Abstract

In the framework of models with Higgs triplets, doubly and singly charged triplet Higgs boson production in the processes $e^-e^-\rightarrow \delta_{L,R}^\pm Z^0$ and $e^-e^-\rightarrow \delta_L W_L^-$ are considered.

keywords:Higgs 12.15.Cc,triplet 14.80.-j,collisions
13.,neutrino 13.15.-f,violation 11.30.Hv

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Yerevan 1998
1. Introduction

As known, isospin triplets (with \( T = 1 \) and hypercharge \( Y = 2 \)) of Higgs bosons provide a natural explanation of the smallness of the left-handed neutrino masses by See-Saw mechanism (see e.g. [1]-[5] and references therein).

Another consequence of such Higgs triplet introduction is the presence of some new phenomena with lepton number violation such as neutrinoless \( \beta \)-decays, \( \mu \to 3e \) decay, muonium-antimuonium conversion mediated by doubly charged Higgs bosons and/or Majorana neutrinos (see refs. [5]-[8] and references therein).

In \( e^- e^- \) collisions doubly charged Higgs bosons may be produced at lower order in resonance [4],[9]:

\[
e^- e^- \to \delta_{L,R}^-, 
\]

with subsequent decays \( \delta_{L,R}^- \to l^- l^-, W_{L,R}^- W_{L,R}^- , W_L^- \delta_L^- \).

It must be noted however, that the mass of the Higgs bosons is the free parameter of the theory and, thus, we don’t know which energies are necessary for \( \delta_{L,R}^- \)-bosons production in resonance whereas doubly charged Higgs bosons production in association with other particles may soften the above resonanse condition. For instance in ref. [10] has been considered\[1\] the process (see Fig.1):

\[
e^- e^- \to \delta_{L,R}^- \gamma, 
\]

\[1\]Previously the process (2) has been considered in ref.[4] however in this reference the third s-channel diagramm of Fig.1 which is necessary for gauge invariance of the process (2) has been neglected and the cross section of this reference (formula (3.31)) differs from result of ref.[10] (formula(4)).
which is allowed if $\sqrt{s} > m_H$.

Here we study doubly charged Higgs bosons production in the process:

$$e^- e^- \rightarrow \delta^-_{L,R} Z^0,$$

and the process of singly charged triplet Higgs boson (which is also a member of the left Higgs triplet) production:

$$e^- e^- \rightarrow \delta^-_{L} W^-.$$

These processes are described by the three diagrams of Fig.1. Previously, the process (4) has been considered in ref.[11] however, there the third $s$-channel diagram of Fig.1 with virtual $\delta^-_{L}$-boson exchange has not been considered and the cross section (formula (5.1)) differs from our result expressed by formulas (9)-(15) below, where this $s$-channel $\delta^-_{L}$-boson exchange is taken into account.

It must be noted that whereas reaction (1) is the lowest order of the doubly charged Higgs bosons, reaction (4) is the lowest order of singly charged Higgs bosons production.

Singly charged triplet Higgs bosons, produced in reaction (4) as well as doubly charged Higgs bosons may decay into leptons ($\delta^- \rightarrow l^- \nu$) or into gauge bosons (e.g. into $W^- L \nu$) or into gauge bosons and/or more light Higgs bosons (these decays are considered in ref. [3]).

If $Z^0$-bosons produced in reaction (3) decay into neutrino pairs the final state of the process (3) is the same as in reaction of $W^- W^-$-fusion [12]:

$$e^- e^- \rightarrow \delta^-_{L} \nu_e \nu_e.$$
The cross section of the process (5) via $W_L^+ W_L^-$-fusion is of the same order as the cross section of the standard $H^0$-boson production by $W_L^+ W_L^-$-fusion considered in ref.\[13]-\[16] multiplied by $(\frac{\alpha}{k_L})^2$ because the vertex $W_L^+ W_L^- H^0$ multiplied by $\frac{\nu}{k_L}$ is of the same order as the vertex $W_L^- W_L^- \delta^{++}_L$.

