Distributed CTL Model Checking in the Cloud

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Abstract—The recent extensive availability of “big data” platforms calls for a more widespread adoption by the formal verification community. In fact, formal verification requires high performance data processing software for extracting knowledge from the unprecedented amount of data which come from analyzed systems. Since cloud based computing resources have become easily accessible, there is an opportunity for verification techniques and tools to undergo a deep technological transition to exploit the new available architectures. This has created an increasing interest in parallelizing and distributing verification techniques. In this paper we introduce a distributed approach which exploits techniques typically used by the “big data” community to enable verification of Computation Tree Logic (CTL) formulas on very large state spaces using distributed systems and cloud computing facilities. The outcome of several tests performed on benchmark specifications are presented, thus showing the convenience of the proposed approach.

I. INTRODUCTION

Ensuring the correctness of software and hardware products is an issue of great importance. This has led to an increased interest in applying formal methods and verification techniques in order to ensure correctness of developed systems. Among the most successful techniques that are widely used in both research and industry is model checking. Model checking of dynamic, concurrent and real-time systems has been the focus of several decades of software engineering research. One of the most challenging task in this context is the development of tools able to cope with the complexity of the models needed in the analysis of real word examples. In fact, the main obstacle that model checking faces is the state explosion problem [1]: The number of global states of a concurrent system with multiple processes can be enormous. It increases exponentially in both the number of processes and the number of components per process. The most significant contributions the research has provided in order to cope with this problem are symbolic model checking with ordered binary decision diagrams [2], partial order reduction techniques [3], and bounded model checking [4].

These breakthrough techniques have enabled the analysis of systems with a fairly big number states. Nevertheless, taking advantage of a distributed environment is still important to cope with real world problems. The idea is to increase the computational power and a larger available memory, by using a cluster of computers. The use of networks of computers can provide the resources required to achieve verification of models representing real world examples. Unfortunately, this last approach requires several skills which—while common in the “big data” community—are still rather rare in the “formal methods” community.

In fact, our recent works were focused on the connection between formal methods in software engineering and big data approaches [5], [6], [7]. The analysis of very complex systems certainly falls in this context, although formal verification has so far poorly explored by big data scientists. We believe, however, the challenges to be tackled in formal verification can benefit a lot from results and tools available for big data access and management. In fact formal verification requires several different skills: On the one hand, one needs an adequate background on formal methods in order to understand specific formalisms and proper abstraction techniques for modeling and interpreting the analysis results; On the other hand, one should also strive to deploy this techniques into software tools able to analyze large amount of data very reliably and efficiently similarly to “big data” projects. Recent approaches have shown the convenience of employing distributed memory and computation to manage large amount of reachable states, but unfortunately exploiting these results requires further skills in developing complex applications with knotty communication and synchronization issues. In particular, adapting an application for exploiting the scalability provided by cloud computing facilities as the Amazon Cloud Computing platform [8] might be a daunting task without the proper knowledge of the subtleties of data-intensive and distributed analyses.

In this paper, we try to further reduce the gap between these different areas of expertise by providing a distributed CTL (Computation Tree Logic) model checker, which exploits computational models typically used to tackle big data problems. Our software tool is built on top of HADOOP MAPREDUCE [9], [10] and can be easily specialized to deal with the verification of CTL formulas on very large state spaces coming from different kinds of formalisms (e.g., different kinds of Petri Nets, Process Algebra etc.), thus it is suitable for simplifying the task of dealing with a large amount of reachable states by exploiting large clusters of machines. The MapReduce programming model, which has become the de facto standard for large scale data-intensive applications, has provided researchers with a powerful tool for tackling big-data problems in different areas [11], [5], [12], [13]. We firmly believe that explicit state model checking could benefit from a distributed MapReduce based approach, but the topic has not been yet explored as far as we know. Exposing this issue to scientists with different backgrounds could stimulate the development of new interesting and more efficient solutions.

