CP Violation in $B^\pm \to \rho^0\pi^\pm$ and $B^\pm \to \sigma\pi^\pm$ Decays

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Abstract

The decay amplitude of $B^+ \to \pi^+\pi^-\pi^+$ in the Dalitz plot has been analyzed by the LHCb using three different approaches for the $S$-wave component. It was found that the mode with $\sigma$ (or $f_0(500)$) exhibited a $CP$ asymmetry of 15% in the isobar model, whereas the $f_2(1270)$ mode had a 40% asymmetry. On the contrary, $CP$ asymmetry for the dominant quasi-two-body decay $B^- \to \rho^0\pi^-$ was found to be consistent with zero in all three approaches, while all the existing theoretical predictions lead to a negative $CP$ asymmetry ranging from $-7\%$ to $-45\%$. We show that the nearly vanishing $CP$ violation in $B^- \to \rho^0\pi^-$ is understandable in the framework of QCD factorization (QCDF). It arises from the $1/m_b$ power corrections to the penguin amplitudes due to penguin annihilations and to the color-suppressed tree amplitude due to hard spectator interactions. Penguin annihilation and hard spectator interactions contribute destructively to $A_{CP}(B^- \to \rho^0\pi^-)$ to render it consistent with zero. The branching fraction and $CP$ asymmetry in $B^- \to \sigma/f_0(500)\pi^-$ are investigated in QCDF with results in agreement with experiment.
I. INTRODUCTION

In 2013 and 2014 LHCb has measured direct CP violation in charmless three-body decays of B mesons [1,2] and found evidence of inclusive integrated CP asymmetries in $B^+ \rightarrow \pi^+\pi^+\pi^-$, $K^+K^+K^-$, $K^+K^-\pi^+$ and a 2.8σ signal of CP violation in $B^+ \rightarrow K^+\pi^+\pi^-$. Besides the integrated CP asymmetry, LHCb has also observed large asymmetries in localized regions of phase space, such as the low invariant mass region devoid of most of known resonances and the rescattering regions of $m_{\pi^+\pi^-}$ or $m_{K^+K^-}$ between 1.0 and 1.5 GeV.

Recently LHCb has analyzed the decay amplitudes of $B^+ \rightarrow \pi^+\pi^-\pi^+$ in the Dalitz plot [4, 5]. In the LHCb analysis, the $S$-wave component of $B^- \rightarrow \pi^+\pi^-\pi^-$ was studied using three different approaches: the isobar model, the $K$-matrix model and a quasi-model-independent (QMI) binned approach. In the isobar model, the $S$-wave amplitude was presented by LHCb as a coherent sum of the $\sigma$ (or $f_0(500)$) meson contribution and a $\pi\pi \leftrightarrow KK$ rescattering amplitude in the mass range $1.0 < m_{\pi^+\pi^-} < 1.5$ GeV. The fit fraction of the $S$-wave is about 25% and predominated by the $\sigma$ resonance.

A clear CP asymmetry was seen in the $B^- \rightarrow \pi^+\pi^-\pi^-$ decay in the following places: (i) the $S$-wave amplitude at values of $m_{\pi^+\pi^-}$ below the mass of the $\rho(770)$ resonance. In the isobar model, the $S$-wave amplitude is predominated by the $\sigma$ meson. Hence, a significant CP violation of 15% in $B^- \rightarrow \sigma\pi^-$ is implied in this model. (ii) the $f_2(1270)$ component with a CP violation of 40% exhibited, and (iii) the interference between $S$- and $P$-waves which is clearly visible in Fig. 12 of [5] where the data are split according to the sign of $\cos \theta$ with $\theta$ being the angle between the momenta of the two same-sign pions measured in the rest frame of the dipion system. The significance of CP violation in the interference between $S$- and $P$-waves exceeds 25σ in all the $S$-wave models.

