Comparison of Model Predictive and Reinforcement Learning Methods for Fault Tolerant Control

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Abstract: A desirable property in fault-tolerant controllers is adaptability to system changes as they evolve during systems operations. An adaptive controller does not require optimal control policies to be enumerated for possible faults. Instead it can approximate one in real-time. We present two adaptive fault-tolerant control schemes for a discrete time system based on hierarchical reinforcement learning. We compare their performance against a model predictive controller in presence of sensor noise and persistent faults. The controllers are tested on a fuel tank model of a C-130 plane. Our experiments demonstrate that reinforcement learning-based controllers perform more robustly than model predictive controllers under faults, partially observable system models, and varying sensor noise levels.

Keywords: Reinforcement learning control, model predictive control, fault tolerance, model-based control, hierarchical reinforcement learning

1. INTRODUCTION

Complex systems are expected to operate efficiently in diverse environments. A controller that can function within bounds despite malfunctions and degradation in the system, or changes in the environment is safer. With the rise in automation and increasing complexity of systems, direct or timely human supervision may not always be possible. Fault Tolerant Control (FTC) seeks to guarantee stability and performance in nominal and anomalous conditions.

Blanke et al. (1997) delineates the difference between fail-safe and fault-tolerant control in their treatment of faults. Fail-safe control employs robust control approaches to avoid degradation from faults in the first place. But it comes at the cost of additional redundancy. Fault-tolerance combines redundancy management with controller reconfiguration and adaptivity to avoid system-wide failures, while keeping the system operational (to the extent possible). A fault-tolerant controller allows for graceful degradation, i.e., it may be sub-optimal after encountering faults, but it will keep the system operational for extended periods of time.

Patton (1997) surveys constituent research areas of FTC. We explore supervision methods in this paper. Supervision uses a priori knowledge of system faults to choose optimal actions. Fault-tolerant supervision has been explored in several fields including probabilistic reasoning, fuzzy logic, and genetic algorithms. This paper focuses on the evaluation of two supervision approaches for FTC: model predictive (MP) and reinforcement learning (RL)-based control. We assume system models are reasonably accurate, the fault diagnoser is correct, and the system can reconfigure in response to the supervisor.

MP and RL-based control rely on sampling the state space in real-time to find a control policy. However, such FTC approaches may be subject to additional constraints due to faults and constraints on operation. Controllers can be limited by processing power, poor observability of states due to sensor failures, and loss of functionality because of faulty components. We explore how the two control approaches fare under various conditions.

Some work has been done in designing RL-based fault adaptive controllers. Lewis et al. (2012) provides a theoretical discussion of using RL in real-time to find a control policy that converges to the optimum. Liu et al. (2017) applies RL based control to a discrete time system and uses a fully connected shallow neural network to approximate the control policy.

The main contribution of this work is to benchmark RL-based controllers against MP control. We implement discrete-time MP control and propose two approaches for applying RL principles towards FTC. The controllers are tested on a model of fuel transfer system of a C-130 cargo plane where the controllers attempt to maintain a balanced fuel distribution in the presence of leaks.

The rest of the paper is organized as follows. Section 2 provides a background of MP and RL-based control. Section 3 discusses online and offline RL controller design. Our case study is presented in Section 4. The results are presented and discussed in Section 5. Sections 6 and 7
discuss future work, open problems in this domain, and the conclusions.

2. BACKGROUND

2.1 Model Predictive Control

Model Predictive Control (MPC) optimizes the current choice of action by a controller that drives the system state towards the desired state. It does this by modeling future states of the system+environment and a finite receding horizon over which to predict state trajectories and select the “best” actions. At each time step, a sequence of actions is selected up until the horizon such that a cost or distance measure to a desired goal is minimized. For example, for a system with state variables \((x_1, x_2, \ldots, x_N)\) and the desired state \((d_1, d_2, \ldots, d_N)\) the cost may be a simple Euclidean distance measure between the two vectors.

A model predictive controller operates over a set of states \(s \in S\), actions \(a \in A\), and a state transition model of the system \(T: S \times A \to S\). It generates a tree of state trajectories rooted at the system’s current state. The tree has a depth equal to the specified lookahead horizon for the controller. The shortest path trajectory, that is, the sequence of states and actions producing the smallest cost is chosen, and the first action in that trajectory is executed. The process repeats for each time step.

