Rock Burst Classification Prediction Method Based on Weight Inverse Analysis Cloud Model

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Abstract. A cloud model comprehensive evaluation method based on weight inverse analysis is proposed and applied to the prediction of rock burst grade. At first, the objective function with weight as independent variable is derived and established. The objective weight of each evaluation factor is obtained by genetic algorithm. Then the concrete steps of the coupling of weight inverse analysis and cloud model are given. After that, $\sigma_0/\sigma_c$, $\sigma_c/\sigma_t$ and Wet are selected as the evaluation factors. $\sigma_0/\sigma_c$ is the ratio of the maximum tangential stress in the cavern to the compressive strength of rock. $\sigma_c/\sigma_t$ is the ratio of tensile strength to the compressive strength of rock. Wet is the elastic energy index. According to the rock burst engineering example, the inverse analysis and calculation of the factor weights are carried out. Finally, this method is applied to the rock burst grade prediction of Jiangbian Hydropower Station and Maluping Mine. Compared with the cloud model prediction results of other weighting methods, its feasibility and effectiveness has been verified. The research shows that the cloud model for rock burst based on weighted inverse analysis is less subjective in the weighting process and the prediction effect is better.

1. Introduction

The prediction and evaluation of rock burst occurrence and its intensity grade is a research hotspot in the field of underground rock engineering. In recent years, scholars have put forward some Rock burst classification prediction method, such as fuzzy comprehensive evaluation (WANG Yuanhan 1998), distance discriminant analysis (GONG Fengqiang 2007), extension evaluation (QIU Daohong 2010), grey classification (PEI Qutai 2013), projection pursuit method (XU Fei 2010), and neural network method (FENG XT 1994). These methods are all multi-index comprehensive evaluation methods, which overcome the one-sidedness of single index. However, there are deficiencies in these methods because of the randomness and fuzziness of the prediction and evaluation process of rock burst.

In the process of rock burst grade prediction and evaluation, on the one hand, because of the complicated geological conditions of rock strata, the errors of testing instruments and the level of operators, there are more or less random errors in the measured values of evaluation factors. On the other hand, the measured values of multiple evaluation factors often fail to meet a certain grade standard at the same time in practice. In order to comprehensively consider the above randomness and fuzziness, the cloud model proposed by Academician Li Deyi has been introduced into the prediction and evaluation of rock burst grade (WANG Yingchao 2015, HAO Jie 2016, LIU ZB 2013).
Reasonable weighting method is very important to multi index comprehensive evaluation method. However, the existing cloud models for rock burst grade prediction mostly use simple equivalence weights or weights based on expert experience, which disturbs the evaluation results of cloud models by subjective factors and affects the accuracy of prediction. Therefore, it is necessary to further study the objective weighting method of cloud model factors.

In this paper, the cloud model theory is coupled with the weighted inverse analysis method. On the basis of deducing and establishing the optimized objective function, the genetic algorithm is used to search the factor weight vector when the objective function value is maximal. Thus, the cloud model of rock burst grade prediction based on weighted inverse analysis is established. After compared with cloud model prediction effect of other weighted methods, we prove the feasibility and effectiveness of the method in this paper.

2. Coupling Method Between Cloud Model and Weight Inverse Analysis

2.1. Coupling Method between Cloud Model and Weight Inverse Analysis

Determining factor weight is an important step and component of multi factor comprehensive evaluation method. The method of determining factor weights can be divided into subjective weighting method, objective weighting method and combination weighting method. In order to avoid the influence of subjective factors on the determination of factor weights, inverse analysis method (QIU Daohong 2010) is used to determine weights in this paper.

Assuming that the evaluation index vector is $x$, the weight vector is $\omega$, and the mapping relationship between the evaluation index and the evaluation result $y$ is $f$, then there is

$$ y = f(x, \omega) $$

Suppose there are $m$ samples in the sample set, the predictive grade vector of the sample is $y=(y_1, y_2, ..., y_m)$, the actual grade vector of the sample is $Y=(Y_1, Y_2, ..., Y_m)$, $g=(g_1, g_2, ..., g_m)$ is the indicator vector. If $y_i = Y_i$, the corresponding element of the indicator vector $g_i=1$, otherwise $g_i =0$, where $i=1, 2, ..., m$.

