Renormalization of the SU(2)-symmetric model of hadrodynamics

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It is proved that the SU(2)-symmetric model of hadrodynamics can well be set up on the gauge-invariance principle. The quantization of the model can readily be performed in the Lagrangian path-integral formalisms by using the Lagrangian undetermined multiplier method. Furthermore, it is shown that the quantum theory is invariant with respect to a kind of BRST-transformations. From the BRST-symmetry of the theory, the Ward-Takahashi identities satisfied by the generating functionals of full Green functions, connected Green functions and proper vertex functions are successively derived. As an application of the above Ward-Takahashi identities, the Ward-Takahashi identities obeyed by the propagators and various proper vertices are derived. Based on these identities, the propagators and vertices are perfectly renormalized. Especially, as a result of the renormalization, the Slavnov-Taylor identity satisfied by renormalization constants is naturally deduced. To demonstrate the renormalizability of the theory, the one-loop renormalization of the theory is carried out by means of the mass-dependent momentum space subtraction and the renormalization group approach, giving an exact one-loop effective coupling constant and one-loop effective nucleon, pion and $\rho$-meson masses.

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In the early time, a SU(2)-symmetric model of hadrodynamics was proposed by Sakurai [1,2] based on the non-Abelian gauge-field theory which was first initiated by C. N. Yang and R. L. Mills [3]. A tempting feature of this model is that the model gives a complete description of the interactions among nucleons, pions and $\rho$-mesons. However, the model was beset with two serious difficulties: one is the non-gauge-invariance of the $\rho$-meson mass term because according to the prevailing viewpoint, the requirement of gauge-invariance does not admit the $\rho$-meson mass term to enter the Lagrangian; another is the unrenormalizability of the model as argued in the previous literature [4-9]. Due to these difficulties, the model was eventually relinquished even though a part of the interactions between nucleons and $\rho$-mesons and the interaction between nucleons and pions which are all included in the model have been widely applied in nuclear physics.

Against the difficulties mentioned above, this paper attempts to answer the questions: whether the SU(2)-symmetric model of hadrodynamics could be set up on the gauge-invariance principle and whether the model could be renormalizable? The first question has been answered in our published papers [10-12]. In the papers, it is argued that a non-Abelian massive gauge field theory in which the masses of all gauge fields are the same can actually be set up on the principle of gauge-invariance without need of introducing the Higgs mechanism. This means that the model under consideration can exactly be made up of the massive gauge field theory with SU(2) gauge symmetry. The essential points to achieve this conclusion are as follows. (a) The gauge boson fields such as the $\rho$-meson fields must be viewed as a constrained system in the whole space of vector potentials and the Lorentz condition, as a necessary constraint, must be introduced from the beginning and imposed on the massive Yang-Mills Lagrangian; (b) The gauge-invariance of a gauge field theory should be generally examined from its action other than from the Lagrangian because action is of more fundamental dynamical meaning than Lagrangian. Particularly, for a constrained system such as the $\rho$-meson field, the gauge-invariance should be seen from its action given in the physical subspace defined by the Lorentz condition because the field exists and moves only in the physical subspace; (c) In the physical subspace, only infinitesimal gauge transformations are possibly allowed and necessary to be considered in examination of whether the theory is gauge-invariant or not; This fact was clarified originally in Ref. [13]; (d) To construct a correct gauge field theory, the residual gauge degrees of freedom existing in the physical subspace must be eliminated by a constraint condition on the gauge group. This constraint condition may be determined by requiring the action to be gauge-invariant [10]. Based on these points of view, as will be shown in this paper, the SU(2)-symmetric model of hadrodynamics can exactly be set up on the basis of gauge-invariance and the quantization of the model can readily be performed in the path-integral formalism by means of the Lagrange undetermined multiplier method.

The main purpose of this paper is to show that the quantum theory of the SU(2)-symmetric hadrodynamics built up on the gauge-invariance principle, as the $U(1)$-symmetric hadrodynamics [14], can perfectly be renormalized. Since a correct renormalization should be performed by exactly respecting Ward-Takahashi (W-T) identities [15-17] which follow from the gauge-symmetry of the theory, we first show that the quantum theory established here has an important property that the effective action appearing in the generating functional of Green functions is invariant with respect to a kind of BRST-transformations [18]. From the BRST-symmetry of the theory, we will derive various W-T identities satisfied by the generating functionals of Green functions and proper vertex functions. Furthermore, from the W-T identities obeyed by the generating functionals, we will derive W-T identities satisfied by the $\rho-$meson and ghost particle propagators and various proper vertices which appear in the perturbative expansion

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of S-matrix elements. Based on these W-T identities, the propagators and vertices will be perfectly renormalized. As a result of the renormalizations, the Slavnov-Taylor (S-T) identity satisfied by the renormalization constants \[19, 20\] will be derived. This identity is much useful for practical calculations of the renormalization carried out by the approach of renormalization group equation \[21-23\]. It should be mentioned that the previous conclusion for the unrenormalizability of the model was drawn from the quantum theory which was not established correctly \[4-9\] because the unphysical degrees of freedom involved in the theory are not eliminated by introducing appropriate constraint conditions. In our theory, the unphysical degrees of freedom appearing in the theory, i.e., the unphysical longitudinal components of vector potentials for the $\rho$-meson fields and the residual gauge degrees of freedom existing in the subspace defined by the Lorentz condition are respectively eliminated by the introduced Lorentz condition and the ghost equation which acts as the constraint condition on the gauge group. This guarantees that the quantum theory set up in this paper must be renormalizable. To demonstrate further the renormalizability of the theory, in this paper, the one loop renormalization will specifically be carried out by means of the mass-dependent momentum space subtraction scheme and the renormalization group equation (RGE). In this renormalization, we derive an exact one-loop effective coupling constant and one-loop effective nucleon, $\rho$-meson and pion masses without any ambiguity.

The arrangement of this paper is as follows. In Sec. 2, we will formulate the quantization of the model in the path-integral formalism and derive the BRST-transformations under which the effective action of the model is invariant. In Sec. 3, we will derive the W-T identities satisfied by various generating functionals. In Sec. 4, the W-T identity obeyed by the $\rho$-meson propagator and ghost propagator will be derived and the renormalization of these propagators will be discussed. In Sec. 5, the W-T identity obeyed by the $\rho$-meson three-line proper vertex will be derived and the renormalization of the vertex will be discussed. In Sec. 6, the same thing will be done for the $\rho$-meson four-line proper vertex. In section 7, the W-T identity satisfied by the nucleon-$\rho$-meson vertex will be derived and the renormalization of the vertex will be discussed. In Sec. 8, the W-T identity obeyed by the pion-$\rho$-meson three-line proper vertex will be derived and the renormalization of the vertex will be discussed. In Sec. 9, the same thing will be done for the pion-$\rho$-meson four-line proper vertex. Section 10 serves derive the one-loop effective coupling constant. Section 11 is used to discuss the renormalization of pion propagator and derive the one-loop effective boson ($\rho$-meson and pion) masses. In Sec. 12, we will discuss the renormalization of nucleon propagator and derive the one-loop effective nucleon mass. In the last section, some conclusions and discussions are made. In Appendix, the Feynman rules derived from the model action will be listed for convenience of perturbative calculations.

### I. QUANTIZATION AND BRST-TRANSFORMATIONS

The $SU(2)$-symmetric model of hadrodynamics is described by the following Lagrangian \[1,2\]:

\[ \mathcal{L} = \mathcal{L}_N + \mathcal{L}_\rho + \mathcal{L}_\pi + \mathcal{L}_{N\pi} \]  

(2.1)

where

\[ \mathcal{L}_N = \bar{\psi} \{ i\gamma^\mu D_\mu - M \} \psi, \]  

(2.2)

\[ \mathcal{L}_\rho = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \frac{1}{2} m_\rho^2 A_\mu A_\mu, \]  

(2.3)

\[ \mathcal{L}_\pi = \frac{1}{2} (D_\mu \pi)^+ (D_\nu \pi) - \frac{1}{2} m_\pi^2 \pi^2 \]  

(2.4)

and

\[ \mathcal{L}_{N\pi} = ig \bar{\psi} \sigma_5 \tau^a \psi \pi^a. \]  

(2.5)

In Eq. (2.2),

\[ \psi = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix} \]  

(2.6)

is the nucleon isospin doublet in which $\psi_p$ and $\psi_n$ denote the proton and neutron field functions respectively, $M$ is the nucleon mass and

\[ D_\mu = \partial_\mu - ig T^a A_\mu^a \]  

(2.7)
is the covariant derivative in which $T^a = \frac{\theta^a}{\pi} (a = 1, 2, 3)$ are the generators of $SU(2)$ gauge group with $\tau^a$ being the isospin Pauli matrices, $g$ is the coupling constant and $A^a_\mu(x)$ represent the vector potentials of $\rho$-meson fields. In Eq. (2.3),

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon^{abc} A^b_\mu A^c_\nu$$  \hspace{1cm} (2.8)

stand for the strength tensors in which $\varepsilon^{abc}$ are the structure constants of $SU(2)$ group (the 3-rank Levi-Civita tensor) and $m_\rho$ is the $\rho$-meson mass. In Eq. (2.4),

$$\pi = \left( \begin{array}{c} \pi_1 \\ \pi_2 \\ \pi_3 \end{array} \right)$$  \hspace{1cm} (2.9)

designates the pion isospin triplet and $m_\pi$ denotes the pion mass. It is noted that the covariant derivative $D_\mu$ in Eq. (2.4) is still represented in Eq. (2.7), but the matrix $T^a$ is now given in the adjoint representation with matrix elements $(T^a)_{bc} = -i\varepsilon_{abc}$.

The Lagrangian described above, except for the $\rho$-meson mass term, was derived from the requirement of $SU(2)$ gauge symmetry. The $\rho$-meson mass term is added from the physical requirement and is obviously not gauge-invariant. Therefore, the model described by the Lagrangian was considered to be gauge-non-invariance in the past. In the model, the nucleon fields are spinor fields and the pion fields are pseudoscalar fields which are all independent physical field variables, while, the $\rho$-meson fields, acting as the $SU(2)$ massive gauge fields, are vector fields for which the four Lorentz components of a vector potential $A^{\mu a}$ (for a certain $a$) are not all independent physical variables. Since a massive vector field has only three degrees of freedom of polarization which can completely be described by the transverse part $A^T_\mu$ of the vector potential $A^{\mu a}$, the longitudinal part $A^L_\mu$ of the vector potential appears to be a redundant degree of freedom which must be eliminated by introducing the Lorentz condition (the constraint condition on the gauge field)

$$\partial^\mu A^a_\mu = 0$$  \hspace{1cm} (2.10)

whose solution is

$$A^a_\mu = 0.$$  \hspace{1cm} (2.11)

Therefore, the $\rho$-meson fields must be viewed as a constrained system and the Lorentz condition must be introduced from the beginning and imposed on the Lagrangian so as to eliminate the redundant unphysical variables. As mentioned in Introduction, the gauge-invariance of a system should be generally examined from its action other than from its Lagrangian because action is of more fundamental dynamical meaning than Lagrangian. Particularly, for a constrained system, the gauge-invariance should be seen from its action given in the physical subspace defined by the constraint condition because the fields exist and move only in the physical subspace. And, as pointed out originally in Ref. [13], in the physical subspace, only infinitesimal gauge transformations of gauge fields are possibly allowed and necessary to be considered in examination of whether a field theory is gauge-invariant or not. In accordance with these point of view, it is easy to prove that the action given by the Lagrangian in Eqs. (2.1)-(2.9) is locally gauge-invariant under the Lorentz condition. In fact, when we perform the following infinitesimal $SU(2)$ local gauge transformations in the action given by the Lagrangian in Eqs. (2.1)-(2.9):

$$\delta A^a_\mu(x) = D^a_\mu(x)\theta^b(x),$$
$$\delta\psi(x) = igT^a\theta^b(x)\psi(x),$$
$$\delta\bar{\psi}(x) = -ig\bar{\psi}(x)T^a\theta^b(x),$$
$$\delta\pi^a(x) = g\varepsilon^{abc}\pi^b(x)\theta^c(x)$$  \hspace{1cm} (2.12)

where $\theta^a(x) (a = 1, 2, 3)$ are the parametric functions of the local gauge group $U(x) = \exp\{igT^a\theta^a(x)\}$ and

$$D^a_\mu = \delta^{ab}\partial_\mu - g\varepsilon^{abc} A^c_\mu,$$  \hspace{1cm} (2.13)

and apply the Lorentz condition to the action, it is found that

$$\delta S = -m_\rho^2 \int d^4 x \theta^a \partial^\mu A^a_\mu = 0$$  \hspace{1cm} (2.14)

which implies that the model constructed by the Lagrangian in Eqs. (2.1)-(2.9) and the Lorentz condition in Eq. (2.10) is gauge-invariant.
where the number $\xi$ to give a more generalized Lagrangian as follows

$$L_\lambda = L - \frac{1}{2} \alpha (\lambda^a)^2$$  \hspace{1cm} (2.15)

and

$$\partial^\mu A^a_\mu + \alpha \lambda^a = 0$$  \hspace{1cm} (2.16)

where $L$ was given in Eqs. (2.1)-(2.9). When the constraint in Eq. (2.16) is incorporated into the Lagrangian in Eq. (2.15) by the Lagrange multiplier method, we obtain

$$L_\lambda = L - \frac{1}{2} \alpha (\lambda^a)^2 + \lambda^a (\partial^\mu A^a_\mu + \alpha \lambda^a)$$

$$= L + \lambda^a \partial^\mu A^a_\mu + \frac{1}{2} \alpha (\lambda^a)^2.$$  \hspace{1cm} (2.17)

This Lagrangian is obviously not gauge-invariant. However, to build up a correct gauge field theory, it is necessary to require the action given by the Lagrangian (2.17) to be invariant under the gauge transformations denoted in Eqs. (2.12). By this requirement, noticing the identity $\varepsilon^{abc} A^a_{\mu} A^b_{\mu} = 0$ and applying the constraint condition in Eq. (2.16), we find

$$\delta S_\lambda = -\frac{1}{\alpha} \int d^4x \partial^\mu A^a_\mu (x) \partial^\nu (D^a_\mu (x) \theta^b (x)) = 0$$  \hspace{1cm} (2.18)

where

$$D^a_\mu (x) = \delta^{ab} \sigma^2 \frac{\partial x}{\partial x} + D^a_\mu (x)$$  \hspace{1cm} (2.19)

in which $\sigma^2 = \alpha m^2$ and $D^a_\mu (x)$ was defined in Eq. (2.13). From Eq. (2.16), we see $\frac{1}{\alpha} \partial^\nu A^a_\mu = -\lambda^a \neq 0$. Therefore, to ensure the action to be gauge-invariant, the following constraint condition on the gauge group is necessary to be required

$$\partial^\mu (D^a_\mu (x) \theta^b (x)) = 0.$$  \hspace{1cm} (2.20)

When we introduce, as usual, the ghost field variables $C^a (x)$ in such a manner

$$\theta^a (x) = \xi C^a (x),$$  \hspace{1cm} (2.21)

where $\xi$ is an infinitesimal Grassmann’s number, the constraint condition in Eq. (2.20) can be rewritten as

$$\partial^\mu (D^a_\mu C^b) = 0,$$  \hspace{1cm} (2.22)

where the number $\xi$ has been dropped. This constraint condition usually is called ghost equation. Certainly, the condition in Eq. (2.22) may also be incorporated into the Lagrangian in Eq. (2.17) by the Lagrange multiplier method to give a more generalized Lagrangian as follows

$$L_\lambda = L + \lambda^a \partial^\mu A^a_\mu + \frac{1}{2} \alpha (\lambda^a)^2 + \overline{C^a} \partial^\mu (D^a_\mu C^b)$$  \hspace{1cm} (2.23)

where $\overline{C^a} (x)$, acting as Lagrange undetermined multipliers, are the new scalar variables conjugate to the ghost variables $C^a (x)$.

At present, we are in a position to proceed the quantization of the model. As we learn from the Lagrange undetermined multiplier method, the dynamical and constrained variables as well as the Lagrange multipliers in the Lagrangian (2.23) can all be treated as free ones, varying independently. Therefore, in the Lagrangian path-integral formalism of quantization [10, 16, 17], we are allowed to use this kind of Lagrangian to construct the generating functional of Green functions

$$Z[\overline{\eta}, \eta, J^{a\mu}, K^a, \xi^a, \lambda^a] = \frac{1}{\lambda} \int D(\overline{\psi}, \psi, A^a_\mu, \pi^a, \overline{C^a}, C^a, \lambda^a) \exp \{ i \int d^4x [L_\lambda (x) + \overline{\psi} (x) \eta (x) + \overline{\eta} (x) \psi (x) + J^{a\mu} (x) A^a_\mu (x) + K^a \pi^a + \xi^a (x) C^a (x) + \overline{C^a} (x) \xi^a (x)] \}$$  \hspace{1cm} (2.24)
where \( D(A^a_{\mu}, \ldots, \lambda^a) \) denotes the functional integration measure, \( \eta, \eta, J^a_{\mu}, K^a, \xi^a \) and \( \xi^a \) are the external sources coupled to the nucleon, \( \rho \)-meson, pion and ghost fields respectively and \( N \) is the normalization constant. The integral over \( \lambda^a(x) \) in Eq. (2.24), as seen from Eq. (2.23), is of Gaussian-type. After calculating this integral, we arrive at

\[
Z[\eta, \eta, J^a_{\mu}, K^a, \xi^a] = \frac{1}{Z} \int D(\bar{\psi}, \psi, A^a_{\mu}, C^a, \bar{C}^a) \exp \{ i \int d^4x [\mathcal{L}_{\text{eff}}(x) + \bar{\psi}(x)\gamma(x) + \bar{\eta}(x)\psi(x) + J^a_{\mu}(x)A^a_{\mu}(x) + K^a\pi^a + \bar{C}^a(x)C^a(x) + \bar{\bar{C}}^a(x)\xi^a(x)] \}
\]

where

\[
\mathcal{L}_{\text{eff}} = \mathcal{L} - \frac{1}{2\alpha} (\partial^\mu A^a_{\mu})^2 - \partial^\mu \bar{C}^a \mathcal{D}^{ab}_\mu C^b
\]

is the effective Lagrangian given in arbitrary gauges in which \( \mathcal{L} \) is the Lagrangian written in Eqs. (2.1)-(2.9), and the second and third terms usually are named as gauge-fixing term and ghost term, respectively.

Similar to other gauge field theories such as the standard model, for the quantum hadrodynamics described above, there is a set of BRST-transformations including the infinitesimal gauge transformations shown in Eq. (2.12) and the transformations for the ghost fields under which the effective action is invariant. The transformations for the ghost fields may be found from the stationary condition of the effective action under the BRST-transformations. By applying the transformations in Eq. (2.12) to the action given by the Lagrangian in Eq. (2.26), one can derive

\[
\delta S = \int d^4x \{[\delta C^a - \frac{\xi}{\alpha} \partial^\nu A^a_{\nu}] \partial^\mu (\mathcal{D}^{ab}_\mu C^b) + \bar{C}^a \partial^\mu \delta(\mathcal{D}^{ab}_\mu C^b) \} = 0.
\]

This expression suggests that if we set

\[
\delta C^a = \frac{\xi}{\alpha} \partial^\nu A^a_{\nu}
\]

and

\[
\partial^\mu \delta(\mathcal{D}^{ab}_\mu C^b) = 0.
\]

The action will be invariant. Eq. (2.28) gives the transformation law of the ghost field variable \( C^a(x) \) which is similar to the one in quantum chromodynamics (QCD). From Eq. (2.29), we may derive a transformation law of the ghost field variables \( C^a(x) \). Noticing the relation in Eq. (2.19), we can write

\[
\delta(\mathcal{D}^{ab}_\mu C^b(x)) = \frac{\sigma^2}{\Box} \partial^\mu \delta C^a(x) + \delta(\mathcal{D}^{ab}_\mu C^b(x)).
\]

Paralleling to the proof in QCD [9, 18], it can be found that

\[
\delta(\mathcal{D}^{ab}_\mu C^b(x)) = D^{ab}_\mu(x)[\delta C^b(x) + \frac{\xi}{2} g \varepsilon^{bcd} C^c(x) C^d(x)].
\]

With this result, Eq. (2.30) can be represented as

\[
\delta(\mathcal{D}^{ab}_\mu C^b(x)) = D^{ab}_\mu(x)\delta C^b(x) - D^{ab}_\mu(x)\delta C^b_0(x)
\]

where

\[
\delta C^a_0(x) = \frac{\xi g}{2} \varepsilon^{abc} C^b(x) C^c(x).
\]

On substituting Eq. (2.32) into Eq. (2.29), we have

\[
M^{ab}(x)\delta C^b(x) = M^{ab}_0(x)\delta C^b_0(x)
\]

where we have defined

\[
M^{ab}(x) \equiv \partial^\mu \mathcal{D}^{ab}_\mu(x) = \delta^{ab}(\Box + \sigma^2) - g \varepsilon^{abc} A^c_{\mu}(x) \partial^\mu
\]
\[ M_0^{ab}(x) \equiv \partial_\mu D_\mu^{ab}(x) = M^{ab}(x) - \sigma^2 \delta^{ab}. \]  

(2.36)

It is noted that the operator in Eq. (2.35) is just the operator appearing in Eq. (2.22). Corresponding to Eq. (2.22), we may write an equation satisfied by the Green function \( \Delta^{ab}(x - y) \),

\[ M^{ac}(x) \Delta^{ch}(x - y) = \delta^{ab} \delta^4(x - y). \]

(2.37)

The function \( \Delta^{ab}(x - y) \) just is the exact propagator of the ghost field which is the inverse of the operator \( M^{ab}(x) \). With the help of Eq. (2.37) and noticing Eq. (2.36), we may solve out the \( \delta C^a(x) \) from Eq. (2.34)

\[ \delta C^a(x) = (M^{-1} M_0 \delta C_0)^a(x) = \{M^{-1}(M - \sigma^2)\delta C_0\}^a(x) = \delta C_0^a(x) - \sigma^2 \int d^4 y \Delta^{ab}(x - y) \delta C_0^b(y). \]

(2.38)

This just is the transformation law for the ghost field variables \( C^a(x) \). It is interesting to note that in the Landau gauge (\( \alpha = 0 \)), due to \( \sigma = 0 \), the above transformation will reduce to the form similar to the one given in QCD. This result is natural since in the Landau gauge, the \( \rho \)-meson field mass term in the action is gauge-invariant. However, in general gauges, the mass term is no longer gauge-invariant. In this case, to maintain the action to be gauge-invariant, it is necessary to give the ghost field a mass \( \sigma \) so as to counteract the gauge-non-invariance of the \( \rho \)-meson field mass term. As a result, in the transformation given in Eq. (2.38) appears a term proportional to \( \sigma^2 \).

**II. WARDA-TAKAHASHI IDENTITIES**

This section is devoted to deriving the W-T identities satisfied by generating functionals based on the BRST-symmetry of the theory. When we make the BRST-transformations shown in Eqs. (2.12), (2.28) and (2.38) to the generating functional in Eq. (2.25) and consider the invariance of the generating functional, the action and the integration measure under the transformations (the invariance of the integration measure is easy to check), we obtain an identity such that [9, 17]

\[ \frac{1}{\xi} \int \mathcal{D}(A_\mu^a, \bar{C}^a, C^a, \pi^a, \bar{\psi}, \psi) \int d^4 x \{ J^{\mu a}(x) \delta A_\mu^a(x) + \bar{\eta}(x) \delta \bar{\psi}(x) + \delta \bar{\psi}(x) \eta(x)

\]

\[ + K^a(x) \delta \pi^a(x) + \delta \bar{C}^a(x) \xi^a(x) + \bar{\xi}(x) \delta C^a(x) \} e^{iS + EST} = 0 \]

(3.1)

where EST is an abbreviation of the external source terms appearing in Eq. (2.25). The Grassmann number \( \xi \) contained in the BRST-transformations in Eq. (3.1) may be eliminated by performing a partial differentiation of Eq. (3.1) with respect to \( \xi \). As a result, we get a W-T identity as follows

\[ \frac{1}{\xi} \int \mathcal{D}(A_\mu^a, \bar{C}^a, C^a, \bar{\psi}, \psi) \int d^4 x \{ J^{\mu a}(x) \Delta A_\mu^a(x) - \bar{\eta}(x) \Delta \bar{\psi}(x) + \Delta \bar{\psi}(x) \eta(x)

\]

\[ + K^a(x) \Delta \pi^a(x) + \Delta \bar{C}^a(x) \xi^a(x) - \bar{\xi}(x) \Delta C^a(x) \} e^{iS + EST} = 0 \]

(3.2)

where

\[ \Delta A_\mu^a(x) = D_\mu^{ab}(x) C^b(x), \]

\[ \Delta \bar{\psi}(x) = ig T^a C^a(x) \psi(x), \]

\[ \Delta \bar{\psi}(x) = ig \bar{\psi}(x) T^a C^a(x), \]

\[ \Delta \pi^a(x) = g \varepsilon^{abc} A^b(x) C^c(x), \]

\[ \Delta C^a(x) = \frac{1}{\alpha} \delta_{\mu} A_\mu^a(x), \]

\[ \Delta C^a(y) = \int d^4 y [\delta^{ab} \delta^4(x - y) - \sigma^2 \delta^{ab} \delta^4(x - y)] \Delta C_0^b(y). \]

(3.3)

The functions defined above are finite. Each of them differs from the corresponding BRST-transformation written in Eqs. (2.12), (2.28) and (2.38) by an infinitesimal Grassmann parameter \( \xi \).

