Constraining the neutron star equation of state using multi-band independent measurements of radii and tidal deformabilities

Margherita Fasano,† Tiziano Abdelsalhin,‡ Andrea Maselli,§ and Valeria Ferrari¶
Dipartimento di Fisica, Sapienza Università di Roma & Sezione INFN Roma1, P.A. Moro 5, 00185, Roma, Italy

Using a Bayesian approach, we combine measurements of neutron star macroscopic observables obtained by astrophysical and gravitational observations, to derive joint constraints on the equation of state (EoS) of matter at supranuclear density. In our analysis we use two sets of data: (i) the masses and tidal deformabilities measured in the binary neutron star event GW170817, detected by LIGO and Virgo; (ii) the masses and stellar radii measured from observations of nuclear bursts in accreting low-mass X-ray binaries. Using two different parametrizations of the equation of state, we compute the posterior probability distributions of the EoS parameters, using which we infer the posterior distribution for the radius and the mass of the two neutron stars of GW170817. The constraints we set on the radii are tighter than previous bounds.

Introduction. The detection of the gravitational wave (GW) signal emitted in the coalescence of the binary neutron star (BNS), GW170817 [1], offers a unique opportunity to probe the properties of matter at the extreme densities occurring in a neutron star (NS) core. This detection has stimulated a number of studies in this direction, both by the LIGO/Virgo collaboration (LVC) and by independent groups. As a result, new constraints on NS internal composition have been derived, according to which the equation of state (EoS) of nuclear matter is more likely to be soft, leading to stellar configurations with high compactness and small radii [2–22].

The GW signal emitted in a BNS coalescence carries the imprint of NS structure in both the inspiral and merger/post-merger phases [23–29]. During the inspiral, the information on the stellar composition is encoded at the leading order in the quadrupolar tidal deformability Λ, which describes how the shape of one star changes in response to the external tidal field [33]. For a given equation of state, this parameter depends solely on the stellar compactness \( C = M/R \), i.e., on the ratio between mass and circumferential radius of the NS at equilibrium. Moreover, \( Λ \) is a monotonic function of \( C \), and decreases as the compactness grows, i.e. for more compressible (soft) matter.

The analysis of the data of GW170817 has allowed to estimate the average parameter\(^1\) \( \tilde{Λ} = \frac{16}{13} (M_1 + 12M_2)M_1^{1/4}M_2^{1/4} + (M_2 + 12M_1)M_1^{7/4}M_2^{7/4} \), where \( Λ_{1,2} \) are the individual NS tidal deformabilities and \( M_{1,2} \) are the NS masses [12, 13]. Assuming low-spin priors, the analysis carried out by the LVC collaboration yields a value of \( \tilde{Λ} = 306^{+420}_{−230} \) at 90% confidence level. Combined with the posterior distributions of the inferred masses, this result leads to a constraint on the NS radius of \( 10 \lesssim R \lesssim 13 \) km, which excludes a class of EoS predicting stiff matter, i.e., less compact stars.

\(^1\) \( \tilde{Λ} \) is the actual parameter that enters at the leading order into the gravitational waveform [34–36].

The first BNS event also marks the dawn of multimessenger astronomy, which will combine observations in the electromagnetic (EM) and in the gravitational bandwidths, expected to cover a broad range of wavelengths, and different stages of the evolution of the observed sources.

Following the discovery of the EM counterpart of GW170817, the information obtained from the GW data has been complemented by that inferred from the electromagnetic observations associated with the merger/post-merger phase of the binary event [37–39]; in particular, the properties of the gamma ray burst and the kilonova light curves have been exploited [3, 4, 6, 8–11, 15]. Thus, in these studies EM and GW observations of the same event have been fully exploited.

In this paper we combine independent measurements of NS macroscopic observables obtained from EM and GW data, to derive joint constraints on the EoS of matter at supranuclear density. We consider two distinct datasets based on: (i) masses and tidal deformabilities extracted from the the data analysis of GW170817; (ii) masses and radii measured through spectroscopic observations of NS thermonuclear bursts in low-mass X-ray binaries [40]. Using two different parametrization of the EoS, we infer the posterior probability distribution of the EoS parameters through a fully Bayesian analysis, and derive new bounds on the radius of the two NS coalescing in GW170817. Our results show how our understanding of the EoS of matter at supranuclear density can benefit from the synergy of data coming from astrophysical phenomena spanning very different dynamical regimes, and detected with distinct experimental setups.

