On Dp-Dp+4 systems, QCD dual and phenomenology

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Abstract

D4-D8 and D3-D7 systems are studied and a possible holographic dual of large N QCD (SU(N) gauge fields and fundamental quarks) is sought. A candidate system is found, for which however no explicit solution is available. Susy is broken by having a $D_7 - \bar{D}_7$ condensing to a D5. The mechanism for supersymmetry breaking is then used to try to construct a Standard Model embedding. One can either obtain too few low energy fields or too many. The construction requires TeV scale string theory.
1 Introduction

One of the reasons why people are interested in gravity-field theory dualities is the hope that we can describe QCD via gravity. The original paper [1] treated the duality between $\mathcal{N}=4$ SU(N) SYM at large N and $AdS_5 \times S^5$ string theory, followed shortly thereafter by the paper of Witten [2] describing how to get a description of the pure glue theory (SU(N) Yang-Mills) via gravity. Other developments afterwards involve breaking supersymmetry e.g. [3] and/or conformal invariance e.g. [4, 5, 6, 7], and also introducing bifundamental fields via D7 branes or D5 branes e.g. [8, 9, 10], but the question of having a theory without conformal invariance and supersymmetry and with dynamical massless quarks has eluded attempts to solve it. For a review of the developments until 1999 see [11]. In this paper I will address this question, and find that while the decoupled D3-D7 theory describes a supersymmetric version of QCD, there is a modification of this system which describes the (large N) QCD, but unfortunately it is not possible to write down explicitly (not even implicitly).

The observation which allows us to do this is that one can write down the solution for a $D8 - \bar{D}8$ system, and that if this system condenses to a D6, one can write that solution down as well. I use the embedding of massive 10d IIA theory in M theory defined in [12] and extended in [13] to give a UV definition of the massive IIA solutions. It is then found that one has to T dualize and go to a D3-D7 system, smeared over an overall transverse coordinate. The holographic dual to QCD is then a system of $D3 - (D7 - \bar{D}7(D5)) - D7'$ branes, which however doesn’t have a known supergravity solution. The $D3 - D7 - \bar{D}7$ decoupled solution can be written explicitly (up to a one dimensional integral). The solution for the $D3 - D7 - \bar{D}7 - D5$ can be found up to an integro-differential equation for a function $H_{4,pq}(x)$, but for the $D3 - D7 - \bar{D}7 - D5 - D7'$ even the ansatz cannot be written.

An obvious question then is can one lift this D brane construction for the holographic dual of QCD to a Standard Model embedding? I study this question in the context of D-brane-world GUT models and find that one needs to have TeV-scale string theory. In the context of an SU(5) susy GUT we can obtain massles states corresponding to the 5 fermions, Higgses and gauge fields, but no $\bar{10}$ fermions (which contain the fields in the $(3, 2)$ of $SU(3) \times SU(2)$). By adding orientifolds, one is able to obtain the required fields, but much more on top of that, and the corresponding masses seem to be wrong anyway. In any case, that system is very hard to analyze.

The paper is organized as follows. In section 2 I study D4-D8 solutions, in 2.1. previous solutions and in 2.2 solutions which will be used thereafter. In section 3 I try to define the D4-D8 system and its M theory embedding using the construction in [12, 13]. Section 4 is devoted to motivating the supergravity-field theory correspondence for the D4-D8 system and identifying the gauged supergravity describing it. In section 5.1 I describe the proposed set-up for the holographic dual of QCD, analyze the brane-antibrane condensation process and then in section 5.2 study the susy breaking using the $D8 - \bar{D}8$ system. In section 6 I try to embed the QCD description into a Standard Model description using D-brane worldvolumes. After a study of supersymmetric lagrangians for D3-D7-O(7) systems in 6.1, I try to build a model. In 6.2 systems without orientifolds are studied and section 6.3 introduces orientifolds. I finish in section 7 with discussion and conclusions. The appendix reviews GUTs for our
purposes.

2 D4-D8 systems and QCD

From the perspective of a braneworld scenario or holographic duality, if one wants to realize a QCD-like system one has to introduce fundamental quarks, and the most obvious way is to look at Dp-Dp+4 systems. Since it is not clear how a D5-D9 system would be useful for either holography or braneworld phenomenology, we are left with D3-D7 systems and D4-D8. Let’s start out with D4-D8 systems for their simplicity and then notice that one needs to go back to D3-D7, but keep the advantages of the D4-D8.

The D4 brane theory dimensionally reduced to 4d is $\mathcal{N}=4$ SYM, which has fermions in the adjoint representation of $SU(N_c)$. That theory is conformal, however in 5d the D4 theory has a dimensionful coupling constant and is getting strongly coupled in the UV, therefore the theory is not well defined. We will get back later to the question of defining the theory.

By introducing $N_f$ D8s we have quarks and scalars in the bifundamental representation $(N_c, \bar{N}_f) + (\bar{N}_c, N_f)$, forming an $\mathcal{N}=2$ hypermultiplet. In the holographic context, since $g_{D_5}^2 = g^2_{D_9} - 3g_s l_s$, in the decoupling limit $l_s \to 0$, keeping $g_{D_4}^2$ fixed means $g_{D_8}^2 \to 0$, so we are left with $N_f$ fundamental hypermultiplets.

The D8 has the string metric and dilaton ($d\sigma^2_{8,1}$ is the 8 + 1-dimensional Minkowski metric)

\[
\begin{align*}
    ds^2 &= H^{-1/2}(d\sigma^2_{8,1}) + H^{1/2} dx^2 \\
    e^\phi &= H^{-5/4} \\
    H &= c + |\tilde{M}||x| = c + \frac{m}{l_s}|x| \tag{2}
\end{align*}
\]

where $c$ is an arbitrary constant of integration or (by the usual rescaling for p-branes)

\[
\begin{align*}
    ds^2 &= \tilde{H}^{-1/2}(d\sigma^2_{8,1}) + \tilde{H}^{1/2} d\tilde{x}^2 = \left[\frac{3}{2}(1 + \frac{g_s m}{l_s}|z|)\right]^{-1/3}(d\sigma^2_{8,1} + dz^2) \\
    e^\phi &= e^{\phi_0} \tilde{H}^{-5/4} = g_s \tilde{H}^{-5/4} \\
    \tilde{H} &= H/c = 1 + g_s |\tilde{M}||\tilde{x}| = 1 + \frac{g_s m}{l_s}|\tilde{x}| \tag{3}
\end{align*}
\]

where $g_s$ is defined as the coupling constant at the position of the D8 brane. Here $\tilde{M}(x) = \pm H'$, so the mass is piecewise constant, and jumps at the positions of the D8 branes. The $\pm$ in the mass corresponds to D8 branes vs. anti-D8 branes. One can obviously redefine $x$ such that the harmonic function appears just as a conformal factor for flat space.
2.1 D4-D8 in the literature

The solution for a D4-D8 system can be written as

\[ ds^2 = H_8^{-1/2} [H_4^{-1/2} dx^4_{1,1} + H_4^{1/2} dy^2] + H_8^{1/2} H_4^{1/2} dz^2 \]

\[ e^{\phi - \phi_0} = H_8^{-5/4} H_4^{-1/4} \]

\[ F_{(4)} = Q_4 vol(\Omega_{transv}) \]

\[ H_8(z) = 1 + g_s |\vec{M}||z| \]

\[ M = \pm H_8' \tag{4} \]

and here also \( \pm \) corresponds to the D8 vs. anti-D8. In the literature, people have considered a “D4 inside D8” solution\(^{[14]}\) and a “partially localized D4-D8 system\(^{[15]}\). The “D4 inside D8” is just a D4 delocalized over the transverse coordinates to the D8 and is given by (in Einstein frame)

\[ ds_{E}^2 = W^{2/25} [H_4^{-3/8} (-dt^2 + dx_4^2) + H_4^{5/8} (dr^2 + r^2 d\Omega_3^2)] + H_4^{5/8} dz^2 \]

\[ \hat{A}_{(3)} = \frac{\bar{Q}}{4m} W^{32/25} \Omega_3 \]

\[ e^\phi = W^{-4/5} H^{-1/4} \]

\[ H_4 = 1 + \frac{\bar{Q}}{r^2} \quad W = 1 + k|z| \tag{5} \]

Here \( \bar{Q} \) is the \( Q_4 \) density on the unit of \( z \) direction. It was obtained by lifting the D4 in 9d solution via KK reduction on the D8 domain wall (that paper introduced the notion of dimensional reduction on a domain wall; by contrast one can always dimensionally reduce on a coordinate parallel to the domain wall), with ansatz

\[ ds_{10,E}^2 = e^{-\frac{5}{16}} \sqrt{\frac{2}{3}} W^{\frac{2}{25}} ds_{9,E}^2 + e^{\frac{35}{16}} \sqrt{\frac{2}{3}} e^{\frac{1}{2}} d\Omega_3 \]

\[ \hat{A}_{(1)} = 0, \quad \hat{A}_{(2)} = \frac{1}{2m} W^{\frac{16}{25}} F_{(2)}, \quad \hat{A}_{(3)} = \frac{1}{4m} W^{\frac{16}{25}} F_{(3)} \]

\[ e^\phi = W^{-\frac{2}{5}} e^{-\frac{7}{8}} \sqrt{\frac{2}{3}} e^{\frac{1}{2}} \tag{6} \]

The partially localized solution of Youm\(^{[15]}\) reads

\[ ds^2 = \Omega^2(z) (H_4^{-1/2} (-dt^2 + dx_4^2) + H_4^{1/2} (dy^2 + dz^2)) \]

\[ \Omega(z) = \left( \frac{3 g_s m}{2 l_s} z \right)^{-1/6} \]

\[ e^\phi = g_s \left( \frac{3 g_s m}{2 l_s} z \right)^{-5/6} \]

\[ H_4 = \frac{Q_4}{l_s^{10/3} (y^2 + z^2)^{5/3}} \tag{7} \]

and corresponds to the case \( H_8 = \frac{2 m}{l_s} |\vec{x}| = \left( \frac{3 g_s m}{2 l_s} z \right)^{3/2} \) (that is, for \( g_s \to \infty \), see\(^{[13]}\)).

Through a change of coordinates \( z = rsin\alpha, y = rcos\alpha \) in was shown in\(^{[10]}\) that one finds...
a metric
\[ ds^2 = \left( \frac{3}{2} C \sin \alpha \right)^{-1/3} (Q_4^{-1/2} r^{4/3} dr^2 + Q_4^{1/2} \frac{dr^2}{r^2} + Q_4^{1/2} d\Omega_3^2) \] (8)

which is a form where now \( \alpha \) (compact coordinate, in \( S_1/Z_2 \), since \( 0 < \alpha < \pi/2 \)) can be interpreted as being the transverse coordinate to the D8 due to the lucky coincidence that there is no \( r \) dependence in the transverse metric. Note that just for the D8 solution it would not be true, but since we have a D4 harmonic function with the correct \( r \) dependence, the total \( r \) dependence cancels. Because of that cancellation, (8) can be interpreted as the metric in the presence of an O(8) (and at infinite coupling; presumably there is a finite coupling O(8) one could add to the D4-D8 system, and it should limit to this). One needs to add O(8) planes at the fixed planes to cancel the D8 brane charge (16 D8 branes to cancel -16 units of charge from two O8’s). The authors of [16] found that the corresponding dual D4-D8 theory is a fixed point with global \( SU(2) \times E_{N_f+1} \) global symmetry, derived from a \( Sp(Q_4) \) gauge theory theory at infinite bare coupling. So, although the D4 theory is not conformal, by adding the D8 and O(8), the theory flows to a nontrivial conformal fixed point.

2.2 D4-D8 solutions

However, finding a localized solution is not so difficult after all. The point is that there are so-called partially localized intersections, where brane 1 with harmonic function \( H_1 \) lives on \( t, \vec{w}, \vec{x} \), and brane 2 with harmonic function \( H_2 \) lives on \( t, \vec{w}, \vec{y} \), with overall transverse space \( \vec{z} \). They are written in terms of harmonic functions \( H_1 \) and \( H_2 \) in the usual way, except that now \( H_1 \) and \( H_2 \) satisfy the equations (e.g. [15], [17])

\[
\partial_z^2 H_1(z, y) + H_2(z) \partial_y^2 H_1(z, y) = 0, \partial_y^2 H_2 = 0 \quad \text{or} \quad \partial_z^2 H_2(z, x) + H_1(z) \partial_x^2 H_2(z, x) = 0, \partial_z^2 H_1 = 0
\] (9)

In other words, we delocalize one brane (say brane 2) over the worldvolume coordinates of the other brane (1), and then \( H_1 \) is harmonic (obeys the laplace equation) in the background of brane 2. In the case of a Dp-Dp+4 system, this condition is automatically satisfied, and then \( H_4 \) obeys the equation

\[
\partial_z^2 H_4(z, \vec{y}) + H(z) \partial_y^2 H_4(z, \vec{y}) = Q \delta(z) \delta^4(\vec{y})
\] (10)

where \( Q = a g_s N l_3^3 \), with \( a \) some numerical constant, and then

\[
H_4(y, z) = 1 + \int \frac{d^4 p}{(2\pi)^4} e^{i \vec{p} \vec{y}} H_p(z) = 1 + \frac{1}{4\pi^2 y} \int_0^\infty dp p^2 J_1(py) H_p(z)
\] (11)

If we put \( H(z) = c + m|z| \), then the resulting equation

\[
H_p''(z) - (c + m|z|) p^2 H_p(z) = Q \delta(z)
\] (12)

is solved by

\[
H_p(z) = c_p z^{1/2} K_{1/3}(\frac{2}{3} \bar{x}^{3/2}), \quad \bar{x} = (\frac{p}{m})^{2/3}(c + m|z|)
\] (13)
The constant \( c_p \) is fixed by matching with the normalization of the \( \delta \) function source, and one gets
\[
c_p = \frac{Q \sqrt{c}}{2p^{1/3}m^{2/3}[K_{1/3}(\frac{2p}{3m}c^{3/2}) - \frac{2c^{3/2}K_{4/3}(\frac{2p}{3m}c^{3/2})}{m}]}
\]
and so
\[
H_4(y, z) = 1 + \frac{Q \sqrt{c}}{4\pi^2m^{2/3}} \frac{\beta^{1/3}}{y} \int dpp^2 \frac{J_1(py)K_{1/3}(\frac{2p}{3m}c^{3/2})}{K_{1/3}(\frac{2p}{3m}c^{3/2}) - \frac{p^3/2}{m}K_{4/3}(\frac{2p}{3m}c^{3/2})}
\]
with
\[
\beta = \frac{(c + m|x|)^{2/3}}{m}
\]
Let us now define the decoupling limit. As I mentioned, we want to keep the D4 SYM coupling fixed, \( g_{D4}^2 = g_s l_s \), so that
\[
H_8 = 1 + \frac{g_s N_f}{l_s} |z| = 1 + g_{D4}^2 N_f |Z| = \text{fixed}
\]
(I have rescaled as usual \( z = l_s^2 Z \) and so \( m = M/l_s^2 \) and the number of D8’s is \( N_f \). Then by rescaling also \( y = l_s^2 U \) and the integration variable \( p = P/l_s^2 \), we get in the limit \( l_s \to 0 \)
\[
H_{D4}(U, Z) \approx \frac{1}{l_s^4} \frac{ag_{D4}^2 N \beta^{1/3}}{4\pi^2 \alpha^{2/3} U} \int_0^\infty dPP^2 \frac{J_1(PU)K_{1/3}(\frac{2\beta P}{3})}{K_{1/3}(\frac{2\beta P}{3}) - \frac{P^3/2}{3\alpha}K_{4/3}(\frac{2\beta P}{3})} = \frac{h_{D4}(U, Z)}{l_s^4}
\]
and the decoupled D4-D8 system is
\[
ds^2 = \alpha' H_8^{-1/2}(Z)[h_{D4}^{-1/2}(U, Z)dx_\mu^2 + h_{D4}^{1/2}(U, Z)(H_8(Z)dZ^2 + dU^2 + U^2 d\Omega_3^2)]
\]
A similar analysis for the case of the D2-D6 system was done in \[18\] and for a D1-NS5 system it was done in \[13\]. Notice that the near core D8 can be always trusted, independent of the number of D8 branes (the curvature in string units is always small, curvature scalar \( R \sim M^2 H^{-5/2} \), so one needs actually \( g_s N_8 \ll 1 \), which can be satisfied for \( N_f \sim 1 \)). The number of D4’s however, has to be very large, as usual.

