Robustness of baryon-strangeness correlation and related ratios of susceptibilities

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Using quenched lattice QCD simulations we investigate the continuum limit of baryon-strangeness correlation and other related conserved charge-flavour correlations for temperatures $T_c < T \leq 2T_c$. By working with lattices having large temporal extents ($N_\tau = 12, 10, 8, 4$) we find that these quantities are almost independent of the lattice spacing, i.e., robust. We also find that these quantities have very mild dependence on the sea quark mass and acquire values which are very close to their respective ideal gas limits. Our results also confirm robustness of the Wroblewski parameter.

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I. INTRODUCTION

The recent results from the Relativistic Heavy Ion Collider (RHIC) [1] indicate the formation of a thermalized medium endowed with large collective flow and very low viscosity [2]. These findings suggest that Quark Gluon Plasma (QGP) is a strongly interacting system for temperatures close to its transition temperature ($T_c$). Apart from the experimental indications, the most convincing evidence in favour of the existence of a strongly interacting QGP comes from the lattice QCD simulations [3, 4]. These non-perturbative studies show that the thermodynamic quantities, like pressure and energy density, deviate from their respective ideal gas (of free quarks and gluons) values by about 20% even at temperature $T = 3T_c$. On the other hand, other lattice studies indicate the smallness of the viscous forces in QGP [5]. All these results point to the fact that close to $T_c$ nature of QGP is far from a gas of free quarks and gluons.

In order to uncover the nature of QGP in the vicinity of $T_c$ and also to understand the underlying physics of these lattice results many different suggestions have been made over the last decade. Descriptions in terms of various quasi-particles [6, 7], resummed perturbation theories [8], effective models [9] etc. are few among many such attempts. Apart from all these, the newly proposed model of Shuryak and Zahed [10] has generated considerable amount of interest in the recent years. Motivated by the lattice results for the existence of charmonium in QGP [11], this model proposed a strongly interacting chromodynamic system of quasi-particles (with large thermal masses) of quarks, anti-quarks and gluons along with their numerous bound states. As different conserved charges, e.g., baryon number ($B$), electric charge ($Q$), third component of isospin ($I$) etc., are carried by different flavours ($u, d, s$) of quarks, in the conventional quasi-particle models, conserved charges come in strict proportion to number of $u, d, s$ quarks. Thus conserved charges are strongly correlated with the flavours and the flavours have no correlations among themselves. On the other hand, in the model of [10], presence of bound states demand correlations among different flavours. Hence correlations between conserved charges and flavours depend on the mass-spectrum of the bound states and the strong correlations among them are lost.

Based on the above arguments, in [12], it has been suggested that the quantity

\[ C_{BS} = -3 \frac{\langle BS \rangle - \langle B \rangle \langle S \rangle}{\langle S^2 \rangle - \langle S \rangle^2}, \]

(1)

can be used to probe the degrees of freedom of QGP. Here $B = (U + D - S)/3$ is the net baryon number and $U, D, S$ are the numbers of net (quarks minus anti-quarks) up-quarks, down-quarks and strange-quarks respectively. The notation $\langle \cdot \rangle$ denotes average taken over a suitable ensemble. It has been argued in [12] that for QGP where quarks are the effective degrees of freedom, i.e., where correlations among $U, D$ and $S$ are absent, $C_{BS}$ will have a value of 1 for all temperature $T > T_c$. On the other hand, for the model of [10] $C_{BS} = 0.62$ at $T = 1.5T_c$, while for a gas of hadron resonances $C_{BS} = 0.66$. Thus the knowledge of $C_{BS}$ helps to identify the degrees of freedom in QGP.