II. COMPUTATION TREE LOGIC

CTL [14] is a branching-time logic which models time as a tree-like structure where each moment can be followed by several different possible futures. In CTL each basic temporal
operator (i.e., either \(X, F, G\)) must be immediately preceded by a path quantifier (i.e., either \(A\) or \(E\)). In particular, CTL formulas are inductively defined as follows:

\[
\phi := p \mid \neg \phi \mid \phi \lor \psi \mid A\psi \mid E\psi \text{ (state formulas)}
\]

\[
\psi := X\phi \mid F\phi \mid G\phi \mid \phi U \psi \text{ (path formulas)}
\]

Where \(p \in AP\), the set of atomic propositions. The universal path operator \(A\) and the existential path operator \(E\) express respectively that a property is valid for all paths and for some paths. The temporal operators next \(X\) and until \(U\) express respectively that a property is valid in the next state, and that a property is valid until another property becomes valid. The interpretation of a CTL formula is defined over a Kripke structure (i.e., a state transition system) \(\sigma\). A Kripke structure is made up by a finite set of states, a set of transitions (i.e., a relation over the states), and a labeling function which assigns to each state the set of atomic propositions that are true in this state. Such a model describes the system at any point in time represented by states; the transition relation describes how the system evolves from a state to another over one time step. The formal definition is the following.

**Definition 1 (Kripke structure):** A Kripke structure \(T\) is a quadruple \((S, S_0, R, L)\), where:

1) \(S\) is a finite set of states.
2) \(S_0\) is the set of initial states.
3) \(R \subseteq S \times S\) is a total transition relation, that is: \(\forall s \in S \exists s' \in S\) such that \((s,s') \in R\).
4) \(L : S \rightarrow 2^{AP}\) labels each state with the set of atomic propositions that hold in that state.

Note that the third point imposes the *seriality* of the transition relation. This means that the system cannot have deadlock states. This condition can be always achieved easily by adding into the system a state of error (with one outgoing transition directed to itself) from which the system cannot get out once reached.

A path \(\sigma\) in \(T\) from a state \(s_0\) is an infinite sequence of states \(\sigma = s_0s_1s_2\ldots\) where \(\forall i \geq 0, (s_i, s_{i+1}) \in R\).

**Definition 2 (Satisfiability):** Given a CTL formula \(\phi\) and a state transition system \(T\) with \(s \in S\), we say that \(T\) satisfies \(\phi\) in the state \(s\) (written as \(T \models_s \phi\)) if:

- \(T \models_s p\) iff \(p \in L(s)\).
- \(T \models_s \neg \phi\) iff \(T \nvDash_s \phi\).
- \(T \models_s \phi \land \psi\) iff \(T \models_s \phi \land T \models_s \psi\).
- \(T \models_s \phi \lor \psi\) iff \(T \models_s \phi \lor T \models_s \psi\).
- \(T \models_s EX\phi\) iff \(\exists t\) such that \(R(s, t) \land T \models_t \phi\).
- \(T \models_s EG\phi\) iff \(\exists a\) path \(s_0s_1s_2\ldots\) such that:
  - \(\forall i \geq 0, T \models_{s_i} \phi\).
  - \(T \models_s E[\phi U \psi]\) iff \(\exists a\) path \(s_0s_1s_2\ldots\) such that:
    - \(\exists i \geq 0, T \models_{s_i} \phi\) \(\land\) \((T \models_{s_j} \phi\) \(\forall j < i\).

We can also write \(T \models \phi\) which means that \(T\) satisfies \(\phi\) in all the initial states of the system.

It can be shown that any CTL formula can be written in terms of \(\neg, \lor, EX, EG,\) and \(EU\), for example \(AX\phi\) is \(\neg EX \neg \phi, EG \phi\) is \(E[T \land U \phi]\), and so forth. The possible combinations are only eight:

\[
AX, EX, AF, EF, AG, EG, AU, EU
\]

The semantics of some widely used CTL operators is exemplified in Figure 1.