Garcia et al. (1989) discuss MPC in further detail particularly in reference to continuous systems. Abdelwahed et al. (2005) presents a case study for MPC of a hybrid system. The control algorithm for the discrete subsystem is essentially a limited breadth-first search of state space. They use a distance map, which uses the euclidean distance between the controller state and the closest goal state as the cost function. The MPC algorithm implemented for this work is adapted from Abdelwahed et al. (2005) and described in Algorithm 1.

2.2 Reinforcement Learning

Reinforcement learning is the learning of behaviour by an agent, or a controller, from feedback through repeated interactions with its environment. Kaelbling et al. (1996) divide RL into two broad approaches. In the first, the genetic programming approach, an agent explores different behaviours to find ones that yield better results. In the second, the dynamic programming approach, an agent estimates the value of taking individual actions from different states in the environment. The value of each state and action dictates how an agent behaves. This section explores methods belonging to the latter approach.

An agent (controller) is connected to (interacts with) the system+environment through its actions and perceptions. Actions an agent takes cause state transitions of the system in the environment. The change is perceived as a reinforcement signal from the system+environment, and the agents measurement of the new state.

The standard RL problem can be represented as a Markov Decision Process (MDP). A MDP consists of a set of states \(S\), a set of actions \(A\), a reward function \(R: S \times A \times S \to \mathbb{R}\) which provides reinforcement after each state change, and a state transition function \(T: S \times A \to \Pi(S)\), which determines the probabilities of going to each state after an action. This is the model for the system operating in an environment. For the purposes of this paper, the transition function is assumed to deterministically map \(s \in S\) and \(a \in A\) to the next state \(s' \in S\).

In a MDP, everything an agent needs from the environment is encoded in the state. The history of prior states and actions can exploit either value function to derive a policy that can exploit either value function to derive a policy that

\[
V(s) = \max_{a \in A} \{E[R(s,a,s') + \gamma V(s')]\},
\]

where \(\gamma\) is the discount factor that weighs delayed rewards against immediate reinforcement. Alternatively, an agent can learn the value of each action, the q-value, from a state:

\[
Q(s,a) = E[R(s,a,s') + \gamma \max_{a' \in A} Q(s',a')].
\]

We use the q-value formulation for this paper. An agent can exploit either value function to derive a policy that governs its behaviour during operation. To exploit the learned values, a policy greedily or stochastically selects actions that lead to states with the highest value:

\[
\text{Algorithm 1 Model Predictive Control}
\]

\begin{table}[h]
\centering
\begin{tabular}{ll}
\hline
Require: & system model \(T: S \times A \to S\) \\
Require: & distance map \(D: S \to \mathbb{R}\) \\
Require: & horizon \(N\) \\
\hline
1: & \(s_0 \leftarrow \text{CurrentState}\) \\
2: & Initialize state queue \(\text{Queue} = \{s_0\}\) \\
3: & Initialize state set \(\text{Visited} = \{s_0\}\) \\
4: & Initialize optimal state \(\text{Optimal} \leftarrow \text{Null}\) \\
5: & \(\text{MinDistance} \leftarrow \infty\) \\
6: & \text{while} \text{Queue.length} > 0 \text{ do} \\
7: & \quad \text{state} = \text{Queue.pop()} \\
8: & \quad \text{if} \text{state.depth} > N \text{ then} \\
9: & \quad \quad \text{break} \\
10: & \quad \text{end if} \\
11: & \text{newStates} = \text{state.neighbourStates} \cap \text{Visited} \\
12: & \text{for} \ s \in \text{newStates} \text{ do} \\
13: & \quad \text{Visited.add}(s) \\
14: & \quad \text{Queue.insert}(s) \\
15: & \quad \text{if} \ D(s) < \text{MinDistance} \text{ then} \\
16: & \quad \quad \text{MinDistance} = D(s) \\
17: & \quad \quad \text{Optimal} = s \\
18: & \quad \text{end if} \\
19: & \text{end for} \\
20: & \text{end while} \\
21: & \text{while} \text{OptimalState.previous} \neq s_0 \text{ do} \\
22: & \quad \text{Optimal} \leftarrow \text{Optimal.previous} \\
23: & \text{end while} \\
24: & \text{action} = T(\text{Optimal.previous}, \cdot) \to \text{Optimal}\n\end{tabular}
\end{table}
An improvement on $TD(\lambda)$ methods is achieved by making the return $G$ more representative of the state space. Values of actions are weighed by the agent’s action selection policy. This is known as $n$-step Tree Backup (Precup (2000)). Defining $\pi_{exp}(a|s)$ as the probability of exploratory actions, $s_T$ as the terminal time, and:

$$V_t = \sum_a \pi_{exp}(a|s_t)Q_{t-1}(s_t, a)$$

$$\delta_t = R_{t+1} + \gamma V_{t+1} - Q_{t-1}(s_t, a_t)$$

where $V_t$ is the expected value of $s_t$ under the action selection policy $\pi_{exp}$, and $\delta_t$ is the error between the return and prior value estimate. Then the $n$th-step return is given by:

$$G_{t}^{(n)} = Q_{t-1}(s_t, a_t) + \min_{n+1} \gamma \pi_{exp}(a_t|s_t) \sum_{k=1}^{n} \prod_{i=t+1}^{k} \gamma \pi_{exp}(a_t|s_i).$$

The value function can then be updated using the error $G_{t}^{(n)} - Q_{t-1}(s_t, a_t)$. A longer lookahead gives a more representative estimate of value. A greedy action selection policy with $n = 0$ reduces this to the $TD(0)$. The following section presents a modified $n$-step Tree Backup algorithm tailored for adaptive fault-tolerant control.

3. REINFORCEMENT LEARNING-BASED CONTROLLER DESIGN

Temporal difference based RL methods iteratively derive a controller’s behaviour by estimating the value of states and actions in a system through exploration. During operation, the controller exploits the knowledge gained through exploration by selecting actions with the highest value. This works well if the system dynamics is stationary. The agent is able to achieve optimal control by exploring over multiple episodes and iteratively converging on the value function. Conservative learning rates can be used such that the value approximation is representative of the agent’s history.

When a fault occurs in a system, its dynamics change. The extant value function may not reflect the most rewarding actions in the new environment model. The agent seeks to optimize actions under the new system dynamics. The agent must, therefore, estimate a new value function by exploration every time a fault occurs. The learning process can be constrained by deadlines and sensor noise. This paper proposes design of a RL-based controller subject to the following requirements:

Adaptivity: The controller should be able to remain functional under previously unseen faults (Goldberg et al. (1993)).

Speedy convergence: The agents behaviour should be responsive to faults as they occur.

Sparse sampling: The system model may lose reliability following a fault due to sensor noise or incorrect diagnosis. The agent should be able to calculate value

(1), known as the Bellman equation, and (2) can be solved recursively for each state using dynamic programming. However, the complexity of the solution scales exponentially with the number of states and actions. Various approaches have been proposed to approximate the value function accurately enough with minimal expenditure of resources.

Monte Carlo (MC) methods (Sutton and Barto (2017)) alleviate the Curse of Dimensionality by approximating the value function. Instead of traversing the entirety of state-space, they generate episodes of states connected by actions. For each state in an episode, the total discounted reward received after the first occurrence of that state is stored. The value of the state is then the average total reward across episodes. Generating an episode requires a choice of action for each state. The exploratory action selection policy $\pi_{exp}$ can be uniform, partially greedy with respect to the highest valued state ($\epsilon$-greedy), or proportional to the relative values of actions from a state (softmax). The MC approach still requires conclusion of an episode before updating value estimates. Additionally, it may not lend itself to problems that cannot be represented as episodes with terminal or goal states.

Temporal Difference (TD) methods provide a compromise between DP and MC approaches. Like MC, they learn episodically from experience and use different action selection policies during learning to explore the state space. Like DP they do not wait for an episode to finish to update value estimates. TD updates the value function iteratively at each time step $t$.

$$G_t = R_{t+1} + \gamma \max_{a \in A} Q_{t-1}(s_{t+1}, a)$$

$$Q_t(s_t, a_t) = Q_{t-1}(s_t, a_t) + \alpha (G_t - Q_{t-1}(s_t, a_t))$$

where $\alpha$ is the learning rate for the value function. $G_t$, called the return, is a new estimate for the value. The value function is updated using the error between the new estimate and the existing approximation. Note that $G$ estimates the new value by only using the immediate reward and backing up the discounted value of the next state. This is known as $TD(0)$. TD methods can be expanded to calculate better estimates of values by explicitly calculating rewards several steps ahead, and only backing up value estimates after that. These are known as $TD(\lambda)$ methods. For example, the return for $TD(2)$ is:

$$G_{t}^{(2)} = R_{t+1} + \gamma (R_{t+2} + \gamma \max_{a \in A} Q_{t-1}(s_{t+2}, a))$$

$$Q_t(s_t, a_t) = Q_{t-1}(s_t, a_t) + \alpha (G_{t}^{(2)} - Q_{t-1}(s_t, a_t))$$

In the extreme case, when $G$ is calculated for all future steps until a terminal state $s_T$, the TD method becomes a MC method because value estimates then depend on the returns from an entire episode.
estimates representative of its local state space from minimal (but sufficient) sampling. Generalization: The agent should be able to generalize its behaviour over states not sampled in the model during learning.

