Understanding Hawking radiation in the framework of open quantum systems

Hongwei Yu and Jialin Zhang

Department of Physics and Institute of Physics, Hunan Normal University, Changsha, Hunan 410081, China

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Abstract

We study the Hawking radiation in the framework of open quantum systems by examining the time evolution of a detector (modelled by a two-level atom) interacting with vacuum massless scalar fields. The dynamics of the detector is governed by a master equation obtained by tracing over the field degrees of freedom from the complete system. The nonunitary effects are studied by analyzing the time behavior of a particular observable of the detector, i.e., its admissible state, in the Unruh, Hartle-Hawking, as well as Boulware vacua outside a Schwarzschild black hole. We find that the detector in both the Unruh and Hartle-Hawking vacua would spontaneously excite with a nonvanishing probability the same as what one would obtain if there is thermal radiation at the Hawking temperature from the black hole, thus reproducing the basic results concerning the Hawking effect in the framework of open quantum systems.

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I. INTRODUCTION

Black holes are intriguing objects and are worth studying in all possible varieties. The idea of black holes has been proven to be highly fruitful. In particular, Hawking’s discovery that black holes are not, after all, completely black, but quantum mechanically, emit radiation with a thermal spectrum \[1\], has provided us with the understanding that black holes may play the role of “Rosetta stone” to relate gravity, quantum theory and thermodynamics. Therefore, Hawking radiation, as one of the most striking effects that arise from the combination of quantum theory and general relativity, has attracted widespread interest in physics community and it has been extensively examined from different perspectives, yielding different derivations of it. These derivations include (but not limited to) Hawking’s original one which calculates the Bogoliubov coefficients between the quantum scalar field modes of the in vacuum states and those of the out vacuum \[1, 2\], an Euclidean quantum gravity derivation \[3\] which has been interpreted as a calculation of tunnelling through classically forbidden trajectory \[4\], an approach based upon string theory \[5, 6\], an interesting proposal which ties its existence to the cancellation of gravitational anomalies at the horizon \[7\], and a recent study which reveals an interesting relationship between the existence of Hawking radiation and the spontaneous excitation of atoms using the DDC formalism \[8\] that separates the contributions of vacuum fluctuations and radiation reaction to the rate of change of the mean atomic energy \[9\].

In the current paper, we shall try to understand the Hawking radiation by examining the time behavior of a static detector (modelled by a two-level atom) outside a Schwarzschild black hole immersed in vacuum massless scalar fields using the well-known techniques developed in the study of open quantum systems. As for any open system, the full dynamics of the detector can be obtained from the complete time evolution describing the total system (detector plus external fields) by integrating over the field degrees of freedom, which are in fact not observed. It is worth noting here that an examination of a similar issue, i.e., the Unruh effect associated with uniformly accelerated atoms in the paradigm of open quantum system has been already been carried out \[10\].
The paper is organized as follows. In next Section, we shall review the basic formalism, the derivation of the master equation describing system of the detector plus external vacuum scalar fields in weak coupling limit and the reduced dynamics it generates for the finite time evolution of the detector. In Section III, we apply the method and results of the preceding Section to discuss the probability of spontaneous transition of detector from the ground state to the excited states outside a Schwarzschild black hole. Finally, we conclude with some discussions in Section IV.

II. THE MASTER EQUATION

We shall consider the evolution in the proper time of a static detector (two-level atom) interacting with vacuum massless scalar fields outside a Schwarzschild black hole and assume the combined system (detector + external vacuum fields) to be initially prepared in a factorized state, with the detector held static in the exterior region of the black hole and the fields in their vacuum states. Our derivation of the master equation in this Section follows closely to that in Ref. [10]. The atom is assumed to be fully described in terms of a two-dimensional Hilbert space, so that its states can be represented by a $2 \times 2$ density matrix $\rho$, which is Hermitian $\rho^+ = \rho$, and normalized $\text{Tr}(\rho) = 1$ with $\det(\rho) \geq 0$. In order to achieve a rigorous, mathematically sound, derivation of the reduced dynamics of the detector, we shall assume that the interaction between the detector and the scalar fields are weak so that the finite-time evolution describing the dynamics of the detector takes the form of a one-parameter semigroup of completely positive maps [11, 12]. Without loss of generality, we take the total Hamiltonian for the complete system to have the form

$$H = H_s + H_\phi + \lambda H'.$$  \hspace{1cm} (1)

