A joint reconstruction of dark energy and modified growth evolution

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We present the first combined non-parametric reconstruction of the three time-dependent functions that capture departures from the standard cosmological model, ΛCDM, in the expansion history and gravitational effects on matter and light from the currently available combination of the background and large scale structure data. We perform the reconstruction with and without a theory-informed prior, built on the general Horndeski class of scalar-tensor theories, that correlates the three functions. We find that the combination of all data can constrain 15 combined eigen-modes of the three functions with respect to the prior, allowing for an informative reconstruction of the cosmological model featuring non-trivial time-dependences. We interpret the latter in the context of the well-known tensions between some of the datasets within ΛCDM, along with discussing implications of our reconstruction for modified gravity theories.

I. INTRODUCTION

Rapidly improving data from cosmological surveys is opening new opportunities for testing the pillars of the Λ Cold Dark Matter (ΛCDM) model. Along with probing the nature of dark matter and dark energy (DE), it is becoming possible to examine the foundational principles of General Relativity (GR), such as the universal geometric nature of gravity and the precise way in which matter distorts spacetime. Since the discovery of cosmic acceleration [1, 2], significant effort went into constraining the dynamics of DE, primarily by looking for deviations of its equation of state from \( w_\Lambda = -1 \). The past decade and a half also witnessed the emergence and maturing of the field of cosmological tests of GR, which led to identifying broad classes of potentially interesting modified gravity (MG) theories (see [3–7] for reviews) and developing phenomenological frameworks for non-model-specific tests [8–17] along with their numerical implementations [18–22]. Testing gravity and the physics of DE is one of the primary science goals of the ongoing and upcoming surveys, such as DESI [23], Euclid [24] and Vera Rubin Observatory [25], which will take these tests to qualitatively higher levels [26–28].

So far, the majority of phenomenological tests of DE dynamics and departures from GR were conducted independently from each other. Namely, GR would be assumed when constraining the evolution of \( w \) with redshift \( z \), or \( w = -1 \) would be assumed when constraining the MG effects in the growth of structure, parameterized, e.g., by phenomenological functions \( \mu(k, z) \) and \( \Sigma(k, z) \) [29]. In addition, in most cases, fixed simple parametric forms were used for \( w(z) \), \( \mu(z) \) and \( \Sigma(z) \) or their equivalents. Such a simplification may be justified when the constraining power of the data is limited – e.g. measurements of the cosmic microwave background (CMB) temperature and polarization anisotropies only constrain a weighted average of \( w(z) \), hence it makes sense to fit a constant \( w \) to CMB alone. However, measurements of the baryon acoustic oscillations (BAO), supernovae (SN) magnitudes, as well as the galaxy counts and galaxy shear surveys, offer measures of the background expansion and the growth of large scale structure at multiple redshifts. Using simple parameterizations when analyzing these datasets is likely to bias the outcome and result in a loss of potentially important information. In addition, in any specific MG theory, the dynamics of the effective DE, which impacts the background expansion, is correlated with the changes in the growth of perturbations. Thus, rather than assuming that only one of the two is modified, it makes more physical sense to vary them simultaneously when performing fits to the data. As we demonstrate in this work, current datasets already allow us to simultaneously reconstruct the effective DE density and the two modified growth functions using flexible non-parametric forms.

There are several approaches to the non-parametric reconstruction of cosmological functions such as \( w(z) \). Popular methods include binning the functions at several redshifts, using Gaussian Processes (GP) [30–32], and the correlated prior approach [33–36]. A simple binning, with the function assumed to be constant and independent in each bin, or with a smooth interpolation between the redshift nodes, makes the results dependent on an unphysical implicit smoothness prior. Also, one is typically restricted in the number of bins they can use by...
the MCMC convergence times. Using a small number of bins might, in turn, bias the reconstruction. The GP method is not restricted in the number of bins, introducing instead a Gaussian prior that correlates the function at neighbouring redshifts. However, the choice of the GP prior is essentially phenomenological (without any relation to a physical theory) and the parameters of the prior are typically marginalized over, thus obscuring the Bayesian interpretation of the resultant reconstruction. The correlated prior approach also introduces a correlation between the neighboring redshifts but, unlike the GP method, it uses a fixed prior covariance matrix which is meant to be derived from theory. Having an explicit prior makes it possible to clearly state how much the data improves on the prior, and to compute the Bayesian evidence that can be compared to that of the ΛCDM model.

In this work, for the first time, we jointly reconstruct the redshift dependence of the effective DE fractional energy density Ω_X(z) and the phenomenological functions µ and Σ, which parameterize possible modifications of the Poisson equation relating the matter density contrast to the Newtonian and the lensing potentials, respectively, from the combination of the current CMB, BAO, Redshift Space Distortion (RSD), SN, galaxy counts and galaxy weak lensing data. Since the phenomenological parameterization by µ and Σ is valid only for linear perturbations, we will restrict our analysis to linear scales. We perform this reconstruction with and without using a prior covariance of the three functions derived from the Horndeski class of theories [37]. As we show, the theoretical prior has a significant smoothing effect on the allowed variation of these functions with redshift. Importantly, we find that current data can constraint 15 independent degrees of freedom (DOF) (combined eigenmodes) of the three functions relative to the prior. This allows us to learn much more from the current data than one would if using simple ad hoc parametric forms. A reconstruction of the EFT functions was performed using a similar method in [38, 39]. The reconstruction in this work has the benefit of not being restricted to Horndeski theories and allows us to check the overall consistency of the class of Horndeski theories with data.

We identify the imprints of the few tensions, present within the ΛCDM model, on the dark energy and MG phenomenology. These include the “Hubble tension”, i.e. the 4.2σ disagreement between the Hubble constant H_0 deduced from the Planck cosmic microwave background (CMB) data within the ΛCDM model [40] and that determined in a less cosmology-dependent way from Cepheid calibrated type IA Supernova (SN) [41]. There is also some tension in the estimates of the galaxy clustering amplitude, quantified by the parameter S_8, predicted by the Planck best fit ΛCDM model and that obtained from galaxy weak lensing surveys, such as the Dark Energy Survey (DES) [42, 43], the Kilo-Degree Survey (KiDS) [44, 45] and the Subaru Hyper Suprime-Cam (HSC) survey [46]. Thirdly, the weak lensing effect on the CMB temperature anisotropy, quantified by the parameter A_L, appears to be stronger than expected from ΛCDM [40]. Many extensions of ΛCDM, including modifications of GR, have been proposed with the aim of relieving these tensions [47]. Our reconstruction allows us to reveal the properties that DE and MG would need to have in order to (at least partially) resolve the H_0, S_8 and A_L tensions. We also discuss the implications of the reconstructed phenomenology for the MG theories considered in the literature.

The key takeaways from analysis in the context of the cosmic tensions and implications for alternative gravity theories are presented in an accompanying Letter [48]. This paper presents the details of our analysis and the complete set of our results. The rest of the paper is organized as follows. In Sec. II we review the phenomenological functions Ω_X, µ and Σ and their expected behaviour in Horndeski theories. Sec. III details the datasets, the key technical details of our method. The reconstruction results are presented in Sec. IV, followed by a discussion of theoretical implications in Sec. V and a concluding summary in Sec. VI.

II. THE MODEL

For the datasets analyzed in this work, it suffices to consider linear scalar perturbations to the Friedmann-Lemaître-Robertson-Walker (FLRW) metric. Working in the Newtonian gauge, the line-element reads:

\[
\text{d}s^2 = -(1 + 2\psi)\text{d}t^2 + a^2(1 - 2\phi)\text{d}x^2,
\]

where \(a\) is the scale factor, \(t\) is the cosmic time, and \(\psi\) and \(\phi\) are the scalar perturbations to the metric. The dynamics of the expansion is set by the Friedmann equation,

\[
\frac{H^2}{H_0^2} = \Omega_r(1 + z)^4 + \Omega_m(1 + z)^3 + \Omega_X(z),
\]

where \(H = d\ln a/dt\) is the Hubble parameter, \(H_0\) is its current value, \(z = 1/a - 1\) is the redshift, \(\Omega_r\) and \(\Omega_m\) are the fractional energy densities of radiation and matter. The time evolution of the fractional energy density of dark energy is described by \(\Omega_X(z)\) with \(\Omega_{DE} \equiv \Omega_X(z = 0)\), the fractional energy density of dark energy today. We assume spatial flatness, hence \(\Omega_r + \Omega_m + \Omega_{DE} = 1\). It is worth noting that the DE component is defined quite broadly through this equation, to capture not only a dynamical DE field, but also, e.g., modifications to gravity or non-minimal interactions with matter. In other words, \(\Omega_X(z)\) represents an effective DE fluid that encodes all contributions to the Friedmann equation different from radiation and minimally coupled matter, with \(\Omega_X(z) = \Omega_{DE}\) corresponding to the cosmological constant \(\Lambda\). As emphasized in [36], using the DE equation of state to describe such an effective fluid could potentially bias the reconstruction because it prohibits the effective
DE density from changing its sign, which is not uncommon in theories with new interactions. This is the reason for choosing to work with $\Omega_X(z)$ instead of $w(z)$.

