Glueballs, a fulfilled promise of QCD?

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Abstract. This is a contribution to the review "50 Years of Quantum Chromodynamics" edited by F. Gross and E. Klempt, to be published in EPJC. The contribution remembers the early searches and explains how to find a glueball, based on its properties. The results of a coupled-channel analysis are presented that provides evidence for the scalar glueball and first hints for the tensor glueball. Data on radiative decays of ψ(2S) and Υ(1S) show scalar intensity that is likely due to glueball production.

1 Introduction

At the Workshop on QCD: 20 Years Later held 1992 in Aachen, Heusch [1] reported on searches for glueballs, gluonium, or glue states as Fritzsch and Gell-Mann [2,3] had called this new form of matter. Glueballs are colorless bound states of gluons and should exist when their newly proposed quark-gluon field theory yields a correct description of the strong interaction. The title of Heusch’s talk Gluonium: An unfilled promise of QCD? expressed the disappointment of a glueball hunter: At that time there was some - rather weak - evidence for glueball candidates but there was no convincing case. In 1973, the e⁺e⁻ storage ring SPEAR at the Stanford Linear Accelerator Center had come into operation and one year later, the J/ψ resonance was discovered [4] - this was the very first SPEAR publication on physics. The J/ψ resonance and its radiative decay became and still is the prime reaction for glueball searches.

One of the first glueball candidates was the ρ(1440) [5,6]. The name ρ stood for the “number one” of all glueballs to be discovered. It was observed as very strong signal with pseudoscalar quantum numbers in the reaction J/ψ → γK¯Kπ. Its mass was not too far from the bag-model prediction (1290 MeV) [7]. Now the ρ(1440) is supposed to be split into two states, η(1405) and η(1475), where the lower-mass meson is still discussed as glueball candidate even though its mass is incompatible with lattice gauge calculations. They find the mass of the pseudoscalar glueball above 2 GeV.

A second candidate was a resonance called Θ(1640) [8,9]. It was seen in the reaction J/ψ → γηη and confirmed as G(1590) - by the GAMS collaboration in π⁻p → ηηp [10]. Its quantum numbers shifted from JPC = 2++ to 0++, and its mass changed to 1710 MeV. This resonance still plays an important role in the glueball discussion.

A third candidate, or better three candidates, were observed in the OZI rule violating process π⁻p → ϕn

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2 QCD predictions

2.1 Glueball masses

First estimates of the masses of glueballs were based on bag models. The color-carrying gluon fields were required to vanish on the surface of the bag. Transverse electric and transverse magnetic gluons were introduced populating the bag. The lowest excitation modes were predicted to have quantum numbers JPC = 0++ and 2++ and to be degenerate in mass with M = 960 MeV [7,15]. A very early review can be found in Ref. [16].

The bag model is obsolete nowadays. Most reliable are presumably simulation of QCD on a lattice (see Ref. [25] for an introduction). In lattice gauge theory, the spacetime is rotated into an Euclidean space by the transformation

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\begin{table}
\centering
\begin{tabular}{|c|c|c|c|c|c|c|}
\hline
Glueball & Ref. [17] & Ref. [18] & Ref. [19] & Ref. [20] & Ref. [21] & Ref. [22] & Ref. [24] \\
\hline
\(0^{++}\) & 1710 \pm 50 \pm 80 & 1653 \pm 26 & 1795 \pm 60 & 1980 & 1780 +140 \pm 170 & 1850 \pm 130 & 1920 \\
\(2^{++}\) & 2390 \pm 30 \pm 120 & 2376 \pm 32 & 2620 \pm 50 & 2420 & 1860 +140 \pm 170 & 2610 \pm 180 & 2371 \\
\(0^{-+}\) & 2560 \pm 40 \pm 120 & 2561 \pm 40 & \text{--} & 2220 & 2170 \pm 110 & 2580 \pm 180 & \text{--} \\
\hline
\end{tabular}
\end{table}

The study of radiative decays of the \(J/\psi\) meson is the prime path to search for glueballs with masses of less than \(\sim 2500\) MeV. Gui \textit{et al.} \cite{32} calculated the yield of a scalar glueball having a mass of 1710 MeV on lattice and found

\begin{equation}
BR_{J/\psi \rightarrow \gamma G_{0^+}} (TH) = (3.8 \pm 0.9) \cdot 10^{-3}.
\end{equation}

For higher glueball masses the yield increases.

