Measurement-induced quantum coherence recovery

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Abstract. We show that measurement can recover the quantum coherence of a qubit in a pure dephasing environment. The experimental demonstration of this in an optical system by comparing the visibilities (and fidelities) of the final states with and without measurement is provided here. This method can be extended to other two-level quantum systems and entangled states in a dephasing evolution environment. It may also be used to implement other types of quantum information processing.

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1. Introduction

During the early years of the development of quantum mechanics, the measuring problem was treated with the project measurement model given by Von Neumann [1]. In this model, the measuring process leads to the irrevocable collapse of the quantum system into eigenstates and the coherence is destroyed. During the last few decades, with the development of quantum information theory, quantum measurement has been understood in the framework of quantum decoherence theory [2] and has been used to construct some quantum information processes, such as the Knill, Laflamme and Milburn (KLM) scheme [3], one-way quantum computation [4], etc. In particular, the quantum Zeno effect with continuous measurement can be used to preserve the coherence of specific states [5, 6]. Recently, it has been shown that weak measurement can erase the collapse effect induced by a previous weak measurement and the initial quantum state can be recovered [7]. Katz et al have experimentally verified this idea using superconducting phase qubits [8].

In this paper, we show that measurement can recover the quantum coherence of a single qubit evolved in a dephasing environment. A theoretical description of this method, which predicts that the visibility of the qubit can be recovered from 0 to 50%, is given. Then, we demonstrate this phenomenon experimentally in an optical system with photons produced by the process of spontaneous parametric down-conversion (SPDC). The recovery can be seen clearly by comparing the visibilities (and fidelities) of the final states with and without measurement. We also provide a simplified picture to understand this phenomenon and compare our technique with other methods used to suppress decoherence in the discussion section. Finally, we give the conclusion.

2. Theoretical description

The polarization of a single photon is used as the information carrier, and birefringent elements, which can couple the photon’s frequency with its polarization, are adopted as the adjustable dephasing environment. Consider an arbitrary input pure polarization state

\[ |\psi\rangle = \alpha |H\rangle + \beta |V\rangle, \]  

(1)

where \( \alpha \) and \( \beta \) are complex numbers that obey \( |\alpha|^2 + |\beta|^2 = 1 \). \( |H\rangle \) and \( |V\rangle \) represent the horizontal and vertical polarization states, respectively. As a result, the output after interaction time \( t \) in the birefringent crystal can be written as

\[ |\psi(\omega, t)\rangle = \alpha |H\rangle + e^{i\kappa\omega t} \beta |V\rangle \]  

(2)

when the optic axis of the birefringent crystal is set to be horizontal. The parameter \( \kappa \) is proportional to \( \Delta n = n_o - n_e \), which is the difference between the indices of refraction of ordinary \( (n_o) \) and extraordinary \( (n_e) \) light. Because \( \Delta n \neq 0 \), different frequencies will introduce different phase shifts \( \kappa \omega t \) in the output state. Considering the contributions of all the frequencies, the relative phase between the information carrier bases \( |H\rangle \) and \( |V\rangle \) may become truly uncorrelated, which will destroy the coherence of the qubit [9].

This phenomenon is quite similar to the Rabi oscillation of a qubit in an external field. For a Rydberg atom in a cavity [10], the overall Rabi oscillation should be integrated over the photon number distribution, which corresponds to the frequency distribution \( f(\omega) \) of the photon in our
The two projected states are separated into two paths, 1 and 2, without disturbing the photon’s subsequent dynamics. It should be noted that the frequency distributions of the projected states after measurement are different from that of the initial state, and the Pauli \(\sigma_x\) operation is employed to reverse \(H\) and \(V\) of the final output state. If the interaction time is \(t\) before measurement and the residual interaction time is \(t'\) after measurement, then the output density operator of the polarization reads (the subscripts represent paths 1 and 2)

