Renormalization of the three flavor Lagrangian in heavy baryon chiral perturbation theory

G. Müller†#2, Ulf-G. Meißner‡#3

†Universität Bonn, Institut für Theoretische Kernphysik
Nussallee 14-16, D-53115 Bonn, Germany

‡Forschungszentrum Jülich, Institut für Kernphysik (Theorie)
D-52425 Jülich, Germany

Abstract

The complete renormalization of the generating functional for Green functions of quark currents between one–baryon states in three flavor heavy baryon chiral perturbation theory is performed to order $q^3$. As an example, we study the kaon loop induced divergences in neutral pion photoproduction off protons.

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#2 email: mueller@pythia.itkp.uni-bonn.de
#3 email: Ulf-G.Meissner@kfa-juelich.de
1 Introduction

Chiral perturbation theory (CHPT) is the effective field theory of the standard model at energies below the scale of spontaneous chiral symmetry breaking QCD is believed to undergo. The basic degrees of freedom are the eight (almost) massless pseudoscalar Goldstone bosons. In the framework of the non–linear realization of the chiral symmetry, it is also straightforward to implement matter fields (like e.g. baryons) in the pertinent effective field theory [1] [2]. The first systematic discussion for the two flavor sector, i.e. the pion–nucleon system, implementing the ideas of a chiral power counting [3], was performed in ref.[4]. However, treating the nucleons as relativistic spin–1/2 (Dirac) fields does not allow for a one–to–one correspondence between the expansion in small momenta and quark masses on one and the pion loop expansion on the other side. As pointed out in ref.[5], this short–coming can be overcome if one makes use of methods borrowed from HQEFT, namely to consider the baryons as extremely heavy, static sources. This is called heavy baryon CHPT (HBCHPT). One considers a particular frame in which any baryon field can be characterized by a four–velocity $v_\mu$. This allows to transform the troublesome baryon mass term in the propagator into a string of $1/m$ suppressed meson–baryon interaction vertices. A more formal approach based on path integral methods was proposed in ref.[6]. In that scheme, it is particularly simple to systematically construct all $1/m$ suppressed vertices with fixed and with free coupling constants. Stated differently, Lorentz invariance is automatically ensured since one starts from the fully relativistic pion–nucleon Lagrangian to perform the frame–dependent decomposition of HBCHPT. To one loop order, divergences appear. Some of these were treated e.g. in [3]. A systematic treatment of the leading divergences of the generating functional for Green functions of quark currents between one–nucleon states was given in ref.[7]. This allows for a chiral invariant renormalization of all two–nucleon Green functions of the pion–nucleon system to order $q^3$ in the low–energy expansion, were $q$ denotes a small momentum or Goldstone boson mass. This complete divergence structure listed in [7] was e.g. heavily used as a check in the calculation of the reaction $\pi N \rightarrow \pi \pi N$ to $O(q^3)$ [8]. Furthermore, in ref.[9] nucleon field transformations were used to bring the renormalized pion–nucleon Lagrangian in standard form, i.e. all finite and infinite one–particle–irreducible vertices were obtained. In that paper, it was also stressed that one has to consistently construct the meson and the meson–baryon Lagrangians since otherwise one is left with unwanted divergences in the reducible functional diagrams at order $q^3$ proportional to the equations of motion for the meson fields. Further applications to the pion–nucleon system to one loop accuracy are summarized in the reviews [10] [11] [12].

The situation is very different in SU(3). Although the original heavy fermion approach was formulated for three flavors [3], most of the corresponding calculations of baryon masses, magnetic moments, hyperon polarizabilities and decays and so on are either finite to order $q^3$ or only the leading non–analytic pieces (at a scale of 1 GeV) were accounted for. The only works in which renormalization at order $q^3$ has been performed are concerned with the kaon–nucleon interaction [13], kaon photo– and electroproduction [14] and strong and electromagnetic decays of the decuplet (where the EFT is extended to include the
spin–3/2 fields). It is our aim to work out all leading divergences in the generating functional of the three flavor meson–baryon interaction, thus extending the work of Ecker making use of the same methods. The main difference to the two flavor case is the richer structure of the corresponding Lie algebra, thus leading to much more allowed terms at a given order in the chiral Lagrangian. Stated differently, the main difference between the two calculations lies in the fact that the nucleons are in the fundamental representation of SU(2), while the baryons are in the adjoint representation of SU(3). This leads to some algebraic consequences for the construction of the one-loop generating functional to be discussed below. We remark that all terms to order $q^3$ in relativistic SU(3) baryon CHPT were enumerated in [10]. In that paper, mass and wavefunction renormalization was performed. Obviously, only a subset of the operators listed in [10] contains divergences.

The manuscript is organized as follows. In section 2 we review the path integral formalism of heavy baryon CHPT extended to the three flavor case. In section 3, we work out the generating functional to one loop, i.e. to order $q^3$. Here and in what follows, our work closely parallels the one of ref.[7]. However, we give a more detailed exposition of the method. Some formalism related to the heat kernel technique is spelled out in section 4. Then, in section 5 we work out the renormalization of the irreducible tadpole graph. The much more involved renormalization of the irreducible self–energy graph is spelled out in section 6. In section 7, we write down the full counterterm Lagrangian at order $q^3$ and tabulate the pertinent operators and their $\beta$–functions. This table constitutes the main result of this investigation. Section 8 contains a sample calculation for $\pi^0$ photoproduction off protons. Here, kaon loops lead to a $q^3$ divergence. We give straightforward Feynman diagram evaluation of this divergence and show how to use table 1. A summary and a discussion of the various checks on our calculation is given in section 9. The appendices contain sufficiently detailed technicalities to check the calculation at various intermediate steps.

## 2 Heavy baryon formalism: path integral approach

The interactions of the Goldstone bosons with the ground state baryon octet states are severely constrained by chiral symmetry. The generating functional for Green functions of quark currents between single baryon states, $Z[j, \eta, \bar{\eta}]$, is defined via

$$\exp \{ i Z[j, \eta, \bar{\eta}] \} = N \int [du][dB][\bar{B}] \exp \left[ i \left( S_M + S_{MB} + \int d^4x \left< \bar{\eta}B > + < \bar{B}\eta > \right) \right] , \quad (1)$$

with $S_M$ and $S_{MB}$ denoting the mesonic and the meson–baryon effective action, respectively, to be discussed below. $\eta$ and $\bar{\eta}$ are fermionic sources coupled to the baryons and $j$ collectively denotes the external fields of vector ($v_\mu$), axial–vector ($a_\mu$), scalar ($s$) and pseudoscalar ($p$) type. These are coupled in the standard chiral invariant manner. In particular, the scalar source contains the quark mass matrix $\mathcal{M}$, $s(x) = \mathcal{M} + \ldots$. Traces in flavor space are denoted by $< ... >$. The underlying effective Lagrangian can be decomposed into a purely mesonic ($M$) and a meson–baryon ($MB$) part as follows (we only
consider processes with exactly one baryon in the initial and one in the final state

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_M + \mathcal{L}_{MB} \]  

subject to the following low-energy expansions

\[ \mathcal{L}_M = \mathcal{L}_M^{(2)} + \mathcal{L}_M^{(4)} + \ldots, \quad \mathcal{L}_{MB} = \mathcal{L}_{MB}^{(1)} + \mathcal{L}_{MB}^{(2)} + \mathcal{L}_{MB}^{(3)} + \ldots \]  

where the superscript denotes the chiral dimension. The lowest order meson Lagrangian takes the form [17]

\[ \mathcal{L}_M^{(2)} = \frac{F_0^2}{4} < u_\mu u^\mu + \chi >, \]
\[ u_\mu = i [u^\dagger (\partial_\mu - ir_\mu) u - u (\partial_\mu - il_\mu) u^\dagger] = i [\xi_R^\dagger (\partial_\mu - ir_\mu) \xi_R - \xi_L^\dagger (\partial_\mu - il_\mu) \xi_L], \]
\[ \chi_\pm = u^\dagger \chi u^\dagger \pm u \chi u^\dagger. \]  

The pseudoscalar Goldstone fields (\( \phi = \pi, K, \eta\)) are collected in the 3 \times 3 unimodular, unitary matrix \( U(x) \),

\[ U(\phi) = u^2(\phi) = \exp\{i\phi/F_0\} \]  

with \( F_0 \) the pseudoscalar decay constant (in the chiral limit), and

\[ \phi = \sqrt{2} \left( \begin{array}{ccc} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & \pi^+ & K^+ \\ \pi^- & \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta & K^0 \\ K^- & K^0 & - \frac{2}{\sqrt{6}} \eta \end{array} \right). \]  

Under \( \text{SU(3)}_L \times \text{SU(3)}_R \), \( U(x) \) transforms as \( U \rightarrow U' = LUR' \), with \( L, R \in \text{SU(3)}_{L,R} \). Furthermore, \( \xi_L, \xi_R \) are elements of the chiral coset space \( \text{SU(3)}_L \times \text{SU(3)}_R / \text{SU(3)}_V \) with \( U(\phi) = \xi_R(\phi) \xi_L^\dagger(\phi) \). The more familiar choice for the \( \xi_{L,R} \) is \( u(\phi) = \xi_R = \xi_L^\dagger \), compare Eq.(5). The external fields appear in Eq.(4) in the following chiral invariant combinations,

\[ r_\mu = v_\mu + a_\mu, \quad l_\mu = v_\mu - a_\mu, \quad \chi = 2B_0 (s + ip), \]  

and \( B_0 \) is related to the quark condensate in the chiral limit, \( B_0 = |<0|\bar{q}q|0>|/F_0^2 \). We adhere to the standard chiral counting,

\[ \mathcal{O}(1): U, u, \xi_L, \xi_R, \quad \mathcal{O}(q): \partial_\mu, l_\mu, r_\mu, u_\mu, \quad \mathcal{O}(q^2): s, p, F^{\mu\nu}, \]  

with \( q \) denoting a small momentum or meson mass.

The effective meson–baryon Lagrangian starts with terms of dimension one,

\[ \mathcal{L}_{MB}^{(1)} = \langle \bar{B} [i\nabla, B] \rangle - m \langle \bar{B} B \rangle 
+ \frac{D}{2} \langle \bar{B} \{\gamma_5, B\} \rangle + \frac{F}{2} \langle \bar{B} [\gamma_5, B] \rangle, \]  

with \( D \) and \( F \) denoting the meson mass and decay constant, respectively.
with $m$ the average octet mass in the chiral limit. The $3 \times 3$ matrix $B$ collects the baryon octet,

$$B = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & \frac{1}{\sqrt{2}} \Sigma^+ & p \\ -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda & n \\ \Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda \end{pmatrix}. \quad (10)$$

Under $SU(3)_L \times SU(3)_R$, $B$ transforms as any matter field,

$$B \rightarrow B' = KBK^\dagger, \quad (11)$$

with $K(U, L, R)$ the compensator field representing an element of the conserved subgroup $SU(3)_V$. $\nabla_\mu$ denotes the covariant derivative,

$$[\nabla_\mu, B] = \partial_\mu B + [\Gamma_\mu, B] \quad (12)$$

and $\Gamma_\mu$ is the chiral connection,

$$\Gamma_\mu = \frac{1}{2} [u^\dagger(\partial_\mu - ir_\mu)u + u(\partial_\mu - il_\mu)u^\dagger] \nonumber$$

$$= \frac{1}{2} [\xi_R^\dagger(\partial_\mu - ir_\mu)\xi_R + \xi_L^\dagger(\partial_\mu - il_\mu)\xi_L]. \quad (13)$$

Note that the first term in Eq. (9) is of dimension one since $[i\nabla, B] - mB = O(q)$ [4]. The lowest order meson–baryon Lagrangian contains two axial–vector coupling constants, denoted by $D$ and $F$. The dimension two and three terms have been enumerated by Krause [16]. Treating the baryons as relativistic spin–1/2 fields, the chiral power counting is no more systematic due to the large mass scale $m$, $\partial_0 B \sim mB \sim \Lambda_\chi B$. This problem can be overcome in the heavy mass formalism proposed in [5]. We follow here the path integral approach developed in [3]. Defining velocity–dependent spin–1/2 fields by a particular choice of Lorentz frame and decomposing the fields into their velocity eigenstates (sometimes called 'light' and 'heavy' components),

$$H_v(x) = \exp\{imv \cdot x\} P_v^+ B(x), \nonumber$$

$$h_v(x) = \exp\{imv \cdot x\} P_v^- B(x), \nonumber$$

$$P_v^\pm = \frac{1}{2}(1 \pm \phi), \quad v^2 = 1, \quad (14)$$

the mass dependence is shuffled from the fermion propagator into a string of $1/m$ suppressed interaction vertices. In this basis, the three flavor meson–baryon action takes the form

$$S_{MB} = \int d^4x \left\{ \bar{H}_v^a A^{ab} H_v^b - \bar{k}_v^a C^{ab} k_v^b + \bar{h}_v^a B^{ab} H_v^b + \bar{H}_v^a \gamma_0 B^{ab}_{\gamma_0} h_v^b \right\}, \quad (15)$$

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with $a, b = 1, \ldots, 8$ flavor indices. The $8 \times 8$ matrices $A$, $B$ and $C$ admit low energy expansions, in particular

\begin{align}
A_{(1)}^{ab} &= A_{(1)}^{ab} + A_{(2)}^{ab} + A_{(3)}^{ab} + \ldots \\
A_{(2)}^{ab} &= < \lambda^a i v \cdot \nabla, \lambda^b > + D < \lambda^a \{ S \cdot u, \lambda^b \} > + F < \lambda^a [ S \cdot u, \lambda^b ] > , \\
B_{(1)}^{ab} &= \frac{D}{F} < \lambda^a ( \chi_+ , \chi_+ ) > + b_0 < \lambda^a \lambda^b > < \chi_+ > \\
C_{(1)}^{ab} &= A_{(1)}^{ab} + 2m \delta^{ab}
\end{align}

where $\lambda^a$ denotes the SU(3) matrices in the physical basis,

\begin{equation}
O(x) = O^a(x) \lambda^a , \quad O = \{ B, H_v, h_v \} , \quad < \lambda^a \lambda^b > = \delta^{ab},
\end{equation}

and $S^\mu$ is the covariant spin–operator à la Pauli–Lubanski,

\begin{equation}
S^\mu = \frac{i}{2} \gamma_5 \sigma^{\mu\nu} v_\nu,
\end{equation}

subject to the constraint $S \cdot v = 0$, with

\begin{equation}
[S_\mu, S_\nu] = i \epsilon_{\mu\nu\rho} v^\rho S_\rho , \{ S_\mu, S_\nu \} = \frac{1}{2} ( v_\mu v_\nu - g_\mu\nu ) , \quad S^2 = - \frac{d - 1}{4} .
\end{equation}

Furthermore, $\nabla^\perp = \gamma^\mu ( v_\mu v_\nu - g_\mu\nu ) \nabla^\nu$. We have introduced the compact notation

\begin{equation}
D/F < \ldots ( Q, O ) \perp \ldots > = \frac{D}{F} < \ldots ( Q, O ) \ldots > + F < \ldots ( Q, O ) \ldots >
\end{equation}

for any $3 \times 3$ matrices $Q$ and $O$. Notice that the (anti)commutators ($\ldots \perp \ldots$) only act in flavor space and that spin–matrices appearing in the operators have all to be taken to the left in the appropriate order. To reduce the number of terms in $A_{(2)}^{ab}$, we have made use of the Cayley-Hamilton relation for traceless matrices $X = \{ u, v \}$

\begin{equation}
< \lambda^a X > < X \lambda^b > = < \lambda^a \{ X^2, \lambda^b \} > + < \lambda^a X \lambda^b X > - \frac{1}{2} < \lambda^a \lambda^b > < X^2 > .
\end{equation}

We also have $F^{\mu\nu}_+ = u F^{L\mu\nu}_+ + u^\dagger F^{R\mu\nu}_+$, with $F^{L,R\mu\nu}$ the field strength tensors related to $l_\mu$ and $r_\mu$, respectively. Similarly, we split the baryon source fields $\eta^a(x)$ into velocity eigenstates,

\begin{align}
R_v(x) &= \exp \{ i m v \cdot x \} P^+_v \eta(x) , \\
\rho_v(x) &= \exp \{ i m v \cdot x \} P^-_v \eta(x),
\end{align}

