Target tracking algorithms for multi-UAVs formation cooperative detection

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ABSTRACT
This paper considers the problem of the ground target positioning and tracking algorithm for multi-UAVs formation cooperative detection, and a real time and fast algorithm is proposed based on UAV airborne electro optical sensors. One state estimation problem for nonlinear stochastic system is studied by means of the unscented Kalman filter algorithm from target tracking process. To extend the single target tracking to multiple target tracking, one improved unscented Kalman filter algorithm is advised based on iterative multiple models. Furthermore, to relax the strict condition on white noise in Kalman filtering, the target tracking or state estimation is reduced to derive the inner and outer ellipsoidal approximations for the state in case of unknown but bounded noise. Finally, one simulation example confirms our theoretical results.

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1. Introduction
Unmanned aerial vehicles (UAVs) are widely used as an efficient detection, attack and defense devices, and in these recent years, they are gained great superiority in our military field. To improve their detection and attack efficiency, one novel concept of multi-UAVs formation is proposed in America. Multi-UAVs formation means that many UAVs fly in a special flight mode throughout the entire task completion. The aim of the formation flight is to make each UAV maintain the relative position and attitude, thus expanding the vision field and achieving the full ground or controlling a target. When formation task or battlefield environment change with time increases, the entire multi-UAVs formation will also adjust adaptively. Now on basis of the international development of UAVs and their related technologies, US military believes UAVs are capable of meeting the needs of the future missions, due to many properties for UAVs reconnaissance show up, for example, a short intelligence period, real-time reconnaissance, etc. Generally, UAVs can obtain a large amount of reconnaissance image information, while depending on the surveillance information about the moving targets, then a deep research on multi-UAVs formation is necessary from the theoretical perspective.

Research regarding to multi-UAVs formation primarily concentrate on formation control, formation aerodynamic coupling and formation communication positioning systems (Zhang et al., 2015). These three categories can be further divided into trajectory planning, trajectory maintenance, trajectory control and autonomous reconfiguration. Some statistical hypothesis testing methods are put forth to solve the problem of anomaly detection for multi-UAVs formation flight in Zhang et al. (2017), including the cluster method, classification method, nearest neighbour method, information entropy method and the spectral analysis method, etc. The cluster method is used for image compression sensing (Alamo et al., 2009), further reference (Bertsekas, 2019) considers the credibility of applying minimum variance control in Gaussian communication channels and the information entropy method is applied to determine the probability of failure with the application of that minimum variance control strategy. In Bertsekas and Goyal (2012), the spectral analysis method is proposed to judge the hypothesis testing probability regarding the system stability when data drop information is added to the communication channel. In Blackmore et al. (2011), the optimal control of dynamical systems over unreliable communication links is considered, it is pointed out that when the number of hypothesis is greater than two, it becomes difficult to solve multi-hypothesis testing problem, and the number of hypothesis will increase with the number of systems increase (Calafiore, 2010). Then multi-UAVs formation cooperative
can be benefitted in cooperative detection, cooperative positioning and cooperative interference, etc., so that the research on information processing and command-control corresponding to multi-UAVs formation cooperative reconnaissance is one central issue in US military (Campi & Garatti, 2016).

Roughly speaking, multi-UAVs formation cooperative can not only extend their mission scopes but also achieve lots of merits, such as multi-UAVs cooperative management, cluster tactics and multi-tasking requirements (Campi et al., 2009). Multi-UAVs formation cooperative depends on the transmission of information and sharing in case of implementing the task. Task coupling and information sharing make the formation cooperative as a distributed networked intelligence system, where each UAV is regarded as one knot and one wireless communication network is proposed to realize the communication among all UAVs (Callawy & Hiskens, 2011). Based on this distributed networked intelligence system, each UAV can perform the monitoring task according to the different locations for mission planning (Farina & Giulioni, 2016). Network module sends control requirement to cooperative mission planning, so that each UAV is controlled by its network goal. By using this network module, there are many kinds of function modules for each UAV, for example, cooperative mission planning, cooperative trajectory planning, tracking, attacking and searching, etc. (Garatti & Campi, 2013). All of these function modules will communicate with their neighbouring UAVs to share information resources (Abdelrahim et al., 2017). Kalman filter theory is one basis for modern filter theory. For the special linear system with Gaussian noise, Kalman filter algorithm can be proposed to obtain the minimum mean square estimate about the system state (Pillonetto, 2016), and this corresponding estimate is named as the optimal filter value. Further in extending Kalman filter algorithm, the idea of state space model is introduced in the optimal filter theory (Jianhong & Ramirez-Mendoza, 2020). The dynamic model and observation model correspond to the state equation and observation equation, respectively, then Kalman filter algorithm can be extended in dealing with the time-variant system Jianhong et al. (2021). Due to its recursive computation iteratively, Kalman filter algorithm is easy to implement. However, Kalman filter algorithm is suitable under one condition that the considered system is a linear time-invariant system with Gaussian white noise. This corresponds to the classical Kalman filter algorithm (Hostettler et al., 2016). To relax this strict assumption, unscented Kalman filter algorithm is proposed to solve the state estimation problem for the nonlinear stochastic systems. One core idea of unscented Kalman filter algorithm is that unscented transformation Care et al. (2018). It means that the probability density of the considered state can be described by a finite number of sampled points, which can be fully expressed as their means and covariances. After these sampled points are mapper by using of state or observation equation, the updated mean and covariance are given through the weighted summation (Weyer et al., 2017).