On the contrary, CP asymmetry for the dominant quasi-two-body decay mode $B^- \rightarrow \rho^0\pi^-$ was found by the LHCb to be consistent with zero in all three $S$-wave approaches (see Table I), which was already noticed by the LHCb previously in 2014 [2]. ¹ Indeed, if this quasi-two-body CP asymmetry is nonzero, it will destroy the aforementioned interference pattern between $S$- and $P$-waves. However, the existing theoretical predictions based on QCD factorization (QCDF) [7, 8], perturbative QCD (pQCD) [9], soft-collinear effective theory (SCET) [10], topological diagram approach (TDA) [11] and factorization-assisted topological-amplitude (FAT) approach [12] all lead to a negative CP asymmetry for $B^- \rightarrow \rho^0\pi^-$, ranging from $-7\%$ to $-45\%$ (see Table II).

|          | isobar          | $K$-matrix       | QMI            |
|----------|-----------------|------------------|----------------|
| $\rho(770)^0$ | 0.7 ± 1.1 ± 0.6 ± 1.5 | 4.2 ± 1.5 ± 2.6 ± 5.8 | 4.4 ± 1.7 ± 2.3 ± 1.6 |

¹ There was a measurement of $A_{CP}(\rho^0\pi^-)$ by BaBar with the result $0.18 \pm 0.07^{+0.05}_{-0.15}$ from the Dalitz plot analysis of $B^- \rightarrow \pi^+\pi^-\pi^-$ [6].
TABLE II: Theoretical predictions of CP violation (in %) for the $B^- \to \rho^0 \pi^-$ decay in various approaches.

|        | QCDF [7]   | QCDF [8]   | pQCD [9]   | SCET [10] | TDA [11] | FAT [12] |
|--------|------------|------------|------------|-----------|----------|----------|
| cp     | $-9.8^{+3.4+11.4}_{-2.6-10.2}$ | $-6.7^{+0.2+3.2}_{-0.2-3.7}$ | $-27.5^{+2.3+0.9}_{-3.1-1.0} \pm 1.4 \pm 0.9$ | $-19.2^{+15.5+1.7}_{-13.4-1.9}$ | $-23.9 \pm 8.4$ | $-45 \pm 4$ |

The purpose of this work is twofold. First, we would like to resolve the long-standing puzzle in regard to the CP asymmetry in $B^- \to \rho^0 \pi^-$. Second, we will present a study of $B^- \to \sigma \pi^-$ in QCDF. For CP violation in $B^- \to f_2(1270) \pi^-$, it has been studied in [13–15] before the LHCb experiment. As for CP asymmetry induced by interference, we will give a detailed study elsewhere.

II. $B^\pm \to \rho^0 \pi^\pm$ DECAYS

As stressed in the Introduction, we are concerned about the discrepancy between theory and experiment in regard to CP asymmetry in the tree-dominated mode $B^- \to \rho^0 \pi^-$. It has been argued in [16] that in $B \to PV$ decays with $m_V < 1$ GeV, CP asymmetry induced from a short-distance mechanism is suppressed by the CPT constraint. Normally, CPT theorem implies the same lifetimes for both particle and antiparticle. When partial widths are summed over, the total width of the particle and its antiparticle should be the same. Final-state interactions are responsible for distributing the CP asymmetry among the different conjugate decay channels. In the three-body $B$ decays, the “2+1” approximation is usually assumed so that the resonances produced in heavy meson decays do not interact with the third particle. In $B \to PV$ decays with $m_V < 1$ GeV, for example, $V = \rho(770)$ or $K^*(892)$, there do not exist other states below the $K\bar K$ threshold which can be connected to $\pi\pi$ or $\pi K$ rescattering through final-state interactions. As stressed in [16], the absence of final-state interactions is a hadronic constraint and therefore, the impossibility to observe CP asymmetry in those processes is independent from the relative short-distance contribution from tree and penguin diagrams. As elucidated in [16], there are three other possibilities that can produce CP violation, for example, a three-body rescattering including the third particle.

If we take this argument seriously to explain the approximately vanishing CP asymmetry in $B^+ \to \rho^0 \pi^+$, it will be at odd with the CP violation seen in other PV models [17]: $A_{CP}(B^+ \to \rho^0 K^+) = 0.37 \pm 0.11$, $A_{CP}(B^0 \to K^* \eta) = 0.19 \pm 0.05$ and $A_{CP}(B^0 \to K^{*+}\pi^-) = -0.271 \pm 0.044$, especially CP violation in the last mode was first observed by the LHCb [18]. In general, the agreement between theory and experiment for these three modes is good (see e.g. [18]). Therefore, it seems to us that the smallness of $A_{CP}(B^+ \to \rho^0 \pi^+)$ probably has nothing to do with the CPT constraint.

