, \( \gamma_i \) denotes the anomalous dimensions of \( \phi^i \) and \( C_a^0(\phi^i) \) denotes the quadratic Casimir corresponding to the representation of the matter field \( \phi^i \). In the right hand side, \( a_{ijk} M_0 \) and \( c_i M_0^2 \) denote the \( A \)-term and soft scalar mass squared due to the pure modulus mediation. These coefficients, \( a_{ijk} \) and \( c_i \), are determined by modulus-dependence of the Kähler metric of \( \phi^i, \phi^j \) and \( \phi^k \) as well as Yukawa couplings. Indeed, by using the tree-level Kähler metric, the coefficient \( c_i \) is explicitly calculated as a fractional number such as 0, 1, 1/2, 1/3. We would have \( O(1/\pi^2) \) of corrections on \( c_i \) due to the one-loop corrections on the Kähler metric. Such a correction would be important when \( c_i = 0 \), but that is model-dependent. Here, we consider the case with

\[ a_{ijk} = c_i + c_j + c_k. \]

It is convenient to use the following parameter \( \alpha \),

\[ \alpha \equiv \frac{m_{3/2}}{M_0 \ln(M_{\text{pl}}/m_{3/2})}, \]

representing the ratio of the anomaly mediation to the modulus mediation. Here \( M_{\text{pl}} \) is the reduced Planck scale.

The mirage mediation has a very important energy scale, that is, the mirage scale defined by

\[ M_{\text{mir}} = \left( M_{\text{GUT}} / M_{\text{pl}} \right)^{\alpha/2}. \]

The above spectrum of the gaugino masses at \( M_{\text{GUT}} \) leads to

\[ M_a(M_{\text{mir}}) = M_0, \]

at the mirage scale. That is, the anomaly mediation contributions and the radiative corrections cancel each other, and the pure modulus mediation appears at the mirage scale. Furthermore, the \( A \)-terms and the scalar mass squared also satisfy

\[ A_{ijk}(M_{\text{mir}}) = (c_i + c_j + c_k) M_0, \quad m_i^2(M_{\text{mir}}) = c_i M_0^2, \]

if the corresponding Yukawa couplings are small enough or if the following conditions are satisfied,

\[ a_{ijk} = c_i + c_j + c_k = 1, \]

for non-vanishing Yukawa couplings, \( y_{ijk} \).

When \( \alpha = 2 \), the mirage scale \( M_{\text{mir}} \) is around 1 TeV. Then, the above spectrum \( \mathbf{(10)} \) and \( \mathbf{(11)} \) is obtained around the TeV scale. That is the TeV scale mirage mediation scenario. In particular, there would appear a large gap between \( M_0 \) and the scalar mass \( m_i \) with \( c_i \approx 0 \). We will apply the TeV scale mirage scenario to the NMSSM in the next section.
III. NMSSM IN TEV SCALE MIRAGE MEDIATION

A. NMSSM

Here, we briefly review on the NMSSM, in particular its Higgs sector. We extend the MSSM by adding a singlet chiral multiplet $S$ and imposing a $Z_3$ symmetry. Then, the superpotential of the Higgs sector is written as Eq. (3). Here and hereafter, for $S$, $H_u$ and $H_d$ we use the convention that the superfield and its lowest component are denoted by the same letter.

The following soft SUSY breaking terms in the Higgs sector are induced,

$$
V_{\text{soft}} = m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 - \lambda A_S S H_u H_d + \frac{\kappa}{3} A_s S^3 + \text{h.c.}
$$

(13)

Then, the scalar potential of the neutral Higgs fields is given as

$$
V = \chi^2 |S|^2 (|H_u^0|^2 + |H_d^0|^2) + |\kappa S^2 - \lambda H_u^0 H_d^0|^2 + V_D + m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 - \lambda A_S S H_u H_d + \frac{\kappa}{3} A_s S^3 + \text{h.c.,}
$$

(14)

with

$$
V_D = \frac{1}{8} (g_1^2 + g_2^2) (|H_d^0|^2 - |H_u^0|^2)^2,
$$

(15)

where $g_1$ and $g_2$ denote the gauge couplings of $U(1)_Y$ and $SU(2)$. Similarly, there appear the soft SUSY breaking terms including squarks and sleptons as well as gaugino masses. These are the same as those in the MSSM.

