Universal and deterministic manipulation of the quantum state of harmonic oscillators: a route to unitary gates for Fock State qubits

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We present a simple quantum circuit that allows for the universal and deterministic manipulation of the quantum state of confined harmonic oscillators. The scheme is based on the selective interactions of the referred oscillator with an auxiliary three-level system and a classical external driving source, and enables any unitary operations on Fock states, two-by-two. One circuit is equivalent to a single qubit unitary logical gate on Fock states qubits. Sequences of similar protocols allow for complete, deterministic and state-independent manipulation of the harmonic oscillator quantum state.

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In the last two decades, state-of-the-art experiments on both cavity QED and trapped ions have been exploring the quantized nature of different spatially confined harmonic oscillators. As examples of important experimental demonstrations we can cite the production and detection of Fock states, Schrödinger-cat like states, as well as the complete measurement of non-classical quasi-probability distributions in quantum phase space.

Recently, understanding and operating on those quantized harmonic oscillators has also become an important issue for quantum information theory both from a fundamental point of view as well as from practical implementations. Quantized light and vibrational modes play essential roles in many different proposals and experiments of quantum protocols, be it as memory, information buses or even as computational qudits.

The possibility to investigate in a more controllable fashion different quantized confined harmonic oscillators and to use them as qudits has prompted a renewed theoretical and experimental effort towards the engineering and manipulation of more complex quantum states in these systems. These proposals and experiments are based on coupling the quantized harmonic oscillator to classical sources and low-dimensional systems such as few selected electronic levels of neutral atoms or ions. All of them explore important features of quantized harmonic oscillators. However, none of them allows for a complete, state-independent, universal manipulation of quantum states in these systems. These proposals and experiments are based on coupling the quantized harmonic oscillator to classical sources and low-dimensional systems such as few selected electronic levels of neutral atoms or ions. All of them explore important features of quantized harmonic oscillators. However, none of them allows for a complete, state-independent, universal manipulation of quantum states in these systems. In fact, even the apparently simple task of rotating a Fock state qubit remained solvable only in the \{\ket{0}, \ket{1}\} subspace.

In this paper, we combine those two interactions and the concept of spin-echo to present a quantum circuit that implements any state-independent, unitary transformation on arbitrary quantum states of harmonic oscillators. Our proposal is based on the so-called selective interactions, described in for Cavity QED and in for trapped ions. In these interactions, an external classical source is used to control the effective coupling between the harmonic oscillator and auxiliary electronic levels of a neutral atom or an ion. The scheme here presented is general and can be applied to any harmonic oscillator in the presence of an external driving classical source and an auxiliary three-level system.

In the first part of this manuscript we show how to combine two selective interactions and an intermediate spin flip to build an universal deterministic logical gate for a chosen Fock state qubit. In other words, we show how to implement an arbitrary rotation in a chosen two-dimensional \{\ket{m}, \ket{m-1}\} energy subspace of the harmonic oscillator. This three-step circuit (see Fig.1), which we will refer to as UG\_m will, then, constitute the fundamental block for arbitrarily manipulating any state of the harmonic oscillator, i.e. sequences of UG\_m gates, adjusted to rotate different pairs of Fock states (different “m’s”), can be used to manipulate more general states of the harmonic oscillator. Note that in order to illustrate the scheme for practical applications, we analyze it in the context of Cavity QED, but always keeping in mind that similar setups are available for manipulating the vibrational modes of trapped ions.

The building block of our circuit involves two types of operations as shown in Fig(1): a selective coupling \(\hat{H}(\theta)\) between two Fock states of the harmonic oscillator (in our example, a cavity mode) and an auxiliary low dimensional system (a three-level atom) intercalated by a local operation on the auxiliary system. By selective, we mean a linear interaction \(\hat{H}(\theta)\), between the atom and the light field that is resonant for a chosen joint subspace and dispersive for all the remaining ones. For example, if we denote the atomic states by \{\ket{g}, \ket{e}\} and we choose a particular Fock state \(m\) for the cavity field, then \(\hat{H}(\theta)\) is engineered so that only states \{\ket{gm}, \ket{em - k}\} perform Rabi oscillations. All the other doublets evolve dispersively and all the remaining states just acquire phases. We show bellow how selectivity allows for the rotation of an arbitrary harmonic oscillator qubit \(\ket{qb} = \alpha(m - 1) + \beta|m\). The extension to more general two-dimensional subspaces \{\ket{m}, \ket{m - k}\}
is immediate for the vibration of trapped ions.

