A New Estimation Method of Ensemble Forecast Error in ETKF Assimilation with Nonlinear Observation Operator

Chengcheng Huang¹,²,³, Guocan Wu¹,³, and Xiaogu Zheng⁴

¹College of Global Change and Earth System Science, Beijing Normal University, Beijing, China
²Institute of Urban Meteorology, China Meteorological Administration, Beijing, China
³Joint Center for Global Change Studies, Beijing, China
⁴Key Laboratory of Regional Climate-Environment Research for East Asia, Institute of Atmospheric Physics, Chinese Academy of Sciences, Beijing, China

Abstract

The estimation accuracy of ensemble forecast errors has a key influence on the assimilation results of all ensemble-based schemes. The ensemble transform Kalman filter (ETKF) assimilation scheme can assimilate nonlinear observations without using the adjoint of a dynamical model; however, the initially estimated ensemble forecast errors must be further adjusted. In this paper, the estimation of forecast error is improved using a self-generated analysis and a corresponding iterative procedure is then established for the ETKF with nonlinear observation operators. The improved assimilation scheme is validated using the Lorenz-96 model with a nonlinear and spatially correlated observation system as a test bed. The experiment results demonstrate that the improved ETKF assimilation scheme can effectively reduce the analysis error.

(Citation: Huang, C., G. Wu, and X. Zheng, 2017: A new estimation method of ensemble forecast error in ETKF assimilation with nonlinear observation operator. SOLA, 13, 63–68, doi: 10.2151/sola.2017-012.)

1. Introduction

Data assimilation is a technique used to combine numerical model forecasts and observations to obtain an analysis state that can best estimate the true state (Talagrand 1997). The analysis state can be treated as a weighted average of forecasts and observations, while the corresponding weights are roughly inversely proportional to their covariance matrices. If these matrices are estimated properly, the analysis state can be generated by technically minimizing a cost function. Therefore, the estimation accuracy of error covariance matrix has a key influence on the assimilation results (Reichle 2008).

For ensemble-based data assimilation schemes, the ensemble forecast errors are represented by the perturbed forecast states minus their mean. The ensemble transform Kalman filter (ETKF) is a commonly used method that can address generally nonlinear and correlated observations using a cost function (Bishop and Toth 1999; Bishop et al. 2001; Wang and Bishop 2003). Hunt et al. (2007) further described a practical method for ETKF assimilation with nonlinear observation operators by minimizing a cost function in the weight vector space. However, their emphasis was on the easy application and computational speed of the scheme rather than on reducing the analysis errors. In addition to the low computational cost, the ETKF scheme proposed by Hunt et al. (2007) can assimilate nonlinear, uncorrelated observations (Chen et al. 2009; Miyoshi et al. 2012; Steward et al. 2012).

Despite the aforementioned development of ETKF, the method of estimating forecast error statistics in ETKF is still not perfect, specifically for nonlinear observation operators. The forecast error is estimated in the observation space using a nonlinear observation operator. The initially estimated ensemble forecast errors are perturbed forecast states minus their mean, which requires further inflation (Miyoshi and Kunii 2011; Wu et al. 2014). The multiplicative inflation technique can be used to mitigate filter divergence by inflating the empirical covariance and increasing the robustness of the filter (Luo and Hoteit 2011). However, the commonly used forecast error inflation method is still limited to linear observation operators (Li et al. 2012; Miyoshi 2011; Wang and Bishop 2003; Zheng 2009) or tangent-linear observation operators (Li et al. 2009). For the general nonlinear observation operator, the error of using its tangent-linear operator can be very large. In this case, the second-order least squares (SLS, (Wang and Leblanc 2008)) estimation method can be applied to calculate the inflation factor of ensemble forecast errors avoiding the tangent-linear operator (Wu et al. 2014). The corresponding cost function is the second-order distance between the expectations of the squared innovation and its realization, which can be treated as a way to reduce the mismatch between simulated and real observations (Luo and Hoteit 2013, 2014).

