Analysis of the Reliability of the Butter-Oil Processing Plant using CAS Mathematica and Maxima

Shalini Jindal, Reena Garg, Tarun Garg

Abstract: The aim of the present paper is to analysis the reliability of the system by using CAS Mathematica and also, it’s comparative study with CAS Maxima. The butter oil manufacturing plant consists of seven units i.e. separator, pasteurizer, continuous butter making, melting vats, butter oil clarifier, packaging and standby state. Model is developed by using Markov birth-death process. The first order differential equations are derived by using model and then solved for comparative study of reliability to give accuracy in result. By using Mathematica, we calculate probability of each state, which is beneficial to plant owners because better accuracy in result enhance reliability of the plant. Graphs are plotted and tables are developed with the help of CAS Mathematica and Maxima, graphs show the variation and tables shows the fluctuations in reliability in comparative manner.

Keywords: Reliability, Chapman Kolmogorov Differential Equations, Markov Process, Mathematica, Maxima, Butter-oil Processing Plant.

I. INTRODUCTION

In industry, issue of system reliable becomes major term because of production of system are not affected with this issue, with increasing demand of this era so to overcome this issue from past, now and in future there are many techniques. But to consider the value of time and accuracy in result, now there are also some software available to find the reliability of the system. In this research paper, we have not only evaluated the reliability of the system but also analyses the comparative study for its justification and improvement in results, by utilizing time. The overall concern is to increase the reliability of the system and decrease its failure rate so our system becomes more reliable. In present research paper, we have compared the reliability of the system by using CAS Mathematica and CAS Maxima. The butter oil manufacturing plant consists of seven units and System is working in standby state, to increase the redundancy of the system. We also analyses variation of the failure and repair rates on the reliability of system. In [1] Jai Singh, Arvind Kumar Lal, Rajendra Kumar Sharma and Pawan Kumar have analyses the “Reliability of butter oil processing plant by using R-K Method”. In present paper we will analyses the “Reliability of butter oil processing plant using Mathematica” So we take the System, its transition diagram from [1].

II. SYSTEM CONFIGURATION

Butter oil manufacturing plant consist of following seven units:

1. Unit A (Separator): In this unit, the fat from chilled milk (chilled by using chiller) is separated in form of cream & remaining skimmed milk is stored in milk silos.

2. Unit B (Pasteuriser): In this unit, separated cream is pasteurised. When the pasteurised milk goes out of this unit some particles in form of sludge stuck around the outlet, which block the flow of milk resulting in failure of unit.

3. Unit C (Continuous Butter Making): Cream is pumped into the CBM to form homogenous butter in this unit. The buttermilk produced in this process pumped back to the milk silos.

4. Unit Ė (Continuous Butter Making): In this unit, unit C is working in standby state.

5. Unit D (Melting Vats): In this unit, butter is melted to evaporate water at 107 degree.

6. Unit E (Butter-Oil Clarifier): In this unit, butter oil is allowed to settle down few hours to remove fine particle of butter oil residue.

7. Unit F (Packaging): Packets filling machine produce the packets of butter oil in this unit.
Analysis of the Reliability of the Butter-Oil Processing Plant using CAS Mathematica and Maxima

III. NOTATIONS AND ASSUMPTIONS

Notations:
- a, b, c, d, e, f: Represent the failed state of the sub-systems A, B, C, D, E and F respectively.
- B: Represents that sub-system B is working in reduced state.
- ai (i=1,2, ...7): Represents the constant failure rates of sub-systems A, B, C, D, E, F, B and B.
- βi (i=1,2, ...6): Represents, respectively, the constant repair rates of sub-systems A, C, D, E, F and B.
- Pj(t) (j=1, 2, 3, ..., 13): Represents the probability that the system is in jth state at time t.
- Pj′(t): Represents derivative of Pj(t) with respect to t.

Assumptions:
- Repair and failure rates are independent of each other.
- There are no simultaneous failures among the sub-systems.
- Sub-system B fails through reduced state only.
- Repaired unit is as good as new one.
- System is failed when either of unit is completely failed.

