RADIAL REDSHIFT SPACE DISTORTIONS

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ABSTRACT

The radial component of the peculiar velocities of galaxies causes displacements in the galaxies' positions in redshift space. We study the effect of the peculiar velocities on the linear redshift space two-point correlation function. Our analysis takes into account the radial nature of the redshift space distortions, and it highlights the limitations of the plane-parallel approximation. We consider the problem of determining the value of $\beta$ and the real space two-point correlation function from the linear redshift space two-point correlation function. The inversion method proposed here takes into account the radial nature of the redshift space distortions and can be applied to magnitude-limited redshift surveys that have only partial sky coverage.

Subject headings: cosmology: theory — galaxies: clusters: general — galaxies: distances and redshifts — large-scale structure of universe

1. INTRODUCTION

The redshift of a galaxy has information about both its position and the radial component of its peculiar velocity, and as a consequence galaxy-galaxy correlations in redshift space differ from the correlations among their real positions. Using linear theory and the plane-parallel approximation, Kaiser (1987) showed that the redshift space power spectrum is the real space power spectrum amplified by the factor $(1 + \Omega_b \Omega_c \mu^2)^2$, where $\mu$ is the cosine of the angle between the line of sight and the wavevector. He also pointed out that the anisotropy of the redshift space power spectrum can be used to measure the value of $\Omega_b$.

The plane-parallel approximation assumes that the pairs of galaxies between which the correlation is being measured are sufficiently far away that their separation subtends a very small angle at the observer, and the displacements in redshift space caused by their peculiar motions may be treated as parallel. Most of the subsequent work (Lilje & Efstathiou 1989; McGill 1990; Loveday et al. 1992; Hamilton 1992; Gramman, Cen, & Bahcall 1993; Bromley 1994; Fry & Gaztañaga 1994; Fisher et al. 1994a; Fisher 1995; Cole, Fisher, & Weinberg 1994, 1995; Matsubara & Suto 1996) are based on the plane-parallel approximation.

A proper analysis, however, requires that the radial nature of the displacements in redshift space is taken into account. This is required if pairs of galaxies which sub tend a large angle are also to be included in the analysis. This was first taken into account in the analysis of Fisher, Scharf, & Lahav (1994b), who decomposed the angular behavior of the density field (in redshift space) into spherical harmonics and integrated out the radial dependence after multiplying the density field with a Gaussian radial window function. The power spectrum of the coefficients of this expansion was then used to determine the value of $\beta = \Omega_b \Omega_c / b$, and they obtained the value $\beta = 1.0 \pm 0.3$ on applying this technique to the 1.2 Jy IRAS redshift survey. This method was improved by Heavens & Taylor (1995), who expanded the radial dependence of the density field into spherical Bessel functions, but both these analyses have the drawback that they require prior knowledge of the linear power spectrum $P(k)$. These methods were further refined by Ballinger, Heavens, & Taylor (1995), who do not fix the shape of $P(k)$ but allow it to vary in six bins in $k$-space. All these methods have the limitation that they require the galaxy survey to have full sky coverage, and they cannot be applied to two-dimensional redshift surveys.

Hamilton & Culhane (1996) and Zaroubi & Hoffman (1996) have calculated the linear two-point correlation in redshift space taking into account the radial nature of the distortions. Zaroubi & Hoffman (1996) have also investigated the mode-mode coupling that arises when the analysis is done in Fourier space, but they have not addressed the problem of determining the value of $\beta$ and the real space correlation in detail. This problem has been discussed in detail by Hamilton & Culhane (1996), who propose a method for determining $\beta$ in a manner that does not require any prior assumption about the real space correlation function. They have studied the eigenfunctions of the “spherical distortion operator,” which relates the real space correlation to its redshift space counterpart. The observed redshift space correlation is expanded in terms of these eigenfunctions, and the ratio of the values of these coefficients can be used to obtain $\beta$.

In this paper we have investigated the effect of the radial nature of the redshift space distortions on the linear two-point correlation function. The redshift space two-point correlation is a function of the triangle formed by the observer and the pair of galaxies for which the correlation is being measured. In order to get a better understanding of the effects of the redshift space distortions, we study in some detail how the redshift space correlation function changes with the shape of the triangle, the value of $\beta$, and the slope of the real space correlation. We also address the question as to when the radial nature of the distortions is important and when they may be ignored and the plane-parallel approximation may be used instead. In § 3 we address the problem of determining the value of $\beta$ and the real space correlation function from the observed redshift space correlation function taking into account the radial nature of the distortions. The analysis of Hamilton & Culhane (1996) is based on the assumption that the selection function has a power-law spatial dependence. We have investigated whether this assumption is justified for a magnitude-limited sample. In this paper we discuss the inverse problem for two
different situations, (1) assuming that the selection function is a power law and (2) for a more realistic form of the selection function that can be used in a magnitude-limited survey. Both the inversion methods proposed here can be applied to redshift surveys that have partial sky coverage.

2. THE LINEAR REDSHIFT SPACE CORRELATION

A large amount of our information about the spatial distribution of galaxies is inferred from redshift surveys that provide angular positions and redshifts of a large number of galaxies. The distance to galaxies is very hard to measure, and the analysis of redshift surveys has to rely on the redshift as an indicator of the distance to the galaxies. This has a drawback because, in addition to the Hubble expansion, the redshift has contributions from the radial component of the peculiar velocity of our Galaxy and the galaxy being observed. The peculiar velocity of our Galaxy has been determined from the dipole anisotropy observed in the cosmic microwave background radiation (CMBR) (Kogut et al. 1993), and this can be corrected for in all observations made from our Galaxy, but the contribution from the radial component of the peculiar velocity of the galaxy being observed remains in its redshift.

Using the vector $z$ to denote the position of a galaxy in the 3-dimensional redshift space formed by the angular positions and redshifts, the relation between $z$ and the actual position of the galaxy $x$ can be written as

$$ z = x + \hat{z}(v \cdot \hat{z}). \tag{1} $$

Here the caret denotes a unit vector ($\hat{z} = \hat{z}/|\hat{z}|$), and $v \cdot \hat{z}$ is the radial component of the peculiar velocity of the galaxy. The units have been chosen so that the speed of light $c$, and the present value of the scale factor $a$ and its time derivative $\dot{a}$ are all unity.

The problem is how to use quantities measured from the distribution of galaxies in redshift space to draw inferences about the actual distribution of the galaxies. In addressing this problem, it is also necessary to take into account the fact that usually the galaxies in a redshift survey are not selected uniformly from the region of space surveyed. For magnitude-limited surveys the selection criterion is a function of the actual distance from the observer, and it is represented by the selection function $\Phi(x)$, which gives the fraction of galaxies selected in the survey as a function of the distance from the observer.

Taking these effects into account, the observed number density of galaxies in redshift space, $n^R(z)$, can be related to the number density of galaxies in real space, $n(x) = \langle n \rangle [1 + \delta(x)]$ and the peculiar velocity field $v(x)$ according to (Kaiser 1987)

$$ n^R(z) = \Phi(z) \langle n \rangle \times \left( 1 + \delta(z) - \left\{ 1 + \left[ 2 + \frac{d \log \Phi(z)}{d \log z} \right] \hat{z} \cdot v(z) \right\} \right), \tag{2} $$

where $\hat{z} = \hat{z} \cdot V_c$ is used to denote the derivative in the radial direction.

This equation is valid at linear order in $v$, and it has the assumption that $v \ll z$ and $\Phi(x)$ is a slowly varying function. It is convenient to define a function

$$ \alpha(z) = 2 + \frac{d \log \Phi(z)}{d \log z}, \tag{3} $$

where $\alpha(z) = 2$ for a volume-limited sample for which the selection function is a constant.

