How to check the one-count operator experimentally

A. V. Dodonov,† S. S. Mizrahi,‡ and V. V. Dodonov

Departamento de Física, CCET, Universidade Federal de São Carlos, Via Washington Luiz km 235, 13565-905, São Carlos, São Paulo, Brazil

Instituto de Física, Universidade de Brasília, PO Box 04455, 70910-900, Brasília, Distrito Federal, Brazil

(Dated: January 5, 2022)

We propose an experimental scheme to probe the form of one-count operation used in the theory of continuous photodetection in cavities. Two main steps are: 1) an absorption of a single photon by an atom passing through a high-Q cavity containing electromagnetic field in a thermal or coherent state, 2) a subsequent measurement of the photon statistics in the new field state arising after the photon absorption. Then comparing the probabilities of finding 0 and 1 photons in the initial and final states of the field, one can make conclusions on the form of the one-count operation. This method can be readily applied in the microwave cavity QED with present technology.

PACS numbers: 03.65.Ta, 42.50.Ar, 42.50.Lc

It is well known [1] that the probability of absorbing one photon per unit time from a quantized electromagnetic field is proportional to the average value of the ordered product of the negative and positive frequency electric field operators in the given quantum state of the field. In the simplest case of the single-mode field, this probability can be written in terms of the standard bosonic lowering and raising operators ˆa and ˆa†, satisfying the commutation relation [ˆa, ˆa†] = 1, as

\[ p_{ab} = \gamma \text{Tr} (\hat{a} \hat{p}_i \hat{a}^\dagger) \]  

where ˆp_i is the statistical operator of the field before absorption and \( \gamma \) is a coefficient with the dimensionality s^{-1}. Due to an interaction with a ‘detector’ (which absorbs a photon), the field makes a ‘quantum jump’ to a new state, which can be described mathematically by an action of the one-count operator (OCO) ˆJ as

\[ \hat{\rho}_f = \hat{J} \hat{\rho}_i / \text{Tr}(\hat{J} \hat{\rho}_i) \]  

where ˆ\rho_f is the statistical operator of the field immediately after the absorption of one photon. Operator \( \hat{J} \) is frequently called also quantum jump superoperator (QJS). However, this term is usually associated with random processes and the so called ‘quantum trajectories approach’ (see e.g. [2, 4]). In order to avoid a confusion, we shall use the term OCO throughout the paper.

The hermiticity of operator \( \hat{\rho}_f \) can be ensured if one uses the decomposition

\[ \hat{J} \hat{\rho} = \hat{O} \hat{\rho} \hat{O}^\dagger \]  

where \( \hat{O} \) is some ‘lowering’ operator responsible for the subtraction of one photon from the field. Obviously, the explicit form of operators ˆJ or ˆO depends on the details of the interaction between the field and a detector, and concrete calculations based on different models were performed by many authors since the 1960s [3, 6, 7, 8] (other references can be found in [9]). A very common form of OCO, first proposed in [2] and considered for applications in quantum-counting quantum nondemolition (QND) measurements in [10], consists in the identification \( \hat{O} = \hat{a} \) (we shall refer to it as ‘A-model’):

\[ \hat{J}^A \hat{\rho}_i = \hat{a} \hat{\rho}_i \hat{a}^\dagger \]  

Such a form seems quite natural, if not obvious, in view of equation (11). However, we would like to emphasize that this choice is, as a matter of fact, intuitive (phenomenological), although it can be derived from some ‘microscopical’ models under certain assumptions [7, 11], where the most important are the weak coupling and short interaction time limits. Nonetheless, if these assumptions are replaced by others, one can obtain different operators \( \hat{J} \). In particular, the OCO \( \hat{J}^n \hat{\rho}_i = \hat{a}^\dagger \hat{\rho}_i \hat{a}^n \), where \( n \equiv \hat{a}^\dagger \hat{a} \) is the photon number operator, was considered in [12] in connection with continuous quantum nondemolition measurements of photon number. A family of OCO based on the ‘nonlinear lowering operators’ of the form \( \hat{J} = (1 + \hat{n})^{-\beta} \) was derived in Ref. [11]. Its special case with \( \beta = 1/2 \) corresponds to the so-called ‘E-model’, which was proposed within the frameworks of phenomenological considerations in [13, 14]:

\[ \hat{J}^E \hat{\rho}_i = \hat{E}^- \hat{\rho}_i \hat{E}^+, \quad \hat{E}^- \equiv (1 + \hat{n})^{-1/2} \hat{a}. \]  

The operator \( \hat{E}^- \) is known under the name ‘exponential phase operator’ [13, 16, 17, 18].

In some special cases, e.g., if a detector is a resonant two-level atom passing through a cavity, one can deduce an exact form of the one-count operator, using some known atom–field interaction Hamiltonian. Indeed, if one can describe the interaction by means of the Jaynes–Cummings model, then the exact form of the OCO is [19]

\[ \hat{J}^H \hat{\rho}_i = \sin(y\sqrt{n} + 1) \hat{E}^- \hat{\rho}_i \hat{E}^+ \sin(y\sqrt{n} + 1) \]  

\[ \hat{J}^H \rho_i = \sin(y\sqrt{n} + 1) \hat{E}^- \rho_i \hat{E}^+ \sin(y\sqrt{n} + 1) \]
where \( y = gt \), \( t \) being the atom transit time through the cavity and \( g \) the atom-field coupling constant related to the Rabi frequency. We shall call operator (14) the ‘H-model’. Common values in cavity QED in the microwave regime are (see, e.g., table I in [20]): \( g \sim 100 \text{kHz}, t \sim 100 \mu \text{s}, \) so \( y \sim 1 - 10 \). Notice that the exact OCO (1) is a bounded superoperator, as expected from physical point of view.

Although the OCO in the form (1) was used ad hoc for more than three decades in numerous papers devoted to different applications, it seems that its validity was never verified in direct experiments. However, such a verification cannot be considered as unnecessary for several reasons. First, it is possible that in some realistic situations, the approximations under which the phenomenological operator (1) was derived can fail. Second, since \( J^A \) is an unbounded operator, some inconsistencies in the theoretical treatment appear (they were noticed already in the original paper [2]; see also \([14, 19, 21]\)). Third, applying (1) to some states, one arrives at predictions which look counterintuitive, thus deserving an experimental verification.

For example, it is easy to check that if the mean number of photons in the state \( \hat{\rho}_f \) (before the detection of one photon) was \( \langle \hat{n} \rangle_i \), then the mean number of photons in the state \( \hat{\rho}_f \) with operator (1) must be

\[
\langle \hat{n} \rangle_f = \langle \hat{n}^2 \rangle_i / \langle \hat{n} \rangle_i - 1 = \langle \hat{n} \rangle_i + Q
\]

where \( Q \) is the known Mandel’s Q-factor describing the type of photon statistics in the initial state \( \hat{\rho}_i \). Only for the initial Fock states one has \( \langle \hat{n} \rangle_f = \langle \hat{n} \rangle_i - 1 \), whereas equation (1) yields \( \langle \hat{n} \rangle_f = 2 \langle \hat{n} \rangle_i \) for the initial thermal state and \( \langle \hat{n} \rangle_f > 2 \langle \hat{n} \rangle_i \) for the initial squeezed vacuum state. In contrast, using OCO in the form (1) one obtains instead of (17) the formula

\[
\langle \hat{n} \rangle_f = \frac{\langle \hat{n} \rangle_i}{1 - \chi_0} - 1, \quad \chi_0 \equiv \langle 0 | \hat{\rho}_i | 0 \rangle
\]

where \( \chi_0 \) is the probability of occupation of the vacuum state in the initial state \( \hat{\rho}_i \). In particular, for the thermal state equation (8) yields \( \langle \hat{n} \rangle_f = \langle \hat{n} \rangle_i \).