**Definition 3 (Model Checking):** Let \(T\) be a Kripke structure and let \(\phi\) be a CTL formula. The model checking problem is to find all the states \(s \in S\) such that \(T \models_s \phi\).

### III. Fixed-Point Algorithms

One of the existing model-checking algorithms is based on fixed-point characterizations of the basic temporal operators of CTL (similar ideas can be used for LTL model checking) [15]. Let \(T = (S, S_0, R, L)\) be a Kripke structure. The set \(\mathcal{P}(S)\) of all subsets of \(S\) forms a lattice under the set inclusion ordering. For convenience, we identify each state formula with the set of states in which it is true. For example, we identify the formula \(false\) with the empty set of states, and we identify the formula \(true\) with \(S\) (the set of all states). Each element of \(\mathcal{P}(S)\) can be viewed both as a set of states and as a state formula (a predicate). Formally, given a CTL formula \(\phi\) we can define:

\[
[\phi]_T := \{s \in S : T \models_s \phi\}
\]

This way, we can associate set operators to boolean connectors:

\[
[\phi_1 \land \phi_2] = [\phi_1] \cup [\phi_2], \quad [\phi_1 \lor \phi_2] = [\phi_1] \cap [\phi_2],
\]

\[
[\neg \phi] = S \setminus [\phi]
\]

The set of states identified by the temporal operator \(EX\), can be defined trivially if we consider the counterimage with respect to the relation \(R\). Given \(W \in \mathcal{P}(S)\):

\[
R^-(W) := \{s \in S : \exists s'(R(s, s') \land s' \in S)\}
\]

Thus we can verify easily that the following holds:

\[
[EX \phi]_T = R^-( [\phi]_T)
\]

Let’s now consider a function \(\tau : \mathcal{P}(S) \rightarrow \mathcal{P}(S)\) called predicate transformer.

**Definition 4 (Fixed-Point):** We say that a state formula \(X\) is the least fixed-point \(\mu_X\) (or respectively the greatest fixed-point \(\nu_X\)) of a predicate transformer \(\tau\) iff (1) \(X = \tau(X)\), and (2) for all state formulas \(X'\), if \(X' = \tau(X')\), then \(X \subseteq X'\) (respectively \(X \supseteq X'\)).

**Definition 5 (Monotonic Predicate Transformer):** A predicate transformer \(\tau\) is monotonic iff for all \(X, X' \in \mathcal{P}(S)\) \(X \subseteq X'\) implies \(\tau(X) \subseteq \tau(X')\).

A monotonic predicate transformer on \(\mathcal{P}(S)\) always has a least fixed-point and a greatest fixed-point (by Tarski’s Fixed-Point Theorem [16]). The temporal operators \(EG\) and \(EU\) can each be characterized respectively as the greatest and the least fixed-point of two different monotonic predicate transformers:

\[
[EG \phi]_T = \nu_X ([\phi]_T \land R^-(X))
\]

\[
[E[\phi U \psi]]_T = \mu_X ([\psi]_T \cup ([\phi]_T \land R^-(X)))
\]
We can calculate the least fixed-point of a monotonic predicate transformer: $\mu_X(\tau(X))$ as follows. We define $X_0 = \emptyset$ and $X_i = \tau(X_{i-1})$ for $i \geq 1$. We first compute $X_1$, then $X_2$, then $X_3$, and so forth, until we find a $k$ such that $X_k = X_{k-1}$. It can be proved that the $X_k$ computed in this manner is the least fixed-point of $\tau$. To compute the greatest fixed-point, we follow a similar procedure but starting from $S$. Pseudocode for this procedure is shown by Algorithm 1.

