A cooperative fault-tolerant control method for the coupled system based on interaction effect utilization

Jing Chang * Jérôme Cieslak ** Zongyi Guo *** David Henry **

* Xidian University, Xi’an, China (e-mail: jchang@xidian.edu.cn).
** University of Bordeaux, Talence 33400, France (e-mail: jerome.cieslak@ims-bordeaux.fr; david.henry@u-bordeaux.fr)
*** Northwestern Polytechnical University, Xi’an, China (e-mail: guozongyi@nwpu.edu.cn)

Abstract: An active fault-tolerant control scheme is proposed in this paper for the strongly coupled MIMO systems which subject to total actuator failures. A control-oriented interaction indicator is defined in the Lyapunov stability sense and then is utilized to design the cooperative fault-tolerant control law. The proposed scheme can achieve robust tracking performance with a globally uniformly ultimate boundlessness and is capable of improving transient performance (fault-tolerant ability) by wisely using the interactions. Simulation results obtained on a flight attitude control system illustrates the benefit of the proposed techniques.

Keywords: Fault tolerant control, cooperative control, interaction effect utilization.

1. INTRODUCTION

With the rapid development of unmanned autonomous systems, the past several decades have witnessed an explosive growth of Fault Detection and Diagnosis (FDD), Fault Detection and Isolation (FDI), Fault-Tolerant Control (FTC) and Fault-Tolerant Guidance (FTG) in most disciplines of engineering to enhance the safety of the system in case of the occurrence of unknown fault/failure, see Zolghadri et al. (2014). And particularly, safety is a critical issue in the area of aircraft and aerospace industry, where the FDI, fault-tolerant flight control have been widely developed (Gao and Wang (2014)). Reliable control systems are needed more than ever in the face of rising autonomous and intelligent levels, increasing advancement and complexity of aircraft and aerospace vehicles, the opportunities and challenges presented by new technologies (distributed, networked and cooperative). Despite the high number of published works about fault-tolerant control, too few works take into account the total actuator failures.

Control Allocation (CA) is an efficient approach for dealing with total actuator failures without the reconfiguration of the controller (Alwi and Edwards (2008)). However, it is hard to consider the uncertain control effectiveness and obtain the optimized CA. On the other hand, the CA method required the invertible of the control distribution matrix, which is not always held for the serious actuator faults. As for a faulty system within total actuator failures, it sometimes becomes an underactuated system. The existence of state interactions and control interactions in the MIMO system is a form of coupling. The strong coupling exists in some MIMO systems (see Weiland et al. (2004)); Tian et al. (2013)) which results in difficulty to control the system in fast maneuvering motions, such as rapid descent and fast lateral maneuver of flight. For the normal case, the decoupling control scheme (Dhadekar and Patre (2017)) and decentralized control scheme (Dickeson et al. (2009)) can help in easily and directly designing the controller, since it allows the designer to set the controller for each state independent of the other states. When applicable, the advantage of completely decentralized control is that one can apply the simpler SISO theory (Dhadekar and Patre (2017)). While this traditional design scheme essentially regards the coupling as a detrimental element in the system and eliminates or suppresses its effect on the system directly (Guo et al. (2017, 2018a)). In fact, the existence of interactions between the states or inputs of MIMO systems could be helpful to stabilize the actual motion in under-actuated systems. As for couplings acting on the MIMO systems, the fault-tolerant property of interactive effect has not been sufficiently investigated yet, nor the relationship with the expected system trajectory. In recent work (Guo et al. (2017, 2018b); Chang et al. (2018)), a novel coupling effect indicator is proposed to demonstrate the coupling effects on the system. This proposed control scheme achieved a better dynamic performance by explicitly utilizing the system couplings in the controller design. The improvements in the dynamic performance of this strategy thanks to the switching behavior triggered by the coupling effect. This control strategy is similar to the phase-based gain-modulation control that improves damping while the error is increasing and reducing the control gain while transitioning toward the desired output. This motivates the work of this paper, wherein the controller is modulated according to interaction indicators to attain a certain cooperative fault-tolerant method for MIMO systems. More specifically, this paper will revisit

Copyright lies with the authors 4175
the state and input coupling effect and consider whether
the interactions can be utilized in the fault-tolerant control
design more wisely.

In this context, a cooperative FTC scheme is presented for
the coupled MIMO system subjected to actuator failures.
Firstly, the effect of the couplings on system stability and
performance is analyzed to help with building a control-
oriented interaction indicator. Then, the cooperative fault-
tolerant controller based on these indicators is proposed,
which can enforce and maintain the system tracking per-
formance in the presence of actuator failures. The idea
behind this scheme is to utilize the interactions to derive a
control law with virtual control in Lyapunov stability sense
for the coupled system. The resulting solution en-
sures the dynamic performance of MIMO systems in the
presence of actuator failures. Simulation results obtained
on the nonlinear lateral model of the flight attitude system
illustrate the benefits of the proposed scheme.

