CP violation in charged Higgs boson decays into tau and neutrino

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Abstract: We calculate the CP-violating rate asymmetry of $H^\pm$ decays into tau and neutrino at one loop in the MSSM with complex parameters. We find that the asymmetry is typically of the order of $10^{-3}$, depending mainly on the phases of the trilinear coupling $A_\tau$ and the gaugino mass $M_1$.

Keywords: Supersymmetric Standard Model, Higgs Physics, CP violation.

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1. Introduction

The CP violation observed in the kaon system and in $B$ meson decays appears to be consistent with the Standard Model (SM). However, the baryon asymmetry in the universe requires a new source of CP violation beyond the CKM phase of the SM. Indeed, most extensions of the SM provide additional sources of CP violation. Searching for new effects of CP violation has become one of the most interesting ways to test physics beyond the SM [1].

Within the Minimal Supersymmetric Standard Model (MSSM) with complex parameters, the new sources of CP violation are the phase of the higgsino mass parameter $\mu$, two phases of the gaugino masses $M_i$, $i = 1, 2, 3$, and the phases of the trilinear couplings $A_f$. Especially the latter ones are practically unconstrained. Recently, we pointed out [2] that CP violation in the MSSM may lead to a difference in the partial decay widths of $H^+$ and $H^-$. More precisely, we showed that large phases of $A_t$, $A_b$ and/or $M_3$ can give a CP-violating asymmetry $\delta_{tb}^{CP} = \left[\frac{\Gamma(H^+ \to t\bar{b}) - \Gamma(H^- \to \bar{t}b)}{\Gamma(H^+ \to t\bar{b}) + \Gamma(H^- \to \bar{t}b)}\right]$ of $10-15\%$ for $m_{H^\pm} > m_t + m_b$.

In this article, we consider the lepton decay channels of the charged Higgs bosons, $H^+ \to \tau^+\nu_\tau$ and $H^- \to \tau^-\bar{\nu}_\tau$. In particular, we calculate the CP-violating asymmetry

$$\delta_{\tau\nu}^{CP} = \frac{\Gamma(H^+ \to \tau^+\nu_\tau) - \Gamma(H^- \to \tau^-\bar{\nu}_\tau)}{\Gamma(H^+ \to \tau^+\nu_\tau) + \Gamma(H^- \to \tau^-\bar{\nu}_\tau)},$$

(1.1)

at the one-loop level in the MSSM with explicit CP violation and discuss its parameter dependence. The decay into $\tau\nu$ may be important for relatively low masses of $H^\pm$ and large $\tan\beta$ as its branching ratio increases with increasing $\tan\beta$. For example, for $m_{H^+} = 250$ GeV, we have $Br(H^+ \to t\bar{b}) = 0.90$ and $Br(H^+ \to \tau^+\nu_\tau) = 0.06$ for $\tan\beta = 5$, and $Br(H^+ \to \bar{t}b) = 0.64$ and $Br(H^+ \to \tau^+\nu_\tau) = 0.36$ for $\tan\beta = 30$. The asymmetry $\delta_{\tau\nu}^{CP}$ is sensitive to the phases of the trilinear coupling $A_t$ and of the gaugino mass $M_1$. Although one expects $\delta_{\tau\nu}^{CP}$ to be smaller than $\delta_{tb}^{CP}$ due to the missing gluino exchange, it is an interesting quantity in the case $m_\tau + m_{\tilde{\tau}_1} < m_{H^\pm} < m_t + m_b$.

The article is organized as follows: in Sect. 2 we derive the basic formulae for the $H^\pm \to \tau\nu$ decay widths and define $\delta_{\tau\nu}^{CP}$ in terms of CP-violating form factors $\delta Y^{CP}_\tau$. In Sect. 3, we perform a numerical analysis. In Sect. 4, we summarize our results and comment on the measurability of $\delta_{\tau\nu}^{CP}$. Appendices A and B contain the explicit formulae for the form factors, masses and couplings.

