Sum rules for asymptotic form factors
in $e^+e^- \rightarrow W^+W^-$ scattering

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Abstract
At very large energies and in $SU(2)_L \otimes U(1)_Y$ gauge theories, the trilinear gauge boson vertices relevant for $e^+e^- \rightarrow W^+W^-$ scattering are related in a simple way to the gauge boson self-energies. We derive these relations, both from the requirement of perturbative unitarity and from the Ward identities of the theory. Our discussion shows that, in general, it is never possible to neglect vector boson self-energies when computing the form factors that parametrize the $e^+e^- \rightarrow W^+W^-$ helicity amplitudes. The exclusion of the self-energy contributions would lead to estimates of the effects wrong by orders of magnitude. We propose a simple way of including the self-energy contributions in an appropriate definition of the form factors.
1. It has become customary to analyse the $W$ pair production in $e^+e^-$ machines by parametrizing the scattering amplitude in terms of a set of form factors characterizing the most general trilinear gauge vertex (TGV) involving a neutral vector boson $V$ ($V = \gamma, Z$) and two on-shell $W's$ \[1, 2\]. By studying the angular distributions of the $W$ it is possible to extract some informations on the form factors, thus providing new tests of the underlying electroweak theory \[2, 3\]. Any SM extension can be easily constrained, by analysing directly the contribution of the new particles and/or interactions to the different anomalous couplings. Indeed, in any recent analysis of $e^+e^- \rightarrow W^+W^-$ \[4, 5\], the above mentioned form factors play a crucial role and represent an important ingredient for a meaningful comparison between theory and experiment. For this reason any new information on the trilinear gauge vertices is welcome, particularly if it comes from the theory.

Purpose of this note is to show that, at large energies, the TGV relevant to the $e^+e^- \rightarrow W^+W^-$ process satisfy a set of relations that will be called sum rules. These relations can be viewed as a direct consequence of perturbative unitarity, that is the physical requirement that, at a given loop-order, the bad unitarity behaviour, which potentially affects the scattering amplitudes in massive Yang-Mills theories, disappears thanks to a cancellation among the dangerous contributions. Indeed, in the first part of this note we derive the sum rules by discussing the one-loop renormalization of the $e^+e^- \rightarrow W^+W^-$ scattering, and by imposing unitarity to the amplitude obtained.

It has been known since a long time that there is a deep relation between unitarity and gauge invariance \[6\] and in the second part of this work we re-derive the sum rules starting from the Ward identities of a spontaneously broken $SU(2) \otimes U(1)$ theory.

Although the sum rules presented here are valid only in the limit of very large energies, we will see how they qualitatively constrain also the region of experimental interest, namely the one between the electroweak scale $m_W$ and the scale $M$ associated to new physics. An important feature of the sum rules is that they involve combinations of the three-point and the two-point functions among vector bosons. This emphasizes the importance of including the vector boson self-energies for a correct evaluation of the form factors which parametrize the $e^+e^- \rightarrow W^+W^-$ process. Use of form factors explicitly involving only the TGV may easily lead to wrong conclusion about the corresponding cross-section. We will discuss these issues in a concluding paragraph.

2. We start the discussion by summarizing the standard parametrization of the $e^+e^- \rightarrow W^+W^-$ helicity amplitudes \[2\] to which we will refer in the following. We denote with $\sigma$ ($\bar{\sigma}$) and $\lambda$ ($\bar{\lambda}$) the helicities of the electron (positron) and of the $W^-$ ($W^+$), with $\Theta$ the scattering angle of the $W^-$ with respect to the $e^-$ direction in the $e^+e^-$ center of mass frame. The polarization amplitude reads:

$$M_{\sigma;\bar{\lambda}}(\Theta) = \sqrt{2}e^2\tilde{M}_{\sigma;\bar{\lambda}}(\Theta) \epsilon \, d^{J_0}_{\Delta \sigma, \Delta \lambda}(\Theta)$$

where $\epsilon = \Delta \sigma (-1)^{\bar{\lambda}}$, $\Delta \sigma = (\sigma - \bar{\sigma})/2$, $\Delta \lambda = \lambda - \bar{\lambda}$, $J_0 = max(|\Delta \sigma|, |\Delta \lambda|)$ and $d^{J_0}_{\Delta \sigma, \Delta \lambda}$ are the $d$ functions \[7\].
When $\Delta \lambda = \pm 2$, the reduced amplitude $\tilde{M}$ contains only the contribution from the neutrino exchange and is given by:

$$\tilde{M} = -\sqrt{2} \frac{\delta_{\Delta \sigma,-1}}{\sin^2 \theta} \frac{1}{1 + \beta^2 - 2\beta \cos \Theta}$$  \hspace{1cm} (2)

Here $\beta = \sqrt{1 - 4m_W^2/s}$ is the $W$ velocity and $\theta$ is the weak mixing angle. For $|\Delta \lambda| \leq 1$, the amplitude is a sum of three contributions:

$$\tilde{M} = \tilde{M}^\gamma + \tilde{M}^Z + \tilde{M}^\nu$$  \hspace{1cm} (3)

where

$$\tilde{M}^\gamma = -\beta \delta_{|\Delta \sigma|,1} \left[A_{\lambda \lambda}^{\gamma} + \delta A_{\lambda \lambda}^{\gamma}\right]$$

$$\tilde{M}^Z = \beta \frac{s}{s - m_Z^2} \left[\delta_{|\Delta \sigma|,1} - \frac{\delta_{\Delta \sigma,-1}}{2 \sin^2 \theta}\right] \left[A_{\lambda \lambda}^{Z} + \delta A_{\lambda \lambda}^{Z}\right]$$

$$\tilde{M}^\nu = \frac{\delta_{\Delta \sigma,-1}}{2 \sin^2 \theta} \beta \left[B_{\lambda \lambda} - \frac{1}{1 + \beta^2 - 2\beta \cos \Theta} C_{\lambda \lambda}\right]$$  \hspace{1cm} (4)