The rest of the paper is organized as follows. Section II
lays down problem formulation and the main results are
listed in Section III. Simulation studies are illustrated in
Section IV. The paper concludes in Section V.

2. PROBLEM STATEMENT

Considering the following class of nonlinear, multi-variable
and strongly coupled systems, which are suitable to repre-
sent many mechatronic systems

\[ \ddot{q} = f(q, \dot{q}) + g(q, \dot{q}) + B(q)u \]  

where \( q, \dot{q} \in \mathbb{R}^n \) denotes position and velocity, \( u \in \mathbb{R}^m \)
denotes control input; \( f(q, \dot{q}) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n \), \( g(q, \dot{q}) : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^m \), \( B(q) : \mathbb{R}^n \to \mathbb{R}^{m \times n} \) represent
the system dynamics terms. Let \( u^k_\sigma \) represent the signal from
the k-th actuator that has failed. Then the actuator fault
can be described as follows:

\[ u^k_\sigma(t) = \lambda^k_\sigma(t)u(t) + d^k_\sigma(t)(q, \dot{q}) \]  

where \( 0 \leq \lambda^k_\sigma(t) \leq 1, \ k = 1, 2, \ldots, m \), \( d^k_\sigma(t)(q, \dot{q}) \)
denotes a bounded signal, \( \sigma(t) \) is a switching function
representing the healthy and faulty controller cases. Here,
when \( 0 \leq \lambda^k_\sigma(t) \leq 1 \), it represents the loss of effectiveness
of the actuators and \( \lambda^k_\sigma(t) = 0 \) if the ith actuator fails
completely. The \( d^k_\sigma(t)(q, \dot{q}) \) is the actuator bias fault. For
the sake of simplicity, in this study \( \lambda^k_\sigma(t) \) and \( d^k_\sigma(t) \)
are used to replace \( \lambda^k_\sigma(t) \) and \( d^k_\sigma(t)(q, \dot{q}) \), respectively. Define \( \Lambda^\sigma = diag(\lambda^1_\sigma, \lambda^2_\sigma, \ldots, \lambda^m_\sigma) \) and \( d^\sigma(q, \dot{q}) = [d^1, d^2, \ldots, d^m]^T \).

Let \( x_1 = q, x_2 = \dot{q}, \) the dynamics of system (1) can be further written as

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = f(x_1, x_2) + g(x_1, x_2)x_2 + B(x_1)u \]  

**Assumption 1.** The \( f(q, \dot{q}), g(q, \dot{q}) \) and \( d(q, \dot{q}) \) are assumed to satisfy the local Lipschitz and the linear growth conditions,
where \( f(0, 0) = 0, \ d(0, 0) = 0 \).

**Assumption 2.** The matrix \( B(q) \) is invertible and the system (1) with fault-free is fully-actuated in configuration
\( (q, \dot{q}, t) \), which means \( \text{rank}[B(q)] = n \).

First of all, since the system (1) is in strict-feedback form,
controller. Introduce a new variable \( s = Kx_1 + x_2 \), where \( K = \text{diag}\{k_1, k_2, \ldots, k_n\} \) is a positive definite matrix.
Using Eq,(1), the time derivative of (3) yields

\[ \dot{s} = f(x_1, x_2) + Kx_2 + g(x_1, x_2)x_2 + B(x_1)u \]  

The nominal controller with fault-free can be easily de-
designed as:

\[ u = B(x_1)^{-1}[-f(x_1, x_2) - g(x_1, x_2)x_2 - Ks] \]  

Then we have

\[ \begin{bmatrix} \dot{x}_1 \\ \dot{s} \end{bmatrix} = \begin{bmatrix} -K & I \\ 0 & -K \end{bmatrix} \begin{bmatrix} x_1 \\ s \end{bmatrix} \]  

Therefore, the system (3) is asymptotically stable in the
healthy condition. If the actuator fault occurs, the dynam-
ics of \( s \) become

\[ \dot{s} = f(x_1, x_2) + Kx_2 + g(x_1, x_2)x_2 + B(x_1)d^\sigma(q, \dot{q}) + B(x_1)\Lambda^\sigma B(x_1)^{-1}[-f(x_1, x_2) - (K + g(x_1, x_2))s - Ks] \]  

In general, after the fault occurs, the fault-tolerant control
is activated as soon as actuator faults are detected and
isolated. On the basis of the desired control \( u(t) \) and the
estimation of \( \lambda^k_\sigma \) and \( d^k_\sigma \), the fault-tolerant control \( u_\sigma \) is constructed as

\[ u_\sigma = u_k - \frac{d^k_\sigma}{\lambda^k_\sigma} \]  

where \( \lambda^k_\sigma, d^k_\sigma \) are the estimates of \( \lambda^k_\sigma \), \( d^k_\sigma \) respectively.

