Healing of damaged metal by a pulsed high-energy electromagnetic field

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Abstract. The processes of defect (intergranular micro-cracks) transformation are investigated for metal samples in a high-energy short-pulsed electromagnetic field. This investigation is based on a numerical coupled model of the impact of high-energy electromagnetic field on the pre-damaged thermal elastic-plastic material with defects. The model takes into account the melting and evaporation of the metal and the dependence of its physical and mechanical properties on the temperature. The system of equations is solved numerically by finite element method with an adaptive mesh using the arbitrary Euler–Lagrange method. The calculations show that the welding of the crack and the healing of micro-defects under treatment by short pulses of the current takes place. For the macroscopic description of the healing process, the healing and damage parameters of the material are introduced. The healing of micro-cracks improves the material healing parameter and reduces its damage. The micro-crack shapes practically do not affect the time-dependence of the healing and damage under the treatment by the current pulses. These changes are affected only by the value of the initial damage of the material and the initial length of the micro-crack. The time-dependence of the healing and the damage is practically the same for all different shapes of micro-defects, provided that the initial lengths of micro-cracks and the initial damages are the same for these different shapes of defects.

1. Introduction
The micro-defects of linear dimensions of order 10 \(\mu\)m present the most widely spread dimension of defects in polycrystalline metals. They are mostly formed near the surfaces of the neighboring monocrys-tals (grains). A number of such defects are always formed between the grains after casting, and they can also arise and develop in metals in the process of their plastic deformation. From the standpoint of the further macro-fracture of a piece of metal, the most dangerous defects are the plane micro-cracks.

The hypothesis that the defects in conducting materials can be healed (changed) under the action of short pulses of a high-energy electromagnetic field (HEMF) was suggested by several authors [1–5] who tried to explain the phenomenon of electroplasticity of metals.

The hypotheses of the defect healing in the material can currently be treated as an experimentally observed phenomenon [6–10].

In this cases, the experimental data testify that not only the compressing stresses arise with crater (pore) melting at the defect tips, but the defect shape itself is changed and the continuous structure of the material is reconstructed, which is accompanied by variations in the bulk content of micro-defects (up to the complete disappearance of some micro-defects). This results in a
decrease in the material damage and in an increase in the limit plastic strain till the fracture. But the mathematical models [3–5, 11, 12], which have been proposed until recently, cannot explain these experimental facts. Naturally, under such conditions, the problems of damage variations and, moreover, of the laws of these variations under the HEMF action on the material remained unstudied.

To study the physical processes which occur near micro-cracks under the action of intensive current pulses and to explain the healing effect, a model of the pulse HEMF action on a preliminary damaged material with defects was proposed in [13]. This model allowed one mathematically to describe the experimentally observed process of micro-defect transportation and the decrease in the damage of the metal. The problems of choosing the domain of integration and revealing the influence of the distance between the cracks and their mutual location on the processes of deformation and micro-defect healing were investigated in [14].

In the present paper, we study the influence of the micro-defect shape on the physical fields near them and on the processes of healing of micro-defects. We consider how the healing and damage of material vary in time under the action of a pulse high-energy electromagnetic field.

2. Electro-thermo-mechanical model
We consider the model of electromagnetic field action on a pre-damaged material with defects [13, 14] and use the finite element method for calculations.

We consider a damaged electro-conductive material containing uniform (in shape, size, and orientation) defects like plane thin \((h_0 \ll l_0)\) micro-cracks with rounded tips (figure 1a). We assume that the defects in the material are arranged at the nodes of a rectangular lattice (figure 1b). For this arrangement, we can easily identify the representative volume cell (periodicity cells) as is shown in figure 1a.

The material is subjected to the effect of a short-pulse high-energy electromagnetic field by applying a potential difference to the outer boundaries of the sample, which, on these boundaries, induces an electric current with the density vector perpendicular to the \(xz\)-plane of micro-cracks (figure 1b). We consider high electromagnetic fields inducing an electric current with density varying from \(10^8\) to \(10^{11}\) A/m\(^2\) and duration \(10^{-5}–10^{-4}\) s in the sample.

