Method for Theoretical Line Loss Calculation of 10kV Distribution District based on Actual Electric Energy of Distribution Transformer Secondary Side

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Abstract. The 10kV distribution district serves the electricity customer, and its theoretical loss can reflect the real situation of the technical line loss. Adopting the equivalent resistance method based on average current, on the basis of the remotely read active and reactive electricity at the head of the 10kV line, as well as the actual copying electricity on the secondary side of distribution transformer, this paper establishes mathematical models for calculating the electricity loss in both forward and backward directions. The case analysis shows that the results of the backward mathematical model accurately reflect the actual value of the theoretical line loss, and by comparing the theoretical line loss rate calculated by the two mathematical models, the unknown loss of the line can be identified.

1. Introduction

Line loss refers to the electricity loss in the process of transmission and distribution. According to the proportion of electricity loss in various level of power grid, the electricity loss of 10kV distribution line accounts for 27% of the total loss, which is relatively high among voltage levels and directly affect the economic benefits of power companies [1]. Therefore, one of the basic tasks of the power department is to manage the electricity loss of 10kV distribution lines, finding out the reasons for the large loss and taking measures to reduce loss.

The actual line loss can be divided into theoretical line loss and management line loss including meter error, power theft, leakage and other unknown loss [2]. The theoretical line loss is the sum of variable and invariable line loss, where variable line loss is divided into wire loss and loss of transformer winding on the line. For the 10 kV distribution lines with long running time and high actual line loss rate, theoretical line loss rates obtained by different methods are compared to identify unknown loss, which needs to obtain the theoretical line loss first. There are many methods to calculate the theoretical line loss, such as root-mean-square current method, average current method, loss factor method, equivalent resistance method, power flow algorithm and other methods. Because of the influence of factors such as multiple branch lines in the 10kV distribution district and load fluctuations on the secondary side of distribution transformer [3], it is difficult to accurately calculate the theoretical line loss. The calculation process of power flow algorithm is complicated, which is difficult to obtain accurate data and the error is large [4], so it has many limitations in practical application. The root-mean-square current method and average current method can be used to...
calculate the theoretical line loss roughly, but can not find the unknown loss according to the results. On the premise of calculating theoretical line loss of low-voltage distribution district with load electricity, reference [5] finds unknown line loss by comparing the difference between actual line loss rate and theoretical line loss rate.

At present, the power distribution system has a high level of automation, especially the application of smart meters and remote automatic meter reading system [6], making it more convenient and accurate to collect the active electricity, reactive electricity and load records on the secondary side of distribution transformer, and realizing the automatic calculation of actual line loss, so as to monitor the operation of the line in real time.

Based on the above situation, this paper establishes mathematical models for calculating the theoretical line loss in both forward and backward directions, discusses the results of the forward and backward models and the respective electricity line loss and distribution transformers through cases. By comparing the theoretical line loss rates and their changes calculated by forward and backward models, the real situation of technical line loss is reflected and the method can be used to identify unknown line loss, which provide reference for power department to take effective measures to improve the level of line loss management.

2. Mathematical models of calculating the theoretical line loss in 10kV distribution district
The theoretical line loss is the sum of the variable line loss and the invariable line loss. The invariable line loss is the no-load loss of the distribution transformer. After determining the type of the transformer, the value of invariable line loss can be calculated, and the expression is as follows:

\[ \Delta A_{LD,io} = \sum_{i=1}^{m} \Delta P_{io} \times 10^{-3} \]  

where \( \Delta P_{io} \) is no-load loss power of the \( i \)-th distribution transformer, \( m \) is the number of transformers on the whole line, \( t \) is the line power supply time.

2.1. The basic principle of equivalent resistance method
This paper mainly discusses the variable line loss, which is divided into line loss and transformer winding loss. The equivalent resistance method approximately calculate the variable line loss [7] and its theoretical basis is the average current method. The equivalent resistance method regards the variable loss of the distribution district as the loss caused by the average current \( I_{av} \) at the head of the line passing through the sum of equivalent resistance of the line and the distribution transformer \( R_{eq} = R_{DZL} + R_{DZB} \), and the equivalent resistance model of distribution lines is shown in figure 1.

\[ R_{DZL} \quad R_{DZB} \]

Fig 1. Equivalent resistance model.

The calculation of line equivalent resistance is based on the plotted line diagram and the actual copying electricity on the secondary side of distribution transformer. It is calculated from the end to the head of the line, from the branch to the trunk, which is the order of increasing the line electricity quantity, and the common expression is as follows [8]:

\[ R_{eq} = R_{DZL} + R_{DZB} \]
2018 International Conference on Civil, Architecture and Disaster Prevention
IOP Publishing
IOP Conf. Series: Earth and Environmental Science 218 (2019) 012152
doi:10.1088/1755-1315/218/1/012152

\[
R_{DZ_L} = \frac{\sum_{j=1}^{n} (A_{by}^2 R_j)}{\left(\sum_{i=1}^{m} A_{bi}\right)^2}
\]  \hspace{1cm} (2)

where \(A_{by}\) is the sum of actual copying electricity on the secondary side of distribution transformer for line \(j\) power supply, \(n\) is the total segment number for line segmentation, \(A_b\) is the actual copying electricity on the secondary side of distribution transformer, \(m\) is the number of transformers on the whole line, \(R_j\) is calculation resistance of line wire for section \(j\): \(R_j = r_0 L\), \(r_0\) is unit length resistance and \(L\) is the length of the line.