The above conventional FTC design is restrictive in the
following two aspects (Shen et al. (2017)):

1) The denominator of the fault-tolerant control input (8)
contains the estimation of the gain fault. If the denomina-
tor is equal to zero, such as \( \lambda^k_\sigma = 0 \), a controller singularity
tends. To avoid such singularity, the FTC scheme can be
modified as \( u_\sigma = \lambda^k_\sigma(u_k - d^k_\sigma) / (\lambda^k_\sigma + \epsilon) \), where \( \epsilon (\epsilon < 1) \) is a small
positive constant (Shen et al. (2017)). Then the system (1)
is asymptotically stable for the case \( \lambda^k_\sigma > 0 \) and the state
\( x_1, x_2 \) converging asymptotically to a small neighborhood.

2) In most fault estimation-based FTC design schemes, it
assumes that \( \lambda^k_\sigma \neq 0 \) or \( B(x_1)\Lambda^\sigma \) is invertible. However,
as \( n = m \), the stuck and float of actuator result in
\( \lambda^k_\sigma = 0, \ t > t_f \), thus the irreversibility of \( B(x_1)\Lambda^\sigma \).
The circuit system failure of the controller unit often
causes loose connection faults, which will result in the
intermittent deviation of controller output with \( \lambda^k_\sigma(t_f) = 0, \ t \in \big[^{t_1}_1, T_1^{T_k}\big) \). Here, \( T_k \)
is the k-th switching instant due to the failure and the system switches to the healthy case at \( T_k \).
Application of adaptive FTC in nonlinear system
for the case \( \lambda^k_\sigma = 0 \) and \( \text{rank}[B(x_1)\Lambda^\sigma] < n \) is still a
challenge, to the best of the author’s knowledge.

In view of the pertaining issues of the above FTC design,
the control objective is re-defined as follows. To simply
illustrate the design problem, we consider the following
two-input-two-output system in this paper:
In order to make sure the system is controllable, we assume that only one actuator totally failed at any given moment for system (9).

This work investigates the problem of fault-tolerant control for system (9), with the objective to provide a solution for stabilizing system in serious actuator faulty situations (stuck, float of the actuator). More specifically, for a given fault \( u_f(t) \) and an allowed ultimate tracking accuracy \( |x_1(t)| \leq \varepsilon \), the purpose of this paper is to find a cooperative fault-tolerant control \( u(t) \) such that the closed-loop system is stable to face \( \lambda_k^f = 0 \). An interesting question arises that is how to use these interactions to stabilize the serious faulty system.

**Remark 1.** In general, the control distribution matrix \( B(x_1) \) to be diagonally dominant matrix, where \(|b_{11}(x_1)| > |b_{12}(x_1)| \) and \(|b_{22}(x_1)| > |b_{21}(x_1)| \).

### 3. MAIN RESULTS

The basic idea of the proposed method in this paper is to use coupling effects between different subsystems to compensate for the total loss of effectiveness of specific actuators. The coupling effect is utilized as auxiliary input for the subsystem under fault to achieve the desired goal.

Before introducing the fault-tolerant controller, the effect of the interactions between the states or inputs of MIMO systems is studied in the fault recoverability analysis. First of all, to analyze how couplings influence the system in the Lyapunov stability sense, we design the baseline control as

\[
\begin{align*}
    u_{10} &= \frac{1}{b_{11}(x_1)}(-k_{1,s} s_1 - f_1(x_{11}, x_{21}) - k_{1} x_{21}) \\
    u_{20} &= \frac{1}{b_{22}(x_1)}(-k_{2,s} s_2 - f_2(x_{12}, x_{22}) - k_{2} x_{22})
\end{align*}
\]  

(10)

where \( s_1 = k_1 x_{11} + x_{21}, s_2 = k_2 x_{12} + x_{22} \). Substitution of (10) into (9) yields

\[
\begin{align*}
    \dot{x}_{11} &= \begin{bmatrix} -k_{1, s} & 1 \\ 0 & -k_{1} \end{bmatrix} \begin{bmatrix} x_{11} \\ s_1 \end{bmatrix} + \begin{bmatrix} g_{1}(x_1, x_2) x_{22} + b_{12}(x_1) u_2 \\ 0 \end{bmatrix} \\
    \dot{s}_1 &= A_{11} z_1 + G_1(x, u_2) \\
    \dot{x}_{12} &= \begin{bmatrix} -k_{2, s} & 1 \\ 0 & -k_{2} \end{bmatrix} \begin{bmatrix} x_{12} \\ s_2 \end{bmatrix} + \begin{bmatrix} g_2(x_1, x_2) x_{21} + b_{21}(x_1) u_1 \\ 0 \end{bmatrix} \\
    \dot{s}_2 &= A_{22} z_2 + G_2(x, u_1)
\end{align*}
\]  

