Abstract Resource allocation is an essential aspect of successful Product Development (PD). In this paper, we formulate the dynamic resource allocation of the PD process as a convex optimization problem. Specially, we build and solve two variants of this issue: the budget-constrained problem and the performance-constrained problem. By using convex optimization, we propose a framework to optimally solve large problem instances at a relatively small computational cost. The solutions to both problems exhibit similar trends regarding resource allocation decisions and performance evolution. Furthermore, we show that the product architecture affects resource allocation, which in turn affects the performance of the PD process. By introducing centrality metrics for measuring the location of the modules and design rules within the product architecture network, we find that resource allocation decisions correlate to their metrics. These results provide simple, but powerful, managerial guidelines for efficiently designing and managing the PD process. Finally, for validating the model and its results, we introduce and solve two design case studies for a mechanical manipulator and for an automotive appearance design process.

1 Introduction

Successful Product Development (PD) requires careful allocation of development resources. Allocating resources to various subsystems and modules within the product system requires a deep understanding of many complex interactions. These interactions arise from various sources; namely, due to the physical interdependencies between the different subsystems in the product itself (i.e., the product architecture), the arrangements of organizations that will carry out the development process (i.e., the social network behind the organization), and the structure of the development process (i.e., predecessor relationships between development activities) [32]. In particular, this paper is focused on obtaining an understanding of the product architecture and its role in resource allocation decisions during PD.

Product architecture is the scheme by which the functional elements of the product are arranged into physical chunks and by which the chunks interact [28]. Product architecture plays a significant role in every aspect of the product lifecycle from influencing how the product is designed, manufactured, marketed, experienced, serviced, and retired [29, 34]. Additionally, the product architecture has profound implications for many product behavior properties from robustness [12] to evolvability [20]. It also influences how resources are allocated in the PD process [23, 33].

Product architecture is usually described by a continuum between an integral product architecture to a modular one. In integral architectures, the product functions are shared by product modules (i.e., physical elements), and in modular architectures, each function is delivered by a sep-
nature element or module. Thus, integrality creates interdependence between product elements or modules. This interdependence, in turn, results in complexity. That is, some of the interdependencies may not be known in advance, or their influence on product and PD process performance may also be unknown. Within this complex PD environment, several studies have argued that the product architecture may evolve from integral to modular \cite{13,33}.

In this paper, we want to investigate how the product architecture may influence the resource allocation decision to various modules using an optimization framework. Using this framework, we can investigate the tendency for product architectures to evolve form integral to modular architectures. The main objective of the paper is to check whether the location of a module within the product architecture can offer PD managers insights into optimal resource allocation decisions.

Several authors have formulated and analyzed the PD problem by analogy to dynamic linear systems (e.g., \cite{23,25,31}). In their analysis, they assumed that all tasks in the design structure matrix (DSM) proceed in parallel, where the DSM is a matrix representation of the development network. At any iteration stage, one unit of work on one task results in a fraction of rework for the other dependent tasks during the next iteration stage. The dependency between tasks is captured by the numerical values in the DSM. As such, the work completed in a current design iteration is a linear function of the work completed in the previous design iteration, with the linear weights being the numerical values in the DSM.

Other authors have used complexity theory to describe and analyze the PD process \cite{11}. For instance, Braha and Bar-Yam \cite{12} introduced the NK-based model and analysis of product development project networks. They showed how the underlying network topologies and statistical structural properties provide direct information about the functionality, dynamics, robustness, and fragility of these PD projects. Also, the authors in \cite{17} argued that modules could be optimized independently if interface standards between modules are left unchanged. Similarly, Luo \cite{20} used the NK framework to show how different product architectural patterns can influence product evolvability.

Network analysis has also been used for analyzing PD project network \cite{6,15}. For example, the analysis of the network structure (i.e., statistical properties) for various software and hardware development projects in \cite{12} revealed that these networks have both small-world and scale-free network patterns. Additionally, they demonstrated that complex design networks are highly robust to the failure of randomly selected design components, but weak for failures targeting specific components (such as hub components). Similarly, Sosa et al. \cite{26} found that the analysis of the network structure of complex product designs (particularly, the existence of hubs in the design network) impacts the quality of the product being developed.

More recently, the authors in \cite{33} have formulated the PD resource allocation problem as a nonlinear optimization problem. Furthermore, the authors proposed a dynamic model in which there are several investment runs (or rounds) during the PD process. This formulation allowed the investigation of several interesting hypotheses, including the impact of architecture on performance evolution from integral to modular systems.

However, in this paper, we offer a more efficient optimization approach based on convex optimization techniques. The contribution of this paper can be stated as follows. First, we adopt a discrete-time linear system to represent the work transition feature in the PD process. Then, we propose an optimization framework where the resource allocation problem of the PD process can be transformed into a convex optimization problem. Finally, from analyzing the experimental results, we gain an insight into the resource allocation problem and provide a guide for designing and managing the PD process.

The following sections are organized as follows. In Section 2, we describe the work transition feature of the PD process by a discrete-time linear system. Then, we formulate the budget-constrained problem and the performance-constrained problem for optimal resource allocation. In Section 3, we propose the framework that both the problems can be transformed into the convex optimization problems. In Section 4, we perform an analysis for the decision variables and performance of the PD process. Finally, we make two case studies to illustrate the result of this paper in Section 5.

2 Proposed model

In this section, we first review the dynamic model of the PD process proposed in \cite{33}. Then, from the perspective of system and control, we show that the work transition feature in the PD process can be expressed by a discrete-time linear system. Finally, we formulate the optimal resource allocation problem of the PD process as the budget-constrained optimization and the performance-constrained optimization problem separately.

2.1 Work transformation matrix

In PD, the product architecture is built not only by the constituent parts that define the product system (i.e., modules or components), but also by defining the interaction relationships between these parts (i.e., dependency structure) \cite{28}.

In this paper, we assume that the product architecture has
been determined in the early design stage. That is, the modules and their dependency structure (i.e., design rules) have been established. In this situation, we focus on improving the performance of the development project through allocating the development resources to the various modules and design rules over several investment rounds (i.e., design iterations).

