A Nearly Incompressible Turbulence-Driven Solar Wind Model

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Abstract. We develop a 1D steady-state turbulence driven-solar wind model by coupling recently developed nearly incompressible magnetohydrodynamic turbulence transport model equations and 1D hydrodynamic solar wind equations. The dissipation of majority component quasi-2D turbulence and minority slab turbulence generated by the emergence of the “magnetic carpet” from the photosphere is responsible for heating the coronal plasma. We solve the coupled quasi-2D and slab turbulence transport equations and the solar wind equations from the base of the solar corona until 20 solar radii. We find that i) the coronal temperature increases to \(\sim 2.5 \times 10^6\) K near the base of the solar corona; ii) the solar wind accelerates rapidly within a few solar radii; iii) turbulence energy densities decrease and correlation lengths increase with distance; iv) balanced quasi-2D turbulence at the coronal base becomes imbalanced, and imbalanced slab turbulence becomes more imbalanced with distance, and v) the normalized quasi-2D and slab residual energy becomes positive near the coronal base, and both energies become negative with increasing distance.

1. Introduction
The heating of the solar corona to millions of degrees Kelvin and the acceleration of the solar wind are attracting the attention of the science community after the launch of the Parker Solar Probe (PSP). It is thought that the heating of active regions, closed loops in the quiet corona, and open field regions, accelerates the solar wind from subsonic to supersonic speed. The models so far developed to describe the heating of the solar corona have been classified broadly by Cranmer et al [1, 2] into two physical models, i) “wave/turbulence-driven” (W/T) and ii) “reconnection/loop-opening” (RLO) models.

The W/T models assume that the transverse motions of the magnetic footpoints in the photosphere generate Alfvén waves with a speed of \(> 10^3\) km s\(^{-1}\) that propagate outward through the chromosphere, which partially reflect and dissipate in
the chromosphere, and are then transmitted into the lower corona with a smaller flux. The outwardly propagating Alfvén waves produced by the super-Alfvénic transverse photospheric motions are thought to undergo partial non-WKB reflection [3 – 5]. The partially reflected inward Alfvén wave then interacts nonlinearly with an outwardly propagating Alfvén wave [5] resulting in a non-propagating quasi-2D mode. In the W/T models, dissipation of large-scale energy via the generation of quasi-2D nonpropagating fluctuations [6] is thought to be responsible for the heating of coronal plasma. Also, see papers [7 – 12], which employed the ideas of Matthaeus et al [5] to model coronal heating and the origin of fast solar wind in open magnetic field regions. The magnetic field in the photosphere concentrates in intergranular regions or lanes in the quiet Sun ([13, 14]), from which magnetic flux tubes can form. These regions of concentrated and large magnetic field are sometimes called “kilo-Gauss patches” [15]. The flux tubes guide the solar wind and Alfvén waves and expand quickly with increasing height.

The concept of RLO was initially developed to explain plasma heating within closed loops, and was then extended to open magnetic field regions. RLO models require exchange reconnection between an open flux tube and a closed loop [16–18], which can lead to the injection of heated loop plasma into an open flux tube. Of course, the question of how the loop plasma is heated remains to be addressed.

The magnetic flux in the photosphere is mostly concentrated in small tubes of kilogauss strength, and is “stirred” with a characteristic speed of order ~ 1 – 2 km s$^{-1}$ [19–20]. A continual flow at the boundaries of photospheric granules of different sizes significantly twist and braid the magnetic fields [21], leading to the release of energy via the generation of current sheets [21]. Rappazzo & Parker [22] used simulations to show that quasi-2D turbulence and current sheets form in a closed magnetic field representing a loop. This is consistent with the nearly incompressible description of MHD [23–25], which shows that the fluctuating fields are dominated by quasi-2D structures in the presence of strong guide field. The variance of the amplitude of the emerging magnetic carpet determines the scale height at which quasi-2D turbulence is driven by the dynamical emergence and interaction of magnetic carpet loops, so that the dissipation of quasi-2D turbulence heats the plasma above the photosphere (Zank et al [26]).

