Probing Dark Matter in the Economical 3-3-1 Model

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ABSTRACT: We show that the economical 3-3-1 model has a dark matter candidate. It is a real scalar \( H_1^0 \) in which main part is bilepton (with lepton number 2) and its mass is in the range of some TeVs. We calculate the relic abundance of \( H_1^0 \) dark matter by using micrOMEGAs 2.4 and figure out parameter space satisfying the WMAP constraints. Direct and indirect searches are studied for a special choice of parameters in the WMAP-allowed region.

KEYWORDS: 3-3-1 Model, Dark Matter, Extensions of Electroweak Higgs sector, WMAP.

ArXiv ePrint: 1110.1482
1 Introduction

Now is a golden age of cosmology and astrophysics. Many abstractive notions such as black holes, dark matter (DM), dark energy etc. have become step by step more scientifically feasible and widely accepted subjects. According to the WMAP [1] the non baryonic dark matter, which is called the cold dark matter (CDM), must exist and contain approximately 22% of all energy density of the universe. The characteristic of the CDM is thought to be massive and rarely interact with ordinary matter. There is yet little astrophysical data which bear on the CDM. However, there are a few proposals to explain the DM in the context of particle physics [2]. The most popular particles in this class are the sterile neutrino, the axion, the lightest supersymmetric particle and etc.

The DM does not exist within the SM. It has to be realized beyond the SM at the electroweak scale or above, so that newly introduced particles in those models are potentially good candidates for the DM. In most supersymmetric models, there is a conserved multiplicative quantum number R-parity, which implies that the lightest superpartner is stable and can be a DM candidate. This kind of model can not only explain the original DM but also represent the greatest expectations in particle physics at TeV scale to be probed by the LHC. However, there is yet no experimental evidence to support the models. The other way to extend the SM is the enlargement of the gauge symmetry group from
SU(3)\textsubscript{C} \otimes SU(2)\textsubscript{L} \otimes U(1)\textsubscript{Y} to larger groups. In particular, there exists a simple extension of the SM gauge group to SU(3)\textsubscript{C} \otimes SU(3)\textsubscript{L} \otimes U(1)\textsubscript{X}, the so called 3-3-1 models [3, 4].

Depending on the electric charge of particle at the bottom of the lepton triplet, the 3-3-1 models are classified into two main versions: the minimal model [3] with the lepton triplet (ν, l, l^c)\textsubscript{L} and the version with right-handed (RH) neutrinos [4], where the RH neutrinos place at the bottom of the triplet: (ν, l, ν^c)\textsubscript{L}. In the 3-3-1 model with right-handed neutrinos, the scalar sector requires three Higgs triplets. It is interesting to note that two Higgs triplets of this model have the same U(1)\textsubscript{X} charges with two neutral components at their top and bottom. In the model under consideration, the new charge X is connected with the electric charge operator through a relation

\[ Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X. \]  

(1.1)

Assigning these neutral component vacuum expectation values (VEVs) we can reduce the number of Higgs triplets to two. Therefore, we have a resulting 3-3-1 model with two Higgs triplets [5, 6]. As a consequence, the dynamical symmetry breaking also affects the lepton number. Hence it follows that the lepton number is also broken spontaneously at a high scale of energy. Note that the mentioned model contains a very important advantage, namely, there is no new parameter, but it contains very simple Higgs sector; therefore, the significant number of free parameters is reduced. To mark the minimal content of the Higgs sector, this version that includes right-handed neutrinos (RHν) is going to be called the economical 3-3-1 model.

We would like here to emphasize that by choosing different electric charge operators, we can get a few different versions of 3-3-1 model such as the minimal 3-3-1 model, the 3-3-1 model with right handed neutrino, the economical 3-3-1 model and etc. However, all those models have the same motivations such as

1. The family number must be a multiplicative of three.
2. It could explain why the value \( \sin^2 \theta_W < \frac{1}{4} \) is observed.
3. It can solve the strong CP problem.
4. It is the simplest model that includes bileptons of both types: scalar and vectors ones.
5. The model has several sources of CP violation.

Besides those motivations, the 3-3-1 models can contain a candidate for the dark matter. As an example, the 3-3-1 model with right handed neutrino exhibits that the scalar bilepton is a candidate for the dark matter as shown in Ref. [7], in which the authors provided a new quantum number to forbid the interaction of bilepton with the SM particles. However, in the economical 3-3-1 model, which we are considering now, the model naturally contains a candidate for the dark matter even without introducing a new quantum number or a discrete symmetry.
The paper is organized as follows: Section 2 is devoted to a brief review of the model. The Higgs sector is considered in the effective approximation \(w \gg v, u\) in section 3. Here, we also find out stable Higgs - a candidate for the dark matter. The relic abundance as well as its dependence/indepence of parameters is figured out in section 4. We study direct and indirect searches for dark matter in section 5. Conclusions are given in the last one - section 6.

2 A brief review of the model

The particle content in this model, which is anomaly free, is given as follows:

\[
\begin{align*}
\psi_aL &= (\nu_aL, l_aL, (\nu_aR)^c) \sim (3, -1/3), & l_aR &\sim (1, -1), & a &= 1, 2, 3, \\
Q_{1L} &= (u_{1L}, d_{1L}, U_L) \sim (3, 1/3), \\
Q_{aL} &= (d_{aL}, -u_{aL}, D_{aL})^T \sim (3^*, 0), & D_{aR} &\sim (1, -1/3), & \alpha &= 2, 3, \\
u_{aR} &\sim (1, 2/3), & d_{aR} &\sim (1, -1/3), & U_R &\sim (1, 2/3),
\end{align*}
\]

(2.1)

where the values in the parentheses denote quantum numbers based on the \(\text{SU}(3)_L \otimes \text{U}(1)_X\) symmetry. Unlike the usual 3-3-1 model with right-handed neutrinos, where the third family of quarks should be discriminating [8], in this model under consideration the first family has to be different from the two others [9]. The electric charges of the exotic quarks \(U\) and \(D_\alpha\) are the same as of the usual quarks, i.e., \(q_U = 2/3, q_{D_\alpha} = -1/3\).

