Control of magnetoelectric antenna by electric field

Aleksandr O. Nikitin¹, Roman V. Petrov², and Marina A. Havanova²,*

¹AO “OKB-Planeta”, Veliky Novgorod, 173004, Russian Federation
²Institute of Electronic and Information Systems of Novgorod State University
 n.a. Yaroslav-the-Wise, Veliky Novgorod, 173003, Russian Federation

Abstract. The results of researching on the microwave processes in the microstrip antenna in which the substrate is consist of composite structure: ferroelectric – ferromagnetic – dielectric was presented. The paper shows that unified wave can be present in this structure, which combines the properties of electromagnetic and spin types of waves – this is a hybrid electromagnetic-spin wave. The dispersion characteristic of this wave was obtained and its dependence on the applied electric field to the ferroelectric layer was established. This dependence is a consequence of two effects: magnetoelectric effect and dependence dielectric constant from applied electric field. The total effect is a change in the resonant frequency of antenna under study about 2 GHz with the applied electric field changed on 40 kV/cm. It was also found that, depending on the value of the applied electric field, the mode of standing waves can be achieved.

1 Introduction

Practical benefit or practical interest is the keynote of any scientific research. Best confirm of this – research in the field of composite materials. Direct evidence is their extremely wide range of application: from spacecraft to home appliances. Magnetostriction-piezoelectric materials occupy an important place among the known composite materials, the unique properties of which is determine by a large number of effects, and one of which is a magnetoelectric effect (ME) [1].

The task of controlling the properties of ME antenna by electric field focuses on a number of important points: 1) a series of experiments [2] showed the dependence of the microwave properties of antenna array on the electric field applied to the ME-element; 2) experiments showed that the intensity of the dependence of microwave characteristics of antenna array on the applied electric field is relate to the design parameters of the ME-structure and the antenna; 3) in addition to the ME-effect, a variety of effects take place in this composite, including the dependence of the dielectric constant on the applied electric field.

The aspects mentioned above actualize the work of researching processes, which occur in the ME-antenna under the influence of an electric field, identifying the dependencies of its microwave characteristics on design features and selected mode. Results of research that determined these dependencies can be the basis for the further research work on the design of ME antennas and antenna arrays with electronic control.

* Corresponding author: Marina.Havanova@novsu.ru

© The Authors, published by EDP Sciences. This is an open access article distributed under the terms of the Creative Commons Attribution License 4.0 (http://creativecommons.org/licenses/by/4.0/).
2 The object of study and solution

Fig.1. The structure of the investigated magnetolectric antenna: (a) design; (b) current distribution.

The object of study is a simple microstrip antenna (MSA) (Fig.1(a)). The substrate of the MSA is a composite structure, which consist of the ferroelectric lead zirconate titanate (PZT) layer, and the ferromagnetic yttrium iron garnet (YIG) layer placed on the gadolinium gallium garnet (GGG) substrate.

This structure is characterized by ME-effect, which is the so-called “secondary” effect in the chain “magnetostriction – elastic deformation – piezoelectric effect”. It manifests itself in the induction of electric polarization under action of magnetic field, or vice versa, the induction of magnetization under action of an electric field. In the latter case, we are dealing with the reverse ME-effect, which in microwave range, in an area of a ferromagnetic resonance (FMR), determines the practical interest. In other words, on the reverse ME-effect, it is possible to design a microwave device with electronic control [3].

Type of devices in which a magnetolectric composite material acts as an active element is considered to be magnetolectric.

From the above follows the mechanism for controlling the properties of MSA: an electric field is applied to the ferroelectric layer. On the one hand, this field, because of the ME-effect, leads to a change in the resonant magnetic field of the ferrite; on the other hand, it changes the dielectric constant of the ferroelectric layer. This leads to a change in the microwave properties of the MSA.

To study the above processes, a calculation method was used, the essence of which is:
- at the first stage MSA is considered, and the dependence of the microwave characteristics of antenna on the field structure in the substrate is derived by one of the known methods;
- after, the field structure in ME-composite is considered by solution of the microwave problem taking into account the corresponding boundary condition.

