Baryon structure in chiral effective field theory on the light front

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Outline

- Introduction
- General formalism: covariant light-front dynamics
- Chiral effective field theory
- Vertex functions
- Observables
- Perspectives
Introduction

Need of a non-perturbative framework to calculate bound state properties

easy with $\pi NN$ coupling

to be generalized for $\pi\pi NN$ case
Light-front dynamics

- Relativistic description of bound states
- The state vector is expanded in Fock components
- The state vector is defined on a surface in 4-dimensional space-time

Standard version of LFD: plane \( t + z = \text{const} \)

Rotational invariance is broken!
Covariant light-front dynamics

Plane: \(\omega \cdot x = 0\)
\[\omega^2 = 0\]

(Any plane tangential to the red cone)
\(\omega = (1, 0, 0, -1)\) corresponds to standard LFD

Advantages:

- no vacuum fluctuation contributions
- transparent connection with non-relativistic approaches
- clear separability of unphysical components in approximate calculations
Fock representation of the state vector

The state vector

\[ \varphi(p) = |1\rangle + |2\rangle + \ldots + |N\rangle + \ldots \]

\(N\) – the maximal number of Fock sectors under consideration
\(n\) – number of constituents in a given Fock sector, \(n \leq N\)

The many-body vertex function

\[ \Gamma_n^{(N)} \]

\( (n - 1) \) bosons
Renormalization scheme

Contribution to the physical fermion propagator

\[ \frac{1}{\Sigma(p)} + \frac{\delta m}{\delta m_{N-n+1}} \]

The general case: dependence on the Fock sector

(maximal number of particles in which the fermion line can fluctuate)
Renormalization scheme

Bare coupling constant: the same strategy

- Interaction with internal bosons

\[ \Gamma_n^{(N)} \rightarrow g_0(N-n+2) \]
the number of particles in which the fermion line can fluctuate after absorption

- Interaction with external bosons

\[ \Gamma_n^{(N)} \rightarrow g_0(N-n+1) \]
... before emission
Covariant light-front graph technique

- Particles are on their mass shell
- New fictitious particles – spurions
- Conservation laws include all physical particles and spurions

Conservation law: \( P + \omega \tau = k_1 + k_2 \)

Integration over \( \tau \) and unfixed 4-momentum
ChPT Lagrangian

- Lagrangian is formulated in terms of $u$ fields

$$u = e^{i \frac{\pi \cdot \pi}{2 F_0}}$$

$F_0$ is a pion decay constant

- Expansion in a finite number of pion fields

$$\mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + ... + \mathcal{L}^{(N)} + ...$$

- $N$-body Fock space truncation: $2(N - 1)$ pion fields

\[(N - 1) \text{ bosons} \quad \begin{array}{c} \text{braid} \end{array} \quad \begin{array}{c} \text{braid} \end{array} \quad (N - 1) \text{ bosons}\]
ChPT Lagrangian

2-body Fock space truncation: \[ \mathcal{L} = \mathcal{L}^{(1)} + \mathcal{L}^{(2)} \]

\[ \mathcal{L}^{(1)}_{\pi N} = \bar{\Psi}(i \not \! D - \bar{m} + \frac{g_A}{2} \gamma^\mu \gamma^5 u_\mu) \Psi, \]

\[ \mathcal{L}^{(2)}_{\pi N} = c_1 \text{Tr}(\chi_+) \bar{\Psi} \Psi - \frac{c_2}{4M^2} \text{Tr}(u_\mu u_\nu)(\bar{\Psi} D^\mu D^\nu \Psi + \text{H.c.}) + \frac{c_3}{2} \text{Tr}(u^\mu u_\mu) \bar{\Psi} \Psi \]

\[ - \frac{c_4}{4} \bar{\Psi} \gamma^\mu \gamma^\nu [u_\mu, u_\nu] \Psi + c_5 \bar{\Psi} \left( \chi_+ - \frac{1}{2} \text{Tr}(\chi_+) \right) \Psi + \bar{\Psi} \left( \frac{c_6}{2} f^+_{\mu \nu} + \frac{c_7}{2} v^{(s)}_{\mu \nu} \right) \sigma^{\mu \nu} \Psi \]

In the absence of external fields:

\[ D_\mu \Psi = (\partial_\mu + \Gamma_\mu) \Psi \]

\[ \Gamma_\mu = \frac{1}{2}(u^\dagger \partial_\mu u + u \partial_\mu u^\dagger) \]

\[ u_\mu = i(u^\dagger \partial_\mu u - u \partial_\mu u^\dagger) \]

\[ \chi_+ = u^\dagger \chi u^\dagger + u \chi^\dagger u \]

\[ \chi = \mu^2 \hat{I} \]

\[ f^+_{\mu \nu} = v^{(s)}_{\mu \nu} = 0 \]
ChPT Lagrangian

In terms of pion fields

\[ \mathcal{L}^{(1)} = -\frac{g_A}{2F_0} \bar{\Psi} \gamma^\mu \gamma_5 \vec{\pi} \cdot \partial_\mu \vec{\pi} \Psi - \frac{1}{4F_0^2} \bar{\Psi} \vec{\pi} \cdot (\vec{\pi} \times \partial_\mu \vec{\pi}) \gamma^\mu \Psi \]

\[ \mathcal{L}^{(2)} = 2c_1 \mu^2 (2 - \frac{\vec{\pi}^2}{F_0^2}) \bar{\Psi} \Psi - \frac{c_2}{2M^2 F_0^2} \partial_\mu \vec{\pi} \cdot \partial_\nu \vec{\pi} (\bar{\Psi} \partial^\mu \partial^\nu \Psi + H.c.) \]

