Phase structure of the quartic-cubic generalized two dimensional Yang Mills $U(N)$ on the sphere

L. Lavaei-Yanesi* and M. Khorrami†
Department of Physics, Alzahra University, Tehran 1993891167, Iran.

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Abstract

The large-$N$ behavior of the quartic-cubic generalized two dimensional Yang Mills $U(N)$ on the sphere is investigated, for small cubic couplings. It is shown that single transition at the critical area which is present for the quartic model, is split into two transitions, both of them are third order. The phase diagram of the system for small cubic couplings is obtained.

1 Introduction

During recent years, there has been extensive studies of the two-dimensional Yang-Mills theory ($YM_2$) and generalized Yang-Mills theories ($gYM_2$’s) [1–18]. A review on this topic is [19]. These are important integrable models which are expected to shed light on some basic features of pure QCD in four dimensions. Besides, there are certain relations between these theories and string theories. To be more specific, there relations are between the large-gauge-group limit of $YM_2$ and string theory. For example, it is shown in [3], [6], and [7], that a gauge theory based on $SU(N)$ is split at large $N$ into two copies of a chiral theory, encapsulating the geometry of the string maps. The chiral theory associated to the Yang-Mills theory on a two–manifold $M$ is a summation over maps from the two–dimensional world sheet (of arbitrary genus) to the manifold $M$. This leads to a $1/N$ expansion for the partition function and observables, which is convergent for all of the values of area×coupling constant on the target space $M$, if the genus is one or greater.

The partition function and the expectation values of the Wilson loops of $YM_2$ were obtained for theories on a lattice [1, 20] and continuum [4, 11, 12]. The partition function and the expectation values of the Wilson loops of $gYM_2$’s were calculated in [9,10]. All of these results are in terms of summations over the irreducible representations of the corresponding gauge group. When the group is large, these summations are dominated by some specific representations (the

*yalda57l@yahoo.com
†mamwad@mailaps.org
so called classical representations) in some cases, and one can obtain closed-form expressions for that representation and the observables of the theory.

The large-$N$ limit of $U(N)$ YM$_2$ on a sphere was first studied in [21]. There the classical representation was calculated and it was shown that the free energy of theory has a logarithmic behavior with respect to the area of the sphere. In [13], it was shown that that behavior is correct provided the area of the sphere is smaller than some critical area, and it was further shown that at that area a third order phase transition occurs. This transition is similar to the well-known Gross-Witten-Wadia phase transition for the lattice two dimensional multicolor gauge theory [22, 23]. The phase structure of the large-$N$ YM$_2$, generalized YM$_2$’s, and nonlocal YM$_2$’s on a sphere were further discussed in [16, 18, 24, 25]. The large-$N$ limit of the partition function of YM$_2$ on orientable compact surfaces with boundaries was discussed in [26], and the large-$N$ behavior of Wilson loops of YM$_2$ and gYM$_2$ on sphere were investigated in [27]. The critical behaviors of these quantities have been also studied.

A gYM$_2$ with the gauge group $U(N)$ on a sphere is characterized by a function $G$, as introduced in [18], for example. Most of these investigations on gYM$_2$’s have been on theories with even $G$. In fact, as it was pointed out in [18], if $G$ is a polynomial then its degree should be even in order that the partition function be convergent. So if one takes $G$ a monomial, it should be a monomial of even degree. So to study theories based on non-even $G$’s, the corresponding $G$’s should contain at least two terms. A quadratic-linear combination gives nothing new, as the linear term can be absorbed in a shifted variable. So the simplest nontrivial non-even $G$ must contain a quartic term.

In this paper the behavior of a gYM$_2$ based on the gauge group $U(N)$ on a sphere is studied, for which the function $G$ is a binomial containing a quartic term as well as a cubic one. The study is performed for small values of the cubic coupling. In section 2 the behavior of this model for areas smaller than the critical area (the weak phase) is investigated and compared with the behavior of a theory without the cubic term. In section 3 the transitions properties at the first transition area are investigated. In section 4 it is seen that there exists another transition, and the relation between this transition to the first one is investigated. Section 5 is devoted to the concluding remarks.

2 The weak phase

To fix notation, let us first quickly review the expression for the partition function of a $U(N)$-gYM$_2$ on the sphere, in the large $N$ limit [18].

The partition function of a $gYM_2$ on the sphere of area $A$ is

$$ Z = \sum_r d_r^2 e^{-AA(r)}, $$

where $r$’s label the irreducible representations of the gauge group, $d_r$ is the dimension of the representation $r$, and $A$ is a combination of the Casimirs of
the group. For the gauge group U(N), one can take \( \Lambda \) as:

\[
\Lambda(r) = \sum_{k=1}^{p} \frac{a_k}{N^{k-1}} C_k(r),
\]

(2)
in which \( C_k \) is the \( k \)'th Casimir of the group, and \( a_k \)'s are arbitrary constants.