In order to represent the composite field functions \( \Delta A_\mu^a, \Delta \bar{\psi}, \Delta \pi^a \) and \( \Delta C^a \) in Eq. (3.2) in terms of differentials of the functional \( Z \) with respect to external sources, we may, as usual, construct a generalized generating functional by introducing new external sources (called BRST-sources later on) into the generating functional written in Eq. (2.25), as shown in the following [9, 17]

\[ Z[J^{\mu a}, \bar{C}^a, C^a, \bar{\psi}, \psi; \bar{u}^a, u^a, \bar{\nu}^a, \nu^a, \bar{\chi}^a, \chi^a, \bar{\zeta}, \zeta] \]

\[ = \frac{1}{\xi} \int \mathcal{D}[\bar{\psi}, \psi, A_\mu^a, \pi^a, \bar{C}^a, C^a, \chi^a, \zeta^a] \exp \{ i S + i \int d^4 x [u^{\mu a} \Delta A_\mu^a + \Delta \bar{\psi} \bar{\psi} + \Delta \bar{\psi} \bar{\psi} + \bar{\chi}^a \Delta \pi^a + \bar{\nu}^a \Delta C^a + J^{\mu a} A_\mu^a + \bar{\xi}^a C^a + C^a \bar{\xi}^a + K^a \pi^a + \bar{\eta}(x) + \bar{\psi}(x)] \}

(3.4)
where $u^a$, $v^a$, $\chi^a$ and $\zeta$ and $\xi$ are the sources of the functions $\Delta A^a_\mu$, $\Delta C^a$, $\Delta \pi^a$, $\Delta \Psi$ and $\Delta \bar{\Psi}$, respectively. Obviously, the $u^a$, $\Delta A^a_\mu$, $\chi^a$ and $\Delta \pi^a$ are anticommuting quantities, while, the $v^a$, $\zeta$, $\xi$, $\Delta C^a$, $\Delta \Psi$ and $\Delta \bar{\Psi}$ are commuting ones. We may start from the above generating functional to re-derive the W-T identity. In order that the identity thus derived is identical to that as given in Eq. (3.2), it is necessary to require the BRST-source terms $u_i \Delta \Phi_i$, where $u_i = u^{a\mu}$, $v^a$, $\chi^a$, $\bar{\chi}$ or $\xi$ and $\Delta \Phi_i = \Delta A^a_\mu$, $\Delta C^a$, $\Delta \pi^a$, $\Delta \Psi$ or $\Delta \bar{\Psi}$ to be invariant under the BRST-transformations. How to ensure the BRST-invariance of the source terms? For illustration, let us introduce the source terms in such a fashion

$$
\int d^4x [u^{a\mu} \delta A^a_\mu + v^a \delta C^a + \bar{\chi}^a \delta \pi^a + \bar{\xi} \delta \Psi + \delta \bar{\Psi} \zeta]
$$

where

$$
u^{a\mu} = \tilde{u}^{a\mu} \xi, \quad v^a = \tilde{v}^a \xi, \quad \chi^a = \bar{\chi}^a \xi, \quad \bar{\chi} = \bar{\xi} \xi, \quad \xi = -\bar{\xi} \zeta.
$$

These external sources are defined by including the Grassmann number $\xi$ and hence products of them with $\xi$ vanish. This suggests that we may generally define the sources by the following condition

$$u_i \xi = 0.
$$

Considering that under the BRST-transformation, the variation of the composite field functions given in arbitrary gauges can be represented in the form $\delta \Delta \Phi_i = \xi \bar{\Phi}_i$, where $\bar{\Phi}_i$ are functions without including the parameter $\xi$. Clearly, the definition in Eq. (3.7) for the sources would guarantee the BRST-invariance of the BRST-source terms. When the BRST-transformations in Eqs. (2.12), (2.28) and (2.38) are made to the generating functional in Eq. (3.4), due to the definition in Eq. (3.7) for the sources, we have $u_i \delta \Delta \Phi_i = 0$ which means that the BRST-source terms give a vanishing contribution to the identity in Eq. (3.1). Therefore, we still obtain the identity as shown in Eq. (3.1) except that the external source terms is now extended to include the BRST-external source terms. This fact indicates that we may directly insert the BRST-source terms into the exponent in Eq. (3.1) without changing the identity itself. When performing a partial differentiation of the identity with respect to $\xi$, we obtain a W-T identity which is the same as written in Eq. (3.2) except that the BRST-source terms are now included in the identity. Therefore, Eq. (3.2) may be expressed as

$$
\int d^4x [J^{a\mu}(x) \frac{\delta}{\delta u^{a\mu}(x)} - \bar{\eta}(x) \frac{\delta}{\delta v^a(x)} + \eta(x) \frac{\delta}{\delta \chi^a(x)} + K^a(x) \frac{\delta}{\delta \bar{\chi}^a(x)}]
$$

This is the W-T identity satisfied by the generating functional of full Green functions.

On substituting in Eq. (3.8) the relation [9, 17]

$$Z = e^{iW}
$$

where $W$ denotes the generating functional of connected Green functions, one may obtain a W-T identity expressed by the functional $W$

$$\int d^4x [J^{a\mu}(x) \frac{\delta}{\delta u^{a\mu}(x)} - \bar{\eta}(x) \frac{\delta}{\delta v^a(x)} + \eta(x) \frac{\delta}{\delta \chi^a(x)} + K^a(x) \frac{\delta}{\delta \bar{\chi}^a(x)}]
$$

This from identity, one may get another W-T identity satisfied by the generating functional $\Gamma$ of proper (one-particle-irreducible) vertex functions. The functional $\Gamma$ is usually defined by the following Legendre transformation [9, 17]

$$\Gamma[A^{a\mu}, \bar{C}^a, C^a, \pi^a, \bar{\psi}, \psi; u^a_\mu, v^a, \chi^a, \bar{\chi}, \xi, \zeta] = W[J^{a\mu}, \bar{C}^a, C^a, \pi^a + K^a \bar{\psi} + \bar{\psi} \eta]$$

where $A^{a\mu}, \bar{C}^a, C^a, \pi^a, \bar{\psi}$ and $\psi$ are the field variables defined by the following functional derivatives

$$A^{a\mu}(x) = \frac{\delta W}{\delta J^{a\mu}(x)}, \quad \bar{C}^a(x) = -\frac{\delta W}{\delta \pi^a(x)}, \quad C^a(x) = \frac{\delta W}{\delta \bar{\psi}(x)}, \quad \psi(x) = -\frac{\delta W}{\delta \bar{\psi}(x)}.
$$

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From Eq. (3.11), it is not difficult to get the inverse transformations \[9, 17\]

\[
\begin{align*}
J^{a\mu}(x) &= -\frac{\delta \Gamma}{\delta \bar{C}^{a\mu}(x)}; \quad \bar{\xi}^{a}(x) = \frac{\delta \Gamma}{\delta \bar{C}^{a\mu}(x)}; \quad \xi^{a}(x) = -\frac{\delta \Gamma}{\delta C^{a\mu}(x)}; \\
K^{a}(x) &= \frac{\delta \Gamma}{\delta \bar{\pi}^{a}(x)}, \quad \bar{\eta}(x) = \frac{\delta \Gamma}{\delta \bar{\psi}(x)}, \quad \eta(x) = -\frac{\delta \Gamma}{\delta \psi(x)}.
\end{align*}
\]  

(3.13)

It is obvious that

\[
\frac{\delta W}{\delta u^{a\mu}} = \frac{\delta \Gamma}{\delta u^{a\mu}}; \quad \frac{\delta W}{\delta \bar{u}^{a\mu}} = \frac{\delta \Gamma}{\delta \bar{u}^{a\mu}}; \quad \frac{\delta \Gamma}{\delta \chi^{a}} = \frac{\delta W}{\delta \bar{\chi}^{a}}; \quad \frac{\delta \Gamma}{\delta \bar{\chi}^{a}} = \frac{\delta W}{\delta \chi^{a}}; \quad \frac{\delta \Gamma}{\delta \bar{\chi}^{a}} = \frac{\delta W}{\delta \chi^{a}}.
\]

(3.14)

Employing Eqs. (3.13) and (3.14), the W-T identity in Eq. (3.10) will be written as \[9, 17\]

\[
\int d^{4}x \left\{ \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} + \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} + \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} \right\} = 0.
\]

(3.15)

This is the W-T identity satisfied by the generating functional of proper vertex functions.

The above identity may be represented in another form with the aid of the so-called ghost equation of motion. The ghost equation may easily be derived by first making the translation transformation: \(C^{a} \rightarrow \bar{C}^{a} + \bar{\lambda}^{a}\) in Eq. (2.25) where \(\bar{\lambda}^{a}\) is an arbitrary Grassmann variable, then differentiating Eq. (2.25) with respect to the \(\bar{\lambda}^{a}\) and finally setting \(\bar{\lambda}^{a} = 0\). The result is \[9, 17\]

\[
\frac{1}{N} \int D(A^{a\mu}, \bar{C}^{a\mu}, C^{a\mu}, K^{a}, \bar{\nu}, \bar{\psi}) \{ \xi^{a}(x) + \partial^{\mu}(D^{\nu}(x)C^{\nu}(x)) \} e^{iS+EST} = 0.
\]

(3.16)

When we use the generating functional defined in Eq. (3.4) and notice the relation in Eq. (2.19), the above equation may be represented as \[9, 17\]

\[
\left[ \xi^{a}(x) - i\partial^{\mu}(\frac{\delta}{\delta u^{a\mu}(x)}) - i\sigma^{2}(\frac{\delta}{\delta \bar{u}^{a\mu}(x)}) \right] Z[J^{a\mu}, \ldots, \zeta] = 0.
\]

(3.17)

On substituting the relation in Eq. (3.9) into the above equation, we may write a ghost equation satisfied by the functional \(W\) such that

\[
\xi^{a}(x) + \partial^{\mu}(\frac{\delta W}{\delta u^{a\mu}(x)}) + \sigma^{2}(\frac{\delta W}{\delta \bar{u}^{a\mu}(x)}) = 0.
\]

(3.18)

From this equation, the ghost equation obeyed by the functional \(\Gamma\) is easy to be derived by virtue of Eqs. (3.12)-(3.14) \[9, 17\]

\[
\frac{\delta \Gamma}{\delta C^{a}(x)} - \partial^{\mu}(\frac{\delta \Gamma}{\delta u^{a\mu}(x)}) - \sigma^{2}C^{a}(x) = 0.
\]

(3.19)

Upon applying the above equation to the last term in Eq. (3.15), the identity in Eq. (3.15) will be rewritten as

\[
\int d^{4}x \left\{ \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} + \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} + \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} \frac{\delta \Gamma}{\delta \bar{\chi}^{a}(x)} \right\} = 0.
\]

(3.20)

Now, let us define a new functional \(\hat{\Gamma}\) in such a manner

\[
\hat{\Gamma} = \Gamma + \frac{1}{2\alpha} \int d^{4}x (\partial^{a\mu}A^{a\mu})^{2}.
\]

(3.21)

From this definition, it follows that

\[
\frac{\delta \Gamma}{\delta A^{a\mu}} = \frac{\delta \hat{\Gamma}}{\delta A^{a\mu}} + \frac{1}{\alpha} \partial^{\nu}(\partial^{\mu}A^{a\nu}).
\]

(3.22)
When inserting Eq. (3.21) into Eq. (3.20) and considering the relation in Eq. (3.22), we arrive at

\[
\int d^4x \left\{ \frac{\delta \hat{\Gamma}}{\delta A^a_\mu} \frac{\delta \hat{\Gamma}}{\delta \bar{\psi} \dot{\psi}} + \frac{\delta \hat{\Gamma}}{\delta C^a} \frac{\delta \hat{\Gamma}}{\delta \dot{\psi}} + \frac{\delta \hat{\Gamma}}{\delta \bar{\psi}} \frac{\delta \hat{\Gamma}}{\delta \dot{\psi}} + \frac{\delta \hat{\Gamma}}{\delta \pi^a} \frac{\delta \hat{\Gamma}}{\delta \dot{\chi}^a} + m^2_\rho \partial^\mu A^a_\mu C^a \right\} = 0. \tag{3.23}
\]

The ghost equation represented through the functional \( \hat{\Gamma} \) is of the same form as that in Eq. (3.19)

\[
\frac{\delta \hat{\Gamma}}{\delta C^a(x)} - \partial_\mu \frac{\delta \hat{\Gamma}}{\delta u^\mu(x)} - \sigma^2 C^a(x) = 0. \tag{3.24}
\]

In the Landau gauge, since \( \sigma = 0 \) and \( \partial^\mu A^a_\mu = 0 \), Eqs. (3.23) and (3.24) respectively reduce to \([9, 17]\)

\[
\int d^4x \left\{ \frac{\delta \hat{\Gamma}}{\delta A^a_\mu} \frac{\delta \hat{\Gamma}}{\delta \bar{\psi} \dot{\psi}} + \frac{\delta \hat{\Gamma}}{\delta C^a} \frac{\delta \hat{\Gamma}}{\delta \dot{\psi}} + \frac{\delta \hat{\Gamma}}{\delta \bar{\psi}} \frac{\delta \hat{\Gamma}}{\delta \dot{\psi}} + \frac{\delta \hat{\Gamma}}{\delta \pi^a} \frac{\delta \hat{\Gamma}}{\delta \dot{\chi}^a} \right\} = 0 \tag{3.25}
\]

and

\[
\frac{\delta \hat{\Gamma}}{\delta C^a} - \partial_\mu \frac{\delta \hat{\Gamma}}{\delta u^\mu} = 0. \tag{3.26}
\]

which are homogeneous equations.

From the W-T identities formulated above, we may derive various W-T identities obeyed by Green functions and vertices, as will be illustrated in the next sections.

III. \( \rho \)-MESON AND GHOST PARTICLE PROPAGATORS

In this section, we plan to derive the W-T identities satisfied by the \( \rho \)-meson and ghost particle propagators by starting from the W-T identity represented in Eq. (3.8) and the ghost equation shown in Eq. (3.17) and then discuss their renormalization. Let us perform differentiations of the identities in Eqs. (3.8) and (3.17) with respect to the external sources \( \xi^a(x) \) and \( \bar{\xi}^b(y) \) respectively and then set all the sources except for the source \( J^a_{\mu}(x) \) to be zero. In this way, we obtain the following identities

\[
\frac{1}{\alpha} \partial^\mu_x \frac{\delta Z[J]}{\delta J^\mu(x)} + \int d^4y J^{\mu
\nu}(y) \frac{\delta^2 Z[J, \xi, \bar{\xi}]}{\delta \xi^a(x) \delta u^{\nu\mu}(y)}|_{\xi = u = 0} = 0 \tag{4.1}
\]

and

\[
i\partial^x_{\mu} \frac{\delta^2 Z[J, \xi, \bar{\xi}]}{\delta u^{\mu\nu}(x) \delta \xi^a(y)}|_{\xi = u = 0} + i\sigma^2 \frac{\delta^2 Z[J, \xi, \bar{\xi}]}{\delta \xi^a(x) \delta \bar{\xi}^b(y)}|_{\xi = u = 0} + \delta^{ab} \delta^4(x-y) Z[J] = 0. \tag{4.2}
\]

Furthermore, on differentiating Eq. (4.1) with respect to \( J^a_{\mu}(y) \) and then letting the source \( J \) vanish, we may get an identity which is, in operator representation, of the form \([9, 17]\)

\[
\frac{1}{\alpha} \partial^\mu_y < 0^+ | T[A^a_\mu(x) \hat{A}^a_\mu(y)] 0^- > = < 0^+ | T^* [\hat{C}^a(x) \hat{D}^{ab}_\nu(y) \hat{C}^d(y)] 0^- > \tag{4.3}
\]

where \( \hat{A}^a_\nu(x) \), \( \hat{C}^a(x) \) and \( \hat{\bar{C}}^a(x) \) stand for the gluon field and ghost field operators and \( T^* \) symbolizes the covariant time-ordering product. When the source \( J \) is set to vanish, Eq. (4.2) gives such an equation \([9, 17]\)

\[
i \partial^\nu_y < 0^+ | T^* \{ \hat{C}^a(x) \hat{D}^{ab}_\nu(y) \hat{C}^d(y) \} 0^- > + i\sigma^2 < 0^+ | T[\hat{C}^a(x) \hat{C}^b(y)] 0^- > = \delta^{ab} \delta^4(x-y). \tag{4.4}
\]

Upon inserting Eq. (4.3) into Eq. (4.4), we have

\[
\partial^\mu_y \partial^\nu_y D^{ab}_{\mu\nu}(x-y) - \alpha \sigma^2 \Delta^{ab}(x-y) = -\alpha \delta^{ab} \delta^4(x-y) \tag{4.5}
\]

where
\[ i D^{ab}_{\mu\nu}(x-y) = \langle 0^+ | T\{ \hat{A}^a_\mu(x)\hat{A}^b_\nu(y)\} | 0^- \rangle \] (4.6)

which is the full \( \rho \)-meson propagator and

\[ i \Delta^{ab}(x-y) = \langle 0^+ | T\{ \hat{C}^a(x)\hat{C}^b(y)\} | 0^- \rangle \] (4.7)

which is the full ghost particle propagator. Eq. (4.5) just is the W-T identity respected by the \( \rho \)-meson propagator which establishes a relation between the longitudinal part of \( \rho \)-meson propagator and the ghost particle propagator. Particularly, in the Landau gauge \((\alpha = 0)\), as we see, Eq. (4.5) reduces to the form which exhibits the transversity of the \( \rho \)-meson propagator. By the Fourier transformation, Eq. (4.5) will be converted to the form given in the momentum space as follows

\[ k^\mu k^\nu D^{ab}_{\mu\nu}(k) - \alpha \sigma^2 \Delta^{ab}(k) = -\alpha \delta^{ab}. \] (4.8)

The ghost particle propagator may be determined by the ghost equation shown in Eq. (4.4). However, we would rather here to derive its expression from the Dyson-Schwinger equation [24] satisfied by the propagator which may be established by the perturbation method.

\[ \Delta^{ab}(k) = \Delta^{ab}_0(k) + \Delta^{ab}_{0\prime}(k)\Omega^{a\prime b\prime}(k)\Delta^{b\prime}(k) \] (4.9)

where

\[ i\Delta^{ab}_0(k) = i\delta^{ab}\Delta^{ab}_0(k) = \frac{-i\delta^{ab}}{k^2 - \sigma^2 + i\varepsilon} \] (4.10)

is the free ghost particle propagator which can be derived from the generating functional in Eq. (2.25) by a perturbative calculation and \(-i\Omega^{ab}(k) = -i\delta^{ab}\Omega(k)\) denotes the proper self-energy operator of ghost particle. From Eq. (4.9), it is easy to solve that

\[ i\Delta^{ab}(k) = \frac{-i\delta^{ab}}{k^2[1 + \Omega(k^2)] - \sigma^2 + i\varepsilon} \] (4.11)

where the self-energy has properly been expressed as

\[ \Omega(k) = k^2\Omega(k^2). \] (4.12)

Similarly, we may write a Dyson-Schwinger equation for the \( \rho \)-meson propagator by the perturbation procedure [24]

\[ D_{\mu\nu}(k) = D^{0}_{\mu\nu}(k) + D^{0}_{\mu\lambda}(k)\Pi^{\lambda\nu}(k)D_{\rho\nu}(k) \] (4.13)

where the color indices are suppressed for simplicity and

\[ iD^{(0)ab}_{\mu\nu}(k) = i\delta^{ab}D^{(0)}_{\mu\nu}(k) = -i\delta^{ab}\sigma\sigma^{\mu\nu} - \frac{k_\mu k_\nu/k^2}{k^2 - m^2_\rho + i\varepsilon} + \frac{\alpha k_\mu k_\nu/k^2}{k^2 - \sigma^2 + i\varepsilon} \] (4.14)

is the free \( \rho \)-meson propagator which can easily be derived from the perturbative expansion of the generating functional in Eq. (2.25) and \(-i\Pi^{ab}_{\mu\nu}(k) = -i\delta^{ab}\Pi^{\mu\nu}(k)\) stands for the \( \rho \)-meson proper self-energy operator. Let us decompose the propagator and the self-energy operator into a transverse part and a longitudinal part:

\[ D^{\mu\nu}(k) = D^{\mu\nu}_T(k) + D^{\mu\nu}_L(k), \quad \Pi^{\mu\nu}(k) = \Pi^{\mu\nu}_T(k) + \Pi^{\mu\nu}_L(k) \] (4.15)

where

\[ D^{\mu\nu}_T(k) = \mathcal{D}^{\mu\nu}_T(k)D_T(k^2), \quad D^{\mu\nu}_L(k) = \mathcal{D}^{\mu\nu}_L(k)D_L(k^2), \quad \Pi^{\mu\nu}_T(k) = \mathcal{P}^{\mu\nu}_T(k)\Pi_T(k^2), \quad \Pi^{\mu\nu}_L(k) = \mathcal{P}^{\mu\nu}_L(k)\Pi_L(k^2) \] (4.16)

here \( \mathcal{P}^{\mu\nu}_T(k) = (g^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}) \) and \( \mathcal{P}^{\mu\nu}_L(k) = \frac{k^\mu k^\nu}{k^2} \) are the transverse and longitudinal projectors respectively. Considering these decompositions and the orthogonality between the transverse and longitudinal parts, Eq. (4.13) will be split into two equations

\[ D_{T\mu\nu}(k) = D^0_{T\mu\nu}(k) + D^0_{T\mu\lambda}(k)\Pi^{\lambda\nu}_T(k)D_{T\rho\nu}(k) \] (4.17)
\[ D_{\mu\nu}(k) = D_{\mu\nu}^0(k) + D_{\mu\lambda}^0(k)\Pi_L^{\mu\lambda}(k)D_{\lambda\nu}(k). \]  

(4.18)

Solving the equations (4.17) and (4.18), one can get

\[ iD_{\mu\nu}^{ab}(k) = -i\delta^{ab}\{ \frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 + \Pi_T(k^2) - m_\rho^2 + i\varepsilon} + \frac{\alpha k_\mu k_\nu/k^2}{k^2 + \alpha\Pi_L(k^2) - \sigma^2 + i\varepsilon} \}. \]

(4.19)

With setting

\[ \Pi_T(k^2) = k^2\Pi_1(k^2) + m_\rho^2\Pi_2(k^2) \]

(4.20)

which follows from the Lorentz-covariance of the operator \( \Pi_T(k^2) \) and

\[ \alpha\Pi_L(k^2) = k^2\Phi_L(k^2), \]

(4.21)

Eq. (4.19) will be written as

\[ iD_{\mu\nu}^{ab}(k) = -i\delta^{ab}\{ \frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2[1 + \Pi_1(k^2)] - m_\rho^2[1 - \Pi_2(k^2)] + i\varepsilon} + \frac{\alpha k_\mu k_\nu/k^2}{k^2[1 + \alpha\Pi_L(k^2)] - \sigma^2 + i\varepsilon} \}. \]

(4.22)

We would like to note that the expressions given in Eqs. (4.12), (4.20) and (4.21) can be verified by practical calculations and are important for the renormalization of the propagators and the \( \rho \)-meson mass.

Substitution of Eqs. (4.11) and (4.22) into Eq. (4.8) yields

\[ \Phi_L(k^2) = \frac{\sigma^2\Omega(k^2)}{k^2[1 + \Omega(k^2)]}. \]

(4.23)

From this relation, we see, either in the Landau gauge or in the zero-mass limit, the \( \Phi_L(k^2) \) vanishes.

