A NEW APPROACH TO DETERMINE THE INITIAL MASS FUNCTION IN THE SOLAR NEIGHBORHOOD

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ABSTRACT

Oxygen to iron abundance ratios of metal-poor stars provide information on nucleosynthesis yields from massive stars that end in Type II supernova (SN II) explosions. Using a standard model of chemical evolution of the Galaxy we have reproduced the solar neighborhood abundance data and estimated the oxygen and iron yields of genuine SN II origin. The estimated yields are compared with the theoretical yields to derive the relation between the lower and upper mass limits in each generation of stars and the initial mass function (IMF) slope. Independent of this relation, we furthermore derive the relation between the lower mass limit and the IMF slope from the stellar mass-to-light ratio in the solar neighborhood. These independent relations unambiguously determine the upper mass limit of $m_u = 50 \pm 10 M_\odot$ and the IMF slope index of 1.3–1.6 above 1 $M_\odot$. This upper mass limit corresponds to the mass beyond which stars end as black holes without ejecting processed matter into the interstellar medium. We also find that the IMF slope index below 0.5 cannot be much shallower than 0.8.

Subject headings: Galaxy: evolution — nuclear reactions, nucleosynthesis, abundances — stars: luminosity function, mass function — supernovae: general

1. INTRODUCTION

The initial mass function (IMF) of stars is a key ingredient in modeling the evolution of galaxies. The IMF shape, together with the lower and upper bounds of stellar mass, largely influences the resulting abundance pattern of heavy elements ejected through supernova explosions, the total amount of mass contained in stellar remnants, and the integrated photometric properties of galaxies (cf. Tinsley 1980).

As a conventional practice the IMF is assumed to be a time-invariant mass spectrum having a power law of the form $n(m) dm \propto m^{-(1+\alpha)} dm$ ($m_\nu \leq m \leq m_u$), where $n(m) dm$ is the number of stars in the mass interval $m$ to $m + dm$. The IMF is usually derived from the observed luminosity function of stars, but even the local IMF in the solar neighborhood is still controversial (e.g., Salpeter 1955; Scalo 1986; Hunter 1995), ranging between Salpeter's IMF ($x = 1.35$) and a much steeper IMF ($x \sim 1.9$) for $m \gtrsim 1 M_\odot$. The integrated photoionization rates and colors of nearby disk galaxies are fitted by a Salpeter-like shallower IMF (Kennicutt 1983; Kennicutt, Tamblyn, & Congdon 1994; Sommer-Larsen 1996). Such a global IMF for disk galaxies could be compared to the solar neighborhood IMF, because theoretical arguments indicate that the IMF originates from fragmentation of the gas cloud in a way nearly independent of local physics in the gas, although the IMF slope might change in starbursts (Silk 1977, 1995; Yoshii & Saio 1985; Price & Podsiadlowski 1995).

This controversial situation for determining the IMF slope holds also for the Large Magellanic Cloud (LMC).

Through transformation of the photometric stellar data to the theoretical H-R diagram, Hill, Madore, & Freedman (1994) have obtained a steep IMF ($x = 2$), whereas a significantly shallower slope is derived by Massey et al. (1995) using a similar method ($x = 1.3$), by Hunter et al. (1995) from recent Hubble Space Telescope (HST) observation ($x = 1.22$), and by Will, Bomans, & de Boer (1995a) and Will et al. (1995b) from the stellar luminosity function ($x = 1.1–1.2$). Such a difference in the reported IMFs for either the solar neighborhood or the LMC suggests that the previous methods cannot enable a precise determination of the IMF slope.

In addition, the upper mass limit $m_u$ remains undetermined. The ratio $Y/Z$ (where $Y$ is the helium mass fraction and $Z$ is the mass fraction of metals more massive than He) is a strongly decreasing function of progenitor mass. Then the change in the helium relative to the metal abundance, $\Delta Y/\Delta Z$, from the big bang to the present time is strongly dependent on $m_u$. Maeder (1992, 1993), on the basis of his stellar models with the empirical mass-loss function, found that the observed high ratio $\Delta Y/\Delta Z$ in extragalactic H II regions (Walker et al. 1991; Pagel et al. 1992) can only be matched with $m_u$ as low as $20–25 M_\odot$. This is significantly smaller than the estimate of $m_u \sim 40–80 M_\odot$ by van den Heuvel (1992) from the presence of black hole X-ray binaries and X-ray pulsars in high-mass X-ray binaries.

