Tunneling magnetoresistance in junctions composed of ferromagnets and time-reversal invariant topological superconductors

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Abstract

Tunneling magnetoresistance between two ferromagnets is an issue of fundamental importance in spintronics. In this work, we show that tunneling magnetoresistance can also emerge in junctions composed of ferromagnets and time-reversal invariant topological superconductors without spin-rotation symmetry. Here the physical origin is that when the spin-polarization direction of an injected electron from the ferromagnet lies in the same plane of the spin-polarization direction of Majorana zero modes, the electron will undergo a perfect spin-equal Andreev reflection, while injected electrons with other spin-polarization directions will be partially Andreev reflected and partially normal reflected, which consequently has a lower conductance, and therefore, the magnetoresistance effect emerges. Compared to conventional magnetic tunnel junctions, an unprecedented advantage of the junctions studied here is that arbitrary high tunneling magnetoresistance can be obtained even when the magnetization of the ferromagnets are weak and the insulating tunneling barriers are featureless. Our findings provide a new fascinating mechanism to obtain high tunneling magnetoresistance.

1. Introduction

The superconducting phase and the ferromagnetic phase are the two most familiar and classic symmetry-broken phases. The former one breaks the U(1) gauge symmetry, while the latter one breaks the full spin-rotation symmetry (SRS) down to an axis-fixed rotation symmetry, as a consequence of symmetry breaking, the order parameter characterizing the superconductor (SC) takes a fixed phase, while the one characterizing the ferromagnet (FM) takes a fixed direction. Interestingly, the break down of the two symmetries has a direct impact on the tunneling behavior of junctions composed of SCs or FMs. For junctions formed by two SCs sandwiching a thin insulator (also known as an insulating tunneling barrier (ITB) (SC-I-SC junction), the tunneling current depends on the phase-difference [1], while for junctions formed by two FMs sandwiching a thin insulator (FM-I-FM junction, known as magnetic tunnel junction (MTJ)), tunneling current depends on the angle-difference of the magnetization directions [2]. The former phenomenon is known as the Josephson effect, while the latter one is known as the magnetic valve effect (MVE) [3]. Both effects are very fascinating and have very wide applications, for the latter one, there is a quantity named as tunneling magnetoresistance (TMR) to characterize it. Higher TMR has always been pursued since the concept was proposed because a higher TMR implies a better performance of the effect in real applications, such as field sensor and magnetic random access memory [4–9].

SCs of nontrivial topological properties are known as topological superconductors (TSCs) [10]. Due to hosting of the Majorana zero modes [11, 12] which have potential application in topological quantum computation [13, 14], TSCs and topological superfluids (TSFs) have been among the central themes of both condensed matter and cold atom physics in recent years [15–35]. According to the existence or absence of time-
reversal symmetry, particle-hole symmetry and sublattice symmetry (or chiral symmetry), TSCs can be classified in a ten-fold way [36–38]. It is interesting to find that TSCs in some classes, e.g. BDI class and DIII class, also break the SRS. Based on this observation, it leads us to expect that a FM-I-TSC junction will also exhibit TMR if the TSC breaks SRS. A direct investigation confirms our expectation and what interesting is that the nontrivial topological property of the SC endows nontrivial property to the TMR. For example, we find that for a generalized FM-I-FM-I-TSC junction, arbitrary high TMR can be obtained even the magnetization of the ferromagnets are weak and the insulating tunneling barriers are featureless.

The paper is organized as follows; in section 2, the theoretical model and main picture are given. In sections 3 and 4, the tunneling spectroscopies of a one-dimensional FM-I-TSC junction and a one-dimensional FM-I-FM-I-TSC junction are studied in detail, and based on the tunneling spectroscopies, TMR’s dependence on parameters are obtained. In section 5, the higher dimensional case of the FM-I-FM-I-TSC junction is studied, and similar results like the one-dimensional case are obtained. In section 6, discussions and conclusions about the results obtained in previous sections are given.

2. The theoretical model and main picture

The general picture of the junction structure we will study is illustrated in figure 1(a). But to obtain a clear physical picture, we will first study the simplest case, a one-dimensional FM-I-TSC junction. Before we write down the Hamiltonian, it is worth stressing the fact that in one dimension, if the Cooper pairs of the superconductor is spin-polarized (\(S_0 z \neq 1\), \(S_0 z\) is the total spin angular momentum of the Cooper pair), the zero-bias conductance (ZBC) of a FM-I-TSC is always quantized no matter what magnetization direction the FM chooses. Therefore, to obtain remarkable TMR, the TSC needs to be time-reversal invariant and the Cooper pairs need to be un-spin-polarized (\(S_0 z = 0\)) [39–42].

