Energy conditions in $f(R)$ gravity and Brans-Dicke theories

K. Atazadeh, A. Khaleghi, H. R. Sepangi and Y. Tavakoli
Department of Physics, Shahid Beheshti University, Evin, Tehran 19839, Iran

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Abstract

The equivalence between $f(R)$ gravity and scalar-tensor theories is invoked to study the null, strong, weak and dominant energy conditions in Brans-Dicke theory. We consider the validity of the energy conditions in Brans-Dicke theory by invoking the energy conditions derived from a generic $f(R)$ theory. The parameters involved are shown to be consistent with an accelerated expanding universe.

1 Introduction

Recent observations have revealed that the present state of the universe is undergoing an accelerated expansion [1]. There are, in general, a number of different approaches towards explaining this acceleration. One such approach utilizes what is known as $f(R)$ modification of gravity which, in effect is equivalent to Brans-Dicke (BD) type theories. The assumption of the existence of dark energy is another approach often used in this respect. In all the above theories, energy conditions impose stringent constraints whose validity should be studied in the light of their ability to explain the observational data. From a theoretical viewpoint, energy conditions in their various forms, namely strong energy condition (SEC), weak energy condition (WEC), dominant energy condition (DEC), and null energy condition (NEC) have been used in different contexts to derive general results that would hold for a variety of situations [2]. For example, the Hawking-Penrose singularity theorems invoke the WEC and SEC [3], whereas the proof of the second law of black hole thermodynamics requires the NEC [4]. Another example comes from cosmology [5] where energy conditions are studied by using red shifts.

The equivalence between BD theories and $f(R)$ gravity is a subject that has been studied by various authors, as an example see [6]. The study of energy conditions may thus benefit from such equivalence in that knowing the energy conditions in one theory would point to the energy conditions in the other. For example, in the BD theory case $V(\phi) = V_0 \phi^2$ [7] where $V_0$ cannot be found easily, one can use the equivalent $f(R)$ theory to facilitate the calculation of $V_0$ which, could then be used in the BD theory.

*email: k-atazadeh@sbu.ac.ir
†email: ahadkhaleghi@gmail.com
‡email: hr-sepangi@sbu.ac.ir
§email: y-tavakoli@std.sbu.ac.ir
Even though the goal of this paper is to study energy conditions in modified theories of gravity and consequently in Brans-Dicke theory, much of the techniques will be borrowed from the analysis of energy conditions in Einstein’s gravity. Therefore, we shall briefly review the derivation of energy conditions in general relativity. For a pedagogical review, see for example [3].

2 Energy conditions in general relativity

Let $\mathbf{\nu}^\mu$ be a tangent vector to a congruence of time-like geodesics. For a hypersurface orthogonal congruence, Raychaudhuri’s equation reads

$$\frac{d\theta}{d\tau} = -\frac{1}{3}\theta^2 - \sigma^\mu_{\nu\sigma}^\mu_{\nu} - R^\mu_{\nu\sigma}^\mu_{\nu},$$

(1)

where $\theta$ and $\sigma^\mu_{\nu\sigma}$ are the expansion and sheer of two nearby tangent vectors, respectively. In Einstein’s gravity, SEC and the ensuing singularities theorem follow from requiring that

$$R^\mu_{\nu\sigma}^\mu_{\nu} = 8\pi G(T^\mu_{\nu\sigma} - \frac{1}{2}g^\mu_{\nu\sigma}T)v^\mu v^\nu \geq 0,$$

(2)

for all time-like $v^\mu$ for which $\frac{d\theta}{d\tau} < 0$. A pair of nearby time-like geodesic vectors converge and will eventually intersect. The stress-energy tensor at each point $p \in M = \mathbb{R}^4$ obeys the inequality $T^\mu_{\nu\sigma}^\mu_{\nu\sigma} \geq 0$ for any time-like vector $v^\mu \in T_p$ of an observer whose world-line at $p$ has the unit tangent vector $\mathbf{v}$ and the local energy density appears to be $T^\mu_{\nu\sigma}^\mu_{\nu\sigma}$. This assumption is thus equivalent to the energy density being non-negative as measured by any observer which, of course, is physically reasonable.

3 Energy condition in $f(R)$ gravity

In this section we use the metric formalism in $f(R)$ gravity and derive the strong, weak and dominant energy conditions for a general form of $f(R)$. In doing so we will follow the formalism recently developed in [8]. We take the Freedman-Roberston-Walker (FRW) metric to study the cosmological implications of the models studied here.