Algorithm 1 Least Fixed-Point Procedure

1: function LFP($\tau$)
2: \hspace{1em} $X := \emptyset$
3: \hspace{1em} while $X \neq \tau(X)$ do
4: \hspace{2em} $X := \tau(X)$
5: \hspace{1em} end while
6: \hspace{1em} return $X$
7: end function

IV. DISTRIBUTED MODEL CHECKING ALGORITHMS

We now recall briefly the MapReduce computational model (the basis on top of which our application is built) and later on we present our distributed approach in which we used the fixed-point algorithms to exploit distributed and “cloud” facilities. The distributed algorithms presented in this section aim just at computing formulas of type $EX$, $EG$, and $EU$ because any CTL formula can be reformulated in terms of these three basic operators (see [1]).

A. MapReduce

MapReduce relies on the observation that many information processing activities have the same basic design: a same operation is applied over a large number of records (e.g., database records, or vertices of a graph) to generate partial results, which are then aggregated to compute the final output. The MapReduce model consists of two functions: The “map” function turns each input element into zero or more key-value pairs. A “key” is not unique, in fact many pairs with a given key could be generated from the Map function; The “reduce” function is applied, for each key, to its associated list of values. The result is a key-value pair consisting of whatever is produced by the Reduce function applied to the list of values. Between these two main phases the system sorts the key-value pairs by key, and groups together values with the same key. This two-step processing structure is presented in Figure 2.

The execution framework handles transparently all non-functional aspects of execution on big clusters. It is responsible, among other things, for scheduling (moving code to data), handling faults, and the large distributed sorting and shuffling needed between the map and reduce phases since intermediate key-value pairs must be grouped by key. The “partitioner” is responsible for dividing up the intermediate key space and assigning intermediate key-value pairs to reducers. The default partitioner computes a hash function on the value of the key modulo the number of reducers.

B. Distributed State Space Generation

This task builds the reachability graph $T$ of a given model in a distributed fashion. The idea underlying a distributed algorithm for state space exploration is to use multiple computational units to perform the exploration of different parts of the whole state space in parallel. The task is typically performed by using classical parallel Workers algorithm [17]: States are partitioned among workers by means of a static hash function. The workers explore successor states and assign them to the proper computational units. Communication among different machines is implemented through message passing. Since the partitioning of the state space is a critical issue, different load balancing techniques and compact states representation [18], [19], [20] were studied. Recent approaches have shown also the convenience of exploiting big data approaches and cloud computing facilities in order to accomplish this task. In particular the MaRDiGraS [5] framework could be
employed to implement distributed state space builders for different formalisms. Given a cluster size of \( n \) machines, a \( MaRDiGaS \) based application generates \( n \) files \( F_1, F_2, ..., F_n \) containing the whole state space, partitioned into \( n \) different sets. The set of states emitted by the \( i \)th computational unit is \( S_i = \{ s \in S : \text{Hash}(f(s)) = i \} \), where \( S \) is the set of reachable states, \( f \) is a user supplied function and \( f(s) \) computes specific features on states such that the equality of the evaluation of these features is a necessary condition for having an inclusion/equality relationship among states. What makes this representation interesting and suitable for further analysis by using our distributed approach is in the transitions management (the \( R \) relation). In particular each state stores locally all incoming transitions as a list of state identifiers, therefore, given a set of state \( W \), \( R^{-1}(W) \) can be easily computed:

\[
\bigcup_{i=1}^{n} s_i \in S : (id(s_i) \in R^{-1}(s_j), \forall s_j \in W)
\]

It is worth noting that the set of predecessor states’ identifiers should be immediately available inside state definition because our MapReduce based approach exploits the evaluation of \( R^{-1} \) as a basic operation without communication among computational units.

In order to apply our distributed fixed-point algorithms the analyzed transition system must preserve the seriality of the transition relation (introduced in section \[ \text{[1]} \]). If not, the \( MaRDiGaS \) framework can add an output file containing a single “error” state where the list of incoming transitions is made up by itself and all deadlock states.

