Observational imprints of our lost twin anti-universe

Samuel Barroso Bellido\textsuperscript{1,a}, Mariusz P. Dąbrowski\textsuperscript{1,2,3,b}

\textsuperscript{1} Institute of Physics, University of Szczecin, Wielkopolska 15, 70-451 Szczecin, Poland
\textsuperscript{2} National Centre for Nuclear Research, Andrzeja Sołtana 7, 05-400 Otwock, Poland
\textsuperscript{3} Copernicus Center for Interdisciplinary Studies, Szczepańska 1/5, 31-011 Kraków, Poland

Received: 30 May 2022 / Accepted: 21 October 2022 / Published online: 3 November 2022
© The Author(s) 2022

Abstract We consider observational consequences of the entanglement between our universe and a hypothetical twin anti-universe in the third quantization scheme of the canonical quantum gravity in order to make such a scenario falsifiable. Based on our recent and unique investigations we select some special form of the interuniversal interaction which allows the entanglement entropy of the pair of universes to diverge at the critical points of their classical evolution where the specific behaviour of the Hubble parameter plays the key role. We find that the modification of the cosmic microwave background (CMB) power spectrum due to the entanglement with our twin anti-universe for such models is enlarged for small modes $k$ and small multipole numbers $l$. Our specific form of the interaction is then constrained by the Planck satellite data giving the interaction coupling constant bound to be $\lambda_o \lesssim O(10^{-56}) s^{-3}$ which is small, but not negligible compared to the inflationary contribution. Some other coupling functions which allow more critical points are also briefly commented on in the context of their observational effect on CMB and other prospective observational data.

1 Introduction

Year by year, the multiverse scenario reaches more and more important status in theoretical physics [1–10]. The question whether the multiverse exists or not is still open and under debate. In Ref. [11], a level III multiverse scenario [12] is put to the test for which the imprints on the Cosmic Microwave Background (CMB) spectrum are found. It seems then appealing to apply its strategy as viable and proceed to find the observable signals from the CMB for other scenarios.

The method relies on the third quantization scheme [13–16] of canonical quantum gravity [17,18] which now is widely considered as a field theory of universes over the multiverse, that, assuming they are Friedmann–Lemaître–Robertson–Walker (FLRW) universes, each universe is described by its wave function $\Psi(a, \{\phi\})$, where $a$ is the scale factor and $\{\phi\}$ the matter content. Such wave function fulfils its own renowned Wheeler–DeWitt (WDW) equation $\mathcal{H}\Psi = 0$. In that kind of the multiverse, the individual universes interact as particles of any other field theory. In particular, they follow the most natural process of quantum field theory which is the pair creation of universes [19,20] “from nothing” as an analogy with particle-antiparticle pair creation [21].

The reason for that comes from a novel founding which arose when the entanglement entropy of such pair of universes was calculated [22–24] as the von Neumann entropy. It was then found that the entanglement entropy was divergent at any critical point along the classical evolution of the universes, or equivalently, when the Hubble parameter $H$ vanished (corresponding to turning points of possible cyclic evolution – cf. Refs. [25,26]). This novel scenario should be subject to the observational verification which motivates the current study.

As we know from quantum mechanics, the Hamiltonian description of a bipartite system with a certain interaction between the parts can be written as

$$\mathcal{H} = \mathcal{H}_1 + \mathcal{H}_2 + \mathcal{H}_{\text{int}},$$

where $\mathcal{H}_i$ is the free Hamiltonian of each subsystem, and $\mathcal{H}_{\text{int}}$ is the contribution to the Hamiltonian by their interaction. Since the entanglement entropy diverges when the Hubble parameter vanishes, the interaction must also diverge. One is then compelled to write the Hamiltonian of interaction as
the function of the Hubble parameter and not solely as the function of the scale factor \( a \), as it was considered in Ref. [11].

It is strictly necessary to include the entanglement in any interacting scenario because one cannot get rid of it without an external measurement [27] or an environment, which is not included into the theory. Even if the environment is added [28], the system may undergo decoherence (see Ref. [29] and the references therein) through some period of time which suppresses the entanglement after a while but not instantaneously, so the entanglement must be taken into account at the earliest times. In the case one does not include the entanglement, one neglects an important contribution to the energy of the system. Furthermore, the bigger is the number of created pairs, the more energy due to the entanglement comes into play, hence the more complicated is to describe it and the more likely is to be mistaken it when one considers a specified shape of the Hamiltonian of interaction \( \mathcal{H}_{\text{int}} \). That is the reason why we consider a simplified scenario, where we can recognize the origin of the interaction unambiguously given by von Neumann entanglement entropy of the pair, whose properties have been lately analysed further than they were before [23,24].

In order to find the Hamiltonian of the third quantized theory, we need to find the action for which each universe fulfils its WDW equation and interacts with the other. In order to do so, we find the individual actions, sum them up, and include the yet unknown interaction term. Our proposal is to include a pretty general form for the Hamiltonian of the interaction based on the nearest neighbour interaction process

\[
\mathcal{H}_{\text{int}}(a, H) = -a \lambda^2(a, H)(\Psi_2 - \Psi_1)^*(\Psi_2 - \Psi_1),
\]

where \( \lambda^2(a, H) \) is a coupling function whose constraints are to be based on two conditions. Following our important recent studies [22–24], we assume that this function must depend on the Hubble parameter (and not just on the scale factor as in Ref. [11]) in such a way that the coupling function diverges when the Hubble parameter vanishes, i.e.

\[
\lim_{H \to 0} \lambda(a, H) \to \infty,
\]

and secondly (cf. Ref. [30]), the probability of the false vacuum to decay which is computed for such kind of interaction

\[
\mathcal{P} \sim \exp \left\{-\frac{a^4}{\lambda^2(a, H)} \right\}^3,
\]

must be suppressed at large values of the scale factor \( a \to \infty \).