2. Decay widths at tree level and one loop

At lowest order, the widths of the $H^\pm \to \tau^\pm\bar{\nu}_\tau$ decays are given by

$$\Gamma^{LO}(H^\pm \to \tau\nu) = \frac{(m_{H^\pm}^2 - m_\tau^2)^2 y_\tau^2}{16\pi m_{H^\pm}^4},$$

(2.1)

where $y_\tau = h_\tau \sin\beta$, $h_\tau$ being the tau Yukawa coupling, $h_\tau = g m_\tau / (\sqrt{2} m_W \cos\beta)$. At tree level, the amplitude is real and the decay widths are always equal, $\Gamma^{LO}(H^+ \to \tau^+\nu_\tau) = \Gamma^{LO}(H^- \to \tau^-\bar{\nu}_\tau)$. The article is organized as follows: in Sect. 2 we derive the basic formulae for the $H^\pm \to \tau\nu$ decay widths and define $\delta_{\tau\nu}^{CP}$ in terms of CP-violating form factors $\delta Y^{CP}_\tau$. In Sect. 3, we perform a numerical analysis. In Sect. 4, we summarize our results and comment on the measurability of $\delta_{\tau\nu}^{CP}$. Appendices A and B contain the explicit formulae for the form factors, masses and couplings.
However, once loop corrections with complex couplings are included, we have \( y_\tau \to Y_\tau^\pm = y_\tau + \delta Y_\tau^\pm \) and thus a difference in the decay widths appears. At next-to-leading order we get:

\[
\Gamma^{NLO}(H^\pm \to \tau \nu) = \frac{(m^2_{H^\pm} - m^2_\tau)^2 y_\tau^2}{16\pi m^3_{H^\pm}} \left( 1 + \frac{2 \Re \delta Y_\tau^\pm}{y_\tau} \right). \tag{2.2}
\]

Here \( \delta Y_\tau^+ \) stands for the decay of \( H^+ \) and \( \delta Y_\tau^- \) for the decay of \( H^- \). The radiative corrections \( \delta Y_\tau^\pm \) have, in general, both CP-invariant and CP-violating contributions:

\[
\delta Y_\tau^\pm = \delta Y_\tau^{inv} \pm \frac{1}{2} \delta Y_\tau^{CP}. \tag{2.3}
\]

Both the CP-invariant and the CP-violating contributions have real and imaginary parts. Using eqs. (2.2) and (2.3), we can write the CP-violating asymmetry \( \delta_{\tau\nu}^{CP} \) of eq. (1.1) in the simple form:

\[
\delta_{\tau\nu}^{CP} = \frac{\Re \delta Y_\tau^{CP}}{y_\tau + 2 \Re \delta Y_\tau^{inv}} \simeq \frac{\Re \delta Y_\tau^{CP}}{y_\tau} \tag{2.4}
\]

In the MSSM, \( \delta_{\tau\nu}^{CP} \) gets contributions from loop exchanges of charginos, neutralinos, sfermions, W bosons, and neutral Higgs bosons. The relevant Feynman diagrams are shown in Fig. 1. Note that the various diagrams contribute to \( \delta_{\tau\nu}^{CP} \) only if they have absorptive parts. The form factors \( \delta Y_\tau^{CP} \) can be obtained from \( \delta Y_b^{CP} \) in [2] by the replacements \((s)\text{bottom} \to (s)\text{tau} \) and \((s)\text{top} \to (s)\text{neutrino} \). Since \( m_\nu \sim h_\nu \sim 0 \), many terms vanish and \( \delta Y_\tau^{CP} \) becomes much simpler than \( \delta Y_b^{CP} \). The explicit expressions for \( \delta Y_\tau^{CP} \) due to the various diagrams of Fig. 1, together with the masses and couplings of staus and sneutrinos, are given in the Appendix. All other necessary formulae can be found in [2].

