Vacuum Fluctuations and the Cosmological Constant

Shi Qi
Department of Physics, Nanjing University, Nanjing 210093, China
and Joint Center for Particle, Nuclear Physics and Cosmology, Nanjing University, Nanjing 210093, China
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The hypothesis is proposed that under the approximation that the quantun equations of motion reduce to the classical ones, the quantum vacuum also reduces to the classical vacuum—the empty space. The vacuum energy of QED is studied under this hypothesis. A possible solution to the cosmological constant problem is provided and a kind of parameterization of the cosmological "constant" is derived.

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I. INTRODUCTION

If the physics of the vacuum looks the same to any inertial observer, as it should be, its contribution to the energy-momentum tensor must be invariant under Lorentz transformation. The only class of tensors that have such a property is a multiple of the metric tensor.

The proper volume evolves as the scale factor

$$a(t)$$

evolves. Increasing $$a(t)$$ corresponds to creation of space. If one treats the vacuum as an ideal fluid, then in the momentarily comoving reference frame(MCRF) its energy-momentum tensor takes the form

$$T_{\mu\nu}^{vac} = \rho_{vac} \delta \mathcal{V}$$

Unlike other ideal fluids, the vacuum associates with the space. It is produced immediately after creation of space. Filling the created space of proper volume $$\delta \mathcal{V}$$ with the vacuum would cause an increase of energy of $$\rho_{vac} \delta \mathcal{V}$$. Energy conservation requires $$-\rho_{vac} \delta \mathcal{V} = \rho_{vac} \delta \mathcal{V}$$, or $$\rho_{vac} = -\rho_{vac}$$.

The energy-momentum tensor of the vacuum

$$\rho_{vac}$$

a multiple of the metric tensor. The vacuum fluctuations themselves are real. However, a straightforward calculation of the energy density of the vacuum leads to a infinitely large cosmological constant. A cutoff at reasonable momentum yields a vacuum energy density that is still many orders of magnitude larger than observation. This is the well-known cosmological constant problem, as reviewed by many authors [1] [2] [3] [4] [5] [6] [7] [8] [9] [10] [11].

With the development of supersymmetry, Zumino [12] pointed out that unbroken supersymmetry implies a vanishing vacuum energy density. But the trouble is that supersymmetry is broken at low temperatures such as in today's universe. See [3] for a detailed discussion of this aspect of the problem.

Due to the huge discrepancy between theory and observation, a lot of possible solutions have been proposed. See [3] [4] [6] [7] [9] [10] [11] for reviews. See also [13] for another possible cancellation mechanism.

In this letter, we present another possible way of thinking about the problem.

II. THE BASIC HYPOTHESIS

Consider a theory of a real scalar field with Lagrangian density

$$\mathcal{L} = \frac{1}{2} \partial_{\mu} \phi \partial^{\mu} \phi - \frac{1}{2} m^{2} \phi^{2} - \frac{1}{4!} g \phi^{4}$$

The energy-momentum density of the vacuum

$$T_{\mu\nu}^{\nu} = \langle 0 | T^{\nu\mu} | 0 \rangle$$

where $$T^{\nu\mu} = \partial_{\nu} \phi \partial^{\mu} \phi - g^{\mu\nu} \phi^{4}$$, can be written in terms of Green functions by making use of

$$\langle 0 | \phi^{2} | 0 \rangle = \lim_{x \to y} \langle 0 | T \{ \phi(x) \phi(y) \} | 0 \rangle$$

$$\langle 0 | \partial^{\mu} \phi \partial_{\nu} \phi | 0 \rangle = \lim_{x \to y} \partial_{x} \partial_{y} \langle 0 | T \{ \phi(x) \phi(y) \} | 0 \rangle$$

$$\langle 0 | \phi^{4} | 0 \rangle = \lim_{x_{1}, x_{2}, x_{3}, x_{4} \to x} \langle 0 | T \{ \phi(x_{1}) \phi(x_{2}) \phi(x_{3}) \phi(x_{4}) \} | 0 \rangle$$

