Reconstruction of cosmological matter perturbations in Modified Gravity

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The analysis of perturbative quantities is a powerful tool to distinguish between different Dark Energy models and gravity theories degenerated at the background level. In this work, we generalise the integral solution of the matter density contrast for General Relativity gravity [1,2] to a wide class of Modified Gravity (MG) theories. To calculate this solution is necessary prior knowledge of the Hubble rate, the density parameter at the present epoch ($\Omega_m$) and the functional form of the effective Newton’s constant that characterises the gravity theory. We estimate in a model-independent way the Hubble expansion rate by applying a non-parametric reconstruction method to model-independent cosmic chronometer data and high-$z$ quasar data. In order to compare our generalised solution of the matter density contrast, using the non-parametric reconstruction of $H(z)$ from observational data, with purely theoretical one, we choose a parameterisation of the Screened MG and the $\Omega_{m0}$ from WMAP-9 collaborations. Finally, we calculate the growth index for the analysed cases, finding very good agreement between theoretical values and the obtained ones using the approach presented in this work.

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I. INTRODUCTION

The present cosmic accelerated phase of the Universe discovered at the end of the 90’s [3,4] can be explained by the standard cosmological model, the so-called ΛCDM. In this model, the content of the Universe is mainly composed by Dark Energy (DE) characterised by the cosmological constant (∼70%), dark matter (∼25%) and ordinary matter (∼5%) [5]. However, there are some inconsistencies in the standard model from the theoretical point of view (e.g. fine tuning and cosmic coincidence problems [6]) and some tensions in the observational constraints (see Ref [7] for a brief review). For these reasons, many models describing the behaviour of dynamical DE [8,9] and modified gravity (MG) (see Ref. [10-15] for reviews) theories at large scales have been proposed to explain satisfactorily the cosmic acceleration. The nature of these two approaches is essentially different. The first one constitutes a modification of the r.h.s. of the Einstein Equations (EE) where it is included a highly negative pressure fluid to the Universe content. The second one corresponds to a modification of the geometrical side of the EE. In both of them, the DE or the MG can be described by the inclusion of scalar fields (e.g chameleons [16], symetrons [17], dilatons [18], galileons [19]). The effect of these fields is tightly constrained by local experiments, which implies that they are screened in dense environments [20-21].

The accuracy and the number of geometrical cosmological observables have grown considerably. Nevertheless, this is not enough to distinguish between some DE and MG theories because they can predict the same cosmic expansion and the same values for background observables like the luminosity and angular diameter distance. Hence, it is necessary to explore some quantities at the perturbation level where the background degeneracy between different explanations of the cosmic acceleration may be broken [22]. For instance, in the screened MG theories the cosmic expansion is the same as the one in the ΛCDM but the perturbative quantities growth presents an anomalous behaviour inside the Compton radius of the scalar field [21].

Each DE model or MG theory assumes a specific functional behaviour of dynamical and kinematic variables, being the parameters of the theory constrained by observational data. In contrast, it is possible to obtain model-independent information directly from the observational data using non-parametric methods [23-36]. In the latter scenario, it is assumed a correlation between each data point but it is not required prior information about the functional form of the observable.

In this work, we generalise the integral solution of the linear matter density contrast valid for General Relativity gravity in a homogeneous and isotropic universe, presented in Ref. [1,2], for MG theories where the matter decays proportional to $a^{-3}$ and also the effect of the MG is encoded in the effective Newton’s constant, $G_{eff}(k,a)$.

This solution requires the knowledge of the Hubble parameter, the matter density contrast at the present epoch and the functional form of the MG effect. In order to obtain a model-independent reconstruction of the Hubble parameter, we apply the NPS method [37,38] to the cosmic chronometer data at the redshift range [0.07, 1.04] [37,42] and the high-$z$ quasar data at $z \approx 2.3$ [33]. The cosmic chronometer data, obtained from the differential age method for passively evolving galaxies, within its redshift range is aimed to be model-independent both cosmological and stellar population synthesis. The quasar data is obtained by the correlation function of the transmitted flux fraction in the Lyα-forest of high-$z$ quasars.
We combine these two data sets to obtain information of the behaviour of $H(z)$ up to the matter dominated epoch.

