Recoiling D-branes

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Abstract

We propose a new method to describe a recoiling D-brane that is elastically scattered by closed strings in the non-relativistic region. We utilize the low-energy effective field theory on the worldvolume of the D-brane, and the velocity of the D-brane is described by the time derivative of the expectation values of the massless scalar fields on the worldvolume. The effects of the closed strings are represented by a source term for the massless fields in this method. The momentum conservation condition between the closed strings and the D-brane is derived up to the relative sign of the momentum of the D-brane.

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1 Introduction

Studies of interactions between D-branes and closed strings are quite important from various points of view. The interactions play a crucial role in finding non-trivial relationships between open strings and closed strings such as AdS/CFT [1] and recently-proposed open-closed dualities [2, 3, 4], for example. Studies of the interactions are also important to analyze the dynamics of the systems of multiple D-branes.

However, almost all analyses of the interactions have been carried out by considering static D-branes, and it is a long-standing problem to describe the back reaction of the D-branes. A D-brane in the worldsheet description is just a boundary of the worldsheet with Dirichlet boundary condition, and the D-brane is treated as an infinitely heavy classical source of closed strings; the position, or the collective coordinate, of the D-brane is fixed at a point in the target space in the Dirichlet direction.

There are several attempts to describe the back reaction of D-branes. Some of them are based on the conformal field theory on the worldsheet [5, 6, 7, 8]. In Ref. [5], the disk amplitude for the scattering of closed string states from a D-particle is computed in the bosonic string theory in which the collective coordinate of the D-particle is quantized. The current conservation condition of the D-particle is obtained by demanding the conformal invariance of the amplitude in that work. Momentum conservation condition between the closed strings and the D-particle is described by using the zero-mode integral in the path integral of the trajectory of the D-particle.

Attempts to obtain the momentum conservation condition between closed strings and a D-brane from the viewpoint of conformal invariance can be found in Refs. [6, 7, 8]. An annulus amplitude for the scattering of the closed strings from the D-brane is considered and a variant of the Fischler-Susskind mechanism is proposed there; the infrared (IR) divergence in the open string channel of the annulus amplitude is canceled by adding an appropriate operator to the boundary of the worldsheet. The momentum conservation condition between the closed strings and the D-particle is explicitly obtained in Refs. [7, 8] by demanding the conformal invariance of the total amplitude. In Ref. [8], the IR divergence is canceled by adding a logarithmic operator [9] that represents the recoil of the D-particle [10, 11]. However, it is also pointed out in Ref. [8] that the divergence does not exist in the case of Dp-brane with $p > 1$. In Ref. [7], the energy conservation condition is also obtained as well as the momentum conservation condition in the case of D-particle, explicitly. However, there is still room to clarify how to define the initial momentum of the D-particle there. Further investigation along the ideas of Refs. [6, 7, 8] is still important for deeper understanding of

\footnote{For other related studies on D-brane recoil with the logarithmic operators, see Refs. [12, 13, 14, 15, 16, 17, 18], for example.}
recoil of D-branes. Some applications of D-brane recoil to other topics, and related works are found in Refs. [19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30].

A target-space theory that handles second-quantized closed strings with dynamical D-branes may provide us a description of back reaction of D-branes. Some arguments on back reaction and recoil of D-branes along this approach is found in Ref. [31].

In the present work, we propose an alternative method to describe the scattering process between a D-brane and closed strings together with the back reaction of the D-brane in the bosonic string theory. The impact of the closed strings in this method is represented by a source term of the low-energy effective worldvolume theory of the D-brane, and the initial and the final velocity of the D-brane are described by the time derivative of the expectation values of the massless scalar fields of the worldvolume theory. We utilize the following approximations to justify our approach:

1. Field theory limit, namely \( k^2 \alpha' \ll 1 \) where \( k \) is the typical momentum of the open strings on the D-brane.

2. Elastic limit, namely the momenta of the closed strings are small enough and no massive open-string mode is excited on the D-brane. We also assume that the closed strings do not lose their total momentum in the worldvolume directions of the D-brane, and no internal field on the D-brane gets momentum from the closed strings.

3. Non-relativistic limit, namely the velocity of the D-brane is very small.

4. Tree level approximation in the string theory, namely the string coupling is very small.

The above conditions 3 and 4 means that the tension of the D-brane is very large. This is also consistent with the condition 1.

A nontrivial problem is how to represent the source term of the worldvolume theory in terms of the quantities of the closed strings. The basic idea is as follows. We consider a scattering process between the D-brane and the closed strings that creates \( n \) massless scalar particles on the worldvolume of the D-brane. We calculate the probability of the creation of the \( n \) massless scalar particles in the two different frameworks: one of them is the worldvolume theory of the D-brane with the source term and the other is the perturbative string theory. By comparing the two results, we obtain the relationship between the source term and the momenta of the closed strings, and we obtain the momentum conservation condition between the closed strings and the D-brane up to the relative sign of the momentum of the D-brane.

One of the distinction between the present work and those in Refs. [6, 7, 8] is that all the diagrams we consider in the string theory are disk diagrams and we need not annulus diagrams.
The organization of this article is as follows. We consider bosonic strings, and we start by considering a recoiling D-particle for simplicity. In section 2, we consider the worldvolume theory of the D-particle and review some basic facts necessary for the later discussions. We calculate the amplitude of the creation of the \( n \) massless particles explicitly. In section 3, we consider the scattering process between the D-particle and the closed strings and calculate the amplitude of the process that creates \( n \) massless open string modes on the D-particle in the framework of the string theory. In section 4, we compare the results obtained in section 2 with those in section 3. We show that the absolute value of the total momentum transfer from the closed strings is exactly equal to the absolute value of the change of the D-particle’s momentum, within the above approximations. We also comment on the distribution of the probability of the \( n \) massless particle creation. We see that the expectation value of the total energy of the created particles gives the kinetic energy of the D-particle, correctly. In section 5, we generalize the results obtained for the D-particle to the case of higher dimensional D-branes. We compactify the worldvolume of the D-brane to make its mass finite, and we obtain the momentum conservation condition between the closed strings and the D-brane up to the relative sign of the momentum of the D-brane, again. We provide conclusion, several open problems and discussions in the last section.

2 Effective field theory on D-particle

2.1 Effective action

We begin with the Dirac-Born-Infelt (DBI) action of a D-particle. It is just an ordinary action for a point particle:

\[
S_{\text{DBI}} = -\tau \int dt \sqrt{1 - \dot{X}^2},
\]

(2.1)

where \( \tau \) is the tension of the D-particle, and \( \dot{X}^\mu \) denotes the time derivative of the space coordinate \( X^\mu \) of the D-particle. We consider a non-relativistic D-particle in this work. Then \( \dot{X}^2 \ll 1 \) and the DBI action is written as

\[
S_{\text{DBI}} = \tau \int dt \left\{ \frac{1}{2} (\partial_\tau X)^2 \right\},
\]

(2.2)

where we have dropped the constant term and the \( O(\dot{X}^4) \) contributions.