The linearly perturbed Einstein equations provide us with a set of equations relating metric perturbations to the perturbations in the energy-momentum tensor of the matter fields. As shown in [8–10], the phenomenology of linear perturbations in many extensions of $\Lambda$CDM can be effectively captured by introducing two functions of time and scale, defined through the Poisson equations in Fourier space for the Newtonian potential $\psi$ and the lensing (Weyl) potential $\phi + \psi$, as

\[
k^2\psi = -4\pi G\mu(a, k)a^2\left[\rho\Delta + 3(\rho + P)\sigma\right], \tag{3}
k^2(\phi + \psi) = -4\pi G\Sigma(a, k)a^2\left[2\rho\Delta + 3(\rho + P)\sigma\right], \tag{4}
\]

where $k$ is the Fourier number, $G$ is the gravitational constant, $\rho$ is the background matter density, and $\Delta$ is the comoving density contrast, $\Delta \equiv \delta + 3aH\nu/k$, where $\delta \equiv \delta \rho/\rho$ is the density contrast in the Newtonian gauge and $\nu$ is the irrotational component of the peculiar velocity. The anisotropic stress $\sigma$ due to relativistic components is included for consistency but is negligible during matter and dark energy dominated epochs. Since $\Sigma$ directly controls the Weyl potential, it is best constrained by weak lensing (WL) measurements. On the other hand, $\mu$ sets the Newtonian potential, which determines the peculiar velocities of galaxies. Thus, combining the WL data with RSD allows us to effectively break the degeneracy between the two functions [8, 10, 49].

Analytical expressions for $\mu$ and $\Sigma$ in specific theories can only be obtained after adopting the quasi-static approximation (QSA), which restricts to sub-horizon scales and ties the time-dependence of all perturbations to the growth of matter perturbations. As shown in [50], under the QSA, the $k$-dependence of $\mu$ and $\Sigma$ in all local theories of gravity takes the form of a ratio of polynomials. Further, the vast majority of DE and alternative gravity theories involve additional scalar degrees of freedom, and most of them are a particular subset of the broad class of scalar-tensor models of gravity with second order equations of motion derived by Horndeski [51] and rediscovered more recently in the context of generalized Galileons [52]. In all scalar-tensor theories, there is an effective length scale $\lambda_f$ below which the scalar field mediates an attractive ("fifth") force. Specific Horndeski theories studied to date fall into two broad categories, according to the type of the mechanism that screens the fifth force and allows them to comply with the tight Solar System tests of GR. In models with the Vainshtein screening [53], $\lambda_f$ tends to be comparable to the horizon scale, i.e. $\lambda_f \sim H^{-1}$, while in models with the chameleon type screening [54–56], it is constrained to $\lambda_f \lesssim 1$ Mpc [5, 57], where perturbations are in the non-linear regime. Hence, when analyzing the large scale structure data on linear scales, one is likely to be probing modified gravity either well above or well below $\lambda_f$. In either case, the scale-dependence would be negligible. Hence, we opt to ignore the scale-dependence and reconstruct the redshift dependences of these functions, $\mu(z)$ and $\Sigma(z)$.

In what follows, we present a joint reconstruction of $\Omega_X(z)$, $\mu(z)$ and $\Sigma(z)$ from the currently available data. We will do it with and without an explicit theoretical prior, derived from Horndeski theories, that correlates the values of these functions at different redshifts. The prior was derived in [37] by generating many solutions in the EFT representation of Horndeski models and projecting them onto $\Omega_X$, $\mu$ and $\Sigma$. The EFT framework makes it possible to separately sample particular representative sub-classes of Horndeski, such as Generalized Brans Dicke (GBD) and models that have $c_T(z) = 1$. The resultant correlation between the three functions is different in each case. In particular, it was found that the correlation of $\mu$ and $\Sigma$ with $\Omega_X$ is strong in GBD, while it is non-negligible in models with $c_T(z) = 1$, but quite weak in general Horndeski. In this work, we will use the least constraining prior representing samples of all Horndeski theories with $c_T(z = 0) = 1$. Note that, while the speed of gravitational waves has been measured to be the same as the speed of light at present [58, 59], it could, in principle, be different in the past.

The considerations above imply that discovering an anomalous evolution of $\Omega_X$, $\mu$ or $\Sigma$ would not only put pressure on $\Lambda$CDM, but could also rule out large subclasses of Horndeski. It should also be noted that Horndeski theories assume a universal coupling of the metric to all matter, and many of the above-mentioned theoretical arguments would not apply to theories of dark sector interactions, e.g. theories in which the scalar field only coupled to dark matter.

### III. METHODS

In what follows, we describe the methods and the datasets used to reconstruct the three phenomenological functions introduced in the previous Section.

#### A. Discretizing and reconstructing functions with a correlation prior

The starting point of our reconstruction is to parameterize $\Omega_X$, $\mu$ and $\Sigma$ in terms of their values at 11 discrete values (nodes) of $a$. From the 11 nodes, 10 values are distributed uniformly in the interval $a \in [1, 0.25]$ (corresponding to $z \in [0, 3]$) with another one at $a = 0.2$ ($z = 4$). We make the functions approach their $\Lambda$CDM values at higher redshifts, because the theoretical prior is obtained by looking at models that deviate from GR at late times only, though studying earlier times deviations from GR is generally possible within the same framework [60]. To allow for a smooth transition between their values at $z = 4$ and $z = 1000$, we add a set of 9 anchor nodes arranged along a tanh pattern, and then use cubic spline to interpolate between all the nodes to obtain...
FIG. 1. a) The implicit correlation prior, as a function of redshift, induced by using the cubic spline to connect the 11 redshift nodes. All three functions, $\Omega_X$, $\mu$ and $\Sigma$, are subject to the same implicit prior, with no cross-correlation between different functions. b) The Horndeski prior correlating the nodes of $\Omega_X$, $\mu$ and $\Sigma$. The correlation between the nodes of each function is much stronger than that introduced by the cubic spline. The Horndeski prior also introduces a strong correlation between $\mu$ and $\Sigma$. c) The correlation obtained from our “Baseline” data posterior covariance of the nodes, i.e. that determined by the data and the implicit prior correlation in Panel (a). d) The correlation corresponding to the posterior covariance derived from the Baseline data with the help of the Horndeski prior in Panel (b).

Our results do not depend on how many nodes we use, since, with 10 nodes, we already include many nodes per prior correlation length and additional nodes will be made redundant by the correlation prior. In addition, the BAO, RSD, DES and SN data probe $z \lesssim 3$, while CMB constrains the integrated effect over $z \lesssim 1000$, with no data in the $3 < z < 1000$ range.

The cubic spline introduces an implicit smoothness prior into the reconstruction that suppresses sharp changes of the functions between nodes. Panel (a) of Fig. 1 shows the correlation between the nodes of $\Omega_X$, $\mu$ and $\Sigma$ imposed by the cubic spline. As one can see by comparing to Panel (b), this prior is substantially weaker than that derived from the Horndeski theories, as discussed below. Panels (c) and (d) show, respectively, the correlation imposed by data only (which includes the implicit prior), and by data in combination with the Horndeski prior. These are further discussed at the beginning of Sec. IV A.

We use an appropriately modified version of MGCosmoMC\textsuperscript{1} [18, 19, 61], based on CosmoMC\textsuperscript{2} [62], to sample the parameter space, which, in addition to the node parameters $\Omega_X$, $\mu$, $\Sigma$, introduced earlier, includes the

\textsuperscript{1} https://github.com/sfu-cosmo/MGCosmoMC
\textsuperscript{2} http://cosmologist.info/cosmomc/
usual cosmological parameters: $\Omega_b h^2$, $\Omega_c h^2$, $\theta_s$, $\tau$, $A_s$, $n_s$, $N$, where $\Omega_b h^2$ and $\Omega_c h^2$ are the physical densities of baryons and CDM, $\theta_s$ is the angular size of the sound horizon at the decoupling epoch, $\tau$ is the reionization optical depth, $A_s$ and $n_s$ are the amplitude and the spectral index of primordial fluctuations, and $N$ collectively denotes the nuisance parameters that appear in various data likelihoods. We note that the last node of $\Omega_X$, corresponding to $a = 1$, is not varied as it is the same as the derived parameter $\Omega_{DE}$.

In addition to performing the reconstruction of $\Omega_X(a)$, $\mu(a)$ and $\Sigma(a)$ by determining the best fit node parameters from data alone, we use the method of [33, 63] to add the Horndeski prior that correlates the nodes $\{\Omega_{X_1}, \mu_1, \Sigma_1\} \equiv f$. It is introduced as a Gaussian prior

$$P_{\text{prior}} \propto \exp[-(f - f_{\text{fid}})^T C^{-1} (f - f_{\text{fid}})]$$

where $C$ is the correlation matrix derived from the joint covariance of the three functions obtained in [37]. While we have the full covariance at our disposal, along with the mean values, we opt not to use the latter as our fiducial values $f_{\text{fid}}$ in order to avoid biasing the outcome of the reconstruction, and also use the normalized correlation matrix for $C$. In practice, the prior is implemented as a new contribution to the total $\chi^2$, with $f_{\text{fid}}$ determined during sampling using the so-called “running average” method [33]. The theory prior acts much like a Wiener filter, discouraging (but not completely prohibiting) abrupt variations of the functions.

