Parameter Adaptive Control for a Quadrotor with a Suspended Unknown Payload under External Disturbance

YING WU1, YUN XIE2, AND SEN LI3

1School of Automation, Guangdong University of Technology, Guangzhou 510006, China (e-mail: wuying277727@gmail.com)
2School of Automation, Guangdong University of Technology, Guangzhou 510006, China(e-mail: xieyun@gdut.edu.cn)
3School of Automation, Guangdong University of Technology, Guangzhou 510006, China(e-mail: senswm@foxmail.com)

Corresponding author: Yun Xie (e-mail: xieyun@gdut.edu.cn).

This work was supported by the Natural Science Foundation of Guangdong Province under Grants 2016A030313706, and in part by the College Students’ Innovative Entrepreneurial Training Plan Program of Guangdong Province, China under Grant 201811845089.

ABSTRACT This paper focuses on the disturbance attenuation control of the quadrotor suspended payload system with unknown payload mass. Since the quadrotor transportation system is an eight-degree-of-freedom system with only four actuated degrees, it is difficult to achieve rapid stabilization of the quadrotor under varying payload and external disturbances. In this paper, a nonlinear controller based on rotation matrix is designed to rapid stabilization of attitude. By analyzing the effect of load mass on the trajectory tracking control, a parameter adaptive controller is proposed for position control. A mass estimation algorithm is established to estimate the payload mass in the air, which improves the robustness of the system. This approach can maximize load-bearing capacity within the thrust range of the quadrotor. The Lyapunov-based analysis proves the exponential convergence of the attitude error and the stability of the whole system. The contrast simulations demonstrate that the controller is superior to the sliding-mode controller incorporating input shaping on mass estimation, trajectory tracking, and disturbance resistance.

INDEX TERMS Adaptive control, disturbance attenuation, quadrotor, suspended payload.

I. INTRODUCTION

In recent years, Unmanned Aerial Vehicles (UAVs) have been widely studied and utilized in both military [1] and civilian [2] applications due to their simple structure and great maneuverability. With the deepening research on UAV flight control algorithms, the research hotspot of UAVs has gradually changed from attitude and position control to UAV applications such as UAV formation flight [3], fault-tolerant control [4], and load transportation control [5]. This leads to the study of UAVs in spraying [6], surveillance [7], and fire rescue [8] field becomes more popular.

With the rapid development of the delivery industry, UAV logistics receive much attention [9]. UAV logistics is of great importance for courier delivery in remote and low-traffic areas. Zipline, a US company, has developed a high-safety, autonomous fixed-wing drone for aeromedical transportation. The company has saved many lives by transporting blood and medical supplies in Rwanda [10]. Research about UAV logistics includes drone lift [11], drone obstacle avoidance [12], collaborative drone robotic arm [13]-[14], and drone trajectory tracking [15]. Among them, robustness to environmental disturbances and an unknown payload is necessary for UAV transportation applications. The quadrotor UAV is the simplest structure of vertical take-off and landing UAV, with algorithm universality. Therefore, this paper studies anti disturbance control of the quadrotor suspended payload system.

The quadrotor transportation system is an underactuated system with the high coupling of quadrotor position and payload swing angle. The payload oscillation also harms the position control of the payload transportation system [16]. So many studies have been done for stability control of the quadrotor transportation system [17]-[26]. Reference [17] designs a proportional plus derivative controller which will spend a lot of time to reach the balance point and have large steady-state error. In [25], a nonlinear controller is designed for the point-to-point flight which ensures the position error and swing angle converge exponentially to zero. However, the system robustness against perturbations is not considered in these studies. Since the real environment is complex
and changeable, the existence of external disturbance is inevitable. There are some studies to estimate and compensate the external disturbances [27]- [30]. Reference [27] presents a computational Geometric method to estimate the effects of wind. An uncertainty and disturbance estimator (UDE) is proposed in [29] for a quadrotor with a suspended payload. Though the uncertainty and disturbance estimator is suitable to attenuate unknown perturbations, it requires more computational power. And complex environment makes it very difficult to obtain an accurate model of external disturbance. The adaptive PID-SMC method in [31] is shown that the quadrotor’s states can be stabilized in finite time when facing external disturbance. The adaptive position controller in [32] guarantees global asymptotic stability of UAV in the presence of disturbances. Hence, the controller with high robustness is more appropriate for disturbance rejection control of the quadrotor transportation system.