The aim of this article is to show how the form of the OCO can be verified by detecting single photons in high-Q cavities (where one can use the single-mode approximation for the quantized electromagnetic field). We are inspired by the recent progress in experiments described in [23]. The scheme that we propose employs both destructive and nondemolition measurements, that can be realized with the present available technology [23, 26].

In quantum nondemolition experiments realized recently (based on a proposal made in [27]), the Rydberg atoms, initially prepared in the ground state \( |q \rangle \) of an effective two-level configuration, were sent through an interferometer composed of a high-Q cavity (with the damping time \( \sim 0.1 \text{s} \)) and resonant classical fields. On the exit they were detected by a state selective field ionization detector. Besides, the experiments were performed under the conditions where the mean number of photons in the cavity was much smaller than unity. In such a case, due to the nondemolition nature of measurements (because the cavity field eigenfrequency is chosen in such a way that the atomic transitions are out of resonance with the field), if the atom is detected in the excited state \( |e \rangle \), then one may conclude that there is only one photon in the cavity, so the field state within the cavity is projected into the 1-photon state. Similarly, if the atom is detected in the state \( |g \rangle \), this means that there are no photons in the cavity, and the field state is the vacuum state. If one sends more atoms through the cavity, the outcomes of the measurements will be the same and the state within the cavity will not be altered. In rare cases when there is more than 1 photon in the cavity, the atom will be in a superposition of states \( |g \rangle \) and \( |e \rangle \) after passing through the cavity, so in consecutive measurements the outcome will not be always the same, but will alternate probabilistically between \( |g \rangle \) and \( |e \rangle \). Thus, using consecutive nondemolition measurements, an experimenter can distinguish between 0, 1 and more than 1 photon in the cavity.

Our experimental proposal is based on the assumption that one can prepare a field state \( \hat{\rho}_f \) in the cavity with known statistical properties. Actually, we have in mind either a thermal or a coherent state with a small mean photon number \( \langle n \rangle_i < 5 \), in order to ensure a negligibly small influence of multiphoton Fock states. The methods of preparation of such ‘classical’ states seem to be well known. (Note that the Fock states themselves cannot distinguish between the OCO’s – one needs superpositions or mixtures of these states.) If the nature of the state is known, then it can be characterized by measuring the ensemble probabilities \( \chi_0 \) and \( \chi_1 \) of having initially 0 and 1 photons. So, the first step of the experiment is the QND measurement of the photon statistics in the initial state. After this, one should send through the cavity an atom in the ground state of another effective two-level configuration, tuned in resonance with the cavity mode (e.g., using Rydberg atoms, whose quantum states are different from those used in the first step), in order to change the quantum state of the field due to the absorption of one photon. If the atom absorbs a photon (which is signaled by a detection of atom in the excited state), this means that the field state makes a quantum jump to the state \( \hat{\rho}_f \), whose statistical properties are determined by the form of OCO \( \hat{J} \). Consequently, measuring the probabilities \( P_n = \langle n | \hat{\rho}_f | n \rangle \) of finding \( n \) photons in the state \( \hat{\rho}_f \) after the quantum jump and comparing the results with theoretical predictions, one can verify the form of \( \hat{J} \). It is sufficient to measure only the probabilities \( P_0 \) and \( P_1 \).

The predictions for the A-model are as follows,

\[
P^A_n = \frac{\langle n | \hat{a} \hat{a} \hat{\rho}_f | n \rangle}{\langle \hat{n} \rangle_i} \frac{(n + 1)!}{\langle \hat{n} \rangle_i} \chi_{n+1},
\]

where \( \chi_n = \langle n | \hat{\rho}_f | n \rangle \). Analogously, for the E-model we
have

\[ P_n^E = \frac{\chi_{n+1}}{1-\chi_0} \quad (10) \]

and for the H-model

\[ P_n^H = \frac{\sin^2(y\sqrt{n+1})\chi_{n+1}}{(\sin^2(y\sqrt{n}))_i}. \quad (11) \]

Thus, we see that the resulting probabilities are fundamentally different. Let us illustrate these different behaviors for the A- and E-models for two different initial states (for H-model the expressions are more lengthy, so we do not put them here).

