Microlensing Constraints on the Mass of Single Stars from HST Astrometric Measurements*

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Abstract

We report on the first results from a large-scale observing campaign aiming to use astrometric microlensing to detect and place limits on the mass of single objects, including stellar remnants. We used the Hubble Space Telescope to monitor stars near the Galactic Center for three years, and we measured the brightness and positions of ~2 million stars at each observing epoch. In addition to this, we monitored the same pointings using the VIMOS imager on the Very Large Telescope. The stars we monitored include several bright microlensing events observed from the ground by the OGLE collaboration. In this paper, we present the analysis of our photometric and astrometric measurements for six of these events, and derive mass constraints for the lens in each of them. Although these constraints are limited by the photometric precision of ground-based data, and our ability to determine the lens distance, we were able to constrain the size of the Einstein ring radius thanks to our precise astrometric measurements—the first routine measurements of this type from a large-scale observing program. This demonstrates the power of astrometric microlensing as a tool to constrain the masses of stars, stellar remnants, and, in the future, extrasolar planets, using precise ground- and space-based observations.

Key words: astrometry – gravitational lensing: microlensing: stars: black holes – stars: general

1. Introduction

When stars and compact objects move within close alignment, both with one another and with respect to an observer, the gravitational deflection of light from the background “source” object by the “lens” object leads to the formation of multiple images of the source, an effect known as gravitational lensing. In the case of microlensing, individual images cannot be resolved, due to their small separation, but the total brightness of the images is larger than that of the unlensed source, leading to a temporary magnification of the source. This photometric effect has been used extensively, most notably in the search for extrasolar planets (e.g., Beaulieu et al. 2006; Gaudi et al. 2008; Kains et al. 2013a; Street et al. 2016), as well as in the direct mass measurement of several isolated stars (e.g., Ghez et al. 2004; Jiang et al. 2004), thanks to second-order effects that can be observed under certain conditions.

In addition to this photometric effect, a gravitational microlensing event produces an astrometric deflection as the event unfolds. This is because the images produced by the lens are not symmetrically distributed, leading to the observed centroid of the source shifting during the event (e.g., Dominik & Sahu 2000). Measuring this shift can then allow us to constrain, or measure directly, the lens mass.

Achieving routine measurements of the masses of isolated objects would prove particularly useful in the context of microlensing exoplanet surveys, because they would allow for the planet masses to be tightly constrained. Without mass measurements of the planets’ host stars, only the ratio of a planet’s mass to that of its host is typically known, with some additional probabilistic constraints derived using Galactic models. Better constraints on planet masses are crucial to our understanding of planet populations, and to our knowledge of exoplanet demographics, particularly for cold, low-mass exoplanets that are best probed with microlensing. Furthermore, mass constraints from microlensing are also an excellent technique to investigate populations of black holes, especially stellar-mass (e.g., Lu et al. 2016) and intermediate-mass black holes (Kains et al. 2016). Photometric and astrometric microlensing is currently the only known way to probe isolated stellar-mass black holes, with all current information on these objects coming from their effect on companions.

In this paper, we first recall the main elements of photometric and astrometric microlensing (Section 2), before describing our observations, taken with the Hubble Space Telescope (HST) and the Very Large Telescope (VLT) in Section 3, and

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discussing data reductions in Section 4. We present the modeling of our photometric and astrometric data in Section 5, and discuss the corresponding constraints they provide on the properties of the lenses, as well as implications for the potential of future space-based microlensing surveys in Section 6. Our findings are summarized in Section 7.

2. Astrometric and Photometric Microlensing

In this section, we outline the key equations of photometric and astrometric microlensing. For an in-depth discussion of these effects, we refer the reader to Paczyński (1996) and Dominik & Sahu (2000), respectively.

When an observer, a source at distance $D_S$, and a lens of mass $M$ at a smaller distance $D_L$, move into close enough projected alignment, a microlensing event can occur. The angular separation of the lens and source $\phi$ is usually expressed, as a function of time $t$, in units of the Einstein ring radius, $\theta_E$, as $u(t) = u = \phi / \theta_E$, where

$$\theta_E = \sqrt{\frac{4GM}{c^2}}(D_L^{-1} - D_S^{-1}).$$  \hspace{1cm} (1)

The lens leads to the production of multiple images of the source, whose total integrated luminosity is larger than that of the unlensed source. This leads to the magnification of the source by a factor (e.g., Paczyński 1986)

$$\mu(u) = \frac{u^2 + 2}{u^2 + 4}. \hspace{1cm} (2)$$

Because the images of the source are not symmetrically distributed, the apparent centroid of the source also shifts during the event, corresponding to the astrometric part of the microlensing event. The shift $\delta(u)$ can be expressed as (Hog et al. 1995)

$$\delta(u) = \frac{u}{u^2 + 2} \theta_E, \hspace{1cm} (3)$$

and points away from the lens, from the observer’s standpoint. The parallel and perpendicular components of the displacement, relative to the relative motion of source and lens, can be expressed (e.g., Dominik & Sahu 2000) as