Recent accumulation of observed heavy-element abundances for the solar neighborhood stars offers an opportunity of using a new method for the IMF determination based on the supernova nucleosynthesis argument. In a supernova explosion, specific nucleosynthetic features will be revealed, and the features averaged over an IMF are incorporated in the gas from which stars of next generations are born. The stars below 1 $M_\odot$ do not eject the synthesized heavy elements into the interstellar medium, but confine the heavy elements produced by massive stars when stars are formed. From the viewpoint of chemical evolution, it does not matter how stars below 1 $M_\odot$ are distributed. The important point is therefore the total stellar mass of such
chemical evolution models for the solar neighborhood

| k    | \( f(T_0) \) | [O/H]_g | [Fe/H]_g | \( P_{\text{II,O}} \) | \( P_{\text{II,Fe}} \) | \( P_{\text{III,Fe}} \) |
|------|-----------|---------|---------|----------------|----------------|----------------|
| 1…… | 0.19      | -0.03   | 0.05    | \( 8.03 \times 10^{-4} \) | \( 3.81 \times 10^{-4} \) | \( 7.76 \times 10^{-4} \) |
|      | 0.25      | 0.00    | 0.08    | \( 9.54 \times 10^{-3} \) | \( 4.52 \times 10^{-4} \) | \( 9.56 \times 10^{-4} \) |
| 2…… | 0.19      | -0.03   | 0.05    | \( 8.21 \times 10^{-3} \) | \( 3.89 \times 10^{-4} \) | \( 7.63 \times 10^{-4} \) |
|      | 0.25      | 0.00    | 0.08    | \( 9.54 \times 10^{-3} \) | \( 4.52 \times 10^{-4} \) | \( 9.23 \times 10^{-4} \) |
| 3…… | 0.19      | -0.03   | 0.05    | \( 8.42 \times 10^{-3} \) | \( 3.99 \times 10^{-4} \) | \( 7.66 \times 10^{-4} \) |
|      | 0.25      | 0.00    | 0.08    | \( 9.66 \times 10^{-3} \) | \( 4.58 \times 10^{-4} \) | \( 9.19 \times 10^{-4} \) |

Note—\( f(T_0) \) is the present mass fraction of the gas, [O/H], and [Fe/H], are the present abundances of oxygen and iron in the gas, respectively. The parameters used in common are \( t_{\text{in}} = 1.5 \) Gyr (the lifetime of Type Ia supernova progenitors), \( t_{\text{in}} = 5 \) Gyr (the timescale of the gas infall), and \( [\text{O/Fe}]_{\text{II}} = +0.4 \) (the oxygen-to-iron abundance ratio for metal-poor Population II stars).

low-mass stars. In other words, we can assume a single-slope IMF over the whole mass range and can know the mass fraction below 1 \( M_\odot \) from the combination of the lower mass limit \( m_1 \), and the IMF slope \( x \) that can reproduce the observed chemical properties.

The mass of ejecta of elements like O, Ne, and Mg from Type II supernovae (SNe II) increases strongly with progenitor mass. On the other hand, the mass of products from explosive burning Si, S, Ar, Ca, and Fe remains close to constant as a function of progenitor mass or even slightly decreases in the case of Fe (Nomoto & Hashimoto 1988; Thielemann, Nomoto, & Hashimoto 1993; Hashimoto et al. 1993b, 1996). Thus, the average ratios O/Fe, Ne/Fe, and Mg/Fe, as they result from a whole population of SNe II integrated over the IMF, will also depend strongly on \( m_1 \), while a weaker dependence is expected for Si/Fe through Ca/Fe.

The O/Fe abundance ratio of nearby stars is among the best observed of these ratios (e.g., Barbuy 1988; Barbuy & Erdelyi-Mendes 1989; Gratton 1991; Nissen & Edvardsson 1992; Edvardsson et al. 1993). Yoshii, Tsujimoto, & Nomoto (1996, hereafter YTN) used the standard chemical evolution model and determined the O and Fe yields from SNe II. These empirical yields are compared with the

