Longitudinal dielectric permeability into quantum non-degenerate and maxwellian plasma with frequency of collisions proportional to the module of a wave vector

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Abstract

Formulas for the longitudinal dielectric permeability in quantum non-degenerate and maxwellian collisional plasma with the frequency of collisions proportional to the module of the wave vector, in approach Мермин, are received. Equation of Shrödinger—Boltzmann with integral of collisions relaxation type in Mermin’s approach is applied.

It is spent numerical and graphic comparison of the real and imaginary parts of dielectric function of non-degenerate and maxwellian collisional quantum plasma with a constant and a variable frequencies of collisions. It is shown, that the longitudinal dielectric function weakly depends on a wave vector.

Key words: Klimontovich, Silin, Lindhard, Mermin, quantum collisional plasma, conductance, non-degenerate and maxwellian plasmas.

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1. Introduction

In Klimontovich and Silin’s work [1] expression for longitudinal and transverse dielectric permeability of quantum collisionless plasmas has been received.

Then in Lindhard’s work [2] expressions has been received also for the same characteristics of quantum collisionless plasma.

By Kliewer and Fuchs [3] it has been shown, that direct generalisation of formulas of Lindhard on a case of collisionless plasmas, is incorrectly. This lack for the longitudinal dielectric permeability has been eliminated in work

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of Mermin [4] for collisional plasmas. In this work of Mermin [4] on the basis of the analysis of a nonequilibrium matrix density in $\tau$-approach expression for longitudinal dielectric permeability of quantum collisional plasmas in case of constant frequency of collisions of particles of plasma has been announced.

For collisional plasmas correct formulas longitudinal and transverse electric conductivity and dielectric permeability are received accordingly in works [5] and [6]. In these works kinetic Wigner—Vlasov—Boltzmann equation in relaxation approximation in coordinate space was used.

In work [7] the formula for the transverse electric conductivity of quantum collisional plasmas with use of the kinetic Shrödinger—Boltzmann equation in Mermin’s approach (in space of momentum) has been deduced.

In work [8] the formula for the longitudinal dielectric permeability of quantum collisional plasmas with use of the kinetic Shrödinger—Boltzmann equation in approach of Mermin (in space of momentum) with any variable frequency of collisions depending from wave vector has been deduced.

In the present work on the basis of results from our previous work [8] formulas for longitudinal dielectric permeability in quantum collisional plasma with frequency of collisions, proportional to the module of a wave vector are received. The modelling is thus used Shrödinger—Boltzmann equation in relaxation approximation.

In our work [9] formulas for longitudinal and transverse electric conductivity in the classical collisional gaseous (maxwellian) plasma with frequency of collisions of plasma particles proportional to the module particles velocity have been deduced.

Research of skin-effect in classical collisional gas plasma with frequency of collisions proportional to the module particles velocity has been carried out in work [10].

Let’s notice, that interest to research of the phenomena in quantum plasma grows in last years [11] – [24].

1. Longitudinal dielectric function of quantum collisional plasma with variable collisional frequency

In work [5] longitudinal dielectric function of the quantum collisional
plasmas with frequency of collisions, proportional to the module of a wave vector has been received

\[ \varepsilon_l(q, \omega, \nu) = 1 + \frac{4\pi e^2}{q^2} \left[ B(q, \omega + i\nu) + ib_\nu(q, \omega + i\nu) \frac{b(q, 0) - b(q, \omega + i\nu)}{\omega b(q, 0) + i b_{\omega, \nu}(q, \omega + i\nu)} \right]. \quad (1.1) \]