Previous studies have demonstrated that the analysis state can be used to further improve the estimation accuracy of ensemble forecast errors (Wu et al. 2013; Zheng et al. 2013), which can be treated as the multiplicatively inflated sampling error covariance matrix plus an additive inflation matrix, representing a systematic component of model error (Bai and Li 2011; Picollo 2011). However, this self-generated analysis was only effectively validated for the assimilation schemes with linear observation operators. Whether the analysis state can be used to improve the estimation accuracy of the ensemble forecast errors in ETKF with generally nonlinear observation operators is currently unclear.

This paper addresses the aforementioned problem. For the ETKF with general nonlinear observation operators, the SLS inflation method is applied to address the nonlinearity in observation operator and an iterative estimation procedure using the self-generated analysis is proposed to account for model error and sampling error. The new estimation method is validated using the Lorenz-96 model to quantify how assimilation results are improved. The conceptual aspects are shed on to illustrate the main methodology in this study. The performance of the proposed method in realistic data assimilation problems will be discussed in future work.

The remainder of this paper is organized as follows. Section 2 describes the proposed iterative estimation procedure. Section 3 provides the assimilation results. Section 4 presents a discussion and our conclusions are presented in Section 5.

2. Methodology

2.1 ETKF with forecast error inflation

Using the notations similar to those of Ide et al. (1997), a nonlinear discrete-time forecast and observation system is as follows:
where \( i \) stands for time step, \( x_i = [x_i^1, x_i^2, \ldots, x_i^n]^T \) and \( x_{i,j} = [x_{i,j}^1, x_{i,j}^2, \ldots, x_{i,j}^n]^T \) are the \( n \)-dimensional true and analysis state vectors, respectively; \( M_{i,j} \) is a nonlinear forecast operator; \( y_j = [y_j^1, y_j^2, \ldots, y_j^p]^T \) is the \( p \)-dimensional observation vector; \( H_i \) is the nonlinear observation operator; \( \eta \) and \( e \) are the forecast and observation error vectors, which are assumed to be time-un correlated and statistically independent. The ETKF assimilation result is a series of analysis state \( x_i \); that is, a better estimate of the true state \( x_i \), than either model forecasts or observations.

Suppose the perturbed analysis state at previous time step \( x_{i-1} \) has been estimated \((j = 1, \ldots, m \) stands for the ensemble member), the ETKF assimilation scheme with forecast error inflation is as follows:

**Step (1). Forecast step**

The perturbed forecast states at time step \( i \) is calculated by running the forecast model forward

\[
x_i^j = M_{i-1,i}(x_{i-1}^j) + \eta_j,
\]

(3)

Then, \( x_i^j \) and \( y_i^j \) are defined as the ensemble mean of \( x_{i,j} \) and \( H(x_{i,j}) \), respectively (Hunt et al. 2007). The realizations of ensemble forecast errors are initially generated as

\[
X_i^j = (x_{i,j}^1 - x_i^j, x_{i,j}^2 - x_i^j, \ldots, x_{i,j}^n - x_i^j)
\]

(4)

and then inflated to the form \( \sqrt{\lambda}X_i^j \). Following Eq. (2) of Wang and Leblanc (2008), the inflation factor \( \lambda \) is estimated by minimizing the following cost function:

\[
L_i(\lambda) = \text{Tr}[(d_i - C_i(\lambda) - I)(d_i - C_i(\lambda) - I)^T],
\]

(5)

where \( I \) is the \( p \times p \) identity matrix,

\[
d_i = R_i^{-1/2}(y_i - y_i^j)
\]

(6)

is the normalized innovation statistic (Wang and Bishop 2003), and

\[
C_i(\lambda) = \frac{1}{m-1} \sum_{j=1}^m R_i^{-1/2} \left( H_i(x_i^j + \sqrt{\lambda}(x_{i,j}^j - x_i^j)) - y_i^j \right) - y_i^j \left( R_i^{-1/2} \right)^T.
\]

(7)

In fact, the cost function defined by Eq. (5) is the SLS function of the squared innovation statistic (see Appendix A for details). It also suggests approximating \( H_i(x_i^j + \sqrt{\lambda}(x_{i,j}^j - x_i^j)) - y_i^j \) as \( \sqrt{\lambda}H_i(x_{i,j}^j - x_i^j) \) to reduce the computational cost. Then, \( L_i(\lambda) \) is just a polynomial function of \( \sqrt{\lambda} \), which can easily be used to determine the minima \( \lambda_i \).