Next, we consider a scattering process between the D-particle and closed strings. We attempt to include the effects of the closed strings in the above action. A simple conjecture is that a source term may effectively represent the impact of the closed strings. Therefore, we
consider the following effective action

\[ S = \tau \int dt \left\{ \frac{1}{2} (\partial_t X)^2 - \frac{1}{2} m^2 X^2 + J(t) \cdot X(t) \right\}, \quad (2.3) \]

and let us examine how this action represents the nature of the recoiling D-particle. Here, \( m \) is an IR cut-off that should be sent to zero finally, and we assume that \( J(t) \) includes relevant effects of the closed strings.

2.2 Expectation value of \( \dot{X}^\mu \)

Let us calculate the time dependence of the expectation value of \( X^\mu \) at the classical level. We choose the target space coordinate so that the initial momentum of the D-particle is equal to zero and its final momentum is in the \( x^{25} \) direction. We assume that the source term is switched on only for the duration \( t_i \leq t \leq t_f \). The expectation value of the field \( X^{25} \) is given by

\[ X^{25}(t) = X_0^{25}(t) + \int_{-\infty}^{\infty} dt' G_{\text{ret}}(t-t') J^{25}(t'), \quad (2.4) \]

where \( G_{\text{ret}}(t-t') \) is the retarded Green’s function which satisfies

\[ (\partial_t^2 + m^2) G_{\text{ret}}(t-t') = \theta(t-t') \delta(t-t'), \quad (2.5) \]

and \( X_0^{25}(t) \) satisfies the equation of motion without the source term,

\[ (\partial_t^2 + m^2) X_0^{25}(t) = 0. \quad (2.6) \]

\( \dot{X}_0^{25} \) is the initial velocity of the D-particle and \( \dot{X}_0^{25} = 0 \), in fact. Substituting the explicit form of the retarded Green’s function,

\[ X^{25}(t) = X_0^{25}(t) + \int_{-\infty}^{\infty} dq \int_{-\infty}^{\infty} dt' \frac{e^{iq(t-t')}}{(q - i\epsilon + m)(q - i\epsilon - m)} J^{25}(t'), \quad (2.7) \]

where we should take the limit \( \epsilon \to +0 \). Then,

\[ \dot{X}^{25} = \dot{X}_0^{25} + \int_{-\infty}^{\infty} dq \frac{q e^{iqt}}{2\pi i (q - i\epsilon + m)(q - i\epsilon - m)} \tilde{J}^{25}(q), \quad (2.8) \]

where \( \tilde{J}^{25}(q) \) is the source term in the momentum space.\(^2\) Note that \( \dot{X}^{25} = \dot{X}_0^{25} = 0 \) for \( t \leq t_i \).

\(^2\)Our definition of the Fourier transformation is

\[ F(t) = \int_{-\infty}^{\infty} \frac{dq}{2\pi} e^{iqt} \hat{F}(q), \quad (2.9) \]

\[ \hat{F}(q) = \int_{-\infty}^{\infty} dt e^{-iqt} F(t). \quad (2.10) \]
For $t \geq t_f$, 
\[ \dot{X}^{25} \rightarrow \frac{1}{2} e^{imt} \tilde{J}^{25}(m) + \frac{1}{2} e^{-imt} \tilde{J}^{25}(-m) \quad (\epsilon \rightarrow +0) \]
\[ \rightarrow \tilde{J}^{25}(0) \quad (m \rightarrow 0). \]  
(2.11)

The limit $\epsilon \rightarrow +0$ should be taken before we take the limit $m \rightarrow 0$. Note that $\tilde{J}^{25}(0)$ is real. $t_f - t_i$ should be regarded as the duration of that the closed strings affect the D-particle in the scattering process. We assume that the effects of the closed strings disappear at the infinitely far future and we define the final velocity of the D-particle $\dot{X}_f^{25}$ in this region ($t \geq t_f$). In the same way, we assume that the effects of the closed strings disappears at the infinitely past and the initial velocity of the D-particle $\dot{X}_i^{25}$ should be defined in the region of $t \leq t_i$.

Then, the change of the velocity of the D-particle is given by $\dot{X}_f^{25} = \tilde{J}^{25}(0)$, and the problem we should consider is how to rewrite $\tilde{J}^{25}(0)$ in terms of the quantities of the closed strings. We will come back to this problem in section 4.

### 2.3 $n$ particle creation amplitude

We calculate the $n$ particle creation amplitude with the source term in order to compare it with the corresponding amplitude in the string theory. We treat the source term as a perturbation.

The action without the source term is
\[ S_0 = \tau \int dt \left\{ \frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} m^2 \phi^2 \right\}. \]  
(2.12)

We consider only $X^{25}$ and we define a dimension-less field $\phi(t)$ as $X^{25} = \sqrt{2\pi\alpha'}\phi$. Then the Lagrangian and the canonical momentum are
\[ L_0 = 2\pi\alpha'\tau \left\{ \frac{1}{2} (\partial_t \phi)^2 - \frac{1}{2} m^2 \phi^2 \right\}, \]  
(2.13)
\[ \mathcal{P} = 2\pi\alpha'\tau \phi. \]  
(2.14)

We define the creation operator $a$ and the annihilation operator $a^\dagger$ as
\[ \phi = \frac{1}{\sqrt{2\pi}} (a + a^\dagger), \]  
(2.15)
\[ \mathcal{P} = -i\sqrt{\frac{h}{2}} (a - a^\dagger), \]  
(2.16)

where $h \equiv 2\pi\alpha'\tau m$. The Hamiltonian is given by
\[ H_0 = m \left( a^\dagger a + \frac{1}{2} \right). \]  
(2.17)
Including the source term, the Hamiltonian is

\[ H = H_0 + V, \]  
\[ V = -j(t)(a + a^\dagger), \]  

where we have defined \( j(t) \) as

\[ j(t) \equiv \sqrt{\frac{\tau}{2m}} j^{25}(t). \]  

We define the \( n \) particle state as \( |n\rangle = \frac{1}{\sqrt{n!}} (a^\dagger)^n |0\rangle \).

By using the above notations, the \( n \) particle creation amplitude \( A^{(n)}_{\text{field}} \) is given by

\[ A^{(n)}_{\text{field}} = \lim_{t_0 \to -\infty, t \to \infty} \langle n | e^{-iH(t-t_0)} | 0 \rangle = \frac{1}{\sqrt{n!}} (i)^n j^{(0)} n e^{-\frac{1}{2} |j^{(0)}|^2} e^{i\theta}, \]  

at the limit \( m \to 0 \). \( \theta \) is a real number defined in (A.9). Note that \( |A^{(n)}_{\text{field}}|^2 = \frac{1}{n!} \lambda^n e^{-\lambda} \) gives a Poisson distribution where \( \lambda = |j^{(0)}|^2 \), and the final state is a coherent state. The detailed calculation to obtain (2.21) is reviewed in appendix A.