We start the problem formulation by reviewing the dynamic PD model presented in [33]. Suppose that there are \( n \) modules and \( T \) investment rounds during the PD process, we let \( P_i(k) \) represent the amount of the remaining work in the \( i \)th module after finishing the \( k \)th round. The vector of the remaining work is defined by

\[
P(k) = \begin{bmatrix} P_1(k) \\ \vdots \\ P_n(k) \end{bmatrix}.
\]

The performance of the PD process is evaluated by the sum of the remaining work in each module [19]. This implies that the less the total remaining work left, the higher performance the product system has. Thus, the total performance after the \( k \)th round is expressed by the following equation:

\[
\sum_{i=1}^{n} P_i(k).
\]  

\hspace{1cm} (1)

At each iteration stage, the module finishes a certain amount of remaining work, and sends/receives the produced work (i.e., a fraction of rework) to/from its dependent modules. To describe this work transformation process, we use a discrete-time linear system with the following equation:

\[
P(k+1) = A_k(\phi_k, \gamma_k) P(k), \quad k = 0, \ldots, T,
\]  

\hspace{1cm} (2)

where \( A_k(\phi_k, \gamma_k) \) is the work transformation matrix (WTM),

\[
\phi_k = \{ \phi_{1,k}, \ldots, \phi_{n,k} \}
\]

represent the work completion rate of the modules, and

\[
\gamma_k = \{ \gamma_{j,k} \} \quad (i, j = 1, \ldots, n, \ i \neq j)
\]

are the updated value of design rules. For an established product architecture, the performance of the product system can be further improved by investing in both modules (i.e., resulting in performance improvement) and design rules (i.e., reducing the ratio of the produced work between two modules). We assume that \( \phi_k, \gamma_k \) can be tuned within the following intervals:

\[
0 < \phi_{2,k} \leq \phi_{i,k} \leq \phi_{1,k}, \quad 0 < \gamma_{i,j,k} \leq \gamma_{j,k}.
\]  

\hspace{1cm} (3)

where \( \phi_{1,k}, \gamma_{j,k} \) are the initialized parameter values of the WTM, and \( \phi_{2,k}, \gamma_{i,j,k} \) are lower bounds of the parameters. For the PD process with multiple investment rounds, the authors in [33] showed that the value of the design rules \( \gamma_{j,k} \) in the \( k \)th round is updated to include the values of the design rules that resulted from the investment in the \((k-1)\)th round. Therefore, the updated value of the design rules at the \( k \)th iteration \( \gamma_{j,k} \) is the multiplication of the updated value of the design rules from the 0th to the \( k \)th, which is expressed as \( \prod_{\ell=1}^{k} \gamma_{j,\ell} \). The specific form of the \( A_k(\phi_k, \gamma_k) \) is given by

\[
A_k(\phi_k, \gamma_k) = \begin{bmatrix} \phi_{1,k} & \prod_{\ell=1}^{k} \gamma_{1,\ell} & \cdots & \prod_{\ell=1}^{k} \gamma_{n,\ell} \\ \prod_{\ell=1}^{k-1} \gamma_{2,\ell} & \phi_{2,k} & \cdots & \prod_{\ell=1}^{k-1} \gamma_{n,\ell} \\ \vdots & \vdots & \ddots & \vdots \\ \prod_{\ell=1}^{k-1} \gamma_{n,\ell} & \cdots & \cdots & \phi_{n,k} \end{bmatrix}.
\]  

\hspace{1cm} (4)

Suppose that we can use the development resources to update the value of \( \phi_k, \gamma_k \). That is, we can use the resources to improve the work completion rate \( \phi_k \) on the module, also we can use the resources to reduce the dependency \( \gamma_k \). Moreover, we assume that there is an associated cost \( f_i(\phi_{i,k}) \) for tuning the value from \( \phi_{i,k} \) to \( \phi_{i,k} \). Likewise, \( g_{ij}(\gamma_{j,k}) \) is the cost for tuning the value of the dependency from \( \gamma_{i,k} \) to \( \gamma_{j,k} \). Then, the total cost in the \( k \)th investment round equals

\[
B_k(\phi_k, \gamma_k) = \sum_{i=1}^{n} f_i(\phi_{i,k}) + \sum_{i=1}^{n} \sum_{j \neq i} g_{ij}(\gamma_{j,k}).
\]  

\hspace{1cm} (5)

Making resource allocation decisions based on intuition or heuristic methods to achieve the goal of PD project is not trivial due to shear size of the problem that can include thousands of decision variables. Thus, a mathematical programming formulation for finding the optimal investment strategy is essential.

2.2 Problem formulation

As mentioned in Section 2.1 at each iteration of the PD process, PD managers can use a certain amount of development resources to improve the performance of the product system. Particularly, the resources can be allocated on a module for improving its work completion rate or on a specific dependency for reducing its strength between two modules.[Chengyan: see pdf] Assuming that we are given a set of budgets for each investment round, and the associated cost for the development resources, how should we make the investment decisions to minimize the total remaining work of the PD process? Based on this question, we formulate the budget-constrained optimization problem as follows:

**Problem 1 (Budget-constrained optimization)** Assume that, given \( P(0) \), there are \( T \) investment rounds with the corresponding budgets \( B_k > 0 \) (\( k = 1, \ldots, T \)) for resource allocation during the PD process, as well as the cost functions \( f_i(\phi_{i,k}) \) and \( g_{ij}(\gamma_{j,k}) \). Find a sequence of decision variables for allocating the investment resources in modules \( \phi = \{ \phi_{i,k} \}_{i=1}^{n} \) (\( i = 1, \ldots, n \)) and design rules \( \gamma = \{ \gamma_{j,k} \}_{j=1}^{n} \) (\( i = 1, \ldots, n, \ i \neq j \)) to minimize the total remaining work while
satisfying the budget constraints on the investment resources in each round. Mathematically, we formulate Problem 1 as:

\[
\begin{align*}
\text{minimize} & \quad \sum_{i=1}^{n} P_i(T) \quad (6a) \\
\text{subject to} & \quad B_i(\phi_i, \gamma_i) \leq \bar{B}_i, \quad (6b) \\
& \quad \gamma \quad \in \quad [3]. \\
& \quad \gamma \quad \in \quad (6c)
\end{align*}
\]

