On the Phase Covariant Quantum Cloning

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It is known that in phase covariant quantum cloning the equatorial states on the Bloch sphere can be cloned with a fidelity higher than the optimal bound established for universal quantum cloning. We generalize this concept to include other states on the Bloch sphere with a definite z component of spin. It is shown that once we know the z component, we can always clone a state with a fidelity higher than the universal value and that of equatorial states. We also make a detailed study of the entanglement properties of the output copies and show that the equatorial states are the only states which give rise to separable density matrix for the outputs.

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I. INTRODUCTION

Universal quantum cloning refers to the possibility of constructing unitary transformations which approximately copy an arbitrary quantum state and hence partially alleviate the limitations of the no-cloning theorem [1] (see also [2] and [3]). It was first achieved by Bužek and Hillery in [6] in which they proposed a cloning transformation which cloning arbitrary states with equal fidelity \( \frac{5}{6} \approx 0.83 \). Their pioneering work stimulated a lot of intense research in quantum cloning, a sample of which includes works on proofs of optimality [5, 6, 7, 8], generalizations to \( N \rightarrow M \) cloning [9], cloning of \( d \)-level states [10, 11, 12, 13], and finally experimental realization of cloning by various techniques [14, 15, 16].

Since the optimal bound of 5/6 for fidelity was set for universal cloning, attempts were also made to go beyond this limit by cloning special subsets of states for which we have some a priori partial information. This search was indeed successful and led to the so-called phase covariant quantum cloning [7, 17, 18, 19, 20]. For two level states, indeed successful and led to the so-called phase covariant universal cloning, attempts were also made to go beyond this increase in the final density matrix of the clones in the cloning transformation, which is of the form \( \sum_k |\alpha_k|^2 |k\rangle\langle k| \), becomes automatically state independent (universal), hence no need for making its coefficient vanish by tuning the parameters of the cloning transformation. With the automatic disappearance of this term and one more parameter at hand we find the chance to obtain higher fidelity than the optimal one. This is all the technical point of the phase covariant quantum cloning. There is of course one motivation for studying these states which comes from quantum cryptography, since at least in the BB84 protocol, the states in transfer between the legitimate parties are of this form and an eavesdropper needs only to clone these kinds of states to threat the security of the communication.

However, when we think in terms of physical properties, the partial information that we have about these states is that the z component of their spin is zero. Therefore, it is natural to ask a more general question, that is, how well we can clone a spin states \( |\psi\rangle \) if we know the third component of its spin \( \langle \psi| \sigma_z |\psi\rangle \). This question is specially interesting for those who try to achieve optimal cloning by NMR techniques [14]. In fact this is precisely the state of a nuclear spin which is precessing in magnetic field with a definite energy. In this sense we not only generalize the concept of phase covariant quantum cloning, but describe it in a physically and experimentally interesting context.

We show that there exist a one-parameter family of cloning transformations in which by tuning the parameter one can always clone such states with higher fidelity than the optimal one. Furthermore we show that within this class, the case of equatorial states give a lower fidelity of cloning compared to other states. However they are unique in the sense that they are the only states in this class which give rise to separable density matrix for the outputs copies. We also show that our consideration can be readily generalized to \( d \)-level states.

The crucial property of this class which allows for this higher fidelity is that all the coefficients in their expansion have equal norm. Due to this property a state dependent term in the final density matrix of the clones in the cloning transformation, which is of the form \( \sum_k |\alpha_k|^2 |k\rangle\langle k| \), becomes automatically state independent (universal), hence no need for making its coefficient vanish by tuning the parameters of the cloning transformation.
The structure of this paper is as follows: In Sec. II we study the general properties of a one parameter family of cloning transformations of qubits. In Sec. III we make detailed comparison between different cloning transformations, namely the universal cloning machine proposed by Bužek and Hillery, the phase covariant cloning proposed in [4] and the one proposed in this paper. In Sec. IV we briefly discuss the phase covariant cloning of d-level states [9] in this new context. The paper ends with a conclusion.

