Effect of the electric field on a superconducting powder

Keshav N. Shrivastava*
School of Physics, University of Hyderabad, Hyderabad 500 046, India.

Abstract.
We find that in the presence of an electric field there is an attractive intergrain interaction in superconductors which is small. When charge on the ball is permitted to vary with the ball radius, very large balls can be formed. The pairing energy makes the ball compact and hence reduces the size of the ball compared with the classical value. The ball radius depends on the gap of the superconductor due to Josephson tunneling.

Keywords: Electric fields; Josephson interaction
1. Introduction.
Some time ago, it was found that application of an electric field produced a second-order interaction which led to anisotropic resistivity with respect to a change in the sign of the electric field [1]. Hence the change in sign of the applied voltage which can be achieved by reversing the battery, resulted into two different values of the resistivity for two different polarities [2]. The geometry of the experimental configuration for the application of the electric field plays an important role. Therefore, another measurement was carried out by Frey et al [3]. The second-order term in the potential due to the dipole moment induced by the electric field leads to a resistivity which is linear in electric field. At small values of the electric field, $E \leq 0.3E_{BD}$, this prediction [4] is in accord with the experimental measurement. Here $E_{BD}$ is the break-down field. The break-down voltage is $V_{BD} = E_{BD}d$ with $d$ as the width of the film along the $c$ direction. At larger values of the applied voltage, $0.16 < E/E_{BD} < 0.89$, the resistivity is found [4] to depend on the square root of the applied electric field, $R \propto E^{1/2}$. It is found [4,5] that the density of states depend on the dimensionality so that one-dimensional conduction normal to the surface of the film produces the resistivity proportional to the square root of the electric field. The same result is obtained by more elementary considerations of the Thomas-Fermi screening length.

In the present paper, we show that there is an intergrain interaction due to the electric dipole moment which is attractive so that the grains tend to aggregate to make a ball. The Josephson tunneling plays an important role to determine the superconducting surface tension. We find that pairing interaction helps in making a compact ball from superconducting grains. The binding of Cooper pairs here, is the same as in the B.C.S. theory except that large grains bind into a big ball.

2. Theory

We assume that the electron coordinates are $r_i$ and the charge is $-e$ so that the dipole moment of one ion is $p = -e r_i$. Since different electrons are located at different coordinates in an ion, we replace the dipole moment by the average value, $<p>$. The potential energy
of a grain due to dipole moment of all of the ions within a grain is given by,

\[ V_g^{(1)} = \sum_{i=1}^{N} V^{(1)} = -\sum_{i=1}^{N} E < p_i > \]

where the sum is over all ions within a grain and \( N \) is the number of ions within the grain. In the case of two grains,

\[ \sum_{i} V^{(1)} = -\sum_{i=1}^{N(1)} E < p_{i1} > - \sum_{j=1}^{N(2)} E < p_{j2} > \]

is the potential energy where \( N(1) \) is the number of electrons within the first grain and \( N(2) \) is the number of electrons within the second grain. The states of the grains are given by \( |0_2, 0_1>, |n_2, n_1> \), etc. the second-order energy of the system of two grains is,

\[ V^{(2)} = \sum_{i} \sum_{j} \left[ \frac{< n_1, n_2 | E p_{i1} | 0_1, n_2 > < 0_1, n_2 | E p_{j2} | 0_2, 0_1 >}{E(0_1, n_2) - E(0_2, 0_1)} \right] \]

with one more term in which the subscripts 1 and 2 are interchanged. This interaction varies as \( E^2 \) and attracts two grains. Since \( E \) is the electric field, it can be expanded into photon operators. Then it means that one grain emits a photon which is absorbed by other and vice versa. This interaction travels with the speed of light and grains become attractive except that it is small compared with the electromagnetic energy \( E^2/8\pi \).

