Measuring the Galactic Binary Fluxes with LISA: Metamorphoses and Disappearances of White Dwarf Binaries

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The space gravitational wave detector LISA is expected to detect $\sim 10^4$ of nearly monochromatic binaries, after $\sim 10$yr operation. We propose to measure the inspiral/outspiral binary fluxes in the frequency space, by processing tiny frequency drifts of these numerous binaries. Rich astrophysical information is encoded in the frequency dependencies of the two fluxes, and we can read the long-term evolution of white dwarf binaries, resulting in metamorphoses or disappearances. This measurement will thus help us to deepen our understanding on the strongly interacting exotic objects. Using a simplified model for the frequency drift speeds, we discuss the primary aspects of the flux measurement, including the prospects with LISA.

Introduction.— Galactic ultra-compact binaries (orbital periods less than $\sim 10$min) are secure and important observational targets for the space gravitational wave (GW) interferometer LISA [1, 2]. They are also promising systems for multi-messenger observations [3, 4]. By efficiently analyzing their data, we will be able to obtain fruitful information on strongly interacting exotic objects.

Most of these ultra-compact binaries would be detached white dwarf binaries (WDBs) and AM CVn-type systems, both emitting nearly monochromatic GWs [5–8]. The formers are at the inspiral phase ($\frac{\dot{f}}{f} > 0$, GW frequency), and eventually their less massive white dwarfs fill the Roche-lobes, initiating the mass transfer. In the basic picture, after this stage, the subsequent evolution bifurcates into two branches; survival or disappearance [9, 10]. If the mass transfer is stable, a WDB turns into an AM CVn-type system, and its frequency drifts $\frac{\dot{f}}{f}$, keeping the Roche-lobe overflow. If the mass transfer is unstable, a WDB merges shortly, possibly accompanying an explosion event (e.g. type Ia supernova). But, at present, our understanding on the bifurcation (e.g. branching ratio) is quite limited, due to the lack of observational knowledge and the theoretically formidable physics on the strongly interacting compact objects [11, 12].

Even operating LISA for ten years, we are unlikely to observe a single WDB merger in the Galaxy, given its estimated merger rate $O(10^{-2})$ yr $^{-1}$ [8]. However, after such an operation period, LISA will measure small frequency drifts $\frac{\dot{f}}{f}$ for $\sim 10^4$ of the ultra-compact binaries [2, 4].

In this letter, we propose to observationally determine the inspiral/outspiral binary fluxes, by using these swarm of binaries. We point out the importance of the frequency dependencies of the two fluxes, to statistically follow the destinies of the WDBs. Below, combining the basic picture for WDB evolution and a simplified model for the drift speed $\frac{\dot{f}}{f}$, we clarify the primary aspects of the flux measurement at $f \gtrsim 5$ mHz.

In fact, AM CVn systems are considered to be generated also from hybrid binaries of white dwarfs and non-degenerate helium stars. Since they will emits GWs at most $\sim 3$ mHz [8–7], we ignore this component below.

Binary fluxes.— In the basic picture, by tracing flows of inspiral binaries (see Fig. 1), we can easily understand their continuity equation in the frequency space, at the large number limit

$$\frac{\partial \rho_+(f, t)}{\partial t} + \frac{\partial \rho_+(f, t)}{\partial f} F_+(f, t) = \Sigma_{IJ}(f, t) - \Sigma_M(f, t) - \Sigma_T(f, t).$$  \hspace{1cm} (1)

Here $\rho_+$ is the number density of inspiral binaries and $F_+$ is their flux. The three non-negative quantities $\Sigma_{IJ}$, $\Sigma_M$ and $\Sigma_T$ are the injection, merger and turnover rates (in units of Hz$^{-1}$s$^{-1}$). In the basic picture, the outspiral flux $F_- (\leq 0)$ is sourced by the turnover rate (see Fig. 1) and described by $\frac{\partial \rho_-}{\partial t} + \frac{\partial \rho_-}{\partial f} F_- = \Sigma_T$, (ignoring potential disappearances after turnovers).

