Generality of Inflation in a Planar Universe
(Revised version)

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abstract
We study a generality of an inflationary scenario by integrating the Einstein equations numerically in a plane-symmetric spacetime. We consider the inhomogeneous spacetimes due to (i) localized gravitational waves with a positive cosmological constant $\Lambda$, and (ii) an inhomogeneous inflaton field $\Phi$ with a potential $\frac{1}{2}m^2\Phi^2$. For the case (i), we find that any initial inhomogeneities are smoothed out even if waves collide, so that we conclude that inhomogeneity due to gravitational waves do not prevent the onset of inflation. As for the case (ii), if the mean value of the inflaton field is initially as large as the condition in an isotropic and homogeneous inflationary model (i.e., the mean value is larger than several times Planck mass), the field is soon homogenized and the universe always evolves into de Sitter spacetime. These support the cosmic no hair conjecture in a planar universe. We also discuss the effects of an additional massless scalar field, which is introduced to set initial data in usual analysis.

Key Words: General Relativity, Cosmology, Inflationary Universe.

1 Introduction

Over a decade, the inflationary universe model is widely accepted as a standard scenario in the history of the early universe. The fundamental idea of inflation is very similar to the standard big-bang model. One of the current searches is to study the generality and universality of this model itself. That is, to investigate the model in anisotropic and/or inhomogeneous spacetime, and to judge whether or not such a generic spacetime supports the cosmic no hair conjecture.

The rapid expansion in the inflationary era is caused by a vacuum energy described by an inflaton field, or equivalently by a positive cosmological constant $\Lambda$. For spacetimes with $\Lambda$, the cosmic no hair conjecture has been proposed. The conjecture is: All initially expanding universes with $\Lambda$ approach the de Sitter spacetime asymptotically. The de Sitter spacetime is stable against linear perturbations and many models also support this conjecture. There, however, exist some counter examples. So that we expect we may prove this conjecture with some additional constraints.

For homogeneous but anisotropic spacetimes, Wald showed that if the mean value of the inflaton field is initially as large as the condition in an isotropic and homogeneous inflationary model, the field is soon homogenized and the universe always evolves into de Sitter spacetime. These support the cosmic no hair conjecture in a planar universe. We also discuss the effects of an additional massless scalar field, which is introduced to set initial data in usual analysis.

Assuming a plane symmetric space-time, Matzner and Wilson studied a phase transition induced by the inflaton field...
field under the existence of initial thermal fluctuations using new inflationary model. They found that inflation occurs only when the potential is flat enough. Since their motivation was different from studying the cosmic no hair conjecture, they did not consider the case which include large inhomogeneities of spacetime. Although they found de Sitter space is a final state, their initial inhomogeneity is not enough large to study the cosmic no hair conjecture.

Assuming a spherically symmetric spacetime, Goldwirth and Piran (GP) studied the behavior of inhomogeneous distributions of scalar field. From an estimation of an energy balance between potential energy and scalar field gradient term, they proposed a criterion for the onset of inflation such that

\[
a \frac{\text{scale length of inhomogeneity}}{\text{horizon scale}} > \text{a few times} \frac{\delta \phi}{m_{pl}}
\]

(1)

where \(\delta \phi\) is a spatial deviation of the scalar field from the mean value and \(m_{pl}\) is Planck mass. They found that a short scale inhomogeneity compared to the horizon scale leads spacetime to collapse in closed universe, and concluded that (1) is sufficient condition for chaotic inflationary models. GP’s simulations, however, are done under assumptions on the isocurvature initial data with additional massless scalar field, which may cause homogeneous expansion of the spacetime in the very initial stage. This additional scalar field may change a homogenization process of spacetime as we will see later.

In this paper, we study the validity of the cosmic no hair conjecture in a plane symmetric spacetime. We have mainly two reasons why we assume a planar universe. First, we investigate the behaviors of the inhomogeneities due to the gravitational waves, which cannot exist in a spherically symmetric spacetime. It is well studied that any linear gravitational waves decay and disappear immediately in de Sitter spacetime [1], so our main interests are the spacetimes with strong gravitational waves. As some numerical study in flat spacetimes shows, if the strong gravitational waves are localized, those come to have apparent horizons and collapse into black holes [11]. If such a strong gravitational field exists in a planar spacetime, then the spacetime may evolve to a naked singularity. Szekeres and Khan-Penrose [1] found that the gravitational waves in a Minkowski background spacetime make the spacetime is expanding, on the other hand, the forces to extend localized waves, which causes the collapse of the singularity due to nonlinear effect of gravity. The behavior of those competitive two effects acts also for a localized inhomogeneity of a scalar field or a inhomogeneous inflaton in a plane symmetric spacetime. We will compare the effects to those in spherical symmetric case.

Secondly, we are interested in the formation of a naked singularity, since a singularity formed in a plane symmetric spacetime is always naked. Recently, Nakao, Maeda, Nakamura and Oohara examined time-symmetric initial data for Brill waves in a background with cosmological constant and found that a dust sphere with large gravitational mass provide trapped surfaces [12]. It is also shown with large gravitational mass in a background with cosmological constant, the dust sphere will not collapse into a black hole spacetime but evolve into a naked singularity. If such a strong gravitational field is localized in a plane symmetric spacetime, then the spacetime may evolve to a naked singularity. Szekeres and Khan-Penrose [11] found that a strong gravitational field in a planar spacetime, then the spacetime may evolve to a naked singularity. Szekeres and Khan-Penrose [1] found that a strong gravitational field in a planar spacetime, then the spacetime may evolve to a naked singularity. From these results, the inhomogeneity driven inflationary era, inhomogeneities will not necessarily collapse into a black hole in a background de Sitter universe [13], and it is also shown that a naked singularity might form. This situation may thus become the main problem in the study of the cosmic no hair conjecture. Our 1-dimensional case seems well suited to address this problem.

In Section 2, we describe basic equations and numerical procedures.
We show the effects of inhomogeneities due to gravitational waves in Section 3, and those due to inflaton field in Section 4. Conclusions and remarks are in Section 5. As units, we use $8\pi G = 1$ and $c = 1$.