The action for $f(R)$ gravity is [9]

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g}f(R) + S_m,$$

(3)

where we have set $\kappa = 8\pi G = 1$. The field equations resulting from this action in the metric approach, assuming the connections are that of the Levi-Civita, are given by

$$G^\mu_{\nu\sigma} = R^\mu_{\nu\sigma} - \frac{1}{2}g^\mu_{\nu\sigma}R = T^c_{\mu\nu} + \tilde{T}_\mu^\nu,$$

(4)

where $\tilde{T}_\mu^\nu = \frac{T^\mu_{\nu\sigma}}{f'(R)}$ represents the energy-momentum tensor of ordinary matter considered as perfect fluid given by

$$T^m_{\mu\nu} = (\rho + p)u^\mu u^\nu + pg^\mu_{\nu},$$

(5)

and $T^c_{\mu\nu}$ is the stress energy tensor of the gravitational fluid

$$T^c_{\mu\nu} = \frac{1}{f'(R)} \left[ \frac{1}{2}g^\mu_{\nu\sigma}f(R) - Rf'(R) \right] + (g^\alpha_{\mu\sigma}g^\beta_{\nu\sigma} - g^\mu_{\nu\sigma}g^\alpha_{\beta\sigma})\nabla^\alpha \nabla^\beta f'(R),$$

(6)

where a prime represents differentiation with respect to $R$. The field equation (4) now reads

$$f'(R)R^\mu_{\nu\sigma} - \frac{1}{2}f(R)g^\mu_{\nu\sigma} - (\nabla^\mu \nabla_{\nu\sigma} - g^\mu_{\nu\sigma}\Box) f'(R) = T^m_{\mu\nu}.$$
Contracting the above equation we obtain
\[ f'(R)R - 2f(R) + 3\Box f'(R) = T^m. \] (8)

Now, let us briefly review the energy conditions in \( f(R) \) gravity. We begin by defining an effective stress-energy tensor using equation (7) as follows
\[ T^\text{eff}_{\mu\nu} = \frac{1}{f'(R)} \left( T^m_{\mu
u} + \frac{1}{2} \left( f(R) - Rf'(R) \right) g_{\mu\nu} + \left( \nabla_\mu \nabla_\nu - g_{\mu\nu} \Box \right) f'(R) \right), \] (9)
with
\[ T^\text{eff} = \frac{1}{f'(R)} \left( T^m + 2f(R) - Rf'(R) \right) - 3\Box f'(R) \right]. \] (10)

Therefore, we can write \( R_{\mu\nu} \) in the terms of an effective stress-energy tensor and its trace, that is
\[ R_{\mu\nu} = \kappa \left( T^\text{eff}_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T^\text{eff} \right). \] (11)

Using the spatially flat \((k = 0)\) FRW metric as
\[ ds^2 = -dt^2 + a(t)^2 dx^2, \] (12)
the effective energy density and pressure are given by
\[ \rho^\text{eff} = \frac{1}{f'(R)} \left( \rho - \frac{1}{2} \left( f(R) - Rf'(R) \right) - 3H \dot{R} f''(R) \right), \] (13)
and
\[ p^\text{eff} = \frac{1}{f'(R)} \left( p + \frac{1}{2} \left( f(R) - Rf'(R) \right) + 2\dot{H} + \ddot{R} \right) f''(R) + \dot{R}^2 f'''(R) \right), \] (14)
where \( \dot{R} = dR/dt \) and \( H(t) = \frac{\dot{R}(t)}{a(t)} \) is the Hubble parameter. Now, using these equations, we can write the NEC and SEC, given by \( \rho^\text{eff} + p^\text{eff} \geq 0 \) and \( \rho^\text{eff} + 3p^\text{eff} \geq 0 \) respectively as
\[ \rho + p + (\dot{R} - \ddot{H}) f''(R) + \dot{R}^2 f'''(R) \geq 0 \] (15)
and
\[ \rho + 3p + (f(R) - Rf'(R)) + 3(\ddot{R} + \dot{H}) f''(R) + 3\dot{R}^2 f'''(R) \geq 0. \] (16)

To compare our results here with that of general relativity for a given \( f(R) \), we use the FRW metric which, for WEC \((\rho^\text{eff} \geq 0)\) leads to
\[ \rho - \frac{1}{2} (f(R) - Rf'(R)) - 3H \dot{R} f''(R) \geq 0. \] (17)

For DEC \((\rho^\text{eff} - p^\text{eff} \geq 0)\), we find
\[ \rho - p - (5\dot{R}H + \ddot{R}) f''(R) - \dot{R}^2 f'''(R) - (f(R) - Rf'(R)) \geq 0. \] (18)

4 Energy conditions in Brans-Dicke theory

Let us now investigate a non-minimally coupled self interacting scalar-tensor field theory such as the Brans-Dicke (BD) theory and find the various energy conditions for this type of modified gravity. In the context of BD theory \([10]\) with a self interacting potential and a matter field, the action is given by
\[ S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[ \phi R - \frac{\omega}{\phi} \nabla^\alpha \phi \nabla_\alpha \phi - V(\phi) \right] + S_m, \] (19)
where $\omega$ is the usual BD parameter and we have chosen units such that $8\pi G = c = 1$. The gravitational field equations can be derived from action (18) by varying the action with respect to the metric

$$G_{\mu\nu} = \frac{T^m_{\mu\nu}}{\phi} + \omega \left( \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla^\alpha \phi \nabla_\alpha \phi \right) + \frac{1}{\phi} \left( \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi \right) - g_{\mu\nu} \frac{V(\phi)}{2} \phi,$$

(20)

where $T^m_{\mu\nu}$ is the stress-energy tensor of the normal matter as expressed in equation (5). Variation of action (19) with respect to $\phi$ gives

$$\Box \phi = \frac{T^m_{\mu\nu}}{2 \omega + 3} + \frac{1}{2 \omega + 3} \left[ \phi \frac{dV(\phi)}{d\phi} - 2V(\phi) \right],$$

(21)

where the expression $\Box \phi = g^{\mu\nu} \nabla_\mu \nabla_\nu \phi$, using (12) is given by

$$\Box \phi = -\left( \ddot{\phi} + 3H \dot{\phi} \right).$$

(22)

Using the equation of motion, we can write $G_{\mu\nu} = \kappa' \left( T^m_{\mu\nu} + T^\phi_{\mu\nu} \right)$ where $\kappa' = \frac{1}{\phi}$. The stress-energy tensor and its trace for the BD theory may now be calculated with the result

$$T^\phi_{\mu\nu} = \omega \frac{\dot{\phi}^2}{\phi} \left[ \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2} g_{\mu\nu} \nabla^\alpha \phi \nabla_\alpha \phi \right] + \left[ \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \Box \phi \right] - g_{\mu\nu} \frac{V(\phi)}{2},$$

(23)

and

$$T^\phi = \frac{\omega}{\phi} \left[ (\nabla \phi)^2 - 2 \nabla^\alpha \phi \nabla_\alpha \phi \right] - 3 \Box \phi - 2V(\phi).$$

(24)

Comparison of these equations with equation (2) leads to a similar equation for the SEC,

$$R_{\mu\nu\nu'} = \kappa' \left[ (T^m_{\mu\nu} + T^\phi_{\mu\nu}) - \frac{1}{2} g_{\mu\nu} (T^m + T^\phi) \right] \geq 0.$$

(25)

We can now write the relations for NEC and SEC analogously as $(\rho + \rho^\phi + p + p^\phi) \geq 0$ and $(\rho + \rho^\phi + 3(p + p^\phi)) \geq 0$ respectively [3] so that we may first derive $\rho^\phi$ and $p^\phi$ for the spatially flat FRW metric as follows

$$\rho^\phi = \frac{\omega}{2\phi} \dot{\phi}^2 - 3H \dot{\phi} + \frac{V(\phi)}{2},$$

(26)

and

$$p^\phi = \frac{\omega}{2\phi} \dot{\phi}^2 + (2H \dot{\phi} + \ddot{\phi}) - \frac{V(\phi)}{2}.$$

(27)

Thus, the NEC and SEC for the BD theory are given by

$$\rho + p + \frac{\omega}{\phi} \dot{\phi}^2 + (\ddot{\phi} - H \dot{\phi}) \geq 0,$$

(28)

and

$$\rho + 3p + \frac{2\omega}{\phi} \dot{\phi}^2 + 3(\ddot{\phi} + H \dot{\phi}) - V(\phi) \geq 0.$$

(29)

Now, following and expanding on the GR approach to include WEC and DEC, as has been employed in $f(R)$ gravity theories, we may obtain similar equations in the BD theory. Therefore, the WEC and DEC in the BD theory are respectively given by

$$\rho + \frac{\omega}{2\phi} \dot{\phi}^2 - 3H \dot{\phi} + \frac{V(\phi)}{2} \geq 0,$$

(30)

and

$$\rho - p - (\ddot{\phi} + 5H \dot{\phi}) + V(\phi) \geq 0.$$  

(31)
5 Equivalence of the energy conditions in $f(R)$ gravity and Brans-Dicke theory

Considering action (3) within the context of the metric formulation of $f(R)$ gravity, one can introduce a new field $\chi$ and write a dynamically equivalent action [6]

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[ f(\chi) + f'(\chi)(R - \chi) \right] + S_m(g_{\mu\nu}, \psi).$$  \hfill (32)

Variation with respect to $\chi$ leads to equation $\chi = R$ provided $f''(R) \neq 0$, which reproduces action (3). Redefining the field $\chi$ by $\phi = f'(R)$ and setting

$$V(\phi) = \phi \chi(\phi) - f(\chi(\phi)), \quad \hfill (33)$$

the action takes the form

$$S = \frac{1}{2\kappa} \int d^4x \sqrt{-g}(\phi R - V(\phi)) + S_m(g_{\mu\nu}, \psi).$$  \hfill (34)

Comparison with action (19) reveals that this is the action of a BD theory with $\omega = 0$. Therefore, metric $f(R)$ theories, as has been observed long ago, are fully equivalent to a class of BD theories with a vanishing kinetic term [6]. Now, taking $\chi = R$ and $\phi = f'(R)$ we have

$$\dot{\phi} = \dot{R}f''(R) \quad \text{and} \quad \ddot{\phi} = \ddot{R}f''(R) + \dddot{R}f''(R).$$  \hfill (35)

Substituting these relations into equations for NEC, SEC, WEC and DEC in BD theory, namely equations (28), (29), (30) and (31) with $\omega = 0$, one can easily derive respectively NEC, SEC, WEC and DEC for the $f(R)$ modification of gravity, that is equations (15), (16), (17) and (18).

5.1 Examples

To see how equation (17) can be used to put constraints on a given $f(R)$ and equivalently on the BD potential, let us examine two examples. First, we consider $f(R)$ as having a general power-law form, given by

$$f(R) = \sigma R^n.$$  \hfill (36)

Let us now concentrate on the vacuum sector i.e. $\rho = p = 0$. Substituting in equation (17) we have the following condition for WEC

$$\sigma(n - 1)(1 - nA) \geq 0,$$  \hfill (37)

where $A = \frac{q_0 - q_0^2}{(1 - q_0)q_0}$ and the deceleration ($q_0$), jerk ($j_0$) and snap ($s_0$) parameters for the present-day values are defined in [8]. I what follows, we examine two values of the exponent $n$, namely $n = -1$ and $n = 2$ which satisfy the inequality (37).

Case I: $n = -1$

Taking $q_0 \sim -0.81$ and $j_0 \sim 2.16$ given in [8], so that $A \sim 0.29$. Equation (37) for $n = -1$ reduces to

$$-2.58\sigma \geq 0.$$  

This relation is satisfied for $\sigma < 0$ which in turn satisfies equation (37) with $n = -1$. The potential in BD theory with $\omega = 0$ for $V(\phi) = V_0\phi^m$, corresponding to $f(R) = \sigma R^n$ can thus be obtained simply as

$$V_0 = \frac{(n - 1)\sigma}{(n\sigma)^m},$$

where $m = \frac{n}{n - 1}$, so that for $n = -1$ the corresponding BD potential is $V(\phi) = -2\sqrt{-\sigma}\phi^{1/2}$.

Case II: $n = 2$

As a second case we consider $n = 2$, so that $f(R) = \sigma R^2$, leading to a WEC given by $-0.06\sigma \geq 0$ which requires $\sigma < 0$. The corresponding BD potential is $V(\phi) = \frac{1}{4\sigma}\phi^2$ with $\sigma$ being negative. One can therefore come to the conclusion that $V_0$ must also have a negative value.
6 Redshift and energy conditions

Let us now take the potential $V(\phi) = V_0 \phi^m$ in BD theory with $\omega = 0$ and use the following power-low ansätze
\[
\phi = \phi_0 \left( \frac{t}{t_0} \right)^\beta \quad \text{and} \quad a(t) = a_0 \left( \frac{t}{t_0} \right)^\alpha. \tag{38}
\]
Inserting these relations into equation (21) in the vacuum sector, one then finds that
\[
\beta = \frac{2}{1 - m}. \tag{39}
\]
Substituting the spatially flat FRW metric (12) in the field equations (4) we get
\[
3 \left( \frac{\dot{a}}{a} \right)^2 = \rho^{\text{eff}}, \tag{40}
\]
and
\[
2 \frac{\ddot{a}}{a} + \left( \frac{\dot{a}}{a} \right)^2 = -p^{\text{eff}}. \tag{41}
\]
Now, from equations (38) and (40) we can write
\[
\rho^{\text{eff}} = 3H_0^2(1 + z)^2, \tag{42}
\]
where we have used the relations $\frac{a(t_0)}{a(t)} = 1 + z$, $H_0 = \frac{\dot{a}(t_0)}{a(t_0)}$ and $\alpha = t_0H_0$ where $z$ is the redshift of a luminous source [11]. From this equation one may conclude that in the very early universe $\alpha$ would have been small and therefore $\rho^{\text{eff}}$ large. Now, the WEC is given by
\[
3H_0^2(1 + z)^2 \geq 0. \tag{43}
\]
This equation for the effective energy density clearly satisfies the WEC. If we follow the same method as in section 3 or in [8] and substitute the total energy density and pressure by $\rho^{\text{eff}}$ and $p^{\text{eff}}$, we will find the same relation between the distance modulus and redshift parameter which is studied in [11] where the energy conditions have been used. Now let us write the NEC, DEC and SEC with respect to the redshift parameter as follows
\[
\frac{2H_0}{t_0}(1 + z)^2 \geq 0, \tag{44}
\]
\[
(6\alpha - 2) \frac{H_0}{t_0}(1 + z)^2 \geq 0, \tag{45}
\]
\[
6(1 - \alpha) \frac{H_0}{t_0}(1 + z)^2 \geq 0, \tag{46}
\]
respectively. From equation (44) one can see that it is not possible to extract more information from NEC. As far as the DEC and SEC are concerned however, they require $\alpha \geq \frac{1}{3}$ and $\alpha \leq 1$ respectively. The first is in agreement with the observation that the universe is undergoing an accelerated expansion phase, $\alpha > 1$. A glance at the last equation reveals that it is completely in contradiction with an accelerated expanding universe, but we know that the SEC ensures gravity to be always attractive. Violation, as discussed in [12], allows for the late time accelerated cosmic expansion as suggested by the combination of recent astronomical observations.
7 Energy condition in the Einstein frame

What we have done so far in the previous sections has been in the so-called Jordan frame. However, it would also be instructive to study these relations in the Einstein frame. As is well known, the usual procedure, going from one frame to the other, is to use a conformal transformation. A problem then arises in that whether the tensor representing the physical metric structure of space-time is the one belonging to the Jordan frame or to the Einstein frame. However, what we are concerned with in this work is the relation between the energy conditions in these frames and will not deal with the question posed above. Under the conformal transformation

\[ \tilde{g}_{\mu\nu} \rightarrow e^{\phi} g_{\mu\nu}, \]  

(47)

and taking

\[ \phi = - \ln f'(R), \]  

(48)

the action (3) is rewritten as

\[ S_E = \int d^4x \sqrt{-g} \left( R - \frac{3}{2} g^{\alpha\beta} \partial_\alpha \phi \partial_\beta \phi - V(\phi) \right). \]  

(49)

Here

\[ V(\phi) = \frac{R}{f(R)} - \frac{f(R)}{f'(R)^2}. \]  

(50)

As a result, one finds the field equations for the metric in the form

\[ \tilde{G}_{\mu\nu} = \frac{3}{2} \nabla_\mu \phi \nabla_\nu \phi - \frac{3}{4} \tilde{g}_{\mu\nu} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi - \frac{1}{2} V(\phi) \tilde{g}_{\mu\nu}. \]  

(51)

Also, the equation of motion for \( \phi \) becomes

\[ \Box \phi = \frac{1}{3} \frac{dV}{d\phi}. \]  

(52)

For the FRW metric we find

\[ \Box \phi = - e^{-\phi} (\ddot{\phi} + \dot{\phi}^2 + 3H \dot{\phi}). \]  

(53)

In order to solve these equations for the case \( f(R) = \sigma R^n \) we use the following power-low ansätze

\[ \tilde{a}(\tilde{t}) \propto \tilde{t}^\beta \quad \text{ and } \quad \phi = \alpha \ln \tilde{t}, \]  

