Elementary proof of the Syracuse’ conjecture
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Syracuse’ conjecture (Collatz’ conjecture)
Algorithm of Collatz (C):
Let x a positive integer number.
1 - if x is even then x := x/2
2 - if x is odd then x := x * 3 + 1
We repeat 1 - 2 until obtain a cycle (is only cycle?) or x tends to infinity.
The symbol := means : assign value on right to variable on left.
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Representation of numbers :
Let V a variable which, added to variable x, gives the successor x + V.
The variable V is a variable of adjustment.
Variables x and V are written in the form :
x := a(y) with a := 2^α and α is integer >= 0, y is an odd variable.
V := b(z) où b = 2^β and β is integer >= 0, z is an odd variable.
x + V := a(y) + b(z) ; a, b, y, z are positive integer variables.

Application of the algorithm of Collatz :
The algorithm is applied to the odd part y of x := a(y) giving a sequence of
Syracuse C(x) = 1 and the odd part z of V := b(z) is multiplied by 3 plus an
adjustment.
The aim is to prove if C(x) = 1 then C(x + V) = 1.
In operation 3* + 1 , x := a(3*y+1) = a’(y’), x is increased by (a - 1) to subtract
from V and we have for V in x + V : V := b(3*z) - (a-1) = b'(z').
We have the equality
a(3*y+1) + b(3*z) - (a-1) = a(3*y) +1 + b(3*z) = 3 * (a(y) + b(z)) + 1
according to the rule 2 of the algorithm.
a’ et b’ are power of 2 which can be equal to unity, y’ and z’ are odd numbers.

In the line a'(y') + b'(z'), a’ and b’ are divided by gcd(a’,b’) according to the rule1
of the algorithm.
If gcd(a’,b’) = 1, the division by 2 is deferred.
Let's give an example of calculation:
As initial data of the algorithm:
x = 13, V = 2 et x + 2 = 15 is the successor number of x at the first step.

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\begin{align*}
x & := & V & := & x + V & := \\
1(13) & + 2(1) & & = 15 & = 1(15) \\
1(40) & + 2(3) & & = 46 & = 2(23) \\
8(5) & + 2(3) & & = 46 & = 2(23) \\
4(5) & + 1(3) & & = 23 & = 1(23) \\
4(16) & + 1(9) - 3 & & = 70 & = 2(35) \\
64(1) & + 2(3) & & = 70 & = 2(35) \\
32(1) & + 1(3) & & = 35 & = 1(35) \\
32(4) & + 1(9) - 31 & & = 106 & = 2(53) \\
128(1) & + 2(-11) & & = 106 & = 2(53) \\
64(1) & + 1(-11) & & = 53 & = 1(53) \\
64(4) & + 1(-33) - 63 & & = 160 & = 32(5) \\
256(1) & + 32(-3) & & = 160 & = 32(5) \\
8(1) & + 1(-3) & & = 5 & = 1(5) \\
8(4) & + 1(-9) - 7 & & = 16 & = 16(1) \\
32(1) & + 16(-1) & & = 16 & = 16(1) \\
2(1) & + 1(-1) & & = 1 & = 1(1)
\end{align*}
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When x is multiplied by 3 then + 1, V is multiplied by 3.
When x is divided by 2, V is divided by 2.
When \(x = a(3*y+1)\), x is increased by (a - 1), V is decreased by (a - 1).
We deduce that V is always less than x for \(x > 1\).
In the application of the algorithm of Collatz:
\(x > 0\ , \ x + V > 0\ et \ V < x\).
As x gives a sequence of Syracuse, when \(C(x) \in [4, 2, 1]\) (« trivial cycle»), it implies \(V < 4\) and, as \(x + V > 0\, \, V > -4\) and \(C(x + V) \in [7, 6, 5, 4, 2, 1]\), and ultimately \(C(x + V) \in [4, 2, 1]\).
So by recurrence, every positive integer gives a sequence of Syracuse.