Isospin Violations in the Pion-Nucleon System

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Abstract

We examine the effect of isospin-violating meson-nucleon coupling constants on low-energy pion-nucleon scattering. We compute the couplings in the context of a nonrelativistic quark model. The difference between the up and down constituent masses induces a coupling of the neutral pion to the proton that is slightly larger than the corresponding one for the neutron. This difference generates a large isospin-violating correction—proportional to the isospin-even contribution arising from the nucleon Born terms—to the charge-exchange ($\pi^- p \rightarrow \pi^0 n$) amplitude. In contrast to the isospin-conserving case, this correction is not cancelled by $\sigma$-meson exchange; in our model there is no isospin-violating $NN\sigma$ coupling at $q^2 = 0$. As a result, we find a violation of the triangle identity consistent with the one reported by Gibbs, Ai, and Kaufmann from a recent analysis of pion-nucleon data.

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I. INTRODUCTION

Low energy pion-nucleon ($\pi N$) scattering is one of the best available tools for testing small violations to approximate symmetries of nature. Such violations are expected to be amplified in low-energy $\pi N$ scattering because of the constraints imposed on the symmetry-conserving amplitudes by chiral symmetry. At low energies (i.e., in the soft-pion limit) the pions couple very weakly to the nucleons as a direct consequence of chiral symmetry. Thus, although the violations to the symmetry might be small, they must be considered relative to intrinsically small symmetry-conserving amplitudes.

An example of such a scenario has been reported recently by Gibbs, Ai, and Kaufmann [1]. They have analyzed low-energy pion-nucleon data in search of isospin violations. From very precise data on elastic ($\pi^\pm p$) and charge-exchange ($\pi^- p \to \pi^0 n$) reactions they have extracted $\pi N$ scattering amplitudes from which they have computed violations to the “triangle identity”

$$D \equiv f(\pi^- p \to \pi^0 n) - \frac{1}{\sqrt{2}} \left[ f(\pi^+ p) - f(\pi^- p) \right].$$

(1)

They observed a large isospin violation—of the order of 7%—even after accounting for Coulomb effects and hadronic mass differences. This is particularly interesting since isospin-breaking mechanisms, having their origin in the up-down quark mass difference and electromagnetic effects, are expected to be present at the $\sim 1\%$ level.

Evidence for the loss of isospin symmetry in the nucleon-nucleon ($NN$) system is well documented. The difference in the $pp$ and $nn$ scattering lengths [2], the Nolen-Schiffer anomaly [3,4], and the neutron-proton analyzing-power difference [5–7] are all well known examples. Most theoretical efforts directed at understanding isospin-violating observables in the $NN$ system proceed from a two-body interaction constrained from fits to two-nucleon data and incorporate isospin-violating corrections from a variety of sources. These can be classified as arising from: (i) isovector-isoscalar mixing in the meson propagator—such as $\rho$-$\omega$ mixing, (ii) isospin-breaking in the nucleon wave function—through the neutron-proton mass difference, and (iii) isospin-breaking in the meson-nucleon and photon-nucleon vertices—as in the case of electromagnetic scattering. It is important to note that all these isospin-breaking mechanisms also operate in the pion-nucleon system. Thus, a clear understanding of their role in $NN$ scattering could be of great value to the analysis of low-energy $\pi N$ data. A particularly important—and timely—example is $\rho$-$\omega$ mixing. Naively, one would expect large violations to the triangle identity (also known as the “triangle discrepancy”) to arise from $\rho$-$\omega$ mixing because of the strong $NN\omega$ and $\pi\pi\rho$ couplings. Note, however, that in computing near-threshold $\pi N$ observables it is the mixing amplitude near $q^2 = 0$ that is relevant. The traditional mechanism of $\rho$-$\omega$ mixing, with the mixing amplitude fixed at the on-shell point, has been called recently into question [8]. Indeed, a large number of calculations using a variety of models have found a value of the $\rho$-$\omega$ mixing amplitude at $q^2 = 0$ that is strongly suppressed relative to its on-shell value [9–13]. Moreover, for models in which the vector mesons couple to conserved currents, the $\rho$-$\omega$ mixing amplitude is identically zero at $q^2 = 0$ [9,13]. Thus, we believe that $\rho$-$\omega$ mixing should play a small role in low-energy pion-nucleon scattering.