In the linear regime, in the presence of only the growing mode of perturbations (Peebles 1980), it is possible to express the perturbation and the peculiar velocity in terms of a potential as

$$ \delta(x) = \nabla^2 \Psi(x) \quad \text{and} \quad v(x) = -\beta \nabla \Psi(x). \tag{4} $$

Here $\beta = \Omega_0^{0.6}/b$, where $\Omega_0$ is the density parameter and $b$ is the bias parameter, which takes into account the fact that the galaxies may be a biased tracer of the underlying matter density that determines the peculiar velocities.

We use these relations in equation (2) to write the number density of galaxies at the point $z$ in redshift space as

$$ n^R(z) = \Phi(z) \langle n \rangle \left\{ 1 + \left[ \nabla^2 + \beta \left( \frac{\alpha(z)}{z} \hat{z} \cdot v(z) + \frac{\partial^2}{\partial z^2} \right) \right] \Psi(z) \right\}, \tag{5} $$

and we use this to calculate the linear redshift space two-point correlation function $\xi^R(z_1, z_2)$ defined as

$$ \xi^R(z_1, z_2) = \frac{\langle n^R(z_1) n^R(z_2) \rangle - \langle n^R(z_1) \rangle \langle n^R(z_2) \rangle}{\langle n^R(z_1) \rangle \langle n^R(z_2) \rangle}, \tag{6} $$

where the angular brackets denote an ensemble average. In evaluating this, we encounter the ensemble average $\langle \Psi(z_1) \Psi(z_2) \rangle$. As the universe is statistically homogeneous and isotropic we can define $\phi(z_1, z_2) = \langle \Psi(z_1) \Psi(z_2) \rangle$, which is a function of only the magnitude of the vector $z_1 = z_2 - z_1$. Using this, we obtain

$$ \xi^R(z_1, z_2) = \left[ \nabla^2 + \beta \left( \frac{\alpha(z_1)}{z_1} \hat{z}_1 \cdot v(z_1) + \frac{\partial^2}{\partial z^2} \right) \right] \phi(z_1, z_2) \left[ \nabla^2 + \beta \left( \frac{\alpha(z_2)}{z_2} \hat{z}_2 \cdot v(z_2) + \frac{\partial^2}{\partial z^2} \right) \right] \phi(z_1, z_2), \tag{7} $$

for the linear redshift space two-point correlation function. This should be compared with the real space two-point correlation function

$$ \xi(z_1, z_2) = \langle \delta(z_1) \delta(z_2) \rangle = \nabla^4 \phi(z_1, z_2), \tag{8} $$

which depends only on $z_{21}$.

Equations (7) and (8) together are equivalent to the expression for the linear redshift space correlation function derived by Hamilton & Culphe (1996). Equation (8) can be inverted to relate various derivatives of the potential $\phi$, which appear in equation (7) to integrals of $\xi(x)$, and this is described in Appendix A.

The expression for the redshift space two-point correlation function presented here is valid in the regime where $\xi(z_1, z_2) \ll 1$. In addition, there are the restrictions that the redshifts $z_1$ and $z_2$ are in a range where they are much larger than $\langle v^2 \rangle^{1/2}$ (the rms peculiar velocity) and where the selection function does not vary too rapidly.

Unlike the real space two-point correlation $\xi([z_1 - z_2])$, which depends on just on the distance between the points $z_1$ and $z_2$, the redshift space counterpart depends on the triangle formed by the observer $O$ and the points $z_1$ and $z_2$, and we next investigate this behavior in some detail.

The behavior of $\xi^R(z_1, z_2)$ is relatively simple in the situation where the two edges of the triangle $z_1$ and $z_2$ are made very large keeping $z_{21}$ fixed. In this limit $\hat{z}_1$ and $\hat{z}_2$ are
nearly parallel, i.e., \( \lim_{z_2 \to \infty} \frac{\dot{z}_1}{\dot{z}_2} = \hat{a}, \) and the peculiar velocities of the galaxies at \( z_1 \) and \( z_2 \) can be treated as being parallel. In addition, if the selection function is such that \( \lim_{z_2 \to \infty} [a(z)/z] = 0 \), then the terms involving \( a(z) \) can be dropped, and equation (7) becomes

\[
\xi^R(z_1, z_2) = \left[ \langle \nabla_{z_2}^2 - \beta (\hat{a} \cdot \nabla_{z_2}) \rangle \right]^2 \phi(z_1, z_2) \tag{9}
\]

and we have the linear redshift space two-point correlation in the plane-parallel approximation (PPA). In this limit the redshift space two-point correlation depends on the length of just one edge of the triangle \( z_2 \), and it depends on the angle between \( z_2 \) and the line of sight \( \hat{a} \). This angular dependence introduces anisotropy in the redshift space correlation, and this is well understood in the PPA (Hamilton 1992). Here we investigate the behavior of the redshift space correlation function in a more general situation where the plane-parallel approximation cannot be applied and the radial nature of the distortions has to be taken into account. In the rest of the discussion in this section we use the value \( c = 2 \), which corresponds to a volume-limited sample where the selection function is a constant.

In order to study separately the dependence of \( \xi^R(z_1, z_2) \) on the shape and the size of the triangle formed by the observer \( O \) and the points \( z_1, z_2 \), we consider a situation where the real space correlation function has a power-law behavior \( \xi(x) \propto x^{-\gamma} \). In this case the effect of changing the size of the triangle is very simple, \( \xi^R(z_1, z_2) \) is \( y^{-1} \xi^R(z_1, z_2) \), and the ratio

\[
w(z_1, z_2) = \frac{\xi^R(z_1, z_2)}{\xi^R(z_{21})} \tag{10}
\]

depends only on the shape of the triangle. We have used the function \( w(z_1, z_2) \) to study how \( \xi^R(z_1, z_2) \) varies with the shape of the triangle.

We parameterize triangles of all possible shapes by first carrying out the following operations which leave \( w(z_1, z_2) \) unchanged: (1) Label the larger of the two sides that originate from \( O \) as \( z_1 \). (2) Rotate the triangles around \( O \) so that they all lie in the \( x-y \) plane with \( z_1 \) along the \( x \)-axis. (3) Scale the triangles so that \( z_1 = 1 \). At the end of these operations, for all the triangles, \( z_1 \) corresponds to a unit vector in the \( x \)-direction, while \( z_2 \) lies in the \( x-y \) plane and it is restricted to be inside a circle of unit radius centered around \( O \). Triangles which lie in the lower half-plane can be related to triangles with the same shape in the upper half-plane by reflecting \( z_2 \) on the \( x \)-axis. It is thus possible to parameterize triangles of all possible shapes by using the vector \( z_2 \), which is restricted to lie inside the upper half of a circle of unit radius centered around \( O \). Figure 1 shows the observer \( O \) at the point \((0, 0)\) and the point \( z_1 \) at \((1, 0)\), and the semicircle shows the region inside which the point \( z_1 \) must lie. Every point in the semicircle corresponds to a triangle with a different shape, and one possible triangle is shown in Figure 1. We have used this parameterization to study how \( w(z_2) \) varies with the shape of the triangle, and the results of this study are presented in the form of contour plots that show contours of equal \( w \) plotted at equal intervals of \( w \) for triangles of all possible shapes.