First, let us briefly summarize the selective interaction in cavity QED. Much like in [12] let us consider an off-resonance Raman Hamiltonian described, in the interaction picture, by

\[
\hat{H}_{\text{int}} = \hbar g \hat{\sigma}_g \hat{a} e^{-i\delta t} + \hbar \Omega \hat{\sigma}_e \hat{a} e^{-i\delta t} + \text{h.c.}
\]

(see Fig(2) for the levels scheme). The first term couples dispersively, with coupling constant \(g\), the lowest atomic energy level \(|g\rangle\) to the higher one \(|h\rangle\) through the quantized mode described by annihilation operator \(\hat{a}\). The second term couples dispersively, with coupling constant \(\Omega\), the intermediate atomic energy level \(|e\rangle\) with level \(|h\rangle\) through an external driving source (for example, an intense laser field). \(\delta = \omega_{gh} - \omega_0 = \omega_{he} - \omega_L\) is the detuning between both atomic transitions and their respective interacting fields frequencies.

When \(\delta \gg |\Omega_L|\), \(g\) (considering \(g\) real), we can adiabatically eliminate level \(|h\rangle\) and approximate \(\hat{H}_{\text{int}}\) by the effective Hamiltonian (h = 1) [14]:

\[
\hat{H}_{\text{eff}}(\theta) = \frac{g^2}{\delta} \hat{\sigma}_g \hat{a}^\dagger + \frac{g^2}{\delta} \hat{\sigma}_e \hat{a} + \lambda (e^{i\theta} \hat{\sigma}_g \hat{a}^\dagger + e^{-i\theta} \hat{\sigma}_e \hat{a}),
\]

where \(\theta\) is the phase of \(\Omega_L\), \(\lambda = \frac{2 \Omega L}{\hbar}\), and we have already included an energy shift \(\Delta_m = \frac{g^2}{\delta} \hat{m}\) to level \(|e\rangle\) implementable through the action of an external classical source, where \(m\) is an integer number. This Hamiltonian splits the joint atom-cavity mode Hilbert space into two-dimensional subspaces spanned by the doublets \(|\{g, n\}, |e, n - 1\}\). It describes a dispersive (or resonant) dynamics whenever \(\Delta(n) \gg \lambda\) (or \(\Delta(n) < \lambda\)), just like the typical Jaynes-Cummings interaction [14]. However, unlike the JC model, now, the effective detuning \(\Delta(n) = \frac{g^2}{\delta}(n-m)\) between levels \(|g, n\rangle\) and \(|e, n - 1\rangle\) depends on the number \(n\) of excitations in the cavity field. Selectivity is achieved when \(\Delta(n) \gg \lambda\) for all \(n \neq m\), which means that all the doublets evolve dispersively except for a chosen one \(|\{g, m\}, |e, m - 1\}\) that evolves resonantly (\(\Delta(m) = 0\)). In our case, selectivity holds for \(|\Omega_L|\).

In the selective regime, second order Hamiltonian \(\hat{H}_{\text{eff}}(\theta)\) unfolds as \(\hat{H}_0 + \hat{H}(\theta)\). The first part, given by

\[
\hat{H}_0 = \sum_{n \neq m, m=1} \left[ \frac{g^2}{\delta} |g\rangle \langle g| + \frac{g^2}{\delta} |e\rangle \langle e| \right] \otimes |n\rangle \langle n|,
\]

describes the dispersive dynamics for all the doublets that do not involve Fock states \(|m\rangle, |m-1\rangle\), whereas \(\hat{H}(\theta) = \hat{H}_0 + \hat{H}_c(\theta)\) describes the interaction between these Fock states and the atomic levels. \(\hat{H}_0\) contains the self-energy correction terms for the joint states \(|\{g, m\}, |g, m-1\rangle, |e, m\rangle, |e, m-1\rangle\},

\[
\hat{H}_0 = \frac{g^2}{\delta} \hat{I}_s - \frac{g^2}{\delta} |g, m-1\rangle \langle g, m-1|,
\]

where \(\hat{I}_s\) is the identity in this subspace, and,

\[
\hat{H}(\theta) = \lambda \sqrt{m} (e^{i\theta} |g, m\rangle \langle e, m-1| + h.c.),
\]

describes the selective coupling itself. Note that \(\hat{H}(\theta)\) allows for excitation exchanges between the atom and the harmonic oscillator to happen only inside the chosen subspace \(|\{g, m\}, |e, m - 1\}\). As a consequence, if the harmonic oscillator is initially prepared in a superposition of Fock states \(|m\rangle\) and \(|m-1\rangle\), no other Fock state gets populated during the interaction with the atom. This property turns out to be the only necessary condition for Fock states qubits deterministic manipulation.

