Relativistic Astronomy

Bing Zhang1 and Kunyang Li2,3

1Department of Physics and Astronomy, University of Nevada Las Vegas, NV 89154, USA; zhang@physics.unlv.edu
2Department of Physics, 837 State St NW, Georgia Institute of Technology, Atlanta, GA 30332, USA
3Center for Relativistic Astrophysics, Georgia Institute of Technology, Atlanta, GA 30332, USA

Received 2017 October 22; revised 2018 January 19; accepted 2018 January 19; published 2018 February 20

Abstract

The “Breakthrough Starshot” aims at sending near-speed-of-light cameras to nearby stellar systems in the future. Due to the relativistic effects, a transrelativistic camera naturally serves as a spectrograph, a lens, and a wide-field camera. We demonstrate this through a simulation of the optical-band image of the nearby galaxy M51 in the rest frame of the transrelativistic camera. We suggest that observing celestial objects using a transrelativistic camera may allow one to study the astronomical objects in a special way, and to perform unique tests on the principles of special relativity. We outline several examples that suggest transrelativistic cameras may make important contributions to astrophysics and suggest that the Breakthrough Starshot cameras may be launched in any direction to serve as a unique astronomical observatory.

Key words: methods: observational

1. Introduction

The Breakthrough Initiatives (https://breakthroughinitiatives.org/) is a program of scientific and technological exploration, probing some big questions of life in the universe. Among these initiatives is the “Breakthrough Starshot” program (https://breakthroughinitiatives.org/Challenges/3/), which aims at proving the concept of developing an unmanned space flight (probe) at a good fraction of the speed of light, c. Such a probe is designated to reach nearby stellar systems (such as Alpha Centauri) within decades, allowing humankind to explore extrasolar systems for the first time. The first prototype, “Sprites,” which are 3.5 cm × 3.5 cm chips that weigh just 4 grams each and are the precursors to eventual “starChip” probes, has recently been launched at a low-Earth orbit (https://breakthroughinitiatives.org/News/12). Here we point out that, due to the relativistic effects, a transrelativistic camera naturally serves as a spectrograph, a lens, and a wide-field camera while traveling in space, allowing humankind to study the astrophysical objects in a unique manner and to conduct tests on special relativity.

2. Relativistic Astronomy

2.1. Relativistic Effects

When a camera travels in space with a speed close to c, some interesting relativistic effects would occur. For example, Christian & Loeb (2017) suggested that an interferometer moving at a relativistic speed offers a sensitive probe of acceleration making use of the temporal Terrell effect (Penrose 1959; Terrell 1959).

Here we focus on the observational distortions of emission from distant astronomical objects. In the comoving frame of the probe, all astronomical objects undergo a unique Doppler boost (Doppler factor D > 1) or deboost (D < 1) depending on the Lorentz factor of the probe and the angular between the object with respect to the direction of probe motion. For the problem involving a flying probe, one can define two rest frames4: the Earth rest frame or the laboratory frame (which is also the rest frame of astronomical objects), K, and the probe comoving frame, K′. Let us define the Lorentz factor of the probe as

\[ \Gamma = \frac{1}{\sqrt{1 - \beta^2}}, \]

where \( \beta = v/c \) is the normalized speed of the probe. In Frame K′, all the astronomical objects move with the same Lorentz factor \( \Gamma \), but with different angles with respect to the opposite direction of the probe motion. The Doppler factor of the source is defined as (Rybicki & Lightman 1979)

\[ D \equiv \frac{1}{\Gamma(1 - \beta \cos \theta')} = \Gamma(1 + \beta \cos \theta), \]

(1)

where the angle between the object moving direction and the line of sight in two different frames are related through

\[ \cos \theta' = \frac{\cos \theta + \beta}{1 + \beta \cos \theta}. \]

(2)

Some characteristic angles and Doppler factors are

1. \( \theta' = 0 \) and \( \theta = 0 \): \( D = D_{\text{max}} \equiv (1 + \beta) \Gamma = \frac{1 + \beta}{\sqrt{1 - \beta^2}} \).
2. \( \theta' = \cos^{-1} \beta \) and \( \theta = \pi/2 \): \( D = \Gamma \).
3. \( \theta' = \theta' \equiv \cos^{-1} \left( \frac{1 - \beta}{\sqrt{1 + \beta^2}} \right) \) and \( \theta = \theta_0 \equiv \cos^{-1} \left( \frac{1}{\sqrt{1 + \beta^2}} \right) \): \( D = 1 \).
4. \( \theta' = \pi/2 \) and \( \theta = \cos^{-1}(-\beta) \): \( D = 1/\Gamma \).
5. \( \theta' = \pi \) and \( \theta = \pi \): \( D = \frac{1}{(1 + \beta)^2} = \frac{1 - \beta}{\sqrt{1 + \beta^2}} \).

