Fuzzy Foldness of P-Ideals in BCI-Algebras

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Abstract

This paper aims to introduce new notions of (fuzzy) n-fold P-ideals and (fuzzy) n-fold weak P-ideals in BCI-algebras, and investigate several properties of the foldness theory of P-ideals in BCI-algebras. Finally, we construct a computer-program for studying the foldness theory of P-ideals in BCI-algebras.

Keywords

BCK/BCI Algebras, P-Ideals of BCI-Algebras, Fuzzy P-Ideal of BCI-Algebra, Fuzzypoint, (Fuzzy) n-Fold P-Ideals, (Fuzzy) n-Fold Weak P-Ideals

1. Introduction

The study of BCK/BCI-algebras was initiated by Iséki [1] as a generalization of the concept of set-theoretic difference and propositional calculus. Since then, a great deal of theorems has been produced on the theory of BCK/BCI-algebras. In (1965), Zadeh [2] was introduced the notion of a fuzzy subset of a set as a method for representing uncertainty. In 1991, Xi [3] defined fuzzy subsets in BCK/BCI-algebras.

Huang and Chen [4] introduced the notions of n-fold implicative ideal and n-fold (weak) commutative ideals. Y. B. Jun [5] discussed the fuzzification of n-fold positive implicative, commutative, and implicative ideal of BCK-algebras.

In this paper, we redefined a P-ideal of BCI-algebras and studied the foldness theory of fuzzy P-ideals, P-weak ideals, fuzzy weak P-ideals, and weak P-weak ideals in BCI-algebras. This theory can be considered as a natural generalization of P-ideals. Indeed, given any BCI-algebras \( X \), we use the concept of fuzzy point to characterize n-fold P-ideals in \( X \). Finally, we construct some algorithms for studying the foldness theory of P-ideals in BCI-algebras.
2. Preliminaries

Here we include some elementary aspects of BCI that are necessary for this paper. For more detail, we refer to [4] [6].

An algebra \((X; *, 0)\) of type \((2, 0)\) is called BCI-algebra if 
\(\forall x, y, z \in X\) the following conditions hold:

BCI-1. \((x * y) * (x * z) = (y * z) * x\);

BCI-2. \((x * (x * y)) * y = y\);

BCI-3. \(x * x = 0\);

BCI-4. \(x * y = 0\) and \(y * x = 0 \Rightarrow x = y\).

A binary relation \(\leq\) can be defined by

BCI-5. \(x \leq y \Leftrightarrow x * y = 0\).

Then \((X, \leq)\) is a partially ordered set with least element 0.

The following properties also hold in any BCI-algebra [7] [8]:

1) \(x * 0 = x\);
2) \(x * y = 0\) and \(y * z = 0 \Rightarrow x * z = 0\);
3) \(x * y = 0 \Rightarrow (x * z) * (y * z) = 0\) and \((z * y) * (z * x) = 0\);
4) \((x * y) * z = (x * z) * y\);
5) \((x * y) * x = 0\);
6) \((x * (x * y)) = x * y\); let \((X, *, 0)\) be a BCI-algebra.

Definition 2.1. A fuzzy subset of a BCK/BCI-algebra \(X\) is a function
\[\mu : (0, 1) \rightarrow \mathbb{X}.\]

Definition 2.2. (C. Lele, C. Wu, P. Weke, T. Mamadou, and C.E. Njock [9]).

Let \(\xi\) be the family of all fuzzy sets in \(X\). For \(x \in X\) and \(\lambda \in (0, 1], \ x_\lambda \in \xi\) is a fuzzy point if
\[x_\lambda(y) = \begin{cases} \lambda & \text{if } x = y, \\ 0 & \text{otherwise}. \end{cases}\]

We denote by \(\bar{X} = \{x_\lambda : x \in X, \lambda \in (0, 1]\}\) the set of all fuzzy points on \(X\) and we define a binary operation on \(\bar{X}\) as follows
\[x_\lambda * y_\mu = (x * y)_{\min(\lambda, \mu)}\]

It is easy to verify \(\forall x_\lambda, y_\mu, z_\alpha \in \bar{X}\), the following conditions hold:

BCI-1’. \((x_\lambda * y_\mu) * (x_\alpha * z_\mu) = (z_\alpha * y_\mu) * (x_\lambda * z_\mu)\);

BCI-2’. \((x_\lambda * (x_\mu * y_\mu)) * y_\mu = 0_{\min(\lambda, \mu, \mu)}\);

BCI-3’. \(x_\lambda * x_\mu = 0_{\min(\lambda, \mu)}\);

BCK-5’. \(0_{\mu} * x_\lambda = 0_{\min(\lambda, \mu)}\).

