A thermal theory for charged leptons

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We propose that charged leptons are the ‘baryons’ of the gauge dynamics subject to a SU(2)_L×SU(2)_R×SU(2)_R symmetry. The ‘mesons’ of this theory are neutral and thus candidates for dark matter. There is, in addition, a gauge group SU(2)_{CMB} generating the CMB photon. We compute the value of \( \alpha_{em} \) and the contribution to \( \Lambda_{\text{cosm}} \) coming from SU(2)_{CMB}. We qualitatively explain the occurrence of dark matter, large intergalactic magnetic fields, a lepton asymmetry, the stability of \( T_{\text{CMB}} \), and the deviation from thermal QED behavior of hot plasmas present in the interior of the sun and terrestrial nuclear fusion experiments.

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In \( \Box \) we have developed an analytic, inductive approach to hot SU(N) pure gauge dynamics. Here we apply this approach. In the approach \( \Box \) we have observed that an SU(2) gauge theory develops an electric phase \( (2\text{nd order thermal phase transition}) \), presumably at a temperature \( T_E^\ast \) tremendously higher than \( T_c \) (deconfinement temperature), maybe we have \( T_E^\ast \sim M_P \) (\( M_P \) the Planck mass). In the electric phase SU(2) is spontaneously broken to U(1) by an adjoint Higgs field \( \phi \) describing the condensation of calorons \( \Box \). The tree-level massless (TLM) gauge boson - the photon - acquires mass by a one-loop radiative tadpole correction, see Fig. 1. This mass is given as

\[
m_{\text{TLM}} = 0.433/\sqrt{8\pi} \sqrt{\Lambda_E^2/(2\pi T)} + O(1/e^2),
\]

where \( \Lambda_E \) denotes the mass scale governing the electric phase, and \( e \) is the (large) SU(2) gauge coupling. As a function of \( T m_{\text{TLM}} \) has a global maximum at the critical temperature \( T_M^\ast \) where a mild 1st order thermal phase transition towards the magnetic phase takes place. The mass \( m_{\text{TLM}} \) decreases with \( T^{-1/2} \) as \( T \) grows. Taking \( T \) to be room temperature, \( T \sim 6.5 \times 10^{-2} \text{eV} \), the temperature of visible radiation, \( T \sim 6.5 \text{eV} \), the temperature in the interior of the sun, \( T \sim 10^7 \text{eV} \) and assuming \( \Lambda_E \sim 3.91 \times 10^{-4} \text{eV} \) (see below) we arrive at a thermal photon mass in the electric phase of about \( 5.9 \times 10^{-7} \text{eV} \), \( 5.9 \times 10^{-8} \text{eV} \), \( 4.8 \times 10^{-11} \text{eV} \), respectively. The thermal photon mass is not to be confused with the only valid bound on the photon mass \( (\sim 10^{-14} \text{eV}) \) coming from precision measurements of the Coulomb potential \( \Box \), see below. In unitary gauge off-diagonal gauge bosons \( (V^\pm) \) have large tree-level masses (TLH mode \( \Box \)) with small radiative corrections throughout the entire electric phase. There is one species of quasiclassical, stable BPS magnetic monopoles and antimonopoles in the electric phase \( \Box \Box \Box \) to which the massive gauge bosons \( (V^\pm) \) do not couple.

Under the intact U(1) gauge symmetry in the electric phase the monopole charge \( g \) is given as \( g = 2\pi/e \). As the system cools down the electric coupling \( e \) strongly rises from a value very close to zero at \( T_E^\ast \) and then quickly relaxes to a value \( e \sim 17.15 \). Close to a temperature \( T_M^\ast \ll T_E^\ast \) a regime of logarithmic blow-up is reached. At \( T_M^\ast \) \( V^\pm \) gauge bosons decouple kinematically.

The value \( e \sim 17.15 \), see Fig. 2, is independent of the initial condition \( e(T_E^\ast) = 0 \) as long as \( T_E^\ast \) is sufficiently larger than \( T_M^\ast \). Once the system has undergone the transition to the magnetic phase by condensing monopoles, the left-over photon is exactly massless \( \Box \) on the magnetic side of the phase boundary. In the magnetic phase the gauge coupling \( g \) and the photon mass quickly increases with decreasing temperature, see Fig. 1. The ground state of the system becomes superconducting with respect to the spontaneously broken U(1) gauge symmetry. Considerably far away from the transition center vortices form due to a growing size of monopoles in the condensate \( \Box \). In the magnetic phase, single crossings of center vortices (topological charge \( 1/2 \)) have charge \( 2\pi/g \) with respect to the massive photon \( \Box \).