Here $H_s$ is the Hamiltonian of the atom, which in the most generic case takes the form

$$H_s = \frac{\omega_0}{2} \sum_{i=1}^{3} n_i \sigma_i,$$  \hspace{1cm} (2)

where $\sigma_i$ ($i = 1, 2, 3$) are the Pauli matrices, $\omega_0$ the energy level spacing and $\mathbf{n} = (n_1, n_2, n_3)$ a unit vector. In the present paper, we will take, for simplicity, $\mathbf{n}$ to be along the third axis.
such that $H_s$ simplifies to

$$H_s = \frac{\omega_0}{2} \sigma_3 .$$

(3)

$H_\phi$ is the standard Hamiltonian of massless, free scalar fields, details of which need not be specified here and $H'$ is the interaction Hamiltonian of the atom with the external scalar fields and is assumed to be given by

$$H' = \sigma_3 \Phi(x) .$$

(4)

It should be pointed out that the coupling constant $\lambda$ in (1) is small, and this is required by our assumption that the interaction of the atom with the scalar fields is weak.

Initially, the complete system is described by the total density $\rho_{tot} = \rho(0) \otimes |0\rangle \langle 0|$, where $\rho(0)$ is the initial reduced density matrix of the atom, and $|0\rangle$ is vacuum state of the field $\Phi(x)$. In the frame of the atom, the evolution in the proper time $\tau$ of the total density matrix, $\rho_{tot}$, of the complete system satisfies

$$\frac{\partial \rho_{tot}(\tau)}{\partial \tau} = -iL_H[\rho_{tot}(\tau)] ,$$

(5)

where the symbol $L_H$ represents the Liouville operator associated with $H$

$$L_H[S] \equiv [H, S] .$$

(6)

The dynamics of the atom can be obtained by tracing over the field degrees of freedom, i.e., by applying the trace projection to the total density matrix $\rho(\tau) = \text{Tr}_\Phi[\rho_{tot}(\tau)] .$

In the limit of weak-coupling which we assume in the present paper, the reduced density is found to obey an equation in the Kossakowski-Lindblad form [13, 14]

$$\frac{\partial \rho(\tau)}{\partial \tau} = -i[H_{\text{eff}}, \rho(\tau)] + \mathcal{L}[\rho(\tau)] ,$$

(7)

where

$$\mathcal{L}[\rho] = \frac{1}{2} \sum_{i,j=1}^{3} a_{ij} [2 \sigma_j \rho \sigma_i - \sigma_i \sigma_j \rho - \rho \sigma_i \sigma_j] .$$

(8)

The matrix $a_{ij}$ and the effective Hamiltonian $H_{\text{eff}}$ are determined by the Fourier and Hilbert transforms of the field vacuum correlation functions

$$G^+(x - y) = \langle 0|\Phi(x)\Phi(y)|0\rangle ,$$

(9)
which are defined as
\[ G(\lambda) = \int_{-\infty}^{\infty} d\tau \ e^{i\lambda \tau} \ G^+(x(\tau)) , \]  
\[ K(\lambda) = \frac{P}{\pi i} \int_{-\infty}^{\infty} d\omega \ \frac{G(\omega)}{\omega - \lambda} . \]  
Then the coefficients of the Kossakowski matrix \( a_{ij} \) can be written explicitly as
\[ a_{ij} = A\delta_{ij} - iB\epsilon_{ijk}\delta_k3 + C\delta_{i3}\delta_{j3} \]  
with
\[ A = \frac{1}{2} [G(\omega_0) + G(-\omega_0)] , \quad B = \frac{1}{2} [G(\omega_0) - G(-\omega_0)] , \quad C = G(0) - A . \]  
Meanwhile, the effective Hamiltonian, \( H_{\text{eff}} \), contains a correction term, the so-called Lamb shift, and one can show that it can be obtained by replacing \( \omega_0 \) in \( H_s \) with a renormalized energy level spacing \( \Omega \) as follows [10]
\[ H_{\text{eff}} = \frac{\Omega}{2} \sigma_3 = \{\omega_0 + i[K(-\omega_0) - K(\omega_0)]\} \sigma_3 , \]  
where a suitable subtraction is assumed in the definition of \( K(-\omega_0) - K(\omega_0) \) to remove the logarithmic divergence which would otherwise be present.