For Problem 1, our goal is to make the optimal resource allocation strategy to minimize the total remaining work. However, there exists another situation where the manager plans to meet the prescribed target on the remaining work at \( T \) by using the minimum resources, where the remaining work can be regarded as a proxy for judging the progress of the PD process. In this case, the performance-constrained optimization problem can be formulated as follows:

**Problem 2 (Performance-constrained optimization)**

Assume that, given \( P(0) \), there are \( T \) investment rounds and the prescribed remaining work constraint \( P_T \) > 0, as well as the cost functions \( f_i(\phi_i, k) \) and \( g_{ij}(\gamma_{ij}, k) \). Find a sequence of decision variables for allocating the development resources in modules \( \phi = \{\phi_{ik}\}_{i=1}^{n} \) and design rules \( \gamma = \{\gamma_{ijk}\}_{i=1}^{n} \) to minimize the total investment resource while satisfying the constraint on remaining work.

As in [6], we can mathematically build the performance-constrained optimization problem as the following:

\[
\begin{align*}
\text{minimize} & \quad \sum_{k=1}^{n} B_i(\phi_k, \gamma_k) \quad (7a) \\
\text{subject to} & \quad \sum_{i=1}^{n} P_i(T) \leq P_T, \quad (7b) \\
& \quad \gamma \quad \in \quad [3]. \\
& \quad \gamma \quad \in \quad (7c)
\end{align*}
\]

The difficulty of solving Problems 1 and 2 mainly stems from the nonlinearity of the functions [6a–7a] and constraints [6b–7b]. That is, Problems 1 and 2 become nonlinear optimization problems. Although there are some numerical solutions for this case based on heuristic methods [2,7], such techniques can cause the solution to be trapped in a local optimal point. Moreover, the computation cost (time) of the heuristic solver grows rapidly with the increase in problem size (i.e., the number of modules, design rules and the investment rounds). Thus, there exists a need for developing a framework that can deliver the optimal solution for Problems 1 and 2 using a relatively small computational cost.

### 3 Solution using convex optimization

In this section, we present an optimization framework for efficiently solving the budget-constrained problem and the performance-constrained problem. Under the relatively mild assumption on the cost function, we show that problems can be transformed into convex optimization problems.

We first show that Problems 1 and 2 can be optimally solved if the cost functions \( f_i(\phi_i, k) \) and \( g_{ij}(\gamma_{ij}, k) \) belong to a wide class of functions named posynomials, which is commonly used in modeling mathematical problem. We start this section by reviewing the definition of posynomials with the following:

**Definition 1** ([8]) Let \( v_1, \ldots, v_n \) denote \( n \) real positive variables.

1. We say that a real function \( g(v) \) is a **monomial** if there exist \( c > 0 \) and a set of real numbers \( a_1, \ldots, a_n \) such that \( g(v) = c a_1^{v_1} \cdots v_n^{a_n} \).
2. We say that a real function \( f(v) \) is a **posynomial** if \( f \) is a sum of monomials of \( v \).

To precisely model the cost features, the nonlinearity in practice affects the performance of the model, which cannot be ignored. From Definition 1, we can see that the posynomials are nonlinear functions which can be used for fitting real data from practical PD problems. For specific techniques to fit posynomials to real data, we refer readers to [9]. Also, the following lemma shows the convexity property of posynomials, which is essential in transforming Problems 1 and 2 into convex optimization problems.

**Lemma 1** ([9]) If \( f \) is a posynomial, then, the function

\[
\begin{align*}
x \mapsto \log f(\exp(x))
\end{align*}
\]

is convex.

As mentioned earlier, the nonlinearity of the real data can be fitted by posynomials. From Definition 1 we can see that the range of the posynomials is in the nonnegative number field. However, in practice, real data may run out of the nonnegative area. Thus, normalizing the range of the cost function to the nonnegative field is necessary (i.e., adjust the minimum value of the cost function larger than 0). For this reason, we make the following assumption for ensuring the nonnegativity of the cost function. We assume that the cost function has the following form:

\[
\begin{align*}
f_i(\phi_{ik}) &= f_i^+(\phi_{ik}) - f_i^-(\phi_{ik}), \\
g_{ij}(\gamma_{ijk}) &= g_{ij}^+(\gamma_{ijk}) - g_{ij}^-(\gamma_{ijk}).
\end{align*}
\]

The essential part of the cost function is the first term \( f_i^+(\phi_{ik}) \), while the second term \( -f_i^-(\phi_{ik}) \) is for normalizing the cost function as \( f_i(\phi_{ik}) = 0 \), similarly for \( g_{ij}(\gamma_{ijk}) \), which means that the zero investment yields no cost. Furthermore,
for the total cost function in (5), we let
\[ B_k^+(\phi_k, y_k) = \sum_{i=1}^{n} f_i^+ (\phi_i, k) + \sum_{i=1}^{n} \sum_{i \neq j} g_{ij}^+ (\gamma_{ij, k}), \]
\[ B_k^- (\phi_k, y_k) = \sum_{i=1}^{n} f_i^- (\phi_i, k) + \sum_{i=1}^{n} \sum_{i \neq j} g_{ij}^- (\gamma_{ij, k}). \]

Then, we can rewrite the total cost function (5) as:
\[ B_k (\phi_k, y_k) = B_k^+ (\phi_k, y_k) - B_k^- (\phi_k, y_k). \]

With the above preliminaries, we can now present the main result of this paper; namely, we can show that Problems 1 and 2 can be solved via convex optimization.