The spontaneous symmetry breaking in this model is obtained by two stages:

\[
\text{SU}(3)_L \otimes \text{U}(1)_X \rightarrow \text{SU}(2)_L \otimes \text{U}(1)_Y \rightarrow \text{U}(1)_Q.
\]

(2.2)

The first stage is achieved by a Higgs scalar triplet with a VEV given by

\[
\chi = (\chi_0^1, \chi_2^0, \chi_3^0) \sim (3, -1/3), \quad \langle \chi \rangle = \frac{1}{\sqrt{2}} (u, 0, \omega)^T.
\]

(2.3)

The last stage is achieved by another Higgs scalar triplet needed with the VEV as follows

\[
\phi = (\phi_1^0, \phi_2^0, \phi_3^0) \sim (3, 2/3), \quad \langle \phi \rangle = \frac{1}{\sqrt{2}} (0, v, 0)^T.
\]

(2.4)

The Yukawa interactions which induce masses for the fermions can be written in the most general form:

\[
\mathcal{L}_Y = \mathcal{L}_{\text{LNC}} + \mathcal{L}_{\text{LNV}},
\]

(2.5)

in which, each part is defined by

\[
\begin{align*}
\mathcal{L}_{\text{LNC}} &= h^U_{1L}Q_1^U & \quad & D_{1R} \\
&+ h^D_{a\beta} &Q_{aL} &\phi_d &\phi^*_D &\phi^*_d, \\
&+ h^d_{aeta} &Q_{aL} &\phi^*_d &\phi^*_D &\phi^*_d, &\phi^*_d, \\
&+ h^u_{a\alpha} &Q_{aL} &\phi^*_u &\phi^*_U &\phi^*_u, &\phi^*_u, \\
&+ h^u_{a\alpha} &Q_{aL} &\phi^*_u &\phi^*_U &\phi^*_u, &\phi^*_u, \\
&+ H.c., \\
\mathcal{L}_{\text{LNV}} &= s_{\alpha}^U Q_{1L} &Q_{aL} &\phi^*_d &\phi^*_U &\phi^*_d, \\
&+ s_{\alpha}^D &Q_{1L} &\phi^*_d &\phi^*_U &\phi^*_d, &\phi^*_d, \\
&+ s_{\alpha} &Q_{aL} &\phi^*_d &\phi^*_U &\phi^*_d, &\phi^*_d, \\
&+ H.c.,
\end{align*}
\]

(2.6) (2.7)
Table 1. Nonzero lepton number $L$ of the model particles.

| Field | $\nu_{aL}$ | $l_{aL,R}$ | $\nu'_{aR}$ | $\chi^0_1$ | $\chi^\pm_1$ | $U_{L,R}$ | $D_{aL,R}$ |
|-------|-------------|-------------|-------------|------------|-------------|-----------|-------------|
| $L$   | 1           | 1           | -1          | 2          | 2           | -2        | -2          |

Table 2. $B$ and $L$ charges of the model multiplets.

| Multiplet | $\chi$ | $\phi$ | $Q_{1L}$ | $Q_{aL}$ | $u_{aR}$ | $d_{aR}$ | $U_R$ | $D_{aR}$ | $\psi_{aL}$ | $l_{aR}$ |
|-----------|--------|--------|----------|----------|----------|----------|-------|----------|-------------|----------|
| $B$-charge | 0      | 0      | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ | $0$ | $0$ |
| $L$-charge | $\frac{2}{3}$ | $-\frac{2}{3}$ | $-\frac{2}{3}$ | $\frac{2}{3}$ | $0$ | $0$ | $-2$ | $2$ | $\frac{1}{3}$ | $1$ |

where the subscripts $p$, $m$ and $n$ stand for SU(3)$_L$ indices.

The VEV $\omega$ gives mass for the exotic quarks $U$, $D_\alpha$ and the new gauge bosons $Z'$, $X$, $Y$, while the VEVs $u$ and $v$ give mass for the quarks $u_a$, $d_a$, the leptons $l_a$ and all the ordinary gauge bosons $Z$, $W$ [9]. To keep a consistency with the effective theory, the VEVs in this model have to satisfy the constraint

$$u^2 \ll v^2 \ll \omega^2. \tag{2.8}$$

In addition we can derive $v \approx v_{\text{weak}} = 246 \text{ GeV}$ and $|u| \leq 2.46 \text{ GeV}$ from the mass of $W$ boson and the $\rho$ parameter [6], respectively. From atomic parity violation in cesium, the bound for the mass of new natural gauge boson is given by $M_{Z'} > 564 \text{ GeV}$ ($\omega > 1400 \text{ GeV}$) [10]. From the analysis on quark masses, higher values for $\omega$ can be required, for example, up to 10 TeV [9].

The Yukawa couplings of (2.6) possess an extra global symmetry [11, 12] which is not broken by $v, \omega$ but by $u$. From these couplings, one can find the following lepton symmetry $L$ as in Table 1 (only the fields with nonzero $L$ are listed; all other fields have vanishing $L$). Here $L$ is broken by $u$ which is behind $L(\chi^0_1) = 2$, i.e., $u$ is a kind of the SLB scale.

It is interesting that the exotic quarks also carry the lepton number (so-called leptoquarks); therefore, this $L$ obviously does not commute with the gauge symmetry. One can then construct a new conserved charge $\mathcal{L}$ through $L$ by making a linear combination $L = x T_3 + y T_8 + \mathcal{L} I$. Applying $L$ on a lepton triplet, the coefficients will be determined

$$L = \frac{4}{\sqrt{3}} T_8 + \mathcal{L} I. \tag{2.9}$$

Another useful conserved charge $\mathcal{B}$, which is exactly not broken by $u$, $v$ and $\omega$, is usual baryon number: $B = BI$. Both the charges $\mathcal{L}$ and $\mathcal{B}$ for the fermion and Higgs multiplets are listed in Table 2.

Let us note that the Yukawa couplings of (2.7) conserve $\mathcal{B}$, however, violate $\mathcal{L}$ with $\pm 2$ units which implies that these interactions are much smaller than the first ones [9]:

$$s_a^u, s_{aa}^d, s^D_\alpha, s^U_\alpha \ll h^U_\alpha, h^D_\alpha, h^d_\alpha, h^u_\alpha. \tag{2.10}$$
In this model, the most general Higgs potential has very simple form [13]

\[ V(\chi, \phi) = \mu_1^2 \chi^\dagger \chi + \mu_2^2 \phi^\dagger \phi + \lambda_1 (\chi^\dagger \chi)^2 + \lambda_2 (\phi^\dagger \phi)^2 + \lambda_3 (\chi^\dagger \chi)(\phi^\dagger \phi) + \lambda_4 (\chi^\dagger \phi)(\phi^\dagger \chi). \] (2.11)

It is noteworthy that \( V(\chi, \phi) \) does not contain trilinear scalar couplings and conserves both the mentioned global symmetries, this makes the Higgs potential much simpler and discriminative from the previous ones of the 3-3-1 models [11, 12, 14]. The non-zero values of \( \chi \) and \( \phi \) at the minimum value of \( V(\chi, \phi) \) can be obtained by

\[ \chi^\dagger \chi = \frac{\lambda_3 \mu_2^2 - 2 \lambda_2 \mu_1^2}{4 \lambda_1 \lambda_2 - \lambda_3^2} \equiv \frac{u^2 + \omega^2}{2}, \] (2.12)

\[ \phi^\dagger \phi = \frac{\lambda_3 \mu_2^2 - 2 \lambda_1 \mu_3^2}{4 \lambda_1 \lambda_2 - \lambda_3^2} \equiv \frac{v^2}{2}. \] (2.13)

Any other choice of \( u, \omega \) for the vacuum value of \( \chi \) satisfying (2.12) gives the same physics because it is related to (2.3) by an SU(3)\(_L \otimes U(1)_X \) transformation. It is worth noting that the assumed \( u \neq 0 \) is therefore given in a general case. This model of course leads to the formation of Majoron [13].