Far from the threshold the cross section of the $W_L^- W_L^-$-fusion increases as:

$$\sigma = \frac{1}{16 \sin^6 \theta_W} \frac{\alpha^3}{m_W^2} (\frac{\nu}{k_L})^2 (\log(\frac{s}{m_H^2}) - 2),$$

(6)

whereas the cross section of the processes (3),(4) decreases as $s^{-1}$ with growth of $\sqrt{s}$.

In the left-right symmetric model [17]-[20], [2]-[5],[26] with Higgs triplets [2]-[3],[23] (see Appendix), however, the vertex $W_L^- W_L^- \delta^{++}_L$ which is responsible for $W_L^- W_L^-$-fusion is suppressed by the factor of $\frac{\nu}{k_L}$, which must be small to preserve the true relation between $W_L^-, Z^0$-bosons masses and Weinberg’s angle. Besides, because $Z^0$-boson is on shell, the process (5) mediated by the mechanism (3) with subsequent decay of $Z^0$-boson into neutrino pairs has in fact a 2-body final state and that is why it does not decrease with the growth of $m_H$ as fast as the process (5) mediated by $W_L^- W_L^-$-fusion which has a 3-particle final state.

Thus, at sufficiently small $\frac{\nu}{k_L}$ [3], sufficiently large Yucawa couplings $h_{ee}$

2In other models with Higgs triplets factor $\frac{\nu}{k_L}$ in vertex $W_L^- W_L^- \delta^{++}_L$ may be not small (e.g. in the standard model where Higgs sector contains also two Higgs triplets with $Y = 2; 0$, see refs.[23]-[26]) and the role of $W_L^- W_L^-$-fusion in the process (5) increases in comparison with the left-right symmetric model with Higgs triplets case. Excluding large factor $\frac{\nu}{k_L}$ case all our results are also applicable to any model with left and/or right Higgs triplets.
and not very high energies the studied contribution dominates over $W_L^{-}W_L^{-}$-fusion.

For instance, at $\frac{v}{k_L} = 10^{-2}$, $h_{ee} = 10^{-2}$, $\sqrt{s} = 1$ TeV and $m_H = 100$ GeV the contribution of the diagrams of Fig.1 exceeds the contribution of $W_L^{-}W_L^{-}$-fusion approximately by a factor 100.

Analogously, if $W_L^{-}$-bosons produced in reaction (4) decay into $e^-\nu_e$ we must in general consider jointly the process (4) with subsequent decays $W_L^- \rightarrow e^-\nu_e$ and $\delta_L^-$-bosons production via vertex $W_L^-Z^0\delta_L^{++}$ in $Z^0W_L^-$-fusion [21], [22] (which is of the same order as vertex $W_L^-W_L^-\delta_L^{++}$) as part of the same process:

$$e^-e^- \rightarrow \delta_L^-e^-\nu_e.$$  

(7)

All conclusions concerning reaction (5) are also true in the case of reaction (7), because vertex $W_L^-Z^0\delta_L^{++}$ is of the same order as vertex $W_L^-W_L^-\delta_L^{++}$ also small in the framework of the left-right symmetric model.

**2. Results**

Using formula (A5) in Appendix A for the $\delta_{L,R}^-$-interaction with electrons we obtain the following amplitudes of processes (3),(4):

$$M = 2eh_{ee}a_{L,R}\bar{u}^c(k_1) \left( \frac{\hat{k}_\gamma \hat{Z}}{t} + \frac{\hat{k}_\gamma \hat{k}_4}{u} + 4 \frac{(k_4Z)}{s - m_H^2} \right) P_{L,R}u(k_2),$$  

(8)

$$M = \frac{eh_{ee}}{\sin \theta_W} \bar{u}^c(k_1) \left( \frac{\hat{k}_4 \hat{W}}{t} + \frac{\hat{W} \hat{k}_4}{u} + 4 \frac{(k_4W)}{s - m_H^2 + im_H \Gamma_H} \right) P_Lu(k_2).$$  

(9)

Here we neglect the electron mass and use the following notation: $Z_\mu, W_\mu$ is the polarization 4-vector of the $Z^0$- and $W_L^{\pm}$-bosons, $a_L = \frac{-\frac{1}{2} + \sin^2 \theta_W}{\sin \theta_W \cos \theta_W}, a_R = \tan \theta_W$, $s = (k_1 + k_2)^2$, $t = (k_1 - k_4)^2$, $u = (k_2 - k_4)^2$, $m_H$ is the mass of $\delta_{L,R}^-$-bosons, $\Gamma_H$ is the width of the decay $\delta_L^- \rightarrow \delta_L^-W_L^-$.  