**Theorem 1** If the solution of the following convex optimization problem is given by \( x \in \{x_k\}_{k=1}^T \) and \( y = \{y_k\}_{k=1}^T \), where \( x_k = \{x_{ij, k}\}_{i=1}^n \), \( y_k = \{y_{ij, k}\}_{i=1}^n \), \( i, j = 1, \ldots, n, i \neq j \), and \( x, y \) belong to the real number field.

\[
\begin{align*}
\text{minimize} & \quad \Psi \quad (11a) \\
\text{subject to} & \quad \log \sum_{i=1}^{n} P_i (T) \leq \log \hat{P}_T, \quad (11b) \\
& \quad \log \sum_{k=1}^{T} B_k^+ (x_k, y_k) \leq \log (\Psi + \sum_{k=1}^{T} B_k^-), \quad (11c) \\
& \quad \log \phi_{ik} \leq x_{ik} \leq \log \hat{\phi}_{ik}, \quad (11d) \\
& \quad \log \gamma_{ij, k} \leq y_{ij, k} \leq \log \bar{\gamma}_{ij, k}. \quad (11e)
\end{align*}
\]

Then, the solution of Problem 1 is given by
\[ \phi = \exp [x], \quad \gamma = \exp [y], \quad (10) \]
where \( \exp [\cdot] \) is the entrywise exponential function of the variables.

**Proof** Under Lemma 1, it can easily be seen that (6a), (6b), and (6c) in Problem 1 are equivalent to (9b), (9c), (9d), and (9e) of Theorem 1, respectively. Therefore, the solution of Theorem 1 given by (10) is the solution of Problem 1. Under this equivalence, we show the convexity of Theorem 1. It is sufficient to show that constraints (9b) and (9c) are convex if the performance functions (6a), (6b) and the cost function (5) follow Definition 1. Thus, we complete the proof of Theorem 1.

Similarly, the next theorem shows that Problem 2 can also be solved with the same optimization framework.

**Theorem 2** If the solution of the following convex optimization problem is given by \( x \in \{x_k\}_{k=1}^T \) and \( y = \{y_k\}_{k=1}^T \), where \( x_k = \{x_{ij, k}\}_{i=1}^n \), \( y_k = \{y_{ij, k}\}_{i=1}^n \), \( i, j = 1, \ldots, n, i \neq j \), and \( x, y \) belong to the real number field.

\[
\begin{align*}
\text{minimize} & \quad \Psi x \text{subject to} \quad \log \sum_{i=1}^{n} P_i (T) \leq \log \hat{P}_T, \quad (11b) \\
& \quad \log \sum_{k=1}^{T} B_k^+ (x_k, y_k) \leq \log (\Psi + \sum_{k=1}^{T} B_k^-), \quad (11c) \\
& \quad \log \phi_{ik} \leq x_{ik} \leq \log \hat{\phi}_{ik}, \quad (11d) \\
& \quad \log \gamma_{ij, k} \leq y_{ij, k} \leq \log \bar{\gamma}_{ij, k}. \quad (11e)
\end{align*}
\]

Then, the solution of Problem 2 is given by (10).

**Proof** Under Lemma 1, it can easily be seen that (7a), (7b), and (7c) in Problem 2 are equivalent to (11b), (11c), (11d), and (11e) in Theorem 2, respectively. Therefore, the solution of Theorem 2 given by (10) is the solution of Problem 2. Let us show the convexity of Theorem 2. It is sufficient to show that constraints (11b) and (11c) are convex if the performance functions (7a), (7b), and the cost function (5) follow Definition 1. Thus, we complete the proof of Theorem 2.

4 Experimental setup, analysis and discussion of results

In this section, we show the effectiveness of the proposed framework by solving relatively large-size PD problems with different product architectures. Furthermore, by investigating the solution, we reveal the trends, structure, and relationship of the decision variables. In Section 4.1, we introduce four typical DSM architectures embedded in our simulation experiments. In Section 4.2, we give the specific form of the cost function. Then, in Section 4.3, we present the optimal solution of Problem 1, perform its analysis, and discuss the results. Likewise, in Section 4.4, we present the optimal solution of Problem 2, and its corresponding analysis and discussion are carried out. In Section 4.5, we investigate the impact of the product architecture on the resource allocation and performance of the PD process.

4.1 DSM architecture

As mentioned earlier, the design structure matrix (DSM) is a matrix representation of the development network which can have a particular architecture [24]. For this reason, the DSM architecture in our experiment is determined by the following network models: the Block-diagonal [24], the Erdős-Rényi (random) [16], the Watz-Strogatz (small world) [30] and the Barabási-Albert (scale-free) [4] graphs.

Fig. 1 shows the four DSM architectures used in this paper. On one end, the Block-diagonal network represents...
Fig. 1: Four DSM architectures. All with 50 modules and 100 design rules: (a) Block-diagonal network, (b) Erdős-Rényi network (random), (c) Watz-Strogatz network (small world), (d) Barabási-Albert network (scale-free). The the diagonals in the DSM represent the location of modules, and the off-diagonals show the dependencies between modules.

Fig. 2: Three cost functions with $p = 1, 10, \text{ and } 50$. $f_{ij}(\Omega_{ij}) = 0$ represents that no resource is allocated, where $\Omega_{ij}$ denotes the initial value of the certain entry in WTM. $f_{ij}(\epsilon\Omega_{ij}) = 1$ indicates the upper bound of the allocated resources where we can obtain the fully improved value.

As mentioned in Section 2.1, the resource allocated in the modules and design rules result in a reduction of the values in WTM. Based on this, we claim that the cost function should be a decreasing function, and satisfy Definition 1. Thus, we use the following cost function:

$$f_{ij}(\gamma_{ij}) = c_{ij} \left( \frac{1}{(\gamma_{ij})^p} - \frac{1}{(\Omega_{ij})^p} \right),$$

(12)

where $\gamma_{ij}$ is the updated value of the parameter in the WTM, $p$ is a positive number for tuning the shape of the concerned cost function, and $c_{ij}, \Omega_{ij}$ are positive numbers for fitting the value of the cost function to satisfy Definition 1. Then, we make the following assumption that shows the diminishing return property, which ensures the convexity of the cost function as well. Suppose that there is a fixed increment $\epsilon_{ij} > 0$ on $\gamma_{ij}$, and let $\Delta f_{ij}(\gamma_{ij}) = f_{ij}(\gamma_{ij} - \epsilon_{ij}) - f_{ij}(\gamma_{ij})$ represent the cost for tuning $\gamma_{ij}$ to $\gamma_{ij} - \epsilon_{ij}$. The diminishing return property means that the parameter tuning cost $\Delta f_{ij}(\gamma_{ij})$ increases with $\gamma_{ij}$, and also implies the convexity of $f_{ij}$. In practice, the parameters of the cost function are carefully assigned by the managers and the work teams (e.g., see [27,33]). Fig. 2 shows three realizations of the cost function under different values of $p$.