Although we plan to study the one-dimensional FM-I-TSC junction firstly, here for compactness, we will write down the general Hamiltonian describing the system illustrated in figure 1(a). Under the representation \(\hat{\psi}_i^\dagger (x) = (\hat{\psi}_i^\dagger (x), \hat{\psi}_i^\dagger (x), \hat{\psi}_i^\dagger (x), \hat{\psi}_i^\dagger (x))\), the general one-dimensional (higher dimensions will be studied in lateral section) Hamiltonian is given as (\(\hbar = m = 1\)) [43].
\[
\mathcal{H} = \tau_z \left[ -\frac{\partial^2}{2} - \mu(x) + V(x) \right] + \tau_x \Delta(x),
\]

where \( \tau_i = (\tau_x, \tau_y, \tau_z) \) are Pauli matrices in particle-hole space, \( V(x) \) is potential induced by external field, etc., here we assume it takes the form

\[
V(x) = -\tau_x M_1 (\hat{n}_1 \cdot \vec{\sigma}) \Theta(x_3 - x) - \tau_x M_2 (\hat{n}_2 \cdot \vec{\sigma}) \Theta(x - x_0) - \sum_{i=1}^4 Z_i \delta(x_i - x),
\]

the terms in the first line denote the magnetization of FM1 and FM2, \( M_1 \) and \( M_2 \) denote the magnetization strength of FM1 and FM2, while \( \hat{n}_1 = (\cos \phi_1 \sin \theta_1, \sin \phi_1 \sin \theta_1, \cos \theta_1) \) and \( \hat{n}_2 = (\cos \phi_2 \sin \theta_2, \sin \phi_2 \sin \theta_2, \cos \theta_2) \) denote the magnetization directions of FM1 and FM2, respectively; \( \sigma_{x,y,z} \) are Pauli matrices acting on the spin space, \( \Theta(x) \) is the Heaviside function, i.e., \( \Theta(x < 0) = 1 \), \( \Theta(x > 0) = 0 \); the terms in the second line denote the scattering potential at the interfaces; \( \mu(x) \) is the chemical potential, we set \( \mu(x) = \mu_f \) (or \( E_f \)) for the two FMMs, \( \mu(x) = -\mu_f \) for the two insulating tunneling regions, and \( \mu(x) = \mu_j > 0 \) for the SC. \( \Delta(x) = -i \Delta_0 \delta(x - x_0) \) is the pairing potential, which is assumed to be \( p \)-wave type (then as \( \mu_j > 0 \) is assumed, the SC is a TSC with Majorana zero modes located at the boundary [44, 45]) and homogeneous at \( x > x_1 \) and vanish at \( x < x_1 \) for the sake of theoretical simplicity.

To see that the TSC breaks the SRS, we transform the Hamiltonian corresponding to the superconducting part into momentum space, then under the representation \( \hat{\Psi}^\dagger = (\hat{\psi}^\dagger_1(k), \hat{\psi}^\dagger_2(-k), \hat{\psi}^\dagger_3(k), \hat{\psi}^\dagger_4(-k)) \)

\[
\mathcal{H}_{sc}(k) = \left[ \frac{k^2}{2} - \mu_0 + \Delta(k) \tau_z \sigma_0 \right]
\]

where \( \Delta(k) = \Delta_0 k, \sigma_0 \) is the \( 2 \times 2 \) unit matrix in spin space. By making a spin-rotation: \( \hat{\psi}_1(k) \rightarrow \cos \theta \hat{\psi}_1(k) + \sin \theta \hat{\psi}_3(k), \hat{\psi}_3(k) \rightarrow -\sin \theta \hat{\psi}_1(k) + \cos \theta \hat{\psi}_3(k) \), it is easy to check that the superconducting term, \( \Delta_0 k \hat{\psi}_3^\dagger(k) \hat{\psi}_1^\dagger(-k) \) and h.c., is not invariant, and therefore breaks the SRS. The time-reversal symmetry of the Hamiltonian is easy to check: \( \mathcal{H}_{sc}(k) T^{-1} = \mathcal{H}_{sc}(-k) \) with \( T = \tau_z K \), where \( K \) is the complex conjugate operator.

To obtain the tunneling spectroscopies of the junction, here we follow the Slonczewski [3] and Blonder–Tinkham–Klapwijk (BTK) approach [46–48]. The first step of the approach is to write down the wave functions of each part of the junctions. If we consider that a majority electron with energy \( E \) (relative to \( \mu_f \)) is injected from FM1, the wave function in FM1 is given as

\[
\psi_{FM1} = \chi_1 e^{i k_1 x} + b_1 + \chi_2 e^{-i k_1 x} + a_1 + \chi_4 e^{i k_1 x}
\]

\[
+ b_1 - \chi_3 e^{-i k_1 x} + a_1 - \chi_4 e^{i k_1 x}
\]

where \( \chi_i = (\eta_1, 0, \eta_2 e^{i \theta_1}, 0)^T, \chi_2 = (0, \eta_2, 0, \eta_1 e^{i \theta_1})^T, \chi_3 = (-\eta_2, 0, \eta_1 e^{i \theta_1}, 0)^T, \chi_4 = (0, -\eta_1, 0, \eta_2 e^{i \theta_1})^T \)

\[
k_{i+,+} = \sqrt{2(\mu_f + M_i + (-\varepsilon))}, k_{i,-,-} = \sqrt{2(\mu_f - M_i + (-\varepsilon))}.
\]

The coefficients \( b_{1+(-)}, a_{1+(-)} \) denote the spin-equal (spin-opposite) normal reflection (a majority electron reflected as a majority (minority) electron) amplitude and spin-equal (spin-opposite) Andreev reflection (a majority electron reflected as a majority (minority) hole [49]) amplitude, respectively. The wave functions in other parts can be obtained easily but as their forms are tedious, their concrete expressions will be given in the appendix.

