Amplification of a surface electromagnetic wave by running over plasma surface ultrarelativistic electron bunch as a new scheme for generation of Terahertz radiation

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The amplification of a surface electromagnetic wave by means of ultrarelativistic monoenergetic electron bunch running over the flat plasma surface in absence of a magnetic field is studied theoretically. It is shown that when the ratio of electron bunch number density to plasma electron number density multiplied by a powered to 5 relativity factor is much higher than 1, i.e. \( \gamma^5 n_b / n_p >> 1 \), the saturation field of the surface electromagnetic wave induced by trapping of bunch electrons gains the magnitude: \( E_x = B_y \approx 0.16 \omega_p mc_e ( \frac{\gamma^5 n_b}{n_p} )^{1/7} \) and does not approach the surface electromagnetic wave front breakdown threshold in plasma. The surface electromagnetic wave saturation energy density in plasma can exceed the electron bunch energy density. Here, we discuss the possibility of generation of superpower Terahertz radiation on a basis of such scheme.

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I. INTRODUCTION

The surface electromagnetic waves (SEW) on plasma surface and plasma-like media (gaseous plasma, dielectric and conducting media, etc.) attract special attention of researchers due to their unique properties. First of all, due to its high phase and group velocities close to light speed in vacuum at high media conductivity what makes them the most valuable in radiophysics. The SEW are widely applied in physical electronics due to its high phase velocity leading to its uncomplicated generation by relativistic electron bunches and output from plasma.

Below we discuss the problem of SEW amplification with the help of electron bunch running over flat plasma surface. We consider the case of ultrarelativistic monoenergetic electron bunch which remains relativistic in the frame of reference of SEW generated by this bunch compared to the works\(^2\),\(^3\), where the bunches were non-relativistic. Such a problem of generation of three-dimensional electromagnetic wave (wakefields) in plasma with the help of ultrarelativistic electron and ion bunches through Cherenkov resonance radiation was solved in\(^4\), where it was shown that bunch ultrarelativity influences significantly the nonlinear stage of plasma-bunch interaction, in particular, the saturation amplitude of the generated wave.

In the present work we apply the method developed for the case of amplification of a surface electromagnetic wave by means of ultrarelativistic monoenergetic electron bunch running over the flat plasma surface. The interest to the SEW amplification was aroused by its uncomplicated output from plasma compared to that of the three-dimensional wave generated by the bunch as well and high magnitudes of SEW energy density. The latter is related to the field structure. Thus, as it’ll be shown below, the SEW saturation energy density can exceed the bunch energy density.

It is noteworthy that the real SEW amplification device should be cylindrical what we do comprehend very well. However, the problem taking into account the cylindrical geometry is much more complex compared to that of plane geometry from the mathematical point of view and is not appropriate for illustrative purposes. This is why we restrict ourselves to the plane geometry problem. Soon, we are planning to finish an article considering the real cylindrical SEW bunch-plasma amplifier and will present it for publication.

II. DESCRIPTION OF THE MODEL. DISPERSION RELATION

Let us start our description with the schematic illustration of interaction of the ultrarelativistic monoenergetic electron bunch with cold isotropic plasma (no thermal...
motion) being in a rest, which generates the plane wave \( E = E_0 \exp(-i \omega t + i \vec{k} \cdot \vec{r}) \), and put the external field as absent.

Over the collisionless plasma, filling in the half-plane \( x < 0 \), with the dielectric permittivity

\[
\varepsilon_p = 1 - \frac{\omega_p^2}{\omega^2},
\]

the ultrarelativistic monoenergetic electron bunch, filling in the space \( x \geq a \), with the dielectric permittivity

\[
\varepsilon_b = 1 - \frac{\omega_b^2}{(\omega - k_z u)^2},
\]

propagates on a distance \( a \). Here \( \omega_p = \sqrt{4 \pi e^2 n_p/m} \), \( \omega_b = \sqrt{4 \pi e^2 n_b/m} \) are Langmuir plasma electron and bunch frequencies respectively (in GSU units) with \( n_p \), \( n_b \) being the plasma and bunch number densities in the laboratory frame of reference (plasma in a rest) \( n_b \ll n_p \), \( k_z \) is the longitudinal (directed along the velocity of the bunch \( \vec{u} \)) component of the SEW wave vector \( \vec{k} = (k_x, 0, k_z) \), \( e \) = the electron charge, \( m \) = its mass. The bunch is considered to be an ultrarelativistic when

\[ \gamma = \frac{1}{\sqrt{1 - u^2/c^2}} \gg 1. \] (#3)

