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Fatigue Performance of CRTS III Slab Ballastless Track Structure under High-speed Train Load Based on Concrete Fatigue Damage Constitutive Law

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Abstract

The mechanical characteristics of CRTS III (China Railway Track System III) slab ballastless track structure under high-speed train load were analyzed by both the full-size fatigue test and numerical simulation in commercial FEM (finite element method) software Ansys. Considering the damage characteristics of concrete, a concrete fatigue damage constitutive model with the concept of mode-II microcracks for concrete was selected and applied in the FEM model. Based on this model, the fatigue FEM model of CRTS III slab ballastless track system was established and relative numerical simulations were conducted. Also, the simulation results were verified by full sized fatigue test on full sized CRTS III slab ballastless track structure. In addition, through the FEM analysis of CRTS III slab ballastless track structure under fatigue loading, the damage evolution law of CRTS III slab ballastless track structure under high-speed train load was also explored and concluded. It was illustrated that the numerical simulation results by FEM model established in this work was in well agreement with the fatigue test results. Therefore, the relative findings and conclusions from the tests and FEM analysis in this work were able to provide significant references for relevant researchers.

1. Introduction

Ballastless track system is widely applied in high-speed railway structure due to its advantages in riding comfort and high stability. Generally, relate to the ballastless track system, there are mainly 4 types utilizing in China, which are: CRTS (China Railway Track System) I, CRTS II, CRTS III and Double-block structure. Specifically, the ballastless track used in CRTS III is developed based on the main characteristics of that used in the CRTS I, CRTS II and double-block structure with certain improvements which has Chinese independent intellectual property rights. Until now, in China, the total length of CRTS III slab ballastless track line under construction and operation is nearly 4000 kilometers (Wang et al. 2017). However, since the service time of CRTS III slab ballastless track system is comparative short, there are several problems appeared on structure performance during the processes of construction and operation (Gao et al. 2013; Liu et al. 2013; Liu et al. 2014; Yang 2014; Zeng et al. 2016; Zhou and Liu 2011; Zhou et al. 2013). Among those problems, the structure response analysis during the fatigue loading due to the high-speed trains is the most attractive topic for certain researchers. In detail, according to present literature, the studies and analysis of fatigue performance and fatigue life prediction are mainly based on the empirical S-N curve and linear fatigue cumulative damage theory. Sun (2013) discussed the static mechanical characteristics of CRTS III slab ballastless track structure and proposed the critical loading position and the most disadvantageous position in the structure under static loading. For numerical simulation, by applying the damage constitutive model coupled with plasticity in concrete, Wang (2012) discussed the maximum crack width and fatigue characteristics of track structures, and further predicted the fatigue life of CRTS III slab ballastless track based on the S-N curve method. He (2011) has predicted the fatigue life of CRTS III slab ballastless track structure considering both the train load and temperature load by using Palmgren-Miner fatigue damage theory and the S-N curve method. However, in those literature, the build-in constitutive models utilized for numerical simulation in common commercial FE software are not able to accurately reflect the mechanical response of the concrete in ballastless track structure under high speed train load, especially the damage development in the structure during the fatigue loading. Therefore, a proper fatigue damage model for concrete should be selected and relative application in the FEM software should be developed in order to reveal the initiation and evolution of damages, and results in a reasonable analysis for the mechanical properties of CRTS III slab ballastless track structure during the fatigue loading.

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2. Concrete fatigue damage constitutive model