The coefficients $A$, $\delta A$, $B$ and $C$ contain the dependence on the TGV. The terms $A$, $B$ and $C$ represent the SM, tree-level contribution and they are explicitly listed in Table 1.

| $\lambda \bar{\lambda}$ | $A_{\lambda \lambda}^{\gamma} = A_{\lambda \lambda}^{Z}$ | $B_{\lambda \lambda}$ | $C_{\lambda \lambda}$ |
|--------------------------|---------------------------------|-----------------|-----------------|
| $++$, $--$               | 1                               | 1               | $1/\gamma^2$    |
| $+0$, $0-$               | $2\gamma$                       | $2\gamma$       | $2(1 + \beta)/\gamma$ |
| $0+$, $-0$               | $2\gamma$                       | $2\gamma$       | $2(1 - \beta)/\gamma$ |
| $00$                     | $2\gamma^2 + 1$                 | $2\gamma^2$     | $2/\gamma^2$    |

Table 1: Standard Model coefficients expressed in terms of $\gamma^2 = s/4m_W^2$.

On the contrary, any additional contribution is given by $\delta A$ which, assuming exact CP invariance, can be decomposed in terms of four form factors:

$$\delta A_{++}^V = \delta A_{--}^V = \delta f_1^V$$

$$\delta A_{+0}^V = \delta A_{0+}^V = \gamma (\delta f_3^V + \beta \delta f_5^V)$$

$$\delta A_{00}^V = \gamma^2 \left[-(1 + \beta^2)\delta f_1^V + 4\gamma^2 \beta \delta f_2^V + 2\delta f_3^V \right]$$  \hspace{1cm} (5)

The form factors $\delta f_i^V$ ($i = 1, 2, 3, 5$) come from the parametrization of the most general $VWW$ vertex ($V = \gamma, Z$), compatible with Lorentz and CP symmetries and with off-shell components projected out:

$$\Gamma_{\mu \alpha \beta}^V(p, q, \bar{q}) = (f_1^V + \delta f_1^V)k_\mu g_{\alpha \beta} - \delta f_2^V k_\mu \frac{p_\alpha p_\beta}{p^2}$$

$$+ (f_3^V + \delta f_3^V)(p_\alpha g_{\mu \beta} - p_\beta g_{\mu \alpha}) + i\delta f_5^V \epsilon_{\mu \alpha \beta \rho} k^\rho$$  \hspace{1cm} (6)
where \((p, q, \bar{q})\) and \((\mu, \alpha, \beta)\) are the momenta and Lorentz indices of the \((V, W^-, W^+)\) lines, \(k_\mu = q_\mu - \bar{q}_\mu\) and we have explicitly separated in \(f^V_{1,3}\) the SM, tree-level contribution:

\[
f^V_1 = 1, \quad f^V_3 = 2
\]

Notice that we have also redefined the form factor \(f_2\) of ref. [2] according to:

\[
4\gamma^2 f^V_2 \rightarrow \delta f^V_2
\]

We recall that a factor \(-e (-e \cot \theta)\) is conventionally extracted from the \(WW\gamma (WWZ)\) vertex. The reduced amplitudes of eqs. (2,4) allow to analyse quite clearly the high-energy behaviour of the process. Referring to the SM, one immediately sees that, due to the dependence on \(\gamma\) of the \(A\) and \(B\) coefficients, the separate amplitudes of eqs. (4) with one or both \(W\)’s longitudinally polarized diverge in the large energy limit. In the sum of eq. (3), those terms cancel as a result of the asymptotic equalities:

\[
A^\gamma_{\lambda\bar{\lambda}} = A^Z_{\lambda\bar{\lambda}} = B_{\lambda\bar{\lambda}}
\]

as required by unitarity [1].

All the above formulae and properties are standard and well known and we have reported them here only to keep our presentation self-contained.

We also recall that, when discussing non standard contributions to \(e^+e^- \rightarrow W^+W^-\), it is current practice to assume that any deviation from the SM predictions is concentrated in the TGV. Departures from the SM at the level of vector boson self-energies or in \(e^+e^- V \quad (V = \gamma, Z)\) vertices are assumed to be negligibly small in comparison to the deviations which can occur in TGV. This assumption is certainly well supported by the overall amount of data accumulated by LEP1 and SLC, which have made possible to test vector boson two-point functions and fermion-antifermion-gauge boson vertices at the per mille level. However, in any SM extension one can think of, it is quite difficult, barring unnatural cancellation, to satisfy the previous assumption. Deviations from the SM naturally occur in two- as well as three-point functions for the electroweak bosons. Indeed, this fact has been exploited to point out that it is implausible to expect large deviations from the SM in \(W\) pair production at LEP2, since the modifications of the TGV required to produce an observable effect would have probably had a visible counterpart at LEP1/SLC [3]. In the present note we will assume that the underlying theory possesses the following properties:

(i) Gauge invariance under \(SU(2)_L \otimes U(1)_Y\) and discrete CP invariance.

(ii) The deviations from the SM induced by the new particles or interactions are all of oblique type, that is they only affects the \(n\)-point gauge vector boson functions \((n = 2, 3, 4, \ldots)\).