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and shift variables
\[ h_v^{a'} = h_v^a - (C^{ac})^{-1} (B^{cd} H_v^d + \rho_v^c) \]
so that the generating functional takes the form
\[ \exp[iZ] = \mathcal{N} \Delta_h \int [dU][dH_v][d\bar{H}_v] \exp\{iS_M + iS_{MB}'\} \]
in terms of the meson–baryon action \( S_{MB}' \),
\[ S_{MB}' = \int d^4x \bar{H}_v^a (A^{ab} + \gamma_0 [B^{ac}]^\dagger \gamma_0 [C^{cd}]^{-1} B^{db}) H_v^b + \bar{H}_v^a (R_v^a + \gamma_0 [B^{ac}]^\dagger \gamma_0 [C^{cd}]^{-1} \rho_v^c) + \bar{\rho}_v^c [C^{cb}]^{-1} B^{ba} H_v^a. \]
The determinant \( \Delta_h \) related to the 'heavy' components is identical to one as first noted in [18]. This can be understood from the observation that this determinant is related to anti–particle propagation which decouples completely to the order we are working. The generating functional is thus entirely expressed in terms of the Goldstone bosons and the 'light' components of the spin–1/2 fields. The action is, however, highly non–local due to the appearance of the inverse of the matrix \( C \). To render it local, one now expands \( C^{-1} \) in powers of \( 1/m \), i.e. in terms of increasing chiral dimension,
\[ [C^{ab}]^{-1} = \frac{1}{2m} \sum_{n=0}^\infty (-1)^n \left( \left[ \frac{C - 2m \gamma_0}{2m} \right]^{ab} \right)^n, \]
\[ = \frac{\delta^{ab}}{2m} - \frac{A_{(1)}^{ab}}{(2m)^2} + O(q^2). \]
For calculating the higher order corrections to \( C^{-1} \) not exhibited in Eq.(26), it is advantageous to make use of the completeness relation
\[ < \lambda^a O \lambda^b > < \lambda^b Q \lambda^c > = < \lambda^a O Q \lambda^c > - \frac{1}{3} < \lambda^a O > < Q \lambda^c >, \quad O, Q \in SU(3) \]
to bring the pertinent expressions in a more compact form. To any finite power in \( 1/m \), one can now perform the integration of the 'light' baryon field components \( N_v \) by again completing the square,
\[ H_v^{a'} = [T^{ac}]^{-1} (R_v^c + \gamma_0 [B^{ac}]^\dagger \gamma_0 [C^{cd}]^{-1} \rho_v^d) \]
\[ T^{ab} = A^{ab} + \gamma_0 [B^{ac}]^\dagger \gamma_0 [C^{cd}]^{-1} B^{db}. \]
Notice that the second term in the expression for \( T^{ab} \) only starts to contribute at chiral dimension two (and higher). To be more precise, we give the chiral expansion of \( T^{ab} \) up to and including all terms of order \( q^3 \),
\[ T^{ab} = A_{(1)}^{ab} + A_{(2)}^{ab} + A_{(3)}^{ab} + \frac{1}{2m} \gamma_0 [B^{ac}]^\dagger_{(1)} \gamma_0 B^{cb}_{(1)} \]
\[ + \frac{1}{2m} \left( \gamma_0 [B^{ac}]^\dagger_{(1)} \gamma_0 B^{cb}_{(2)} + \gamma_0 [B^{ac}]^\dagger_{(2)} \gamma_0 B^{cb}_{(1)} \right) \]
\[ - \frac{1}{(2m)^2} \gamma_0 [B^{ac}]^\dagger_{(1)} \gamma_0 C^{cd}_{(1)} B^{db}_{(1)} + O(q^4). \]
We thus arrive at
\[ \exp[iZ] = N' \int [dU] \exp\{iSM + iZ_{MB}\} , \] (30)
with \( N' \) an irrelevant normalization constant. The generating functional has thus been reduced to the purely mesonic functional. \( Z_{MB} \) is given by
\[
Z_{MB} = - \int d^4x \left\{ \bar{\rho}_v^a ([C^{-1}]_{B}^{cd} [T^{de}]^{-1} \gamma_0 [B^{ef}]^\dagger \gamma_0 [C'^{-1}]_{B}^{ab} - [C^{-1}]_{B}^{ab} ) \rho_v^b \\
+ \bar{\rho}_v^a ([C^{-1}]_{B}^{ac} ) B^{cd} [T^{de}]^{-1} ) R_v^b + \bar{R}_v^a ([T^{ac}]^{-1} \gamma_0 [B^{cd}]^\dagger \gamma_0 [C'^{-1}]_{B}^{db} ) R_v^b \\
+ \bar{R}_v^a [T^{ab}]^{-1} R_v^b \right\} .
\] (31)

At this point, it is important to determine the chiral dimension with which the various terms in Eq.(31) start to contribute. The first term in the first line obviously only starts at order \( q^3 \) since both \( B^{ab} \) and \( T^{ab} \) start out at \( O(q) \). The second term in the first line is more tricky. As shown in Eq.(29), \( C^{-1} \) starts with terms of order one. However, these are exactly the contributions from anti–particle propagation, i.e. \( C^{-1} \) is related to the anti–baryon propagator as shown in Eq.(32). While the baryon propagator does not contain the mass any more, the anti–baryon propagator picks up exactly the factor \( 2m \) which is nothing but the gap between the particle and the anti–particle sectors. A more formal argument is given below. The terms in the second line of Eq.(31) are of order \( q^2 \) (and higher) and the term in the third line starts out at \( O(q) \). Consequently, to order \( q^3 \), only this last term in Eq.(31) generates the Green functions related to the 'light' fields. This also means that extending the calculation one order further, i.e. to \( O(q^4) \), will lead to a much more complicated structure than the one discussed here. We hope to come back to this point at a later stage. Let us return to the question of anti–particle propagation. We conjecture that the propagator \( C^{-1}(x) \) describes backward propagation along the time–like vector \( v \). This implies that fermion loops in the small momentum expansion can never be closed in coordinate space. Consider first the lowest order. With \( C^{(1)} = (iv \cdot \partial + 2m) \) one gets the free propagator
\[
S^{(1)}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{v \cdot k + 2m - i\epsilon} e^{-ikx} \\
= i e^{2mi v \cdot x} \Theta(-v \cdot x) \int \frac{d^3k}{(2\pi)^3} \delta(k \cdot v) e^{-ikx} , \] (32)
or in the rest–frame of the antiparticle,
\[
S^{(1)}(x) = i e^{2imt} \Theta(-t) \delta(\bar{r}) \] (33)
which shows that the anti–baryon is sharply located in space. At next order,
\[
C^{(2)} = - A^{(2)} , \] (34)
and the free propagator is given by Eq. (32) which shows that \( C_{(1)} + C_{(2)} \) corresponds to an antiparticle. On the other hand, for the lowest two orders of the 'light' fields we find

\[
S^{(1)}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{v \cdot k + i\epsilon} e^{-ikx}
\]

\[
= -i \Theta(v \cdot x) \int \frac{d^4k}{(2\pi)^3} \delta(k \cdot v) e^{-ikx},
\]

(35)

and with the contribution to the next order

\[
A^{(2)} = \frac{1}{2m} (v \cdot \partial)^2 - \frac{1}{2m} \partial^2,
\]

(36)

the corresponding free propagator is given by

\[
S^{(2)}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{v \cdot k - (v \cdot k)^2/2m + k^2/2m + i\epsilon} e^{-ikx}
\]

\[
= -i \Theta(v \cdot x) \int \frac{d^4k}{(2\pi)^3} \delta(k \cdot v) e^{-ikx} e^{ik^2(v \cdot x)/2m},
\]

(37)

which again shows that the particle propagates forward in time. It is, however, no longer sharply located at the point \( \vec{r} \) in coordinate space but rather shows some spread. Higher orders can be treated along the same lines, i.e. to any finite order in \( 1/m \) the particle and anti–particle sectors do not interact (see also Ecker’s proof that there are no closed baryon loops [19]).

To end this section, we give the chiral dimension \( D \) for processes with exactly one baryon line running through the pertinent Feynman diagrams,

\[
D = 2L + 1 + \sum_{d=4,6,...} (d - 2) N^M_d + \sum_{d=2,3,...} (d - 1) N^{MB}_d \geq 2L + 1
\]

(38)

with \( L \) denoting the number of (meson) loops, and \( N^M_d (N^{MB}_d) \) counts the number of mesonic (meson–baryon) vertices of dimension \( d \) (either a small momentum or meson mass). This means that tree graphs start to contribute at order \( q \) and \( L \)-loop graphs at order \( q^{2L+1} \). Consequently, the low energy constants appearing in \( L^{(2)}_{MB} \) are all finite.

## 3 Generating functional to one loop

In this section, we turn to the calculation of \( S_M[j] + Z_{MB}[j, R_v] \) to one loop, i.e. to order \( q^3 \) in the small momentum expansion. For doing that, we follow essentially the method used in ref. [4]. To be specific, one has to expand

\[
L^{(2)}_M + L^{(4)}_M - R^a_v \left[ A^{ab}_{(1)} \right]^{-1} R^b_v
\]

(39)

in the functional integral Eq. (30) around the classical solution, \( u_{cl} = u_{cl}[j] \), obtained by the variation \( \delta \int d^4x L^{(2)}_M / \delta U \) to lowest order. We need the first term in Eq. (39) to order
ξ^3, the second one to order ξ and the last one to order ξ^2. All other terms in Eq.(31) start at order q^2 or higher and thus can not contribute to the order q^3 one loop generating functional. The calculation for the purely mesonic sector is identical to the one given in ref.[17]. One thus arrives at a set of irreducible and reducible diagrams as shown in Fig. 1. The sum of the reducible diagrams is finite. Let us consider the irreducible ones. As in the mesonic sector, we chose the fluctuation variables ξ in a symmetric form [17],

\[ ξ_R = u_{cl} \exp\{i ξ/2\}, \quad ξ_L = u_{cl}^\dagger \exp\{-i ξ/2\}, \]

with ξ^† = ξ traceless 3×3 matrices. Consequently, we have also

\[ U = u_{cl} \exp\{i ξ\} u_{cl}. \]

To second order in ξ, the covariant derivative ∇µ, the chiral connection Γµ and the axial–vector uµ take the form

\[ Γ_µ = Γ_{cl}^µ + \frac{1}{4} [u_{cl}^µ, ξ] + \frac{1}{8} ξ \hat{∇}_µ ξ + O(ξ^3) \]

\[ [∇_{cl}^µ, ξ] = ∂_µ ξ + [Γ_{cl}^µ, ξ], \quad [∇_{cl}^µ, ξ] = ξ [∇_{cl}^µ, ξ] - [∇_{cl}^µ, ξ] ξ \]

\[ u_µ = u_{cl}^µ - [∇_{cl}^µ, ξ] + \frac{1}{8} [ξ, [u_{cl}^µ, ξ]] + O(ξ^3). \]

Notice that while ∇_{cl}^µ defined here acts on the fluctuation variables (fields) ξ(x), the covariant derivative ∇_µ defined in Eq.(12) acts on the baryon fields. Inserting this into the expression for \( A_{ab}^{(1)} \) and retaining only the terms up to and including order ξ^2 gives

\[ A_{ab}^{(1)} = A_{ab}^{(1), cl} + \frac{i}{4} < λ^{a\dagger} [v \cdot u_{cl}, ξ], λ^b > - D/F < λ^{a\dagger} ([S \cdot ∇_{cl}, ξ], λ^b) > \]

\[ + \frac{i}{8} < λ^{a\dagger} [ξ v \cdot ∇_{cl} ξ, λ^b] > + \frac{1}{8} D/F < λ^{a\dagger} ([ξ, [S \cdot u_{cl}, ξ]], λ^b) > + O(ξ^3). \]

We have not made explicit in Eq.(44) the dependence of all operators on the external sources j (see below).

We are now in the position to expand the fermion propagator S to quadratic order in the fluctuations making use of the relation (for the moment, we suppress the flavor indices)

\[ A_{1} S_{1} = 1. \]

Following Gasser et al. [1], we split \( A_{1} \) into the free and the interaction part,

\[ A_{1} = d_0 + d_I = d_0 [1 + d_0^{-1} d_I]. \]

We tentatively assume the existence of the inverse of the free fermion propagator. The interaction term \( d_I \) admits a low energy expansion starting at order q,

\[ d_I = d_I^q + d_I^q + d_I^q + \ldots. \]
Therefore, the fermion propagator to order $\xi^2$ reads

$$S_{(1)} = S_{(1)}^{cl} - S_{(1)}^{cl} d_1 S_{(1)}^{cl} - S_{(1)}^{cl} d_2 S_{(1)}^{cl} + S_{(1)}^{cl} d_1 S_{(1)}^{cl} d_1 S_{(1)}^{cl} + \mathcal{O}(\xi^3, q^3) .$$

(47)

Here, $S_{(1)}^{cl}$ denotes the full classical fermion propagator, i.e. with all possible tree structures of the external sources attached,

$$S_{(1)}^{cl} = (1 + d_0^{-1} d_1)^{-1} d_0^{-1} ,$$

(48)

where $d_0$ denotes the interactions with the Goldstone bosons after expanding around the classical solution. The expanded fermion propagator in Eq. (17) leads to an irreducible and a reducible part in the generating functional. Let us first consider the irreducible graphs. These are generated from the third and fourth term in Eq. (47), respectively, and referred to as the tadpole and the self-energy contribution, in order. Both are of $\mathcal{O}(\xi^2)$. The corresponding generating functional reads

$$Z_{irr}[j, \bar{R}_v] = \int d^4x \, d^4x' \, d^4y \, d^4y' \, \bar{R}_v(x) S_{(1)}^{de,cl}(x, y) \times$$

$$[\Sigma_{2}^{ab}(y, y') \delta(y - y') + \Sigma_{1}^{ab}(y, y')] S_{(1)}^{de,cl}(y', x') \bar{R}_v(x')$$

(49)

in terms of the self-energy functionals $\Sigma_{1,2}$. These read

$$\Sigma_1^{ab} = -\frac{2}{F_0^2} V_i^{ac} G_{ij} [A_{(1)}^{de,cl}]^{-1} V_j^{db} = -\frac{2}{F_0^2} V_i^{ac} G_{ij} S_{(1)}^{de,cl} V_j^{db}$$

$$\Sigma_2^{ab} = \frac{1}{8F_0^2} \left\{ \frac{D}{F} < \lambda^a \uparrow ([\lambda^i_G, [S \cdot u^{cl}, \lambda^j_G]], \lambda^b) \downarrow > G_{ij}$$

$$+ i < \lambda^a \uparrow [\lambda^i_G G_{ij} v \cdot d_{jk} - v \cdot d_{ij} G_{jk}) \lambda^b, \lambda^b] > \right\}$$

$$V_i^{ab} = V_i^{(1) ab} + V_i^{(2) ab}$$

$$V_i^{(1) ab} = \frac{i}{4\sqrt{2}} < \lambda^a \uparrow [v \cdot u^{cl}, \lambda^i_G], \lambda^b >$$

$$V_i^{(2) ab} = -\frac{D}{F} \frac{1}{\sqrt{2}} < \lambda^a \uparrow (\lambda^j_G S \cdot d_{ji}, \lambda^b) \downarrow >$$

(50)

with $i, j, k = 1, \ldots, 8$ and $\lambda^i_G$ denote Gell–Mann’s SU(3) matrices, which are related to the ones in the physical basis by $\lambda_{\mu} = (\lambda_{\mu}^c + i\lambda_{\mu}^7)/2, \lambda_n = (\lambda_{\mu}^6 + i\lambda_{\mu}^7)/2$, and so on. $G_{ij}$ is the full meson propagator

$$G_{ij} = (d_\mu d^\mu \delta_{ij} + \sigma^{ij})^{-1}$$

(51)

with

$$[\nabla^{\mu}_{\cl}, \xi] = \frac{1}{\sqrt{2}} \lambda^i_G d^\mu_{jk} \xi_k , \quad \xi = \frac{1}{\sqrt{2}} \lambda^i_G \xi_i ,$$

$$d^\mu_{ij} = \delta_{ij} \partial^\mu + \gamma^\mu_{ij}, \quad d^\mu_{ij} = \delta_{ij} \frac{\partial}{\partial \xi^\mu} - \gamma^\mu_{ij} ,$$

$$\gamma^\mu_{ij} = -\frac{1}{2} < \Gamma^\mu_{[ij]} \lambda^\ell_G, \lambda^{\ell}_G > ,$$

$$\sigma^{ij} = \frac{1}{8} < [u^\ell_{\cl}, \lambda^i_G] [\lambda^\ell_G, u^\mu_{\cl}] + \chi_+ \{ \lambda^i_G, \lambda^j_G \} > .$$

(52)
Note that the differential operator $d_{ij}$ is related to the covariant derivative $\nabla^\mu_{\cl}$ and it acts on the meson propagator $G_{ij}$. The connection $\gamma_\mu$ defines a field strength tensor,

$$
\gamma_{\mu \nu} = \partial_\nu \gamma_\mu - \partial_\mu \gamma_\nu + [\gamma_\mu, \gamma_\nu],
$$

$$
[d_\lambda, \gamma_{\mu \nu}] = \partial_\lambda \gamma_{\mu \nu} + [\gamma_\lambda, \gamma_{\mu \nu}],
$$

where we have omitted the flavor indices. For the calculations, it helps to make use of the Bianchi–identity,

$$
[d_\lambda, \gamma_{\mu \nu}] + [d_\nu, \gamma_{\lambda \mu}] + [d_\mu, \gamma_{\nu \lambda}] = 0.
$$

## 4 Heat kernel techniques

In this section, we collect the heat kernel technique formulae which are necessary to extract the divergent parts of the self-energy functionals $\Sigma_{1,2}(x, y)$ in the coincidence limit $x \to y$. This method is particularly useful since it maintains the underlying symmetries. One considers the propagators in $d$–dimensional Euclidean space. In the heat kernel representation, the divergences appear as simple poles in $\epsilon = 4 - d$ with residua that are local polynomials of order $q^3$ in the fields. The latter can easily be transformed back to Minkowski space. The reader familiar with these techniques might skip this section. Detailed expositions of the method can be found in the reviews [20][21].