The solution is sought in the domains of integration (figure 1c–d) containing from one to
four micro-cracks or from one to four parts of representative cells. This choice of the domains of integration is stipulated by the following results. In [14], we showed that the use of other domains of integration (and with more representative cells) does not increase the accuracy of the solution near the micro-crack (centered at $x = 0, y = 0$) and does not affect the healing processes.

During the considered treatment, the following physical processes occur in the material: electromagnetic, mechanical, and thermal. The characteristic time of each of these processes is approximately inversely proportional to the speed of propagation of the respective perturbations. Therefore, the time required for steady-state electromagnetic and mechanical processes is determined from the equilibrium equations written in the form of the virtual work principle [15].

Strains are assumed to be finite. The rates of the elastic, plastic, and temperature strains are assumed to be additive. We also assume that Hooke’s law for an isotropic body and the associated flow rule with the von Mises plasticity condition hold for the rates of elastic and plastic strains. The relationship between the rates of temperature strains and the time-derivative of the temperature is assumed to be linear.

The temperature field $T$ is determined from the energy conservation law. The time of electromagnetic action on the material is small ($10^{-13} - 10^{-12}$ s) and the temperature gradients near the crack tips are very high ($10^7^\circ$C/m) [16], and we ignore the heat conduction and consider the process to be adiabatic.

The complete system of equations for the displacement vector $\mathbf{u}$, the electric potential $\phi$, and the temperature $T$ in the considered electro-thermo-mechanical model has the form [13]

$$
\int_V \nabla \delta \phi \sigma^E(T) \nabla \phi \, dV = \int_S \delta \phi \, j \, dS, \quad j = \sigma^E \mathbf{E} = -\sigma^E \frac{\partial \phi}{\partial \mathbf{x}}, \quad j = -j \cdot \mathbf{n},
$$

(1)

$$
\int_V \sigma : \varepsilon \, dV = \int_S \mathbf{t} \cdot \delta \mathbf{u} \, dS + \int_V \mathbf{f} \cdot \delta \mathbf{u} \, dV, \quad \dot{\varepsilon} = \dot{\varepsilon}^{el} + \dot{\varepsilon}^{pl} + \dot{\varepsilon}^{th},
$$

(2)

$$
\dot{\sigma} = \lambda(T) \dot{\varepsilon}^{el} : \mathbf{I} + 2 \mu(T) \dot{\varepsilon}^{el}, \quad \dot{\varepsilon}^{pl} = \frac{\partial \Phi}{\partial \sigma} = \dot{\Lambda} \mathbf{s}, \quad \dot{\sigma} = \sigma_Y(T), \quad \dot{\varepsilon} = \sqrt{\frac{2}{3}} \mathbf{s} : \dot{\varepsilon}^{th}, \quad \rho(T) c(T) \dot{T} = r^E + r^{pl} + r^{mel} + r^{eval}, \quad r^E = \eta^E \mathbf{j} \cdot \mathbf{E} = \eta^E \nabla \phi \cdot \sigma^E \cdot \nabla \phi, \quad r^{pl} = \eta^{pl} \sigma : \dot{\varepsilon}^{pl},
$$

(3)

$$
T = T_{melt}, \quad t_{sol} \leq t \leq t_{liq}, \quad \int_{t_{sol}}^{t_{liq}} (r^E + r^{pl}) \, dt = \rho \Lambda_{melt},
$$

(4)

$$
T = T_{evap}, \quad t_{liq} \leq t \leq t_{evap}, \quad \int_{t_{liq}}^{t_{evap}} (r^E + r^{pl}) \, dt = \rho \Lambda_{evap},
$$

(5)