FIG. 1: Radiative corrections arising in the electric phase if gauge-boson fluctuations are assumed to be noninteracting thermal quasiparticles. (a) - (c) thermal mass corrections and (d) - (g) corrections to the vacuum pressure. Thin lines are associated with tree-level massless (TLM) modes and thick ones with tree-level heavy (TLH) modes.
The electric charge of a lepton? what went wrong in our naive approach to the electric phase boundary, where the photon is almost massless, see below. The scale of the first factor is set by the temperature was higher than the muon but lower than the pion mass.

Using this input, we now derive the value of \( \alpha_{em} \). Since the \( \tau \) lepton has a very small life-time (\( \delta t_\tau / \delta t_\mu \approx 10^{-7} \) but \( m_\mu / m_\tau = 6 \times 10^{-2} \), decay by electroweak interactions) the photons of \( SU(2)_\tau \) scattered by this particle were most probably never thermalized. Moreover, the gauge dynamics due to \( SU(2) \), is contaminated by strong interactions for temperatures larger than the pion mass. But we only want to consider pure electromagnetism here. At \( m_\mu < T < m_\pi \) we thus consider the gauge group \( SU(2)_{\text{CMB}} \times SU(2)_\epsilon \times SU(2)_\mu \) only. We then have three species of thermalized photons \( a_\mu^i \) (\( i = \text{CMB}, \epsilon, \mu \)) in the plasma which mix quantum mechanically by a thermal mass matrix (recall, that the tree-level massless modes acquire a mass by radiative corrections in the electric phase, Fig. 1 and 3). In a thermal state it is very reasonable to assume that photon wave functions \( \tilde{\alpha}_\mu \) are generated by maximal mixing of the photon states originating from the respective dynamics of each \( SU(2) \) factor, for example

\[
\tilde{\alpha}_\mu^{\text{CMB}} = 1/\sqrt{3}(a_\mu^{\text{CMB}} + a_\mu^{\epsilon} + a_\mu^\mu). \tag{2}
\]

and similar for the orthogonal superpositions \( \tilde{\alpha}_\mu^\epsilon \) and \( \tilde{\alpha}_\mu^\mu \). When the temperature falls below \( m_\mu \), an epoch of exponential cosmological inflation of scale \( m_\mu \), which is terminated by a nonthermal 1st order transition, renders the \( SU(2)_\mu \) gauge theory confining. The photon \( a_\mu^\mu \) decouples kinematically and is almost instantaneously removed from 2 as a fluctuating degree of freedom. This changes the wave function normalization of the remaining two photons by a factor \( \sqrt{2}/\sqrt{3} \). The new maximal-mixing superposition still sees the charge of the muon because of a quantum entanglement with the kinematically decoupled photon \( a_\mu^\mu \). The coherent superposition of \( a_\mu^\mu \) and \( a_\mu^\mu \) is now normalized as

\[
\tilde{a}_\mu^{\text{CMB}} = 1/\sqrt{2}\sqrt{2/3}(a_\mu^{\text{CMB}} + a_\mu^\epsilon) \tag{3}.
\]

The further removal of \( a_\mu^\epsilon \) from the thermal spectrum by a period of exponential inflation of scale \( T \sim m_\mu \) does not imply a new normalization since only the CMB photon survives kinematically. In the electric phase and away from the boundary to the magnetic phase monopoles are much heavier than the photons, \( m_{\text{mon}} / m_{\text{photon}} = 64\pi^2/(\sqrt{2}e) = 25.5 \). Shortly before the transition to the magnetic phase they experience a strong drop in their mass and condense subsequently in the magnetic phase. This condensation slowly rotates a superposition of monopoles belonging to different \( SU(2) \) factors to the monopole of the \( SU(2) \) factor which undergoes the electric-magnetic transition (monopole excitations, which disappear in the magnetic phase, re-appear as charged leptons in the center phase). We conclude,
that the above argument for photons does not apply to monopoles. The factor $\sqrt{2}/\sqrt{3}$ in (2) reduces the wave function of the original photon or, equivalently, the gauge coupling $g \rightarrow g_\tau = \sqrt{2}/\sqrt{3} g$. As a consequence, the real value of $\alpha_{em}$ is