Here, we consider the problem of ground target tracking for multi-UAVs formation cooperative reconnaissance mission, i.e. the fusion problem of multi-UAVs formation cooperative detection within the network environment. The goal of target tracking process is to use the observed data to estimate those motion parameters, which include the location and speed of the target. The target tracking process can be deemed as a process of continuously acquiring target motion information (for example, position, velocity and size, etc.) in a vide sequence. Target tracking is difficulty in the field of computer vision, and it depends on target detection and recognition technology. In the common practical engineering, we always firstly detect the interested target, and then accurately convert the target into an automatic tracking mode. During our target tracking process, we analyse the target shape, scale, motion law and other information, then the target attributes are evaluated or classified. Generally target tracking technology plays an important role in both military and civilian applications. The classical target tracking algorithms are divided as follows: algorithm based on region, algorithm based on deformation template and algorithm based on Bayesian filtering theory. As the algorithms based on region and deformation template are not widely used in practical engineering, because of the instability. When considered the target tracking algorithm based on Bayesian filtering theory, it is well known that this kind of target tracking problem can be modelled as an optimal estimation problem based on Bayesian filtering theory. The detailed process of target tracking is formulated as that after the prior probability of the target state is known, then the maximum posterior probability of the target state is obtained iteratively by using the new observations.

Throughout this paper, as the contribution of Kalman filter algorithm is in estimating the system state, and some motion parameters are included in the system state, so we apply Kalman filter algorithm to estimate the system state, which corresponds to a nonlinear stochastic system, coming form multi-UAVs formation cooperative detection. It is convenient to define that target tracking process, which emphasizes the fusion estimation and prediction about the system state. Through modelled the tracked target real time, the current target state is obtained accurately in order to predict the next target state within a future time domain.
The main contributions of this paper are formulated as follows.

1. Unscented Kalman filter algorithm is proposed to study the problem of state estimation for a nonlinear stochastic system at a series of points, where this nonlinear stochastic system coincides with our multi-UAVs formation cooperative detection.

2. For this unscented Kalman filter algorithm, the state estimation is influenced by two design parameters – the scaling parameter and a covariance matrix. The choice of scaling parameter may lead to the increased quality of the state estimation, so here one different criterion function is constructed, so that the scaling parameter is chosen adaptively by minimizing this established criterion function. The property of this criterion function is shown from its own different observed information and computational complexity.

3. Based on our considered unscented Kalman filter algorithm, we see that it is impossible to use only one model to describe the target motion in the target tracking process, different models would be used in different motion segments. When considered these different models, one improved unscented Kalman algorithm based on the idea of iterative multi-models is studied here.

4. To give a further result on the problem of state estimation with a given discrete time-invariant linear system, we are interested to build ellipsoidal approximation of the state by a control satisfying the norm bound. Thus we come to the problem as follows: Given two ellipsoids, find the best inner and outer ellipsoidal approximations of their arithmetic sum. In fact this arithmetic sum corresponds to such an ellipsoidal approximations of the state system. This further result can be a supplement to our considered unscented Kalman filter algorithm.

The paper is organized as follows. In Section 2, the problem formulation is addressed, further ground target positioning methods are explained for multi-UAVs reconnaissance platform. Unscented Kalman filter algorithm is used to solve the state estimation problem for the nonlinear stochastic system in Section 3, where the detailed process are also given. In Section 4, one different criterion function is constructed to update the scaling parameter adaptively, and the computational complexity of this adjustment is covered. One improved unscented Kalman filter algorithm based on iterative multi-models is proposed to consider different models within different motion segments in Section 5. A supplement to the unscented Kalman filter algorithm is provided to build ellipsoidal approximation of the state in Section 6. In Section 7, one numerical example illustrates the effectiveness of our proposed target tracking algorithm for multi-UAVs formation cooperative detection. Section 8 ends the paper with final conclusion and points out the next topic.

2. Ground target positioning system

The process of multi-UAVs formation cooperative detection applies the image-vide information of UAV reconnaissance to obtain the detailed location information for the ground target. At present, the ground target positioning methods includes: (1) non-real-time positioning based on image matching mode in case of reference image, (2) real-time positioning based on some telemetry data, such as attitude, location and reconnaissance turntable parameters.

