Color Screening and Quark-Quark Interactions in Finite Temperature QCD

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We analyze the screening of static diquark sources in 2-flavor QCD and compare results with the screening of static quark-antiquark pairs. We show that a two quark system in a fixed color representation is screened at short distances like a single quark source in the same color representation whereas at large distances the two quarks are screened independently. At high temperatures we observe that the relative strength of the interaction in diquark and quark-antiquark systems, respectively, obeys Casimir scaling. We use this result to examine the possible existence of heavy quark-antiquark bound states in the high temperature phase of QCD. We find support for the existence of $bb$ states up to about $2T_c$, while $cc$ states are unlikely to be formed above $T_c$.

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I. INTRODUCTION

Correlation functions of static quark and antiquark sources, represented by the Polyakov loop and its hermitean conjugate, have proven to be very useful observables that allow to characterize thermal modifications of the interaction among partons at high temperature. At short distances Polyakov loop correlation functions are sensitive to the interaction between quarks and antiquarks which is transmitted through gluon exchange. The thermal modification of gluons propagating in a hot medium is reflected in the screening of the interaction among static quark and antiquark sources. This allows, for instance, the study of the running of the QCD coupling [1] and its temperature dependence [2, 3]. The large distance behavior of static quark correlation functions is sensitive to the confining properties of the QCD vacuum, the breaking of the string due to pair creation in the vacuum as well as the transition to a deconfined phase at high temperature and density. While at low temperature the correlation functions calculated in a $SU(3)$ gauge theory approach zero at large distances and thus reflect strict confinement of quarks, they approach constant values in QCD with light dynamical quarks indicating the onset of string breaking at some characteristic distance. At zero temperature string breaking arises through the creation of a quark-antiquark pair from the QCD vacuum. With increasing temperature more complex mechanisms involving quarks and gluons from the thermal medium also start playing a role. In the high temperature phase static quark correlation functions approach constant values in QCD as well as in pure gauge theories suggesting that screening of static sources through a gluon cloud is the dominant mechanism for the neutralization of the color charge of static external sources.

Information deduced from the temperature dependence of the change of free energy induced in a thermal medium due to the presence of external sources has been used to model the thermal modification of the potential between heavy quarks. Through an analysis of the non-relativistic Schrödinger equation, which may be a good approximation for heavy quarks, it has been concluded that some heavy quark bound states may also survive the transition to the high temperature phase of QCD [4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Recently it has been argued that also more exotic, colored bound states made up of quark antiquark pairs or quark-quark pairs may exist in a thermal medium at high temperature [11]. To quantify such a scenario for quark-quark systems requires a better understanding of the interaction among quarks at high temperature and the screening of colored diquark states.

First studies of quark-quark interactions at finite temperature have been performed in quenched QCD [16] and some results for QCD with light staggered fermions have been reported at the Lattice conference in 2005 [17, 18]. In this paper we will focus on an analysis of static quark-quark correlation functions in 2-flavor QCD. We will calculate free energies of diquark systems at finite temperature and compare with results obtained for quark-antiquark systems at the same temperature. We will also calculate the induced quark number that arises from the presence of a diquark source that has non-vanishing triality in a thermal bath of quarks and gluons. This provides information on the response of the thermal medium to the presence of external sources and allows to distinguish between the quark dominated screening mechanisms at low temperature (string breaking) and the gluon dominated screening at high temperature.

This paper is organized as follows. In the next Section we introduce the framework for our study of diquark correlation functions, discuss the basic setup for our numerical calculations and present some basic results on thermal properties of diquark and quark-antiquark free energies. In Section III we present results on free energies of color anti-triplet and sextet diquark configurations calculated in Coulomb gauge and discuss the gradual change from string breaking at low temperature to screening at high temperature which becomes transparent in the behavior of the net quark number induced in a thermal medium due to the presence of static quark sources. In Section IV
we discuss consequences of the observed Casimir scaling in diquark and quark-antiquark systems for the possible existence of colored quark-quark bound states at high temperature. We finally give our conclusions in Section V.