\[ + \frac{c_3}{F_0^2} \partial_\mu \vec{\pi} \cdot \partial_\mu \vec{\pi} \bar{\Psi} \Psi - \frac{ic_4}{2F_0^2} \bar{\Psi} \gamma^\mu \gamma^\nu \vec{\pi} \cdot \partial_\mu \vec{\pi} \times \partial_\nu \vec{\pi} \Psi \]
Vertex functions

\[ \mathcal{L} = -\frac{g_A}{2F_0} \bar{\Psi} \gamma^\mu \gamma_5 \vec\tau \cdot \partial_\mu \vec\pi \Psi - \frac{1}{4F_0^2} \bar{\Psi} \vec\tau \cdot (\vec\pi \times \partial_\mu \vec\pi) \gamma^\mu \Psi \]

Equations for the $\pi N$ vertex functions in the case of the two-body Fock space truncation:
Regularization scheme

Pauli-Villars scheme: extension of Fock space

We use

\[ i = 0 \text{ (physical) and } 1 \text{ (Pauli-Villars) fermions} \]

\[ j = 0 \text{ (physical) and } 1, 2 \text{ (Pauli-Villars) bosons} \]
Vertex functions

System of equations:

\[
\begin{align*}
\Gamma_1^i &= \Gamma_1^i - \delta m_2 V_1 + \Gamma_2^{ij} V_2 \\
\Gamma_2^{ij} &= \Gamma_1^i - \Gamma_2^{ij} V_3 + \Gamma_2^{ij} V_4 \\
\bar{u}(p_{1i})\Gamma_1^i u(p) &= \bar{u}(p_{1i})(V_1 + V_2) u(p), \\
\bar{u}(k_{1i})\Gamma_2^{ij} u(p) &= \bar{u}(k_{1i})(V_3 + V_4) u(p).
\end{align*}
\]
Vertex functions representation

\[ \bar{u}(k_{1i}) \Gamma^i_1 u(p) = (m_i - m^2) a^i_1 \bar{u}(k_{1i}) u(p), \]

\[ \bar{u}(k_{1i}) \Gamma^{ij}_2 u(p) = i \bar{u}(k_{1i}) \left( (k_{2j} - \phi \tau) b^{ij}_1 (R_\perp, x) + \frac{m}{\omega \cdot p} b^{ij}_2 (R_\perp, x) \right) \gamma_5 u(p). \]

\[ \tau = \frac{s - m^2}{2 \omega \cdot p} \quad x = \frac{\omega \cdot k_{2j}}{\omega \cdot p} \]

\[ s = (k_{1i} + k_{2j})^2 \quad R = k_{2j} - x p, \quad R = (R^0, R_\perp, R_\parallel) \]

In our case \( a^i_1, b^{ij}_1, b^{ij}_2 \) are constants depending on \( i \) only

Necessary condition: the physical on-mass shell vertex function should not depend on \( \omega \)

\[ b^{i=0, j=0}_2 (s = m^2) = 0 \]
Solution without contact term

Equations:

\[
\Gamma_1^i = \Gamma_1^r + \Gamma_2^{r,i} \\
\Gamma_2^{i,j} = \Gamma_1^r
\]

Solution:

\[
b_1^{ij} = \frac{g_A}{F_0} m a_1^0 - \frac{g_A}{2F_0} \frac{(m_1 + m)(m_1 + m_i)}{(m + m_i)} a_1^1, \\
b_2^{ij} = \frac{g_A}{2F_0} \frac{(m_1^2 - m^2)(m - m_i)}{2m} a_1^1.
\]

The condition \( b_2^{i=0,j=0} (s = m^2) = 0 \) is satisfied automatically.
Solution with contact term

\[ b_{2}^{i=0, j=0} (s = m^2) = \text{const} \neq 0 \]

We need a new \( \omega \)-dependent counterterm to kill it

\[
L_{\text{int}} = -\frac{1}{2} \frac{g_A}{F_0} \bar{\Psi} \gamma^\mu \gamma_5 \tau^b \partial_\mu \phi^b \Psi - \frac{1}{4F_0^2} \bar{\Psi} \gamma^\mu \vec{\nabla} \cdot \vec{\phi} \times \partial_\mu \vec{\phi} \Psi + i Z_3 \bar{\Psi} \gamma_5 \tau^b \phi^b \Psi + \ldots
\]

With this counterterm \( b_{2}^{i=0, j=0} (s = m^2) = 0 \)
Nucleon mass as a function of pion mass

\[ \Sigma = A + \frac{\varphi}{m} B + \frac{\varphi m}{\omega p} C \]

\( C = 0 \)  (unphysical contribution)

\[ m_N = \bar{m} + (f(m_N, \mu) - f(\bar{m}, 0)) \]

\[ f = A + B \]

\[ m_N = \bar{m} + \frac{3g_A^2 \bar{m}}{32F_0^2 \pi^2} \mu^2 - \frac{3g_A^2}{32F_0^2 \pi} \mu^3 \]
Scalar form factor

Relation to the pion-nucleon sigma-term:

\[ \sigma = \mu^2 \frac{\partial \Sigma}{\partial \mu^2} \]

Analysis in progress…
Perspectives

- Finish test for 1 nucleon and 1 pion
  
  scalar and electromagnetic form factors calculation

- Calculations for 1 nucleon and 2 pions

\[ \Delta \text{ and Roper resonance contributions} \]

Calculations are already done for the Yukawa model (scalar meson)