3 Calculations in string theory

In this section, we calculate the amplitude of the creation of the \( n \) massless scalar open strings on the D-particle from \( N \) closed strings, at the tree level. We choose the spacetime coordinate so that the D-particle recoils in the \( x^{25} \) direction as we did in section 2, and the massless open strings we consider here correspond to the zero mode of the D-particle in the \( x^{25} \) direction.\(^3\)

We also choose the spacetime coordinate so that the initial velocity of the D-particle is zero. The amplitude consists of several disk diagrams as shown in Fig. 1 for example. We calculate a single disk amplitude to begin with, and we consider the full diagrams by using the results on the single disk amplitude.

3.1 Single disk amplitude

We calculate a disk amplitude \( A^{(N,n)} \) of \( N \) closed strings and \( n \) massless scalar open strings on the D-particle. We consider a unit disk \( \mathcal{M} \) with Neumann boundary condition \( \partial_r X^0 |_{r=1} = 0 \) and Dirichlet boundary conditions \( X^\nu |_{r=1} = 0 \) for \( \nu \neq 0 \), where \( z = r e^{i\phi} \) is the complex

\(^3\)The massless open strings discussed in the following should always be understood as the zero modes of the D-brane in the \( x^{25} \) direction.
coordinate on the unit disk. We explicitly calculate for the case that all the closed strings are closed-string tachyons, for simplicity, however the results should be easily generalized to the cases of arbitrary closed strings. The open strings cannot carry momenta in the Dirichlet directions and thus their momenta in the space directions are zero. The energy of the massless open string is also zero due to the on-shell condition. Therefore, the massless open string labeled by $l$ is represented by the vertex operator $i\zeta^{(l)}_2 \cdot \partial_r X(e^{i\phi_l})$ where $\zeta^{(l)}_2$ is the polarization vector which satisfies $\zeta^{(l)}_2 = 1$ and $\zeta^{(l)}_\nu = 0$ for $\nu \neq 25$.

### 3.1.1 $N \geq 2$ case

The amplitude is given by

$$A^{(N,n)} = ig_c^N g_o^N \int_0^1 dr_2 \left( \prod_{i=3}^N \int d^2 z_i \right) \left( \prod_{l=N+1}^{N+n} \int_0^{2\pi} d\phi_l \right)$$

$$\left\langle c(0)\bar{c}(0)e^{i\phi_1}X(0)e^{i\phi_2}X(2)\prod_{i=3}^N e^{i\phi_i}X(z_i) \prod_{l=N+1}^{N+n} i\zeta^{25}_l \partial_r X^{25}(e^{i\phi_l}) \right\rangle_{\mathcal{M}}$$

$$= ig_c^N g_o^N C \delta(\sum_i k^0_{(i)})$$

$$\times \int_0^1 dr_2 \left( \prod_{i=3}^N \int d^2 z_i \right) Z_{gh}(r_2) \prod_{i<j\leq N} |z_i - z_j|^{\alpha \kappa^{(i)}(\kappa^{(j)})} \prod_{i,j\leq N} |1 - z_i \bar{z}_j|^{\alpha(i,j)}$$

$$\times \left( \prod_{l=N+1}^{N+n} \int_0^{2\pi} d\phi_l \right) \prod_{l=N+1}^{N+n} \prod_{i=1}^N \exp\{-k^{25}_{(i)} \zeta^{25}_l X^{25}(z_i) \partial_r X^{25}(e^{i\phi_l})\}$$

$$\times \prod_{l,m=N+1, l<m} \exp\{-\zeta^{25}_{(m)} \zeta^{25}_l \partial_r X^{25}(e^{i\phi_m}) \partial_r X^{25}(e^{i\phi_l})\},$$

(3.1)

where only the terms that are linear with respect to every $\zeta^{25}_l$ ($l = N+1, \cdots, N+n$) should be taken and the other terms should be discarded in [3.1]. $C$ is a constant, $g_o$ is the coupling constant for the massless open strings and $g_c$ is the closed-string coupling constant. $c$, $\bar{c}$ and $c^\phi$ is the ghost fields of holomorphic, anti-holomorphic and the $\phi$-component respectively. We have fixed the positions of the closed string vertices as $z_1 = 0$ and $z_2 = r_2$, where $z_j = r_je^{i\phi_j}$ is
the position of the \( j \)-th vertex operator.\footnote{\( \phi_2 = 0 \) is fixed and \( r_2 \) can still move in the region \( 0 \leq r_2 \leq 1 \).} The worldsheet coordinates are assigned as follows: \( z_j \)'s with \( 1 \leq j \leq N \) are for the closed strings and \( z_j \)'s with \( N + 1 \leq j \leq N + n \) are for the open strings. We have also defined \( a(i,j) \) as

\[
a(i,j) \equiv \frac{\alpha'}{2} \left( k^0_{(i)} k^0_{(j)} - \sum_{\nu \neq 0} k^\nu_{(i)} k^\nu_{(j)} \right),
\]

and \( Z_{gh}(r_2) \) as

\[
Z_{gh}(r_2) \equiv \langle c(0) c(0) e^{\phi}(r_2) \rangle_M. \tag{3.3}
\]

We can easily find that \( Z_{gh}(r_2) \) is a constant for the present case. Note that

\[
\int d\phi_m \int d\phi \zeta^{25}_i \zeta^{25}_j \langle X^{25}(z_i) \partial_r X^{25}(e^{i\phi}) \rangle = 0 \tag{3.4}
\]
due to (B.7), and the last line of (3.1) becomes 1. We can calculate the contribution of the third line of (3.1) by using

\[
\int_0^{2\pi} d\phi_l \sum_{i=1}^N k^{25}_{(i)} \zeta^{25}_{(i)} \langle X^{25}(z_i) \partial_r X^{25}(e^{i\phi_l}) \rangle = -2\pi \alpha' \sum_{i=1}^N k^{25}_{(i)}, \tag{3.5}
\]

where \( w = e^{i\phi_l} \), \( |z_i| \leq 1 \) and we have substituted \( \zeta^{25}_{(i)} = 1 \). See appendix B for the details. Therefore the amplitude is given by

\[
A^{(N,n)} = i g_c^N g_o^n C \delta(\sum_i k^0_{(i)}) Z_{gh} \times \prod_{i=3}^N \int d^2z_i \prod_{i<j\leq N} |z_i - z_j|^{\alpha'_k k_{(i)}^\mu} \prod_{i,j\leq N} |1 - z_i z_j|^{|a(i,j)|} \bigg|_{z_1=0, z_2=r_2} \times \prod_{l=N+1}^{N+n} \left( 2\pi \alpha' \sum_{i=1}^N k^{25}_{(i)} \right) \tag{3.6}
\]

\[
= \left( \frac{1}{\sqrt{\tau}} \sum_{i=1}^N k^{25}_{(i)} \right)^n \times A^{(N,0)},
\]
where we have used the relationship $\tau (2\pi \alpha' g_o)^2 = 1$ in the last line. $A^{(N,0)}$ is the disk amplitude of $N$ closed-string tachyons without open strings. Note that the momentum conservation condition among the strings appears due to the zero-mode integral. We do not have the momentum conservation condition among the strings in the Dirichlet directions due to lack of the zero-mode integral and thus $\sum_{i=1}^{N} k_{(i)}^{25} \neq 0$ is allowed.