The attraction between grains helps grains to aggregate together, but is not sufficient for the ball formation. As proved by theoretical calculations and numerous experiments, the dipolar attraction only leads to form chains and columns along the field direction, never leads to macroscopic balls (see reference 7). Therefore, the ball formation reveals new and deep physics. The surface tension is \( \sigma \) so that the surface energy of a ball of radius \( a \) is \( 4\pi a^2 \sigma \). The ball is near an electrode of charge \( q \). The charge on the ball is \( q \) and the dielectric constant of the medium is \( \epsilon_o \). The Coulomb energy is then \( q^2/2a\epsilon_o \) and the electromagnetic energy is \( -3(4\pi a^3/3)\epsilon_o E^2/8\pi = -(1/2)\epsilon_o E^2 a^3 \). The energy of the ball is then given by,

\[ U = 4\pi a^2 \sigma + q^2/(2\epsilon_o a) - (1/2)\epsilon_o E^2 a^3 \]

**Case I.** We discuss two cases of this free energy separately. For constant \( q \) or when charge is independent of the size, the above is minimized with respect to the radius of the ball by
setting \(dU/da = 0\) so that,

\[
\sigma = q^2/(16\pi\epsilon_o a^3) + 3E^2a\epsilon_o/(16\pi) .
\] (5)

The charge, \(q\), of the ball is proportional to \(E\epsilon_o a^2\). Therefore we assume that \(q = \gamma E\epsilon_o a^2\) which substituted in the above gives a relation between applied electric field \(E\) and the radius of the ball as,

\[
E^2a = 16\pi\sigma/\left[\epsilon_o(3 + \gamma^2)\right]
\] (6)

so that the radius of the ball depends on the inverse square of the electric field, \(a \propto 1/E^2\).

This relation seems to be marginally satisfactory for \(\text{NdBa}_2\text{Cu}_3\text{O}_{7-\delta}\) but in the case of \(\text{Bi}_2\text{Sr}_2\text{CaCu}_2\text{O}_{8+\delta}\), the relation is obeyed only for large values of \(E\). For \(E < 0.94 \text{ kV/mm}\) there is a deviation between the measured values and those calculated from \(E^2a = \text{constant}\). The measured [7] values of the radius of the ball are smaller than those calculated. None of the three terms in (4) are explicitly dependent on quantum effects. The surface energy is the product of the surface area \(4\pi a^2\) and the surface tension but does not have explicit dependence on quantum nature of superconductivity. The Coulomb energy is just the square of the charge divided by the distance and the electrostatic energy also does not involve any quantum effects. If the electrostatic energy was dominant, there will be crystal growth according to the crystallographic symmetry. Since the growth is spherical, according to the crystal symmetries, there will be a texture which minimizes the energy.

**Case II.** We consider that in the free energy (4) the charge depends on the radius of the ball, \(a\). Therefore, we treat the charge as dependent on the radius. The charge \(q = \gamma E\epsilon_o a^2\) is eliminated from (4) so that the free energy becomes,

\[
U = 4\pi a^2\sigma + \frac{\gamma^2E^2\epsilon_o a^3}{2} - \left(\frac{1}{2}\right)\epsilon_o E^2 a^3 .
\] (7)

Minimizing this free energy with respect to the radius of the ball we set \(dU/da = 0\) which gives,

\[
E^2a = \frac{16\pi\sigma}{3\epsilon_o(1 - \gamma^2)} .
\] (8)

This gives maximum, if \(\gamma^2 \ll 1\). In fact, the minimum is at \(a = \infty\) when \(\gamma^2 \ll 1\). This will lead to the formation of large balls. If \(\gamma^2 \gg 1\), the minimum is at \(a = 0\) so that balls can
not be formed.

Case III. Pairing with constant charge. We introduce the pairing energy which is important for superconductivity and explains the experimentally measured values of the radius of the superconducting ball. The electron-phonon interaction is given by

$$\sum_{k,k'} D \delta_{k,s} c_{k,s}^\dagger c_{k',s}^\dagger a_{k-k'} + \text{h.c.}$$

where \(h.c\) stands for the hermitian conjugate of the previous term. The \(c_{k,s}^\dagger(c_{k,s})\) are the creation (annihilation) operators for electrons of wave vector \(k\) and spin \(\sigma\) and \(a_{q}^\dagger(a_{q})\) for phonons of wave vector \(q\) and frequency \(\omega\). Here \(D\) is the interaction constant. This interaction leads to the attractive potential,

$$V_{\text{eff}} = \frac{2D^2 \hbar \omega_q c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger c_{k\downarrow} c_{k\uparrow}}{(\epsilon_k - \epsilon_{k'})^2 - (\hbar \omega_q)^2}$$

