Lattice Boltzmann simulation of the sedimentation of two spheres in a vertical tube

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Abstract. In this work the three-dimensional lattice Boltzmann simulations are performed to study the settling patterns of two spheres with different densities in a 5d (d is the diameter of sphere) tube of square-cross section. The focus is on the hydrodynamic interaction between the two spheres at the low Reynolds numbers. It has been found that the spheres will migrate to the reverse-diagonal plane of the tube even if they are initially placed in the centerline plane of the tube. The pattern suggests that the particles can only find their equilibrium positions in the diagonal plane when the Reynolds number is low. In addition, the effect of the Reynolds number on this pattern is also discussed.

1. Introduction
The behaviour of systems involving the motion of particles immersed in fluids exists in a wide range of phenomena of interest to both scientists and engineers. Understanding the behaviour and characteristics of particle suspensions in fluids is important for many separation processes. The complexity of relative particle-fluid motions may be due to the mutual interaction of particles as well as the interaction between the particles and walls. For instance, the well-known phenomenon of “drafting-kissing-tumbling” (DKT) that was first observed by Fortes et al. [1], was also numerically studied by Feng et al. [2]. Recently, Wang et al. [3] used the lattice Boltzmann Method (LBM) to investigate the DKT phenomenon of two non-identical particles. They demonstrated that the effect of the diameter difference on the DKT process was significant. Yacoubi et al. [4] numerically studied the two-dimensional dynamics of horizontal arrays of settling particles in a container for the Reynolds number of Re = 200 via an immersed-interface method. They focused on the effect of the particle-particle interaction on the settling pattern in the intermediate-Reynolds-number range.[4] It has been found that in the case of odd-numbered arrays, the middle one was always leading, whereas in the case of even-numbered arrays, the steady-state shape was concave-down. Similar work has also been conducted by Nie et al. [5], who studied the influence of inter-particle distance on the DKT process. Nie et al. [6] reported the grouping behaviours of multiple particles settling along their line-of-centers in a narrow channel. They showed that the settling particles separated into several groups resulting from the particle-particle interaction, with each group settling at the same velocity. Furthermore, their work demonstrated that this type of grouping behavior strongly depended on the number of particles and the Reynolds number. [6] More recently, Nie et al. [7] studied the settling of two circular particles in a narrow channel and revealed some new features of the settling behaviour of particles.

The two-particle sedimentation system is simple but rich in dynamics and worthy of extensive examination. In general, the magnitude of the interaction between the particles is governed by several variables, among which the density difference is a significant factor characterizing the difference of
inertia between the particles. The attention paid to the effect of the density difference on the motion of particles is very limited, especially in three dimensions. A better understanding of this settling problem is needed because it provides valuable insights into the hydrodynamic interactions among multiple particles at a finite Reynolds number. Therefore this work aims to investigate the settling behaviour of two spheres with different densities in a vertical channel at various Galileo number values via the LBM.

2. Numerical Method
In this work the motion of fluid is solved using the LBM. The discrete lattice Boltzmann equations of a single-relaxation-time model are expressed as,

\[ f_i(x + e_i \Delta t, t + \Delta t) - f_i(x, t) = -\frac{1}{\tau} [f_i(x, t) - f_i^{(0)}(x, t)] \]  

where \( f_i(x, t) \) is the distribution function for the microscopic velocity \( e_i \) in the \( i \)th direction, \( f_i^{(0)}(x, t) \) is the equilibrium distribution function, \( \Delta t \) is the time step of the simulation, \( \tau \) is the relaxation time, \( c_s \) is the speed of sound, and \( w_i \) are weights related to the lattice model. The fluid density \( \rho_f \) and velocity \( u \) are determined by the distribution function

\[ \rho = \sum_i f_i, \quad \rho u = \sum_i f_i e_i \]  

For the three-dimensional study, the D3Q19 lattice model is used here, and the discrete velocity vectors are shown in Fig. 1.