2 \ Equations and Numerical Methods

2.1 \ The models

We consider following two effects on the evolution of the spacetime; [I] inhomogeneities due to gravitational waves and [II] inhomogeneities due to an inflaton field. To make the problem simple, we set models as follows:

[I] Initial large inhomogeneities due to gravitational waves exist in the spacetime with positive cosmological constant $\Lambda$ (no inflaton field). We are interested in the competitive processes between the expansion of the background spacetime and the attractive forces of the gravity.

[II] Introduce an inhomogeneous inflaton field $\Phi$ in the context of the chaotic inflationary model. In a realistic inflationary model, it is natural to assume such an inhomogeneous inflaton field. Our interest is the effects of the spatial gradient term of the field on the evolution of the spacetime, which may prevent the onset of the inflation.

For the second model, we make initial data in two ways. One is the similar setting of initial data as those of GP’s, i.e., the spacetime with an additional scalar field $\Psi$ to obtain an isocurvature initially. The other is without such an additional scalar field. We examine the contributions of $\Psi$ to evolutions of the spacetime.

In the chaotic inflationary scenario, we usually assume that the average of the inflaton, $\Phi$, is larger than a few $m_{pl}$ in order for the Universe to expand sufficiently. Even under an existence of large inflaton field as an average ($\bar{\Phi} > a few \ m_{pl}$), it is not trivial how the large inhomogeneity of $\Phi$ contributes to the evolution of the Universe. We analyzed whether or not such an gradient term in the energy momentum density prevents from the onset of inflation.

2.2 \ Field Equations

We use the Arnowitt-Deser-Misner (ADM) formalism to solve the Einstein equation

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = 8\pi G (\rho + \frac{1}{2}T_{\mu\nu}T^{\mu\nu})$$

with the metric

$$ds^2 = -(\alpha^2 - \beta^2/\gamma_{11})dt^2 + 2\beta dx + \gamma_{ij}dx^i dx^j$$

where the lapse function $\alpha$, the shift vector $\beta$, and the $\gamma_{ij}$ depend only on time $t$ and one spatial coordinate $x^1$. The Ricci tensor $R_{ij}$ becomes in a matrix form

$$R_{ij} = \begin{pmatrix} K_{11} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

and evolution equations are

$$\partial_t \gamma_{ij} = -2\alpha K_{ij} + D_j \beta_i + D_i \beta_j + \alpha (\gamma_{ij} - \gamma_{kl})K_{kl} + (D_m K^m + D_k \gamma_{ij} - 2\gamma_{ik}D_{jk})$$

where Greek indices $i, j, k, ...$ move 1 to 3, the Ricci tensor and $(3)R$ is its Ricci scalar, $K_{ij}$ is the momentum density and stress tensor, $\gamma_{ij}$ is the 3-dimensional hypersurface $\Sigma$ determined by a world line $n_{\mu} = (-\alpha, 0, 0, 0)$. They are given as

$$\rho_H \equiv T_{\mu\nu}n^\mu n^\nu, \quad J_i \equiv -T_{i\mu}n^\mu$$
where $h^\nu_\mu = \delta^\nu_\mu + n^\nu n_\mu$ is the projection operator onto $\Sigma$. In section 3, we will treat vacuum spacetime, i.e. $T_{\mu\nu} = 0$, under the assumption of the existence of a positive cosmological constant $\Lambda$, while in section 4, we will treat a scalar field $\Phi$ with potential $V(\Phi) = \frac{1}{2}m^2\Phi^2$ as a source of the inflation rather than $\Lambda$. The energy momentum tensor of the scalar field is given by

$$T_{\mu\nu} = \partial_\mu \Phi \partial_\nu \Phi - g_{\mu\nu} \left[ \frac{1}{2} (\nabla \Phi)^2 + V(\Phi) \right].$$

(10)

The scalar field evolution, then, obeys the Klein-Gordon equation

$$\Box \Phi = \frac{dV}{d\Phi}.$$  

(11)

To integrate (11), we introduce the conjugate momentum

$$\Pi = \sqrt{\frac{\gamma}{\alpha}} \left( -\partial_t \Phi + \frac{\beta}{\gamma_{11}} \partial_x \Phi \right),$$

(12)

where $\gamma = \text{det} \gamma_{ij}$, and write down eq.(11) into two first-order partial differential equations:

$$\partial_t \Phi = \frac{\beta}{\gamma_{11}} \partial_x \Phi - \frac{\alpha}{\gamma} \Pi,$$

$$\partial_t \Pi = \alpha \sqrt{\gamma} \frac{dV}{d\Phi} + \partial_x \frac{1}{\gamma_{11}} \left[ \beta \Pi - \alpha \sqrt{\gamma} \partial_x \Phi \right].$$

(13)

(14)

The dynamical variables are $\gamma_{ij}$ and $K_{ij}$ (and $\Phi$ and $\Pi$, when a scalar field exists).

We impose periodic boundary condition, i.e., $f(x_1, t) \equiv f(x_{N+1}, t)$, where $N$ is the grid number in $x$-direction.

Our simulation procedure follows Nakamura, Maeda, Miyama and Sasaki [17]:

(i) Determine initial values by solving two constraint equations.

(ii) Evolve time slices by using the dynamical equations.

(iii) Check the results of (i) and (ii) using the two constraint equations on every time slice.

Explicit procedure is described in the section 2.4.

So far, the Texas group [20] has constructed a numerical code for a planar cosmology. Their metric is given by a diagonalized spatial metric, which is different from ours. Their time developing procedure is also different from ours. They use both constraint and dynamical equations to get the stable solutions.

### 2.3 Initial Value Problem

We use the York-O’Murchadha’s conformal approach [21] to get initial values. Defining conformal factor $\phi(x)$ as

$$\gamma_{ij} = \phi^4 \tilde{\gamma}_{ij},$$

$$K_{ij} = \phi^{-2} \tilde{K}_{ij},$$

$$\rho = \phi^{-6} \tilde{\rho},$$

where $\tilde{K}^{TF}$ is the trace-free part of the extrinsic curvature  and $\text{tr} K$ is a trace part of $K_{ij}$. The quantities with a caret denote physical variables in the conformal frame.

