Steel Beams and their Plastic Bending-share Resistance

Pavol Juhas1, Ingrid Juhasova Senitkova1

1Institute of Technology and Business in České Budějovice, Department of Civil Engineering, Okružní 517/10, 370 01 České Budějovice, Czech Republic

14667@mail.vstecb.cz

Abstract. The theoretical analysis, experimental investigation and calculation of the plastic bending-shear resistance of the steel cross-section beams are presented within the paper. The original theoretical analysis of the plastic bending-shear resistance of the hybrid steel beams with the compact I cross-sections is also presented. They can have cross-sections combined from different steels, where flanges are forms of the higher strength steels and the web is from the middle steel. The theoretical calculations take into consideration the actual international standards for the design of the steel structures. The experimental program has comprised the tests of 24 simple rolled beams with IPE160 cross-section. The obtained experimental results are evaluated and compared with the corresponding theoretical results.

1. Introduction

Cross-sections of steel beams are usually subjected simultaneously to bending and shear. In these cases, their resistance depends on the efficiency of bending and shear loading. The bending loading is mostly decisive. But in some cases, the shear loading can be also important or even decisive. Therefore, the shear effect of the loading should be evaluated and considered for the safe and economical design of steel beams.

The identification of elastic bending–shear resistance of the steel beams is not difficult by simple bending theory and adequate strength theory. From supposing distribution of the normal stresses \(\sigma\) and shear stresses \(\tau\) across decisive cross-section is possible to accurately define the final stress distribution and the elastic bending–shear resistance of the steel beams.

But in the elastic-plastic region of loading, distribution of the normal stresses \(\sigma\) and shear stresses \(\tau\) across decisive cross-section is not cleared, and so the consequent determination of the bending-shear resistance of the steel beams is rather complicated. Many theoretical and experimental works have been done for analysis of the elastic-plastic behaviour of steel beams subjected simultaneously to bending a shear. For the representative knowledge and results about the real elastic-plastic mechanisms of behaviour and failure of the steel cross-sections beams are very important from scientific and practical aspects [1, 2].

The paper contains selected information and results about realized theoretical analysis, experimental investigation and calculation of the elastic-plastic bending-shear resistance of the steel cross-sections beams.
2. Theoretical analysis

The theoretical analysis assumes typical welded compact I cross-section. It means that the web and flanges of the cross-section are stable and simple bending theory can be used. For theoretical analysis is also assumed:

- Ideal elastic-plastic material
- Huber-Misses-Hencky strength theory of the constant deformation energy
- The normal stresses $\sigma$ along cross-section are linear changes. The shear stresses $\tau$ are assumed constant along the web of the cross-section, in flanges there can be reached normal stress $f_y$, where $f_y$ is the yield stress in tension.

The full plastic bending moment $M_{pl}$ may be assumed in analysed compact cross-section. The shear loading influences the plasticization of the cross-section and reduces its plastic bending moment $M_{pl}$. Then the problem is how to assume the plastic stress distribution in the cross-section and how to calculate its reduced bending moment $M_{u,V}$.

More theoretical solutions of the plastic bending-shear load-carrying capacity of the steel cross-sections exist at present. All these theoretical solutions are based on specific assumes of the stress distribution in the limit plastic stage. The appropriate interactive equations of the bending-shear resistance are derived for assumed distribution of the normal and shear stresses $\sigma$ and $\tau$ [1, 3, 4].

The simplest cases of stress distribution in the plastic stage of loading are shown in Figure 1.

![Figure 1. Assumed distribution of the normal stresses $\sigma$ and shear stresses $\tau$](image)

In the first case (Figure 1a) the shear stresses $\tau$ are assumed along whole depth of the web, at the same time the shear stresses $\tau \leq f_s$, where $f_s$ is the yield stress in shear. Therefore, the normal stresses in the web at limit plastic stage $\sigma \leq f_y$. The normal stresses in the flanges can rich full yield stress $f_y$. The equivalent interactive equation has this form:

$$\frac{M}{M_{pl}} + \frac{M_{pl,V}}{M_{pl}} \left( \frac{2V}{V_{pl}} - 1 \right)^2 = 1$$ (1)
This interactive equation is applied at the actual European Standard EN 1993-1-1.