In the formula (1.1) \( e \) is the electron charge, \( q \) is the wave vector, \( \omega \) is the frequency of oscillations of an electromagnetic field, \( \nu(k) \) is the frequency of collisions of particles of plasma,

\[ \nu = \nu(k, q) = \nu(k + \frac{q}{2}, k - \frac{q}{2}) = \frac{\nu(k + \frac{q}{2}) + \nu(k - \frac{q}{2})}{2}, \quad (1.2) \]

\[ B(q, \omega + i\nu) = \int \frac{d^3k}{4\pi^3} \left( f_{k+q/2} - f_{k-q/2} \right) \Xi(\omega + i\nu(k + q/2, k - q/2)), \quad (1.3) \]

\[ b(q, \omega + i\nu) = \int \frac{d^3k}{4\pi^3} \left( f_{k+q/2} - f_{k-q/2} \right) \Xi(\omega + i\nu(k + q/2, k - q/2)) \times \]

\[ \times \frac{\nu(k + q/2, k - q/2)}{\omega + i\nu(k + q/2, k - q/2)}, \quad (1.4) \]

\[ b(q, 0) = \int \frac{d^3k}{4\pi^3} \left( f_{k+q/2} - f_{k-q/2} \right) \Xi(0) \frac{\nu(k + q/2, k - q/2)}{\omega + i\nu(k + q/2, k - q/2)}, \quad (1.5) \]

\[ b_{\nu}(q, \omega + i\nu) = \int \frac{d^3k}{4\pi^3} \left( f_{k+q/2} - f_{k-q/2} \right) \Xi(\omega + i\nu(k + q/2, k - q/2)) \times \]

\[ \times \nu(k + q/2, k - q/2), \quad (1.6) \]

\[ b_{\omega, \nu}(q, \omega + i\nu) = \int \frac{d^3k}{4\pi^3} \left( f_{k+q/2} - f_{k-q/2} \right) \Xi(\omega + i\nu(k + q/2, k - q/2)) \times \]
\[ \times \frac{\tilde{\nu}^2(k + q/2, k - q/2)}{\omega + i\tilde{\nu}(k + q/2, k - q/2)}, \]  

(1.7)

In integrals (1.3) – (1.7) the following designations are accepted

\[ \Xi(\omega + i\tilde{\nu}(k + q/2, k - q/2)) = \]

\[ = \frac{1}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2} + \hbar[\omega + i\tilde{\nu}(k + q/2, k - q/2)]}; \]

\[ f_k = \frac{1}{1 + \exp \left( \frac{\varepsilon_k - \mu}{k_B T} \right)}, \]

\[ \varepsilon_{k\pm q/2} = \frac{\hbar^2}{2m} \left( k \pm \frac{q}{2} \right)^2. \]

Here \( m \) is the electron mass, \( k_B \) is the Boltzmann constant, \( \mu \) is the chemical potential of molecules of gas, \( \hbar \) is the Planck’s constant.

Let’s show, that at \( \nu(k) = \nu = \text{const} \), i.e. at a constant collisional frequency the formula (1.1) passes in the known Mermin’s formula [4]

\[ \varepsilon_i^{\text{Mermin}} = 1 + \frac{4\pi e^2}{q^2} \frac{(\omega + i\bar{\nu})B(q, \omega + i\nu)B(q, 0)}{\omega B(q, 0) + i\nu B(q, \omega + i\nu)}. \]  

(1.8)

In (1.8) the following designations are used

\[ B(q, \omega + i\nu) = \int \frac{d^3k}{4\pi^3} (f_{k+q/2} - f_{k-q/2}) \Xi(\omega + i\nu), \]

(1.9)

\[ B(q, 0) = \int \frac{d^3k}{4\pi^3} (f_{k+q/2} - f_{k-q/2}) \Xi(0), \]

\[ \Xi(\omega + i\nu) = \frac{1}{\varepsilon_{k-q/2} - \varepsilon_{k+q/2} + \hbar(\omega + i\nu)}. \]