Furthermore, the proposed approach is a promising tool to exploit the data of high-precision surveys which, in the near future, will be available from space satellites [41] and from advanced and third generation GW interferometers [42, 43].

Parametrized EoS. The thermodynamical properties of matter inside a cold neutron star can be described by a barotropic relation between the pressure \( p \) and the en-
ergy density $\epsilon$, i.e., by the equation of state, $p = p(\epsilon)$. At densities $\rho \lesssim \rho_0$, where $\rho_0 \sim 2.7 \times 10^{14}$ g/cm$^3$ is the equilibrium density of nuclear matter, the EoS has been determined by extrapolating the results of terrestrial experiments on atomic nuclei [44–47], and there is a general consensus on its properties. For densities above the saturation point, typical of a NS core, the EoS is less certain; indeed, due to the complexity of quantum chromo-dynamics and to the difficulty of testing these regimes with experiments on Earth, hadronic interactions are described by a variety of models based on different approaches and assumptions [48]. These EoS, when used to describe a NS, lead to different observables and to different relations between radius, or tidal deformability, and mass. Thus, astrophysical and gravitational wave measurements of these observables can be exploited to constrain the EoS, solving the so-called inverse stellar problem [40, 49–59]. Phenomenologically parametrized EoS are particularly useful in this respect, as they allow us to describe a large class of theoretical EoS through a relatively small set of coefficients [60–64]. Moreover, they provide a unique tool to combine stellar parameters obtained from NS observations in different wavebands, to infer features of the true EoS which may not be predicted by current models.

In this work we focus onto two parametrizations:

(i) The piecewise polytropic model by Read et al. [60]. In this representation, the EoS of the NS core is modelled by three polytropic segments. The parametrization is completely specified by 4 parameters, namely, the three adiabatic indices $(\Gamma_1, \Gamma_2, \Gamma_3)$ of the polytropic branches, and the value of the pressure $p_1$ at approximately twice the saturation density.

(ii) The spectral representation by Lindblom [61]. This model is based on a series expansion of the adiabatic index $\Gamma(p)$. It has been shown that most theoretical EoS are well approximated including the first four terms in the expansion, which correspond to the 4 free parameters of the model $(\gamma_0, \gamma_1, \gamma_2, \gamma_3)$, i.e., $\Gamma(p) = \sum_{k=0}^{3} \gamma_k (p/p_0)^k$, where $p_0$ is the pressure at the crust-core interface, which we choose as in [13].

The Bayesian framework. The goal of this work is to combine measurements of NS observables obtained in different astrophysical channels, such as masses, radii and tidal deformabilities, and to reconstruct the parameters of a phenomenological EoS. The mass and the radius of a NS can be measured through electromagnetic observations in the X-, optical and radio wavebands of low-mass binary systems [65]. The detection of gravitational waves emitted in BNS merging allows to estimate masses and tidal deformabilities of the coalescing bodies [1, 12, 13, 66]. We shall now show how, using a Bayesian scheme of inference, such complementary information can be combined to put stronger constraints on the NS EoS.

Both the piecewise polytropic and the spectral representation models are characterized by $m = 4$ free parameters; in addition, in order to determine the structure of a NS we need to set the central pressure $p_c$. Each NS observation provides 2 observables, either the mass and the radius $(M, R)$, or the mass and the tidal deformability $(M, \Lambda)$, obtained in the EM and GW channel, respectively. Therefore, to fully characterise the parametrised EoS we need at least $N = m = 4$ observations, which would allow to determine the 8 unknown quantities

$$\theta = \begin{cases} (p_1, \Gamma_1, \Gamma_2, \Gamma_3, p_4, \ldots), & \text{piecewise} \\ (\gamma_0, \gamma_1, \gamma_2, \gamma_3, p_4, \ldots), & \text{spectral} \end{cases}.$$