We reconstruct the matter perturbation quantities following the approach presented in Ref. [20, 21] for the same MG parameter values used in Ref. [21]. For this, we assume the model-independent reconstruction of the cosmic rate and the parameterisation of screened MG proposed in Ref. [20, 21]. This approach allows us to explore the validity of this screened MG using only background data, by comparing the purely theoretical calculations and the ones obtained in this work.

This paper is organised as follows: in Sec. I we generalised the treatment of linear matter perturbations of Ref. [2, 34, 35] and present the considered screened MG to reconstruct the perturbative quantities. We also introduce the basic equations of the matter perturbation theory. In Sec. II we discuss the observational data and present the non-parametric method used to reconstruct the cosmic expansion history. In Sec. III we present the results of our reconstructed cosmic expansion and the matter perturbation analysis. We end this paper with the main conclusions in Sec. IV.

II. MATTER PERTURBATION EQUATIONS

In order to study the behaviour of the matter perturbations in MG theories at large scales, we consider the parameterisation \( \{m(a), \beta(a)\} \) of screened MG presented in Ref. [20, 21]. These models are defined by the effect on the perturbation analysis. We end this paper with the main conclusions in Sec. IV.

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where $G_N$, $\rho_m$ and $\delta_k$ correspond to the Newton’s constant, the background matter density and the density contrast \( \langle \delta(x, t) \rangle \equiv (\rho(x, t) - \rho(t))/\rho(t) \), respectively.

Finally, it is found the second order differential equation for matter perturbations \[ \delta'' + \frac{a'}{a} \delta' - \frac{3\Omega_m(a)}{2} \left( \frac{a'}{a} \right)^2 \mu(k, a) \delta = 0 , \] or \[ \ddot{\delta} + 2H\dot{\delta} - \frac{3\Omega_m(a)}{2} H^2 \mu(k, a) \delta = 0 , \]
the prime denotes the derivative with respect to the conformal time and the dot the derivative with respect to the cosmic time. The $\Omega_m(a)$ is the matter density parameter defined by:
\[ \Omega_m(a) \equiv \frac{\Omega_m H_0^2}{a^2 H^2(a)} , \]
and $H$ is the Hubble parameter. In this case, the evolution of the matter structures is totally determined by the functions $H$ and $\mu(k, a)$, and the value of the matter density today $\Omega_{m0}$. In these equations, any effect of the MG is encoded in the function $\mu(k, a)$.

Defining the dimensionless linear physical distance,
\[ D = H_0 \int_t^{z_0} \frac{dt}{a(t)} = H_0 \int_0^z \frac{dz_1}{H(z_1)} , \]
we can rewrite the Eqs. (4-5) in terms of the redshift and $D$ as:
\[ \frac{d}{dD} \left( \frac{d\delta}{dD} \right) = \frac{3}{2} \Omega_{m0} \mu(k, a) \delta , \]
assuming that $\rho_m \propto a^{-3}$.

The solution of the previous equation can be written as an integral function as:
\[ \delta(D) = 1 + \delta_0 \int_0^D \left[ 1 + z(D_1) \right] dD_1 + \frac{3}{2} \Omega_{m0} \times \int_0^D \left[ 1 + z(D_1) \right] \left( \int_0^{D_1} \mu(k, a) \delta(D_2) dD_2 \right) dD_1 , \]
\[ \delta'(D) = \delta_0' \left[ 1 + z(D) \right] + \frac{3}{2} \Omega_{m0} \left[ 1 + z(D) \right] \int_0^D \mu(k, a) \delta(D_1) dD_1 . \]

In the rest of the text we ignore the Fourier space subindex $k$.
which are easily calculated if we know the Hubble parameter and \( \mu(k, a) \).