To obtain the tunneling conductance, the coefficients in the wave functions need to be determined by matching the wave functions at the interfaces according to the boundary conditions [3, 46–48]

\[
\psi_L(x = x_i) = \psi_R(x = x_i),
\]

\[
\psi_{L,R} \psi_R(x_i^+) - \nu_{L,R} \psi_R(x_i^-) = -i 2 Z_i \tau_z \sigma_0 \psi_R(x_i),
\]

where \( \psi_L(x = x_i) \) (\( \psi_R(x = x_i) \)) denotes the wave function in the left (right) neighbouring part of the interface located at \( x = x_i \), \( \nu_{L,R}(1) = \partial \mathcal{H}_{L,R}(1)/\partial k \) is the velocity operator corresponding to the right (left) neighbouring part of the interface [44].

After the coefficients are obtained, the zero temperature tunneling conductance can be determined according to the BTK formula [46]

\[
G(E = eV, \hat{n}_1, \hat{n}_2) = \frac{e^2}{h} (1 + A_+ + A_+ - B_+ - B_-),
\]
where $A_+ = k_{i+2} \langle n_{i+2}^2 \rangle / k_{i+2}$, $B_+ = k_{i-2} \langle n_{i-2}^2 \rangle / k_{i-2}$, and $B_+ = k_{i+2} \langle n_{i+2}^2 \rangle / k_{i+2}$. Similar procedures can obtain $G_+ (eV, n_i, n_2)$, the tunneling conductance for a minority electron, and the total tunneling conductance $G (eV, n_i, n_2)$ is given as the summation of $G_+ (eV, n_i, n_2)$ and $G_- (eV, n_i, n_2)$.

### 3. One-dimensional FM-I-TSC junction

Now, we are going to study the one-dimensional FM-I-TSC junction. For the FM-I-TSC junction, we consider that it is only composed of FM2, ITB2 and TSC, and $x_3$ is set to infinity. For this structure, the wave function in FM2 takes the same form as $\psi_{\text{FM2}}$, but with a substitution of the parameters: $(k_{i+2})_{(a,b)} = \sqrt{2} \mu_{F} + M_2 + (-E)$, $(k_{i-2})_{(a,b)} = \sqrt{2} \mu_{F} - M_2 + (-E)$.

By a simple numeric calculation of the coefficients of the wave functions, the tunneling conductance of the FM-I-TSC junction is shown in figures 1(b) and (c). There are two extraordinary characteristics in the tunneling spectroscopy. The first one is that the tunneling conductance is angle-dependent, the second one is that the tunneling conductance at zero-bias voltage is of topological feature in the sense that it is independent of the thickness of ITB2 and the interface scattering potential. In fact, at zero-bias voltage, the four key quantities $A_+$, $A_-$, $B_+$ and $A_-$ can be analytically obtained. When we consider that a majority electron is injected, their analytical forms are

\[
A_+ = \frac{16k_{i+2}^2 k_{i-2}^2 \sin^2 \theta_2}{[k_{i+2} + k_{i-2}]^2 \cos^2 \theta_2 + 4k_{i+2} k_{i-2} \sin^2 \theta_2^2} - 1
\]

\[
A_- = \frac{4k_{i+2} k_{i-2} (k_{i+2} + k_{i-2}) \cos^2 \theta_2}{[k_{i+2} + k_{i-2}]^2 \cos^2 \theta_2 + 4k_{i+2} k_{i-2} \sin^2 \theta_2^2}
\]

\[
B_+ = \frac{(k_{i+2}^2 - k_{i-2}^2) \cos^2 \theta_2}{[k_{i+2} + k_{i-2}]^2 \cos^2 \theta_2 + 4k_{i+2} k_{i-2} \sin^2 \theta_2^2}
\]

\[
B_- = \frac{4k_{i+2} k_{i-2} (k_{i+2} - k_{i-2}) \sin^2 \theta_2 \cos \theta_2}{[k_{i+2} + k_{i-2}]^2 \cos^2 \theta_2 + 4k_{i+2} k_{i-2} \sin^2 \theta_2^2}
\]

(8)

where $k_{2+(-)} = \sqrt{2} (\mu_{F} + (-E))$. If a minority electron is injected, the four key quantities have such an exchange, $A_+ \leftrightarrow A_-$. Then according to the equation (7), it is direct to obtain

\[
G (0, n_i) = \frac{e^2}{h} \frac{16k_{i+2} k_{i-2}}{(k_{i+2} + k_{i-2})^2 \cos^2 \theta_2 + 4k_{i+2} k_{i-2} \sin^2 \theta_2^2}
\]

(9)

The zero-bias conductance (ZBC) only depends on $\theta_2$ and the parameters of FM2. From the formula we can see that $G (0, n_i)$ is quantized as $4e^2/h$ at $\theta_2 = \pi / 2$ and takes its minimum value at $\theta_2 = 0$ and $\theta_2 = \pi$, as shown in figure 1(d). In a recent work [43], we have proven that by making use of this minimum value, it is very convenient to determine the polarization of FM. From equation (8) or more directly from figures 2(a) and (b), we can see that $\theta_2 = \pi / 2, A_+ = 1$, while the other three quantities are all equal to zero, which indicates a perfect spin-equal Andreev reflection. As this perfect spin-equal Andreev reflection is unaffected by the ITB2 and the scattering potential, here the only possible origin is the resonant tunneling due to the topological Majorana zero modes located at the boundary of the TSC. In [50], by using scattering matrix, the authors have shown that perfect spin-equal Andreev reflection will occur when the injected electron takes certain spin-polarization direction. In the following, we will show that here the magic direction is just the spin-polarization direction of the Majorana zero modes.