Consider first an elliptic second–order differential operator of the form (in $d$ Euclidean dimensions)

$$
A = -d_\mu d_\mu + a(x) + \mu^2, \quad d_\mu = \partial_\mu + \gamma_\mu,
$$

where $\gamma_\mu$ and $\sigma = a(x) + \mu^2$ are $C^\infty$ valued matrix functions and $\gamma_\mu$ is anti–hermitian whereas $A$ is hermitian. The heat kernel $G(t) = \exp(-At)$ satisfies the (diffusion) equation

$$
\frac{\partial}{\partial t} G(t) + AG(t) = 0
$$

subject to the boundary condition $G(t = 0) = 1$. Splitting the heat kernel into its free and interaction part,

$$
G = G_0 H, \quad A_0 = -\partial_\mu \partial_\mu + \mu^2,
$$

the coordinate space representation of the free heat kernel $G_0$ reads

$$
G_0(x, y, t) = \langle x | G_0(t) | y \rangle = \langle x | \exp(-A_0 t) | y \rangle = \frac{1}{(4\pi t)^{d/2}} \exp\left\{ -\mu^2 t - \frac{z_\mu z_\mu}{4t} \right\},
$$

with $z_\mu = x_\mu - y_\mu$. $G_0$ satisfies the heat equation Eq.(58) in terms of the free operator $A_0$ and consequently we have for the interaction part $H$

$$
\left[ \frac{\partial}{\partial t} + (A - \mu^2) + \frac{1}{t} z_\mu d_\mu \right] H(x, y, t) = 0.
$$
Notice that the differential operator $d_\mu$ only acts on $x$. The boundary condition in coordinate space reads $G(x,y,0) = \delta^d(x-y)$. Eq.(59) can be solved using the ansatz

$$H(x,y,t) = \sum_{n=0}^{\infty} h_n(x,y) t^n$$

(60)

which leads to the recurrence relation for the coefficient functions $h_n(x,y)$ (the so-called Seeley-deWitt coefficients)

$$(n+z_\mu d_\mu) h_n(x,y) = -[a(x)-d_\mu d_\mu] h_{n-1}(x,y) \ (n \geq 1)$$

$$z_\mu d_\mu h_0(x,y) = 0 \ .$$

(61)

The task is now to determine the first few coefficients in the coincidence limit, $h_n \equiv h_n(x,y)|_{z=0}$. It is easy to show that

$$d_\alpha z_\mu d_\mu h_n| = d_\alpha h_n| \ .$$

(62)

Similarly, for a string of $m$ differential operators $d_\alpha d_\beta \ldots d_\omega$, one finds

$$d_\alpha \ldots d_\omega h_n(x,y)|_{z=0} = -\frac{1}{m+n}\left\{d_\alpha \ldots d_\omega (a-d_\mu d_\mu) h_{n-1}(x,y) + P_{\alpha \ldots \omega} h_n(x,y)\right\}|_{z=0}$$

$$P_{\alpha \ldots \omega} = d_\alpha \ldots d_\omega z_\mu d_\mu - m d_\alpha \ldots d_\omega$$

(63)

The first three heat coefficients $h_{0,1,2}$ follow as

$$h_0| = 1 \ , \ h_1| = -a \ , \ h_2| = 1/2 a^2 - 1/6 [d_\mu, [d_\mu, a]] + 1/12 (\gamma_{\mu\nu})^2 \ ,$$

(64)

with

$$\gamma_{\mu\nu} = \partial_\mu \gamma_{\nu} - \partial_\nu \gamma_{\mu} + [\gamma_{\mu},\gamma_{\nu}] = [d_\mu, d_\nu] \ .$$

(65)

The first few derivatives in the coincidence limit read

$$d_\mu d_\nu h_0| = 1/2 \gamma_{\mu\nu} \ , \ d_\lambda d_\mu d_\nu h_0| = 1/3 \left\{[d_\lambda, \gamma_{\mu\nu}] + [d_\mu, \gamma_{\lambda\nu}]\right\}$$

$$d_\alpha d_\alpha d_\mu d_\mu h_0| = 1/2 \gamma_{\mu\alpha} \gamma_{\mu\alpha} \ , \ d_\mu h_1| = -1/2 [d_\mu, a] + 1/6 [d_\nu, \gamma_{\mu\nu}]$$

$$d_\mu d_\mu h_1| = -1/3 [d_\mu, [d_\mu, a]] + 1/6 \gamma_{\mu\alpha} \gamma_{\mu\alpha} \ .$$

(66)

We also need the differential operator acting from the right. It is defined in complete analogy to Eq.(59) via

$$\left[ \frac{\partial}{\partial t} + (A - \mu^2) + \frac{1}{t} z_\mu d_\mu^2 \right] (H \overset{\rightarrow}{d_\nu}) = \frac{1}{t} d_\nu H$$

(67)

and the pertinent recurrence relations read

$$(n+z_\mu d_\mu) \left(h_n(x,y) \overset{\leftarrow}{d_\nu} \right) = d_\nu h_n(x,y) - (A - \mu^2) h_{n-1}(x,y) \overset{\rightarrow}{d_\nu} \ (n \geq 1)$$

$$z_\mu (d_\mu h_0(x,y) \overset{\leftarrow}{d_\nu}) = d_\nu h_0(x,y) \ ,$$

(68)
and in the coincidence limit we have for the first derivatives
\[
\begin{align*}
   d_\mu h_0 & \sim d_\nu h_0, \quad \{d_\lambda, d_\mu\} h_0 \sim d_\lambda d_\mu h_0, \\
   h_0 & \sim d_\nu d_\mu h_0, \quad d_\mu h_0 \sim d_\nu d_\lambda h_0 = \frac{1}{6} ([d_\lambda, \gamma_{\mu\nu}] + [d_\nu, \gamma_{\mu\lambda}]), \\
   h_1 & \sim d_\mu h_1 - \frac{1}{3} [d_\nu, \gamma_{\mu\nu}] = -\frac{1}{2} [d_\mu, a] - \frac{1}{6} [d_\nu, \gamma_{\mu\nu}].
\end{align*}
\]

We can now construct the propagator corresponding to Eq.\((51)\),
\[
G(x, y) = \int_0^\infty dt G(x, y, t),
\]
which is a well–defined operator for all \(x \neq y\). At large distances the behavior of \(G(x, y)\) will be controlled by the form of the interaction part at large \(t\). One has a well defined behavior in the infrared region, since the meson propagator in Eq.\((51)\) is nothing less than the free propagator. The short–distance (UV) behavior will be controlled by the form of the interaction part \(H\) at small \(t\). \(G(x, y)\) admits an asymptotic expansion for \(t \to 0\) in terms of the Seeley–deWitt coefficients,
\[
G(x, y) = \sum_{n=0}^\infty G_n(x, y) h_n(x, y)
\]
\[
G_n(x, y) = \int_0^\infty dt \ (4\pi t)^{-d/2} \exp\{|x-y|^2/4t\} \ t^n
= \frac{1}{(4\pi)^{d/2}} \left[ \frac{4}{|x-y|^2} \right]^{\frac{d}{2} - n - 1} \Gamma\left(\frac{d}{2} - n - 1\right).
\]

Observe that \(H\) tends to 1 as \(t \to 0\) so that the proper–time integration fails to converge at the lower limit when \(x = y\) for \(d \geq 2\). This is reflected in the singular behavior of \(G(x, y)\) as \(|x-y| \to 0\). In particular, the functions \(G_n\) contain divergences for \(d = 4\). Using an \(\overline{MS}\) subtraction scheme, one rewrites the expansion for \(G(x, y)\) as \([23]\),
\[
G(x, y) = G_0(x, y) h_0(x, y) + \sum_{n=1}^2 R_n(x, y) h_n(x, y) + \tilde{G}(x, y)
\]
with
\[
G_0(x) = \frac{1}{4\pi^{d/2}} \Gamma\left(\frac{d}{2} - 1\right) |x|^{2-d} \to \frac{1}{4\pi^2 |x|^2} \quad \text{for} \quad d \to 4
\]
\[
R_1(x) = \frac{1}{16\pi^2} \left[ 2 \mu^{-\epsilon} + \frac{\Gamma(d/2 - 2)}{\pi^{d/2-2}} |x|^{4-d} \right] \to -\frac{1}{16\pi^2} \{\gamma + \ln \pi + \ln(\mu^2 |x|^2)\},
\]
\[
R_2(x) = -\frac{1}{32\pi^2} \left[ \frac{|x|^2}{\epsilon^\mu} - \frac{\Gamma(d/2 - 3)}{2\pi^{d/2-2}} |x|^{6-d} \right] \to \frac{1}{64\pi^2} |x|^2 \left[ \gamma + \ln \pi - 1 + \ln(\mu^2 |x|^2) \right],
\]
in the coincidence limit \(x = y\). Here, \(\mu\) is a mass scale introduced on dimensional grounds and \(\gamma = 0.5772\) is the Euler–Mascheroni constant. Notice that \(R_1\) and \(R_2\) are regular as
\( \epsilon \to 0 \) and \( \bar{G} \) has no poles in \( \epsilon \) and is regular for \( x = y \), even for two derivatives. \( G(x, y) \) is independent of the scale \( \mu \) and any arbitrariness due to the regularization scheme is compensated by \( \bar{G}(x, y) \). By analytic continuation in \( d \) from \( d < 2 \) it follows that

\[
G(x, x) = \frac{2 - \epsilon}{\epsilon 16\pi^2} h_1(x, x) + \text{finite} = \frac{2 - \epsilon}{\epsilon 16\pi^2} h_1(x, x) + G(x, x) .
\]

(74)

Similarly, we need find for the first derivative of \( G(x, y) \),

\[
d_\mu G(x, y) = d_\mu \sum_{n=0}^{\infty} G_n h_n = \sum_{n=0}^{\infty} (\partial_\mu G_n) h_n + \sum_{n=0}^{\infty} G_n (d_\mu h_n) .
\]

(75)

In the coincidence limit, this simplifies to

\[
d_\mu G(x, x) = \frac{2 - \epsilon}{\epsilon 16\pi^2} d_\mu h_1(x, x) + \text{finite} = \frac{2 - \epsilon}{\epsilon 16\pi^2} d_\mu h_1(x, x) + d_\mu G(x, x) ,
\]

(76)

which means that all divergences are encoded in the coefficient \( h_1(x, x) = h_1 \) and its first derivative, \( d_\mu h_1 \). This technology is sufficient to determine the leading divergences in the meson sector and the ones related to the tadpole graph.

In case of the self–energy graph, we need a modification as proposed in ref.[7]. Consider the following differential operator,

\[
\Delta = -(v \cdot d)^2 + a(x) + \mu^2 , \quad d_\mu = \partial_\mu + \gamma_\mu .
\]

(77)

The heat kernel \( J(t) = \exp\{-\Delta t\} = J_0 K \) can be split again into its free part,

\[
J_0(t) = \frac{1}{\sqrt{4\pi t}} \exp\left\{-\mu^2 t - \frac{(v \cdot (x - y))^2}{4t}\right\} ,
\]

(78)

and the interaction part \( K \), which satisfies the equation,

\[
\left[ \frac{\partial}{\partial t} - (v \cdot d)^2 + a(x) + \frac{1}{t} v \cdot (x - y) v \cdot d \right] K(x, y, t) = 0 ,
\]

(79)

using \( (v \cdot v) = 1 \). It is important to stress the difference to the previous case. Because \( v \cdot d \) is a scalar, one has essentially reduced the problem to a one–dimensional one, i.e. \( v \cdot d \) is a one–dimensional differential operator in the direction of \( v \). Because of this, it is advantageous to modify the heat kernel expansion for \( K \),

\[
K(x, y, t) = g(x, y) \sum_{n=0}^{\infty} k_n(x, y) t^n ,
\]

(80)

where the function \( g(x, y) \) is introduced so that one can fulfill the boundary condition in the coordinate–space representation (see below). The explicit form of the function \( g \) is not needed to derive the recurrence relations for the heat kernel. It is given later when
products of singular operators are constructed (see appendix B). The pertinent recurrence relations are

\[ \begin{align*}
[n + v \cdot (x - y) v \cdot d] g(x, y) k_n(x, y) &= -[a(x) - (v \cdot d)^2] g(x, y) k_{n-1}(x, y) \quad (n \geq 1) \\
[v \cdot (x - y) v \cdot d] g(x, y) k_0(x, y) &= 0 \ .
\end{align*} \]  

(81)

This can be cast into the form

\[ \begin{align*}
(v \cdot d)^m g(x, y) k_n(x, y) &= \frac{-1}{m+n} \left\{ (v \cdot d)^m[-(v \cdot d)^2 + a(x)] g(x, y) k_{n-1}(x, y) \\
&\quad + P^m g(x, y) k_n(x, y) \right\} \\
P^m &= (v \cdot d)^m (v \cdot z)(v \cdot d) - m (v \cdot d)^m .
\end{align*} \]  

(82)

It is easy to show that \( P^m g k \) vanishes in the coincidence limit for all \( m \geq 0 \). Demanding now

\[ v \cdot \partial g(x, y) = 0 \ , \]  

(83)

the recurrence relation takes the particular simple form

\[ \begin{align*}
(v \cdot d)^m k_n(x, y)|_{z=0} &= \frac{-1}{m+n} \left\{ (v \cdot d)^m[-(v \cdot d)^2 + a(x)] k_{n-1}(x, y) \right\} |_{z=0} .
\end{align*} \]  

(84)

It is straightforward to read off the lowest Seeley–deWitt coefficients and their derivatives,

\[ \begin{align*}
k_0 &= 1 \ , \ (v \cdot d)^m k_0 = 0 \ , \ (v \cdot d)^m k_0 (v \cdot d)^n | = 0 \ , \ k_1 = -a \ , \\
(v \cdot d) k_1 &= -\frac{1}{2} [v \cdot d, a] \ , \ (v \cdot d)^2 k_1 = -\frac{1}{3} [v \cdot d, [v \cdot d, a]] \ , \\
k_2 &= \frac{1}{2} a^2 - \frac{1}{6} [v \cdot d, [v \cdot d, a]] \ , \ k_1 v \cdot d | = v \cdot d k_1 .
\end{align*} \]  

(85)

The propagator is given as the integral

\[ \begin{align*}
J(x, y) &= \Delta^{-1}(x, y) = \int_0^\infty dt \ J(x, y, t) \\
J(x, y) &= \sum_{n=0}^{\infty} J_n(x, y) k_n(x, y) \\
J_n(x, y) &= g(x, y) \int_0^\infty \frac{dt}{\sqrt{4\pi t}} \ \exp \left\{ -\mu^2 t - \frac{[v \cdot (x - y)]^2}{4t} \right\} t^n .
\end{align*} \]  

(86)

Since the particle propagator \( A_{(1)}^{ab} \) is massless, one must keep \( \mu^2 \neq 0 \) in intermediate steps to get a well defined heat kernel representation without infrared singularities. For later use, we also need the operator \( v \cdot d \) acting on \( J(x, y) \),

\[ v \cdot d J(x, y) = \sum_n (v \cdot \partial J_n(x, y)) k_n(x, y) + \sum_n J_n(x, y) v \cdot d k_n(x, y) . \]  

(87)

We are now in the position to apply these methods to the problem under investigation.
5 Renormalization of the tadpole graph

In this section, we consider the renormalization of the tadpole contribution \( \Sigma^b_{ab}(y, y) \). This is done in Euclidean space letting \( x^0 \to -ix^0, v^0 \to iv^0, v \cdot \partial \to -v \cdot \partial, S^0 \to -i S^0 \) and \( S \cdot u \to S \cdot u \). To show how this calculation works, we split the tadpole into two terms as in Eq.(50), \( \Sigma^b_{ab} = \Sigma^b_{ab}(1) + \Sigma^b_{ab}(2) \), with the first term being proportional to the meson propagator \( G_{ij} \). The meson propagator is an operator of the type Eq.(55) and using the result of Eq.(74), we can identify

\[ h^i_{ij} = -\sigma^i_{ij} \]  

(88)

with \( \sigma^i_{ij} \) given in Eq.(52). The divergent part of the tadpole can be cast in the form

\[ \Sigma^b_{2, div}(y, y) = \frac{1}{(4\pi F_0)^2} \frac{2}{\epsilon} \hat{\Sigma}^b_{ab}(y, y) \]  

(89)

with \( \hat{\Sigma}^b_{2}(y, y) \) a finite monomial in the fields of chiral dimension three. As an example, let us take a closer look at \( \Sigma^b_{2}(1) \). For that, consider the object

\[ I^{ab} = \langle \lambda^a \dagger \{ [\lambda^i_G, [A, \lambda^j_G]], \lambda^b \} \rangle \pm > - B^{ij} > \]  