The Hamiltonian and momentum constraints are

$$\Delta \phi = (3)R \phi - \tilde{K}_{ij}^{TF} \tilde{K}_{ij}^{TF},$$

$$\tilde{D}_j \tilde{K}_{ij}^{TF} = \frac{2}{3} \phi^6 \tilde{D}^j \text{tr} \tilde{K} + \tilde{J}^j,$$

(15)

(16)

where $(3)\Delta$ is the 3-dimensional Laplacian operator, $\text{tr} K$ is the trace of the extrinsic curvature $\tilde{K}_{ij}^{TF}$ (and $\rho$ is left to our choice).

We impose constant-mean-curvature boundary condition on the initial hypersurface. We also assume that the matter density ($\tilde{\rho} = 0$). Then, for the initial data, constraint equation (17) becomes trivial, and we just have to give an arbitrary $\tilde{K}_{ij}^{TF}$. The trace part of $\tilde{K}_{ij}^{TF}$ vanishes.
In order to get the periodic solution of (18), the value of \(\text{tr} \hat{K}\) which we give on the initial hypersurface should be chosen so that such a solution exists. The condition for \(\text{tr} \hat{K}\) depends on the models. Details are shown in the following sections. We use the 5th order Runge-Kutta method (Fehlberg method) to solve \(\phi\) in (18).

### 2.4 Time Developing Scheme

For coordinate conditions, we impose a geodesic slicing condition such as \(\alpha = 1\) and \(\beta = 0\). This slicing condition entails the risk that our numerical hypersurface may hit a singularity and stop there. If no singularity appears, however, then it may be the best coordinate condition for revealing whether de Sitter space emerges as a result of time evolution.

We use finite differential scheme to integrate the Einstein equations (7) and (8) with 400 grids in the spatial direction \(x\). We use a simple central finite difference with second order scheme to determine derivatives.

The time step \(\Delta t\) on each slice is determined, as to keep the accuracy of the computations. We set \(\Delta t\) by imposing that a volume element \(\gamma\) and an expansion rate \(\text{tr} \hat{K}\) do not change too much (\(\pm 0.1\%\) change) in each step.

The accuracy is checked by comparing both sides of constraint equations. For Hamiltonian constraint equation in the case of vacuum, for example, we calculate

\[
Err(t, x) = \frac{(R^{(3)} + K^2) - (K_{ij}K^{ij} + 2\Lambda + 2\rho_H)}{|R^{(3)}| + K^2 + K_{ij}K^{ij} + 2\Lambda + |2\rho_H|}
\]

at the point \(x\) on \(\Sigma(t)\) and regard its maximum value \(Error(t) = \max\{Err(t, x) \mid x \in \Sigma(t)\}\) on each hypersurface as the error on that slice. A similar definition is performed for momentum constraint equation. An example of \(Error(t)\) is shown in Fig. 2.1. In our calculation, the maximum error in the Hamiltonian constraint equation is a few times of \(O(10^{-4})\) on the initial hypersurface, and this accuracy is maintained even after time evolutions.

Our overall procedures to evolve the system are as follows. At first stage, we solve the initial value on a hypersurface \(\gamma_{ij}(1)\) and \(K_{ij}(1)\).

1. Evaluate the time step \(\Delta t\).
2. Define next hypersurface \(\Sigma_2\) and \(\Sigma_3\) at \(t = t + \frac{\Delta t}{2}\) and \(t = t + \Delta t\), respectively.
3. Evolve the metric on \(\Sigma_2\) \((\gamma_{ij}(2)\) and \(K_{ij}(2))\) using (7) and (8) on \(\Sigma_1\).
4. If the scalar field exists, evolve them onto \(\Sigma_2\) \((\Phi(2)\) and \(\Pi \Phi(2))\) using (13) and (14) on \(\Sigma_1\).
5. If the massless scalar field exists (in section 4.1), evolve them onto \(\Sigma_2\) \((\Psi(2)\) and \(\Pi \Psi(2))\) for \(\Psi\) on \(\Sigma_1\).
6. Calculate r.h.s. of (7), (8), (13) and (14) for \(\Psi\) on \(\Sigma_1\).
7. Evolve values on \(\Sigma_3\), using leap-frog method.
8. Check the increasing ratios of \(\gamma\) and \(K\). If one of them exceeds a certain value (\(\pm 0.1\%\) change), then control \(\Delta t\) and return to 2. again.
9. Check the accuracy of the computations by constraint equations.
10. Replace the values on \(\Sigma_3\) by those on \(\Sigma_1\), and turn to 1.

### 3 Inhomogeneities due to gravitational waves

In this section, we show the evolution of inhomogeneities due to gravitational waves in the spacetime with positive cosmological constant \(\Lambda\). We expect two competitive effects: one is the expansion of space due to the cosmological constant, the other is the attractive forces due to the nonlinearity of the gravity. We examine whether such a spacetime leads to an inflationary era, and whether such initial inhomogeneities and anisotropies smooth out during inflation periods.
As we already mentioned in the introduction, linear gravitational waves are always decay in the de Sitter background spacetime. This result is also confirmed in our numerical calculations as one of our code tests. We, therefore, concentrate our attentions to the large inhomogeneities due to gravitational waves. The summary has been reported in [23].

Since we have \( \Lambda \) in this system, we adopt the Hubble expansion time \( \tau_H = (\Lambda/3)^{-1/2} \) as our time unit, which is a characteristic expansion time of the universe. Our unit of length is also normalized to the horizon length \( l_H = (\Lambda/3)^{-1/2} \).