$$\alpha_{em} = g_\tau^2/(4\pi) \sim 1/140.433. \quad (4)$$

The deviation from the measured value $\alpha_{em} \sim 1/137$ is 2.4%! We attribute this small mismatch to the omission of radiative corrections in our evolution of the gauge coupling $\epsilon$ in the electric phase (recall that there is a zeroth-order in $1/\epsilon^2$ radiative correction to the photon mass). The result (4) is an impressive experimental confirmation for the validity of the analytic approach to thermal SU(N) Yang-Mills theory in [2]. The reader may wonder why we could predict the value of $\alpha_{em}$ from an analysis in the electric phase while charged leptons, ‘baryons’ of SU(2)$_c \times$SU(2)$_b$×SU(2)$_r$, are solitons in the center phase of this theory. The solution to this apparent puzzle is indicated in Fig. 3 and explained in its caption. The electric charge is quantized by the single monopole residing in a ‘baryon’ (or charged lepton) of spin $1/2$ as the crossing point of two center vortices. This ‘baryon’ has precisely the same charge as a monopole in the electric phase due to the topological equivalence of a center-vortex crossing and an isolated monopole [11].

In the magnetic phase monopoles condense, the magnetic coupling $g$ is precisely zero at the magnetic-electric transition and the electric coupling $\epsilon$ is infinite. The critical temperature $T_E^c$ is the only point where a magnetic description is precisely dual to an electric description of the underlying SU(2) gauge theory. In the course of the magnetic-center transition, when center vortices condense, monopoles re-appear on the crossings of center vortices. In a given spatial region, charged lepton number is not conserved. This leads us to a new mechanism for the generation of lepton asymmetry. Recall Sakharov’s conditions [11] for this: (i) lepton number violation, (ii) violation of thermal equilibrium, and (iii) a CP violation in the interaction. We just discussed the violation of electric charge (in the sense of the Standard Model) during the magnetic-center transition. So condition (i) is met. The transition to the center phase is truely 1$^\text{st}$ order. It is preceded by an epoch of exponential inflation (a photon decouples kinematically shortly before the transition and thus the energy density $\rho_{vac}$ is the one of the ground state, the equation of state is $P_{vac} = -\rho_{vac}$). So condition (ii) is met. We know that weak interactions violate CP due to a 3×3 CKM matrix for the three visible charged leptons. So there is a CP violating phase, condition (iii) is also met. The magnitude of lepton asymmetry depends on the mass of the charged lepton since this sets the scale for the strength of the 1$^\text{st}$ order transition to the center phase. It also depends on the structure of the CKM matrix. Lepton asymmetry should then be computable from the measured CKM matrix in the Standard Model and the possibility to estimate the strength of the 1$^\text{st}$ order transition. Let us now derive a boundary condition for the gauge theory SU(2)$_{CMB}$ from experimental information. On cosmological scales we do not see neither center vortices nor do we see light-mass electric charges at any temperature in laboratory experiments [12] (monopoles and antimonopoles are extremely dilute in the electric phase). We do, however, see a massless photon in the CMB radiation which has an (almost) perfect thermal spectrum. This leads us to the conclusion that the CMB temperature $T_{CMB}$ is slightly lower than the critical temperature $T_E^c$ of the electric-magnetic transition. Only for $T_{CMB} + 0 = T_E^c$ do we have a condensate of monopoles (no explicit monopoles!) and not yet an explicit center vortex in our theory. Moreover, in our theory the CMB photon is exactly massless at $T_{CMB} = T_E^c - 0$ due to a logarithmic blow-up of $\epsilon$ on the electric side of the phase boundary rendering $g = 0$. As a consequence, the Abelian Higgs mechanism is not operative close to the phase boundary. So the boundary condition is: $T_{CMB} = T_E^c$.

