Non-equilibrium properties of the $S = \frac{1}{2}$ Heisenberg model in a time-dependent magnetic field

V. Turkowski$^a$, V.R. Vieira$^a$ and P.D. Sacramento$^a$

$^a$CFIF, Instituto Superior Tecnico, Av.Rovisco Pais, 1049-001 Lisbon, Portugal

The time-dependent behavior of the Heisenberg model in contact with a phonon heat bath and in an external time-dependent magnetic field is studied by means of a path integral approach. The action of the phonon heat bath is taken into account up to the second order in the coupling to the heat bath. It is shown that there is a minimal value of the magnetic field below which the average magnetization of the system does not relax to equilibrium when the external magnetic field is flipped. This result is in qualitative agreement with the mean field results obtained within $\phi^4$-theory.

PACS: 75.10.Jm, 75.40Gb, 05.70.Ln

Keywords: Spin Systems, Heat Bath, External Fields, Nonequilibrium Dynamics

1. INTRODUCTION

The problem of the behavior of a real quantum system is complicated in many cases by the fact that it is coupled to the environment degrees of freedom. Therefore, the adequate description of the non-equilibrium time-dependent properties of quantum systems in contact with a thermal reservoir and in external time-dependent fields is an interesting and important problem of modern theoretical physics. In particular, this description is necessary for studying stochastic processes in mathematical physics and for studying the effective potentials at finite temperatures in particle physics. In condensed matter physics examples of this type of problems are two-state systems coupled to a dissipative environment, spin systems in a heat bath environment, superconductors in contact with a heat bath and many others.

The standard method for studying quantum systems out of equilibrium is the closed-time-path Green function formalism. For spin systems in general, this method can’t be applied directly, since the commutators of arbitrary spin operators are not $c$-numbers, and for this case a standard Wick theorem does not exist. However, for the particular case of $S = \frac{1}{2}$, when the operators anti-commute, the corresponding path integral technique has been developed. This technique is based essentially on standard field theory methods with the introduction of Majorana fermions. In this paper we study the non-equilibrium properties of the $S = 1/2$ ferromagnetic Heisenberg model in contact with a phonon heat bath by means of this method (the Heisenberg model in equilibrium was studied with this technique in [8,26], for example).

There are several reasons for studying spin systems out of equilibrium. For example, recently discovered big magnetic molecules show relaxation times which are extremely large (see also [31], where the experimental results for the case of external magnetic field are described). Theoretical studies of this phenomenon were performed in [30,12], but more quantitative analysis is still lacking.

The behavior of a spin system in the presence of an external magnetic field flip $\vec{H} \rightarrow -\vec{H}$ is also an interesting problem. In particular, the question whether there exists a critical value of the magnetic field below which the system does not relax to equilibrium and is in a quasi-periodic regime, as it takes place in the case of a $\phi^4$-theory
is interesting.

In this paper the questions mentioned above are studied in the case of the Heisenberg model coupled to a phonon heat bath and in an external time-dependent magnetic field. We consider the interacting spin system in a magnetic field on the mean field level, and the phonon heat bath is studied as a perturbation up to the second order in the spin-phonon coupling. We introduce the formalism specific to the spin problem and apply it to several examples of a single spin in a magnetic field and then take the leading approximation to study interacting spins.

2. FORMALISM

The Heisenberg Hamiltonian with a spin-phonon coupling in an external magnetic field can be written as:

\[
\hat{H} = -\frac{1}{2} \sum_{ij} J_{ij} \vec{S}_i \cdot \vec{S}_j - \sum_i \vec{H}_i \cdot \vec{S}_i + \sum_{qi} c_{qi} \vec{S}_i \cdot \vec{X}_{qi} + \frac{1}{2} \sum_{qi} \left( \frac{\vec{p}_{qi}^2}{m_{qi}} + m_{qi} \omega_{qi}^2 \vec{X}_{qi}^2 \right)
\]

(1)

where \(J_{ij} > 0\) is the nearest neighbor ferromagnetic coupling, \(i, j\) are site coordinates, \(q\) is a bath mode with coordinate \(\vec{X}_{qi}\), momentum \(\vec{p}_{qi}\) and frequency \(\omega_{qi}\) and \(c_{qi}\) is the spin-phonon coupling.