Removing $\rho$-$\omega$ mixing as a viable source of isospin-breaking has important phenomenological consequences; on-shell $\rho$-$\omega$ mixing accounts for a substantial fraction of the neutron-
proton analyzing-power difference at 183 MeV \[5,14,15\]. Hence, if $\rho$-$\omega$ mixing is no longer important at $q^2 \lesssim 0$, additional sources of isospin violation must be found. In a recent study of hadronic structure Dmitrašinović and Pollock have computed isospin-violating corrections to the electroweak form factors of the nucleon \[16\]. Motivated by their findings we have investigated new sources of charge-symmetry violation in the $NN$ potential which resulted from isospin-violating meson-nucleon coupling constants \[17\]. The resulting class IV contribution to the charge-symmetry-breaking $NN$ potential is comparable in magnitude and identical in sign to the one obtained from on-shell $\rho$-$\omega$ mixing. We showed that this new contribution—without on-shell $\rho$-$\omega$ mixing—is consistent with the measured value of $\Delta A$ at 183 MeV \[18\]. It is the purpose of this paper to estimate the effect of isospin-violating meson-nucleon coupling constants on low-energy pion-nucleon scattering.

II. LOW-ENERGY PION-NUCLEON SCATTERING

We approach the study of low-energy pion-nucleon scattering in a conventional way; we include contributions arising from the (s- and u-channel) nucleon Born terms and from (t-channel) meson exchanges \[19\]. These contributions—particle-exchange poles—give a good representation of the amplitude when the poles are close to the physical region, such as in low-energy $\pi N$ scattering in the chiral ($m_\pi \to 0$) limit. The linear $\sigma$-model \[20\] and Quantum Hadrodynamics (QHD-II) \[21\] are appropriate theoretical frameworks to generate these tree-level contributions. The models differ, at tree level, in the allowed t-channel exchanges and, hence, in the prediction of low-energy $\pi N$ parameters. However, as we shall see, they generate the same isospin-violating contributions in our model.

The $\pi N$ scattering matrix can be written in terms of two sets (one for each isospin combination) of two Lorentz invariant amplitudes ($A$ and $B$) which contain all dynamical information about the reaction \[19\]

\[
\mathcal{T} = \left[ A^{(+)}(s,t) + \frac{1}{2}(\bar{k} + \bar{k}')B^{(+)}(s,t) \right] - \left[ A^{(-)}(s,t) + \frac{1}{2}(k + k')B^{(-)}(s,t) \right] (\mathbf{T} \cdot \mathbf{\tau}) .
\]

(2)

Note, the Lorentz invariant amplitudes are written in terms of the relevant Mandelstam variables ($t \equiv q^2$)

\[
\begin{align*}
    s &= (p + k)^2 = (p' + k')^2 , \\
    t &= (k - k')^2 = (p' - p)^2 , \\
    u &= (p - k')^2 = (p' - k)^2 ,
\end{align*}
\]

(3a)

(3b)

(3c)

where $k(k')$ and $p(p')$ are the initial(final) four-momenta of the pion and nucleon, respectively. The Mandelstam variables are related by $s + t + u = 2m_\pi^2 + 2M^2$. We have also introduced pion ($\mathbf{T}$) and nucleon ($\mathbf{\tau}$) isospin matrices [note, $(T_a)_{bc} \equiv -i\epsilon_{abc}$]. Isospin invariance, which is still assumed unbroken, allows for only two isospin combinations: isospin even [denoted by $(\pm)$] and isospin odd [denoted by $(\mp)$]. The connection to the reaction amplitudes is given through the following relations:

\[
\begin{align*}
    \mathcal{T}(\pi^+ p \to \pi^+ p) &= \mathcal{T}^{(+)} - \mathcal{T}^{(-)} , \\
    \mathcal{T}(\pi^- p \to \pi^- p) &= \mathcal{T}^{(+)} + \mathcal{T}^{(-)} , \\
    \mathcal{T}(\pi^- p \to \pi^0 n) &= -\sqrt{2} \mathcal{T}^{(\mp)} .
\end{align*}
\]

(4a)

(4b)

(4c)
From these, the triangle identity [see Eq. (11)] follows by inspection.