The particular form of the function \( \mu(k, a) \) depends on the values of the parameters \( \{m(a), \beta(a)\} \). In this parameterisation, \( m(a) \) represents the mass of the scalar field at the background level and \( \beta(a) \) represents the coupling function between the field and the CDM particles. In this paper, we consider the same family of parameterisation as that of Ref. [20, 21], where the screened MG is characterised by

\[
\mu(k, a) = \frac{1 + 2\beta(a)^2)k^2 + m(a)^2a^2}{k^2 + m(a)^2a^2}, \quad (10)
\]

with \( \beta = 1/\sqrt{6} \) and the mass of the field given by:

\[
m(a) = m_0a^{-3(n+2)/2}, \quad (11)
\]

where \( m_0 \) is a free scale close to 1 Mpc\(^{-1} \) and \( n > 1 \).

Now, if we identify

\[
G_{\text{eff}}(k, a) = \frac{G_N}{(1 + 2\beta(a)^2)k^2 + m(a)^2a^2}, \quad (12)
\]

the Eq. (10) constitutes a generalization of the solution for the density contrast shown in Ref [2] for any MG theory satisfying the second order differential equation

\[
\ddot{\delta} + 2H\dot{\delta} - 4\pi G_{\text{eff}} \rho_m = 0, \quad (13)
\]

where \( G_{\text{eff}} \) is the effective Newton’s constant.

Thus, we define the growth rate as:

\[
f(k, z) = \frac{d\ln \delta(k, z)}{d\ln a} = -\frac{(1+z)H_0 \delta'(k, z)}{H(z) \delta(k, z)}, \quad (14)
\]

where \( z \) is the redshift. This quantity has crucial importance because it is directly estimated by the observations of the large-scale structure (LSS) of the Universe. Due to the growth rate constitutes a cosmological observable, it is possible to compare the one obtained from solving the previous equation using background information with the one obtained from the LSS observations. This comparison allows us to prove the hypothesis of the background dynamics of the Universe used to calculate the Eq. (1).

These hypotheses are:

- The correct theory of gravity is the Screened MG characterised by the function \( \mu(k, a) \) at the perturbation level (Eq. 10) and behaved as a LCDM cosmology at the background level. As mentioned below, we can generalise this condition to any MG theory satisfying the Eq. (13).

Finally, we complete the set of matter perturbation quantities defining the growth index: [49–50]

\[
\gamma(k, z) = \ln f(k, z) \ln \Omega_m(z), \quad (15)
\]

which is an excellent tool to characterise modified gravity theories. For instance, we highlight the growth index value for the following models: LCDM model, \( \gamma_0 = 6/11; \) slow varying wCDM DE models, \( \gamma \approx \frac{3(w-1)}{6w-5} \) and DGP model, \( \gamma_0 = 11/16 \). In these cases, the growth index is well described by a constant function. However, in a general way, it can evolve with the redshift and for future accurate data its evolution must be taken into account [22].

III. DATA AND HUBBLE PARAMETER RECONSTRUCTION

A. Data

In the approach presented in this work to calculate the cosmological matter perturbations, the expansion history of the Universe plays a crucial role. Therefore, it is very important to use a model-independent data to obtain the cosmic expansion rate. For this purpose, we use the cosmic chronometers estimates based in the differential age inferences [53].

The basic assumption of this method is that the relative age of two passively-evolving elliptical red galaxies at approximately the same redshift can be used to estimate the age variation of the Universe and, consequently, the Hubble expansion rate at the redshift of the observation. The difference between the two galaxies ages corresponds to the variation of the age of the Universe, \( \Delta t \), and with the redshift difference, \( \Delta z \), it is possible to approximate the Hubble rate as: \( H(z) \approx -1/(1 + z)\Delta z/\Delta t \).

The differential age approach estimates the Hubble rate directly from the data without assuming a specific spatial geometry or any other cosmological model.

The cosmic chronometers estimates are cosmological model-independent, but they can depend on the stellar population synthesis models at high redshift. As pointed out in Ref [54], the Hubble parameter measurements are almost stellar population synthesis model-independent until \( z \approx 1.2 \). Following the discussion in Ref [54], we use the cosmic chronometer data up to \( z = 1.04 \) [37,41] and we increase (20%) the error bar of the data point at \( z = 1.04 \). Note that we also use the cosmic chronometer data \( H(z = 0.4293) = 91.8 \pm 5.3 \) [42].
Finally, we complete the dataset with two measurements of the Hubble parameter at high-z, $z = 2.34$ [55] and $z = 2.36$ [56], obtained from the correlation function of Lyα forest systems from quasar data. The complete data set is shown in Fig. 1.

