Coherent control of a self-trapped Bose-Einstein condensate

C.E. Creffield

Department of Physics and Astronomy, University College London, Gower Street, London WC1E 6BT, United Kingdom
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We study the behavior of a Bose-Einstein condensate held in an optical lattice. We first show how a self-trapping transition can be induced in the system by either increasing the number of atoms occupying a lattice site, or by raising the interaction strength above a critical value. We then investigate how applying a periodic driving potential to the self-trapped state can be used to coherently control the emission of a precise number of correlated bosons from the trapping-site. This allows the formation and transport of entangled bosonic states, which are of great relevance to novel technologies such as quantum information processing.

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Introduction Ultracold atoms held in optical lattices are currently at the center of intense theoretical and experimental investigation. The experimental parameters of these systems can be controlled extremely precisely, and in addition, the high degree of isolation from the environment permits their quantum dynamics to remain coherent over long timescales. Consequently such systems provide an attractive way of investigating quantum many-body physics, and it has been proposed [1] to use them to simulate strongly correlated quantum systems of interest in other areas of physics such as high-temperature superconductivity [2]. They also provide an excellent starting point for engineering and manipulating entangled states, which are vital for implementations of quantum information processing.

Bosons confined in an optical lattice provide an almost ideal realization of the Bose-Hubbard (BH) model, and a recent pioneering experiment directly observed the quantum phase transition between a superfluid and a Mott insulator [3]. Theoretical work [3, 4] has shown how this transition may be induced in an alternative way: by applying an additional oscillatory driving field to suppress the inter-site tunneling by means of the quantum interference effect termed coherent destruction of tunneling (CDT) [7]. In this work we show how such an oscillating driving field can be used to manipulate the dynamics of a different ground-state of the BH model – the self-trapped state. In particular we demonstrate an effect analogous to photon-assisted tunneling, in which certain driving frequencies induce a coherent oscillation of an integer number of bosons between the trapping-site and its nearest-neighbors. Combining this effect with CDT allows us to control the emission of a definite number of bosons from the trapping site and manipulate their speed of propagation through the optical lattice, and thus enables the self-trapped state to be used as a quantum beam-splitter or as a coherent source of entangled bosons.

Static properties The BH model is described by the Hamiltonian

\[
H_{BH} = -J \sum_{\langle j,k \rangle} [a_j^\dagger a_k + H.c.] + \frac{U}{2} \sum_j n_j(n_j-1),
\]

where \(a_j^\dagger\) and \(a_j\) are the standard annihilation/creation operators for a boson on site \(j\), \(n_j = a_j^\dagger a_j\) is the number operator, \(J\) is the tunneling amplitude between neighboring sites, and \(U\) is the repulsion between a pair of bosons occupying the same site. Here we initialize the system in a state in which all the bosons occupy a single lattice site. As the Hubbard interaction is repulsive, it might be thought that such a state would be extremely unstable. Surprisingly, however, this is not necessarily the case and, depending on the strength of the interaction and the filling of the lattice site, this highly-localized configuration is able to persist for long times. In such cases the bosons are said to be “self-trapped” [8].

Self-trapping has been recently observed experimentally in Bose-Einstein condensates of roughly 1000 atoms [9, 10], and can be understood qualitatively by an energetics argument. The presence of the optical lattice causes the energy spectrum of non-interacting bosons to be confined to a Bloch band of width \(4J\). Consequently, if the potential energy per particle of the trapped condensate is much higher than this, it cannot be converted into the kinetic energy of free bosons and the trapped state cannot decay. Its stability thus depends critically on the absence of dissipative processes in optical lattice systems.

To describe this effect quantitatively, we consider a 2-site BH model holding \(N\) bosons. If the system is initialized in the state \(|N,0\rangle\) (that is, a Fock state with \(N\) bosons occupying one site with the other site empty), then the primary tunneling process will be to the state \(|N-1,1\rangle\). If we truncate the Hilbert space to just these two states, we obtain an effective two-level model

\[
H_{2-lev} = \begin{pmatrix} V(N) & \sqrt{N} \\ J\sqrt{N} & V(N-1) \end{pmatrix}.
\]

where \(V(n) = U(n-1)/2\) is the potential energy of \(n\) bosons occupying one site. It is useful to visualize the time evolution of the system geometrically by making use of the Bloch sphere representation. Parameterizing \(H_{2-lev}\) in terms of the Pauli matrices

\[
H_{2-lev} = \frac{U(N-1)^2}{2} I + J \sqrt{N} \sigma_z + \frac{U(N-1)}{2} \sigma_z,
\]

