What drives the nominal yield curve in Brazil?

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Abstract
This paper describes the dynamics of the level, slope and curvature of the Brazilian nominal yield curve using only observable macroeconomic indicators. The model is able to explain 94.5% of the variation in the yield curve. We find that the main drivers of the level factor is the Brazil risk premium (5-year CDS spread) and the unemployment rate. In turn, the slope steepens with increases either in the SELIC rate or in the spot exchange rate, and flattens with increases in unemployment rate and commodity returns. Lastly, the curvature increases with the unemployment, inflation and SELIC rates, but decreases with changes in the exchange rate.

Keywords: curvature, level, Nelson–Siegel factors, slope, yield curve
JEL Codes: E43, E44, E47

1 Introduction
Understanding the dynamics of the term structure of interest rates is extremely important for economists and policy makers, as well as for risk and fixed-income managers. In particular, it is key to examine how the dynamics of the yield curve depends on the macroeconomic environment. Otherwise, how to evaluate monetary policy experiments and risk scenarios? This paper proposes describing the dynamics of the level, slope and curvature factors of the nominal yield curve in Brazil using exclusively observable variables by employing Huse’s (2011) modeling strategy.
One could to some extent group the models that investigate the macroeconomic drivers of the term structure of interest rates into three classes. The first examines the yield curve dynamics from the general equilibrium perspective given agents’ optimal strategy. See, among others, Vasicek (1977), Cox, Ingersoll, and Ross (1985), and Campbell (1986). The second class assumes only no arbitrage conditions. See, for instance, Ang and Piazzesi (2003), Ang, Piazzesi, and Wei (2006), Bikbov and Chernov (2010), and Christensen, Diebold, and Rudebusch (2011). These models combine both observable and latent state variables to describe the yield curve dynamics and to extract asset pricing implications.

The third group employs models à la Nelson and Siegel (1987) and Diebold and Li (2006) to track how the level, slope and curvature factors of the yield curve evolve over time, and then augment the information set with macroeconomic variables to increase predictive power, not necessarily imposing no arbitrage conditions. See, inter alia, Diebold, Rudebusch, and Aruoba (2006), Hördahl, Tristani, and Vestin (2006), Moench (2008), Huse (2011), Joslin, Priebsch, and Singleton (2014), Cieslak and Povala (2015), Coroneo, Giannone, and Modugno (2016), Bauer and Hamilton (2018), and Fernandes and Vieira (2019).

The model we employ builds heavily on Huse (2011), and hence falls into the third class. The idea is to estimate a Nelson–Siegel panel model in which the level, slope and curvature factors are affine functions of observable macroeconomic variables. In contrast to Ang and Piazzesi (2003) and Ang et al. (2006), we do not consider any latent component to describe the yield curve. Our model inherits from the panel structure not only more robustness to measurement errors but also more parcimony. In particular, the number of parameters to estimate does not grow with the number of maturities in the term structure, but only linearly in the number of state variables.

The literature on the estimation of the Brazilian yield curve is very extensive. One strand restricts attention to Diebold and Li’s (2006) latent approach: see, among others, Lima, Luduviche, and Tabak (2006), Vicente and Tabak (2008), and Caldeira, Moura, and Portugal (2010). Another strand employs Bayesian estimation methods, such as Laurini and Hotta (2010) who apply Markov Chain Monte Carlo (MCMC) techniques. In turn, Leite, Gomes Filho, and Vicente’s (2009) suggest forecasting the yield curve using both observable and latent variables in a Svensson’s (1994) model, whereas Vieira, Fernandes, and Chague (2017) employ a FAVAR model to predict the Nelson–Siegel factors using macroeconomic information. The latter strategy is in three steps. First, they extract the principal components of a large panel of financial and macroeconomic
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(mainly leading) indicators. Second, they estimate a Nelson–Siegel model to recover the level, slope and curvature factors of the term structure of nominal interest rates. Third, they estimate a FAVAR model to predict the future path of the entire yield curve.