The Doppler factor connects the quantities of the source rest frame (frame K in our convention) and those in the probe comoving frame (frame K′ in our convention). In particular, for the camera on board the probe all the emission is blueshifted (redshifted) for \( D > 1 \) (\( D < 1 \)), i.e.,

\[ \nu' = D \nu, \]

(3)

For an isotropic, point source, the specific flux and flux transformations are (Appendix A)

\[ F'_\nu(\nu') = D^2 F_\nu(\nu), \]

\[ F(\nu) = D^4 F(\nu'). \]

(4)
For an extended source, the specific flux and flux transformations for an emission pixel reads (Appendix A)

\[
F'(\nu') = D F(\nu), \\
F'(\nu') = D^2 F(\nu).
\] (5)

These salient relativistic effects provide a unique opportunity to study the universe and to test the principle of special relativity, which may be discussed under the broad umbrella of “relativistic astronomy.”

2.2. Universe Seen from the Probe’s Frame \(K'\)

For a wide-field camera moving with a Lorentz factor \(\Gamma\), all the objects in the field of view undergo relativistic distortions, including shift of position (Equation (2)), shift of frequency (Equation (3)), and change of specific flux and flux (Equations (4) and (5)). The degree of distortion, characterized by the Doppler factor \(D\) (Equation (1)), solely depends on the angle \(\theta\) with respect to the direction of motion for a constant \(\Gamma\). In general, an extended object is bluer and more compact as observed in Frame \(K'\). Since astronomers already have a detailed view in Frame \(K\), a measurement of the differences in the observed properties between Frames \(K\) and \(K'\) offers valuable information about the astronomical sources.

For a probe with a constant velocity, based on the position shifts of three point sources (background stars) close to the direction of motion (where the sources are squeezed), one can uniquely determine the direction of motion and the Lorentz factor \(\Gamma\) (or dimensionless velocity \(\beta\)) of the probe (Appendix B).

Once the direction of motion and \(\Gamma\) are determined, one can calculate the Doppler factor of all the celestial objects. Given the same observing frequency of the camera, the intrinsic frequencies of different astronomical objects are different (corrected by the respective \(D\) of the source). For a direction with \(\theta < \theta_\ast\), one has \(D > 1\), so that the intrinsic frequency of the source that the camera records is redder than the observed frequency. Conversely, for \(\theta > \theta_\ast\) (\(D < 1\)), the intrinsic frequency of the source of the camera records is bluer than the observed frequency. As a result, for a camera in a particular band (e.g., R band), one may study other frequencies (e.g., infrared (IR) or ultraviolet (UV)) of different sources without the need of using technically challenging IR/UV cameras. In a sense, a relativistically moving camera is a natural spectrograph.

In the \(D > 1\) regime, the fluxes of the sources are enhanced. So a relativistically moving camera is also a natural lens. So, in practice, astronomical objects are better studied in the \(D > 1\) regime, where the source is studied in an intrinsically redder band.

If a camera continuously observes an astronomical source as the camera is accelerated, it would record emission of the source in a span of frequencies, so that one can obtain a detailed spectrum of the source in the frequency range between \(\nu_{\text{camera}}\) and \(\nu_{\text{camera}}/D_{\text{max}}\). The higher the achievable \(D_{\text{max}}\), the wider the spectrograph is. For different frequencies, one should properly correct for the respective flux Doppler boosting factors to get the intrinsic flux at those frequencies in order to retrieve the intrinsic spectrum of the source.

The camera itself may be designed to have a grism spectrograph. In this case, after determining the direction of motion and the probe Lorentz factor using photometry observations, one may turn on the grism mode to capture the fine spectra of the sources in a different (redder) spectral regime.

Due to the light aberration effect, the objects in the moving direction are more packed. The entire hemisphere in Frame \(K\) is combed into a cone defined by the angle \(\theta = \cos^{-1}\beta\). Given the same field of view, the camera can observe more objects. This effectively increases the field of view of the camera.

