Evolution of a Magnetic Flux Tube in a Sunspot Penumbra

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The motion of an individual magnetic flux tube inside the penumbra of a sunspot is studied numerically. Here, we present preliminary results. The thin flux tube approximation together with a simplified radiative heat exchange with the surroundings is used to study the evolution of a flux tube embedded into a background given by a global magneto-static sunspot model. The investigation is undertaken in order to verify the conjecture that convection in sunspot penumbrae occurs by an interchange of magnetic flux tubes. The code being developed can be used to study dynamic aspects of filamentary structure in the penumbra: the temporal and spatial fluctuations of the temperature and the magnetic field, the motion of bright penumbral grains, or the Evershed effect. Here we present the evolution of a wave formed by the tube whose fragment emerges in the penumbral photosphere and migrates towards the umbra. The properties of this wave show qualitative features of the observed bright penumbral grains with corresponding upward velocity and its correlation with brightness and the inclination of the magnetic field, and also of the Evershed effect.

1. INTRODUCTION

High resolution observations of filamentary sunspot penumbrae give some detailed hints at a particular type of convection that seems to occur there (see an extensive discussion in Jahn & Schmidt, 1994). In summary, the inclination of the magnetic field exhibits substantial azimuthal fluctuations which preserve their radial coherence and are well correlated with the brightness of filaments (Title et al., 1993): the dark ones are permeated by an almost horizontal field whereas the bright ones contain a more vertical field. The velocity field shows similar correlations: the horizontal Evershed effect occurs in the dark regions and upward flow in brighter ones. Additionally, an inward motion of bright penumbral grains towards the umbra is observed (Muller, 1992), which may be associated with the inward part of a balanced two-way migration of the magnetic flux. Those features observed in the
penumbral photosphere concur with the concept of an interchange convection in the penumbra which appears to have a deep structure (Schmidt 1987, Jahn 1989, Solanki & Schmidt 1993). This hypothetical convection would be accomplished by a non-local exchange of azimuthally narrow but radially elongated magnetic flux tubes that extend many scale heights below the photosphere (Schmidt 1991, Jahn 1992). It would set in when the total flux and with it the inclination of the field lines at the edge of a magnetic flux concentration exceeds some critical value. Rucklidge et al. (1995) studied an abrupt development of the penumbra with a simple model based on this concept. Some anticipated properties of interchanging flux tubes have been used to approximate the average structure of a deep penumbra by a one dimensional stratification of the matter, and to construct a tripartite magneto-static model for a sunspot (Jahn & Schmidt 1994). Now we intend to construct a 2-D model for the penumbra based on a more detailed study of the dynamics of the interchange convection. This might be achieved in an iterative manner by following the non-linear time-dependent evolution of a single magnetic flux tube embedded into a background which asymptotically assumes the thermodynamic and magnetic properties determined by the properties of the flux tube itself averaged over all phases of its motion.

In this progress report we present preliminary results of a study of the migration of a flux tube embedded into the fixed pregiven penumbra of the tripartite model, and compare its dynamic properties with the observed evolution of bright penumbral grains.

FIGURE 1. Parameters of the tripartite sunspot model (cf. Jahn & Schmidt 1994)
2. THE MODEL

The structure of the magnetic flux element has been approximated by a 1-D thin flux tube, and the Lagrangian formalism has been adapted to describe the 2-D dynamics of the tube. The equations governing the time evolution of the tube are basically the same as those used by Moreno-Insertis (1986) except that we allow for the radiative heat exchange between the tube and the surroundings and calculate the rate of the entropy change due to radiation. This is made in a similar manner as by Montesinos & Thomas (1993) who distinguished between large optical depths ($\tau > 2/3$), where the entropy change is proportional to the temperature difference between the tube and the surroundings (according to Spiegel's (1957) analysis), and small optical depths ($\tau < 2/3$), where this change is proportional to the difference in the local values of the Planck function (Kalkofen & Ulmschneider 1977).

Another difference is related to the 2-D nature of the force-free magnetic field of the background in which our tube is embedded. Although the stratification of the gas in the tripartite model is one dimensional, the magnetic pressure has a lateral gradient, so that the distribution of the total pressure (gas plus magnetic) is two-dimensional. This leads to additional terms in the equation of motion and in the condition of mechanical equilibrium of the tube with the surroundings. Finally, the set of equations describing the tube's evolution is completed by the same equation of state as that used in the tripartite model, i.e. with partial ionization effects included for a gas of solar chemical composition.

All parameters of the particular tripartite model used as background in the present simulations are given in Fig. 1 (see also Fig. 1 in Jahn & Schmidt, 1994).

3. RESULTS

A thin tube with a magnetic flux equal to $10^{15}$ Mx is originally placed on the inner side of the magnetopause of the tripartite model. The corresponding diameter of the tube at the penumbral photosphere is equal to about 10 km. Such a tube is much too small to represent directly any of the fine-structure elements observed in the penumbra, but our simplified treatment of the radiative transfer does not allow to consider much larger flux tubes. Otherwise, we would have to take into account an internal radiative diffusion. The initial stratification along the tube is identical to that in the penumbra. The end points of the tube have been placed far from the photosphere of the penumbra: the lower one at a depth of 15 Mm and the upper one on a sphere 25 Mm away from the center of the spot. For the time being both points are kept at fixed positions on the magnetopause during the evolution of the tube.