Remark 2.3. (C. Lele, C. Wu, P. Weke, T. Mamadou, and C.E. Njock [9]). The condition BCI-4 is not true \((\bar{X}, *)\). So the partial order \(\leq\) cannot be extended to \((\bar{X}, *)\).

We can also establish the following conditions \(\forall x_\lambda, y_\mu, z_\alpha \in \bar{X}\):

1’) \(x_\lambda * 0_\mu = x_{\min(\lambda, \mu)}\);

2’) \(x_\lambda * y_\mu = 0_{\min(\lambda, \mu)}\) and \(y_\mu * z_\alpha = 0_{\min(\mu, \alpha)} \Rightarrow x_\lambda * z_\alpha = 0_{\min(\lambda, \alpha)}\);
3') \( x_\lambda \ast y_\mu = 0_{\min(\lambda, \mu)} \Rightarrow (x_\lambda \ast z_\alpha) \ast (y_\mu \ast z_\alpha) = 0_{\min(\lambda, \mu, \alpha)} \) and
\( (z_\alpha \ast y_\mu) \ast (z_\alpha \ast x_\lambda) = 0_{\min(\lambda, \mu, \alpha)} \);
4') \( x_\lambda \ast y_\mu \ast z_\alpha = (x_\lambda \ast z_\alpha) \ast y_\mu \);
5') \( x_\lambda \ast y_\mu \ast x_\lambda = 0_{(\lambda, \mu)} \);
6') \( x_\lambda \ast (x_\lambda \ast y_\mu) = x_\lambda \ast y_\mu \).

We recall that if \( A \) is a fuzzy subset of a BCK/BCI algebra \( X \), then we have the following:

\[
\tilde{A} = \left\{ x_\lambda \in \tilde{X} : A(x) \geq \lambda, \lambda \in (0,1] \right\}.
\]

\( \forall \lambda \in (0,1], \tilde{X}_\lambda = \left\{ x_\lambda : x \in X \right\} \), and \( \tilde{A}_\lambda = \left\{ x_\lambda \in \tilde{X}_\lambda : A(x) \geq \lambda \right\} \)

We also have \( \tilde{X}_\lambda \subset \tilde{X}, \tilde{A} \subset \tilde{X}, \tilde{A}_\lambda \subset \tilde{A}, \tilde{A}_\lambda \subset \tilde{X}_\lambda \), and one can easily check that \( \left( \tilde{X}_\lambda, \ast, 0_\lambda \right) \) is a BCK-algebra.

**Definition 2.4** (Isèki [10]). A nonempty subset of BCK/BCI-algebra \( X \) is called an ideal of \( X \) if it satisfies
1) \( 0 \in I \);
2) \( \forall x, y \in X, (x \ast y \in I \text{ and } y \in I) \Rightarrow x \in I \).

**Definition 2.5.** A nonempty subset \( I \) of BCI-algebra \( X \) is P-ideal if it satisfies:
1) \( 0 \in I \);
2) \( \forall x, y, z \in X \)
\[
((x \ast z) \ast (y \ast z) \in I \text{ and } y \in I) \Rightarrow x \in I,
\]

**Definition 2.6** (Xi [11]). A fuzzy subset \( A \) of a BCK/BCI algebra \( X \) is a fuzzy ideal if
1) \( \forall x \in X, A(0) \geq A(x) \);
2) \( \forall x, y \in X, A(x) \geq \min(A(x \ast y), A(y)) \).

