Use of Integrated Accounting Methods for Calculation of the Profile Volume of Embankments

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Abstract. The article presents a fundamentally new approach to the calculation of the volume of profile cubic capacity of embankments, excavations, as well as reserves and drainage ditches. New analytical dependences for calculation of volumes of profile cubic capacity of embankments, excavations, and also reserves and drainage ditches, on the basis of use of methods of integral calculation are deduced. Determined the relative error of calculation of the mentioned volumes of the relevant cubic capacity compared to the traditional methods of calculating these quantities. The results of the research can be used in the creation of information systems that quickly implement the proposed analytical dependences of more effective calculation indicators, as well as in the planning of large volumes of excavation. A rigorous proof of the proposed mathematical dependencies is carried out. Conducted an evaluation of the effectiveness of the proposed analytical dependencies in the calculation of the amount of the relevant cubic capacity of the embankments.

1. Introduction
Currently, analytical dependences, nomograms, graphs, special counting lines, tables and implemented computer programs are used to calculate the volume of profile cubic capacity of embankments, pits, cavaliers, drainage ditches and other structures [1-11].

These methods based on geometric relationships do not allow obtaining accurate results from the point of view of mathematics due to the complex geometric shapes of the subgrade areas.

2. Relevancy
In practical terms, these circumstances lead to an increase in the time and cost of excavation, including the execution of calculations of the volume of work.

Therefore, the development of methods, analogues of which are not found in the publications, allowing to reduce these costs due to greater accuracy of calculations in determining the volume of excavation and simply implemented in the environment of existing application packages, is quite relevant.

Note that due to the limited scope of the article, these methods are given only for embankments.
3. Problem statement
Determine the volume of profile cubic capacity of straight and curved sections of embankments.

The volume of the profile cubature of the straight section of the embankment will be determined by the formula

\[ V = \int_{0}^{L} S(x) \, dx \]

where \( S(x) \) – variable cross-sectional area of the straight section of the embankment of length \( L \).

The volume of the profile cubature of the curved section of the embankment will be determined by the formula

\[ V = \int_{\alpha}^{\beta} S(\phi) \, d\phi \]

where \( S(x) \) – the variable cross-sectional area of the curved section of the embankment, \( \alpha \) and \( \beta \) – the initial and final angles that specify the coordinates of the beginning and end of the embankment section in the polar coordinate system, \( R \) – is the distance from the origin of the coordinate system to the mound axis.

As a limitation, we assume that the calculations are carried out for the sections of the roadbed on the terrain having a longitudinal slope of the embankment base.

4. Theoretical part

4.1. Calculation of the volume of the straight section of the embankment
Let us calculate the volume of the plot of the roadbed (embankment) which is a complex geometric figure – a prismatic with a cross section of variable area (Fig. 1), which is divided into two parts: a prismatic with a cross section of a constant area \( V_2 \); a trapezoidal wedge \( V_1 \) (Fig. 1).

![Figure 1. Main indicators of the embankment area (without drain prism).](image)

To calculate the volume \( V_2 \) it is necessary to calculate the value \( m_2 \) which according to the definition \( \tan \alpha \) is calculated by the formula

\[ m_2 = \frac{H_1}{\tan \alpha} \]
where $\tan \alpha = \sqrt{m}$. 

Next, the values of the values $b_1$ and $V_2$ are defined:

$$b_1 = b + 2 \cdot m_2, \quad V_2 = \frac{b + b_1}{2} \cdot H_1 \cdot L.$$ 

To calculate the volume $V_1$, it should be calculated a certain integral of the form [12]

$$\int_0^L S(x) dx,$$ 

where $S(x)$ - is a variable area of the orthogonal section of the trapezoidal wedge. For this the variable is the height, which is defined based of the similarity of the triangles shown in Fig. 2.