The normalized generating functional of the system is

\[
Z = \frac{\text{Tr} \left[ \hat{T} \exp \left( -i \int \hat{H}_s \, dt \right) \right]}{\text{Tr} \left[ \hat{T} \exp \left( -i \int \hat{H}_b \, dt \right) \right]} \cdot \text{Tr} \left[ \hat{T} \exp \left( -i \int \hat{H}_s \, dt \right) \right] \cdot \text{Tr} \left[ \hat{T} \exp \left( -i \int \hat{H}_b \, dt \right) \right]
\]

(2)

where \(\hat{H}_s\) and \(\hat{H}_b\) are the spin and bath parts of the Hamiltonian, respectively. Initially the spins and the phonons are decoupled. The spin system is at a temperature \(T_i = \frac{1}{\beta_i}\) and the phonon bath is at a temperature \(T_f = \frac{1}{\beta_f}\). At time \(t_i\) the spin-phonon coupling is turned on and it is assumed that the spin system will reach thermodynamic equilibrium with the phonons. In the closed-time path Green function formalism the numerator of the generating functional can be written in this case in terms of the path integral:

\[
\int D\zeta D\vec{X} \exp \left( -i \int_C \left[ \frac{1}{2} \sum \zeta_i \frac{d}{dt} \zeta_i - \frac{i}{2} \sum_{ij} J_{ij} \zeta_i \zeta_j - \sum_i \vec{S}_i \cdot \vec{H}_s \right] \right)
\]

where \(\vec{D}\) is the phonon propagator and the time integration contour \(C\) is presented in Fig.1.

In a Majorana fermion operator \(\zeta_i\) is introduced. In the case of the spin \(\frac{1}{2}\) the following relation holds:

\[
\vec{S}_i = -\frac{i}{2} \zeta_i \times \zeta_i.
\]

(4)
After the integration over the phonon modes, the numerator of the generating functional becomes

$$Z = \int \frac{D\tilde{\zeta} D\tilde{\phi}}{\sqrt{\text{det}(2\pi J)}} \exp \left( -i \int_C dt \left[ -\frac{i}{2} \sum_i \tilde{\zeta}_i \frac{d}{dt} \tilde{\zeta}_i \right. \right.$$  

\begin{align*}
&\left. + \frac{1}{2} \sum_{ij} \tilde{\phi}_i \tilde{J}_{ij} \tilde{\phi}_j - \sum_i \tilde{\phi}_i \tilde{S}_i - \sum_i \tilde{S}_i \tilde{H}_i \right] \right)
\end{align*}

and with the shift \( \tilde{\phi}_i \rightarrow \tilde{\phi}_i - \tilde{H}_i \):

$$Z = \int \frac{D\tilde{\zeta} D\tilde{\phi}}{\sqrt{\text{det}(2\pi J)}} \exp \left( -i \int_C dt \left[ -\frac{i}{2} \sum_i \tilde{\zeta}_i \frac{d}{dt} \tilde{\zeta}_i \right. \right.$$  

\begin{align*}
&\left. + \frac{1}{2} \sum_{ij} (\tilde{\phi}_i - \tilde{H}_i) \tilde{J}_{ij} (\tilde{\phi}_j - \tilde{H}_j) - \sum_i \tilde{\phi}_i \tilde{S}_i \right] \right)
\end{align*}

In the case of the substitution \( \tilde{\phi}_i \rightarrow \tilde{\phi}_i - \tilde{H}_i \), the effective fermion Hamiltonian is

$$\tilde{H}_{eff} = -\sum_i \tilde{\phi}_i \tilde{S}_i = \frac{i}{2} \sum_{imn} \phi_i \varepsilon^{lmn} s^m s^n,$$

where \( l, m, n \) are vector components. The action in the exponent of \( \tilde{\phi}_i \) is quadratic on the \( \zeta \)-variables. Therefore, it can be formally integrated,

$$Z = \int \frac{D\tilde{\phi}}{\sqrt{\text{det}(2\pi J)}} \times e^{-\frac{i}{2} \int_C dt \sum_i (\tilde{\phi}_i - \tilde{H}_i) J_{ij} (\tilde{\phi}_j - \tilde{H}_j)} \prod_i K(\tilde{\phi}_i),$$

where

$$K(\tilde{\phi}_i) = \int D\tilde{\zeta} \exp \left( -i \int_C dt \left[ \frac{1}{2} \tilde{\zeta}_i \frac{d}{dt} \tilde{\zeta}_i + i\tilde{H}_{eff} \right] \right)$$

is a function of the order parameter \( \tilde{\phi} \). The functional \( K(\tilde{\phi}_i) \) is the Helmholtz free energy for a single spin in an external effective field \( \tilde{H}_i \).