$$\delta_{||} = \frac{u_0^2 + p^2 + 2}{u_0^2 + p^2 + 2} \frac{u_0}{ \theta_E},$$

$$\delta_\perp = \frac{u_0}{u_0^2 + p^2 + 2} \theta_E, \hspace{1cm} (4)$$

where $u_0$ is the minimum angular separation of source and lens, in units of $\theta_E$, also referred to as the impact parameter. This occurs at time $t_0$, and

$$p \equiv p(t) = \frac{t - t_0}{t_0}, \hspace{1cm} (5)$$

where $t$ is the time, and $t_0$ is the Einstein timescale, with $t_0 = \theta_E / \mu_{LS}$, where $\mu_{LS}$ is the relative motion of source and lens. Note that Equation (4) assumes a rectilinear uniform relative motion of source and lens, and is independent of the observational point-spread function (PSF). As the source moves relative to the lens, the components of the astrometric shift lead to a characteristic elliptical motion of the source’s centroid, as shown in Figure 1. These ellipses have eccentricity $\epsilon = [2/(u_0^2 + 2)]^{1/2}$ (Dominik & Sahu 2000).

From Equation (4), measuring the astrometric shift can enable us to measure $\theta_E$, which can, via Equation (1), allow us to make a direct measurement of the lens mass, provided the lens and source distances can be estimated. This is possible by measuring the effect of annual parallax caused by the Earth’s orbit around the Sun on the photometric light curve of the microlensing event (e.g., Dominik 1998; An et al. 2002; Gould 2004). By fitting the components of the parallax vector $\pi_E$ projected onto the sky along the east and north equatorial coordinates, $\pi_{E,E}$ and $\pi_{E,N}$ respectively, we can calculate $\pi_E$ as

$$\pi_E = \sqrt{\frac{\pi_{E,E}^2}{\pi_{E,N}^2}} = \frac{D_L^{-1} - D_S^{-1}}{\theta_E}, \hspace{1cm} (6)$$

which then allows us to estimate the mass of the lens by reworking Equation (1) as

$$M = \frac{\theta_E c^2}{4G\pi_E}. \hspace{1cm} (7)$$

3. Observations

3.1. OGLE

Each of the events presented in this paper was initially observed and alerted by the OGLE Early Warning System (Udalski 2003) as part of the OGLE-IV survey (Udalski et al. 2015a). Observations were taken with the 1.3 m Warsaw University Telescope at Las Campanas Observatory, Chile. For full details on the OGLE telescope and CCD setup, as well as observing cadences, see Udalski et al. (2015a).

3.2. HST

Our HST observations were taken in 2012, 2013, and 2014, from mid-March to mid-October, at a cadence of ~2 weeks, as well as an initial epoch in 2011 October, totalling 192 orbits. This cadence was chosen to provide sufficient time coverage of the astrometric deflection caused by microlensing of a background source by a massive lens, as well as the photometric signature of such events, which would unfold over the course of months. Detecting events caused by massive, nonluminous lenses, such as black holes or neutron stars, was the program’s primary science goal, although in this paper we will not discuss such events, but rather “regular” microlensing events lasting days to weeks that were detected by OGLE, and took place within our HST pointings.

These observations were taken as part of HST programs GO-12586, GO-13463, and GO-13057 (PI: K. C. Sahu), using the Advanced Camera for Surveys’ Wide Field Channel (ACS/WFC, hereafter ACS) and the UVIS channel of the Wide Field Camera 3 (WFC3/UVIS, hereafter WFC3) in parallel. Each epoch consisted of a pair of observations in each of the F606W and F814W filters, with exposure times varying from 350 to 400 s. A few short exposures of 50 s were also taken each season, to allow us to derive a photometric catalogue of bright stars in our field, which are useful for the photometric and astrometric reductions of HST data. Due to HST’s mid-year flip in orientation, the ACS fields were covered over the entire year from April to October, whereas the WFC3 pointings were only covered for half of the time each. A summary of the data set is given in Table 1 and the entire data set is available in MAST (doi:10.17909/T993OF).
A separate publication (K. C. Sahu et al. 2017, in preparation) will discuss the details of this large observing program and will focus on an in-depth discussion of the techniques and methods developed in the last decade that enabled us to carry out this large program with HST and perform the scientific analysis presented in this paper.

### 3.3. VIMOS

In addition to our HST observations, we monitored the same pointings with the VIMOS imager at the VLT at Paranal Observatory, Chile (hereafter referred to as VIMOS data), proposals 091.D-0489(A) and 093.D-0522(A) (PI: M. Zoccali). We covered our HST footprint with three VIMOS pointings, detailed in Table 2, in a manner pictured in Figure 2. VIMOS is a wide-field imager made up of four quadrants, each with a field of view of 7̊ by 8̊, separated by a cross-shaped gap 2̊ wide. Each quadrant is imaged with a deep-depletion E2V CCD with 2048 × 2440 pixels and a pixel scale of 0̊.205. Further details on this instrument can be found in the instrument reference paper of Le Fèvre et al. (2003); see also Hammersley et al. (2010). These observations were obtained in 2013 and 2014, from early April to early October, with a higher cadence (∼4 days) than the HST observations, with the aim of obtaining light curves sampled densely enough to provide constraints on the microlensing parallax (see Section 2) of long microlensing events likely to be caused by massive lenses. Images were obtained mostly in the Bessel-V filter, with a small number of Bessel-V band images also taken during the 2014 season in order to provide color information on stars in our fields. Exposure times were 30 s for most images, with a smaller number of exposures of 10 s in order to be able to construct a reference image with fewer saturated stars. Table 2 summarizes the number of images obtained in each band.

