Nonlinear optics in strongly magnetized pair plasma, with applications to FRBs

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ABSTRACT

Intense radiation field can modify plasma properties, the corresponding refractive index, and lead to such nonlinear propagation effects as self-focusing. We estimate the corresponding effects in pair plasma, both in unmagnetized and strongly magnetically dominated case. First, in the unmagnetized pair plasma the ponderomotive force does not lead to charge separation, but to density depletion. Second, for astrophysically relevant plasmas of pulsar magnetospheres, (and possible loci of Fast Radio Bursts), where cyclotron frequency \( \omega_B \) dominates over plasma frequency \( \omega_p \) and the frequency of the electromagnetic wave, \( \omega_B \gg \omega_p, \omega \), we show that (i) there is virtually no nonlinearity due to changing effective mass in the field of the wave; (ii) ponderomotive force is \( F_p^{(B)} = -m_e c^2/(4B_0^2) \nabla E^2 \); it is reduced by a factor \((\omega/\omega_B)^2\) if compared to the unmagnetized case \( (B_0 \) is the external magnetic field and \( E \) is the electric field of the wave); (iii) for radiation beam propagating along constant magnetic field in pair plasma with density \( n_\pm \), the ponderomotive force leads to appearance of circular currents that lead to the decrease of the field within the beam by a factor \( \Delta B/B_0 = 2\pi n_\pm m_e c^2 E^2/B_0^4 \). Applications to the physics of FRBs are discussed; we conclude that for parameters of FRB’s the dominant magnetic field completely suppresses nonlinear radiation effects.

1. Introduction

Neutron stars posses magnetic fields that can approach quantum critical magnetic field \((e.g.\) Thompson & Duncan 1993; Thompson et al. 2002; Kaspi & Beloborodov 2017). In addition, pulsars produce high intensity coherent emission (giant pulses are especially intense Popov et al. 2006) that may modify the properties of the background plasma. The
effects of the back-reaction of the radiation field on the background plasma are becoming even more important with the recent discoveries related to fast radio bursts (Lorimer et al. 2007; Petroff et al. 2019; Cordes & Chatterjee 2019), in particularly identifications of the Repeater FRB121102 (Hessels et al. 2019), FRB180814 (The CHIME/FRB Collaboration et al. 2019b), and lately numerous FRBs detected by CHIME (The CHIME/FRB Collaboration et al. 2019a; Josephy et al. 2019). Magnetospheres of neutron stars are one of the main possible loci of the FRBs Lyutikov et al. (2016); Lyutikov (2019b).

Lyutikov (2019a) discusses new limitations on the plasma parameters that FRBs impose if compared with pulsars. High inferred radiation energy densities at the source renewed interest in non-linear radiative phenomena in plasmas (Machabeli et al. 2019; Gruzinov 2019). Both of the above cited works consider non-magnetized/weakly magnetized plasma. As discussed by Lyutikov (2017, 2019a), the properties of first Repeater FRB121102 requires large magnetic field at the source. For given observed flux and known distance the equipartition magnetic field energy density at the source evaluates

\[ B_{eq} = \sqrt{8\pi \frac{\sqrt{\nu F_{\nu}} D}{c^{3/2}\tau}} = 3 \times 10^8 \tau^{-1/3} \text{ G}. \]  

The resulting cyclotron frequency \( \omega_B \) is much larger than the observational frequency and, mostly likely larger than the local plasma frequency \( \omega_p \). (The inherent assumption is that the duration of the bursts \( \tau \approx 1 \text{ msec} \equiv \tau_{-3} \) is an indication of the emission size.) Specifically, as Lyutikov (2019a) argued, such high magnetic fields are needed to avoid high “normal” (non-coherent) radiative losses.