4.3 Analysis and Discussion of Problem 1

In this subsection, we optimally solve Problem 1 through our proposed framework. Then, we investigate the evolution...
of decision variables during the budget-constrained PD process. Finally, we introduce the centrality metrics for measuring the importance of elements (i.e., module and design rules) in the DSM, and study whether the allocated resources or the remaining work in the module (or their dependencies) correlates with its centrality.

In this simulation experiment, for testing the effectiveness of solving a relatively large scale PD problem [3], we produce the DSMs of size 50 and hold the total number of dependencies to 100 for each DSM architecture. We set the number of investment rounds \( T = 5 \), and the budget \( B_k = 300 \) for each investment round. For initializing the parameters of the WTM, we unify \( \phi_{jk} = 0.5 \) (\( k = 1, \ldots, 5, i = 1, \ldots, 50 \)) and \( \gamma_{ij,k} = 0.05 \) (\( i, j = 1, \ldots, 50, i \neq j \)) for all the experiments. For all the cost functions, we unify the parameters with \( c_{ij} = 1, p = 1, \Omega_{ij} = 1, \) and \( \epsilon = 0.1 \), which indicates that the \( \phi \) and \( \gamma \) can be updated between \([0\%-90\%]\) of the initial value. From the parameter initialization, we can see that the values of \( \phi_{jk} \) and \( \gamma_{ij,k} \) can be tuned within the intervals \([0.05, 0.5]\) and \([0.005, 0.05]\), respectively. For measuring the evolution of PD process, we use the percentage of the finished work in each investment round given by

\[
\frac{\sum_{i=1}^{n} P_i(k) - \sum_{i=1}^{n} P_i(k + 1)}{\sum_{i=1}^{n} P_i(k)}
\]

(13)
to represent the performance at each iteration stage. We conducted the experiment with the selected DSM architectures, and observed the following response variables: remaining work in modules, investment modules and design rules, and the dependency values of the design rules.

For problem solving, we adopt the commonly used off-the-shelf software for convex optimization problem: fmincon routine in MATLAB. From the initialization of Problem 1, we can see that the total number of the decision variables is \((50 + 100) \times 5 = 750\), which can be seen as a relatively large problem size. Through running the experiment on the desktop with common configuration (i.e., Intel Core i7-7700 and 8GB memory), the average time for solving the optimization problem is 210 minutes, which illustrates that our framework has great potential in solving large-scale problems.

Fig. shows the evolution of the decision variables and the remaining work of Problem 1. Particularly, Fig. (a) shows the decreasing trends of the remaining work in each module, which indicates that the PD process is in progress. However, from Fig. (a), we cannot clearly see the improved performance of the product system through resource allocation. Thus, we adopt the evolution of the performance as seen in Fig. where it can be seen that the performance of product system is monotonously improved with successive investment. Compared with the performance with no investment (dashed line), we observe that the PD process is accelerated with the allocated resources. As seen in Fig. in future investment rounds, the system performance cannot be further improved (reaches its saturation point). Then, we also notice that the performance limit reached in the different DSM architectures are not the same, which implies that for further improving the performance, the DSM architecture must be taken into consideration.

Figs. (b) and (c) show the evolution of investment in modules and design rules, where we can see that the investment in modules increases, while the investment in design rules decreases. This phenomenon is in line with the result in [33] that there is a shift in resource allocation from design rules to modules as the development process progresses. Moreover, it is worth noting that the modular architecture consumed more resources on the modules compared with design rules, while integral architecture consumed more resources on the design rules. Thus, the evolution of the decision variables also confirms that the product architecture evolves from integral to modular as the product matures. In Fig. (d), we can see that the dependency values of the design rules tends to 0, which indicates that the strength of dependency between modules is reduced or nearly eliminated by successive investments in design rules.

Next, we carry out a further investigation on whether there is a relationship between the investment in a specific module \( \sum_{k=1}^{T} f_k(\phi_{ik}) \) (\( i = 1, \ldots, n \)) and its related design rules \( \sum_{k=1}^{T} \gamma_{i,k} \). Fig. (e) shows a positive correlation between the investment in certain modules and their related design rules. This observation can be used as a managerial guideline for resource allocation: if a module is assigned resources, then corresponding amount of resource must be allocated to its relevant design rules.

After discussing the results in Fig. 1 we introduced the centrality metrics (i.e., importance measures) for the modules and design rules in the DSM to investigate whether there is a relationship between the investment in module (design rules) and its centrality. For describing the importance of the elements in the DSM, we adopted three centrality metrics: the Eigenvector, the PageRank, and the Closeness centrality (see, e.g., [21]). Figs. 5-8 show the total investment (remaining work) in the modules (design rules) versus its centrality. For simplicity, we normalize each centrality metric to 1. From these figures, we can see that the extent of correlation varies with different centrality metrics. To decide which centrality metric is the best in describing the correlation, we adopt Pearson correlation to help us select the proper centrality metric. A perfect Pearson correlation 1 occurs when each of the variables is a perfect monotone function of the other. On the contrary, zero means that there is completely no correlation between the two set of numbers.

Table 1 shows the result of Pearson correlation for Figs. 5-8. From Table 1 we can see that PageRank performs the best except for the Block-diagonal case because the defi-
Fig. 3: The optimal solution of Problem 1. Color lines distinguish the importance of module/design rules via the PageRank.
Optimal Resource Allocation for Dynamic Product Development Process via Convex Optimization

4.4 Analysis and Discussion of Problem 2

In this subsection, we optimally solve Problem 2. Although we have revealed the trends of the decision variables and the internal relations for the budget-constrained problem, we cannot conclude that the same situation also exists in the performance-constrained problem.