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Fig. 1.—Features of chemical quantities in the solar neighborhood. Shown are the models with \( t_{\text{in}} = 1.5 \) Gyr, \( t_{\text{in}} = 5 \) Gyr and \( f(T_0) = 0.19 \) for \( k = 1 \sim 3 \). The symbols represent the data taken from various papers. The model metallicity distributions and the observed one by Wyse & Gilmore (1995) are normalized to coincide with the total number of the sample stars used by Edvardsson et al. (1993). The models for \( f(T_0) = 0.25 \) are indistinguishable from those for \( f(T_0) = 0.19 \) and are therefore not shown except in the lower right-hand panel.
updated supernova nucleosynthesis yields (Hashimoto, Nomoto, & Shigeyama 1989; Hashimoto et al. 1996; Thielemann, Nomoto, & Hashimoto 1996) to derive the relation between the lower and upper mass limits ($m_u, m_l$) and the IMF slope $x$. (Wang & Silk 1993 derived the $m_x$ relation using a fixed value of $m_u = 60 \, M_\odot$ and the averaged O yield over those given by Arnett 1978, Woosley & Weaver 1986, Maeder 1992, and Thielemann, Nomoto, & Hashimoto 1994.) In order to determine a unique combination of ($m_u, m_l, x$), we need another relation among these IMF parameters.

We note that the stellar mass-to-light ratio $M/L$ provides an independent constraint on the IMF (Tinsley 1980; Scalo 1986). As in the case of chemical evolution, we can assume a single-slope IMF in calculating this ratio, because the luminosity from stars below $1 \, M_\odot$ is negligible and therefore how stars below $1 \, M_\odot$ are distributed is not important. Moreover, as far as an IMF decreases toward high masses, the $M/L$ ratio depends sensitively on $x$ and $m_u$, but only very weakly on $m_u, m_l \geq 30 \, M_\odot$. Thus, we obtain the $m_x$ relation that reproduces the present value of $M/L$ in the solar neighborhood.

In this paper we construct the chemical evolution model that reproduces the major observational features in the solar neighborhood ($\S$ 2). Thereby we derive the $m_x$ and $m_x$ relations from the oxygen to iron abundance ratios of metal-poor stars ($\S$ 3) and the $m_x$ relation from the stellar mass-to-light ratio ($\S$ 4). These relations are used to determine $m_u, m_l$, and $x$ for the local IMF in the solar neighborhood ($\S$ 5). The result of the paper is discussed in $\S$ 6.

2. THE CHEMICAL EVOLUTION MODEL

SNe II produce most of the oxygen after a short lifetime $t_{II} \sim 10^{-6}$--$10^{-7}$ yr due to their massive progenitor stars, while SNe Ia produce most of the iron delayed by the considerably longer lifetime $t_{II} \sim 1$ Gyr of their less massive progenitor stars. We employ an instantaneous recycling approximation $t_{II} \ll t$ only for SN II progenitors, and two nucleosynthesis approximations such as (1) $y_{II,O}/y_{II,Fe} = (Z_O/Z_{Fe})_{II} \times [O/Fe]_{II} = 10^{0.04}$ for $[Fe/H] < -1$ (Gratton 1991), and (2) $y_{II,O}/y_{III,O} < 1$ (Tsujimoto et al. 1995). Hereafter we use the effective yield defined as $p_i = y_i/x$, where $y_i$ is the mass of ejected heavy element $i$ and $x$ is the mass fraction locked up in stellar remnants and low-mass stars in each generation of stars. (We note that this fraction $x$ and a rate coefficient $v$ of star formation always appear as their product in the chemical evolution equations.) The use of these approximations in the chemical evolution enables an empirical estimate of $p_{II,O}$, $p_{II,Fe}$, and $p_{III,Fe}$ without advance knowledge of the IMF.

Following the procedure described in YTN, we construct a simplified model of chemical evolution, allowing for material infall from outside the solar neighborhood. The chemical evolution equations describing the time variation of gas fraction $f_g$ and the abundances of oxygen, $Z_O$, and iron, $Z_{Fe}$, are then solved under the boundary condition that these quantities at $T_g = 15$ Gyr should coincide with those observed in the present disk. We assume that the star formation rate is proportional to some power $k$ of the gas fraction $C(t) = v (f_g(t))^k$ and that the infall rate has a modified exponential form with a timescale $t_{in}$. For a possible range of $k = 1$--3, we fix $t_{in} = 1.5$ Gyr and $t_{in} = 5$ Gyr, because both the $[O/Fe]$--$[Fe/H]$ diagram and the $[Fe/H]$ abundance distribution function of long-lived stars are well reproduced by this combination of $t_{in}$ and $t_{in}$ (YTN).

