Probability based global sensitivity analysis of fatigue reliability of steel structures

Z Kala\textsuperscript{1,2}

\textsuperscript{1}Department of Structural Mechanics, Brno University of Technology, Faculty of Civil Engineering, Czech Republic

E-mail: kala.z@fce.vutbr.cz

Abstract. This article focuses on reliability-oriented global sensitivity analysis of the fatigue limit state of a steel member stressed by many times repeated loading. The fatigue limit state is associated with fatigue failure, which is caused by brittle fracture due to propagation of a fatigue crack from initial to critical size. The fatigue crack propagation is analysed using linear fracture mechanics. The fundamental question in terms of structural reliability is how significant is the effect of input random quantities on the probability of failure. A new type of global sensitivity analysis subordinated to a contrast identified the equivalent stress range and initial edge crack length as random quantities that most significantly influence the failure probability. The new findings obtained using the contrast-based global sensitivity analyses show interaction effects that are unusually strong in comparison with some results of Sobol’s sensitivity analysis aimed at the reliability of structures. The sensitivity indices are estimated using double-nested-loop simulation of the Latin Hypercube Sampling method.

1. Introduction

Numerous bridges worldwide are exposed to an increasingly stronger influence of traffic, which causes damage and reduces their service life. In particular, the fatigue phenomenon is one of the most common mechanisms of damage of steel bridges that lead to failure. Society continuously seeks measures to extend the functionality and operability of old bridges that are both economical and efficient, providing a basic framework for planning interventions [1]. The issue of fatigue analysis and evaluation of old steel bridges is, therefore, still the subject of research of reliability-based models in many countries [2].

Currently, a number of researches are focused on the experimental investigation of fatigue characteristics of steel of old bridges [3,4] as well as on the development of probabilistic approaches for assessing fatigue of these bridges [5,6]. The study of the propagation of fatigue damage of steel load-carrying structures is an essential part of identification of crucial input random quantities [7]. The results of experiments allow the verification of numerical models of bridges used in the fatigue assessment of load-carrying bridge structures [8].

Great interest has been devoted to fatigue cracks in steel girders for many years [9,10]. A procedure based on linear elastic fracture mechanics was used to evaluate the growth of a fatigue crack in [11]. The introduction, application and development of fracture mechanics combined with stochastic models has been a subject of interest in technical sciences [12,13]. A computational model based on fracture mechanics can be part of the stochastic evaluation of the fatigue characteristics of materials of steel girders, see for e.g. [5,8]. The stochastic model can be applied to study the sensitivity of the resulting
reliability to the variability of input random quantities.

The presented paper deals with global sensitivity analysis investigating the effects of input quantities on the failure probability of a steel bridge member subjected to many times repeated bending loading. The sensitivity analysis of reliability can be evaluated in a number of ways, see e.g. [14-16]. The sensitivity measure of failure probability evaluating common structural reliability can be computed using unconditional and conditional probabilities, see for e.g. [17,18]. An advanced kind of global sensitivity analysis of failure probability [19], which identifies all the main and interaction effects of input random quantities on time-dependent reliability of a steel bridge member with a propagating fatigue crack [20], is applied in the paper.

A recently published innovative sensitivity measurement [19] applied to failure probability, which identifies all important effects of input random quantities on the time-dependent reliability of a steel bridge member with a propagating fatigue crack [20], is applied in the paper. The novelty of the results of the case study is the consistent analysis of the influence of five random variable factors on the fatigue reliability of a bridge member using the decomposition of the failure probability. The results of the work can be primarily used in probability studies of reliability of steel bridge girders. It is shown that the effects of input factors on the fatigue reliability differ over time. However, the weight of passing vehicles represents a permanently dominant factor clearly influencing the reliability of both new and old bridge members.