To evaluate the inhomogeneities on each hypersurfaces, we use three different curvature invariants. First one is the 3-dimensional Riemann invariant scalar \( (3)R_{ijkl} \equiv (3)R_{ijkl}^{(3)} \), where \( (3)R_{ijkl} \) is the Riemann tensor of the 3-metric on \( \Sigma \). We use its dimensionless value normalized by the cosmological constant,

\[
C(t, x) \equiv \frac{(3)R_{ijkl}^{(3)}R_{ijkl}^{(3)}}{\Lambda} \quad \text{on} \quad \Sigma(t),
\]

and call it the “curvature” hereafter. We estimate the magnitude of the inhomogeneities in the 3-space \( \Sigma \) by the maximum value of this “curvature” on each slice, i.e. \( C_{\text{max}}(t) = \max\{C(t, x) \mid x \in \Sigma(t)\} \).

Second one is an invariant scalar induced from the Weyl tensor \( C_{\mu\nu\rho\sigma} \). For the vacuum spacetime, if the Weyl tensor vanishes and no singularity appears, the spacetime is homogeneous and isotropic. Hence this can be used to check whether de Sitter universe is recovered or not. We use the decomposition of the Weyl tensor:

\[
E_{\rho\sigma} = C_{\mu\rho\sigma\nu}n^\mu n^\nu, \quad B_{\rho\sigma} = *C_{\mu\rho\sigma\nu}n^\mu n^\nu,
\]

where \( *C_{\mu\rho\sigma\nu} \equiv \frac{1}{2}C^\alpha_{\rho\sigma\nu}n^\mu n^\nu \) is the dual of the Weyl tensor and \( n^\mu \) is a timelike vector orthogonal to the hypersurface \( \Sigma \). In analogy to the electromagnetism, the 3-dimensional variables \( E_{\rho\sigma} \) and \( B_{\rho\sigma} \) are called an electric and a magnetic parts of the Weyl tensor. We can reconstruct the Weyl tensor completely from this pair of tensors. Moreover, those two variables are easy to drive out using equations:

\[
E_{\rho\sigma} = (3)R_{\rho\sigma}^{(3)}, \quad B_{\rho\sigma} = \varepsilon_{\rho\sigma\mu\nu}D^\mu F_{\mu\nu}
\]

To estimate the field’s inhomogeneities, we introduce a sort of gravitational “super-energy”. In fact, it is the purely timelike component of the Bel-Robinson tensor, unless the hypersurface becomes null.

Third invariant is 4-dimensional Riemann invariant scalar

\[
R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = C_{\mu\nu\rho\sigma} C^{\mu\nu\rho\sigma}
\]

In the de Sitter case (isotropic and homogeneous case), since \( C_{\mu\nu\rho\sigma} = 0 \), \( R_{\mu\nu} = \Lambda g_{\mu\nu} \) and \( R = 3\Lambda \).

The initial values are determined as follows. We treat pure gravitational waves with following two cases: [case 1] \( \gamma_{ij} = 0 \) and \( \hat{K}_{ij}^{TT} = 0 \). We set a pulse-like distortion initially expressed by

\[
\text{diag}(\hat{\gamma}_{ij}) = (1, 1 - a(x)), \quad \text{diag}(\hat{\mathbf{K}}_{ij}^{TT}) = \tilde{a} e^{-x/\tilde{x}_0}
\]

where \( a \) and \( x_0 \) are free parameters. Here, the spacetime is flat and the inhomogeneities reside in the

\[
\text{diag}(\tilde{\mathbf{K}}_{ij}^{TT}) = \tilde{a} e^{-x/\tilde{x}_0}
\]

where \( \tilde{a} \) and \( \tilde{x}_0 \) are also free parameters. Initial data we investigated are listed in Table 3.1 and 3.2. The results for \( \gamma_{ij} \) are quite similar to those of those of [case 1].

In order to get a consistent data under periodic boundary condition, we set \( \hat{K} \) as

\[
\text{tr} \hat{K} = -\sqrt{3}
\]
on the initial hypersurface $\Sigma(t = 0)$. Here, $\delta_K$ is a positive constant, which should be introduced because the periodic pulse increases expansion rate of the universe besides $\Lambda$. By integrating $[18]$, we get conformal factor $\phi$. There is one free parameter $\lambda$, which fixes the scale. If $\phi$ is multiplied by $\lambda$, we find another solution by replacing $\delta_K$ with $\delta_K/\lambda^2$, which is physically the same as the previous solution. Hence, if we impose $\phi \to 1$ at the numerical boundary, then the value of $\delta_K$ is fixed; it is less than $O(10^{-2})$.

A pulse-like wave has two characteristic physical dimensions, a width and an amplitude. We use $C_{\text{max}}(t)$ as an amplitude measure, and we define the width $l(t)$ by the proper distance between two points where $\gamma$ (the square of the 3-volume) decreases by half from its maximum value $\gamma_{\text{max}}$. An example of the initial data is shown in Fig. 3.1. This is the case of $l(t = 0) = 0.086 l_H, C_{\text{max}}(t = 0) = 9.67$.

The behaviors of the time evolutions are following. Initial distortion begins to propagate both in the $\pm x$ directions. We see the initial curvature $C(t = 0, x)$ localized at the center with three peaks [Fig. 3.1(b)] separates just after $t = 0$, and propagate at the light speed. In Fig. 3.2, we show both “curvature” $C(t, x)$ and “super-energy” $H(t, x)$. Two waves moves to the numerical boundaries and collides each other on the boundary, since we assume periodic boundary condition. After the collision, two waves proceed again in their original directions. This behavior is reminiscent of solitonic waves. We also show, in Fig. 3.3, $C_{\text{max}}(t)$ and the invariant $I(t, x) = \sqrt{R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}} / \Lambda$. We can see clearly that the collision of the waves occurs around $t = 0.2 \tau_H$, when the “curvature” is superposed. At the final stage, the spacetime is homogenized by the expansion of the universe. We can see $I(t, x) \to \sqrt{8/3}$ (homogeneous de Sitter spacetime) within one Hubble expansion time $\tau_H$ as expected [see eq.(23)]. We see the spacetime finally succumbs to the overall expansion driven by the cosmological constant, and becomes indistinguishable from de Sitter spacetime.