We have studied the behavior of the function \( w(z_2) \) for three different cases, \( \gamma = 4, 3.5, \) and \( 2.5 \), with \( \beta = 1 \), and the corresponding contour plots are shown in Figures 2a, 3a, and 4a, respectively. The value of \( w(z_2) \) along the 45° cuts shown in Figures 2a, 3a, and 4a are plotted in Figures 2b, 3b, and 4b, and the purpose of these graphs is to show the value of \( w \) corresponding to the different contours. Figures 2b, 3b, and 4b also show \( w(z_2) \) for other values of \( \beta \) for which we have not shown contour plots.

Going back to the contour plots, we expect PPA to be valid in the region near the lower right-hand corner of the figures where \( z_2 \) is nearly equal to \( z_1 \) and \( z_{21} \) is less than \( z_1 \). For comparison we have calculated \( w(z_2) \) using PPA for the case with \( \gamma = 4 \) and \( \beta = 1 \) for triangles of all shapes (i.e., also beyond the region where we would expect PPA to be valid), and this is shown in Figure 2c. A comparison of Figures 2a and 2c shows that, as expected, the figures match in the region around \( z_1 \), but we start seeing considerable differences from the predictions of PPA as either (1) the angle between \( z_1 \) and \( z_2 \) is increased or (2) the sizes of \( z_1 \) and \( z_2 \) start to differ significantly. We find that PPA correctly describes the effect of the redshift space distortions on the linear two-point correlation function provided that the two vectors \( z_1 \) and \( z_2 \) do not differ by more than 30%, i.e., \( z_{21} \leq 0.3 z_1 \). The function \( w(z_2) \) behaves quite differently from the predictions of PPA once the two redshift space vectors \( z_1 \) and \( z_2 \) differ by more than 30%, and the radial nature of the distortions becomes important in this situation.

A comparison of Figures 2a, 3a, and 4a shows how the effect of the redshift space distortion changes with the slope of the real space correlation \( \gamma \). For \( \gamma = 4 \) and 3.5 the redshift space correlation is of the same sign as the real space correlation in the forward direction (when \( z_1 \) and \( z_2 \) are in the same direction), and the redshift space correlation function changes sign as \( z_2 \) is turned away from \( z_1 \), whereas for \( \gamma = 2.5 \) the behavior is just the opposite. We also find that the shape of the contour lines changes significantly as the value of \( \gamma \) is changed. For all the cases the effect of redshift space distortions becomes stronger as \( z_2 \) approaches the observer and \( w(z_2) \) diverges in the limit \( z_2 \to 0 \). This behavior is due to the factor \( a(z_2)/z_2 \) which appears in equation (7), and the cause of the divergence can be related to the fact that under the map from real space to redshift space a nonzero volume element in real space collapsed to a point at \( z = 0 \).

In Figure 2d we have shown \( w(z_2) \) for the case where \( \gamma = 4 \) and \( \beta = 0.2 \). Comparing this with Figure 2a, we see that a change in the value of \( \beta \) can make a qualitative difference in the behavior of the redshift space two-point correlation function. Although the shape of the contour...
3. INVERSE PROBLEM

In this section we address the problem of using the linear redshift space correlation function measured from redshift surveys to determine the value of \( \beta \) and the real space correlation function.

The inverse problem is very easily solved if \( \zeta^R(z_1, z_2) \) is restricted to a region where PPA is valid. The function \( \zeta^R(z_1, z_2) \) can then be written as a function of \( z_{21} = |z_2 - z_1| \) and \( \mu \), which is the cosine of the angle (shown in Fig. 1) between \( z_{21} \) and \( \hat{u} \), the line of sight to the pair of galaxies. In PPA the angular dependence of \( \zeta^R(z_21, \mu) \) is very simple, and it can be expressed in terms of Legendre polynomials \( P_l(\mu) \) as

\[
\zeta^R(z_{21}, \mu) = \sum_{l=0}^{\infty} P_l(\mu) \xi^R_l(z_{21}) ,
\]

where the \( \xi^R_l(z_{21}) \) are different angular moments of \( \zeta^R(z_{21}, \mu) \). As shown by Hamilton (1992), only the first three even moments have nonzero values, and the value of \( \beta \) can be obtained from the ratios of these angular moments determined from redshift surveys.

The situation is changed if the radial nature of the redshift space distortion is taken into account as the behavior of \( \zeta^R(z_1, z_2) \) now depends on the shape of the triangle formed by \( z_1, z_2 \) and \( O \). This has been studied by Hamilton & Culhane (1996), who have expressed the shape dependence of \( \zeta^R(z_1, z_2) \) in terms of five shape functions and have proposed a method for measuring the value of \( \beta \) based on this. In their work Hamilton & Culhane (1996) have made the simplifying assumption that the selection function can be described by a power law \( \Phi(z) \propto z^{\alpha - 2} \), which implies that \( \alpha(z) = \alpha \) is a constant.

Here we propose a different method for analyzing the linear redshift space two-point correlation function and determining the value of \( \beta \). In the first part of the analysis
presented below we assume that $\alpha(z) = \alpha$ is a constant, and later on in a separate subsection we treat the inverse problem for a more realistic selection function.

In this analysis $z_1$ and $z_2$, the two sides of the triangle which originate from $O$, are not treated on an equal footing. Writing one of the sides (say $z_1$) as $z_1 = z_1 \hat{n}$, all possible triangles can be parameterized using the lengths of two of the sides, $z_1$ and $z_{21}$, and $\mu$, the cosine of the angle (shown in Fig. 1) between $z_{21}$ and $\hat{n}$. This way of parameterizing is very similar to that used in PPA, except that in PPA $\hat{n}$ is the common direction along which both $z_1$ and $z_2$ lie, whereas now $\hat{n}$ refers to the direction of one of the sides $z_1$. Also, in PPA two parameters, $z_{21}$ and $\mu$, suffice to describe the behavior of $\xi^R$, whereas we require an additional third parameter, $z_1$, if the radial nature of the distortions is taken into account. For a fixed value of $z_{21}$, PPA corresponds to the limit $z_1 \to \infty$, and hence the value of $z_1$ does not appear in $\xi^R$ in PPA.

It is convenient to express $z_1$ in terms of the dimensionless ratio

$$s = \frac{z_{21}}{z_1},$$

and we use $(s, z_{21}, \mu)$ to parameterize $\xi^R$. For a fixed value of $z_{21}$, the limit $s \to 0$ corresponds to PPA, and the effects of the radial nature of the redshift space distortions become more important as $s$ increases.

The set of parameters $(s, z_{21}, \mu)$ has the feature that any triangle can be described by two sets of values of $s$ and $\mu$. For example, for a triangle formed by a pair of galaxies (which we call A and B) and O, we get different values of $s$ and $\mu$ depending on whether we label A as $z_1$ or label B as $z_1$. Here we take the point of view that both labelings should be used, and as a consequence each pair of galaxies contributes to $\xi(s, z_{21}, \mu)$ for two different values of $s$ and $\mu$.