In order to visually show how the astronomical objects look differently in Frame \(K'\), we carry out some simulations based on true observational images. The upper panel of Figure 1 shows an observed image of the nearby galaxy M51. We use the HST images in three filters: F435W, F555W, and F814W.5 The filter information is from the SVO filter profile service website.6 In our simulation, we adopt \(\beta = 0.5\) (\(\Gamma \approx 1.7321\)). The simulation code loops through all pixels of the input image, and uses the counts of a pixel in different bands to generate a simple spectrum for that particular pixel. Then the spectrum undergoes the relativistic transformation according to Equations (3) and (5). Integration of the product of transformed spectrum and the response functions of the filters give rise to the new counts in different bands of that pixel in Frame \(K'\). For each pixel, the polar coordinate angles \((\theta, \phi)\) are calculated relative to the direction of motion, which is set to (1075, 1525) pixel in the image. Using Equation (2) the observed polar angle from the moving direction in the probe frame \((\theta')\) can be calculated. With the image scale (0.2 arcsec/pixel), we then obtain the new position of the pixel in the simulated image. In order to simulate the color scheme of the image, we use an image editor named GIMP (GNU Image Manipulation Program) that enables the simulation with an image in more than three bands. The color of each layer of the image is represented by a hue value in GIMP that can be set manually by the user. However, the hue value of images in the same band should be kept the same to show the true color difference between the field of view in Frames \(K\) and \(K'\) due to the relativistic effects. The hue values of F435W, F555W, and F658N images after relativistic transformation are set to 42, 159, and 232, which are the same as the hue values in the original image. Due to the relativistic effects, the spectrum of M51 is shifted toward blue, meaning that the IR band spectrum is shifted into the optical band. In our simulation, we have used the M51 image in the F814W filter as the IR data, and shifted it to the H\(_\alpha\) (F658N) filter range in the simulated image.

The simulated image is presented in the lower panel of Figure 1. One can see that the simulated image is indeed bluer, brighter, and more compact (i.e., each extended source becomes smaller, and the two extended sources have a smaller spatial separation).

2.3. Examples of Astronomical Applications

The impacts of transrelativistic cameras on astronomy should be multi-fold and may not be fully appreciated until they become available. In the following, we discuss several examples that suggest these cameras may make important contributions to astrophysics.

---

5 https://archive.stsci.edu/prepds/m51/dataList.html
6 http://ivoa.net/documents/Notes/SVOFPS/index.html
Comparison of the image of a nearby galaxy M51 in Frames Figure 1. The Astrophysical Journal, ’Bottom: simulated false color image of M51 as observed in the probe and K images (HST 1075, 1525 854:123 pixel in the image). Top: false color K image (F435W, F555W, and F814W) observed in the rest frame of Earth, i.e., Frame K. Bottom: simulated false color image of M51 as observed in the probe’s comoving frame K′. The dimensionless velocity is set to β = 0.5 (Γ ≃ 1.7321), and the pixel scale of the image is 0.2 arcsec/pixel. The direction of motion is set to the (1075, 1525) pixel in the image (cross in both images).

Table 1

| β | Γ | D_{max} | D_{max}^1 | z_{Lyα}(λ) |
|---|---|---|---|---|
| 0 | 1 | 1 | 1 | 8.2259 (λ/1 μm) −1 |
| 0.1 | 1.0050 | 1.1055 | 1.3512 | 9.0941 (λ/1 μm) −1 |
| 0.2 | 1.0206 | 1.2247 | 1.8371 | 10.0746 (λ/1 μm) −1 |
| 0.3 | 1.0487 | 1.3628 | 2.5309 | 11.2190 (λ/1 μm) −1 |
| 0.4 | 1.0911 | 1.5275 | 3.5642 | 12.5653 (λ/1 μm) −1 |
| 0.5 | 1.1547 | 1.7321 | 5.1962 | 14.2477 (λ/1 μm) −1 |
| 0.6 | 1.2500 | 2.0000 | 8.0000 | 16.4518 (λ/1 μm) −1 |
| 0.7 | 1.4003 | 2.3805 | 13.4894 | 19.5816 (λ/1 μm) −1 |
| 0.8 | 1.6667 | 3.0000 | 27.0000 | 24.6777 (λ/1 μm) −1 |
| 0.9 | 2.2942 | 4.3589 | 82.8191 | 35.8559 (λ/1 μm) −1 |
| 0.95 | 3.2026 | 6.2450 | 243.555 | 51.3708 (λ/1 μm) −1 |
| 0.99 | 7.0888 | 14.1067 | 2807.20 | 116.0408 (λ/1 μm) −1 |