The partial-wave decomposition of the scattering amplitude is simplest if carried out after the Lorentz-invariant scattering matrix has been evaluated between on-shell spinors in the center-of-mass (CM) frame. Thus, as an operator in the spin space of the nucleon the \( \pi N \) scattering amplitude can be written as,

\[
\hat{f}(\pm) = f_1^{(\pm)}(W, \theta) + f_2^{(\pm)}(W, \theta) \frac{(\sigma \cdot k')(\sigma \cdot k)}{k^2},
\]

where the connection to the Lorentz-invariant amplitudes is given through the relations

\[
f_1^{(\pm)}(W, \theta) = \left( \frac{E_k + M}{8\pi W} \right) \left[ A^{(\pm)}(s, t) + (W - M)B^{(\pm)}(s, t) \right], \quad (6a)
\]

\[
f_2^{(\pm)}(W, \theta) = \left( \frac{E_k - M}{8\pi W} \right) \left[ -A^{(\pm)}(s, t) + (W + M)B^{(\pm)}(s, t) \right]. \quad (6b)
\]

Here \( \theta \) denotes the CM scattering angle and \( W = (\epsilon_k + E_k) \) is the total energy of the system in the CM frame; it is written in terms of the individual pion \( (\epsilon_k) \) and nucleon \( (E_k) \) contributions. Finally, by introducing the partial-wave amplitudes \( f_l^{\pm} \), appropriate for scattering in a total angular-momentum channel \( j = l \pm 1/2 \), the amplitudes \( f_1 \) and \( f_2 \) can be expanded in a partial-wave series:

\[
f_1^{(\pm)}(W, \theta) = \sum_l \left[ f_{l+}^{(\pm)}(W)P_l(\cos \theta) - f_{l-}^{(\pm)}(W)P_{l-1}(\cos \theta) \right], \quad (7a)
\]

\[
f_2^{(\pm)}(W, \theta) = \sum_l \left[ f_{l+}^{(\pm)}(W) - f_{l-}^{(\pm)}(W) \right]P_l(\cos \theta). \quad (7b)
\]

We compute the Lorentz invariant amplitudes \( A \) and \( B \) in the linear sigma model [19]. The connection to other models, specifically to QHD, will be done below. At tree-level, the amplitudes receive contribution from only three Feynman diagrams: the two nucleon Born terms and \( \sigma \)-meson exchange. That is,

\[
A^{(+)}(s, t) = -\frac{g_{\pi NN}^2}{M} \frac{m_\sigma^2 - m_\pi^2}{|k| \to 0} \frac{2}{M} \left( 1 - \frac{m_\pi^2}{m_\sigma^2} \right), \quad (8a)
\]

\[
A^{(-)}(s, t) = 0, \quad (8b)
\]

\[
B^{(+)}(s, t) = -\frac{g_{\pi NN}^2}{s - M^2} + \frac{g_{\pi NN}^2}{u - M^2} \to \frac{g_{\pi NN}^2}{Mm_\pi} \left( 1 - \frac{m_\pi^2}{4M^2} \right)^{-1}, \quad (8c)
\]

\[
B^{(-)}(s, t) = -\frac{g_{\pi NN}^2}{s - M^2} - \frac{g_{\pi NN}^2}{u - M^2} \to \frac{g_{\pi NN}^2}{2M^2} \left( 1 - \frac{m_\pi^2}{4M^2} \right)^{-1}. \quad (8d)
\]

where the limit follows from evaluating the amplitudes at threshold: \( t = 0, s = (M + m_\pi)^2 \), and \( u = (M - m_\pi)^2 \). The extraction of the \( \pi N \) scattering lengths, defined by

\[
a_0^{(\pm)} = \lim_{|k| \to 0} f_1^{(\pm)} = \frac{1}{4\pi(1 + m_\pi/M)} \left[ A^{(\pm)} + m_\pi B^{(\pm)} \right], \quad (9)
\]

is now straightforward. We obtain,
\[ a_{0}^{(+)} = \frac{1}{4\pi(1 + m_\pi/M)} \frac{g_{NN\pi}^2}{M} \left[ \left( 1 - \frac{m_\pi^2}{m_\sigma^2} \right) - \left( 1 - \frac{m_\pi^2}{4M^2} \right)^{-1} \right] \xrightarrow{m_\pi \to 0} 0, \quad (10a) \]

\[ a_{0}^{(-)} = \frac{1}{4\pi(1 + m_\pi/M)} \frac{g_{NN\pi}^2}{M} \left( \frac{m_\pi}{2M} \right) \left( 1 - \frac{m_\pi^2}{4M^2} \right)^{-1} \xrightarrow{m_\pi \to 0} 0. \quad (10b) \]

The \( \sigma \)-exchange contribution is a direct consequence of the underlying chiral symmetry of the model; it is essential for effecting the sensitive cancellation of the isospin-even scattering length. Indeed, each individual contribution to \( a_{0}^{(+)} \) is approximately two orders of magnitude larger than the experimental value. Instead, the isospin-odd scattering length vanishes in the chiral limit without the need for sensitive cancellations; in the linear \( \sigma \) model no additional t-channel exchanges are included.