We first consider the angular dependence of $\xi^R(s, z_{21}, \mu)$, and following the analysis used in PPA (eq. [11]) we decompose $\xi^R$ in terms of Legendre polynomials. We find that unlike the situation in PPA, now all the moments have nonzero values, and this does not provide a convenient way
of analyzing $\xi$. The analysis becomes considerably simpler if we use the redshift-weighted correlation function $\xi^R$ defined as

$$\xi^R(s, z_{21}, \mu) = \frac{\mu^2}{z_1^2} \xi^R(s, z_{21}, \mu) ,$$  \hspace{1cm} (13)

where $\frac{(z_2/z_1)^2}{2}$ can be written as $(1 + s^2 + 2\mu)$. The angular dependence of $\xi^R(s, z_{21}, \mu)$ is much simpler, and expanding it in terms of Legendre polynomials,

$$\xi^R(s, z_{21}, \mu) = \sum_{n=0}^{\infty} \xi^R_n(s, z_{21}) P_n(\mu) ,$$ \hspace{1cm} (14)

we find that only the first five moments have nonzero values, which are shown below:

$$\xi^R_0(s, z_{21}) = (1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2)\xi(z_{21})$$
$$+ s^2[(1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2)\xi(z_{21})]$$
$$- \frac{1}{2} \beta(3 - 6)\xi_4(z_{21})] ,$$

$$\xi^R_1(s, z_{21}) = s[(1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2)\xi(z_{21})]$$
$$+ s^3\beta(3 - 6)\xi_4(z_{21})] ,$$

$$\xi^R_2(s, z_{21}) = (1 + \beta)\xi(z_{21})$$
$$+ s^3(1 + \beta)\xi(z_{21})] ,$$

$$\xi^R_3(s, z_{21}) = s(1 + \beta)\xi(z_{21})$$
$$+ s^3(1 + \beta)\xi(z_{21})] ,$$

$$\xi^R_4(s, z_{21}) = \frac{1}{2} \beta(3 - 6)\xi_4(z_{21})] .$$

We have used the relations given in Appendix A to express the derivatives of the potential $\phi(z_{21})$ in terms of volume integrals of the real space two-point correlation function $\xi_\nu$, which are defined as

$$\xi_\nu(z_{21}) = \frac{2}{z_{21}} \int_0^{z_{21}} \xi(r)dr ,$$  \hspace{1cm} (20)

and for $n < 1$,

$$\xi_n(z_{21}) = \frac{n + 1}{z_{21}} \int_0^{z_{21}} \xi(r)r^n dr ,$$ \hspace{1cm} (21)

The first point to note is that for a fixed value of $z_{21}$, in the limit $s \to 0$ we recover the results calculated by Hamilton (1992) in the plane-parallel approximation. As expected, the odd moments all vanish in this limit.

The effect of including the radial nature of the distortions manifests itself through the $s$-dependent terms. The different angular moments all have a very simple dependence on $s$, and this involves at most a cubic polynomial in $s$. The angular moment $\xi^R_2$ has no $s$ dependence, and the expression calculated for this moment using PPA remains unchanged if the radial nature of the distortions is taken into account.

The procedure for determining the value of $\beta$ from redshift surveys is now quite straightforward in principle. The first step is to estimate the redshift-weighted correlation function $\xi^R(s, z_{21}, \mu)$ using all the pairs of galaxies in the survey. For a fixed value of $z_{21}$, for a survey which extends from a redshift $z_a$ to a redshift $z_b$, the variable $s$ will lie in the range $z_{21}/z_a \geq s \geq z_{21}/z_b$.

The second step is to decompose the angular dependence of the observationally determined $\xi^R(s, z_{21}, \mu)$ in terms of a Legendre polynomial. This is possible only for those values of $(s, z_{21})$ where there are observations of $\xi^R(s, z_{21}, \mu)$ for both positive and negative $\mu$, and in general the range of $(s, z_{21})$ where this is possible will depend on the geometry of the redshift survey. The angular decomposition can be used to obtain estimates for the first five angular moments, $\xi^R_0(s, z_{21})$ to $\xi^R_4(s, z_{21})$. We expect all the higher moments to be zero in the linear regime.

For a fixed value of $z_{21}$, the $s$ dependence of the angular moments $\xi^R_0(s, z_{21})$ is very simple in the linear regime. The third step is to do a least-squares fit for the $s$ dependence of the $\xi^R_0(s, z_{21})$ using polynomials in $s$ of the form predicted by equations (15)–(19). For example, the monopole determined from redshift surveys can be fitted using a quadratic function of the form $\xi^R_0(s, z_{21}) = \xi^R_0(0, z_{21})s^0 + \xi^R_2(0, z_{21})s^2$, where $\xi^R_0(0, z_{21})$ and $\xi^R_2(0, z_{21})$ are the unknown quantities which have to be determined from the fit.

The coefficient of the terms $s^0$ in the fits for the even moments gives $\xi^R_0(0, z_{21})$, $\xi^R_2(0, z_{21})$, and $\xi^R_4(0, z_{21})$ which correspond to the values of these moments in PPA. Effectively, this allows us to use pairs of galaxies for which the radial nature of the redshift space distortion is important (large values of $s$) to predict the behavior of the angular moments at $s = 0$ which corresponds to PPA. Once these are known we can use the method proposed by Hamilton (1992) for determining $\beta$ and $\xi(z_{21})$ using results calculated in the plane-parallel approximation.

There are various possible ways of doing this, and one way is to use the relations

$$\left(1 + \frac{2}{3} \beta + \frac{1}{5} \beta^2\right)[\xi(z_{21}) - \xi_2(z_{21})]$$

$$= \xi^R_0(0, z_{21}) - \frac{3}{z_{21}} \int_0^{z_{21}} \xi^R(0, r)r^2 dr ,$$ \hspace{1cm} (22)

$$\left(\frac{4}{3} \beta + \frac{4}{7} \beta^2\right)[\xi(z_{21}) - \xi_2(z_{21})] = \xi^R_0(0, z_{21}) .$$ \hspace{1cm} (23)

The right-hand side of equations (22) and (23) involve quantities which can be determined from the redshift surveys by using the procedure discussed above. The ratio of these two equations can then be used to calculate $\beta$, which can in turn be used to determine $\xi(x)$.

3.1. A Realistic Selection Function

Until now we have discussed the inverse problem for a situation where the selection function has a specific form $\Phi(z) \propto z^{\alpha - 2}$, which implies that $\alpha(z) = \alpha$ is a constant. In this subsection we consider a realistic situation where we have a magnitude-limited survey. We first investigate the validity of the assumption that $\alpha(z)$ is a constant.

For a magnitude-limited sample with a lower apparent magnitude limit $m_{\text{min}}$ and an upper magnitude limit $m_{\text{max}}$, the selection function is related to $N(M)$, the differential
galaxy luminosity function, by
\[ \Phi(z) = C_1 \int_{M_{\text{min}(z)}}^{M_{\text{max}(z)}} N(M) dM , \]  
(24)
where the limits in the integral can be related to the limits in the apparent magnitude using the relation between absolute and apparent magnitudes, and the galaxy luminosity function \( N(M) \) is usually modeled using the Schechter function (Schechter 1976). The normalization constant \( C_1 \) in equation (24) is of no interest in this discussion, as it does not affect \( \alpha(z) \), and it is only the shape of the selection function which is of interest.

As an example we have used the apparent magnitude limits and the luminosity function given for the N112 subsample of the Las Campanas Redshift Survey (LCRS; Lin et al. 1996) to calculate \( \Phi(z) \) and \( \alpha(z) \) (shown in Figs. 5 and 6) for \( q_0 = 0.5 \) and \( H_0 = 100 \text{ km s}^{-1} \text{ Mpc}^{-1} \). We have set the normalization \( C_1 \) to an arbitrary number chosen for the convenience of plotting \( \Phi(z) \).