where $V$ is an arbitrary volume bounded by the piecewise-smooth surface $S$, $\mathbf{n}$ is the outer normal to $S$, $T$ is the temperature, $\mathbf{E} = -\partial \phi/\partial \mathbf{x}$ is the vector of the electric field intensity determined as the negative gradient of the electric potential, $\sigma^E(T)$ is the electrical conductivity, $j = -j \cdot \mathbf{n}$ is the current density along the normal to the surface $S$, $\delta \phi$ is the variation in the electric potential satisfying the boundary conditions, $\mathbf{u}$ is the displacement vector, $\sigma$ is the stress tensor, $\varepsilon$ is the total strain tensor, $\mathbf{f}$ is the body force vector, $\mathbf{t}$ is the surface force vector, $\delta \mathbf{u}$ and $\delta \sigma$ are the variations in displacements and their associated total strains, $\dot{\varepsilon}^{el}$, $\dot{\varepsilon}^{pl}$ and $\dot{\varepsilon}^{th}$ are tensors of the rates of elastic, plastic, and temperature strains, respectively, $\lambda(T)$ and $\mu(T)$ are Lamé’s moduli of elasticity, $\sigma_Y(T)$ is the yield strength, $\mathbf{s} = \sigma - \frac{2}{3} \sigma : \mathbf{I}$ is the stress tensor deviator, $\Phi$ is the
Figure 2. Dependence of the electrical conductivity $\sigma_E^E$ (a), Young’s modulus $E$ (b), and the yield stress $\sigma_Y$ (c) on the temperature $T$ (vertical dashed lines are the melting point $T_{\text{melt}} = 419^\circ C$ and the evaporation point $T_{\text{evap}} = 906^\circ C$) for a zinc monocrystal.

yield function, $I$ is the unit tensor, $\alpha(T)$ is the coefficient of thermal expansion, $\rho(T)$ is the density, $c(T)$ is the specific heat capacity, $\dot{T}$ is the material derivative of the temperature, $r^k$ is the heat released per volume unit in the current configuration of the body per unit time ($r^E$ is the heat released due to the electric current flow, $r^{\text{pl}}$ is the heat released during the plastic deformation, $r^{\text{melt}}$ is the heat absorbed in the process of melting, and $r^{\text{evap}}$ is the heat absorbed in the process of evaporation), $\eta^E$ is an empirical coefficient defined as the fraction of power of electric current per volume unit dissipated as heat, $\eta^{\text{pl}}$ is an empirical coefficient determined as the fraction of plastic power per volume unit dissipated as heat, $T_{\text{melt}}$ is the temperature of the material melting, $t_{\text{sol}}$ is the time at which the material begins to melt, $t_{\text{liq}}$ is the time at which the material is totally melted, $\Lambda_{\text{melt}}$ is the latent heat of melting, $T_{\text{evap}}$ is the temperature of the material evaporation, $t_{\text{eliq}}$ is the time at which the material begins to evaporate, $t_{\text{evap}}$ is the time at which the material is totally evaporated, and $\Lambda_{\text{evap}}$ is the latent heat of evaporation.

The evolution equation for the temperature takes into account the heat generated per unit volume in the current configuration of the body per unit time due to the electric current (Joule heating), the heat generated during plastic deformation, and the latent heat absorbed in the processes of melting and evaporation. To calculate the temperature field, we supplement the evolution equation for the temperature with additional equations which are the conditions of latent heat absorption during the matter transition from one aggregate state into another (during melting and evaporation).

The temperature in these processes varies from room temperature to the material evaporation temperature [16]. Therefore, all physical and mechanical properties of the material in the proposed model (density, specific heat, electrical conductivity, thermal expansion coefficient, elastic moduli, yield stress, etc.) depend on the temperature (until the evaporation temperature is attained).

At the points where the material is melted, its physical properties are changed sharply, including the conductivity, specific heat, density, linear thermal expansion coefficient, and all other mechanical characteristics of the material. Such a change in the properties of the material is consistent with the available experimental data [17–19]. Figure 2 shows the temperature-dependence of some of these characteristics for zinc.

Thus, after attaining the melting point, the material does not lose its ability to conduct the electric current (figure 2a), and the further heating of the melt occurs. The further fall in the elastic moduli and the yield stress vanishing (figure 2b, c) allow us to describe the melted material behavior at all subsequent moments by constitutive relations which degenerate into thermo-viscoelastic ones with a nonlinear viscosity.
Figure 3. Shapes of microdefects: (a) horizontal microcracks with rounded tips; (b) narrow microcracks with rounded tips; (c) elliptical microcracks; (d) dumbbell-shaped microdefects.

At the points where the material has evaporated entirely, at all subsequent moments, it is assumed that the current density $j = 0$, the stress tensor $\sigma = 0$, and the temperature is constant $T = T_{\text{evap}}$. Thus, in the framework of the proposed model, when the material attains the evaporation temperature, it loses its ability to conduct electric current, and it is not heated further. The metal loses the properties of a viscous fluid and is considered as a rarefied gas.