Our target band is $f \gtrsim 5$ mHz and almost all the WDBs there are expected to be generated at lower frequencies [8]. We thus put $\Sigma_{IJ} = 0$. Then, from Eq. (1), we have

$$F_+(f_1, t) - F_+(f_2, t) = \int_{f_1}^{f_2} \left[ \Sigma_T(f, t) + \Sigma_M(f, t) \right] df$$

$$+ \int_{f_1}^{f_2} \frac{\partial \rho_+}{\partial f} \left[ \rho_+(f, t) df \right] \hspace{1cm} (2)$$

The last terms is the correction caused by the time variation of the inspiral flux $F_+$. Considering the Galaxy-wide binary formation and the delay time distribution before chirping up to $f \sim 5$ mHz, the flux $F_+(f, t)$ (after suppressing the Poisson fluctuation) is expected to change slowly at the Hubble timescale $t_H \sim 10^{10}$yr [8, 12]. Then, in Eq. (2), the last correction term will be $\sim \frac{t_d}{t_H} \sim 10^{-4}$ times smaller than the term $F_+(f_1, t)$. Here $t_d \sim 10^3$yr is the characteristic transition time from $f_1$ to $f_2$ (above $\sim 5$ mHz). As we see later, the Poisson fluctuation of the fluxes (more than $\sqrt{10^{-4}} = 10^{-2}$) will completely mask the correction term of this level. We thus drop the time dependence of variables, and obtain

$$F_+(f_1) - F_+(f_2) = \int_{f_1}^{f_2} \left[ \Sigma_T(f) + \Sigma_M(f) \right] df \hspace{1cm} (3)$$

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imprinted in the frequency dependencies of the two fluxes \( F_{\pm}(f) \). The information of the merger and turnover is clearly embedded in the frequency dependencies of the two fluxes \( F_{\pm}(f) \).

and similarly \( F_{-}(f_1) - F_{-}(f_2) = - \int_{f_1}^{f_2} \Sigma_T(f) df \).

By observationally measuring the frequency dependencies of the two fluxes \( F_{\pm}(f) \), we can separately estimate the two rates \( \Sigma(f) \) and \( \Sigma_T(f) \) at some frequency resolutions. This is the central part of the present proposal. Note that the conservation equations have been sometimes used theoretically, mainly for estimating the number densities of binaries (see e.g. \[7, 13, 14\]). But we use these equations in a completely different way.

**Evolution of individual binaries.**—Next, to discuss the flux measurement more concretely, we introduce a simplified model for the drift speed \( \dot{f} \) \[3, 9\] (see also \[15–17\]). This model would be a workable approximation to the steady drifting binaries, except for the stages close to the merger or turnover frequencies. In fact, the binaries around the turn over \( f \approx 0 \) would show somewhat complicated time evolution \[18, 19\]. But these binaries individually have small contributions (\( \propto \dot{f} \)) to the overall fluxes \( F_{\pm}(f) \). We should also stress that, at actual observational measurement of the fluxes, we do not need detailed theoretical models for the drift speeds.

We first describe our simplified drift model for a circular inspiraling WDB \( (\dot{f} > 0) \). We denote its two initial masses by \( m_1 \) and \( m_2 \) with \( m_1 \leq m_2 \), and define the initial mass ratio \( q \equiv m_1/m_2 \leq 1 \).

Under the point particle approximation with the orbital separation \( a \), the GW frequency \( f \) is given by

\[
f = \pi^{-1}[G(m_1 + m_2)a^{-2}]^{1/2}
\]

and orbital angular momentum by

\[
J = G^{1/2}a^{3/2}(m_1 + m_2)^{-1/2}m_1m_2.
\]

Due to the angular momentum loss by GW emission, we have

\[
\frac{\dot{f}}{3f} = \frac{1}{2a} = \frac{\tilde{J}}{(J)_{gw}} = \frac{32G^{5/3}\pi^{8/3}M^{5/3}f^{8/3}}{5e^5} \equiv \tau^{-1}_{gw}
\]

with the chirp mass \( M \equiv (m_1m_2)^{3/5}(m_1 + m_2)^{-1/5} \).

We use the mass-radius relation \( r(m) \) for completely degenerate helium in \[20\] originally given by P. Eggleton (see e.g. \[21\] for thermal effects). The Roche lobe radius of the less massive one is roughly given by

\[
R_L \simeq 3^{-4/3}2a m_1^{1/3}(m_1 + m_2)^{-1/3}.
\]

It shrinks, as the orbital separation \( a \) decreases. Eventually the WD fills the Roche lobe at the separation with \( R_L = r(m_1) \). The GW frequency at this moment is given by

\[
f_R(m_1) = \frac{2^{3/2}}{9\pi} \sqrt{\frac{Gm_1}{r(m_1)^3}}
\]

as a function of the initial mass \( m_1 \) \[22\]. Now the less massive WD becomes a donor of the mass transfer to the more massive WD. If the initial mass ratio satisfies the following inequality

\[
q = \frac{m_1}{m_2} > \frac{3\zeta(m_1) + 5}{6},
\]

the mass transfer is unstable and two WDs merge \[5, 9\]. Here we assumed the conservative mass transfer and efficient angular momentum redistribution to the orbital component. The related physical parameters are not well understood at present \[11, 12, 14\], and our flux approach would provide us with useful information. For idealized cold Fermi gas at the non-relativistic limit, we have \( \zeta = -1/3 \) and \( q > 2/3 \) for Eq. \[5\].

