Nonlocal Condensates in Hadrons and Multiquark States

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Abstract

The definition of QCD vacuum is an important issue in the description of hadronic properties. Recent researches on the vacuum condensates have resulted in the suggestion of in-hadron condensates which can be taken as a paradigm shift concerning the viewpoints on the QCD vacuum. In this Letter, we will try to define the in-hadron regions and to classify the hadronic and multiquark states. Topological classifications are naturally introduced and by considering nonlocal measure we can estimate the variations of dimension 2 condensate $\langle A^2 \rangle$. The calculational techniques can be easily applied to multiquark states and are expected to be applied to the more complex nuclear states.

Keywords: In-hadron region, Vacuum condensates, Connection amplitude

PACS: 12.38.Lg, 14.80.-j, 24.85.+p

1. Introduction

Condensates are defined as the non-zero vacuum expectation values of normal ordered products of field operators. Since the effects of normal orderings are to remove the infinite zero-point energy and to define the zero of energy as the energy of the vacuum state, it is indispensable to relate the values of various condensates to the precise definition of the vacuum state itself. Recent progresses in the research of these condensates have resulted in the suggestion of in-hadron condensates \cite{1} which can be taken as a paradigm shift concerning the viewpoints on the QCD vacuum \cite{2}.

For the in-hadron condensates, we assume that the strong interaction condensates are the properties of well-defined wavefunctions of hadrons rather than the hadron-less ground state of QCD. This viewpoint imposes the problem of determining the shape of hadronic wavefunctions which corresponds partly to figuring out the quark and gluon distributions \cite{3}. In general, the values of hadronic wavefunctions are not constants so that we can induce that the condensate values can vary from point to point. Conversely, if we assume that condensates have constant values inside hadrons, then there should be sharp boundaries between the outside and the inside of hadrons. Although there exist some hard-wall models, it is not natural to draw the sharp boundary surfaces to define the in-hadron regions. Therefore we need to consider smoothly changing functions for the values of strong interaction condensates.

2. The model : connection amplitude

The motivation to introduce the in-hadron condensates is to account for the fact that the vacuum states in a confining theory is not defined relative to the fields in the Lagrangian but to the actual physical, color-singlet, states \cite{4}. So in order to define the in-hadron regions, it is better to start with the classifications of the color-singlet states. Since the condensates effectively originate from $q\bar{q}$ and gluon contributions to the higher Fock states, we have to include these states from the beginning. Then it is convenient to represent the color-singlet states by their number of quarks and antiquarks. For the color-singlet states with $a$ quarks and $b$ antiquarks forming some in-hadron regions, the difference between $a$ and $b$ is a multiple of 3 in SU(3)$_c$ case and is related to the baryon number of the given color-singlet state. Because the baryon number is conserved in strong interactions, we need to categorize the color-singlet states into the sets of correlated ones with fixed baryon number. The correlations between different color-singlet states can be established by tracing the changes of in-hadron regions which are represented by the numbers of quarks and antiquarks. With the constraint of baryon number conservation, the methods to change the in-hadron regions are provided only through the quark pair creation and the quark pair annihilation. These changes of in-hadron regions can be systematically described by introducing the notion of open sets for in-hadron regions and the operations of union and intersection of the open sets. The union operation corresponds to the quark pair annihilation and the intersection operation to the quark pair creation. Now we can write down our starting assumptions as \cite{5}

\begin{itemize}
  \item Open sets are the stable in-hadron regions.
  \item The union of stable in-hadron regions becomes a stable in-hadron region.
  \item The intersection between a connected stable in-hadron
\end{itemize}
region and disconnected stable in-hadron regions is the reverse operation of the union.