![Discrete velocity vectors of D3Q19.](image)

The equilibrium distribution function is chosen as,

\[ f_i^{(0)}(x, t) = w_i \rho_f \left[ 1 + \frac{3 e_i \cdot u}{c_s^2} + \frac{9 (e_i \cdot u)^2}{2 c_s^4} - \frac{3 u^2}{2 c_s^2} \right] \]  

where the \( w_i \) are set to \( w_0 = 1/9, w_{1-6} = 1/36, \) and \( w_{7-18} = 1/72. \) In this model, the fluid viscosity is computed using the equation \( \nu = c_s^2 (\tau - 0.5) \Delta t. \)

In the LBM, special treatment for a moving boundary is usually needed to ensure the no-slip boundary condition on the surface of the particle. In this work, we adopted the schemes proposed by Lallemand & Luo [8] and Aidun et al. [9]. The method proposed by Lallemand & Luo [8] is based on the simple bounce-back boundary scheme and interpolations, which will be briefly described as follows.

The force and torque on the solid particle exerted by the fluid-boundary nodes are computed through the momentum exchange scheme. In addition, to account for the influence of a solid particle entering or leaving the fluid region, the method proposed by Aidun et al. [9] is used to calculate the added force and torque due to the covered and uncovered fluid nodes. Using the net force and torque, the motion of a particle is determined by solving the Newton's equations.
Figure 2. Schematic diagram of the present problem.

In this work, the sedimentation of two spheres in a tube is simulated by the LBM. As shown in Fig. 1, two spheres with a diameter of $d$ are horizontally aligned in a tube of width $L$. Initially both spheres are located in the centerline plane, i.e. $y=0$, which are $2d$ apart from each other. The densities of the two spheres are denoted as $\rho_s$ and $\rho_s'$, respectively. For the following, the parameters are fixed at $\rho = 1$, $\rho_s = 1.5$, $d = 16$ and $g = 9.8 \times 10^{-4}$. Note that all the parameters are in lattice units. The computational domain is $L \times L \times H = 80 \times 80 \times 480$. In addition, a moving computational domain is used to simulate an infinite channel. The upstream boundary of the computational domain is $10d$ upstream of the heavy sphere, whereas the downstream boundary is $20d$ from the heavy sphere. The normal derivative of velocity is zero at the downstream boundary and the velocity at the upstream boundary is zero.

3. Numerical Results

In order to examine the interaction between two spheres with different densities, the value of $\rho_s'$ is chosen as 1.51, which is a little larger than that of $\rho_s$. By doing this the red sphere shown in Fig. 1 is heavier than the blue one. Then both spheres are released at a height of $10d$ away from the bottom boundary. Due to the gravitational force, the two spheres are settling in the tube. As is known, the heavy particle will leave the light one behind if their density difference is large enough because the heavy particle goes down faster. In this case the two spheres will settle separately in the tube. However, things are quite different if their density difference is small. Fig. 3 shows the trajectories of both spheres which are represented by their horizontal ($X^\prime = X/d$) and lateral ($Y^\prime = Y/d$) displacements. Note that in this work the reference velocity is chosen as:

$$U_0 = \sqrt{\left(\frac{\rho_s}{\rho} - 1\right)gd}$$

(4)

where $g$ is the gravity acceleration. Then the time scale is defined as $T_0 = d/U_0$. Therefore in Fig. 2 the time is normalized through $t^\prime = t/T_0$. As shown in the figure, the two spheres are initially oscillating in the centerline plane $(y=0)$ when $t^\prime < 3000$. The oscillation is seen to be damping with time. It is expected that the two spheres finally reach a steady state in this plane, as is observed in the two dimensional computations. However, this is not the case for the spheres. The spheres are seen to leave the centerline plane when $t^\prime > 3000$, which is clearly evidenced by the variation of the spheres’ lateral displacements, i.e. $Y_1^\prime$ and $Y_2^\prime$. Eventually, either of them is almost equal to the negative value of its horizontal counterpart, i.e. $Y_1^\prime = -X_1^\prime$ and $Y_2^\prime = -X_2^\prime$. This indicates that the two spheres reach a steady state in the reverse-diagonal plane of the tube, as shown in Fig. 2. It is an interesting pattern of motion for the spheres because they are initially symmetrical with respect to the centerline plane $(y=0)$. Numerical results show that the spheres are likely to leave the centerline plane at low Reynolds numbers. For the
case of $\rho_s'=1.51$, the particle Reynolds number, which is based on the terminal velocity of spheres, is found to be $Re_p \approx 1.5$.

![Figure 3](image1)

Figure 3. Time history of the horizontal displacement ($X' = X/d$) and the lateral displacement ($Y' = Y/d$) for $\rho_s'=1.51$ for (a): the light sphere and (b) the heavy sphere, respectively.

