Numerical simulation and experimental research of fractal suspended carrier based on nonlinear equation

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Abstract
The geometric structure of the suspended carrier is an important factor that directly affects the effluent quality of the moving bed biofilm reactor, and it should be a valuable mathematical solution to solve the nonlinear equation through numerical simulation and experimental research. Therefore, this study has designed and prepared a coral-shaped fractal suspension carrier based on nonlinear equations and verified the effectiveness of the new carrier for sewage treatment through FLUENT numerical simulation and domestic sewage treatment experiments. The experimental results show that the coral-shaped fractal suspension carrier has a significant effect on the velocity, vortex distribution, and gas-phase distribution of the flow field in the reactor. The mass transfer dead area in the reactor is reduced, the number of vortices is significantly increased, and the fractal dimension of the carrier is negatively correlated with the flow velocity and pressure drop of the fluid. After stabilization, the average removal rates of COD and \( \text{NH}_4^+ \)-N by the reactor are 89.5% and 93.21%, respectively; the effluent quality reaches the national first-class A standard; and the sewage treatment performance is good. At the same time, this research provides a preliminary research basis for the method of solving nonlinear equations through numerical simulation and experimental research.

Keywords Nonlinear equation suspension carrier · Numerical simulation · Biological denitrification · 3D printing

Introduction
As the core of the integrated treatment equipment of the moving bed biofilm reactor, the suspended carrier is the main factor limiting the effect of sewage treatment. The preferred suspension carrier is very important for the start-up and operation of a moving bed biofilm reactor (MBBR) (Zhu et al. 2022). However, due to the unclear mathematical law of the influence of the existing suspended carrier on the fluid, the disturbance of the fluid in the reactor is inconsistent with the mass transfer mechanism of the multiphase flow hydrodynamics, and the demand of the microorganisms for oxygen and substrate cannot be fully satisfied in time, and the reaction The mass transfer efficiency in the reactor is low, and the growth and development of the biofilm is limited, which leads to the low processing efficiency of the moving bed biofilm reactor, which directly affects the effluent quality of the reactor. Therefore, by changing and optimizing the spatial structure of the suspended carrier, the problem of mass transfer of the suspended carrier is solved, and the mathematical law of the influence of fluid on the carrier is further explored. The shedding and renewal are
of great significance to the improvement of the efficiency of the moving bed biofilm reactor and the improvement of the effluent quality.

The research on suspended carriers began in the 1980s. The German LINDE company first used a block-shaped porous plastic foam with a size of 12–15 mm and a density close to water. The specific gravity of the porous plastic foam after the completion of biofilm growth is slightly larger than that of water, and its settling speed is lower than the speed of aeration and stirring, and it is in a fluidized state in the reactor through aeration or mechanical stirring. (Liu et al. 2020; Wang et al. 2018). The geometry of the carrier directly affects the effluent quality of MBBR (Leyva-Díaz et al. 2017; Barwal and Chaudhary 2015), which further directly affects the efficiency of the reactor. Most of the current research is aimed at numerical simulation using kinematic equations and statistical data (Achilli et al. 2011; Almomani et al. 2014). Izabela et al. (2018) studied the effect of MBBR carrier geometry on reactor fluid volatility. The experimental results show that there is a strong correlation between the shape of the carrier and the flow rate in the inner region of the carrier, and the optimization of the carrier shape is beneficial to improving the biofilm stability on the carrier surface.