Here we only consider the second real-time positioning method, as it can compute the instantaneous position of the ground target rapidly. The airborne photoelectric measurement system can realize the positioning function, and it involves the position and attitude of the flight platform. Further ground target positioning system depends on the resolution of the reconnaissance sensor, as well as the pointing accuracy for the stable platform and other factors, such as the delay of the data link, and some kinds of coordinate system transformation. More specifically, ground target positioning system includes laser range finder, airborne gyro system, inertial measurement equipment, inertial navigation system, attitude measuring device and photoelectric stabilization platform. It tells us that UAV reconnaissance platform acquires a frame of image, coinciding with ground target and landscape. Two cases exist for extracting target location information from the received image. During the process of the ground target positioning process, the primary target is in the centre of the field of the airborne optoelectronic platform, and the secondary target is not in the centre of the image. The primary target obtains a relatively accurate target position through the laser ranging value. The explicit coordinate transformation process of ground target positioning is plotted in Figure 1, coming from the target reconnaissance image of airborne photoelectric reconnaissance platform.

The main steps for ground target positioning process are formulated as follows.

1. Calculate the coordinate value of the ground target in earth rectangular coordinate system. Set the coordinate value of the ground target in camera rectangular coordinate system as $C_c = (x_c, y_c, z_c)^T = (0, 0, r)^T$, where $r$ is the laser ranging value. After a
sequence of coordinate transformation in Figure 4, the coordinate value of the ground target in earth rectangular coordinate system is $C_g = (x_g, y_g, z_g)^T$.

(2) Transform the coordinate value of the ground target in earth rectangular coordinate system into the coordinate value of the ground target in earth coordinate system.

3. Unscented Kalman filter algorithm for target tracking process

3.1. Preliminary

Multi-UAVs reconnaissance platform is modelled as one dynamic system with implicit state space form, and its implicit information can be obtained through observed data iteratively. The state vector in the state space form may contain location, velocity and acceleration of the ground target, and these observed data are collected by inertial gyro.

Consider the following discrete time nonlinear stochastic system as that.

$$
\begin{align*}
    x_{k+1} &= f_k(x_k) + w_k \\
    z_k &= h_k(x_k) + v_k, \quad k = 0, 1, 2, \ldots
\end{align*}
$$

where in equation (1) $x_k \in \mathbb{R}^{n_x}$ and $z_k \in \mathbb{R}^{n_z}$ denote the state vector and measurement vector at instant $k$, respectively. Two maps $f_k : \mathbb{R}^{n_x} \to \mathbb{R}^{n_x}$ denote two unknown nonlinear functions, $w_k \in \mathbb{R}^{n_w}$ and $v_k \in \mathbb{R}^{n_v}$ are two state and measurement noises with zero mean. These white noises are independent and identically distributed between each other, and their covariance matrices are $\Sigma_w$ and $\Sigma_v$. $X_0$ is the initial state, its mean and covariance matrix are $\bar{x}_0$ and $P_0$, respectively. The initial state $x_0$ is independent of these two white noises $w_k$ and $v_k$.

After observing equation (1), our goal is to infer the state estimation from observed data, it corresponds to the filter process for that nonlinear stochastic system. In the framework of Bayesian theory, the state estimation is equivalent to complete our approximate the posterior probability distribution of the state vector, in case of the observed data. It is well known that this posterior probability distribution is named as the conditional probability density function on the basis of the observed data. Our unscented Kalman filter algorithm in Bayesian nonlinear filtering is to obtain a series of points in state space form and to match the Gaussian distribution in each update step. Due to the fact that the state estimation depends on minimizing one given criterion function, for example, the commonly used minimum square error criterion.

$$
J_k = E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T | Z^k]
$$

where $E[ ]$ is the expectation operation, and $Z^k$ is the set of all observed data until to instant $k$, i.e.

$$
Z^k = [z_0, z_1, \ldots, z_k]^T
$$

$\hat{x}_k$ is the state estimation of state $x_k$, and $\hat{x}_k$ is a function of $Z^k$.

Minimizing criterion function (2), then the state estimation $\hat{x}_k$ is get as that.

$$
\hat{x}_k = \hat{x}_{k|k} = E[x_k | Z^k]
$$

where equation (3) is the conditional mean, its expectation operation can be approximated by stochastic sample strategy. For linear system, this conditional mean is simplified to the classical Kalman filter algorithm. But on the contrary of the nonlinear system, it is difficult to compute the expectation operation.

3.2. Unscented Kalman filter algorithm

Unscented Kalman filter algorithm calculate the mean and covariance matrix on the filtering and prediction process iteratively. Set

$$
\chi^{a|b} = [\chi^a, \chi^{a+1}, \ldots, \chi^b]^T
$$

and $I_{a \times b}$ and $0_{a \times b}$ are diagonal matrix and zero matrix with dimension $a \times b$. Factorizing the matrix $P$ as that.

$$
P = \sqrt{P} \sqrt{P}^T
$$

Then the detailed unscented Kalman filter algorithm can be formulated as follows.