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By extending the idea of [12], recently in [13], many ratios like

\[ C_{(KL)/L} = \frac{\langle KL \rangle - \langle K \rangle \langle L \rangle}{(L^2) - \langle L \rangle^2} \equiv \frac{\chi_{KL}}{\chi_L}, \]  

(2)

have been calculated using lattice QCD simulations with two flavours of dynamical light quarks and three flavours (two light and one heavy) of valance quarks. Here \( \chi_L \) and \( \chi_{KL} \) denote the susceptibilities corresponding to conserved charge \( L \) and correlation among conserved charges \( K \) and \( L \) respectively. The physical meaning of the ratios like \( C_{(KL)/L} \) can be interpreted as follows— Create an excitation with quantum number \( L \) and then observe the value of a different quantum number \( K \) associated with this excitation. Thus these ratios identify the quantum numbers corresponding to different excitations and hence provide information about the degrees of freedom. The calculations of [13] found no evidence for the existence of bound states [10] even at temperatures very close to \( T_c \). These findings are consistent with the results of [14], where the hypothesis of [10] has been tested by investigating the ratios of higher order baryon number susceptibilities obtained from lattice simulations.

As these lattice studies [13] involved simulations with dynamical quarks, they were done using small lattices having temporal lattice size \( N_T = 4 \). By comparing with the results from quenched simulations it has been shown [13] that \( C_{(KL)/L} \) do not depend on \( N_T \) for temperature \( T = 2T_c \). It is clearly important to verify whether the same conclusion holds even close to \( T_c \). Furthermore, it is known that in the case of quenched QCD with standard staggered quarks the diagonal quark number susceptibilities (QNS) have strong dependence on the lattice spacing even for the free theory [15, 16]. On the other hand, the off-diagonal QNS are identically zero for an ideal gas and acquires non-zero value only in the presence of interactions. So the lattice spacing dependence of the off-diagonal QNS is likely to be more complicated, as opposed to that for the diagonal QNS where these corrections are dominated by the lattice artifacts of the naive staggered action. Thus if these two QNS become comparable the ratios mentioned in eq. (2) can have non-trivial dependence on the lattice spacing \( a \) and hence the continuum limit of these ratios can be different from that obtained using small lattices. Since the perturbative expressions for diagonal and off-diagonal QNS (for vanishingly small quark mass and chemical potential) are respectively [17]—

\[ \frac{\chi_{ff}}{T^2} \approx 1 + \mathcal{O}(g^2) \quad \text{and} \quad \frac{\chi_{ff'}}{T^2} \approx -\frac{5}{144\pi^6} g^6 \ln g^{-1}, \]

(3)

it is reasonable to expect that the off-diagonal QNS may not be negligible at the vicinity of \( T_c \) where the coupling \( g \) is large. As the contributions of the bound states in the QNS become more and more important as one approaches \( T_c \) [18], on the lattice it is necessary to investigate the continuum limit of the these ratios of in order to verify the existence of bound states in a strongly coupled QGP. At present a continuum extrapolation of this kind can only be performed using quenched approximation due to the limitations of present day computational resources. A quenched result for these ratios will also provide an idea about the dependence of these ratios on the sea quark mass.

The aim of this work is to carefully investigate the continuum limit of the ratios of the kind \( C_{(KL)/L} \) for temperatures \( T_c < T \leq 2T_c \) using quenched lattice QCD simulations. The plan of this paper is as follows — In Section II we will give the details of our simulations and present our results. In the Section III we will summarise and discuss our results.

II. SIMULATIONS AND RESULTS

The partition function of QCD for \( N_f \) flavours, each with chemical potential \( \mu_f \) and mass \( m_f \), at temperature \( T \) has the form

\[ Z(T, \{\mu_f\}, \{m_f\}) = \int \mathcal{D}\mathcal{U} \ e^{-S_G(\mathcal{U})} \prod_f \det M_f(T, \mu_f, m_f), \]