These assumptions are just those needed for constructing the topological spaces of in-hadron regions. Let’s use the notation $R_{a,b}$ as representing the in-hadron regions formed between $a$ quarks and $b$ antiquarks. Thus $R_{1,1}$ represents the in-hadron regions of mesons, and $R_{3,0}$ and $R_{0,3}$ correspond to those of baryons and antibaryons. Then the simplest non-trivial set of in-hadron regions satisfying the above three assumptions is given by

$$T_{0,0} = \{ \phi, R_{1,1}, R_{0}^{2}, R_{1,1}^{2}, \cdots, R_{0}^{n}, \cdots \},$$

(1)

where $R_{0}^{n}$ denotes the in-hadron regions of $n$ mesons which can be obtained by repeated quark pair creations. The next baryon number 1 topological space for baryon–meson system is

$$T_{1,0} = \{ \phi, R_{3,0}, R_{3,0} R_{1,1}, R_{3,0} R_{0}^{2}, \cdots, R_{3,0} R_{1,1}^{2}, \cdots \},$$

(2)

and the baryon number $-1$ space for antibaryon–meson system becomes

$$T_{0,1} = \{ \phi, R_{0,3}, R_{0,3} R_{1,1}, R_{0,3} R_{0}^{2}, \cdots, R_{0,3} R_{1,1}^{2}, \cdots \}.$$  

(3)

We can easily see that $R_{1,1}$ can be multiplied at any time without the violation of baryon number conservation, and therefore we may omit these multiplications in our notation writing as

$$T_{1,0} = \{ \phi, R_{3,0} \}, \quad T_{0,1} = \{ \phi, R_{0,3} \}.$$ 

(4)

In general, we have

$$T_{i,j} = \{ \phi, R_{3,0}^{i} R_{0,3}^{j}, R_{3,0}^{i} R_{0,3}^{j} R_{2,2}^{k}, \cdots \},$$

(5)

where we can take $i$ as the number of incoming 3-junctions and $j$ that of outgoing 3-junctions if we represent the in-hadron region as originating from quarks and terminating at antiquarks. For a given space, the baryon number $B = i - j$ is conserved and the number of boundary points encompassing the quarks and antiquarks is reduced by $(1, 1)$ pair through one union operation. As examples we can write down

$$T_{3,1} = \{ \phi, R_{3,0} R_{0,3}, R_{3,0} R_{2,2}, R_{3,0} R_{4,1}, R_{6,0} \},$$

(6)

and

$$T_{5,2} = \{ \phi, R_{3,0}^{2} R_{0,3}^{2}, R_{3,0}^{2} R_{0,3} R_{2,2}, R_{3,0}^{2} R_{1,4}, R_{3,0}^{2} R_{0,3} R_{4,1}, R_{3,0}^{2} R_{0,3} R_{6,0}, R_{2,2}^{2}, R_{2,2}^{2} R_{7,1}, R_{3,0}^{2} R_{2,2}^{2}, R_{2,2}^{2} R_{4,1}, R_{3,0}^{2} R_{6,0}, R_{6,0} R_{4,1}, R_{9,0}, \}$$

(7)

and so on.

With the defined topological spaces of in-hadron regions, we are now to try to deduce the spatial variations of in-hadron condensates. As is well-known, there exist various kinds of condensates such as $(\langle A_{\mu} A^{\mu} \rangle, \langle \bar{q} q \rangle, \langle G_{\mu\nu} G^{\mu\nu} \rangle, \langle \bar{q} q \rangle)$ and higher dimension condensates. All these combinations are made from gluon and quark fields, and in order to figure out the condensate structures it is better to consider firstly the simpler ones $(\langle \bar{q} q \rangle)$ and $(\langle \bar{q} q \rangle)$. The effects of these condensates have been discussed in many ways. One of the common conclusions is that they are related to the generation of dynamical masses. The dimension 2 condensate $(\langle \bar{q} q \rangle)$ gives rise to the gluon mass and the momentum dependent dynamical quark mass $M(p^2)$ [42], and the non-zero value of $(\langle \bar{q} q \rangle)$ is usually held to signal the dynamical chiral symmetry breaking which is an efficient mass-generating mechanism resulting in the constituent quark masses. The momentum dependences of mass function $M(p^2)$ can be interpreted as the position dependences in coordinate space, which, if possible, have to be deduced from basic properties of in-hadron regions or their correlations. Since the correlations between in-hadron regions are induced by quark pair creations or annihilations, it is natural for us to consider the bilocal scalar condensate of quark pair

$$\langle \bar{q}(x) U(x,0) q(0) \rangle \approx \langle \bar{q}(0) q(0) \rangle Q(x^2),$$

(8)

where $U(x,0)$ represents the connection through in-hadron region. The function $Q(x^2)$ is nonlocal and can be named as the connection amplitude [43].