2.1 The bundle-chain model

In nearly a century, concerning to the concrete, a number of constitutive models have been proposed. Specifically, the modeling of heterogeneity in the stress-strain relationship of concrete during the loading process is a key characteristic in numerical simulation. By considering the random pre-existing micro-flaws and cracks in concrete, a method named fiber bundle models (FBMs) (Wu et al. 2006, 2013; Li and Ren 2009; Le et al. 2011; Le and Bazant 2011) focus on the stochastic properties with the approach of materials’ micro-mechanics is developed. In detail, with the randomly scattered micro-cracks and voids, when concrete is subjected to external loads, the evolution and the propagation of the initial damage will cause the degradation of materials’ stiffness and finally results in the observation on the nonlinearity in the stress-strain relationship during the experiment. Based on above mechanics, the FBMs is widely accepted in both physical and engineering communities. In addition, during the loading process, the appearance of irreversible strain of concrete is also a significant factor which influences the mechanical properties in structures. Generally, there are two main physical mechanisms which cause the production of irreversible strain in concrete: mode-II microcracks and irreversible-frictional sliding (Abu et al. 2003; Bazant and Hubler 2014; Dascalu et al. 2010; Faria et al. 1998; Lee and Fenves 1998; Paskin et al. 1985; Dienes 1987; Gumbsch 1995; Gumbsch and Beltz 1995; Zhou et al. 1995). In order to characterize the irreversible strains in concrete due to above mechanisms, the irreversible chain model (ICM) is introduced into normal FBMs in order to couple the elastic and irreversible strains (Hidalgo et al. 2001, 2002, 2008; Pradhan et al. 2002, 2010; Raischel et al. 2006). However, in those proposed literature, the coupling of irreversible deformation and damage is lacking for clear physical definition. Therefore, by considering the advantages of Chain model (CM), a model called Bundle-chain model (BCM) based on the combination of irreversible micro-elements and normal elastic-elements in FBMs has been introduced (Shan and Yu 2015). In this model, the irreversible strains in concrete are modeled by a chain made up of several irreversible micro-elements connected together (Fig. 1). During the loading process, when the strain exceeds the yield strain threshold of a plastic micro-element, this element will slip and result in generation of irreversible strains. Compared to the normal CM, the plastic chain will not break and the accumulation of irreversible strains is able to be illustrated. In this model (Fig. 2), the fracture threshold and probability density of the fiber bundle is considered to be \( \varepsilon_{\text{th,e}} \) \((0 \leq \varepsilon_{\text{th,e}} \leq \varepsilon_{\text{th,e max}}) \) and \( P(\varepsilon_{\text{th,e}}) \). The yield threshold and probability density of the irreversible micro-element is considered to be \( \varepsilon_{\text{th,i}} \) \((0 \leq \varepsilon_{\text{th,i}} \leq \varepsilon_{\text{th,i max}}) \) and \( P(\varepsilon_{\text{th,i}}) \).

2.2 The Fatigue damage constitutive model

Based on BCM, for a certain fatigue loading process (a certain stress level \( S_{\text{max}} \)), the scalar damage constitutive model of concrete is given as follows (Shan and Yu 2015; Yu et al. 2016):

\[
\sigma^+ = (I - D^+) E_0 \varepsilon
\]  

Fig. 1 Bundle-chain model (BCM) for concrete under uniaxial loading.
where + and – denote tension and compressive uniaxial cases. \( n \) denotes fatigue load cyclic numbers. \( E_0 \) represents initial Young’s modulus. \( I \) is the identity tensor. \( D \) denotes the scalar damage variable in uniaxial cases. \( D_n \) denotes the scalar irreversible-damage variable in uniaxial cases. \( \varepsilon \) is the irreversible-damage strain.

During the fatigue loading, the damage evolution process is strongly influenced by the fatigue load cyclic numbers. Considering both the influence of fatigue load cyclic numbers and characteristics of BCM, the definition of damage variables in uniaxial fatigue loading with constant amplitude is given as follows (Fig. 3):

\[
D_n(n) = D_u(n) + D_i(n) = \frac{\sigma_n - \sigma_{\text{max}}}{\sigma_v} = \frac{\varepsilon_n - \varepsilon_{\text{max}}}{\varepsilon_v}
\]

where \( D_u(n) \) denotes fatigue elastic-damage variable. \( D_i(n) \) denotes fatigue irreversible-damage variable. \( \sigma_n \), \( \sigma_{\text{max}} \) and \( \varepsilon_n \) denote the stress reduction of concrete after \( n \) times fatigue loading due to the total damage, elastic-damage and irreversible-damage. \( \sigma_v \) denotes effective stress and \( \sigma_{\text{max}} \) denotes maximum fatigue stress. \( \varepsilon_n \), \( \varepsilon_{\text{max}} \) and \( \varepsilon_n \) denote the strain evolution of concrete after \( n \) times fatigue loading due to the total damage, elastic-damage and irreversible-damage. \( \varepsilon_n \) denotes total strain and \( E_0 \) represents initial Young’s modulus. Therefore, with the application of BCM, above definitions are able to illustrate the development of damage during the fatigue loading with clear physical meaning. Then, the fatigue damage constitutive model of concrete is given as follows:

\[
\sigma_n = \left[ I - D(n) \right] E_0 \varepsilon_n
\]

The verification of this model is conducted by comparison with the mean value of the experimental results and showed it is able to be effectively modeling the stress-strain relationship of concrete under fatigue loading, as shown in Fig. 4 (the stress-average denotes the mean value of the experimental results and the stress-model denotes value calculated by this model). Therefore, the tensile/compressive constitutive curve of self-compacting concrete and the concrete used as bed plate can be determined, as shown in Figs. 4 and 5.