(90)

with \( A = S \cdot u \) and \( B^{ij} = \chi_+ \{ \lambda^i_G, \lambda^j_G \} \) for a typical case. Using the completeness and various trace relations (collected in appendix A), this can be cast into the form

\[ I^{ab} = 8 \langle \lambda^a \dagger \{ [A, \lambda^b] \} \rangle \pm > - A B > + 8 \langle \lambda^a \dagger (B, \lambda^b) \rangle \pm > - A > \] 

\[ - 8 \langle \lambda^a \dagger (A, \lambda^b) \rangle \pm > - B > - 12 \langle \lambda^a \dagger (\{ A, B \}, \lambda^b) \rangle \pm > . \]  

(91)

Using now the trace relations

\[ < u_\mu >= 0, \; < S \cdot u >= 0, \]  

(92)

we find

\[ \hat{\Sigma}^b_{2}(1)(y, y) = -\frac{1}{8} D/F \left\{ -\frac{3}{2} \langle \lambda^a \dagger \{ [S \cdot u, u \cdot u + \chi_+] \}, \lambda^b \rangle \pm > \right. \]

\[ \left. - \langle \lambda^a \dagger (S \cdot u, \lambda^b) \rangle \pm > - u \cdot u + \chi_+ > - 2 \langle \lambda^a \dagger (u_\mu, \lambda^b) \rangle \pm > - u^\mu S \cdot u > \right\} \]

\[ - \frac{1}{4} D < \lambda^a \dagger \lambda^b > - S \cdot u (u \cdot u + \chi_+) > . \]  

(93)

We turn to the divergence structure of \( \Sigma^b_{2}(2)(y, y) \). For that, we need the following heat kernel expansion (omitting all traces and SU(3) matrices)

\[ G_{ij}(v \cdot d^y)_{jk} - (v \cdot d^x)_{ij} G_{jk} x \rightarrow y \sum_n G_n[(h_n)_{ij} v \cdot d^y]_{jk} - G_n[v \cdot d_{ij}(h_n)_{jk}] . \]  

(94)
Using now Eq. (66), Eq. (73) and Eq. (53), this turns into
\[
\frac{1}{16\pi^2} \left[ (h_1)_{ij} v^e d_{jk} - v \cdot d_{ij} (h_1)_{jk} \right] = \frac{1}{16\pi^2} \left[ \frac{1}{3} v^\mu [d_\nu, \gamma_{\mu\nu}]_{ik} \right].
\] (95)

Reinstating the prefactors and SU(3) matrices, we find after some lengthy but straightforward algebra for \( \hat{\Sigma}^{ab(2)}(y, y) \)
\[
\hat{\Sigma}^{ab(2)}(y, y) = -\frac{i}{4} < \lambda^a | [\nabla^\mu, \Gamma_{\mu\nu} v^\nu], \lambda^b > ,
\] (96)
with \( \Gamma_{\mu\nu} \) the standard field strength tensor related to \( \Gamma_\mu \) and we dropped the indices 'cl', see Eqs. (52). The divergences encoded in the tadpole, cf. Eq. (89), are therefore determined, and the finite polynomial of order \( q^3 \) reads
\[
\hat{\Sigma}^{ab(2)}(y, y) = \hat{\Sigma}^{ab(1)}(y, y) + \hat{\Sigma}^{ab(2)}(y, y) ,
\] (97)
as given in Eqs. (93, 96).

6 Renormalization of the self–energy graph

In this section, we consider the renormalization of the self–energy contribution \( \Sigma^{ab}(y, y) \). The divergences are due to the singular behavior of the product of the meson and the baryon propagators
\[
G_{ij}(x, y) \left[ A_{(1)}^{ab} \right]^{-1}(x, y)
\] (98)
in the coincidence limit \( x \to y \). This expression is directly proportional to the full classical fermion propagator \( S^{ab, cl}_{(1)} \) as discussed in section 3.\#4 However, this differential operator is not elliptic and thus not directly amenable to the heat–kernel expansion. Consider therefore the object \( S^{ab}_{(1)} \)
\[
S^{ab}_{(1)} = i \left[ i A^{ac}_{(1)} \right]^\dagger \left[ (i A^{cd}_{(1)}) (i A^{ef}_{(1)}) \right]^{-1}.
\] (99)

In fact, the operator in the square brackets in Eq. (99) is positive definite and hermitian. Furthermore, it is a one–dimensional operator in the direction of \( v \) and we can use the heat kernel methods spelled out in section 4 for such type of operators. In Euclidean space, we have
\[
i A_{(1)}^{ab} = -< \lambda^a | v \cdot \nabla, \lambda^b > - i D/F < \lambda^a (S \cdot u, \lambda^b)_\pm >
\]
\[
(i A_{(1)}^{ab})^\dagger = +< \lambda^a | v \cdot \nabla, \lambda^b > - i D/F < \lambda^a (S \cdot u, \lambda^b)_\pm > ,
\] (100)
or in a more convenient form for later use
\[
i A_{(1)}^{ab} = -v \cdot d^{ab} - i D/F < \lambda^a (S \cdot u, \lambda^b)_\pm > = -v \cdot d^{ab} + \rho^{ab}
\]
\[
v \cdot d^{ab} = \delta^{ab} v \cdot \partial + v \cdot \gamma^{ab} , v \cdot \gamma^{ab} = < \lambda^a [v \cdot \Gamma, \lambda^b] > .
\] (101)\#4From now on, we drop the index 'cl'.

18
We note that it is important that $\rho^{ab}$ does not contain any differential operator. After some algebra one gets

\[
(i A_{(1)}^{ac})(i A_{(1)}^{db})^\dagger = -v \cdot d^{ac} v \cdot d^{eb} + a^{ab} \tag{102}
\]

with

\[
a^{ab} = a_1^{ab} + a_2^{ab},
\]

\[
a_1^{ab} = iD/F < \lambda^a([v \cdot \nabla, S \cdot u], \lambda^b)_+ >
\]

\[
a_2^{ab} = -< \lambda^a(D/F S \cdot u, (D/F S \cdot u, \lambda^b)_+ ) + \frac{4}{3} D^2 < \lambda^a S \cdot u < S \cdot \lambda^b > .
\]

The last term in this equation is typical for SU(3), i.e. such a type of term does not appear in the analogous SU(2) calculation [6]. It can be traced back to the use of the completeness relation, Eq.(27), and that fact that $< \lambda^a, O > = 0$ for $O = O^b \lambda^b \in$ SU(3),

\[
< \lambda^a (O, \lambda^b)_+ > < \lambda^c (O', \lambda^b)_+ >^\dagger
\]

\[
= < \lambda^a (O, (O'^\dagger, \lambda^b)_+) > - \frac{D^2}{3} < \{ \lambda^a \dagger, O \} > .
\]

The corresponding singularities of $\Sigma^{ab}_{i1}$ can be extracted in terms of singular products of $G_{0,1}(x)$ and $J_0(x)$ as listed in detail in Eqs.(44-51) in Ecker’s paper [7]. In appendix B, we give these for completeness and show how one derives them (for one particular example). The heat kernel expansion of $G_{ij}(x, y) [A_{(1)}^{ab}]^{-1}(x, y)$ takes the form

\[
X^{ij,cd} = i \left\{ G_n h^{ij}_n (v \cdot d^{cd} + \rho^{cd}) J_m k_m \right\}
\]

\[
= i \left\{ (G_n v \cdot \partial J_m) h^{ij}_n \delta^{cd} k_m + G_n J_m h^{ij}_n (v \cdot d^{cd} + \rho^{cd}) k_m \right\}. \tag{105}
\]

This operator has to be sandwiched between the different interactions $V_i^{ab(1,2)}$ defined in Eq.(10) and one evaluates the corresponding products. This procedure is very general, i.e. it holds for all interactions of a structure like given in Eq.(103). In particular, the two– and three–flavor case can be treated on the same footing. We have to differentiate between three types of products,

1) $V^{ac(1)}(x) X^{ij,cd}(x, y) V_j^{db(1)}(y)$

2) $V^{ac(1)}(x) X^{ij,cd}(x, y) V_j^{db(2)}(y) + (1 \leftrightarrow 2)$

3) $V^{ac(2)}(x) X^{ij,cd}(x, y) V_j^{db(2)}(y) . \tag{106}$

Notice that the interactions $V_i$ depend on different space–time points. Consider case 1). It is straightforward to show (using Eqs.(44)-(51) of ref.[7]) that the divergent part of this operator takes the form

\[
\frac{i}{(4\pi)^2} \epsilon V^{ac}_{i}(x) \left\{ -2v \cdot \delta(x - y) h^{ij}_0(x, y) \delta^{cd} k_0(x, y)
\right.
\]

\[
+ 2 \delta(x - y) h^{ij}_0(x, y) (v \cdot d^{cd} + \rho^{cd}) k_0(x, y) \right\} V_j^{db}(y) . \tag{107}
\]
After partial integration, one can perform the coincidence limit and is left with the two operators \( O_{ab}^{1} \) given below. In the second and third case, at least one of the interactions contains a spin–operator \( S_\mu \) and one covariant derivative acting on the meson propagator. It is convenient to rewrite the interaction in a way that it only contains flavor matrices, \( V^{ac(2)}_i(x) \equiv V^{ac(2)}_k S_\mu d^{ki}_\mu \).

In the coincidence limit, we get for case 2):

\[
\frac{i}{(4\pi)^2} \frac{2}{\epsilon} \delta(x-y) \left[ -2 \left\{ V^{ac(1)}_i(x) S_\cdot d \ h_0 \ d \cdot v|^{ij} V^{cb(2)}_j(x) \right. \\
+ V^{ac(2)}_i(x) v_\cdot d \ h_0 \ d \cdot S|^{ij} V^{cb(1)}_j(x) \right\} \right].
\]

This leads to the operator \( O_{ab}^{3} \). Case 3) is more involved. We find

\[
i V^{ac(2)}_i(x) S_\mu \left\{ \partial_{x\mu} G_n(x,y) \stackrel{\leftarrow}{\partial_{y\nu}} v \cdot \partial J_m(x,y) h^{ij}_n(x,y) k^{cd}_m(x,y) \\
+ G_n(x,y) v \cdot \partial J_m(x,y) (d_\mu h_n(x,y) \stackrel{\leftarrow}{\partial_{y\nu}})^{ij}_m k^{cd}_m(x,y) \\
+ \partial_{x\mu} G_n(x,y) \stackrel{\leftarrow}{\partial_{y\nu}} J_m(x,y) h^{ij}_n(x,y) (v \cdot d + \rho) k^{cd}_m(x,y) \right\} S_\nu V^{cd(2)}_j(y).
\]

Notice that terms linear in \( \partial_\mu G_n \) vanish in the coincidence limit. In what follows, we need the terms with \( n = m = 0 \) or \( n = 0, m = 1 \) or \( n = 1, m = 0 \), whereas in the previous cases only \( m = n = 0 \) was relevant. Case 3) thus generates much more terms and leads to the operators \( O_{ab}^{i} \), \( i = 4, \ldots, 16 \), as listed below. We remark that the divergences are given by the singular products \( G_n J_m \) and derivatives thereof. This destroys covariance since only partial derivatives are involved. However, by appropriately combining terms one can restore covariance. The \( O_{ab}^{i} \) given below are such combinations. This restoration of covariance serves as an important check on the calculation. Rotating back to Minkowski space, one finds after some lengthy algebra a local functional \( \Sigma_{ab}^{1}(y) \),

\[
\Sigma_{1,div}^{ab}(y,y) = \frac{1}{(4\pi F_0)^2} \frac{2}{\epsilon} \delta^4(x-y) \sum_{i=1}^{16} \hat{\Sigma}_{i,1}^{ab}(y) ,
\]

where we have decomposed the lengthy expression for \( \Sigma_{1,div}^{ab}(y,y) \) in such a way that one can most easily recover the SU(2) result, compare Eq.(53) of ref.[7]. In appendix C, we list the operators corresponding to the three cases discussed above and the resulting contributions to the divergent part of the self–energy functional.

### 7 The counterterm Lagrangian

We are now in the position to enumerate the full counterterm Lagrangian at order \( q^3 \). To bring it in a more compact form, powers of the spin matrix are reduced with the help of
Eq. (19). We also use the curvature relation
\[ \Gamma_{\mu\nu} = \frac{1}{4} [u_\mu, u_\nu] - \frac{i}{2} F^+_{\mu\nu}. \] (112)

To separate the finite parts in dimensional regularization, we follow the conventions of [17] to decompose the irreducible one–loop functional into a finite and a divergent part. Both depend on the scale \( \mu \):
\[
\delta^4(x - y) \Sigma^{ab}_2(x, y) + \Sigma^{ab}_1(x, y) = \delta^4(x - y) \sum^{ab,\text{fin}}_2(x, y, \mu) + \sum^{ab,\text{fin}}_1(x, y, \mu) - \frac{2L(\mu)}{F_0^2} \delta^4(x - y) [\hat{\Sigma}^{ab}_2(y) + \hat{\Sigma}^{ab}_1(y)], \quad (113)
\]

with
\[
L(\mu) = \mu^{d-4} \left\{ \frac{1}{d-4} - \frac{1}{2} \left[ \log(4\pi) + 1 - \gamma \right] \right\}. \quad (114)
\]

The generating functional can then be renormalized by introducing the counterterm Lagrangian
\[
\mathcal{L}^{(3)}_{c\text{t}}(x) = \frac{1}{(4\pi F_0)^2} \sum_i d_i H^{ab}_v(x) \tilde{\phi}^{bc}_i(x) H^{ca}_v(x), \quad (115)
\]

where the \( d_i \) are dimensionless coupling constants and the field monomials \( \tilde{\phi}^{bc}_i(x) \) are of order \( q^3 \). The low–energy constants \( d_i \) are decomposed in analogy to Eq.(113),
\[
d_i = d_i^r(\mu) + (4\pi)^2 \beta_i L(\mu). \quad (116)
\]
The \( \beta_i \) are dimensionless functions of \( F \) and \( D \) constructed such that they cancel the divergences of the one–loop functional. They are listed in table 1 together with the corresponding operators \( \tilde{\phi}^{bc}_i(x) \). Notice that in this table the quantity \( \chi - = u^\dagger \chi u^\dagger - u \chi^\dagger u \) never appears because of the relation
\[
\left[ \nabla^\mu, u_\mu \right] = \frac{i}{2} \chi - - \frac{i}{4} \langle \chi - \rangle. \quad (117)
\]
The operators listed in table 1 constitute a complete set for the renormalization of the irreducible tadpole and self–energy functional for off–shell baryons. These are the terms where the covariant derivative acts on the baryon fields. As long as one is only interested in Green functions with on–shell baryons, the number of terms can be reduced considerably by invoking the baryon equation of motions. In particular, all equation of motion terms of the form
\[
[i v \cdot \nabla, H] = -D/F (S \cdot u, H)_\pm + \frac{2}{3} D < S \cdot u H > 1 \quad (118)
\]
can be eliminated by appropriate field redefinitions in complete analogy to the two–flavor case [11]. The last term in Eq.(118) is due to the fact that the baryons are in the adjoint representation of \( SU(3) \). A further reduction in the number of terms could be achieved by use of the Cayley–Hamilton relation, compare Eq.(21). Also, many of the terms given in
the table refer to processes with at least three Goldstone bosons. These are only relevant in multiple pion or kaon production by photons or pions off nucleons [8]. The renormalized LECs $d_i^r(\mu)$ are measurable quantities. They satisfy the renormalization group equations

$$\mu \frac{d}{d\mu} d_i^r(\mu) = -\beta_i .$$

(119)

Therefore, the choice of another scale $\mu_0$ leads to modified values of the renormalized LECs,

$$d_i^r(\mu_0) = d_i^r(\mu) + \beta_i \log \frac{\mu}{\mu_0} .$$

(120)

We remark that the scale–dependence in the counterterm Lagrangian is, of course, balanced by the scale–dependence of the renormalized finite one–loop functional for observable quantities.