Let us consider an initial product state \(|\Psi\rangle = |\psi_{at}\rangle |qb\rangle\) between the atom and the cavity mode, where the atom is prepared in the symmetric (or anti-symmetric) superposition of its internal electronic states, \(|\psi_{at}\rangle = \frac{|g\rangle + \epsilon |e\rangle}{\sqrt{2}}\). This choice for the atomic state is justified later on when it also becomes clear that both symmetric and anti-symmetric states are equally good for this single qubit rotation protocol. Now, let us analyze in details the following sequence of operations: first the atom interacts selectively with the harmonic oscillator for a chosen time \(\tau\). Then an external driving field flips the atomic state (spin-echo technique), and finally atom and harmonic oscillator interact selectively again for the same time \(\tau\) but at a slightly rotated angle \(\theta_0 = \frac{\pi}{4\tau}\). The final joint state is, then, given by:

\[
|\Psi_f\rangle = e^{-i\hat{H}(\theta_0)\tau} \sigma_x e^{-i\hat{H}(0)\tau} |\psi_{at}\rangle |qb\rangle,
\]

where \(\hat{H}(\theta) = \hat{H}_0 + \hat{H}_c(\theta)\) and \(\sigma_x = |g\rangle \langle e| + |e\rangle \langle g|\).

In order to rewrite this time evolution in a clearer version, first note that \(\sigma_x e^{-i\hat{H}(0)\tau} \sigma_x = e^{-i\hat{H}(0)\tau}\), where

\[
\hat{H}_c(0) = \frac{g^2}{\Delta} I_s - \frac{g^2}{\Delta} |e, m-1\rangle \langle e, m-1| + \lambda \sqrt{m} |g, m-1\rangle \langle g, m-1| + h.c.
\]

Also note that \([\hat{H}_0, \hat{H}_c] = \hat{H}_c, [\hat{H}_c, \hat{H}_c] = 0\) (where \(\hat{H}_c = \sigma_x \hat{H}_s \sigma_x\)) and that \(e^{-i\hat{H}(\theta_0)\tau} e^{-i\hat{H}(\theta_0)\tau} \tau \hat{H}_0 \tau = e^{-i(\hat{H}_c(\theta_0) + \hat{H}_c(\theta_0))}\). Using these properties of the effective Hamiltonian and the Baker-Hausdorff theorem, we can rearrange this product of exponentials as

\[
|\Psi_f\rangle = e^{-i(\hat{H}_c(\theta_0) + \hat{H}_c(\theta_0))\tau} e^{-i(\hat{H}_0 + \hat{H}_0 \tau)\tau} |\psi_{at}\rangle |qb\rangle
\]

After simple algebra it is straightforward to show that \((\hat{H}_0 + \hat{H}_0 \tau)\tau = 2\eta I_s + \beta I_{at} \otimes |m-1\rangle \langle m-1|\), where \(\hat{I}_{at}\) is the identity in the atomic subspace and \(\eta = \frac{g^2}{\delta} m\). This part represents the addition of a global phase \(\eta\) to state \(|\Psi\rangle\) and the addition of a small phase \(\theta_0\) to the qubit state \(|m-1\rangle\). It takes state \(|\Psi\rangle\) into state \(|\Psi'\rangle\) of \(e^{-i\eta |\psi_{at}\rangle |qb\rangle}, |g\rangle = \alpha e^{-i\theta_0} |m-1\rangle + \beta |m\rangle\). Similar calculations lead to the relation

\[
\left[ \hat{H}_c(\theta_0) + \hat{H}_c(\theta_0) \right] \tau = \{|+\rangle \langle + | - |\rangle \langle - |\}
\times \phi(e^{i\theta_0} |m\rangle \langle m-1| + h.c.),
\]

where \(\phi = \lambda \tau \sqrt{m}\). This part represents a direct linear coupling between qubit states \(|m\rangle\) and \(e^{-i\theta_0} |m-1\rangle\) provided the atom is initially prepared in an eigenstate of
with coupling constant $g_L$, similar to the one proposed in [15].

