On CP violation and the measurement of the dimuon charge asymmetry at hadron colliders

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26 July 2006

Abstract

B factories measure the CP violation parameter of $B^0\bar{B}^0$ mixing and decay. Hadron colliders measure the dimuon charge asymmetry of an admixture of $B$ hadrons. In this note we discuss a subtle point on how the CP violation parameter of $B^0_s\bar{B}^0_s$ mixing and decay can be extracted from these measurements.

1 Introduction

$B$ factories measure the CP violation parameter of $B^0\bar{B}^0$ mixing and decay. Hadron colliders measure the dimuon charge asymmetry of an admixture of $B$ hadrons. In this note we discuss a subtle point on how the CP violation parameter of $B^0_s\bar{B}^0_s$ mixing and decay can be extracted from these measurements. This discussion is particularly relevant since the parameters of mixing and decay in the $B^0_s$ system, $\Delta M_s$, $\Delta \Gamma_s$ and $\alpha_s \equiv \Re(\epsilon_{B^0_s})/(1 + |\epsilon_{B^0_s}|^2)$ have come within experimental reach.

Let me summarize our arguments. We agree with the derivation in [1] of the decay rates up to a normalization factor. This normalization factor determines the total number of decays. For a given number of decays, the normalization factor may, or may not, depend on CP violation. In this note we present in detail two alternatives to illustrate this point.

The outline is as follows. In Section 2 we briefly review the standard formalism of $B^0\bar{B}^0$ mixing and decay. In Sections 3 and 4 we present two alternative calculations of the time-integrated probabilities of $B$ and $\bar{B}$ to decay to flavor specific final states. In Section 5 we present a discussion on these alternatives. The difference between them is significant for the $B^0_s$ system. Our conclusions are collected in Section 6.
2 $B^0 \bar{B}^0$ mixing and decay

Let us review the standard formalism of CP violation in mixing and decay.\[1\] We take the Hamiltonian in the $(\bar{B}^0, B^0)$, or $(\bar{B}^0_s, B^0_s)$, basis as

$$H \equiv M - \frac{i}{2} \Gamma \equiv \begin{bmatrix} m & M_{12} \\ M_{12}^* & m \end{bmatrix} - \frac{i}{2} \begin{bmatrix} \Gamma & \Gamma_{12} \\ \Gamma_{12}^* & \Gamma \end{bmatrix},$$

(1)

where the matrices $M$ and $\Gamma$ are hermitian. The Hamiltonian $H$ itself is not hermitian since the $B$ mesons do decay. This $2 \times 2$ Hamiltonian is an approximate description of a system that has many more dimensions.

The solution of the Schrödinger equation $i \partial \psi/\partial t = H \psi$ with

$$\psi \equiv \begin{pmatrix} \bar{B}^0(t) \\ B^0(t) \end{pmatrix}$$

(2)

is

$$\bar{B}^0(t) = \frac{1}{2} \{ s_+(t) + s_-(t) \} \bar{B}^0(0) + \frac{1 + \epsilon}{1 - \epsilon} \cdot \frac{1}{2} \{ s_+(t) - s_-(t) \} B^0(0),$$

$$B^0(t) = \frac{1 - \epsilon}{1 + \epsilon} \cdot \frac{1}{2} \{ s_+(t) - s_-(t) \} \bar{B}^0(0) + \frac{1}{2} \{ s_+(t) + s_-(t) \} B^0(0),$$

(3)

where

$$s_-(t) = \exp(-imt) \exp(-\Gamma t/2) \exp(i\Delta M t/2) \exp(\Delta \Gamma t/4),$$

$$s_+(t) = \exp(-imt) \exp(-\Gamma t/2) \exp(-i\Delta M t/2) \exp(-\Delta \Gamma t/4),$$

(4)

$$\frac{1 + \epsilon}{1 - \epsilon} = \frac{\Delta M - \frac{i}{2} \Delta \Gamma}{2 \left( M_{12} - \frac{i}{2} \Gamma_{12} \right)} = \frac{2 \left( M_{12} - \frac{i}{2} \Gamma_{12} \right)}{\Delta M - \frac{i}{2} \Delta \Gamma}.$$  

(5)

The phase of $(1 + \epsilon)/(1 - \epsilon)$ is arbitrary: it can be changed by redefining the phase of $\bar{B}^0(0)$. Observables depend on the absolute value of $(1 + \epsilon)/(1 - \epsilon)$, or equivalently, on the CP violation parameter of $B^0$ mixing and decay:

$$\alpha_q \equiv \frac{\Re(\epsilon)}{1 + |\epsilon|^2},$$

(6)

where $q = d, s$ (we write the subscript $q$ explicitly only on quantities that are needed later). For the same reason, we can multiply $M_{12}$ and $\Gamma_{12}$ by a common phase-factor. Only the relative phase is observable:

$$\angle \begin{bmatrix} -\Gamma_{12} \\ M_{12} \end{bmatrix} \equiv \phi.$$  

(7)
We introduce the standard notation
\[ x \equiv \frac{\Delta M}{\Gamma}, \quad y \equiv \frac{\Delta \Gamma}{2\Gamma}. \quad (8) \]

We now calculate the time integrated probabilities for \( B \) and \( \bar{B} \) to decay to flavor specific final states. Two alternatives are considered.

