Didactical design work sheet of complex variable function based on epistemology, didactical, and learning trajectory to enhance student’s ability for representation and communication

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Abstract. One of the important keys to improve undergraduate students’ learning process is by using a good learning material, especially for those who are from mathematics department. In this study, the learning material was developed not merely by considering the characteristic of the materials and supported by appropriate models, methods and approaches. Further than that, it also pays attention on learning obstacles faced by the undergraduate students from both internal and external perspectives. By this way, the learning material was made based on epistemology, didactic, and learning trajectory. This study was undertaken by applying Didactical Design Research (DDR) which has three analytical phases: didactical situation analysis prior; metapedidactical analysis; and retrospective analysis. This study was beginning with problems (concept founding or even concept reinforcement), then the problems was analysed by the students by referring textbook for students and lastly making the teaching guide based on the learning obstacle, didactical, and learning trajectory. After learning process and learning obstacle test was done, an analysis was carried out and it shows that the didactical design successfully improved the undergraduate students’ representation and communication skill. However, it seems that the undergraduate students’ representation skill does not significantly affect to their communication skill.

1. Introduction
Complex variable function is one of analytic lesson studies considered as mandatory course for the prospective-teacher undergraduate students from department of mathematics and mathematics education. It demands the students to think analytically and correlatively to its numerous prerequisite topics so that they can deliver their materials clearly, verbally and literally. It is important to improve learning quality through significant learning process.

Suryadi [1] claimed that a learning process correlated to three aspects which were teacher, student, and material. Moreover, Suryadi [1] also claimed that if the learning activity depend only on the textual understanding, it would lead to insignificant learning process in which it lacks of meanings and contexts, and a result-oriented learning which could contribute to passively participative students. Insignificant learning also affects to a condition in which students understood concepts partially and were not able to
integrate between concepts meaningfully. These conditions were seemingly caused by the textbook used by students, or unstructured learning process, or both of them [2].

In the accordance to this situation, we have found several findings based on our observation which show several obstacles that the undergraduate students faced when dealing with complex variable problems or related prerequisite topics. These findings can be found in the table 1.

**Table 1. Students’ obstacle on solving complex variable problems or related prerequisite topics.**

| Problem                                                                 | Student Respond                                                                 |
|------------------------------------------------------------------------|---------------------------------------------------------------------------------|
| True or false by giving reasons                                         | Students failed to interrelate definite positive concept with comparison theorem |
| If \( f(x) = 3 + x^2, x \in (a, b) \), then \( \int_a^b f(x) \, dx > 0 \) |                                                                                 |
| Let one of the roots of \( z = (1 - i\sqrt{3})^{1/4} \) is \( \beta = \sqrt[4]{2} \left( \frac{1}{2} \right)^{1/4} \) | Students were not able to find relationship between the roots of complex number and double angle rule and they were not able to communicate algebraic idea to geometric one. |
| State the other roots in the form of \( \beta \).                       |                                                                                 |
| Draw all of the roots in the complex plane and find their meaning geometrically. |                                                                                 |
| Find \( z \in \mathbb{C} \) that satisfies \( \cosh z = 3i \)           | Students faced difficulties to find the relationship between sine and cosine concepts. |
| Find \( z \in \mathbb{C} \) that satisfies \( \sinh z = 3 \)          |                                                                                 |

Taking the above into account, students still couldn’t think mathematically. Students could only count without knowing why. It is important for a teacher or a lecturer to solve this problem with various approaches. One of the ways to do it is by constructing or developing the learning materials. It is convenient to create the materials based on materials characteristic approach equipped with appropriate models, methods and approaches. Hence, the actual problems which students deal with were not solved. Because of that, in this study, the learning material which is called by didactical design was made based on epistemology, didactic, and learning trajectory. This didactical design aims to improve the undergraduate students’ representation and communication skill. Since it is focused, it can be used to develop and improve the learning quality. This effort is in line with Dedy and Encum’s research [3] which revealed that the didactical design used in their research successfully minimize errors that students made during learning obstacle test. Certainly, this effort can flaw Rukmana’s claim [4] that there is no significant improvement of the undergraduate students’ performance on calculus course over years.

