TeV Neutrinos from SuperNova Remnants embedded in Giant Molecular Clouds

Vincenzo Cavasinni\textsuperscript{1,2}†, Dario Grasso\textsuperscript{2,3}‡, Luca Maccione\textsuperscript{2,4} ∗

August 4, 2018

1 Dip. di Fisica “E. Fermi”, Università di Pisa, Largo B. Pontecorvo, 3, Pisa
2 I.N.F.N. Sezione di Pisa
3 Scuola Normale Superiore, P.zza Dei Cavalieri, 7, I-56126 Pisa
4 S.I.S.S.A., Via Beirut, 2- I-34014, Trieste

† Vincenzo.Cavasinni@pi.infn.it; ‡ d.grasso@sns.it; ∗ luca.maccione@pi.infn.it

Abstract

The recent detection of γ-rays with energy up to 10 TeV from dense regions surrounding some Supernova Remnants (SNR) provides strong, though still not conclusive, evidence that the nucleonic component of galactic Cosmic Rays is accelerated in the supernova outflows. Neutrino telescopes could further support the validity of such scenario by detecting neutrinos coming from the same regions. We re-evaluate the TeV range neutrino-photon flux ratio to be expected from pion decay, finding small differences respect to previous derivations. We apply our results and the recent HESS measurements of the very high energy γ-ray flux from the molecular cloud complex in the Galactic Centre, to estimate the neutrino flux from that region discussing the detectability perspectives for Mediterranean Neutrino Telescopes. We also discuss under which conditions neutron decay may give rise to a significant TeV antineutrino flux from a SNR embedded in Molecular Cloud complexes.

1 Introduction

The existence of a correlation between the position of supernova remnants (SNRs) and that of giant molecular clouds (GMCs) is supported by theoretical arguments and observational evidences. Infrared observations confirmed
the theoretical expectation that stars, especially massive ones, form mainly in the womb of GMs. Having a relatively short timelife, massive stars can complete their evolution and explode within the parent GMC before this is swept-out by stellar winds. SN explosions within, or in the nearby of, GMCs may trigger the formation of new stars explaining the bursting star formation activity observed in several dense regions of the Galaxy, e.g. in the Orion giant cloud, and in starburst galaxies.

From the point of view of high energy astrophysics, SNRs interacting with MCs are very interesting systems. Since SN explosions are expected to be the main source of the bulk of cosmic rays (CRs) in the InterStellar Medium (ISM) [1], a MC which is embedding, or just lay in the nearby of, an active SNR, should be crossed by a flux of relativistic particles which is significantly larger than in the rest of the Galaxy. Furthermore, due to their relatively high gas density \(n_H \sim 10^2 \div 10^6 \text{ cm}^{-3}\), GMCs should act as efficient astrophysical beam-dumps converting a relevant fraction of the primary particle energy into secondary \(\gamma\)-ray and neutrino emissions (see e.g. [2] [3]). In the following we will generally call SNRs interacting with MCs as embedded SNRs, also in those cases in which the SNR is not completely surrounded by the molecular gas.

Evidences of a correlation between \(\gamma\)-ray emission and dense molecular gas have been found by several satellites and particularly by EGRET (see [4]) and have been extensively studied by many authors (see e.g. [5]). Those observations, however, may generally be explained without invoking any local excess in the CR energy density respect to the value observed on the Earth. Only recently, a new generation of atmospheric Cherenkov telescopes, especially HESS, were able to find the smoking-gun of the interaction of CRs accelerated by an active SNR with a close MC. The most interesting HESS observations in this respect are those of RXJ1713.7-3946 [6] and the very recent measurements of the Galactic Centre (GC) ridge emission [7].

Several arguments suggest that the origin of the very high energy (HE) radiation coming from these sources may be hadronic. One of the most solid among these arguments is based on the observation of strong (up to several milliGauss) magnetic fields in MCs [8] (see [9] for the GC region). Such strong field should give rise to dramatic electron synchrotron energy losses above the TeV suppressing a possible HE Inverse Compton contribution to the total \(\gamma\)-ray emission from those regions (see e.g. [7]). Although the previous argument is rather convincing, a more direct evidence that HE \(\gamma\)-rays from embedded SNRs are produced by \(\pi^0\) decay would be extremely precious to confirm the validity of the entire scenario. Such an evidence may come from the detection of neutrinos coming from the same sources.

Several Neutrino telescopes are already operating, or under construc-
tion, around the world (see e.g. [10]). Due to their large detection volume and observation time, these instruments may have the capability to detect at least few neutrinos from some SNRs, especially if they are embedded in a dense medium. It is important to observe that the energy threshold above which Neutrino Telescopes may identify astrophysical neutrino sources (\(\gtrsim 100 \text{ GeV}\)) is approximately the same above which atmospheric Cherenkov telescopes can detect photons. This coincidence offers the interesting possibility to directly test if HE photons and neutrinos have the same hadronic origin confirming the half-century pending hypothesis that HECRs are accelerated by SNRs. Having that purpose in mind, in this paper we estimate the expected neutrino/photon flux ratio in the TeV energy range from embedded SNRs.

Our computation of the \(\nu/\gamma\) ratio from pion decay is similar to that followed by the authors of [11] for RXJ1713-3946 (see also [12]) and confirm their main results, apart small differences which we discuss in Sec 3.

In Sec 4 we discuss the production of TeV antineutrinos by neutron decay in GMC complexes. In [13] it was proposed that this process may give rise to a detectable antineutrino flux from the Cyg OB2 association and from the GC. That analysis was based on Ultra High Energy CR anisotropy measurements which, however, have a very low statistical significance or, in the case of the GC, have not been confirmed by other experiments. We will show that unless the primary particle spectrum is considerably different from that expected from ordinary shock acceleration, an observable contribution of neutron decay to the total neutrino flux can hardly be expected. Finally, in Sec 5 we apply the results of Sec 3 to estimate the muon neutrino flux from the MC complex in the GC on the basis of the new HESS measurements [7].