3. Fatigue test of CRTS III slab ballastless track structure

3.1 Specimen fabrication

The CRTS III slab ballastless track structure consists of rail, fastening system, track slab, self-compacting concrete layer, isolation layer, bed plate, support layer and other parts, as shown in Fig. 6. The functions and basic significations of main members are shown in Tables 1 and 2. According to China National Standard (2017): Provisional technical regulations of prestressed concrete track slab of CRTS III slab ballastless track in high speed railway (including the rebar arrangement and prestressing condition), the CRTS III track slab for test is selected as the type P5600 (size is 5600 mm × 2500 mm × 200 mm) slab, which is widely used in the passenger dedicated line of Zhengzhou to Xuzhou in China. The slab is produced and cured according to the standardize production process in the track slab factory of China Railway No.3 Engineering Group Co. Ltd. After the
concrete material reached required strength and maturity, the track slab is transported to the laboratory for further usage. In addition, in order to ensure the construction of bed plate and self-compacting concrete layer are identical to the actual projects, in this test, the bed plate and the self-compacting concrete layer are produced by the professional construction team of China Railway No. 4 Engineering Group Co. Ltd. The dimensions and fabrication process of these two layers is strictly obey the requirements of China National Standard (2013a, 2014): Provisional technical regulations of self-compacting concrete of CRTS III slab ballastless track in high speed railway and Code for design of high speed railway. The isolation layer is set up before pouring the self-compacting concrete based on China National Standard (2013b): Provisional technical regulations of geotextile for isolation layer of CRTS III slab ballastless track in high speed railway. In detail, the fabrication process of the specimen is shown in Fig. 7. Furthermore, in this test, the rail and the fastening system is selected according to China National Standard (2014, 2015a): Code for design of high speed railway and Fastening systems for high speed railway.

3.2 Test content

The test has been completed with PMS-500 digital display pulsating machine in National Engineering Laboratory for High-Speed Railway Construction. The schematic diagram of the testing system is shown in Fig. 8. The specimen was consolidated with the support layer which was fixed on a rigid foundation. The test is consisted of two parts: fatigue test and static load test. For the fatigue

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| Table 1 The functions of main members. |
|----------------------------------------|
| **Main members** | **Materials** | **Main functions** | **Auxiliary functions** |
|-------------------|--------------|--------------------|------------------------|
| Track slab        | Concrete (C60) | 1. Provide fastener interface and keep track geometry. | 1. Provides an interface for track circuits and integrated grounding. |
|                   |              | 2. Become a Composite structure with self-compacting concrete layer to bear load. | 2. Improve the structural durability. |
| Self-compacting concrete layer | Self-compacting concrete (C40) | 1. Construction adjustment. | 3. To form a limit convex platform. |
|                   |              | 2. Become a Composite structure with self-compacting concrete layer to bear load. | 1. It has better liquidity and homogeneity than normal concrete |
| Isolation layer   | Geotextile   | 1. Interlayer isolation, release of temperature difference and contraction stress. | 1. Buffer action. |
|                   |              | 2. Reduced stress concentration | 2. Provide conditions for maintenance and maintenance. |
| Bed plate         | Concrete (C30) | 1. Bear and disperse train loads. | 2. Coordinated foundation deformation. |

| Table 2 The basic specifications of main members. |
|-----------------------------------------------|
| **Rail** | **Track slab** | **Self-compacting concrete layer** | **Bed plate** | **Isolation layer** | **Support layer** |
| Length  | 5600 mm | 5600 mm | 5600 mm | 5600 mm | |
| Width   | 2500 mm | 2500 mm | 2900 mm | 2600 mm | |
| Height  | 200 mm  | 90 mm   | 200 mm  | 4 mm    | |
| Concrete strength | C60 | C40 | C30 | |
| Elasticity modulus | 210 GPa | 36000 MPa | 34000 MPa | 32000 MPa | 3.32 MPa |
| Poisson’s ration | 0.3 | 0.2 | 0.2 | 0.2 | 0.35 |
| Density | 7800 kg/m³ | 2500 kg/m³ | 2500 kg/m³ | 2500 kg/m³ | 700 g/m² |
| Stress      | 1000 MPa/m | |

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Fig. 5 The tensile (a)/compressive (b) constitutive curve of bed plate concrete.
The upper limit of the fatigue load is set up as 408 kN and the lower limit of fatigue load is 40.8 kN according to China National Standard (2014): Code for design of high speed railway. The fatigue load is set up as a sine form load with 5 Hz frequency. In addition, the static load test is carried out after a certain fatigue load cyclic numbers being applied to the structure for studying the characteristic of damage evolution in concrete. Specifically, there are five load levels applied in the static load test: 82 kN, 164 kN, 246 kN, 328 kN and 408 kN. For each load level, the relative displacements of each structural layer and the strains of concrete are measured. Furthermore, it is noted that the destruction test was not carried out in this work, the information of static capacity for each member concerning the failure can be referenced from relevant literature (Jin, 2016).

