An assertion language for slicing constraint logic languages

Moreno Falaschi\textsuperscript{1} and Carlos Olarte\textsuperscript{2}

\textsuperscript{1} Dept. Information Engineering and Mathematics, Università di Siena, Italy.
moreno.falaschi@unisi.it.
\textsuperscript{2} ECT, Universidade Federal do Rio Grande do Norte, Brazil
carlos.olarte@gmail.com.

Abstract. Constraint Logic Programming (CLP) is a language scheme for combining two declarative paradigms: constraint solving and logic programming. Concurrent Constraint Programming (CCP) is a declarative model for concurrency where agents interact by telling and asking constraints in a shared store. In a previous paper, we developed a framework for dynamic slicing of CCP where the user first identifies that a (partial) computation is wrong. Then, she marks (selects) some parts of the final state corresponding to the data (constraints) and processes that she wants to study more deeply. An automatic process of slicing begins, and the partial computation is “depurated” by removing irrelevant information. In this paper we give two major contributions. First, we extend the framework to CLP, thus generalizing the previous work. Second, we provide an assertion language suitable for both, CCP and CLP, which allows the user to specify some properties of the computations in her program. If a state in a computation does not satisfy an assertion then some “wrong” information is identified and an automatic slicing process can start. We thus make one step further towards automatizing the slicing process. We show that our framework can be integrated with the previous semi-automatic one, giving the user more choices and flexibility. We show by means of examples and experiments the usefulness of our approach.

Keywords: Concurrent Constraint Programming, Constraint Logic Programming, Dynamic slicing, Debugging, Assertion language.

1 Introduction

Constraint Logic Programming (CLP) is a language scheme \cite{20} for combining two declarative paradigms: constraint solving and logic programming (see an overview in \cite{19}). Concurrent Constraint Programming (CCP) \cite{28} (see a survey in \cite{25}) combines concurrency primitives with the ability to deal with constraints, and hence, with partial information. The notion of concurrency is based upon the shared-variables communication model. CCP is intended for reasoning, modeling and programming concurrent agents (or processes) that interact with each other and their environment by posting and asking information in a medium, a so-called store. CCP is a very flexible model and has been applied to an increasing number of different fields such as probabilistic and stochastic, timed and mobile systems \cite{269}, and more recently to social...
networks with spatial and epistemic behaviors [25], as well as modeling of biological systems [11,10,24,6].

One crucial problem with constraint logic languages is to define appropriate debugging tools. Various techniques and several frameworks have been proposed for debugging these languages. Abstract interpretation techniques have been considered (e.g. in [12,13,16,17]) as well as (abstract) declarative debuggers following the seminal work of Shapiro [30]. However, these techniques are approximated (case of abstract interpretation) or it can be difficult to apply them when dealing with complex programs (case of declarative debugging) as the user should answer to too many questions.

In this paper we follow a technique inspired by slicing. Slicing was introduced in some pioneer works by Mark Weiser [33]. It was originally defined as a static technique, independent of any particular input of the program. Then, the technique was extended by introducing the so called dynamic program slicing [22]. This technique is useful for simplifying the debugging process, by selecting a portion of the program containing the faulty code. Dynamic program slicing has been applied to several programming paradigms (see [21] for a survey). In the context of constraint logic languages, we defined a tool [15] able to interact with the user and filter, in a given computation, the information which is relevant to a particular observation or result. In other words, the programmer could mark (select) the information (constraints, agents or atoms) that she is interested to check in a particular computation that she suspects to be wrong. Then, a corresponding depurated partial computation is obtained automatically, where only the information relevant to the marked parts is present.

In a previous paper [15] we presented the first formal framework for debugging CCP via dynamic slicing. In this paper we give two major contributions. First, we extend our framework to CLP. Second, we introduce an assertion language which is integrated within the slicing process for automatizing it further. The extension to CLP is not immediate, as while for CCP programs non-deterministic choices give rise to one single computation, in CLP all computations corresponding to different non-deterministic choices can be followed and can lead to different solutions. Hence, some rethinking of the the framework is necessary. We show that it is possible to define a transformation from CLP programs to CCP programs, which allows us to show that the set of observables of a CLP program and of its translation to a CCP program correspond. This result also shows that the computations in the two languages are pretty similar and the framework for CCP can be extended to deal with CLP programs.

Our framework [15] consists of three main steps. First the standard operational semantics of the sliced language is extended to an enriched semantics that adds to the standard semantics the needed meta-information for the slicer. Second, we consider several analyses of the faulty situation based on the program wrong behavior, including causality, variable dependencies, unexpected behaviors and store inconsistencies. This second step was left to the user’s responsibility: the user had to examine the final state of the faulty computation and manually mark/select a subset of constraints that she wants to study further. The third step is an automatic marking algorithm that removes the information not relevant to derive the constraints selected in the second step. This algorithm is flexible and applicable to timed extensions of CCP [27]. Here, for CLP programs we introduce also the possibility to mark atoms, besides constraints.
We believe that the second step above, namely identifying the right state and the relevant information to be marked, can be difficult for the user and we believe that it is possible to improve automatization of this step. Hence, one major contribution of this paper is to introduce a specialized assertion language which allows the user to state properties of the computations in her program. If a state in a computation does not satisfy an assertion then some “wrong” information is identified and an automatic slicing process can start. We show that assertions can be integrated in our previous semi-automatic framework [15], giving the user more choices and flexibility. The assertion language is a good companion to the already implemented tool for the slicing of CCP programs to automatically detect (possibly) wrong behaviors and stop the computation when needed. The framework can also be applied to timed variants of CCP.

Organization and Contributions Section 2 describes CCP and CLP and their operational semantics. We introduce a translation from CLP to CCP programs and prove a correspondence theorem between successful computations. In Section 3 we recall the slicing technique for CCP [15] and extend it to CLP. The extension of our framework to CLP is our first contribution. As a second major contribution, in Section 4 we present our specialized assertion language and describe its main operators and functionalities. In Section 4.2 we show some examples to illustrate the expressiveness of our extension, and the integration into the former tool. Within our examples we show how to automatically debug a biochemical system specified in timed CCP and one classical search problem in CLP. Finally, Section 5 discusses some related work and concludes.

2 Constraint Logic Languages

In this section we define an operational semantics suitable for both, CLP [19] and CCP programs [28]. We start by defining CCP programs and then we obtain CLP by restricting the set of CCP operators.