Panel (b) of Fig. 1 shows the Horndeski correlation prior used in our work. One can clearly see that the correlation “length” is much longer than that of the implicit prior due to the cubic spline shown in Panel (a). This ensures that the prior aided reconstruction is independent of the binning scheme. Also notable is the nearly perfect correlation between $\mu$ and $\Sigma$, in line with the $(\Sigma - 1)(\mu - 1) \geq 0$ conjecture made in [64]. The correlation between $\Omega_X$ and $\mu$ or $\Sigma$ is nearly absent, although, as mentioned above, it can be strong in certain subclasses of Horndeski theories.

**B. Data**

We consider combinations of the following datasets:

- “Planck”: the 2018 release of the Planck CMB temperature, polarization and the reconstructed CMB weak lensing spectra [65];
- “BAO”: the eBOSS DR16 BAO compilation from [66] that includes measurements at multiple redshifts from the samples of Luminous Red Galaxies (LRGs), Emission Line Galaxies (ELGs), clustering quasars (QSOs), and the Lyman-α forest [67–70], along with the SDSS DR7 MGS [71] data. We also add the BAO measurement from 6dF [72]. This compilation covers the BAO measurements at $0.07 < z < 3.5$. Note that the BAO data considered here are the “tomographic” version of the DR12 BOSS BAO at $0.20 < z < 0.75$ [73] (not the “consensus” version using effective redshifts presented in [66]).

- “SN”: the Pantheon SN sample at $0.01 < z < 2.3$ [74];
- “RSD”: the eBOSS joint measurement of BAO and RSD for LRGs, ELGs and QSOs [69, 75–77], using it instead of the eBOSS BAO-only measurement. For LRGs, it combines eBOSS LRGs and BOSS CMASS galaxies spanning the redshift range $0.6 < z < 1$, at an effective redshift of $z_{\text{eff}} = 0.698$. QSOs cover $0.8 < z < 2.2$ with an effective redshift of $z_{\text{eff}} = 1.48$, while ELGs cover $0.6 < z < 1.1$ with an effective redshift of $z_{\text{eff}} = 0.845$. In addition, we add BAO-only measurements from 6dF and MGS.
- “DES”: the Dark Energy Survey Year 1 measurements of the angular two-point correlation functions of galaxy clustering, cosmic shear and galaxy-galaxy lensing with source galaxies at $0.2 < z < 1.3$ [42]; since our formalism has no nonlinear prescription for structure formation, the angular separations probing the nonlinear scales were removed using the “aggressive” cut option of MGCAMB described in [61], which uses the method introduced in [42, 78].
- “M$_{SN}$”: the SH0ES determination of the intrinsic SN type Ia brightness magnitude as obtained by [41], included in the Pantheon SN likelihood as in [79]. This provides a measurement of $H_0 = 73.2 \pm 1.3$ km/s/Mpc in $\Lambda$CDM.

Our baseline dataset combination (labelled “Baseline” from now on) includes Planck, BAO and SN. In addition, we also consider the additional Baseline+RSD+DES and Baseline+RSD+DES+M$_{SN}$. Note that, when RSD is included in the combination, the BAO data do not coincide with the one used in Baseline for the eBOSS LRGs BAO measurement, as we replace it with the joint RSD-BAO measurement.

For brevity, we sometimes refer to RSD+DES as simply “LSS”, and to Baseline+RSD+DES+M$_{SN}$ as “All”.

**IV. RESULTS**

**A. Reconstructed functions**

Fig. 2 shows the functions $\Omega_X(z)$, $\mu(z)$ and $\Sigma(z)$ reconstructed from the Baseline and Baseline+LSS data combinations with and without the Horndeski correlation prior. The first observation one can make is that the correlation prior smooths out the oscillations seen in the data-only reconstructions of all three functions. These oscillations are dependent on the number of nodes and affected by the implicit correlation shown in Fig. 1(a).
FIG. 2. Reconstruction of $\Omega_X(z)$ (top panels), $\mu(z)$ (middle panels) and $\Sigma(z)$ (bottom panels) from the Baseline (red lines) and Baseline+LSS (blue lines) data, without (left panels) and with the Horndeski correlation prior (right panels). The shaded regions show the 68% confidence levels. The two vertical lines show the redshifts of equality between the matter and DE densities, $z_{eq}$, and the beginning of cosmic acceleration, $z_{acc}$, in the best fit $\Lambda$CDM model.

The impact of the implicit prior is apparent for the data-only case shown in Fig. 1(c), where the pattern of the rings evidently has the same frequency as the features in Fig. 1(a). One can also see from Fig. 1(c) that data (the Baseline dataset in this case) introduces correlations among the three functions, with $\Sigma$ being more strongly correlated with $\Omega_X$ than $\mu$. The data also introduces correlations between $\mu$ and $\Sigma$. The artifacts of the implicit prior are not present when data is combined with the Horndeski prior, as shown in Fig. 1(d). The theory prior suppresses correlations introduced by the cubic spline, while retaining the correlation introduced by the data. This shows the important role played by the theory prior in the reconstruction, as it prevents an overfitting of the data by favouring the reconstruction of only those features that are consistent with the theory.

The smoothing by the prior is particularly evident for $\Omega_X(z)$, whose oscillations around the constant $\Lambda$CDM value are completely erased. For this function we find a different behaviour depending on the data combination considered. For our Baseline dataset combined with the correlation prior, we find that $\Omega_X(z)$ is close to a constant and perfectly consistent with the $\Lambda$CDM limit. When RSD and DES data are included in the analysis, $\Omega_X(z)$

is still close to a constant at low redshifts, but there is a trend for slight preference of higher values at higher redshifts. This contributes to a shift of $S_8$ to a lower value (see Fig. 4) due to a lower amount of matter at these redshifts. However, $S_8$ is not the parameter directly constrained by DES, as the weak lensing is also affected by $\Sigma$. As we will show below, this lowering of $S_8$ does not lead to an improvement of the fit to the DES data and there is no statistical significance for a deviation of the background function $\Omega_X(z)$ from $\Lambda$CDM.

Fig. 2 also shows that $\mu(z)$ is compatible with the GR limit $\mu(z) = 1$ both with and without the inclusion of the correlation prior. The increase of this function at low redshift is caused by its correlation with $\Sigma(z)$, with such a correlation introduced by the data and made stronger by the Horndeski prior. There is also a hint for $\mu(z) < 1$ at high redshift, which compensates for the higher values at low redshift and helps to keep the clustering amplitude $S_8$ consistent with the data.

The most striking outcome of our reconstruction is the behavior of $\Sigma(z)$; the function is systematically above the GR limit $\Sigma = 1$. The high redshift deviation accounts for the lensing anomaly in the Planck CMB temperature anisotropy spectra (TT) [40], (see the discussion of the $A_L$ anomaly in the following subsection) as the higher $\Sigma(z)$ boosts the lensing power at the redshifts that most affect the CMB spectra. The low redshift deviation is instead due to the deficit of CMB TT power at low $\ell$. A larger $\Sigma(z)$ enhances the growth of the Weyl potential, thus offsetting the decay of gravitational potentials during the accelerated expansion and reducing the ISW contribution to TT. The high redshift deviation does not change by adding RSD and DES to the data combination. On the other hand, the low redshift deviation is suppressed after adding DES, as $\Sigma$ at lower redshift is directly constrained by this survey.

**B. The role of CMB and weak lensing anomaly**

In order to assess if the high redshift departure of $\Sigma(z)$ from the GR limit is indeed due to the CMB lensing anomaly, we perform our analysis including the lensing amplitude $A_L$ [80] as an extra free parameter. This is
a completely phenomenological parameter that rescales the CMB lensing power spectrum as

\[ C^\text{lens}_\ell = A_L C^\text{lens}_\ell. \]  

(6)

Fig. 3 shows the reconstruction of \( \Sigma(z) \) with and without the correlation prior, for both the Baseline and Baseline+LSS data combinations, and with the \( A_L \) parameter both fixed and free to vary. Comparing the fixed and free \( A_L \) reconstructions shows how the inclusion of \( A_L \) completely removes the high \( z \) departure from GR, with \( \Sigma(z) \) now fully consistent with one.

We further explore the correlations between \( \Sigma(z) \) and \( A_L \) and its implication for the \( S_8 \) parameter in Fig. 4. As discussed above, the reconstructed shapes of \( \Omega_X \) and \( \mu \) allow for slightly lower values of \( S_8 \) than \( \Lambda \text{CDM} \). However, \( S_8 \) is also related to the amplitude of the lensing potential, which in our analysis is modulated by \( \Sigma \). The parameter that is constrained by DES is \( \langle \Sigma \rangle S_8 \) where \( \langle \Sigma \rangle \) is an average of \( \Sigma \) in the redshift range relevant for DES, and it is equal to one in the \( \Lambda \text{CDM} \) limit. Despite the lowering of \( S_8 \), this parameter remains the same as that in \( \Lambda \text{CDM} \) when \( A_L = 1 \), as it can be seen in the top panels of Tables IV and V, where the results for this case are shown. This is due to the enhancement of \( \Sigma \) from the GR limit driven by the CMB lensing anomaly as discussed above. As a consequence, if one keeps a fixed \( A_L = 1 \), the quality of the fit to DES data is not better than \( \Lambda \text{CDM} \) even when \( \Omega_X \), \( \mu \) and \( \Sigma \) are free to vary (see Table V). If one instead allows for \( A_L \) to be free, an anomalous value of this parameter “solves” the CMB lensing anomaly, and \( \Sigma \) becomes consistent with the GR limit. In this case, the lowering of \( S_8 \) leads to lower values of \( \langle \Sigma \rangle S_8 \) and the fit to the DES data is improved compared with \( \Lambda \text{CDM} \). Table V shows the values of the \( \chi^2 \) for the different data considered in the analysis. One can see that, with respect to \( \Lambda \text{CDM} \), the \( A_L = 1 \) analysis improves the DES \( \chi^2 \) by 0.4 (1.2 without the Horndeski prior), while this improvement increases to 2.8 (4.7 without the Horndeski prior) if \( A_L \) is free. Overall, this analysis reveals that the late time modifications alone are not able to improve the fit to CMB and DES weak lensing simultaneously.