(Eq. 5 and Eq. 7 are self-evident. The proof of Eq. 6 is put in the appendix). For example, for the theory of free real scalar field ($g = 0$ in Eq. 6)

$$\langle 0 | T \{ \phi(x) \phi(y) \} | 0 \rangle = \int \frac{d^{4}k}{(2\pi)^{4}} \frac{i}{k^{2} - m^{2} + i\epsilon} e^{-ik(x-y)}$$

*Electronic address: qishi@chenwang.nju.edu.cn*
the vacuum energy-momentum density is
\[ T_{\text{vac}}^{\mu\nu} = \lim_{x \to y} \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 + i\epsilon} \times [k^\mu k^\nu - \frac{1}{2}g^{\mu\nu}(k^2 - m^2)]e^{-ik(x-y)} \]
\[ = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} [k^\mu k^\nu - \frac{1}{2}g^{\mu\nu}(k^2 - m^2)] \left| k^0 = \pm \omega_k = \pm \sqrt{k^2 + m^2} \right. \]
\[ = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} k^\mu k^\nu \left| k^0 = \pm \omega_k = \pm \sqrt{k^2 + m^2} \right. \]  
(9)

(Here we have assumed there is a well behaved cutoff function in the integrand, which is not written out explicitly, so that we can carry out the contour integration over \( k^0 \) which is the same result as just to substitute
\[ \phi(x) = \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_k} [\alpha(\vec{k})e^{-ik\cdot x} + a^\dagger(\vec{k})e^{ik\cdot x}] \]  
(10)
to Eq. (11). In particular,
\[ T_{\text{vac}}^{00} = \int \frac{d^3k}{(2\pi)^3} \frac{\omega_k^3}{2} \]  
(11)
is the zero point energy density. In particle physics, what matters is the relative energy, not the absolute value, and such infinities are eliminated by normal ordering. However, in general relativity, energy-momentum tensor is the source of space-time curvature and vacuum energy-momentum should be taken into account. Unfortunately, Eq. (11) is divergent and any reasonable cutoff leads to a cosmological constant which is unsatisfactorily high.

Recall that if only tree level Feynman graphs are considered, the quantum effective Lagrangian is the same as the classical Lagrangian and quantum field equations as classical field equations. We put it forward as a hypothesis that under the approximation that the quantum equations of motion reduce to the classical ones, the quantum vacuum also reduces to the classical one. In classical theories, the vacuum is just the empty space and there is no vacuum fluctuations. So for the case of vacuum energy-momentum, it is equivalent to assuming that the part of vacuum energy-momentum that arises from tree level Green functions is unobservable in principle, and therefore does not contribute to gravitation. If we view the \( \Lambda \) term as a quantum correction of matter fields to Einstein field equation of general relativity without \( \Lambda \) term, then it follows from the above hypothesis that such a \( \Lambda \)-term quantum correction vanishes for tree level Green functions, which is analogous to the fact that quantum corrections to classical theories arise from loop Feynman graphs in quantum field theory.

After excluding tree level Green functions, the lowest order contribution in \( g \) (quadratic in \( g \)) of vacuum energy-momentum density arises only from \( \partial^\mu \phi \partial^\nu \phi - g^{\mu\nu}(\frac{1}{4}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2 \phi^2) \) in \( T^{\mu\nu} \), which has the same appearance as the free field energy-momentum density operator. In its calculation, only two-point Green function is involved. What remarkable here is that the quadratic nature in \( g \) of the leading term makes the vacuum energy more dependent on the form of field interactions.

Such a hypothesis has another appealing feature. In a quantum field theory without gravitation, a totally free field and its quanta cannot be observed in principle because all experiments are based on interactions. So we can extend the physical Hilbert space by taking the direct product with any free particle Hilbert space without causing any observable effects. Employing the above hypothesis leads to the conclusion that the vacuum of free fields does not contribute to gravitational interactions, and it follows that direct product extension of physical Hilbert space with free particle Hilbert spaces also does not cause any observable gravitational effects as long as we assume no quanta are excited in such spaces considering that there are no interactions to cause such excitations.

