Simplifying Neural Networks with the Marabou Verification Engine

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Abstract. Deep neural network (DNN) verification is an emerging field, with diverse verification engines quickly becoming available. Demonstrating the effectiveness of these tools on real-world DNNs is an important step towards their wider adoption. We focus here on the recently proposed Marabou verification tool, and demonstrate its usage for a novel application: simplifying neural networks, by reducing the size of a DNN without harming its accuracy. We report on the workflow of the simplification process, and on its potential significance and applicability to domains of interest.

Keywords: Deep Neural Networks, Simplification, Verification, Marabou

1 Introduction

Deep neural networks (DNNs) are revolutionizing the way complex software is produced, obtaining unprecedented results in domains such as image recognition [1], natural language processing [2], game playing [3], and many others. There is now even a trend of using DNNs as controllers in autonomous cars and unmanned aircraft [4, 5]. With DNNs becoming prevalent, it is highly important to develop automatic techniques to assist in creating, maintaining and adjusting them.

As DNNs are used in tackling increasingly complex tasks, their sizes (i.e., number of neurons) are also increasing – to a point where modern DNNs can have millions of neurons [6]. DNN size is thus becoming a liability, as deploying larger networks takes up more space, increases energy consumption, and prolongs response times. Consequently, researchers have started working on DNN minimization and simplification. A common approach is to start with a large network, and reduce its size by removing some of its components (i.e., neurons and edges) [7,18]. The parts to be removed from the network are determined heuristically, and network accuracy may be harmed, sometimes requiring additional training after the simplification process has been performed [7].

Here, we propose a novel simplification technique that harnesses recent advances in DNN verification [8-11,13-17,19-25]. Using verification queries, we propose to iden-
tify components of the network that never affect its output. A major benefit of verification-based simplification is that it is accurate, i.e. the simplified network is completely equivalent to the original. Thus, retraining of the simplified network, which may be expensive, is not required. Our technique can be applied using existing DNN verification tools as a backend, and we report here on our experience using the recently published Marabou framework [11]. We propose a work-flow in which we (1) perform lightweight simulations to identify parts of the network that are candidates for removal; (2) use Marabou to dispatch verification queries that determine which of those parts that can indeed be removed without affecting the network’s outputs; and (3) construct the simplified network, which is equivalent to the original. We evaluate our approach on the ACAS Xu family of DNNs for airborne collision avoidance in unmanned aircraft [5], and report a significant reduction of up to 10% in network sizes.

2 Background: DNNs, Verification and Simplification

DNNs are comprised of an input layer, an output layer, and multiple hidden layers in between. A layer is comprised of multiple nodes (neurons), each connected to nodes from the preceding layer using a predetermined set of weights (see Fig. 1). By assigning values to inputs and then feeding them forward through the network, values for each layer can be computed from the values of the previous layer, finally resulting in values for the outputs.

As DNNs are increasingly used in safety-critical applications [4, 5], there is a surge of interest in verification methods that can provide formal guarantees about DNN behavior [8-11,13-17,19-25]. A DNN verification query consists of a neural network and a property to be checked; and it results in either a formal guarantee that the network satisfies the property, or a concrete input for which the property is violated (a counter-example). Verification queries can encode various properties about DNNs; e.g., that slight perturbations to a network’s inputs do not affect its output. Recently, there has been significant progress on DNN verification (see a recent survey [12]), although scalability remains a major limitation of existing approaches. It has been shown that the DNN verification problem is NP-complete, and becomes exponentially harder as the network size increases [8].