\[
\rho' = \int f(\omega)(K_+|\varphi⟩_1⟨\varphi| + K_-|\varphi⟩_2⟨\varphi|) d\omega, \tag{5}
\]

where \(K_+ = \frac{1}{2}(1 + 2|\alpha||\beta| \cos(\phi + \kappa \omega t))\) and \(K_- = \frac{1}{2}(1 - 2|\alpha||\beta| \cos(\phi + \kappa \omega t))\) are the probabilities of projecting into the + and - polarization states, respectively. \(\phi\) is the relative phase between \(|H⟩\) and \(|V⟩\) of the initial state. \(|\varphi⟩_1 = \frac{1}{\sqrt{2}}(|V⟩_1 + e^{i\kappa \omega t}|H⟩_1\) and \(|\varphi⟩_2 = \frac{1}{\sqrt{2}}(|V⟩_2 - e^{i\kappa \omega t}|H⟩_2)\) are the residual evolution states in paths 1 and 2, respectively. As a result, the total probability of finding \(|\psi⟩\) is

\[
P'_{|\psi⟩} = \frac{1}{2} + 2|\alpha|^2|\beta|^2 \int f(\omega) \cos(\phi + \kappa \omega t) \cos(\phi + \kappa \omega t') d\omega. \tag{6}
\]

At the limit of a long enough interaction time and with \(t' = t\), \(P'_{|\psi⟩}\) tends to \(\frac{1}{2} + |\alpha|^2|\beta|^2\) and we can get coherence recovery for a set of pure states, which can be seen from figure 1. In the range \(\frac{3}{8}(3 - \sqrt{3}) < |\beta|^2 < \frac{1}{6}(3 + \sqrt{3})\), the fidelity with measurement is larger than the one without measurement and it reaches its maximal recovery at \(|\beta|^2 = \frac{1}{2}\) where the fidelity restores to 75%. The states that are close to the eigenstates of the decoherence environment \(H/V\) are less decohered, and measuring the qubit cannot improve the fidelity. For the set of maximally recovered states with the form \(\frac{1}{\sqrt{2}}(|H⟩ + e^{i\phi}|V⟩)\), the recovered fidelity of 75% is larger than the classical limit 66.7%,\(^3\) which shows their quantum effect.

### 3. Experimental demonstration

In order to experimentally demonstrate this phenomenon, we choose the initial state \(|+⟩ = \frac{1}{\sqrt{2}}(|H⟩ + |V⟩)\) from the set of maximally recovered states. The set-up of the experiment is shown

\(\dagger\)

For the continuous frequency distribution \(f(\omega), \lim_{\omega \to \infty} f(\omega) \exp(i\omega t) d\omega = 0.\)

\(^3\) The projective probability of any orthogonal measurement basis of the subspace spanned by these maximally recovered states distributes on \([0, 1]\). Therefore, the fidelity allowed by classical optics is 66.7%, see [12].
Figure 1. The fidelity for different cases with the interaction time long enough and \( t = t' \) for the case with measurement during the evolution.

schematically in figure 2. The second harmonic ultraviolet (UV) pulses are frequency doubled from a mode-locked Ti:sapphire laser with the center wavelength mode locked to 0.78 \( \mu \text{m} \) (with 130 fs pulse width and 76 MHz repetition rate). These UV pulses are focused into a beam-like cut beta-barium-borate (BBO) crystal [13, 14] to produce highly bright SPDC photons with special polarizations. We get about 28 000 coincidence events per second and the integral time is 10 s for each measurement. One of the SPDC photons (path b) is set to \( |+\rangle \) to demonstrate the coherence recovery, while the other (path a) is used as a trigger.