2.3.1. Reionization History of the Universe

The first example is to study the reionization history of the universe (Loeb & Barkana 2001; Fan et al. 2006). The hot Big Bang theory predicts that the universe became neutralized around z ∼ 1100, when electrons and protons recombined to produce neutral hydrogen. This is the epoch of the cosmic microwave background radiation. The universe was later reionized by the first objects (first stars, galaxies, and quasars) in the universe that shine in UV and X-rays (above 13.6 eV). The epoch between recombination and reionization is sometimes called the “cosmic dark ages.” Quasar observations suggest that reionization was nearly complete around z ∼ 6 (Fan et al. 2006). However, the exact detail of the reionization history is not known. Different models predict different neutron fractions as a function of redshift (e.g., Holder et al. 2003 and references therein). In order to map the reionization history in detail, one needs to populate bright beacons in the redshift range from 6 to ∼20, and measure the spectrum blueward of the redshifted Lyα line, i.e., λ ≲ (1 + z)1215.67 Å (or hν ≥ 10.2/(1+z) eV). A mostly neutral intergalactic medium would essentially absorb emission in this regime, forming the so-called Gunn–Peterson trough (Gunn & Peterson 1965). The shape of the red damping wing carries the important information of the neutral fraction of the IGM in that epoch (Miralda-Escudé 1998). However, such studies are hindered by the fact that the feature is moved to progressively more infrared bands as z increases, and that the sources are typically fainter at higher z as well.

Sending high-β probes toward high-z galaxies or quasars would make it more convenient to measure their redshifts and to study the detailed red damping wing of these objects. This is because the Gunn–Peterson trough is shifted to the bluer bands in the probe’s frame and the source flux is enhanced.

Table 1 presents the relevant parameters for relativistic astronomy for different β values. The parameters include Lorentz factor Γ, maximum Doppler factor D_{max}, its third power D_{max}^3, and the relevant redshift, z_{Lyα}(λ), for the Lyα wavelength (1215.67 Å) to be at a particular wavelength λ in Frame K′. For example, with β ∼ 0.3, z ∼ 10 can be probed with a 1 μm camera, and the source is brighter by ∼1 magnitude. If β ∼ 0.6 is achieved, z ∼ 10 can even be probed with an R-band (λ ∼ 0.658 μm) camera, and the source is brighter by more than 2 mag. Since an optical grism is much easier to build, one can use relativistic cameras with different
termination speeds to probe a range of redshifts to most thoroughly study the reionization history of the universe.

### 2.3.2. Redshift Desert

The redshift interval $1.4 \lesssim z \lesssim 2.5$ has been described by some authors as the “redshift desert” due to the lack of strong spectral lines in the optical band (4300–9000) Å. Since this redshift range coincides with the epoch of significant star formation, the lack of a large sample of galaxies in this redshift range hinders an unbiased mapping of the star formation history of the universe (Steidel et al. 2004 and references therein). Observations with transrelativistic cameras can easily fill this gap. One does not need a very high $\beta$ in order to achieve this goal. For example, with $\beta = 0.2$, one has a Doppler factor range from $D_{\text{min}} = 0.8165$ to $D_{\text{max}} = 1.2247$ (depending on the observational direction $\theta$). This is already enough to remove the redshift desert. In particular, for the redshift range (1.9, 2.5), one can use the $\theta' = 0$ mode, so that the effective redshift range in frame $K'$ is changed to $1.2247 \times (2.9, 3.5) - 1 = (2.55, 3.29)$, which is outside the desert. Similarly, for the redshift range (1.4, 1.9), one can use the $\theta = \pi$ mode, so that the effective redshift range is $0.8165 \times (2.4, 2.9) - 1 = (0.96, 1.37)$, again outside the desert.