(iii) The deviations from the SM are small and they can be treated perturbatively in some parameter. In practice we will discuss \(W\) pair production in the one-loop approximation.

\[^1\text{Moreover, tree-level unitarity constraints can be obtained on combinations of } \delta f^V \text{ [8].}\]
To specify two- and three-point functions for the gauge vector bosons we will perform the computation in the framework of the background field gauge, by choosing the t’Hooft-Feynman gauge for the quantum fields. Then, in any theory fulfilling the properties (i)-(iii), at one-loop level the amplitude for $e^+ e^- \rightarrow W^+ W^-$ can be cast in the following form [10]:

$$\Delta \lambda = \pm 2$$

$$\Delta \lambda \leq 1$$

$$\hat{M} = -\frac{\sqrt{2}}{\sin^2 \theta} \delta_{\Delta \sigma, -1} \left[ 1 + \frac{\sin^2 \bar{\theta}}{\cos 2 \theta} \Delta r_W - e_6 \right] \frac{1}{1 + \beta^2 - 2 \beta \cos \Theta}$$

$$\hat{M}^\rho = -\beta \delta_{\Delta \sigma, 1} \left[ 1 + \Delta \alpha(s) \right] \left[ A^\rho_{\lambda \bar{\lambda}} + \delta A^\rho_{\lambda \bar{\lambda}}(s) \right]$$

$$\hat{M}^Z = \frac{\beta - s}{s - m_Z^2} \left[ \delta_{\Delta \sigma, 1} \left[ 1 + \Delta \rho(s) \right] + \frac{\cos^2 \bar{\theta}}{\cos^2 \theta} \Delta k(s) \right] \left[ A^Z_{\lambda \bar{\lambda}} + \delta A^Z_{\lambda \bar{\lambda}}(s) \right]$$

$$\hat{M}^\nu = \frac{\delta_{\Delta \sigma, -1}}{2 \sin^2 \theta} \beta \left[ 1 - \frac{\sin^2 \bar{\theta}}{\cos 2 \theta} \Delta r_W - e_6 \right] \left[ B_{\lambda \bar{\lambda}} - \frac{1}{1 + \beta^2 - 2 \beta \cos \Theta} C_{\lambda \bar{\lambda}} \right]$$

Here the coefficients $\delta A^V_{\lambda \bar{\lambda}}$ are still given by eq. (5) where the form factors $\delta f^V_i(s) (i = 1, 2, 3, 5)$, which in our convention include only the contribution coming from the unrenormalized, irreducible, 1-loop correction to the vertex $V W W$, should be replaced by combinations $\delta F^V_i(s)$ which account for the wave function renormalization of the external $W$ legs. This replacement is explicitly given by:

$$\delta F^V_1(s) = \delta f^V_1(s) - \Pi'_W(m^2_W)$$
$$\delta F^V_2(s) = \delta f^V_2(s)$$
$$\delta F^V_3(s) = \delta f^V_3(s) - 2 \Pi'_W(m^2_W)$$
$$\delta F^V_5(s) = \delta f^V_5(s)$$

Whereas the form factors $\delta f^V_i(s)$ are ultraviolet divergent, the quantities $\delta F^V_i(s)$ are all finite.

The quantities $\Delta \alpha(s), \Delta k(s), \Delta \rho(s), \Delta r_W$ and $e_6$ appearing in eqs. (III) and (IV) are finite self-energy corrections, defined by:

$$\Delta \alpha(s) = \Pi'_{\gamma \gamma}(s) - \Pi'_{\gamma \gamma}(0)$$
$$\Delta k(s) = -\frac{\cos^2 \bar{\theta}}{\cos 2 \theta} (e_1 - e_4) + \frac{1}{\cos 2 \theta} e_3(s)$$
$$\Delta \rho(s) = e_1 - e_5(s)$$
$$\Delta r_W = -\frac{\cos^2 \bar{\theta}}{\sin^2 \theta} e_1 + \frac{\cos 2 \bar{\theta}}{\sin^2 \theta} e_3(m^2_Z) + e_2 + 2 e_3(m^2_Z) + e_4$$
$$e_6 = \Pi'_{WW}(m^2_W) - \Pi'_{WW}(0)$$

(13)
where

\[ e_1 = \frac{\Pi_{ZZ}(0)}{m_Z^2} - \frac{\Pi_{WW}(0)}{m_W^2} \]

\[ e_2 = \Pi'_{WW}(0) - \cos^2 \theta \Pi'_{ZZ}(0) - 2 \cos \theta \sin \theta \frac{\Pi'_{ZZ}(m_Z^2)}{m_Z^2} \]

\[ - \sin^2 \theta \Pi'_{\gamma\gamma}(m_Z^2) \]

\[ e_3(s) = \frac{\cos \bar{\theta}}{\sin \theta} \left\{ \sin \bar{\theta} \cos \bar{\theta} \left[ \Pi'_{\gamma\gamma}(m_Z^2) - \Pi'_{ZZ}(0) \right] + \cos 2 \bar{\theta} \frac{\Pi'_{ZZ}(s)}{s} \right\} \]

\[ e_4 = \Pi'_{\gamma\gamma}(0) - \Pi'_{\gamma\gamma}(m_Z^2) \]

\[ e_5(s) = \Pi'_{ZZ}(s) - \Pi'_{ZZ}(0) \]

(14)

In the previous eqs. \( \Pi_{ij} \ (i, j = \gamma, Z \text{ or } W) \) stands for the transverse part of the unrenormalized gauge vector bosons self-energy and

\[ \Pi'_{VV'}(s) = \frac{\Pi_{VV'}(s) - \Pi_{VV}(m_{VV'}^2)}{(s - m_{VV'}^2)} \quad (V, V' = \gamma, Z, W) \]

(15)

with \( m_{\gamma\gamma} = m_{\gamma Z} = 0, m_{ZZ} = m_Z \) and \( m_{WW} = m_W \).

