Efficient Federated Learning over Multiple Access Channel with Differential Privacy Constraints

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Abstract—In this paper, the problem of federated learning (FL) over a multiple access channel (MAC) is considered. More precisely, we consider the FL setting in which clients are prompted to train a machine learning model by simultaneous communications with a parameter server (PS) with the aim of better utilizing the computational resources available in the network. We also consider the additional constraint in which the communication between the users and the PS is subject to a privacy constraint. To minimize the training loss while also satisfying the privacy rate constraint over the MAC channel, the distributed transmission of digital variants of stochastic gradient descents (D-DSGD) is performed by each client. Additionally, binomial noise is also added at each user to preserve the privacy of the transmission. The optimum levels of quantization in the D-DSGD and the binary noise parameters to achieve efficiency in terms of convergence are investigated, subject to privacy constraint and capacity limit of the MAC channel.

Index Terms—Federated Learning, Differential privacy, Multiple access channel, Distributed Stochastic Gradient Quantization.

I. INTRODUCTION

The ever-increasing need to process large amounts of data being transferred between users in a communication network, such as wireless sensor networks and mobile networks, has spurred the development of joint data processing algorithms and network protocols [1]. As learning is one of the fundamental application motivating data transfer in large networks, federated learning (FL) has recently emerged as a promising paradigm for decentralized machine learning (ML). In the FL paradigm, a parameter server (PS) aims at training a global model with the federated collaboration of distributed users, also referred to as local learners [2]. Unlike the centralized learning model that requires clients to share their local data with the central server, the FL model alleviates the needs of communicating local data toward the PS by opting to, instead, perform the optimization locally and use consensus over multiple iterations to converge to the optimal solution [3]. One way for this paradigm to be implemented, is by having the PS share the current model estimate with the network users at each communication round. Upon receiving an updated model estimate, the local learners compute the gradients over the local datasets and update the model estimate using stochastic gradient descent (SGD). The locally-updated models are then communicated to the PS. Upon receiving the different local gradients, the PS merges these estimates to produce an updated central estimate. By iteratively repeating these two update steps, i.e. the local model update and the global model update, the clients and the PS can converge to the optimal solution.

In practical scenarios, the communication between the PS and the local learners is constrained in either the communication rate, reliability, or connectivity: for this reason authors have considered variation of the basic FL setting also encompassing such communication constraints. For instance, when a rate constraint over the communication links is considered, one can resort to quantization of the local gradients. This approach is proposed in [4] where it is referred to as quantized SGD (QSGD).

The FL model, not only allows a better utilization of computational resources, but also provides some security enhancements. Since the row data is not communicated to the central server, the users’ data is never exposed to the security vulnerability which are intrinsic in communication. Despite this, it has been shown that local gradients may leak some information about the data endangering the privacy of the model [5]. This possible information leakage has made the investigation of privacy for the FL model an issue of considerable significance. A formal way of studying privacy is through consideration of differential privacy (DP) which has been introduced to quantify the amount of ambiguity in distinguishing the use of a particular client’s data over neighboring data-sets, [6], [7]. To ensure privacy in FL model, a decentralized approach is preferred since it allows for learning under the coordination of an untrusted PS in expense of some additional rate consumption. For differential privacy to be guaranteed, each client performs a local perturbation process on the released gradient and sends it to the PS. Further results including DP analysis of FL model can be found in [8]–[10].

In the following, we focus on an information-theoretical formulation of the communication link between the local users and the PS in the FL model by modelling it as a MAC channel. In this framework, the privacy constraint is described in terms of the secure transmission rates attainable over MAC channel. By exploiting the underlying MAC structure of wireless communication in FL model through simultaneous transmission of clients, it is also possible to improve the accuracy and privacy of the FL over that of orthogonal transmission as in [11], [12]. Furthermore, this can cause clients for a better management of
bandwidth and power resources to propose greater efficiency in communication. This can be seen in [11] where a multi-level QSGD is applied by taking different channel conditions and dynamic ranges of clients’ gradients into account.

