Michel parameter in 3-3-1 model with three lepton singlets

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Abstract

We show that the mass matrix of electrically neutral gauge bosons in the recently proposed model based on SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U(1)$_X$ group with three lepton singlets [1] has two exact eigenvalues: a zero corresponding the photon mass and the second one equaling the mass of the imaginary component $A_5^\mu$. Hence the neutral non-Hermitian gauge boson $X_\mu^0$ (defined as $\sqrt{2}X_\mu^0 = A_{4\mu} - iA_{5\mu}$) is properly determined. With extra vacuum expectation value of the Higgs field $n_2$, there are mixings among the Standard Model $W$ boson and the extra charged gauge boson $Y$ carrying lepton number 2 (bilepton) as well as among neutral gauge bosons $Z$, $Z'$ and $X^0$. These mixings lead to very rich phenomenology of the model. The leading order of the Michel parameter ($\rho$) has quite special form requiring an equality of the vacuum expectation values in the second step of spontaneous symmetry breaking, namely, $k_1 = k_2$.

PACS numbers: 12.10.Dm, 12.60.Cn, 12.60.Fr, 12.15.Mm

1 Introduction

At present, it is well known that neutrinos are massive that contradicts the Standard Model (SM). The experimental data [2] show that masses of neutrinos are tiny small and neutrinos mix with special pattern in approximately tribimaximal form [3]. The neutrino masses, dark matter and the baryon asymmetry of Universe (BAU) are the facts requiring extension of the SM.

Among the extensions beyond the SM, the models based on SU(3)$_C$ $\otimes$ SU(3)$_L$ $\otimes$ U(1)$_X$ (3-3-1) gauge group [4, 5] have some interesting features including the ability to explain why there exist three families of quarks and leptons [4, 5] and the electric charge quantization [6]. In this scheme the gauge couplings can be unified at the scale of order TeV without supersymmetry [7].

Concerning the content in lepton triplet, there exist two main versions of 3-3-1 models: the minimal version [4] without extra lepton and the model with right-handed neutrinos [5] without exotic charged particles. Due to the fact that particles with different lepton numbers lie in the same triplet, the lepton number is violated and it is better to deal with
a new conserved charge $\mathcal{L}$ commuting with the gauge symmetry $^8$  

$$L = \frac{4}{\sqrt{3}} T_8 + \mathcal{L}.$$  (1.1)  

In the framework of the 3-3-1 models, almost issues concerning neutrino physics are solvable. In the minimal 3-3-1 model where perturbative regime is trustable until 4-5 TeV, to realize idea of seesaw, the effective dimension-5 operator is used $^9$. In regard to the 3-3-1 model with right-handed neutrinos, effective-5 operators are sufficient to generate light neutrino masses. The effective dimension-5 operator may be realized through a kind of type-II seesaw mechanism implemented by a sextet of scalars belonging to the GUT scale $^{10}$. There are some ways to explain smallness of neutrino masses: the radiative mechanism, the seesaw one or their combination - radiative seesaw. The seesaw mechanism is the most easy and elegant way of generating small neutrino masses by using the Majorana neutrinos with mass belonging to GUT scale. With such high scale, the Majorana neutrinos are unavailable for laboratory searches. The existence of sextet is unfavorableness because of lack predictability associated with it. There are attempts to improve the situation.

In the recently proposed model $^{11}$, the authors have introduced three lepton/neutrino singlets and used radiative mechanism to get a model, where the seesaw mechanism is realized at quite low scale of few TeVs. We remind that in the 3-3-1 model with right-handed neutrinos, there are two scalar triplets $\eta, \chi$ containing two electrically neutral components lying at top and bottom of triplets: $\eta^0_1, \chi^0_1$ and $\eta^0_3, \chi^0_3$. In the previous version $^5$, only $\eta^0_1$ and $\chi^0_3$ have vacuum expectation values (VEVs). However, in the new version, the $\eta^0_3$ carrying lepton number 2 has larger VEV of new physics scale. This leads to the mixings in both charged and neutral gauge boson sectors. In the neutral gauge boson sector, the mass mixing matrix is $4 \times 4$. In general, the diagonalizing process for $4 \times 4$ matrix is approximate only. However, in this paper, we show that the matrix has two exact eigenvalues and eigenstates. As a result, the diagonalization is exact!