2.1 Radiation characteristics of the investigated microstrip antenna

A modified resonator method was used to analyze the characteristics of the investigated microstrip antenna [4]. Its essence is as follow. Initially, electromagnetic fields in the rectangular resonator with magnetics \((x = 0, a; y = 0, b)\) and electric \((z = 0, d)\) sides are defined (Fig.1(б)):

\[ J^M = n \times E, \]  

где \(n\) – normal to side, \(E\) – electric field strength in the resonator.
After, conduction of radiating currents are determined taking into account the excitation of both spatial and surface waves. In the end, the expressions for the radiation pattern near the antenna resonance (given that $kb \approx \pi$, where $k$ – propagation constant in the substrate) will be as follows [4]:

in the plane $H(z0x)$

$$F_\phi(\theta) = \frac{2 \cos \theta}{\sqrt{\cos^2 \theta + (\frac{\xi}{\cos(k_0d)} \sin \theta)^2}} \left[ \frac{\sin(0.5k_0a \sin \theta)}{0.5k_0a \sin \theta} \right].$$ (2)

in the plane $E(z0y)$

$$F_\theta(\theta) = \frac{2 \cos \theta \cos(0.5k_0b \sin \theta)}{\sqrt{\xi^2 + (\frac{\epsilon'_1 \cos \theta \cot(\frac{\xi}{k_0d})}{\xi})^2}} \frac{\epsilon'_1}{\xi},$$ (3)

where:

$$\xi = \sqrt{\epsilon'_1 - \sin^2 \theta};$$ (4)

$\epsilon'_1$ – relative dielectric constant of the substrate MSA; $k_0$ – propagation constant in a vacuum.

The resonator method considers the MSA substrate as a uniform dielectric. Consequently, an electromagnetic wave (EMW) that propagates in the substrate can be described by a simple dispersion equation [5]:

$$\omega(k) = \frac{c_0k}{\epsilon'_1}$$ (5)

where $\omega$ – EMW frequency; $c_0$ – light speed in a vacuum. In other words, $\epsilon'_1$ is a so-called connection measure of the EMW frequency propagating in a homogeneous dielectric medium with propagation constant in the same environment, and can be expressed through them:

$$\epsilon'_1 = \left( \frac{c_0k}{\omega} \right)^2.$$ (6)

2.2 The structure of the field in the magnetoelectric composite

For a qualitative description of the microwave processes in the given substrate, we consider each layer separately.

The GGG layer is an isotropic dielectric, the field in which depends on the dielectric constant and its described by the dispersion equation (5).

The properties of the ferrite YIG layer on the microwave are determined by the FMR region. They are manifested in the resonant absorption of the electromagnetic field energy at the frequencies that coincide with the precession frequencies of the magnetic moments, and formation magnetostatic (spin) waves. This type belongs to the class of slow waves.

Finally, the ferroelectric layer PZT. Here two things are important to us: the first is that this layer has a high value of dielectric constant; the second – there is a dependence of the dielectric constant on the applied electric field.

Therefore, the task of the subsection is to find a single wave solution for all three layers, in which both the different nature of the oscillations and the influence of the applied electric field on the properties of the ferrite and ferroelectric would be taken into account. At the stage of analysis, the existence of such a decision is confirmed based on following: EMW that propagates in the PZT layer with a large dielectric constant, as well as the spin wave, can be classified as slow. This is a manifestation of the electromagnetic delay effect, which
follows from the expression (5). That is, situations are possible in which the phase velocities of an electromagnetic wave in the ferroelectric and a spin wave in the ferrite will be comparable – electromagnetic interaction of these types of waves are occurs, in other words, hybridization.

One way to solve this problem may be the theory of spin-wave modes, which is a solution method of the integrodifferential equation derived from a join consideration of the equation of electrodynamics of a layered structure and linearized magnetization motion taking into account electrodynamic and exchange boundary conditions [6]. According to the method, first the expressions specifying the relationship between the distributions of the variable magnetic field and the variable magnetization over the thickness of the layered structure are found – tensor Green function of Maxwell equations for the layered structure. Then, the obtained expressions are substituted the linearized equation of the magnetization motion that is solved for three cases of exchange boundary conditions: free surface spins, fixed and arbitrary fixing.