**Contributions:** Motivated by the benefits of MAC communication in FL model, this paper studies privacy issue of communication in FL model over MAC. The model considers the learning process is done by sending digital version of the local gradients over MAC subject to transmission rate and a specific privacy level constraints. The proposed communication scheme generalizes that of [11] by ensuring the privacy through adding a discrete perturbation of binomial distribution at each client to meet the specific privacy level. Our achievable scheme also generalizes the one in [8] by enabling efficient communication over MAC through multi-level QSGD that uses different quantization levels allowing clients to meet the capacity constraint of MAC. Hence, the goal is to optimize the accuracy of the FL model in terms of maximizing convergence rate of the quadratic loss of PS as a function of the quantization levels and noise parameters such that privacy and transmission limits of communication are guaranteed.

**Organization:** The paper is organized as follows: in Sec. II we introduce the model and formulation of the problem under consideration. In Sec. III the main result is presented: the MSE and privacy level analysis of the FL model assuming an average estimation by the PS. Also, the formulation of the optimization problem for this FL model over MAC is presented in Sec. IV. In Sec. V numeric solution to this problem and related discussions with respect to the privacy-utility trade-off are provided. Sec. V concludes the paper.

**Notation:** The following notation is used throughout the paper. Calligraphic and boldface letters $\mathcal{X}$ and $\mathbf{x}$ denote sets and vectors, respectively. $Pr(.)$ and $E[.]$ denote the probability and expectation of a random variable. Also, $\| \cdot \|_q$ is the $q$-norm and $\nabla$ denotes the gradient operator. The superscript in brackets indicates the client, while the time index is indicated through a subscript e.g. $x_t^{(i)}$ is the value of the variable $x$ at client $i$ at time $t$.

**II. SYSTEM MODEL AND PROBLEM FORMULATION**

**A. FL model with efficiency and privacy**

A distributed learning model over MAC, as introduced in [11], [12] and shown in Fig. 1, is characterized by a set $\mathcal{N} \triangleq \{1, \ldots, N\}$ of clients communicating with a remote PS over a MAC with the aim of training a machine learning model that minimizes a chosen loss function during $T$ iterations. Assume that the local private database used for training at client $i$, $i \in \mathcal{N}$, is the set $D_i = \left\{ (d_k^{(i)}, v_k^{(i)}) \right\}_{k=1}^{|D_i|}$ of size $|D_i|$ data pairs consisting of a data point $d_k^{(i)}$, which denotes the $k$-th data point, and its corresponding label, $v_k^{(i)}$, in database $D_i$ of client $i$. Then, the goal is to minimize the following loss function with respect to the training vector $w \in \mathbb{R}^d$

$$
\mathcal{L}(w) \triangleq \frac{1}{N} \sum_{i \in \mathcal{N}} \frac{1}{|D_i|} \sum_{k=1}^{|D_i|} g^{(i)}(d_k^{(i)}, v_k^{(i)}, w),
$$

where $g^{(i)}(.)$ is the loss function at user $i \in \mathcal{N}$.

![Fig. 1. FL over MAC with privacy and efficiency](image)

The solution $w^* = \arg\min_w \mathcal{L}(w)$ for this optimization problem is carried out collaboratively by the clients and the PS through the iterative use of distributed gradient descent (DGD) algorithm. To this end, at iteration $t = 1, \ldots, T$, the parameter vector $w_t$ updated by the PS from the previous iteration is first broadcasted to all clients. Next, each client $i$ computes the average local gradient of its loss function over all the data points of the database $D_i$ as

$$
g_t^{(i)} = g^{(i)}(w_t) = \frac{1}{|D_i|} \sum_{k=1}^{|D_i|} \nabla l^{(i)}(d_k^{(i)}, v_k^{(i)}, w_t). \quad (2)
$$

Due to the limitations imposed by resources, such as bandwidth and transmit power, and also to preserve privacy of the local gradients aggregated at the PS, we consider the FL scenario in which each client needs to send its local gradient to the PS through an efficient and private communication in the FL model. These two additional requirements are mathematically formulated as follows.

The first constraint, the communication constraint, is concerned with the limit imposed by the capacity of communication over MAC which limits the transmission rate of clients. For this constraint to be met, each client $i$ applies a digital transmission approach (D-DGD), as in [13], by quantizing the local gradient through computing a function $q^{(i)} : \mathcal{G}^{(i)} \to \mathcal{Q}^{(i)}$ of its gradient as $Q_t^{(i)} = q^{(i)}(g_t^{(i)})$.