For the present gas fraction, we set $f_g(T_g) = 0.19$ or 0.25, assuming that the surface mass density of the gas component near the Sun is 10 or 11.5 $M_\odot$ $pc^{-2}$, compared with the total mass density of 54 or 46 $M_\odot$ $pc^{-2}$, respectively (see $\S$ 4). For the present abundances of oxygen and iron, we set $([O/Fe]_p, [Fe/H]_p) = (-0.03, +0.05)$ for $f_g(T_g) = 0.19$, and $(0.00, +0.08)$ for $f_g(T_g) = 0.25$, which are adjusted to give the best agreement with the observed chemical properties near the Sun.

Using the above inputs in the chemical evolution equations, we have derived the values of $p_{II,O}$, $p_{II,Fe}$, and $p_{III,Fe}$. The results are tabulated in Table 1 and are shown in Figure 1. The models for $k = 1$--3 in the case of $f_g(T_g) = 0.19$ are shown in the $[O/Fe]$--$[Fe/H]$ diagram (upper left panel), the $[Fe/H]$ distribution function of long-lived disk stars (lower left panel), the age-metallicity relation (upper right panel), and the evolutionary behavior of the gas fraction (lower right panel). The data taken from various papers are shown by symbols. The models shown in the lower left panel are normalized to coincide with the total number of the sample stars used by Edvardsson et al. (1993). We note that the models for $f_g(T_g) = 0.25$ are indistinguishable from those for $f_g(T_g) = 0.19$ in the first three panels and are therefore not shown in these panels. In the last panel, however, the model for $k = 1$ and $f_g(T_g) = 0.25$ is also shown for reference.

The empirical oxygen yield $p_{II,O}$ and the star formation rate $C(t)$ derived here will be used to determine the IMF in the following sections.

3. APPROACH FROM THE NUCLEOSYNTHESIS ARGUMENT

Comparing the empirical O and Fe yields with the updated theoretical supernova yields, we can derive the lower and upper mass limits ($m_u, m_l$) as a function of the IMF slope index $x$. These IMF parameters are subject to the following two relations:

$$p_{II,O} = \frac{\int_{10}^{m_u} M_{II,O}(m)dm}{\int_{10}^{m_l} M_{II,O}(m)dm}$$

and

$$p_{II,Fe} = \frac{\int_{10}^{m_u} M_{II,Fe}(m)dm}{\int_{10}^{m_l} M_{II,Fe}(m)dm}.$$  

The ejected oxygen and iron masses ($M_{II,O}, M_{II,Fe}$) from SNe II are taken from explosive nucleosynthesis calculations for massive stars with $m = 10$--70 $M_\odot$ by Hashimoto et al. (1989), Thielemann, Nomoto, & Hashimoto (1990, 1996). The nucleosynthesis contribution from $m = 8$--10 $M_\odot$ stars is assumed to be negligible (Nomoto 1984, 1987; Hashimoto, Iwamoto, & Nomoto 1993a). These ejected masses for each main-sequence star are tabulated in Tsujimoto et al. (1995) and are shown in Figure 2. It is seen from this figure that the produced oxygen increases steeply as the stellar mass increases.

The calculated oxygen mass is subject to the combined uncertainties involved in the $^{12}C(x, \gamma)^{16}O$ rate and convective overshooting near the end of core He burning. The higher rate and smaller overshooting result in the smaller C/O ratio after He burning and thus smaller Ne/O and
Mg/O ratios after C burning. The range of combined uncertainties is constrained by the requirement that the resultant SN II yields reproduce the solar ratios of Ne/O and Mg/O. The allowed range of oxygen masses thus obtained lies between our standard masses and those by Woosley & Weaver (1995), where the latter calculation produces ~1.3 times larger oxygen masses than our standard case. Taking into account the above uncertainty of the oxygen product, we calculate two cases for the oxygen product, that is, the standard oxygen product obtained by our nucleosynthesis calculations (solid line) and the high-O case in which the standard oxygen products for each main-sequence mass are multiplied by a factor of 1.3 (dotted line).

There also exists an uncertainty concerning the iron product, but this uncertainty is relatively small as far as an IMF decreases toward high masses, because (1) the dependence of the ejected iron mass on the progenitor mass is very weak and therefore (2) the integrated iron product is mainly determined by a lower mass range 10–20 $M_\odot$ as constrained by recent observations of Type II, Ib/c supernovae (SN 1983N, SN 1987A, SN 1990E, SN 1993J, SN 1994I; Nomoto, Iwamoto, & Suzuki 1995). Therefore we take into account only the uncertainty of the oxygen product.