(11)

where \( z_1 = [x_{11}, s_1]^T, z_2 = [x_{12}, s_2]^T, x = [x_{1}, x_2]^T \). The \( A_1, A_2 \) are Hurwitz matrix. For the Lyapunov candidate \( V_1 = \frac{1}{2} z_1^T z_1 \), its derivative is given by

\[
\dot{V}_1 = -k_{1, s} s_1^2 - f_1(x_{11}, x_{21}) x_{21} - k_{1} x_{11} x_{21} + s_1 g_1 x_{22} + s_1 b_{12} u_2
\]

(12)

Take the derivative of \( V_2 = \frac{1}{2} z_2^T z_2 \), we have

\[
\dot{V}_2 = -k_{2, s} s_2^2 - k_{2} x_{12} x_{22} + s_2 g_2 x_{21} + s_2 b_{21} u_1
\]

(13)

**Definition 1.** Consider the system (9), the control-oriented state and control input coupling interaction indicators (COI) are defined as:

\[
\begin{align*}
    J_{s1} &= s_1 g_1 x_{22}, J_{u1} = s_1 b_{12} u_2 \\
    J_{s2} &= s_2 g_2 x_{21}, J_{u2} = s_2 b_{21} u_1
\end{align*}
\]  

(14)

It can be seen that the state couplings and control input couplings on system will determine the sign of the \( V_{1,2} \). If \( J_{s1}, J_{u1} < 0 \), couplings help the state toward zero. If \( J_{s1}, J_{u1} > 0 \), it will prevent the state convergence.

In this paper, borrowing the idea from coupling utilization and adaptive backstepping control technique developed in (Guo et al. (2018a)), we will use the cooperative control based on interaction utilization to deal with the serious actuator faults. For the two-input-two-output system (9), we only can tolerant one actuator total failure. Suppose that the actuator fault happened in \( u_1 \). When \( \lambda_k^f = 0 \), the state \( x_{11} \) is uncontrollable under (10). From (11), the state \( x_{22} \) can be regarded as a virtual control source for state \( x_{11} \). Thus, the cooperative fault-tolerant control strategy is firstly designing the virtual control \( x_{22}^d \) to maintain \( x_{11} \) to a small bound, then using \( u_2 \) to control the state \( x_{12} \). In order to make \( x_{12} \) also converge to a small value, the virtual control \( x_{22}^d \) should be zero after the state \( |x_{11}| < \varepsilon \).

In terms of cooperative fault-tolerant control, the COI can be reformulated as

\[
\begin{align*}
    J_{s1} &= s_1 g_1 x_{22}, J_{u1} = s_1 b_{12} u_2 \\
    J_{s2} &= s_2 g_2 x_{21}, J_{u2} = s_2 b_{21} u_1
\end{align*}
\]  

(15)

where \( \hat{s}_2 = x_{22} - x_{22}^d \). Despite the fact that estimate value \( \lambda_k^f, d_k^f \) will not be a perfect estimate of the real one \( \lambda_k^f, d_k^f \). Here, we consider an exogenous estimate for \( \lambda_k^f \) and an biased estimate of \( d_k^f \), where \( \lambda_k^f = (1 - \Delta_k) \lambda_k^f \), \( d_k^f = d_k^f + \hat{d}_k^f \). Then, using the \( C_n \)-class functions in (Chen et al. (2015)), we will design virtual control variable and the actual control law in the following forms:

\[
\begin{align*}
    x_{22} &= \frac{1}{s_1} [-\eta_1 D_{s,2} s_1(s) - M_{s,2} s_1(k_{1,s} s_1 + S_f(J_{s1}) g_1 \hat{s}_2 + S_f(J_{s1}) b_{12} u_2 + f_1 + b_{11} u_1^f) \\
    &\quad + k_{2} x_{21})] + (1 - M_{s,2} s_1(x_{22}^d) x_{22}^d) \\
    x_{22} &= -k_{2} x_{21} \\
    u_1 &= \frac{1}{b_{11} \lambda_k^f} [-f_1 - k_{1,s} x_{21} - k_{1,s} s_1 - \eta_1 \text{sign}(s_1) \\
    &\quad - S_f(J_{s1}) g_1 \hat{s}_2 - S_f(J_{s1}) b_{12} u_2 - \frac{d_k^f}{\lambda_k^f} \hat{\lambda}_k^f \neq 0
\end{align*}
\]  

(16)