The interaction between these levels and the quantized mode, interaction, i.e. producing an anti-Jaynes-Cummings shift in one (or both) of them inverting their roles in the diate step of the spin flip can be replaced by an energy

As it is shown above, if the atom is initially prepared in the ground state, since $U_{G_n}$ operates only on the subspace of the harmonic oscillator.

Given any initial state $|\Phi\rangle = |\psi_{init}\rangle \sum_n c_n |n\rangle$, one can produce any other state by applying rotations involving different subspaces $\{ |m\rangle, |m - k\rangle \}$, each one of them selected by its respective shift $\Delta_m$ to atomic level $|e\rangle$. Fig. (3) shows the quantum circuit for this qudit manipulation using only one atomic qubit, operating on different pairs of Fock states, one pair at a time. Note that the whole operation can be done with only one auxiliary system, due to the fact that after each one qubit gate there is no entanglement between the harmonic oscillator and the auxiliary system. For example, beginning with the harmonic oscillator in its ground state $|0\rangle$, one can use a sequence of $n$ gates, to prepare the superposition $|\alpha(0) + \beta|n\rangle$. First we shift level $|e\rangle$ by $\Delta_1 = \frac{g \Omega}{\Delta} - \frac{|\Omega|^2}{\Delta}$ corresponding to the primary circuit $U_{G_1}$ which operates on the subspace $\{ |0\rangle, |1\rangle \}$ of the harmonic oscillator. This first gate is used to prepare the quantum superposition $|\alpha(0) + \beta|1\rangle$. Then, we select the energy shift $\Delta_2$ to coherently transfer the population of Fock state $|1\rangle$ to Fock state $|2\rangle$, with no changes to the population of the ground state, since $U_{G_2}$ operates only on the subspace $\{ |1\rangle, |2\rangle \}$. This second operation prepares state $|\alpha(0) + \beta|2\rangle$. It is clear that the sequence of operations $U_{G_n}(\tau_n)\ldots U_{G_2}(\tau_2)U_{G_1}(\tau_1)|0\rangle$, where $\alpha = \cos(\frac{g \Omega |\tau_1|}{\Delta})$, $\beta = \sin(\frac{g \Omega |\tau_1|}{\Delta})$ and $\Omega = \frac{\Delta}{2}$ prepares the quantum superposition $|\alpha(0) + \beta|n\rangle$.

In this particular example, the gates must be sequential in time, i.e. first we operate $U_{G_1}$, then $U_{G_2}$ and so forth. However, depending on the desired transformation, more than one auxiliary qubit can be used in order to accelerate the process, since each one of them can be adjusted (by different shifts) to rotate a particular subspace of the harmonic oscillator qubit, as shown in Fig (4). In this case, if $N$ is the higher Fock state to be manipulated, then $N/2$ auxiliary systems can be used to fasten the whole operation. This second approach is particularly interesting for the manipulation of vibrations of

FIG. 1: Primary circuit: three steps quantum circuit that rotates Fock states $\{ |m\rangle, |m + 1\rangle \}$ into $\{ \cos \theta |m\rangle + e^{i\phi} \sin \theta |m + 1\rangle, -\sin \theta |m\rangle + e^{-i\phi} \cos \theta |m + 1\rangle \}$

$U_{G_n}$

FIG. 2: Atomic level scheme: lower-level $|g\rangle$ is dispersively coupled to higher-level $|h\rangle$, through the cavity mode $\omega_{car}$, with coupling constant $g$, while intermediate level $|e\rangle$ is dispersively coupled to level $|h\rangle$ through the external source field $\omega_L$ with coupling constant $\Omega_L$.

the described $\sigma_x$ operator. It takes state $|\Psi\rangle$ into the final state $|\Psi'\rangle = |\psi_{init}\rangle |q_{bf}\rangle$, where $|q_{bf}\rangle = (\alpha \cos \phi + i\beta \sin \phi) e^{-i\theta} |m-1\rangle + (\beta \cos \phi + i\alpha \sin \phi) |m\rangle$.

As it is shown above, if the atom is initially prepared in an eigenstate of $\sigma_z$, then the proposed three step process implements a unitary rotation in the Fock states basis $\{ |m\rangle, |m - 1\rangle \}$ of the harmonic oscillator, without entangling it with the atomic state. Note that after the first interaction atom and harmonic oscillator may get highly entangled. Being a local operation, the spin-flip does not change this degree of entanglement, but the final two-qubit operation implements the final rotation disentangling both systems.