The payloads with unknown masses are also a challenge of the stable flight. Indeed, only a few literatures investigated stable control of unknown payload mass [33]- [35]. A linear Proportional-Integral-Derivative controller is designed to probe the effect of a helicopter under the disturbance of increased payload mass in [33]. Under this control method, a helicopter can reduce added load offsets within a relatively large range. And successful helicopter loading deployments are not susceptible to substantial changes in mass. For the variable load, [34] proposed a mass estimation algorithm to estimate the quadrotor mass in the air. Combining with a novel robust sliding mode, the mass estimation algorithm can improve the quadrotor’s performance in suppressing the variable payload disturbance. Reference [35] presents a fixed-gain nonlinear proportional-derivative controller to meet the expected performance for an effective payload mass and a retrospective cost adaptive controller to compensate for the uncertain payload mass. Unlike external disturbances, the payload mass change is relatively small or unchanged. Therefore, mass estimation is more suited to stable control of the quadrotor transportation system. However, due to the complicated nonlinear model of the quadrotor with a suspended payload, it is difficult to estimate the payload mass in the air. To the best knowledge of the authors, few previous works have achieved mass estimation of a quadrotor with a suspended payload.

In this paper, the quadrotor transportation system is modeled based on the Lagrange equation, and the influence of the payload mass on the trajectory tracking control performance is explored through the analysis of the model. The proposed controller is divided into three parts: attitude control, position control, and oscillation suppression. A nonlinear attitude controller based on a rotation matrix and feedback is designed. However, the position control and the oscillation suppression of the quadrotor are coupled together. Therefore, we designed a nonlinear control method and a mass estimation algorithm, which can adjust the control parameters based on the mass of the payload. By Lyapunov stability theory analysis, the system can be shown to be exponentially stable in attitude, with the convergence of both position error and payload swing angle. By the performance analysis method proposed in [24], the simulation results show that the proposed trajectory tracking controller is effective in improving robustness to external disturbance and unknown payload mass. Besides, payload mass can be accurately estimated and the maximum load mass can reach 1 kg. The main contributions of this work are as follows:

1) A mass estimation algorithm is designed for a quadrotor with a suspended payload, which is proven to be able to precisely estimate the payload mass of a quadrotor transportation system in the air.
2) A novel parameter adaptive control law is proposed, which is combined with the estimated payload mass and state feedback to improve the robustness of the traditional adaptive control law. The proposed trajectory tracking controller can enhance the trajectory tracking control performance and increase the maximum payload capacity under external disturbances.

This paper is organized as follows. The dynamics of the quadrotor with a slung payload is demonstrated in Section II. Section III details the design of attitude controller, position controller, and swing suppression. The simulation results are analyzed in Section IV. Finally, concluding remarks are provided in Section V.

II. MODEL

This section introduces the nonlinear model of the quadrotor with a suspended load derived by Lagrange’s equation. The quadrotor transportation system, the inertial coordinate frame \( \{x_e, y_e, z_e\} \), and the body-fixed coordinate frame \( \{x_b, y_b, z_b\} \) are shown in Fig. 1. And the body-fixed coordinate frame is fixed to the mass center of the quadrotor.
A. NOTATION

In this paper, $\mathbb{R}^{m \times n}$ represents the $m \times n$ dimensional real vector space. $\| \cdot \|$ is the Euclidean norm of $\ast \in \mathbb{R}^n$. The symbols $c_\ast$ and $s_\ast$ denote $\cos(\ast)$ and $\sin(\ast)$, respectively.

The symbol $y = \text{diag}(x) \in \mathbb{R}^{n \times n}$ is a diagonal matrix whose $N^{th}$ diagonal element is the $N^{th}$ element of vector $x \in \mathbb{R}^n$. The "wedge" operator produces a skew-symmetric matrix which satisfies $\mathbf{a} \mathbf{b} = \mathbf{a} \times \mathbf{b}$ for $\forall \mathbf{a}, \mathbf{b} \in \mathbb{R}^3$. And " vee" operator, $(\ast)^\vee$ is the inverse operator of the "wedge" operator. The symbols used in the paper are summarized in Table 1.

B. SYSTEM DYNAMICS

The motion equation of our cargo transportation system is derived from the following assumptions.

- Assumption 1: The payload is connected to the mass center of the quadrotor through a cable.
- Assumption 2: The position of quadrotor is always above the payload.
- Assumption 3: The cable connecting the quadrotor to the payload is inelastic, weightless, and always taut.

From Fig. 1, the position of the load is given by the following equations.

\[
p_l = p_q + p_b^h \quad (1)
\]

\[
p_b^h = \begin{bmatrix} L \sin \beta & -L \cos \beta \sin \alpha & -L \cos \beta \cos \alpha \end{bmatrix}^T \quad (2)
\]