Now let us discuss the renormalization. The function \( \Omega(k^2) \) in Eq. (4.11) and the functions \( \Pi_1(k^2), \Pi_2(k^2) \) and \( \Phi_L(k^2) \) in Eq. (4.22) are generally divergent in higher order perturbative calculations. According to the conventional procedure of renormalization, the divergences included in the functions \( \Omega(k^2), \Pi_1(k^2), \Pi_2(k^2) \) and \( \Phi_L(k^2) \) may be subtracted at a renormalization point, say, \( k^2 = \mu^2 \). Thus, we can write [9, 16, 17]

\[ \tilde{\Omega}(k^2) = \Omega(\mu^2) + \Omega^c(k^2), \quad \Pi_1(k^2) = \Pi_1(\mu^2) + \Pi_1^c(k^2), \]

\[ \Pi_2(k^2) = \Pi_2(\mu^2) + \Pi_2^c(k^2), \quad \Phi_L(k^2) = \Phi_L(\mu^2) + \Phi_L^c(k^2) \]

(4.24)

where \( \tilde{\Omega}(\mu^2), \Pi_1(\mu^2), \Pi_2(\mu^2), \Phi_L(\mu^2) \) and \( \Omega^c(k^2), \Pi_1^c(k^2), \Pi_2^c(k^2), \Phi_L^c(k^2) \) are respectively the divergent parts and the finite parts of the functions \( \Omega(k^2), \Pi_1(k^2), \Pi_2(k^2) \) and \( \Phi_L(k^2) \). The divergent parts can be absorbed in the following renormalization constants defined by [9, 16, 17]

\[ Z_3^{-1} = 1 + \Omega(\mu^2), \quad Z_3^{-1} = 1 + \Pi_1(\mu^2), \quad Z_3^{-1} = 1 + \Phi_L(\mu^2), \]

\[ Z_{m_\rho}^{-1} = \sqrt{Z_3[1 - \Pi_2(\mu^2)]} = \sqrt{[1 - \Pi_1(\mu^2)][1 - \Pi_2(\mu^2)]} \]

(4.25)

where \( Z_3 \) and \( \tilde{Z}_3 \) are the renormalization constants of \( \rho \)-meson and ghost particle propagators respectively, \( Z_3^\prime \) is the additional renormalization constant of the longitudinal part of gluon propagator and \( Z_{m_\rho} \) is the renormalization constant of gluon mass. With the above definitions of the renormalization constants, on inserting Eq. (4.24) into Eqs. (4.11) and (4.22), the ghost particle propagator and the gluon propagator can be renormalized, respectively, in such a manner

\[ i\Delta^{ab}(k) = \tilde{Z}_3i\Delta_{R}^{ab}(k) \]

(4.26)

and

\[ iD_{\mu\nu}^{ab}(k) = Z_{m_\rho}D_{R\mu\nu}^{ab}(k) \]

(4.27)
written in the operator form as follows
\[ i\Delta_R^{ab}(k) = \frac{-i\delta^{ab}}{k^2[1 + \Omega_R(k^2)] - \sigma_R^2 + i\varepsilon} \]  
(4.28)

and
\[ iD_{R\mu
u}^{ab}(k) = -i\delta^{ab}\left\{ \frac{g_{\mu\nu} - k_\mu k_\nu/k^2}{k^2 - m_R^2 + \Pi_{R}^T(k^2) + i\varepsilon} + \frac{Z_3\alpha_R k_\mu k_\nu/k^2}{k^2[1 + \Pi_{R}^T(k^2)] - \sigma_R^2 + i\varepsilon} \right\} \]  
(4.29)

are the renormalized propagators in which \( m_R^2, \sigma_R \) and \( \sigma_R^2 \) are the renormalized masses, \( \Omega_R(k^2), \Pi_{R}^T(k^2) \) and \( \Pi_{R}^L(k^2) \) denote the finite corrections coming from the loop diagrams. They are defined as
\[ m_R^2 = Z_m^{-1} m_p, \quad \alpha_R = Z_3^{-1} \alpha, \quad \sigma_R = \sqrt{Z_3} \sigma, \quad \Omega_R(k^2) = \tilde{Z}_3 \Omega^c(k^2), \quad \Pi_{R}^T(k^2) = Z_3[k^2\Pi_{R}^T(k^2) + m_R^2\Pi_{R}^L(k^2)], \quad \Pi_{R}^L(k^2) = Z_3\Pi_{R}^L(k^2). \]  
(4.30)

From the definitions in Eqs. (4.24) and (4.30), it is clearly seen that at the renormalization point \( \mu \), the finite corrections \( \Omega_R(k^2), \Pi_{R}^T(k^2) \) and \( \Pi_{R}^L(k^2) \) vanish. In this case, the propagators reduce to the form of free ones. As we see from Eq. (4.29), the longitudinal part of the gluon propagator, except for in the Landau gauge, needs to be renormalized and has an extra renormalization constant \( Z_3' \). This fact coincides with the general property of the massive vector boson propagator (see Ref. (9), Chap.V). From Eqs. (4.23)-(4.25), it is easy to find that the longitudinal part in Eq. (4.22) can be renormalized as
\[ \frac{\alpha}{k^2[1 + \Pi_{R}^T(k^2)] - \sigma_R^2 + i\varepsilon} = Z_3\alpha_R[1 + \Omega_R(k^2)]\Delta_R(k^2) \]  
(4.31)

where
\[ \Delta_R(k^2) = \frac{1}{k^2[1 + \Omega_R(k^2)] - \sigma_R^2 + i\varepsilon} \]  
(4.32)

which appears in Eq. (4.28) and the renormalization constant \( Z_3' \) can be expressed as
\[ Z_3' = [1 + \sigma_R^2(1 - \tilde{Z}_3)]^{-1}. \]  
(4.33)

If choosing \( \mu = \sigma_R \), we have
\[ Z_3' = \tilde{Z}_3. \]  
(4.34)

### IV. \( \rho \)-MESON THREE-LINE PROPER VERTEX

The aim of this section is to derive the W-T identity satisfied by the \( \rho \)-meson three-line proper vertex and discuss the renormalization of the vertex. For this purpose, we first derive a W-T identity satisfied by the \( \rho \)-meson three-point Green function. Let us begin with the derivation from the W-T identity in Eq. (4.1) and the ghost equation in Eq. (4.2). By taking successive differentiations of Eq. (4.1) with respect to the sources \( J_\nu^x(y) \) and \( J_\lambda^x(z) \) and then setting the sources to vanish, one may obtain the W-T identity obeyed by the \( \rho \)-meson three-point Green function which is written in the operator form as follows
\[ \frac{1}{\alpha} \partial^\mu G^{ab\lambda}_{\mu
u}(x, y, z) = < 0^+ | T^* [ \hat{C}^a(x) \hat{D}^{bd}_\nu(y) \hat{C}^d(y) \hat{A}^\lambda_\mu(z) ] | 0^- > \]
\[ + < 0^+ | T^* [ \hat{C}^a(x) \hat{A}^b_\nu(y) \hat{D}^{cd}(z) \hat{C}^d(z) ] | 0^- > \]  
(5.1)

where
\[ G^{abc}_{\mu
u\lambda}(x, y, z) = < 0^+ | T \hat{A}^a_\mu(x) \hat{A}^b_\nu(y) \hat{A}^c_\lambda(z) | 0^- > \]  
(5.2)

is the three-point Green function mentioned above. The identity in Eq. (5.1) will be simplified by a ghost equation which may be derived by differentiating Eq. (4.2) with respect to the source \( J_\lambda^x(z) \)
\[ 
\partial^\mu \rho \rho_0^\mu(x,y,z) = \alpha \omega^2 \{ \partial^\nu G^{abc}_\lambda(x,y,z) + \partial^\lambda G^{abc}_\nu(y,x,z) \} 
\]

where

\[ 
G^{abc}_\mu(x,y,z) = \langle 0^+ | T \{ \hat{C}^a(x) \hat{A}^b(y) \hat{A}^c(z) \} | 0^- \rangle 
\]

is the three-point \( \rho \)-meson-ghost particle Green function. In the Landau gauge, Eq. (5.4) reduces to

\[ 
\partial^\mu \partial^\nu \partial^\lambda G^{abc}_\mu(x,y,z) = 0 
\]

which shows the transversity of the Green function. From Eq. (5.4), we may derive a W-T identity for the \( \rho \)-meson three-line vertex. For this purpose, it is necessary to use the following one-particle-irreducible decompositions of the Green functions which can easily be obtained by the well-known procedure [9, 17]

\[ 
G^{abc}_{\mu \nu \lambda}(x,y,z) = \int d^4 x' d^4 y' d^4 z' \delta^{(4)}(x-x') 
\times iD^{abc}_{\nu \mu'}(y-y')iD^{\nu \alpha}_{\lambda}(z-z') \Gamma^{abc}_{\mu' \nu' \lambda'}(x',y',z') 
\]

and

\[ 
G^{abc}_\nu(x,y,z) = \int d^4 x' d^4 y' d^4 z' \delta^{(4)}(x-x') \Delta^{(4)} \Gamma^{abc}_{\nu' \nu' \lambda'}(x',y',z') 
\times iD^{\nu' \lambda}(y'-y)iD^{\nu \nu'}(z'-z) 
\]

where \( iD^{abc}(x-x') \) and \( \Delta^{aa}(x-x') \) are respectively the \( \rho \)-meson and the ghost particle propagators discussed in the preceding section, \( \Gamma^{abc}_{\mu \nu \lambda}(x,y,z) \) and \( \Gamma^{abc}_{\mu \nu \lambda}(x,y,z) \) are the three-line \( \rho \)-meson proper vertex and the three-line \( \rho \)-meson-ghost particle proper vertex respectively. They are defined as [9, 17]

\[ 
\Gamma^{abc}_{\mu \nu \lambda}(x,y,z) = \frac{\delta^3 \Gamma}{\delta A^\mu(x) \delta A^\nu(y) \delta A^\lambda(z)} |_{J=0} 
\]

and

\[ 
\Gamma^{abc}_{\mu \nu \lambda}(x,y,z) = \frac{\delta^3 \Gamma}{i \delta C^a(x) \delta C^b(y) \delta C^c(z)} |_{J=0} 
\]

where \( J \) stands for all the external sources. Substituting Eqs. (5.7) and (5.8) into Eq. (5.4) and transforming Eq. (5.4) into the momentum space, one can derive an identity which establishes the relation between the longitudinal part of the three-line \( \rho \)-meson vertex and the three-line ghost vertex as follows

\[ 
p^{\mu}q^{\nu}k^{\lambda} \Lambda^{abc}_{\mu \nu \lambda}(p,q,k) = -\frac{g^2}{\alpha} \chi(p^2)[\chi(k^2)q^{\nu} \Lambda^{abc}(k,p,q) 
+ \chi(q^2)k^{\lambda} \Lambda^{abc}(q,k,p)] 
\]

where we have defined

\[ 
\Gamma^{abc}_{\mu \nu \lambda}(p,q,k) = (2\pi)^4 \delta^{(4)}(p+q+k) \Lambda^{abc}_{\mu \nu \lambda}(p,q,k), 
\Gamma^{abc}_{\mu \nu \lambda}(p,q,k) = (2\pi)^4 \delta^{(4)}(p+q+k) \Lambda^{abc}_{\mu \nu \lambda}(p,q,k) 
\]

and

\[ 
\chi(p^2) = \{ k^2[1 + \tilde{\Pi}_L(p^2)] - \sigma^2 + i\epsilon \} \{ k^2[1 + \Omega(p^2)] - \sigma^2 + i\epsilon \}^{-1} 
= [1 + \Omega(k^2)]^{-1} 
\]

here \( \tilde{\Pi}_L(p^2) \) and \( \Omega(p^2) \) are the self-energies defined in Eqs. (4.12) and (4.21). The second equality in Eq. (5.13) is obtained by inserting the relation in Eq. (4.23) into the first equality.

Obviously, in the Landau gauge, Eq. (5.11) reduces to
\[ p^\mu q^\nu k^\lambda \Lambda_{\mu\nu\lambda}^{abc}(p, q, k) = 0 \]  \hspace{1cm} (5.14)

which implies that the vertex is transverse in this case. In the lowest order approximation, owing to

\[ \chi(p^2) = 1 \]  \hspace{1cm} (5.15)

and

\[ \Lambda^{(0)abc}_\mu(p, q, k) = g \varepsilon^{abc} p_\mu, \]  \hspace{1cm} (5.16)

the right hand side (RHS) of Eq. (5.11) vanishes, therefore, we have

\[ p^\mu q^\nu k^\lambda \Lambda_{\mu\nu\lambda}^{(0)abc}(p, q, k) = 0. \]  \hspace{1cm} (5.17)

This result is consistent with that for the bare three-line \( \rho \)-meson vertex given by the Feynman rule.

Now, let us discuss the renormalization of the three-line gluon vertex. From the renormalization of the \( \rho \)-meson and ghost particle propagators described in Eqs. (4.26) and (4.27) and the definitions of the propagators written in Eqs. (4.6) and (4.7), one can see

\[ A^{\alpha\mu}(x) = \sqrt{Z_3} A^{\alpha\mu}_R(x), \]
\[ C^{\alpha}(x) = \sqrt{Z_4} C^{\alpha}_R(x), \quad \tilde{C}^{\alpha}(x) = \sqrt{Z_3} \tilde{C}_R^{\alpha}(x) \]  \hspace{1cm} (5.18)

(hereafter the subscript \( R \) marks renormalized quantities). According to above relations and the definitions given in Eqs. (5.9), (5.10) and (5.12), we find

\[ \Lambda^{abc}_{\mu\nu\lambda}(p, q, k) = Z_3^{-3/2} \Lambda^{abc}_{R\mu\nu\lambda}(p, q, k), \]
\[ \Lambda^{abc}_\lambda(p, q, k) = \tilde{Z}_3^{-1} Z_3^{-1/2} \Lambda^{abc}_{R\lambda}(p, q, k). \]  \hspace{1cm} (5.19)

Applying these relations, the renormalized version of the identity written in Eq. (5.11) will be

\[ p^\mu q^\nu k^\lambda \Lambda_{R\mu\nu\lambda}^{abc}(p, q, k) = -\frac{\sigma_R^2}{\alpha_R} \chi_R(p^2) [\chi_R(k^2) q^\nu \Lambda_{R\mu\nu\lambda}^{abc}(p, q, k) \]
\[ + \chi_R(q^2) k^\lambda \Lambda_{R\lambda}^{abc}(p, q, k) \]  \hspace{1cm} (5.20)

where \( \alpha_R \) and \( \sigma_R \) were defined in Eq. (4.30) and

\[ \chi_R(k^2) = \frac{1}{1 + \Omega_R(k^2)} \]  \hspace{1cm} (5.21)

is the renormalized expression of the function \( \chi(k^2) \). In the above, we have considered

\[ \chi(k^2) = \tilde{Z}_3 \chi_R(k^2) \]  \hspace{1cm} (5.22)

which follows from \( \tilde{\Omega}(k^2) = \tilde{\Omega}(\mu^2) + \tilde{\Omega}(k^2) \), \( \tilde{Z}_3^{-1} = 1 + \tilde{\Omega}(\mu^2) \) and \( \Omega_R(k^2) = \tilde{Z}_3 \tilde{\Omega}(k^2) \) defined in the preceding section.

At the renormalization point chosen to be \( p^2 = q^2 = k^2 = \mu^2 \), we see, \( \chi_R(\mu^2) = 1 \). In this case, the renormalized ghost vertex takes the form of the bare vertex so that the RHS of Eq. (5.20) vanishes, therefore, we have

\[ p^\mu q^\nu k^\lambda \Lambda_{R\mu\nu\lambda}^{abc}(p, q, k)|_{p^2=q^2=k^2=\mu^2} = 0. \]  \hspace{1cm} (5.23)

Ordinarily, one is interested in discussing the renormalization of such three-line vertices that they are defined from the vertices defined in Eqs. (5.9) and (5.10) by extracting a coupling constant \( q \). These vertices are denoted by \( \Lambda^{abc}_{\mu\nu\lambda}(p, q, k) \) and \( \Lambda^{abc}_\lambda(p, q, k) \). Commonly, they are renormalized in such a fashion \cite{9, 17}

\[ \tilde{\Lambda}^{abc}_{\mu\nu\lambda}(p, q, k) = Z_1^{-1} \Lambda^{abc}_{R\mu\nu\lambda}(p, q, k), \]
\[ \tilde{\Lambda}^{abc}_\lambda(p, q, k) = \tilde{Z}_1^{-1} \Lambda^{abc}_{R\lambda}(p, q, k) \]  \hspace{1cm} (5.24)

where \( Z_1 \) and \( \tilde{Z}_1 \) are referred to as the renormalization constants of the \( \rho \)-meson three-line vertex and the \( \rho \)-meson-ghost particle vertex, respectively. It is clear that the W-T identity shown in Eq. (5.11) also holds for the vertices \( \Lambda^{abc}_{\mu\nu\lambda}(p, q, k) \) and \( \Lambda^{abc}_\lambda(p, q, k) \). So, when the vertices \( \Lambda^{abc}_{\mu\nu\lambda}(p, q, k) \) and \( \Lambda^{abc}_\lambda(p, q, k) \) in Eqs. (5.11) are replaced by
\( \tilde{\Lambda}_{\mu\nu\lambda}^{abc}(p, q, k) \) and \( \tilde{\Lambda}_{\mu\nu\lambda}^{abc}(p, q, k) \) respectively and then Eq. (5.24) is inserted to such an identity, we obtain a renormalized version of the identity as follows

\[
p^{\mu}q^{\nu}k^{\lambda} \tilde{\Lambda}_{R \mu\nu\lambda}^{abc}(p, q, k) = -\frac{Z_1\tilde{Z}_3}{Z_3} \frac{s^2}{\alpha R} \chi_R(p^2)[\chi_R(k^2)] \\
\times q^{\rho} \tilde{\Lambda}_{R \rho}^{abc}(k, p, q) + \chi_R(q^2)k^{\lambda} \Lambda_{abc}^{R \mu\nu\lambda}(p, q, k).
\]

(5.25)

When multiplying the both sides of Eq. (5.25) with a renormalized coupling constant \( g_R \) and absorbing it into the vertices, noticing

\[
\Lambda_{R \mu\nu\lambda}^{abc}(p, q, k) = g_R \tilde{\Lambda}_{R \mu\nu\lambda}^{abc}(p, q, k), \\
\Lambda_{\mu\nu\lambda}^{abc}(p, q, k) = g_R \Lambda_{\mu\nu\lambda}^{abc}(p, q, k),
\]

(5.26)

we have

\[
p^{\mu}q^{\nu}k^{\lambda} \Lambda_{R \mu\nu\lambda}^{abc}(p, q, k) = -\frac{Z_1\tilde{Z}_3}{Z_3} \frac{s^2}{\alpha R} \chi_R(p^2)[\chi_R(k^2)] \\
\times q^{\rho} \Lambda_{R \rho}^{abc}(k, p, q) + \chi_R(q^2)k^{\lambda} \Lambda_{abc}^{R \mu\nu\lambda}(p, q, k).
\]

(5.27)

In comparison of Eq. (5.27) with Eq. (5.20), we see, except for the factor \( Z_1\tilde{Z}_3Z_3^{-1}\tilde{Z}_1^{-1} \), the both identities are identical to each other. From this observation, we deduce

\[
\frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3}
\]

(5.28)

This is the S-T identity similar to that given in QCD [9, 16, 17, 19].

V. \( \rho \)-MESON FOUR-LINE PROPER VERTEX

By the similar procedure as deriving Eqs. (5.1) and (5.3), the W-T identity obeyed by the \( \rho \)-meson four-point Green function may be derived by differentiating Eq. (4.1) with respect to the sources \( J_{\mu}^b(y) \), \( J_{\lambda}^c(z) \) and \( J_{\tau}^d(u) \). The result represented in the operator form is as follows

\[
\frac{1}{\alpha x} \partial_x G_{\mu\nu\lambda\tau}^{abcd}(x, y, z, u) \\
= 0^+|T^*[\hat{C}^a(x)\hat{D}^b(y)\hat{C}^c(y)\hat{A}_\lambda^d(u)]|0^-
\]

(6.1)

\[
+ 0^+|T^*[\hat{C}^a(x)\hat{A}_\lambda^c(z)\hat{D}^d(y)\hat{C}^c(y)]|0^-
\]

\[
+ 0^+|T^*[\hat{C}^a(x)\hat{A}_\lambda^c(z)\hat{D}^d(y)\hat{C}^c(u)]|0^-
\]

where

\[
G_{\mu\nu\lambda\tau}^{abcd}(x, y, z, u) = 0^+|T[\hat{A}_\mu^a(x)\hat{A}_\lambda^c(z)\hat{A}_\tau^d(u)]|0^-
\]

(6.2)

is the \( \rho \)-meson four-point Green function. The accompanying ghost equation may be obtained by differentiating Eq. (4.2) with respect to the sources \( J_{\lambda}^c(z) \) and \( J_{\tau}^d(u) \). The result is

\[
\partial_x <0^+|T[\hat{A}_\mu^a(x)\hat{D}^b(y)\hat{C}^b(y)\hat{A}_\tau^d(u)]|0^- > \\
+ \sigma^2 G_{\lambda\tau}^{abcd}(x, y, z, u) = -\delta^{ab}\delta^a(x-y)D_{\lambda\tau}^{cd}(z-u)
\]

(6.3)

where

\[
G_{\mu\nu\lambda\tau}^{abcd}(x, y, z, u) = 0^+|T[\hat{C}^a(x)\hat{C}^c(z)\hat{A}_\tau^d(u)]|0^->
\]

(6.4)

is the four-point \( \rho \)-meson-ghost particle Green function. Differentiation of Eq. (6.1) with respect to the coordinates \( y, z \) and \( u \) and use of Eq. (6.3) lead to

\[
\frac{1}{\alpha x} \partial_y \partial_y \partial_\tau \partial_\tau G_{\mu\nu\lambda\tau}^{abcd}(x, y, z, u) = \delta^{ab}\delta^4(x-y)\partial_\lambda^c \partial_\tau^d D_{\lambda\tau}^{cd}(z-u) \\
+ \delta^{ac}\delta^4(x-z)\partial_\nu^a \partial_\tau^d D_{\nu\tau}^{cd}(y-u) + \delta^{ad}\delta^4(x-z)\partial_\mu^a \partial_\lambda^d D_{\mu\lambda}^{bd}(y-z) \\
+ \sigma^2 \{ \delta^{ab} \partial_\lambda^c \partial_\tau^d G_{\lambda\tau}^{abcd}(x, y, z, u) \delta^a y^c \partial_\tau^d G_{\mu\nu\lambda\tau}^{abcd}(x, y, z, u) \\
+ \partial_\lambda^a \partial_\tau^d G_{\mu\nu\lambda\tau}^{abcd}(x, y, z, u) \}. 
\]

(6.5)
It is noted that the four-point Green functions appearing in the above equations are unconnected. Their decompositions to connected Green functions are not difficult to be found by making use of the relation between the generating functionals $Z$ for the full Green functions and $W$ for the connected Green functions as written in Eq. (3.9). The result is

\[
G^{abcd}_{\mu\nu\lambda\tau}(x, y, z, u) = G^{abcd}_{\mu\nu\lambda\tau}(x, y, z, u) - D^{ab}_{\mu\nu}(x - y)D^{cd}_{\lambda\tau}(z - u) - D^{ac}_{\mu\lambda}(x - z)D^{bd}_{\nu\tau}(y - u) - D^{ad}_{\mu\tau}(x - u)D^{bc}_{\nu\lambda}(y - z)
\]

(6.6)

and

\[
G^{abcd}_{\lambda\tau}(x, y, z, u) = G^{abcd}_{\lambda\tau}(x, y, z, u) - \Delta^{ab}(x - y)D^{cd}_{\lambda\tau}(z - u).
\]

(6.7)

The first terms marked by the subscript "c" in Eqs. (6.6) and (6.7) are connected Green functions. When inserting Eqs. (6.6) and (6.7) into Eq. (6.5) and using the W-T identity in Eq. (4.5), one may find

\[
\partial^\mu y^\nu \partial^\lambda y^\sigma \partial^\mu y^\rho G^{abcd}_{\mu\nu\lambda\tau}(x, y, z, u) = \alpha_\sigma^2 \{\delta^\nu \partial^\lambda y^\rho G^{abcd}_{\nu\lambda}(u, x, y, z) + \partial^\rho y^\mu G^{abcd}_{\mu\nu\lambda\tau}(x, y, z, u)\},
\]

(6.8)

This is the W-T identity satisfied by the connected four-point Green functions. In the Landau gauge, we have

\[
\partial^\mu y^\nu \partial^\lambda y^\sigma \partial^\mu y^\rho G^{abcd}_{\mu\nu\lambda\tau}(x, y, z, u) = 0
\]

(6.9)

which shows the transversity of the Green function.

The W-T identity for the four-line proper $\rho$-meson vertex may be derived from Eq. (6.8) with the help of the following one-particle-irreducible decompositions of the connected Green functions which can easily be found by the standard procedure [9, 17],

\[
G^{abcd}_{\mu\nu\lambda\tau}(x_1, x_2, x_3, x_4) = \int \prod_{i=1}^{4} d^4y_i \Gamma^{a'b'c'd'}_{\mu\nu\lambda\tau}(y_1 - y_i) \Gamma^{b'd'}(y_1, y_2, y_3, y_4)
\]

(6.10)

\[
G^{abcd}_{\lambda\tau}(x_1, x_2, x_3, x_4) = \int \prod_{i=1}^{4} d^4y_i \Delta^{a'b'}(y_1 - y_i) \Gamma^{b'd'}_{\lambda\tau}(y_1, y_2, y_3, y_4) \Delta^{c'd'}(y_2 - x_2)
\]

(6.11)

where $\Gamma^{abcd}_{\mu\nu\lambda\tau}(x_1, x_2, x_3, x_4)$ is the four-line $\rho$-meson proper vertex and $\Gamma^{abcd}_{\lambda\tau}(x_1, x_2, x_3, x_4)$ is the four-line $\rho-$meson-ghost particle proper vertex. They are defined as [9, 16, 17]

\[
\Gamma^{abcd}_{\mu\nu\lambda\tau}(x_1, x_2, x_3, x_4) = i \frac{\delta^{\mu\nu}_{\lambda\tau}}{\delta C^{a'b'c'd'}(x_1)\delta A^{a'b'}(x_2)\delta A^{a'b'}(x_3)\delta A^{a'b'}(x_4)}|_{J=0},
\]

(6.12)

\[
\Gamma^{abcd}_{\lambda\tau}(x_1, x_2, x_3, x_4) = \frac{\delta^{\mu\nu}_{\lambda\tau}}{\delta C^{a'b'c'd'}(x_1)\delta C^{a'b'}(x_2)\delta A^{a'b'}(x_3)\delta A^{a'b'}(x_4)}|_{J=0}.
\]
When substituting Eqs. (6.10) and (6.11) into Eq. (6.8) and transforming Eq. (6.8) into the momentum space, one can find the following identity satisfied by the four-line $\rho$-meson proper vertex

\[
k^{\mu}k^{\nu}k^{\lambda}k^{\tau}\Lambda^{abcd}_{\mu\nu\lambda\tau}(k_1, k_2, k_3, k_4) = \Psi \left( \begin{array}{cccc} a & b & c & d \\ k_1 & k_2 & k_3 & k_4 \end{array} \right) + \Psi \left( \begin{array}{cccc} a & c & d & b \\ k_1 & k_3 & k_4 & k_2 \end{array} \right) + \Psi \left( \begin{array}{cccc} a & d & b & c \\ k_1 & k_4 & k_2 & k_3 \end{array} \right) \tag{6.13}
\]

where

\[
\Psi \left( \begin{array}{cccc} a & b & c & d \\ k_1 & k_2 & k_3 & k_4 \end{array} \right) = -ik^{\mu}k^{\nu}_{2}k^{\lambda}_{3}k^{\tau}_{4}\Delta^{abe}_{\mu\nu\sigma}(k_1, k_2, -(k_1 + k_2))D^{\sigma\rho}_{ef}(k_1 + k_2)k^{\lambda}_{3}k^{\tau}_{4}\Lambda^{fcd}_{\rho\lambda\tau}(-(k_3 + k_4), k_3, k_4)
\]

\[
+ \frac{1}{2} \chi(k^\mu_{1})\chi(k^\nu_{2})(ik^{\lambda}_{3}k^{\tau}_{4}\Lambda_{\rho\lambda\tau}(k_2, k_1, k_3, k_4))\tag{6.14}
\]

\[
-k^{\mu}_{1}\Lambda^{bed}_{\sigma\tau}(k_2, -(k_2 + k_4), k_3)\Delta^{ef}_{\sigma\tau}(k_2 + k_4)k^{\lambda}_{3}k^{\tau}_{4}\Lambda^{fac}_{\lambda\tau}(-(k_1 + k_3), k_1, k_3)
\]

\[
-k^{\mu}_{1}\Lambda^{dec}_{\tau\lambda}(k_2, -(k_2 + k_3), k_4)\Delta^{ef}_{\tau\lambda}(k_2 + k_3)k^{\lambda}_{3}k^{\tau}_{4}\Lambda^{fcd}_{\lambda\tau}(-(k_1 + k_4), k_1, k_4)]
\]

The second and third terms in Eq. (6.13) can be written out from Eq. (6.14) through cyclic permutations. In the above, we have defined

\[
\Gamma^{abcd}_{\mu\nu\lambda\tau}(k_1, k_2, k_3, k_4) = (2\pi)^4 \delta^{4}(\sum_{i=1}^{4} k_i)\Lambda^{abcd}_{\mu\nu\lambda\tau}(k_1, k_2, k_3, k_4), \tag{6.15}
\]

\[
\Gamma^{abcd}_{\lambda\tau}(k_1, k_2, k_3, k_4) = (2\pi)^4 \delta^{4}(\sum_{i=1}^{4} k_i)\Lambda^{abcd}_{\lambda\tau}(k_1, k_2, k_3, k_4).
\]

In the lowest order approximation, we have checked that except for the first term in Eq. (6.14) which was encountered in the massless theory, the remaining mass-dependent terms are cancelled out with the corresponding terms contained in the second and third terms in Eq. (6.13). Therefore, the identity in Eq. (6.13) leads to a result in the lowest order approximation which is consistent with the Feynman rule.