If the external field is uniform in space and time, the propagator of the \( \zeta \)-fields

$$G^{ab}_{ij}(t, t') = -i < \hat{T}_z^a(t) \hat{T}_z^b(t') >$$

can be obtained analytically. In the case of an external magnetic field \( \tilde{H} = (0, 0, Hz) \), this can be done most conventionally in the spherical basis with unit vectors \( \hat{a}_\pm = \frac{1}{\sqrt{2}} (\hat{e}_z \pm i\hat{e}_y) \), \( \hat{a}_0 = \hat{e}_z \).

It becomes diagonal and satisfies the equation

$$(i \frac{d}{dt} - \alpha H_z) G^{z}_{\alpha}(t, t') = \delta(t-t'),$$

where \( \alpha = \pm, 0 \) denotes the spherical components. The times \( t \) and \( t' \) take values on the contour \( C \), and therefore \( G \) can be presented as

$$G^{z}_{\alpha}(t, t') = G^{z}_{\alpha}(t-t') \theta_C(t-t') + G^{z}_{\alpha}(t-t') \theta_C(t'-t),$$
where $\theta_C(t' - t)$ is the theta-function on the contour $C$.

The solution of (10) in the case of a time-independent magnetic field $\tilde{H}$ with the Kubo-Martin-Schwinger boundary conditions [31]

$$G_\alpha^>(t - i\beta - t') = -G_\alpha^<(t - t')$$

is

$$G_\alpha^>(t - t') = -i \frac{1}{e^{-\beta\alpha H^z} + 1} e^{-i\alpha H^z(t - t')}, \quad (12)$$

$$G_\alpha^<(t - t') = i \frac{1}{1 + e^{\beta\alpha H^z}} e^{-i\alpha H^z(t - t')}.$$

However, in general, since the field may be both time and space dependent there is no simple analytical solution. Therefore, in practice, the $\zeta$-integration in (10) can be performed using the discrete path integral approximation, with the time contour divided on a finite number $N$ of time intervals [41] and Eq. (10) written as:

$$K(\phi) = \frac{1}{2^{3N/2}} \int \cdots \int e^x T \dot{\zeta}^{-1} \sum_{k=0}^{N-1} \hat{H}_{eff}(t_k) \Delta t \times d\zeta_1 \cdots d\zeta_N, \quad (14)$$

where

$$\hat{A} = \left( \begin{array}{cccc} 0 & -\hat{1} & \hat{i} & \cdots & \hat{i} \\ -\hat{i} & 0 & -\hat{1} & \cdots & \hat{i} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \hat{i} & -\hat{i} & \hat{i} & \cdots & 0 \end{array} \right), \quad \zeta = \left( \begin{array}{c} \zeta_1 \\ \zeta_2 \\ \cdots \\ \zeta_N \end{array} \right),$$

and $\hat{0}$ and $\hat{1}$ are diagonal $3 \times 3$ zero- and unit-matrices, respectively. The discrete inverse propagator in the presence of a general magnetic field $\phi$ is then

$$G^{-1} = -2i \left( \begin{array}{cccc} \hat{C} & -\hat{1} & \hat{i} & \cdots & \hat{i} \\ \hat{i} & \hat{C} & -\hat{1} & \cdots & \hat{i} \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \hat{i} & \hat{i} & \hat{i} & \cdots & \hat{C} \end{array} \right), \quad (15)$$

with

$$\hat{C} = \left( \begin{array}{cc} 0 & \phi_i^z - \phi_i^y \\ -\phi_i^z & -\phi_i^y \end{array} \right) \times \Delta t, \quad (16)$$

The integration over the $\zeta$-fields in (14) gives

$$Z = \frac{1}{2^{3N/2}} \int \frac{D\phi}{\sqrt{det(2\pi J)}}$$

$$\times \exp \left( -\frac{i}{2} \int_C dt \sum_{ij} (\phi_i - \bar{H}_i) J_{ij}^{-1} (\phi_j - \bar{H}_j) + \frac{1}{2} \sum_i Tr \ln G^{-1}(\phi_i) \right), \quad (17)$$

where $G(\phi_i)$ is calculated numerically. The loop expansion on the field $\phi$ in the thermodynamic potential can be performed to take into account the fluctuations of the order parameter [29].