Recently, Zank et al [26] developed coupled nearly incompressible magnetohydrodynamic (NI MHD) quasi-2D and slab turbulence transport model equations for the quiet solar corona in open field regions. The NI MHD turbulence transport equations of Zank et al [26, 27] are appropriate to a low plasma beta $\beta_p$. NI MHD in the plasma beta $\beta_p \sim 1$ and $\beta_p \ll 1$ regimes ($\beta_p \equiv 2nk_BT/(B^2/\mu_0)$, where $n$ is the solar wind density, $k_B$ is Boltzmann’s constant, $T$ is the solar wind temperature, $B$ is the large-scale magnetic field, and $\mu_0$ is the magnetic permeability) describes solar wind turbulence as a nonlinearly coupled superposition of a majority quasi-2D and minority slab fluctuations [23, 24, 27]. In Zank et al [26], the coronal turbulence transport model and the background plasma do not interact with each other than through the dissipation of turbulence that heats the background plasma. Zank et al [26] use a prescribed linearly increasing velocity profile, a density profile that conserves mass flux, and a radial average magnetic field. In this paper, we couple the original Parker solar wind model [21] to the Zank et al [26, 27] turbulence transport and heating model so that the turbulence transport model equations couple with the background plasma flow. In Section 2, we present the coupled
turbulence transport-driven solar wind model. Section 3 discusses the theoretical results of our solar wind and turbulence transport model equations. Finally, Section 4 presents a discussion and conclusions.

2. The solar wind and turbulence transport model equations

In this section, we present the coupled NI MHD turbulence transport-driven solar wind model equations. The steady-state mass and momentum conservation equations and the solar wind temperature equation in a spherical coordinate system $r$ are given by

$$\frac{d}{dr}(r^2 \rho U) = 0; \quad (1)$$

$$\rho U \frac{dU}{dr} = -\frac{dP}{dr} - \frac{GM_\odot \rho}{r^2}; \quad (2)$$

$$U \frac{dT}{dr} = -(\gamma - 1)T \frac{dU}{dr} - (\gamma - 1) \frac{2UT}{r} + \frac{1}{3} \frac{m_p \alpha}{k_B} \left[ 2 \frac{z^{*+2} z^{\infty+2} z^{\infty-2}}{L_\infty} \right]$$

$$+ \frac{z^{*+2} z^{\infty-2}}{L_\infty^+} + \frac{z^{\infty+2} z^{\infty-2}}{L_\infty^-}$$

$$+ \frac{z^{\infty+2} z^{\infty-2}}{L_\infty^0} + \frac{z^{\infty+2} z^{\infty-2}}{L_\infty^0}$$

$$+ \frac{z^{\infty+2} z^{\infty-2}}{L_\infty^0} \right], \quad (3)$$

where $\rho(\equiv m_p n)$ is the mass density, $U$ the solar wind speed, $P(\equiv 2nk_B T)$ the thermal pressure, $G$ the gravitational constant, $M_\odot$ the solar mass, $m_p$ the proton mass, $n$ the solar wind density, and $k_B$ Boltzmann’s constant. The parameter $\alpha$ is the von Kármán-Taylor constant. The parameter $\alpha$ determines the nonlinear interaction rate between the forward and backward propagating modes. The dissipation of the turbulence energy heats the coronal/solar wind plasma. Following Matthaeus et al [5] and Zank et al [26], we use $\alpha = 1$. In our future work, we will investigate coronal heating and acceleration of the solar wind as well as the turbulence energy with different values of $\alpha$ ($0 < \alpha < 1$). The nonlinear dissipative terms become strong or weak depending on whether $\alpha$ is large or small. Therefore, the turbulence energy decays more rapidly when $\alpha$ is large compared to small values of $\alpha$. Since we are re-considering the Parker problem by including the heating term, we only include the gas pressure gradient and the gravitational force in the momentum equation, and neglect the magnetic field in the momentum equation. In future, we will include the magnetic field and the ponderomotive forces in the momentum equation (see [28–36]). The third term on the right-hand side of Equation (3) is the turbulence heating term, which distinguishes Equation (3) from the original Parker model [21]. Prior heating models used an exponential heating function of the form $h = h_0 \exp(-(r - r_1)/\lambda)$ (Habbal et al [37]), where $h_0$ is the strength of the heating term, $\lambda$ is a heating deposition length scale, $r$ heliocentric distance, and $r_1 \sim 2$ solar radii. Habbal et al [37] found that the corona could be heated to millions of degrees Kelvin based on this phenomenological model. However, the form of the exponential heating term is difficult to justify physically.
The residual energy per permeability. Similarly, the quasi-2D and slab variances of the Els"asser variables and \(u\) incompressible corrections, i.e., slab turbulence. The parameters \( \lambda_{\infty} \) respectively. Here, the superscript \("\infty\) refers to core incompressible variables, i.e., quasi-2D turbulence, and the superscript \("\pm\) or subscript \("1\) denotes the higher order incompressible corrections, i.e., slab turbulence. The parameters \( u \) and \( \mathbf{B} \) are the fluctuating velocity and magnetic field, \( \rho \) is the solar wind density, and \( \mu_0 \) is the magnetic permeability. Similarly, the quasi-2D and slab variances of the Els"asser variables and the residual energy \( E_D \) can be formed from Equation (5) as [26]