The hypothesis excludes the reality of zero point energy. Criticism may be raised based on the Casimir effect, which has been measured to about 1% precision [14]. There are different opinions on taking the Casimir effect as the evidence of zero point energy. In ref. [15] R. L. Jaffe argued that we can calculate Casimir forces as a dynamical effect of QED, instead of introducing zero point energy, and it vanishes as \( \alpha \) approaches zero. And the conclusion is drawn out that the reality of zero point energy is not demonstrated by any known phenomenon, including the Casimir effect. While V. V. Nesterenko etc. [16] think the Casimir effect should be treated as the manifestation of the reality of zero point energy because of the corresponding simpler and clearer theoretical analysis. The criticism does not stand up if we admit the first opinion. If we accept the second opinion, according to the theoretical calculation of Casimir effect [17], we can still argue that what Casimir effect manifests is the zero point energy with boundaries, i.e. in experiments that measure Casimir effect, the zero point energy itself is not observed, what we observe is the excess of it caused by boundary conditions. The experimental confirmation of the Casimir effect does not manifest the reality of the zero point energy in unbounded homogeneous space, so it does not necessarily mean that such zero point energy has gravitational effects.

**III. THE VACUUM ENERGY OF QED**

Now let’s turn to a real theory — quantum electrodynamics (QED) with Lagrangian density
\[ \mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\Psi} (i\gamma^\mu \partial_\mu - m_B) \Psi_B - \epsilon_B \bar{\Psi} \gamma^\mu \Psi_B A_{B\mu} \]  
(12)
where the subscript $B$ indicates bare or unrenormalized quantities. In the following, symbols without subscript $B$ implicitly denote quantities renormalized on mass shell. As discussed above, to first order, we only need the free energy-momentum tensor

$$T^{\mu\nu} = T^{\mu\nu}_{\gamma,0} + T^{\mu\nu}_{e,0}$$  \hspace{1cm} (13)$$
where

$$T^{\mu\nu}_{\gamma,0} = F^{\sigma\mu} F^\nu_{\sigma} - g^{\mu\nu} \mathcal{L}_{\gamma,0}$$
$$= F^{\sigma\mu} F^\nu_{\sigma} - \frac{1}{4} g^{\mu\nu} F_{\sigma\rho} F^{\sigma\rho}$$  \hspace{1cm} (14)$$
$$T^{\mu\nu}_{e,0} = \bar{\psi} i\gamma^\mu \partial^\nu \psi - g^{\mu\nu} \mathcal{L}_{e,0}$$
$$= \bar{\psi} i\gamma^\mu \partial^\nu \psi - g^{\mu\nu} \bar{\psi} (i\gamma^\sigma \partial_\sigma - m) \psi$$  \hspace{1cm} (15)$$

Using equations like Eq. (5) and our hypothesis, we can begin to estimate the vacuum energy of QED. The estimation is done in Feynman gauge, and considering an infrared cutoff (for example, caused by the cosmological horizon, see [18] for related discussion) we introduce the effective photon mass $\eta$ in its propagator.

To first order [17]

$$\langle 0 | T \{ A^\mu(x) A^\nu(y) \} | 0 \rangle_l$$
$$= \frac{i}{(2\pi)^4} \int d^4 p e^{-i p \cdot (x - y)}$$
$$\times \left( \frac{1}{p^2 - \eta^2 + i\epsilon} \right)^2 \left( (p^\mu p^\nu - p^2 g^{\mu\nu}) \pi(p^2) \right)$$  \hspace{1cm} (16)$$

(the subscript $l$ indicates that the Green function only includes loop graphs and corresponding counter terms) where

$$\pi(p^2) = -\frac{e^2}{12\pi^2} \int_{4m^2}^{+\infty} \frac{dk^2}{k^2} \frac{1}{k^2 - p^2 - i\epsilon}$$
$$\times \left( \frac{1}{1 - \frac{4m^2}{k^2}} \right)^{\frac{1}{2}} \left( 1 + \frac{2m^2}{k^2} \right)$$  \hspace{1cm} (17)$$