2. Fatigue reliability assessment of steel structures

The fatigue reliability is investigated using linear elastic fracture mechanics, which uses mathematical equations to describe the propagation of the (considered edge) crack. Fatigue crack growth is generally described using Paris’ law (also known as the Paris-Erdogan law), which was suggested by Paris and Erdogan [21]. Paris’ law can be mathematically expressed as a linear dependence provided that log vs log plot of the range of the stress intensity factor during the fatigue cycle $\Delta K$ vs the crack growth rate $daldN$ is introduced.

$$\log(\frac{da}{dN}) = m \cdot \log(\Delta K) + \log C \quad (1)$$

where $a$ is the length of the crack, $N$ is the integral number of loading cycles and $C, m$ are experimentally determined Paris’ constants that depend on the characteristics of material, environment and stress ratio. Equation (1) can be simplified as equation (2).

$$\frac{da}{dN} = C \cdot (\Delta K)^m \quad (2)$$

Parameter $C$ is determined as

$$\log(C) = c_1 + m \cdot c_2 \quad (3)$$

where $c_1, c_2$ can be considered for common structural steel as $c_1=-11.141, c_2=-0.507$ [8]. The value of $\Delta K$ can be determined by Broek [22]:

$$\Delta K = \Delta \sigma_E \cdot \sqrt{\pi a} \cdot F\left(\frac{a}{W}\right) \quad (4)$$

Where $\Delta \sigma_E$ is the equivalent stress range [23]. The stress range is obtained as the difference between maximal and minimal stress values. $F(a/W)$ is the calibration function, which is obtained from experimental research [3,4]. In the case study presented here, results obtained from experimental research [3,4] on fatigue-loaded rectangular steel bodies, which cannot be applied to other geometries, are used for the introduction of equation(5). Equation (5) is sometimes interpreted and written as the course of crack growth. The calibration function for uniform bending (5) is applied in the presented case study.
Equation (2) can be integrated with the aim of obtaining a mathematical equation that links the crack growth from size \( a_0 \) to \( a_{cr} \) with the integral number of cycles from \( N_0 \) to \( N \).

\[
\int_{a_0}^{a_{cr}} \frac{1}{F\left(\frac{a}{W}\right) \cdot \sqrt[3]{\pi \cdot a}} \cdot \frac{m}{m} \, da = \int_{N_0}^{N} C \cdot \Delta \sigma_E^m \, dN
\]

(6)

The crack increases its length in nonlinear dependence from \( a_0 \) to \( a_{cr} \) during change in the integral number of cycles from \( N_0 \) to \( N \), where \( a_0 \) is the initial length of the crack and \( a_{cr} \) is the critical length of the crack. If we assume that the crack propagates from the first cycle, then we can consider \( N_0 = 0 \) and the right side of the equation can be written as

\[
A = C \cdot N \cdot \Delta \sigma_E^m
\]

(7)

The left side of equation (6) represents the resistance \( R_{cr} \), which can be expressed as equation (8).

\[
R_{cr} = \int_{a_0}^{a_{cr}} \frac{da}{F\left(\frac{a}{W}\right) \cdot \sqrt[3]{\pi \cdot a}} \cdot \frac{m}{m}
\]

(8)

Then the reliability function can be expressed as equation (9).

\[
G = R_{cr} - A
\]

(9)

Equation (9) represents a general mathematical formulation of the reliability function with five random quantities \( \Delta \sigma_E, a_0, m, W, N \), whose probability density function can be considered according to [5-8]. Probabilistic analysis of reliability is based on the probabilistic approach to the reliability condition equation (9):

\[
P_f = P(G \leq 0)
\]

(10)

Equation (10) expresses the probability of failure due to brittle fracture. This is the probability with which the crack propagated from \( a_0 \) to \( a_{cr} \) after \( N \) load cycles. Equation (10) is a general mathematical notation of the calculation of probability that has five basic random quantities listed in Table 1 in this article.