**Figure 1:** Diagrams contributing to CP violation in \( H^+ \to \tau^+ \nu_\tau \) in the MSSM with complex couplings \((j = 1, 2; k = 1, \ldots, 4; l = 1, 2, 3) \).
For \( \phi \) thus space by \( \text{CP}-\text{violating effects. In order not to vary too many parameters, we fix part of the parameter space by}

\[
M_2 = 200 \text{ GeV}, \quad \mu = 300 \text{ GeV}, \quad M_{\tilde{E}} = M_{\tilde{L}} - 5 \text{ GeV}, \quad |A_\tau| = 400 \text{ GeV},
\]

\[
M_{\tilde{Q}} = 500 \text{ GeV}, \quad M_{\tilde{U}} = 450 \text{ GeV}, \quad M_{\tilde{D}} = 550 \text{ GeV}, \quad A_t = A_b = -500 \text{ GeV}.
\]

For \( M_1 \), we assume \(|M_1| = \frac{1}{2} \tan \theta_W |M_2|\), keeping \( \phi_1 \), the phase \( M_1 \), as a physical phase. The phase of \( M_2 \) can be rotated away. Since according to the measurements of the electron and neutron electric dipole moments we have \( \phi_\mu < O(10^{-2}) \) [3] for SUSY masses of the order of a few hundred GeV, we take \( \phi_\mu = 0 \). The remaining phases in our analysis are thus \( \phi_t, \phi_b \) and \( \phi_\tau \) (the phases of \( A_1, A_0 \) and \( A_\tau \) and \( \phi_1 \)). These phases also induce, at one-loop level, a mixing of the CP-even and CP-odd neutral Higgs boson states to form mass eigenstates \( H_0^i \), \( i=1,2,3 \) [4]. We take this mixing into account using [5].

Figure 2 shows \( \delta_{\tau\nu}^{CP} \) as a function of \( m_{H^\pm} \) for the two cases \( \phi_\tau = \pi/2, \phi_1 = 0 \) and \( \phi_\tau = 0, \phi_1 = \pi/2 \) (all other phases zero) and three values of \( \tan \beta: \tan \beta = 5, 10, \) and 30. \( M_\tilde{L} \) is chosen such that the lighter stau mass is \( m_{\tilde{\tau}_1} = 135 \text{ GeV} \). The corresponding values for \( m_{\tilde{\nu}}, m_{\tilde{\tau}_2} \) and \( \theta_\tilde{\tau} \) are listed in Table 2.

| \( \tan \beta \) | \( M_\tilde{L} \) | \( m_{\tilde{\nu}} \) | \( m_{\tilde{\tau}_2} \) | \( \theta_\tilde{\tau} \) |
|---|---|---|---|---|
| 5 | 138 | 123 | 150 | 56° |
| 10 | 147 | 132 | 166 | 50° |
| 30 | 180 | 168 | 221 | 47° |

Table 1: Parameters used in the analysis, masses in \( \text{GeV} \), \( m_{\tilde{\tau}_1} = 135 \text{ GeV} \).

The dominant source of CP violation in \( H^\pm \to \tau^\pm \tilde{\nu}_\tau \) decays is the sneutrino-stau-neutralino loop of Fig. 1b: For \( m_{H^\pm} < m_{\tilde{\tau}_1} + m_{\tilde{\nu}} \), \( \delta_{\tau\nu}^{CP} \) is negligibly small, while it sharply rises once the \( H^\pm \to \tilde{\tau}_1 \tilde{\nu}_\tau \) channel is open. In Fig. 2, \( |\delta_{\tau\nu}^{CP}| \) goes up to \( \sim 3.5 \times 10^{-3} \); in our analysis, we have not found values larger than 0.5% (though we do not exclude them for some extreme values of MSSM parameters). Here note that we have taken a rather large value for \( |A_\tau| \) compared to \( M_\tilde{L} \). For smaller \( |A_\tau| \), \( \delta_{\tau\nu}^{CP} \) typically decreases. \( \delta_{\tau\nu}^{CP} \) also decreases with increasing \( \tan \beta \).

It is interesting to note that maximal \( \phi_\tau \) and maximal \( \phi_1 \) lead to very similar values of \( \delta_{\tau\nu}^{CP} \) but with opposite signs. However, if both phases are maximal, i.e. \( \phi_1 \sim \phi_1 \sim \pi/2 \) or \( 3\pi/2 \), they compensate each other and \( \delta_{\tau\nu}^{CP} \) practically vanishes. In Fig. 3, \( \delta_{\tau\nu}^{CP} \) is shown as a function of \( \phi_\tau \) for \( m_{H^\pm} = 350 \text{ GeV}, \tan \beta = 5 \), and various values of \( \phi_1 \). One sees that \( \phi_\tau \) and \( \phi_1 \) are of equal importance for \( \delta_{\tau\nu}^{CP} \).