In QCDF, the decay amplitude of $B^- \to \rho^0 \pi^-$ is given by [19]

$$A(B^- \to \rho^0 \pi^-) = \frac{1}{\sqrt{2}} \left[ \delta_{\rho\pi}(a_2 - \beta_2) - a_4^\rho a_6^\pi + \frac{3}{2}(a_7^\rho + a_9^\rho) + \frac{1}{2}(a_{10}^\rho + r_\chi a_8^\pi) - \beta_3^\rho - \beta_3^{EOF} \right]_{\pi\rho}$$

$$\times X(B^- \pi, \rho) + \frac{1}{\sqrt{2}} \left[ \delta_{\rho a_1} + a_4^\rho r_\chi a_6^\pi + a_{10}^\rho - r_\chi a_8^\pi + \beta_3^\rho + \beta_3^{EOF} \right]_{\pi\rho} X(B^- \rho, \pi),$$

(2.1)
where the chiral factors $r_{\chi}^{\pi, \rho}$ are given by
\[ r_{\chi}^{\pi}(\mu) = \frac{2m_{\pi}^2}{m_b(\mu)(m_u + m_d)(\mu)}, \quad r_{\chi}^{\rho}(\mu) = \frac{2m_{\rho}}{m_b(\mu)} \frac{f_{\pi}^{(\rho)}}{f_{\rho}}, \quad (2.2) \]
and the factorizable matrix elements read
\[ X^{(B^-, \pi, \rho)}(2.1) = 2f_{\pi}\alpha_{BPc}F_1^{B\pi}(m_{\rho}^2), \quad X^{(B^-, \rho, \pi)}(2.1) = 2f_{\pi}\alpha_{BPc}F_0^{B\rho}(m_{\pi}^2), \quad (2.3) \]
with $p_c$ being the c.m. momentum. Here we have followed \[20\] for the definition of form factors. In Eq. (2.1), the order of the arguments of the $a_i^p(M_1M_2)$ and $\beta_i(M_1M_2)$ coefficients is dictated by the subscript $M_1M_2$.

The flavor operators $a_i^p$ are basically the Wilson coefficients in conjunction with short-distance nonfactorizable corrections such as vertex corrections and hard spectator interactions. In general, they have the expressions \[19, 21\]
\[ a_i^p(M_1M_2) = \left( c_i + \frac{c_{i+1}}{N_c} \right) N_i(M_2) + \frac{c_{i+1}}{N_c} \frac{C_F\alpha_s}{4\pi} \left[ V_i(M_2) + \frac{4\pi^2}{N_c} H_i(M_1M_2) \right] + P_i^p(M_2), \quad (2.4) \]
where $i = 1, \ldots, 10$, the upper (lower) signs apply when $i$ is odd (even), $c_i$ are the Wilson coefficients, $C_F = (N_c^2 - 1)/(2N_c)$ with $N_c = 3$, $M_2$ is the emitted meson and $M_1$ shares the same spectator quark with the $B$ meson. The quantities $V_i^h(M_2)$ account for vertex corrections, $H_i^h(M_1M_2)$ for hard spectator interactions with a hard gluon exchange between the emitted meson and the spectator quark of the $B$ meson and $P_i(M_2)$ for penguin contractions.

In the $m_b \to \infty$ limit, the decay amplitudes of charmless two-body decays of $B$ mesons are factorizable and can be described in terms of decay constants and form factors. However, it is well known that the short-distance contribution to $a_4^{ca} + r_{\chi}^{\pi} a_6^{ca}$ will yield CP asymmetries for $B^0 \to K^-\pi^+, K^+\pi^-, \pi^+\pi^-$, $B^- \to K^-\pi^0$ and $\bar{B}_s \to K^+\pi^-$, etc., which are wrong in signs when confronted with experiment \[1, 22\]. Beyond the heavy quark limit, it is thus necessary to introduce $1/m_b$ power corrections. In QCDF, power corrections to the penguin amplitudes are described by the penguin annihilation characterized by the parameters $\beta_{BP, 2, 3}$ and $\beta_{BP, EW}$ given in Eq. (2.1). Penguin annihilation is also responsible for the rate deficit problems with penguin-dominated modes encountered in the heavy quark limit.

