Methodology for optimization of preload in a bolted-flange connection based on Markov theory

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Abstract. The bolted-flange connection is one of the most popular detachable mechanical joints in mechanical flange connection. It is well known that bolted-flange joints need to be tightened to meet the assembly performance in practical engineering applications. However, the dispersion of the pre-tightening force caused by the elastic interaction between the bolts has a serious impact on the sealing and safety of the product, and therefore the optimization of bolt preload has always been a hot spot in the assembly research field. Combining mathematical theory and experimental data, this paper proposes an optimization model for uniformly distributed preload based on the Markov theory. The torque method was utilized to fasten the bolt-flange joint. The bolt pre-tightening force in the final state was collected and used as sample data. A state transition model was constructed to determine the state transition matrix. Ultimately, an iterative model of the preload was deduced, and the rationality of the model was verified through experimentation. The proposed optimization method of bolt preload can be applied to determine the initial preload of each bolt. This method improves the assembly efficiency and accuracy, which provides a theoretical approach to effectively guide the tightening work of bolted-flange connections.

1. Introduction

The bolt-flange connection is one of the most popular connections in mechanical flange connection due to its features such as easy installation and disassembly, low maintenance cost and so on. To ensure the tightness of the equipment during service, the bolt-flange connection must be tightened beforehand. At present, torque method and angle method are often used to tighten bolts in practical engineering. However, due to the influence of elastic interaction between bolts, the pre-tightening force of some fastened bolts inevitably changes. The study of this problem and its optimization method has become a hot spot for high performance assembly of the bolt-flange connection.

Bibel and Ezell extended the elastic interaction coefficient method proposed by Van Campen, and pointed out that the elastic interaction is repetitive and seems to be predictable[1,2]. Coria et al. further verified the effectiveness of the elastic interaction coefficient method in optimizing the fastening sequence of assembly bolts of ring structures through experimentation[3]. Nassar and Alkelani analyzed the influence of gasket material and thickness, bolt spacing, preload level, bolt clamping
length, tightening order and number of times on preload distribution[4]. The results show that the simultaneous fastening strategy can effectively solve the problem of uneven distribution of bolt pretension caused by elastic interaction. However, this method is expensive and requires a large operation space. Zhu et al. used 3d nonlinear finite element modelling and experimental methods to study the influence of elastic interaction during the tightening process of bolt and flange connections[5,6]. Based on the elastic interaction coefficient method and using the nonlinear finite element model, the tightening load of bolts under different number of tightening was determined, and the uniform pre-tightening force was finally obtained. Nassar et al. proposed a new method to study the elastic interaction of multi-bolt connected structures[7,8]. The influence of gasket thickness, bolt spacing, initial preload and tightening sequence on elastic interaction was investigated by using this method. Based on the research of Abasolo et al., Coria et al. verified the effectiveness of the four-parameter method by experiment and finite element method, extended it to the two-time fastening strategy, and expanded the application scope of the method[9-11]. By analyzing the stress field of a multi-bolt connection, Wang et al. established an analytical model of elastic interaction stiffness[12]. Li et al. extended the work of Wang et al. and established the pre-tightening force distribution prediction model for linear and ring bolts[13].

At present, the elastic interaction coefficient, reverse sequence, four-parameter and elastic interaction stiffness/flexibility methods are mainly used to evaluate the preload distribution of bolt-flange connections. The optimal theoretical model for its initial preload has not yet been established. In this paper, based on the Markov theory, a method for uniform distribution of preload in a bolt-flange connection carried out in tightening sequence of diagonal is proposed. The finite element model of a bolt-flange connection with real thread characteristics is established. Based on the relationship between torque and preload, the optimized torque of each bolt is obtained. Finally, the torque loading process is simulated, the bolt-flange connection pre-tightening force distribution is analyzed, and the effectiveness of the proposed method is verified.

2. Optimization method of preload distribution

2.1. Transfer matrix model

The number of bolts in a bolt-flange connection is generally an even number. In engineering practice and specification, the bolts are generally tightened in diagonal sequence.

