Against Supersymmetry

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Abstract

We consider the massless supersymmetric vector multiplet in a purely quantum framework and propose a power counting formula. Then we prove that the interaction Lagrangian for a massless supersymmetric non-Abelian gauge theory (SUSY-QCD) is uniquely determined by some natural assumptions, as in the case of Yang-Mills models, however we do have anomalies in the second order of perturbation theory. The result can be easily generalized to the case when massive multiplets are present, but one finds out that the massive and the massless Bosons must be decoupled, in contradiction with the standard model. Going to the second order of perturbation theory produces an anomaly which cannot be eliminated. We make a thorough analysis of the model working only with the component fields.

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1 Introduction

A quantum field theory should provide two items: the Hilbert space of the physical states and the (perturbative) expression of the scattering matrix. In perturbation theory the Hilbert space is generated from the vacuum by some set of free fields i.e. it is a Fock space. In theories describing higher spin particles one considers a larger Hilbert space of physical and unphysical degrees of freedom and gives a rule of selection for the physical states; this seems to be the only way of saving unitarity and renormalizability, in the sense of Bogoliubov. In this case one should check that the interaction Lagrangian (i.e. the first order of the $S$-matrix) leaves invariant the physical states. If the preceding picture is available in all detail then one can go very easily to explicit computations of some scattering process. Other constructions of a quantum field theory, as those based on functional integration, are incomplete in our opinion if they are not translated in the operatorial language in such a way that the consistency checks can be easily done.

The construction of the QCD Lagrangian in the causal approach goes as follows [7], [16]. The Hilbert space of the massless vector field $v_\mu$ is enlarged to a bigger Hilbert space $\mathcal{H}$ including two ghost fields $u, \tilde{u}$ which are Fermi scalars of null mass; in $\mathcal{H}$ we can give a Hermitian structure such that we have

$$v_\mu^\dagger = v_\mu, \quad u^\dagger = u, \quad \tilde{u}^\dagger = -\tilde{u}. \quad (1.1)$$

Then one introduces the gauge charge $Q$ according to:

$$Q \Omega = 0, \quad Q^\dagger = Q, \quad [Q, v_\mu] = i \partial_\mu u, \quad \{Q, u\} = 0, \quad \{Q, \tilde{u}\} = -i \partial^\mu v_\mu; \quad (1.2)$$

deepath where $\Omega \in \mathcal{H}$ is the vacuum state. Because $Q^2 = 0$ the physical Hilbert space is given by $\mathcal{H}_{phys} = \text{Ker}(Q)/\text{Im}(Q)$.

The gauge charge is compatible with the following causal (anti)commutation relation:

$$[v_\mu(x), v_\nu(y)] = i \ g_{\mu\nu} \ D_0(x - y) \quad \{u(x), \tilde{u}^\dagger(y)\} = -i \ D_0(x - y) \quad (1.3)$$

and the other causal (anti)commutators are null; here $D_m(x - y)$ is Pauli-Jordan causal distribution of mass $m \geq 0$. In fact, the first relation together with the definition of the gauge charge, determines uniquely the second relation as it follows from the Jacobi identity

$$[v_\mu(x), \{\tilde{u}(y), Q\}] + \{\tilde{u}(y), [Q, v_\mu(x)]\} = \{Q, [v_\mu(x), \tilde{u}(y)]\} = 0. \quad (1.4)$$

We can then assume that all the fields $v_\mu, u, \tilde{u}$ have the canonical dimension equal to 1 so the gauge charge raises the canonical dimension by 1. It is usefull to convince the reader that the gauge structure above gives the right physical Hilbert space. We do this for the one-particle Hilbert space. The generic form of a state $\Psi \in \mathcal{H}^{(1)} \subset \mathcal{H}$ from the one-particle Hilbert subspace is

$$\Psi = \left[ \int f_\mu(x) v_\mu(x) + \int g_1(x) u(x) + \int g_2(x) \tilde{u}(x) \right] \Omega. \quad (1.5)$$
with test functions \( f_\mu, g_1, g_2 \) verifying the wave equation equation. We impose the condition 
\[ \Psi \in \text{Ker}(Q) \iff Q\Psi = 0; \] we obtain \( \partial^\mu f_\mu = 0 \quad g_2 = 0 \) i.e. the generic element \( \Psi \in \mathcal{H}^{(1)} \cap \text{Ker}(Q) \) is

\[ \Psi = \left[ \int f_\mu(x)v^\mu(x) + \int g(x)u(x) \right] \Omega \quad (1.6) \]

with \( g \) arbitrary and \( f_\mu \) constrained by the transversality condition \( \partial^\mu f_\mu = 0 \); so the elements of \( \mathcal{H}^{(1)} \cap \text{Ker}(Q) \) are in one-one correspondence with couples of test functions \( (f_\mu, g) \) with the transversality condition on the first entry. Now, a generic element \( \Psi' \in \mathcal{H}^{(1)} \cap \text{Im}(Q) \) has the form

\[ \Psi' = Q\Phi = \left[ -\int \partial^\mu f_\mu'(x)u(x) + \int \partial_\mu g'(x)v^\mu(x) \right] \Omega \quad (1.7) \]

so if \( \Psi \in \mathcal{H}^{(1)} \cap \text{Ker}(Q) \) is indexed by \( (f_\mu, g) \) then \( \Psi + \Psi' \) is indexed by \( (f_\mu + \partial_\mu g', g - \partial^\mu f_\mu') \).

If we take \( f_\mu' \) conveniently we can make \( g = 0 \). We introduce the equivalence relation \( f_\mu^{(1)} \sim f_\mu^{(2)} \iff f_\mu^{(1)} - f_\mu^{(2)} = \partial_\mu g' \) and it follows that the equivalence classes from \( \text{Ker}(Q)/\text{Im}(Q) \) are indexed by equivalence classes of wave functions \( [f_\mu] \); we have obtained the usual one-particle Hilbert space \( \otimes \): the idea comes from Hodge theory and amounts in finding an homotopy operator \( Q \) such that the spectrum of the “Laplace” operator \( \{Q, \bar{Q}\} \) can be easily determined.

By definition quantum chromodynamics assumes that we have \( N \) copies \( v_j^\mu, u_j, \bar{u}_j \quad j = 1, \ldots, N \) verifying the preceding algebra for any \( j = 1, \ldots, N \).

The interaction Lagrangian \( t(x) \) is some Wick polynomial acting in the total Hilbert space \( \mathcal{H} \) and verifying the conditions: (a) canonical dimension \( \omega(t) = 4 \); (b) null ghost number \( gh(t) = 0 \) (where by definition we have \( gh(v_j^\mu) = 0 \), \( gh(u_j) = 1 \) \( gh(\bar{u}_j) = -1 \) and the ghost number is supposed to be additive); (c) Lorentz covariant; (d) gauge invariance in the sense:

\[ [Q, t(x)] = i\partial_\mu t^\mu(x) \quad (1.8) \]

for some Wick polynomials \( t^\mu \) of canonical dimension \( \omega(t^\mu) = 4 \) and ghost number \( gh(t^\mu) = 1 \).

The gauge invariance condition guarantees that, after spatial integration the interaction Lagrangian \( t(x) \) factorizes to the physical Hilbert space \( \text{Ker}(Q)/\text{Im}(Q) \) in the adiabatic limit, i.e. after integration over \( x \); the condition \( (1.8) \) is equivalent to the usual condition of (free) current conservation. Expressions of the type

\[ d_Q b + \partial_\mu t^\mu \quad (1.9) \]

with

\[ \omega(b) = \omega(t^\mu) = 3 \quad gh(b) = -1 \quad gh(t^\mu) = 0 \quad (1.10) \]

are called trivial Lagrangians because they induce a null interaction after space integration (i.e. the adiabatic limit) on the physical Hilbert space. One can prove that the condition \( (1.8) \) restricts drastically the possible form of \( t \) i.e. every such expression is, up to a trivial Lagrangian, equivalent to

\[ t = f_{jkl}(\vdots v_j^\nu v_k^\nu \partial_\nu v_{l\mu} : - : v_j^\nu u_k \partial_\mu \bar{u}_l :) \quad (1.11) \]
where the (real) constants $f_{jkl}$ must be completely antisymmetric. In the the rest of the paper we will skip the Wick ordering notations. Going to the second order of the perturbation theory produces the Jacobi identity. So we see that, starting from some very natural assumptions, we obtain in an unique way the whole structure of Yang-Mills models. We expect the same thing to happen for more complex models as supersymmetric theories.

If we want to generalize to the supersymmetric case we must include all the fields $v_\mu, u, \bar{u}$ in some supersymmetric multiplets. By definition [2] a supersymmetric multiplet is a set of Bose and Fermi fields $b_j, f_A$ together with the supercharge operators $Q_a$ such that the commutator (resp. the anticommutator) of a Bose (resp. Fermi) field with the supercharges is a linear combination of Fermi (resp. Bose) fields; the coefficients of these linear combinations are partial derivative operators. We must also suppose that the supercharges are part of an extension of the Poincaré algebra called the supersymmetric algebra; essentially we have (for $N = 1$ supersymmetry):

$$Q_a \Omega = 0, \quad Q_a \Omega = 0 \quad Q_a = (Q_a)^\dagger$$

$$\{Q_a, Q_b\} = 0, \quad \{Q_a, \bar{Q}_b\} - 2\sigma^\mu_{ab} P_\mu = 0$$

and

$$[Q_a, P_\mu] = 0, \quad U_A^{-1} Q_a U_A = A_a^b Q_b.$$  

Here $U_A$ is a unitary representation of the Poincaré group and $P_\mu$ are the infinitesimal generators of the space-time translations. There are not many ways to do this. We will show that for the $v_\mu$ we must use the vector multiplet and for the ghost fields we must use chiral multiplets. Then we must impose that $t$ is also supersymmetric invariant. A natural definition is:

$$[Q_a, t] = d_Q s_a + \partial_\mu t^\mu_a \quad \omega(s_a) = 7/2 \quad gh(s_a) = -1 \quad \omega(t^\mu_a) = 7/2 \quad gh(t^\mu_a) = 0;$$

this means that after space integration (i.e. the adiabatic limit) we obtain on the physical Hilbert space an expression commuting with the supercharges.

In the supersymmetric framework one usually makes a supplementary requirement, namely that the basic supersymmetric multiplets should be organized in superfields [1], [8], [17], [18] i.e. fields dependent on space-time variables and some auxiliary Grassmann parameters $\theta^a, \bar{\theta}^\alpha$. It is showed in [2] that there is a canonical map $w \mapsto s w \equiv W$ mapping a ordinary Wick monomial $w(x)$ into its supersymmetric extension

$$W(x, \theta, \bar{\theta}) \equiv \exp (i\theta^a Q_a - i\bar{\theta}^\alpha \bar{Q}_{\alpha});$$

in particular this map associates to every field of the model a superfield. Moreover, one postulates that the interaction Lagrangian $t$ should be of the form

$$t(x) \equiv \int d\theta^2 d\bar{\theta}^2 T(x, \theta, \bar{\theta})$$

for some supersymmetric Wick polynomial $T$. We expect that the preceding expression is of the form (1.11) plus other monomials where the super-partners appear.
One can hope to have an uniqueness result for the coupling if one finds out a supersymmetric generalization of (1.8). A natural candidate would be the relation:

\[ [Q, T(x, \theta, \bar{\theta})] = \mathcal{D}T(x, \theta, \bar{\theta}) - H.c. = \mathcal{D}T + \bar{\mathcal{D}}\bar{T} \]  

(1.18)

where

\[ \mathcal{D}_a \equiv \frac{\partial}{\partial \theta^a} - i\sigma^\mu_{ab} \bar{\theta}^b \partial_\mu \quad \bar{\mathcal{D}}_\bar{a} \equiv -\frac{\partial}{\partial \bar{\theta}^\bar{a}} + i\sigma^\mu_{\bar{a}b} \theta^b \partial_\mu. \]  

(1.19)

We have

\[ (\mathcal{D}_a T)^\dagger = \pm \bar{\mathcal{D}}_{\bar{a}} T^\dagger, \]

\[ \{\mathcal{D}_a, \mathcal{D}_b\} = 0, \quad \{\mathcal{D}_a, \bar{\mathcal{D}}_\bar{b}\} = 0, \quad \{\mathcal{D}_a, \bar{\mathcal{D}}_\bar{b}\} = -2i\sigma^\mu_{ab} \partial_\mu. \]  

(1.20)

where in the first formula the sign +(-) corresponds to a super-Bose (-Fermi) field. The last relations is used to eliminate space-time divergences \( \partial^\mu T^\mu(x, \theta, \bar{\theta}) \) in the right-hand side of the relation (1.18). It is clear that (1.18) implies (1.8).

The most elementary and general way of analyzing supersymmetries is to work in components and to see later if the solution can be expressed in terms of superfields. We will prove that in the massless case there is an unique solution for SUSY-QCD if we consider a weaker form of (1.15) i.e. we require that this relation is true only on physical states:

\[ \langle \Psi_1, ([Q_a, t] - d_Q s_a - \partial_\mu t_a^\mu)\Psi_2 \rangle = 0 \]  

(1.21)

where \( \Psi_1, \Psi_2 \in Ker(Q) \) modulo \( Im(Q) \). We remark that (1.15) implies (1.21) but not the other way round.

In the next Section we give the structure of the multiplets of the model and we present the gauge structure. In Section 3 we determine the most general form of the interaction Lagrangian compatible with gauge invariance and prove that we have supersymmetric invariance also. The expression for the ghost coupling seems to be new in the literature. The details of the computation are given in the Appendix.

We also investigate in what sense one can rephrase the result using superfields. An immediate consequence of the analysis in terms of component fields is that one cannot impose (1.18). However we can establish a contact with traditional literature based on the so-called Wess-Zumino gauge.

In Section 4 we extend the result to the massive case hoping to obtain the minimal supersymmetric extension of the standard model. We obtain a curious obstruction, namely the sector of massive gauge fields and the sector of massless gauge fields must decouple; this does not agree with the standard model.

Unfortunately, if we proceed to the second order of perturbation theory, we obtain a supersymmetric contribution to the anomaly which cannot be eliminated by redefinitions of the chronological products.

