A Modified Control Method of Robot Path Curve Deviation Based on Terminal Trajectory Tracking

Yongtao Zheng¹, Xinghua Lu²*  
¹Guangzhou Huali Science and Technology Vocational College, Guangzhou, China  
²Huali College Guangdong University of Technology, Guangdong Guangzhou, China  

*Corresponding author e-mail: xhlu@gdut.edu.cn  

Abstract. The robot is affected by small disturbance in the path following motion, which can easily cause the path curve to deviate. A modified control method of robot path curve deviation based on terminal trajectory tracking is proposed. A path trajectory tracking grid model of robot is constructed in the hyperredundancy space, and the force control of the robot is analyzed by quantitative fusion. The adaptive admittance controller is used to correct the deviation of robot path curve, and the mechanical loading is carried out according to the robot and load transfer mechanism model, and the force closed loop is placed in the outer layer of the position control loop. The desired path trajectory is corrected by force feedback, and a linear interpolation optimization method is used to track the trajectory of robot path curve deviation, and the cubic spline curve is used to fit the control points. The robot path curve deviation correction control model is transformed into force following mode, and the robot path curve deviation correction is realized by using multi-degree-of-freedom coordinated control method. The simulation results show that, the proposed method has the advantages of high precision, good stability, good anti-disturbance ability, it can improve path optimization accuracy effectively.

1. Introduction  

With the rapid development of robot technology, all kinds of robots are replacing artificial intelligent operation. There are many kinds of robots, but the main design methods of robots are bionic design, robots imitate the movement of nature. In order to improve the environmental adaptability of the robot, the bionic robot can achieve intelligent operation by tracking and writing, imitating human beings and other animals. In the industrial process, the trajectory tracking control of the robot is the key technology to ensure the robot can walk accurately and operate independently. The control and conversion of multi-mode motion has always been a difficult problem for mobile robot. In the course of robot walking, path curve is easily affected by environment disturbance and external factors, which can easily cause trajectory deviation and result in motion accuracy. It has great significance to study the modified control of robot path curve deviation in improving the control stability and motion accuracy of the robot [1].

The robot path curve deviation correction control also plays an important role in the realization of robot obstacle avoidance and hyperredundancy motion planning. Traditionally, the artificial potential field method, neural network algorithm, genetic algorithm and fuzzy control algorithm are mainly...
used to control the deviation of robot path curve. The above robot path curve deviation correction algorithms can guarantee the robot end to complete the assigned task to a certain extent, at the same time make the robot link and joint avoid obstacles, emphasize the terminal operation ability, it is not strictly restricted by the movement of connecting rod and joint [2-4]. In order to further enhance the control ability of hyperredundancy and the ability of crossing obstacles, the terminal trajectory tracking control is needed, in reference [5], a path following motion method called following end trajectory is proposed. The characteristic of following end trajectory motion is that the linkage and joint of super redundant robot move along the path curve in the restricted area that is, the whole robot can do everything in motion. It can approach the path curve and achieve the goal of crossing the smallest area in the space. The behavior space of the robot is greatly improved, but the control method has the problem of poor anti-disturbance ability and high computational cost. In reference [6], a modified control method of robot path curve deviation based on artificial neural network is proposed, which solves the stability problem of robot loading force on static curtain wall. Combined with sensor quantization fusion tracking, the robot can be improved. Path recognition and control ability, but this method needs to use a large number of contact force experimental data to estimate impedance parameters, so it is difficult to adapt to the environment.

In order to solve the above problems, this paper presents a modified control method of robot path curve deviation based on terminal trajectory tracking. Firstly, a path trajectory tracking grid model of robot is constructed in the hyperredundancy space. The force control of human is analyzed by quantitative fusion, and the robot path curve deviation correction is carried out by adaptive adjusting admittance controller. The mechanical loading is carried out according to the robot and load transfer mechanism model, and the force closed loop is placed in position. In the outer layer of the control loop, the desired path trajectory is corrected by force feedback, and a linear interpolation optimization method is used to track the end trajectory of the robot path curve deviation. The cubic spline curve is used to fit the control point and the machine. The modified control model of human path curve deviation is transformed into force following mode, and the robot path curve deviation correction is realized by using multi-degree-of-freedom coordinated control method. Finally, the experiment results show that the proposed method can improve the robot performance. The superior performance of path curve deviation correction control ability is shown.