For the convenience of description, the state parameter \( S_i \) is defined during the fastening of bolted-flange connection, which describes the state of tightening the i-th bolt. In other words, the bolt-flange connection is in the state of \( S_{N-1} \) after bolt \( N-1 \) is tightened. The state of the bolted-flange connection after tightening screw \( N \) is \( S_N \). The number of bolts shall not be less than 2, that is, \( N \geq 2 \). It's not hard to see that state \( S_N \) is only related to state \( S_{N-1} \), not the front \( N-2 \) states. In addition, when the torque method controls the bolt pre-tightening force, it inevitably introduces method error, assembly geometric deviation and other uncertain factors, resulting in differences in the state \( S_N \) under the same initial tightening torque. The above analysis shows that the bolt-flange connection conforms to the basic principle of the Markov theory. However, the difficulty is the construction of a transfer matrix model.

Figure 1 shows the circular state transfer in the Markov process of the pre-tightening force of a flange connection with \( N \) bolts. The change in the pre-tightening force of the \( N-1 \) bolts fastened in the state \( S_N \) is due to the elastic interaction between the bolts. Therefore, a change in the pre-tightening force of the fastened bolt is considered as a transfer of state \( S_{N-1} \).

If the initial tightening torque of the bolt-flange connection is \( M \), the pre-tightening force under its control is \( F_0 = M / kd \). \( k \) is the nut factor, which is assumed to be 0.2. \( d \) is the nominal diameter of the bolt. The bolts are tightened using the digital torque wrench. The ring-type pressure sensor is applied to detect the bolt preload in the state \( S_N \). An experimental platform was built. A digital torque wrench with an accuracy of \( \pm 2\% \) was applied to tighten the bolts, which was carried out in tightening sequence of diagonal. The pre-tightening force of each bolt was monitored. The same experiment was
performed twice, and the results are shown in table 1. The preload transfer amount of the bolt-flange connection in different states is denoted as $x_{ij}$, and the Markov state transfer amount constructed is shown in table 2.

![Figure 1. Transfer of preload for bolted-flange connections.](image)

**Table 1. Experimental values of each bolt preload in final state.**

| Number of experiments | Bolt 1 | Bolt 2 | Bolt 3 | Bolt 4 | ...... | Bolt N |
|-----------------------|--------|--------|--------|--------|-------|--------|
| $F^{(0)}$ in Experiment 1 | $F_1$ | $F_2$ | $F_3$ | $F_4$ | ...... | $F_N$ |
| $F^{(1)}$ in Experiment 2 | $F_{11}$ | $F_{12}$ | $F_{13}$ | $F_{14}$ | ...... | $F_{1N}$ |

**Table 2. State transition matrix.**

| Current state | $S_1$ | $S_2$ | $S_3$ | ...... | $S_N$ | Supply |
|---------------|-------|-------|-------|-------|-------|--------|
| $S_1$         | $x_{11}$ | $x_{12}$ | $x_{13}$ | ...... | $x_{1,N}$ | $F_1$    |
| $S_2$         | $x_{21}$ | $x_{22}$ | $x_{23}$ | ...... | $x_{2,N}$ | $F_2$    |
| $S_3$         | $x_{31}$ | $x_{32}$ | $x_{33}$ | ...... | $x_{3,N}$ | $F_3$    |
| ......         | ...... | ...... | ...... | ...... | ...... | ...... |
| $S_{N-1}$     | $x_{N-1,1}$ | $x_{N-1,2}$ | $x_{N-1,3}$ | ...... | $x_{N-1,N}$ | $F_{N-1}$ |
| $S_N$         | $x_{N,1}$ | $x_{N,2}$ | $x_{N,3}$ | ...... | $x_{N,N}$ | $F_N$    |
| Demand        | $F_{11}$ | $F_{12}$ | $F_{13}$ | ...... | $F_{1N}$ | --      |