(17)
\[ u_2 = \frac{1}{b_{22}} \left[ -f_2 - k_x \tilde{s}_2 - \eta_2 \text{sign}(\tilde{s}_2) + \dot{x}_2 \right] \]
\[ - S_f(J_{\tilde{s}_2}) g_{2x2} x_1 - S_f(J_{u_2}) b_{21} \dot{u}_1 \]
(18)

where the function \( S_f(z) \) and \( S_f(z) \) follow
\[ S_f(z) = \begin{cases} S_{\epsilon,2}(z), & z \geq 0 \\ 0, & z < 0 \end{cases} \]
\[ S_f(z) = \frac{\text{sign}(z) + 1}{2} \begin{cases} 1, & z \geq 0 \\ 0, & z < 0 \end{cases} \]
(19)

and \( \epsilon, \eta, k_1, k_2 \) are the design parameters, \( \dot{u}_1 = \dot{x}_1 \), \( \dot{u}_2 = \dot{x}_2 \).

It can be verified that \( x_{2c} \) is continuous and \( \dot{x}_{2c} \) can be obtained by any differentiator. We used the sliding mode control term in \( u_1 \) and \( u_2 \) to achieve fast convergence. Then, the dynamics of the closed-loop system is expressed as
\[ \dot{x}_{11} = -k_1 x_{11} + s_1 \]
\[ \dot{s}_1 = -k_1 s_1 - \eta_1 \text{sign}(s_1) + G_1(s_1, \dot{s}_1, \dot{u}_1) \]
\[ \dot{u}_1 = -k_2 x_{12} + s_2 \]
(20)

where
\[ G_1(s_1, \dot{s}_1, \dot{u}_1) = (1 - S_f(J_{s_1})) g_{1x22} + (1 - S_f(J_{u_1})) b_{12} u_2 \]
\[ \Delta_1(1 - b_{11}) \dot{u}_1 - \Delta_1 b_{11} \dot{u}_1 \]
\[ G_2(s_1, \dot{s}_2, \dot{u}_1) = (1 - S_f(J_{s_1})) g_{2x21} + (1 - S_f(J_{u_2})) b_{21} \dot{u}_1 \]
\[ \Delta_1 b_{11} \dot{u}_1 \]

Remark 2. Based on the definition of \( M_{e,2}(x), D_{e,2}(x) \), \( S_f(z) \) and \( S_f(z) \), it follows that:
\[ z D_{c}(z) = \begin{cases} \left[ z \right], & if \ |z| \geq \epsilon \\ \frac{z}{\cos^2\left(\frac{\pi}{2} - \frac{z^2}{4\epsilon^2}\right)}, & if \ |z| > \epsilon \ \epsilon > 0 \end{cases} \]
(21)
\[ z M_{e}(z) = \begin{cases} z, & if \ |z| \geq \epsilon \\ \frac{z}{\cos^2\left(\frac{\pi}{2} - \frac{z^2}{4\epsilon^2}\right)} \cos^n(\frac{\pi}{2} - \frac{z^2}{4\epsilon^2}), & if \ |z| > \epsilon \ \epsilon > 0 \end{cases} \]
(22)
\[ z S_f(z) = \begin{cases} z S_{\epsilon,2}(z), & if \ z > 0 \\ 0, & if \ z < 0 \end{cases} \]
(23)
\[ z S_f(z) = \begin{cases} z S_{\epsilon,2}(z), & if \ z > 0 \\ 0, & if \ z < 0 \end{cases} \]
(24)

where \( D_{c}(x), M_{e}(x) \) are the short hand of \( D_{e,2}(x) \) and \( M_{e,2}(x) \).

Theorem 1. Consider the system (9) subjected to actuator failure \( u_i(t) \) modeled as (2), if the control law is designed as (17)-(18) with the virtual control variable (16), and the control gains satisfy the following relationship
\[ \eta_1 > \max\{\delta_1, b_{11}|\delta_1^2|\}, \ if \ |s_1| > \epsilon; \]
\[ \eta_1 > \frac{(1 - \epsilon_1)}{\epsilon_2} \delta_1 - \delta_2, \ if \ |s_1| < \epsilon; \]
\[ \eta_2 > b_{21}|\delta_2| \]
(25)

where \( \delta_1, \epsilon_1, \delta_2, i = 1, 2 \) will be defined in the following.

Then, for any bounded initial conditions, we have the following statements:

- all the signals of the closed-loop system (9) remain bounded all the time.
- the error variable \( \tilde{s}_2 \) converges to zero in finite time, the state \( s_1 \) satisfies \( |s_1| \leq \epsilon \) in finite time, and the state \( x_{11}, x_{12} \) are converging to a small bound.

Proof. Note that there are two kind of faults considered here, thus, the proof will be provided for two cases.

a Loss of effectiveness fault, \( \tilde{s}_1 \neq 0 \).