As we discussed in the present paper, the radiation-plasma interaction in the case of FRBs takes place in an unusual, compared to the more well studied laboratory laser plasma, regime. First, the plasma is likely to be composer of electron-positron pairs. That eliminates/modifies many effects that arise due to different masses of charge carries even in the unmagnetized case. For example, in the electron-ion plasma the ponderomotive force
leads to electrostatic charge separation. In unmagnetized pair plasma it leads to density depression, §2.1, while in the highly magnetized plasma it leads to the modification of the background magnetic field, §2.1) Most importantly, astrophysical plasma are often magnetically dominated, so the cyclotron motion plays the leading role.

Nonlinear plasma effects in this regime remain unexplored. Highlighting these differences is the main goal of the paper. (Current reviews on relativistically strong lasers, e.g., Mourou et al. 2006; Bulanov et al. 2016; Shukla et al. 1986, do not address this specific regime).

In this Letter we consider non-linear propagation effects in pair plasma, both non-magnetic and magnetically dominated plasma with \( \omega_B \gg \omega_p, \omega \).

2. Non-linear effects in pair plasmas

There are two main types of non-linear effects that we will consider. First, strong EM wave can induce large relativistic velocities of the plasma particles; this modifies the effective mass and thus changes the dispersion relation. Second, transverse (with respect to the direction of wave propagation) gradients of wave intensity produce ponderomotive force that modifies the plasma density (or magnetic field! - see §3.2), also changing the dispersion relations. We do not consider plasma effects due to changing intensity of the wave, that lead to longitudinal ponderomotive effects like wake fields.

2.1. Non-linear effects in absence of magnetic field

In the absence of external magnetic field a particle in strong radiation field experiences oscillations (quiver) with dimensionless transverse momentum (Roberts & Buchsbaum 1964;
Akhiezer et al. 1975; Kaw et al. 1973; Kennel & Pellat 1976; Max et al. 1974; Pukhov 2002)

\[
a_0 \equiv \frac{p_\perp}{m_e c} = \frac{e E}{m_e c \omega}
\]

(2)

where \( E \) is the electric field in the wave, \( \omega \) is the frequency of the wave and other notations are standard. When \( a \geq 1 \) the transverse oscillating momentum of a particle in a wave becomes relativistic. This corresponds to wave’s intensity

\[
P = a_0^2 \frac{c E^2}{4 \pi} = a_0^2 \frac{m_e^2 c^3 \omega^2}{4 \pi e^2} = 3 \times 10^{14} \text{ergs}^{-1} \text{cm}^{-2} a_0^2 \nu_9
\]

(3)

where \( \nu_9 \) is the frequency in GHz.

### 2.1.1. Non-linear effects due to changing mass

In unmagnetized plasma the non-linear effects of the strong laser light can be first accommodated into changing effective mass of particles (Akhiezer et al. 1975), so that the refractive index \( n \) for a circularly polarized electromagnetic wave becomes

\[
n^2 = 1 - \frac{\omega_p^2}{\sqrt{1 + a_0^2}}
\]

\[
\omega_p^2 = \frac{4 \pi n_\pm e^2}{m_e}
\]

(4)

where \( n_\pm \) is plasma density.\(^1\)

Consider a beam of radiation propagating in plasma. The jump of the refractive index between the core of the beam and the background due to the changing mass is then

\[
\Delta n \approx \frac{1}{2} \frac{\omega_p^2}{\omega^2} \left( 1 - \frac{1}{\sqrt{1 + a_0^2}} \right) \approx \begin{cases} \frac{1}{4} \frac{\omega_p^2}{\omega^2} a_0^2, & a_0 \ll 1 \\ \frac{1}{2} \frac{\omega_p^2}{\omega^2}, & a_0 \gg 1 \end{cases}
\]

(5)

\(^1\)Note that in an electron-ion plasma with a density \( n_\pm \), a displacement of the electrons with respect to the ions generates the electric field \( E_{\text{disp}} \approx (\omega_p/\omega)^2 E \). This does not happen in pair plasma.
Refractive index is larger in the core of the beam. If the radiation pattern forms a beam with decreasing power away from the central axis (this can occur also due to fluctuations on the beam intensity), parameter $a_0$ as well as $n_\pm$ also decrease away from the center, so that a converging lens is formed.