The second constraint, the security constraint, is concerned with the client-based privacy which entails that the privacy of the clients’ gradients aggregated at the PS should be provided. For this constraint to be met, one promising way is via the notion of differential privacy (DP) for the average gradient, as introduced in [8], which is a strong measure of privacy that can ensure privacy for the FL model even after post-processing performed by the PS. In order to alleviate the restrictive need to trust the PS, a local differential privacy (LDP) approach is proposed in which a randomized mechanism is applied by each client contributing some randomness to the quantized local gradient $q^{(i)}(g_t^{(i)})$ before transmission toward the PS. More precisely, this process can be carried out by a $(\epsilon, \delta)$-differentially private mechanism $\mathcal{M}^{(i)} : \mathcal{Q}^{(i)} \to \mathcal{A}^{(i)}$ such
that

$$\Pr\left( \mathcal{M}^{(i)} \left( q^{(i)} \left( \mathbf{g}^{(i)} \right) \right) \in \mathcal{X}^{(i)} \right)$$

is satisfied for any measurable subset $\mathcal{X}^{(i)} \subseteq \mathcal{X}^{(i)}$, and for any $\mathbf{g}^{(i)}, \mathbf{g}^{(i)} \in \mathcal{G}^{(i)}$. Here, it is assumed that $\mathcal{G}^{(i)}, \mathcal{Q}^{(i)}, \mathcal{X}^{(i)} \subseteq \mathbb{R}^d$.

As a result of the above procedure, at iteration $t$, client $i$ sends back the randomized quantized variant $\mathcal{M}^{(i)} \left( q^{(i)} \left( \mathbf{g}^{(i)} \right) \right)$ as a function of the local gradient to the PS through $n$ channel uses of the MAC. It should be noted that since the rate of transmission by client $i$, $R_i$, should not exceed the capacity of the MAC then it is required that

$$\sum_{i \in S} R_i \leq C_S = C_S = \sum_{i \in S} \frac{1}{2} \log_2 \left( 1 + S_i \right), \quad \forall S \subseteq N. \quad (4)$$

where $C_S$ is the sum-capacity of the given MAC from the subset $S$ of clients to the PS. For a Gaussian $N$-user MAC with received SNRs $S_i$, the capacity region is a polymatroid in $\mathbb{R}^N$. Finally, the PS should first recover the aggregated noisy version of local gradients $\mathbf{g}^{(i)}$ from the noisy received signal $y_t$ using a post-processing function $p_t(\cdot)$ which extracts the average of these noisy local gradients as

$$p_t(y_t) = \mathbf{g}_t = \frac{1}{N} \sum_{i \in N} \mathbf{g}^{(i)}_t, \quad (5)$$

provided that it releases an unbiased estimation of the local gradients i.e. $E[\mathbf{g}_t] = \mathbf{g}_t = \frac{1}{N} \sum_{i \in N} \mathbf{g}^{(i)}_t$. The estimation error is evaluated using the mean square error (MSE) as

$$E_t = E \left( \left( \mathbf{g}_t - \mathbf{g}_t \right)^2 \right). \quad (6)$$

Next, the PS proceeds to update the parameter vector $\mathbf{w}_{t+1}$ to be used for the next iteration by clients according to

$$\mathbf{w}_{t+1} = \mathbf{w}_t - \gamma_t \mathbf{g}_t, \quad (7)$$

with $\gamma_t$ representing the learning rate of DGD at iteration $t$.

The convergence of the FL model is measured using

$$E \left[ \left( \mathbf{w}_t \right) - \tilde{\mathbf{w}}(\mathbf{w}^*) \right],$$

which is expected to vanish as $t$ grows, as the model is learned at the PS and, consequently, at the local learners.