**Step (2). Analysis step**

The analysis state is estimated as

\[
x_{i,j} = x_i^j + \sqrt{\lambda}X_i^j \hat{w}_i^j,
\]

where \( \hat{w}_i^j \) is a weight vector that is estimated by minimizing the cost function

\[
J_f(w) = \frac{1}{2} (m-1)w^T \hat{w} + \frac{1}{2} y_i^j - H_i(x_i^j + \sqrt{\lambda}X_i^j \hat{w})^T \cdot R_i^{-1/2}(y_i^j - H_i(x_i^j + \sqrt{\lambda}X_i^j \hat{w})).
\]

(9)

The perturbed analysis state is calculated as

\[
x_{i,j}^a = x_{i,j} + \sqrt{\lambda} X_i^j \hat{w}_{i,j}^a,
\]

(10)

where \( W_{i,j}^a \) is the \( j \)-th column vector of the matrix \( W_i = \sqrt{m-1}(J_i(\hat{w}))^{-1/2} \) and \( J_i(\hat{w}) \) is the second-order derivative of \( J_i(w) \) at \( \hat{w}_i \) (see Appendix B for details). Finally, set \( i = i + 1 \) and return to Step (1) at next time step.

### 2.2 Constructing error statistics using analysis state

Because \( x_i \) is the true state, the forecast error should be defined as \( x_{i,j} - x_i \). However, \( x_i \) is not known and is initially estimated as the forecast state \( x_i \) in Eq. (4). Because the analysis state \( x_i \) is derived based on observations and forecasts, it should be a better estimate of \( x_i \) than \( x_i \), especially when the model error is large (Wu et al. 2013; Zheng et al. 2013). Therefore, after calculating the analysis state (Eq. 8) in analysis step (2), substitute the \( x_i \) in Eqs. (4) and (7) with \( x_i \) to obtain

\[
C_i(\lambda) = \frac{1}{m-1} \sum_{j=1}^m \left( R_i^{-1/2}(H_i(x_i^j + \sqrt{\lambda}(x_{i,j}^j - x_i^j)) - H_i(x_i^j)) \right)^T \left( R_i^{-1/2} \right)^T.
\]

(11)

This procedure is repeated until the corresponding cost function (Eq. 5) converges; then, an iterative estimation procedure is constructed (see Fig. 1). The proposed method combines the merit from SLS inflation and iterative procedure, which account for nonlinearity in observation operator, and model error and sampling error, respectively.

Furthermore, Eq. (11) can be approximately expressed as

\[
C_i(\lambda) \approx \frac{\lambda}{m-1} \sum_{j=1}^m \left( R_i^{-1/2}H_i(x_i^j - x_i^j)H_i(x_i^j - x_i^j)^T \right) \left( R_i^{-1/2} \right)^T.
\]

(12)

The first term of Eq. (12) is the multiplicatively inflated forecast error covariance matrix in the observation space, and the second term is a measurement of the departure of the ensemble mean from the analysis, which can be treated as a systematic component of model error ((Piccolo 2011), see Appendix C for details).

### 2.3 Validation statistics

In the following experiments, the “true” state \( x_i \) can be obtained by numerical solution of partial differential equations, and is non-dimensional. Therefore, the root mean square error

**Fig. 1. Flowchart of our proposed assimilation scheme.**
(RMSE) of the analysis state can be used to evaluate the accuracy of the assimilation results. The RMSE at the i-th time step is defined as

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{k=1}^{n} [x_i'(k) - x_i(k)]^2},$$

(13)

where \(x_i'(k)\) and \(x_i(k)\) are the k-th component of the analysis state and true state at the i-th time step, respectively. In principle, the smaller the RMSE is, the better performance of the assimilation scheme.