To obtain the zero modes in the TSC, we need to calculate the BdG equation, which is given as

\[
Eu_1 (x) = \left( -\frac{\partial^2 x}{2} - \mu \right) u_1 (x) - i\Delta \partial_x v_1 (x),
\]

\[
Ev_1 (x) = -i\Delta \partial_x u_1 (x) + \left( \frac{\partial^2 x}{2} + \mu \right) v_1 (x),
\]

\[
Eu_1 (x) = \left( -\frac{\partial^2 x}{2} - \mu \right) u_1 (x) - i\Delta \partial_x v_1 (x),
\]

\[
Ev_1 (x) = -i\Delta \partial_x u_1 (x) + \left( \frac{\partial^2 x}{2} + \mu \right) v_1 (x),
\]

(10)

with the boundary condition that the wave functions $u_1 (x), v_1 (x), u_1 (x)$ and $v_1 (x)$ all need to vanish at $x = 0$ and $x = +\infty$. For the zero modes, i.e., $E = 0$, a direct calculation gives
\[ u_i(x) = i\psi_i(x) = N \sin(\kappa x) e^{-\gamma x}, \]
\[ u_i(x) = i\psi_i(x) = N \sin(\kappa x) e^{-\gamma x}, \]

where \( \kappa = \sqrt{2\mu - \Delta^2}, \gamma = \Delta, \) and \( N \) is the normalization constant which guarantees
\[ \int_{0}^{+\infty} dx (|u_{i(1)}(x)|^2 + |\psi_i(x)|^2) = 1, \]
then the zero modes located at the \( x = 0 \) boundary is given as
\[ C_1 = |N| \int dx [e^{i\varphi}\psi_1(x) + e^{-i\varphi}\psi_i^*(x)] \sin(\kappa x) e^{-\gamma x}, \]
\[ C_2 = |N| \int dx [e^{i\varphi}\psi_1(x) + e^{-i\varphi}\psi_i^*(x)] \sin(\kappa x) e^{-\gamma x}, \]

here we have chosen \( N = |N| e^{i\varphi/4} \) for the convenience of the following discussions. By using the normalization condition, it is direct to verify \( C_1, C_2 \) are related with each other by the charge conjugate, i.e., \( C_1^\dagger = C_2 \), but \( C_1^\dagger \neq C_1 \) and \( C_2^\dagger \neq C_2 \), this indicates that the two fermionic zero modes \( C_1 \) and \( C_2 \) are not of Majorana characteristic. However, from the Kitaev Majorana chain model we already know that a fermion operator with its conjugate can construct two Majorana operators \[11\], here as \( C_1^\dagger = C_2 \), the two Majorana zero modes can be constructed as
\[ \gamma_1 = C_1 + C_2^\dagger \sqrt{2}, \quad \gamma_2 = C_1 - C_2^\dagger \sqrt{2}, \]

the Majorana zero modes are an equal-weight superposition of the two fermionic zero modes. Based on equations \[12\] and \[13\], the spin-polarization of the zero modes can be directly calculated,
\[ \langle C_1|\hat{S}_z|C_1^\dagger \rangle = \langle C_1|\hat{S}_y|C_1^\dagger \rangle = 0, \quad \langle C_1|\hat{S}_z|C_2^\dagger \rangle = -\frac{\hbar}{2}, \]
\[ \langle C_1|\hat{S}_y|C_2^\dagger \rangle = \langle C_1|\hat{S}_z|C_2^\dagger \rangle = -\frac{\hbar}{2}, \]
\[ \langle \gamma_1|\hat{S}_z|\gamma_1^\dagger \rangle = \langle \gamma_1|\hat{S}_y|\gamma_1^\dagger \rangle = 0, \quad \langle \gamma_1|\hat{S}_z|\gamma_2^\dagger \rangle = -\frac{\hbar}{2}, \]
\[ \langle \gamma_1|\hat{S}_y|\gamma_2^\dagger \rangle = -\frac{\hbar}{2}, \quad \langle \gamma_2|\hat{S}_z|\gamma_2^\dagger \rangle = 0, \quad \langle \gamma_2|\hat{S}_y|\gamma_2^\dagger \rangle = -\frac{\hbar}{2}, \]

where \( |\gamma_1 > = (e^{-i\varphi}(i\psi_1 + \psi_i^*)|^0> \), \( |\gamma_2 > = (e^{-i\varphi}(i\psi_1 + \psi_i^*)|^1> \), \( \gamma_1 > = (e^{-i\varphi/\sqrt{2}})(i\psi_1 + \psi_i^*)|^0> \), \( \gamma_1 > = (e^{-i\varphi/\sqrt{2}})(i\psi_1 + \psi_i^*)|^1> \), and
\[ \hat{S}_z = \frac{\hbar}{2} (\psi_1^* \psi_i + \psi_i^* \psi_1), \quad \hat{S}_y = -\frac{i\hbar}{2} (\psi_1^* \psi_i - \psi_i^* \psi_1), \quad \hat{S}_z = \frac{\hbar}{2} (\psi_1^* \psi_i - \psi_i^* \psi_1). \]

From equation \[14\], it is direct to
see that the spin-polarization directions of the two fermionic zero modes \( C_1 \) and \( C_2 \) are along the positive \( z \)-direction and negative \( z \)-direction, respectively, while the spin-polarization directions of the two Majorana zero modes \( \gamma_1 \) and \( \gamma_2 \) are along the positive \( x \)-direction and negative \( x \)-direction, respectively, which is a natural result according to equation (13). Projecting the spin-polarization on the Bloch sphere, the spin-polarization directions of the two fermionic zero modes \( C_1 \) and \( C_2 \) point to the north pole \( (\theta_2 = 0) \) and south pole \( (\theta_2 = \pi) \), respectively, while the spin-polarization directions of the two Majorana zero modes are lying in the equatorial plane, which corresponds to \( \theta_2 = \pi/2 \), as shown in figure 2(c).