II. CLONING TRANSFORMATION OF QUBITS: GENERAL PROPERTIES

Consider the following cloning transformation

\[ U : |0\rangle_a \rightarrow \nu|0\rangle_{a,b}|0\rangle_x + \mu(|01\rangle_{a,b} + |10\rangle_{a,b})_x \]
\[ U : |1\rangle_a \rightarrow \nu|1\rangle_{a,b}|1\rangle_x + \mu(|01\rangle_{a,b} + |10\rangle_{a,b})_x \]

(4)

where on the left hand side we have not shown the blank state and initial state of the cloning machine and on the right hand side, the states from left to right correspond respectively to the input \( (a) \), the copy \( (b) \) and the machine states \( (x) \). The states \( |0\rangle \) and \( |1\rangle \), are also orthonormal regardless of their indices. The only requirement for this transformation to be unitary is that \( \mu \) and \( \nu \) be related as

\[ \nu^2 + 2\mu^2 = 1. \]

(5)

Consider now a general two level state, i.e. a state with a definite spin in the direction \( \mathbf{n} = (\sin \theta \cos \phi, \sin \theta \sin \phi, \cos \theta) \), where \( \theta \) and \( \phi \) are the polar coordinates on the unit sphere. This state has the following form in the \( z \)-basis \( (\sigma_z |0\rangle = |0\rangle, \sigma_z |1\rangle = -|1\rangle) \)

\[ |\mathbf{n}\rangle = \cos \frac{\theta}{2} |0\rangle + e^{i\phi} \sin \frac{\theta}{2} |1\rangle \]

(6)

The output state of the composite system \( ab \) is obtained by tracing out the states of the machine \( x \), that is

\[ \rho_{ab}^{(\text{out})} = \text{Tr}_x(U|\mathbf{n}\rangle\langle \mathbf{n}|U^\dagger). \]

(7)

When acted on by the cloning machine \( U \) this state gives rise to the following density matrix for the output \( a \)

\[ \rho_a^{(\text{out})} = \mu^2 \ 1 + 2\mu \nu |\psi\rangle\langle \psi| + (\nu^2 - 2\mu \nu)(\cos^2 \frac{\theta}{2} |0\rangle\langle 0| + \sin^2 \frac{\theta}{2} |1\rangle\langle 1|). \]

(8)

The new copy \( b \) will also have the same density matrix. The fidelity of cloning defined by \( F := \langle \mathbf{n}|\rho_a^{(\text{out})}|\mathbf{n}\rangle \) is found to be

\[ F(\theta) = \mu^2 + 2\mu \nu + (\nu^2 - 2\mu \nu)(\cos^2 \frac{\theta}{2} + \sin^2 \frac{\theta}{2}) \]

(9)

which after a little algebra using, the fact that \( \cos \theta = \langle \mathbf{n}|\sigma_z|\mathbf{n}\rangle = \langle \sigma_z \rangle \), and the normalization condition \( \nu^2 + 2\mu^2 = 1 \), takes the form

\[ F(\theta) = \frac{1}{2} + \mu \nu + \left( \frac{\nu^2}{2} - \mu \nu \right) \cos^2 \theta \]

(10)

\[ = \frac{1}{2} + \mu \nu + \left( \frac{\nu^2}{2} - \mu \nu \right) \langle \sigma_z \rangle^2. \]

(10)

The last term clearly depends on the input state. All the states on the Bloch sphere with the same value of \( \phi \) are cloned with equal fidelity, a special subclass of these states are the so-called equatorial states, those with \( \langle \sigma_z \rangle = 0 \).

Following Bužek and Hillery [4] it is useful to define and calculate two distances which characterize further the quality of cloning, namely

\[ D_{ab}^{(1)} := \text{Tr}\left[ (\rho_{ab}^{(\text{out})} - \rho_a^{(\text{out})} \otimes \rho_b^{(\text{out})})^2 \right] \]

(11)

which measures the degree of entanglement of the two output copies and

\[ D_{ab}^{(2)} := \text{Tr}\left[ (\rho_{ab}^{(\text{out})} - \rho_a^{(\text{id})} \otimes \rho_b^{(\text{id})})^2 \right], \]

(12)

which measures the distance of the two mode output density matrix with the ideal case of having two disentangled exact copies of the input states.