(9)

where \(\hbar \omega_q\) is the phonon single-particle energy and \(\epsilon_k\) are the electron single-particle energies. The attractive interaction is achieved when \(\epsilon_k - \epsilon_{k'} \ll \hbar \omega_q\). This condition introduces the negative sign with respect to the kinetic energy terms. The pairing operators are averaged as \(\langle c_{k\uparrow}^\dagger c_{k\downarrow}^\dagger \rangle = \langle c_{k\downarrow} c_{k\uparrow} \rangle = \frac{\Delta}{V}\) where \(V\) is the attractive potential \(V = -2D^2/(\hbar \omega_q)\). The kinetic energy terms of the hamiltonian are slightly renormalized in going from the normal to the superconducting state. However ignoring this small renormalization effect, we can write the average value of the B.C.S. hamiltonian as

$$-\frac{2\Delta^2}{V} = \Delta^2 \hbar \omega / D^2$$

where \(\Delta\) is the gap energy. Therefore eq.(4) is subject to a quantum correction due to pairing energy. The volume of the ball is \(4\pi a^3/3\) and that of a Cooper pair is \(4\pi \xi^3/3\) so that the number of Cooper pairs is \((a/\xi)^3\). Since \(\Delta\) is the gap in the single-particle dispersion relation the pairing energy inside the superconducting ball is \((\Delta^2 \hbar \omega / D^2)(a/\xi)^3\). Therefore the energy of the ball becomes,

$$U = 4\pi \sigma a^2 + q^2/(2\epsilon_o a) - a^3[\Delta^2 \hbar \omega / (D^2 \xi^3) + (1/2)E^2 \epsilon_o] .$$

(10)

Minimizing \(U\) with respect to the radius of the ball, we set \(dU/da = 0\) which we solve for the surface tension to find,

$$\sigma = \frac{q^2}{16\pi \epsilon_o a^3} + \frac{3a\Delta^2 \hbar \omega}{8\pi D^2 \xi^3} + \frac{3aE^2 \epsilon_o}{16\pi} ,$$

(11)

5
for a constant charge on the ball. Substituting the charge of the ball \( q = \gamma E \epsilon_o a^2 \) in eq.(11) and solving for the radius we obtain,

\[
E^2 a = 16\pi \sigma / [\epsilon_o \{3 + \gamma^2 + a_1\}] \tag{12}
\]
where

\[
a_1 = 6\Delta^2 \hbar \omega / [\epsilon_o D^2 E^2 \xi^3] = a_2 (\Delta^2 / \xi^3) . \tag{13}
\]

When \( E \) is reduced, \( a_1 \) increases and hence the denominator in (12) increases and \( a \) reduces. We assume that \( a_1 \) is a small number, \( a_1 / (3 + \gamma^2) \ll 1 \). The eq.(12) then can be written by using the binomial theorem expansion and retaining only the first two terms as,

\[
E^2 a = \left[16\pi \sigma / \{\epsilon_o (3 + \gamma^2)\}\right] \{1 - a_1 / (3 + \gamma^2)\} . \tag{14}
\]

The radius of the superconducting ball given by this expression is smaller than that given by (6) due to pairing of electrons. The classical ball is thus compressed by the pairing energy. Since the mass of the ball is independent of the pairing of electrons and the volume is reduced we find that the density of the ball increases due to pair formation. In the case of strong pairing, \( a_1 \gg 3 + \gamma^2 \), the eq.(12) gives,

\[
a = 8\pi \sigma D^2 \xi^3 / (3\Delta^2 \hbar \omega) \tag{15}
\]
and the radius of the superconducting ball becomes independent of the applied electric field. This means that when charge on the ball is small and the tunneling current is small, then the ball can move in the electric field with a constant radius. It is also clear that when the surface tension vanishes, \( \sigma = 0 \), the ball collapses with zero radius, \( a = 0 \).

The surface tension on the superconducting ball is caused by the Josephson tunneling along the surface of the ball which means that the \( c \) axis is tangential to the radius so that

\[
\sigma' = Jc_o \tag{16}
\]
where \( c_o \) is the unit cell dimension along the \( c \) axis and the Josephson coupling energy is

\[
\mathcal{H}' = -J \cos \theta \tag{17}
\]
where
\[ \theta = \theta_1 - \theta_2 - (2e/hc) \int \mathbf{A} \cdot d\mathbf{l} \]  
(18)
is the phase factor. All the grains are aligned in such a way that the c axis is always on the surface. Keeping the c axis on the surface can be achieved by rotations along the a or b axis which are equivalent. Therefore the superconducting ball develops a texture.

