We have examined the physics and the experimental feasibility of studying various kaon decay processes in which the polarization of a muon in the final state is measured. Valuable information on CP violation, the quark mixing (CKM) matrix, and new physics can be obtained from such measurements. We have considered muon polarization in $K_L \to \mu^+\mu^-$ and $K \to \pi \mu^+\mu^-$ decays. Although the effects are small, or difficult to measure because of the small branching ratios involved, these studies could provide clean measurements of the CKM parameters. The experimental difficulty appears comparable to the observation of $K \to \pi \nu \bar{\nu}$. New sources of physics, involving non-standard CP violation, could produce effects observable in these measurements. Limits from new results on the neutron and electron electric dipole moment, and $\epsilon'/\epsilon$ in neutral kaon decays, do not eliminate certain models that could contribute to the signal. A detailed examination of muon polarization out of the decay plane in $K^+ \to \mu^+\pi^0\nu$ and $K^+ \to \mu^+\nu\gamma$ decays also appears to be of interest. With current kaon beams and detector techniques, it is possible to measure the $T$-violating polarization for $K^+ \to \mu^+\pi^0\nu$ with uncertainties approaching $\sim 10^{-4}$. This level of sensitivity would provide an interesting probe of new physics.

1. Introduction

We have examined the possibility of measuring muon polarization asymmetries that are sensitive to $P$, $T$ or CP symmetries; these are tabulated in Table 1. Observation of the possible effects requires high fluxes ($> 10^{12}$ K decays per year) of kaons, now available at several accelerator facilities, notably at Brookhaven National Laboratory AGS and at the KEK-PS. In the near future, the Fermilab main injector, as well as the Japanese Hadron Factory, could deliver much higher intensities of separated as well as unseparated kaon beams. The $\phi$ meson factory, DAPHNE, is also a new facility with an intense pure-kaon flux. Thus far, CP violation has been observed conclusively only in the neutral kaon system. Although a theoretical description of CP-violation in the neutral kaon system is available in the single complex phase of the standard-model Cabibbo-Kobayashi-Maskawa (CKM) matrix, part of, or the entire phase, can have origin in deeper causes that have so far eluded experimental scrutiny. During this past decade, experiments at FNAL and at CERN, focusing
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Decays Correlations Symmetries tested

(1) \( K^+ \rightarrow \pi^0 \mu^+ \nu \) \( \vec{s}_\mu \cdot (\vec{P}_\mu \times \vec{P}_\pi) \) T

(2) \( K^+ \rightarrow \mu^+ \nu \gamma \) \( \vec{s}_\mu \cdot (\vec{P}_\mu \times \vec{P}_\pi) \) T

(3) \( K_L \rightarrow \mu^+ \mu^- \) \( \vec{s}_\mu \cdot \vec{P}_\mu \) P, CP

(4) \( K^+ \rightarrow \pi^+ \mu^+ \nu \) \( \vec{s}_\mu \cdot \vec{P}_\mu \) P

(5) \( K^+ \rightarrow \pi^+ \mu^+ \nu \) \( (\vec{s}_\mu^+ \cdot \vec{P}_\mu^+) (\vec{s}_\mu^- \cdot (\vec{P}_\mu^- \times \vec{P}_\mu^-)) \) T

(6) \( K^+ \rightarrow \pi^+ \mu^- \nu \) \( \vec{s}_\mu \cdot (\vec{P}_\mu^+ \times \vec{P}_\mu^-) \) P, T

(7) \( K^0_L \rightarrow \pi^0 \mu^+ \mu^- \) \( \vec{s}_\mu \cdot \vec{P}_\mu \) P

(8) \( K^0_L \rightarrow \pi^0 \mu^+ \mu^- \) \( \vec{s}_\mu \cdot (\vec{P}_\mu^+ \times \vec{P}_\mu^-) \) T

Table 1: Decay modes and polarization asymmetries (or correlations) of interest in K decays. The symbols \( \vec{s} \) and \( \vec{p} \) refer to the spin and momentum vectors in the decays.

Since the measurement of the direct \( K^0_L \rightarrow \pi^0 \pi^0 \) transition, or \( \epsilon' \), have reported ever-improved results, the latest being \( \text{Re}(\epsilon'/\epsilon) = (28.0 \pm 3.0 \pm 2.8) \times 10^{-4} \) (FNAL) and \( \text{Re}(\epsilon'/\epsilon) = (14.0 \pm 4.3) \times 10^{-4} \) (CERN). These provide conclusive evidence for the presence of direct CP violation in \( K^0 \) decays, and there is great theoretical effort in progress to interpret these numbers. We will not attempt here to review \( \epsilon'/\epsilon \); but it is clear that it is not yet certain if the standard model description of CP violation can accommodate these results. In addition to \( \epsilon'/\epsilon \), rare kaon decays are also of interest for understanding the CKM matrix. For a recent review of rare kaon decay processes see Ref. 7.

Over the next decade, ambitious efforts towards gaining a better understanding CP-violation and the CKM matrix elements will be pursued at B-factories. The importance of these efforts is undeniable, but it is also worthwhile to investigate the possibility that some or all of the CP-violation arises from effects outside of the minimal standard model, particularly outside of the current CKM matrix.

It should be recalled that CP-violation is required to generate the observed baryon asymmetry in the universe, and it is now accepted that the CP-violation embodied in the CKM matrix does not have sufficient strength to serve this purpose. It is important to examine if sources of physics beyond the standard model that could generate the baryon asymmetry can also generate CP or T violating muon polarizations in the kaon decay modes given in Table 1.

We will now briefly consider the measurement of muon polarization. This has been discussed before in the literature. Muon polarization in the experiments under consideration would be observed by stopping the muons in some appropriate material, and measuring the direction of the decay electron (positron in the case of \( \mu^+ \)). The muon decay spectrum is given by:

\[
\frac{dN}{dzd\Omega} = \frac{\varepsilon^2}{2\pi} \left( (3 - 2z) \pm |\vec{P}| \cos \theta (2z - 1) \right)
\]

where the positive sign in the brackets is for \( \mu^+ \) decay and the negative sign is for
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| Mode Branch. | Standard Model Fraction | Final State Int. | Non-SM value | Ref. |
|--------------|-------------------------|------------------|--------------|-----|
| $K^+ \to \pi^0 \mu^+ \nu$ | 0.032 | 0.0 | $\sim 10^{-6}$ | $\leq 10^{-3}$ | [11, 12] |
| $K^+ \to \mu^+ \nu \gamma$ | $5.5 \times 10^{-3}$ | 0.0 | $\sim 10^{-3}$ | $\leq 10^{-3}$ | [13] |
| $K_L \to \mu^+ \mu^-$ | $7.2 \times 10^{-9}$ | $\sim 0.002$ | 0.0 | $\leq 10^{-2}$ | [14, 15] |
| $K^+ \to \pi^+ \mu^+ \mu^-$ | $7.6 \times 10^{-8}$ | $\sim 10^{-2}$ | – | – | [16, 21] |
| $K_L^0 \to \pi^0 \mu^+ \mu^-$ | $\sim 5 \times 10^{-12}$ | – | – | – | [23, 24] |
| $K_L^0 \to \pi^0 \mu^+ \mu^-$ | $\sim 0.5$ | – | – | – | [23] |

Table 2: The decay modes and asymmetries of interest; the row numbers correspond to those in Table 1. The other columns are: the known standard-model branching ratio (note that the $K_L \to \pi^0 \mu^+ \mu^-$ branching ratio is not yet measured, the present 90% C.L. limit is $< 3.8 \times 10^{-10}$), the estimated standard-model value of the asymmetry, the value due to final-state interactions, the maximum possible value allowed by non-standard physics, and the theoretical reference. Some of the results have been adjusted to account for more recent values of the top quark mass (174 GeV/c²). In the case of $K_L \to \mu^+ \mu^-$ and $K^+ \to \pi^+ \mu^+ \mu^-$, the theoretical estimates for maximum possible non-standard contributions to $T$ violation do not agree; we chose the mean value of the different estimates. The “–” means that there is no reliable prediction.

For the $\mu^-$ decay, $z = 2E_e/m_\mu$, and $\theta$ is the angle between the polarization $\vec{P}$ and the direction of the positron or electron. We will now restrict ourselves to only $\mu^+$ decays. The magnitude of the polarization $\vec{P}$, multiplied by a dilution factor $D(z_0)$, is the asymmetry $A$ in the number of decays that produce positrons forwards versus backwards with respect to a plane normal to the polarization vector.

$$A = \frac{N_1 - N_2}{N_1 + N_2} = D(z_0)|\vec{P}|$$

where $N_1$ and $N_2$ are the number of forward and backward decays, respectively. $D(z_0)$ is the asymmetry dilution factor that depends on the lower-energy cutoff $z_0 = 2E_0/m_\mu$, where $E_0$ is the minimum observable energy of the electron from $\mu$ decays. The uncertainty on the asymmetry is given by $\delta A = \sqrt{\frac{1-A^2}{N}}$, where $N = N_1 + N_2$. When the asymmetry is small, the error on the polarization is given by $\delta P = \frac{1}{D(z_0)\sqrt{N}}$. In practical devices, there is usually a lower cutoff on the positron energy. Integrated over the entire spectrum, the asymmetry is $|\vec{P}|/3$. Clearly, both the total number of detected decays $N$, and the dilution factor $D$, depend on the low-energy cutoff. Best performance is reached when the lower cutoff is at about $z_0 \sim 0.75$, which keeps approximately 1/2 of the spectrum and corresponds to a dilution factor of $\sim 0.5$. Often, the muons are precessed by a small magnetic field (at a rate of 42.5 kHz per Gauss) perpendicular to the direction of the spin. This makes it possible to measure two components of the polarization, as
well as eliminate systematic differences in detector efficiencies, at only a cost of $\sqrt{2}$ loss in the dilution factor.

Other considerations that determine the asymmetry are depolarization of the stopped $\mu^+$, and confusion from the multiple scattering of the decay positron. It is well known that multiple Coulomb scattering of the muon, as it comes to rest, does not cause depolarization; however, after it slows down to atomic velocities, there are many processes that can lead to depolarization. The amount of depolarization is strongly material dependent. Carbon (in graphite form) and aluminum are considered good materials for muon polarimeters because they preserve the polarization. Detectors such as wire chambers or scintillators must be placed next to blocks of graphite or aluminum to detect the decay positrons. No practical active materials such as plastic scintillator or scintillating crystals have been found that would preserve polarization. Dilution of the asymmetry from multiple scattering of the positron depends on the geometry and the energy cutoff. Ultimately, the dilution factor must be measured in each experiment to determine the sensitivity of the result.

2. $K^+ \rightarrow \pi^0 \mu^+ \nu$

The transverse, or out-of-plane, muon polarization in this decay has been analyzed previously. The decay amplitude, $\mathcal{M}$, can be written as follows:

$$\mathcal{M} = \frac{G_F}{2} \sin \theta_c f_D (q^2) \left( (p_K + p_\pi)^\lambda + \xi (q^2) (p_K - p_\pi)^\lambda \right) \langle \bar{u}_\mu \gamma_\lambda (1 - \gamma_5) u_\nu \rangle$$ (3)

where $G_F$ is the Fermi constant, $\sin \theta_c$ is the Cabibbo angle, and $q$, $p_K$, $p_\pi$ are the momentum transfer, the kaon and the pion 4-momenta, respectively. The out-of-plane polarization ($P^T_\mu$) of the muon is non-zero when the form factor $\xi$ has an imaginary component. This polarization is expected to vanish to first order in the standard model, because of the absence of the CKM phase in the decay amplitude. Irreducible backgrounds, such as from final-state interactions (FSI) in this decay are expected to be small ($\sim 4 \times 10^{-6}$), and can be ignored. It has been shown that any model involving only effective V or A interactions cannot produce this type of polarization. The existence of a non-zero value of this polarization will therefore provide a definite signature of physics beyond the standard model. $P^T_\mu$ is a function of the two Dalitz variables that define the 3-body $K^+ \rightarrow \mu^+ \pi^0 \nu$ decay. In most experimental situations, one averages over some portion of the Dalitz plot. Therefore, the average $P^T_\mu$ has two components: (1) a kinematic factor ($f_D$) that describes the average over the Dalitz plot, including experimental acceptance, and (2) the imaginary part of an amplitude or combination of amplitudes ($\text{Im}\xi$). We will often use the same $P^T_\mu$ to denote the average over an ensemble of events:

$$P^T_\mu = f_D \text{Im}\xi$$
$f_D$ is estimated by setting $\xi = 0$, and ignoring terms of $O\left(\frac{m^2}{m_K}\right)$.

\[ f_D \approx \left(\frac{m_\mu}{m_K}\frac{E_\mu}{|p_\mu|} \sin \theta_{\mu\nu} \right) \]

where $E_\mu$ is the muon energy and $\theta_{\mu\nu}$ is the angle between the muon and the neutrino in the kaon rest frame. The dependence over the Dalitz plot is averaged with the experimental acceptance. If we choose a typical point on the Dalitz plot, e.g., $E_\mu \approx 170\, \text{MeV}$ and $\theta_{\mu\nu} = 90^\circ$, then the value of $f_D$ calculated at that point is $f_D \approx 0.17$. The value of $\text{Im}\xi$ is model dependent; for example, in the case of a non-standard effective scalar interaction, $\text{Im}\xi$ is proportional to the imaginary part of the scalar coupling strength.

In particular, multi-Higgs and leptoquark models can produce non-zero out-of-plane polarization. In multi-Higgs models, a charged Higgs particle mediates a scalar interaction that interferes with the standard model decay amplitude; in such models, the polarization can be as large as $10^{-3}$, without conflicting with other experimental constraints, for example, the measurements of the neutron electric dipole moment and the branching fractions for $B \to X\tau\nu$, and $b \to s\gamma$. The indirect limits on $P'_\mu$ from other measurements have been examined in the context of the minimal 3 Higgs Doublet Model (3HDM).

The transverse polarization for 3HDM is given by

\[ P'_\mu \approx f_D \frac{m_\mu^2}{m_h^2} s'_3 s'_3 c'_3 \left( \frac{\nu_2}{\nu_1} \frac{\nu_3}{\nu_1} \right) \]

\[ \nu^2 = \nu_1^2 + \nu_2^2 + \nu_3^2 \]

where $s'_3$, $c'_3$, and $c'_3$ are unknown parameters from the unitary $3 \times 3$ mixing matrix of the 3 Higgs doublets. It is assumed that the ratios of the 3 vacuum expectation values are the same as the ratios of the 3 generation masses: $v_1 : v_2 : v_3 :: m_b : m_t : m_\tau$. We have updated the calculation of constraints on the 3HDM model to include new experimental results and collected them in Table 3. The constraints are calculated in terms of two assumptions on the mass of the charged Higgs: $m_h \sim m_W$ or $m_h \sim 2m_W$. Both of these assumptions are above the current lower limit of $m_h \sim 78\, \text{GeV}$ on the mass of a charged Higgs boson from LEP. The best constraints are from the branching ratio measurements $B(b \to s\gamma)$ and $B(b \to X\tau\nu)$. These are unlikely to improve in the near future because the errors have large theoretical components. Furthermore, these constraints have the requirement that the real part of the 3HDM amplitude cancels with the standard model amplitude. Without this assumption, the constraints would be quite weak. Nevertheless, as can be seen in the table, even these optimistic limits allow any value of $P'_\mu$ below the current direct bound, which is described below. T-violation could occur in the minimal supersymmetric models through an effective scalar interaction involving the charged Higgs particle, nevertheless, the effect is considered too small to observe, except in models that contain R-parity violation.
Table 3: Constraints on $P_T^\mu$ of $K^+ \rightarrow \mu^+\pi^0\nu$ decay for 3HDM from other measurements. The "-" means that there is no significant constraint. All measurements have been scaled to correspond to the same 95% confidence level to get consistent constraints.

| Measurement | Value | $P_T^\mu < m_h \sim m_W$ | $m_h \sim 2m_W$ |
|-------------|-------|--------------------------|-----------------|
| $d_n$ [13] | $< 7.5 \times 10^{-26} e - cm$ (95% C.L.) | 0.039 | 0.039 |
| $d_e$ [34] | $< 1.6 \times 10^{-26} e - cm$ (95% C.L.) | – | – |
| $|\tau_\rho|$ [1, 4, 7] | $(1.9 \pm 0.24) \times 10^{-3}$ | – | – |
| $m_{K_L} - m_{K_S}$ [32, 36] | $(0.5301 \pm 0.0014) \times 10^{10} \text{fs}^{-1}$ | – | – |
| $B(b \rightarrow s \gamma)$ [3] | $(2.54 \pm 0.56) \times 10^{-4}$ | 0.09 | 0.02 |
| $B(b \rightarrow X\tau\nu)$ [38] | $(4.08 \pm 0.98) \times 10^{-2}$ | 0.009 | 0.009 |

The best limit on this process was obtained recently by an experiment at the KEK-PS, E246. They measured $P_T^\mu = -0.0042 \pm 0.0049 \pm 0.0009$, which corresponds to a value of the T violating parameter: $\text{Im}\xi = -0.013 \pm 0.016 \pm 0.003$, where the errors are statistical and systematic, respectively. This measurement was performed using approximately $3.9 \times 10^6$ events. Additional data may increase the total sample by a factor of two.

Experimental limits were obtained almost 20 years ago with both neutral and charged kaons at the BNL-AGS. The experiment with $K^+$ decays made a measurement of the transverse polarization, $P_T^\mu = 0.0031 \pm 0.0053$. The combination of the neutral and charged experiments could be interpreted as a limit on $\text{Im}\xi = -0.01 \pm 0.019$.

Measurements from the 1980s were based on $1.2 \times 10^7 K_L^0$ and $2.1 \times 10^7 K^+$ decays to $\mu^+\pi\nu$, and were limited by low analyzing power and by backgrounds. The KEK-E246 measurement relied on the new technique of using a stopped $K^+$ beam, and measuring the muon decay direction in an aluminum absorber, without spin precession. The BNL measurements used in-flight kaon decays, stopped the muons in an aluminum absorber, and measured the muon spin by its precession in a weak magnetic field. Both techniques relied on the cylindrical symmetry of the apparatus to suppress systematic errors due to non-uniform efficiencies. The systematic error in KEK-E246 was reduced further by collecting events with forward and backward going pions. The BNL experiment alternated the direction of the precessing magnetic field to cancel the detection efficiency to second order. The remaining systematic errors in both techniques are quite different: the largest systematic error in the stopped kaon experiment comes from the knowledge of the fringe magnetic field in the muon stopping region. The largest systematic error for the in-flight experiment is thought to be due to the mechanical misalignment of the azimuthal segments of the muon absorber with respect to the precessing axis of the magnetic field.

A new experiment should reach much higher statistics and have high analyzing power for the muon polarization. KEK-246 has demonstrated that a well designed muon stopper with low background can have an analyzing power of $0.197 \pm 0.005$. 
It is possible to extend the sensitivity of the KEK-246 experiment with a more intense beam\textsuperscript{43} such as the low energy separated beam used by BNL experiment E787\textsuperscript{44} which has been able to obtain kaon stopping rates of approximately 1 million per sec. A stopped kaon experiment, with a field for precession has the potential advantages of having better systematic control using cancelations from both the forward/backward $\pi^0$ direction, as well as by alternating field direction\textsuperscript{45}.

A new experiment has been designed at the BNL-AGS to perform this measurement, with an error on the polarization approaching $10^{-4}$\textsuperscript{46}. The design is based on the 1980 experiment, however, it uses a 2 GeV/c separated charged kaon beam to reduce the background counting rate. Other improvements will involve higher acceptance and analyzing power, with a larger apparatus and a more finely segmented polarimeter made of graphite. The experiment will collect approximately 550 events per AGS pulse (3.6 seconds). Thus the statistical accuracy of the polarization measurement in a 2000 hr ($2 \times 10^6$ pulses) run could be $\delta P_T \sim 1.3 \times 10^{-4}$. Through a precise alignment of the polarimeter and through measurements of null asymmetries from $K^+ \rightarrow \mu^+\nu\gamma$ decays the experimenters expect to control the systematic error in the apparatus to below the statistical uncertainty. Other studies carried out for experiments at the Japanese Hadron Factory and the $\phi$ factory at DAPHNE also show that statistical sensitivities of the order $\sim 10^{-4}$ are possible\textsuperscript{47},\textsuperscript{48}.

Finally, we note that it could be easier to measure the transverse polarization in the case of $K_L \rightarrow \pi^-\mu^+\nu_\mu$ decays, because it is easier to detect a charged pion. However, the transverse polarization in this case is contaminated by final state interactions due to the presence of a charged pion. This effect, averaged over the phase space, has recently been estimated to be $P_T^\mu = 2.4 \pm 0.1 \times 10^{-3}$\textsuperscript{49}.

3. $K^+ \rightarrow \mu^+\nu\gamma$

The branching ratio for $K^+ \rightarrow \mu^+\nu\gamma$ decay is $(5.5 \pm 0.28) \times 10^{-3}$\textsuperscript{50}. The decay is dominated by inner bremsstrahlung. Recently, the structure-dependent part of the branching ratio (mostly for positive-helicity photons) has been measured to be $(1.33 \pm 0.12 \pm 0.18) \times 10^{-5}$\textsuperscript{50}. The structure-dependent form factors have also been measured recently in decays of $K^+ \rightarrow e^+\nu e^-\nu$ and $K_L \rightarrow e^+\nu e^-\nu$, and are sensitive to the form factor for negative-helicity photons\textsuperscript{50}.

In $K^+ \rightarrow \mu^+\nu\gamma$ decay, the transverse muon polarization, which is T-violating, can arise from interference between inner bremsstrahlung and the structure-dependent part of the decay. It is sensitive to new pseudo-scalar, vector, and axial-vector interactions, in contrast to $K^+ \rightarrow \mu^+\pi^0\nu$ decay’s sensitivity to new scalar interactions\textsuperscript{51}. Recent studies have shown that in certain extensions of the standard model, including SUSY, there could be a T-violating muon polarization in $K^+ \rightarrow \mu^+\nu\gamma$ decays as large as $10^{-2}$\textsuperscript{52}. Others argue that it is unnatural to generate a contribution to transverse polarization in $K^+ \rightarrow \mu^+\nu\gamma$ decay larger than $10^{-4}$ in SUSY, unless R-parity is broken\textsuperscript{53}. Finally, final-state interactions can induce a non-zero transverse polarization. This has been calculated recently to be of the order of $10^{-4}$, and varies on the Dalitz plot\textsuperscript{54}. It can be as large as $5 \times 10^{-4}$ at the high end of the...
muon energy spectrum, for \( E_\gamma \sim M_K/4 \).

Following the calculations in Ref. 53, the \( K^+ \to \mu^+ \nu \gamma \) decay is described by “inner bremsstrahlung” (IB) and “structure-dependent” (SD) contributions. The parameters are the kaon decay constant \( f_K \), axial vector form factor \( F_A \), and a vector form factor \( F_V \). Physics beyond the SM is introduced in additional terms in the Lagrangian

\[
\mathcal{L} = -\frac{G_F}{\sqrt{2}} \sin \theta_c \bar{s} \gamma^\alpha (1 - \gamma_5) u \bar{\nu} \gamma_\alpha (1 - \gamma_5) \mu + G_S \bar{s} u \bar{\nu} (1 + \gamma_5) \mu + G_P \bar{s} \gamma_5 u \bar{\nu} (1 + \gamma_5) \mu + G_V \bar{s} \gamma^\alpha \gamma_5 u \bar{\nu} \gamma_\alpha (1 - \gamma_5) \mu + \text{h.c.,}
\]

where \( G_F \) is the Fermi constant, \( \theta_c \) is the Cabibbo mixing angle, and \( G_S, G_P, G_V, \) and \( G_A \) are parameters of the new interactions due to scalar, pseudo-scalar, vector, and axial vector exchange, respectively.

The new interactions will modify the decay constants and form factors in the following way:

\[
\begin{align*}
    f_K & = f^0_K (1 + \Delta_P + \Delta_A), \\
    F_A & = F^0_A (1 + \Delta_A), \\
    F_V & = F^0_V (1 - \Delta_V),
\end{align*}
\]

and

\[
\Delta_{(P,A,V)} = \frac{\sqrt{2}}{G_F \sin \theta_c} \left( \frac{G_P m_K^2}{(m_s + m_u) m_\mu}, G_A, G_V \right). \tag{8}
\]

The T-violating muon polarization (\( P_T \)) comes from the interference of IB and the imaginary part of the SD. Only the pseudo-scalar (\( G_P \)) and right-handed current (\( G_R = G_V + G_A \)) terms contribute:

\[
P_T(x, y) = P^V_T(x, y) + P^A_T(x, y) \tag{9}
\]

with

\[
\begin{align*}
    P^V_T(x, y) & = \sigma_V(x, y) [\text{Im}(\Delta_A + \Delta_V)], \\
    P^A_T(x, y) & = [\sigma_V(x, y) - \sigma_A(x, y)] \text{Im}(\Delta_P),
\end{align*}
\]

where \( \sigma_V(x, y) \) and \( \sigma_A(x, y) \) are analytic functions of the Dalitz plot variables \( x = 2E_\gamma/M_K, \ y = 2E_\mu/M_K \). Figure 1 shows the contours of muon polarization along and perpendicular to the muon momentum within the decay plane, and the contours of \( \sigma_V \) and \( \sigma_V - \sigma_A \) as a function of the Dalitz plot variables, \( E_\mu \) and \( E_\gamma \). Events with high muon momentum have higher sensitivity to the T-violating parameters.

The detectors designed for \( K^+ \to \mu^+ \pi^0 \nu \) muon-polarization measurement can be used for studying \( K^+ \to \mu^+ \nu \gamma \) decays, with only minimal modification. In the near future, the first measurement of the T-violating polarization in \( K^+ \to \mu^+ \nu \gamma \) is expected from KEK-E246. It is expected to have sensitivity of the order of 5%.\[5\]
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A $K^+ \to \mu^+ \nu \gamma$ event is usually identified in a detector through a coincidence of a stopped muon with an energetic acollinear photon. To reject background from $K^+$ decays with $\pi^0$ (mainly $K^+ \to \mu^+ \pi^0 \nu$), events with additional photons are rejected. This requires a hermetic photon veto system surrounding the $K^+$ decay region. With a precise measurement of the photon energy and the muon momentum, the $K^+ \to \mu^+ \nu \gamma$ decay can be fully reconstructed. This helps to further reduce background. A precision calorimeter with good spatial resolution is essential. The muon momentum can be measured in a magnetic field, as in KEK-E246, or the muon energy can be derived from the muon range in the polarimeter, as proposed in the new BNL experiment. A study from the latter showed that with sufficient shielding around the decay volume, and photon veto down to about 20MeV, one could obtain a sample of $3 \times 10^7 K^+ \to \mu^+ \nu \gamma$ events, with a signal to background ratio of about 2:1, in 2000 hours of running. This would provide constraints on the new interactions at sensitivities of:

$$\delta_{Im(\Delta A + \Delta V)} = 7 \times 10^{-3}$$
$$\delta_{Im(\Delta P)} = 20 \times 10^{-3}$$

An experiment with the sensitivity of Eq. (11) will clearly probe new physics. Events with high muon energy have even higher sensitivity to T-violation; they are also beyond the kinematic limit of $K^+ \to \mu^+ \pi^0 \nu$ events. They have a much better signal to background ratio, but at the expense of efficiency for signal. This is partially compensated by the higher sensitivity. For example, selecting events with high muon energy ($2E_\mu/M_K > 0.95$), the average $\sigma_V$ increases from 0.11 (for all events) to 0.15; the effect is larger for $(\sigma_V - \sigma_A)$, which increases from 0.04 to 0.25. We note that this is also the region where the asymmetry from FSI may be maximal. The selection of events on the Dalitz plot can be optimized differently for a search for pseudo-scalar or right-handed currents.

It should be noted that the T-conserving components of the muon polarization in $K^+ \to \mu^+ \nu \gamma$ and $K^+ \to \mu^+ \pi^0 \nu$ decays are of opposite sign. By measuring the T-conserving polarization of the accepted $K^+ \to \mu^+ \nu \gamma$ sample, the background from $K^+ \to \mu^+ \pi^0 \nu$ can be easily evaluated.

4. $K_L \to \mu^+ \mu^-$

This decay has been measured recently with high statistics. Experiment E871 at BNL has collected more than 6200 events with about 1% background contamination. The new branching ratio is:

$$\frac{\Gamma(K_L \to \mu^+ \mu^-)}{\Gamma(K_L \to \pi^+ \pi^-)} = (3.474 \pm 0.057) \times 10^{-6}$$

Using the measured branching ratio $B(K_L \to \pi^+ \pi^-) = (2.056 \pm 0.033) \times 10^{-3}$ from Ref. [35], we obtain $B(K_L \to \mu^+ \mu^-) = (7.14 \pm 0.17) \times 10^{-9}$. The phenomenology of this reaction has been studied extensively in the past.
The longitudinal muon polarization in this decay violates CP invariance. In general, the correlation between the $\mu^+$ and $\mu^-$ polarizations also contains information about CP violation and new physics, however this is much more difficult to measure. The decay amplitude for $K_L \rightarrow \mu^+\mu^-$ is known to be dominated by the two photon intermediate state. Interference of this amplitude with any other flavor-changing neutral scalar interaction can produce a longitudinal polarization. Following Herczeg, we set the amplitude for $K_L \rightarrow \mu^+\mu^-$ to:

$$M(K_L \rightarrow \mu^+\mu^-) = a\bar{u}(p_-)\gamma_5v(p_+) + bu(p_-)v(p_+)$$

where $a_2$ and $b_2$ are the CP conserving and CP violating amplitudes of $K_2$, respectively. Similarly, $a_1$ and $b_1$ are the CP violating and CP conserving amplitudes for $K_1$, respectively. The decay rate and the longitudinal polarization ($P_L$) are then given by:

$$\Gamma = \frac{m_K\beta}{8\pi}(|a|^2 + \beta^2|b|^2)$$

$$P_L = \frac{2\beta\text{Im}(ba^*)}{|a|^2 + \beta^2|b|^2}$$

$$\beta = (1 - 4m_\mu^2/m_K^2)^{1/2} \approx 0.905$$

The expression for the decay rate has several parts: terms of $O(\epsilon^2$ can be neglected, terms of $O(\epsilon$ multiply amplitudes with direct CP violation in $K_1$ or $K_2$ and are therefore small. The remaining terms are $\text{Re}(a_2)^2 + \text{Im}(a_2)^2 + \text{Re}(b_2)^2 + \text{Im}(b_2)^2$. Each of these reflects a sum of contributions from electroweak interactions and possible non-electroweak physics. The lowest-order standard-model electroweak contributions arise from the intermediate two photon diagram and the short-distance diagrams involving mainly the top quark, W, and Z exchange.

The largest contribution to the total branching ratio is known to be the absorptive contribution ($\text{Im}(a_2)$) from two photon exchange. This can be calculated in a model independent way:

$$\Gamma(K_L \rightarrow \gamma\gamma \rightarrow \mu\mu) = \frac{\alpha^2(m_\mu)^2}{m_K^2} \frac{1}{2\beta} \left( \frac{\log 1 + \beta}{1 - \beta} \right) = 1.195 \times 10^{-5}$$

Eq. 15 is also known as the unitarity bound. All other contributions to $\text{Im}(a_2)$ are much smaller. $\text{Im}(b_2)$ in the standard model is due to $K_2 \rightarrow \pi\pi \rightarrow \gamma\gamma(\text{CP}=+1) \rightarrow \mu^+\mu^-$: this is constrained to be very small by $\epsilon'/\epsilon$. The short-distance electroweak diagrams contribute to $\text{Re}(a_2)$. The branching ratio from this contribution is also well calculated in the standard model to be:

$$B_{SD}(K_L \rightarrow \mu^+\mu^-) = \frac{\tau_L}{\tau_{K^+}} \frac{\alpha^2 B(K^+ \rightarrow \mu\nu)}{\pi^2\sin^4\theta_W |V_{us}|^2} |Y_c\text{Re}(\lambda_c) + Y_t\text{Re}(\lambda_t)|^2$$

(16)
$Y_q$ are Inami-Lim functions of $x_q = M_q^2/M_W^2$; $\lambda_j = V_{pj}^* V_{jd}$ are combinations of the CKM matrix elements.\[\text{Or, in terms of the Wolfenstein parameters } A, \rho \text{ and } \lambda:\]

$$B_{SD}(K_L \rightarrow \mu^+\mu^-) = 1.51 \times 10^{-9} A^4 (\rho_0 - \bar{\rho})^2$$ (17)

where $\bar{\rho} = \rho(1 - \lambda^2/2)$, $\rho_0 = 1.2$ is the charm contribution.

The second contribution to Re($a_2$) comes from $K_L \rightarrow \gamma^*\gamma^* \rightarrow \mu^+\mu^-$, in which the photons are off mass-shell. The sign and magnitude of this contribution are still uncertain; this uncertainty limits the precision with which the experimental measurement can be used to obtain a limit on polarization, as well as on the fundamental standard-model parameter $\rho$. First order amplitudes involving new flavor-changing vector bosons, leptoquarks, or scalar particles, such as a light Higgs, contribute to Re($b_2$); these contributions would be responsible for a possibly large longitudinal polarization.

The unitarity bound from the absorptive amplitude can be subtracted from the measured branching ratio to obtain the sum of all the remaining parts. In Ref. [56, this subtraction is performed using the measured ratio $\Gamma(K_L \rightarrow \mu^+\mu^-)/\Gamma(K_L \rightarrow \pi^+\pi^-)$ and the unitarity bound calculated for the ratio $\Gamma(K_L \rightarrow \gamma\gamma)/\Gamma(K_L \rightarrow \pi^+\pi^-)$, with proper account of correlations in errors.\[\text{They obtain a dispersive contribution to the branching ratio of } (0.11 \pm 0.18) \times 10^{-9}:\]

$$\frac{m_K \beta}{8\pi} (\text{Re}(a_2^{SD}) \pm \text{Re}(a_2^{LD}))^2 + \beta^2 \text{Re}(b_2)^2 = (0.11 \pm 0.18) \times 10^{-9} \times \Gamma(K_L)$$ (18)

Here Re($a_2^{SD}$) is the magnitude of the amplitude due to the electroweak short-distance diagrams and Re($a_2^{LD}$) is the magnitude of the dispersive contribution from the two photon diagram. We will now consider the implications of this result on the value of the polarization:

Concentrating only on the direct CP violating contribution, the longitudinal polarization can be written:

$$P_L = -\frac{m_K \beta^2}{4\pi \Gamma(K_L \rightarrow \mu^+\mu^-)} \text{Re}(b_2) \text{Im}(a_2)$$ (19)

One can limit the magnitude of this polarization using the measurement in Eq. [18, by assuming Re($a_2$) = 0. Then, for the square of the polarization, we can write:

$$|P_L|^2 < 4 \times \left(1 - \frac{\Gamma(K_L \rightarrow \gamma\gamma \rightarrow \mu^+\mu^-)}{\Gamma(K_L \rightarrow \mu^+\mu^-)}\right) \frac{\Gamma(K_L \rightarrow \gamma\gamma \rightarrow \mu^+\mu^-)}{\Gamma(K_L \rightarrow \mu^+\mu^-)}$$ (20)

Using the Particle Data Group prescription, we obtain an upper limit\[ for the polarization of:

$$|P_L| < 0.44 \quad 90\%C.L.$$ (21)

\[\text{Following the PDG prescription, the value of } |P_L|^2 \text{ is } 0.058 \pm 0.094. \text{ However, it should be noted that a previous result from E791 has an unphysical central value of } -0.06 \pm 0.21 \text{ for } |P_L|^2.\]
Clearly, a better limit can be obtained if \( \text{Re}(a_2) \) can be specified more accurately.

There are several mechanisms that could contribute towards generating a significant value of \( \text{Re}(b_2) \) and a large polarization. In general, \( P_L \) is quite sensitive to the presence of light scalars with CP violating Yukawa couplings. The polarization can originate in the standard CKM model because of the induced \( s - d - H \) coupling, where \( H \) is the standard-model Higgs. Very large polarizations can be expected for a light Higgs boson with a mass comparable to \( K_L \). Results from LEP that limit the Higgs boson masses to be larger than 115 GeV, rule out detectable polarization from this mechanism. In Ref. \(^{63}\) multi-Higgs, leptoquark, as well as left-right symmetric models are considered. Polarizations of the order of few percent are possible, without violating bounds from neutron or electron electric dipole moments, or from lepton-flavor violation. Supersymmetric contributions to \( P_L \) are expected to be small (\( \sim 10^{-3} \)), without fine-tuning of parameters.

The dominant contribution to \( P_L \) within the standard model is through the indirect CP violation parameterized by \( \epsilon \). The direct CP violating amplitudes \( (b_2 \) and \( a_1) \) can be neglected, and using the value of \( \epsilon \) \(^{63}\): \( \epsilon = 2.3 \times 10^{-3} e^{i\phi} \) \(^{(22)}\)

\[
\phi = 43.7^\circ \pm 0.1^\circ \approx \frac{\pi}{4} \quad (23)
\]

The polarization can be written as

\[
P_L = \frac{m_K^2 \beta^2}{4\pi \Gamma(K_L \to \mu^+\mu^-) \sqrt{2}} |\epsilon| \left[ \text{Re}(a_2)(\text{Re}(b_1) - \text{Im}(b_1)) + \text{Im}(a_2)(\text{Re}(b_1) + \text{Im}(b_1)) \right] \quad (24)
\]

Ecker and Pich have calculated the value of \( b_1 \) using chiral perturbation theory (CHPT), to obtain a rather good prediction for \( P_L \).\(^{64}\) They obtain:

\[
(1.9)1.5 < |P_L| \cdot 10^{-3} \sqrt{\frac{2 \times 10^{-6}}{B(K_S \to \gamma\gamma)}} < 2.5(2.6) \quad (25)
\]

The numbers without (with) brackets correspond to 1 standard deviation, \( \sigma \), (2 \( \sigma \)) errors. There are three sources of uncertainty in the above estimate. The uncertainty on the octet coupling strength \( (G_8) \) in CHPT is largely eliminated by normalizing to the measured \( B(K_S \to \gamma\gamma) = (2.4 \pm 0.9) \times 10^{-6} \).\(^{43}\) The uncertainty in the upper limit is small because it comes from \( \text{Im}(a_2) \). The uncertainty in the lower limit comes from the unknown sign and magnitude of \( \text{Re}(a_2) \). Ecker and Pich point out that there is a constructive interference in the term multiplying \( \text{Im}(a_2) \), which makes their estimate larger than a previous estimate.\(^{63}\) To summarize, the polarization due to indirect CP violation is estimated to be \( \sim 2 \times 10^{-3} \) with 30-40\% uncertainty.

The main experimental difficulty in this measurement is the small branching fraction of \( (7.14 \pm 0.17) \times 10^{-9} \) for the decay. Much effort must therefore be put
into separating these events from background, before a polarization analysis can be performed. Experiment E871, with \( \sim 6200 \) events, was optimized to look for \( K_L \rightarrow \mu^+ e^- \). We have estimated that, if the experiment were optimized for \( K_L \rightarrow \mu^+ \mu^- \), and the beam intensity were increased, E871 could collect about 20,000 events in two years of running. Appropriate upgrades to the marble muon-range detector will allow approximately 50% of the positive muon decays to be analyzed. A study by the E791 collaboration (previous version of E871), indicated that, with 20,000 events, they could achieve a sensitivity of 11% on the longitudinal polarization asymmetry of the positive muons.

The \( \sim 1\% \) background to \( K_L \rightarrow \mu^+ \mu^- \) arises mainly from \( K_L \rightarrow \pi^+ \mu^+ \nu \) events, in which the charged pion decays in flight or is misidentified as a muon, and the momentum of one of the charged particles is mismeasured so that the \( \mu \mu \) invariant mass is higher than the kinematic limit of 489 MeV. The positive muons in the \( K_L \rightarrow \pi^- \mu^+ \nu \) background events at the kinematic endpoint will be almost completely longitudinally polarized; the positive muons in the background \( K_L \rightarrow \pi^+ \mu^- \nu \) will come from \( \pi^- \) decay, and will be polarized (with strength dependent on the \( \pi^- \) decay angle within the experimental acceptance) in the opposite direction. Given these considerations, this background will not be problematic for an experiment with a sensitivity of \( \sim 0.1\% \), however, a measurement of the \( K_L \rightarrow \mu^+ \mu^- \) polarization better than 1% will require less background.

5. \( K^+ \rightarrow \pi^+ \mu^+ \mu^- \)

This decay has a very rich structure that can lead to important measurements. Table 1 shows three different asymmetries that could be interesting to measure: longitudinal muon polarization, transverse muon polarization, and transverse \( \mu^\pm \) polarization in correlation with \( \mu^\mp \) longitudinal polarization. The decay has recently been analyzed extensively. The different processes that govern the decay are: the one-photon intermediate state, a two-photon intermediate state, the short-distance “Z-penguin” and “W-box” graphs, and potential contributions from extensions to the Higgs sector.

The dominant amplitude from the one photon intermediate state (with a vector form factor) is best understood in the framework of chiral perturbation theory, although experimental inputs are needed to fully describe the decay. The \( K^+ \rightarrow \pi^+ l^+ l^- \) decays can be discussed using the following variables:

\[
x = \frac{(p_1 + p_2)^2}{m_K^2},
\]

\[
y = \frac{2P \cdot (p_1 - p_2)}{m_K^2 \lambda^{1/2}(1, x, x_\pi)},
\]

\[
y = \frac{2k \cdot (p_1 - p_2)}{m_K^2 \lambda^{1/2}(1, x, x_\pi)}
\]

where \( p_1 \) and \( p_2 \) are the four-momenta of the negative and positive lepton. \( P \) and \( k \) are the four-momenta of the kaon and the pion, respectively. \( \lambda(a, b, c) = \)
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\[a^2 + b^2 + c^2 - 2(ab + bc + ac)\] and \[x_\pi = m_\pi^2/m_K^2\]. The limits of the phase space are given by:

\[
\frac{4m_l^2}{m_K^2} < x < \frac{(m_K - m_\pi)^2}{m_K^2} \\
|y| < (1 - \frac{4m_l^2}{xm_K^2})^{1/2}
\] (27)

In the kaon rest frame, the energy of the pion is given by: \[E_\pi = (M_K/2)(1 - x - x_\pi)\]. In terms of these variables, the decay rate is given by

\[
\frac{d\Gamma}{dx dy} = \frac{m_K}{64(2\pi)^3}\lambda^{1/2}(1, x, x_\pi)|\mathcal{M}|^2
\] (28)

where \(\mathcal{M}\) is the matrix element. For a vector interaction model, the decay rate is given by the following:

\[
\frac{d\Gamma}{dx dy} = \frac{\alpha^2|G_8|^2m_K^5}{16\pi}\lambda^{3/2}(1, x, x_\pi)(1 - y^2)|\phi(x)|^2
\] (29)

where \(G_8\) is the octet coupling constant. The general features of \(K \to \pi l^+l^-\) decay spectrum can be understood in terms of angular momentum conservation. In the vector interaction model, the quantum numbers of the \(l^+l^-\) pair are \(J^{PC} = 1^{--}\). The lepton pair must therefore be in a p state relative to the pion, and it must be longitudinally polarized. This has two consequences: the angle between the lepton and the pion in the rest frame of the lepton pair has a distribution that goes as \((1 - \cos^2 \theta)\) (This is the reason for the \((1 - y^2)\) term; \(y \approx \cos \theta\), is the difference in the energies of the lepton pair divided by the pion momentum in the kaon rest frame); as the mass of the lepton pair gets large, the orbital angular momentum barrier forces the decay spectrum to fall faster than phase space, and to vanish at the kinematic endpoint.

The form factor \(\phi(x)\) has been measured very well by experiment E865, with more than 10,000 events in the \(K^+ \to \pi^+e^+e^-\) decay mode.\(^7\) The form factor is linear in \(x\), with a large slope parameter that has been measured with high precision. The dominance of the vector current in the decay has been demonstrated experimentally. The measurement puts significant constraints on models containing long-distance contributions to \(K \to \pi l^+l^-\) decays. The \(K^+ \to \pi^+\mu^+\mu^-\) decays have been measured in two different experiments, \(\sim 200\) events in E787\(^\text{12}\) and \(\sim 400\) events in E865\(^\text{13}\). The average branching ratio is \(7.6 \times 10^{-8}\), but the two experiments disagree by \(3.3\sigma\).

\[
B(K^+ \to \pi^+\mu^+\mu^-, E787) = (5.0 \pm 0.4(stat) \pm 0.7(syst) \pm 0.6(th)) \times 10^{-8}; \\
B(K^+ \to \pi^+\mu^+\mu^-, E865) = (9.22 \pm 0.60(stat) \pm 0.49(syst)) \times 10^{-8}.
\] (30)

Using the more recent \(K^+ \to \pi^+e^+e^-\) measurement of the form factor, the \(K^+ \to \pi^+\mu^+\mu^-\) branching ratio is predicted to be \((8.7 \pm 0.4) \times 10^{-8}\). The E787 result...
used an old value for the form factor; the theoretical error in their measurement corresponds to this form factor. The disagreement between the two measurements is large even if we correct the E787 result using the new form factor.

In general, the decay process can have contributions from scalar, vector, pseudo-scalar and axial-vector interactions, with corresponding form factors, $F_S$, $F_V$, $F_P$, and $F_A$ (following the notation in Ref. 22):

$$
\mathcal{M} = F_S \bar{u}(p_1, s)v(\bar{p}_1, \bar{s}) + F_P \bar{u}(p_1, s)\gamma_5 v(\bar{p}_1, \bar{s}) + F_V p_\mu^\mu \bar{u}(p_1, s)\gamma_\mu v(\bar{p}_1, \bar{s}) + F_A p_\mu^\mu \bar{u}(p_1, s)\gamma_\mu \gamma_5 v(\bar{p}_1, \bar{s})
$$

(31)

Here $p_k$, $p_\pi$, $p_l$, and $\bar{p}_l$ are the kaon, pion, lepton, and antilepton 4-momenta. Any interference between the terms with a complex phase difference leads to polarization effects. Within the standard model, the largest contribution is from the one-photon intermediate state to the vector form factor ($F_V$), which is expected to be almost real. The scalar form factor is expected to get a small contribution from only the two-photon intermediate state. $F_P$ and $F_A$ get contributions from the short-distance “Z-penguin” and “W-box” diagrams, where the dominant term arises from t-quark exchange, both form factors are therefore proportional to $V_{ts}V_{td}^*$, with a small contribution from the charm quark.

The parity-violating longitudinal muon polarization (asymmetry (4) in Table 1) has terms proportional to $\text{Re}(F_PF_V^*)$ and to $\text{Re}(F_VF_A^*)$. It is therefore sensitive to the Wolfenstein parameter $\rho$. The value of this polarization within the standard model is estimated beyond the leading logarithms to be $0.003 - 0.0096$, for $-0.25 < \rho < 0.25$, and depends on the experimentally accessible region of phase space. Neglecting the dependence of the form factor, the polarization asymmetry varies as $(1 - 4m_\mu^2/m^2_k)\lambda^{3/2}(1, x, x_\pi)$. It has a maximum at a $\mu\mu$ invariant mass of approximately $250 \text{MeV}/c^2$. There is a non-negligible contribution to this polarization from the long-distance two photon process, which has not as yet been calculated accurately. The T-violating transverse polarization (asymmetry (5) in Table 1) is proportional to $\text{Im}(F_SF_V^*)$, and it is therefore expected to be small within the standard model, and the final-state interaction correction to this polarization is expected to be $\sim 10^{-3}$. The T-violating spin-spin correlation that involves both $\mu^+$ and $\mu^-$ polarizations (asymmetry (6) in Table 1) has terms proportional to $-\text{Im}(F_VF_V^*)$ and $\text{Im}(F_VF_A^*)$. This is expected to have much smaller final-state interaction corrections, and is theoretically sound. Within the standard model, such asymmetry is proportional to the CKM parameter $\eta$, and it is expected to increase with $\mu\mu$ invariant mass, becoming as large as $\sim 0.06$ in some parts of the decay phase space (see Fig. 2). A good measurement would certainly be valuable in understanding CP violation. Similar to $K_L \rightarrow \mu^+\mu^-$, this asymmetry could also receive contributions from non-standard physics, such as from charged Higgs or leptoquarks. The consensus in the literature on the numerical results is summarized in Table 2. It should be noted that these estimates were made before the precise measurement of the form factor $F_V$ in Ref. 70. Nevertheless, that should not affect substantially the values in Table 2.
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Figure 2: T-violating spin-spin correlation, or asymmetry (6) in Table 1, for $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ over the Dalitz plot. (After Agrawal, Ng, Belanger, and Geng22).

The main experimental difficulty is in isolating the rare $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ decays from background. For both E787 and E865, the main background is the $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ decay, which has a branching ratio 6 orders of magnitude higher. This background must be suppressed at the trigger level as well as in the final analysis. In E865, the charged pions can be misidentified as muons due to pion decays in flight. In E787, where the unique total kinetic energy is used as the signature, the additional energy release in $\pi^-$ nuclear capture can increase the apparent kinetic energy of $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ events. Neither experiment was optimized for $K^+ \rightarrow \pi^+ \mu^+ \mu^-$, and both experiments took dedicated $K^+ \rightarrow \pi^+ \mu^+ \mu^-$ triggers in only a limited running period.

Scaled from E865, a dedicated and optimized experiment, with improved acceptance and trigger capability for muons, increased kaon flux with reduced beam halo, longer running time, and an excellent muon polarimeter, could yield $\sim 50,000$ events. This could result in a measurement of $\mu^+$ polarization with $\delta P_L \sim 0.05$, assuming the same analyzing power as the estimate for the polarization measurement of $K_L \rightarrow \mu^+ \mu^-$. An interesting option for this measurement could be to rely on the decay-at-rest technique used by E787: the $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ background could be suppressed by selecting the $\pi^+$ momentum to be above the kinematic limit of $K^+ \rightarrow \pi^+ \pi^+ \pi^-$ decays (125 MeV/c); then both $\mu^+$ and $\mu^-$ would have low kinetic energies, and could be stopped and analyzed in the same active target that serves to stop incoming kaons. Clearly, measuring asymmetries that require analyzing both
μ⁺ and μ⁻ polarizations will be difficult because μ⁻ decays have a lower analyzing power due to muon capture into atomic orbits around nuclei in the muon stopper.

Authors of Ref. 22 point out that there are only a few interesting kaon measurements that offer clean theoretical interpretation in terms of the standard weak-interaction parameters. The measurement of the branching ratio of \( K^+ \rightarrow π^+\nu\bar{ν} \) provides a measurement of \([V_{ts}^∗V_{td}]\), the branching ratio \( K^0_L \rightarrow π^0\nu\bar{ν} \) is proportional to \( η^2 \). The measurement of the polarization asymmetries (4) and (6) from Table 1 could be of comparable significance in terms of the overall theoretical uncertainties. The experimental difficulty should be similar to other proposed investigations of the K and B systems.

6. \( K^0_L \rightarrow π^0μ^+μ^- \)

The structure of \( K_L \rightarrow π^0l^+l^- \) decays is more complex than that of \( K^+ \rightarrow π^+l^+l^- \) because of CP suppression. There are three possible contributions to the decay amplitude: 1) direct CP-violating contribution from electroweak penguin and W-box diagrams, 2) indirect CP-violating amplitude from the \( K_1 \rightarrow π^0l^+l^- \) component in \( K_L \), and 3) CP-conserving amplitude from the \( π^0\gamma\gamma \) intermediate state. The sizes of the three contributions depend on the final-state lepton. The CP-conserving two-photon contribution to the electron mode is expected to be \((1 - 4) \times 10^{-12}\), based on \( K_L \rightarrow π^0\gamma\gamma \) data. Although suppressed in phase space, this contribution to the muon mode is comparable to the electron mode because of a term proportional to lepton mass. The CP-violating contribution to the electron mode is expected to be

\[
B(K_L \rightarrow π^0e^+e^-)_{CPV} = \left[ 15.3a_S^2 - 6.8\frac{Imλ_t}{10^{-4}}a_S + 2.8\left(\frac{Imλ_t}{10^{-4}}\right)^2 \right] \times 10^{-12} \tag{32}
\]

and the muon mode is suppressed by a factor of 5 due to phase space. \( a_S \) is an unknown parameter in the \( K_S \rightarrow π^0l^+l^- \) vector form factor, and \( λ_t = V_{ts}^∗V_{td} \).

The modes \( K_L \rightarrow π^0e^+e^- \) and \( K_L \rightarrow π^0μ^+μ^- \) have not as yet been observed; the current best limits on the branching ratios for \( K_L \rightarrow π^0l^+l^- \) were obtained by the KTeV experiment at FNAL; \( B(K_L \rightarrow π^0μ^+μ^-) < 3.8 \times 10^{-10} \) and \( B(K_L \rightarrow π^0e^+e^-) < 5.1 \times 10^{-10} \). These limits were based on 2 observed events in each case, and expected backgrounds of 0.87 ± 0.15 for the muon mode and 1.06 ± 0.41 for the electron mode.

The main backgrounds for the muon mode were estimated to be from \( μ^+μ^-γγ \) (0.37 ± 0.03) and \( π^+π^-π^0 \) (0.25 ± 0.09), in which both charged pions decay in flight. Of these, the former background could be irreducible and therefore of great concern. The decay \( K_L \rightarrow μ^+μ^-γγ \) proceeds via the Dalitz decay \( K_L \rightarrow μ^+μ^-γ \), with an internal bremsstrahlung photon. KTeV has detected 4 such events, with expected background of 0.16 ± 0.08 resulting in a branching ratio of \( B(K_L \rightarrow μ^+μ^-γγ) = (10.4^{+3.9}_{-2.5} ± 0.7) \times 10^{-9} \), consistent with the expectation of \( (9.1 ± 0.8) \times 10^{-9} \) obtained from QED and the measurement of \( K_L \rightarrow μ^+μ^-γ \). Such decays were analyzed by Greenlee as background to \( K_L \rightarrow π^0e^+e^- \). He showed that the background
from $e^+e^-\gamma\gamma$ can be suppressed by removing events with photon-photon invariant masses near the $\pi^0$ mass, and by constraining the energies and angles of accepted photons. Unfortunately, the criteria that can be used for suppressing $e^+e^-\gamma\gamma$ are not effective for the $\mu^+\mu^-\gamma\gamma$ channel because the invariant mass of the photons is restricted by the energy available in the latter decay. In KTeV, the $\pi^0$ mass resolution was expected to be 2.4 MeV; they applied a 2.5 $\sigma$ cut around the $\pi^0$ mass to obtain the estimated background of $0.37 \pm 0.03$ from $\mu^+\mu^-\gamma\gamma$. For any future experiment wishing to measure the polarization of the muon, a good range measurement of the muons will be required, which will most likely suppress the other backgrounds found in KTeV ($\pi^+\pi^-\pi^0$ with pion decay to muons, and $\pi^\pm\mu^\mp\nu$ with an accidental $\pi^0$). However, it seems unlikely that the background due to $\mu^+\mu^-\gamma\gamma$ can be lowered. The single-event sensitivity of the KTeV result is quoted as $7 \times 10^{-11}$; the signal to background ratio, assuming that only $\mu^+\mu^-\gamma\gamma$ will contribute in a future experiment, will therefore be around 1/5, if the standard-model signal is taken as $B(K_L \rightarrow \pi^0\mu^+\mu^-) \approx 5 \times 10^{-12}$.

The interesting direct CP-violating component must be extracted from any signal found for $K_L \rightarrow \pi^0l^+l^-$ in the presence of two formidable obstacles: 1) the theoretical uncertainty on contamination from indirect CP-violating and CP-conserving contributions and 2) the experimental background from the $l^+l^-\gamma\gamma$. A complete discussion of this situation is beyond the scope of this article. In other reviews, it is stated that, in the case of $K_L \rightarrow \pi^0e^+e^-$ other measurements may be needed to understand the direct CP-violating contribution: e.g., the branching ratio for $K_S \rightarrow \pi^0e^+e^-$ and the energy asymmetry between $e^+$ and $e^-$. The branching ratio measurement of the $\pi\mu\mu$ mode provides additional information, such as an alternative measurement of the contribution due to the single-photon and two-photon intermediate states. The measurement of muon polarization in the $\pi\mu\mu$ case could also provide important additional information. This can be seen as follows: the decay amplitude can be divided into scalar (S), pseudo-scalar (P), vector (V), and axial-vector (A) parts. Of these, the scalar piece, which comes from the two-photon intermediate state is CP-conserving and the indirect CP-violating contribution is mostly vector. The short-distance direct CP violating amplitude has both vector and axial-vector parts. The longitudinal muon polarization (asymmetry (7) in Table 1), while not strictly CP-violating, can arise only from interference of P and A with S and V, and therefore it provides information on the direct CP-violating amplitudes. Unlike for $K^+$, in the case of $K_L$ decay, all the amplitudes should be of the same order of magnitude, and polarization effects should therefore be very large ($\sim 1$), unless there are strong cancelations. The interference of scalar and vector components can give rise to transverse muon polarization (asymmetry (8) in Table 2). This was studied in the context of a CHPT calculation of $O(p^4)$. The transverse polarization is displayed in Fig. 3 as a function of $z = m_{\mu\mu}^2/m_K^2$, for several values of the renormalization parameter $w_s$. In this calculation, $\text{Im}(w_s)$ quantifies the direct CP-violation in the vector part of the matrix element. At the time of that study, the parameter $\text{Re}(w_s)$ was thought to be constrained by the
measurement of $K^+ \to \pi^+ e^+ e^-$. Later, it was argued that the constraint is model dependent, and $\text{Re}(w_s)$ should be considered as unknown as $a_S$. Although a new detailed calculation is needed, especially to incorporate the new knowledge of the CP-conserving contribution, it is clear that the transverse polarization can be large.

These large asymmetries should be easy to measure with sufficient statistics. This is no longer out of question, if one considers that the proponents of the BNL experiment KOPIO expect to measure $\sim 50 K_L \to \pi^0 \nu \bar{\nu}$ events. Measuring the muon polarization asymmetries in $K_L \to \pi^0 \mu^+ \mu^-$, together with the branching ratio and the lepton energy asymmetry, could be a good way of defeating the intrinsic background from CP-conserving and indirect CP-violating amplitudes and the experimental background from $\mu^+ \mu^- \gamma \gamma$.

7. Conclusion

Muon polarization from kaon decays has a rich phenomenology. In the case of $K_L \to \mu^+ \mu^-$, $K^+ \to \pi^+ \mu^+ \mu^-$ and $K_L \to \pi^0 \mu^+ \mu^-$, new measurements could lead to important constraints on the CKM parameters, in particular the Wolfenstein parameters $\rho$ and $\eta$. The experimental difficulties should be comparable to those facing the rare kaon decay measurements of $K^+ \to \pi^+ \nu \bar{\nu}$ and $K_L \to \pi^0 \nu \bar{\nu}$, which are considered the best modes for understanding short distance physics in the kaon sector (for a recent review, see Ref. 7). In particular, the flux available for the new experiment E926 for $K_L \to \pi^0 \nu \bar{\nu}$, could be sufficient for a measurement of polarization in $K_L \to \pi^0 \mu^+ \mu^-$. As shown in Tables 1 and 2, for many cases, limits on the muon polarization will probe new physics beyond the standard model. In particular, the polarization will be sensitive to the physics of a more complicated Higgs sector, or leptoquarks, that could give rise to CP or T violation from sources outside of the standard model.

We have examined the measurement of the out-of-plane muon polarization in $K^+ \to \mu^+ \pi^0 \nu$ and $K^+ \to \mu^+ \nu \gamma$ decays. Such measurements will not be sensitive to sources of CP violation in the standard model. Nevertheless, the measurements can be performed with sensitivity approaching $\delta P \sim 10^{-4}$ for $K^+ \to \mu^+ \pi^0 \nu$, and $\delta P \sim 10^{-3}$ for $K^+ \to \mu^+ \nu \gamma$. For $K^+ \to \mu^+ \pi^0 \nu$ decays, this is well beyond the current direct limit of $(4.2 \pm 4.9) \times 10^{-3}$, and the indirect limit of $\sim 10^{-3}$, available from other experimental constraints. Although the electric dipole moments of the neutron and electron are considered more favorably for T violation outside the standard model, they do not cover the entire spectrum of possibilities beyond the standard model. At the moment, the measurement of T-violating polarization in $K^+ \to \mu^+ \pi^0 \nu$ and $K^+ \to \mu^+ \nu \gamma$ decays is well justified and should be considered complementary to other efforts in understanding CP violation.

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Figure 3: Up-down asymmetry in $K_L \rightarrow \pi^0 \mu^+ \mu^-$ decay as a function of $z$ (defined as $z = m_{\mu\mu}^2/m_K^2$), for two possible values of Re$(w_s)$ and three different values of Im$(w_s)$ covering the expected ranges for this parameter. The different curves correspond to the following values of (Re$(w_s)$, Im$(w_s)$): double-dot-dashed (0.73, $-10^{-3}$), long-dashed (0.73, 0), dot-dashed (0.73, $+10^{-3}$), dashed (-1.00, $-10^{-3}$), solid (-1.00, 0), dotted (-1.00, $+10^{-3}$), (after Ecker, Pich, and de Rafael[23]).
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