The renormalization of the four-line vertices is similar to that for the three-line ones. From the definitions given in Eqs. (6.12), (6.15) and (5.18), it is clearly seen that the four-line vertices should be renormalized in such a manner

\[
\Lambda^{abcd}_{\mu\nu\lambda\tau}(k_1, k_2, k_3, k_4) = Z_3^{-2}\Lambda^{abcd}_{\mu\nu\lambda\tau}(k_1, k_2, k_3, k_4),
\]

\[
\Lambda^{abcd}_{\lambda\tau}(k_1, k_2, k_3, k_4) = Z_3^{2}Z_3^{-2}\Lambda^{abcd}_{\lambda\tau}(k_1, k_2, k_3, k_4).
\]

On inserting these relations into Eqs. (6.13) and (6.14), one can obtain a renormalized identity similar to Eq. (5.20), that is

\[
k^{\mu}k^{\nu}k^{\lambda}k^{\tau}\tilde{\Lambda}^{abcd}_{\mu\nu\lambda\tau}(k_1, k_2, k_3, k_4) = \Psi \left( \begin{array}{cccc} a & b & c & d \\ k_1 & k_2 & k_3 & k_4 \end{array} \right) + \Psi \left( \begin{array}{cccc} a & c & d & b \\ k_1 & k_3 & k_4 & k_2 \end{array} \right) + \Psi \left( \begin{array}{cccc} a & d & b & c \\ k_1 & k_4 & k_2 & k_3 \end{array} \right) \tag{6.17}
\]

where

\[
\Psi \left( \begin{array}{cccc} a & b & c & d \\ k_1 & k_2 & k_3 & k_4 \end{array} \right) = -ik^{\mu}k^{\nu}_{2}k^{\lambda}_{3}k^{\tau}_{4}\tilde{\Lambda}^{abc}_{\mu\nu\sigma}(k_1, k_2, -(k_1 + k_2))D^{\sigma\rho}_{ef}(k_1 + k_2)k^{\lambda}_{3}k^{\tau}_{4}\tilde{\Lambda}^{fcd}_{\rho\lambda\tau}(-(k_3 + k_4), k_3, k_4)
\]

\[
+ \frac{1}{2} \chi(k^\mu_{1})\chi(k^\nu_{2})(ik^{\lambda}_{3}k^{\tau}_{4}\tilde{\Lambda}_{\rho\lambda\tau}(k_2, k_1, k_3, k_4))\tag{6.18}
\]

\[
-k^{\mu}_{1}\tilde{\Lambda}^{bed}_{\sigma\tau}(k_2, -(k_2 + k_4), k_3)\Delta^{ef}_{\sigma\tau}(k_2 + k_4)k^{\lambda}_{3}k^{\tau}_{4}\tilde{\Lambda}^{fac}_{\lambda\tau}(-(k_1 + k_3), k_1, k_3)
\]

\[
-k^{\mu}_{1}\tilde{\Lambda}^{dec}_{\tau\lambda}(k_2, -(k_2 + k_3), k_4)\Delta^{ef}_{\tau\lambda}(k_2 + k_3)k^{\lambda}_{3}k^{\tau}_{4}\tilde{\Lambda}^{fcd}_{\lambda\tau}(-(k_1 + k_4), k_1, k_4)].
\]

We can also define vertices \(\tilde{\Lambda}^{abcd}_{\mu\nu\lambda\tau}(k_1, k_2, k_3, k_4)\) and \(\tilde{\Lambda}^{abcd}_{\lambda\tau}(k_1, k_2, k_3, k_4)\) from the vertices \(\Lambda^{abcd}_{\mu\nu\lambda\tau}(k_1, k_2, k_3, k_4)\) and \(\Lambda^{abcd}_{\lambda\tau}(k_1, k_2, k_3, k_4)\) by taking out the coupling constant squared, respectively. The renormalization of these vertices are usually defined by [9, 17]
\[ \tilde{\Lambda}_{abcd}^{\mu \nu \lambda \tau}(k_1, k_2, k_3, k_4) = Z_4^{-1} \tilde{\Lambda}_{abcd}^{\mu \nu \lambda \tau}(k_1, k_2, k_3, k_4),\]
\[ \tilde{\Lambda}_{\lambda \tau}^{abcd}(k_1, k_2, k_3, k_4) = \tilde{Z}_4^{-1} \tilde{\Lambda}_{\lambda \tau}^{abcd}(k_1, k_2, k_3, k_4). \]

(6.19)

where \( Z_4 \) and \( \tilde{Z}_4 \) are the renormalization constants of the four-line \( \rho \)-meson vertex and the four-line \( \rho \)-meson-ghost particle one respectively. Obviously, the identity in Eqs. (6.13) and (6.14) remains formally unchanged if we replace all the vertices \( \Lambda_i \) in the identity with the ones \( \tilde{\Lambda}_i \). Substituting Eqs. (5.24), (6.19), (4.26) and (4.27) into such an identity, one may write a renormalized identity similar to Eq. (5.25), that is

\[ k_1^\alpha k_2^\beta k_3^\gamma k_4^\delta \tilde{\Lambda}_{R}^{abcd}(k_1, k_2, k_3, k_4) = \tilde{\Psi}_R \left( \frac{a}{k_1} \frac{b}{k_2} \frac{c}{k_3} \frac{d}{k_4} \right) = \tilde{\Psi}_R \left( \frac{a}{k_1} \frac{b}{k_2} \frac{c}{k_3} \frac{d}{k_4} \right) \]

(6.20)

where

\[ \tilde{\Psi}_R \left( \frac{a}{k_1} \frac{b}{k_2} \frac{c}{k_3} \frac{d}{k_4} \right) = \frac{Z_4}{Z_2} \{ -i k_1^\alpha k_2^\beta k_3^\gamma k_4^\delta \tilde{\Lambda}_{R}^{abcd}(k_1, k_2, k_3, k_4) \} \]

Multiplied by the both sides of Eqs. (6.20) and (6.21) by \( g_\text{Ref}^2 \), according to the relations given in Eqs. (5.26) and in the following

\[ \Lambda_{abcd}^{\mu \nu \lambda \tau}(k_1, k_2, k_3, k_4) = g_\text{Ref}^2 \tilde{\Lambda}_{R}^{abcd}(k_1, k_2, k_3, k_4), \]

(6.22)

we have an identity which is of the same form as the identity in Eqs. (6.20) and (6.21) except that the vertices \( \tilde{\Lambda}_i \) in Eqs. (6.20) and (6.21) are all replaced by the vertices \( \Lambda_i \). Comparing this identity with that written in Eqs. (6.17) and (6.18), one may find

\[ \frac{Z_3 Z_4}{Z_2^2} = 1, \quad \frac{Z_3 Z_4}{Z_1 Z_2} = 1, \quad \frac{Z_4 Z_2^2}{Z_3 Z_1} = 1, \quad \frac{Z_4 Z_2^2}{Z_3 Z_1} = 1 \]

(6.23)

which lead to

\[ \frac{Z_1}{Z_3} = \frac{Z_1}{Z_3}, \quad \frac{Z_1}{Z_3}, \quad \frac{Z_1}{Z_3}, \quad \frac{Z_1}{Z_3} = \frac{Z_1}{Z_3} \]

(6.24)

This just is the S-T identity analogous to that for QCD \[9, 16, 17, 19\].

VI. NUCLEON-\( \rho \)-MESON VERTEX AND NUCLEON PROPAGATOR

This section is used to derive the W-T identity for nucleon-\( \rho \)-meson vertex and discuss its renormalization. First we derive a W-T identity satisfied by the nucleon-\( \rho \)-meson three-point Green function. This identity can easily be derived by differentiating the W-T identity in Eq. (3.8) or (3.10) with respect to the sources \( \xi^b(z), \eta(y) \) and \( \Psi(x) \) and then setting all the sources to be zero. The result written in the operator form is as follows

\[ \partial_x^b G^a_\mu(x, y, z) = i g [G^b a_1(x, y, z) T^b - T^b G^b a_2(x, y, z)] \]

(7.1)

where

\[ G^a_\mu(x, y, z) = \langle 0^+ \mid \tilde{\psi}(x) \tilde{\psi}(y) \tilde{\Lambda}(z) \mid 0^- \rangle \]

(7.2)
is the nucleon-ρ-meson three-point Green function,

\[ G^\rho_{1}(x, y, z) = \left\langle 0^+ | \tilde{\psi}(x) \tilde{\psi}(y) \bar{C}^b(y) \tilde{C}^a(z) | 0^- \right\rangle \]  

(7.3)

and

\[ G^\rho_{2}(x, y, z) = \left\langle 0^+ | \tilde{\psi}(x) \tilde{\psi}(y) \bar{C}^b(x) \tilde{C}^a(z) | 0^- \right\rangle \]  

(7.4)

are the nucleon-ghost particle mixed Green functions. The Green functions in Eqs. (7.3) and (7.4) are connected because a nucleon field and a ghost field are of a common coordinate.

The W-T identity for nucleon-ρ-meson vertex can be derived from Eq. (7.1) with the help of one-particle irreducible decompositions of the Green functions shown in Eqs. (7.2)-(7.4). The decompositions can easily be obtained by the standard procedure \cite{9, 17}. The results are given in the following.

\[ G^\rho_{\mu}(x, y, z) = \int d^4x' d^4y' d^4z' iS_F(x - x') \Gamma^{\nu \mu}(x', y', z') iS_F(y' - y) iD^{\nu \rho}_{\nu \mu}(z' - z) \]  

(7.5)

where \( D^{\nu \rho}_{\nu \mu}(z' - z) \) is the ρ-meson propagator defined in Eq. (4.6),

\[ iS_F(x - x') = \left\langle 0^+ | \tilde{\psi}(x) \tilde{\psi}(x') | 0^- \right\rangle \]  

(7.6)

is the nucleon propagator and

\[ \Gamma^{\nu \mu}(x', y', z') = \frac{\delta^3 \Gamma}{i \delta \tilde{\psi}(x') \delta \tilde{\psi}(y') \delta A^\rho_\nu(z')} |_{J=0} \]  

(7.7)

is the nucleon-ρ-meson proper vertex.

\[ G^{ba}_{1}(x, y, z) = \int d^4x' d^4z' S_F(x - x') \gamma^{bc}_{1}(x', y, z') \Delta^{ca}_{0}(z' - z) \]  

(7.8)

where \( \Delta^{ca}_{0}(z' - z) \) is the ghost particle propagator defined in Eq. (4.7) and

\[ \gamma^{bc}_{1}(x', y, z') = \int d^4u d^4v \Delta^{bd}_{0}(y - u) \Gamma^{cd}_{0}(x', v, u, z') S_F(v - y) \]  

(7.9)

in which

\[ \Gamma^{cd}_{0}(x', v, u, z') = i \frac{\delta^4 \Gamma}{\delta \tilde{\psi}(x') \delta \tilde{\psi}(v) \delta \bar{C}^c(u) \delta C^d(z')} |_{J=0} \]  

(7.10)

is the nucleon-ghost particle vertex. Similarly,

\[ G^{ba}_{2}(x, y, z) = \int d^4y' d^4z' \gamma^{bc}_{2}(x, y', z') S_F(y' - y) \Delta^{ca}_{0}(z' - z) \]  

(7.11)

where

\[ \gamma^{bc}_{2}(x, y', z') = \int d^4u d^4v S_F(x - u) \Delta^{bd}_{0}(x - v) \Gamma^{dc}_{0}(u, y', v, z'). \]  

(7.12)

On substituting Eqs. (7.5), (7.8) and (7.11) into Eq. (7.1) and then transforming Eq. (7.1) into the momentum space, we have

\[ S_F(p) \Gamma^{ba}_{0}(p, q, k) S_F(q) k^\mu D^{\mu \rho}_{ab}(k) = -i \delta \left[ S_F(p) \gamma^{\alpha}_{1}(p, q, k) - \gamma^{\alpha}_{2}(p, q, k) S_F(q) \right] \Delta^{ab}(k) \]  

(7.13)

where we have defined

\[ \gamma^{\alpha}_{0}(p, q, k) = \gamma^{ab}_{0}(p, q, k) T^b, \]  

\[ \gamma^{\alpha}_{2}(p, q, k) = T^b \gamma^{ba}_{2}(p, q, k). \]  

(7.14)
where \( \Lambda^{a\mu}(p, q, k) \) and \( \tilde{\gamma}^{a}_i(p, q, k) \) are the new vertex functions in which \( k = q - p \). Noticing the above relations and the expressions of \( \rho \)-meson and ghost particle propagators as given in Eqs. (4.11) and (4.22), the W-T identity in Eq. (7.13) can be rewritten via the functions \( \Lambda^{a\mu}(p, q, k) \) and \( \tilde{\gamma}^{a}_i(p, q, k) \) in the form

\[
k_{\mu}\Lambda^{a\mu}(p, q, k) = ig\chi(k^2)[S_F^{-1}(p)\tilde{\gamma}^{a}_2(p, q, k) - \tilde{\gamma}^{a}_1(p, q, k)S_F^{-1}(q)]
\]  

(7.16)

where \( \chi(k^2) \) was defined in Eq. (5.13).

Let us turn to discuss the renormalized form of the above W-T identity. It is well-known that the nucleon propagator can be expressed in the form

\[
S_F(p) = \frac{1}{p - M - \Sigma(p) + i\varepsilon}
\]  

(7.17)

where \( p = \gamma^\mu p_\mu \) and \( \Sigma(p) \) denotes the nucleon self-energy. The above expression can easily be derived from the Dyson equation [24]. Usually, the nucleon propagator is renormalized in such a fashion

\[
S_F(p) = Z_2 S_F^R(p)
\]  

(7.18)

which implies

\[
\psi(x) = \sqrt{Z_2}\psi_R(x), \quad \overline{\psi}(x) = \sqrt{Z_2}\overline{\psi}_R(x).
\]  

(7.19)

From the relations in Eqs. (7.19) and (5.18), it is clearly seen that the vertex defined in Eq. (7.7) is renormalized as

\[
\Gamma^{a\mu}(x, y, z) = Z_2^{-1}Z_3^{-1}\gamma^\mu_R(x, y, z)
\]  

(7.20)

which leads to

\[
\Lambda^{a\mu}(p, q, k) = Z_2^{-1}Z_3^{-1}\gamma^\mu_R(p, q, k).
\]  

(7.21)

While, for the functions \( \tilde{\gamma}^{a}_i(p, q, k) \), according to the relations in Eqs. (5.18) and (7.19), they seem to be renormalization-invariant. But, these functions actually are divergent in the perturbative calculation. We assume that they are renormalized in such a manner

\[
\tilde{\gamma}^{a}_i(p, q, k) = Z_\gamma^{-1}\tilde{\gamma}^{a}_iR(p, q, k).
\]  

(7.22)

where \( Z_\gamma \) is the renormalization constant of the functions \( \tilde{\gamma}^{a}_i(p, q, k) \). Based on the relations in Eqs. (5.22), (7.18), (7.21), and (7.22), Eq. (7.16) can be represented in terms of the renormalized quantities

\[
k_{\mu}\Lambda^{a\mu}_R(p, q, k) = ig\chi(k^2)[S_F^{R^{-1}}(p)\tilde{\gamma}^{a}_2(p, q, k) - \tilde{\gamma}^{a}_1(p, q, k)S_F^{R^{-1}}(q)]
\]  

(7.23)

where \( g_R \) is the renormalized coupling constant defined by

\[
g_R = \tilde{Z}_3\tilde{Z}_3^\dagger Z_\gamma^{-1}g
\]  

(7.24)

It is well-known that

\[
g_R = \tilde{Z}_3\tilde{Z}_3^\dagger Z_1^{-1}g
\]  

(7.25)

where \( \tilde{Z}_1 \) is the ghost vertex renormalization constant as defined in Eq. (5.24). The relation in Eq. (7.25) ordinarily is determined from the renormalization of S-matrix elements. In comparison of Eq. (7.24) with Eq. (7.25), we see

\[
\tilde{Z}_\gamma = \tilde{Z}_1
\]  

(7.26)
which means that the functions $\tilde{\gamma}_i^a(p,q,k)$ are renormalized in the same way as for the ghost vertex (the three-line $\rho-$meson-ghost particle vertex). This result arises from the fact that the functions $\tilde{\gamma}_i^a(p,q,k)$ contains a ghost vertex as easily seen from perturbative calculations.

In the conventional discussion of the vertex renormalization, one considers such a vertex denoted by $\tilde{\Lambda}^{a\mu}(p,q,k)$ that it is defined from $\Lambda^{a\mu}(p,q,k)$ by taking out a coupling constant. Obviously, the W-T identity obeyed by the $\tilde{\Lambda}^{a\mu}(p,q,k)$ can be written out from (7.16) by taking away the coupling constant on the RHS of Eq. (7.16), that is

$$k_\mu \tilde{\Lambda}^{a\mu}(p,q,k) = i\chi(k^2)[S_F^{-1}(p)\tilde{\gamma}_2^a(p,q,k) - \tilde{\gamma}_1^a(p,q,k)S_F^{-1}(q)].$$  (7.27)

The renormalization of the vertex $\tilde{\Lambda}^{a\mu}(p,q,k)$ usually is defined by

$$\tilde{\Lambda}^{a\mu}(p,q,k) = Z_F^{-1}\tilde{\Lambda}_R^{a\mu}(p,q,k)$$  (7.28)

where $Z_F$ is the nucleon-$\rho$-meson vertex renormalization constant. When Eqs. (5.22), (7.18), (7.22) and (7.28) are inserted into Eq. (7.27) and then multiplying the both sides of Eq. (7.27) with a renormalized coupling constant, we arrive at

$$k_\mu \Lambda_R^{a\mu}(p,q,k) = iZ_F\tilde{Z}_2Z_2^{-1}\tilde{Z}^{-1}_1g_{\mu\nu}(k^2)[S_F^{-1}(p)\tilde{\gamma}_2^a(p,q,k) - \tilde{\gamma}_1^a(p,q,k)S_F^{-1}(q)]$$  (7.29)

where

$$\Lambda_R^{a\mu}(p,q,k) = g_{\mu\nu}\Lambda_R^{a\nu}(p,q,k).$$  (7.30)

In comparison of Eq. (7.29) with Eq. (7.23) and considering the equality in Eq. (7.26), we find, the following identity must hold

$$\frac{Z_F}{Z_2} = \frac{\tilde{Z}_1}{\tilde{Z}_3}.$$  (7.31)

Combining the relations in Eqs. (5.28), (6.24) and (7.31), we have

$$\frac{Z_F}{Z_2} = \frac{\tilde{Z}_1}{\tilde{Z}_3} = \frac{Z_1}{Z_3} = \frac{Z_1}{Z_1}.$$  (7.32)

This just is the well-known S-T identity which is formally identical to that given in QCD [9, 17, 19].

VII. PION-$\rho-$MESON THREE-LINE VERTEX AND PION PROPAGATOR

In this section, we are devoted to deriving the W-T identity obeyed by the pion-$\rho-$meson three-line vertex and discuss its renormalization. Taking derivative of Eq. (3.10) with respect to the source $\xi^c(z)$ and then setting $\bar{\eta} = \eta = \xi^a = 0$, we have

$$\frac{1}{\alpha}\frac{\partial^\mu}{\partial J^{\mu}(z)} - \int d^4x[K^{b}(x)\frac{\delta^2W}{\delta \chi^b(x)\delta \xi^c(z)} + J^{a\mu}(x)\frac{\delta^2W}{\delta u^{a\mu}(x)\delta \xi^c(z)}] = 0$$  (8.1)

Furthermore, differentiating the above identity with respect to $K^{b}(y)$ and $K^{a}(x)$ and then turning off all the sources, one may obtain an identity satisfied by the $\pi - \rho$ three-point Green function. Witten in the operator formulation, it is represented as

$$\partial^\mu \tilde{G}^{abc}_\mu(x,y,z) = -\alpha g^{bde}\tilde{G}^{ade\mu}(x,y,z)$$  (8.2)

where

$$\tilde{G}^{abc}_\mu(x,y,z) = \langle 0^+ \mid T[\tilde{\pi}^a(x)\tilde{\pi}^b(y)\tilde{A}_\mu^c(z)] \mid 0^- \rangle$$  (8.3)

is the $\pi - \rho$ three-point Green function and

$$\tilde{G}^{ade\mu}(x,y,z) = \langle 0^+ \mid T[\tilde{\pi}^a(x)\tilde{\pi}^d(y)\tilde{C}_\mu^c(z)] \mid 0^- \rangle$$  (8.4)
is the pion-ghost particle three-point Green function with a pion operator and a ghost particle operator being put on the same position. Since the identity in Eq. (8.2) is derived from the W-T identity in Eq. (3.10), the Green functions in Eq. (8.2) are connected. In the following, we will start from the identity in Eq. (8.2) to derive the identity satisfied by the \( \pi - \rho \) three-line vertex without recourse to the ghost equation because the ghost equation will not simplify our discussion. For this purpose, we need the following one-particle irreducible decompositions:

\[
\tilde{\Gamma}^{abc}_{(a)}(x, y, z) = \int d^4 x' d^4 y' d^4 z' i \Delta^{a'}_{\pi}(x - x') \Gamma^{a'b'c'}(x', y', z') i \Delta^{b'}_{\pi}(y' - y) i D_{\mu}(z' - z) \tag{8.5}
\]

where

\[
i \Delta^{a'}_{\pi}(x - x') = \left\langle 0^+ \left| T[\tilde{\pi}^{a'}(x)\tilde{\pi}^{a'}(x')] \right| 0^- \right\rangle \tag{8.6}
\]

is the pion propagator and

\[
\tilde{\Gamma}^{a'b'c'}_{\mu}(x', y', z') = i \frac{\delta^3 \Gamma}{\delta \pi^{a'}(x') \delta \pi^{b'}(y') \delta A^{c'}(z')} \mid_{J=0} \tag{8.7}
\]

denotes the \( \pi - \rho \) three-line proper vertex. And

\[
\tilde{\Gamma}^{abc}_{(a)}(x, y; z) = \int d^4 x' d^4 y' d^4 z' \Delta^{a'}_{\pi}(x - x') \Delta^{c'}_{\rho}(z - z') \gamma^{a'b'c'}(x', y', z) \tag{8.8}
\]

where \( \Delta^{c'}_{\rho}(z - z') \) is the ghost particle propagator defined in Eq. (4.7) and

\[
\gamma^{a'b'c'}(x', y', z) = \int d^4 y' d^4 u \Delta^{dd'}_{\pi}(y - y') \Delta^{ee'}_{\rho}(y - u) \tilde{\Gamma}^{a'd'e'}_{\mu}(x', y', u, z') \tag{8.9}
\]

in which the propagators \( \Delta^{dd'}_{\pi}(y - y') \) and \( \Delta^{ee'}_{\rho}(y - u) \) have a common coordinate \( y \) and

\[
\tilde{\Gamma}^{a'd'e'}_{\mu}(x', y', u, z') = \frac{\delta^4 \Gamma}{i \delta \pi^a(x') \delta \pi^{d'}(y') \delta A^{e'}(u) \delta A^{e'}(z')} \mid_{J=0} \tag{8.10}
\]

is the pion-ghost particle four-line vertex.