\[
\langle z^{\infty,\pm^2} \rangle = \langle z^{\infty,\pm^2} \cdot z^{\infty,\pm^2} \rangle; \quad E_D^{\infty,*} = \langle z^{\infty,*^+} \cdot z^{\infty,*^-} \rangle,
\]

where \( \langle z^{\infty,\pm^2} \rangle \) denote the ensemble-averaged quasi-2D/slab energy in backward and forward propagating modes with respect to the inward orientation of the magnetic field. Similarly, the correlation functions corresponding to backward/forward propagating modes, and the residual energy can be written as

\[
L_{\infty,*}^+ = \int (z^{\infty,*^+} \cdot z^{\infty,*^-}) dy \equiv \langle z^{\infty,\pm^2} \rangle \lambda_{\infty,*}^+; \\
L_{\infty,*}^- = \int (z^{\infty,*^+} \cdot z^{\infty,*^-} + z^{\infty,*^+'} \cdot z^{\infty,*^-'}) dy \equiv E_D^{\infty,*} \lambda_{\infty,*}^-,
\]
where \( y = |y| \) is the spatial lag between fluctuations, and \( z^{\infty, *=} \) denotes the lagged Eötvös variables. The quantities \( \lambda_{L^*}^{+} \) and \( \lambda_{D^*}^{\infty} \) are the quasi-2D/slab correlation lengths corresponding to backward/forward propagating modes, and the residual energy.

The 1D steady-state quasi-2D turbulence transport model equations in a spherical coordinate system \( r \) can be expressed as [26, 27, 38]

\[
U \frac{d\langle z^{\infty, \pm 2} \rangle}{dr} + \frac{1}{2} \left( \langle z^{\infty, 2} \rangle + E_D^\infty \right) \left( \frac{dU}{dr} + \frac{2U}{r} \right) = -2 \frac{\langle z^{\infty, +2} \rangle \langle z^{\infty, +2} \rangle}{L_\infty^+} + 2 \frac{\langle z^{\infty, 2} \rangle \langle z^{\infty, 2} \rangle}{L_\infty^+};
\]

\[
U \frac{dE_D^\infty}{dr} + \frac{1}{2} (E_D^\infty + E_T^\infty) \left( \frac{dU}{dr} + \frac{2U}{r} \right) = -E_D^\infty \left( \frac{\langle z^{\infty, +2} \rangle \langle z^{\infty, -2} \rangle}{L_\infty^-} + \frac{\langle z^{\infty, -2} \rangle \langle z^{\infty, 2} \rangle}{L_\infty^-} \right)
+ E_D^\infty \left( \frac{\langle z^{\infty, 2} \rangle \langle z^{\infty, -2} \rangle}{L_*^+} + \frac{\langle z^{\infty, -2} \rangle \langle z^{\infty, +2} \rangle}{L_*^+} \right);
\]

