Laboratory study of forced rotating shallow water turbulence

Stefania Espa, Gabriella Di Nitto and Antonio Cenedese
DICEA, Sapienza Università di Roma, via Eudossiana 18, 00184, Italy
E-mail: stefania.espa@uniroma1.it, gabriella.dinitto@uniroma1.it, antonio.cenedese@uniroma1.it

Abstract. During the last three decades several authors have studied the appearance of multiple zonal jets in planetary atmospheres and in the Earth’s oceans. The appearance of zonal jets has been recovered in numerical simulations (Yoden & Yamada, 1993), laboratory experiments (Afanasyev & Wells, 2005; Espa et al., 2008, 2010) and in field measurements of the atmosphere of giant planets (Galperin et al., 2001). Recent studies have revealed the presence of zonation also in the Earth’s oceans, in fact zonal jets have been found in the outputs of Oceanic General Circulation Models-GCMs (Nakano & Hasumi, 2005) and from the analysis of satellite altimetry observations (Maximenko et al., 2005). In previous works (Espa et al., 2008, 2010) we have investigated the impact of the variation of the rotation rate and of the fluid depth on jets organization in decaying and forced regimes. In this work we show results from experiments performed in a bigger domain in which the fluid is forced continuously. The experimental set-up consists of a rotating tank (1m in diameter) where the initial distribution of vorticity has been generated via the Lorentz force in an electromagnetic cell. The latitudinal variation of the Coriolis parameter has been simulated by the parabolic profile assumed by the free surface of the rotating fluid. Flow measurements have been performed using an image analysis technique. Experiments have been performed changing the tank rotation rate and the fluid thickness. We have investigated the flow in terms of zonal and radial flow pattern, flow variability and jet scales.

1. Introduction

The presence of alternating zonal jets is a very common feature of atmospheres and oceans. In freely decaying conditions, the spontaneous appearance of zonal jets has been highlighted, firstly, by Rhines (Rhines, 1975). He showed that the latitudinal variation of the Coriolis parameter, induces an anisotropization of the inverse energy cascade and the energy is transferred towards slowest modes accumulating at a wavenumber, called Rhines scale $k_{Rh}$, where it is channelled into zonal jets and Rossby waves. In this sense this scale has said to arrest the inverse energy cascade and the resulting flow is anisotropic and characterized by a steep power law $k^{-5}$ for $k > k_{Rh}$. The concept of cascade arrest has been recently revisited by Sukoriansky and co-workers (Sukoriansky et al., 2007) considering a continuously forced flow in presence of a $\beta$—effect and a large scale energy sink. They studied barotropic flows with inverse energy cascade and they demonstrated that a $\beta$—effect causes only the anisotropization of the inverse cascade rather then its halting. In consequence a small-scale forced, barotropic, dissipative, two-dimensional turbulence, under $\beta$—effect develops an anisotropic inverse energy cascade and may attain several
steadystateregimes which can be classified in terms of the characteristic wavenumbers, $k_f$, $k_d$, $k_\beta$ and $k_{fr}$, which are associated with the forcing, the small-scale dissipation, the $\beta$-effect and the large-scale drag, respectively. Among these regimes an universal behaviour has been recovered in the so-called zonostrophic regime, that can be considered as a subset of the geostrophic turbulence. Its main characteristics are a strongly anisotropic kinetic energy spectrum and a slowly changing system of alternating zonal jets spanning the entire flow domain. A zonally banded flow pattern can be found also in intermediate regimes. In the context of continuously forced flow we study, experimentally, the appearance of zonal jets in turbulent, rotating, shallow water fluid, and how the variation of parameters, like fluid thickness and rotation rate, impacts on their formation/evolution.

2. Material and methods

The experiments have been performed in a square tank whose internal dimensions are 69cm $\times$ 68cm $\times$ 15cm, placed on a rotating table. In order to simulate flows in the Northern hemisphere, we imposed a rotation in a counterclockwise direction ranging between $2s^{-1}$ and $3s^{-1}$. The parabolic free surface assumed by the fluid under rotation is used to model in laboratory the variation of the Coriolis parameter with latitude ($f(y)$, where $y$ is the meridional coordinate), near the poles. In fact, it can be shown that due to the potential vorticity conservation, there is an exact dynamical equivalence between the variation of the Coriolis parameter with latitude, and the variation of height in the presence of constant $f$ (Pedlosky, 1987). The dynamics associated with the Coriolis parameter in the polar region is captured by a quadratic variation of it in $r$, the radial distance from the pole, assuming the pole as the reference point (this is the so-called $\gamma$-plane or polar $\beta$-plane approximation):

$$f(y) = f_0 + \gamma r^2$$

where $f_0 = 2\Omega \sin \varphi_0$, $\varphi_0$ is the central latitude of the domain considered, $\gamma = \Omega / R^2$, $\Omega$ and $R$ are the rotation and the radius of the Earth. In a rotating free-surface fluid the parabolic shape of the free surface is (Afanasyev et al., 2011):