The temporal difference RL approach is inherently adaptive as it frequently updates its policy from exploration. Its responsivity can be enhanced by increasing the learning rate $\alpha$. In the extreme case, $\alpha = 1$ replaces the last value of an action with the new estimate at that time step. However, larger values of $\alpha$ may not converge the value estimate to the global optimum.

RL controllers have to balance exploratory actions to discover new optima in the value function against exploitative actions that use the action values learned so far. An exploration parameter $\epsilon \in [0, 1]$ can be set, which determines the periodicity of value updates and hence the controller’s responsivity to system faults.

Hierarchical RL can be used to sample the state space at various resolutions (Lampton et al. (2010)). An agent can use a hierarchy function $H : S \rightarrow \mathbb{R}^+$ that gives the step size of actions an agent takes to sample states. $H$ can yield smaller time steps for states closer to goal and coarser sampling for distant states. This allows the agent to update the value function more frequently with states where finer control is required.

TD RL methods can use function approximation to generalize the value function over the state space instead of using tabular methods. Gradient descent is used with the error computed by the RL algorithm to update parameters of the value function provided at design time. Function bases like radial, Fourier, and polynomial can be used to generalize a variety of value functions.

Algorithm 2 implements Variable n-step Tree Backup for a single episode. It updates the value estimate after exploring a sequence of states up to a finite depth $d$ of actions with varying step sizes. Value estimates for each state in the sequence depend on following states up to $n$ steps ahead. A RL-based controller explores multiple episodes to converge on a locally optimum value for actions. During operation it exploits the value function to pick the most valuable actions as shown in (3).

The following subsections discuss two approaches to approximating values for a RL-based controller.

### 3.1 Offline Control

An offline RL-based controller derives the value function by dense and deep sampling of the state space. The learning is done once each time a new value approximation is required. The agent learns from episodes over a larger sample of states in the model. Each episode lasts till a terminal state is reached. Offline control is computationally expensive. However, by employing a partially greedy exploratory action selection policy $\pi$ and a suitable choice of learning rate $\alpha$, it is liable to converge to the global optimum.

#### Algorithm 2 Variable n-Step Tree Backup Episode

1. Get action $a_0$ from $\pi_{exp}(s_0)$
2. $Q_0 \leftarrow Q(s_0, a_0)$
3. $t \leftarrow \infty$, $t \leftarrow 0$, $\tau \leftarrow 0$
4. while $t < d$ and $\tau < T - 1$
5. if $t < T$
6. Take action $a_t$ for $H(s_t)$ steps
7. $s_{t+1} \leftarrow T(s_t, a_t)$
8. $R_t \leftarrow R(s_t, a_t, s_{t+1})$
9. if $s_{t+1}$ is in goal states
10. $T \leftarrow t + 1$
11. $\delta_t \leftarrow R_t - Q_t$
12. else
13. $\delta_t \leftarrow R_t + \gamma \sum_a \pi_{exp}(a | s_{t+1})Q(s_{t+1}, a) - Q_t$
14. $a_{t+1} \leftarrow \pi_{exp}(s_{t+1})$
15. $Q_{t+1} \leftarrow Q(s_{t+1}, a_{t+1})$
16. $\pi_{t+1} \leftarrow \pi_{exp}(a_{t+1} | s_{t+1})$
17. $\tau \leftarrow t - n + 1$
18. if $\tau \geq 0$
19. $E \leftarrow 1$
20. $G \leftarrow Q_1$
21. for $k = \tau, \ldots, \min(\tau + n - 1, T - 1)$ do
22. $G \leftarrow G + E \cdot \delta_k$
23. $E \leftarrow \gamma E \cdot \pi_{k+1}$
24. end for
25. $Error \leftarrow Q(s_{\tau}, a_{\tau}) - G$
26. Update $Q(s_{\tau}, a_{\tau})$ with $\alpha \cdot Error$
27. end if
28. $t \leftarrow t + 1$
29. end while

#### Algorithm 3 Offline RL Control

1. $d \leftarrow \infty$
2. for $s \in S$ do
3. Call Episode
4. end for
5. while True do
6. $s_0 \leftarrow \text{Current State}$
7. $a_k = \arg \max_{a \in \mathcal{A}} Q(s_0, a)$
8. end while

### 3.2 Online Control

Online RL-based control interleaves operation with multiple shorter learning phases. The agent sporadically explores via limited-depth episodes starting from the controller’s current state. The sample size is small and local. Each exploratory phase is comparatively inexpensive but may only converge to a local optimum.