### 3.1.2 $N = 1$ case

In this case, we should fix both the position of the closed-string vertex and that of one of the open-string vertices.\(^5\) We will fix the position of the open-string vertex labeled by $l = 2$ as $z_2 = e^{i\phi_2}$, where $\phi_2$ is a fixed value. The position of the closed-string vertex will be fixed at the center of the unit disk, again.

The amplitude is given by

$$A^{(1,n)} = ig_c g_o^6 C \delta(k_{(1)}^0) Z_{gh} \frac{1}{2\pi} \prod_{l=2}^{1+n} \left(2\pi \alpha' k_{(l)}^{25}\right)$$

$$= \left( \frac{1}{\sqrt{\tau}} k_{(1)}^{25}\right)^n \times A^{(1,0)},$$

(3.7)

where we have identified $A^{(1,0)}$ as

$$A^{(1,0)} = ig_c C \delta(k_{(1)}^0) Z_{gh} \frac{1}{2\pi}.$$  

(3.8)

The factor $1/(2\pi)$ in the first line of (3.7) appears because of the absence of the $\phi_2$ integral. The identification (3.8) can be justified as follows. In the calculation of $A^{(1,0)}$, we fix the position of the closed-string vertex at the center of the disk. Although the degrees of freedom of the modular transformation on the disk have not fixed completely, the residual symmetry is just a rotational symmetry the volume of which is $2\pi$. Then let us fix the rotational degree of freedom by hand. More precisely, let us mark at the point $z = e^{i\phi}$, for example, and fix the value of the $\phi$-coordinate of the marked point by hand. This corresponds to divide the amplitude by $2\pi$. Now we have fixed the position of the closed string vertex and the $\phi$-coordinate of the marked point on the disk. Thus the Faddeev-Popov determinant is the same as the case we have more than one vertex; it is $Z_{gh}$. As a result, we obtain (3.8).

### 3.1.3 General results

After all, we obtain

$$A^{(N,n)} = \left( \frac{1}{\sqrt{\tau}} \sum_{i=1}^{N} k_{(i)}^{25}\right)^n \times A^{(N,0)},$$

(3.9)

\(^5\)Alternative method of the calculation is to leave the positions of the open-string vertices unfixed and divide the amplitude by the volume of the rotational symmetry as mentioned below (3.8).
for all positive integers $N$. All the closed-string vertices we have used in the calculations are closed-string tachyon vertices, however we can easily find that (3.9) holds for arbitrary closed strings if we regard $A^{(N,n)}$ as the disk amplitude of the corresponding $N$ closed strings and the $n$ massless open strings.

### 3.2 Total amplitude

Now we can calculate the total amplitude of the creation of the $n$ massless scalar open strings by using the results obtained in the previous subsection. We define the total amplitude as $A^{(N,n)}$. For simplicity, we demonstrate the calculation of the amplitude in the case of $N = 2$ and $n = 1$ that is given schematically in Fig. 1. We can easily calculate the total amplitude by using (3.9) as

$$A^{(2,1)} = A^{(2,1)} + A^{(1,1)} A^{(1,0)} A^{(1,1)} + A^{(1,0)} A^{(1,1)}$$

$$= \sum_{i=1}^{2} \frac{k_{25}^{(i)}}{\sqrt{T}} A^{(2,0)} + \frac{k_{25}^{(1)}}{\sqrt{T}} A^{(1,0)} A^{(1,0)} + A^{(1,0)} \left( \frac{k_{25}^{(2)}}{\sqrt{T}} A^{(1,0)} A^{(1,1)} + A^{(1,0)} \right)$$

$$= \sum_{i=1}^{2} \frac{k_{25}^{(i)}}{\sqrt{T}} A^{(2,0)},$$

(3.10)

where $A^{(2,0)}$ is the total amplitude for no open-string creation and $A^{(1,n)}$ is the disk amplitude with $n$ massless open-string vertices and $i$-th closed-string vertex. The above calculation can be easily extended into the case of $N > 2$, and we obtain

$$\frac{A^{(N,1)}}{A^{(N,0)}} = \sum_{i=1}^{N} \frac{k_{25}^{(i)}}{\sqrt{T}} ,$$

(3.11)

in general. Furthermore, we can show that

$$\frac{A^{(N,n)}}{A^{(N,0)}} = \left( \frac{\sum_{i=1}^{N} k_{25}^{(i)}}{\sqrt{T}} \right)^{n} .$$

(3.12)

The derivation of (3.11) and (3.12) is given in appendix C.

### 4 Momentum conservation

#### 4.1 The source term from the string amplitude and the momentum conservation

In order to rewrite $\tilde{J}_{25}^{25}(0)$ in terms of the quantities of the closed strings, we compare the amplitude obtained in section 2 with that in section 3. Before comparing them explicitly, we
should notice the following facts:

1. **Normalization of the states**
   If we have \(n\) same particles in the final state, we should divide the cross section by \(n!\). In other words, we should multiply \(1/\sqrt{n!}\) to the string amplitude in order to compare it with the amplitude in the worldvolume theory where the factor \(1/\sqrt{n!}\) is already included in the definition of the \(n\)-particle state.

2. **Dimension of the spacetime**
   In the worldvolume theory, the massless particle exists in the one-dimensional worldvolume and the calculation in section 2 is based on non-relativistic quantum mechanics. On the other hand, the strings considered in section 3 exist in 26-dimensional spacetime, and the center-of-mass coordinates of the massless open strings are confined in the one-dimensional subspace of it. Because of the above difference, we also need the following consideration to obtain the correct normalization.