(a) For the thermal state (which is an eigenstate of superoperator \( \hat{J}^E \)) with the mean photon number \( \bar{n} \) we have

\[ \chi_n = \frac{\bar{n}^n}{(n+1)n+1} = \chi_0 (1-\chi_0)^n \quad (12) \]

so we obtain \( P_n^E = \chi_n \),

\[ P_0^A = \chi_0^2, \quad P_1^A = 2\chi_0^2 (1-\chi_0). \]

(b) For the coherent state (an eigenstate of \( \hat{J}^A \)) with

\[ \chi_n = e^{-\bar{n}}\frac{\bar{n}^n}{n!} = \chi_0 \frac{(-\ln \chi_0)^n}{n!} \]

we have \( P_n^A = \chi_n \),

\[ P_0^E = \frac{\chi_0(-\ln \chi_0)}{1-\chi_0}, \quad P_1^E = \frac{\chi_0(-\ln \chi_0)^2}{2(1-\chi_0)} \]

We see that \( P_1^E \) is twice smaller than \( P_1^A \) if \( 1-\chi_0 \ll 1 \), for both initial coherent and thermal quantum states.

In figures 1 and 2 we plot \( P_0 \) and \( P_1 \) as function of \( \chi_0 \) for A and E models and the both states. In figure 3 we plot the same probabilities as functions of \( \chi_1 \) for the initial thermal state (in the case of \( \bar{n} < 5 \)). We choose \( \chi_0 \) and \( \chi_1 \) as possible independent variables, because these quantities can be determined experimentally in the most direct way. Two branches in figure 3 are the consequence of two signs in the dependence \( \chi_0(\chi_1) \): solving equation

**FIG. 1:** Probabilities of finding 0 and 1 photons after the quantum jump from the initial coherent state, characterized by the initial probability of having zero photons \( \chi_0 \). The superscripts A and E correspond to predictions of A-model and E-model, respectively.

**FIG. 2:** The same as in figure 1 but for the initial thermal state.

**FIG. 3:** The same as in figure 2 but as functions of the probability \( \chi_1 \).
The upper sign should be chosen if \( n < 0 \) for the set-up described in [25, 26, 27, 28] varied from 0 to 0.7. In the case of initial thermal states, the values of \( \chi \) and \( \eta \) can be varied by changing the temperature of the cavity or by some other means [25]. Before passing any atom, the mean number of (initial) thermal photons in the set-up described in [25] varied from 0.7 to 1. This range of temperatures corresponds to the variations of \( \chi_0 \) from 0.6 to 0.9 and \( \chi_1 \) from 0.24 to 0.09. Figures [2] and [3] show that these are just the intervals where the functions \( P^H_n(\chi_0) \) and \( P^E_n(\chi_0) \) are quite distinguishable from each other (\( k, j = 0, 1 \)). Moreover, for \( \chi_1 = 0.1 \), the probability of detecting more than one photon becomes less than 0.01, and the scheme described in [25] is quite reliable.

In the H-model the OCO depends on the parameter \( y \), i.e., the atom transit time. Thus, the resulting probabilities \( P^H_n \) oscillate as functions of transit time, attaining zero values for certain values of \( y \). In figure 1, we plot functions \( P^H_n(y) \) and \( P^H_1(y) \) for the thermal state with \( \chi_0 = 0.6 \) and \( y \) ranging from 1 to 10, corresponding to achievable values in microwave cavity QED experiments. Such a peculiar behavior of probabilities as functions of the transit time could also be checked experimentally. Consequently, by performing ensemble experiments in an accessible interval of temperatures one can easily verify which one of the OCO’s holds, or whether neither of them is observed in practice.