The form of the function $Q(x^2)$ was parameterized by introducing vacuum distribution functions and estimated within single instanton approximation [44]. However, the vacuum distribution functions are not known and they have to be calculated from QCD vacuum theory. Since we are now considering smoothly changing functions for the values of in-hadron condensates, it is plausible to introduce a measure $\mathfrak{M}(Q)$ to account for an intuitive picture of in-hadron regions. First, we can assume

$$\mathfrak{M}(Q) \text{ decreases as } Q \text{ increases.}$$

(9)

This condition implies that a smaller in-hadron region is more likely to be connected than a larger one. The second condition can be drawn from the relation

$$\langle \bar{q}(x) U(x,y) q(y) \rangle_0 U(y,0) q(0) \rangle 
= \langle \bar{q}(0) q(0) \rangle_0^2 Q((x - y)^2) \rangle_0 Q(y^2),$$

(10)

where one quark pair creation at $y$ divides the original in-hadron region into two regions. Thus we have

$$\mathfrak{M}(Q_1) + \mathfrak{M}(Q_2) = \mathfrak{M}(Q_1 Q_2),$$

(11)

where $Q_1$ and $Q_2$ are taken to be independent. This condition states that two independent in-hadron regions can be joined to form a single in-hadron region. Then with these two conditions, we get the solution

$$\mathfrak{M}(Q) = -k \ln \frac{Q}{Q_0}$$

(12)

where $Q_0$ is a normalization constant and $k$ is an appropriate parameter.

The measure $\mathfrak{M}(Q)$ should be a metric function defined on the in-hadron region. For the in-hadron region with a quark pair at position $\vec{x}$ and $\vec{y}$, the distance function between $\vec{x}$ and $\vec{y}$ can be written as

$$d(\vec{x}, \vec{y}) = |\vec{x} - \vec{y}|$$

(13)

with $v$ being an arbitrary number. This distance function can be made metric for points $\vec{z}$ satisfying

$$|\vec{x} - \vec{z}|^v + |\vec{y} - \vec{z}|^v \geq |\vec{x} - \vec{y}|^v.$$  

(14)
where all possibilities from the line shape with \( \nu = 1 \) to another shape with \( \nu = \alpha \) have been included. The weight factor \( F(\nu) \) has been introduced to account for possible different contributions from different \( \nu \)'s, and the variable \( r \) is

\[
r = \frac{1}{\ell} |\vec{r} - \vec{y}|
\]

with \( \ell \) being a scale parameter. If we take \( \alpha = 2 \), which corresponds to a spherical shape of in-hadron region, and considering the case of equal weight \( F(\nu) = 1 \), we have

\[
Q = Q_0 \exp \left( -\frac{1}{k} \frac{\nu^2 - r^2}{\ln r} \right). \tag{17}
\]

This functional form describes smooth changes of the connectedness of in-hadron region for large enough values of \( r \), however, for short ranges, we need to include perturbative local effects. These effects can be accounted for if we substitute \( r^2 Q \) for \( Q \), in which case the conditions on \( \Re \) still hold with \( \beta > 0 \). Then the final form of \( Q \) becomes

\[
Q = \frac{Q_0}{r^\beta} \exp \left( -\frac{1}{k} \frac{\nu^2 - r^2}{\ln r} \right). \tag{18}
\]