In this paper, we take the opposite direction to Vieira et al. (2017), who prime for predictive power without paying too much attention to interpretability of the principal components. Our interest lies exactly on identifying the main macroeconomic drivers of the nominal yield curve in Brazil, so that interpretability is paramount. We do not focus on forecast performance, but on the economic/statistical significance of the partial effects of each macroeconomic indicator. Our model is more appropriate for dynamic stochastic general equilibrium (DSGE) analyses. It allows us to impose internally consistent paths for the variables of interest, and hence to assess the risk of different macroeconomic scenarios. In fact, this is the same motivation of previous works using Huse’s (2011) model to estimate term structures in Brazil. In particular, Thiele and Fernandes (2015) investigate how the term structure of inflation expectations varies with observable macroeconomic variables, whereas Fernandes, Munhoz, and Nunes (2021) focus on the USD-denominated yield curve in Brazil.

Our empirical findings show that the model fits well the nominal yield curve dynamics between January 2004 and June 2013. The most relevant drivers of interest rate levels are the unemployment rate and Brazil risk, as measured by the 5-year CDS spread. As for the slope, the reference interest rate (SELIC) and the exchange rate have positive partial effects, whereas they are negative for the unemployment rate and commodity prices. In turn, curvature increases with the unemployment, inflation and SELIC rates and decreases with the exchange rate. Our estimates of the level and slope factors highly correlate with their empirical proxies, namely, the long rate and the difference between the long and short rates, respectively. There are two reasons why the same does not apply to the curvature factor. First, the empirical proxy given by the difference between long and medium rates minus the difference between the medium and short rates is too crude and noisy. Second, it could well be the case that the macroeconomic indicators help explain mainly changes in level and slope, rather than curvature changes.

The remainder of this work proceeds as follows. Section 2 discusses term-structure models, whereas section 3 describes not only the methodology but also the data we employ. Section 4 then documents our empirical results. Section 5 concludes.
2 Term-structure models

The first generation of yield curve models rest on a general equilibrium approach (Vasicek, 1977; Cox et al., 1985), with the short rate dynamics driving the entire yield curve. Despite their elegance, these models have poor predictive power (see discussion in Duffee (2002). This is in line with the strong empirical evidence that there are at least three factors that govern the changes in the yield curve (Litterman & Scheinkman, 1991). Longstaff and Schwartz (1992) consider a second factor in the determination of the yield curve. Apart from the short rate, they also entertain a stochastic volatility. Balduzzi, Das, Foresi, and Sundaram (1996) add as a third factor in the model the short rate level in the long run. As in Piazzesi (2005), one could always add more factors to improve predictive power, but at the risk of overfitting (Joslin, Singleton, & Zhu, 2011).

Another strand of term structure models is purely statistical, focusing essentially on curve fitting. As there is a limited number of maturities, it makes sense to determine the yield curve using interpolation and smoothing techniques. McCulloch (1971, 1975) and Vasicek and Fong (1982) respectively use quadratic and cubic splines to fit the yield curve, whereas Schaefer (1981) employs Bernstein polynomials. These smoothing methods are problematic, though. Not only do they fit poorly the very short and long ends of the yield curve, but also lacks economic interpretation.

Nelson and Siegel (1987) propose an exponential polynomial whose terms reflect well the level, slope and curvature factors:

$$y(\tau_i) = \beta_1 + \beta_2 \left( \frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} \right) + \beta_3 \left( \frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} - e^{-\lambda \tau_i} \right) + \nu(\tau_i)$$

(1)

where $y(\tau_i)$ denotes the zero-coupon yields with maturity $\tau_i$, for $i = 1, \ldots, N$. The coefficients $\beta_1$, $\beta_2$ e $\beta_3$ correspond respectively to the level, slope and curvature factors, whereas $\lambda$ defines the decay of the exponential polynomial or, equivalently, the point at which curvature is maximum. Finally, $\nu(\tau_i)$ is a white noise. Svensson (1994) adds a second curvature factor to the mix in order to improve goodness of fit.