### B. Non-parametric Smoothing

To obtain a smooth function in a non-parametric way that represents the expansion rate of the Universe until very high redshift, we apply the NPS method, firstly proposed in Ref. [30], to the cosmic chronometer and high-z quasar data. This method has been widely used in the literature to reconstruct the luminosity distance, the physical distance and the Hubble parameter.

The main goal of this NPS method is to smooth the noise of the data. For this, we subtract an initial guess model to the data and then apply the smoothing kernel. We recover the quantity adding back the guess model. The general form of the method taking into account the physical distance and the Hubble parameter.

The data in Sec. III A obtaining a smooth function for the cosmic expansion rate. As the reconstruction method and the data are model-independent, it is expected that our estimate of the Hubble parameter is not biased by any cosmological model. In Fig. 1 we show the final result of the NPS method and, for comparison, we also plot the result obtained using the non-parametric method Gaussian Process (GP) [24, 26, 28, 34]. As it was shown in Ref. [35], the NPS reconstruction of these data presents a decelerated phase at high redshift which is expected in the standard cosmology when the matter domains the Universe dynamics. We emphasize that this behaviour is required because the assumptions used to solve the differential equation of the density contrast. We also present the matter density parameter calculated with the Eq. (6) using the reconstruction of $H(z)$ and the value of the density parameter at the present epoch from WMAP-9 collaborations [57].

Now, to solve the Eq. (9a), we need to choose a value for $\Omega_{m0}$ and a parameterisation for the MG. We employ the parameterisation shown in Eq. (10). In order to perform a direct comparison of our results from a non-parametric reconstruction of the Hubble parameter and the theoretical prediction of the matter perturbations in this screened MG scenario, we explore the same values as in Ref. [21] for $(m_0,n)$ = (1 Mpc$^{-1}$,0), (0.1 Mpc$^{-1}$,0), (1 Mpc$^{-1}$,1), (0.1 Mpc$^{-1}$,1) and the same $\Omega_{m0}$ value from WMAP-9 [57]. With a fixed gravity theory and matter density prior, we solve the Eq. (9a) for many values of $\delta_0'$ and select the one that produces $(1+z)\delta \propto \cos$ at the redshift of the last $H(z)$ observation (see Refs. [33, 35] to know more details). This condition is required by the standard theory of cosmological perturbations in a matter domination epoch and compatible with the considered screened MG.

We obtain the growth rate for four values of the MG parameters using the calculations of the density contrast. These results are shown in Fig. 2a). We find a good agreement between the results herein obtained and the theoretical ones in the Fig. 5 of the Ref. [21]. Nevertheless, we recognize a difference in the interception of the curves with $(m_0,n)$ ={(1,0), (1,1)} and their behaviours at high-z. In Fig. 2b), we plot the growth rate for the MG parameters $(m_0,n)$ ={(1,0), (0.1,0)} with their respective 1σ confidence level follows the approach proposed in this work. It shows that the Hubble parameter information can distinguish between these two MG parameters at 1σ level. The comparison of these results with the observational data of the growth rate can constrain the parameters of the considered gravity theory or, in a more general way, other MG theories.

Finally, we calculate the growth index (Eq. (15)) and plot it in Fig. 3. Note that the growth index for the parameters $(m_0,n)$ ={(1,0), (1,1)} does not evidence redshift (time) evolution and in this case $f \approx \Omega_m(z)^{\gamma}$ would be a good approximation of the growth rate. In contrast, for the screened MG parameters $(m_0,n)$ ={(0.1,0), (0.1,1)}, there is evolution of the growth index which is compatible with what is expected in theories beyond GR.