We find that the assumption that \( \alpha(z) \) is a constant does not correctly describe the behavior of the calculated values of \( \alpha(z) \) shown in Figure 6. The behavior of \( \alpha(z) \) in the redshift range \( 0.05 \leq z \leq 0.19 \) can be fitted using a function of the form \( \alpha(z) = \alpha - z^2/q^2 \), with \( \alpha = 1.7 \) and \( q = 0.072 \). This corresponds to a selection function of the form \( \Phi(z) = C_1 z^{-2}e^{-z^2/2q^2} \) (shown in Fig. 5), and we find that this fits the calculated selection function to better than 2% in the range \( 0.05 \leq z \leq 0.19 \). Figure 5 also shows \( n(<z) \), the fraction of the galaxies we expect to find at redshifts less than \( z \), and we see that about 90% of the galaxies are expected to lie in the redshift range over which this fit is valid.

In the rest of this paper we assume that the selection function is of the form
\[ \Phi(z) = C_1 z^{-2}e^{-z^2/2q^2} \]  
(25)
with
\[ \alpha(z) = \alpha - z^2/q^2 , \]  
(26)
where \( \alpha \) and \( q \) are constants which have to be determined for the particular galaxy survey being considered. We expect the selection function to behave as predicted by equation (25) for any magnitude-limited survey where the luminosity function is of the Schechter form, which is a product of a power law and an exponential. It may also be noted that a fit of the form \( \alpha(z) = -z^2/q^2 \), which corresponds to a selection function \( \Phi(z) = C_1 e^{-z^2/2q^2} \), is reasonably accurate (\( \sim 10\% \)), and this may also be used instead of equation (25).

The form of the selection function in equation (25) introduces a new length scale \( q_i \) and \( \xi^R(z_1, z_2) \) now behaves quite differently from the case where \( \alpha(z) \) is a constant. The most important difference is that now the limit \( \lim_{z \to 0} \alpha(z)/z \) diverges and hence \( \xi^R(z_1, z_2) \) as given by equation (7) diverges in the plane-parallel approximation, which corresponds to the limit \( z_1 \to \infty \) and \( z_2 \to \infty \) with \( z_{12} \) held fixed. Before proceeding further, it is necessary to clarify the fact that this divergence in the behavior of \( \xi^R(z_1, z_2) \) is not a physical effect, and we do not expect to find a divergent behavior in the \( \xi^R(z_1, z_2) \) determined from redshift surveys when \( z_1 \) or \( z_2 \) is made very large. The divergent behavior predicted by equation (7) arises because the derivation of this equation involves Taylor-expanding \( \Phi(z + v_p) \) (\( v_p \) is the radial component of the peculiar velocity) in powers of \( v_p \). For the selection function given in equation (25) this involves expanding the function \( e^{-(z + v_p)^2/2q^2} = e^{-(z^2 + 2zv_p + v_p^2)/2q^2} \) in powers of \( v_p \). Such an expansion is valid only if \( zv_p \ll q^2 \) and \( v_p \ll q \), and the expansion is invalid if we take the limit \( z \to \infty \). As a consequence equation (7) also becomes invalid for very large values of \( z_1 \) or \( z_2 \), and this gives rise to a divergent behavior in \( \xi^R(z_1, z_2) \). Equation (7) is valid provided \( \langle v^2 \rangle^{1/2} z_1/q^2 \ll 1 \), \( \langle v^2 \rangle^{1/2} z_2/q^2 \ll 1 \), and \( \xi(z_{12}) \) is in the linear regime, and it does not correctly describe the behavior of \( \xi^R \) if any of these conditions are not satisfied.

We next check over what redshift range equation (7) is valid for the LCRS. Using the values \( \langle v^2 \rangle^{1/2} \sim 0.004 \) and \( q = 0.07 \), we find that equation (7) can be used at linear scales over the entire redshift range for which the fit given by equation (25) is valid. Having ascertained the fact that equations (7) and (25) are both valid over the redshift range in which most of the galaxies lie, we proceed to investigate the inverse problem for a situation where the linear \( \xi^R \) is described by these equations.

As a consequence of the divergent behavior discussed above, it is not possible to extrapolate the \( \xi^R \) determined from pairs of galaxies for which equations (7) and (25) are
valid to obtain $\xi^R$ in the plane-parallel approximation. Hence, it is not possible to obtain the value of $\beta$ using the method discussed earlier for the case where $\alpha(z)$ is a constant, and a different inversion method has to be used for the selection function given by equation (25).

We find that in this case it is convenient to use $(z_1, z_2, \mu)$ to parameterize the redshift-weighted two-point correlation function:

$$\xi^R(z_1, z_2, \mu) = \frac{z_2^2 - z_1^2}{z_1^2} \xi^R(z_1, z_2, \mu),$$

where $z_2$ can be written as $z_2^2 = z_1^2 + z_{21}^2 + 2z_1 z_{21} \mu$.

Decomposing $\xi^R(z_1, z_2, \mu)$ in terms of Legendre polynomials (eq. 14), we again find that only the first five moments are nonzero. In this case it is more convenient to work directly in terms of the potential $\phi(z_{21})$ instead of expressing the derivatives of $\phi(z_{21})$ in terms of volume averages of $\xi(z_{21})$. The expressions for the angular moments $\xi^R$ are given below:

$$\xi^R(z_1, z_2) = \frac{4\beta z_{21}}{5z_1} \left[ 1 + \beta \right] \frac{d^4}{dz_{21}^4} \left[ \frac{8\beta}{5z_1} \frac{6\beta z_{21}}{5q^2 z_1} \frac{dz}{dz_{21}} \right] + \left[ \frac{4\beta z_{21}}{5q^2 z_1} - \frac{4\beta z_{21}^2}{5q^2} \right] \frac{dz^2}{dz_{21}^2} \phi(z_{21}),$$

$$\xi^R(z_1, z_2) = \frac{8}{35} \frac{d^3}{dz_{21}^3} \phi(z_{21}),$$

The expressions for the other nonzero $\xi^R$ are very similar to equation (28), and these are presented in Appendix B. The equation for $\xi^R(z_1, z_2)$ is much simpler than the equations for the other angular moments, and it does not involve $z_1$. This equation can be integrated once to obtain $d^3\phi(z_{21})/dz_{21}^3$ in terms of the $\xi^R(z_1, z_2)$ determined from the redshift survey, and this gives us the relation

$$\frac{d^3}{dz_{21}^3} \phi(z_{21}) = \frac{35}{\beta^2 8} \exp \left( -\frac{z_{21}^2}{q^2} \right) \int d^2 y e^{-q^2/4} \xi^R(y)$$

$$+ \exp \left( -\frac{z_{21}^2 - z_{2L}^2}{q^2} \right) A_1,$$

where we only use the values of $\xi^R(21)$ at separations $z_{21} \geq z_L$, where we expect the correlations to be in the linear regime. The constant of integration $A_1$ corresponds to $d^2\phi(z_{21})/dz_{21}^2$ at the point $z_{21} = z_L$. Equation (30) can be integrated once more to obtain

$$\frac{d^2}{dz_{21}^2} \phi(z_{21}) = \frac{35}{\beta^2 8} \int d^2 y e^{-q^2/4} \int_y \int d^2 y e^{-q^2/4} \xi^R(y) + \exp \left( -\frac{z_{21}^2 - z_{2L}^2}{q^2} \right) A_1 + A_2,$$

where $A_2$ is a constant of integration that corresponds to the value of $d^2\phi(z_{21})/dz_{21}^2$ at $z_{21} = z_L$. Equation (30) can also be used to obtain the relation

$$\frac{d^4}{dz_{21}^4} \phi(z_{21}) = \frac{35}{\beta^2 8} \xi^R(z_{21}) + \frac{2z_{21}^2}{q^2} \frac{dz}{dz_{21}} \phi(z_{21}).$$