The test elements include strain gauges, displacement sensors and acceleration sensors. The arrangements of the strain gauges are shown in Fig. 9. The $q$, $f$, $h$, $s$ represent the quarter of the length of the plate, vertical loading points, the middle of the lengthwise direction or width direction of the plate, the edge of plate in lengthwise direction. The $l$ and $t$ represent longitudinal and transverse direction. For example, $h-s-l$ represent the longitudinal direction strain/stress at the middle of plate edge in lengthwise direction of the plate. In addition, in order to measure relative displacement and acceleration at loading point and midspan, the arrangements of displacement sensors and acceleration sensors are shown in Fig. 10.

### 3.3 Test results

According to the strain results obtained from the test, the stress can be calculated by equation (3) - (5) which are shown in Table 3 (+/- means tension/compression), the measure points of track slab on upper surface are in the compressive state during the fatigue loading process. Due to the high strength of the concrete material and the two-dimensional prestressing structure used in the track board, as the fatigue load cyclic numbers increasing, the

![Fig. 7 The production of the test specimen.](image)

![Fig. 8 Schematic diagram of the testing system.](image)
compressive stresses were decreasing slightly. When the fatigue load cyclic numbers reached 4 million times, the maximum compressive stresses ranged from 0.63 MPa to 0.70 MPa which were reduced by less than 7%. Therefore, after 4 million times of fatigue loading cyclic numbers, the track slab was considered to still maintain excellent mechanical properties. The measure points in self-compacting concrete layer were under tensile state during the fatigue loading process. As the fatigue load cyclic numbers increasing, the tensile stresses were increasing significantly. When the fatigue load cyclic numbers reached 4 million times, the maximum tensile stresses ranged from 0.4 MPa to 0.75 MPa were increased by 183% to 241%. The measure points of bed plate on upper surface were under tension during the fatigue loading process. As the fatigue load cyclic numbers increasing, the tensile stresses were growing. When the fatigue load cyclic numbers reached 4 million times, the maximum tensile stresses ranged from 0.28 MPa to 0.42 MPa which were increased by 154% to 167%.

The development of the acceleration of the CRTS III slab ballastless track system under the fatigue loading process is shown in Fig. 11. As the fatigue load cyclic numbers increasing, the accelerations of the track slab were decreasing. The acceleration at the loading point was 4.7% higher on average than that at the mid-span of the track slab. When the fatigue load cyclic numbers reached 4 million times, the acceleration at the loading point was reduced from 308 mg to 271 mg, and the acceleration at the mid-span was reduced from 292 mg to 261 mg. However, as the fatigue load cyclic numbers increasing, the accelerations of the bed plate were growing. The acceleration at the loading point was 11.7% higher on average than that at the mid-span of the bed plate. When the fatigue load cyclic number was 4 million times, the acceleration at the loading point was increasing from 151 mg to 174 mg, and the acceleration at the mid-span was reduced from 137 mg to 154 mg. This phenomenon was concluded as resulting from the changing of the stiffness of the isolation layer during the fatigue loading. As the fatigue load cyclic numbers increasing, the isolation layer was gradually compacting to a denser state which caused the increase of the stiffness. Then, larger amount of vibration was passed from the track slab to the bed plate through the isolation layer and the vibration of each layer tends to be aligned. Therefore, the acceleration of the track slab decreased, and the ac-