In the process of elastic interaction, the pre-tightening force of the bolt increases or decreases. The increase in the pre-tightening force is defined as the input, and the decrease in the pre-tightening force is defined as the output. The increase of the preloading force of each bolt means that the preload of the other bolts is reduced or unchanged, so the output is taken as the analysis quantity here. The output of pre-tightening force should be less than the supply, and the input of pre-tightening force should be greater than the demand. Excessive elastic interaction is likely to cause excessive warpage of the connection in bolt tightening. Therefore, it is better to keep the total transfer amount of pre-tightening force caused by the elastic interaction to a minimum, which could enhance the stability of the connection. It’s assumed that the sum function of the preload transfer amount is $z$. The solution model of the state transition matrix representing the elastic interaction between bolts is established as:

$$
\begin{align*}
\text{min} & \quad z = \sum_{i,j} x_{ij} \\
\text{s.t.} & \quad x_{11} + x_{12} + x_{13} + x_{14} + x_{15} + \cdots + x_{1,N-1} + x_{1N} \leq F_1 \\
& \quad x_{i1} + x_{i2} + x_{i3} + x_{i4} + x_{i5} + \cdots + x_{iN-1} + x_{iN} \geq F_i \\
& \quad x_{ij} \geq 0, \quad i, j = 1, 2, 3, \cdots, N.
\end{align*}
$$

(1)
The transition probability matrix between states is denoted as:

\[
P = \begin{bmatrix}
\sum_{j=1,3,\cdots,N} x_{1j} & \sum_{j=1,3,\cdots,N} x_{1j} & \cdots & \sum_{j=1,3,\cdots,N} x_{1j} \\
\sum_{j=1,3,\cdots,N} x_{2j} & \sum_{j=1,3,\cdots,N} x_{2j} & \cdots & \sum_{j=1,3,\cdots,N} x_{2j} \\
\vdots & \vdots & \ddots & \vdots \\
\sum_{j=1,3,\cdots,N} x_{nj} & \sum_{j=1,3,\cdots,N} x_{nj} & \cdots & \sum_{j=1,3,\cdots,N} x_{nj}
\end{bmatrix}
\]  

(2)

2.2. Acquisition of initial tightening torque

Based on the above analysis, in the bolt-flange connection, after tightening all the bolts, the optimization model of the initial preload of each bolt is:

\[
\begin{align*}
F^{(2)} &= F^{(1)} P, \\
& \quad \cdots \\
F^{(m)} &= F^{(1)} P^{m-1}.
\end{align*}
\]  

(3)

The initial preload of a single bolt is calculated by empirical formula \( F_0 = (0.5~0.6) \sigma_w A \). Let the initial pre-tightening force of multiple bolts be \( F_0 = [F_{01} \ F_{02} \ F_{03} \ \ldots \ F_{0N}] \), and the initial tightening torque be \( M_0 \). Through \( m \) experiments, it is known from the preload optimization model that the experiments are grouped in pairs to deduce the state transition probability matrix \( P_1 (l=1, 2, \ldots, N) \) when \( m=2N \). When \( m=2N+1 \), the experiments are grouped according to the number of combinations \( C(m,2) \), and the probability matrix \( P_l (l=1, 2, \ldots, C_m) \) of state transition is derived. The average column value of the transition probability matrix is the probability vector \( P_{\text{final}} \) of the initial preload on each bolt. Then, the optimization value of the corresponding bolt pre-tightening force is respectively:

\[
F^{(w)}_{0j} \mid_{m=2l-1,2l} = [F_{001} \ F_{002} \ F_{003} \ \ldots \ F_{00N}]^T = F_0^T P^{m-1}.
\]  

(4)

where, \( F_{00j} \) is the pre-tightening force of No. \( j \) \( j=1, 2, 3, \ldots, N \). \( T \) is the symbol of matrix or vector transpose. This equation is used to calculate the optimal value of bolt pre-tightening force corresponding to each experiment.