2 Photon emissivity by \(\pi_0\) decay

The process under consideration is \(pp \rightarrow NN + n_\pi \pi^{\pm,0}\) where \(n_\pi\) is the pion multiplicity and \(N\) is either a proton or a neutron. The general expression of the photon emissivity is

\[
Q_{\pi^0}(E_{\pi^0}) = c \, n_H \int_{E_{\pi^0}^{\min(N)}}^{E_{\pi^0}^{\max}} dE_p \, n_p(E_p) \frac{d\sigma}{dE_{\pi^0}}(E_p, E_{\pi^0}) ,
\]

where we assumed that hydrogen, having density \(n_H\), is the main proton target. When the proton energy is much larger than the proton mass, i.e. \(E_p \gtrsim\) several GeV, the differential pion production cross section is well ap-
proximated by a scaling expression

\[ \frac{d\sigma(E_p, E_{\pi^0})}{dE_{\pi^0}} = \frac{\sigma_0}{E_{\pi^0}} f_{\pi^0}(x) , \]  

(2)

where \( x \equiv E_{\pi^0}/E_p \) and \( \sigma_0 \simeq 3 \times 10^{-26} \, \text{cm}^{-2} \). Using the numerical package \textsc{pythia} [15] we verified that the expression of the scaling function adopted by the authors of [14], \( f_{\pi^0}(x) = 0.67(1-x)^{3.5} + 0.5e^{-18x} \), provides a good fit of simulated \( pp \) scattering data up to 500 TeV.

In all cases in which the proton spectrum is well approximated by a power spectrum

\[ n_p(E_p) = n_0 \left( \frac{E_p}{E_0} \right)^{-\alpha} , \]  

(3)

the \( \pi^0 \) emissivity becomes

\[ Q_{\pi^0}(E_{\pi^0}) \simeq n_p(E_{\pi^0})\sigma_0 c n_H Y(\alpha) , \]  

(4)

where

\[ Y(\alpha) = \int_0^1 dx x^{-2} f_{\pi^0}(x, \alpha) \]  

(5)

Hence the photon emissivity is

\[ Q_\gamma(E_\gamma) = \int_{E_{\pi^0}^{\text{min}}(E_\gamma)}^{E_{\pi^0}} dE_{\pi^0} \frac{2}{E_{\pi^0}} Q_{\pi^0}(E_{\pi^0}) \]  

\[ \simeq \frac{2}{\alpha} \sigma_0 c n_H Y(\alpha) n_p(E_\gamma) , \]  

(6)

where we used \( E_{\pi^0}^{\text{min}}(E_\gamma) = E_\gamma \left( 1 + \frac{m_{\pi^0}^2}{4E_\gamma^2} \right) \).

It is also useful to write the mean value of the ratio between photon energy and that of the parent proton (photon elasticity), which is

\[ \langle E_\gamma/E_p \rangle = \frac{1}{2} \int_{-\pi}^{\pi} d\cos \theta \ (1 - \cos \theta) \int_0^1 dx f_{\pi^0}(x) \]  

\[ = \int_0^1 dx \ f_{\pi^0}(x) = 0.18 \]  

(7)

where \( \theta \) is the angle of the photon momentum in the lab frame respect to that of the primary proton. We conclude this section by observing that by writing the proton spectrum in the form Eq. (3), we implicitly assumed that the ultra-violet cut-off is well above the maximal energy relevant for \( \gamma \)-ray observations. This is well motivated by the fact that the energy spectra observed by HESS for RXJ1713.7-3946 [6] and the GC [7], which we assume to be representative of all embedded SNRs, are steady power laws up to \( \sim 10 \, \text{TeV} \).
3 Muon and Electron Neutrinos from charged pion decay

Inelastic $pp$ collisions lead to roughly the same number of $\pi^0$’s, $\pi^+$’s and $\pi^−$’s. If, as we are assuming here, $\pi^0$ decay is the dominant source of high energy photons, a comparable emission of $\nu_\mu$ and $\nu_e$ has to be expected. The emissivity of muon neutrinos and antineutrinos produced by direct $\pi^+$ and $\pi^−$ decay is

$$Q_{\pi\nu}(E_\nu) \simeq \frac{m_\pi^2}{m_\pi^2 - m_\mu^2} \int_{E_{\min}^\pi(E_\nu)}^{\infty} \frac{dE_\pi}{E_\pi} Q_{\pi^\pm}(E_\pi) ,$$  \hspace{1cm} (8)

where $Q_{\pi^\pm}(E_\pi) = Q_{\pi^0}(E_\pi) \times (1 \pm \epsilon)$, $\epsilon$ being the asymmetry between the positive and negative pions effectively produced in $pp$ collisions:

$$\epsilon \equiv \frac{N_{\pi^+} - N_{\pi^-}}{N_{\pi^+} + N_{\pi^-}} .$$  \hspace{1cm} (9)

For $\langle E_\nu \rangle \gg m_\pi$,

$$E_{\min}^\pi(E_\nu) = \frac{m_\pi^2}{m_\pi^2 - m_\mu^2} E_\nu + \frac{m_\pi^4}{m_\pi^2 - m_\mu^2} \frac{1}{4 E_\nu} \simeq \frac{1}{1 - r^2} E_\nu ,$$  \hspace{1cm} (10)

where $r \equiv m_\mu/m_\pi$. Since we don’t know the details of the interaction, the parameter $\epsilon$ cannot be evaluated analytically and we have to recourse to numerical simulations. We performed a simulation of proton-proton interactions up to an incident energy of about 500 TeV with PYTHIA \cite{hy}. Our best fit for $\epsilon$ is

$$\epsilon(s) = 0.1 - 6 \cdot 10^{-3} \ln(s/s_0) - 0.15 \cdot \sqrt{s_0/s} ,$$  \hspace{1cm} (11)