### Table 3 The stress of each measure points (MPa).

| Measuring point                  | 1               | 0.1 Million | 0.5 Million | 1.5 Million | 3.0 Million | 4.0 Million |
|---------------------------------|-----------------|-------------|-------------|-------------|-------------|-------------|
| Track slab f-s-l                | -0.68           | -0.67       | -0.66       | -0.65       | -0.63       | -0.63       |
| Track slab f-s-t                | -0.74           | -0.73       | -0.72       | -0.71       | -0.70       | -0.70       |
| Track slab h-s-l                | -0.71           | -0.71       | -0.70       | -0.69       | -0.69       | -0.68       |
| Track slab h-s-t                | -0.72           | -0.72       | -0.71       | -0.70       | -0.70       | -0.70       |
| Self—compacting concrete layer f-s-l | 0.22         | 0.24       | 0.28       | 0.36       | 0.51       | 0.75       |
| Self—compacting concrete layer f-s-t | 0.18         | 0.21       | 0.24       | 0.29       | 0.43       | 0.67       |
| Self—compacting concrete layer h-s-l | 0.12         | 0.14       | 0.18       | 0.22       | 0.31       | 0.40       |
| Self—compacting concrete layer h-s-t | 0.12         | 0.14       | 0.16       | 0.20       | 0.28       | 0.34       |
| Bed plate f-s-l                 | 0.16            | 0.18       | 0.22       | 0.27       | 0.32       | 0.42       |
| Bed plate f-s-t                 | 0.15            | 0.15       | 0.18       | 0.23       | 0.29       | 0.41       |
| Bed plate h-s-l                 | 0.11            | 0.13       | 0.16       | 0.21       | 0.26       | 0.32       |
| Bed plate h-s-t                 | 0.11            | 0.13       | 0.15       | 0.19       | 0.24       | 0.28       |
The development of the relative displacement between rail and track slab at the loading point is shown in Fig. 12. Under the action of load, the relative displacement between rail and track slab is consisted by two parts: the flexural deformation of the track slab and the compressive deformation of the fastener system. Under maximum load level (408 kN), the CRTS III slab ballastless track system will bend which make the track slab in the compressive state and the self-compacting concrete layer and the bed plate in the tensile state. As the fatigue load cyclic numbers increasing, the flexural deformation increased, and the compressive deformation of the fastener system decreased. At the loading point, due to the flexural deformation of the structure is apparent small, the compressive deformation of the fastener system was dominant. Therefore, as the fatigue load cyclic numbers increasing, the relative displacement between rail and track slab was decreasing. When the fatigue load cyclic numbers reached 4 million times, under maximum load level (408 kN), the relative displacement between rail and track slab was reduced by 10.8%, as shown in Fig. 12 (b).

The development of the relative displacement between track slab and bed plate at the loading point is shown in Fig. 13. During the fatigue, the relative displacement between track slab and bed plate is consisted by two parts: the flexural deformation of the structure and the compressive deformation of the isolation layer. As the fatigue load cyclic numbers increasing, the flexural deformation of the structure was increasing caused by the damage accumulation and the compressive deformation of the isolation layer was decreasing caused by dense increasingly. At the loading point, due to the flexural deformation of the structure is comparative small, the compressive deformation of the isolation layer was considered as dominant. Therefore, as the fatigue load cyclic numbers increasing, the relative displacement between track slab and bed plate was decreasing. When the fatigue load cyclic numbers reached 4 million times, under maximum load level (408 kN), the relative displacement between track slab and bed plate was reduced by 16.9%, as shown in Fig. 13 (b).
4. Numerical Simulation

4.1 The selection of parameters

In the finite element (FE) model, the main structure components of CRTS III slab ballastless track system are consisted of rail, track board, self-compacting concrete layer, isolation layer, bed plate, support layer and other parts. The parameters of those components are listed in Table 2. In detail, the parameters of rail and the fastener are set up refer to the properties of CHN60 type rail and WJ-8 type fastener listed in China National Standard (2011, 2015a): Rails for high speed railway and Fastening systems for high speed railway. The dynamic stiffness of fastener is determined as 50 kN/mm, which is 1.5 times of its static stiffness. In addition, according to the China National Standard (2015b): Code for design of concrete structures, the fatigue load is set up as 1.5 times of the static load applied by train to the track system (255 kN/per wheel pair) at the design speed 350 km/h. In this work, there are two loading types used in the numerical simulation: in the part of the model verification, the loading type is selected as the same as the fatigue test. In the part of damage prediction, based on the verified model, the loading type is selected by only considering the load produced by single shaft with double wheels, referred to the method stipulated in the China National Standard (2015b): Code for design of concrete structures. Furthermore, for both loading types, the loading method is choose as a sine curve with 5 Hz frequency.

4.2 The selection of constitutive models

In this work, only concrete material and steel bars are considering utilizing the fatigue constitutive models. The constitutive models for other materials in the finite element model are considering as the static model. Specifically, the model of concrete is selected as the fatigue damage constitutive model which is described in section 2. The fatigue constitutive model of the steel bars is selected according to China National Standard (2015b): Code for design of concrete structures. In detail, the elastic modulus of the steel bars is assumed to be constant under fatigue load. The fatigue constitutive model for the steel bars is shown as follow:

\[ f_y(N) = \left( \sigma_{min} + \Delta\sigma \right) \left[ 1 - \frac{\lg N}{\lg N_f} \left( 1 - \frac{\sigma_{min} + \Delta\sigma}{f_y} \right) \right] \quad (6) \]

\[ \sigma(N) = \left\{ \begin{array}{ll} E_y \varepsilon(N), & \Delta\varepsilon(N-1) < \varepsilon(N) \\ f_y(N), & \varepsilon(N) > \varepsilon(N) \end{array} \right. \quad (7) \]

\[ \varepsilon(N) = \Delta\varepsilon(N-1) + \frac{f_y}{E_y} \quad (8) \]

\[ \Delta\varepsilon(N-1) = \frac{f_y(N) - f_y(N-1)}{E_y} \quad (9) \]

where \( E_y \) denotes the initial Young’s modulus of rebar, \( f_y \) and \( f_y(N) \) denote initial yield strength and yield strength after \( N \) times fatigue load. \( \varepsilon(N) \) and \( \Delta\varepsilon(N) \) denote yield strain and residual strain after \( N \) times fatigue load.