reveals that we can interpret it as an interaction between the Bloch vector \(\sigma\) and a fictitious magnetic field \(B = (J\sqrt{N},0,U(N-1)/2)\). Thus under the influence of the Hamiltonian the Bloch vector will simply make a Larmor orbit about \(B\). This form of time evolution is shown in Fig[1] for a...
strongly interacting \((U = 8J)\) 7-boson system. It can be clearly seen that, as expected, the Bloch vector traces out a periodic circular orbit centered on \(\hat{B}\). Since for these parameters \(\hat{B}\) is almost parallel to the \(s_x\) axis, the radius of this orbit is rather small, and thus the system exhibits a high degree of self-trapping.

To assess the degree of self-trapping more precisely we measure the overlap of the system’s state with the initial state, \(p_i(t) = |\langle \psi(t) | \psi_i \rangle|^2\), since when self-trapping occurs this quantity is restricted to values close to unity. The circular motion shown in Fig.1 equates to a very small-amplitude sinusoidal oscillation of \(p_i\). As \(N\) is reduced, the angle between \(\hat{B}\) and the \(s_x\) axis increases, and as a result the radius of the Larmor orbit made by the Bloch vector increases (Fig.1b). Consequently the degree of trapping is reduced, and the amplitude of oscillations of \(p_i\) is larger. If \(N\) is decreased even further (Fig.1c) the self-trapping effect is lost, and the two-level approximation breaks down. In this case the Bloch vector rapidly leaves the surface of the Bloch sphere, and its erratic time-evolution corresponds to an irregular quasi-periodic behavior of \(p_i\), which can take very low values.

In the inset of Fig.2 we plot the minimum value of \(p_i\) attained in the 7-boson system as a function of the interaction strength. For small values of \(U\) no self-trapping occurs, and thus \(p_i^{\text{min}}\) takes a value of zero. As \(U\) is increased, however, the oscillations in \(p_i\) are quenched, and for \(U/J > 3\) it can be seen that the effective two-level model describes the dynamics extremely well. From Eq.3 we obtain a criterion for the crossover to the self-trapped regime, by defining the transition to occur when \(p_i^{\text{min}}\) drops below a value \(\alpha\). This yields a value for the critical value of \(U\),

\[
\frac{U_c}{J} = \frac{2}{N-1} \sqrt{\frac{N\alpha}{1-\alpha}}. 
\]

This boundary is plotted in Fig.2 for \(\alpha = 0.99\) – that is, when less than 1% of the initial state is able to leak to the other site. Although the self-trapping regime is more easily achievable for large boson numbers since \(U_c \sim 1/\sqrt{N}\), it is in principle possible in a system of just two bosons, if the interaction strength can be raised sufficiently high. This has recently been achieved experimentally \([11]\) in a gas of trapped rubidium atoms.

**Dynamical properties** We now consider dynamically controlling the self-trapped state by applying a harmonic driving potential

\[
H(t) = H_{BH} + K \sin \omega t \sum_j j n_j, 
\]

where \(K\) is the amplitude of the driving field, and \(\omega\) is its frequency. We first consider the case of extracting a single boson from the trapping site. As the occupation of the trap site changes from \(N\) to \(N-1\) there is a corresponding loss of potential energy \(\Delta E = V_N - V_{N-1} = U(N-1)\), where \(V_n\) is defined as in Eq.2. When a system possesses such a large energy gap,
Floquet analysis may be used to show \[ \text{[6, 12]} \] that extremely fine control over the tunneling dynamics is possible at multi-photon resonances, that is, when \( m \omega = \Delta E \), \( m = 1, 2, \ldots \). In general when this condition is satisfied, the system is able to exchange energy with the driving field to overcome the energy gap, and so tunneling is restored \[ \text{[13]} \]. However, at particular values of the amplitude of the driving field, CDT will occur when the Floquet quasienergies of the system become degenerate, and correspondingly the dynamics of the system will be frozen. For a sinusoidal driving potential these degeneracies occur at the zeros of \( \mathcal{J}_m(K/\omega) \), the \( m \)th Bessel function of the first kind. Thus at a photon resonance it is possible to produce dramatic differences in the tunneling rate by making small changes in the amplitude of the driving field to move the system between CDT and photon assisted tunneling.