### 4. Data Reduction

#### 4.1. HST

We reduced our HST images using the state-of-the-art suite of algorithms by Jay Anderson (e.g., Anderson & King 2003). For this work, we used flc images, which have been corrected for losses due to charge-transfer efficiency (CTE). As discussed extensively in the literature, CTE losses arise because of detector damage from cosmic rays, and it is important to correct for their effect when trying to determine the precise positions of stars.

Stars in each image were detected and measured using the Fortran routine hst2xy (Anderson & King 2006). We used a standard PSF array for each filter, accounting for spatial variation across the detector; these standard PSF libraries are also provided by Anderson & King (2006). For each star location, the four nearest PSFs are interpolated, to create a PSF that is then used for the local measurement. In addition to this, the time dependence of the PSF is also taken into account by calculating perturbations to the standard PSFs for each image. Accounting for dependence on both the location and time yields PSFs that represent the real stellar profiles accurately, allowing for good flux and position measurements. Finally, we obtained deep photometry using the PSF for each image, with the KS2 algorithm developed by Jay Anderson (e.g., Anderson & King 2000).

Position measurements from ACS images are also affected by significant geometric distortion. Solutions for the distortion were derived by Anderson & King (2006) and are applied to improve the precision of star position measurements.

The photometric measurements were calibrated to the VEGAmag system using the zero-points for the instruments and filters published by STScI (see Bohlin 2012 for the relevant discussion of the ACS and WFC3 zero-points).

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**Table 2**

| Pointing | R.A. (J2000.0) | Decl. (J2000.0) | F606W Epochs | F814W Epochs |
|----------|---------------|----------------|--------------|--------------|
| VIMOS-1  | 17:58:42.9    | −29:15:14      | 815          | 9            |
| VIMOS-2  | 17:59:11.8    | −29:13:33      | 811          | 6            |
| VIMOS-3  | 17:59:18.2    | −29:18:56      | 818          | 6            |

**Note.** The table shows the coordinates of the centre of each pointing and the number of epochs in $I$ and $V$.

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13 In the following discussion, we will refer to "HST data" instead of ACS and WFC3 data whenever common procedures were used for both data sets, in the interest of simplifying language; we will otherwise refer to data sets by instrument name.
4.1.1. Additional Corrections

We applied additional corrections to both photometric and astrometric measurements from the initial KS2 reduction of our HST data, in order to correct for systematics and residual trends.

For each target star, we selected reference stars within a 200 pixel radius, each with median magnitude $\xi_k$. At each epoch $t_i$, we then calculated the offset between this and the measured magnitude of each star, $\omega_{k,i} = m_{k,i} - \xi_k$. The photometric offset $\kappa_i$ to be applied at $t_i$ to the target star is then the median of the reference stars’ offsets at $t_i$, i.e., $\kappa_i = \text{med} \{\omega_{k,i}\}$. For a field of constant stars with well-measured magnitudes, no systematics, and Gaussian errors, $\kappa_i \sim 0$; however, systematics are important, in particular in the ACS data due to the mid-year change in orientation of the telescope, meaning that $\kappa_i$ can be significant. For typical stars in our ACS data, the median photometric offset is around $\pm 2.5$ mmag, with the sign changing seasonally (i.e., a typical star’s brightness is either over- or underestimated by $\pm 2.5$ mmag, depending on the telescope’s orientation). Stars in WFC3/UVIS fields, on the other hand, are only observed for half-seasons, removing the element of seasonal variation; for these, the median offset is typically 1 mmag.

For astrometry, we selected reference stars in the Bulge within a radius of 200 pixels and with a brightness within 1 mag of the target. Bulge stars were identified on the basis of their color and magnitude, with color selection given in Table 3, based on the color–magnitude diagram from observations of the Sagittarius Window Eclipsing Extrasolar Planet Search (SWEEPS) project (Sahu et al. 2006; Clarkson et al. 2008), the location of which is a subfield of the observations discussed in this paper. For each star, we then derived mean proper motions over the three years of observations, and subtracted these from the astrometric time-series to obtain residual astrometric shifts $\beta_{k,i}$. For each epoch $t_i$, we performed a quadratic fit to these residuals for all reference stars, to account for 2D trends with star position

$$(x_k, y_k),$$

in the form

$$F_i(x_k, y_k) = a_0 + a_1 x_k + a_2 y_k + a_3 x_k^2 + a_4 y_k^2,$$

where $F$ is the model and $a_k$ are fitted parameters. We then subtracted the modeled trends from astrometric measurements at each epoch to obtain corrected residuals $\beta_{k,i} = \beta_{k,i} - F_i(x_k, y_k)$. The median correction for typical ACS stars is large, of the order of $\pm 0.02$ pixels, with the sign changing seasonally. For WFC3/UVIS stars, the median correction is much smaller, of the order of 0.002 pixels, and there is no seasonal variation.