C. The role of the SH0ES prior

We now turn our attention to the impact of including the SH0ES prior on the SN magnitude in the data combination analyzed. As it can be seen in Table VI, the main impact of such an addition is an enlargement of the uncertainties and an increase of the estimated mean value of \( H_0 \). We find \( H_0 = 69.44 \pm 1.30 \text{ km/s/Mpc} \), which is consistent with the value obtained by SH0ES within 2\( \sigma \). Fig. 5 shows \( \Omega_m \), \( H_0 \) and the sound horizon at baryon decoupling \( r_{\text{drag}} \), with and without the SH0ES prior and with and without the Horndeski prior. It is possible to notice how, for the Baseline+LSS combination, the additional freedom given by the \( \Omega_X \), \( \mu \) and \( \Sigma \) functions only produces an enlargement of the error on \( H_0 \), with its mean value being the same as in \( \Lambda \text{CDM} \). When the SH0ES prior is included we obtain instead the increase of the mean value of \( H_0 \) as well as a slight shift of \( r_{\text{drag}} \) with respect to \( \Lambda \text{CDM} \).

Fig. 6 shows the difference of the luminosity distance prediction with the SH0ES prior from the prediction of the best-fit \( \Lambda \text{CDM} \) without the SH0ES prior. The Pantheon SN data points are calibrated using the SH0ES measurement of \( M_{\text{SN}} \) while the BAO data points are converted to the luminosity distance from the angular diameter distance using \( d_L = (1 + z)^2 d_A \). There are two main issues. The first problem is that the luminosity distance calibrated from CMB in \( \Lambda \text{CDM} \) does not agree with the SN data while it agrees with the BAO data. The second problem is the discrepancy between the BAO and SN data. The latter makes it hard for late modifications to resolve this tension fully as it is not possible to fit BAO and SN data simultaneously unless we change \( r_{\text{drag}} \) by an early time modification. This is also the case in our reconstruction. Due to the freedom in \( \Omega_X(z) \), the luminosity distance at lower redshifts becomes closer to the SN data compared with \( \Lambda \text{CDM} \), which leads to a larger \( H_0 \). However, it is still not possible to reproduce the luminosity distance calibrated by the SH0ES measurement of \( M_{\text{SN}} \) fully.
FIG. 5. 68% and 95% confidence level contours Ω_m, H_0 and the sound horizon at baryon decoupling r_{drag}, with and without the SH0ES prior (yellow and red contours, respectively) and with and without the Horndeski prior (left and right panels, respectively). The blue contours show the ΛCDM fit to Baseline+LSS, while the grey band shows the constrain on H_0 obtained by the SH0ES collaboration.

D. Implications for the tensions and the significance of measured departures from ΛCDM

In order to summarize the extent to which the well known H_0 and S_8 tensions can be resolved by allowing for time-dependent Ω_X, μ and Σ, we show in Fig. 7 the constraints on Ω_m, H_0 and ⟨Σ⟩_{S_8} and compare the results obtained with and without the SH0ES prior. The results shown in this figure refer to the case where A_L is considered as a free parameter. As discussed before, without this parameter, the CMB lensing would prevents us from addressing the S_8 tension, thus achieving a better fit to DES. On the other hand, allowing for a varying A_L removes the CMB anomaly, making it possible to fit simultaneously to the SH0ES and DES data better than ΛCDM. As shown in Table VI, the total improvement of χ^2 is Δχ^2 = −25.8 without the Horndeski prior and Δχ^2 = −14.1 with the prior.

Table I lists the signal-to-noise ratios (SNR) in the detection of departures of Ω_X, μ and Σ from their ΛCDM values:

\[
SNR^2 \equiv (\theta - \theta_{GR})^T C_p^{-1} (\theta - \theta_{GR})
\]  

where the vector θ includes parameters related to the three MG functions, C_p their covariance and θ_{GR} represents their GR limit. The table also shows the SNR for the gravitational slip γ, which will be discussed in Sec. V. One can see that the significance of the detection generally drops after including the Horndeski prior, most notably, from ~3σ to ~1σ for Ω_X when the SH0ES data is not used. The most persistent deviation is in Σ, which
FIG. 7. The 68% and 95% confidence level contours for the tension parameters, $S_8$, $H_0$, and $\Omega_M$, from the Baseline+LSS data with and without the SH0ES prior on the SN magnitude $M_{SN}$ (yellow and red contours, respectively), and with and without the Horndeski prior (left and right panels, respectively). The blue contours show the results obtained in the ΛCDM limit, while the grey band shows the $H_0$ measured by the SH0ES collaboration.

| SNR               | $\Omega_X$ | $\mu$ | $\Sigma$ | $\gamma$ |
|-------------------|------------|-------|----------|----------|
| no theory prior   |            |       |          |          |
| Baseline          | 3.0        | 1.6   | 2.0      | 1.0      |
| Baseline+LSS      | 2.9        | 1.7   | 2.4      | 1.9      |
| All               | 3.8        | 2.1   | 2.6      | 2.3      |
| Baseline+$A_L$    | 3.1        | 1.5   | 1.9      | 1.0      |
| Baseline+LSS+$A_L$| 3.3        | 1.5   | 1.8      | 1.0      |
| All+$A_L$         | 4.0        | 1.9   | 2.2      | 1.3      |
| with Horndeski prior |          |       |          |          |
| Baseline          | 1.1        | 0.6   | 1.8      | 0.6      |
| Baseline+LSS      | 1.3        | 1.6   | 2.3      | 1.8      |
| All               | 2.3        | 2.2   | 2.7      | 2.4      |
| Baseline+$A_L$    | 1.2        | 0.5   | 1.7      | 0.5      |
| Baseline+LSS+$A_L$| 1.7        | 1.3   | 1.3      | 0.9      |
| All+$A_L$         | 2.6        | 2.1   | 1.7      | 1.2      |

TABLE I. The signal-to-noise ratio (SNR) in the detection of departure of $\Omega_X$, $\mu$, $\Sigma$ and $\gamma$ from their ΛCDM values. The gravitational slip $\gamma$ is discussed in Sec. V.

is above 2σ with and without the prior and, to a lesser extent, in $\mu$, because of its strong correlation with $\Sigma$. One can also see that this is largely driven by the CMB lensing anomaly, as the inclusion of $A_L$ as a free parameter brings the SNR in $\Sigma$ down to ~1σ. Interestingly, the inclusion of the SH0ES prior does not only increase the SNR in $\Omega_X$ but also in $\mu$ and $\Sigma$, due to a non-negligible correlation between the background expansion and the growth rate, as one can also see from Panel (c) of Fig. 1.

E. The best constrained eigenmodes

To gain further insight into the features and the number of DOF of the three functions constrained by the data, we use the Karhunen-Loeve (KL) decomposition of the prior and posterior covariances, as discussed in [38, 81]. We decompose the prior $C_\Pi$ and posterior $C_p$ covariances as

$$C_\Pi \Psi = C_p \Psi \Lambda,$$

(8)

where the matrix $\Psi$ has as columns the KL eigenmodes of the posterior with respect to the prior, and the matrix $\Lambda$ is diagonal and quantifies the improvement of the posterior with respect to the prior. While the KL modes are not orthonormal in the Euclidean sense, they are orthogonal in the metrics induced by the prior and posterior covariances:

$$\Psi^T C_\Pi \Psi = \Lambda,$$
$$\Psi^T C_p \Psi = I,$$

(9)

so that parameters projected along the KL modes are statistically independent for both the prior and the posterior.

The posterior Fisher matrix can also be decomposed into the same KL modes:

$$C_p^{-1} = \Psi \Psi^T,$$

(10)
TABLE II. The number of well-constrained eigenmodes of \( \Lambda \), \( N_{\text{eff}} \), for each function and for the three combined, along with the trace, \( T = \text{Tr}(\Lambda) \), that quantifies the net constraining power of the data.

| Function | \( N_{\text{eff}} \) | \( \mu \) | \( \Sigma \) | Combined |
|----------|-----------------|------|------|-----------|
| Baseline | 5.6             | 1.7  | 7.6  | 14.9      |
| Baseline+LSS | 5.6           | 2.0  | 8.3  | 15.3      |
| Baseline \( T \) | 1834.4         | 22.7 | 141.7| 2603.6    |
| Baseline+LSS \( T \) | 1816.8        | 40.3 | 170.3| 2755.6    |

Fig. 8 shows the eigenvalues of \( \Lambda \) ordered from highest to lowest, thus corresponding to the best-to-worst constrained eigenmodes \( \Psi \). They can be written as \( \lambda_i = \sigma_{p,i}^2 / \sigma_i^2 - 1 \), where \( \sigma_i^2 \) and \( \sigma_{p,i}^2 \) are the eigenvalues of the prior and the posterior covariances, so that a mode can be considered “constrained” relative to the prior when \( \lambda_i \gtrsim 1 \). We show the eigenvalues of the “combined” modes, corresponding to the joint covariance of all three functions, as well as those for the individual functions, after marginalizing over the other two. The former tells us how many independent DOF (roughly) quantifying departures from \( \Lambda\text{CDM} \) can be measured without asking what function they correspond to. The latter quantify the ability of data to constrain the specific functions. One can see that the number of constrained combined modes is quite large, around 15, and that the top highest eigenvalues are the same as those for \( \Omega_X \) that probes the background expansion. Also, it is clear that \( \Sigma \) is much better constrained than \( \mu \).