In Fig. 4 we plot \( p_i^{\text{min}} \) obtained in a 5-boson system driven at a frequency of \( \omega = 4U \). This corresponds to the \( m = 1 \) photon resonance. For \( K = 0 \) the system is self-trapped, and consequently \( p_i \) remains near unity. Increasing \( K \), however, causes the value of \( p_i^{\text{min}} \) to rapidly drop to zero, demonstrating how photon assisted tunneling overcomes the self-trapping effect. As \( K \) is increased further, \( p_i^{\text{min}} \) exhibits a number of extremely sharp peaks centered on \( K/\omega = 3.83, 7.01 \) and \( 10.17 \) – the zeros of \( \mathcal{J}_1(K/\omega) \).

Away from these zeros, the driving field causes a single boson to diffuse symmetrically from the trapping site to its neighboring lattice sites, and from there to continue propagating through the optical lattice with a renormalized tunneling rate \( \mathcal{J}_{\text{eff}} = \mathcal{J}_0(K/\omega) \). If we therefore choose a value of \( K \) such that \( \mathcal{J}_{\text{eff}} = 0 \), the particle will not be able to propagate further, and will thus just make a Rabi-like oscillation between the trapping site and its neighbors. This situation is illustrated schematically in Fig. 3. We show in Fig. 3 the time dependence of the occupation of the trapping site and its neighbor. Initially we set \( K/\omega = 3.83 \) (a zero of \( \mathcal{J}_1 \)), and no tunneling occurs: for this value of \( K/\omega \) CDT reinforces the self-trapping effect. At \( t = 10 \) we alter the amplitude of the driving to \( K/\omega = 2.40 \) (a zero of \( \mathcal{J}_0 \)) and a very clear particle oscillation occurs, in which the occupation of the trapping site cycles between 5 and 4, and that of the neighboring sites moves between 0 and 0.5. In this sense the optical lattice is acting as a particle beam-splitter, dividing a single boson into a superposition of left and right-moving components.

We have so far considered the case of extracting a single boson from the trapping site. We can however apply a similar method to induce the emission of an integer number of bosons, \( N_{\text{em}} \), by noting that the energy difference per particle takes the remarkably simple form

\[
\Delta E/N_{\text{em}} = (V_N - (V_{N-N_{\text{em}}} + V_{N_{\text{em}}})/N_{\text{em}} = U(N - N_{\text{em}})
\]

The self-trapped state thus contains a ladder of equally-spaced energy levels, separated by \( U \), and so by driving the system at the correct frequency we can induce the emission of a given number of particles. In Fig. 5 we show the response of the 5-boson system to a driving field of frequency \( \omega = 3U \), thereby inducing the emission of two bosons. As before we can observe peaks in \( p_i^{\text{min}} \) centered on the zeros of \( \mathcal{J}_1(K/\omega) \) at which CDT occurs, while between them photon-assisted tunneling causes \( p_i^{\text{min}} \) to take low values. Fig. 5 shows the effect...
of switching the amplitude of the driving field to a value of $K/\omega = 2.40$. We can again see that this has the effect of inducing a Rabi-like oscillation between the trapping site and its neighbors, but in this case the oscillation indeed consists of two bosons, and has a longer period.

In Fig. 5 we show how combining the PAT effect with CDT allows the population of the self-trapped state to be reduced step-by-step. The system is initialized with 5 bosons in the central site, and is initially driven at $\omega = 4U$. Driving the system at $K/\omega = 3.80$ induces CDT which suppresses the number variance of the trapping site and stabilizes the self-trapped state. At $t = 5$ the value of $K/\omega$ is changed to 2.40 which induces the Rabi-type oscillation between the trapping site and its neighbors, as schematically shown in Fig. 4a. After a half-integer number of these oscillations we then alter the driving parameters to $\omega = 3U$, $K/\omega = 3.80$ which traps the remaining 4 bosons in the trapping site. For these parameters, however, the single-particle tunneling is not quenched and so the ejected boson is able to propagate through the optical lattice away from the trapping site, as shown in Fig. 4b. The emitted particle smears out to an extent as it moves through the lattice (see Fig. 5 lower panel), but nonetheless the atom-pulse remains quite clearly defined after propagating through several lattice spacings. By repeating this procedure with appropriate driving frequencies we can successively reduce the occupation of the trapping site in integer steps, and thereby produce a sequence of well-defined, phase-coherent atom-pulses.

Conclusions
We have shown how self-trapping arises in a Bose-Einstein condensate confined to an optical lattice. Applying a resonant driving field to the self-trapped state can either stabilize the trapping (when CDT occurs), or can induce a Rabi-like particle oscillation. The interplay between these effects makes it possible to control the emission and propagation of a precisely-defined number of particles, and thus enables the self-trapped state to be used as a particle beam-splitter or a source of mesoscopic entangled states, which have many possible applications in quantum information.

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