\[
U \frac{dL^+}{dr} + \frac{1}{2} (L^+ + L_D^-) \left( \frac{dU}{dr} + \frac{2U}{r} \right) = 0;
\]

\[
U \frac{dL_D^-}{dr} + \frac{1}{2} \left( \frac{dU}{dr} + \frac{2U}{r} \right) \left( L_D^- + L_D^+ + L_\infty^- \right) = 0,
\]

where \( E_T^\infty = \left( \langle z^{\infty, +2} \rangle + \langle z^{\infty, -2} \rangle \right) / 2 \) is the total quasi-2D turbulent energy. We assume \( L_D^+ = L_D^- = L^* \) in the second terms on the right-hand sides of Equations (5) and (6). The assumption \( L_D^+ = L_D^- = L^* \) reduces the complexity of the equations. Similarly, the 1D steady-state slab model equations can be expressed as ([26, 27, 38])

\[
(U \mp V_A) \frac{d\langle z^{\pm, 2} \rangle}{dr} + \frac{1}{2} \frac{dU}{dr} \left( \langle z^{\pm, 2} \rangle - E_D^\star \right) - (2b - 1) \frac{U}{r} \langle z^{\pm, 2} \rangle + (6b - 1) \frac{U}{r} E_D^\star
+ 4 b \frac{V_A}{r} E_D^\star + \frac{1}{2} \frac{dU}{dr} \left( \langle z^{\pm, 2} \rangle - E_D^\star \right) = -2 \frac{\langle z^{\pm, 2} \rangle \langle z^{\pm, 2} \rangle \langle z^{\pm, 2} \rangle}{L_\infty^+} - 2 \frac{\langle z^{\pm, 2} \rangle \langle z^{\mp, 2} \rangle \langle z^{\pm, 2} \rangle}{L_*^+};
\]

\[
U \frac{dE_D^\star}{dr} + \frac{1}{2} \frac{dU}{dr} (E_D^\star - E_T^\star) - (2b - 1) \frac{U}{r} E_D^\star + (6b - 1) \frac{U}{r} E_T^\star - 4 b \frac{V_A}{r} E_C^\star - \frac{1}{2} \frac{dU}{dr} E_C^\star = - E_D^\star \left( \frac{\langle z^{\pm, +2} \rangle \langle z^{\pm, -2} \rangle}{L_\infty^-} + \frac{\langle z^{\pm, -2} \rangle \langle z^{\pm, +2} \rangle}{L_\infty^-} \right)
- E_D^\star \left( \frac{\langle z^{\pm, +2} \rangle \langle z^{\pm, -2} \rangle}{L_*^-} + \frac{\langle z^{\pm, -2} \rangle \langle z^{\pm, +2} \rangle}{L_*^-} \right);
\]
\[ U \frac{dL_s}{dr} + \frac{1}{2} \frac{dU}{dr} \left( L_s - \frac{L_D^*}{2} \right) - (2b - 1) U \frac{L_s}{r} + \left( 3b - \frac{1}{2} \right) U L_D^* = 0; \tag{14} \]
\[ U \frac{dL_D^*}{dr} + \frac{dU}{dr} \left( \frac{L_D^*}{2} - L_s \right) - (2b - 1) U \frac{L_D^*}{r} + 2(6b - 1) U L_s = 0, \tag{15} \]

where \( E_T^* = (\langle z^{*+2} \rangle + \langle z^{*-2} \rangle)/2 \) is the total slab turbulent energy, \( E_T^C = (\langle z^{*+2} \rangle - \langle z^{*-2} \rangle)/2 \) the slab cross-helicity, and \( V_A = B/\sqrt{\mu_0 \rho} \) the Alfvén velocity. The parameter \( b \) reflects the assumed geometry of the slab turbulence and is related to the covariance of the slab form of the Elsässer variables [26, 39]. We use \( b = 0.3 \).