Minimization of the thermodynamic potential with respect to $\phi$ gives the saddle-point equation for the order parameter:

$$\tilde{\phi}_i(t) = \tilde{H}_i(t) + \int_C dt' \tilde{J}_{ij}(t, t') \tilde{M}_j^{(0)}(t'), \quad (18)$$
Taking the time limit properly, one gets

\[ M_j^{(0)}(t') = \frac{\delta}{i\delta \phi_j(t')} \frac{1}{2} Tr \ln iG^{-1}(\vec{\phi}) \]

where the mean-field magnetization \( M_j^{(0)}(t') \) is

\[ M_j^{(0)}(t') = \frac{1}{2} e^{isr} G^{sr}_{jj}(t', t'). \]

Taking the time limit properly, one gets

\[ M_j^{(0)}(t) = \frac{1}{4} e^{isr} (G^{sr}_{jj}(t^+, t) + G^{sr}_{jj}(t^-, t)). \]

The self-consistent system of the equations [18], [19] together with [15]-[16] can be solved numerically, giving the \( \phi \)-dependence of the Green function [19], which allows us to analyze the magnetic properties of the system. In this paper we consider the spatially homogeneous magnetic field case, so the Green’s function for the \( \zeta \)-field can be found analytically in some cases of the field time dependence.

The magnetization \( M_j^{(1)}(t) \) calculated to the order of one-loop in the boson fluctuations is [20],

\[ M_j^{(1)}(t) = \frac{\delta(-\beta F^{(1)})}{\delta H_j^{(0)}} \]

where

\[ \beta F^{(1)} = I(\vec{\phi}) + \frac{1}{2} Tr \ln \bar{D}^{-1} \]

is the free energy calculated to the order of one loop in the effective RPA interaction \( D \):

\[ D^{pm-1}_{sj}(t, t') = J_{sj}^{pm-1}(t, t') - i\chi_{sj}^{pm}(t, t'), \]

with the free magnetic susceptibility

\[ \chi_{sj}^{pm}(t, t') = -\frac{1}{2} \varepsilon_{ll}(t' 2\omega_{l}m_{2}G^{pm}_{ij}(t, t')G^{m_{2}l_{i}}(t', t). \]

evaluated at the saddle-point (here and below the summation over repeated indices is assumed).

The function \( I(\vec{\phi}) \) is the mean-field action

\[ I(\vec{\phi}) = -\frac{i}{2} \int d\phi \sum_{ij} (\delta \phi_i - H_i) \bar{J}^{-1}_{ij}(\vec{\phi} - \vec{H}) \]

\[ -\frac{1}{2} \sum_i Tr \ln iG^{-1}(\phi_i) \]

The expression for \( M^{(1)} \) is

\[ M_j^{(1)}(t) = M_j^{(0)}(t) \]

\[ + [\delta_{js} n^{sr} \delta(t_2 - t_3) + i\chi_{js}^{nm}(t_2, t_4)D^{nm}_{ts}(t_4, t_3)] \]

\[ \times B^{sr}_{m_{1}}(t_3, t_2, t_1) D^{kl}_{ji}(t_1, t), \]

where the third order vertex is

\[ B^{sr}_{m_{1}}(t_3, t_2, t_1) = \frac{i}{2} \frac{\delta \chi_{sr}^{nm}(t_3, t_2)}{\delta \phi_j(t_1)} = \frac{1}{4} e^{\nu_{r} \nu_{l} \nu_{u} n_{m_{1}} n_{2} e^{\nu_{m_{1}} \nu_{1}(t_3, t_2)} \}

\[ \times (\nu_{r}^{\nu_{u} \nu_{l}}(t_1, t_2) G_{\nu_{r} \nu_{l}}^{\nu_{u} \nu_{l}}(t_3, t_2) G_{\nu_{r} \nu_{l}}^{\nu_{u} \nu_{l}}(t_3, t_2))) \]

\[ + G_{\nu_{r} \nu_{l}}^{\nu_{u} \nu_{l}}(t_1, t_2) G_{\nu_{r} \nu_{l}}^{\nu_{u} \nu_{l}}(t_3, t_2) G_{\nu_{r} \nu_{l}}^{\nu_{u} \nu_{l}}(t_2, t_1)). \]

