The optical phenomena in multiple-photon interactions (I)

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Abstract

In the recent experiment, the phenomena of superluminal and slow-light propagation in dispersive medium were found, and there are various explanation in theory. We find the phenomenon can be explained by multiple-photon interaction. Otherwise, the multiple-photon interaction can also appear other optical phenomenon: doubling frequency, sum frequency, difference frequency which are found in nonlinear optics.

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1. Introduction

Since laser appeared in 1960’s, many new optical phenomena were found in nonlinear optics, e.g. frequency doubling, sum frequency, many-photo absorption, light condense, ultra-short impulse, laser self-interaction and so on [1]. These phenomena are generated when strong light interact with the nonlinear medium. In the past years, the new optical phenomena of superluminal and slow-light were found in the dispersion medium [2][3][4][5][6], and there are various explanation in theory. In this paper, we research these new optical phenomena with multiple-photon interaction, photon-photon interaction belongs to electromagnetic interaction, and it must satisfy some conservation laws, one of them is C parity conservation. For photon, C parity $\eta_c = -1$. So, photon-photon interaction should satisfy [7]:

(1) if the results of photon-photon interaction generate photons, then even photons will generate even photons and odd photons will generate odd photons.

(2) if the results of photon-photon generate material particle ($m_0 \neq 0$), then they will be generate neutral particle $n_0$ or neutral particle system ($n + \bar{n}$).

In quantum field theory, there isn’t direct-acting among photons, and they interact indirectly by Dirac vacuum. More recently, the noncommutative QED theory has been developed [10][11]. In the theory, photon has self-interaction unlike the case of ordinary QED, and Many new predictions [10][11] can be obtained from the noncommutative QED theory: multiple-photon direct-acting, superluminal photon, the photons of bound states and so on. In this paper, we can also obtain many new optical phenomena from multiple-photon interaction, and they are discussed only in kinematics. The research from dynamics will be appeared in the next paper.

2. Doubling frequency, sum frequency and difference frequency

In the following, we can obtain the optical phenomena of doubling frequency, sum frequency and difference frequency from three-photon collision. We consider two kinds of collision of three-photon.

case (a):
where $\nu_i$ ($i = 1, 2, \cdots$) is frequency of $i$-th photon, the arrows represent moving direction of photon. In the photon collision, the energy and momentum are conservative.

$$h(\nu_1 + \nu_2 + \nu_3) = h(\nu_4 + \nu_5 + \nu_6) \quad (1)$$

$$h\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_3}\right) = h\left(\frac{1}{\lambda_4} + \frac{1}{\lambda_5} - \frac{1}{\lambda_6}\right) \quad (2)$$

Combining the Eq.(1) and (2)

$$\nu_3 = \nu_6 \quad (3)$$

$$\nu_1 + \nu_2 = \nu_4 + \nu_5 \quad (4)$$

From Eq.(4) if $\nu_4 = 2\nu_1$ then $\nu_5 = \nu_2 - \nu_1$ and if $\nu_4 = 3\nu_1$ then $\nu_5 = \nu_2 - 2\nu_1$. So three-photon collision may produce frequency doubling, triplex frequency and difference frequency light phenomena.

**case (b):**

$$h\nu_1 \rightarrow h\nu_2 \rightarrow h\nu_3 = h\nu_4 \rightarrow h\nu_5 \rightarrow h\nu_6$$

From the conservation law of energy and momentum

$$h(\nu_1 + \nu_2 + \nu_3) = h(\nu_4 + \nu_5 + \nu_6) \quad (5)$$

$$h\left(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_3}\right) = h\left(\frac{1}{\lambda_4} - \frac{1}{\lambda_5} - \frac{1}{\lambda_6}\right) \quad (6)$$

From Eq.(5) and Eq.(6)

$$\nu_6 = \nu_1 + \nu_2 \quad (7)$$

So three-photon collision can also produce sum frequency light. In experiment, frequency doubling, many-times frequency, sum frequency and difference frequency light are usually found in nonlinear medium [1]. We think these optical phenomena can be produced from collision three-photon or more photons.
3. Phenomena of superluminal

The superluminal phenomena have been found in many experiments [2]. We think the phenomena can be observed in three-photon collision, as follows:

**case (a):**

\[
\frac{h\nu_1}{h\nu_2} - \frac{h\nu_3}{h\nu_4} = 0
\]