This paper is organized as follows. In Sect.2 we briefly give particle content of the model. Sect.3 is devoted for gauge boson sector. Mass mixing matrices for charged and neutral gauge bosons are presented. The exact solutions of $4 \times 4$ with some special feature are presented. In Sect.4 we present the $\rho$ parameter of the model and the equality of two VEVs at the second step of spontaneous symmetry breaking. We give the conclusions in the last section - Sect.5.

2 The model

As usual $^5$, the left-handed leptons are assigned to the triplet representation of $SU(3)_L$

$$f^\ell_L = (\nu_\ell, \ell^-, N^c_\ell)^T_L \sim (1, 3, -\frac{1}{3})^c, \ell_R \sim (1, 1, -1, 1),$$  (2.1)

$^1$In this work, the Higgs triplets are labeled as $\rho, \chi, \eta$ instead of $\phi_1, \phi_2, \phi_3$ as in Ref. $^1$.  

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where $\ell = 1, 2, 3 \equiv e, \mu, \tau$. The numbers in bracket are assignment in $SU(3)_C, SU(3)_L, U(1)_X$ and $\mathcal{L}$.

The third quark generation is in triplet

$$Q^3_L = (t, b, T)_L^T \sim \left(3, 3, \frac{1}{3}, -\frac{2}{3}\right), T_R \sim \left(3, 1, \frac{2}{3}, -2\right),$$

$$t_R \sim \left(3, 1, \frac{2}{3}, 0\right), b_R \sim \left(3, 1, -\frac{1}{3}, 0\right).$$

Two first quark generations are in antitriplet

$$Q^i_L = (d_i, -u_i, D_i)_L^T \sim \left(3, 3^*, 0, -\frac{2}{3}\right), i = 1, 2,$$

$$D_{iR} \sim \left(3, 1, -\frac{1}{3}, 2\right), u_{iR} \sim \left(3, 1, \frac{2}{3}, 0\right), d_{iR} \sim \left(3, 1, -\frac{1}{3}, 0\right).$$

In addition to the new two-component neutral fermions present in the lepton triplet $N^c_L \equiv (N^c)_L \equiv (\nu_R)^c$ where $\psi^c = -C\psi^T$, ones introduce new sequential lepton-number-carrying gauge singlets $S = \{S_1, S_2, S_3\}$ with the following number

$$S_i \sim (1, 1, 0, -1).$$

With the above $\mathcal{L}$ assignment the electric charge operator is given in terms of the $U(1)_X$ generator $X$ and the diagonal generators of the $SU(3)_L$ as

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + X. \quad (2.2)$$

Note that in the electric charge operator given in Ref.[1], here, the sign in front of $T_8$ is opposite, because the leptons lie in antitriplet. If so the electrically neutral gauge bosons in the gauge matrix below [see Eq.(3.3)] are $A_4, A_5$ instead of $A_6, A_7$.

In order to spontaneously break the weak gauge symmetry, ones introduce three scalar triplets with VEVs

$$\chi = \left(\chi^0, \chi^-, \chi^0\right)^T \sim \left(1, 3, -\frac{1}{3}, \frac{4}{3}\right); \quad \langle \chi \rangle = (0, 0, n_1)^T, \quad (2.3)$$

$$\eta = \left(\eta^0, \eta^-, \eta^0\right)^T \sim \left(1, 3, -\frac{1}{3}, -\frac{2}{3}\right); \quad \langle \eta \rangle = (0, 0, n_2)^T, \quad (2.4)$$

$$\rho = \left(\rho^+, \rho^0, \rho^+\right)^T \sim \left(1, 3, \frac{2}{3}, -\frac{2}{3}\right); \quad \langle \rho \rangle = (0, k_1, 0)^T. \quad (2.5)$$

With this VEVs structure, as we see below, the simplest consistent neutrino mass, avoiding the linear seesaw contribution is realized [11]. Remind that $n_2$ is a VEV of the lepton number carrying scalar, while all of other VEVs do not.