The decoherence evolution of the signal photon in path b is the controllable birefringent ‘environment’ using quartz plates with thickness \( L \), which are distributed into two sets (set 1 with thickness \( L_1 \) and set 2 with thickness \( L_2 \)). The measurement apparatus (M), which contains three half-wave plates (\( \lambda/2 \)) with the optical axes set at the same angle of 22.5° according to the axis of quartz and a polarization beam splitter (PBS), can project a photon state onto + or − linear polarization, which corresponds to path 1 or 2, respectively. We use a polarizer (P) in path b to choose the final detecting polarization of the signal photon. Both photons are then coupled by multi-mode fibers to single-photon avalanche detectors that are equipped with long pass filters (LP) to minimize the influence of the pump light. Any successful detection is given by the coincidence of the trigger photon (D1) and the photon concerned (D2 or D3).

The frequency spectrum of the photon is considered a Gaussian amplitude function \( G(\omega) \) with frequency spread \( \sigma \) and it is peaked at the central frequency \( \omega_c \) corresponding to the central wavelength \( \lambda_c = 0.78 \mu\text{m} \).\(^4\) According to equation (4), which is the case without measurement, the total probability of detecting \( |+\rangle \) is

\[
P_+ = \frac{1}{2} + \frac{1}{2} \cos(\gamma \omega_c) e^{-\gamma^2 \sigma^2/16},
\]

where \( \gamma = L \Delta n/c \) and \( c \) represents the velocity of the photon in vacuum. In our experiment, we can treat \( \Delta n \) as a constant of 0.01 for the small frequency distribution and the thickness of quartz plates \( L \) is represented by the corresponding retardation \( x \), which obeys the equation

\(^4\) Deduced from the Gauss-like pulse pumping laser.
Figure 2. The experimental set-up. The decoherence evolution (DE) is denoted by a dashed pane. The measuring apparatus (M) is inserted depending on different cases. We use half-wave plates ($\lambda/2$) to reverse $H$ and $V$ of the output state. The final detecting bases are chosen by a polarizer (P). Long pass filters (LP) are placed in front of single-photon detectors to minimize the influence of the pump light. Any successful detection is given by the coincidence of single-photon detectors D1 and D2 (path 2) or D1 and D3 (path 1).

$L = \Delta n / x$. The visibility of the final state without measurement can be calculated as $V = \cos(\gamma \omega_0) e^{-\gamma^2 \sigma^2/16}$. We can see that it will tend to zero with an increase in the thickness of quartz crystals.

However, if we measure the photon by inserting M between $L_1$ and $L_2$, we can obtain a certain coherence recovery. According to equation (6), we get the final total probability of detecting $|+\rangle$ (for $L > L_1$)

$$P'_{+} = \frac{1}{4} \left( 1 + \cos(\gamma \omega_0) e^{-\gamma^2 \sigma^2/16} + \cos(\xi \omega_0) e^{-\xi^2 \sigma^2/16} \right),$$

where $\xi = (2L_1 - L) \Delta n / c$.

It can be seen that for large $L$ and $L_1 = L_2 = L/2$, we still have a probability of 0.75 to detect $|+\rangle$ compared with 0.5 in the case without measurement. The visibility in this case is $V'_+ = \frac{1}{2} + \frac{1}{2} \cos(\gamma \omega_0) e^{-\gamma^2 \sigma^2/16}$, which can finally tend to 0.5 with an increase in $L$.

The visibility of the final state as a function of thickness $L$ is presented in figure 3, where the dots represent the data obtained with measurement during the decoherence evolution and the squares represent the data obtained without measurement. For each thickness $L$, we let $L_1 = L_2 = L/2$ to get the corresponding visibility in the case with measurement. We tilt the quartz plates with the optic axes set to the horizontal so that the relative phase is the integral multiple of 360° [9]. The solid lines are the theoretical fitting using the equations $V$ (without measurement) and $V'_{+}$ (with measurement) mentioned above. In our experiment, the frequency spread is about $6.9 \times 10^{12}$ Hz. It is seen that when the total thickness of the quartz plates is
Figure 3. Experimental results for the visibility in different cases. The solid lines are the theoretical results. The thickness of quartz plates is represented by the retardation. $\lambda_c = 0.78 \mu\text{m}$. The error bars, which are due to counting statistics, are less than the size of the symbols.

