Neighbourhood Bootstrap for Respondent-Driven Sampling

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Abstract

Respondent-Driven Sampling (RDS) is a form of link-tracing sampling, a sampling technique for ‘hard-to-reach’ populations that aims to leverage individuals’ social relationships to reach potential participants. While the methodological focus has been restricted to the estimation of population proportions, there is a growing interest in the estimation of uncertainty for RDS as recent findings suggest that most variance estimators underestimate variability. Recently, Baraff et al. (2016) proposed the tree bootstrap method based on resampling the RDS recruitment tree, and empirically showed that this method outperforms current bootstrap methods. However, some findings suggest that the tree bootstrap (severely) overestimates uncertainty. In this paper, we propose the neighbourhood bootstrap method for quantifying uncertainty in RDS, and empirically show that our method outperforms the tree bootstrap in terms of bias and coverage under realistic RDS sampling assumptions.

Keywords: Hidden population sampling; Resampling; Respondent-Driven Sampling; Simulations; Social networks.

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1 Introduction

Respondent-Driven Sampling (RDS) is a variant of link-tracing sampling that relies on social connections to reach members of hard-to-reach populations (Heckathorn [1997]). The RDS recruitment process starts off with the selection of initial seed participants and runs through a number of recruitment waves within which each selected individual is given a fixed number of coupons and asked to recruit peers into the study.

Inference for RDS has primarily focused on estimating population means and proportions. There is a rich literature on inference for population prevalence of viral diseases (Gile and Handcock 2010, Gile et al. 2015, Volz and Heckathorn 2008) and risk behaviors (Malekinejad et al. 2008, Johnston et al. 2010, 2008, Heckathorn 2002) among stigmatized populations. The vast majority of these studies use two classical (and widely used) approaches to inference for RDS, which rely on approximations to the true (and unknown) RDS sampling process. Volz and Heckathorn (2008) approximated the RDS sampling process as a random walk on the nodes of an undirected graph and treated RDS samples as independent draws from its stationary distribution. The resulting inclusion probabilities are used to compute design weights known as RDS-II weights, applied to yield Horvitz-Thompson type estimators along with model-based estimates of uncertainty. This approach is implemented in the R package RDS. Alternative estimators include the RDS-I and successive sampling estimators (Gile 2011, Heckathorn 1997), however these are less commonly used and so we focus only on RDS-II in what follows.

There is a growing interest in the estimation of uncertainty for RDS (Spiller et al. 2018) as recent findings show that most variance estimators (greatly) underestimate variability (Goel and Salganik 2010), thus yielding confidence intervals with coverage rates that are below expected nominal values. As an alternative variance estimation strategy, Baraff et al. (2016) proposed the tree bootstrap method based on resampling the RDS recruitment tree. The method generates bootstrap trees from the observed recruitment tree in a hierarchical fashion. First, we sample with (or without) replacement from the initial seed participants. Then, we resample from each of the selected seeds’ recruits. In the third level of the procedure, we resample from the second-level recruits’ recruits. The sampling process continues until there are no recruits remaining. By mimicking the branching structure of the recruitment tree, this method aims to take into account the underlying network structure of the RDS sample, which other existing methods generally fail to achieve. Through simulation studies, Baraff et al. (2016) empirically demonstrated that the tree bootstrap method outperforms existing bootstrap methods, and yield confidence intervals with coverage rates at or above expected nominal values. Moreover, being the only known RDS bootstrap method...
for which resampling relies on the structure of the observed network and not on respondents’ attributes, the tree bootstrap can yield estimates of any number of attributes from a single bootstrap sample, making it more computationally efficient than existing methods. However, recent findings suggest that this method can severely overestimates uncertainty (Gile et al. [2018]) so that the cost of covering at or above the nominal level comes at a significant cost in terms of power.

In this paper, we propose a bootstrap method for quantifying uncertainty in RDS that relies on the structure of the partially observed RDS network. The method is based on resampling recruited individuals and their observed neighbours, i.e. individuals with whom they are directly connected within the RDS tree. The paper is organized as follows. In Section 2, we present RDS as a sampling design with an underlying network structure. We then present the neighbourhood bootstrap method and its corresponding variance estimator. In Section 3, we compare our method to the tree bootstrap method via a simulation study.