The calculation of these distances are straightforward but rather lengthy. We give only the final results

\[ D_{ab}^{(1)}(\theta) = A_8(\mu) \cos^8 \frac{\theta}{2} + A_6(\mu) \cos^6 \frac{\theta}{2} + A_4(\mu) \cos^4 \frac{\theta}{2} + A_2(\mu) \cos^2 \frac{\theta}{2} + A_0(\mu) \]

(13)

where

\[ A_8(\mu) = 576 \mu^8 - 768 \mu^6 + 352 \mu^4 - 64 \mu^2 + 4 \]
\[ A_6(\mu) = -1152 \mu^8 + 1536 \mu^6 - 704 \mu^4 + 128 \mu^2 - 8 \]
\[ A_4(\mu) = 672 \mu^8 - 928 \mu^6 + 424 \mu^4 - 72 \mu^2 + 4 \]
\[ A_2(\mu) = -96 \mu^8 + 160 \mu^6 - 72 \mu^4 + 8 \mu^2 \]
\[ A_0(\mu) = 4 \mu^8 + 2 \mu^4 \]

(14)

and

\[ D_{ab}^{(2)}(\theta) = 8 \mu^4 - (6 \mu^4 + \mu^2 + 2 \mu \nu - 1) \sin^2 \theta \]

(15)

III. COMPARISON OF CLONING MACHINES

Until now the value of \( \mu \) has been kept arbitrary. We should now fix it and hence complete definition of our cloning transformation [4]. In the sequel, we consider three different cases.
A. Universal quantum cloning

Looking at Eq. (10), we find that universality, in the sense of Bužek and Hillery, is achieved only by setting \( \frac{\mu^2}{2} - \mu \nu = 0 \) which together with normalization yields

\[
\mu = \frac{1}{\sqrt{6}} \quad F_{\text{universal}} = \frac{5}{6} \simeq 0.833.
\]

Here no optimization should be performed, since the demand of universality has fixed completely the parameter \( \mu \). It is interesting to note that in this case the two distances \( D_{ab}^{(1)} \) and \( D_{ab}^{(2)} \) are also state independent. In fact, by inserting the above value for \( \mu \) in Eqs. (15) and (13) one finds that

\[
D_{ab}^{(1)}(\theta) = \frac{19}{324} \quad D_{ab}^{(2)}(\theta) = \frac{2}{9} \quad \forall \theta.
\]

\[
\mu = \frac{1}{2} F_{\text{opt.}}^{\frac{\pi}{2}} = \frac{1}{2}(1 + \frac{1}{\sqrt{2}}) \simeq 0.854
\]

which is slightly higher than the value for universal quantum cloning.

The distances are found to be

\[
D_{ab}^{(1)}(\frac{\pi}{2}) = \frac{9}{64} \quad D_{ab}^{(2)}(\frac{\pi}{2}) = \frac{7}{8} - \frac{1}{\sqrt{2}}.
\]

Although the distance \( D_{ab}^{(1)} \) for the equatorial states is appreciably higher than the universal value, as we will see below the equatorial states are separable when cloned phase covariantly \([18]\) while in universal cloning machine of Bužek and Hillery the output states are not separable.