Let us now look at various ways of breaking supersymmetry. The most used is the method of Witten \[19\], of putting the system at finite temperature. This corresponds to compactifying on a supersymmetry breaking circle. The fermions acquire masses of the order of the compactification scale at tree level, and the scalars at quantum level, by fermion loops.

For the holographic dual, Witten’s solution had the AdS black hole as a starting point. Then scale the mass \( M \) to infinity, together with \( r \) to infinity and \( t \) to 0 in a particular way. The resulting solution has only one parameter (the radius of AdS).

But equivalently \[20\] one can just take the near horizon limit of the nonextremal solution. Although this solution has apparently two parameters (the AdS radius and the nonextremality parameter \( M \), or temperature \( T \), calculations in this background will not depend on \( T \) alone. Indeed there exists a rescaling (with no parameters going to infinity!) which takes the D3 nonextremal near-horizon solution to Witten’s metric, namely
$U = \rho(TR), t = t'/(TR), \bar{y} = \bar{x}/T \ (R=\text{AdS radius}).$ In particular, for the Wilson loop calculation of $q\bar{q}$ potential in [21], this means that $E(L,R,T)=E'(LT,R) \ (TR)$ or equivalently $EL=f(LT,R)$. The bottom line is that when one computes either $q\bar{q}$ potential or glueball masses, one can use either Witten’s type of construction, or a near horizon nonextremal solution, in which case by scaling of the coordinates one gets the desired nonsusy theory. Both ways were used in glueball calculations [23, 24, 25, 26, 27], but when one starts with a nonconformal theory before the compactification, there is no analog of the AdS black hole solution (since there is no AdS background), so one has to use the scaling in the nonextremal solution. A similar case, of glueballs in the N=1 nonconformal cascade theory of Klebanov and Strassler [5], was treated in [28].

So let us try to put the D4-D8 system at finite T by making it nonextremal. It is easy to do so for the “D4 inside D8” solution. Just lift the solution for D4 in 9d at finite temperature on the D8 with the ansatz (6) and get

$$ds^2 = H_8^{-1/2}[H_4^{-1/2}(-dt^2 + dx_4^2) + H_4^{1/2}(\frac{dy^2}{f(y)} + y^2 d\Omega_3^2 + H_8 dz^2)]$$ (20)

where

$$f(y) = 1 - \frac{\mu}{y^2}$$ (21)

But this is again the delocalization over z of the full nonextremal D4-D8 solution, which however now is hard to find.

If one could find the finite temperature localized D4-D8 solution, it would still not be so useful, since the corresponding field theory will be the same as for pure D4 branes at finite temperature: pure 4d Yang-Mills theory. Maybe though by comparing the two descriptions one would be able to find out the spurious effects of the construction (by seeing if there are quantities which do change).

We want however to keep the fundamental fermions after susy breaking. When compactifying the D4-D8 field theory, we would like therefore to put antiperiodic boundary conditions for the (4,4) fermions, so that they become massive, and periodic boundary conditions for the (4,8) fermions, so that they remain massless. One would have to check whether such boundary conditions are consistent with the interactions, and whether unitarity is preserved in such a system.

But let’s see whether one can find a holographic dual to such a system. By the general argument in [29], if one dimensionally reduces on a euclidian black hole spacetime, all the fermions will be antiperiodic around the KK coordinate, so they will get a mass. The argument is that there is only one spin structure available to the spinors around the KK coordinate. In general, the possible phases around it are dictated by the invariances of the lagrangian. At large distances from the black hole, the space is topologically flat (times the KK circle), so that all phases are allowed. Near the horizon however, the spacetime is flat space times the transverse sphere and admits a unique spin structure, which becomes the antiperiodic one at infinity. The same argument can be extended to nonextremal branes, for instance nonextremal D4 branes, as in [19]. The spacetime is flat at infinity and has a transverse sphere near the horizon. Only the antiperiodic spin structure is valid over the whole spacetime.
So in the case of the euclidian black hole or nonextremal D4, fermions defined over the whole space become massive. They couple to fermionic operators on the D4. Therefore the D4 fermions are antiperiodic and become massive.

But we also see a way out. If there are fermions which are defined only over a part of the holographically dual spacetime, they can be mapped to fermionic operators remaining in the spectrum, whereas the ones defined over the whole spacetime are mapped to operators dissappearing from the spectrum.

In the context of compactification, there doesn’t seem to be a solution of this type, but there is a domain wall type solution (“alternative to compactification”) which has the necessary properties.

The nice thing about D8 branes is that one can also write down a $D_8 - \bar{D}_8$ solution (unlike for other branes), in the particular case where in between the two branes we have flat space. This can be achieved by writing

$$H_8 = \begin{cases} 
1 - 2g_s |\tilde{M}|z, & z < 0, \quad M = -g_s \tilde{M} \\
1, & 0 < z < z_0 \\
1 + 2g_s |\tilde{M}|(z - z_0), & z_0 < z \quad M = -g_s \tilde{M}
\end{cases}$$

(22)

If one would have $M = +g_s \tilde{M}$ for $z > z_0$ it would be a D8-D8, but now it is a $D_8 - \bar{D}_8$, and if the D8 can be trusted -curvature small in string units means

$$g_s N_8 \ll 1$$

(23)

and $g_s$ small implies no quantum corrections- the $D_8 - \bar{D}_8$ can be trusted as well. All we did was change the sign of the mass on one side, which changes the sign in the Killing spinor equation, therefore the Killing spinor on one side is not valid on the other. So there is no globally defined fermion in this background.

One can still write down a localized D4 inside the D8, even in the presence of the $\bar{D}_8$. Piecewise, the $H_4$ equation is still (12), with c and m derived from (22), and then we just have to match the solutions over the branes.

In the case $z_0 \to 0$, we have a $D_8 - \bar{D}_8$ on top of each other. From the string theory point of view, that gives a gravitational solution, but can also (depending on the K theory class of the system) give a lower dimensional brane, e.g. a D6. From the gravity point of view, the holographic dual is the same as for the D4-D8, just that now $M = -g_s \tilde{M}$ on both sides of the brane.

I will postpone the discussion of a specific set-up for later, but let us note that whereas there is no globally defined fermion, there are fermions defined on the D8 (and at x=0 we still have supersymmetry)-or rather on the z=0 slice of the holographic dual (19), corresponding to the D8.

So in the case of the $D4 - D8 - \bar{D}8$ solution, fermions defined over the whole spacetime couple to field theory operators which will dissappear from the spectrum. These will be operators with no D8 charges. On the other hand, fermions defined only on the D8 will couple to fermionic operators in the field theory which remain in the spectrum. These are operators charged under the D8 global symmetry.
Note that the fact that there are fermions which are defined only on a subset of the holographic dual is not a new concept. The $\mathcal{N}=2$ superconformal theory of D3-D7-O(7) described in [30] has bulk modes coupling to operators with no $N_f$ charges and vector modes defined on an $AdS_5 \times S_3$ orientifold fixed plane inside $AdS_5 \times S_5$ coupling, to operators with $N_f$ (vector) charges. It is in fact very similar to this system: without the O(7) and after a T duality it becomes the D4-D8, so the coupling of the operators with $N_f$ charges to vector modes defined on D8 (or rather the z=0 slice of (19)) is an established fact. The only new observation is that therefore the uncharged fermionic operators disappear from the spectrum (get high anomalous dimensions), whereas the charged fermionic operators don’t.

3 Defining the D4-D8 system

As I mentioned, the D4 field theory (and the D8 field theory actually, but that is now “frozen”) is not well defined in the UV, so one must allow for a UV completion. In the UV, the effective D4 coupling is large, and the theory must be described by M theory, therefore the UV completion of the (nonrenormalizable) D4 theory is given by the M5 brane field theory. But what about the D4-D8 case?

Let us start with seeing how to embed the D8 in M theory. There have been attempts to embed the D8 directly in M theory, as an “M9” domain wall. One of these is solution in [31], where the 11th direction is an isometry direction for the metric. But the D8 is a solution to Romans’ massive supergravity [32], and a fully covariant M9 would be also a solution to an 11d supergravity with a mass parameter (cosmological constant).

It is unknown how to lift the massive IIA theory and its D8 background solution directly into M theory. The point is that M theory doesn’t seem to admit a cosmological constant, and if one wanted to lift massive 10d supergravity directly, one would get a cosmological constant in 11d. The only possible way around this is if the 10d mass arises via a Scherk-Schwarz generalized reduction on a circle. But for that, one would need a global symmetry in 11d, and the action doesn’t have such a symmetry. The equations of motion have however a scaling symmetry, which was exploited in [33] to reduce to a massive 10d sugra. However, that is a different massive supergravity in 10d (one that admits, in particular, the de Sitter space as a background), and moreover, it amounts in 11d to a compactification on the euclidian radial direction. That massive supergravity can also be obtained as a usual reduction of a modified M theory, as in [34].

Instead, the most conservative embedding in M theory was done by Hull [12], who was able to embed the massive supergravity and the D8 background in M theory by introducing two extra T dualities, one of which was a “massive T duality” as defined in [35]. M theory on a $T^2$ of zero area is type IIB and IIB can be compactified on $S^1$ via Scherk-Schwarz. After a “massive T duality,” we get massive IIA.

The endpoint is that massive IIA supergravity is equivalent to M theory on the space $B(A,R)$, in the limit $A \rightarrow 0, R \rightarrow 0$, with metric

$$ds_B^2 = R_3^2(dx_3)^2 + \frac{A}{Im(\tau)}|dx_1 + \tau(x_3)x_2|^2 = R_3^2(dx_3)^2 + R_2^2(dx_2)^2 + R_1(dx_1 + mx_3dx_2)^2 \quad (24)$$
and all the radii going to zero, and the x’s have periodicity 1, $x_i \sim x_i + 1$. In the limit, we should keep the massive IIA quantities fixed, so

$$g_s = \frac{l_s}{\Im(\tau_0) R_3} = \frac{R_1 l_s}{R_2 R_3} = \text{fixed}, \quad l_s = \frac{l_p^{3/2}}{R_1^{1/2}} = \text{fixed (and } m \text{ fixed)} \quad (25)$$

So how does the Hull duality help us in defining the D8 and the D4-D8 field theories? The correct description of the D8 brane field theory is not clear, but the D8 goes over to a gravitational background in the (Hull) dual M theory, so the field theory on that soliton in M theory will be the correct description.

In the D4-D8 case, the D8 field theory decouples, but one is left with (4,8) fields, which can’t be lifted to usual M theory, so one needs to look for the Hull dual. The field theory description is given by the lift of the D4-D8 system to M theory. One is T dualizing twice to get to M theory, so it matters where are located the dualized coordinates. There are 3 choices: T dualize along two transverse coordinates, two parallel coordinates, or along one each. The latter situation is the most useful, since then after two T dualities, one is still describing a D4 brane, albeit with two small radii. The D8 background has now become a 6-brane smeared over two transverse directions. Then the lift to M theory gives as usual a M5 brane, in the background dual to the D8 (i.e. a 7+1-dimensional worldvolume, described in (33)). The decoupling of the D8 field theory corresponds to decoupling of the degrees of freedom localized at the M theory 7+1d background, but one still has degrees of freedom coming from M2’s stretched between the M5 and the “M7” (corresponding to (4,8) strings). Unfortunately, it is unclear how to describe these degrees of freedom, but at least it is possible in principle. In the next subsection I will analyze in more detail the embedding in M theory via Hull duality.

### 3.1 Embedding in M theory

Let us now derive the embedding of supergravity solutions of massive IIA into 11d supergravity solutions (if the solutions are BPS, the embeddings are still valid, even if the space is singular, and one can’t use quantum perturbation theory).

Dimensionally reducing M theory to massless IIA on $r_1$ one has

$$ds_{11}^2 = e^{-2\phi/3}(ds_8^2 + r_2^2 A^2 dz_2^2 + r_3^2 B^2 dz_3^2) + e^{4\phi/3} r_1^2 (dz_1 + A_2 dz_2 + A_\mu dz^\mu)^2$$

$$C_{(3)} = A_{(3)} + dz^1 \wedge B_{(2)} \quad (26)$$

Now one has to perform a T duality on $z_2$ to get to IIB and then a massive T duality on $z_3$ to get to massive IIA.