These equations (eqs. [30], [31], and [32]) allow us to determine the value of the derivatives of the potential $\phi(z_{21})$ in terms of the observed $\xi^R(z_{21})$ and three unknown parameters, $\beta$, $A_1$, and $A_2$. Using these relations in equation (28), we obtain the expression for $\xi^R(z_{21})$ presented below:

$$\xi^R(z_{21}) = \frac{7z_{21}}{2z_1} \left( 1 + \beta \right) \xi^R(z_{21}) \left( 1 + \frac{z_{21}^2}{4q^2} \right)$$

$$+ \frac{7}{z_1} \left( 1 + \beta \right) \frac{z_{21}^2}{4q^2} \frac{z_{21}^2}{\beta q^2}$$

$$\times \left[ \int d^2 y e^{-q^2/4} \xi^R(y) \right] + \frac{7z_{21}}{2q^2} \left( \frac{\alpha}{z_1} - \frac{z_{21}^2}{q^2} \right)$$

$$\times \left[ \int d^2 y e^{-q^2/4} \int d^2 y e^{-q^2/4} \xi^R(y) \right]$$

$$+ \frac{4\beta^2 z_{21}}{5q^2} \left( \frac{\alpha}{z_1} - \frac{z_{21}^2}{q^2} \right)$$

$$\times \left[ \int d^2 y e^{-q^2/4} \int d^2 y e^{-q^2/4} \xi^R(y) \right]$$

$$+ \frac{2\beta}{5z_1} \left( 4 + \frac{x^2}{q^2} \right) A_2 \phi \left( \frac{z_{21} - z_{2L}}{q^2} \right).$$

This equation allows us to use the $\xi^R(z_{21})$ determined from observations to predict the values of $\xi^R(z_{21}, z_{2L})$. The quantities which involve the observed values of $\xi^R(z_{21}, z_{2L})$ have been enclosed in square brackets in the above equation. The relation between the observed $\xi^R$ and the predicted values for $\xi^R$ also involves three parameters, $\beta$, $A_1$, and $A_2$, which are the only unknown quantities in equation (33). By comparing the values of $\xi^R(z_{21}, z_{2L})$ predicted by equation (33) with the observed values of $\xi^R(z_{21}, z_{2L})$, it is possible to determine the value of these parameters $\beta$, $A_1$, and $A_2$ for which the predicted $\xi^R$ best fits the observed $\xi^R$.

A similar procedure can also be carried out using the angular moments $\xi^R$, $\xi^R$, and $\xi^R$, and the expressions for these quantities in terms of the observed $\xi^R$ and the parameters $\beta$, $A_1$, and $A_2$ are presented in Appendix B. The best-fitting values of the parameters $\beta$, $A_1$, and $A_2$ determined using the four different angular moments should be consistent, and this provides a way of checking the validity of the method proposed here.

Once the values of the parameters $\beta$, $A_1$, and $A_2$ are known, it is quite straightforward to determine the real space correlation $\xi(z_{21}) = V^4 \phi(z_{21})$.

4. SUMMARY AND DISCUSSION

We have studied how the peculiar velocities distort the linear two-point correlation function in redshift space. Our analysis takes into account the radial nature of the distortion. We have compared this with the linear two-point correlation calculated in the plane-parallel approximation, and we find that there are significant differences in the behavior of $\xi^R(z_{21}, z_{2L})$ if the two redshift space vectors $z_1$ and $z_2$ differ by more than 30%. The effect of the radial nature of the redshift space distortions becomes important when
either the angle between \( z_1 \) and \( z_2 \) becomes large, or the lengths of \( z_1 \) and \( z_2 \) differ significantly and the plane-parallel approximation does not correctly describe the effect of redshift space distortions under these circumstances.

We have also addressed the problem of extracting the value of \( \beta \) and the real space correlation function from the redshift space correlation function, taking into account the radial nature of the distortions. This problem was studied earlier by Hamilton & Culhane (1996), who have assumed that \( \alpha(z) \) is a constant. We have tested the validity of this assumption for a magnitude-limited survey, and we find that such an assumption is not justified. The inversion scheme proposed by Hamilton & Culhane (1996) can be applied only to volume-limited samples where \( \alpha(z) = 2 \).

In the first part of our analysis we have followed Hamilton & Culhane (1996) in assuming that \( \alpha(z) \) is a constant. The inversion procedure proposed here is quite different from that proposed by Hamilton & Culhane (1996). The main difference is that we propose the use of a redshift-weighted two-point correlation function \( \xi^R \) instead of the correlation function \( \xi \). The function \( \xi^R \) has the advantage that if we decompose its angular dependence in terms of Legendre polynomials, only the first five angular moments are nonzero, and all the higher angular moments are zero. This is not the case if we use \( \xi \) instead. The expressions we obtain for the angular moments of \( \xi^R \) are very closely related to the results one gets in the plane-parallel approximation, and this makes the proposed inversion procedure very simple.

For a magnitude-limited sample the selection function can be very well approximated by \( \Phi(z) = C z^a e^{-z^{1/2}a^2} \), and we have analyzed the inverse problem for such a situation. We find that in this situation also the redshift-weighted correlation function has only five nonzero angular moments, and the procedure proposed for determining \( \beta \) in § 3.1 is based on this. This procedure can be applied to magnitude-limited samples. It also has the feature that it uses the values of the redshift space correlation only at separations larger than some separation \( z_L \), which can be chosen so that all scales larger than \( z_L \) are in the linear regime. This has the advantage that the inversion procedure uses only the values of the redshift space correlation function from scales that are definitely in the linear regime, and the inversion procedure is not affected in any way by the nonlinear scales.

Both the inversion schemes discussed in this paper can be applied to redshift surveys with partial sky coverage; for example, they can be applied to the Las Campanas Redshift Survey, where the observations are restricted to six thin conical slices.

APPENDIX A

The equation

\[
\xi(z) = \nabla^4 \phi(z) ; \tag{A1}
\]

can be inverted to obtain

\[
\partial_i \partial_j \phi(z) = \frac{1}{3} \delta_{ij} \left[ \int_0^z \xi(y) dy + C \right] - \frac{1}{2} \partial_i \partial_j \left( \int_0^z \xi(y) dy \right)
- \frac{1}{6} \delta_i \delta_j \left( \frac{1}{z} \right) \int_0^z \xi(y) y^4 dy , \tag{A2}
\]

where \( z_i \) refers to the components of \( z \), \( \partial_i \) refers to the components of \( \nabla_z \), and \( C \) is a constant whose value is fixed by the boundary conditions. It is most natural to choose the condition

\[
\lim_{z \to \infty} \partial_i \partial_j \phi(z) = 0 , \tag{A3}
\]

which gives us

\[
C = -\int_0^\infty \xi(y) dy . \tag{A4}
\]

We also have

\[
\partial_i \partial_j \partial_k \phi(z) = -\frac{1}{2} \partial_i \partial_j \partial_k \left( \int_0^z \xi(y) y^2 dy \right) - \frac{1}{6} \partial_i \partial_j \partial_k \left( \frac{1}{z} \right) \int_0^z \xi(y) y^4 dy, \tag{A5}
\]

and

\[
\partial_i \partial_j \partial_k \partial_l \phi(z) = \frac{z_i z_j z_k z_l}{z^4} \xi(z) - \frac{1}{2} \partial_i \partial_j \partial_k \partial_l \left( \int_0^z \xi(y) y^2 dy \right)
- \frac{1}{6} \partial_i \partial_j \partial_k \partial_l \left( \frac{1}{z} \right) \int_0^z \xi(y) y^4 dy . \tag{A6}
\]
APPENDIX B