C. \( EX \) Formulas

To compute \( [EX\phi]_T \), we assume that the set of states satisfying \( \phi \) is already computed. Thus \( \phi \) can be either a formula locally evaluable or a more complex sub-formula evaluated previously. We can deploy this operation into a single MapReduce job where the predecessor states of the \( [\phi]_T \) set are evaluated in parallel. The input of this distributed computation is two different sets of files. The first set contains all states belonging to \( S \setminus [\phi]_T \), the second contains all states belonging to \( [\phi]_T \). This way all the mappers can evaluate and emit in parallel the identifiers of the states belonging to \( R^{-1}([\phi]_T) \). As shown by Algorithm 2, the Map function emits the identifiers of these states associated with an empty value \( \bot \). Then the shuffle phase groups together all the values with the same identifier, so that the Reduce function can emit the final result by checking whenever the empty value was passed into the input list.

D. \( EG \) Formulas

As for the previous formula, to compute \( [EG\phi]_T \), we assume that the set of states satisfying \( \phi \) is already computed. The evaluation of the final result is a bit more complex than the previous case. Our approach is based on the greatest fixed-point characterization of the monotonic predicate transformer introduced in \[ \text{[1]} \]. Thus we apply an iterative MapReduce algorithm, where at each iteration we compute the predicate transformer on the output of the previous iteration until we reach the fixed-point. Algorithm 3 shows the Map and the Reduce functions employed within the job iterations. The input of each MapReduce job is made up by a set of files containing \( [\phi]_T \) and another set of files \( X \) representing the current evaluation of the formula. Since the first iteration should start from \( X = S \) and \( R^{-1}(S) = S \), we already know the result of the first evaluation of the predicate transformer introduced in \[ \text{[1]} \), thus we start directly from the second iteration by posing \( X \) to \( [\phi]_T \). As shown by Algorithm 3, the map phase computes in parallel all the predecessor states and the reduce phase verifies and emits in parallel all predecessors belonging to \( [\phi]_T \). The iterations keep going until the number of key-value pairs given in output by two consecutive jobs becomes equal or we reach the empty set.

Algorithm 2 MapReduce algorithm for evaluating \( EX\phi \)

1: function MAP(k, s)
2: if \( s \in \phi \) then
3: for \( e \in R^{-1}(s) \) do
4: emit(e, \bot)
5: end for
6: end if
7: emit(k, s)
8: end function
9: function REDUCE(k, list := [s_1, s_2, ...])
10: if \( \bot \in list \) then
11: \( s := s' \in list \) s.t. \( s' \neq \bot \)
12: emit(k, s)
13: end if
14: end function

E. \( EU \) Formulas

As for the previous formulas, to compute \( [E[\phi U\psi]]_T \), we assume that the set of states satisfying the two sub-formulas \( \phi \) and \( \psi \) are already computed. The approach employed to evaluate this formulas is similar to the previous one, in fact our distributed algorithm is based on the least fixed-point characterization of the monotonic predicate transformer introduced in \[ \text{[2]} \]. The iterative map-reduce algorithm, which uses the Map and the Reduce functions presented by the algorithm 4 is employed in order to reach the fixed-point. The input of each iteration is made up by a set of files \( X \) containing the current evaluation of the formula and another set of files
containing $[\psi]_T$. Since the first iteration should start from the empty set, but we already know that the predicate transformer $[2]$ computed on the input $X = \emptyset$ is $[\psi]_T$, we start directly from the second iteration posing $X$ to $[\psi]_T$. The map phase emits in parallel all predecessor states of $X$ and sets forth all states of $[\psi]_T$ to reducers. The reduce phase emits in parallel all predecessor states of $[\phi]_T$ and all states of $[\psi]_T$.

Algorithm 4 MapReduce algorithm for evaluating $E[\phi U \psi]$

1: function MAP($k, s$
2:     if $s \in X$ then
3:         for $e \in R^-(s)$ do
4:             emit(e, $\bot$
5:         end if
6:     if $s \in [\phi]_T \lor s \in [\psi]_T$ then
7:         emit($k, s$
8:     end if
9: end function
10: function REDUCE($k, list := [s_1, s_2, ...]$)
11:     $s := s' \in list$ s.t. $s' \neq \bot$
12:     if $(\exists s \in list \land s \neq null) \lor (s \in [\psi]_T)$ then
13:         emit($k, s$
14:     end if
15: end function

$R^-(X_{i-1}) \subseteq R^-(X_i)$, since Algorithm 4 computes $R^-(X)$ for each iteration and $X_{i-1} \subseteq X_i$. For this reason we implemented an optimized version which computes, for each iteration, just $R^-(X_i \setminus X_{i-1})$.