B. Phase covariant quantum cloning

In this part we are interested in cloning only the states with \( \langle \sigma_z \rangle = \cos \theta = 0 \). Thus the parameter \( \mu \) is free and we can fix it by maximizing the value of \( F(\frac{\pi}{2}) = \frac{1}{2} + \mu \nu = \frac{1}{2} + \mu \sqrt{1 - 2\mu^2} \). One thus finds

\[
\mu = \frac{1}{2} F_{\text{opt.}}^{\frac{\pi}{2}} = \frac{1}{2}(1 + \frac{1}{\sqrt{2}}) \simeq 0.854
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C. Cloning of states with a definite component of spin along the \( z \) direction

In this case, we fix the value of \( \theta \) and find from Eq. (10) that \( F \) is extremized by two values of \( \mu \) obtained from

\[
\tan^2 \theta = \frac{2\mu \sqrt{1 - 2\mu^2}}{1 - 4\mu^2} \quad \text{or} \quad \mu^2 = \frac{1}{4}(1 \pm \frac{1}{\sqrt{1 + 2\tan^2 \theta}}).
\]

It turns out that the negative sign corresponds to the maximum fidelity. Inserting this value of \( \mu \) in Eqs. (10), (13) and (15) will give us the optimal fidelity and the distances for this class of states. The results are shown in Figs. 1, 2 and 3. It is seen clearly that for each fixed \( \theta \), one can clone the spin state with a fidelity greater than the universal value and for most angles a higher fidelity can be obtained than for the equatorial states. If judged on the basis of the distances \( D_{ab}^{(1)} \) and \( D_{ab}^{(2)} \), it also appears from Figs. 2 and 3 that there are other states which are closer to a product state than the equatorial ones. However the equatorial states are unique in one important respect which is discussed in the next subsection.
D. Separability properties

For the universal cloning case, using Peres-Horodecki’s positive partial transposition (PPT) criterion [21, 22], it has been shown that two output modes are inseparable, while the phase covariant cloning of equatorial states lead to separable copies [18]. To check separability for general angles, we have numerically computed the eigenvalues of the partial transpose of the output density matrix which is of the following form

\[
\begin{bmatrix}
\rho^{out}_{ab}\end{bmatrix}_{Ta} = \begin{pmatrix}
\nu^2 \cos^2 \theta & \mu \nu e^{i\phi} \sin \theta \cos \theta & \mu e^{-i\phi} \sin \theta \cos \theta & \mu^2 \sin^2 \theta \\
\mu \nu e^{i\phi} \sin \theta \cos \theta & \mu^2 \cos^2 \theta & \mu \nu e^{i\phi} \sin \theta \cos \theta & 0 \\
\mu e^{-i\phi} \sin \theta \cos \theta & \mu \nu e^{i\phi} \sin \theta \cos \theta & \mu^2 \cos^2 \theta & \mu \nu e^{i\phi} \sin \theta \cos \theta \\
0 & \mu \nu e^{i\phi} \sin \theta \cos \theta & \mu \nu e^{i\phi} \sin \theta \cos \theta & \nu^2 \sin^2 \theta \\
\end{pmatrix},
\]

(21)

and have found that three of the eigenvalues are always positive, while one of them is marginally negative and becomes zero only for the states on the north pole \(|0\rangle\), the south pole \(|1\rangle\) and the equator of the Bloch sphere. The values of this negative eigenvalue is shown in Fig. [4] and is compared to the negative eigenvalue of the universal machine which is \(\frac{1}{6} - \frac{\sqrt{3}}{6} \approx -0.04\), (while the other eigenvalues of the universal cloning machine being \(\frac{1}{6}, \frac{1}{6}\) and \(\frac{1}{6} + \frac{\sqrt{3}}{6}\), all independent of \(\theta\)).

![Fig. 4: The nonpositive eigenvalue of \([\rho^{out}_{ab}]_{Ta}\) vs. \(\theta\). The dashed line shows its value in universal cloning.](image)

Thus also in this general class of states, the equatorial states are special in that they are completely separable. However, if one considers the multiple criteria of high fidelity and approximate separability then it may be concluded from all the above figures that the states with angles less than \(\theta \leq 0.5\) radians around the north and south poles can be cloned with sufficiently high (larger than 0.9 fidelity) and rather good separability properties.