#### 2.3.3. Gamma-Ray Bursts (GRBs)

GRBs are the most luminous astrophysical objects in the universe. In the case of observing transient relativistic events such as GRBs, relativistic astronomy would allow humankind to study a relativistically moving source by a relativistically moving observer for the first time, a scenario previously only imagined in a thought experiment. Catching the early laboratory-frame IR afterglow of a GRB using an optical camera on board a high-\(\beta\) probe would help to identify very high-z GRBs. Theoretical models suggest that GRBs might form as early as $z \sim 20$ when the first-generation stars die (e.g., Toma et al. 2009; Mészáros & Rees 2010). A $\beta \sim 0.7$ camera will probe $z \sim 18.6$ with a 1 \(\mu\)m camera. A systematic study of these explosions very early in the universe will help to probe the deep dark ages of the universe (Tanvir et al. 2011).

A good fraction of GRBs (30\%–50\%) are “optically dark” GRBs. Even though high-z GRBs may comprise a portion, most of them may be embedded in dusty star-forming regions, so that the optical emission is absorbed via dust extinction (e.g., Perley et al. 2009). If a relativistic camera is launched after the trigger of a GRB and an observation is carried out during the acceleration of the camera, given the same observational frequency, the camera would continuously observe a range of frequency toward the IR regime, which will catch the characteristic features of dust extinction. Combining afterglow modeling, one may also precisely map the extinction curve of the GRB host galaxy, which is currently poorly constrained (e.g., Scaldh et al. 2012).

#### 2.3.4. Electromagnetic Counterpart of Gravitational Waves (GWs)

The new era of multi-messenger astrophysics just arrived recently with the detection of the first double neutron star (NS–NS) merger system GW170817 and its associated GRB 170817A and multiband electromagnetic counterpart (e.g., Abbott et al. 2017a, 2017b). One important phenomenon is the so-called “kilonova,” a type of IR/optical transient arising from the r-process of neutron-rich materials dynamically ejected during the merger (Li & Paczyński 1998; Metzger et al. 2010). The kilonova associated with GW170817 appeared to have a “red” component and a “blue” component (e.g., Villar et al. 2017), with the former likely associated with the high-opacity ejecta, possibly involving heavy elements such as lanthanides. Understanding these events will greatly benefit from a careful study of the spectra in a broad range from IR to UV. In the future relativistic astronomy era, NS–NS and NS–BH mergers will be regularly detected by the next-generation GW detectors. Observations with the transrelativistic cameras in the directions toward and away from the GW trigger direction, together with ground-based observations, will help to uncover the broadband spectra of kilonovae, leading to an in-depth study of the NS–NS and NS–BH merger physics.

### 2.4. Testing Special Relativity

One can also use the observations of transrelativistic cameras to test the principles of spectral relativity. There are two ways to do so.

1. As seen from Figure 2 and discussed in Appendix B, the measurement of relative positions of three point sources in Frame $K'$ as compared with those measured in $K$ (on Earth) can uniquely solve the motion of the camera (the direction and the Lorentz factor). With this information, one can predict the positions of the fourth, fifth ... point sources in the sky in Frame $K'$. If within the field of view of the camera there are more than three sources, a comparison between the observed and predicted positions in Frame $K'$ offers a unique test of the effect of aberration of light in special relativity. So far, a direct test of aberration of light has been made via observing the parallaxes of distant stars (through the very nonrelativistic motion of Earth orbiting the Sun; e.g., Hirshfeld 2001) or via Earth-based experiments to measure a small gravitational aberration of light (e.g., Kopeikin & Fomalont 2007). A transrelativistic camera will open the window to test this effect in the relativistic regime.

2. A comparison of the observations of the same bright object in two different frames ($K$ and $K'$) at the same intrinsic frequency offers another way to test the principle of special relativity. In nonstandard theories, such as massive electrodynamics, the Doppler factor $D$ may take a form that slightly deviates from the simplest form Equation (1). A tight upper limit on the deviation of the measured (specific) flux at frequency $\nu' = D\nu$ in Frame $K'$ and that at $\nu$ in Frame $K$ (properly correcting for the Doppler boosting effect) would give a tight constraint on the deviation of $D$ from Equation (1), and hence, the violation of the principle of special relativity. No similar test has been performed so far.

### 3. Conclusions and Discussion

If indeed a transrelativistic camera can be launched in the near future as envisaged by the ambitious Starshot project, besides the exciting prospects of sending cameras directly to Alpha Centauri, one can observe the universe with these cameras in flight. We have shown that unique observations can be carried out thanks to several relativistic effects. In particular, due to Doppler blueshift and intensity boosting, one can use a camera sensitive to the optical band to study the near-IR bands. The light aberration effect also effectively increases the field of view of the camera since astronomical objects are packed in the
direction of the camera motion, allowing a more efficient way of studying astronomical objects. These observations also offer unique ways to test the principles of special relativity. We have discussed several examples of astrophysics research directions in which relativistic astronomy can play an important role to advance the field.