### IV. RESULTS

We apply the NPS method described in Sec. III B to the data in Sec. III A obtaining a smooth function for the cosmic expansion rate. As the reconstruction method and the data are model-independent, it is expected that our estimate of the Hubble parameter is not biased by any cosmological model. In Fig. 1 we show the final result of the NPS method and, for comparison, we also plot the result obtained using the non-parametric method Gaussian Process (GP) [24, 26, 28, 34]. As it was shown in Ref. [35], the NPS reconstruction of these data presents a decelerated phase at high redshift which is expected in the standard cosmology when the matter domains the Universe dynamics. We emphasize that this behaviour is required because the assumptions used to solve the differential equation of the density contrast. We also present the matter density parameter calculated with the Eq. (6) using the reconstruction of $H(z)$ and the value of the density parameter at the present epoch from WMAP-9 collaborations [57].

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FIG. 1: a) The reconstruction of the Hubble parameter using the cosmic chronometer and high-z quasar data. The solid line correspond to the reconstruction applying the NPS method (Sec. 111B) and the dashed line correspond to a reconstruction using GP method. b) The matter density parameter using the NPS reconstruction of $H(z)$ and the $\Omega_{m0}$ prior from WMAP-9 collaborations [57]. The shaded regions represent the 1σ and 2σ confidence levels.

FIG. 2: a) The growth factor on sub-Hubble scale obtained solving Eq. 9a using the WMAP-9 $\Omega_{m0}$ value and the parameterisation of the screened MG in Eq. 10. b) The same as in the previous panel for just two values of $(m_0, n)$. The solid line corresponds to the reconstruction whereas the shaded regions represent the 1σ confidence level. The unit of $m_0$ is Mpc$^{-1}$.

| $\Omega_{m0}$ | $n$ | $\gamma_0^{\text{rec}}$ | $\gamma_0^{\text{teo}}$ |
|-------------|-----|-----------------|-----------------|
| 1 Mpc$^{-1}$ | 1   | 0.54            | 0.54 ± 0.06     |
| 0.1 Mpc$^{-1}$ | 1   | 0.47            | 0.46 ± 0.06     |
| 1 Mpc$^{-1}$ | 0   | 0.54            | 0.53 ± 0.06     |
| 0.1 Mpc$^{-1}$ | 0   | 0.44            | 0.44 ± 0.06     |

TABLE I: Calculations of the growth index via the reconstruction of the matter density perturbation and the theoretical value by [20].

In Table I we present the values of $\gamma_0$ for the analytical approach [21] and the ones obtained with the calculations of the matter density perturbations via non-parametric reconstruction of the expansion rate.

V. CONCLUSIONS

In this work we have generalised the integral solution of the matter density contrast at the linear regime. This generalisation is valid for a MG theory that satisfies the Eq. (13) and that conserves a matter dominated epoch in the past of the Universe history.

We have performed a non-parametric reconstruction of gravity theories. In Table I we present the values of $\gamma_0$ for the analytical approach [21] and the ones obtained with the calculations of the matter density perturbations via non-parametric reconstruction of the expansion rate.
FIG. 3: The growth index calculated solving Eq. 9a for four combinations of the parameters $m_0$ and $n$ using the WMAP-9 $\Omega_{m0}$ value. The solid line corresponds to the reconstruction whereas the shaded regions represent the 1σ and 2σ confidence levels.

As shown in Fig 2b) different values of the parameters of $\mu(k, a)$ can be distinguished by $H(z)$ data at 1σ confidence level analysing the growth rate. We will be able to test the validity of the fundamental hypotheses involved in the analysis (Sec. II) by comparing the matter perturbation reconstruction with future scale dependent growth rate estimates.

We have calculated the growth index and plotted it in Fig. 3. We have found that for the cases $(m_0, n) = \{(1, 0), (1, 1)\}$ the growth index is degenerated and it can be well described by a constant. The other two considered cases are univocally characterised and revealing a possible evolution of the growth index. The calculated values of the growth index at the present epoch are compatible with the theoretical ones (see Table. I). Just for comparison, we calculate the growth index using a non-parametric reconstruction of $H(z)$ via GP method. We found that the differences in the growth indexes are not higher than 5%. This exposes the robustness of the methods applied.

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