Projection Classification Based Iterative Algorithm

Ruiqiu Zhang¹, Chen Li¹ and Wenhua Gao²
¹School of Design, South China University of Technology, Guangzhou 510640, Guangdong, China
²School of Mathematics, South China University of Technology, Guangzhou 510640, Guangdong, China

E-mail: whgao@scut.edu.cn

Abstract. Iterative algorithm has good performance as it does not need complete projection data in 3D image reconstruction area. It is possible to be applied in BGA based solder joints inspection but with low convergence speed which usually acts with x-ray Laminography that has a worse reconstruction image compared to the former one. This paper explores to apply one projection classification based method which tries to separate the object to three parts, i.e. solute, solution and air, and suppose that the reconstruction speed decrease from solution to two other parts on both side lineally. And then SART and CAV algorithms are improved under the proposed idea. Simulation experiment result with incomplete projection images indicates the fast convergence speed of the improved iterative algorithms and the effectiveness of the proposed method. Less the projection images, more the superiority is also founded.

BGA (Ball Grid Array) package as a high-density and high thermal performance package form has been widely used in CPU manufacturing, memory manufacturing and other manufacturing. Because of its contacts distributed in the packages below, it is very difficult to detect solder joint quality. Traditional artificial visual, automated optical inspection (AOI) and flying probe test technology are no longer applicable, although you can use X-rays penetrate the board to get the information of hidden solder joints distributed in the packages below, X-rays detection can only make a rough detection of solder joints if there is a bridge or weld defects ,but for little tin, viod ,weld and other smaller defects which impact the quality of the product ,is helpless because the projected image contains many overlapping information. Therefore, many scholars explore the use of hierarchical scanning method (Laminography) to reconstruct the joints [3][4], but the reconstructed image quality and is poor compared with CT (Computed Tomography) reconstructed image.

Although CT technology has been widely used in various industrial NDT field, its electronic detection for BGA chip solder joints was ineffective. The reason of high attenuation characteristics of the ray of flat shaped circuit board and solder joints makes the traditional algorithm of cone beam CT (such as FDK [1]) not play it more accurate and fast reconstruction of objects advantages, because Ray beam hardening [13] specific direction leads to the inaccurate projection data which can not reflect the actual material attenuation coefficient distribution. The application of CT technology in BGA solder joint inspection need to be considered: on the one hand, due to testing requirements of high efficiency in solder circuit board production line, a complete projection data algorithm must be abandoned, because it would waste a lot of time to acquire projection images. On the other hand, as far as possible to avoid the inaccurate projection data of the circuit board in the horizontal direction. Therefore, short scan system and less angle reconstruction seems to be a feasible solution, the former is to reduce the
number of projection and consider short scan form analysis type of algorithm of complete projection data\[10\][11], the latter is more focused on the research for the incomplete projection data algorithm, such as iterative algorithm.

Iterative algorithm which is antithetical to analytic algorithm was introduced firstly into the field of image reconstruction by Gordon et al. The theory of reconstruction field has made great progress after a lot of scholars' efforts. Objects are usually discretized by iterative algorithm into a grid in accordance with the pixel or voxel (2D) (3D) size of objects. ART, EM, SIRT, SART, CAV are common iterative algorithm, the first is the basic iterative algorithm, the second is statistical iterative algorithm, the latter three are classified as synchronous iterative algorithm, they are the improved algorithm of ART algorithm in convergence speed and the quality of the reconstructed image. Usually the iterative algorithm is more suitable for incomplete reconstruction projection data, but these types of algorithm such as ART algorithm, SART algorithm, and CAV algorithm have large amount of calculation and the slow convergence speed, researchers have been exploring to accelerate the convergence speed of the iterative algorithm. For example, Mueller [5] studied the effect of access order to projection data on convergence speed, Herman [6] studied the effect of different relaxation factors on the convergence rate of the reconstruction, Wang studied that how OS technique can be applied to the SART algorithm to accelerate the convergence [12].

In conclusion, the analytical algorithm is not suitable for the detection of BGA solder joints on the circuit board. This paper is to improve the convergence speed of iterative algorithm and tells us that the objects of different materials have different backprojection speed, and then discusses the method of classification of projection data, the reconstructed objects are classified according to the solute, solution and air, the backprojection speed of iterative algorithm and material of objects are relevant. In addition, this paper describes the design of a virtual 3D cone beam circular orbit scan system and simulates the projection and reconstruction of BGA solder joint.