**Step 1**: (Initialization). Set time instant $k = 0$ and define the predictive mean and covariance matrix in case of prior
computing the following second-order moment for the state.

\[
\hat{x}_{k+1|k} = \sum_{i=0}^{2n_x} w_{i}^k (\hat{x}_{k+1|k} - \hat{x}_{k+1|k})^T
\]

\[
P_{k}^{xz} = \sum_{i=0}^{2n_x} w_{i}^k (\hat{x}_{k+1|k} - \hat{x}_{k+1|k})^T (\hat{x}_{k+1|k} - \hat{x}_{k+1|k})^T
\]

\[
\Sigma^z_k
\]

Set \( k = k + 1 \), and continue to step 2.

After the unscented transformation, the position of point \( \sigma \) is determined by the mean and covariance matrix of one transformed variable. Then the position of point \( \sigma \) will affect the denominator of the covariance matrix and the scaling parameter. More specifically, in the predictive step, the position of point \( \sigma \) is chosen in the control of one super ellipsoid, where \( \hat{x}_{k|k} \) is one interior point. As the primary transformation direction as \( \hat{x}_{k|k} \) is given by an initial feature vector of that covariance matrix \( P_{k}^{xz} \), so in the filtering step, the primary transformation direction at \( \hat{x}_{k|k} \) is determined by one feature vector of that covariance matrix \( P_{k}^{xz} \). The size of super ellipsoid is judged by the scaling parameter and the position of point \( \sigma \) simultaneously. The scaling parameter \( \mu \) may affect the accuracy, and it always is set as \( \mu = 3 - n_x \). The choice of that scaling parameter can be achieved by series expansion error, and this series expansion error represents the difference between the true mean and its unscented transformation approximation. The first three terms of the series expansion will be zero through the approximate selection of the weights, and the fourth term can also be guaranteed to be zero on the basis of the scaling parameter. Moreover, the determination of the scaling parameter is related with the criterion function. But in the unscented transformation of our considered unscented Kalman filter algorithm, no fixed scaling parameter is given to ensure high accuracy of the state estimation. Because the position of the working point or the expected state of the target will change with the time-invariant system. For this reason, one optimization strategy based on minimizing the approximate maximum likelihood function is applied to adjust the scaling parameter adaptively.

4. Adjustment of scaling parameter

The choice of scaling parameter depends on one criterion function about some estimations in unscented transformation. But in our above state estimation for unscented Kalman filter algorithm, no any true variables can be acquired. The only information available for the state estimation is the sequence of observations. This limitation emphasizes the importance of adjusting the scaling parameter.
parameter adaptively. In this section, maximum likelihood criterion is proposed to obtain one suitable scaling parameter. From the theoretical perspective, the maximum likelihood criterion coincides with the probability density function within the unscented Kalman filter algorithm, so the maximum likelihood criterion requires a prior knowledge about the state and two probability density functions $p(w_k)$ and $p(v_k)$ of the observed noises.

When the maximum likelihood criterion is used to design the optimal scaling parameter $\mu_k$, its explicit form is given that:

$$\mu^k_k = \arg \min_{\mu} p(z_k | Z^{k-1}, \mu)$$

If two probability density functions $p(x_k | Z^{k-1})$ and $p(z_k | x_k) = p(w_k)(z_k - h_k(x_k))$ are all Gaussian distributions, then we have.

$$p(z_k | Z^{k-1}, \mu) = N(\hat{z}_{k|k-1}(\mu), P_{k|k-1}^z(\mu))$$

where $N(\hat{z}_{k|k-1}(\mu), P_{k|k-1}^z(\mu))$ is one Gaussian normal distribution with mean $\hat{z}_{k|k-1}(\mu)$ and covariance matrix $P_{k|k-1}^z(\mu)$, and the mean and covariance matrix are all functional of the scaling parameter $\mu$.

To obtain one closed and analytic solution for equation (12), some numerical optimization methods can be applied to achieve the goal, for example, numerical grid method or global adaptive method. Numerical grid method covers a feasible optimization area $[\mu_{\min}, \mu_{\max}]$, then $\mu$ is obtained by equal space mesh point. After the optimization function is calculated at the equal space grid, the optimal scaling parameter $\mu^*$ is chosen by selecting the maximum or minimum grid point. In the global adaptive random search algorithm, the minimum value of the scaling parameter is set as the lower bound of the adaptive interval, i.e. $\mu_{\min} = 0$. This value guarantees that the covariance matrix of the random variable in unscented Kalman filter process is a positive form. The upper bound $\mu_{\max}$ of the adaptive interval can be set as one probability level, it means that the probability level of the stochastic variable $x$ lies in one region is that.

$$P^* = \frac{2^{-n_x/2}}{\Gamma(n_x/2)} \int_0^{\sqrt{n_x + \mu_{\max}}} e^{-t^2/2} t^{n_x-1} dt$$

where in equation (14), $P^*$ is the designed parameter, $\Gamma(n_x/2)$ is the Gram function. When dimension $n_x$ is a special case, $n_x = 2$, then $\mu_{\max}$ is chosen as that.

$$\mu_{\max} = -2 \log(1 - P^*) - 2$$

If we set $P^* = 0.999$, then $\mu_{\max} = 11.8$. But this global adaptive process for choosing the optimal scaling parameter will increase the computational complexity for unscented Kalman filter algorithm. This adaptive adjustment of scaling parameter can be applied to all time instant, instead of being limited to nonlinear function $h_k(x_k)$ of state estimation $\hat{x}_{k|k-1}$. And for the special case of linear function $h_k(x_k)$, the scaling parameter does not give any performance improvement for the unscented transformation, but the computational complexity can be greatly reduced.