**Definition 2.7** (Xi [11]). A fuzzy subset \( A \) of a BCI-algebra \( X \) is called a fuzzy P-ideal of \( X \) if
1) \( \forall x \in X, A(0) \geq A(x) \);
2) \( \forall x, y, z \in X \)
\[
A(x) \geq \min\left( A((x \ast z) \ast (y \ast z)) \ast A(y) \right).
\]

**Definition 2.8** [12]. \( \tilde{A} \) is a weak ideal of \( \tilde{X} \) if
1) \( \forall \nu \in \text{Im}(A); 0_\nu \in \tilde{A} \);
2) \( \forall x_\lambda, y_\mu \in X \). Such that \( x_\lambda \ast y_\mu \in \tilde{A} \) and \( y_\mu \in \tilde{A} \), we have
\[
x_{\min(\lambda, \mu)} \in \tilde{A}.
\]

**Theorem 2.9** [13]. Suppose that \( A \) is a fuzzy subset of a BCK-algebra \( X \), then the following conditions are equivalent:
1) \( A \) is a fuzzy ideal;
2) \( \forall x_\lambda, y_\mu \in \tilde{A}, (z_\alpha \ast y_\mu) \ast x_\lambda = 0_{\min(\lambda, \mu, \alpha)} \Rightarrow z_{\min(\lambda, \mu, \alpha)} \in \tilde{A} \);
3) \( \forall t \in (0,1] \), the \( t \)-level subset \( A^t = \{ x \in X : A(x) \geq t \} \) in an ideal when \( A^t \neq \emptyset \);
4) \( \tilde{A} \) is a weak ideal.

3. Fuzzy n-Fold P-Ideals in BCI-Algebras

Throughout this paper \( \tilde{X} \) is the set of fuzzy points on BCI-algebra \( X \) and \( n \in \mathbb{N} \) (where \( \mathbb{N} \) the set of all the natural numbers).

Let us denote \( \left( \cdots (x \ast y) \ast \cdots ) \ast y \right) \) by \( x \ast y^n \).

Moreover, \( \left( \cdots (x_{\min(\lambda,\mu)} \ast 0_\mu) \ast 0_\mu \right) \ast \cdots ) \ast 0_\mu \) by \( x_\lambda \ast y_\mu^n \) (where \( y \) and \( y_\mu \) occurs respectively \( n \) times) with \( x, y \in X, x_\lambda, y_\mu \in \tilde{X} \).

**Definition 3.1.** A nonempty subset \( I \) of a BCI-algebra \( X \) is an n-fold P-ideal of \( X \) if it satisfies:
1) \( 0 \in I \);
2) \( \forall x, y, z \in X, \left((x \ast z) \ast (y \ast z) \in I \text{ and } y \in I \right) \Rightarrow x \ast z \ast \in I \).

**Definition 3.2.** A fuzzy subset \( A \) of \( X \) is called a fuzzy n-fold P-ideal of \( X \) if it satisfies:
1) \( \forall x \in X, A(0) \geq A(x) \);
2) \( \forall x, y, z \in X, A(x \ast z \ast) \geq \min(A((x \ast z) \ast (y \ast z)) \ast A(y)) \).

**Definition 3.3.** \( \tilde{A} \) is P-weak ideal of \( \tilde{X} \) if
1) \( \forall \nu \in \text{Im}(A), 0_\nu \in \tilde{A} \);
2) \( \forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}, \left((x_\lambda \ast z_\alpha) \ast (y_\mu \ast z_\alpha) \in \tilde{A} \text{ and } y_\mu \in \tilde{A} \right) \Rightarrow x_{\min(\lambda,\mu)} \ast z_\alpha \tilde{A} \).

**Definition 3.4.** \( \tilde{A} \) is an n-fold P-weak ideal of \( \tilde{X} \) if
1) \( \forall \nu \in \text{Im}(A), 0_\nu \in \tilde{A} \);
2) \( \forall x_\lambda, y_\mu, z_\alpha \in \tilde{X}, \left((x_\lambda \ast z_\alpha) \ast (y_\mu \ast z_\alpha) \in \tilde{A} \text{ and } y_\mu \in \tilde{A} \right) \Rightarrow x_{\min(\lambda,\mu)} \ast z_\alpha^n \in \tilde{A} \).

**Example 3.5.** Let \( X = \{0, a, b, c\} \) with * defined by Table 1.

By simple computations, one can prove that \( (X, \ast, 0) \) is BCI-algebra. Define \( \mu : X \to [0,1] \) by \( \mu(0) = 1, \mu(a) = \mu(b) = \mu(c) = t \), where \( t \in [0,1] \).

**Table 1.** Example 3.5.

|       | 0     | a     | b     | c     |
|-------|-------|-------|-------|-------|
| 0     | 0     | 0     | 0     | c     |
| a     | a     | 0     | 0     | c     |
| b     | b     | b     | 0     | c     |
| c     | c     | c     | c     | 0     |

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One can easily check that for any \( n \geq 3 \).