As in Section 4.3, we performed the simulation experiments on a controlled set of product architectures. For initializing Problem 2, we adopted the same parameters setting as Problem 1. Based on the formulation of Problem 2, we set constraint for the total remaining work of the final investment round to $P_T = 0.01$, which can be regarded as a threshold for judging the accomplishment of the PD process. For example, suppose that the total remaining work at the beginning is normalized to 1, if we set $P_T = 0.01$, it means that when the total remaining work is 1% its initial value, we can say that the project is finished. Then, we ran the experiments and observed the following response variables: remaining work in the module, investment in the module and the design rules, the value of design rules, and the total investment in each round.

Fig. 9 shows the evolution of decision variables and the remaining work for Problem 2. From Figs. 9(a)-(c), we can observe a very different phenomenon that hundred of variables are overlapped to five points, and are strongly correlated with the Eigenvector and the Closeness centrality. We notice that, in Fig. 1, the Block-diagonal network used in our experiment is produced by several independent sub-blocks, which all the sub-blocks are independent with each other, and all the modules in the sub-block are fully interacted (i.e., numbers of design rules in the sub-blocks are different). Based on the special structure of Block-diagonal network and our observation, we conclude that, for each sub-block, the resources allocated on its design rules is independently determined by the complexity of sub-block (i.e., the number of design rules).

| DSM          | Remaining work | Investment in module | Investment in DRs |
|--------------|----------------|----------------------|-------------------|
| Block-diagonal | Remaining work | Investment in module | Investment in DRs |
|              | 0.993          | 0.992                | 0.993             |
| Random       | Remaining work | Investment in module | Investment in DRs |
|              | 0.561          | 0.518                | 0.443             |
| Small-world  | Remaining work | Investment in module | Investment in DRs |
|              | 0.501          | 0.483                | 0.377             |
| Scale-free   | Remaining work | Investment in module | Investment in DRs |
|              | 0.493          | 0.427                | 0.666             |

Table 1: Pearson correlation analysis for Problem 1 (Figs. 5-8)
see that the solution of Problem 2 exhibits similar trends to Problem 1. Particularly, the solution shows that the product architecture evolves from an integral to a modular as successive investments are made on modules and design rules. In Fig. 9 (d), it is worth noting that there is a decreasing tendency on the total investment during the PD process, which contradicts our intuition that the resource should be equally allocated for each investment round. From Fig. 9 (e), we can see that the positive correlation between the investment in module and its dependent design rules does not change in Problem 2.

To perform the analysis for the relationship between the investment in module (design rules) and its centrality, we use the same method as in Section 4.3. Figs. 10-13 show the correlation between the investment and remaining work under three different centrality metrics, which reveals the same correlation as discussed for Problem 1. From Table 2, we can see that the PageRank is also suitable for measuring the correlation for Problem 2.

4.5 Analysis of different DSM architectures

In this subsection, we carry out an analysis of variance on the product architecture to investigate whether the product architecture affects the resource allocation and the performance of the designed PD system.

In this experiment, we used the four DSM architectures introduced in Fig. 1 and selected the three response variables: total remaining work, total investment in modules, and total investment in design rules. To detect any statistical difference, we randomly generate 50 sample networks for each type of product architecture. In all the problems, we unify the parameters of the WTM and the cost functions as in the previous sections. We solve these problems with the proposed framework in this paper. The results of
Fig. 6: The remaining work, investment in modules and design rules of Problem 1 versus their centrality measures in the Watz-Strogatz (small world) network. Dash line: Linear regression line.

Table 2: Pearson correlation analysis for Problem 2 (Figs. 10, 13)

| DSM               | Remaining work | Investment in module | Investment in DRs | Eigenvector centrality | PageRank | Closeness centrality |
|-------------------|----------------|----------------------|-------------------|------------------------|----------|----------------------|
| Block-diagonal    | 0.991          | 0.993                | 0.993             | 1.000                  | 0.999    | 0.980                |
| Random            | 0.624          | 0.598                | 0.467             | 0.895                  | 0.899    | 0.785                |
| Small-world       | 0.549          | 0.535                | 0.270             | 0.974                  | 0.975    | 0.521                |
| Scale-free        | 0.493          | 0.446                | 0.632             | 0.995                  | 0.991    | 0.946                |
Figs. 14 and 15, we can observe that the architecture affects the resource allocation, which in turn affects the remaining work (performance) of the PD process.

For the remaining work of Problem 1 (Fig. 14 (a)), we can see that the Block-diagonal architecture has the minimum remaining work (the best performance) compared with the other three architectures, which indicates that the modular architecture performs better than integral architecture (i.e., small-world and scale-free). Besides, we also notice that although the Block-diagonal has the best performance, the variability is larger than the Small-world case. This result implies that the stability of product architecture cannot be neglected in designing the DSM structure. From Figs. 14 (b) and (c), we confirm that there exists a variance on the investment in modules and design rules with different product architectures.

Fig. 15 shows the analysis for Problem 2. The result validates that for meeting the same target of the remaining work, the Block-diagonal architecture costs the minimum resources among the four architectures, which also indicates that the modular architecture performs better than integral architecture. In Figs. 15 (b) and (c), it shows that the product architecture also has an effect on the investment in the performance-constrained PD problem.

5 Case study

In this section, we demonstrate and validate our proposed model using two case studies considering the resource allocation problems for the PD processes presented in [18,31].

5.1 Case 1: Mechanical manipulator design

In this section, we consider the case study problem form the tire and wheel manipulator design process presented in [18].
Table 3 lists all the tasks for the design process and the best/worst duration time (in hours) per investment round. The dependency structure of the design process, illustrating the interactions among the tasks, is demonstrated in the DSM (or WTM) shown in Table 4. In Table 4, the diagonal entries represent the completion rate for handling the remaining work of the task, while the off-diagonal element stands for the ratio of the fractional work exchanged among the tasks. In this paper, we adopt the inverse of average duration as the value of the diagonal entries of WTM. Table 4 shows the value of dependency strength between the tasks which are assigned in three numerical values: 0.5, 0.25 and 0.05 for strong (S), medium (M), and weak (W) dependencies, respectively.