To include dynamics in the Nelson–Siegel model, Diebold and Li (2006) estimate the level, slope and curvature factors using the cross-section of yields at each day:

$$y_t(\tau_i) = \beta_{1t} + \beta_{2t} \left( \frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} \right) + \beta_{3t} \left( \frac{1 - e^{-\lambda \tau_i}}{\lambda \tau_i} - e^{-\lambda \tau_i} \right) + \nu(\tau_i)$$

(2)
where $i = 1, \ldots, N$, and $t = 1, \ldots, T$.

To simplify matters, they fix the value of $\lambda$ to 0.0609, so as to maximize curvature at the 30-month maturity. Given a fixed $\lambda$, they estimate betas at each time period $t$ by least squares. Next, they estimate an autoregressive (AR) model for each beta. Note however that the beta estimation in the first step brings about a measurement error that contaminates the AR model estimation in the second stage.

Diebold et al. (2006) consider a vector AR model for the level, slope and curvature factors augmented by a few macroeconomic variables. As they keep free, they resort to a Kalman filter to estimate the VAR model by maximum likelihood. Empirically, they find that the level and slope factors highly correlate with inflation and real activity, respectively. In contrast, curvature seems orthogonal to the macroeconomic indicators they control for.

Huse (2011) employs only observable macroeconomic indicators to describe the dynamics of the level, slope and curvature factors of the yield curve. In particular, he assumes that $eta_{jt} = \beta_{j0} + \beta_{j1}X_{1t} + \cdots + \beta_{jp}X_{pt}$ (for $j = 1, 2, 3$), where $(X_1, \ldots, X_p)$ denote the vector of observable macroeconomic indicators. Note that the sensitivity of the yield curve factors to each state variable is time invariant, so that only changes in pre-determined macroeconomic variables affect the betas and, accordingly, the yield curve dynamics. This reduces the dimensionality of the problem in a considerable manner given that the number of free parameters does not depend on the number of maturities. In addition, taking advantage from both the cross-section and time dimensions should also yield more precise beta estimates.

There are many papers that attempt to understand the influence of macroeconomic conditions on the term structure of the nominal interest rates. Using a model with latent and observable variables, Ang and Piazzesi (2003) find that real activity shocks have a significant impact on medium-term rates, mainly through the curvature factor. In turn, unexpected inflation affect mostly the level factor. Piazzesi (2005) shows that changes in the monetary policy entail a strong impact on the slope factor by moving the short rates more than the long rates. Ang, Bekaert, and Wei (2008) examine the effect of inflation (and two additional latent variables) on the yield curve, whereas Rudebusch and Wu (2008) also investigate the link with GDP growth (apart from inflation and latent factors).

To estimate the Brazilian yield curve, Lima et al. (2006), Almeida, Gomes, Leite, and Vicente (2008), and Caldeira et al. (2010) employ Diebold and Li's
Laurini and Hotta (2010) apply MCMC methods to fit, interpolate and predict the term structure of nominal interest rates using an extension of the Diebold–Li model with stochastic volatility. Vicente and Tabak (2008) study the forecasting performance of affine models relative to variations of the Diebold–Li model. In turn, Leite et al. (2009) estimate Svensson’s (1994) model to predict the yield curve controlling for term premia and inflation expectations.