Return to our task, the similar structure (Fig.2), metal-ferroelectric-ferromagnetic-dielectric-metal, was investigated [7]. The approaching dispersion equation for a hybrid electromagnetic-spin wave (HEMSW) was obtained as a result that had the following form:

\[
(\Omega_n - \omega_M A_{n}^{xx})[(\Omega_n - \omega_M (A_{n}^{yy} \cos^2 \varphi + A_{n}^{zz} \sin^2 \varphi)) - (\omega + \omega_M A_{n}^{xx} \sin \varphi)\omega - \omega_M A_{n}^{xx} \sin \varphi) = 0
\]

Fig.2. Layered geometry.

where:

\[
A_{n}^{xx} = \begin{bmatrix}
\frac{1}{\sinh(\gamma_n b)N_2 (\gamma_n^2 + k_n^2)} \times \left\{ C_n^3 [D_b (-1)^n + T_b] + C_n^4 [D_a - T_a (-1)^n] \right\} \\
\frac{1}{\sinh(\gamma_n b)N_2 (\gamma_n^2 + k_n^2)} \times \left\{ C_n^3 [D_b (-1)^n + T_b] + C_n^4 [D_a - T_a (-1)^n] \right\}
\end{bmatrix}
\]

\[
A_{n}^{yy} = \begin{bmatrix}
\frac{k_n^2}{\gamma_n^2 + k_n^2} - \frac{1}{\sinh(\gamma_n a)N_2 (\gamma_n^2 + k_n^2)} \times \left\{ C_n^3 \frac{\gamma_n \gamma_a}{\epsilon_n \epsilon_a \gamma_b} \tanh(\gamma_n a) D_b \tanh(\gamma_n a) T_b + C_n^4 \epsilon_n \epsilon_a \gamma_b \tanh(\gamma_n a) T_a \right\}
\end{bmatrix}
\]

\[
A_{n}^{zz} = \begin{bmatrix}
\frac{k_n^2}{\gamma_n^2 + k_n^2} - \frac{1}{\sinh(\gamma_n a)N_2 (\gamma_n^2 + k_n^2)} \times \left\{ C_n^3 \frac{\gamma_n \gamma_a}{\epsilon_n \epsilon_a \gamma_b} \tanh(\gamma_n a) D_b \tanh(\gamma_n a) T_b + C_n^4 \epsilon_n \epsilon_a \gamma_b \tanh(\gamma_n a) T_a \right\}
\end{bmatrix}
\]
\[ A_i^{xx} = -\frac{\gamma_i^2}{\gamma_i^2 + \kappa_n^2} + \frac{1}{\sinh(\gamma_i^2 + \kappa_n^2) L(1 + \delta_{on})} \left( C_i^2 \left[ \frac{\gamma_i}{\gamma_a} \tanh(\gamma_i^2 b) T_b \right] - C_i^4 \left[ \frac{\gamma_i}{\gamma_b} \tanh(\gamma_i^2 a) T_a \right] \right) \] (12)

\[ C_i^1 = \cosh(\gamma_i^2 (a + L))(-1)^n - \cosh(\gamma_i^2 a), \] (13)

\[ C_i^2 = \cosh(\gamma_i^2 b) (-1)^n - \cosh(\gamma_i^2 (b + L)), \] (14)

\[ C_i^3 = \sinh(\gamma_i^2 (a + L))(-1)^n - \sinh(\gamma_i^2 a), \] (15)

\[ C_i^4 = \sinh(\gamma_i^2 b) (-1)^n - \sinh(\gamma_i^2 (b + L)). \] (16)

\[ D_a = \sinh(\gamma_i^2 (a + L)) + \frac{\gamma_i}{\gamma_a} \tanh(\gamma_i^2 a) \cosh(\gamma_i^2 (a + L)), \] (17)

\[ D_b = \sinh(\gamma_i^2 (b + L)) + \frac{\gamma_i}{\gamma_b} \tanh(\gamma_i^2 b) \cosh(\gamma_i^2 (b + L)), \] (18)

\[ T_a = \sinh(\gamma_i^2 a) - \frac{\gamma_i}{\gamma_b} \tanh(\gamma_i^2 b) \cosh(\gamma_i^2 a), \] (19)

\[ T_b = -\sinh(\gamma_i^2 b) + \frac{\gamma_i}{\gamma_a} \tanh(\gamma_i^2 a) \cosh(\gamma_i^2 b). \] (20)

\[ N = \sinh(\gamma_i^2 L) \left[ 1 + \frac{\gamma_i^2}{\gamma_a \gamma_b} \tanh(\gamma_i^2 a) \tanh(\gamma_i^2 b) \right] + \cosh(\gamma_i^2 L) \left[ \frac{\gamma_i}{\gamma_b} \tanh(\gamma_i^2 b) \right] \] (21)