II. DIQUARK FREE ENERGIES

The general framework for the analysis of thermal properties of QCD in the presence of \( n \) static quarks and \( m \) static antiquarks has been formulated by McLerran and Svetitsky [19]. We concentrate here on 2 quark systems for which the lattice regularized QCD partition function with \( n_f \) light dynamical quark degrees of freedom and static sources represented by the operator \( L^{(c)}_{QQ}(r) \) is given by,

\[
Z^{(c)}_{QQ}(\beta, m, r) = \int \prod_{x,\nu} dU_{x,\nu} L^{(c)}_{QQ}(r) (\det D(\hat{m}, \mu))^{n_f/4} \times e^{-\beta S_G(U)} ,
\]

where \( U_{x,\nu} \in SU(3) \) are gauge field variables defined on the links of a 4-dimensional lattice of size \( N_x^3 N_r \); \( \beta = 6/g^2 \) denotes the gauge coupling, \( \hat{m} \) is the bare quark mass for the \( n_f \) degenerate quark flavors. Furthermore, \( D(\hat{m}, \mu) \) denotes the fermion matrix, for which we use the staggered fermion discretization scheme. Although all our calculations have been performed at vanishing quark chemical potential (\( \mu \)), we have indicated in Eq. 1 the dependence of the fermion determinant on \( \mu \). We will make use of this in our discussion of the quark number induced in a thermal medium due to the presence of external sources.

In Eq. 1 \( L^{(c)}_{QQ}(r) \) denotes the operator for two static quark sources separated by a distance \( r \). The superscript \((c)\) indicates the color representation of the diquark system which we will specify later. A static quark source is represented by the Polyakov loop,

\[
P(\vec{x}) = \langle z_{n_f}(\beta, \hat{m}) \rangle \prod_{x_0=1}^{N_r} U(\vec{x}, \vec{0}) ,
\]

and the source for antiquarks is given by the hermitean conjugate, \( P^\dagger(\vec{x}) \). In the definition of the Polyakov loop we have introduced a renormalization constant, \( z_{n_f}(\beta, \hat{m}) \), that removes divergent self energy contributions to the static quark sources and insures that the diquark partition function normalized with the partition function in the absence of any sources, has a well defined continuum limit. The renormalization constants for static quark sources have been determined previously for \( n_f = 0 \) (quenched QCD) [20] and 2-flavor QCD (\( n_f = 2 \)) [3] for the action and for values of the gauge coupling used also in this study.

The normalized \( QQ \) partition function,

\[
C^{(c)}_{QQ}(r, T) = \frac{Z^{(c)}_{QQ}(\beta, \hat{m}, r)}{Z(\beta, \hat{m})} = \langle L^{(c)}_{QQ}(r) \rangle ,
\]

is the correlation function for two static quark sources separated by a distance \( r \). Its logarithm defines the free energy of the static diquark system, i.e. the change in free energy of a thermal medium at temperature \( T \) that arises form the presence of two static sources,

\[
F^{(c)}_{QQ}(r, T) = -T \ln C^{(c)}_{QQ}(r, T) .
\]

We note that the discussion of diquark partition functions and free energies given so far is completely analogous to the discussion usually given for the expectation value of the Polyakov loop\(^1\),

\[
L_Q = \frac{1}{N_x} \sum_{\vec{x}} \text{Tr} P(\vec{x}) ,
\]

and the related static quark free energy,

\[
F_Q(T) = -T \ln \langle L_Q \rangle .
\]

The Polyakov loop operator \( L_Q \) simply has been replaced by the diquark operator \( L^{(c)}_{QQ}(r) \).

Similarly one may, of course, also discuss the partition function and free energies of quark-antiquark systems. To be specific we introduce here the so-called color averaged operators\(^2\) for a diquark and quark-antiquark system,

\[
L^{(av)}_{QQ}(r) = \text{Tr} P(0) \text{Tr} P(r) ,
\]

\[
L^{(av)}_{QQ}(r) = \text{Tr} P^\dagger(0) \text{Tr} P(r) .
\]

Like the Polyakov loop also the operator defining the diquark partition function is not invariant under global \( Z(3) \) transformations. The static quark and diquark sources represent states of non-vanishing triality. Due to the \( Z(3) \) symmetry of the \( SU(3) \) gauge action partition functions with non-vanishing triality will be zero in quenched QCD. To insure a proper definition of diquark partition functions also for the pure \( SU(3) \) gauge theory one should introduce a symmetry breaking term which is removed again after taking the thermodynamic limit. In the presence of dynamical quarks the \( Z(3) \) symmetry is, however, explicitly broken and the partition functions with non-zero triality are well defined. The fermion sector of the QCD partition function, Eq. 3 provides the appropriate number of quark and antiquark contributions.