Generally the adjustment for the scaling parameter in the unscented Kalman filter algorithm is formulated as follows, where the maximum likelihood criterion is used here.

**Step 1:** (Initialization) Set $\mu_{\min} = 0$ and compute $\mu_{\max}$ from equation (15), define that nonlinear measurement threshold as $T$, the initial time instant $k = 0$. The mean and covariance matrix at initial condition are defined as that.

$$\begin{align*}
\hat{x}_{0|0} &= E[x_0] = \bar{x}_0 \\
P_{x|0} &= \text{cov}[x_0] = P_x
\end{align*}$$

**Step 2:** (Adjustment) Define the scaling parameter as that.

$$\mu_k = \begin{cases} 
\mu_{k-1}^* & \text{if } \lambda_{\max}(z^TP_{k|k-1}z) > T \\
3 - n_x & \text{otherwise}
\end{cases}$$

**Step 3:** (Filtering) Implement the filtering step in the unscented Kalman filter algorithm and substitute the optimal scaling parameter $\mu_k$ into step 2.

**Step 4:** (Prediction) Implement the prediction step in the unscented Kalman filter algorithm and substitute the optimal scaling parameter $\mu_k$ into step 2.

Then set $k = k + 1$, continue the above steps and turn to step 2.

5. One improved unscented Kalman filter algorithm

To extend above-mentioned Kalman filter algorithm, we find that it is impossible to use only one model to describe the target motion in the target tracking process. In this section, different models would be applied in different motion segments, and one improved unscented Kalman filter algorithm is studied based on iterative multiple models.

The basic idea of multiple models is explained firstly. The possible motion mode of the target is mapped into one model set, then each model in this model set
indicates the different mode. Through sing multiple filters based on different modes in parallel, the final state estimation of the output will be chosen as the fusion result, corresponding to the local state estimation from each filter. Each filter corresponds to its own state space model, while different state space model describes different motion mode, so the state estimation, coming from each filter, is also different. Roughing speaking, iterative multiple models algorithm assigns different weights to these different estimation, and these different weights are determined by probability level.

The improved unscented Kalman filter algorithm is plotted in Figure 2. where this recursive algorithm includes four steps, i.e. initialization, conditional filter, probability undate and combined output.

Let \( M^{(i)}_k \) signifies the effective event at the \( t \)th sampled period for model \( M^{(t)}_k \), then \( M^{(i)}_{k-1} \) is the effective event at the \( k-1 \)th sampled period for model \( M^{(j)}_k \). For the case of \( r \) models, the improved unscented Kalman filter algorithm based on iterative multiple models is formulated as follows.

1. Apply the estimation \( \hat{x}^{(i)}(k-1|k-1) \) of model \( j \) and covariance matrix \( P^{(i)}(k-1|k-1) \) to compute the hybrid initialization, matching to model \( M^{(t)}_k \). Assume the considered models satisfy the Markov property, then

\[
\hat{x}^{(i)}(k-1|k-1) = \sum_{j=1}^{r} \hat{x}^{(j)}(k-1|k-1) \\
\times \mu^{(j)}(k-1|k-1) \\
p^{(i)}(k-1|k-1) = \sum_{j=1}^{r} p^{(j)}(k-1|k-1) \\
+ (\hat{x}^{(i)}(k-1|k-1) - \hat{x}^{(i)}(k-1|k-1)) \mu^{(j)}(k) \\
\times (k-1|k-1)
\]

\[
\mu^{(j)}(k-1|k-1) = p(M^{(i)}(k-1)|M^{(i)}(k), Z_{k-1}) \\
= \frac{1}{\bar{c}_t} \pi^{(j)}_t \mu^{(j)}(k-1)
\]

where \( \mu^{(j)}(k-1) \) is the probability level for model \( M^{(j)}_k \), \( \bar{c}_t = \sum_{j=1}^{r} \pi^{(j)}_t \mu^{(j)}(k-1) \) is one constant, and \( \mu^{(j)}_t \) is the transition probability from model \( M^{(j)}_k \) to model \( M^{(t)}_k \).
(2) [Unscented Kalman filter algorithm on each model]

Unscented Kalman filter algorithm is used in $\hat{x}^{(t)}(k - 1 | k - 1)$ and $p^{(t)}(k - 1 | k - 1)$ for all models $t = 1, 2, \ldots r$. Without loss of generality, the above adjustment of scaling parameter is also used here.

(a) [Initialization] Apply $\bar{x}^{(t)}(k-1 | k - 1)$ and $p^{(t)}(k - 1 | k - 1)$ to solve many sigma points $\{x^{(t)}_i\}$ and weights $\{w^{(t)}_i\}$.