In Fig. 2, with the green curve, we show the model prediction \( f \) for WDB with initial masses \( 0.35M_\odot + 0.42M_\odot \) and similarly \( f_{-}(f_1) - f_{-}(f_2) = - \int_{f_1}^{f_2} \Sigma_T(f) df \).

FIG. 1: The schematic picture for the inspiral/outspiral binary fluxes in the frequency space. The black curve shows the total inspiral flux \( F_+(f) = F_0^I(f) + F_+^T(f) \) that is subdivided into the merger (green) and turnover (orange) components. The latter generates the outspiral flux \( F_-(f) = -F_0^I(f) \) (blue). The information of the merger and turnover is clearly imprinted in the frequency dependencies of the two fluxes \( F_\pm(f) \).

FIG. 2: The drift speeds \( |\dot{f}| \) for the binaries with initial masses \( 0.35M_\odot + 0.46M_\odot \) (green) and \( 0.15M_\odot + 0.3M_\odot \) (orange: inspiral, blue: outspiral phases). The dotted and dashed curves show the estimations errors \( \Delta \dot{f} \) for corresponding binaries at \( d = 20kpc \) with the observational periods \( T_0 = 10yr \) and \( 4yr \).
mass transfer, the frequency derivative of the outspiral following [7] (23). For its mass distribution, we fix the initial rate of Eq. (5) [22]. For the accretor, we have

\[ \frac{\dot{f}}{f} = -\frac{3a}{2a} = \frac{3}{2} \left( \zeta(m_{1e}) - \frac{1}{3} \right) \left( \frac{\zeta(m_{1e})}{2} + \frac{5}{6} - \frac{m_{1e}}{m_{2e}} \right) t_{gw}^{-1} \]

where the chirp mass in \( t_{gw} \) should be evaluated with the two evolved masses.

In Fig. 2, we show \( \dot{f} \) for a WDB with initial masses 0.15\( M_\odot \) + 0.3\( M_\odot \) \((q = 1/2) \) and \( \zeta(m_{1e}) = -0.33 \). In the basic picture, this binary initially moves on the orange curve up to \( f_R(m_1) = 7.3\) mHz. With stable mass transfer, it starts outspiral along the blue curve. Due to the effects of the chirp mass and the second parenthesis in Eq. (7), the outspiral rate is much smaller than the inspiral rate. It takes \( 1.1 \times 10^6 \) yr for this binary to move from 5.0 mHz up to 7.3 mHz, and \( 5.9 \times 10^6 \) yr to go back from 7.3 mHz down to 5.0 mHz.

In reality, a relatively diffuse envelope of the donor could be stripped at the late inspiral phase [18, 19]. But this would not change the concept of the flux approach (e.g. by using an appropriate relation \( r(m) \) at the stage of interpreting the measured fluxes).

**Flux model.**—We now discuss the frequency dependence of the Galactic inspiral and outspiral fluxes. For the former, we put the total value

\[ F_+(3\text{mHz}) = 0.02\text{yr}^{-1} \]

at 3 mHz with no additional injection above this frequency [1]. This flux is divided into the merger and turn-over components \( F^M_+ \) and \( F^T_+ \) (see Fig. 1).

For the merger flux, we set \( F^M_+(3\text{mHz}) = 0.015\text{yr}^{-1} \), following [1] (\( \sim 10^3 \) larger than double neutron star stars [23]). For its mass distribution, we fix the initial ratio at \( q_M = 5/6 \) and assume a flat profile for \( m_1 \) in the range \( 0.25M_\odot \leq m_1 \leq 0.54M_\odot \). Here we set the massive end \( 0.54M_\odot \) so that the characteristic frequency \( f_R(0.54M_\odot) = 30\) mHz is close to the highest WDB frequency predicted in [1]. The lower end was chosen somewhat arbitrarily with \( f_R(0.25M_\odot) = 12\) mHz. Actually, the green curve in Fig. 1 shows the flux \( F^M_+(f) \) obtained for the present setting. The nearly straight-line structure at 12-30 mHz is due to the approximately linear relation \( f_R(m) \) in the relevant mass range.