### 3 Stable Higgs bosons in 3-3-1 model

This section is to show that the economical 3 − 3 − 1 model furnishes a good candidate for self interacting dark matter. The important properties are that dark matter must be stable and neutral. Hence, we are going to consider the scalar sector of the model and specially neutral scalar sector, and we can specify whether any of them can satisfy the self interacting dark matter conditions.

Let us review the Higgs states and coupling constants. In this model, the most general Higgs potential has very simple form given in (2.11). As usual, we first shift the Higgs fields as follows:

\[ \chi = \begin{pmatrix} \chi_1^P + \frac{u}{\sqrt{2}} \\ \chi_2^P \\ \chi_3^P + \frac{\omega}{\sqrt{2}} \end{pmatrix}, \quad \phi = \begin{pmatrix} \phi_1^P + \frac{u}{\sqrt{2}} \\ \phi_2^P \\ \phi_3^P \end{pmatrix}. \] (3.1)

The subscript \( P \) denotes \textit{physical} fields as in the usual treatment. The constraint equations at the tree level are given as

\[ \mu_1^2 + \lambda_1 (u^2 + \omega^2) + \lambda_3 \frac{u^2}{2} = 0, \] (3.2)

\[ \mu_2^2 + \lambda_2 v^2 + \lambda_3 \frac{(u^2 + \omega^2)}{2} = 0. \] (3.3)

Note that \( u \) is a parameter of lepton-number violation, therefore the terms linear in \( u \) violate the latter. Applying the constraint equations (3.2) and (3.3) we get the minimum value, mass terms, lepton-number conserving and violating interactions as follows

\[ V(\chi, \phi) = V_{\text{min}} + V_{\text{mass}}^N + V_{\text{mass}}^C + V_{\text{LNC}} + V_{\text{LNV}}, \] (3.4)
where

\[
V_{\text{min}} = -\frac{\lambda_2}{4} v^4 - \frac{1}{4} (u^2 + \omega^2)[\lambda_1 (u^2 + \omega^2) + \lambda_3 v^2],
\]

\[
V_{\text{mass}}^N = \lambda_1 (u S_1 + \omega S_3)^2 + \lambda_2 v^2 S_2^2 + \lambda_3 v (u S_1 + \omega S_3) S_2,
\]

\[
V_{\text{mass}}^C = \frac{\lambda_4}{2} (u \phi_1^+ + v \chi_2^+ + \omega \phi_3^+) (u \phi_1^- + v \chi_2^- + \omega \phi_3^-),
\]

\[
V_{\text{LNC}} = \lambda_1 (\chi^+ \phi)^2 + \lambda_2 (\phi^+ \phi)^2 + \lambda_3 (\chi^+ \chi)(\phi^+ \phi) + \lambda_4 (\chi^+ \phi) (\phi^+ \chi)
\]

\[
+ 2 \lambda_1 \omega S_3 (\chi^+ \chi) + 2 \lambda_2 v S_2 (\phi^+ \phi) + \lambda_3 v S_2 (\chi^+ \chi) + \lambda_3 \omega S_3 (\phi^+ \phi)
\]

\[
+ \frac{\lambda_4}{\sqrt{2}} (v \chi_2^- + \omega \phi_3^-) (\chi^+ \phi) + \frac{\lambda_4}{\sqrt{2}} (v \chi_2^+ + \omega \phi_3^+) (\phi^+ \chi),
\]

\[
V_{\text{LNV}} = 2 \lambda_1 u S_1 (\chi^+ \chi) + \lambda_3 u S_1 (\phi^+ \phi) + \frac{\lambda_4}{\sqrt{2}} u \left[ \phi_1^- (\chi^+ \phi) + \phi_1^+ (\phi^+ \chi) \right].
\]

In the above equations, we have dropped the subscript \(P\) and used \(\chi = (\chi^0_1, \chi^0_2, \chi^0_3)^T\), \(\phi = (\phi^+_1, \phi^0_2, \phi^+_3)^T\). Moreover, we have expanded the neutral Higgs fields as

\[
\chi_1^0 = \frac{S_1 + i A_1}{\sqrt{2}}, \quad \chi_3^0 = \frac{S_3 + i A_3}{\sqrt{2}}, \quad \phi_2^0 = \frac{S_2 + i A_2}{\sqrt{2}}.
\]

In the pseudoscalar sector, all the fields are Goldstone bosons: \(G_1 = A_1, \ G_2 = A_2\) and \(G_3 = A_3\) (cl. Eq. (3.5)). The scalar fields \(S_1, \ S_2\) and \(S_3\) gain masses via (3.5), thus we get one Goldstone boson \(G_4\) and two neutral physical fields the standard model \(H^0\) and the new \(H_1^0\) with masses

\[
m_{H^0}^2 = \lambda_2 v^2 + \lambda_1 (u^2 + \omega^2) - \sqrt{[\lambda_2 v^2 - \lambda_1 (u^2 + \omega^2)]^2 + \lambda_3^2 v^2 (u^2 + \omega^2)}
\]

\[
\simeq \frac{4 \lambda_1 \lambda_2 - \lambda_3^2}{2 \lambda_1} v^2,
\]

\[
M_{H_1^0}^2 = \lambda_2 v^2 + \lambda_1 (u^2 + \omega^2) + \sqrt{[\lambda_2 v^2 - \lambda_1 (u^2 + \omega^2)]^2 + \lambda_3^2 v^2 (u^2 + \omega^2)}
\]

\[
\simeq 2 \lambda_1 \omega^2.
\]

In term of original fields, the Goldstone and Higgs fields are given by

\[
G_4 = \frac{1}{\sqrt{1 + t_2^2}} (S_1 - t_2 S_3),
\]

\[
H^0 = c_\zeta S_2 - \frac{s_\zeta}{\sqrt{1 + t_2^2}} (t_2 S_1 + S_3),
\]

\[
H_1^0 = s_\zeta S_2 + \frac{c_\zeta}{\sqrt{1 + t_2^2}} (t_2 S_1 + S_3),
\]

where

\[
t_2 = \frac{\lambda_3 M_W M_X}{\lambda_1 M_W^2 - \lambda_2 M_W^2}.
\]
From Eq. (3.11), it follows that mass of the new Higgs boson $M_{H_1^0}$ is related to mass of the bilepton gauge $X^0$ (or $Y^\pm$ via the law of Pythagoras) through

$$M_{H_1^0}^2 = \frac{8\lambda_1}{9\alpha} M_X^2 \left[ 1 + \mathcal{O} \left( \frac{M_W^2}{M_X^2} \right) \right]$$

$$= \frac{2\lambda_1 s_W^2}{\pi \alpha} M_X^2 \left[ 1 + \mathcal{O} \left( \frac{M_W^2}{M_X^2} \right) \right] \approx 18.8\lambda_1 M_X^2.$$  (3.16)

Here, we have used $\alpha = \frac{1}{128}$ and $s_W^2 = 0.231$.