The remnant masses $M_{\text{rem}}$ are taken as the masses of white dwarfs for the initial low-mass stars, and as the masses of neutron stars for the initial massive stars. The initial-final mass relations for white dwarfs and neutron stars are described in YTN. (For the 70 $M_\odot$ star, $M_{\text{rem}} = 1.57 M_\odot$, which is not given in YTN.) As for the lower mass limit below which stars do not experience the mass loss, we set $m_{\text{wd}, l} = 0.7 M_\odot$, and therefore $M_{\text{rem}} = m$ below $m = m_{\text{wd}, l}$.

Using the empirical O/Fe yield ratio on the left-hand side and the nucleosynthesis product masses on the right-hand side, we derive the upper mass limit $m_u$ as a function of the IMF slope index $x$ from equation (1). The result is shown in the upper panel of Figure 3, where solid and dotted lines represent the standard and high-O cases of the oxygen product, respectively. For the standard case a Salpeter IMF ($x = 1.35$) corresponds to $m_u = 50 M_\odot$ and the extremely steep IMF ($x = 1.9$) to $m_u = 80 M_\odot$, whereas these are $m_u = 40$ and $50 M_\odot$ for the high-O case. We note that an uncertainty of $m_u$ arising from the uncertainty in the oxygen products becomes larger for a steeper IMF.

Similarly, using the obtained $m_{\text{u}, x}$ relation in equation (2), we derive the lower mass limit $m_l$ as a function of $x$. Since we assume a single IMF slope over the whole mass range, the lower mass limit derived here might not represent the real $m_l$ if the IMF slope changes below 1 $M_\odot$. The derived $m_l$ is a parameter to determine the total stellar mass for $m < 1 M_\odot$, as mentioned in § 1. The result of the $m_{\text{u}, x}$ relation is shown in the lower panel of Figure 3, where two thin lines correspond to those for the lowest and highest values of $p_{\text{u}, x}$ in Table 1. Contrary to the $m_{\text{u}, x}$ relation, different choices of $M_{\text{u}, x}$ hardly change the $m_{\text{u}, x}$ relation because the IMF steeply decreases toward high masses, so that the right-hand side of equation (2) is not very sensitive to $m_l$. From the obtained $m_{\text{u}, x}$ and $m_{\text{u}, x}$ relations, the mass range covers 0.04–50 $M_\odot$ for $x = 1.35$ and 0.4–80 $M_\odot$ for $x = 1.9$.

In the present analysis we do not include a possible metallicity dependence of the nucleosynthesis yields.
Oxygen and iron are primary yields and do not depend on seed abundances from previous stellar populations. They could be affected only by a metallicity dependence of stellar structure, which is not expected to be large (Woosley & Weaver 1982). More significant is the influence of metallicity-dependent stellar wind losses. If the O output from SNe II is caused almost entirely by yields from hydrostatic stellar evolution, mass loss would cause a drastic change of O/Fe with respect to calculations which neglect this mass loss (Woosley, Langer, & Weaver 1993). In particular, the oxygen yield integrated over an appropriate IMF becomes smaller than that without mass loss, but the difference is not very significant (Wang & Silk 1993; Prantzos, Vangioni-Flam, & Chauveau 1994). This effect of mass loss sets in only for progenitor masses of 40–60 $M_\odot$ with metallicities [Fe/H] $\geq -0.4$ (Langer 1989; Maeder 1990, 1992; Charbonnel et al. 1993; Schaerer et al. 1993). Therefore, the $m_{\ast}$ relation stays unaltered because our analysis of the O/Fe ratio is based on the metal-poor stars before such metallicities are attained. On the other hand, given the empirical value of $P_{\ast}$, the induced reduction in the oxygen yield in the numerator of equation (2) requires lowering the stellar remnant mass in the denominator, which corresponds to an upward shift of the $m_{\ast}$ relation in Figure 3.

4. APPROACH FROM THE M/L ARGUMENT

The total stellar mass $M_\ast$ is very sensitive to the lower mass limit $m_l$, and the total stellar luminosity $L_g$ is sensitive to the IMF slope $x$ rather than $m_l$. For the decreasing IMFs the upper mass limit $m_u$ hardly affects either $M_\ast$ or $L_g$ as far as $m_u \approx 30 M_\odot$ because such massive stars are very few in number and have very short lifetimes. This indicates that the $(M/L)_g$ ratio is essentially determined by a combination of $m_u$ and $x$.