The quadrotor suspended payload system is segmented in the quadrotor attitude subsystem, the quadrotor position subsystem, and slung payload subsystem. The quadrotor position subsystem and slung payload subsystem influence each other. Since the dynamic model of our cargo transportation system is more complex than the quadrotor dynamic model, we obtain the motion equation via Lagrange’s equation. The derivation of our system model considers only gravity and the translational kinetic energy and potential energy of the system, respectively. The generalized coordinates include the quadrotor’s position and the slung load’s swing angle. Defining three unit orthogonal vectors as $p_1 = [s_\beta -c_\beta s_\alpha -c_\beta c_\alpha]^T$, $p_2 = [0 \ c_\beta -s_\beta]^T$, and $p_3 = [-c_\beta -s_\beta s_\alpha -s_\alpha s_\beta]^T$, the motion equation of the system is given by:

\[
\ddot{q} = M(q) \ddot{\ell} + C(q, \dot{q}) \quad (4)
\]

\[
\tau = J \dot{\Omega} + \Omega \times J \Omega \quad (5)
\]

\[
\dot{\ell} = \dot{q} \quad (6)
\]

\[
R = R_{\Omega} \quad (7)
\]

where $q = [p_q^T \ y^T]^T \in \mathbb{R}^5$, $\ell = [F^T \ 0_{2 \times 1}]^T \in \mathbb{R}^5$, and $F = R [0 \ 0 \ f]^T \in \mathbb{R}^3$. The matrix in (4) is defined as:

\[
M(q) = \begin{bmatrix} M_{11} & 0_{3 \times 2} \\ 0_{2 \times 3} & M_{22} \end{bmatrix} \quad (8)
\]

\[
C(q, \dot{q}) = \begin{bmatrix} C_1(q, \dot{q}) \\ C_2(q, \dot{q}) \end{bmatrix} \quad (9)
\]

\[
M_{11} = \frac{1}{m_q} \begin{bmatrix} I_3 - \frac{m_p}{m_q + m_p} p_1 p_1^T \end{bmatrix} \quad (10)
\]

\[
M_{22} = \frac{1}{m_q L} \begin{bmatrix} 1_{c_\beta} p_2 & p_3 \end{bmatrix}^T \quad (11)
\]

\[
C_1(q, \dot{q}) = \frac{m_p L c_\beta^2 \dot{\alpha}^2}{m_q + m_p} p_1 - G \quad (12)
\]

\[
C_2(q, \ddot{q}) = \begin{bmatrix} \frac{2c_\beta d_\beta s_\alpha}{c_\beta} & -c_\beta s_\beta \dot{\alpha}^2 \end{bmatrix}^T \quad (13)
\]

Therefore, (4) can be transformed into the following equations after some mathematical operations.

\[
\ddot{p}_q = \frac{1}{m_q} (I_3 - \frac{m_p}{m_q + m_p} p_1 p_1^T) F + C_1 \quad (14)
\]

\[
\ddot{\ell} = \frac{1}{m_q L} \begin{bmatrix} 1_{c_\beta} p_2 & p_3 \end{bmatrix}^T F + C_2 \quad (15)
\]

From (14), the dynamic model of the quadrotor transportation system is affected by the payload mass and the payload swing angle. And the greater the scale of the payload mass to the quadrotor mass, the more the quadrotor position is greatly influenced by the load swing.

III. CONTROL DESIGN

In this section, a parameter adaptive controller is presented to enhance the performance of the tracking control in the face of large unknown payload mass and environmental disturbance.

A. ATTITUDE CONTROL

Denote the rotation matrix representing the desired attitude of the quadrotor as $R_d$. Then the attitude error matrix of the quadrotor is defined as:

\[
R_e = R^T R_d \quad (16)
\]
where the error rotation matrix can be expressed as:

\[ R_e = \exp(\dot{\Omega}_e \theta) = I + \dot{\Omega}_e \sin \theta + \dot{\Omega}_e^2 (1 - \cos \theta). \] (17)

The above equation and the "vee" operator translate the error rotation matrix into an error vector representing the attitude error of the quadrotor.

\[ e_R = \log(R_e) \] (18)

Therefore, the angular velocity error of the body is defined as:

\[ e_\Omega = \dot{e}_R = \Omega_d - \Omega \] (19)

where \( \Omega_d \) is the desired angular velocity. From (19), the attitude error dynamics is

\[ J \dot{e}_\Omega = J\dot{\Omega}_d - J\dot{\Omega}. \] (20)

Theorem 1: Considering the desired attitude, desired angular velocity, and desired angular acceleration are known and bounded, the control torque is designed as

\[ \tau = K_R e_R + K_\Omega e_\Omega + J \dot{\Omega}_d + \Omega \times J \dot{\Omega}. \] (21)

where \( K_R = \text{diag}( [K_{Rx} \ K_{Ry} \ K_{Rz}] ) \in \mathbb{R}^{3 \times 3} \) and \( K_\Omega = \text{diag}( [K_{\Omega x} \ K_{\Omega y} \ K_{\Omega z}] ) \in \mathbb{R}^{3 \times 3} \). Then the attitude error and angular velocity are globally exponentially stable at \( R_e = I_3 \) and \( e_\Omega = [0 \ 0 \ 0] \).