The full set of T duality rules giving the (hatted) massless IIA fields in terms of the IIB
ones are
\[
\begin{align*}
\hat{g}_{00} &= \frac{1}{g_{00}}, \quad \hat{g}_{0i} = \frac{B_{0i}}{g_{00}}, \quad \hat{g}_{ij} = g_{ij} - \frac{g_{0i}g_{0j} - B_{0i}B_{0j}}{g_{00}}, \\
\hat{B}_{0i} &= \frac{g_{0i}}{g_{00}}, \quad \hat{B}_{ij} = B_{ij} + \frac{g_{0i}B_{0j} - B_{0i}g_{0j}}{g_{00}}, \\
\hat{\phi} &= \phi - \frac{1}{2} \log(g_{00}), \\
\hat{A}_{ijk} &= \frac{8}{3} D_{0ijk}^+ + B_{0[i}B_{jk]} \frac{(B_{0i})^2}{g_{00}} - \frac{B_{0[i}B_{0jk]}g_{k0}}{g_{00}} - B_{0[i}^2 B_{0jk]}g_{k0}/g_{00}, \\
\hat{A}_{0ij} &= \frac{2}{3} B_{ij}^2 + 2 \frac{B_{0[i}g_{j0]}^2}{g_{00}}, \\
\hat{A}_i &= -B_{0i}^2 + a B_{0i}, \quad \hat{A}_0 = a
\end{align*}
\]

The inverse T duality rules, this time with the added complication of them being massive, are (this time the hatted quantities are IIB and unhatted massive IIA)
\[
\begin{align*}
\hat{g}_{00} &= \frac{1}{g_{00}}, \quad \hat{g}_{0i} = \frac{B_{0i}}{g_{00}}, \quad \hat{g}_{ij} = g_{ij} - \frac{g_{0i}g_{0j} - B_{0i}B_{0j}}{g_{00}}, \\
\hat{B}_{0i} &= \frac{g_{0i}}{g_{00}}, \quad \hat{B}_{ij} = B_{ij} + \frac{g_{0i}B_{0j} - B_{0i}g_{0j}}{g_{00}}, \\
\hat{\phi} &= \phi - \frac{1}{2} \log(g_{00}), \\
\hat{D}_{0ijk}^+ &= \frac{3}{8} [A_{ijk} - A_{ij}B_{jk}] + \frac{g_{0[i}B_{jk]}A_0}{g_{00}} - \frac{3 g_{0[i}A_{jk]}}{2}, \\
\hat{a} &= A_0 + m x^0, \\
\hat{B}_{ij}^2 &= \frac{3}{2} A_{ij0} - 2 A_{ij}B_{0j} + 2 g_{0[i}B_{0j]A_0}{g_{00}} + m x^0 (B_{ij} + 2 g_{0[i}B_{0j]}g_{00}), \\
\hat{B}_{0i}^2 &= -A_i + \frac{A_0 g_{0i}}{g_{00}}
\end{align*}
\]

Applying the above T duality rules going from IIA to IIB on $z_2$ and then to massive IIA on $z_3$ one gets for the 11d metric (keeping only the fields relevant for our discussion)
\[
\begin{align*}
ds_{11}^2 &= e^{-2\phi/3} (ds_8^2 + r_3^2 B^2 d^2 z_3^2) \\
&\quad + e^{4\phi/3} r_1^2 [(d^2 z_1 + ad z_2 + B_{\mu^2}^2 d^2 z_2^2 + e^{-2\phi + 2\phi_0} d^2 z_2^2] \\
&\quad + e^{4\phi/3} r_1^2 [(d^2 z_1 + (m + A_3) d^2 z_2 + A_{234} d^2 z_2^2 + e^{-2\phi} A_{4}^2 r_2^2 d^2 z_2^2] \\
&\quad + e^{4\phi/3} r_1^2 [(d^2 z_1 + (m + A_3) d^2 z_2 + A_{234} d^2 z_2^2 + e^{-2\phi} A_{4}^2 r_2^2 d^2 z_2^2]
\end{align*}
\]

and the corresponding massive IIA metric is then
\[
ds_{10mA}^2 = ds_8^2 + A^{-2} r_2^2 d z_2^2 + B^2 r_3^2 d z_3^2
\]
while the dilaton is ($\hat{\phi}$ is the massive IIA dilaton, $\hat{\phi}$ is the IIB dilaton and $\phi$ the massless IIA dilaton)

$$e^{\hat{\phi}} = \frac{e^{\hat{\phi}}}{B r_3} = \frac{e^{\phi}}{A B r_2 r_3}$$  \hspace{1cm} (31)

When one applies this prescription to the D8 solution of massive type IIA

$$ds^2_{10m,A} = H^{-1/2}(d\sigma^2_{8,1}) + H^{1/2}dx^2$$

$$e^\phi = H^{-5}$$

$$H = c + |\tilde{M}||x| = c + \frac{m}{\ell_s}|x|$$  \hspace{1cm} (32)

one indeed finds the 11d gravitational metric

$$ds^2_{11} = H^{1/2}(H^{-1/2}d\bar{\sigma}^2_{6,1} + H^{1/2}dx^2) + ds^2_B = d\bar{\sigma}^2_{6,1} + H dx^2 + ds^2_B$$  \hspace{1cm} (33)

with

$$ds^2_B = H(r_3^2 dx_3^2 + r_2^2 dx_2^2) + \frac{r_1^2}{H}(dx_1 + mx_3 dx_2)^2$$  \hspace{1cm} (34)

Hull [12] has also found this solution as the correct 11d gravitational background corresponding to the D8 background. We should note here that the massive IIA sugra does not admit flat space as a solution, the background with maximal supersymmetry is the D8.

Now when one lifts a solution of massive IIA to M theory, it matters where one chooses to make the two T dualities, i.e. where one puts $z_2$ and $z_3$. The best choice is of course to arrange $z_2$ and $z_3$ such as to get the same type of solution after the two T dualities.

In the particular case of the D4-D8 solution, the best choice is to have one direction parallel to the D4, one perpendicular. Then after two T dualities, one still has the D4 solution, and it will lift to an M5.

Let us however first treat the case where both T dualities are parallel to the D4. We will reach a D2 which lifts to an M2 in the gravitational background. Indeed, one gets

$$ds^2_{10m,A} = H^{-1/2}(d\sigma^2_{8,1} + \frac{dz_2^2}{r_2^2} + \frac{dz_3^2}{r_3^2}) + H^{1/2}d\bar{\sigma}^2_{6,1} + H^{1/2}dx^2$$

$$e^{\hat{\phi}} = H^{-5/4} H^{1/4}$$  \hspace{1cm} (35)

which implies

$$ds^2_{11} = H^{-2/3}d\bar{\sigma}^2_{2,1} + H^{1/3}[d\bar{r}^2_4$$

$$+ H_5(dx^2 + r_3^2 dz_3^2 + r_2^2 dz_2^2) + H_8^{-1}r_1^2(dx_1 + m z_3 dx_2)^2]$$  \hspace{1cm} (36)

Restricting the D4-D8 to the “D4 inside D8” solution corresponds as before just to dropping the x dependence of $H_4$.

When one T duality is parallel and one perpendicular, the same solution as above, but with $z_3$ a transverse coordinate, lifts to

$$ds^2_{11} = H^{1/3}(d\bar{\sigma}^2_{3,1} + H_8 r_3^2 dz_3^2 + H_8^{-1}r_1^2(dx_1 + m z_3 dx_2)^2)$$

$$+ H^{2/3}(d\bar{r}^2_3 + H_5(dx^2 + r_2^2 dz_2^2))$$  \hspace{1cm} (37)
which corresponds to an M5 in the gravitational background \( \text{(33)} \). Note that in both cases there is also a nontrivial \( F(4) \) field.

In \( [13] \), a procedure was developed for getting a Matrix model \( [36, 37, 38] \) corresponding to the massive IIA supergravity, and was applied to the D8 background. I will apply it now to the D4-D8 system.

Since massive 10d IIA string theory is equivalent to M theory on the singular background \( \text{(33)} \), one defines Matrix theory in that background and compactifies it. After T dualities in all the \( r_i \), one gets a Matrix model of D3 branes. As an intermediate step necessary to decouple gravity from the D3 brane theory, following Sen \( [39] \) and Seiberg \( [40] \), an \( \tilde{M} \) theory was introduced, such that

\[
\frac{R_s}{\tilde{l}_p} = \frac{R}{l_p}, \quad \frac{R_i}{\tilde{l}_p} = \frac{R_i}{l_p}
\]

are held fixed in the \( \tilde{l}_p \to 0 \) limit, and the \( l_p \to 0 \) limit is imposed afterwards.

The metric \( \text{(34)} \) for B(A,R) is invariant under the isometries (we have put \( R_i = 1 \) for simplicity)

\[
T_1 : \quad x_1 \rightarrow x_1 + a_1, x_2 \rightarrow x_2, x_3 \rightarrow x_3 \\
T_2 : \quad x_2 \rightarrow x_2 + a_2, x_1 \rightarrow x_1, x_3 \rightarrow x_3 \\
T_3 : \quad x_3 \rightarrow x_3 + a_3, x_1 \rightarrow x_1 - m x_2 a_3, x_2 \rightarrow x_2
\]

with Killing vectors \( V_1 = \partial_1, V_2 = \partial_2 \) and \( V_3 = \partial_3 - m x_2 \partial_1 \). One also notes that \([T_2, T_3] \neq 0\). Since \( T_2 \) and \( T_3 \) don’t commute, it matters in which order one makes the T dualities. We choose to do \( T_1 \), then \( T_2 \), then \( T_3 \).

After \( T_1 \) one has:

\[
ds^2 &= (dx_3^2 + dx_2^2 + dx_1^2) \\
B_{12} &= m x_3 \rightarrow H_{123} = m \\
e^\phi &= e^{\phi_0}
\]

(40)

After \( T_2 \) one has

\[
ds^2 &= (dx_3^2 + dx_2^2) + (dx_2 - m x_3 dx_1)^2 \\
e^\phi &= e^{\phi_0}
\]

(41)

\( T_3 \) is generated by the vector \( V_3 = \partial_3 + m x_1 \partial_2 \). Making this \( = \partial'_3 \) so that one can apply the T duality rules means the coordinate transformation

\[
x'_3 = x_3, x'_2 = x_2 + m x_1 x_3
\]

(42)

The metric in the new coordinates is (after dropping primes on coordinates)

\[
ds^2 &= (dx_3^2 + dx_1^2) + (dx_2 + m x_1 dx_3)^2 \\
e^\phi &= e^{\phi_0}
\]

(43)
After the third $T$-duality, we have

\[
    ds^2 = dx_1^2 + \frac{(dx_2^2 + dx_3^2)}{1 + m^2 x_1^2}
\]

\[
    B_{23} dx^2 \wedge dx^3 = \frac{m x_1}{1 + m x_1^2} dx_2 \wedge dx_3
\]

\[
    e^\phi = \frac{e^\phi_0}{(1 + m x_1^2)^{1/2}} \quad (44)
\]

So let us apply this procedure for the D4-D8 solution. One goes to an $\hat{M}$ theory to decouple string theory, compactifies on a lightcone coordinate, and then $T$ dualizes on all 3 $r_i$'s.

We will drop the bars from all quantities (in the end, nothing will depend on the $\hat{M}$ theory anyway). Let’s start with the background corresponding to an M2. The IIA metric after dimensional reduction on the lightlike coordinate will be (we choose that coordinate to be perpendicular to the M2, thus getting a D2 brane)

\[
    ds^2 = H_4^{-1/2} d\tilde{\sigma}_{2,1}^2 + H_4^{1/2} [d\tilde{r}_3^2] H_8 (dx^2 + r_3^2 dz_3^2 + r_2^2 d\tilde{z}_2^2) + H_8^{-1} r_1^2 (dz_1 + m z_3 d\tilde{z}_2)^2]
\]

\[
    e^\phi = g_s H_4^{1/2} \quad (45)
\]

After the $T$ duality on $T_1$ it will become a D3 brane ending on a NS5 brane in the $z_1$ direction. The metric is

\[
    ds^2 = H_4^{-1/2} d\tilde{\sigma}_{2,1}^2 + H_4^{-1/2} H_8 \frac{dz_1^2}{r_1^2} + H_4^{1/2} d\tilde{r}_3^2 + H_4^{1/2} H_8 [dx^2 + r_3^2 dz_3^2 + r_2^2 d\tilde{z}_2^2]
\]

\[
    e^{\phi} = \frac{e^{\phi_0}}{r_1} H_8^{1/2} \quad (46)
\]

After a $T$ duality on $T_2$ we get a D4 brane again

\[
    ds^2 = H_4^{-1/2} d\tilde{\sigma}_{2,1}^2 + H_4^{-1/2} H_8 \frac{dz_1^2}{r_1^2} + H_4^{1/2} d\tilde{r}_3^2 + H_4^{1/2} H_8 (dx^2 + r_3^2 dz_3^2) + H_4^{-1/2} H_8^{-1} \frac{1}{r_2^2} (dz_2 + m z_3 d\tilde{z}_1)^2
\]

\[
    e^\phi = \frac{e^{\phi_0}}{r_1} H_4^{-1/4} \quad (47)
\]

and finally after the coordinate transformation and $T$ duality on $T_3$ one gets a D5 brane (coming from the original D4) in the “7-brane” background (corresponding to the original
\[ ds^2 = H_4^{-1/2}d\sigma_{2,1}^2 + H_4^{-1/2}H_8 \frac{dz_1^2}{r_1^2} + H_4^{1/2}dr_3^2 + H_4^{1/2}H_8dx^2 \]
\[ + H_4^{-1/2}H_8 \frac{d\bar{z}_2^2}{r_2^2} + \frac{dz_3^2}{r_3^2} + \frac{dz_2^2/r_2 + dz_3^2/r_3}{1 + H_4^{-1}H_8^{-2}m^2z_1^2/r_2^2r_3^2} \]
\[ e^\phi = \frac{e^{\phi_0}}{r_1r_2r_3H_8^{-1/2}H_4^{-1/2}[1 + H_4^{-1}H_8^{-2}m^2z_1^2]^{-1/2}} \]
\[ B_{23}dz^2 \wedge dz^3 = -\frac{mr_1}{r_2r_3H_8^2} \frac{z_1/r_1}{1 + H_8^2(m/r_2r_3)^2z_2^2/r_1^2}dz_2/r_2 \wedge dz_3/r_3 \]

(48)

String theory in the D4-D8 background corresponds to D3 Matrix theory in the above (D5-“7-brane”) background. Note that one has a transverse “lightcone” coordinate, which really means that we have boosted the M2 in a transverse direction, so one has to approach the \( R_s \to 0 \) limit with care.

Let us now analyze what happens in M theory when one decouples the D4-D8 theory.