Also note that if levels $|g\rangle$ and $|e\rangle$ have approximately the same energy, for example, if they form a hyperfine structure of the same electronic level, then the intermediate step of the spin flip can be replaced by an energy shift in one (or both) of them inverting their roles in the interaction, i.e. producing an anti-Jaynes-Cummings coupling between these levels and the quantized mode, similar to the one proposed in [15].

The same selectivity is also available in the trapped ions setup. For example, Ref. [17] shows how to produce the same effective Hamiltonian in this context. In fact, due to the nature of the coupling between the internal levels of a trapped ion and its vibrational motion, in the resolved sideband regime, it is possible to engineer selectivity for the so-called multi-quantum-Jaynes-Cummings model [18], described by the coupling term:

$$\tilde{H}_{int}(\theta) = \lambda_k(\sigma_x \hat{a}^k + e^{-i\theta} \sigma_z \hat{a}^k),$$

In this case, the selected doublet is $\{ |g, m\rangle, |e, m - k\rangle \}$. Now that we showed a quantum gate that rotates arbitrary Fock states qubits, it is trivial to extend the idea to higher dimensional qudits in order to engineer and manipulate any state of quantized harmonic oscillators. Note that the basic circuit $U_{G_m}$ is a combination of two and one qubit gates. In fact, what we present next is a practical application of the known fact that any quantum computation can be executed with those two elements [19].

Given any initial state $|\Phi\rangle = |\psi_{init}\rangle \sum_n c_n |n\rangle$, one can produce any other state by applying rotations involving different subspaces $\{ |m\rangle, |m - k\rangle \}$, each one of them selected by its respective shift $\Delta_m$ to atomic level $|e\rangle$. Fig. (3) shows the quantum circuit for this qudit manipulation using only one atomic qubit, operating on different pairs of Fock states, one pair at a time. Note that the whole operation can be done with only one auxiliary system, due to the fact that after each one qubit gate there is no entanglement between the harmonic oscillator and the auxiliary system. For example, beginning with the harmonic oscillator in its ground state $|0\rangle$, one can use a sequence of $n$ gates, to prepare the superposition $|\alpha(0) + \beta|n\rangle$. First we shift level $|e\rangle$ by $\Delta_1 = \frac{g \Omega}{\Delta} - \frac{|\Omega|^2}{\Delta}$ corresponding to the primary circuit $U_{G_1}$ which operates on the subspace $\{ |0\rangle, |1\rangle \}$ of the harmonic oscillator. This first gate is used to prepare the quantum superposition $|\alpha(0) + \beta|1\rangle$. Then, we select the energy shift $\Delta_2$ to coherently transfer the population of Fock state $|1\rangle$ to Fock state $|2\rangle$, with no changes to the population of the ground state, since $U_{G_2}$ operates only on the subspace $\{ |1\rangle, |2\rangle \}$. This second operation prepares state $|\alpha(0) + \beta|2\rangle$. It is clear that the sequence of operations $U_{G_n}(\tau_n)\ldots U_{G_2}(\tau_2)U_{G_1}(\tau_1)|0\rangle$, where $\alpha = \cos(\frac{g \Omega |\tau_1|}{\Delta})$, $\beta = \sin(\frac{g \Omega |\tau_1|}{\Delta})$ and $\Omega = \frac{\Delta}{2}$ prepares the quantum superposition $|\alpha(0) + \beta|n\rangle$.
FIG. 3: Quantum circuit that implements deterministic transformations in arbitrary states of the harmonic oscillator, taking state $|qd\rangle = \sum_n c_n |n\rangle$ into state $|qd_f\rangle = \sum_n c'_n |n\rangle$ using just one auxiliary system. In this case, the short circuits $UG_m$ must be sequential in time.

FIG. 4: Quantum circuit that implements deterministic transformations in arbitrary states of the harmonic oscillator, taking state $|qd\rangle = \sum_n c_n |n\rangle$ into state $|qd_f\rangle = \sum_n c'_n |n\rangle$ using $N/2$ auxiliary systems. In this case, $N/2$ short circuits $UG_m$ can be processed in parallel, since each auxiliary system can be used to manipulate a pair of consecutive Fock states at the same time.

trapped ions, given that all the ions in the same trap are coupled to their different collective vibrational modes.

In this manuscript, we present a simple quantum circuit, based on a selective interaction, that implements arbitrary unitary transformations on Fock states qubits. We also show that sequences of this circuit can be used to operate any unitary transformation on the quantum state of harmonic oscillators.

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