### B. Problem Formulation

For the FL setting with MAC communication subject to privacy and efficiency, as described in Sec. [I-A] we formulate the following optimization problem. For the FL model with a given D-DGD learning algorithm, we aim at determining the optimum quantization levels $\{l_t\}_{t=1}^T$ of local gradients and parameters of the LDP mechanism that maximizes the convergence rate subject to the transmission rates and privacy constraints of clients. This can be formulated as follows:

$$\mathcal{P} : \min_{\{l_t\}_{t=1}^T} \left[ \mathbb{E} \left[ \tilde{\mathcal{L}}(\mathbf{w}_T) - \mathcal{L}(\mathbf{w}^*) \right] \right],$$

s.t. $\mathcal{C}_1 : \epsilon \leq \Delta$, $\mathcal{C}_2 : \sum_{i \in S} R_i \leq nC_S$, $\mathcal{C}_3 : l_t \geq 0$, $\forall t \in N$,

$C_4 : l_t \leq 1$, $\forall t \in N$,

$C_5 : l_t \geq 0.5$, $\forall t \in N$,

$C_6 : l_t \leq 1$, $\forall t \in N$,

$C_7 : l_t \leq 1$, $\forall t \in N$,

$C_8 : l_t \leq 1$, $\forall t \in N$.

Therefore, we seek the amount of quantization and hence the transmission rates allocated to the clients so that the maximum convergence rate for the D-DGD algorithm is achieved such that the prescribed level of privacy and reliable transmission of gradients are satisfied for clients. Also, it is of great interest to exploit the possible trade-offs between the desired issues in terms of the known and given factors. These questions will be addressed in the remainder of the paper.

### C. Preliminaries

A random function $\tilde{\mathbf{g}}(\mathbf{w})$ is a stochastic gradient for $f$ provided that $E[\tilde{\mathbf{g}}(\mathbf{w})] = \nabla f(\mathbf{w})$ implying that it is a subgradient of $f$ at $\mathbf{w}$. A function $f$ is a $\lambda$-strong convex if $\forall \mathbf{w}, \mathbf{w}' \in \mathcal{W}$ we have $f(\mathbf{w}') \geq f(\mathbf{w}) + \nabla f(\mathbf{w}) \cdot (\mathbf{w}' - \mathbf{w}) + \frac{\lambda}{2} \| \mathbf{w}' - \mathbf{w} \|^2$. A function $f$ is $\mu$-smooth with respect to the optimum point $\mathbf{w}^*$ i.e. $\forall \mathbf{w} \in \mathcal{W}$ we have $f(\mathbf{w}) - f(\mathbf{w}^*) \leq \frac{\mu}{2} \| \mathbf{w} - \mathbf{w}^* \|^2$ which normally appear in least square regressions. A function $f$ is $L$-Lipschitz if $\forall \mathbf{w}, \mathbf{w}'$ we have $\| f(\mathbf{w}) - f(\mathbf{w}') \| \leq L \| \mathbf{w} - \mathbf{w}' \|$.

### III. MAIN RESULTS

For the problem formulation in Sec. [II], we provide an upper bound on the convergence of rate in [8]. To do so, we design a communication and computation scheme which is comprised of a number of elements, each covered in this section in a separate subsection: stochastic gradient descent, in Sec. [III-A] a quantization scheme, in Sec. [III-B] and a privacy preserving mechanism, in Sec. [III-C]. The performance of the overall scheme is presented in Th. [2] in Sec. [III-D].

**Data:** Databases $\{D_i\}_{i \in N}$, constraints $\Delta$, $C_S$, number of iterations $T$.

**Result:** Maximum convergence rate and optimum quantization levels $\{l_t\}_{t=1}^T$ and LDP mechanism parameters $\{m_t\}_{t=1}^T$.

1: Initialize iteration $t = 0$;
2: while $t < T$ do
3: \hspace{1cm} $t = t + 1$;
4: \hspace{1cm} Client $i$ calculates local gradients $\mathbf{g}^{(i)}_t, i \in N$;
5: \hspace{1cm} Client $i$ calculates quantized gradients $\mathbf{Q}^{(i)}_t, i \in N$;
6: \hspace{1cm} Client $i$ calculates and sends $x_t^{(i)} = M_t^{(i)}(\mathbf{Q}^{(i)}_t), i \in N$ over MAC;
7: \hspace{1cm} PS receives $y_t$ and calculates $\mathbf{g}_t$ from [5];
8: \hspace{1cm} PS updates parameter vector $\mathbf{w}_{t+1}$ from [7] and sends back to clients;
9: \hspace{1cm} Calculate loss function at $\mathbf{w}_{t+1}$ from [1];
10: \hspace{1cm} Minimize convergence rate and find the optimum values $l_{t+1}$ and $m_{t+1}$ from [8];
end

