Assessing the Deep Interior Dynamics and Magnetism of A-type Stars

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Abstract. A-type stars have both a shallow near-surface zone of fast convection that can excite acoustic modes and a deep zone of core convection whose properties may be studied through asteroseismology. Many A stars also exhibit large magnetic spots as they rotate. We have explored the properties of core convection in rotating A-type stars and their ability to build strong magnetic fields. These 3-D simulations using the ASH code may serve to inform asteroseismic deductions of interior rotation and magnetism that are now becoming feasible. Our models encompass the inner 30\% by radius of a 2 solar mass A-type star, capturing both the convective core and some of the overlying radiative envelope. Convection can drive a column of strong retrograde differential rotation and yield a core prolate in shape. When dynamo action is admitted, the convection is able to generate strong magnetic fields largely in equipartition with the dynamics. Introducing a modest external field (which may be of primordial origin) into the radiative envelope can substantially alter the turbulent dynamics of the convective core, yielding magnetic fields of remarkable super-equipartition strength. The turbulent convection involves a complex assembly of helical rolls that link distant portions of the core and stretch and advect magnetic field into broad swathes of strong toroidal field. These simulations reveal that supercomputing is providing a perspective of the deep dynamics that may become testable with asteroseismology for these stars.

1. Observed Character of Magnetic A-type Stars

The peculiar A stars (Ap) have continued to engender much interest since their initial discovery in the late 19th century [12]. Broadly speaking, these stars exhibit strong and variable spectral lines (relative to solar values) in Si and certain rare earth metals (e.g. Sr and Hg). Of the Si and Sr-Cr-Eu peculiarity classes, most, if not all, possess equally variable and unusually strong magnetic fields [e.g., 13]. Typically, the magnetic Ap stars possess field strengths of a few hundred Gauss, but field strengths ranging from a lower threshold of \( \sim 300 \) G up to 20,000 G have been observed [1].

When variable, these fields appear to change at the stellar rotation rate [e.g. 8; 13], suggesting that the magnetic fields are “frozen in” to the radiative envelope of the star. Since a typical Ohmic decay time is much longer than the lifetime of an A star, these magnetic fields are likely...
to be remnants of the primordial magnetic field which threaded the molecular cloud whose collapse ultimately formed the star [7]. Such primordial fields likely evolve over long-time scales, ultimately relaxing into a stable, twisted field configuration [2]. A dynamo operating within the core of the A star might also influence the surface magnetic fields. That the cores of these stars likely harbor magnetic dynamos has been suggested by Krause & Oetken [10]. More recently, 3-D numerical simulations have demonstrated that nuclear burning cores of A-type stars can drive vigorous convection capable of generating equipartition strength magnetic fields [3; 4].

Understanding the strong surface fields of these stars in terms of a core-dynamo faces some difficulties. The circulations and magnetic buoyancy effects (on equipartition strength fields) are likely to be too weak for the transmission of interior field to the surface [e.g., 11]. Nevertheless, if the surface fields are indicative of a global-scale magnetic field with roots in the core, then the primordial field must exert some influence on the core dynamo, and vice versa. We explore this possibility by modeling a core dynamo with a large-scale field of primordial origin threading the radiative envelope.

2. Modeling A-type Star Core Dynamos

We model convection and dynamo action in a main-sequence A-type star of 2\(M_\odot\) rotating at four times the solar rate (with a rotation period of seven days) using the 3-D anelastic spherical harmonic (ASH) code. ASH solves the three-dimensional MHD equations in a rotating, spherical frame under the anelastic approximation, thereby filtering out sound waves which would otherwise severely limit the time steps [e.g., 5]. The anelastic approximation is particularly appropriate in the deeper interiors of A-type stars where the fluid motions are distinctly subsonic and thermodynamic variables are small compared to their mean, horizontally averaged values at a given depth.

Our computational domain extends from the inner 0.02 to 0.3 of the star by radius, the inner 0.15 of which is convectively unstable, with an overlying stable, radiative envelope comprising the remainder of the domain. The computational domain is decomposed into 82 Chebyshev polynomials in the radial and horizontally into spherical harmonics up to degree \(l\) of 170. ASH simulations explicitly follow the largest scales of motion, while employing a sub-grid scale (SGS) treatment of the unresolved motions using eddy diffusivities. For this study, our diffusivities scale as the inverse square root of the mean density. We adopt a Prandtl number of 0.25 and a magnetic Prandtl number of 5 throughout the domain. Based on the average rms velocity at midcore and using the core radius as our length scale, the Reynolds number following equilibration is \(\sim 136\) and the magnetic Reynolds number is \(\sim 680\). Further details of this model may be found in Featherstone et al. [9].

We have initialized our fossil field system by augmenting the magnetic fields from a mature A-star dynamo simulation (case C4m) of Brun et al. [4]. Their dynamo was started using a well equilibrated hydrodynamic simulation from Browning et al. [3] by adding a small dipole seed field to this system. Persistent dynamo action realized in that simulation yielded equipartition magnetic energies with respect to the convective flows. The magnetic energy was largely comprised of fluctuating (non-axisymmetric) fields, with the mean (axisymmetric) poloidal and toroidal fields constituting only a small fraction (\(\sim 5\%\)) of the magnetic energy.