Concluding, we are proposing a simple scheme of an experiment, which could decide in an unambiguous way the form of the one-count operator. This scheme only needs a cavity with initial thermal or coherent state of the electromagnetic field containing a small mean number of photons. The available experimental level seems to be quite sufficient for this purpose. This method can also be applied to other physical systems in which one can perform both destructive and QND (or instantaneous destructive photon number) measurements.

AVD and SSM acknowledge financial support from FAPESP (SP, Brazil, Contract No. 04/13705-3). SSM and VVD acknowledge partial financial support from CNPq (DF, Brazil).

---

**References**

[1] Glauber R J 1963 Phys. Rev. 130 2529

[2] Srinivas M D and Davies E B 1981 Opt. Acta 28 981

[3] Carmichael H J 1997 Phys. Rev. A 56 5065

[4] Plenio M B and Knight P L 1998 Rev. Mod. Phys. 70 101

[5] Kelley P L and Kleiner W H 1964 Phys. Rev. 136 A316

[6] Mollow B R 1968 Phys. Rev. 168 1896

[7] Imoto N, Ueda M and Ogawa T 1990 Phys. Rev. A 41 4127

[8] Kelley P L and Kleiner W H 1964 Phys. Rev. 136 A316

[9] Peˇ rinov´ a V and Lukˇ s A 2000 Progress in Optics 40 ed E Wolf (Amsterdam: Elsevier) p 115

[10] Milburn G J and Walls D F 1984 Phys. Rev. A 30 56

[11] Dodonov A V, Mizrahi S S and Dodonov V V 2005 Phys. Rev. A 72 023816

[12] Ueda M, Imoto N, Nagaoka H and Ogawa T 1992 Phys. Rev. A 46 2859

[13] Ben-Aryeh Y and Brif C 1995 Preprint quant-ph/9504009 (unpublished)

[14] de Oliveira M C, Mizrahi S S and Dodonov V V 2003 J. Opt. B: Quantum. Semiclass. Opt. 5 S271

[15] Susskind L and Glogower J 1964 Physics 1 49

[16] Carruthers P and Nieto M 1968 Rev. Mod. Phys. 40 411

[17] Wünsche A 2001 J. Opt. B: Quantum. Semiclass. Opt. 3 206

[18] Kastrup H A 2003, Fortschr. Phys. 51 975

[19] Dodonov A V, Mizrahi S S and Dodonov V V 2005 J. Opt. B: Quantum. Semiclass. Opt. 7 99

[20] Blais A, Huang R-S, Wallraff A, Girvin S M and Schoelkopf R J 2004 Phys. Rev. A 69 062320

[21] Dodonov A V, Mizrahi S S and Dodonov V V 2006 Phys. Rev. A 74 033823

[22] Ueda M, Imoto N and Ogawa T 1990 Phys. Rev. A 41 3891

[23] Lee C T 1993 Phys. Rev. A 48 2285

[24] Mizrahi S S and Dodonov V V 2002 J. Phys. A: Math. Gen. 35 8847

[25] Gleyzes S, Kuhr S, Gerlin C, Bernu J, Deléglise S, Hoff U B, Brune M, Raimond J-M and Haroche S 2007 Nature 446 297 Preprint quant-ph/0612031

[26] Raimond J-M, Brune M and Haroche S 2001 Rev. Mod. Opt. 446 297
Phys. 73 565
Brune M, Haroche S, Lefèvre V, Raimond J-M and Zagury N 1990 Phys. Rev. Lett. 65 976
Brune M, Haroche S, Raimond J-M, Davidovich L and Zagury N 1992 Phys. Rev. A 45 5193
Nogues G, Rauschenbeutel A, Osnaghi S, Brune M, Raimond J-M and Haroche S 1999 Nature 400 239