Narison gave a mass dependent formula derived from sum rules. For a mass of 1865 MeV, a yield of about \(10^{-3}\) is predicted \cite{28}.

The tensor glueball is expected \cite{33} to be observed with a branching ratio

\begin{equation}
BR_{J/\psi \rightarrow \gamma G_{1^+}} (TH) = (11 \pm 2) \cdot 10^{-3}.
\end{equation}

Production of the pseudoscalar glueball seems to be considerably smaller. For a mass of 2395 (or 2560) MeV, the authors of Ref. \cite{34} find

\begin{equation}
BR_{J/\psi \rightarrow \gamma G_{0^-}} (TH) = (0.231 \pm 0.080) \cdot 10^{-3} \quad \text{or} \quad (0.107 \pm 0.037) \cdot 10^{-3}.
\end{equation}
These are very significant yields, and the glueballs must be found provided they can be identified convincingly as glueballs amidst their $qar{q}$ companions.

3 How to identify a glueball

Figure 1 shows the prime reactions in which glueballs have been searched for.

3.1 $N\bar{N}$ annihilation

A decisive step forward in the search for glueballs was the discovery of two scalar isoscalar states in $p\bar{p}$ annihilation at rest. With the large statistics available at the Low Energy Antiproton Ring (LEAR) at CERN, $f_0(1370)$ and $f_0(1500)$ were identified in several final states. A large fraction of the data taken at LEAR is still used jointly with data on radiative $J/\psi$ decays in a coupled-channel analysis. Glueballs decay via $q\bar{q}$ pair creation. Hence they can be produced via $q\bar{q}$ annihilation. Meson production in $p\bar{p}$ annihilation was studied by the ASTERIX, OBELEX and Crystal Barrel experiments at LEAR and is a major objective of the PANDA collaboration at the GSI.

3.2 Central production

In central production, two hadrons (e.g. two protons) scatter in forward direction via the exchange of Pomerons. Pomerons are supposed to be glue-rich, hence glueballs can be formed in Pomeron-Pomeron fusion. This process was studied extensively at CERN by the WA76 and WA102 collaborations and is now investigated with the STAR detector at BNL. In the WA102 experiment, $f_0(1370)$ and $f_0(1500)$ were confirmed and $f_0(1710)$ was added to the number of scalar resonances.

3.3 radiative $J/\psi$ decays

In radiative $J/\psi$ decays, the primary $c\bar{c}$ pair converts into two gluons and a photon. The two gluons are mainly produced in S-wave, the two gluons can form scalar and tensor glueballs which should be produced abundantly. The large statistics available from BESIII at Beijing makes this reaction the most favorable one for glueball searches. Radiative decays of heavy mesons is the only process for which glueball yields have been calculated. The data will be discussed below in more detail.

3.4 Decay analysis

The decay of mesons into two pseudoscalar mesons is governed by SU(3)$_F$. In a meson nonet, there are two isoscalar mesons, one lower in mass the other one higher, which both contain a $n\bar{n} = (u\bar{u} + d\bar{d})/\sqrt{2}$ and a $s\bar{s}$ component and are mixed with the mixing angle $\varphi$. Figure 2 shows the SU(3)$_F$ squared matrix elements for meson decays into two pseudoscalar mesons as a function of the scalar mixing angle.

$$
\begin{align*}
| \langle n\bar{n} | f^H \rangle |^2 &= \left( \begin{array}{c} x_{11} \ x_{12} \ x_{13} \\
 x_{21} \ x_{22} \ x_{23} \\
 x_{31} \ x_{32} \ x_{33} \end{array} \right) \left( \begin{array}{c} |n\bar{n}> \\
 |s\bar{s}> \\
 |g\bar{g}> \end{array} \right) \\
\end{align*}
$$

3.5 Supernumery

The three scalar isoscalar mesons $f_0(1370)$, $f_0(1500)$ and $f_0(1710)$ played an important role in the glueball discussion. Amsler and Close [36,37] suggested to interpret these three states as the result of mixing of the two expected isoscalar states with the scalar glueball.