Proof: Consider the Lyapunov function as

\[ V_1 = e_\Omega^T K_R e_R + e_\Omega^T J e_\Omega. \] (22)

Then the derivative of \( V_1 \) is

\[ \dot{V}_1 = e_\Omega^T \dot{K}_R e_R + e_\Omega^T J \dot{e}_\Omega. \] (23)

Substituting (5) and (21) for (20), the attitude error dynamic becomes

\[ J \dot{e}_\Omega = -K_R e_R - K_\Omega e_\Omega. \] (24)

Therefore, the derivative of \( V \) is

\[ \dot{V}_1 = -e_\Omega^T K_\Omega e_\Omega \leq 0. \] (25)

Since the Lyapunov function is bounded, while the rotation matrix group is compact, then \( e_R \) and \( e_\Omega \) are bounded. The derivative of \( V_1 \) is

\[ \dot{V}_1 = -e_\Omega^T K_\Omega e_\Omega \] (26)

Since all signals on the right-hand side of (26) are bounded, \( \dot{V}_1 \) is bounded. While the \( V_1 \) is uniformly continuous, \( \dot{V}_1 \) converges to zero applying Barbalat’s lemma. Therefore \( e_\Omega \) is globally asymptotically stable to the equilibrium point zero. The (24) can be rewritten as

\[ K_R e_R = -K_\Omega e_\Omega - J \dot{e}_\Omega. \] (28)

Therefore,

\[ \| K_R e_R \| = \| -K_\Omega e_\Omega - J \dot{e}_\Omega \| \leq \| K_\Omega e_\Omega \| + \| J \dot{e}_\Omega \|. \] (29)

Since \( \| K_\Omega e_\Omega \| \to 0 \) and \( \| J \dot{e}_\Omega \| \to 0 \), it follows that \( \| K_R e_R \| \to 0 \). Then the attitude error system is globally asymptotically stable.

To prove globally exponential stability, we define \( q_1 = [e_\Omega^T R^T \dot{e}_\Omega]^T \). Equation (22) can be rewritten as

\[ V_1 = q_1^T P_1 q_1 \] (30)

where

\[ P_1 = \begin{bmatrix} K_R & 0 \\ 0 & J \end{bmatrix}. \] (31)

It follows that \( \lambda_{min} \| q_1 \|^2 \leq V_1 \leq \lambda_{max} \| q_1 \|^2 \) where \( \lambda_{max} \) and \( \lambda_{min} \) are the maximum eigenvalue and the minimum eigenvalue of \( P_1 \). \( \lambda_0 \) is the minimum eigenvalue of \( K_\Omega \). Then \( V_1 \) satisfies the following inequality.

\[ \dot{V}_1 = -\lambda_0 \| q_1 \|^2 \leq -\frac{\lambda_0}{\lambda_{max}} V_1 \] (32)

Base on the upper of \( V_1 \) is derived as

\[ V_1 \leq V_1(0) \exp(-\frac{\lambda_0}{\lambda_{max}} t). \] (33)

It can be concluded that

\[ \| q_1 \| \leq \sqrt{\frac{\lambda_{max}}{\lambda_{min}}} \| q_1(0) \| \exp(-\frac{\lambda_0}{2\lambda_{max}} t). \] (34)

Hence, the attitude error system is globally exponential stability.

**B. POSITION CONTROL**

Denote the desired position of the quadrotor as \( p_d \). The position error and velocity error of the quadrotor are defined as

\[ e_p = p_q - p_d \] (35)

\[ \dot{e}_p = \dot{p}_q - \dot{p}_d. \] (36)

Therefore, the position error dynamics and the payload’s swing angle dynamics are derived as

\[ \dot{e}_p = \omega_q + g_q F \] (37)

\[ \dot{\gamma} = \omega_L + g_L F. \] (38)

where \( g_q, \omega_q, g_L, \) and \( \omega_L \) in (37) and (38) are

\[ g_q = \frac{1}{m_q} (I_3 - \frac{m_p}{m_q + m_p} p_1 p_1^T) \] (39)

\[ \omega_q = \frac{m_p L \dot{\alpha}^2}{m_q + m_p} - p_1 - G - \ddot{p}_d \] (40)

\[ g_L = \frac{1}{m_q L} \begin{bmatrix} p_2^T \\ p_3 \end{bmatrix} \] (41)

\[ \omega_L = C_2. \] (42)
First, consider the desired thrust as

$$F_d = f R_d [0, 0, 1]^T. \quad (43)$$

We denote the control input $f$ as

$$F = f R_d [0, 0, 1]^T + \kappa f. \quad (44)$$

Then we take $b_3$ to be the third column vector of $R$ and $b_{d3}$ to be the third column vector of $R_d$. While $\theta_3$ is the variation of the angle between the vector $b_3$ and the vector $b_{d3}$, it follows that

$$\kappa = (b_3 - b_{d3}) \quad (45)$$

$$\|b_3 - b_{d3}\| < |\theta_3| \leq \|e_R\|. \quad (46)$$

Based (45) and (46), we can conclude that

$$\|\kappa\| \leq \|e_R\|. \quad (47)$$

Equation (37) and (38) are formatted as

$$\dot{e}_p = g_q F_d + \omega_q + g_q \kappa f \quad (48)$$

$$\dot{\gamma} = g_L F_d + \omega_L + g_L \kappa f. \quad (49)$$

Suppose that the estimated mass of payload is identical to actual mass, the desired control thrust is designed as

