Lifetime of quasiparticles in hot gauge theories

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Abstract

The perturbative calculation of the lifetime of charged excitations in ultrarelativistic plasmas is plagued with infrared divergences which are not eliminated by the screening corrections. The physical processes responsible for these divergences are the collisions involving the exchange of longwavelength, quasistatic, magnetic gluons (or photons), which are not screened by plasma effects. In QED, the leading divergences can be resummed in a non-perturbative treatment based on a generalization of the Bloch-Nordsieck model at finite temperature. The resulting expression of the fermion propagator is free of infrared problems, and exhibits a non-exponential damping at large times: $S_R(t) \sim \exp\{-\alpha T \ln \omega_p t\}$, where $\omega_p = eT/3$ is the plasma frequency and $\alpha = e^2/4\pi$.
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1. INTRODUCTION

The study of the elementary excitations of ultrarelativistic plasmas, such as the quark-gluon plasma, has received much attention in the recent past. (See [1,2] for recent reviews and more references.) The physical picture which emerges is that of a system with two types of degrees of freedom: i) the plasma quasiparticles, whose energy is of the order of the temperature $T$; ii) the collective excitations, whose typical energy is $gT$, where $g$ is the gauge coupling, assumed to be small: $g \ll 1$ (in QED, $g = e$ is the electric charge).

For this picture to make sense, however, it is important that the lifetime of the excitations be large compared to the typical period of the modes.

Information about the lifetime is obtained from the retarded propagator. A usual expectation is that $S_R(t, p)$ decays exponentially in time, $S_R(t, p) \sim e^{-iE(p)t}e^{-\gamma(p)t}$, where $E(p) \sim T$ or $gT$ is the average energy of the excitation, and $\gamma(p)$ is the damping rate. Therefore, $|S_R(t, p)|^2 \sim e^{-\Gamma(p)t}$ with $\Gamma(p) = 2\gamma(p)$, which identifies the lifetime of the single particle excitation as $\tau(p) = 1/\Gamma(p)$. The exponential decay may then be associated to a pole of the Fourier transform $S_R(\omega, p)$, located at $\omega = E - i\gamma$. The quasiparticles are well defined if their lifetime $\tau$ is much larger than the period $\sim 1/E$ of the field oscillations, that is, if the damping rate $\gamma$ is small compared to the energy $E$. If this is the case, the respective damping rates can be computed from the imaginary part of the on-shell self-energy, $\Sigma(\omega = E(p), p)$.

Previous calculations [3] suggest that $\gamma \sim g^2 T$ for both the single-particle and the
collective excitations. In the weak coupling regime \( g \ll 1 \), this is indeed small compared to the corresponding energies (of order \( T \) and \( gT \), respectively), suggesting that the quasiparticles are well defined, and the collective modes are weakly damped. However, the computation of \( \gamma \) in perturbation theory is plagued with infrared (IR) divergences, which casts doubt on the validity of these statements [3—9].

The first attempts to calculate the damping rates were made in the early 80’s. It was then found that, to one-loop order, the damping rate of the soft collective excitations in the hot QCD plasma was gauge-dependent, and could turn out negative in some gauges (see Ref. [11] for a survey of this problem). Decisive progress on this problem was made by Braaten and Pisarski [3] who identified the resummation needed to obtain the screening corrections in a gauge-invariant way (the resummation of the so called “hard thermal loops” (HTL)). Such screening corrections are sufficient to make IR-finite the transport cross-sections [11,12], and also the damping rates of excitations with zero momentum [3,13].

At the same time, however, it has been remarked [3] that the HTL resummation is not sufficient to render finite the damping rates of excitations with non vanishing momenta. The remaining infrared divergences are due to collisions involving the exchange of longwavelength, quasistatic, magnetic photons (or gluons), which are not screened in the hard thermal loop approximation. Such divergences affect the computation of the damping rates of charged excitations (fermions and gluons), in both Abelian and non-Abelian gauge theories. Furthermore, the problem appears for both soft (\( p \sim gT \)) and hard (\( p \sim T \)) quasiparticles. In QCD this problem is generally avoided by the ad-hoc introduction of an IR cut-off (“magnetic screening mass”) \( \sim g^2 T \), which is expected to appear dynamically from gluon self-interactions [2]. In QED, on the other hand, it is known that no magnetic screening can occur [14], so that the solution of the problem must lie somewhere else.