(4)

where \( S_G \) is the gauge part of the action and \( M \) is the Dirac operator. We have used standard Wilson action for \( S_G \) and staggered fermions to define \( M \). The temperature \( T \) and the spatial volume \( V \) are expressed in terms of lattice spacing \( a \) by the relations \( T = 1/(aN_T) \) and \( V = (aN_s)^3 \), \( N_s \) and \( N_T \) being the number of lattice sites in the spatial and the Euclidean time directions respectively. The flavour diagonal and the flavour off-diagonal quark number susceptibilities (QNS) are given by—

\[ \chi_{ff} = \left( \frac{T}{V} \right) \frac{\partial^2 \ln Z}{\partial \mu_f^2} = \left( \frac{T}{V} \right) \left\langle \text{Tr} \left( M_f^{-1} M_f' - M_f^{-1} M_f' M_f^{-1} M_f' \right) \right\rangle + \left\langle \left\{ \text{Tr} \left( M_f^{-1} M_f' \right) \right\}^2 \right\rangle, \quad \text{and} \]

(5)

\[ \chi_{ff'} = \left( \frac{T}{V} \right) \frac{\partial^2 \ln Z}{\partial \mu_f \partial \mu_{f'}} = \left( \frac{T}{V} \right) \left\langle \text{Tr} \left( M_f^{-1} M_f' \right) \text{Tr} \left( M_{f'}^{-1} M_{f'}' \right) \right\rangle, \]

(6)
respectively. Here the single and double primes denote first and second derivatives with respect to the corresponding \( \mu_f \) and the angular bracket denote averages over the gauge configurations.

In this paper we report results of these susceptibilities on lattices with \( N_T = 4, 8, 10, \) and 12, for the temperatures \( 1.1T_c \leq T \leq 2T_c \), chemical potential \( \mu_f = 0 \) and using quenched approximations. The details of our scale setting procedure are given in [3]. We have generated quenched gauge configurations by using the Cabibbo-Marinari pseudo-heatbath algorithm with Kennedy-Pendleton updating of three \( SU(2) \) subgroups on each sweep. Since for \( m_q/T_c \leq 0.1 \) QNS are almost independent of the bare valance quark mass \( \langle m_q \rangle \) [15], we have used \( m_q/T_c = 0.1 \) for the light \( u \) and \( d \)-flavours. Motivated by the fact that for the full theory \( m_s/T_c \sim 1 \) we have used \( m_q/T_c = 1 \) for the heavier \( s \)-flavour. The fermion matrix inversions were done by using conjugate gradient method with the stopping criterion \( |r_n|^2 < \epsilon |r_0|^2 \), \( r_n \) being the residual after the \( n \)-th step and \( \epsilon = 10^{-4} \) [19]. The traces have been estimated by the stochastic estimator—
\[
\text{Tr} A = \sum_{i=1}^{N_v} R_i^1 AR_i/2N_v,
\]
where \( R_i \) is a complex vector whose components have been drawn independently from a Gaussian ensemble with unit variance. The square of a trace has been calculated by dividing \( N_v \) vectors into \( L \) non-overlapping sets and then using the relation—
\[
(\text{Tr} A)^2 = 2 \sum_{j=1}^{L} (\text{Tr} A)_j (\text{Tr} A)_j/L(L-1).
\]
We have observed that as one approaches \( T_c \), from above these products, and hence \( \chi_{ff'} \), become more and more noisy for larger volumes and smaller quark masses. So in order to reduce the errors on \( \chi_{ff'} \) number of vectors \( N_v \) have been increased (for the larger lattices and the smaller quark masses) with decreasing temperature. Details of all our simulations are provided in Table I.