Finally, the effective weak angle \( \bar{\theta} \) is defined by:

\[ \sin^2 \bar{\theta} = \frac{1}{2} - \sqrt{\frac{1}{4} - \frac{\pi \alpha(s)}{\sqrt{2} G_F m_Z^2}} \]

(16)

where \( \alpha(s) \) is the electromagnetic coupling with all the effects coming from SM particles included at the given energy \( s \).

Some comments are in order. First of all we will neglect SM radiative corrections in this analysis. They constitute a gauge-invariant set of corrections and the partial amplitudes they give rise satisfy unitarity by themselves. Here we would like to focus on the contribution coming from new physics. We have made use of an on-shell renormalization scheme based on the renormalized parameters \( \alpha, G_F \) and \( m_Z \). The function \( \Delta \alpha(s) \) describes the running of \( \alpha \) due to the new particles; \( \Delta k(s) \) represents an energy dependent shift of the weak effective angle; \( \Delta \rho(s) \) is related to the different strengths of the neutral and charged current interactions; \( e_6 \) is due to the wave-function renormalization of the external \( W \)'s in the amplitude with neutrino exchange and \( \Delta r_W \) arises when expressing the electron-neutrino-\( W \) coupling in terms of \( \alpha \) and \( \sin^2 \bar{\theta} \). If \( s = m_Z^2 \) then \( \Delta \alpha(s), \Delta k(s), \Delta \rho(s) \) coincides with the corrections \( \Delta \alpha, \Delta k, \Delta \rho \) which characterize the electroweak observables at the \( Z \) resonance [12, 13, 14].

We now consider the high-energy limit of the above amplitudes. When the energy is much larger than the characteristic scale \( M \) of the considered theory, the tree-level asymptotic relations \( A_{\lambda\lambda}^\gamma = A_{\lambda\lambda}^Z = B_{\lambda\lambda} \) are not sufficient to guarantee the correct behaviour for the amplitude with one or both longitudinally polarized \( W \)'s. With the inclusion of one-loop contributions, one has new terms proportional to \( \gamma^2 \) and \( \gamma \) (see \( \delta A_{\lambda\lambda}^\gamma \) and \( \delta A_{\lambda\lambda}^Z \) [15].

\[ \delta A_{\lambda\lambda}^\gamma \]
in eq. (11) and the cancellation of those terms in the high-energy limit entails relations among oblique and vertex corrections. By requiring the asymptotic cancellation of the terms proportional to positive powers of $\gamma$, separately for the $\Delta\sigma = 1$ and the $\Delta\sigma = -1$ amplitudes, we obtain:

\[
\left[ (\Delta\alpha(s) - \Delta\rho(s) - \cos\frac{2\theta}{\cos^2\theta}\Delta k(s))A_{\lambda\bar{\lambda}}^y + \delta A_{\lambda\bar{\lambda}}^z \right]_\infty = \text{constant}
\]

\[
\left[ (\Delta\rho(s) - \frac{\sin^2\theta}{\cos^2\theta}\Delta k(s)) + \frac{\sin^2\theta}{\cos 2\theta}\Delta r_W(e_6)A_{\lambda\bar{\lambda}}^y + \delta A_{\lambda\bar{\lambda}}^Z \right]_\infty = \text{constant}
\] (17)

where the suffix $\infty$ indicates that the relations hold only for asymptotically large energies. In eq. (17) at least one of the helicities $\lambda$, $\bar{\lambda}$ is required to vanish; when CP invariance is assumed, this occurs for three independent helicity combinations ($\lambda\bar{\lambda}$). This makes a total of six independent relations, hereafter termed sum rules. These relations can be also expressed in terms of unrenormalized self-energies and TGV. Using eqs. (5), (12), (13), (14), (15), it is possible to write the sum rules in eq. (17) in the equivalent form:

\[
[2\delta f_1^y(s) - \delta f_2^y(s)]_\infty = \left[ \delta f_3^y(s) \right]_\infty = \frac{2}{s} \left[ \Pi_{\gamma\gamma}(s) + \frac{\cos\theta}{\sin\theta}\Pi_{\gamma\gamma}(s) \right]_\infty
\]

\[
[\delta f_5^y(s)]_\infty = 0
\]

\[
[2\delta f_1^Z(s) - \delta f_2^Z(s)]_\infty = \left[ \delta f_3^Z(s) \right]_\infty = \frac{2}{s} \left[ \Pi_{ZZ}(s) + \frac{\sin\theta}{\cos\theta}\Pi_{\gamma\gamma}(s) \right]_\infty
\]

\[
[\delta f_5^Z(s)]_\infty = 0
\] (18)

Before illustrating how, in specific models, these sum rules are satisfied, we will discuss how they are related to the symmetry properties of the underlying theory.

Green functions of spontaneously broken gauge theories satisfy Ward Identities (WI), which take a particularly simple form in the background field gauge formalism \[15\]. The case of $SU(2)_L \otimes U(1)_Y$ has been explicitly worked out in ref. \[16\] \footnote{Analogous WI can be derived in one-loop approximation in the framework of the pinch-technique \[17\].}. Relevant to our discussion are the WI relating TGV, gauge vector boson self-energies and vertex functions with two gauge vector bosons and a goldstone boson $\varphi$:

\[
q^\alpha \Gamma_{\mu\nu\rho}^{VVW}(p, q, \bar{q}) + im_W \Gamma_{\mu\nu}^{W\varphi}(p, q, \bar{q}) = \left[ \Pi_{\mu\nu}(p) - \Pi_{\mu\nu}^W(\bar{q}) \right]
\] (19)

where $V = \gamma, Z$ and the functions $\Pi_{\mu\nu}^V(p)$ are defined by:

\[
\Pi_{\mu\nu}^y(p) = \Pi_{\mu\nu}^{\gamma}(p) + \frac{\cos\theta}{\sin\theta}\Pi_{\mu\nu}^{Z}(p)
\]

\[
\Pi_{\mu\nu}^Z(p) = \Pi_{\mu\nu}^{ZZ}(p) + \frac{\sin\theta}{\cos\theta}\Pi_{\mu\nu}^{Z}(p)
\] (20)