Is a fuzzy \( n \)-fold P-ideal.

**Remark 3.6.** \( \tilde{A} \) is a 1-fold P-weak ideal of a BCK-algebra \( \tilde{X} \) if \( \tilde{A} \) is P-weak ideal of \( \tilde{X} \).

**Theorem 3.7.** If \( A \) is a fuzzy subset of \( X \), then \( A \) is a fuzzy \( n \)-fold P-ideal if \( \tilde{A} \) is an \( n \)-fold P-weak ideal.

**Proof.** \( \Rightarrow \)

- Let \( \lambda \in \text{Im}(A) \), it is easy to prove that \( 0 \lambda \in \tilde{A} \);
- Let \( (x \lambda * z_a) * (y \mu * z_a) \in \tilde{A} \) and \( y \mu \in \tilde{A} \)

\[
A((x \lambda * z_a) * (y \mu * z_a)) \geq \min(\lambda, \mu, \alpha) \quad \text{and} \quad A(y) \geq \mu.
\]

Since \( A \) is a fuzzy \( n \)-fold P-ideal, we have

\[
A(x \lambda * z_a) \geq \min(A((x \lambda * z_a) * (y \mu * z_a)) * A(y))
\]

\[
\geq \min(\min(\lambda, \mu, \alpha), \mu) = \min(\lambda, \mu, \alpha)
\]

Therefore \( (x \lambda * z_a)_{\text{min(\lambda, \mu, \alpha)}} = x_{\text{min(\lambda, \mu)}} * z_a \in A \).

\( \Leftarrow \)

- Let \( x \in X \), it is easy to prove that \( A(0) \geq A(x) \);
- Let \( x, y, z \in X \) and let \( A((x \lambda * z_a) * (y \mu * z_a)) = \beta \quad \text{and} \quad A(y) = \alpha \), then

\[
((x \lambda * z_a) * (y \mu * z_a))_{\min(\beta, \alpha)} = ((x \lambda * z_a) * (y \mu * z_a)) = \tilde{A} \quad \text{and} \quad y \mu \in \tilde{A}.
\]

Since \( \tilde{A} \) is P-weak ideal, we have

\[
x_{\text{min(\beta, \alpha)}} * z_a = (x \lambda * z_a)_{\text{min(\beta, \alpha)}} \in \tilde{A}
\]

Thus \( A(x \lambda * z_a) \geq \min(\beta, \alpha) = \min(A((x \lambda * z_a) * (y \mu * z_a)), A(y)) \). \( \square \)

**Proposition 3.8.** An \( n \)-fold P-weak ideal is a weak ideal.

**Proof.** \( \forall x \lambda, y \mu \in \tilde{X} \) let \( x \lambda * y \mu = (x \lambda * 0 \mu) * (y \mu * 0 \mu) \in \tilde{A} \) and \( y \mu \in \tilde{A} \), since \( \tilde{A} \) is an \( n \)-fold P-ideal, we have

\[
x_{\text{min(\beta, \alpha)}} = x_{\text{min(\lambda, \mu)}} * 0 \mu \in \tilde{A}
\]

Thus \( \tilde{A} \) is a weak ideal.

**Corollary 3.9.** A fuzzy \( n \)-fold P-ideal is a fuzzy ideal.

**Theorem 3.10.** Let \( \{\tilde{A}_i\} \) be a family of \( n \)-fold P-weak ideals and \( \{A_i\} \) be a family of fuzzy \( n \)-fold P-ideals. Then: 1) \( \bigcap_{i=1}^{n} \tilde{A}_i \) is an \( n \)-fold P-weak ideal.

2) \( \bigcup_{i=1}^{n} \tilde{A}_i \) is an \( n \)-fold P-ideal.

3) \( \bigcap_{i=1}^{n} A_i \) is a fuzzy \( n \)-fold P-ideal.

4) \( \bigcup_{i=1}^{n} A_i \) is a fuzzy \( n \)-fold P-ideal.