Together with Eq. (14), we can derive the energy-momentum tensor of the vacuum of electromagnetic field

$$T^{\mu\nu}_{\gamma, vac} = \lim_{x \rightarrow y \rightarrow 0} \frac{i}{(2\pi)^4} \int d^4 p f(|\vec{p}|, |p^0|) e^{-i p \cdot (x - y)}$$
$$\times \left( \frac{1}{p^2 - \eta^2 + i\epsilon} \right)^2 \pi(p^2)$$
$$\times p^2 (2p^\mu p^\nu - \frac{1}{2} g^{\mu\nu} p^2)$$  \hspace{1cm} (18)$$

where $f(|\vec{p}|, |p^0|)$ is a cutoff function introduced by, say, supersymmetry and/or any other unknown extremely high energy physics. It is expected to approximately equal 1 if $|\vec{p}|$ and $|p^0|$ are small and 0 if $|\vec{p}|$ or $|p^0|$ are large. For simplicity we take

$$f(|\vec{p}|, |p^0|) = \begin{cases} 
1 & \text{if } |\vec{p}| \leq \Lambda \text{ and } |p^0| \leq \Lambda \\
0 & \text{if } |\vec{p}| > \Lambda \text{ or } |p^0| > \Lambda 
\end{cases}$$  \hspace{1cm} (19)$$

and assume it has no poles in the $p^0$ complex plane. For the vacuum energy, i.e. $\mu = \nu = 0$, carrying out the integral over $p^0$, we get

$$T^{00}_{\gamma, vac} = \frac{e^2}{12\pi^2 (2\pi)^4} \int d^4 p \int_{4m^2}^{+\infty} dk^2 f(|\vec{p}|, \sqrt{p^2 + k^2})$$
$$\times \frac{\bar{p}^0 + \frac{1}{4} k^2}{k^2}$$
$$\sqrt{p^2 + k^2} (k^2 - \eta^2 + i\epsilon)^2$$
$$\times \left( 1 + \frac{2m^2}{k^2} \right)$$  \hspace{1cm} (20)$$

Substitute Eq. (19) into it

$$T^{00}_{\gamma, vac} = \frac{e^2}{12\pi^2 (2\pi)^4} \int d^4 p \int_{4m^2}^{+\infty} \frac{dk^2}{k^2} \frac{\bar{p}^0 (\bar{p}^0 + \frac{1}{4} k^2)}{\sqrt{p^2 + k^2} (k^2 - \eta^2 + i\epsilon)^2}$$
$$\times \left( 1 + \frac{2m^2}{k^2} \right)$$
$$\times \left( 1 + \frac{2m^2}{k^2} \right)$$  \hspace{1cm} (21)$$

Similarly, for the electron field, to first order [17]

$$\langle 0 | T \{ \bar{\psi}(x) \psi(y) \} | 0 \rangle_l$$
$$= \frac{i}{(2\pi)^4} \int d^4 p e^{-i p \cdot (x - y)} \frac{(\hat{p} + m) \Sigma^* (p) (\hat{p} + m)}{(p^2 - m^2 + i\epsilon)^2}$$  \hspace{1cm} (22)$$

and

$$\Sigma^*(p) = -\frac{e^2}{16\pi^2} \left\{ \hat{p} \left( \frac{m^4}{(p^2)^2} - 1 \right) L(p^2) + \frac{m^2}{p^2} - 1 \right\}$$
$$+ 4m \left( 1 - \frac{m^2}{p^2} \right) L(p^2)$$
$$- 2 (\hat{p} - m) \left( 1 + \ln \frac{\eta^2}{m^2} \right)$$  \hspace{1cm} (23)$$

where

$$L(p^2) = \begin{cases} 
\ln \left( 1 - \frac{p^2}{m^2} \right) & \text{if } p^2 \leq m^2 \\
\ln \left( \frac{p^2}{m^2} - 1 \right) - i\pi & \text{if } p^2 > m^2 
\end{cases}$$  \hspace{1cm} (24)$$