The development to find the Wheeler–DeWitt equation and the semiclassical Friedmann equation which corresponds to that model is the purpose of Sect. 2. Using the new dynamics, we find the spectra of the CMB in Sect. 3. A comment of the choice of a certain coupling function \( \lambda^2(\alpha, \phi) \) is given in Sect. 4. The conclusion follows in Sect. 5.

## 2 Third quantized Wheeler–DeWitt equation and the semiclassical Friedmann equation

Considering a pair of flat FLRW universes containing a single homogeneous scalar field \( \phi \), the mode expansion of its perturbations \( v_k \), and the perturbations of the metric, the Lagrangian of each universe of the pair can be expressed, in terms of the conformal time \( d\eta = dt/\alpha(t) \), like [11,31,32]

\[
\mathcal{L} = \frac{1}{2} \left[ -(a')^2 + a^2(\phi')^2 - 2a^4 V(\phi) + \sum_k (v_k v_k^* + \omega_k^2 v_k v_k^*) \right].
\]

where the prime denotes the derivative with respect to \( \eta \), we have assumed \( 3/4\pi G = 1 \), \( G \) is the gravitational constant, \( V(\phi) \) is the potential of the scalar field, and the frequency is given by

\[
\omega_k^2 = k^2 - \frac{z''}{z}, \quad z = \frac{\phi'}{\mathcal{S}}, \quad \mathcal{S} = \frac{a'}{a}.
\]

where \( z \) is the Mukhanov–Sasaki variable, and the perturbations \( v_k \) are the solutions to the Mukhanov–Sasaki equation

\[
v_k'' + \left( k^2 - \frac{z''}{z} \right) v_k = 0.
\]

The conjugated momenta are found from Eq. (5) to be

\[
p_a = -a', \quad p_\phi = a^2 \phi', \quad p_{v_k} = v_k' \cos \omega_k t + v_k \sin \omega_k t,
\]

and a Legendre transformation yields the Hamiltonian

\[
\mathcal{H} = \frac{1}{2} \left[ -p_a^2 + 2a^4 \rho(\phi) + \sum_k (p_{v_k} p_{v_k}^* - \omega_k^2 v_k v_k^*) \right],
\]

where we kept the scalar field as classical through the relation [33]

\[
\rho(\phi) = \frac{1}{2} \frac{\phi''}{a^2} + V(\phi).
\]

In order to find the Wheeler–DeWitt equation \( \mathcal{H}(\Psi) = 0 \) of a single universe, where \( \Psi(a, \{v_k\}) \) is the wave function of each one, we now quantize the momenta, for a certain factor ordering, as

\[
p_a^2 = -\frac{1}{a} \frac{\partial}{\partial a} \left( a \frac{\partial}{\partial a} \right), \quad p_{v_k} = -i \frac{\partial}{\partial v_k},
\]
and replace them into the Hamiltonian (9), so that the kinetic part could be expressed covariantly as the Laplace–Beltrami operator [18]. Thus, one finds the Wheeler–DeWitt equation

\[
\frac{1}{\dot{a} a} \frac{\partial}{\partial a} \left( a \frac{\partial}{\partial a} \right) + 2a^4 \rho(\phi) + \sum_k \left( \frac{\partial^2}{\partial v_k^2} + \omega_k^2 v_k^2 \right) \Psi(a, \{v_k\}) = 0,
\]

(12)

where \(\Psi(a, \{v_k\})\) is the wave function of the universe, which is a function on the scale factor and the modes of the perturbations.

In the third quantization picture, Eq. (12) is the equation of motion for a certain field \(\Psi^*\), meaning that the universe described by \(\Psi\) in Eq. (12) is just an excitation of a field of universes. The action from where we recover it, reads

\[
S_{3Q} = \int da \sum_k d v_k a \left[ -\frac{\partial \Psi^*}{\partial a} \frac{\partial \Psi}{\partial a} + 2a^4 \rho(\phi) \Psi^* \Psi 
+ \sum_k \left( \frac{\partial \Psi^*}{\partial v_k} \frac{\partial \Psi}{\partial v_k} + \omega_k^2 v_k^2 \Psi^* \Psi \right) \right].
\]

Identifying the momentum of the universe as

\[
P_{\Psi} = -\frac{\partial \Psi}{\partial a},
\]

we find the third quantized Hamiltonian

\[
H_{3Q} = a \left[ -\frac{1}{a^2} P_{\Psi} P_{\Psi^*} - 2a^4 \rho(\phi) \Psi^* \Psi 
- \sum_k \left( \frac{\partial \Psi^*}{\partial v_k} \frac{\partial \Psi}{\partial v_k} + \omega_k^2 v_k^2 \Psi^* \Psi \right) \right],
\]

(15)

which accounts for one of the universes of the bipartite system of two interacting universes we want to focus on. The whole system is described by a Hamiltonian like the one in Eq. (1)

\[
H_{\text{pair}} = H_{3Q}^1 + H_{3Q}^2 -a \lambda^2(a, H)(\Psi_2 - \Psi_1)^* (\Psi_2 - \Psi_1),
\]

(16)

where the interaction term has been taken as the one in Eq. (2), controlled by a coupling function \(\lambda(a, H)\) which here is the clue to justify the divergence of the entanglement entropy at the critical points of the classical evolution as it was recently found in Ref. [23], and to maintain the vacuum stability.