**Proof.** 1) \( \forall \lambda \in \text{Im}\left(\bigcap_{i=1}^{n} \tilde{A}_i\right) \), then \( \lambda \in \text{Im}(\tilde{A}_i), \forall i \), so, \( 0 \lambda \in \tilde{A}_i, \forall i \), i.e. \( 0 \lambda \in \bigcap_{i=1}^{n} \tilde{A}_i \). For every \( x \mu, y \mu, z_a \in \tilde{X} \), if \( (x \lambda * z_a) * (y \mu * z_a) \in \bigcap_{i=1}^{n} \tilde{A}_i \) and \( y \mu \in \bigcap_{i=1}^{n} \tilde{A}_i \), then

\[
(x \lambda * z_a) * (y \mu * z_a) \in \tilde{A}_i \quad \text{and} \quad y \mu \in \tilde{A}_i \quad \forall i \), thus
\[ x_{\min(I,\mu)} \ast z^a_n \in \tilde{A}_i, \forall i \]

So \( x_{\min(I,\mu)} \ast z^a_n \in \bigcap_{i \in I} \tilde{A}_i \). Thus \( \bigcap_{i \in I} \tilde{A}_i \) is an n-fold P-weak ideals.

2) \( \forall \lambda \in \operatorname{Im}(\bigcup_{i \in I} \tilde{A}_i) \), then \( \exists \theta \in I \), such that \( \lambda \in \tilde{A}_\theta \), so, \( 0 = \tilde{A}_\theta \), i.e. \( 0 \in \bigcup_{i \in I} \tilde{A}_i \). For every \( x_\mu, y_\lambda, z_\alpha \in \tilde{X} \), if \( (x_\mu \ast z_\alpha) \ast (y_\mu \ast z_\alpha) \in \bigcup_{i \in I} \tilde{A}_i \) and \( y_\mu \in \bigcup_{i \in I} \tilde{A}_i \), then \( \exists \delta \in I \) such that

\[ ((x_\mu \ast z_\alpha) \ast (y_\mu \ast z_\alpha)) \in \tilde{A}_\delta \] and \( y_\mu \in \tilde{A}_\delta \), thus \( x_{\min(I,\mu)} \ast z^a_n \in \tilde{A}_\delta \).

So \( x_{\min(I,\mu)} \ast z^a_n \in \bigcap_{i \in I} \tilde{A}_i \). Thus \( \bigcap_{i \in I} \tilde{A}_i \) is an n-fold P-weak ideals.

3) Follows from 1) and Theorem 3.7.

4) Follows from 2) and Theorem 3.7.

4. Fuzzy-Fold Weak P-Ideals in BCI-Algebras

In this section, we define and give some characterizations of (fuzzy) n-fold weak P-weak ideals in BCI-algebras.

**Definition 4.1.** A nonempty subset \( I \) of \( X \) is called an n-fold weak P-ideal of \( X \) if it satisfies

1) \( 0 \in I \);

2) \( \forall x, y, z \in X (x \ast z) \ast (y \ast z) \in I \) and \( y \in I \) \( \Rightarrow x \in I \).

**Definition 4.2.** A fuzzy subset \( A \) of \( X \) is called a fuzzy n-fold weak P-ideal of \( X \) if it satisfies

1) \( \forall x \in X, A(0) \geq A(x) \);

2) \( \forall x, y, z, A(x) \geq \min \{ A((x \ast z) \ast (y \ast z)), A(y) \} \).

**Definition 4.3.** \( \tilde{A} \) is a weak P-weak ideal of \( \tilde{X} \) if

1) \( \forall \nu \in \operatorname{Im}(A), 0 \in \tilde{A} \);

2) \( \forall x_\lambda, y_\mu, z_\alpha \in \tilde{X} \),

\[ ((x_\lambda \ast z_\alpha) \ast (y_\mu \ast z_\alpha)) \in \tilde{A} \] and \( y_\mu \in \tilde{A} \) \( \Rightarrow x_{\min(I,\mu,a)} \in \tilde{A} \).

**Definition 4.4.** \( \tilde{A} \) is an n-fold a weak P-weak ideal of \( \tilde{X} \) if

1) \( \forall \nu \in \operatorname{Im}(A), 0 \in \tilde{A} \);

2) \( \forall x_\lambda, y_\mu, z_\alpha \in \tilde{X} \),

\[ ((x_\lambda \ast z_\alpha) \ast (y_\mu \ast z_\alpha)) \in \tilde{A} \] and \( y_\mu \in \tilde{A} \) \( \Rightarrow x_{\min(I,\mu,a)} \in \tilde{A} \).