4
For the differential cross section we obtain the following result:

\[
\frac{d\sigma}{dt} = \frac{1}{2} \frac{\alpha h_{ee}^2}{s} \frac{1}{4 \sin^2 \theta_W} \left( a \left( \frac{1}{t} + \frac{1}{u} \right) + (2b - 4) - m_W^2 m_h^2 \left( \frac{1}{t^2} + \frac{1}{u^2} \right) \right),
\]

(10)

\[
a = m_W^2 + m_h^2 - s - \frac{4 s m_h^2 (s - m_H^2)}{(s - m_H^2)^2 + m_H^2 \Gamma_H^2} - \frac{2(s - m_W^2)m_H^2}{m_W^2 + m_h^2 - s},
\]

(11)

\[
b = \frac{s}{m_W^2} \left( \frac{(m_H^2 - m_h^2 - m_W^2)^2 - 4 m_h^2 m_W^2}{(s - m_H^2)^2 + m_H^2 \Gamma_H^2} + m_H^2 \Gamma_H^2 \right),
\]

(12)

\[
t_- < t < t_+,
\]

(13)

\[
t_\pm = \frac{m_h^2 + m_W^2 - s \pm \sqrt{(m_h^2 + m_W^2 - s)^2 - 4 m_h^2 m_W^2}}{2}.
\]

(14)

Here \( m_h \) denotes the mass of the singly triplet charged Higgs bosons.

Integrating within the limits (13),(14) we obtain for the total cross section the following result:

\[
\sigma = \frac{\alpha h_{ee}^2}{s} \frac{1}{2 \sin^2 \theta_W} (a \log \left( \frac{t_+^2}{m_h^2} \right) + (b - 3)(t_+ - t_-)).
\]

(15)

At \( \sqrt{s} \gg m_H, m_h, m_W \) the previous formula is reduced and we have:

\[
\sigma = \frac{\alpha h_{ee}^2}{s} \frac{1}{2 \sin^2 \theta_W} (2 \log \left( \frac{s}{m_h^2 m_W^2} \right) - 3).
\]

(16)

The differential and the total cross section of the processes (3) may be obtained from the formulas (10)-(16) at \( m_H = m_h, \Gamma_H = 0 \) by the following replacements:

\[
\frac{1}{2 \sin^2 \theta_W} \to a_{L,R}^2, m_W \to m_Z.
\]

(17)
On Fig. 2,3 we present the number of events \( \delta_{L}^{-}Z^{0} \) and \( \delta_{L}^{-}W_{L}^{-} \) per year for the processes (3) and (4) versus \( m_{H} \) at fixed \( \sqrt{s} \) and fixed \( m_{h} \) at yearly luminosity \( L = 100 fb^{-1} \) (\( e^{-}e^{-}\)-colliders with yearly luminosity \( L = 100 fb^{-1} \) has been considered e.g. in ref.[27]).

The number of events \( \delta_{R}^{-}Z^{0} \) may be easily obtained from Fig.2 by multiplying the number of events \( \delta_{L}^{-}Z^{0} \) by \( a_{L}^{2}a_{L}^{2}L = 0.742 \). On the other hand at the \( m_{H} = m_{h} \) \( \delta_{L}^{-} \)-bosons may be produced more efficiently (by \( \frac{1}{4\sin^{2}\theta_{W}a_{L}^{2}} = 2.661, 3.586 \) times) in reaction (4) than \( \delta_{L,R}^{-} \)-bosons in reaction (3).

Indeed, at \( m_{H} = m_{h} \) the cross sections of processes (3) and (4) are different from each other only by coefficients and the influence of the \( m_{W} \) and \( m_{Z} \) mass difference far from the threshold is negligible.