The LIGO/Virgo collaboration has detected one binary NS merger so far, GW170817, and the mass and tidal deformabilities of the two stars have been estimated [12, 13]. Estimates of NS masses and radii based on electromagnetic observations have been obtained using a wide variety of methodologies applied to different astrophysical environments [65]. In this work we consider the results of a study carried out by Ozel and collaborators, who analysed the spectroscopic observations of twelve NSs, during thermonuclear bursts or in quiescence [40]. GW170817 and two (out of the twelve) EM measurements provide enough information to solve the inverse stellar problem described before, with a joint set of data given by $d = \{(M_1, \Lambda_1, M_2, \Lambda_2), (M_3, R_3), (M_4, R_4)\}$.

Using a Bayesian approach, we compute the posterior probability density function (PDF) of the EoS parameters given the experimental data, $P(\theta | d) \propto L(d | \theta) P_0(\theta)$, where $P_0(\theta)$ is the prior on the parameters, and $L(d | \theta)$ is the likelihood function, which, in our case, reads:

$$L(d | \theta) = L^{GW}(M_1, \Lambda_1, M_2, \Lambda_2) \times L^{EM}(M_3, R_3) \times L^{EM}(M_4, R_4),$$

where $L^{GW}(M_1, \Lambda_1, M_2, \Lambda_2)$ and $L^{EM}(M_i, R_i)_{i=3,4}$ are the probability computed by the LIGO/Virgo collaboration [67, 68] and Ozel et al. [69], respectively (both publicly available). We sample the posterior distribution using Markov chain Monte Carlo (MCMC) simulations based on the Metropolis-Hastings algorithm [70]. The MCMC convergence is enhanced by a Gaussian adaptation algorithm [71, 72] (see [57] and reference therein for a detailed discussion on this approach).

Numerical setup. For the GW data, the likelihood $L^{GW}(M_1, \Lambda_1, M_2, \Lambda_2)$ is given by the joint probability distribution for the masses and tidal deformabilities of GW170817 inferred by the LVC using a parametrised EoS [13]. For the EM sector, $L^{EM}(M_i, R_i)_{i=3,4}$ is given
by the joint probability distributions of the most accurate measurements of masses and radii provided in [40]. The latter correspond to two NSs in low-mass X-ray binaries observed during thermonuclear bursts, namely 4U 1724-207 and EXO 1745-248 [40]. Table I shows the median and the 90% confidence intervals for the GW and EM data.

| M1 [M⊙] | M2 [M⊙] | M3 [M⊙] | M4 [M⊙] |
|----------|----------|----------|----------|
| 1.46^{+0.14}_{-0.09} | 2.55^{+0.16}_{-0.17} | 1.26^{+0.09}_{-0.12} | 661^{+885}_{-375} |

| A1 | A2 |
|----|----|
| 11.46^{+2.54}_{-2.49} | 1.60^{+0.36}_{-0.42} |
| 10.33^{+2.75}_{-2.38} |

TABLE I. Median and 90% intervals for the masses M1,2 and the tidal deformabilities $\Lambda_{1,2}$ of the two NS observed in GW170817 [13] and for the masses M3,4 and the radii R3,4 of 4U 1724-207 and EXO 1745-248 [65].

Having inferred the probability distributions for the parameters (1), we can compute the posterior distributions for radius and mass of the two NS of GW170817, by solving the TOV equations for each of the model sampled by the MCMC.

Figure 2 shows the joint distribution of mass and radius of the two stars of GW170817, constructed using the piecewise polytropic EoS and the spectral representation. The distributions obtained through the two parametrised EoS are compatible with each other, but a difference appears in the width of the distributions. Indeed, the spectral approach leads to tighter constraints. This is due to how the two phenomenological parameterisations model the energy density profiles of theoretical EoS. Both the piecewise and the spectral representation reproduce macroscopic observables, such radii and tidal deformabilities, with comparable accuracy, of the order of few percent. However, locally, the two approaches behave differently. As shown in Fig. 6 of [57], for low-compact NS some of the piecewise coefficients may not contribute significantly to the stellar structure, and therefore are poorly constrained by the Bayesian analysis. This broadens the posterior distribution of all the parameters, and then affects the reconstructed radii. Further studies on the intrinsic features of each parametrisation, together with a detailed comparison with other models available in literature, will be presented in a forthcoming publication.