The representations of the group U(N) are parametrized by \( N \) integers \( n_1 \geq n_2 \geq \ldots \geq n_N \). In terms of this parametrization, one has

\[
d_r = \prod_{1 \leq i < j \leq N} \left( 1 + \frac{n_i - n_j}{j-i} \right),
\]

\[
C_k = \sum_{i=1}^{N} [(n_i + N - i)^k - (N - i)^k].
\]

(3)

In order that the partition function (1) be convergent, it is necessary that \( p \) in Eq. (2) be even and \( a_p \) be positive.

For the case of large \( N \), the summation can be rewritten in the form of a path integral over continuous parameters. These parameters are introduced as

\[
0 \leq x := \frac{i}{N} \leq 1, \\
n(x) := \frac{n_i}{N}, \\
\phi(x) := -n(x) - 1 + x.
\]

(4)
The partition function (1) then becomes

\[
Z = \int \prod_{0 \leq x \leq 1} d\phi(x) \exp[S(\phi)],
\]

(5)
where

\[
S(\phi) := N^2 \left\{ -A \int_{0}^{1} dx \ G[\phi(x)] + \int_{0}^{1} dx \int_{0}^{1} dy \ \log |\phi(x) - \phi(y)| \right\},
\]

(6)
apart from an unimportant constant, and

\[
G(z) := \sum_{k=1}^{p} (-1)^k a_k z^k.
\]

(7)
Introducing the density

\[
\rho[\phi(x)] := \frac{dx}{d\phi(x)},
\]

(8)
one would have

\[
\int_{b}^{a} dz \ \rho(z) = 1,
\]

(9)
(the so called normalization condition for the density), where $b$ and $a$ are the lower and upper limits of $\phi(x)$, respectively. The condition that $n_i$’s be non-increasing with respect to $i$ imposes the following condition on the density.

$$0 \leq \rho(z) \leq 1.$$  \hspace{1cm} (10)

The path integral in the right-hand side of (5) is dominated by the classical representation, which maximizes the function $S$, or minimizes the free energy $F$ defined through

$$F := -\frac{1}{N^2} \ln Z.$$  \hspace{1cm} (11)

Denoting the density corresponding to this representation by $\rho_{cl}$, it is seen that the derivative of the free energy with respect to the area is

$$F'(A) = \int_0^1 dx \ G[\phi_{cl}(x)],$$

$$= \int_b^a dz \ \rho_{cl}(z) \ G(z),$$

$$= \oint_{C_\infty} \frac{dz}{2\pi i} H_{cl}(z) \ G(z),$$  \hspace{1cm} (12)

where the function $H$ of a complex variable is defined through

$$H(z) := \int_b^a d\zeta \ \frac{\rho_{cl}(\zeta)}{z - \zeta},$$  \hspace{1cm} (13)

and $C_\infty$ is a counterclockwise contour outside of which $H$ is analytic. It is seen from (9) that $H(z)$ behaves like $z^{-1}$ for large $z$.

The classical density $\rho_{cl}$ is the weak density $\rho_w$ satisfying

$$g(z) = P \int_{b_w}^{a_w} d\zeta \ \frac{\rho_w(\zeta)}{z - \zeta}, \quad b_w \leq z \leq a_w,$$  \hspace{1cm} (14)

where $P$ means principal value and

$$g(z) := \frac{A}{2} G'(z),$$  \hspace{1cm} (15)

provided the conditions (10) are not violated by $\rho_w$. This is the weak phase. One obtains

$$H_w(z) = g(z) - \sqrt{(z - a_w)(z - b_w)}$$

$$\times \sum_{m,n,q=0}^{\infty} \frac{(2n - 1)!!(2q - 1)!!}{2^{n+q} n! q! (n + q + m + 1)!} a_w^n b_w^q z^m g^{(n+m+q+1)}(0),$$  \hspace{1cm} (16)

where $g^{(n)}$ is the $n$’th derivative of $g$, and

$$\rho_w(z) = \frac{\sqrt{(a_w - z)(z - b_w)}}{\pi}$$

$$\times \sum_{m,n,q=0}^{\infty} \frac{(2n - 1)!!(2q - 1)!!}{2^{n+q} n! q! (n + q + m + 1)!} a_w^n b_w^q z^m g^{(n+m+q+1)}(0).$$  \hspace{1cm} (17)
The condition that $H(z)$ behaves like $z^{-1}$ for large $z$, is equivalent to the following equations.

$$
\sum_{n,q=0}^{\infty} \frac{(2n-1)!! (2q-1)!!}{2^n n! q! (n+q)!} a_w^n b_w^q g^{(n+q)}(0) = 0,
$$

(18)

$$
\sum_{n,q=0}^{\infty} \frac{(2n-1)!! (2q-1)!!}{2^n n! q! (n+q-1)!} a_w^n b_w^q g^{(n+q-1)}(0) = 1.
$$

(19)

These equations are used to obtain $a_w$ and $b_w$.