3 Methodology and data

Huse (2011) assume that a vector $M_t$ of macroeconomic variables governs the yield curve dynamics. Let $\beta_t$ denote a $3 \times 1$ vector that collects the level, slope and curvature factors at time $t$ and $\lambda$ a scalar. The model specification reads

$$y_t(\tau) = X_t(\lambda)\beta_t + \nu(\tau),$$

where $y_t(\tau)$ is a column vector of yields at time $t$,

$$X(\lambda) = \begin{bmatrix}
1 & 1 - e^{-\lambda \tau_1} & 1 - e^{-\lambda \tau_N} \\
1 - e^{-\lambda \tau_1} & \frac{1}{\lambda \tau_1} & - e^{-\lambda \tau_1} \\
1 - e^{-\lambda \tau_2} & \frac{1}{\lambda \tau_2} & - e^{-\lambda \tau_2} \\
\vdots & \vdots & \vdots \\
1 - e^{-\lambda \tau_N} & \frac{1}{\lambda \tau_N} & - e^{-\lambda \tau_N}
\end{bmatrix}$$

and $\nu(\tau)$ is the error term. The factors are affine functions of the state variables: $B_0 + B_1 M_t$. To ensure the pre-determination of the state variables $M_t$, we entertain their values at the beginning of the period, as opposed to yields that we record at the end of the period.

We fix $\lambda$ in order to make the model linear in the parameter vector. This allows estimation by pooled least squares (PLS). In particular, we let $\lambda = 0.064$ in order to maximize the weight of the curvature factor at the 2-year maturity. Figure 1 illustrates the resulting level, slope and curvature factor loadings for $\lambda = 0.064$. The level factor loading is constant, regardless of $\lambda$ and maturity. By construction, the slope factor loading monotonically decreases to zero as the maturity increases. In turn, a change in curvature has little effect on the short and long ends of the yield curve, but great impact on the medium-term rates.
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Among the macroeconomic state variables, we include expected inflation, unemployment rate, expected reference rate (SELIC), commodity price index, spot exchange rate, and the 5-year CDS spread measure of Brazil risk.

The dynamics of the level, slope and curvature factors of the yield curve are as follows:

\[
\begin{align*}
\beta^L_t &= \beta^L_0 + \beta^L_1 e_t + \beta^L_2 \pi_t + \beta^L_3 CDS_t + \beta^L_4 CRB_t + \beta^L_5 u_t + \beta^L_6 i_t, \\
\beta^S_t &= \beta^S_0 + \beta^S_1 e_t + \beta^S_2 \pi_t + \beta^S_3 CDS_t + \beta^S_4 CRB_t + \beta^S_5 u_t + \beta^S_6 i_t, \\
\beta^C_t &= \beta^C_0 + \beta^C_1 e_t + \beta^C_2 \pi_t + \beta^C_3 CDS_t + \beta^C_4 CRB_t + \beta^C_5 u_t + \beta^C_6 i_t,
\end{align*}
\]

where \( \beta^k = (\beta^k_0, \beta^k_1, \beta^k_2, \beta^k_3, \beta^k_4, \beta^k_5, \beta^k_6)' \) denote the vector of betas associated with factor \( k \) of the yield curve, and the vector \( (e_t, \pi_t, CDS_t, CRB_t, u_t, i_t) \) gathers the information about exchange rate, inflation, Brazil risk, commodity prices, unemployment, and SELIC rate.

To bootstrap the nominal yield curve in Brazil, we use rates implied by interbank deposit (DI) futures contracts traded on the Brazilian Exchange (B3). Data correspond to biweekly annualized rates for the maturities of 1, 2, 3, 6, 9, 12, 24, 36, 60, 84, and 120 months from January 2004 to June 2013, amounting to 245 time-series observations. For the less liquid maturities, we also employ the DI swaps that trade over the counter at B3. The underlying asset of the DI futures contract corresponds to the daily average interbank deposit rates,
as measured by CETIP (now part of B3). The DI futures contract trades at a
discount over a face value of 100,000 BRL according to the interest rate agreed
between the parties. Figure 2 shows that the shape of the yield curve varies
substantially over time.

To compute the empirical proxies for the level, slope and curvature factors, we
consider linear combinations of the short, medium and long rates, as in Diebold
and Li (2006). More specifically, we approximate the level factor by the long
rate. As in Frankel and Lown (1994), we proxy the slope factor by the difference
between the long and short rates. Finally, we gauge the curvature factor by
taking the difference between the long and medium rates minus the difference
between the medium and short rates. We proxy the short and long rates by
the most liquid contracts in the short and long ends of the yield curve, namely,
3-month and 5-year futures. As for the medium-term rate, we interpolate the
28.5-month rate from the existing futures contracts.