In the second case (Figure 1b) the shear stresses $\tau$ are assumed centered about the neutral axis of the cross-section, at the same time the shear stresses $\tau = f_s$. Therefore, the normal stresses in the odd partition of the web and in flanges at limit plastic stage $\sigma \leq f_y$. The equivalent interactive equation has form:

$$\frac{M}{M_{pl}} + \frac{M_{pl,w}}{M_{pl}} \left( \frac{V}{V_{pl}} \right)^2 = 1 \tag{2}$$

This interactive equation is applied at the international standard ISO 10721-1:1997.

Several theoretical solutions assume more exactly distribution of the normal stresses $\sigma$ and shear stresses $\tau$ in the plastic stage of the cross-section loading.

The original distribution of the normal stresses $\sigma$ and shear stresses $\tau$ through hybrid I cross-section in the elastic and plastic stage of the loading by the first author of this paper is presented in Figure 2.

Figure 2. Assumed distribution of the normal stresses $\sigma$ and shear stresses $\tau$ in the elastic stage (a) and plastic stage (b and c) over the hybrid I cross-section.

The ultimate plastic stage is here considered as the perfect plasticization of the web in cross-section. In accordance with this assumption two cases of the limit stage may occur in assumed I cross-section [1, 4]:

- the whole cross-section is plasticised, the bending moment $M_{u,V} \geq M_b$, (Figure 2b)
- the web of cross-section only is plasticised, the bending moment $M_{u,V} < M_b$, (Figure 2c).

$M_b$ means the boundary bending moment of the cross-section at simultaneous plasticisation of the web and flanges.
It is not generally known, which limit case is correct. Therefore, it is needed to determine the ultimate shear load $V_{u,M}$ for both cases. $V_{u,M}$ depends on bending moment $M$ acting in the cross-section. The ultimate bending moment $M_{u,V}$ depends on ultimate shear load $V_{u,M}$.

The equivalent equations for calculation of the ultimate shear load $V_{u,M}$ or limit ratios $V/V_{pl}$ were derived:

In the case, if the whole cross-section is plasticised, the following equation is valid:

$$\left(\frac{V}{V_{pl}}\right)^2 + \frac{V}{V_{pl}} \left(\frac{(1,5 + 9\gamma\delta^2_0)\delta^2_0 - 6\delta^2_0}{\delta^2_0 + 3\delta^2_0} - \frac{(2,25 + 9\gamma\delta^2_0,9m)\delta^2_0 - 3\delta^2_0}{9^2} \right) = 0$$

(3)

If the web of cross-section is plasticised only, then the following cubic equation is valid:

$$\sqrt{3}\left(\frac{2,25\delta_0 - 4,5\gamma\delta^2_0\delta_0}{9^3} \right) = 0$$

(4)

For dedicated $V/V_{pl}$ the ultimate bending moment $M_{u,V}$, or limit ratio $M/M_{pl}$ can be calculated from the following equation:

$$\frac{M}{M_{pl}} = \frac{V}{V_{pl}} \left(\frac{2}{3\sqrt{3}\delta_0} 1 + 6\gamma\delta^2_0 \right)$$

(5)

The presented theoretical solution and results are rather complicated for practical application. Practical design procedures and standards contain only simple interaction formulas for the calculation of bending-shear load-carrying capacity of steel beam cross-sections.

3. Experimental research and results

The experimental program has comprised the tests of 24 simple rolled beams, IPE160 cross-section, from steel S235. The experimental program by the loading was divided into two parts, the first part contents the tests with static loading of the beams (12 beams) and the second part contents the tests with repeated loading of the beams (12 beams). The paper presents some information and results from the first part of the experimental program only.

Considering the different shear effect of the loading the tested beams have various length or span, from 550 to 1800 mm. The beams by span were divided into four groups (NS11-NS13, NS21-NS23 NS31-NS33 and NS41-NS43. The spans for these beams were 550, 750, 1100 and 1800 mm. The real material properties of the tested beams were investigated by standard tension tests. Determined values are: $f_y = 292,88$ MPa, $f_u = 405,25$ MPa and $A = 35,45\%$.