Let’s notice, that at \( \nu(k) \equiv \nu, \tilde{\nu}(k, q) \equiv \nu \), and we receive following equalities

\[ B(q, \omega + i\tilde{\nu}) \equiv B(q, \omega + i\nu), \]

\[ b(q, \omega + i\tilde{\nu}) = \frac{\nu}{\omega + i\nu} B(q, \omega + i\nu); \]

\[ b(q, 0) = \frac{\nu}{\omega + i\nu} B(q, 0), \]
\[
b_{\nu}(q, \omega + i\nu) = \nu B(q, \omega + i\nu),
\]
\[
b_{\omega,\nu}(q, \omega + i\nu) = \frac{\nu^2}{\omega + i\nu} B(q, \omega + i\nu).
\]

It is as a result received, that
\[
\epsilon_l(q, \omega, \nu) = 1 + \frac{4\pi e^2}{\omega + i\nu} \left[ 1 + \frac{B(q, 0) - B(q, \omega + i\nu)}{\omega B(q, 0) + i\nu B(q, \omega + i\nu)} \right] \equiv \epsilon_{\text{Mermin}}^l(q, \omega, \nu).
\]

Each of integrals (1.3) – (1.7) we will break into a difference of two integrals. In each of two integrals it is realizable the obvious linear replacement of variables. It is as a result received, that
\[
B(q, \omega + i\nu) = \int \frac{d^3k}{4\pi^3} f_k \left[ \Xi(\omega + i\nu(k, k - q)) - \Xi(\omega + i\nu(k + q, k)) \right],
\]
\[
b(q, \omega + i\nu) = \int \frac{d^3k}{4\pi^3} f_k \left[ \frac{\nu(k, k - q)}{\omega + i\nu(k, k - q)} \Xi(\omega + i\nu(k, k - q)) - \frac{\nu(k + q, k)}{\omega + i\nu(k + q, k)} \Xi(\omega + i\nu(k + q, k)) \right],
\]
\[
b(q, 0) = \int \frac{d^3k}{4\pi^3} f_k \left[ \frac{\nu(k, k - q)}{\omega + i\nu(k, k - q)} \Xi(\omega + i\nu(k, k - q)) - \frac{\nu(k + q, k)}{\omega + i\nu(k + q, k)} \Xi(\omega + i\nu(k + q, k)) \right],
\]
\[
b_{\nu}(q, \omega + i\nu) = \int \frac{d^3k}{4\pi^3} f_k \left[ \frac{\nu^2(k, k - q)}{\omega + i\nu(k, k - q)} \Xi(\omega + i\nu(k, k - q)) - \frac{\nu^2(k + q, k)}{\omega + i\nu(k + q, k)} \Xi(\omega + i\nu(k + q, k)) \right].
\]
In integrals (1.10) – (1.14) following designations are accepted
\[ \bar{\nu}(k, k - q) = \frac{\nu(k) + \nu(k - q)}{2}, \]
\[ \bar{\nu}(k + q, k) = \frac{\nu(k + q) + \nu(k)}{2}, \]
\[ \Xi(\omega + i\bar{\nu}(k, k - q)) = \frac{1}{\varepsilon_{k-q} - \varepsilon_k + \hbar[\omega + i\bar{\nu}(k, k - q)]}, \]
\[ \Xi(\omega + i\bar{\nu}(k + q, k)) = \frac{1}{\varepsilon_k - \varepsilon_{k+q} + \hbar[\omega + i\bar{\nu}(k + q, k)]}. \]

2. Longitudinal dielectric function of the quantum collisional non-degenerate plasmas with frequency of collisions, proportional to the module of a wave vector

Let’s consider the frequency of collisions proportional to the momentum module, or, that all the same, to the module of a wave vector:
\[ \nu(k) = \nu_0 |k|. \]

Then
\[ \bar{\nu}(k_1, k_2) = \frac{\nu(k_1) + \nu(k_2)}{2} = \frac{\nu_0}{2} (|k_1| + |k_2|) \]
and
\[ \bar{\nu}(k, q) = \bar{\nu}(k + \frac{q}{2}, k - \frac{q}{2}) = \frac{\nu_0}{2} \left(|k + \frac{q}{2}| + |k - \frac{q}{2}|\right). \]