2. Methods

This study used a qualitative method called DDR (Didactical Research Design) through three stages of analysis, namely: (a) Analysis of the situation before didactical learning in the form of a didactical design hypotheses including anticipation of didactic pedagogic analysis (ADP), (b) Metapedidactic analysis which meant teachers’ analysis capability, three integrated components, namely unity, flexibility, and coherence in learning included, and (c) Retrospective analysis, which meant analysing the results of the analysis of the situation about didactic linking hypothesis with the results of the analysis metapedidactic.[5-6]

This study began with making mathematically representable and communicative learning obstacles (LO) instrument (as epistemology obstacles) from several concepts. LO instrument contained previous difficulties faced by undergraduate students as they tried to resolve problems (concepts findings or affirmations), prerequisite topics (differential calculus, integral calculus, many calculus variables, real
analysis, and topology). The steps applied in this research were analysing mathematically representable and communicative LO instrument as epistemology obstacles, source textbooks, learning videos, analysing learning trajectory in form of concepts flow from chosen materials, compiling early didactic design, implementing early didactic design, testing implementation results of early didactic design to finally establish revised didactic design (if there’s any or new LO’s been found) according to learning trajectory so that the revised didactical design could fix and improve learning process to be better thus solving learning obstacles faced by undergraduate students.

This study focused on analysing the learning trajectory on limit, continuity, differential, Cauchy-Riemann’s equation, and analytic function topics.

3. Result and Discussion
This study was begun with analysing obstacles which students faced on complex variable course and its prerequisite topics. We have found that most of problems that students deal with are concerning with mathematical representation and communication skill. Mathematical representation skill includes external, shared (cf. negotiated) and internal. External representations are symbols, signs and signals while shared representation are the representations those occur in shared mode. Moreover, internal representations are verbal/syntactic, imagistic, formal notational and affective [7]. On the other hand, mathematical communication skill measured in this study is only written-text aspect.

Next, the essential materials that students must master well were decided before the materials are used to create the didactical design. This didactical design also considers obstacles students faced which were found from learning obstacle tests analysis, and learning trajectory from two text books which are *Fungsi Variabel Kompleks* by R Soemantri and Complex Variables and Applications by R V Churchill and J W Brown. These two books have similar characteristic, especially on the essential materials which are limit, continuity, differential, Cauchy-Riemann’s equation, and analytic function topics. [8-9]

When the learning activity occurs, the students are divided into several groups to discuss the materials provided in the didactical design. After several minutes, each group presents their findings. Based on their presentations, several difficulties regarding mathematical connection skill were found. Moreover, most of the students faced difficulties when they tried to communicate their finding verbally or by writing. A review was conducted after each presentation to discuss about what should be corrected and improved. The result of analysis of the undergraduate students’ mathematical representation and communication skill from mid-term and final-term exam can be seen in table 2.

Table 2. Students’ Mathematical Representation and Communication Skill on Mid-and-Final-Term Exam (SN: Student Number; REP: Representation; COM: Communication; MT: Mid Term test; Final Term test).

| NO | SN   | NAME | REP. | COM. | REP (%) | COM. (%) |
|----|------|------|------|------|---------|----------|
|    |      |      | MT   | FT   | MT      | FT       | MT       | FT       |
| 1  | 1301487 | ANA  | 14   | 14   | 0       | 3        | 28.7%    | 41.07%   |
| 2  | 1500198 | MDR  | 23   | 23   | 10      | 10       | 18.7%    | 32.85%   |
| 3  | 1500752 | ANS  | 32   | 32   | 0       | 5        | 17.5%    | 45.71%   |
| 4  | 1500941 | EFM  | 12   | 12   | 10      | 8        | 17.5%    | 17.14%   |
| 5  | 1501077 | NU   | 41   | 41   | 10      | 3        | 37.5%    | 58.57%   |
| 6  | 1501319 | TS   | 36   | 36   | 10      | 8        | 18.7%    | 51.43%   |
| 7  | 1501412 | AFS  | 38   | 38   | 2       | 0        | 17.5%    | 54.28%   |
| 8  | 1501589 | RA   | 34   | 34   | 2       | 5        | 22.5%    | 48.57%   |
From table above, we could conclude that mathematical representation skill seemed improved. Twenty-four from twenty-seven undergraduate students had improvements. It didn’t mean that there were no obstacles faced by undergraduate students seen from LO results (Mid Semester Exam), it just meant that the percentage was so little. Nevertheless, only 9 undergraduate students (30%) passed the minimal mathematical representation score which was 55.

Students’ mathematical communication skill by the percentage were increased with minimal score of 55. Three from twenty-seven students had passed. It meant that obstacles faced by undergraduate students, as seen from LO results (mid-semester exam), had decreased even if it just by 11%.

The established didactical design seemed to affect students’ mathematical skills especially at representation and communication skill even if it seemed that communication skills still needs fixing and didactical design improvement. Also, representation mathematical skills weren’t affecting.

4. Conclusion
The analysis of students’ error after learning process and learning obstacle test on the complex variable topic shows that the didactical design can improve the undergraduate students’ representation and communication skill, although the representation skill does not significantly affect to the communication skill.
5. References

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