Effects of pore water pressure on cohesive-frictional slope stability by limit analysis

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Abstract. In the region there was generally influenced by seasonal weather such as highly rainfall intensity, the slope failure will be influenced by the existence of water and these conditions may trigger the changing of failure surface on the slope. The increasing pore water pressure causes shear strength reduction and shear strength enhancement. Consequently, the water level changes need to be evaluated conscientiously. In this paper, the various water levels are applied to perform the different pore water pressure conditions below the ground surface. Many methods are commonly used to predict the slope stability failure mechanism, including limit analysis. Limit analysis has been more widely employed for stability assessment in recent years because of its accuracy. In limit analyses, gravity multiplier will be adopted in this paper by using two-dimensional (2D) numerical approach to identify the failure mechanism. The results will be verified by comparing this study to previous studies

1. Introduction
Slope stability is one of the most important cases that should be assessed in a geotechnical problem because the slope stability established the safety of surrounding highways, housing, building, and other constructions. Many types of slopes which are encountered in the field such as natural slopes, embankment, and cut and fill slopes were investigated by some researchers [1]. However, natural slopes generally have been discovered in slope stability more than any other slopes. Those slope types used long-term stability analysis to predict the safety factor during climate changes including rainfall effects. The saturated condition is still used in conventional slope stability analysis although most of the unsaturated slope stability analysis has been developed. This condition causes the soil suction effect is neglected and the rainfall effects is assumed by raising groundwater level [2].

Changing of groundwater level (GWL) leads to slope instability because when GWL increase, it will increase pore water pressure within saturated soil slopes and it will reduce the shear strength of the soil. GWL in slope stability analyses is applied to present water in slopes by using water table elevation method. Ratios of water height to slope height (Hw/H) were adopted from Eid [3]. Whereas this manner served the simplest way to account the presence of water, the result should be proved with other common manners [4-6].

Many numerical methods have been conducted to investigate slope stability, including limit equilibrium method [7,8], finite element method [9], and limit analysis method [10,11]. In some cases,
the finite element limit analysis provided a more realistic result than the limit equilibrium method because the failure surface was not assumed. Limit analysis method permitted the failure surface was placed precisely [12]. In addition, by using the LEM, the factor of safety can be overestimated at more than 10\% compared to limit analysis upper bound [13]. Then this study has adopted limit analysis gravity multiplier (LAGM) to predict the failure surface of slope subjected to pore water pressure.

In addition to the slope stability soil condition also can influence the failure mechanism. Cohesive-frictional soil is generally found in the field and takes advantages to be developed. So, the objectives of this study are (1) to present limit analysis gravity multiplier (LAGM) subjected to pore water pressure in cohesive-frictional slope stability comparing with another studies, (2) to present the effect of some various parameters in failure mechanism for cohesive-frictional slope stability subjected to pore water pressure.

Based on the previous study by Michalowski [14], presenting a stability chart is more convenient to apply because repetitive processes are not required. Many studies regarding stability charts have been developed by Eid [3], Gao, Zhang [15], and Sun and Zhao [16] to ease the slope stability analysis. The input parameters in this paper which were undertaken are $c'$, $\phi'$, saturated unit weight ($\gamma_{\text{sat}}$), dry unit weight ($\gamma_d$), slope angle ($\beta$), the height of water ($H_w$), and height of slope ($H$) as in Figure 1. Later, this paper also performs the stability charts to present the result in an easy way.

![Figure 1. Geometry of slope stability subjected to pore water pressure which is adopted in this study.](image1)

2. Numerical analysis of slope stability

2.1. Limit analysis gravity multiplier (LAGM)

The limit analysis method assumed the soil as a perfectly plastic material (Figure 2(a)) and an associated flow rule (Figure 2(b)) [17]. Limit analysis satisfied force and moment equilibrium by upper and lower bound solutions. Employing both of them, limit analysis bracketed the exact collapse load. Upper bound works on kinematically admissible velocity fields and an associated ratio of plastic dissipation. Therefore, lower bound complies statically admissible stress fields.

![Figure 2. (a) perfectly plastic material (b) associated flow rule in limit analysis.](image2)
Kim, Salgado [4] calculated the factor of safety according to finite element limit analysis lower and upper bound, assuming that pore water pressure acted at each node in a triangular element for lower and upper bound analysis as shown in Figure 3. Hence in that study, the pore water pressure has been considered as external load rather than internal stress. This study is applying this mechanism by expressing pore water pressure as external load.

\[
\begin{align*}
\sigma\xi_1, \sigma\gamma_1, \tau_{xy1}, pp_1, \sigma\xi_2, \sigma\gamma_2, \tau_{xy2}, pp_2, \sigma\xi_3, \sigma\gamma_3, \tau_{xy3}, pp_3, \alpha \\
(\sigma\xi_1, \sigma\gamma_1, \tau_{xy1}, pp_1) & \quad (\sigma\xi_2, \sigma\gamma_2, \tau_{xy2}, pp_2) & \quad (\sigma\xi_3, \sigma\gamma_3, \tau_{xy3}, pp_3) \quad (u_1, v_1, pp_1) & \quad (u_2, v_2, pp_2) \quad (u_3, v_3, pp_3)
\end{align*}
\]

**Figure 3.** Triangular 3-noded assumption subjected to pore water pressure as external load (a) lower bound (b) upper bound.