These identities are direct consequence of the $SU(2)_L \otimes U(1)$ gauge invariance and we are naturally lead to explore the relation between the sum rules of eq. (18) and the WI.
listed above. Indeed, as we will now show, the sum rules discussed here can be directly
derived from the WI of the theory. To this purpose we consider the general, off-shell,
decomposition of the VWW and VφW vertices:

\[ \Gamma_{\mu\alpha\beta}^{VWW}(p, q, \bar{q}) = \delta f^V_1 k_{\mu} g_{\alpha\beta} - \delta f^V_2 k_{\mu} \frac{p_{\alpha} p_{\beta}}{p^2} + \delta f^V_3 (p_{\alpha} g_{\mu\beta} - p_{\beta} g_{\mu\alpha}) + \]

\[ i \delta f^V_5 \epsilon_{\mu\alpha\beta\rho} k^{\rho} - \delta h^V_2 \frac{k_{\alpha} k_{\beta} k_{\mu}}{p^2} + \delta h^V_3 (k_{\alpha} g_{\mu\beta} + k_{\beta} g_{\mu\alpha}) - \delta h^V_4 (k_{\alpha} p_{\beta} - k_{\beta} p_{\alpha}) \frac{k_{\mu}}{p^2} \]

\[ + i \delta h^V_5 \epsilon_{\mu\rho\sigma} \frac{p^\rho k^\sigma}{p^2} [(p + k)_{\alpha} \delta_{\beta\gamma} - (p - k)_{\beta} \delta_{\alpha\gamma}] + i \delta k^V_6 \epsilon_{\alpha\beta\rho\sigma} \frac{p^\rho k^\sigma}{p^2} p_{\mu} \quad (21) \]

\[ i \Gamma_{\mu\beta}^{V\phi W}(p, q, \bar{q}) = \delta \varphi^V g_{\mu\beta} + \delta \varphi^V \frac{p_{\mu} p_{\beta}}{p^2} + \delta \varphi^V \frac{\bar{q}_{\mu} \bar{q}_{\beta}}{p^2} + \delta \varphi^V \frac{\bar{q}^\rho p^\sigma}{p^2} + i \delta \varphi^V \epsilon_{\mu\beta\rho\sigma} \frac{p^\rho k^\sigma}{p^2} \quad (22) \]

We also make use of the standard parametrization for the vector boson self-energies:

\[ \Pi_{\mu\nu}^{ij}(p) = (g_{\mu\nu} - \frac{p_{\mu} p_{\nu}}{p^2}) \Pi^{ij}(p^2) + \frac{P_{\mu} P_{\nu}}{p^2} \Pi_L^{ij}(p^2) \quad (23) \]

Notice that all the form factors of eq. (21) are dimensionless function of \( p^2 \), \( q^2 \) and \( \bar{q}^2 \).
Moreover, when the terms proportional to \( p_{\mu} \), \( q_{\alpha} \) and \( \bar{q}_{\beta} \) are neglected, as for the case of
the on-shell amplitude of interest, the above decomposition collapses on that given in eq.
(19), provided one makes the following identifications:

\[ \delta f^V_1 = \delta f^V \]
\[ \delta f^V_2 = \delta f^V - \delta h^V_2 - 2 \delta h^V_4 \]
\[ \delta f^V_3 = \delta f^V - \delta h^V_3 \]
\[ \delta f^V_5 = \delta f^V \]

\[ \delta \varphi^V = \frac{p^\rho k^\sigma}{p^2} \quad (24) \]

When we combine eqs. (21), (22) and (23) with eqs. (19), after identifications of the
independent tensor structures, we obtain the following relations:

\[ \frac{p^2 + p k}{2} \delta f^V_3 + \frac{p k + k^2}{2} \delta h^V_3 + m_w \delta \varphi^V = \Pi^V(p^2) - \Pi^{WW}(\bar{q}^2) \]

\[ \frac{p^2 + p k}{2 p^2} \delta f^V_2 - \frac{p k + k^2}{2 p^2} \delta h^V_2 + \frac{p^2 - k^2}{2 p^2} \delta h^V_4 - \delta f^V_3 + \delta h^V_3 - \frac{p^2 + 2 p k + k^2}{2 p^2} \delta h^V_5 + \frac{m_w}{p^2} \delta \varphi^V = \frac{\Pi^V_L(p^2)}{p^2} - \frac{\Pi^V(p^2)}{p^2} \]

\[ -2 \delta f^V_1 + \frac{p^2 + p k}{p^2} \delta f^V_2 + \frac{p k + k^2}{p^2} \delta h^V_2 - \frac{p^2 - k^2}{p^2} \delta h^V_4 + \]

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\[ p^2 + 2pk + k^2 \delta f_3^V - \delta f_6^V + \frac{m_W}{p^2} \delta \phi_3^V = 0 \]

\[ - \delta f_1^V + \frac{p^2 + 2pk + k^2}{p^2} \delta h_2^V - 2\delta h_3^V - \frac{p^2 + pk}{p^2} \delta f_4^V + \frac{p^2 + pk}{p^2} \delta h_5^V + \frac{m_W}{p^2} \delta \phi_4^V = 0 \]

\[ 2\delta f_1^V - 2\frac{p^2 + pk}{p^2} \delta h_2^V + 2\delta h_3^V + \frac{2p^2 + pk}{p^2} \delta h_4^V + \frac{m_W}{p^2} \delta \phi_5^V = \frac{\Pi^{WW}(q^2)}{q^2} - \frac{\Pi^{WW}(\bar{q}^2)}{\bar{q}^2} \]