According to the “Breakthrough Starshot” website, technology is being developed such that, in the near future, launches of low weight cameras will be possible at a low cost. An ambitious goal of these launches is to send probes to Alpha Centauri. From the above discussion, one can see that another powerful application of the technology would be to launch these cameras in any direction to study astronomical objects as needed. Starshot may become an observatory to launch cameras in desired directions with desired Lorentz factors in order to carry out astronomical observations.

According to Table 1, the relativistic effect is mild at 20% speed of light, which is the target speed of StarChips to visit Alpha Centauri. The maximum Doppler factor $D_{\text{max}}$ is $\sim 1.2247$. Nonetheless, observable relativistic effects would take place. One can already make interesting tests of special relativity by comparing the images and rest-frame fluxes of bright objects as observed in Frames K and K', respectively. If one drops the goal of reaching Alpha Centauri, cameras with even higher Doppler factors may be designed and launched. Doppler factors of 2 and 3 (which give factors of 2 and 3 for the shift of the spectrum) are available at 60% and 80% speeds of light, respectively. More interesting astronomical observations can be carried out at these speeds.

For the Starshot project to send cameras to nearby stellar systems, two main challenges include how to shield cosmic rays during the long journey to the destination and how to transmit image data back to Earth from a large distance. For the probes launched for astronomical observations, these two challenges are somewhat alleviated, since there is no need to operate the camera for a long period of time or for a very long distance. In fact, given the same emitting power from the probe, the transmission signal received from Earth by a $\beta \sim 0.8$ probe at a light-hour distance is much stronger than that by a $\beta \sim 0.2$ probe at a light-year distance. The main challenge, on the other hand, is how to achieve a higher $\beta$ than the nominal value 0.2 (e.g., $\beta \sim 0.6$ needed to study $z \sim 10$ universe at R-band). This requires a laser accelerator with even higher power. For the purpose of studying the high-$z$ universe, another issue may be the limiting magnitude due to the small size of the cameras (even with the proper Doppler boosting). Technology for building large-area, thin and light chips is encouraged. If the obstacles to launch these chips to transrelativistic speed are overcome in the future, unprecedented information about our universe can be obtained in the era of relativistic astronomy.

We thank an anonymous referee for valuable suggestions. This work is partially supported by NASA through grant NNX15AK85G.

Software: Python, https://www.python.org, matplotlib, https://matplotlib.org, GIMP, https://www.gimp.org.

**Appendix A**

**Doppler Transformations**

The standard Doppler transformation relations include (Rybicki & Lightman 1979; Begelman et al. 1984; Zhang 2018):

\[
\begin{align*}
\frac{dt'}{dt} &= D^{-1} \Delta t, \\
\frac{\nu'}{\nu} &= D \nu, \\
E' &= DE, \\
\frac{ds'}{ds} &= \frac{D}{2} E, \\
\frac{dv'}{dv} &= D \Delta v, \\
\frac{d\Omega'}{d\Omega} &= D^{-2} \Delta \Omega, \\
I_{\nu}^1(\nu') &= D^2 I_{\nu}(\nu), \\
n_j(\nu') &= D^2 n_j(\nu), \\
\alpha_{\nu}(\nu') &= D^{-1} \alpha_{\nu}(\nu).
\end{align*}
\]
Here \( dt \) is the time interval differential, \( \nu \) is the frequency, \( E \) the is energy, \( ds \) is the length differential at the source, \( dV \) is the volume differential at the source, \( d\Omega \) is the solid angle differential, \( L_\nu(\nu) \) is the specific intensity of the emitting source at frequency \( \nu \), \( j_\nu(\nu) \) is the specific emission coefficient of the source at frequency \( \nu \), and \( \alpha_\nu(\nu) \) is the specific absorption coefficient at frequency \( \nu \). The primed and unprimed quantities are with respect to Frames \( K \) and \( K' \), respectively.

Astronomy measurements concern the specific luminosity \( L_\nu(\nu) \) and flux \( F_\nu(\nu) \). The transformations of these parameters between the two frames depend on the source properties.