$$F_d = g_q^{-1}(-\omega_q + u_q + L[p_2 c_b p_3] u_L) \quad (50)$$

where $u_q$ and $u_L$ are designed for the position control and payload swing suppression. $g_q^{-1}$ is calculated as

$$g_q^{-1} = m_q(I_3 + \frac{m_p}{m_q} p_1 p_1^T). \quad (51)$$

And based on (50), the desired attitude rotation matrix is calculated as

$$R_d = [b_{1d} b_{2d} b_{3d}] \quad (52)$$

where $b_{3d} = \frac{F_d}{\|F_d\|}, b_{2d} = b_{d3} \times b_{1d},$ and $b_{1d} = b_{d3} \times b_{3d}.$

Since $p_1, p_2,$ and $p_3$ are orthogonal to each other, substituting (50) into (48) and (49) yields

$$\dot{e}_p = u_q + L[p_2 c_b p_3] u_L + g_q \kappa f \quad (53)$$

$$\dot{\gamma} = -g_L g_q^{-1} \omega_q + g_L g_q^{-1} u_q + u_L + \omega_L + g_L \kappa f. \quad (54)$$

Denoting $\Gamma(\gamma, \dot{\gamma}) = \omega_L - g_L g_q^{-1} \omega_q + \frac{g}{L} \gamma,$ the position error dynamics and the payload’s swing angle dynamics are formatted as

$$\dot{e}_p = u_q + L[e_2, e_3] u_L + h_q + \kappa_q \quad (55)$$

$$\dot{\gamma} = u_L - \frac{g}{L} \gamma + \frac{1}{L} \begin{bmatrix} e_2^T \\ e_3^T \\ e_1^T \end{bmatrix} u_q + h_L + \kappa_L. \quad (56)$$

with unit vectors $e_1 = [0 \ 0 \ -1]^T,$ $e_2 = [0 \ 1 \ 0]^T,$ and $e_3 = [-1 \ 0 \ 0].$ And the functions defined in (55) and (56) are

$$h_q = L \dot{h_q} u_L \quad (57)$$

$$\kappa_q = g_q \kappa f \quad (58)$$

$$h_L = \frac{1}{L} \dot{h_L} u_q + \Gamma \quad (59)$$

$$\kappa_L = g_L \kappa f \quad (60)$$

$$h_q = [c_b p_2 - e_2, p_1 - e_3] \quad (61)$$

$$\dot{h}_L = \left[ \frac{1}{c_b} p_2^T - e_2^T \right] \quad (62)$$

Based on assumption 2, there exist constants $\rho_0 > 0$ and $\rho_L > 0$ satisfying the inequalities $\|h_q\| < \rho_0 \|\gamma\|$ and $\|h_L\| < \rho_L \|\gamma\|$. Since $\|p_1\| = 1,$ $\|p_2\| = 1,$ and $\|p_3\| = 1,$ it can be concluded that

$$\|g_q\| \leq \frac{1}{m_q} (\|I_3\| + \|\frac{m_p}{m_q} p_1 p_1^T\|) \leq \bar{g}_q \quad (63)$$

$$\|g_L\| \leq \frac{1}{m_q L} \left( \|\frac{1}{c_b} p_2^T\| \right) \leq \bar{g}_L. \quad (64)$$

where $\bar{g}_q$ and $\bar{g}_L$ are two positive constants. Considering $q_{\gamma} = [\gamma^T, \dot{\gamma}^T]^T,$ $\|\Gamma\|$ is upper bounded as

$$\|\Gamma\| \leq \bar{p}_\gamma (\dot{\bar{p}}_q) + \rho_{\gamma} (\bar{p}_q) \|q_{\gamma}\|^2 \quad (65)$$

with two constants $\rho_\gamma (\bar{p}_q) > 0$ and $\bar{p}_\gamma (\bar{p}_q) > 0.$ The above analysis can be concluded that

$$\begin{cases} 
\|h_q\| \leq L \rho_q \|\gamma\| \|u_L\| \\
\|\kappa_q\| \leq \bar{g}_q \|e_R\| f \\
\|h_L\| \leq \frac{1}{L} \rho_L \|\gamma\| \|u_q\| + \bar{p}_\gamma (\bar{p}_d) + \rho_{\gamma} (\bar{p}_d) \|q_{\gamma}\|^2 \\
\|\kappa_L\| \leq \bar{g}_L \|e_R\| f. 
\end{cases} \quad (66)$$