**Example 4.5.** Let \( X = \{0,1,2,3\} \) in which \( * \) is given by **Table 2**.

|   |   |   |   |   |
|---|---|---|---|---|
| * | 0 | a | b | c |
| 0 | 0 | 0 | 0 | 0 |
| a | a | 0 | 0 | 0 |
| b | b | b | 0 | 0 |
| c | c | c | c | 0 |
Then \((X;*,0)\) is a BCI-algebra. Let \(t_1, t_2 \in \{0,1\}\) and let us define a fuzzy subset \(A : X \to \{0,1\}\) by
\[
t_i = A(0) = A(a) = A(b) > A(c) = t_2
\]
It is easy to check that for any \(n > 2\)
\[
\tilde{A} = \{0_\lambda : \lambda \in (0, t_1]\} \cup \{a_\lambda : \lambda \in (0, t_2]\} \cup \{b_\lambda : \lambda \in (0, t_1]\} \cup \{c_\lambda : \lambda \in (0, t_1]\}
\]
is an n-fold weak P-weak ideal.

**Remark 4.6.** \(\tilde{A}\) is a 1-fold weak P-weak ideal of a BCK-algebra \(X\) if \(\tilde{A}\) is a weak P-weak ideal.

**Theorem 4.7.** [13] If \(A\) is a fuzzy subset of \(X\), then \(A\) is a fuzzy n-fold weak P-ideal if \(\tilde{A}\) is an n-fold weak P-weak ideal.

**Proof.** \(\Rightarrow\)
- Let \(\lambda \in \text{Im}(A)\) obviously \(0_\lambda \in \tilde{A}\);
- Let \((x_\lambda \ast z_\lambda) \ast (y_\mu \ast z_\mu) \in \tilde{A}\) and \(y_\mu \in \tilde{A}\), then
\[
A((x_\lambda \ast z_\lambda) \ast (y_\mu \ast z_\mu)) \geq \min(\lambda, \mu, \alpha) \quad \text{and} \quad A(y) \geq \mu.
\]
Since \(A\) is a fuzzy n-fold weak P-ideal, we have
\[
\forall x, y, z, A(x) \geq \min\left(A\left((x \ast z) \ast (y \ast z^n)\right), A(y)\right)
\]
\[
geq \min(\min(\lambda, \mu, \alpha), \alpha) = \min(\lambda, \mu, \alpha)
\]
Therefore \(x_{\min(\lambda, \mu, \alpha)} \in \tilde{A}\).

\(\Leftarrow\)
- Let \(x \in X\), it is easy to prove that \(A(0) \geq A(x)\);
- Let \(\forall x, y, z, A((x \ast z) \ast (y \ast z^n)) = \beta\) and \(A(y) = \alpha\).
Then \((x \ast z) \ast (y \ast z^n) = \min(\beta, \alpha) = (x_\beta \ast z_\beta) \ast (y_\alpha \ast z_\alpha) \in \tilde{A}\) and \(y_\alpha \in \tilde{A}\).
Since \(\tilde{A}\) is n-fold weak P-weak ideal, we have
\[
x_{\min(\beta, \alpha)} \in \tilde{A}
\]
Hence \(A(x) \geq \min(\beta, \alpha) = \min(A((x \ast z) \ast (y \ast z^n)), A(y))\).

**Proposition 4.8.** An n-fold weak P-weak ideal is a weak ideal.

**Proof.** Let \(x_\lambda, y_\mu \in \tilde{X}\) and \(x_\lambda \ast y_\mu = (x_\lambda \ast 0_\mu) \ast (y_\mu \ast 0_\mu) \in \tilde{A}\) and \(y_\mu\).
Since \(\tilde{A}\) is n-fold weak P-weak ideal, we have \(x_{\min(\lambda, \mu)} \in \tilde{A}\).