Based on these results, the physical picture is clear: when the spin-polarization direction of the injected electron is along the \( z \)-direction, \( \theta_2 = 0 \) or \( \theta_2 = \pi \), the injected electron only feels one of the fermionic zero modes and the only possible Andreev reflection is the spin-opposite Andreev reflection. For the spin-opposite Andreev reflection, the Fermi-surface mismatch in the FM due to the magnetization will suppress it and therefore, it can not be perfect and spin-equal normal reflection will take place, as shown in figures 2(a) and (b). But as the fermionic zero modes are of topological nature, the insulating tunneling barrier and the interface scattering potential can not affect the spin-opposite Andreev reflection. When the spin-polarization direction of the injected electron is lying in the equatorial plane, i.e., \( \theta_2 = \pi/2 \), it feels the two fermionic zero modes equally and is equivalent to couple with the Majorana zero modes, and then the only possible Andreev reflection is the spin-equal Andreev reflection (because the electron coupled with either one of the two Majorana zero modes will undergo a perfect spin-equal Andreev reflection, the azimuthal angle \( \phi_2 \) has no effect to the tunneling process). For the spin-equal Andreev reflection, however, the magnetization has no effect to it and therefore, the electron will undergo a perfect Andreev reflection and consequently results in a quantized conductance. For the general case that the spin-polarization direction of the injected electrons is neither along the fermionic zero modes nor the majorana zero modes, the electron can be divided into two parts, one along the spin-polarization direction of the fermionic zero modes and the other one along the Majorana zero modes. Based on this division, the conductance given in equation (9) can also correspondingly be divided into two parts,

\[
\frac{1}{G(0, \vec{n_2})} = \frac{1}{G_F} \cos^2 \theta_2 + \frac{1}{G_M} \sin^2 \theta_2, \tag{15}
\]

where \( G_F = 16k_2, k_2, G_0/(k_{2+} + k_{2-})^2 \) and \( G_M = 4G_0 \) with \( G_0 = e^2/h \), \( G_F \) corresponds to the process that the injected electron is coupled with only one of the fermionic zero modes, and \( G_M \) corresponds to the process that the injected electron is coupled with the Majorana zero modes. The physical meaning of equation (15) is that the two processes form a parallel circuit as shown in figure 2(d). An even more clear picture can be obtained if the conductance is substituted by resistance, i.e., \( R = 1/G \), then equation (15) is correspondingly rewritten as

\[
R(0, \vec{n_2}) = R_F \cos^2 \theta_2 + R_M \sin^2 \theta_2. \tag{16}
\]

The physical meaning of equation (16) is that the total resistance of the tunneling process is a sum of the resistances of the two possible tunneling processes, which is obviously physical right.

As \( G(0, \vec{n_2}) \) has an explicit angle dependence, the junction will naturally exhibit TMR. Because low-bias voltage corresponds to low-energy consumption which is very important for real applications, the low-bias regime is of central interest. Therefore, in the following, when we consider the TMR, we will restrict ourselves to the zero-bias voltage. In the low-bias regime, \( eV \ll \Delta \), where \( \Delta \) is the energy gap of TSCs, the effect of increasing the bias voltage is a reduction of the TMR, but the reduction will be quite limited and the main physics obtained at zero-bias voltage will still hold.

Generally, the TMR is defined as

\[
\text{TMR} = \frac{R_{ap} - R_p}{R_p} = \frac{R_{\text{max}} - R_{\text{min}}}{R_{\text{min}}}, \tag{17}
\]

where \( R_{ap} \) is the electrical resistance in the anti-parallel state, whereas \( R_p \) is the resistance in the parallel state. However, here a better definition is given as

\[
\text{TMR} = \frac{G_{\text{max}} - G_{\text{min}}}{G_{\text{min}}}. \tag{18}
\]

According to the formula (18) and (9), the TMR of the junction is given as

\[
\text{TMR} = \frac{(k_{2+} - k_{2-})^2}{4k_{2+}k_{2-}}. \tag{19}
\]

Since the ZBC is of topological feature, here the TMR is also of topological feature, which is fundamentally different from the usual cases [3].

In equation (19), with the increase of magnetization strength, \( k_{2\pm} \) will continue to decrease to zero, and then TMR will go to diverge, as shown in the inset of figure 1(d). The divergent behavior indicates that if the FM turns to be a half metal, the MVE is perfect even there is only one FM. However, as shown in figure 1(d), the increase of
TMR is quite slow, even when $M_2/\mu_t$ reaches 90%, the TMR is still smaller than 100%, this implies that only when the FM is close to perfect polarization, a very high TMR can be obtained. The request of strong magnetization will greatly limit the applicable ferromagnetic materials and consequently reduces the novelty of the new MVE. In the following section, we will show that a generalized FM-I-FM-I-TSC junction overcomes this shortcoming.