(54)

where \( \alpha \) and \( \beta \) are arbitrary constants. The Hubble parameter, \( \tilde{H} \), then reads

\[ \tilde{H} = \frac{\beta}{\tilde{t}}. \]  

(55)

Now, using equations (48) and (50) we have

\[ V(\phi) = \frac{n}{n^2 \sigma} \left( \frac{e^{-\phi}}{n\sigma} \right)^{\frac{2-n}{n-1}}. \]  

(56)

Matching the exponents of \( t \) in equation (51) we arrive at the following expression for \( \alpha \)

\[ \alpha = \frac{2(1-n)}{2n - 3}. \]  

(57)

If we define \( T^\phi_{\mu\nu} \) as

\[ T^\phi_{\mu\nu} = \frac{3}{2} \tilde{\nabla}_\mu \phi \tilde{\nabla}_\nu \phi - \frac{3}{4} \tilde{g}_{\mu\nu} g^{\alpha\beta} \tilde{\nabla}_\alpha \phi \tilde{\nabla}_\beta \phi - \frac{1}{2} V(\phi) \tilde{g}_{\mu\nu}, \]  

(58)
then we obtain the following relations for $\rho^\phi$ and $P^\phi$

$$\rho^\phi = \frac{3\dot\phi^2}{4} + \frac{1}{2}e^{\phi}V(\phi) \quad \text{and} \quad p^\phi = \frac{3a^2\dot\phi^2}{4} - \frac{1}{2}e^{\phi}V(\phi)a^2.$$  \hfill (59)

Now let us write the WEC, SEC, DEC, and NEC

$$\frac{3\dot\phi^2}{2} + e^{\phi}V(\phi) \geq 0,$$  \hfill (60)

$$\frac{3(1 + 3a^2)\dot\phi^2}{2} + e^{\phi}V(\phi)(1 - 3a^2) \geq 0,$$  \hfill (61)

$$\frac{3(1 - a^2)\dot\phi^2}{2} + e^{\phi}V(\phi)(1 + a^2) \geq 0,$$  \hfill (62)

$$\frac{3(1 + a^2)\dot\phi^2}{2} + e^{\phi}V(\phi)(1 - a^2) \geq 0.$$  \hfill (63)

Using our ansätze (54), we find the following expression for the WEC

$$\frac{6(1 - n)^2}{(2n - 3)^2} + \left(\frac{n - 1}{n^2\sigma}\right) \left(\frac{1}{n\sigma}\right)^\frac{2-n}{n-1} \geq 0.$$  \hfill (64)

As we can see in equation (64), the WEC always holds. However, in the case that either $n < 0$ or $\sigma < 0$, we have a constraint on $n$, namely that $n$ must be even. For the other energy conditions at late times, $a(t) \to \infty$, we have

$$6 \left(\frac{1 - n}{2n - 3}\right)^2 - \left(\frac{n - 1}{n^2\sigma}\right) \left(\frac{1}{n\sigma}\right)^\frac{2-n}{n-1} \geq 0 \quad \text{SEC and NEC},$$  \hfill (65)

$$6 \left(\frac{1 - n}{2n - 3}\right)^2 - \left(\frac{n - 1}{n^2\sigma}\right) \left(\frac{1}{n\sigma}\right)^\frac{2-n}{n-1} \leq 0 \quad \text{DEC}.$$  \hfill (66)

Equation (65) tells us that there is no guarantee that it remains positive. As can be seen, when the SEC and NEC hold, the DEC does not. For these equations we also have a constraint on $n$, that is, when either $n < 0$ or $\sigma < 0$, then $n$ must be even. In the case of $n < 0$ another constraint appears which is important for both equations. There are ranges of $n$ for which either SEC or NEC do not hold. Figure 1 shows this situation. Also, for the case $n > 0$, we see that the DEC does not hold at all, regardless of the sign of $\sigma$.

8 Conclusions

In this work we have studied the energy conditions in the BD theory and compared the results with that of $f(R)$ gravity, benefiting from the ease with which the parameters of interest can be derived in the latter and subsequently used in the former. This would help us to check the validity of the energy conditions in BD theory by invoking the energy conditions derived from a generic $f(R)$ theory. The parameters involved were shown to be consistent and compatible with an accelerated expanding universe.

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Figure 1: The behavior of SEC and NEC as a function of $n$ for $n < 0$ and $\sigma > 0$, left and of SEC and NEC for $n < 0$ and $\sigma < 0$, right.

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