Simulations on the Heavy Hadron Transport at RHIC

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Based on the hadron transport frames, detailed simulations are carried out to discuss $\phi$, $\Omega$ productions and the significant enhancements in the very low $p_T$ region for some of the soft spectra in RHIC. Elastic interactions are introduced in the simulations. The elastic cross sections vary from different hadrons and energy scales, which can qualitatively explain the different collective motions of various hadrons.

PACS numbers: 12.38.Mh, 24.10.Lx, 24.10.Nz, 25.75.-q

INTRODUCTION

In the recent progress [1], we have developed a transport model to describe the hadron production in relativistic heavy ion collisions. In the model, a decoupling hypersurface of the fluid which consists of a group of splitting QGP droplets [2] is used to emit hadrons. The model satisfies most of light hadrons' spectra, but disagrees with the distributions of $\phi$ [3, 4], $\Omega$ [5] and some heavy resonances [6, 7]. As another remarkable phenomenon, there seems to be significant enhancements at very low transverse momentum in some of the spectra for $\Lambda$, $\Sigma'(1385)$ and $\phi$, to change the slopes. On the contrary, the model predicts a decrease when $p_T$ drops.

The idea of the hypersurface hadronization locks the emitted hadrons in the local rest frames of the hypersurface. This is right when the fluid is dense enough in the early stage of the evolution. In the later stage, when the inelastic free path becomes as long as $4$ fm, the emission fixed in the hypersurface might be less strict. Considering the spectra of light hadrons are well satisfied, as other hydrodynamic methods with decoupling hypersurface did, there must be some reasons to push the inside light hadrons to participate in the collective motions. One of the possible candidate is the elastic collision [8]. Although elastic collisions do not change the hadron abundances themselves, the effect of preventing hadrons from entering the more dense regions and being absorbed, will enhance their abundances or momenta.

Thus, detailed simulations on hadron transport are applied to some of the heavy hadrons, such as $\phi$ and $\Omega$ in the central events in RHIC Au-Au 200 GeV. The advantages for $\phi$ and $\Omega$ are based on the condition that almost no hadrons could decay to them and their widths are large enough to neglect the affections of decays in the expanding medium. As a simplification, only selected hadrons for the measurements are simulated. The medium evolution and the feedbacks to the medium are not simulated in microscopic frames. As the cross sections of some heavy or multi-strange hadrons may be much smaller than those of light hadrons [10], it will provide us a possible solution to explain the various collective behaviors between different hadrons and $p_T$ regions.

MEDIUM EVOLUTION

The decoupling of the fluid is a gradual progress in principle. As the feedback of the hadron simulation to the fluid is unjustified, the medium is separated to an ideal hydrodynamic part and a free expanding part with evaporation by a cut $\epsilon_b$. The 1 + 1 hydrodynamic evolution with an invariant boost for the central events in RHIC Au-Au 200 GeV is determined by the method in ref [1]. The decoupling surface here is similar to the results of surface evaporation [1] except in the very early stage which is considered to be less important. For a group of decoupled droplets, there are no collective motions and local rest frames indeed. Approximately, the whole evolution is defined,

$$\frac{\partial \epsilon}{\partial t} = \int \{ \epsilon V_R \frac{\partial n_{vR}}{\partial t} + \epsilon v n_{vR} \frac{\partial V_R}{\partial t} \} dR dv,$$

where the former term in the integral stands for the free flights of the droplets and the latter term is the evaporation. Then,

$$\frac{\partial n_{vR}}{\partial t} = -v \frac{\partial n_{vR}}{\partial r} - (N - 1) \frac{n_{vR} v}{r},$$

where $n_{vR}(r, v) = \frac{d^2 n}{d\phi dr}$ is the velocity and radius distribution of the number density of the droplets.