The lower bound principle under drained condition is explained in this following equation:

Maximize

\[
\alpha
\]

Subject to

\[
\begin{align*}
\nabla^T(\sigma') + a\alpha p_s + ab = 0 & \quad \text{in } V \\
\mathbf{p}^T(\sigma') + a\alpha p_s = t_{\text{ext}} + at_{\text{ext}} & \quad \text{in } S_\sigma \\
\mathbf{t}^T\sigma' - k + S = 0 & \quad s \geq 0
\end{align*}
\]

The upper bound under drained condition is also described below:

Minimize

\[
\int_V k^T \lambda dV - \int_V [\nabla^T(\alpha p_s) + b]^T \dot{u} dV + \int_{S_\sigma} t_{\text{ext}}^T \lambda dS
\]

Subject to

\[
\nabla \dot{u} = F\lambda, \lambda \geq 0 \\
\int_{S_\sigma} t_{\text{ext}}^T \dot{u} dS = 1
\]

**Figure 4.** Soil mass subject to body forces and surface system.

The basic concept of LAGM, the gravity is amplified to reach a state of collapse condition whilst maintaining all external tractions constant. The factor of safety which is denoted as FS, is defined by:

\[
FS_\gamma = \frac{\gamma_{cr}}{\gamma}
\]
where \( b \) is the body forces, \( p_s \) is seepage pressures, \( t_s \) is tractions due to seepage, \( \alpha \) is the maximum stress, \( t_{ext} \) is the external load, \( \gamma \) is the unit weight, and \( \gamma_s \) is the unit weight at the collapsed state. Natural slopes with cohesive-frictional soil stand as drained material, so it can be analyzed by long term or short term condition. However, generally natural slopes are recognized as long term condition.

### 2.2. Numerical model and input parameters

The materials used in this study are given in Table 1 then the geometry set is shown in Figure 1. This paper included 2 cases, the first case comparing both ways to account pore water pressure that is drawn in Figure 1 and Figure 4 and the second case describing the influence of various parameters. In the first case, the geometry values were used are \( D = 2 \), \( H = 10 \) m, \( \beta = 45^\circ \), and \( H_w = 2 \) m, 4 m, and 6 m, respectively. Then, for the second case, the geometry is also defined in Table 1.

**Table 1.** Materials and geometries of cohesive-frictional slopes subjected to pore water pressure for the first case and the second case.

| Material parameter | Unit | Materials for the first case | Materials for the second case | Geometry Set | Unit | Geometry Values |
|--------------------|------|-----------------------------|-------------------------------|--------------|------|----------------|
| \( \gamma_{sat} \) | kN/m\(^3\) | 18              | 22                           | D            | m    | 2              |
| \( \gamma_{dry} \) | kN/m\(^3\) | 18              | 19                           | H            | m    | 8              |
| \( c' \)          | kN/m\(^2\) | 20              | \( \leq 20 \)                | \( \beta \)  | degrees | 25, 35        |
| \( \phi' \)        | degrees | 10, 15           | \( \leq 40 \)                | \( H_w/H \)  | \( \circ \) | 1, 0.8, 0.6   |

This study considers long term condition and associated flow rule in the analysis. Although associated flow rule is a basic concept of limit analysis, some studies concerned to carry on non-associated flow rule that because associated flow rule is not applicable for frictional materials. Tschuchnigg, F. Schweiger [19] have explained that non-associated flow rule is influenced by dilatancy angle and those give a slight impact in slope stability issue. Neglecting the flow rule is not suitable employed if the friction angle more than \( 40^\circ \) and the slope condition is steep enough because it will significantly influence the results. Therefore, this study assigned the highest friction angle (\( \phi' \)) is equal to \( 40^\circ \) so that is not violating the rule.

### 3. Result and discussion

#### 3.1. Factor of safety from LAGM compared with previous study

Adopting from the previous study by Eid [3], the pore water pressure on the slope in this study is drawn in a linear water surface as shown in Figure 1. Then it will be compared with a parabolic free water surface by Kim, Salgado [4] as in Figure 5 to illustrate the presence of water. This comparison will be used to evaluate the accuracy of both methods. The 2D factor of safety for various water table conditions with a parabolic free water surface is described by Kim, Salgado [4] and Bishop [20]. This paper adopts those comparisons and proposes the factor of safety by using upper and lower bound limit analysis gravity multiplier (LAGM) with linear water surface that is shown in Table 2.