We have also examined the dependence on \( \phi_{t,b} \). Here notice that in the considered range of \( m_{H^\pm} \), \( 200 \text{ GeV} < m_{H^\pm} < 600 \text{ GeV} \), the diagram with \( \tilde{t}b \), Fig. 1e, does not contribute.
Figure 2: $\delta^{CP}$ as a function of $m_{H^\pm}$ for $\phi_\tau = \pi/2$, $\phi_1 = 0$ ($\delta^{CP} < 0$), and for $\phi_1 = \pi/2$, $\phi_\tau = 0$ ($\delta^{CP} > 0$). The full, dashed, and dotted lines are for $\tan \beta = 5$, 10, and 30, respectively.

Figure 3: $\delta^{CP}$ as a function of $\phi_\tau$ for $m_{H^\pm} = 350$ GeV and $\tan \beta = 5$. The full, dashed, and dotted lines are for $\phi_1 = 0$, $\pi/4$, and $\pi/2$, respectively.

Thus the parameters of the squark sector, eq. (3.2), enter only through radiative corrections to the neutral Higgs sector, see diagrams 1c and 1f, which turn out to be negligible in the case $m_{H^\pm} > m_{\tilde{\tau}_3} + m_{\tilde{\nu}}$. For completeness we note that also a non-zero $\phi_\mu$ has only little influence on $\delta^{CP}_{\tau\nu}$.

4. Conclusions

We have calculated the one-loop contributions to the decays $H^\pm \to \tau^\pm (\nu_\tau)$ within the MSSM with complex parameters. They lead to a CP-violating asymmetry $\delta^{CP}_{\tau\nu}$, eq. (1.1), different from zero. The relevant phases in our analysis are those of the trilinear coupling $A_\tau$ and
the gaugino mass $M_1$. For $m_{H^+} > m_{\tilde{\tau}_1} + m_{\tilde{\nu}}$, the asymmetry is typically of order $10^{-3}$, the dominant source being the sneutrino–stau–neutralino loop.

Some comments are in order on the feasibility of measuring such an asymmetry. As already mentioned, the branching ratio for $H^\pm \rightarrow \tau^\pm \nu_\tau$ is sizeable only for $\tan \beta > 10$. At the LHC, the dominant production channel for $H^+$ is $g b \rightarrow H^+ t$. One expects [6] 1560 events for a Higgs mass of $m_{H^\pm} = 400$ GeV and $\tan \beta = 50$ with a ratio signal over background $S/\sqrt{B} = 19.8$ with an integrated luminosity of 100 fb$^{-1}$. (Here several cuts were already applied and $b$ tagging assumed.) With a branching ratio of 22% one then has 343 events of $H^+ \rightarrow \tau^+ \nu_\tau$.

At a linear $e^+ e^-$ collider at $\sqrt{s} = 1$ TeV, the cross section of $e^+ e^- \rightarrow H^+ H^-$ for $m_{H^\pm} = 400$ GeV is 6.5 fb. Assuming a luminosity of $\mathcal{L} = 500$ fb$^{-1}$ and again a branching ratio of 22% for $H^+ \rightarrow \tau^+ \nu_\tau$ at $\tan \beta = 50$, one gets 715 events. At CLIC with $\sqrt{s} = 3$ TeV the cross section for $e^+ e^- \rightarrow H^+ H^-$ for $m_{H^\pm} = 400$ GeV is 3 fb. With $\mathcal{L} = 800$ fb$^{-1}$ one gets 528 $H^+ \rightarrow \tau^+ \nu_\tau$ events. Therefore, in all cases a higher luminosity would be necessary to observe the CP–violating asymmetry $\delta_{CP}$.