In the charged Higgs sector, the mass terms for $(\phi_1, \chi_2, \phi_3)$ are given by (3.6), thus there are two Goldstone bosons and one physical scalar field:

$$H_2^+ = \frac{1}{\sqrt{u^2 + v^2 + \omega^2}} (u \phi_1^+ + v \chi_2^+ + \omega \phi_3^+)$$  (3.17)

with mass

$$M_{H_2^+}^2 = \frac{\lambda_4}{2} (u^2 + v^2 + \omega^2) = 2\lambda_4 \frac{M_Y^2}{\alpha} = \frac{s_W^2 \lambda_4}{2\pi \alpha} M_Y^2 \approx 4.7\lambda_4 M_Y^2.$$  (3.18)

The two remaining Goldstone bosons are

$$G_5^+ = \frac{1}{\sqrt{1 + t_\theta^2}} (\phi_1^+ - t_\theta \phi_3^+),$$  (3.19)

$$G_6^+ = \frac{1}{\sqrt{(1 + t_\theta^2)(u^2 + v^2 + \omega^2)}} \left[ v(t_\theta \phi_1^+ + \phi_3^+) - \omega(1 + t_\theta^2) \chi_2^+ \right].$$  (3.20)

Thus, all the pseudoscalars are eigenstates and massless (Goldstone). Other fields are related to the scalars in the weak basis by the linear transformations:

$$\begin{pmatrix} H^0 \\ H_1^0 \\ G_4 \end{pmatrix} = \begin{pmatrix} -s_\xi s_\theta & c_\xi & -c_\xi c_\theta \\ c_\xi s_\theta & s_\xi & c_\xi c_\theta \\ c_\theta & 0 & -s_\theta \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \\ S_3 \end{pmatrix},$$  (3.21)

$$\begin{pmatrix} H_2^+ \\ G_5^+ \\ G_6^+ \end{pmatrix} = \frac{1}{\sqrt{\omega^2 + c_\theta^2 v^2}} \begin{pmatrix} -s_\xi \sqrt{\omega^2 + c_\theta^2 v^2} & 0 & \omega c_\theta \\ c_\theta \sqrt{\omega^2 + c_\theta^2 v^2} & 0 & -s_\theta \sqrt{\omega^2 + c_\theta^2 v^2} \end{pmatrix} \begin{pmatrix} \phi_1^+ \\ \chi_2^+ \\ \phi_3^+ \end{pmatrix}. $$  (3.22)

Let us comment on our physical Higgs bosons. In the effective approximation $w \gg v, u$, from Eqs. (3.21), and (3.22) it follows that

$$H^0 \sim S_2, \quad H_1^0 \sim S_3, \quad G_4 \sim S_1,$$

$$H_2^+ \sim \phi_3^+, \quad G_5^+ \sim \phi_1^+, \quad G_6^+ \sim \chi_2^+. $$  (3.23)

From the Higgs gauge interactions given in [13], the coupling constants of $H_1^0$ Higgs and SM gauge bosons depend on $s_\xi$ with $t_\xi = \frac{\lambda_3 M_W M_X}{\lambda_1 M_Y^2 - \lambda_2 M_W^2}$. In the $w \gg v, u$ limit, $M_X \gg M_W$ or $|t_\xi| \to 0$. Therefore, the $H_1^0$ Higgs does not interact with the SM gauge bosons $W^\pm, Z^0, \gamma$. 

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However, there are couplings of \( H_1^0 \) Higgs with the Bilepton \( Y \) and \( Z' \). In order to forbid the decay of \( H_1^0 \), we assume that \( M_{H_1^0}^2 \leq M_Y^2 \). It means that \( 2\lambda_1 \omega^2 \leq \frac{1}{4}g^2\omega^2 \) or \( \lambda_1 \leq 0.051 \).

The interactions of \( H_1^0 \) Higgs with new gauge boson \( Z' \) is \( Z' - H_1^0 - G_3 \) interaction. But \( G_3 \) is a Goldstone bosons, this interaction can be gauged away by a unitary transformation.

Let us consider the interaction of dark matter to Higgs bosons. From the Higgs potential (2.11), we can obtain the coupling of the new Higgs \( H_0^1 \) to \( H_0^1 H_0^1 \). The decay rate of the \( H_1^0 \rightarrow H_0^0 H_0^0 \) is written as

\[
\Gamma_{H_1^0 \rightarrow H_0^0 H_0^0} = \frac{\lambda_3^2}{16\pi} \frac{w^2}{M_{H_0^1}^2} \left( 1 - \frac{2M_{H_0^1}^2}{M_{H_0^1}^2} \right).
\]

The lifetime is the inversion of decay rate \( \tau = \frac{\hbar}{\Gamma} \), with \( \hbar = 6.6 \times 10^{-25} \) GeV \( \times \) s. If taking \( \tau > 10^{20} \) s (the life time longer than our universe’s age), \( M_{H_0^1} = 7000 \) GeV, \( w = 10000 \) GeV, \( M_{H_0^0} = 120 \) GeV, then we get

\[
\Gamma_{H_1^0 \rightarrow H_0^0 H_0^0} = \frac{\lambda_3^2}{16\pi} \frac{10^8}{7.10^3} \left( 1 - \frac{2 \times 120^2}{49 \times 10^6} \right) \simeq 284 \times \lambda_3^2.
\]

In order to get the constraint on the lifetime of \( H_1^0 \) longer than our universe’s age, it is easy to see that the value of \( \lambda_3 \) is approximately order of \( 10^{-24} \). It is to be emphasized that the limit of \( \lambda_3 \) makes sure that \( t_\zeta \) is small.

To avoid \( H_1^0 \) decaying to \( H_2^\pm \), we need the constraint for the mass of two Higgs, namely \( M_{H_1^0}^2 < 4M_{H_2^\pm}^2 \), which means \( \lambda_1 < \lambda_4 \). From the Lagrangian given in (2.5), it is easy to see that the \( H_1^0 \) does not interact with the SM leptons but it interacts with exotic quarks. As we know the exotic quarks are heavy ones, we assume that their masses are heavier than that of \( H_1^0 \). Hence, \( H_1^0 \) can be stable and be candidate for dark matter.

