Synthesis of quasioptimal control of the technical systems with the direct current drive

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Abstract. The paper presents an approximate solution of the optimal control problem of the technical system driven by the direct current motor. The braking mode of the system is under consideration. The optimization criterion was a nonlinear integral functional, which reflects the root-mean-square value of the electric drive power during the system’s braking mode. It was found, that the necessary condition of the criterion minimum – Euler-Poisson equation – is a sixth-order nonlinear differential equation. It is impossible to find its analytical solution. In order to find the approximate solution a direct variational method was applied. Proper selection of the class of the basis functions allowed to reach the absolute minima of the terminal criteria and meeting the boundary conditions of the system’s movement. The first provides decreasing of the dynamical forces in the system transmission; the latter is connected with the statement of the problem. Among the class of functions, the one was found, which minimizes the integral criterion. In order to illustrate the approximate solution of the optimal control problem, the plots have been built. They show the smoothness of the system movement and practicability of the obtained result. All calculations have carried out in an analytical form.

1. Introduction

A considerable number of technical systems are driven by direct current (DC) motors. During transient modes of DC motors operation a significant amount of electrical energy is lost. Thus, the problem of reducing of energy losses of the DC motors arises. Such problems are especially significant for the technical systems, which are exploited in intermittent mode: urban railway electrical transport, lifting machines, robotic systems, etc. The mathematical statement of the problem is connected with the theory of optimal control [1].

In most of the carried out studies, researchers have considered closed-loop optimal control problems [2-3]. In the article [2] the problem of the optimal energy recovery during regenerative braking of a vehicle was solved. The authors have taken into account variations in the vehicle mass, the inclination of the road and the road type. The similar aim was presented in the work [3]. In addition to that, the braking performance was under consideration.

One of the trends in optimal control methods development is connected with different metaheuristic approaches [4-6]. They are extremely effective for problems of PID controllers tuning, which control DC motors velocity or torque. It should be noted, that the linear quadratic regulator (LQR) also applicable to DC motors’ velocity optimal control [7]. PID, fuzzy and LQR approaches to energy optimal control of DC motors’ velocity was researched in the article [8]. Based on unit-step response, disturbance rejection and signal tracking tests the superiority of LQR was stated.
In the cases of determined disturbances in the DC motors operation, the program optimal control is relevant. For such class of problems other approaches should be considered. One of them is presented in the current investigation.

The article is arranged as follows: in the second subsection all the necessary elements of optimal control problem are grounded, which forms the problem statement; the third subsection reveals the problem complexity due to the nonlinearity of the criterion to minimize; in the fourth subsection based on the defined basis function, the approximate solution of the problem was found; the last subsection gives a graphical illustration of the results and confirms its practicability.

2. The optimal control problem statement

2.1. Mathematical model of the system

In the current research, the unconstrained optimal control problem is under consideration. It includes a mathematical model of the technical system, boundary conditions of the system motion, and criterion to minimize. The mathematical model of the system is as it follows:

\[
\begin{align*}
U &= \dot{x}A_1 + FA_2 + \dot{F}A_3; \\
F - \mu m g &= m \ddot{x},
\end{align*}
\]  

(1)

where \( U \) – the armature winding voltage; \( F \) – the reduced to the linear motion drive force \( (F = \frac{M_{dr}i}{r}) \); \( M_{dr} \) – the motor torque \( (M_{dr}=cI_d) \); \( I_a \) – the armature winding current; \( i \) – the gear ratio of the transmission; \( \eta \) – the drive efficiency; \( r \) – the radius of reduction, for example, the radius of a trolley wheel; \( A_1, A_2, A_3 \) – the coefficients \( (A_1 = \frac{cI}{r}, A_2 = \frac{R_a r}{\eta c f}, A_3 = \frac{L r}{\eta c f}) \); \( R_a \) – the resistance of the armature winding; \( f \) – the magnetic flux of the motor (it considered as a constant value, which requires to support the constant value of the current in the excitation winding); \( L \) – the armature winding induction coefficient; \( c \) – the coefficient which is determined by the motor parameters \( (c = \frac{pN}{2\pi a}) \); \( p \) – the number of motor poles pairs; \( N \) – the number of the armature winding active conductors; \( a \) – the number of parallel rings of the armature winding; \( g \) – the gravitational acceleration; \( \mu \) – the friction coefficient; \( x \) – the linear generalized coordinate of the system; \( m \) – the reduced to a linear motion mass of the system.

A dot above a symbol indicates time differentiation. The first equation (1) presents the second Kirchhoff’s law, and the second equation of system (1) is composed with the aid of Newton’s second law and the Coulomb’s law.