Finally, we iterated the entire procedure to obtain final residual astrometric changes not due to proper motion. This was done separately for each dimension, i.e., we fitted residuals in the $x$ direction as a function of $x$ and $y$, and then repeated this for the residuals in the $y$ direction. The resulting time-series are shown in Section 5.4, while astrometric series of sample reference stars are plotted in Figure 3.

4.2. VIMOS

We reduced the VIMOS images using the difference image analysis (DIA) pipeline DanDIA (Bramich 2008; Bramich et al. 2013). This pipeline is particularly good at dealing with crowded fields such as the cores of globular clusters (e.g., Kains et al. 2012, 2013a) and the Galactic Bulge (e.g., Kains et al. 2013b). Each VIMOS quadrant was reduced separately, resulting in 12 separate independent reduction processes.

We produced a stacked reference using the short (10 s) exposures taken at seeing within 10% of the best-seeing images, in order to minimize the number of saturated stars, which are present in the longer-exposure (30 s) images. This resulted in a reference image with the effective exposure times and FWHM of the PSF listed in Table 4.

We measured the position and flux of each star in the reference images by extracting a third-degree polynomial empirical PSF from each image and fitting this PSF to each star. A linear transformation was derived using a triangulation algorithm, in order to register it with the reference. The reference image, convolved with a spatially variable kernel, was subtracted from each image in the series, and resulting difference fluxes were measured at each previously determined star position. Finally, we used the method of Bramich & Freudling (2012) to derive photometric offsets to be applied to each epoch, in order to correct for errors in fitted values of the photometric scale factor. This step was shown by Kains et al. (2015) to lead to significant improvements in the photometric

| $V$ | Bulge | $a$ | $b$ |
|-----|-------|-----|-----|
| $16.5 < V < 19.5$ | $(V - I) \geq f(V)$ | 0.0167 | 0.933 |
| $20 < V < 21$ | $(V - I) \leq f(V)$ | 0.06 | $-$0.04 |
| $21 < V < 22$ | $(V - I) \leq f(V)$ | 0.16 | $-$2.14 |
| $22 < V < 23$ | $(V - I) \leq f(V)$ | 0.22 | $-$3.46 |
| $23 < V < 25$ | $(V - I) \leq f(V)$ | 0.367 | $-$6.83 |

Note. Bulge stars are those satisfying the conditions in the first two columns, with the function $f(V) = a V + b$, and coefficients $a$ and $b$ given in columns 3 and 4. For $19.5 < V < 20$, it is not possible to separate Bulge and Disk stars using color alone. Note that these color selections are valid only in the vicinity of the SWEEPS field, and are not corrected for extinction.
4.2.1. Photometric Calibration

We calibrated the VIMOS photometry using stars in common with OGLE, by matching them using their coordinates. We derived a color-dependent calibration relation for each field in the form

\[ I_{\text{cal}} = I_{\text{ins}} + a + b(V_{\text{ins}} - I_{\text{ins}}), \]

where \( I_{\text{cal}} \) (\( V_{\text{cal}} \)) is the calibrated magnitude, \( I_{\text{ins}} \) (\( V_{\text{ins}} \)) is the instrumental magnitude measured by the DIA photometry, and \( a \) and \( b \) are the transformation coefficients.

Figure 4 shows the rms scatter of the light curves of stars in both our \( HST \) and VIMOS photometry. The range of magnitudes covered overlaps by only \( \sim 1.5 \) mag, with \( HST \) observations saturated at \( I \sim 18 \) and VIMOS observations being no deeper than \( I \sim 19.5 \). The typical photometric rms scatter is less than 0.01 for stars down to \( I \sim 20.5 \) for \( HST \) observations, whereas that is the case for stars brighter than \( I \sim 18 \) in the VIMOS data.

The OGLE observations were reduced with their optimized offline pipeline, which is built from the standard OGLE pipeline (Udalski 2003), but optimized for use on OGLE-IV data after determining the centroid for the source star of each event manually.

5. Modeling

5.1. Event Selection

We selected OGLE events based on the quality of the \( HST \) astrometric measurements, the value of \( t_E \), and the event reaching its photometric peak within our \( HST \) observing campaigns, to ensure good coverage of any astrometric signal and thus yield the best possible mass constraint. Events with a \( t_E \) much shorter than our observing cadence of \( \sim 2 \) weeks were not analyzed, given the small size of their expected astrometric signals; neither were events where residuals showed residual “seasonal” trends with the telescope orientation. The selected events are listed in Table 5. Of the events satisfying these conditions, one event, OGLE-2014-BLG-0442, was also rejected because the photometry showed clear deviations from point source–point lens (PSPL) microlensing. One event, OGLE-2014-BLG-1045, was rejected due to large residual seasonal trends that we were unable to correct for, and one event was saturated due to close proximity to a 15th magnitude star. The remaining six events are the bright single-lens events in our program that could be followed from the ground and space simultaneously. We also detected a number of events with faint sources that could only be observed from space; these events will be discussed in a separate publication.