It was found by us [6] that the normal effects can be changed to superconducting properties by introducing the factor of \( l_s/\xi \) where \( l_s \) is the mean free path of normal electrons and \( \xi \) is the coherence length. The superconducting ball gets charged by the electrodes which is a normal effect. Therefore, due to this normal charge the superconducting Josephson current is reduced by the factor \( l_s/\xi \). We suppose that \( n \) is the concentration of normal electrons so that the mean free path is \( l_s = (\pi/3)^{1/6} [a_o/(4n^{1/3})]^{1/2} \) with \( a_o = \hbar^2/me^2 \) as the Bohr radius.

Hence, we can write the surface tension on the surface of the charged ball as,
\[ \sigma = Jc_o l_s/\xi , \]  
(19)
which replaces (16). It is sufficient for the present purpose to write \( J \approx J_o \approx \pi \Delta/2SR_N \) where \( S \) is the surface area and \( R_N \) is the normal resistivity and \( \Delta = \Delta_o[1 - T/T_c]^{1/2} \) so that we can estimate the temperature dependence of the ball radius from,
\[ a = [\pi c_o l_s D^2 \xi_o^2/(3\hbar \omega R_N)]^{1/3} \]  
(20)
where we have used (15), (19) and \( S = 4\pi a^2 \). Using the fact that \( \xi \) diverges as \( \xi = \xi_o/(1 - T/T_c)^\nu \) where \( \nu \approx 0.7 \) is the exponent for the divergence of the coherence length, the above expression can be written as,
\[ a = [\pi c_o l_s D^2 \xi_o^2/(3\hbar \omega R_N \Delta_o)]^{1/3}/(1 - T/T_c)^{1/6+2\nu/3} . \]  
(21)
At \( T = T_c \), the coherence length diverges and hence, the ball radius for strong pairing shows strong divergence.

Case IV. As noted in (7), for the charge depending on radius we rewrite the eq.(10) as
\[ U = 4\pi \sigma a^2 + \frac{\gamma^2 E^2 \epsilon_o a^3}{2} - a^3[\Delta^2 \hbar \omega/(D^2 \xi^3) + (\frac{1}{2})E^2 \epsilon_o] \]  
(22)
which gives results similar to those already discussed.

A scanning electron micrograph of the superconducting ball formed by the application of an electric field on a powder of superconducting material contains small grains dispersed in liquid nitrogen is shown by Tao et al [7]. It is quite clear that the ball is not perfectly spherical. If the pairing of electrons is an $s$-wave type, we would expect the formation of a perfectly spherical ball. Therefore, we think that the gap has $d$-wave symmetry. In which case the gap in (10) and in subsequent relations should be replaced by $\Delta = \Delta_o \cos 2\varphi$. It is also possible that the gap is of complex nature in which the symmetry changes from $d(x^2 - y^2)$ to $d(xy)$ or from $s$ to $d(x^2 - y^2)$ when temperature or magnetic field is varied. The high temperature phase may have higher symmetry than the low temperature phase as found earlier [8]. A detailed study of the effect of the electric field on superconductors is given in a recent book[9].

3. Conclusions.

In conclusion, we find that superconducting powder forms a ball when subjected to an electric field. The pairing interaction plays an important role while Josephson interaction provides the surface tension. The temperature dependence of the ball radius arises from the divergence in the coherence length.

References. [1] K.N. Shrivastava, J. Phys.: Condens. Matter 5 (1993) L597.
[2] J. Mannhart, D.G. Schlom, J.G. Bednorz and K.A. Müller, Phys. Rev. Lett. 67 (1991) 2099.
[3] T. Frey, J. Mannhart, J.G. Bednorz and E.J. Williams, Phys. Rev. B51 (1995) 3257.
[4] L.S. Lingam and K.N. Shrivastava, Mod. Phys. Lett. B10 (1996) 1123.
[5] L.S. Lingam and K. N. Shrivastava, Physica B 223(1996)577.
[6] K.N.Shrivastava, J. Phys.(Paris)Colloq 49(1988)C8-2239.
[7] R. Tao, X. Zhang, X. Tang and P.W. Anderson, Phys. Rev. Lett. 83 (1999) 5575.
[8] N.M. Krishna and K.N. Shrivastava, Physica B230 (1997) 939.
[9] K. N. Shrivastava, Superconductivity:Elementary Topics, World Scientific Pub., New Jersey, London, Hong Kong, Singapore, (2000).