IV. FLOW DIAGRAM OF THE SYSTEM

V. MATHEMATICAL FORMULATION OF THE SYSTEM

The mathematical modelling of the system is carried out with the help of Fig.-1, to determine the reliability of butter oil processing plant and following Chapman Kolmogorov differential equations are developed on basis of Markov birth-death process are given as:

\[ P_i'(t) = H_i P_i(t) + b_i P_{i+1}(t) + \sum_{j=1}^{i-1} b_j P_j(t) + b_{i+1} P_{i+1}(t) - b_i P_i(t) \]

Therefore,

\[ P_1'(t) = H_1 P_1(t) + b_2 P_2(t) + b_3 P_3(t) + b_4 P_4(t) + b_2 P_5(t) + b_3 P_6(t) + b_4 P_7(t) \]

\[ P_2'(t) = H_2 P_2(t) + b_3 P_3(t) + b_4 P_4(t) + b_5 P_5(t) + b_2 P_6(t) + b_3 P_7(t) + b_4 P_8(t) + b_5 P_9(t) \]

Similarly, we get

\[ P_i'(t) + \beta_i P_i(t) = \alpha_i P_i(t) \]

Where, i=1,2,...5

\[ P_i'(t) + \beta_i P_i(t) = \alpha P_i(t) \]

Where, i=1,2,...5

\[ P_{13}'(t) + \beta P_{13}(t) = \alpha P_2(t) \]

With initial conditions at time t = 0

\[ P_i(t) = \begin{cases} 1 & \text{for } i = 1 \\ 0 & \text{for } i \neq 1 \end{cases} \]

The system of differential equations with initial conditions are solved by using CAS Mathematica and Runge kutta method using maxima.

Values of P0, P1, P2,......, P13 given by Mathematica at point t=t0:

\[ P_0(t) = 0.6419695170869428 \exp (-6.011199999999989) + 0.00010873876123088067 \exp (5.011266880652618) + 0.005829038084812744 + 0.0008121347847161149 \exp (5.335604731401927) + 7.78117019844699 * 10^{-30} + 0.014304666420200689 \exp (5.59256086254593) + 0.022072008786365106 + 0.0018681727716482193 \exp (5.608411654317564) + 0.003106122635421983 + 0.0012367208829568373 \exp (5.608389289568373) + 0.0019847779825256558 + 0.4994464581994554 \exp (5.995171432069307) + 1.0 \exp (6.011199999999989) \]

\[ P_1(t) = 0.318093039620148 \exp (-6.011199999999989) + 0.00000126703186509244 \exp (5.011266880652618) + 0.005773745186247931 \exp (5.335604731401927) + 1.18007061624946 * 10^{-29} \exp (5.341199999999989) + 0.0029270674193962 \exp (5.59256086254593) + 0.018220922125274613 \exp (5.593097971588801) + 0.00269616111056674 \exp (5.608411654317564) + 0.002594153466182855 \exp (5.6408748995529875) + 0.001919035393238276 \exp (5.680388295668373) + 0.0017275040717069402 \exp (5.680430024268092) - 0.9971512179394846 \exp (5.995171432069307) + 1.0 \exp (6.011199999999989) \]

\[ P_2(t) = (935 \exp (5.680430024268092) + 0.5197662333550848 \exp (5.995171432069307) + 0.60111999999999998) \exp (-6.011199999999989) \exp (0.003934675859809094 \exp (-6.011199999999989) \exp (0.0005757729061392627 \exp (5.011266880652618) - 0.008989890776328033 \exp (5.335353928254821) - 0.01236835017731519 \exp (5.335604731401927) - 1.456814338273868 * 10^{-29} \exp (5.341199999999989) - 0.6816564422412152 \exp (5.59256086255493) + 1.146682218404302 \exp (5.593097971588801) + 0.1062101430056183 \exp (5.608411654317564) + 1.0 \exp (5.601199999999989) \]
Analysis of the Reliability of the Butter-Oil Processing Plant using CAS Mathematica and Maxima

\[ P_{12}(t) = 0.0012818673299391762 \exp (-6.011199999999999) t + 0.66171087927163 \exp (5.011626880652618) t + 0.66171087927163 \exp (5.33553928524821) t - 0.6338512285013291 \exp (5.335604731401927) t + 6.655070831674937 \times 10^{-14} \exp (5.341199999999999) t - 0.05409406784917304 \exp (5.59259606255493) t + 0.04846143745002845 \exp (5.59309791588801) t - 0.006762089793393138 \exp (5.60841645317564) t + 0.006502874858298912 \exp (5.608478955292878) t - 0.003790677634071463 \exp (5.60838295668373) t + 0.00341192596539984 \exp (5.60843002468092) t - 1.02159097975006 \exp (5.95917143269037) t + 1.0 \exp (6.01119999999999999) t, \]

The reliability of the system can be computed by

\[ R(t) = P_{1}(t) + P_{2}(t) \] .......................... (7)

VI. COMPARATIVE ANALYSIS OF THE PERFORMANCE OF SYSTEM BY VICTIMIZATION CAS & Mathematica

The reliability of system is calculated by using equation (7) for various values of failure and repair rates and given in tables as:

1. Effect of failure rate of Separator (α) on reliability of the system:

In Table 1, the reliability of the system is calculated by varying their values as α_{1}=0.006, 0.007, 0.008, 0.009, 0.010 and all other values α_{2}=0.0050, α_{3}=0.0027, α_{4}=0.0009, α_{5}=0.0027, α_{6}=0.0055, α_{7}=0.0111; b_{1}=0.41, b_{2}=0.40, b_{3}=0.67, b_{4}=0.33, b_{5}=1.0 are kept fixed.