The minimum of the Higgs potential is obtained by analyzing the stationary conditions of the Higgs potential,

$$
\frac{\partial V}{\partial H_u^0} = \frac{\partial V}{\partial H_d^0} = \frac{\partial V}{\partial S} = 0.
$$

(16)

Here, we denote VEVs as

$$
v^2 = \langle |H_d^0|^2 \rangle + \langle |H_u^0|^2 \rangle, \quad \tan \beta = \frac{\langle H_u^0 \rangle}{\langle H_d^0 \rangle}, \quad s = \langle S \rangle.
$$

(17)

Using the above stationary conditions, we obtain the $Z$ boson mass $m_Z^2 = \frac{1}{2} g^2 v^2$ as

$$
m_Z^2 = 1 - \cos 2\beta m_{H_u} - 1 + \cos 2\beta m_{H_d} - 2\mu^2,
$$

(18)

where $\mu = \lambda s$. For $\tan \beta \gg 1$, this equation becomes Eq. (1). That is, this relation is the same as the one in the MSSM. Thus, the natural values of $|m_{H_u}|$ and $|\mu|$ would be of $\mathcal{O}(100)$ GeV. Furthermore, the natural value of $|m_{H_d}|/\tan \beta$ would be of $\mathcal{O}(100)$ GeV or smaller. Alternatively, $|\mu|$ and $|m_{H_u}|/\tan \beta$ could be larger than $\mathcal{O}(100)$ GeV when $\mu^2$ and $m_{H_u}^2$ are canceled each other in the above relation at a certain level. Even in such a case, $|m_{H_d}|$ would be naturally of $\mathcal{O}(100)$ GeV. On the other hand, other sfermion masses as well as gaugino masses must be heavy as the recent LHC results suggested. To realize such a spectrum, we apply the TeV scale mirage mediation in the next section, where we take $c_{H_u} = 0$ to realize a suppressed value of $|m_{H_u}|$ compared with $M_0$.

B. TeV scale mirage mediation

Here, we apply the TeV scalar mirage mediation scenario to the NMSSM and study its phenomenological aspects. Soft SUSY breaking terms are obtained through the generic formulas (1) and (2) with taking $\alpha = 2$. We concentrate on the Higgs sector as well as gauginos and stops.

A concrete model in the mirage mediation is fixed by choosing $c_i$. We consider the following values of $c_i$,

$$
c_{H_d} = 1, \quad c_{H_u} = 0, \quad c_S = 0, \quad c_{t_L} = c_{t_R} = \frac{1}{2},
$$

(19)
up to one-loop corrections. The above assignment of $c_i$ (19) satisfies the condition (12) for the top Yukawa coupling, and the coupling $\lambda$, but not for the coupling $\kappa$. However, we do not consider a large value of $\kappa$ to avoid the blow-up of $\kappa$ and $\lambda$. Thus, we obtain the following values,

$$A_t \approx A_\lambda \approx M_0, \quad m_{H_u}^2 \approx M_0^2, \quad m_{H_d}^2 \approx m_{t_R}^2 \approx \frac{1}{2} M_0^2,$$

up to $\mathcal{O}(\kappa^2/8\pi^2)$ at the TeV scale. Similarly, at the TeV scale we can obtain

$$m_{H_u}^2 \approx 0, \quad m_{S}^2 \approx 0, \quad |A_\kappa|^2 \approx 0,$$

up to $\mathcal{O}(M_0/8\pi^2)$. That is, values of $|A_\kappa|^2$, $m_{H_u}^2$ and $m_S^2$ are suppressed compared with $M_0^2$, and their explicit values depend on the one-loop corrections on the Kähler metric. Thus, we use $A_\lambda$ as a free parameter, which must be small compared with $M_0$. In addition, we determine the values of $m_{H_u}^2$, $m_S^2$ and $\mu (= \lambda \kappa)$ at the electroweak scale from the stationary conditions, (16), where we use the experimental value $m_Z = \sqrt{2} g v = 91.19$ GeV and $\tan \beta$ as a free parameter.

Through the above procedure, the parameters, $m_{H_u}^2$, $m_S^2$ and $\mu$, at the electroweak scale are expressed by $\tan \beta$, $m_{H_u}^2$, $A_\lambda$. For $\tan \beta \gg \max(1, \kappa/\lambda)$, these parameters are approximated as,

\begin{equation}
\mu = \lambda(S) \sim \frac{m_{H_u}^2}{A_\lambda \tan \beta},
\end{equation}

\begin{equation}
m_S^2 \sim -2 \left( \frac{\kappa}{\lambda} \right)^2 \left( \frac{m_{H_d}^2}{A_\lambda \tan \beta} \right)^2 - \left( \frac{\kappa}{\lambda} \right) A_\kappa \left( \frac{m_{H_u}^2}{A_\lambda \tan \beta} \right) + \frac{\lambda^2}{2} \frac{A_\lambda^2}{m_{H_d}^2} m_Z^2,
\end{equation}

\begin{equation}
m_{H_u}^2 \sim \frac{m_{H_d}^2}{\tan^2 \beta} - \frac{m_{H_d}^2}{A_\lambda^2 \tan^2 \beta} = \frac{m_Z^2}{2}.
\end{equation}