For the turnover flux, we assume \( F^T_+(3\text{mHz}) = 0.005\text{yr}^{-1} \) as a model parameter [1, 2]. For its mass distribution, we fix \( q_T = 1/2 \) with a flat profile for \( m_1 \) in the range \( 0.15M_\odot \leq m_1 \leq 0.3M_\odot \). As a precaution, we set the lower end to the very small value with \( f_R(0.15M_\odot) = 7.3\) mHz which corresponds to the minimum turnover/merger frequency \( f_1 \) in Fig. 1. This frequency \( f_1 \) is important for the flux analysis and worth further study. We also have \( f_R(0.13M_\odot) = 6.3\) mHz and \( f_R(0.17M_\odot) = 8.2\) mHz for two different masses. The upper mass 0.3\( M_\odot \) was selected to match the highest frequency of the Galactic AM CVn-type system, \( f_R(0.3M_\odot) = 15\) mHz. In Fig. 1, the orange curve shows the resultant turnover flux \( F^T_+(f) \). The outspiral flux (blue curve) is given by \( F_-(f) = -F^T_+(f) \) in the simplified picture. We should comment that the magnitude of the turnover flux \( F^T_+(3\text{mHz}) \) is more uncertain than the merger flux \( F^M_+(3\text{mHz}) \). In a pessimistic model with an inefficient angular momentum redistribution [1, 2], the flux \( F^T_+(3\text{mHz}) \) could be two order of magnitude smaller than the value adopted above.

From the fluxes \( F_\pm(f) \) and the drift speeds \( \dot{f} \) for the composing binaries, we can evaluate the number densities of binaries per unit frequency interval \( \rho_\pm(f) = F_\pm(f)/\dot{f} \) with the number weighted mean drift speed \( \dot{f}_+ \). In Fig. 3, we present our numerical results. In contrast to the fluxes in Fig. 1, the magnitudes of the orange and blue curves are not the same, reflecting the difference between the inspiral/outspiral speeds \( \dot{f} \) as in Fig. 2. At \( f < f_1 = 7.3\) mHz, we have the well-known form \( \rho_+(f) \propto f^{-11/3} \) simply determined by Eq. (7) [1, 3]. Above 5 mHz, the total numbers of the inspiral and outspiral binaries are estimated to be 6700 and 11800. If we decrease the mass ratios \( q_M, q_T \) by 10% from the original setting \( (5/6, 1/2) \), the larger components \( m_2 \) are increased by 10%. With this modification, the profiles \( F_\pm(f) \) in Fig. 1 are unchanged, depending basically on the distribution of \( m_1 \). But the total numbers of binaries above 5 mHz shrink to 6200 and 10400 respectively for inspiral and outspiral binaries, because of higher chirp rates \( |\dot{f}| \).

**GW observation and flux measurement.**—We now discuss how to measure the two binary fluxes \( F_\pm(f) \) with LISA. Our basic procedure will be to firstly identify a large number of binaries by fitting their parameters including \( f \), and subsequently calculate the fluxes using the detected binaries.

The angular-averaged strain amplitude of a nearly monochromatic binary at the distance \( d \) is given by

\[ h = \frac{8(MM)^{5/3}r^{2/3}f^{2/3}}{5^{1/2}e^{8/3}d} . \] 

Note that for an outspiral binary with \( m_1e \ll m_2e \), we have \( M^{5/3} \sim m_1e m_2^{2/3} \sim m_1e m_2^{2/3} \) and the amplitude depends weakly on the assumption on the mass conser-
and depends strongly on \( T_o \) \([24]\). In Fig. 2, with the dotted and dashed curves, we show the errors \( \Delta \dot{f} \) for binaries identical to those in Fig. 4. For \( T_o \sim 10\)yr, LISA is likely to have a resolution \( \Delta \dot{f}/\dot{f} \lesssim 0.2 \) at \( f \gtrsim 5\)mHz, even for the smallest steady speed \( \dot{f} \) (blue curve).

Here we briefly comment on the potential signal overlapping. For each drifting binary, the number of fitting parameters is eight, and we need two frequency bins to determine them (using four complex numbers from two data channels) \([22]\). Therefore, the densities should be \( \rho_+ (f) + \rho_- (f) \lesssim T_o/2 \) for resolving binaries. As shown in Fig. 3, for \( T_o \gtrsim 4\)yr, the signal confusion would not be a fundamental problem at \( f \gtrsim 5\)mHz (see also \([12]\)).