V. EXPERIMENTS

The experiments described in this section were executed using the Amazon Elastic MapReduce [8] on the Amazon Web Service cloud infrastructure. They were supported by an “AWS in Education Grant award” [21]. In particular all runs have been performed on clusters of various sizes made up by m2.2xlarge computational units [8].

As a proof of concept we generated three different state spaces, sized with different order of magnitude. Successively, we applied our distributed algorithms in order to verify three different CTL formulas (of type $EX, EG$ and $EU$) for each state space. Both models and formulas used during the experiments were introduced in [22]. The models are three Petri Net benchmarks and their state space were generated by means of a MaRDiGraS based tool.

A. Shared Memory

This P/T net models a system composed of 10 processors which compete for the access to a shared memory by using a unique shared bus. The number of reachable states of this model is $1.831 \times 10^6$. Given the function $m : \text{Place} \rightarrow \mathbb{N}$ which computes the number of tokens for a given place, the three properties verified on this model are:

$$EX[A], \ EG[A], \ E[True \ U \ A] = EF[A]$$

where:

$$A := m(\text{Active}) \neq m(\text{Memory}) \lor m(\text{Queue}) = m(\text{Active})$$

Despite the generated state space is relatively small, the benefit gained from our distributed approach grows as the number of states involved in the verification grows (as shown in Table I): indeed, the verification of the last formula $E[\text{True U A}]$ scales better than the previous two.

| property | $[\text{property}]_T$ | # machines | time (s) |
|----------|------------------------|------------|----------|
| EX[A]    | $2.135 \times 10^3$    | 1          | 70       |
| EX[A]    | $2.135 \times 10^3$    | 2          | 67       |
| EX[A]    | $2.135 \times 10^3$    | 4          | 50       |
| EX[A]    | $2.135 \times 10^3$    | 8          | 38       |
| EG[A]    | 0                      | 1          | 67       |
| EG[A]    | 0                      | 2          | 55       |
| EG[A]    | 0                      | 4          | 58       |
| E[True U A] | $1.831 \times 10^4$ | 1          | 1898     |
| E[True U A] | $1.831 \times 10^4$ | 2          | 1124     |
| E[True U A] | $1.831 \times 10^4$ | 4          | 839      |
| E[True U A] | $1.831 \times 10^4$ | 8          | 564      |
| E[True U A] | $1.831 \times 10^4$ | 16         | 569      |

B. Dekker

This model represents a 1-safe P/T net of a variant of the Dekker’s mutual exclusion algorithm [23] for $N = 20$ processes. The state space generated by this model is an order of magnitude higher than the previous example ($1.53 \times 10^7$ reachable states). The three properties verified on this model are:

$$EX[B], \ EG[B], \ E[C \ U \ D]$$

where:

$$B := m(p_{1,18}) \neq m(p_{1,13}) \lor m(p_{0,15}) = m(p_{3,18})$$

$$C := m(flag_{1,8}) \neq m(p_{0,4}) \land m(p_{0,17}) = m(flag_{1,11})$$

$$D := m(p_{0,17}) = m(flag_{1,11})$$

In this case, as shown by Table I and by the graph shown in Figure 3(b), the benefits deriving from our distributed approach are clearer. In fact, the evaluation of both the three formulas gets substantially faster by increasing the number of computational units. The graph shown by Figure 3(b) (and Figure 3(d) for the next model) plots the function $\text{cheat}$ defined as follow:

$$\text{cheat}(n) = \frac{\text{exec. time of parallel version with 1 node}}{\text{exec. time of parallel version with n nodes}}$$