The decoupling limit of the D4-D8 is \( l_s \to 0 \), with \( g_s l_s = g_{D4}^2 \) fixed. But we have been a bit cavalier about the \( l_s \) dependence. In the massive IIA metric, the radii are \( l_i^2/r_i \), \( i=2,3 \), and are supposed to go to infinity (or be very large). Taking \( r_i = l_s^2R_i \), \( R_i \) are still very small (though finite). In order for this to be a good decoupling limit in M theory, it is clear that \( r_1 \) has to be treated as \( r_2, r_3 \) namely \( r_1 = l_s^2R_1 \) also, and one indeed finds that. Then

\[ g_{D4}^2 = g_s l_s = \frac{l_p^3}{r_2r_3} = \frac{R_1}{R_2R_3} \to l_p^3 \sim r_2r_3 \sim l_s^4 \]

(49)

and as usual \( \bar{r}_4 = l_s^2\bar{U}_4, x = l_s^2X \)

\[ H_4 \sim \frac{g_s l_s^3}{r_3} \sim \frac{1}{l_p^4U_4^3} \sim \frac{1}{l_p^4} \]

(50)

Therefore the decoupling limit of the string theory corresponds to a decoupling limit of the M theory, with metric (in the M2 case)

\[ ds_{11}^2 \sim l_p^2\left[ h_4^{-2/3}d\bar{\sigma}_{2,1}^2 + h_4^{1/3}\left[ d\bar{U}_4^2 + H_8(dX^2 + R_2^2dz_2^2 + R_3^2dz_3^2) \right] + H_8^{-1}R_1^2(dz_1 + mzd_2z_2)^2 \right] \]

(51)

Since in the decoupling limit the D4-D8 field theory is equivalent to string theory in the background [19] with harmonic function [18], by Hull duality the corresponding M2- “M7” field theory (“M7” is the gravitational background [33]) is dual to M theory in the background [51].

In the M5 case, one would get a M5- “M7” duality in a similar manner.

In this analysis, depending on the position of the \( z_2, z_3 \) and lightcone coordinate \( R \) (parallel or transverse to the M theory brane corresponding to D4), one has different endpoints. One
other possible complication arises if one wants to compactify this system to get to a 4d field theory. Then one would need to choose that coordinate v as well.

Nondecoupled D4-D8 theory we saw can be lifted in 3 ways to M theory (M2, M5 or KK monopole always in the gravitational background), which in turn can be described by a decoupled theory of D3 branes (describing gravity), with some added branes (describing the D4-D8 background). I have analyzed the case of M2 with perpendicular lightcone coordinate. Another case is obtained if one chooses \( x_{11} \) (the lightcone coordinate) and \( z_2 \) in D4 and \( z_3 \) perpendicular to it and so lift to an M5 brane. Going to string theory on \( x_{11} \), one gets a D4 with \( z_1, z_2 \) parallel and \( z_3 \) perpendicular to it. After all 3 T dualities, one gets a D3' with only \( z_3 \) parallel, whereas the D3s coming from the Matrix D0 branes have parallel \( z_1, z_2, z_3 \). The (4,8) strings are mapped to strings going from D3' to the 8d plane transverse to x and \( x_1 \) (“7-brane”).

So I have mapped the D4-D8 system to \( D3 \perp D3' \) (1) in the presence of a 7-brane. Gravity comes from the moduli space of the D3 and one still has the full field theory on D3. Decoupling of the gravity corresponds to decoupling of the D3 theory, and one is left with the D3' theory, with string modes ending on the 7-brane. One has therefore described the decoupled D4-D8 theory by a decoupled D3- “7-brane” theory, which seems consistent, since the T dual to the D4-D8 should be a D3-D7. At an intermediate step, the 11d description of the decoupled theory was in terms of a M5- “M7” theory, so the D4 theory was correctly lifted to the M5 theory, and there is an implicit (even if not useful) definition of the UV completion of the D4-D8 field theory.

Finally, just as a curiosity, let us see how far can we go in writing the metric for the holographic dual of the D3 theory giving the Matrix model in the presence of the D5 and “7-brane” (the M2 picture for string theory in the D4-D8 background). The simplest place to start is after T duality on \( T_1 \), when the Matrix model will be in terms of D1 branes in the background of D3s ending on smeared NS5s. D1 is in \( t \) and \( z_1 \) direction, D3 in \( \vec{\sigma}_2, 1 \) and \( z_1 \), and NS5 in \( \vec{\sigma}_2, 1 \) and \( \vec{r}_3 \). The coordinates \( z_1, z_2, z_3 \) are smeared over and \( x \) is overall transverse. The solution is written in the obvious way, but it satisfies the criterion of partially localized multiple intersections (since with the smearing in \( z_1 \perp NS5 \) we have \( D1 \in D3 \) inside smeared NS5). Then the harmonic functions obey the equations

\[
\begin{align*}
\partial_x^2 H_5(x) &= 0 \\
\partial_x^2 H_3(x, \vec{r}_3) + H_5(x)\partial_{\vec{r}_3}^2 H_3(x, \vec{r}_3) &= 0 \\
\partial_x^2 H_1(x, \vec{r}_3, \vec{\sigma}_2) + H_5(x)\partial_{\vec{r}_3}^2 H_1(x, \vec{r}_3, \vec{\sigma}_2) + H_3(x, \vec{r}_3)\partial_{\vec{\sigma}_2}^2 H_1(x, \vec{r}_3, \vec{\sigma}_2) &= 0
\end{align*}
\] (52)

To obtain the holographic dual one would need to make the \( T_2 \) T duality, coordinate transformation and \( T_3 \) T-duality, and then take a decoupling limit. But for that one would need an explicit solution of (52), and one can solve just the first two equations (similar to (10)), and then the third (the equation for \( H_1 \)) is too complicated to solve.
By defining

\[ H_3(\vec{r}_3, x) = 1 + \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p}\cdot\vec{r}_3} H_{p;3}(x) = 1 + \frac{1}{(2\pi)^2 r_3} \int_0^\infty dp \sin(pr_3) H_{p;3}(x) \]

\[ H_1(\vec{r}_3, \vec{\sigma}_2, x) = 1 + \int \frac{d^3p}{(2\pi)^3} \int \frac{d^2q}{(2\pi)^2} e^{i\vec{p}\cdot\vec{r}_3 + i\vec{q}\cdot\vec{\sigma}_2} H_{p,q;1}(x) \]

\[ = 1 + \frac{1}{(2\pi)^2 r_3} \int_0^\infty dp \sin(pr_3) \int \frac{d^2q}{(2\pi)^2} e^{i\vec{q}\cdot\vec{\sigma}_2} H_{p,q;1}(x) \] (53)

and setting \( H_5(x) = c + m|x| \) one gets the equations

\[ H''_{p;3}(x) - (c + m|x|) p^2 H_{p;3}(x) = Q_3 \delta(x) \]

\[ H''_{p,q;1}(x) - (c + m|x|) p^2 H_{p,q;1}(x) \]

\[ -q^2 (H_{p,q;1}(x)) + \int \frac{d^3p'}{(2\pi)^3} H_{p'-q;1}(x) H_{p';3}(x)) = Q_1 \delta(x) \] (54)

The first one is the same as (12), so is solved in the same way, but the second one is too hard.

4 DW-QFT for D4-D8 and motivating the field theory-supergavity correspondence

In this section I will analyze the field theory-gravity correspondence for the D4-D8 system. I would like to argue that the correct supergravity description of the D4-D8 field theory is in terms of a 6d \( N=2 \) supergravity.

First notice that the decoupled D4-D8 solution in (19) is not of the \( AdS_n \times S_m \) type. It is not even a fibration of \( AdS_6 \) over \( S_4 \) like the D4-D8 solution of Brandhuber and Oz in (8), hence one will not have a corresponding conformal field theory, but rather a nonconformal quantum field theory. The theory in [16] became conformal due to the presence of the orientifold planes, together with the near-horizon limit. It was conjectured in [16] that the nonlinear KK reduction on the \( AdS_6 \) fibered over \( S_4 \) (\( AdS_6 \times S_4 \) with a warp factor)

\[ ds_{10}^2 = (\sin\alpha)^{-1/3}(ds_{AdS_6}^2 + \text{const.}(d\alpha^2 + (\cos\alpha)^2 d\Omega_3^2)) \] (55)

will give the \( N=4 \) F(4) gauged sugra in 6d of Romans [41], with an susy \( AdS_6 \) ground state and an SU(2) gauge group. The conjecture was later proven in [12]. The F(4) sugra is the only \( N=4 \) gauged sugra in 6d with and \( AdS_6 \) ground state. But there are other \( N=4 \) gauged sustras in 6d with no \( AdS_6 \) ground state. In particular, there is an SU(2) gauged sugra which arises as an \( S_1 \) reduction of the (minimal) \( N=2 \) gauged sugra in 7d [43], with an SU(2) gauge group and a topological mass term. I will try to argue that this is the theory on the supergravity side of the correspondence. It is a good guess since the D4-D8 system has \( N=2 \) susy and so has SU(2) R-symmetry, related by the correspondence to the sugra gauge group.
Let us see what kind of supergravities are available, and start in 7d, where one knows the
dual to the M5 field theory. In seven dimensions there is a maximal (N=4) gauged sugra,
with gauge group SO(5). It is obtained as a KK reduction of 11d sugra on $S_4$ and as such
it gives the gravity dual to the M5 brane (the flat brane corresponds to its $AdS_7$
vacuum solution). Then there are the minimal (N=2) gauged supergravity with SU(2) gauge group
of $S_3$ and the coupled N=2 sugra+ vector multiplet, with gauge group SO(4) (SU(2) in the
sugra multiplet and another SU(2) in the vector multiplet) of $S_1$. They were obtained as
KK reduction of 10d N=1 sugra on $S_3$ and $S_1$ and as such give the gravity dual of
the NS5 brane theory. The pure SU(2) sugra can be written using a 2-form field or its 3-form
field dual, but it is in the latter formulation only that one can add a “topological mass” term
$h\epsilon(7)dA(3)A(3)$, which can be made supersymmetric. Since the SO(4) sugra+matter can only
be written in the 2-form formulation, it doesn’t admit a topological mass term deformation.
By truncation of the maximal gauged sugra, one can obtain an SU(2) gauged sugra with a
fixed topological mass term $h$ (related to the gauge coupling $g$). The maximal sugra does
contain a sugra multiplet and a vector multiplet, but has also a topological mass term, hence
it cannot be consistently truncated to the SO(4) sugra.

Going down to six dimensions, one has several gauged sugra models too. First, there is
the dimensional reduction of the N=4 model, which gives an N=8 gauged sugra with SO(5)
gauged group, written in $S_4$. It is natural therefore to associate it with the theory on a D4
brane, as was done indeed in $S_4$. If one dimensionally reduces the pure SU(2) 7d gauged
theory with topological mass $h$, one generates an N=4 sugra with gauge group SU(2) coupled
to an U(1) vector multiplet as in $S_4$. When $h=0$ one can consistently truncate the vector,
resulting in pure N=4 gauged sugra. Since the pure N=4 sugra theory is also a consistent
truncation of the maximal N=8 theory in 6d, it should also be related to the D4 brane theory,
but with half the supersymmetry. But there is yet another gauged SU(2) N=4 sugra, found
by Romans $S_4$, which is just a different mass deformation of the pure case, not involving
any new fields, but with a new parameter $m$. The Romans theory admits a supersymmetric
ground state with the full $AdS_6$ symmetry group, $F(4)$, if the gauge coupling is related to
the mass $m$ by $g=3m$. Having F(4) as symmetry group, it is not surprising that it was found
in $S_4$ to correspond to the conformal field theory on a D4+D8+O8 system.

Let us now match supergravities with brane systems. The M5 corresponds to the 7d
SO(5) supergravity, obtained by $S_4$ reduction of 11d sugra, and by a further $S_1$ reduction
one relates the 6d SO(5) sugra to the D4 theory. The type IIA NS5 brane theory is matched
to the 7d SO(4) sugra, by reduction on $S_3$ of 10d type IIA sugra (with the type I NS5 subset
related to the 7d SU(2) sugra by reduction of the 10d type I sugra. The 7d SU(2) vector
multiplet couples to operators charged under the vector multiplet on the IIA NS5). The 6d
SU(2) sugra corresponds to a D4 with half the supersymmetry, that is to a D4 of type I in
9d (M5 corresponds to 7d SO(5) sugra, and we compactify on a transverse circle, thereby
modifying the transverse sphere as $S_4 \rightarrow S_3 \times S_1$ and a parallel circle, giving in the end a D4
in 9d).

The deformation with mass parameter $m = g/3$ in 6d gives the F(4) sugra, corresponding
to the D4-D8-O(8) system. It is not clear to what corresponds the deformation with $m$
independent of $g$, but maybe it means moving away from the orientifolds (away from the
conformal point). The mass parameter \( m \) is related by \( S_4 \) reduction of the massive 10d sugra \[ m \propto (8 - N_f) \], where the 8 comes from the O8 charge. So the 6d mass deformation with parameter \( m \) corresponds to adding O8 and \( N_f \) D8 branes.

The deformation with mass parameter \( h \) in 7d corresponds to a deformation of the type I NS5 brane theory, but a deformation outside the massless type IIA NS5. The only possibility is that the \( h \) deformation corresponds to the mass deformation of 10d sugra, that is, to the NS5 parallel to D8 system. (The NS5-D8(5) is a particular case of the “overlapping brane system” NS5-Dq(q-3), \( 3 \leq q \leq 8 \), which has the M theory solution \( M5 \perp M5(1) \) as a prototype).

This would mean in particular that the massive type IIA sugra in 10d compactified on \( S_3 \) would reduce to \( h \)-deformed 7d SU(2) sugra, with \( h \) related to the 10d mass. While this is not proven, it seems very likely given the precedent: the massive 10d IIA compactified on \( S_4 \) gives the 6d F(4) sugra, with \( g = 3m \) related to the 10d mass. Yet another argument is the fact that 11d sugra on \( S_4 \) can be consistently truncated to the SU(2) sugra with a fixed \( h \) (proportional to \( g \) \[ 16 \]). The NS5-D8 solution will be obtained in a manner completely analogous to the D4-D8 solution \[ 19 \]. One can easily do it as an exercise. Compactifying 10d sugra to 7d on the decoupled NS5-D8 solution should give the same \( h \)-deformed sugra.

On the other hand, compactifying NS5-D8 on an \( S_1 \) parallel to NS5 one gets a D4-D7 solution of massive 9d sugra (which could be oxidized back to a D4-D8), related to \( h \)-deformed 6d SU(2) sugra.

Let’s mention now that in the nonconformal cases discussed (like the D4 and the D4-D8 systems) one has to turn to the so-called Domain Wall-Quantum Field Theory correspondence (DW-QFT), rather than AdS-CFT, a particular case of gravity-field theory correspondence where the backgrounds for sugra are domain walls rather than Minkowski or (anti) de Sitter.