The spontaneous symmetry breaking follows the pattern

$$SU(3)_L \otimes U(1)_X \xrightarrow{n_1, \rho} SU(2)_L \otimes U(1)_Y \xrightarrow{k_1, \eta} U(1)_Q.$$
The Yukawa Lagrangian of quark sector is as follows \[1, 5\]

\[
\mathcal{L}_{\text{quarks}} = y^T Q^L T_R \chi + y^D_{ij} Q^L D^R \chi^* + y^u_{i\alpha} Q^L u^R \rho + y^d_{3,\alpha} Q^L d^R \eta + y^d_{3,\alpha} Q^L d^R \eta + \text{H.c.}.
\]  

(2.6)

The VEV \( n_1 \) provides masses for exotic quarks, while \( n_2 \) causes mixing among exotic quarks \( T, D_i \) and ordinary ones.

For the lepton sector, we have \[1\]

\[
\mathcal{L}_{\text{leptons}} = y^{\ell}_{ij} f^i_L \rho + y^A_{ij} \epsilon^{abc} (f^i_L)_a (f^j_L)_b (\rho^*)_c + y^{s}_{ij} f^i_L S^j \chi + \text{H.c.}
\]  

(2.7)

where \( i, j = 1, 2, 3 \) is the flavor index and \( a, b, c = 1, 2, 3 \) is the \( \text{SU}(3) \) index. Note that only \( y^A \) is antisymmetric and \( \eta \) does not couple to leptons. The charged leptons get masses the same as in the 3-3-1 model with right-handed neutrino \[5\]. The neutrino mass matrix at the tree level, in the basis \( (\nu_L, N^c, S) \) is given by \[1\]

\[
M_\nu = \begin{pmatrix}
0 & m_D & 0 \\
0 & M & 0 \\
0 & 0 & 0
\end{pmatrix},
\]

(2.8)

where \( m_D = k_1 y^A \), and \( M = n_1 y^s \). At this level, one state \( \nu_1 \) is massless. The one-loop radiative corrections, with gauge bosons in the loop, yield a calculable Majorana mass term \[1\]. Note that the radiative seesaw is implied for the minimal version in Ref.\[12\], where the scalar bilepton is in the loop. The obtained neutrino mass matrix and the charged lepton masses have a strong correlation leading to leptogenesis of the model. However, in this work, we focus our attention only in the gauge boson sector.

### 3 Gauge boson sector

The kinetic term for the scalar fields is

\[
\mathcal{L}_{\text{kin}} = \sum_{H=\chi, \eta, \rho} (D^\mu H)^\dagger (D_\mu H).
\]

(3.1)

The covariant derivative is

\[
D_\mu = \partial_\mu - ig A_{\mu a} T_a - ig' X B_\mu T_g,
\]

(3.2)

where \( X \) is the \( U(1)_X \) charge of the field, \( A_{\mu} \) and \( B_\mu \) are the gauge bosons of \( SU(3)_L \) and \( U(1)_X \), respectively. The above equation applies for triplet as is follows: \( T_a \to \lambda_a/2, T_g \to \)
\[ \lambda_0/2, \] where \( \lambda_i \) are the Gell-Mann matrices, and \( \lambda_9 = \sqrt{\frac{2}{3}} \text{diag}(1, 1, 1) \). The matrix \( A_\mu \equiv \sum_a A^a_\mu \lambda_a \) is

\[
A^\mu = \begin{pmatrix}
A^\mu_3 + \frac{1}{\sqrt{3}} A^\mu_8 & \sqrt{2} W^\mu_{12} & A^\mu_4 - i A^\mu_5 \\
\sqrt{2} W^\mu_{12} & -A^\mu_3 + \frac{1}{\sqrt{3}} A^\mu_8 & \sqrt{2} W^\mu_{67}
\end{pmatrix}.
\]

The charged states are defined as

\[
W^\mu_{12} = \frac{1}{\sqrt{2}} (A^\mu_1 - i A^\mu_2), \quad W^\mu_{67} = \frac{1}{\sqrt{2}} (A^\mu_6 \pm i A^\mu_7).
\]

The mass Lagrangian of gauge fields is given by

\[
\mathcal{L}_{\text{mass}} = \sum_{H=\chi,\eta,\rho} (D^\mu \langle H \rangle)^\dagger (D_\mu \langle H \rangle).
\]

In the charged gauge boson sector, the mass Lagrangian in (3.5) gives one decoupled \( A^\mu_5 \) with mass

\[
m^2_{A_5} = \frac{g^2}{4} (n_1^2 + n_2^2 + k_2^2),
\]

and two others with the mass matrix given in the basis of \( (W^\mu_{12}, W^\mu_{67}) \) as

\[
M_{\text{charged}} = \frac{g^2}{2} \begin{pmatrix}
k_1^2 + k_2^2 & n_2 k_2 \\
n_2 k_2 & n_1^2 + n_2^2 + k_1^2
\end{pmatrix}.
\]