4 Thermal relic abundance

4.1 Constraints

Before considering the relic abundance of dark matter, let us summarize the constraints on the couplings \( \lambda_{1,2,3,4} \), the VEV \( w \), and exotic quarks masses:

1. From Eqs. (3.10) and (3.11), we obtain the constraints as follows

\[
\lambda_1 > 0, \quad \lambda_2 > 0, \quad 4\lambda_1 \lambda_2 > \lambda_3^2.
\]

2. The mass of the charged Higgs boson \( H_2^\pm \) is proportional to that of the charged bilepton \( Y \) through a coefficient of Higgs self-interaction \( \lambda_4 > 0 \). Analogously, this happens for the standard-model-like Higgs boson \( H^0 \) and the new \( H_1^0 \). Combining (4.1) with the constraint equations (3.2), (3.3) we get a consequence: \( \lambda_3 \) is negative (\( \lambda_3 < 0 \)).
3. In order to get the stable Higgs particle $H_1^0$, we need the constraints as follows

$$
\lambda_1 < \lambda_4, \quad \lambda_1 \leq 0.051, \quad |\lambda_3| \sim 10^{-24}, \quad M_{H_1^0} \leq M_U. \tag{4.2}
$$

Since $\lambda_3 < 0$, we get $\lambda_3 \sim -10^{-24}$.

4. In the limit of $\lambda$ given in (4.2), the SM Higgs mass can be estimated as $M_{H_1^0}^2 = \frac{4\lambda_1\lambda_2 - \lambda_2^2}{2\lambda_3} v^2 \sim 2\lambda_2 v^2$. Combining with the constraint $80 < M_{H_1^0} < 160 \text{ GeV}$, we can obtain the constraints on $\lambda_2$ as follows: $0.053 < \lambda_2 < 0.212$.

### 4.2 Implication for parameter space from the WMAP constraints.

In this subsection, we discuss constraints on the parameter space of the 3-3-1 model originating from the WMAP results on dark matter relic density [15],

$$
\Omega h^2 = 0.1120 \pm 0.0056.
$$

In order to calculate the relic density, we use micrOMEGAs 2.4 [16] after implementing new model files into CalcHEP [17]. The parameters of our model are the self-Higgs couplings, $\lambda_1, \lambda_2, \lambda_3, \lambda_4$, the VEV $w$ and exotic quarks masses. Note that the usual quarks $u, s, b$ gain masses at one-loop level [9]. We express the couplings of Higgs with exotic quarks $s^u_1, s^d_2, s^d_3$ as functions of $\lambda_1$, $w$ and exotic quarks masses.

All Feynman diagrams contributing to the annihilation of $H_1^0$ Higgs are listed in the Appendix (A). At the tree level, the annihilation of $H_1^0$ dark matter can be done through s-, or t-channel, or direct annihilation. Since there is neither coupling of $H_1^0H_1^0$ to one neural gauge boson nor coupling of $H_1^0H_1^0$ with one fermion, the propagator in the s-channel can be $H_1^0$ or $H_0^0$ Higgs only. To draw Feynman diagrams contributing to the annihilation of $H_1^0H_1^0$ through s- and t-channels, we list all non-zero couplings $H_0^0 AB$ and $H_0^0 AB$, where $A, B$ can be Higgs, or gauge boson, or fermion. We see that $H_1^0$ couples to one usual quark (anti-usual quark) $u, s, b$ and one anti-exotic quark (exotic quark) while $H_0^0$ couples to $c, d, t$ quarks. Therefore, the annihilation of $H_1^0H_1^0$ into $u\bar{u}, ss, bb$ are done through t-channel through exotic quark exchange while the contributions of the remaining usual quarks are done via s-channel through $H_0^0$ exchange. Since the coupling $H_1^0H_0^0 \sim v\lambda_3$, the contribution of $\bar{c}c, \bar{d}d, \bar{t}t$ channels to $\frac{1}{m_h^2}$ is very small. $H_1^0$ Higgs can also annihilate into two Higgs bosons or two gauge bosons directly. Theses vertices arise from the Higgs potential and Higgs-gauge interactions.

First we study the behavior of $\Omega h^2$ as a function of one parameter each time. Table 3 shows the dependence of $\Omega h^2$ on $\lambda_2, \lambda_3, \lambda_4$ corresponding to the point 1, 2, 3. In all three cases, we fix $\lambda_1 = 0.04, w = 10 \text{ TeV}$, and exotic quarks masses $M_U = 36 \text{ TeV}, M_D = M_{D_3} = 100 \text{ TeV}$ as a special choice of parameters in the WMAP allowed band (please look at Fig. 1), the green dot-dashed line. We can see that neither $\Omega h^2$ nor the contribution of channels change when varying $\lambda_2$ in the range $0.053 \sim 0.212$, or $\lambda_3$ from $-10^{-33}$ to $-10^{-20}$, which regions satisfy the constraints given in 4.1. The couplings $H_1^0H_1^0H_0^0$ and $H_0^0H_1^0\bar{H}_1^0$ are proportional to $v\lambda_3$, and these contribute to the annihilation of $H_1^0$ dark matter through s-channel $H$ exchange. The coupling $H_1^0H_1^0H_0^0$ is $v\lambda_3 \sim \lambda_3$, ...
where $\lambda_3$ is very small. That is why $\Omega h^2$ does not depend on $\lambda_2$. The relic density changes negligibly if we vary $\lambda_4$. It gets the minimum value $\Omega h^2 = 0.1116$ at the point $\lambda_4 = 0.15$ and the maximum value $\Omega h^2 = 0.1128$ at the two points $\lambda_4 = 0.08$ and $\lambda_4 = 0.19$. With $\lambda_4 \geq 0.2$, the relic density keeps constant value $\Omega h^2 = 0.1095$.

Now we consider the dependence of the relic density of $H_3^0$ dark matter on the remaining parameters $\lambda_1, w, M_U, M_{D_2}$ and $M_{D_3}$. First, we fix the values of $\lambda_2, \lambda_3, \lambda_4$ satisfying the constraints given in (4.2), especially taking $\lambda_2 = 0.12$, $\lambda_3 = -10^{-24}$, $\lambda_4 = 0.06$ and varying the masses of exotic quarks. We consider the relic density as a function of $\lambda_1$. Fig. 1 compares WMAP data to the theoretical prediction. The red dashed line presents predictions from our theory by fixing $M_{D_2} = M_{D_3} = 100$ TeV, $w=10$ TeV and $M_U = 24$ TeV. In order to meet fully the WMAP data, the value of $\lambda_1$ must be different from the allowed value in (4.2). However, if we change the masses of exotic quarks, we can obtain allowed region, namely the green dot-dashed line given by taking $w = 10$ TeV and $M_U = 36$ TeV. The allowed region of $\lambda_1$ satisfy both the WMAP data and the stable Higgs constraints (4.2) is $0.0393 < \lambda_1 < 0.0406$. The orange full line is obtained by fixing $w = 30$ TeV and $M_U = 36$ TeV. In this case, the constraints on $\lambda_1$ is $0.0424 < \lambda_1 < 0.0436$. On the other hand, if we change the mass of exotic $D$-quarks, we can find the other allowed region of $\lambda_1$. For example, if we take $M_{D_2} = M_{D_3} = 12$ TeV, the allowed region of $\lambda_1$ is $0.0502 < \lambda_1 < 0.051$. Hence, we could conclude that the mass of exotic $U$-quark can be larger or smaller than that of $D$-quarks in order to come to agreement with the WMAP data.