## 3 Alternative A

From (3) we obtain the time integrated probability that a \( \bar{B}_q^0 \) decays as a \( B_q^0 \):

\[
\chi_q = \frac{\int_0^\infty \left| \frac{1+\epsilon}{1-\epsilon} \right|^2 \frac{1}{4} \left| s_+ - s_- \right|^2 dt}{\int_0^\infty \left| \frac{1+\epsilon}{1-\epsilon} \right|^2 \frac{1}{4} \left| s_+ - s_- \right|^2 dt + \int_0^\infty \frac{1}{4} \left| s_+ + s_- \right|^2 dt}. \quad (9)
\]

The denominator in (9) normalizes the probabilities, so the probability that a \( \bar{B}_q^0 \) decays as a \( B_q^0 \) is \( 1 - \chi_q \). This normalization of probabilities defines alternative A.

The integrals are

\[
\int_0^\infty \left| s_+ \pm s_- \right|^2 dt = \frac{2}{\Gamma} \left[ \frac{1}{1-y^2} \pm \frac{1}{1+x^2} \right], \quad (10)
\]

so

\[
\chi_q = \frac{(x^2 + y^2) \left( \frac{1}{2} + \alpha_q \right)}{1 + x^2 - 2\alpha_q (1 - y^2)}, \quad (11)
\]

Similarly, the probability that a \( B_q^0 \) decays as a \( \bar{B}_q^0 \) is

\[
\bar{\chi}_q = \frac{(x^2 + y^2) \left( \frac{1}{2} - \alpha_q \right)}{1 + x^2 + 2\alpha_q (1 - y^2)}, \quad (12)
\]

and the probability that a \( B_q^0 \) decays as a \( B_q^0 \) is \( 1 - \bar{\chi}_q \). These equations can be found in [2].

Let us now obtain the dimuon charge asymmetry in the limit \( \alpha_q \ll 1 \):

\[
A_q \equiv \frac{N^{++} - N^{--}}{N^{++} + N^{--}} = \chi_q(1 - \bar{\chi}_q) - \bar{\chi}_q(1 - \chi_q) \approx 4\alpha_q = 3 \left\{ \frac{\Gamma_{12}}{M_{12}} \right\}. \quad (13)
\]

In this limit, \( \Delta M \approx 2|M_{12}| \). The single muon charge asymmetry is:

\[
a_q \equiv \frac{N^+ - N^-}{N^+ + N^-} = \frac{[\chi_q + (1 - \bar{\chi}_q)] - [\bar{\chi}_q + (1 - \chi_q)]}{[\chi_q + (1 - \chi_q)] + [\bar{\chi}_q + (1 - \chi_q)]} = \chi_q - \bar{\chi}_q = A_q \xi_q, \quad (14)
\]
where $\xi_q \equiv \chi_q + \bar{\chi}_q - 2\chi_q\bar{\chi}_q$. The asymmetry of tagged “wrong-sign” decay rates induced by oscillations is equal to the dimuon charge asymmetry $A_q$:

$$a_{SL}(t) \equiv \frac{\Gamma(\bar{B}_q^0 \to B_0^0 \to \mu^+ X) - \Gamma(B_q^0 \to B_0^0 \to \mu^- X)}{\Gamma(B_0^0 \to B_q^0 \to \mu^- X) + \Gamma(B_q^0 \to B_0^0 \to \mu^- X)}$$

$$= \frac{\frac{1+\epsilon}{1-\epsilon} s_+ - s_-}{\frac{1+\epsilon}{1-\epsilon} s_+ - s_-}$$

$$\approx 4\alpha_q. \quad (15)$$

Note that $a_{SL}(t)$ is independent of time $t$.

The preceding equations apply to the $B^0$ or $B_s^0$, systems separately. Let us now consider $N$ dimuon events with a $B$ and a $\bar{B}$ hadron at production. For simplicity we consider only direct semileptonic decays $B \to \mu X$. The sample has $N_{f_d}$ mesons $B^0$, $N_{f_s}$ mesons $B_s^0$, and $N(1 - f_d - f_s)$ other $B$ hadrons that do not mix. Similarly, the sample has $N_{f_d}$ mesons $B^0$, $N_{f_s}$ mesons $B_s^0$, and $N(1 - f_d - f_s)$ other $\bar{B}$ hadrons that do not mix.