\[ d = a + b + L, \] (22)

\[ \Omega_n = \omega_H + \omega_M \alpha (k_i^2 + \kappa_n^2), \] (23)

\[ \omega_M = \gamma M_0, \quad \omega_H = \gamma H_{eff}, \] (24)

\[ \kappa_n = \frac{n \pi}{L}, \] (25)

\[ \gamma_i^2 = k_i^2 - k_{BL}^2, \quad k_i^2 = \frac{\omega^2}{c_0^2} \varepsilon_i, \] (26)

\[ \gamma_a^2 = k_a^2 - k_{0a}^2, \quad k_a^2 = \frac{\omega^2}{c_0^2} \varepsilon_a, \] (27)

\[ \gamma_b^2 = k_b^2 - k_{0b}^2, \quad k_b^2 = \frac{\omega^2}{c_0^2} \varepsilon_b. \] (28)

\( n \) – the number of the spin-wave mode; \( \gamma = 2\pi \times 2.8 \times 10^6 \text{ psec}^{-1} \) – gyromagnetic ratio; \( \alpha \) – constant of nonuniform exchange interaction (for YIG \( \alpha = 3.1 \times 10^{-12} \text{ cm}^3 \)); \( M_0 \) – the saturation magnetization of ferrite.

The terms \( N^Y, T_i^Y, D_i^Y \) are derived from terms \( N, T_i, D_i \) by replacing

\[ \frac{\gamma_i}{\gamma_a} \rightarrow \frac{\varepsilon_i Y_a}{Y_L}, \quad \frac{\gamma_i}{\gamma_b} \rightarrow \frac{\varepsilon_i Y_b}{Y_L}, \quad \frac{\gamma_i}{\gamma_{0a}} \rightarrow \frac{\varepsilon_i Y_{0a}}{Y_L}, \quad \frac{\gamma_i}{\gamma_{0b}} \rightarrow \frac{\varepsilon_i Y_{0b}}{Y_L}. \] (29)

As can be seen from the expression for HEMS (7), the dispersion characteristic can be influenced both by changing the effective internal magnetic field of the ferrite \( H_{eff} \) (23), (24) and by changing the dielectric constant of the ferroelectric (PZT) (27). Therefore, in expression (7), it is necessary to introduce their dependencies \( H_{eff}(E) \) (by ME-effect) and \( \varepsilon_{0}(E) \) on the applied external electric field.
2.2.1 The influence of the ME-effect on the dispersion characteristic HEMSW

The effective internal constant magnetic field of ferrite is the sum of an external constant magnetic field, demagnetizing field and anisotropy fields, in particular, her magnetic crystallographic and magnetoelastic types. The magnetoelastic type of anisotropy is determined by the ME-effect. Then, provided that electric field is directed along the axis of symmetry of the structure, the expression for the effective constant magnetic field will take the form [8]:

\[ H_{\text{eff}}(E) = H_0 - M_0 + \frac{2K_1}{M_0} + 2M_0[(B_{31} - B_{33})E_0 + (b_{31} - b_{33})E_0^2], \tag{30} \]

where \( H_0 \) – the external constant magnetic field, \( E_0 \) – the external constant electric field, and \( B_{ij} \) and \( b_{ij} \) – linear and nonlinear ME-constants. When only linear ME-constants are taken into account, the value for the composite PZT-YIG [1]:

\[ 2M_0(B_{31} - B_{33}) = 0.125 \ [\text{Oe} \times \text{cm}/\text{kV}], \tag{31} \]

2.2.2 The influence of the external constant electric field on the properties of the ferroelectric

To describe the theoretical dependence \( \varepsilon(E) \) can use one of the ways through the model of the multipolarization mechanism [9]

\[ \varepsilon_r(E) = \frac{\varepsilon_r(0)}{[1 + \varepsilon_r(0)[\varepsilon_0(0)]^b E^2]^{1/3}} + \sum(P_0 x/\varepsilon_0) \times [\cosh(E x)]^{-2}, \tag{32} \]

\[ x = P_0 V/k_B T, \lambda = 3\beta \sim 10^{10}, \tag{33} \]

where \( \varepsilon_r(0) \) – relative dielectric constant at zero electric field, \( \varepsilon_0 \) – absolute dielectric constant of vacuum, \( T \) – temperature, \( P_0 \) – effective polarization of one cluster, \( V \) – volume of cluster, \( k_B \) – Boltzmann constant, \( \beta \) – generally temperature-independent coefficient. In most practical cases, the first term of expression (32) is enough for calculations, which can be expanded in a power series subject to small changes in the field \( E \) (up to 60 kV/cm):

\[ \varepsilon_r(E) = \frac{\varepsilon_r(0)}{[1 + \varepsilon_r(0)[\varepsilon_0(0)]^b E^2]^{1/3}} = \varepsilon_1 - \varepsilon_2 E^2 + \varepsilon_3 E^4 - \varepsilon_4 E^6 + \varepsilon_5 E^8 - \ldots, \tag{34} \]