To facilitate the discussion of the properties of solutions for (7) and (8), let us express the density matrix \( \rho \) in terms of the Pauli matrices,
\[ \rho(\tau) = \frac{1}{2} \left( 1 + \sum_{i=1}^{3} \rho_i(\tau) \sigma_i \right) . \]  
Substituting Eq. (15) into Eq. (8), it is easy to show that the Bloch vector \( |\rho(\tau)\rangle \) of components \( \{\rho_1(\tau), \rho_2(\tau), \rho_3(\tau)\} \) satisfies
\[ \frac{\partial}{\partial \tau} |\rho(\tau)\rangle = -2\mathcal{H} |\rho(\tau)\rangle + |\eta\rangle , \]  
where \( |\eta\rangle \) denotes a constant vector \( \{0, 0, -4B\} \). The exact form of the matrix \( \mathcal{H} \) reads
\[ \mathcal{H} = \begin{pmatrix} 2A + C & \Omega/2 & 0 \\ -\Omega/2 & 2A + C & 0 \\ 0 & 0 & 2A \end{pmatrix} . \]  

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Eq. (16) can be solved formally, with the solution being written as

$$|\rho(\tau)\rangle = e^{-2\mathcal{H}\tau}|\rho(0)\rangle + (1 - e^{-2\mathcal{H}\tau})|\rho_\infty\rangle,$$

(18)

with

$$|\rho_\infty\rangle = \frac{1}{2}\mathcal{H}^{-1}|\eta\rangle = -\frac{B}{A} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}.$$  

(19)

Here the matrix (operator) $e^{-2\mathcal{H}\tau}$ is defined by series expansion as usual. However, since $\mathcal{H}$ obeys a cubic eigenvalue equation, so powers of $\mathcal{H}$ higher than 2 can always be written in terms of combinations of $\mathcal{H}^2$, $\mathcal{H}$ and $I$, the $3 \times 3$ unit matrix. Actually, three eigenvalues of $\mathcal{H}$ can be found explicitly: $\lambda_1 = 2A$, $\lambda_\pm = (2A + C) \pm i\Omega/2$. One can then show that

$$e^{-2\mathcal{H}\tau} = \frac{4}{\Omega^2 + 4C^2} \left\{ e^{-4A\tau} \Lambda_1 + 2e^{-2(2A+C)\tau} \left[ \Lambda_2 \cos(\Omega\tau) + \Lambda_3 \frac{\sin(\Omega\tau)}{\Omega} \right] \right\},$$

(20)

where

$$\Lambda_1 = [(2A + C)^2 + \frac{\Omega^2}{4}] I - 2(2A + C)\mathcal{H} + \mathcal{H}^2,$$

$$\Lambda_2 = -2A(A + C)I + (2A + C)\mathcal{H} - \frac{1}{2}\mathcal{H}^2,$$

$$\Lambda_3 = 2A\left[ \frac{\Omega^2}{4} - C(2A + C) \right] I + [C(4A + C) - \frac{\Omega^2}{4}] \mathcal{H} - C\mathcal{H}^2.$$  

(21)

Eq. (20) reveals the effects of decoherence and dissipation on the atom characterized by the exponentially decaying factors involving the real parts of the eigenvalues of $\mathcal{H}$ and oscillating terms associated with the imaginary part. These nonunitary effects can be analyzed by examining the evolution behavior in time of suitable atom observables. For any observable of the atom represented by a Hermitian operator $\mathcal{O}$, the time behavior of its mean value is determined by

$$\langle \mathcal{O}(\tau) \rangle = \text{Tr}[\mathcal{O}\rho(\tau)]$$

(22)