If only part of the control effectiveness lost in the \( u_1 \) we consider the Lyapunov function candidate \( V_1 = \frac{1}{2} x_1^2 + \frac{1}{2} b_{11} \dot{u}_1^2 \), whose derivative is easily computed as follows
\[ \dot{V}_1 = -k_1 s_1 x_1 - \eta_1 |s_1| + s_1 [1 - S_f(J_{s_1})] g_{1x22} + s_1 [1 - S_f(J_{u_1})] b_{12} u_2 + s_2 [\Delta_1(1 - b_{11})] \dot{u}_1 \]
\[ - k_2 s_2 \dot{u}_1 - \eta_2 |s_2| + s_2 [1 - S_f(J_{s_2})] g_{2x21} + s_2 [1 - S_f(J_{u_2})] b_{21} \dot{u}_1 \]
(26)

Suppose the bounded fault estimation error is small, and \( \Delta_1(1 - b_{11}) |\dot{u}_1| - \Delta_1 b_{11} |\dot{u}_1| < \delta_1^2 \) and \( |\dot{u}_1| < \delta_2^2 \). Then, with Remark 2, we have
\[ \dot{V}_1 \leq -k_1 s_1^2 - \eta_1 |s_1| - k_2 s_2^2 \]
\[ -(\eta_2 - b_{21}|\tilde{s}_2^2|) |s_2| \]
(27)

where \( \eta_1 \geq \delta_1^2, \eta_2 > b_{21}|\tilde{s}_2^2| \) and \( \gamma_1, \gamma_2 > 0 \). Thus, the tracking errors \( s_1, \tilde{s}_2 \) will converge to zero in finite time, see Chang et al. (2017). After \( s_1 \) converge to zero, it is observed from (16) that \( x_{22} \neq x_{22} \). Then, we have
\[ \dot{x}_{11} = -k_1 \tilde{s}_1, \quad \dot{x}_{12} = -k_2 x_{12} \]
(28)

Therefore, the states \( x_{11}, x_{12} \) will asymptotically converge to zero in fault-free case.

b Serious actuator fault, \( \lambda_1^2 = 0 \).

Then, the analysis is divided into three phases.

Phase 1: \( |s_1| > \epsilon \)

Consider the Lyapunov function candidates \( V_s = \frac{1}{2} x_1^2 + \frac{1}{2} b_{11} \dot{u}_1^2 \), whose derivative is easily computed as follows
\[ \dot{V}_s = -k_1 s_1 x_1 - \eta_1 |s_1| + s_1 [1 - M_s(s_1)] g_{1x22} + s_1 [1 - M_s(s_1)] S_f(J_{s_1}) g_{1x22} + s_1 [1 - M_s(s_1)] b_{12} u_2 + s_1 [1 - M_s(s_1)] b_{11} \dot{u}_1 \]
\[ - k_2 s_2 \dot{u}_1 - \eta_2 |s_2| + s_2 [1 - S_f(J_{u_2})] g_{2x21} + s_2 [1 - S_f(J_{u_2})] b_{21} \dot{u}_1 \]
(29)

Based on (21), then Eqs.(29) can be updated as
\[ \dot{V}_s \leq -k_1 s_1^2 - \eta_1 |s_1| - k_2 s_2^2 \]
\[ - (\eta_2 - b_{21}|\tilde{s}_2^2|) |s_2| \]
(30)

It can be confirmed that the \( \tilde{s}_2 \) will converge to zero in finite time, and the variable \( s_1 \) will converge to \( \epsilon \) in finite time, if the following relationship is satisfied
\[ \eta_1 > \max\{\delta_1, b_{11}|\delta_1^2|\}, \quad \eta_2 > b_{21}|\delta_2^2| \]
(31)
Furthermore, $x_{22}^2$ is bounded in finite time due to its definition. The boundness of $x_{22}^2$ and $\bar{s}_2$ further implies that $x_{22} = x_{22}^2 + \bar{x}_2$ is bounded. Choose the Lyapunov function candidates for $x_{11}, x_{12}$ as $V_{11} = \frac{1}{2}x_{11}^2, V_{12} = \frac{1}{2}x_{12}^2$. From (20), we have $V_{11} \leq -(k_1|x_{11} - s_1||x_{11} - \bar{s}_1|)$ and $V_{12} \leq -(k_2|x_{12} - \bar{s}_2 - \bar{x}_{22}|x_{12})$. It is obvious that $V_{11} < 0$ if $|x_{11}| \geq |s_1| / k_1$, and $V_{12} < 0$ if $|x_{12}| \geq \frac{\bar{s}_2^2}{k_2}$. Thus, for this case $x_{11}$ will converge to $\bar{s}_1$ and $x_{12}$ will be bounded. Therefore, we conclude that all states are bounded.