According to the test, under the action of high-speed train load, the track structure is mainly under pressure, and the stress is indistinctive. Therefore, the slip between rebar and concrete may not occur, and the influence of the bond between rebar and concrete can be neglected.

In addition, the interfaces between slabs should be considered. Due to the effects of the rebar, the track slab and self-compacting concrete layer are able to be considering as a whole composite structure with no slip effect in the interfaces between these two slabs. The interfaces between self-compacting concrete layer and isolation layer and the interfaces between isolation layer and bed plate can be model by contact interface element with detailed parameters determined based on the internal frictional force of these two interfaces (Jin 2016).

Furthermore, in numerical simulation, the support layer is simulated by the spring element. The upper nodes of the spring element are consolidated with the base plate and all degrees of freedom of the lower nodes are con-
strained. The degrees of freedom in two horizontal direction of the bed plate are constrained and all degrees of freedom of the rail are constrained.

4.3 Development of fatigue FE model of CRTS III slab ballastless track structure

The cyclic numbers of fatigue load are usually millions of times and results in huge amounts of work if calculate step by step in the numerical simulation. Therefore, in order to improve the computation efficiency, the continuous fatigue damage process can be divided into a number of discrete damage processes by certain step sizes based on the characteristics of the fatigue damage evolution and required accuracy.

Based on the fatigue test and the S-N curve of concrete, a simplified method can be formulated to determine certain step sizes. At the first stage of fatigue damage (5 million times fatigue loading before), the step size should be as small as possible (10 thousand) which can reflect the initiation of fatigue damage. At the second stage of fatigue damage (5 million to 30 million times fatigue loading), the step size can be slightly larger than the first stage because of the steady development of fatigue damage (0.1 million). Due to the low stress level of the track structure, the fatigue damage did not reach the third stage under 30 million times fatigue loading in the numerical simulation. In addition, by applying the interpolation function, the fatigue performance between two adjacent step sizes selected in the numerical simulation can be correlated, and the whole fatigue damage accumulation process of concrete is continuous. Therefore, the fatigue performance of concrete under the fatigue load is obtained and the whole process of fatigue numerical simulation can be simplified.

The fatigue FE model of CRTS III slab ballastless track system is developed on the commercial software ANSYS. The FE units of main structural layers are determined as shown in Table 4. The whole process of fatigue FE analysis of CRTS III slab ballastless track structure under high speed train load is divided into three steps, as shown in Fig. 14.

Step A: By consideration of the complexity for calculation of the structure response under extensive fatigue loading cyclic numbers, the total fatigue loading cyclic numbers are divided into \( i \) parts. In detail, the selected intervals for \( i \) parts are different in three stages for the structure response under fatigue loading. For \( n \)th (1 \( \leq n \leq i \)) interval, calculate the structure response under fatigue loading including the deterioration of the stiffness, residual strength and the accumulation of the residual strain. By analyzing the results, determine whether the structure is reaching the fatigue failure, if is, the simulation is ceasing.

Step B: If the structure is not failing, uploading the relative material properties in model with the effects of the \( n \)th fatigue loading (deteriorated material stiffness, stress-strain relationship) and repeating the calculation procedures of step A.

Step C: Repeat step A and step B until structure reach fatigue failure.

| Structural layers       | Finite element units |
|-------------------------|----------------------|
| Concrete                | Solid 65             |
| Rail                    | Beam 188             |
| Support layer           | Combin 14            |
| Fastening system        | Combin 14            |
| Isolation layer         | Solid 45             |
| Interface condition     | Target 170           |

Table 4 The finite element units of main structural layers.

![Fig. 14 The whole process of fatigue analysis.](image-url)
5. Model verification

In the numerical simulation, the loads to the track system model are setting up to five classes (82 kN, 164 kN, 246 kN, 328 kN and 408 kN). The position of the loading point is setting up on the 3th and 7th sleepers pairs under the rail as shown in Fig. 15. Generally, in the track system, due to the high strength of the concrete material and the two-dimensional prestressing structure used in the track board compared to the self-compacting concrete layer and bed plate, the latter two components are required to be carefully examined on the performance under the fatigue loading.