The quantity \( \nu_0 \) we take in the form \( \nu_0 = \frac{\nu}{k_T} \), where \( k_T \) is the thermal wave number, \( k_T = \frac{mv_T}{\hbar} \), \( \hbar \) is the Planck’s constant, \( v_T \) is the thermal electron velocity. Now
\[ \nu(k) = \frac{\nu}{k_T} |k|. \quad (2.1) \]

Let’s notice, that at \( k = k_T \): \( \nu(k_T) = \nu. \) So, further in formulas (1.1) – (1.7) frequency of collisions according to (2.1) is equal:
\[ \bar{\nu}(k, q) = \frac{\nu}{2k_T} \left(|k + \frac{q}{2}| + |k - \frac{q}{2}|\right). \quad (2.2) \]
Instead of a vector \( \mathbf{k} \) we will enter the new dimensionless wave vector of integration

\[
\mathbf{K} = \frac{\mathbf{k}}{k_T}, \quad d^3k = k_T^3 d^3 K.
\]

Let’s enter also a new wave vector

\[
\mathbf{Q} = \frac{\mathbf{q}}{k_T}.
\]

At the specified replacement of variables we have

\[
f_k = \frac{1}{1 + e^{K^2 - \alpha}} = f_K.
\]

According to the specified replacement of variables further it is received

\[
\tilde{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q}) = \frac{\nu}{2} \left( |\mathbf{K}| + |\mathbf{K} - \mathbf{Q}| \right),
\]

\[
\tilde{\nu}(\mathbf{k} + \mathbf{q}, \mathbf{k}) = \frac{\nu}{2} \left( |\mathbf{K} + \mathbf{Q}| + |\mathbf{K}| \right),
\]

\[
\mathcal{E}_{\mathbf{k}-\mathbf{q}} - \mathcal{E}_{\mathbf{k}} + \hbar[\omega + i\tilde{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})] = \\
= \frac{\hbar^2}{2m} \left[ (\mathbf{k} - \mathbf{q}) - \mathbf{k}^2 \right] + \hbar[\omega + i\tilde{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})] = \\
= -2\mathcal{E}_T Q \left( K_x - \frac{Q}{2} \right) + \hbar[\omega + i\tilde{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})] = \\
= -2\mathcal{E}_T Q \left( K_x - \frac{Q}{2} - \frac{z^-}{Q} \right).
\]

Here

\[
\mathbf{Q} = Q(1, 0, 0), \quad z^- = x + iy\rho^-, \quad x = \frac{\omega}{k_T v_T}, \quad y = \frac{\nu}{k_T v_T},
\]

\[
\rho^- = \frac{1}{2} \left( |\mathbf{K}| + |\mathbf{K} - \mathbf{Q}| \right) = \\
= \frac{1}{2} \left[ \sqrt{K_x^2 + K_y^2 + K_z^2} + \sqrt{(K_x - Q)^2 + K_y^2 + K_z^2} \right].
\]

Similarly we receive, that

\[
\mathcal{E}_{\mathbf{k}} - \mathcal{E}_{\mathbf{k}+\mathbf{q}} + \hbar[\omega + i\tilde{\nu}(\mathbf{k}, \mathbf{k} - \mathbf{q})] = \\
= -2\mathcal{E}_T Q \left( K_x + \frac{Q}{2} - \frac{z^+}{Q} \right), \quad z^+ = x + iy\rho^+,
\]
\[
\rho^+ = \frac{1}{2}(|K| + |K + Q|) = \frac{1}{2}\sqrt{K_x^2 + K_y^2 + K_z^2 + (K_x + Q)^2 + K_y^2 + K_z^2}.
\]