8 A sample calculation

Here, we present a sample calculation to demonstrate the use of table 1. Consider the reaction $\gamma(p_1) p(p_2) \to \pi^0(q) p(p_3)$ in the threshold region calculated in SU(3) (for details, see ref.[14]). The T–Matrix can be decomposed into S– and P–wave multipoles as detailed in [23]. Whereas in the two–flavor case the loops are all finite to order $q^3$, in the three–flavor calculation one encounters divergences already at this order. The transition amplitude has the form

$$\frac{m}{4\pi W} \tilde{T} = i\tilde{\sigma} \left( E_0 + \hat{q} \cdot \hat{k} P_1 \right) + i\tilde{\sigma} \cdot \hat{k} \hat{q} P_2 + \hat{q} \times \hat{k} P_3 ,$$

(121)

with $W$ the total cms energy, $m$ the nucleon mass and the multipoles are complex functions of the pion cms energy $\omega$. To be specific, consider the multipole $P_3$. Straightforward evaluation of the irreducible Feynman graphs shown in Fig. 3a,b leads to [14]

$$P_3^a = -C \frac{(D + F)^3}{2} (\gamma_3(\omega) - \gamma_3(-\omega)) = -C (D + F)^3 L \omega + \text{finite}$$

$$P_3^b = C D \left( F^2 - \frac{2}{3} F D - \frac{1}{3} D^2 \right) (\gamma_3^K(\omega) - \gamma_3^K(-\omega))$$

$$= C D \left( F^2 - \frac{2}{3} F D - \frac{1}{3} D^2 \right) 2L \omega + \text{finite}$$

(122)

with $C = (e |\vec{q}| )/(4\pi F_0^3)$ and $\vec{q}$ the three-momentum of the produced neutral pion. The loop functions $\gamma_3(\pm \omega)$ are given in the review [14]. Similarly, the reducible graphs 3c,d and their crossed partners lead to

$$P_3^c = C \frac{(D + F)^3}{2} (\gamma_3(\omega) - \gamma_3(-\omega)) = C (D + F)^3 L \omega + \text{finite}$$

$$P_3^d = C \frac{1}{3} (D + F) (3F^2 + D^2) (\gamma_3^K(\omega) - \gamma_3^K(-\omega))$$

$$= C \frac{1}{3} (D + F) (3F^2 + D^2) 2L \omega + \text{finite} .$$

(123)
We now show how these results can be recovered using table 1. The counterterms 91, 93, 94, 95 and 96 lead to the structure multiplying $P_3$ for irreducible graphs. Straightforward calculation leads to

$$P_3^{\text{irr, ct}} = -C \left( 2\beta_{91} + \frac{4}{3} \beta_{93} + 4\beta_{94} + \frac{4}{3} \beta_{95} + 4\beta_{96} \right) 2L\omega + \text{finite}$$

$$= C \frac{1}{3} (5D^3 + 13D^2F + 3DF^2 + 3F^3) L\omega + \text{finite}$$

(124)

which exactly cancels the divergences from the direct calculation. The reducible graphs are of course proportional to the equations of motion and thus the corresponding counterterms are 97 and 98. We find

$$P_3^{\text{red, ct}} = -C (D + F) \left( \frac{1}{3} \beta_{97} + \beta_{98} \right) 2L\omega + \text{finite}$$

$$= -C (D + F) \left( DF + \frac{5D^2 + 9F^2}{6} \right) 2L\omega + \text{finite} ,$$

(125)

in agreement with the direct Feynman graph calculation, Eq.(123). We note that the reducible graphs could be eliminated using the baryon equations of motion. This would, of course, also modify the coefficient appearing in Eq.(124).

9 Summary and conclusions

In this paper, we have performed the chiral–invariant renormalization of the effective three–flavor meson–baryon field theory and constructed the complete counterterm Lagrangian to leading one–loop order $q^3$. To describe the ground state baryon octet, we have used heavy baryon chiral perturbation theory in the path integral formulation [6]. This extends previous work by Ecker [7], who considered the pion–nucleon system, i.e. the two–flavor case. Since there exist very few explicit calculations in SU(3) where divergences at $O(q^3)$ were evaluated, we list the following checks on the rather involved manipulations:

(1) Reducing our expressions to SU(2) and taking into account that $g_A = F + D$ and $F - D = 0$, we recover the results of the table in ref.[1].

(2) As stressed before, the method destroys covariance in some intermediate steps (see section 6, e.g. Eq.(107), and the formulae in appendix C). Of course the final results are covariant. This is achieved by forming appropriate combinations of the operators $O_{i}^{ab}$, see appendix C.

(3) The self–energy contributions $\Sigma_{1,i}^{ab}$ given in appendix C are covariant but not hermitian. Hermiticity is restored by again combining appropriate terms. This leads to the complete counterterm Lagrangian given in section 7 in terms of the operators $\tilde{O}_{i}^{ab}$, compare table 1.
(4) Some of the trace relations given in appendix A are rather involved. We have checked these with the help of standard analytical packages.

(5) For the case of pion/kaon photoproduction, we have compared our results with the one obtained by direct Feynman graph calculation \[14\] as detailed in section 8.

In summary, we remark that the method used has the disadvantage of leading to very lengthy and complicated expressions in the intermediate steps due to the loss of covariance. There should exist an improved method which does not share this complication. On the other hand, this method is very general, i.e. it can be applied to any differential operator that has the structure given in Eq.\[50\] straightforwardly. In particular, the necessary inclusion of virtual photons in the pion–nucleon (or meson–baryon) system can be treated along these lines. We hope to report on the results of such an investigation in the near future.

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A  Trace relations and other useful identities

In this appendix, we collect some useful trace relations used throughout the text. The completeness relation of the generators in the Gell-Mann basis reads,

\[
\lambda^i_{\alpha\beta} \lambda^i_{\alpha'\beta'} = 2\delta_{\alpha\beta}\delta_{\alpha'\beta'} - \frac{2}{N} \delta_{\alpha\beta} \delta_{\alpha'\beta'} .
\] (A.1)

The superscripts \'a, b, \ldots' refer to the physical basis whereas the \(\lambda_{i,j,\ldots}\) are the generators in the Gell-Mann basis (called \(\lambda^i_G\) in the main text). Consider now matrices \(A, B, C, \ldots \in SU(N)\) (which are not necessarily traceless).

\[
\begin{align*}
[\lambda_i, [A, \lambda_j]] &< B\lambda_i, \lambda_j > = 8 < AB > - 8A < B > + 8 < A > B \\
&- 4N[A, B] \\
[\lambda_i, [A, \lambda_j]] &< B\lambda_i, \lambda_j > = - 4N[A, B] \quad (A.2) \\
[\lambda_i, [A, \lambda_j]] &< B, \lambda_i][\lambda_j, C] > = 4 \left\{ - 2B < AC > - 2C < AB > - 2A < BC > + < A > \{B, C\} + < B \{A, C\} + < C \{A, B\} + < ABC > + < ACB > \\
&- N ACB - N BCA \right\} \quad (A.3)
\end{align*}
\]
\[ A \lambda_i B \lambda_j C = 2AC < B > - \frac{2}{N} ABC \]  \hspace{1cm} (A.5)

\[ X \lambda_i A[B, \lambda_j] Y = 2XY < AB > - 2XYB < A > \]  \hspace{1cm} (A.6)

\[ X [A, \lambda_i] B[C, \lambda_j] Y = 2XAY < BC > - 2XACY < B > - 2XY < ABC > + 2XCY < AB > \]  \hspace{1cm} (A.7)

We frequently use

\[ < (\lambda^{a \dagger}, \lambda_i) \pm (\lambda_j, \lambda^b)_\pm > = \left\{ \left( 4N - \frac{8}{N} \right) D^2 + 4NF^2 \right\} < \lambda^{a \dagger} \lambda^b > . \]  \hspace{1cm} (A.8)

Now we specialize to the case that the matrices \( A, B, \ldots \) are traceless, i.e. \( < A > = < B > = \ldots = 0 \). Useful relations are

\[ < (\lambda^{a \dagger}, \lambda_i) \pm (\lambda^j, \lambda^b) \pm > = \left( 2ND(D^2 - F^2) - \frac{8}{N} D^3 \right) < \lambda^{a \dagger} \{ A, \lambda^b \} > \]  \hspace{1cm} (A.9)

\[ + \left( -2NF(D^2 - F^2) - \frac{8}{N} D^2 F \right) < \lambda^{a \dagger} [A, \lambda^b] > \]  \hspace{1cm} (A.9)

\[ < (\lambda^{a \dagger}, \lambda_i) \pm (\lambda^j, \lambda^b) \pm > = 2((D + F)^2 + (D - F)^2) < \lambda^{a \dagger} \lambda^b > < AB > \]  \hspace{1cm} (A.10)

\[ + 4(D^2 - F^2)^2 < \lambda^{a \dagger} A > < B \lambda^b > + < \lambda^{a \dagger} B > < A \lambda^b > \]  \hspace{1cm} (A.10)

\[ + \left( N(D^2 - F^2)^2 - \frac{8}{N} D^4 \right) < \lambda^{a \dagger} \{ A, \{ B, \lambda^b \} \} > \]  \hspace{1cm} (A.10)

\[ + \left( N(D^2 - F^2)^2 - \frac{8}{N} D^2 F^2 \right) < \lambda^{a \dagger} [A, \{ B, \lambda^b \}] > \]  \hspace{1cm} (A.10)

\[ - \left( \frac{8}{N} D^3 F \right) \left\{ < \lambda^{a \dagger} \{ A, [B, \lambda^b] \} > + < \lambda^{a \dagger} [A, \{ B, \lambda^b \}] > \right\} \]  \hspace{1cm} (A.10)

Other more complicated relations can be derived along similar lines.

To arrive at the terms given in table 1, one makes use of various relations derived from the Cayley–Hamilton identity,

\[ A^3 - < A > A^2 + \frac{1}{2} \left( < A >^2 - < A^2 > \right) A = \det(A) . \]  \hspace{1cm} (A.11)

For traceless matrices \( A_1, A_2, \ldots \) one can derive from that

\[ \sum_{6 \text{ perm.}} < \lambda^{a \dagger} A_1 A_2 \lambda^b > - < \lambda^{a \dagger} \lambda^b > < A_1 A_2 > \]  \hspace{1cm} (A.12)

\[ = < \lambda^{a \dagger} A_2 > < A_1 \lambda^b > + < \lambda^{a \dagger} A_1 > < A_2 \lambda^b > , \]  \hspace{1cm} (A.12)

which for \( A_1 = A_2 = X \) gives the relation Eq.(21). Similarly, we find

\[ \sum_{24 \text{ perm.}} < \lambda^{a \dagger} A_1 A_2 A_3 \lambda^b > - \sum_{20 \text{ perm.}} < \lambda^{a \dagger} \lambda^b > < A_1 A_2 A_3 > = 0 . \]  \hspace{1cm} (A.13)

For more relations see [24].
B Products of singular operators

In this appendix we consider the singularities arising from the product of the baryon and meson propagators in the evaluation of the self–energy diagram. The corresponding singularities can best be extracted in Euclidean $d$-dimensional Fourier space \[22\]. There is no difference between SU(2) and SU(3) because the structure of divergences is related to the coordinate space and not to the flavor space. Before we show how this calculation proceeds, some general remarks are in order. The $G_n(x, y)$ given in Eq.(71) lead to singular terms of the type

$$\frac{1}{|x - y|^{2\alpha}}. \tag{B.1}$$

The singular behavior of these terms can be studied by use of the d–dimensional Fourier-transform,

$$\int d^d x \frac{1}{|x|^{2\alpha}} e^{ikx} = \pi^{d/2} \frac{\Gamma(d/2 - 2)}{\Gamma(\alpha)} \left[ \frac{k}{4} \right]^{\alpha - d/2}, \tag{B.2}$$

which has poles at $\alpha - d/2 = 0, 1, 2, \ldots$. In an analogous manner, we will treat the products of singular operators, related to the baryon and meson propagators in the self–energy loop. These have the form $G_n(x, y) J_m(x, y)$ and can be treated by Fourier–transforms involving now more complicated integrands of the type $\exp(-\alpha t - \beta/t)$, compare Eqs.(78,86). To be specific, consider

$$\int d^d x G_n(x) J_m(x) e^{ikx}. \tag{B.3}$$

To evaluate it, we need a specific representation of the function $g(x)$ \[7\]

$$g(x) = \int \frac{d^d p}{(2\pi)^{d-1}} \delta(k \cdot v) e^{-ipx}. \tag{B.4}$$

Choosing the Euclidean rest-frame, $v = (0, \ldots, 0, 1)$, we have $g(x) = \delta^{d-1}(x)$ and the integral Eq.(B.3) takes the form

$$\frac{2}{(4\pi)^{d/2+1/2}} \Gamma(\alpha) \Gamma(\beta) \int_{-\infty}^{\infty} dx x^{-2\alpha}[x^2 - 2ikx + \mu^2]^{-\beta} \tag{B.5}$$

with

$$\alpha = \frac{d}{2} - 1 - n, \quad \beta = 4 - d + 2n + m - \frac{1}{2}, \quad \alpha + \beta = 2 - \frac{d}{2} + n + m + \frac{1}{2}. \tag{B.6}$$

Only for $m = n = 0$ one finds divergences. Using standard methods, it can be brought into the form

$$\frac{1}{(4\pi)^{2-\epsilon/2}} \int_{-\infty}^{\infty} dx \int_0^1 dt \frac{(1 - t)^{\alpha-1} t^{\beta-1}}{[x^2 + k^2 t^2 + t\mu^2]^{\alpha+\beta}}. \tag{B.7}$$

which leads to the divergence

$$-\frac{2}{(4\pi)^2} \Gamma\left(\frac{\epsilon}{2}\right) + \text{finite}. \tag{B.8}$$
Expanding the $\Gamma$ function and going back to coordinate space, one is finally left with

$$-\frac{2}{(4\pi)^2} \frac{2}{\varepsilon} \int d^d x \delta^d(x) + \text{finite}. \quad (B.9)$$

In a more transparent way one can evaluate the integral \( [B.3] \) by transforming it into typical heavy baryon one–loop integrals in momentum space. Straightforward algebra leads to the following list \( [7] \) that contains all singular products appearing in $\Sigma$ (with $\varepsilon = 4 - d$):

$$G_0(x - y) J_0(x - y) \sim -\frac{2}{(4\pi)^2} \frac{2}{v^2} \delta^d(x - y) \quad (B.10)$$

$$G_0(x - y) v \cdot \partial J_0(x - y) \sim \frac{2}{(4\pi)^2} \frac{2}{\varepsilon} v \cdot \partial \delta^d(x - y) \quad (B.11)$$

$$S_\mu S_\nu \partial_\mu \partial_\nu G_0(x - y) J_0(x - y) \sim \frac{2}{(4\pi)^2} \frac{2}{\varepsilon} S_\mu S_\nu \delta_{\mu\nu} (v \cdot \partial)^2 \delta^d(x - y) \quad (B.12)$$

$$S_\mu S_\nu \partial_\mu \partial_\nu G_0(x - y) v \cdot \partial J_0(x - y) \sim -\frac{2}{3(4\pi)^2} \frac{2}{\varepsilon} S_\mu S_\nu \delta_{\mu\nu} (v \cdot \partial)^3 \delta^d(x - y) \quad (B.13)$$

$$S_\mu S_\nu \partial_\mu \partial_\nu G_0(x - y) J_1(x - y) \sim -\frac{2}{3(4\pi)^2} \frac{2}{\varepsilon} S_\mu S_\nu \delta_{\mu\nu} \delta^d(x - y) \quad (B.14)$$

$$S_\mu S_\nu \partial_\mu \partial_\nu G_0(x - y) v \cdot \partial J_1(x - y) \sim \frac{2}{(4\pi)^2} \frac{2}{\varepsilon} S_\mu S_\nu \delta_{\mu\nu} v \cdot \partial \delta^d(x - y) \quad (B.15)$$

$$S_\mu S_\nu \partial_\mu \partial_\nu G_1(x - y) J_0(x - y) \sim \frac{1}{(4\pi)^2} \frac{2}{v^2} \frac{2}{\varepsilon} S_\mu S_\nu \delta_{\mu\nu} \delta^d(x - y) \quad (B.16)$$

$$S_\mu S_\nu \partial_\mu \partial_\nu G_1(x - y) v \cdot \partial J_0(x - y) \sim -\frac{1}{(4\pi)^2} \frac{2}{v^2} \frac{2}{\varepsilon} S_\mu S_\nu \delta_{\mu\nu} v \cdot \partial \delta^d(x - y). \quad (B.17)$$

We explicitly make use of the identity $S \cdot v = 0$. In the case of a more general differential operator acting on the meson propagator we get a richer structure of divergences.