To simplify a bit the calculations, we expand the wave functions in its frequency modes like

\[
\Psi_{1,2} = \frac{1}{\sqrt{2}} (\tilde{\Psi}_2 \mp \tilde{\Psi}_1), \quad P_{\Psi_{1,2}} = \frac{1}{\sqrt{2}} (\tilde{P}_{\Psi_2} \mp \tilde{P}_{\Psi_1}),
\]

(17)

since there are only two universes. Substituting (17) into (16), we get the third quantized Hamiltonian of the system as

\[
\tilde{H}_{3Q}^{\text{pair}} = \tilde{H}_{3Q}^1 + \tilde{H}_{3Q}^2 -2a \lambda^2(a, H) \tilde{\Psi}_1^* \tilde{\Psi}_1,
\]

(18)

where

\[
\tilde{H}_{3Q}^l = a \left[ -\frac{1}{a^2} \tilde{P}_{\Psi} \tilde{P}_{\Psi^*} - 2a^4 \rho(\phi) \tilde{\Psi}_l^* \tilde{\Psi}_l 
- \sum_k \left( \frac{\partial \tilde{\Psi}_l^*}{\partial v_k} \frac{\partial \tilde{\Psi}_l}{\partial v_k} + \omega_k^2 v_k^2 \tilde{\Psi}_l^* \tilde{\Psi}_l \right) \right], \quad l = \{1, 2\}.
\]

(19)

The equation of motion for \(\tilde{\Psi}_l\), which is the Wheeler–DeWitt equation for \(\tilde{\Psi}_l\), after a parameterization of the scale factor as \(a = e^{\alpha}\), is

\[
-e^{-2\alpha} \frac{\partial^2}{\partial \alpha^2} + 2e^{2\alpha} \rho(\phi) \frac{\partial}{\partial \alpha} + \omega_k^2 v_k^2 + 2\lambda^2(a, H) \delta(l - 1) \times \tilde{\Psi}_l(a, \{v_k\}) = 0,
\]

(20)

where \(\delta(l - 1)\) is a Dirac delta function. Here, \(\tilde{\Psi}_l\) is the wave function with mode \(l\) of the bipartite system. From (20), we can see that the mode \(l = 2\) does not feel the interaction at all, hence the dynamics of the classical universe related to it is equivalent to a non-interacting one. The question one may arises is in what combination of them is our universe, and this is impossible to answer. We will, therefore, focus our attention over the mode \(l = 1\) for which the dynamics differs, and from now on, no index will be used.

Using the semiclassical ansatz

\[
\tilde{\Psi}(\alpha, \{v_k\}) = e^{i S_0(\alpha)} \prod_k \psi_k(\alpha, \{v_k\}),
\]

(21)

we find the approximate relation\(^1\)

\[
-2e^{2\alpha} \left( \frac{\partial S_0}{\partial \alpha} \right)^2 = -2e^{4\alpha} \rho(\phi) - 2\lambda^2(a, H).
\]

(22)

Recognizing (22) as the Hamilton–Jacobi equation for the slowly moving background, we use it together with

\[
\frac{\partial S_0}{\partial a} = p_a = -\frac{da}{dt},
\]

(23)

in order to obtain the modified Friedmann equation

\[
H^2 = 2\rho(\phi) + 2\frac{\lambda^2(a, H)}{a^4}.
\]

(24)

\(^1\) For the explicit calculation, see the Appendix A of the analogous case in Ref. [11].
where we have recovered the scale factor $a$ and the cosmological time $t$. For the fluctuations $v_k$, it is found that they obey the Schrödinger-like equation [11], as expected.

### 3 Observational consequences: CMB angular power spectrum

The new dynamics ruled by the modified Friedmann equation (24) which takes into account a small interaction between our universe and its twin by means of the coupling function $\lambda(a, H)$, may have an impact on the anisotropies of the CMB. The intention of this section is to find out how it makes a difference from the best fit found by Planck satellite [34], where $\Lambda$CDM model is assumed. We will consider a pretty general coupling function fulfilling the condition (3), which reads

$$\lambda^2(a, H) = \frac{\lambda_o}{2} \frac{a^q}{H^n} = \frac{\lambda_o}{2} \frac{a^{q+n}}{\dot{a}^n},$$

(25)

where $n > 0$, and $q$ are some real constants, and $\lambda_o$ is a constant small enough to see the interaction between the pair as a perturbation for large values of the scale factor, whose units are $[T^{-2-n}]$. However, a priori it is not valid to perform any approximation around $\lambda_o$ since such a term contributes mainly to the dynamics close to the initial singularity.

In order to also adopt other extreme points of the universe discussed in Ref. [23] such as exotic singularities [35,36] which may have regular energy density but irregular the pressure and its derivatives, one may also postulate a more general ansatz

$$\lambda^2(a, H, \tilde{q}) = \frac{\lambda_o}{2} \frac{a^q}{H^n \tilde{q}^m} = (-1)^m \frac{\lambda_o}{2} \frac{a^{q+n-m}}{\tilde{a}^n - 2m \tilde{q}^m},$$

(26)

where $\tilde{q} \equiv -(\dot{a}a)/\dot{a}^2$ is the deceleration parameter, and the constant $m > 0$. For example, in a sudden future singularity (SFS) [37] the scale factor $a = a_s = \text{const.}$, and its first derivative (proportional to the energy density) $\dot{a} = \dot{a}_s = \text{const.}$, while the second derivative (proportional to the pressure) $\ddot{a} \to -\infty$. This procedure may further be extended into the singularities in higher derivatives of pressure (for example generalized sudden future singularities (GSFS) [38]) by inserting higher-order kinematic quantities in the denominator such as jerk, snap, and pop [39–41] which would involve higher order derivatives of the scale factor into (26).