3. Comparison with other mechanisms of triplet Higgs bosons production

The processes

\[ e^{+}e^{-} \rightarrow \delta_{L,R}^{++}\delta_{L,R}^{--}, \delta_{L}^{+}\delta_{L}^{-}. \]  

(18)

have a large cross section \([4]-[6],[8],[28],[29]\), however, it becomes kinematically allowed at energies \( \sqrt{s} > 2m_{H} \), whereas process (3),(4) is kinematically allowed at lower energies \( \sqrt{s} > m_{H} + m_{Z}, m_{h} + m_{W} \).

From formula (2.1) of the ref.[3] and from Fig.2 we see that at \( \sqrt{s} = 1 \) TeV, far from the threshold the number \( \delta_{L,R}^{++}\delta_{L,R}^{--} \) pairs produced in reaction (18) exceeds by approximately 1500 times the number of \( \delta_{L}^{-} \) bosons produced in reaction (3) and the number of \( \delta_{L}^{+}\delta_{L}^{-} \) pairs produced in reaction (19) as seen from Fig.14 of the ref.[8] and from Fig.3 exceeds by approximately 100 times the number of \( \delta_{L}^{-} \) bosons produced in reaction (4) at \( \sqrt{s} = 1 \) TeV.
It must be noted, however, that with the growth of $h_{ee}$ the number of the triplet Higgs bosons produced in reactions (3),(4) quadratically increases. Besides, the resonant enhancement (if $m_H > m_h + m_W$) in the process (4), as seen from Fig.3, may considerably increase the cross section of the process (4) in comparison with the process (19) even sufficiently far from resonance.

It must also be noted, that the processes (18),(19) decrease with the growth of $\sqrt{s}$ as $s^{-1}$ whereas, as seen from formula (16) the processes (3,4) decrease more slowly as $s^{-1} \log(\frac{s}{m_h m_W})$.

At LHC doubly charged Higgs bosons may be produced in pairs [9, 10]; at integrated luminosity $L = 100 fb^{-1}$, $\sqrt{s} = 14$ TeV, and $m_H = 800$ GeV we have about $100 \delta^{++}_L \delta^{-}_L$ pairs per year which is comparable with the number of $\delta^{-}_L$-bosons produced in reaction (3) at $\sqrt{s} = 1$ TeV, $L = 100 fb^{-1}$, $h_{ee} = 10^{-2}$ and the same mass of the doubly charged Higgs bosons. The number of $\delta^{+}_L \delta^{-}_L$ pairs produced at LHC several times is smaller than number of $\delta^{++}_L \delta^{-}_L$ bosons whereas about 200 of $\delta^{-}_L$-bosons may be produced in reaction (4) at $\sqrt{s} = 1$ TeV, $L = 100 fb^{-1}$, $h_{ee} = 10^{-2}$, and the same mass of the triplet charged Higgs bosons if resonance is absent.

Triplet Higgs bosons may be produced also in $e^{-\gamma}$-collisions in reaction

$$e^{-\gamma} \rightarrow \delta^{-}_L \nu,$$  \hspace{1cm} (19)
As seen from Fig.2 of the ref. [30], at $L = 100 fb^{-1}$, $h_{ee} = 10^{-2}$ about 700-350, 200-90 (at $\sqrt{s} = 0.5, 1$ TeV respectively) doubly charged Higgs bosons with $0.1$ TeV $< m_H$ may be produced per year in reaction (20). The number of $\delta_{L}^{-}$-bosons produced in reaction (21) at the same energies, luminosities (and far from resonance) is of the same order.

4.Acknowledgements

The author express his sincere gratitude to I.G.Aznauryan for helpful discussions and to F. Cuypers for helpful discussions and for kind hospitality at Paul Scherrer Institute.