By analyzing the error dynamics of (14) and (15), the adaptive control law $u_q$ and $u_L$ are designed as

$$u_q = -K_{pq} (m_p) e_p - K_{dq} (m_p) \dot{e}_p \quad (67)$$

$$u_L = -K_{pl} (m_p) e_p - K_{dl} (m_p) \dot{e}_L \quad (68)$$

where $K_{pq} (m_p) = diag([K_{pqx}, K_{pqy}, K_{pqz}]) \in \mathbb{R}^{3\times3}$, $K_{dq} (m_p) = diag([K_{dqx}, K_{dgy}, K_{dqz}]) \in \mathbb{R}^{3\times3}$, $K_{pl} (m_p) = diag([K_{plx}, K_{ply}, K_{plz}]) \in \mathbb{R}^{2\times2}$, and $K_{dl} (m_p) = diag([K_{dlx}, K_{dlz}]) \in \mathbb{R}^{2\times2}.$ And the mass estimation algorithm is defined as:

$$\dot{m}_{eq} = -k_m u_q (3). \quad (69)$$

The closed-loop dynamics of (55) and (56) is formatted as

$$q_\gamma = \Lambda_2 q_\gamma + \kappa_\gamma + h_2. \quad (70)$$

In (70), the vector $q_\gamma = [e_2^T, e_3^T, q_{\gamma}^T]^T$ represents the state of the closed-loop system, and the matrix $\Lambda_2$ is

$$\Lambda_2 = \begin{bmatrix}
0_{3\times3} & I_3 & 0_{3\times2} & 0_{3\times2} \\
-K_{pq} & -K_{dqz} & L_{23} K_{plx} & L_{23} K_{dlx} \\
0_{2\times3} & 0_{2\times2} & I_2 & 0_{2\times2} \\
0_{2\times3} & 0_{2\times2} & 0_{2\times2} & A_2 \\
0_{2\times3} & 0_{2\times2} & 0_{2\times2} & -K_{dl} \\
\end{bmatrix}. \quad (71)$$
Based on the conditions \(\|q_2\| \leq (1-\mu_h)/(2\rho_h \|P_2\|), \mu_h \geq 2\rho_h \|P_2\|,\) and \(\mu_c \geq 2\rho_c f \|P_2\|,\) the following inequality holds.

\[
\dot{V}_2(t) \leq -\mu_h \|q_2\|^2 + \bar{\mu}_h \|q_2\| + \mu_c \|q_1\| \|q_2\| \tag{85}
\]

Define the Lyapunov function of the whole system as

\[
V = \varepsilon V_1 + V_2. \tag{86}
\]

Since \(\zeta_{min}\) and \(\zeta_{max}\) are the minimum and maximum eigenvalue of \(\text{diag}\{\varepsilon P_1, P_2\},\) \(V(t)\) satisfies the following inequality.

\[
\zeta_{min} \|q\|^2 \leq V \leq \zeta_{max} \|q\|^2 \tag{87}
\]

Differentiating (86), and substituting (32) and (85), it follows that

\[
\dot{V}(t) \leq -\varepsilon \lambda_1 \|q_1\|^2 - \mu_h \|q_2\|^2 + \bar{\mu}_h \|q_2\| + \mu_c \|q_1\| \|q_2\| \tag{88}
\]

For the condition that \(\bar{\mu}_d = 0, (\bar{\rho}_d, \bar{\rho}_d) = 0\), (88) can be transformed as follow:

\[
\dot{V}(t) \leq -\mu \|q\|^2. \tag{89}
\]

Based on (82) and (87), (89) is concluded as

\[
\dot{V}(t) \leq -\frac{2\mu}{\zeta_{max}} - V(t). \tag{90}
\]

The upper bound of \(V(t)\) is obtained as

\[
\dot{V}(t) \leq V(0) \exp\left(-\frac{2\mu}{\zeta_{max}}\right). \tag{91}
\]

Then, the upper bound of \(q(t)\) is derived as

\[
\|q\| \leq \sqrt{\frac{\zeta_{max}}{\zeta_{min}}} \|q(0)\| \exp\left(-\frac{\mu}{\zeta_{max}}\right). \tag{92}
\]

Thus, for trajectory whose acceleration is zero, the system is locally exponentially stable. For the acceleration of desired trajectory is nonzero, there exists \(n > 0\) satisfy

\[
\dot{V}(t) \leq -\mu \|q\|^2 + \bar{\mu}_h \|q_2\| \leq -n \|q\|^2 \tag{93}
\]

with \(\|q\| \geq \frac{\mu}{\bar{\mu}_h}.\) Thus, \(\|q\|\) is converge to a constant with exponential convergence rate.

IV. NUMERICAL SIMULATIONS AND RESULTS

This section presents the simulations to verify the control performance of the proposed control method. The contrastive analysis of the proposed controller and a sliding-mode controller (SMC) is conducted.