Processes in CCP interact with each other by telling and asking constraints (pieces of information) in a common store of partial information. The type of constraints is not fixed but parametric in a constraint system (CS), a central notion for both CCP and CLP. Intuitively, a CS provides a signature from which constraints can be built from basic tokens (e.g., predicate symbols), and two basic operations: conjunction $\sqcap$ (e.g., $x \neq y \sqcap x > 5$) and variable hiding $\exists$ (e.g., $\exists x. y = f(x)$). As usual, $\exists x. c$ binds $x$ in $c$. The CS defines also an entailment relation ($\models$) specifying inter-dependencies between constraints: $c \models d$ means that the information $d$ can be deduced from the information $c$ (e.g., $x > 42 \models x > 37$). We shall use $\mathcal{C}$ to denote the set of constraints with typical elements $c, c', d, d', \ldots$. We assume that there exist $\tau, \tau' \in \mathcal{C}$, such that for any $c \in \mathcal{C}, c \models \tau$ and $\tau' \models \mathcal{C}$. The reader may refer to [25] for different formalizations and examples of constraint systems.

The language of CCP processes. In process calculi, the language of processes in CCP is given by a small number of primitive operators or combinators. Processes are built from constraints in the underlying constraint system and the following syntax:
\[
\begin{align*}
\pi & \vdash c_k & k \in I \\
(X, \text{tell}(c), F; d) & \rightarrow (X, \text{skip}, F; c \cup d) & \text{RTELL} \\
(X, \sum_{i \in I} \text{ask}(c_i) \text{ then } P_i; F; d) & \rightarrow (X, P_{\sum}; F; d) & \text{RSUM} \\
x \notin X \cup f\epsilon(d) \cup f\epsilon(\Gamma) & \rightarrow (X \cup \{x\}; F; d) & \text{RLOC} \\
p[\gamma] & \equiv P \in D & \text{RCALL} \\
(X, F; c) & \equiv (X; F'; c') \rightarrow (Y; \Delta'; d') \equiv (Y; \Delta; d) & \text{REQUIV} \\
\end{align*}
\]

Fig. 1: Operational semantics for CCP calculi

\[P, Q ::= \text{skip} \mid \text{tell}(c) \mid \sum_{i \in I} \text{ask}(c_i) \text{ then } P_i \mid \text{if } Q \mid (\text{local } x) P \mid p(\pi)\]

The process skip represents inaction. The process tell(c) adds c to the current store d producing the new store c \cup d. Given a non-empty finite set of indexes I, the process \(\sum_{i \in I} \text{ask}(c_i) \text{ then } P_i\) non-deterministically chooses \(P_k\) for execution if the store entails \(c_k\). The chosen alternative, if any, precludes the others. This provides a powerful synchronization mechanism based on constraint entailment. When I is a singleton, we shall omit the “\(\sum\)” and we simply write \(\text{ask}(c) \text{ then } P\).

The process \(P | Q\) represents the parallel (interleaved) execution of P and Q. The process \((\text{local } x) P\) behaves as P and binds the variable x to be local to it.

Given a process definition \(p[\gamma] \equiv P\), where all free variables of P are in the set of pairwise distinct variables \(\gamma\), the process \(p(\pi)\) evolves into \(P[\pi/\gamma]\). A CCP program takes the form \(D.P\) where D is a set of process definitions and P is a process.

The Structural Operational Semantics (SOS) of CCP is given by the transition relation \(\gamma \rightarrow \gamma'\) satisfying the rules in Figure 1. Here we follow the formulation in [14] where the local variables created by the program appear explicitly in the transition system and parallel composition of agents is identified by a multiset of agents. More precisely, a configuration \(\gamma\) is a triple of the form \((X; \Gamma; c)\), where \(c\) is a constraint representing the store, \(\Gamma\) is a multiset of processes, and \(X\) is a set of hidden (local) variables of \(c\) and \(\Gamma\). The multiset \(\Gamma = P_1, P_2, \ldots, P_n\) represents the process \(P_1 \parallel P_2 \parallel \cdots \parallel P_n\). We shall indistinguishably use both notations to denote parallel composition. Moreover, processes are quotiented by a structural congruence relation \(\equiv\) satisfying: (STR1) \(P \equiv Q\) if P and Q differ only by a renaming of bound variables (alpha conversion); (STR2) \(P \parallel Q \equiv Q \parallel P\); (STR3) \(P \parallel (Q \parallel R) \equiv (P \parallel Q) \parallel R\); (STR4) \(P \parallel \text{skip} \equiv P\). We denote by \(\rightarrow^*\) the reflexive and transitive closure of a binary relation \(\rightarrow\).

Definition 1 (Observables and traces). A trace \(\gamma_1, \gamma_2, \gamma_3, \cdots\) is a sequence of configurations s.t. \(\gamma_1 \rightarrow \gamma_2 \rightarrow \gamma_3 \cdots\). We shall use \(\pi, \pi'\) to denote traces and \(\pi(i)\) to denote the i-th element in \(\pi\). If \((X; \Gamma; d) \rightarrow^* (X'; \Gamma'; d')\) and \(\exists X'.d' \models c\) we write \((X; \Gamma; d) \triangleleft c\). If \(X = \emptyset\) and \(d = \varepsilon\) we simply write \(\Gamma \triangleleft c\).

Intuitively, if \(P\) is a process then \(P \triangleleft c\) says that \(P\) can reach a store \(d\) strong enough to entail \(c\), i.e., \(c\) is an output of \(P\). Note that the variables in \(X'\) above are hidden from \(d'\) since the information about them is not observable.
2.1 The language of CLP

A CLP program [20] is a finite set of rules of the form
\[ p(\overline{t}) \leftarrow A_1, \ldots, A_n \]
where \( A_1, \ldots, A_n \), with \( n \geq 0 \), are literals, i.e. either atoms or constraints in the underlying constraint system \( C \), and \( p(\overline{t}) \) is an atom. An atom has the form \( p(t_1, \ldots, t_m) \), where \( p \) is a user defined predicate symbol and the \( t_i \) are terms from the constraint domain.

The top-down operational semantics is given in terms of derivations from goals [20]. A configuration takes the form \((\Gamma; c)\) where \( \Gamma \) (a goal) is a multiset of literals and \( c \) is a constraint (the current store). The reduction relation is defined as follows.