The cooling of the ground state into superconductivity for $T < T_{CMB}$ will change our universe drastically. The onset of this regime is already seen in the existence of large intergalactic magnetic fields. The generation of these magnetic fields is driven by one-sided temperature fluctuations which drive the associated, intergalactic region away from the electric-magnetic phase boundary into the magnetic phase, see Fig. 4. What is the mechanism that prevents us from running deep into the superconducting regime? The thermal mass of the CMB photon in the electric phase jumps from $m_{\gamma,CMB} = 0.433/(\sqrt{8\pi})\sqrt{\Lambda_E}/(2\pi T_{CMB}) \sim 10^{-5}$ eV to zero across the phase boundary. In the magnetic phase the CMB photon acquires mass as temperature drops. As a function of temperature this creates a potential for the photon mass with a wall of height $\sim 10^{-5}$ eV to the right and smoothly increasing mass to the left of the minimum at $T = T_{CMB}$, see Fig. 4. This effect keeps the CMB photon from decaying.
the dynamics of SU(2) are seen in a lattice simulation of the energy density $\rho$ actually taken from the ground state. The stabilizing effect is unreasonably large for about another present age of the universe. The energy needed for gravitational expansion will put to an end the existence of the CMB photon.

Since the masses of charged leptons are much higher than $T_{\text{CMB}}$ ($m_e = 5 \times 10^5$ eV $\sim 2.3 \times 10^9 T_{\text{CMB}} \sim \Lambda_c$) the dynamics of SU(2)$_c$, SU(2)$_\mu$, and SU(2)$_e$ are all in their confining phase at $T_{\text{CMB}}$. The vacuum energy densities and the vacuum pressures in all three theories are precisely zero [1] (this is protected by local $Z_2$ symmetries). Let us now calculate the electromagnetic, homogeneous contribution to the cosmological constant. At the electric-magnetic phase boundary we have $\lambda_M(0) = (1/4)^{1/3} \times 9.193$ where due to our above boundary condition $\lambda_M(0) \equiv 2\pi T_{\text{CMB}}/\Lambda_M$ and $\Lambda_M$ denotes the magnetic scale of SU(2)$_{\text{CMB}}$, see [1]. Prescribing $T = T_{\text{CMB}} = 2.728 K = 2.1824 \times 10^{-4}$ eV, yields $\Lambda_M = 2.368 \times 10^{-4}$ eV. The resulting vacuum energy density is $1/2 V_M = \pi T_{\text{CMB}} \Lambda_M^3 = (3.1 \times 10^{-44} \text{eV})^4$. This is smaller than the commonly accepted value $\Lambda_{\text{cosmo}} \sim (10^{-3} \text{eV})^4$. There are two possible reasons for this. First, we have used a tree-level analysis to determine the critical value $T_E$ in our approach. Including radiative corrections in the running of the electric gauge coupling $e$ should yield a slightly smaller value than $\lambda_c = 9.193$ since massive TLM modes slow down the evolution, see [1]. This should imply a slightly higher value of $1/2 V_M$. Second, there are local regions with negative pressure induced by high-energy particle collisions subject to SU(N) gauge dynamics. After coarse-graining this effect also contributes to $\Lambda_{\text{cosmo}}$.

Let us now discuss the validity of thermal QED. The additional photon belonging to SU(2)$_c$ can not be seen in terrestrial experiments with hot plasmas up to $T \sim m_e = 5 \times 10^5$ eV. The highest temperatures reached in nuclear fusion experiments are about $T \sim 10^7$ eV - the temperature in the interior of the sun. Due to the occurrence of a second photon, the strong dilution of the electron density in the electric phase of SU(2)$_c$, and a considerable contribution of vacuum energy these plasmas should show a strong deviation from standard thermal QED behavior.

All solar models rely on standard thermal QED and thus should be revised. This should have a major impact on the prediction of neutrino fluxes. As for Tokamaks we predict a strong increase of the magnetic field at temperatures close to the electron mass and a disappearance of electronic electric charge at about 1.8 times this temperature. This effect could be measured by applying a large and homogeneous electric field to the Tokamak. Positively charged ions should then move to one end and no negative charge should be measured at the other end of the Tokamak.

To summarize, we have proposed a strongly interacting gauge theory underlying electrodynamics, and we have explored some of the consequences of this theory. The apparent structurelessness of the electron and the muon, as it is seen in high-energy experiments for the modulus of momentum transfers away from the electron and muon mass, is explained in [8].

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