An easy reading of modern ether-drift experiments

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Abstract

Modern ether-drift experiments look for a preferred reference frame searching for modulations of the beat note of two optical resonators that might be induced by the Earth’s rotation. We present a compact formalism to evaluate the signal for most experiments where two arbitrary gaseous media fill the resonating cavities. Our predictions can provide useful hints to optimize the experimental set up and the data taking.

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1. In modern ether-drift experiments with optical resonators, the search for the possible existence of a preferred reference frame is performed by looking for modulations of the signal that might be induced by the Earth’s rotation. Descriptions of this important effect are already available in the literature. For instance, within the SME model [1] the relevant formulas are given in the appendix of Ref.[2] and for the RMS test theory [3] one can look at Ref.[4]. However, either due to the great number of free parameters (19 in the SME model) and/or to the restriction to a definite experimental set up, it is not always easy to adapt these papers to the various experimental conditions. For this reason, in this Letter, we will present a set of compact formulas that can be immediately used by the reader to evaluate the signal when two arbitrary gaseous media fill the resonating cavities. The formalism covers most experimental set up including the very recent type of experiment proposed in Ref.[5].

In our presentation one clearly understands that the Earth’s rotation enters only through two quantities, $v = v(t)$ and $\theta_0 = \theta_0(t)$, respectively the magnitude and the angle associated with the projection of the unknown cosmic Earth’s velocity $V$ in the plane of the interferometer. At the same time, our predictions can provide useful hints to optimize the experimental set up and the data taking.

2. Once the angle $\theta_0$ is conventionally defined when one of the arms of the interferometer is oriented to the North point in the laboratory (counting $\theta_0$ from North through East), we can immediately use the formulas given by Nassau and Morse [6]. These are valid for short-term observations, say 3-4 days, where there are no appreciable changes in the cosmic velocity due to changes in the Earth’s orbital velocity around the Sun so that the only time dependence is due to the Earth’s rotation.

In this approximation, introducing the magnitude $V$ of the full Earth’s velocity with respect to a hypothetic preferred frame $\Sigma$, its right ascension $\alpha$ and angular declination $\gamma$, we get

$$\cos z(t) = \sin \gamma \sin \phi + \cos \gamma \cos \phi \cos(\tau - \alpha)$$  \hspace{1cm} (1)

$$\sin z(t) \cos \theta_0(t) = \sin \gamma \cos \phi - \cos \gamma \sin \phi \cos(\tau - \alpha)$$  \hspace{1cm} (2)

$$\sin z(t) \sin \theta_0(t) = \cos \gamma \sin(\tau - \alpha)$$  \hspace{1cm} (3)

$$v(t) = V \sin z(t),$$  \hspace{1cm} (4)

where $z = z(t)$ is the zenithal distance of $V$, $\phi$ is the latitude of the laboratory and $\tau = \omega_{\text{sid}} t$ is the sidereal time of the observation in degrees ($\omega_{\text{sid}} \sim \frac{2\pi}{23\text{h}56\text{m}}$).
Let us now consider two orthogonal cavities oriented for simplicity to North (cavity 1) and East (cavity 2) in the laboratory frame. They are filled with two different gaseous media with refractive indices $N_i$ ($i=1,2$) such that $N_i = 1 + \epsilon_i$, and $0 \leq \epsilon_i \ll 1$. The frequency in each cavity is

$$\nu_i(\theta_i) = \bar{u}'_i(\theta_i) k_i$$

(5)

and the frequency shift is

$$\Delta \nu = \nu_1(\theta_1) - \nu_2(\theta_2)$$

(6)

In the above relations we have introduced the parameters $k_i$

$$k_i = \frac{m_i}{2L_i}$$

(7)

where $m_i$ are integers fixing the cavity modes, $L_i$ are the cavity lengths and $\bar{u}'_i(\theta_i)$ denote the two-way speeds of light, as measured in the Earth’s frame, $\theta_i$ being the angle between $\mathbf{V}$ and the axis of the $i$-th cavity.

Following the point of view of Refs.\[7,8,9\], that no observable Fresnel’s drag has ever been detected in the gaseous regime, we shall assume that the two speeds of light $\frac{c}{N_i}$ are seen isotropic in the preferred frame $\Sigma$. Using Lorentz transformations to connect to the Earth’s frame, one then obtains to $O(V^2/c^2)$ \[7\]

$$\bar{u}'_i(\theta) = \frac{c}{N_i}[1 - (A_i + B_i \sin^2 \theta)\frac{V^2}{c^2}]$$

(8)

with

$$A_i = \frac{N_i^2 - 1}{N_i^2}, \quad B_i = -\frac{3}{2} A_i$$

(9)

We emphasize that the structure in Eq.(8), although obtained in connection with Eqs.(9) by using Lorentz transformations, remains also valid under the more general assumptions of the RMS test theory \[3\]. As such, if $A_i$ and $B_i$ are considered as free parameters, it provides a physical framework that is equivalent to the RMS model.