Another validation statistic is the consistency ratio to diagnose the accuracy of ensemble spread, which is defined as

$$r = \frac{\text{Tr}(d(d^T))/\text{Tr}(C(\lambda) + I)}.$$  

(14)

If the forecast error is correctly estimated, the expectation of \(d(d^T)\) is \(C(\lambda) + I\) (see Appendix A). The closer r is to 1, the better the estimation of forecast error is.

3. Experiments

3.1 The forecast model and observation system

The Lorenz-96 model (Lorenz 1996) is a strongly nonlinear dynamical system with properties relevant to realistic forecast problems; it is governed by

$$\frac{dX}{dt} = (X_{k+1} - X_k)X_{k-1} - X_k + F,$$  

(15)

where \(k = 1, 2, \ldots, 40\), \(X_1 = X_{40}, X_2 = X_{41}, \ldots, X_{40} = X_1\). The three terms on the right-hand side of Eq. (15) can be treated as a nonlinear advective term, a damping term, and an external forcing term, respectively, which can be thought of as an atmospheric quantity (e.g. zonal wind speed) distributed on a latitude circle.

Therefore, it is “atmosphere-like” and used as a test bed to evaluate the performance of assimilation schemes in many studies (Wu et al. 2013; Zheng et al. 2013).

The time step is set as 0.05 non-dimensional units when we generate the numerical solution, which is roughly equivalent to 6 hours in real time, assuming that the characteristic time-scale of the dissipation in the atmosphere is 5 days (Lorenz 1996). The true state is generated using a fourth-order Runge-Kutta time integration scheme (Butcher 2003), and the forcing term is set as \(F = 8\) (Lorenz and Emanuel 1998). The initial condition is \(X_0 = F\) when \(k \neq 20\) and \(X_{40} = X_1 = 1.001F\).

Because model error is inevitable in practical dynamic models, it is reasonable to add model error to the Lorenz-96 model in the assimilation process. Because the forcing term is set as \(F = 8\) when generating the true state, a wide range of model errors are introduced by setting \(F = 4, \ldots, 12\). A larger distance of \(F\) from 8 indicates a larger model error.

The synthetic observation at the i-th time step and k-th model grid point is generated by the following observation equation:

$$y_i'(k) = x_i'(k) \exp \{\alpha x_i'(k)\} + e_i(k),$$

(16)

where \(k = 1, \ldots, p\), and \(e_i = [e_i(1), e_i(2), \ldots, e_i(p)]^T\) is the p-dimensional observation error vector with mean zero and covariance matrix \(R\). The parameter \(\alpha\) is used to indicate the nonlinearity of the observation operator and \(\alpha = 0\) refers to the linear observation operator. In our experimental design, the variables are assumed to be observed in all of the 40 model grids every 4 time steps and the observation errors are set to be spatially correlated. The variance of the observation error on each grid point is \(\sigma_e^2 = 1\), and the covariance between the j-th and k-th grid points is

$$R_j(j, k) = \sigma_e^2 \times 0.5^{|j-k|} \times 0.5^{|j-k|}.$$  

(17)

That is, the correlation coefficient between any two different grid points is proportional their spatial distance. The forecast model is run for 100,000 steps to ensure robust results (Oke et al. 2009; Sakov and Oke 2008) and the ensemble size is selected as 30.

3.2 Experimental results

The iterative process is stopped when the magnitude of the cost function defined by Eq. (5) falls below one thousandth of its initial value. Because each iteration requires the computation of the trace of a \(p \times p\) matrix (Eq. 5), this step is computationally inexpensive. In the following, the performances of assimilation results are evaluated among the proposed ETKF with iterative inflation, SLS inflation and tangent-linear inflation schemes (see Appendix D for details).