Table 1: Effect of failure rate of separator (α) on reliability of the system.

2. Effect of failure rate of CBM (α_{2}) on reliability of the system:

In Table 2, The reliability of the system is calculated by varying their values as α_{2}=0.0050, 0.0055, 0.0060, 0.0065, 0.0070 and other values α_{1}=0.006, α_{3}=0.0027, α_{4}=0.0009, α_{5}=0.0027, α_{6}=0.0055, α_{7}=0.0111; b_{1}=0.41, b_{2}=0.40, b_{3}=0.67, b_{4}=0.33, b_{5}=1.0 are kept fixed.

Table 2: Effect of failure rate of CBM (α_{2}) on reliability of the system.

3. Effect of failure rate of Melting vats (α_{4}) on reliability of the system:

In Table 3, The reliability of the system is calculated by varying their values as α_{4}=0.0022, 0.0024, 0.0026, 0.0028, 0.0030 and all other values α_{1}=0.006, α_{2}=0.0050, α_{3}=0.0027, α_{5}=0.0055, α_{6}=0.0111; b_{1}=0.41, b_{2}=0.40, b_{3}=0.67, b_{4}=0.33, b_{5}=1.0 are kept fixed.

Table 3: Effect of failure rate of Melting vats (α_{4}) on reliability of the system.

4. Effect of repair rate of Separator (β_{1}) on reliability of the system:

In Table 4, The reliability of the system is calculated by varying their values as b_{1}=0.30, 0.35, 0.40, 0.45, 0.50 and all other values α_{1}=0.006, α_{2}=0.0050, α_{3}=0.0027, α_{4}=0.0009, α_{5}=0.0027, α_{6}=0.0055, α_{7}=0.0111; b_{1}=0.40, b_{2}=0.67, b_{3}=0.33, b_{4}=0.67, b_{5}=1.0 are kept fixed.

Table 4: Effect of repair rate of Separator (β_{1}) on reliability of the system.
Table 4: Effect of repair rate of separator ($\beta_1$) on reliability of the system.

| $t$  | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 |
|------|------|------|------|------|------|------|
| 30   | 0.89750 | 0.89675 | 0.89600 | 0.89525 | 0.89450 | 0.89375 |
| 40   | 0.89500 | 0.89425 | 0.89350 | 0.89275 | 0.89200 | 0.89125 |

Table 5: Effect of repair rate of CBM ($\beta_2$) on reliability of the system.

| $t$  | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 |
|------|------|------|------|------|------|------|
| 30   | 0.89750 | 0.89675 | 0.89600 | 0.89525 | 0.89450 | 0.89375 |
| 40   | 0.89500 | 0.89425 | 0.89350 | 0.89275 | 0.89200 | 0.89125 |

Table 6: Effect of repair rate of Melting vats ($\beta_3$) on reliability of the system.

| $t$  | 0.40 | 0.45 | 0.50 | 0.55 | 0.60 | 0.65 |
|------|------|------|------|------|------|------|
| 30   | 0.89750 | 0.89675 | 0.89600 | 0.89525 | 0.89450 | 0.89375 |
| 40   | 0.89500 | 0.89425 | 0.89350 | 0.89275 | 0.89200 | 0.89125 |

VII. GRAPHICAL STUDY

Fig. 2

Fig. 3
Analysis of the Reliability of the Butter-Oil Processing Plant using CAS Mathematica and Maxima

VIII. DISCUSSION AND CONCLUSION

Analysis of reliability of butter-oil manufacturing plant can help in increasing the production of the butter-oil. The tables and graphs show the variation of reliability with change in failure rates ($\alpha_i$) and repair rates ($\beta_i$) of separator, pasteuriser, continuous butter making, melting vats, butter oil clarifier and packaging. We observed that reliability of the system decreases with increase in failure rate and with passage of time it approaches the steady state. Graphical study respective to their tables such as graph-2 represents table-1, graph-3 represents table-2, and so on, clears the variation with different values of failure rate on reliability of system. We analysed from the tables that reliability of the system decreases with increase in failure rate but increases with increase in repair rate, also with passage of time reliability is decreased slightly and after that it reaches the steady state. In comparative study, Mathematica is a method of better accuracy than Maxima. Values given by Mathematica are exact than approximates values given by Maxima. Also, Mathematica gives individual values of $P_1, P_2, ..., P_{13}$ at point $t$ which is not possible in Maxima. In Mathematica, $P_1, P_2, ..., P_{13}$ are continuous functions of time so $R(T)$ is also a continuous function of time not discrete. Mathematica gives exact values closer to the solution rather than approximates values given by Maxima. This software not only gives accuracy in results but also saves computation time. Such results might be useful for optimization in performance of butter-oil processing plant.