As pointed out in \[1, 22\], while the signs of CP asymmetries in aforementioned modes are flipped to the right ones in the presence of power corrections from penguin annihilation, the signs of $A_{CP}$ in $B^- \to K^-\pi^0$, $K^-\eta$, $\pi^-\eta$ and $B^0 \to \pi^0\pi^0$, $K^*0\eta$ will also get reversed in such a way that they disagree with experiment. This CP puzzle is resolved by invoking power corrections to the color-suppressed tree topology as all the above-mentioned five modes receive contributions from $a_2^n$ \[1, 22\]. An inspection of Eq. (2.4) reveals that hard spectator contributions to $a_i$ are usually very small except for $a_2$ and $a_{10}$ as $c_1 \sim O(1)$ and $c_9 \sim O(-1.3)$ in units of $\alpha_{em}$. Explicitly,
\[ a_2(M_1M_2) = c_2 + \frac{c_1}{N_c} \frac{C_F\alpha_s}{4\pi} \left[ V_2(M_2) + \frac{4\pi^2}{N_c} H_2(M_1M_2) \right], \quad (2.5) \]
where the hard spectator term $H_2(M_1M_2)$ reads
\[ H_2(M_1M_2) = \frac{if_{BP_c} f_{M_1} f_{M_2}}{X(B^{M_1M_2})} \alpha_{BP_c} \int_0^1 dx dy \left( \frac{\Phi_{M_1}(x)\Phi_{M_2}(y)}{xy} + r_{\chi}^{M_1} \frac{\Phi_{M_1}(x)\Phi_{M_2}(y)}{xy} \right), \quad (2.6) \]
with $\bar{x} = 1 - x$. Subleading $1/m_b$ power corrections arise from the twist-3 amplitude $\Phi_m$. As shown in detail in [22], power corrections to $a_2$ not only resolve the aforementioned CP puzzles (including the so-called $\pi \bar{K}$ puzzle) but also account for the observed rates of $B^0 \to \pi^0 \pi^0$ and $\rho^0 \pi^0$.

In the QCD factorization approach, power corrections often involve endpoint divergences. We shall follow [21] to model the endpoint divergence $X \equiv \int_0^1 dx/\bar{x}$ in the penguin annihilation and hard spectator scattering diagrams as

$$X^i_A = \ln \left( \frac{m_B}{\Lambda_h} \right) (1 + \rho_A^i e^{i \phi_A^i})$$

$$X_H = \ln \left( \frac{m_B}{\Lambda_h} \right) (1 + \rho_H e^{i \phi_H}), \quad (2.7)$$

with $\Lambda_h$ being a typical hadronic scale of 0.5 GeV, where the superscripts 'i' and 'f' refer to gluon emission from the initial and final-state quarks, respectively. In principle, one can also add the superscripts 'VP' and 'PV' to distinguish penguin annihilation effects in $B \to VP$ and $B \to PV$ decays [19]:

$$A_1^i \approx -A_2^i \approx 6 \pi \alpha_s \left[ 3 \left( X^V_{iA} - 4 + \frac{\pi^2}{3} \right) + r^V X^P \left( \left( X^V_{iA} \right)^2 - 2 X^P_{iA} \right) \right],$$

$$A_4^i \approx 6 \pi \alpha_s \left[ -3r^V X^P \left( \left( X^V_{iA} \right)^2 - 2 X^P_{iA} + 4 - \frac{\pi^2}{3} \right) + r^V \left( \left( X^V_{iA} \right)^2 - 2 X^P_{iA} + \frac{\pi^2}{3} \right) \right],$$

$$A_5^i \approx 6 \pi \alpha_s \left[ 3r^V \left( 2X^V_{iA} - 1 \right) \left( 2 - X^V_{iA} \right) - r^V \left( 2 \left( X^V_{iA} \right)^2 - X^V_{iA} \right) \right], \quad (2.8)$$

for $M_1 M_2 = VP$ and

$$A_1^i \approx -A_2^i \approx 6 \pi \alpha_s \left[ 3 \left( X^V_{iA} - 4 + \frac{\pi^2}{3} \right) + r^V X^P \left( \left( X^V_{iA} \right)^2 - 2 X^P_{iA} \right) \right],$$

$$A_4^i \approx 6 \pi \alpha_s \left[ -3r^V X^P \left( \left( X^V_{iA} \right)^2 - 2 X^P_{iA} + 4 - \frac{\pi^2}{3} \right) + r^V \left( \left( X^V_{iA} \right)^2 - 2 X^P_{iA} + \frac{\pi^2}{3} \right) \right],$$

$$A_5^i \approx 6 \pi \alpha_s \left[ -3r^P \left( 2X^P_{iA} - 1 \right) \left( 2 - X^P_{iA} \right) + r^V \left( 2 \left( X^P_{iA} \right)^2 - X^P_{iA} \right) \right], \quad (2.9)$$

for $M_1 M_2 = PV$. Nevertheless, for simplicity we shall assume that the parameters $X^V_{iA}$ and $X^P_{iA}$ are the same. So we shall drop the superscripts $VP$ and $PV$ hereafter.