Because of the symmetry of representative cells, the used integration domains consisting of half or quarters of representative cells (figure 1c, d). On the horizontal and vertical boundaries of the integration domains, we posed the conditions that the normal component of the displacement vector and the tangential components of the stress tensor were absent and that the potential was unperturbed by the presence of a defect. The crack did not conduct a current, so the normal derivative of the potential at its boundary was assumed to be zero. The crack surface was assumed to be free of stresses. The initial fields of temperature, displacements and electric potential were assumed to be homogeneous ($T_0 = 20^\circ\text{C}$, $u_0 = 0$, $\varphi_0 = 0$).

In [13, 14], it was in particular shown that, under the action considered in this model, the volume of micro-defects decreases with time. Following the first works of the theory of damaged media [20], we associate the damage with the formation of voids in the material (porosity) due to the formation and development of microcracks and microvoids in the process of loading. We introduce the notation: $V(t)$ is the volume of the unit micro-defect, $V_0 = V(0)$ is the initial volume of the defect, and $V_{\text{re}}$ is the volume of the representation cell containing such a microdefect (figure 1). We define the damage $f = f(t)$ and healing $\chi = \chi(t)$ parameters as follows:

$$f(t) = \frac{V(t)}{V_{\text{re}}}, \quad \chi(t) = \frac{V(0) - V(t)}{V(0)}. \quad (7)$$

The initial damage of the material at time $t = 0$ is equal to $f_0 = f(0) = V_0/V_{\text{re}}$. Thus, the damage of the material decreases and the healing increases due to the action of the electromagnetic field.

3. Results of numerical simulation

The coupled equations of the model are solved numerically together with boundary, contact, and initial conditions. The computations were performed for the plane strain using liner four-node isoparametric and three-node finite elements. The mesh was reconstructed on the basis of the arbitrary Euler–Lagrange method. The temperature is not a degree of freedom of the problem. The temperature evolution equation was solved directly at each point of integration using the temperature derivative approximation by the inverse difference in time.

The modeling was performed for samples manufactured of zinc, it physical and mechanical properties were temperature dependent [17–19].

The dimensions (a and b) of the representative cells varied from 7.5 to 37.5 $\mu$m due to the spread of the grain dimensions of the polycrystalline zinc.

The considered forms of microcracks are shown in figure 1a–c. To complete the picture, a dumbbell-shaped microdefect (figure 3d) was also considered (figure 1d). All defects had the same initial length (the size on the axis $x$) $l_0 = 10 \mu$m and the same area (in the plane $xy$). Thus, the initial damage $f_0$ for all forms of microdefects was the same. In the calculations, it varied
in the range \( f_0 = 0.27 - 2.45\% \). It seems that, for large initial damage, it is already impossible to treat the material as undamaged in the sense of macro- (meso-) fracture.

The potential difference (per unit length) used in the computations was 534.3 mV/mm, which corresponds to the current of density 8.95 kA/mm\(^2\) in materials without defects. The potential difference was assumed to be constant during the entire time of the pulse action \( \tau_0 = 90 \mu s \).

In [13, 14], it was shown that when the electric current flows through a sample with micro-cracks, there arise large gradients of the electric field near the cracks and this leads to a significant increase in the current density near the crack tips compared with the density of the current applied to the sample. This results in fast inhomogeneous local heating near the micro-crack tip accompanied by its thermal expansion (at the same time, there is no heating near the crack center and on its shores). The inhomogeneous thermal expansion generates large compression stresses (the pressure can exceed 100 MPa) near the crack and, as a consequence, results in simultaneous healing of the crack (narrowing the crack) thus decreasing the crack length and releasing the melted metal into the crack (figures 4 and 5).

Further, we study how the micro-defect shape affects the variations in the healing \( \chi(t) \) and damage \( f(t) \) of the material under the action of the current. Figure 6 shows the dependence of the healing \( \chi(t) \) and damage \( f(t) \) on the time \( t (\mu s) \) for squared representative cells of different dimensions with micro-defect shaped as horizontal, narrowing, and elliptic micro-cracks and as a dumbbell (the corresponding shapes are shown in figure 3a–c and figure 3d).

