Improved numerical model for fatigue cumulative damage of mechanical structure considering load sequence and interaction

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Abstract
To more accurately predict the fatigue life of components under the action of random loads, it is necessary to explore the influence of the interaction between the load sequence and the load on the life prediction. Based on the Manson-Halford method and Corten-Dolan model, this paper establishes a fatigue cumulative damage model that takes into account both the load order and the interaction between loads, and also takes into account the loads near the fatigue limit. The fatigue life of mechanical parts under random load can be calculated through this model, which provides a theoretical basis for life prediction under random load spectrum. The fatigue life of mechanical parts under random load can be calculated through this model, which provides a theoretical basis for life prediction under random load spectrum. Comparing the calculation results of the proposed model with the results of Palmgren Miner, Manson-Halford method, and Corten-Dolan model, it is found that the fatigue damage model established can reasonably predict the fatigue life of parts. Comparison and verification of examples further prove the accuracy and reliability of the proposed model.

Keywords
Cumulative fatigue damage, load sequence, interaction between loads, fatigue life

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Introduction
The fatigue strength is one of the important evaluation indexes in the reliability analysis of parts. Components in mechanical systems are mainly subjected to cyclic load. Under cyclic load, fatigue cracks on components will continue to expand, and fatigue failure will occur when the cracks expand to a certain degree. When scholars study the failure behavior of components under cyclic load, they generally determine the fatigue damage state of the component by studying the relationship between the number of cyclic loads times and the cumulative damage amount.

Traditionally, the Palmgren-Miner rule of linear cumulative damage theory has been used to assess the fatigue state of components. The Palmgren-Miner rule has the characteristics of simple calculation and easy to use, and has been widely used for fatigue life prediction of parts. However, the Palmgren-Miner rule ignores the load sequence and the interaction between loads, so it cannot guarantee the prediction accuracy of fatigue life, it will not work in some engineering problems. To ensure that the predicted fatigue life is

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accurate, some scholars modified the linear cumulative fatigue damage method according to the Palmgren-Miner rule. In practice, crack propagation is related to the number of load cycles times. The relationship between the crack propagation process and the Palmgren-Miner rule was studied by Scholars. Mansor et al.26 divided the fatigue damage development process into crack formation stage and crack propagation stage, and analyzed the fatigue state of parts at different stages by the line cumulative damage method. Although this method considers the influence of load sequence by dividing the fatigue damage process into two stages, it is difficult to determine the position of the boundary point between the two stages, and it is difficult to directly apply it to engineering.

With the development of fatigue cumulative damage theory, nonlinear fatigue cumulative damage theory is also widely used in fatigue life estimation. According to the existing research, non-linear damage theory can be divided into three categories: non-linear cumulative damage theory considering the interaction between loads; Nonlinear cumulative damage model based on damage curve method (Marco-Starkey model, etc.); Non-linear cumulative damage based on energy. Manson-Halford established the complete Manson-Halford model by using the relationship between the crack length and the load cycle ratio and the parameter calculation method. This model can better describe the effect of load loading order, but the interaction between loads is not considered, and the accuracy of prediction cannot be guaranteed. Maierhofer et al.24 established an overall fatigue crack propagation model including short crack behavior and load sequence effects. Richter and Sander investigated the influence of underloads on crack arrest. The experimental tests are performed with the tool steel AISI/SAE 1045. Mansor et al.26 presents a predictive model for fatigue crack growth under loading sequences. The applicability of the crack closure model was studied to predict the fatigue life span resulting from the continuous effect of loading blocks. Calderon and Biezma develop a practical and simple correction factor to ensure the sum of linear cumulative damage is conservative. The effects of loading sequence on the calculation of non-Gaussian random vibration fatigue damage are studied by Wang through the virtual experiments. Through the calculation results of fatigue damage in the notch sensitive area of test part, it was confirmed that the damage calculated by linear and nonlinear cumulative damage formula was improved with the increase of kurtosis. Benkabouchea et al.29 propose a modified nonlinear cumulative fatigue damage model, extended to multiaxial loading, taking into account the effects of the amplitude and sequence of variable amplitude loading.