As we do not know the distribution of the droplets $n_{vR}$ and can not simulate all of the droplets either, the expansion is simplified as,

$$\frac{\partial \lambda}{\partial t} = -\frac{\partial \lambda}{\partial r} - \frac{\lambda v}{r} - (N - 1) \frac{\lambda v}{r},$$

$$\frac{\partial v}{\partial t} = -\frac{v}{r},$$

where $\lambda = \gamma \epsilon$ and $N = 2$ is the symmetric dimension. Considering that the droplet evolution by emitting and absorbing hadrons is not clear, the evaporation term is omitted in our calculations.
HADRON TRANSPORT

The classical transport equation for the evolution of hadron phase space distribution \( f(t,r,p) \) can be written as

\[
\frac{\partial f}{\partial t} + v \cdot \nabla f = Q + \alpha - \beta f,
\]

where the second term on the left hand side is the free streaming part, and on the right hand side the lose and gain terms \( \beta(p)f \) and \( \alpha(p) \) indicate hadron absorption and production in the medium, and \( Q(p,f) \) stands for the elastic term.

In the simulations, hadrons generated in QGP or decayed from heavier hadrons, will pass through the medium and decay at the same time. Thus, the gain term includes two parts, \( \alpha = \alpha_{QGP} + \alpha_{dc}. \) The decay contribution is easy to obtain, but the term \( \alpha_{QGP} \) is complicated. The reason is that the quantitative temperature dependencies for some parameters are unknown [11], especially in a rapidly expanding system. Therefore, the production is simplified as ideal QGP with massless quarks and gluons plus the contributions of bound states which will never change via \( T \), as well as the coupling constants. The simplification is based on such an idea that in large \( T \) regions, where the inelastic free path is short and the produced hadrons may travel more distance before they escape from the medium, the affections may be less important for the simulations. Thus, the approximation will be sufficient enough if the productions are well satisfied near the critical temperature \( T_c \). Then,

\[
\alpha_{QGP} = \begin{cases} 
\alpha_0 \left( \frac{\epsilon}{\epsilon_c} \right)^{\frac{2}{3}}, & \epsilon > \epsilon_c, \\
\alpha_0 \eta, & \epsilon \leq \epsilon_c,
\end{cases}
\]

where \( l = 2 \) for mesons and \( l = 1 \) for baryons due to the quark-diquark frame [12]. \( \eta \) is the ratio of the occupation volume of all the QGP droplets to the whole volume of the system, \( \alpha_0 \) is the production rate at \( T_c \) and

\[
\epsilon' = \epsilon - \epsilon_{Bound}, \quad \epsilon'_c = \epsilon_c - \epsilon_{Bound},
\]

where \( \epsilon_{Bound} \) is contributions of bound states in QGP.

The loss rate includes the decay rate and the medium absorption, \( \beta = \beta_{dc} + \beta_{abs} \), where \( \beta_{dc} = \gamma^{-1} \Gamma \) and

\[
\beta_{abs} = \begin{cases} 
\beta_0 \left( \frac{\epsilon}{\epsilon_c} \right)^{\frac{2}{3}}, & \epsilon > \epsilon_c, \\
\beta_0 \eta, & \epsilon \leq \epsilon_c,
\end{cases}
\]

with \( \beta_0 \approx (1+e^2/3)n_q(\sigma)_{inelastic} \) at the critical temperature \( T_c \). As the expression belongs in the local rest frame of the medium, a Lorentz transformation on time step is required when the simulation is applied in the selected frame. Details on \( \alpha_0 \) and \( \beta_0 \) are discussed in ref [1, 12].

It is noted, (6) and (7) are not independent, even the droplets are assumed under random distribution. When \( \eta < 1 \), the location of production and absorption is correlated. If the QGP droplets are large enough, a factor \( \kappa(R) \) should be inserted to the independent productions (6) to remove the absorption by the droplet where the hadron is produced by QGP combination in principle. \( \kappa(R) \) can be fitted as,

\[
\kappa(R) \sim \frac{1}{1 + a \Delta + b \Delta^2},
\]

where

\[
\Delta \sim (1 - \eta) \frac{\beta_0 R}{v + v_s}.
\]

As discussed in above sections, the evolutions of the droplets are unjustified so far. Thus we set \( R \sim 0 \) and \( \kappa(R) \sim 1 \) to minimize the parameter dependency.