**Definition 2 (Semantics of CLP [20]).** Let \( \mathcal{H} \) be a CLP program. A configuration \( \gamma = (L_1, \ldots, L_i, \ldots, L_n; c) \) reduces to \( \psi \), notation \( \gamma \rightarrow_{\text{CLP}(\mathcal{H})} \psi \), by selecting and removing a literal \( L_i \) and then:

1. If \( L_i \) is a constraint \( d \) and \( d \sqcup c \neq \emptyset \), then \( \gamma \rightarrow_{\text{CLP}(\mathcal{H})} (L_1, \ldots, L_{i-1}, L_{i+1}, \ldots, L_n; c \sqcup d) \).
2. If \( L_i \) is a constraint \( d \) and \( d \sqcup c = \emptyset \) (i.e., the conjunction of \( c \) and \( d \) is inconsistent), then \( \gamma \rightarrow_{\text{CLP}(\mathcal{H})} (\emptyset; \emptyset) \) where \( \emptyset \) represents the empty multiset of literals.
3. If \( L_i \) is an atom \( p(t_1, \ldots, t_k) \), then \( \gamma \rightarrow_{\text{CLP}(\mathcal{H})} (L_1, \ldots, L_{i-1}, \Delta, L_{i+1}, \ldots, L_n; c) \) where one of the definitions for \( p \), \( p(s_1, \ldots, s_k) \leftarrow A_1, \ldots, A_n \), is selected and \( \Delta = A_1, \ldots, A_n, s_1 = t_1, \ldots, s_k = t_k \).

A computation from a goal \( G \) is a (possibly infinite) sequence \( \gamma_1 = (G; \emptyset) \rightarrow_{\text{CLP}(\mathcal{H})} \gamma_2 \rightarrow_{\text{CLP}(\mathcal{H})} \cdots \). We say that a computation finishes if the last configuration \( \gamma_n \) cannot be reduced, i.e., \( \gamma_n = (\emptyset; c) \). In this case, if \( c = \emptyset \) then the derivation fails otherwise we say that it succeeds.

Given a goal with free variables \( \overline{x} = \text{var}(G) \), we shall also use the notation \( G \parallel^H \overline{x} \) to denote that there is a successful computation \((G; \emptyset) \rightarrow_{\text{CLP}(\mathcal{H})}^\ast (\emptyset; d) \) s.t. \( \exists \overline{x}.d \models c \).

We note that the free variables of a goal are progressively “instantiated” during computations by adding new constraints. Finally, the answers of a goal \( G \), notation \( G \parallel^H \) is the set \( \{ \exists \overline{x}.(\overline{x} \setminus \text{var}(G))(c) \mid (G; \emptyset) \rightarrow_{\text{CLP}(\mathcal{H})}^\ast (\emptyset; c), c \neq \emptyset \} \) where “\( \setminus \)" denotes set difference.

**From CLP to CCP.** CCP is a very general paradigm that extends both Concurrent Logic Programming and Constraint Logic Programming [23]. However, in CLP, we have to consider non-determinism of the type “don’t know” [29], which means that each predicate call can be reduced by using each rule which defines such a predicate. This is different from the kind of non-determinism in CCP, where the choice operator selects randomly one of the choices whose ask guard is entailed by the constraints in the current store (see \( R_{\text{SUM}} \) in Figure 1).

It turns out that by restricting the syntax of CCP and giving an alternative interpretation to non-deterministic choices, we can have an encoding of CLP programs as CCP agents. More precisely, we shall remove the synchronization operator and we shall consider only blind choices of the form \( Q = \sum_{i \in I} \text{ask}(\tau) \text{ then } P_i \). Note that \( c \models \tau \) for
any \( c \) and then, the choices in the process \( Q \) are not guarded/constrained. Hence, any of the \( P_i \) can be executed regardless of the current store. This mimics the behavior of CLP predicates (see (3) in Definition 2), but with a different kind of non-determinism. The next definition formalizes this idea.

**Definition 3 (Translation).** Let \( C \) be a constraint system, \( \mathcal{H} \) be a CLP program and \( G \) be a goal. We define the set of CCP process definitions \( \llbracket \mathcal{H} \rrbracket = \mathcal{D} \) as follows. For each user defined predicate symbol \( p \) of arity \( j \) and \( 1..m \) defined rules of the form

\[
p(t_1, \ldots, t_j) \leftarrow A_1, \ldots, A_n,
\]

we add to \( \mathcal{D} \) the following process definition

\[
p(x_1, \ldots, x_j) \overset{A}{=} \text{ask}(t) \text{ then} \left( \left( \text{local} \ x_i \right) \prod D_i \ | \ \prod A_i \right) + \ldots + \text{ask}(t) \text{ then} \left( \left( \text{local} \ x_i \right) \prod D_i \ | \ \prod A_i \right)
\]

where \( \prod t_i = \text{var}(t_1, \ldots, t_j) \cup \text{var}(A_1, \ldots, A_n) \), \( D_i \) is the set of constraints \( \{ x_1 = t_1, \ldots, x_j = t_j \} \), \( \prod D_i \) means \( \text{tell}(x_1 = t_1) \ | \ \cdots \ | \ \text{tell}(x_j = t_j) \) and literals are translated as \( A(\overline{x}) = A(\overline{t}) \) (case of atoms) and \( [c] = \text{tell}(c) \) (case of constraints). Moreover, we translate the goal \( \llbracket A_1, \ldots, A_n \rrbracket \) as the process \( \llbracket A_1 \rrbracket \ | \ \cdots \ | \ \llbracket A_n \rrbracket \).

We note that the head \( p(\overline{t}) \) of a process definition \( p(\overline{t}) \overset{A}{=} P \) in CCP can only have variables while a head of a CLP rule \( p(\overline{t}) \leftarrow B \) may have arbitrary terms with (free) variables. Moreover, in CLP, each call to a predicate returns a variant with distinct new variables (renaming the parameters of the predicate) \[^{[20]}\]. These two features of CLP can be encoded in CCP by first introducing local variables ((local \( z_i \)) in the above definition) and then, using constraints \( (D_i) \) to establish the connection between the formal and the actual parameters of the process definition.