For $a_0 \ll 1$ we can write the refractive index in the form

$$n = n_0 + n_2 E^2$$

$$n_0 = 1 - \frac{\omega_p^2}{2\omega^2} \approx 1$$

$$n_2 = a_0^2 \frac{\omega_p^2}{4\omega^2} = \pi \frac{n_\pm e^4}{m_e^3 c^2 \omega^4}$$  \hspace{1cm} (6)$$

If the beam diameter is $d$, the beam might be expected to expand by diffraction with an angular divergence of $\theta \sim \lambda/(d)$. But higher refractive index inside the beam may lead to internal reflection if the total power of the beam satisfies (Akhmanov et al. 1968)

$$P > P_c = \pi a_0^2 E^2 c \frac{1.22^2 c}{256 n_2} = \frac{7 \times 10^{-4} m_e^3 c^5 \omega^2}{e^4 n_\pm}$$  \hspace{1cm} (7)$$

This is total power for self-focusing in unmagnetized plasma, taking only modification of mass; weakly non-linear regime is assumed $a_0 \ll 1$.

The corresponding focal length and lensing angle are

$$R_f \approx \frac{d}{2} \sqrt{\frac{n_0}{n_2 E^2}} \approx \frac{d \omega}{a_0 \omega_p}$$

$$\theta_f = \frac{d}{R_f} = 2 \sqrt{\frac{n_2 E^2}{n_0}} \approx \frac{n_2 E^2 \omega_p}{d \omega}$$  \hspace{1cm} (8)$$

In the highly non-linear regime, $a_0 \gg 1$, the refractive index inside the beam becomes $\approx 1$, while outside it is still $\approx 1 - \frac{\omega_p^2}{2\omega^2}$. Equating the diffraction angle $\sim 1.22\kappa_{GJ}/(2a)$ to the critical angle of internal total reflection gives a condition on the width of the self-collimating beam

$$d \leq 7.6 \frac{c \omega}{\omega_p^2} = 0.6 \frac{c m_e \omega}{e^4 n_\pm}$$  \hspace{1cm} (9)$$
2.1.2. Non-linear effects in unmagnetized pair plasma due to ponderomotive force

In addition to changing mass the plasma particles experiences ponderomotive force due to the transverse gradient of the intensity of the wave. Separating particle motion into fast oscillations along, e.g., $x$ direction with coordinate-dependent amplitude,

$$\ddot{x} = g(x) \cos \omega t$$

$$g = \frac{eE}{m_e}$$

and averaging over fast oscillation, the slow coordinate $x_0$ evolves according to

$$\ddot{x}_0 = -\frac{1}{\omega^2} \frac{d}{dx} (g(x)^2) = -\frac{e^2}{4m_e^2\omega^2} \nabla E^2$$

$$F_p = -\frac{e^2}{4m_e\omega^2} \nabla E^2$$

Relations (11) give the drift motion of a charged particle under the effect of a non-uniform oscillating field and the corresponding ponderomotive force $F_p$.

Typically, in electron-ion laboratory plasmas the ponderomotive force on electrons is balanced by electrostatic forces, giving rise to the ponderomotive electrostatic potential

$$\Phi = \frac{m_e}{4\omega^2} g^2 = \frac{e^2E^2}{4\omega^2}$$

As the electric fields are screened on Debye/skin depth, this gives the relativistic critical power $W_c$

$$W_c \sim P(a = 1) \left( \frac{c}{\omega_p} \right)^2 = \frac{m_e^2c^5\omega^2}{e^2\omega_p^2}$$

In unmagnetized pair plasma the situation is very different: the ponderomotive force (11) acts both on electrons and positrons, creating a density depression within the beam. This will be balanced by pressure gradients. In plasma of temperature $T$ the relative density depression within the beam is then

$$\frac{\delta \rho}{\rho} = \frac{e^2E^2}{m_e T \omega^2} = a_0^2 \frac{m_e c^2}{T} = \frac{a_0^2}{\theta_T}$$
where $\theta_T = T/m_e c^2$. Thus we expect that in a sufficiently cold pair plasma, with $\theta_T \leq 1$, a strong radiation beam creates a density cavity for $a_0 \sim \theta_T^{1/2}$.