The beams were loaded by two gradually increased concentrated loads. During the tests were measured strains $\varepsilon$ by tensometers (1 – 9) in the middle cross-section and deflections $v$ by electrical deflect meters in the middle cross-section and at the supports of the beams. The all tested beams were
also horizontally supported in the middle cross-section to prevent lateral buckling. The static scheme and measure arrangement of the tested beams is clear from Figure 3.

Every test continued till total failure. The behaviour of tested beams during loading is well characterized by measured strains $\varepsilon$ and deflection $v$ in the most loaded middle cross-section. In the elastic state, the ascertained dependences $F - \varepsilon$ and $F - v$ were linear, in the elastic-plastic and particularly in the plastic state were evident nonlinear. The failure of all tested beams inducted at state when the strains $\varepsilon$ and deflections $v$ rich large values. The failure of tested beams started up in consequence of continually plasticization and increasing strains $\varepsilon$ and deflections $v$ without marked stability effects. The local vertical deformations of the flanges in the places of concentrate loads but accrued, too.

![Figure 3. Static scheme and measure arrangement of the tested beams](image)

The typical failure mechanism of tested beams is presented in Figure 4 and the dependences $F - v$ for individual groups of the tested beams are presented in Figure 5.

![Figure 4. Loading and typical failure of the tested beams](image)
The all calculated relative theoretical limit loads ($F_{el,M}$ – bending elastic, $F_{el,MV}$ – bending-shear elastic, $F_{pl,M}$ – bending plastic, $F_{pl,V}$ – shear plastic, $F_{pl,MV,EN}$ – bending-shear plastic by EN 1993-1-1:1995 and $F_{pl,MV,ISO}$ – bending-shear plastic by ISO 10721-1:1997 for the individual groups of tested beams are assigned in Table 1. There are presented average values of ultimate experimental loads $F_{u,exp}$ for the individual groups of tested beams. The evaluation and comparison the theoretical limit loads $F_{pl,MV,i}$ by reflected standards with equivalent ultimate experimental ultimate loads $F_{u,exp}$ are presented in Table 2.

Table 1. Theoretical limit loads and experimental ultimate loads

|        | $F_{el,M}$ | $F_{el,MV}$ | $F_{pl,M}$ | $F_{pl,MV,EN}$ | $F_{pl,MV,ISO}$ | $F_{u,exp}$ |
|--------|------------|-------------|------------|----------------|-----------------|-------------|
| NS11-NS13 | 134.03     | 92.15       | 153.22     | 122.41         | 120.85          | 126.77      |
| NS21-NS23 | 92.60      | 74.73       | 105.83     | 98.30          | 92.63           | 110.02      |
| NS31-NS33 | 60.12      | 54.33       | 68.72      | 68.62          | 64.58           | 73.49       |
| NS41-NS43 | 35.49      | 34.18       | 40.57      | 40.59          | 39.65           | 47.03       |

Table 2. Theoretical limit loads and the experimental ultimate loads evaluation and comparison

|        | $F_{u,exp}/F_{pl,M}$ | $F_{u,exp}/F_{pl,MV,EN}$ | $F_{u,exp}/F_{pl,MV,ISO}$ |
|--------|-----------------------|---------------------------|-----------------------------|
| NS11-NS13 | 0.83                 | 1.04                      | 1.05                        |
| NS21-NS23 | 1.03                 | 1.12                      | 1.20                        |
| NS21-NS33 | 1.06                 | 1.07                      | 1.14                        |
| NS21-NS43 | 1.15                 | 1.16                      | 1.19                        |
| Average values | 1.10             | 1.10                      | 1.14                        |

4. Conclusions
Some differences result from comparing the theoretical analysis and corresponding standard bending – shear relations $M/M_{pl} - V/V_{pl}$. They are relatively significant, especially if the effect of the shear...
stresses is significant. The effect of shear loading on the deflection of the beams is evident from Figure 5.

The obtained experimental results confirm the reliability of the considered standard interaction relations. But they also pointed that the theoretical analyses and resulted interaction relations for standards and practical application should be more correct taking account the adequate experimental results and real behaviour of steel beams subjected to bending and shear in the elastic-plastic and plastic stage.

References
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