Interestingly, the number of constrained modes does not change appreciably with the inclusion of the LSS data. This is, in part, because our \( N_{\text{eff}} \) is only a coarse measurement of improvement. Also, since we have cut the LSS data to exclude nonlinear scales, the CMB and LSS are probing a similar range of scales. Hence, most of the modes that can be constrained on linear scales are already constrained, to some extent, by the Baseline data. The addition of the LSS data, however, makes a notable difference in how well the individual modes of \( \mu \) and \( \Sigma \) are constrained. As one can see from Table II, the trace \( T \) is increased by a factor of \( \sim 2 \) for \( \mu \) and by 20% for \( \Sigma \). This illustrates that combining the RSD data and WL helps to break the degeneracy between \( \mu \) and \( \Sigma \).

Further insight can be gained by considering the shapes of the best constrained eigenmodes \( \Psi \) when plotted as functions of redshift. They can be interpreted as the window functions representing sensitivity of the data to the redshift evolution of \( \mu \), \( \Sigma \) and \( \gamma \). As one can see from Fig. 9, the modes derived from Baseline and Baseline+LSS appear rather similar. For \( \Omega_X \), in particular, the change is difficult to detect by eye. This is because the LSS constraint on the expansion history is much weaker than that of Baseline. The redshifts covered by the top three modes of \( \Omega_X \), in the order from best to worst constrained, are low-\( z \), high-\( z \), and in between.

The two constrained eigenmodes of \( \mu \) can be identified with the overall growth of structure and the ISW effect. In both cases, the impact of \( \mu \) is an integrated effect, i.e., via the change of the gravitational coupling in the differential equation that determines the growth of density perturbations. Hence, both modes have a broad support in redshift. Interestingly, the addition of LSS flips the two modes – the “ISW” mode, best constrained by Baseline, becomes the second best, since LSS includes additional measurements of WL and \( f\sigma_8 \).
FIG. 9. The shapes of the best constrained individual eigenmodes of $\mu$, $\Sigma$ and $\gamma$ plotted vs redshift for the Baseline (left) and Baseline+LSS (right). The amplitude and the overall sign of modes are arbitrary and are rescaled in the figure so that the maximum of each mode is one. The eigenmodes can be interpreted as the window functions representing sensitivity of the data to the redshift evolution of the three functions.

The top two modes of $\Sigma$ mirror those of $\mu$, but with different pivot points, since the impact of $\Sigma$ on the Weyl potential is direct, not integrated like in the case of $\mu$. The higher order modes match quite well the lensing kernels that contribute to the lensing of the CMB temperature and polarization anisotropies (see Fig. 11 of [82]). With the addition of LSS, the CMB kernels that correspond to the large scale CMB lensing, i.e. occurring at lower redshifts, become mixed with the galaxy lensing kernels of DES, but the first few best modes are largely unchanged in shape. As mentioned earlier, the ability to constrain the modes of $\mu$ and $\Sigma$ increases appreciably with the addition of LSS.

Finally, Fig. 10 shows the SNR of deviations from $\Lambda$CDM for each mode. In the case of $\Omega_X$, for both Baseline and Baseline+LSS, the most anomalous modes are the first and the fifth, which have support at low and high redshifts. As discussed earlier, these are representative of the low TT power at low multipoles and the $A_L$ anomaly, respectively. The most anomalous mode of $\mu$ is also the one that is best constrained, corresponding to the net growth of the large scale structure and, therefore, most affected by the $S_8$ tension. Essentially the same mode is also the most anomalous for $\Sigma$, where it is the the second best constrained. In all cases, the significance of detection is increased with the addition of the LSS.

V. IMPLICATIONS FOR THEORY

Any statistically significant departure of $X(z)$, $\mu(z)$ or $\Sigma(z)$ from unity would imply either a break down of the $\Lambda$CDM model or a problem, e.g. a systematic effect, with
A. Implications for Horndeski

As mentioned in Sec. II, under the QSA, the expressions for $\mu$ and $\Sigma$ in local theories of gravity take the form of ratios of polynomials in $k$. In Horndeski theories, with second order equations of motion, the polynomials are quadratic in $k$. The scale dependence of $\mu$ and $\Sigma$, however, is unlikely to manifest itself in the range of $k$ probed by large scale structure surveys for which linear perturbation theory is valid. The $k$-dependence marks the transition from the $k \ll \lambda_f^{-1}$ limit, where perturbations of the scalar field can be neglected, to the $k \gg \lambda_f^{-1}$ regime in which the scalar field perturbations mediate a fifth force. As mentioned in Sec. II, the known screening mechanisms in Horndeski theories place the range of $k$ probed by our reconstruction in one of these two limits.

While the QSA is not guaranteed to be accurate in all circumstances, especially on near-horizon scales, it has been found to work quite well for identifying the key phenomenological signatures of Horndeski theories [83]. Even though we cannot be certain which regime, $k \ll \lambda_f^{-1}$ or $k \gg \lambda_f^{-1}$, is being probed by our scale-independent parametrization, we can still make several useful deduction by comparing our reconstructions to the QSA expressions for $\mu$ and $\Sigma$ in the two limits.

It is useful to define the gravitational slip $\gamma$, which can be derived from $\mu$ and $\Sigma$ (neglecting the anisotropic stress $\sigma$ sourced by radiation) as

$$\gamma \equiv \frac{\phi}{\psi} = \frac{2\Sigma}{\mu} - 1. \quad (14)$$

In the $k \ll \lambda_f^{-1}$ limit, the QSA expressions for our functions are [64]

$$\mu \rightarrow \mu_0 = \frac{m_0^2}{M_2^2} c_T^2 \quad (15)$$
$$\Sigma \rightarrow \Sigma_0 = \frac{m_0^2}{M_2^2} \left(1 + c_T^2 + \beta_\xi \right) \quad (16)$$
$$\gamma \rightarrow \gamma_0 = c_T^{-2}, \quad (17)$$

while, for $k \gg \lambda_f^{-1}$, one has [64, 84]

$$\mu \rightarrow \mu_\infty = \frac{m_0^2}{M_2^2} \left(c_T^2 + \beta_\xi^2 \right) \quad (18)$$
$$\Sigma \rightarrow \Sigma_\infty = \frac{m_0^2}{M_2^2} \left(1 + c_T^2 + \frac{\beta_\xi^2 + \beta_B \beta_\xi}{2} \right) \quad (19)$$
$$\gamma \rightarrow \gamma_\infty = \frac{1 + \beta_B \beta_\xi}{c_T^2 + \beta_\xi^2}, \quad (20)$$

where $m_0$ is the Planck mass, $M_2$ is the modified Planck mass, $c_T$ is the propagation speed of tensor metric modes, i.e. the speed of gravitational waves, while $\beta_B$ and $\beta_\xi$ are two functions, originating from different ways of coupling the metric and the scalar field, that represent the fifth force contribution.
The limiting expressions above highlight the close relationship between the gravitational slip and the tensor speed $c_T$ \cite{85}. In particular, $\gamma$ must approach 1 in the large scale limit if $c_T = 1$ \cite{64}. In the small scale limit, on the other hand, $\gamma \neq 1$ could be either due to a fifth force or $c_T \neq 1$, or both. Since $c_T \neq 1$ is disfavoured by the multi-messenger observations of gravitational waves from binary neutron stars at $z \sim 0$ \cite{58, 59}, one would conclude that there is evidence for a fifth force at low redshifts.

The subclass of the Horndeski theories with $\gamma = 1$, other than $\Lambda$CDM, include the Cubic Galileon \cite{86} and KGB \cite{87} models, along with the so-called “no-slip gravity” \cite{88}, for which $c_T = 1$, $\beta_\xi = 0$ so that

$$\mu_0 = \Sigma_0 = \mu_\infty = \Sigma_\infty = \frac{m^2}{M^2} \tag{21}$$

Fig. 11 shows the gravitational slip $\gamma(z)$ derived from the reconstructions of $\mu(z)$ and $\Sigma(z)$. It is important to note that the Baseline dataset is not capable of break-
ing the degeneracy between $\mu$ and $\Sigma$. Correspondingly, $\gamma(z)$ reconstructed from the Baseline data has a large uncertainty and is strongly prior dependent. Using Baseline+RSD+DES, on the other hand, allows for the degeneracy between $\mu(z)$ and $\Sigma(z)$ to be partially broken. As a result, $\gamma(z)$ is better constrained and the trends in its time-evolution are essentially the same with and without the Horndeski prior. In both cases, one finds $\gamma > 1$ at higher $z$ and $\gamma < 1$ at lower $z$, with the transition between these two limits happening around the redshift at which cosmic acceleration sets in. In the case with the Horndeski prior, the uncertainties are reduced, making the trend significant at more than $2\sigma$, as can be read from Tab. I.