A model that allows for additional t-channel exchanges is QHD-II [21]. Note, even though QHD-II is not a chiral model, a reasonable description of low-energy \( \pi N \) scattering has been achieved through a “fine tuning” of parameters [21,22]. A potentially important (t-channel) isospin-breaking contribution to \( \pi N \) scattering might come via \( \rho \)-meson exchange. Indeed, recently we have computed a large isospin violation in the \( NN\rho \) coupling constant [17]. This, combined with the large isospin-conserving \( \pi\pi\rho \) coupling, could have a substantial impact on the triangle discrepancy. However, as we shall see below, in our model all isospin violations arising from the vector-meson sector must vanish as \( q^2 \to 0 \).

### III. ISOSPIN-VIOLATING MESON-NUCLEON COUPLING CONSTANTS

In this section we concentrate on isospin violations to the triangle identity which arise, exclusively, from isospin-violating meson-nucleon coupling constants. Additional isospin-breaking mechanisms, particularly those associated with Coulomb effects and hadronic mass differences, have been treated elsewhere [1]. Recently, we have estimated the effect of isospin-violating meson-nucleon coupling constants on the \( NN \) potential [17]. We have reported a large contribution from vector-meson exchange to the class IV nucleon-nucleon potential. The isospin-violating couplings that we have computed emerged from evaluating matrix elements of quark currents between nucleon states; the violations are driven by the up-down quark mass difference.

The isospin violations that we have computed arise on rather general grounds; we have assumed that the vector mesons (\( \omega \) and \( \rho \)) couple to appropriate isospin components of the quark electromagnetic current. Moreover, at \( q^2 = 0 \) our results are insensitive to the quark-momentum distribution; they depend merely on the spin and flavor structure of the nucleon wave function. As a result, some important constraints emerge at \( q^2 = 0 \). In particular, only isospin violations in the tensor (or anomalous) couplings are allowed at \( q^2 = 0 \); the vector couplings are “protected” by gauge invariance and remain unchanged. However, since all tensor-driven contributions to \( \pi N \) scattering vanish in the soft-pion limit (\( q_\mu \to 0 \)) isospin-violating vector-meson-nucleon coupling constants can not contribute to the triangle discrepancy. Moreover, there is no contribution from \( \rho-\omega \) mixing at \( q^2 = 0 \), [3,13]. Note that, contrary to the claim of Ref. [23], the momentum-dependence of the \( \rho-\omega \) mixing amplitude can not be absorbed into the vertex without violating gauge invariance. Thus, in our model, all three sources of isospin breaking in the vector-meson sector must vanish at \( q^2 = 0 \). In
our model, there is no isospin-violating $NN\sigma$ coupling either; the $NN\sigma$ vertex, which has the same nonrelativistic limit as the timelike component of the vector, is also protected at $q^2 = 0$.

However, there is no symmetry that protects the $NN\pi$ coupling at $q^2 = 0$. We are interested in computing the coupling of the neutral pion to the nucleon in a nonrelativistic quark model. At $q^2 = 0$ the coupling is determined from the spin and flavor content of the nucleon wave function. In contrast, the isospin-violating coupling of the nucleon to the charged pions is sensitive to the quark momentum distribution and, therefore, more uncertain [4]. It seems, however, that under reasonable assumptions the quark model is able to generate isospin-violating ($NN\pi^\pm$) couplings of comparable strength as those obtained in conventional hadronic treatments based on the neutron-proton mass difference. Presumably, these effects have been included in Ref. [1].