Let the observable $\mathcal{O}$ be an admissible atom state $\rho_f$, then Eq. (22) yields the probability, $P_{i\rightarrow f}(\tau)$, that the atom evolves to the expected state represented by density matrix $\rho_f(\tau)$ from an initial one $\rho(0) \equiv \rho_i$

$$P_{i\rightarrow f}(\tau) = \text{Tr}[\rho_f \rho(\tau)].$$

(23)
If initially the atom is in the ground state, so that its Bloch vector \( |\rho(0)\rangle \) is \( \{0, 0, -1\} \), and the final state \( \rho_f \) is the excited state given by the Bloch vector \( |\rho_f\rangle = \{0, 0, 1\} \), then one has, by making use of Eqs. (18-23),

\[
P_{i\rightarrow f}(\tau) = \frac{1}{2} \left( 1 - e^{-4A\tau} \right) \left( 1 - \frac{B}{A} \right).
\]

The probability per unit time of the transition from the ground state to the excited state, in the limit of infinitely slow switching on and off the atom-field interaction, i.e., the spontaneous excitation rate can be calculated by taking the time derivative of \( P_{i\rightarrow f}(\tau) \) at \( \tau = 0 \),

\[
\Gamma_{i\rightarrow f} = \left. \frac{\partial}{\partial \tau} P_{i\rightarrow f}(\tau) \right|_{\tau=0} = 2A - 2B = 2G(-\omega_0).
\]

III. PROBABILITY OF SPONTANEOUS TRANSITION OF THE DETECTOR OUTSIDE A SCHWARZSCHILD BLACK HOLE

Now, we apply the open quantum system formalism developed in the preceding Section to address the issue of finite time evolution of a static detector (two-level atom) interacting with vacuum scalar fields outside a spherically symmetric black hole and calculate the probability of spontaneous transition and the spontaneous excitation rate of the detector from the ground state to the excited state. The metric of a spherically symmetric black hole can be written, in the Schwarzschild coordinates, as

\[
ds^2 = \left( 1 - \frac{2M}{r} \right) dt^2 - \frac{dr^2}{1 - 2M/r} - r^2 (d\theta^2 + \sin^2 \theta d\phi^2).
\]

However, a delicate issue that we have to address first before moving on is how the vacuum state of the quantum fields is determined. Normally, a vacuum state is associated with non-occupation of positive frequency modes. However, the positive frequency of field modes are defined with respect to the time coordinate. Therefore, to define positive frequency, one has to first specify a definition of time. In a spherically symmetric black hole background, one definition is the Schwarzschild time, \( t \). The vacuum state, defined by requiring normal modes to be positive frequency with respect to the Killing vector \( \partial/\partial t \) with respect to which the exterior region is static, is called the Boulware vacuum \([15]\). Other possibilities that have
been proposed are the Unruh vacuum [16] and the Hartle-Hawking vacuum [17]. The Unruh vacuum is defined by taking modes that are incoming from $\mathcal{J}^-$ to be positive frequency with respect to $\partial/\partial t$, while those that emanate from the past horizon are taken to be positive frequency with respect to the Kruskal coordinate $\bar{u}$, the canonical affine parameter on the past horizon. The Hartle-Hawking vacuum, on the other hand, is defined by taking the incoming modes to be positive frequency with respect to $\bar{v}$, the canonical affine parameter on the future horizon, and outgoing modes to be positive frequency with respect to $\bar{u}$.

Therefore, in what follows, we shall calculate the probability of spontaneous transition of a static detector from the ground state to its excited state and spontaneous excitation rate in these three vacuum states.

A. The Unruh vacuum

We start with the Unruh vacuum. The Wightman function for massless scalar fields in the Unruh vacuum can be computed as follows [18, 19, 20]

$$G_U^+(x, x') = \sum_{ml} \int_{-\infty}^{\infty} \frac{e^{-i\omega \Delta t}}{4\pi \omega} |Y_{lm}(\theta, \phi)|^2 \left[ \frac{|\tilde{R}_l(\omega, r)|^2}{1 - e^{-2\pi \omega/\kappa}} + \theta(\omega) |\tilde{R}_l(\omega, r)|^2 \right] d\omega ,$$  

(27)

where $\kappa = 1/(4M)$ is the surface gravity of the black hole. The Fourier transform with respect to the proper time, which is related with the coordinate time through,

$$d\tau = \sqrt{g_{00}} dt = \sqrt{1 - \frac{2M}{r}} dt ,$$  

(28)

reads

$$G_U(\lambda) = \int_{-\infty}^{\infty} e^{i\lambda \tau} G_U^+(x, x') d\tau$$

$$= \frac{1}{8\pi \lambda} \sum_{l=0}^{\infty} \left[ \theta(\lambda \sqrt{g_{00}})(1 + 2l) |\tilde{R}_l(\lambda \sqrt{g_{00}}, r)|^2 + \frac{(1 + 2l) |\tilde{R}_l(\lambda \sqrt{g_{00}}, r)|^2}{1 - e^{-2\pi \lambda \sqrt{g_{00}/\kappa}}} \right] ,$$  