Keeping in mind that the significance of the $\gamma \neq 1$ detection in Fig. 11 is relatively low, one could ask what such a time-dependence would imply for Horndeski theories. Aside from ruling out models with $\gamma = 1$, like the no-slip gravity, Cubic Galileons and KGB, the fact that we observe $\gamma > 1$ would rule out the generalized Brans-Dicke models (GBD), which predicts $c_T = 1$ and $\beta_B = -\beta_2$, i.e. all models with a canonical form of the scalar field kinetic energy term. The latter conclusion follows from the fact that one should have $\gamma \leq 1$ in GBD on all scales.

Finally, as pointed out in [64] based on analytical considerations in the QSA limit, and later confirmed by a numerical sampling of Horndeski solutions [37, 83], one expects a strong correlation between $\mu$ and $\Sigma$, with $(\Sigma - 1)(\mu - 1) \geq 0$. To violate the latter condition, independent sectors/terms of the Horndeski theory would need to conspire to evolve in just the right way for no apparent reason. Thus, looking for signs of violation of the $(\Sigma - 1)(\mu - 1) \geq 0$ conjecture is an important test of Horndeski. Fig. 12 shows the evolution of $(\Sigma - 1)(\mu - 1)$ derived from our reconstructions. With or without the Horndeski prior, the reconstructions show a good consistency with $(\Sigma - 1)(\mu - 1) \geq 0$.

**B. Simpler parameterizations**

It is interesting to check how well our reconstructed functions can be fit by simple parameterizations. For $\mu$ and $\Sigma$, we consider the following forms:

1. constant $\mu$ and $\Sigma$, although this parametrization is not commonly used;

2. the “DE fraction” parametrization used by DES [89] and Planck [78], where the time dependence of $\mu$ and $\Sigma$ is determined to the fraction of the total energy density in DE, i.e.

   \[
   \mu, \Sigma = 1 + \alpha_{\mu, \Sigma} \rho_{\text{DE}}(a)/\rho_{\text{tot}}(a) ;
   \]

3. the “linear model” used by Planck [78], where

   \[
   \mu, \Sigma = 1 + \alpha_{\mu, \Sigma} + \beta_{\mu, \Sigma}(1 - a) ;
   \]

For the effective DE evolution, we consider two commonly used parametrization DE equation of state:

1. constant $w$;

2. the Chevallier-Polarski-Linder (CPL) parametrization [90, 91]

   \[
   w(a) = w_0 + w_a(1 - a).
   \]

We then take the reconstruction MCMC chains and, sample by sample, project them on the above parameterizations.

| no theory prior | SNR$^2$ | Bias |
|-----------------|--------|------|
| $w$ constant    | 4.7    | 4.6  |
| $w = w_0 + w_a (1 - a)$ | 4.7 | 4.6 |
| $\mu$ constant | 3.1    | 2.7  |
| $\mu = 1 + \alpha_{\text{DE}}$ | 3.1 | 2.7 |
| $\mu = 1 + \alpha + \beta(1 - a)$ | 3.1 | 0.7 |
| $\Sigma$ constant | 6.2    | 4.3  |
| $\Sigma = 1 + \alpha_{\text{DE}}$ | 6.2 | 4.4 |
| $\Sigma = 1 + \alpha + \beta(1 - a)$ | 6.2 | 3.0 |

| with Horndeski prior | SNR$^2$ | Bias |
|----------------------|--------|------|
| $w$ constant | 1.0    | 1.0  |
| $w = w_0 + w_a (1 - a)$ | 1.0 | 1.0 |
| $\mu$ constant | 2.5    | 1.9  |
| $\mu = 1 + \alpha_{\text{DE}}$ | 2.5 | 2.5 |
| $\mu = 1 + \alpha + \beta(1 - a)$ | 2.5 | 0.4 |
| $\Sigma$ constant | 5.3    | 2.4  |
| $\Sigma = 1 + \alpha_{\text{DE}}$ | 5.3 | 4.3 |
| $\Sigma = 1 + \alpha + \beta(1 - a)$ | 5.3 | 1.8 |

**TABLE III.** Bias introduced by using a simple parameterization for the Baseline+LSS dataset, as defined by Eq. 22 and the SNR$^2$ for the corresponding reconstruction, which is the upper bound of Bias. The closer the two values are, the worse is the ability of the parameterization to represent the function. For $w$, to avoid singularities caused by negative effective DE densities, the comparison was restricted to $z \in [0, 1]$ and [0, 2] for the cases without and with the Horndeski prior, respectively.

As mentioned earlier, the projection of $\Omega_X(a)$ onto $w(a)$ is not always well-defined, as the effective DE density can be negative in MG theories and in particular MCMC samples in our reconstruction. In fact, we found that such occurrences were too frequent at $z > 1$ in reconstructions without the Horndeski prior, and at $z > 2$ when using the prior. Thus, in the case of $w$, we restrict our projections to $z \in [0, 1]$ and [0, 2], respectively.

To quantify the bias introduced by simple parameterizations, we compute the “Bias”, defined as

\[
\text{Bias} = d^T C_p^{-1} d
\]
where \( \mathbf{d} \) is the vector of differences between the reconstructed values of the nodes of a given function, and the values given by the best fit simple parameterization. Bias is bounded from above by the square of the SNR, given in Table I. The closer Bias is to its upper bound, the less adequate is the parametrization. Table III shows the two values for the parameterizations listed above. As one can see, both the constant \( w \), and the CPL forms are a very poor representation of DE evolution if no theory prior is used. However, with the Horndeski prior, they perform quite well, as \( \Omega_X(a) \) in this case was quite consistent with a constant.

In the case of \( \mu \) and \( \Sigma \), most parameterizations perform quite poorly, with the exception of the linear model of \( \mu(a) \). The DE fraction parametrization, which is one of the most popular ones in the literature, performs the worst. To visualize the bias, we plot the differences between the reconstructed \( \mu \) and \( \Sigma \) and the best fit DE fraction parameterizations in Fig. 13. Fig. 14 shows the difference plot for \( w(z) \) in the case of the CPL parametrization.

VI. SUMMARY

In this paper, we presented, for the first time, the reconstruction of three time-dependent functions that capture departures from the standard cosmological model, \( \Lambda \)CDM, at the level of the expansion history as well as gravitational effects on matter and light on linear scales. The background expansion is described in terms of the effective dark energy fractional energy density \( \Omega_X(z) \), while gravitation effects in the large scale structure are described by \( \mu(z) \) and \( \Sigma(z) \), which parametrise the relation of the Newton potential and the lensing potential to the density contrast, respectively. The reconstruction was performed with and without a theoretical prior derived from the Horndeski theory, both for a Baseline dataset (Planck, BAO and SN) and an extended one including DES and RSD. Without the Horndeski prior, the reconstruction is affected by an implicit prior imposed by the redshift binning and is prone to high frequency oscillations that are poorly constrained by the data. We demonstrated that the theoretical prior successfully suppressed oscillations in the reconstructed functions.

All reconstructed functions are consistent with the GR
predictions within 2–3σ. At the same time, we found interesting features in these functions that are driven by the data. Firstly, ΩX is larger than ΩDE and μ is smaller than one at higher redshifts when RSD and DES data are included in the analysis. These two effects lower the amplitude of the perturbations, leading to smaller $S_8$ values. On the other hand, Σ shows an interesting behaviour depending on the datasets. For the Baseline (Planck, BAO, SN) data, Σ increases at low redshifts, which allows for a better fit to the dip in the CMB TT power spectrum at low ℓ by suppressing the ISW effect. On the other hand, Σ is larger than unity at higher redshifts, which is driven by the CMB lensing anomaly at high ℓ. The latter behaviour disappears once we include the $A_L$ parameter to account for the CMB lensing anomaly.

Despite the fact that the reconstructed functions offer possibilities to lower $S_8$, we found that this did not lead to an improvement of the fit to DES unless $A_L$ was included in the model. This is because the amplitude of weak lensing observed by DES is determined by $⟨Σ⟩S_8$, where $⟨Σ⟩$ is an average of Σ in the redshift range relevant for DES. Σ is driven to be larger than one due to the CMB lensing anomaly and this compensates the suppression of $S_8$, giving a similar $⟨Σ⟩S_8$ with ΛCDM. Once $A_L$ is included, Σ becomes closer to one and the reconstructed functions provide lower values of $⟨Σ⟩S_8$, which improve the fit to DES.

We also investigated the effect of adding the SH0ES prior on the SN absolute magnitude. The primary effect of the reconstructed functions is to enlarge uncertainties in $H_0$, but the central value is also shifted to a larger value obtaining $H_0 = 69.44 ± 1.30$ km/s/Mpc, which is consistent with the values quoted by SH0ES within 2σ. We also showed that it was not possible to fully resolve the $H_0$ tension due to the discrepancy between the SH0ES calibrated SN and the CMB calibrated BAO data. Once the $A_L$ parameter is included, the reconstructed functions offer a possibility to fit DES and SH0ES better than ΛCDM simultaneously, improving the total $χ^2$ by 14.1 and 25.8 with and without the theoretical prior, respectively.