Let’s pass to new variables in integrals (1.10) – (1.14). We receive following equalities. For integral (1.10) it is had

\[
B(q, \omega + i\bar{\nu}) = -\frac{k_T^3}{8\pi^3E_TQ}B(Q, z^\pm),
\]

where

\[
B(Q, z^\pm) = \int f_K \left[ \frac{1}{K_x - Q/2 - z^-/Q} - \frac{1}{K_x + Q/2 - z^+/Q} \right] d^3K.
\]

For integral (1.11) it is received

\[
b(q, \omega + i\bar{\nu}) = -\frac{yk_T^3}{8\pi^3E_TQ}b(Q, z^\pm),
\]

where

\[
b(Q, z^\pm) = \int f_K \left[ \frac{\rho^-}{z^-(K_x - Q/2 - z^-/Q)} - \frac{\rho^+}{z^+(K_x + Q/2 - z^+/Q)} \right] d^3K.
\]

For integral (1.12) it is received

\[
b(q, 0) = -\frac{yk_T^3}{8\pi^3E_TQ}b(Q, 0^\pm),
\]

where

\[
b(Q, 0^\pm) = \int f_K \left[ \frac{\rho^-}{z^-(K_x - Q/2)} - \frac{\rho^+}{z^+(K_x + Q/2)} \right] d^3K.
\]

For integral (1.13) it is received

\[
b_\nu(q, \omega + i\bar{\nu}) = -\frac{yk_T^3\nu_T}{8\pi^3E_TQ}b_\nu(Q, z^\pm),
\]

where

\[
b_\nu(Q, z^\pm) = \int f_K \left[ \frac{\rho^-}{K_x - Q/2 - z^-/Q} - \frac{\rho^+}{K_x + Q/2 - z^+/Q} \right] d^3K.
\]

At last, for integral (1.14) it is similarly received

\[
b_{\omega,\nu}(q, \omega + i\bar{\nu}) = -\frac{y^2k_T^4\nu_T}{8\pi^3E_TQ}b_{\omega,\nu}(Q, z^\pm),
\]
where
\[ b_{\omega,\nu}(Q, z^\pm) = \int f_K \left[ \frac{\rho^{-2}}{z^{-}(K_x - Q/2 - z^{-}/Q)} - \frac{\rho^{+2}}{z^{+}(K_x + Q/2 - z^{+}/Q)} \right] d^3 K. \]

Let’s substitute the received equalities in the formula (1.1). We receive the expression for longitudinal dielectric function
\[ \varepsilon_l(Q, x, y) = 1 - \frac{3x_p^2}{4\pi Q^2} B(Q, z^\pm) + iy b_{\nu}(Q, z^\pm) \left[ \frac{b(Q, 0^\pm) - b(Q, z^\pm)}{xb(Q, 0^\pm) + iy b_{\omega,\nu}(Q, z^\pm)} \right]. \]  

(2.3)

Here \( x_p \) is the dimensionless plasma (Langmuir) frequency,
\[ x_p = \frac{\omega_p}{k_T v_T}, \quad \omega_p^2 = \frac{4\pi^2 e N}{m}, \]
\( \omega_p \) is the dimension plasma (Langmuir) frequency.

Let’s notice, that in case of constant frequency of electron collisions the quantity \( \rho^{\pm} \) passes in unit. Then
\[ B(Q, z^\pm) = QB(Q, z), \quad b(Q, 0) = \frac{Q}{z} B(Q, 0), \]
\[ b_{\nu}(Q, z^\pm) = QB(Q, z), \quad b_{\omega,\nu}(Q, z^\pm) = \frac{Q}{z} B(Q, z), \]

where
\[ B(Q, z) = \int \frac{f_K d^3 K}{(K_x - z/Q)^2 - (Q/2)^2}. \]