From the conservative laws of energy and momentum

\[
E = h(\nu_1 + \nu_2 + \nu_3) \quad (8)
\]
\[
P = \frac{h}{\lambda_1} + \frac{h}{\lambda_2} - \frac{h}{\lambda_3} \quad (9)
\]

Where \(E\) and \(P\) are energy and momentum of photon \(\nu_4\)

The speed of photon \(\nu_4\) is \(\tilde{c}\):

\[
\tilde{c} = \frac{E}{P} = \frac{h(\nu_1 + \nu_2 + \nu_3)}{h(\frac{1}{\lambda_1} + \frac{1}{\lambda_2} - \frac{1}{\lambda_3})} = \frac{c(\nu_1 + \nu_2 + \nu_3)}{\nu_1 + \nu_2 - \nu_3} > c
\]

**case (b):**

\[
\frac{h\nu_1}{h\nu_2} - \frac{h\nu_3}{h\nu_4} = 0
\]

Let \(\nu_2 = \nu_3\), the energy and momentum of photon \(\nu_4\) is

\[
E = h\nu_1 + h\nu_2 + h\nu_3 \quad (10)
\]
\[
P = \frac{h}{\lambda_1} \quad (11)
\]

The speed of photon \(\nu_4\) is

\[
\tilde{c} = \frac{E}{P} = c\frac{\nu_1 + \nu_2 + \nu_3}{\nu_1} > c
\]

Obviously, many-photon \((n > 3)\) collision can also produce superluminal photon.
4. Phenomena of slow-light

More recently, the phenomena of slow-light were found in many experiments\cite{6}\cite{7}\cite{8}\cite{9}. We think it can also be observed in many-photon collision.

\[
\begin{align*}
\frac{h\nu_1}{h\nu_2} + h\nu_3 &= h\nu_4 + n_0 \\
\end{align*}
\]

The photon $\gamma_4$ energy and momentum are

\[
E = h\nu_1 + h\nu_2 + h\nu_3 - mc^2
\]

\[
P = \frac{h}{\lambda_1} + \frac{h}{\lambda_2} - \frac{h}{\lambda_3} \pm P_0
\]

where $mc^2$ and $P_0$ are energy and momentum of neutral particle $n_0$. In Eq.(13), ” $\pm$” is from the same or opposite momentum direction of photon $\nu_4$ and particle $n_0$.

The speed of photon $\gamma_4$ is

\[
\tilde{c} = \frac{E}{P} = c \frac{\nu_1 + \nu_2 + \nu_3 - \frac{mc^2}{h}}{\nu_1 + \nu_2 - \nu_3 \pm \frac{P_0c}{h}}
\]

if

\[
\frac{\nu_1 + \nu_2 + \nu_3 - \frac{mc^2}{h}}{\nu_1 + \nu_2 - \nu_3 \pm \frac{P_0c}{h}} < 1
\]

then

\[
\tilde{c} < c
\]

The Eq. (15) is

\[
2h\nu_3 < mc^2 \pm P_0c
\]

When the momentum direction of photon $\nu_4$ and particle $n_0$ is same, the Eq. (17) is

\[
2h\nu_3 < mc^2 - P_0c^2
\]

On this condition, photon $\gamma_4$ is slow-light.

When the momentum direction of photon $\nu_4$ and $n_0$ is opposite, the Eq. (17) is
\[ 2h\nu_3 < mc^2 + P_0c \]  

(19)

On this condition, photon \( \gamma_4 \) is slow-light.

We think that the phenomena of slow-light can also been found in multiple-photon interaction, which means there are slow-light photons in the vacuum.

5. Particle mass problem

In particle physics, particle mass is from spontaneous symmetry breaking and Higgs mechanism and particles can be produced by photon-photon interaction and they can be annihilation into photons. We can give particle rest mass \( m_0 \) in photon-photon interaction. We can consider it as follows:

case (a): two-photon collision

\[
\begin{array}{c}
\hline
\nu_1 \quad \nu_2 \\
\hline
\end{array} = n_0
\]

\[ \nu_1 + \nu_2 \rightarrow n_0 \]

From the conservative laws of energy and momentum

\[
E = h\nu_1 + h\nu_2 \\
\]

(20)

\[
P = \frac{h}{\lambda_2} - \frac{h}{\lambda_1} \\
\]

(21)

Where \( E \) and \( P \) are energy and momentum of particle \( n_0 \). From special relatively, the relation between the total energy \( E \) of particle \( n_0 \) and its momentum \( P \) and rest mass \( m_0 \) is

\[
E^2 = m_0^2c^4 + c^2P^2 \\
\]