Table 2 explains when \( \phi' \) is small (\( \phi'=10^\circ \)) the safety factors which are obtained from LAGM similar with Bishop, but when the friction angle is increased (\( \phi'=15^\circ \)) the similarities will go further and LAGM presents the higher safety factor. Simulating linear water surface should have lower factor of safety than parabolic free water surface because the volume of water above the failure surface by parabolic water surface should be less than linear water surface. Actually contrary conditions were found, it might be the points of hydraulic head determination were different between Kim, Salgado [4] and Krabbenhoft and Lyamin [18]. LAGM gives highest safety factor comparing with Bishop and Kim when friction angle (\( \phi' \)) equal to \( 15^\circ \), but the factor of safety difference decreases as the water
level increases. Instead of the friction angle is changed to 10°, the degree of difference between this method and other methods becomes smaller.

Limit Analysis Gravity Multiplier (LAGM) by accounting pore water pressure in a linear water surface delivers almost the similar result with the previous studies. This way confirmed that exhibiting the water level in the linear method has a good agreement and can be proposed for the next study because the parabolic water surface in presenting pore water pressure shows a more complicated way to define the GWL. In fact, GWL is determined by site investigations. Generally speaking, certain assumptions should be made because limited underground data are available. So the right GWL curve will be very difficult to perform.

![Figure 5. The geometry of slope stability subjected to pore water pressure by Kim, Salgado [4].](image)

Table 2. Comparison of 2D safety factor subjected to pore water pressure with different water level ($H_w$) ($D = 2$, $H = 10$ m, $\beta = 45^\circ$, $\gamma = 18$ kN/m$^3$, $c = 20$ kPa).

| $\phi'$  | $H_w$ (m) | Safety factor, FS | Average FS | Difference |
|---------|-----------|------------------|------------|------------|
| $c'$, $\phi'$ | Kim et al (FELA, lower-bound) | Kim et al (FELA, upper-bound) | Bishop (LAGM, lower-bound) | This study (LAGM, upper-bound) | This study compared with Kim, et al |
| 2       | 0.957     | 1.052            | 0.988      | 0.996      | 1.002      | 0.999±0.3% | 0.5%       |
| 10°     | 0.892     | 1.001            | 0.957      | 0.954      | 0.959      | 0.957±0.3% | 1.1%       |
| 6       | 0.829     | 0.892            | 0.924      | 0.907      | 0.913      | 0.910±0.3% | 5.8%       |
| 2       | 1.101     | 1.230            | 1.166      | 1.297      | 1.306      | 1.302±0.4% | 11.7%      |
| 15°     | 0.936     | 1.166            | 1.101      | 1.209      | 1.218      | 1.214±0.4% | 10.2%      |
| 6       | 0.971     | 1.068            | 1.036      | 1.109      | 1.116      | 1.113±0.3% | 9.1%       |

3.2. Failure Mechanism of $c'$-$\phi'$ slopes subjected to pore water pressure

The failure can occur if the shear stress increases because it will increase the weight above the failure surface because of the pores filled by water. In this study, pore water pressure has been evaluated by steady seepage conditions because both hydrostatic and steady seepage have good accuracy to be implemented in drained conditions [21].

Varying strength parameters including $c'$ and $\phi'$ tend to produce the different failure surface. When $\phi'$ increases the failure surface is imminent to the slope crest and it is getting disguised as shown in Figure 6. The $\phi'$ variation changes the failure surface higher than the $c'$ variation in part of $c'$-$\phi'$ soils. The increasing of $H_w/H$ causes the failure surfaces approaching the horizontal baseline over the slope toe.

The various water level contributes to the slope failures. As shown in Figure 7, for the same geometry and soil parameters ($H = 8$ m, $\beta = 35^\circ$, $c' = 16$ kPa and $\phi' = 15^\circ$) when GWL increase from 0.6 to 0.8 the safety factor decrease from 1.3 to 1.1. Consequently, if the water level fluctuation during
rainfall and dry season too high, as for instance 20% of the height (0.2H) or more it will lead to slope failures. This study also demonstrates that the soil parameters (c’ and φ’) give substantial effects on slope stability. For the same geometry and GWL that is defined by Hw/H (H = 8 m, β = 35°, c’ = 16 kPa and Hw/H=0.8), but different friction angle (φ’ = 15° and φ’ = 20°), the safety factor significantly changes from 1.5 to 1.1.

![Figure 6. Failure mechanism of LAGM for the cohesive-frictional homogeneous slope. (D=2, H=8 m, c’=10 kPa, Hw=6 m, β=35°), (a) φ’=20° (b) φ’=30°.](image)

3.3. Charts for c’-φ’ slopes subjected to pore water pressure

Stability charts are presented in this paper to pronounce the safety factor solution as shown in Figure 7.

![Figure 7. Stability charts for homogeneous cohesive-frictional soil subjected to pore-water pressure with different water level elevation (Hw).](image)
4. Conclusion
The various water level (Hw), c’, and φ’ can influence the slope stability substantially. When φ’ and c’ increases the failure surfaces shift nearby to the crest and when c’ decreases the failure surface that is close from the toe becomes more clear. The increasing of Hw/H causes the failure surfaces attaining the horizontal baseline through the toe of the slope. Slope stability analysis by LAGM with linear water surface delivers good result and has the highest difference approximately 12% for H/Hw from 0 to 0.8. The increasing of H/Hw causes the failure surfaces attain more clear. By using this chart, the GWL can be measured periodically to prevent the failures.

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