\(^1\) Here and everywhere else we use the normalization of traces in color space \( \text{Tr} \mathbf{1} = 1 \).

\(^2\) Note that we always use renormalized operators for static quark sources.
such that only states with vanishing triality will finally contribute to the partition function. We will elaborate further on this in Section III.B.

The analysis of diquark free energies presented in this paper has been performed on data samples that have been used previously to analyze free energies of static quark-antiquark systems [3]. These data samples have been generated on $16^3 \times 4$ lattice for 2-flavor QCD using an improved staggered fermion action and quark masses, that correspond to a light pseudo-scalar (pion) mass of about 770 MeV. For further details on this data sample and further details on the action and simulation parameters we refer to [3] and references quoted therein. The SU(3) pure gauge data used in Section III.C have been obtained on lattices of size $32^3 \times 4$, where we used the tree-level Symanzik improved gauge action [4]. For further details see [21].

In Fig. 1 we compare the color averaged free energy of a static diquark system with that of a static quark-antiquark system both calculated in 2-flavor QCD. The figure shows free energies as function of the separation of the two sources measured in units of the zero temperature string tension $r\sqrt{\sigma}$. In both cases renormalized Polyakov loop operators, Eq. 2 have been used; no further normalization of the relative magnitude of $QQ$ and $QQ$ free energies thus is needed nor has it been performed. Apparently the free energies of a diquark and a quark-antiquark system approach the same large distance limit at all temperatures. In fact, as noted previously [2, 3], this asymptotic value equals twice the free energy of a static quark represented by the renormalized Polyakov loop, Eq. 4. This observation confirms our picture of screening at large distance; the two static sources are screened independently and the thermal bath does not distinguish between quark and antiquark sources.

At short distances the $QQ$ free energies drop faster than the $QQ$ free energies. This is expected from a perturbative analysis of the interaction in 2 quark-systems; the difference in Casimir factors for one gluon-exchange in quark-quark and quark-antiquark systems suggests that the attractive interaction in an anti-triplet $QQ$ system is half as strong as in a singlet $QQ$ system. The color averaged free energies receive also contributions from octet and sextet representations of a $QQ$ and $QQ$ system, respectively. These contributions are, however, repulsive and thus are exponentially suppressed in Polyakov loop correlation functions at short distances. An additional difference in free energies of $QQ$ and $QQ$ arises from the fact that due to the non-vanishing triality of a $QQ$ state the free energy of a diquark system will receive an additional screening contribution which is not present in a $QQ$ system.

To quantify the differences seen in Fig. 1 and to explore in more detail the mechanism that leads to screening of color charges in a hot medium it is advantageous to study diquark and quark-antiquark correlation functions in definite color channels which can be defined in fixed gauges.

### III. ANTI-TRIPLET AND SEXTET DIQUARK FREE ENERGIES

In addition to the color averaged diquark source introduced in Eq. 9 we will consider here sources in different color channels. This is analogous to studies of color singlet and octet free energies performed for static quark-antiquark systems. A system of two quarks can be either in a color anti-triplet (antisymmetric) or color sextet (symmetric) state,

$$3 \otimes 3 = \bar{3} \oplus 6$$

The corresponding operators defining the anti-triplet and sextet partition functions are given by [3, 4]:

$$L_{QQ}^{(\bar{3})}(r) = \frac{3}{2} Tr P(0) P(r) - \frac{1}{2} Tr P(0) P(r)$$

$$L_{QQ}^{(6)}(r) = \frac{3}{4} Tr P(0) P(r) + \frac{1}{4} Tr P(0) P(r)$$

These operators are, however, gauge dependent and the corresponding partition functions can only be studied in a fixed gauge. We will analyze them in Coulomb gauge. The partition function for the color averaged diquark system can be written as a superposition of partition functions for color anti-triplet and sextet systems,

$$Z_{QQ}^{(\text{av})}(\beta, \tilde{m}, r) = \frac{1}{3} Z_{QQ}^{(\bar{3})}(\beta, \tilde{m}, r) + \frac{2}{3} Z_{QQ}^{(6)}(\beta, \tilde{m}, r)$$

It has already been observed in analogous studies of singlet and octet free energy in quark-antiquark systems that the large distance behavior of free energies does not depend on the relative color orientation of the two static sources. As shown in Fig. 2 this is also reflected in the large distance behavior of diquark free energies. The same gauge invariant asymptotic value is approached by $QQ$ free energies calculated with sources in the anti-triplet and sextet representation.
quark-antiquark system, respectively, this clearly has to the same amount to the free energy of a diquark and that at infinite distances the screening clouds contribute long distance behavior of free energies shown in Fig. 2.

