1. INTRODUCTION

There are well-based, theoretical, smeared models for designing FRC beams [1, 2, 3, 4, 5 and 6], however, these models cannot be used well in the analysis of experimental results because of the high deviation of the data. The existing smeared models can provide only the approximate average of the load-displacement curves which does not fit well to the discrete results of the experimental samples. Another disadvantage of the existing models is that because of the high deviation of the input data numerous experiments have to be carried out to gain an appropriate approximation for the real load-displacement curves. To overcome these drawbacks of the existing models a new analytical beam model was suggested according to Tóth, Pluzsik and Juhász [7] which takes into consideration the real distribution of the fibres in the cross-section of the FRC beam. With the help of this new model experimental
beam results can be compared to each other by a fictive pull-out force eliminating the high deviation caused by the different amounts and distribution of fibres in the critical cross-section. The pull-out behaviour is one of the distinctive features of fibre reinforced concrete. The more we know the phenomenon of the pulling out the better we understand the mechanical behaviour of the FRC material. It can usually be observed that fibres are mainly pulled out rather than torn in the cracked cross-section of a FRC beam [7]. The higher ductility of FRC compared to plain concrete and the residual tensional strength after the first crack is due to the frictional stresses acting on the interface of the fibres and the concrete during the slipping of the fibres in the cracked zone.

In the new beam model according to Tóth, Pluzsik and Juhász [7] the fictive pull-out force was assumed to be constant. The aim of this work is to choose a more appropriate analytical model for the pull-out of synthetic fibres which can be built in the beam model to improve it and to gain a more precise approximation of the experimental load-displacement curves for FRC beams.

Few analytical models for the pull-out of synthetic fibres can be found in the literature [8 and 9]. The existing models are not supported by comprehensive experimental investigations. A more detailed analysis can be found for steel fibres [10], however, the question arises whether the existing models can be applied for synthetic fibres.

2. TESTING METHOD, EXPERIMENTAL RESULTS

Although the pull-out of fibres in a FRC beam is determined mainly by the bonding of the fibre surface and the mortar in the concrete, it is affected by a lot of other factors (aggregate type, porosity, efficiency of the compaction, chemical admixtures...). Thus, there is no simple, determinable relation between the concrete strength and the maximum pull-out force. To eliminate the uncertainties mentioned above, mortar samples were used in the experiments. The samples were prepared based on the requirements of EN 196-1 standards in the Miskolc Cement Laboratory of CRH. Two series of the samples were prepared with five different cement contents in both.

Table 1. Experimental samples

| Sign of the sample     | Cement content of the mix [g] | Limestone powder content of the mix [g] | Fibre type       | Anchoraged length [mm] | Anchoragement type   |
|------------------------|-------------------------------|----------------------------------------|------------------|------------------------|----------------------|
| 150_20                 | 150                           | 300                                    | waved/ribbed     | 20                     | one-sided/ two-sided |
| 225_20                 | 225                           | 225                                    | waved/ribbed     | 20                     | one-sided/ two-sided |
| 300_20                 | 300                           | 150                                    | waved/ribbed     | 20                     | one-sided/ two-sided |
| 375_20                 | 375                           | 75                                     | waved/ribbed     | 20                     | one-sided/ two-sided |
| 450_20                 | 450                           | -                                      | waved/ribbed     | 20                     | one-sided/ two-sided |
| 150_15/150_20/150_25  | 150                           | 300                                    | waved/ribbed     | 15/20/25               | one-sided            |
| 225_15/225_20/225_25  | 225                           | 225                                    | waved/ribbed     | 15/20/25               | one-sided            |
| 300_15/300_20/300_25  | 300                           | 150                                    | waved/ribbed     | 15/20/25               | one-sided            |
| 375_15/375_20/375_25  | 375                           | 75                                     | waved/ribbed     | 15/20/25               | one-sided            |
| 450_15/450_20/450_25  | 450                           | -                                      | waved/ribbed     | 15/20/25               | one-sided            |
Experimental Investigations of Pull-out Behaviour of Synthetic Fibres

The experiments took place in the polymer laboratory of TUB according to the measure method used in case of fibre reinforced plastic. The speed of the pull-out was 10 mm/min. The pull-out tester was the ZWICK/ROELL Z005 universal material testing machine. Because of the restriction of the length of this paper not all the experimental results are presented here, only representative samples are shown. The curves in the following figures are calculated as an average of 3–5 similar samples.

Two fibre types which had different surface characteristics (ribbed, waved) were examined (Fig. 1). In Fig. 1 the pull-out curves of the two types of fibres are compared in case of one-sided samples, with 300 g cement content and 20 mm anchoraged length. Although the ribbed fibres had higher maximal force in all cases, after the maximum force the waved fibres behaved more favourably in the descending part of the pull-out function. The speciality of ribbed fibres furthermore was the systematic jumps in the descending part of the curves as a result of the surface shaping. Aside from the above differences the curves for both fibre-types had similar main features in all examined cases. Ultimately, these fibre-types can be modelled by the same theoretical model.

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A one-side anchored sample can more easily be carried out in the laboratory, however, according to the literature [9] the phenomenon of the two-sided pull-out, which really occurs in the cracked cross-section of an FRC beam, has significantly different features. In the present experiment fifty two-sided and fifty one-sided samples were made with various cement contents as well (Table 1, Fig. 2) to examine the differences between the one-sided and two-sided phenomena.

In Fig. 3 as a typical example the one-sided and two-sided samples with 300 g cement content and 20 mm anchoraged length are compared to each other for both ribbed and waved fibres. The maximal forces of one-sided samples were higher, which can be caused by the eccentric pull-out. However, the main features of the curves were similar in all examined cases. Contrary to [9] the results of the present experiments show that the two-sided pull-out phenomenon can be modelled by using one-sided pull-out samples with the reduction of the maximum force.

Then the effect of the mortar strength (compressive strength) was examined.

Increasing the amount of cement in the mortar resulted in higher mortar strength (Fig. 4). The increase was approximately linear for lower cement content. In case of higher cement content the increase in the strength was smaller. However, the relationship between the maximal slipping force and the cement content (or the mortar strength as well) cannot be modelled linearly (Fig. 4). Moreover, in case of high cement content adding more cement resulted in lower maximal slipping force. The dispersion of the samples was high. In this experiment mortar samples were made in laboratory circumstances. In case of not mortar but concrete matrix the dispersion of the results would be even higher. So, there is no general rule to describe the relationship between the maximal slipping force and the cement content, laboratory test is required in all cases.

To examine the applicability of the existing theoretical models 150 one-side anchored samples were made with different mortar strength, fibre surface and anchoraged fibre length (Table 1).
3. THEORETICAL MODEL

Few analytical models for the pull-out of synthetic fibres can be found in the literature [8 and 9]. A more detailed analysis can be found for steel fibres [10]. In Fig. 7 the existing models are compared to the experimental results for the one-sided case, with 300 g cement content and 20 mm anchored length.

To choose the appropriate model, the relation of the maximal slipping force and the length of the anchored fibre segment were examined in Fig. 5 by comparing the theoretical curves with the experimental results. It can be seen in Fig. 5 that the suggestion of Zhan and Meschke [10] did not predict well the maximal slipping forces.

The model of [10] gave a better approximation for the maximal force (first part of the slipping). However, none of the existing analytical models could be fitted well to the descending part (second part of the slipping) of the experimental curves (an example is shown in Fig. 7). The theoretical curve of [10] was calculated by solving the differential equation (1, 2).

\[ dP(\xi) = nd\tau(\xi)d\xi, \quad P(\xi = L) = F \quad \text{and} \quad s(\xi = L) = u \]  

(2)

Where:

The feature of the resulting F-u curve depends on the friction law (τ-s relation) which is substituted in the differential equation in (1, 2). The friction law for steel fibres according to Zhan and Meschke [10] is given in (3, 5) and in Fig. 6. According to Lin, Li and Kanda [8] a constant \( t_0 \) is assumed while according to Wang and Backer [9] the function of the slipping stress is a second order parabola. Instead of these friction laws a modified model of Zhan and Meschke [10] is suggested in (4, 5). Substituting (4) to (1), the resulted F-u curve fits better the experimental curves (Fig. 7).

\[ \tau(s) = Gs, \text{if } s \leq s_0; \]

\[ \tau(s) = \tau_{\text{max}}, \text{if } s_0 < s \leq s_1 \quad \text{and} \]

\[ \tau(s) = \tau_0 + (\tau_{\text{max}} - \tau_0)\exp \left(\frac{s_1-s}{s_{\text{ref}}}\right), \text{if } s > s_1 \]  

(3)

\[ \tau(s) = \frac{G \mu s}{\sqrt{1 + \left(\frac{s}{s_{\text{ref}}}\right)^2}} \]

(5)

\[ L_{\text{crit}} = \frac{\tau_{\text{max}} - \tau_0}{\pi d_{\text{eff}} \tau_{\text{max}}} \]

(6)

Table 2 contains the parameters used in the calculation.

In the experiments some fibres were torn (Fig. 8). With the help of the modified theoretical model, the critical length of anchorage could be calculated (Table 3) (6).
4. CONCLUSIONS

Detailed experimental investigation was performed to examine the pull-out phenomenon of synthetic fibres. Contrary to the literature, the experiments revealed that the two-sided pull-out problem has the same features as the one-sided pull-out one, but the maximum pull-out force is bigger in the one-sided case. None of the available analytical models for synthetic fibres [8 and 9] could be fitted well to the experimental data. The model suggested for steel fibres according to Zhan and Meschke [10] also failed the prediction of the experimental results. This model [10] with a modified friction law (τ-s relation) is suggested to be used in beam modelling [7] to gain the most precise approximation not only for the pull-out of synthetic fibres but for the load-displacement curves for FRC beams. The suggested model gave acceptable approximation in all examined cases for different mortar strength, fibre surface and anchored fibre length.

5. LIST OF NOTATION

- $a$ [-]: free parameter of parabola
- $A$ [mm$^2$]: cross sectional area of fibre
- $df$ [mm]: diameter of fibre
- $E$ [MPa]: elastic modulus of fibre
- $E_m$ [MPa]: elastic modulus of mortar matrix
- $F_{\text{max}}$ [N]: maximal force
- $G$ [N/mm$^3$]: relative bond modulus
- $L$ [mm]: embedment length
- $L_{\text{crit}}$ [mm]: critical length of embedment
- $P$ [N]: Axial force of the fibre

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Table 3. Critical length of anchorage

| Fibre types  | Cement content [g] | $F_{\text{max}}$ [N] | $\tau$ [MPa] | $L_{\text{crit}}$ [mm] |
|--------------|--------------------|----------------------|--------------|-----------------------|
| Waved fibre  | 150                | 280                  | 2.5          | 45                    |
|              | 225                |                      | 3.4          | 33                    |
|              | 300                |                      | 4            | 28                    |
|              | 375                |                      | 4            | 24                    |
|              | 450                |                      | 4            | 28                    |
| Ribbed fibre | 150                | 180                  | 2            | 37                    |
|              | 225                |                      | 3.4          | 24                    |
|              | 300                |                      | 3.4          | 21                    |
|              | 375                |                      | 3.8          | 19                    |
|              | 450                |                      | 3.8          | 19                    |
$R/r[-]$: matrix-fibre size ratio

$s$ [mm]: slip (relative displacement)

$s_{\text{ref}}$ [mm]: reference slip

$u$ [mm]: displacement of the fibre

$v_m[-]$: Poisson’s ratio

$\xi$ [mm]: Local coordinate of the fibre

$\tau_0$ [MPa]: asymptotic frictional strength

$\tau_{\text{max}}$ [MPa]: bond strength

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