A Learning Design Analysis of the Pre-service Teachers' Mathematics Pedagogical Content Knowledge

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Abstract. A number of international assessments result in Indonesian students' low achievement in mathematics. One substantial factor that affects to the inadequate achievement is the quality of the mathematics teachers. In 2007, teacher certification was launched by the government concerning the improvement program for teachers' qualifications. Nevertheless, the program still struggling to foster the teachers' quality. One fundamental aspect that constitutes to the teachers' qualification is a Pedagogical Content Knowledge (PCK). This study aims to analyze the pre-service teacher PCK from the designed learning materials. This content analysis uses the learning materials which are the final product developed by the pre-service mathematics teachers. The framework from both Maher et.al. and Chick et.al. are used for analyzing the Clearly PCK categorization. Document analysis was used to obtain the data. The result suggests that pre-service teachers lack the ability to consider the students thinking and their misconception to develop effective teaching strategies.

1. Introduction
The Indonesian Government has conducted massive reform in the development of the quality of teachers in Indonesia. The program consists of Teacher Certification, for an in-service teacher, and Teacher Professional Program, for pre-service teachers [1]. The programs develop the teachers; Pedagogical Competency and Professional Competency described in the Permendiknas 16 2007 [2]. The qualification of pedagogical and professional are what should have administered by teachers properly. These qualifications are referred to as Pedagogical Knowledge and Content Knowledge [3]. However, most of the implementation of this knowledge is applied separately. Shulman suggests that the knowledge of pedagogy and content should be applied in the classroom practice by supporting each other by which this idea is called Pedagogical Content Knowledge (PCK) [4,5]. The idea of PCK is how the teachers transfer the content materials from their understanding of the instructional matter to their students [6].

Despite the government program that theoretically brings the vital qualification for the teaching, a recent study revealed that these programs still unable to improve the quality of the teaching by the teachers. The certification program that aimed as the incentive of teachers’ professional development is failed to invite the teacher in improving their knowledge of teaching [7]. The program does success to make the teachers in focusing their routine on the school activities. However, even most of the prerequisite program includes the implementation of the PCK as theoretically and practically, most of the teachers have already unwilling to improve their knowledge especially the knowledge on how to appropriately deliver the content of subject into their students. This matter had to do with the teachers’ belief that has been developed since their pre-service program [8]. Therefore, finding out to what extent the pre-service teachers can implement PCK is crucial.
It is undeniable that PCK has an impact more on mathematics education from teaching and learning activities. PCK help the teachers to deliver the content of mathematics by considering the students' thinking such that it makes sense [6]. The new curriculum of a teacher training program for pre-service teachers has already highlighted the pedagogical competence as well as professional competency [9]. Therefore, finding out how pre-service teachers can implement their knowledge into the lesson plan. Several frameworks have been developed to elicit to what extend the pre-service teachers can implement PCK in their design. Several studies also have discussed this matter although their absence in detailed categories [8,10–12]. Therefore in this study, we used the framework of PCK in mathematics teaching developed by Maher et al. [6]. Table 1 illustrate the framework developed by Maher et al. by which it is not only analysis the PCK categories, but also the interrelation between PK and CK.

Table 1, The framework of PCK on Mathematics Teaching

| PCK Category |
|--------------|
| Clearly PCK  |
| Teaching Strategy |
| Student Thinking |
| Misconception |
| Cognitive Demand |
| Representation of Concept |
| Explanation |
| Knowledge of Example |
| Knowledge of Resource |
| Curriculum Knowledge |
| Purpose of Content Knowledge |
| Content Knowledge in Pedagogical Context |
| Profound Understanding of Fundamental Mathematics |
| Deconstructing Content to Key Content |
| Mathematical Structure and Connection |
| Procedural Knowledge |
| Methods of Solution |
| Pedagogical Knowledge in Content Context |
| Goals of Learning |
| Getting and Maintain Student Focus |
| Classroom Technique |

Pre-service teachers still have no regular class that they have taught. This matter may not be trivial for finding the pre-service teachers qualification on pedagogic and professional. Regarding the importance of eliciting the PCK on the pre-service teachers, in this study, we looked into their designed learning activity. The framework, then, adapted from Maher et al. to the basis of mathematics lesson design. Therefore, in this study, we do a PCK analysis of learning design on the pre-service teachers' mathematics lesson.

2. Method
This study used content analysis technique to elicit the pre-service teachers PCK from their designed mathematics lesson. The data were drawn from the lesson designed by the pre-service teacher as part of their thesis for the undergraduate program. In this study, we focus on the geometry lesson design as the content of the lesson. An instrument was developed based on the framework mentioned in Table 1 on the geometry content and became the primary term in analyzing PCK (see table analysis on each category). The validated analysis instrument based on Maher et al. The framework, then, was used to elicit the PCK quality on three different lesson design. The three learning design consists of geometry content Three-dimensional Figures (G1), Circle (G2), and Volume measurement (G3) as mentioned in Table 2.