E. Optimality

In this section we address the question of optimality of the transformations [17]. The general form of these transformations are the same as the original cloning transformations found by Bužek and Hillery [4] and proved to be optimal for universal cloning [1, 2, 17, 18]. Here we have shown that by adjusting the single parameter of these transformations, one can clone states with definite \(z\) components of spin, with a higher than universal fidelity. However it may be possible to go beyond these one parameter family of transformations and obtain even higher fidelity. There is in fact a constructive procedure for deriving the trace preserving completely positive (CP) maps which perform a given task like cloning, to the best approximation. However we think that by following the procedure of [18], our transformations may not retain their simple form that they have now.

IV. GENERALIZATION TO D-LEVEL STATES

Since phase covariant quantum cloning has been achieved for \(d\)-level states, again with a fidelity which is higher than the universal value, it is natural to ask how the above considerations extend to \(d\)-level states. Consider the following cloning transformation [19], which is a simple and natural generalization of (17)

\[
|j\rangle \rightarrow \nu |j, j\rangle_{a,b|x} + \mu \sum_{l \neq j} \langle j, l| + |l, j\rangle_{a,b}|l\rangle_{x}. \quad (22)
\]

It can be easily verified that this transformation is unitary provided that we have

\[
\nu^2 + 2(d-1)\mu^2 = 1. \quad (23)
\]

In particular for 3-level states or qutrits, the transformation is

\[
\begin{align*}
|0\rangle & \rightarrow \nu |0, 0\rangle |0\rangle + \mu(|0, 1\rangle + |1, 0\rangle)|1\rangle + \mu(|0, 2\rangle + |2, 0\rangle)|2\rangle \\
|1\rangle & \rightarrow \nu |1, 1\rangle |1\rangle + \mu(|0, 1\rangle + |1, 0\rangle)|0\rangle + \mu(|1, 2\rangle + |2, 1\rangle)|2\rangle \\
|2\rangle & \rightarrow \nu |2, 2\rangle |2\rangle + \mu(|0, 2\rangle + |2, 0\rangle)|0\rangle + \mu(|1, 2\rangle + |2, 1\rangle)|1\rangle
\end{align*}
\]

(24)

The cloning transformation [22] transforms a pure state

\[
|\psi\rangle = \sum_{j=0}^{d-1} \alpha_j |j\rangle
\]

(25)
where have equal amplitude we have \( \cos \theta \) 
and in phase covariant cloning, one gets rid of the final term by considering only the equatorial states of the form given in Eq. (2). Clearly this is a heavy restriction on the states. To see what this implies for the states in terms of observables we note that the Lie algebra of \( SU(d) \) is spanned by traceless hermitian matrices. The Cartan subalgebra of this Lie algebra which generalizes the \( \sigma_z \) Pauli matrix for spins, is spanned by diagonal traceless matrices, \( H_1, H_2, \ldots H_{d-1} \) normalized to \( \text{tr}H_iH_j = \delta_{ij} \). One convenient choice is \( H_k = \frac{1}{\sqrt{k(k+1)}} \) diagonal \( (1, 1, \ldots, 1, -k, 0, 0, \ldots 0) \). For example, for qutrits we have

\[
H_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & -1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \quad \text{and} \quad H_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 0 & -1 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}
\]

(27)

Thus phase covariant qutrit states are precisely those states for which \( \langle \psi | H_1 | \psi \rangle = \langle \psi | H_2 | \psi \rangle = 0 \). In fact the most general state of a qutrit is given by

\[
| \psi \rangle = \cos \theta | 0 \rangle + \sin \theta \cos \phi e^{i \alpha} | 1 \rangle + \sin \theta \sin \phi e^{i \beta} | 1 \rangle
\]

(28)

and the fidelity of cloning of this state by the transformation \([31]\) is found from Eq. (28) to be equal to

\[
F = \langle \psi | \rho_{out}^{(\text{out})} | \psi \rangle = 2\mu^2 + 2\mu \nu + (\nu^2 - 2\mu \nu)A_\psi
\]