To verify the effectiveness of the proposed control method, we assume a quadrotor for simulations equipped with sensors that can get the measurement of the quadrotor’s state as well as a motion capture system that estimates the payload’s swing angle. The dynamics equations of the quadrotor suspended payload system is given by (4)-(7). And according
to a quadrotor developed in [25], the model parameters are chosen as follow:

\[ m_q = 1.008 \text{ kg}, \quad L = 1 \text{ m} \]

\[ J = \text{diag}([11.9, 11.9, 20.8] \times 10^{-3}) \text{ kgm}^2. \]

Based on the experiment in [36], the external disturbance is chosen as follow:

\[ F_c = -(\mu_1 \| \dot{\bar{p}}_q \| + \mu_2 \| \ddot{\bar{p}}_q \|) \frac{\ddot{\bar{p}}_q}{\| \bar{p}_q \|} \quad (94) \]

where \( \mu_1 = 0.172 \) and \( \mu_2 = 0.0025 \). The proposed parameter adaptive controller is labeled as var-param controller. The SMC combined with input shaping theory and var-param controller are implemented for simulations. For the sliding-mode controller combined with input shaping theory, the position control law and the transfer function of the input shaper are as follows:

\[ U = G_s(p_{sd} - (m_q + m_p) \ddot{\bar{p}}_s - \overline{K}_s \text{sign}(\overline{K}q_s + \dot{q}_s)) \]

\[ G = A_1e^{t_1s} + A_2e^{t_2s} + A_3e^{t_3s} \]

where \( q_s = [e^T_1 e^T_2 0_{3 \times 1}]^T, p_{sd} = [p^T_2 0_{3 \times 1}]^T, \) and \( \overline{G} = [G^T \ 0_{5 \times 1}]^T. \ \overline{K}, \ K_s, \) and \( G_s \) are the gain matrix of sliding-mode controller. \( A_i \) and \( t_i \) are the parameters of the input shaper. For our controller, the control parameters are designed in Table 2.

The simulations require the quadrotor to track the given trajectory with a payload mass of 1 kg and 0.5 kg. We consider two different trajectories including a) move from its initial position to a target position and b) a helix trajectory. The desire trajectory and the initial condition of the first trajectory are as follow:

\[ p_d = [1 \ 1 \ 1]^T m, \quad b_{1d} = [1 \ 0 \ 0]^T \]

\[ p_0 = [0 \ 0 \ 0]^T m, \quad R_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \]

The acceleration of the second given trajectory is nonzero. The desired trajectory and the initial condition of the system are set as:

\[ p_d = [\cos(0.3t) \ \sin(0.3t) \ 0.3t]^T m, \quad b_{1d} = [1 \ 0 \ 0]^T \]

\[ p_0 = [1 \ 0 \ 0]^T m, \quad R_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \]

The simulation results of the first trajectory are shown in Fig. 2 and Fig. 3. From Fig. 2 and Fig. 3, whatever the load mass is 0.5 kg or 1 kg, the position and the swing angle converge converges in 5 s by both the parameter adaptive controller and the SMC. And the steady-state position error for both two controllers are smaller than 0.002 m. For the steady-state error, the estimated mass of the var-param controller is smaller than that of the SMC. Although the swing angle of the var-param controller is larger than that of the SMC, the attitude error and position error of the parameter adaptive controller has smaller oscillation regardless of the load mass of 0.5kg or 1kg. In addition, SMC shows more dramatic oscillations of the z-axis position with the payload mass of 1 kg in Fig. 3. Hence, for point-to-point flight, both two controllers deliver good performance under environmental perturbation, but the parameter adaptive controller induces more robust effects than SMC for unknown mass.

### Table 2. Parameters of the Proposed Controller

|         | K_{pq} | K_{dq} | K_{pl} | K_{dl} |
|---------|--------|--------|--------|--------|
| var-param | diag([1, 1, 6]) | diag([0.6(5 - m_p), 0.6(5 - m_p), 7]) | diag([16m_p/5, 16m_p/5]) | diag([2m_p, 2m_p]) |
| k_m     | 1/3    |        |        |        |
The simulation results of the second trajectory are shown in Fig. 4 and Fig. 5. From Fig. 4 and Fig. 5, at a payload mass of both 0.5 kg and 1 kg, the position error, attitude error, and the swing angle for the var-param controller converge within 5 s. For the steady state, the error of the var-param controller on position, attitude, and estimated mass is all less than that of SMC. Furthermore, for the var-param controller, the response curves of position error, attitude error, and swing angle are smoother. In Fig. 5, SMC shows more dramatic oscillations on the z-axis position when payload mass is 1 kg. As a result, for trajectory tracking flight, the var-param controller achieves better performance than SMC when subjected to unknown mass and air disturbance.

In addition, refer to the method proposed in [24], we conduct a comparative analysis for position error and swing angle of the var-param control system and the sliding-mode control system. The integral comparative measure of two controllers performance is define as follow:

$$\eta = \frac{\int |e_{var}| dt}{\int |e_{smc}| dt}$$  (95)

where $e_{var}$ and $e_{smc}$ represent the errors of the var-param controller and the SMC. After that calculations, the performance indexes are concluded in Table 3.

In Table 3, $\eta_x$, $\eta_y$, and $\eta_z$, represent the performance of the controllers in x-axis, y-axis, and z-axis of position. $\eta_{\alpha}$ and $\eta_{\beta}$ represent the performance of the controllers in swing angle. $\eta$ performance criteria is defined as follows. If the $\eta$ is less than 1, it means that the error of the var-param controller is smaller. Also, if this $\eta$ is greater than 1, it indicates that the performance of the var-param controller is poorer than the SMC.