**Algorithm 1:** Achievable scheme for the FL over MAC

### A. Stochastic gradient descent (SGD)

One simple first-order method for solving a convex learning problem is stochastic gradient descent (SGD) [14]. SGD can be used to optimize a convex function $\tilde{\mathcal{L}}$ using its unbiased estimates of gradients. More formally, assume that $\mathcal{W} \subseteq \mathbb{R}^d$ is a convex set and $\tilde{\mathcal{L}} : \mathcal{W} \rightarrow \mathbb{R}^d$ is a differentiable, convex
and smooth function. Applying SGD for a finite number of iterations $T$, the goal is to bound the convergence $\tilde{L}(w_T) - \tilde{L}(w^*)$ in the expectation sense assuming that $\tilde{L}$ achieves its minimum $w^* \in W$. A standard convergence analysis assumes three properties for $\tilde{L}$ as in Sec. II-C: (i) $\tilde{L}$ is a $\lambda$-strong convex, (ii) $\tilde{L}$ is $\mu$-smooth with respect to the optimum point $w^*$, and $\tilde{L}$ has $L$-Lipschitz gradients.

### B. Multi-Level QSGD

When communication among PS and client is subject to a cardinality constraint, one can consider a variation of SGD in which gradients are quantized before transmission. Such variation of the classic set up is termed quantized SGD (QSGD): in the proposed scheme, we consider a multilevel version of QSGD described as follows. At iteration $t$, each client $i$ quantizes the variation range of its local gradient vector $Q_{i,t}=g_{i,t}^{max} - g_{i,t}^{min}$ to $l_{i,t}$ levels where $g_{i,t}^{max} = \max_j g_{i,t}^{(j)}(j)$ and $g_{i,t}^{min} = \min_j g_{i,t}^{(j)}(j)$, $j = 1, \ldots, d$. Then, a stochastic procedure for assigning the quantized value $Q_{i,t}^{(j)}(j) = Q_{i,t}^{(j)}(\mathbf{g}_{i,t}^{(j)})$ to the $j$-th component of local gradient vector $\mathbf{g}_{i,t}^{(j)}$ is considered as

$$Q_{i,t}^{(j)}(\mathbf{g}_{i,t}^{(j)}) = \begin{cases} B_t^{(i)}(r_{i,t}+1) & \text{w.p.} \frac{g_{i,t}^{(j)}(j)-B_t^{(i)}(r_{i,t})}{B_t^{(i)}(r_{i,t}+1)-B_t^{(i)}(r_{i,t})} \\ B_t^{(i)}(r_{i,t}) & \text{w.p.} \frac{B_t^{(i)}(r_{i,t}+1)-g_{i,t}^{(j)}(j)}{B_t^{(i)}(r_{i,t}+1)-B_t^{(i)}(r_{i,t})} \end{cases}$$

(9)

where $r_{i,t} \in [0, l_{i,t})$, $l_{i,t} \geq 2$, and $B_t^{(i)}(r_{i,t})$ denote respectively the index and the bin to which the local gradient of client $i$ belongs. $r_{i,t}$ is such that $g_{i,t}^{(j)}(j) \in [B_t^{(i)}(r_{i,t}), B_t^{(i)}(r_{i,t}+1)]$ and $B_t^{(i)}(r_{i,t})$ is given as

$$B_t^{(i)}(r_{i,t}) = g_{i,t}^{min} + \frac{r_{i,t} + 1}{l_{i,t} - 1}. \quad (10)$$

As a result of this element-wise quantization scheme, the quantized vector of the local gradient is produced as

$$\mathbf{Q}_{i,t}^{(j)} = \left[ Q_{i,t}^{(j)}(\mathbf{g}_{i,t}^{(j)}(1)) \quad Q_{i,t}^{(j)}(\mathbf{g}_{i,t}^{(j)}(2)) \ldots \quad Q_{i,t}^{(j)}(\mathbf{g}_{i,t}^{(j)}(d)) \right].$$

It is assumed that the variation range $\Delta_{i,t}$ of local gradients are sent before the iteration to the PS.