Fig. 1. A small neural network with 2 hidden nodes in one hidden layer. Weights are denoted over the edges. Hidden node values are typically determined by computing a weighted sum according to the weights, and then applying a non-linear activation function to the result.
In recent years, enormous DNNs have been appearing in order to tackle increasingly complex tasks – to a point where DNN size is becoming a liability, because large networks take longer to train and even to evaluate when deployed. Techniques for neural network minimization and simplification have thus started to emerge: typically, these take an initial, large network, and reduce its size by removing some of its components [7]. The pruning phase involves the removal of edges from the network. The selection of which edges to remove is done heuristically, often by selecting edges that have very small weights, because these edges are less likely to affect the network’s outputs significantly. If all edges connecting a node to the preceding layer or to the succeeding layer are removed, then the node itself can be removed. After the pruning phase, the reduced network is retrained. This approach has been shown to produce a smaller network whose performance is on par with that of the original network [7,18].

3 Simplification using Verification

Despite the demonstrated usefulness of pruning-based DNN simplification [7,18], heuristic-based approaches might sometimes miss edges that can be removed, if these edges do not have particularly small weights. However, such edges can be identified by a verification-based approach. For example, consider the small network shown in Fig. 2. As all edges have weights with identical magnitudes, none of them would be pruned by a heuristic-based approach. However, using a verification engine, it is possible to check the property: “does there exist an input for which \(v_4\) takes a non-zero value?”. If the verification tool answers “no”, as is the case for the network in Fig. 2 (because \(v_4 = v_2 - v_3\) and \(v_2 = v_3\)), then we are guaranteed that \(v_4\) is always assigned 0, regardless of the input. In turn, this means that \(v_4\) can never affect nodes in subsequent layers. In this case, \(v_4\) and all its edges can be safely removed from the network (rendering the network’s output constant). Due to the soundness of the verification process, we are guaranteed that the simplified DNN is completely equivalent to the original DNN, and thus no retraining is required.

![Fig. 2. Using verification, node \(v_4\) can safely be removed from the network.](image)

The approach of using verification to identify nodes that are always assigned 0 for every possible input and can be safely removed is the core of our technique. However, because verification is costly, posing this query for every node of the DNN might take a long time. To mitigate this difficulty, we propose the following workflow:

1. Use lightweight simulations to identify nodes that are candidates for removal. Initially, all hidden nodes are such candidates. We then evaluate the network for random input values, and remove from the list of candidates any hidden node
that is assigned a non-zero value for some input. With each simulation, the number of candidates for removal decreases.

2. For each remaining candidate node \( v \), we create a separate verification query stating that \( v \neq 0 \), and use the underlying verification engine to dispatch it. If we get an UNSAT answer, we mark node \( v \) for removal. The candidates are explored in a layer-by-layer order, which allows us to only examine a part of the DNN for every query. For example, when addressing a candidate in layer \#2, we do not encode layers \#3 and on as part of our verification query, because a node’s assignment can only be affected by nodes in preceding layers. Because verifying smaller networks is generally easier, this layer-by-layer approach accelerates the process as a whole. In addition, this process naturally lends itself to parallelization, by running each verification query on a separate machine.

3. Finally, we construct the simplified network, in which the nodes marked for removal and all their incoming and outgoing edges are deleted. We can also remove any nodes that subsequently become irrelevant due to the removal of all of their incoming or outgoing edges (e.g., for the DNN in Fig. 2, after removing \( v_4 \) we can also remove \( v_2 \) and \( v_3 \), as neither has any remaining outgoing edges.)

We note that our technique can be extended to simplify DNNs in additional ways, by using different verification queries. For example, it can identify separate nodes that are always assigned identical values (duplicates) and unify them, thus reducing the overall number of nodes. It can also identify (and remove) nodes that can be expressed as linear combinations of other nodes.

4 Evaluation

Our approach is general, in two senses: (1) it can be applied to simplify any DNN, regardless of its application domain; and (2) it can use any DNN verification engine as a backend, thus benefiting from any future improvement in verification technology. In practice, it is required that the DNN in question be supported by the backend verification engine – for example, some engines may not support certain network topologies. For our evaluation here, we focused on the ACAS Xu family of 45 DNNs for airborne collision avoidance [5], and on the Marabou DNN verification tool [11]. Our code will be made publicly available with the final version of this paper.