C Contributions to the self–energy

Here we list the operators corresponding to the three cases discussed in section 6 and the resulting contribution to the divergent part of the self–energy functional.

$$O_1^{ab} = V^{ac(1)}_i \left\{ v \cdot \gamma^j V^{cb(1)}_j + v \cdot d^{cd} V^{db(1)}_i + V^{cb(1)}_i v \cdot \partial \right\}$$

$$\Sigma_{1,1}^{ab} = \frac{i}{4} \left\{ 2 < \lambda^a \cdot [v \cdot \nabla, \lambda^b] > < (v \cdot u)^2 > +4 < \lambda^a v \cdot u > < v \cdot u [v \cdot \nabla, \lambda^b] > 
+ 2 < \lambda^a \lambda^b \cdot v \cdot u > [v \cdot \nabla, v \cdot u] > +2 < \lambda^a v \cdot u > [v \cdot \nabla, v \cdot u] \lambda^b > 
+ 2 < \lambda^a \cdot [v \cdot \nabla, v \cdot u] > < v \cdot u \lambda^b > +3 < \lambda^a \cdot \{ (v \cdot u)^2, [v \cdot \nabla, \lambda^b] \} > 
+ 3 < \lambda^a v \cdot u [v \cdot \nabla, v \cdot u] \lambda^b > +3 < \lambda^a \lambda^b [v \cdot \nabla, v \cdot u] v \cdot u > \right\} \quad (C.1)$$
\[
O_{2}^{ab} = V_{i}^{ac(1)} < \lambda^{\dagger} (S \cdot u, \lambda^{b})_{\pm} > V_{i}^{db(1)}
\]
\[
\Sigma_{1, 2}^{ab} = -\frac{1}{2} D/F \left[ < \lambda^{\dagger} (v \cdot u, \lambda^{b})_{\pm} > < v \cdot u S \cdot u > - < \lambda^{\dagger} (S \cdot u, v \cdot u)_{\pm} > < v \cdot u \lambda^{b} > - < \lambda^{\dagger} v \cdot u < (v \cdot u, S \cdot u)_{\pm} > \frac{1}{2} < \lambda^{\dagger} (S \cdot u, \lambda^{b})_{\pm} > < (v \cdot u)^{2} > \right] + \frac{3}{4} (D + F) < \lambda^{\dagger} S \cdot u \lambda^{b} (v \cdot u)^{2} > + \frac{3}{4} (D - F) < \lambda^{\dagger} (v \cdot u)^{2} \lambda^{b} S \cdot u > + \frac{D}{2} \left[ < \lambda^{\dagger} \lambda^{b} > < (v \cdot u)^{2} S \cdot u > - < \lambda^{\dagger} (v \cdot u)^{2} > < S \cdot u \lambda^{b} > - < \lambda^{\dagger} S \cdot u > < (v \cdot u)^{2} \lambda^{b} > \right] \quad (C.2)
\]
\[
O_{3}^{ab} = V_{i}^{ac(1)} (\gamma_{\mu \nu})_{ij} V_{j}^{cb(2)} + (1 \leftrightarrow 2)
\]
\[
\Sigma_{1, 3}^{ab} = -\frac{3}{2} D/F < \lambda^{\dagger} ([\Gamma_{\mu \nu}, v \cdot u], \lambda^{b})_{\pm} > S_{\mu} v_{\nu} \quad (C.3)
\]
\[
O_{4}^{ab} = V_{i}^{ac(2)} \left\{ \delta^{ij} (v \cdot d)^{3 \cdot cd} + 3 v \cdot \gamma^{ij} (v \cdot d)^{2 \cdot cd} + 3 v \cdot d \cdot v \cdot \gamma^{ij} (v \cdot d)^{cd} \right\} V_{j}^{cb(2)}
\]
\[
\Sigma_{1, 4}^{ab} = -i \left( \frac{10}{3} D^{2} + 6 F^{2} \right) < \lambda^{\dagger} [v \cdot \nabla, [v \cdot \nabla, [v \cdot \nabla, \lambda^{b}]]] > \quad (C.4)
\]
\[
O_{5}^{ab} = V_{i}^{ac(2)} \left[ (\gamma_{\mu \nu} v \cdot \gamma)^{ij} \delta^{cd} + \gamma_{\mu \nu} v \cdot d^{cd} \right] V_{j}^{db(2)}
\]
\[
\Sigma_{1, 5}^{ab} = S^{\mu} \left\{ \left( \frac{10}{3} D^{2} + 6 F^{2} \right) < \lambda^{\dagger} [\Gamma_{\mu \nu}, [v \cdot \nabla, \lambda^{b}]] > + 12 D F < \lambda^{\dagger} \{ \Gamma_{\mu \nu}, [v \cdot \nabla, \lambda^{b}] \} > \right\} S^{\nu} \quad (C.5)
\]
\[
O_{6}^{ab} = V_{i}^{ac(2)} \left[ \delta_{\mu \nu} (d_{\lambda} \gamma_{\kappa})^{ij} v^{\kappa} + 2 (v \cdot \gamma_{\mu \nu})^{ij} + 2 (d_{\nu} \gamma_{\mu \kappa})^{ij} v^{\kappa} \right] V_{j}^{cb(2)}
\]
\[
\Sigma_{1, 6}^{ab} = i \left\{ \left( \frac{-25}{36} D^{2} - \frac{5}{4} F^{2} \right) < \lambda^{\dagger} [[\nabla^{\mu}, \Gamma_{\mu \nu} v^{\nu}], \lambda^{b}] > - \frac{5}{2} D F < \lambda^{\dagger} \{ [\nabla^{\mu}, \Gamma_{\mu \nu} v^{\nu}], \lambda^{b} \} > + i \epsilon^{\mu \nu \rho \sigma} v_{\rho} S_{\sigma} \left\{ 3 D F < \lambda^{\dagger} \{ [v \cdot \nabla, \Gamma_{\mu \nu}], \lambda^{b} \} > + \left( \frac{5}{6} D^{2} + \frac{3}{2} F^{2} \right) < \lambda^{\dagger} [[v \cdot \nabla, \Gamma_{\mu \nu}], \lambda^{b}] > \right\} \right\} \quad (C.6)
\]
\[ O_{i}^{ab} = V_{i}^{ac(2)} \left[ 2v \cdot \gamma^{ij} (v \cdot d)^{cd} + < \lambda^{\alpha \dagger} (S \cdot u, \lambda^{d})_{\pm} > (v \cdot d)^{2ed} \delta^{ij} \\
+ (v \cdot d v \cdot \gamma)^{ij} < \lambda^{\alpha \dagger} (S \cdot u, \lambda^{d})_{\pm} > \right] V_{j}^{db(2)} \]

\[ \Sigma_{1,7}^{ab} = -(D^{3} + 3D^{2}F^{2}) < \lambda^{\alpha \dagger} \{ S \cdot u, [v \cdot \nabla, [v \cdot \nabla, \lambda^{b}]] \} > \\
+(3F^{3} - \frac{5}{3}D^{2}F) < \lambda^{\alpha \dagger} [S \cdot u, [v \cdot \nabla, [v \cdot \nabla, \lambda^{b}]]] > \quad (C.7) \]

\[ O_{8}^{ab} = V_{i}^{ac(2)} \left[ v \cdot \gamma^{ij} a_{1}^{cd} + (a_{1} v \cdot d)^{cd} \delta^{ij} \right] V_{j}^{db(2)} \]

\[ \Sigma_{1,8}^{ab} = -D(3F^{2} + D^{2}) < \lambda^{\alpha \dagger} \{ [v \cdot \nabla, S \cdot u], [v \cdot \nabla, \lambda^{b}] \} > \\
+(3F^{3} - \frac{5}{3}D^{2}F) < \lambda^{\alpha \dagger} [[v \cdot \nabla, S \cdot u], [v \cdot \nabla, \lambda^{b}]] > \quad (C.8) \]

\[ O_{9}^{ab} = V_{i}^{ac(2)} [v \cdot d, a_{1}]^{cd} V_{j}^{db(2)} \]

\[ \Sigma_{1,9}^{ab} = -\frac{1}{3}D(D^{2} + 3F^{2}) < \lambda^{\alpha \dagger} \{ [v \cdot \nabla, [v \cdot \nabla, S \cdot u]], \lambda^{b} \} > \\
-\frac{1}{9}F(5D^{2} - 9F^{2}) < \lambda^{\alpha \dagger} [[v \cdot \nabla, [v \cdot \nabla, S \cdot u]], \lambda^{b}] > \quad (C.9) \]

\[ O_{10}^{ab} = V_{i}^{ac(2)} \left[ \gamma^{ij} < \lambda^{\alpha \dagger} (S \cdot u, \lambda^{d})_{\pm} > \right] V_{j}^{db(2)} \]

\[ \Sigma_{1,10}^{ab} = S^{u} \left\{ -4F(F^{2} + 3D^{2}) < \lambda^{a \dagger} \lambda^{b} > < S \cdot u \Gamma_{\mu \nu} > \\
+ 4F(D^{2} - F^{2}) < \lambda^{a \dagger} \Gamma_{\mu \nu} > < S \cdot u \lambda^{b} > \\
+ 4F(D^{2} - F^{2}) < \lambda^{a \dagger} S \cdot u > < \Gamma_{\mu \nu} \lambda^{b} > \\
-6(D - F)(D^{2} - F^{2}) < \lambda^{a \dagger} S \cdot u \lambda^{b} \Gamma_{\mu \nu} > \\
+ 6(D + F)(D^{2} - F^{2}) < \lambda^{a \dagger} \Gamma_{\mu \nu} \lambda^{b} S \cdot u > \\
-\frac{8}{3}D^{2}D/F \left[ < \lambda^{a \dagger} [\Gamma_{\mu \nu}, (\lambda^{b}, S \cdot u)_{\pm}] > + < \lambda^{a \dagger} ([\Gamma_{\mu \nu}, \lambda^{b}], S \cdot u)_{\pm} > \right] \right\} S^{u} \quad (C.10) \]

\[ O_{11}^{ab} = V_{i}^{ac(2)} \left[ (\sigma v \cdot \gamma)^{ij} \delta^{cd} + \sigma^{ij} v \cdot d^{cd} \right] V_{j}^{db(2)} \]

\[ \Sigma_{1,11}^{ab} = i \left\{ \left( \frac{3}{8}D^{2} - \frac{9}{8}F^{2} \right) < \lambda^{a \dagger} \{ \chi_{+}, [v \cdot \nabla, \lambda^{b}] \} > - \frac{5}{4}DF < \lambda^{a \dagger} [\chi_{+}, [v \cdot \nabla, \lambda^{b}]] > \\
- \left( \frac{13}{12}D^{2} + \frac{3}{4}F^{2} \right) < \lambda^{a \dagger} [v \cdot \nabla, \lambda^{b}] > < \chi_{+} > + \frac{1}{2}D^{2} < \lambda^{a \dagger} [u_{\mu}, [u^{\mu}, [v \cdot \nabla, \lambda^{b}]]] > \right\} \]
\[
-\frac{3}{4}(D^2 + F^2) < \lambda^a \{ v \cdot \nabla, \lambda^b \} > u^2 > -\frac{9}{8}(D^2 + F^2) < \lambda^a \{ u^2, [v \cdot \nabla, \lambda^b] \} > \\
-\frac{9}{4}DF < \lambda^a \{ u^2, [v \cdot \nabla, \lambda^b] \} > +\frac{3}{2}(D^2 - F^2) < \lambda^a u_\mu > u^\mu [v \cdot \nabla, \lambda^b] > \}
\]

\[
O^a_{12} = V_{i}^{ac(2)} [v \cdot d, \sigma]^ij V_j^{cb(2)}
\]

\[
\Sigma^a_{1,12} = i\left\{ \frac{3}{16} (D^2 - 3F^2) < \lambda^a \{ [v \cdot \nabla, \chi_+], \lambda^b \} > -\frac{5}{8}DF < \lambda^a \{ [v \cdot \nabla, \chi_+], \lambda^b \} > \\
-\frac{1}{24}(13D^2 + 9F^2) < \lambda^a \lambda^b > [v \cdot \nabla, \chi_+] > \\
-\frac{3}{4}(D^2 + F^2) < \lambda^a \lambda^b > u_\mu [v \cdot \nabla, u^\mu] > \\
+\frac{1}{4}D^2 \left( < \lambda^a \{ u_\mu, [v \cdot \nabla, u^\mu], \lambda^b \} > + < \lambda^a \{ [v \cdot \nabla, u_\mu], [u^\mu, \lambda^b] \} > \right) \\
-\frac{9}{16}(D^2 + F^2) < \lambda^a \{ [v \cdot \nabla, u_\mu], u^\mu \} \lambda^b > \\
-\frac{9}{8}DF < \lambda^a \{ [v \cdot \nabla, u_\mu], u^\mu \} \lambda^b > + < \lambda^a u_\mu > u^\mu [v \cdot \nabla, u^\mu] \lambda^b \right\} \]

\[
O^a_{13} = V_{ac}^{(2)} \left[ \sigma^ij < \lambda^c (S \cdot u, \lambda^d)_{\pm} > \right] V_j^{cb(2)}
\]

\[
\Sigma^a_{1,13} = \frac{2}{9}D^3 + \frac{1}{4}D(D^2 - F^2) \left( < \lambda^a \chi_+ > S \cdot u \lambda^b > + < \lambda^a S \cdot u > \chi_+ \lambda^b > \right) \\
+ \left( \frac{1}{9}D^2 D/F + \frac{1}{8}(D^2 - F^2) D/(-F) \right) < \lambda^a (S \cdot u, \lambda^b)_{\pm} > \chi_+ > \\
+ \frac{1}{4}D(D^2 + 3F^2) < \lambda^a \lambda^b > S \cdot u \chi_+ > \\
- \left( \frac{1}{3}D^3 - \frac{3}{32}D(D^2 - F^2) \right) < \lambda^a \{ \chi_+, \{ S \cdot u, \lambda^b \} \} > + < \lambda^a \{ S \cdot u, \chi_+, \lambda^b \} > \\
- \left( \frac{3}{32}F(D^2 - F^2) \right) < \lambda^a \{ \chi_+, [S \cdot u, \lambda^b] \} > + < \lambda^a [S \cdot u, \chi_+, \lambda^b] > \\
- \left( \frac{1}{6}DF^2 + \frac{3}{32}D(D^2 - F^2) \right) < \lambda^a [\chi_+, [S \cdot u, \lambda^b]] > + < \lambda^a [S \cdot u, [\chi_+, \lambda^b]] > \\
- \left( \frac{1}{6}D^2 F - \frac{3}{32}F(D^2 - F^2) \right) < \lambda^a [\chi_+, \{ S \cdot u, \lambda^b \} > + < \lambda^a \{ S \cdot u, [\chi_+, \lambda^b] > \\
+ \frac{1}{8}D/(-F)(D^2 - F^2) < \lambda^a (S \cdot u, \lambda^b)_{\pm} > u^2 > \\
+ \frac{D}{4}(D^2 + 3F^2) < \lambda^a \lambda^b > S \cdot u u \cdot u > \right\} \]

\[
(C.11)
\]

\[
(C.12)
\]

\[
(C.13)
\]
\[- \frac{D}{4} (D^2 + 3F^2) < \lambda^a \{ u_\mu, \lambda^b \} > < u^\mu S \cdot u > \]
\[- \frac{F}{4} (3D^2 + F^2) < \lambda^a [u_\mu, \lambda^b] > < u^\mu S \cdot u > \]
\[- \frac{1}{4} D/F (D^2 - F^2) \left[ < \lambda^a u_\mu > (u^\mu, S \cdot u)_\pm \lambda^b > + < \lambda^a (S \cdot u, u_\mu)_\pm > < u^\mu \lambda^b > \right] \]
\[+ \frac{D}{4} (D^2 - F^2) \left[ < \lambda^a S \cdot u > u^\mu \lambda^b > + < \lambda^a u \cdot u > < S \cdot u \lambda^b > \right] \]
\[- \frac{D^2}{6} D/(-F) \left[ < \lambda^a [u_\mu, [u^\mu, (S \cdot u, \lambda^b)_\pm]] > + < \lambda^a (S \cdot u, [u_\mu, [u^\mu, \lambda^b]])_\pm > \right] \]
\[+ \frac{3}{8} (D - F)(D^2 - F^2) < \lambda^a S \cdot u \lambda^b u \cdot u > \]
\[+ \frac{3}{8} (D + F)(D^2 - F^2) < \lambda^a u \cdot u \lambda^b S \cdot u > \]

\[
O_{14}^{ab} = V_i^{ac(2)} \left[ (a_2 v \cdot d)^{cd} \delta^{ij} + v \cdot \gamma^{ij} a_2^{cd} \right] V_j^{db(2)}
\]

\[
\Sigma_{1,14}^{ab} = -2S^\mu S^\kappa S^\tau S_\mu \left\{ -\frac{16}{3} D^2 (D/F)^2 < \lambda^a ([i v \cdot \nabla, \lambda^b], (u_\kappa, \tau)_\pm) > \right. \]
\[+ \left( \frac{32}{9} D^4 + 4(D^4 + 6D^2 F^2 + F^4) \right) < \lambda^a [i v \cdot \nabla, \lambda^b] > u_\kappa u_\tau > \]
\[+ \left( \frac{32}{9} D^4 + 4(D^2 - F^2)^2 \right) < \lambda^a u_\kappa > u_\tau [i v \cdot \nabla, \lambda^b] > \]
\[+ \left( 4(D^2 - F^2)^2 \right) < \lambda^a u_\tau > u_\kappa [i v \cdot \nabla, \lambda^b] > \]
\[+ \left( 6(D^2 - F^2)^2 - \frac{16}{3} D^2 (D^2 + F^2) \right) < \lambda^a u_\kappa u_\tau [i v \cdot \nabla, \lambda^b] > \]
\[+ \left( 6(D^2 - F^2)^2 - \frac{16}{3} D^2 (D^2 + F^2) \right) < \lambda^a [i v \cdot \nabla, \lambda^b] u_\tau u_\kappa > \]
\[+ \left( -\frac{16}{3} D^2 (D^2 - F^2) \right) < \lambda^a u_\kappa [i v \cdot \nabla, \lambda^b] u_\tau > \]
\[+ \left( -\frac{16}{3} D^2 (D^2 - F^2) \right) < \lambda^a u_\tau [i v \cdot \nabla, \lambda^b] u_\kappa > \right\} \quad (C.14)
\]

\[
O_{15}^{ab} = V_i^{ac(2)} \left[ < \lambda^c (S \cdot u, \lambda^e) > a_1^{cd} + [v \cdot d, a_2]^{cd} \right] V_j^{db(2)}
\]

\[
\Sigma_{1,15}^{ab} = -\frac{2}{3} i S^\mu S^\kappa S^\tau S_\mu \left\{ -\frac{32}{3} D^2 (D/F)^2 < (\lambda^a, \lambda^b)_\pm (u_\kappa, [v \cdot \nabla, u_\tau])_\pm > \right. \]
\[- \frac{16}{3} D^2 (D/F)^2 < (\lambda^a, \lambda^b)_\pm ([v \cdot \nabla, u_\kappa], u_\tau)_\pm > \]
\[+ \left( \frac{64}{9} D^4 + 8(D^4 + 6D^2 F^2 + F^4) \right) < \lambda^a \lambda^b > u_\kappa [v \cdot \nabla, u_\tau] > \]