3.6 Conclusions

Identifying a glueball is a difficult task. The main argument in favor of a glueball interpretation is an anomalously large production rate in $J/\psi$ decays. It turns out that scalar mesons are organized like pseudoscalar mesons, into mainly singlet and mainly octet mesons. A large production rate of a mainly-octet scalar isoscalar meson in radiative $J/\psi$ decays directly points to a significant glueball content in its wave function. A second argument relies on meson decays into pseudoscalar mesons. In presence of

Fig. 1. Reactions most relevant for glueball searches. Left: $p\bar{p}$ annihilation; middle: Pomeron-Pomeron fusion; right: radiative $J/\psi$ decays. The glueball is supposed to decay into $K^0\bar{K}^0$.

Fig. 2. Decay probabilities of mesons for decays into two pseudoscalar mesons as a function of the scalar mixing angle [35].
a glueball, a pair of mesons assigned to the same multiplet should have a decay pattern that is incompatible with a qq interpretation for any mixing angle. Supernumery is a weak argument. It requires a full understanding of the regular excitation spectrum. Further studies are required to learn if central production is gluon-rich. The large production rates of \( f_0(1500) \), \( f_0(1710) \) and \( f_0(2100) \) in \( pp \) annihilation at collision energies above 3 GeV encourages glueball searches at the FAIR facility with the PANDA detector.

4 Evidence for glueballs from coupled-channel analysis

We have performed a coupled-channel partial wave analysis of radiative \( J/\psi \) decays into \( \pi^0 \pi^0, K_0^0 \eta_0, \) and \( \omega \phi \), constrained by the CERN-Munich data on \( \pi N \) scattering, data from the GAMS collaboration at CERN, data from BNL on \( \pi \pi \rightarrow K_0^0 K_s^0 \), and 15 Dalitz plots on \( pp \) annihilation at rest from LEAR. Data on \( K_{e4} \) decays constrain the low-energy region. Fitting details and references to the data can be found in Ref. [38]. Figure 3 shows the data on radiative \( J/\psi \) decays into \( \pi^0 \pi^0, K_0^0 K_s^0 \) and the fit. Ten scalar isoscalar resonances were included in the fit. Oller [40] has shown that \( f_0(500) \) is singlet-like, the \( f_0(980) \) octet-like (see also [41]). The \( f_0(1500) \) is seen in Figure 3 as a dip. This pattern was reproduced in Ref. [38] assuming that \( f_0(1370) \) is a singlet state and \( f_0(1500) \) an octet state. Hence we assumed that the ten mesons can be divided into two series of states, mainly-singlet states with lower masses and mainly-octet states with higher masses.

In a \((M^2, n)\) plot, the masses of singlet and octet states follow two linear trajectories (see Fig. 4). Remarkably, the slope \((1.1 \text{ GeV}^{-2})\) is close to the slope of standard Regge trajectories. The separation between the two trajectories is given by the mass square difference between \( \eta' \) and \( \eta \) meson as suggested by instanton-induced interactions [44]. The figure includes a meson reported by the BESIII collaboration studying \( J/\psi \rightarrow \gamma \eta' \eta' \) [45]. As \( \eta' \eta' \) resonance, \( f_0(2480) \) is very likely a SU(3) singlet state. Indeed, its mass is compatible with the "mainly-singlet" trajectory. The figure gives the pole positions of the eleven resonances as small inserts.

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1 The BESIII data were fitted by Rodas et al. [39] with four scalar and three tensor resonances only. I have several objections against the fit. i) It uses an amplitude in which the \( J/\psi \) converts into three gluons which hadronize. A final-state meson radiates off the photon. Since the photon is not an isospin eigenstate, this amplitude can produce isovector mesons. This process is highly suppressed and experimentally absent. ii) the \( f_0(1370) - f_0(1500) \) interference region is not well described, neither in the mass distribution nor in the \( S - D \) phase difference. iii) The fit is not constrained by the \( \pi \pi \) S-wave from the CERN-Munich data nor by the data on \( K_{e4} \) decays. A fit with the seven resonances used in Ref. [40] without an isospin breaking amplitude leads to a \( \pi \pi \) S-wave that is extremely incompatible with the known \( \pi \pi \) S-wave (A.V. Sarantsev, private communication, October 2021.)