The surface wave is a wave of \( E \)-type with the nonzero field components \( E_x, E_z, B_y \), which satisfy the following system of equations:

\[ \frac{\partial^2 E_z}{\partial x^2} - k_z^2 E_z + \frac{\omega^2}{c^2} \varepsilon(x) E_z = 0 \]

\[ E_x = \frac{ik_z}{\kappa^2} \frac{\partial E_z}{\partial x}, \quad B_y = -\frac{i \omega}{\kappa x} \varepsilon(x) \frac{\partial E_z}{\partial x}, \]

where \( \kappa^2 = k_z^2 - \varepsilon \omega_b^2/c^2 \). The system (#4) is valid for all domains shown in Fig. (#1) with the corresponding substitutions \( \varepsilon = \varepsilon_p, \varepsilon = \varepsilon, \varepsilon = \varepsilon_b \). The electric fields are the following functions of the time and the coordinates

\[ E_z = E_0 z(x) \exp(-i \omega t + ik_z z). \] (#5)

Dependence on \( x \) is defined by the system (#4) and can be represented as follows

\[ E_0 z(x) = \begin{cases} C_1 e^{\kappa_p x} & \text{at } x \leq 0, \\ C_2 e^{\kappa_b x} + C_3 e^{-\kappa_b x} & \text{at } 0 \leq x \leq a, \\ C_4 e^{-\kappa_b x} & \text{at } x \geq a, \end{cases} \]

where \( \kappa_p = \sqrt{k_z^2 - \omega_b^2 \varepsilon_b/c^2}, \kappa_b = \sqrt{k_z^2 - \omega_b^2 \varepsilon_b/c^2} \).

The boundary conditions can be obtained from the field equations by integrating over a thin layer near the interface between two corresponding media and have the following view:

\[ E_{zp}\big|_{x=0} = E_{zp}\big|_{x=a}, \quad E_{z \nu}\big|_{x=0} = E_{z \nu}\big|_{x=a}, \quad B_{y p}\big|_{x=0} = B_{y p}\big|_{x=a}, \quad B_{y \nu}\big|_{x=0} = B_{y \nu}\big|_{x=a}. \] (#6)

In addition to these boundary conditions the following condition must be satisfied:

\[ E_{zp}\big|_{x \to \pm \infty} = E_{z b}\big|_{x \to \pm \infty} = 0, \quad B_{y p}\big|_{x \to \pm \infty} = B_{y b}\big|_{x \to \pm \infty} = 0. \]

Having solved the system of equations (#4-#6) we can finally obtain the following dispersion relation:

\[ \varepsilon_p \kappa_\nu + \kappa_p = -\frac{\kappa_p}{2} \exp(-2a \kappa_\nu)(\varepsilon_b - 1)(1 + k_z^2/\kappa_\nu^2). \] (#7)

When the bunch is absent, i.e. \( n_b = 0 \) and \( \varepsilon_0 = 1 \), one can get the dispersion relation of surface plasma wave from the following equation:

\[ \varepsilon_p \sqrt{k_z^2 - \omega^2/c^2} + \sqrt{k_z^2 - \omega^2 \varepsilon_p/c^2} = 0, \] (#8)

which was studied with the solution \( \omega = \omega_0 \) in detail in Sec. III A. The bunch leads to the amplification of this wave and solution of Eq. (#6) should be found in the following form:

\[ \omega = k_z u(1 + \delta) = \omega_0(1 + \delta), \quad \delta \ll 1. \] (#9)

Since we took into account that \( n_b \ll n_p \), the highest bunch effect on the surface wave occurs when the following Cherenkov resonance condition is satisfied

\[ \omega_0 = k_z u. \] (#10)

### III. Analysis of the Dispersion Relation.

**Bunch-Assisted Generation of the Surface Electromagnetic Wave Under the Cherenkov Resonance Condition.**

Let us first determine the SEW frequency in a bunch absence, i.e. find solution of Eq. (#1). We are interested in the frequency range of high-speed waves with \( \omega_0 \approx k_z c \) which can be generated by an ultrarelativistic bunch under Cherenkov resonance condition, i.e. \( \omega_0 \approx k_z u \). From Eq. (#8) follows that such waves can exist only in dense plasmas when \( \omega \ll \omega_p \) and hence \( \varepsilon_p \approx -\omega_p^2/\omega^2 \). From Eq. (#8) we can easily find

\[ \omega_0^2 = \frac{\omega_p^2}{\gamma^2 + 1} \approx \frac{\omega_p^2}{\gamma^2}, \] (#11)

where the inequality (#3) was taken into account.