On substituting Eqs. (8.5) and (8.8) into Eq. (8.2) and then transforming the equation thus obtained into the momentum space, it is easy to get

\[
k^{\mu}_{\mu} \tilde{\Gamma}^{abc}_{\mu}(k_1, k_2, k_3) = -i g^{bde} \Delta^{c}_{\pi}(k_2)^{-1} \chi(k_3) \gamma^{acde}(k_1, k_2, k_3) \tag{8.11}
\]

where \( \chi(k_3) \) was defined in Eq. (5.13),

\[
\gamma^{acde}(k_1, k_2, k_3) = \int d^4 l \Delta^{l}_{\pi}(l) \Delta(k_2 - l) \tilde{\Gamma}^{acde}(k_1, l; k_2 - l, k_3) \tag{8.12}
\]

and

\[
\Delta^{ab}_{\pi}(k) = \delta^{ab} \Delta_{\pi}(k) \tag{8.13}
\]

with

\[
\Delta_{\pi}(k) = \frac{1}{k^2 - m^2_{\pi} - \Omega_{\pi}(k) + i\varepsilon} \tag{8.14}
\]

is the pion propagator given in the momentum space. In the above, the pion self energy is defined by \( -i \Omega^{ab}_{\pi}(k) = -i \delta^{ab} \Omega_{\pi}(k) \). With the definitions:

\[
\Gamma^{abc}_{\mu}(k_1, k_2, k_3) = (2\pi)^4 \delta^4(k_1 + k_2 + k_3) \tilde{\Lambda}^{abc}_{\mu}(k, k_2, k_3),
\gamma^{acde}(k_1, k_2, k_3) = (2\pi)^4 \delta^4(k_1 + k_2 + k_3) \gamma^{acde}(k_1, k_2, k_3), \tag{8.15}
\]

the identity in Eq. (8.11) can be written in the form

\[
k^{\mu}_{\mu} \tilde{\Lambda}^{abc}_{\mu}(k_1, k_2, k_3) = -i g^{bde} \Delta^{c}_{\pi}(k_2)^{-1} \chi(k_3) \gamma^{acde}(k_1, k_2, k_3) \tag{8.16}
\]
Let us proceed to discuss the renormalization of the pion propagator and the \( \pi - \rho \) three-line vertex. In accordance with the Lorentz-covariance, the self-energy can be written in the form
\[
\Omega_\pi(k) = k^2\omega_1(k^2) + m_\pi^2\omega_2(k^2).
\] (8.17)
The divergence in the function \( \omega_i(k^2) \) \((i = 1, 2)\) can be subtracted at the renormalization point \( \mu \), \( \omega_i(\mu^2) = \omega_i(k^2) + \omega_i^\varepsilon(k^2) \) where \( \omega_i(\mu^2) \) and \( \omega_i^\varepsilon(k^2) \) are the divergent and finite parts of the function \( \omega_i(k^2) \) respectively. Defining the renormalization constant \( \tilde{Z}_3 \) of the pion propagator as
\[
\tilde{Z}_3^{-1} = 1 - \omega_1(\mu^2)
\] (8.18)
the pion propagator will be renormalized as
\[
\Delta_\pi(k) = \tilde{Z}_3\Delta_R^\pi(k)
\] (8.19)
where
\[
\Delta_R^\pi(k) = \frac{1}{k^2 - m_\pi^2 - \Omega_\pi^R(k) + i\varepsilon}
\] (8.20)
in which \( m_\pi^R = Z_m^{-1}m_\pi \) is the renormalized pion mass with
\[
Z_m = \{\tilde{Z}_3[1 + \omega_2(\mu^2)]\}^{-1/2}
\] (8.21)
being the pion mass renormalization constant and \( \Omega_\pi^R(k) = \tilde{Z}_3[k^2\omega^\varepsilon_1(k^2) + m_\pi^2\omega_2(k^2)] \) represents the finite correction to the propagator.

From the definition in Eq. (8.6) and the relation in Eq. (8.19), we see
\[
\pi^a(x) = \tilde{Z}_3^{1/2}\pi_R^a(x)
\] (8.22)
where \( \pi_R^a(x) \) stands for the renormalized pion field function. According to the definitions in Eq. (8.7) and (8.15) and the relations in Eqs. (5.18) and (8.22), it is clear to see that the vertex \( \Lambda_{abc}^{\mu}(k_1, k_2, k_3) \) is renormalized as
\[
\tilde{\Lambda}_{abc}^{\mu}(k_1, k_2, k_3) = \tilde{Z}_3^{-1}\tilde{Z}_3^{-1/2}\Lambda_{abc}^{\mu R}(k_1, k_2, k_3). \] (8.23)

Analogous to the analysis mentioned in Eqs. (7.22)-(7.26), it can be proved that the vertex \( \hat{\gamma}^{acde}(k_1, k_2, k_3) \) is renormalized in such a fashion
\[
\hat{\gamma}^{acde}(k_1, k_2, k_3) = \tilde{Z}_1^{-1}\hat{\gamma}^{acde}(k_1, k_2, k_3). \] (8.24)
This means that the vertex \( \hat{\gamma}^{acde}(k_1, k_2, k_3) \), as the vertex \( \hat{\gamma}^{acde}(k_1, k_2, k_3) \) defined in Eq. (7.15), is renormalized in the same way as the ghost vertex \( \Lambda_{acde}^{\mu}(k_1, k_2, k_3) \) because the divergence occurring in the vertex \( \hat{\gamma}^{acde}(k_1, k_2, k_3) \), as the vertex \( \hat{\gamma}^{acde}(k_1, k_2, k_3) \), comes from the \( p \)-meson-ghost particle vertex which is included in the vertex \( \hat{\gamma}^{acde}(k_1, k_2, k_3) \). This fact can also be seen from a perturbative analysis of the vertex. When Eqs. (8.19), (8.23), (8.24) and (5.22) are inserted into Eq. (8.16), one may obtain a renormalized version of the identity in Eq. (8.16)
\[
k_3^\mu\tilde{\Lambda}_{abc}^{\mu R}(k_2, k_3) = -i\varepsilon^{bde}\frac{g_R\Delta_R^\mu(k_2)}{k_3^\mu}\hat{\gamma}_{acde}(k_1, k_2, k_3). \] (8.25)
According to the usual discussion of vertex renormalization, we introduce a vertex denoted by \( \hat{\Lambda}_{abc}^{\mu}(k_1, k_2, k_3) \) which is defined from \( \tilde{\Lambda}_{abc}^{\mu R}(k_1, k_2, k_3) \) by extracting a coupling constant. This vertex is renormalized as
\[
\hat{\Lambda}_{abc}^{\mu}(k_1, k_2, k_3) = \tilde{Z}_1^{-1}\hat{\Lambda}_{abc}^{\mu R}(k_1, k_2, k_3)
\] (8.26)
where \( \tilde{Z}_1 \) is the renormalization constant of the \( \pi - \rho \) vertex. Clearly, the W-T identity for the vertex \( \hat{\Lambda}_{abc}^{\mu}(k_1, k_2, k_3) \) can be written from Eq. (8.16) by taking out a coupling constant on the right hand side of Eq. (8.16),
\[
k_3^\mu\tilde{\Lambda}_{abc}^{\mu R}(k_1, k_2, k_3) = -i\varepsilon^{bde}\Delta_\pi(k_3)\hat{\gamma}_{acde}(k_1, k_2, k_3) \] (8.27)
Upon substituting Eqs. (8.19), (8.24), (8.26) and (5.22) into Eq. (8.27) and then multiplying Eq. (8.27) by a renormalized coupling constant, noticing \( \tilde{\Lambda}_{abc}^{\mu R}(k_1, k_2, k_3) = g_R\hat{\Lambda}_{abc}^{\mu R}(k_1, k_2, k_3) \), we have
\[ k_3^{\mu} \tilde{A}_{R, \mu}(k_2, k_2, k_3) = -A_3 \tilde{Z}_1 e^{bde} g R(t_2) \Delta R(k_2) \chi R(k_3) \tilde{R}_{R}^{a, \alpha}(k_1, k_2, k_3). \]  

(8.28)

In comparison of Eq. (8.28) with Eq. (8.25), we see, it must be

\[ \frac{\tilde{Z}_3 \tilde{Z}_1}{Z_1 Z_3} = 1 \]  

(8.29)

which leads to the S-T identity such that

\[ \frac{\tilde{Z}_4 \tilde{Z}_1}{Z_1 Z_3} = \frac{\tilde{Z}_1}{Z_3}. \]  

(8.30)

VIII. PION-\( \rho \)-MESON FOUR-LINE VERTEX

In this section, we plan to derive the W-T identity satisfied by the \( \pi - \rho \) four-line proper vertex. For this purpose, we first derive the W-T identity obeyed by the \( \pi - \rho \) four-point Green function by starting the identity shown in Eq. (8.1), then differentiating Eq. (8.1) with respect to the sources \( K^a(x), K^b(y) \) and \( J^{\alpha \nu}(z) \) and finally setting all the sources to be zero, we obtain an identity which, written in the operator form, is

\[ \frac{\partial^\nu}{\partial u} \left[ 0^+ \left| T[\hat{\pi}^a(x)\hat{\pi}^b(y)\tilde{A}_\mu^c(z)\tilde{A}^d_\mu(u)] \right| 0^- \right] = -\sigma^2 \left[ 0^+ \left| T[\hat{\pi}^a(x)\hat{\pi}^b(y)\tilde{A}^c_\mu(z)\tilde{C}^d_\mu(u)] \right| 0^- \right]. \]  

(9.1)

This identity may be simplified with the help of a ghost equation which can be derived from the ghost equation in Eq. (3.18). Changing the variable \( x \) and the index \( a \) in Eq. (3.18) to \( z \) and \( c \), then differentiating Eq. (3.18) successively with respect to the sources \( \xi^d(u), K^b(y) \) and \( K^a(x) \) and finally setting all the sources to vanish, one can obtain the ghost equation. Written in the operator form, it reads

\[ \frac{\partial^\mu}{\partial z} \left[ 0^+ \left| T^*[\hat{\pi}^a(x)\hat{\pi}^b(y)\Delta \tilde{A}_\mu^c(z)\tilde{C}^d_\mu(u)] \right| 0^- \right] = -\sigma^2 \left[ 0^+ \left| T^*[\hat{\pi}^a(x)\hat{\pi}^b(y)\tilde{C}^c_\mu(z)\tilde{C}^d_\mu(u)] \right| 0^- \right]. \]  

(9.2)

Differentiating Eq. (9.1) with respect to \( z \), applying Eq. (9.2) and noticing the definition of \( \Delta \pi^b(y) \) in Eq. (3.3), we arrive at

\[ \frac{\partial^\mu}{\partial z} \frac{\partial^\nu}{\partial x} \tilde{C}^{abcd}(x_1, x_2, x_3, x_4) = -\alpha g^{bij} \partial^\mu x_3 \tilde{C}^{ajdc}(x_1, x_2, x_3, x_4) + \sigma^2 \tilde{G}^{abcd}(x_1, x_2, x_3, x_4). \]  

(9.3)

where we have changed the position variables for later convenience and the Green functions are defined by

\[ \tilde{G}^{abcd}(x_1, x_2, x_3, x_4) = \left[ 0^+ \left| T[\hat{\pi}^a(x_1)\hat{\pi}^b(x_2)\tilde{A}^c_\mu(x_3)\tilde{A}^d_\mu(x_4)] \right| 0^- \right], \]  

(9.4)

\[ \tilde{G}^{ajdc}(x_1, x_2, x_3, x_4) = \left[ 0^+ \left| T[\hat{\pi}^a(x_1)\hat{\pi}^b(x_2)\tilde{A}^c_\mu(x_3)\tilde{C}^d_\mu(x_4)] \right| 0^- \right], \]  

(9.5)

and \( \tilde{C}^{abcd}(x_1, x_2, x_3, x_4) \) was defined in Eq. (8.4). These Green functions are all connected.

The W-T identity for \( \pi - \rho \) four-line vertex can be derived from Eq. (9.3) with the aid of one-particle irreducible decompositions of the Green functions which can easily be found by the standard procedure and are shown below. For the Green function \( \tilde{G}^{abcd}(x_1, x_2, x_3, x_4) \), we have the one-particle irreducible decompositions as follows

\[ \tilde{G}^{abcd}(x_1, x_2, x_3, x_4) = \sum_{\text{one-particle}} \tilde{G}^{abcd}(x_1, x_2, x_3, x_4). \]  

(9.10)
where
\[
\Gamma_{\mu
u}^{abcd}(y_1, y_2, y_3, y_4) = i \delta^{4}\Gamma_{\mu
u}^{abcd}(y_1, y_2, y_3, y_4) \big|_{J=0} \tag{9.11}
\]
is the \(\pi - \rho\) four-line proper vertex, while the \(\pi - \rho\) three-line vertex and the \(\rho\)-meson three-line vertex were already defined in Eqs. (8.7) and (5.9) respectively. For the Green function \(G^{a_1j_{d,c}}_{\mu}(x_1, x_2, x_3, x_4)\), its irreducible decomposition is
\[
G^{a_1j_{d,c}}_{\mu}(x_1, x_2, x_3, x_4) = -i \int \prod_{i=1}^{3} dy_i \Delta_{\pi}(x_1 - y_1) \Delta_{\pi}(x_2 - y_2) D_{\mu\nu}(x_3 - y_3) \sum_{\alpha=1}^{5} \gamma_{a_1j_{d,c}}^{\alpha}\gamma_{a_1j_{d,c}}^{\alpha}(x_2, y_1, y_2, y_3) \tag{9.12}
\]
where
\[
\gamma_{a_1j_{d,c}}^{\alpha}\gamma_{a_1j_{d,c}}^{\alpha}(x_2, y_1, y_2, y_3) = \int d^4z_1d^4z_2\Delta_{\pi}(x_2 - z_1)\Delta_{\pi}(x_2 - z_2)\Gamma_{\mu
u}(y_1, y_2, y_3, z_1, z_2) \tag{9.13}
\]
and
\[
\Gamma_{\mu
u}(y_1, y_2, y_3, z_1, z_2) = \int \frac{\delta^{4}\Gamma}{i\delta\pi(y_1)\delta\pi(y_2)\delta C^\mu(y_3)\delta C^\nu(y_4) \delta A_{\gamma}^{\nu}(z_2)} \big|_{J=0} \tag{9.18}
\]
which contains a \(\rho\)-meson-ghost particle vertex in it, and the three-line \(\pi - \rho\) vertex, the three-line \(\rho\)-meson-ghost particle vertex and the four-line pion-ghost particle vertex were respectively defined Eqs. (8.7), (8.10) and (8.10). The typical feature of the functions \(\gamma_{a_1j_{d,c}}^{\alpha}\gamma_{a_1j_{d,c}}^{\alpha}(x_2, y_1, y_2, y_3)\) is that there are a pion propagator and a ghost particle propagator in the functions which have a common coordinate \(x_2\). As for the Green function \(G^{abcd}(x_1, x_2, x_3, x_4)\), its one-particle irreducible decomposition can be found to be
\[
G^{abcd}(x_1, x_2, x_3, x_4) = \int \prod_{i=1}^{3} dy_i \Delta_{\pi}(x_1 - y_1) \Delta_{\pi}(x_4 - y_2) \Delta_{\pi}(x_3 - y_3) \Delta_{\pi}(x_4 - y_4) \Gamma^{abcd}(y_1, y_2, y_3, y_4) \tag{9.19}
\]
When we define
\[
\Gamma^{abcd}_{\mu
u}(k_1, k_2, k_3, k_4) = (2\pi)^4\delta^4(\sum_{i=1}^{4} k_i)\Lambda^{abcd}_{\mu
u}(k_1, k_2, k_3, k_4),
\]
\[
\tilde{\Gamma}^{abc}(k_1, k_2, k_3) = (2\pi)^4\delta^4(\sum_{i=1}^{3} k_i)\Lambda^{abc}_{\mu}(k_1, k_2, k_3),
\]
\[
\gamma^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4) = (2\pi)^4\delta^4(\sum_{i=1}^{4} k_i)\tilde{\gamma}^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4),
\]
\[
\tilde{\Gamma}^{abcd}(k_1, k_2, k_3, k_4) = (2\pi)^4\delta^4(\sum_{i=1}^{4} k_i)\tilde{\Lambda}^{abcd}(k_1, k_2, k_3, k_4),
\]
the identity in Eq. (9.20) can be written as
\[
\begin{align*}
&-k^\mu_1 k^\nu_1 \tilde{\Lambda}^{abcd}_{\mu\nu}(k_1, k_2, k_3, k_4) \\
&= -k^\mu_1 \tilde{\Lambda}^{abc}_{\mu}(k_2, k_3) \Delta_\pi(k_1 + k_4) k^\nu_1 \tilde{\Lambda}^{ed}_{\nu}(k_1, k_4) \\
&-k^\mu_1 \Lambda^{abc}_{\mu}(k_1, k_3) \Delta_\pi(k_1 + k_4) k^\nu_1 \tilde{\Lambda}^{ed}_{\nu}(k_2, k_4) \\
&-\tilde{\Lambda}^{abcd}_{\mu}(k_1, k_3) D^{\mu\nu}_{\tau}(k_1 + k_2) k^\nu_1 k^\rho_1 \tilde{\Lambda}^{ed}_{\rho\tau}(k_3, k_4) \\
&-g_\pi^{bij} \Delta_\pi^{-1}(k_2) \chi(k_4) \sum_{i=1}^{5} k^\mu_3 \tilde{\gamma}^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4) \\
&-\frac{g_\pi^2}{\alpha_R} \chi(k_3) \chi(k_4) \tilde{\Lambda}^{abcd}(k_1, k_2, k_3, k_4).
\end{align*}
\] (9.22)

We are now in a position to discuss the renormalized version of the above identity. According to the definitions in Eqs. (9.11) and (8.10) and the relations in Eqs. (5.18) and (8.22), it is easy to see
\[
\begin{align*}
\tilde{\Lambda}^{abcd}_{\mu\nu}(k_1, k_2, k_3, k_4) &= Z_3^{-1} Z_3^{-1} \tilde{\Lambda}^{abcd}_{\mu\nu}(k_1, k_2, k_3, k_4) \\
\tilde{\Lambda}^{abcd}_{\mu}(k_1, k_2, k_3, k_4) &= Z_3^{-1} Z_3^{-1} \tilde{\Lambda}^{abcd}_{\mu}(k_1, k_2, k_3, k_4) \\
\tilde{\gamma}^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4) &= Z_3^{-1/2} Z_3^{-1/2} \tilde{\gamma}^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4). 
\end{align*}
\] (9.23)

In the last equality, the factor $Z_3^{-1/2}$ is given by applying the relations in Eq. (5.18) and (8.22) to all the propagators and vertices contained in the functions $\gamma^{aijdc}_{\alpha\mu}(x_2, y_2, y_3)$ shown in Eqs. (9.13)-(9.17), while, the factor $Z_3^{-1}$ arises from the consideration that the functions $\tilde{\gamma}^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4)$, as the function $\gamma^{acde}(k_1, k_2, k_3, k_4)$, includes the contribution given by the ghost vertex which is contained in functions $\gamma^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4)$ (more explanations will be given soon later). Substituting Eqs. (9.23), (5.19), (8.22), (8.19), (8.23) and (4.27) into Eq. (9.22), we obtain the renormalized version of the identity
\[
\begin{align*}
&-k^\mu_1 \tilde{\Lambda}^{abcd}_{\mu}(k_1, k_2, k_3, k_4) \\
&= -k^\mu_1 \tilde{\Lambda}^{abc}_{\mu}(k_2, k_3) \Delta_\pi(k_1 + k_4) k^\nu_1 \tilde{\Lambda}^{ed}_{\nu}(k_1, k_4) \\
&-k^\mu_1 \Lambda^{abc}_{\mu}(k_1, k_3) \Delta_\pi(k_1 + k_4) k^\nu_1 \tilde{\Lambda}^{ed}_{\nu}(k_2, k_4) \\
&-\tilde{\Lambda}^{abcd}_{\mu}(k_1, k_3) D^{\mu\nu}_{\tau}(k_1 + k_2) k^\nu_1 k^\rho_1 \tilde{\Lambda}^{ed}_{\rho\tau}(k_3, k_4) \\
&-g_\pi^{bij} \Delta_\pi^{-1}(k_2) \chi_R(k_4) \sum_{i=1}^{5} k^\mu_3 \tilde{\gamma}^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4) \\
&-\frac{g_\pi^2}{\alpha_R} \chi(k_3) \chi(k_4) \tilde{\Lambda}^{abcd}(k_1, k_2, k_3, k_4).
\end{align*}
\] (9.24)

where $g_R, \sigma_R$ and $\alpha_R$ were defined in Eqs. (7.25) and (4.30).

As usual, we redefine the vertices in Eq. (9.22) by extracting a coupling constant or its squared one as shown in the following
\[
\begin{align*}
\tilde{\Lambda}^{abcd}_{\mu\nu}(k_1, k_2, k_3, k_4) &= g^2 \tilde{\Lambda}^{abcd}_{\mu\nu}(k_1, k_2, k_3, k_4), \\
\tilde{\Lambda}^{abc}_{\mu}(k_1, k_2, k_3) &= g\Lambda^{abc}_{\mu}(k_1, k_2, k_3), \\
\Lambda^{abc}_{\mu\lambda}(k_1, k_2, k_3) &= g\Lambda^{abc}_{\mu\lambda}(k_1, k_2, k_3), \\
\tilde{\Lambda}^{abcd}(k_1, k_2, k_3, k_4) &= g^2 \Lambda^{abcd}(k_1, k_2, k_3, k_4), \\
\tilde{\gamma}^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4) &= g^2 \gamma^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4), \\
\gamma^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4) &= g\gamma^{aijdc}_{\alpha\mu}(k_1, k_2, k_3, k_4), \\
\tilde{\Lambda}^{abcd}(k_1, k_2, k_3, k_4) &= g\tilde{\Lambda}^{abcd}(k_1, k_2, k_3, k_4). 
\end{align*}
\] (9.25)

With these definitions, the W-T identity in Eq. (9.22) will be replaced by
\[
\begin{align*}
&k_4^{\mu}k_3^{\nu}\hat{\Lambda}^{abcd}(k_1, k_2, k_3, k_4) \\
&= -k_4^{\mu}\hat{\Lambda}_4^{bc}(k_2, k_3)\Delta_{\pi}(k_1 + k_4)k_4^{\nu}\hat{\Lambda}_3^{abcd}(k_1, k_4) \\
&-k_4^{\mu}\hat{\Lambda}_3^{ac}(k_1, k_3)\Delta_{\pi}(k_1 + k_3)k_4^{\nu}\hat{\Lambda}_2^{abcd}(k_2, k_4) \\
&-\hat{\Lambda}_2^{\mu}(k_1, k_2, k_3)D_{\pi}(k_1 + k_2)k_4^{\nu}\hat{\Lambda}_{R\mu\nu}(k_3, k_4) \\
&-\epsilon^{bij}\Delta_{\pi}^{-1}(k_2)\chi(k_3)\sum_{\alpha=1}^{5}k_4^{\alpha}\hat{\gamma}_{\alpha\mu}(k_1, k_2, k_3, k_4) \\
&-\alpha^{2}\chi(k_3)\chi(k_4)\hat{\Lambda}^{abcd}(k_1, k_2, k_3, k_4).
\end{align*}
\] (9.26)

The vertices in Eq. (9.26) are renormalized in the manner as shown in Eqs. (5.24), (8.24), (8.26) and in the following

\[
\hat{\Lambda}^{abcd}(k_1, k_2, k_3, k_4) = Z_4^{-1}\hat{\Lambda}^{abcd}(k_1, k_2, k_3, k_4)
\] (9.27)

\[
\hat{\Lambda}^{abcd}(k_1, k_2, k_3, k_4) = Z_5^{-1}\hat{\Lambda}^{abcd}(k_1, k_2, k_3, k_4)
\] (9.28)

\[
\hat{\gamma}_{\alpha\mu}^{aijdc}(k_1, k_2, k_3, k_4) = Z_3Z_1^{-1}\hat{\gamma}_{\alpha\mu}^{aijdc}(k_1, k_2, k_3, k_4), \quad \text{if } \alpha = 2, 3
\] (9.29)

\[
\hat{\gamma}_{\alpha\mu}^{aijdc}(k_1, k_2, k_3, k_4) = Z_3Z_1^{-1}\hat{\gamma}_{\alpha\mu}^{aijdc}(k_1, k_2, k_3, k_4), \quad \text{if } \alpha = 4, 5.
\] (9.30)

It should be noted that the renormalization constant \(Z_1^{-1}\) in Eqs. (9.30) and (9.31) arises from the relation in Eq. (8.24) because the functions \(\hat{\gamma}_{\alpha\mu}^{aijdc}(k_1, k_2, k_3, k_4)\) with \(\alpha = 2, 3, 4\) and 5 contain a function \(\hat{\gamma}^{abcd}(k_1, k_2, k_3)\). For example, by the Fourier transformation, one can obtain from Eq. (9.14) that

\[
\hat{\gamma}_{2\mu}^{aijdc}(k_1, k_2, k_3, k_4) = i\int \frac{d^4q}{(2\pi)^4}\Delta_{\pi}(q)\hat{\Lambda}^{ace}(k_1, k_3, q)\gamma^{eijd}(k_2, k_4, q)
\] (9.32)

where \(\gamma^{eijd}(k_2, k_4, q)\) has an expression similar to that as shown in Eq. (8.12). It is easy to see that use of Eqs. (8.19), (8.24) and (8.26) in Eq. (9.32) directly gives rise to Eq. (9.30). Similarly, from Eq. (9.17), one can get

\[
\hat{\gamma}_{5\mu}^{aijdc}(k_1, k_2, k_3, k_4) = i\int \frac{d^4q}{(2\pi)^4}\Delta_{\pi}(q)\hat{\Lambda}^{ace}(k_4, k_3, q)\gamma^{eijd}(k_1, k_2, q)
\] (9.33)

Apparently, substitution of Eqs. (4.26), (5.24) and (8.24) in Eq. (9.33) leads to the relation in Eq. (9.31). The function \(\hat{\gamma}_{3\mu}^{aijdc}(k_1, k_2, k_3, k_4)\) is a kind of exchange term of \(\hat{\gamma}_{2\mu}^{aijdc}(k_1, k_2, k_3, k_4)\) and therefore the renormalization of \(\hat{\gamma}_{3\mu}^{aijdc}(k_1, k_2, k_3, k_4)\) should be as the same as \(\hat{\gamma}_{2\mu}^{aijdc}(k_1, k_2, k_3, k_4)\). The discussion for the function \(\hat{\gamma}_{4\mu}^{aijdc}(k_1, k_2, k_3, k_4)\) is the same because the \(\hat{\gamma}_{4\mu}^{aijdc}(k_1, k_2, k_3, k_4)\) is also a kind of exchange term of \(\hat{\gamma}_{5\mu}^{aijdc}(k_1, k_2, k_3, k_4)\).