2. Path trajectory distribution model of robot and description of controlled object

2.1. Path trajectory distribution model of the robot

In order to realize the path curve deviation correction control and the end trajectory tracking, it is necessary to construct the robot controlled object model, assuming that the robot studied is a humanoid robot. The kinematics of the robot is analyzed, and the relationship between force and position is established by admittance control theory [7]. There are 12 joints, 13 connecting rods and 14 key points of the manipulator.

Let the humanoid robot kinematic chain composed of the waist and the left (right) arm be described as \( \{A^0, A^1\} \), \( A^0 \) is regarded as the root of the kinematic chain, when all the connecting rods are collinear and the angle of all joints is 0, including describing the motion of the lumbar joint. The offset angle \( \alpha \), the steering angle \( \beta \), and the rotation angle \( \gamma \), based on the above three degrees freedom of rotation, it is expressed as \( \alpha_0 = [\alpha, \beta, \gamma] \). The position and attitude can be expressed as \( \theta_1 = [q_1, \cdots, q_7] \). The total number of degrees of freedom consolidated in the connecting rod \( n \) is 10. The configuration of the coordinate system can be expressed as \( \theta = [q_0^T, q_1^T] \equiv [\theta_1, \cdots, \theta_{10}] \), the initial configuration is \( \theta_{start} = [\theta_1^{start}, \cdots, \theta_{10}^{start}] \), and the target position on the connecting rod \( j \) is \( p_{obj} \in \mathbb{R}^6 \). The feasible trajectory set of robot in superredundancy space is \( \mathcal{G}_c = \{g_0, \cdots, g_N\} \). The forward kinematics
The motion tracking planning problem with discrete trajectory points is defined as [8]:

Given the initial configuration of the robot \( \theta_{\text{start}} \in C_{\text{free}} \) (free C-space), the locus discrete point \( p_{\text{obj}} \) and the target tracking trajectory set \( g_{c} \), the target configuration \( \theta_{\text{goal}} \) is unknown, the path curve can be generated offline or online, and a collision avoidance can be found in the C-space. The continuous path mapping: \( \tau : [0,1] \rightarrow C_{\text{free}}, \tau[0] = \theta_{\text{start}}, \tau[1] = \theta_{\text{goal}}, (p_{\text{obj}}, g_{c}) \rightarrow f(\theta_{\text{goal}}) \), which avoids singularity and satisfies the condition of joint constraint, makes the motion space trajectory distribution model of humanoid robot based on D-H convention 7 degrees of freedom is shown in Fig. 1.

![Figure 1. Path trajectory distribution model of robot](image)

According to the trajectory distribution grid structure shown in figure 1, the path deviation correction control is carried out to realize the terminal trajectory tracking [9].

### 2.2. Controlled object description

Set the paths at the sampling time is known, expressed as \( q_{i} = [q_{1i}, ..., q_{7i}]^{T} \), \( \sin q_{i} \) and \( \cos q_{i} \) are respectively abbreviated as \( s_{qi} \) and \( c_{qi} \). According to the initial attitude of the robot, the homogeneous matrix \( i^{-1} T_{i}(q_{i}) \) between coordinate system \( i \) and \( i-1 \) can be expressed \( i^{-1} T_{i}(q_{i}) \):

\[
 i^{-1} T_{i}(q_{i}) = \begin{bmatrix}
 c_{i} & -c_{a_{i}}s_{i} & s_{a_{i}}s_{i} & a_{a_{i}}c_{i} & \n_{a_{i}} \\
 s_{i} & c_{a_{i}}c_{i} & s_{a_{i}}c_{i} & a_{i}s_{i} & \n_{i} \\
 0 & s_{a_{i}} & c_{a_{i}} & d_{i} & \n_{0} \\
 0 & 0 & 0 & 1 & \end{bmatrix} \tag{1}
\]