Now consider a function $G$ like $G(z) = z^4 + \lambda z^3$.

(20)

The conditions (18) and (19) become

$$
\tau_w^2 \left( 3 \sigma_w + \frac{3}{4} \lambda \right) + \sigma_w^2 \left( 2 \sigma_w + \frac{3}{2} \lambda \right) = 0,
$$

(21)

$$
\frac{3}{4} \tau_w^4 + \tau_w^2 \left( 3 \sigma_w^2 + \frac{3}{2} \sigma_w \lambda \right) = \frac{1}{A},
$$

(22)

where

$$
\sigma := \frac{a + b}{2},
$$

$$
\tau := \frac{a - b}{2}.
$$

(23)

Using (16), one obtains

$$
H_w(z) = \frac{A}{2} \left\{ 4 z^3 + (3 z^2) \lambda - \sqrt{(z - \sigma_w)^2 - \tau_w^2} \right. \\
\left. \times \left[ 4 \sigma_w^2 + 2 \tau_w^2 + 4 \sigma_w z + 4 z^2 + 3 (\sigma_w + z) \lambda \right] \right\}.
$$

(24)

The solution to (21) and (22) for small values of $\lambda$ is

$$
\sigma_w = -\frac{1}{4} \lambda - \frac{(3 A)^{1/2}}{96} \lambda^3 + O(\lambda^5),
$$

$$
\tau_w^2 = \frac{2}{(3 A)^{1/2}} + \frac{1}{8} \lambda^2 + O(\lambda^4).
$$

(25)

Using these, (12), and (24), one obtains a perturbative expression for the derivative of the free energy with respect to the area:

$$
F_w'(A) = \frac{1}{4 A} - \frac{1}{8 (3 A)^{1/2}} \lambda^2 + O(\lambda^4).
$$

(26)

To investigate the conditions (10) for $\rho_w$, one obtains $\rho_w$ and its minima and maxima. From (17), one has

$$
\rho_w(z) = \frac{A}{2 \pi} \sqrt{\tau_w^2 - (z - \sigma_w)^2} \left[ 4 \sigma_w^2 + 2 \tau_w^2 + 4 \sigma_w z + 4 z^2 + 3 (\sigma_w + z) \lambda \right].
$$

(27)
Using this, one obtains three points where the derivative of $\rho_w$ vanishes:

$$
\begin{align*}
z_0 &= -\frac{1}{4} \lambda + \frac{(3A)^{1/2}}{96} \lambda^3 + O(\lambda^5), \\
z_\pm &= \pm \frac{1}{(3A)^{1/4}} \left( 1 - \frac{1}{4} \lambda \pm \frac{(3A)^{1/4}}{16} \lambda^2 - \frac{(3A)^{1/2}}{96} \lambda^3 + O(\lambda^4) \right),
\end{align*}
$$

and the corresponding values for the density $\rho_w$:

$$
\begin{align*}
\rho_w(z_0) &= \sqrt{\frac{\pi}{8}} \frac{(3A)^{1/4}}{3 \pi} - \frac{(3A)^{3/4}}{\sqrt{128 \pi}} \lambda^2 + O(\lambda^4), \\
\rho_w(z_\pm) &= \frac{4 (3A)^{1/4}}{3 \pi} \pm \frac{A}{16 \pi} \lambda^3 + O(\lambda^5).
\end{align*}
$$

The absolute maximum of $\rho_w$ exceeds one at $A = A_c$. This gives the critical area $A_c$:

$$
A_c = A_c(0) \left[ 1 - \frac{A_c(0)}{4 \pi} |\lambda|^3 + O(|\lambda|^5) \right],
$$

where

$$
A_c(0) = \frac{27 \pi^4}{256}.
$$

It is seen that in the limit $\lambda = 0$ one recovers the result obtained in [18].