When the relations in Eqs. (4.27), (5.22), (5.24), (8.19), (8.26) and (9.27)-(9.31) are inserted into Eq. (9.26) and then multiplying Eq. (9.26) by \(g_{R}^{2}\), we arrive at

\[
\begin{align*}
&k_4^{\alpha}k_3^{\nu}\hat{\Lambda}^{abcd}(k_1, k_2, k_3, k_4) \\
&= -k_4^{\mu}\hat{\Lambda}_4^{bc}(k_2, k_3)\Delta_{\pi}(k_1 + k_4)k_4^{\nu}\hat{\Lambda}_3^{abcd}(k_1, k_4) \\
&-k_4^{\mu}\hat{\Lambda}_3^{ac}(k_1, k_3)\Delta_{\pi}(k_1 + k_3)k_4^{\nu}\hat{\Lambda}_2^{abcd}(k_2, k_4) \\
&-\hat{\Lambda}_2^{\mu}(k_1, k_2, k_3)D_{\pi}(k_1 + k_2)k_4^{\nu}\hat{\Lambda}_{R\mu\nu}(k_3, k_4) \\
&-\epsilon^{bij}\Delta_{\pi}^{-1}(k_2)\chi(k_3)\sum_{\alpha=1}^{5}k_4^{\alpha}\hat{\gamma}_{\alpha\mu}(k_1, k_2, k_3, k_4) \\
&-\alpha^{2}\chi(k_3)\chi(k_4)\hat{\Lambda}^{abcd}(k_1, k_2, k_3, k_4)
\end{align*}
\] (9.34)

where we have considered the relations in Eq. (9.25) which also hold for the renormalized vertices. In comparison of Eq. (9.34) with Eq. (9.24), we have
\[
\frac{\tilde{Z}_4 \tilde{Z}_3}{Z_1^2} = 1, \quad \frac{\tilde{Z}_4 Z_3}{Z_1 Z_1} = 1, \quad \frac{\tilde{Z}_4 \tilde{Z}_3}{\tilde{Z}_3 Z_1^2} = 1
\]  
(9.35)

and

\[
\frac{Z_4 \tilde{Z}_3}{Z_3 Z_5} = 1, \quad \frac{\tilde{Z}_4 \tilde{Z}_3}{\tilde{Z}_3 Z_5} = 1.
\]  
(9.36)

From the four identities in Eq. (9.35), it is found that

\[
\frac{Z_4}{Z_1} = \frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{Z_3} = \frac{\tilde{Z}_1}{\tilde{Z}_3}, \quad \frac{Z_3}{Z_5} = \frac{Z_5}{Z_5}.
\]  
(9.37)

while, from the two identities in Eq. (9.36), one gets

\[
\frac{Z_3}{Z_5} = \frac{Z_5}{Z_5}.
\]  
(9.38)

Combining the identities given in Eqs. (5.28), (6.24), (7.32), (8.30), (9.37) and (9.38), we finally obtain the S-T identities as follows

\[
\frac{Z^F}{Z_2} = \frac{Z_1}{Z_3} = \frac{\tilde{Z}_1}{Z_3} = \frac{\tilde{Z}_4}{Z_1} = \frac{Z_4}{Z_1}
\]  
(9.39)

and

\[
\frac{Z_1}{Z_4} = \frac{Z_3}{Z_4}, \quad \frac{Z_1}{Z_1} = \frac{Z_3}{Z_3} = \frac{Z_5}{Z_5}.
\]  
(9.40)

IX. EFFECTIVE COUPLING CONSTANT

In this section, we plan to perform the one-loop renormalization of the SU(2)-symmetric hadrodynamics by using the renormalization group approach. As argued in our previous paper [14, 25, 26], when the renormalization is carried out in the mass-dependent momentum space subtraction scheme, the solutions to the RGEs satisfied by renormalized propagators and vertices can be uniquely determined by the boundary conditions of the renormalized propagators and vertices. In this case, an exact S-matrix element can be written in the form as given in the tree-diagram approximation provided that the coupling constant and particle masses in the matrix element are replaced by their effective (running) ones which are given by solving their renormalization group equations. Therefore, the task of renormalization is reduced to find the solutions of the RGEs for the renormalized coupling constant and particle masses. Suppose \(F_R\) is a renormalized quantity. In the multiplicative renormalization, it is related to the unrenormalized one \(F\) in such a way

\[
F = Z_F F_R
\]  
(10.1)

where \(Z_F\) is the renormalization constant of \(F\). The \(Z_F\) and \(F_R\) are all functions of the renormalization point \(\mu = \mu_0 e^t\) where \(\mu_0\) is a fixed renormalization point corresponding the zero value of the group parameter \(t\). Differentiating Eq. (10.1) with respect to \(\mu\) and noticing that the \(F\) is independent of \(\mu\), we immediately obtain a renormalization group equation (RGE) satisfied by the function \(F_R\) [21-23]

\[
\mu \frac{dF_R}{d\mu} + \gamma_F F_R = 0
\]  
(10.2)

where \(\gamma_F\) is the anomalous dimension defined by

\[
\gamma_F = \mu \frac{d}{d\mu} \ln Z_F.
\]  
(10.3)

Since the renormalization constant is dimensionless, the anomalous dimension can only depend on the ratio \(\beta = \frac{m}{\mu}\) where \(m_R\) denotes a renormalized mass and \(\gamma_F = \gamma_F(g_R, \beta)\) in which \(g_R\) is the renormalized coupling constant and
depends on \( \mu \). Since the renormalization point is a momentum taken to subtract the divergence, we may set \( \mu = \mu_0 \lambda \) where \( \lambda = e^t \) which will be taken to be the same as in the scaling transformation of momentum \( p = p_0 \lambda \). In the above, \( \mu_0 \) and \( p_0 \) are the fixed renormalization point and momentum respectively. When we set \( F \) to be the coupling constant \( g \) and noticing \( \mu \frac{d}{d\mu} = \lambda \frac{d}{d\lambda} \), one can write from Eq. (10.2) the RGE for the renormalized coupling constant

\[
\lambda \frac{dg_R(\lambda)}{d\lambda} + \gamma_g(\lambda)g_R(\lambda) = 0
\]  

(10.4)

with

\[
\gamma_g = \mu \frac{d}{d\mu} \ln Z_g.
\]  

(10.5)

According to the definition in Eq. (10.1) and the relation in Eq. (7.25), we may take,

\[
Z_g = \frac{\tilde{Z}_1}{\tilde{Z}_3 Z_3^3}
\]  

(10.6)

to calculate the anomalous dimension. As denoted in Eqs. (4.25) and (5.24), the renormalization constants \( Z_3, \tilde{Z}_3 \) and \( \tilde{Z}_1 \) are determined by the \( \rho \)-meson self-energy, the ghost particle self-energy and the ghost particle \( \rho \)-meson vertex correction, respectively. At one-loop level, the \( \rho \)-meson self-energy is depicted in Figs. (1a)-(1f), the ghost particle self-energy is shown in Fig. (2) and the ghost vertex correction is represented in Figs. (3a) and (3b). According to the Feynman rules listed in Appendix and noticing that the symmetry factors of the diagrams in Figs. (1a), (1c), (1e) and (1f) are 1/2 and the symmetry factors of the other diagrams are 1, the expressions of the self-energies and the vertex correction are easily written out. For the gluon one-loop self-energy denoted by \( -i \Pi^{ab}_{\mu\nu}(k) \), one can write

\[
\Pi^{ab}_{\mu\nu}(k) = \sum_{i=1}^{6} \Pi^{(i)ab}_{\mu\nu}(k)
\]  

(10.7)

where \( \Pi^{(1)ab}_{\mu\nu}(k) - \Pi^{(6)ab}_{\mu\nu}(k) \) represent the self-energies given in turn by Figs.(1a)-(1f). They are separately represented in the following:

\[
\Pi^{(1)ab}_{\mu\nu}(k) = i\delta^{ab}g^2 \int \frac{d^d l}{(2\pi)^d} \frac{g^{\lambda\rho}(l+k)}{[l-k]^2 - m^2 + i\epsilon}[g_{\mu\lambda}(l+2k)_\rho - g_{\rho\lambda}(2l+k)_\mu + g_{\rho\mu}(l-k)\lambda] - g_{\lambda\rho}(2l+k)\nu + g_{\lambda\nu}(l+2k)\rho],
\]  

(10.8)

\[
\Pi^{(2)ab}_{\mu\nu}(k) = -i\delta^{ab}2g^2 \int \frac{d^d l}{(2\pi)^d} \frac{(l+k)_\mu l_\nu}{[l-k]^2 - m^2 + i\epsilon][l^2 - m^2 + i\epsilon]},
\]  

(10.9)

\[
\Pi^{(3)ab}_{\mu\nu}(k) = -i\delta^{ab}2g^2 \int \frac{d^d l}{(2\pi)^d} \frac{g^{\lambda\rho}}{(l^2 - m^2 + i\epsilon)}(g_{\mu\nu}g_{\lambda\rho} - g_{\mu\rho}g_{\lambda\nu}),
\]  

(10.10)

\[
\Pi^{(4)ab}_{\mu\nu}(k) = -i\delta^{ab}2g^2 \int \frac{d^d l}{(2\pi)^d} \frac{1}{[(l-k)^2 - m^2 + i\epsilon][l^2 - m^2 + i\epsilon] \times Tr[\gamma_\mu(1 - k + M)\gamma_\nu(1 + M)]},
\]  

(10.11)

\[
\Pi^{(5)ab}_{\mu\nu}(k) = i\delta^{ab}g^2 \int \frac{d^d l}{(2\pi)^d} \frac{(2l+k)_\mu(2l+k)_\nu}{[(l+k)^2 - m^2 + i\epsilon][(k+l)^2 - m^2 + i\epsilon]},
\]  

(10.12)

and

\[
\Pi^{(6)ab}_{\mu\nu}(k) = -i\delta^{ab}2g^2 \int \frac{d^d l}{(2\pi)^d} \frac{g_{\mu\nu}}{(l^2 - m^2 + i\epsilon)}
\]  

(10.13)

where \( I = \gamma^\lambda l_\lambda \), \( k = \gamma^\lambda k_\lambda \) with \( \gamma^\lambda \) are the 8 \( \times \) 8 block-diagonal \( \gamma \)-matrices. In the above, \( \varepsilon^{abcd} \varepsilon_{bcd} = 2\delta^{ab} \) and \( Tr(T^a T^b) = \frac{1}{2} \delta^{ab} \) have been considered. It should be noted that in writing Eqs. (10.8)-(10.10), we choose to work in
the Feynman gauge for simplicity. This choice is based on the fact that the SU(2)-symmetric model of hadrodynamics, as a non-Abelian gauge field theory, has been proved to be an unitary theory [27], that is to say, the S-matrix elements evaluated from the model are independent of gauge parameter. Therefore, we are allowed to choose a convenient gauge in the calculation. From Eqs. (10.8)-(10.13), it is clearly seen that

\[ \Pi^{ab}_{\mu\nu}(k) = \delta^{ab} \Pi_{\mu\nu}(k) = \delta^{ab} \sum_{i=1}^{6} \Pi_i^{ab}(k). \]  

(10.14)

By the dimensional regularization approach [28-32], the divergent integrals over \( l \) in Eqs. (10.8)-(10.13) can be regularized in a \( n \)-dimensional space and thus are easily calculated. The results are

\[ \Pi_{\mu\nu}^{(1)}(k) = -\frac{g^2}{(4\pi)^2} \int_0^1 dx \frac{x}{x^2} \{ \frac{1}{2} \mu^{\mu\nu}\{g_{\mu\nu}[11x(x-1) + 5k^2 + 9m^2] + 2[5x(x-1) - 1]k_\mu k_\nu \}, \]  

(10.15)

\[ \Pi_{\mu\nu}^{(2)}(k) = \frac{g^2}{(4\pi)^2} \int_0^1 dx \frac{x}{x^2} \{ \frac{1}{2} \mu^{\mu\nu}\{[k^2x(x-1) + m^2]g_{\mu\nu} + 2x(x-1)k_\mu k_\nu \}, \]  

(10.16)

\[ \Pi_{\mu\nu}^{(3)}(k) = \frac{6g^2}{(4\pi)^2} \frac{m^2}{\varepsilon} g_{\mu\nu}, \]  

(10.17)

\[ \Pi_{\mu\nu}^{(4)}(k) = -\frac{8g^2}{(4\pi)^2} \int_0^1 dx \frac{k^2x(x-1) + M^2}{x^2} \{ [k^2x(x-1) - m^2]g_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \}, \]  

(10.18)

\[ \Pi_{\mu\nu}^{(5)}(k) = -\frac{g^2}{(4\pi)^2} \int_0^1 dx \frac{x}{x^2} \{ \frac{1}{2} \mu^{\mu\nu}\{[k^2x(x-1) + m^2]g_{\mu\nu} + [2-2x]k_\mu k_\nu \}, \]  

(10.19)

and

\[ \Pi_{\mu\nu}^{(6)}(k) = \frac{2g^2}{(4\pi)^2} \frac{m^2}{\varepsilon} g_{\mu\nu} \]  

(10.20)

where \( \varepsilon = 2 - \frac{d}{2} \to 0 \) when \( n \to 4 \). In Eqs. (10.15)-(10.20), except for the \( \varepsilon \) in the factor \( 1/\varepsilon[k^2x(x-1) + m^2]^\varepsilon \) where \( m = M, m_\rho \) or \( m_\pi \), we have set \( \varepsilon \to 0 \) in the other factors and terms by the consideration that this operation does not affect the calculated result of the anomalous dimension. According to the decomposition shown in Eqs. (4.15) and (4.16) and noticing \( g_{\mu\nu} = \mathcal{P}_{\mu\nu}^T + \mathcal{P}_{\mu\nu}^F \), it is easy to get the transverse part of \( \Pi_{\mu\nu}(k) \) from Eqs. (10.15)-(10.20) and furthermore, based on the decomposition denoted in Eq. (4.20), the functions \( \Pi_1(k^2) \) and \( \Pi_2(k^2) \) can be written out. The results are

\[ \Pi_1(k^2) = -\frac{g^2}{(4\pi)^2} \int_0^1 dx \{ \frac{5[2x(x-1)+1]}{x^2} \mu^{\mu\nu}\{5x(x-1) + 8x(x-1)2x(x-1) + M^2\} \} \]  

(10.21)

and

\[ \Pi_2(k^2) = -\frac{g^2}{(4\pi)^2} \int_0^1 dx \frac{8}{x^2} \mu^{\mu\nu}\{5x(x-1) + 8x(x-1)2x(x-1) + M^2\} \]  

(10.22)

It is clear that the both functions \( \Pi_1(k^2) \) and \( \Pi_2(k^2) \) are divergent in the four-dimensional space-time. When the divergences are subtracted in the mass-dependent momentum space subtraction scheme [30-33], in accordance with the definition in Eq. (4.25), we immediately obtain from the expression in Eq. (10.21) the one-loop renormalization constant \( Z_3 \) as follows

\[ Z_3 = 1 - \Pi_1(\mu^2) \]  

(10.23)
Next, we turn to the ghost particle one-loop self-energy denoted by \(-i\Omega^{ab}(q)\). From Fig. (2), in the Feynman gauge, one can write

\[
\Omega^{ab}(q) = i\delta^{ab}2q^2 \int \frac{d^4l}{(2\pi)^4} \frac{q \cdot (q - l)}{[(q - l)^2 - m_p^2 + i\epsilon][l^2 - m_p^2 + i\epsilon]}
\]

By the dimensional regularization, it is easy to get

\[
\Omega^{ab}(q) = \delta^{ab}q^2\hat{\Omega}(q^2)
\]

where

\[
\hat{\Omega}(q^2) = \frac{g^2}{(4\pi)^2} \int_0^1 dx \frac{2(x - 1)}{\varepsilon[x^2(1 - x) + m_p^2]^\gamma}
\]

According to the definition given in Eq. (4.25) and the above expression, the one-loop renormalization constant of ghost particle propagator is of the form

\[
\tilde{Z}_d = 1 - \hat{\Omega}(\mu^2) = 1 - \frac{g^2}{(4\pi)^2} \int_0^1 dx \frac{2(x - 1)}{\varepsilon[\mu^2 x(1 - x) + m_p^2]^\gamma}
\]

Now, let us discuss the ghost vertex renormalization. In the one-loop approximation, the vertex defined by extracting out a coupling constant is expressed as

\[
\tilde{A}_{abc}(p, q) = \varepsilon^{abc} p_\lambda + A^{abc}_{1\lambda}(p, q) + A^{abc}_{2\lambda}(p, q)
\]

where the first term is the bare vertex, the second and the third terms stand for the one-loop vertex corrections shown in Figs. (3a) and (3b) respectively. In the Feynman gauge, the vertex corrections are expressed as

\[
A^{abc}_{1\lambda}(p, q) = -i\varepsilon^{abc} g^2 \int \frac{d^4l}{(2\pi)^4} \frac{p \cdot (q - l)(p - l)\lambda}{[l^2 - m_p^2 + i\epsilon][(p - l)^2 - m_p^2 + i\epsilon][(q - l)^2 - m_p^2 + i\epsilon]}
\]

and

\[
A^{abc}_{2\lambda}(p, q) = i\varepsilon^{abc} g^2 \int \frac{d^4l}{(2\pi)^4} \frac{l \cdot (p - q - l)p_\lambda - p \cdot lq_\lambda + p \cdot (2q - p + l)\lambda}{[l^2 - m_p^2 + i\epsilon][(p - l)^2 - m_p^2 + i\epsilon][(q - l)^2 - m_p^2 + i\epsilon]}
\]

where \(\varepsilon^{acd}\varepsilon^{bef}\varepsilon^{dfc} = -\varepsilon^{abc}\) has been noted. By employing the dimensional regularization to compute the above integrals, it is not difficult to get

\[
A^{abc}_{1\lambda}(p, q) = \varepsilon^{abc} \frac{g^2}{(4\pi)^2} \int_0^1 dx \int_0^1 dy \left\{ \frac{4g^{\lambda\lambda}}{\Theta^{\lambda}_{xy}} - \frac{1}{\Theta^{\lambda}_{xy}} [p_\lambda A_1(p, q) + q_\lambda B_1(p, q)] - \frac{1}{8} \right\}
\]

where

\[
\Theta_{xy} = p^2 xy(x - y) + q^2[(x - y) + (x - y)]y - 2p \cdot qx(x - y)^2 + m_p^2
\]

\[
A_1(p, q) = \{ p \cdot q[1 + (x - y)] - p^2 xy\} (1 - xy)y,
\]

\[
B_1(p, q) = \{ p \cdot q[1 + (x - y)] - p^2 xy\} (x - y)^2
\]

and

\[
A^{abc}_{2\lambda}(p, q) = \varepsilon^{abc} \frac{g^2}{(4\pi)^2} \int_0^1 dx \int_0^1 dy \left\{ \frac{3g^{\lambda\lambda}}{\Theta^{\lambda}_{xy}} + \frac{1}{\Theta^{\lambda}_{xy}} [p_\lambda A_2(p, q) + q_\lambda B_2(p, q)] - \frac{3}{8} \right\}
\]

where

\[
A_2(p, q) = \{ p^2 (2xy - x^2 y^2 - 1) - q^2[(x - y) + (x - y)]y + p \cdot q[2 - (3x - 2)y + 2x(x - y)^2]\} y,
\]

\[
B_2(p, q) = [p \cdot q(x - y) - p^2 xy] y^2.
\]
The divergences in the both vertices $\Lambda^{abc}_{\alpha}(p, q)$ and $\Lambda^{abc}_{\beta}(p, q)$ may be subtracted at the renormalization point $p^2 = q^2 = \mu^2$ which implies $k = p - q = 0$, being consistent with the momentum conservation held at the vertices. Upon substituting Eqs. (10.31) and (10.33) in Eq. (10.28), at the renormalization point, one can get

$$\tilde{\Lambda}^{abc}_{\Lambda}(p, q) |_{p^2=q^2=\mu^2} = \varepsilon^{abc}p_\lambda(1 + \tilde{L}_1) = Z_{\mu}^{-1}\varepsilon^{abc}p_\lambda$$  \hspace{1cm} (10.35)

where

$$Z_\mu = 1 - \tilde{L}_1 = 1 - \frac{g^2}{4\pi^2} \int_0^1 dx \left[ \frac{2x}{\varepsilon[m^2x(x-1) + m^2_\rho]} - \frac{2x^2(x-1)\mu^2}{\mu^2x(x-1) + m^2_\rho} - \frac{1}{2} \right]$$  \hspace{1cm} (10.36)

which is the one-loop renormalization constant of the ghost vertex.

Now we are ready to calculate the anomalous dimension $\gamma_\lambda(\lambda)$. Substituting the expressions in Eqs. (10.6), (10.23), (10.27) and (10.36) into Eq. (10.5), it is easy to find an analytical expression of the anomalous dimension $\gamma_\lambda(\lambda)$. When we set $\frac{m_\rho}{\mu} = \frac{\alpha}{\Lambda}$, $\frac{m_\sigma}{\mu} = \frac{\beta}{\Lambda}$ and $\frac{m_\beta}{\mu} = \frac{\gamma}{\Lambda}$ (here we have set $\mu_0 = \Lambda$), the expression of $\gamma_\lambda(\lambda)$, in the approximation of order $g^2$, is given by

$$\gamma_\lambda(\lambda) = \tilde{\gamma}_1(\lambda) - \tilde{\gamma}_3(\lambda) - \frac{1}{2} \gamma_3(\lambda)$$  \hspace{1cm} (10.37)

where

$$\tilde{\gamma}_1(\lambda) = \lim_{\varepsilon \to 0} \frac{d}{d\mu} \ln Z_\mu = \frac{\alpha R}{2\pi} \left[ 1 + \frac{2\rho^2}{\lambda^2 - 4\rho^2} - \frac{4\rho^4}{\lambda(\lambda^2 - 4\rho^2)} I(\lambda, \rho) \right],$$  \hspace{1cm} (10.38)

$$\tilde{\gamma}_3(\lambda) = \lim_{\varepsilon \to 0} \frac{d}{d\mu} \ln Z_3 = -\frac{\alpha R}{2\pi} \left[ 1 + \frac{2\rho^2}{\lambda(\rho^2 - 4\rho^2)} I(\lambda, \rho) \right]$$  \hspace{1cm} (10.39)

and

$$\gamma_3(\lambda) = \lim_{\varepsilon \to 0} \frac{d}{d\mu} \ln Z_3 = -\frac{\alpha R}{2\pi} \left[ 1 + \frac{2\rho^2}{\lambda(\rho^2 - 4\rho^2)} I(\lambda, \rho) \right]$$  \hspace{1cm} (10.40)

here $\alpha R = g^2 R^2/4\pi$ and

$$I(\lambda, a) = \frac{1}{\sqrt{\lambda^2 - 4a^2}} \ln \frac{\lambda + \sqrt{\lambda^2 - 4a^2}}{\lambda - \sqrt{\lambda^2 - 4a^2}} \begin{cases} \frac{1}{\sqrt{a^2 - \lambda^2}} \cot^{-1} \frac{\lambda}{\sqrt{a^2 - \lambda^2}}, & \text{if } \lambda \leq 2a \\ \frac{1}{\sqrt{\lambda^2 - a^2}} \coth^{-1} \frac{\lambda}{\sqrt{\lambda^2 - a^2}}, & \text{if } \lambda \geq 2a \end{cases}$$  \hspace{1cm} (10.41)

with $a = \rho, \sigma$ or $\beta$. With the expressions given in Eqs. (10.38)-(10.40), the $\gamma_\lambda(\lambda)$ can be represented as

$$\gamma_\lambda(\lambda) = \frac{\alpha R}{2\pi} \left[ \frac{10}{3} - \frac{10\rho^2}{\lambda^2} + \frac{\rho^2}{\lambda^2} + (8 - \frac{10\rho^2}{\lambda^2} + \frac{\rho^2}{\lambda^2}) \frac{2\rho^2}{\lambda} I(\lambda, \rho) \right] - \frac{1}{\lambda} \left[ 1 + \frac{6\rho^2}{\lambda^2} + \frac{12\rho^4}{\lambda^2} I(\lambda, \rho) \right]$$  \hspace{1cm} (10.42)

We would like to note that the fixed renormalization point $\Lambda$ in $\rho, \sigma$ and $\beta$ can be taken arbitrarily. For example, the $\Lambda$ may be chosen to be the mass of nucleon. In this case, $\beta = 1$, $\rho = m_\rho/M$ and $\sigma = m_\sigma/M$. Apparently, $\beta = 1$ implies $\lambda = \sqrt{p^2/M^2}$. In practice, the $\Lambda$ will be treated as a scaling parameter of renormalization.