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\[ \begin{align*}
&+ \left( \frac{32}{9} D^4 + 4(D^4 + 6D^2F^2 + F^4) \right) < \lambda^a \lambda^b > [v \cdot \nabla, u_\tau] u_\tau > \\
&+ \left( \frac{64}{9} D^4 + 8(D^2 - F^2)^2 \right) < \lambda^a u_\kappa > [v \cdot \nabla, u_\tau] \lambda^b > \\
&+ 8(D^2 - F^2)^2 < \lambda^a [v \cdot \nabla, u_\tau] > u_\kappa \lambda^b > \\
&+ \left( \frac{32}{9} D^4 + 4(D^2 - F^2)^2 \right) < \lambda^a [v \cdot \nabla, u_\kappa] > u_\tau \lambda^b > \\
&+ 4(D^2 - F^2)^2 < \lambda^a u_\tau > [v \cdot \nabla, u_\kappa] \lambda^b > \\
&+ \left( -\frac{32}{3} D^2(D^2 + F^2) + 12(D^2 - F^2)^2 \right) < \lambda^a u_\kappa [v \cdot \nabla, u_\tau] \lambda^b > \\
&+ \left( -\frac{16}{3} D^2(D^2 + F^2) + 6(D^2 - F^2)^2 \right) < \lambda^a [v \cdot \nabla, u_\kappa] u_\tau \lambda^b > \\
&+ \left( -\frac{32}{3} D^2(D^2 + F^2) + 12(D^2 - F^2)^2 \right) < \lambda^a \lambda^b [v \cdot \nabla, u_\tau] u_\kappa > \\
&+ \left( -\frac{16}{3} D^2(D^2 + F^2) + 6(D^2 - F^2)^2 \right) < \lambda^a \lambda^b u_\tau [v \cdot \nabla, u_\kappa] > \\
&- \frac{32}{3} D^2(D^2 - F^2) < \lambda^a u_\kappa \lambda^b [v \cdot \nabla, u_\tau] > \\
&- \frac{16}{3} D^2(D^2 - F^2) < \lambda^a [v \cdot \nabla, u_\kappa] \lambda^b u_\tau > \\
&- \frac{32}{3} D^2(D^2 - F^2) < \lambda^a [v \cdot \nabla, u_\tau] \lambda^b u_\kappa > \\
&- \frac{16}{3} D^2(D^2 - F^2) < \lambda^a u_\tau \lambda^b [v \cdot \nabla, u_\kappa] >
\end{align*} \]

\[ O_{16}^{ab} = V_{i}^{ac(2)} \left[ < \lambda^a \lambda^b (S \cdot u, \lambda^d)_{\pm} > \alpha^{de}_{2} \right] V_{i}^{cb(2)} \]

\[ \Sigma_{1,16} = \]

\[ -\frac{2}{3} S^\kappa \left\{ \left[ S^\mu S^\nu S^\tau \right] \left( D^5 + 10D^3F^2 + 5DF^4 \right) < \lambda^a \lambda^b > [u_\mu, u_\nu] u_\tau > \right. \]

\[ + \left[ S^\mu S^\nu S^\tau \right] \left( 5D^4F + 10D^2F^3 + F^5 \right) < \lambda^a \lambda^b > [u_\mu, u_\nu] u_\tau > \]

\[ + \left[ S^\nu S^\mu S^\tau \right] \left( 6(D + F)(D^2 - F^2)^2 - \frac{8}{3} D^2(D + F)^3 \right) < \lambda^a u_\mu u_\nu u_\tau \lambda^b > \]

\[ + \left[ S^\tau S^\mu S^\nu \right] \left( 6(D - F)(D^2 - F^2)^2 - \frac{8}{3} D^2(D - F)^3 \right) < \lambda^a \lambda^b u_\mu u_\nu u_\tau > \]

\[ + \left[ S^\mu \left\{ S^\nu, S^\tau \right\} + S^\tau S^\mu S^\nu \right] \left( -\frac{8}{3} D^2(D + F)(D^2 - F^2) \right) < \lambda^a u_\mu u_\nu \lambda^b u_\tau > \]

\[ + \left[ S^\mu S^\tau S^\nu + S^\tau \left\{ S^\mu, S^\nu \right\} \right] \left( -\frac{8}{3} D^2(D - F)(D^2 - F^2) \right) < \lambda^a u_\mu \lambda^b u_\nu u_\tau > \]

\[ + \left[ S^\nu \left\{ S^\tau, S^\mu \right\} + S^\mu S^\nu S^\tau \right] \left( 2(D + F)^4(D - F) \right) < \lambda^a \lambda^b u_\mu > u_\nu u_\tau > \]
\[+
\left[ S^\nu, S^\mu \right] S^\tau + S^\tau S^\nu S^\mu \right] \left( 2(D + F)(D - F)^4 \right) < \lambda^{a\dagger} u_\mu \lambda^b > < u_\nu u_\tau >
\]
\[+ S^\mu \left[ S^\nu, S^\tau \right] + S^\tau S^\mu S^\nu \right] \left( 2(D + F)(D^2 - F^2)^2 \right) < \lambda^{a\dagger} u_\mu u_\nu > < u_\tau \lambda^b >
\]
\[+ S^\tau S^\nu S^\mu + S^\nu S^\mu S^\tau \right] \left( 2(D - F)(D^2 - F^2)^2 \right) < \lambda^{a\dagger} u_\mu u_\nu > < u_\tau \lambda^b >
\]
\[+ S^\mu \left[ S^\frac{\tau}{S^\tau} + S^\mu S^\tau S^\nu \right] \left( 2(D + F)(D^2 - F^2)^2 \right) < \lambda^{a\dagger} u_\mu > < u_\nu u_\tau \lambda^b >
\]
\[+ \left[ S^\mu S^\tau S^\nu + S^\tau \left[ S^\nu, S^\mu \right] \right] \left( 2(D - F)(D^2 - F^2)^2 \right) < \lambda^{a\dagger} u_\mu > < u_\nu u_\tau \lambda^b > \right\} S_\kappa
\]
\[\frac{-2}{3} S^\kappa S^\mu S^\nu S^\tau S_\kappa \left\{ -\frac{8}{3} D^2(D/F)^3 < (\lambda^{a\dagger}, \lambda^b)_\pm (u_\mu, (u_\nu, u_\tau)_\pm)_\pm >
\right.
\]
\[\frac{-8}{3} D^2(D/F)^3 < (u_\mu, \lambda^{a\dagger})_\pm (\lambda^b, (u_\nu, u_\tau)_\pm)_\pm >
\]
\[\frac{8}{3} D^4(D/F) < \lambda^{a\dagger} \lambda^b > < u_\mu (u_\nu, u_\tau)_\pm >
\]
\[\frac{32}{9} D^4(D/F) < (\lambda^{a\dagger}, \lambda^b)_\pm u_\mu > < u_\nu u_\tau >
\]
\[\frac{32}{9} D^4(D/F) < (\lambda^{a\dagger}, \lambda^b)_\pm u_\tau > < u_\mu u_\nu >
\]
\[\frac{32}{9} D^4(D/F) < \lambda^{a\dagger} u_\mu > < (u_\nu, u_\tau)_\pm \lambda^b >
\]
\[\frac{32}{9} D^4(D/F) < \lambda^{a\dagger} (u_\mu, u_\nu)_\pm > < u_\tau \lambda^b > \right\} \quad (C.16)\]
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Table 1: Counterterms and their $\beta$–functions as defined in Eqs.(115,116)

| $i$ | $\tilde{O}^{ab}_i$                                                                 | $\beta_i$                  |
|-----|----------------------------------------------------------------------------------|-----------------------------|
| 1   | $< \lambda^a \, u_\mu > < [i v \cdot \nabla, u_\mu] \lambda^b > + \text{h.c.}$ | $-4D^4/9$                  |
| 2   | $< \lambda^a \, v \cdot u > < [i v \cdot \nabla, v \cdot u] \lambda^b > + \text{h.c.}$ | $4D^4/9$                   |
| 3   | $v_\rho \, F^{\rho\mu\sigma\tau} \, S_\sigma < \lambda^a \{ [v \cdot \nabla, u_\mu], \{ u_\nu, \lambda^b \} \} > + \text{h.c.}$ | $-2D^2(D^2 - F^2)/9$       |
| 4   | $v_\rho \, F^{\rho\mu\sigma\tau} \, S_\sigma < \lambda^a \{ [v \cdot \nabla, u_\mu], [u_\nu, \lambda^b] \} > + \text{h.c.}$ | $2D^2(D^2 - F^2)/9$       |
| 5   | $< \lambda^a \{ (v \cdot u)^2, [i v \cdot \nabla, \lambda^b] \} > + \text{h.c.}$ | $3/4$                     |
| 6   | $< \lambda^a \{ [v \cdot u, [i v \cdot \nabla, v \cdot u]], \lambda^b \} >$ | $3/4 + (D^4 - 10D^2F^2 + 9F^4)/12$ |
| 7   | $< \lambda^a \{ [u_\mu, [i v \cdot \nabla, u_\mu]], \lambda^b \} >$ | $-(D^4 - 10D^2F^2 + 9F^4)/12$ |
| 8   | $< \lambda^a \{ [v \cdot u, [i v \cdot \nabla, v \cdot u]], \lambda^b \} >$ | $-4D^3F/3$                 |
| 9   | $< \lambda^a \{ [u_\mu, [i v \cdot \nabla, u_\mu]], \lambda^b \} >$ | $4D^3F/3$                  |
| 10  | $i < \lambda^a \{ [u_\mu, [i v \cdot \nabla, u_\mu]], \lambda^b \} >$ | $-(9 + 25D^2 + 45F^2)/72$ |
| 11  | $i < \lambda^a \{ [\nabla_\mu, v \cdot u], \lambda^b \} >$ | $-(9 + 25D^2 + 45F^2)/72$ |
| 12  | $i < \lambda^a \{ [u_\mu, [\nabla_\mu, v \cdot u]], \lambda^b \} >$ | $-5DF/4$                   |
| 13  | $i < \lambda^a \{ [\nabla_\mu, [u_\mu, v \cdot u]], \lambda^b \} >$ | $-5DF/4$                   |
| 14  | $v_\rho \, F^{\rho\mu\sigma\tau} \, S_\sigma < \lambda^a \{ [u_\mu, [v \cdot \nabla, u_\nu]], \lambda^b \} >$ | $-(15D^4 + 26D^2F^2 - 9F^4)/18$ |
| 15  | $v_\rho \, F^{\rho\mu\sigma\tau} \, S_\sigma < \lambda^a \{ [u_\mu, [v \cdot \nabla, u_\nu]], \lambda^b \} >$ | $8D^3F/9$                  |
| 16  | $v_\rho \, F^{\rho\mu\sigma\tau} \, S_\sigma < \lambda^a \{ [\nabla_\mu, v \cdot u], \lambda^b \} >$ | $(34D^4 + 108D^2F^2 + 18F^4)/27$ |
| 17  | $v_\rho \, F^{\rho\mu\sigma\tau} \, S_\sigma < \lambda^a \{ [u_\mu, v \cdot \nabla, u_\nu], \lambda^b \} >$ | $8D^4/9$                   |
| 18  | $v_\rho \, F^{\rho\mu\sigma\tau} \, S_\sigma < \lambda^a \{ [v \cdot \nabla, u_\nu], \lambda^b \} > + \text{h.c.}$ | $2(13D^4 - 18D^2F^2 + 9F^2)/27$ |
| 19  | $v_\rho \, F^{\rho\mu\sigma\tau} \, S_\sigma < \lambda^a \{ [u_\mu, v \cdot \nabla, \lambda^b] \} > + \text{h.c.}$ | $-3DF/2 - 4D^3F/3$         |
| 20  | $v_\rho \, F^{\rho\mu\sigma\tau} \, S_\sigma < \lambda^a \{ [u_\mu, [v \cdot \nabla, \lambda^b]], \lambda^b \} > + \text{h.c.}$ | $-(5D^2 + 9F^2)/12$       |
| 21  | $iv_\rho \, F^{\rho\mu\sigma\tau} < \lambda^a \lambda^b > < [u_\mu, u_\nu] u_\sigma >$ | $(D^4 - 10D^2F^2 + 9F^4)/12$ |
| 22  | $iv_\rho \, F^{\rho\mu\sigma\tau} < \lambda^a \{ [u_\mu, u_\nu], \{ u_\sigma, \lambda^b \} \} > + \text{h.c.}$ | $-F(3D^2 + F^2)/4$       |
| 23  | $iv_\rho \, F^{\rho\mu\sigma\tau} < \lambda^a \{ [u_\mu, u_\nu], \{ u_\sigma, \lambda^b \} \} > + \text{h.c.}$ | $F(D^2 - F^2)/4$         |
| 24  | $iv_\rho \, F^{\rho\mu\sigma\tau} < \lambda^a \{ [u_\mu, u_\nu], [u_\sigma, \lambda^b] \} > + \text{h.c.}$ | $3F(D^2 - F^2)/32$       |
| 25  | $iv_\rho \, F^{\rho\mu\sigma\tau} < \lambda^a \{ [u_\mu, u_\nu], [u_\sigma, \lambda^b] \} > + \text{h.c.}$ | $-3D(D^2 - F^2)/32$      |
| 26  | $iv_\rho \, F^{\rho\mu\sigma\tau} < \lambda^a \{ [u_\mu, u_\nu], [u_\sigma, \lambda^b] \} > + \text{h.c.}$ | $-D(7D^2 + 9F^2)/96$     |
| 27  | $i < \lambda^a \{ [v \cdot \nabla, [v \cdot \nabla, [v \cdot \nabla, \lambda^b]]] \} >$ | $F(7D^2 + 9F^2)/96$      |
|     |                                                                                 | $-(20D^2 + 36F^2)/3$      |
Table 1: continued