| \( T/T_c \) | \( \beta \) | Lattice size | \( N_{\text{stat}} \) | \( N_v \) |
|----------|---------|------------|---------|-------|
|          |         | \( m_q/T_c = 0.1 \) | \( m_q/T_c = 1 \) |
| 5.7000   | 4 × 10^3 | 44         | 250     | 100   |
|          | \times 16^3 | 50         | 250     | 100   |
| 6.1250   | 8 × 18^3 | 30         | 250     | 100   |
| 6.2750   | 10 × 22^3 | 38         | 250     | 100   |
| 6.4200   | 12 × 26^3 | 41         | 250     | 100   |
| 5.7880   | 4 × 10^3 | 52         | 100     | 100   |
| 6.2100   | 8 × 18^3 | 49         | 200     | 100   |
| 6.3600   | 10 × 22^3 | 46         | 200     | 100   |
| 6.5050   | 12 × 26^3 | 45         | 200     | 100   |
| 5.8941   | 4 × 10^3 | 51         | 100     | 100   |
| 6.3384   | 8 × 18^3 | 49         | 150     | 100   |
| 6.5250   | 10 × 22^3 | 49         | 150     | 100   |
| 6.6500   | 12 × 26^3 | 48         | 150     | 100   |
| 6.0625   | 4 × 10^3 | 51         | 100     | 100   |
| 6.5500   | 8 × 18^3 | 50         | 100     | 100   |
| 6.7500   | 10 × 22^3 | 46         | 100     | 100   |
| 6.9000   | 12 × 26^3 | 49         | 100     | 100   |

| \( T/T_c \) | \( \beta \) | Lattice size | \( N_{\text{stat}} \) | \( N_v \) |
|----------|---------|------------|---------|-------|
| 1.25     | 6.6100  | 8 × 18^3  | 49       | 200   |
| 1.25     | 6.6100  | 8 × 18^3  | 49       | 200   |
| 1.25     | 6.6100  | 8 × 18^3  | 49       | 200   |
| 1.25     | 6.6100  | 8 × 18^3  | 49       | 200   |

| \( T/T_c \) | \( \beta \) | Lattice size | \( N_{\text{stat}} \) | \( N_v \) |
|----------|---------|------------|---------|-------|
| 2.0      | 6.0655  | 8 × 18^3  | 50       | 100   |
| 2.0      | 6.0655  | 8 × 18^3  | 50       | 100   |
| 2.0      | 6.0655  | 8 × 18^3  | 50       | 100   |
| 2.0      | 6.0655  | 8 × 18^3  | 50       | 100   |

| \( T/T_c \) | \( \beta \) | Lattice size | \( N_{\text{stat}} \) | \( N_v \) |
|----------|---------|------------|---------|-------|
| 2.0      | 6.0655  | 8 × 18^3  | 50       | 100   |
| 2.0      | 6.0655  | 8 × 18^3  | 50       | 100   |
| 2.0      | 6.0655  | 8 × 18^3  | 50       | 100   |
| 2.0      | 6.0655  | 8 × 18^3  | 50       | 100   |

TABLE I: The couplings (\( \beta \)), lattice sizes (\( N_T \times N_s^3 \)), number of independent gauge configurations (\( N_{\text{stat}} \)) and number of vectors (\( N_v \)) that have been used for our simulations are given for each temperature. The gauge configurations were separated by 100 sweeps.

In the following sections we present our results. The notations we use are same as in [13]. Since we use equal masses for the two light \( u \) and \( d \) flavours, the flavour diagonal susceptibilities in this context are \( \chi_{uu} = \chi_{dd} = \chi_u \) and the flavour off-diagonal susceptibilities are \( \chi_{du} = \chi_{ud} \). For the heavy flavour \( s \) the flavour diagonal susceptibility is denoted as \( \chi_{ss} \equiv \chi_s \) and the flavour off-diagonal susceptibilities are \( \chi_{ds} = \chi_{us} \). Expressions for all the susceptibilities used here have been derived in the appendix of [13].