In order to make the damping rate \( \gamma \) finite, Lebedev and Smilga proposed a self-consistent computation of the damping rate, by including \( \gamma \) also in internal propagators [4]. However, the resulting self-energy is not analytic near the complex mass-shell, and the logarithmic divergence actually reappears when the discontinuity of the self-energy is evaluated at \( \omega = E - i\gamma \) [4]. More thorough investigations along the same lines led to the conclusion that the full propagator has actually no quasiparticle pole in the complex energy plane [5]. These analyses left unanswered, however, the question of the large time behavior of the retarded propagator.

As we have shown recently for the case of QED [15], the answer to this question requires a non perturbative treatment, since infrared divergences occur in all orders of perturbation theory. We have identified the leading IR divergences in all orders, and solved exactly an effective theory which reproduces all these leading divergences. The resulting fermion propagator \( S_R(\omega) \) turns out to be analytic in the vicinity of the mass-shell. Moreover, for large times \( t \gg 1/gT \), the Fourier transform \( S_R(t) \) does not show the usual exponential decay alluded to before, but the more complicated behavior \( S_R(t) \sim e^{-\omega p t} \exp\{-\alpha T t \ln \omega_p t\} \), where \( \alpha = g^2/4\pi \) and \( \omega_p \sim gT \) is the plasma frequency. This corresponds to a typical lifetime \( \tau^{-1} \sim g^2 T \ln(1/g) \), which is similar to the one provided by the perturbation theory with an IR cut-off of the order \( g^2 T \).
2. THE INFRARED PROBLEM

Let me briefly recall how the infrared problem occurs in the perturbative calculation of the damping rate $\gamma$. For simplicity, I consider an Abelian plasma, as described by QED, and compute the damping rate of a hard electron, with momentum $p \sim T$ and energy $E(p) = p$.

To leading order in $g$, and after the resummation of the screening corrections, $\gamma$ is obtained from the imaginary part of the effective one-loop self-energy in Fig. 1. The blob on the photon line in this figure denotes the effective photon propagator in the HTL approximation, commonly denoted as $^* D_{\mu\nu}(q)$. In the Coulomb gauge, the only non-trivial components of $^* D_{\mu\nu}(q)$ are the electric (or longitudinal) one $^* D_{00}(q) \equiv ^* \Delta_l(q)$, and the magnetic (or transverse) one $^* D_{ij}(q) = (\delta_{ij} - \hat{q}_i \hat{q}_j) ^* \Delta_t(q)$, with

$$^* \Delta_l(q_0, q) = \frac{-1}{q^2 - \Pi_l(q_0, q)}, \quad ^* \Delta_t(q_0, q) = \frac{-1}{q_0^2 - q^2 - \Pi_t(q_0, q)},$$

where $\Pi_l$ and $\Pi_t$ are the respective pieces of the photon polarisation tensor [1,2]. Physically, the on-shell discontinuity of the diagram in Fig. 1 accounts for the scattering of the incoming electron (with four momentum $p^\mu = (E(p), \mathbf{p})$) off a thermal fermion (electron or positron), as mediated by a soft, dressed, virtual photon. (See Fig. 2.)

The interaction rate corresponding to Figs. 1 or 2 is dominated by soft momentum transfers $q \ll T$. It is easily computed as

$$\gamma \simeq \frac{g^4 T^3}{12} \int_0^{q^*} dq \int_{-q}^q \frac{dq_0}{2\pi} \left\{ |^* \Delta_l(q_0, q)|^2 + \frac{1}{2} \left( 1 - \frac{q_0^2}{q^2} \right)^2 |^* \Delta_t(q_0, q)|^2 \right\},$$

where the upper cut-off $q^*$ distinguishes between soft and hard momenta: $gT \ll q^* \ll T$. Since the $q$-integral is dominated by IR momenta, its leading order value is actually independent of $q^*$.