With the $\gamma_\lambda(\lambda)$ given above, the equation in Eq. (10.4) can be solved to give the effective coupling constant as follows

$$\alpha_R(\lambda) = \frac{\alpha_R}{1 + \frac{\alpha_R}{2\pi} G(\lambda)}$$  \hspace{1cm} (10.43)

where $\alpha_R = \alpha_R(1)$ and

$$G(\lambda) = \int_1^\lambda d\Lambda \frac{4\pi\gamma_\lambda(\lambda)}{\alpha_R} = \frac{2}{3} \left[ \varphi_1(\lambda, \rho) - \varphi_1(1, \rho) \right] - \frac{1}{3} \left[ \varphi_2(\Lambda, \sigma) - \varphi_2(1, \sigma) \right] - \frac{1}{3} \left[ \varphi_2(\Lambda, \beta) - \varphi_2(1, \beta) \right]$$  \hspace{1cm} (10.44)
in which

\[ \varphi_1(\lambda, \rho) = \frac{5 \rho^2}{\lambda^2} + \left( \frac{19}{2} - \frac{5 \rho^2}{\lambda^2} \right) \frac{\lambda^2 - 4 \rho^2}{2 \lambda} + \frac{3 \lambda}{4} I(\lambda, \rho), \]  

(10.45)

\[ \varphi_2(\lambda, \sigma) = -\frac{2 \sigma^2}{\lambda^2} + \left( 1 + \frac{2 \sigma^2}{\lambda^2} \right) \frac{\lambda^2 - 4 \sigma^2}{2 \lambda} I(\lambda, \sigma) \]  

(10.46)

and

\[ \varphi_2(\lambda, \beta) = -\frac{3 \beta^2}{\lambda^2} + \left( 1 + \frac{2 \beta^2}{\lambda^2} \right) \frac{\lambda^2 - 4 \beta^2}{2 \lambda} I(\lambda, \beta). \]  

(10.47)

here \( \varphi_1(\lambda, \rho) \) arises from the \( \rho - \)meson self-interaction, \( \varphi_2(\lambda, \sigma) \) and \( \varphi_2(\lambda, \beta) \) come from the interaction between pion and \( \rho - \)meson and the one between nucleon and \( \rho - \)meson, respectively.

In the large momentum limit \( (\lambda \to \infty) \), we have

\[ G(\lambda) = \frac{17}{3} \ln \lambda. \]  

(10.48)

Therefore, in the limit mentioned above, we have \( \alpha_R(\lambda) \to 0 \). This exhibits that the interaction given by the SU(2)-symmetric model is also of the asymptotically free behavior. It should be noted that the expressions in Eqs. (10.42) and (10.45)-(10.47) are obtained at the timelike subtraction point where the \( \lambda \) is a real variable. We may also take spacelike momentum subtraction. For this kind of subtraction, corresponding to \( \mu \to i \mu \), the variable \( \lambda \) in Eqs. (10.42), (10.45)-(10.47) should be replaced by \( i \lambda \) where \( \lambda \) is still a real variable. In this case, the function in Eq. (10.41) will be replaced by

\[ I(\lambda, a) = \frac{1}{\sqrt{\lambda^2 + 4a^2}} \ln \frac{\sqrt{\lambda^2 + 4a^2} + \lambda}{\sqrt{\lambda^2 + 4a^2} - \lambda}, \]  

(10.39)

It is easy to see that the function in Eq. (10.48) is the same for the both subtractions.

The behavior of the function \( \alpha_R(\lambda) \) is graphically described in Fig. (4). In our test, we find that the behavior of the \( \alpha_R(\lambda) \) sensitively depends on the choice of the constant \( \alpha_R \) and the scaling parameter \( \Lambda \). In this paper, we only take \( \alpha_R = 0.5 \) and \( \Lambda = M \) as an illustration. Figs. (4a) and (4b) represent respectively the effective coupling constants obtained at the timelike subtraction point and the spacelike subtraction point. To exhibit the effects of the \( \rho - \)meson self-interaction, the pion-\( \rho - \)meson interaction and the nucleon-\( \rho - \)meson interaction on the effective coupling constant, in each figure, besides the total effective coupling constant, we also separately show the effective coupling constants given by the functions \( \varphi_1(\lambda, \rho), \varphi_2(\lambda, \sigma) \) and \( \varphi_2(\lambda, \beta) \). These effective coupling constants are represented by the solid, dotted and dashed lines respectively in the subfigures within Figs. (4a) and (4b). Fig. (4a) shows that the effective coupling constant given by the timelike momentum subtraction has a peak with the maximum value \( \alpha_R(\lambda)_{\text{max}} = 1.49222 \alpha_R \) at \( \lambda = 1.5303 \). When \( \lambda \to 0 \), the \( \alpha_R(\lambda) \) abruptly falls down to zero, while, when \( \lambda \) goes to infinity, the \( \alpha_R(\lambda) \) rather smoothly decreases from its maximum and tends to zero. Fig. (4b) tells us that the effective coupling constant given by the spacelike momentum subtraction has a different behavior in the low-energy region. This coupling constant keeps almost a constant near the value of \( \alpha_R \) in the region \([0, 1] \) of \( \lambda \) and then decreases and tends to zero with the growth of \( \lambda \). From the subfigures, it is clearly seen that at one-loop level, only the \( \rho - \)meson self-interaction is of the behavior of asymptotic freedom, while the interactions between nucleon and \( \rho - \)meson and between pion and \( \rho - \)meson have no such a behavior. The asymptotically free behavior of the total effective coupling implies that the \( \rho - \)meson self-interaction plays an overwhelming role for the one-loop interaction.

### X. EFFECTIVE MESON MASSES

In this section, we proceed to derive the one-loop effective \( \rho - \)meson and pion masses. First, we derive the \( \rho - \)meson effective mass. Setting \( F_R = m_{\rho}^R \) in Eq. (10.2), we have the RGE for the renormalized \( \rho - \)meson mass

\[ \lambda \frac{dm_{\rho}^R(\lambda)}{d\lambda} + \gamma_{\rho}(\lambda)m_{\rho}^R(\lambda) = 0 \]  

(11.1)

where
\[ \gamma_{m_\rho}(\lambda) = \mu \frac{d}{d\mu} \ln Z_{m_\rho}. \]  

(11.2)

From the last equality in Eq. (4.25) and Eqs. (10.21) and (10.22), in the approximation of order \( g^2 \), we can write

\[ Z_{m_\rho} = 1 - \frac{g^2}{(4\pi)^2} \int_0^1 dx \left\{ \frac{5x(\lambda^2 - x) + 13/2}{\epsilon(x(x-1)+m_{m_\rho}^2)} + \frac{4x(\lambda^2 - x)}{\epsilon(x(x-1)+m_{m_\rho}^2)} \right\} + \frac{z(x(x-1)+m_{m_\rho}^2/m_{m_\rho}^2)}{\epsilon(x(x-1)+m_{m_\rho}^2)} + \frac{1}{2}(m_{m_\rho}^2/m_{m_\rho}^2 + \frac{\gamma}{2}). \]  

(11.3)

On inserting Eq. (11.3) into Eq. (11.2) and completing the differentiation with respect to \( \mu \) and the integration over \( x \), we find

\[ \gamma_{m_\rho}(\lambda) = \frac{5g^2}{(4\pi)^2} \left\{ \frac{29}{12} + \frac{\sigma^2}{2\lambda^2} - \frac{4\beta^2}{\lambda^2}(5\rho^2 + \sigma^2 + 4\beta^2) \right\} + \frac{\rho^2}{\lambda^2}(13\lambda^2 - 10\rho^2)I(\lambda, \rho) + \frac{\sigma^2}{\lambda^2}I(\lambda, \sigma) - \frac{4\beta^2}{\lambda^2}I(\lambda, \beta). \]  

(11.4)

where the functions \( I(\lambda, \cdot) \) were defined in Eq. (10.41). With this anomalous dimension, the RGE in Eq. (11.1) can be solved to give an effective \( \rho \)-meson mass such that

\[ m_{\rho_\text{eff}}^R(\lambda) = m_{\rho_\text{eff}}^R e^{-S_\rho(\lambda)} \]  

(11.5)

where \( m_{\rho_\text{eff}}^R = m_{\rho_\text{eff}}^R(1) \) and

\[ S_\rho(\lambda) = \int_1^{\lambda} \frac{d\lambda}{\lambda} \gamma_{m_\rho}(\lambda). \]  

(11.6)

In general, the coupling constant \( g_R \) in Eq. (11.5) may be taken to be the effective one. If the coupling constant is taken to be the constant \( g_R \), the function \( S_\rho(\lambda) \) can be explicitly represented as

\[ S_\rho(\lambda) = \frac{2g^2}{(4\pi)^2} \left\{ A_1(\lambda, \rho) - A_1(1, \rho) + A_2(\lambda, \sigma) - A_2(1, \sigma) + A_3(\lambda, \beta) - A_3(1, \beta) \right\} \]  

(11.7)

where

\[ A_1(\lambda, \rho) = \frac{5\rho^2}{\lambda^2} + (17 - \frac{5\rho^2}{\lambda^2}) \frac{4\rho^2 - \lambda^2}{2\lambda}I(\lambda, \rho), \]  

(11.8)

\[ A_2(\lambda, \sigma) = \frac{\sigma^2}{\lambda^2} - (1 - \frac{6\sigma^2}{\rho^2}) + \frac{2\sigma^2}{\lambda^2} \frac{4\sigma^2 - \lambda^2}{4\lambda}I(\lambda, \sigma) \]  

(11.9)

and

\[ A_3(\lambda, \beta) = \frac{4\beta^2}{\lambda^2} - (1 + \frac{2\beta^2}{\lambda^2}) \frac{4\beta^2 - \lambda^2}{\lambda}I(\lambda, \beta). \]  

(11.10)

In the large momentum limit \( (\lambda \to \infty) \), we have

\[ S_\rho(\lambda) \to \frac{\alpha_R}{\pi} \left\{ \frac{29}{2} + \frac{3m_{\rho_\text{eff}}^2}{m_{\rho_\text{eff}}^2} \right\} \ln \lambda, \]  

(11.11)

therefore,

\[ \lim_{\lambda \to \infty} m_{\rho_\text{eff}}^R(\lambda) = 0. \]  

(11.12)

which exhibits the asymptotically free behavior.

The behavior of the effective \( \rho \)-meson mass \( m_{\rho_\text{eff}}^R(\lambda) \) in the whole range of momenta is displayed in Fig. (5) where the solid line represents the effective mass given by the timelike momentum subtraction and the dashed line represents the one obtained by the spacelike momentum subtraction. In comparison of Fig. (5) with Fig. (4), one can see
that the behaviors of the effective masses are much similar to the behaviors of the corresponding coupling constants. Saying concretely, the timelike momentum effective mass has a peak with the maximum $m^R_\rho(\lambda)_{\max} = 1.06674m^R_\rho$ at $\lambda = 1.16675$ and also abruptly falls down to zero when $\lambda \to 0$ and rather smoothly decreases from its maximum and tends to zero when $\lambda$ goes to infinity. While, the spacelike momentum effective mass almost behaves as a constant near the value $m^R_\mu$ in the region $[0,1]$ of $\lambda$ and then decreases and tends to zero with the growth of $\lambda$.

Next, we turn to the effective pion mass. With setting $F_R = m^R_\pi$ in Eq. (10.2), we can write the RGE for the renormalized pion mass

$$\lambda \frac{dm^R_\pi(\lambda)}{d\lambda} + \gamma_{m^R_\pi}(\lambda)m^R_\pi(\lambda) = 0$$

(11.13)

where

$$\gamma_{m^R_\pi}(\lambda) = \mu \frac{d}{d\mu} \ln Z_{m^R_\pi}.$$  

(11.14)

From Eq. (8.21), the one-loop renormalization constant $Z_{m^R_\pi}$ can be written as

$$Z_{m^R_\pi} = 1 - \frac{1}{2} [\omega_1(\mu^2) + \omega_2(\mu^2)]$$

(11.15)

where $\omega_1(\mu^2)$ and $\omega_2(\mu^2)$ are contributed from the one-loop self-energies $-i\Omega_1^{ab}(k)$ and $-i\Omega_2^{ab}(k)$ as depicted in Figs. (6a) and (6b) respectively. According to the Feynman rules shown in Appendix, in the Feynman gauge, one can write

$$\Omega_1^{ab}(k) = \delta^{ab}\Omega_1(k) = -i\delta^{ab}2g^2 \int \frac{d^4l}{(2\pi)^4} \frac{(l - 2k)^2}{[(k - l)^2 - m^2_\pi + i\epsilon][l^2 - m^2_\rho + i\epsilon]}$$

(11.16)

and

$$\Omega_2^{ab}(k) = \delta^{ab}\Omega_2(k) = -i\delta^{ab}16g^2 \int \frac{d^4l}{(2\pi)^4} \frac{l^2 - k \cdot l - M^2}{[(k - l)^2 - M^2 + i\epsilon][l^2 - M^2 + i\epsilon]}$$

(11.17)

where $\varepsilon^{abcd}t^{ab} = 2\delta^{ab}$, $Tr(t^a t^b) = 2\delta^{ab}$ and $Tr[\gamma_5(i\gamma^\mu - \gamma^\mu + M)\gamma_5(i\gamma^\mu + M)] = 8(\epsilon \cdot l - l^2 + M^2)$ have been used in writing the above expressions. The integrals in Eqs. (11.16) and (11.17) are divergent. They can easily be calculated in the dimensional regularization scheme. The results are

$$\Omega_1(k) = \frac{2g^2}{(4\pi)^2} \int_0^1 dx \frac{k^2(3x^2 - 6x + 4) + 2[(m^2_\pi - m^2_\rho)x + m^2_\rho]}{\varepsilon[k^2x(x - 1) + m^2_\pi + m^2_\rho(1 - x)]}$$

(11.18)

and

$$\Omega_2(k) = \frac{16g^2}{(4\pi)^2} \int_0^1 dx \frac{3k^2x(x - 1) + M^2}{\varepsilon[k^2x(x - 1) + M^2]}$$

(11.19)

Substituting $\Omega_x(k) = \Omega_1(k) + \Omega_2(k)$ with $\Omega_1(k)$ and $\Omega_2(k)$ given above into Eq. (8.17), we find

$$\omega_1(\mu^2) = \frac{2g^2}{(4\pi)^2} \int_0^1 dx \left\{ \frac{3x^2 - 6x + 4}{\varepsilon[k^2x(x - 1) + (m^2_\pi - m^2_\rho)x + m^2_\rho]} \right\}$$

(11.20)

and

$$\omega_2(\mu^2) = \frac{2g^2}{(4\pi)^2} \int_0^1 dx \left\{ \frac{2[(1 - m^2_\rho/m^2_\pi)x + m^2_\rho]}{8M^2/m^2_\rho} \right\}$$

(11.21)

where we have set $k^2 = \mu^2$. On inserting Eqs. (11.20) and (11.21) into Eq. (11.15), we obtain an explicit expression of the renormalization constant $Z_{m^R_\pi}$. Substituting such a renormalization constant in Eq. (11.14), through a lengthy derivation, we get

$$\gamma_{m^R_\pi}(\lambda) = \frac{\alpha}{2\pi} \{ \xi_1(\lambda) + \xi_2(\lambda) \ln \frac{\xi_1}{\xi_3} + \xi_3(\lambda)J(\lambda; \rho, \sigma) + \xi_4(\lambda) + \xi_5(\lambda)I(\lambda, \beta) \}$$

(11.22)
where
\[\xi_1(\lambda) = \frac{1}{2\sigma^2\lambda^4} [6\sigma^2(\rho^2 - \sigma^2)^2 - (4\rho^4 + \sigma^4 + \rho^2\sigma^2)\lambda^2 + 2(\rho^2 + 3\sigma^2)\lambda^4],\]
\[\xi_2(\lambda) = \frac{1}{\sigma^2\lambda^2} [3\sigma^2(\rho^2 - \sigma^2)^3 - 2(\rho^2 - \sigma^2)(\rho^4 + \sigma^4 + \rho^2\sigma^2)\lambda^2 + (2\rho^4 - 3\sigma^4 + 4\rho^2\sigma^2)\lambda^4],\]
\[\xi_3(\lambda) = \frac{1}{2\sigma^2\lambda^2} [3\sigma^2(\rho^2 - \sigma^2)^4 - (\rho^2 - \sigma^2)^2(2\rho^4 + 5\sigma^4 + 5\rho^2\sigma^2)\lambda^2 + (4\rho^6 + 5\sigma^6 + 6\rho^4\sigma^2 - 9\rho^2\sigma^4)\lambda^4 - (2\rho^4 + 3\sigma^4 + 4\rho^2\sigma^2)\lambda^6],\]
\[\xi_4(\lambda) = \frac{4}{\lambda^2\sigma^2} [(2\beta^2 - \sigma^2)\lambda^2 - 6\beta^2\sigma^2],\]
\[\xi_5(\lambda) = \frac{16\beta^4}{\lambda^4\sigma^2}(\lambda^2 - 3\sigma^2),\]
\[J(\lambda; \rho, \sigma) = \frac{1}{\sqrt{K(\lambda; \rho, \sigma)}} \ln \frac{\lambda^2 - (\rho^2 + \sigma^2)}{\lambda^2 - (\rho^2 + \sigma^2) + \sqrt{K(\lambda; \rho, \sigma)}},\]
in which
\[K(\lambda; \rho, \sigma) = \lambda^4 - 2(\rho^2 + \sigma^2)\lambda^2 + (\rho^2 - \sigma^2)^2\]
and \(I(\lambda, \beta)\) was defined in Eq. (10.41).

With the anomalous dimension given above, the one-loop effective pion mass will be obtained by solving the RGE in Eq. (11.13). The result is
\[m_\pi^R(\lambda) = m_\pi^R e^{-S_\pi(\lambda)}\]
(11.30)
where
\[S_\pi(\lambda) = \int_1^\lambda \frac{d\lambda}{\lambda} \gamma_{m_\pi}(\lambda)\]
(11.31)
If the coupling constant in Eq. (11.22) is taken to be a constant, the integral over \(\lambda\) in Eq. (11.31) can partly be calculated analytically, giving the result as follows
\[S_\pi(\lambda) = \frac{\alpha_\pi}{2\pi} \{B_1(\lambda) - B_1(1) + B_2(\lambda) - B_2(1) + B_3(\lambda) - B_3(1) + B_4(\lambda)\}\]
(11.32)
where
\[B_1(\lambda) = \frac{8\beta^2}{\lambda^2} - 2(1 - \frac{2\beta^2}{\sigma^2} + \frac{2\beta^2}{\lambda^2})(\lambda^2 - 4\beta^2)I(\lambda, \beta),\]
\[B_2(\lambda) = \frac{1}{2\lambda^2} \{4(\rho^2 + 3\sigma^2)\ln \lambda + \frac{1}{4}[4\rho^4 + \sigma^4 + \rho^2\sigma^2]\lambda^2 - 3\sigma^2(\rho^2 - \sigma^2)^2\},\]
\[B_3(\lambda) = -\frac{1}{2\lambda^2} \ln \lambda \frac{1}{4}[2(\rho^4 - 3\sigma^4 + 4\rho^2\sigma^2)\lambda^4 - (\rho^2 - \sigma^2)(\rho^2 + \sigma^2 + \rho^2\sigma^2)\lambda^2 + \sigma^2(\rho^2 - \sigma^2)^3]\]
(11.35)
and
\[B_4(\lambda) = \int_1^\lambda \frac{d\lambda}{\lambda} \xi_5(\lambda)J(\lambda; \rho, \sigma).\]
(11.36)
The integral in Eq. (11.36) with the functions \(\xi_5(\lambda)\) and \(J(\lambda; \rho, \sigma)\) given respectively in Eqs. (11.25) and (11.28) can only be evaluated numerically.

The effective pion mass \(m_\pi^R(\lambda)\) is graphically represented in Fig. (7). In the figure, the solid line and the dashed one represent the effective mass given by the timelike momentum subtraction and the spacelike momentum subtraction, respectively. The figure indicates that the effective mass given by the timelike momentum subtraction has a sharp peak around \(\lambda = 0.43057\) at which we have a maximum \(m_\pi^R(\lambda)_{\text{max}} = 24.0411\). Departing from the maximum, the effective mass rapidly falls to zero when either \(\lambda \to 0\) or \(\lambda \to \infty\). In contrast, the effective mass given by the spacelike momentum subtraction has a low peak around \(\lambda = 0.49384\) at which there is a maximum \(m_\pi^R(\lambda)_{\text{max}} = 1.49159\). From the maximum, the effective mass smoothly tends to zero.
XI. EFFECTIVE NUCLEON MASS

Before deriving the one-loop effective nucleon mass, we need first to discuss the subtraction of the nucleon one-loop self-energy on the basis of the W-T identity represented in Eq. (7.16). For later convenience, the identity in Eq. (7.16) will be given in another form. Introducing new vertex functions $\hat{\Lambda}^{\mu\nu}(p, q)$ and $\hat{\gamma}^a_i(p, q)$ defined by

\[
\begin{align*}
\Gamma^{\mu\nu}(p, q, k) &= (2\pi)^4\delta^4(p - q + k)ig\hat{\Lambda}^{\mu\nu}(p, q) \\
\gamma^a_i(p, q, k) &= -\frac{1}{(2\pi)^4}\delta^4(p - q + k)\hat{\gamma}^a_i(p, q)
\end{align*}
\]

where $i = 1, 2$ and $\hat{\gamma}^a_i(p, q) = -\hat{\gamma}^a_i(p, q)$ and considering $k = q - p$, Eq. (7.16) can be rewritten as

\[
(p - q)_\mu\hat{\Lambda}^{\mu\nu}(p, q) = \chi(k^2)[S_F^{-1}(p)\hat{\gamma}^a_2(p, q) - \hat{\gamma}^a_1(p, q)S_F^{-1}(q)].
\]

From the perturbative calculation, it can be found that in the lowest order of perturbation, we have

\[
\begin{align*}
\hat{\Lambda}^{(0)\mu}(p, q) &= \gamma^a\hat{T}_a, \\
\hat{\gamma}^{(0)\mu}_i(p, q) &= \hat{\gamma}^{(0)\mu}_2(p, q) = T^a.
\end{align*}
\]

In the one-loop approximation, the nucleon-gluon vertex denoted by $\hat{\Lambda}^{(1)\mu}_i(p, q)$ is of order $g^2$. The nucleon-ghost vertex functions $\hat{\gamma}^{(1)\mu}_i(p, q)$ ($i = 1, 2$) are contributed from Figs. (8a) and (8b) and can be represented as

\[
\hat{\gamma}^{(1)\mu}_i(p, q) = T^aK_i(p, q)
\]

where

\[
K_1(p, q) = ig^2\int \frac{d^4l}{(2\pi)^4}F_{\mu\nu}(l)(q - l)^\nu D_{\mu\nu}(p - l)\Delta(q - l)
\]

and

\[
K_2(p, q) = ig^2\int \frac{d^4l}{(2\pi)^4}F_{\mu\nu}(l)\gamma^\nu D_{\mu\nu}(q - l)(p - l)^\nu\Delta(p - l).
\]

It is clear that the above functions are logarithmically divergent. In the one-loop approximation, the function $\chi(k^2)$ can be written as $\chi(k^2) = 1 - \hat{\Omega}^{(1)}(k^2)$ where the one-loop ghost particle self-energy $\hat{\Omega}^{(1)}(k^2)$ was represented in Eq. (10.24). Thus, up to the order of $g^2$, with setting $\hat{\Lambda}^{(1)\mu}_i(p, q) = T^a\hat{\Lambda}^{(1)\mu}_i(p, q)$, we can write

\[
\hat{\Lambda}_\mu^a(p, q) = T^a[\gamma_\mu + \hat{\Lambda}^{(1)\mu}_i(p, q)]
\]

and

\[
\chi(k^2)\hat{\gamma}^a_i(p, q) = T^a[1 + I_i(p, q)]
\]

where

\[
\begin{align*}
I_i(p, q) &= K_i(p, q) - \hat{\Omega}^{(1)}(k^2).
\end{align*}
\]

Upon substituting Eqs. (12.7) and (12.8) and the inverse of the nucleon propagator denoted in Eq. (7.17) into Eq. (12.2), then differentiating the both sides of Eq. (12.2) with respect to $\mu^\nu$ and finally setting $q = p$, in the order of $g^2$, we get

\[
\overline{\Lambda}_\mu(p, p) = -\frac{\partial \Sigma(p)}{\partial \mu^\nu}
\]

where

\[
\begin{align*}
\overline{\Lambda}_\mu(p, p) &= \Lambda^{(1)}_\mu(p, p) - \gamma_\mu I_2(p, p) - (p - M)\frac{\partial I_2(p, p)}{\partial \mu^\nu} |_{q=p} \\
&+ \frac{\partial I_1(p, p)}{\partial \mu^\nu} |_{q=p} (p - M).
\end{align*}
\]
It is emphasized that at one-loop level, the both sides of Eq. (12.11) are of the order of \( g^2 \). The terms of orders higher than \( g^2 \) have been neglected in the derivation of Eq. (12.11). The identity in Eq. (12.10) formally is the same as we met in QED. By the subtraction at \( p = \mu \), the vertex \( \Lambda_{\mu}(p,p) \) will be expressed in the form

\[
\Lambda_{\mu}(p,p) = L\gamma_{\mu} + \Lambda'_{\mu}(p) \tag{12.12}
\]

where \( L \) is a divergent constant defined by

\[
L = \Lambda_{\mu}(p,p) \bigg|_{p=\mu} \tag{12.13}
\]

and \( \Lambda'_{\mu}(p) \) is the finite part of \( \Lambda_{\mu}(p,p) \) satisfying the boundary condition

\[
\Lambda_{\mu}(p) \bigg|_{p=\mu} = 0. \tag{12.14}
\]

On integrating the identity in Eq. (12.10) over the momentum \( p_\mu \) and considering the expression in Eq. (12.12), we obtain

\[
\Sigma(p) = A + (p - \mu)[B - C(p^2)] \tag{12.15}
\]

where

\[
A = \Sigma(\mu), \tag{12.16}
\]

\[
B = -L \tag{12.17}
\]

and \( C(p^2) \) is defined by

\[
\int_{p_0^0}^{p_\mu} dp_\mu \Lambda_{\mu}(p) = (p - \mu)C(p^2). \tag{12.18}
\]

Clearly, the expression in Eq. (12.15) gives the subtraction version of the nucleon self-energy which is required by the W-T identity and correct at least in the approximation of order \( g^2 \). With this subtraction, the nucleon propagator in Eq. (7.17) will be renormalized as

\[
S_F(p) = \frac{Z_2}{p - M_R - \Sigma_R(p)} \tag{12.19}
\]

where \( Z_2 \) is the renormalization constant defined by

\[
Z_2^{-1} = 1 - B, \tag{12.20}
\]

\( M_R \) is the renormalized nucleon mass defined as

\[
M_R = Z_M^{-1}M \tag{12.21}
\]

in which

\[
Z_M^{-1} = 1 + Z_2[A^{-1}M^{-1} + (1 - \mu M^{-1})B], \tag{12.22}
\]

\( Z_M \) is the nucleon mass renormalization constant and \( \Sigma_R(p) \) is the finite correction of the self-energy satisfying the boundary condition \( \Sigma_R(p) \big|_{p^2=\mu^2} = 0 \).