$$h(x) = \frac{(H_2 - H_1) \cdot x}{L}.$$ 

Next, using the obvious dependencies, the values are calculated $m_1$, $b_2$, $S(x)$ and $V_1$ (see Fig. 1):

$$m_1 = \frac{(H_2 - H_1)}{\tan \alpha}, \quad b_2 = b + 2 \cdot (m_1 + m_2),$$

$$S(x) = \frac{b_1 + b_2}{2} \cdot h(x), \quad V_1 = \int_0^L S(x) dx.$$ 

Thus, the volume of the section of the embankment is defined by the formula

$$V_n = V_1 + V_2.$$ 

**Figure 2.** Longitudinal section of a section of the embankment.

In the case of need for construction of drain prisms, the formula (2) is adjusted for the amount of drain volume of the prism

$$V_{cn} = \frac{b + b_1}{2} \cdot h_3 \cdot L,$$

where $b$, $b_3$ and $h_3$ accordingly are the width of the main site, the width of the base and the height of the drain prism (in m), and has the form

$$V_n = V_1 + V_2 + V_{cn}.$$
4.2. Proof of the validity of the formula (1)

The proof of the validity of the proposed formula (1), which is used to calculate the volume of the earth massif, will be used for the case of prismatic (see Fig. 1).

Consider the part of the prismatic (V) contained between the planes \( x = a \) and \( x = b \) (Fig. 3), and cut it by planes perpendicular to the axis \( x \). The cross-sectional area corresponding to the abscissa \( x \), indicated by \( S(x) \), will be a continuous function from \( x \) (for \( a \leq x \leq b \)).

If projected (without distortion) the two of such sections on a plane perpendicular to the \( x \)-axis, they would contain one another. In this assumption it can be argued that the body has a volume that is expressed by the formula

\[
V = \int_{a}^{b} S(x) \, dx .
\]

(3)

Figure 3. Illustration to the proof.

To prove this, we divide the segment \([a,b]\) on the \( x \)-axis on parts by points \( a = x_0 < x_1 < \ldots < x_i < x_{i+1} < \ldots < x_n = b \), and dissect the whole body into layers with the planes \( x = x_i \), drawn through the points of division. Consider the \( i \)-layer contained between the planes \( x = x_i \) and \( x = x_{i+1} (i = 0, 1, \ldots, n-1) \).

In between of \([x_i,x_{i+1}]\), the function \( S(x) \) has the largest area value of \( M_i \) and the smallest value of \( m_i \). If the sections corresponding to different \( x \) in this interval are placed on one plane, for example \( x = x_i \), then all of them under the assumption will be in the largest, within the area of \( M_i \), and contain the smallest, within the area of \( m_i \). If to build straight cylinders of \( \Delta x_i = x_{i+1} - x_i \) height on these largest and smallest sections, the larger of them will include the considered layer of the body, and the smaller itself will be in this layer. The volumes of such cylinders will be respectively \( M_i \Delta x_i \) and \( m_i \Delta x \).

From the incoming cylinders a body \( T \), and from the outgoing the body \( U \) are composed, their volumes are, respectively, \( \sum M_i \Delta x_i \) and \( \sum m_i \Delta x_i \); when \( \lambda = \max \Delta x_i \) tends to zero, they have a common limit, determined by the formula (3). In [12, p. 225] it is proved: "If for the body \((V)\) the two sequences can be constructed, respectively to entering and leaving bodies \( \{T_n\} \) and \( \{U_n\} \),

\[ \lambda \to 0 \]

1 The proof for other types of earth massifs is carried out similarly.
that have volumes, and these volumes tend to a common limit, \( \lim T_n = \lim U_n = V \) then the body \((V)\) has a volume equal to the said limit." By virtue of the above, the following formula will be just
\[
V = \int_{0}^{b} S(x)\,dx = \int_{a}^{b} S(x)\,dx.
\]

4.3. Calculation of the volume of the curved section of the embankment

Calculate the volume of the embankment area taking into account its broadening in the curve (Fig. 4).