However, just focusing our attention on the coupling function (25), in order to recover the spectrum in a customary way, we should use a scalar field whose equation of state is $p = \omega \rho$, where $\omega$ is expected to be very close to $\omega_\Lambda = -1$, since the expansion must be almost de Sitter [31]. Let us then write

$$\omega = -1 + \frac{2\alpha}{3}, \quad \alpha > 0.$$ 

(27)

For a universe with a scalar field fulfilling the above condition, its energy density [33] scales like $\rho(\phi) = (1/2)H^2 \rho_{\phi}(\phi/a)^{2\alpha}$, and thus Eq. (24) is written as

$$H^2 = H_{DS}^2 \left( \frac{a_d}{a} \right)^{2\alpha} + \frac{\lambda_o}{2H_{DS}^2 a^{1-q}},$$

(28)

where $a_d$ is a constant to be determined.

The condition for the probability (4) is fulfilled when $4 - q - n > 0$. Hence, at the very beginning, the universe is dominated by a term which is essentially different from the one in Ref. [11], since it depends on the Hubble parameter in the denominator.

The analysis of an arbitrary $q$ and $n$ is left for future research. Here we will only consider the values $q = 1$ and $n = 1$, which is the extreme case since the Hubble parameter goes like $1/H$ in the interaction term, and it is positive during inflation. Therefore, any other value of $n$ will decrease the interaction and the effects would be less noticeable. This case is one of the family of cases for which the analysis is simplified fulfilling the condition $2 - q - n = 0$, so that the Friedmann equation can be written as

$$\dot{a}^2 + 2H_{DS}^2 \dot{a} a^2 a^2 + \lambda_o = 0,$$

(29)

The asymptotic behaviour one can obtain from Eq. (29) of the scale factor close to the singularity is

$$a(t) \approx \sqrt{\lambda_o} t,$$

(30)

which is an extremely slowly expanding (Milne) universe. Using Eq. (30) as feedback for Eq. (28), we infer that a more precise expansion is still slightly accelerated.

Even if there is a short period before inflation, it should have been in an almost constant thermal equilibrium. Indeed, one finds that $(aH) \sim \sqrt{\lambda_o}$ there, which is constant throughout the initial stage. Furthermore, the dynamics of the very beginning of the universe is not perfectly obtained from canonical quantum gravity due to the problem of the factor ordering [18]. For these reasons, we will not repeat the method used in Ref. [11], and we will neglect the effects at the very beginning of the universe assuming that $\lambda_o$ can be taken as a perturbation at any point of the evolution.

The Eq. (28) can be expanded at first order around $\lambda_o$ as

$$H \approx H_{DS} \left( \frac{a_d}{a} \right)^\alpha + \frac{\lambda_o}{2H_{DS}^2 a_{\dot{a}}^2} \dot{a}^2 \approx H_{DS} \left( \frac{a_d}{a} \right)^\alpha + \frac{\lambda_o}{2H_{DS}^2 a^2},$$

(31)

Trying the ansatz

$$a(t) = a_o(t) \left[ 1 + \xi(t, \lambda_o) \right],$$

(32)
where \(a_\alpha(t)\) is the solution for Eq. (31) setting \(\lambda_\alpha\) to zero, and \(\xi(t, \lambda_\alpha)\) is a function such that \(\xi \ll 1\), after cancelling second order derivatives of \(\xi\), we find

\[
a(t) \approx a_d (\alpha H_{\text{dS}} t)^{1/\alpha} \left[1 - \frac{\lambda_\alpha}{6a_d^3 H_{\text{dS}}^3 t^{1+3/\alpha}(\alpha t)^{-1+3/\alpha}}\right].
\]

Using the slow-roll parameter [31]

\[
\epsilon = -\frac{\dot{H}}{H^2} \approx \alpha - \lambda_\alpha \frac{(3 - \alpha)(3 - 2\alpha)(1 - \alpha)}{6H^2 a^2(\eta)},
\]

the Mukhanov–Sasaki equation (7), at first order in \(\lambda_\alpha\), is just

\[
v''_k(\eta) + \left( k^2 - \frac{2}{\eta^2} - \frac{3\alpha}{\eta^2} + \lambda_\alpha \frac{\alpha d'_{\text{dS}}(1 - \alpha)}{6H^2_{\text{dS}}} \right) v_k(\eta) = 0,
\]

where we have included the relations

\[
\eta \approx -\frac{\alpha t}{(1 - \alpha) a(t)} \approx -\frac{1}{(1 - \alpha) a H},
\]

\[
\frac{\dot{\zeta}}{\dot{z}} = (a H)^2 (2 - \epsilon).
\]

As expected, the Bunch–Davies vacuum [42]

\[
v_k(\eta) = \frac{e^{-ik\eta}}{\sqrt{2k}},
\]

cannot be recovered at \(\eta \to -\infty\), since the third term into the parenthesis in Eq. (35), given by the interaction between universes, dominates, and hence it cannot be used as a good boundary condition. However, since \((a H) \sim \sqrt{\lambda_\alpha}\) during the initial phase, the corresponding conformal time for the late time evolution is

\[
\eta_{\text{knee}} \approx -\frac{1}{(1 - \alpha) \sqrt{\lambda_\alpha}} \approx -\frac{1}{\sqrt{\lambda_\alpha}}.
\]

We will use a solution to Eq. (35) such that it behaves like (37) at \(\eta_{\text{knee}}\). This approximation is thought to be very good since \(|\eta_{\text{knee}}|\) must be very large.