On the premise that the evaluation sample set grade $Y$, evaluation index $x$ and function relation $f$ are know, weight inverse analysis based on optimization algorithm is to obtain the most consistent weight vector $\omega$ with the actual grade (assuming the number of vector elements is $n$). The mathematical expression for calculating index weight from sample set is as follows:

$$ \sum_{i=1}^{2} \omega_i = 1, \max(\text{fitness} = \sum_{i=1}^{n} g_i) $$

Among them, $\sum_{i=1}^{2} \omega_i = 1$, $m$ is the number of samples, $n$ is the number of evaluation indicators, $g_i$ is the $i$th element of the indicator vector defined above, $\text{fitness}$ is the optimal fitness function.

Using the optimization ability of the optimization algorithm, the objective weight of the evaluation index can be obtained by finding the factor weight vector $\omega$ when fitness is maximized.

2.2. Cloud Model

Based on Cloud Model Theory, in the field of rock burst grade prediction, $E_x$, $E_n$ and $H_e$ are defined as the average value, discrete range and range uncertainty of rock burst state data in different engineering examples. According to the above three digital characteristics, Formula 1 can be used to calculate that a certain index measured value $x$ belongings to a certain certainty degree $x$. The Gauss cloud model is used in this paper.
In the formula above, \( \mu \in [0,1] \), \( \sigma^2_{En} \) is a random value, obeys the Gauss distribution with \( En \) as expectation and \( H^2_e \) as variance, i.e. \( \sigma^2_{En} \sim N(En, H^2_e) \).

After calculating the certainty degree \( \mu_i \) of a certain level from formula 3, the comprehensive determination degree \( \Omega \) should be calculated from formula 4 combined with the weight of each factor. The rock burst grade should be judged according to the comprehensive determination value.

\[
\Omega = \sum_{i=1}^{n} \mu_i \omega_i
\]  

(4)

2.3. Comprehensive Evaluation Method of Cloud Model Based on Weight Inverse Analysis

In order to reduce the interference of subjective factors on the factors weighted results and optimize the prediction effect of cloud model, a comprehensive evaluation method based on weighted inverse analysis is proposed. This method couples cloud model theory with weighted inverse analysis method. Its core is to establish inverse analysis fitness function. Based on formula (1)-(4), fitness functions with weight vectors \( \omega \) as independent variables can be derived as follows formula (5)

In the above formula, \( i, j \) and \( k \) represent the \( i \)th category, the \( j \)th evaluation index and the \( k \)th sample, respectively. \( p, n \) and \( m \) are the total number of categories, the total number of evaluation indexes and the total number of samples, respectively. \( x(k,j) \) is the predicted values of the \( j \)th evaluation index of the \( k \)th sample, \( y_i \) is the predicted category of the \( k \)th sample. \( Y_i \) is the actual category of the \( k \)th sample.

\[
\mu(i,j) = \exp\left(-\frac{(x(k,j)-E(i,j))^2}{2\sigma^2_{E(i,j)}}\right)
\]

\[
\Omega = \sum_{j=1}^{m} \mu(i,j) \omega_j
\]

\[
\max_{i=1}^{p} \Omega = \Omega_i \Rightarrow y_i = i
\]

\[
g_i = \begin{cases} 
1 & (y_i = Y_i) \\
0 & (y_i \neq Y_i) 
\end{cases}
\]

\[
\text{fitness} = \sum_{i=1}^{p} g_i
\]

(5)

From the above fitness function, it can be found that the index weight is determined by inverse analysis method, aiming at the consistency between the predicted results of cloud model and the actual situation of samples. This helps to reduce the interference of subjective factors in the process of building cloud model and optimize the evaluation effect of cloud model.

The implementation process of this method is as follows: (1) Selection of evaluation indicators. (2) Collect and sort out sample examples, calculate the digital features \( E_x, E_n \) and \( H_e \) of cloud model, and establish cloud model preliminarily. (3) According to formula (5), the optimization objective function code is compiled in MATLAB. (4) The analysis object is sample set. The formula (5) is the objective function. Under the constraint condition that the sum of the indicators weights equals 1, the genetic algorithm is used to optimize the indicators weight vector \( \omega \). The index weights are obtained, which are most consistent with the actual situation. (5) According to formula (4), the comprehensive
certainty degree \( \Omega \) is calculated, and then the prediction results are given by \( \Omega \). (6) The new cloud model is applied to engineering practice to verify its feasibility and effectiveness.