Fig. 2 shows the dependence of the relic density on the VEV $w$ for $\lambda_1 = 0.04$, $\lambda_2 = 0.12$, $\lambda_3 = -10^{-24}$ and $\lambda_4 = 0.06$. This figure shows that the VEV $w < 15.33$ TeV is in the WMAP-allowed region for $M_U = 36$ TeV, $M_{D_2} = M_{D_3} = 100$ TeV. However, if the values of

| point | 1 | 2 | 3 |
|-------|---|---|---|
| $\lambda_1$ | 0.04 | 0.04 | 0.04 |
| $\lambda_2$ | 0.053 to 0.212 | 0.12 | 0.12 |
| $\lambda_3$ | $-10^{-24}$ | $-10^{-33}$ to $-10^{-29}$ | $-10^{-24}$ |
| $\lambda_4$ | 0.06 | 0.06 | 0.004 to 200 |
| $w$ (TeV) | 10 | 10 | 10 |
| $M_U$ (TeV) | 36 | 36 | 36 |
| $M_{D_2}$ (TeV) | 100 | 100 | 100 |
| $M_{D_3}$ (TeV) | 100 | 100 | 100 |
| $\Omega h^2$ | 0.1127 | 0.1127 | (0.1116, 0.1128) |
| channels | $u\bar{u}(97.40\%)$ | $u\bar{u}(97.40\%)$ | $u\bar{u}(97.36 - 97.45\%)$ |
| | $s\bar{s}(1.26\%)$ | $s\bar{s}(1.26\%)$ | $s\bar{s}(1.26\%)$ |
| | $b\bar{b}(1.28\%)$ | $b\bar{b}(1.28\%)$ | $b\bar{b}(1.28\%)$ |
| | $H_3^0 H_3^0$ (0.05%) | $H_3^0 H_3^0$ (0.05%) | $H_3^0 H_3^0$ (0 - 0.09%) |
| | rest (0.01%) | rest (0.01%) | rest (0.01 - 0.02%) |
Figure 1. $\Omega h^2$ vs $\lambda_1$ for $\lambda_2 = 0.12$, $\lambda_3 = -10^{-24}$, $\lambda_4 = 0.06$, $M_{D_2} = M_{D_3} = 100$ TeV, and for $w = 10$ TeV, $M_U = 24$ TeV (red dashed line), $w = 10$ TeV, $M_U = 36$ TeV (green dot-dashed line), $w = 30$ TeV, $M_U = 36$ TeV (orange full line), and $M_{D_2} = M_{D_3} = 12$ TeV, $w = 10$ TeV, $M_U = 70$ TeV (brown large dashing line). The blue dotted vertical line corresponds to $\lambda_1 = 0.051$.

$M_U = 24$ TeV or $M_D = M_U = 36$ TeV, there is no allowed region of $\omega$ in agreement with the WMAP data. The situation becomes totally different for $M_U = 70$ TeV, $M_{D_2} = M_{D_3} = 12$ TeV (brown large dashing line). The relic density at first increases then decreases as a function of $w$. In the WMAP band, $w$ is in the range $8.752 - 13.85$ GeV or $23.3 - 24.61$ GeV.

Similarly, we can figure out the region of $M_U$, $M_{D_2}$, and $M_{D_3}$ in agreement with the WMAP data by fixing $\lambda_1 = 0.04$, $\lambda_2 = 0.12$, $\lambda_3 = -10^{-24}$, $\lambda_4 = 0.06$, and $w = 10$ TeV. The value of $M_U$ is in the narrow band $34.93 \sim 36.73$ TeV for $M_{D_2} = M_{D_3} = 100$ TeV, and $M_U$ is should be heavier, $66.71 < M_U < 85.01$ TeV if we take $M_{D_2} = M_{D_3} = 12$ TeV. In case of $M_U = 36$ TeV and $M_{D_3} = 100$ TeV, $M_{D_2}$ is in the region of $37.99 < M_{D_2} < 259.7$ TeV. For $M_U = 36$ TeV and $M_{D_2} = 100$ TeV, the relic density is always in the WMAP-allowed region for $M_{D_3} > 40$ TeV. There are many choices of $(M_U, M_{D_2}, M_{D_3})$ set satisfying the WMAP result, and we can see that the WMAP constraints do not require the order of exotic quarks masses.

To give an overview of the behavior of the relic density in the $M_{D_2}$ - $M_{D_3}$ plane, as shown in Fig. 3, we consider the model with $\lambda_1 = 0.04$, $\lambda_2 = 0.12$, $\lambda_3 = -10^{-24}$, $\lambda_4 = 0.06$ and $w = 10$ TeV. The region of $M_{D_2}$ - $M_{D_3}$ in agreement with the WMAP is very wide for $M_U = 36$ TeV (red), while it seems to be two narrow bands for $M_U = 70$ TeV (grey bands).

From now on we take $M_D = M_{D_2} = M_{D_3}$ for convenience. In the $M_U$ - $M_D$ plane (see Fig. 4), we can see that to satisfy the WMAP constraints, $M_U$ must be heavier than 35.2 TeV and $M_D$ must be heavier than 11.8 TeV. For $M_U = 36$ TeV and $w = 10$ TeV, the relic density as slowly varying function of $M_D$. The situation is similar to that of $M_U = 40$
Figure 2. $\Omega h^2$ vs $w$ for $\lambda_1 = 0.04$, $\lambda_2 = 0.12$, $\lambda_3 = -10^{-24}$, $\lambda_4 = 0.06$, and for $M_U = 24$ TeV, $M_{D_2} = M_{D_3} = 100$ TeV (red dashed line), $M_U = 36$ TeV, $M_{D_2} = M_{D_3} = 100$ TeV (green dot-dashed line), $M_U = M_{D_2} = M_{D_3} = 36$ TeV (orange full line) and $M_U = 70$ TeV, $M_{D_2} = M_{D_3} = 12$ TeV (brown large dashing line).

Figure 3. Contour plots for $0.1064 < \Omega h^2 < 0.1176$ (WMAP constraints) in $M_{D_2} - M_{D_3}$ plane for $\lambda_1 = 0.04$, $\lambda_2 = 0.12$, $\lambda_3 = -10^{-24}$, $\lambda_4 = 0.06$, $w = 10$ TeV, and for $M_U = 36$ TeV (red) and $M_U = 70$ TeV (grey).