4. One-dimensional FM-I-FM-I-TSC junction

The tunneling spectroscopies of the one-dimensional FM-I-FM-I-TSC junction are shown in figures 3 and 4, the most remarkable property of the tunneling spectroscopies is that when $22_2 \equiv 0$, the ZBC keeps the topological feature (as shown in figures 3(a) and 3(b)) and is found to take the same form as equation (9) but with a substitution of $k_{+\pm(-)}$ to $k_{1\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\pm\p
is of most interest. Based on this recognition, without loss of the main physics, we set $\theta_1 = \phi_1 = \phi_2 = 0$, while keeping $\theta_2$ as the only variable.

In figures 3(c) (d), it has already shown that with the increase of $d_3$, the tunneling conductance at low-bias voltage will be greatly suppressed. Figure 4(a) provides a more complete picture of the ZBC’s dependence on $\theta_2$ and the effect of $d_3$ to the ZBC. It is clear that the minimum ZBC decreases monotonically with $d_3$, but for the ZBC at $\theta_2 = \pi/2$, the increase of $d_3$ has no effect to it, as shown in figure 4(b). From the expression of $G(0, n, \theta_2 = \pi/2)$ we know it takes the value $16k_{1+}k_{1-}G_{00}/(k_{1+} + k_{1-})^2$. Figures 4(c) (d) show the effect of the interface scattering potential to the ZBC, it is easy to see that the results are similar to figures 4(a) and (b).

Since the increase of $d_3$ and the increase of interface scattering potential both have no effect to the ZBC at $\theta_2 = \pi/2$, but will greatly suppress the ZBC away from $\theta_2 = \pi/2$, it is obvious that the increase of $d_3$ and the increase of interface scattering potential will greatly increase the TMR. In figure 5(a), the blue square-line shows that the TMR grows exponentially with the increase of $d_3$. This is a remarkable property, because it indicates that the TMR is easy to tune and can be tuned to arbitrary high value even the two FMs are both weak. Unlike the conventional MVE between two FMs that the increase of the thickness of ITB will exponentially suppress the tunneling conductance (or say tunneling current) for all possible angle-difference, here due to the nontrivial topological property of the TSC, the tunneling conductance at the neighbourhood of $\theta_2 = \pi/2$ is topologically stable and therefore, even $d_3$ is large, the tunneling current can still be large. This is also important for real applications because the strength of the tunneling current is proportional to the strength of signal, therefore a high signal-to-noise ratio, which is necessary for storage applications [7], can be guaranteed in this system. The TMR’s explicit dependence on the interface scattering potential of each interface is neglected here, the only thing we need to stress again is that the TMR monotonically increases with the increase of interface scattering potential, and therefore, the interface roughness emerging in the process of synthesizing the junction is not bad here.

Other parameters of the junction also may affect the TMR. The red dot-line in figure 5(a) shows the TMR’s dependence on the thickness of ITB1, $d_1$, we can see that the TMR is almost unchanged with the variation of $d_1$. Although an increase of $d_1$ has small impact on the TMR, it can greatly suppress the magnetic proximity effect of FM2 to the TSC. Figure 5(b) shows that the TMR has an oscillating dependence on the length of the FM2, $d_2$, this oscillating behavior is due to interference of wave functions in FM2. In figure 5(c), the TMR shows a dependence on $M_f$ which is similar to the inset of figure 1(d), where the TMR monotonically increases with the increase of the magnetization strength. The inset of figure 5(c) shows a remarkable property of the junction that even the
FM1 turns to be a normal metal (NM), the TMR also grows exponentially with the increase of $d_2$. Therefore, a NM-I-FM-I-TSC junction is also an ideal structure for applications. In figure 5, it is shown that when the length of FM2 is large, i.e., $k_d > \pi$, the TMR will exhibit an oscillating dependence on $M_2$, which is also due to interference of wave functions in FM2. However, when $k_d < \pi$, the TMR will exhibit a monotonically increasing dependence on $M_2$, as shown in the inset of figure 5. From figure 5(b) and (d), we see that the interference of wave functions in FM2 may greatly suppressed the TMR, to avoid the suppression in real applications, therefore, the length of FM2 is better to choose to satisfy $k_d < \pi$.

5. Higher-dimensional case

Here we only consider a two-dimensional FM-I-FM-I-TSC junction for illustration, the three-dimensional case can be directly generalized from it.

The two-dimensional Hamiltonian we consider under the representation

\[ \hat{\Psi}(\vec{r}) = (\psi_1(\vec{r}), \psi_0(\vec{r}), \psi_1^*(\vec{r}), \psi_0^*(\vec{r})) \]

is given as

\[ H_{2D} = \tau_z \left( -\frac{\partial_x^2 + \partial_y^2}{2} - \mu(x) + V(x) \right) + \tau_x \Delta(x) + \tau_y \Delta(y), \]

here we have assumed that the chemical potential $\mu(\vec{r})$, the potential $V(\vec{r})$ and the pairing potential $\Delta(x)$ take the same form as $\mu(x)$, $V(x)$ and $\Delta(x)$ given previously, respectively. The other pairing potential $\Delta(y) = i \Delta_0 (x - x_i) \partial_y$.