Let us note that the functions \( G, \chi \) and \( B \) are diagonal on site indices. The Green’s functions \( G \) in these expressions are calculated self-consistently by means of equations [18] and [19].
3. ONE SPIN IN A TIME-DEPENDENT MAGNETIC FIELD

As a first example we consider the problem of the behavior of the magnetization of one spin in the presence of the homogeneous time dependent magnetic field $\vec{H}(t) = (0, 0, H_z(t))$. We will consider the effect of performing spin rotations by $\pi$ or $\pi/2$. Let us consider first the case of magnetic field flipping. It is supposed that the system is in equilibrium at initial time $t = t_i$ at temperature $T_i$ and at $t = t_i + \delta$ the magnetic field suddenly changes its direction: $\vec{H} \rightarrow -\vec{H}$.

Also at this time the coupling to the phonons at temperature $T_f$ is switched on, otherwise the relaxation to equilibrium will not take place. The heat bath is necessary to get magnetization relaxation to equilibrium with the magnetic field $-\vec{H}$. Otherwise, the magnetization will precess around the old field $\vec{H}$, as it follows from the equation for magnetization in the case of no spin-phonon coupling:

$$i \frac{d}{dt} \vec{M} = -\vec{M} \times \vec{H}.$$  

Let us get the equation for $\phi_i(t)$ when there is spin-phonon coupling. Since the magnetic field is homogeneous in space, the field $\phi_i(t)$ and the magnetization $\vec{M}_i(t)$ do not depend on site $i$, and the mean-field equation (18) can be written as

$$\phi_i(t) = \vec{H}_i(t) + i \int_{t_i}^{t} dt' \Delta(t, t') \vec{M}_i(t'), \quad(25)$$

This last equation is equivalent to

$$\phi_i(t) = \vec{H}_i(t) - 2 \int_{t_i}^{t} dt' \Im \alpha(t - t') \vec{M}_i(t'), \quad(26)$$

where

$$\Im \alpha(t - t') = -A \int_{0}^{\infty} d\omega \omega^2 e^{-\omega/\omega_c} \sin(\omega(t - t'))$$

This is a simple renormalization of the magnetic field induced by the heat bath and does not depend on the heat bath temperature $T_f$. Therefore, a higher correction beyond the saddle-point level must be considered.

We shall study the behavior of the magnetization $M(1)$ of one spin calculated by expressions
Following expression for the $z$-component of magnetization:

$$M_z^{(1)}(t) \simeq M_z^{(0)}(t) + \frac{i}{4} \int_{C_1+C_2} dt_1 dt_2 \Delta_{nn}(t_1-t_2) e^{n_1 n_2 \epsilon_{kk_1 k_2}} \times [G^{k_2 n_1}(t_1) G^{n_2 r_1}(t_1, t_2) G^{r_2 k_1}(t_2, t) + G^{k_2 r_1}(t_2, t_2) G^{r_2 n_1}(t_2, t_1) G^{n_2 k_1}(t_2, t_1)].$$ \hspace{7cm} (27)

Since we are interested in the correction to the magnetization linear in $\Delta$, the Green's function $G(t_1, t_2)$ in equation (27) can be calculated at $\tilde{\Phi}(t) = -\tilde{H}(t)$. When the magnetic field flipping is applied, the expressions for free Green's functions \[12\] and \[13\] remain correct with the only substitution $H_z = -H$ in the complex exponents. The Fermi-factors in these expressions do not change with the magnetic field flipping, since they are defined by the initial conditions, which define the dynamics of the spins, are changed accordingly. The overall expression does not get modified because both fields are along the same direction.