The two terms within the parentheses in eq. (3) correspond to the exchange of an electric and of a magnetic photon respectively. For a bare (i.e., unscreened) photon, we have $|\Delta_l(q_0, q)|^2 = 1/q^4$ and $|\Delta_t(q_0, q)|^2 = 1/(q_0^2 - q^2)^2$, so that the $q$-integral in eq. (2) shows a quadratic IR divergence:

$$\gamma \simeq \frac{g^4 T^3}{8\pi} \int_0^{q^*} \frac{dq}{q^3}.$$
This divergence reflects the singular behaviour of the Rutherford cross-section for forward scattering. As well known, however, the quadratic divergence is removed by the screening corrections contained in the photon polarization tensor. We shall see below that the leading IR contribution comes from the domain $q_0 \ll q \ll T$, where we can use the approximate expressions \((1,2)\) (with $\omega_p = eT/3$)

$$
\Pi_l(q_0 \ll q) \approx 3\omega_p^2 \equiv m_D^2,
\Pi_t(q_0 \ll q) \approx -i\frac{3\pi}{4}\omega_p^2 \frac{q_0}{q}.
$$

(4)

We see that screening occurs in different ways in the electric and the magnetic sectors. In the electric sector, the familiar static Debye screening provides an IR cut-off $m_D \sim gT$. Accordingly, the electric contribution to $\gamma$ is finite, and of the order $\gamma_l \sim g^4 T^3/m_D^2 \sim g^2 T$.

Its exact value can be computed by numerical integration \([7]\). In the magnetic sector, screening occurs only for nonzero frequency $q_0$ \([11]\). This comes from the imaginary part of the polarisation tensor, and can be associated to the Landau damping \([16]\) of space-like photons ($q_0^2 < q^2$). This “dynamical screening” is not sufficient to completely remove the IR divergence of $\gamma_t$, which is only reduced to a logarithmic one:

$$
\gamma_t \approx \frac{g^4 T^3}{24} \int_0^{\omega_p} dq \int_{-q}^{q} dq_0 \frac{1}{q^4 + (3\pi\omega_p^2 q_0/4q)^2}
\approx \frac{g^2 T}{4\pi} \int_\mu^{\omega_p} dq \frac{q}{q} = \frac{g^2 T}{4\pi} \ln \frac{\omega_p}{\mu}.
$$

(5)

The unphysical lower cut-off $\mu$ has been introduced by hand, in order to regularize the IR divergence of the integral over $q$. The upper cut-off $\omega_p \sim gT$ accounts approximately for the terms which have been neglected when going from the first to the second line of eq. \((5)\). As long as we are interested only in the coefficient of the logarithm, the precise value of this cut-off is unimportant. The scale $\omega_p$ however is uniquely determined by the physical process responsible for the existence of space-like photons, i.e., the Landau damping. As we shall see later, this is the scale which fixes the long time behavior of the retarded propagator.

The remaining IR divergence in eq. \((5)\) is due to collisions involving the exchange of very soft ($q \to 0$), \textit{quasistatic} ($q_0 \to 0$) magnetic photons, which are not screened by
plasma effects. To see that, note that the IR contribution to $\gamma_t$ comes from momenta $q \ll gT$, where $|\ast \Delta_t(q_0, q)|^2$ is almost a delta function of $q_0$:

$$
|\ast \Delta_t(q_0, q)|^2 \simeq \frac{1}{q^4 + (3\pi \omega_n^2 q_0/4q)^2} \rightarrow_{q \rightarrow 0} \frac{4}{3q\omega_n^2} \delta(q_0).
$$

This is so because, as $q_0 \rightarrow 0$, the imaginary part of the polarisation tensor vanishes linearly (see the second equation (4)), a property which can be related to the behaviour of the phase space for the Landau damping processes. Since energy conservation requires $q_0 = q \cos \theta$, where $\theta$ is the angle between the momentum of the virtual photon ($\mathbf{q}$) and that of the incoming fermion ($\mathbf{p}$), the magnetic photons which are responsible for the singularity are emitted, or absorbed, at nearly 90 degrees.