Since

$$T^{\mu\nu}_{e,0} = \lim_{x \rightarrow y \rightarrow 0} \text{Tr} \left[ -i \gamma^\mu \frac{\partial}{\partial x_\nu} T \{ \bar{\psi}(x) \psi(y) \} \right]$$
$$+ g^{\mu\nu} \left( i\gamma^\sigma \frac{\partial}{\partial x_\sigma} - m \right) T \{ \bar{\psi}(x) \psi(y) \}$$  \hspace{1cm} (25)$$
according to our hypothesis, we have

$$T_{e,\text{vac}}^{\mu\nu} = \lim_{x \to y} \text{Tr} \left[ -i \gamma^\mu \frac{\partial}{\partial x^\nu} \langle 0 | T \{ \Psi(x) \Psi(y) \} | 0 \rangle_{\text{T}} + g^{\mu\nu} \left( i \gamma^\sigma \frac{\partial}{\partial x^\sigma} - m \right) \langle 0 | T \{ \Psi(x) \Psi(y) \} | 0 \rangle_{\text{T}} \right]$$

$$= \lim_{x \to y} \text{Tr} \left\{ \frac{i}{(2\pi)^4} \int d^4p g(|\vec{p}|, |p^0|) e^{-i\vec{p} \cdot (x-y)} \times \gamma^\nu p^\nu + g^{\mu\nu} (\vec{p} - m) \right. \left( \frac{p^2 - m^2 + i\epsilon}{p^2 - m^2 + i\epsilon} \right) \times (\vec{p} + m) \Sigma^*(p) (\vec{p} + m) \right\} \quad (26)$$

where \( g(|\vec{p}|, |p^0|) \) is also a cutoff function like \( f(|\vec{p}|, |p^0|) \). Again we take a simple form of it

$$g(|\vec{p}|, |p^0|) = \left\{ \begin{array}{ll} 1 & \text{if } |\vec{p}| \leq \Lambda \text{ and } |p^0| \leq \sqrt{\Lambda^2 + m^2} \\ 0 & \text{if } |\vec{p}| > \Lambda \text{ or } |p^0| > \sqrt{\Lambda^2 + m^2} \end{array} \right. \quad (27)$$

and assume it has no poles in the \( p^0 \) complex plane. After some derivation similar to the case for the electromagnetic field, we get

$$T_{e,\text{vac}}^{00} = -\frac{e^2}{16\pi^2} \int_{m^2}^{\Lambda^2 + m^2} \frac{dk^2}{k^2 (k^2 - m^2)}$$

$$\times \left[ \frac{m^4}{k^2} \int_0^{\sqrt{\Lambda^2 + m^2 - k^2}} d|\vec{p}| \frac{|\vec{p}|^4}{|\vec{p}|^2 + k^2} \right. - 2m^2 \int_0^{\sqrt{\Lambda^2 + m^2 - k^2}} d|\vec{p}| \frac{|\vec{p}|^2}{|\vec{p}|^2 + k^2} \right]$$

$$+ 4m^2 \int_0^{\Lambda} d|\vec{p}| |\vec{p}|^2 \sqrt{|\vec{p}|^2 + m^2}$$

$$- \frac{e^2}{16\pi^2} \left( 4 + 2 \ln \frac{n^2}{m^2} + 4 \ln \frac{\Lambda^2 + m^2}{\Lambda^2} \right)$$

$$\times \int_0^{\Lambda} d|\vec{p}| |\vec{p}|^2 \sqrt{|\vec{p}|^2 + m^2} \quad (28)$$

Since the first order vacuum energy depends on the interaction, we should view the vacuum energy of the electromagnetic field and that of electron field as a whole, i.e. the first order QED vacuum energy

$$T_{QED,\text{vac}}^{00} = T_{\gamma,\text{vac}}^{00} + T_{e,\text{vac}}^{00} \quad (29)$$

Fig. 1 and Fig. 2 show the dependence of QED vacuum energy of different effective photon mass \( \eta \) on the cutoff \( \Lambda \). We see that QED vacuum energy crosses zero at some cutoff. This property makes it possible for us to get an arbitrarily small vacuum energy or cosmological constant. The cosmological constant problem could probably be resolved in this way. What we need to do is just to show that the vacuum energy with the correct infrared and ultraviolet cutoff is consistent with the observed cosmological constant. Of course, the real situation, in which many other fields should be considered, would be much more complicated.
vestigating the dependence of the vacuum energy on \( \eta \), since for small \( \eta \)

\[
\left( \frac{1}{p^2 - \eta^2 + \mathcal{V}} \right)^2 \approx \left( \frac{p^2}{p^2 + \mathcal{V}} \right)^2 + 2p^2 \eta^2 \frac{\eta^2}{m^2} \frac{1}{p^2 + \mathcal{V}}
\]

we have the vacuum energy

\[
\rho_{\text{vac}}(\eta) = \rho_0 + \rho_1 \frac{\eta^2}{m^2} + \rho_2 \frac{\ln \frac{\eta^2}{m^2}}{m^2}
\]  

or the cosmological “constant”

\[
\lambda(\eta) = \lambda_0 + \lambda_1 \frac{\eta^2}{m^2} + \lambda_2 \ln \frac{\eta^2}{m^2}
\]

We see that the cosmological “constant” is no more a constant, but a parameter depends on the infrared cutoff. Taking the cosmological horizon that blocks information as the cause of the infrared cutoff, we will get a time-dependent cosmological “constant” because of the time-dependence of the cosmological horizon.

IV. CONCLUSION

From the hypothesis that under the approximation that the quantum equations of motion reduce to the classical ones, the quantum vacuum also reduces to the classical vacuum, we propose that the part of vacuum energy-momentum that arises from tree level Green functions is unobservable in principle, hence does not contribute to gravitation. As a result, two usually accepted facts, i.e. the quantum corrections to classical theory only come from loop Feynman graphs, and the direct product extension of physical Hilbert space with free particle Hilbert spaces does not cause any observable effects, are extended from quantum field theory to theories with gravitation included. The hypothesis does not contradict the Casimir effect, which has been proved experimentally. The evaluation of the QED vacuum energy under the hypothesis shows that it crosses zero at some cutoff, which provides a possible solution to the cosmological constant problem. In the calculation we use very simple cutoff functions, which should in principle be derived from theories beyond the standard model in further studies. From the dependence of QED vacuum energy on the infrared cutoff, we derive a kind of parameterization of the cosmological “constant”.

APPENDIX A

For the scalar field theory, the time-ordered product is defined as

\[
T \{ \phi(x)\phi(y) \} = \theta(x^0 - y^0)\phi(x)\phi(y) + \theta(y^0 - x^0)\phi(y)\phi(x)
\]

We will show that

\[
\lim_{x-y \to 0} \frac{\partial}{\partial x^\mu} \frac{\partial}{\partial y^\nu} T \{ \phi(x)\phi(y) \} = \lim_{x-y \to 0} T \left\{ \frac{\partial}{\partial x^\mu} \phi(x) \frac{\partial}{\partial y^\nu} \phi(y) \right\}
\]

If both \( \mu \) and \( \nu \) are space indices, \( \mu = i, \nu = j \), obviously

\[
\frac{\partial}{\partial x^i} \frac{\partial}{\partial y^j} T \{ \phi(x)\phi(y) \} = T \left\{ \frac{\partial}{\partial x^i} \phi(x) \frac{\partial}{\partial y^j} \phi(y) \right\}
\]

If one of \( \mu \) and \( \nu \) stands for the time index, say \( \mu = 0, \nu = i \), then

\[
\frac{\partial}{\partial x^0} \frac{\partial}{\partial y^i} T \{ \phi(x)\phi(y) \} = T \left\{ \frac{\partial}{\partial x^0} \phi(x) \frac{\partial}{\partial y^i} \phi(y) \right\}
\]