The object of nonlinear equation fractal has a high degree of complexity, and the differences and similarities coexist in this high complexity. The research shows that the complex difference and similarity fractal boundaries have a significant effect on the generation of turbulence and the uniform distribution of the flow field. Stuart et al. (2007) used a large eddy current simulation to study the flow of fractal shapes as the branching algebra increased. Numerical simulations were used to study turbulence near an idealized fractal tree. Applying high-resolution, branch-analytic LES showed that the dependence of the drag coefficient on the fractal internal cutoff scale is very strong, which is obvious when the number of iterative branches is small and medium. At the same time, a new technique, RNS, is proposed to effectively calculate the interaction between the fluid and the fractal boundary. Relevant studies have shown that in addition to fractal trees, objects with a fractal structure such as leaves, corals, and animal lungs have efficient transport, ideal mass transfer, and heat transfer characteristics. (Dong and Shun 2012). Chen et al. (2015) investigated the seepage properties of the Sierpinski carpet-structured fractal porous media through numerical simulations, and the results showed that pore structure and connectivity have a significant effect on the permeability of porous media with the same porosity. Huang et al. (2020) studied the effect of fractal dimension on the flow characteristics of the carrier through numerical simulation, and the experimental results showed that the increase of fractal dimension leads to the enhancement of pore connectivity and the reduction of channel tortuosity, which reduces the flow resistance and improves the performance of porous media. transport capacity. Research shows that relative roughness and roughness details at different length scales on the nonlinear flow behavior due to inertia associated with the formation of eddy flows, an increase in surface roughness results in an increase in hydraulic resistance and a decreased inflow (Aghajanezhad and Sellier 2022; Wang et al. 2022).

This research is based on nonlinear equations, by combining 3D design and 3D printing, the mathematical model of the fractal theory is applied to the structural design of the carrier, and a coral-shaped fractal suspension carrier is designed and prepared. The motion of the suspended carrier in the moving bed biofilm reactor is simulated by FLUENT. By analyzing the velocity, vortex distribution, and gas-phase distribution of the flow field in the reactor, the influence of the spatial structure of the carrier on the integrated sewage treatment equipment is analyzed, and the sewage treatment performance of the new suspension carrier is verified by sewage treatment experiments finally. This will provide a preliminary research basis for studying methods for solving nonlinear equations through numerical simulations and experiments.

**Materials and methods**

**Experimental setup**

The parallel test device of a small biological mobile bed reactor was used in the experiment. The flow chart of the test device and the transparent organic glass reactor are shown in Fig. 1.

The four small reactors are aerated by an oxygen booster pump, and the aeration amount is controlled by a gas flow meter to keep the aeration amount of each reactor the same. The material of the reactor is transparent plexiglass, the effective volume is 1.8 L, and the volume of the filling suspension carrier is 720 mL (filling rate 40%).

The study adopts the activated sludge method to carry out the biofilm formation experiment. First, the appropriate and equal amount of activated sludge was added to each moving bed biofilm reactor, and then, 40% of the carrier was added. The prepared simulated domestic sewage is added to the reactor through the bottom water inlet, and the nutrient solution is prepared according to the ratio of C:N:P = 100:5:1. The method of continuous water inflow and gradually increasing the inflow water flow rate is adopted. The initial inflow flow was 0.15 L/h, the flow was increased to 0.2 L/h after 3 days of operation, the flow was increased to 0.25 L/h after 3 days of operation, and the inflow flow was adjusted to 300 mL/h after 3 days of operation, run for 7 days. During the 16-day continuous operation, the COD and NH₄⁺-N
concentrations of the influent and effluent were measured every day, and the growth of biofilm on the surface of the suspended carrier was observed.

**Suspended carrier structure design and preparation**

Based on the simulation of the differential growth of different regions on the initial surface, the initial surface was transformed into a triangular mesh structure using the grasshopper. The initial surface grows over time and with each time step, the forces are calculated on each vertex of the triangle mesh while integrating new positions for each vertex, and the growth rate is extended to each edge of the triangle mesh.