Table 1 display some descriptive statistics of the yield curve. There is
substantial variation in the level of the yield curve over time. For the 1-month
maturity, rates vary from 6.93% to 19.81% per year, with mean and standard
development of 12.44% and 3.53% per annum, respectively. The yield curve is
typically upward sloping, with increasing term premia. Interest rate volatility
decreases with maturity: the standard deviations of the 1-month and 10-year
rates are of 3.53% and 2.68% per annum, respectively. Importantly, statistical
tests reject the null hypothesis of unit root at the 5% significance level.

As macroeconomic indicators, we gather information on the unemployment
rate (from IBGE), the BRL/USD spot exchange rate, expected inflation (IPCA)
in the next 12 months (mean rates from the FOCUS survey), expected SELIC
rate in the next 12 months (mean rates from the FOCUS survey), the CRB
index of commodity futures prices (in BRL), and the Brazil risk implied by the
5-year CDS spread. We use the unemployment rate as our real activity indicator
because it is at the monthly frequency. The remaining variables are financial
indicators, and hence it is easy to collect data at the biweekly frequency. The
lower frequency of the unemployment rate is not so problematic given that it
appears only as a control, and not as a dependent variable (Ghysels, 2016).

Table 2 documents the descriptive statistics of the macroeconomic and
financial indicators. Macroeconomic uncertainty is very strong during the
governments by Luís Inácio Lula da Silva and Dilma Rousseff. For instance,
exchange rate varies from 1.56 in July 2008 to 3.21 in May 2004. Expected
inflation moves from 6.48% in July 2004 to 3.37% per year in June 2007, whereas
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Figure 2. Term structure of the nominal interest rates in Brazil over time.

| maturity (in months) | mean   | minimum | maximum | standard deviation |
|----------------------|--------|---------|---------|--------------------|
| 1                    | 12.44  | 6.93    | 19.81   | 3.53               |
| 2                    | 12.42  | 6.96    | 19.84   | 3.53               |
| 3                    | 12.42  | 6.98    | 19.85   | 3.52               |
| 6                    | 12.46  | 7.03    | 19.73   | 3.47               |
| 9                    | 12.52  | 7.05    | 19.51   | 3.40               |
| 12                   | 12.59  | 7.08    | 19.31   | 3.31               |
| 24                   | 12.90  | 7.59    | 20.24   | 3.00               |
| 36                   | 13.06  | 8.14    | 21.49   | 2.84               |
| 60                   | 13.20  | 8.64    | 23.06   | 2.74               |
| 84                   | 13.27  | 9.01    | 23.75   | 2.71               |
| 120                  | 13.32  | 9.31    | 24.28   | 2.68               |

Empirical proxy

| factor   | level  | slope  | curvature |
|----------|--------|--------|-----------|
| mean     | 13.20  | −0.70  | 0.06      |
| standard deviation | 2.74 | 1.67   | 1.22      |
Table 2. Macroeconomic and financial indicators from January 2004 to June 2013.

| indicator                              | mean | maximum | minimum | standard deviation |
|----------------------------------------|------|---------|---------|--------------------|
| BRL/USD exchange rate                  | 2.08 | 3.21    | 1.56    | 0.39               |
| expected inflation (% per annum)       | 4.94 | 6.48    | 3.37    | 0.73               |
| Brazil risk (% per annum)              | 2.06 | 8.57    | 0.62    | 1.51               |
| CRB index                              | 11.29| 13.91   | 9.93    | 0.75               |
| unemployment rate                      | 8.37 | 12.33   | 4.93    | 2.07               |
| expected SELIC rate (% per annum)      | 12.25| 18.00   | 7.25    | 3.06               |

the Brazil risk changes from 8.56% in May 2004 to 0.61% per annum in May 2007. As for the unemployment rate, it drops from 12.32% in August 2004 to 4.93% in March 2013. Finally, the expected SELIC rate decreases from 18% in April 2005 to 7.25% per year in November 2012.