When the tube is slightly perturbed locally, its near photospheric portion at depths less than about 500 km, starts to oscillate across the magnetopause, because of two competing forces: the buoyancy which expels the tube from the more dense stratification of the quiet sun, and the curvature force acting in the opposite direction. But each time the tube intrudes into the quiet sun it heats up by radiative exchange which is still efficient at those depth in a nonlinear fashion, so that the amplitude of the oscillation grows with each cycle. Eventually, the tube gains a buoyancy large enough to overcome the curvature force, then emerges in the penumbral photosphere and migrates further towards the umbra.

Figure 2 shows the geometry of the outer part of the penumbra and the shape of the tube at sample times differing from each other by five minutes. The arrows
FIGURE 2. Migration of a flux tube in the penumbra plotted with a time step of 5 minutes. Dashed line corresponds to the magnetopause, and arrows show the flow of the matter. Insertions show the time history of the three points crossing the photosphere: the emerging (continuous line) and sinking one (dotted line). Shown are: fluctuations of the density, temperature, and the magnetic field relatively to the corresponding background values, the inclination of the tube to the vertical direction, vertical component of the velocity of plasma in the tube, and the distance of the points from the spot’s center along the tube indicate the velocity of the matter. The length of arrows corresponding to the Alfvén and sound speed at the photosphere are also marked in the figure. Initially the tube crosses the penumbral photosphere only once, but as it proceeds inwards its part laying above the photosphere near the edge of the spot sinks, so that the tube assumes a sine-wave shape with three intersections with the photosphere. The time history of different parameters of the two inner intersection points are plotted in the insertions, where continuous and dotted lines correspond to the ascending and the descending intersection respectively. The third intersection retracted after about 10 minutes and never intruded further than 500 km into the penumbra. For a short time two more intersections appeared near the penumbral
boundary, because the tube continuously oscillates across the magnetopause in this region. The last shape of the tube shown in the Fig. 2 may suggest that most probably there will soon be generated a second wave crossing the surface and traveling inward. However, problems with our oversimplified lower boundary condition do not allow to continue the run. Therefore, we concentrate at present on the features of the first wave which traverses about 2.5 Mm in the penumbra during the simulation. It forms a flattened arch above the photosphere with a typical height of about 50 km which has a length that slowly increases with time from about 300 to 800 km (see insertions in Fig. 2).

The flow of the gas in the arch is directed opposite to the migration of the tube. It assumes speeds close to sonic, or even supersonic in the descending intersection. It should be noted that our code does not yet allow a proper treatment of shocks. This streaming along the arch has a nature similar to a siphon flow (Meyer & Schmidt, 1968). The velocity of this flow in our model, of about 7 km/s, is larger than the observed maximum velocity of the Evershed effect, i.e. 6 km/s, reached at the outer penumbra boundary (see e.g. Muller, 1992).

Properties of both intersection points show some qualitative similarities to those observed in penumbral fine-structure, i.e. flow velocities, magnetic field directions, and their correlations with brightness. Thus, the ascending intersection has a more vertical magnetic field than the mean penumbral field in the model. The matter flows upward there and is hotter than the ambient medium. That concurs with the upward velocity of the bright matter measured in penumbrae (Beckers 1981, Degenhardt & Wiehr 1991). The other intersection is more horizontal, but shows a downflow with a substantial vertical component of the velocity as the gas is cooler than the surrounding background in the photosphere of the penumbra.

It is tempting to identify the ascending intersection of the arch with a bright penumbral grain, despite the fact that it does not possess all the quantitative features of the grains. It moves inward with a speed of about 2 km/s, the descending one somewhat slower. These values are by a factor of four larger than the observed velocities of the penumbral grains (Muller, 1992). The large velocity may be ascribed to the mechanism of heating of the flux tube in our present model. It is achieved by the radiative exchange whenever the tube intrudes into the quiet sun. The radiative time scale increases abruptly with depth below the photosphere due to increasing opacity. Thus, the tube can acquire the energy efficiently only over a limited range of depths, so that the buoyancy can be increased in a short fragment of the tube only, which then rises thereby forming a relatively short and fast-moving wave. One might expect that more efficient heating at larger depths might be achieved by complex magneto-convective processes. This then would make the tube buoyant over larger lengths, and thereby would induce longer and slower waves.

4. CONCLUSIONS

We have developed a code that allows investigations of the motion of a thin flux tube embedded in a pregiven magnetic field and background stratification extending over many scale heights in the solar convection zone and atmosphere. Preliminary simulations presented here show the effect of radiative heating of the flux tube immediately outside the magnetopause occurring over a relatively small range of depths. It induces a wave-like inward migration of the tube, and the properties
of the migrating wave correspond in general to the observed features of the fine structure in the penumbra, although the particular values differ by a factor of a few from the observed quantities. Probably, a better agreement can be achieved with a more efficient heating of the tube at larger depths. That requires, however, a model for the heat exchange across an inclined magnetopause in convectively unstable stratification, like that of Schmidt et al. (1986). Also a correct treatment of shocks should be included in the code, because supersonic velocities along the tube appear in the photospheric layers.

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