### A. Susceptibilities

In order to understand the cut-off dependence of \( C_{(KL)/L} \) let us start by examining the same for the diagonal and off-diagonal QNS. We have found that for the all the temperatures the diagonal QNS (\( \chi_u \) and \( \chi_s \)) depend linearly on \( a^2 \propto 1/N_T^2 \), i.e., the finite lattice spacing corrections to the diagonal QNS have the form \( \chi_{ff}(a, m_f, T) = \)
For the sake of completeness we also present our continuum extrapolated results for the two very important quantities, the baryon number susceptibility ($\chi_B$) and the electric charge susceptibility ($\chi_Q$). These quantities are related to the event-by-event fluctuations of baryon number and electric charge [21] which have already been measured at RHIC [22]. The definitions that we use for $\chi_B$ and $\chi_Q$ are [13]

$$\chi_B = \frac{1}{9} (2\chi_u + \chi_s + 2\chi_{ud} + 4\chi_{us}), \quad \text{and} \quad \chi_Q = \frac{1}{9} (5\chi_u + \chi_s - 4\chi_{ud} - 2\chi_{us}).$$

(7)

In Fig. 3 we show the continuum results for $\chi_B/T^2$ and $\chi_Q/T^2$. Continuum extrapolations have been performed by making linear fits in $a^2 \propto 1/N_t^2$. Continuum limit of these quantities were also obtained in [15] for $T \geq 1.5T_c$, though using different definitions for these quantities. Nevertheless, given the compatibility of our diagonal QNS with that
FIG. 2: $N_f$ dependence of the off-diagonal QNS $\chi_{ud}/T^2$ at $1.5T_c$ (top panel) and $\chi_{us}/T^2$ at $1.1T_c$ (bottom panel) have been shown.

FIG. 3: The continuum results for $\chi_B/T^2$ (squares) and $\chi_Q/T^2$ (circles) have been shown.

of [15] and the smallness of the off-diagonal QNS for $T \geq 1.5T_c$ our continuum results for $\chi_B$ and $\chi_Q$ are compatible with that of Ref. [15], for any chosen definitions for these quantities.

B. Ratios

Wroblewski parameter ($\lambda_s$) [23] is a quantity of extreme interest due to its relation to the enhancement of strangeness production in QGP [24]. The rate of production of quark pairs in a equilibrated plasma is related to the imaginary part of the complex QNS by fluctuation-dissipation theorem. If one assumes that the plasma is in chemical (and thermal) equilibrium and the typical energy scales for the production of $u$, $d$ and $s$ quarks are well separated from the inverse of the characteristic time scale of the QCD plasma, then using Kramers-Kroing relation one can relate $\lambda_s$ to the ratio of QNS [25]—

$$\lambda_s = \frac{2\langle s\bar{s} \rangle}{\langle u\bar{u} + d\bar{d} \rangle} = \frac{\chi_s}{\chi_u}. \quad (8)$$

(In the above equation $\langle f\bar{f} \rangle$ should be interpreted as quark number density and not as quark anti-quark condensates.) We have found that $\lambda_s$, which is a ratio of two diagonal QNS, remains constant (within $\sim 5\%$) with varying lattice
FIG. 4: In the top panel robustness of the Wroblewski parameter ($\lambda_s$) with changing lattice spacings has been shown for $1.1T_c$. The lines indicate the 5% error band of a constant fit to this data. In the bottom panel we show our continuum results for $\lambda_s$ (see text for details).

spacings for all temperatures in $1 < T/T_c \leq 2$. We have illustrated this in the top panel of Fig. 4 by plotting $\lambda_s$ with $1/N^2$ for the temperature $1.1T_c$. These results are somewhat surprising since the order $a^2$ corrections are not negligible for the individual diagonal QNS. But for the ratio of the diagonal QNS for two different bare valance quark masses these order $a^2$ corrections happen to be negligible and thus seems to be quark mass independent. This indicates that the finite lattice spacing corrections to the diagonal QNS is constrained to have the form $\chi_{ff}(a, m_f, T) = \chi_{ff}(0, m_f, T)[1 + b(T)a^2 + \cdots]$, as opposed to the more general form $\chi_{ff}(a, m_f, T) = \chi_{ff}(0, m_f, T) + b(m_f, T)a^2 + \cdots$.