Arguments concerning the geometry most likely to support a primordial field against decay have long suggested that such fields require a poloidal and toroidal component if they are to remain stable over the lifetime of a star [e.g., 14]. Recent numerical simulations have shown that such fields may relax into a twisted toroidal shape with a strong dipolar component to the field [2].

We have adopted such a twisted field geometry for our model fossil field, taking \(B_\phi\) to be symmetric in longitude with a Gaussian cross-section of amplitude 30 kG, centered along the equator at \(r_0 = 0.17R_{\rm star}\) with a halfwidth of 0.2\(r_0\). A poloidal field component consistent
Figure 1. Evolution of volume-averaged energy densities following the imposition of an external mixed magnetic field spanning 20,000 days (or about 2,900 rotation periods). Magnetic energy ME (red) has grown in strength to become about ten-fold greater than the kinetic energy KE (black). The mean toroidal field magnetic energy MTE (blue) has also increased prominently in strength, attaining roughly equipartition levels with KE at ∼9,000 days, and becoming slightly super-equipartition at ∼13,000 days.

with a current threading through the center of our magnetic torus was then added to give some twist to the field, with the total magnetic energy associated with this field constituting a 10% increase to the magnetic energy of the overall system. The strength of this field was adjusted so that the ratio of energy in the poloidal field to that in the toroidal field was 1:9; this ratio was suggested by the calculations of Braithewaite & Spruit [2] as being the likely one for a stable twisted torus residing in the radiative zone of an A-type star. The twist, however, is not crucial to the discussion that follows. We have explored other fossil field configurations involving purely poloidal fields with a spherical harmonic degree \( \ell \) up to 8, and found similar long-term behavior of the core-fossil field system in all cases.

3. Nature of Super-Equipartition Hydromagnetic State

One of the more striking effects arising from the inclusion of a fossil field in a core dynamo model is readily apparent in the temporal evolution of the system’s (globally averaged) energy densities. The time histories of the kinetic energy density (KE), magnetic energy density (ME), and the energy associated with the mean (axisymmetric) toroidal fields (MTE) are presented in Figure 1. The evolution of these quantities is shown over the course of 20,000 days, approximately three magnetic diffusion times across the core. ME and KE vary on the convective turnover timescale (100-200 days), and MTE evolves more gradually on timescales of 500-1000 days.

The inclusion of a fossil magnetic field in our system has led to a notable departure from the energy balance achieved in [4]. A fivefold increase in ME develops over approximately one magnetic diffusion time (∼7000 days) following the imposition of the external field. This rise in magnetic energy is accompanied by a halving in the KE of the system, yielding a ratio of ME/KE ∼ 10 that is maintained (with some variation) throughout the remainder of the simulation. The core dynamo also shows a propensity for building strong mean fields in the presence of a fossil field. Energies associated with the mean toroidal fields attain levels roughly 20% of the total magnetic energy, growing steadily in time alongside ME, reaching equipartition levels at about 9,000 days and eventually attaining slightly super-equipartition levels of its own. Such a change in the energy balance is remarkable given that the imposed fossil field constituted but only a small perturbation to the magnetic energy of the system.

3.1. Structure of Flows and Fields

The bulk of the ME change seen in Figure 1 is associated with non-axisymmetric magnetic fields largely contained within the core. Typical magnetic field strengths have transitioned
Sampling the evolving (a) flow streamlines and (b) accompanying magnetic energy density close to the equatorial plane at day 9,000. Violet tones indicate positive motions in the $y$-direction, and yellow tones negative motions. Regions of strong ME are shown in yellow/blue tones. Convective motions freely cross the rotation axis, stretching and advecting magnetic field as they do so.

from $\sim 67$ kG in the absence of a fossil field (based on rms values at mid-core) to $\sim 80$ kG, and typical flow speeds are $\sim 20$ m s$^{-1}$ (vs. $\sim 29$ m s$^{-1}$ initially). Magnetic fields and flows in this super-equipartition state are topologically different from those in the initial state as well. Magnetic fields are larger in scale on average and substantial mean fields have developed that are characterized by a coherent dipolar component and strong (100 kG) toroidal bands encircling the equator in the outer core and lower radiative zone. Along with the strength of the convective motions, mean flows have diminished. Differential rotation of the convective core, once characterized by a slow retrograde column at mid-core aligned with the rotation axis, is virtually non-existent in the super-equipartition field state.