Using the rate of star formation $C(t)$ that reproduces the solar-neighborhood chemical quantities (§ 2), we calculate the present stellar mass $M_\ast(T_g)$ and luminosity $L_{V'}(T_g)$ according to the equations below:

$$M_\ast(T_g) = \int_{m_u}^{m_l(T_g)} dt C(T_g - t) \times \left[ \int_{m_u}^{m_m} mn(m) dm + \int_{m_m}^{m_l} M_{\text{rem}}(m)n(m) dm \right],$$

and

$$L_{V'}(T_g) = \int_{m_u}^{m_l} dt C(T_g - t) \int_{m_m}^{m_l} l_V(m)n(m) dm .$$

The turnoff mass $m_t$ corresponding to an age of $t$ is given by (Renzini & Buzzoni 1986)

$$\log (m_t/M_\odot) = 0.0558 \left[ \log (t/\text{yr}) \right]^2 - 1.338 \log (t/\text{yr}) + 7.764,$$

and $t_{m_l}$ is the lifetime of stars with mass $m_l$. In equation (3) the first and second terms in the brackets denote the contribution from main-sequence stars and stellar remnants, respectively. The stellar mass-luminosity relation $l_V(m)$ in the $V$ band, which is used in equation (4), is taken from the theoretical solar-abundance $m$-$M_{V'}$ relation compiled by Tinsley (1980). This relation is consistent with the recent theoretical relation by Tout et al. (1996) and agrees with the empirical relation by Scalo (1986). In equation (4) we do not consider a possible metallicity effect in the $m$-$M_{V'}$ relation or the luminosity from giant stars. This is because the majority of long-lived disk stars of our concern have metallicities as high as [Fe/H] $\sim -0.1$ (see the lower left-hand panel of Fig. 1), and there is a large uncertainty in estimating the luminosity of giant stars.

The observed $(M/L_{V'})_g$ ratio, which will be compared with the calculated ratio, is obtained from the surface mass density of nearby disk stars and their surface luminosity density of only main-sequence stars. Since the dynamical mass in the solar neighborhood is a sum of masses of stars and gas (Kuijken & Gilmore 1989, 1991), the stellar contribution is evaluated from more direct measurements of the other two, e.g., subtraction of the gas mass from the dynamical mass. Various estimates of the dynamical mass from analyses of vertical motion of disk stars are converged to give $\Sigma_{dy} = 46-54 M_\odot$ pc$^{-2}$ (Kuijken & Gilmore 1989, 1991; Gould 1990; Flynn & Fuchs 1994). Much larger values of 67 and 84 $M_\odot$ pc$^{-2}$ (Bahcall 1984; Bahcall, Flynn, & Gould 1992) are attributed to inadequate choice of tracer stars and restrictive analysis technique (Kuijken 1995). The mass of gas is investigated in detail by Kuijken & Gilmore (1989), and their estimate leads to $\Sigma_{gas} = 13 \pm 3 M_\odot$ pc$^{-2}$. Straightforward application of this estimate of $\Sigma_{gas}$ gives the gas fraction of 19%–35%, the upper bound of which is unrealistically high. Indeed, Young & Scoville (1991) study the gas fraction for all types of spiral galaxies and report that Sb galaxies like our Galaxy have a definite upper bound of 25%. Accordingly we adopt the gas fraction of 19%–25%, which corresponds to $\Sigma_{gas} = 35-44 M_\odot$ pc$^{-2}$ for the surface mass density of nearby disk stars. We note that stellar remnants, brown dwarfs, and giant stars are not subtracted out of the above estimate of $\Sigma_{\ast}$.

Star-count analyses in the direction perpendicular to the Galactic disk provide an estimate of the surface luminosity density of all stars in the $V$ band, such as $\Sigma_{V} = 23.3 L_{V'}$ pc$^{-2}$ (Bahcall 1984), 23.8 $L_{V'}$ pc$^{-2}$ (van der Kruit 1986), and 25.3 $L_{V'}$ pc$^{-2}$ (Yoshii, Ishida, & Stobie 1987). These values are consistent with the value of 24 $L_{V'}$ pc$^{-2}$, which is evaluated from 2.4$\mu$m observations assuming $V-K = 3.2$ (Fazio, Dame, & Kent 1990). By subtracting a contribution from giant stars, these authors have also estimated the surface luminosity density of only main-sequence stars as 9.7 $L_{V'}$ pc$^{-2}$ (Bahcall 1984), 10.2 $L_{V'}$ pc$^{-2}$ (van der Kruit 1986), and 11.4 $L_{V'}$ pc$^{-2}$ (Yoshii et al. 1987), respectively, leading to 9.7–11.4 $L_{V'}$ pc$^{-2}$ for a reliable range.