| i  | $\mathcal{O}_{i}^{ab}$                                                                                     | $\beta_i$                                                                 |
|----|-------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------|
| 28 | $\langle \lambda^\dagger u_\mu [i v \cdot \nabla, \lambda^b] u^u \rangle + \text{h.c.}$                  | $-D^2 + 4D^2(D^2 - F^2)$                                                 |
| 29 | $\langle \lambda^\dagger \{u^2, [i v \cdot \nabla, \lambda^b]\} \rangle + \text{h.c.}$                   | $-\frac{5D^2 + 9F^2}{8} + \frac{(15D^4 + 26F^2D^2 - 9F^4)}{4}$          |
| 30 | $\langle \lambda^\dagger [u^2, [i v \cdot \nabla, \lambda^b]] \rangle + \text{h.c.}                     | $-9D/4 - 4D^3F$                                                          |
| 31 | $\langle \lambda^\dagger [i v \cdot \nabla, \lambda^b] \rangle < u \cdot u > + \text{h.c.}$              | $-3(D^2 + F^2)/4 - (17D^4 + 54D^2F^2 + 9F^4)/6$                          |
| 32 | $\langle \lambda^\dagger u_\mu \rangle < u^u [i v \cdot \nabla, \lambda^b] \rangle + \text{h.c.}$        | $3(D^2 - F^2)/2 - (13D^4 - 18D^2F^2 + 9F^4)/3$                           |
| 33 | $\langle \lambda^\dagger \{\chi_+, [i v \cdot \nabla, \lambda^b]\} \rangle + \text{h.c.}               | $3(D^2 - 3F^2)/8$                                                       |
| 34 | $\langle \lambda^\dagger [\chi_+, [i v \cdot \nabla, \lambda^b]] \rangle + \text{h.c.}                 | $-5DF/4$                                                                 |
| 35 | $\langle \lambda^\dagger [i v \cdot \nabla, \lambda^b] \rangle < \chi_+ > + \text{h.c.}               | $-(13D^2 + 9F^2)/12$                                                    |
| 36 | $\langle [\lambda^\dagger, v \cdot \nabla] \{S \cdot u, [v \cdot \nabla, \lambda^b]\} \rangle >$      | $2D(D^2 + 3F^2)$                                                        |
| 37 | $\langle [\lambda^\dagger, v \cdot \nabla] [S \cdot u, [v \cdot \nabla, \lambda^b]] \rangle >$       | $-2F(9F^2 - 5D^2)/3$                                                    |
| 38 | $\langle \lambda^\dagger \{[v \cdot \nabla, [v \cdot \nabla, S \cdot u]], \lambda^b\} \rangle >$     | $-2D(D^2 + 3F^2)/3$                                                     |
| 39 | $\langle \lambda^\dagger [[v \cdot \nabla, [v \cdot \nabla, S \cdot u]], \lambda^b] \rangle >$      | $-2F(5D^2 - 9F^2)/9$                                                    |
| 40 | $\langle \lambda^\dagger \{S \cdot u, \lambda^b\} \rangle < \chi_+ >$                                | $D/4 + 2D^3/9 + D(D^2 - F^2)/4$                                          |
| 41 | $\langle \lambda^\dagger [S \cdot u, \lambda^b] \rangle < \chi_+ >$                                  | $F/4 + 2D^2F/9 - F(D^2 - F^2)/4$                                         |
| 42 | $\langle \lambda^\dagger \lambda^b \rangle < S \cdot u \chi_+ >$                                     | $-D/2 + D(D^2 + 3F^2)/2$                                                 |
| 43 | $\langle \lambda^\dagger \{v \cdot u, \lambda^b\} \rangle < v \cdot u S \cdot u >$                   | $-D - D(D^4 + 2D^2F^2 - 3F^4)/3$                                         |
| 44 | $\langle \lambda^\dagger [v \cdot u, \lambda^b] \rangle < v \cdot u S \cdot u >$                    | $-F - F(-3D^4 + 2D^2F^2 + F^4)/3$                                        |
| 45 | $\langle \lambda^\dagger \{S \cdot u, v \cdot u\} \rangle < v \cdot u \lambda^b > + \text{h.c.}     | $D - D(D^2 - F^2)^2/3$                                                   |
| 46 | $\langle \lambda^\dagger [S \cdot u, v \cdot u] \rangle < v \cdot u \lambda^b > + \text{h.c.}      | $F - 16D^4F/27$                                                          |
| 47 | $\langle \lambda^\dagger \{S \cdot u, \lambda^b\} \rangle < (v \cdot u)^2 >$                           | $D/2 - D(D^4 + 2D^2F^2 - 3F^4)/6 - 16D^5/27$                            |
| 48 | $\langle \lambda^\dagger [S \cdot u, \lambda^b] \rangle < (v \cdot u)^2 >$                           | $F/2 - F(-3D^4 + 2D^2F^2 + F^4)/6 + 16D^4F/27$                           |
| 49 | $\langle \lambda^\dagger S \cdot u \rangle < (v \cdot u)^2 \lambda^b > + \text{h.c.}$                | $-D - D(D^2 - F^2)^2/3 - 16D^5/27$                                       |
| 50 | $\langle \lambda^\dagger \lambda^b \rangle < (v \cdot u)^2 S \cdot u >$                               | $D - D(D^4 + 10D^2F^2 + 5F^4)/6 - 16D^5/27$                             |
| 51 | $\langle \lambda^\dagger \{v \cdot u S \cdot u v \cdot u, \lambda^b\} \rangle$                      | $3D/2 + D(5D^4 - 30D^2F^2 + 9F^4)/18$                                    |
| 52 | $\langle \lambda^\dagger [v \cdot u S \cdot u v \cdot u, \lambda^b] \rangle$                       | $3F/2 + F(-19D^4 - 6D^2F^2 + 9F^4)/18$                                   |
| 53 | $\langle \lambda^\dagger \{v \cdot u, S \cdot u\} \lambda^b v \cdot u \rangle > + \text{crossed}$        | $2D^3(D^2 - F^2)/9$                                                     |
| 54 | $\langle \lambda^\dagger \{v \cdot u, S \cdot u\} \lambda^b v \cdot u \rangle > - \text{crossed}$   | $2D^2F(D^2 - F^2)/9$                                                    |
| i  | $\hat{O}_{i}^{ab}$                                                                 | $\beta_{i}$                                                                                           |
|----|----------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------|
| 55 | $< \lambda^{\dagger} (S \cdot u, \lambda^{b}) > < u^{2} >$                      | $D/4 + D(D^{2} - F^{2})/4$                                                                             |
| 56 | $< \lambda^{\dagger} [S \cdot u, \lambda^{b}] > < u^{2} >$                     | $+ D(D^{4} + 2D^{2}F^{2} - 3F^{4})/6 + 16D^{5}/27$                                                  |
| 57 | $< \lambda^{\dagger} \lambda^{b} > < S \cdot u u \cdot u >$                    | $F/4 - F(D^{2} - F^{2})/4$                                                                             |
| 58 | $< \lambda^{\dagger} \{u_{\mu}, \lambda^{b}\} > < u^{\mu} S u >$              | $+ F(-3D^{4} + 2D^{2}F^{2} + F^{4})/6 - 16D^{4}F/27$                                                |
| 59 | $< \lambda^{\dagger} \{u_{\mu}, \lambda^{b}\} > < u^{\mu} S u >$              | $- D/2 + D(D^{2} + 3F^{2})/2$                                                                         |
| 60 | $< \lambda^{\dagger}\{S \cdot u, \{(v \cdot u)^{2}, \lambda^{b}\}\} > + h.c.$ | $+ D(D^{4} + 10D^{2}F^{2} + 5F^{4})/6 + 16D^{5}/27$                                                |
| 61 | $< \lambda^{\dagger} [S \cdot u, [(v \cdot u)^{2}, \lambda^{b}]] > + h.c.$     | $D/2 - D(D^{2} + 3F^{2})/2$                                                                           |
| 62 | $< \lambda^{\dagger} [S \cdot u, [(v \cdot u)^{2}, \lambda^{b}]] > + h.c.$     | $+ D(D^{4} + 2D^{2}F^{2} - 3F^{4})/3$                                                                  |
| 63 | $< \lambda^{\dagger} [S \cdot u, [(v \cdot u)^{2}, \lambda^{b}]] > + h.c.$     | $F/2 - F(3D^{2} + F^{2})/2$                                                                           |
| 64 | $< \lambda^{\dagger} v \cdot u [iv \cdot \nabla, \lambda^{b}] v \cdot u > + h.c.$ | $+ F(-3D^{4} + 2D^{2}F^{2} + F^{4})/3$                                                                |
| 65 | $< \lambda^{\dagger} v \cdot u > < v \cdot u [iv \cdot \nabla, \lambda^{b}] > + h.c.$ | $D(29D^{4} + 28D^{2}F^{2} - 9F^{4})/36$                                                             |
| 66 | $< \lambda^{\dagger} [(v \cdot u)^{2}, [iv \cdot \nabla, \lambda^{b}]] > + h.c.$ | $- 3F/4 + F(-19D^{4} + 12D^{2}F^{2} - 9F^{4})/36$                                                 |
| 67 | $< \lambda^{\dagger} [(v \cdot u)^{2}, [iv \cdot \nabla, \lambda^{b}]] > + h.c.$ | $F(-23D^{4} + 16D^{2}F^{2} - 9F^{4})/36$                                                            |
| 68 | $< \lambda^{\dagger} [i v \cdot \nabla, \lambda^{b}] > < (v \cdot u)^{2} > + h.c.$ | $- 3D/4 + D(3D^{4} + 16D^{2}F^{2} - 3F^{4})/12$                                                   |
| 69 | $< \lambda^{\dagger} u_{\mu} > < \{u^{\mu}, S \cdot u \} \lambda^{b} > + h.c.$ | $- 4D^{2}(D^{2} - F^{2})$                                                                             |
| 70 | $< \lambda^{\dagger} u_{\mu} > < \{u^{\mu}, S \cdot u \} \lambda^{b} > + h.c.$ | $1 + (13D^{4} - 18D^{2}F^{2} + 9F^{2})/3$                                                            |
| 71 | $< \lambda^{\dagger} S \cdot u > < u \cdot u \lambda^{b} > + h.c.$              | $4D^{3}F$                                                                                             |
| 72 | $< \lambda^{\dagger} \{S \cdot u, \{u^{2}, \lambda^{b}\}\} > + h.c.$          | $- (15D^{4} + 26F^{2}D^{2} - 9F^{4})/4$                                                                |
| 73 | $< \lambda^{\dagger} \{S \cdot u, [u^{2}, \lambda^{b}]\} > + h.c.$          | $(3 + 17D^{4} + 54D^{2}F^{2} + 9F^{4})/6$                                                             |
| 74 | $< \lambda^{\dagger} [S \cdot u, \{u^{2}, \lambda^{b}\}] > + h.c.$          | $- D(D^{2} - F^{2})/2 + D(D^{2} - F^{2})^{2}/3$                                                      |
| 75 | $< \lambda^{\dagger} \{S \cdot u, [u^{2}, \lambda^{b}]\} > + h.c.$          | $- F(D^{2} - F^{2})/2 + 16D^{4}F/27$                                                                 |
|    |                                                                                 | $D(D^{2} - F^{2})/2 + D(D^{2} - F^{2})^{2}/3 + 16D^{5}/27$                                             |
|    |                                                                                 | $3D(1 + D^{2} - F^{2})/16 - D^{3}/3$                                                                   |
|    |                                                                                 | $- D(29D^{4} + 28D^{2}F^{2} - 9F^{4})/36$                                                             |
| 76 |                                                                                 | $3F(1 + D^{2} - F^{2})/16$                                                                             |
| 77 |                                                                                 | $- F(-19D^{4} + 12D^{2}F^{2} - 9F^{4})/36$                                                            |
| 78 |                                                                                 | $- 3F(-1 + D^{2} - F^{2})/16 + D^{2}F/3$                                                              |
| 79 |                                                                                 | $- F(-23D^{4} + 16D^{2}F^{2} - 9F^{4})/36$                                                            |
| 80 |                                                                                 | $- 3D(-1 + D^{2} - F^{2})/16$                                                                          |
| 81 |                                                                                 | $- D(3D^{4} + 16D^{2}F^{2} - 3F^{4})/12$                                                               |
| i  | $O^{ab}_i$                                                                 | $\beta_i$                                                                 |
|----|---------------------------------------------------------------------------|---------------------------------------------------------------------------|
| 76 | $\lambda^\dagger \{u^\mu \cdot (S \cdot u) \cdot u^\mu, \lambda^b\} >$ | $-D(5D^4 - 30D^2F^2 + 9F^4)/18$                                         |
| 77 | $\lambda^\dagger \{u^\mu \cdot (S \cdot u) \cdot u^\mu, \lambda^b\} >$ | $-F(-19D^4 - 6D^2F^2 + 9F^4)/18$                                         |
| 78 | $\lambda^\dagger \{u^\mu \cdot S \cdot u \cdot \lambda^b u^\mu\} > +$ crossed | $2D^3/3 - 2D^3(D^2 - F^2)/9$                                             |
| 79 | $\lambda^\dagger \{u^\mu \cdot S \cdot u \cdot \lambda^b u^\mu\} > -$ crossed | $-2D^2F(D^2 - F^2)/9$                                                   |
| 80 | $\lambda^\dagger \{S \cdot u \cdot \{\chi^+, \lambda^b\}\} > + h.c.$     | $-D(-9 + 23D^2 + 9F^2)/48$                                              |
| 81 | $\lambda^\dagger \{S \cdot u \cdot [\chi^+, \lambda^b]\} > + h.c.$        | $-F(-9 + 7D^2 + 9F^2)/48$                                               |
| 82 | $\lambda^\dagger \{S \cdot u \cdot \{\chi^+, \lambda^b\}\} > + h.c.$     | $-3F(-1 + D^2 - F^2)/16$                                                |
| 83 | $\lambda^\dagger \{S \cdot u \cdot [\chi^+, \lambda^b]\} > + h.c.$        | $-D(-9 + 9D^2 + 7F^2)/48$                                               |
| 84 | $\lambda^\dagger \{S \cdot u \cdot \chi^+ \lambda^b\} > + h.c.$          | $4D^3/9 + D(D^2 - F^2)^2/2$                                              |
| 85 | $i\varphi^\mu_{\nu} \varepsilon_{\nu \mu \tau \tau} < \lambda^\dagger \chi^+ \lambda^b < [u^\mu, u^\nu] u^\tau >$ | $-F(5D^4 + 10D^2F^2 + F^4)/8 - 4D^4F/9$                                   |
| 86 | $i\varphi^\mu_{\nu} \varepsilon_{\nu \mu \tau \tau} < \lambda^\dagger \{[u^\mu, u^\nu], \{u^\tau, \lambda^b\}\} > + h.c.$ | $F(71D^4 + 18D^2F^2 - 9F^4)/96$                                          |
| 87 | $i\varphi^\mu_{\nu} \varepsilon_{\nu \mu \tau \tau} < \lambda^\dagger \{[u^\mu, u^\nu], \{u^\tau, \lambda^b\}\} > + h.c.$ | $D(-9D^4 - 30D^2F^2 - 9F^4)/96$                                         |
| 88 | $i\varphi^\mu_{\nu} \varepsilon_{\nu \mu \tau \tau} < \lambda^\dagger \{[u^\mu, u^\nu], \{u^\tau, \lambda^b\}\} > + h.c.$ | $D(-D^4 - 38D^2F^2 - 9F^4)/96$                                          |
| 89 | $i\varphi^\mu_{\nu} \varepsilon_{\nu \mu \tau \tau} < \lambda^\dagger \{[u^\mu, u^\nu], \{u^\tau, \lambda^b\}\} > + h.c.$ | $F(31D^4 + 58D^2F^2 - 9F^4)/96$                                         |
| 90 | $i\varphi^\mu_{\nu} \varepsilon_{\nu \mu \tau \tau} < \lambda^\dagger \{[u^\mu, u^\nu], \{u^\tau, \lambda^b\}\} > + h.c.$ | $-(D^2 - F^2)^2F/4 - 4D^4F/9$                                           |
| 91 | $\varphi^\mu_{\nu} \varepsilon_{\mu \nu \sigma} < \lambda^\dagger \chi^+ \lambda^b > F^\mu_\nu u^\sigma >$ | $-F(3D^2 + F^2)/2$                                                       |
| 92 | $\varphi^\mu_{\nu} \varepsilon_{\mu \nu \sigma} < \lambda^\dagger \chi^+ \lambda^b > F^\mu_\nu u^\sigma > + h.c.$ | $F(D^2 - F^2)/2$                                                         |
| 93 | $\varphi^\mu_{\nu} \varepsilon_{\mu \nu \sigma} < \lambda^\dagger \{u^\sigma, \{F^\mu_\nu, \lambda^b\}\} > + h.c.$ | $3F(D^2 - F^2)/16$                                                       |
| 94 | $\varphi^\mu_{\nu} \varepsilon_{\mu \nu \sigma} < \lambda^\dagger \{u^\sigma, \{F^\mu_\nu, \lambda^b\}\} > + h.c.$ | $-D(7D^2 + 9F^2)/48$                                                     |
| 95 | $\varphi^\mu_{\nu} \varepsilon_{\mu \nu \sigma} < \lambda^\dagger \{u^\sigma, \{F^\mu_\nu, \lambda^b\}\} > + h.c.$ | $-3D(D^2 - F^2)/16$                                                      |
| 96 | $\varphi^\mu_{\nu} \varepsilon_{\mu \nu \sigma} < \lambda^\dagger \{u^\sigma, \{F^\mu_\nu, \lambda^b\}\} > + h.c.$ | $F(7D^2 + 9F^2)/48$                                                      |
| 97 | $i\varphi^\mu_{\nu} \varepsilon_{\mu \nu \sigma} S^\sigma < \lambda^\dagger \{F^\mu_\nu, [v \cdot \nabla, \lambda^b]\} > + h.c.$ | $3DF$                                                                   |
| 98 | $i\varphi^\mu_{\nu} \varepsilon_{\mu \nu \sigma} S^\sigma < \lambda^\dagger \{F^\mu_\nu, [v \cdot \nabla, \lambda^b]\} > + h.c.$ | $(5D^2 + 9F^2)/6$                                                        |
| 99 | $i\varphi^\mu_{\nu} v^\nu < \lambda^\dagger \{[ F^\mu_\nu, v \cdot [v \cdot \nabla, \lambda^b]\} >$ | $3D/2$                                                                   |
| 100| $i\varphi^\mu_{\nu} v^\nu < \lambda^\dagger \{[ F^\mu_\nu, v \cdot [v \cdot \nabla, \lambda^b]\} >$ | $3F/2$                                                                   |
| 101| $\lambda^\dagger \{[\nabla^\nu, F^\mu_\nu v^\nu], \lambda^b\} >$ | $-(9 + 25D^2 + 45F^2)/36$                                               |
| 102| $\lambda^\dagger \{[\nabla^\nu, F^\mu_\nu v^\nu], \lambda^b\} >$ | $-5DF/2$                                                                 |
Figure Captions

Fig. 1 Contributions to the one–loop generating functional \((\Sigma_1, \Sigma_2, \Sigma_3)\) and the tree level mesonic generating functional at order \(\bar{h}\), \(\Sigma_4\). The solid (dashed) double lines denote the baryon (meson) propagator in the presence of external fields, respectively. The circle–cross in \(\Sigma_4\) denotes the counter terms from \(\mathcal{L}_M^{(4)}\). The contributions \(\Sigma_{1,2}\) are called irreducible where as \(\Sigma_{3,4}\) are reducible.

Fig. 2 Some physical processes encoded in the irreducible generating functional. For the self–energy \((\Sigma_1)\) and the tadpole graph \((\Sigma_2)\), typical contributions from single and double pion photo/electroproduction are shown. Solid, dashed and wiggly lines denote baryons, pions and photons, in order.

Fig. 3 Feynman graphs which lead to the divergence in the P–wave multipole \(P_3\) for \(\gamma p \to \pi^0 p\) as discussed in the text. \(Y\) and \(Y'\) can either be the \(\Lambda\) or the \(\Sigma^0\). Crossed graphs are not shown.
Figure 2

Figure 3