For the forcing term \(F\) varying from 4 to 12, the corresponding time-mean analysis RMSEs and consistency ratios of these Lorenz-96 schemes are shown in Fig. 2. When the model error is small, all the methods present similar RMSE values, but differences among different methods increases with increasing model error. However, for the model with large errors (when \(F\) is increasingly distant from 8), the analysis RMSE and consistency ratio of the tangent-linear inflation method are greater than those of the SLS inflation method, indicating that the tangent-linear inflation method enhances computational efficiency, at a cost of losing assimilation accuracy. The analysis RMSE and consistency ratio of the iterative method become progressively smaller than that of the SLS inflation method, indicating that the proposed
iterative method using the analysis state has good performance in further improving the ETKF assimilation precision.

The time-mean analysis RMSEs for the perfect model \((F = 8)\) and a model with large error \((F = 12)\) are listed in Table 1. The time-mean cost function values and consistency ratios are also listed. It indicates that for the perfect model, the time-mean analysis RMSE and cost function are almost indistinguishable. However, for the model with large error (this represents the practical case), the time-mean values of the proposed iterative inflation method are less than those of the SLS inflation method and tangent-linear inflation method. All the estimated consistency ratios are all larger than 1 for the three methods. The value using the iterative method is closer to 1 than the other two methods illustrating that the proposed method can obtain the most accurate estimated forecast error.

For the case \(F = 12\), the mean values of inflation factor using tangent-linear, SLS and iterative methods are 6.91, 7.32 and 3.15, and the standard deviations are 3.37, 3.14 and 2.05, respectively. The inflation factor at the first iteration with the self-generated analysis is about 20% smaller than the inflation factor obtained with the SLS method, indicating that the iterative procedure is effective to further estimate the inflation factor.

4. Discussion

Correctly representing the ensemble forecast errors is crucial for any ensemble-based data assimilation schemes. Traditionally, the forecast state \(x_i\) is used to estimate the true model state \(x_i\) in presenting the ensemble forecast errors (Anderson 2007, 2009). However, it is widely recognized that the initially estimated forecast error should be inflated, owing to model errors and the limited ensemble size. Hence, the ensemble forecast errors are usually adjusted to \(\sqrt{A_{XX}^T}\), and the inflation factor \(\lambda\) can be calculated via SLS estimation. The advantage of SLS inflation compared with the tangent-linear inflation is that the former does not require calculating the tangent-linear form of the nonlinear observation operator.

Actually, the true forecast error is the forecast state minus the true state. However, because the true state is not available in real problems, it is replaced by the mean of the perturbed forecast states in ETKF. Then, the ensemble forecast errors are represented as the perturbed forecast states minus their ensemble mean, which may be not accurate enough, especially for the model with large error. Consequently, these initially estimated ensemble forecast errors will remain far from the truth, regardless of the inflation technique. Therefore, the estimation of ensemble forecast errors can be further improved using analysis state.

To test the influence of model error, the ratio of the reduced RMSE of the ETKF assimilation with iterative inflation to the RMSE of SLS inflation is calculated. As shown in Fig. 3, the ratio is nearly 20% for the model with large error \((F = 12)\), whereas is just approximately 5% for the perfect model \((F = 8)\). It illustrates that the larger the model error is, the greater the magnitude of improvement the ETKF assimilation with iterative inflation.
The iterative method uses the same observation multiple times in each assimilation cycle. This is a potential caveat because it may result in over fitting the observation. The constraint approach can be applied to mitigate this problem (Gharamti et al. 2014; Hamill and Whitaker 2011). For the model with large error, covariance inflation may not be effective enough to improve the assimilation results, and the localization technique can further improve the assimilation quality (Constantinescu et al. 2007; Houtekamer and Mitchell 2001; Miyoshi 2011). Another problem that requires attention is that the computational cost will increase for the proposed iterative method. Actually, the convergence speed of Eq. (5) largely depends on the threshold. At least for the selected threshold in our experiments, the cost function (Eq. 5) converges after approximately three iterations; thus, computation is feasible. For other experiments, especially strongly nonlinear observation operator, the convergence speed may be different. The experimental results obtained in this study can provide a foundation for future assimilation research.