C. Simple Load Balancing

This P/T net represents a simple load balancing system composed of 10 clients, 2 servers, and between these, a load balancer process. The reachability graph generated is very large: $4.060 \times 10^8$ states and $3.051 \times 10^9$ arcs for a total size of 120 GB of data. The three properties verified on this model are:

$$EX[H], \ EG[J], \ E[K \ U \ H]$$
TABLE II: Dekker report

| property | |property| | # machines | time (s) |
|----------|-----------------|------------|------------|------------|---------|
| EX[B]    | 1.15 × 10^7    | 1          | 660        |
| EX[B]    | 1.15 × 10^7    | 2          | 532        |
| EX[B]    | 1.15 × 10^7    | 4          | 241        |
| EX[B]    | 1.15 × 10^7    | 8          | 144        |
| EX[B]    | 1.15 × 10^7    | 16         | 120        |
| EG[B]    | 7.40 × 10^6    | 2          | 1356       |
| EG[B]    | 7.40 × 10^6    | 4          | 517        |
| EG[B]    | 7.40 × 10^6    | 8          | 391        |
| EG[B]    | 7.40 × 10^6    | 16         | 287        |
| E(C U D) | 7.57 × 10^6    | 1          | 1357       |
| E(C U D) | 7.57 × 10^6    | 2          | 1063       |
| E(C U D) | 7.57 × 10^6    | 4          | 585        |
| E(C U D) | 7.57 × 10^6    | 8          | 454        |
| E(C U D) | 7.57 × 10^6    | 16         | 372        |

TABLE III: Simple load balancing report

| property | |property| | # machines | time (s) |
|----------|-----------------|------------|------------|------------|---------|
| EX[B]    | 1.71 × 10^6    | 1          | 2908       |
| EX[B]    | 1.71 × 10^6    | 2          | 2401       |
| EX[B]    | 1.71 × 10^6    | 4          | 937        |
| EX[B]    | 1.71 × 10^6    | 8          | 693        |
| EX[B]    | 1.71 × 10^6    | 16         | 251        |
| EG[J]    | 4.06 × 10^6    | 1          | 21678      |
| EG[J]    | 4.06 × 10^6    | 2          | 17147      |
| EG[J]    | 4.06 × 10^6    | 4          | 6525       |
| EG[J]    | 4.06 × 10^6    | 8          | 2983       |
| EG[J]    | 4.06 × 10^6    | 16         | 1226       |
| E(K U H) | 7.52 × 10^4    | 1          | 1821       |
| E(K U H) | 7.52 × 10^4    | 2          | 1714       |
| E(K U H) | 7.52 × 10^4    | 4          | 692        |
| E(K U H) | 7.52 × 10^4    | 8          | 377        |
| E(K U H) | 7.52 × 10^4    | 16         | 203        |

where:

\[ H := m(\text{server\_processed}) \neq m(\text{server\_notification}) \land m(\text{server\_waiting}) = m(\text{server\_idle}) \]

\[ J := m(\text{client\_idle}) \neq m(\text{client\_waiting}) \]

\[ K := m(\text{client\_idle}) \neq m(\text{client\_waiting}) \land m(\text{client\_idle}) = m(\text{client\_request}) \]

As shown by Table III and by the graph shown in Figure 3(d), the benefits deriving from our distributed approach are greater with respect to both previous examples. This points out a clear trend: the major is the complexity of the model to be analyzed, the major is the scalability of our distributed algorithm. In fact, the cheat gained during the analysis of this last example greatly overcome the one gained in the analysis of the Dekker model (5.5 using 16 machines to evaluate EX[B]). As shown in Figure 3(d), in this model we reach a super-linear speedup during the evaluation of EG[J].

VI. RELATED WORK

The use of distributed and/or parallel processing to tackle the state explosion problem gained interest in recent years. In fact, for very complex models, the state space may not completely fit into the main memory of a single computer and hence model-checking tools becomes very slow or even crash as soon as the memory is exhausted. 