Similarly, the general formula for calculating the total equivalent resistance of transformer winding is \([8]\):

\[
R_{DZ_B} = \frac{\sum_{i=1}^{m} (A_{bi}^2 R_{B,i})}{\left(\sum_{i=1}^{m} A_{bi}\right)^2}
\]  \hspace{1cm} (3)

where \(R_{B,i}\) is the equivalent resistance of the \(i\)-th distribution transformer winding to the primary side, which is calculated by the following formula:

\[
R_{B,i} = \Delta P_{k,i} \left(\frac{U_{1N}}{S_{ei}}\right)^2 \times 10^3
\]  \hspace{1cm} (4)

where \(\Delta P_{k,i}\) is the rated short-circuit loss power of the \(i\)-th distribution transformer, \(S_{ei}\) is the rated capacity of the \(i\)-th distribution transformer, and \(U_{1N}\) is the rated voltage for primary side of distribution transformer.

2.2. The forward mathematical model
Because the head of 10kV line is equipped with active energy meter, reactive energy meter, voltmeter and other meters, the active and reactive electricity can be read for a period of time. So the theoretical line loss can be calculated by the "electricity method", that is, the method of energy meter counting and the formula is as follows:

\[
\Delta A_{KB} = I_{av}^2 K^2 R_{eq}^2 t \times 10^{-3}
\]

\[= \left(A_{pg}^2 + A_{Q,g}^2\right) \frac{K^2 R_{eq}^2}{U_{1e}^2 t} t \times 10^{-3}
\]  \hspace{1cm} (5)

where \(A_{pg}\) and \(A_{Q,g}\) are the active and reactive electricity copied at the head of the line, respectively, \(K\) is the shape coefficient of the load curve, and after calculating the average current at the head of the line, the shape coefficient of the load curve is determined by looking up the relationship curve \(K = f(U_{av})\), \(R_{eq}\) is total equivalent resistance of line, \(U_{1e}\) is average operating voltage at the head of line, and \(t\) is the line power supply time.

It can be seen from the above formula that the basis of forward calculation of theoretical line loss by equivalent resistance method is the average load current calculated by active and reactive electricity which is read from the head of 10kV line. Since the active and reactive electricity measured by the
head meter of the line is the total amount of electricity actually supplied to the distribution district, which means the active electricity contains the unknown loss when the line fault occurs. Therefore, the error of theoretical line loss determined by the data is large, and the unknown line loss can-not be identified.

When the 10kV distribution line is normally operating, its theoretical line loss should match the actual copying electricity on the secondary side of distribution transformer. If the unknown loss is produced in the line, the electricity supply on the secondary side of the distribution transformer should be kept unchanged to meet the demand of electricity customer, while the total active power at the head of the line should be changed with the change of unknown loss. Therefore, the changes of theoretical line loss rate calculated by using the actual electricity on the secondary side of distribution transformer are different from that calculated by using the electricity at the head of the line. So this information can be used to identify if there is any unknown loss in the line.

2.3. Backward mathematical model

Based on the basic phenomenon that the theoretical line loss in 10kV distribution district matches the actual copying electricity on the secondary side of the distribution transformer, this section establishes a mathematical model for calculating the theoretical line loss by using the actual copying electricity on the secondary side of the distribution transformer, and the formula for calculating the average current at the head of the line is:

$$I_{av} = \frac{\left( \sum_{i=1}^{n} A_{li} \right) + \Delta A_{KB}}{U_{li} \cos \phi \cdot t}$$ \hspace{1cm} (6)

The theoretical line loss formula is:

$$\Delta A_{KB} = \frac{I_{av}^2 K^2 R_{eq} t \times 10^{-3}}{U_{li} \cos \phi \cdot t} = \left[ \left( \sum_{i=1}^{n} A_{li} \right) + \Delta A_{KB} \right]^2 \frac{K^2 R_{eq} t \times 10^{-3}}{U_{li} \cos \phi \cdot t}$$ \hspace{1cm} (7)

where $\Delta A_{KB}$ is the theoretical line loss for 10kV distribution district and $\cos \phi$ is the average power factor for the head of the line.

When calculating theoretical line loss with forward model, the average current $I_{av}$ is determined by the data of active and reactive electricity supply at the head of the line firstly, and the theoretical line loss can be obtained by: $\Delta A_{KB} = I_{av}^2 K^2 R_{eq} t \times 10^{-3}$.

However, it can be seen from (6) that the average load current at the head of the line in the backward mathematical model can-not be obtained until the theoretical line loss is obtained. Therefore, the average current is substituted into the theoretical line loss calculation formula, to obtain the quadratic equation with the theoretical line loss of the 10kV distribution line as a variable as shown in (7), and the theoretical line loss calculated by the root formula.

