Gauge potential singularities and the gluon condensate at finite temperatures

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The continuum limit of SU(2) lattice gauge theory is carefully investigated at zero and at finite temperatures. It is found that the continuum gauge field has singularities originating from center degrees of freedom being discovered in Landau gauge. Our numerical results show that the density of these singularities properly extrapolates to a non-vanishing continuum limit. The action density of the non-trivial $Z_2$ links is tentatively identified with the gluon condensate. We find for temperatures larger than the deconfinement temperature that the thermal fluctuations of the embedded $Z_2$ gauge theory result in an increase of the gluon condensate with increasing temperature.

1. INTRODUCTION

A precise definition of a Quantum Field Theory (QFT) is provided by the critical limit of a lattice model. Thereby, the QFT is solely specified by the number of space-time dimensions and symmetries. Here, we will critically re-investigate the lattice gauge theory with Wilson action which is assumed to reduce to continuum SU(2) Yang-Mills theory in the critical limit.

It is usually assumed that all SU(2) link variables $U_\mu(x)$ can be expanded in the vicinity of the unit element for sufficiently small lattice spacing $a$, i.e. $U_\mu(x) = \exp\{iW_\mu(x) a\}$, such that the Wilson action density reduces to the Yang-Mills Lagrangian

$$\frac{\beta}{2} \text{tr}\{1 - P_{\mu\nu}[U]\} \rightarrow \frac{a^4}{2g^2} F^b_{\mu\nu}[W](x) F^b_{\mu\nu}[W],$$

where $\beta = 4/g^2$, $P_{\mu\nu}[U](x)$ is the plaquette calculated in terms of the link elements $U_\mu(x)$ and

\begin{equation}
F^b_{\mu\nu}[W] \text{ is the usual field strength tensor. We will find that this Taylor expansion is not always justified. We will propose to relate the action density of the corresponding singularities to the gluon condensate.}
\end{equation}

2. THE LATTICE THEORY OF GAUGE POTENTIAL SINGULARITIES

Before we can localize the links where the above expansion eventually fails, we must bring the link elements as close as possible to the unit element by exploiting the gauge freedom $\Omega = \prod_x \omega_x$. We are thus led to implement an algorithm which puts the Monte Carlo configurations into the Landau gauge, accomplishing $\sum_{x,\mu} \text{tr} \ U_\mu^\Omega(x) \rightarrow \text{max}$, which is most suitable for our purpose. We have used an improved simulated annealing algorithm in order to find the maximum of the gauge fixing functional (details will be presented elsewhere). We then decompose the appropriately gauged link elements $U^\Omega$ into a center part and a coset part

$$U^\Omega_\mu(x) = Z_\mu(x) \exp\left\{i A^b_\mu(x)t^b a\right\},$$

where $A^b_\mu(x)$ is the gauge field, $t^b$ are the generators of the $SU(2)$ group, and $Z_\mu(x)$ is the center element.

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3. THE FINITE TEMPERATURE GLUON CONDENSATE

The Operator Product Expansion (OPE) somewhat artificially distinguishes between contributions from perturbative gluons and "other" contributions to Green functions, which are not further specified. In lattice regularization, the perturbative gluon contribution is recovered by expanding the link variables (in Landau gauge) around the unit elements. Thus, the singular component, originating from the non-trivial center elements, contributes to the "other parts" which are parameterized by the condensates of the OPE. In particular, the gluon condensate is defined as the expectation value of the action density where the (divergent) perturbative gluon contribution has been subtracted. In practice, this subtraction uses the result of a high order calculation in lattice perturbation theory \[3\]. Alternatively, the gluon contribution can be removed by a cooling procedure which reduces the action of the coset (gluon) fields \[\Omega(x)\]. The latter approach suggests that the gluon condensate \[G\] gets contributions from the energy density stored in the underlying \[Z_2\] fields, which are revealed by the coset cooling mechanism. We therefore find \[G a^4 \propto c = 12 \rho a^4\].

It was already observed above (see figure \[3\]) that \(G\) is lattice spacing independent and non-vanishing close to the continuum limit.

Let us now study the temperature dependence of the gluon condensate \(G\) which is defined by \(3\) and \(4\). Temperature is introduced by varying the number \(N_t\) of grid points in time direction, \(T = 1/N_t a(\beta)\). We used the one loop formula for \(a(\beta)\) and the string tension \(\sigma = (440 \text{ MeV})^2\) as the reference scale. Simulations were performed on \(16^3 \times N_t, N_t = 4...16\) lattices for \(\beta = 2.3, 2.4, 2.5\). Due to the space-time asymmetry induced by the finite temperature, it is convenient to independently measure the spatial and the time-like parts of the gluon condensate, i.e. \(G_s\) and \(G_t\). These condensates are calculated from \(3\) where \(c\) is separately evaluated with spatial-spatial and spatial-time-like plaquettes, respectively. The result is shown in figure \[3\].
Below the deconfinement temperature for SU(2) gluodynamics, $T_c \approx 300$ MeV, the condensates $G_s$ and $G_t$ are weakly temperature dependent. Above $T_c$, the time-like component $G_t$ is rapidly increasing with $T$ while the variations of $G_s$ with $T$ are still moderate.

We interpret this behavior as follows: at zero temperature, the vacuum energy density of the underlying $Z_2$ gauge fields generates the gluon condensate. In the deconfined phase at high temperatures, these fields start thermally fluctuating in addition to the fluctuations of the coset (gluon) fields. The thermal energy density stored in the $Z_2$ system induces the rise of the gluon condensate with temperature while the gluonic black body radiation does not contribute by definition.

5. CONCLUSIONS

Landau gauge fixing of SU(2) lattice gauge theory reveals a $Z_2$ vacuum texture. In the continuum limit, this texture consists of point-like gauge potential singularities with a density of $\rho \approx 0.7/fm^4$. The action density which is stored in the embedded $Z_2$ gauge system is identified with (part of) the gluon condensate. At temperatures above the deconfinement transition we observed that the $Z_2$ system carries thermal energy density leading to a gluon condensate which increases with temperature.

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