We also note that only three events have VIMOS photometry: since VIMOS observations were taken in 2013 and 2014 only, there are no data for OGLE-2012-BLG-0645. OGLE-2014-BLG-0117 occurred during the gap between 2013
and 2014 observations. OGLE-2013-BLG-1547 could not be found in the VIMOS data, possibly because of its position near the edge of the image, where photometric scatter is larger, and its location near two bright stars affected our ability to detect this event. Lastly, OGLE-2013-BLG-0182 does have a VIMOS light curve, but it covers only the very end of the event.

5.2. Photometry

For each event, we first fitted the available light curves, comprising HST, VIMOS, and OGLE data. We used a Markov chain Monte Carlo (MCMC) algorithm to fit the simple PSPL parameters, $t_0$, $t_f$, and $u_0$, as well as parallax parameters $\pi_E$ and $\pi_{E,N}$ (see Equation (6)). The latter allow us to fit for the effect of the annual parallax due to Earth’s orbit around the Sun, for which we used the geocentric formalism (Dominik 1998; Gould 2004). The advantage of this approach is that good estimate for the parameters $t_0$, $t_f$, and $u_0$ can be obtained from a simple PSPL fit that does not include parallax. We also fitted the source and blend flux parameters, $F^i_S$ and $F^i_B$, for each telescope and filter combination, such that the model flux at time $t$ and site $i$ is given by

$$F^i(t) = F^i_S \mu(t) + F^i_B$$

These flux parameters are fitted by linear regression for each set of PSPL parameters explored by the MCMC, and take into account blended flux detected that is not being lensed. This produced a first fit of the PSPL parameters, along with associated uncertainties from the posterior distribution of each parameter.

5.3. Astrometry

As detailed in Section 2, an astrometric trajectory is described by the parameters $t_0$, $u_0$, $t_f$, as well as $\theta_E$. In order to describe the elliptical motion fully, an additional parameter $\alpha$ is needed, corresponding to the angle of relative source–lens motion in the plane of the sky. In addition, we fit two parameters $x_0$ and $y_0$, which correspond to the arbitrary baseline reference position of the source at $t = t_0$, when no astrometric deflection due to microlensing is present. Since $t_0$, $u_0$, and $t_f$ are well constrained by the photometry, the posteriors obtained from MCMC fits to the light curves can be used to set priors on these parameters for the subsequent astrometric fits. Prior information on $\alpha$ can also be obtained by considering the proper motion of the source and the proper-motion distribution of Disk stars. As shown in Figure 5, Disk stars lie in a subregion of the $(\mu_l, \mu_b)$ plane, where $\mu_l$ and $\mu_b$ are the two components of the proper motions along the $l$ and $b$ directions, respectively. To obtain this distribution, we first measured the proper motions of stars down to $V \sim 28$ mag, and then overplotted Disk and Bulge stars selected according to their color and magnitude. We then derived median values for the proper motion of each population, along each direction, and 1σ error bars corresponding to the limit of the 68.3% confidence interval. We find values of $\langle (\mu_l), (\mu_b) \rangle = (-0.35, 0.06)$ mas yr$^{-1}$ and $\langle \sigma_l, \sigma_b \rangle = (2.97, 2.77)$ mas yr$^{-1}$ for Bulge stars, and $\langle (\mu_l), (\mu_b) \rangle = (2.89, -0.40)$ mas yr$^{-1}$ and $\langle \sigma_l, \sigma_b \rangle = (3.12, 2.09)$ mas yr$^{-1}$ for Disk stars. This is in good agreement with values found by Clarkson et al. (2008) for the SWEEPS field.

For stars with parallax measurements yielding a lens distance corresponding to a Disk star, one can therefore derive a probability distribution for the proper-motion components of the lens, and transform this using the measurements of the proper motion of the source into a probability distribution for the relative angle of motion $\alpha$, which can then be used as a prior. On the other hand, for stars with no parallax measurements, the probability that the lens as well as the source is a Bulge star is high, and we therefore keep the prior on $\alpha$ uniform for these events. Only $\theta_E$, $x_0$, and $y_0$ are left as parameters with uniform (non-informative) priors. Finally, we note that, although parallax does have a small effect on the astrometry, it is smaller than what can be detected with our data, and we therefore ignore this by not refitting $\pi_E$ and $\pi_{E,N}$ as
part of the astrometric model, even when it is well constrained by the photometry.

5.4. Final Parameters

The photometric fits are shown in Figure 6 and the astrometric fits in Figures 7 and 8, with the parameters given in Table 6.

Only two events, OGLE-2013-BLG-0804 and OGLE-2013-BLG-0547, have well-constrained parallax parameters, despite the former being a low-magnification event ($\mu_0 \sim 0.9$); in this case, the precise HST photometry was crucial to constraining the parallax, as well as VIMOS observations.

6. Results and Discussion

The photometric models are well constrained by the photometry, particularly by our VIMOS and HST photometry, thanks to the precision of the photometry from these data sets (Figure 4), which have scatter approximately five and ten times smaller than OGLE data for a typical $V \sim 20$ star. However, OGLE data are also important in constraining the baseline of the source star. The photometric parameters enable us to derive tight priors for the astrometric modeling.

In the absence of detections of astrometric microlensing signals, only upper limits on the mass can be derived, corresponding to the largest value of $\theta_E$ that is allowed by the data. The astrometric fit parameters in four of our six events are consistent with $\theta_E = 0$ at the $3\sigma$ level, corresponding to a no-microlensing model, which is also clear from the plots of astrometric fits in Figures 7 and 8, meaning that no lower limit can be derived for the lens mass. Because we know from photometry that microlensing did occur, the posterior distributions from the MCMC astrometric fit allow us to place limits on the size of $\theta_E$, and therefore on lens masses, when combined with distance constraints. The mass limits derived from these constraints are given in Table 7. For two other events, the $3\sigma$ lower limit on $\theta_E$ is larger than zero, but the lack of parallax measurements means that this does not translate into significant lower limits on the lens mass.

The astrometric models are well constrained, thanks to the astrometric precision of our measurements, which is consistent with the expected astrometric precision $z_i$ at the $i$th epoch, expressed by Kuijken & Rich (2002),

$$z_i = 0.7 \frac{\text{FWHM}}{(S/N) \times \sqrt{N_i}},$$

where FWHM is the full width at half-maximum of the star’s PSF, $S/N$ is the signal-to-noise ratio, and $N_i$ is the number of images at the $i$th epoch. Therefore, the constraints on $\theta_E$ that we obtain from the astrometric measurements are as good as what can be expected for the depth and number of observations in our program. Figure 9 shows the rms scatter of astrometric light curves for a representative sample of stars in our observations.

The effect of parallax is constrained in just two of the six events, meaning that the distance to the lens $D_L$ is not constrained for the other four; for these, we must therefore use probabilistic distances derived from each event’s fitted time-scale $t_E$ and Galactic models. Here we use the algorithm detailed in Tsapras et al. (2016), which is itself based on the approach of Dominik (2006); resulting distributions of $D_L$ for these four events are shown in Figure 10. Using these probabilistic distances leads to essentially unconstrained lens masses, due to the large spread of allowed values of $D_L$; this is reflected in large error bars and very large upper limits on mass.

6.1. Individual Events

6.1.1. OGLE-2012-BLG-0645

This event is short, with $t_E = 7.4 \pm 0.3$ days, meaning that parallax is not constrained. HST data only cover the event before and after the peak, at small magnification, but the small value of $\mu_0$ means that the OGLE data are sufficient to constrain the photometric parameters well. The astrometric model is consistent with $\theta_E = 0$ at $3\sigma$, with a 99.7% confidence interval of $\theta_E = [0, 3.18]$ mas, but is poorly constrained due to large $x$ and $y$ astrometric scatter of 1.26 and 0.92 mas, respectively. Combined with the statistical distance of $7.0^{+0.7}_{-0.6}$ kpc, the astrometric parameters do not allow us to derive a meaningful upper mass limit for the lens; nevertheless, the formal mass limit estimated for all events is quoted in Table 7.

6.1.2. OGLE-2013-BLG-0182

The photometric parameters for this event are well constrained, because it is relatively bright with a deblended source magnitude $I_S = 19.2$ mag, and the impact parameter is small, $\mu_0 = 0.07$, but the parallax is not measured. The astrometric model is tightly constrained by the WFC3 observations, with a scatter of 0.39 mas in the $x$ position and 0.53 mas in the $y$ position. The resulting 99.7% confidence interval for $\theta_E$ is $[0.01, 1.43]$ mas, but no meaningful mass limits are derived due to the lack of parallax constraints.
6.1.3. OGLE-2013-BLG-0547

This event has the longest timescale in our sample, at \( t_E = 50 \) days, which, combined with \( u_0 = 0.2 \), allowed us to constrain the parallax parameters from the light curve. The scatter in the astrometric measurements is 1.56 and 0.84 mas for the \( x \) and \( y \) positions, respectively, and the astrometric model yields edges of the 99.7% confidence interval at \( \theta_E = [0, 1.02] \) mas; combining the various parameters gives a distance of 3.0 ± 0.6 kpc and an upper limit for the lens mass of 0.66\( M_\odot \).