2 Methods

2.1 Sampling design and assumptions

Consider a finite population of $N$ individuals. We assume that the individuals in the population are connected with social ties, or through a network.

Assumption 1 (The network). The population network is an undirected, finite and connected graph.

This implies that (i) social connections are reciprocal and (ii) an individual within the network can reach another individual through a finite set of connections. The RDS process takes place within the population network and progresses across individuals’ social connections.

Assumption 2 (No multiple recruitments). Individuals cannot be recruited more than once into the study.

Assumption 2 aligns with actual RDS recruitment processes. In this work, we make all comparisons under this assumption. In contrast, Baraff et al. (2016) assumed that sampling is done with replacement, and subsequently approximated the RDS process as a random walk on a connected, finite social network.
In practice, RDS samples are generated by sampling from simulated population networks using the following assumed procedure. First, we sample \( s \) individuals sequentially as seeds; these are selected without replacement, and with probability proportional to their number of connections (i.e. to their degree). The selection regime is not dependent on the variable of interest. Each selected seed is then given \( c \) coupons to distribute among its neighbours. We assume that coupons are distributed (uniformly) at random among neighbours. Second, successive sampling waves are obtained by sampling up to \( c \) individuals from among the unsampled neighbours of each selected individual. The sampling process is stopped once the sample size reaches \( n \). This sampling procedure will be used in Section 3 to simulate RDS samples from a population network.

### 2.2 The RDS-II estimator

Let \( a \) be an \( N \times N \) adjacency matrix representing social ties in the population, with entries \( a_{ij} = a_{ji} \) such that \( a_{ij} = 1 \) if there is a tie between individuals \( i \) and \( j \), and \( a_{ij} = 0 \) otherwise. Let \( d_i = \sum_{j=1}^{N} a_{ij} \) be the degree of the \( i \)th individual or the number of connections that she/he shares with other individuals in the network. Let \( z \in \{0, 1\} \) be a two-valued variable of interest indicating the presence or absence of an attribute, with mean \( \mu \). Under the assumption that sampling is done with replacement, the RDS sampling process can be approximated as a random walk on the nodes of an undirected graph and RDS samples as independent draws from its stationary distribution. The resulting RDS-II inclusion probability for the \( i \)th individual is defined as

\[
\pi_i = \frac{1}{d_i} \frac{\sum_{i=1}^{n} d_i}{n}.
\]

The VolzHeckathorn estimator for the mean (Volz and Heckathorn 2008) is

\[
\hat{\mu} = \frac{\sum_{i=1}^{n} z_i \pi_i^{-1}}{\sum_{i=1}^{n} \pi_i^{-1}}. \quad (1)
\]

Without the assumption that sampling is done with replacement, estimator (1) is asymptotically unbiased (Volz and Heckathorn 2008). For practical purposes, the VolzHeckathorn estimator will be used as a basis of comparison for bootstrap variance estimators discussed in Section 2.3.
2.3 The neighbourhood bootstrap variance estimator

Baraff et al. (2016) proposed a variance estimator for \( \hat{\mu} \) based on resampling the RDS seed-induced trees. The method is described as follows. First, we sample with (or without) replacement from the seeds. Then, we resample from each of the seeds’ recruits. In the third level of sampling, we resample from the second-level recruits’ recruits. The process continues until there are no more recruits from which to sample. This method aims to take into account the underlying network structure of the RDS sample by mimicking the branching structure of the observed tree.

We propose a bootstrap method based on sequentially resampling recruited individuals and their observed neighbours. First, we uniformly select \( n/c_r \) individuals from the set of recruits, where \( c_r \) is the average number of (observed) connections within the resampled RDS tree. We then include the neighbours of all selected individuals in the bootstrap sample. The network component of the resampled dataset is the subgraph induced by the selected individuals and their neighbours. This is illustrated in Figure 2.3.