3. A NON PERTURBATIVE CALCULATION

The IR divergence of the leading order calculation invites to a more thorough investigation of the higher orders contributions to $\gamma$. Such an analysis [15] reveals strong, power-like, infrared divergences, which signal the breakdown of the perturbation theory. (A similar breakdown occurs in the computation of the corrections to the non-Abelian Debye mass [17].) To a given order in the loop expansion, the most singular contributions to $\gamma$ arise from self-energy diagrams of the kind illustrated in Fig. 3. These diagrams have no internal fermion loops (quenched QED), and all the internal photons are of the magnetic type (the electric photons, being screened, give no IR divergences). Furthermore, the leading divergences arise, in all orders, from the same kinematical regime as in the one loop calculation, namely from the regime where the internal photons are soft ($q \rightarrow 0$) and quasistatic ($q_0 \rightarrow 0$). This is so because of the specific IR behaviour of the magnetic photon propagator, as illustrated in eq. (3). Physically, these divergences come from multiple magnetic collisions.

This peculiar kinematical regime can be conveniently exploited in the imaginary time formalism (see, e.g., [4]), where the internal photon lines carry only discrete (and purely imaginary) energies, of the form $q_0 = i\omega_n = i2\pi nT$, with integer $n$ (the so-called Matsubara frequencies). The non-static modes with $n \neq 0$ are well separated from the static one $q_0 = 0$ by a gap of order $T$. I have argued before that the leading IR divergences come from the kinematical limit $q_0 \rightarrow 0$. Correspondingly, it can be verified [15] that,
in the Matsubara formalism, all these divergences are concentrated in diagrams in which the photon lines are static, i.e., they carry zero Matsubara frequency. (To one loop order, this has been also notified in Refs. \[5\].) In what follows, we shall restrict ourselves to these diagrams, and try to compute their contribution to the fermion propagator near the mass-shell, in a non perturbative way. Note that, for these diagrams, all the loop integrations are three-dimensional (they run over the three-momenta of the internal photons), so that the associated IR divergences are those of a three-dimensional gauge theory. This clearly emphasizes the non perturbative character of the leading IR structure.

As we shall see now, this “dimensional reduction” brings in simplifications which allows one to arrive at an explicit solution of the problem \[15\]. The point is that three-dimensional quenched QED can be “exactly” solved in the Bloch-Nordsieck approximation \[18\], which is the relevant approximation for the infrared structure of interest. Namely, since the incoming fermion is interacting only with very soft \((q \to 0)\) static \((q_0 = 0)\) magnetic photons, its trajectory is not significantly deviated by the successive collisions, and its spin state does not change. This is to say, we can ignore the spin degrees of freedom, which play no dynamical role, and we can assume the fermion to move along a straightline trajectory with constant velocity \(v\) (for the ultrarelativistic hard fermion, \(|v| = 1\); more generally, for the soft excitations, \(v(p) \equiv \partial E(p)/\partial p = v(p) \hat{p}\) is the corresponding group velocity, with \(|v(p)| < 1\)). Under these assumptions, the fermion propagator can be easily computed as \[15\]

\[
S_R(t, p) = i \theta(t)e^{-iE(p)t} \Delta(t),
\]

where

\[
\Delta(t) = \exp \left\{ -g^2 T \int_{q_0}^{\omega_p} \frac{d^3q}{(2\pi)^3} \frac{1 - \cos t(v(p) \cdot q)}{q^2 (\hat{p} \cdot q)^2} \right\},
\]

contains all the non-trivial time dependence. The integral in eq. (8) is formally identical to that one would get in the Bloch-Nordsieck model in 3 dimensions. Note, however, the upper cut-off \(\omega_p \sim gT\), which occurs for the same reasons as in eq. (5). Namely, it reflects the dynamical cut-off at momenta \(\sim gT\), as provided by the Landau damping.

The integral over \(q\) has no infrared divergence, but one can verify that the expansion of \(\Delta(t)\) in powers of \(g^2\) generates the most singular pieces of the usual perturbative expansion for the self-energy \[13\]. Because our approximations preserve only the leading infrared behavior of the perturbation theory, eq. (8) describes only the leading large-time behavior of \(\Delta(t)\). Since the only energy scale in the momentum integral of eq. (8) is the upper cut-off, of order \(gT\), the large-time regime is achieved for \(t \gg 1/gT\). Note that, strictly speaking, eq. (8) holds only in the Feynman gauge. However, its leading large time behaviour — which is all what we can trust anyway! — is actually gauge independent \[13\] and of the form (we set here \(\alpha = g^2/4\pi\) and \(v(p) = 1\) to simplify writing)

\[
\Delta(\omega_p t \gg 1) \approx \exp\left( -\alpha T t \ln \omega_p t \right).
\]