Under the algorithm of automatic path planning, the forward kinematics (FK) equation of a robot can be controlled by a curve, and the pose of the end track effector \( \Sigma_{7} \) is described as the homogeneous coordinate transformation matrix relative to the reference coordinate system \( \Sigma_{0} \):

\[
 0 T_{7}(q_{1}) = \prod_{i=1}^{7} i^{-1} T_{i}(q_{i}) = \begin{bmatrix}
 0 & 0 & a & p \\
 0 & 0 & 0 & 1 \\
\end{bmatrix}
\]

The upper description of the robot's right arm has redundancy, which can be described by the rotation of the center of the elbow joint around the wrist and \( \Sigma_{0} \) origin axis. The force control of the robot is analyzed by quantitative fusion [10], and adaptive admittance control is adopted. The
analytical form of inverse kinematics of robot can be deduced by modifying the deviation of robot path curve:

\[
0^T_4 = \prod_{i=1}^{4} i^{-1} T_i(q_i) = \begin{bmatrix}
    n_i & o_i & a_i & p_i \\
    0 & 0 & 0 & 1
\end{bmatrix}
\]

(3)

Wherein, \(a_i = r_i - p_i \parallel r_i - p_i\), \(r_i = p - l_i\), \(n_i = a_i \times (r_i - p_i) \parallel a_i \times (r_i - p_i)\), \(n_i = o_i \times a_i\).

The desired position is corrected by the force feedback, and the feedback position and posture of the trajectory deviation are as follows:

\[
4^T_1 = \prod_{i=5}^{7} i^{-1} T_i(q_i) = \prod_{i=4}^{7} i^{-1} T_i^{-1}(q_i) \cdot 4T_4 = \begin{bmatrix}
    n_x & o_x & a_x & p_x \\
    0 & 0 & 0 & 1
\end{bmatrix}
\]

(4)

The force vector \(0^t_1^{-1}\) loaded by the robot are obtained, and the deviation correction of the robot path curve is carried out by adaptive adjusting admittance controller [11]. According to the mechanical loading model of the robot and the load transfer mechanism, it is obtained as:

\[
0^T_1^{-1}(q_1) \cdot 0^T_4 = \prod_{i=2}^{4} i^{-1} T_i(q_i)
\]

(5)

Under the control of the load transfer mechanism, the position correction vector of the robot is:

\[
q_1 \equiv \theta_4 = \text{atan2} (\pm p_{4y}, \pm p_{4x})
\]

(6)

\[
q_2 \equiv \theta_5 = \text{atan2} (-p_{4z}, c_1 p_{4x} + s_1 p_{4y} - l_s)
\]

(7)

Left multiplicative inverse matrix on both sides \(2^{-1} T_1(q_2)\):

\[
\prod_{i=2}^{4} i^{-1} T_i^{-1}(q_i) \cdot 0^T_4 = \prod_{i=3}^{4} i^{-1} T_i(q_i)
\]

(8)

According to the above controlled object model, the robot path curve deviation correction control problem is transformed into the load transfer mechanism model problem of the robot [13].

3. Control algorithm optimization

3.1. Deviation correction of robot path curve

Based on the quantitative fusion analysis of the force control of the robot, the adaptive adjusting admittance controller is used to correct the deviation of the robot path curve, and the position correction vector elements of the robot are designed:

\[
q_3 = \text{atan2} (-s_1 o_{4x} + c_1 o_{4y} - s_2 c_1 o_{4x} + s_2 s_1 o_{4y} - c_2 o_{4z})
\]

(9)

\[
q_4 \equiv \theta_7 \text{ atan2} (c_2 c_1 n_{4x} + c_2 s_1 n_{4y} - s_2 n_{4z}, c_2 c_1 a_{4x} + c_2 s_1 a_{4y} - s_2 a_{4z})
\]