Using the Karhunen-Loève (KL) decomposition of the prior and posterior covariances, we determined the number of modes that are constrained by the data relative to the Horndeski prior. Function μ is the least constrained, with only 2 such modes, while 8 and 6 modes are constrained for Σ and $Ω_X$, respectively. We examined the redshift dependence of the eigenmodes, identifying the features in the data that determines them, and quantified their contributions to deviations from ΛCDM. When looking for any departure from ΛCDM, without identifying the function responsible for it, we find that data can constrains 15 combined eigenmodes of μ, Σ and $Ω_X$.

Finally, we studied the implication of the reconstructed functions for the Horndeski theory. Combining the reconstructions of μ and Σ, we derived the reconstructed gravitational slip, $γ(z)$. The function shows a trend for $γ > 1$ at higher redshift and $γ < 1$ at lower redshifts, with the transition around the onset of cosmic acceleration, but it is overall consistent with ΛCDM within 2σ. Such a trend, if detected at a higher statistical significance, would rule out ΛCDM and all models with $γ = 1$ (e.g. cubic galileon, KGB, no-lsip gravity), as well as GBD models for which one expects $γ ≤ 1$ on all scales. In previous works, a conjecture was made that μ and Σ satisfy $⟨μ − 1⟩(Σ − 1) > 0$. We find no violations of this condition.

Overall, our work shows that current data allow for an informative, data-driven, reconstruction of the cosmological model, with some interesting hints of non-standard trends. Adopting a non-parametric approach, we have shown that current data can constrain much more than a few parameters of the ad hoc simple parametrizations that, as we have demonstrated, would lead to significantly biased results. Furthermore, the method we employ allows us to capture the non-trivial trends in the functions, while unphysical features due to statistical noise are filtered away by the theoretical prior.

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TABLE IV. The best fitting maximum posterior model parameters, in parentheses, mean and 68% C.L. constraints for the Baseline data combination. Data likelihoods at maximum posterior are compared to their reference ΛCDM values in parenthesis.

| Baseline data | ΛCDM (reference) | No theory prior | With Horndeski prior |
|---------------|------------------|-----------------|----------------------|
| $H_0$         | (67.44) 67.63$^{+0.72}_{-0.81}$ | (69.86) 69.2 ± 1.4 | (67.70) 67.6$^{+1.1}_{-1.2}$ |
| $\Omega_m$    | (0.3141) 0.314$^{+0.011}_{-0.011}$  | (0.2890) 0.295 ± 0.012 | (0.3108) 0.310$^{+0.010}_{-0.014}$ |
| $\langle \Sigma \rangle S_8$ | (0.8319) 0.831$^{+0.021}_{-0.020}$ | (0.8529) 0.893 ± 0.050 | (0.8555) 0.901 ± 0.044 |

| $\chi^2_{\text{CMB}}$ | 2765.7 | 2760.7$^{(-5)}$ | 2761.92$^{(-3.8)}$ |
| $\chi^2_{\text{CMBL}}$ | 8.7 | 9.9$^{(+1.2)}$ | 9.58$^{(+0.9)}$ |
| $\chi^2_{\text{SN}}$ | 1035.2 | 1030.9$^{(-4.3)}$ | 1035.5$^{(+0.3)}$ |
| $\chi^2_{\text{BAO}}$ | 20.6 | 15.3$^{(-5.3)}$ | 21.08$^{(+0.5)}$ |
| $\chi^2_{\text{tot}}$ | 3830.2 | 3816.9$^{(-13.3)}$ | 3828.1$^{(-2.1)}$ |

| Baseline + RSD + DES | ΛCDM (reference) | No theory prior | With Horndeski prior |
|---------------------|------------------|-----------------|----------------------|
| $H_0$               | (67.396) 67.83 ± 0.46 | (69.78) 69.3 ± 1.4 | (67.15) 67.8$^{+1.1}_{-1.4}$ |
| $\Omega_m$          | (0.3079) 0.3088 ± 0.0061 | (0.2902) 0.294$^{+0.011}_{-0.013}$ | (0.3122) 0.308 ± 0.012 |
| $\langle \Sigma \rangle S_8$ | (0.8114) 0.814 ± 0.013 | (0.800) 0.883$^{+0.067}_{-0.061}$ | (0.817) 0.870 ± 0.070 |
| $A_L$               | (1.0739) 1.069 ± 0.036 | (1.084) 1.027$^{+0.093}_{-0.11}$ | (1.117) 1.060$^{+0.093}_{-0.13}$ |

| $\chi^2_{\text{CMB}}$ | 2760.2$^{(-5.5)}$ | 2759.9$^{(-5.8)}$ | 2759.6 (6.1) |
| $\chi^2_{\text{CMBL}}$ | 10.0$^{(+1.3)}$ | 9.7$^{(+1.0)}$ | 9.7$^{(+1.0)}$ |
| $\chi^2_{\text{SN}}$ | 1034.9$^{(-0.3)}$ | 1030.8$^{(-4.4)}$ | 1035.7$^{(+0.5)}$ |
| $\chi^2_{\text{BAO}}$ | 21.0$^{(+0.4)}$ | 15.7$^{(-4.9)}$ | 20.1$^{(-0.5)}$ |
| $\chi^2_{\text{tot}}$ | 3826.1$^{(-4.1)}$ | 3816.2$^{(-14.0)}$ | 3825.1$^{(-5.1)}$ |

TABLE V. The best fitting maximum posterior model parameters, in parentheses, mean and 68% C.L. constraints for the Baseline + RSD + DES data combination. Data likelihoods at maximum posterior are compared to their reference ΛCDM values in parenthesis.

| Baseline + RSD + DES | ΛCDM (reference) | No theory prior | With Horndeski prior |
|---------------------|------------------|-----------------|----------------------|
| $H_0$               | (67.632) 67.68 ± 0.41 | (70.13) 69.3 ± 1.5 | (68.23) 67.9 ± 1.2 |
| $\Omega_m$          | (0.3115) 0.3107 ± 0.0054 | (0.2877) 0.293$^{+0.012}_{-0.014}$ | (0.3040) 0.306$^{+0.010}_{-0.012}$ |
| $\langle \Sigma \rangle S_8$ | (0.8255) 0.822 ± 0.010 | (0.8236) 0.844$^{+0.032}_{-0.037}$ | (0.8247) 0.838 ± 0.025 |

| $\chi^2_{\text{CMB}}$ | 2766.3 | 2761.3$^{(-5.0)}$ | 2764.4$^{(-1.9)}$ |
| $\chi^2_{\text{CMBL}}$ | 8.8 | 9.3$^{(+0.5)}$ | 8.9$^{(+0.1)}$ |
| $\chi^2_{\text{SN}}$ | 1035.0 | 1029.6$^{(-5.4)}$ | 1034.6$^{(-0.4)}$ |
| $\chi^2_{\text{BAO}}$ | 19.3 | 13.8$^{(-5.5)}$ | 17.9$^{(-1.4)}$ |
| $\chi^2_{\text{DES}}$ | 322.5 | 321.3$^{(-1.2)}$ | 322.1$^{(-0.4)}$ |
| $\chi^2_{\text{tot}}$ | 4151.8 | 4135.3$^{(-16.5)}$ | 4147.9$^{(-3.9)}$ |

| Baseline + RSD + DES | ΛCDM (reference) | No theory prior $+A_L$ | With Horndeski prior $+A_L$ |
|---------------------|------------------|-----------------|----------------------|
| $H_0$               | (68.029) 68.20 ± 0.46 | (69.64) 69.2$^{+1.5}_{-1.3}$ | (67.64) 67.7 ± 1.3 |
| $\Omega_m$          | (0.3061) 0.3038 ± 0.0059 | (0.2898) 0.292$^{+0.012}_{-0.014}$ | (0.3071) 0.307 ± 0.012 |
| $\langle \Sigma \rangle S_8$ | (0.8084) 0.800 ± 0.013 | (0.7672) 0.786 ± 0.040 | (0.7765) 0.781$^{+0.031}_{-0.036}$ |
| $A_L$               | (1.0777) 1.093$^{+0.034}_{-0.038}$ | (1.219) 1.199$^{+0.082}_{-0.096}$ | (1.248) 1.208 ± 0.090 |

| $\chi^2_{\text{CMB}}$ | 2760.3$^{(-6.0)}$ | 2760.5$^{(-5.8)}$ | 2761.8$^{(-4.5)}$ |
| $\chi^2_{\text{CMBL}}$ | 10.0$^{(+1.2)}$ | 10.1$^{(+1.3)}$ | 10.2$^{(+1.4)}$ |
| $\chi^2_{\text{SN}}$ | 1034.8$^{(-0.2)}$ | 1030.2$^{(-4.8)}$ | 1034.6$^{(-0.4)}$ |
| $\chi^2_{\text{BAO}}$ | 19.3(0.0) | 12.4$^{(-6.9)}$ | 15.1$^{(-4.2)}$ |
| $\chi^2_{\text{DES}}$ | 321.2$^{(-1.3)}$ | 317.8$^{(-4.7)}$ | 319.7$^{(-2.8)}$ |
| $\chi^2_{\text{tot}}$ | 4145.6$^{(-6.2)}$ | 4131.0$^{(-20.8)}$ | 4141.4$^{(-10.4)}$ |