Here we present the expressions for the angular moments \( \xi_0(z_1, z_{21}) \), \( \xi_1(z_1, z_{21}) \), and \( \xi_2(z_1, z_{21}) \) of the redshift-weighted linear two-point correlation function \( \xi^R(z_1, z_{21}, \mu) \) calculated for the selection function described by equation (25).

\[
\xi_0(z_1, z_{21}) = \left[ 1 + \frac{2b}{3} + \frac{b^2}{3} + \frac{z_{21}^2}{z_1^2} \left( \frac{1}{3} + \frac{b^2}{3} + \frac{1}{3} \right) \right] \frac{d^4}{dz_{21}^4} \phi(z_{21}) + \left[ \frac{4}{3z_{21}^2} (3 + \beta) - \frac{b z_{21}}{15q} (15 + \beta) + \frac{z_{21}^2}{3z_1^2} (12 + 8\beta + \alpha \beta - \alpha \beta^2) - \frac{\beta z_{21}^3}{3q^2 z_1^2} (3 + \beta) \right] \frac{d^3}{dz_{21}^3} \phi(z_{21}) + \left[ \frac{2\beta}{3q^2} (\alpha \beta - 3) + \frac{4}{z_{21}^2} \left( 12 + 2\alpha \beta - \alpha \beta^2 \right) + \frac{\beta z_{21}^2}{q^2 z_1^2} (\alpha \beta - 2) - \frac{b^2}{3q^2} (3z_{21}^2 + z_1^2) \right] \frac{d^2}{dz_{21}^2} \phi(z_{21}), \tag{B1}
\]

\[
\xi_1(z_1, z_{21}) = \left[ \frac{z_{21}}{5z_1} (5 + 8\beta + 3\beta^2) \right] \frac{d^4}{dz_{21}^4} \phi(z_{21}) + \left[ \frac{8}{5z_1} (5 + 4\beta) - \frac{2b z_{21}}{5q^2 z_1} (5 + 2\beta) - \frac{\alpha \beta z_{21}}{z_1^2} (1 + \beta) \right] \frac{d^3}{dz_{21}^3} \phi(z_{21}) + \left[ \frac{8}{z_{21}^2 z_1} + \frac{4\beta z_{21}}{5q^2 z_1} (4\alpha \beta - 5) \right] \frac{d^2}{dz_{21}^2} \phi(z_{21}) + \left[ \frac{\alpha \beta z_{21}}{z_1^2} (2 + \alpha \beta) + \frac{\alpha \beta z_{21}}{q^2 z_1^3} \right] \frac{d}{dz_{21}} \phi(z_{21}), \tag{B2}
\]

\[
\xi_2(z_1, z_{21}) = \left[ 4\beta \left( \frac{1}{3} + \frac{\beta}{7} \right) + \frac{2b z_{21}}{3z_1^2} (1 + \beta) \right] \frac{d^4}{dz_{21}^4} \phi(z_{21}) + \left[ \frac{8}{3z_{21}} - \frac{10b z_{21}}{21q^2} \right] \frac{d^3}{dz_{21}^3} \phi(z_{21}) + \left[ \frac{4\alpha \beta^2}{3q^2} - \frac{2b^2 z_{21}}{q^2} \right] \frac{d^2}{dz_{21}^2} \phi(z_{21}) + \left[ \frac{2\alpha \beta z_{21}}{3z_1^2} (4 + \alpha \beta) + \frac{2\alpha \beta z_{21}}{q^2 z_1^3} - \frac{2b^2 z_{21}}{3q^4} \right] \frac{d}{dz_{21}} \phi(z_{21}). \tag{B3}
\]

Equations (30), (31) and (32) can be written as

\[
\frac{d^4}{dz_{21}^4} \phi(z_{21}) = \frac{35}{\beta^2} F_3(z_{21}) + f_3(z_{21}) A_3, \tag{B4}
\]

\[
\frac{d^3}{dz_{21}^3} \phi(z_{21}) = \frac{35}{\beta^2} F_2(z_{21}) + f_2(z_{21}) A_3 + A_2, \tag{B5}
\]

\[
\frac{d^2}{dz_{21}^2} \phi(z_{21}) = \frac{35}{\beta^2} \xi^R + \frac{2z_{21}}{q^2} \frac{d^3}{dz_{21}^3} \phi(z_{21}), \tag{B6}
\]

where the functions \( f_2(z_{21}) \) and \( f_3(z_{21}) \) are defined as

\[
f_3(z_{21}) = \exp \left[ (z_{21}^2 - z_1^2)/q^2 \right], \tag{B7}
\]

\[
f_2(z_{21}) = \int_{z_{21}}^{z_{21}} dr \exp \left[ (r^2 - z_1^2)/q^2 \right], \tag{B8}
\]

and \( F_2(z_{21}) \) and \( F_3(z_{21}) \) are integrals of the observed \( \xi^R(z_{21}, \mu) \):

\[
F_3(z_{21}) = \exp \left( \frac{z_{21}^2}{q^2} \right) \int_{z_{21}}^{z_{21}} dy e^{-y^2/q^2} \xi^R(y), \tag{B9}
\]

\[
F_2(z_{21}) = \int_{z_{21}}^{z_{21}} dr e^{-r^2/q^2} \int_{z_{21}}^{z_{21}} dy e^{-y^2/q^2} \xi^R(y). \tag{B10}
\]
Using equations (B4), (B5), and (B6) for the derivatives of \( \phi(z) \) in equations (B1), (B2), and (B3), we obtain the following expressions for \( \xi_{0}^{R}(z_{1}, z_{2}) \) and \( \xi_{1}^{R} \) in terms of the observed \( \xi_{0}^{R} \) and three parameters, \( \beta, \alpha, \) and \( A_{2} \):

\[
\xi_{0}^{R}(z_{1}, z_{2}) = \left( \frac{-2\beta}{q_{1}} + \frac{2\alpha \beta}{3q_{1}^{2}} + \frac{4}{z_{1}} - \frac{2\beta z_{21}}{z_{1}^{2}} + \frac{4}{z_{1}} + \frac{2\alpha \beta}{3z_{1}} - \frac{2\beta z_{21}}{z_{1}^{2}} + \frac{4}{3z_{1}} + \frac{\alpha \beta^{2} z_{21}}{q_{1}^{2}} - \frac{\beta^{2} z_{21}^{2}}{3q_{1}^{3}} \right) \\
\times \left[ A_{2} + f_{3}(z_{21})A_{3} \right] + \left( \frac{4}{z_{1}} + \frac{4\beta}{3z_{1}} + \frac{2\beta z_{21}}{q_{1}^{2}} + \frac{\beta^{2} z_{21}^{2}}{3q_{1}^{3}} + \frac{2\beta z_{21}}{z_{1}^{2}} + \frac{\beta^{2} z_{21}^{2}}{3q_{1}^{3}} \right) f_{3}(z_{21})A_{3} \\
+ 35 \left( \frac{\alpha}{12q_{1}} - \frac{1}{4\beta q_{1}} + \frac{\beta}{12\beta^{2} q_{1}} - \frac{\beta^{2}}{24z_{1}^{2}} - \frac{\alpha}{4\beta^{2} q_{1}} - \frac{\beta^{2}}{24z_{1}^{2}} + \frac{\beta^{2} z_{21}}{24z_{1}^{2}} + \frac{\beta^{2} z_{21}^{2}}{24z_{1}^{2}} \right) f_{3}(z_{21})A_{3} \\
+ 35 \left( \frac{\alpha}{12q_{1}} - \frac{1}{4\beta q_{1}} + \frac{\beta}{12\beta^{2} q_{1}} - \frac{\beta^{2}}{24z_{1}^{2}} - \frac{\alpha}{4\beta^{2} q_{1}} - \frac{\beta^{2}}{24z_{1}^{2}} + \frac{\beta^{2} z_{21}}{24z_{1}^{2}} + \frac{\beta^{2} z_{21}^{2}}{24z_{1}^{2}} \right) f_{3}(z_{21})A_{3} \\
+ 35 \left( \frac{\alpha}{12q_{1}} - \frac{1}{4\beta q_{1}} + \frac{\beta}{12\beta^{2} q_{1}} - \frac{\beta^{2}}{24z_{1}^{2}} - \frac{\alpha}{4\beta^{2} q_{1}} - \frac{\beta^{2}}{24z_{1}^{2}} + \frac{\beta^{2} z_{21}}{24z_{1}^{2}} + \frac{\beta^{2} z_{21}^{2}}{24z_{1}^{2}} \right) f_{3}(z_{21})A_{3} \\
+ \left( \frac{7}{8} + \frac{35}{8\beta^{2} q_{1}^{2}} + \frac{35\beta z_{21}}{8\beta^{2} q_{1}^{2}} + \frac{35\beta z_{21}^{2}}{8\beta^{2} q_{1}^{2}} \right) f_{3}(z_{21})A_{3} \\
(B11)
\]