While we just have deduced from the interaction part in the free energies obeys Casimir scaling. However, the gluonic screening cloud in a $QQ$ depends on in-medium properties of the exchanged gluons, such as strong as in a $QQ$ and $\bar{Q}\bar{Q}$ systems, which should be valid at least at high temperature as well as at large and short distances, respectively,

$$F_{QQ,pert}(r, T) \approx 2(\frac{F_{QQ}(r, T) - F_{Q}(T)}{r}) .$$

In Fig. 2 we compare the left and right hand sides of Eq. 15 at various temperatures. As can be seen the equality indeed holds very well at all distances for temperatures above the transition temperature, while some differences show up at intermediate distances below $T_c$. Nonetheless, at short distances the equality in Eq. 15 holds at all temperatures which shows that indeed a well localized anti-triplet diquark system is screened in the same way as a single antiquark. Moreover, we note here that Eq. 15 is found to hold also in the deconfined phase of SU(3) pure gauge theory.

**B. Screening and string breaking**

We want to elaborate here a bit more in detail on the differences seen in the screening of color singlet quark-antiquark and anti-triplet diquark free energies below $T_c$. As noted before the contribution of states with at least one additional quark is needed in the diquark partition function to project on the sector of vanishing triality. At low temperature it is, in fact, a single quark which dominates the additional contributions in a diquark partition function for small separations of the two quarks. At large distances, on the other hand, two antiquarks will be needed to screen the color charges of the two well separated quarks. The situation in a quark antiquark system is quite different. No additional quarks are needed for screening of the sources at short distances, while a quark-antiquark pair is needed for screening at large distances, i.e. the net quark number will always vanish.

At low temperature the screening mechanism at large distances generally is referred to as string breaking. At high temperature such a screening mechanism based on the presence of dynamical quarks in the thermal medium or the vacuum becomes less relevant; screening with gluons becomes the dominant mechanism. To explore the features of string breaking at low and high temperature on a more quantitative level we have analyzed the net quark number induced in a thermal medium due to the presence of a diquark system separated by a distance $r$. 

\[ F_{QQ,pert}(r, T) = \frac{1}{2} F_{QQ,pert}(r, T) + \text{const.} \]

\[ \sim -\frac{2}{3} \frac{\alpha(T)}{r} e^{-m_D(T)r} + \text{const.} \]

This suggest that at least at high temperatures the $r$-dependent $QQ$ and $\bar{Q}\bar{Q}$ interaction part in the free energies obeys Casimir scaling. However, the gluonic screening cloud in a $QQ$ and $\bar{Q}\bar{Q}$ system will differ as a function of distance. While we just have deduced from the long distance behavior of free energies shown in Fig. 2 that at infinite distances the screening clouds contribute the same amount to the free energy of a diquark and quark-antiquark system, respectively, this clearly has to be different at short distances. A $\bar{Q}Q$ system in a color singlet state does not require a screening cloud at short distances; it is color neutral by construction. However, an anti-triplet $QQ$ state needs to be screened even when the separation between both sources vanishes. In fact, we expect that at short distances the free energy of the screening cloud for an anti-triplet $QQ$ state is identical to that of a single static antiquark, $F_{Q}(T)$.
The net quark number can be obtained from the static diquark free energy, Eq. [5] by considering its parametric dependence on the quark chemical potential through the fermion determinant as indicated in Eq. [3]. A derivative of $\ln Z_{QQ}(r, T)$ with respect to $\mu/T$ evaluated at $\mu/T = 0$ then yields the net quark number,

$$ N_{QQ}^{(c)}(r, T) = \langle N_q \rangle_{QQ} = \frac{\langle N_q L_{QQ}^{(c)}(r, T) \rangle}{\langle L_{QQ}^{(c)}(r, T) \rangle} . $$

where $N_q$ is the quark number operator in 2-flavor QCD,

$$ N_q = \frac{1}{2} \mathrm{Tr} \left[ D^{-1}(\vec{m}, 0) \left( \frac{\partial D(\vec{m}, \mu)}{\partial \mu} \right)_{\mu=0} \right] . $$