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A. Masses and couplings of staus and sneutrinos

The mass matrix of the staus in the basis ($\tilde{\tau}_L$, $\tilde{\tau}_R$),

$$
\mathbf{M}_\tilde{\tau}^2 = \begin{pmatrix}
M_{\tilde{\tau}_L}^2 - m_Z^2 \cos 2\beta (\frac{1}{2} - \sin^2 \theta_W) + m_{\tilde{\tau}}^2 & (A_\tau^* - \mu \tan \beta) m_{\tau} \\
(A_\tau - \mu^* \tan \beta) m_{\tau} & M_{\tilde{\tau}_E}^2 - m_Z^2 \cos 2\beta \sin^2 \theta_W + m_{\tilde{\tau}}^2
\end{pmatrix}.
$$

(A.1)

It is diagonalized by a rotation matrix $R_{\tilde{\tau}}$,

$$
R_{\tilde{\tau}} = \begin{pmatrix}
e^{\frac{i}{2} \phi} \cos \theta & e^{\frac{i}{2} \phi} \sin \theta \\
e^{-\frac{i}{2} \phi} \sin \theta & e^{-\frac{i}{2} \phi} \cos \theta
\end{pmatrix},
$$

(A.2)

such that $R_{\tilde{\tau}}^T \mathbf{M}_\tilde{\tau}^2 R_{\tilde{\tau}} = \text{diag}(m_{\tilde{\tau}_1}^2, m_{\tilde{\tau}_2}^2)$ and $(\tilde{\tau}_1, \tilde{\tau}_2) = R_{\tilde{\tau}} (\tilde{\tau}_1, \tilde{\tau}_2)$. The mass of the left-sneutrino is given by

$$
m_{\tilde{\nu}}^2 = M_{\tilde{\tau}_L}^2 + \frac{1}{2} m_Z^2 \cos 2\beta.
$$

(A.3)

If neutrinos have non-zero masses there is also a right-sneutrino. However, since it is electrically neutral and $h_{\nu} \sim 0$, it does not take part in the phenomenology discussed here. We thus neglect this state and only consider $\tilde{\nu} \equiv (\tilde{\nu}_L)$.

In the following, we give the Lagrangian for the interactions of (s)taus and (s)neutrinos. The other necessary parts of the interaction Lagrangian are given in [2]. We start with the interaction of Higgs bosons with leptons and sleptons:

$$
\mathcal{L}_{H\ell\ell} = H^+ \tilde{\nu} (y_\tau P_R) \tau^- + H^- \tau^+ (y_\tau P_L) \nu + H^0 \tau^0 (s^{\tau,R}_i P_R + s^{\tau,L}_i P_L) \tau^-,
$$

(A.4)

$$
\mathcal{L}_{H\tilde{\nu}\tilde{\nu}} = G^{\tilde{\tau}}_{ij} H^+ \tilde{\nu}^*_j \tilde{\tau}_i + G^{\tilde{\tau}*}_{ij} H^- \tilde{\tau}^*_i \tilde{\nu}_j,
$$

(A.5)

with $j = 1, 2$, $l = 1, 2, 3$ and

$$
P_L = \frac{1}{2} (1 - \gamma_5), \quad P_R = \frac{1}{2} (1 + \gamma_5).
$$

(A.6)

For the Higgs boson couplings to leptons we have

$$
y_\tau = h_\tau \sin \beta, \quad h_\tau = \frac{g m_\tau}{\sqrt{2} m_W \cos \beta},
$$

(A.7)

and

$$
s^{\tau,R}_i = \frac{g m_\tau}{2 m_W} (g_{H_{i,\tau\tau}} + i g_{P_{H_{i,\tau\tau}}}),
$$

(A.8)

$$
s^{\tau,L}_i = - \frac{g m_\tau}{2 m_W} (g_{H_{i,\tau\tau}} - i g_{P_{H_{i,\tau\tau}}}),
$$

(A.9)

The $H^\pm$ couplings to stau and sneutrino are given by

$$
G^{\tilde{\tau}}_{ij} = \begin{pmatrix}
h_\tau m_\tau \sin \beta - \sqrt{2} g m_W \sin \beta \cos \beta \\
h_\tau (A^*_\tau \sin \beta + \mu \cos \beta)
\end{pmatrix} R_{\tilde{\tau}}.
$$

(A.10)
The interactions with charginos and neutralinos are described by

\[
\mathcal{L}_{\ell\tilde{\chi}^\pm} = \bar{\nu}_\tau (l_j^\ell P_R + k_j^{\tilde{\chi}^+_j} P_L) \tilde{\chi}_{\tau}^+ \tau_i + \bar{\tau} (l_j^\ell P_R + k_j^{\tilde{\chi}^-_j} P_L) \tilde{\chi}_{\tau}^- \tau_i + \bar{\chi}_{\tau} (l_j^\ell P_R + k_j^{\tilde{\chi}^0_j} P_L) \tilde{\chi}_{\tau}^0 \tau_i,
\]