где

\[ \varepsilon_1 = \varepsilon_r(0), \varepsilon_2 = \frac{1}{3} \lambda \varepsilon_0^3, \varepsilon_3 = \frac{2}{9} \lambda^2 \varepsilon_0^6, \varepsilon_4 = \frac{4}{81} \lambda^3 \varepsilon_0^9, \varepsilon_5 = \frac{35}{243} \lambda^4 \varepsilon_0^{12}, \varepsilon_r(0)^{13} \]

(35)

As can be seen from (35), the first two terms of the series have the main influence. Then, dependence \( \varepsilon(E) \) for PZT will have form:

\[ \varepsilon_r(E) = 1870 - 0.277 \times E^2, \tag{36} \]

where \( E \) has dimension kV/cm.

Thus, we can influence on the dispersion characteristic of the composite structure by changing the applied constant electric field to the ferroelectric layer by two mechanisms (30) and (36), and consequently, on the microwave properties MSA.

3 Calculation fo the dependence of the microwave properties of MSA on the applied electric field

The following parameters of the composite structure were used to calculate (Fig.2): layer \( a \) – PZT: thickness \( a = 100 \mu m \), relative dielectric constant \( \varepsilon_a(0) = 1870 \); layer \( b \) –
GGG: thickness $b = 500$ um, relative dielectric constant $\varepsilon_b = 11.6$; layer L – YIG: thickness $L = 5$ um, relative dielectric constant $\varepsilon_L = 14$, saturation magnetization $M_0 = 1750$ Gs; external constant magnetic field $H_0 = 4113$ Oe. Angle $\varphi = 0^\circ$ – the magnetizing external magnetic field lies in the plane of the sample and is directed with the propagation constant $k_\zeta$. Proportions of MSA: $50 \times 50$ (mm), thickness $d = a + L + b = 605$ um.

The dispersion characteristic of the HEMSW for the first spin-wave mode was obtained (Fig.3) by graphoanalytical method for solving the equation (7) based on derived dependencies (30), (31) and (36) for two values of an external constant electric field $E = 0$ kV/cm и $E = 40$ kV/cm.

The points A (f = 18.92 GHz, $k_\zeta = 20$ cm$^{-1}$) и B (f = 18.92 GHz, $k_\zeta = 55.3$ cm$^{-1}$) show the dependence of propagation constant of the structure on the constant electric field at a fixed frequency. According to the obtained characteristics, our composite structure at a frequency of 18.92 GHz will be characterized be the following: at zero field (point A) the value of the propagation constant will be equal to $20$ cm$^{-1}$, an increase in the constant electric field leads to a decrease $k_\zeta$ up to the formation of standing waves ($k_\zeta = 0$ cm$^{-1}$) at $E = 18.5$ kV/cm, a future increase in the field will lead to a subsequent decrease $k_\zeta$ in the range from $+\infty$ to the point B at $E = 40$ kV/cm. This process is illustrated more clearly in Fig.4(a).

![Fig.3](image-url) The dispersion characteristic of the HEMSW for two values of the applied electric field 0 kV/cm и 40 kV/cm

As can be seen from Fig.3, the so-called “repulsion” of dispersion branches and the appearance of frequency “gaps” occur in the obtained spectrum as a result of hybridization of spin and electromagnetic waves [6].

However, to study MSA, the graph of the dependence of the resonance frequency on the applied electric field at $k_\zeta = const$ is more important (Pic.4(6)). This dependence was calculate under the condition of antenna resonance ($kb \approx \pi$), that is, a characteristic is obtained for the wave number $k_\zeta \approx 63$ cm$^{-1}$ for a given value of $b = 50$ mm. As can be seen at Fig.4(b) , in our case, an increase in applied electric field leads to an almost linear increase in the resonance frequency MSA.