The matrix in Eq. (3.7) has two eigenvalues

\[
\lambda_{1,2} - k_1^2 = \frac{1}{2} (n_1^2 + n_2^2 + k_2^2 \pm \sqrt{\Delta}),
\]

where

\[
\Delta = (n_1^2 + n_2^2 - k_2^2)^2 + 4n_2^2 k_2^2 \\
= (n_1^2 + n_2^2)^2 \left( 1 + \frac{k_2^2}{(n_1^2 + n_2^2)^2} \left[ 2(n_2^2 - n_1^2) + k_2^2 \right] \right).
\]

In the limit \( n_1 \sim n_2 \gg k_1 \sim k_2 \), one has

\[
\sqrt{\Delta} = n_1^2 + n_2^2 + k_2^2 - \frac{2n_1^2 k_2^2}{n_1^2 + n_2^2} + \frac{k_4 n_1^2}{(n_1^2 + n_2^2)^2} \\
- \frac{k_6}{2(n_1^2 + n_2^2)^2} \left[ 1 - \frac{2n_2^2}{n_1^2 + n_2^2} \right] + O(k_8)
\]

\[ (3.10) \]
We will identify the light eigenvalue with square mass of the SM $W$ boson, while the heavy one with that of the new charged gauge boson $Y$ carrying lepton number 2 (bilepton):

\[
m^2_W = \frac{g^2}{2} \lambda_1 = \frac{g^2}{2} \left[ k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2} - \frac{n_1^2 k_2^4}{(n_1^2 + n_2^2)^2} \right] + O(k^6),
\]

\[
m^2_Y = \frac{g^2}{2} \lambda_2 = \frac{g^2}{2} \left[ n_1^2 + n_2^2 + k_1^2 + k_2^2 - \frac{n_1^2 k_2^2}{n_1^2 + n_2^2} + \frac{n_1^2 k_2^4}{(n_1^2 + n_2^2)^2} \right] + O(k^6)
\]

\[
\simeq \frac{g^2}{2} (n_1^2 + n_2^2).
\]

In the limit $n_1 \sim n_2 \gg k_1 \sim k_2$, our result is consistent with that in [1].

Two physical bosons are determined as [13]

\[
W^- = \cos \theta W_{\mu 12} - \sin \theta W_{\mu 67},
\]

\[
Y^- = \sin \theta W_{\mu 12} + \cos \theta W_{\mu 67},
\]

where the $W - Y$ mixing angle $\theta$ characterizing lepton number violation is given by

\[
\tan 2\theta \equiv \epsilon \sim \frac{2n_2 k_2}{n_1^2 + n_2^2 - k_2^2}.
\]

We emphasize that due to $W - Y$ mixing, both the $W$ boson of the SM and the bilepton $Y$ contribute to the neutrinoless double beta decay [14].

Now we turn to the electrically neutral gauge boson sector. Four neutral fields, namely, $A_3^\mu$, $A_8^\mu$, $B^\mu$, $A_4^\mu$ mix

\[
M^2 = \frac{g^2}{4} \begin{pmatrix}
 k_1^2 + k_2^2 & -t \sqrt{\frac{2}{3}} (k_2^2 + 2k_1^2) & \frac{1}{\sqrt{3}} (4(n_1^2 + n_2^2) + (k_1^2 + k_2^2)) \\
\frac{1}{\sqrt{3}} (4(n_1^2 + n_2^2) + (k_1^2 + k_2^2)) & M_{23} & -\frac{1}{\sqrt{3}} n_2 k_2 \\
-\frac{1}{\sqrt{3}} n_2 k_2 & M_{33} & \frac{2t \sqrt{2/27} n_2 k_2}{n_1^2 + n_2^2 + k_2^2}
\end{pmatrix},
\]

where we have denoted $M_{23} \equiv \frac{\sqrt{3}}{3} t [2(n_1^2 + n_2^2) + (2k_1^2 - 2k_2^2)]$, $M_{33} \equiv \frac{2t^2}{27} [(n_1^2 + n_2^2) + (4k_1^2 + k_2^2)]$ and $t$ is given by (see the last paper in Ref. [3])

\[
t = \frac{g'}{g} \frac{3 \sqrt{2} \sin \theta_W (m'_Z)}{\sqrt{3 - 4 \sin^2 \theta_W (m'_Z)}}.
\]