Like MPC, online RL employs a limited lookahead horizon periodically to learn the best actions. Unlike MPC, online RL remembers and iteratively builds upon previously learned values of actions.
Sensor noise is simulated by scaling the measurements of tank levels with values derived from a Gaussian distribution centered at 1 with a standard deviation $\sigma$. The noise is assumed to be inherent to the sensors.

A trial begins with all tanks filled to capacity with a possible fault in one tank. The trial concludes when no fuel is left in tanks. For each trial, the maximum imbalance and the time integral of imbalance are used as performance metrics.

Goal states in the system are configurations where there is zero moment about the central axis. The magnitude of the moment is the imbalance in the system. The distance map $D$ used by MP controller is a measure of imbalance. \[ D(s) = \text{Abs}(3(T_1 - T_4) + 2(T_2 - T_3) + (T_{LA} - T_{RA})) \]

RL controllers are supplied with a reward function $R$. The controller is rewarded for reaching low-imbalance states and for doing it fast when there is more fuel to spare. \[ R(s,s') = \frac{\left( T_1' + T_2' + T_{LA}' + T_{RA}' + T_3' + T_4' \right)}{600} + \frac{1}{1 + D(s')} \]

RL controllers also hierarchically sample state space. The further a state is from goal, the more imbalanced the tanks are, therefore, the longer the action duration is at that state. \[ H(s) = 1 + \log_{10}(1 + D(s)) \]

Value function approximation is done via a linear combination $Q(s,a)$ of normalized fuel level and valve state products with a bias term. At each episode, the Error is used to update weights $w$ by way of gradient descent. \[ Q(s,a) = w \cdot \left\{ \frac{T_1(1 + V_1)}{200}, \ldots, \frac{T_4(1 + V_4)}{200}, 1 \right\}^T \]

\[ \nabla Q_w = \left\{ \frac{T_1(1 + V_1)}{200}, \ldots, \frac{T_4(1 + V_4)}{200}, 1 \right\}^T \]

RL controllers employ a softmax action selection policy $\pi_{exe}$ during exploration (Doya et al. (2002)). The probability of actions is proportional to the values of those actions at that instant. This ensures that during learning, states which yield more reward are prioritized for traversal.

5. RESULTS AND DISCUSSION

For each of the three control schemes, thirty trials were carried out with random faults (Ahmed (2017)). Table 1 shows the values of the baseline parameters used. For these parameters, MP and online RL controllers sampled the same number of states at each step on average.

Under baseline conditions, Figure 2 shows the performance of various controllers using two metrics. (1) Maximum imbalance is the average maximum value $D(s)$ reached during trials. (2) Total imbalance is the average time integral of $D(s)$ over each trial.
Fig. 1. Simplified C-130 fuel system schematics. The controller manages valves (dotted ovals) and can observe fuel tank levels. Net outflow to engines via pumps (solid ovals) is controlled independently.

| Name            | Symbol | MP | Offline RL | Online RL |
|-----------------|--------|----|------------|-----------|
| Learning rate   | $\alpha$ | N/A | 0.1        | 0.1       |
| Discount        | $\gamma$ | N/A | 0.75       | 0.75      |
| Depth           | $d$    | N/A | 30         | 5         |
| Lookahead steps | $n$    | N/A | 10         | 5         |
| Exploration rate| $\epsilon$ | N/A | N/A        | 0.4       |
| Sampling density| $\rho$ | 1   | 1          | 0.5       |
| Horizon         | $N$    | 1   | N/A        | N/A       |

Table 1. Controller baseline parameters

These results are representative of how controllers sample the state space to select actions. MP control samples all reachable states within a horizon at each step. It finds the locally optimal action. The choice of each action depends on the instantaneous values associated with the states in the model. In case of sensor noise, sub-optimal actions may be chosen.