Substituting these equalities in (2.3), we receive expression of dielectric function for quantum non-degenerate collisional plasmas with constant frequency of collisions
\[ \varepsilon_l(Q, x, y) = 1 - \frac{3x_p^2}{4\pi Q^2} B(Q, z) \left[ 1 + iy \frac{B(Q, 0) - B(Q, z)}{xB(Q, 0) + iy B(Q, z)} \right]. \]

Let’s result the formula (2.3) in the calculation form. For this purpose in the plane \((K_y, K_z)\) we will pass to polar coordinates
\[ K_y^2 + K_z^2 = r^2, \quad dK_y dK_z = r dr d\varphi. \]

Then
\[ \varepsilon_l(Q, x, y) = 1- \]
\[-\frac{3x^2}{2Q^3} \left[ D(Q, z^\pm) + iyd_{\rho}(Q, z^\pm) \frac{d(Q, 0) - d(Q, z^\pm)}{xd(Q, 0) + iyd_{\omega, \rho}(Q, z^\pm)} \right]. \quad (2.4)\]

Here

\[
D(Q, z^\pm) = \int_{-\infty}^{\infty} dK_x \int_0^{\infty} \left( \frac{1}{K_x - Q/2 - z^-/Q} - \frac{1}{K_x + Q/2 - z^+/Q} \right) f_F(K_x, r, \alpha) r dr,
\]

\[
z^- = x + iy\rho^- , \quad \rho^- = \frac{1}{2} \left( \sqrt{(K_x - Q)^2 + r^2} + \sqrt{K_x^2 + r^2} \right),
\]

\[
z^+ = x + iy\rho^+ , \quad \rho^+ = \frac{1}{2} \left( \sqrt{(K_x + Q)^2 + r^2} + \sqrt{K_x^2 + r^2} \right),
\]

\[
f_F(K_x, r, \alpha) = \frac{1}{1 + \exp \left( K_x^2 + r^2 - \alpha \right)}.
\]

Besides,

\[
d(Q, z^\pm) = \int_{-\infty}^{\infty} dK_x \int_0^{\infty} \left( \frac{\rho^-}{z^-(K_x - Q/2 - z^-/Q)} - \frac{\rho^+}{z^+(K_x + Q/2 - z^+/Q)} \right) f_F(K_x, r, \alpha) r dr,
\]

\[
d(Q, 0) = \int_{-\infty}^{\infty} dK_x \int_0^{\infty} \left[ \frac{\rho^-}{(x + iy\rho^-)(K_x - Q/2)} - \frac{\rho^+}{(x + iy\rho^+)(K_x + Q/2)} \right] f_F(K_x, r, \alpha) r dr,
\]

\[
d_{\rho}(Q, z^\pm) = \int_{-\infty}^{\infty} dK_x \int_0^{\infty} \left( \frac{\rho^-}{K_x - Q/2 - z^-/Q} - \frac{\rho^+}{K_x + Q/2 - z^+/Q} \right).
\]
\[-\frac{\rho^+}{K_x + Q/2 - z^+/Q}\] \(f_F(K_x, r, \alpha)rdr,

and, at last,

\[d_{\omega, \nu}(Q, z^\pm) = \int_{-\infty}^{\infty} dK_x \int_{0}^{\infty} \left(\frac{\rho^{-2}}{z^-(K_x - Q/2 - z^-/Q)} - \frac{\rho^{+2}}{z^+(K_x + Q/2 - z^+/Q)}\right) f_F(K_x, r, \alpha)rdr.

For comparison we take the formula for longitudinal dielectric functions in case of constant frequency of collisions of plasma particles \(\nu(k) = \nu = \text{const} :\)

\[\varepsilon_l(Q, x, y) = 1 - \frac{3x_p^2}{4\pi Q^2} \frac{(x + iy)B(Q, z)B(Q, 0)}{xB(Q, 0) + iyB(Q, z)}.