The total yields of scalar mesons in radiative \( J/\psi \) decays - including decay modes not reported by the BESIII collaboration - was determined from the coupled-channel analysis [38] that included also other data. The yield of mainly-octet and mainly-singlet mesons as a function of their mass is shown in Fig. 5. Mainly-octet mesons should not be produced (or at most weakly) in radiative \( J/\psi \) decays. However, they are produced abundantly, in a limited mass range centered at about 1865 MeV. Mainly-singlet mesons are produced over the full mass range but show a peak structure at the same mass. This enhancement must be due to the scalar glueball mixing into the wave functions of scalar mainly-octet and mainly-singlet mesons. A Breit-Wigner fit to these distributions gives mass and width

\[
M_G = (1865 \pm 25^{+10}_{-30}) \text{ MeV} \quad \Gamma_G = (370 \pm 50^{+30}_{-20}) \text{ MeV},
\]

(8)
and the (observed) yield is determined to

\[ Y_{J/\psi \rightarrow \gamma G} = (5.8 \pm 1.0) \times 10^{-3}. \] (9)

5 Meson-glueball mixing

Earlier attempts to identify the glueball have in common that the full glueball is distributed among the three states \( f_0(1370), f_0(1500) \) and \( f_0(1710) \). Inspecting Fig. 3 this seems not to be obvious: Above 1 GeV, four peaks with three valleys are seen, and there is no reason why one particular region should be more gluish than the other ones. The yield of scalar mesons sees the glueball contribution distributed over several resonances.

We did not impose that the full glueball should be seen in these three states nor that we must see the full glueball at all. We fitted the decay modes of pairs of scalar mesons, one mainly-singlet one mainly-octet, and allowed for a glueball component [35].

\[ f_0^{nH}(xxx) = (n \sin \varphi_n^{s} - s \sin \varphi_n^{s}) \cos \varphi_n^{G} + G \sin \varphi_n^{G} \]
\[ f_0^{nL}(xxx) = (n \sin \varphi_n^{s} + s \sin \varphi_n^{s}) \cos \varphi_n^{G} + G \sin \varphi_n^{G} \]

(10)

\( \varphi_n^{s} \) is the scalar mixing angle, \( \varphi_n^{G} \) and \( \varphi_n^{G} \) are the meson-glueball mixing angles of the high-mass state H and of the low-mass state L in the nth nonet. The fractional glueball content of a meson is given by \( \sin^2 \varphi_n^{G} \) or \( \sin^2 \varphi_n^{G} \).

With this mixing scheme and the SU(3) coupling constant (see Fig. 2), we have fitted the meson decay modes and have thus determined the glueball content of the eight high-mass scalar mesons. Figure 6 shows the glueball fraction in the scalar mesons.

The glueball fractions derived from the decay analysis of pairs of scalar mesons add up to a sum that is compatible with 1. The distribution of the glueball fraction in Fig. 3 is identical to the distribution of yields in Fig. 5. This is a remarkable confirmation that the scalar glueball of lowest mass has been identified and has mass and width as given in Eqn. (8) and a yield as given in Eqn. (9).

6 Comparison with LHCb data

Most striking is the mountain landscape above 1500 MeV in the data on radiative \( J/\psi \) decays. In these decays a \( c\bar{c} \) pair converts into gluons which hadronize (see Fig. 7, left). The huge peak in the \( K \bar{K} \) mass spectrum at 1750 MeV and the smaller one at 2100 MeV decay are produced with two gluons in the initial state. This is to be contrasted with data on \( B^0 \) and \( B^0 \) decays into \( J/\psi + \pi^+ \pi^- \) and \( K \bar{K} \). In this reaction, a primary \( s\bar{s} \) pair – recoiling against the \( J/\psi \) – converts into the final state mesons (see Fig. 7, right). We have included the spherical harmonic moments in the coupled channel analysis that describes the radiative \( J/\psi \) decays [48]. High-mass scalar mesons are only weakly produced in \( B^0 \) decays with \( s\bar{s} \) in the initial state. The strong peak in the \( K \bar{K} \) invariant mass at 1750 MeV in Fig. 5 is nearly absent in \( B^0 \) → \( J/\psi K \bar{K} \)!