### Table 1. Input random quantities.

| Name of quantity  | Density      | Arith. mean | Stand. dev. |
|-------------------|--------------|-------------|-------------|
| \( \Delta \sigma_E \) | Equivalent stress range | Gauss | 31 MPa | 3 MPa |
| \( a_0 \) | Initial crack size | log-normal | 0.2 mm | 0.06 mm |
| \( m \) | Parameter \( m \) | Gauss | 3 | 0.03 |
| \( N \) | Stress peaks per year | Gauss | 1E6 | 1E5 |
| \( W \) | Specimen width | Gauss | 326 mm | 15 mm |

All input quantities listed in Table 1 have zero correlation between each other. The failure probability \( P_f \) is computed using a time step of one tenth of a year; see full line in figure 1. \( P_f \) was evaluated at each time step using two million samples of the Monte Carlo method.
3. Global sensitivity analysis of failure probability subordinated to contrasts

The failure probability $P_f$ is a subject of interest in the reliability analysis of steel structures. Let us consider a computational model in the form $Y = f(X_1, X_2, \ldots, X_M)$, with $Y$ a scalar. The input quantities ($X_1, X_2, \ldots, X_M$) are random quantities described using probability distributions reflecting the uncertainty of information on material parameters, load actions, etc. Failure ensues if $Y < 0$. The contrast function $\psi$ for the probabilistic appraisement ($P_f$) of the reliability can be expressed using real parameter $\theta$ as

$$\psi(\theta) = E(\psi(Y, \theta)) = E(1_{Y \leq 0} - \theta)^2$$  \hspace{1cm} (11)$$

The sensitivity measurement is based on the assumption that $\psi(\theta)$ is a contrast associated with parameter $\theta^*$, where

$$\theta^* = \text{Argmin}_\theta \psi(\theta)$$  \hspace{1cm} (12)$$

Using [19], the main effect (first order sensitivity index) based on the contrast is defined as

$$P_i = \frac{\min_{\theta} \psi(\theta) - E[\min_{\theta} E(\psi(Y, \theta | X_i)]}{\min_{\theta} \psi(\theta)}$$  \hspace{1cm} (13)$$

The parameter $\theta^*$ has the significance of $P_f$ and can be estimated from equation (10) using the Latin Hypercube Sampling (LHS) method [24,25]. For the unconditional failure probability, the minimal value of equation (11) can be estimated using $K$ samples of the LHS procedure as

$$\min_{\theta} \psi(\theta) \approx \frac{1}{K} \sum_{k=1}^{K} (1_{Y_k \leq 0} - \theta^*)^2$$  \hspace{1cm} (14)$$

In the numerator of equation (13), the second member can be computed similarly using double-nested-loop algorithm of LHS, where the conditional failure probability $\theta^*$ is calculated for $K$ runs of $Y$ using random quantities $X_i$ (nested loop), where $X_i$ is a fixed (constant) run from $L$ runs of LHS (outer loop). The second order contrast index $P_{ij}$ can be expressed as

$$P_{ij} = \frac{\min_{\theta} \psi(\theta) - E[\min_{\theta} E(\psi(Y, \theta | X_i, X_j)]}{\min_{\theta} \psi(\theta)} - P_i - P_j$$ \hspace{1cm} (15)$$
and estimated using LHS method analogously. The other probability contrast indices can be expressed similarly. After computing all sensitivity indices, their sum must be 1.

\[
\sum_i P_i + \sum_{j \neq i} P_{ij} + \sum_{j \neq i, k \neq j} P_{ijk} + \ldots + P_{123...M} = 1
\]

(16)

Global sensitivity analysis based on the contrast measures the effect of the variability of five input random quantities \( \Delta \sigma_E, a_0, m, N, W \) (table 1) on the failure probability \( P_f \) (10). The global sensitivity analysis is applied to the fatigue limit state of a steel member with a small initial edge crack.

4. Sensitivity analysis results

Sensitivity analysis based on the contrast measures the global effect of the variability of five input random quantities: \( \Delta \sigma_E, a_0, m, N, W \) (table 1) on the failure probability \( P_f \) (10). The numbers of LHS samplings are \( K=2000000, L=2000 \).