Similarly, the 1D steady-state transport equation for the variance in the density fluctuations is [26, 27, 38]
\[ U \frac{d(\rho^\infty)}{dr} + \frac{2(\rho^\infty)}{dr} + 4 \frac{U}{r} (\rho^\infty) = -\frac{\langle u^\infty \rangle^2 (\rho^\infty)}{l_a^\infty}, \tag{16} \]
where \( \langle u^\infty \rangle \) and \( l_a^\infty \) are the fluctuating quasi-2D kinetic energy and the corresponding correlation length, and are given by
\[ \langle u^\infty \rangle = \frac{\langle z^{*+2} \rangle + \langle z^{*-2} \rangle + 2E_D^\infty}{4} \quad \text{and} \quad l_a^\infty = \frac{(E_T^\infty + E_T^C)\lambda_+ + (E_T^\infty - E_T^C)\lambda_- + E_D^\infty \lambda_D^\infty}{2(E_T^\infty + E_T^C)}. \tag{17} \]

Notice that the transport equation for backward propagating modes, Equation (12), exhibits a singular point at \( U = V_A \). We replace the mixing dissipative term in Equation (12) by an advective term \( \langle z^{*-2} \rangle^{1/2} \mathbf{s} \cdot \nabla \langle z^{*+2} \rangle \) to avoid the Alfvénic singularity [Zank et al 26]. In the vicinity of the Alfvén surface, we solve the modified transport equation
\[ \left( U - V_A + \langle z^{*-2} \rangle^{1/2} \right) \frac{d(\langle z^{*+2} \rangle^2)}{dr} = \frac{1}{2} \frac{dU}{dr} (\langle z^{*+2} \rangle - E_D^* + (2b - 1) U \langle z^{*+2} \rangle) \tag{18} \]
\[ - (6b - 1) \frac{U}{r} E_D^* - 4b \frac{V_A}{r} E_D^* + \frac{1}{2} \frac{\rho}{dr} (\langle z^{*+2} \rangle - E_D^*) \frac{2(\rho^{*+2})^2(\rho^{*-2})^{1/2}}{L_s} \]
\[ U \frac{dT}{dr} = -(\gamma - 1) T \frac{dU}{dr} - (\gamma - 1) \frac{2UT}{r} + \frac{1}{3} \frac{m_p}{k_B} \left[ \frac{2(\rho^{*+2})^2(\rho^{*-2})^{1/2}}{L_s^+} \right] + \frac{2(\rho^{*+2})^2(\rho^{*-2})^{1/2}}{L_s^+} \]
\[ + \frac{2(\rho^{*+2})^2(\rho^{*-2})^{1/2}}{L_s^-} + E_D^* \left( \frac{\langle z^{*+2} \rangle^2(\rho^{*-2})^{1/2}}{L_s^+} + \frac{\langle z^{*+2} \rangle^2(\rho^{*-2})^{1/2}}{L_s^-} \right). \tag{19} \]

We solve the coupled solar wind equations and turbulence transport equations using a 4th order Runge-Kutta method from 1 to 20 R_\odot. Since these equations are coupled, the background solar wind speed, density and temperature are influenced by the evolving MHD turbulence and vice versa.
3. Results

We solve the self-consistent 1D steady-state NI MHD turbulence transport model equations and background solar wind equations for the quiet solar corona from 1 to 20 R⊙ using the boundary conditions shown in Table 1. Here we assume balanced quasi-2D turbulence, and imbalanced slab turbulence at the base of the solar corona. The boundary conditions for the quasi-2D Elsässer energies are obtained by assuming 2.7 G magnetic field fluctuations and a solar coronal density of $4 \times 10^8$ cm$^{-3}$, which yields the turbulent Alfvén speed as $V_A \sim 295$ km s$^{-1}$, or $V_A^2 \sim 8.7 \times 10^4$ km$^2$ s$^{-2}$ [26]. To obtain the boundary condition for the energy in forward propagating modes (slab), we assume an 80:20 ratio between quasi-2D and slab turbulence. The boundary condition for the correlation function is obtained by multiplying the energy density and the corresponding correlation length. The correlation length of the quasi-2D turbulence is taken from case 2 of Zank et al [26], where $\lambda_\perp = 5 \times 10^4$ km was assumed. This correlation length is 10 times larger than the correlation length suggested by Abramenko et al [40]. For the residual energy, we assume approximate equipartition between the fluctuating kinetic and magnetic energy, slightly dominated by fluctuating magnetic energy at the coronal base.