We first consider an isotropic point source. In Frame \( K \), one has

\[
L_\nu(\nu) = \int \int j_\nu(\nu)d\Omega dV = 4\pi j_\nu(\nu),
\]

where \( V \) is the emitting volume of the source, which for an optically thin source (e.g., a quasar) is the entire volume; whereas, for an optically thick source (e.g., a star), it is the volume in the optically thin region (i.e., within the skin-depth of the last Thomson scattering).

In Frame \( K' \), the source moves relativistically. One can write

\[
\frac{dL_\nu(\nu)}{d\Omega'} = \int j'_\nu(\nu')dV' = \int D^2 j_\nu(\nu)DdV = \frac{D^4 L_\nu(\nu)}{4\pi},
\]

where the isotropic condition in Frame \( K \) has been applied for the last equality.

For a point source, all the emitter materials are considered to move toward one direction in Frame \( K' \) (no \( \theta \)-dependence of the source material in terms of motion). An observer mostly cares about the isotropic equivalent specific luminosity, i.e., the specific luminosity, assuming that the source is isotropic in Frame \( K' \). For a point source, the isotropic specific luminosity is simply Equation (8) multiplied by \( \int d\Omega' = 4\pi \), so that

\[
L_{\nu,iso}'(\nu') = D^3 L_\nu(\nu).
\]

The isotropic total luminosity at the frequency \( \nu' \) is

\[
L_{iso}'(\nu') = \nu' L_{\nu,iso}'(\nu') = D^3(\nu L_\nu(\nu)).
\]

For an extended source, one should consider different spatial elements with different angles with respect to the direction of motion and line of sight in Frame \( K' \), the specific luminosity of a unit emitting element at a particular frequency \( \nu' \) reads

\[
L_{\nu}'(\nu') = DL_\nu(\nu).
\]

The luminosity of a unit emitting element at a particular frequency \( \nu' \) reads

\[
L'(\nu') = \nu' L_{\nu}'(\nu') = D^2(\nu L_\nu(\nu)) = D^2 L(\nu).
\]

When considering the observed specific flux and flux of cosmological objects, one should differentiate the emission frequencies at the source \( \nu'_s(\nu'_s) \) and the observed frequencies \( \nu'(\nu') \), which are related through the cosmological expansion factor \((1+z)\). Specifically, one has \( \nu = \nu'_s/(1+z) \) and \( \nu' = \nu'_s/(1 + z) \). We restore these subscripts and still define the observed frequencies as \( \nu \) and \( \nu' \) in Frame \( K \) and \( K' \), respectively. For the case of a point source, in Frame \( K \), the observed specific flux and flux at frequency \( \nu \) for an Earth observer can be written as

\[
F_\nu(\nu) = \frac{(1 + z)L_\nu(\nu)}{4\pi D_L^2},
\]

\[
F'(\nu') = \nu F_\nu(\nu) = \frac{L(\nu)}{4\pi D_L^2},
\]

where \( z \) and \( D_L \) are the redshift and luminosity distance of the object. In Frame \( K' \), the observed specific flux and flux at frequency \( \nu' = D\nu \) for an observer in the probe (the camera) would be

\[
F'_\nu(\nu') = \frac{(1 + z)L_{\nu,iso}'(\nu'_s)}{4\pi D_L^2} = \frac{(1 + z)D^3 L_{\nu}(\nu)}{4\pi D_L^2} = D^3 F_\nu(\nu),
\]

\[
F'(\nu') = \frac{L'(\nu')}{4\pi D_L^2} = \frac{D^2 L(\nu)}{4\pi D_L^2} = D^2 F(\nu).
\]

Similarly, for an extended source, specific flux and flux for an emitting element are given by

\[
F'_\nu(\nu') = DF(\nu),
\]

\[
F'(\nu') = D^2 F(\nu).
\]

### Appendix B

#### Solving the Parameters of Motion with Three Bright Point Sources

The parameters of the motion for a probe include the direction of motion and the dimensionless velocity \( \beta \) (or Lorentz factor \( \Gamma \)).

For a constant velocity probe, one needs to measure the sky positions of at least three point sources in Frame \( K' \) (e.g., \( 1', 2', 3' \) in Figure 2) and compare them against the sky positions of the same three objects in Frame \( K \) (\( 1, 2, 3 \)) in order to determine the parameters of motion. Let us suppose that the direction of motion is at points 0 and 0', respectively, in the two frames.