REFERENCES:

1. Jai Singh, Arvind Kumar Lal, Rajendra Kumar Sharma and Pawan Kumar Gupta (2005), “Numerical analysis of reliability and availability of the serial processes in butter-oil processing plant”, Article in International Journal of Quality & Reliability Management
2. Tarun Garg (2015), “Application of CAS Maxima to the Availability Analysis of Screening Unit in a paper plant”.
3. Practical Mathematics Using Maxima, G. S. Tuteja, International Book House Pvt. Ltd., Delhi.
4. Dayal, B. and Singh, J. (1992), “Reliability analysis of a system in a fluctuating environment”, Microelectron Reliability, Vol. 32, pp. 601-3.
5. Dhillon, B.S. and Natesan, J. (1983), “Stochastic analysis of outdoor power system in fluctuating environment”, Microelectron Reliability, Vol. 23, pp. 867-81.
6. Goel, P. and Singh, J. (1995b), “Reliability analysis of a standby complex system having imperfect switch-over device”, Microelectron Reliability, Vol. 35, pp. 285-8.
7. Kumar, D., Singh, J. and Singh, I.P. (1988a), “Reliability analysis of the feeding system in paper industry”, Microelectron Reliability, Vol. 28, pp. 213-15.
8. Kumar, D., Singh, J. and Singh, I.P. (1988b), “Availability of the feeding system in the sugar industry”, Microelectron Reliability, Vol. 28, pp. 867-71.
9. Mahajan, P. and Singh, J. (1999), “Reliability of utensils manufacturing plant – a case study”, Operation research, Vol. 36, pp. 260-71.
10. Singh, J. (1984), “Reliability of a fertilizer production supply problem”, Proceedings of ISPTA, Wiley Eastern Ltd, New Delhi, pp. 95-8. Singh, J. (1990a), “A warm stand-by redundant system with common cause failures”, Reliability Engineering and System Safety, Vol. 26, pp. 135-41.
11. Singh, J. (1989b), “Reliability analysis of a biogas plant having two dissimilar units”, Microelectron Reliability, Vol. 29, pp. 779-81.
12. Olga Bunukhina, Anna Bushinskaya, Irina Maltceva and Svyatoslav Timashev (2018), “Mechanical system reliability analysis using reliability matrix method”, : Materials Science and Engineering.

13. Khanduja, R., Tewari, P.C., and Chauhan, R.S.(2012), “Performance Modeling and Optimization for the Stock Preparation Unit of a Paper Plant using Genetic Algorithm”, International Journal of Quality and reliability Management, Vol. 28, No. 6, pp. 688-703.

14. Kumar, S. and Tewari, P.C.(2011), “Mathematical Modeling and Performance Optimization of CO2 Cooling System of a Fertilizer Plant”, International Journal of Industrial Engineering Computations , vol. 2, pp. 689-695.

AUTHOR PROFILE

I am Shalini Jindal Research Scholar in Department of mathematics, currently pursuing PhD in mathematics in J. C. Bose University of Science and Technology, YMCA Faridabad. E mail: shalinjindal91@gmail.com

Dr. Reena Garg, M.Sc Mathematics (GOLD MEDALIST) M.Phil, Ph.D is an Assistant Professor (Sr Scale) Mathematics in J.C. BOSE UNIVERSITY OF SCIENCE And TECHNOLOGY, YMCA FARIDABAD, HARYANA( NAAC “A’ GRADE STATE UNIVERSITY). Three Ph.D scholars are pursuing Ph.D under her kind supervision. She is working in University for the last twelve years. Her teaching experience of more than a decade in the University has made this book more valuable for the knowledge seekers. She has published / presented more than 80 research papers in various International Journals / International conferences / national conferences.

Dr. Tarun Kumar Garg tk garg@satyawati.du.ac.in has been working as an associate professor since last 20 years in the department of Mathematics, Satyawati College, University of Delhi, Delhi. His field of research is Wavelet Analysis, Reliability and availability of Mechanical systems and Optimization. He has published many research papers in international journals.