   Let us consider the probability \(P(n)\) of that \(n\) open-string particles of mass \(m\) whose momenta lie in a small region \(d^{25}\vec{k}_{(1)} \cdots d^{25}\vec{k}_{(n)}\) are created from the closed strings.\(^6\) \(P(n)\) is given as

   \[
   P(n) = \frac{d^{25}\vec{k}_{(1)}}{(2\pi)^{25}} \frac{1}{2E_{k_{(1)}}} \cdots \frac{d^{25}\vec{k}_{(n)}}{(2\pi)^{25}} \frac{1}{2E_{k_{(n)}}} \left| \frac{1}{\sqrt{n!}} A^{(N,n)} \right|^2 \left(2\pi\right)^{25} \delta(\vec{k}_{(1)}) \cdots \left(2\pi\right)^{25} \delta(\vec{k}_{(n)})
   \]

   \[
   = \left( \prod_{i=1}^{n} \frac{1}{2E_{k_{(i)}}} \right) \left| \frac{1}{\sqrt{n!}} A^{(N,n)} \right|^2
   \]

   \[
   = \left( \frac{1}{2m} \right)^n \left| \frac{1}{\sqrt{n!}} A^{(N,n)} \right|^2. \quad (4.1)
   \]

   We have the delta functions in the first line because the open strings do not have momenta in the Dirichlet directions. Therefore, we should also multiply the factor \(\left( \frac{1}{2m} \right)^{n/2}\) to the amplitude in the string theory to compare it with the amplitude in the worldvolume theory. Note that the factor \(\left( \frac{1}{2m} \right)^n\) in (4.1) is necessary to make \(P(n)\) dimensionless.

3. **Difference of the Hilbert spaces**
   The Hilbert space of string theory in section 3 includes the closed-string sector and the massive open-string sector, too. On the other hand, the Hilbert space of the effective field theory on the D-particle considered in section 2 includes only massless open-string

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\(^6\)We have introduced the same IR cut-off as that of the worldvolume theory here; now the mass of the open-string particles is \(m\). On the other hand, we have calculated the string amplitudes for the case of \(m = 0\) in order to maintain the conformal invariance of the worldsheet. This discrepancy disappears because we take the limit \(m \to 0\) eventually.
states. Due to the above difference of the Hilbert spaces, the absolute normalization of the amplitudes cannot be compared directly with each other.

With the above things in mind, we propose the following identification between the amplitudes in the effective field theory and those in the string theory:

\[
\frac{A^{(1)}_{\text{field}}}{A^{(0)}_{\text{field}}} = \frac{P(1)}{P(0)} = \frac{1}{\sqrt{2m}} \frac{1}{\sqrt{1!}} A^{(N,1)} = \frac{1}{2m} \frac{1}{\sqrt{1!}} A^{(N,0)}.
\]

The physical meaning of the right-hand side is the following. The probability of the creation of the one open string through the scattering between the D-particle and the closed strings should be proportional to \(\frac{1}{2m} |A^{(N,1)}|^2\) and that of no creation of the massless open string should be proportional to \(|A^{(N,0)}|^2\). We can cancel the ambiguity of the normalization of the probability which arises due to the problem 3 above, by considering the ratio of the probability.

Let us substitute the above results into (4.2). From Eqs. (2.11), (2.20), (2.21) and (3.11), we obtain

\[
\frac{A^{(1)}_{\text{field}}}{A^{(0)}_{\text{field}}} = i \tilde{j}(0),
\]

\[
\frac{A^{(N,1)}}{A^{(N,0)}} = \frac{1}{\sqrt{\tau}} \sum_{i=1}^{N} k^{25}_{(i)},
\]

\[
\tau \dot{X}^{25} = \sqrt{2m \tau} \dot{j}(0).
\]

Then we obtain the following relationship from (4.2):

\[
|\tau \dot{X}^{25}| = \left| \sum_{i=1}^{N} k^{25}_{(i)} \right|.
\]

This is the momentum conservation condition between the D-particle and the closed strings up to the relative sign. The result (4.6) is independent of the IR cut-off and of the types of the closed strings.

### 4.2 Poisson distribution from string theory and the kinetic energy of the D-particle

We have considered only \(P(1)/P(0)\) in the previous subsection. Let us consider \(P(n)/P(0)\) for general \(n\) and see what happens. In the framework of the effective field theory, \(P(n)\) is
given from (2.21) as

\[ P(n) = \frac{1}{n!} \lambda^n e^{-\lambda}, \]  \hspace{1cm} (4.7)

\[ \lambda = |\tilde{j}(0)|^2, \]  \hspace{1cm} (4.8)

and \( P(n)/P(0) = \frac{1}{n!} \lambda^n. \) The above gives a Poisson distribution. In the framework of the string theory, \( P(n)/P(0) \) is given as

\[ \frac{P(n)}{P(0)} = \frac{1}{n!} \left( \frac{1}{\sqrt{2m}} \frac{\sum_i k_{(i)}^{25}}{\sqrt{\tau}} \right)^{2n}, \]  \hspace{1cm} (4.9)

from (3.12) and the discussions in section 4.1. Thus the identification

\[ |\tilde{j}(0)| = \frac{1}{\sqrt{2m}} \frac{\sum_i k_{(i)}^{25}}{\sqrt{\tau}}, \]  \hspace{1cm} (4.10)

that is made in (4.2) is still valid for \( n \geq 2. \) In other words, (4.9) indicates that we can also derive the Poisson distribution for the massless particles from the string theory. The above observation strongly suggests that the effective action (2.3) with the source term captures the phenomena of the scattering between the D-particle and the closed strings well.

It is interesting to see that the total energy of the massless scalar particles on the world-volume of the D-particle gives the correct kinetic energy of the D-particle. The expectation value of the total energy of the scalar particles after the scattering is given by using (4.9) as

\[ m \sum_{n=0}^{\infty} n P(n) \frac{\sum_{n=0}^{\infty} P(n)}{\sum_{n=0}^{\infty} P(n)} = \frac{(\sum_i k_{(i)}^{25})^2}{2\tau}, \]  \hspace{1cm} (4.11)

that is exactly the expected value of the kinetic energy of the D-particle after the scattering. This quantity is also independent of the IR cut-off.

5 Generalization to the higher dimensional D-branes

We generalize the results in the previous sections to the cases of higher dimensional D-branes. We consider a Dp-brane which extends in the \( x^0, \cdots, x^p \) directions. We choose the spacetime coordinate so that the initial momentum of the D-brane is zero and the final momentum of it is in the \( x^{25} \) direction. We consider an elastic scattering between the Dp-brane and closed strings; we assume that the momenta of the closed strings parallel to the Dp-brane are conserved within the closed strings, and the total momentum coming from the closed strings is exactly perpendicular to the Dp-brane. In this case, the Dp-brane does not get any internal energy from the closed strings.
5.1 Results from the worldvolume theory

In this section, we consider the following effective scalar field theory on the Dp-brane:

\[ S = \tau_p \int d^{p+1}x \left\{ \frac{1}{2} \partial_\rho X^\rho \partial^\rho X^\rho - \frac{1}{2} m^2 X^2 + J(x) \cdot X(x) \right\}, \]  

(5.1)

where \( m \) is an IR cut-off and \( \tau_p \) is the tension of the Dp-brane. \( \rho \) runs from \( p + 1 \) to 25. We have chosen the worldvolume coordinate \( x^a \) so that it is equal to the zero mode of \( X^a \), where \( a \) runs from 0 to \( p \). We have \( U(1) \) gauge field on the worldvolume and the gauge field may contribute to the recoiling process of the D-brane, too. However, we assume that all the contributions of the gauge field are implicitly included in the source term.