The values of mass and radius for the two stars, at 90% confidence level for the piecewise polytropic EoS are

- $M_1 = 1.45^{+0.12}_{-0.07} M_\odot$, $R_1 = 10.56^{+0.84}_{-0.75}$ km and $M_2 = 1.27^{+0.07}_{-0.10} M_\odot$, $R_2 = 11.39^{+1.67}_{-1.45}$ km. For the spectral representation EoS we obtain

- $M_1 = 1.47^{+0.07}_{-0.09} M_\odot$, $R_1 = 10.56^{+0.84}_{-0.75}$ km and $M_2 = 1.26^{+0.08}_{-0.09} M_\odot$, $R_2 = 10.58^{+0.90}_{-0.82}$ km.

In Fig. 3 we plot, for the same GW170817 stars, the corresponding marginalised distribution of the two radii.

Note that, as shown in Table III of [60] the values of $\Gamma_0$ of the EoS proposed in the literature range within $\sim [1.3,4.0]$. 

Note that $p_1$ is the base-ten logarithm of the pressure expressed in CGS units.
FIG. 1. Posterior distributions of the parameters of the piecewise polytropic (top panels) and of the spectral representations (bottom row), derived through the multi-messenger analysis. Dashed vertical lines identify 90% confidence intervals, also shown on top of each panel with the median.

FIG. 2. 90% confidence regions for the posterior distribution of mass $M$ and radius $R$ of the two neutron stars of GW170817, built using the spectral (solid curves) and piecewise polytropic (dashed curves) EoS. Black curves identify the mass-radius profiles for some theoretical EoS [74–77].

The dashed and the solid vertical lines identify the 90% credible interval determined by LVC in [13], and by our analysis, respectively. In both cases the spectral EoS has been used. The analysis we perform, which combines GW and EM observations, seems to prefer configurations which are more compact than those inferred by the LVC data alone, indicating a softer equation of state in the core. This is a very interesting result: including in our analysis more massive stars, i.e. those with mass and radius estimated through EM observations (Table I), we are probing the EoS in a region where the energy density is larger with respect to that probed by the LVC analysis.

Conclusions. Multi-wavelength observations of relativistic sources provide an arena where the joint efforts of the astrophysics, high-energy and particle physics community convey to provide new insights on the fundamental laws of Nature. Neutron stars are among the primary targets of this quest, as unique laboratories to investigate the behaviour of matter at densities not reproducible in experiments on Earth. The detection of the first coalesc-
ing binary composed of two neutron stars has allowed the LIGO/Virgo collaboration to derive the first GW constraint on the equation of state of matter in the inner core of neutron stars. A large variety of follow-up analyses have been pursued to further exploit the observation of the electromagnetic counterpart in coincidence with the gravitational event. As a result, new bounds have been derived, which indicate that the EoS of matter in the inner core of a neutron star is in the soft sector, and produces more compact stars.

In this paper we have made a step forward in this search, by combining independent measurements of NS macroscopic parameters, namely the radius and the tidal deformabilities from the LIGO/Virgo event, and the mass and the radius derived from EM observations of low mass X-ray binaries, in the spirit of multi-messenger astrophysics. Using this approach, we have been able to set tighter constraints on the radius of the two neutron stars coalescing in GW170817, thus supporting, and strengthening, the observational evidence that neutron star cores are composed of soft nuclear matter.

We thank Giovanni Camelio and Francesco Panunzio for useful discussions and advices on the methods used in this work. A.M. acknowledges financial support provided under the European Union’s H2020 ERC, Starting Grant agreement no. DarkGRA–757480. We acknowledge support from the Amaldi Research Center funded by the MIUR program “Dipartimento di Eccellenza” (CUP: B81118001170001). The authors would like to acknowledge networking support by the COST Action CA16104.

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* margherita.fasano@roma1.infn.it  
† tiziano.abdelsalhin@roma1.infn.it  
‡ andrea.maselli@roma1.infn.it  
§ valeria.ferrari@uniroma1.it  
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