In Section 5 we do the same analysis for the new vector multiplet [9] working in components also. In conclusion \( N = 1 \) supersymmetry and gauge invariance do not seem to be compatible in quantum theory.
2 The Quantum Superfields of the Model

2.1 The Vector Multiplet

The vector multiplet is the collection of fields $C, \phi, v_\mu, d, \chi, \lambda$ where $C$, is real scalar, $\phi$ is a complex scalar, $v_\mu$ is a real vector and $\chi, \lambda$ are spinor fields. We suppose that all these fields are of mass $m \geq 0$. We can group them in the superfield

$$V = C + \theta \chi + \bar{\theta} \bar{\chi} + \theta^2 \phi + \bar{\theta} \phi^\dagger + (\theta \sigma^\mu \bar{\theta}) v_\mu + \theta^2 \bar{\theta} \lambda + \bar{\theta}^2 \theta \lambda + \theta^2 \bar{\theta}^2 d.$$  \hspace{1cm} (2.1)

It is convenient to define the new field:

$$\lambda'_a \equiv \lambda_a + \frac{i}{2} \sigma^\mu_{ab} \partial_\mu \bar{\chi}^\dagger \hspace{1cm} d' \equiv d - \frac{m^2}{4} C$$  \hspace{1cm} (2.2)

and then the action of the supercharges is given by

$$i \left[ Q_a, C \right] = \chi_a$$

$$\{ Q_a, \phi \} = 2i \epsilon_{ab} \phi$$

$$\{ Q_a, \bar{\chi}_b \} = -i \sigma^\mu_{ab} (v_\mu + i \partial_\mu C)$$

$$\left[ Q_a, \phi \right] = 0$$

$$i \left[ Q_a, \phi^\dagger \right] = \lambda'_a - i \sigma^\mu_{ab} \partial_\mu \bar{\chi}^\dagger$$

$$i \left[ Q_a, v_\mu \right] = \sigma^\mu_{ab} \bar{\chi}^\dagger \hspace{-1cm} - \bar{\theta} \theta \lambda$$

$$\{ Q_a, \lambda'_b \} = 2i \epsilon_{ab} \bar{d}' - 2i \sigma^\mu_{ab} \partial_\mu \bar{\chi}$$

$$\{ Q_a, d' \} = -\frac{1}{2} \sigma^\mu_{ab} \partial_\mu \bar{\chi}^\dagger.$$  \hspace{1cm} (2.3)

It is a long but straightforward exercise to verify that the supersymmetric algebra is valid [10]. In [10] we have also determined the generic form of the causal (anti)commutation relations:

$$[C(x), C(y)] = -i \ c_1 \ D_m(x - y)$$

$$[C(x), d(y)] = -i \ c_2 \ D_m(x - y)$$

$$[C(x), \phi(y)] = -i \ (c_4 - i c_3) \ D_m(x - y)$$

$$[\phi(x), \phi^\dagger(y)] = -i \ \left( \frac{m^2}{4} \ c_1 + c_2 \right) \ D_m(x - y)$$

$$[\phi(x), d(y)] = \frac{m^2}{4} (c_4 - i c_3) \ D_m(x - y)$$

$$[\phi(x), v_\mu(y)] = (c_3 + i c_4) \ \partial_\mu D_m(x - y)$$

$$[d(x), d(y)] = -\frac{im^4}{16} \ c_1 \ D_m(x - y)$$

$$[v_\mu(x), v_\nu(y)] = i \ c_1 \ \partial_\mu \partial_\nu D_m(x - y) + i \ \left( \frac{m^2}{2} \ c_1 - 2c_2 \right) g_{\mu\nu} D_m(x - y)$$
\[ \{ \chi_a(x), \chi_b(y) \} = 2(c_4 - ic_3) \epsilon_{ab} D_m(x - y), \]
\[ \{ \chi_a(x), \bar{\chi}_b(y) \} = c_1 \sigma^\mu_{ab} \partial_\mu D_m(x - y), \]
\[ \{ \lambda_a(x), \lambda_b(y) \} = -\frac{m^2}{2} (c_4 - ic_3) \epsilon_{ab} D_m(x - y), \]
\[ \{ \lambda_a(x), \bar{\lambda}_b(y) \} = \frac{m^2}{4} c_1 \sigma^\mu_{ab} \partial_\mu D_m(x - y), \]
\[ \{ \chi_a(x), \lambda_b(y) \} = -2i c_2 \epsilon_{ab} D_m(x - y), \]
\[ \{ \chi_a(x), \bar{\lambda}_b(y) \} = -i (c_4 + ic_3) \sigma^\mu_{ab} \partial_\mu D_m(x - y) \] (2.4)

and the rest of the (anti)commutators are null; here \( c_j \) \( j = 1, \ldots, 4 \) are some real coefficients. However, if we want that the commutation relations for the vector field remain unchanged \([1, 3]\) then we must require \( c_1 = 0, c_2 = -\frac{1}{2} \). This choice implies in particular that \( v_\mu \) and \( \phi \) have the canonical dimensions 1 so the causal commutator between them should have the order of singularity −2. But this is compatible only with the choice \( c_3 = c_4 = 0 \).

In the end we find out that the causal (anti)commutation relations:
\[
[C(x), d'(y)] = \frac{i}{2} D_m(x - y)
\]
\[
[d'(x), d'(y)] = -\frac{im^2}{4} D_m(x - y)
\]
\[
[\phi(x), \phi^\dagger(y)] = \frac{i}{2} D_m(x - y)
\]
\[
[v_\mu(x), v_\nu(y)] = i g_{\mu\nu} D_m(x - y)
\]
\[
\{ \chi_a(x), \chi_b(y) \} = i \epsilon_{ab} D_0(x - y), \]
\[
\{ \chi_a(x), \bar{\lambda}_b(y) \} = \sigma^\mu_{ab} \partial_\mu D_0(x - y) \] (2.5)

and the rest of the (anti)commutators are null. More compactly
\[
[V(X), V(Y)] = -\frac{1}{2} D_2(X; Y) \] (2.6)

where we are using the notations from \([10]\) for the possible causal super-distributions. The canonical dimension of the component fields are
\[
\omega(v_\mu) = 1 \quad \omega(\lambda') = 3/2 \quad \omega(d) = 2 \quad \omega(\phi) = 1 \quad \omega(\chi) = \frac{1}{2} \quad \omega(\lambda') = \frac{3}{2}. \] (2.7)

It is natural to assume that
\[
\omega(\theta) = -1/2 \quad \omega(\mathcal{D}) = 1/2 \] (2.8)
as it is usually done in the literature. In this way one can make sense of the notion of canonical dimension for the vector superfield; more precisely we have:
\[
\omega(V) = 0. \] (2.9)
2.2 The Ghost Chiral Multiplets

We require that the ghost fields are also members of some multiplets of chiral type. We admit that these ghost multiplets are also massless. The generic forms of a chiral ghost and anti-ghost superfields are

\[ U(x, \theta, \bar{\theta}) = a(x) + 2i \bar{\theta} \zeta(x) + i (\theta \sigma^\mu \bar{\theta}) \partial_\mu a(x) + \bar{\theta}^2 g(x) + \bar{\theta}^2 \theta \sigma^\mu \partial_\mu \bar{\zeta}(x) \]  

and respectively

\[ \tilde{U}(x, \theta, \bar{\theta}) = \tilde{a}(x) - 2i \bar{\theta} \tilde{\zeta}(x) + i (\theta \sigma^\mu \bar{\theta}) \partial_\mu \tilde{a}(x) + \bar{\theta}^2 \tilde{g}(x) - \bar{\theta}^2 \theta \sigma^\mu \partial_\mu \tilde{\zeta}(x) \]  

where \( a, g, \tilde{a}, \tilde{g} \) are Fermi scalar fields and \( \zeta_a, \tilde{\zeta}_a \) are Bose spinor fields. Let us remind that we choose them such that

\[ (\zeta_a) \dagger = \bar{\zeta}_{\tilde{a}} \quad (\tilde{\zeta}_a) \dagger = -\bar{\zeta}_a \]  

- see 1.1; the chirality condition means

\[ \mathcal{D}_a U = 0 \quad \mathcal{D}_a \tilde{U} = 0. \]  

The action of the supercharges on these fields is

\[ \{Q_a, a\} = 0 \quad \{Q_a, a\} = 2\zeta_a \]

\[ \{Q_a, g\} = -2i \sigma^\mu_{ab} \partial_\mu \bar{\zeta}_b \quad \{Q_a, g\} = 0 \]

\[ [Q_a, \zeta_b] = \epsilon_{ab} g^\dagger \quad i[Q_a, \tilde{\zeta}_b] = \sigma^\mu_{ab} \partial_\mu a \]  

and respectively:

\[ \{Q_a, \tilde{a}\} = 0 \quad \{Q_a, \tilde{a}\} = 2\tilde{\zeta}_a \]

\[ \{Q_a, \tilde{g}\} = 2i \sigma^\mu_{ab} \partial_\mu \bar{\zeta}_b \quad \{Q_a, \tilde{g}\} = 0 \]

\[ [Q_a, \tilde{\zeta}_b] = \epsilon_{ab} \tilde{g}^\dagger \quad i[Q_a, \zeta_b] = -\sigma^\mu_{ab} \partial_\mu \tilde{a}. \]  

It is convenient to work with the Hermitian (resp. anti-Hermitian) fields

\[ u \equiv a + a^\dagger \quad v \equiv -i(a - a^\dagger) \]

\[ \bar{u} \equiv \bar{a} - \bar{a}^\dagger \quad \bar{v} \equiv -i(\bar{a} - \bar{a}^\dagger) \]  

such that we have

\[ u^\dagger = u \quad v^\dagger = v \quad \bar{u}^\dagger = -\bar{u} \quad \bar{v}^\dagger = -\bar{v}. \]  

Then we have the following action of the supercharges:

\[ \{Q_a, u\} = 2\zeta_a \quad \{Q_a, v\} = 2i \zeta_a \]

\[ \{Q_a, g\} = -2i \sigma^\mu_{ab} \partial_\mu \bar{\zeta}_b \quad \{Q_a, g\} = 0 \]

\[ [Q_a, \zeta_b] = \epsilon_{ab} g^\dagger \quad i[Q_a, \tilde{\zeta}_b] = \frac{1}{2} \sigma^\mu_{ab} \partial_\mu (u + iv) \]  

\[ 7 \]
and respectively:

\[
\begin{align*}
\{Q_a, \tilde{u}\} &= -2\tilde{\zeta}_a \\
\{Q_a, \tilde{v}\} &= -2i \tilde{\zeta}_a \\
\{Q_a, \tilde{g}\} &= 2i \sigma^\mu_{ab} \partial_\mu \tilde{\zeta} \\
\{Q_a, \tilde{g}^\dagger\} &= 0 \\
& \quad \left[ Q_a, \tilde{\zeta}_b \right] = \epsilon_{ab} \tilde{g} \dagger \\
& \quad i[Q_a, \tilde{\zeta}_b] = -\frac{1}{2} \sigma^\mu_{ab} \partial_\mu (\tilde{u} + i\tilde{v}).
\end{align*}
\]

These relations are consistent with the following canonical dimensions for the fields:

\[
\begin{align*}
\omega(u) &= 1 & \omega(v) &= 1 & \omega(g) &= 2 & \omega(\zeta) &= \frac{3}{2} \\
\omega(\tilde{u}) &= 1 & \omega(\tilde{v}) &= 1 & \omega(\tilde{g}) &= 2 & \omega(\tilde{\zeta}) &= \frac{3}{2}.
\end{align*}
\]

One can define the so-called \( R \) symmetry by:

\[
R\Omega = 0 \quad R\dagger = R
\]

and

\[
\begin{align*}
[R, C] &= 0 & [R, \phi] &= -2\phi & [R, \phi^\dagger] &= 2\phi^\dagger & [R, d] &= 0 \\
[R, \chi] &= -\chi & [R, \tilde{\chi}] &= \tilde{\chi} & [R, \chi^\dagger] &= \chi & [R, \tilde{\chi}^\dagger] &= -\tilde{\chi}^\dagger \\
[R, u] &= 0 & [R, v] &= 0 & [R, g] &= -2g & [R, g^\dagger] &= 2g^\dagger & [R, \zeta_a] &= -\zeta & [R, \tilde{\zeta}_a] &= \tilde{\zeta} \\
[R, \tilde{u}] &= 0 & [R, \tilde{v}] &= 0 & [R, \tilde{g}] &= 0 & [R, \tilde{g}^\dagger] &= 0 & [R, \tilde{\zeta}_a] &= -\tilde{\zeta}_a & [R, \tilde{\zeta}_a] &= \tilde{\zeta}_a.
\end{align*}
\]

One usually imposes this invariance on the interaction Lagrangian.

### 2.3 The Gauge Charge

The purpose is to generalize the construction from the Introduction and to extend naturally the formulas (1.2) to the supersymmetric case. We define the gauge charge \( Q \) postulating the following properties:

- We have

\[
Q\Omega = 0, \quad Q^\dagger = Q.
\]

- The (anti)commutator of \( Q \) with a Bose (resp. Fermi) field is a linear combination of Fermi (resp. Bose) fields; the coefficients of this linear combinations are partial differential operators.

- The (anti)commutator of \( Q \) with a field raises the canonical dimension by an unit;

- The (anti)commutator of \( Q \) with a field raises the ghost number by an unit;
The gauge charge commutes with the action of the Poincaré group; in this way the Poincaré group induces an action on the physical space $H_{\text{phys}} \equiv \text{Ker}(Q)/\text{Im}(Q)$.

The gauge charge anticommutates with the supercharges:

$$\{Q, Q_a\} = 0; \quad (2.24)$$

in this way the supersymmetric algebra induces an action on the physical space $H_{\text{phys}}$.

The gauge charges squares to zero as in the Yang-Mills case

$$Q^2 = 0. \quad (2.25)$$

If one makes the most general ansatz for the action of $Q$ compatible with the preceding conditions one finds out an important result: making some convenient rescaling of the fields, the gauge charge is uniquely determined preceding assumptions. In the massless case we have $d' = d$ and $Q$ is uniquely determined by:

$$Q\Omega = 0 \quad Q^\dagger = Q \quad (2.26)$$

and

$$[Q, C] = i \, v \quad [Q, v^a] = i \partial^\mu u \quad [Q, \phi] = -g^\dagger \quad [Q, \phi^\dagger] = g \quad [Q, d] = 0 \quad (2.27)$$

One can express everything in terms of superfields also [8]:

$$[Q, V] = U - U^\dagger \quad \{Q, U\} = 0$$

$$\{Q, U\} = -\frac{1}{16} \mathcal{D}^2 \mathcal{D}^2 V \quad (2.28)$$

As in the Introduction we postulate that the physical Hilbert space is the factor space $\mathcal{H}_{\text{phys}} = \text{Ker}(Q)/\text{Im}(Q)$. Using this gauge structure it is easy to prove that the one-particle Hilbert subspace contains the following type of particles: a) a particle of null mass and helicity 1 (the photon); b) a particle of null mass and helicity $1/2$ (the photino); c) the ghost states generated by the fields $\tilde{g}$ from the vacuum. These states must be eliminated by imposing the supplementary condition that the physical states have null ghost number. Only the transversal degrees of freedom of $v_\mu$ and $\lambda'_a$ are producing physical states.

We can now determine the causal (anti)commutation relations for the ghost fields. As in the Yang-Mills case - see relation (1.4) - one uses the Jacobi identities

$$[b(x), \{f(y), Q\}] + \{f(y), [Q, b(x)]\} = \{Q, [b(x), f(y)]\} = 0 \quad (2.29)$$
where $b = C, \phi, \phi^\dagger, v, \chi, \lambda, \bar{\chi}, \bar{\lambda}$ and $f = \tilde{u}, \tilde{g}, \tilde{g}^\dagger, \tilde{\zeta}, \bar{\tilde{\zeta}}$; if we take into account the particular choice we have made for the causal (anti)commutation relations of the vector multiplet one finds out [10] that we have

$$\{a(x), \tilde{a}(y)\} = \frac{i}{2} D_0(x - y) \quad [\zeta_a(x), \bar{\zeta}_b(y)] = -\frac{1}{4} \sigma^\mu_{ab} \partial_\mu D_0(x - y)$$

(2.30)

and all other causal (anti)commutators are null. So, like for ordinary Yang-Mills models, the causal (anti)commutators of the ghost fields are determined by the corresponding relations of the vector multiplet fields.