In our simulation, $l$ and $C_{\text{max}}(0)$ range for [case 1] between $0.080 l_H \leq l \leq 2.5 l_H$ and $0.020 \leq C_{\text{max}}(0) \leq 125.0$. We also computed the evolution of [case 2], and found that all initial inhomogeneities decay and disappeared within one Hubble expansion time. We conclude that for any large Riemann inhomogeneity on the initial hypersurface, a little effect and the spacetime always succumbs to the overall expansion driven by the cosmological constant.

One of our motivation is to see what happens in the presence of a cosmological constant. Collisions occur at the boundary, which may avoid such numerical difficulties, we assume two nearby pulse waves in one numerical range. Evolution of the curvature has been shown in Figure 3.4. We find again that all inhomogeneities decay and disappear, widths and “curvatures” ($0.080 l_H \leq l \leq 10 l_H$, and the periodic distance $d$ is $0.20 l_H$). decay below $1 \%$ of their initial “curvature”.

The results that all the spacetimes we consider include collision of waves occurs first at one time and the periodic distance $\lambda$ exceeds $10 l_H$. By integrating $[19]$, they examined the collision of gravitational shock waves in an expanding Kasner background both analytically and numerically, and concluded that such a collision leads to no singularity. The preliminary results of the time evolutions show that such a periodic wave propagate with decreasing amplitude $[C_{\text{max}}(t) \sim \gamma^{-1/3}]$ if the background spacetime expand or contract. This suggests a plane-symmetric expanding spacetime.
olutions for the case of colliding waves and for the case in a contracting background are still under studying. The details will be reported in the elsewhere.

4 Inhomogeneities due to scalar field

In this section, we introduce a scalar field $\Phi(x,t)$ as a source of inflation instead of cosmological constant, and show the results of the behavior of its inhomogeneities to the evolution of the spacetime. In the usual analytic approach of inflationary scenario, the inflaton field is assumed homogeneous. By introducing inhomogeneities, the energy density of the scalar field $\rho_\Phi$ is written as

$$
\rho_\Phi = \frac{1}{2\gamma} \Pi^2 + \frac{1}{2\gamma_{11}} (\partial_x \Phi)^2 + V(\Phi)
$$
\begin{align}
\equiv \rho_\Phi + \rho_{\Phi'} + V(\Phi),
\end{align}

(30)

where $V(\Phi)$ is the potential term of the scalar field. Our interests is how the gradient term $\rho_{\Phi'}$ affects on the expansion of the universe. The initial kinetic term $\rho_{\Phi'}$ gives less effects on the inflationary scenario.

The same-aimed simulations have been already done by GP using a spherically symmetric code [8], and one of their results is that the spacetime with large $\rho_{\Phi'}$ do not evolve into the inflationary era. In order to compare the results between ours and theirs, i.e., the difference of the symmetry of the spacetime, we first set similar initial situations and parameters with GP, that is with 'isocurvature initial data' (section 4.1). This condition makes us easy to determine initial data with constant mean curvature slicing, but also makes us unclear the effects of inflaton $\Phi$ itself, because the spacetime may expand homogeneously in the very initial stage by introducing another scalar field $\Psi$, and such an expansion might reduce an initial large inhomogeneity of inflaton field. So that we also prepare the initial data without $\Psi$, and computed their time evolutions (section 4.2).

4.1 Evolution of the isocurvature initial data

We study the evolutions of inhomogeneities with the initial data following to GP. In addition to the inflaton field, we introduce a massless scalar field $\Psi$ such that the initial total energy density

$$
\rho_{\text{total}} = \rho_\Phi + \rho_\Psi
$$

becomes uniform, where $\rho_\Psi = \rho_{\Psi'} + \rho_{\Psi''}$. The constraint equation (3) is initially trivial under constant mean curvature slicing (33). This method was proposed by GP. We think that it will help to compare our results with theirs to work with same initial situations. The massless field $\Psi$ is introduced as an additional radiation field, and is expected to dissipate immediately by the expansion of the universe. As we show later in Fig. 4.5, only inflaton $\Phi$ effects to the spacetime evolution at the late stage. The initial spacetime, however, is set to have uniform expansion rate (since $\rho_{\text{total}}$=constant.), so that the initial-isocurvature setting may not produce an exact model for our aims to search the effects of inhomogeneity. As we show in the next sections, we also prepare a fully inhomogeneous initial data and followed those time evolutions.

In order to define dimensionless variables, we introduce the effective cosmological constant as $\Lambda_{\text{eff}} \equiv \rho_{\text{total}}$, and scale by the units $\tau_H$ and $l_H$, where $\tau_H$ and $l_H$ are the Hubble parameter and Hubble radius, respectively. We set the initial spacetime to be flat and have no gravitational waves ($\hat{\gamma}_{ij} = \delta_{ij}$, $\hat{K}_{ij} = 0$). Initial scalar distribution is set as

$$
\Phi_{\text{initial}} = \Phi_0 + \delta \Phi \cos(2\pi x/\lambda),
$$

(32)

where $\Phi_0$, $\delta \Phi$ and $\lambda$ are parameters of the scalar field, amplitude of inhomogeneity and scale length of inhomogeneity, respectively. We set the initial spatially homogeneous gradient of $\Phi$ is zero.
We choose $\Phi_0$ several times of $m_{pl}$ in order to get ‘suitable’ expansion of the Universe when $\delta \Phi = 0$. This is from a condition for inflation in isotropic and homogeneous universe. We are interested in effects of gradient term of the scalar field $\rho_{\Phi'}$ to the time evolution of spacetime, especially large $\rho_{\Phi'}$ compare to $\rho_{\text{total}}$. We get large $\rho_{\Phi'}$, if we choose small $\lambda$ and large $\delta \Phi$. However, the total energy density $\rho_{\text{total}} = \rho_{\Phi'} + V(\Phi)$ at the inflationary era is order of the Planck scale $m_{pl}^4$, so that if we introduce ‘suitable’ $\Phi_0$, then acceptable values of $\lambda$ and $\delta \Phi$ are limited. But we can get large enough $\rho_{\Phi'}$ compare to $\rho_{\text{total}}$ as will be shown in Table.