Consider for instance this simple CLP program dealing with lists:

\[
p([], []). \quad p([h_1 | L_1], [h_2 | L_2]) :- c(h_1, h_2), p(L_1, L_2).
\]

and its translation

\[
p(x, y) \overset{A}{=} \text{ask}(t) \text{ then} \left( \left( \text{local} \ x \right) \prod D \ | \ \text{tell}(x = []) \ | \ \text{tell}(y = []) \right)+ \text{ask}(t) \text{ then} \left( \left( \text{local} \ X \right) \prod D \ | \ c(H_1, H_2) \ | \ p(L_1, L_2) \right)
\]

where \( D = \{ x = [H_1 | L_1], y = [H_2 | L_2] \} \) and \( X = \{ H_1, H_2, L_1, L_2 \} \). Note that the CCP process \( p(l_a, l_b) \) can lead to 2 possible outcomes:

- Using the first branch, the store becomes \( l_a = [] \cup l_b = [] \).
- In the second branch, due to rule \( \text{R} \), four local distinct variables are created (say \( h_1, h_2, l_1, l_2 \), the store becomes \( l_a = [h_1 | l_1] \cup l_b = [h_2 | l_2] \cup c(h_1, h_2) \) and the process \( p(l_1, l_2) \) is executed on this new store.

These two CCP executions match exactly the behavior of the CLP goal \( p(LA, LB) \).

We emphasize that one execution of a CCP program will give rise to a single computation (due to the kind of non-determinism in CCP) while the CLP abstract computation model characterizes the set of all possible successful derivations and corresponding answers. In other terms, for a given initial goal \( G \), the CLP model defines the full set of answer constraints for \( G \), while the CCP translation will compute only one of them, as only one possible derivation will be followed.

**Theorem 1 (Adequacy).** Let \( C \) be a constraint system, \( c \in C \), \( \mathcal{H} \) be a CLP program and \( G \) be a goal. Then, \( G \vdash^C \mathcal{H} \iff [G] \vdash c \).
3 Slicing CCP and CLP programs

Dynamic slicing is a technique that helps the user to debug her program by simplifying a partial execution trace, thus depurating it from parts which are irrelevant to find the bug. It can also help to highlight parts of the programs which have been wrongly ignored by the execution of a wrong piece of code. In [15] we defined a slicing technique for CCP programs that consisted of three main steps:

**S1** Generating a (finite) trace of the program. For that, a new semantics is needed in order to generate the (meta) information needed for the slicer.

**S2** Marking the final store, to select some of the constraints that, according to the wrong behavior detected, should or should not be in the final store.

**S3** Computing the trace slice, to select the processes and constraints that were relevant to produce the (marked) final store.

We shall briefly recall the step S1 in [15] which remains the same here. Steps S2 and S3 need further adjustments to deal with CLP programs. In particular, we shall allow the user to select processes (literals in the CLP terminology) in order to start the slicing. Moreover, in Section 4, we provide further tools to automatize the slicing process.

**Enriched Semantics (Step S1).** The slicing process requires some extra information from the execution of the processes. More precisely, (1) in each operational step \( \gamma \rightarrow \gamma' \), we need to highlight the process that was reduced; and (2) the constraints accumulated in the store must reflect, exactly, the contribution of each process to the store. In order to solve (1) and (2), we introduced in [15] the enriched semantics that extracts the needed meta information for the slicer. Roughly, we identify the parallel composition \( Q = P_1 \parallel \cdots \parallel P_n \) with the sequence \( \Gamma_Q = P_1 : i_1, \cdots , P_n : i_n \) where \( i_j \in \mathbb{N} \) is a unique identifier for \( P_j \). The use of indexes allow us to distinguish, e.g., the three different occurrences of \( P \) in \( "T_1, P : i, T_2, P : j, (\text{ask} (c) \text{ then } P) : k". \) The enriched semantics uses transitions with labels of the form \([i]k\rightarrow\) where \( i \) is the identifier of the reduced process and \( k \) can be either \( \perp \) (undefined) or a natural number indicating the branch chosen in a non-deterministic choice (Rule \( R'_\text{SUM} \)). This allows us to identify, unequivocally, the selected alternative in an execution. Finally, the store in the enriched semantics is not a constraint (as in Figure 1) but a set of (atomic) constraints where \( \{d_1, \cdots , d_n\} \) represents the store \( d_1 \sqcup \cdots \sqcup d_n \). For that, the rule of \texttt{tell}(c) first decomposes \( c \) in its atomic components before adding them to the store.

**Marking the Store (Step S2).** In [15] we identified several alternatives for marking the final store in order to indicate the information that is relevant to the slice that the programmer wants to recompute. Let us suppose that the final configuration in a partial computation is \( (X; \Gamma; S) \). The user has to select a subset \( S_{\text{sliced}} \) of the final store \( S \) that may explain the (wrong) behavior of the program. \( S_{\text{sliced}} \) can be chosen based on the following criteria:

1. **Causality:** the user identifies, according to her knowledge, a subset \( S' \subseteq S \) that needs to be explained (i.e., we need to identify the processes that produced \( S' \)).
translating replacing pairs in
we shall consider also markings on the set of processes, i.e., the marking can be also a fresh constant symbol
) is replaced with
specified (constraint)
where \( \gamma_{t} \) = \((X_{t}; \Gamma_{t}; S_{t}) \)
Output: a sliced trace \( \gamma_{n}^{0} \rightarrow \cdots \rightarrow \gamma_{n} \)
begin
1  let \( \theta = \{[\bullet/i] \mid P; i \in \Gamma_{n} \setminus \Gamma_{s}\} \) in
2  \( \gamma_{n}^{0} \leftarrow (X_{n} \cup \text{vars}(S_{\text{sliced}}, \Gamma_{\text{sliced}}); \Gamma_{n}; S_{\text{sliced}}) \);
3  for \( \ell = n - 1 \) to 0 do
4    let \( \theta', c = \text{sliceProcess}(\gamma_{\ell+1}; \ell+1; \ell, \Gamma_{t}; S_{t}) \) in
5    \( S_{\text{sliced}} \leftarrow S_{\text{sliced}} \cup \text{minimal}(S_{\ell}, c) \);
6    \( \theta \leftarrow \theta' \circ \theta \);
7  \( \gamma_{\ell} \leftarrow (X_{\ell} \cap \text{vars}(S_{\text{sliced}}, \Gamma_{\text{sliced}}); \Gamma_{\ell}; S_{\ell} \cap \text{sliced}) \);
end

Algorithm 1: Trace Slicer.

\[ S_{\text{minimal}}(S, c) = \emptyset \text{ if } c = t; \text{ otherwise, } S_{\text{minimal}}(S, c) = \bigcup\{S' \subseteq S \mid \bigcup S' \models c \text{ and } S' \text{ is set minimal}\}. \]

2. Variable Dependencies: The user may identify a set of relevant variables \( V \subseteq \text{freeVars}(S) \) and then, we mark \( S_{\text{sliced}} = \{c \in S \mid \text{vars}(c) \cap V \neq \emptyset\} \).