Appendix A

In the left-right symmetric model with Higgs triplets the matrix of left and right Higgs triplets with $(T = 1, Y = 2)$ may be written as:

$$
\Delta_{L,R} = \begin{pmatrix}
\delta_{L,R}^{+}/\sqrt{2} & \delta_{L,R}^{++} \\
\delta_{L,R}^{0} & -\delta_{L,R}^{+}/\sqrt{2}
\end{pmatrix}
$$

(A.1)

and their interaction with the left- and right-handed lepton fields $\psi_{L,R}^{T} = (\nu_{L,R}^{T}, e_{L,R}^{T})$ is described by the lagrangian:

$$
\mathcal{L} = i h_{ij} \left( \psi_{iL}^{T} C \tau_{2} \Delta_{L} \psi_{jL} + \psi_{iR}^{T} C \tau_{2} \Delta_{R} \psi_{jR} \right) + h.c.
$$

(A.2)

Here $i,j = e, \mu, \tau$ are generations indices, $C$ is the charge conjugation matrix, and $\tau_{2}$ is the Pauli matrix. After symmetry breaking Majorana masses of the heavy approximately right-handed neutrinos are expressed through the Yucawa couplings $h$ and the neutral component of the right triplet vacuum expectation value ($v_{R}$) in the following way:

$$
m_{N} = \sqrt{2} h v_{R}.
$$

(A.3)
Also, large right triplet vacuum expectation \( v_L \ll k_L, k_R \ll v_R, k_L, k_R \) are vacuum expectations of the left and right doublets, \( v_L \)-vacuum expectation of the left triplet) provides the mass of the \( W^\pm_R \)-bosons:

\[
m_{W^R} = \frac{1}{2} g v_R
\]  

(A.4)

whereas the doublet vacuum expectation is responsible for the mass of the \( W^\pm_L \)-bosons.

From (A2) we derive charged triplet Higgs bosons interactions with leptons:

\[
L = -\sqrt{2} h_{ee} \bar{\ell} e P_L \nu \delta^+_L + h.c. - h_{ee} \bar{\ell} e P_{L,R} \ell \delta^{++}_{L,R}
\]

(A.5)

where \( \ell^c = C\ell^T \).

Vertexes \( \gamma \delta^{++}_{L,R} \delta^{--}_{L,R}, Z^0 \delta^{++}_{L,R} \delta^{--}_{L,R}, W^- \delta^- L \delta^+_L \) are described by the following Feynmann rules: \( i2e (p - q)^\mu \cdot A_\mu, i2a_{L,R} e (p - q)^\mu \cdot Z_\mu, i\frac{e}{\sin \theta_W} (p - q)^\mu \cdot W_\mu \)

where \( p, q \)- denotes incoming momentums of the Higgs bosons.

Doubly charged Higgs bosons mass matrix is nondiagonal, however, in a wide range of Yukawa coupling and vacuum expectations however, in more general case left and right doubly charged Higgs bosons may be, in principle, mixed.

Mass matrix of the doubly and singly charged Higgs bosons in general is nondiagonal (see formula (A4),(A5) in ref.\[5\]), however in the left-right symmetric model \( \delta^{++}_L - \delta^{--}_R \) and doublet-triplet mixing is negligible because it is proportional to the \( v_L \).

In the above mentioned models where \( v_L \) is not restricted by experiment there may a large mixing among the doublet and triplet singly charged Higgs
bosons.

In the end we must remind that in the limit \( v_L \ll k_L, k_R \ll v_R \delta R \)-

boson is approximately the charged Goldstown boson and after spontaneous

symmetry breaking it becomes the longitudinal part of the \( W_R^\pm \)-

bosons.

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Figures captions:

Fig.1 Diagrams corresponding to the processes (2)-(4).

Fig.2 Number of events $\delta_{L,R}Z^0$ per year ($\sigma L$) versus $m_H$ at fixed $\sqrt{s}$ (at yearly luminosity $L = 100 fb^{-1}$) produced in reaction (3) as a function of $m_H$ with $h_{ee} = 10^{-2}$. Curves 1,2 correspond to the $\sqrt{s} = 0.5, 1$ TeV respectively.

Fig.3 Number of events $\delta_L W_L^-$ per year ($\sigma L$) versus $m_H$ at fixed $\sqrt{s}$ produced in reaction (4) (at yearly luminosity $L = 100 fb^{-1}$) as a function of $m_H$ with $h_{ee} = 10^{-2}$. Curves 1,2 correspond to the number of events at $m_H = 1.3 m_h$ and $\sqrt{s} = 0.5, 1$ TeV respectively. Curves 3,4 correspond to the number of events $\delta_L W_L^-$ events per year at $m_H = m_h$ and energies $\sqrt{s} = 0.5, 1$ TeV respectively.
Figure 2:
