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Design Charts for the Stability Analysis of Unsaturated Soil Slopes

Kenneth Gavin · Jianfeng Xue

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Abstract Simple limit equilibrium analyses can be performed to determine the Factor of Safety (FOS) against slope failure of unsaturated soil slopes. However, many of the input parameters needed for these analyses are highly variable, and the FOS value obtained is critically dependent on assumptions made by the designer. This paper describes a suite of reliability analyses on unsaturated soil slopes performed using an invariant reliability model. The results are presented in design charts from which a designer can choose the FOS value required to ensure a given target reliability index for a slope. The approach ensures that despite the variability of input parameters the slope will have a probability of failure of 2.23% or less.

Keywords Slope stability · Reliability analyses · Design charts

Abbreviations

\begin{align*}
C & \text{ Total cohesion} \\
\text{COV} & \text{ Coefficient of variation} \\
E(X) & \text{ Mean value of vector } X \\
\text{FOS} & \text{ Factor of Safety} \\
P_f & \text{ Probability of failure} \\
X & \text{ Vector of variables} \\
\bar{X} & \text{ Vector of reduced variables} \\
W & \text{ The weight of a slice of the slope} \\
C' & \text{ Effective cohesion of soil} \\
g(X) & \text{ Limit state function defined with vector } [X] \\
g(\bar{X}) & \text{ Limit state function defined with reduced variables} \\
h & \text{ Wetting front depth} \\
r & \text{ Radius in polar coordinate} \\
\rho_a & \text{ Pore air pressure} \\
\rho_w & \text{ Pore water pressure} \\
\alpha & \text{ Slope angle} \\
\beta & \text{ Reliability index} \\
\beta_{HL} & \text{ Reliability index defined with Hasofer–Lind method} \\
\gamma & \text{ Unit weight of soil} \\
\phi & \text{ Friction angle of soil} \\
\phi' & \text{ Angle indicating the rate of increase in shear strength relative to the matric suction} \\
\theta & \text{ Angle in polar coordinates} \\
\sigma(X) & \text{ Standard deviation of vector } X \\
\sigma_n & \text{ Total normal stress on the failure plane} \\
\tau & \text{ Shear strength of unsaturated soils} \\
\partial & \text{ Partial differential}
\end{align*}

1 Introduction

Rainfall induced slope failures are caused by the development of a wetting front (downward percolating...
water) which causes a reduction in near surface suctions and thereby reduces the soil strength. The rate of development and depth of a wetting front is dependent on the initial soil state (initial water content/suction) and the amount of the applied rainfall which infiltrates into the slope. The initial soil state largely depends on the soil type, water table level and antecedent rainfall. The amount of infiltration depends on additional factors such as the slope angle, vegetation type, rainfall intensity and rainfall duration. It is clear that to perform even a simple analysis of the stability of a slope; the designer must first make a number of assumptions regarding the current soil state and likely future infiltration. A number of these assumptions are not straightforward and differ from other geotechnical design situations in that they are not based on knowledge of soil characteristics and superstructure loading and therefore involve expertise in areas beyond those normally developed by geotechnical engineers. Given changing climatic conditions the structure loading and therefore involve expertise in areas beyond those normally developed by geotechnical engineers. Given changing climatic conditions the quantification of the effects of future rainfall events which may be heavier and of longer duration is an area of particular uncertainty.

Analysis of the stability of unsaturated slopes is a complex problem and a range of approaches from simple limit equilibrium solutions to advanced finite element method analyses have been proposed. The more advanced approaches, presented by (Ng and Shi 1998; Gasmo et al. 2000; Cho and Lee 2001) although more rigorous, require input data such as parameters derived from the Soil Water Characteristic Curves (SWCC) which are, in some cases difficult to obtain. Probabilistic analysis offers a rational way to engineers to include uncertainty in their designs. Uncertainty may be caused by natural variability evident in soil properties, or in the range of likely rainfall events (duration and intensity) experienced by a slope. This paper describes a framework whereby the FOS value required from a deterministic analysis can be chosen from design charts. The design charts summarise the results of reliability analyses which account for uncertainty in the design values of parameters used in the analysis. As such they provide a simple and rational method of ensuring a meaningful FOS for a given slope.

2 Background

A broad range of stability analyses are used in practice ranging from deterministic approaches where single design values are assumed for the variables, to probabilistic approaches where variability is considered.

2.1 Deterministic Methods

Fredlund et al. (1978) expanded the Mohr–Coulomb model to incorporate negative porewater pressure (matric suction) effects:

\[
\tau = c' + (\sigma_n - u_a) \tan \phi' + (u_a - u_w) \tan \phi_b
\]

\[
\tau = C + (\sigma_n - u_a) \tan \phi'
\]  

(1)

where \(\tau\) = shear strength of unsaturated soils, \(c'\) is the effective cohesion', \(\sigma_n\) is the total normal stress on the failure plane, \(u_a\) is the pore-air pressure on the failure plane, \(\phi'\) is the angle of internal friction associated with the net normal stress state variable \(\sigma_n\), \(u_w\) is the pore-water pressure on the failure plane, \(u_a - u_w\) is the matric suction on the failure plane, and \(\phi_b\) is the angle indicating the rate of increase in shear strength relative to the matric suction. \(\tau, c', \sigma_n, u_a, u_w\) are typically given in kPa, whilst \(\phi'\) and \(\phi_b\) are given in degrees. By combining the effects of \(c'\) and the contribution of matric suction \((u_a - u_w)\) into a single parameter \((C)\), (Fredlund and Rahardjo 1993) note that Eq. 1 takes the form of the conventional Mohr–Coulomb model, and thus can be implemented easily into existing slope stability software developed to analyse saturated soil slopes. Using the method of slices, they suggest that the variation of suction with depth in an unsaturated soil slope can be modelled by dividing the soil into discrete layers, and varying \(C\) within these layers. They note that the drawbacks of this approach are that \(C\) must be computed manually, remain constant throughout the analysis and be set by the user.

Fourie et al. (1999) note that in most slope failures caused by infiltration, the failure plane forms parallel to the existing slope surface. For this reason they suggest using an infinite slope model in which the FOS is given by:

\[
FOS = \frac{C + \gamma h \cos^2 \theta \tan \phi}{\gamma h \cos \phi \sin \theta}
\]

in which \(\gamma\) is the unit weight of soil (kN/m³), \(h\) is the wetting front depth in metres and \(\theta\) is the slope angle in degrees.

They suggest that Eq. 2 can provide useful information on the contribution of suction to the stability of an unsaturated soil slope. This is illustrated in Fig. 1.
where the stability of an embankment with a slope angle of 40°, soil friction angle of 30°, and a wetting front depth which varies from 0.25 to 1.5 m is presented. The effect of increasing the total cohesion (C) from 0 to 12 kPa is considered. When C is zero, the slope angle exceeds the friction angle of the soil, and the FOS is below 1. Assuming the effective cohesion of the soil is zero, the role of even a small suction in maintaining the stability of steep slopes is evident. For a given total cohesion, the FOS value of a slope reduces as the wetting front depth increases, for example when C = 4 kPa, the slope will fail when the wetting front depth reaches 1 m. The limitations of this approach are that the soil properties are assumed to be constant in the zone above the wetting front.

The drawbacks associated with the method of slices and infinite slope method can be overcome by performing a Finite Element Analysis (FEA). Using a constitutive model, which accounts for the effect of suction on soil strength, and allows full discretisation of the slope, thus permitting the soil properties to vary in the wetted zone, a coupled seepage and stability calculation can be performed. Although these FEA approaches are possible, the complex soil properties needed for the constitutive models are difficult to measure accurately and are rarely available for routine design situations.

2.2 Probabilistic Methods

There are many uncertainties associated with the choice of variables to use in the analysis of the stability of a soil slope. All soils are heterogeneous, and therefore soil properties vary with depth and location. Additional uncertainties associated with the analysis of unsaturated slope stability problems include; the variation of total cohesion when suctions reduce as a result of changes in water content during infiltration and difficulties in assessing the rate at which wetting front development occurs. The depth to which infiltrating water will penetrate into a slope, which represents the depth from ground surface to the slip surface, depends on many parameters, a number of which are highly variable. These include the initial suction profile, the geometry of the slope, permeability of the soil, rainfall intensity and duration amongst other factors. Traditional deterministic design methods, in which the soil properties are assigned fixed (conservative) values, cannot adequately model the dynamic process of wetting front development, and its effect on the disturbing forces causing slope failure, and the resistance (soil strength) preventing failure. For this reason (Zhang et al. 2005; Babu and Murthy 2005) and others propose the use of probabilistic design approaches.

In an attempt to improve commercial software, which typically performs uncoupled analysis, (Zhang et al. 2005) describe the development of a coupled hydro-mechanical model and slope stability analysis implemented using the finite element approach. Using the FE method they determined the FOS of a slope where the porosity, saturated permeability parameters derived from the SWCC and the shear strength values used in the assessment were varied.

Babu and Murthy (2005) carried out a probabilistic analysis of a slope using a planar slip surface model, with c', φ', γ, φh, K (hydraulic conductivity), the wetting front depth and the matric suction as variables. The reliability index (β) was calculated using a First Order Second Moment (FOSM) approach. In this method, the FOS is determined at a design point (normally the mean values of these variables). By considering the variation of these variables, the reliability of the slope is obtained. A sensitivity analysis was performed which identified matric suction, K and φh as the critical variables controlling slope stability.
2.3 Invariant Reliability Analysis

Whilst the probabilistic approaches described are a significant advance on traditional deterministic methods, they do not fully utilise the power of probabilistic analysis. Due to its simplicity the FOSM approach is widely used. However, there are several disadvantages associated with FOSM, the most significant being the lack of invariance of the approach for non-linear performance functions (USACE 1997).

Hasofer and Lind (1974) propose a method to determine an invariant estimate of \( \beta \). By transforming the random variables \( x \) into non-dimensional form using the mean \( (\mu) \) and standard deviation \( (\sigma) \), an invariant estimate of \( \beta \) is determined:

\[
\bar{x}_i = \frac{x_i - \mu_i}{\sigma_i} \quad (i = 1, 2, \ldots, n).
\]

When the limit-state (performance) function which compares the Capacity and Demand is given by:

\[
g(\bar{X}) = \text{Capacity} - \text{Demand}. \tag{4}
\]

In the non-dimensional (reduced variable) space the limit-state function can be rewritten for a number of variables which affect both Capacity and Demand as:

\[
g(\bar{X}) = g(\bar{x}_1, \bar{x}_2, \bar{x}_3, \ldots, \bar{x}_n). \tag{5}
\]

The limit-state surface \( g(\bar{X}) = 0 \) is the boundary between safe and unsafe regions (see Fig. 2). The reliability index estimated using the Hasofer–Lind approach (\( \beta_{HL} \)) is the minimum distance from the origin of the reduced variable space to the limit-state surface (design point). Many workers have used a cosine directional search approach to determine this distance, however (Val et al. 1996) demonstrate that because of the highly non-linear form of the performance functions considered, there is a tendency for this approach to identify local minima. In such circumstance the entire limit-state surface is not examined and the method may fail to locate the true reliability index. They show that by transforming the normalised variables from rectangular to polar coordinates this problem is overcome. Although the problem could also be addressed by using a full population, iterative search technique such as the Monte–Carlo or Genetic Algorithm methods, (Xue and Gavin 2007) adopted the technique of transforming the variables to polar coordinates as it facilitated the formulation of the complex objective function and allowed them to develop a model which simultaneously determined the critical slip surface and reliability index of slopes using Bishop’s simplified method applied to non-circular failure surfaces. The variables considered in the analyses included the soil properties \((c', \phi', \gamma)\) and the coordinates of the slip surface. The reliability index \( \beta_{HL} \) is given by (see Fig. 2):

\[
\beta_{HL} = \min_{X \in \Psi} (r)
\]

where \( r = \{(X')^T X'\}^{1/2} \).

This constrained optimisation problem was solved through the use of the Genetic Algorithm (GA) method.

Gavin and Xue (2009) extended this model to consider planar slip surfaces which develop during the downward migration of a wetting front in unsaturated soil slopes. The variables which describe the soil properties were \( C, \phi, \gamma \) and \( h \). At the ultimate limit-state (FOS = 1) Eq. 2 becomes:

\[
g(X) = C + \gamma h \cos^2 \alpha \tan \phi - \gamma h \cos \alpha \sin \phi. \tag{7}
\]

The coordinates of the design point in Fig. 2 can be described using the radial distance \( r \) and the polar angle \( \theta \):
To avoid mathematical complexities when substituting expressions describing the variables in Eq. 8, into Eq. 2, it is convenient to group some of the variables:

\[ Y_1 = \gamma h \]
\[ Y_2 = Y_1 \cos^2 x \tan \phi. \]  

The limit-state or performance function can now be rewritten as:

\[ g(X) = C + Y_2 - Y_1 \cos x \sin x. \]  

Assuming \( C, \phi, \gamma \) and \( h \) are normally distributed and un-correlated, according to the Taylor series of multi-variables we have:

\[
\begin{align*}
E(Y_1) &= E(\gamma)E(h) \\
\sigma(Y_1) &= \left[ (E(h)\sigma(\gamma))^2 + (E(\gamma)\sigma(h))^2 \right]^{1/2} \\
E(Y_2) &= E(Y_1) \cos^2 x \tan(E(\phi)) \\
\sigma(Y_2) &= \left[ \cos^2 x \tan(E(\phi))\sigma(Y_1) \right]^2 \\
&\quad + \left[ E(Y_1)\sigma(\tan \phi) \cos^2 x \right]^2 \right]^{1/2}.
\end{align*}
\]

and the reduced variables \( C, Y_1 \) and \( Y_2 \) can be expressed as:

\[
\begin{align*}
C &= E(C) - r\omega_1\sigma(C) \\
Y_1 &= E(Y_1) - r\omega_2\sigma(Y_1) \\
Y_2 &= E(Y_2) - r\omega_3\sigma(Y_2)
\end{align*}
\]

where: \( \omega_1 = \cos \theta_1 \cos \theta_2 \), \( \omega_2 = \cos \theta_1 \sin \theta_2 \) and \( \omega_3 = \sin \theta_1 \). Substituting Eq. 12 into Eq. 10 and rewriting we have:

\[
r = \frac{E(C) + E(Y_2) - E(Y_1) \cos x \sin x}{\omega_1\sigma(C) + \omega_2\sigma(Y_2) - \omega_3\sigma(Y_1) \cos x \sin x}. \]  

Because of the assumption that the variables are normally distributed, we must set lower bound values for these variables:

\[
\begin{align*}
C &= E(C) - r\omega_1\sigma(C) \geq 0 \\
Y_1 &= E(Y_1) - r\omega_2\sigma(Y_1) \geq 0 \\
Y_2 &= E(Y_2) - r\omega_3\sigma(Y_2) \geq 0.
\end{align*}
\]

This constrained optimisation problem was solved through the use of a computationally efficient Genetic Algorithm method Gavin and Xue (2009), known as Genetic Algorithm for Slope Stability Analysis (GASSA). The reliability index \( \beta \), computed using GASSA, Monte–Carlo analysis and FOSM are compared in Fig. 3. The stability of embankment slope, considered in Fig. 1, was calculated using the three approaches with the wetting front depth increasing from 0.02 to 1 m below ground level. Whilst all methods produce similar results for shallow failure surfaces (wetting front depth < 0.2 m), below this depth the Monte–Carlo and GASSA methods produce comparable results, however, the FOSM method predicts significantly higher \( \beta \) values, with the difference increasing with wetting front depth. Since higher \( \beta \) values suggest larger FOS, it is clear that the FOSM approach does not determine the minimum (critical) FOS and is therefore unconservative. The similarity between the \( \beta \) values computed using the full-population Monte–Carlo analysis and the computationally more efficient GASSA model for the whole
range of wetting front depths considered are in
keeping with similar calibration exercises performed
on saturated slope profiles by (Xue 2007)

3 Hybrid Design approach

Probabilistic design approaches such as that
described in the previous section are not yet in
widespread use in geotechnical engineering practice.
Deterministic approaches are taught widely in the
majority of undergraduate engineering design courses
and this coupled with their simplicity and perceived
transparency means such methods remain the industry
standard. In deterministic design, the target FOS is
normally set as a minimum of value, e.g. 1.3. The soil
properties are usually chosen based on characteristic
values as defined in (Eurocode 7 2004), often referred
to as moderately conservative parameters. In practice
this means adopting design values which are below
the actual mean value of the available measurements
of soil properties such as shear strength. The design
values are often chosen without regard for the actual
variation of soil properties, which means that
although the FOS value is fixed, the reliability index
will vary, reducing as the variability of the soil
properties increase. In these situations the adoption of
conservative input parameters does not in fact
suggest a safe design. An alternative approach
would be to set a fixed minimum target reliability
index ($\beta_T$), which would represent an acceptable
probability of failure for a slope. For a given slope a
rational design approach which does not involve a
full probabilistic analysis would be to determine the
FOS required to obtain the set target reliability index.
The FOS required to meet this $\beta_T$ value will naturally
increase as uncertainties regarding the input param-
ters for the slope stability analysis increase.

Considering Eq. 2, routine laboratory tests are
used to obtain values of $\phi$ and $\gamma$. In common soils
these values are relatively well defined are therefore
mean design values can be assigned with some confidence. Because of significant recent interest in
the behaviour of partly saturated soils, major develop-
ments in laboratory and field measurement com-
plimented by modelling have been achieved. It is
relatively straightforward to measure the total cohe-
sion $C$, in the laboratory or field (Springman et al.
2003). An estimate of the wetting front depth $h$, can
be obtained using simple empirical methods (Gavin
and Xue 2008) or more accurate complex numerical
analyses (Ng and Shi 1998). By making a number of
such determinations the variability of the critical
input parameters can be determined.

3.1 Target Reliability Index ($\beta_T$)

The use of reliability theory to select a FOS consistent
with a target reliability index has been discussed by
(Benjamin and Cornell 1970; Whitman 1984) in his
seminal paper on the application of reliability analysis
in geotechnical engineering. The choice of $\beta_T$ is
typically based on (Benjamin and Cornell 1970); (1)
historical data, (2) mathematical modelling based on
probability theory or (3) quantification of expert
systems (Paikowsky et al. 2004). Although for most
geotechnical problems the choice of $\beta_T$ is not
straightforward (Chowdhury and Flentje 2003), sug-
gest that for most slopes (outside of those in urban
areas) $\beta_T$ can be set at 2.0, which corresponds to a
probability of failure of 2.23%. Although this reliability
index would classify the performance as poor in
accordance with (USACE 1999), it is consistent with
$\beta_T$ which is implied by conventional practice in
geotechnical engineering (Whitman 1984) and will
be adopted in this paper. A framework is proposed in
which a designer can use a set of design charts, which
have been developed based on the invariant reliability
model, to choose a target FOS value to ensure a
consistent reliability index regardless of the uncer-
tainty surrounding the input parameters. To minimise
the number of variables which need to be considered in
the design charts, the critical input parameters which
most strongly affect the reliability index are deter-
mined in the following section.

3.2 Sensitivity Analysis of Input Parameters

The influence of the absolute depth of the wetting
front ($h$) and the degree of uncertainty with respect to
the estimation of this value, i.e. the coefficient of
variation ($\text{COV} = \text{standard deviation}/\text{mean value}$),
on the FOS required to maintain the target reliability index ($\beta_T$) of a $30^\circ$ slope is illustrated in Fig. 4. As
would be expected from the analyses of (Fourie et al.
1999) and others (see Fig. 1), as the wetting front
depth increases from 0.7 to 1.5 m, the mean cohesion $E(C)$ required to maintain the target reliability index

\[ E(C) = \beta_T \times \frac{h}{\text{mean cohesion}} \]
For a given COV(h) value increased. It is evident from Fig. 4, that for the range of the wetting front depths considered, as the level of uncertainty regarding the evaluation of the wetting front depth increased, the mean cohesion and the FOS value required to achieve $\beta_T$ increased. For example for a given wetting front depth, the FOS value required increased from 1.36 to 1.68 when the COV(h) increased from 0.05 to 0.15. The analyses suggest that for a given value of $E(C)$, the FOS required to achieve $\beta_T$ is influenced significantly by the level of uncertainty with respect to the estimate of the wetting front depth.

To study the influence of uncertainty with respect to the mean value of cohesion COV(C) on the FOS required to maintain the target reliability index, analyses were performed on three slopes, with slope angles (z) of 30°, 35° and 45°. The soil properties adopted in the analyses were $E(\phi) = 34^\circ$ and COV(\phi) = 0.1. The COV(h) was varied from 0.05 in Fig. 5a to 0.15 in Fig. 5b to examine the effect of COV(C) on the FOS value required to achieve a target reliability index. The results shown in Fig. 5 reveal that:

1. The FOS required to achieve the target reliability index increased gradually until the COV(C) exceeded 0.3, after which a significant increase in FOS was required to meet the target reliability index.

2. The rate of increase of FOS depended on the slope angle and the COV(h). Considering Fig. 5a, only a small increase in FOS was required to ensure stability of a 30° slope, with COV(h) = 0.05.
when the COV(C) increased from 0.1 to 0.3. In contrast, when the COV(h) increased to 0.15 (see Fig. 5b), the rate of increase of FOS was much more significant.

3. In all cases, the largest increase in FOS required, occurred for slopes where \( \phi \) was highest (i.e. the difference between the slope angle and the mean friction angle was greatest).

The effect of varying the water content of a soil on the total cohesion \( (C) \) can be established using simple in situ or laboratory tests (Springman et al. 2003; Doherty et al. 2007) whilst the wetting front depth can be estimated using either simple empirical models or Finite Element Analyses. Once estimates of the two parameters \( C \) and \( h \) can be determined the FOS value of the slope can be determined using Eq. 2 or a design chart such as Fig. 1.

The sensitivity analyses performed using GASSA in Figs. 4 and 5 suggests the uncertainty associated with the determination of \( C \) and \( h \), significantly influences the FOS value required to achieve a target reliability index. It is necessary to confirm this observed behaviour using more established numerical techniques such as Monte–Carlo Analyses. In order to perform this calibration excercies the FOS required to maintain a \( \beta_r \) value of 2.0 was calculated for a 30° slope, with a fixed wetting front depth of 0.7 m, soil unit weight of 20 kN/m³ (COV = 0.05). A Monte–Carlo analysis was performed on a slope with a slope angle of 35°, soil friction angle \( \phi = 30^\circ \), COV(\( \phi \)) = 0.1, mean wetting front depth = 0.9 m, and mean cohesion = 4 kPa (see Fig. 6a). The FOS value was fixed at 1.3 and the probability of failure (\( P_f \), which is the probability that the demand will equal or exceed the capacity) for a range of COV(C) between 0.1 and 0.4, and the COV(h) between 0.05 and 0.15 weas calculated. The data shown in Fig. 6b show that for all COV(C) values considered, \( P_f \) increased approximately linearly as COV(h) increased from 0.01 to 0.15. Thereafter, \( P_f \) increased sharply. This response is entirely comparable with the predictions of GASSA.

4 Design Charts

The sensitivity analyses performed in the previous section suggests that uncertainty associated with the determination of \( C \) and \( h \) significantly influences the FOS value required to achieve a target reliability index. By considering different values for the COV of \( C \) and \( h \), design charts can be constructed for a range of slopes. In the charts the target reliability index is fixed at 2, and the FOS required to obtain this value was investigated for different values of COV of the total cohesion and wetting front depth. Four values of COV(C), 0.1, 0.2, 0.3 and 0.4 were considered. Since in the sensitivity analyses the FOS was found to increase almost linearly with COV(h), for values between 0.05 and 0.15, only the upper and lower values in this range were included. Slope angles of 25°, 30°, 35°, 40° and 45° were assumed and for each of these slopes analyses were performed for soil friction angles of 26°, 30° and 40° to give a range of values typically encountered in design practice. The coefficient of variation of the soil friction angle was assumed to be 0.1, and a mean soil unit weight of 20 kN/m³ was assigned to the soil with a COV of 0.05. The FOS values required for the conditions considered are shown in Fig. 7. It should be noted that the results for three or four slopes are plotted on each design chart merely for brevity and this analysis approach could be applied to a wider range of slope angles and soil properties. The FOS values for all slopes angles at a given COV(C) are not directly comparable as the mean values for cohesion were not equal. The values of the parameters \( \phi \) and \( \alpha \) were chosen such that linear interpolation could be performed to consider intermediate values not specifically shown in Fig. 7.

5 Application of the Design Charts

A deterministic analysis of the stability of an unsaturated soil slope, with a slope angle of 40° was presented in Fig. 1. In this analysis, the soil friction angle and unit weight were taken as 30° and 20 kN/m³ respectively. When the total cohesion was assumed to be 8 kPa, and the wetting front depth was 1 m, the FOS value calculated using Eq. 2 was 1.45. Although this value is significantly above the value of 1.3 which is normally assumed to be the minimum allowable, the actual safety of the slope depends on the assumptions made by the designer in their choice of design values for the variables.

We could consider determine a more meaningful value of FOS if we quantify the variability of the
542 parameters used in the analysis. Assuming the soil
543 properties and wetting front depth given above are
544 the mean values obtained from a number of mea-
545 surements or estimates of \( \phi \), \( \gamma \), \( C \) and \( h \), and the COV
546 of \( \phi \) and \( \gamma \) were 0.1 and 0.05 respectively, the
547 uncertainty with respect to the measured or estimated
548 values of \( C \) and \( h \) can be investigated using the design
549 charts given in Fig. 7.
550 If we consider in the first instance, good confi-
551 dence in the values of \( C \) and \( h \), and assign COV
552 values of 0.1 and 0.05 respectively, then a target
553 value for FOS = 1.25 is obtained from Fig. 7c. This
554 is lower than the value (1.45) achieved using just the
555 mean values of the variables and the minimum
556 allowable value (1.3). The slope therefore has a low
557 probability of failure and a reliability index signifi-
558 cantly higher than 2.0. An alternative view of its
559 stability can be considered in Fig. 8, where for the
560 FOS value of 1.25 (associated with a target reliability
561 index of 2), suggest the safety of the slope is adequate
562 until the wetting front depth reaches 1.4 m below
563 ground level (i.e. much deeper than the wetting front
564 depth of 1 m identified in the deterministic analysis).
565 However, if there was greater uncertainty about
566 the estimate of cohesion and the variability of this
567 parameter COV(\( C \)) increased to 0.3, then from
568 Fig. 7c the FOS value required to meet the target
569 reliability index increases to 1.75. This is larger than
570 the value of 1.45 achieved in the deterministic
571 analysis and this indicates that the reliability index
572 is less than 2. According to the probabilistic analysis,
573 the slope performance would become unacceptable if
574 the wetting front depth reach 0.75 m below ground
575 level. Whilst the reduction in FOS is fundamentally
576 caused by the increased variability of the parameters
577 used in the stability analysis, the relatively large
578 increase in FOS required to meet the target reliability
579 index, from 1.25 to 1.75 is exacerbated because for

Fig. 6 a Slope layout for
Monte–Carlo analyses.
(b) Sensitivity analyses
 calibrated using
Monte–Carlo analysis
Fig. 7 Hybrid stability analysis design charts

![Graphs showing Factor of Safety (FOS) vs Coefficient of Variation (C) for different angles and COVs]

- For 25 degrees, COV(h) = 0.05, φ = 26°, COV(φ) = 0.1
- For 30 degrees, COV(h) = 0.15, φ = 30°, COV(φ) = 0.1
- For 35 degrees, COV(h) = 0.05, φ = 26°, COV(φ) = 0.1
- For 40 degrees, COV(h) = 0.15, φ = 30°, COV(φ) = 0.1
the slope considered the slope angle is $10^\circ$ higher than the soil friction angle. We see from Fig. 7c, when the difference between the slope angle and soil friction angle is within $5^\circ$, the FOS required is relatively insensitive to increases in COV(C) until the latter value exceeds 0.3. Although it is clear from charts 7a to f, that even when the slope angle is lower than the mean friction angle, the FOS value required to obtain target reliability index, exceeds the value of 1.3 often used in practice.

6 Conclusions

A hybrid method of assessing the stability of unsaturated soil slopes is presented. The method utilises the benefits of the rational treatment of the variability of input parameters provided by reliability based design and the simplicity afforded by deterministic approaches. The FOS value required to ensure a target reliability index of 2 is achieved can be chosen from a design chart which accounts for the COV of the total cohesion and wetting front depth. If the FOS value achieved using the deterministic approach with design values for the variables chosen at the mean value exceeds the FOS determined from the design chart, the probability of failure of the slope will be lower than $2.23\%$. The hybrid design approach incorporates a reliability model in which an invariant measure of the reliability index, defined using the Hasofer–Lind method, is determined. The approach assumes no knowledge of probabilistic analysis and requires the designer only to determine the mean value and the COV of the variables. The key benefit of the approach is its simplicity in that it provides a mechanism for considering parameter variability in routine design. Its usefulness will of course depend on the amount of information available to describe parameter variability at a given site and some of the assumptions inherent in the reliability model.

Whilst simple deterministic analyses of a slope identified the effect of total cohesion and wetting front depth on the FOS achieved, the sensitivity analyses performed using the reliability model identified the critical effect of uncertainty with respect to the design values of both C and h to the value of FOS required to achieve the target reliability index. Such information will allow the designer to concentrate on the determining a representative value of these materials and suggests that effort spent investigating the likely variability of these parameters, for example performing additional laboratory tests to determine C or additional numerical analyses to determine the sensitivity of the wetting front depth to the choice of variables used in its determination would be beneficial.

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