Figure 6 shows that, in the entire considered range of distances between the micro-cracks (initial damage), the curves of variation in the healing \( \chi(t) \) and damage \( f(t) \) practically coincide with each other for the same initial damage \( f_0 \). This effect (\( \chi(t) \) and \( f(t) \) are independent of the micro-defect shape) also takes place even for micro-defects of a cardinally different shape even for the dumbbell-shaped micro-defect.

The change of the representative cell shape from square to rectangle for the same \( f_0 \) does not practically change the dependence \( \chi(t) \) and \( f(t) \) shown in figure 6. In this case, the variations in the length and width of the representative cells did not exceed 25% of the dimensions shown in figure 6.

Now we consider the physical fields generated by the action of the current near the micro-defects.

Figure 4 presents the temperature fields induced by the current flow in the representative cell of size \( a = b = 10 \mu m \) with defects of different shapes at time \( t = 25.4 \mu s \). This figure shows that the temperature field is practically independent of the defect shape in the whole domain of integration except for the extreme points of the micro-defects on the axis \( x \) (which we call “micro-defect tips” for brevity). The temperature fields (figure 4) induce similar states in the entire domain of integration (except for the neighborhoods of the micro-defect tips).

Figure 7 illustrate the results of computation of the displacement fields \( u_y \) in the representative cell of size \( a = b = 10 \mu m \) with defects of different shapes at time \( t = 25.4 \mu s \).

At the other times, the obtained fields of temperature, stress, and displacement are also quantitatively close to one another for different shapes of defects. The considered stress-strain states show that the variations in the defect volume in the course of time depend on their shape.
Figure 4. Healing of various micro-defects at time $t = 25.4 \mu s$ (the dashed line shows the initial boundaries of the defects) and the temperature contours ($^\circ C$), the area of melted metal is colored gray.

Figure 5. Boundaries of various micro-defects (the dashed line shows the initial boundaries of the defects) at time $t = 48.0 \mu s$ and the contours of the equivalent plastic strain.

very weakly (for the same initial volumes and lengths of the defects).

The above-described deformation process is accompanied by very intensive plastic flow near the tips of micro-defects. Figure 5 shows the equivalent plastic strain (the second invariant of the plastic strain tensor) at time $t = 48.0 \mu s$.

The residual plastic strains in these regions attain hundred percents. This fact allows us to believe that the shape of the micro-defect cannot be reconstructed in its initial dimensions.

There may be no viscous fracture of the material and formation of a new discontinuity (defect) at the site of the healed micro-crack, because the plastic deformation occurs under the conditions of strong uniform compression.

As was already noted, the experiments, see [1, 2, 6–10], confirmed that the continuity of the material structure is reconstructed in the material and this phenomenon is accompanied
Figure 6. Dependence (a) healing $\chi$ and (b) damage $f$ on time $t$ (\(\mu s\)) for microdefects of different shapes (horizontal, narrow, elliptical microcracks and dumbbells) and various initial damage $f_0$.

by variations in the bulk content of micro-defects (till the total disappearance of several micro-defects). The results obtained above qualitatively agree with these experimental data.

Conclusions
In the modeling of the process of micro-crack healing, one can, without loss of generality, consider only one quarter of the representative cell (figure 1c) as the domain of integration. In this case, in the entire range of distances between the defects (or equivalently, for any initial damage),
Figure 7. Contours of displacements $u_y$ (mm) at time $t = 25.4 \mu s$ for defects of different shapes (initial damage $f_0 = 2.45\%$).

the healing $\chi(t)$ and the damage $f(t)$ vary identically in time under the action of current and their variation is independent of whether the domains of integration, where these functions are calculated, consist of one or several representative cells.

The decisive factor for variations in the healing and damage of the material in the course of time under the action of a current pulse is not the micro-defect shape (for the same length of the micro-defect) but the parameter of the material damage introduced above. The modeling shows that, for the same equal initial damage of the material $f_0$, no significant variations in the shape of the micro-defect can practically affect the dependence $\chi(t)$ and $f(t)$.

The obtained fields of temperature, stress, and displacement near micro-defects of different shapes turn out to be quantitatively close to one another.

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