3. Unexpected behaviors: there is a constraint \( c \) entailed from the final store that is not expected from the intended behavior of the program. Then, one would be interested in the following marking \( S_{\text{sliced}} = \bigcup\{S' \subseteq S \mid \bigcup S' \models c \text{ and } S' \text{ is set minimal}\} \), where “\( S' \) is set minimal” means that for any \( S'' \subseteq S', S'' \models c \).

4. Inconsistent output: The final store should be consistent with respect to a given specification (constraint) \( c \), i.e., \( S \) in conjunction with \( c \) must not be inconsistent. In this case, we have \( S_{\text{sliced}} = \bigcup\{S' \subseteq S \mid \bigcup S' \models c \text{ and } S' \text{ is set minimal}\} \).

For the analysis of CLP programs, it is important also to mark literals (i.e., calls to procedures in CCP). In particular, the programmer may find that a particular goal \( p(x) \) is not correct if the parameter \( x \) does not satisfy certain conditions/constraints. Hence, we shall consider also markings on the set of processes, i.e., the marking can be also a subset \( \Gamma_{\text{sliced}} \subseteq \Gamma \).

Trace Slice (Step S3). Starting from the then the pair \( \gamma_{\text{sliced}} = (S_{\text{sliced}}, \Gamma_{\text{sliced}}) \) denoting the user’s marking, we define a backward slicing step. Roughly, this step allows us to eliminate from the execution trace all the information not related to \( \gamma_{\text{sliced}} \). For that, the fresh constant symbol \( \bullet \) is used to denote an “irrelevant” constraint or process. Then, for instance, \( c \sqcup \bullet \) results from a constraint \( c \sqcup d \) where \( d \) is irrelevant. Similarly in processes as, e.g., \( \text{ask}(c) \) then \( \{P \parallel \bullet\} + \bullet \). A replacement is either a pair of the shape \([T/i]\) or \([T/c]\). In the first (resp. second) case, the process with identifier \( i \) (resp. constraint \( c \)) is replaced with \( T \). We shall use \( \theta \) to denote a set of replacements and we call these sets as “replacing substitutions”. The composition of replacing substitutions \( \theta_{1} \) and \( \theta_{2} \) is given by the set union of \( \theta_{1} \) and \( \theta_{2} \), and is denoted as \( \theta_{1} \circ \theta_{2} \).

Algorithm 1 extends the one in [15] to deal with the marking on processes \( \Gamma_{\text{sliced}} \). The last configuration \( \gamma_{n}^{0} \in \Gamma_{\text{sliced}} \) means that we only observe the local variables of interest, i.e., those in \( \text{vars}(S_{\text{sliced}}, \Gamma_{\text{sliced}}) \) as well as the relevant processes \( (\Gamma_{\text{sliced}}) \) and constraints \( (S_{\text{sliced}}) \). The algorithm backwardly computes the slicing by accumulating replacing pairs in \( \theta \) (line 7). The new replacing substitutions are computed by the function \( \text{sliceProcess} \) that returns both, a replacement substitution and a constraint needed in the case of ask agents as explained below.

8
Marking algorithms. Let us explain how the function sliceProcess works. Consider for instance the process \(P = \text{ask}(c') \text{then } P\) + \(\text{ask } (c) \text{ then } \text{tell}(d \sqcup e)\) and assume that we are backwardly slicing the trace \(\gamma = \gamma_1 \gamma_2 \cdots\gamma_n\). The procedure sliceProcess is applied to \(\gamma\) and it determines that only \(c\) is relevant in \(\text{tell}(d \sqcup e)\). Hence, the replacement \(\text{tell}(\bullet \sqcup e) / j\) is returned (see line 7 in Algorithm 1). The procedure is then applied to \(\gamma\). We already know that the ask agent \(P\) (partially) relevant since \(\text{tell}(d \sqcup e) \neq \bullet\) (i.e., the selected branch does contribute to the final result). Thus, the replacement \(\bullet + \text{ask } (c) \text{ then } \text{tell}(\bullet \sqcup e) / i\) is accumulated in order to show that the first branch is irrelevant. Moreover, since the entailment of \(e\) was necessary for the reduction, the procedure returns also the constraint \(c\) (line 5 of Algorithm 1) and the constraints needed to entail \(c\) are added to the set of relevant constraints (line 6 of Algorithm 1).

Example 1. Consider the following (wrong) CLP program:

\[
\text{length}([0], 0). \\
\text{length}([A | L], M) :- M = N, \text{length}(L, N). 
\]

The translation to CCP is similar to the example we gave in Section 2.1. An excerpt of a possible trace for the execution of the goal \(\text{length}([10, 20], \text{Ans})\). is:

```ccp
[135x-405]agent
tell
[135x-457][... H2 L2 N2 M2 ; M2=0 ; [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-405][... H2 L2 N2 M2 ; [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-435][135x-427][... H2 L2 N2 M2 ; [20]=[H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-405][... H2 L2 N2 M2 ; [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
[135x-429][ ... H2 L2 N2 M2 ; [20]= [H2 | L2], Ans=N1, N1=M1, [10,20]= [H1|L1], Ans=N1, N1=M1, [20]= [H2 | L2], M1=N2, N2=M2, L2=[]] ->
```

9
In this trace, we can see how the calls to the process definition `length` are unfolded and, in each state, new constraints are added. Those constraint relate, e.g., the variable `Ans` and the local variables created in each invocation (e.g., `M1` and `M2`).