We therefore conclude that the observed $(M/L_{V'})_g$ ratio in the solar neighborhood is constrained to be 3.1–4.5 $(M/L_{V'})_g$. Using this empirical ratio and theoretical inputs in equations (3) and (4), we obtain the $m_{\ast}$ relation as shown by thick lines in the lower panel of Figure 3. In equation (3) we tentatively use $m_u = 50 M_\odot$, but different choices hardly change the result. The allowable range of the $m_{\ast}$ relation is confined between two lines corresponding to $(M/L_{V'})_g = 3.1$ and 4.5 $(M/L_{V'})_g$.

5. THE IMF DETERMINATION

We have obtained two $m_{\ast}$ relations that are independent of each other; one relation, shown by thin lines in the lower panel of Figure 3, is from the nucleosynthesis argument (§ 3), and another relation, shown by thick lines, is from the mass-to-light ratio argument (§ 4). It is evident that two allowable regions overlap in a narrow range of $x = 1.3–1.6$.
Instead of displaying two independent $m_p$-$x$ relations separately, we substitute the nucleosynthesis $m_p$-$x$ and $m_p$-$x$ relations in equations (3) and (4) to calculate $(M/L)_{a,x}$ as a function of only $x$. The resulting $(M/L)_{a,x}$ relations for different choices of empirical oxygen yield are shown by thin lines in Figure 4. Considering the uncertainty of this relation and an observed range of $(M/L)_{a,x}$, we can obtain $x = 1.3 - 1.6$, which gives the best estimate of $m_p = 50 \pm 10 \, M_{\odot}$ from Figure 3.

If we adopt the metallicity-dependent yield, the $m_p$-$x$ relations, shown by thin lines in the lower panel in Figure 3 (§ 3), move upward, leading to a slightly shallower IMF.

6. DISCUSSION

The IMF slope derived here is consistent with the slope $x = 1.35 - 1.5$ inferred from the photoionization properties of nearby galaxies (Kennicutt 1983; Kennicutt et al. 1994; Sommer-Larsen 1996). Much steeper IMFs are ruled out because they fail to reproduce the O/Fe ratio of metal-poor stars and the $(M/L)_a$ ratio in the solar neighborhood simultaneously.

The upper mass limit of stars corresponds to the maximum supernova progenitor mass beyond which the stars form black holes without ejecting nucleosynthesis products into the interstellar medium rather than neutron stars, the formation of which is associated with the ejection of substantial amounts of metal-enriched material. While the details of the supernova mechanism still await a final explanation, an observational constraint from the O/Fe ratio of metal-poor stars allows us to put a limit of $m_p = 50 \pm 10 \, M_{\odot}$ on the progenitor mass where the transition from neutron star to black hole remnants occurs. This agrees with the estimate of $m_u = 40 - 80 \, M_{\odot}$ by van den Heuvel & Habets (1984) and van den Heuvel (1992) from the presence of black hole X-ray binaries and X-ray pulsars in high-mass X-ray binaries.

The IMF, having a single slope over the whole mass range, extends down to $m_p = 0.03 - 0.16 \, M_{\odot}$, which is comparable to the hydrogen-burning minimum mass ($m \approx 0.08 \, M_{\odot}$). It is very difficult to determine the IMF below $1 \, M_{\odot}$ because it is hard to constrain the luminosity function and there are large uncertainties in the mass–luminosity relation.

The IMF slope might be shallower below 1 and $0.5 \, M_{\odot}$ (Kroupa, Tout, & Gilmore 1993). Taking this possibility into account, we first consider that the IMF slope $x$ below $1 \, M_{\odot}$ could be shallower than that for the massive part of the IMF, provided that the mass fraction below $1 \, M_{\odot}$ is kept unchanged. Since $m_p$ for $x \lesssim 1$ becomes smaller than the theoretical estimate of the minimum Jeans mass 0.007–0.01 $M_{\odot}$ (Low & Lynden-Bell 1976; Silk 1977), we restrict ourselves to $x = 1.1 - 1.2$ and estimate how much mass is contained in nonradiating objects (brown dwarfs). For reference the mass fraction $f_{BD}$ of such objects below 0.08 $M_{\odot}$ but more massive than $m_p$ is tabulated in Tables 2 and 3. Second, we consider an IMF that is shallower below