The number of $\bar{B}$ hadrons that mix and decay as $\mu^+$ is $N_{f_d}\chi_d + N_{f_s}\chi_s \equiv N\chi$. The number of $B$ hadrons that mix and decay as $\mu^-$ is $N_{f_d}\bar{\chi}_d + N_{f_s}\bar{\chi}_s \equiv N\bar{\chi}$. So $\chi = f_d\chi_d + f_s\chi_s$. The number of $\bar{B}$ hadrons that decay as $\mu^-$ is $N_{f_d}(1 - \chi_d) + N_{f_s}(1 - \chi_s) + N(1 - f_d - f_s) = N(1 - \chi)$. Finally, the number of $B$ hadrons that decay as $\mu^+$ is $N_{f_d}(1 - \bar{\chi}_d) + N_{f_s}(1 - \bar{\chi}_s) + N(1 - f_d - f_s) = N(1 - \bar{\chi})$.

The dimuon charge asymmetry is

$$A = \frac{\chi(1 - \bar{\chi}) - \bar{\chi}(1 - \chi)}{\chi(1 - \bar{\chi}) + \bar{\chi}(1 - \chi)} = f_d A_d \frac{\xi_d}{\xi} + f_s A_s \frac{\xi_s}{\xi}, \quad (16)$$

where $\xi \equiv \chi + \bar{\chi} - 2\chi\bar{\chi}$. The dimuon charge asymmetry can be written approximately as

$$A \approx f_d A_d \frac{\chi_0}{\chi_0} \frac{1 - \chi_0}{1 - \chi_0} + f_s A_s \frac{\chi_0}{\chi_0} \frac{1 - \chi_0}{1 - \chi_0}, \quad (17)$$

where $\chi_0$ is the value of $\chi$ or $\bar{\chi}$ in the absence of CP violation, and similarly for $\chi_{q0}$.

## 4 Alternative B

Alternative B is the same as Alternative A, except that $\epsilon$ is set to zero in the denominator of (10). In other words, the probabilities add up to 1 only in the case of no CP violation, i.e. $\alpha_q = 0$. The time integrated probability that a
\( \bar{B}_q^0 \) decays as a \( B_q^0 \) is

\[
\chi_q = \frac{1 + 2\alpha_q}{1 - 2\alpha_q} \cdot \frac{x^2 + y^2}{2(1 + x^2)} \equiv \frac{1 + 2\alpha_q}{1 - 2\alpha_q} \cdot \chi_{q0}. \tag{18}
\]

The probability that a \( B_q^0 \) decays as a \( \bar{B}_q^0 \) is

\[
\bar{\chi}_q = \frac{1 - 2\alpha_q}{1 + 2\alpha_q} \chi_{q0}. \tag{19}
\]

The probability that a \( B_q^0 \) decays as a \( \bar{B}_q^0 \), and the probability that a \( B_q^0 \) decays as a \( B_q^0 \), are equal to \( (1 - \chi_{q0}) \), independent of CP violation. This is the definition of alternative B.

The dimuon charge asymmetry \( A_q \) is:

\[
A_q \equiv \frac{N^{++} - N^{--}}{N^{++} + N^{--}} = \frac{\chi_q(1 - \chi_{q0}) - \bar{\chi}_q(1 - \chi_{q0})}{\chi_q(1 - \chi_{q0}) + \bar{\chi}_q(1 - \chi_{q0})} \approx 4\alpha_q = \Re \left\{ \frac{\Gamma_{12}}{M_{12}} \right\}. \tag{20}\]

The single muon charge asymmetry is:

\[
a_q = \frac{N^+ - N^-}{N^+ + N^-} = \frac{[\chi_q + (1 - \chi_{q0})] - [\bar{\chi}_q + (1 - \chi_{q0})]}{[\chi_q + (1 - \chi_{q0})] + [\bar{\chi}_q + (1 - \chi_{q0})]} \approx \frac{\chi_q - \bar{\chi}_q}{2} = A_q \chi_{q0}. \tag{21}\]

The asymmetry of tagged “wrong-sign” decay rates induced by oscillations is \( a_{SL}(t) \approx 4\alpha_q \approx A_q \), as for alternative A. The dimuon charge asymmetry for the admixture of \( B \) hadrons is:

\[
A = f_d A_d \frac{\chi_{d0}}{\chi_0} + f_s A_s \frac{\chi_{s0}}{\chi_0}. \tag{22}\]

Equation (22) can be found in [3].

5 Discussion

We agree with the derivation in [1] of the decay rates up to a normalization factor. This normalization factor determines the total number of decays. For a given number of decays, the normalization factor may, or may not, depend on CP violation.