Figure 8 shows the ratio of the decay frequencies of \( J/\psi \to \gamma f_0 \) and \( B^0 \to J/\psi f_0 \) with \( f_0 \) decaying into \( \pi^\pi \) or \( \bar{K}K \). The \( f_0(980) \) is likely a mainly-octet state, little produced in radiative \( J/\psi \) decays but strongly with \( s\bar{s} \) in the initial state. On the contrary, \( f_0(1770) \) is seen as strong peak in radiative \( J/\psi \) but very weakly only in \( B^0 \) decays. The uncertainties are large, but the ratio of the decay frequencies is fully compatible with the shape of the glueball derived above.

This is highly remarkable: the two gluons in the initial state must be responsible for the production of resonances that decay strongly into \( K \bar{K} \) but are nearly absent when \( s\bar{s} \) pairs are in the initial state. Also, there is a rich structure in the \( \pi \pi \) mass spectrum produced in radiative \( J/\psi \) decays but little activity only when the initial state is an \( s\bar{s} \) pair. The rich structure stems from gluon-gluon dynamics. Similar conclusions can be drawn [41] from a comparison.
son of the invariant mass distributions from radiative $J/\psi$ decays with the pion and kaon form factors. Their square is proportional to the cross sections. The $f_0(980)$ resonance dominates both form factors but is nearly absent in radiative $J/\psi$ decays: The $f_0(980)$ has large $n\bar{n}$ and $s\bar{s}$ components mixed to a dominant SU(3) octet component. The large intensity above 1500 MeV in radiative $J/\psi$ decays is absent when not two gluons but an $s\bar{s}$ pair is in the initial state: the mountainous structure in radiative $J/\psi$ decays is produced by gluons and not by $q\bar{q}$ pairs: The structure is due to the scalar glueball.

7 A trace of the tensor glueball

The tensor glueball is predicted with an even higher yield \[ \Gamma_{J/\psi \to \gamma f_2} / \Gamma_{\text{tot}} = (11 \pm 2) \times 10^{-3}. \] The yield of $f_2(1270)$ in radiative $J/\psi$ decays is $(1.64 \pm 0.12) \times 10^{-3}$, about six times weaker than the predicted rate for the tensor glueball! Bose symmetry implies that the $\pi^0\pi^0$ or $K_sK_s$ pairs are limited to even angular momenta, practically, only $S$ and $D$-wave contribute. The scalar intensity originates from the electric dipole transition $E0$. Three electromagnetic amplitudes $E1, M2,$ and $E3$ excite tensor mesons. Figure 9 shows these three amplitudes and the relative phases.

Two fits were performed \[50\]. One fit describes the mass distribution only. Apart from the well known $f_2(1270)$ and $f_2(1525)$ the fit needs one high-mass resonance with \[ M = (2210 \pm 60) \text{ MeV}; \quad \Gamma = (360 \pm 120) \text{ MeV}, \]

where the error includes systematic studies with or without additional low-yield resonances. The enhancement was called $X_2(2210)$. In this fit, the phases are not well described. Figure 9 shows a fit in which the 2200 MeV region is described by three tensor resonances with masses and widths of about $(M, \Gamma) = (2010, 200), (2300, 150),$ and $(2340, 320)$ MeV. These resonances had been observed by Etkin et al. \[12\] in the reaction $\pi^- p \to \phi\omega n$. The unusual production characteristics were interpreted in Ref. \[12\] as evidence that these states are produced by $1 \to 3$ glueballs.

The total observed yield of $X_2(2210)$ in $\pi\pi$ and $K\bar{K}$ is $(0.35 \pm 0.15) \times 10^{-3}$, far below the expected glueball yield. We assume the glueball is – like the scalar glueball – distributed over several tensor mesons. Adding up all contributions from tensor states above 1900 MeV seen in radiative $J/\psi$ decays, one obtains

\[ \sum_{M=1.9 \text{ GeV}}^{M=2.5 \text{ GeV}} Y_{J/\psi \to \gamma f_2} = (3.1 \pm 0.6) \times 10^{-3}, \]

which is a large yield even though still below the predicted yield.