| 2D Core Model Equations | Slab Model Equations |
|-------------------------|----------------------|
| $\langle z^{+\infty} \rangle$ | 22000 km$^2$s$^{-2}$ |
| $\langle z^{-\infty} \rangle$ | 20000 km$^2$s$^{-2}$ |
| $E^{\infty}_D$ | -2200 km$^2$s$^{-2}$ |
| $L^+_\infty$ | $1.0 \times 10^9$ km$^3$s$^{-2}$ |
| $L^-_\infty$ | $1.0 \times 10^9$ km$^3$s$^{-2}$ |
| $L^+_D$ | -1.1 $\times 10^8$ km$^3$s$^{-2}$ |
| $U$ | 29.26 km s$^{-1}$ |
| $n$ | $4 \times 10^8$ cm$^{-3}$ |
| $T$ | $5 \times 10^4$ K |
| $\langle \rho^{\infty} \rangle$ | $1.6 \times 10^{15}$ cm$^{-6}$ |
| $E^+_D$ | -115.79 km$^2$s$^{-2}$ |
| $L^+_s$ | $1.92 \times 10^6$ km$^3$s$^{-2}$ |
| $L^+_D$ | -2.89 $\times 10^6$ km$^3$s$^{-2}$ |

Table 1. Assumed base values at 1 R⊙ used to solve the turbulence transport and the background (large-scale) solar wind equations.

The numerical solutions of the 1D hydrodynamic solar wind equations when coupled to the turbulence transport equations are shown in Figure 1. The top left panel of Figure 1 shows the solar wind speed and the Alfvén velocity as a function of normalized heliocentric distance. The solar wind plasma accelerates rapidly near the base of the solar corona, and reaches $\sim 500$ km s$^{-1}$ at 20 R⊙. Similarly, the Alfvén velocity increases rapidly to a peak value of $\sim 2600$ km s$^{-1}$ and then decreases with increasing heliocentric distance. Here the acceleration of the solar wind plasma and the increase in the Alfvén velocity are associated with the decay rate of turbulence, which can be adjusted by the von Kármán-Taylor constant $\alpha$. The numerical solutions of the solar wind speed and the Alfvén velocity show that the solar wind speed and the Alfvén velocity intersect at $\sim 7.5$ R⊙, which identifies the Alfvén critical point. The Alfvén surface is the location at which the solar wind and the Alfvén speed become equal, i.e., $|U| = |V_A|$. The Alfvén
Figure 1. Solar wind parameters as a function of heliocentric distance normalized to the solar radius. Top left: The solar wind speed, and the Alfvén velocity. Top right: The solar wind temperature. Bottom left: The solar wind density. Bottom right: The Alfvén and sonic Mach number. The horizontal blue dotted line shows $M_A = M_S = 1.$

surface distinguishes the sub-Alfvénic region $|U| \ll |V_A|$ and the super-Alfvénic region $|U| \gg |V_A|$. Recently, Adhikari et al [41] used the low plasma beta turbulence transport model equations to show that the solar wind turbulence at the Alfvén surface does not turn off.

The top right panel of Figure 1 shows the coronal plasma temperature as a function of normalized heliocentric distance. It shows that the coronal temperature increases to $\sim 2.5 \times 10^6$ K near the coronal base, and then decreases with increasing heliocentric distance. The bottom left panel of Figure 1 illustrates the solar wind density as a function of normalized heliocentric distance. The solar wind density decreases rapidly near the coronal base, and then slowly with distance. The rapid decrease in the solar wind density corresponds to a rapid increase in the solar wind speed to conserve mass flux.

The bottom right panel of Figure 1 shows the Alfvén Mach number and the sonic Mach number as a function of normalized heliocentric distance. Both Mach numbers are less than 1 at 1 $R_\odot$, become equal to 1 at a certain distance, and then become larger than 1 with increasing distance. The sonic Mach number (dashed curve) intersects the horizontal blue line at $\sim 1.14 R_\odot$, which is the sonic point. The Alfvén Mach number (solid curve) intersects the horizontal line at $\sim 7.5 R_\odot$, which is the Alfvén surface, also shown in the top panel of Figure 1 (intersection point of the solar wind speed and the
Alfvén velocity). The locations of the sonic point and the Alfvén surface depend on the heating rate due to the dissipation of turbulence.