3. The results and analysis of case

The forward and backward mathematical models are used to calculate the theoretical line loss respectively, the factors affecting the value of line loss are analyzed, and the difference and cause of the calculation results of the two mathematical models when the line produces unknown loss are explored, thus the unknown line loss can be identified based on this. The following is a single line diagram of this case.
3.1. The effect of load rate

Because the load rate of 10kV distribution line varies with the load within 24 hours of a day, in order to reflect the actual line loss and theoretical line loss under different load rates and the difference between them, the actual and theoretical line loss rate of the line are calculated respectively with the single line diagram of the distribution district in figure 2 as an example.

From figure 3, it can be seen that the actual line loss rate of 10kV distribution district is greater than the theoretical line loss rate under different loads. Because the actual line loss is the difference between the actual power supply and the electricity sales, including meter reading error, meter error and so on, so the theoretical line loss rate is an important reference for the power department to consider the loss reduction measures.

According to the blue curve in figure 3, the theoretical line loss rate decreases first and then increases when the load rate increases from 10% to 100%, because the invariable loss and variable loss are almost equal when the load rate is about 20%. In this situation, the line load current is called the economic load current. If the line can operate in an area around the economic load current, the corresponding theoretical line loss is relatively small.

However, in order to improve the utilization rate of distribution transformer, meet the demand of electricity customer and consider the economy of line operation, 10kV distribution district often work under heavy load or even full load conditions and the corresponding line loss will be higher. Therefore, the power department should take effective measures to reduce the loss of power distribution lines during heavy operation.

As can be seen from figure 4, the theoretical line loss calculated by the forward mathematical model is greater than that calculated by the backward mathematical model in the process of increasing...
the line load rate from small to full load. Because the active electricity at the head of the line read by
the forward calculation of the line loss includes not only the actual power supply of the secondary
meter, but also the loss of the line and transformer windings, so the result of line loss will be larger
when adding the active electricity to (5). The backward mathematic model for calculating line loss
based on the actual power supply on the secondary side of distribution transformer matches the
theoretical line loss of the 10kV distribution district, and the calculation results can truly reflect the
line loss in normal operation of the line.

The losses of wire and distribution transformer under different load rates are calculated
respectively and their changes are shown in the following figure.

![Fig 5. Variation of wire loss and winding loss of distribution transformer.](image)

As can be seen from figure 5, the wire loss accounts for a large proportion of the total loss, and it
increases more rapidly than the transformer winding loss. When the load rate is more than 90%, the
winding loss of distribution transformer is about 300 kWh, while the wire loss is as high as 700 kWh,
which is the main reason for the higher theoretical line loss rate during heavy operation of the 10kV
distribution district.

### 3.2. Influence of power factor

Taking the backward mathematic model of calculating line loss by using the actual copying electricity
on the secondary side of distribution transformer as an example, the line loss under typical conditions
of light and heavy loads is calculated. The initial power factor of the line is 0.6, and the variable losses
are 60.15 kWh and 199.43 kWh respectively when the line load rate is 20% and 80%. The variation of
line loss obtained by substituting the increased average power factor of multiple groups into (6) is
shown in the table below.

| cosφ | 0.65 | 0.7 | 0.75 | 0.8 | 0.85 | 0.9 | 0.95 |
|------|------|-----|------|-----|------|-----|------|
| ΔA₁ (kWh) | 10 | 17 | 23 | 28 | 32 | 35 | 38 |
| ΔA₂ (kWh) | 245 | 416 | 541 | 637 | 713 | 773 | 823 |

ΔA₁ and ΔA₂ are loss changes with load rates of 20% and 80%, respectively. From the above table,
it can be seen that the line loss decreases with the increase of power factor, and the line loss reduction
is especially obvious when the load rate is 80%, which shows that increasing the power factor is of
great significance to reduce the loss of overloaded lines.

### 3.3. Identification of unknown line loss

Because the distribution lines with long running time often produce unknown loss, two mathematical
models are used to calculate the theoretical line loss rate in the process of increasing unknown line
loss and the changes of line loss rate is shown in the following figure.
Fig 6. Graph of theoretical line loss rate varying with unknown line loss.

It can be seen from the variation of the curve in figure 6 that the theoretical line loss rate calculated by the forward mathematical model increases with the increase of the unknown loss power, which makes use of the characteristic that the active electricity at the head of the line read by the forward mathematical model contains unknown loss, while the theoretical line loss rate calculated by the backward mathematical model decreases gradually. Therefore, we can identify the unknown line loss based on this information.

4. Conclusion
This paper presents a method for calculating the theoretical line loss of 10kV distribution district based on the actual electricity on the secondary side of distribution transformer. The results truly reflect the theoretical line loss of distribution district. Line loss increases with the increase of load rate, but decreases with the increase of power factor, and compared with the distribution transformer winding loss, the wire loss accounts for higher in the total line loss and the growth is more rapid. When the unknown loss occurs in the line, it can be identified by the different loss rates calculated by the forward and backward models.

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