See for more precise results Refs. [1, 2]. When $\tan \beta = \mathcal{O}(10)$, the values of $\mu$, $|m_{H_u}|$ and $|m_S|$ are smaller than $M_0$ by the factor $\tan \beta$ because $m_{H_d} \sim A_\lambda \approx M_0$. Thus, the values of $\mu$ and $|m_{H_u}|$ could be of $\mathcal{O}(100)$ GeV while the other masses of the superpartners are of $\mathcal{O}(M_0)$ = $\Omega(1)$ TeV. Then, the fine-tuning problem can be ameliorated. Furthermore, one can see that the first and the second terms in the last equation cancel each other for our choice of $c_i$. The next leading contributions are of $\mathcal{O}(m_{H_u}^2/\tan^4 \beta)$ or $\mathcal{O}(m_{H_u}^2/\mu/\tan^2 \beta A_\lambda)$. Thus, $m_Z^2$ is almost determined by $m_{H_u}^2$ alone and insensitive to the value of $\mu$. This means that actually $\tan \beta \approx 3$ is enough to obtain the fine-tuning of $|\partial \ln m_Z^2/\partial \ln m_{H_u}^2|^{-1} = m_Z^2/2 m_{H_u}^2 = \mathcal{O}(100)\%$ for $M_0 \approx 1$ TeV. In this case, $\mu$ can be as heavy as $\mathcal{O}(400)$ GeV without introducing further fine-tuning.

C. Higgs sector and fine-tuning

Here, we show some numerical results on the Higgs sector and fine-tuning. (See in detail Refs. [1, 2]) Figure 1 shows masses of the two light CP-even Higgs bosons, $h_1$ and $h_2$, as a function of $\lambda$ and $\kappa$ for $\tan \beta = 10$ and $M_0 = 1500$ GeV. The red curve in the figure indicates the boundary where $\lambda$ and $\kappa$ blow up at the Planck scale. Inside this curve the model remains perturbative until the Planck scale. The yellow region is disfavored due to the false vacuum (see in detail [13]), the lightest CP-even Higgs boson mass can not reach 125 GeV. Instead, the second lightest CP-even Higgs boson mass can be 125 GeV. In this region, the coupling squared to the gauge boson indicates that $h_1$ almost consists of the singlet and hence $h_2$ is almost doublet. For small $\tan \beta$, e.g. $\tan \beta = 3$, there is a parameter region, where the lightest Higgs boson has 125 GeV mass, too.

We numerically estimate the degree of fine-tuning of the electroweak symmetry breaking in our model. Following the standard lore, we define the fine-tuning measure of a observable $y$ against an input parameter $x$ as,

$$\Delta y = \frac{\partial \ln y}{\partial \ln x}.$$
To evaluate the electroweak symmetry breaking, we set $y = M_Z^2$ and as $x$, we take $\lambda, \kappa$ and the small parameters, $m_{H_u}, A_\lambda$ and $A_\kappa$ at the SUSY scale. Note that the large parameters such as $m_{H_d}, A_\lambda$ are not free parameters in our model and fixed by the ultraviolet physics. Figure 2 shows the fine-tuning measures $\Delta M^2_z$ for the $\tan\beta = 10$ case in the parameter region leading to $124 \text{GeV} < m_{h_1} < 126 \text{GeV}$. In the left panel, we take $M_0 = 1500 \text{ GeV}$ and $A_\kappa = -100 \text{ GeV}$. The fine-tuning measures $\Delta M^2_z$ is of $\mathcal{O}(10)$ or smaller. The worst value is obtained for $\Delta M^2_z$ with $x = m_{H_u}^2$. However, the severe fine-tuning is not required, but $\mathcal{O}(10)\%$ tuning is enough for any parameter $x$. The right panel shows $\Delta M^2_z$ for $M_0 = 3 \text{ TeV}$ and $M_0 = 5 \text{ TeV}$ ($\tan\beta = 20$) compared with $M_0 = 1.5 \text{ TeV}$ ($\tan\beta = 10$) case. It is remarkable that only $\mathcal{O}(1)\%$ fine-tuning is required even for such a heavy spectrum. That is, even the 5 TeV case is acceptable in the standard of the conventional models build around 1 TeV such as the constrained MSSM.

For small $\tan\beta$, e.g. $\tan\beta = 3$, we can realize a similar value of fine-tuning measures, $\Delta M^2_z \leq \mathcal{O}(10)$ in the parameter region leading to $124 \text{GeV} < m_{h_1} < 126 \text{GeV}$ for the lightest Higgs mass. (See in detail Refs. [1, 2].)

All of three gaugino masses are equal to $M_0$ and heavy, and squark and slepton masses are similarly heavy. The value of $\mu$ can vary from 100 GeV to about 500 GeV without introducing further fine-tuning. The singlino mass is similar to $\mu$. Then, their mixing would be the lightest supersymmetric particle.
IV. CONCLUSION

We have studied the NMSSM in the TeV scale mirage mediation. We can realize the 125 GeV Higgs mass without severe fine-tuning. $\mathcal{O}(10)\%$ tuning is sufficient in our model, where cancellation between the first and the second terms in Eq. (23c) is significant. The size of $\mu$ is insensitive to $m_Z$, and it can be of $\mathcal{O}(400)$ GeV without introducing further fine-tuning.

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