We now impose the on-shell conditions \( q^2 = \bar{q}^2 = m_W^2 \) and we proceed to expand the above equations in inverse power of \( p^2 \). In doing that, we assume that for each form factor the leading term of this expansion is provided by naive dimensional analysis. In other words, the dimensionless functions \( \delta f_i^V \), \( \delta h_i^V \) and \( \delta k_i^V \) are assumed not to grow with positive powers of \( p^2 \). The two-point functions \( \Pi^{ij} \) and \( \Pi_L^{ij} \) are required to scale at most like \( p^2 \) and the form factors \( \delta \phi_i^V \) may scale as \( \sqrt{p^2} \). Logarithmic corrections to this behaviour are also admitted. By identifying, order by order, the expanded expressions one obtains an infinite set of relations. Those corresponding to the leading terms, after using eq. (24) are given by

\[
\begin{align*}
\left[ \delta f_3^V(p^2) \right]_\infty &= \frac{2}{p^2} \left[ \Pi^V(p^2) \right]_\infty \\
\left[ \delta f_4^V(p^2) + \delta f_5^V(p^2) + \delta f_6^V(p^2) \right]_\infty &= 0 \\
\left[ \delta f_1^V(p^2) - \frac{\delta f_2^V(p^2)}{2} - \delta f_3^V(p^2) \right]_\infty &= -\frac{1}{p^2} \left[ \Pi^V(p^2) - \Pi_L^V(p^2) \right]_\infty \\
\left[ \delta f_4^V(p^2) \right]_\infty &= 0 \\
\left[ \delta f_5^V(p^2) + \delta h_2^V(p^2) + 2\delta h_3^V(p^2) - \delta h_5^V(p^2) \right]_\infty &= 0 \\
\left[ 2\delta f_1^V(p^2) + 2\delta h_2^V(p^2) + 2\delta h_3^V(p^2) + 2\delta h_4^V(p^2) \right]_\infty &= \frac{1}{q^2} \left[ \Pi^{WW}(q^2) - \Pi_L^{WW}(\bar{q}^2) \right]
\end{align*}
\]

From these equations one can immediately recover the sum rules of eq. (26). One also finds \( \left[ \Pi_L^V(p^2) \right]_\infty = 0 \) and two conditions on the form factors \( \delta h_i^V \) \( (i = 2, \ldots, 5) \). Had we used the less general parametrization for the vertex functions with \( \delta h_i^V = 0 \) \( (i = 2, \ldots, 5) \), we would have obtained inconsistent equations. We notice that, in the derivation which

\(^3\)Notice that the factors of \( p^2 \) inserted in eq. (21) to obtain dimensionless form factors are consistent with this assumption. A similar comment applies to eq. (23).

\(^4\)One can check that some of the relations obtained at the next-to-leading order coincide with those predicted by the so-called equivalence theorem.
make use of the WI of eq. (19), the assumption (ii) is inessential. We are thus led to
conjecture that in the presence of corrections involving the electron-positron lines, such
corrections should satisfy a set of independent asymptotic relations which do not interfere
with those obtained here.

3. Eq. (18), or its equivalent form eq. (17), represents the main result of the present
work. Their meaning can be better elucidated with some examples.

For instance, the 1-loop contribution due to an additional, heavy quark doublet with
a degenerate mass $M$
reads:

$$
\delta f_1^\gamma(s) = \frac{g^2}{16\pi^2} \left[A + \frac{7}{6}\right] + ... \\
\delta f_2^\gamma(s) = -\frac{g^2}{16\pi^2} + ... \\
\delta f_3^\gamma(s) = \frac{g^2}{16\pi^2} \left[2A + \frac{10}{3}\right] + ... \\
\delta f_5^\gamma(s) = 0 \\
\Pi_{\gamma\gamma}(s) = \frac{g^2}{16\pi^2} \sin^2 \theta \left[\frac{20}{9}A + \frac{100}{27}\right] s + ... \\
\Pi_{\gamma Z}(s) = \frac{g^2}{16\pi^2} \frac{\sin \theta(9 - 20 \sin^2 \theta)}{\cos \theta} \left[\frac{1}{9}A + \frac{5}{27}\right] s + ... \\
\Pi_{Z Z}(s) = \frac{g^2}{16\pi^2} \frac{(9 - 18 \sin^2 \theta + 20 \sin^4 \theta)}{\cos^2 \theta} \left[\frac{1}{9}A + \frac{5}{27}\right] s + ... \\
$$

(27)

where

$$
A = \frac{2}{d-4} + i\pi - \text{Log} \left(\frac{s}{\mu^2}\right) \\
$$

(28)

and dots stand for terms of order $M^2/s$ or $m_W^2/s$. The leading terms of $\delta f_1^Z(s)$ in the
large $s$ limit coincide with those of $\delta f_1^\gamma(s)$ given above. The divergence contained in the
expression $A$ is cancelled in the combinations $\Delta \alpha(s), \Delta k(s), \Delta \rho(s), \Delta r_W$ and $e_6$ or when
including the $W$ wave function renormalization in the $VWW$ vertices, as explicitly shown
in eq. (12).