The nonconformal cases of D-branes were studied first in \[ 54 \], and later in \[ 50 \], where the term “Domain Wall-QFT correspondence” was coined. The reason for the name is that the authors of \[ 50 \] realized that the near-horizon solution of nonconformal D-branes in Einstein frame gives known domain wall solutions of gauged supergravities. These domain wall solutions become in the dual p-brane frame just \( AdS_n \times S_m \) vacua, and the correspondence becomes simpler (in particular, the UV-IR relation becomes just \( E \sim u \)). Further treatments of the DW-QFT correspondence can be found in \[ 55, 56, 57 \].

In particular, the analysis revealed that the D4 brane theory is dual to type IIA sugra on \( S_4 \), which has as a massless mode a N=8 (maximal) SO(5)-gauged sugra which was later \[ 49 \] obtained as an \( S_1 \) reduction of the maximal (N=4) SO(5)-gauged sugra in 7d. It was also found that the compactification on tori transverse to the brane produces as effective sugras ones with noncompact gaugings, for \( T^k \) reduction of the M2 brane one has a CSO(8-k,k) gauged N=8, D=4 theory, for a \( T^k \) reduction of the M5 brane one has a CSO(5-k,k) gauged D=7 sugra and for a \( T^k \) reduction of the D3 brane one has a CSO(6-k,k) gauged D=5 sugra (the C stands here for contraction, which eliminates unphysical gauge fields of negative metric from the spectrum). In particular, the D=5 CSO(6-k,k) series was later found in \[ 58 \], whereas the D=7 CSO(5-k,k) series is still not constructed.
Finally, let us note that the h=0 6d SU(2) model should not only be a truncation of the 6d SO(5) model. It should also be obtained as an alternative 1/2 susy truncation of a CSO(4,1) model in 6d. The CSO(4,1) model in 6d is the circle reduction of the corresponding model in 7d, and corresponds to the near-horizon theory of the D4 brane in N=2, D=9 theory (circle reduction of the usual type IIA D4 brane in D=10).

In conclusion, one expects a DW-QFT correspondence to relate the 6d SU(2)-gauged sugra, coupled with a mass parameter h to an U(1) vector multiplet, to a D4-D8 system. The decoupled solution should dimensionally reduce on S^3 to a solution of h-deformed 7d sugra.

Unfortunately, just the S^3 reduction of the massive IIA theory would be a hard task, one that merits a whole new paper.

5 Towards a holographic dual for QCD

In this section I would like to take what we have learned about D4-D8 systems and see if one can write down a holographic dual for QCD. The D4-D8 system contains all the fields necessary for QCD, so one needs to generate mechanisms for getting rid of the extra fields.

5.1 Set-up, condensation

First, one needs to break supersymmetry. As we saw, one way was to compactify with susy breaking conditions for the fermions, which corresponds on the gravity dual side to making the solution nonextremal, but this kills unfortunately all fermions. Another way was to introduce a D8, which was very easy to do in supergravity. Just flip the sign of the mass on one side of the D8, and you get a D8 – D8 background. The string theory Dp – Dp system is however not necessarily purely gravitational, but depending on the charges, can contain also lower (Dp-2, Dp-4,...) branes. So after the D8 – D8 condensation, one could be left with a D6 (One would not like to be left with another D4, since then that D4 field theory will become again relevant, i.e. will not decouple. The D6 field theory is still decoupled). The solution for D4 inside D8 – D8 does not yet holographically describe the QCD-like field theory. A priori it is an unstable point in the dynamics, but we will argue that it should take a very large time to decay. However, the D8 – D8 by itself doesn’t have any overall gauge fields left, so we need a lower brane to take the role of D8 in the dual. After the condensation has ended a D6 will be formed, so it is natural to represent it in the holographic dual even if the D8 – D8 is still there. It is understood as a core around which the condensation will eventually take place.

In conclusion, one would like to have both the D8 – D8 and the D6 in the gravity solution, the D8 – D8 since the condensation should take infinite time, and the D6 since we want bifundamental fields.

If a Dp brane (p ≤ 6) collides with a Dp brane, we expect to form as an intermediate stage a black uncharged p-brane (the extension of the Schwarzschild solution to a p+1 dimensional worldvolume), which might then decay to a Dp-2 brane, but the gravitational description of this string process seems hard to obtain. But as we saw, in the particular
case of \( p=8 \) there is no collapse of the gravitational solution when the \( Dp \) touches the \( \bar{D}p \). Even if the \( D8 - \bar{D}8 - D6 - D4 \) solution is still unstable on a very large timescale, it will be holographically related shortly to massless QCD, so it is what we want. The issue of black holes and understanding the susy breaking process will be postponed for the next subsection.

Let us try to write down this \( D4 \) parallel to \( D6 \) parallel to \( D8 - \bar{D}8 \) solution. It can be obtained implicitly, since the harmonic functions should satisfy (again invoking the criterion of partially localized multiple interesections)

\[
\begin{align*}
\partial_2^2 H_8(x) &= 0 \\
\partial_2^4 H_6(x, \bar{r}_2) + H_8(x) \partial_{r_3}^2 H_6(x, \bar{r}_2) &= 0 \\
\partial_2^4 H_4(x, \bar{r}_2, \bar{\sigma}_2) + H_8(x) \partial_{r_2}^2 H_4(x, \bar{r}_2, \bar{\sigma}_2) + H_6(x, \bar{r}_2) \partial_{r_2}^2 H_4(x, \bar{r}_2, \bar{\sigma}_2) &= 0 
\end{align*}
\]

Solving it would be identical to solving \([52]\), so the discussion is the same. Thus the solution is

\[
\begin{align*}
H_8(x) &= c + m|x| \\
H_6(x, \bar{r}_2) &= 1 + \frac{1}{(2\pi)^2 r_2} \int_0^\infty dpp \sin(pr_2) H_{p,6}(x) \\
&= 1 + \frac{Q\sqrt{e}}{4\pi^2 m^2 \frac{\beta}{3}} \frac{\beta^{1/3}}{r_2} \int dpp \frac{\sin(pr_2) K_{1/3}(\frac{2}{3} \frac{p}{m} \frac{\beta}{3})}{K_{1/3}(\frac{2}{3} \frac{p}{m} \frac{\beta}{3})} \\
H_4(x, \bar{r}_2, \bar{\sigma}_2) &= 1 + \frac{1}{(2\pi)^2 r_2} \int_0^\infty dpp \sin(pr_2) \int d^2q e^{i\bar{q} \bar{\sigma}_2} H_{p,q;4}(x) 
\end{align*}
\]

and where the equation for \( H_{p,q;4}(x) \) is

\[
\begin{align*}
H_{p,q;4}''(x) - (c + m|x|)p^2 H_{p,q;4}(x) \\
-q^2(H_{p,q;4}(x) + \int \frac{d^2p'}{(2\pi)^3} H_{p'-q;4}(x) H_{p';6}(x)) &= Q_1 \delta(x) 
\end{align*}
\]

So what field theory does this solution (or rather its decoupling limit) describe? Since one still has the \( D6 \) brane in place of the \( D8 \), one still has the bifundamental fields, but there are now differences.

First of all, susy is broken at the string scale (by the \( \bar{D}8 \)), so the \((4,4)\) (and \((8,8)\)) adjoint fermions get a string scale mass.

One comment is in order here. This statement doesn’t imply any knowledge of nonperturbative physics, it is just meant to parametrize our ignorance. At the string scale we don’t have control anymore (in particular, the \( \bar{D}8 \) might evaporate, say), so it is natural to put the susy breaking scale there (at least at the string scale would be more appropriate).

But note that there still is supersymmetry on the \( D8 \) (at \( x=0 \)), so if fermions are defined only on the \( D8 \), they will remain massless. This is indeed the case for the \((4,8)\) (bifundamental) fermions. Why is this so? There are no globally defined fermions in the bulk, and that translates to string-scale mass for the bulk fermions. By the AdS-CFT, via the coupling of the closed string bulk modes to the \((4,4)\) bilinear operators, the \((4,4)\) fermionic operators will get a very large anomalous dimension and decouple. But the fermions defined on the
D8 couple to fermionic bilinear operators with \((4,8)\) quantum numbers \(SO(2N_f)\) quantum numbers), and these fermions will remain massless, therefore the \((4,8)\) fermionic operators remain in the theory. This still does not say anything yet about the dynamics of the theory, just that the starting point has fundamental fermions, and no adjoints. The dynamics would be derived from the final decoupled holographic dual.

But in order to have a holographic dual for QCD, one still has to say what happens to the scalars (adjoint and bifundamental), how to get rid of the fermions in the conjugate representation (we need \(\mathcal{N}=1\) fermions, not \(\mathcal{N}=2\)), and also how to get to 4d.

One should note here that an \(\mathcal{N}=1\) scalar superfield has a complex scalar, related to two transverse coordinates, and one complex fermion. To get to 4d, it would seem that one needs to compactify one coordinate. But one can easily check (by looking at the D4 holographic dual analyzed in [54]) that if one compactifies (on a small radius) the D4 in 10d to a D3 in 9d and still insist on the decoupling of gravity, one is forced to go to the T dual description of the holographic dual, namely the near-horizon D3 brane in 10d with one transverse coordinate compactified. Indeed, the compactified D4 holographic dual is

\[
\begin{align*}
    ds^2 &= l_s^2 \left[ \frac{U^{3/2}}{\sqrt{g_{D4}^2}} \left( dx_{3+1}^2 + R^2 dx_5^2 \right) + \sqrt{g_{D4}^2} \frac{N}{U^{3/2}} \left( dU^2 + U^2 d\Omega_4^2 \right) \right] \\
    e^{\phi} &= \left[ \frac{U^{3/2} g_{D4}^2}{N} \right]^{1/4} \\
    g_{D3}^2 &= \frac{g_{D4}^2}{R} 
\end{align*}
\]

To get the same from the D3 brane in 10d,

\[
\begin{align*}
    ds^2 &= l_s^2 \left[ \frac{\bar{U}^2}{\sqrt{g_{D3}^2}} \left( dx_{3+1}^2 + \frac{d\bar{U}^2 + \bar{U}^2 d\Omega_5^2}{U^2} \right) \right] 
\end{align*}
\]

with \(\bar{r} = l_s^2 \bar{U}, r = l_s^2 U\), one needs a well defined decoupling limit. For that, since

\[
H_3 = \frac{g_{D3}^2 NL_s^4}{\bar{r}^4} = \frac{g_{D3}^2 NL_s^4}{R^3} = \frac{g_{D3}^2 N}{U^3} \frac{1}{l_s^4} \frac{l_s^2}{R}
\]

one need to have \(R = \frac{l_s^2}{R}\) (the T dual radius in the 9d D3 brane) fixed. The D3 brane scalar is identified with

\[
\phi^9 \sim \phi^9 + \frac{\bar{R}}{l_s^2} = \phi^9 + \frac{1}{R}
\]

Then, if \(1 \gg Ul_s\) and \(l_s/R \gg Ul_s\), one can’t see the identification of \(\phi^9\) (we don’t probe it). So for \(R\) fixed and small the field theory is with all the scalars noncompact, and the dual of this \(\mathcal{N}=4\) SYM is D3 in 10d. In the regime where one begins to probe the D4, we have to have a transverse scalar compact, and average over it in the holographic dual as was done in [51]. The D3 field theory should have only one holographic dual in a given energy regime, so it is not surprising that the compactified D4 holographic dual is not valid if \(R\) is sufficiently small. Hence compactifying on a small circle the holographic dual forces you to
go to the T dual description, even if $R \gg l_s$ (the point being that for transverse coordinates, the distances are rescaled by $l_s^2$, so even though $R/l_s = l_s/R \ll 1$, still $1/R = R/l_s^2 \rightarrow \infty$).

Since we are interested in a 4d field theory, we should then look for the T dual description of the D4-D8: a D3-D7 system smeared over an overall transverse direction. I leave for the next section the details of this construction, but from now on I will be talking about D3-D7 systems.

But one still needs to get rid of the scalars (both adjoint and bifundamental) and the conjugate fermions. The adjoint scalars correspond to the positions of the D3 branes, and the bifundamental to the relative D3-D7 positions.

Getting rid of the scalars can be done by introducing an effective potential for them, or equivalently by fixing the motion of the D3 inside the D7. It is so since by putting a potential for the D3 (by modifying the metric), we change the D3 theory from pure SYM to a modified SYM dictated by the DBI action in that background. For instance, if the D3 is stuck at a metric singularity, the DBI action will imply a term

$$\partial_a X^\mu \partial_a X^\nu g_{\mu\nu}(X) \rightarrow 0, \text{as} X \rightarrow X_0$$

(63)

and so the kinetic term will be null, or by rescaling to a canonical form, the potential will be infinitely steep, and thus the corresponding scalar(s) will disappear from the spectrum.

By condensing the $D7 - \bar{D7}$ to the D5, one has effectively insured that the motion of the D3 is fixed in those directions (since the D5 position will have a singular metric in its transverse directions). That means that one still needs to fix the position in the directions inside the D5, transverse to the D3.

A comment is in order here. If there is supersymmetry -e.g. between a parallel D3 and a D7 in flat space- then the above argument is not true. Anyway, the argument above is for the supergravity approximation, and we should consider the full string theory. If there is susy, there is no potential between D3 and D7. If there is no susy (susy broken at the string scale), there will be string scale masses for the scalars separating the two. Of course, the D3-D5 background is still supersymmetric inside the 7-plane, but there will be a potential in between the two, since the (3,3) (and (5,5)) fermions are massive, so there will be no cancellation of forces.

But one still needs to fix the position of the D3 inside the D5, (as well as to get rid of the fundamental fermions in the conjugate representation), so one needs to have a special point inside the D5. It can be either a singularity, or a brane.

The simplest way to do that is to put an additional D7', perpendicular on the D7. This is a supersymmetric configuration, where $D7 \perp D7'(5)$, such that the common worldvolume with the D5 is the D3. This configuration still preserves $\mathcal{N}=1$ supersymmetry, as does the D3-D5-D7 (the D7' doesn’t break any additional supersymmetry). Of course, the $\bar{D7}$ breaks the susy completely.

When the $D7 - \bar{D7}$ condenses to the D5, the $\mathcal{N}=1$ (D3,D7) bifundamental superfield describing the condensation directions will dissipate (become massive), so we will be left only with the (D3,D5) $\mathcal{N}=1$ bifundamental superfield. D7' is still needed to make the bifundamental scalar massive by giving it a potential as argued above.

One could correctly argue that by introducing D7' we generate (D3,D7') fields, but these become massive since their operators couple to fields which can propagate in the whole
spacetime, which is not supersymmetric. It would seem like we would need to have \( N_f \gg N'_f \), in order to treat \( D7' \) as a perturbation, but the system before the introduction of the \( \bar{D}7 \) is supersymmetric, and the \( D7 \) breaks the susy only outside the brane, on it is still valid. And in any case, all fermions propagating outside the \( \bar{D}7 \) will become massive. The \( D3-D7' \) bifundamental scalars are massive for the same reasons that the \( D3-D7 \) scalars are.