Our continuum results for the Wroblewski parameter have been shown in the bottom panel of Fig. 4. In view of the constancy of $\lambda_s$ we have made the continuum extrapolations by making a constant fit to $a^2 \propto 1/N^2$. Our Continuum limit for $\lambda_s$ are consistent with the previously reported [15] continuum values for $T \geq 1.5T_c$. Our continuum results for $\lambda_s$ are very close to the results of [13] for the whole temperature range of $T_c < T \leq 2T_c$. Closeness of our quenched results with the results from the dynamical simulations of [13] suggest that the Wroblewski parameter has practically no dependence on the mass of the sea quarks. These observations along with the fact that $\lambda_s$ has very mild dependence on the valance quark mass [26] shows that the present day lattice QCD results for the Wroblewski parameter are very reliable. The robustness of the Wroblewski parameter is very encouraging specially since in the vicinity of $T_c$ the lattice results for this quantity almost coincides with the value ($\lambda_s \approx 0.43$) extracted by fitting the experimental data of RHIC with a hadron gas fireball model [27].

FIG. 5: The top panel shows the $N_f$ dependence of $\chi_{ud}/\chi_u$ at $1.5T_c$. The bottom panel shows the same for $\chi_{us}/\chi_s$ at $1.1T_c$. 
After examining the ratio of the diagonal QNS let us focus our attention on the ratios of off-diagonal to diagonal QNS. Given our results for the diagonal and off-diagonal QNS it is clear that these will have the form—\(\frac{\chi_{ff}(a, m_f, m_{f'}, T)}{\chi_{ff}(a, m_f, T)}\approx\frac{\chi_{ff}(0, m_f, m_{f'}, T)}{\chi_{ff}(0, m_f, T)}[1-b(T)a^2]\). Since \(b(T)\) is positive, i.e., \(\chi_{ff}\) decreases with decreasing lattice spacing, this ratio is expected to decrease (as \(\chi_{ff}\) is negative) and move away from zero. However, due smallness of these ratios itself, within our numerical accuracies, we have been unable to identify any such effect. This has been exemplified in Fig. 5 where \(\chi_{ud}/\chi_s\) at \(1.5 T_c\) (top panel) and \(\chi_{us}/\chi_s\) at \(1.1 T_c\) (bottom panel) have been shown.

Following the main theme of this paper we now present the lattice spacing dependence of ratios the like \(C_{(KL)/L}\). Two such ratios that can directly probe the degrees of freedom in a QGP are \([12, 13]\)

\[
C_{BS} \equiv -3\frac{\chi_{BS}}{S} = \frac{\chi_s + 2\chi_{us}}{\chi_s} = 1 + \frac{2\chi_{us}}{\chi_s}, \quad \text{and} \tag{9a}
\]

\[
C_{QS} \equiv 3\frac{\chi_{QS}}{S} = \frac{\chi_s - \chi_{us}}{\chi_s} = 1 - \frac{\chi_{us}}{\chi_s}. \quad \tag{9b}
\]
These quantities probe the linkages of the strangeness carrying excitations to baryon number ($C_{BS}$) and electric charge ($C_{QS}$) and hence give an idea about the average baryon number and the average electric charge of all the excitations carrying the $u$ flavours. These ratios are normalized such that for a pure quark gas, i.e., where unit strangeness is carried by excitations having $B = -1/3$ and $Q = 1/3$, $C_{BS} = C_{QS} = 1$. A value of $C_{BS}$ and $C_{QS}$ significantly different from 1 will indicate that the QGP phase may contain some other degrees of freedom apart from the quasi-quarks.