Hadrons produced in time, position and momentum bins \((i)\) are tracked when they pass through the medium. They are recorded separately, as

\[
N_i = N_{escape} + N_{reco},
\]

where \( N_{escape} \) is the direct and reconstructed hadrons escaped from the medium by cuts and \( N_{reco} \) is the reconstructed hadrons which may decay in the medium,

\[
N_{reco} = \sum_{\text{step}} \sum_i \left\{ N_i \prod_j \frac{n_j}{N_I} \right\},
\]

with recorded decay products \( n_j \) and

\[
N_I = \text{Decayed hadrons} \times \text{Branch ratio}.
\]

There is a small difference between our calculation and the experimental manipulations. In the experiments, usually one channel is measured for the reconstruction to provide the total abundance by dividing the corresponding branch ratio.

Inelastic simulations are applied with elastic cross sections omitted \((Q = 0)\). For the consistency with [1], no parameter is changed. There is no new parameter either. The results for \( \phi \) and \( \Omega \) are shown in Fig. [1] and Fig. [2]. The simulated results look much better than the original estimations from hypersurface emissions [1]. The productions in very low \( p_T \) region are enhanced and the collective flows are reduced. The perfect \( \Omega \) spectrum shows the elastic collisions for \( \Omega \) could be neglected as they may not participate in the collective motions. As the enhancement in very low \( p_T \) region for \( \phi \) is still smaller than expected, elastic collisions are required in our considerations.

ELASTIC COLLISIONS

The elastic process can be divided to two effects approximately. The acceleration pushed by the pressure
FIG. 1: Model estimations for $m_T$ distribution of $\phi$(1020) comparing with the data from STAR Collaboration [3] and the PHENIX Collaboration [4]. (a) Comparison between hypersurface hadronization with transport emit distributions [1] and inelastic transport simulation. (b) Demonstration of the push and drag effects by elastic collisions. (c) Demonstration of drag effects at different mean elastic cross sections. Subscripts for the mean elastic cross sections are removed for clarity.

FIG. 2: Model estimations for $p_T$ distribution of $\Omega^-$ + $\Omega^+$ comparing with the data from STAR Collaboration [3].

gradient and the deceleration dragged by the relative motion. Different from the inelastic cross sections, which is nearly stable via energy scales and parton species, the elastic cross sections vary from $10^4$ mb to $10^3$ mb [13], which makes it difficult to obtain a strict calculation with limited information of parton-hadron or hadron-hadron elastic differential cross sections, especially in the condition that only a few selected hadrons are simulated.

Heavy hadrons are discussed here to present a rough estimation on elastic process. As they are heavy, the changes of their momenta after each collision and the thermalization are supposed to be small. Thus, the push and drag effects can be estimated by integrating all of the collisions or the probabilities of them.

The push effect can be described in the local rest frame of the medium,

$$\left. \frac{dp_i}{dt} \right|_{\text{push}} = -\langle V \rangle_i \gamma_i^{-1} \nabla P,$$

(13)

where $\langle V \rangle_i$ is the mean hadron volume of elastic collisions. As detailed differential cross sections are not clear, it is estimated by a rigid ball approximation,

$$\langle V \rangle_i = \frac{4\pi}{3} \left( \frac{\langle \sigma \rangle_{ip}}{\pi} \right)^{3/2}.$$

On the other hand, the drag effect for a flat plate moving in normal direction is presented as,

$$\left. \frac{dp_i}{\Delta Sdt} \right|_{\text{drag}} = \sum_j \int d^3q_i f_j(q_j) |v_j - v_i| \Delta p_{ij}(p_i, q_j),$$

where $\Delta p_{ij}(p_i, q_j)$ is the momentum shift after each collision. The integral could be simplified by assuming the medium particles to be massless and fixing some of the momentum symbols to a kind of average $\langle p_j \rangle$. Thus,

$$\left. \frac{dp_i}{dt} \right|_{\text{drag}} = -2w_i P(3 + v_i^2) \langle \sigma \rangle_{id} v_i,$$

(14)

where,

$$w_i = c_f \frac{E_i(E_i + \langle p_j \rangle)}{E_i(1 + v_i) [E_i(1 - v_i) + \langle p_j \rangle]},$$

(15)

with $\langle p_j \rangle \approx 500$ MeV for medium particles near $T_c \approx 166$ MeV. $c_f$ is a correction function for the "shape" of the hadrons.