In our case study, we specify the manipulator design problem with an initial work vector \( P(0) = 1 \) (i.e., all the tasks at the beginning of the design process have 100% work remaining). We adopt the mean value of the best and worst duration as a measure of the complexity of each task (i.e., \( A_0(\phi_0, \gamma_0) = (t_{\max} + t_{\min})/2 \)).

| Table 3: Task duration of tire and wheel manipulator design |
|------------------|-----------------|------------------|
| Tasks                        | Duration (h) | best and worst case |
| A. Design arm                       | 4, 10      |                    |
| B. Design arm joints                 | 2, 10      |                    |
| C. Design grip mechanism             | 5, 10      |                    |
| D. Design rotating base             | 4.5, 5     |                    |
| E. Finite element analysis           | 6, 10      |                    |
| F. Dynamic simulation                | 1.5, 10    |                    |
| G. Factor of safety                  | 1, 15      |                    |
| H. Failure mode analysis             | 1, 4       |                    |
| I. Build prototype                   | 27, 40     |                    |
| J. Evaluated prototype               | 2, 5       |                    |
(a) Remaining work (performance) in module versus investment round.

(b) Investment in modules versus investment round.

(c) Investment in design rules versus investment round.

(d) The total resource cost versus investment round.

(e) The correlation between the investment in module and its related design rules (X-axis: module; Y-axis: design rules).

Fig. 9: The optimal solution of Problem 2. Color lines distinguish the importance of module/design rules via the PageRank.
Fig. 10: The remaining work, investment in modules and design rules of Problem 2 versus their centrality measures in the Erdős-Rényi (random) network. Dash line: Linear regression line.

Table 4: The WTM of the manipulator design process

|    | A     | B     | C     | D     | E     | F     | G     | H     | I     | J     |
|----|-------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| A  | *     | S     | W     | W     | W     | W     | W     | W     | W     | W     |
| B  | M     | *     | W     | W     | W     | S     | W     | W     | S     | W     |
| C  | M     | W     | *     | M     | W     | S     | W     | W     | S     | W     |
| D  | W     | W     | W     | *     | S     | W     | W     | W     | S     | W     |
| E  | W     | W     | W     | W     | *     | S     | W     | W     | S     | W     |
| F  | M     | M     | M     | W     | W     | W     | W     | W     | W     | W     |
| G  | W     | W     | W     | W     | S     | W     | *     | W     | W     | W     |
| H  | W     | W     | W     | M     | W     | M     | *     | W     | W     | W     |
| I  | W     | W     | W     | W     | M     | W     | W     | W     | *     | W     |
| J  | W     | W     | W     | W     | W     | W     | W     | S     | *     | W     |

S-strong dependency between tasks
M-medium dependency between tasks
W-weak dependency between tasks

Based on the analysis result from Section 3, we adopt the PageRank centrality for measuring the importance of the task/dependency in the WTM network. From Fig. 16, we can see that both problems reveal the same trends and struc-

eters of all cost functions with $c_{ij} = 1$, $p = 1$, $\Omega_{ij} = 1$, in that case, the unit of investment in the individuals has the same effect. Also, we let the variable $\phi$ and $\gamma$ of cost function vary within the interval $[0, 1]$, which means that, after each investment round, the rate of dealing process in each module and design rule can be accelerated between $[0\%-90\%]$. Also, we set investment round $T = 5$ for the PD process. In the budget-constrained problem, we let the investment budget for each round equally fixed by $\bar{B}_k = 200 \ (k = 1, \ldots, T)$. For the work-constrained problem, we set the target for the remaining work $P(T) = 0.01$, which means that if the remaining work exceeds the threshold 1%, we can assume the completion of the PD process. From Section 4, we have shown the trends and structure of the optimal solution, so in the case study, we only focus on analyzing the relations between the importance of modules and design rules (i.e., their centrality) in the WTM network and their allocated investment/performance.
Eigenvector centrality  
PageRank  
Closeness centrality

(a) Remaining work of module versus its centrality measures in the network.

(b) Investment in module versus its centrality measures in the network.

(c) Investment in design rule versus its centrality measures in the network.

Fig. 11: The remaining work, investment in modules and design rules of Problem 2 versus their centrality measures in the Watz-Strogatz (small world) network. Dash line: Linear regression line.

ture. In Figs. [16] (a) and (e), the regression line shows that there is a correlation but not strong between the remaining work in the module and its PageRank centrality. We can still conclude that the task with higher centrality tends to have a larger amount of remaining work. In Figs. [16] (b) and (f), we get nearly the same trend that the task with higher central-

ity tends to receive larger investment. However, in Figs. [16] (c) and (g), there is still an ambiguous correlation between the investment in dependency and its corresponding centrality, which coincides with the analysis result in Section 4.

From Figs. [16] (d) and (h), we can see there is a clear pos-

Table 5: WTM for Automotive appearance design

|      | $L_1$ | $L_2$ | $L_3$ | $L_4$ | $L_5$ | $L_6$ | $L_7$ | $L_8$ | $L_9$ |
|------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| $L_1$ | 0.85  | 0.12  | 0.02  | 0.06  | 0.06  | 0.06  | 0.06  |       |       |
| $L_2$ | 0.1   | 0.53  | 0.04  | 0.3   | 0.02  | 0.24  | 0.02  |       |       |
| $L_3$ | 0.02  | 0.04  | 0.47  | 0.08  | 0.24  | 0.02  | 0.18  | 0.02  |       |
| $L_4$ | 0.06  | 0.18  | 0.68  | 0.14  | 0.1   | 0.02  | 0.08  |       |       |
| $L_5$ | 0.04  | 0.3   | 0.26  | 0.16  | 0.28  | 0.06  | 0.02  | 0.2   |       |
| $L_6$ | 0.02  | 0.02  | 0.1   | 0.06  | 0.76  | 0.06  | 0.04  |       |       |
| $L_7$ | 0.1   | 0.06  | 0.83  | 0.16  |       |       |       |       |       |
| $L_8$ | 0.08  | 0.24  | 0.18  | 0.08  | 0.04  | 0.04  | 0.16  | 0.63  | 0.2   |
| $L_9$ | 0.02  | 0.02  | 0.26  | 0.2   | 0.2   |       |       |       |       |
| $L_{10}$ |       |       |       |       |       |       |       |       | 0.7   |
5.2 Case 2: Automotive appearance design

In this section, we adopt the automotive appearance design presented in [31], which focuses on the design process of the interior and the exterior surface of the automotive for achieving a balance among the appearance, quality, and operational interface. However, we only focus on the local team which contains the following tasks: 1) carpet, 2) center console, 3) door trim panel, 4) garnish trim, 5) overhead system, 6) instrument panel, 7) luggage trim, 8) package tray, 9) seats, and 10) steering wheel.