We also assume that the source term is switched on only for the duration \( t_i \leq x^0 \leq t_f \). We compactify the \( x^1, \ldots, x^p \) directions on a torus of radius \( R \) to make the mass of the Dp-brane finite. Let us consider momentum conservation in the \( x^{25} \) direction and we consider only the field \( X^{25} \). Standard calculations in the field theory lead us to the following results:

\[ P(n)_{D_p} = \frac{1}{n!} \lambda_p^n e^{-\lambda_p}, \]  

(5.2)

\[ \lambda_p \equiv \frac{\tau_p}{V_p} \sum_{\vec{n}} \frac{1}{2 E_{\vec{q}}} |\tilde{J}^{25}(E_{\vec{q}}, \vec{q})|^2, \]  

(5.3)

\[ \dot{X}^{25}_f = \frac{1}{2 V_p} \sum_{\vec{n}} \left \{ \tilde{J}^{25}(E_{\vec{q}}, \vec{q}) e^{i E_{\vec{q}} x^0} e^{-i \vec{q} \cdot \vec{x}} + \tilde{J}^{25}(-E_{\vec{q}}, -\vec{q}) e^{-i E_{\vec{q}} x^0} e^{i \vec{q} \cdot \vec{x}} \right \}, \]  

(5.4)

where \( P(n)_{D_p} \) is the probability of that the source creates \( n \) massless scalar particles that correspond to the zero mode of the Dp-brane in the \( x^{25} \) direction, and \( \dot{X}^{25}_f \) is the time derivative of the expectation value of \( X^{25} \) at \( x^0 \geq t_f \). We regard \( \dot{X}^{25}_f \) as the final velocity of the D-brane in the \( x^{25} \) direction as we did in the case of D-particle. The integrals over the momenta in the uncompactified theory have been replaced with the summation over the Kaluza-Klein momenta \( \vec{q} = (\frac{n_1}{R}, \ldots, \frac{n_p}{R}) \) which are labeled by \( \vec{n} \equiv (n_1, \ldots, n_p) \). Note that the volume of the Dp-brane \( V_p = (2\pi R)^p \) has appeared in order to get the correct measure of the summation. We have defined \( E_{\vec{q}} \) and \( \vec{x} \) as \( E_{\vec{q}} = \sqrt{|\vec{q}|^2 + m^2} \) and \( \vec{x} = (x^1, \ldots, x^p) \). \( \tilde{J}^{25}(E_{\vec{q}}, \vec{q}) \) is the source term in the momentum space that is given by

\[ \tilde{J}^{25}(E_{\vec{q}}, \vec{q}) = \int_{-\infty}^{\infty} dx^0 \int_0^{2\pi R} dx^1 \cdots \int_0^{2\pi R} dx^p e^{-i E_{\vec{q}} x^0 + i \vec{q} \cdot \vec{x}} J(x^0, \vec{x}). \]  

(5.5)

The probability that the source creates the one massless scalar particle of momentum \( \vec{k} \) is given by

\[ P(1)_{D_p} = \lambda_{p,\vec{k}} e^{-\lambda_p}, \]  

(5.6)

\[ \lambda_{p,\vec{k}} \equiv \frac{\tau_p}{V_p \ 2 E_{\vec{k}}} |\tilde{J}^{25}(E_{\vec{q}}, \vec{k})|^2. \]  

(5.7)
We are considering the elastic case in which the Dp-brane does not get any internal energy from the closed strings. Then \( \tilde{J}^{25}(E_{k}, \tilde{k}) = 0 \) for \( \tilde{k} \neq 0 \), because only the massless particle with \( \tilde{k} = 0 \) can be created in this process. Therefore Eqs. (5.3) and (5.4) become

\[
\lambda_{\mu} = \frac{\tau_{\mu}}{V_{\mu}} \frac{1}{2m} |\tilde{J}^{25}(0, 0)|^{2},
\]

(5.8)

\[
\dot{X}_{i}^{25} = \frac{1}{V_{\mu}} \tilde{J}^{25}(0, 0).
\]

(5.9)

in the present case.

### 5.2 Comparison with the amplitude of string theory

In the case that the massless scalar open string does not carry the momentum, we can easily see that the calculations in the string theory gives a similar result as that of the D-particle case:

\[
\left| \frac{A^{(N,1)}_{Dp}}{A^{(N,0)}_{Dp}} \right|^{2} = \frac{1}{\tau_{\mu}} \left( \sum_{\rho=1}^{N} k_{(\rho)}^{25} \right)^{2},
\]

(5.10)

where \( A^{(N,n)}_{Dp} \) is the total tree-level amplitude of the creation of the \( n \) massless scalar open strings on the Dp-brane from \( N \) closed strings.\(^{7}\) Next, we generalize the relationship (4.2) to the present case. Let us consider the probability of that one scalar particle of mass \( m \) whose momenta lies in a small region \( d^{25} \tilde{k} \) is created from the closed strings.\(^{8}\) The probability written in terms of the string amplitude up to the normalization is

\[
\frac{d^{25} \tilde{k}}{(2\pi)^{25}} \frac{1}{2E_{\tilde{k}}} |A^{(N,1)}_{Dp}|^{2} \prod_{\rho=p+1}^{25} \left( 2\pi \delta(k_{(\rho)}) \right) = \frac{d^{p} \tilde{k}}{(2\pi)^{p}} \frac{1}{2E_{\tilde{k}}} |A^{(N,1)}_{Dp}|^{2},
\]

(5.11)

where we have considered uncompactified spacetime here. The delta functions come from the fact that the scalar particle do not have the momentum perpendicular to the Dp-brane. In the present case, the spacetime is compactified and \( 1/V_{\mu} \) appears when we convert the momentum integrals into the summation over the Kaluza-Klein momenta. Then the probability (5.11) should be written as

\[
\frac{1}{V_{\mu}} \frac{1}{2m} |A^{(N,1)}_{Dp}|^{2},
\]

(5.12)

\(^{7}\)These massless open strings are those corresponding to the zero mode of the Dp-brane in the \( x^{25} \) direction.