It should be noted, as the elastic cross sections vary in a large range for different energy scales, different averages of elastic cross sections $\langle \sigma \rangle_{ip}$ and $\langle \sigma \rangle_{id}$ will be reached by equations (13) and (14). The relation between them is not clear. As Eq (13) affects slow hadrons more intensively and Eq (14) affects fast hadrons more significantly, $\langle \sigma \rangle_{ip}$ might be slightly larger than $\langle \sigma \rangle_{id}$. To make the value $\langle \sigma \rangle_{id}$ similar to $\langle \sigma \rangle_{ip}$, we estimate $c_f \sim 0.25$ as a rough approximation.

The elastic corrections are only applied in the medium before it decouples. Equations (13) and (14) are inserted to the simulation separately. The results are troublesome when the push and drag effects are inserted together. They disturb each other in their invalid regions and the problem is not resolved so far. As shown in Fig. 1, $\phi$ production at very low $p_T$ is enhanced by the drag effect of preventing hadrons from moving inside. The enhancement [3] could be well satisfied by fitting $\langle \sigma \rangle_{id} = 20$ mb. In higher $p_T$ regions, push effect will effectively accelerate the hadrons, like the hydrodynamics does, to participate in the collective motions until the medium decouples, when $\langle \sigma \rangle_{ip} = 20 \sim 25$ mb. They don’t work well in opposite regions. The effects of different $\langle \sigma \rangle_{id}$ are
shown in Fig. [1]. The result for $\Omega$ implies the elastic cross section for $\Omega$ is no more than 20 mb.

As the momentum of each surviving daughter hadron may be shifted by elastic collisions, it is hard in current simulations to determine the efficiency of the reconstruction and the changed momenta of the reconstructed hadrons, which are different from those when they decay, until all of the related hadrons including backgrounds are simulated to commit a real reconstruction. Further more the elastic cross sections may be much different for hadrons and their decay products. Thus, the spectra for some strong decay hadrons, can not be simulated strictly. The spectrum of $\Sigma^*(1385)$ is not satisfied at very low $p_T$ so far, as shown in Fig. [5]; $\Xi^*(1530)$ has the same problem but looks more serious. Other possibilities may be the lack of information for some of heavier baryons which may decay to $\Sigma^*(1385)$ or $\Xi^*(1530)$, and inelastic hadron re-scattering.

Although the equations (13) and (14) are not suitable for light hadrons, we still applied them to test the push effect for pions. When $\langle \sigma \rangle_{ip} \sim 75$ mb, the inclusive pion distribution (14) can be reproduced, as shown in Fig. [4]. For direct pions without any decay contributions, $\langle \sigma \rangle_{ip} \sim 250$ mb. Local thermalization as a Boltzmann distribution was tried for strongly interacting regions for all the simulations and no significant difference or better results were found.

**SUMMARY**

Beyond the approximation of hypersurface hadronization developed from the transport model [1], we have applied transport simulations for heavy hadrons to reproduce the significant enhancement in very low $p_T$ region in some of transverse momentum spectra. Although the method is neither strict nor complete, and the values of mean elastic cross sections are just qualitative, the reason for the enhancement and therefore the different slopes in the spectra may be successfully explained by the push and drag effects. The simulation shows that the elastic cross sections of $\phi$ and $\Omega$ are much smaller than those of other hadrons. The simulation can also explain the reason why the hypersurface hadronization works well for light hadrons by the push effect.

**Acknowledgements:** We thank Professor Pengfei Zhuang for useful discussions and Xianglei Zhu for numerical supports. This work is supported in part by the grants No. NSFC10547001 and 10425810.

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