![Figure 2](image)

**Figure 2.** Turbulence quantities plotted as a function of normalized heliocentric distance. First row: Left, middle and right panels are the energy in backward and forward propagating modes, and the total turbulent energy. Second row: Left, middle and right panels are the normalized residual energy, the normalized cross-helicity and the Alfvén ratio. Third row: Left, middle and right panels are the fluctuating kinetic energy, fluctuating magnetic energy, and the variance of the density fluctuations. In all panels, the solid curves depict quasi-2D turbulence quantities, the dashed curves slab turbulence quantities, and the dashed-dotted-dashed curves the sum of quasi-2D and slab turbulence quantities.

Unlike the Zank et al. [26] model, the radial profiles of the turbulence quantities shown in Figure 2 and the background plasma shown in Figure 1 influence each other. In Figure 2, the solid curves depict majority quasi-2D turbulence, the dashed curves minority slab turbulence, and the dashed-dotted-dashed the total turbulence, i.e., the sum of quasi-2D and slab turbulence. In Figure 2, the left, middle, and right panels of the first row illustrate the energy in backward and forward propagating modes, and the total turbulent energy as a function of normalized heliocentric distance. The energy in quasi-2D turbulence is larger than that in slab turbulence between 1 and 20 $R_\odot$. 


both energies decrease with increasing heliocentric distance.

The left, middle, and right panels of the second row of Figure 2 show the normalized residual energy, the normalized cross-helicity, and the Alfvén ratio as a function of normalized heliocentric distance. Here the left panel of the second row shows that the quasi-2D and slab normalized residual energies become positive near the coronal base and then become negative with increasing heliocentric distance. The figure also shows that the quasi-2D fluctuations are more magnetically dominated compared to slab fluctuations. As illustrated in the normalized cross-helicity plot (middle panel of the second row), the balanced quasi-2D turbulence at 1 R⊙ evolves to an imbalanced state beyond 1 R⊙, while the imbalanced slab turbulence becomes more imbalanced with increasing heliocentric distance. The slab normalized cross-helicity is larger than the quasi-2D normalized cross-helicity, suggesting that it is primarily outward propagating waves that are present. The right panel of the second row displays the quasi-2D and slab Alfvén ratios as a function of heliocentric distance. It shows that both Alfvén ratios decrease with increasing distance, indicating that both quasi-2D and slab turbulence are dominated by the fluctuating magnetic energy, consistent with the result of the normalized residual energy.

The left panel of the third row of Figure 2 is the fluctuating kinetic energy. Both the quasi-2D and slab fluctuating kinetic energies decrease with increasing heliocentric distance. Here the fluctuating quasi-2D energy is larger than the fluctuating slab kinetic energy between 1 and 20 R⊙. The middle panel of the third row of Figure 2 shows that the quasi-2D fluctuating magnetic energy is the dominant component in the region between 1 and 20 solar radii. The quasi-2D and slab fluctuating magnetic energies decrease with increasing heliocentric distance. Similarly, the right panel of the third row describes the variance of density fluctuations as a function of heliocentric distance. Similar to the turbulence energy, the variance of density fluctuations decreases with increasing distance.

Numerical solutions of the transport equations for the quasi-2D and slab correlation functions are shown in top left and middle panels of Figure 3. Here the quasi-2D correlation functions corresponding to forward and backward modes L±∞ (black and red curves) and the residual energy LD∞ decrease initially and then become approximately constant with increasing distance. Notice that the L±∞ are positive and LD∞ negative throughout the heliosphere, and that L+∞ and L−∞ overlap with each other. Similarly, the correlation function associated with slab turbulence L* decreases rapidly and then increases slightly with distance, whereas LD* gradually decreases with heliocentric distance.