Finally we mention that we can impose that the vector field is part of the so-called rotor multiplet [10]. The fields of this are $(d, v, \lambda, \bar{\lambda})$ and the supercharges defined through

$$\{Q_a, d\} = \sigma^\mu_{ab} \partial_\mu \bar{\lambda}^b.$$

(2.31)

where $F_{\mu\nu} \equiv \partial_\mu v_\nu - \partial_\nu v_\mu$.

However in this case the general form of the gauge charge

$$[Q, v_\mu] = i \partial^\mu u \quad [Q, d] = 0 \quad \{Q, \lambda_a\} = \alpha \sigma^\mu_{ab} \partial_\mu \bar{\lambda}^b$$

is not compatible with the relation $\{Q, Q_a\} = 0$. Also if we consider that the ghost multiplets are Wess-Zumino we obtain a contradiction.

### 3 Supersymmetric QCD

#### 3.1 Supersymmetric QCD in Terms of Component Fields

It is better to illustrate the method we use to find the most general gauge invariant Lagrangian on the simplest case, namely ordinary QCD, i.e. we will briefly show how to obtain the expression (1.14) as the unique possibility. So, we consider a Wick polynomial $t$ which is tri-linear in the fields $v_\mu^j, u_j, \tilde{u}_j$ has canonical dimension 4 and null ghost number, is Lorentz covariant and gauge invariant in the sense (1.8). First we list all possible monomials compatible with all these requirements; they are:

$$f^{(1)} = f^{(1)}_{jkl} v^\mu_j v^\nu_k \partial_\mu v_\nu_l$$

and

$$g^{(1)} = g^{(1)}_{jkl} v^\mu_j u_k \partial_\mu \tilde{u}_l \quad g^{(2)} = g^{(2)}_{jkl} \partial_\mu v^\mu_j u_k \tilde{u}_l \quad g^{(3)} = g^{(3)}_{jkl} v^\mu_j \partial_\mu u_k \tilde{u}_l.$$

(3.1)

(3.2)

We now list the possible trivial Lagrangians. They are total divergences of null ghost number

$$t^{(1)} = t^{(1)}_{jkl} v^\nu_j v^\kappa_k v_\mu_l \quad t^{(2)} = t^{(2)}_{jkl} v^\nu_j u_k \tilde{u}_l.$$

(3.3)
and the co-boundary terms of ghost number $-1$:

\[ b^{(1)} = b^{(1)}_{jkl} v^\mu_j v^\nu_k u^\nu_l \]
\[ b^{(2)} = b^{(2)}_{jkl} u^\mu_j u^\nu_k u^\nu_l \]

(3.4)

Now we proceed as follows: using $\partial^\mu t^{(1)}_\mu$ it is possible to make

\[ f^{(1)}_{jkl} = - f^{(1)}_{lkj} \]

(3.5)

using $d_Q b^{(1)}$ we can make

\[ f^{(2)}_{jkl} = 0 \]

(3.6)

using $\partial^\mu t^{(2)}_\mu$ it is possible to take

\[ g^{(3)}_{jkl} = 0 \]

(3.7)

finally, using $d_Q b^{(2)}$ we can make

\[ g^{(2)}_{jkl} = g^{(2)}_{kjl} \]

(3.8)

So we are left only with three terms. If we compute $d_Q t$ and use the known identity:

\[ \partial^2 f_j = 0, \quad j = 1, 2, 3 \implies (\partial^\mu f_1)(\partial_\mu f_2) f_3 = \frac{1}{2} \partial_\mu \left[ (\partial^\mu f_1) f_2 f_3 + f_1 (\partial^\mu f_2) f_3 - f_1 f_2 (\partial^\mu f_3) \right] \]

(3.9)

the result is

\[ d_Q t = i u_j A_j + \text{total div} \]

(3.10)

where:

\[ A_j = - 2 f^{(1)}_{jkl} \partial^\nu v_k^\mu \partial_\nu v_l^\mu + (f^{(1)}_{lkj} + g^{(2)}_{kjl}) \partial_\mu v_k^\mu \partial_\nu v_l^\nu \]
\[ + ( - f^{(1)}_{jkl} + f^{(1)}_{lkj} + f^{(1)}_{klj} + g^{(1)}_{kjl}) u^\mu_k \partial_\nu v_l^\nu . \]

(3.11)

Now the gauge invariance condition (1.8) becomes

\[ u_j A_j = \partial_\mu t^\mu . \]

(3.12)

From power counting arguments it follows that the general form for $t^\mu$ is

\[ t^\mu = u_j t^\mu_j + (\partial_\nu u_j) t^{\mu \nu}_j . \]

(3.13)

We can prove that $t^{\mu \nu}_j = g^{\mu \nu} t_j$ from where $A_j = - \partial^2 t_j$; making a general ansatz for $t_j$ we obtain that we must have in fact

\[ A_j = 0 \]

(3.14)

i.e. the following system of equations:

\[ f^{(1)}_{jkl} = - f^{(1)}_{kjl} \]
\[ f^{(1)}_{lkj} + g^{(2)}_{kjl} = 0 \]
\[ - f^{(1)}_{jkl} + f^{(1)}_{lkj} + f^{(1)}_{klj} + g^{(1)}_{kjl} = 0; \]

(3.15)
the first equation, together with \( (3.14) \) amounts to the total antisymmetry of the expression \( f_{jkl} \equiv f_{jk}^{(1)} \). The second equation from the preceding system gives then \( g_{kjl}^{(2)} = 0 \). As a result we obtain the (unique) solution:

\[
t^{(1)} = f_{jkl}^{(1)}(v^\mu_j v^\nu_k \partial_\nu v_{l\mu} - v^\mu_j u_k \partial_\mu \bar{u}_l);
\]

\( (3.16) \)

it can be easily be proved, using the formula \( (3.9) \) that \( d_Q t^{(1)} \) is indeed a total divergence.

The supersymmetric case goes on the same lines only the computational difficulties grow exponentially because now we have many more terms of the type \( f, g, t \) and linear in \( \lambda' \).

The supersymmetric case goes on the same lines only the computational difficulties grow exponentially because now we have many more terms of the type \( f, g, t \) and linear in \( \lambda' \).

We would expect to obtain the expression \( (3.16) \) together with terms with supersymmetric partners. The details are given in the Appendix. It is remarkable that we obtain only two possible solutions namely the usual Yang-Mills solution for QCD \( t^{(1)} \) given above by \( (3.16) \) and

\[
t^{(2)} = f_{jkl}^{(1)}[(\lambda'_j \sigma_\mu \bar{\lambda}_k^l)v^\mu_j + 2i(\lambda'_j \bar{\zeta}_k)u_l + 2i(\bar{\lambda}_k^l \bar{\zeta}_j)u_l].
\]

\( (3.17) \)

We impose now supersymmetric invariance condition \( (1.15) \) on

\[
t \equiv t^{(1)} + t^{(2)}
\]

\( (3.18) \)

and we hope to obtain a non-trivial solution. If we are successful we must also go the the second order of perturbation theory. First we consider only the terms bilinear in \( v_\mu \) and linear in \( \lambda' \) from the commutator \([Q_a, t]\) and impose that condition that they are a sum of a co-boundary and a total divergence. It is not very hard to prove that we obtain the restriction \( f_{jkl}^{(2)} = -i f_{jk}^{(1)} \) i.e. the interaction Lagrangian should be:

\[
t = f_{jkl}[v^\mu_j v^\nu_k \partial_\nu v_{l\mu} - i(\lambda'_j \sigma_\mu \bar{\lambda}_k^l)v^\mu_j - v^\mu_j u_k \partial_\mu \bar{u}_l + 2(\lambda'_j \bar{\zeta}_k)u_l + 2(\bar{\lambda}_k^l \bar{\zeta}_j)u_l]
\]

\( (3.19) \)

where we have simplified the notation: \( f_{jkl} \equiv f_{jk}^{(1)}. \) We note that this interaction Lagrangian is \( R \)-invariant. The first two terms are standard in the literature - see for instance \( [14, 15] \). The next contribution is the standard ghost coupling from the Yang-Mills theory \( [6, 16] \). The last two contributions to the ghost coupling in seems to be absent from this analysis. So, gauge invariance in the first order is not true for the expression from \( [15] \).

We can prove after some computation that the preceding expression verifies the following equation:

\[
[Q_a, t] = d_Q s_a + \partial^\mu t_{\mu a} - 2 f_{jkl}(v^\mu_j u_k \partial_\mu \bar{\zeta}_l + 2i(\sigma_\mu^a v_j \partial_\mu \bar{\zeta}_l)u_l)
\]

\( (3.20) \)

where

\[
s_a \equiv f_{jkl}[-2\sigma_\mu^a v_j \partial_\mu \bar{\zeta}_l - i(\lambda'_j \sigma_\mu \bar{\lambda}_k^l)v^\mu_j - 2i(\lambda'_j \bar{\zeta}_k)\zeta_l - 2i(\bar{\lambda}_k^l \bar{\zeta}_j)\zeta_l + \bar{v}_j \sigma_\mu^a \bar{\lambda}_k^l v_\mu - 2\bar{v}_j \bar{\zeta}_k u_l - 2\lambda'_j \phi_l u_l]
\]

\( (3.21) \)

and

\[
t_{\mu a} \equiv f_{jkl}[\partial_\mu \chi_{ja} u_k \bar{u}_l - \chi_{ja} \partial_\mu u_k \bar{u}_l + \chi_{ja} v^\nu_k \partial_\nu v_{l\mu} - i\bar{\epsilon}_{\mu \nu \rho \sigma} \sigma_\rho^a \bar{\lambda}_k^l v^\nu_k v^\rho_l - \chi_{ja} v_\nu^k \partial_\nu v_{l\mu} + i(\lambda'_j \sigma_\mu \bar{\lambda}_k^l)\chi_{la} + g_{\mu \nu} \sigma_\rho^a \bar{v}_j \bar{\lambda}_k^l u_l + 4i g_{\mu \nu} \sigma_\rho^a \bar{v}_j \bar{\lambda}_k^l u_l]
\]

\( (3.22) \)
So, it seems that the last two terms from (3.20) - which cannot be rewritten as $d_Q s_a + \text{total divergence}$ - are apparently spoiling the supersymmetric invariance condition (1.15).

However, as we have said in the Introduction, there is a natural way to save supersymmetric invariance namely to impose instead of (1.15) the weaker form (1.21)

$$< \Psi_1, ([Q_a,t] - d_Q s_a - \partial_\mu t^\mu) \Psi_2 > = 0$$

with $\Psi_1, \Psi_2 \in Ker(Q)$ modulo $Im(Q)$. We first take $\Psi$ to be generated from the vacuum by the physical fields $v^\mu_j, \lambda'_j, d'_j$; if we consider as before, the terms bilinear in $v_\mu$ and linear in $\lambda'$ from the commutator $[Q_a,t]$ we obtain the interaction Lagrangian should be given by (3.19). It follows that we also have (3.20); however, if we apply this relation on a physical states $\Psi_j$ we obtain that (1.21) is true; indeed the extra terms give zero in this average. If we substitute in (1.21) $\Psi_j \rightarrow \Psi_j + Q\Phi_j$ the relation stays true (one has to use the anticommutativity of $Q$ with the supercharges.) So we have obtained an unique solution for supersymmetric QCD as in the Yang-Mills case.

### 3.2 Supersymmetric QCD in Terms of Superfields

From the preceding Subsection we conclude that we cannot express the interaction Lagrangian in terms of superfields such that (1.17) and (1.18) are true; indeed if this would be true then we would have (1.15) with $s_a = 0$. Even the possibility of obtaining the expression (3.19) in the form (1.17) is very unlikely. Nevertheless let us start from the superfield expression of the interaction suggested by the classical analysis [10].

Arguments from classical field theory suggests that the interaction should be an expression of the type

$$t_{\text{classical}} = t^{(1)}_{\text{classical}} + t^{(2)}_{\text{classical}}$$  \hspace{1cm} (3.23)

where

$$t^{(1)}_{\text{classical}} \equiv -\frac{i}{8} \int d\theta^2 d\bar{\theta}^2 \ f_{jkl} \left[ V_j D^a V_k \ D^2 D_a V_l - \text{H.c.} \right]$$

$$t^{(2)}_{\text{classical}} \equiv 2i \int d\theta^2 d\bar{\theta}^2 \ f_{jkl} V_j (U_k + U_k^\dagger) (\bar{U}_l + \bar{U}_l^\dagger)$$  \hspace{1cm} (3.24)

(see [10]). After a tedious computation one obtains up to a total divergence

$$t^{(1)}_{\text{classical}} = f_{jkl} \left[ C_j (\partial_\nu \chi_k \sigma^{\mu\nu} \partial_\mu \chi_l) - C_j (\partial_\nu \bar{\chi}_k \sigma^{\mu\nu} \partial_\mu \bar{\chi}_l) - 2 \partial_\mu C_j v^\mu_k dl 
+ \frac{1}{2} (\chi_j \sigma^{\mu} \partial_\mu \chi_k) dl - \frac{1}{2} d_j (\partial_\mu \chi_j \sigma^{\mu} \bar{\chi}_k) dl - 4i \phi_j \phi_k^\dagger dl 
+ v_{jkv} \phi_k \partial^{\nu} v^\nu_l - i (\lambda'_j \sigma_\mu \lambda'_k) v^\mu_l 
- i (\chi_j \sigma_\mu \partial^{\nu} \lambda'_k) v^\nu_l - i (\bar{\chi}_j \sigma_\mu \partial^{\nu} \bar{\lambda}_k) v^\nu_l 
+ 2i (\chi_j \lambda'_k) dl - 2i (\bar{\chi}_j \bar{\lambda}_k) dl \right] + \text{total divergence};$$  \hspace{1cm} (3.25)
the third line is suggested in the literature e.g. [13] see formula (C.1c) and appears in (3.19) also. The expression of the ghost coupling is much more complicated:

\[
t^{(2)}_{\text{classical}} = f_{ijkl} \left[ -v^\mu u_k \partial_i \bar{u}_l + v^\mu \partial_{i} \mu v_k \bar{v}_l - 2d_j u_k \bar{v}_l - 2 \phi_j g_k \bar{v}_l - 2 \phi_j^\dagger g_k^\dagger \bar{v}_l \\
+ 2i \phi_j u_k \bar{g}_l + 2i \phi_j^\dagger u_k^\dagger \bar{g}_l + 2i C_j g_k^\dagger \bar{g}_l + 2i C_j g_k \bar{g}_l^\dagger \\
- i (\chi_j \sigma^\mu \bar{\zeta}_k) \partial_{i} \mu \bar{u}_l + i (\zeta_j \sigma^\mu \bar{X}_k) \partial_{i} \mu \bar{u}_l + 2i (\lambda_j \zeta_k \bar{v}_l + 2i (\bar{\lambda}_j \zeta_k) \bar{v}_l \\
- (\partial_{i} \mu \chi_j \sigma^\mu \bar{\zeta}_k) \bar{v}_l + (\chi_j \sigma^\mu \partial_{i} \mu \bar{\zeta}_k) \bar{v}_l + (\partial_{i} \mu \zeta_j \sigma^\mu \bar{X}_k) \bar{v}_l - (\zeta_j \sigma^\mu \partial_{i} \mu \bar{X}_k) \bar{v}_l \\
+ 2 (\lambda_j \zeta_k) u_l + 2 (\bar{\lambda}_j \zeta_k) u_l + i (\partial_{i} \mu \chi_j \sigma^\mu \bar{\zeta}_k) u_k - i (\chi_j \sigma^\mu \partial_{i} \mu \bar{\zeta}_k) u_k \\
+ i (\partial_{i} \mu \bar{\zeta}_j \sigma^\mu \bar{X}_k) u_k - i (\zeta_i \sigma^\mu \partial_{i} \mu \bar{X}_k) u_k - (\chi_j \sigma^\mu \partial_{i} \mu \bar{z}_k) u_k + (\zeta_j \sigma^\mu \partial_{i} \mu \bar{z}_k) u_k \\
- 4i \phi_j (\zeta \zeta_k \bar{v}_l) + 4i \phi_j (\bar{\zeta}_k \bar{v}_l) + 2 C_j (\partial_{i} \mu \zeta_j \sigma^\mu \bar{z}_k) + 2 C_j (\zeta_j \sigma^\mu \partial_{i} \mu \bar{z}_k) \\
- 2 C_j (\partial_{i} \mu \zeta_j \sigma^\mu \bar{z}_k) + 2 C_j (\zeta_j \sigma^\mu \partial_{i} \mu \bar{z}_k) - 2i v^\mu_j (\zeta \zeta \bar{v}_l) - 2i v^\mu_j (\zeta \zeta \bar{v}_l) + 2 (\chi_j \zeta_k) \bar{g}_l + 2 (\bar{\chi}_j \bar{\zeta}_k) g_l^\dagger \right] + \text{total divergence} \tag{3.26}
\]