Let us now take into account the bunch effect, i.e. find solution of Eq. (#7) when the Cherenkov resonance...
condition (10) is satisfied. Here, we would like to restrict ourselves to consideration of the ultrarelativistic bunch, when $|\delta|\gamma^2 > 1$, running over and closely to the flat plasma surface, $2^{3/2}a\omega_0\sqrt{\pi}/c << 1$, for this $\exp(-2a\kappa_p) \simeq 1$. Then one can get the following solution for $\delta$

$$\delta^{7/2} = \frac{\omega_0^2}{4\sqrt{2}\omega_p^2\gamma^2}. \quad (12)$$

The solution of Eq. (12), we are interested in, has the following form

$$\delta = \delta' + i\delta'' = \left(\cos\left(\frac{6\pi}{7}\right) + i\sin\left(\frac{6\pi}{7}\right)\right)\left(\frac{n_b}{4\sqrt{2}\omega_p^2}\right)^{2/7}. \quad (13)$$

It is obvious that the saturation of instability can occur when the kinetic energy of electrons, in the SEW frame of reference, will become less than the amplitude of the potential of the plasma wave measured in the same frame. In this case the bunch electrons get trapped by the SEW, i.e. there will be no relative motion between the bunch and SEW, thus, no energy exchange between the bunch and SEW occurs, the bunch and the SEW become stationary. For determination of the saturation amplitude of the potential of the plasma SEW, generated by the bunch and amplified with the time with the increment $3m\delta = \delta''$, we will apply the same method used in [5]. Let us choose the SEW frame of reference in which the wave is purely potential and its stationary saturation amplitude $\Phi'$ can be determined by condition of the bunch electrons trapping in the wave field $\tilde{E}$.

$$\frac{e\Phi'}{mc^2} = \frac{1}{\sqrt{1 - u_1^2/c^2}} - 1, \quad (14)$$

where $\Phi'$ is the SEW potential and $u_1$ is the bunch electrons velocity, both measured in the SEW frame of reference. In accordance with Lorentz transformations the speed of bunch electron in the chosen frame will be

$$u_1 = -\frac{\omega\gamma^2}{1 - 2\omega\delta'\gamma^2} \approx -\frac{\omega\gamma^2}{1 - 2\delta'(\gamma^2 - 1)} = \left\{ \begin{array}{ll} u_1 \approx -u\delta'\gamma^2 << u, & \text{at } 2|\delta'|\gamma^2 << 1 \\ u_1 \approx u/2, & \text{at } 2|\delta'|\gamma^2 >> 1, \end{array} \right.$$ 

here the real part of $\delta (\delta')$ is considered.

In the laboratory frame of reference the potentials $\Phi_0$ and $A_z$ are not zero and

$$\Phi_0 = \Phi'\gamma, \quad A_z = \frac{u}{c}\Phi_0 \simeq \gamma\Phi'.$$ \quad (15)$$

Knowing $\Phi_0$ and $A_z$ we can determine the fields in the laboratory frame of reference:

$$E_z = -\frac{1}{\epsilon_0}\frac{\partial A_z}{\partial x} - \frac{\partial \Phi_0}{\partial z} = -ik_z\Phi_0 + i\frac{\gamma}{\epsilon_0}A_z = -ik\gamma\Phi'(1 - \frac{\omega\gamma^2}{\omega_p^2}),$$

$$E_x = -\frac{\partial \Phi_0}{\partial x}, \quad B_y = -\frac{\partial A_z}{\partial x} \simeq E_x. \quad (16)$$

It is obvious that the amplified by the bunch SEW wave can be easily radiated out of the vacuum domain, in which the bunch is running (see Fig. 2). At the same time, the transverse fields $E_x$ and $B_y$ are much higher than the longitudinal fields. This is why we restrict ourselves to calculation of the transverse fields in the vacuum domain, also because these components form the much higher longitudinal radiating component of Poynting vector (energy density flux) compared to the transverse one.

$$P = \frac{c}{4\pi}E_xB_y = \frac{c}{4\pi}E_x^2 \quad (17)$$

It is quite easy to calculate the fields $E_x$ and $B_y$ in the plasma domain from Eq. (16). It is noteworthy that

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FIG. 1. Schematic illustration of interaction of the ultrarelativistic monoenergetic electron bunch with plasma. Here, the corresponding dielectric permittivities are presented as well.
in our previous work \(n_b\) the electric field generated by the ultrarelativistic bunch in the range \(|\delta|\gamma^2 \gg 1\) was decreasing with an increase of \(\gamma\) and we had to determine the \(\gamma\) at which the field is the highest. In this case, the behaviour is similar: they increase with \(\gamma\) at \(|\delta|\gamma^2 \ll 1\) and decrease in the range \(|\delta|\gamma^2 \gg 1\). In the considered case the ultrarelativistic range \(|\delta|\gamma^2 \gg 1\) is of high interest.