(A.11) with \(i, j = 1, 2\) and \(k = 1, ..., 4\). The chargino–slepton–lepton couplings are

\[
l_j^\ell = -g V_{j1}, \quad k_j^\ell = h_\tau U_{j2}, \quad k_{ij}^\ell = 0.
\]

(A.13) (A.14)

The neutralino couplings to slepton and lepton are

\[
a_k^\tilde{\chi}^- = \sqrt{2} \tan \theta_W N_{k1} - N_{k2}, \quad b_k^\tilde{\chi}^- = 0,
\]

(A.15) with

\[
f_{Lk}^\tilde{\chi}^- = \frac{1}{\sqrt{2}} (\tan \theta_W N_{k1} + N_{k2}) ,
\]

(A.17)

\[
f_{Rk}^\tilde{\chi}^- = -\sqrt{2} \tan \theta_W N_{k1}^* ,
\]

(A.18)

\[
h_{Lk}^\tilde{\chi}^- = -h_\tau N_{k3} = (h_{Rk}^\tilde{\chi}^-)^*.
\]

(A.19)

The interaction with \(W\) bosons is given by

\[
\mathcal{L}_{\ell\tilde{\chi}W} = -\frac{g}{\sqrt{2}} (W^+ \bar{\nu}_\tau \gamma^\mu P_L \tau + W^- \bar{\tau} \gamma^\mu P_L \nu_\tau),
\]

(A.20)

\[
\mathcal{L}_{\ell\tilde{\chi}W} = -i \frac{g}{\sqrt{2}} \left[ R_{1i}^\tilde{\chi}^- W^+ (\bar{\nu}_\tau^\dagger \partial^\mu \tilde{\tau}_i) + R_{1i}^\tilde{\chi}^- W^- (\bar{\tau}_i \partial^\mu \tilde{\bar{\nu}}_\tau) \right],
\]

where

\[
A \partial^\mu B = A (\partial_\mu B) - (\partial_\mu A) B.
\]

(A.22)

B. Vertex graphs

B.1 Neutralino–chargino–sneutrino (stau) loop

The graph of Fig. 1a, with a neutralino, a chargino, and a sneutrino in the loop, leads to

\[
\Re \delta Y_\tau^{CP} (\tilde{\chi}_{\tau}^0 \tilde{\chi}_{\tau}^0 \tilde{\nu}) = \frac{1}{8 \pi^2} \left\{ \Im (F_{Lk}^{\tilde{\chi}^-_j} a_{kL}^j) \Im (B_0 (m_{\tilde{\chi}^-_{Lk}}^2, m_{\tilde{\chi}^-_{Lj}}^2)) + \left[ m_{\chi_0^0} \Im (F_{jL}^{\tilde{\chi}^+_j} a_{Lj}^{\tilde{\chi}^+_j}) + m_{\chi_0^0} \Im (F_{jL}^{\tilde{\chi}^+_j} a_{Lj}^{\tilde{\chi}^+_j}) + m_{\chi_0^0} \Im (F_{jL}^{\tilde{\chi}^+_j} a_{Lj}^{\tilde{\chi}^+_j}) \right] \Im (C_0) + m_{\chi_0^0} \Im (F_{jL}^{\tilde{\chi}^+_j} a_{Lj}^{\tilde{\chi}^+_j}) + m_{\chi_0^0} \Im (F_{jL}^{\tilde{\chi}^+_j} a_{Lj}^{\tilde{\chi}^+_j}) \right\} \Im (C_2), \]

(B.1)
with \( C_X = C_X(0, m_{H^+}^2, m_{\tau}^2, m_{\tilde{\chi}_0^0}^2, m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_j^+}^2) \), \( X = 0, 2 \), the three-point functions [7] in the notation of [8]. The contribution from the neutralino–chargino–stau loop has exactly the same structure. Therefore, \( \Re \delta Y^{\text{CP}}_\tau (\tilde{\chi}_k^0 \tilde{\chi}_j^+ \tilde{\tau}_\tau) \) is obtained from Eq. (B.1) by the following substitutions: for the masses of the loop particles \( m_{\tilde{\chi}_0^0} \to m_{\tilde{\chi}_k^0} \), \( m_{\tilde{\chi}_j^+} \to m_{\tilde{\chi}_k^0} \), \( m_\nu \to m_\tau \) and for the couplings \( a_k^l \to b_{ij} \), \( b_{k}^l \to k_i^j \), \( k_j^i \to b_{ik}^j \), and \( l_i^j \to a_{ik} \).