TeV and $w = 30$ TeV. On the other hand, for the value of $M_D$ around 12 TeV, the relic density varies very slowly as a function of $M_U$. For $12 < M_D < 22$ TeV, the relic density at $w = 10$ TeV is the same as that at $w = 30$ TeV.
Similarly we investigate contour plots for $0.1064 < \Omega h^2 < 0.1176$ (WMAP constraints) in the $M_U - M_D$ plane. For an example, if $15 < M_D < 21$ TeV, the allowed region of $M_U$ is in $40 < M_U < 64$ TeV. Finally we study contour plots for $0.1064 < \Omega h^2 < 0.1176$ (WMAP constraints) in the $\lambda_1 - M_U$ plane. Combining with the constraint on $\lambda_1$ given in (4.2), we can see that the allowed bands are: $0.028 < \lambda_1 < 0.051$ for $M_{D_2} = M_{D_3} = 100$ TeV, and for $M_{D_2} = M_{D_3} = 12$ TeV, $0.039 < \lambda_1 < 0.051$ if $w = 10$ TeV, and no region of $\lambda_1$ allowed if $w = 30$ TeV.

In next section we will study how to search for the DM in the WMAP - allowed region, in direct and indirect searches. We would like to analyze the results as functions of $M_{H_1^0}$, which is expressed in term of $\lambda_1$ and $w$. With the WMAP constraints, we use the best parameter space, $\lambda_1 = 0.04$, $\lambda_2 = 0.12$, $\lambda_3 = -10^{-24}$, $\lambda_4 = 0.06$, $M_U = 36$ TeV, $M_{D_2} = M_{D_3} = 100$ TeV, and we vary $5 < w < 15.3$ TeV, which requires $M_{H_1^0}$ to be few TeVs.

5 Direct and indirect searches for the dark matter

Experimentalists worldwide are actively chasing searches for DM candidates either directly through detection of elastic scattering of the weakly interacting massive particles with the nuclei in a large detector or indirectly through detection of products of the dark matter annihilation (photons, positrons, neutrinos or antiprotons) in the galaxy or in the sun.

5.1 Direct search

In direct search, the recoil energy deposited by scattering of WIMPs with the nuclei is measured. In general, WIMP-nuclei interactions can be split into two types: a spin independent interaction and a spin dependent interaction. In our model, scalar $H_1^0$ Higgs DM can only contribute to spin independent interaction. To calculate the direct detection rate

![Contour plots](image.png)

**Figure 4.** Contour plots for $0.1064 < \Omega h^2 < 0.1176$ (WMAP constraints) in $M_U - M_D$ plane for $\lambda_1=0.04$, $\lambda_2=0.12$, $\lambda_3 = -10^{-24}$, $\lambda_4=0.06$ and for $w=10$ TeV (red) and $w=30$ TeV (green).
we use the method, as mentioned in [18]; The direct detection rate depends on the WIMP nucleus cross section. To derive the $H_1^0$-nucleus cross section one has to compute first the interactions at the quark level then convert them into effective couplings of WIMPs to protons and neutrons. Finally, we have to sum the proton and neutron contributions and turn this summation into a cross section at the nuclear level.

The calculation of the cross section for WIMP scattering on a nucleon is obtained at the tree level. The normalized cross section on a point-like nucleus is given as

$$
\sigma_{H_1^{0}N}^{SI} = \frac{4\mu_{H_1^{0}}^2}{\pi} \left( Z f_p + (A - Z) f_n \right)^2 A^2,
$$

(5.1)

where $\mu_{H_1^{0}}$ is the $H_1^{0}$ - nucleus reduced mass, $f_p$ and $f_n$ are amplitudes for protons and neutrons, respectively. For Xenon, $A = 131$, $Z = 54$, while for Germanium $A = 76$, $Z = 32$. The recoil spectrum of the nuclei depends on the velocity distribution and is contained in the elastic form factor of the nucleus. Using micrOMEGAs 2.4, we get the amplitudes and cross sections for WIMP-nucleon elastic scattering calculated at zero momentum as well as the total number of events/day/kg if we consider detector made of Xe or Ge.

![Figure 5. $H_1^0$ DM-nucleon cross sections vs $M_{H_1^0}$ for $\lambda_1 = 0.04$, $\lambda_2 = 0.12$, $\lambda_3 = -10^{-24}$, $\lambda_4 = 0.06$, $M_U = 36$ TeV and $M_{D_2} = M_{D_3} = 100$ TeV.](image)

Fig. 5 shows the values of $\sigma_{\text{WIMP-nucleon}}$ as a function of dark matter mass by fixing the nucleon form factors, $\sigma_0 = 40$ MeV and $\sigma_{\pi N} = 55$ MeV. The value of $\sigma_{\text{WIMP-nucleon}}$ is in order of $10^{-6}$ (pb), which is allowed by experimental constraints of CDMs 2009 (Ge). However, in the limit the dark matter mass is smaller than 2.5 TeV or larger than 3.5 TeV, the result given by our theoretical prediction is somehow different from experiment of CDMs 2009 (Ge). We would like to emphasize that the form factors of nucleons can be reset by changing the pion-nucleon sigma term, $\sigma_{\pi N} = 55-73$ MeV and from the SU(3) symmetry breaking effect, $\sigma_0 = 35 \pm 5$ MeV [19], however, the final WIMP-nucleon cross section predicted by our model does not change much.
Figure 6. Number of events/day/kg vs \( M_{H^0_1} \) for \( \lambda_1 = 0.04, \lambda_2 = 0.12, \lambda_3 = -10^{-24}, \lambda_4 = 0.06, M_U = 36 \) TeV and \( M_{D_2} = M_{D_3} = 100 \) TeV.

Let us deal with the number of the nucleon recoil in Ge and Xe detectors. The predicted number of nucleon recoil is given in Fig. 6. The Xe detector is more sensitive than Ge detector. In the limit, \( 2.5 < M_{H^0_1} < 3.5 \) TeV, the theoretical predictions are 22.3 and 14.6 nucleon recoils those are observed in the Xe and Ge detectors for 1 kg per year, respectively. It is worth mentioning that the number of the nucleon recoils exposure between 10 keV ∼ 50 keV approximately equals one half of total of that number.