In the last configuration, it is possible to mark only the equalities dealing with numerical expressions (i.e., `Ans=N1, M1=N1, M1=N2, N2=M2, M2=0`) and the resulting trace will abstract away from all the constraints and processes dealing with equalities on lists:

```
[0 ; length([10,20],Ans) ; t] -->
[0 ; * + ask() ... ; t] -->
[0 ; local ... ; t] -->
[N1 M1 ; Ans=N1 || N1=M1 || length(L1, M1) ; t] -->
[N1 M1 ; Ans=N1 || N1=M1 || length(L1, M1) ; ] -->
[N1 M1 ; length(L1, M1) ; Ans=N1] -->
[N1 M1 ; length(L1, M1) ; Ans=N1, N1=M1] -->
...
```

The fourth line should be useful to discover that `Ans` cannot be equal to `M1` (the parameter used in the second invocation to `length`).

### 4 An assertion language for logic programs

The declarative flavor of programming with constraints in CCP and CLP allows the user to reason about (partial) invariants that must hold during the execution of her programs. In this section we give a simple yet powerful language of assertion to state such invariants. Then, we give a step further in automatizing the process of debugging.

**Definition 4 (Assertion Language).** Assertions are built from the following syntax:

\[ F ::= \text{pos}(c) \mid \text{neg}(c) \mid \text{cons}(c) \mid \text{icons}(c) \mid F \land G \mid p(x)(F) \mid p(x)[F] \mid p(x)[F] \]

where \( c \) is a constraint \((c \in C)\), \( p(-) \) is a process name and \( \oplus \in \{\land, \lor, \rightarrow\} \).

The first four constructs deal with partial assertions about the current store. These constructs check, respectively, whether the constraint\( c \): (1) is entailed, (2) is not entailed, (3) is consistent wrt the current store or (4) leads to an inconsistency when added to the current store. Assertions of the form \( F \land G \) have the usual meaning. The assertions \( p(x)[F] \) states that all instances of the form \( p(\bar{t}) \) in the current configuration must satisfy the assertion \( F \). The assertions \( p(x)[F] \) is similar to the previous one but it checks for the existence of an instance \( p(\bar{t}) \) that satisfies the assertion \( F \).

Let \( \pi(i) = (X_i; I_i; S_i) \). We shall use \( \text{store}(\pi(i)) \) to denote the constraint \( \exists X_i. \bigcup S_i \) and \( \text{procs}(\pi(i)) \) to denote the sequence of processes \( I_i \). The semantics for assertions is formalized next.

**Definition 5 (Semantics).** Let \( \pi \) be a sequence of configurations and \( F \) be an assertion. We inductively define \( \pi, i \models_F F \) (read as \( \pi \) satisfies the formula \( F \) at position \( i \)) as:

- \( \pi, i \models_F \text{pos}(c) \) if \( \text{store}(\pi(i)) \models c \).
- \( \pi, i \models_F \text{neg}(c) \) if \( \text{store}(\pi(i)) \not\models c \).
- \( \pi, i \models_F \text{cons}(c) \) if \( \text{store}(\pi(i)) \cup c \not\models f \).
- \( \pi, i \models_F \text{icons}(c) \) if \( \text{store}(\pi(i)) \cup c \models f \).
- \( \pi, i \models_F F \land G \) if \( \pi, i \models_F F \) and \( \pi, i \models_F G \).
- \( \pi, i \models_F F \lor G \) if \( \pi, i \models_F F \) or \( \pi, i \models_F G \).
\[ \pi, i \models F \rightarrow G \text{ if } \pi, i \models F \implies \pi, i \models G. \]

\[ \pi, i \models p(\overline{t})[F] \text{ if for all } p(\overline{t}) \in \text{procs}(\pi(i)), \pi, i \models F[\overline{t}/\overline{t}]. \]

\[ \pi, i \models p(\overline{t})(F) \text{ if there exists } p(\overline{t}) \in \text{procs}(\pi(i)), \pi, i \models F[\overline{t}/\overline{t}]. \]

If it is not the case that \( \pi, i \models F \), then we say that \( F \) does not hold at \( \pi(i) \) and we write \( \pi(i) \not\models F \).

The above definition is quite standard and reflects the intuitions given above. Moreover, let us define \( \sim F \) as \( \sim \text{pos}(c) = \text{neg}(c) \) (and vice-versa), \( \sim \text{cons}(c) = \text{icons}(c) \) (and vice-versa), \( \sim (F \oplus F) \) as usual and \( \sim p(\overline{t})[F(\overline{t})] = p(\overline{t})(\sim F(\overline{t})) \) (and vice-versa). Note that, \( \pi(i) \models F \) iff \( \pi(i) \not\models \sim F \).

Example 2. Assume that the store in \( \pi(1) \) is \( S = x \in 0..10 \). Then,

- \( \pi, 1 \models \text{cons}(x = 5) \), i.e., the current store is consistent wrt the specification \( x = 5 \).
- \( \pi, 1 \not\models \text{icons}(x = 5) \), i.e., the store is not inconsistent wrt the specification \( x = 5 \).
- \( \pi, 1 \not\models \text{pos}(x = 5) \), i.e., the store is not “strong enough” in order to satisfy the specification \( x = 5 \).
- \( \pi, 1 \models \text{neg}(x = 5) \), i.e., store is “consistent enough” to guarantee that it is not the case that \( x = 5 \).

Note that \( \pi, i \models \text{pos}(c) \) implies \( \pi, i \models \text{cons}(c) \). However, the other direction is in general not true (as shown above). We note that CCP and CLP are monotonic in the sense that when the store \( c \) evolves into \( d \), it must be the case that \( d \models c \) (i.e., information is monotonically accumulated). Hence, \( \pi, i \models \text{pos}(c) \) implies \( \pi, i + j \models \text{pos}(c) \). Finally, if the store becomes inconsistent, \( \text{cons}(c) \) does not hold for any \( c \).

Temporal \( \{23\} \) and linear \( \{14\} \) variants of CCP remove such restriction on monotonicity.

We note that checking assertions amounts, roughly, to testing the entailment relation in the underlying constraint system. Checking entailments is the basic operation CCP agents perform. Hence, from the implementation point of view, verification of assertions does not introduce a significant extra computational cost.

Example 3 (Conditional assertions). Let us introduce some patterns of assertions useful for verification.

- Conditional constraints: The assertion \( \text{pos}(c) \rightarrow F \) checks for \( F \) only if \( c \) can be deduced from the store. For instance, the assertion \( \text{pos}(c) \rightarrow \text{neg}(d) \) says that \( d \) must not be deduced when the store implies \( c \).