Similarly we obtain the net quark number induced by a single static quark source,

$$ N_{Q}(T) = \langle N_q \rangle_{Q} = \frac{\langle N_q \mathrm{Tr} P(\vec{0}) \rangle}{\langle \mathrm{Tr} P(\vec{0}) \rangle} . $$

For a quark-antiquark system it is straightforward to see that the induced net quark number indeed is strictly zero for all temperatures, i.e. $N_{QQ}^{(c)}(r, T) = 0$. However, as argued above, for a single quark or diquark system a net quark number has to be induced to generate a state with vanishing triality. In Fig. 4 we show the net quark number induced by color averaged diquark sources separated by a distance $r \sqrt{\tau}$ at temperatures below (upper plot) and above (lower plot) the transition temperature in 2-flavor QCD.

The properties of $N_{QQ}$ become more apparent when we consider also the net quark numbers induced by static sources in an anti-triplet state. In Fig. 5 we show the values for the induced net quark number in color averaged and anti-triplet diquark systems in the large distance limit as well as for the shortest distance available, $rT = 1/N_T$. At all temperatures we find that the net quark number induced for small separations of the sources in the anti-triplet channel is positive and equals the net quark number needed to 'neutralize' a static antiquark source. At large separations, on the other hand, it is minus twice that needed to 'neutralize' a static antiquark source.
thus is 'neutralized' by a light quark from the thermal medium, while at large distances both quark sources are individually 'neutralized' by a light antiquark. While at finite temperature $N_{Q\bar{Q}}$ also receives contributions from other (multi-quark) states, this mechanism becomes increasingly dominant at low temperature. In fact, the data shown in Figs. 4 and 5 support the expected limiting behavior,

$$\lim_{T \to 0} N_{Q\bar{Q}}(r, T) = \begin{cases} 1, & r < r_c \\ -2, & r > r_c \end{cases},$$

(19)

with $r_c$ being a typical hadronic scale, $r_c \simeq 1.5/\sqrt{s} \simeq 0.8$ fm, that characterizes the string breaking radius at zero temperature.

In all cases considered here the induced net quark number rapidly becomes small at high temperature. In fact, already for $T \gtrsim 1.2$ the deviations from zero are smaller than 0.02 at all distances. Thermal fluctuations of the quark number and the largely available number of thermal gluons thus seem to be sufficient to screen the static color sources at high temperature.

C. Screening of sextet diquarks

The considerations of the previous section can also be carried over to sextet diquark systems. The perturbative one-gluon exchange among quarks in a sextet representation is repulsive. Also this interaction is screened at finite temperature. At high temperature we thus expect to find for the genuine 'interaction part' in the sextet free energy,

$$F_{QQ}^{(6)}(r, T) \sim -\frac{1}{4} F_{QQ}^{(1)}(r, T) \sim \frac{1}{3} \frac{\alpha(T)}{r} e^{-m_D(T)r},$$

(20)

$$\lim_{r \to 0} (2F_{QQ}^{(6)}(r, T) + F_{QQ}^{(5)}(r, T)) = 2F_Q(T) + F_Q(T)$$

(21)

where $F_Q(T)$ is the free energy of a fermion in a color sextet representation given by the corresponding Polyakov loop expectation value,

$$e^{-F_Q(T)/T} = \left\langle 1/N^3 \sum_{\vec{x}} L_6(\vec{x}) \right\rangle,$$

(22)

where

$$L_6(\vec{x}) = \frac{3}{2} (\text{Tr} P(\vec{x}))^2 - \frac{1}{2} \text{Tr} P^4(\vec{x})$$

(23)

is obtained from standard group theoretical relations. Eq. (21) is indeed fulfilled at all temperatures in 2-flavor QCD as well as in the deconfinement phase in SU(3) pure gauge theory, as can be seen in Fig. 6. We note that the expectation value of a Polyakov loop in the sextet representation is, of course, gauge invariant. The results on the short distance behavior of sextet as well as anti-triplet quark-quark free energies thus reflect gauge invariant screening properties of colored diquark systems.
IV. HEAVY DIQUARK BOUND STATES IN THE QUARK GLUON PLASMA?