The most general form for the on-shell $NN\pi^0$ vertex function consistent with Lorentz covariance and parity invariance is given by

$$g_{NN\pi}\Lambda_{NN\pi}^5 = g_{NN\pi}\left[g_\pi\gamma^5\right].$$

Here $g_{NN\pi}$ is the isospin-conserving $NN\pi$ coupling constant known phenomenologically from fits to $NN$ phase shifts and to the properties of the deuteron: $g_{NN\pi}^2/4\pi = 14.21$ [24,25]. The isospin-violating component is assumed to emerge from evaluating matrix elements of a flavor odd, pseudoscalar quark current between nucleon states, i.e.,

$$\langle N(p',s') | \left[ \frac{1}{5} \bar{u}\gamma^5 u - \frac{1}{5} \bar{d}\gamma^5 d \right] | N(p,s) \rangle = \bar{U}(p',s')\Lambda_{NN\pi}^5 U(p,s).$$

Here $U(p,s)$ denotes an on-shell nucleon spinor of mass $M_N$, momentum $p$ and spin $s$. Moreover, the constituent quarks are assumed elementary as no quark form factors are introduced. The coupling constants are computed at $q^2 = 0$ by examining the nonrelativistic reduction of Eq. (12); this is the essence of the quark-pion model of Mitra and Ross [26]. In particular, in this limit the derivation closely resembles that which is used in computing the nucleon magnetic moments [27]. We obtain,

$$\frac{g_p^\pi}{2M_p} = \frac{4}{3} \left( \frac{+1/5}{2m_u} \right) - \frac{1}{3} \left( \frac{-1/5}{2m_d} \right) = \frac{4}{30m_u} + \frac{1}{30m_d},$$

$$\frac{g_n^\pi}{2M_n} = \frac{4}{3} \left( \frac{-1/5}{2m_d} \right) - \frac{1}{3} \left( \frac{+1/5}{2m_u} \right) = -\frac{4}{30m_d} - \frac{1}{30m_u},$$

where $m_u$ and $m_d$ are the up and down constituent quark masses. Alternatively, one can construct nucleon isoscalar and isovector combinations:

$$\frac{g_p^\pi}{2M_p} \frac{1}{2}(1 + \tau_z) + \frac{g_n^\pi}{2M_n} \frac{1}{2}(1 - \tau_z) = \frac{1}{6m} \left( \frac{3 \Delta m}{10m} + \tau_z \right) \equiv \frac{1}{2M} \left( g_0 + g_1^\pi \tau_z \right).$$

Note that we have introduced the following definitions:

$$M \equiv \frac{1}{2}(M_n + M_p) ; \quad m \equiv \frac{1}{2}(m_d + m_u) ; \quad \Delta m \equiv (m_d - m_u).$$
The above relations are correct to leading order in $\Delta m/m$. Moreover, they reveal an isospin-violating component ($g_0^\pi$) in the $NN\pi^0$ coupling constant. In particular, by selecting $m = M/3 = 313$ MeV and $\Delta m = 4.1$ MeV [28] we obtain:

$$g_0^\pi = \frac{3}{10} \frac{\Delta m}{m} \approx 0.004.$$  \hfill (16)

Ultimately, this isospin-violation can be traced back to the up-down quark mass difference; the up quark, which is lighter, generates a stronger coupling of the neutral pion to the proton than to the neutron. Note that the isospin breaking computed in the quark model is substantially larger—by about a factor of six—than in the nucleon model of Ref. [29] where the scale of the breaking is set by the neutron-proton mass difference. In contrast, for the coupling of the nucleon to charged pions both models seem to generate an isospin violation of comparable strength [4].

Incorporating the isospin-violating correction from $g_0^\pi$ into the evaluation of the triangle discrepancy is straightforward. First, the elastic $\pi^\pm p$ amplitudes remain unchanged. Second, it modifies the charge-exchange (CEX) amplitude $f(\pi^- p \rightarrow \pi^0 n)$ through a simple renormalization of the nucleon Born terms; the $s$-channel, which has a neutron in the intermediate state, gets reduced relative to the $u$-channel, which contains a proton in the intermediate state. Thus, in computing the charge-exchange amplitude one must use an isospin-odd contribution given by [see Eq. (8d)]:

$$\tilde{B}^{(-)}(s, t) \equiv -\frac{g_{NN\pi}^2 (1 - g_0^\pi)}{s - M^2} - \frac{g_{NN\pi}^2 (1 + g_0^\pi)}{u - M^2} = B^{(-)}(s, t) - g_0^\pi B^{(+)}(s, t).$$  \hfill (17)