One sees now an added reason for going to the \( D4-D8 \) system. A \( D8 \) can’t intersect with a transverse \( D8 \) and still preserve susy, since \( D8 \) preserves say \( \Gamma_9 \epsilon_0 = \pm \epsilon_0 \), and a transverse \( D8 \) will preserve say, \( \Gamma_8 \epsilon_0 = \pm \epsilon_0 \), but that would mean that one needs \( [\Gamma_8, \Gamma_9] \epsilon_0 = 0 \) which implies \( \epsilon_0 = 0 \). A \( Dp \) brane can supersymmetrically self-intersect over a \( p-2 \) brane (as e.g., \( D7 \perp D7' \)), because, e.g., \( [\Gamma_8, \Gamma_9] = 0 \).

Finally, let us address the question of the condensation of the \( D7 - \bar{D}7 \) system to \( D5 \). There are two ways this can happen. One is due to Sen [59] and formalized in the language of K theory by Witten [60] (for more details see the lectures [61]). The point is that on a \( Dp-\bar{D}p \) worldvolume there is a tachyon field which condenses. At the minimum of its potential, the tachyon potential and the tension of the \( Dp - \bar{D}p \) cancel each other, \( g^{-1}V(T_0) + 2T_D = 0 \) and one has vacuum. But the complex tachyon might have a vortex solution which behaves like

\[
T \simeq T_0 e^{i \theta}, A_\theta^{(1)} - A_\theta^{(2)} \simeq 1 \quad \text{at} \quad r \to \infty
\]

where the tachyon kinetic term is

\[
|D_\mu T|^2 = |\partial_\mu - iA^{(1)}_\mu + iA^{(2)}_\mu T|^2
\]

and so one gets a magnetic flux for \( A^{(1)}_\mu - A^{(2)}_\mu \) at the core, which means that one has a \( Dp-2 \) brane.

The other way by which one could get an \( Dp-2 \) endpoint for the condensation is if one has an explicit flux from the beginning. Either way, one can view the supergravity solution as the one before tachyon condensation, when the tachyon field is still at \( T=0 \) throughout most of the \( Dp \) brane, except around a \( Dp-2 \) core.

Note that, as I mentioned, \( g_s \) is small enough to ensure that we can trust the supergravity approximation (unlike for other \( Dp \) branes, where one needs an extra condition, like large \( N \) for susy branes and large \( r \) for nonsusy ones), so even though the system may be unstable, there is an interval of time when the \( Dp - \bar{D}p \) solution is correct. And that time should go to infinity as \( g_s \to 0 \). There was over the last year a flurry of activity in the analysis of time dependence on the unstable \( Dp \) branes and brane-antibrane pair, started by [62]. For instance, the analysis in [63], done in the context of classical field (and string field) theory notes that the timescale of quantum effects would go to infinity as \( g_s \to \infty \). But in our case the string field theory effects themselves (\( \alpha' \) effects) will be proportional to \( g_s \).

Also note that this mechanism only works for \( D8 \) branes (and in general smeared \( Dp \) branes with only one nontrivial transverse coordinate), since a \( Dp - \bar{D}p \) will be a black brane, with a singularity; so for that there will be significant stringy corrections. Another way of seeing this is that radiating away energy in one transverse dimension is much harder than in higher dimensions, so the decay timescale can be actually made infinite. A further discussion of general \( Dp - \bar{D}p \) brane systems will be done in the next subsection.
So the holographic dual for QCD would have been the $D7 \perp D7'(5)$, with an extra $\bar{D}7$ parallel to the $D7$, and smearing over one of its transverse coordinates, with a D5 remnant intersecting $D7'$ over a D3. The problem is that now, we can’t write down this solution, not even implicitly, but one has at least defined the system.

5.2 Understanding susy breaking: black holes, Randall-Sundrum, compactification and going down to D7 branes

Why were we able to write down the $D8 - \bar{D}8$ solution? In general, if we approach a D$p$-brane and a anti-D$p$-brane in flat spacetime (otherwise, there are many nontrivial stable solutions) we expect that the $Dp - \bar{D}p$ brane solution (which will have gravitational mass, but no charge) will be a uncharged black-p-brane (a generalization of the Schwarzschild black hole by adding p flat coordinates). That solution would be

$$
\begin{align*}
    ds^2 &= -dt^2 + dx_\perp^2 + f^{-1}(r)dr^2 + r^2d\Omega_n^2 \\
    f(r) &= 1 - \frac{\mu}{r^{n-1}}
\end{align*}
$$

(66)

A charged black p-brane solution will be

$$
\begin{align*}
    ds_s^2 &= H_p^{-1/2}(-dt^2 + dx_\perp^2) + H_p^{1/2}(f^{-1}(r)dr^2 + r^2d\Omega^2) \\
    e^\phi &= g_s H_p^{\frac{3-p}{2}}, \quad F_{p+2} = Q_{p+1} vol(\Omega_{8-p}) \\
    H_p &= 1 + g_s N l_7^{8-p}
\end{align*}
$$

(67)

But one can also have nontrivial dilaton, and hence have a dilatonic “black hole”-type solution. But this solution can only appear in the presence of a cosmological constant in string frame. That cosmological constant is supplied by the constant mass in massive IIA, but can be obtained by any constant field strength for a RR field, since the action is

$$
S_{\text{string},10d} = \frac{1}{4k_{10}^2} \int d^{10}x \sqrt{g}(2e^{-2\phi}R + \bar{M}^2 + \sum_p F_{p+2}^2 + \ldots)
$$

(68)

The “dilatonic black hole” is the generalization for nontrivial dilaton of the Randall-Sundrum domain wall inside AdS space (positive tension domain wall with nontrivial cosmological constant). The Randall-Sundrum set-up [64, 65] in d dimensions (as opposed to 5) has the equations of motion

$$
R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = -\lambda g_{ij}\delta(z)\delta^i_\mu \delta^j_\nu - \lambda' g_{\mu\nu}
$$

(69)

which has a solution of the type

$$
ds^2 = A(z)(dx_\perp^2 + dz^2)
$$

(70)

with scalar curvature

$$
R = \frac{d-1}{A}[(lnA)'' + \frac{d-2}{4}((lnA)')^2]
$$

(71)
if
\[ A = (1 - \frac{\lambda}{2(d - 2)})^{-2} \] (72)
and
\[ \lambda' = \lambda^2 \frac{d - 1}{8(d - 2)} \] (73)

The dilatonic version of this is obtained in the presence of a constant field strength and is given by
\[ ds^2_s = H(z)^{-1/2}d\vec{x}^2_{p+1} + H^{1/2}(z)(dz^2 + d\vec{y}_n) \]
\[ e^\phi = g_s H^{3-p}/4 \]
\[ F_{(n)} = m \]
\[ H = 1 + m|z| \] (74)

and this solution breaks supersymmetry and is uncharged, so it is the analog of a black hole (or rather, Randall-Sundrum).

It is interesting to note that RS would break susy anyway (while keeping susy on the brane), but in string theory one doesn’t have a cosmological constant, but at most constant field strength, as we saw, which provides a potential (which at constant dilaton can be interpreted as a cosmological constant). So this solution is as close as one can get to a Randall-Sundrum type scenario. Also note that (74) generates $Dp$–$\bar{D}p$ solutions for $p=4,...,8$ (we need a magnetic-type solution, since $F_{(8-p)}=ct.$), and could be also extended for $p=3$ (with self-dual field strength $F_{(5)} = *F_{(5)}$), and this list exhausts all interesting configurations (i.e. configurations where our 4 dimensions live on the brane).

Schwarzschild black holes (non-dilatonic, no cosmological constant) can exist in $p+1$ dimensions, $p > 2$, with solution (66). For $p=2$, we can still have a solution, but of a different form than (66). For convenience, we embed it in 4d as the “cosmic string” solution. It is
\[ ds^2 = -dt^2 + dr^2 + r^2(1 - 8G\mu)d\theta^2 + dz^2 \] (75)
The fact that the solution looks different can be understood as the manifestation of the fact that 2+1 gravity is of Chern-Simons type, hence topological, and thus the black hole is just a conical defect.

However, there are no 1+1 dimensional black holes, the reason being that the Einstein action is purely topological, $\int R$ is just the Euler invariant in 2d, so one can’t put a source into the Einstein equations. There is however a solution for gravity coupled to a dilaton, which is just the dimensional reduction of the dilatonic black hole.

This fact implies in 4d the statement that a gravitating infinite plane (domain wall) can’t have static metric (unlike a cosmic string), but rather the plane is inflating. The solution (Villenkin) is
\[ ds^2 = -(1 - k|z|)^2dt^2 + dz^2 + (1 - k|z|)^2e^{2kt}(dx^2 + dy^2) \] (76)
When a higher dimensional black hole solution is compactified on a transverse direction, and the radius \( R \) of the compact dimension \( y \) is smaller than the gravitational radius \( R_G \) of the black hole, then

\[
f = 1 - \frac{R_G^m}{(\bar{z}^2 + y^2)^{m/2}} \tag{77}
\]

gets averaged over \( y \) and we get the lower dimensional black hole, with

\[
\tilde{f} = 1 - \frac{R_G^m}{\bar{z}^{m-1}} \tag{78}
\]

Let us see in more detail how the \( D_p - \bar{D}_p \) solution works for \( D_7 \), which we saw is our preferred choice.

One would write a \( D_7 \) solution as

\[
ds_s^2 = H^{1/2}(r, \theta) d\bar{x}^2 + H^{1/2}(dr^2 + r^2 d\theta^2)
\]

\[
e^\phi = H^{-1}, \quad \partial_r a \sim \partial_r H \tag{79}
\]

and the usual \( D_7 \) is the oxidation of the stringy cosmic string solution, i.e.

\[
ds_E^2 = d\bar{x}^2 + H(r, \theta)(dr^2 + r^2 d\theta^2), \quad H = \Omega^2
\]

\[
H = \Omega^2 = \tau_2 = e^{-\phi}, \quad \tau = \tau(z)
\]

\[
ag(j(\tau(z))) = \frac{P(z)}{Q(z)} \tag{80}
\]

but on the other hand the T dual solution to the \( D_8 \), which is just the \( D_7 \) averaged over one transverse direction, is

\[
ds_s^2 = H^{-1/2}(z) d\bar{x}^2 + H^{1/2}(z)(dz^2 + dx^2)
\]

\[
e^{-\phi} = H(z), \quad H(z) = 1 + m|z|
\]

\[
a = \pm H'x \Rightarrow F_x = \pm H' = ct.
\]

\[
\epsilon = H^{-1/8} \epsilon_0, \quad \Gamma_{xx} \epsilon_0 = \pm i \epsilon_0 \tag{81}
\]

where as usual the \( \pm \) refers to \( D_7 \) versus \( \bar{D}_7 \). We can easily see that a \( D_7 - \bar{D}_7 \) solution exists and the only modification is that it has \( F_x = m = \text{const.} \), and as a consequence no global \( \epsilon \).

One can generalize this \( D_7 \) solution to the (multiply T dualized \( D_8 \))

\[
ds_s^2 = H^{-1/2}(z) d\bar{x}^2_{p+1} + H^{1/2}(z)(dz^2 + dx^2_n)
\]

\[
e^{\phi} = H^{3-p}(z)
\]

\[
F_{(n)} = \pm H', \quad H(z) = 1 + m|z| \tag{82}
\]

and the \( D_p - \bar{D}_p \) solution is again obtained by having \( F_{(n)} = m = \text{constant} \), exactly the “dilatonic black hole” solution (74).
6 Phenomenology-trying to embed the Standard Mo-
del in string theory via Dp-Dp+4 systems

Finally, in this section I will try to see whether we can use the susy breaking mechanism used
for the QCD holographic dual to lift it to an embedding of the Standard Model in string
theory in the braneworld approach. I will try to fit the model into a GUT type scenario,
including SU(5). In the appendix I review GUTs from our point of view. The ingredients
used for the model building are D3-D7-O(7) systems, so I will analyze these first, and move
to model building in the next subsections.

A few comments are in order about the procedure. The goal is not to apply the gravity-
gauge duality, but just to take the string theory system and lift it to the Standard Model,
and see what is obtained in the 4d field theory. Since $N_c$ is now small and $\alpha'$ is finite but
nonzero, on the D3 branes we will have string corrections to the YM action. The fact that $N_c$
is small will only affect the geometry of the compact space, but I will not make any precise
statements about that (there will be a significant backreaction).

As an example of the procedure, let’s take the $\mathcal{N} = 4$ $SU(N_c)$ SYM - $AdS_5 \times S_5$ holography
and try to make a braneworld model (by which in this example I just mean adding gravity
and making $N_c$ finite). One would make the space transverse to the D3 branes compact.
Since $N_c$ is small, that space would not be approximated by any version of $AdS_5 \times S_5$ with
the radial AdS direction compactified, but that is not what one is after. The corrections to
the D3 brane theory (in the form of the DBI action) will still be small, so the analysis of
the field theory should carry through. The $1/N_c$ effects will affect the quantitative physics
(correlators, etc.), but not the qualitative physics (low energy fields and possible interaction
terms). The question deserves further study, but I am going to assume that the qualitative
physics is unmodified by the small $N_c$.

6.1 Supersymmetric lagrangeians for D3-D7-O(7) systems

The Dp-D(p+4) lagrangeian is, dimensionally reduced to 4d (the notation used is for D5-D9,
but the rest -D3-D7 and D4-D8- are the same modulo a relabeling of fields).

$$L^{N=2}_{4d} = \frac{1}{8\pi} ImTr[\tau(\int d^2\theta W_a W^a + 2 \int d^4\theta \Phi_i^+ e^{-2V} \Phi_i e^{2V} + \frac{1}{3!} \int d^2\theta \epsilon_{ijk} \Phi^i \Phi^j \Phi^k]_{(5,5)+(9,9)} + \int d^4\theta(Q^+ e^{-2V_{5,5}} Q e^{2V_{9,9}} + \bar{Q} e^{-2V_{5,5}} \bar{Q}^+ e^{-2V_{9,9}}) + \int d^2\theta \sqrt{2}(\bar{Q} \Phi_{5,5}^1 Q + \bar{Q} \Phi_{9,9}^1 Q + h.c.)]$$

(83)
where by \((5, 5) + (9, 9)\) we understand the sum over both kind of indices. The component fields are, as usual:

\[
V = -\theta \gamma^\mu \partial A_\mu + i \theta^2 \bar{\partial} \lambda - i \bar{\theta}^2 \theta \lambda + 1/2 \theta^2 \bar{\theta}^2 D
\]

\[
\Phi^i(y, \theta) = A^i(y) + \sqrt{2} \theta \Psi^i(y) + \theta \theta F^i(y)
\]

\[
Q = a(y) + \sqrt{2} \theta q(y) + \theta \theta f(y)
\]

\[
\bar{Q} = \tilde{a}(y) + \sqrt{2} \theta \tilde{q}(y) + \theta \theta \tilde{f}(y)
\]

and \(V, \Phi^1 = \Phi\) make up an \(\mathcal{N} = 2\) vector, whereas \(\Phi^2\) and \(\Phi^3\) make up a hypermultiplet, together making up an \(\mathcal{N} = 4\) vector (one for \((5, 5)\) fields and one for \((9, 9)\) fields), and \(Q\) and \(\bar{Q}\) make up a hypermultiplet \((5, 9) + (9, 5)\).