A measure of the decay time \(\tau\) is given by

\[
\frac{1}{\tau} = \alpha T \ln \omega_p \tau = \alpha T \left( \ln \frac{\omega_p}{\alpha T} - \ln \ln \frac{\omega_p}{\alpha T} + \ldots \right).
\]
Since $\alpha T \sim g\omega_p$, $\tau \sim 1/(g^2T \ln(1/g))$. This corresponds to a damping rate $\gamma \sim 1/\tau \sim g^2T \ln(1/g)$, similar to that obtained in a one loop calculation with an IR cut-off $\mu \sim g^2T$ (cf. eq. (5)).

However, contrary to what perturbation theory predicts, $\Delta(t)$ is decreasing faster than any exponential. It follows that the Fourier transform

$$S_R(\omega, p) = \int_{-\infty}^{\infty} dt \ e^{-i\omega t} S_R(t, p) = i \int_0^{\infty} dt \ e^{it(\omega - E(p) + i\eta)} \Delta(t), \quad (11)$$

exists for any complex (and finite) $\omega$. Thus, the retarded propagator $S_R(\omega)$ is an entire function, with sole singularity at $\text{Im} \omega \to -\infty$. The associated spectral density $\rho(\omega, p)$ (proportional to the imaginary part of $S_R(\omega, p)$) retains the shape of a resonance strongly peaked around the perturbative mass-shell $\omega = E(p)$, with a typical width of order $\sim g^2T \ln(1/g)$ [15].

4. CONCLUSIONS

The previous analysis, and, in particular, the last conclusion about the resonant shape of the spectral density, confirm that the quasiparticles are well defined, even if their mass-shell properties cannot be computed in perturbation theory. The infrared divergences occur because of the degeneracy between the mass shell of the charged particle and the threshold for the emission or the absorption of $n$ ($n \geq 1$) static transverse photons. Note that the emitted photons are virtual, so, strictly speaking, the physical processes that we have in mind are the collisions between the charged excitation and the thermal particles, with the exchange of quasistatic magnetic photons. The resummation of these multiple collisions to all orders in $g$ modifies the analytic structure of the fermion propagator and yields an unusual, non-exponential, damping in time.

This result solves the IR problem of the damping rate in the case of QED. Since a similar problem occurs in QCD as well, it is natural to ask what is the relevance of the present solution for the non-Abelian plasma. It is generally argued — and also supported by lattice computations [19] — that the self-interactions of the chromomagnetic gluons may generate magnetic screening at the scale $g^2T$ (see [2] and Refs. therein). As a crude model, we may include a screening mass $\mu \sim g^2T$ in the magnetostatic propagator in the QED calculation. This amounts to replacing $1/q^2 \to 1/(q^2 + \mu^2)$ for the photon propagator in eq. (5). After this replacement, the latter equation provides, at very large times $t \gtrsim 1/g^2T$, an exponential decay: $\Delta(t) \sim \exp(-\gamma t)$, with $\gamma = \alpha T \ln(\omega_p/\mu) = \alpha T \ln(1/g)$. However, in the physically more interesting regime of intermediate times $1/gT \ll t \ll 1/g^2T$, the behavior is governed uniquely by the plasma frequency, according to our result (9): $\Delta(t) \sim \exp(-\alpha T t \ln(\omega_p t))$. Thus, at least within this limited model, which is QED with a “magnetic mass”, the time behavior in the physical regime remains controlled by the Bloch-Nordsieck mechanism. But, of course, this result gives no serious indication about the real situation in QCD, since it is unknown whether, in the present problem, the effects of the gluon self-interactions can be simply summarized in terms of a magnetic mass.

To conclude, the results of Refs. [13,17] suggest that the infrared divergences of the ultrarelativistic Abelian plasmas can be eliminated by soft photon resummations, à la Bloch-Nordsieck. For non-Abelian plasmas, on the other hand, much work remains to be
done, and this requires, in particular, the understanding of the non-perturbative sector of the magnetostatic gluons.

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