5.2 Indirect search

DM annihilation in the galactic halo produces pairs of the SM particles that hadronize and decay into stable particles. These particles then evolve freely in the interstellar medium. The final states with \( \gamma, e^+ \) and \( \bar{p} \) are particularly interesting as they are the subject of indirect searches. From Feynman diagrams, we can see that the annihilation of \( H^0_1 H^0_1 \) into \( t\bar{t}, c\bar{c}, d\bar{d}, b\bar{b}, \nu_{l}\bar{\nu}_{l} \) and \( ZZ \) are done through s-channel \( H^0 \) exchange. The annihilation of \( H^0_1 H^0_1 \) into \( H^0 H^0 \) is done through s-, t-channel \( H^0_1, H^0 \) exchange or quartic couplings. Since the couplings \( H^0_1 H^0_1 H^0_1 H^0_1, H^0_1 H^0_1 H^0_1 H^0_1, H^0_1 H^0_1 H^0_1 H^0_1 \) are proportional to \( \lambda_3 \), the contributions of those channels are very small. With the choice of parameters \( \lambda_1 = 0.04, \lambda_2 = 0.12, \lambda_3 = -10^{-24}, \lambda_4 = 0.06, M_U = 36 \) TeV, \( M_{D_2} = M_{D_3} = 100 \) TeV and \( 5 < w < 15.3 \) TeV, the dominant channel is \( u\bar{u} \). For example, in case of \( w = 10 \) TeV, the relative contribution in % are displayed as following: (97.40% : \( H^0_1 H^0_1 \rightarrow u\bar{u} \)); (1.28% : \( H^0_1 H^0_1 \rightarrow b\bar{b} \)); (1.26% : \( H^0_1 H^0_1 \rightarrow s\bar{s} \)); (0.05% : \( H^0_1 H^0_1 \rightarrow H^+_2 H^+_2 \)); and (0.01% : the rest).

The total annihilation cross section times the relative velocity of incoming dark matter particles is shown in Fig. 7. With the allowed region of the dark matter mass satisfied the WMAP constraints, we find \( 2.15 \times 10^{-26} < \sigma v < 2.4 \times 10^{-26} \) cm\(^3\)/s. This result is in the same order of that given in [20], which is said that away from the Higgs resonance and the \( W \) threshold, \( < \sigma v > \) is essentially constant and equal to the so-called typical annihilation cross section, \( < \sigma v > \sim 3.10^{-26} \) cm\(^3\)/s. The AMS-2 experiment given in [21] predicted for
dark matter mass up to 600 GeV. With dark matter mass from 100 GeV to 600 GeV, \( \sigma v \) keeps constant value in order of \( 10^{-26} \text{cm}^3/\text{s} \). We expect that our result for heavy dark matter can be covered by the future experiment of AMS-2.

**Figure 7.** The annihilation cross section times the relative velocity of incoming DM particles vs \( M_{H^0} \) for \( \lambda_1 = 0.04, \lambda_2 = 0.12, \lambda_3 = -10^{-24}, \lambda_4 = 0.06, M_U = 36 \text{ TeV} \) and \( M_{D_2} = M_{D_3} = 100 \text{ TeV} \).

**Figure 8.** Photon flux vs \( E \) for \( \lambda_1 = 0.04, \lambda_2 = 0.12, \lambda_3 = -10^{-24}, \lambda_4 = 0.06, w = 10 \text{ TeV}, M_U = 36 \text{ TeV} \) and \( M_{D_2} = M_{D_3} = 100 \text{ TeV} \).

Let us proceed with the discussion of the photon flux and energy spectrum for dark matter with mass 2828.4 GeV. The spectrum for photon flux is predicted in Fig. 8. It is easy to see that the photon flux in the energy range from a few MeV to 10 GeV is much larger than that of higher energy ranges. These results can be understood as following: As previously mentioned, our model predicts that the annihilation of the dark matter and anti-dark matter to \( u \)-quark and \( \pi \)-quark is the dominating channel. Therefore, the dominating jet is the neutral pion jet, composed of pairs of \( u \)-quark and anti-quark in this case. The \( \gamma \)-rays from particle annihilation processes have spectra bounded by the rest mass energy...
of the annihilation particle. The $\gamma$-rays are dominated by pion decay at low energy from a few MeV to 10 GeV. The additional contribution to photon spectrum at higher energy due to other annihilation processes such as polarization of the gauge bosons final state, photon radiation, etc., which are predicted to be tiny. Antiproton flux and positron flux also can be calculated by MicrOMEGAs 2.4. The fluxes go down fast as functions of energy and their values are significant at low energy as the same as photon case.

6 Conclusions

We have shown that the economical 3-3-1 model provides a candidate for dark matter without any discrete symmetry; and it just requires some constraints on Higgs coupling constant. The scalar Higgs $H_1^0$ is a good candidate for self interacting dark matter. To forbid the decay of $H_1^0$, we require that $\lambda_1 \leq 0.051$, $\lambda_1 < \lambda_4$, and $|\lambda_3| \sim 10^{-24}$. The constraint on the Higgs coupling $\lambda_3$ looks unnatural, which could be canceled by introducing a discrete symmetry $S_3$. However, by introducing new symmetry the Higgs sector becomes more complicated. Therefore, we do not consider such scheme in this work. The parameter space has been studied in detail and the results satisfying the WMAP observation are summarized as following:

- The relic density does not change much when varying $\lambda_2$ from 0.053 to 0.212, $\lambda_3$ around $-10^{-24}$ value, $\lambda_4$ from 0.004 to 200.
- $\lambda_1$ should be around 0.04.
- The region of $\omega$ is narrow compared to those of $M_U$, $M_{D_2}$ and $M_{D_3}$.
- $U$-quark mass can be smaller or larger than $D$-quarks masses.

We have studied direct and indirect searches for $H_1^0$ dark matter. The dark matter–nucleon cross section is in agreement with CDMs 2009 (Ge). The total number of events observed in Xe and Ge detectors is quite small because of the heavy dark matter. We hope that these results can be covered in future by experiments. Dark matter annihilation is considered with special choice of parameters. In case $M_U < M_{D_2} = M_{D_3}$, the dominant channel is $u\bar{u}$, while the dominant channel is $s\bar{s}$ for $M_{D_2} = M_{D_3} > M_U$, because the interactions of quarks with other particles depend much on exotic quarks masses. However, choosing $U$-quark mass smaller or larger than $D$-quark mass does not affect our results on cross section times relative velocity as well as photon flux. The value of $\sigma v$ is in order of $10^{-26} cm^3/s$ in agreement with typical annihilation cross section. Photon flux is dominated at low energy below 10 GeV.

Acknowledgments

N. T. T. would like to thank S. Kraml at LPSC, Grenoble, France for kind help and hospitality during her visit where this work was initiated and P. V. Dong at Institute of Physics,
Hanoi, Vietnam for comments and H. Sung Cheon at Yonsei University, Seoul, Korea for discussion. The work H. N. L. was supported in part by the National Foundation for Science and Technology Development (NAFOSTED) of Vietnam under Grant No. 103.01-2011.63. The work of C. S. K. and N. T. T. was supported in part by the National Research Foundation of Korea (NRF) grant funded by the Korea government of the Ministry of Education, Science and Technology (MEST) (No. 2011-0027275), (No. 2011-0017430) and (No. 2011 -0020333).

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A APPENDIX: Feynman diagrams contributing to the annihilation of $H_0^1$ dark matter