The asymptotic expressions of the previous equations satisfy the sum rules given in
eq. (17). The form factors $\delta f_{1,2,3}^{\gamma,s}(s)$ do not vanish at large energies, not even when
combined with the $W$ wave function renormalization. Rather, the correct high-energy
behaviour of the $e^+e^- \rightarrow W^+W^-$ amplitudes is assured by the interplay between vector
boson self-energies and TGV. For a new heavy quark doublet, we have explicitly verified
that the cancellation implied by eq. (17) holds only asymptotically. In this example, at
lower energies, the relations in eq. (17) are corrected by terms of order $M^2/s$. Indeed
no unitarity argument forbids such terms, which, enhanced by the additional longitudinal
factors $\gamma$ or $\gamma^2$, may lead to large and observable deviations from the SM amplitude, a
behaviour known as delayed unitarity [18].
As a second example we consider the effect of heavy electroweak gauginos, with a common mass $M$, in the minimal supersymmetric standard model. We assume that squarks, sleptons, higgsinos and additional higgses decouple from the low-energy theory due to their masses which we take much larger than $M$. In this case, we find it useful to give the complete result, up to terms of order $m_W^2/s$:

$$
\delta f_1^{\gamma,Z}(s) = \frac{g^2}{16\pi^2} \left[ \frac{4}{3} \left( A' - 2 \frac{s - 4M^2}{s} B \right) - \frac{2}{9s} \left( 48M^2 - 7s - 36M^2C \right) \right] 
$$

$$
\delta f_2^{\gamma,Z}(s) = \frac{g^2}{16\pi^2} \left[ 32 \frac{M^2}{s} B - \frac{4}{3s} \left( 24M^2 + s - 12M^2C \right) \right] 
$$

$$
\delta f_3^{\gamma,Z}(s) = \frac{g^2}{16\pi^2} \left[ \frac{8}{3} \left( A' - 2 \frac{s + 2M^2}{s} B \right) + \frac{8}{9s} \left( 12M^2 + 5s \right) \right] 
$$

$$
\delta f_5^{\gamma,Z}(s) = 0 
$$

$$
\Pi_{ZZ}(s) = \frac{g^2}{16\pi^2} \cos^2 \theta \left[ \frac{4}{3} \left( A' - 2(s + 2M^2)B \right) s + \frac{4}{9} \left( 12M^2 + 5s \right) \right] 
$$

$$
= \frac{\cos^2 \theta}{\sin^2 \theta} \Pi_{\gamma\gamma}(s) = \cos \theta \sin \theta \Pi_{\gamma Z}(s) 
$$

(29)

where

$$
A' = \frac{2}{4 - d} - \log \frac{M^2}{\mu^2} 
$$

$$
B = \sqrt{-1 + \frac{4M^2}{s}} \text{ArcTan} \left( \frac{1}{\sqrt{-1 + \frac{4M^2}{s}}} \right) 
$$

$$
C = \text{ArcTan} \left( \frac{1}{\left( -1 + \frac{4M^2}{s} \right)} \right) 
$$

(30)

We notice that in this case, the sum rules of eq. (17) are satisfied not only asymptotically, for $s$ much larger than $M$, but also at lower energies. Actually, neglecting terms of order $m_W^2/s$, they are satisfied identically in $s$. This means that in this case the cancellation dictated by unitarity take place before the asymptotic regime. The enhancement factors provided by the longitudinal $W$ components get now multiplied by terms of order $m_W^2/s$ in the overall amplitude and deviations from the SM larger than few per cent cannot be expected [10]. Quite a different result would have been obtained if one had neglected the contribution from the vector boson self-energies. In this case no unitarity cancellation would have occurred and, due to the unbalanced $\gamma$ or $\gamma^2$ contributions, one would have obtained large, unrealistic departures from the SM cross-section, even at energies below the threshold for production of new particles. This is exemplified in fig. 1 where we compare the cross-section for longitudinally polarized $W$'s (LL) obtained, for gauginos, with or without including the neutral gauge boson self-energies. This should sound as a warning against the assumption that the amplitudes for $e^+e^- \rightarrow W^+W^-$ are dominated by the irreducible TGV corrections.
Figure 1: Relative deviation $\Delta R$ from the SM cross-section $d\sigma/d\cos\Theta$ versus $\cos\Theta$ in the LL polarization channel at $\sqrt{s} = 500$ GeV. The figure on the left shows the contribution due to gauginos of mass respectively $M = 300, 600, 1000$ GeV (full, dotted, dashed) when all the contributions (bilinear and trilinear) are included. The one on the right is without self-energy contributions.

4. The parametrization given in eqs. (1-6) is quite efficient and widely used in the literature. Indeed, as will be shown in this section, it is possible to cast the results of our one-loop computation in a form which is very close to that displayed in eqs. (1-6). By inspecting eqs. (9-10), we are led to introduce an effective, $s$-dependent, Weinberg angle defined by:

$$\sin^2 \theta_{\text{eff}}(s) = \sin^2 \bar{\theta}(1 + \Delta k(s))$$

Then we proceed by absorbing the overall correction to the neutrino-exchange amplitude in a redefinition of the electric charge:

$$e^2_{\text{eff}}(s) = e^2(1 + \Delta k(s)) - \frac{\sin^2 \bar{\theta}}{\cos 2\bar{\theta}} \Delta r_W - e_6$$

When expressed in terms of $\sin^2 \theta_{\text{eff}}(s)$ and $e^2_{\text{eff}}(s)$, the amplitudes of eqs. (1)[11] will coincide with those of eqs. (1)[11], provided one defines new coefficients $\delta A$, which, to avoid confusion, we will denote by $\Delta A$:

$$\Delta A^\gamma_{\lambda\lambda}(s) = \delta A^\gamma_{\lambda\lambda}(s) + A^\gamma_{\lambda\lambda} \left(\frac{\sin^2 \bar{\theta}}{\cos 2\bar{\theta}} \Delta r_W + e_6 + \Delta \alpha(s) - \Delta k(s)\right)$$

$$\Delta A^Z_{\lambda\lambda}(s) = \delta A^Z_{\lambda\lambda}(s) + A^Z_{\lambda\lambda} \left(\Delta \rho(s) - \frac{\sin^2 \bar{\theta}}{\cos 2\bar{\theta}} \Delta k(s)\right) + \frac{\sin^2 \bar{\theta}}{\cos 2\bar{\theta}} \Delta r_W + e_6$$