The sensitivity analysis results are shown in figures 2 and 3 for the 50th, 75th, 100th and 120th year of the lifetime of the bridge member. It is evident that \( P_f \) is strongly influenced by \( \Delta \sigma_E \) in each reference period. It is not only influenced by the main effect of \( \Delta \sigma_E \), but also by the higher-order interactions of \( \Delta \sigma_E \) with other quantities. The second dominant quantity is \( a_0 \) in the main and interaction effects. The third influential quantity is Paris parameter, which depends on the material, environment and stress ratio.

![Figure 2](image1.png)

Figure 2. Sensitivity analysis results for year 50 and 75.

![Figure 3](image2.png)

Figure 3. Sensitivity analysis results for year 100 and 120.

The interaction effects are relatively strong at all monitored times of bridge operation, which is in contrast with the results of Sobol’s sensitivity analysis in structural reliability studies [26,27]. The relatively strongest interactions are observed at 50 years of bridge operation, see figure 2. Conversely, the weakest interactions are observed at 100 years of bridge operation, see figure 3.
Similarly strong interactions were observed in the results of quantile-based sensitivity analysis of the design value of resistance, where the design (lower quantile) resistance is computed as 0.1 percentile [28,29].

The variability of quantities \( N, W \) does not have a great influence on \( P_f \), neither in the main effects nor in the interaction effects. These quantities can be fixed at any value of their domain without any significant effect on \( P_f \). In practical terms, it is not mandatory to identify the types of probability density functions and statistical characteristics of quantities \( N, W \) and as precisely as the dominant quantities \( \Delta \sigma, a_0 \). This knowledge can simplify and streamline stochastic computational models and contribute to a deeper understanding of the limit states of steel bridges exposed to the effects of fatigue.

It can be noted that the presented concept of global sensitivity analysis of the reliability (failure probability) can be applied to other limit states of steel structures. For instance, applications studying the interaction effects between fatigue behaviour and material degradation due to corrosion may present one of the possible goals of further fundamental or applied research, which will further elaborate the results of the presented study [30].

5. Conclusion
The probabilistic reliability assessment of a steel bridge member subjected to fatigue phenomena is presented in the case study in which time-dependent effects of input random quantities are taken into account. The probability of failure \( P_f \) was investigated using an advanced kind of contrast-based global sensitivity analysis. The outcomes identify the effects of input quantities on \( P_f \) in the 50\(^{th}\), 75\(^{th}\), 100\(^{th}\) and 120\(^{th}\) year of the lifetime of the bridge member. The input random quantities are divided into groups: influential and non-influential, according to their effects on the fatigue limit state.

The equivalent stress range influences \( P_f \) most significantly at all monitored times. The reliability of the steel members under bending can be improved by restricting very heavy vehicles, which can prolong the lifetime of an aging bridge in cases when repairs are no longer economical. Such justified limitation of extreme vehicle weights is common in old, very old and historic bridges, where the number of lanes is intentionally reduced or shuttle services are introduced if no other alternatives are economical or possible. Nevertheless, overloading does not benefit the lifetime of middle-aged bridge members. This article not only contributes to understanding the reasons for weight limitations, but also shows mathematically quantified sensitivities between the influence of five factors including their combinations on the fatigue limit state.

The initial edge crack length is the second random quantity with a strong influence on \( P_f \). Both random quantities contribute strongly to the interaction effects between each other and with other quantities. The strongest interaction effects were found during fifty years of bridge operation, when the value of \( P_f \) is the lowest of all four times in which the sensitivity analysis was evaluated.

The sensitivity of \( P_f \) to the width of the specimen and load cycles in the considered year of solution is relatively very low to negligible. Fixing one, the other or both quantities at any value of their domain will have very little influence on \( P_f \). The uncertainty of such minor to stochastically negligible quantities can be identified approximately (expert judgement) or not at all. The introduction of these quantities as deterministic can be considered during optimization of stochastic computational models.

