Thermodynamics and fluctuations of conserved charges in Hadron Resonance Gas model in finite volume

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The thermodynamics of hot and dense matter created in heavy-ion collision experiments are usually studied as a system of infinite volume. Here we report on possible effects for considering a finite system size for such matter in the framework of the Hadron Resonance Gas model. The bulk thermodynamic variables as well as the fluctuations of conserved charges are considered. We find that the finite size effects are insignificant once the observables are scaled with the respective volumes. The only substantial effect is found in the fluctuations of electric charge which may therefore be used to extract information about the volume of fireball created in heavy-ion collision experiments.

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Our present day universe contains a significant fraction of matter in hadronic form. In the very early universe — a few microseconds after the Big Bang [1] when the temperature was extremely high, the strongly interacting matter is expected to have existed in the partonic form. Similar exotic state of matter may exist inside compact stars due to extremely high matter density attained by gravitational compression [2]. For the last few decades various experimental efforts are being made to recreate such exotic matter through the collisions of heavy-ions at ultra-relativistic energies. Experimental facilities at CERN (France/Switzerland), BNL (USA) and the upcoming facility at GSI (Germany) are at the forefront of efforts taken to create these exotic states of matter. One of the major goals in the experiments is to study thermodynamic properties of strongly interacting matter at high temperatures and densities. Present experimental data as well as lattice QCD simulations seem to indicate a smooth cross over from hadronic to quark gluon matter at low density and high temperature [3, 4]. At high density and low temperature a first order transition is expected [5–10].

Usually any thermodynamic study assumes the system volume to be infinite. However the fireball created in the relativistic heavy ion collision experiments has a finite spatial volume. The size of such spatial volume critically depends on three parameters: the size of the colliding nuclei, the center of mass energy (\(\sqrt{s}\)) and the centrality of collisions. Analysis of experimental data could reveal the freeze out volume of the system. One way to carry out such an analysis is the study of HBT radii which has been done in Ref. [11]. The major finding of this study indicates that the freeze out volume increases as the \(\sqrt{s}\) increases and the estimated freeze out volume varies from 2000 \textit{fm}^3 to 3000 \textit{fm}^3. Another way of estimating the system size is through the comparison of simulation results with the experimental data as done in Ref. [12] where the UrQMD model [13] is used for this purpose. A study of the \textit{Pb – Pb} collisions at different energies and centralities resulted in a freeze-out volume in the range 50 \textit{fm}^3 to 250 \textit{fm}^3. We note that these volumes as quoted above, are the freeze out volumes and as one looks back to the early times in the evolution of the system, a smaller volume is expected. So both in the quark phase and in the hadron phase (after the putative phase transition) finite volume could play an important role in the quantitative measurement of the
energy density $\varepsilon$ From partition function we can calculate various thermodynamic quantities of interest. The partial pressure factor and $T$ where $E$ potentials. The (+) and ($-$) cut-off $p_i$ and $S_i$.

Our purpose is to consider HRG in a system of finite volume. We incorporate this by considering a lower momentum cut-off $p_{\text{min}} = \pi / R = \lambda$(say) where $R$ is the size of a cubic volume. With this cut-off the partition function for particle $i$ becomes

$$\ln Z_{id}^i = \pm \frac{V g_i}{2\pi^2} \int_0^\infty p^2 \, dp \ln \left[ 1 \pm \exp\left( - (E_i - \mu_i) / T \right) \right],$$

where sum is over all the hadrons, $id$ refers to ideal $i.e.$, non-interacting HRG. For the $i$'th species,

$$\ln Z_{id}^i = \frac{V g_i}{2\pi^2} \int_0^\infty p^2 \, dp \ln \left[ 1 \pm \exp\left( - (E_i - \mu_i) / T \right) \right],$$

where $E_i = \sqrt{p^2 + m_i^2}$ is the single particle energy, $m_i$ is the mass, $V$ is the volume of the system, $g_i$ is the degeneracy factor and $T$ is the temperature. In the above expression $\mu_i = B_i \mu_B + S_i \mu_S + Q_i \mu_Q$ is the chemical potential and $B_i, S_i, Q_i$ are respectively the baryon number, strangeness and charge of the particle, $\mu$'s being corresponding chemical potentials. The (+) and (−) sign corresponds to fermions and bosons respectively.

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From partition function we can calculate various thermodynamic quantities of interest. The partial pressure $P_i$, the energy density $\varepsilon_i$ and the entropy density $s_i$ can be calculated using the standard definitions,
The specific heat at constant volume $C_V$ is given by,

$$C_V^{id} = \left( \frac{\partial E_i^{id}}{\partial T} \right)_V = \frac{g_i V}{2 \pi^2} \int_0^\infty \frac{p^2 dp \exp\left((E_i - \mu_i)/T\right)}{T^2(\exp((E_i - \mu_i)/T) \pm 1)^2} (E_i - \mu_i) E_i,$$

(7)

Here we consider the grand canonical partition function given in equation (3) to incorporate the effect of finite volume and have incorporated all the hadrons listed in the particle data book [55] up to mass of 3 GeV.