5. Conclusions

In this study, the iterative method of using the analysis state to iteratively construct the ensemble forecast errors in the ETKF with general nonlinear observation operator can indeed reduce the analysis error in our experiments. We presented the methodology and validated it using the Lorenz-96 model to illustrate the feasibility of the proposed approach. In the future, we will further test the method using more idealized experiments and apply it to assimilate real observations into real models.

Acknowledgments

This work was supported by the National Program on Key Basic Research Project of China (Grant No. 2015CB953703) and the National Natural Science Foundation of China (Grant Nos. 91647202 and 41405098).

Edited by: J. Ruiz

References

Anderson, J. L., 2007: An adaptive covariance inflation error correction algorithm for ensemble filters. Tellus A, 59, 210–224.
Anderson, J. L., 2009: Spatially and temporally varying adaptive covariance inflation for ensemble filters. Tellus A, 61A, 72–83.
Bai, Y., and X. Li, 2011: Evolutionary algorithm-based error parameterization methods for data assimilation. Mon. Wea. Rev., 139, 2668–2685.
Bishop, C. H., and Z. Toth, 1999: Ensemble transformation and adaptive observations. J. Atmos. Sci., 56, 1748–1765.
Bishop, C. H., B. J. Etherton, and S. J. Majumdar, 2001: Adaptive sampling with the ensemble transform kalman filter. Mon. Wea. Rev., 129, 420–435.
Butcher, J. C., 2003: Numerical Methods for Ordinary Differential Equations. JohnWiley & Sons, 425 pp.
Chen, Y., D. S. Oliver, and D. Zhang, 2009: Data assimilation for nonlinear problems by ensemble Kalman filter with repa-parameterization. J. Petrol. Sci. Eng., 66, 1–14.
Constantinescu, E. M., A. Sandu, T. Chai, and G. R. Carmichael, 2007: Ensemble-based chemical data assimilation. II: Covariance localization. Quart. J. Roy. Meteor. Soc., 133, 1245–1256.
Gharamti, M. E., A. Kadoura, J. Valstar, S. Sun, and J. Hoteit, 2014: Constraining a compositional flow model with flow-chemical data using an ensemble- based Kalman filter. Water Resour. Res., 50, 2444–2467.
Hamill, T. M., and J. S. Whitaker, 2011: What constrains spread growth in forecasts initialized from ensemble Kalman filters? Mon. Wea. Rev., 139, 117–131.
Houtekamer, P. L., and H. L. Mitchell, 2001: A sequential ensemble Kalman filter for atmospheric data assimilation. Mon. Wea. Rev., 129, 123–137.
Hunt, B. R., E. J. Kostelich, and I. Szunyogh, 2007: Efficient data assimilation for spatiotemporal chaos A local ensemble transform Kalman filter. Physica D, 230, 112–126.
Ide, K., P. Courtier, M. Ghil, and A. C. Lorenz, 1997: Unified notation for data assimilation operational sequential and variational. J. Meteor. Soc. Japan, 75, 181–189.
Li, H., E. Kalnay, and T. Miyoshi, 2009: Simultaneous estimation of covariance inflation and observation errors within an ensemble Kalman filter. Quart. J. Roy. Meteor. Soc., 135, 523–533.
Liang, X., X. Zheng, S. Zhang, G. Wu, Y. Dai, and Y. Li, 2012: Maximum likelihood estimation of inflation factors on error covariance matrices for ensemble Kalman filter assimilation. Quart. J. Roy. Meteor. Soc., 138, 263–273.

Fig. 4. Time-mean values of the analysis RMSE (a) and the constancy ratio (b) as a function of parameter $\alpha$ for different assimilation methods in the Lorenz-96 model with $F=12$: tangent-linear inflation (red), SLS inflation (green), and iterative inflation (blue). The ensemble size is 30.
Lorenz, E. N., 1996: Predictability—a problem partly solved. *Proc. Seminar on Predictability*.