Now we are in a position to discuss the one-loop renormalization of nucleon mass. The RGE for the renormalized nucleon mass can be written from Eq. (10.2) by setting \( F = M \),

\[
\lambda \frac{dM_R(\lambda)}{d\lambda} + \gamma_M(\lambda)M_R(\lambda) = 0 \tag{12.23}
\]

where

\[
\gamma_M(\lambda) = \mu \frac{d}{d\mu} \ln Z_M. \tag{12.24}
\]
It is clear that to determine the one-loop renormalization constant $Z_M$, we first need to determine the divergent constants $A$ and $B$ from the nucleon one-loop self-energy which is represented in the form as shown in Eq. (12.15). The one-loop self-energy denoted by $-i\Sigma(p)$ contains two terms which can be written out from Figs. (9a) and (9b) respectively. In the Feynman gauge, it is represented as

$$\Sigma(p) = \Sigma_1(p) + \Sigma_2(p)$$  \hspace{1cm} (12.25)

where

$$\Sigma_1(p) = -i\gamma^2 g^2 \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma^\mu(k+p+M)\gamma_\nu}{[(k+p)^2-M^2+i\varepsilon][(k-p)^2-M^2+i\varepsilon]}$$

$$\Sigma_2(p) = i\gamma^2 g^2 \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma^\mu(k+p+M)\gamma_\nu}{[(k+p-M)^2+i\varepsilon][(k-p-M)^2+i\varepsilon]}$$  \hspace{1cm} (12.26)

here $k = \gamma^\mu k_\mu$ and $p = \gamma^\mu p_\mu$. By making use of the dimensional regularization to calculate the above integral, it is found that

$$\Sigma_1(p) = \frac{3\gamma^2}{(4\pi)^2} \int_0^1 dx \frac{(x-1)p+2M}{x\mu^2x(x-1)+M^2x+m_0^2(1-x)}$$

$$\Sigma_2(p) = \frac{3\gamma^2}{(4\pi)^2} \int_0^1 dx \frac{(x-1)p+M}{x\mu^2x(x-1)+M^2x+m_0^2(1-x)}$$  \hspace{1cm} (12.27)

According to Eqs. (12.15), (12.25) and (12.27), we have

$$A = \Sigma(p) |_{p=\mu} = \frac{3\gamma^2}{(4\pi)^2} \int_0^1 dx \left\{ \frac{(x-1)\mu+2M}{2[(x-1)\mu+M]} \right\}$$  \hspace{1cm} (12.28)

With the aid of the following formula

$$\frac{1}{a^2} - \frac{1}{b^2} = \varepsilon \int_0^1 dx \frac{b-a}{[a x + b(1-x)]^{1+\varepsilon}},$$

one can get from Eqs. (12.15) and (12.25)-(12.28) that

$$B = \frac{\Sigma(p) - A(p-M)^{-1}}{\mu=\mu} = \frac{3\gamma^2}{(4\pi)^2} \int_0^1 dx \left\{ \frac{2x(x-1)[(x-1)\mu+2M]}{x\mu^2x(x-1)+M^2x+m_0^2(1-x)} \right\}$$  \hspace{1cm} (12.30)

where $C(\mu^2) = 0$ has been considered. On inserting Eqs. (12.28) and (12.30) into Eq. (12.22) and noting that in the approximation of order $g^2$, $Z_2 \simeq 1$ should be taken in Eq. (12.22), it can be found that

$$Z_M = 1 - \frac{A}{M} - \frac{B}{M}$$

$$= 1 - \frac{3\gamma^2}{(4\pi)^2} \frac{3}{2} \int_0^1 dx \left\{ \frac{2x(x-1)[(x-1)\mu+2M]}{x\mu^2x(x-1)+M^2x+m_0^2(1-x)} \right\}$$  \hspace{1cm} (12.31)

When substituting Eq. (12.31) in Eq. (12.24) and applying the familiar integration formulas, through a lengthy calculation, we obtain

$$\gamma_M(\lambda) = \gamma^{(1)}_M(\lambda) + \gamma^{(2)}_M(\lambda)$$  \hspace{1cm} (12.32)

where $\gamma^{(1)}_M(\lambda)$ and $\gamma^{(2)}_M(\lambda)$ are contributed from the self-energies depicted in Figs. (9a) and (9b) respectively. They are represented as follows.

$$\gamma^{(1)}_M(\lambda) = \frac{3\gamma^2}{4\pi} \{ \eta_1(\lambda) + \eta_2(\lambda) \ln \frac{\rho}{\beta} + \frac{3}{4\pi K(\lambda;\beta;\rho)} \eta_3(\lambda) \}$$

$$+ \frac{1}{2\pi} \{ \eta_4(\lambda) + \frac{4}{K(\lambda;\beta;\rho)} \eta_5(\lambda) \} J(\lambda; \beta, \rho)$$  \hspace{1cm} (12.33)

where

$$\eta_1(\lambda) = \frac{1}{\lambda^2} \left[ \frac{3\lambda}{2\beta} + \left( \rho^2 - 3\beta^2 \right) \frac{\lambda}{\beta} + \beta^2 - \rho^2 \right],$$  \hspace{1cm} (12.34)
\[ \eta_2(\lambda) = \frac{1}{\lambda} \left[ \frac{1}{3} \lambda^3 + (6 \beta^2 - 7 \rho^2) \lambda^2 \right. \\
\left. - \frac{1}{3} (\beta^2 - \rho^2)(3 \beta^2 - \rho^2) \lambda + 3(\beta^2 - \rho^2)^2 \right], \]

\[ \eta_3(\lambda) = \frac{d^2}{d \lambda^2} (\lambda^5 - (2 \beta^3 + 3 \rho^2) \lambda^4 + \frac{1}{4} (3 \beta^4 + 3 \beta^2 \rho^2 - 2 \rho^4) \lambda^3 \\
+ (\beta^2 - \rho^2)(\beta^2 - 4 \rho^2) \lambda^2 - \frac{1}{3} (\beta^2 - \rho^2)^2 (3 \beta^2 - \rho^2) \lambda + (\beta^2 - \rho^2)^3, \] (12.35)

\[ \eta_4(\lambda) = \frac{d^2}{d \lambda^2} (\lambda^5 - (6 \beta^2 + 7 \rho^2) \lambda^4 + \frac{1}{3} (9 \beta^4 + 11 \rho^2 \beta^2 - 8 \rho^2) \lambda^3 \\
+ 11(\beta^2 - \rho^2)(\beta^2 - 2 \rho^2) \lambda^2 - \frac{5}{3} (\beta^2 - \rho^2)^2 (3 \beta^2 - \rho^2) \lambda + 7(\beta^2 - \rho^2)^3, \] (12.36)

\[ \eta_5(\lambda) = \frac{d^2}{d \lambda^2} (\lambda^5 - (2 \beta^4 + 3 \rho^4) \lambda^6 + \frac{1}{6} (3 \beta^6 + 4 \beta^2 \rho^4 - 3 \rho^6) \lambda^5 \\
+ (\beta^2 - \rho^2)(3 \beta^4 - 3 \beta^2 \rho^2 - 7 \rho^4) \lambda^4 - \frac{1}{4} (\beta^2 - \rho^2)^2 (6 \beta^4 + 5 \beta^2 \rho^2 - 3 \rho^4) \lambda^3 \\
+ 5 \rho^2 (\beta^2 - \rho^2)^3 \lambda^2 + \frac{1}{3} (\beta^2 - \rho^2)^4 (3 \beta^2 - \rho^2) \lambda - (\beta^2 - \rho^2)^5, \] (12.37)

\[ K(\lambda; \beta, \rho) \text{ and } J(\lambda; \beta, \rho) \text{ are the functions defined in Eqs. (11.28) and (11.29) with } \rho \text{ being replaced by } \beta \text{ and } \rho. \]

\[ \gamma_M^{(2)}(\lambda) = \frac{2 \rho}{\sqrt{\lambda}} \left\{ \zeta_1(\lambda) + \zeta_2(\lambda) \ln \frac{\lambda}{\beta} + \frac{2}{\lambda^2} \zeta_3(\lambda) \right\} \\
+ \frac{2}{\lambda^3} \left[ \zeta_4(\lambda) + \frac{4}{\lambda^2} \zeta_5(\lambda) \right] J(\lambda; \beta, \sigma) \] (12.39)

where

\[ \zeta_1(\lambda) = \frac{1}{\lambda^2} \frac{\lambda^3}{2 \beta^2} + \frac{\lambda}{2} + \frac{1}{\beta} (\sigma^2 - 2 \beta^2) + \beta^2 - \sigma^2, \] (12.40)

\[ \zeta_2(\lambda) = \frac{1}{\lambda^2} \frac{\lambda^3}{2 \beta^2} + \frac{\lambda}{2} (\beta^2 - \sigma^2 - 2 \beta^2 + \beta^2 - \sigma^2)^2, \] (12.41)

\[ \zeta_3(\lambda) = \frac{d^2}{d \lambda^2} (\lambda^5 - (\beta^2 - 2 \sigma^2) \lambda^4 + \frac{1}{3} (3 \beta^4 + \beta^2 \sigma^2 - \sigma^4) \lambda^3 \\
+ 3 \sigma^2 (\beta^2 - \sigma^2) \lambda^2 - \frac{1}{3} (\beta^2 - \sigma^2)^2 (2 \beta^2 - \sigma^2) \lambda + (\beta^2 - \sigma^2)^3, \] (12.42)

\[ \zeta_4(\lambda) = \frac{d^2}{d \lambda^2} (\lambda^5 - (3 \beta^2 + 4 \sigma^2) \lambda^4 + \frac{2}{3} (3 \beta^4 + 4 \beta^2 \sigma^2 - 4 \sigma^2) \lambda^3 \\
- (\beta^2 - \sigma^2)(2 \beta^2 + 7 \sigma^2) \lambda^2 - \frac{2}{3} (\beta^2 - \sigma^2)^2 (2 \beta^2 - \sigma^2) \lambda + 7(\beta^2 - \sigma^2)^3, \] (12.43)

\[ \zeta_5(\lambda) = \frac{d^2}{d \lambda^2} (\lambda^7 - (\beta^4 + 2 \sigma^4) \lambda^6 + \frac{7}{3} (2 \beta^6 + 3 \beta^2 \sigma^4 - 3 \sigma^6) \lambda^5 \\
+ (\beta^2 - \sigma^2)(\beta^4 + 3 \beta^2 \sigma^2 - 5 \sigma^4) \lambda^4 - \frac{1}{3} (\beta^2 - \sigma^2)^2 (4 \beta^4 + 3 \beta^2 \sigma^2 - 3 \sigma^4) \lambda^3 \\
+ (\beta^2 - \sigma^2)^3 (\beta^2 + 4 \sigma^2) \lambda^2 + \frac{1}{3} (\beta^2 - \sigma^2)^4 (2 \beta^2 - \sigma^2) \lambda - (\beta^2 - \sigma^2)^5, \] (12.44)

\[ K(\lambda; \beta, \sigma) \text{ and } J(\lambda; \beta, \sigma) \text{ are the those defined in Eqs. (11.28) and (11.29) with } \rho \text{ being replaced by } \beta. \]

With the anomalous dimension given above, the equation in Eq. (12.23) can be solved and gives the nucleon effective mass as follows

\[ M_R(\lambda) = M_{Re}^{-S_M(\lambda)} \] (12.45)

where

\[ S_M(\lambda) = \int_1^\lambda \frac{d \lambda}{\lambda} \gamma_M(\lambda) \] (12.46)

This integral can only be calculated numerically when the coupling constant in Eqs. (12.33) and (12.39) is taken to be the effective one. Even though the coupling constant is taken to be a constant, due to that the last terms in Eq.
identities respected by the connected Green functions and proper vertex functions. Furthermore, from the above identities, we derived the W-T BRST-invariance, we derived a set of W-T identities satisfied by the generating functionals for full Green functions, effective action and the generating functional are invariant with respect to a set of BRST-transformations. From the model action is exactly gauge-invariant. For the quantum theory, the gauge-invariance is embodied in the fact that the in the physical subspace defined by the Lorentz condition. As shown in Sec.2, under the constraint conditions, the model, we need to prove that all divergences occurring in perturbative calculations can be removed by introducing a complex one. the real part and imaginary part of the effective mass are represented in Fig. (10) by the dashed and dashed-dotted lines respectively. The figure shows that either the real part or the imaginary part behaves as an oscillating function with a damping amplitude. It is noted that in the most of practical applications to both of scattering and bound state problems, only the effective nucleon mass given by the timelike momentum subtraction is concerned.

XII. CONCLUSIONS AND DISCUSSIONS

In this paper, it has been shown that the SU(2)-symmetric model of hadrodynamics, as a massive non-Abelian gauge field theory, can surely be set up on the basis of gauge-invariance. This conclusion is achieved by the consideration that the model built up Lorentz-covariantly actually describes a constrained interacting system since the vectorial $\rho$-meson fields included in the model contains redundant unphysical degrees of freedom. Therefore, to establish a correct quantum theory of the model, it is necessary to introduce appropriate constraint conditions to eliminate the unphysical degrees of freedom, i.e., to introduce the Lorentz condition to remove the unphysical longitudinal components of the $\rho$-meson vector potentials and the ghost equation to constrain the residual gauge degrees of freedom which exist in the physical subspace defined by the Lorentz condition. As shown in Sec.2, under the constraint conditions, the model action is exactly gauge-invariant. For the quantum theory, the gauge-invariance is embodied in the fact that the effective action and the generating functional are invariant with respect to a set of BRST-transformations. From the BRST-invariance, we derived a set of W-T identities satisfied by the generating functionals for full Green functions, connected Green functions and proper vertex functions. Furthermore, from the above identities, we derived the W-T identities respected by the $\rho$-meson propagator, the $\rho$-meson three-line and four-line proper vertices, the pion-$\rho$-meson three-line and four-line proper vertices and the nucleon-$\rho$-meson proper vertex. Based on these identities, we discussed the renormalization of the propagators and the vertices. In particular, from the renormalized forms of the W-T identities obeyed by propagators and vertices, the S-T identity for the renormalization constants is naturally deduced. This identity is helpful for the renormalization by means of the renormalization group approach.

From the derivations given in this paper, it is clearly seen that there is no any difficulty to appear in performing the renormalization of the propagators and vertices as well as their W-T identities. This indicates that the SU(2)-symmetric model of hadrodynamics is renormalizable. Certainly, to give a complete proof of the renormalizability of the model, we need to prove that all divergences occurring in perturbative calculations can be removed by introducing a finite number of counterterms. For the massive non-Abelian gauge field theory without Higgs mechanism in which all the gauge bosons have the same masses such as the model under consideration, this kind of proof has actually been given in Ref. [33]. As argued in Refs. [14, 25], when we work in the renormalization group approach, an exact renormalized S-matrix element can be given by writing out the expressions of its tree diagrams provided that the coupling constant and particle masses in the S-matrix element are replaced by their effective ones which are determined by solving their RGEs. In this paper, to demonstrate the renormalizability of the model, the one-loop renormalization is performed by the renormalization group method. In this renormalization, the analytical expressions of the one-loop effective coupling constant and the effective particle masses have been derived. Since the renormalization was carried out by employing the mass-dependent momentum space subtraction scheme and exactly respecting the W-T identities, the results obtained are faithful and allow us to discuss the physical behaviors of the effective coupling constant and masses in the whole range of momentum (or distance), unlike the results given in the minimal subtraction scheme which are only suitable in the large momentum limit.

As shown in sections 10-12, in the mass-dependent renormalization, it is necessary to distinguish the results given by the timelike momentum subtraction from the corresponding ones obtained by the spacelike momentum subtraction. For example, as one can see from Fig. (4), the effective coupling constants given in the timelike and spacelike subtraction schemes have different behaviors in the low and intermediate energy region although in the large momentum limit, the difference between the both coupling constants disappears. Obviously, the both results obtained in the
timelike and spacelike momentum subtraction schemes are meaningful and suitable for different physical processes. For instance, when we study the nucleon-nucleon scattering taking place in the t-channel, the transfer momenta in the $\rho$–meson and pion propagators are spacelike. In this case, it is suitable to take the effective coupling constant and the effective $\rho$–meson and pion masses given by the spacelike momentum subtraction. If we investigate the nucleon-antinucleon annihilation process which takes place in the s-channel, since the transfer momenta are timelike, the effective coupling constant and the effective $\rho$–meson and pion masses given in the timelike momentum subtraction scheme should be used.

As mentioned in section 10-12, the one-loop effective coupling constant and the effective masses tend to zero in the large momentum (or short distance) limit. This shows that the interactions given by the SU(2)-symmetric hadrodynamics, as QCD and other non-Abelian gauge field theories, exhibits an asymptotically free behavior at least in the one-loop approximation of perturbation [34-36]. This behavior arises from the $\rho$–meson self-interactions which seems to be stronger than both of the interaction between $\rho$–mesons and nucleons and the interaction between pions and nucleons. But, we are not sure whether the asymptotically free property is still preserved beyond the one-loop approximation. To give a definite answer to this question, it is necessary to perform a nonperturbative calculation. Nuclear force is a complicated problem. At the level of hadrodynamics, it can not be solved by using only the SU(2)-symmetric model. However, if one attempts to solve the problem within the framework of gauge field theory, besides the $\sigma-\omega$ model and some others, the SU(2)-symmetric model, as an effective field theory, is necessarily to be taken into account.

XIII. ACKNOWLEDGMENT

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XIV. APPENDIX A: FEYNMAN RULES

For the application of the SU(2)-symmetric model of hydrodynamics to perturbative calculations, in this appendix, we list the Feynman rules of the model which can easily be derived from the effective action given by the effective Lagrangian in Eq. (2.26) with the Lagrangian $L$ written in Eqs. (2.1)-(2.9). In the momentum space, they represented as follows (Note: In the following, the propagators and vertices in Eqs. (A1)-(A11) are in turn represented graphically in Figs. 11a-11k, each figure should be put on the right of the corresponding formula):

Nucleon propagator:

$$i S_F(p) = \frac{i}{p - M + i \varepsilon}.$$  \hfill (A1)

Pion propagator:

$$i \Delta^{ab}_\pi(k) = \frac{i \delta^{ab}}{k^2 - m^2_\pi + i \varepsilon}.$$ \hfill (A2)

$\rho$–meson propagator:

$$i D^{ab}_{\mu \nu}(k) = \frac{-i \delta^{ab}}{k^2 - m^2_\rho + i \varepsilon} \left[ g_{\mu \nu} - (1 - \alpha) \frac{k_\mu k_\nu}{k^2 - \sigma^2 + i \varepsilon} \right]$$ \hfill (A3)

where $\sigma = \sqrt{\alpha} m_\rho$.

Ghost particle propagator:

$$i \Delta^{ab}(k) = \frac{-i \delta^{ab}}{k^2 - \sigma^2 + i \varepsilon}.$$ \hfill (A4)

Nucleon-pion vertex:

$$\Lambda^a(p, q, k) = -g_\gamma_5 \tau^a.$$ \hfill (A5)

Nucleon-$\rho$–meson vertex:

$$\Lambda^a_\mu(p, q, k) = ig_\gamma_\mu T^a.$$ \hfill (A6)
where $T^a = \tau^a / 2$.

Pion-$\rho$-meson three-line vertex:
\[
\bar{\Lambda}_{\mu}^{abc}(k_1, k_2, k_3) = g\varepsilon^{abc}(k_1 - k_2)_{\mu}.
\] (A7)

(corresponding to Fig. 11g)

Pion-$\rho$-meson four-line vertex:
\[
\bar{\Lambda}_{\mu\nu}^{abcd}(k_1, k_2, k_3, k_4) = ig^2(\varepsilon^{ace\varepsilon^{bde}} + \varepsilon^{ade\varepsilon^{bce}})g_{\mu\nu}.
\] (A8)

$\rho$-meson three-line vertex:
\[
\Lambda_{\mu\nu\lambda}^{abc}(k_1, k_2, k_3) = -g\varepsilon^{abc}[g_{\mu\nu}(k_1 - k_2)_{\lambda} + g_{\nu\lambda}(k_2 - k_3)_{\mu} + g_{\lambda\mu}(k_3 - k_1)_{\nu}].
\] (A9)

$\rho$-meson four-line vertex:
\[
\Lambda_{\mu\nu\rho\sigma}^{abcd}(k_1, k_2, k_3, k_4) = -ig^2[\varepsilon^{abc\varepsilon^{cde}}(g_{\mu\sigma}g_{\nu\rho} - g_{\mu\rho}g_{\nu\sigma})
+ \varepsilon^{ace\varepsilon^{bde}}(g_{\mu\rho}g_{\nu\sigma} - g_{\mu\sigma}g_{\nu\rho})]
+ \varepsilon^{ade\varepsilon^{bce}}(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\sigma}g_{\rho\nu}).
\] (A10)

Ghost vertex:
\[
\bar{\tilde{\Lambda}}_{\mu}^{abc}(k_1, k_2, k_3) = g\varepsilon^{abc}k_{2\mu}.
\] (A11)

XV. REFERENCES

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XVI. FIGURE CAPTIONS

Fig. (1): The $\rho$–meson one-loop self-energy. The solid, helical, wavy and dashed lines represent the nucleon, $\rho$–meson, pion and ghost particle free propagators, respectively.
Fig. (2): The one-loop ghost particle self-energy. The lines represent the same as in Fig. (1).
Fig. (3): The one-loop ghost-$\rho$–meson vertices. The lines mark the same as in Fig. (1).
Fig. (4): (a) The one-loop effective coupling constants $\alpha_R(\lambda)$ given by the timelike momentum space subtraction; (b) the one-loop effective coupling constants $\alpha_R(\lambda)$ given by the spacelike momentum space subtraction. In the subfigures, the solid, dashed and dashed-dotted lines represent the coupling constants arising from the $\rho$–meson self-interaction, the interaction between $\rho$–mesons and pions and the interaction between $\rho$–mesons and nucleons, respectively.
Fig. (5): The one-loop effective $\rho$–meson masses $m_R^\rho(\lambda)$. The solid and the dashed lines represent the effective masses given in the timelike and spacelike momentum subtractions respectively.
Fig. (6): The pion self-energy. The lines represent the same as mentioned in Fig. (1).
Fig. (7): The one-loop effective pion masses $m_R^\pi(\lambda)$. The solid and the dashed lines represent the effective masses given in the timelike and spacelike momentum subtractions, respectively.
Fig. (8): The one-loop nucleon-ghost particle vertices. The lines represent the same as in Fig. (1).
Fig. (9): The one-loop nucleon self-energy. The lines represent the same as in Fig. (1).
Fig. (10): The one-loop effective nucleon masses $M_R(\lambda)$. The solid line represents the effective mass given by the timelike momentum space subtraction. The dashed and dashed-dotted lines respectively represent the real part and the imaginary part of the effective mass given by the spacelike momentum subtraction.
Fig. (11): The figures for Feynman rules. There are 11 figures in Fig. (11). These figures numbered as fig.11a-fig. 11k in turn correspond to the formulas (A1)-(A11) in Appendix. Each figure should be put on the right of the corresponding formula.
(a) \(- (l + k)\)
(d)

\[ l \]

\[ k \alpha \mu \]

\[ l - k \]

\[ k \beta \nu \]
\((\epsilon)\) \quad -(l + k)\n
\[ k \ a \ \mu \quad c \quad d \quad d' \quad c' \quad k \ b \ v \]

\( l \)
\[(a) \quad k = p - q \]
effective $\rho$ meson mass $m_\rho(\lambda)/m_\rho^0$
(a)

\[
\begin{array}{c}
\mu \\
k a \quad k - l \quad k b
\end{array}
\]

\[
\begin{array}{c}
\nu' \\
c' \quad c
\end{array}
\]
\((b)\)
effective pion mass $m_{\pi}(\lambda)/m_{\pi}^0$
\( p \rightarrow p - l \rightarrow k = q - p \)

\( l \rightarrow q - l \)

\( (a) \)

\( p \rightarrow l \rightarrow l - p \)

\( e \rightarrow k = q - p \)

\( (b) \)
\[(a) \quad b \nu \quad (p + k) \quad a \mu \quad p\]
effective nucleon mass $M_N(\lambda)/M_N^0$
$\mathbf{p}$
\[ a \rightarrow k \rightarrow b \]
\[ k_4 d \nu \xrightarrow{k_3 c \mu} k_1 a \xrightarrow{k_2 b} \]
$k_4 d \sigma \times k_3 c \rho$

$k_1 a \mu \times k_2 b v$
$k_3 c \mu$

$k_1 a$  $k_2 b$