Influence of Extending the Height on the Vortex Structures Generated Inside A Lid Driven Cavity

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Abstract. A numerical study is conducted to study the effect of aspect ratio (extending the height) on the natural convection flow inside a square cavity. 2D incompressible equations are solved using the open CFD package OpenFOAM. A domain and grid independence study is conducted for getting the optimum dimensions and the grid size used in the problem. The validation is done for benchmark problems before applying on to the problem. The vortices formed inside the enclosure is identified by the help of streamlines. This paper clearly discusses the different regimes formed by changing the aspect ratio.

1. Introduction
The lid driven cavity is a classic problem, which is studied extensively by great many researchers. It is a standard benchmark problem to understand the shear flow generated due to the top lid of the cavity. Even if in appliance, the appearance of cavities might be non-rectangular, major researches in the theory are exercised only with the rectangular and square cavity flows. A narrow division also studies about handling of flows inside arched cavities directed by a traversing top.

On the lid-driven flow, Reyad Omari [1] took a numerical and experimental observation at moderate Reynolds Number. Numerical solutions of flow behaviour in cavities driven by lid at uplifted Reynolds Number has been studied by Fudhail A.M et al [2]. The spectral effects of cavities driven by lid flows were undergone by O.Botella et al. [3]. Later, for wall-driven flow in a semi-circular cavity, a finite element method has been carried out and the formation of vortices at increased Reynolds numbers has been published by Glowinski et al. [4]. Numerical and experimental buoyancy effects and following heat conduction upon internal flow structures of arc-shaped cavities were brought forward by Ching et al. [5]

So far, peoples were only concentrating on the formation of the primary vortex at moderate Reynolds Number of a lid-driven cavity. This paper aims at the study of formation of secondary vortex and the tertiary vortex at moderate Reynolds number, Re (defined as the relative dominance of inertia force over the viscous force) for $1 \leq AR \leq 3.1$ (AR is the aspect ratio of the cavity, defined as $D/L$, where $D$ is the height and $L$ is the length of the cavity) by using an open source CFD software, OpenFOAM.
2. Methodology
The non-dimensional governing equations for the 2D laminar, isothermal, incompressible flow (equations 1-3) are solved using the icoFoam solver in OpenFOAM package. We have analysed the flow for various Reynolds number with aspect ratios 1.0 to 3.1.

2.1 Computational domain
A square domain is selected as the computational domain after a domain independence test and is shown in Figure 1. The velocity components (u and v) are non-dimensionalised by the velocity of the lid and the pressure, p is non-dimensionalised with the dynamic pressure in the computations. The boundary conditions used for the simulation of the problem is also indicated in this figure. At the top lid u = 1 and v = 0 is enforced. In addition, the three walls are assigned a no-slip condition (u = v = 0).

![Figure 1. Schematic of Lid Driven Cavity](image)

2.2 Numerical details
The non-dimensional governing equations for the 2D laminar, isothermal, incompressible flow (equations 1-3) are solved using the icoFoam solver in OpenFOAM package. We have analysed the flow for various Reynolds number with aspect ratios 1.0 to 3.1.

Continuity equation:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]  

(1)
x-momentum equation:
\[
\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re} \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right]
\]  
(2)

y-momentum equation:
\[
\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{\partial p}{\partial y} + \frac{1}{Re} \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right]
\]  
(3)

Given
\[
Re = \frac{\rho ul}{\mu}
\]  
(4)

Where,
\(\mu\) is the viscosity of fluid, in (kilogram / metre-second)
\(u\) is the velocity of the fluid, in (metre/second)
\(L\) is the length or diameter of the fluid, in (metre)
\(\rho\) is the density of the fluid, in (kilogram / metre\(^3\))

3. Results and Discussions
The validation is done for the lid driven cavity problem and is compared with the previous works in the literature and is shown in figures 2a and 2b. Figure 2a represents the \(u\) velocity plotted along the vertical centreline and Figure 2b represents the \(v\) velocity plotted along the horizontal centreline of the cavity. It is seen from these figures that the velocity profiles are in good agreement with the existing literature.

Figure 2a. Computed \(u\)-velocity profile through a vertical line which passes along the geometric half-way point of the cavity \((x=0.5)\) for \(Re=100\) at various aspect ratios.

Figure 2b. Computed \(v\)-velocity profiles through a vertical line passing along the geometric half-way point of the cavity \((X=0.5)\) for \(Re=100\) at various aspect ratios.
The simulations are performed for a range of Reynolds numbers \((5 \leq \text{Re} \leq 1000)\) and aspect ratio \(1 \leq \text{AR} \leq 3.1\) for lid driven cavity. The lid is moving with a non-dimensional velocity \(u=1\) and the temperature of fluid is maintained to be constant (isothermal). We have used streamline plots to identify the vortical structures inside the cavity.

It is well known that the lid driven cavity would result in a single primary vortex structure for \(\text{Re}=1000\) (\(\text{AR}=1\)). However, in the present study, we found that the number of vortical structures formed inside the square cavity shows a strong dependency on AR. When the AR increases above a critical value, a secondary vortex is found to be generated. This critical value is found to be between 1.1 and 1.2 for \(\text{Re}=1000\). Figure 3 represents the streamlines plotted inside the cavity at \(\text{Re}=1000\) and \(\text{AR}=1.1\) and 1.2. A secondary vortex is seen for \(\text{AR}=1.2\) whereas it was absent for \(\text{AR}=1.1\). Similarly a tertiary vortex is seen for \(\text{AR}=2.3\), which was completely not visible for \(\text{AR}=2.2\), as shown in figure 4. Hence the present study reveals that there exist a critical value for AR at which the secondary and tertiary vortex develop.

**Figure 3.** Streamline plots inside the cavity for \(\text{Re}=1000\) and for different \(\text{AR}=1.1\) (fig 3a) and 1.2 (fig 3b). The Secondary vortex is not seen in fig 3a (AR 1.1) while it is seen in fig 3b (AR 1.2).
Figure 4. Streamline plots inside the cavity for Re=1000 and for different AR=2.2 (fig 4a) and 2.3 (fig 4b). The tertiary vortex is not seen in fig 4a (AR=2.2) while it is seen in fig 4b (AR=2.3).

Figure 5. Critical values of Aspect ratio for the development of secondary and tertiary vortex is plotted against the Reynolds number.
The number of vertical structures formed inside the cavity has great influence on the Reynolds number of the flow and the aspect ratio used. Hence, it is meaningful to plot the critical value of AR and the Re in this kind of a problem. Figure 5 represents the critical aspect ratio vs Reynolds number graph. This creates three different regimes for primary vortex, secondary vortex and tertiary vortex for the lid driven cavity problem. So this can be used in future for the prediction of the number of vortical structures formed inside the lid driven cavity.

4. Conclusions
The 2D incompressible laminar fluid flow inside a lid driven cavity is simulated for different values of Reynolds number and aspect ratio using an open source CFD package known as OpenFOAM. The vortical structures formed inside the cavity are identified with the help of streamline plots. It is found that for Re=1000, above a critical value for the aspect ratio, additional vortices are being generated (a secondary vortex between AR=1.1 and 1.2, a tertiary vortex at AR=2.2 and 2.3). In addition, we have found the different regimes of primary, secondary and tertiary vortices in the AR-Re plot.

5. References
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