$$H(r) = H_0 \left(1 + \hat{\gamma} \left(r^2 - \left(\frac{L^2 + W^2}{12}\right)\right)\right)$$

where $H_0$ is the initial depth of the fluid, $\hat{\gamma} = f_0^2 / 8gH_0$ is the analogue parameter for the polar-$\beta$ plane given by (1), $f_0 = 2\Omega$, $L$ and $W$ are the dimensions of the tank. The point of maximum depression of the fluid surface represents the pole, while the periphery of the domain corresponds to the lower latitudes. Using this experimental configuration the resulting flow is then aimed to simulate the dynamics in correspondence to the polar region of a rotating sphere (indicatively $45^\circ \leq \varphi \leq 90^\circ$). In this frame of reference, the expression of $\beta$ (that in the linear approximation represents the northward gradient of the Coriolis parameter) for intermediate latitudes is (Afanasyev & Wells, 2005; Espa et al., 2008):

$$\beta = \frac{2sr_m f_0}{H(r_m)}$$

where $r_m = r_{max}/2$ is a mid-latitude reference point, $s = \Omega^2 / 2g$.

In our experiments turbulence was produced nonintrusively by means of electromagnetic forcing. The tank was filled with an electrolyte solution (C NaCl = 80g/l) to a depth of 3cm or 4cm. An array of axially magnetized permanent (neodymium) magnets was placed underneath the fluid layer with a spacing of 3cm and alternating polarity. We used a combination of circular
(δ = 1.1cm) and rectangular magnets (Lx × Ly × H = 2 × 1 × 0.5cm³), whose magnetic field strengths were, respectively, of approximately 1191.5G and 1232.3G at the center of the magnet surface. Since the depth of the rotating fluid varied parabolically with the radius, in the inner part of the tank we placed the circular magnets, while in the external part the intenser rectangular ones. When the fluid was spun up to the solid-body rotation at a constant rate Ω, a constant voltage was applied to two electrodes placed inside the tank, on opposite sidewalls. The interaction between the horizontal current and the vertical magnetic field generated a Lorentz force perpendicular to both fields that set the fluid in motion. The initial vorticity distribution was characterized by the formation of opposite signed eddies, positive or negative according to the phase of the resulting Lorentz force, and whose initial horizontal length scales were related to the dimensions of the magnets and the distance between them. To investigate the turbulence by means of a non intrusive image analysis technique called Feature Tracking, the solution was seeded with styrene particles with a mean diameter of ∼ 50µm. The tank was covered with a transparent lid to prevent interaction with air. The free surface was lit with two lateral lamps to have a high contrast between the white particles and the black bottom. A video camera corotating with the system, perpendicular to the tank and with the optical axis parallel to the rotation axis, recorded the experiments with a frame rate of 20 frame/s and a resolution of 1023 × 1240 pixel. The FT technique allowed to reconstruct the velocity field evolution in a Lagrangian framework, and then the field interpolated onto a regular Eulerian grid of 128 × 128. More technical details on the measuring technique are given in (Espa et al., 2010). We performed several experiments changing some parameters in order to analyse their impact on the characteristics of the flow. For a fixed initial depth (H₀ = 3cm – 4cm) and an input current of 8A, we changed the rotation rate of the table. The forcing is applied continuously for all the duration of the experiments (∼ 6min). In Table 1 all the experimental parameters are listed. The value of β is computed assuming as a reference point r = r_{max}/2. The Ekman spindown time is evaluated as τ_{E_k} = H₀/(νΩ)⁰.⁵, where ν is the kinematic viscosity of the fluid. For all the experiments we estimated nondimensional parameters: the aspect ratio, the Ekman number, the Reynolds number and the Rossby number. The aspect ratio is 0.06 for experiments A–F and 0.04 for experiments G–M. The Ekman number (Ek = ν/(ΩH²)) is of the order of 10⁻⁴; the Reynolds number (Re = UL/ν, where U is the mean absolute value of the total velocity and L is the lenght of the tank), is of the order of 10³; and the Rossby number (Ro = U/(2OL)) is of the order of 10⁻³. In this work we will focus on the experiment A, E and G, corresponding results are shown in the next section.

3. Results

3.1. Radial and zonal flow pattern

The parabolic profile of the rotating fluid suggets to use a polar coordinate system with the pole in corrispondence of the center of the tank. In a polar coordinate system each point on the plane is determined by a distance from the pole, and an angle from a fixed direction (center of the tank-East). The distance from the pole is called the radial coordinate or radius (r), and the angle is the angular coordinate, polar angle, or azimuth (θ). The velocity field is interpolated onto a polar grid created using 360 radii and 60 circles. For each point of the grid the velocity is decomposed into the radial (parallel to the radius) and azimuthal components (tangential to the curve). The azimuthal component of the velocity is referred also as the zonal component. In a rotating fluid points with the same height are aligned along circles. These circles correspond to planetary parallels (points with the same latitude in a rotating sphere), thus they repesent the zonal direction. In Figure 1—2 are shown, respectively, the instantaneous radial-azimuthal pattern of the zonal and radial velocity for the experiment E (t = 3min). The azimuthal velocity shows alternating zonal bands, more homogeneous along intervals of θ. In the remainder of the
paper we consider $\theta = [0–180^\circ]$ for zonal averaging. We observe similar patterns in all the experiments with some differences in terms of jet spacing.