2.2. Boundary conditions

As the braking mode under consideration the boundary conditions of the problem may be presented as follows:

\[
\begin{align*}
\dot{x}(0) &= 0, \quad \dot{x}(0) = v; \\
\dot{x}(T) &= s, \quad \dot{x}(T) = 0,
\end{align*}
\]  

(2)

where \( v \) – the steady-state velocity of the system; \( s \) – the final position of the system (at the moment when it stops).

2.3. The set of optimization criteria

In order to optimize the energy losses during the system’s braking mode the following criterion was exploited:

\[
I = \left( \frac{1}{T} \int_0^T p^2 dt \right)^{1/2} = \left( \frac{1}{T} \int_0^T (UI_a)^2 dt \right)^{1/2} = \left( \frac{1}{T} \int_0^T (m \ddot{x} + \mu m g)^2 (\dot{x}A_1(m \ddot{x} + \mu m g)A_2 + \dot{m} \ddot{x}A_3) dt \right)^{1/2} \rightarrow \min, 
\]  

(3)

2
where \( P \) – the power consumed by the motor; \( T \) – the duration of the braking mode; \( A_0 \) – the coefficient \( A_0 = \frac{r}{c \eta} \). It reflects the root-mean-square value of the consumed (or generated) electric power of the motor (during braking mode). Minimization of the value \( I \) will allow to improve the energy efficiency of the technical system’s braking mode.

In addition to the criterion (3) the following terminal functionals should be minimized:

\[
\begin{align*}
(F(0) - \mu mg)^2 & \to \min; \\
F^2(T) & \to \min; \\
\dot{F}^2(0) & \to \min; \\
\dot{F}^2(T) & \to \min.
\end{align*}
\]

The first system of criteria taken into consideration is based on the requirement of minimization of dynamic forces in the system’s transmission elements. Indeed, the dynamic impact in the elements is proportional to the value of excessive (dynamic) drive force. Thus, the lesser they are, the lower the overall level of dynamic impact.

The second system of criteria is justified by the requirement of minimization of the armature winding voltage at the beginning and at the end of the controlled mode. In addition to that, minimization of the drive force derivatives allows reducing the „soft” dynamic impacts in the transmission.

3. Analysis of the problem complexity

In order to illustrate the stated problem complexity, the necessary condition of criterion’s (3) minimum has been studied. In the carried out calculation the set of criteria (4) was temporarily ignored.

The necessary condition of criterion’s (3) minimum is the Euler-Poisson equation [9]. It is represented of the nonlinear homogeneous sixth-order differential equation. It might be obtain as a sum of all of the elements of the matrix:

\[
(\mathbf{x}^T \times \mathbf{x})^j \mathbf{Q} = 0,
\]

where \( \mathbf{x} \) – the column vector of the generalized coordinate and its derivatives up to sixth order \( (x=[x], j \in (0, 6)) \); \( \mathbf{Q} \) – the 7-by-7 matrix of the coefficients, which are expressed via \( A_1, A_2, A_3 \) (some of the matrix \( \mathbf{Q} \) elements are equal to zero). Symbol „\( \times \)” in the expression (5) represents the dot product, and the symbol „\( \circ \)” represents Hadamard product of the matrices.

The nonlinearity of the differential equation prevents of finding its analytical solution. Moreover, it can be shown that applying to the problem of Bellman’s dynamic programming or Pontryagin’s maximum principle [10] does not lead to the solvable equations, as well. They are nonlinear and it is impossible to find their analytical solution. In addition to that, the solution of the problem (1)-(4) must meet the boundary conditions (2) and minimize the set of terminal criteria (4). It complicates the problem a lot.

4. Approximate solution

4.1. Selection of the class of basis functions

In order to obtain acceptable result, a direct variational method has been applied [11]. Note, it has been used to the energy optimization problem of a mine winder [12], and the quasioptimal solution has been found.

The first step of the method is setting the class of the basis functions. Among the selected class, the approximate solution of the problem will be searching. Within the framework of the current study, the
differentiable class of functions was used. Such class has desired features. In a practical sense, it provides the smoothness of the braking mode.

The class of basis functions may be set as a solution of the boundary problem:

\[
\begin{align*}
    y(0) = 0, \\
    x(0) = 0, \\
    x(T) = s, \\
    \dot{x}(T) = 0, \\
    \ddot{x}(T) = -\mu \gamma, \\
    \dot{x}(0) = v, \\
    \ddot{x}(0) = 0;
\end{align*}
\]

(6)

The solution of the boundary problem (6) reaches the absolute minima of the criteria (4), which are equal to zero.