### 3. Estimation of nonlocal condensates

With the above connection amplitude, we can estimate the values of dimension 2 condensate \( \langle A^2 \rangle \) by assuming that

\[
\langle \phi(x)U(x,y)A^2(y)U(y,0)\phi(0) \rangle \quad \propto \quad \langle \phi(x)U(x,y)\phi(y)U(y,0)\phi(0) \rangle,
\]

which implies that the probability amplitude to have a quark pair is taken to be proportional to the value of \( \langle A^2 \rangle \) at that point. For a meson with quarks at \( \vec{r}_1 \) and \( \vec{r}_2 \), the condensate value at \( \vec{r} \) becomes

\[
\gamma(\vec{r}) = \gamma_m \prod_{i=1}^{2} |\vec{r} - \vec{r}_i|^{-\beta} \exp \left( -\frac{1}{k} \frac{|\vec{r} - \vec{r}_i|^2 - |\vec{r} - \vec{r}_j|^2}{\ln |\vec{r} - \vec{r}_i|} \right) \tag{20}
\]

with an appropriate normalization factor \( \gamma_m \). If we fix the quark positions at \( \vec{r}_1 = (-\frac{i}{2}, 0) \) and \( \vec{r}_2 = (\frac{i}{2}, 0) \), we can draw the values of dimension 2 condensate as in Fig. [1]. We have changed the values of \( \beta \) and \( k \), and shown the results here for the values \( \beta = 1.0 \) and \( k = 1.0 \). For a baryon, the quark-to-quark connections are three fold and therefore the condensate value at \( \vec{r} \) becomes

\[
\gamma(\vec{r}) = \gamma_b \sum_{i=1}^{3} |\vec{r} - \vec{r}_1|^{-\beta} \exp \left( -\frac{1}{k} \frac{|\vec{r} - \vec{r}_1|^2 - |\vec{r} - \vec{r}_j|^2}{\ln |\vec{r} - \vec{r}_1|} \right) \prod_{i=1}^{3} |\vec{r} - \vec{r}_k|^{-\beta} \exp \left( -\frac{1}{k} \frac{|\vec{r} - \vec{r}_k|^2 - |\vec{r} - \vec{r}_j|^2}{\ln |\vec{r} - \vec{r}_k|} \right) \tag{21}
\]

where \( \vec{r}_i \) are the positions of three quarks. The structure of dimension 2 condensates can be drawn as in Fig. 2 with quark positions at \((0, 1), (0,-1), \) and \((\sqrt{3}, 0)\). We can see the Y-type connections which have been obtained in lattice calculations.
through the estimate of gluon field components [11]. Of course the field components are squared to get the energy density or the action density, however, the structures defined by constant densities are not changed by squaring the components.

The calculational techniques using the connection amplitude can be easily applied to systems with complex boundary conditions such as multiquark states. For tetraquark states, we have two possible combinations of color-singlet states with one quark pair creation. One is the meson-tetraquark combination and the other is the baryon-antibaryon combination. The meson-tetraquark combination has 4 amplitudes composed of 7 connections and the baryon-antibaryon combination has 1 amplitude composed of 6 connections. The calculated results are shown in Fig. 3 where only one surface with constant condensate value has been presented. The quark positions are fixed at (0, 1, 0), (0, −1, 0), (√2, 0, 1), and (√2, 0, −1). As another example, we have calculated the case of hexaquark states or deuteron for which we have 6 amplitudes with 16 connections for the meson-hexaquark combination and 15 amplitudes with 13 connections for the baryon-pentaquark combination. The calculated results are shown in Fig. 4 with quark positions at (0, 0, √2), (1, 1, 0), (1, −1, 0), (−1, −1, 0), (−1, 1, 0), and (0, 0, −√2).

Figure 3: Equi-γ surface for a tetraquark state with quarks at (0, 1, 0), (0, −1, 0), (√2, 0, 1), and (√2, 0, −1).

Figure 4: Equi-γ surface for a hexaquark state with quarks at (0, 0, √2), (1, 1, 0), (1, −1, 0), (−1, −1, 0), (−1, 1, 0), and (0, 0, −√2).

4. Summary

In summary, we have introduced the notion of in-hadron region and constructed the topological spaces of in-hadron regions. For a given space, it is possible to parameterize the quark-to-quark connections with nonlocal measure and to estimate the variations of dimension 2 condensate $\langle A_2^2 \rangle$. The estimated results are impressive for the systems of mesons and baryons compared with the results of lattice gauge calculations.

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