Trying a solution of the form

\[
v_k(\eta) = v_k^{(0)}(\eta) f_k(\eta, \lambda_\alpha),
\]

where \(v_k^{(0)}(\eta)\) is the solution to Eq. (35) when the third term in the parenthesis is vanishing, and \(f_k(\eta, \lambda_\alpha)\) is a slow varying function whose value is expected to be small enough in its whole domain. Introducing (39) into (35), and cancelling the terms containing \(f''\) and \((f')^2\), we find the solution going like

\[
v_k(\eta) = v_k^{(0)}(\eta) \exp\left[ -\lambda_\alpha \frac{\alpha d'_{\text{dS}}(1 - \alpha)}{12H^2_{\text{dS}}} \int_\eta^{\tilde{\eta}} \frac{v_k^{(0)}(\tilde{\eta})}{v_k^{(0)}(\eta)} d\tilde{\eta} + \Delta_k \right],
\]

where \(\Delta_k\) is a constant of integration. This constant \(\Delta_k\) is very close to be purely imaginary, and for the future calculation of the power spectrum the phase to which it contributes is not relevant. The solution \(v_k^{(0)}(\eta)\) is well-known as [31]

\[
v_k^{(0)}(\eta) = \frac{\sqrt{\pi |\eta|}}{2} H^{(1)}_\mu(k|\eta|), \quad \mu \approx \frac{3}{2} + \frac{5}{9} \alpha,
\]

where \(H^{(1)}_\mu(z)\) is the Hankel function of the first kind (for special functions, see [43]), since it fulfills the condition (37) at \(\eta \to -\infty\).

The anisotropies of the CMB are expressed via the power spectrum of the curvature perturbations [42]

\[
P_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \frac{|v_k|^2}{z^2}.
\]

One expects to adjust the spectrum to a power law like

\[
P_{\mathcal{R}}(k) = A_s \left( \frac{k}{k_s} \right)^{n_s - 1},
\]

when \(a H \sim k\), or equivalently, \(k|\eta| \sim 1\), where \(A_s\), and \(n_s\) depend on the given value \(k_s\) set by the experiment. From the Planck analysis [44], where they considered \(k_s = 0.05\) Mpc\(^{-1}\), it was obtained \(A_s = (2.105 \pm 0.30) \times 10^{-9}\), and \(n_s = (0.9665 \pm 0.0038)\). That way, the power spectrum is found to have the known form [11,31,32]

\[
P_{\mathcal{R}}(k \lesssim a H) \approx \left[ 1 - \alpha - \frac{\Gamma(\nu)}{\Gamma(3/2)(2\pi)} \right]^2 \left( \frac{k|\eta|}{2} \right)^{2\alpha - 3} |e^{f_k(\eta, \lambda_\alpha)}|^2,
\]

which is written, in our case, at first order in \(\epsilon \sim \alpha\), and at the moment when the modes cross the horizon \(k_\alpha \approx a_H H_{\text{dS}}\), to be like

\[
P_{\mathcal{R}}(k \lesssim a H) \approx \left\{ 1 - \alpha + \Psi \left( \frac{3}{2} \right) \right\}^2 |e^{f_k(\eta, \lambda_\alpha)}|^2,
\]

where \(\Psi(z) := \Gamma'(z)/\Gamma(z)\) is the digamma function. The absolute value of the exponential can be considered very
close to unity such that we can derive, comparing with the fit (43), the values\(^2\)
\[
\alpha = 0.0168 \pm 0.0019, \quad a_d \equiv a_s = (9.052 \pm 0.067) \cdot 10^{-56}, \\
H_{\text{ds}} = (2.896 \pm 0.021) \cdot 10^{-4} = (5.372 \pm 0.039) \cdot 10^{35} \text{ s}^{-1},
\]
which we can compare with the actual values
\[
a_0 \equiv 1, \quad H_0 \approx 1.2 \cdot 10^{-61}. \tag{47}
\]

The power spectrum (42) is used then to find the angular power spectrum, given in terms of the coefficients
\[
C_l = 2T_0^2 l(l + 1) \int_0^\infty \frac{dk}{k} P_R(k) \Delta_l^2(k), \tag{48}
\]
where \(T_0 = (2.72548 \pm 0.00057) \text{ K}\) is the temperature of the CMB [45], and CAMB has been used to obtain the transfer functions \(\Delta_l(k)\). For us, those coefficients depend on \(\lambda_o\), and so the power spectrum (42) and the angular power spectrum are shown in Fig. 1 for different values of \(\lambda_o\). As expected, the interaction with our hypothetical partner universe affects only the smaller values of \(k\) and the multipoles. The lighter the interaction, the more suppressed the effects. Since the spectra differ too fast from the standard one because of the strong impact of the other universe or, a priori, the approximation we performed to get Eq. (40), we can easily constraint the value of the coupling constant \(\lambda_o\), for a kind of coupling function as (25), to be
\[
\lambda_o \lesssim O(10^{-56}) \text{ s}^{-3}. \tag{49}
\]

A fast inspection of the Friedmann equation (28), shows that during inflation \(H^2 \sim H_{\text{ds}}^2 \sim O(10^{55}) \text{ s}^{-2}\), and the term due to the interaction is of the order of \(O(10^{60}) \text{ s}^{-2}\), where we have used the values of \(H_{\text{ds}}\) and \(a_d\) from Eq. (46) in order to get such estimation. Thus, we claim that the value of the coupling constant can be as a small perturbation. However, even if it looks a really small perturbation, we can see from Fig. 1 that taking \(\lambda_o = 10^{-55} \text{ s}^{-3}\), which is just an order of magnitude larger, changes the lowest multipoles of the spectrum significantly, so the effect is not negligible at all. This also confirms that all approximations we performed are then justified.