In addition, radiation patterns for the studied MSA were calculated using expressions (2) and (3), based on the following considerations: with point of view of the hybrid wave, the layered structure can be represented as a homogeneous nonlinear medium with a calculated characteristic (Fig.3). Consequently, the effective replacement of the relative
Fig. 4. Dependences on the value of constant electric field applied to the PZT layer: (a) for propagation constant of composite structure; (b) for resonance frequency MSA.

dielectric constant in expressions (2) and (3) by expression (6) with the subsequent substitution of pairs of value of $\omega$ and $k$ from the obtained dispersion characteristic HEMSW (Fig.3) is justified. In our case, radiation patterns were calculated for points C ($f = 16.98$ GHz, $k_\zeta = 63$ cm$^{-1}$) and D ($f = 19.2$ GHZ, $k_\zeta = 63$ cm$^{-1}$), which corresponds to two values of the applied electric field 0 kV/cm и 40 kV/cm.

Fig. 5. Radiation patterns of the investigated MSA: (a) in plane $H (z0x)$; (b) in plane $E (z0y)$.

As can be seen at Fig.5(a) and Fig.5(b), an increase in the applied constant electric field to the ferroelectric layer leads to a narrowing of the radiation. This is consistent with the dependence of the MSA resonance frequency on the field value $E$ (Fig.4(b)), since this leads to a decrease in the ratio of the resonance wavelength to the geometric dimensions of the MSA, that is equivalent to increasing the effective size of the antenna. This effect can be used to create a class of electronic frequency-tunable antenna devices.

4 Conclusions

Thus, in this work the single wave solution that is fully capable of taking into account the processes occurs in the composite structure under study, when an external constant electric field is applied to the ferroelectric component was obtained. This wave solution corresponds to a hybrid electromagnetic-spin wave, dispersion characteristic of which was obtained by calculation method (Fig.3)
For the case, when the magnetizing constant field lies in the plane of structure in the direction of propagation of hybrid wave, an increase in the constant electric field leads to an almost linear increase in the resonance frequency of the structure. This is due to two mechanisms: ME-effect – through a change in the effective magnetic field of ferrite, and due to the dependence of the dielectric constant of the ferroelectric layer on the applied constant electric field. The total effect gives a change in the resonant frequency of about 2 GHz with a change in the constant electric field by 40 kV/cm. As a result, there is a change in the wave properties of the investigated MSA – narrowing of the radiation pattern.

The following conclusion may be equally important in meaning: at a constant oscillation frequency, a change in the applied electric field can lead to the formation of standing waves in a layered structure (Fig.4(a)). This is due to the frequency “gaps” formed during the hybridization process.

Acknowledgements

The research at Novgorod State University was supported by the Ministry of Education and Science of Russia in the state task of №11.7069.2017/8,9.

References

1. M.I. Bichurin, V.M. Petrov, D.A. Filippov, G. Srinivasan Magnetoelectric effect in composite materials NovSU n.a. Yaroslav-the-Wise, Veliky Novgorod (2005)
2. R.V. Petrov, G. Srinivasan Design of magnetoelectric phased antenna array, Vestnik NovSU, Issue: Engineering Sciences 50, pp. 61-65 (2009)
3. M.I. Bichurin, V.M. Petrov, R.V. Petrov, A.S. Tatarenko Magnetoelectric microwave devices, Ferroelectrics 280(1), pp. 211-218 (2011)
4. B.A. Panchenko, E.I. Nefedov Microstrip antennas, Radio and Communication, Moscow (1986)
5. B.A. Kalinikos Spin waves in ferromagnetic films, SOJ 5, pp 93-100 (1996)
6. N.J. Grigireva, B.A. Kalinikos Theory of spin waves in a ferromagnetic film layered structures, SPbGETU, Sankt-Petersburg (2008)
7. B.E. Demidiv, B.A. Kalinikos, JTF 71(2), pp. 89-93 (2001)
8. M. I. Bichurin, V. M. Petrov, and Yu. V. Kiliba Magnetic and magnetoelectric susceptibilities of a ferroelectric-ferromagnetic composite at microwave frequencies, Physical review B 66, 134404, pp 1-10 (2002)
9. Ang Ch., Yu Zh. DC electric-field dependence of the dielectric constant in polar dielectrics: Multipolarization mechanism model, Physical review B 69, 174109, pp 1-8 (2004)