To prevent the curvature of any surface from being too small, make the surface continue to grow, and introduce bending resistance, the difference before and after deformation is the square difference of the average curvature. The formula is shown in Formula 1,

\[
\left[ Tr(\varphi^*S) - Tr(S) \right]^2 = 4 \left( H_{\text{dvp}} - H \right)^2
\]

\(S\) and \(\bar{S}\) are the shape configurations after deformation and before deformation, respectively. \(H\) and \(\bar{H}\) are the mean curvature before and after deformation, respectively. \(\varphi^*S\) is the callback of \(S\) to the initial configuration. \(\varphi\) represents the change of the surface in three-dimensional space.

\[
Tr(\varphi^*S) = \varphi^*Tr(S) = Tr(S)_{\text{dvp}} = H_{\text{dvp}}
\]  

(2)

Discretizing the integral over a piecewise linear grid, we express the discrete bending energy as the sum of the grid edges, given in Formula 3,

\[
W_{\text{B}}(x) = \sum \left( \theta_e \bar{\theta}_e \right)^2 \| \bar{\tau} \| / \bar{h}_e
\]

(3)

A series of coral-shaped fractal suspension carriers with different diameters as shown in Fig. 2 are designed: \(C_{21-1}, C_{22-2}, C_{23-3}, C_{24-4}\) (respectively grown for 1 ms, 2 ms, 3 ms, 4 ms).

The suspension carrier filler is prepared by 3D printing, and the filler is printed based on the fusion layer molding technology. The printing speed is 80 mm/s, the nozzle diameter is 0.4 mm, and the nozzle temperature is 220 °C. The basic physical parameters such as the weight, diameter, and specific surface area of the coral-shaped fractal suspension carrier are directly read by Rhino, and the porosity is calculated by dividing the actual volume by the volume of a sphere of equal diameter. The detailed data are shown in Table 1; the printed object is shown in Fig. 3.
Experimental water and testing methods

The measurement methods of the main water quality indicators involved in the test refer to Water and Wastewater Monitoring and Analysis Methods (fourth edition). The specific water quality indicators and analysis methods are shown in Table 1, and the detection reagents are shown in Table 2. Metrics that need to be tested during the experiment include COD, NH₄⁺-N, dissolved oxygen, and temperature. The main reagents used in the configuration of simulated sewage, nutrient solution, and dissolved oxygen detection are shown in Table 2. Table 2 lists the reagents used for the detection. Refer to Water and Wastewater Monitoring and Analysis Methods for measurement methods.

Results and discussion

Influence of fractal dimension of carrier on the internal flow field of carrier

The dimensions of different coral-shaped fractal suspension carriers of the same diameter are shown in Table 4.

The velocity and vector distribution inside the coral-shaped fractal suspension carrier of different dimensions are shown in Fig. 4.

The pressure and streamline distribution inside the carrier is shown in Fig. 5. The volume pressure of the air inlet at the bottom of the reactor is 16.6 pa.
Fig. 4 Velocity and vector distribution in different dimension carrier. a $C_{14-41}$; b $C_{14-42}$; c $C_{14-43}$; d $C_{14-44}$

Fig. 5 Distribution map of pressure and streamline of different dimension carriers. a $C_{14-41}$; b $C_{14-42}$; c $C_{14-43}$; d $C_{14-44}$
It can be seen from Figs. 4 and 5 that with the increase of fractal dimension, the maximum equivalent pore size in the coral-shaped fractal suspension carrier gradually decreases, while the peak velocity of the liquid flows through the carrier decreases, but the overall permeability inside the carrier gradually increases. In addition, the fractal dimension is negatively correlated with the pressure drop of the liquid inside the carrier. That is, as the fractal dimension increases, the pressure loss through the fractal carrier decreases gradually. The reason is that the roughness of the inner surface of the carrier increases with the increase of the fractal dimension, and the streamline of the carrier surface tends to be curved and rough; that is, the tortuosity of the carrier’s profile curve increases. This directly leads to the reduction of the correlation of the internal structure of the carrier, the enhancement of the connectivity of the internal channels of the carrier, and the improvement of the transport capacity inside the carrier. Therefore, the coral-like fractal suspension carrier with a higher fractal dimension can obtain better internal flow characteristics and a more stable internal environment. This characteristic of the coral-like fractal suspension carrier is more conducive to providing a stable environment for the growth of microorganisms.

**Influence of different particle size carriers on velocity field in reactor**

The vector distribution of flow velocity in each reactor of the coral-shaped fractal suspension carriers with different particle sizes is shown in Fig. 6.