One can also obtain the critical exponent relating $\lambda$ and $A_c$. For a specific value of $A_c$, one obtains two values for $\lambda$. Denoting the difference of these values by $\Delta \lambda$, it is seen that

$$
\Delta \lambda \sim [A_c(0) - A_c]^{1/3}.
$$

## 3 The strong phase

For $A > A_c$, there is a region where the density $\rho_w$ exceeds one. Then $\rho_{cl}$ cannot be $\rho_w$ and one uses another ansatz for $\rho_{cl}$, which is $\rho_s$:

$$
\rho_s(z) = \begin{cases} 
1, & z \in [d_s, c_s] \\
\tilde{\rho}_s(z), & z \in [b_s, d_s] \cup [c_s, a_s].
\end{cases}
$$

This is the strong phase. An argument similar to that used in [18] shows that

$$
H_s(z) = \frac{A}{2} \left( 4 z^3 + (3 z^2) \lambda \right) \\
- \sqrt{[(z - \sigma_s)^2 - \tau_s^2] [(z - s_s)^2 - \tau_s^2]} \left[ 4 (\sigma_s + s_s + z) + 3 \lambda \right] \\
- \sqrt{[(z - \sigma_s)^2 - \tau_s^2] [(z - s_s)^2 - \tau_s^2]} \int_{d_s}^{c_s} \frac{d\zeta}{(z - \zeta) R(\zeta)},
$$

where
where

\[ s := \frac{c + d}{2}, \]
\[ t := \frac{c - d}{2}, \]
\[ R(z) := \sqrt{\tau^2 - (z - \sigma_s)^2} \left[ t^2 - (z - s)^2 \right]. \] (35)

\( H_s(z) \) should behave like \( z^{-1} \) for \( z \to \infty \). Expanding \( H_s(z) \) for large \( z \), It is seen that

\[ H_s(z) = \alpha_1 z + \alpha_0 + \alpha_{-1} z^{-1} + O(z^{-2}). \] (36)

Hence one arrives at three equations

\[ \alpha_1 = 0, \]
\[ \alpha_0 = 0, \]
\[ \alpha_{-1} = 1. \] (37)

One also has

\[ \int_{d_s}^{c_s} dz \left[ g(z) - H_s(z) \right] = 0, \] (38)

which is

\[ \frac{A}{2} \int_{d_s}^{c_s} dz \left[ 4 (\sigma_s + s_s + z) + 3 \lambda \right] R(z) + P \int_{d_s}^{c_s} dz \int_{d_s}^{c_s} d\zeta \frac{R(z)}{R(\zeta)} \frac{1}{z - \zeta} = 0. \] (39)

Using this equation and the equations (37), one can obtain the four unknowns \( \sigma_s, \tau_s, s_s, \) and \( t_s \). Expanding these in terms of \( (A - A_c) \), and using (12), one can obtain an expansion for the derivative of the free energy with respect to the area.

However, to obtain the behavior of the free energy in the strong phase at the limit \( \lambda \to 0 \), there exists a simpler way based on [18] and [27]. It is known from [27] that the transition is of third order, meaning that the difference between \( H_s(z) \) and \( H_w(z) \) for large \( z \) behaves like \( (A - A_c)^2 \):

\[ \lim_{A \to A_c^+ (\lambda)} \left( [A - A_c(\lambda)]^{-2} [H_s(z) - H_w(z)] \right) = B \left[ z, \lim_{A \to A_c(\lambda)} \rho_w \right]. \] (40)

Increasing \( A \) further, \( \rho_s \) itself would exceeds one in some region. That is, a second phase transition occurs. Denoting the classical density after this transition by \( \rho_{s2} \), and the area corresponding to this transition by \( A_{c2} \), one has

\[ \lim_{A \to A_{c2} (\lambda)} \left( [A - A_{c2}(\lambda)]^{-2} [H_{s2}(z) - H_s(z)] \right) = B \left[ z, \lim_{A \to A_{c2}(\lambda)} \rho_s \right]. \] (41)

Now consider the limit \( \lambda \to 0 \). At this limit the two transition occur at the same area \( A_c(0) \). One also has

\[ \lim_{\lambda \to 0} \left[ \lim_{A \to A_{c2}(\lambda)} \rho_s \right] = \lim_{\lambda \to 0} \left[ \lim_{A \to A_c(\lambda)} \rho_w \right]. \] (42)
So, adding (40) and (41) at the limit \( \lambda \to 0 \), one arrives at

\[
\lim_{A \to A^+} \{ [A - A_c(0)]^2 \lim_{\lambda \to 0} [H_{s2}(z) - H_w(z)] \} = 2 B \left[ z, \lim_{\lambda \to 0} \lim_{A \to A_c} \rho_w \right].
\] (43)

Defining

\[
I := \lim_{A \to A^+} \{ [A - A_c(0)]^2 \lim_{\lambda \to 0} [F'_s(A) - F'_w(A)] \},
\] (44)

it is then seen from (40) and (43) that

\[
\lim_{A \to A^+} \{ [A - A_c(0)]^2 \lim_{\lambda \to 0} [F'_s(A) - F'_w(A)] \} = \frac{1}{2} I,
\] (45)

where (12) has been used the relate \( F' \) to \( H \).