Here

\[B(Q, z) = \int \frac{f_K d^3K}{(K_x - z/Q)^2 - (Q/2)^2} = \pi \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - K_x^2})dK_x}{(K_x - z/Q)^2 - (Q/2)^2},\]

\[B(Q, 0) = \pi \int_{-\infty}^{\infty} \frac{\ln(1 + e^{\alpha - K_x^2})dK_x}{K_x^2 - (Q/2)^2}, \quad z = x + iy.

3. Quantum maxwellian collisinal plasmas

In case of quantum maxwellian plasmas \(f_K = e^{-K^2}\). Hence, longitudinal dielectric function it is calculated again under the formula (2.4) in which now are accepted following designations

\[D(Q, z^\pm) = \int_{-\infty}^{\infty} e^{-K_x^2}dK_x \int_{0}^{\infty} \left(\frac{1}{K_x - Q/2 - z^-/Q} - \frac{1}{K_x + Q/2 - z^+/Q}\right)e^{-r^2}rdr,

\[z^- = x + iy\rho^- , \quad \rho^- = \frac{1}{2}\left(\sqrt{(K_x - Q)^2 + r^2} + \sqrt{K_x^2 + r^2}\right),\]
\[ z^+ = x + iy \rho^+, \quad \rho^+ = \frac{1}{2} \left( \sqrt{(K_x + Q)^2 + r^2} + \sqrt{K_x^2 + r^2} \right). \]

Besides,

\[
d(Q, z^\pm) = \int_{-\infty}^{\infty} e^{-K_x^2} dK_x \int_0^\infty \left( \frac{\rho^-}{z^-(K_x - Q/2 - z^-/Q)} - \frac{\rho^+}{z^+(K_x + Q/2 - z^+/Q)} \right) e^{-r^2} r dr,
\]

\[
d(Q, 0) = \int_{-\infty}^{\infty} e^{-K_x^2} dK_x \int_0^\infty \left[ \frac{\rho^-}{(x + iy \rho^-)(K_x - Q/2)} - \frac{\rho^+}{(x + iy \rho^+)(K_x + Q/2)} \right] e^{-r^2} r dr,
\]

\[
d_\rho(Q, z^\pm) = \int_{-\infty}^{\infty} e^{-K_x^2} dK_x \int_0^\infty \left( \frac{\rho^-}{K_x - Q/2 - z^-/Q} - \frac{\rho^+}{K_x + Q/2 - z^+/Q} \right) f_F(K_x, r, \alpha) e^{-r^2} r dr,
\]

and, finally,

\[
d_{\omega, \rho}(Q, z^\pm) = \int_{-\infty}^{\infty} e^{-K_x^2} dK_x \int_0^\infty \left( \frac{\rho^{-2}}{z^-(K_x - Q/2 - z^-/Q)} - \frac{\rho^{+2}}{z^+(K_x + Q/2 - z^+/Q)} \right) e^{-r^2} r dr.
\]

For comparison we take the formula for the longitudinal dielectric function in case of constant frequency of collisions of plasma particles \( \nu(k) = \nu = \text{const} \):

\[
\varepsilon_l(Q, x, y) = 1 - \frac{3x_p^2}{4\pi Q^2} \frac{(x + iy)B(Q, z)B(Q, 0)}{xB(Q, 0) + iyB(Q, z)}.
\]

Here

\[
B(Q, z) = \int \frac{f_K d^3K}{(K_x - z/Q)^2 - (Q/2)^2} = \pi \int_{-\infty}^{\infty} \frac{e^{-K_x^2} dK_x}{(K_x - z/Q)^2 - (Q/2)^2},
\]
\[ B(Q, 0) = \pi \int_{-\infty}^{\infty} \frac{e^{-K^2}dK_x}{K^2 - (Q/2)^2}, \quad z = x + iy. \]

On Figs. 1-4 comparison of the real and imaginary parts of dielectric function depending on quantity of the dimensionless wave vector \( Q \) (Figs. 1,2) and depending on the dimensionless quantity of frequency of an electromagnetic field \( x \) (Figs. 3,4) is carry out. Thus curves 1 correspond to values of frequency the collisions, proportional to the module of a wave vector; curves 2 correspond to constant frequency of collisions of particles of plasma. Both curves are constructed at \( y = 0.01 \). All graphics answer to non-degenerate quantum plasma. The case of maxwellian plasmas is considered on Figs. 5-8.