(b) [Sigma points] Use each state model to predict state estimations $\{\bar{x}^{(t)}(k | k - 1)\}$ and sigma points $\{(z^{(t)}_i)\}$, then compute some prediction values $\{\hat{x}^{(t)}(k | k - 1)\}$ and $\{Z^{(t)}(k)\}$.

(c) [Covariance matrix] Apply $\{\hat{x}^{(t)}(k | k - 1)\}$, $\{Z^{(t)}(k)\}$ and $\{w^{(t)}_i\}$ to compute the covariance matrix $p^{(t)}(k | k - 1)$, cross covariance matrix $R^{(t)}(k)$ and information covariance matrix $S^{(t)}(k)$.

(d) [Updated strategy] The filtering gain is that.

$$K^{(t)} = \frac{P^{(t)}_Z(k)}{S^{(t)}(k) + R(k)}$$  

(18)

The updated state is that.

$$\hat{x}^{(t)}(k | k) = x^{(t)}(k | k - 1) + K^{(t)}(z^{(t)}(k) - z^{(t)}(k))$$  

(19)

The updated covariance matrix is that.

$$P^{(t)}(k | k) = p^{(t)}(k | k - 1) + K^{(t)} \{S^{(t)}(k) + R(k)\}(K^{(t)})^T$$  

(20)

(3) [Model probability undated]

$$\Lambda^{(t)}(k) = p(Z(k) | M^{(t)}(k), Z_{k-1}) = p(\bar{Z}(k) | M^{(t)}(k), Z_{k-1}) = [2\pi S^{(t)}(k)]^{-\frac{1}{2}} \times \exp \left\{ -\frac{1}{2} (\bar{Z}^{(t)}(k))^T \left( S^{(t)}(k)^{-1} (\bar{Z}^{(t)}(k)) \right) \right\}$$

$$\mu^{(t)}(k) = P(M^{(t)}(k) | Z^k) = \frac{1}{c} \Lambda^{(t)}(k) \bar{c}_t$$  

(21)

where $\Lambda^{(t)}(k)$ is the likelihood function for filter, and

$$\bar{Z}^{(t)}(k) = Z(k) - Z^{(t)}(k)$$

$$\bar{c}_t = \sum_{j=1}^r \pi_{Rj} \mu^{(j)}(k - 1)$$

$$c = \sum_{j=1}^r \Lambda^{(j)}(k) \bar{c}_j$$  

(22)

(4) [State estimation fusion]

$$\hat{x}(k | k) = \sum_{j=1}^r \hat{x}^{(j)}(k | k) \mu^{(j)}(k)$$

$$P(k | k) = \sum_{j=1}^r \left\{ p^{(j)}(k | k) + (\hat{x}^{(j)}(k | k) - \hat{x}^{(j)}(k | k)) \times (\hat{x}(k | k) - \hat{x}^{(j)}(k | k))^T \right\} \mu^{(j)}(k)$$  

(23)

6. Building ellipsoidal approximation

Observing equation (2) again, unscented Kalman filter algorithm holds on the basis of white noise with independent and identically distributed property. This property corresponds to the classical probabilistic description on noise. To relax this strict assumption on noise, we investigate to build ellipsoidal approximation of the state, where a given discrete time-invariant linear system without measurement equation.

$$\begin{align*}
X_{k+1} &= A X_k + B w_k \\
X_0 &= 0, \quad k = 0, 1, \ldots, N - 1
\end{align*}$$  

(24)

This special discrete time-invariant linear system is driven by $w_k$ satisfying the norm bound.

$$\|w_k\| \leq 1, \quad k = 0, 1, \ldots, N - 1$$

Our goal is to build ellipsoidal approximation of the state recursively. Let $X_k$ be the set of all states where the system can be driven in instant $k | \lambda \in \lambda$ and assume that we have build inner and outer ellipsoidal approximations $E_{in}^k$ and $E_{out}^k$ of the set $X_k$.

$$E_{in}^k \subset X_k \subset E_{out}^k$$  

(25)

Let also

$$E = \{x = Bw | w^T w \leq 1\}$$  

(26)

Then the set

$$F_{in}^{k+1} = AE_{in}^k + E$$

$$\{x = Ay + z \; | \; y \in E_{in}^k, z \in E\}$$  

(27)

clearly is contained in $X_{k+1}$, so that a natural recurrent way to define an inner ellipsoidal approximation of $X_{k+1}$ is to take as $E_{in}^{k+1}$ the largest volume ellipsoid contained in $F_{in}^{k+1}$. Similarly, the set

$$F_{out}^{k+1} = AE_{out}^k + E$$

$$\{x = Ay + z \; | \; y \in E_{out}^k, z \in E\}$$  

(28)

clearly covers $X_{k+1}$, and a natural recurrent way to define an outer ellipsoidal approximation of $X_{k+1}$ is to take as
Due to the fact that for every symmetric positive semidefinite matrix $X$ such that $\text{Tr}(B'X) \leq 1$, $i = 1, 2 \ldots m$, one has $\text{Tr}(M[Z]X) \leq 1$. Then we arrive at the following result.