with $s \equiv 2m_\mu^2 + 2m_\mu E_\pi$ and $s_0 \simeq 1$ GeV$^2$. In Fig\ref{fig} we show the energy dependence of our $\epsilon$ as well as our best fit. We can now write

$$Q_{\nu_\mu}(E_\nu) \simeq \left(\frac{1}{1 - r^2}\right)^{1-\alpha} \frac{1 + \langle \epsilon \rangle}{2} Q_\gamma(E_\nu) ,$$  \hspace{1cm} (12)

where $\langle \epsilon \rangle$ represents the average of $\epsilon$ over the relevant energy range which, in our case, is 10 GeV – 500 TeV. The computation of the contribution of secondary muon decay $\mu \rightarrow e\nu_\mu\nu_\mu$ to the neutrino emissivity is slightly more involved. The contribution of this process to the neutrino emissivity is

$$Q_{\nu_\mu}^\mu(E_\nu) = \int_{E_{\min}^\mu(E_\nu)}^{E_{\max}^\mu(E_\nu)} dE_\mu \: Q_\nu^\mu(E_\mu) \frac{dP(E_\mu, E_\nu)}{dE_\nu} ,$$  \hspace{1cm} (13)
where from the kinematics it follows $E_{\mu}^{\text{min}} \simeq E_{\nu}$ and $E_{\mu}^{\text{max}}$ can be safely posed to $\infty$ if the muon spectrum is broad enough. The muon emissivity is

\begin{equation}
Q_\mu^\pi(E_{\mu}) \simeq \left( 1 - E_{\mu}/r^2 \right) \frac{dE_{\pi}}{E_{\pi}} Q_{\pi\pm}(E_{\pi}) = \left( 1 - r^2 \right) \frac{1}{2} \left( 1 \pm \langle \epsilon \rangle \right) Q_\gamma(E_{\mu}) .
\end{equation}

The neutrino emissivity can thus be estimated as done in [16] by using the spectrum-weighted momenta

\begin{equation}
Q_\nu^\mu(E_{\nu}) \simeq \frac{1}{2} \frac{\langle \epsilon \rangle}{Q_\gamma(E_{\nu})} \frac{1}{\alpha(1 - r^2)} \times \left( \frac{\langle y^{\alpha-1} \rangle_0}{1 - r^2} + \frac{1}{1 - r^2} \left( 1 + r^2 - \frac{2}{\alpha - 1} \frac{r^2}{1 - r^2} \right) \langle y^{\alpha-1} \rangle_1 \right) ,
\end{equation}

where $\langle y^{\alpha-1} \rangle_0$ and $\langle y^{\alpha-1} \rangle_1$ are given in the same textbook for the different flavours. The total neutrino emissivity by both positive and negative pion decay is given by the sum of (12) and of (13).

We can account for kaons contribution as well, provided that we can relate their flux to that of pions. Again, a simulation performed with the help of PYTHIA allows us to determine that the mean ratio $K/\pi$ is about few percent. Thus, accounting for the well known branching ratio for the decay $K \to \mu \nu$, that is 63.5%, ¹ and keeping in mind that no significant dynamical

¹We neglect here three-body decay which have only a few percent BR.
differences arise between pions and kaons, we found the correction to the neutrino emissivity due to pion. This come out to be few percent.

Finally, using the expressions for the spectrum-weighted momenta and a mean asymmetry \( \epsilon \) of about 5% we find the values of the ratios \( Q_{\nu_i}/Q_\gamma \) which we shown in Tab.1 for several values of the spectral index \( \alpha \).

| \( \alpha \)     | \( \nu_\mu \) | \( \bar{\nu}_\mu \) | \( \nu_e \) | \( \bar{\nu}_e \) |
|-----------------|--------------|----------------|------------|----------------|
| 2.0             | 0.52         | 0.52           | 0.28       | 0.26           |
| 2.2             | 0.46         | 0.46           | 0.25       | 0.23           |
| 2.4             | 0.40         | 0.40           | 0.22       | 0.21           |
| 2.6             | 0.36         | 0.36           | 0.20       | 0.18           |

Table 1: Ratio between neutrino and \( \gamma \)-ray emissivities for several values of the primary proton spectral index \( \alpha \).

These values are close, though not coincident, with those given in [11]. The percentage difference ranges between 10% and 30%, being maximal for electron antineutrinos. Most likely, such a discrepancy is due to a value of the pion charge asymmetry that in [11] is larger than the value adopted here. We think that the value of \( \epsilon \) used in [11] is correct only at low energies (\( \sim 100 \) GeV) and that the authors may have not taken into account the decreasing behaviour of \( \epsilon \) at larger energy. Indeed, from Fig.1 we see that the scaling behaviour is reached only above several hundred GeV. Nevertheless, it should be noted that being the uncertainty in the value of \( \epsilon \) provided by PYTHIA of the order of 20%, the values of the neutrino-photon relative emissivities given in Tab. 1 are almost compatible with those given in [11] within errors.

3.1 The effect of Neutrino oscillation

As well established by several experiments, neutrinos undergo flavor oscillations during their propagation in vacuum. Since the typical SNRs distance from the Earth is much larger than the neutrino oscillation length around the TeV, the phase of oscillations is very large so that we can safely deal with averaged vacuum oscillations. In this case the flavor oscillation probability can be written

\[
P_{\nu \nu} = \sum_j |U_{lj}^2| \cdot |U_{l'j}^2|, \tag{16}
\]
Table 2: Oscillation probability in the average vacuum oscillation hypothesis, calculated with the mixing angles reported in the text.

|      | $\nu_e$ | $\nu_\mu$ | $\nu_\tau$ |
|------|---------|-----------|-----------|
| $\nu_e$ | 57%     | 21%       | 21%       |
| $\nu_\mu$ | 21%     | 39%       | 39%       |
| $\nu_\tau$ | 21%     | 39%       | 39%       |

where $U_{ll'}$ is the unitary mixing matrix among the lepton flavours $l,l'=e,\mu,\tau$. After the propagation the neutrino flux is

$$F_i = \sum_{j=e,\mu,\tau} P_{ij} F_j^0 ,$$

(17)

$F_j^0$ being the expected neutrino flux in the absence of oscillations.