$$V_z (cm s^{-1})$$

![Figure 1. Instantaneous radial-azimuthal map of zonal velocity (Exp E, t = 3min)](image1)

$$V_r (cm s^{-1})$$

![Figure 2. Instantaneous radial-azimuthal map of radial velocity (Exp E, t = 3min)](image2)
3.2. Flow variability

To evidence the temporal evolution of the zonal structures we plot the Hovmöller diagrams of the zonally averaged azimuthal velocity in a time interval of 120s (Figure 3–4–5–6). On the right of each diagram there is the corresponding mean profile of the mean azimuthal velocity. We considered the experiments A and E characterized by a depth of 3cm and different rotation rate, and the case G in which the rotation rate is the same as A but the depth is 4cm. In Figure 3, the case A with a low $\beta$, the flow shows three alternating bands at smaller radii, with a broader westward band near $r = 0$cm. Even if magnets are placed from $r = 5$cm, the flow organizes itself, in the inner part of the tank, like an anticyclone vortex. On the other hand, in the higher $\beta$ case E, the first band seems a jet-like structure, localized around $r = 5$cm. The flow exhibits four opposite bands at smaller radii. In both cases the flow shows irregular patterns in the external part of the tank, even if the case E shows a better but weaker zonal structure. Comparing cases with the same rotation rate and different depths (Figure 4–5), we can observe that a higher depth induces a smaller $\beta$ and consequently less defined zonal bands. In Figure 6 we plot the Hovmöller diagram for the case M with the highest value of $\beta$ considered in our experiments. The flow shows a clear zonal structure in the whole domain.

Figure 3. Hovmöller diagram and the time mean profile of the mean zonal velocity (Exp A)

Figure 4. Hovmöller diagram and the time mean profile of the mean zonal velocity (Exp E)
3.3. Mean and instantaneous zonal velocity profiles
An insight into the degree of anisotropy of the flow is gained by performing zonal averaging, that is averaging over $\theta$. In Figure 7–8 are shown instantaneous and time-mean radial profiles of the mean azimuthal velocity for three experiments (A–C–E). The profile are plotted, from left to right, with increasing values of $\beta$. Instantaneous profiles show a stronger variability for larger radii also for the lower $\beta$ case (Figure 7–a), even if they are weaker due to the forcing intensity. Comparing these profiles, we can observe qualitatively differences in zonal jet scales. As expected we found broader jets in lower $\beta$ case. Figure 8 shows radial profiles of the time and azimuthal mean azimuthal velocity, averaged over approximately 5 minutes of the late phase of the experiments. It is possible to perform time averages since we checked the establishing of a steady state with a statistical tool, the so-called reverse arrangement test (Bendat & Piersol, 2000). From these profiles, it is clear that there are some zonal jetlike structures that survive time averaging, especially in correspondence of the inner part of the tank. We evaluated the root mean square of the zonal velocity and we found values between 0.48cms$^{-1}$ and 0.80cms$^{-1}$.
3.4. Zonal energy and jet scale

According to Huang and Robinson (Huang & Robinson, 1998), the time-mean total energy can be decomposed by $E = E_{SZ} + E_{TZ} + E_{SE} + E_{TE}$, where the terms on the right-hand size of the equation represent the time mean of the stationary zonal energy, the transient zonal energy, the stationary eddy energy and the transient eddy energy, respectively. The term "eddy" means deviation from the zonal mean. The ratio of the zonal to the total energy does not vary greatly from case to case, with a typical value of around 0.5, thus a rough equipartition between the eddy and zonal energy. A measure of the scale of the zonal jets is given by the square root of the ratio between the zonal enstrophy and the zonal energy (Huang & Robinson, 1998). The scale of the jets generally decreases with decreasing energy, or increasing $\beta$, consistent with the phenomenology described by Vallis and Maltrud (Vallis & Maltrud, 1993) and by Panetta (Panetta, 1993). In our experiments we estimated $1.36cm^{-1} \leq k_j \leq 1.66cm^{-1}$, with increasing values for increasing $\beta$, as expected. Thus the number of jets increases with increasing $\beta$.

4. Conclusion

We have investigated in laboratory the appearance of zonal jets in forced rotating shallow water turbulence. From the obtained results we were able to show the formation of alternating bands characterized by positive/negative mean zonal velocities. We highlighted the dependence of the jet scales and their spatial distribution on the strenght of the $\beta$–effect. As expected the flow show a clear zonally banded structure for higher values of $\beta$. We are actually performing further analysis to investigate the dependence of jet characteristics on a wider range of experimental parameters.
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