If we express the quantities $\Delta \alpha(s)$, $\Delta k(s)$, $\Delta \rho(s)$, $\Delta r_W$ and $e_6$ in terms of unrenormalized self-energy corrections, as in eqs. (1)[13] and (1)[14], and if we relate the coefficients $\delta A$ to the
unrenormalized vertex corrections, we find that eqs. (33) are equivalent to the following definition of form factors:

\[
\begin{align*}
\Delta f_V^1(s) &= \delta f_V^1(s) - \hat{\Pi}^V(s) \\
\Delta f_V^2(s) &= \delta f_V^2(s) \\
\Delta f_V^3(s) &= \delta f_V^3(s) - 2\hat{\Pi}^V(s) \\
\Delta f_V^5(s) &= \delta f_V^5(s)
\end{align*}
\] (34)

where the functions \(\hat{\Pi}^V(s)\) \((V = \gamma, Z)\) are explicitly given by:

\[
\begin{align*}
\hat{\Pi}^\gamma(s) &= \Pi_{\gamma\gamma}'(s) + \frac{\cos \bar{\theta} \Pi_{\gamma Z}(s)}{\sin \bar{\theta}} \\
\hat{\Pi}^Z(s) &= \Pi_{ZZ}'(s) + \frac{\sin \bar{\theta} \Pi_{\gamma Z}(s)}{\cos \bar{\theta}}
\end{align*}
\] (35)

To summarize, by replacing the electric charge \(e\), the Weinberg angle \(\theta\) and the form factors \(\delta f_V^i(s)\) of eqs. (1-6) with the quantities \(e_{\text{eff}}(s)\), \(\theta_{\text{eff}}(s)\), \(\Delta f_V^i(s)\) introduced above, one reproduces exactly the one-loop amplitude discussed in the present paper. In fact, eqs. (34-35) provide a concise and practical prescription to account for the self-energy effects in \(e^+e^- \rightarrow W^+W^-\), by including them in appropriately defined form factors. Moreover, it is immediate to show that the sum rules discussed before, when expressed in terms of the form factors \(\Delta f_V^i(s)\), read:

\[
\begin{align*}
[2\Delta f_V^1(s) - \Delta f_V^2(s)]_\infty &= [\Delta f_V^3(s)]_\infty = [\Delta f_V^5(s)]_\infty = 0
\end{align*}
\] (36)

When discussing the low-energy limit of the process, an effective lagrangian description turns out to be useful. The departures from the tree-level SM predictions are described by a set of operators (organized in a dimensional or derivative expansion) whose coefficients can be related to physical observables. In particular the low-energy limit of the form factors \(\Delta f_V^i\) can be expressed in terms of the coefficients \(a_i\) \((i = 0, \ldots, 14)\) characterizing the so-called electroweak chiral lagrangian \([19, 20, 21]\) [5]:

\[
\begin{align*}
\Delta f_1^\gamma(0) &= g^2(a_1 - a_8) \\
\Delta f_2^\gamma(0) &= 0 \\
\Delta f_3^\gamma(0) &= g^2(a_1 - a_8 + a_2 - a_3 - a_9) \\
\Delta f_5^\gamma(0) &= 0 \\
\Delta f_1^Z(0) &= -g^2(a_8 + a_{13}) - g^2 \tan^2 \bar{\theta}(a_1 + a_{13}) - \frac{g^2}{\cos^2 \bar{\theta}} a_3 \\
\Delta f_2^Z(0) &= 0 \\
\Delta f_3^Z(0) &= -g^2(a_8 + a_3 + a_9 + a_{13}) - g^2 \tan^2 \bar{\theta}(a_1 + a_2 + a_{13}) - \frac{g^2}{\cos^2 \bar{\theta}} a_3 \\
\Delta f_5^Z(0) &= -\frac{g^2}{\cos^2 \bar{\theta}} a_{14}
\end{align*}
\] (37)

---

5We follow the conventions of the first paper in ref. [5].
We notice that the combinations $\Delta f_3^V(0) - \Delta f_1^V(0)$ \quad ($V = \gamma, Z$) depend only on the coefficients $a_2$, $a_3$ and $a_9$ which parametrize the directions that are blind to the LEP1 precision tests [9].

5. In conclusion, we have shown that, at asymptotically large energies, the trilinear gauge boson vertices in $e^+e^- \rightarrow W^+W^-$ amplitudes obey a set of sum rules connecting them to gauge vector boson self-energies. We have derived the sum rules by performing an explicit one-loop computation in a certain class of theories and by demanding that the resulting amplitudes satisfy the requirement of perturbative unitarity. We have also demonstrated that these sum rules are a direct consequence of the Ward identities of spontaneously broken $SU(2)_L \otimes U(1)_Y$ gauge theories and therefore they are verified in a more general context than the one considered in the first derivation. Our discussion shows that, in general, it is never possible to neglect vector boson self-energies when computing the form factors which parametrize the $e^+e^- \rightarrow W^+W^-$ helicity amplitudes. The exclusion of the self-energy contributions would lead to estimates of the effects which are wrong by orders of magnitude, as we have explicitly discussed in some examples. Finally we have shown how the self-energy effects can be suitably included in the formalism by a simple redefinition of the form factors which allow to use the existing parametrizations of the considered process.

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