As shown in Fig. 6, carriers $C_1$, $C_2$, $C_3$, and $C_4$ are located inside the four reactors: the liquid flow velocity near the air inlet at the bottom of each reactor is in the range of $0.16–0.21 \text{ m/s}$. The airflow at the air inlet drives the liquid in the reactor to move upward, and the liquid velocity decreases by about $0.02 \text{ m/s}$ before it contacts the carrier, the liquid velocity in the inner and outer areas of the carrier changes significantly after flowing through the carrier. The color of the outer and bottom areas of the carrier is darker, and the color of the inner and upward areas of the carrier is lighter, indicating that the speed of the liquid slows down after passing through the carrier, and the flow rate of the liquid inside the carrier decreases more obviously. Comparing the inside of each reactor, the change range of the liquid velocity inside the carriers $C_1$, $C_2$, $C_3$, and $C_4$ with different particle sizes are $0.21–0.16 \text{ m/s}$, $0.19–0.16 \text{ m/s}$, $0.20–0.04 \text{ m/s}$, and $0.19–0.00 \text{ m/s}$; the liquid flow velocity from the bottom of the four carriers to the top of the carrier decreased by about $0.02 \text{ m/s}$, $0.03 \text{ m/s}$, $0.16 \text{ m/s}$, and $0.19 \text{ m/s}$, respectively.

Inside the coral-shaped fractal suspension carrier reactor, the velocity of the gas–liquid mixed flow has a significant downward trend during the upward movement. The gas–liquid goes deep into the carrier with a mixed flow and finally flows out from the carrier, and the flow rate of the liquid decreases continuously. The reason for the preliminary analysis is that when the liquid starts to contact the carrier, it is subjected to the shear force and resistance on the surface of the carrier. And when the liquid flows through the inner channel of the carrier, it is constantly subjected to the frictional resistance and blocking effect of the inner surface of the channel. The reduction of the flow rate of the liquid inside and outside the carrier is conducive to the formation of a more stable internal and external environment, which is conducive to the attachment and stable growth of microorganisms.

**Influence of different particle size carriers on the vortex distribution in the reactor**

The liquid streamline distribution in each reactor of the coral-shaped fractal suspension carriers with different particle sizes is shown in Fig. 7.

As shown in Fig. 7, many vortices of different sizes are formed in different reactors, and four large-scale vortices are formed around the carrier and are symmetrically distributed.
The shape and number of vortices inside the reactor are approximately the same. At the same time, vortices of different sizes are distributed around the carrier, and the streamline in the upper region of the reactor is relatively gentle. It shows that the fractal carrier has a strong disturbance to the internal flow field of the reactor.

To study and analyze the movement of the fluid inside the carrier in more detail, a streamlined diagram of the fluid inside the carrier is extracted separately, as shown in Fig. 8. The streamlines on the outer surface of the carriers with different particle sizes are relatively smooth, and the size and number of the internal vortices are different. When the particle size is 20 mm, many tiny-sized vortices are formed inside the carrier. Compared with the outer area of the carrier, the vortex size in the central area is smaller and the number is less. When the particle size is 26 mm and 32 mm, the vortices are concentrated in the upper and middle regions of the vector, the vortex size is small, and the number of vortices inside the carrier is almost 0. When the particle size is 38 mm, a large number of small-scale vortices are distributed in the outer region of the carrier, and the number of vortices in the central region is 0. With the increase of the
particle size of the carrier, the number of vortices outside the carrier gradually increases, and the size of vortices also increases, but the change is small. The reason is that as the particle size of the carrier continues to increase, the internal structure becomes more and more complex, the blocking effect on the liquid becomes more and more obvious, and it is difficult for the liquid to flow through the entire carrier. At the same time, as the particle size of the carrier continues to increase, the number of surface wrinkles on the coral-shaped fractal suspension carrier increases, and the disturbance to the fluid gradually increases. The blocking effect of the large-diameter carrier on the fluid is significantly enhanced, and the increase of the wrinkle on the carrier surface is conducive to the generation of numerous small-sized vortices. However, the fluidity inside the carrier will be weakened, and the fractal dimension is very important for a carrier with a large diameter.