The simulation and test results of the self-compacting concrete layer are shown in Fig. 16. It is illustrated that, during the five different loading levels, the difference of the stresses between the simulation and the test results in both longitudinal and transverse direction is in the range of 5% to 15% at point f-s-l and f-s-t at upper surface of the self-compacting concrete layer, as shown in Figs. 16 (a) and 16 (b). The difference of the stresses between the simulation and the test results in both longitudinal and
transverse direction is in the range of 4% to 14% at point h-s-l and h-s-t at upper surface of the self-compacting concrete layer, as shown in Figs. 16 (c) and 16 (d).

The simulation and test results of the bed plate are shown in Fig. 17. It is illustrated that, during the five different loading levels, the difference of the stresses between the simulation and the test results in both longitudinal and transverse direction is in the range of 4% to 13% at point f-s-l and f-s-t at upper surface of the bed plate, as shown in Figs. 17 (a) and 17 (b). The difference of the stresses between the simulation and the test results in both longitudinal and transverse direction is in the range of 6% to 14% at point h-s-l and h-s-t at upper surface of the bed plate, as shown in Figs. 17 (c) and 17 (d).

In addition, with regard to the relative displacement between the track slab and base plate, the results are shown in Fig. 18. The maximum difference between the simulation and test results is around 7.3% and 5.8% at mid-plate and end-plate respectively.

It is revealed that the results of the numerical simulation are agree well with the test results for CRTS III slab ballastless track system.

6. Fatigue damage law

6.1 The mechanics characteristic of CRTS III slab ballastless track structure

In order to explore the damage law under fatigue loading of CRTS III slab ballastless track structure, it is required the mechanics characteristic of the structure should be defined first. In general, firstly, the deformation of each layer and the stress distribution over the whole structural system under specific loading condition should be determined; Secondly, according the analysis conducted by first step, the worst case scenarios (positions of the maximum tensile stress occurred) are determined. Lastly, analyzed the fatigue load reactions in those positions.

The numerical simulation results of the deformation in the self-compacting concrete layer is shown in Fig. 19. In detail, according to the numerical simulation results, it is found that the maximum tensile stress in longitudinal direction (0.35 MPa) is greater than that in lateral direction (0.28 MPa). In addition, the maximum tensile stress (0.35 MPa) in longitudinal direction of this layer is appeared in the mid-span of the lower surface and the position in a quarter of whole slab on upper surface in lengthwise direction. The maximum transverse tensile stresses (0.28 MPa) are found on the position in the mid-span of the lower surface and the corner of the slab.

Fig. 17 The simulation and test results of the fatigue damage development of the bed plate.
of the upper surface.

The numerical simulation results of the deformation in the bed plate layer is shown in Fig. 20. The stress distribution of this layer is similar to the self-compacting concrete layer. The maximum longitudinal tensile stress (0.31 MPa) is greater than that in lateral direction (0.26 MPa). In addition, the maximum tensile stress (0.31 MPa) in longitudinal direction of this layer is appeared in the mid-span of the lower surface and the position in a quarter of whole slab on upper surface in lengthwise
direction. The maximum transverse tensile stresses (0.26 MPa) are found on the position in the mid-span of the lower surface and the corner of the slab of the upper surface.

6.2 The distribution of fatigue damage

For the self-compacting concrete layer, as the fatigue load cyclic numbers increasing, the stress distribution in worst case scenarios are varied in both the mid-span on lower surface and the quarter-span on upper surface as shown in Figs. 21 and 22. In general, the longitudinal tensile stress increases as the fatigue load cyclic numbers increasing. The maximum longitudinal tensile stress occurred in the edge of this layer, and gradually decrease widthwise in the position of edge layer, under sleepers and middle plate. According to the numerical results, the damage evolution is concluded in four stages. In detail, when the fatigue load cyclic numbers are between 1 and 5 million times, the increase of stress is significant (0.025 MPa/1 million). When the fatigue load cyclic numbers are between 5 million and 15 million times, the stresses increase is comparatively slow (0.013 MPa/1 million). After 20 million times, the increase in stress is significant again (0.02 MPa/1 million). Meanwhile, during this period, it is found that the longitudinal tensile stress at the edge of this layer decreased to zero, which reveals that in this position, the tensile stress reached the damage tensile strength and the self-compacted concrete has been cracked. When the fatigue load cyclic numbers are more than 25 million times, the cracks began to develop from the edge to the centre of the plate widthwise. Relate to cracking times, it is found that when the fatigue load cyclic numbers are 19 million times, the self-compacted concrete cracks at the mid-span on lower surface, as shown in Fig. 21 (b). When the fatigue load cyclic number is 17 million times, the self-compacted concrete cracks at the quarter of plate lengthwise direction on surface, as shown in Fig. 22 (b).

