Calculation and modeling of harmonic lenses with a variable height of microrelief

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Abstract. In this paper the profile of harmonic lens with a variable height of microrelief was calculated. Moreover, structure of harmonic lens with a constant height of microrelief was displayed. Also, the point-spread function (PSF) for different types of the harmonic lens was shown. The point-spread function for selected types of harmonic lenses was calculated. Comparison of the main maximums sizes in area 40x40 microns when falling 3 wavelengths and 11 wavelengths was made. Finally, the point-spread function of three zones lens is shown.

1. Introduction
The use of classical lenses and mirrors is difficult in the manufacture of multi-structure optical elements and systems. It is also impossible to create a light complex field using only classical optical elements (lens, mirrors, etc.). There is a question about minimizing size of optical elements allowing to form the same distribution of the light field like when using a complex optical system. This optical task has a good solution by using diffractive optical elements (DOE). Using diffractive optical elements considered the wave property of light we have a chance to transform incident laser beam with variety distribution of amplitude and phase [1,2].

Diffraction optical elements (DOE) divide a light beam into a large number of rays with different refractive index. This feature is the cause of appearing chromatic dispersion effect [3,4]. The influence of this effect strongly noticeable in imaging [5-7] and focusing [8-10] optical systems. To reduce or completely eliminate chromatic dispersion effect we suggest using mixed optical systems, i.e. systems with refraction and diffraction elements where the opposite chromaticity is observed.

An example of DOE combining both diffractive and refractive elements are harmonic lenses [12-16]. Transformation of the phase to value $2\pi N$ is called harmonic interval (where $N$ – the order of harmonic lens). The larger a value of $N$, the closer properties of harmonic DOE to the refractive element, where chromaticity depends only on properties of the lens material.

In this work theoretical calculations of structure harmonic lens illuminated by light beams with wavelengths 450 nm, 550 nm and 650 nm were showed. After calculation of structure harmonic lens, we displayed a point-spread function of each lens, which was made under certain baseline (450 nm, 550 nm, and 650 nm). Further comparing of radiuses the main maximum for different harmonic lens were made.
2. Theoretical describing in radial-symmetric case
Consider a task of diffraction on the optical element which has a radial complex transmission function \( g(r) \). In paraxial case this task is being decided using an equation (1):

\[
E(\rho, z) = \left(\frac{ik}{z}\right) \exp(ikz) \exp(ik\frac{\rho^2}{2z}) \int_{0}^{\infty} g(r) \exp \left(ik\frac{kr^2}{2z}\right) r dr.
\]

(1)

The height of microrelief of a harmonic lens with order \( N \) calculated under certain wavelength \( \lambda \) is given by next equation:

\[
h_N(\rho, \lambda) = \left[\text{mod}_{2\pi N} \varphi(\rho, \lambda)\right] \frac{\lambda_i}{2\pi[n-1]},
\]

(2)

where \( \varphi(\rho, \lambda) \) - the phase function of the spherical lens.

Thus, the phase of the element under illumination with wavelength \( \lambda_i \) is given by the formula (3):

\[
\varphi_N(\rho, \lambda_i) = h_N(\rho) \frac{2\pi[n-1]}{\lambda_i}.
\]

(3)

Based on the phase obtained above, the transmission function of such diffractive optical element is given by equation (4):

\[
g(\rho, \lambda_i) = e^{i\varphi_N(\rho, \lambda_i)}.
\]

(4)

3. Results of modeling and comparing radius of beams
Following the equation (4) complex transmission function \( g(\rho, \lambda_i) \) of harmonic lens is calculated under one base wavelength (550 nm) and three zones’ harmonic lens calculated under 3 wavelengths (450, 550 and 650 nm) were made. After modeling of microrelief flat waves with wavelengths equal to 450, 550 and 650 nm were falling on these lenses. Further, the intensity of radiation in the focal plane of the lens was calculated (\( f = 100 \text{ mm}, \ d = 10 \text{ mm} \)).

In figure 1 point-spread function depending on a wavelengths number falling on the harmonic lens with a constant height of microrelief is shown. Note that in figure 1a falling radiation focuses in the area closed to the center and central maximum is clearly expressed unlike figure 1b where many falling depicted wavelengths. Using an FWHM in a necessary area, we calculated diameters of beams. In the first case (figure 1a) this diameter of beam equals to 5.4 microns, in the second case equals to 11.32 microns. Thus, we can conclude that for larger wavelengths numbers harmonic lens with a constant height of microrelief has many peaks (maximums) shifted relatively to each other. This a spect maybe decrease the image quality in the focal plane.

Describing of 3 zones’ harmonic lens was made. To maintain a balance of intensity in the simulation such microstructure we need to consider a case of equality zones calculated under certain wavelength (in this paper for wavelength \( \lambda_1 = 450 \text{ nm}, \ \lambda_2 = 550 \text{ nm}, \ \lambda_3 = 650 \text{ nm} \)). The condition of squares equality zones is given by next equation
When solving equality (5) we find the ranges of heights for each zone:

1. For range $0 \leq r \leq \frac{R}{\sqrt{3}}$:

$$h_N(r) = \left[ \text{mod}_{2\pi N} \varphi(r, \lambda) \right] \frac{\lambda_1}{2\pi [n - 1]}.$$ 

2. For range $\frac{R}{\sqrt{3}} < r < \frac{\sqrt{2}}{3}R$:

$$h_N(r) = \left[ \text{mod}_{2\pi N} \varphi(r, \lambda) \right] \frac{\lambda_2}{2\pi [n - 1]}.$$ 

3. For range $\frac{\sqrt{2}}{3}R \leq r(i) \leq R$:

$$h_N(r) = \left[ \text{mod}_{2\pi N} \varphi(r, \lambda) \right] \frac{\lambda_3}{2\pi [n - 1]}.$$ 

The obtained height of microlief is shown in figure 2.