3. Prediction of Rock Burst Grade

3.1. Evaluation Indicators and Classification Criteria

It is generally believed that the main factors affecting the rock burst grade are \( \sigma_\theta \), \( \sigma_c \), \( \sigma_c/\sigma_r \), \( \sigma_\theta/\sigma_c \), \( \sigma_c/\sigma_t \) and \( W_e \). \( \sigma_\theta \) is the maximum tangential stress of the cave. \( \sigma_c \) is the rock compressive strength of c, \( \sigma_r \) is the ratio tensile strength of rock. \( W_e \) is the elastic energy index. Literature (LIU ZB 2013) shows that the weights of factors such as the \( \sigma_\theta \), \( \sigma_c \), \( \sigma_t \) are relatively small. Therefore, three indices of the \( \sigma_\theta/\sigma_c \), \( \sigma_c/\sigma_t \) and \( W_e \) are selected as evaluation factors for the prediction and evaluation of rock burst grade. The rock burst grade is divided into four categories, including non-rock burst (I), slight rock burst (II), medium rock burst (III) and strong rock burst (IV). The classification table of rock burst is shown in Table 1.

| Rock burst grade | \( \sigma_\theta/\sigma_c \) | \( \sigma_c/\sigma_t \) | \( W_e \) |
|------------------|----------------|----------------|------|
| I                | 0.00–0.30     | 40.00–55.00    | 0.00–2.00 |
| II               | 0.30–0.50     | 40.00–26.70    | 2.00–3.50 |
| III              | 0.50–0.70     | 26.70–14.50    | 3.50–5.00 |
| IV               | 0.70–1.00     | 0.00–14.50     | 5.00–6.50 |

3.2. Digital Characteristics of Cloud Model

According to the principle of cloud model, the cloud numerical characteristics of rock burst grade can be calculated by formula 6. The specific results are shown in Table 2.

\[
E_x = \left( \frac{\alpha_{max} + \alpha_{min}}{2} \right) \\
E_n = \left( \frac{\alpha_{max} - \alpha_{min}}{3} \right) \\
H_e = 0.01
\]

In the above formula, \( \alpha_{max} \) and \( \alpha_{min} \) are respectively the upper and lower limit values corresponding to a certain rock burst grade of the evaluation factor.

| Rock burst grade | \( E_x \) | \( E_n \) |
|------------------|---------|---------|
| I                | 0.15    | 0.100   |
| II               | 0.40    | 0.033   |
| III              | 0.60    | 0.033   |
| IV               | 0.85    | 0.050   |

* \( H_e = 0.01 \)

3.3. Weight Inverse Analysis Result

After obtaining the digital features of cloud model, the fitness function of optimization is compiled in Matlab according to formula 5. The eighteen rock burst cases of tunnel engineering at home and abroad given in reference (WANG Yuanhan 1998) are taken as analysis objects. The genetic algorithm toolbox in matlab is used to search the weight vector \( \omega \) when fitness is maximal. The weight result of
each factor is shown in Table 3. For comparison, the factor weights by other weighting methods are also listed in Table 3.

Table 3 shows that although the factors weights given by different methods are different, the factor weights of Wet are smaller than those of the two other factors. The influencing factors of rock burst grade mainly include stress, lithology and impact tendency of surrounding rock. $\sigma/\sigma_c$ is the main index to characterize the stress state of surrounding rock, $\sigma/\sigma_t$ is the main index to characterize the lithology of surrounding rock. While the elastic energy index Wet based on complete rock samples can only partially characterize the impact tendency of rock mass containing joints and fissures. So its weight value is relatively small.

In Table 3, using prediction cloud model under different weighting methods, the rock burst grade prediction accuracy of 18 examples in reference (WANG Yuanhan 1998) is also given. Obviously, the prediction accuracy of cloud model in this method is significantly higher than that of other weighting methods.

Table 3. Rock burst classification based on cloud model with different weighting methods.

| Weighting method               | Weighting method | Weight value | Prediction accuracy |
|-------------------------------|------------------|--------------|---------------------|
| Equivalent weight method      |                  | $\sigma/\sigma_c$ | 0.333 0.333 | 0.333 50.0 |
| Delphy method (WANG Yingchao 2015) |                  | $\sigma/\sigma_t$ | 0.163 0.674 | 0.163 66.7 |
| Expert experience (WANG Yuanhan 1998) |                  | $W_{et}$ | 0.400 0.300 | 0.300 61.1 |
| Inverse analysis method       |                  | $W_{et}$ | 0.318 0.422 | 0.260 88.9 |

4. Engineering Applications

This method is applied to the rock burst grade prediction of Jiangbian Hydropower Station and Maluping Mine. Compared with the cloud model prediction results of other weighting methods, its feasibility and effectiveness can be verified.