Everywhere more low \( x_p = 1 \), and value of chemical potential equally: \( \alpha = 0 \).

On Figs. 9-12 the case of maxwellian plasmas is considered. On Figs. 9 and 10 comparison of the real parts is considered at \( x = 1, 0 \leq Q \leq 3 \) (fig. 9) and imaginary parts of dielectric function at \( Q = 1, 0 \leq x \leq 3 \) (fig. 10) of quantum collisional plasmas with the frequency of collisions proportional to the module of a wave vector. Curves 1,2,3 answer according to values \( y = 0.1, 0.05, 0.001 \).

On Figs. 11 and 12 comparison is carry out of a relative deviation of real (curves 1) and imaginary parts (curves 2) of dielectric function from the present work (with frequency of collisions, proportional to the module of a wave vector) with the corresponding dielectric Mermin’s function (with constant collision frequency) at the same parametres, and quantity \( y = \frac{\nu}{k_F v_F} = 0.01 \) is the same. Curves 1 on Figs. 11 and 12 are defined by function

\[ O_r(Q, x, y) = \frac{\text{Re} \varepsilon^\text{Mermin}_{\text{r}}(Q, x, y) - \text{Re} \varepsilon_l(Q, x, y)}{\text{Re} \varepsilon^\text{Mermin}_{\text{r}}(Q, x, y)}, \]

and curves 2 are defined by function

\[ O_i(Q, x, y) = \frac{\text{Im} \varepsilon^\text{Mermin}_{\text{i}}(Q, x, y) - \text{Im} \varepsilon_l(Q, x, y)}{\text{Im} \varepsilon^\text{Mermin}_{\text{i}}(Q, x, y)}. \]

5. Conclusions

In the present work formulas for the longitudinal dielectric permeability (dielectric function) in the quantum collisional non-degenerate plasma (with
any degree of degeneration) and maxwellian plasma are deduced. Frequency of collisions plasma particles it is supposed proportional to the module of a wave vector (or an momentum plasma particles). Graphic research of behaviour the real and imaginary parts of the found dielectric functions is carried out. Comparison of the real and imaginary parts is spent also the found dielectric function with the corresponding characteristics of dielectric function with constant collision frequency.
Fig. 1. Real part of dielectric function, $x = 1, y = 0.01$.

Fig. 2. Imaginary part of dielectric function, $x = 1, y = 0.01$. 
Fig. 3. Real part of dielectric function, $Q = 1, y = 0.01$.

Fig. 4. Imaginary part of dielectric function, $Q = 1, y = 0.01$. 
Fig. 5. Real part of dielectric function, $x = 1, y = 0.01$.

Fig. 6. Imaginary part of dielectric function, $x = 1, y = 0.01$. 
Fig. 7. Real part of dielectric function, $Q = 1, y = 0.01$.

Fig. 8. Imaginary part of dielectric function, $Q = 1, y = 0.01$. 
Fig. 9. Real part of dielectric function, $x = 1$. Maxwellian plasma.

Fig. 10. Imaginary part of dielectric function, $Q = 1$. Maxwellian plasma.
Fig. 11. Relative deviation of the real and imaginary parts of dielectric function. Graphics $O_r(Q, 1, 0.01)$ (curve 1) и $O_i(Q, 1, 0.01)$ (curve 2). Maxwellian plasma.

Fig. 12. Relative deviation of the real and imaginary parts of dielectric function. Graphics $O_r(1, x, 0.01)$ (curve 1) and $O_i(1, x, 0.01)$ (curve 2). Maxwellian plasma.
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