The density depression (14) will create a variation of the refractive index of the order

$$
\Delta n^{(p)} \approx \frac{1}{2} \frac{\omega_p^2 \delta \rho}{\omega^2} \rho \approx \frac{a_0^2 \omega_p^2}{\theta_T \omega^2}
$$

(15)

where superscript $(p)$ stresses this relation apply to the effects produced by the ponderomotive force.

The refractive index due to the ponderomotive force is higher in the core of the beam: effects of increasing effective mass and density depletion due to the ponderomotive force amplify each other. Thus, high intensity radiation field in unmagnetized pair plasma creates nonlinear lens that focus the light rays, and lead to further amplification of the energy density of radiation.

Using Eq. (7) for the critical power in terms of $n_2$, the expression (8) for focal length, and Eq. (15) for the change of the refraction index, we find in this case

$$
n_2^{(p)} = \frac{\Delta n}{E^2} = \pi \frac{n e^4}{m_e^2 \omega^4 T}$$

$$
P_c^{(p)} = 7 \times 10^{-2} \frac{m_e^3 c^5 \omega^2}{e^4} n \theta_T = 3 \times 10^8 \text{ergs}^{-1} \nu_0^2 \lambda_0^{-1} b_q^{-1} P \theta_T$$

$$
R_f^{(p)} = \frac{m_e^{1/2} \omega}{2 \pi^{1/2} c n_0^{1/2} a_0^{1/2}} \frac{\theta_T^{1/2}}{d}$$

$$
\theta_t^{(p)} = \frac{2 \pi^{1/2} c n_0^{1/2}}{m_e^{1/2} \omega} \frac{a_0}{\theta_T^{1/2}}
$$

(16)
3. Nonlinearity in magnetically-dominant plasma

3.1. No nonlinear effects due to quiver momentum

If there is external magnetic field $B_0$, such that $\omega_B \gg \omega$, the plasma dynamics changes dramatically. Most importantly, the leading nonlinear effects in the unmagnetized plasmas - induced by the variation of effective mass - disappears.

For $\omega_B \gg \omega$ a particle in a wave experiences linear acceleration not for a fraction of wave period, but for a fraction of the cyclotron gyration. The magnetic nonlinearity parameter is then

$$a_{(B)}^0 \equiv \frac{p_\perp}{m_e c} = \frac{eE}{m_e c \omega_B} = a_0 \frac{\omega}{\omega_B} = \frac{eE}{m_e c \omega_B} = a_0 \omega_B,$$

the ratio of the electric field in the wave to the external magnetic field.

For a wave with energy flux $P$, the ratio of the electric field in the wave to the external field is

$$\frac{E}{B_0} = 2\sqrt{\frac{\pi}{\sqrt{c}B_0}}$$

It becomes unity for

$$P = \frac{B_0^2 c}{4\pi} = 4 \times 10^{36} b_q^2 \text{erg cm}^{-2} \text{ s}^{-1}$$

where we normalized the magnetic field to the quantum critical magnetic field, $B_0 = b_q B_Q$, $B_Q = m_e^2 c^3 / (e\hbar)$. This is unrealistically high energy flux, not likely to be reached: the electric field in the wave is much smaller than external magnetic field: $a_{(B)}^0 \ll 1$ (this corrects a typo in Lyutikov 2017, Eq. (5)).

Thus, instead of large amplitude oscillations a particle experiences an $E \times B$ drift with non-relativistic velocity

$$\frac{v_\perp}{c} = a_{(B)}^0 = \frac{E}{B_0} \ll 1$$
The magnetic non-linearity is always small, $a_0^{(B)} \ll 1$, quiver velocity is non-relativistic, and the mass modification in the regime $\omega_B \gg \omega$ is negligible.