From Table 3, with the trajectory whose accelerate is nonzero, the parameter adaptive controller has a very good performance.
control under unknown mass and varying mass in the air. The simulations corresponding to different payload mass and two different trajectories are carried on. Simulation results demonstrate that the parameter adaptive controller is greatly superior to the SMC controller under unknown mass and air disturbance, with a maximum payload mass of up to 1kg.

The contribution of this work is improving the tracking accuracy for the quadrotor with a suspended payload under large unknown load and air disturbance conditions. With this control method, the robustness and load-bearing capacity of a quadrotor transportation system are substantially enhanced. This is an important difference from most methods that require known payload mass to achieve stable control. The future work will focus on validation and optimization of the parameter adaptive controller via outdoor experiments.

V. CONCLUSIONS

In this paper, a novel control method with anti-disturbance capability is presented to quickly track trajectory for a quadrotor transporting a suspended payload. First, based on linear feedback and estimated payload mass, the parameter adaptive control system is designed for position control and swing suppression. In addition, we add a mass estimation algorithm to the parameter adaptive controller for stable effect in position control with good suppression of swing. For the flight to a target position, although the swing angle of the var-param controller is larger than that of the SMC, the var-param controller indicates better performance on position control, specifically the z-axis position. Hence, it can be concluded that the parameter adaptive controller improves the performance of trajectory tracking control compared to the SMC when facing unknown mass and air disturbance.

TABLE 3. Performance Index of Two Controllers

| $P_d = 0$ | $P_d \neq 0$ |
|---|---|
| $\eta_x$ | 0.65 | 0.67 |
| $\eta_y$ | 0.65 | 0.67 |
| $\eta_z$ | 0.39 | 0.10 |
| $\eta_p$ | 1.38 | 0.52 |
| $\eta_{\dot{p}}$ | 1.39 | 0.74 |

REFERENCES

[1] V. Roberge, M. Tarbouchi, and G. Labonte, “Fast genetic algorithm path planner for fixed-wing military UAV Using GPU,” IEEE Trans. Aerosp. Electron. Syst., vol. 54, no. 5, pp. 2105–2117, 2018.

[2] Y. Mualla et al., “Agent-based simulation of unmanned aerial vehicles in civilian applications: A systematic literature review and research directions,” Futur. Gener. Comput. Syst., vol. 100, pp. 344–364, 2019.

[3] H. Qiu and H. Duan, “A multi-objective pigeon-inspired optimization approach to UAV distributed flocking among obstacles,” Inf. Sci. (Ny.), vol. 509, pp. 515–529, 2020.

[4] S. Mallavalli and A. Fekih, “A fault tolerant tracking control for a quadrotor UAV subject to simultaneous actuator faults and exogenous disturbances,” Int. J. Control, vol. 93, no. 3, pp. 655–668, 2020.

[5] M. E. Guerrero-Sánchez, D. A. Mercado-Ravell, R. Lozano, and C. D. García-Beltrán, “Swing-attenuation for a quadrotor transporting a cable-suspended payload,” ISA Trans., vol. 68, pp. 433–449, 2017.

[6] D. K. Giles and R. C. Billing, “Deployment and performance of a uav for crop spraying,” Chem. Eng. Trans., vol. 44, pp. 307–320, 2015.

[7] H. Huang and A. V. Savkin, “An algorithm of reactive collision free 3-d deployment of networked unmanned aerial vehicles for surveillance and monitoring,” IEEE Trans. Ind. Informatics, vol. 16, no. 1, pp. 132–140, 2020.

[8] H. X. Pham, H. M. Lu, D. Feil-Seifer, and M. C. Deans, “A distributed control framework of multiple unmanned aerial vehicles for dynamic wildfire tracking,” IEEE Trans. Syst. Man, Cybern. Syst., vol. 50, no. 4, pp. 1537–1548, 2020.

[9] B. D. Song, K. Park, and J. Kim, “Persistent UAV delivery logistics: MILP formulation and efficient heuristic,” Comput. Ind. Eng., vol. 120, no. May, pp. 418–428, 2018.

[10] E. Ackerman and E. Strickland, “Medical delivery drones take flight in east Africa,” IEEE Spectr., vol. 55, no. 1, pp. 34–35, 2018.

[11] P. J. Cruz and R. Fierro, “Cable-suspended load lifting by a quadrotor UAV: hybrid model, trajectory generation, and control,” Auton. Robots, vol. 41, no. 8, pp. 1629–1643, 2017.

[12] L. S. Tang and V. Kumar, “Mixed Integer Quadratic Program trajectory generation for a quadrotor with a cable-suspended payload,” 2015 IEEE International Conference on Robotics and Automation (ICRA), 2015, pp. 2216-2222.