Several experiments have been performed, and are now in progress, to determine the values of the mixing parameters. Adopting the standard parametrization [17] for the mixing matrix $U_{ij}$, we can describe our present knowledge as:

- $\theta_{12} \simeq \theta_\odot \simeq (33.9^{+2.4}_{-2.2})^\circ$ by solar and KamLAND data [18].
- $\theta_{23} \simeq \theta_\text{atm} \simeq 45^\circ \pm 10^\circ$ by atmospheric data (see e.g. [11]).
- $\theta_{13} \simeq 0 \pm 10^\circ$ from CHOOZ data [19].

The CP violating phase $\delta_{CP}$ is still unknown and we assume here that it is negligible. Under this hypothesis the oscillation probability matrix $P_{ij}$ is symmetric and equal for both neutrinos and antineutrinos. We show it in Tab.2

Using Eq. (17) we calculate the $\nu/\gamma$ ratios after propagation which we shown in Tab. 3. As an effect of oscillations the flavor composition is almost isotropized: the expected fluxes for each flavor are the same within a 20%. Another important effect is the appearing of a $\nu_\tau$ component, even if it is not generated at the source.

Neutrino telescope looking for up-going muons produced in the Earth are mainly sensitive to the incoming flux of $\nu_\mu$ and $\bar{\nu}_\mu$. As a reference we write here the relation between the arrival $\nu_\mu + \bar{\nu}_\mu$ and $\gamma$-ray fluxes from a SNR accelerating primary protons with a power-law spectrum with $\alpha = 2.2$

$$F_{\nu_\mu + \bar{\nu}_\mu}(E_\nu)|_{\alpha=2.2} = 0.46F_\gamma(E_\nu) .$$

(18)

We will not discuss here the errors introduced by the uncertainties in our knowledge of the mixing angles. As showed in [11], they are much smaller than the expected experimental errors.
\[
\alpha = 2.0 \quad \alpha = 2.2 \quad \alpha = 2.4 \quad \alpha = 2.6
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\nu_\mu & 0.27 & 0.23 & 0.21 & 0.18 \\
\bar{\nu}_\mu & 0.26 & 0.23 & 0.20 & 0.18 \\
\nu_e & 0.27 & 0.24 & 0.22 & 0.19 \\
\bar{\nu}_e & 0.26 & 0.23 & 0.20 & 0.18 \\
\nu_\tau & 0.27 & 0.23 & 0.21 & 0.18 \\
\bar{\nu}_\tau & 0.26 & 0.23 & 0.20 & 0.18 \\
\hline
\end{array}
\]

Table 3: \( \nu/\gamma \) ratios as seen at Earth after propagation.

It is important to notice that since “averaged” oscillations are energy independent they do not affect the power-law energy spectrum expected for neutrino at the source. As it has been pointed out in [20], a spectral distortion in the flux of astrophysical neutrinos might be the signal of long-wavelength energy-dependent oscillations, possibly due to the existence of new families of sterile neutrinos.

4 Antineutrinos from neutron decay

The interest about this antineutrino production mechanism has been stimulated by claims of possible detections of CR anisotropy in the EeV range. Namely, the HIRES [21] and AGASA [22] collaborations found an excess of CRs with energy close to the EeV coming from the direction of Cygnus-OB2, though with low statistical significance. CR excesses was also detected by the AGASA [22] and SUGAR [23] experiments in directions close to the Galactic Centre (GC) and in the same energy range. Even if the existence of this anisotropy has not been confirmed by AUGER [24], theoretical arguments suggest that a positive signal may be found when the statistics of that experiment will be improved [25]. Both the Cyg-OB2 star association, laying in the Galactic plane at a distance of 1.7 kpc from the Earth, and the region within 100 pc from the GC, at a distance of about 8.5 kpc, are known to be active star forming regions rich of molecular gas and dust. The origin of the CR anisotropy in their directions have been imputed to a neutron emission from those systems with energy up to few EeV. At that energy, in fact, neutrons can travel over galactic distances and, differently from protons and composite nuclei which are isotropised by the galactic magnetic field, they should keep the information about the angular position of their source unspoiled. It was proposed that if primary nuclei are accelerated above the EeV in that system, their PD onto the Far Infra Red (FIR) radiation which is present
in Cyg-OB2 and in the GC regions could efficiently produce neutrons of the required energy. The presence of a FIR background is a common feature of star forming regions. It is due to the absorption and reprocessing of optical and UV stellar radiation by the thick dust which is generally present in those systems. The spectrum of this radiation is almost that of a black-body with temperatures typically ranging between 10 and 100 K.

The authors of [13] suggested that, would the detection of a neutron flux from Cyg-OB2 and the GC be confirmed, a “guaranteed” flux of antineutrino in the TeV energy range has also to be expected from those sources. Their argument is based on the hypothesis that the primary nuclei spectrum between 1 PeV and few EeV is a rather steep power law (they assumed $\alpha \sim 3$ as for the CR spectrum observed on the Earth) and that TeV antineutrinos are produced by the decay of neutrons, with energy close to the PeV, which are produced by the PD of nuclei onto the UV radiation in the proximity of massive stars. Although this is an interesting possibility, it should be noted that the neutron spectrum was not measured and that, since neutrons should be produced in the nearby of the source and do not diffuse in the Galactic magnetic field, it is questionable that its slope coincides with that of ordinary CRs. Diffusion of primary nuclei in the MC region should be almost absent since, even assuming a magnetic field strength of $\sim 100 \mu G$, their giration radius at energies above the EeV is comparable to the cloud size. Furthermore, only few, if any, SNRs may be able to accelerate nuclei beyond the EeV.