First, we prepare initial data of the scale $\lambda \approx l_H$ and the amplitude up to $\delta \Phi \sim 0.1 \Phi_0$ (model [II-1a ~ 1c] in Table 4.1). Corresponding ratio of $\rho_\Psi \equiv \rho_{\Phi'} + \rho_{\Psi}$ to $\rho_{\text{total}}$ are also shown in Table 4.1. Those model have large contributions of $\rho_\Psi$, we may regard that gradient term is locally dominated. As we can see from Fig. 4.1, in which the ways of evolutions (model [II-1b]) is shown, all the inhomogeneous scalar field are first homogenized and the field go to the slow-rolling phase along to the ordinary chaotic inflationary scenario. We find the spacetime expands sufficiently in the region where the inflaton has large enough $\Phi_0$. This confirms the criterion [1] proposed in spherically symmetric case by GP.

Next, we prepare initial data of the small $\lambda$. In these cases, we may expect a formation of the collapse of the spacetime, as an example shown by GP. Our case has planar symmetry, so if the collapse occurs, then the naked singularity appears. Hence from this calculation we can also discuss the so-called cosmic censorship in the spacetime with inflaton field. We prepare data with the same $\delta \Phi$ and different $\lambda$‘s such as $\lambda = l_H, \frac{1}{2} l_H, \frac{1}{4} l_H$ and $\frac{1}{6} l_H$, when $\Phi_0 = 8.0 m_{pl}$ and $\delta \Phi = 0.06 m_{pl}$ (model [II-1d ~ 1g] in Table 4.1). In the last case ($\lambda = \frac{1}{6} l_H$), $\rho_\Psi$ is locally 86% of the $\rho_{\text{total}}$, we can regard gradient term is locally dominated again.

The typical time evolution of $\Phi$ is shown in Fig. 4.2 (model [II-1g]). The evolution is quite similar to Fig. 4.1. That is, in all the case, the field $\Phi$ is homogenized and the spacetime expands sufficiently.

For various scale of the deviation $\lambda$ (model [II-1d ~ 1g], we show the trajectories of the evolution at $x = 0$, $-\Pi/\gamma$ (Fig. 4.3). At the beginning, the trajectory around the mean value of the field in the $\Pi$ phase space, where $\Pi$ is locally $86\%$ of the $\rho_{\text{total}}$. At the very end, the system will soon approach that of the homogeneous case when $\Phi$ scalar field begins damped oscillation around the potential minimum.

To compare the effects of $\rho_\Psi$ on the evolutions of a universe, we prepare data with the same $\rho_{\Phi'}$ and compare using the maximum value of $\rho_\Psi$. In Fig. 4.5, the homogeneous initial data has $\rho_{\Phi'} \sim 8.0 m_{pl}$ and maximum value $\Phi_{\text{initial}} = \Phi_0$ are drawn in Fig. 4.4(b). Effects of $\rho_\Psi$ are shown in Fig. 4.6, the maximum value of $\rho_\Psi$ becomes less effect within $1.0 \tau_H$, maximum value $\Phi_{\text{initial}} = \Phi_0$, with solid line, which are almost planar inhomogeneous initial data. So we can judge the inhomogeneity of the universe. In Fig. 4.5, we show the energy density $\rho_\Psi$, $\rho_{\Phi'}$, $\rho_{\Psi'}$, $\rho_\Psi = \rho_{\Phi'} + \rho_{\Psi'}$, becomes less effect when $\lambda = \frac{1}{6} l_H$. The kinetic energy of the scalar field dominates, so that we may say that the collapse is stopped by the field $\Psi$. Hence, to search the effects of the inhomogeneity, we may better survey initial data without such an additional field.

Since we are working in a planar spacetime, the gravitational waves also exist in this system. We can discuss the evolution of the gravitational waves on each hypersurface for various $\lambda$ in four dimensions, and the initial Weyl invariant become large in planar inhomogeneous case. We can also discuss the gravitational waves homogenized by the expansion of the universe as in Section 3.
4.2 Evolutions of the inhomogeneous scalar field

Next, we show the time evolution of the initial data without introducing a massless scalar field. This setting makes clear the effects of inhomogeneous scalar field, since initial expansion rate are not uniform over the spacetime.

To show the difference with the previous model, we set the same chaotic potential with \( m^2 = 0.01 \), and the same initial scalar distribution form \( \Phi_{\text{initial}} \). To define a unit, we set the effective cosmological constant as \( \tilde{\Lambda}_{\text{eff}} \equiv \frac{1}{2} m^2 \Phi_0^2 \), where \( \Phi_0 \) is the mean value of the initial scalar field [see eq. (32)], and fix unit \( t_H \) and \( \tau_H \) using \( \tilde{\Lambda}_{\text{eff}} \). We set the background spacetime is flat on the initial hypersurface, and choose \( \text{tr} \hat{K} \) as

\[
\text{tr} \hat{K} = -\sqrt{3\tilde{\Lambda}_{\text{eff}}(1 + \delta_K)}.
\]

Since our present model is not isocurvature, \( \delta_K \) must be introduced in order to take into account the inhomogeneity effect, and is determined by the same condition as in Sec.3.

As like the cases in the previous section, we first prepare initial data for large amplitude \( \delta \Phi \) and the scale \( \lambda \) to be larger than a horizon scale (model [II-2a \sim 2c] in Table 4.2). Initial rate of \( \rho_\Phi \) to \( \rho_{\text{total}} \) and each \( \delta_K \) are shown in Table 4.2. The gradient energy rate \( \frac{\rho_\Phi}{\rho_{\text{total}}} \) are large enough to treat our problem. The inflaton in such a case behaves like a homogeneous field in one horizon region, and induce a sufficient expansion if the average value of \( \Phi \) is ‘suitably’ large for the inflationary model. These results are again agree with GP’s results as we expected.

We next prepare initial data for small \( \lambda \) compare to the horizon scale. With the same reason in the previous section, if we concentrate our attentions for the small scale deviations \( \lambda \) compare to the horizon scale, then only small amplitude of \( \delta \Phi \) is acceptable. We set \( \lambda = 0.2l_H \) and change \( \delta \Phi \) (model [II-2d \sim 2h] in Table 4.2).