Initially, it was expected that $\rho_A^i = \rho_A^f \sim 1$ and $\phi_A^i = \phi_A^f$. The two unknown parameters $\rho_A$ and $\phi_A$ were fitted to the data of $B \to PP, VP, PV$ and $VV$ decays. The values of $\rho_A$ and $\phi_A$ are given, for example, in Table III of [14], where the results are very similar to the so-called “S4 scenario” presented in [19]. Now a surprise came from the measurement of the pure annihilation process $B^0 \to \pi^+ \pi^-$ by the CDF [23] and LHCb [24]. The world average $B(B_s \to \pi^+ \pi^-) = (0.671 \pm 0.083) \times 10^{-6}$ [17] is much higher than the QCDF prediction of $(0.26^{+0.10}_{-0.09}) \times 10^{-6}$ [22]. Since this mode proceeds through the penguin-annihilation amplitudes $A_1^i$ and $A_5^i$, it is natural to expect that $\rho_A^i \neq \rho_A^f$ and that $\rho_A^i \sim 3$ is needed to accommodate the data [26, 27]. That is,

2 At first sight, the new measurement of another pure annihilation process $B(B^0 \to K^+ K^-) = (7.80 \pm 1.27 \pm 0.84) \times 10^{-5}$ by the LHCb [28] seems to be at odd with a large $\rho_A^i$ in the PP sector. As can be seen from Fig. 3 in [29] for the dependence of $B(B^0 \to K^+ K^-)$ on $(\rho_A^i, \phi_A^i)$, a large $\rho_A^i$ is still allowed so long as $\phi_A^i$ is not in the region of $[-100^\circ, 100^\circ]$. The constraint on the phase $\phi_A^i$ arises mainly from CP violation in $B \to \pi K$ decays. It follows that $\phi_A^i \sim [-140^\circ, -60^\circ]$ with a large $\rho_A^i$ is favored by the data of CP asymmetries. Putting all together, a large $\rho_A^i$ with $\phi_A^i \sim [-140^\circ, -100^\circ]$ is still favored by the data even when the new measurement of $B^0 \to K^+ K^-$ is take into account [30].
TABLE III: The branching fraction and CP asymmetry of $B^- \to \rho^0\pi^-$ within the QCDF approach. Experimental data are taken from [17]. The theoretical errors correspond to the uncertainties due to the variation of (i) Gegenbauer moments, decay constants, form factors, the strange quark mass, and (ii) $\rho_{A,H}$, $\phi_{A,H}$, respectively. In (ii) we assign an error of $\pm 0.4$ to $\rho$ and $\pm 4^\circ$ to $\phi$.

| $\mathcal{B}(10^{-6})$ | $\mathcal{A}_{CP} (%)$ | Comments |
|-------------------------|--------------------------|----------|
| 8.3$^{+1.2}_{-1.3}$    | 0.7 $\pm$ 1.9           | Expt     |
| 8.9$^{+2.0}_{-1.0}$    | 6.3$^{+0.5}_{-0.8}$     | (1) Heavy quark limit |
| 9.3$^{+1.8}_{-1.0}$    | $-13.0^{+10.0}_{-8.8}$  | (2) $\rho_H = 0$ and $\phi_H = 0$ with $\rho_A$ and $\phi_A$ given by Eq. (2.12) |
| 6.7$^{+0.6}_{-0.4}$    | $-4.8^{+4.3}_{-2.4}$    | (3) $\rho_H = 3.08$, $\phi_H = -145^\circ$, $\rho_A$ and $\phi_A$ given by Eq. (2.12) |
| 8.4$^{+1.6}_{-0.8}$    | $-0.7^{+3.2}_{-2.8}$    | (4) $\rho_H = 3.15$, $\phi_H = -113^\circ$, $\rho_A$ and $\phi_A$ given by Eq. (2.12) |
| 6.4$^{+0.6}_{-0.4}$    | 14.4$^{+2.2}_{-1.3}$    | (5) $\rho_H = 3.08$, $\phi_H = -145^\circ$, $\rho_A^{i,f} = 0$, $\phi_A^{i,f} = 0$ |
| 8.1$^{+1.7}_{-0.8}$    | 15.2$^{+1.3}_{-1.1}$    | (6) $\rho_H = 3.15$, $\phi_H = -113^\circ$, $\rho_A^{i,f} = 0$, $\phi_A^{i,f} = 0$ |