(10)

Combined with the position analysis of the load transfer mechanism, it is concluded that:
The position vector of the relative motion of the robot is solved as:

\[ q_5 \equiv \theta_8 = \tan(\pm \alpha_{cy}, \pm \alpha_{ex}) \]  

(12)

\[ q_6 = \theta_9 = \tan(\pm \alpha_{ex}, -c_y \alpha_{ex} - s_y \alpha_{cy}) \]  

(13)

\[ q_7 \equiv \theta_{10} = \tan(\pm \alpha_{ex} n_{ex} + c_y n_{ex}, s_y \alpha_{ex} - c_y \alpha_{cy}) \]  

(14)

\[ 4T_{x}^{-1}(q_i) \cdot 4T_{y} = \prod_{i=6}^{7} i^{-1}T_{i}(q_i) \]  

(11)

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Trajectory curve deviation from modified control space distribution of robot}
\end{figure}

3.2 Correction of path curve deviation of robot based on coordinated control of multi-degree of freedom

A linear interpolation optimization method is used to track the end trajectory of the robot path curve deviation. In the inertial reference system \( \Sigma_i \), the inertial motion of the robot can be represented by a homogeneous coordinate matrix \( T_i(\alpha_0, \beta_0, \gamma_0) \) of \( 4 \times 4 \) \((\equiv T_i(\theta_1, \theta_2, \theta_3))\), which includes three directions of translation \( p_i = [x_i, y_i, z_i]^T \). The decoupling matrix \( R_z(\alpha_0) \) in each axis direction is expressed as:

\[ R_z(\alpha_0) = \begin{bmatrix}
    c_{\alpha_0} & -s_{\alpha_0} & 0 \\
    s_{\alpha_0} & c_{\alpha_0} & 0 \\
    0 & 0 & 1
\end{bmatrix} \equiv R_z(\theta_1) \]  

(15)

In the space of multiple degrees of freedom, \( \beta_0 \) is the rotation angle around the \( y \) axis, and the rotation matrix \( R_y(\beta_0) \) is expressed as:

\[ R_y(\beta_0) = \begin{bmatrix}
    c_{\beta_0} & 0 & s_{\beta_0} \\
    0 & 1 & 0 \\
    -s_{\beta_0} & 0 & c_{\beta_0}
\end{bmatrix} \equiv R_y(\theta_2) \]  

(16)
By fitting the control points with cubic spline curve, the homogenous transformation matrix $\Sigma_e(=\Sigma_f)$ of the end effector coordinate $\Sigma_f$ relative to the inertial coordinate system $^{i}T_i(\theta)$ is obtained:

$$^{i}T_i(\theta)=^{i}T_0(\theta_1,\theta_2,\theta_3)\cdot T_f(q_1)$$  \hspace{1cm} (17)

Thus, the forward kinematics relationship between the kinematic chain joint $\theta$ and the terminal effector position $p_e$ of humanoid robot from the waist to the end of the arm can be obtained:

$$p_e = f(\theta)$$  \hspace{1cm} (18)

Then the forward kinematics differential equation of the deviation of the path curve of the robot is expressed as:

$$\dot{p}_e = J(\theta)\dot{\theta}$$  \hspace{1cm} (19)

Wherein, $J(\theta) \in \mathbb{R}^{6 \times 10}$ is Jacobian Matrix of humanoid robot. The inverse kinematics solutions of redundant kinematics equations are obtained as follows:

$$\dot{\theta} = J^+\dot{p}_e + (I - J^+J)\ddot{\xi}$$  \hspace{1cm} (20)

Wherein, $J^+ = J^T(JJ^T)^{-1}$ is the Moore-Penrose generalized inverse matrix of Jacobian matrix $J$. The robot path curve deviation correction is realized by using multi-degree-of-freedom coordinated control method, and the optimized control equation is written as follows:

$$\dot{\theta} = J^+\dot{p}_e + k(I - J^+J)\nabla H(\theta)$$  \hspace{1cm} (21)