**Proposition 6.1:** Let a positive definite matrix $Z$ be such that the optimal value in the semidefinite programme.

$$\max_{X} \{ \text{Tr}(M[Z]X) | \text{Tr}(B'X) \leq 1, i = 1, 2, X \succeq 0 \}$$

is $\leq 1$. Then the ellipsoid

$$W[Z] = \{ x | x^T Z x \leq 1 \}$$

contains $W_1 + W_2$ of the ellipsoids $W_i = \{ x | x^T B_i x \leq 1 \}$.

The above proposition is the first step to build a parametric family of ellipsoids, which contains the arithmetic sum $W_1 + W_2$. Then the second problem of finding the smallest volume ellipsoid within the parametric family can be reduced to one semidefinite program as that.

**Proposition 6.2:** Given two centred at the origin full-dimensional ellipsoids,

$$W_i = \{ x | x^T B_i x \leq 1 \}, \quad i = 1, 2.$$

Let us associate with these two ellipsoids the semidefinite program.

$$\max_{t, Z, \lambda} \left\{ t \mid t \leq \text{Det}^{1/2}(Z), \lambda_1 B_1 + \lambda_2 B_2 \succeq M[Z], \lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_1 + \lambda_2 = 1, Z \succeq 0 \right\}$$

(34)

Every feasible solution $(t, Z, \lambda)$ to this semidefinite program with positive value of the objective produces ellipsoid.

$$W[Z] = \{ x | x^T Z x \leq 1 \}$$

which contains $W_1 + W_2$, and the smallest volume ellipsoid is given by optimal solution of the semidefinite program (34).

### 6.2. Inner ellipsoidal approximation

Let us represent the given centred at the origin ellipsoids $W_i$ as.

$$W_i = \{ x | x = A_i w | w^T w \leq 1 \}, \quad i = 1, 2$$

Due to the fact that an ellipsoid $E[Z] = \{ x = Z w | w^T w \leq 1 \}$ is contained in the sum $W_1 + W_2$ of the ellipsoids $W_i$ if and only if one has

$$\forall x : \| Z^T x \|_2 \leq \sum_{i=1}^{2} \| A_i^T x \|_2$$

(35)

A natural way to generate ellipsoids satisfying equation (35) is to note that whenever matrix $X_i$ satisfying
the following the property about its special norms.

\[ |X_i| = \sqrt{\lambda_{\max}(X_i^T X_i)} \]

\[ = \max_{\|x\|_2 = 1} \|X_i^T x\|_2 \leq 1, \quad i = 1, 2 \]  \hspace{1cm} (36)

Then the matrix

\[ Z = Z(X_1, X_2) = A_1 X_1 + A_2 X_2 \]  \hspace{1cm} (37)

satisfies equation (35).

\[ \|Z^T x\|_2 = \|\|A_1 X_1 + A_2 X_2 x\|_2 \]

\[ \leq 2 \sum_{i=1}^{2} \|X_i^T A_i^T x\|_2 \leq 2 \sum_{i=1}^{2} \|A_i^T x\|_2 \]

\[ \leq 2 \sum_{i=1}^{2} \|A_i^T x\|_2 \]  \hspace{1cm} (38)

Thus every collection of square matrices \( X_i \) with spectral norms not exceeding 1 produces an ellipsoid satisfying equation (35) and thus contained in \( W \).

Similarly, the largest volume ellipsoid within the parametric family can also be reduced to the following semidefinite program.

**Proposition 6.3:** Let \( W_i = \{ x \mid x = A_i w \mid w^T w \leq 1 \}, A_i > 0, i = 1, 2, \) consider the following semidefinite program.

\[ \max \quad t \]

subject to \[ t \leq \left( \text{Det} \left( \frac{1}{2} \sum_{i=1}^{2} [X_i^T A_i^T x] \right) \right)^{\frac{1}{nx}} \]

\[ \left( \begin{array} {ccc} I_n & -X_i^T \\ -X_i & I_n \end{array} \right) \geq 0, \quad i = 1, 2 \]  \hspace{1cm} (39)

with design variables \( X_1, X_2 \). Every feasible solution \( (X_1, X_2, t) \) to this problem produces the ellipsoid.

\[ E(X_1, X_2) = \{ x \mid \left( \sum_{i=1}^{2} A_i x \right) w \mid w^T w \leq 1 \} \]

contained in the arithmetic sum \( W_1 + W_2 \) of the original ellipsoids, and the largest volume ellipsoid which can be obtained in this way is associated with optimal solution to the semidefinite program (39).

After solving these two semidefinite programs (34) and (39), then we build inner and outer ellipsoidal approximations \( E_{in}^k \) and \( E_{out}^k \) of the set \( X_k \), i.e.

\[ E_{in}^k \subset X_k \subset E_{out}^k \]

**7. Simulation**

Here in this section, one example is given to prove the efficiency of this unscented Kalman filter algorithm for tracking one ground target. Our goal is to track one continuous time acceleration motion model with white noise.