Alternatives A and B differ only in the normalizing factor for the probabilities: for alternative B we have set \( \epsilon = 0 \) in the denominator of (9).

Note that in both alternatives A and B we obtain the same (well known) dimuon charge asymmetry \( A_q = 4\alpha_q \), where \( q = d, s \). We also obtain the
same (well known) asymmetry of tagged “wrong-sign” decay rates induced by oscillations $a_{SL}(t) = 4\alpha_q$. However, the single muon charge asymmetries $a_q$ are different, and the dimuon charge asymmetries $A$, corresponding to an admixture of $B$ hadrons, are different.

About 99% of the branching fractions of $B^0$ and $\bar{B}^0$ are to flavor specific final states. Let us consider the approximation in which we neglect the branching fraction to non flavor specific final states (such as $J/\psi K_s$). In this approximation, a $\bar{B}^0_q$ can decay either as a $B^0_q$ or as a $\bar{B}^0_q$. Therefore the sum of the corresponding probabilities is 1. If the probability of the former is $\chi_q$, then the probability of the latter is $(1 - \chi_q)$. This approximation is alternative A.

Now consider alternative B. By construction, the normalizing factor is independent of CP violation. Therefore, the time integrated probabilities for $\bar{B}^0_q \to B^0_q$ and $B^0_q \to B^0_q$ are independent of CP violation. Consider a given number $N$ of events with a $B^0_q \bar{B}^0_q$ pair at production. The $N B^0_q$s can decay either to flavor specific final states as a $B^0_q$ or as a $\bar{B}^0_q$, or to non flavor specific final states $f_{nfs}$. Similarly, the $N \bar{B}^0_q$s can decay either to flavor specific final states as a $\bar{B}^0_q$ or as a $B^0_q$, or to non flavor specific final states $f_{nfs}$. To be specific, consider an increase of the number of decays $\bar{B}^0_q \to B^0_q$ due to CP violation. Then, for a given $N$, there must be a corresponding decrease of the number of decays $\bar{B}^0_q \to f_{nfs}$. Also, the number of $B^0_q \to \bar{B}^0_q$ decreases by approximately the same amount due to CP violation. Then, there must be a corresponding increase of the number of decays $B^0_q \to f_{nfs}$.

In summary, if alternative B is the correct description of nature, CP violation in “wrong sign” decays to flavor specific final states must be exactly compensated by CP violation in “mixing and decay” to non flavor specific final states. Such compensation implies a delicate balance between many branching fractions to, and CP violation asymmetries of, flavor specific and non flavor specific final states. To my knowledge, such relations have not been established for the standard model and new physics.

6 Conclusions

$B$ factories measure $\alpha_d$ (equal to $\frac{1}{4}A_d$, where $A_d$ is the dimuon charge asymmetry of direct decays of $B^0 \bar{B}^0$'s, independently of the alternative A or B). Hadron colliders measure the dimuon charge asymmetry $A$ of an admixture of $B$ hadrons. $A_s$ can be extracted from these measurements using equation \text{(17)} of alternative A, or equation \text{(22)} of alternative B. These equations differ by factors $(1 - \chi_{sd})/(1 - \chi_0) \approx (1 - 0.186)/(1 - 0.127) = 0.93$ for $A_d$, and $(1 - \chi_{sd})/(1 - \chi_0) \approx (1 - 0.5)/(1 - 0.127) = 0.57$ for $A_s$. 

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Our conclusion is that *we do not know* how the normalizing factor depends on CP violation. It depends on how well CP violation to non flavor specific final states compensates CP violation to flavor specific final states. We have considered two benchmark alternatives A and B. The difference is relatively small for $A_d$, but becomes important for the extraction of $A_s$ from the measurement of the dimuon charge asymmetry at hadron colliders.

Alternative A is a well defined approximation. Alternative B assumes a dubious cancelation of CP violation in flavor specific and non flavor specific decays. Until “theory catches up”, we advocate using the dimuon charge asymmetry (17) of alternative A, instead of (22) of alternative B, as it is a better defined approximation, and results in a more conservative error of $A_s$.

References

[1] “CP violation in meson decays”, “The Review of Particle Physics”, Particle Data Group, W.-M. Yao et al., Journal of Physics G 33, 1 (2006).

The PDG notation is related to ours by $g_\pm = \frac{1}{2}(s_+ \pm s_-)$, and $-q/p = (1 - \epsilon)/(1 + \epsilon)$. In this note we consider decays to flavor specific final states so $A_f = \bar{A}_f$, $\bar{A}_f = A_f = 0$.

[2] B. Hoeneisen and C. Marín, hep-ph/0009226 (2000).

[3] Yuval Grossman, Yosef Nir, and Guy Raz, hep-ph/0605028, (2006), equation (16) to the required approximation.