To make the regression coefficients comparable, we normalize every macroeconomic and financial indicators by their respective mean and standard deviation.

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Table 3 report the PLS estimates of the coefficients that appear in equations (4) to (6). The sample period runs from January 2004 to June 2013. The model fits very well the data, with an adjusted $R^2$ of 94.5%. This is not surprising given that both yields and macroeconomic indicators are very persistent. Accordingly, we must certify ourselves that the regression indeed is genuine, and not spurious due to unit roots; the exchange rate and the commodity index are clearly nonstationary, for instance. As it turns out, residuals are very well behaved, with little evidence of unit root behavior. This seems to indicate there are cointegration relationships among the macroeconomic indicators at play. We next discuss our main empirical findings given that we find no evidence of spurious regression.

Our findings reveal that commodity prices and the expected reference rate (SELIC) do not affect the yield curve level. Apart from the negative effect of the exchange rate, the remaining indicators affect positively the level factor. In particular, the main drivers of the yield curve level are the CDS spread and unemployment rate. As for slope, the SELIC and exchange rates entail significantly positive partial effects, whereas the coefficient estimates for unemployment,
Table 3. Yield curve factors as a function of macroeconomic conditions.

|                        | estimate | standard deviation | t-statistics | p-value  |
|------------------------|----------|--------------------|--------------|----------|
| **Level**              |          |                    |              |          |
| intercept              | 13.5505  | 0.0687             | 197.1508     | 0.0000   |
| exchange rate          | −0.4507  | 0.1973             | −2.2838      | 0.0225   |
| expected inflation     | 0.3143   | 0.1306             | 2.4070       | 0.0162   |
| Brazil risk            | 1.7124   | 0.1734             | 9.8765       | 0.0000   |
| commodities            | −0.0532  | 0.0942             | −0.5648      | 0.5722   |
| unemployment           | 1.3367   | 0.1960             | 6.8210       | 0.0000   |
| expected SELIC         | 0.0346   | 0.1311             | 0.2638       | 0.7920   |
| **Slope**              |          |                    |              |          |
| intercept              | −1.2080  | 0.1111             | −10.8707     | 0.0000   |
| exchange rate          | 2.4495   | 0.3003             | 8.1564       | 0.0000   |
| expected inflation     | −1.2415  | 0.2034             | −6.1028      | 0.0000   |
| Brazil risk            | 0.3862   | 0.1660             | 2.3258       | 0.0201   |
| commodities            | −1.9106  | 0.2414             | −7.9136      | 0.0000   |
| unemployment           | −2.1456  | 0.2826             | −7.5937      | 0.0000   |
| expected SELIC         | 3.1166   | 0.1993             | 15.6386      | 0.0000   |
| **Curvature**          |          |                    |              |          |
| intercept              | −0.2702  | 0.1584             | −1.7063      | 0.0881   |
| exchange rate          | −3.4550  | 0.4314             | −8.0093      | 0.0000   |
| expected inflation     | 1.3209   | 0.2773             | 4.7635       | 0.0000   |
| Brazil risk            | −0.0618  | 0.2252             | −0.2745      | 0.7837   |
| commodities            | −0.0990  | 0.4677             | −0.2117      | 0.8323   |
| unemployment           | 1.5960   | 0.4177             | 3.8205       | 0.0001   |
| expected SELIC         | 1.7360   | 0.2727             | 6.3659       | 0.0000   |
expected inflation and commodity prices are negative. In addition, curvature does not respond to commodity prices and CDS spread in a significant manner. In turn, not only does it increase with unemployment, expected inflation and SELIC rates, but it also decreases with the exchange rate.