Lorenz, E. N., and K. A. Emanuel, 1998: Optimal sites for supplementary weather observations simulation with a small model. *J. Atmos. Sci.*, 55, 399–414.

Luo, X., and I. Hoteit, 2011: Robust ensemble filtering and its relation to covariance inflation in the ensemble Kalman filter. *Mon. Wea. Rev.*, 139, 3938–3953.

Luo, X., and I. Hoteit, 2013: Covariance inflation in the ensemble kalman filter: A residual nudging perspective and some implications. *Mon. Wea. Rev.*, 141, 3360–3368.

Luo, X., and I. Hoteit, 2014: Ensemble Kalman filtering with residual nudging: An extension to state estimation problems with nonlinear observation operators. *Mon. Wea. Rev.*, 142, 3696–3712.

Miyoshi, T., 2011: The Gaussian approach to adaptive covariance inflation and its implementation with the local ensemble transform Kalman filter. *Mon. Wea. Rev.*, 139, 1519–1534.

Miyoshi, T., and M. Kunii, 2011: The local ensemble transform kalman filter with the Weather Research and Forecasting model: Experiments with real observations. *Pure & Applied Geophysics*, 169, 321–333.

Miyoshi, T., E. Kalnay, and H. Li, 2012: Estimating and including observation-error correlations in data assimilation. *Inverse Problems in Science & Engineering*, 32, 1–12.

Oke, P. R., P. Sakov, and E. Schulz, 2009: A comparison of shelf observation platforms for assimilation in an eddy-resolving ocean model. *Dyn. Atmos. Oceans*, 48, 121–142.

Piccolo, C., 2011: Growth of forecast errors from covariances modeled by 4DVAR and ETKF methods. *Mon. Wea. Rev.*, 139, 1505–1518.

Reichle, R. H., 2008: Data assimilation methods in the Earth sciences. *Adv. Water Res.*, 31, 1411–1418.

Sakov, P., and P. R. Oke, 2008: A deterministic formulation of the ensemble Kalman filter: An alternative to ensemble square root filters. *Tellus A*, 60A, 361–371.

Steward, J. L., I. M. Navon, M. Zupanski, and N. Karmitsa, 2012: Impact of non-smooth observation operators on variational and sequential data assimilation for a limited-area Shallow-Water equation model. *Quart. J. Roy. Meteor. Soc.*, 138, 323–339.

Talagrand, O., 1997: Assimilation of observations, an introduction. *J. Meteor. Soc. Japan*, 75, 191–209.

Wang, L., and A. Leblanc, 2008: Second-order nonlinear least squares estimation. *Annals of the Institute of Statistical Mathematics*, 60, 883–900.

Wang, X., and C. H. Bishop, 2003: A comparison of breeding and ensemble transform Kalman filter ensemble forecast schemes. *J. Atmos. Sci.*, 60, 1140–1158.

Wu, G., X. Zheng, L. Wang, S. Zhang, X. Liang, and Y. Li, 2013: A new structure for error covariance matrices and their adaptive estimation in EnKF assimilation. *Quart. J. Roy. Meteor. Soc.*, 139, 795–804.

Wu, G., X. Yi, X. Zheng, L. Wang, X. Liang, S. Zhang, and X. Zhang, 2014: Improving the ensemble transform Kalman filter using a second-order Taylor approximation of the nonlinear observation operator. *Nonlinear Proc. Geophys.*, 21, 955–970.

Zheng, X., 2009: An adaptive estimation of forecast error statistic for Kalman filtering data assimilation. *Adv. Atmos. Sci.*, 26, 154–160.

Zheng, X., G. Wu, S. Zhang, X. Liang, Y. Dai, and Y. Li, 2013: Using analysis state to construct forecast error covariance matrix in EnKF assimilation. *Adv. Atmos. Sci.*, 30, 1303–1312.

Manuscript received 6 December 2016, accepted 27 February 2017

SOLA: https://www.jstage.jst.go.jp/browse/sola/