During the tightening process of the bolt-flange connection, each bolt is usually tightened with the same actual preload within the appropriate range. In this case, according to the above optimization analysis, the actual initial preload applied to the group bolts is:

\[
F_{\text{bd}} = [F_0^T (I - P_{\text{final}})]^T.
\]  

(5)

In the Equation (5), \( I = [1 \ 1 \ 1 \ 1 \ 1] \).

The elastic interaction between bolts is considered and the initial preload is calculated with the output as the analysis quantity. On the other hand, the actual tightening should be based on the original tightening torque \( M_0 \), and then input the tightening torque of the same proportion as \( P_{\text{final}} \) to make the initial pre-tightening force more evenly distributed. Combined with the torque-preload relation, the initial tightening moment of bolts in practical engineering is obtained as:

\[
M_{0d} = M_0 + k[F_0^T P_{\text{final}}]^T d.
\]  

(6)
3. Finite element analysis and discussion based on experimental data

3.1. Optimization results of preload

The design criterion specifies the functional relationship between the pretension of bolts of different materials, yield strength and cross section area, as shown in equation (7). The yield strength of the stainless-steel bolt is \( \sigma_s = 450 \text{N/mm}^2 \), and the section area is \( A = 157 \text{mm}^2 \). Thus, the range of the bolt preload is

\[
F_0 = (0.5 \sim 0.6) \sigma_s A = (35.325 \sim 42.390) \text{kN}
\]

The required tightening torque range of the bolts is

\[
M = kF_0d = (113.04 \sim 135.648) \text{N} \cdot \text{m}
\]

Flange samples of the 4 bolts were processed. Bolts were made of A2-70 stainless steel, M16×2 in specification. The initial tightening torque of the design bolt is 125Nm. The digital torque wrench with accuracy of +2% was used to tighten the bolts, which was carried out in tightening sequence of diagonal. The experiment was conducted twice, and the pre-tightening force distribution of the bolted-flange connection after fastening is shown in figure 2. This state is denoted as \( S_4 \).

Based on the above experimental values of bolt preload, a mathematical optimization model of state transition matrix is established as:

\[
\begin{align*}
\begin{array}{cccc}
11 & 12 & 13 & 14 \\
21 & 22 & 23 & 24 \\
31 & 32 & 33 & 34 \\
41 & 42 & 43 & 44 \\
\end{array}
\begin{array}{cccc}
x_{11} + x_{12} + x_{13} + x_{14} & \leq 17.6 \\
x_{21} + x_{22} + x_{23} + x_{24} & \leq 23.55 \\
x_{31} + x_{32} + x_{33} + x_{34} & \leq 41.4 \\
x_{41} + x_{42} + x_{43} + x_{44} & \leq 40.7 \\
11 & 21 & 31 & 41 \\
12 & 22 & 32 & 42 \\
13 & 23 & 33 & 43 \\
14 & 24 & 34 & 44 \\
s.t. x_{11} + x_{21} + x_{31} + x_{41} & \geq 31.15 \\
x_{12} + x_{22} + x_{32} + x_{42} & \geq 27.6 \\
x_{13} + x_{23} + x_{33} + x_{43} & \geq 32.6 \\
x_{14} + x_{24} + x_{34} + x_{44} & \geq 21.68 \\
x_{ij} & \geq 0, i, j = 1,2,3,4.
\end{array}
\end{align*}
\]

Using equation (9), the state transition matrix \( T \) and transition probability matrix \( P \) between states of the Markov chain are calculated as:
Based on equation (5), the actual initial preload of the four bolts is

\[ F_{\text{bd}} = [28.254525 \ 29.475225 \ 27.757275 \ 31.512975]. \]  \hspace{1cm} (12)

Therefore, according to equation (6), the actual initial tightening torque of the 4 bolts in the actual project should be

\[ M_{\text{bd}} = [159.38552 \ 155.47928 \ 160.97672 \ 148.95848]. \]  \hspace{1cm} (13)

3.2. Finite element analysis

Based on the parametric modelling method, the bolted flange connection finite element model with real thread characteristics was established, as shown in figure 3. The torque method is simulated to control the initial preload of the bolts.