- Conditional predicates: Let \( G = p(\overline{t})(\text{cons}(\overline{t})) \). The assertion \( G \rightarrow F \) states that \( F \) must be verified whenever there is a call/goal of the form \( p(\overline{t}) \) in the context. Moreover, \( (\sim G) \rightarrow F \) verifies \( F \) when there are no calls of the form \( p(\overline{t}) \) in the context.

4.1 Dynamic slicing with assertions

Assertions allow the user to specify conditions that her program must satisfy during execution. If this is not the case, the program should stop and start the debugging process. In fact, the assertions may help to give a suitable marking pair \( (\text{Sliced}, \Gamma_{\text{Sliced}}) \) for the step \( S2 \) of our algorithm as we show in the next definition.
Definition 6. Let $F$ be an assertion, $\pi$ be a partial computation, $n > 0$ and assume that $\pi, n \not\models F$, i.e., $\pi(n)$ fails to establish the assertion $F$. Let $\pi(n) = (X; \Gamma; S)$. As testing hypotheses, we define $\text{symp}(\pi, F, n) = (S_{sl}, \Gamma_{sl})$ where

1. If $F = \text{pos}(c)$ then $S_{sl} = \{d \in S \mid \text{vars}(d) \cap \text{vars}(c) \neq \emptyset\}$, $\Gamma_{sl} = \emptyset$.
2. If $F = \text{neg}(c)$ then $S_{sl} = \bigcup\{S' \subseteq S \mid \bigcup S' = c \text{ and } S' \text{ is set minimal}\}$, $\Gamma_{sl} = \emptyset$.
3. If $F = \text{cons}(c)$ then $S_{sl} = \bigcup\{S' \subseteq S \mid \bigcup S' \cup c = \top \text{ and } S' \text{ is set minimal}\}$, $\Gamma_{sl} = \emptyset$.
4. If $F = \text{Icons}(c)$ $S_{sl} = \{d \in S \mid \text{vars}(d) \cap \text{vars}(c) \neq \emptyset\}$ and $\Gamma_{sl} = \emptyset$.
5. If $F = F_1 \land F_2$ then $\text{symp}(\pi, F_1, n) \cup \text{symp}(\pi, F_2, n)$.
6. If $F = F_1 \lor F_2$ then $\text{symp}(\pi, F_1, n) \cap \text{symp}(\pi, F_2, n)$.
7. If $F = F_1 \rightarrow F_2$ then $\text{symp}(\pi, \sim F_1, n) \cup \text{symp}(\pi, F_2, n)$.
8. If $F = p(\vec{t})(F_1)$ then $S_{sl} = \emptyset$ and $\Gamma_{sl} = \{p(\vec{t}) \in \Gamma \mid \pi, n \not\models F_1[\vec{t}/\vec{x}]\}$.
9. If $F = p(\vec{t})(F_1)$ then $S_{sl} = \{d \in S \mid \text{vars}(d) \cap \text{vars}(F_1) \neq \emptyset\}$, $\Gamma_{sl} = \{p(\vec{t}) \in \Gamma\}$

Let us give some intuitions about the above definition. Consider a (partial) computation $\pi$ of length $n$ where $\pi(n) \not\models F$. In the case (1) above, $c$ must be entailed but the current store is not strong enough to do it. A good guess is to start examining the processes that added constraints using the same variables as in $c$. It may be the case that such processes should have added more information to entail $c$ as expected in the specification $F$. Similarly for the case (4): $c$ in conjunction with the current store should be inconsistent but it is not. Then, more information on the common variables should have been added. In the case (2), $c$ should not be entailed but the store indeed entails $c$. In this case, we mark the set of constraints that entails $c$. The case (3) is similar. In cases (5) to (7) we use $\cup$ and $\cap$ respectively for point-wise union and intersection in the pair $(S_{sl}, \Gamma_{sl})$. These cases are self-explanatory (e.g., if $F_1 \land F_2$ fails, we collect the failure information of either $F_1$ or $F_2$). In (8), we mark all the calls that do not satisfy the expected assertion $F(\pi)$. In (9), if $F$ fails, it means that either (a) there are no calls of the shape $p(\vec{t})$ in the context or (b) none of the calls $p(\vec{t})$ satisfy $F_1$. For (a), similarly to the case (1), a good guess is to examine the processes that added constraints with common variables to $F_1$ and see which one should have added more information to entail $F_1$. As for (b), we also select all the calls of the form $p(\vec{t})$ from the context. The reader may compare these definitions with the information selected in Step 82 in Section 8 regarding possibly wrong behavior.

Classification of Assertions. As we explained in Section 2.1, computations in CLP can succeed or fail and the answers to a goal is the set of constraints obtained from successful computations. Hence, according to the kind of assertion, it is important to determine when the assertions in Definition 4 must stop or not the computation to start the debugging process. For that, we introduce the following classification:

- **post-conditions**, $\text{post}(F)$ **assertions**: assertions that are meant to be verified only when an answer is found. This kind of assertions are used to test the “quality” of the answers wrt the specification. In this case, the slicing process begins only when an answer is computed and it does not satisfy one of the assertions. Note that assertions of the form $p(\vec{t})(F(\pi))$ and $p(\vec{t})(F(\pi))$ are irrelevant as post-conditions since the set of goals in an answer must be empty.
- path invariants, $\text{inv}(F)$ assertions: assertions that are meant to hold along the whole computation. Then, not satisfying an invariant must be understood as a symptom of an error and the computation must stop. We note that due to monotonicity, only assertions of the form $\text{neg}(c)$ and $\text{cons}(c)$ can be used to stop the computation (note that if the current configuration fails to satisfy $\text{neg}(c)$, then any successor state will also fail to satisfy that assertion). Constraints of the form $\text{pos}(c), \text{icons}(c)$ can be only checked when the answer is found since, not satisfying those conditions in the partial computation, does not imply that the final state will not satisfy them.

4.2 Experiments

We conclude this section with a series of examples showing the use of assertions. Examples 4 and 5 deal with CLP programs while Examples 6 and 7 with CCP programs.