Now consider the case \( \lambda = 0 \), which has been investigated in [18]. When \( \lambda = 0 \), the density \( \rho_w \) has two equal maxima and at the transition both maxima exceed one. It is then seen that \( \lim_{A \to A_c} \rho_w \) is in fact the derivative of the free energy in the strong phase for the case \( \lambda = 0 \) studied in [18]. So the difference between \( F'_s(A) \) and \( F'_w(A) \) in the limit \( \lambda \to 0 \), is half what obtained in [18]:

\[
\lim_{\lambda \to 0} [F'_s(A) - F'_w(A)] = \frac{1}{54 A_c(0)} \left( \frac{A - A_c}{A_c} \right)^2 + O \left( \left( \frac{A - A_c}{A_c} \right)^3 \right).
\] (46)

4 The second transition

As it was pointed out in the previous section, \( \rho_w \) has two maxima which are equal when \( \lambda = 0 \). If \( \lambda \) is not zero, then one of these becomes larger and the absolute maximum of \( \rho_w \). The transition point is where this maximum is going to exceed one. After this transition, the classical density is \( \rho_s \), for which there is a region where the density is equal to one. However, in the region around the point the other maximum of \( \rho_w \) occurs, the density \( \rho_s \) continues to increase and there may be some point where this second maximum exceeds one as well. Here the second transition occurs. For small values of \( \lambda \) it is easy to find this point. Once again the result of [27] is used. There it is proved that the difference between the \( \rho_s(z) \) and \( \rho_w(z) \) vanishes faster than \( (A - A_c) \), provided the distance of \( z \) from the region where \( \rho_s \) is one is large compared to the width of the region. This criterion is satisfied for \( z \) around the point where the second maximum of \( \rho_w \) occurs. So,

\[
\rho_s(z) = \rho_w(z) + o(1) \left( A - A_c \right), \quad z \approx z_m,
\] (47)

where \( z_m \) is the point where the second maximum of \( \rho_w \) occurs. This shows that up to leading order, the second transition occurs where the second maximum of \( \rho_w \) approaches one. So,

\[
A_{c2} = A_c(0) \left[ 1 + \frac{A_c(0)}{4 \pi} |\lambda|^3 \right] + \cdots
\] (48)
The phase picture of the system for small $\lambda$ is now complete. There are three phases: weak, strong, and stronger.

- **The weak phase.** Here one has

  \[ A < A_c(0) \left[ 1 - \frac{A_c(0)}{4\pi |\lambda|^3} \right], \quad (49) \]

  the classical density is $\rho_w$, and the classical density is everywhere less than one.

- **The strong phase.** Here one has

  \[ A_c(0) \left[ 1 - \frac{A_c(0)}{4\pi |\lambda|^3} \right] < A < A_c(0) \left[ 1 + \frac{A_c(0)}{4\pi |\lambda|^3} \right], \quad (50) \]

  the classical density is $\rho_s$, and there is one interval where the classical density is equal to one. For fixed $\lambda$, the area interval corresponding to this phase is seen to be proportional to $\lambda^3$. This phase consists of two parts. For positive (negative) $\lambda$ the region where $\rho_{\text{cl}}$ is one is around $z_-$ ($z_+$).

- **The stronger phase.** Here one has

  \[ A_c(0) \left[ 1 + \frac{A_c(0)}{4\pi |\lambda|^3} \right] < A, \quad (51) \]

  the classical density is $\rho_{s^2}$, and there are two intervals where the classical density is equal to one.

It is seen that if $\lambda$ vanishes, the strong phase disappears and only two phases remain, which agrees with the result of [18].

## 5 Concluding remarks

A gYM$_2$ with quartic and cubic couplings was studied. The effect of the cubic coupling on the classical density and the free energy was investigated. The quantitative changes in the weak phase were determined for small cubic couplings. The strong phases of the system were also investigated. It was seen that for small but nonvanishing cubic couplings there are two transitions, compared to a single transition if there is no cubic coupling. Both of these transitions are third order (in area). It was also seen that for small but nonvanishing cubic couplings, the jump in the third derivative of the free energy in each of these transitions is half the jump corresponding to the single transition which occurs if there is no cubic coupling.

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