1. The measured quantities include the angular separations among the three points, i.e., \( \theta_{12}, \theta_{13}, \theta_{23} \) in \( K \) and \( \theta_{1'2'}, \theta_{1'3'}, \theta_{2'3'} \) in \( K' \), and the respective angles among the three sources, i.e., \( \angle 123, \angle 231, \angle 312 \) in \( K \) and \( \angle 1'2', \angle 2'3', \angle 3'1' \) in \( K' \).

2. There are \( 3 + 3 + 2 + 1 = 11 \) unknown quantities in order to solve the problem: the angular separations between the moving direction 0 (or 0') and the three sources, i.e., \( \theta_{01}, \theta_{02}, \theta_{03} \) in \( K \) and \( \theta_{0'1'}, \theta_{0'2'}, \theta_{0'3'} \) in \( K' \); the opening angles between 0 (or 0') and two of the three sources (that of the third can be uniquely determined if the first two are solved), e.g., \( \angle 012, \angle 023 \) in \( K \) and \( \angle 0'1'2', \angle 0'2'3' \) in \( K' \); and the dimensionless velocity \( \beta \) (or Lorentz factor \( \Gamma \)).

3. There are \( 4 + 4 + 3 = 11 \) equations making use of the above mentioned known and unknown parameters: In Frames \( K \) and \( K' \), there are four spherical triangles in each frame. Each triangle gives an independent equation according to the standard spherical geometry. Finally, there are three equations due to the light aberration effect,
i.e., $\cos \theta_{0i}^\prime = (\cos \theta_{0i} + \beta)/(1 + \beta \cos \theta_{0i})$ (i = 1, 2, 3 and $i' = 1', 2', 3'$).

4. As a result, the direction of motion and the dimensionless velocity of motion can be uniquely solved with the observations of three bright point sources in both frames. Such a solution is within the framework of special relativity.

5. After solving the motion, one can add the observational data of the 4th, 5th... celestial objects to the problem to test the validity of the above assumptions, e.g., predicting the positions in Frame $K'$ and use the observations to test the predictions. This leads to a test of the principle of special relativity. The precision of the test would increase with the increasing number of the sources observed.

**ORCID iDs**

Bing Zhang © https://orcid.org/0000-0002-9725-2524

**References**

Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017a, PhRvL, 119, 161101
Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2017b, ApJL, 848, L12

Begelman, M. C., Blandford, R. D., & Rees, M. J. 1984, RvMP, 56, 255
Christian, P., & Loeb, A. 2017, ApJL, 834, L20
Fan, X., Carilli, C. L., & Keating, B. 2006, ARA&A, 44, 415
Gunn, J. E., & Peterson, B. A. 1965, ApJ, 142, 1633
Hirshfeld, A. W. 2001, Parallax: The Race to Measure the Cosmos (New York: Freeman)

Holder, G. P., Haiman, Z., Kaplinghat, M., & Knox, L. 2003, ApJ, 595, 13
Kopeikin, S. M., & Fomalont, E. B. 2007, GrReGr, 39, 1583
Li, L.-X., & Paczyński, B. 1998, ApJL, 507, L59
Loeb, A., & Barkana, R. 2001, ARA&A, 39, 19
Mészáros, P., & Rees, M. J. 2010, ApJ, 715, 967
Metzger, B. D., Martínez, G., Darbha, S., et al. 2010, MNRAS, 406, 2650
Miralda-Escudé, J. 1998, ApJ, 501, 15
Penrose, R. 1959, PCPS, 55, 137
Perley, D. A., Cenko, S. B., Bloom, J. S., et al. 2009, AJ, 138, 1690
Rybicki, G. B., & Lightman, A. P. 1979, Radiative Processes in Astrophysics, 1979 (New York: Wiley-Interscience), 393
Schady, P., Dwelly, T., Page, M. J., et al. 2012, A&A, 537, A15
Steidel, C. C., Shapley, A. E., Pettini, M., et al. 2004, ApJ, 604, 534
Tanvir, K., Ioka, K., Sakamoto, T., & Nakamura, T. 2011, ApJ, 731, 127
Toma, N. R., Fox, D. B., Levan, A. J., et al. 2009, Natur, 461, 1254
Villar, V. A., Guillochon, J., Berger, E., et al. 2017, ApJL, 851, L21
Zhang, B. 2018, The Physics of Gamma-ray Bursts (Cambridge: Cambridge Univ. Press) in press