Offline RL control samples a large fraction of potential successor states once, and derives a single value function and control policy that is applied to select actions in the future. Online RL periodically samples a small, local fraction of successor states to make recurring corrections to its policy. It starts off being sub-optimal, but with each exploratory phase improves on its choice of actions till it converges to the lowest cost choices. It is possible in scenarios where response time is critical, online RL may not have time to sample enough states to converge to an acceptable policy.

Both RL methods rely on accumulated experience. Having a learning rate means that state values, and hence the derived control policy, depend on multiple measurements of the model. For Gaussian sensor noise, the average converges to the true measurement of the state variable. Therefore, the controller is influenced little by random variations in measurements. Furthermore, the choice of value function can regularize the policy derivation so it does not over-fit to noise.

Fig. 2. Performance of controllers under varying sensor noise. RL-based controllers are more robust to sensor noise than MPC. Lower imbalances are better.

RL methods are also sensitive to the form of the value approximation function, which may under-fit the true value of states or over-fit to noise. They forego some accuracy in state valuation by using approximations in exchange for the ability to interpolate values for unsampled states. The use of neural networks as general function approximators...
may allow for a more accurate representation of true state values, and therefore, better control.

Another set of 30 trials was carried out with online RL and MP controllers to simulate sampling and processing constraints during operation. State sample sizes were restricted by lowering the sampling density $\rho$. At each step, the MP and RL controllers could only advance a fraction $\rho$ of available actions to their neighbouring states to explore. The exploration rate was set to $\epsilon = 0.2$ to ensure equal sample sizes for both controllers. Sensor noise was reset to $\sigma = 0$. Results are shown in Figure 3.

Online RL control is quick to recover from a fault and maintains smaller imbalance for the duration of the trial. By having a value function approximation, the controller estimates utilities for actions it cannot sample in the latest time step based off of prior experience. MP control, however, is constrained to choose only among actions it can observe in the model. It is, therefore, more likely to make suboptimal choices. With a decreasing sample density, MP control’s performance deteriorates whereas online RL control is less sensitive to the change.

![Figure 3](image)

**Fig. 3.** Performance of controllers under varying sample densities. RL-based controllers are more robust to smaller state samples than MPC. Lower imbalances are better.

Finally, combining sensor noise and low sampling densities yielded a significant disparity in performance between the RL and MP control approaches as shown in Table 2.

| Control     | Max Imbalance | Total Imbalance |
|-------------|---------------|-----------------|
| MP          | 147.66        | 1972.89         |
| Online RL   | 75.17         | 721.57          |

Table 2. MP vs Online RL control ($\sigma = 0.05, \rho = 0.5$)

The results show that reinforcement learning-based control is more robust. When the model is insufficiently observable, either due to sensor noise or due to sampling constraints, RL controllers are able to generalize behaviour and deliver consistently better performance than MP control. The complexity of both MP and RL approaches is linear in the number of states sampled. However, by discarding prior experience and sampling states anew at each step, model predictive control forfeits valuable information about the environment that RL methods exploit.

6. FUTURE WORK

The fuel tank model made a simplifying assumption of discrete-time dynamics. The problem of continuous-time reinforcement learning-based control remains open and has been discussed in some detail by Doya (2000) and Lewis et al. (2012).

The RL-based controllers approximated the value of actions based on a second order polynomial function, which was learned using stochastic gradient descent. The universal approximation theorem as described by Cybenko (1989) shows that single hidden-layer neural networks with sigmoidal non-linearities can approximate any continuous and real-valued function under some constraints. Using such a basis for the value function may lead to more optimal control policies that reflect the true values of actions.

Using stochastic gradient descent is computationally efficient as a single observation is used to compute changes to the value approximation. However, it is prone to instability in convergence in case of noisy data. Experience replay, as applied to control systems by Adam et al. (2012), maintains a history of prior observations which can be randomly sampled in batches to calculate value updates. By aggregating observations, the effects of sampling errors and noise may be mitigated and an optimal control policy may be derived sooner.

7. CONCLUSION

Reinforcement learning-based control provides a competitive alternative to model predictive control, especially in situations when the system degrades over time, and faults may occur during operations. RL control’s independence of explicitly defined goals allows it to operate in environments where faults may render nominal goal states unfeasible. The ability to generalize behavior over unseen states and actions, and to derive control policies from accumulated experience, make RL control the preferred candidate for system models subject to computational constraints, partially observable environments, and sensor noise. There are several venues for exploration in the choice of hyperparameters for RL based controllers which may yield further performance gains.
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