Now one can eliminate terms following the procedure given in the table from the Appendix. As a result the total expression is of the following form:

\[
t_{\text{classical}} = t + d_Q b + \partial^\mu t^\mu + \bar{t} \tag{3.27}
\]

where \( t \) is given by (3.19) and

\[
\bar{t} \equiv f_{ijkl} \left[ \frac{i}{2} (\partial_{i} \mu \chi_j \sigma^\mu \bar{X}_k) \partial_{i} \mu v^\mu_j + \frac{i}{2} (\chi_j \sigma^\mu \partial_{i} \mu \bar{X}_k) \partial_{i} \mu v^\mu_j + \phi_j (\chi_k \sigma^\mu \partial_{i} \mu \bar{X}_k) - \phi_j (\partial_{i} \mu \chi_l \sigma^\mu \bar{X}_k) \\
+ 2i \phi_j u_k \bar{g}_l + 2i \phi_j^\dagger u_k^\dagger \bar{g}_l + 2i C_j g_k^\dagger \bar{g}_l + 2i C_j g_k \bar{g}_l^\dagger \\
+ i (\partial_{i} \mu \chi_j \sigma^\mu \bar{z}_k) u_k - i (\zeta_j \sigma^\mu \partial_{i} \mu \bar{z}_k) u_k + i (\partial_{i} \mu \bar{\zeta}_j \sigma^\mu \bar{X}_k) u_k - i (\zeta_j \sigma^\mu \partial_{i} \mu \bar{X}_k) u_k \\
+ (\chi_j \sigma^\mu \partial_{i} \mu \bar{z}_k) \bar{v}_l - (\partial_{i} \mu \zeta_j \sigma^\mu \bar{X}_k) \bar{v}_l - 2 (\chi_j \zeta_k) \bar{g}_l + 2 (\bar{\chi}_j \bar{\zeta}_k) g_l^\dagger \right] \tag{3.28}
\]

is not trivial. So we do not have an exact match between classical and quantum analysis. However, there is a way of eliminating the last term used in the literature, namely to compute \( t_{\text{classical}} \) in the so-called Wess-Zumino gauge [20]. This means that the superfield \( V \) can be written as

\[
V = A + A^\dagger + V' \tag{3.29}
\]

where:

\[
V' = (\theta \sigma^\mu \bar{\theta}) v^\mu + \theta^2 \bar{\theta} \lambda' + \bar{\theta}^2 \theta \lambda' + \theta^2 \bar{\theta}^2 \ d' \tag{3.30}
\]

and

\[
A = \frac{1}{2} C + \theta \bar{X} + \bar{\theta}^2 + \frac{i}{2} (\theta \sigma^\mu \bar{\theta}) \partial_{\mu} C - \frac{i}{2} \bar{\theta}^2 (\theta \sigma^\mu \partial_{\mu} \bar{X}). \tag{3.31}
\]

If instead of \( V \) one uses \( V' \) then we have \( \bar{t} \rightarrow 0 \) so we have

\[
t_{\text{classical}} \rightarrow t + d_Q b + \partial^\mu t^\mu. \tag{3.32}
\]
The conclusion is that our expression for the interaction Lagrangian $t$ can be obtained from (3.23) - (3.26) if we substitute $V_j \rightarrow V'_j$.

Gauge invariance and supersymmetric invariance are not very conveniently expressed in terms of $V'_j$ so one must work with the interaction Lagrangian (3.23) in every order of the perturbation theory and make at the end $V_j \rightarrow V'_j$. However, going to the second order of perturbation theory might be more difficult in this superfield formalism than working in component fields.

### 3.3 Second Order of Perturbation Theory

One can go to the second order of perturbation theory very easily in components and compute the anomaly as in [7]; we have in (3.23)

$$t^\mu = f_{jkl} \left[ \frac{1}{2} u_j v_k (\partial^\nu v^\mu_l - \partial^\mu v^\nu_l) - \frac{1}{2} u_j u_k \partial^\mu \tilde{u}_l - i (\lambda'_j \sigma^\mu \tilde{\lambda}'_k) u_l \right].$$

(3.33)

From here we obtain

$$d_Q \left[ t(x), t(y) \right] = i \frac{\partial}{\partial x^\mu} \left[ t^\mu(x), t(y) \right] + i \frac{\partial}{\partial y^\mu} \left[ t(x), t^\mu(y) \right].$$

(3.34)

The anomalies are obtained in the process of causal splitting of this identity; we obtain in general

$$d_Q T(t(x), t(y)) = i \frac{\partial}{\partial x^\mu} T(t^\mu(x), t(y)) + i \frac{\partial}{\partial y^\mu} T(t(x), t^\mu(y)) + A(x, y)$$

(3.35)

where $T(a(x), b(y))$ is the chronological product associated to the Wick monomials $a$ and $b$ and $A(x, y)$ is the anomaly which spoils the gauge invariance in the second order. One finds out that

$$A(x, y) = A_{YM}(x, y) + A_{susy}(x, y)$$

(3.36)

where the first term already appears in the pure Yang-Mills case and can be eliminated (as a co-boundary plus a total divergence) if and only if one imposes Jacobi identity on the constants $f_{jkl}$. The second term from the anomaly is of purely supersymmetric nature:

$$A_{susy}(x, y) = a_{susy}(x) \delta(x - y)$$

(3.37)

with

$$a_{susy} = f_{jkl} f_{mn} \left[ -\frac{i}{2} u_j v_k (\lambda'_m \sigma_\mu \tilde{\lambda}'_n) + 2 u_j u_k (\lambda'_m \tilde{\lambda}'_n) + 2 u_j u_k (\lambda'_m \tilde{\lambda}'_n) \right].$$

(3.38)

Apparently one cannot write this expression as co-boundary plus a total divergence.

### 4 Extension to the Massive Case

The first thing we must do is to remind how one can give mass to the photon in the causal approach [7] [16]. One modifies the framework from the Introduction as follows. The Hilbert
space of the massive vector field $v_\mu$ is enlarged to a bigger Hilbert space $\mathcal{H}$ including three ghost fields $u, \tilde{u}, h$. The first two ones are Fermi scalars and the last is a Bose real scalar field; all the ghost fields are supposed to have the same mass $m > 0$ as the vector field. In $\mathcal{H}$ we can give a Hermitian structure such that we have

$$v_\mu^\dagger = v_\mu, \quad u^\dagger = u, \quad \tilde{u}^\dagger = -\tilde{u}, \quad h^\dagger = h$$

(4.1)

and we also convene that $gh(h) = 0$. Then one introduces the gauge charge $Q$ according to:

$$Q\Omega = 0, \quad Q^\dagger = Q, \quad [Q, v_\mu] = i\partial_\mu u, \quad [Q, h] = imu$$

$$\{Q, u\} = 0, \quad \{Q, \tilde{u}\} = -i(\partial^\mu v_\mu + mh)$$

(4.2)

and, because $Q^2 = 0$, the physical Hilbert space is given, as in the massless case, by $\mathcal{H}_{phys} = Ker(Q)/Im(Q)$.

The gauge charge is compatible with the following causal (anti)commutation relation:

$$[v_\mu(x), v_\nu(y)] = i \ g_{\mu\nu} D_m(x-y) \quad \{u(x), \tilde{u}^\dagger(y)\} = -i \ D_m(x-y) \quad [h(x), h(y)] = -i \ D_m(x-y)$$

(4.3)

and the other causal (anti)commutators are null. We see that the canonical dimension of the scalar ghost field must be $\omega(h) = 1$. It is an easy exercise to determine the physical space in the one-particle sector: the equivalence classes are indexed by wave functions of the form

$$\Psi_0 = \int f_\mu(x)v_\mu(x) \quad \partial^\mu f_\mu = 0.$$  

(4.4)

The general argument can be found in [7].

To go to the supersymmetric case we must include the field $h$ into a supersymmetric multiplet. Again, the natural candidate is a chiral field we take:

$$B(x, \theta, \bar{\theta}) = b(x) + 2 \bar{\theta}\bar{\psi}(x) + i (\theta\sigma^\mu\bar{\theta}) \partial_\mu b(x) + \theta^2 f(x)$$

$$-i \bar{\theta}^2 \theta\sigma^\mu\partial_\mu\bar{\psi}(x) + \frac{m^2}{4} \theta^2\bar{\theta}^2 b(x)$$

(4.5)

where $b, f$ are complex scalars and $\psi$ is a spinor field. The chirality conditions is:

$$\mathcal{D}_a B = 0.$$  

(4.6)

Some mass-dependent extra-terms $\frac{m^2}{4} \theta^2\bar{\theta}^2 u(x)$ and $\frac{m^2}{4} \theta^2\bar{\theta}^2 \tilde{u}(x)$ should be included in the expressions of (2.10) and (2.11) of $U$ and $\tilde{U}$ from subsection 2.2 respectively.

The action of the supercharges can be taken to be:

$$i \ [Q_a, b] = 0 \quad i \ [Q_a, b^\dagger] = 2\psi_a$$

$$[Q_a, f] = -2 \sigma^\mu_{ab} \partial_\mu \bar{\psi}^{\dagger}_b \quad [Q_a, f^\dagger] = 0$$

$$\{Q_a, \psi_b\} = i \epsilon_{ab}f^\dagger \quad \{Q_a, \bar{\psi}_b\} = \sigma^\mu_{ab} \partial_\mu b.$$  

(4.7)
These relations are compatible with the following canonical dimensions:

\[
\omega(b) = 1 \quad \omega(\psi) = 3/2 \quad \omega(f) = 2. \tag{4.8}
\]

It is convenient to work with the self-adjoint bosonic ghost fields:

\[
h \equiv b + b^\dagger \quad h' \equiv -i(b - b^\dagger). \tag{4.9}
\]

Next we must find the general form of the gauge charge. We proceed as in Subsection 1.2 but we do not assume that the (anti)commutator with the gauge charge raises the canonical dimension by an unit; we find out that in the massive case \(Q\) is uniquely determined by:

\[
Q\Omega = 0 \quad Q^\dagger = Q \tag{4.10}
\]

and

\[
\{Q, C\} = i\, v \quad\{Q, \nu^\mu\} = i\partial^\mu v \quad \{Q, \phi\} = -g^\dagger \quad \{Q, \phi^\dagger\} = g \quad \{Q, \sigma^\prime\} = 0 \]
\[
\{Q, \chi\} = 2i\zeta \quad \{Q, \tilde{\chi}\} = -2i\tilde{\zeta} \quad \{Q, \lambda\} = 0 \]
\[
\{Q, u\} = 0 \quad \{Q, v\} = 0 \quad \{Q, g\} = 0 \quad \{Q, g^\dagger\} = 0 \quad [Q, \zeta_a] = 0 \quad [Q, \tilde{\zeta}_a] = 0 \]
\[
\{Q, \tilde{u}\} = -i \,(\partial^\mu v^\mu + mh) \quad \{Q, \tilde{v}\} = i \,(2\tilde{v} + mh + m^2 C) \]
\[
\{Q, \tilde{g}\} = -m^2 \phi^\dagger - im f \quad \{Q, \tilde{g}^\dagger\} = -m^2 \phi + im f^\dagger \]
\[
[Q, \tilde{h}] = im \, u \quad [Q, h'] = im \, v \quad [Q, f] = -im g \quad [Q, \psi_a] = m\zeta_a \quad [Q, \tilde{\psi}_a] = m\tilde{\zeta}_a. \tag{4.11}
\]

One can express everything in terms of superfields also \[10\]:

\[
\{Q, V\} = U - U^\dagger \quad \{Q, U\} = 0
\]
\[
\{Q, \tilde{U}\} = -\frac{1}{16} \, D^2 \, \mathcal{D}^2 V - i m B \quad [Q, B] = i m U. \tag{4.12}
\]

As in the Introduction we postulate that the physical Hilbert space is the factor space \(\mathcal{H}_{phys} = Ker(Q)/Im(Q)\). Using this gauge structure it is easy to prove that the one-particle Hilbert subspace contains the following type of particles: a) a particle of mass \(m\) and spin 1 (the massive photon); b) a particle of mass \(m\) and spin 1/2 (the massive photino); c) a scalar particle of mass \(m\). Only the transversal degrees of freedom of \(v_\mu\) and \(\lambda^\prime_a\) are producing physical states.

We can now determine the causal (anti)commutation relations for the ghost fields. If we take into account the particular choice we have made for the causal (anti)commutation relations of the vector multiplet one finds out \[10\] that we have

\[
\{a(x), \tilde{a}^\dagger(y)\} = \frac{i}{2} \, D_m(x - y) \quad \{\zeta_a(x), \tilde{\zeta}_b(y)\} = -\frac{1}{4} \, \sigma^\mu_{ab} \partial_\mu D_m(x - y)
\]
\[
[b(x), b^\dagger(y)] = -\frac{i}{2} \, D_m(x - y) \quad \{\psi_a(x), \tilde{\psi}_b(y)\} = -\frac{1}{4} \, \sigma^\mu_{ab} \partial_\mu D_m(x - y) \tag{4.13}
\]
and all other causal (anti)commutators are null. So, like for ordinary Yang-Mills models, the causal (anti)commutators of the ghost fields are determined by the corresponding relations of the vector multiplet fields. So we see that the causal (anti)commutators are uniquely fixed by some natural requirements. We also can prove that it is not possible to use rotor multiplet instead of the full vector multiplet and it is not possible to use Wess-Zumino multiplets instead of the ghost multiplets.