A typical example of the time evolution of scalar field is shown in Fig. 4.7. We see the field \( \Phi \) is rolling down along the chaotic inflationary potential. In the figure, only the configuration until \( 12\tau_H \) is drawn, but we can see differences with that of the isocurvature model (Fig. 4.2). Since there is no uniform expansion in the initial data, the field is not homogenized immediately but has different expansion rate at each point, and drags the initial distribution to the homogeneity.

In the later stage, the field of initially larger \( \Phi \) gets larger kinetic energy faster than that from initial smaller one, and we find again inhomogeneous distribution of the scalar field. And then the spacetime goes to the inflationary stage following the usual slow rolling scenario. We can see this by using phase diagram of this model (Fig. 4.8). For the different initial data shown in Table 4.2, the field seems to become homogeneous around \( 5\tau_H \), this is because the field of initially larger \( \Phi \) gets large kinetic energy and goes down the potential faster than that from initially smaller \( \Phi \). In the last stage, we find again inhomogeneous distribution similar to the initial one. Although the configuration of the scalar field is inhomogeneous, the evolution of the spacetime goes to the inflationary stage, since the field at each point follows the usual slow rolling scenario of the inflationary model.

An important fact is that we did not find a collapse of the spacetime even if we set the initial inhomogeneity is large enough. All the initial effect of the gradient energy term \( \rho_\nabla \) do not work on preventing an onset of inflation in a planar universe. If the \( \Phi_0 \) is large enough to derive an inflation just as same as that in homogeneous and isotropic spacetime, then the planar inhomogeneous spacetime go into inflationary era.

5 Conclusion

We analyzed the validity of the cosmic no hair conjecture in a plane symmetric spacetime. We integrated the Einstein equations numerically using ADM formalism to see following two
time; strong inhomogeneity [I] due to localized gravitational waves in the spacetime with positive cosmological constant and [II] due to inhomogeneous inflaton field.

For the model [I], we see all initial inhomogeneities are smoothed out within one Hubble expansion time $\tau_H$ including their collisions' effect. So that we conclude that the nonlinearity of the gravity do not cause to prevent the onset of the inflation under the existence of positive cosmological constant $\Lambda$.

For the model [II], we prepare two initial situations; isocurvature initial data and fully inhomogeneous initial data. In both cases, we find that initial gradient term of the energy density of the scalar field may not lead to collapse of the spacetime. If the initial mean value of the inflaton field, $\Phi_0$, is large enough to derive inflation in homogeneous and isotropic spacetime, then we can conclude such a universe will always be homogenized and expand sufficiently.

For the case of the inhomogeneous scale $\lambda$ is larger than the horizon scale, our calculations confirm the criterion [I] by Goldwirth and Piran, and also their calculations for spherical spacetime.

Even for the case of small $\lambda$, we could not find the collapsing universe. We did not face to a formation of a naked singularity at least in a planar universe. This may support a cosmic censorship conjecture. Therefore, we conclude that inflation is generic, and the usual analysis with isotropy and homogeneity may be justified in a plane-symmetric spacetime. We also partially confirm our inflationary scenario of inhomogeneous universe, which we mentioned in the introduction. That is, a large inhomogeneity does not necessarily prevent cosmological constant-driven expansion, instead the spacetime evolve into a de Sitter spacetime with many small black holes induced by gravitational collapses.

As we denoted in section 4.2, the condition of isocurvature initial data by introducing an additional massless field may lead to the different configuration on the evolution of the spacetime. The critical difference did not appear in the plane symmetric spacetime, but it is necessary to treat precisely also in a spherical universe.

In order to clear out the generality— for example, those present in cylindrical and more general spacetimes—are required.

**Acknowledgment**

We would like to acknowledge L. Gunnarsen, T. Nakamura, K. Nakao and K. Oohara for helpful discussions. We also like to thank to the Yukawa Institute for this work was done. This work was supported by Grant-in-Aid for Scientific Research Fund of the Ministry of Education, Science and Culture Nos. (04640312 and 05218010), by a Waseda University Grant for Special Research Projects and by The Sumitomo Foundation.

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Table Captions

Table 3.1:
Examples of initial parameters for model [I] [case 1]. $a$ and $x_0$ are parameters to define $\hat{\gamma}_{ij}$ in eq.(27), $L$ is a periodic boundary length, and $\delta_K$ is a constant in eq.(29). $l(t = 0)$ and $C_{\max}(t = 0)$ are width and amplitude, respectively, on the initial hypersurface.

Table 3.2:
Examples of initial parameters for model [I] [case 2]. $\tilde{a}$ and $\tilde{x}_0$ are parameters to define $\hat{\gamma}_{ij}$ in eq.(28), and the rest are the same with Table 3.1.

Table 4.1:
Examples of initial parameters for the isocurvature model. $\Phi_0$, $\delta \Phi$, $\lambda$ means average, amplitude of deviations and scale of the deviation, respectively [in eq.(28)]. Initial ratio of gradient term $\rho \nabla \Phi$ to the $\rho_{\text{total}}$ are also shown. The mass term of the potential, $m$, is set as $m^2 = 0.01$.

Table 4.2:
Initial parameters for the fully inhomogeneous model. $\tilde{\delta}_K$ is a constant in eq.(34). The mass term of the potential is set as $m^2 = 0.01$.
| Model | $\Phi_0$ | $\delta\Phi$ | $\lambda$ | $(\text{max}) \frac{\rho V}{\rho_{\text{total}}}$ | $\frac{\rho V}{\rho_{\text{total}}}$ |
|-------|--------|-------------|--------|-----------------|-----------------|
| II-1a | 5.00   | 0.75        | 1.00   | 92.5%           | 46.1%           |
| II-1b | 8.00   | 0.72        | 1.00   | 85.3%           | 42.5%           |
| II-1c | 8.00   | 0.36        | 0.50   | 85.3%           | 42.5%           |
| II-1d | 8.00   | 0.06        | 1.00   | 2.37%           | 1.18%           |
| II-1e | 8.00   | 0.06        | 0.50   | 9.47%           | 4.73%           |
| II-1f | 8.00   | 0.06        | 0.250  | 37.9%           | 18.9%           |
| II-1g | 8.00   | 0.06        | 0.167  | 85.2%           | 42.5%           |
| II-2a | 8.00   | 1.50        |        |                 |                 |
| II-2b | 8.00   | 2.40        |        |                 |                 |
| II-2c | 8.00   | 3.60        |        |                 |                 |
| II-2d | 8.00   | 0.03        |        |                 |                 |
| II-2e | 8.00   | 0.06        |        |                 |                 |
| II-2f | 8.00   | 0.18        |        |                 |                 |
| II-2g | 8.00   | 0.30        |        |                 |                 |
| II-2h | 8.00   | 0.60        |        |                 |                 |