When the dimension of the system is higher than one, the injected electron’s momentum $\vec{k}$ can be decomposed as $(k_x, k_y)$, for the two-dimensional case we consider here, $k_z = k_0$. If $k_0$ is conserved in the process of electrons transporting across the junction, the tunneling process of an injected electron with nonzero $k_0$ at zero-bias voltage no longer exhibits the topological feature. This can be simply explained under the Majorana picture [51, 52]. In higher dimension, the boundary Majorana zero modes require $k_z = 0$, this indicates that due to the momentum conservation, an injected electron with nonzero $k_z$ will no longer directly couple with the Majorana zero modes, and then the ZBC will be parameter-dependent (topological feature is absent) no matter what value $\theta_2$ takes. As the final conductance includes contributions from every direction, it is obvious that the new added dimension will generally mask the topological effect. However, as the conductance
curve should be continuous, this implies that if the ratio of the transverse momentum to the longitudinal momentum in TSC decreases, the effect of the non-zero transverse momentum will decrease, and then the topological effect will be strengthened. One way to reduce the momentum ratio in TSC is to increase $s_m$, since $k_{k\perp}$ is independent of $s_m$, while $|k_{x\perp}|$ monotonically increases with $s_m$.

The junction itself has an intrinsic effect to enhance the topological effect. The intrinsic effect is the so-called momentum filtering effect (MFE) \[53\] that is if $|k_{\perp}|$ is fixed, then injected electrons with larger $k_x$ will feel a lower tunneling barrier because the effective potential barrier is given as $U_{k_{\parallel}} = \mu_1 + |k_x|^2 - k_{\perp}^2 = \mu_1 + |k| \sin^2 \theta$ with $\theta = \arccos(k_{\perp}/|k|)$. As the MFE reduces the contributions from injected electrons with larger $k_{\perp}$, the ratio of the contribution from the neighbourhood of $k_{\parallel} = 0$ increases, consequently, the topological effect is also enhanced. Therefore, when the MFE is very strong, the TMR is expected to be very large. From the expression of $U_{k_{\parallel}}$, we know that the most direct way to enhance the MFE is to reduce $l_m$, this can be realized by choosing small gap insulating materials as the ITB.

For the sake of discussion, we introduce a dimensionless quantity $\sigma_A$ which is defined as

$$\sigma_A(\theta_2) = \frac{\sum_{k_y} G(0, \theta_2, k_y)}{\sum_{k_y} 2e^2h}$$

(20)

where $G(0, \theta_2, k_y)$ is the summation of ZBC for spin-up and spin-down electron with transverse momentum $k_y$. Figures 6(a) and (b) shows that with the increase of $d_3$, ZBC will be greatly suppressed no matter what value $\theta_2$ takes, this agrees with our previous argument. However, due to a combination of the topological effect and MFE, the suppression at the neighbourhood of $\theta_2 = \pi/2$ is still much smaller than the ones at the neighbourhood of $\theta_2 = 0$ or $\pi$. The angle-dependent suppression behavior results in an approximately exponential dependence between the TMR (now given as $(\sigma_{A_{\max}} - \sigma_{A_{\min}})/\sigma_{A_{\min}}$) and $d_3$, as shown in figure 6(c), therefore, the remarkable property of the FM-I-FM-I-TSC junction that the TMR is easy to tune and can be tuned to arbitrary high value is still hold in two-dimension. Compared figures 6(b) to (a), we can see that an increase of $\mu_3$ indeed results in an enhancement of the topological effect and consequently a slower decrease of $\sigma_A$ at the neighbourhood of $\theta_2 = \pi/2$. Figures 6(c) and (d) show that the slower decrease of $\sigma_A$ results in a larger TMR. The effects of other parameters are similar to the ones shown in figure 5, we do not plan to discuss them here again.

Figure 6. Tunneling spectroscopy of FM-I-FM-I-TS junction in two dimension and TMR’s dependence on parameters, logarithmic coordinates are used in all figures. Common parameters in all figures, $\mu_A = 1, \mu_B = 0.1, \Delta_0 = 0.05$, $M_k = M_L = 0.4$, $d_1 = 0.2$, $d_2 = 1$, $\theta_1 = \phi_1 = \phi_2 = 0$. Here choosing a small $\mu_1$ is to enhance the momentum filtering effect. (a)–(b) show the ZBC’s dependence on $\theta_2$ and $d_3$ with (a) $\mu_1 = 1$, (b) $\mu_1 = 3$. (c) shows the TMR-$d_3$ relation corresponding to (a) and (b). (d) shows the TMR-$\mu_3$ relation for different $d_3$. 
6. Discussions and conclusions

In summary, because the tunneling process in junctions composed of FM and time-reversal invariant TSC without SRS strongly depends on the spin-polarization direction of the injected electrons, high TMR is found to exist in these junctions. Compared to conventional TMR shown in MTJs, the TMR shown in the FM-I-TSC and FM-I-FM-I-TSC junctions exhibits several extraordinary characteristics: (i) so far, the best mechanism to obtain high TMR in conventional MTJs is to take use of a MgO tunneling barrier [54–57], however, for junctions we have considered, high TMR is obtained even the ITB is featureless; (ii) for the FM-I-FM-I-TSC junction in one dimension, the TMR only depends on the magnetization strength of the FM, and it goes to infinity when the FM turns to be a half metal; (iii) for the FM-I-FM-I-TSC junction, the TMR shows a remarkable property that it grows exponentially with the thickness of ITB between the two FMs and also grows monotonically with the interface scattering potential, this remarkable property makes it possible to tune the TMR to arbitrary high value even the magnetization strength of the two FMs are both weak, in fact, the magnetization strength of FM1 can even be zero. Even with a consideration of the effect of finite temperature, these characteristics will still hold if $k_B T$ is much smaller than the energy gap of the TSC.