In addition, according to the Chinese Code for Design of Concrete Structures, the maximum crack width considering the effect of long-term load can be calculated as follows:

$$\omega_{max} = \alpha_c \psi \sigma_c \frac{1.9 c_s + 0.08 d_u}{\rho_c}$$

where $\alpha_c$ denotes force characteristic coefficient. $\psi$ denotes nonuniform coefficient of strain of tensile rebar in
crack. $E_s$ denotes the Young’s modulus of rebar. $\sigma_s$ denotes the stress of tensile rebar. $c_s$ denotes the distance from the outer edge of the outermost tensile rebar to the bottom edge of tensile zone. $\rho_{te}$ denotes the reinforcement ratio of longitudinal tensile reinforcement based on the effective area of concrete in tension. $d_{eq}$ denotes the equivalent diameter of the longitudinal rebar in the tensile zone. Furthermore, when the fatigue load cyclic numbers reached 30 million times, the stresses of longitudinal reinforcement in self-compacting concrete layer are shown in Fig. 25 (a). Therefore, based on equation (10), the crack width is calculated to be 0.068 mm.

For the bed plate, as the fatigue load cyclic numbers increasing, the stress distribution in worst case scenarios are varied in both the mid-span on lower surface and the quarter-span on upper surface as shown in Figs. 23 and 24. In general, the longitudinal tensile stress increases as the fatigue load cyclic numbers increasing. The maximum longitudinal tensile stress occurred in the edge of this layer, and gradually decrease widthwise in the position of edge layer, under sleepers and middle plate. According to the numerical results, the damage evolution is concluded in four stages. In detail, When the fatigue load cyclic numbers are between 1 and 5 million times, the increase of stress is significant (0.02 MPa/1 million). When the fatigue load cyclic numbers are between 5 million and 15 million times, the stresses increase is comparatively slow (0.01 MPa/1 million). After 20 million times, the increase in stress is significant again (0.022 MPa/1 million). Meanwhile, during this period, it is found that the longitudinal tensile stress at the edge of this layer decreased to zero, which reveals that in this position, the tensile stress reached the damage tensile strength and the bed plate concrete has been cracked. When the fatigue load cyclic numbers are more than 25 million times, the cracks began to develop from the edge to the centre of the plate widthwise. Relate to cracking times, it is found that when the fatigue load cyclic numbers are 21 million times, the bed plate concrete cracks at the mid-span on lower surface, as shown in Fig. 23 (b). When the fatigue load cyclic number is 23 million times, the bed plate concrete cracks at the quarter of plate lengthwise direction on surface, as shown in Fig. 24 (b).

In addition, when the fatigue load cyclic numbers reached 30 million times, the stresses of longitudinal reinforcement in bed plate are shown in Fig. 25 (b).
Therefore, based on equation (10), the crack width is calculated to be 0.042 mm.

7. Conclusion

In this work, a new concrete fatigue damage constitutive model based on the bundle-chain model is applied for analyzing the reactions of CRTS III slab ballastless track system under fatigue loading. Based on this damage model, the fatigue finite element model of CRTS III slab ballastless track system is established. In order to verify this model and illustrate the fatigue performance of the CRTS III slab ballastless track structure, a structural fatigue experiment has been carried out. According to the experiment results, it is found that after 4 million times fatigue loading, the track slab is considered to maintain excellent mechanical properties, and the damage in the self-compacting concrete layer and the bed plate layer is continuing to evolve. In addition, at the early stage during the fatigue loading, the vibration of each structure layer is significantly influenced by the evolution of the isolation layer’s stiffness.

Based on the three stages of fatigue damage development, the whole process of fatigue analysis of CRTS III slab ballastless track structure was presented and verified. Through this measure, the deformation and the stress distribution in worst case scenarios (positions of the maximum tensile stress occurred) for both the self-compacting concrete layer and the bed plate under the specified condition can be determined. Therefore, the mechanics characteristic of the structure can be defined. In general, according to the numerical simulation results, the maximum tensile stress of the self-compacting concrete layer and the bed plate in longitudinal direction is greater than that in lateral direction. The maximum stresses are found in the mid-span on lower surface, the quarter plate in lengthwise direction and the plate corner on upper surface. Furthermore, the fatigue damage development of CRTS III slab ballastless track structure is studied. It is shown that the self-compacting concrete layer cracked earlier than the bed plate, and for crack mode, the transverse crack will first appear. In summary, this work can provide certain references for the development and perfection of CRTS III slab ballastless track structure system.

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