the parameters $X_i^A$ and $X_f^A$ should be treated separately. A large $\rho_A^{i,f}$ is also a good news for the hard spectator interactions because $\rho_H > 3$ together a large phase $\phi_H$ are required to solve the CP puzzle together with the rate deficit issue of $B^0 \to \pi^0\pi^0$ and $\rho^0\pi^0$. Hence, it is pertinent to set $\rho_H = \rho_A^{i,f}$ and $\phi_H = \phi_A^{i,f}$ to the first order approximation.

For $B \to PV$ decays, when $(\rho_H, \phi_H)$ and $(\rho_A^{i,f}, \phi_A^{i,f})$ are treated as free parameters, it was found in [8] that the allowed regions of $(\rho_A^{i,f}, \phi_A^{i,f})$ are small and tight, while those of $(\rho_A^{i,f}, \phi_A^{i,f})$ are big and loose. Moreover, the allowed $(\rho_H, \phi_H)$ regions are significantly separated from those of $(\rho_A^{i,f}, \phi_A^{i,f})$ and overlap partly with the regions of $(\rho_A^{i,f}, \phi_A^{i,f})$. When $(\rho_H, \phi_H)$ are set to $(\rho_A^{i,f}, \phi_A^{i,f})$ as a first order approximation, a fit of the four parameters $(\rho_A^{i,f}, \phi_A^{i,f})$ to the $B \to PV$ data yields [8]

$$(\rho_A^{i,f}, \phi_A^{i,f})_{PV} = (2.87^{+0.06}_{-0.05}, 0.91^{+0.12}_{-0.04})^{\circ}, \quad (\phi_A^{i,f}, \phi_A^{i,f})_{PV} = (-145^{+14}_{-21}, -37^{+10}_{-9})^{\circ},$$

where the allowed regions of $(\rho_A^{i,f}, \phi_A^{i,f})$ shrink considerably. For comparison, they are close to the solutions obtained in the PP sector [31]

$$(\rho_A^{i,f}, \phi_A^{i,f})_{PP} = (2.98^{+1.12}_{-0.86}, 1.18^{+0.20}_{-0.23}), \quad (\phi_A^{i,f}, \phi_A^{i,f})_{PP} = (-105^{+34}_{-24}, -40^{+11}_{-8})^{\circ}.$$  

In this work, we shall follow [32] to take

$$(\rho_A^{i,f}, \phi_A^{i,f})_{PV} = (3.08, 0.83), \quad (\phi_A^{i,f}, \phi_A^{i,f})_{PV} = (-145^{\circ}, -36^{\circ}),$$

for calculations.

We are now ready to compute the branching fraction and CP asymmetry for $B^- \to \rho^0\pi^-$. In the heavy quark limit, its CP asymmetry is positive with a magnitude of order 0.06. We then turn on power corrections induced from penguin annihilation. It is clear that the sign of $\mathcal{A}_{CP}(\rho^0\pi^-)$ is flipped and in the meantime its magnitude is enhanced. We next switch on $1/m_b$ corrections from hard spectator interactions. Under the simplification with $\rho_H = \rho_A^{i}$ and $\phi_H = \phi_A^{i}$, we will have $\mathcal{B}(\rho^0\pi^-) \approx 6.7 \times 10^{-6}$ and $\mathcal{A}_{CP}(\rho^0\pi^-) \approx -0.05$. However, the resultant branching fraction is too small by 20% when compared with experiment. This implies that the realistic values of $\rho_H$ and
\( \phi_H \) should have some deviation from \( \rho_A^j \) and \( \phi_A^j \), respectively. Indeed, we find that the data can be accommodated by having \( \rho_H = 3.15 \) and \( \phi_H = -113^\circ \), for instance, shown in case (4) of Table III. To see the effect of hard spectator interactions alone, we turn off \( \rho_A \) and \( \phi_A \). It is evident that \( A_{CP}(\rho^0\pi^-) \) will be enhanced from \( O(6) \) to \( O(15) \) in the presence of hard spectator effects. If the heavy quark limit of \( A_{CP}(\rho^0\pi^-) \) is considered as a benchmark, hard spectator interactions will push it up further, whereas penguin annihilation will pull it to the opposite direction. Therefore, the nearly vanishing \( A_{CP}(\rho^0\pi^-) \) arises from two destructive \( 1/m_b \) power corrections.