The pre-tightening force distribution on the contact surface of the connected parts was analyzed, and the material properties of the connected parts were set to alloy steel. The bolts and nuts were made of 304 stainless steel. The parameters of material property were set in the finite element model. The torque method was simulated to control the bolt preload. The tightening sequence of diagonal was applied to bolted flange connections. HyperMesh was used to divide fine mesh. The finite element model is shown in figure 3. The contact type of mating threads between bolts and nuts was set to bonded. The others were set to frictional.

In order to verify the feasibility of the proposed optimization method, the initial tightening torques of the two groups are set: the tightening torques are the same, which are 125Nm; Tightening torques are the result in equation (11). The stress distribution around the bolt hole was taken as the evaluation index, and the results of the finite element analysis are shown in figure 4 and figure 5. By comparison, two results were reported:

(1) Before optimization, there is one stress concentration area on the bearing surface of the No. 3 bolt hole and three stress concentration areas on the bearing surface of the No. 4 bolt hole. After
optimization, the stress concentration area on the bearing surface of No. 3 bolt hole disappeared, and the stress on the bearing surface of No. 4 bolt hole decreased from $1.835 \times 10^8$Pa to $1.483 \times 10^8$Pa.

(2) The dispersion of stress distribution on the bearing surface of the bolted-flange connection decreased by about 19.18%.

![Figure 4](image_url)

**Figure 4.** Stress distribution on bearing surfaces before optimization (a) overall distribution, (b) bolt 1, (c) bolt 2, (d) bolt 3, (e) bolt 4.

![Figure 5](image_url)

**Figure 5.** Optimized stress distribution on bearing surfaces (a) overall distribution, (b) bolt 1, (c) bolt 2, (d) bolt 3, (e) bolt 4.
Nine nodes are selected in the stress concentration area of the bolt 4. Node 1, node 2, and node 3 are in Area 1, node 4, node 5, and node 6 are in Area 2, and node 7, node 8, and node 9 are in Area 3. The stress values corresponding to the existing method and the optimized method are collected, as shown in figure 6. Compared the stress of the two methods, the maximum stress of the three regions shows a decreasing trend. It indicates that the optimization method can make the overall stress distribution develop in the direction of a certain equilibrium position, that is, it can effectively suppress the spread of stress concentration.

![Figure 6](image_url)

**Figure 6.** Analysis for stress concentration (a) the stress concentration area of the bolt 4, (b) nodal stress of existing method, (c) nodal stress of optimized method.

### 3.3. Discussion

In fact, in view of factors such as tightening speed, tightening torque, tightening sequence, the elastic interaction between bolts occurs during the pre-tightening process, so the experimental results have great random uncertainties. Due to the elastic interaction, the pre-tightening force of some bolts may be lower than the target preload and the stress distribution around the bolt hole is uneven. Therefore, this paper proposes a method of torque optimization considering the experimental uncertainty, so that the stress distribution around the bolt hole is more uniform.

In order to further verify the effectiveness of the model, two groups of experiments were carried out. The tightening torque of each bolt in group 1 is the same as that in experiment 1 and 2. The tightening torque of each bolt in group 2 is set according to the calculated result (13). Eight experiments were conducted in each group. The experimental results are shown in table 3 and table 4.
According to the data in above tables, the pre-tightening force distribution is shown in figure 7. The overall distribution of the bolt pre-tightening force before optimization is relatively dispersed. Especially, the bolt pre-tightening force has the largest fluctuation in the first four experiments, while the bolt pre-tightening force is relatively stable in the last six experiments. Obviously, the dispersibility of the bolt pre-tightening force of the first four experiments has been improved by the optimization method. In addition, the results in figure 7(b) show that the pre-tightening forces of bolt 2 and bolt 4 have similar changing trends, while the pre-tightening forces of bolt 1 and bolt 3 have similar changing trends. According to this rule, the bolts were tightened, the distribution of the bolt pre-tightening force is more uniform, and the degree of dispersion is smaller.