**Corollary 4.9.** A fuzzy n-fold weak P-ideal is a fuzzy ideal.

**Theorem 4.10.** Let \(\{\tilde{A}_i\}\) be a family of n-fold weak P-weak ideals and \(\{A_i\}\) be a family of fuzzy n-fold weak P-ideals. then 1) \(\bigcap_{i} \tilde{A}_i\) is an n-fold weak P-weak ideal.
2) \(\bigcap_{i} A_i\) is a fuzzy n-fold weak P-ideal.
3) \(\bigcup_{i} A_i\) is a fuzzy n-fold weak P-ideal.
4) \(\bigcup_{i} \tilde{A}_i\) is a fuzzy n-fold weak P-ideal.
Proof. 1) ∀ λ ∈ Im \left( \bigcap_{i \in I} \tilde{A} \right), then λ ∈ Im(\tilde{A}), ∀ i, so, 0, ∈ \tilde{A}, ∀ i, i.e.

0, ∈ \bigcap_{i \in I} \tilde{A}. For every x, y, z ∈ X, if

(\text{and } y, ∈ \bigcap_{i \in I} \tilde{A}, \text{ then }

\text{and } y, ∈ \bigcap_{i \in I} \tilde{A}, \text{ thus }

x_{min(, , , , )} \in \bigcap_{i \in I} \tilde{A}. Thus \bigcap_{i \in I} \tilde{A} \text{ is an n-fold weak P-weak ideal.}

2) ∀ λ ∈ Im(\bigcup_{i \in I} \tilde{A}), then ∃ λ, ∈ I, such that λ ∈ \tilde{A}, so, 0, ∈ \tilde{A}, i.e.

0, ∈ \bigcup_{i \in I} \tilde{A}. For every x, y, z ∈ X, if (\text{and } y, ∈ \bigcup_{i \in I} \tilde{A}, \text{ then }

(\text{and } y, ∈ \bigcup_{i \in I} \tilde{A}, \text{ thus }

x_{min(, , , , )} \in \bigcup_{i \in I} \tilde{A}. Thus \bigcup_{i \in I} \tilde{A} \text{ is an n-fold weak P-weak ideal.}

3) Follows from 1) and Theorem 4.7.

4) Follows from 2) and Theorem 4.7.

5. Algorithms

Here we give some algorithms for studying the structure of the foldness of (fuzzy) P-ideals in BCI-algebras.

Algorithm for AP-Ideals of BCI-Algebra

Input(X: BCI-algebra, *: binary operation, I: the subset of X);
Output(“I is a P-ideal of X or not”);
Begin
If I = ϕ then
go to (1.);
EndIf
If 0 ∉ I then
go to (1.);
EndIf
Stop := false;
i := 1;
While i ≤ |X| and not (Stop) do
j := 1;
While j ≤ |X| and not (Stop) do
k := 1;
While k ≤ |X| and not (Stop) do
If (x, y, z) ∈ I and y, ∈ I then
If x, ∉ I
Stop := true;
EndIf

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EndIf
Endwhile
Endwhile
If Stop then
Output ("I is aP-ideal of X")
Else
(1.) Output ("I is not aP-ideal of X")
EndIf
End

Algorithm for n-fold P-Ideals of BCI-Algebra
Input(X: BCI-algebra, * : binary operation, I: a subset of X);
Output("I is n-fold P-ideal of X or not");
Begin
If I φ= then
go to (1.);
EndIf
If 0 I ∉ then
go to (1.);
EndIf
Stop:=false;
1: i = 1;
While i ≤ |X| and not (Stop) do
j = 1;
While j ≤ |X| and not (Stop) do
k = 1;
While k ≤ |X| and not (Stop) do
If (x_i * z_k) * (y_j * z_k) ∈ I and y_j ∈ I then
If x_i * z_k ∉ I
Stop:=true;
EndIf
EndIf
EndIf
Endwhile
Endwhile
Endwhile
If Stop then
Output ("I is ann-fold P-ideal of X")
Else
(1.) Output ("I is not ann-fold P-ideal of X")
EndIf
End

Algorithm for Fuzzy P-Ideals of BCI-Algebra
Input(X: BCI-algebra, * : binary operation, A: the fuzzy subset of X);
Output("A is a fuzzy P-ideal of X or not");
Begin
    Stop:=false;
i := 1;
    While $i \leq |X|$ and not (Stop) do
        If $A(0) < A(x_i)$ then
            Stop:=true;
        EndIf
    Endwhile

    j := 1;
    While $j \leq |X|$ and not (Stop) do
        k := 1;
        While $k \leq |X|$ and not (Stop) do
            If $A(x_i * z_k) < A(x_j * y_k)$ then
                Stop:=true;
            EndIf
        Endwhile
    Endwhile