For the matrix in (3.15), using the programming Mathematica9, we get two exact eigenvalues, namely, one massless state

\[
A_\mu = \frac{1}{\sqrt{18 + 4t^2}} \left( \sqrt{3} t A_{3\mu} - t A_{8\mu} + 3 \sqrt{2} B_\mu \right),
\]
which is identified to the photon; and the second eigenvalue defined with

\[ m^2_{A_4} = \frac{g^2}{2} (n_1^2 + n_2^2 + k_2^2), \]

(3.17)

associated with the eigenstate

\[ A'_{4\mu} = \frac{n_2 k_2}{n_1^2 + n_2^2 - k_2^2} A_{3\mu} + \frac{\sqrt{3} n_2 k_2}{n_1^2 + n_2^2 - k_2^2} A_{8\mu} + A_{4\mu}. \]

(3.18)

In a normalized form, the state \( A'_{4\mu} \) is rewritten as

\[ A'_{4\mu} = \frac{t_{2\theta}}{\sqrt{1 + 4t_{2\theta}^2}} A_{3\mu} + \frac{\sqrt{3} t_{2\theta}}{\sqrt{1 + 4t_{2\theta}^2}} A_{8\mu} + \frac{1}{\sqrt{1 + 4t_{2\theta}^2}} A_{4\mu}, \]

(3.19)

where \( t_{2\theta} \equiv \tan \theta \). It is emphasized that, here the angle \( \theta \) has the same value as in the charged gauge boson sector given in (3.14).

Comparing (3.6) with (3.17) we see that two components of \( W_{45} \) have, as expected, the same mass. Hence we can identify

\[ X_\mu^0 = \frac{1}{\sqrt{2}} (A_{4\mu} - iA_{5\mu}) \]

(3.20)

as physical electrically neutral non-Hermitian gauge boson. It is easy to see that this gauge boson \( X_\mu^0 \) carries lepton number 2, hence it is called bilepton gauge boson.

The programming Mathematica9 also gives us two masses of heavy physical bosons:

\[ m^2_{Z_1} = \frac{g^2}{2} \frac{1}{27} \left[ (n_1^2 + n_2^2)(18 + t^2) + k_2^2(18 + 4t^2) + k_1^2(18 + t^2) - \sqrt{\Delta'} \right], \]

(3.21)

\[ m^2_{Z_2} = \frac{g^2}{2} \frac{1}{27} \left[ (n_1^2 + n_2^2)(18 + t^2) + k_2^2(18 + 4t^2) + k_1^2(18 + t^2) + \sqrt{\Delta'} \right]. \]

(3.22)

where

\[ \Delta' = \left[ (n_1^2 + n_2^2 + k_2^2)(18 + t^2) + 2k_1^2(9 + 2t^2) \right] - 108(9 + 2t^2) \left[ n_1^2 k_2^2 + (n_1^2 + n_2^2 + k_2^2) k_1^2 \right] \]

\[ - \left( n_1^2 + n_2^2 \right)^2 (18 + t^2)^2 \left\{ 1 + \frac{2k_2^2}{(n_1^2 + n_2^2)} + \frac{4(9 + 2t^2) k_1^2}{(n_1^2 + n_2^2)(18 + t^2)} \right\} \]

\[ - 108 \frac{(9 + 2t^2)}{(n_1^2 + n_2^2)(18 + t^2)^2} \left[ k_1^2 + \frac{n_1^2 k_2^2}{(n_1^2 + n_2^2)} \right] \]

\[ + \frac{1}{(n_1^2 + n_2^2)^2} \left[ k_1^4 + \frac{4(9 + 2t^2) k_1^2}{(18 + t^2)^2} + \frac{4(9 + 2t^2)(t^2 - 9) k_1^2 k_2^2}{(18 + t^2)^2} \right]. \]

(3.23)