Coming back to a more general coupling function like in (26), the spectrum for an analogous family to the family of solutions we analyzed, fulfilling \(2 - q + m - n = 0\), is expected to be very similar to the one in Fig. 1. Now, the vacuum stability condition (4) is \(4 - q + m - n > 0\), assuming \(\dot{a} > 0\) and taking \(m\) even, which automatically contains the family condition \(2 - q + m - n = 0\). Within this family, we can take the choice \(m = 2, n = 4\) and \(q = 0\), for which the Friedmann equation (24) is
\[
H^2 = H_{\text{ds}}^2 \left[ -\frac{\lambda_o}{a^3} + \frac{\lambda_o}{H_{\text{ds}}^2 a^2 \dot{a}} \right]. \tag{50}
\]

The scale factor goes like \(t^{-3/2}\) close to the initial singularity, and taking \(\lambda_o\) as a perturbation for late times, the scale factor is then evolving like \(t^{1/\alpha}\). The transition between both states is expected to be smooth, like if a single scalar field fills the universe, whose barotropic parameter monotonically varies from \(\omega = -5/9\) to \(\omega \gtrsim -1\). It implies that the power spectrum for the smallest modes crossing the horizon \(k \sim a_H\), during the early states of the universe when the interaction dominates, are the ones which are affected and enlarged. It will not change significantly the results in Fig. 1 since the slopes are quite steep already for small \(k\), and thus the angular power spectrum is not significantly different.

4 A short comment about an interuniversal contribution to the dark energy of the universe at its early stages

Here we shortly present a way for the multiverse interaction to contribute to the dark energy of the universe via its entanglement at the very early times. The Friedmann equation (24) rules the dynamics of the universe for a given interaction \(\lambda(a, H)\) as we have reasoned previously. Now, let us consider the special form of the coupling function
\[
\lambda^2(a, H) = \frac{\lambda_o}{2} a^4 f(H) H^2, \tag{51}
\]
where \(f(H)\) is an explicit and well-behaved function of \(H\), and \(\lambda_o\) is a positive constant. In order to be sure that the interaction term diverges when \(H\) vanishes, we require \(f(H)H^2\) to diverge. The Friedmann equation (24), without any field, yields the trivial result
\[
H = f^{-1}(\lambda_o^{-1}) = \text{constant}. \tag{52}
\]

The dynamics is then equivalent to the one of the de Sitter space
\[
a(t) = a_o \exp(H t), \tag{53}
\]
so the interaction is able to reproduce an exponential expansion just after the creation of the universe. In a certain manner, the interaction is equivalent to a cosmological constant. In case there is already a standard cosmological constant, this contribution just shifts the degree of the de Sitter expansion.

Even if the Friedmann equation (24) has been obtained without assuming that such coupling function was small,
one needs it to be so small since the probability condition (4), which is of the form $P \sim \exp\left(-\lambda_o H^2 f(H)^{-3}\right)$, prevents $\lambda_o$ to be very large in order to keep a small probability at any time. Besides, the probability condition tells us that not considering any kind of matter produces a constant probability along the entire evolution of the universe. There is no other chance but to have some kind of matter contributing to the dynamics (so that the probability decreases) and playing the role of the inflaton after the end of inflation, recovering the anisotropies of the CMB. That is why this scenario should be seen as a specific example of interacting universes in which we are not able to reproduce the angular spectrum of the CMB any other way, but which is worth mentioning for possible future reference.

5 Conclusions

The multiverse scenario derived from the third quantization in quantum cosmology has been considered. In this scenario, the universes behave like particles of the standard quantum field theory. Following our previous paper investigations [23], we have considered the interaction of a pair of universes (our universe and our twin anti-universe), whose entanglement entropy was found to diverge at the critical points of the classical evolution. It has motivated us to consider the coupling function like (25) or a more general one like (26), which both depend on the scale factor, on an arbitrary coupling constant $\lambda_o$ which is to be determined, and in particular, on the Hubble parameter. We have assumed that the functions must fulfill the condition (3), so the coupling function diverges when the Hubble parameter vanishes, and the condition (4) to keep the vacuum stability in the multiverse. For the function (25), we have found the modified Friedmann equation (28) while taking into account the interaction of the pair of universes.

For a certain family of solutions, we have found the modifications of the imprints on the CMB power spectrum due to our anti-universe interaction. The plots have been given in Fig. 1 from which one can see that the interaction with our hypothetical partner enlarges the spectra for small values of $k$ modes and the small monopole numbers $l$. The constraint on the interaction coupling constant which is in agreement with Planck observations [34] has been found to be such as $\lambda_o \lesssim \mathcal{O}(10^{-50})$ s$^{-3}$ which is small, but not negligible in comparison to the inflationary contribution and can be considered as a perturbation.