![Recruitment tree and sampled neighbourhood subgraph](image)

Figure 1: Illustration of the neighbourhood bootstrap method with seed (dotted circle) labelled M, and subsequent participants N, O, ...V, U. The two arrows that point to the sampled nodes represent two different bootstrap resamples. We sampled \( n/c_r = 8/1.75 \approx 4 \) participants in each bootstrap resample, where \( c_r \) is the average number of connections within the recruitment tree.
The neighbourhood bootstrap method is designed with the hope that the sampling distribution of (1) captures the unobserved network-induced dependence of the RDS sample without mimicking the branching structure of the RDS tree. This is motivated by the fact that the RDS tree is a partially observed network of unknown underlying dependence structure. Moreover, network clustering parameters such as homophily, or the tendency for individuals with similar traits to share social connections, cannot be consistently estimated given the RDS tree alone (Crawford et al. 2017; Shalizi and Rinaldo 2013).

3 Simulations

We conducted a simulation study using a real-world dataset of networked individuals from the Colorado Springs Project 90 study (Klovdahl et al. 1994). From 1988 through 1992, data on 13 demographic characteristics and risk behaviors were collected on injecting drug users and their associates (sexual and needle-sharing partners) and sex workers and their partners (paying and non paying). Respondents listed their personal network of contacts within the community, resulting in the construction of the full network of social relationships. Around 600 respondents contributed in building a social network of 5493 individuals and 21644 connections, distributed among 125 connected clusters. The single largest cluster connected 4430 individuals through 18407 ties. The goal of the study was to investigate the effects of the network structure on the dynamics of HIV transmission in a community of high-risk heterosexuals. This dataset was also used by Baraff et al. (2016) while comparing various bootstrap methods for RDS.

We considered the largest connected cluster of the network as the population network and simulated RDS samples using the same setting as Baraff et al. (2016). First, we randomly sample, without replacement, \( s = 10 \) seeds with probability proportional to degree. Each sampled individual recruited uniformly, without replacement, up to \( c = 3 \) individuals, with probabilities of recruiting 0, 1, 2 and 3 individuals set at \( 1/3, 1/6, 1/6 \) and \( 1/3 \), respectively. The process is stopped once the sample size reaches the desired levels; we set the sample size to \( n = 500, n = 800 \) and \( n = 1000 \). We simulated 1000 RDS samples from the Project 90 data. For each simulated sample, we used the tree bootstrap and the neighbourhood bootstrap methods to compute 95% percentile confidence intervals for the population proportions of the 12 demographic characteristics shown in Table 1. The results are presented in the next section.
3.1 Coverage and mean interval width

Figure 3.1 displays the coverages of the 95% (percentile) confidence intervals obtained through the tree bootstrap and the neighbourhood bootstrap methods (coverages of the 80% confidence intervals are displayed in the Appendix). The results show that the tree bootstrap method yield coverages that are above and closer, in some cases, to the nominal values than the neighbourhood bootstrap method when \( n = 500 \). However, as the sample size increases, the neighbourhood method yields coverages that are closer to the nominal values than the tree method across all attributes.

![Figure 2](image)

Figure 2: Project 90 - Coverage probabilities of the 95% confidence intervals obtained through the neighbourhood bootstrap (Nb) and the tree bootstrap (Tree) methods when sampling is done without replacement.

We further compare the mean widths of the 95% and 80% (see Appendix) confidence intervals obtained through both methods by generating 5000 RDS samples and computing the expected widths of the intervals using the sampling distribution of the RDS-II estimators for all attributes. The results, displayed in Figure 3, show that the average widths of our method were far closer to the expected widths than those from the tree bootstrap.
method across all attributes.

Figure 3: Project 90 - Mean interval width of the 95% confidence intervals obtained through the neighbourhood bootstrap (Nb) and the tree bootstrap (Tree) methods when sampling is done without replacement. The gray line in each subplot represents the expected 95% interval widths across all attributes.