Where, $\nabla H(\theta) = \frac{\partial H(\theta)}{\partial \theta} = \begin{bmatrix} \frac{\partial H}{\partial \theta_1} & \cdots & \frac{\partial H}{\partial \theta_{10}} \end{bmatrix}^T$ represents the gradient of the optimization index $H(\theta)$, the scalar real constant $k$ is an automatic magnification factor, and feedback from track deviation correction is:

$$\dot{\theta} = \dot{\theta}_{\min} + k\nabla H(\theta)$$  \hspace{1cm} (22)

Where, the minimum norm solutions are $\dot{\theta}_{\min} = \theta - \dot{\theta}_p - \left[ \begin{array}{c} \frac{\partial H}{\partial \theta_1} \\ \vdots \\ \frac{\partial H}{\partial \theta_{10}} \end{array} \right] \theta_p = \left[ \begin{array}{c} 0 \\ \dot{\theta}_p - k\nabla H(\theta) \end{array} \right]$, $J_M \in \mathbb{R}^{6 \times 6}$.

The robot is converted from loading mode to force following mode, and the optimal solution of end trajectory tracking can be obtained as follows:

$$\dot{\theta} = J^+_{WLN}(\theta) \cdot \dot{p}_e$$  \hspace{1cm} (23)

Wherein, $J^+_{WLN}(\theta) = W^{-1}J^T[J \cdot W^{-1}J^T]^{-1}$, based on $H(\theta)$, relative motion matrix of robot tool coordinate system is $W(i=1\sim10)$:
It can be seen from the analysis that the proposed method is used to correct the path curve deviation of the robot, which makes the robot path curve deviate steadily and converge to zero.

4. Simulation experiment and result analysis

In order to test the application performance of this method in realizing the modified control of robot path curve deviation, the simulation experiment is carried out. The experiment adopts MATLAB design. The model of the robot is SMART II robot. The controller parameters $\lambda_0 = 230, \lambda_1 = 120$, Elastic stiffness of contact surface of load transfer mechanism $K = 300$ N/m, the loading force in the Z axis is set to 10N, the starting point of the end trajectory tracking is [10, 20], the end point is [250,260], the information heuristic factor is $\alpha = 1$, the expected heuristic factor is $\beta = 7$, and the pheromone volatilization coefficient $\rho = 0.3$. According to the above parameters setting, the robot path curve deviation correction control is carried out, and the original path curve of the robot is obtained as shown in figure 3.

$$W = \begin{bmatrix} w_1 & 0 & \cdots & 0 \\ 0 & w_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & w_{10} \end{bmatrix}$$  

Figure 3 shows that the path curve deviates in the end trajectory tracking of the robot. The modified path curve is shown in Figure 4 by using the method in this paper.

Figure 4 shows that the method of this paper is used to control the trajectory of the robot, and the path curve deviation correction is realized. The test results show that the optimal path of the robot is shown in Table 1, and the results in the analysis table show that the proposed method can be compared with the other methods. The optimal path selection is realized with short iterative steps, and the ability of path curve correction control is improved.