Next we discuss how to estimate the inspiral flux \( F_+ (f) \) at a frequency \( f \). We can make almost the same argument for the outspiral flux \( F_+ (f) \). Let us suppose that there are altogether \( N_o^f \) inspiral binaries in the frequency range \([ f - \delta f/2, f + \delta f/2 ]\) with \( \delta f \ll f \). With their label \( i \), the inspiral flux can be estimated as

\[
F_+^i (f) = \sum_{i=1}^{N_o^f} \frac{\dot{f}_i}{\delta f}.
\]

As we mention earlier, the binaries around turnover \( \dot{f} \sim 0 \) individually have small contributions to this expression. We have the expectation values \( \langle N_o^f \rangle \sim \rho_+ (f) \delta f \) and \( \langle F_+^i (f) \rangle \sim F_+ (f) = \rho_+ (f) \dot{f} \omega_+ (f) \) with the mean inspiral speed \( \dot{f} \omega_+ (f) \). The latter \( \langle F_+^i (f) \rangle \) is independent of the width \( \delta f \), but it has a statistical fluctuation \( \Delta F_+^i (f) \) in actual data reduction, due to the finiteness of the sample. More specifically, we can write down

\[
\frac{\Delta F_+^i (f)}{F_+ (f)} \sim \frac{\langle N_o^f \rangle}{10\text{yr}} \frac{1 + \frac{\sigma_{sc}}{\dot{f}_i} + \frac{\sigma_{obs}}{\dot{f}_i}}{f_i}.
\]

The three terms in the last parenthesis originate from (i) the Poisson fluctuation of the sample number, (ii) the intrinsic scatter \( \sigma_{sc} \) of the speed \( \dot{f} \) and (iii) the typical magnitude \( \sigma_{obs} \) of the measurement error \( \Delta \dot{f} \). For \( f \gtrsim 5\)mHz and \( T_o \sim 10\)yr, the third one would be negligible (see Fig. 2). Assuming \( \sigma_{sc} \sim \dot{f}_i \), we have \( \Delta F_+^i (f)/F_+ (f) \sim (\langle N_o^f \rangle)^{-1/2} \) corresponding to the Poisson fluctuation. For example, with our model parameters, we have the inspiral and outspiral binaries of \( N_+ = 2050 \) and \( N_- = 3840 \) in the range \([5.5\)mHz, 6.5mHz\]. We thus measure the fluxes \( F_+ (\dot{f}) \) with Poisson fluctuations less than \( \sim 3\% \). Without injections, mergers and turnovers in \([3\)mHz, 6.5mHz\], we will have \( F_+ (3\)mHz) = \( F_+ (6\)mHz).

As discussed earlier around Eqs. (2) and (3), at \( f > \dot{f}_i \), we also want to finely resolve the frequency dependence of the flux \( F_+^i (f) \) by taking a small width \( \delta f \). But, at the same time, the statistical error should be suppressed.
We can take a balance by choosing the width as $\Delta F^e \simeq |F_+ (f + \delta f/2) - F_+ (f - \delta f/2)|$. Similarly considering the outspiral flux, we have the approximate solutions as

$$\delta f_\pm = \rho_\pm^{1/3} F^e_\pm^{1/3} |dF_\pm/df|^{-2/3}. \quad (14)$$

As shown in Fig. 3, in the range $dF_\pm/df \neq 0$, we have $\delta f_+ \sim 3$ mHz and $\delta f_- \sim 0.2-0.4$ mHz. For deriving Eq. (14), we approximately put $F_+ (f + \delta f/2) - F_+ (f - \delta f/2) \sim \delta f \cdot dF_+/df$. In Fig. 3, this introduces the sharp features, reflecting the discontinuities of $dF_+/df$ as seen in Fig. 1.

For these solutions $\delta f_\pm$, we have the corresponding numbers of binaries $\rho_\pm (f) \delta f_\pm \sim 1000-100$ in the 7-15 mHz range and smaller at higher frequencies. Therefore, the typical magnitude of the Poisson fluctuation is $\sim 10^{-1/2} \sim 0.1$.

Discussion.— In this letter, we proposed to measure the Galactic binary fluxes, by using $\sim 10^4$ of WDBs detected by LISA. By studying the frequency dependencies (closely related to initial mass $m_1$) of the fluxes at $f > f_1$, we can clearly follow how WDBs disappear or survive, affected by physical processes on strongly interacting exotic objects. To examine further details of the binary evolution beyond the basic picture, we could additionally use the distribution of the drift speeds $f$.

While untouched so far, the fluxes at $f < f_1$ would be also useful. We can check the stationary of the fluxes and study potential binary injections and disruptions there. To make a complete Galactic sample at relatively low frequency regime (e.g. $f \lesssim 5$ mHz), other space interferometers (e.g. Taiji [26] and TianQin [27]) could make important contributions, given the expected performance of LISA shown in Figs. 2 and 4. We can also employ Galactic structure models to correct the contributions of distant binaries that have too small rates $|f|$ or even too small amplitudes $h$ [12].

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