If the \(D(p+4)\) theory is decoupled, one just drops the terms involving \((9,9)\) fields in the above. For concreteness, I will talk in D3-D7 language from now on.

So the superpotential for the \(\mathcal{N}=2\) D3-D7 system with \(U(5)\) gauge symmetry is

\[
\mathcal{W} = Tr(\epsilon_{ijk} \Phi^i \Phi^j \Phi^k + q^i \Phi^1 \bar{q}^i)
\]

and the first term is zero for a \(U(1)\) field. The \(U(1)\) in \(U(5)\) has a VEV which gives a mass to all the \((3,7)\) hypermultiplet: \(<\Phi^1_{ab} >= m\delta_{ab}\) implies a mass term \(mq^i \bar{q}^i\) for the hypermultiplet, corresponding to separating the D3 and the D7 (so \(\Phi^1\) correspond to the D3-D7 separation in the overall transverse coordinates). It does not give a mass term for \(\Phi^2\) and \(\Phi^3\), since as we said the \(U(1)\) piece does not have a superpotential. If we separate one D3 from the D7, that would mean giving a VEV to \(\Phi^1\), equal to \(m\delta_{a1}\delta_{b1}\), which does give a mass to the \(\Phi^2_{1a}\) hypermultiplet (strings between D3 brane 1 and the rest) and to the \(q^i_1\) hypermultiplet (strings between D3 brane 1 and all D7's). Separating one D7 from the rest corresponds to an explicit mass term for \(q^i_2\) from the D3 brane theory perspective, and comes from the nonzero VEV for \(\Phi^1_{1(7,7)}\).

When one goes to the D3-D7-O(7) system (which is still \(\mathcal{N}=2\) supersymmetric), the gauge group becomes Sp(10) (for \(N_c = 5\) D3 branes), and the superpotential becomes

\[
\mathcal{W} = Tr(WZ'Z + q^i W \bar{q}^i)
\]

where \((W_\alpha, W)\) form a vector in the symmetric of Sp(10) (adjoint), \((Z', Z)\) form an antisymmetric hypermultiplet of Sp(10) and as before \(q^i_1, \bar{q}^i_1, q^i_2\) form a fundamental hypermultiplet. \(W\) extends \(\Phi^1\) and \(Z', Z\) extend \(\Phi^{2,3}\).

Then the D3-D7-D7' system has \(\mathcal{N}=1\) susy and superpotential

\[
\mathcal{W} = Tr(\epsilon_{ijk} \Phi^i \Phi^j \Phi^k + q_1^i \Phi^1 \bar{q}_1^i + q_2^i \Phi^2 \bar{q}_2^i)
\]

where all are \(\mathcal{N}=1\) scalars. The \(\mathcal{N}=2\) structure was broken by the splitting of the \(\Phi^2, \Phi^3\) hypermultiplet into two.

For the D3-D7-O(7)-D7', the natural guess for the superpotential is

\[
\mathcal{W} = Tr(WZ'Z + q_1^i W \bar{q}_1^i + q_2^i Z' \bar{q}_2^i)
\]
(which is the obvious generalization of the D3-D7-D7’ case, since the presence of the D7’ should only be felt in the bifundamental \((q_2^i, \tilde{q}_2^i)\), coupling to the corresponding coordinates-\(Z’\) again with obvious meaning.

Yet another question is what happens when one goes to the D3-D7-O(7)-D7’-O(7)’ system. The gauge group is \((5 \text{ D3 branes}) \ Sp(10) \times Sp(10)\) a discussion of this model, which is T dual to the original Gimon-Polchinski orientifold and can be described as a \(T^4/(Z_2 \times Z_2)\) orientifold, can be found in [67, 68] and [30]. One would guess that the superpotential is

\[
W = Tr(W_a Z'_a Z_a + q_1^i (W_1 + Z_2) q_2^i + q_2^i (W_2 + Z'_1) \tilde{q}_2^i)
\]  

(89)

However, the analysis of [68] finds that the fields corresponding to coordinates transverse to the D3 brane do not become \((W, Z, Z')\) with indices in \(Sp(10) \times Sp(10)\) (\((W_a, Z_a, Z'_a)\) in the above notation), but instead become two chiral multiplets \(A\) and \(B\) in the \((10,10)\) representation of the gauge group for the 4 coordinates transverse to D7’s \((\Phi_{2}^1\) and \(W\) before). In a \(Sp(2k) \times Sp(2k)\) theory, it would be \((2k,2k)\) for the first 4 coordinates and \((1,1)+(k(2k-1)-1,1)+(1,1)+(1,k(2k-1)-1)\) for the other two (two singlets and two antisymmetric traceless representations, one in each Sp factor; for a single D3, it would just be two singlets, \(S_1\) and \(S_2\)).

A simple argument for the gauge group is as follows (a more detailed study can be found, e.g. in [68]). An orientifold makes the open string vector wavefunction symmetric in the covering space. With 2 intersecting \(O(7)\) planes, we have 4N D3s in the covering space. Taking into account the first \(O(7)\) makes the Chan Patton \(4N \times 4N\) matrix symmetric (symmetric under reflection by the diagonal). The second \(O(7)\) corresponds to rearranging the order of the matrix elements and then making it symmetric, or symmetrizing the matrix under the second diagonal. By this operation, we are left with \(2N(2N+1)\) independent elements, enough to form the adjoint of \(Sp(2N) \times Sp(2N)\). The analysis in [68] did not find a simple superpotential valid everywhere, but for nonzero \(A\) and \(B\) (and zero \(S_3\) and \(S_4\)), it is

\[
W = S_2 AB + q_1^i AB q_1^i / \sqrt{A^2} + q_2^i AB q_2^i / \sqrt{B^2} 
\]  

(90)

Here \(i = 1, \ldots, 2N_f\) (8 at the superconformal point) and \(S_2\) is one of the two singlets. Presumably one should also have a term involving \(S_3\) and \(S_4\), maybe

\[
W = (S_3 + S_4) AB
\]  

(91)

where \(S_3\) and \(S_4\) are the antisymmetric (1,44) and (44,1). The \(Sp(10) \times Sp(10)\) should get broken to a diagonal subgroup when \(O(7)’\) is removed, and \(AB\) probably becomes \(W\) (the rest of the components become massive).

6.2 D3-D7 system; TeV strings

We have seen that the model for QCD was obtained from a D3-D7-D7’ system, so that should be a part of the sought after Standard Model construction. I have also said that we
want an SU(5) GUT (most likely embedded into a larger GUT), so for the beginning, the simplest way to generate an SU(5) is by having 5 D3 branes.

So the system we want should have 5 D3 branes and a number of higher branes, responsible for the fundamental and antisymmetric tensor fields. The gauge group will then be $U(5) = (SU(5) \times U(1))/Z_5$. If the system separates into 2+3 branes, we have SSB to $U(3) \times U(2)$. Out of the remaining 2 U(1)’s, one is the center of mass one and one is the U(1) inside SU(5). Putting this 5 D3 branes inside a $D7 - \bar{D}7$, we generate a hypermultiplet in the fundamental (5), and 4 real scalars in the adjoint remain (the adjoint fermions become massive, as we argued). The hypermultiplet contains 4 real scalars (2 complex) and 2 complex fermions in the (complex) 5 representation. In the SU(5) GUT we have (see appendix for details) the gauge fields in the 24, 3 generations of fermions in the 5 and the 10 (maybe also a right-handed neutrino singlet per generation) and two Higgses, the one responsible for SU(5) breaking in the 24 (real) and the one responsible for electroweak SSB in the 5 (or the 45, but that’s too much)- complex. We see that so far we have enough gauge fields and scalars to contain this, but we still need more fermions in the 5 and more importantly fermions in the 10.

More 5 fermions will be generated by adding more D5s inside the $D7 - \bar{D}7$ and/or more D7’s. But how to generate 10s? The only way I see is through orientifolds. That will be discussed in the next subsection, but let’s see how far can we go without O(7)7.

At this moment one needs to make the following observation. In the $D3 - D7 - \bar{D}7$ type of construction we need the D7 field theory to be decoupled. That was true exactly only in the case of the $\alpha' \to 0$, now we just want it to have a very small coupling. But we saw that $g_{D3}^2 = g_s$ fixed implies $g_{Dp}^2 = g_s l_s^{p-3}$ goes to zero if $l_s \to 0$. But if one compactifies on a radius of the order of $l_s$ we are back to square one, since the effective 4d gauge theory will have a coupling $g_{eff,Ad}^2 = g_{Dp}^2/(\Pi_i R_i) = g_s \Pi_i(l_s/R_i)$. So in order for the Dp ($p > 3$) gauge theory to decouple one needs the volume of the extra dimensions in string units to be very large. If one has just the D3-D5-D7 system, one needs only the D5 compactification to be on a large volume, but if one adds the D7, since the D5 compactification volume is transverse to it, one needs to have some of the other coordinates to be very large.

One can put limits on the size of large dimensions as follows. Whenever one constructs a gauge theory from intersecting branes, one gets extra gauge fields at the compactification scale, and there are strong experimental constraints on that. That is why one needs brane constructions for large extra dimensions: only gravity is 4dimensional up to just a mm scale. There are no new gauge bosons up to the electroweak scale (100 GeV), and depending on their couplings even up to the TeV scale. And when talking about unification, anything other than U(1)’s is hard to introduce below the GUT scale. So in conventional scenarios with intersecting branes, where one needs the 4d field theory to have finite coupling, the only large volume that is allowed is transverse to all the intersecting branes. A large volume brings down the string scale, and then the energy scale of new vector bosons can’t be too small (the length scale too much bigger that the string length). A possible way out of this is by having the D brane wrapping cycles be small, but the overall volume of the (nontrivial) compact space inhabited by branes be large, but a convincing scenario of this type hasn’t appeared yet.
Now, the advantage is that one needs small coupling in 4d, so one actually needs large extra dimensions parallel to the D5 and D7'. There probably is a constraint on how large these can be, but I will not analyze it.

It is refreshing to see that one also needs large extra dimensions from another perspective. We break susy at the string scale, so presumably we need to bring down the string scale to TeV, while keeping gravity at the Planck scale, which can only be done with large extra dimensions.

Let us recap a few numbers associated with large extra dimensions scenarios. The Planck mass in 4d is given by $M_{Pl}^2 = M^2(RM)^n$ ($n=p-3$ is the number of large extra dimensions of radius $R \gg 1/M \gg 1/M_{Pl}$. If $M \sim 10-100$ TeV, which is the present lower limit for $n=2$ (coming from cosmology and astrophysics), then $RM = M_{Pl}/M \sim 10^{14-15}$ ($M_{Pl} \sim 10^{19}$ GeV), then $R \sim 1-100eV^{-1} \sim .2-20\mu m$. If we increase $n$, we can make $R$ even smaller, or equivalently $M$ smaller. For instance, with $n=4$, $R \sim 10^{15/2}M^{-1} \sim 10^{-5}eV^{-1} \sim 10^{-6}\mu m$.

One must also remember that to get the Standard Model one must give masses to the various fermions and scalars. One needs to break the SU(5) GUT by giving a VEV to an adjoint (24) Higgs. But the 5 of SU(5) must remain massless relative to the GUT scale $M_{GUT} \gg M_{2,3}$, which can be realized if the 2 and 3 branes are separated inside the D5, so that the fundamental fields remain massless. Then $r_{2,3} \sim l_s^2M_{GUT}$ ($<\Phi_{2,3}> \sim M_{GUT}$), so that $r_{2,3} \sim 10^{-3}eV^{-1} \sim 10^{-5}R$ ($r_{2,3} \leq R$=maximum, so it is OK), so one doesn’t need warping to increase the energy scales (one of the lessons derived from the Randall-Sundrum I model [64] is that warping dramatically increases energy ratios).

### 6.3 Adding O(7) planes- how far can we go?

As I mentioned in the previous subsection, the only way I see to get a $\bar{10}$ fermion in the above construction is to introduce orientifolds.

The point is that one has a $10$ in the decomposition of the antisymmetric traceless tensor $44$, and there is an antisymmetric tensor in the D3-D7-O(7) gauge theory. It comes from the (3,3) strings, the 4 scalars corresponding to motion inside the D7 become a 44 (antisymmetric traceless) hypermultiplet. Unfortunately, there is only one such hypermultiplet, not $N_{gen} = 3$, as we need. Moreover, it comes from strings stretching between the D3 and its O(7) image, so when Sp(10) is broken to SU(5) they should become massive with mass= symmetry breaking scale. One could see the problem in another way: the $\bar{10}$ contains a (3,2) fermion, but the (3,2) gauge fields are GUT-scale massive. The first problem could be solved though by putting an O(7)’ plane as well, so now we have $S_3$ and $S_4$ in the 44 of each group, but consequently also of the diagonal subgroup (each multiplet containing a 10 and a $\bar{10}$), and also A and B in the (10,10) each containing a 10 in the diagonal subgroup, so we do have $N_{gen} = 3$ fermions in the $\bar{10}$ among the fields. But short of finding some projection which eliminates the extra orientifold bosons, while keeping the fermions, it is hard to see how to solve the second problem.

One would still need to find a way to give Standard Model fields mass. The masses to the 2 and the 3 fundamental fermions can be obtained by separating the D3 from the D5, while remaining inside the D7. With SU(5) restored, this is a mass for the 5 obtained by giving
a VEV to an adjoint of U(5): \( \langle \phi^1_{ij} \rangle = m \delta_{ij} \rightarrow W = q^i \bar{q}^i = mq^i \bar{q}^i \) (giving a VEV to the U(1) of U(5), really). From a Higgs perspective, we would have expected to give a VEV to a Higgs in the 5 though. That is so since the mass term should come from a Yukawa coupling \( \bar{\Psi} H \Psi \), so the Higgs should be in a representation that appears in the tensor product of the fermion and antifermion representations. In the SU(5) GUT, \( \Psi \) is in 5 and \( \bar{\Psi} \) in the \( \bar{10} \), and \( 5 \times \bar{10} = \bar{5} + 45 \). But if the Higgs in the \( \bar{5} \) is one of the \( q^i \)'s, and the fermion in the \( \bar{10} \) comes from a Z field, the superpotential term \( q^i Z \bar{q}^i \) does give the required fermion mass.