The cumulative fatigue damage of a part is related to both the load sequence and the interaction between loads. Many scholars believe that the interaction between loads is reflected by the form of stress ratio, such as Freudenthal-Heller model, Corten-Dolan model, and so on. Gao et al.31 redefined the parameter $d$ in the Corten-Dolan model considering the relationship between the cumulative damage and stress. This model can be practically applied to fatigue life estimation through case verification. Zhu found that the parameter $d$ in the Corten-Dolan model is related to material properties and load spectrum characteristics, and determined the parameter $d$ by analyzing the fatigue damage evolution, and defined the parameter $d$ as a function related to stress. In order to analyze the fatigue life, it is necessary to study the prediction method of fatigue life. Many researchers only consider the effect of loading sequence on fatigue life. The effect of interaction between loads on fatigue life is not discussed. The interaction between loads is considered in the Corten-Dolan model, but the parameters in the model need to be obtained by a large number of experiments. In this paper, the fatigue cumulative damage method is modified based on Manson-Halford method and Corten-Dolan model. Compared with the constant stress ratio in Corten-Dolan model, a cumulative fatigue damage theoretical model that takes into account both the load sequence and the interaction between loads is constructed. Loads near the fatigue limit are also taken into account. This theory can make up for the deficiency of nonlinear cumulative damage theory, improve the accuracy of nonlinear fatigue life estimation, and provide a reasonable method for fatigue life estimation of mechanical parts.

Statement of problem

Palmgren-Miner rule is the theory of linear cumulative damage. Based on the principle of mutual independence of fatigue damage, the single cycle damage is linearly accumulated as the total damage, and the cumulative damage is used to evaluate the state of the part. It is
also assumed that when the damage amount $D$ reaches a critical value of 1, the part will undergo fatigue failure. The linear cumulative damage under multi-stage load could be expressed as

$$D = \sum_{i=1}^{n} \frac{n_i}{N_i} = 1$$

$n_i$ is the number of cycles times at the $i$-th stress level, and $N_i$ is the number of cycles when critical damage is reached at the $i$-th stress level.

The Palmgren-Miner rule uses a linear accumulation method in the calculation process. The cumulative damage caused by different load levels is shown in Figure 1. In Figure 1(a), the damage amounts caused by the stress level $S_1$ after $n_1$ cycles is $D_1$, and the damage caused by the stress level $S_2$ after $n_2$ cycles is $D_2$. The cumulative damage under this two-stage ($S_1 \rightarrow S_2$) load is $D_1 + D_2$. As shown in Figure 1(b), if the stress level $S_2$ is loaded first, the cumulative damage after $n_2$ cycles is $D_2'$, and then the stress level $S_1$ is loaded, the cumulative damage after $n_1$ cycles is $D_1'$. The cumulative damage under this loading order is $D_2' + D_1'$. From the comparison of the two figures, it can be seen that the damage amounts under the different loading orders are equal ($D_1 + D_2 = D_2' + D_1'$).

The Palmgren-Miner rule also ignores the effects of small loads when analyzing component fatigue life. However, in practical situations, loads near the fatigue limit of a part can strengthen or cause damage.

Palmgren-Miner rule fails to consider the influence of interaction between each level load and load sequence, and the calculated results are different from the actual damage. In the actual load process, when the load is from low to high, the fatigue strength of the component is enhanced due to the strengthening effect of the low load, which makes the estimated fatigue life more conservative. When the load is from high to low, the crack occurrence time is advanced due to the high load, which makes the estimated fatigue life tend to be dangerous. The Manson-Halford model considers the influence of load loading sequence on fatigue life, but ignores the interaction between loads. The Corten-Dolan model considers the interaction between loads, but the parameters in the model need to be obtained through a large number of experiments, and it is difficult to determine. In order to solve these problems, this paper modified the Manson-Halford model based on the Corten-Dolan model, and derived the fatigue life estimation model based on load sequence and interaction between loads. A load greater than 0.75 times the fatigue limit was used as the calculation standard.