### TABLE 2

| $x$  | $m_p (\text{STD}/\text{High-O})$ | $m_p$ | $f_{BD}$ |
|------|----------------------------------|------|---------|
| 1.3  | 48/40                            | 0.03 | 0.29    |
| 1.4  | 51/41                            | 0.07 | 0.06    |
| 1.5  | 54/43                            | 0.12 | 0.00    |
| 1.6  | 59/45                            | 0.16 | 0.00    |

Note: The upper and lower mass limits $m_p$ and $m_p$ are in units of solar mass, $M_{\odot}$. The symbol $f_{BD}$ stands for the mass fraction of brown dwarfs below the hydrogen-burning minimum mass of 0.08 $M_{\odot}$ but more massive than $m_p$.

### TABLE 3

| $m \geq 1 \, M_{\odot}$ | $m < 1 \, M_{\odot}$ |
|-------------------------|----------------------|
| $x$ | $m_u$ | $x$ | $m_u$ | $f_{BD}$ |
| 1.3  | 48     | 1.2  | 0.02 | 0.34 |
|      | 1.1    | 0.01 | 0.39 |
| 1.4  | 51     | 1.2  | 0.04 | 0.22 |
|      | 1.1    | 0.02 | 0.28 |
| 1.5  | 54     | 1.2  | 0.05 | 0.09 |
|      | 1.1    | 0.03 | 0.17 |
| 1.6  | 59     | 1.2  | 0.08 | 0.10 |
|      | 1.0    | 0.06 | 0.10 |

| $m \geq 0.5 \, M_{\odot}$ | $m < 0.5 \, M_{\odot}$ |
|-------------------------|----------------------|
| $x$ | $m_u$ | $x$ | $m_u$ | $f_{BD}$ |
| 1.3  | 48     | 0.85 | 0.0004 | 0.41 |
| 1.0  | 1.0    | 0.006| 0.38  |
| 1.4  | 51     | 0.85 | 0.01  | 0.27 |
|      | 1.0    | 0.03 | 0.23  |
| 1.5  | 54     | 0.85 | 0.04  | 0.13 |
|      | 1.0    | 0.06 | 0.07  |
| 1.6  | 59     | 0.85 | 0.08  | 0.01 |
|      | 1.0    | 0.10 | 0.00  |

Note: The upper and lower mass limits $m_u$ and $m_u$ are in units of solar mass, $M_{\odot}$. The symbol $f_{BD}$ stands for the mass fraction of brown dwarfs below the hydrogen-burning minimum mass of 0.08 $M_{\odot}$ but more massive than $m_u$. 

![Figure 4](image-url)
m = 0.5 M_☉, which is suggested from the star-count analysis by Kroupa et al. (1993). The result for the IMF slope x = 0.85–1.0 below m = 0.5 M_☉ is tabulated in Table 3. It is suggested from this table that brown dwarfs may exist in the solar neighborhood (see Carr 1994 for a recent review), but we cannot estimate their exact mass fraction unless the IMF slope below 1 M_☉ is known.

An independent estimate of the mass fraction of brown dwarfs is possible knowing that the dynamical mass is the sum of the gas mass, luminous stars, and brown dwarfs. We here use Σdyn = 50 ± 4 M_☉ pc^–2 (Kuijken & Gilmore 1989, 1991; Gould 1990; Flynn & Fuchs 1994), Σgas = 13 ± 3 M_☉ pc^–2 (Kuijken & Gilmore 1989), and Σ* = 26 ± 2 M_☉ pc^–2 (Bahcall & Soneira 1980; Yoshii et al. 1987; Méra, Chabrier, & Baraffe 1996), where Σdyn stands for the stellar mass excluding the contribution from brown dwarfs. We obtain the mass fraction of brown dwarfs as 30% ± 15%, which is comparable to the other estimates for (m < 0.5 M_☉) or x = 0.85–1.0 (m < 0.5 M_☉) except for the IMF with x = 1.6 for the massive part in Table 3. This indicates that the IMF slope below 1 M_☉ is not much shallower than the slope above 1 M_☉.

Our conclusion derived here is supported by the recent determination of the IMF slope x = 1 ± 0.5 for very low mass stars (m ≤ 0.6 M_☉) by Méra et al. (1996) using both the updated mass-luminosity relation by Chabrier, Baraffe, & Plez (1996) and the luminosity function where a contamination of unresolved binaries is carefully taken into account (Kroupa 1995).