Note that the “small” isospin-odd contribution $B^{(-)}$ is being corrected by the “large” isospin-even term $B^{(+)}$. Indeed, at threshold $|B^{(+)} / B^{(-)}| = 2M/m_\pi \approx 14$. Now, however, there is no cancellation due to chiral symmetry; there is no isospin-violating $NN\sigma$ coupling at $q^2 = 0$. Using the above expression for $\tilde{B}^{(-)}$ we compute the value of the triangle discrepancy at threshold. We obtain,

$$D = -\sqrt{2} \frac{g_{NN\pi}^2 g_0^\pi}{4\pi M} \frac{1}{(1 + m_\pi /M)(1 - m_\pi^2 /4M^2)} \xrightarrow{m_\pi \rightarrow 0} -\sqrt{2} \frac{g_{NN\pi}^2 g_0^\pi}{4\pi M}. \quad (18)$$

This generates an isospin violation to the triangle identity of $D = -0.0145$ fm. The s-wave contribution to the triangle discrepancy shows a very weak energy dependence. Indeed, its contribution at $T_{lab} = 40$ MeV is $D = -0.014$ fm; we obtain a much smaller effect from the p-waves: $1.3 \times 10^{-4}$ fm and $-2.0 \times 10^{-4}$ fm for the $1^+$ and $1^-$ partial waves, respectively. This result is in good agreement with the value reported recently by Gibbs, Ai, and Kaufmann of $D = -0.012 \pm 0.003$ fm from the s-wave alone or $D = -0.011 \pm 0.003$ fm for the sum of $s$ and p waves at 40 MeV [1].

**IV. CONCLUSIONS**

We have examined violations to the triangle identity that arise from isospin-violating meson-nucleon coupling constants. In our model, gauge invariance precludes the contribution from vector-meson exchanges at $q^2 = 0$; these include $\rho$-$\omega$ mixing as well as isospin-violating
$NN\omega$ and $NN\rho$ coupling constants. There is no symmetry, however, that protects the $NN\pi^0$ coupling at threshold. We have computed isospin violations in the $NN\pi^0$ coupling using a nonrelativistic quark model. We have obtained a larger coupling of the neutral pion to the proton than to the neutron as a result of the up quark being lighter than the down quark. The observed isospin violation is about a factor of six larger than the one computed in nucleon models where the breaking is generated by the neutron-proton mass difference. These results were used to modify the relative weights of the s- and u-channel contributions to the charge-exchange reaction $\pi^-p \to \pi^0n$.

The isospin violation in the CEX amplitude became proportional to the large isospin-even amplitude $B^{(+)}$; this amplitude does not vanish in the chiral limit. In chiral models, such as the linear $\sigma$ model used here, the large contribution from $B^{(+)}$ to the isospin-even scattering length is cancelled by an almost equally large and opposite contribution $[A^{(+)}]$ arising from $\sigma$-meson exchange. However, in our model all isospin violations in the $NN\sigma$ coupling must vanish at $q^2 = 0$. As a result, we obtained a large violation to the triangle identity: $D = -0.014$ fm. This value is in good agreement to the one reported from a recent analysis of high-quality $\pi N$ data which yielded $D = -0.012 \pm 0.003$ fm [1].

A particularly interesting test of this mechanism could be a comparison of the “mirror” reactions $\pi^-p \to \pi^0n$ and $\pi^+n \to \pi^0p$ [1]. For the first case, namely, the one treated here, it was the s-channel that was suppressed relative to the u-channel. In contrast, it is the s-channel—now with a proton in the intermediate state—that becomes enhanced in the $\pi^+n \to \pi^0p$ reaction. One could quantify this isospin violation by measuring the difference of these two amplitudes, i.e.,

$$\widetilde{D} \equiv f(\pi^-p \to \pi^0n) - f(\pi^+n \to \pi^0p).$$

Note that the difference between the $pp\pi^0$ and $nn\pi^0$ coupling constants, alone, gives $\widetilde{D} = 2D \approx -0.029$ fm. This value should be compared to a charge-exchange scattering length of $a_0 = -0.19$ fm—it represents an isospin violation of 15%.

Undoubtedly, much work remains to be done before a clear understanding of the underlying mechanism behind the large isospin violation reported in Ref. [1] will emerge. Yet, we believe that isospin violations in the $NN\pi^0$ coupling constant are likely to play an important role in the final analysis.

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