Introducing the unit vectors $\hat{u}_i$ fixing the direction of the two cavities and the projection $\mathbf{v}$ of the full $\mathbf{V}$ in the interferometer’s plane one finds

$$V^2 \sin^2 \theta_i = V^2(1 - \cos^2 \theta_i) = V^2 - (\hat{u}_i \cdot \mathbf{v})^2$$

(10)

so that ($v = |\mathbf{v}|$)

$$V^2 \sin^2 \theta_1 = V^2 - v^2 \cos^2 \theta_0$$

(11)

and

$$V^2 \sin^2 \theta_2 = V^2 - v^2 \sin^2 \theta_0$$

(12)
Therefore, defining the reference frequency \( \nu_0 = \frac{ck_1}{N_1} \) and introducing the parameter \( \xi \) through

\[
\xi = \frac{N_1 k_2}{N_2 k_1}
\]

one finds the relative frequency shift

\[
\frac{\Delta \nu(t)}{\nu_0} = 1 - \xi + \frac{V^2}{c^2} [\xi (A_2 + B_2) - (A_1 + B_1)] + \frac{v^2(t)}{c^2} [B_1 \cos^2 \theta_0(t) - \xi B_2 \sin^2 \theta_0(t)]
\]

(14)

For a symmetric apparatus where \( N_1 = N_2, A_1 = A_2, B_1 = B_2 = B \) and \( \xi = 1 \), one finds

\[
\frac{\Delta \nu(t)}{\nu_0} = B \frac{v^2(t)}{c^2} \cos 2 \theta_0(t)
\]

(15)

On the other hand, for a non-symmetric apparatus of the type considered in Ref.\[5\] with \( L_1 = L_2 = L \), but where one can conveniently arrange \( N_1 = 1 \) (up to negligible terms) so that \( A_1 \sim B_1 \sim 0 \), denoting \( N_2 = \mathcal{N}, A_2 = A, B_2 = B, \frac{m_2}{m_1} = \mathcal{P} \), we find

\[
\frac{\Delta \nu(t)}{\nu_0} = 1 - \frac{\mathcal{P}}{\mathcal{N}} + \frac{\mathcal{P} V^2}{\mathcal{N} c^2} (A + B) - B \frac{\mathcal{P} v^2(t)}{\mathcal{N} c^2} \sin^2 \theta_0(t)
\]

(16)

To consider experiments where one or both resonators are placed in a state of active rotation (at a frequency \( \omega_{\text{rot}} \gg \omega_{\text{sid}} \)), it is convenient to modify Eq.\[14\] by rotating the resonator 1 by an angle \( \delta_1 \) and the resonator 2 by an angle \( \delta_2 \) so that the last term in Eq.\[14\] becomes

\[
\frac{v^2(t)}{c^2} [B_1 \cos^2 (\delta_1 - \theta_0(t)) - \xi B_2 \sin^2 (\delta_2 - \theta_0(t))]
\]

(17)

Therefore, in a fully symmetric apparatus where \( N_1 = N_2, A_1 = A_2, B_1 = B_2 = B \) and \( \xi = 1 \) and both resonators rotate, as in Ref.\[10\], setting

\[
\delta_1 = \delta_2 = \omega_{\text{rot}} t
\]

(18)

one obtains

\[
\frac{\Delta \nu(t)}{\nu_0} = B \frac{v^2(t)}{c^2} \cos 2 (\omega_{\text{rot}} t - \theta_0(t))
\]

(19)

On the other hand, if only one resonator rotates, as in Ref.\[11\], setting \( \delta_1 = 0 \) and \( \delta_2 = \omega_{\text{rot}} t \) one obtains the alternative result

\[
\frac{\Delta \nu(t)}{\nu_0} = B \frac{v^2(t)}{2c^2} [\cos 2 \theta_0(t) + \cos 2 (\omega_{\text{rot}} t - \theta_0(t))]
\]

(20)

By first filtering the signal at the frequency \( \omega = \omega_{\text{rot}} \gg \omega_{\text{sid}} \), the main difference between the two expressions is an overall factor of two.
3. Let us now return to the general case of a non-rotating set up Eq. (11). Using Eqs.(1-4) we obtain the simple Fourier expansion

$$\frac{\Delta \nu(t)}{\nu_0} = 1 - \xi + (f_0 + f_1 \sin \tau + f_2 \cos \tau + f_3 \sin 2\tau + f_4 \cos 2\tau)$$

(21)

where

$$f_0 = \frac{V^2}{c^2}[\xi(A_2 + B_2) - (A_1 + B_1) + B_1(\sin^2 \gamma \cos^2 \phi + \frac{1}{2} \cos^2 \gamma \sin^2 \phi) - \frac{1}{2} \xi B_2 \cos^2 \gamma]$$

(22)

$$f_1 = -\frac{1}{2} \frac{V^2}{c^2} B_1 \sin 2\gamma \sin 2\phi \sin \alpha$$

$$f_2 = -\frac{1}{2} \frac{V^2}{c^2} B_1 \sin 2\gamma \sin 2\phi \cos \alpha$$

(23)

$$f_3 = \frac{1}{2} \frac{V^2}{c^2} (B_1 \sin^2 \phi + \xi B_2) \cos^2 \gamma \sin 2\alpha$$

$$f_4 = \frac{1}{2} \frac{V^2}{c^2} (B_1 \sin^2 \phi + \xi B_2) \cos^2 \gamma \cos 2\alpha$$

(24)

Since the mean signal is most likely affected by systematic effects, one usually concentrates on the daily modulation. In this case, assuming that $f_1, f_2, f_3$ and $f_4$ can be extracted to good accuracy from the experimental data, one can try to obtain a pair of angular variables through the two independent determinations of $\alpha$

$$\tan \alpha = \frac{f_1}{f_2}$$

$$\tan 2\alpha = \frac{f_3}{f_4}$$

(25)

and the relation

$$\tan |\gamma| = \frac{|B_1 \sin^2 \phi + \xi B_2|}{|2B_1 \sin 2\phi|} \sqrt{\frac{f_1^2 + f_2^2}{f_3^2 + f_4^2}}$$

(26)

Notice that, since the ether-drift is a 2nd-harmonic effect, the pair $(\alpha, \gamma)$ cannot be distinguished from the pair $(\alpha + \pi, -\gamma)$. Notice also that two dynamical models that predict the same anisotropy parameters up to an overall re-scaling $B_i \to \lambda B_i$ would produce the same $|\gamma|$ from the experimental data.

Finally for a symmetric apparatus, where $B_1 = B_2 = B$ and $\xi = 1$, one obtains the simpler relation

$$\tan |\gamma| = \frac{1 + \sin^2 \phi}{2 \sin 2\phi} \sqrt{\frac{f_1^2 + f_2^2}{f_3^2 + f_4^2}}$$

(27)

where any reference to the anisotropy parameters drops out.

4. Summarizing: starting from the hypothetical observation of a non-trivial daily modulation of the signal in some ether-drift experiment, one might meaningfully consider the possibility of a preferred reference frame. For instance, for a symmetric apparatus one could try to extract from the data the product $K = B^2\frac{c^3}{V^2}$ and, using Eqs. (25) and (27), a pair
of angular values \((\alpha, \gamma)\). Of course, in this case, by suitably changing the gaseous medium within the cavities, one should also try to check the trend predicted in Eq. (9), namely

\[
\frac{K'}{K''} \sim \frac{N' - 1}{N'' - 1}
\]

(28)

On the other hand, for a non-symmetric apparatus of the type proposed in Ref. [5], where one can conveniently fix the cavity oriented to North to have \(N_1 = 1\) (up to negligible terms), by using Eqs. (9) one would predict \(B_1 \sim 0\) in Eqs. (23) and (24) so that all time dependence should be due to \(B_2\). Thus the modulation of the signal should be a pure \(\omega = 2\omega_{\text{sid}}\) effect with no appreciable contribution at \(\omega = \omega_{\text{sid}}\). This is another sharp prediction that should be preliminarily checked.

For a deeper analysis, it is important to recall that the ether-drift, if it exists, is a 2nd-harmonic effect. Therefore, in a single session, the direction \((\alpha, \gamma)\) cannot be distinguished from the opposite direction \((\alpha + \pi, -\gamma)\). For this reason, a whole set \(j=1,2,..M\) of short-term experimental sessions should be performed in different periods along the Earth’s orbit to obtain an overall consistency check.

Notice that for a complete description of the observations over a one-year period, it is not necessary to modify the simple formulas Eqs. (23) and (24) and introduce explicitly the further modulations associated with the orbital frequency \(\Omega_{\text{orb}} \sim \frac{2\pi}{1\text{ year}}\). Rather, by plotting on the celestial sphere all directions defined by the various \((\alpha_j, \gamma_j)\) pairs obtained in the various short-term observations one can try to reconstruct the Earth’s “aberration circle”. If this will show up, one can determine the mean magnitude of the cosmic velocity \(\langle V \rangle\) from the angular opening of the circle and from the known value of the orbital Earth’s velocity \(\sim 30\) km/s. In this way, given the value of \(\langle K \rangle\), one will be able to disentangle \(\langle V \rangle\) from \(B\) and get a definite test of models that predict the absolute magnitude of the anisotropy parameter.
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