Table 4.1

| Model | $\Phi_0$ | $\delta\Phi$ |
|-------|--------|-------------|
| II-2a | 8.00   | 1.50        |
| II-2b | 8.00   | 2.40        |
| II-2c | 8.00   | 3.60        |
| II-2d | 8.00   | 0.03        |
| II-2e | 8.00   | 0.06        |
| II-2f | 8.00   | 0.18        |
| II-2g | 8.00   | 0.30        |
| II-2h | 8.00   | 0.60        |
Figure Captions

Figure 2.1:  
The accuracy of the Hamiltonian constraint equation. The solid line and the dotted line show the maximum error $\text{Error}(t)$ and the averaged error, respectively, on the hypersurface at every time step for the calculation of the inhomogeneous scalar field (the case of model [II-2f] in Section 4.2).

Figure 3.1:  
An example of the initial configuration for the case of a pulse-like distortion ([case 1]). This is the case of model [I-1a] in Table 3.1. In (a), the conformal factor $\phi$ in equation (15) is shown. In (b), the “curvature” $\mathcal{C}(t, x)$ in (21).

Figure 3.2:  
Time evolutions of propagating plane waves. (a) “curvature” $\mathcal{C}(t, x)$ [eq.(21)] and (b) “super-energy” $\mathcal{H}(t, x)$ [eq.(25)] are shown for the initial data shown in Fig. 3.1.

Figure 3.3:  
The maximal value of $\mathcal{I}(t, x) \equiv \sqrt{R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}}/\Lambda$ (solid line), and $\mathcal{C}(t, x) \equiv \sqrt{(3)R_{ijkl}(3)R^{ijkl}}/\Lambda$ (dotted line) on each hypersurface $\Sigma(t)$ are shown for the same data with Fig. 3.1. We find $\mathcal{I}(t, x) \to \sqrt{8/3}$ within one Hubble expansion time (homogeneous de Sitter spacetime) as expected.

Figure 3.4:  
The time evolution of the “curvature” $\mathcal{C}(t, x)$ resulting from waves that are located closely. Two waves are the same form, $l = 0.10l_H$ and $C_{\max}(t = 0) = 51.0$, and the periodic distance is $0.30l_H$. We see them collide and in the collision region, the “curvature” seems to be superposed, but finally the spacetime is homogenized by the expansion of the universe.

Figure 4.1:  
A typical example of the time evolution of the field (isocurvature model). The model $\Phi$ is rolling down along the chaotic inflationary potential $V(\Phi) = \frac{1}{2}m^2\Phi^2$ with $m^2 = 0.01$. Although the gradient term locally dominated, we see the field $\Phi$ homogenizes immediately.

Figure 4.2:  
The same as Fig. 4.1, but of model [II-1g] in Table 4.1.

Figure 4.3:  
Phase space diagram$(\Phi - \Pi)$ indicating the behavior of the scalar field at $x = 0$ both for the inhomogeneous and homogeneous initial data (solid line). Table 4.1, while the latter initial data, we find the dotted line is coincide with the solid one soon after rolling down, showing that the homogeneous scalar field may work even if we include much inhomogeneities.

Figure 4.4:  
(a) Trajectories in $\Phi - \Pi$ phase space for the isocurvature model. Only the long-dashed, long-dot-dashed, short-dashed line and solid line are that of Model [II-1d, 1e, 1f, 1g] respectively. All of them will coincide with the trajectory of the homogeneous scalar model (the bold line) so on after slowing down the potential.

(b) The evolution of the volume element $\det \gamma_{ij}$ (maximum value on the each hypersurface) for each case. The bold line shows the case of homogeneous $\Phi = 8.06m_{pl}$ (indistinguishable) and the scalar field enhances the expansion.
The proportions of the each terms in energy density to $\rho_{\text{total}}$: $\rho_\Phi$ (bold-solid line), $\rho_\Phi$' (bold-dotted line), $\rho_\Psi$ (thin-solid line), $\rho_\Psi$' (thin-dotted line), and $V(\Phi)$ (three-dot-dashed line). (a) shows until $1.0\tau_H$, while (b) shows until $5.0\tau_H$.

Figure 4.6:
The evolutions of the maximum Weyl invariant $\sqrt{\sqrt{\epsilon_{\mu\nu\rho\sigma}C_{\mu\nu\rho\sigma} \Lambda_{\text{eff}}}}$ on the hypersurface.

Figure 4.7:
A typical example of the time evolution of initially inhomogeneous scalar field. The model [II-2f] in Table 4.2 is shown. We see the field $\Phi$ is rolling down along the chaotic inflationary potential $V(\Phi) = \frac{1}{2}m^2\Phi^2$ with $m^2 = 0.01$. Compare to the Fig. 4.2, we notice the scalar field’s configuration drags its initial form even after the time evolution.

Figure 4.8:
Trajectories in $\Phi - \Pi$ phase space for various kinetic terms’ initial data for the fully inhomogeneous model (at $x = 0$, where the maximum $\Phi$ exists initially). Only the beginning part is drawn. The detail parameters are shown in Table 4.2. The bold line shows the trajectory in the case of homogeneous scalar field ($\Phi = 8.0m_{pl}$). The dotted line, dot-dashed line, dashed line and solid line are those of Model [II-2e, 2f, 2g, 2h] respectively. All of them will coincide to the homogeneous’ line soon after slowing down the potential.
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