[24], [25], [26], [27], [28] discuss parallel/distributed verification of Linear Temporal Logic (LTL) formulas. They aim at increasing the memory available and reducing the overall time required by LTL formulas verification by employing distributed techniques for searching accepting cycles in Büchi automata. Distributed and parallel model checking of CTL logic was also proposed. [29] introduced a CTL model checking technique which works by splitting the given state space into several “partial state spaces”. Each computer involved in the distributed computation owns a partial state space and performs a model checking algorithm on this incomplete structure. To be able to proceed, the border states are augmented by assumptions about truth values of formulas and the computers exchange assumptions about relevant states to compute more precise information. Other approaches were introduced in [30], [31].

The main idea of distributed algorithms for both LTL and CTL model checking is in fact similar: the state graph is partitioned among the network nodes, i.e., each network node owns a subset of the state space. The differences are in the way the state space is partitioned (through a partition function): this is a crucial issue. In order to increase performance of the parallel model checking, it is key to achieve a good load balancing among machines, meaning that each partition should contain nearly the same number of states. The performance of these algorithms depends also on the number of cross-border transitions of the partitioned state space (i.e., transitions having the source state in a component and the target state in another component). This number should be as small as possible, since it has an effect on the number of messages sent over the network during the analysis [32]. In the context of LTL model checking, probabilistic techniques to partition the state space have been used, for example, in [24], [33], and a technique that exploits some structural properties derived from the verified formula has been proposed in [34].

Since our distributed algorithms are quite different from message passing approaches, the number of cross-border transitions is not a crucial issue to cope with. The only synchronization point among computational units is the shuffle phase, where key-value pairs are sorted and transferred from map outputs to reducers input. Reducing the number of cross-border transitions may reduce the data exchanged across the network during this phase. Anyway, this phase is partially overlapped with the map phase, which means that the shuffling starts as soon as data become available from mappers without waiting for the entire map output. Furthermore, since we found experimentally that the time required by this phase does not dominate the overall time required by our algorithms, adding a partitioning phase between each MapReduce iteration could even hurt performances. Nevertheless, we plan to study further this issue in order to understand better how partitioning can impact performances of our MapReduce based approach.

Our contribution is a set of parallel algorithms designed for distributed memory architectures and cloud computing platforms based on a new emerging distributed paradigm. It is worth noting that departing from the current literature on distributed CTL model checking, we considered an important aspect, sometimes understated: we wanted to completely re-
move the costs of deploying our application into an end-to-end solution, for this reason we developed our software on top of the consolidated HADOOP MAPREDUCE framework. As far as we now, the effectiveness of a MapReduce based approach, typically employed to solve big data problems, has been not explored so far by the formal verification community. Thus with our work we aim at further reducing the gap between these two different but related areas of expertise.

VII. Conclusion and Future Work

In this paper we presented a software framework to model check very complex systems by applying iterative MapReduce algorithms based on fixed-point characterizations of the basic temporal operators of CTL.

Our distributed application exploits techniques typically used by the big data community and so far poorly explored for this kind of problem. Therefore we remark a clear connection between formal verification problems and big data problems conveyed by the recent widespread accessibility of powerful computing resources. Despite model checking software tools are so called “push-button”, the setup phase required by a distributed application, is far from being considered such, especially whenever one wants to exploits general purpose “cloud” computing facilities. Our framework aims at re-enabling a “push-button” mode into the distributed verification context even when these (complex on themselves) computing resources are involved.

Our experiments report that our approach can be used effectively to analyze state spaces of different orders of magnitude. In particular, the major is the complexity of the model to be analyzed, the major is the scalability of our distributed algorithms. In some cases we have shown a potential for a super-linear speedup. We believe that this work could be a further step towards a synergy between two very different, but related communities; the “formal methods” community and the “big data” community. Exposing this issue to scientists with different backgrounds could stimulate the development of new interesting and more efficient solutions.

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