First, we eliminate terms independent on the fields $b, f, \psi_a$ of the type $d_Q b + \partial_\mu t^\mu$ exactly as in the massless case. Now comes an important observation: suppose that we have a Wick monomial $t_B$ which has at least one factor $b, f, \psi_a$ and it is of canonical dimension $\omega(t_B) \leq 4$; if we consider the expression $d_Q t_B$ then from (4.11) it follows that $\omega(d_Q t_B) \leq 4$.

This means that the new $(b, f, \psi$-dependent terms) from the interaction Lagrangian cannot produce terms of canonical dimension 5 when commuted with the gauge charge. So, first we can proceed with the elimination of trivial Lagrangians of canonical dimension 4 like in the Appendix. This in turn means that the general solution of the gauge invariance problem must be a sum of two expressions of the type $t^{(1)}$ and $t^{(2)}$ from the preceding Section - see (3.16) and (3.17) - to which one must add $b$-dependent terms such that gauge invariance is restored. These new terms are easy to obtain. For the Yang-Mills coupling $t$ they are well-known \[7\]

\[
t^{(1)}_{\text{massive}} \equiv f_{jkl} (v_{j\mu} v_{k\nu} \partial^\mu v^\nu_l - v^\mu_j u_k \partial_\mu \bar{u}_l) + f'_{jkl} (h_j \partial_\mu h_k v^\mu_l - m_k h_j v_{k\mu} v^\mu_l - m_k h_j \bar{u}_k u_l) + f''_{jkl} h_j h_k h_l
\] (4.14)

and for the second coupling the generic expression is

\[
t^{(2)}_{\text{massive}} \equiv -i f_{jkl} [(\lambda_j^i \sigma_\mu \bar{\lambda}_k^i) v^\mu_l + 2i (\lambda_j^i \bar{\zeta}_k) u_l + 2i (\bar{\lambda}_k^i \zeta_j) u_l] + p_{jkl}^{(1)} (\lambda_j^i \psi_k) h_l + p_{jkl}^{(2)} (\lambda_j^i \bar{\psi}_k) h_l + p_{jkl}^{(3)} (\lambda_j^i \chi_k) h_l + p_{jkl}^{(4)} (\bar{\lambda}_j^i \bar{\chi}_k) h_l.
\] (4.15)

The gauge invariance condition gives well-known constraints on the constants $f', f''$ and:

\[
p_{jkl}^{(3)} = \frac{i}{2} p_{jkl}^{(4)} m_k \\
p_{jkl}^{(4)} = -\frac{i}{2} p_{jkl}^{(2)} m_k
\]

\[
i p_{jkl}^{(1)} m_l = 2 f_{jkl} m_k \\
i p_{jkl}^{(2)} m_l = -2 f_{jkl} m_k.
\] (4.16)

We see from the second set of relations that we must have

\[f_{jkl} = 0 \quad \text{for} \quad m_k \neq 0 \quad m_l = 0 \] (4.17)

which implies that the massive and massless gauge fields must decouple; this does not agree with the standard model.

Regarding supersymmetric invariance, this means that the argument which prevents the equation (1.15) to be true remains valid but (1.21) should stay true.

We note that the anomaly (3.38) will remain in this case also.

**Remark** The solution found in [12] has in fact terms of canonical dimension 5: indeed the third term of formula (4.1) from this paper produces after integration over the Grassmann variables the term $(\psi \zeta)\bar{g}$ which is of canonical dimension 5.
The New Vector Multiplet

The new vector multiplet was introduced in [9]; it is the second possibility of a multiplet which contains a vector field. First we give the definition of a Wess-Zumino multiplet: such a multiplet contains a complex scalar field $\phi$ and a spin $1/2$ Majorana field $f_a$ of the same mass $m$. The supercharges are defined in this case by:

$$\{Q_a, \phi\} = 0, \quad \{\bar{Q}_{\dot{a}}, \phi^\dagger\} = 0$$

$$i \ [Q_a, \phi^\dagger] = 2 f_a, \quad i \ [\bar{Q}_{\dot{a}}, \phi] = 2 \bar{f}_{\dot{a}}$$

$$\{Q_a, f_b\} = -i \ m \epsilon_{ab} \phi, \quad \{\bar{Q}_{\dot{a}}, \bar{f}_{\dot{b}}\} = i \ m \epsilon_{\dot{ab}} \phi^\dagger$$

$$\{Q_a, \bar{f}_{\dot{b}}\} = \sigma_{ab}^\mu \partial_\mu \phi, \quad \{\bar{Q}_{\dot{a}}, f_b\} = \sigma_{\dot{ba}}^\mu \partial_\mu \bar{\phi}^\dagger.$$  \hspace{1cm} (5.1)

The first vanishing commutators are also called (anti) chirality condition. The causal (anti)commutators are:

$$\begin{align*}
\left[\phi(x), \phi(y)^\dagger\right] &= -2i \ D_m(x - y), \\
\{f_a(x), f_b(y)\} &= i \ \epsilon_{ab} \ m \ D_m(x - y), \\
\{f_a(x), \bar{f}_{\dot{b}}(y)\} &= \sigma_{ab}^\mu \partial_\mu D_m(x - y)
\end{align*}$$

and the other (anti)commutators are zero.

To construct the new vector multiplet one and adds a vector index i.e. makes the substitutions $\phi \rightarrow v_\mu$, $f_a \rightarrow \psi_{\mu a}$; here $v_\mu$ is a complex vector field. In the massless case the action of the supercharges is:

$$\begin{align*}
\{Q_a, v_\mu\} &= 0, \quad \{\bar{Q}_{\dot{a}}, v_{\mu}^\dagger\} = 0 \\
i \ [Q_a, v_\mu^\dagger] &= 2 \psi_{\mu a}, \quad i \ [\bar{Q}_{\dot{a}}, v_\mu] = 2 \bar{\psi}_{\dot{a} \mu} \\
\{Q_a, \psi_{\mu b}\} &= 0, \quad \{\bar{Q}_{\dot{a}}, \bar{\psi}_{\dot{b} \mu}\} = 0, \\
\{Q_a, \bar{\psi}_{\dot{b}}\} &= \sigma_{ab}^\nu \partial_\nu v_\mu, \quad \{\bar{Q}_{\dot{a}}, \psi_{\mu b}\} = \sigma_{\dot{ba}}^\nu \partial_\nu v_{\mu}^\dagger.
\end{align*}$$

and the rest of the (anti)commutators are zero. It is more convenient to work with two real vector fields:

$$A_\mu = v_\mu + v_{\mu}^\dagger, \quad B_\mu = -i (v_\mu - v_{\mu}^\dagger)$$

and the gauge structure is done by considering that the ghost and the anti-ghost multiplets are also Wess-Zumino (of null mass) i.e. we can take in the analysis from Subsection 2.2 $g = 0$, $\tilde{g} = 0$.

The gauge charge is defined by

$$Q \Omega = 0 \quad Q^\dagger = Q$$

and

$$\begin{align*}
\{Q, v_\mu\} &= i \partial_\mu u, \quad \{Q, \psi_{\mu a}\} = \partial_\mu \zeta_a, \\
\{Q, a\} &= 0, \quad \{Q, \zeta\} = 0 \\
\{Q, \bar{a}\} &= -i \ \partial^\mu v_\mu, \quad \{Q, \bar{\zeta}\} = -\partial_\mu \psi_{\mu a}^\dagger
\end{align*}$$

and the rest of the (anti)commutators are zero.
such that we have

\[ [Q, A_\mu] = i\partial_\mu u \quad [Q, B_\mu] = i\partial_\mu v \]
\[ \{Q, \tilde{u}\} = -i\partial_\mu A^\mu \quad \{Q, \tilde{v}\} = -i\partial_\mu B^\mu. \] (5.7)

If we have \( r \) such multiplets i.e. the fields \( A^{\mu}_j, B^{\mu}_j, u_j, v_j, \tilde{u}_j, \tilde{v}_j \quad j = 1, \ldots, r \) then it is convenient to define the new fields \( A^{\mu}_{j+r}, u_{j+r}, \tilde{u}_{j+r}, \tilde{v}_{j+r} \quad j = 1, \ldots, 2r \) according to

\[ A^{\mu}_{j+r} = B^{\mu}_j \quad u_{j+r} = u_j \quad \tilde{u}_{j+r} = \tilde{u}_j \quad j = 1, \ldots, r. \] (5.8)

In this way the gauge structure becomes similar to the Yang-Mills case i.e.

\[ [Q, A^{\mu}_J] = i\partial^{\mu} u_J \]
\[ \{Q, u_J\} = 0 \quad \{Q, \tilde{u}_J\} = -i\partial_\mu A^{\mu}_J \] (5.9)

for \( J = 1, \ldots, 2r \).

It is not hard to prove that the gauge invariance fixes the interaction Lagrangian to be of the Yang-Mills type i.e.

\[ t = f_{JKL}(A^{\mu}_J A^{\nu}_K \partial_\nu A_{L\mu} - A^{\mu}_j u_K \partial_\mu \tilde{u}_L) \] (5.10)

where the constants \( f_{JKL} \) are real (from Hermiticity), completely antisymmetric (from first order gauge invariance) and verify Jacobi identity (from second order gauge invariance); no other terms containing spinor fields are compatible with (first order) gauge invariance. One can revert to the old variables defining

\[ f_{jkl}^{(1)} \equiv f_{jkl} \quad f_{jkl}^{(2)} \equiv f_{j+r,k+r,l+r} \]
\[ f_{jkl}^{(3)} \equiv f_{j,k+r,l} \quad f_{jkl}^{(4)} \equiv f_{j+r,k,l+r}. \] (5.11)

Then \( t \) above is a sum of four expressions:

\[ t^{(1)} = f_{jkl}^{(1)}(A^{\mu}_j A^{\nu}_k \partial_\nu A_{l\mu} - A^{\mu}_j u_k \partial_\mu \tilde{u}_l), \]
\[ t^{(2)} = f_{jkl}^{(2)}(B^{\mu}_j B^{\nu}_k \partial_\nu B_{l\mu} - B^{\mu}_j u_k \partial_\mu \tilde{u}_l), \]
\[ t^{(3)} = f_{jkl}^{(3)}(A^{\mu}_j B^{\nu}_k \partial_\nu A_{l\mu} - A^{\mu}_j A^{\nu}_k \partial_\nu B_{l\mu} - B^{\mu}_j A^{\nu}_k \partial_\nu A_{l\mu} - A^{\nu}_j u_k \partial_\mu \tilde{u}_l + A^{\nu}_j u_k \partial_\mu \tilde{v}_l + B^{\nu}_j u_k \partial_\mu \tilde{u}_l); \]
\[ t^{(4)} = f_{jkl}^{(4)}(A^{\mu}_j B^{\nu}_k \partial_\nu A_{l\mu} - 2B^{\mu}_j A^{\nu}_k A_{l\mu} + A^{\mu}_j \partial_\mu u_k \tilde{v}_l - 2A^{\mu}_j \partial_\mu u_k \tilde{u}_l). \] (5.12)

the expression \( t^{(4)} \) can be obtained from \( t^{(3)} \) performing the change \( A^{\mu}_j \leftrightarrow B^{\mu}_j, \quad u_j \leftrightarrow v_j, \quad \tilde{u}_j \leftrightarrow \tilde{v}_j \).

One can show that \( t^{(3)} \) is equivalent to a simple expression, namely:

\[ t^{(3)} \sim f_{jkl}^{(3)}(A^{\mu}_j B^{\nu}_k \partial_\nu A_{l\mu} - 2B^{\mu}_j A^{\nu}_k \partial_\nu A_{l\mu} + A^{\mu}_j \partial_\mu u_k \tilde{v}_l - 2A^{\mu}_j \partial_\mu u_k \tilde{u}_l). \] (5.13)

Now we impose the susy-invariance condition \((1.21)\); after some computations one obtains the restrictions:

\[ f_{jkl}^{(3)} = i f_{jkl}^{(1)}, \quad f_{jkl}^{(2)} = -i f_{jkl}^{(1)}, \quad f_{jkl}^{(4)} = f_{jkl}^{(1)}, \quad f_{jkl} = f_{jkl}^{(1)} \] (5.14)

which, unfortunately, are in contradiction with the reality condition. So, we cannot find a susy-invariant model even in the first order of the perturbation theory.
6 Conclusions

There is another possibility, namely to use extended supersymmetries [11]. However in this case one finds out immediately that one cannot include the usual ghost fields in some supersymmetric ghost multiplet. Our analysis indicates serious obstacles in constructing supersymmetric extensions of the standard model in the causal approach.
7 Appendix

I) We have the following non-trivial possibilities for terms of the type $A_j A_k A_l$ where $A_j$, $A_k$, $A_l$ are fields of the type $C, \phi, v^\mu, d, \chi, \chi'$:

- from the sector $C \chi \lambda$:
  
  \[
  f_{jkl}^{(1a)} = C_j (\partial_\mu \chi_k \sigma^{\mu \nu} \partial_\nu \chi_l) , \quad f_{jkl}^{(1b)} = C_j (\partial_\mu \chi_k \bar{\sigma}^{\mu \nu} \partial_\nu \chi_l) \tag{7.1}
  \]
  and terms with one derivative on $C_j$ which will be denoted generically by $\tilde{F}^{(1)}$;

- from the sector $C v_\mu v_\nu$:
  
  \[
  f_{jkl}^{(2a)} C_j \partial_\mu v_k^\nu \partial_\nu v_{l\mu} , \quad f_{jkl}^{(2b)} C_j \chi^\mu_k \partial_\nu v_{l\nu} , \quad f_{jkl}^{(2c)} \epsilon_{\mu \nu \rho \sigma} C_j \partial_\mu v_k^\nu \partial_\rho v_{l\sigma} , \quad f_{jkl}^{(2d)} C_j \partial_\mu v_k^\mu \partial_\nu v_{l\nu} \tag{7.2}
  \]
  and terms with one derivative on $C_j$ which will be denoted generically by $\tilde{F}^{(2)}$;

- from the sector $C v_\mu d$:
  
  \[
  f_{jkl}^{(3a)} C_j \chi^\mu_k \partial_\mu d_l , \quad f_{jkl}^{(3b)} C_j \partial_\mu v_{k\mu} d_l \tag{7.3}
  \]
  and terms with one derivative on $C_j$ which will be denoted generically by $\tilde{F}^{(3)}$;

- from the sector $\chi \chi \lambda$:
  
  \[
  f_{jkl}^{(4a)} C_j (\chi'_k \sigma^{\mu \nu} \partial_\mu \chi_l) , \quad f_{jkl}^{(4b)} C_j (\partial_\mu \chi'_k \sigma^{\mu \nu} \partial_\nu \chi_l) \tag{7.4}
  \]
  and terms with one derivative on $C_j$ which will be denoted generically by $\tilde{F}^{(4)}$;