In Fig. 1 we have shown the scaled pressure, i.e., the ratio of pressure calculated for a finite system size $R$ to the pressure for infinite volume $i.e., R = \infty$ as a function of temperature $T$. Also shown in the adjacent figures are scaled energy, scaled entropy and scaled specific heat. Five different representative system sizes are chosen: $R = 2 \text{ fm}$, $R = 4 \text{ fm}$, $R = 6 \text{ fm}$, $R = 8 \text{ fm}$ and $R = 10 \text{ fm}$. We find a significant volume dependence in all these quantities. At extremely low temperature ($\sim 0.02 \text{ GeV}$) the finite volume effect is strongest. As the temperature increases the scaled variables approach unity. This is expected since the lowest lying hadrons are most dominant at low temperatures and they feel the finite volume effects the most. With increase in temperature higher mass resonances become important and the dependence on finite system size diminishes.

At around a temperature of 0.1 GeV the scaled quantities for $R = 10 \text{ fm}$ almost reaches one indicating that the results are almost indistinguishable from those at infinite volume. Obviously for smaller volumes the scaled variables will reach unity at higher temperatures. We find that by 0.2 GeV of temperature the scaled quantities for system sizes down to $R = 4 \text{ fm}$ reach the value of one. For smaller volumes the system will still be away from the infinite volume scenario at $T = 0.2 \text{ GeV}$ by which the system is supposed to convert to a partonic phase.

Fluctuations of conserved charges such as net electric charge, baryon number and strangeness have been considered as probes for hadronization and thermalization of the system created in nuclear collisions [56–58]. Moreover, fluctuations are expected to show distinctly different behaviour in a hadron resonance gas and a QGP. From the grand canonical partition function ($Z$) we take derivatives with respect to chemical potential to get the various susceptibilities. The $n^{th}$ order susceptibility is defined as,

$$\chi^n_x = \frac{1}{VT^3} \frac{\partial^n (\ln Z)}{\partial (\mu_x/T)^n},$$

(8)

where $\mu_x$ is the chemical potential for conserved charge $x$. We have considered for our purpose $x = B$ (baryon), $S$ (strangeness) and $Q$ (electric charge).

In Fig. 2 we have shown scaled second order and fourth order susceptibilities for conserved charges namely baryon number, electric charge and strangeness as function of temperature for $\mu_B = 0$ for different system sizes $R$. The
general features of these scaled fluctuations are similar to the previous thermodynamic variables. For all the cases we observe a strong volume dependence especially for low temperature. Though the quantitative features of baryon number and strangeness are very close to those of the thermodynamic variables discussed earlier, the nature of electric charge fluctuations are quite different. We observe that in this case even the systems with volumes larger than 4 fm are significantly different from the infinite volume systems. It may be noted that the lightest hadrons - the pions contribute to this sector and not to the other charge fluctuations. As expected the ensemble with the lightest particles will have the most significant signatures of finite volumes.

This is further reflected in the observables mean ($M_x$), variance ($\sigma_x^2$), skewness ($S_x$) and kurtosis ($\kappa_x$) describing the particle distribution. These observables may be combined to relate to the ratios of susceptibilities as,

$$\frac{\sigma_x^2}{M_x} = \frac{\chi_x^2}{\chi_x^{1}}, \quad S_x \sigma_x = \frac{\chi_x^3}{\chi_x^{2}}, \quad \kappa_x \sigma_x^2 = \frac{\chi_x^4}{\chi_x^{2}}.$$  \hspace{1cm} (9)

These kind of ratios are important when comparing theoretical predictions with experimental results where the volumes may not be estimated reliably and it is expected that the volume factors in the numerator and denominator drop out [59]. However this assumption is valid when the low momentum scales are less important. The variation of these ratios for net proton, net charge and net kaon are shown as a function of the centre of mass collision energy $\sqrt{s}$ in Fig. (3). The parametrization of the temperature and chemical potentials at the freeze-out conditions is taken from Ref. [60]. We find that for protons and kaons the ratios are almost independent of volume, but for the net electric charge sector a significant volume dependence is present. While the proton and kaon masses are in the range of 0.5–1 GeV, the lowest mass in the electric charge sector is that of the pions with is similar to the lowest momentum scale for a finite size system. A similar effect of low momentum scale affecting the results at cross-over has been discussed in [36].

To summarise, we have studied the thermodynamic properties and fluctuations of hot and dense matter in a finite volume. We have used the HRG model for this purpose. The finite volume has a significant effect on the thermodynamic properties and also on the fluctuations. But for most observables the effect diminishes once they are scaled with the respective volumes. The only significant difference even after scaling with the volume is observed for the fluctuations for electric charge. Given that one can relate such volume dependence on the observable in heavy-ion collision experiments, it may be possible to extract information about the system volume from the net electric charge fluctuations.
FIG. 3: (Color online) Energy dependence of $\sigma^2/M$, $S\sigma$ and $\kappa\sigma^2$ for proton (left column), net electric charge (middle column) and net kaon (right column).

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