**Phase 2:** $\epsilon < |s_1| < \varepsilon$

When $|s_1| < \epsilon$, it follows that $M_L(s_1) = t_1$ and $D_L(s_1) = t_2 \text{sign}(s_1)$ where $0 < t_i < 1, i = 1, 2$. Since $\bar{s}_2$ will converge to zero in finite time. Consider the Lyapunov candidate $V_\epsilon = V_{11} + V_{12} + \frac{1}{2}s_2^2$, it follows that

$$V_\epsilon = -k_1x_{11}^2 + x_{11}s_1 - \eta_1t_2|x_1| - k_1t_1s_1^2 + s_1(1 - t_1)\left[f_1 + k_1(s_1 - k_1)x_{11} + b_1u_1 - k_2x_{12} + s_1b_1\right.$$

$$+ s_1[1 - t_1S_f(J_{u1})]b_1u_2 - k_2x_{12} - x_{12}x_{22} \leq -(k_1|x_{11}| - \varepsilon(1 - k_1^2 + t_1k_1^2))|x_{11}|$$

$$- (\eta_1 - (1 - t_1)\bar{s}_1 - \bar{x}_{22})|s_1| - (k_2|x_{12}| - |x_{22}|)|x_{12}|$$

(32)

where $\delta_1 = |f_1 + b_1u_1^{\epsilon} - k_2x_{12}|$ and $\delta_2 = |b_1u_1^{\epsilon} + (1 - t_1S_f(J_{u1}))b_1u_2|$ when $\epsilon < |s_1| < \varepsilon$. If the relationships $|x_{11}| > \frac{1}{(1 - k_1^2 + k_1^3)}\bar{s}_1\bar{s}_2$ and $|x_{12}| \geq \frac{|x_{22}|}{k_2}$ are satisfied, $V_\epsilon < 0$ holds. Recall that the virtual control command $x_{22}$ tends to $x_{22}^2$ as $|s_1|$ decreases in this phase. Thus, $\bar{x}_{22}$ decreases as well, which results in the decreased bound of $x_{12}$. Design positive constants $k_1, k_2$ can guarantee the convergence of $s_1$ to the bound $\epsilon$.

**Phase 3:** $|s_1| \leq \epsilon$

If $|s_1| \leq \epsilon$, it follows that $x_{22} = x_{22}^2$. Thus, as $\bar{s}_2$ converges to zero, $x_{22} = -k_2x_{12}$. The dynamic of $x_{11}, x_{12}$ becomes

$$\bar{x}_{11} = -k_1x_{11} + s_1, \quad \bar{x}_{12} = -k_2x_{12}$$

(33)

Then we have the $x_{12} \rightarrow \frac{\bar{s}_2}{k_2}$ and $x_{12} \rightarrow 0$ as $t \rightarrow \infty$.

4. SIMULATIONS

In this section, in order to demonstrate the efficiency of the proposed method in this paper, we apply the cooperative FTC method into the flight attitude control system (Ra et al. (2013); Chwa and Choi (2001)). The simplified Lateral model of the angular velocity dynamics for the flight is given by

$$\omega_x = -b_1\omega_x - b_2\omega_y - b_3\beta + b_4\delta_x + b_5\delta_y$$

$$\omega_y = -c_1\omega_y - c_2\omega_x - c_3\beta + c_4\delta_x + c_5\delta_y$$

(34)

where $\omega_x, \omega_y$ represent the roll and yaw rates, $\beta$ is yaw angle, and $\delta_x$ and $\delta_y$ are the aileron and rudder deflection angles. Aerodynamic coefficients $b_i, c_i, i = 1, \ldots, 5$ are functions of $V, \alpha, \beta, \omega_x, \omega_y$, where $V$ is the velocity, $\alpha$ is the angle of attack. The relative perturbations in the aerodynamics parameters are 15% of their respective nominal values, respectively. The amplitude and rate of the actuator are restricted as $-30^\circ \leq \delta_x, \delta_y \leq 30^\circ$ and $-200^\circ/s \leq \dot{\delta}_x, \dot{\delta}_y \leq 200^\circ/s$. The failure of aileron deflection $\delta_x$ at $t = 1$ s with $\delta_x = 1^\circ$ is considered in the simulation.

**Fig. 1. Response curves of the angular velocity.**

**Fig. 2. Trajectories of the virtual control $x_{22}$.**

Based on the control method proposed in Section III, the nonlinear controller is designed as

$$u_1(t) = \frac{1}{b_4}\left[b_1\omega_x + b_2\omega_y - k_1\omega_x - b_5\delta_y\right]$$

$$\omega_{yc}(t) = \frac{1}{b_2}\left[\eta_1D_{e,\epsilon}\omega_x + M_{e,\epsilon}\omega_x\right]$$

$$- S_f(J_{u1})b_2\omega_y - S_f(J_{u1})b_1u_2$$

$$+ b_1\omega_x - b_4u_1 - k_1\omega_x]\right]$$

(35)

$$u_2(t) = \frac{1}{c_5}\left[c_1\omega_y - k_2\omega_y - \eta_2\text{sign}(\omega_y) + \omega_{yc}\right]$$

$$+ S_f(J_{u2})c_2x_1 - S_f(J_{u2})c_4u_1]$$

where $w_f(t)$ represents the loss of effectiveness for aileron $\delta_x$ with $w_f(t) = \begin{cases} 0, & t \leq 1 \\ 1, & t > 1 \end{cases}$.