The presented global sensitivity analysis methodology can be used in other case studies, for example welded joints or corrosion. However, such studies will require solving numerous issues, such as how to define random parameters such as the coating effect of the rate of corrosion during modelling. The results of sensitivity analysis can be used to make decisions on inspections, maintenance, repair and the loading capacity of bridges at different periods of its lifetime.

Acknowledgment
Financial support provided by the Czech Science Foundation (grant No. 17-01589S) and by No. LO1408 “AdMAw UP” are greatly appreciated.
References

[1] Leander J, Honfi D, Ivanov O L and Björnsson Í 2018 Eng. Fail. Anal. 91 306-14
[2] Leander J 2018 J. Constr. Steel Res. 141 1-8
[3] Seidl S, Miarka P and Kala Z 2018 Transactions of the VSB – Technical University of Ostrava, Civil Engineering Series 18 44-9
[4] Seidl S, Miarka P, Maliková L and Krejsa M 2017 Key Eng. Mater. 754 353-6
[5] Krejsa M, Koubova L, Flodr J, Protivinsky J and Nguyen Q T 2017 Frattura ed Integrità Strutturale 39 143-59
[6] Krejsa M, Kala Z and Seidl S 2016 Procedia Eng. 142 146-53
[7] Kala Z, Omishore A, Seidl S, Krejsa M and Kala J 2019 Int. J. Mech. 13 69-78
[8] Kala Z 2018 Int. J. Mech. 12 121-30
[9] Marques F, Correia J A F O, de Jesus A M P, Cunha A, Caetano E and Fernandes A A 2018 Eng. Fail Anal. 94 121-44
[10] Alencar G, de Jesus A, da Silva J G S and Calçada R 2019 Eng. Fail Anal. 104 154-76
[11] Omishore A 2017 IOP Conf. Ser. Mater. Sci. Eng. 245 1-6
[12] Omishore A 2019 IOP Conf. Ser. Mater. Sci. Eng. 471 1-10
[13] Leander J and Al-Emrani M 2016 Int. J. Fatigue 93 82-91
[14] Wang Y, Xiao S and Lu Z 2018 Aerosp. Sci. Technol. 79 364-72
[15] Xu L, Lu Z and Xiao S 2019 Appl. Math. Model. 66 592-610
[16] Zhou C, Zhang Z, Liu F and Wang W 2019 Chinese J. Aeronaut. 32 948-53
[17] Wang Y, Xiao S and Lu Z 2019 Mech. Syst. Signal Pr. 115 607-20
[18] Yun W, Lu Z and Jiang X 2019 Reliab. Eng. Syst. Saf. 187 174-82
[19] Fort J C, Kleim T and Rachdi N 2016 Commun Stat–Theor M 45 4349-64
[20] Kala Z 2019 Eng. Struct. 194 36-45
[21] Paris P C and Erdogan F 2963 J. Basic. Eng. 85 528-34
[22] Broek D 1986 Elementary Engineering Fracture Mechanics (The Hague: Springer)
[23] Kwon K and Frangopol D M 1010 Int. J. Fatigue 32 1221-32
[24] Iman R L, Johnson M E and Watson Jr. C C 2005 Risk Anal. 25 1277-97
[25] McKey M D, Conover W J and Beckman R J 1979 Technometrics 1 239-45
[26] Kala Z and Valeš J 2017 Eng. Struct. 134 37-47
[27] Kala Z and Valeš J 2018 Arch. Civ. Mech. Eng. 18 1207-18
[28] Kala Z 2019 J. Civ. Eng. Manag. 25 297-305
[29] Kala Z and Valeš J 2017 J. Constr. Steel Res. 139 110-22
[30] Peng D, Jones R, Cairns K, Baker J and McMillan A 2018 Theor. Appl. Fract. Mech. 97 385-99