### C. Binomial privacy-preserving mechanism

A simple and widely-used method for preserving privacy is to add Gaussian noise to the local gradients. While this mechanism is beneficial in practice, it is not suitable when transmitting quantized local gradients. An alternative privacy mechanism is presented in [8]: here a binomial-distributed noise is added as noise after quantization. Accordingly, at iteration $t$, each client $i$ sends the following vector:

$$x_{i,t}^{(j)} = p_{i,t}^{(j)}(\mathbf{g}_{i,t}^{(j)})$$

$$x_{i,t}^{(j)} = \mathcal{M}_{i,t}^{(j)}(\mathbf{Q}_{i,t}^{(j)}) = \mathbf{Q}_{i,t}^{(j)} + \Delta_{i,t} - \mathbf{Q}_{i,t}^{(j)} = p_{i,t}^{(j)}(\mathbf{g}_{i,t}^{(j)}) \quad \text{where } z_{i,t}^{(j)}(j) \sim \text{Bin}(m_{i,t}, p).$$

For the client $i$ to transmit the vector $\mathbf{x}_{i,t}$, the client needs $\log(l_{i,t} + m_{i,t})$ bits for each component and in total $R_{i,t} = d \log(l_{i,t} + m_{i,t})$ bits for the vector. Hence, for the PS to reliably extract the average gradient, it is required that this amount of information does not exceed the capacity of the MAC. The PS having decoded the transmitted vector $\mathbf{x}_{i,t}$ reliably from $y_{i,t}$, proceeds to compute the average of the local gradients as

$$\mathbf{g}_{i,t} = \frac{1}{N} \sum_{i \in N} x_{i,t}^{(j)} = \frac{1}{N} \sum_{i \in N} Q_{i,t}^{(j)} + \frac{\Delta_{i,t}}{l_{i,t} - 1} \left( z_{i,t}^{(j)} - m_{i,t}p \right). \quad (12)$$

As $E[Q_{i,t}^{(j)}] = g_{i,t}^{(j)}$ and $E[z_{i,t}^{(j)}] = m_{i,t}p$, then $\mathbf{g}_{i,t}$ is an unbiased estimation of $\mathbf{g}_{i,t}$ as desired. Moreover, the MSE of this estimation and the privacy of the mechanism can be bounded as provided in the next theorem.

**Theorem 1**: Using the binomial mechanism $M_{i,t}^{(j)}$ in (11) and multi-level QSGD $Q_{i,t}^{(j)}$ as (9) by clients in the FL model, the PS can estimate an unbiased average of the gradients and the MSE and the LDP at iteration $t$ can be bounded as

$$E_i \leq \frac{d}{N^2} \sum_{i \in N} \frac{(\Delta_{i,t})^2}{p(l_{i,t} - 1)} \left[ 1 + m_{i,t}p(1-p) \right],$$

$$\lambda_{i,t} \leq \frac{\Delta_{u,1}^2}{M_t(p-1)} + \frac{\Delta_{u,2} c_p}{\sqrt{M_t(p-1)}(1 - \frac{\delta}{10})} + \frac{\Delta_{u,\infty} c_p}{\sqrt{M_t(p-1)}(1 - \frac{\delta}{10})},$$

where $\Delta_{u,1}, \Delta_{u,2}, \Delta_{u,\infty}$ and $\lambda_{i,t}$ are given as

$$\Delta_{u,1} = \max_{i \in N} \Delta_{u,1}^{(i)}, \quad \Delta_{u,2} = \max_{i \in N} \Delta_{u,2}^{(i)}, \quad \Delta_{u,\infty} = \max_{i \in N} \Delta_{u,\infty}^{(i)}$$

and

$$\lambda_{i,t} = \max_{i \in N} \lambda_{i,t}^{(i)}.$$

Also, function $u$ and its corresponding sensitivity bounds

$$\lambda_{u,1} = \max_{i \in N} \lambda_{u,1}^{(i)}, \quad \lambda_{u,2} = \max_{i \in N} \lambda_{u,2}^{(i)}, \quad \lambda_{u,\infty} = \max_{i \in N} \lambda_{u,\infty}^{(i)}$$

are given as

$$\lambda_{u,1} = \max_{i \in N} \lambda_{u,1}^{(i)}.$$

Proof: The proof is in Appendix.
Remark 1: It should be noted that if all the clients quantize their local gradients subject to the same level i.e. \( l_i = K \) and that \( \Delta_l^{(i)} = \Delta \) for all \( i \in \mathcal{N} \), then the MSE and LDP of Thm. 1 reduces to the result of [8] Thm. 3.