\(^{8}\)\( \tilde{k} \) is the momentum of the massless particle as that in section 5.1, however it is defined as a 25-dimensional vector in this subsection.
where we have substituted $\vec{k} = 0$ as we did below (5.7). The same probability in terms of the field theory is given by $P(1)_{D^p}$, and then the relationship corresponding to (4.2) for the present case is

$$\frac{P(1)_{D^p}}{P(0)_{D^p}} = \frac{1}{2mV_p} \left| \frac{A^{(N,1)}_{D^p}}{A^{(N,0)}_{D^p}} \right|^2.$$  \hspace{1cm} (5.13)

From Eqs. (5.2), (5.8), (5.9), (5.10) and (5.13), we obtain

$$\tau_p V_p \dot{X}^{25}_f = \sum_{i=1}^{N} k^{25}_{(i)},$$  \hspace{1cm} (5.14)

which is the correct momentum conservation condition in the direction perpendicular to the D$p$-brane up to the relative sign.

We can also show that the Poisson distribution of the probability of the $n$ massless particle creation obtained from the effective field theory is consistent with the calculations of the creation probabilities in the string theory. The expectation value of the total energy of the created massless particles gives the final kinetic energy of the D$p$-brane correctly.

### 6 Conclusion and discussions

We developed a new method to describe an elastic scattering between a D-brane and closed strings in the non-relativistic region. The momentum conservation condition between the D-brane and the closed strings was obtained up to the relative sign of the D-brane’s momentum.

An interesting extension of the present work may be investigation of non-elastic scatterings in which the closed strings lose their total momentum in the worldvolume directions. In this case, the massless modes on the D-brane get non-zero momentum and non-zero energy from the closed strings. How to obtain the energy conservation condition between the D-brane and the closed strings, that have not been discussed in this paper, is also an important problem which we should solve in the future. If we choose a particular spacetime coordinate so that $\dot{X}^{25}_i = -\dot{X}^{25}_i$, energy conservation holds through the scattering process. This suggests that the Lorentz covariant generalization of the present work may solve the problem of the energy conservation. Generalization to superstring theory is interesting, too.

In the present work, the momentum conservation condition between the D-brane and the closed strings was *derived* up to the relative sign and a natural question is how to fix the relative sign. However, we can take a different standpoint in the construction of more generalized formalism to solve the above open problems; the momentum conservation condition with
correct relative sign can be utilized as a prerequisite. For example, the exact momentum conservation condition for a D-particle,

$$\tau X_i^{25} = \sum_{i=1}^N k_{(i)}^{25},$$  \hspace{1cm} (6.1)$$

fixes the relative phase between the amplitudes in the field theory and those in the string theory as

$$\frac{A^{(1)}_{\text{field}}}{A^{(0)}_{\text{field}}} = i \frac{1}{\sqrt{2m}} A^{(N,1)}_{(N,0)},$$  \hspace{1cm} (6.2)$$
in our notation. We can also obtain a similar condition between the amplitudes for a Dp-brane. The new relationship between the amplitudes like (6.2) can be a starting point of solving the open problems. We hope that the present work can be a step to the deeper understanding of the interactions between D-branes and closed strings.

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A Derivation of (2.21)

We review the derivation of (2.21) in this appendix. In the interaction representation,

$$|\Psi(t)\rangle_I = U(t, t_0)|\Psi(t_0)\rangle_I,$$  \hspace{1cm} (A.1)$$

where

$$U(t, t_0) = T \exp \left\{ -i \int_{t_0}^t dt' V_I(t') \right\}$$  
$$= \exp \left\{ ia \int_{t_0}^t dt' j(t') e^{imt'} \right\} \exp \left\{ ia \int_{t_0}^t dt' j(t') e^{-imt'} \right\}$$  
$$\times \exp \left\{ -\int_{t_0}^t dt_1 \int_{t_0}^{t_1} dt_2 \theta(t_1 - t_2) e^{im(t_1-t_2)} j(t_1)j(t_2) \right\}.$$  \hspace{1cm} (A.2)$$
The time evolution of the state in the Schrödinger representation is

$$|\Psi(t)\rangle_S = e^{-iH_0(t-t_0)}U(t,t_0)|\Psi(t_0)\rangle_S. \quad (A.3)$$

The $n$ particle state is defined as $|n\rangle = \frac{1}{\sqrt{n!}}(a^\dagger)^n|0\rangle$ and

$$\langle n|e^{-iH(t-t_0)}|0\rangle = \langle n|e^{-iH_0(t-t_0)}U(t,t_0)|0\rangle$$

$$= e^{-im(n+\frac{1}{2})(t-t_0)}\langle n|e^{ia\dagger\alpha}|0\rangle$$

$$\times \exp\left\{- \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \theta(t_1-t_2)e^{im(t_1-t_2)}j(t_1)j(t_2)\right\}, \quad (A.4)$$

where

$$\alpha = \int_{t_0}^{t_1} dt' j(t')e^{imt'}. \quad (A.5)$$

The following formulae are useful:

$$\langle n|e^{ia\dagger\alpha}|0\rangle = \frac{1}{\sqrt{n!}}(i)^n(\alpha)^n. \quad (A.6)$$

$$\theta(t_1-t_2) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} dq \frac{d}{dq} e^{iq(t_1-t_2)}$$

$$= \frac{1}{2\pi i} \int_{-\infty}^{\infty} dq \left\{ P\frac{1}{q} + i\pi\delta(q) \right\} e^{iq(t_1-t_2)}. \quad (A.7)$$

Therefore

$$- \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \theta(t_1-t_2)e^{im(t_1-t_2)}j(t_1)j(t_2)$$

$$\rightarrow - \frac{1}{2\pi i} P \int_{-\infty}^{\infty} dq \frac{d}{dq} j(q-m)\tilde{j}(-q+m) - \frac{1}{2} \tilde{j}(-m)\tilde{j}(m)$$

$$(t_0 \rightarrow -\infty, t \rightarrow \infty)$$

$$\rightarrow i\theta - \frac{1}{2} |\tilde{j}(0)|^2 \quad (m \rightarrow 0), \quad (A.8)$$

where

$$\theta \equiv \frac{1}{2\pi} P \int_{-\infty}^{\infty} dq \tilde{j}(q)\tilde{j}(-q) \quad (A.9)$$

is a real number. Thus $n$ particle creation amplitude $A_{\text{field}}^{(n)}$ is given as

$$A_{\text{field}}^{(n)} \equiv \lim_{t_0 \rightarrow -\infty, t \rightarrow \infty} \langle n|e^{-iH(t-t_0)}|0\rangle$$

$$= \frac{1}{\sqrt{n!}}(i)^n\tilde{j}(0)^n e^{-\frac{1}{2} |\tilde{j}(0)|^2} e^{i\theta}, \quad (A.10)$$

at the limit $m \rightarrow 0$. 