- from the sector $C d d$:
  
  \[
  f_{jkl}^{(5)} C_j d_k d_l \tag{7.5}
  \]

- from the sector $\chi \chi \phi$:
  
  \[
  f_{jkl}^{(6a)} (\partial_\mu \chi_j \sigma^{\mu \nu} \partial_\nu \chi_k) \phi_l , \quad f_{jkl}^{(6b)} (\partial_\mu \chi_j \sigma^{\mu \nu} \partial_\nu \chi_k) \phi_l , \quad f_{jkl}^{(6c)} (\partial_\mu \chi_j \sigma^{\mu \nu} \partial_\nu \chi_k) \phi_l , \quad f_{jkl}^{(6d)} (\partial_\mu \chi_j \sigma^{\mu \nu} \partial_\nu \chi_k) \phi_l \tag{7.6}
  \]
  and terms with one derivative on $\phi_j$ which will be denoted generically by $\tilde{F}^{(6)}$;

- from the sector $\chi \chi v_\mu$:
  
  \[
  f_{jkl}^{(7a)} (\partial_\mu \chi_j \sigma^{\mu \nu} \partial_\nu \chi_k) v_{l\nu} , \quad f_{jkl}^{(7b)} (\partial_\mu \chi_j \sigma^{\mu \nu} \partial_\nu \chi_k) v_{l\nu} , \quad f_{jkl}^{(7c)} (\partial_\mu \chi_j \sigma^{\mu \nu} \partial_\nu \chi_k) v_{l\nu} , \quad f_{jkl}^{(7d)} (\partial_\mu \chi_j \sigma^{\mu \nu} \partial_\nu \chi_k) v_{l\nu} \tag{7.7}
  \]
  and terms with one derivative on $v_{l\nu}$ which will be denoted generically by $\tilde{F}^{(7)}$;
• from the sector $\chi \chi d$ :

$$f_{jkl}^{(8a)}(\partial_{\mu} \chi_j \sigma^\mu \bar{\chi}_k) d_l, \quad f_{jkl}^{(8b)}(\chi_j \sigma^\mu \partial_{\mu} \bar{\chi}_k) d_l$$

(7.8)

and terms with one derivative on $d_l$ which will be denoted generically by $\tilde{F}^{(8)}$;

• from the sector $\chi \lambda \dot{\phi}$ :

$$f_{jkl}^{(9a)}(\partial_{\mu} \chi_j \sigma^\mu \bar{\chi}_k) \dot{\phi}_l, \quad f_{jkl}^{(9b)}(\chi_j \sigma^\mu \partial_{\mu} \bar{\chi}_k) \dot{\phi}_l, \quad f_{jkl}^{(9c)}(\partial_{\mu} \lambda_j' \sigma^\mu \bar{\chi}_k) \dot{\phi}_l, \quad f_{jkl}^{(9d)}(\lambda_j' \sigma^\mu \partial_{\mu} \bar{\chi}_k) \dot{\phi}_l, \quad f_{jkl}^{(9e)}(\partial_{\mu} \chi_j \sigma^\mu \bar{\chi}_k) \dot{\phi}^\dagger_l, \quad f_{jkl}^{(9f)}(\chi_j \sigma^\mu \partial_{\mu} \bar{\chi}_k) \dot{\phi}^\dagger_l, \quad f_{jkl}^{(9g)}(\partial_{\mu} \lambda_j' \sigma^\mu \bar{\chi}_k) \dot{\phi}^\dagger_l, \quad f_{jkl}^{(9h)}(\lambda_j' \sigma^\mu \partial_{\mu} \bar{\chi}_k) \dot{\phi}^\dagger_l$$

(7.9)

and terms with one derivative on $\dot{\phi}_l, \dot{\phi}_l^\dagger$ which will be denoted generically by $\tilde{F}^{(9)}$;

• from the sector $\phi \phi v_\mu$

$$f_{jkl}^{(10a)} \phi_j \partial_{\mu} \phi_k v_\mu, \quad f_{jkl}^{(10b)} \phi_j \partial_{\mu} \phi^\dagger_k v_\mu, \quad f_{jkl}^{(10c)} \phi_j^\dagger \partial_{\mu} \phi_k^\dagger v_\mu, \quad f_{jkl}^{(10d)} \phi_j^\dagger \partial_{\mu} \phi_k v_\mu$$

(7.10)

and terms with one derivative on $v_\mu$ which will be denoted generically by $\tilde{F}^{(10)}$;

• from the sector $\phi \phi d$

$$f_{jkl}^{(11a)} \phi_j \phi_k d_l, \quad f_{jkl}^{(11b)} \phi_j \phi^\dagger_k d_l, \quad f_{jkl}^{(11c)} \phi_j^\dagger \phi^\dagger_k d_l;$$

(7.11)

• from the sector $\phi \lambda \lambda$

$$f_{jkl}^{(12a)} (\lambda_j' \lambda_k') \phi_l, \quad f_{jkl}^{(12b)} (\lambda_j' \lambda_k') \phi^\dagger_l, \quad f_{jkl}^{(12c)} (\bar{\lambda}_j' \bar{\lambda}_k') \phi_l, \quad f_{jkl}^{(12d)} (\bar{\lambda}_j' \bar{\lambda}_k') \phi^\dagger_l;$$

(7.12)

• from the sector $v_\mu v_\nu v_\rho$

$$f_{jkl}^{(13a)} v_\mu^\nu v_\nu^\rho \partial_{\nu} v_\mu, \quad f_{jkl}^{(13b)} v_\mu^\nu v_\nu^\rho \partial_{\nu} v_\mu^\nu$$

(7.13)

• from the sector $v_\mu \lambda \lambda$

$$f_{jkl}^{(14)} (\lambda_j' \sigma^\mu \bar{\lambda}_k') v_\mu^\nu$$

(7.14)

• from the sector $\chi \lambda v_\mu$

$$f_{jkl}^{(15a)} (\partial^\mu \chi_j \sigma_{\mu \nu} \lambda_k') v_\nu^\rho, \quad f_{jkl}^{(15b)} (\chi_j \sigma_{\mu \nu} \partial^\mu \lambda_k') v_\nu^\rho, \quad f_{jkl}^{(15c)} (\partial^\mu \bar{\chi}_j \bar{\sigma}_{\mu \nu} \bar{\lambda}_k') v_\nu^\rho, \quad f_{jkl}^{(15d)} (\bar{\chi}_j \bar{\sigma}_{\mu \nu} \partial^\mu \bar{\lambda}_k') v_\nu^\rho, \quad f_{jkl}^{(15e)} (\partial_{\mu} \chi_j \lambda_k') v_\mu^\nu, \quad f_{jkl}^{(15f)} (\chi_j \partial_{\mu} \lambda_k') v_\mu^\nu, \quad f_{jkl}^{(15g)} (\partial_{\mu} \bar{\chi}_j \bar{\lambda}_k') v_\mu^\nu, \quad f_{jkl}^{(15h)} (\bar{\chi}_j \partial_{\mu} \bar{\lambda}_k') v_\mu^\nu$$

(7.15)

and terms with one derivative on $v_\mu^\nu$ which will be denoted generically by $\tilde{F}^{(15)}$;
II. We proceed in the same way with the ghost terms. We have to consider terms of the type $A_j A_k A_l$ where $A_j = C, \phi, \phi^\dagger, \nu, d, \chi, \lambda'$ and $A_k = u, v, \zeta$ and $A_l = \tilde{u}, \tilde{v}, \tilde{g}, \tilde{\zeta}$.

- from the sector $\chi \lambda d$:
  \[ f^{(16a)}_{jkl} (\chi_j \chi'_k) d_l, \quad f^{(16b)}_{jkl} (\chi_j \chi'_k) d_l; \]  
  (7.16)

- from the sector $v_{\mu} v_{\nu} d$:
  \[ f^{(17)}_{jkl} v_{\mu} v_{\nu} d_l. \]  
  (7.17)

and terms with the derivative on $u_k$ which will be generically denoted by $\tilde{G}^{(1)}$;

- from the sector $Av\tilde{u}$:
  \[ g^{(1a)}_{jkl} v^\mu_j u_k \partial^\mu \tilde{u}_l \quad g^{(1b)}_{jkl} \partial^\mu v^\mu_j u_k \partial^\mu \tilde{u}_l \quad g^{(1c)}_{jkl} d^\mu_j u_k \partial^\mu \tilde{u}_l \]  
  (7.18)

and terms with the derivative on $v_k$ which will be generically denoted by $\tilde{G}^{(2)}$;

- from the sector $Av\tilde{v}$:
  \[ g^{(2a)}_{jkl} v^\mu_j v_k \partial^\mu \tilde{v}_l \quad g^{(2b)}_{jkl} \partial^\mu v^\mu_j v_k \partial^\mu \tilde{v}_l \quad g^{(2c)}_{jkl} d^\mu_j v_k \partial^\mu \tilde{v}_l \]  
  (7.19)

and terms with the derivative on $v_k$ which will be generically denoted by $\tilde{G}^{(3)}$;

- from the sector $Au\tilde{u}$:
  \[ g^{(3a)}_{jkl} v^\mu_j v_k \partial^\mu \tilde{u}_l \quad g^{(3b)}_{jkl} \partial^\mu v^\mu_j v_k \partial^\mu \tilde{u}_l \quad g^{(3c)}_{jkl} d^\mu_j v_k \partial^\mu \tilde{u}_l \]  
  (7.20)

and terms with the derivative on $\tilde{v}_l$ which will be generically denoted by $\tilde{G}^{(4)}$;

- from the sector $Ag\tilde{u}$:
  \[ g^{(5a)}_{jkl} \phi^\dagger_j g_k \tilde{u}_l \quad g^{(5b)}_{jkl} \phi^\dagger_j g_k \tilde{u}_l \quad g^{(5c)}_{jkl} \phi^\dagger_j g_k \tilde{u}_l \quad g^{(5d)}_{jkl} \phi^\dagger_j g_k \tilde{u}_l \]  
  (7.22)

- from the sector $Ag\tilde{v}$:
  \[ g^{(6a)}_{jkl} \phi^\dagger_j g_k \tilde{v}_l \quad g^{(6b)}_{jkl} \phi^\dagger_j g_k \tilde{v}_l \quad g^{(6c)}_{jkl} \phi^\dagger_j g_k \tilde{v}_l \quad g^{(6d)}_{jkl} \phi^\dagger_j g_k \tilde{v}_l \]  
  (7.23)

- from the sector $Au\tilde{g}$:
  \[ g^{(7a)}_{jkl} \phi^\dagger_j u_k \tilde{g}_l \quad g^{(7b)}_{jkl} \phi^\dagger_j u_k \tilde{g}_l \quad g^{(7c)}_{jkl} \phi^\dagger_j u_k \tilde{g}_l \quad g^{(7d)}_{jkl} \phi^\dagger_j u_k \tilde{g}_l \]  
  (7.24)

- from the sector $Av\tilde{g}$:
  \[ g^{(8a)}_{jkl} \phi^\dagger_j \tilde{g}_k \tilde{g}_l \quad g^{(8b)}_{jkl} \phi^\dagger_j \tilde{g}_k \tilde{g}_l \quad g^{(8c)}_{jkl} \phi^\dagger_j \tilde{g}_k \tilde{g}_l \quad g^{(8d)}_{jkl} \phi^\dagger_j \tilde{g}_k \tilde{g}_l \]  
  (7.25)
• from the sector $A g \tilde{\gamma}$:
\[
g_{jkl}^{(9a)} C_j g_k \tilde{\gamma}_l 
\]
\[
g_{jkl}^{(9b)} C_j g^+_k \tilde{\gamma}_l 
\]
\[
g_{jkl}^{(9c)} C_j g_k \tilde{\gamma}_l 
\]
\[
g_{jkl}^{(9d)} C_j g^+_k \tilde{\gamma}_l 
\]
(7.26)
• from the sector $A \zeta \tilde{\mu}$:
\[
g_{jkl}^{(10a)} (\lambda'_j \zeta_k) \tilde{\mu}_l 
\]
\[
g_{jkl}^{(10b)} (\lambda'_j \zeta_k) \tilde{\mu}_l 
\]
\[
g_{jkl}^{(10c)} (\partial_\mu \chi_j \sigma^\mu \zeta_k) \tilde{\mu}_l 
\]
\[
g_{jkl}^{(10d)} (\chi_j \sigma^\mu \partial_\mu \zeta_k) \tilde{\mu}_l 
\]
\[
g_{jkl}^{(10e)} (\partial_\mu \zeta_k \sigma^\mu \tilde{\chi}_j) \tilde{\mu}_l 
\]
\[
g_{jkl}^{(10f)} (\zeta_k \sigma^\mu \partial_\mu \tilde{\chi}_j) \tilde{\mu}_l 
\]
(7.27)
and terms with the derivative on $\tilde{\mu}_l$ which will be generically denoted by $\tilde{G}^{(10)}$;
• from the sector $A \zeta \tilde{\nu}$:
\[
g_{jkl}^{(11a)} (\lambda'_j \zeta_k) \tilde{\nu}_l 
\]
\[
g_{jkl}^{(11b)} (\lambda'_j \zeta_k) \tilde{\nu}_l 
\]
\[
g_{jkl}^{(11c)} (\partial_\mu \chi_j \sigma^\mu \zeta_k) \tilde{\nu}_l 
\]
\[
g_{jkl}^{(11d)} (\chi_j \sigma^\mu \partial_\mu \zeta_k) \tilde{\nu}_l 
\]
\[
g_{jkl}^{(11e)} (\partial_\mu \zeta_k \sigma^\mu \tilde{\chi}_j) \tilde{\nu}_l 
\]
\[
g_{jkl}^{(11f)} (\zeta_k \sigma^\mu \partial_\mu \tilde{\chi}_j) \tilde{\nu}_l 
\]
(7.28)
and terms with the derivative on $\tilde{\nu}_l$ which will be generically denoted by $\tilde{G}^{(11)}$;
• from the sector $A u \tilde{\zeta}$:
\[
g_{jkl}^{(12a)} (\lambda'_j \zeta_k) u_k 
\]
\[
g_{jkl}^{(12b)} (\lambda'_j \zeta_k) u_k 
\]
\[
g_{jkl}^{(12c)} (\partial_\mu \chi_j \sigma^\mu \zeta_k) u_k 
\]
\[
g_{jkl}^{(12d)} (\chi_j \sigma^\mu \partial_\mu \zeta_k) u_k 
\]
\[
g_{jkl}^{(12e)} (\partial_\mu \zeta_k \sigma^\mu \tilde{\chi}_j) u_k 
\]
\[
g_{jkl}^{(12f)} (\zeta_k \sigma^\mu \partial_\mu \tilde{\chi}_j) u_k 
\]
(7.29)
and terms with the derivative on $u_k$ which will be generically denoted by $\tilde{G}^{(12)}$;
• from the sector $A v \tilde{\zeta}$:
\[
g_{jkl}^{(13a)} (\lambda'_j \zeta_k) \tilde{v}_l 
\]
\[
g_{jkl}^{(13b)} (\lambda'_j \zeta_k) \tilde{v}_l 
\]
\[
g_{jkl}^{(13c)} (\partial_\mu \chi_j \sigma^\mu \zeta_k) \tilde{v}_l 
\]
\[
g_{jkl}^{(13d)} (\chi_j \sigma^\mu \partial_\mu \zeta_k) \tilde{v}_l 
\]
\[
g_{jkl}^{(13e)} (\partial_\mu \zeta_k \sigma^\mu \tilde{\chi}_j) \tilde{v}_l 
\]
\[
g_{jkl}^{(13f)} (\zeta_k \sigma^\mu \partial_\mu \tilde{\chi}_j) \tilde{v}_l 
\]
(7.30)
and terms with the derivative on $v_k$ which will be generically denoted by $\tilde{G}^{(13)}$;
• from the sector $A \zeta \tilde{\zeta}$:
\[
g_{jkl}^{(14a)} \phi_j (\zeta_k \tilde{\zeta}) 
\]
\[
g_{jkl}^{(14b)} \phi^+_j (\zeta_k \tilde{\zeta}) 
\]
\[
g_{jkl}^{(14c)} \phi_j (\zeta_k \tilde{\zeta}) 
\]
\[
g_{jkl}^{(14d)} \phi^+_j (\zeta_k \tilde{\zeta}) 
\]
\[
g_{jkl}^{(14e)} C_j (\partial_\mu \zeta_k \sigma^\mu \tilde{\zeta}_l) 
\]
\[
g_{jkl}^{(14f)} C_j (\tilde{\zeta}_l \sigma^\mu \partial_\mu \zeta_k) 
\]
\[
g_{jkl}^{(14g)} C_j (\tilde{\zeta}_l \sigma^\mu \partial_\mu \zeta_k) 
\]
\[
g_{jkl}^{(14h)} C_j (\zeta_k \sigma^\mu \partial_\mu \tilde{\zeta}_l) 
\]
\[
g_{jkl}^{(14i)} v^\mu_j (\zeta_k \sigma^\mu \tilde{\zeta}_l) 
\]
\[
g_{jkl}^{(14j)} v^\mu_j (\tilde{\zeta}_l \sigma^\mu \zeta_k) 
\]
(7.31)
and terms with the derivative on $C_j$ which will be generically denoted by $\tilde{G}^{(14)}$;
• from the sector $A\zeta \tilde{g}$:

$$
\begin{align*}
&g_{jkl}^{(15a)} (x_j \zeta_k) \tilde{g}_l \\
&g_{jkl}^{(15b)} (x_j \zeta_k) \tilde{g}_l \\
&g_{jkl}^{(15c)} (\bar{x}_j \zeta_l) \tilde{g}_l \\
&g_{jkl}^{(15d)} (\bar{x}_j \zeta_l) \tilde{g}_l
\end{align*}
\quad (7.32)
$$

• from the sector $Ag\tilde{c}$:

$$
\begin{align*}
&g_{jkl}^{(16a)} (x_j \zeta_l) g_k \\
&g_{jkl}^{(16b)} (x_j \zeta_l) g_k \\
&g_{jkl}^{(16c)} (\bar{x}_j \zeta_l) g_k \\
&g_{jkl}^{(16d)} (\bar{x}_j \zeta_l) g_k
\end{align*}
\quad (7.33)
$$

Some of the possible terms have been discarded according to the “magic” formula. The coefficients are subject to various obvious (anti)symmetry properties.

III.) We have total divergence terms $f_{\mu}^{(j)}$, $j = 1, \ldots, 20$ in the sectors

$$
\begin{align*}
f^{(1)} - f^{(4)}, f^{(6)} - f^{(10)}, f^{(13)}, f^{(15)}, g^{(1)} - g^{(4)}, g^{(10)} - g^{(14)}
\end{align*}
$$

respectively.

IV.) We also have the co-boundary terms $d_{\alpha} b$ to eliminate some of the previous expressions; we list the possible expressions $b$:

• of the type $AA'\tilde{u}$:

$$
\begin{align*}
&b_{jkl}^{(1a)} \phi_j \phi_k \tilde{u}_l \\
&b_{jkl}^{(1b)} \phi_j \phi_k \tilde{v}_l \\
&b_{jkl}^{(1c)} \phi_j \phi_k \tilde{u}_l \\
&b_{jkl}^{(1d)} \phi_j \phi_k \tilde{v}_l \\
&b_{jkl}^{(1e)} C_j \tilde{u}_l
\end{align*}
\quad (7.34)
$$

• of the type $AA'\tilde{v}$:

$$
\begin{align*}
&b_{jkl}^{(2a)} \phi_j \phi_k \tilde{v}_l \\
&b_{jkl}^{(2b)} \phi_j \phi_k \tilde{v}_l \\
&b_{jkl}^{(2c)} \phi_j \phi_k \tilde{v}_l \\
&b_{jkl}^{(2d)} \phi_j \phi_k \tilde{v}_l \\
&b_{jkl}^{(2e)} C_j \tilde{v}_l
\end{align*}
\quad (7.35)
$$

• of the type $AA'\tilde{g}$:

$$
\begin{align*}
&b_{jkl}^{(3a)} C_j \phi_k \tilde{g}_l \\
&b_{jkl}^{(3b)} C_j \phi_k \tilde{g}_l \\
&b_{jkl}^{(3c)} C_j \phi_k \tilde{g}_l \\
&b_{jkl}^{(3d)} C_j \phi_k \tilde{g}_l \\
&b_{jkl}^{(3e)} (x_j \chi_k) \tilde{g}_l
\end{align*}
\quad (7.36)
$$

• of the type $AA'\tilde{\zeta}$:

$$
\begin{align*}
&b_{jkl}^{(4a)} \phi_j (x_k \tilde{\zeta}) \\
&b_{jkl}^{(4b)} \phi_j (x_k \tilde{\zeta}) \\
&b_{jkl}^{(4c)} \phi_j (x_k \tilde{\zeta}) \\
&b_{jkl}^{(4d)} \phi_j (x_k \tilde{\zeta}) \\
&b_{jkl}^{(4e)} C_j (x_k \sigma^\mu \tilde{\zeta}) \\
&b_{jkl}^{(4f)} C_j (x_k \sigma^\mu \tilde{\zeta}) \\
&b_{jkl}^{(4g)} C_j (x_k \sigma^\mu \tilde{\zeta}) \\
&b_{jkl}^{(4h)} C_j (x_k \sigma^\mu \tilde{\zeta})
\end{align*}
\quad (7.37)
• tri-linear in the ghost fields:

\[ b_{ijkl}^{(5a)} u_j \bar{u}_k \bar{u}_l \quad b_{ijkl}^{(5b)} u_j \bar{u}_k \bar{v}_l \quad b_{ijkl}^{(5c)} v_j \bar{u}_k \bar{v}_l \quad b_{ijkl}^{(5d)} v_j \bar{u}_k \bar{u}_l \quad b_{ijkl}^{(5e)} v_j \bar{u}_k \bar{v}_l \quad b_{ijkl}^{(5f)} v_j \bar{u}_k \bar{v}_l. \] (7.38)

V.) Now we eliminate terms of the type \( f \) and \( g \) using total divergences. We have to pay
care to the order in which we proceed because if we use a total divergence (or a co-boundary)
we must not modify terms which have already been fixed. We proceed as follows: first we use
\( t_{ij}^{(10)} \equiv t_{ijkl} v^\nu_{jk} v_{i\mu} \quad t_{ijkl} = t_{kjl}; \) (7.39)
because
\[ \partial^\mu t_{ij}^{(10)} = t_{ijkl} (2 \partial^\mu v^\nu_{jk} v_{i\mu} + v^\nu_{jk} \partial^\mu v^\nu_{i\mu}) \] (7.40)
it is possible to take
\[ f_{ijkl}^{(1a)} = -f_{ijkl}^{(1a)}; \] (7.41)
For simplicity we denote from now on: \( f_{ijkl}^{(13)} \equiv f_{ijkl}^{(13a)}. \)
We find convenient to describe the various redefinitions in the following table:

| Nr. crt. | \( t_{ij} \quad b_{ijkl} \) | Restrictions | Modified Terms |
|----------|-----------------|--------------|----------------|
| 1        | \( t_{ij}^{(10)} \) | \( f_{ijkl}^{(13a)} = -(j \leftrightarrow l) \) | \( f_{ijkl}^{(13b)} \) |
| 2        | \( b_{ijkl}^{(1d)} \) | \( f_{ijkl}^{(13b)} = 0 \) | \( G_{ijkl}^{(1)} \) |
| 3        | \( t_{ij}^{(12)} \) | \( G_{ijkl}^{(1)} = 0 \) | \( g_{ijkl}^{(1a)}, g_{ijkl}^{(1b)} \) |
| 4        | \( b_{ijkl}^{(1a)} \) | \( g_{ijkl}^{(5c)} = -(j \leftrightarrow k) \) | \( F_{ijkl}^{(10)} \) |
| 5        | \( b_{ijkl}^{(1b)} \) | \( g_{ijkl}^{(5d)} = -(j \leftrightarrow k) \) | \( F_{ijkl}^{(10)} \) |
| 6        | \( b_{ijkl}^{(1c)} \) | \( g_{ijkl}^{(5e)} = 0 \) | \( F_{ijkl}^{(10)}, g_{ijkl}^{(5a)} \) |
| 7        | \( b_{ijkl}^{(1e)} \) | \( g_{ijkl}^{(5d)} = 0 \) | \( F_{ijkl}^{(10)}, g_{ijkl}^{(5b)} \) |
| 8        | \( b_{ijkl}^{(1f)} \) | \( g_{ijkl}^{(10a)} = 0 \) | \( F_{ijkl}^{(15)} \) |
| 9        | \( b_{ijkl}^{(1g)} \) | \( g_{ijkl}^{(10b)} = 0 \) | \( F_{ijkl}^{(15)} \) |
| 10       | \( t_{ij}^{(16)} \) | \( G_{ijkl}^{(10)} = 0 \) | \( g_{ijkl}^{(10a)}, \ldots, g_{ijkl}^{(10f)} \) |
| 11       | \( b_{ijkl}^{(1h)} \) | \( g_{ijkl}^{(10c)} = 0 \) | \( g_{ijkl}^{(10e)}, F_{ijkl}^{(7)} \) |
| 12       | \( b_{ijkl}^{(1i)} \) | \( g_{ijkl}^{(10c)} = 0 \) | \( g_{ijkl}^{(10e)}, F_{ijkl}^{(7)} \) |
| 13       | \( t_{ij}^{(2j)} \) | \( F_{ijkl}^{(2)} \) | \( f_{ijkl}^{(2a)}, \ldots, f_{ijkl}^{(2d)} \) |
| 14       | \( t_{ij}^{(14)} \) | \( G_{ijkl}^{(3)} = 0 \) | \( g_{ijkl}^{(3a)}, g_{ijkl}^{(3b)} \) |
| 15       | \( b_{ijkl}^{(1j)} \) | \( f_{ijkl}^{(2d)} = 0 \) | \( g_{ijkl}^{(3b)} \) |
| 16       | \( b_{ijkl}^{(1h)} \) | \( f_{ijkl}^{(2b)} = 0 \) | \( g_{ijkl}^{(3a)} \), total div |
| 17       | \( b_{ijkl}^{(2a)} \) | \( g_{ijkl}^{(6)} = -(j \leftrightarrow k) \) | \( f_{ijkl}^{(11a)} \) |
| 18       | \( b_{ijkl}^{(2b)} \) | \( g_{ijkl}^{(6d)} = -(j \leftrightarrow k) \) | \( f_{ijkl}^{(11c)} \) |
| 19       | \( b_{ijkl}^{(2c)} \) | \( g_{ijkl}^{(6b)} = -(j \leftrightarrow k) \) | \( g_{ijkl}^{(6a)}, f_{ijkl}^{(116)} \) |
| 20       | \( t_{ij}^{(15)} \) | \( G_{ijkl}^{(4)} = 0 \) | \( g_{ijkl}^{(4a)}, g_{ijkl}^{(4b)} \) |
Using (2.27) we can compute the expression \( d_Q T \). We exhibit it in the form

\[
d_Q T = d_Q t^{(3)} + iu_j A_j + iv_j B_j + \sum \frac{g_j}{g^{(2c)}} B'_j + i\xi_j X_j + i\xi_j X'_j + \text{total div}
\]

(7.42)

where the first term is tri-linear in the ghost fields and the expressions \( A_j, A'_j, B_j, B'_j, X_j, X'_j \) are independent of the ghost fields. We impose (1.8) and note that the first term above must
be a total divergence by himself. The explicit expression is
\[ d_q t^{(3)} = -g^{(5a)}_{jkl} g^+_j g_k \tilde{u}_l + g^{(5b)}_{jkl} g_j g_k \tilde{u}_l - g^{(5c)}_{jkl} g^+_j g_k \tilde{u}_l - g^{(5d)}_{jkl} g^+_j g_k \tilde{v}_l - g^{(5e)}_{jkl} g^+_j g_k \tilde{v}_l - g^{(5f)}_{jkl} g^+_j g_k \tilde{v}_l - g^{(7a)}_{jkl} g^+_j u_k \tilde{g}_l + g^{(7b)}_{jkl} g^+_j u_k \tilde{g}_l - g^{(7c)}_{jkl} g^+_j u_k \tilde{g}_l + g^{(7d)}_{jkl} g^+_j u_k \tilde{g}_l + i g^{(9a)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9b)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9c)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9d)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9e)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9f)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9g)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9h)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9i)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9j)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9k)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9l)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9m)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9n)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9o)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9p)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9q)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9r)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9s)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9t)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9u)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9v)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9w)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9x)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9y)}_{jkl} v_j g_k \tilde{g}_l + i g^{(9z)}_{jkl} v_j g_k \tilde{g}_l + i g^{(10a)}_{jkl} (\zeta_j \sigma^\mu \partial_\mu \tilde{\zeta}_k) \tilde{u}_l - 2i g^{(10b)}_{jkl} (\partial_\mu \zeta_j \sigma^\mu \tilde{\zeta}_k) \tilde{u}_l + 2i g^{(11c)}_{jkl} (\zeta_j \sigma^\mu \partial_\mu \tilde{\zeta}_k) \tilde{v}_l - 2i g^{(11d)}_{jkl} (\partial_\mu \zeta_j \sigma^\mu \tilde{\zeta}_k) \tilde{v}_l + 2i g^{(12a)}_{jkl} (\partial_\mu \zeta_j \sigma^\mu \tilde{\zeta}_k) u_k + 2i g^{(12b)}_{jkl} (\zeta_j \sigma^\mu \partial_\mu \tilde{\zeta}_k) u_k - 2i g^{(12c)}_{jkl} (\partial_\mu \zeta_j \sigma^\mu \tilde{\zeta}_k) u_k - 2i g^{(12d)}_{jkl} (\zeta_j \sigma^\mu \partial_\mu \tilde{\zeta}_k) u_k + 2i g^{(13e)}_{jkl} (\partial_\mu \zeta_j \sigma^\mu \tilde{\zeta}_k) v_k + 2i g^{(13f)}_{jkl} (\zeta_j \sigma^\mu \partial_\mu \tilde{\zeta}_k) v_k - 2i g^{(13g)}_{jkl} (\partial_\mu \zeta_j \sigma^\mu \tilde{\zeta}_k) v_k - 2i g^{(13h)}_{jkl} (\zeta_j \sigma^\mu \partial_\mu \tilde{\zeta}_k) v_k + 2i g^{(15a)}_{jkl} (\zeta_j \zeta_k) \tilde{g}_l + 2i g^{(15b)}_{jkl} (\zeta_j \zeta_k) \tilde{g}_l - 2i g^{(15c)}_{jkl} (\tilde{\zeta}_j \tilde{\zeta}_k) \tilde{g}_l - 2i g^{(15d)}_{jkl} (\tilde{\zeta}_j \tilde{\zeta}_k) \tilde{g}_l + 2i g^{(16a)}_{jkl} (\zeta_j \tilde{\zeta}_k) g_k + 2i g^{(16b)}_{jkl} (\zeta_j \tilde{\zeta}_k) g_k - 2i g^{(16c)}_{jkl} (\tilde{\zeta}_j \tilde{\zeta}_k) g_k - 2i g^{(16d)}_{jkl} (\tilde{\zeta}_j \tilde{\zeta}_k) g_k \] (7.43)
and it is easy to see that \( d_q t^{(3)} \) is a total divergence iff it is identically zero. This amounts to
\[ g^{(p)} = 0 \quad p = 5, 6, 7, 9, 10, 11, 15, 16 \]
\[ g^{(12a)}_{jkl} = \ldots = g^{(12f)}_{jkl} = 0 \]
\[ g^{(13a)}_{jkl} = 0 \quad \ldots \quad g^{(13f)}_{jkl} = 0. \] (7.44)