**Sneutrino–stau–neutralino loop**

The sneutrino–stau–neutralino loop of Fig. 1b gives

\[
\Re \delta Y^{\text{CP}}_{\tau} (\tilde{\nu} \tilde{\tau}_j \tilde{\chi}_k^0) = \frac{1}{8\pi^2} \left[ m_{\tilde{\chi}_k^0} \Im (G_{ij} \tilde{\nu}_k \tilde{\tau}_j \tilde{\chi}_k^0) \Im (C_0) - m_\tau \Im (G_{ij} \tilde{\nu}_k \tilde{\tau}_j \tilde{\chi}_k^0) \Im (C_2) \right], \tag{B.2}
\]

with \( C_X = C_X(0, m_{H^+}^2, m_{\tau}^2, m_{\tilde{\chi}_0^0}^2, m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_j^+}^2) \).

**W boson–neutral Higgs–tau loop**

For the W boson in the loop we use the \( \xi = 1 \) gauge. We thus have to add the corresponding graph with a charged ghost, i.e. \( W^\pm \to G^\pm \) in Fig. 1c. We get:

\[
\Re \delta Y^{\text{CP}}_{\tau} (W H_l \tau) = -\frac{\sqrt{3} g^2}{32\pi^2} \left[ \Im (X) \left[ (3m_{\tilde{\chi}_k^0}^2 - 2m_{H^+}^2) \Im (C_0) + 2m_\tau^2 \Im (C_2) \right] + \Im (B_0(m_{H^+}^2, m_{W}^2, m_{H^L}^2)) - 2 \Im (B_0(0, m_\tau^2, m_{W}^2)) \right], \tag{B.3}
\]

\[
\Re \delta Y^{\text{CP}}_{\tau} (G H_l \tau) = -\frac{1}{8\pi^2} m_\tau h_\tau \cos \beta \left[ \Im (\hat{X}_R) \Im (C_0) - \Im (\hat{X}_L) \Im (C_2) \right], \tag{B.4}
\]

where \( X_R = g_{H^+H^+} s_l s_j^{R,L}, \hat{\chi}^R_L = g_{H^+H^+} l_l l_j^{R,L} \), and \( C_X = C_X(0, m_{H^+}^2, m_{\tau}^2, m_{\tilde{\chi}_0^0}^2, m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_j^+}^2) \).

**B.2 Self-energy graphs**

**Neutralino–chargino loop**

The self-energy graph with a neutralino and a chargino of Fig. 1d gives

\[
\Re \delta Y^{\text{CP}}_{\tau} (\tilde{\chi}_k^0 \tilde{\chi}_j^+ - W) = \frac{1}{8\pi^2} \frac{g^2 m_\tau}{\sqrt{2m_{H^+}^2 m_{W}^2}} \Im (B_0(m_{H^+}^2, m_{\tilde{\chi}_k^0}^2, m_{\tilde{\chi}_j^+}^2)) \times \left[ \Im (c_{ij}) m_{\tilde{\chi}_j^+} (m_{H^+}^2 + m_{\tilde{\chi}_k^0}^2 - m_{\tilde{\chi}_j^+}^2) - \Im (c_{ij}) m_{\tilde{\chi}_k^0} (m_{H^+}^2 - m_{\tilde{\chi}_k^0}^2 + m_{\tilde{\chi}_j}^2) \right], \tag{B.5}
\]

with \( c_{ij} = F_{jk}^R O_{kj}^R + F_{jk}^L O_{kj}^L \), and \( c_{ij} = F_{jk}^R O_{kj}^L + F_{jk}^L O_{kj}^R \).