[56] K. Hinterbichler and J. Khoury, Phys. Rev. Lett. 104, 231301 (2010), arXiv:1001.4525 [hep-th].
[57] J. Wang, L. Hui, and J. Khoury, Phys. Rev. Lett. 109, 241301 (2012), arXiv:1208.4612 [astro-ph.CO].
### TABLE VI. The best fitting maximum posterior model parameters, in parentheses, mean and 68% C.L. constraints for the Baseline + RSD + DES + $M_{SN}$ data combination. Data likelihoods at maximum posterior are compared to their reference $\Lambda$CDM values in parenthesis.

| Parameter | Baseline + RSD + DES + $M_{SN}$ | $\Lambda$CDM (reference) | No theory prior | With Horndeski prior |
|-----------|---------------------------------|--------------------------|-----------------|---------------------|
| $H_0$     | $(68.039) \pm 0.40$             | $(71.34) \pm 0.16$       | $(69.44) \pm 0.16$ |
| $\Omega_m$| $(0.3062) \pm 0.0052$           | $(0.2759) \pm 0.011$     | $(0.2942) \pm 0.012$ |
| $\langle \Sigma \rangle S_8$ | $(0.8107) \pm 0.0099$ | $(0.8192) \pm 0.0041$ | $(0.8365) \pm 0.0025$ |
| $\chi^2_{CMB}$ | 2768.4                           | 2763.8 (−4.6) | 2764.0 (−4.4) |
| $\chi^2_{CMB}$ | 10.1                             | 8.8 (−1.3)           | 9.0 (−1.1)        |
| $\chi^2_{SN}$ | 1025.9                           | 1022.9 (−3.0)         | 1025.8 (−0.1)     |
| $\chi^2_{BAO}$ | 19.5                             | 15.5 (−4.0)           | 20.6 (−1.1)       |
| $\chi^2_{DES}$ | 321.2                            | 319.2 (−0.2)          | 321.5 (−0.3)      |
| $\chi^2_{M}$ | 18.8                             | 11.9 (−6.9)           | 12.9 (−5.9)       |
| $\chi^2_{M}$ | 4163.9                           | 4142.1 (−21.8)        | 4153.5 (−10.1)    |

| Parameter | Baseline + RSD + DES + $M_{SN}$ | $\Lambda$CDM + $A_L$ | No theory prior + $A_L$ | With Horndeski prior + $A_L$ |
|-----------|---------------------------------|----------------------|--------------------------|-----------------------------|
| $H_0$     | $(68.566) \pm 0.44$             | $(70.96) \pm 0.16$   | $(69.67) \pm 0.16$       |
| $\Omega_m$| $(0.2992) \pm 0.0056$           | $(0.2870) \pm 0.011$ | $(0.2935) \pm 0.013$     |
| $\langle \Sigma \rangle S_8$ | $(0.7922) \pm 0.013$ | $(0.7584) \pm 0.040$ | $(0.7802) \pm 0.033$     |
| $A_L$     | $(1.1067) \pm 0.037$            | $(1.2090) \pm 0.039$ | $(1.118) \pm 0.039$      |
| $\chi^2_{CMB}$ | 2760.7 (−7.7)                    | 2762.1 (−6.3)      | 2762.1 (−6.3)           |
| $\chi^2_{CMB}$ | 10.1 (0.0)                        | 9.8 (−0.3)           | 10.0 (−0.1)             |
| $\chi^2_{SN}$ | 1025.8 (−0.1)                    | 1021.6 (−4.3)       | 1025.7 (−0.2)           |
| $\chi^2_{BAO}$ | 20.7 (1.2)                        | 14.6 (−4.9)          | 19.8 (0.3)               |
| $\chi^2_{DES}$ | 320.2 (−1.0)                      | 317.6 (−3.6)         | 319.2 (−2.0)            |
| $\chi^2_{M}$ | 15.8 (−3.0)                       | 12.4 (−6.4)          | 13.0 (−5.8)             |
| $\chi^2_{M}$ | 4153.3 (−10.6)                    | 4138.1 (−25.8)       | 4149.8 (−14.1)          |

[58] B. Abbott et al. (Virgo, LIGO Scientific), Phys. Rev. Lett. 119, 161101 (2017), arXiv:1710.05832 [gr-qc].

[59] B. P. Abbott et al. (Virgo, Fermi-GBM, INTEGRAL, LIGO Scientific), Astrophys. J. 848, L13 (2017), arXiv:1710.05834 [astro-ph.HE].

[60] M.-X. Lin, M. Raveri, and W. Hu, Phys. Rev. D 99, 043514 (2019), arXiv:1810.02333 [astro-ph.CO].

[61] A. Zucca, L. Pogosian, A. Silvestri, and G.-B. Zhao, JCAP 05, 001 (2019), arXiv:1901.05956 [astro-ph.CO].

[62] A. Lewis and S. Bridle, Phys. Rev. D66, 103511 (2002), astro-ph/0205436.

[63] R. G. Crittenden, L. Pogosian, and G.-B. Zhao, JCAP 0912, 025 (2009), arXiv:astro-ph/0510293 [astro-ph].

[64] L. Pogosian and A. Silvestri, Phys. Rev. D94, 104014 (2016), arXiv:1606.05339 [astro-ph.CO].

[65] N. Aghanim et al. (Planck), (2019), arXiv:1907.12875 [astro-ph.CO].

[66] S. Alam et al. (eBOSS), (2020), arXiv:2007.08991 [astro-ph.CO].

[67] G.-B. Zhao et al., (2020), arXiv:2007.09011 [astro-ph.CO].

[68] Y. Wang et al., (2020), 10.1093/mnras/staa2593, arXiv:2007.09010 [astro-ph.CO].

[69] J. Hou et al., Mon. Not. Roy. Astron. Soc. 500, 1201 (2020), arXiv:2007.08998 [astro-ph.CO].

[70] H. du Mas des Bourboux et al., (2020), arXiv:2007.08995 [astro-ph.CO].

[71] A. J. Ross, L. Samushia, C. Howlett, W. J. Percival, A. Burden, and M. Manera, Mon. Not. Roy. Astron. Soc. 449, 835 (2015), arXiv:1409.3212 [astro-ph.CO].

[72] F. Beutler, C. Blake, M. Colless, D. Jones, L. Staveley-Smith, L. Campbell, Q. Parker, W. Saunders, and F. Watson, Mon. Not. Roy. Astron. Soc. 416, 3017 (2011), arXiv:1106.3366 [astro-ph.CO].

[73] G.-B. Zhao et al. (BOSS), Mon. Not. Roy. Astron. Soc. 466, 712 (2017), arXiv:1607.03153 [astro-ph.CO].

[74] D. Scoccola et al., Astrophys. J. 859, 101 (2018), arXiv:1708.08845 [astro-ph.CO].

[75] J. E. Bautista et al., Mon. Not. Roy. Astron. Soc. 500, 736 (2020), arXiv:2007.08993 [astro-ph.CO].

[76] A. de Mattia et al., Mon. Not. Roy. Astron. Soc. 501, 5616 (2021), arXiv:2007.09008 [astro-ph.CO].

[77] R. Neveux et al., Mon. Not. Roy. Astron. Soc. 499, 210 (2020), arXiv:2007.08999 [astro-ph.CO].

[78] P. A. R. Ade et al. (Planck), Astron. Astrophys. 594, A14 (2016), arXiv:1502.01590 [astro-ph.CO].

[79] G. Buenengo, W. Hu, and M. Raveri, Phys. Rev. D 101, 103517 (2020), arXiv:2002.11707 [astro-ph.CO].

[80] E. Calabrese, A. Slosar, A. Melchiorri, G. F. Smoot, and O. Zahn, Phys. Rev. D 77, 123531 (2008), arXiv:0803.2309 [astro-ph].

[81] M. Raveri and W. Hu, Phys. Rev. D 99, 043506 (2019), arXiv:1806.04649 [astro-ph.CO].

[82] N. Aghanim et al. (Planck), Astron. Astrophys. 641, A8 (2020), arXiv:1807.06210 [astro-ph.CO].
[83] S. Peirone, K. Koyama, L. Pogosian, M. Raveri, and A. Silvestri, Phys. Rev. D97, 043519 (2018), arXiv:1712.00444 [astro-ph.CO].

[84] J. Gleyzes, D. Langlois, M. Mancarella, and F. Vernizzi, JCAP 1602, 056 (2016), arXiv:1509.02191 [astro-ph.CO].

[85] I. D. Saltas, I. Sawicki, L. Amendola, and M. Kunz, Phys. Rev. Lett. 113, 191101 (2014), arXiv:1406.7139 [astro-ph.CO].

[86] C. Deffayet, G. Esposito-Farese, and A. Vikman, Phys. Rev. D 79, 084003 (2009), arXiv:0901.1314 [hep-th].

[87] C. Deffayet, O. Pujolas, I. Sawicki, and A. Vikman, JCAP 1010, 026 (2010), arXiv:1008.0048 [hep-th].

[88] E. V. Linder, JCAP 03, 005 (2018), arXiv:1801.01503 [astro-ph.CO].

[89] T. M. C. Abbott et al. (DES), Phys. Rev. D 99, 123505 (2019), arXiv:1810.02499 [astro-ph.CO].

[90] M. Chevallier and D. Polarski, Int. J. Mod. Phys. D 10, 213 (2001), arXiv:gr-qc/0009008.

[91] E. V. Linder, Phys. Rev. Lett. 90, 091301 (2003), arXiv:astro-ph/0208512.