It is the future problem to fix the parameters of the coupling function and its specific form. However, for the set of solutions fulfilling the condition $2-q-n=0$, where $n > 0$, the results presented in the paper are approximately equivalent. Once those parameters are known, then one can extrapolate our conclusions and say if the CMB angular power spectrum is sensitive to some small interaction term. The need for more observational experiments digging into the very early stages of our universe, like the search for primordial gravitational waves or the cosmic neutrino background, would be of interest in order to falsify the existence of a twin anti-universe or some other entangled universes.

A short comment has also been given for two empty universes whose interaction was defined by a very general coupling function (51). However, with a very special ansatz (52), the dynamics of such universes is similar to the de Sitter universes and it adds to the effect of the cosmological constant at early times. So, in order to recover the spectra, one needs some kind of matter to be introduced into such a simple scenario.
Acknowledgements SBB would like to thank Michele Liguori and Eleonora Di Valentino for providing useful hints to calculate the CMB angular power spectrum and Mar Bastero Gil and Fabian Wagner for some useful discussions and coffees. The work of SBB was supported by the Polish National Research and Development Center (NCBR) project UNIWERSYTET 2.0. STREFA KARIERY, POWR.03.05.00-00-Z064/17-00.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: All data used in this study belong to “ESA” and can be found in “Planck Legacy Archive” (Link:http://pla.esac.esa.int/pla/#home).].

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

Funded by SCOAP3. SCOAP3 supports the goals of the International Year of Basic Sciences for Sustainable Development.