18
B Notations and useful formulae

We use the following worldsheet action in this paper:

\[
S = \frac{1}{2\pi\alpha'} \int d^2z \partial X^\mu \bar{\partial} X^\nu G_{\mu\nu},
\]
\[
G_{\mu\nu} = \text{diag}(+,+,+,\cdots,+).
\]

(B.1)

The Green’s function on the unit disk is given as

\[
\langle X^\mu(z)X^\mu(w) \rangle = -\frac{\alpha'}{2} \left\{ \ln |z - w|^2 + \ln |1 - z\bar{w}|^2 \right\}
\]

if we have Neumann boundary condition in the \(x^\mu\) direction, and

\[
\langle X^\mu(z)X^\mu(w) \rangle = -\frac{\alpha'}{2} \left\{ \ln |z - w|^2 - \ln |1 - z\bar{w}|^2 \right\}
\]

(B.3)

in the case of Dirichlet boundary condition instead.

We assume that we have Dirichlet boundary condition in the \(x^{25}\) direction here. If the point \(w\) is on the boundary of the unit disk,

\[
\langle \partial_r X^{25}(z) \partial_r X^{25}(w) \rangle = -\frac{\alpha'}{2} \left\{ \frac{w}{w-z} - \frac{w}{w-1/z} + \frac{\bar{w}}{\bar{w}-z} - \frac{\bar{w}}{\bar{w}-1/z} \right\},
\]

(B.5)

where \(\partial_r X^\mu(w) = (w\partial_w + \bar{w}\partial_{\bar{w}})X^\mu(w)\). If the point \(z\) is also on the boundary,

\[
\langle \partial_r X^{25}(z) \partial_r X^{25}(w) \rangle = -\alpha' \left\{ \frac{wz}{(w-z)^2} + \frac{\bar{w}\bar{z}}{(\bar{w}-\bar{z})^2} \right\}.
\]

(B.6)

Eq. (3.4) can be shown by using (B.6) and

\[
\int_0^{2\pi} d\phi \frac{e^{i\phi} e^{i\theta}}{(e^{i\phi} - e^{i\theta})^2} = e^{i\theta} \left[ \frac{-1/i}{e^{i\phi} - e^{i\theta}} \right]_{0}^{2\pi} = 0.
\]

(B.7)

C Derivation of (3.11) and (3.12)

We derive (3.11) and (3.12) in this appendix. To begin with, we define \(A(m_1, m_2, \cdots, m_j; n)\) as the disk amplitude with \(m_1\)-th, \(m_2\)-th, \(\cdots\), and \(m_j\)-th closed-string vertices and the \(n\) massless scalar open-string vertices. For example,

\[
A(1, 3, 4; n) = \left( \frac{k_{25}^{(1)} + k_{25}^{(3)} + k_{25}^{(4)}}{\sqrt{7}} \right)^n A(1, 3, 4; 0),
\]

(C.1)
by using (3.9). \( A^{(N,0)} \) is then written by the summation of all the possible connected and disconnected diagrams:

\[
A^{(N,0)} = A(1, 2, \cdots, N; 0) + A(1, 2, \cdots, N - 1; 0)A(N; 0) + A(1, 2, \cdots, N - 2, N; 0)A(N - 1; 0) + \cdots \]

Next, let us calculate \( A^{(N,1)} \) by using a modification of (C.2). In order to obtain \( A^{(N,1)} \), we should modify, for example, \( A(1, 2, \cdots, N - 2, 0)A(N - 1; 0)A(N; 0) \) in the right-hand side of (C.2) as

\[
A(1, 2, \cdots, N - 2, 0)A(N - 1; 0)A(N; 0) \implies A(1, 2, \cdots, N - 2; 1)A(N - 1; 0)A(N; 0) + A(1, 2, \cdots, N - 2; 0)A(N - 1; 1)A(N; 0) + \cdots
\]

\[
= \left\{ \frac{\sum_{i=1}^{N-2} k_{25}^{(i)}}{\sqrt{\tau}} + \frac{k_{25}^{(N-1)}}{\sqrt{\tau}} + \frac{k_{25}^{(N)}}{\sqrt{\tau}} \right\} A(1, 2, \cdots, N - 2, 0)A(N - 1; 0)A(N; 0)
\]

\[
= \frac{\sum_{i=1}^{N} k_{25}^{(i)}}{\sqrt{\tau}} A(1, 2, \cdots, N - 2, 0)A(N - 1; 0)A(N; 0),
\]  

and the extra factor \( \sum_{i=1}^{N} k_{25}^{(i)} \) appears in front of the original term. In the same way, we can show that all the terms in the right-hand side of (C.2) get the same extra factor through the modification and we can conclude that

\[
A^{(N,1)} = \frac{\sum_{i=1}^{N} k_{25}^{(i)}}{\sqrt{\tau}} A^{(N,0)}.
\]  

\( A^{(N,2)} \) is also calculated by a similar modification of \( A^{(N,1)} \). For example, \( A^{(N,1)} \) contains \( A(1, 2, \cdots, N - 2; 1)A(N - 1; 0)A(N; 0) \) and this term should be modified as

\[
A(1, 2, \cdots, N - 2; 1)A(N - 1; 0)A(N; 0) \implies A(1, 2, \cdots, N - 2; 2)A(N - 1; 0)A(N; 0) + A(1, 2, \cdots, N - 2; 1)A(N - 1; 1)A(N; 0) + \cdots
\]

\[
= \left\{ \frac{\sum_{i=1}^{N-2} k_{25}^{(i)}}{\sqrt{\tau}} + \frac{k_{25}^{(N-1)}}{\sqrt{\tau}} + \frac{k_{25}^{(N)}}{\sqrt{\tau}} \right\} A(1, 2, \cdots, N - 2; 1)A(N - 1; 0)A(N; 0)
\]

\[
= \frac{\sum_{i=1}^{N} k_{25}^{(i)}}{\sqrt{\tau}} A(1, 2, \cdots, N - 2; 1)A(N - 1; 0)A(N; 0),
\]
and the extra factor $\frac{\sum_{i=1}^{N} k_{25}^{(i)}}{\sqrt{\tau}}$ appears in front of the original term, again. We can also show that all the terms in $A^{(N,1)}$ get the same extra factor through the modification and we can conclude that

$$A^{(N,2)} = \frac{\sum_{i=1}^{N} k_{25}^{(i)}}{\sqrt{\tau}} A^{(N,1)}.$$  

(C.6)

The same procedure holds for the calculation of $A^{(N,n)}$ from $A^{(N,n-1)}$ and therefore we can conclude that

$$A^{(N,n)} = \left(\frac{\sum_{i=1}^{N} k_{25}^{(i)}}{\sqrt{\tau}}\right)^n A^{(N,0)}.$$  

(C.7)

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