In this case, it is most simple to calculate the volume of the embankment area in the polar coordinate system using the corresponding definite integral [13-15]. Here, by the method of differentials [13], it can be written using the notation Fig. 5. \( dV = S(\varphi)\,dl \), where \( S(\varphi) \) - is the variable area of the radial section of the prismatic to be determined, and \( dl = Rd\varphi \) is the differential of the arc axis of the mound area. Thus, we finally have \( dV = S(\varphi)Rd\varphi \) and, integrating the last expression, we obtain a formula for calculating the embankment volume in the \( V \) polar coordinate system:

\[
V = R \int_{\alpha}^{\beta} S(\varphi)\,d\varphi,
\]

(4)
where $\alpha$ and $\beta$ - are the start and end angles that specify the coordinates of the beginning and end of the embankment. Since the size of this area varies linearly, it is determined by the following dependence

$$S(\phi) = S_\alpha + \frac{(S_\beta - S_\alpha)}{\beta - \alpha} \cdot (\phi - \alpha),$$

where $S_\alpha$ ($S_\beta$) - is the area of the radial section at the beginning (end) of the embankment section.

Thus, the volume of the site of the embankment is defined according to the formula

$$V = \begin{cases} \int_\alpha^\beta R \int S(\phi) d\phi, & \text{without a drain prism} \\ \int_\alpha^\beta R \int S(\phi) d\phi + V_{\text{cn}}, & \text{with a drain prism} \end{cases}$$

4.4. Proof of the validity of the proposed formula (4)

According to [16], the volume of the body in cylindrical coordinates is determined by the triple integral of the form of $V = \iiint_V R dR d\phi dz$, where $z$ is the vertical axis of the Cartesian coordinate system.

Consider a curved prismatic ($V$, see Fig. 4), and assume that the half-plane coming from the $z$ axis, corresponding to $\phi$=const, crosses it over some flat figure $S(\phi)$ when changing $\phi$ from $\alpha$ to $\beta$ (Fig. 6).

Then $V = \iiint_{(V)} R dR d\phi dz = \int_\alpha^\beta d\phi \iiint_{S_\phi} R dR dz$, and the figure $S_\phi$ is conveniently attributed to a rectangular coordinate system $Rz$, which rotates together with the specified half-plane around the $z$ axis.

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1 Detailed calculations proving the validity of the proposed formula (4) will be provided below.
As is known (see [16]), the double integral \( \int \int R dR dz \) represents the static moment of a figure \( S_\phi \) relative to the z axis, which is equal to the product of the area \( S(\phi) \) of this figure at a distance \( R_\phi(\phi) \) from the z axis to its center of gravity \( C \): \( \int \int S(\phi) R dR dz = S(\phi) \cdot R_\phi(\phi) \). Substituting this expression into a formula to calculate the volume, and considering that for Fig. 5. \( R_\phi(\phi) = R \) we get:

\[
V = \int_\alpha^\beta S(\phi) R d\phi = R \int_\alpha^\beta S(\phi) d\phi.
\]

5. Practical significance
Developed on the basis of the proposed formulas in the package of the "Mathcad" software, along with the calculations of the corresponding volumes, also implement estimates of the accuracy of approximate methods for calculating these volumes by traditional methods (table 1).

| Type of calculation | Error value, [%] |
|---------------------|-----------------|
| Calculation of the volume of the embankment area | 1-3 |
| Calculation of the volume of the embankment area with broadening in the curve | 4-7 |
| Calculation of the volume of the embankment area taking into account its broadening in the curve | 6-10 |
| Calculation of dredging volume | 1-4 |

It should be noted that the calculations carried out for one of the objects demonstrated an increase in the volume of excavation by about 12 thousand m³, which corresponds to a relative error of 7%.

6. Summary
Thus, the improvement of the accuracy of the calculation of the volume of work based on the use of formulas (1) – (4), will contribute to the correct conduct of the tender for the execution of concentrated earthworks due to a more accurate description of the objects, structure and types of work in the tender documentation. As a result, a more rational use of the budget allocations can be achieved.

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