Similar ratios can also be formed for the light quark sector [13], e.g., for the $u$ flavour the ratios

\[
C_{BU} \equiv 3C_{(BU)/U} = \frac{3\chi_{BU}}{\chi_U} = \frac{\chi_u + \chi_{ud} + \chi_{us}}{\chi_u} = 1 + \frac{\chi_{ud}}{\chi_u} + \frac{\chi_{us}}{\chi_u}, \quad \text{and} \quad (10a)
\]

\[
C_{QU} \equiv 3C_{(QU)/U} = \frac{3\chi_{QU}}{\chi_U} = \frac{2\chi_u - \chi_{ud} - \chi_{us}}{\chi_u} = 2 - \frac{\chi_{ud}}{\chi_u} - \frac{\chi_{us}}{\chi_u}, \quad (10b)
\]

quantifies the average baryon number ($C_{BU}$) and the average electric charge ($C_{QU}$) of all the excitations carrying $u$ quarks. For a medium of pure quarks, i.e., where the $u$ flavours are carried by excitations with baryon number $1/3$ and electric charge $2/3$, $C_{BU} = 1$ and $C_{QU} = 2$. Similar ratios can also be formed for the $d$ quarks [13]. As can be seen from eqs. (9, 10) the lattice spacing dependence of $C_{BS}$ etc. are governed by the cut-off dependence of the ratios $\chi_{ff}'/\chi_{ff}$. Since we have already emphasised that, within our numerical accuracies, the ratios $\chi_{ff}'/\chi_{ff}$ are almost independent of lattice spacings it is expected that the same will also happen for the ratios $C_{(KL)/L}$. In accordance to this expectation we have found that for temperatures $1.1T_c \lesssim T \lesssim 2T_c$ these ratios are independent of lattice spacings within ~5% errors, see Fig. 6. Note that these ratios are not only independent of the lattice spacings but also acquire values which are very close to their respective ideal gas limits.

In Fig. 7 we present our continuum results for $C_{XS}$ (bottom panel) and $C_{XU}$ (top panel), where $X = B, Q$. Since these ratios remain almost constant with changing $1/N_t^2$ (see Fig. 6) we have made continuum extrapolations by making constant fits of our data to $1/N_t^2$. For the whole temperature range of interests ($T_c < T \leq 2T_c$) these ratios have values which are compatible with that for a gas of pure quarks. This is exactly what has been found in [13] using partially quenched simulations with smaller lattices. For the $d$ quarks also we have found similar results.

III. SUMMARY AND DISCUSSION

In this paper we have made a careful investigation of the continuum limit of different ratios of off-diagonal to diagonal susceptibilities in quenched QCD using lattices with large temporal extents ($N_t = 12, 10, 8$ and $4$), for a very interesting range of temperature ($T_c < T \leq 2T_c$) and for vanishing chemical potential. We have found that for this whole range of temperature the lattice results for the ratios like $C_{BS}, C_{QS}$ etc. are robust, i.e., they are almost independent (within ~5%) of the lattice spacing. We have also arrived at the same conclusion for the Wroblewski parameter which is of interest to the experiments in RHIC and Large Hadron Collider (LHC).
At this point, it is good to have some idea about how unquenching may change our results. It has been found [28] that in the temperature range $T \geq 1.25T_c$ there is only $5 - 10\%$ change in the QNS in going from quenched to $N_f = 2$ dynamical QCD. On the other hand, since the order of the phase transition depends strongly on the number of dynamical flavours the change in QNS is likely to be much larger in the vicinity of the transition temperature for the quenched theory which has a first order phase transition. Though this may be true for the individual QNS, their ratios may have very mild dependence on the sea quarks content of the theory. Given the good compatibility of our results of $C_{BS}$, $C_{QS}$ etc. with the results of [13] it is clear that indeed these ratios have very mild dependence on the sea quark content of the theory. It is also known that [15] for bare valance quark mass of $m_q/T_c \leq 0.1$ the dependence of the QNS on the valance quarks mass is very small. Hence our results show that the ratios like $\chi_{ud}/\chi_s$ are robust not only in the sense that they do not depend on the lattice spacings but also they have very mild dependence on the quark masses.