### Nonlinear fatigue cumulative damage model

#### Manson-Halford model

Manson-Halford\(^{14}\) expressed the damage of a part by the maximum cumulative amount of crack length that can be withstood. According to this theory, the expression of crack propagation was obtained, as shown in equation (2).

$$a = a_0 + (a_f - a_0) \left( \frac{n}{N_f} \right)^{a_f}$$

where, $a_f$ is the crack length of a smooth specimen when fracture occurs, $a_0$ is the initial crack length, and $a$ is the cumulative amount of crack length under cyclic loading. Under a given stress level $S_i$, the cumulative damage amount can be expressed as

$$D = \frac{a}{a_f}$$

where\(^{39}\)
The equations (2) and (4) were brought into (3) to obtain cumulative damage is shown in equation (5)

\[
D = \frac{a}{a_f} + \frac{1}{a_f} \left[ a_0 + (a_f - a_0) \left( \frac{n}{N_f} \right)^{\frac{2}{3} N_f^{0.4}} \right]
\]

According to formula (17), the relationship between the cycle ratio \( n_1/N_1 \) and the equivalent cycle ratio \( n_1/N_f \) can be derived

\[
\frac{n_{12}}{N_f} = \left( \frac{n_1}{N_f} \right)^{(N_1/N_{12})^{0.4}}
\]

The cumulative damage \( D_2 \) after two load cycles \( n_1, n_2 \) is

\[
D_2 = \frac{n_1}{N_f} + \frac{n_2}{N_f} = \left( \frac{n_1}{N_f} \right)^{(N_1/N_f)^{0.4}} + \frac{n_2}{N_f}
\]

The cumulative damage under the third stress level is

\[
D_3 = \left( \frac{n_1}{N_f} \right)^{(N_1/N_f)^{0.4}} + \frac{n_2}{N_f} + \frac{n_3}{N_f}
\]

The cumulative damage under multiple stress levels is

\[
D_n = \left( \frac{n_1}{N_f} \right)^{(N_1/N_f)^{0.4}} + \frac{n_2}{N_f} + \cdots + \frac{n_i}{N_f} + \frac{n_n}{N_f}
\]

**Cartron-Dolan model**

There are many ways to consider load interactions, such as Corten-Dolan model, Bue-Quco model, and Freudenthal-Heller model. Among them, Corten-Dolan model apply to engineering practice well. In 1956, Corten-Dolan\(^{17}\) proposed an exponential damage model. In this model, the crack propagation is divided into three stages: localized work hardening; microcavities or microcracks formed in the localized region; cracks further expanded to fracture. The cumulative fatigue damage amount under a given stress \( S \) can be expressed as

\[
D = m r n^a
\]

\( a \) is the exponential constant related to the stress level, \( m \) is the number of crack cores, \( n \) is the number of stress cycles, and \( r \) is the damage growth factor related to the stress level.

When stresses \( S_1 \) and \( S_2 \) act on the test piece in an alternating manner, if \( S_1 \) is cycled \( \alpha n \) times, then \( S_2 \) is cycled \( (1 - \alpha) n \) times, and the fatigue life under the action of stresses \( S_1 \) and \( S_2 \) is

\[
N_g = \frac{N_1}{\alpha + (1 - \alpha)(S_1/S_1)^d}
\]

The above two-level load can be extended to multi-level load fatigue life estimation, and the multi-level load fatigue life expression is shown in equation (12).

\[
N_g = \frac{N_{\max}}{\sum_{i=1}^r \alpha_i(S_i/S_{\max})^d}
\]

Where, \( N_g \) is the total fatigue life, \( r \) is the stress level, \( \alpha_i \) is the ratio of the number of cycles under the each stress level to the total number of cycles, \( N_{\max} \) is the fatigue life under stress \( S_{\max} \), and \( S_{\max} \) is the maximum stress amplitude in the multi-stage alternating load, \( d \) is the material constant. \( d \) is defined as a function of the number of cycles and stress ratio by Gao et al.,\(^{31}\) the expression as follow.

\[
d = \exp \left[ \left( \frac{n_i}{N_f} \right)^{\frac{\sigma_{\max}}{\sigma_{\max}}} \right] + \gamma
\]

\( \gamma \) is a constant related to the material properties, \( n_i \) is the number of cycles corresponding to the actual cycles, and \( N_f \) is the number of cycles corresponding to the \( i \)-th stress \( \sigma_i \).

**Modification of nonlinear fatigue cumulative damage model**

From the formula (4), it can be seen that the crack length of the smooth specimen when it breaks is \( a_f \). According to Walker’s crack growth model,\(^{40}\) it can be known that the crack growth rate is related to the stress ratio. The conversion expression between damage coefficient and stress ratio in Corten-Dolan model is shown in (14).

\[
\left( \frac{r_i}{r_{\max}} \right)^{\frac{1}{d}} = \left( \frac{\sigma_i}{\sigma_{\max}} \right)^{d}
\]

Here, \( a' \) and \( d \) are constants. It can be known that the damage coefficients are related to the ratio of stress and maximum stress.

In equation (2), \( a \) is the length of crack growth when the part reaches failure at a given stress level, and \( (n/N_f)^{N_f^{0.4}} \) can be regarded as the proportion of damage caused by cycling at a given stress level to the crack
According to this analysis, based on Walker’s crack propagation model and Corten-Dolan model, assuming a parameter related to the interaction between load, this parameter is called the crack propagation effect parameter \( \kappa \), then formula (2) can be rewritten as

\[
a = a_0 + (a_f - a_0) \left( \frac{n}{N_f} \right)^\alpha
\]

where

\[
\alpha = \frac{2}{3} (N_f)^{0.45E}
\]

and

\[
D_n = \left( \left( \frac{n_1}{N_f1} \right)^{(N_f1/N_f2)^{0.45E12}} + \frac{n_2}{N_f2} \right)^{(N_f2/N_f3)^{0.45E23}} + \cdots + \frac{n_{i-1}}{N_{f(i-1)}} \right)^{\left( \frac{n_i}{N_f} \right)^{(N_fi/N_fi-1)^{0.45Ei-1i}}} + \frac{n_i}{N_f}
\]

Equation (23) is the fatigue cumulative damage model that takes into account the interaction between loads and the load loading sequence, and can be applied to the estimation of fatigue life well.

**Case verification**

**Case I**

The feasibility of the model with the data in the fatigue test CFD2 in Tong et al.\(^\text{43}\) is verified. The relevant original data in the test are: the material fatigue limit is \( N_0 = 2.2 \times 10^6 \) times, the material fatigue limit is 173.5MPa, and the measured fatigue life is \( N_f = 2.2 \times 10^7 \) times. CFD2 loading test data is shown in Table1.

The traditional Palmgren-Miner rule does not take into account the load sequence and the interaction between loads. The damage amount is only 0.6174, which is biased against the assumption that damage...
occurs when the damage amount reaches a critical value of 1. The damage amount of the Manson-Halford model and the modified model are relatively similar. According to the research, the large load can accelerate the crack growth, and lead to the failure threshold being reached early. Therefore, the damage amount obtained by the modified cumulative damage calculation model is more reasonable than that calculated by the Manson-Halford model. The revised cumulative damage calculation method could more reasonably calculate the cumulative damage amount of a component under multi-level loads, thereby more accurately estimating the fatigue life of the parts, see Table 2 below:

### Table 2. Damage of multi-level load.

| Model                  | Cumulative damage amount |
|------------------------|--------------------------|
| Miner model            | 0.6174                   |
| Manson-Halford model   | 0.9116                   |
| Modified model         | 0.8949                   |

Case 2

The bogie is an important association mechanism between the train body and the wheels, and it will be affected by the alternating load during the running of the train. The test data of the Type 209 bogie in Liu\(^4\) is used to verify the modified fatigue cumulative damage method. The 209 bogies could run about \(3 \times 10^5\) km per year, vibration frequency is 270 times per kilometer. The 209 bogies can be safely operated for at least 20 years. The actual number of cycles of the bogie is about \(N_r = 1.62 \times 10^9\). Loads and load cycles about bogie are shown in Table 3.

In Table 4, the fatigue life calculated by the Palmgren-Miner rule is approximately three times than the actual fatigue life. The error between the fatigue life calculated by the Manson-Halford model and the actual fatigue life is 28.40%; the error between the fatigue life calculated by the modified cumulative fatigue damage method and the actual fatigue life is 6.4%. The error between actual fatigue life and fatigue life calculated by the modified Corten-Dolan model is -4.41%. Although the absolute value of the error of the

### Table 1. CFD2 test data.

| Load level | \(\sigma_i/\text{MPa}\) | Frequency \(n_i\) | \(N_i\) (times) | \(n_i/N_i\) |
|------------|--------------------------|------------------|----------------|-------------|
| 1          | 305                      | 44               | \(5.60 \times 10^4\) | 0.0008      |
| 2          | 332                      | 352              | \(7.40 \times 10^4\) | 0.0047      |
| 3          | 298                      | 6160             | \(1.30 \times 10^5\) | 0.0475      |
| 4          | 254                      | 59,840           | \(2.80 \times 10^5\) | 0.2140      |
| 5          | 201                      | 440,000          | \(1.25 \times 10^6\) | 0.3520      |
| 6          | 149                      | 2,024,000        | \(\infty\) | 0          |
| 7          | 96                       | 6,160,000        | \(\infty\) | 0          |
| 8          | 44                       | 13,310,000       | \(\infty\) | 0          |