For each of the 45 DNNs, we ran a simple script, external to Marabou, to perform step (1) of our workflow (random simulations), resulting in a list of candidate nodes for removal. For each of the ACAS Xu DNNs we performed 20000 simulations, and this narrowed down the list of nodes that are candidates for removal to about 7% of all hidden nodes (see Fig. 3). For step (2), we ran another script that takes as input a DNN and a node \( v \) that is a candidate for removal, and constructs a smaller DNN, where that node is the only output node (subsequent layers are omitted). These pairs of modified DNNs and nodes are then passed to the Marabou framework, with the query \( v \neq 0 \). Here, we encountered the following issue: the Marabou framework, like many linear-programming based tools, does not provide a way to directly specify that \( v \neq 0 \), but
rather only to state that $v \geq \epsilon$ for some $\epsilon > 0$ (we assume all hidden nodes are, by definition, never negative, which is the case for the ACAS Xu DNNs). We experimented with various values of $\epsilon$ (see Fig. 4), and concluded that the choice of $\epsilon$ has very little effect on the outcome of the experiment – i.e., nodes tend to either be obsolete, or take on large values. Finally, for step (3), we ran yet another script that uses the results of the previous steps to construct the simplified network.

![Graph](image)

**Fig. 3.** Using simulation to identify nodes that are candidates for removal, on one of the ACAS Xu networks.

![Bar chart](image)

**Fig. 4.** Number of removed nodes as a function of the value of $\epsilon$, on one of the ACAS Xu networks. The minor differences are the result of different queries timing out for different values of $\epsilon$.

We performed this process for each of the 45 ACAS Xu networks. We ran the experiments on machines with Intel Xeon E5-2670 CPUs (2.60GHz) and 8GB of memory, and used $\epsilon = 0.01$. Each verification query was given a 4-hour timeout. Out of 1069 verification queries (1 per candidate node), 535 were UNSAT (node marked for removal), 15 were SAT, and 519 timed out (nodes not marked for removal). Thus, on average, 4% of the nodes were marked for removal (535 nodes out of 13500). Fig. 5 depicts their distribution across the 45 DNNs. In most networks, between 11 and 15 nodes (out of 300) could be removed; but for a few networks, this number was
higher. For one of the networks we discovered 29 neurons that could be removed – approximately 10% of that network’s total number of neurons.

![Figure 5](image.png)

**Fig. 5.** Total number of removed nodes of ACAS Xu networks.

In a separate experiment, we used a similar technique to identify pairs of neurons that are always assigned identical values, so that they can be merged. We followed the same general framework: lightweight simulation to identify pairs of neurons that are candidates for merging, followed by verification queries to check the equivalence of each pair. Here, we were unable to find any pairs suitable for merging. Recall that neuron comparison is performed using some small $\epsilon$ tolerance (0.01 in our experiment). It will be interesting to repeat the experiment using larger $\epsilon$ values, which will necessarily lead to some neuron pairs being suitable for merging. We plan to use verification to bound the overall change on the networks’ outputs in these cases, creating a trade-off: a user will be able to specify an acceptable threshold of change to the DNN’s outputs, and this will dictate which neurons can be merged. We leave this, and also the use of more complex queries (e.g., to identify nodes that can be expressed as linear combinations of other nodes) for future work.

## 5 Conclusion

DNN verification is an emerging field, and we are just now beginning to tap its potential in assisting engineers in DNN development. We explored here a new application: using black-box verification engines to simplify neural networks. We demonstrated that simplification is straightforward to achieve using a few simple scripts to wrap a verification engine, and that it can lead to substantial reduction in DNN size.

In the future, we plan to extend this work along several axes. First, we intend to explore additional verification queries, which would allow to simplify DNNs in more sophisticated ways – for example by revealing that some neurons can be expressed as linear combinations of other neurons. In addition, we plan to investigate more aggressive simplification steps, which may change the DNN’s output, while using verification to ensure that these changes remain within acceptable bounds. Finally, we intend to apply the technique to additional real-world DNNs and case studies.
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