D. Results for the solution of the optimization problem

The main result of the paper is the following theorem, which characterizes the convergence of the algorithm in Alg. 1.

**Theorem 2:** Assume that the loss function \( \ell \) is \( \lambda \)-strong convex and \( \mu \)-smooth having \( L \)-Lipschitz gradients. The convergence rate for the FL model with privacy and efficiency constraints can be bounded as

\[
|E\left[\tilde{t}(w_T)\right] - \tilde{t}(w^*)| \leq \frac{2\mu}{\lambda^2 T^2} \sum_{i=1}^{T} \frac{d}{N^2} \sum_{i\in\mathcal{N}} \left( \frac{\Delta_l^{(i)}}{1 - \frac{l_i}{K}} \right)^2 \left[ \frac{1}{4} + m_{i,p}(1 - p) \right] + L^2 \tag{17}
\]

The second-order moment \( \sigma^2_{g,t} \) can be bounded as

\[
\sigma^2_{g,t} = E\left[\|g_t\|^2\right] = E\left[\|g_t - g^*\|^2 + \|g^*\|^2\right] \\
\leq \frac{d}{N^2} \sum_{i\in\mathcal{N}} \left( \frac{\Delta_l^{(i)}}{1 - \frac{l_i}{K}} \right)^2 \left[ \frac{1}{4} + m_{i,p}(1 - p) \right] + L^2 \tag{18}
\]

where (a) follows from MSE result (13) in Thm. 1 and that the Lipschitz condition leads to \( \|g^*\| \leq L, \forall i \in \mathcal{N} \).

**Theorem 3:** The optimum values of quantization levels and noise parameters maximizing the convergence rate in Th. 2 are the solution to the following optimization problem

\[
P_1: \min_{\{l_i,m_i\}} \sum_{i\in\mathcal{N}} \frac{d}{(l_i - 1)^2} \left[ \frac{1}{4} + m_{i,p}(1 - p) \right] \text{ s.t.} \\
\Delta_{n_1} \geq \frac{\sqrt{2 \ln \frac{10}{\delta}}}{\sqrt{3} M_p (1 - p)} + \Delta_{n_2} \geq \frac{2 \ln 10}{3} M_p (1 - p) \left( \frac{1}{1 - \frac{2}{10}} \right) \quad \text{with } M_p = \max_{i\in\mathcal{N}} m_{i,p}(1 - p) \\
\sum_{i\in\mathcal{N}} m_{i,p}(1 - p) \geq \Delta_s \\
\sum_{i\in\mathcal{N}} d \log (m_{i,t} + l_{i,t}) \leq n C_S, S \subseteq \mathcal{N}, \\
m_{i,t},l_{i,t} \in \mathbb{Z}^+, \forall i \in \mathcal{N}. \tag{20}
\]
ensuring a feasible set for all ratios of dynamical range $\Delta_1/\Delta_2$ is $n = 5d$. The blue line in Fig. 2 shows the boundary for the capacity region of Gaussian MAC. The transmission rates for which the convergence rate is optimized subject to efficiency and jointly with privacy are shown by black and red lines, respectively. As it can be seen, to provide the desirable level of privacy and meanwhile work with optimum convergence rate, it is required to transmit with lower rates implying the privacy-utility trade-off. Also, the corresponding ratios of the gradients’ dynamic ranges to achieve this optimum behaviour are characterized in the figure. Fig. 3 shows the changes in privacy and sum-rate to achieve the optimum convergence rate. As the sum-rate increases the level of privacy needed to achieve the optimality decreases (lower privacy level has higher LDP).