8 How to find the pseudoscalar glueball

The BESIII collaboration has studied the reaction $J/\psi \to \pi^+\pi^-\eta' \[51\]$. The left panel of Fig. 10 shows the $\pi^+\pi^-\eta'$ invariant mass distributions with a series of peaks. Assuming that these are all pseudoscalar mesons, two trajectories can be drawn (right panel of Fig. 10). The figure suggests
that the high-mass structures could house two mesons, possibly singlet and octet states in SU(3). If this is true, a cut in the $\pi^+\pi^-$ invariant mass at about 1480 MeV would partly separate the two isobars, $X'(2600)\to f_0(1370)\eta'$ and $X'(2600)\to f_0(1500)\eta'$. We may expect a slight mass shift in the two $\pi^+\pi^-\eta'$ invariant mass distributions. The two mesons $f_0(1370)$ and $\eta'$ are both mainly singlet. The $f_0(1370)\eta'$ isobar as singlet meson in the $X'(2600)$ complex should be slightly higher in mass than the $f_0(1500)\eta'$ mainly octet meson.

The total yields of the high-mass structures – including unseen decay modes – are not known. Nevertheless, their appearance above a comparatively low background is surprising. Personally, I suppose that the pseudoscalar glueball is rather wide, and that the structures are seen so clearly because of a small glueball content. More studies of these data and of different channels are required to substantiate this conjecture.

9 Outlook

The data of the BESIII collaboration presented above are based on $1.3\times10^9$ events taken at the $J/\psi$. Presently available are $10^6$ events. Based on this large statistics, rare radiative decays like $J/\psi \to \gamma\eta\eta'$ [52,53] and $J/\psi \to \gamma\eta'\eta'$ [45] have been analysed. Data on the different charge mode of $J/\psi \to \gamma\pi\pi$ would be extremely important. In an ideal world, these data would be publicly available after publication and would be included in different coupled-channel partial-wave analyses.

Radiative decays of $\psi(2S)$ and of $\Upsilon(1S)$ open a wider range in invariant mass. The authors of Ref. [54] used the data of the CLEO collaboration on radiative $\psi(2S)$ decays into $\pi^+\pi^-$ and $K^+K^-$. The data are shown in Fig. 11. The data are fit with known resonances, no partial-wave analysis was performed. The BaBar collaboration studied radiative $\Upsilon(1S)$ decays into $\pi^+\pi^-$ and $K^+K^-$. The results are shown in Fig. 12. In all four distributions, there is not a single prominent peak in the S-wave contribution which would stick out as glueball candidate. The S-waves rather resembles the distributions observed in radiative $J/\psi$: three major enhancement in the 1500, 1750 and 2200 MeV region separated by dips. (With the larger statistics in $J/\psi$ decays, a fourth enhancement is seen at about 2350 MeV.) In Fig. 11, a peak is found at 1447 MeV and assigned to $f_0(1500)$. At 1500 MeV, there is the dip. The wrong mass is due to the neglect of interference. The phase between $f_0(1500)$ and the “background” (due to the wider $f_0(1370)$) is $180^\circ$ [56]. This phase difference and the significant $f_0(1500)\to \eta\eta'$ branching ratio identify $f_0(1500)$ as mainly SU(3) octet state. The different masses for the high-mass state in the $\pi^+\pi^-$ and $K^+K^-$ invariant mass distributions point again to the neglect of interference between the prominent octet states and the singlet “background”. Inspecting Figs. 11 and 12 shows: there is no striking isolated peak which could be interpreted as “the glueball”. The glueball content must be distributed over a large number of states.

In $\psi(2S)$ radiative decays, the $f_0(1710)\to K\bar{K}$ is observed with a branching fraction of $(6.7\pm0.9)\times10^{-5}$; in $\Upsilon(1S)$ radiative decays, the $f_0(1710)\to K\bar{K}$ is seen with a branching ratio of $(2.02\pm0.51\pm0.35)\times10^{-5}$. The comparison with the yield observed in Ref. [56] allows us to calculate the branching ratio expected for $\psi(2S)$ and $\Upsilon(1S)$ decays when the full scalar glueball is covered, i.e. for $\Upsilon(1S)\to \gamma G_0(1865)$. The values are given in Table 2.

Clearly, a significant increase in statistics is required when these reactions should make in independent impact. The advantage of $\psi(2S)$ and $\Upsilon(1S)$ radiative decays is of course that phase space limitations play no role any more. This is particularly important for the search for the tensor
and pseudoscalar glueball. The scalar glueball seems to be confirmed: there is not much intensity above 2500 MeV.

At the end I would like to give an answer to the question posed in the title: yes, I am convinced, the scalar glueball is identified, and the tensor glueball seems to have left first traces in the data.

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