### 3.2. Ponderomotive force across magnetic field in magnetically-dominant plasma

In pulsars, and presumably FRBs, emission is likely to be produced by relativistic particles propagating approximately along the local magnetic field (Radhakrishnan & Cooke 1969; Sturrock 1971; Lyutikov et al. 1999; Melrose 2000). Let’s assume that the circularly polarized radiation propagates exactly along the external magnetic field. Typically, in pulsar magnetospheres the cyclotron frequency is much higher than the plasma frequency and the radiation frequency (in the plasma frame in the case of relativistic bulk motion).

As we demonstrated in §3.1, in the case $\omega_B \gg \omega$ instead of large amplitude oscillations with $p_\perp \sim am_e c$ particles experience ExB drift with velocity $(E/B_0)c$, where $B_0$ is the external magnetic field. Relations for the ponderomotive force (10) and (11) are then modified

\begin{align*}
\ddot{x}^{(B)} &= \omega E/B_0 c \\
g^{(B)} &= \omega \frac{E}{B_0} \frac{c}{c} \\
F_p^{(B)} &= -\frac{m_e c^2}{4B_0^2} \nabla E^2 \tag{21}
\end{align*}

where superscript $(B)$ indicates that estimate is for the case of strong magnetic field. The expression for $F_p^{(B)}$ is the ponderomotive force in the magnetically dominant plasma.

The ratio of the ponderomotive forces in magnetically dominated plasma and plasma without magnetic field is

\[ \frac{F_p^B}{F_p} = \left( \frac{\omega}{\omega_B} \right)^2 \ll 1 \tag{22} \]
Thus, the ponderomotive force is reduced by a factor $(\omega/\omega_B)^2$ if compared to the unmagnetized case.

Most importantly, the effects of the ponderomotive force on the background particles is qualitatively different in the magnetically dominated case as we demonstrate next. If the radiation beam is propagating along magnetic field and its intensity varies in a perpendicular direction, Eq. (21) gives a force on a particle in a direction perpendicular to magnetic field. As a result, the particle will experience a drift with velocity

$$u_d = \frac{e}{c} \mathbf{F}_p(B) \times \mathbf{B}_0$$

(23)

The drift is in the azimuthal direction (with respect to the background magnetic field), see Fig. 1

Charges of opposite sign rotate in the opposite direction. In a charge-neutral pair plasma with densities $n_\pm$ that will induce a current

$$j_\phi = 2ev_d n_\pm$$

(24)

The magnetic field within the beam will be modified by

$$\Delta B = -2\pi n_\pm m_e c^2 E^2 / B_0^3$$

(25)

(magnetic field is smaller in the core.)

We can then introduce a magnetic non-linear intensity parameter $\eta_0^{(B)}$:

$$\eta_0^{(B)} = \frac{\Delta B}{B_0} = 2\pi n_\pm m_e c^2 E^2 / B_0^3 = \frac{\omega_p^2}{2 \omega_B^2} a_0^{(B),2}$$

(26)

Modification of the field becomes of the order of unity at radiative flux

$$P^{(B)} = \eta_0^{(B)} B_0^4 / 8\pi^2 m_e c n_\pm$$

(27)

dimension of $P^{(B)}$ is erg cm$^{-2}$ s$^{-1}$. 
Fig. 1.— Left Panel: view from the side. Intense radiation beam is propagating along magnetic field. Gradients of field intensity induce ponderomotive force $F_p^{(B)}$. In high external magnetic field $B_0$ the ponderomotive force leads to azimuthal drift of charged particles $\pm v_d$ that creates toroidal current and decrease the background magnetic field. Right panel: view along the direction of the beam. In the core the radiation energy density is high, it induces a ponderomotive force directed away from the center. In the external magnetic field (chosen to be out of the plane) the ponderomotive force leads to charge-dependent drift of particles, generation of the toroidal current (in the clockwise direction). The induced current produces a field counter-aligned with the external field.