(22)

From Eq.(20), (21) and (22)

\[
E_0^2 = m_0^2c^4 = E^2 - c^2P^2 = h^2(\nu_1 + \nu_2)^2 - c^2(\frac{h(\lambda_2 - \lambda_1)}{\lambda_1\lambda_2})^2 = 4h^2\nu_1\nu_2
\]
and so

\[ m_0 = \frac{2h\sqrt{\nu_1\nu_2}}{c^2} = \frac{2\sqrt{\varepsilon_1\varepsilon_2}}{c^2} \]  \tag{23} \]

The kinetic energy \( T \) of particle \( n_0 \) is

\[ T = E - E_0 = h\nu_1 + h\nu_2 - 2h\sqrt{\nu_1\nu_2} = h(\sqrt{\nu_1} - \sqrt{\nu_2})^2 \]  \tag{24} \]

case (b): Multiple-photon collision

\[
\begin{array}{c}
\hline
\hline
h\nu_1 & h\nu_2' \\
\hline
\vdots & \vdots \\
\hline
h\nu_m & h\nu_{n'} \\
\hline
\end{array}
= n_0
\]

\[ (\nu_1 + \nu_2 + \cdots + \nu_m) + (\nu_1' + \nu_2' + \cdots + \nu_{n'}) \rightarrow n_0 \]

The total energy \( E \) and momentum \( P \) of particle \( n_0 \) are:

\[ E = h\nu_1 + h\nu_2 + \cdots + h\nu_m + h\nu_1' + h\nu_2' + \cdots + h\nu_{n'} \]  \tag{25} \\
\[ P = \frac{h}{\lambda_1} + \frac{h}{\lambda_2} + \cdots + \frac{h}{\lambda_m} - \frac{h}{\lambda_1'} - \frac{h}{\lambda_2'} - \cdots - \frac{h}{\lambda_{n'}} \]  \tag{26} \\

and

\[ E^2 = m_0^2c^4 + c^2P^2 \]  \tag{27} \\

From Eq. (25), (26), and (27)

\[ E_0^2 = m_0^2c^4 = E^2 - c^2P^2 = h^2(\nu_1 + \nu_2 + \cdots + \nu_m + \nu_1' + \nu_2') \]
\[ + \cdots + \nu_{n'} \}^2 - h^2(\nu_1 + \nu_2 + \cdots + \nu_m - \nu_1' - \nu_2' - \cdots - \nu_{n'} \}^2 \]
\[ = 4h^2(\nu_1 + \nu_2 + \cdots + \nu_m)(\nu_1' + \nu_2' + \cdots + \nu_{n'}) \]
and so

\[ m_0 = \frac{E_0}{c^2} = \frac{2h\sqrt{(\nu_1 + \nu_2 + \cdots + \nu_m)(\nu_1' + \nu_2' + \cdots + \nu_n')}}{c^2} = \frac{2\sqrt{(\varepsilon_1 + \varepsilon_2 + \cdots + \varepsilon_m)(\varepsilon_1' + \varepsilon_2' + \cdots + \varepsilon_n')}}{c^2} \] (28)

where \( \varepsilon_i = h\nu_i, \varepsilon_j = h\nu_j' \) are energy of photons whose frequency are \( \nu_i, \nu_j' \). The kinetic energy of particle \( n_0 \) :

\[ T = E - E_0 = h(\nu_1 + \nu_2 + \cdots + \nu_m + \nu_1' + \nu_2' + \cdots + \nu_n') - 2h\sqrt{(\nu_1 + \nu_2 + \cdots + \nu_m)(\nu_1' + \nu_2' + \cdots + \nu_n')} = h(\sqrt{\nu_1 + \nu_2 + \cdots + \nu_m - \sqrt{\nu_1' + \nu_2' + \cdots + \nu_n'}})^2 \] (29)

So all matter particle \( (m_0 \neq 0) \) can be produced in photons collisions. Its rest mass \( m_0 \) is from the energy of interaction photons, and its kinetic energy is the rest energy of photons after those photons generate matter particle.

6. Conclusion

There are a number of new optical phenomena were found in nonlinear medium. We think these phenomena can be explained by multiple-photon interaction. Otherwise, matter particle can be obtained by the energy of interaction photons. These predictions can be checked in photon collider, but the dynamics mechanism of multi-photon interaction will be further research.
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