Here we consider a more general scenario in which neutrons may not be produced with the energy needed to reach the Earth and be detectable and try to relate the antineutrino flux from neutron decay to the $\gamma$-ray flux which may be observed coming from the same source. We will determine the neutron production rate in the nearby of the SNR by applying some of the results derived in a previous work [25]. As secondary neutrons can be produced not only by the PD of high energy nuclei but also by hadronic collisions, mainly $p + p_{\text{gas}} \rightarrow n + p + \text{pions}$, both processes need to be considered. We will show that, although PD is expected to be the most effective production process of EeV neutrons in MCs, at lower energies the hadronic scattering channel should be the dominant neutron production channel.

### 4.1 Neutrons from nuclei photo-disintegration

SNRs are expected to accelerate nuclei by a first order shock acceleration mechanism. In the linear shock acceleration theory all nuclear species are expected to have approximately the same power-law energy spectrum up to some UV cut-off depending on the charge number $Z$. 

10
Above a centre of mass energy of a few MeV, a nucleus can photo-disintegrate into a lighter nucleus and one or more nucleons, due to the resonant giant dipole interaction with background photons. This process was studied in details in [26, 27]. Starting from the results presented in those papers, some of us derived the neutron production rate due to the PD of nuclei onto a thermal background of photons at the temperature $T$ [25]. This is

$$\Gamma_A(E, T) \simeq 4 \times 10^{-14} \xi_A \Phi_A(E, T) \left( \frac{n_\gamma}{10^2 \text{ cm}^{-3}} \right) \text{ s}^{-1}, \quad (19)$$

where $\xi_A$ is an order one parameter depending on the nuclear species. The function $\Phi_A(E, T)$, which has been defined in [25], depends on the product $ET$ only. It is peaked at the energy

$$E_A^* \simeq A \times 2 \times 10^{15} \left( \frac{T}{2 \times 10^4 \text{ K}} \right)^{-1} \text{ eV} \quad (20)$$

where it takes a value depending on the nuclear species. For Iron nuclei ($^{56}\text{Fe}$) $\Phi_{56}(E^*) \approx 1$, it is an order of magnitude smaller for $^{16}\text{O}$ and $^{12}\text{C}$, and $3 \times 10^{-2}$ for $^4\text{He}$. Although $^4\text{He}$ are the most abundant composite nuclei, heavier species, especially Iron, give a comparable contribution to the PeV neutron production yield due to their larger PD cross section.

Equation (19) has been derived under the assumption that the photon distribution is that of a black-body, or a grey-body (diluted black-body), with temperature $T$. Since each neutron takes, on average, a $1/A$ fraction of the parent nucleus energy, it follows from (20) that PeV neutrons will be mainly produced by nuclei PD onto a photon background with $T \sim 1 \text{ eV}$. However, due to the huge opacity of the dense molecular gas practically no optical and UV photons should be present in GMC but in small H II blisters produced by the photoionization of the hydrogen gas around massive stars. The radius of these regions, which are generally known as Strömgren spheres, is expected to be roughly (see e.g. [28])

$$R_{\text{H II}} \simeq 0.2 \left( \frac{F}{10^{48} \text{ s}^{-1}} \right)^{1/3} \left( \frac{\beta}{2.6 \times 10^{-23} \text{ cm}^{-3} \text{ s}^{-1}} \right)^{-1/3} \left( \frac{n_H}{10^3 \text{ cm}^{-3}} \right)^{-2/3} \text{ pc}, \quad (21)$$

where $F$ is the UV photon production rate of OB stars and $\beta$ is the recombination coefficient. The size of the Strömgren spheres grows relatively little during the short main sequence life-time of these massive stars [28]. Furthermore, the contribution of dust to the opacity may further reduce $R_{\text{H II}}$. In dense GMCs, having $n_H = 10^3 - 10^5 \text{ cm}^{-3}$ a characteristic size
of 10 pc and a population of hundreds of OB stars, the H II phase filling factor is smaller than $10^{-2}$. Since the photon density in each blister is $n_{\text{UV}}^{\text{HII}} \simeq \frac{F}{\pi c R_{\text{HII}}^2} \simeq 10^2 \text{cm}^{-3}$, the overall mean UV photon density in a dense MC is roughly $n_{\text{UV}}^{\text{MC}} \simeq 1 \text{cm}^{-3}$. An order of magnitude larger photon density may be found in rich systems like the Cyg-OB2 association. As a consequence, in such a kind of systems, the H II phase filling factor is close to unity and the UV photon density $n_{\text{UV}} \sim 10 \text{cm}^{-3}$ which is in agreement with the value adopted in [13]. Using (6) and (19) we find

$$Q_n(E_n) = \sum_A f_A n_p(E_n) \Gamma_A(AE_n, T)$$

$$= 4 \times 10^{-17} \left( \sum_A f_A \Phi_A(E_n, T) \right) n_p(E_n) \left( \frac{n_{\gamma}}{10 \text{ cm}^{-3}} \right) \text{GeV}^{-1}\text{cm}^{-3}\text{s}^{-1}$$

(22)

where $f_A$ is the relative abundance of all the nuclear species with atomic weight $A$ in the SNR which undergo effective PD. For the proton spectrum we used the unit GeV$^{-1}$cm$^{-3}$.

### 4.2 Neutrons from p-p scattering

Besides the PD process discussed in the previous section, EeV neutrons might also be produced by the collision of UHE nuclei with the dense gas in the MC environment. The relevant neutron production channel is the charge exchange $pp$ inelastic collision $pp_{\text{gas}} \rightarrow pn + m_{x}\pi$, where $m_{x}$ is the pion multiplicity. In the case of a power-law spectrum for the HE protons the neutron emissivity due to this process was found to be [25]

$$Q_{n}^{pp}(E_n) = n_{H} n_{p}(E_n) \sigma_0 m_n \ c \ K \ I(\alpha)$$

$$\simeq 2 \times 10^{-15} n_p(E_n) \left( \frac{n_H}{10^3 \text{ cm}^{-3}} \right) \left( \frac{I(\alpha)}{0.1} \right) \text{GeV}^{-1}\text{cm}^{-3}\text{s}^{-1}$$

(23)

where $K = 4/5$ is the inelasticity, and $m_n \simeq 0.24$ the neutron multiplicity. We neglected here a weak logarithmic energy dependence of the cross section. The function $I(\alpha)$ is defined by

$$I(\alpha) \equiv \int_0^1 dx x^{\alpha-2} h(x)$$

(24)

where $h(x) \simeq 0.064 (1 - x)^2 + 0.094 x \sqrt{1 - x}$ and $x \equiv E_n/E_p$. 