Modification of the magnetic field (25) will lead to the changes of the refractive index within a beam (as we argued above there is no contribution from changing oscillatory motion of bulk charges). In the linear approximation, in the limit $\omega_B \gg \omega_p, \omega$, the wave dispersion reads (Arons & Barnard 1986; Kazbegi et al. 1991; Lyutikov 1998, 1999)

$$n^{(B),2} = 1 + \left(\frac{\omega_p}{\omega_B}\right)^2$$

(28)

(for parallel propagation; for simplicity we assume cold plasma in its rest frame.) Expanding
in the wave intensity, we find

\[ n^{(B)} \approx 1 + \frac{1}{2} \left( \frac{\omega_p}{\omega_B} \right)^2 + \frac{\omega_p^4}{2\omega_B^2} a_0^2 = \]

\[ 1 + \frac{1}{2} \left( \frac{\omega_p}{\omega_B} \right)^2 + \frac{\omega_p^4}{2\omega_B^2} a_0^{(B),2} = \]

\[ 1 + \frac{1}{2} \left( \frac{\omega_p}{\omega_B} \right)^2 + \frac{e^2}{2m^2 c^2} \frac{\omega_p^4}{\omega_B^6} E^2 \]  

(29)

where \( \omega_B \) is defined with the initial background field. The plasma lens has larger refractive index in the core and thus is convergent. (The decrease in the magnetic field is due to newly generated internal currents, not expansion, hence the density remains constant.)

Critical power for self-collimation is then

\[ P^{(B)}_c = 3 \times 10^{-3} \frac{B_0^6}{m^2 c n_+ \omega^2}, \]  

(30)

and the focal distance and lensing angle

\[ R^{(B)}_f = \frac{\omega_B^3}{\omega_p^2 \sqrt{2} a_0} = \frac{\omega_p^2}{\omega_B^2 \sqrt{2} a_0^{(B)}} \]

\[ \theta^{(B)}_f = \sqrt{2} a_0 \frac{\omega_p^2}{\omega_B^3} = \sqrt{2} a_0^{(B)} \frac{\omega_p^2}{\omega_B^3} \]  

(31)

### 3.3. Implications for FRBs

Let us use the properties of first Repeater for the estimates of the relevant parameters (Lyutikov 2017): flux \( F_\nu \approx 1 \) Jy, frequency \( \nu = 1 \) GHz, distance to the FRB \( d_{FRB} \approx \) Gpc, duration \( \tau = 1 \) msec. The electric field of the wave at the source of size \( c \tau \) and the beam power are then

\[ E = 2\sqrt{\pi} \frac{d_{FRB}\nu F_\nu}{\nu d_{FRB}^2 c^{3/2} \tau} \approx 2 \times 10^8 \text{ (in cgs units)} \]

\[ P = \frac{\nu F_\nu d_{FRB}^2}{c^{3} \tau^2} = 10^{26} \text{ erg s}^{-1} \text{ cm}^{-2} \]  

(32)
(the estimate of the electric field is also the value of the equipartition magnetic field, Lyutikov 2017) and Eq. (1). The non-linearity parameters then evaluate to

\[
a_{0}^{(B)} = 2\sqrt{\pi} d_{\text{FRB}} \sqrt{\nu F_{\nu}} / c^{3/2} \tau B_{0} = 4 \times 10^{-6} b_{q}^{-1}
\]

\[
a_{0} = 5 \times 10^{5}
\]

(33)

Thus, the nonlinear effects are suppressed by the magnetic field by some ten orders of magnitude (for quantum field \( b_{q} = 1 \)).

To proceed further we need to estimate the plasma density. As the sources of FRBs remain mysterious, below we scale density according to two somewhat oppositely extreme limits: (i) to the Goldreich & Julian (1969) density (with some multiplicity \( \kappa_{GJ} \)); (ii) the quantum density of inverse Compton length cubed, \( n_{\pm} = \kappa_{C} \lambda_{C}^{-3} \), \( \lambda_{C} = \hbar / (m_{e} c) \). These two limits exemplify the clean/light magnetospheres of pulsars, and heavy pair-loaded magnetospheres one expects in magnetar flares.