V. CONCLUSION

In this paper, the problem of efficient and private communication over MAC in FL model was considered. In this model, each client sends a digital version of the local gradient using QSGD with specific level of quantization and meanwhile adds binomial noise of discrete values to these quantized gradients in order to preserve the expected level of DP. For this FL model, the mean estimation error, differential privacy and convergence rate was derived. The problem was formulated as to maximize the convergence rate subject to capacity and privacy constraints for which an analytic approach to the optimal solution was presented. It was shown that to meet both efficiency and privacy in FL model over MAC while having faster convergence to the optimum value, we need to work at lower transmission rates compared with the case where just the efficiency is considered.

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The MSE of estimation follows from the definition and taking into account \((22)\) as
\[
\mathbb{E}
\left[
\|g_t - g_t(t)\|_2^2
\right]
= \mathbb{E}
\left[
\frac{1}{N} \sum_{i \in \mathcal{N}} \left[ Q_i^{(i)} - g_t^{(i)} \right] + \frac{1}{N} \sum_{i \in \mathcal{N}} \frac{\Delta^{(i)}}{l_i - 1} \cdot (z_i^{(i)} - m_ip) \right]^2
\leq \frac{1}{N^2} \sum_{i \in \mathcal{N}} \mathbb{E} \left[ \|Q_i^{(i)} - g_t^{(i)}\|_2^2 \right]
+ \frac{1}{N^2} \sum_{i \in \mathcal{N}} \left( \frac{\Delta^{(i)}}{l_i - 1} \right)^2 \mathbb{E} \left[ \|z_i^{(i)} - m_ip\|_2^2 \right]
= \frac{d}{4N^2} \sum_{i \in \mathcal{N}} \left( \frac{\Delta^{(i)}}{l_i - 1} \right)^2 + \frac{d}{N^2} \sum_{i \in \mathcal{N}} \left( \frac{\Delta^{(i)}}{l_i - 1} \right)^2 . m_ip(1 - p)
= \frac{d}{N^2} \sum_{i \in \mathcal{N}} \left( \frac{\Delta^{(i)}}{l_i - 1} \right)^2 \left[ \frac{1}{4} + m_ip(1 - p) \right]. \tag{22}
\]
However, before addressing the privacy issue of this mechanism, it is required to provide an important result of \cite{8} about the privacy level of the binomial mechanism.

\textbf{Lemma 1:} \cite{8} Thm. 1 (Binomial mechanism) A \((\epsilon, \delta)\) differentially-private binomial mechanism \(M_{\text{Bin}}(X) = f(X) + (Z - mp)_s\) with \(Z \sim \text{Bin}(m, p)\) that is used by a client to release the function \(f(X)\) of the input \(X\) subject to \((\epsilon, \delta)\)-LDP satisfies
\[
\epsilon = \frac{\Delta_{f,2\delta} \sqrt{2 \ln \frac{1}{20} \frac{10d}{\delta} + \Delta_{f,1b} \ln \left( \frac{10d}{\delta} \right)}}{s \sqrt{mp(1 - p)}} + \frac{\Delta_{f,2\epsilon c} \sqrt{2 \ln \frac{10}{\delta} + \Delta_{f,1b} \ln \left( \frac{10}{\delta} \right)}}{smp(1 - p) \left( 1 - \frac{\delta}{10} \right)}
+ \frac{\frac{\Delta_{f,\infty} \ln \frac{1}{20} \frac{10d}{\delta} + \Delta_{f,\infty} \ln \left( \frac{20d}{\delta} \ln \frac{10}{\delta} \right)}}{smp(1 - p)} \tag{23}
\]
for any \(\delta, m, p\) and \(s\) and sensitivity bounds \(\Delta_1, \Delta_2, \Delta_\infty\) such that
\[
mp(1 - p) \geq \max \left( 23 \ln \left( \frac{10d}{\delta} \right) , \frac{2\Delta_\infty}{s} \right), \tag{24}
\]
where \(\Delta_{f,q} \triangleq \max_{X, X'} \|f(X) - f(X')\|_q\).

For the LDP analysis, we consider \cite{12} and use \cite{23} with \(s_i = \frac{\Delta^{(i)}}{l_i - 1}\). It can be shown that \(\Delta_{f,q} = \max_{i \in \mathcal{N}} \frac{2}{N} \left\| u \left( g_t^{(i)} \right) - u \left( g_t^{(i)} \right) \right\|_q = \max_{i \in \mathcal{N}} N \frac{2}{N} \Delta^{(i)}_{u,q}\).