12
By comparing (23) with (22) it is evident that under most common conditions in GMCs, \( pp \) scattering should be the main neutron production channel at energies around the PeV.

An approximated expression for the antineutrino emissivity from neutron decay can then be derived by treating the neutron decay factor as a step function \( \theta \left( \frac{d m_n}{\tau_n} - E_n \right) \) and replacing the antineutrino energy distribution by a delta-function peaked at the mean value \( \bar{E}_\nu \simeq 6 \times 10^{-4} m_n \) (here we followed the same approach adopted in [13]). By doing that, we find

\[
Q_\nu(E_\nu, x) = 0.1 \frac{I(\alpha)}{Y(\alpha)} \left( \frac{m_n}{2 E_\nu} \right)^{1-\alpha} Q_\gamma(E_\nu, x) .
\]  \( (25) \)

Comparing this result with that given in Sec. 3, it is clear that, unless the primary proton spectrum has a quite unusual behaviour, neutron decay should give only a negligible contribution to the total neutrino emissivity of an embedded SNR.

The relative contribution of neutron decay might somewhat be enhanced if the proton energy distribution in the MC has a strong spatial dependence. This may be indeed the case for protons which are shock accelerated by a steady SNR, as the expected energy spectrum upstream the shock front is

\[
n_p(E, r) = n_0(E) \exp \left\{ -\frac{\kappa u_S r}{D(E)} \right\} .
\]  \( (26) \)

Here \( r \) is the distance from the SNR shock; \( n_0(E) \) is the proton spectrum downstream the shock, which we assume to be given by (3); \( u_S \) is the shock velocity; \( D(E) \) is the diffusion coefficient; \( \kappa \) is an order one coefficient. For the sake of simplicity we also assumed that \( u_S \) and \( D(E) \) do not depend on \( r \). By expressing the energy dependence of the diffusion coefficient in the usual form \( D(E) = D(E_0)(E/E_0)\beta \), the meaning of (26) become quite evident: upstream the shock the proton spectrum has an exponential low-energy cutoff at

\[
E_{\text{min}}(r) = E_0 \left( \frac{u_S r}{D(E_0)} \right)^{1/\beta} .
\]  \( (27) \)

Assuming Bohm diffusion in the acceleration region (note that adopting a larger diffusion coefficient in that region would imply a too low maximal acceleration energy), i.e. \( D(E) = \frac{1}{3} c r_L(E) \), where \( r_L(E) \) is the Larmor radius, we find

\[
r_s(E) \simeq 20 \left( \frac{E}{1 \text{ PeV}} \right) \left( \frac{u_S}{500 \text{ km s}^{-1}} \right)^{-1} \left( \frac{B}{10 \mu \text{G}} \right)^{-1} \text{pc} .
\]  \( (28) \)
It is evident that PeV protons can diffuse into the cloud much more deeply than 10 TeV, so that secondary neutrons, as well as TeV ν_e from their decay, are produced in a volume larger than that in which photons and neutrinos are emitted by hadronic scattering. Note, however, that the characteristic distance over which protons propagate before losing their energy due to pp scattering (\(\tau_{\text{loss}} \simeq (c n_H \sigma_0)^{-1}\)), is

\[
L_{\text{loss}}(E) = \sqrt{6D(E)\tau_{\text{loss}}} \simeq 10 \left( \frac{E}{1 \text{ PeV}} \right)^{1/2} \left( \frac{B}{10 \mu\text{G}} \right)^{-1} \left( \frac{n_H}{10^3 \text{ cm}^{-3}} \right)^{-1/2} \text{pc}.
\]

(29)

Therefore, energy losses significantly reduce the effective propagation volume of primary protons in those cases in which the gas density and magnetic field strength are very large. Unfortunately, these are also the most interesting cases from the observational point of view. At the end, we found that in all relevant physical situations the volume gain factor is not sufficient to make the enhancement of the contribution of neutron decay to the total neutrino emissivity observationally relevant.

We conclude this section by stressing that all the previous considerations apply only if the particle primary spectrum is a power law. This will not be the case, for example, if primary nuclei are accelerated by a strong voltage drop around a fast rotating neutron star. In that case, in fact, primary nuclei may have an energy spectrum peaked at the PeV, or above, so that their PD onto stellar radiation be dominant respect to hadronic scattering and neutron decay be the main neutrino production channel.

5 Neutrinos from the Galactic Centre

In this section we apply our previous results to estimate the neutrino flux from the dense molecular cloud complex in the Galactic Centre region. An extended very-high-energy \(\gamma\)-ray emission has recently been discovered by the HESS coming from a region roughly delimited by \(|l| < 0.8^\circ\) and \(|b| < 0.3^\circ\) in galactic coordinates [7]. The coincidence between the angular position of the centroid of this emission and that of the dynamical centre of the Galaxy suggests that both are located at the same distance from the Earth, which is about \(\sim 8.5\) kpc. The energy spectrum of this source was measured to be \((1.73 \pm 0.13 \pm 0.35) \times E_{\text{TeV}}^{-2.29 \pm 0.07 \pm 0.02} \text{TeV}^{-1}\text{cm}^{-2}\text{s}^{-1}\text{sr}^{-1}\). Superimposed to that diffused emission HESS found a point-like source (J1745-290), which was also early detected by HESS [34] (and confirmed by MAGIC [31]) with an energy spectrum \((2.50 \pm 0.2) \times E_{\text{TeV}}^{-2.21 \pm 0.09} \text{TeV}^{-1}\text{cm}^{-2}\text{s}^{-1}\), in a position compatible either with the supermassive black hole in Sgr A* or with the SNR
Sgr A East. This source was subtracted when determining the spectrum of the extended emission.