To better illustrate this pattern, Fig. 3 shows the instantaneous flow field (top view) at different times for the case of $\rho_s'=1.51$. The results are presented separately for the light sphere (upper row) and the heavy one (lower row) because they are located at the different horizontal planes. As shown in Fig. 4 (a), the spheres are seen to settle in the centerline plane of the tube in the initial period of transit. Then they begin to move to the reverse-diagonal plane [Fig. 4(b)-(d)]. This is a very slow process for the spheres (note the normalized time). Furthermore, the velocity vectors are inward in the vicinity of the heavy sphere in the light sphere’s plane (upper row). The opposite is true in the heavy sphere’s plane (lower row). This suggests that the heavy sphere is lower than the light one even they have the same settling speed.

![Figure 4](image2)

Figure 4. Instantaneous fluid velocity field (top view) at different times in the vicinity of the light sphere (upper) and the heavy sphere (lower), respectively. (a) $t'=3044$, (b) $t'=4981$, (c) $t'=6087$ and (d) $t'=8855$.

If increasing the value of $\rho_s'$, the situation becomes different. Fig. 4 shows the time history of particle trajectories for $\rho_s'=1.545$. It is clearly seen that both spheres are periodically oscillating in the centerline plane at $t'<3000$. An examination of Fig. 4 reveals that the heavy sphere quickly goes to the right side of the left one once they are released from rest, i.e. $X_2' > X_1'$. This suggests that the two particles exchange their initial positions in this case. Similar to the case of $\rho_s'=1.51$ (Fig. 3), both spheres begin to leave the centerline plane when $t'>3000$, which eventually reach a steady state in the reverse-diagonal plane of the tube, as shown in Fig. 4. During this process the oscillating is observed to gradually decay in
amplitude towards zero for both spheres owing to the fact that the distance between spheres increases with time which results in the weakening hydrodynamic interaction.

Figure 5. Time history of the horizontal displacement and the lateral displacement for $\rho_s'=1.545$ for: (a) the light sphere and (b) the heavy sphere, respectively.

Figure 6. Final state of the particle motion (top view) for $\rho_s'=1.545$ represented by the fluid velocity in the vicinity of: (a) the light sphere and (b) the heavy sphere, respectively.

Fig. 6 shows the final state of the two spheres for the case of $\rho_s'=1.545$. In comparison with Fig. 4 (the case of $\rho_s'=1.51$), the heavy sphere is located at a position close to the lower right corner this time. Moreover, the fluid velocity in the heavy sphere’s plane is seen to be larger due to the stronger fluid-particle interaction. In addition, the particle Reynolds number is found to be $Re_p \approx 1.6$, which is still within the low-Reynolds-number-regime. However, if further increasing the value of $\rho_s'$, the heavy sphere will leave the light one behind.

4. Conclusion

In this work the three-dimensional lattice Boltzmann method is adopted to simulate the sedimentation of two spheres in a tube of square-cross section. This work focuses on the hydrodynamic interaction between the two settling spheres when their densities are different. Special attention is paid to the unique pattern of motion for the spheres seen at the low Reynolds numbers. It has been found that the spheres will eventually move to the diagonal plane of the tube even if they are initially arranged in the centerline plane of the tube. In addition, the two spheres may exchange their initial positions if the density difference between them is large.

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References

[1] A. Fortes, DD. Joseph, T.S. Lundgren, J. Fluid Mech. 177, 467 (1987).
[2] J. Feng, H.H. Hu, D.D. Joseph, J. Fluid Mech. 261, 95 (1994a).
[3] L. Wang, Z.L. Guo, J.C. Mi, Comput. Fluids 96, 20 (2014).
[4] A.E. Yacoubi, S. Xu, Z.J. Wang, J. Fluid Mech. 705, 134 (2012).
[5] D.M. Nie, J.Z. Lin, M.J. Zheng, Commun. Comput. Phys. 16, 675 (2014).
[6] D.M. Nie, J.Z. Lin, R.Q. Chen, Phys. Rev. E. 93, 013114 (2016).
[7] D.M. Nie, J.Z. Lin, Q.Gao, Comput. Fluids. 156 (2017).
[8] P. Lallemand, L.S. Luo, J. Comput. Phys. 184, 406 (2003).
[9] C.K. Aidun, Y. Lu, E.J. Ding, J. Fluid Mech. 373, 287 (1998).