6.1.4. OGLE-2013-BLG-0804

The timescale \( t_E = 37 \) days also allowed us to constrain the parallax parameters for this event, despite a large impact...
parameter \( u_0 = 0.9 \). The rms scatter of the astrometric measurements is 0.40 and 0.34 mas in the \( x \) and \( y \) directions, respectively, allowing for good constraints on the astrometric model, with a 99.7% confidence interval of \( \theta_E = [0, 0.48] \) mas. Combined with the parallax parameters, this yields a distance of \( 3.7 \pm 0.3 \) kpc, and an upper mass limit of \( 0.43 \, \text{M}_\odot \).

Figure 7. Observations and best-fit model for the astrometric measurements of each of our six events, after subtraction of the mean rectilinear proper motion, along the \( x \) and \( y \) axes (top and middle panels for each event, respectively), and the total shift \( \sqrt{x^2 + y^2} \) (bottom panel for each event), as a function of time. For each event, astrometric measurements are plotted as black filled circles with 1\( \sigma \) error bars, and the best-fit model is plotted as a red solid curve, while the blue shaded areas, delimited by dashed lines, show the 99.7% confidence intervals. Note that the time axis here is in years, rather than days, for easier visualization of the multiyear astrometric curve.
Figure 8. Same as Figure 7, but showing the residual 2D motion of the source, after subtraction of the proper motion. Also plotted are the best-fit astrometric microlensing model (solid black ellipse) and the trajectories allowed at the edges of the 99.7% confidence interval (green dashed ellipses). The symbol sizes and colors change according to the time of the measurement, measured in units of \( t \), as shown by the color bar provided: symbols are redder and larger for measurements closer to \( t_0 \), and smaller and blacker further away. In these plots, the \((x, y)\) baseline reference position of the source at \( t_0 \) is taken to be \((0, 0)\).

Typical error bars for the \( x \) and \( y \) positions are shown on the top and right edges of the plots (black lines), respectively.

Table 6

Best-fit Parameters for the Combined Fits to the Photometry and Astrometry for each Event, with 1σ Error Bars

| Event            | \( t_0 \) | \( \tau_0 \) | \( a_0 \) | \( \tau_{LN} \) | \( \tau_{LE} \) | \( \alpha \) | \( \theta_0 \) | \( V_s \) | \( I_s \) |
|------------------|-----------|-------------|----------|----------------|----------------|-----------|-------------|--------|--------|
| OGLE-2012-BLG-0645 | 6061.976  | 0.007       | 7.40 \( \pm \) 0.05 | 0.087 \( \pm \) 0.004 | \( \ldots \) | \( \ldots \) | 0.95 \( \pm \) 0.58 | 0.54 \( \pm \) 0.37 | 21.69 | 20.28 |
| OGLE-2013-BLG-0182 | 6365.939  | 0.007       | 21.98 \( \pm \) 0.32 | 0.072 \( \pm \) 0.002 | \( \ldots \) | \( \ldots \) | 3.80 \( \pm \) 0.37 | 0.58 \( \pm \) 0.23 | 20.52 | 19.24 |
| OGLE-2013-BLG-0547 | 6418.753  | 0.074       | 50.62 \( \pm \) 0.86 | 0.200 \( \pm \) 0.006 | \(-0.85 \pm 0.25\) | \(-0.17 \pm 0.30\) | 0.28 \( \pm \) 0.17 | 0.22 \( \pm \) 0.13 | 21.88 | 20.69 |
| OGLE-2013-BLG-0804 | 6442.253  | 0.004       | 36.65 \( \pm \) 0.06 | 0.893 \( \pm \) 0.028 | \(-0.41 \pm 0.05\) | \(-0.37 \pm 0.05\) | 1.57 \( \pm \) 0.19 | 0.26 \( \pm \) 0.14 | 20.64 | 19.50 |
| OGLE-2013-BLG-1547 | 6511.909  | 0.004       | 25.09 \( \pm \) 0.39 | 0.412 \( \pm \) 0.039 | \( \ldots \) | \( \ldots \) | 5.51 \( \pm \) 0.30 | 0.27 \( \pm \) 0.21 | 21.46 | 20.33 |
| OGLE-2014-BLG-0117 | 6718.075  | 0.008       | 15.85 \( \pm \) 0.33 | 0.217 \( \pm \) 0.007 | \( \ldots \) | \( \ldots \) | 0.88 \( \pm \) 0.04 | 0.15 \( \pm \) 0.10 | 19.78 | 18.60 |

Note. \( V_s \) and \( I_s \) refer to the deblended baseline magnitudes of the source, in the HST F606W and F814W filters, respectively.
6.1.5. OGLE-2013-BLG-1547

The parallax is not measured for this event, and the large uncertainties in OGLE photometry, sparse HST coverage of the light curve, and lack of a VIMOS light curve mean that the error bars on $t_E$ are large. However, the astrometric model is constrained and is consistent with $\theta_E > 0$ at the $3\sigma$ level. We find a 99.7% confidence interval for $\theta_E$ of [0.19, 3.80] mas. The rms scatter of the astrometric measurements is 0.59 and 0.85 mas in the x and y directions, respectively. Unfortunately, the lack of parallax measurement means that this cannot be translated directly into meaningful mass limits.