The solution of the boundary problem is as it follows:

\[
x = \sum_{i=1}^{7} B_i \mu^i,
\]

(7)

where \( B_i \) – the \( i \)-th coefficient of the polynomial solution of the boundary problem (6).

They are defined in the following manner:

\[
\begin{align*}
    B_1 &= v, \\
    B_2 &= 0, \\
    B_3 &= 0, \\
    B_4 &= -\frac{5(14s + T(gT\mu + 8v))}{2T^4}, \\
    B_5 &= -\frac{84s + T(7gT\mu - 45v)}{T^3}, \\
    B_6 &= \frac{140s - 72Tv + 13gT^2\mu}{2T^6}, \\
    B_7 &= \frac{2(T(5v + gT\mu) - 10s)}{T^4}.
\end{align*}
\]

The basis functions differ from each other only with one parameter \( s \). It provides the opportunity of criterion (3) minimization.

4.2. Minimization of the integral criterion

Basis function will be used for finding the approximate solution of the problem (1)-(4). According to the direct variational method [12], the next step is connected with the determination of the criterion (3) as a function of parameter \( s \). The last step is finding the necessary condition of the function \( I(s) \):

\[
\frac{\partial I}{\partial s} = \sum_{y=0}^{3} C_y s^y = 0,
\]

(8)

where \( C_y \) – the \( y \)-th coefficient of the equation. All the coefficients may be defined as follows:
By using the Cardano’s method [9] three roots of cubic equation (8) were found. Two were complex and one was a real. The real root may be presented in the following manner:

\[
\begin{align*}
\Delta &= \frac{27}{4}m^2 - \frac{3}{2}m^3 \left(3C_1C_3 - C_2^2\right)C_3^{-1}(9C_1C_2C_3 - 27C_0C_3^2 - 2C_3^3 + ((2C_2^3 - 9C_1C_2C_3 + 27C_0C_3^2))^2 - \\
&- 4(C_2^2 - 3C_1C_3)^3) \right)\frac{1}{3}C_3^{-1}(9C_1C_2C_3 - 27C_0C_3^2 - 2C_3^3 + ((2C_2^3 - 9C_1C_2C_3 + 27C_0C_3^2))^2 - \\
&- 4(C_2^2 - 3C_1C_3)^3) \right)^{\frac{1}{3}} - \frac{1}{3}\sqrt[3]{\Delta}.
\end{align*}
\]

Substitution of the expressions (9), (10) to the function (7) gives the approximate solution of the problem (1)-(4).

5. A brief analysis of the obtained results

In order to establish the practicability of the obtained result, its analysis has been carried out. The conducted analysis is based on the built plots (figure 1). All the plots were built for the following numerical values of the system’s parameters: \(m=2000 \text{ kg; } U_0=220 \text{ V; } \eta=0.8, \mu=0.1, r=0.3 \text{ m; } i=9.7, R_s=0.54 \text{ O, } f=9.2 \times 10^{-3} \text{ W; } L=4.6 \times 10^{-3} \text{ H, } p=2, a=1, N=810. \) Duration of the mode \( T \) is set to 2 sec, and the steady-state velocity of the system \( v \) is set to 2.65 m/s.

All the plots show the smooth movement of the system (figure 1, a). It minimizes dynamic loads in transmission and provides increasing of the drive reliability. The plots of the current (figure 1, b) and consumed power (figure 1, d) change the sign. It illustrates the change in energy „flow” direction: negative power reflects the fact, that the drive is operated in generation mode.
Figure 1. Plots of the energetic and dynamic characteristics of the quasioptimal system movement:
a – system’s velocity; b – consumed current; c – armature winding voltage; d – consumed power.

In order to implement the obtained results, the controlled electric power supply source should be exploit (for instance, on the base of power thyristors). Moreover, it must be equipped with the module of energy recuperation or/and power rheostat box. The first option is desirable: it provides the increase of the energy efficiency of the controlled brake mode of the system.

6. Conclusions
In the article, the nonlinear optimal control problem is solved. It has been shown that the applying of traditional variational calculus does not allow finding the solution. The approximate solution was found with the direct variational method. It is based on the use of a proper class of functions (in the investigation it was differentiable functions which met the boundary conditions of the system’s movement). By using the method, the initial problem was reduced to the cubic algebraic equation. Its real root provides the approximate solution of the optimal control problem. By adding more free parameters in basis function will lead to the system of the algebraic equations to solve. It is the direction for further investigations.

Analysis of the result reveals the need for using controlled source for the direct current drive. It must have the option of recuperation or/and releasing the generated energy to the rheostat box.

The character of the system movement is smooth. It provides the reducing of the overall level of the dynamical forces in the system transmission.

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