Then

\[ \sqrt{\Delta'} = \left( n_1^2 + n_2^2 \right)(18 + t^2) + k_2^2(18 + t^2) + 2(9 + 2t^2) k_1^2 - 54 \frac{(9 + 2t^2)}{(18 + t^2)} \left( k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2} \right) \]

\[ + \frac{(18 + t^2)}{(n_1^2 + n_2^2)} \left\{ 54(9 + 2t^2) \left( k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2} \right) \right\} \left[ k_2^2(18 + t^2)^2 + 2k_1^2(18 + t^2)(9 + 2t^2) \right] \]

\[ - 54(9 + 2t^2) \left( k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2} \right) \left( \frac{56(9 + 2t^2) k_1^2 k_2^2}{(18 + t^2)^2} \right) + O(k^0). \]

(3.24)
Substituting (3.24) into (3.21) yields the mass of the physical $Z_1$ boson:

$$m_{Z_1}^2 = \frac{g^2}{54} \left\{ \frac{54(9 + 2t^2)}{18 + t^2} \left[ k_1^2 + \frac{n_1^2 k_2^2}{(n_1^2 + n_2^2)} \right] + 3t^2(k_2^2 - k_1^2) \right\}$$

$$- \frac{(18 + t^2)}{(n_1^2 + n_2^2)} \left\{ \frac{54(9 + 2t^2)}{(18 + t^2)^4} \left[ k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2} \right] \left[ k_2^2(18 + t^2)^2 + 2k_1^2(18 + t^2)(9 + 2t^2) \right] \right\} - 54(9 + 2t^2) \left( k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2} \right) - \frac{56(9 + 2t^2)k_1^2 k_2^2}{(18 + t^2)} + \mathcal{O}(k^6).$$

It is emphasized that, at the leading order, there are two terms (in first line of Eq. (3.25)): one is the main mass term of the $Z$ boson and the second one is the unusual difference of square VEVs: $(k_1^2 - k_2^2)$. This term leads to an interesting equality below.

Similarly, for the physical heavy extra neutral gauge boson $Z_2$, one obtains

$$m_{Z_2}^2 = \frac{g^2}{27} \left\{ (n_1^2 + n_2^2 + k_1^2 + k_2^2)(18 + t^2) + \frac{3}{2}t^2(k_1^2 + k_2^2) - \frac{27(9 + 2t^2)}{18 + t^2} \left( k_1^2 + \frac{n_1^2 k_2^2}{(n_1^2 + n_2^2)} \right) \right\}$$

$$+ \frac{(18 + t^2)}{(n_1^2 + n_2^2)} \left\{ \frac{27(9 + 2t^2)}{(18 + t^2)^4} \left[ k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2} \right] \left[ k_2^2(18 + t^2)^2 + 2k_1^2(18 + t^2)(9 + 2t^2) \right] \right\} - 54(9 + 2t^2) \left( k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2} \right) - \frac{28(9 + 2t^2)k_1^2 k_2^2}{(18 + t^2)} + \mathcal{O}(k^6).$$

$$\simeq \frac{g^2(n_1^2 + n_2^2)(18 + t^2)}{27}.$$

Due to the quark family discrimination in the model, $Z'/Z_2$ couples nonuniversally to the ordinary quarks, it gives rise to tree-level flavour-changing neutral current (FCNC) [15]. This would induce gauge-mediated FCNCs, e.g., $b \to s\mu^+\mu^-$ [16], providing a test of the model. We finish this section by remark that the gauge boson mixing here is completely similar to that of the economical 3-3-1 model (ECN331) [13]. However, the key difference is that, here the lepton number carrying VEV $n_2$ is very large ($n_2 \sim n_1 \gg k_1 \sim k_2$), while in the ECN331 model, the lepton number carrying VEV $u$ is very small ($u \ll v$) with $v \simeq 245$ GeV. Within our result, in the figure 1 of Ref. [1], the unphysical gauge field $W_6$ is replaced by physical field $X^0$, while $W^3, W^8, B$ are replaced by physical neutral gauge bosons $Z_1, Z_2$. However, the result is the same.

The diagonalization process of the mass matrix of neutral gauge bosons, the currents and the model phenomenology will be analyzed in details elsewhere.