In short, a combination of the FM and TSC provides a new fascinating mechanism to obtain high TMR in a convenient way. Besides this remarkable effect, we also note that some works pointing out that spin-polarized current, another important element for spintronics, can also be simply realized by making use of TSCs [50, 58] already exist. All these results suggest that the nontrivial topological property of TSCs may bring new insights in spintronics.

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Appendix

When a majority electron with energy $E$ is injected from the FM1, the wave functions in the two ferromagnetic parts are given as

$$
\psi_{FM1} = \bar{x}_1 e^{i k_{1x} x} + b_1 + \bar{x}_1 e^{-i k_{1x} x} + a_1 + \bar{x}_1 e^{i k_{1x} x} + b_1 - \bar{x}_1 e^{-i k_{1x} x},
$$

$$
\psi_{FM2} = b_{3L} + \bar{x}_1 e^{i k_{2x} x} + b_{3R} + \bar{x}_1 e^{-i k_{2x} x} + a_{3L} + \bar{x}_1 e^{i k_{2x} x} + b_{3R} - \bar{x}_1 e^{-i k_{2x} x},
$$

with $\bar{x}_1 = (\eta_1, 0, \eta_2 e^{i \phi}, 0)^T$, $\bar{x}_2 = (0, \eta_2, 0, \eta_1 e^{i \phi})^T$, $\bar{x}_3 = (\eta_2, 0, \eta_1 e^{i \phi}, 0)^T$, $\bar{x}_4 = (0, -\eta_1, 0, \eta_2 e^{i \phi})^T$, where $\eta_1 = \cos(\theta_1/2)$, $\eta_2 = \sin(\theta_1/2)$; $\bar{x}_1 = \chi_1 (\theta_1, \phi_1 \rightarrow \theta_2, \phi_2)$; the momenta $k_{1, 2+,-}(t) = \sqrt{2}(\mu_j + M_{21} + (-)E)$, $k_{1, 2-,-}(t) = \sqrt{2}(\mu_j - M_{21} + (-)E)$. The wave functions in the two insulating parts are given as

$$
\psi_{II} = b_{2L} \bar{e}_1 e^{-i k_{1x} x} + b_{2R} \bar{e}_1 e^{i k_{1x} x} + a_{2L} \bar{e}_2 e^{-i k_{1x} x} + a_{2R} \bar{e}_2 e^{i k_{1x} x} + a_{2L} \bar{e}_3 e^{-i k_{1x} x} + a_{2R} \bar{e}_3 e^{i k_{1x} x},
$$

$$
\psi_{II} = a_{2L} \bar{e}_4 e^{-i k_{1x} x} + a_{2R} \bar{e}_4 e^{i k_{1x} x},
$$

where $\bar{e}_1 = (1, 0, 0, 0)^T$, $\bar{e}_2 = (0, 1, 0, 0)^T$, $\bar{e}_3 = (0, 0, 1, 0)^T$, $\bar{e}_4 = (0, 0, 0, 1)^T$, the momenta $k_{1, 2+,-}(t) = \sqrt{2}(\mu_j + (-)E)$. The wave functions in the superconducting part are given as

$$
\psi_{TS} = t_{52L} \lambda_1 e^{i k_{1x} x} + t_{52R} \lambda_2 e^{i k_{1x} x} + s_{52L} \lambda_3 e^{i k_{1x} x} + s_{52R} \lambda_4 e^{i k_{1x} x}.
$$

with $\bar{x}_1 = (u_{1k}, v_{1k}, 0, 0)^T$, $\bar{x}_2 = (u_{2k}, v_{2k}, 0, 0)^T$, $\bar{x}_3 = (0, 0, u_{1k}, v_{1k})^T$, $\bar{x}_4 = (0, 0, u_{2k}, v_{2k})^T$, where $u_{1(2)k} = \Delta_k k_{c(\theta)}/N_{1(2)}$, $v_{1(2)k} = (E + \mu_k - k_{s(\theta)}/2)^N_{1(2)}$ with $N_{1(2)} = \sqrt{\left|u_{1(2)k}\right|^2 + \left|v_{1(2)k}\right|^2}$ the normalization coefficients. $k_{c(\theta)} = (-)\sqrt{Q_1 + Q_2} + i\sqrt{Q_1 - Q_2}$ with $Q_1 = (\mu_k - \Delta^2)$ and $Q_2 = \sqrt{\mu_k^2 - E^2}$ (when $E < \Delta\sqrt{2\mu_k - \Delta^2}$), or $k_{c(\theta)} = (-)\sqrt{(Q_1 + Q_3)}$ with $Q_3 = \sqrt{E^2 + \Delta^2 - 2\mu_k \Delta^2}$ (when $E > \Delta\sqrt{2\mu_k - \Delta^2}$).
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