Apart from PLS, we also estimate the model by the generalized method of moments (GMM) using lagged indicators as instruments. Not surprisingly, given data persistence, results are quantitatively very similar in that both coefficient estimates and t-statistics change only marginally. Moreover, Wald tests reject at the 1% significance level null partial effects, regardless of whether we use PLS or GMM. For brevity sake, we do not report these results, but they are obviously available from the authors upon request.

Although the coefficient estimates in Table 3 indicates to some extent how the level, slope and curvature factors react to each macroeconomic variable, it is paramount to assess their impact on each maturity. To this end, as in Thiele and Fernandes (2015), we carry out a reduced-form impulse-response analysis to evaluate how the nominal yield curve changes as we shock each macroeconomic variable. In particular, we give a shock of one standard deviation in one macroeconomic variable, while imposing the mean value of zero (due to the normalization) for all the other macroeconomic indicators, and then observe how the yield curve reacts.

Figure 3 shows that slope and curvature effects dominate after either a devaluation of the Brazilian real or an increase in expected inflation, implying higher short-term yields. In contrast, the negative impact on the level factor is much more apparent for longer rates. Central banking is probably key to explain this pattern. Exchange-rate devaluation increases imported prices, putting pressure on expected inflation, which triggers the Central Bank to increase the SELIC rate. Given the transient nature of the shock, the eventual decrease in inflation calls for a looser monetary policy, with lower interest rates in the future as reflected by the longer rates today.

If the Brazil risk increases, investors would demand higher interest rates as compensation. As we observe in Figure 3, CDS spread shocks affect only the level factor. As for shocks to commodity prices, they reduce interest rates, especially at the shorter maturities. The intuition is simple. The Brazilian real typically appreciates with higher commodity prices, alleviating inflation concerns, and so resulting in lower interest rates. Increasing unemployment (or reducing real activity) in the short run has a similar effect, in that it also mitigates inflation concerns. The Central Bank has more room to adopt anticyclical policies, such as
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Figure 3. Partial effects of each macroeconomic indicator on the nominal yield curve.
a drop in the SELIC rate, aiming at closing the potential output gap. However, anticyclical policies might reveal excessive in the longer run, pushing prices up. To keep the inflation target, the Central Bank is expected to increase future interest rates, explaining why longer rates increase in Figure 3. Finally, if the expected SELIC rate increases, the entire yield curve shifts upwards.

Figure 4 compares our model estimates of the level, slope and curvature factors, as driven by macroeconomic and financial indicators, with their respective empirical proxies (Frankel & Lown, 1994). The fit is excellent, especially for the level and slope factors, with correlation coefficients of 89% and 77%, respectively. As for the curvature factor, there is more discrepancy between the model-implied and empirical measures, yielding a correlation of only 32%. There are two interpretations for such a mismatch. The first is that the empirical proxy is too noise. Indeed, if we change the medium maturity from 28.5 months to 24 months, correlation increases to over 52%. This is not surprising given that we did fix the value of the decay parameter $\lambda$ such that the curvature loading
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...is maximum at the 24-month maturity. The second interpretation is that the macroeconomic variables we employ are more appropriate to fit changes in level and slope than in the curvature of the term structure.

5 Conclusion

We develop a Nelson–Siegel model in which macroeconomic conditions drive the level, slope and curvature of the term structure of nominal interest rates in Brazil. Our empirical findings document that our model fits well the yield curve dynamics between January 2004 and June 2013. The most important drivers of the level factor are the 5-year Brazil CDS spread and unemployment rate. In turn, slope increases with the SELIC reference rate and the spot exchange rate, while decreasing with commodity prices and unemployment rate. Finally, exchange rate, unemployment, expected inflation and SELIC rates entail strong curvature effects.

There are many ways to extend our analysis. One possibility is to assess its forecasting performance relative to the extant models in the literature. Another is to examine how it fares against models with latent variables. More importantly perhaps, one could carry out a more structural impulse-response analysis, given the obvious limitations of the reduced-form approach we take.

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