What about the previous QCDF predictions given in Table III? The results of \( A_{CP}(\rho^0\pi^-) \approx -0.098 \) and \( B(\rho^0\pi^-) \approx 8.7 \times 10^{-6} \) given in [7] were obtained using \( \rho_A \approx 1 \) and \( \phi_A^P = -70^\circ \) and \( \phi_A^{PV} = -30^\circ \), while the power correction to \( a_2 \) was parameterized as \( (1 + 0.8e^{-180^\circ}) \). The QCDF predictions \( A_{CP}(\rho^0\pi^-) \approx -0.067 \) and \( B(\rho^0\pi^-) \approx 6.8 \times 10^{-6} \) given in [8] are very similar to case (3) in Table III. As noticed in passing, one needs to adjust \( \rho_H \) and \( \phi_H \) slightly to render both the branching fraction and \( CP \) asymmetry in agreement with the data.

III. \( B^\pm \to \sigma\pi^\pm \) DECAYS

Charmed hadronic \( B \) decays to scalar mesons have been studied in the approach of QCD factorization [33–35]. For completeness, we shall present a study of \( B^- \to \sigma/f_0(500)\pi^- \) in Eq. (A1) of [35]:

\[
A(B^- \to \sigma\pi^-) = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p^{(d)} \left\{ a_1 \delta_{pu} + a_4^p + a_{10}^p - (a_6^p + a_8^p) f_\chi^{\sigma \pi} X(B\sigma, \pi) \\
+ \left[ a_2 \delta_{pu} + 2(a_3^p + a_5^p) + \frac{1}{2} (a_7^p + a_9^p) + a_4^p - \frac{1}{2} a_{10}^p - (a_6^p - \frac{1}{2} a_8^p) f_\chi^{\sigma \pi} \right] X(B\pi, \sigma) \\
- f_B f_\pi f_\sigma^u \left[ \delta_{pu} b_2(\pi \sigma) + b_3(\pi \sigma) + b_{3,EW}(\pi \sigma) + (\pi \sigma \to \sigma \pi) \right] \right\}, \tag{3.1}
\]

where the factorizable matrix elements read

\[
X(B\sigma, \pi) = -f_\pi F_0^{Br u} (m_\sigma^2)/(m_B^2 - m_\sigma^2), \quad X(B\pi, \sigma) = f_\sigma^u F_0^{B\pi} (m_\sigma^2)/(m_B^2 - m_\sigma^2), \tag{3.2}
\]

with \( \delta_X^u \mu = 2m_\sigma/m_\mu(\mu) \) and \( \lambda_\rho^{(d)} = V_{pb}^* V_{pd} \). The superscript \( u \) in the scalar decay constant \( f_\sigma^u \) and the form factor \( F_{B\sigma^u} \) refers to the \( u \) quark component of the \( \sigma \).

It is known that the neutral scalar meson \( \sigma \) cannot be produced via the vector current. If \( \sigma \) is a 2-quark bound state with the flavor wave function \( (\bar{u}u + \bar{d}d)/\sqrt{2} \), its scale-dependent scalar decay constant can be defined as

\[
\langle \sigma | \bar{u}u | 0 \rangle = m_\sigma f_\sigma^u. \tag{3.3}
\]

For simplicity, we will not consider the mixing of \( \sigma \) and \( f_0(980) \) and hence the strange quark effect in Eq. (3.1). In this work we shall assume that \( \sigma \) has a similar decay constant and light-cone distribution amplitude (LCDA) as \( f_0(980) \). Explicitly, we take \( f_\sigma^u = 350 \text{ MeV} \) at \( \mu = 1 \text{ GeV} \) and \( F_0^{B\sigma^u}(0) = 0.25 \), where the Clebsch-Gordon coefficient \( 1/\sqrt{2} \) is included in \( f_\sigma^u \) and \( F_0^{B\sigma^u} \). Vertex corrections, hard spectator interactions and weak annihilation for \( B \to SP \) and \( B \to SV \).
have been worked out in [33–35]. Since the twist-2 LCDA of the \( \sigma \) meson is dominated by the odd Gegenbauer moments, which vanish for the \( \pi \) mesons, it follows that the flavor operators \( a_1^{\sigma}(\pi \pi) \) and \( a_1^{\sigma}(\pi \pi) \) can be very different numerically except for \( a_6^{\sigma} \) (see Table IV). For example, \( a_1(\pi \sigma) \approx 1 \gg a_1(\pi \pi) \). It appears that \( a_1^{\sigma}(\pi \pi) \) look like the normal ones, but not \( a_1^{\sigma}(\pi \pi) \). Effects of penguin annihilation defined by