The state of this ground target is defined as follows.

\[ x_k = [x_{1k}, x_{2k}, x_{3k}, x_{4k}]^T \]

\[ = [x_k, y_k, \dot{x}_k, \dot{y}_k]^T \]  \hspace{1cm} (40)

where the above target state contains the position and velocity in the \( x \) direction and \( y \) direction, respectively, and the dimension is \( n_x = 4 \). Then the motion equation is that.

\[ x_{k+1} = F x_k + G w_k \]

\[ F = \left[ \begin{array} {cccc} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array} \right], \quad G = \left[ \begin{array} {cc} 0.5 T^2 & 0 \\ 0 & 0.5 T^2 \\ T & 0 \\ 0 & T \end{array} \right] \]  \hspace{1cm} (41)

The ground target is observed by a radar detector, and the observation \( z_k \) at time instant \( k \) from the radar detection is the angle between the ground target and the radar detection. When the radar detector is on \( [x^0_k, y^0_k] \) at time instant \( k \), then the observation \( z_k \) at time instant \( k \) is that.

\[ z_k = \arctan \frac{x_k - x^0_k}{y_k - y^0_k} + \nu_k \]

\[ \Sigma_k = 1 \]  \hspace{1cm} (43)

This ground target is 10 km away from the radar detector with angle \(-135^\circ\) and constant velocity 15 m/s. Define the initial position of the ground target as \( [7, 7] \), and the initial velocity in the \( x \) and \( y \) direction, respectively, and the dimension is \( n_x = 4 \). Then the performance density of the filter is chosen as

\[ P(r) = N(\sqrt{7^2 + 7^2}, 16) \]  \hspace{1cm} (44)

The probability density of the velocity is that.

\[ P(s) = N(\bar{s}, 16) \]  \hspace{1cm} (45)

The largest scaling parameter is set as \( \mu_{\text{max}} = 14 \), then we obtain that \( P^* = 0.999 \). The number of grid used to cover the entire interval is that \( N_s = 20 \). Then the performance
corresponding to our considered filter is measured by one mean square error root, which is defined as that.

\[ RMS_{\mu} = \frac{1}{M} \sum_{i=1}^{M} (\hat{x}_k(i) - x_k(i))^2 + (\hat{y}_k(i) - y_k(i))^2 \]  

(46)

To show the closed relations between mean square error roots and different signal-to-noise ratios, we do some simulations on model (41) and (43), where we take three cases as follows, low signal-to-noise ratio \( \Sigma_{\nu} = (5^2) \); mean signal-to-noise ratio \( \Sigma_{\nu} = (\nu^2) \); and high signal-to-noise ratio \( \Sigma_{\nu} = (0.07)^2 \). The relationship between the performance of the target state estimation and the threshold value in the unscented Kalman filter algorithm is shown in Figure 3, where three curves are represented as the above three cases. From Figure 3, we see that the adjustment of the scaling parameter adaptably does not make any improvement on high signal-to-noise ratio, but instead great improvements for low and medium signal-to-noise ratios.

To compare our proposed unscented Kalman filter algorithm with two adjustable parameters and the existing algorithm for target tracking process, the pitch output angles are used to show the results for state estimation, i.e. the information of pitch angle is included as one element in the considered state vector. The detailed three outputs of the same pitch angle are plotted in Figure 4, where the first is the pitch output of UAV, the second is the pitch output from the existing algorithm, and the third is obtained by our proposed unscented Kalman filter algorithm with two adjustable parameters.

From Figure 4, we see that our obtained pitch output coincides with the first true one, as the shapes between them are the same with low error. But some flutters or deviations exist in the second pitch output, due to the fixed parameters. Moreover, in the process of positioning and tracking ground targets for the entire dual drone formation, the target trajectories fused by our considered algorithm in this paper are smooth, stable and continuous, then further, no local jump exists. When compared with the traditional Kalman filter algorithm, our proposed
unscented Kalman filter algorithm with two adjustable parameters is suitable for multi-UAVs formation tracking system for ground targets with better tracking effect.

8. Conclusion

In this paper, we study the problem of the ground target positioning and tracking algorithm for cooperative detection of multi-UAVs formation. Then a real-time and rapid algorithm is discussed based on UAV airborne electro-optical sensors. The unscented Kalman filter algorithm is proposed to achieve the goal of tracking target, and one adjustment strategy for the scaling parameter adaptively is also advised in the unscented Kalman filter algorithm. Further to extend the single target tracking to multiple target tracking, one improved unscented Kalman filter algorithm is studied based on iterative multiple models. To relax the strict assumption on noise in Kalman filter theory, the problem of target tracking or state estimation is reduced to build ellipsoidal approximation of the considered state, whose inner and outer ellipsoidal approximations are derived through two semidefinite programs. Future research will focus on applying robust optimization or decision into the cooperative detection in the presence of bounded noise.

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Data availability statement

The data used to support the findings of this study are available from the corresponding author upon request.

Disclosure statement

No potential conflict of interest was reported by the author(s).

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