Figure 4. Experimental results for the detecting fidelity. The measurement apparatus (M) and $L_2$ are inserted (denoted by the arrow) when $L_1$ reaches $74\lambda_c$. The solid lines are the theoretical results employing equation (7) (before measurement) and equation (8) (after measurement). The inset presents the oscillation between maximal and minimal results in the dotted pane.

increased to $74\lambda_c$, the visibility with measurement reaches 0.501, whereas it will drop to about zero without measurement. Good agreement between theoretical predictions and experimental data is found.

We further demonstrate this phenomenon in a visualized way by measuring the fidelity of the state in the whole evolution. As shown in figure 4, we insert the measurement apparatus
Figure 5. Simplified picture to understand the coherence recovery phenomenon. All the states at different evolution times are represented by wave packets with special polarizations. E is the decoherence evolution and M represents the measurement apparatus that projects the photon onto + or − polarization.

(M) and $L_2$ when $L_1$ increases to $74\lambda_c$. While $L_2$ increases to $74\lambda_c$ too, we obtain the highest probability 0.773 of getting $|+\rangle$ corresponding to the theory prediction of 0.75. This error is mainly due to the limitation of precision when we calibrate the axes of the quartz plates. We also show the oscillation between the maximal and minimal probability of getting $|+\rangle$ by tilting a quartz plate to get the required angles. It can be seen from the inset of figure 4 that the oscillation is similar to a cosine curve in a small distribution of $L$, which agrees well with the theory prediction. As a result, we have experimentally demonstrated the coherence recovery by measuring the photon in the evolution.

4. Discussion

As measurement preserves the frequency distributions and the relative phases of the two projected states in the dephasing environment, we can choose a suitable interaction time after measurement to erase some of the unwanted effects of the environmental interaction. We may understand this phenomenon in a simplified picture as shown in figure 5 in the time domain. The coherent superposition of the initial state of the photon comes from the overlap in temporal modes of the two eigenstates $|H\rangle$ and $|V\rangle$. The first set of quartz plates with enough thickness, which is represented by E in figure 5, destroys the overlap completely. The projected states in the basis $+/-$ after measurement will preserve their relative phases. After passing through the second set of quartz plates with the same thickness, some of the eigenstate components
will overlap and then we get the coherence recovery. We should only insert a half-wave plate to transfer the state of the recovered part into the initial state acting as a $\sigma_x$ operation. Figure 5 also implies that it is possible to get perfect coherence recovery by employing time bin technology [15] to select only the recovered part.

It has been demonstrated that the quantum Zeno effect, quantum dynamical decoupling and strong continuous coupling can effectively suppress decoherence only if the frequency of the measurements or pulses is large enough or if the coupling is sufficiently strong [16]. In our paper, we demonstrate an interesting phenomenon, namely the measurement-induced recovery of coherence. The measurement used in our experiment is also a nonselective measurement, which is similar to that discussed in [16]. However, for the quantum zero effect, if the frequency of measurement is not extremely large, it will accelerate decoherence. In our case, we can recover the coherence after it has been destroyed completely with only one measurement and the coherence does not become better with an increase in measurement frequency. On the other hand, the suppression of decoherence has been experimentally realized in the pure dephasing environment by employing unitary ‘bang–bang’ control technology [11, 17]. In our experiment, we focus on the measurement-induced recovery of coherence, which gives us a deep understanding of the measurement.

5. Conclusion

In summary, we have demonstrated that by measuring a photon qubit during its evolution in a pure dephasing environment, the destroyed coherence can be recovered. It can be deduced from the theoretical mode we give that this kind of measurement may also be realized in other two-level systems such as a Rydberg atom coupled to a microcavity [18] and an electronic spin coupled to nuclear spins [19, 20]. This technology is also useful to demonstrate entanglement recovery, the Leggett–Garg inequality [21] and some kinds of Bell-like inequalities [22].

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