Example 4. The debugger can automatically start and produce the same marking in Example 1 with the following (invariant) assertion:

\begin{verbatim}
length([A | L], M) :- M = N, length(L, N), inv(pos(M>0)).
\end{verbatim}

Example 5. Consider the following CLP program (written in GNU-Prolog with integer finite domains) for solving the well known problem of posing $N$ queens on a $N \times N$ chessboard in such a way that they do not attack each other.

\begin{verbatim}
queens(N, Queens) :- length(Queens, N), fd_domain(Queens,1,N),
   constrain(Queens), fd_labeling(Queens,[],). 
constrain(Queens) :-fd_all_different(Queens), diagonal(Queens).
    diagonal([]).
    diagonal([Q|Queens]) :- secure(Q, 1, Queens), diagonal(Queens).
secure(_,_,[]).
secure(X,D,[Q|Queens]) :- doesnotattack(X,Q,D),D1 is D+1, secure(X,D1,Queens).
doesnotattack(X,Y,D) :- X + D #\= Y,Y + X #\= D.
\end{verbatim}

The program contains one mistake, which causes the introduction of a few additional and not correct solutions, e.g., $[5,4,3,2]$ for the goal queens(5,X). The user now has two possible strategies: either she lets the interpreter compute the solutions, one by one and then, when she sees a wrong solution she uses the slicer for marking manually the final store to get the sliced computation; or she can define an assertion to be verified. In this particular case, any solution must satisfy that the difference between two consecutive positions in the list must be greater than 1. Hence, the user can introduce the following post-condition assertion:

\begin{verbatim}
secure(X,D,[Q|Queens]) :- doesnotattack(X,Q,D),D1 is D+1, secure(X,D1,Queens),
   post(cons(Q #\= X+1)).
\end{verbatim}

Now the slicer stops as soon as the constraint $X \ #\not\ = \ Q+1$ becomes inconsistent with the store in a successful computation (e.g., the assertion fails on the –partial– assignment “5,4”) and an automatic slicing is performed.

Example 6. In [13] we presented a compelling example of slicing for a timed CCP program modeling the synchronization of events in musical rhythmic patterns. As shown in Example 2 at [http://subsell.logic.at/slicer/](http://subsell.logic.at/slicer/) the slicer for CCP was able to sufficiently abstract away from irrelevant processes and constraints to highlight
the problem in a faulty program. However, the process of stopping the computation to start the debugging was left to the user. The property that failed in the program can be naturally expressed as an assertion. Namely, in the whole computation, if the constraint \( \text{beat} \) is present (representing a sound in the musical rhythm), the constraint \( \text{stop} \) cannot be present (representing the end of the rhythm). This can be written as the conditional assertion \( \text{pos}(\text{beat}) \rightarrow \text{neg}(\text{stop}) \). Following Definition 6, the constraints marked in the wrong computation are the same we considered in [15], thus automatizing completely the process of identifying the wrong computation.

Example 7. Example 3 in the URL above illustrates the use of timed CCP for the specification of biochemical systems (we invite the reader to compare in the website the sliced and non-sliced traces). Roughly, in that model, constraints of the form \( \text{Mdm2} \) (resp. \( \text{Mdm2A} \)) state that the protein \( \text{Mdm2} \) is present (resp. absent). The model includes activation (and inhibition) of biological rules modeled as processes (omitting some details) of the form \( \text{ask(} \text{Mdm2A} \text{)} \text{then next tell(} \text{Mdm2} \text{)} \text{modeling that “if Mdm2 is absent now, then it must be present in the next time-unit”}. The interaction of many of these rules makes the model trickier since rules may “compete” for resources and then, we can wrongly observe at the same time-unit that \( \text{Mdm2} \) is both present and absent. An assertion of the form \( (\text{pos(Mdm2A)} \rightarrow \text{neg(Mdm2)}) \land (\text{pos(Mdm2)} \rightarrow \text{neg(Mdm2A)}) \) will automatically stop the computation and produce the same marking we used to depurate the program in the website.

5 Related work and conclusions

Related work. Assertions for automatizing a slicing process have been previously introduced in [4] for the functional logic language Maude. The language they consider as well as the type of assertions are completely different from ours. They do not have constraints, and deal with functional and equational computations. Another previous work [31] introduced static and dynamic slicing for CLP programs. However, [31] essentially aims at identifying the parts of a goal which do not share variables, to divide the program in slices which do not interact. Our approach considers more situations, not only variable dependencies, but also other kinds of wrong behaviors. Moreover we have assertions, and hence an automatic slicing mechanism not considered in [31]. The well known debugging box model of Prolog [32] introduces a tool for observing the evolution of atoms during their reduction in the search tree. We believe that our methodology might be integrated with the box model and may extend some of its features. For instance, the box model makes basic simplifications by asking the user to specify which predicates she wants to observe. In our case, one entire computational path is simplified automatically by considering the marked information and identifying the constraints and the atoms which are relevant for such information.

Conclusions and future work. In this paper we have first extended a previous framework for dynamic slicing of (timed) CCP programs to the case of CLP programs. We considered a slightly different marking mechanism, extended to atoms besides constraints. Don’t know non-determinism in CLP requires a different identification of the computations of interest wrt CCP. We considered different modalities specified by the
user for selecting successful computations rather than all possible partial computations. As another contribution of this paper, in order to automatize the slicing process, we have introduced an assertion language. This language is rather flexible and allows one to specify different types of assertions that can be applied to successful computations or to all possible partial computations. When assertions are not satisfied by a state of a selected computation then an automatic slicing of such computation can start.

We implemented a prototype of the slicer in Maude and showed its use in debugging several programs. We are currently extending the tool to deal with CLP don’t know non-determinism. Being CLP a generalization of logic programming, our extended implementation could be also eventually used to analyze Prolog programs. Integrating the kind of assertions proposed here with already implemented debugging mechanisms in Prolog is an interesting future direction. We also plan to add more advanced graphical tools to our prototype, as well as to study the integration of our framework with other debugging techniques such as the box model and declarative or approximated debuggers [18,2]. We also want to investigate the relation of our technique with dynamic testing (e.g. concolic techniques) and extend the assertion language with temporal operators, e.g. the past operator (⊖) for expressing the relation between two consecutive states. Another future topic of investigation is a static version of our framework in order to try to compare and possibly integrate it with analyses and semi automatic corrections based on different formal techniques, and other programming paradigms [5,7,8,1].

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