In this range we can obtain the following fields:

\[
|E_x| = |B_y| = \left| - \frac{\partial \Phi_0}{\partial x} \right| = \left| - i \kappa \sqrt{1 - \gamma^2} \right| = \left( \frac{\omega_p}{\sqrt{2}\delta\gamma} \right)^2 / e\gamma^2 n_b^{1/7}. \tag{18}
\]

These fields are lower than the maximum stationary (saturation) plasma wave field obtained in \(u\) being equal to \(E_{\text{max}} = \sqrt{2\omega_p mc}/\gamma e\). It is assumed that at this magnitude the surface electromagnetic wave front breakdown occurs. In the considered case the transverse SEW is generated and its behaviour is not trivial. The only thing we can say is that the fields are determined from condition of the bunch electrons trapping in the wave field and that they are lower than those obtained from SEW front breakdown threshold.

Finally, let us determine the SEW saturation energy density with the corresponding amplitudes (18)

\[
W = \frac{1}{8\pi} (E_x^2 + B_y^2) = \frac{1}{4\pi} E_x^2 = 0.024 mc^2 \left( \frac{2\nu_p n_b^{5/2}}{\gamma^2} \right)^{2/7}. \tag{19}
\]

These magnitudes can exceed the bunch electrons energy density \(E_b \approx mc^2\gamma n_b\) and this fact should not surprise a reader (see the Results and Discussions).

**IV. RESULTS AND DISCUSSIONS**

Let us begin with the concluding words of the last section: The saturation energy density of surface electromagnetic wave (13), generated by means of ultrarelativistic electron bunch running over the flat plasma surface, can exceed the bunch electrons energy density and this fact should not surprise a reader. This is due to the fields structure, i.e. \(|E_x| = |B_y| \gg |E_z|\). The point is that the bunch gets trapped by SEW weak longitudinal field component \(E_z\), whereas the bunch energy is transferred to the whole SEW, i.e. the transverse components \(E_x\) and \(B_y\) get amplified as well forming the longitudinal energy density flux. As a result, the SEW saturation energy density becomes considerably higher than the corresponding bunch energy density.

The schematic view of the real SEW amplifier is presented in Fig. 2. Obviously, the accelerator represents a plasma cylinder (plasma is generated in the glass cylinder with the given gas pressure) of fixed length \(L\) which is blown around by the ultrarelativistic bunch. In the plasma cylinder of radius \(r_p < r_b\) (bunch radius) plasma of given number density \(n_p\) is generated. The ultrarelativistic bunch of radius \(r_b \gg \Delta_b\) (bunch width) and of given number density \(n_b\), the current \(J_b = 2\pi r_b \Delta_b n_b\), penetrates the metallic vacuumed cylindrical chamber of radius \(R\) and length \(L\) from the left end and comes out at the right end being detected by a bunch detector. To the right end of the plasma chamber a metallic coaxial chamber is docked where into the SEW, transformed into the coaxial transverse electric and magnetic mode, is let out. At the junction a partial SEW (longitudinal component) reflection occurs and a quasistatic wave with increasing along Z-axis amplitude gets formed. Only running toward Z-axis wave (forward wave) interacts with the bunch whereas the backward wave - does not.

In conclusion let us make some estimations pursuing the goal of employment of the presented above model for constructing of superpowerful Terahertz radiator \((f_0 \approx 0.5 \cdot 10^{12} \text{ Hz}, \omega_0 = 3 \cdot 10^{12} \text{ s}^{-1})\). Since \(\omega_p = \omega_0/\gamma\) then the plasma and bunch parameters can be chosen respectively. Today, the high-current accelerators are the linear chamber is docked where into the SEW, transformed into the coaxial transverse electric and magnetic mode, is let out. At the junction a partial SEW (longitudinal component) reflection occurs and a quasistatic wave with increasing along Z-axis amplitude gets formed. Only running toward Z-axis wave (forward wave) interacts with the bunch whereas the backward wave - does not.

Finally, let us notice that presented above results are valid only for collisionless plasma, i.e. \(\nu_p >> \nu_e\) being the collisions frequency, satisfying Eq. (1). For plasma of electron number density \(n_p\) and temperature \(T_e\) we have \(\nu_e \sim \hbar m_e / e^2 k T_e\), where \(T_e\) is measured in Kelvin. At \(\omega_0 = 3 \cdot 10^{12} \text{ s}^{-1}\) and \(n_p = 3 \cdot 10^{19} \text{ cm}^{-3}\) the condition \(\omega_0 >> \nu_e\) is satisfied when \(T_e > 2 \cdot 10^5 \text{ K}\). Such temperature can be gained during the ionization of a gas of atmospheric pressure only with the help of powerful lasers.

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FIG. 2. Schematic drawing of surface electromagnetic wave amplifier. Here, $r_b >> \Delta_b$ and $c/\omega_p << r_p$, i.e. the plasma surface can be considered as a flat one.

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