6.1.6. OGLE-2014-BLG-0117

This is the brightest of the six events, and the good OGLE photometry, combined with HST observations, allows us to constrain the photometric parameters well, but not the parallax, which means that no meaningful mass limits can be placed. The astrometric model yields a 99.7% confidence interval for $\theta_E$ of [0, 0.58] mas, thanks to an astrometric scatter of 0.34 and 0.31 mas in the x and y astrometric measurements, respectively.

6.2. Future Prospects for Astrometric Measurements

We are currently limited by ground-based photometry when trying to measure the microlensing parallax in large-scale surveys. Although Gaia is a large space-based astrometric mission, it cannot reach the required precision toward the crowded fields where microlensing is more likely to be detected. In the future, it will be possible to measure parallax routinely from space, without the need for parallel ground-based monitoring, which in any case will struggle to reach the required photometric precision for the fainter sources observed from space. This will be done either with high-precision, dense sampling of photometric light curves to detect the effect of parallax discussed in Section 2, or via measurement of the

| Event            | $D_L$ (kpc) | $M_L/M_\odot$ | $M_{L,up}/M_\odot$ |
|------------------|-------------|---------------|---------------------|
| OGLE-2012-BLG-0645 | 7.0^{+0.7}_{-0.5} | 1.76^{+0.47}_{-0.41} | 403.35              |
| OGLE-2013-BLG-0182 | 6.4^{+1.8}_{-1.8} | 1.25^{+0.20}_{-0.10} | 47.43               |
| OGLE-2013-BLG-0547 | 3.0^{+0.7}_{-0.6} | 0.03^{+0.10}_{-0.03} | 0.66                |
| OGLE-2013-BLG-0804 | 3.7^{+0.3}_{-0.3} | 0.06^{+0.08}_{-0.04} | 0.43                |
| OGLE-2013-BLG-1547 | 6.4^{+0.8}_{-0.9} | 3.59^{+0.37}_{-0.26} | 87.12               |
| OGLE-2014-BLG-0117 | 6.6^{+0.5}_{-0.5} | 0.08^{+0.55}_{-0.07} | 6.88                |

Note. The distance to the source is taken to be $D_L = 8.0 \pm 0.3$ kpc. The lens masses in column 3 are given with 1σ percentile error bars.
so-called microlensing “space parallax.” The former is more easily detected in long ($t_E \gtrsim 40$ days) events, but we carried out some rough simulations of deviations from the PSPL model caused by the effect of parallax in various event configurations. We found that in some cases, the photometric precision afforded by space-based observations could allow us to detect the effect of parallax in some events as short as $t_E \sim 15$ days, with observations from a single space-based observatory. Space parallax, on the other hand, manifests itself as a time shift between a microlensing light curve observed from Earth and observations from space, due to the shift in perspective caused by the distance between Earth and the space-based observatory. This effect has already been used to constrain the distance to lens systems in a few cases using the *Spitzer Space Telescope* (e.g., Udalski et al. 2015b; Yee et al. 2015). Although such measurements usually require a large distance of at least a few thousandths of an au between Earth and the space-based observatory, it will be possible to constrain the parallax in this way using the *Wide-field Infrared Survey Telescope (WFIRST)*, which will be orbiting with an apoapsis of $\sim 0.005$ au. Furthermore, the planned cadence of observations for the *WFIRST* Microlensing Survey (Spergel et al. 2015) will be sufficient to constrain the microlensing parallax in many of the longer microlensing events from the light curve alone (Yee 2013). Additional observations from the ground, for example through the use of target-of-opportunity observations, will also provide further opportunities to measure the effect of parallax when this cannot be achieved from the *WFIRST* light curve alone. Combined with astrometric measurements similar to those presented in this paper, this will yield routine mass measurements for many lenses, enabling us to constrain the properties of many of the detected systems, including exoplanets, single stars, and compact objects.

### 7. Conclusions

In this paper, we have demonstrated that routine measurements of the size of the Einstein ring radius can be achieved using astrometric microlensing, and that this will be a powerful way to constrain the mass of objects detected by space-based microlensing surveys. Currently, only the small fraction of events in which second-order effects are detected have constraints on $\theta_E$, which then becomes a limiting factor in determining lens masses when source stars are faint and have a small angular size. Since most source stars in microlensing events are M dwarfs, this means that most microlenses cannot have their masses constrained strongly. On the other hand, the methods we developed for this study can be applied to all stars for which precise enough astrometry can be measured, and will be applied to a large-scale search for stellar-mass black holes, using the same data set.
In the future, the astrometric precision afforded by the James Webb Space Telescope, and particularly by the WFIRST Microlensing Survey (Spergel et al. 2015), down to \(\sim 50 \mu as\) per measurement for stars with \(V \sim 23\), will mean that masses of exoplanets and black holes will be much better constrained on a routine basis. This is a crucial important step toward deriving large-sample demographics from future microlensing observations.

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Software: hst2xym (Anderson & King 2006), KS2 (e.g., Anderson & King 2000), DanDIA (Bramich 2008; Bramich et al. 2013), OGLE pipeline (Udalski 2003).

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