References

1. K. Langhoffer, C. Muridia, Y. Nomura, Multiverse in an inverted island. Phys. Rev. D 104(8), 086007 (2021). https://doi.org/10.1103/physrevd.104.086007
2. S.E. Aguilar-Gutierrez, A. Chatwin-Davies, T. Hertog, N. Pinzani-Fokeeva, B. Robinson, Islands in multiverse models. J. High Energy Phys. 2021(11), 212 (2021). https://doi.org/10.1007/jhep11(2021)212
3. A. Alonso-Serrano, G. Janess, Conceptual challenges on the road to the multiverse. Universe 5(10), 212 (2019). https://doi.org/10.3390/universe5100212
4. M.P. Dąbrowski, Anthropic selection of physical constants, quantum entanglement, and the multiverse falsifiability. Universe 5(7), 172 (2019). https://doi.org/10.3390/universe5070172
5. A. Linde, V. Vanchurin, How many universes are in the multiverse? Phys. Rev. D 81(8), 083525 (2010). https://doi.org/10.1103/physrevd.81.083525
6. J.B. Hartle, The Quantum Universe (World Scientific, Singapore, 2021). https://doi.org/10.1142/11716
7. J. Garriga, A. Vilenkin, J. Zhang, Black holes and the multiverse. J. Cosmol. Astropart. Phys. 2016(02), 064 (2016). https://doi.org/10.1088/1475-7516/2016/02/064
8. L. Mersini-Houghton, Predictions of the quantum landscape multiverse. Class. Quantum Gravity 34(4), 047001 (2017). https://doi.org/10.1088/1361-6382/34/4/047001
9. A. Vilenkin, A quantum measure of the multiverse. J. Cosmol. Astropart. Phys. 2014(05), 005 (2014). https://doi.org/10.1088/1475-7516/2014/05/005
10. A. Balcerzak, M. Lisaj, Decaying universes and the emergence of Bell-type interuniversal entanglement in varying fundamental constants cosmological model . Eur. Phys. J. C 82, 732 (2022). https://doi.org/10.1140/epjc/s10052-022-10704-3
11. M. Boughmadi-López, M. Krämer, J. Morais, S. Robles-Pérez, The interacting multiverse and its effect on the cosmic microwave back-ground. J. Cosmol. Astropart. Phys. 2019(02), 057 (2019). https://doi.org/10.1088/1475-7516/2019/02/057
12. M. Tegmark, Parallel universes, in Science and Ultimate Reality: Quantum Theory, Cosmology, and Complexity, ed. by J.D. Barrow, P.C.W. Davies, C.L. Harper (Cambridge University Press, Cambridge, 2004), p.459
13. N. Caderni, M. Martellini, Third quantization formalism for Hamiltonian cosmologies. Int. J. Theor. Phys. 23(3), 233–249 (1984)
14. M. McGuigan, Third quantization and the Wheeler–DeWitt equation. Phys. Rev. D 38, 3031–3051 (1988). https://doi.org/10.1103/PhysRevD.38.3031
15. S.B. Giddings, A. Strominger, Baby universes, third quantization and the cosmological constant. Nucl. Phys. B 321, 481–508 (1989). https://doi.org/10.1016/0550-3213(89)90353-2
16. S.J. Robles-Pérez, Quantum cosmology with third quantisation. Universe 7, 404 (2021)
17. B.S. DeWitt, Quantum theory of gravity. I. The canonical theory. Phys. Rev. 160, 1113–1148 (1967). https://doi.org/10.1103/PhysRev.160.1113
18. C. Kiefer, Quantum Gravity, 2nd edn. (Oxford University Press, New York, 2007)
19. S. Robles-Pérez, P.F. González-Díaz, Quantum state of the multiverse. Phys. Rev. D (2010). https://doi.org/10.1103/physrevd.81.083529
20. S.J. Robles-Pérez, Quantum creation of a universe–antiverse pair. Acta Phys. Pol. Suppl. 13, 325 (2020). https://doi.org/10.5506/APhysPolBSupp.13.325, arXiv:2002.09863 [gr-qc]
21. J. Schwinger, On gauge invariance and vacuum polarization. Phys. Rev. 82, 664–679 (1951). https://doi.org/10.1103/PhysRev.82.664
22. S. Robles-Pérez, A. Balcerzak, M.P. Dąbrowski, M. Krämer, Interuniversal entanglement in a cyclic multiverse. Phys. Rev. D (2017). https://doi.org/10.1103/physrevd.95.083505
23. A. Balcerzak, S. Barroso-Bellido, M.P. Dąbrowski, S. Robles-Pérez, Entanglement entropy at critical points of classical evolution in oscillatory and exotic singularity multiverse models. Phys. Rev. D 103, 043507 (2021). https://doi.org/10.1103/PhysRevD.103.043507
24. S.B. Bellido, Effects of a quantum or classical scalar field on the entanglement entropy of a pair of universes. Phys. Rev. D 104, 106009 (2021). https://doi.org/10.1103/PhysRevD.104.106009
25. C. Kiefer, H.D. Zeh, Arrow of time in a recollapsing quantum universe. Phys. Rev. D 51(8), 4145 (1995). https://doi.org/10.1103/physrevd.51.4145
26. M.P. Dąbrowski, A.L. Larsen, Quantum tunneling effect in oscillating Friedmann cosmology. Phys. Rev. D 52(6), 3424–3431 (1995). https://doi.org/10.1103/physrevd.52.3424
27. R. Horodecki, P. Horodecki, M. Horodecki, K. Horodecki, Quantum entanglement. Rev. Mod. Phys. 81(2), 865–942 (2009)
28. S.B. Bellido, F. Wagner, A new guest in the third quantized multiverse. Phys. Rev. D 105, 106001 (2022). https://doi.org/10.1103/PhysRevD.105.106001
29. C. Kiefer, E. Joos, Decoherence: concepts and examples, in Quantum Future From Volta and Como to the Present and Beyond, ed. by P. Blanchard, A. Jadczyk (Springer, Berlin, 1999), pp.105–128
30. S. Robles-Pérez, A. Alonso-Serrano, C. Bastos, O. Bertolami, Vacuum decay in an interacting multiverse. Phys. Lett. B 759, 328–335 (2016). https://doi.org/10.1016/j.physletb.2016.05.091
31. B.A. Bassett, S. Tsujikawa, D. Wands, Inflation dynamics and reheating. Rev. Mod. Phys. 78, 537–589 (2006). https://doi.org/10.1103/RevModPhys.78.537
32. J. Morais, M. Boughmadi-López, M. Krämer, S. Robles-Pérez, Pre-inflation from the multiverse: can it solve the quadrupole problem in the cosmic microwave background? Eur. Phys. J. C 78(3), 5698 (2018). https://doi.org/10.1140/epjc/s10052-018-5698-z
33. S. Weinberg, Cosmology (Oxford University Press, Oxford, 2008)
34. N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A.J. Banday, R.B. Barreiro, N. Bartolo et al., Planck 2018 results. Astron. Astrophys. 641, 5 (2020). https://doi.org/10.1051/0004-6361/201936386
35. M.P. Dąbrowski, Are singularities the limits of cosmology?, in Mathematical Structures of the Universe, ed. by M. Eckstein, M. Heller, S.J. Szybka (Copernicus Center Press, Kraków, 2014), p. 99
36. M.P. Dąbrowski, K. Marosek, Non-exotic conformal structure of weak exotic singularities. Gen. Relativ. Gravit. 50(12), 160 (2018). https://doi.org/10.1007/s10714-018-2482-1
37. J.D. Barrow, Sudden future singularities. Class. Quantum Gravity 21(11), 79 (2004). https://doi.org/10.1088/0264-9381/21/11/003
38. J.D. Barrow, More general sudden singularities. Class. Quantum Gravity 21(23), 5619–5622 (2004). https://doi.org/10.1088/0264-9381/21/23/020
39. R.R. Caldwell, M. Kamionkowski, Expansion, geometry, and gravity. J. Cosmol. Astropart. Phys. 2004(09), 009 (2004). https://doi.org/10.1088/1475-7516/2004/09/009
40. M. Dunajski, G. Gibbons, Cosmic jerk, snap and beyond. Class. Quantum Gravity 25(23), 235012 (2008). https://doi.org/10.1088/0264-9381/25/23/235012
41. M.P. Dąbrowski, Statefinders, higher-order energy conditions, and sudden future singularities. Phys. Lett. B 625(3), 184–188 (2005). https://doi.org/10.1016/j.physletb.2005.08.080
42. V. Mukhanov, Physical Foundations of Cosmology (Cambridge University Press, Cambridge, 2005)
43. F.W. Olver, D.W. Lozier, R.F. Boisvert, C.W. Clark, NIST Handbook of Mathematical Functions, 1st edn. (Cambridge University Press, Cambridge, 2010)
44. N. Aghanim, Y. Akrami, M. Ashdown, J. Aumont, C. Baccigalupi, M. Ballardini, A.J. Banday, R.B. Barreiro, N. Bartolo et al., Planck 2018 results. Astron. Astrophys. 641, 6 (2020). https://doi.org/10.1051/0004-6361/201833910
45. D.J. Fixsen, The temperature of the cosmic microwave background. Astrophys. J. 707(2), 916 (2009). https://doi.org/10.1088/0004-637x/707/2/916