(29)

where we have used the relation

$$\sum_{m=-l}^{l} |Y_{lm}(\theta, \phi)|^2 = \frac{2l + 1}{4\pi} .$$  

(30)
The summation in Eq. (29) is not easy to evaluate into a closed form in general. However, its behavior both close to the event horizon and in the spatial asymptotic region, which, fortunately, are regions we are most interested in, can be found using the following properties of the radial functions

\[
\sum_{l=0}^{\infty} (2l + 1) |\tilde{R}_l(\omega, r)|^2 \approx \begin{cases} \frac{4\omega^2}{1 - \frac{2M}{r}}, & r \to 2M, \\ \frac{1}{r^2} \sum_{l=0}^{\infty} (2l + 1) |B_l(\omega)|^2, & r \to \infty, \end{cases}
\]

\[
\sum_{l=0}^{\infty} (2l + 1) |\tilde{R}_l(\omega, r)|^2 \approx \begin{cases} \frac{1}{4M^2} \sum_{l=0}^{\infty} (2l + 1) |B_l(\omega)|^2, & r \to 2M, \\ 4\omega^2, & r \to \infty, \end{cases}
\]

and

\[
\sum_{l=0}^{\infty} \frac{B_l(\omega)}{\omega} \bigg|_{\omega \to 0} = 4M\delta_{l0}.
\]

With the above expression, it is easy to show that

\[
G_U(\omega_0) \approx \begin{cases} \frac{1}{8\pi\omega_0} \left[ \frac{4\omega_0^2}{1 - e^{-2\pi\omega_0/\kappa_r}} + \frac{\theta(\omega_0)}{4M^2} \sum_{l=0}^{\infty} (1 + 2l) |B_l(\omega_0\sqrt{g_{00}})|^2 \right], & r \to 2M, \\ \frac{1 + \coth \left( \frac{\pi\omega_0}{\kappa_r} \right)}{16\pi\omega_0 r^2} \sum_{l=0}^{\infty} (1 + 2l) |B_l(\omega_0\sqrt{g_{00}})|^2 + \frac{\omega_0 g_{00} \theta(\omega_0)}{2\pi}, & r \to \infty, \end{cases}
\]

where \(\kappa_r = \frac{\kappa}{\sqrt{1 - 2M/r}}\). This leads to the following behaviors of the coefficients of the Kossakowski matrix

\[
A_U \approx \frac{1}{64M^2 \pi \omega_0} \sum_{l=0}^{\infty} (1 + 2l) |B_l(\omega_0\sqrt{g_{00}})|^2 + \frac{\omega_0}{4\pi} \coth \left( \frac{\pi\omega_0}{\kappa_r} \right),
\]

\[
B_U \approx \frac{1}{64M^2 \pi \omega_0} \sum_{l=0}^{\infty} (1 + 2l) |B_l(\omega_0\sqrt{g_{00}})|^2 + \frac{\omega_0}{4\pi},
\]

\[
C_U = G_U(0) - A_U.
\]

\[
r \to 2M : \begin{cases} A_U \approx \frac{\omega_0}{4\pi} + \frac{F^+(\omega_0)}{r^2}, \\ B_U \approx \frac{\omega_0}{4\pi} + \frac{F^-(\omega_0)}{r^2}, \\ C_U = G_U(0) - A_U, \end{cases}
\]

\[
r \to \infty : \begin{cases} A_U \approx \frac{\omega_0}{4\pi} + \frac{F^+(\omega_0)}{r^2}, \\ B_U \approx \frac{\omega_0}{4\pi} + \frac{F^-(\omega_0)}{r^2}, \\ C_U = G_U(0) - A_U, \end{cases}
\]
where the auxiliary function is defined as

$$F(\pm)(\omega_0) = \frac{-1 + \coth\left(\frac{\pi \omega_0}{\kappa}\right)}{32\pi \omega_0} \sum_{l=0}^{\infty} (1 + 2l) \left[ e^{2\pi\omega_0/\kappa} |B_l(\omega_0)|^2 \pm |B_l(-\omega_0)|^2 \right].$$  \hspace{1cm} (37)

Substitute the above coefficients into Eq. (24) and Eq. (25), then the probability of spontaneous transition to the excited state of the ground state detector and the spontaneous excitation rate can be obtained respectively

$$P_{i-f}(\tau) \approx \begin{cases} 
1 - e^{-\tau \omega_0 / \pi \coth\left(\frac{\pi \omega_0}{\kappa r}\right)} & \text{for } r \to 2M, \\
1 - e^{-\tau \omega_0 / \pi} \sum_{l=0}^{\infty} (1 + 2l) \left[ B_l(-\omega_0 \sqrt{g_{00}})^2 \right] / \left(4\omega_0^2 r^2 \right) & \text{for } r \to \infty,
\end{cases}$$  \hspace{1cm} (38)

and

$$\Gamma_{i-f}^U \approx \begin{cases} 
\frac{\omega_0}{\pi \left(e^{2\pi\omega_0/\kappa r} - 1\right)} & \text{for } r \to 2M, \\
\frac{f(-\omega_0, r) \omega_0}{\pi \left(e^{2\pi\omega_0/\kappa} - 1\right)} & \text{for } r \to \infty,
\end{cases}$$  \hspace{1cm} (39)

where

$$f(\omega_0, r) = \sum_{l=0}^{\infty} (1 + 2l) \left[ B_l(\omega_0 \sqrt{g_{00}})^2 \right] / \left(4\omega_0^2 r^2 \right).$$  \hspace{1cm} (40)

The above results reveal that, close to the horizon, the ground state detector in the vacuum would spontaneously excite with an excitation rate same as what one would expect if there is a flux of thermal radiation at the temperature $T = \kappa r / (2\pi)$. However, as the atom is placed away from the horizon, this thermal flux is backscattered by the spacetime curvature, resulting in depletion of part the flux, the effect of which is described by the greybody factor $f(-\omega_0, r)$. The depletion is dependent on the atom’s radial distance from the black hole and becomes greater as the detector is placed farther away. The effective temperature $T$, which can be understood as a result of combined effects of thermal radiation from the black hole and the Unruh effect which results from the acceleration with respect to the local free-falling inertial frame needed to hold the detector static at a finite radial distance, approaches the Hawking temperature $T_H = \kappa / (2\pi)$ in the spatial asymptotic region, suggesting the temperature of the thermal radiation emanating from the horizon is just the Hawking value, since the acceleration needed to hold the detector static vanishes in the spatial infinity. This
understanding we gain for the Unruh vacuum in the paradigm of open quantum systems is consistent with what we have in other different contexts [20, 21, 22].

B. The Hartle-Hawking vacuum

Now, let us turn to the Hartle-Hawking vacuum case. Then the Wightman function for the scalar field becomes [18, 19, 20]

\[ G^+(x, x') = \sum_{ml} \int_{-\infty}^{\infty} \frac{|Y_{lm}(\theta, \phi)|^2}{4\pi \omega} \left[ e^{-i\omega \Delta t} \frac{1}{1 - e^{-2\pi\omega/\kappa}} |\bar{R}_l(\omega, r)|^2 + e^{i\omega \Delta t} e^{2\pi\omega/\kappa} - 1 |\bar{R}_l(\omega, r)|^2 \right] d\omega \] \tag{41}

The Fourier transform is given by

\[ \mathcal{G}_H(\lambda) = \int_{-\infty}^{\infty} e^{i\lambda \tau} G^+(x, x') d\tau \]

\[ = \sum_{l=0}^{\infty} \frac{(1 + 2l)}{8\pi \lambda} \left[ \left| \bar{R}_l(\lambda \sqrt{g_{00}}, r) \right|^2 + \left| \bar{R}_l(-\lambda \sqrt{g_{00}}, r) \right|^2 \right], \] \tag{42}

which can be evaluated approximately at the event horizon and in the spatial asymptotic region to get

\[ \mathcal{G}_H(\omega_0) \approx \begin{cases}
1 + \coth \left( \frac{\pi \omega_0}{\kappa} \right) \frac{1}{16\pi \omega_0} \left[ 4\omega_0^2 + \sum_{l=0}^{\infty} \frac{(1 + 2l) |B_l(-\omega_0 \sqrt{g_{00}})|^2}{4M^2} \right], & r \to 2M, \\
1 + \coth \left( \frac{\pi \omega_0}{\kappa} \right) \frac{1}{16\pi \omega_0} \left[ \sum_{l=0}^{\infty} \frac{(1 + 2l) |B_l(\omega_0 \sqrt{g_{00}})|^2 + 4\omega_0^2 g_{00}}{r^2} \right], & r \to \infty. 
\end{cases} \] \tag{43}

The corresponding coefficients for \( a_{ij} \) then read

\[ \begin{align*}
A_H &\approx \frac{\omega_0 \coth \left( \frac{\pi \omega_0}{\kappa} \right)}{4\pi} + \frac{-1 + \coth \left( \frac{\pi \omega_0}{\kappa} \right)}{128M^2 \pi \omega_0} \sum_{l=0}^{\infty} (1 + 2l) \left| B_l(\omega_0 \sqrt{g_{00}}) \right|^2 \\
+ e^{2\pi\omega_0/\kappa} |B_l(-\omega_0 \sqrt{g_{00}})|^2, \\
B_H &\approx \frac{\omega_0}{4\pi} + \frac{-1 + \coth \left( \frac{\pi \omega_0}{\kappa} \right)}{128M^2 \pi \omega_0} \sum_{l=0}^{\infty} (1 + 2l) \left[ - |B_l(\omega_0 \sqrt{g_{00}})|^2 \\
+ e^{2\pi\omega_0/\kappa} |B_l(-\omega_0 \sqrt{g_{00}})|^2 \right], \\
C_H &= \mathcal{G}_H(0) - A_H,
\end{align*} \tag{44} \]
\[ \longrightarrow \infty : \]
\[
A_H \approx \frac{\omega_0}{4\pi} \coth \left( \frac{\pi \omega_0}{\kappa} \right) + \frac{F^\dagger(\omega_0)}{r^2}, \\
B_H \approx \frac{\omega_0}{4\pi} + \frac{F(-)(\omega_0)}{r^2}, \\
C_H = G_H(0) - A_H.
\]

(45)

Therefore, the probability of spontaneous transition and the spontaneous excitation rate from ground state to exited state are respectively

\[
P_{i-f}^H(\tau) \approx \begin{cases} 
1 - e^{-\frac{\tau \omega_0}{\pi \coth(\pi \omega_0/\kappa)}} & \text{if } r \rightarrow 2M, \\
1 - e^{-\frac{\tau \omega_0}{\pi \coth(\pi \omega_0/\kappa)}} & \text{if } r \rightarrow \infty.
\end{cases}
\]

(46)

and

\[
\Gamma_{i-f}^H \approx \begin{cases} 
\frac{\omega_0}{\pi(e^{2\pi \omega_0/\kappa} - 1)} + \frac{f(\omega_0, r) \omega_0}{\pi(e^{2\pi \omega_0/\kappa} - 1)} & \text{if } r \rightarrow 2M, \\
\frac{\omega_0}{\pi(e^{2\pi \omega_0/\kappa} - 1)} + \frac{f(-\omega_0, r) \omega_0}{\pi(e^{2\pi \omega_0/\kappa} - 1)} & \text{if } r \rightarrow \infty.
\end{cases}
\]

(47)

Here, one can see that the spontaneous excitation rate consists of two pieces. One is the thermal radiation outgoing from the horizon (the first term in \(\Gamma_{i-f}^H\) when \(r \rightarrow 2M\)), which is identical to what we have obtained in the Unruh case, and the other can be viewed as the thermal radiation incoming from infinity (the first term in \(\Gamma_{i-f}^H\) when \(r \rightarrow \infty\)). Both the outgoing and incoming thermal radiation are backscattered by the curvature on their way to a finite radial distance from the horizon, and this backscattering is represented by the greybody factors in the second term in \(\Gamma_{i-f}^H\) for both cases. Therefore, one sees that the Hartle-Hawking vacuum actually corresponds to a black hole immersed in a bath of both incoming and outgoing thermal radiation. Again, this is consistent with our understanding of the Hartle-Hawking vacuum gained in other different contexts.