All the results presented in this paper are for spatial lattice sizes $N_s = 2N_r + 2$, i.e., for aspect ratios $N_s/N_r = 2.5 - 2.17$. In view of the fact that quenched QCD has a first order phase transition it is important to have some idea about the volume dependence of our results, specially in the vicinity of the transition temperature $T_c$. To check this dependence we have performed simulations using lattices having aspect ratios $N_s/N_r = 2.5 - 5$, for our smallest temporal lattice $N_r = 4$ and at temperature $1.1T_c$. In these simulations we have not found any significant volume dependence of any quantity which have been presented in this paper. As an illustration, in Fig. 8, we have shown the dependence of $\chi_{us}/\chi_s$ on the aspect ratio, for $N_r = 4$ at $1.1T_c$. The volume dependence is expected to be even smaller as one goes further away from first order phase transition point. Also the agreement of our results with that of [13], where an aspect ratio of 4 have been used, shows that the these ratios have almost no volume dependence for $N_s \geq N_r + 2$.

While the closeness of $C_{XU}$ and $C_{XS}$ ($X = B, Q$) to their respective ideal gas values do support the notion of quasi-particle like excitations in QGP, a significant deviation of these ratios from their ideal gas values neither rule out the quasi-particle picture nor confirms the existence of the bound states proposed in [10]. Large contributions from the chemical potential dependence of the quasi-particle masses may lead to significant deviation of these ratios, especially in the vicinity of $T_c$. It has already been pointed out [7, 18] that, near $T_c$, the chemical potential dependence of the quasi-particle masses becomes crucial for the baryonic susceptibilities.

Nevertheless, it may be interesting to compare our results with the predictions of the bound state model of [10]. Based on the model of [10] (and assuming that the mass formulae given in [10] hold right down to $T_c$) the predicted values of $C_{BS}$ are approximately 0.62 at $1.5T_c$ [12], 0.11 at $1.25T_c$ and almost zero at $1.1T_c$ [29]. Clearly, as can be seen form Fig. 7 (bottom panel), these values are very much different from our continuum results. However, it has been argued in [18] that apart from all the bound states mentioned in [10], baryon like bound states may also exist in QGP. These baryons make large contributions to the baryonic susceptibilities, especially close to $T_c$ [18]. Taking account the contributions from the strange baryons may increase the value of $C_{BS}$. In [18] it has also been argued that for two light flavours if one considers the contributions of the baryons only then close to $T_c$ the ratio of 2-nd order isospin susceptibility ($d_2^I$) to the 2-nd order baryonic susceptibility ($d_2$) is $d_2^I/d_2 = (\chi_u - \chi_{ud})/(\chi_u + \chi_{ud}) = 0.467$. Clearly this is inconsistent with our results since a value of $d_2^I/d_2 = 0.467$ gives a positive $\chi_{ud}/\chi_u = (0.363)$. Whereas, the lattice results for $\chi_{ud}/\chi_u$ are negative and much smaller in magnitudes. This suggest that the contribution of the

![FIG. 8: Dependence of the ratio $\chi_{us}/\chi_s$ on the aspect ratio has been shown for $N_r = 4$ at temperature $1.1T_c$.](image)
mesons (also possibly of the quarks, diquarks and \( \bar{q}q \)-states) are definitely important in the isospin susceptibility \( d_I^2 \).
If one takes into account of the contributions of the mesons (pions and rhos) and assumes that the Boltzmann weight of the mesons are equal to that of the baryon one gets a lower bound for \( d_I^2/d_2 \), namely \( d_I^2/d_2 \geq 0.644 \) [30]. But this lower bound gives \( \chi_{ud}/\chi_u = 0.217 \) and hence very far from our results. Moreover, very recently it has been argued [31] that one can carefully tune the densities of the baryon and meson like bound states in the model of Refs. [10, 18] to reproduce the lattice results for off-diagonal QNS. But even those carefully tuned values fail to reproduce [31] the lattice results for higher order susceptibilities. In view of all these, the lattice results of [13] favours a quasi-particle like picture of QGP, as opposed to the bound state model of [10, 18]. The results of this paper show that these lattice results are really robust in the sense that they have very mild dependence on the lattice spacing and sea quark content of the theory.

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