    If Stop then
        Output ("A is not a fuzzy P-ideal of X")
    Else
        Output ("A is a fuzzy P-ideal of X")
    EndIf
End

Algorithm for Fuzzy n-fold P-Ideals of BCI-Algebra
Input(X: BCI-algebra, * : binary operation, A: the fuzzy subset of X);
Output("A is a fuzzy n-fold P-ideal of X or not");
Begin
    Stop:=false;
i := 1;
    While $i \leq |X|$ and not (Stop) do
        If $A(0) < A(x_i)$ then
            Stop:=true;
        EndIf
    Endwhile

    j := 1;
    While $j \leq |X|$ and not (Stop) do
        k := 1;
        While $k \leq |X|$ and not (Stop) do
            If $A(x_i * y_k) < \min(A((x_i * z_k) * (y_j * z_k)), A(y_j))$ then
                Stop:=true;
            EndIf
        Endwhile
    Endwhile

    If Stop then
        Output ("A is not a fuzzy P-ideal of X")
    Else
        Output ("A is a fuzzy P-ideal of X")
    EndIf
End
Algorithm for N-Fold weak P-Ideals of BCI-Algebra
Input($X$: BCI-algebra, $k$ subset of $X$, $n \in \mathbb{N}$);
Output(“$I$ is ann-fold weak P-ideal of $X$ or not”);
Begin
    If $I = \emptyset$ then
go to (1.);
    EndIf
    If $0 \notin I$ then
go to (1.);
    EndIf
    Stop := false;
1: $i := 1$;
    While $i \leq |X|$ and not (Stop) do
        $j := 1$;
        While $j \leq |X|$ and not (Stop) do
            $k := 1$;
            While $k \leq |X|$ and not (Stop) do
                If $(x_i * z_k) * (y_j * z_k) \in I$ and $y_j \in I$ then
                    If $x_i \notin I$
                        Stop := true;
                    EndIf
                EndIf
            Endwhile
        Endwhile
    Endwhile
    If Stop then
        Output (“$I$ is ann-fold weak P-ideal of $X$”)
    Else
        (1.) Output (“$I$ is not ann-fold weak P-ideal of $X$”)
    EndIf
End

Algorithm for Fuzzy n-Fold weak P-Ideals of BCI-Algebra
Input($X$: BCI-algebra, $*$: binary operation, A fuzzy subset of $X$);
Output(“$A$ is a fuzzy n-fold weak P-ideal of $X$ or not”);
Begin
    Stop := false;
    $i := 1$;
    While $i \leq |X|$ and not (Stop) do
        If $A$ is a fuzzy n-fold weak P-ideal of $X$
If \( A(0) < A(x_j) \) then
\[
\text{Stop} := \text{true};
\]
EndIf

\( j := 1; \)

While \( j \leq |X| \) and not (Stop) do

\( k := 1; \)

While \( k \leq |X| \) and not (Stop) do

If \( A(x_j) < \min \left( A \left( x_j \ast z_j \right) \ast \left( y_j \ast z_j^* \right) \right) A(y_j) \) then
\[
\text{Stop} := \text{true};
\]
EndIf

Endwhile

Endwhile

Endwhile

If Stop then

Output ("A is not a fuzzy n-fold weak P-ideal of X")

Else

Output ("A is a fuzzy n-fold weak P-ideal of X")

EndIf

End

6. Conclusions and Future Research

In this paper, we introduce new notions of (fuzzy) n-fold P-ideals, and (fuzzy) n-fold weak P-ideals in BCI-algebras. Then we studied relationships between different type of n-fold P-ideals and investigate several properties of the foldness theory of P-ideals in BCI-algebras. Finally, we construct some algorithms for studying the foldness theory of P-ideals in BCI-algebras.

In our future study of foldness ideals in BCK/BCI algebras, maybe the following topics should be considered:

1) Developing the properties of foldness of implicative ideals of BCK/BCI algebras.

2) Finding useful results on other structures of the foldness theory of ideals of BCK/BCI algebras.

3) Constructing the related logical properties of such structures.

4) One may also apply this concept to study some applications in many fields like decision making, knowledge base systems, medical diagnosis, data analysis and graph theory.

Conflicts of Interest

The author declares no conflicts of interest regarding the publication of this paper.

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