\[
\beta^p(M_1 M_2) = -f_B f_\pi \bar f_\sigma \delta_{\rho u} b_2 + b_3 + b_{3,EW} \big|_{M_1 M_2}\]

(3.4)

are also shown in Table IV.

Using the input parameters given in [35] except for the Wolfenstein parameters updated with \( A = 0.8235, \lambda = 0.224837, \bar \rho = 0.1569 \) and \( \bar \eta = 0.3499 \) [36], we obtain

\[
B(B^- \to \sigma \pi^-) = (5.38^{+0.19}_{-0.18} + 1.34^{+0.94}_{-0.90}) \times 10^{-6}, \quad A_{CP}(B^- \to \sigma \pi^-) = (15.95^{+0.29}_{-0.28} + 0.08^{+18.88}_{-21.88})\%.
\]

(3.5)

Theoretical uncertainties come from (i) the Gegenbauer moments \( B_{1,3} \), the scalar meson decay constants, (ii) the heavy-to-light form factors and the strange quark mass, and (iii) the power corrections due to weak annihilation and hard spectator interactions, respectively. The calculated \( CP \) asymmetry agrees well with the LHCb measurement [4,5]

\[
A_{CP}(B^- \to \sigma \pi^-) = (16.0 \pm 1.7 \pm 2.2)\%.
\]

(3.6)

From the fit fraction \((25.2 \pm 0.5 \pm 5.0)\%\) of the \( \sigma \) component in \( B^- \to \pi^+ \pi^- \pi^- \) decay analyzed in the isobar model [4,5] and the total branching fraction \((15.2 \pm 1.4) \times 10^{-6}\) measured by BaBar [6], we obtain

\[
B(B^- \to \sigma \pi^- \to \pi^+ \pi^- \pi^-)_{\text{expt}} = (3.83 \pm 0.76) \times 10^{-6}.
\]

(3.7)

To compute the decay rate of \( B^- \to \sigma \pi^- \to \pi^+ \pi^- \pi^- \) it is necessary to take into account the resonance shape of the \( \sigma \), for example, the standard Breit-Wigner function. If \( \sigma \) were very narrow,
one would have the narrow width approximation

\[ \mathcal{B}(B^- \to \sigma \pi^- \to \pi^+ \pi^- \pi^-) = \mathcal{B}(B^- \to \sigma \pi^-) \mathcal{B}(\sigma \to \pi^+ \pi^-). \]  

Since \( \mathcal{B}(\sigma \to \pi^+ \pi^-) \approx 2/3 \), it appears that the above relation is empirically working. However, as \( \sigma \) is very broad, its finite width effect could be very important [37].

IV. CONCLUSIONS

The decay amplitudes of \( B^+ \to \pi^+ \pi^- \pi^+ \) in the Dalitz plot have been analyzed by the LHCb using three different approaches for the \( S \)-wave component. It was found that the mode with \( \sigma \) (or \( f_0(500) \)) exhibited a \( CP \) asymmetry of 15\% in the isobar model, whereas the \( f_2(1270) \) mode had a 40\% asymmetry. In contrast, \( CP \) asymmetry for the dominant quasi-two-body decay \( B^- \to \rho^0 \pi^- \) was found to be consistent with zero in all three approaches, while all the existing theoretical predictions lead to a negative \( CP \) asymmetry ranging from \(-7\% \) to \(-45\% \). We show that the nearly vanishing \( CP \) violation in \( B^- \to \rho^0 \pi^- \) is understandable in QCDF. The \( 1/m_b \) power corrections penguin annihilation and hard spectator interactions contribute destructively to \( A_{CP}(B^- \to \rho^0 \pi^-) \) to render it consistent with zero. The branching fraction and \( CP \) asymmetry in \( B^- \to \sigma/f_0(500) \pi^- \) are investigated in QCDF with results in agreement with experiment.

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