### C. The Boulware vacuum

Finally, let us briefly discuss the Boulware vacuum case. Now, the Wightman function can be written as \([18, 19, 20]\)

\[
G_B^+(x, x') = \sum_{lm} \int_0^\infty \frac{e^{-i\omega \Delta t}}{4\pi \omega} |Y_{lm}(\theta, \phi)|^2 \left[ |\overrightarrow{R}_l(\omega, r)|^2 + |\overrightarrow{R}_l(-\omega, r)|^2 \right] d\omega,
\]

(48)
The corresponding Fourier transform reads

$$\mathcal{G}_B(\lambda) = \int_{-\infty}^{\infty} e^{i\lambda \tau} G_B^+ [x(\tau)] d\tau$$

$$= \sum_{ml} \int_{0}^{\infty} \frac{d\omega}{4\pi \omega} \int_{-\infty}^{\infty} e^{i(\lambda - \omega/\sqrt{1 - 2M/r})\tau} |Y_{lm}(\theta, \phi)|^2 [\mid \vec{R}_t(\omega, r) \mid^2 + \mid \vec{R}_l(\omega, r) \mid^2] d\tau$$

$$= \sum_{ml} \int_{0}^{\infty} \frac{d\omega}{2\omega} \delta(\lambda - \omega/\sqrt{1 - 2M/r}) |Y_{lm}(\theta, \phi)|^2 [\mid \vec{R}_t(\omega, r) \mid^2 + \mid \vec{R}_l(\omega, r) \mid^2] .$$

The further calculation should proceed in three separate cases

$$\mathcal{G}_B(\lambda) = \begin{cases} 
\sum_{ml} \frac{1}{2\lambda} |Y_{lm}(\theta, \phi)|^2 [\mid \vec{R}_t(\lambda\sqrt{g_{00}}, r) \mid^2 + \mid \vec{R}_l(\lambda\sqrt{g_{00}}, r) \mid^2] , & \lambda > 0 , \\
\sum_{ml} \frac{1}{4\lambda} |Y_{lm}(\theta, \phi)|^2 [\mid \vec{R}_t(\lambda\sqrt{g_{00}}, r) \mid^2 + \mid \vec{R}_l(\lambda\sqrt{g_{00}}, r) \mid^2] \bigg|_{\lambda \to 0} , & \lambda = 0 , \\
0 , & \lambda < 0 . 
\end{cases}$$

This indicates that for a positive $\omega_0$, $\mathcal{G}_B(-\omega_0)$ vanishes. Thus a substitution of Eq. (50) into Eq. (13) yields that $A_B = B_B$, which in turn gives rise to $P_{i \rightarrow f}(\tau) = 0$ and $\Gamma_{i \rightarrow f} = 2A_B - 2B_B = 0$. Therefore, no spontaneous excitation would ever occur in the Boulware vacuum. As a result, the Boulware vacuum corresponds to our familiar notion of a vacuum state.

**IV. CONCLUSION**

In summary, we have examined the Hawking radiation from the point of view of open quantum systems by looking at the time evolution of a detector (modelled by a two-level atom) interacting with vacuum massless scalar fields. The time evolution of the detector is governed by a master equation obtained by tracing over the field degrees of freedom from the complete system. The nonunitary effects have been studied by analyzing the time behavior of the detector’s admissible state in the Unruh, Hartle-Hawking, as well as Boulware vacua outside a Schwarzschild black hole. It is found that the detector in the Unruh and Hartle-Hawking vacua would spontaneously excite with a nonvanishing probability the same as what one would obtain if there is thermal radiation at the Hawking temperature from the
black hole, reproducing the basic results concerning the Hawking effect in the framework of open quantum systems. However, as we have pointed out in Section II, the possibility of a nonvanishing spontaneous excitation rate in fact involves the phenomena of decoherence and dissipation, so the open system approach seems more comprehensive physically than traditional treatments. Our study suggests that the general techniques and results in the theory of open quantum systems not only is applicable to the study of the Hawking effect and possibly even to that of other phenomena in curved space-times, such as particle creation, but also may shed new light on the physical understanding of them. We hope to turn to these issues in the future.

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