If both \( \mu \) and \( \nu \) are time indices, viz. \( \mu = \nu = 0 \), we have

\[
\frac{\partial}{\partial x^0} \frac{\partial}{\partial y^0} T \{ \phi(x)\phi(y) \} = T \left\{ \frac{\partial}{\partial x^0} \phi(x) \frac{\partial}{\partial y^0} \phi(y) \right\}
\]

\[\text{follows from (A3), (A1) and (A6), so we have Eq. (A1)}\]

Similarly, for the spinor field theory

\[
T \{ \Psi_\alpha(x)\Psi_\beta(y) \} = \theta(x^0 - y^0)\Psi_\alpha(x)\Psi_\beta(y) - \theta(y^0 - x^0)\Psi_\beta(y)\Psi_\alpha(x)
\]

where \( \alpha, \beta \) denote Dirac indices, we need to prove

\[
\lim_{x-y \to 0} \frac{\partial}{\partial x^\mu} T \{ \Psi_\alpha(x)\Psi_\beta(y) \} = \lim_{x-y \to 0} T \left\{ \frac{\partial}{\partial x^\mu} \Psi_\alpha(x)\Psi_\beta(y) \right\}
\]
which is obviously true for $\mu = 1, 2, 3$. For $\mu = 0$ so we have Eq. (A7).

$$\begin{align*}
\frac{\partial}{\partial x^0} T \left\{ \Psi_\alpha(x) \bar{\Psi}_\beta(y) \right\} \\
= T \left\{ \frac{\partial}{\partial x^0} \Psi_\alpha(x) \bar{\Psi}_\beta(y) \right\} \\
+ \delta(x^0 - y^0) \left[ \Psi_\alpha(x) \bar{\Psi}_\beta(y) \right]_+ \\
= T \left\{ \frac{\partial}{\partial x^0} \Psi_\alpha(x) \bar{\Psi}_\beta(y) \right\} \\
+ \Gamma^0_{\alpha\beta} \delta^{(4)}(x - y) \quad (A8)
\end{align*}$$

[1] Y. B. Zel’dovich, Sov. Phys. Usp. 11, 381 (1968).
[2] Y. B. Zel’dovich, Sov. Phys. Usp. 24, 216 (1981).
[3] S. Weinberg, Rev. Mod. Phys. 61, 1 (1989).
[4] S. M. Carroll, W. H. Press, and E. L. Turner, Ann. Rev. Astron. Astrophys. 30, 499 (1992).
[5] V. Sahni and A. A. Starobinsky, Int. J. Mod. Phys. D9, 373 (2000), astro-ph/9904398.
[6] S. M. Carroll, Living Rev. Rel. 4, 1 (2001), astro-ph/0004075.
[7] S. E. Rugh and H. Zinkernagel (2000), hep-th/0012253.
[8] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. 75, 559 (2003), astro-ph/0207347.
[9] T. Padmanabhan, Phys. Rept. 380, 235 (2003), hep-th/0212290.
[10] T. Padmanabhan, Curr. Sci. 88, 1057 (2005), astro-ph/0411044.
[11] S. Nobbenhuis (2004), gr-qc/0411093.
[12] B. Zumino, Nucl. Phys. B89, 535 (1975).
[13] J.-Q. Shen, Phys. Lett. A340, 12 (2005), gr-qc/0509009.
[14] U. Mohideen and A. Roy, Phys. Rev. Lett. 81, 4549 (1998), physics/9805038.
[15] R. L. Jaffe, Phys. Rev. D72, 021301 (2005), hep-th/0503158.
[16] V. V. Nesterenko, G. Lambiase, and G. Scarpetta, Riv. Nuovo Cim. 27N6, 1 (2004), hep-th/0503100.
[17] C. Itzykson and J.-B. Zuber, Quantum Field Theory (McGraw-Hill Inc., 1980).
[18] T. Padmanabhan, Class. Quant. Grav. 22, L107 (2005), hep-th/0406060.