Core convection in the presence of such strong magnetic fields is now characterized by four to six evolving cylindrical rolls encircling the rotation axis and aligned with it. A cross section of these rolls in the equatorial plane is displayed in Figure 2a. The force balance in these rolls is largely magneto-geostrophic in nature, with pressure gradients balancing Coriolis and Lorentz forces. Where these cylindrical rolls intersect the spherical edge of the convective core, the stable stratification of the radiative zone works to brake the radial component of their flow, resulting in a “tilting” of the otherwise largely horizontal motions. Such behavior is similar to that found in rotating convection studies of flows in spherical geometries with rigid boundaries [cf. 6] and is visible in detailed rendering of the streamlines in Figure 3b. Pressure gradients between the equatorial plane and the poles set up by this tilting drives flows along the axes of these rolls, imbuing neighboring rolls with opposite senses of helicity. Poleward axial flows from one roll thus link to the equatorward axial flows of their neighbors. Moreover, owing to the diminished differential rotation, downflows from one set of rolls freely cross the rotation axis undeflected, connecting distant regions of the core which would otherwise remain relatively isolated from one another.

Magnetic fields generated by these convective motions exhibit a similar globally-connected topology. Magnetic energy in the equatorial plane, accompanying the streamlines of Figure 2a, is rendered in Figure 2b showing that the cylindrical roll-like convection leaves its imprint in the magnetic structures. These roll-like structures influence the magnetic topology over much
Figure 3. Convective and magnetic structures (with rotation axis vertical) near day 15,000. (a) Magnetic fields lines colored by their magnetic energy density as realized in the convection zone and region of overshooting; orange/green tones indicate high values, and blue/violet tones low values. (b) Interior view of columnar convection in the core of the A-star visualized near day 15,000 using instantaneous streamlines. Streamlines are colored by velocity component along the rotation axis $v_z$. Blue (yellow) tones indicate northward (southward) motion; equatorial plane indicated by light blue. Many rolls in the A-star interior possess axial flows that freely cross the equatorial plane as seen here. These axial flows tend to link neighboring rolls near the edge of the convective core leading to the global magnetic field topology evident in (a).

of the core as can be seen in field line renderings of the full core (Figure 3a). Broad flux ropes of strong ($\sim 300$ kG) field extend from pole to pole encircling the periphery of the cylindrical rolls. Upflows tend to advect these rolls into the region of convective overshooting just outside the core boundary. There, large swathes of field left behind by overshooting convection diffuse away slowly, ultimately joining with other bundles of magnetic flux to create the strong axisymmetric fields observed in that region.

3.2. Diminishing Lorentz Force Feedbacks
One might anticipate that strong Lorentz forces would accompany such high magnetic field strengths, ultimately suppressing the very convective motions that spur their genesis. The super-equipartition state achieved here, in which $\text{ME/KE} \sim 10$ is thus truly remarkable and raises the question of how a system might build and maintain such strong magnetic fields.

The generation of magnetic energy is closely aligned with the large-scale topology of the flow patterns. Near the rotation axis, the cylindrical rolls closely abut one another creating a region of strong shearing. A coherent bundle of streamlines is also visible in Figure 2a extending in the $y$-direction, linking a distant portions of the core across the rotation axis, and possessing a strong magnetic signature as seen in Figure 2b. Magnetic structures advected across the core in this fashion pass through region of strong shearing flows near the rotation axis and serve as the seed for further field generation. A more detailed analysis of the energy generation has been carried out in Featherstone et al. [9], finding that magnetic energy generation is strongest near the rotation axis and in the region of overshooting where motions deposit and shear magnetic field.

As the core convection generates stronger magnetic fields, the flows and magnetic fields tend to arrange themselves in such a way as to minimize the Lorentz forces. Regions of strong flow and strong field tend to exist separately from one another such that flows tend to persist
most strongly in regions where Lorentz forces are already minimal [see 9]. In regions where strong magnetic fields coexist with substantial flows, streamlines and magnetic field lines tend to be nearly parallel. Minimizing the Lorentz force depends on the alignment of the current density $j$ with the magnetic field $B$. The current density $j$, however, depends on $v \times B$ through Ohm’s law, with velocity vector $v$. Minimizing the Lorentz force thus typically implies some alignment of $v$ and $B$. A sense of such alignment can be gathered from Figure 2 where the largest structures in magnetic energy seem to run parallel to streamlines. A time-averaged sampling of the angle between $B$ and $v$ throughout the core shows that typically this angle is smaller in the super-equipartition state than in the progenitor solution.

4. Conclusions and Perspectives
Our work suggests that the presence of a primordial field can subtly influence the dynamics within the core, yielding a stronger core dynamo characterized by global-scale magnetic fields and flow configurations. These results are quite different from those obtained earlier in the absence of a primordial field. We have at best scratched the surface of these systems in terms of rotation rates, diffusivities, and other parameters. Of particular interest is whether the strong mean fields generated near the edge of the convective core might become buoyant, ultimately contributing to the large star spots seen at the surface of the Ap and Bp stars. There may be a wide array of dynamo states accessible to these cores of these stars with peculiarities in their flows and fields beyond those enumerated here. The (nearly) non-existent differential rotation of the core in this study is unlikely to yield any discernible asteroseismic signature at the surface of such a star, and measuring the sub-surface nature of the magnetic fields in Ap stars is fraught with difficulties of its own. Nevertheless, further simulation of the A, and more massive O and B, stars may well yield unexpected surprises in terms of the differential rotation of the core and the nature of the dynamo. These may in turn suggest new routes for making seismic inferences about the deepest interiors of these massive stars.

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