In order to illustrate the performance of the proposed controller, numerical simulations are carried out to compare the performance of the proposed cooperative FTC scheme using interaction effect utilization (shortly called IEU-CFTC method) with standard decentralized control approach from (10) (shortly called standard DC method) and control allocation based FTC method (Alwi and Edwards (2008)) (namely CA-FTC) for system (34). In the simulation, the design parameters for IEU-CFTC are chosen as $\varepsilon = 0.02, \epsilon = 0.01, k_1 = 5, k_2 = 10, \eta_1 = 1, \eta_2 = 2$. 

4179
Chang, J., Guo, Z., and Cieslak, J. (2018). A fault tolerant control scheme for the reentry vehicle with reduced control effort. *IFAC-PapersOnLine*, 51(24), 812–817.

Chen, W., Ge, S.S., Wu, J., and Gong, M. (2015). Globally stable adaptive backstepping neural network control for uncertain strict-feedback systems with tracking accuracy known a priori. *IEEE Transactions on Neural Networks and Learning Systems*, 26(9), 1842–1854.

Chwa, D.K. and Choi, J.Y. (2001). New parametric affine modeling and control for skid-to-turn missiles. *IEEE Transactions on Control Systems Technology*, 9(2), 335–347.

Dhadekar, D.D. and Patre, B.M. (2017). UDE-based decoupled full-order sliding mode control for a class of uncertain nonlinear MIMO systems. *Nonlinear Dynamics*, 88(1), 263–276.

Dickeson, J., Rodriguez, A., Sridharan, S., Benavides, J., and Soloway, D. (2009). Decentralized Control of an Airbreathing Scramjet-Powered Hypersonic Vehicle. In *AIAA Guidance, Navigation, and Control Conference*. American Institute of Aeronautics and Astronautics, Reston, Virginia.

Gao, G. and Wang, J. (2014). Observer-based fault-tolerant control for an air-breathing hypersonic vehicle model. *Nonlinear Dynamics*, 76(1), 409–430.

Guo, Z., Guo, J., Chang, J., and Zhou, J. (2018a). Coupling effect-triggered control strategy for hypersonic flight vehicles with finite-time convergence. *Nonlinear Dynamics*, 1–17.

Guo, Z., Guo, J., and Zhou, J. (2018b). Adaptive Attitude tracking control for hypersonic reentry vehicles via sliding mode-based coupling effect-triggered approach. *Aerospace Science and Technology*, 78, 228–240.

Guo, Z., Zhou, J., Guo, J., Cieslak, J., and Chang, J. (2017). Coupling-Characterization-Based Robust Attitude Control Scheme for Hypersonic Vehicles. *IEEE Transactions on Industrial Electronics*, 64(8), 6350–6361.

Ra, C., Kim, S., Suk, J., Kim, Y., Moon, G., and Jun, B. (2013). Adaptive sliding mode autopilot design for skid-to-turn missile model with uncertainties. *IFAC Proceedings Volumes*, 46(19), 330–335.

Shen, Q., Jiang, B., and Shi, P. (2017). *Fault diagnosis and fault-tolerant control based on adaptive control approach*, volume 91. Springer.

Tian, B., Fan, W., Zong, Q., Wang, J., and Wang, F. (2013). Nonlinear robust control for reusable launch vehicles in reentry phase based on time-varying high order sliding mode. *Journal of the Franklin Institute*, 350(7), 1787–1807.

Weiland, C., Longo, J., Gülhan, A., and Decker, K. (2004). Aerothermodynamics for reusable launch systems. *Aerospace Science and Technology*, 8(2), 101–110.

Zolghadri, A., Henry, D., Cieslak, J., Efimov, D., and Goupil, P. (2014). *Fault Diagnosis and Fault-Tolerant Control and Guidance for Aerospace Vehicles*. Advances in Industrial Control. Springer London, London.

**REFERENCES**

Alwi, H. and Edwards, C. (2008). Fault tolerant control using sliding modes with on-line control allocation. *Automatica*, 44(7), 1859–1866.

Chang, J., Cieslak, J., Dvila, J., Zolghadri, A., and Zhou, J. (2017). Analysis and design of second-order sliding-mode algorithms for quadrotor roll and pitch estimation. *ISA Transactions*, 71(Pt 2), 495.