According to the micro-vortex scale distribution theory, kinetic energy is injected into the flow field from the large-scale vortices, and then, the large-scale vortices are gradually disintegrated. The energy is transferred to the smaller-scale vortex through the non-viscous process, and the energy is dissipated by the smaller-scale vortex. The large-scale vortex is conducive to the formation of a circular flow in the reactor so that the carrier is in a mixed fluidized state. Many tiny-sized vortex structures are formed in the inner region of the coral-shaped fractal suspension carrier. The generation of this local tiny vortex flow can intensify the mutual mixing of liquid and gas inside the carrier, improve the uniformity of the flow field distribution inside the carrier, increase the hydraulic retention time, and improve mass transfer. The tiny-sized vortices can enhance the turbulence of the fluid inside the carrier and reduce the velocity distribution gradient, which is beneficial to the shedding and renewal of the biofilm on the carrier surface.

Effects of different particle size carriers on gas distribution in the reactor

The effect of dissolved oxygen distribution in each reactor of the coral-shaped fractal suspension carriers with different particle sizes is shown in Fig. 9.

As shown in Fig. 9, in the range of 0–0.5 s, the airflow at the bottom air inlet of each reactor moves upward continuously, and the volume fraction gradually decreases from 100% and is continuously dissipated during the movement. The volume fraction is between 60 and 75% until the airflow first contacts the carrier. Within the range of 0.5–0.6 s, the airflow in each reactor begins to contact the carrier and is cut into several parts by the carrier, mainly including the inner and outer parts of the carrier. Within the range of 0.6–0.7 s, the gas flow in each reactor has completely passed through each channel inside the carrier, and the average gas volume fraction of each channel in each carrier is in the range of 30–40%. The gas flow has completely passed through each carrier at 0.7 s, and the average gas volume fraction is used to roughly characterize the gas holdup inside each carrier, and the gas holdup inside the C11-1, C12-2, C13-3, and C14-4 carriers are 0.212, 0.181, 0.162, and 0.131, respectively. Inside each reactor, the gas holdup near the carrier was significantly higher than in the regions inside the reactor.

After contacting the carrier, the partial airflow continuously flows upward along each channel inside the carrier, and the gas volume fraction is dissipated by more than 20%. The reason is that the airflow in the channel is further dissipated during the movement due to the influence of the surface resistance and shape of the carrier. During the upward movement of the airflow outside the carrier along the surface of the carrier, two large-scale vortices are formed on both sides of the carrier, and with the continuous movement of the airflow, the size of the vortex tends to increase continuously. The reason is that the airflow is affected by the resistance of the carrier surface and the shape of the carrier surface. After the airflow passes through the carrier, two vortices with opposite directions are further formed in the upper area of the carrier. The reason is that the internal shape of the carrier has a strong disturbance to the airflow, and the velocity direction of the internal airflow changes greatly after passing through the interior of the carrier.

The removal effect of COD and NH4+ -N in the start-up stage

Figure 10 shows the change of COD concentration in the influent and effluent of each reactor in the start-up stage, and the COD removal rate is shown in Fig. 11. As shown in Fig. 10, the COD concentration in each reactor of the coral-shaped fractal suspension carriers with different particle sizes has decreased slowly by 1–3 days of continuous water inflow and has all decreased by nearly 20 mg/L. Between day 4 and day 6, the COD concentration in each reactor decreased rapidly, and the decreasing range was between 30 and 40 mg/L. Between day 7 and day 16, the decreasing speed of COD concentration in each reactor gradually slowed down, and the decreasing range was between 25 and 45 mg/L. The COD concentration in the reactor of C11-1 has the slowest decreasing rate, while the COD concentration in the reactors of C13-3 and C14-4 has the fastest decreasing rate. After stabilization, the effluent concentrations in the reactors of C11-1, C12-2, C13-3, and C14-4 are 54.81 mg/L, 43.84 mg/L, 37.36 mg/L, and 33.84 mg/L, respectively. Except for the reactor of C11-1, the effluent of the reactors met the Urban Sewage Treatment Plant Pollutant Discharge Standard (GB18918-2002) level A standard.