As the authors of [7] suggest, the detected γ-rays are likely to be produced by the hadronic interaction of locally produced cosmic rays with the dense molecular gas. A relevant support in favour of this hypothesis is given by the strong correlation found between the γ-ray and the millimetric emissions from the CS (Carbon Sulfide) in that region, that compound being a well known tracer of molecular hydrogen. The coincidence, within errors, between the spectral slope of the GC ridge emission detected by HESS and that of J1745-290, suggests that the primary particles responsible for both signals may have the same origin. Indeed, the total CR energy required to explain both of them is compatible with that typically expected from shock acceleration by the remnant of a single SN. These considerations point to Sgr A East as the most plausible primary source (see [33, 25] about the identification of J1745-290 with that SNR).

This elegant scenario may find further support would neutrinos from the same region be detected. The expected neutrino flux can be easily determined by applying the results of Sec. 3. It has to be noted that due the relatively poor angular resolution (of order ≲ 1°) achievable with Neutrino Telescopes and the very low expected neutrino fluxes, it will be quite hard, if not impossible, to disentangle the neutrino point-like emission from that coming from the Galactic Centre ridge. Thus, we have to sum the neutrino flux from both sources. This is straightforward since, as the γ-ray spectral slopes of these sources coincide within errors, the same will be true for neutrinos. We use here the central value of the slope measured by HESS for the GC ridge, i.e. α = 2.29. The total γ-ray flux from the GC ridge source is determined by multiplying the diffused flux measured by HESS by its solid angle opening, i.e. 4 × 0.8° × 0.3° = 3 × 10⁻⁴ sr. It is about the double of the flux from J1745-290. Therefore, the total γ-ray flux, above the TeV, coming from the GC region is roughly (we only write central values in the following)

\[ F_{\gamma}^{GC}(E_{\gamma}) \simeq 7.5 \times 10^{-12} \left( \frac{E_{\gamma}}{1 \text{ TeV}} \right)^{-2.3} \text{ TeV}^{-1} \text{ cm}^{-2} \text{ s}^{-1} . \]  

Using our results of Sec 3, we derive the ν/γ ratios for α = 2.29 at the source and at the Earth are shown in Tab. 5 for the different neutrino species. We notice here that the TeV γ-ray attenuation due to pair-production scattering onto the IR photon background should be negligible. In fact, whenever the radiation temperature is below 100 K, the mean IR photon energy implies a γ-ray threshold energy well above 10 TeV. Gamma-ray scattering onto the UV and X-ray backgrounds present in that region was also showed to give
\[ \alpha = 2.29 \]

| \( \nu_e \) | 0.24 | 0.23 |
| \( \bar{\nu}_e \) | 0.21 | 0.21 |
| \( \nu_\mu \) | 0.43 | 0.22 |
| \( \bar{\nu}_\mu \) | 0.43 | 0.22 |
| \( \nu_\tau \) | 0 | 0.22 |
| \( \bar{\nu}_\tau \) | 0 | 0.22 |

Table 4: \( \nu/\gamma \) ratios before and after oscillations.

It is now straightforward to derive the total expected muon neutrino flux which is

\[
F_{\nu_\mu+\bar{\nu}_\mu}^{GC}(E_\nu) \simeq 3.3 \times 10^{-12} \left( \frac{E_\nu}{\text{TeV}} \right)^{-2.3} \text{TeV}^{-1}\text{cm}^{-2}\text{s}^{-1}. \tag{31}
\]

This flux is about the double of that estimated in [32] using only the 2004 HESS data [34]. Since the detection of secondary up-going muons produced by neutrino interaction in the Earth is the most promising strategy to identify TeV neutrino from astrophysical sources, Neutrino Telescopes in the North hemisphere are best suited to look for a signal from the GC as it is located in the southern Sagittarius constellation. Using the \( \nu_\mu + \bar{\nu}_\mu \) flux given in (31), and the ANTARES effective area for up-going \( \nu_\mu \)s given in [35], we estimated the expected detection rate by this experiment to be 0.07 yr\(^{-1}\) which is, unfortunately, quite small. A km\(^3\) Neutrino Telescope to be built in the Mediterranean sea (e.g. NEMO [36] or NESTOR [37] upgrading) has better chance to get a positive detection. Indeed, since the effective area of such an instrument should be at least 20 times better than that of ANTARES, the expected rate is 1.5 yr\(^{-1}\). As the expected background rate of such device is \( \sim 0.3\) ev/yr [32], the observation time required to detect that source at 95\% C.L is about three years.

6 Conclusions

In this paper we estimated the neutrino/photon flux ratio in the TeV energy range from embedded SNRs. We did that under the hypothesis that SNRs shock accelerate protons and composite nuclei with a power law energy spectrum at least up to the PeV. By assuming that all \( \gamma \)-rays of that energy are produced by \( \pi^0 \) decay, we first computed the relative emissivities of muon
and electron neutrinos produced by charged pion decay. Then, we took into account for vacuum oscillations to determine the $\nu_i/\gamma$ flux ratio to be expected on the Earth for all lepton families. Our results are similar, though not coincident, to those previously derived by other authors [11, 32]. We also considered the possible contribution that neutron decay may give to the TeV antineutrino flux from a GMC embedding an active SNR. Using the results which we derived in a previous work [25], we showed that in dense molecular complexes (e.g. in the GC) hadronic scattering should be the dominant neutrino production mechanism at the relevant energies ($\sim$ PeV). That channel was not considered in [13]. Unfortunately, we found that, although such a neutrino production channel is less severely constrained by $\gamma$-ray observations, it is quite unlikely that under reasonable assumptions it could add a significant contribution to the total neutrino flux from such kind of sources.