3.2 Biases of the variance estimators

We also investigated the accuracy of the competing bootstrap variance estimators. For each replication of the simulation, we ran 1000 replications of the tree bootstrap and the neighbourhood bootstrap procedures to compute the bootstrap variance for \( \hat{\mu} \), \( v(\hat{\mu}) \). For each attribute, the true variance is the mean squared error (MSE) of the corresponding RDS-II estimator. The relative bias for each variance estimator is computed as follows:

\[
RB[E\{v(\hat{\mu})\}] = \frac{E\{v(\hat{\mu})\} - MSE(\hat{\mu})}{MSE(\hat{\mu})},
\]

where \( MSE(\hat{\mu}) = \frac{\sum_i(\hat{\mu}_i - \mu)^2}{1000} \). Table 1 reports the relatives biases for the tree bootstrap and the neighbourhood bootstrap variance estimators across 12 attributes.
The results show that the tree bootstrap variance estimator overestimates variability (relative bias greater 100%) across all attributes and sample sizes. The neighbourhood bootstrap yields a small to negligible bias for most attributes across all sample sizes; the corresponding bias for the variance estimator is negative for $n = 500$ and $n = 800$, explaining coverage probabilities that are below expected nominal values in most cases.

Table 1: Project 90 - Relative biases (in percentages) for the tree bootstrap (Tree) and the neighbourhood bootstrap (Nb) variance estimators for the RDS-II estimates of proportions for 12 variables.

| Variables         | $n = 500$ | $n = 800$ | $n = 1000$ |
|-------------------|-----------|-----------|------------|
|                   | Tree      | Nb        | Tree       | Nb        | Tree      | Nb        |
| Gender            | 5.95      | 0.08      | 9.76       | 0.04      | 11.58     | 0.02      |
| Sex worker        | 3.57      | -0.37     | 7.56       | -0.24     | 11.29     | -0.15     |
| Pimp              | 3.57      | -0.48     | 9.71       | 0.01      | 16.10     | 0.14      |
| Sex work client   | 3.00      | -0.26     | 6.65       | -0.32     | 10.72     | 0.02      |
| Drug dealer       | 4.78      | -0.26     | 8.80       | -0.13     | 13.76     | -0.06     |
| Drug cook         | 7.02      | 0.14      | 10.54      | 0.19      | 12.35     | 0.28      |
| Thief             | 6.30      | -0.09     | 11.30      | -0.08     | 14.74     | 0.00      |
| Retired           | 5.89      | -0.08     | 9.59       | 0.07      | 12.80     | 0.09      |
| Housewife         | 6.46      | -0.13     | 11.03      | -0.04     | 14.75     | 0.28      |
| Disabled          | 6.55      | -0.14     | 10.25      | -0.03     | 11.85     | 0.08      |
| Unemployed        | 6.19      | 0.12      | 10.55      | 0.04      | 14.06     | 0.36      |
| Homeless          | 6.40      | -0.16     | 10.90      | -0.07     | 16.29     | 0.16      |

These results show that while the tree bootstrap method consistently provides coverage at or above the nominal level, it also provides variance estimators that greatly overestimate variability across all sample sizes, implying a significant cost in terms of power. Our method provides coverage at or slightly below the nominal level with increasing sample size, while yielding far less biased variance estimators than those obtained through the tree bootstrap method.
4 Conclusion

Recent findings suggest that the tree bootstrap method, although practical in capturing the variability in RDS estimates, overestimates uncertainty and yield confidence intervals that are too wide in some cases. We proposed a bootstrap method, based on resampling recruited individuals and their neighbours within the RDS tree, that can reasonably capture the variability in the estimates. Our method produces confidence intervals that are narrower than those of the tree bootstrap, with coverage probabilities that are closer to the nominal values as the sample size increases in all simulations that we considered. Further, we empirically showed that the tree bootstrap greatly overestimates the variance associated with RDS estimates (relative biases greater than 100%) while our bootstrap method yields less biased variance estimators across all sample sizes.

We have implemented the neighbourhood bootstrap in an R package, Neighboot, available on CRAN (Yauck and Moodie, 2020).
Figure 4: Project 90 - Coverage probabilities of the 80% confidence intervals obtained through the neighbourhood bootstrap (Nb) and the tree bootstrap (Tree) methods when sampling is done without replacement. RDS samples were drawn from the Project 90 network.
Figure 5: Project 90 - Mean interval width of the 80% confidence intervals obtained through the neighbourhood bootstrap (Nb) and the tree bootstrap (Tree) methods when sampling is done without replacement. The gray line in each subplot represents the expected 80% interval widths across all attributes.

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