### 4 Michel parameter $\rho$

As seen from above, the unusual term in the $Z_1$ boson mass will affect the well-determined parameter $\rho$. Thus, for our purpose we consider the $\rho$ parameter - one of the most important
quantities of the SM, having a leading contribution in terms of the $T$ parameter

$$\rho = 1 + \alpha T. \quad (4.1)$$

In the usual 3-3-1 model, $T$ gets contribution from the $Z - Z'$ mixing and the oblique correction $[17]

$$T = T_{ZZ'} + T_{\text{oblique}},$$

where $T_{ZZ'} \simeq \frac{\tan^2 \phi}{\alpha} \left( \frac{m_{Z'}^2}{m_{Z_1}^2} - 1 \right)$ is negligible for $m_{Z'}$ less than 1 TeV, $T_{\text{oblique}}$ depends on masses of the top quark and the SM Higgs boson.

At the tree level, from (3.11) and (3.25) we get an expression for the $\rho$ parameter in the model under consideration

$$\rho = \frac{m_W^2}{c_w^2 m_{Z_1}^2} \left\{ 1 + \frac{t^2 (18 + t^2)(k_1^2 - k_2^2)}{18(9 + 2t^2)(k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2})} \right\} + \frac{n_2^2 k_1^4}{(n_1^2 + n_2^2)^2 (k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2})} + \frac{1}{(n_1^2 + n_2^2)(18 + t^2)^2} \left[ k_2^2 (18 + t^2)^2 + 2k_1^2 (18 + t^2)(9 + 2t^2) \right] \right\} + O(k^6),$$

where we have denoted $s_w \equiv \sin \theta_w$, $c_w \equiv \cos \theta_w$, $t_w \equiv \tan \theta_w$, and so forth. Two terms in the first line of Eq. (4.2) do not depend on perturbative small value $(k/n)$, where $k \approx k_1, k_2, n \approx n_1, n_2$; and they are the leading order of the $\rho$ parameter.

Experimental data [2] show that the $\rho$ parameter is very close with the unit

$$\rho = 1.01031 \pm 0.00011. \quad (4.3)$$

Hence, at the leading order, the following requirement should be fulfilled

$$\frac{(18 + t^2)}{2(9 + 2t^2)c_w^2} \left[ 1 + \frac{t^2 (18 + t^2)(k_1^2 - k_2^2)}{18(9 + 2t^2)(k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2})} \right] = 1. \quad (4.4)$$

Substituting (3.16) into (4.4) yields

$$\frac{2c_w^2 s_w^2}{(3 - 4s_w^2)} \left[ k_1^2 - k_2^2 \right] \left[ k_1^2 + \frac{n_1^2 k_2^2}{n_1^2 + n_2^2} \right] = 0. \quad (4.5)$$

Thus, we obtain the relation

$$k_1 = k_2. \quad (4.6)$$

Note that, in the first time, the equality in (4.6) exists in the model under consideration. This will helpful in our future study. To get constraint from $\rho$ parameter, we should include oblique corrections; and for more details, the reader is referred to [18].
5 Conclusion

In this paper, we have showed that the mass matrix of electrically neutral gauge bosons in the recently proposed 3-3-1 model with three lepton/neutrino singlets \[1\] has two exact eigenvalues and corresponding eigenvectors. With two determined eigenvalues, the $4 \times 4$ mass matrix is diagonalized exactly. Two components of neutral bilepton boson $X^0_\mu$ have the same mass, hence the neutral non-Hermitian gauge boson $X^0_\mu$ is properly determined. This contradicts to previous analysis in Ref.\[1\]. With extra vacuum expectation values of the Higgs fields, there are mixings among charged gauge bosons $W^\pm$ and $Y^\pm$ as well as among neutral gauge bosons $Z, Z'$ and $X^0$. Due to these mixings, the lepton number violating interactions exist in leptonic currents not only in bileptons $Y$ and $X^0$ but also in both SM $W$ and $Z$ bosons. The mixing of gauge bosons in the model under consideration leads to some anomalous couplings of both $W$ and $Z$ bosons, which are subject of our next works.

The scale of new physics was estimated to be in range of few TeVs. With this limit, masses of the exotic quarks are also not high, in the range of few TeVs. The leading order of the Michel parameter requires the equality: $k_1 = k_2$, which is obtained in the first time. The derived relation will ease our future study. The above mentioned mixings lead to new anomalous currents and very rich phenomenology. The model is interesting and deserves further intensive studies.

Acknowledgment

I thank Phung Van Dong for consultation in Mathematica and useful remarks. This research is funded by Vietnam National Foundation for Science and Technology Development (NAFOSTED) under grant number 103.01-2014.51.

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