1. Iterative algorithm
The following discussion is focused on two representative iterative algorithm and the WP (Weighted Projection) type deformation based on projection data classification.

The SART algorithm proposed by Andersen in paper [7], the following shape type:

\[
f^{k+1}_j = f^k_j + \frac{\lambda_k}{M} \sum_{i=1}^{M} \left( \sum_{j=1}^{N} w_{ij} \left( p_i - w^i f^k \right) \right)
\]

In the formula, \( f^k_j \) represents the image after k iterations, \( \lambda_k \) represents relaxation factor, \( w_{ij} \) is the weight coefficient, which reflects the contribution degree j voxel to article i ray, \( p_i \) represents projection, \( M \) represents pixel number in the total projection, \( N \) is the object voxel number.

CAV algorithm is developed from the synchronous Cimmino algorithm by Censor[8], Censor believes that if the weight coefficient matrix is sparse, then the weight coefficient should not be \( \frac{1}{M} \) of Cimmino algorithm. It should be \( \frac{1}{S_j} \), \( S_j \) is the number of non-zero elements in the j column in the weight coefficient matrix. So the CAV algorithm is obtained and the data update formula is as follows:

\[
f^{k+1}_j = f^k_j + \lambda_k \sum_{i=1}^{M} \left( \frac{p_i - \langle w^i, f^k \rangle}{\sum_{j=1}^{N} S_j w_{ij}^2} w_{ij} \right)
\]
Wherein, \(\langle \cdot, \cdot \rangle\) represents point multiplication. The derivation of this formula can reference [7]

Iterative reconstruction is based on the difference between the projection data and the estimated projection to take backprojection to keep the process of reconstruction of objects constantly. It is generally believed that the weight coefficient matrix is determined by physical configuration of scanning system, effects of all voxels on pixels that arrive on the detector are same, of course, according to other a priori information, such as the density of a known object distribution can be used in the process of back-projection of the weights. Analysis of several common iterative algorithm which contains the following two:

A. Projection deviation, expressed as \(p_i - w^k f^k\)

B. The denominator of the average deviation.

Comparison of several algorithms shows that several iterative algorithms denominator are only concerned with the weight coefficient matrix, the paper [9] put the previous iteration solution into backprojection process to improve the efficiency of the Cimmino algorithm. It is only in a large number of iterations that reflects fast convergence. From the other point of view, this paper proposes that back-projection process should be linked with the projection data categories to accelerate convergence.

For quality detection of circuit board, testing personnel hope to analysis the quality of solder joints to determine whether there are defects weld, bridging and voids. Chip itself and the PCB substrate is not part of the testing personnel concerned, the utilization of fly needle of traditional test method is also based on this starting point, but such an electrical connection testing method for the detection of some defects such as voids, insufficient solder etc is helpless. If you take X ray scanning system, these parts will inevitably reflected in the projected image, a large parts of the amount of calculation will be used for the reconstruction that is not the testing personnel’s concerned substances while in 3D backprojection reconstruction process. Inspired by this, this paper believes that the reconstruction of objects can be divided into three parts: the solute part (solder), solution (such as part of the chip and the substrate) and air part (peripheral reconstruction region), so we should give them different reconstruction speed to accelerate the convergence speed in the backprojection process. Here the specific approach is to solution, threshold for the recovery speed of vertices, respectively to the solute and air threshold linearly decreasing, as shown in Figure 1, to get the improved WP-SART, one of WP type projection algorithm, the form below:

![Figure 1. Penalty function](image-url)
In the formula, \( \eta_i = \frac{p_i}{\max(p) - \min(p)} \), \( \phi_{\eta_i} \) is a threshold value, \( \mu \) is the correction factor, \( \mu \) is a value of 0.1 in this paper. In the formula (3), \( \frac{\phi_{\eta_i} - \eta_i}{\phi_{\eta_i}} + \mu \) can be calculated and stored in advance according to the known projection data. Therefore, this improved method doesn’t increase additional computation in the process of iteration.