The WTM for the appearance design is given in Table 5 in which the diagonal entries represent the completion rate on each module, while the off-diagonal entries represent the ratio of the rework exchanged between each task. In this case study, we specify the automotive appearance design problem with an initial work vector \( P(0) = 1 \) (i.e., all the tasks at the beginning of the design process have 100% work remaining). For the cost function, we unify the parameters of all cost functions with \( c_{ij} = 1, p = 1, Q_{ij} = 1 \), in that case, the unit of investment in the individuals has the same effect. Also, we let the variable \( \phi \) and \( \gamma \) of cost function vary within the interval \([0, 1]\), which means that, after each investment round, the the completion rate on each module and design rule can be accelerated between \([0\%−90\%]\).

We use 5 investment rounds \((T = 5)\) for the design process. In the budget-constraint problem, we let the amount of investment in every round equally fixed to \( B_k = 200 \). While for the work-constraint problem, we set the target for the remaining work 0.01, which means that if percentage of the remaining work is 1% of its initial value, then this will indicate the completion of the PD process. As in the previous case study, we focus on analyzing the relations between the
importance of modules and design rules in the WTM and their allocated investment/performance.

Fig. 17 shows the investments versus the PageRank centrality measures. For the relationship between the performance/remaining work of the module and its importance, we can see from Figs. 17(a) and (e) that the module with a higher centrality has a larger amount of remaining work. This result coincides with our intuition that the module has a lower centrality means it has higher efficiency, which naturally results in the lower level of remaining work. Similarly, in Figs. 17(b) and (f), we can conclude that the module with higher centrality (i.e., lower efficiency) gets the larger investment for improving its performance. As for the design rules, Figs. 17(c) and (g) show the relation between the investment in design rules and its centrality in the WTM network. From the linear regression line, we can see that the correlation is not very strong, but there is still a tendency that the design rule with the higher edge centrality (i.e., connects the two lower efficiency modules) tend to attract more resources. Finally, Figs. 17(d) and (h), we can see that there is a strong correlation that the investment in the module with higher centrality (i.e., lower efficiency) coincide with the total investment in its dependencies.

6 Conclusion and discussion

PD managers are always faced with the problem of making resource allocation decisions. Especially, when there are several investment rounds, making optimal decisions based on intuition or heuristic rules becomes rather difficult. Although, some literatures have proposed analysis methods on complex network theory and dynamic linear systems, they lack of an analytical and mathematical optimization framework similar to the one presented in this paper.

Fig. 13: The remaining work, investment in modules and design rules of Problem 2 versus their centrality measures in the Block-diagonal network. Dash line: Linear regression line.
Our results provide PD managers with an efficient tool to allocate development resources optimally for the budget-constrained problem and performance-constrained problem, where the resources can be allocated on both modules and design rules. Although we carried out the experiments with two types of problems, and with different product architectures for each problem, the evolution of the investment and performance (remaining work) exhibit similar trends, which shows that the evolution property of the PD process is independent of the problem formulation and product architecture. Moreover, the investment and performance in modules also illustrate that certain correlations exist despite the problem formulation and product architecture, which also confirms that these trends and correlations are the intrinsic properties of the PD process.

In the analysis of different PD architectures, we show that the architecture of the product affects resource allocation which in turn affects the performance of the PD process. Design and managerial guidelines can result from the direct analysis of the PD architecture. Specifically, for development engineers, our result can be used for selecting the product architecture which leads to maximum performance. On the other hand, when the PD architecture is fixed, our proposed framework helps PD managers in deciding on the optimal budget proportions to be allocated to modules and design rules.

Furthermore, for making a further utilization of our framework, we discuss the feasibility of adding the linear regression lines for exploring the possibility of making resource allocation decisions without the need for solving the optimization problem (i.e., directly by utilizing the correlation results). In other words, our proposed framework allows us to gain insights into the relationship between the investment and the DSM architecture, which inspires us to further investigate and model the mapping from the centrality of a module to its investment. From the results in Section 4.5, we know that the DSM architecture affects the resource allocation which in turn affects the performance. Specifically, from the regression lines in Figs. 5-8 and 10-13 we observed that the slope in each figure varies among different
Fig. 16: Optimal investment in modules and design rules of Problem 1 (a-d) and 2 (e-h) versus PageRank centrality in the tire and wheel manipulator design. (a, e) Remaining work versus centrality; (b, f) investment in module versus centrality; (c, g) investment in design rules versus centrality; (d, h) investment in module versus the total investment in its dependent design rules.

Fig. 17: Optimal investment in modules and design rules of Problem 1 (a-d) and 2 (e-h) versus PageRank centrality in the automotive appearance design. (a, e) Remaining work versus centrality; (b, f) investment in module versus centrality; (c, g) investment in design rules versus centrality; (d, h) investment in module versus the total investment in its dependent design rules.
DSM architectures, which indicates that modeling a general investment function for all the DSMs is not feasible. However, if we fix the DSM architecture and the problem size (i.e., number of modules and design rules), is it possible for us to address this problem? From the problem formulation in Section[2] we know that there are numerous parameters in the WTM and the cost functions that can affect the shape of the regression line. Currently, we cannot determine what DSM parameters influence the slope of the regression line.

Finally, our proposed framework can serve as a software support tool for calculating optimal resource allocation solution for the dynamic PD process, which reveals a superiority on the feasibility for searching optimal solution and computation cost. With the help of this guideline, we can maximize the performance of the PD architecture in the design stage. In that case, we can get the maximized benefit from the optimally allocated resource.

One limitation of the framework proposed in this paper is that it does not consider the time-delay effect; so, dynamic investment problem with time-delay need to be considered in future work; especially, if the parameters of the PD system are updated after a certain period. Then, the investment decision making problem becomes a more general situation.

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