Example 1: Jiangbian Hydropower Station

Jiangbian Hydropower Station is the last level hydropower station on Jiulong River, a tributary of Yalong River. It is constructed with a dam diversion scheme. The diversion tunnel is situated on the left bank of Jiulong River, with a total length of 8.5 kilometers, a maximum depth of 1690 m. The section with buried depth over 300m accounts for 53% of the total length. The surrounding rock of the tunnel is mainly biotite granite. Its saturated uniaxial compressive strength can reach about 100 MPa and the maximum in-situ stress is about 40 MPa. According to engineering experience, the tunnel has already obtained the basic conditions of rock burst. The rock burst index data obtained from field measurement is shown in Table 4. The rock burst grade prediction for some sections of the tunnel by using the method presented in this paper is also shown in Table 4. We can see that the predicted results of this method are consistent with the actual situation.

Table 4. Prediction of rock burst for Jiangbian Hydropower station (QIU Daohong 2010).

| Mileage | Measured value of evaluation factor | Weighting method of cloud model |
|---------|------------------------------------|--------------------------------|
|         | $\sigma/\sigma_c$ | $\sigma/\sigma_t$ | $W_{et}$ | Equivalent weight method | Delphy method | Expert experience | Inverse analysis method | Actual |
| 300     | 0.47                 | 18.24               | 2.46     | II                     | III           | II               | III                  | II     |
| 400     | 0.52                 | 17.66               | 2.86     | II                     | III           | II               | III                  | III    |
| 500     | 0.54                 | 17.23               | 2.94     | II                     | III           | III              | III                  | III    |
| 600     | 0.58                 | 16.57               | 3.05     | III                    | III           | III              | III                  | III    |
Example 2: Ma Luping Mine

Ma Luping Mine is located in Jinzhong Town, Kaiyang County, Guizhou Province. It has been exploited for more than 40 years. With the increase of mining depth, the crustal stress is also increasing, and the surrounding rock near the working area shows high brittleness and hardness. So the rock burst risk is increasing. This method is used to predict and analyze the rock burst grade in the 600-meter-long middle section, which is one of the main mining sections. The rock burst index data obtained from the test, the prediction results of this method and the actual rock burst grade are all shown in Table 5. It can be seen that the results obtained by this method are in accordance with the actual situation.

| Lithology | Measured value of evaluation factor | Weighting method of cloud model | Actual |
|-----------|------------------------------------|---------------------------------|--------|
|           | $\sigma_0/\sigma_c$ | $\sigma_c/\sigma_t$ | $W_{et}$ | Equivalent weight method | Delphy method | Expert experience | Inverse analysis method |
| A         | 0.74                         | 24.44                         | 6.31     | III                        | III           | III               | IV                | IV                |
| B         | 0.23                         | 6.67                          | 1.39     | I                          | IV            | I                 | I                 | I                 |
| C         | 0.61                         | 24.00                         | 5.10     | III                        | III           | III               | III               | III               |
| D         | 1.00                         | 11.24                         | 2.03     | IV                         | IV            | IV                | IV                | IV                |

*ABCD in the table represents sandstone, dolomite, ore and red shale respectively.

From the analysis of the above 8 groups of data engineering examples, it can be seen that there are deviations between the cloud model prediction results using other weighting methods and the actual situation. And the prediction results of this method are consistent with the actual situation, which proves that the prediction effect of this method is better.

5. Conclusion

Aiming at the objective weighting of evaluation factors in practical application, a cloud model comprehensive evaluation method based on weight inverse analysis is proposed and applied to the prediction of rock burst grade. The main conclusions are as follows:

1. Selecting stress coefficient $\sigma_0/\sigma_c$, brittleness coefficient $\sigma_c/\sigma_t$ and elastic energy index $W_{et}$ as evaluation factors, the objective function with weight as independent variable is derived and established. Then the objective weight of each evaluation factor is obtained by genetic algorithm.

2. Compared with Delphi method, expert experience method and simple equivalence weight method, this method couples weight inverse analysis and cloud model to establish rock burst prediction cloud model. It can not only reduce the interference of subjective factors, but also significantly improve the prediction accuracy of cloud model.

3. This method is essentially a data mining method based on existing knowledge base. So the quantity and quality of sample instances will affect the application effect of this method. Therefore, it is necessary to collect more sample examples and use reasonable methods to optimize data.

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