Similarly, \( \frac{\phi_{\eta_i} - \eta_i}{\phi_{\eta_i}} + \mu \) can be applied to the CAV algorithm and then WP-CAV algorithm can be obtained, here no longer say.

2 The Simulation experiment

To compare the effect of reconstruction of traditional synchronous iterative algorithm with WP algorithm, this paper introduces the design of cone beam CT projection system of a virtual scanning circular orbit and virtual model of the BGA circuit board, setting up that its size is 24 x 64 x 84 (sections such as shown in figure [2]), voxel size is 1mm, the detector size is 128 x 80, pixels size is 1.5mm, source to object rotation center axis distance is 100mm, the distance is 100mm from the ray source to the center of rotation axis of the object, 1 degree projection index.

![Central slice perpendicular to the y-axis](image)

Figure 2. Slice of simulation BGA model

The two kinds of commonly method of reconstructed image evaluation is given to evaluate the convergence speed of the evaluation algorithm in this paper, one is to select key rows or columns of reconstructed image to compare with the original Sliced model, the other is reconstruction of objects and \( MSE \) (Mean Squared Error) [4]of the original model as follows:

\[
MSE = \frac{1}{N} \sum_{j} (f_{j\rho} - f_j)^2
\]

In the formula, \( f_{j\rho} \) is the reconstruction of objects, \( f_j \) is the original model.

This experiment used SART, CAV, WP-SART, WP-CAV algorithm, set up iterative times as 50, coefficient of relaxation 0.3, penalty term correction factor 0.1, the reconstruction results of various data are as follows.

2.1 Reconstruction of 30 degrees to 150 degrees projection angles
The simulation results in Figure 4 obtained by the MSE of various algorithms, the convergence speed of WP algorithm is superior to the same type (WP-SART is better than SART, WP-CAV is better than CAV), CAV algorithm outperforms the SART algorithm. Through nearly 50 iterations, SART and CAV algorithm are not convergent to a small MSE, but WP algorithm from fewer iterations have converged to a steady state. From the analysis of the central slice in Figure 3, we can see that WP algorithm has better detail resilience and the reconstruction effect of the center region of the image is better.

(a) SART

(b) WP-SART

(c) CAV
2.2 Reconstruction of 45 degrees to 135 degrees projection angles

Compared with the projection of 30 degrees to 150 degrees, due to reduced amount of data, the convergence speed of 45 degree to 135 degree projection is slow, but WP algorithm is still better than original algorithm, the convergence speed of WP-CAV algorithm is fastest shown in Figure 5.
2.3 Reconstruction of 60 degrees to 120 degrees projection angles
Compared with the previous two kinds of amount of data, the amount of projection data based on 60 degree to 120 degree is loss seriously, various algorithms are non-adaptive, but WP algorithm is still better than the original algorithm. Figure 6 has shown that WP-SART algorithm convergence speed and CAV algorithm convergence speed are similar, but the WP-CAV algorithm still has a faster convergence speed.

![Figure 6. MSE under 60 to 120 degree projection](image)

3 Conclusion
The iterative algorithm is well suited for this class of flat plate objects like circuit board, it does not need the ray on projection circuit board in the horizontal direction to converge. Thus it avoids the use of the inaccurate projection data. In addition, compared with the traditional analytic algorithm, iterative algorithm can avoid object rotation platform which can be replaced by the rotary imaging sector to meet the needs of production lines of the circuit board.

Because iterative algorithm convergence speed is slow, it has not been applied to BGA solder joint inspection which needs higher demands on the detection time. This paper proposes to deal with different parts of objects (solute, solution, air) to take a different back-projection algorithm to speed up the convergence speed, which is based on the classification of the detected object projection. Based on two typical synchronous iterative algorithm, simulation results show that the SART and CAV algorithm, the WP algorithm by using the algorithm of the same type improved by the method proposed in this paper, it’s convergence speed is faster and can have better detail recovery capability. The less projection data used, the more obvious effect improved, thus the effectiveness of this method is proved.

The MSE chart shows that MSE of WP algorithm were significantly smaller if the iterations is same, the next step will research convergence of the algorithm qualitatively. In addition, the WP method will consider the object shape information to reduce the reconstruction region of objects to further accelerate the convergence speed of the algorithm.

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