The expressions \( A_j, A'_j, B_j, B'_j, X_j, X'_j \) have the following form
\[ A_j = -2 f^{(13)}_{jkl} \partial^\nu v_k^\mu \partial_\mu v_\nu + (-f^{(13)}_{jkl} + f^{(13)}_{ikj} + f^{(1a)}_{kjl} \sigma^\mu \partial_\mu v_\nu + f^{(1b)}_{kjl} \partial_\mu v_\nu) + f^{(14)}_{ikj} + f^{(14a)}_{kjl} \partial^\mu v_k^\nu \partial_\mu v^\nu \]
\[ - f^{(14)}_{ikj} - f^{(12a)}_{kjl} (\partial_\mu X'_k \sigma^\mu \partial_\nu X'_k) - \left( f^{(14)}_{ikj} + f^{(12a)}_{kjl} \right) (\partial_\mu X'_k \sigma^\mu \partial_\nu X'_k) - \left( f^{(15a)}_{kjl} - f^{(15b)}_{kjl} \right) (\partial_\mu X'_k \sigma^\mu \partial_\nu X'_k) - \left( f^{(15c)}_{kjl} - f^{(15d)}_{kjl} \right) (\partial_\mu X'_k \sigma^\mu \partial_\nu X'_k) - 2 (f^{(17)}_{jkl} - g^{(4a)}_{klj}) \partial^\mu v_k^\nu \partial_\mu v_\nu - 2 \left( f^{(17)}_{jkl} - g^{(4a)}_{klj} \right) \partial^\mu v_k^\nu \partial_\mu v_\nu - 2 \left( f^{(17)}_{jkl} - g^{(4a)}_{klj} \right) \partial^\mu v_k^\nu \partial_\mu v_\nu - 2 g^{(4c)}_{klj} \partial^\mu v_k^\nu \partial_\mu v_\nu \] (7.45)

\[ B_j = f^{(1a)}_{jkl} (\partial_\mu \chi_k \sigma^\mu \partial_\nu \lambda'_k) + f^{(1b)}_{jkl} (\partial_\mu \chi_k \sigma^\mu \partial_\nu \lambda'_k) + f^{(2a)}_{jkl} \partial^\nu v_k^\mu \partial_\mu v_\nu + f^{(2c)}_{jkl} \epsilon_{\mu\nu\sigma\tau} \partial_\mu v^\nu \partial_\nu v^\sigma \partial_\sigma v^\tau + f^{(4a)}_{jkl} (\lambda'_k \sigma^\mu \partial_\mu \lambda'_k) + f^{(4b)}_{jkl} (\partial_\mu \lambda'_k \sigma^\mu \lambda'_k) - 2 g^{(2a)}_{jkl} \partial^\mu v_k^\nu \partial_\mu v_\nu - 2 g^{(26a)}_{jkl} \partial^\mu v_k^\nu \partial_\mu v_\nu - 2 g^{(2e)}_{jkl} \partial^\mu v_k^\nu \partial_\mu v_\nu \] (7.46)
\[ A'_j = f^{(6b)}_{ijk} (\partial_{\mu} \chi_k \sigma^{\mu \nu} \partial_\nu \chi_i) + f^{(6d)}_{ijk} (\partial_{\mu} \bar{\chi}_k \bar{\sigma}^{\mu \nu} \partial_\nu \bar{\chi}_i) + f^{(9e)}_{ijk} (\partial_{\mu} \chi_l \sigma^{\mu} \bar{\chi}_k) + f^{(9f)}_{ijk} (\chi_l \sigma^{\mu} \partial_{\mu} \bar{\chi}_k) + f^{(9g)}_{ijk} (\partial_{\mu} \lambda^l_i \sigma^{\mu} \bar{\chi}_k) + f^{(9h)}_{ijk} (\lambda^l_i \sigma^{\mu} \partial_{\mu} \bar{\chi}_k) + f^{(10a)}_{ijkl} \partial_\mu \phi_k \nu_\mu - f^{(10b)}_{ijkl} \phi_k \partial_\mu \nu_\mu + f^{(10c)}_{ijkl} \partial_\mu \nu^\dagger_k \nu^\mu - f^{(10d)}_{ijkl} \phi_k^\dagger \partial_\mu \nu^\mu + f^{(11a)}_{ijkl} \phi_k d_l + 2 f^{(11c)}_{ijkl} \phi^\dagger_k d_l + f^{(12a)}_{ilkj} (\lambda^l_k \lambda^l_i) + f^{(12b)}_{ilkj} (\bar{\lambda}^l_k \bar{\lambda}^l_i) \tag{7.47} \]

\[ B'_j = -f^{(6a)}_{ijk} (\partial_{\mu} \chi_l \sigma^{\mu \nu} \partial_\nu \chi_k) - f^{(6c)}_{ijk} (\partial_{\mu} \bar{\chi}_l \bar{\sigma}^{\mu \nu} \partial_\nu \bar{\chi}_k) + f^{(9a)}_{ijk} (\partial_{\mu} \chi_l \sigma^{\mu} \bar{\chi}_k) - f^{(9b)}_{ijk} (\chi_l \sigma^{\mu} \partial_{\mu} \bar{\chi}_k) - f^{(9c)}_{ijk} (\partial_{\mu} \lambda^l_i \sigma^{\mu} \bar{\chi}_k) + f^{(9d)}_{ijk} (\lambda^l_i \sigma^{\mu} \partial_{\mu} \bar{\chi}_k) + f^{(10a)}_{ijkl} \partial_\mu \phi_k \nu_\mu + f^{(10b)}_{ijkl} \phi_k \partial_\mu \nu_\mu + f^{(10c)}_{ijkl} \partial_\mu \nu^\dagger_k \nu^\mu + f^{(10d)}_{ijkl} \phi_k^\dagger \partial_\mu \nu^\mu - 2 f^{(11a)}_{ijkl} \phi_k d_l - f^{(11b)}_{ijkl} \phi^\dagger_k d_l - f^{(12a)}_{ilkj} (\lambda^l_k \lambda^l_i) - f^{(12c)}_{ilkj} (\bar{\lambda}^l_k \bar{\lambda}^l_i) \tag{7.48} \]

\[ X_j = -2 f^{(1a)}_{ijkl} \partial_\mu C_k \sigma^{\mu \nu} \partial_\nu \lambda^l_i - 4 f^{(6a)}_{jkl} \sigma^{\mu \nu} \partial_\nu \chi_k \partial_\mu \phi_l - 4 f^{(6b)}_{jkl} \sigma^{\mu \nu} \partial_\nu \bar{\chi}_k \partial_\mu \phi_l^\dagger + 2 (-f^{(7a)}_{jkl} + f^{(7b)}_{jkl} + f^{(7c)}_{jkl} + f^{(7d)}_{jkl}) \sigma^{\mu} \partial_\mu \partial_\nu \chi_k \nu^\nu + 2 (-f^{(7a)}_{jkl} + f^{(7c)}_{jkl}) \sigma^{\mu} \partial_\nu \bar{\chi}_k \partial_\mu \nu^\nu + 2 (-f^{(7b)}_{jkl} + f^{(7c)}_{jkl}) \sigma^{\mu} \partial_\mu \bar{\chi}_k \partial_\nu \nu^\nu + 2 f^{(7c)}_{jkl} \sigma^{\mu} \chi_k \partial_\nu \nu^\nu + f^{(7a)}_{jkl} \epsilon_{\mu \nu \rho \lambda} \sigma^{\rho} \bar{\chi}_k \partial_\nu \nu^\lambda \]

\[ + 2 (-f^{(8a)}_{jkl} + f^{(8b)}_{jkl}) \sigma^{\mu} \partial_\mu \bar{\chi}_k d_l - 2 f^{(8a)}_{jkl} \sigma^{\mu} \partial_\mu \bar{\chi}_k d_l + 2 (-f^{(9a)}_{jkl} + f^{(9b)}_{jkl}) \sigma^{\mu} \partial_\mu \lambda^l_i \phi_l + 2 f^{(9a)}_{jkl} \sigma^{\mu} \lambda^l_i \phi_l + 2 (-f^{(9a)}_{jkl} + f^{(9b)}_{jkl}) \sigma^{\mu} \lambda^l_i \phi_l^\dagger + 2 (-f^{(10a)}_{jkl} + f^{(10b)}_{jkl}) \sigma^{\mu} \partial_\mu \lambda^l_i \phi_l^\dagger - 2 f^{(10a)}_{jkl} \sigma^{\mu} \lambda^l_i \phi_l^\dagger + (-2 f^{(15a)}_{jkl} + 2 f^{(15b)}_{jkl}) \sigma_{\mu \nu} \partial_\mu \lambda^l_k \nu^\nu - 2 f^{(15a)}_{jkl} \sigma_{\mu \nu} \lambda^l_k \partial_\mu \nu^\nu + 2 (-f^{(15a)}_{jkl} + f^{(15b)}_{jkl}) \partial_\mu \lambda^l_k \nu^\nu - 2 f^{(15a)}_{jkl} \lambda^l_k \partial_\mu \nu^\nu + 2 f^{(15a)}_{jkl} \lambda^l_k d_l \tag{7.49} \]
\[ X'_j = 2f^{(1b)}_{jkl} \partial_\mu C_k \bar{\sigma}^{\mu\nu} \partial_\nu \bar{\chi}'_l \]
\[ + 4f^{(6c)}_{jkl} \partial_\nu \bar{\chi}_k \bar{\sigma}^{\mu\nu} \partial_\mu \phi_l + 4f^{(6d)}_{jkl} \partial_\nu \bar{\chi}_k \bar{\sigma}^{\mu\nu} \partial_\mu \phi_l \]
\[ + 2(-f^{(7a)}_{jkl} + f^{(7b)}_{jkl} + f^{(7c)}_{jkl} + f^{(7d)}_{jkl}) \partial_\mu \partial_\nu \bar{\chi}_k \sigma^\mu v^\nu_l \]
\[ + 2(-f^{(7a)}_{jkl} + f^{(7d)}_{jkl}) \partial_\mu \chi_k \sigma^\nu \partial_\nu v^\mu_l \]
\[ + 2(-f^{(7b)}_{jkl} + f^{(7d)}_{jkl}) \partial_\mu \chi_k \sigma^\nu \partial_\nu v^\mu_l \]
\[ + 2(-f^{(7b)}_{jkl} + f^{(7d)}_{jkl}) \partial_\mu \chi_k \sigma^\nu \partial_\nu v^\mu_l \]
\[ - 2f^{(7e)}_{jkl} \epsilon_{\mu\nu\rho\lambda} \partial^\mu \chi_j \sigma^\nu \partial^\rho v^\lambda_l \]
\[ + 2(f^{(8a)}_{jkl} - f^{(8b)}_{jkl}) \sigma^\mu \partial_\mu \bar{\chi}_k \partial_\nu \chi^\nu_l - 2f^{(8d)}_{jkl} \partial_\mu \chi_k \sigma^\nu \partial_\nu v^\mu_l \]
\[ + 2(f^{(9c)}_{jkl} - f^{(9d)}_{jkl}) \partial_\mu \chi_k \sigma^\nu \partial_\nu \phi_l - 2f^{(9d)}_{jkl} \chi^\nu_l \sigma^\mu \partial_\mu \phi_l \]
\[ + 2(f^{(9g)}_{jkl} - f^{(9h)}_{jkl}) \partial_\mu \chi_k \sigma^\nu \partial_\nu \phi_l - 2f^{(9h)}_{jkl} \chi^\nu_l \sigma^\mu \partial_\mu \phi_l \]
\[ + 2(f^{(15c)}_{jkl} - f^{(15d)}_{jkl}) \partial_\mu \chi'_k \sigma^\mu v^\nu_l + 2f^{(15g)}_{jkl} \chi^\nu_l \sigma^\nu \partial_\mu v^\lambda_l \]
\[ - 2f^{(15h)}_{jkl} \chi^\nu_l \sigma^\nu \partial_\mu v^\lambda_l \]
\[ - 2f^{(16b)}_{jkl} \chi^\nu_l \sigma^\nu \partial_\mu v^\lambda_l \] (7.50)

If we make a general ansatz

\[ T^\mu = u_j T^\mu_j + (\partial_\nu u_j)T^\mu_{j\nu} + (\partial_\nu \partial_\rho u_j)T^\mu_{j\nu\rho} \] (7.51)

we can prove, as in the Yang-Mills case, that we must have in fact

\[ A_j = 0 \quad A'_j = 0 \quad B_j = 0 \quad B'_j = 0 \quad X_j = 0 \quad X'_j = 0. \] (7.52)

In particular we have from the coefficient of \( \partial^\nu v^\mu_l \partial_\mu v^\nu_l \) that

\[ f^{(13)}_{jkl} = -f^{(13)}_{jkl}; \] (7.53)

together with

\[ g^{(1b)}_{jkl} = g^{(1b)}_{jkl}; \] (7.54)

this amounts to the total antisymmetry of the expression \( f^{(13)}_{jkl} \) and the Yang-Mills solution emerges. We also have

\[ g^{(12a)}_{jkl} = 2i f^{(14)}_{jlk} \quad g^{(12b)}_{jkl} = 2i f^{(14)}_{ljk} \] (7.55)

and the second solution emerges.

One can make a double check of the computation as follows: one eliminates only the terms of the type \( \bar{F}, \bar{G} \) and do not use the co-boundaries. So, we work with the whole set of 132 terms of type \( F, G \). As a result we find 35 solutions of which 33 are trivial and the other two are those already obtained above.

31
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