So let us recap the model so far. We have 5 D3 branes inside the \( D7 - \bar{D7} \) condensed to \( N_1 = N_{gen} = 3 \) D5s. We have added \( N_2 \) D7’s in the direction transverse to the D5 and parallel to D3. Up to now the model has a gauge field in the 24 of SU(5) (the center of mass of the D3 is stuck at the intersection of D5 and D7, so the \( U(1)_{cm} \) is lifted), the fermions in the 24 are massive and there are still the scalars in the 24. Then there are \( N_{gen} = 3 \) hypermultiplets in the fundamental, each composed of 2 complex scalars and 2 complex fermions, one in the 5 and one in the \( \bar{5} \). The complex fermions in the 5 must remain massless, the ones in the \( \bar{5} \) must be massive. This happens for the same reason as in the QCD case (the \( \bar{5} \) fermions correspond to the directions transverse to the D5). The complex scalars in the 5 must become massive, whereas one of the 3 complex scalars in the \( \bar{5} \) must be the electroweak Higgs plus its counterpart, so at least the electroweak Higgs must be massless. The GUT Higgs is one of the 4 scalars in the 24, which means that we must have a separation in a nontrivial direction, and all the adjoint scalars have to be massive:

The SU(5) is broken by separating 2 D3s away from the D7’ center inside the D5 (so that the fundamental fields remain massless). In the absence of warping \( r_{2,3} \sim 10^{-3} \text{eV}^{-1} \) must be (much) smaller than the radius R of the corresponding direction. For instance, one can choose one large dimension inside the D5 and one inside the D7’, in which case one has \( R \sim 20 \mu m \), which is OK. All the scalars are massive since the D3 branes are stuck in all directions, except for the \( q^i \) scalar corresponding to the 2 D3’s in the separation direction, as well as the corresponding adjoint. So one has a massless complex doublet (Higgs) and (complex) triplet in the adjoint of SU(2). One has the 5 fermions, as wanted. One can generate some masses for the 2 and 3 fermions by separating the D3 from the D5, but it is not of the type that we want.

It remains to add the \( \bar{10} \)s, get masses for the fermions and get rid of the triplet scalar. We saw that one could add a O(7) and an O(7)’ at the respective D7s, and increase the gauge group to \( Sp(10) \times Sp(10) \), but in this way one generates 4 fields in the \( \bar{10} \) of SU(5) together with many other: the gauge fields are now in the adjoint of \( Sp(10) \times Sp(10) \), which is 110 dimensional, but also there are 2 antisymmetric\s in the \( 44 = 24 + 10 + 10 \) and two bifundamentals in the \( (10,10) \). Mass terms could come from terms like \( q^i Z \bar{q}^i \), as noted before (q in the 5, \( \bar{q} \) in the \( \bar{5} \), Z in the \( \bar{10} \)). There are unfortunately many fields left over. A detailed analysis would involve understanding better the effect of the anti D-brane \( \bar{D7} \) on the action (susy breaking).

All of this is of course, as I mentioned, in the context of TeV strings, where unification is renormalized in an unknown way by string theory, and it is not clear how much can be said in the context of perturbative physics anyway.

Since one doesn’t have a quantitative understanding of the \( D7 - \bar{D7} \) susy breaking, one
can’t do much to describe the new physics (susy breaking corrections) anyway. All we could do is treat the \( \mathcal{N}=1 \) supersymmetric system of D branes, calculate the superpotential, and qualitatively describe susy breaking and the emergence of the Standard Model-like field content.

### 7 Discussions and conclusion

The first result of this paper was the holographic dual for the D4-D8 system, given in [19]. This is different from the holographic dual of the conformally invariant D4-D8-O(8) system given in [16]. The D4-D8 system is nonconformal and a Domain Wall-QFT correspondence is available, in terms of a 6d SU(2) gauged sugra, coupled with a mass parameter h to a U(1) vector multiplet. The same [19] describes the decoupled \( D4 - D8 - \bar{D}8 \) if we change the sign of M on one side. In order to find a holographic dual to large \( N \) QCD, we have to break susy by adding a \( \bar{D}8 \), which together with the D8 will condense to a D6. The \( D4 - D8 - D8 - \bar{D}8 \) system is described in [57] implicitly (up to one integro-differential equation for the variable \( H_{p,q;4}(x) \)). Also, in order to have a 4d field theory, and get chiral fermions, we need to go to a D3-D7-D7’ system.

The holographic dual \( (l_s \to 0, N \to \infty) \) is then

| coord. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|--------|---|---|---|---|---|---|---|---|---|---|
| D3    |   | x | x | x | x | - | - | - | - | sm |
| D7-D7 |   | x | x | x | x | x | x | - | - | sm |
| D5    |   | x | x | x | x | x | - | - | - | sm |
| D7'   |   | x | x | x | x | x | x | x | x |   |

And the fields left over are the SU(N) adjoint gauge field \( A_{\mu}^{(ab)} \) and the \( (N, N_f) \) fermion \( \psi_{ai} \), the \( \mathcal{N}=1 \) partner of \( \phi_{\bar{4}-\bar{5}} \).

The adjoint fermions are decoupled because their operators couple to fields moving in the nonsupersymmetric bulk, and so get very large anomalous dimensions, while the fundamental fermions are still there, since their operators are constrained to lie on the supersymmetric D7 plane. The adjoint and fundamental scalars get masses because of the D3 being fixed in the extra dimensions. The condensation doesn’t take place because it would take a very large time.

The mechanism for susy breaking could be used to embed the Standard Model into string theory via a SU(5) (or higher) braneworld GUT model, but we find that we come short of that goal. First of all, one would need a TeV scale string theory scenario, which is problematic per se. Without the use of orientifolds, we could not find a 10 fermion in the SU(5) GUT scenario. With orientifolds, there are too many fields and the masses of the Standard Model fields don’t seem to be what we want. Yet it is remarkable that one has a whole new class of nonsupersymmetric theories similar to the Standard model with the gauge group arising on the worldvolume of D branes.

In the context of D6 intersections, remarkable progress has been made towards embedding the Standard Model (see [71, 72] for a review). In particular, [73] gave an embedding of just the Standard Model (non susy). One of the versions in [74] e.g., contains also a \( D3 - D7 - D7' \)
system (but a different version). It is therefore conceivable that by combining the virtues of both one could find a good phenomenological example of Standard Model embedding.

The model discussed in the paper (in one possible parametrization) is (here $l=$large, $s=$small, $sm=$smeared)

| coord. | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|--------|---|---|---|---|---|---|---|---|---|---|
| 2 D3s (A) | x | x | x | x | - | - | - | - | - | - | sm |
| 3 D3s (B) | x | x | x | x | - | - | - | - | - | - | sm |
| D7- $\bar{D}7$ | x | x | x | x | x | x | x | - | - | - | sm |
| $N_1=3$ D5s | x | x | x | x | x | x | - | - | - | - | sm |
| $N_2$ D7's | x | x | x | x | - | - | x | x | x | x |
| size | $\infty$ | $\infty$ | $\infty$ | $\infty$ | l | s | l | s | s | s |

The separations are of order (for a simple model) $r_4(AB) \sim 10^{-3}eV^{-1} \sim r_4(AC)$ ; ($r_4(BC) \sim 0$). The “massless” fields are $q^{ai}_{4-5}(A)$=Higgs doublet, $\phi^{(ab)}_{4-5}(A)$ (extra complex Higgs triplet) and $\psi^{ai}(A,B)$ (fermion matter). When one adds O(7) and O(7)', the model becomes quite complicated.

When trying to apply lessons from the holographic dual theory, we have relaxed two conditions: $N_c$ is now finite (and small), and $l_s$ is nonzero. As a result, gravity is not decoupled, but still lives at a high energy scale, but a lot of the arguments go through. In particular, the analysis of which fermions decouple (or become string-scale massive, in this case), and which fermions remain massless should stay the same.

It was essential that one had the $Dp-\bar{D}p$ condensation to Dp-2 happen in a “frozen” gauge theory sector, giving the fundamental fields, (as opposed to having the worldvolume D3 brane giving the Standard Model gauge theory as the endpoint of condensation), in order to keep the fundamental quarks. It was also essential that there was only one nontrivial transverse coordinate ($D8-\bar{D}8$ or rather $D7-\bar{D}7$ with a coordinate averaged over, being very small), so that we can have a metastable state keeping the condensation process at bay and still describe what happens in the Standard Model field theory. Finally, having a D3-D7 system was important, since it allowed introduction of another D7' breaking the susy to $N'=1$, a requirement for a good phenomenology (having complex representations as opposed to real for $N'=2$).

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Appendix A. Review of SU(5) GUT and higher GUTs

I will review now some relevant facts about unification, see e.g. [75] or [76].

The $SU(3) \times SU(2) \times U(1)$ Standard Model has 3 generations of quarks and leptons,

\[
\begin{align*}
\text{quarks} & \begin{pmatrix} u \\ d \end{pmatrix} & \begin{pmatrix} c \\ s \end{pmatrix} & \begin{pmatrix} t \\ b \end{pmatrix} \\
\text{leptons} & \begin{pmatrix} e \\ \nu_e \end{pmatrix} & \begin{pmatrix} \mu \\ \nu_\mu \end{pmatrix} & \begin{pmatrix} \tau \\ \nu_\tau \end{pmatrix}
\end{align*}
\tag{A.1}
\]

together with a Higgs doublet.

Let us describe the first generation. The creation operators for right-handed particles is

\[
u^+, d^+, e^+, (\bar{u}^+, \bar{d}^+) = \bar{\psi}^+, (\bar{e}^+, \bar{\nu}^+) = \bar{l}^+
\tag{A.2}
\]

with quantum numbers under $(SU(3), SU(2))U(1)_Y$

\[
u^+: (3, 1)_{2/3}, d^+: (3, 1)_{-1/3}, e^+: (1, 1)_{-1}, \bar{\psi}^+: (\bar{3}, 2)_{-1/6}, \bar{l}^+: (1, 2)_{1/2}
\tag{A.3}
\]

and the creation operators for the left handed fields transform in the conjugate representation. These get unified in SU(5) as follows:

\[
(3, 1)_{-1/3} + (1, 2)_{1/2} = 5
\tag{A.4}
\]

and

\[
(3, 1)_{2/3} + (1, 1)_{-1} + (\bar{3}, 2)_{-1/6} = \bar{10}
\tag{A.5}
\]

So a full generation of quarks and leptons fills up a fundamental (5) and a antisymmetric tensor (bar), the $\bar{10} = (\bar{5} \times \bar{5})_a$, all of which must be massless from the point of view of the unification scale. One usually talks about the left-handed operators,

\[
\bar{d}_L, e_L, (\nu_e)_L = 5 \\
u_L, d_L, \bar{u}_L, \bar{e}_L = 10
\tag{A.6}
\]

The gauge field will be in the adjoint of SU(5), the 24, and it will contain also the X, Y bosons, the “leptoquarks”

\[
\begin{pmatrix} Y^{-1/3} \\ X^{-4/3} \end{pmatrix} = (3, 2)_{-5/3} \\
\begin{pmatrix} \bar{X}^{4/3} \\ \bar{Y}^{1/3} \end{pmatrix} = (\bar{3}, 2)_{5/3}
\tag{A.7}
\]

which mediate transitions between quarks and leptons and quarks and antiquarks, therefore violate B and L, hence give proton decay.

There must be an adjoint Higgs (in the 24) which breaks the SU(5). Indeed, the U(1) generator, S, in SU(5) commutes with $SU(3) \times SU(2) \times U(1)$, so an adjoint Higgs with VEV in the S direction does the trick. It will give mass of the order of the unification scale to the X and Y gauge bosons.
Additionally, one must have a Standard Model Higgs doublet, which will give masses to the Standard Model fermions. To have masses for the u, d, and e, this doublet must be contained in either a 5 or a 45 of SU(5). So a complex 5 Higgs will do the job (as will a 45). The problem with that is though that the SM doublet must have the mass of a few 100 GeV for electroweak SSB, whereas the triplet Higgs $H^{\pm 1/3}$ can mediate B and L violation, so it must also have the mass of the order of the GUT scale.

Yet additionally, if we want a neutrino mass, we need a “see-saw mechanism”. It can be added in by a right-handed neutrino, which in SU(5) can only be a singlet $N_R$. We can have a Dirac mass term $m \bar{\nu}_L N_R$, and a large Majorana mass term $M N_R N_R$, with total mass terms

$$\begin{pmatrix} \nu_L & \bar{N}_R \\ \bar{N}_R & \nu_L \end{pmatrix} \begin{pmatrix} 0 & m \\ m & M \end{pmatrix} \begin{pmatrix} \nu_L \\ \bar{N}_R \end{pmatrix}$$

(A.8)

which can be diagonalized to mass eigenstates of $m_1 = m^2 / M$ and $m_2 \simeq M$, with $\nu_1 \simeq \nu_L, \nu_2 \simeq N_R$.

Another unification (most popular at the moment) is given by $SO(10) \rightarrow SU(5) \times U(1)$, under which the spinor representation splits as $16 \rightarrow 10_{-1} + 5_3 + 1_{-5}$, the fundamental as $10 \rightarrow 5 + 5$ and the adjoint (antisymmetric tensor) splits as $45 \rightarrow 24_0 + 10_4 + 10_{-4} + 1_0$. The advantage is that all the quarks in one generation are in one representation (the 16). Moreover, the extra 1 can be attributed to the righthanded neutrino. The gauge field and the SU(5) Higgs live in an adjoint 45, and the Standard Model Higgs in a fundamental 10.

Yet another avenue of research is given by the Sp groups obtained from orientifolds. In particular, $Sp(10) \rightarrow SU(5) \times U(1) = U(5)$, under which again $10 \rightarrow 5 + 5$, and the adjoint (symmetric tensor) $55 \rightarrow 24 + 15 + 15 + 1$ and the antisymmetric traceless tensor $44 \rightarrow 24 + 10 + 10$. The problem is that all these representations are real, but we know that symmetry breaking should work, in the form of the branes going away from the orientifold point. Note that Sp(2n) has the same rank and the same adjoint representation as SO(2n+1) (the Sp(2n) adjoint is symmetric=n(2n+1), and the SO(2n+1) adjoint is antisymmetric=n(2n+1) also, and the rank is n for both). In particular, Sp(10) will be related to SO(11).
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