Table 2, Pre-service Teacher Lesson Design
Learning Design | Geometry Content | Code
--- | --- | ---
P2MRI design in the subject of Prysm | Three-Dimensional Figures | G1
Pyramid Volume PMRI design on the subject of Circle | Three-Dimensional Figures | G2
PMRI design in the subject of Circle | Circle | G3

Content analysis is conducted involving the scaling of the PCK quality from 0-3. Furthermore, in order to simplify the findings, the PCK categories from Maher et.al. were grouped into (1) PCK on Teaching and Learning and (2) PCK on Mathematics Content. Specific categories can be seen in Table 3.

Table 3, The framework of PCK on Mathematics Teaching

| PCK on Mathematics Teaching | Category | Code |
|----------------------------|----------|------|
| PCK on Teaching Strategy   | TS       |      |
| Teaching                   | ST       |      |
| and                        | MS       |      |
| Learning                   | CD       |      |
| Representation of Concept  | RC       |      |
| PCK on Explanation         | EX       |      |
| Mathematics                | KE       |      |
| Content                    | KR       |      |
| Curriculum Knowledge       | CK       |      |
| Purpose of Content         | PC       |      |

3. Result and Discussion

The data from the thesis document of learning design will be analysed using the validated framework for PCK. The result will be described as follow.

3.1. Teaching Strategy (TS)

In this part, the data related to the Teaching strategy will be presented. The document on teaching strategy in the lesson design can be inferences from two sections, which are learning approach and method and Steps on learning activities. The analysis of the three learning design resulted in Table 4.

Table 4, Analysis of Teaching Strategy

| Code | Teaching Strategy | Level |
|------|-------------------|-------|
| G1   | Elaboration on teaching strategy does not match with the chosen approach | 2     |
| G2   | Elaboration on the teaching strategy follow the chosen approach but no example given | 3     |
| G3   | Elaboration on the teaching strategy follow the chosen approach but no example given | 3     |

3.2. Student Thinking (ST)

In this part, the analysis of the students’ thinking will be presented. The document on students’ thinking cannot be found in lesson design. However, the additional learning resource related to the alternative answer towards discussed problems were described. The analysis of the three learning design resulted in Table 5.

Table 5, Analysis of the Students’ Thinking

| Code | Students’ Thinking | Level |
|------|--------------------|-------|
| G1   | Students' thinking was explained on the alternative answer without examples on students' possible conjectures | 2     |
| G2   | Students' thinking was explained on the alternative answer | 2     |
Students’ thinking was described with additional possible teacher’s response to students’ conjectures

3.3. Misconception (MS)
Students’ misconception is mostly neglected in the learning design. In this part, the analysis of the students' misconception will be presented. The document on students’ misconception could be elicited in the same section when the lesson design was discussing the students' thinking. In details, it appears as a further explanation of the students’ conjectures. The analysis of the three learning design resulted in Table 6.

| Code | Misconception                                      | Level |
|------|---------------------------------------------------|-------|
| G1   | No discussion on the misconception                | 0     |
| G2   | No discussion on the misconception                | 0     |
| G3   | Implicitly discussed the misconception in the alternative answers. However, the conjectures mostly unclear | 1     |

3.4. Cognitive Demand (CD)
The cognitive demand related to the instructional competencies carried out during the lesson. This category can be eliciting from comparing the learning between outcomes and evaluation instrument. However, the additional learning resource related to the alternative answer towards discussed problems were described. The analysis of the three learning design resulted in Table 7.

| Code | Cognitive Demand                                      | Level |
|------|-------------------------------------------------------|-------|
| G1   | Cognitive demand was described along with the measuring tool for learning outcomes as mentioned in the evaluation framework | 3     |
| G2   | Cognitive demand was described along with the measuring tool for learning outcomes as mentioned in the evaluation framework | 3     |
| G3   | Cognitive demand was described along with the measuring tool for learning outcomes as mentioned in the evaluation framework | 3     |

3.5. Representation of Concept (RC)

Figure 1. Example of Teaching Strategy Explanation on G1

Figure 2. Alternative answer provided as an attachment on students’ thinking and misconception in G3 learning design

Table 6, Analysis of Misconception

Table 7, Analysis on the Cognitive Demand
The representation of concept was provided as the learning material used in the learning design. The materials are usually found in a particular section after the learning outcomes. Others design used the students' worksheet materials as a medium to provide the concept for learning. However, the representation of the concept on those to different forms has distinguished impact. The concept represented inside the learning design was meant for full topics coverage, meanwhile, the representation at the beginning of the students' worksheets tend to be limited as their function to guide the students in learning the concept. To what extent the concept representation in the worksheet may help the conceptual understanding is essential. The analysis of the three learning design resulted in Table 8.