The analysis of diquark and quark-antiquark free energies presented in the previous section suggests that aside from the different screening mechanisms for $Q\bar{Q}$ and $QQ$ systems the $r$-dependence of the free energies obeys Casimir scaling above $T_c$; i.e. the interaction in a diquark system is only half as strong as in a $QQ$ system. An immediate consequence of this is that the temperature dependent coupling defined in terms of $QQ$ systems,

$$\alpha_{\text{eff}} = \frac{3r^2}{2} \frac{dF_Q^{(3)}(r,T)}{dr},$$

(24)

coincides with that extracted previously from color singlet free energies [3].

The close relation between color singlet quark-antiquark free energies and anti-triplet diquark free energies, Eq. [13] of course also carries over to the corresponding energies ($U$) and entropies ($S$) deduced from the free energy, $F = U - TS$, through appropriate partial derivatives with respect to temperature at fixed separation, $r$, of the sources. Free energies as well as combinations of energy and entropy of static quark systems have been used to construct temperature dependent, effective potentials for heavy quark systems that allow to model the temperature dependence of heavy quark bound states in the high temperature phase of QCD. Although such constructions are not rigorous and will include model assumptions, it seems to be clear from the analysis presented in the previous section that any potential constructed for color singlet quark-antiquark systems also puts stringent constraints on the interaction in a diquark system. We expect to have

$$V_{QQ}(r,T) = \frac{1}{2} V_{Q\bar{Q}}(r,T).$$

(25)

If a tightly bound $Q\bar{Q}$ systems can exist at high temperature it thus is conceivable that also $QQ$ states may persist in some temperature regime above $T_c$. As we have seen in the previous section the color charge of such a $QQ$ system will in addition be screened by a gluon cloud.

Using Eq. [25] we have solved the Schrödinger equation for diquark states formed from charm and bottom quarks respectively,

$$\left[ 2m_a + \frac{1}{m_a} \nabla^2 + V_a(r,T) \right] \Phi^a_i = M^a_i(T) \Phi^a_i, \quad a = c, b.$$  

(26)

As we do not want to go into a detailed modeling of heavy quark bound states but rather want to discuss some generic features expected to hold for $QQ$-systems given some knowledge on the behavior of $Q\bar{Q}$-systems, we have used a generic ansatz for the heavy quark potential, which has been used previously also in studies of charmonium and bottomonium systems,

$$V_{QQ}(r,T) = \frac{\sigma}{\mu} \left( 1 - e^{-\mu(T)r} \right) - \frac{\alpha}{r} e^{-\mu(T)r}.$$  

(27)

Here we have used parameters for the coupling in the Coulomb term and the linear rising confinement term of the zero temperature potential that describe well the lattice results for the static quark potential, $(\alpha, \sigma) = (0.385, 0.224\text{GeV}^{-2})$ and adjusted the screening mass, $\mu(T)$, such that the finite temperature potential reproduces the melting temperature of the charmonium ground state, $J/\psi$, at a temperature consistent with lattice studies of spectral functions, $T_{\text{dis}}(cc) \simeq 1.5T_c$ [22, 23, 24]. With this ansatz we find dissociation temperatures for ground states in the bottomonium system ($\Upsilon$) and the corresponding diquark states given in Table I. As can be seen this analysis suggests that $cc$ states are unlikely to exist in the high temperature phase of QCD. They dissolve already at $T_c$. However, $bb$ states dissociate only at temperatures of about $2T_c$.

| state | $\tilde{c}c$ ($J/\psi$) | $cc$ | $bb$ ($\Upsilon$) | $bb$ |
|-------|----------------|-----|----------------|-----|
| $T_{\text{dis}}$ | 0.06 | 0 | 0.3 | 0.07 |
| $T_{\text{dis}}/T_c$ | 1.5 | 1.0 | 3.2 | 2.1 |

TABLE I: Dissociation temperatures, $T_{\text{dis}}$, for heavy quark-antiquark and diquark bound states in units of the transition temperature $T_c$ and dissociation energies for these states at $T_c$.

V. CONCLUSIONS

We have presented results on free energies of static diquark systems. A detailed comparison with corresponding results for free energies of static quark-antiquark systems shows that the relative magnitude of the $r$-dependent part of these free energies obeys Casimir scaling although at short distances the screening of diquark states is quite different from that of quark-antiquark states; colored diquark systems at short distances are screened like static quark sources in the same color representation. Using these results in a model for heavy diquark bound states we showed that it is likely that in a quark gluon plasma $bb$ states can form at temperatures well above the transition temperature while $cc$ states are unlikely to exist above $T_c$.

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