As shown in Fig. 11, the COD removal rate in each reactor has shown an increasing trend. The COD removal rate
Fig. 9  Distribution of gas phase in reactors with different particle sizes. a C\textsubscript{11,1}; b C\textsubscript{12,2}; c C\textsubscript{13,3}; d C\textsubscript{14,4}
has the fastest increase from day 1 to day 7, while the change of the removal rate has slowed down significantly from day 8 to day 16. The COD removal rate in the reactor of C_{11-1} has the slowest increase, and the average removal rate after stabilization was about 83%, which was significantly lower than that of other carriers. The COD removal rate in the reactor of C_{14-4} has the fastest increase, but the removal rate after stabilization is close to that of C_{13-3}; the removal rates are 89.5% and 88.4% after stabilization, respectively. The highest removal rate of C_{14-4} has reached 91.5%, which is higher than other carriers.

The COD concentration in each reactor has changed significantly from day 1 to day 6, and the removal rate has increased significantly. The reason is that the nutrient source substrate in the reactor was sufficient at that time, resulting in the extensive production of microbial, and the speed of microbial decomposition into organic matter gradually accelerated. From day 12 onwards, the carrier has been in the final stage of microbial film growth, and the COD concentration and COD removal rate of the effluent from the reactor have remained nearly stable. The reason for this is that the growth, shedding, and renewal of microorganisms on the surface of the carrier have reached a dynamic equilibrium, the number of microbial populations has reached stability, and the growth of biofilm on the surface of the carrier has been completed. From the perspective of the
COD removal effect in each reactor during start-up, the C_{14-4} carrier is the best. From the perspective of the COD removal effect after stabilization, C_{14-4} and C_{13-3} carriers have approximately the same effect, and the C_{11-1} carrier is the worst. The reason is that the specific surface area of the C_{14-4} carrier is the largest. Hence, the higher specific surface area has provided enough space for the growth and reproduction of microorganisms in the early stage of biofilm growth, so a large number of microorganisms have been attached, resulting in the fastest growth rate of biofilm, and the highest COD removal efficiency. When each carrier has entered the later stage of the growth of the biofilm, the change of small specific surface area has no obvious effect on the COD removal rate, but the too low specific surface area of C_{11-1} still has a great impact on COD removal, which resulted in the effluent quality is lower than the national first-class A standard.

Figure 12 shows the change of NH\textsubscript{4}\textsuperscript{+}-N concentration in each reactor in the start-up stage, and the NH\textsubscript{4}\textsuperscript{+}-N removal rate is shown in Fig. 13.

As shown in Fig. 12, from day 1 to day 5, the ammonia nitrogen concentration in each reactor has decreased by the amount of 6.6~13.7 mg/L. From day 6 to day 12, ammonia nitrogen in each reactor has decreased by the amount of 3.6~10.8 mg/L. From day 13 to 16,
the ammonia nitrogen concentration in each reactor has decreased by the amount of 1.6–5.1 mg/L. In the first 5 days of biofilm growth, the ammonia nitrogen concentration in the effluent of each reactor has decreased rapidly. From day 13 to day 16 of biofilm growth, the ammonia nitrogen concentration in the effluent of each reactor has remained stable. The general decreasing rate of ammonia nitrogen concentration in the effluent of each reactor is slowing down.

After stabilization, the average concentrations of ammonia nitrogen in the effluent of the reactors of C11-1, C12-2, C13-3, and C14-4 are 6.59 mg/L, 6.33 mg/L, 4.70 mg/L, and 3.22 mg/L, respectively. The ammonia nitrogen concentration in the effluent of reactors of C11-1 and C12-2 is lower than the national first-class A standard, and the effluent ammonia–nitrogen concentration of reactors of C14-4 and C13-3 is higher than the national first-class A standard.