Table 8, Analysis on the Representation of Concept

| Code | Representation of Concept | Level |
|------|---------------------------|-------|
| G1   | Full coverage of concept represented in the lesson design with additional representation in the students' worksheet but lack in guiding conceptual understanding | 2     |
| G2   | The representation mainly provided in the students' worksheet as guided conceptual understanding adequately | 2     |
| G3   | The representation mainly provided in the students' worksheet as guided conceptual understanding adequately | 2     |

Figure 3. Alternative answer provided as an attachment evaluation framework on Cognitive Demand in G2 learning design

Figure 4. Concept representation at the beginning of the students' worksheet from G1 learning Design

3.6. Explanation (EX)
The explanation category related to the representation of the concept. Most of the quality in this category rely on the component used in explaining the concept. Whether the design provides adequate information in various forms like diagrams, algebraic symbols, and others is essential. The explanation can be found in the worksheet or content materials in lesson design (see Fig. 4). The analysis of the three learning design resulted in Table 9.

Table 9, Analysis on the Explanation

| Code | Explanation | Level |
|------|-------------|-------|
| G1   | Explanation cover the content with appropriate diagrams and symbols | 3     |
| G2   | Explanation cover the content with appropriate diagrams and symbols | 3     |
| G3   | Explanation cover the content but less supported diagrams and symbols | 2     |
3.7. Knowledge of Example (KE)
Examples on the topics play an important role for the students as an initial step in understanding the concept implementation. However, most of the examples usually in terms of trivial. Additional learning resource related to the alternative answer towards discussed problems were described (see Error! Reference source not found.). Most of the information on the examples can be seen from the students’ worksheet. The analysis of the three learning design resulted in Table 10.

| Code | Knowledge of Example                                                                 | Level |
|------|-------------------------------------------------------------------------------------|-------|
| G1   | Non-trivial examples were provided along with some trivial ones                      | 2     |
| G2   | Non-trivial examples were provided along with some trivial ones                      | 2     |
| G3   | Non-trivial examples were provided with various context as the background            | 3     |

Table 10, Analysis on the Knowledge of Example

3.8. Knowledge of Resource (KR)
The resource of the content may be formed in various ways. However, most of the design usually only used the worksheet as the primary and only resources. The analysis of the three learning design resulted in Table 11.

| Code | Cognitive Demand                  | Level |
|------|-----------------------------------|-------|
| G1   | Only use the worksheet as a resource | 2     |
| G2   | Only use the worksheet as a resource | 2     |
| G3   | Only use the worksheet as a resource | 2     |

Table 11, Analysis on the Knowledge of Resource

3.9. Curriculum Knowledge (CK)
The curriculum about mathematics, especially on what is the standard on mathematics curriculum, should be mentioned in the learning design. This standard has something to do with the whole package of the mathematics curriculum. The analysis of the three learning design resulted in Table 12.
### Table 12, Analysis on the Curriculum Knowledge

| Code | Cognitive Demand                                              | Level |
|------|---------------------------------------------------------------|-------|
| G1   | Core competencies were mentioned without further explanation | 2     |
| G2   | Core competencies were mentioned without further explanation | 2     |
| G3   | Core competencies were mentioned without further explanation | 2     |

3.10. **Purpose of Content (PC)**

The purpose of the content related to how the concept has a usage in the students' daily life or practical problems. This category can also be elicited from the examples and the implementation provided in the learning design. The analysis of the three learning design resulted in **Table 13**.

### Table 13, Analysis on the Purpose of Content

| Code | Cognitive Demand                                              | Level |
|------|---------------------------------------------------------------|-------|
| G1   | Non-trivial examples were provided                           | 2     |
| G2   | Non-trivial examples were provided                           | 2     |
| G3   | Non-trivial examples were provided with various context as the background | 3     |

3.11. **PCK on Teaching and Learning**

In this part, the summary of PCK related to Teaching and Learning will be presented. The level of each category will be presented in Fig 6. From the category of the three different learning design, the knowledge on the students' misconception tends to be discussed inadequately. Moreover, in the students' thinking category, even though the design provides the alternative of the students' answer, they did not describe more on how the students may end up into those solutions [13,14].
3.12. PCK on Mathematics Content

In this part, the summary of PCK related to Mathematics Content will be presented. The level of each category will be presented in Fig 7. From the category of the three different learning design, the knowledge on the students' five categories was captured adequately. The essential remarks pointed in these categories were the knowledge of examples and purpose of the content. In the provided examples, the two lesson design there is still trivial example without any given context. Furthermore, the purpose of content was provided implicitly that in the design, it cannot be discussed directly without teachers' notification. Teachers knowledge in designing various non-routine example and problem as the implementation of the mathematics concept may be the main problem[15]. Most of the problems were available already in the textbooks.

4. Conclusion

The result suggests the pre-service teachers unable to develop a useful teaching strategy because of their inability to consider the students thinking and their misconception. This knowledge of PCK, especially in students' thinking and misconception, were almost neglected in the lesson design. Besides, relating to PCK on Mathematics Content, the design tends to unable providing various non-routine examples and problem implementation. The facts as mentioned earlier suggest that the pre-service teachers may design well-designed learning activities but lack of knowledge in handling students' thinking and misconception and concept examples and implementation.

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