**Pulsar-like scaling.** Using Goldreich & Julian (1969) scaling for plasma density,

\[
n_{\pm} = \kappa_{GJ} \frac{\Omega B}{2 \pi e c}
\]

(34)

where \( \kappa_{GJ} \) is plasma multiplicity and \( \Omega \) is the spin frequency of a pulsar, we find

\[
P_{c} = 3 \times 10^{3} \kappa_{GJ,6}^{-1} b_{q}^{-1} P_{-3}^{1} \text{ erg s}^{-1} \text{ cm}^{-2}
\]

\[
P_{c}^{(p)} = 3 \times 10^{5} \kappa_{GJ,6}^{-1} b_{q}^{-1} P_{-3}^{1} \text{ erg s}^{-1} \text{ cm}^{-2}
\]

\[
P_{c}^{(B)} = 2 \times 10^{60} b_{q}^{4} P_{-3}^{2} \kappa_{GJ,6}^{-2} \text{ erg s}^{-1} \text{ cm}^{-2}
\]

\[
\theta_{f}^{(B)} = 10^{-16} b_{q}^{-2} P_{-3}^{1} \kappa_{GJ,6}^{-1}
\]

(35)

**Magnetar-like scaling.** Scaling \( n = \kappa_{C} \lambda_{C}^{-3} \), we find, using and (7), (30)

\[
P_{c} = 5.9 \times 10^{-7} \kappa_{C}^{-1} \text{ erg s}^{-1} \text{ cm}^{-2}
\]
\[ P_c^{(p)} = 6 \times 10^{-5} \kappa_C^{-1} \theta_T \text{ erg s}^{-1} \text{ cm}^{-2} \]
\[ P_c^{(B)} = 7 \times 10^{16} \theta_q^{-2} \kappa_C^{-2} \text{ erg s}^{-1} \text{ cm}^{-2} \]
\[ \theta_f^{(B)} = 5 \times 10^{-7} \theta_q^{-3} \kappa_C \]

(36)

where the magnetic field was scaled to the critical quantum field.

The above estimates cover a wide range of densities and magnetic fields. Yet there is a clear conclusion: the nonlinear effects are highly suppressed in the magnetically-dominant plasma, by some fifty orders of magnitude both for magnetar-like and pulsar-like scaling.

4. Conclusion

In this paper we give estimates of the nonlinear optical effects in strongly magnetized pair plasma. Two contributing effects are taken into account: effects of the relativistically strong wave on the effective mass of plasma particles and the ponderomotive effects due to the transverse (with respect to the direction of propagation) gradient of the wave’s intensity. In the unmagnetized case, both the relativistic decrease of the effective particle mass, and the ponderomotive effects lead to the formation of convergent lenses that tend to focus the radiation, further amplifying the non-linearity. This can be considered as an instability of the radiation front to filamentation. In pair plasma the ponderomotive force leads to density depletion (as opposed to formation of electrostatic potential).

In magnetically dominated plasma, e.g. in the case of neutron star’s magnetospheres and presumably FRB’s loci, where \( \omega_B \gg \omega, \omega_p \), the dynamics is very different. We find that in this regime: (i) the relativistic effective mass-changing effects on the wave nonlinearity is completely negligible; (ii) the ponderomotive force is suppressed by a factor \( (\omega/\omega_B)^2 \ll 1 \) if compared with unmagnetized regime; (iii) ponderomotive force induces toroidal currents that modify (decrease) the background magnetic field; the resulting lens is also converging.
Overall, the plasma non-linearity is highly suppressed in the magnetized case. As a result, effects like self-collimation and plasma filamentation are not likely to play an important role in pulsar magnetospheres, and FRBs.

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