**Sneutrino–stau and stop–sbottom loops**

The graph of Fig. 1e with stau and sneutrino leads to

\[
\Re \delta Y^{\text{CP}}_{\tau} (\tilde{\nu} \tilde{\tau}_j - W) = \frac{g^2 m_\tau}{16\pi^2} \frac{m_\tilde{\tau}_j}{m_{H^+}^2 m_{W}^2} (m_{\tilde{\tau}_j}^2 - m_\nu^2) \times \Im (G_{ij} \tilde{R}_{i}^{\tilde{\tau}_j}) \Im (B_0(m_{H^+}^2, m_{\tilde{\chi}_j^+}^2, m_{\tilde{\tau}_j}^2)). \tag{B.6}
\]
The analogous graph with stop and sbottom gives

\[ \Re \delta Y^\text{CP}_\tau (\tilde{t}_i \tilde{b}_j - W) = \frac{3g^2}{16\pi^2} \frac{m_\tau}{m_{H^+}^2 m_W^2} (m_{\tilde{t}_i}^2 - m_{\tilde{b}_j}^2) \oiint (G^i_{4i} R^i_{1i} R^{\tilde{b}_j 4}) \Im \left( B_0 (m_{H^+}^2, m_{\tilde{b}_j}^2, m_{\tilde{t}_i}^2) \right), \tag{B.7} \]

where \(G^i_{4i}\) is the \(\tilde{t}\tilde{b}H^+\) coupling, see eqs. (48)–(49) of [2].

**W±–H± and G±–H± loops**

The self-energy graph with \(W^+\) and \(H^0\) is shown in Fig. 1f. Since we use \(\xi = 1\) gauge for the \(W\) in the loop, we have to add the corresponding graph with a ghost, i.e. \(W^\pm \to G^\pm\) in the loop. (The second \(W\) propagator can be calculated in the unitary gauge. Hence, no ghost is necessary in this case.) The two contributions together give:

\[ \Re \delta Y^\text{CP}_\tau (WH^0 - W) = -\frac{1}{32\pi^2} \frac{g^3 m_\tau}{\sqrt{2} m_{H^+} m_W} (2m_W^2 - 2m_{H^+}^2 - 3m_W^2) \oiint O_3 (\cos \beta O_{11} + \sin \beta O_{21}) \Im \left( B_0 (m_{H^+}^2, m_{H^0}^2, m_W^2) \right). \tag{B.8} \]

**References**

[1] For a recent review, see: T. Ibrahim and P. Nath, hep–ph/0210251.

[2] E. Christova, H. Eberl, S. Kraml and W. Majerotto, Nucl. Phys. B 639 (2002) 263, erratum ibidem to appear, hep-ph/0205227.

[3] P. Nath, Phys. Rev. Lett. 66 (1991) 2565; Y. Kizukuri and N. Oshimo, Phys. Rev. D46 (1992) 3025; R. Garisto and J.D. Wells, Phys. Rev. D 55 (1997) 1611; Y. Grossman, Y. Nir and R. Rattazzi, Adv. Ser. Direct. High Energy Phys. 15 (1998) 755.

[4] A. Pilaftsis, Phys. Rev. D 58 (1998) 096010 [hep-ph/9803297] and Phys. Lett. B 435 (1998) 88 [hep-ph/9805373]; A. Pilaftsis and C. E. Wagner, Nucl. Phys. B 553 (1999) 3 [hep-ph/9902371]; D. A. Demir, Phys. Rev. D 60 (1999) 055006 [hep-ph/9901389]; T. Ibrahim and P. Nath, Phys. Rev. D66 (2002) 015005.

[5] M. Carena, J. R. Ellis, A. Pilaftsis and C. E. Wagner, Nucl. Phys. B 586 (2000) 92 [hep-ph/0003180]; the Fortran program cph.f can be obtained from http://pilaftsi.home.cern.ch/pilaftsi/.

[6] A. Belyaev, D. Garcia, J. Guasch, J. Sola, JHEP 0206, (2002) 059.

[7] G. Passarino and M. J. Veltman, Nucl. Phys. B 160 (1979) 151.

[8] A. Denner, Fortschr. Phys. 41 (1993) 307.