The top right and bottom left panels of Figure 3 describe the quasi-2D and slab correlation lengths (solid and dashed curves, respectively) corresponding to backward and forward propagating modes as a function of normalized heliocentric distance. The correlation lengths of the quasi-2D and slab forward and backward propagating modes increase with increasing distance. Similarly, the bottom middle and right panels of Figure 3 show the correlation lengths corresponding to quasi-2D and slab (solid and dashed curves, respectively) velocity and magnetic field fluctuations, which also increase with increasing heliocentric distance.
Figure 3. Normalized quasi-2D and slab correlation functions and correlation lengths as a function of normalized heliocentric distance. Top left: Quasi-2D correlation functions. Top middle: Slab correlation functions. Top right: Correlation lengths corresponding to backward propagating modes. Bottom left: Correlation lengths corresponding to forward propagating modes. Bottom middle: Correlation length of velocity fluctuations. Bottom right: Correlation length of magnetic field fluctuations.

4. Discussion and Conclusions

We have developed a turbulence driven-solar wind model by coupling the nearly incompressible magnetohydrodynamic (NI MHD) turbulence transport model equations and the solar wind equations. This work is an extension of Zank et al [26]. Unlike the Zank et al [26] model, in which turbulence and plasma flow do not interact with each other, we couple the NI MHD turbulence model to the background plasma flow. The NI MHD turbulence transport model in the plasma beta regime of order $\beta_p \ll 1$ describes majority quasi-2D (nonpropagating) and minority slab turbulence. Since quasi-2D turbulence is the dominant component in the NI MHD model, it plays a major role in heating and driving the coronal/solar wind plasma. The NI MHD model is therefore different from the wave/turbulence-driven (W/T) models [7 – 12], in which Alfvén waves heat and drive the solar wind. The NI MHD model assumes that the small-scale “magnetic carpet” loops in the photosphere continuously pump quasi-2D fluctuations into the region above the photosphere, and the subsequent dissipation of which heats the plasma above the photosphere. We solved the coupled NI MHD quasi-2D and slab turbulence transport model equations and plasma flow equations from the coronal base to 20 solar radii using a Runge Kutta 4th order method. We summarize our findings as follows.

(i) The solar wind flow speed increases rapidly over a height of $\sim 2 - 4 R_\odot$, and then more gradually until $\sim 20$ solar radii. The sonic point is located at $\sim 1.14 R_\odot$, and the Alfvén surface is located at $\sim 7.5 R_\odot$. 
(ii) The coronal plasma temperature rises to $\sim 2.5 \times 10^6$ K within a few solar radii.
(iii) The solar wind density decreases quickly initially, and then more slowly with increasing distance.
(iv) The quasi-2D and slab turbulence energies decrease with distance, and the correlation lengths increase with distance.
(v) The fully balanced quasi-2D turbulence state at 1 R$_\odot$ becomes imbalanced with increasing heliocentric distance. The imbalanced minority slab turbulence at coronal base remains imbalanced between 1 and 20 R$_\odot$, eventually being comprised predominantly of outwardly propagating modes.
(vi) The normalized quasi-2D and slab residual energy becomes positive near the coronal base, and both energies become negative with increasing distance.
(vii) The variance of the density fluctuations decreases with increasing heliocentric distance.

In this paper, we presented a turbulence driven-solar wind model and numerical solutions for a particular set of boundary conditions. In our turbulence driven-solar wind model, the turbulence energy densities decay rapidly with the prescribed boundary conditions shown in Table 1 leaving only a modest turbulence energy at 20 R$_\odot$. The rapid decay of turbulence energy is due to a strong nonlinear dissipation term, and no sources of turbulence have been included in the turbulence transport model equations. In future work, we will investigate the effects of strong and weak dissipation terms and consider sources of turbulence. However, our findings suggest that the dissipation of quasi-2D turbulence is primarily responsible for heating the coronal plasma to several million of degrees Kelvin, and drives the solar wind from a subsonic to a supersonic state. The comparison of the results of our turbulence-driven solar wind model equations with Parker Solar Probe (PSP) measurements will provide a true test of our theory.

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