(29)

where

\[
A_\psi := \sum_{k=0}^{d-1} | \alpha_k |^4 = \cos^4 \theta + \sin^4 \theta (\cos^4 \phi + \sin^4 \phi)
\]

(30)

For a phase covariant state where all the coefficients have equal amplitude we have \( \cos \theta = \frac{1}{\sqrt{3}} \) and \( \cos \phi = \sin \phi = \frac{1}{\sqrt{2}} \). For this very specific class of states with only two free parameters \( \alpha \) and \( \beta \), the fidelity is found from Eq. (29) to be \( F = 2\mu^2 + 3\nu^2 + 2\mu \nu \) which is optimized by taking \( \mu^2 = \frac{1}{6}(1 + \sqrt{17}) \) giving a value of \( F_{\text{optimal}} = \frac{1}{2}(5 + \sqrt{17}) \approx 0.76 \) \([31]\). (There exists also another solution, namely \( \mu^2 = \frac{1}{3}(1 - \sqrt{17}) \), but for this particular situation it is the first solution which gives the higher fidelity, however see below and Fig. \([4]\).) As noted above these states are those for which \( \langle H_1 \rangle = \langle H_2 \rangle = 0 \). However instead of restricting oneself to this very specific class of states we can fix \( A_\psi \) and then optimize the fidelity. In this case one finds that the optimum values of \( \mu^2 \) are obtained from

\[
\mu^2 = \frac{1}{3}(1 \pm \sqrt{\frac{\eta}{9+\eta}}),
\]

(31)

where \( \eta \equiv \frac{(1-2A_\psi)^2}{(1-A_\psi)^2} \). Reinserting these optimal values of \( \mu^2 \) in Eq. (29) we obtain the optimum values of fidelity for each value of \( A_\psi \). The results are shown in Fig. \([5]\), where the two curves correspond to different choices of the signs for optimal \( \mu \) (Eq. (31)).

This equation shows that the equatorial states are a very restricted class of states for which the expectation values of all the observables \( H_1 \) and \( H_{d-1} \) have been fixed to zero. By fixing the value of the quantity \( A_\psi \) which has the above simple expression in terms of these observables one can clone a much larger class of states with higher than universal fidelity. In particular one sees that while for two level states there is no difference in the number of parameters of the equatorial states and states with non-zero \( \sigma_z \), the difference in the number of free parameters in the general \( d \) level case can be quite large depending on the value of \( d \).

V. DISCUSSION

We have described the true physical context for phase covariant quantum cloning, that is we have shown that once we have partial information about a state like the \( z \) component of spin or the energy of a nuclear spin in a magnetic field, we can clone such a state with a fidelity higher than the optimal universal fidelity and higher than equatorial states. We have provided a one parameter family of cloning transformation so that for each value of the \( z \) component, we can tune the parameter to obtain the maximum fidelity. We have also shown in this class the equatorial states are the only ones which give rise to separable density matrix for the outputs. However we have shown that it is possible to clone all the states

FIG. 5: The optimum fidelity of cloning a qutrit as a function of \( A_\psi \). The two curves correspond to the two choices of sign in the expression for \( \mu^2 \). It is seen that for obtaining the best possible cloning one should use either the plus or the minus sign depending on the value of \( A_\psi \). Finally we observe that in general and for \( d \) level states, the quantity \( A_\psi := \sum_{k=0}^{d-1} | \alpha_k |^4 \), can actually be expressed in terms of the expectation values of the operators \( H_k \) in the form

\[
A_\psi = \frac{1}{d} + \sum_{k=1}^{d-1} \langle H_k \rangle^2.
\]

(32)
in the vicinity of the north and south pole, for approximately \((θ < 0.5 \text{ radians or } \pi - θ < 0.5 \text{ radians})\), with sufficiently high (larger than 0.9) fidelity and rather good separability properties. The results of this paper may be useful for those who are interested in experimental realization of quantum cloning by using Nuclear Magnetic Resonance (NMR) techniques. We have also discussed how phase covariant quantum cloning of \(d\)-level states can be generalized in the same way.

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