As shown in Fig. 13, the ammonia nitrogen removal rate in each reactor continued to increase. From day 1 to day 5, the ammonia nitrogen removal rate in each reactor has increased significantly, and the removal rate increased steadily from day 6 to day 12. After the biofilm on the carrier surface grew and matured, the ammonia nitrogen removal rate of each reactor remained nearly stable. At the same time, the removal rates of ammonia nitrogen by C11-1, C12-2, C13-3, and C14-4 are 86.15%, 86.67%, 89.59%, and 93.21%, respectively. The removal rate of ammonia nitrogen by the C14-4 carrier has increased steadily during the biofilm growth, and the highest removal rate is 95.1%. The C14-4 carrier has the best effect on the removal of ammonia nitrogen, followed by the C13-3 carrier, and the effluent has reached the national first-class A standard. The removal rate of ammonia nitrogen by the C11-1 and C12-2 carriers has reached 86%, but the effluent quality is lower than the national first-class A standard, and the reason is that the microbial biomass on the surface of the carrier with a small particle size is still small.

(1) Establish and test a method system for studying the mechanism of sewage treatment or solving nonlinear equations. This paper establishes and explores the system of “carrier mathematical equation design–carrier numerical simulation–carrier 3D printing physical object–carrier sewage treatment experiment,” obtains the corresponding research data of the whole process, and preliminarily proves the feasibility and effectiveness of the system. The new coral fractal suspension carrier designed and prepared by the nonlinear equation has a certain sewage treatment effect.

(2) The fractal dimension of the carrier has a positive correlation effect on the internal flow field of the carrier. With the increase of fractal dimension, the tortuosity of the contour curve of the carrier increases, which leads to the decrease of the correlation of the internal structure of the carrier, the enhancement of the connectivity of the internal channel, and the improvement of the internal transmission capacity.

(3) The fractal structure of the carrier has an obvious influence on the velocity field, vortex distribution, and gas-phase distribution in the reactor. Different iteration times of the new carrier are positively correlated with the removal effect. As the number of iterations increases, the particle size increases and the spatial structure becomes more and more complex, which provides a favorable environment for the attachment and growth of microorganisms. The removal effect of C14-4 is the best. After stabilization, the average removal rate of COD by C14-4 is 88.4%, the highest removal rate is 91.5%, the average removal rate of NH4+-N was 93.21%, and the highest removal rate was 95.1%.

(4) The spatial structure of the new suspension carrier significantly improves the three-phase mass transfer of gas, liquid, and solid in the reactor. The carrier has an obvious blocking effect on the liquid flow. The large-size vortex strengthens the fluidization effect of the carrier, and the small-size vortex effectively promotes the falling off and renewal of the microbial membrane on the surface of the carrier and effectively reduces the mass transfer dead zone inside the reactor.

Here, it is particularly stated that this paper only publishes the first stage of the method system, that is, the design, preparation, numerical simulation results, and water treatment effect of the nonlinear equation carrier. Through this method system, we will systematically explore the mathematical mechanism of sewage treatment, or the systematic study of solving nonlinear equations, which will be written and reported separately.

Acknowledgements At the point of finishing this manuscript, the authors would like to express sincere thanks to all those who have...
lent their hands in the course of our writing this manuscript. At the same time, the authors would like to acknowledge other researchers for assisting in providing references and information during the conduct of this research and manuscript.

Author contribution BZ is the designer of the experiment and the executive of the experimental research in this study, completing data analysis and writing the first draft of the manuscript. YW, RW, DY, and ZZ are involved in the experimental design and analysis of experimental results. AW is the creator and the person in charge of the project, directing the experimental design, data analysis, paper writing, and revision. All authors read and approved the final manuscript.

Data availability All data generated or analyzed during this study are included in this published paper.

Declarations

Ethics approval and consent to participate Not applicable.

Consent for publication Not applicable.

Competing interests The authors declare no competing interests.

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