| Initial point | Object point | Optimal solution of traditional method | Optimal solution in this paper | Optimal solution iteration step number | Cost time/s |
|--------------|--------------|--------------------------------------|-------------------------------|---------------------------------------|-------------|
| [15, 15]     | [250, 250]   | 654                                  | 443                           | 84                                    | 15          |
| [15, 270]    | [250, 250]   | 834                                  | 545                           | 43                                    | 12          |
| [15, 270]    | [250, 15]    | 823                                  | 542                           | 45                                    | 6           |
| [15, 15]     | [250, 15]    | 424                                  | 387                           | 60                                    | 9           |
| [15, 150]    | [250, 150]   | 565                                  | 443                           | 67                                    | 13          |
5. Conclusion
In the course of robot walking, path curve is easily affected by environment disturbance and external factors, and it is easy to cause trajectory deviation, which leads to poor motion precision. In this paper, a robot path curve deviation based on terminal trajectory tracking is proposed. The path trajectory tracking grid model of robot is constructed in the hyperredundancy space, and the force control of robot is analyzed by quantization fusion analysis, and the robot is carried out by adaptive adjusting admittance controller. According to the robot and load transfer mechanism model, the force closed loop is placed in the outer layer of the position control loop. The desired path trajectory is corrected by force feedback, and a linear interpolation is adopted. The method is used to track the end trajectory of the robot path curve deviation. The cubic spline curve is used to fit the control point. The robot path curve deviation correction control model is transformed into force following mode, and the multi-degree-of-freedom coordinated control is adopted. Methods the path curve deviation correction of robot is realized. The research shows that the method in this paper can effectively realize the path curve deviation correction control in the end trajectory tracking of robot, with better precision, lower error and shorter convergence step.

References
[1] Zhang J M, Sun C Y, Zhang R M, et al. Adaptive sliding mode control for re-entry attitude of near space hypersonic vehicle based on backstepping design [J]. IEEE/CAA Journal of Automatic Sinica, 2015, 2 (1): 94-101.
[2] SHI Ruidong, ZHANG Xiuli, YAO Yan'an. A CPG-Based Control Method for the Multi-Mode Locomotion of a Desert Spider Robot. ROBOT, 2018, 40 (2): 146-157.
[3] Si Y Y, He B. Walking control of biped robot based on CPG and CA-CMAC [J]. System Simulation Technology, 2017, 13 (1): 6-10.
[4] Gao Q, Wang Z L, Hu W J, et al. Gait simulation of snake robot based on CPG method [J]. Journal of System Simulation, 2015, 27 (6): 1374-1380.
[5] Zhang X L, Gong J Q, Yao Y A. Effects of head and tail as swinging appendages on the dynamic walking performance of a quadruped robot [J]. Robotics, 2016, 34 (12): 2878-2891.
[6] Shi R D, Zhang X L, Tian Y B, et al. A CPG-based control method for the rolling locomotion of a desert spider [C]//IEEE Workshop on Advanced Robotics and Its Social Impacts. Piscataway, USA: IEEE, 2016: 243-248.
[7] Tunc L T, Shaw J. Experimental study on investigation of dynamics of hexapod robot for mobile machining [J]. International Journal of Advanced Manufacturing Technology, 2016, 84 (5-8): 817-830.
[8] Wang Z L, Gao Q, Zhao H Y. CPG-inspired locomotion control for a snake robot basing on nonlinear oscillators [J]. Journal of Intelligent and Robotic Systems, 2017, 85 (2): 209-227.
[9] Wang M, Yu J Z, Tan M, et al. CPG-based multi-modal swimming control for robotic dolphin [J]. Acta Automatica Sinica, 2014, 40 (9): 1933-1941.
[10] Tian Y B. Research on theories of mobile linkage with multiple locomotion modes [D]. Beijing: Beijing Jiaotong University, 2015.
[11] WANG Jungang, TANG Lei, GU Guoying, ZHU Xiangyang. Tip-following Path Planning and Its Performance Analysis for Hyper-redundant Manipulators. Journal of Mechanical Engineering, 2018, 54 (3): 18-25.
[12] Tang L, Wang J, Zheng Y, et al. Design of a cable-driven hyper-redundant robot with experimental validation [J]. The International Journal of Advanced Robotic Systems, 2017, 14 (5): 1-12.
[13] LIU Changan, YAN Xiaohu, LIU Chunyang, et al. Dynamic path planning for mobile robot based on improved ant colony optimization algorithm [J]. Acta Electronica Sinica, 2011, 39 (5): 1220-1224.
[14] ZHANG Danfeng, LI Bin, WANG Liyan. Tracking Control Method of the Centre-of-Mass Velocity for a Snake-like Robot Based on the Continuum Model. ROBOT, 2017, 39 (6): 829-837.