### Table 3. Load spectrum of welded structure frame of 209 bogie.

| Stress level | Stress (MPa) | Action times \(n_i\) (times) | Fatigue life under single-stage stress \(N_f\) (10^6 times) | \(n_i/N_f\) (10^-6) |
|--------------|--------------|------------------------------|------------------------------------------------------------|-------------------|
| 1            | 241.5        | 1                            | 0.60                                                       | 1.6667            |
| 2            | 228.8        | 1                            | 0.72                                                       | 1.3889            |
| 3            | 214.5        | 3                            | 0.92                                                       | 3.1609            |
| 4            | 203.6        | 2                            | 1.11                                                       | 1.8018            |
| 5            | 203.3        | 7                            | 1.11                                                       | 6.3063            |
| 6            | 181.5        | 14                           | 1.69                                                       | 8.2840            |
| 7            | 172.1        | 40                           | 2.04                                                       | 19.6078           |
| 8            | 154.4        | 97                           | 3.04                                                       | 31.9079           |
| 9            | 136.8        | 283                          | 4.72                                                       | 59.9576           |
| 10           | 122.8        | 800                          | \(\infty\)                                                 | 0                 |
| 11           | 101.6        | 2202                         | \(\infty\)                                                 | 0                 |
| 12           | 89.2         | 6582                         | \(\infty\)                                                 | 0                 |
| 13           | 67.6         | 21901                        | \(\infty\)                                                 | 0                 |
| 14           | 63.3         | 55662                        | \(\infty\)                                                 | 0                 |
| 15           | 34.4         | 912405                       | \(\infty\)                                                 | 0                 |
modified Corten-Dolan model is relatively small, there are more parameters in this model that need to be fitted by experiments. It is more difficult to apply in engineering. According to the error value in Table 4, it can be seen that the error value of the modified fatigue cumulative damage method is relatively small, which indicates that the modified model can reasonably and accurately estimate the fatigue life of the bogie. It does not need to fit the parameters in the calculation process, which can be more conveniently used in actual engineering.

### Conclusion

1. Based on the Manson-Halford method and the Corten-Dolan model, the fatigue cumulative damage method is revised, and the influence of the interaction between loads and the load sequence on the cumulative fatigue damage are considered. It is also considered that a small load between 0.7 and 1 times the fatigue limit will strengthen the components. Two cases are used to verify the revised fatigue life cumulative damage method. The proposed cumulative damage method could more reasonably estimate the fatigue life of components.

2. The Palmgren-Miner rule simplifies the calculation model, and the fatigue life estimation result will deviate greatly from the actual life. In the bilinear damage theory, the boundary between the two stages is difficult to determine. The Manson-Halford model considers the effect of load sequence, but ignores the effect of the interaction between loads; the Corten-Dolan model considers the interaction between loads, but the parameters in this model need a large number of experiments to obtain, and it is difficult to apply in practice.

3. The cumulative fatigue damage model established in this paper is based on the interaction model between loads in the Manson-Halford model and the Corten-Dolan model, and considers small-amplitude loads. This model takes full account of the various influencing factors in load loading, and does not need to fit parameters through experiments. The results are reasonable and easy to apply, and can provide support for the fatigue life estimation of parts under multi-level loads.

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### Supplemental material

Supplemental material for this article is available online.

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**Table 4.** Comparison of different life prediction methods.

| Model                  | Cumulative damage amount | Estimated life | Error (%) |
|------------------------|--------------------------|----------------|-----------|
| Palmgren-Miner rule    | $4.23 \times 10^{-5}$    | $7.45 \times 10^2$ | 360.04    |
| Manson-Halford model   | $4.80 \times 10^{-4}$    | $2.08 \times 10^2$ | 28.40     |
| Modified model         | $6.06 \times 10^{-4}$    | $1.72 \times 10^2$ | 6.4       |
| Corten-Dolan model     | $6.45 \times 10^{-4}$    | $1.55 \times 10^2$ | -4.41     |

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