Table 3. Bolt preload of existing method /kN.

| Number of experiments | Bolt 1 | Bolt 2 | Bolt 3 | Bolt 4 |
|-----------------------|--------|--------|--------|--------|
| 3                     | 29.86  | 37.64  | 34.77  | 38.83  |
| 4                     | 40.23  | 41.15  | 21.75  | 40.61  |
| 5                     | 32.22  | 29.49  | 34.36  | 34.36  |
| 6                     | 33.59  | 21.46  | 34.47  | 27.28  |
| 7                     | 31.9   | 28.18  | 32.36  | 32     |
| 8                     | 30.7   | 22.2   | 30.56  | 26.13  |
| 9                     | 30.21  | 37.08  | 24.9   | 30.36  |
| 10                    | 29.01  | 37.78  | 31.28  | 34.57  |

Table 4. Optimized bolt preload /kN.

| Number of experiments | Bolt 1      | Bolt 2      | Bolt 3      | Bolt 4      |
|-----------------------|-------------|-------------|-------------|-------------|
| 3                     | 29.7844     | 37.6813     | 34.7666     | 38.8941     |
| 4                     | 30.3418     | 38.3864     | 35.4171     | 39.6219     |
| 5                     | 37.5018     | 23.9727     | 38.4857     | 30.4698     |
| 6                     | 33.5828     | 21.4676     | 34.4639     | 27.2857     |
| 7                     | 34.8649     | 25.2118     | 34.7086     | 29.6788     |
| 8                     | 30.7044     | 22.2032     | 30.5667     | 26.1372     |
| 9                     | 30.2105     | 37.0753     | 24.9012     | 30.3631     |
| 10                    | 32.6978     | 40.1278     | 26.9514     | 32.8630     |

In terms of stability, the change of the difference between the maximum and the minimum preload in the optimized method is better than the result of existing method, as shown in figure 8. Analyzing
from figure 4 to figure 6 and figure 8, a phenomenon is shown that the optimization method is more stable than the existing method. In other words, the optimization method is feasible.

4. Conclusion
The elastic interaction between bolts is one of the key factors leading to the non-uniform distribution of preload during the assembly of the bolt-flange connection. Therefore, the optimization method has been studied, and the relevant conclusions are as follows:

1) Based on the Markov theory, an optimization method for preload distribution of bolted flange connection is proposed. A transfer matrix construction method based on experimental data is proposed.

2) The finite element model of the bolted-flange connection with real thread characteristics was established to verify the feasibility and effectiveness of the proposed method.

3) After optimization, the stress distribution on the bearing surface is relatively uniform, with the maximum concentrated stress reduced by 35.2Mpa and the dispersion degree of stress distribution reduced by about 19.18%.

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Appendices
The nomenclature in this article is as follows:

\( M \) Ideal initial tightening torque (N·m)

\( F \) Ideal preload of bolts (kN)

\( F^{(k)} \) Bolt preload before optimization (kN)

\( F_{00}^{(w)} \) Optimized bolt preload (kN)

\( F_0d \) Actual initial preload (kN)

\( M_0d \) Actual initial tightening torque (N·m)

\( P \) State transition probability matrix

\( P_{final} \) Probability vector

\( d \) Nominal diameter of bolt (mm)

\( k \) Nut factor

\( N \) Bolt number

\( S_N \) System state of tightened bolt \( N \)

\( T \) State transition matrix

\( T \) The symbol of matrix or vector transpose

\( I \) Unit vector

\( m \) Number of experiments
\( A \)  
Section area (mm\(^2\))

\( \sigma_s \)  
Yield strength (N/mm\(^2\))

\( x_{ij} \)  
Preload transfer amount from bolt \( i \) to bolt \( j \)

\( l \)  
Number of state transition probability matrix

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