As an application of our results we used the recently released data of the GC observation campaign by the HESS collaboration [7] to estimate the muon neutrino flux from that region. We found that the km$^3$ Neutrino Telescope to be built in Mediterranean sea may have some chances to detect that source. Although the expected neutrino detection rate is rather small, the possible observation of a positional coincidence between the $\gamma$-ray and the neutrino source as well as the measurement of a flux ratio at the level estimated in this paper, would add significant evidence in favour of the hypothesis that hadron acceleration is taking place in that region. Furthermore, since embedded SNRs should be quite common objects in the Galaxy and VHE $\gamma$-ray and neutrino astronomy have just started, other sources of that kind may be discovered in both channels in a not too far future.

Acknowledgements

We thank V. Flaminio, T. Montaruli, T. Sjostrand and F. Vissani for several valuable discussions. The authors are members of the ANTARES collaborations. L.M. thanks G. Usai for help using the PYTHIA package.

References

[1] V.L. Ginzburg and S.I. Syrovatskii, Origin of the Cosmic Rays, Pergamon Press, London, 1964.

[2] T. Montmerle, Astrophys. J. 231 (1979) 95.

[3] F.A. Aharonian and A.M. Atoyan, Astron. Astrophys. 309 (1996) 917.
[4] S.D. Hunter et al., Astrophys. J. 481 (1997) 205.

[5] D. F. Torres, G. E. Romero, T. M. Dame, J. A. Combi and Y. M. Butt, Phys. Rept. 382 (2003) 303 arXiv:astro-ph/0209565.

[6] F. Aharonian et al. [The HESS Collaboration], Nature, 432 (2004) 75.

[7] F. A. Aharonian [HESS Collaboration], Nature 439 (2006) 695, arXiv:astro-ph/0603021.

[8] C. Heiles and R. Crutcher, arXiv:astro-ph/0501550.

[9] F. Yusef-Zadeh et al., Astrophys. J. 466 (1996) L25.

[10] T. Montaruli, Eur. Phys. J. A 24S1 (2005) 103.

[11] M. L. Costantini and F. Vissani, Astropart. Phys. 23 (2005) 477 arXiv:astro-ph/0411761.

[12] M. L. Costantini and F. Vissani, AIP Conf. Proc. 794 (2005) 219 arXiv:astro-ph/0508152.

[13] L. A. Anchordoqui, H. Goldberg, F. Halzen and T. J. Weiler, Phys. Lett. B 593 (2004) 42 arXiv:astro-ph/0311002.

[14] P. Blasi and S. Colafrancesco, Astropart. Phys. 122 (1999) 169 arXiv:astro-ph/9905122.

[15] T. Sjostrand, L. Lonnblad, S. Mrenna and P. Skands, arXiv:hep-ph/0308153.

[16] T.K. Gaisser, Cosmic Rays and Particle Physics, Cambridge University Press, 1990.

[17] S. Eidelman et al. [Particle Data Group], Phys. Lett. B 592 (2004) 1.

[18] B. Aharmim et al. [SNO Collaboration], Phys. Rev. C 72, 055502 (2005) arXiv:nucl-ex/0502021.

[19] M. Apollonio et al., Eur. Phys. J. C 27, 331 (2003) arXiv:hep-ex/0301017.

[20] R. M. Crocker, F. Melia and R. R. Volkas, Astrophys. J. Suppl. 141 (2002) 147 arXiv:astro-ph/0106090.

[21] D. J. Bird et al. [HIRES Collaboration], arXiv:astro-ph/9806096.
[22] N. Hayashida et al. [AGASA Collaboration], Astropart. Phys. **10** (1999) 303 [arXiv:astro-ph/9807045].

[23] J. A. Bellido, R. W. Clay, B. R. Dawson and M. Johnston-Hollitt, Astropart. Phys. **15** (2001) 167 [arXiv:astro-ph/0009039].

[24] A. Letessier-Selvon [Pierre Auger Collaboration], 29th International Cosmic Ray Conference, Pune, 2005, [arXiv:astro-ph/0507331].

[25] D. Grasso and L. Maccione, Astropart. Phys. **24**, (2005) 273 [arXiv:astro-ph/0504323].

[26] J. L. Puget, F. W. Stecker and J. H. Bredekamp, Astrophys. J. **205** (1976) 638.

[27] F. W. Stecker and M. H. Salamon, Astrophys. J. **512** (1992) 521 [arXiv:astro-ph/9808110].

[28] R. I. Diaz-Miller, J. Franco and S. N. Shore, Astrophys. J. **436** (1994) 795.

[29] M. A. Malkov and L. O'C. Drury, Rep. Prog. Phys. **64**, (2001) 429.

[30] F. Aharonian et al. [The HESS Collaboration], Astron. Astrophys. **425** (2004) L13 [arXiv:astro-ph/0408145].

[31] J. Albert et al. [MAGIC Collaboration], Astrophys. J. **638** (2006) L101 [arXiv:astro-ph/0512469].

[32] R. M. Crocker, F. Melia and R. R. Volkas, Astrophys. J. **622** (2005) L37 [arXiv:astro-ph/0411471].

[33] R. M. Crocker, M. Fatuzzo, R. Jokipii, F. Melia and R. R. Volkas, Astrophys. J. **622** (2005) 892 [arXiv:astro-ph/0408183].

[34] F. Aharonian et al. [The HESS Collaboration], Astron. Astrophys. **425** (2004) L13 [arXiv:astro-ph/0408145].

[35] E. Aslanides et al. [ANTARES Collaboration], [arXiv:astro-ph/9907432]

[36] C. N. De Marzo [NEMO Collaboration], Nucl. Phys. Proc. Suppl. **B100** (2001) 344.

[37] S. E. Tzamarias [NESTOR Collaboration], Nucl. Instrum. Meth. A **502** (2003) 150.