\[
\xi_{1}^{R}(z_{1}, z_{2}) = \left( \frac{-2\alpha \beta z_{21}}{z_{1}} - \frac{\alpha \beta^{2} z_{21}}{z_{1}^{2}} + \frac{\alpha \beta^{2} z_{21}}{z_{1}^{2}} + \frac{8}{z_{1} z_{21}} - \frac{4\beta z_{21}}{z_{1}^{2}} + \frac{16\alpha \beta z_{21}}{z_{1}^{2}} - \frac{5\alpha \beta z_{21}}{z_{1}^{2}} \right) [A_{2} + f_{3}(z_{21})A_{3}] \\
\times \left( \frac{-\alpha z_{21}^{2} - \alpha z_{21}^{2} - 8}{z_{1} z_{21}} - \frac{\beta^{2} z_{21}}{q_{1}^{2} z_{1}} + \frac{11\beta^{2} z_{21} z_{21}}{q_{1}^{2} z_{1}} \right) \\
+ \frac{32\beta}{5z_{1}^{2}} + \frac{4\beta z_{21}^{2}}{q_{1}^{2} z_{1}} + \frac{22\beta z_{21}^{2}}{q_{1}^{2} z_{1}} + \frac{8\beta z_{21}^{2}}{q_{1}^{2} z_{1}} + \frac{35\beta z_{21}^{2}}{q_{1}^{2} z_{1}} + \frac{35\beta z_{21}^{2}}{q_{1}^{2} z_{1}} + \frac{77z_{1} z_{21}}{8z_{1}^{2}} \frac{35z_{21} z_{21}}{4\beta z_{21}} \\
\times \left( \frac{-35z_{21} z_{21}}{8z_{1}^{2}} + \frac{35z_{21} z_{21}}{8z_{1}^{2}} + \frac{35z_{21} z_{21}}{8z_{1}^{2}} + \frac{35z_{21} z_{21}}{8z_{1}^{2}} + \frac{77z_{1} z_{21}}{8z_{1}^{2}} \frac{35z_{21} z_{21}}{4\beta z_{21}} \right) f_{3}(z_{21}) \\
+ \left( \frac{21z_{21}}{8z_{1}^{2}} + \frac{35z_{21} z_{21}}{4\beta z_{21}} + \frac{14z_{21} z_{21}}{4\beta z_{21}} \right) f_{3}(z_{21}) \\
(B12)
\]

\[
\xi_{2}^{R}(z_{1}, z_{2}) = \left( \frac{4\alpha \beta^{2}}{3q_{1}} - \frac{2\alpha \beta^{2} z_{21}}{q_{1}} - \frac{8\beta z_{21}}{3z_{1}^{2}} + \frac{2\alpha \beta \beta z_{21}}{3z_{1}^{2}} - \frac{2\beta^{2} z_{21}^{2}}{3z_{1}^{3}} \right) [A_{2} + f_{3}(z_{21})A_{3}] \\
\times \left( \frac{8}{3z_{1}^{2}} + \frac{2\beta z_{21}}{3z_{1}^{2}} + \frac{4\beta z_{21}}{3z_{1}^{2}} + \frac{4\beta z_{21}}{3z_{1}^{2}} + \frac{4\beta z_{21}}{3z_{1}^{2}} + \frac{4\beta z_{21}}{3z_{1}^{2}} \right) f_{3}(z_{21})A_{3} \\
+ \frac{4\beta z_{21}}{3z_{1}^{2}} + \frac{2\beta z_{21}}{3z_{1}^{2}} \right) f_{3}(z_{21})A_{3} \\
+ \left( \frac{8}{3z_{1}^{2}} + \frac{2\beta z_{21}}{3z_{1}^{2}} + \frac{4\beta z_{21}}{3z_{1}^{2}} + \frac{4\beta z_{21}}{3z_{1}^{2}} + \frac{4\beta z_{21}}{3z_{1}^{2}} + \frac{4\beta z_{21}}{3z_{1}^{2}} \right) f_{3}(z_{21})A_{3} \\
+ \left( \frac{7}{8} + \frac{7z_{1} z_{21}}{6z_{1}^2} + \frac{7z_{1} z_{21}}{6z_{1}^2} \right) f_{3}(z_{21})A_{3} \\
(B13)
\]

REFERENCES

Ballinger, W. E., Heavens, A. F., & Taylor, A. N. 1995, MNRAS, 276, L59
Bromley, B. C. 1994 ApJ, 423, L81
Cole, S., Fisher, K. B., & Weinberg, D. H. 1994, MNRAS, 267, 785
Fisher, K. B. 1995, ApJ, 448, 494
Fisher, K. B., Davis, M., Strauss, M. A., Yahil, A., & Huchra, J. P. 1994a, MNRAS, 267, 927
Fisher, K. B., Scharf, C. A., & Lahav, O. 1994b, MNRAS, 266, 219
Fry, J. N., & Gaztañaga, E. 1994, ApJ, 425, 1
Graham, M. Cen, R., & Bahcall, N. A. 1993, ApJ, 419, 440
Hamilton, A. J. S. 1992, ApJ, 385, L5
Hamilton, A. J. S., & Culhane, M. 1996, MNRAS, 278, 73
Heavens, A. F., & Taylor, A. N. 1995, MNRAS, 275, 483
Kaiser, N. 1987, MNRAS, 227, 1
Kogut, A., et al. 1993, ApJ, 419, 1
Lilje, P. B., & Efstathiou, G. 1989, MNRAS, 236, 851
Lin, H., Kirshner, R. P., Shectman, S. A., Landy, S. D., Oemler, A., Tucker, D. L., & Schechter, P. L. 1996, ApJ, 464, 60
Loveday, J., Efstathiou, G., Peterson, B. A., & Maddox, S. J. 1992, ApJ, 400, L43
Matsubara, T., & Suto, Y. 1996, ApJ, 470, L1
McGill, C. 1990, MNRAS, 242, 428
Peebles, P. J. E. 1980, The Large-Scale Structure of the Universe (Princeton: Princeton Univ. Press)
Schechter, P. 1976, ApJ, 203, 297
Zaroubi, S., & Hoffman, Y. 1996, 462, 25