![Figure 2](image1.jpg)

**Figure 2.** The graph of the height of the relief change along the radius.

After that, a microlief and phase of the 3-band harmonic lens were calculated.

![Figure 3](image2.jpg)

**Figure 3.** The image of three-leveled harmonic lens phase.
After computing of microstructure and phase of 3-band harmonic lens, the point-spread function under falling on this element 3 wavelength and 11 wavelengths was modeled. These distributions of intensity are shown in figure 4a and figure 4b, respectively.

Figure 4. The point-spread function of a three-leveled harmonic lens with variable height of microrelief depending on the number of falling wavelength (4a – 3 falling wavelengths, 4b – 11 falling wavelengths). The image scales 40x40 microns.

The point-spread function depending on the number of falling flat wavelengths on 3-band harmonic lens with variable height microrelief is shown in figure 4. From figure 4a, we can observe that falling radiation focuses in the area closed by the center but have a circle shifted relatively to main maximum. In figure 4b, a central peak is wider than the peak as shown in figure 4a. Using an FWHM in a necessary area, we calculated diameters of beams. In the first case (figure 4a) this diameter of beam equals to 4,56 microns, in the second case equals to 7,64 microns. Thus, we can conclude that central maximum of 3-band harmonic lens is tighter than for harmonic lens with a constant height of microrelief. In the first case (by falling 3 wavelengths) the main maximum is tighter on 15,5% and in the second case (by falling 11 wavelengths) is tighter on 32,3%.

Now move to consideration of diffraction given by falling 3 and 11 wavelengths in an area with image scale is 2x2 mm. The point-spread function of a harmonic lens with a constant height of microrelief in an area with image scale 2x2 mm was presented in figures 5a and 5b for different wavelengths.

Figure 5. The point-spread function of a harmonic lens with a constant height of microrelief depending on the number of falling wavelengths (5a – 3 wavelengths, 5b – 11 wavelengths). The image scale is 2x2 mm.

From the obtained images, we can calculate a diameter of the beam using FWHM method. Thus when 3 wavelengths fall on the harmonic lens with a constant height of microrelief, the radius of the beam equals to 5,4 microns. However, when 11 wavelengths fall on the harmonic lens with a constant height of microrelief this value equals to 130 microns.

Consider falling of 3 and 11 wavelengths on the harmonic lens with variable height of microrelief. The results of point-spread function for each case are shown in figures 6a and 6b.
Figure 6. The point-spread function of a three-leveled harmonic lens with a variable height microrelief depending on the number of falling wavelengths (6a – 3 wavelengths, 6b – 11 wavelengths). The image scale is 2x2 mm.

From the obtained images, we can calculate a diameter of the beam using FWHM method. In the first case (by falling 3 wavelengths) this value equals to 4.56 microns, but in the second case (by falling 11 wavelengths) the diameter of the beam is equal to 7.64 microns. Comparing these results with the results of diameters obtained using a harmonic lens with a constant height of the microrelief, it can be concluded that the diameter of the beam decreased in the first case by 15.5%, and in the second case by 95.4%.

After modeling the 3-band harmonic lens, a harmonic lens was simulated with a number of zones equal to the number of incident wavelengths. In our case, an 11-band harmonic lens was considered when falling at 11 wavelengths (within the limits of 450 nm to 650 nm in 20 nm steps). Figure 7 shows the phase of a given harmonic lens.

Figure 7. The image of phase eleven-leveled harmonic lens.

Below we give the results of modeling, the point-spread function in different ranges is presented in figures 8a and 8b. Also, the diameter of the central peak is defined.

Figure 8. The point-spread function of an eleven-leveled harmonic lens with variable height microrelief by falling 11 wavelengths (8a – the image scale is 40x40 microns, 8b – the image scale is 2x2 mm).
Using an FWHM the diameter of central maximum (diameter of the beam) was calculated. This value equals to 5.08. In comparison with the fall of 11 wavelengths into a 3-band harmonic lens with a variable height of the microrelief and a harmonic lens with a constant height of the microrelief, the diameter of the beam decreased in the first case by 15.4%, and compared to a harmonic lens with a constant height of the microrelief decreased by 96.1%. But, as can be seen from the figure, the number of additional rings increased.

4. Conclusion
In this work, the dependence of microrelief height of 3-band harmonic lens on radius was obtained. Also the phase function of harmonic lens was showed.

The point-spread function by falling 3 and 11 wavelengths on the harmonic lens with variable height of microrelief was calculated. Further, phase and point-spread function of 11-band harmonic lens by falling 11 wavelengths were modeled. The diameter of beams using an FWHM method was found. After that, these results of the PSF of the 11-band lens were compared with the result of the PSF of the 3-band lens and revealed a 15.4% reduction in beam diameter in the 11-band compared to the 3-band lens and a reduction in the beam diameter by 96.1% compared to a harmonic lens with a constant height of the microrelief.

Thus, in the case of 3-band harmonic lens, beam diameter decreased to 15.5% relative to the harmonic lens with a constant height of microrelief and decrease of 95.4% by falling 11 wavelengths were observed.

5. References

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