Assuming, for simplicity, that only the $x$-component of the spin is coupled to the heat bath $(\Delta_{nn} \sim \delta_{xz})$, substituting \[11\], \[12\] and \[13\] in (27) and integrating over $t_1$ and $t_2$ leads to the following expression for the $z$-component of the magnetization:

$$M_z^{(1)}(t) = M_z^{(0)} - A \int_0^\infty d\omega \omega^x e^{-\omega/\omega_c} \times [\{\frac{1}{2} + M_z^{(0)} \coth(\frac{\beta \omega}{2})\} \sin^2(\frac{(\omega + H) (t-t_1)}{2}) \sin^2(\frac{\omega (t-t_2)}{2}) + [\frac{1}{2} + M_z^{(0)} \coth(\frac{\beta \omega}{2})] \sin^2(\frac{(\omega - H) (t-t_2)}{2}) \sin^2(\frac{\omega (t-t_1)}{2})],$$ \hspace{7cm} (28)

where $M_z^{(0)} = \frac{1}{2} \tanh(\frac{\beta H}{2})$ is the initial magnetization. Below we shall use an approximation $e^{-\omega/\omega_c} \approx \theta(\omega_c - \omega)$ in the numerical calculations, for convenience. Obviously, expression (28) is correct for small times $(t-t_i)$, when the perturbation theory is valid, which is the regime where the magnetic field flipping is important. The time dependence of the magnetization for different types of the phonon heat bath is presented in Fig.2. As it follows from this figure the phonon density of states plays an important role in the spin dynamics. The behavior of the magnetization after lowering the temperature for the cases of magnetic field flipping and constant magnetic field is presented in Fig.3 for comparison. The magnetization decreases as $(t-t_i)^2$ at short times in both cases and is a linear function of $t-t_i$ when $(t-t_i) \gg 1/H$ (see \[29\] and asymptotic expression \[29\] for long times).

Let us now consider a qualitative scheme of how to obtain from expression (28) the relaxation of the magnetization to the equilibrium value at $T = T_f$. In the case when $\Delta t \equiv (t-t_i) \gg 1/\omega_c, 1/H_z$ the following relation can be used

$$\frac{\sin^2(\omega + H)(t-t_i)/2}{(\omega + H)^2} \approx \frac{\pi}{4} (t-t_i) \delta(\omega + H^z).$$ \hspace{7cm} (29)

Then, equation (28) can be transformed to

$$M_z^{(1)}(t_i + \Delta t) = M_z^{(0)}(t_i) - \lambda \Delta t (M_z^{(0)}(t_i) - M_z^{(0)}),$$ \hspace{7cm} (30)

where

$$\lambda = \frac{\pi A |H_z|^4}{8 |M_z^{(1)}|^2} e^{-\omega/\omega_c}$$ \hspace{7cm} (31)

and the equilibrium value of the magnetization at $T = T_f$ is

$$M_z^{(1)} = \frac{1}{2} \tanh(\frac{\beta_f H_z}{2}).$$ \hspace{7cm} (32)

In the case when $\Delta t$ is not too large, but much bigger than $1/\omega_c$ and $1/H$, and taking $t_i$ as arbitrary, equation (30) can be transformed into the differential equation

$$\frac{d}{dt} M_z^{(1)}(t) = -\lambda (M_z^{(1)}(t) - M_z^{(1)}),$$ \hspace{7cm} (33)

with the initial boundary condition

$$M_z^{(1)}(t_i) = M_z^{(0)} = \frac{1}{2} \tanh(\frac{\beta_i H_z}{2}).$$ \hspace{7cm} (34)

The solution of (33) is

$$M_z^{(1)}(t) = M_z^{(1)} + (M_z^{(0)} - M_z^{(1)}) e^{-\lambda (t-t_i)}.$$ \hspace{7cm} (35)
As it follows from equation (35), the magnetization approaches its equilibrium value exponentially, and the relaxation time $1/\lambda$ is defined by the final temperature, magnetic field, and by the heat bath parameters.

As another example let us now consider the time-dependence of the magnetization under the effect of a $\pi/2$-rotation of the magnetic field: $\vec{H}_i = (0, 0, H) \rightarrow \vec{H}_f = (0, H, 0)$. The dependence calculated by means of expression (27) in the case of the lowest order spin $x$-component-phonon coupling is presented in Fig. 4. The magnetization is precessing and its direction is changing towards the new direction of the magnetic field in the case of non-zero spin-phonon coupling.

4. THE CASE OF MANY SPINS

In the case of many spins the time relaxation dynamics is complicated by the spin-spin interaction. Let us simplify the problem by taking into account the spin-spin interaction at the mean-field level. The spin Hamiltonian in the case of the magnetic field in $z$-direction can be written as

$$H_{\text{spin}}(t) = -2dJM^{(0)}(t)S^z(t) - H^z(t)S^z(t),$$

where $d$ is dimensionality of the system and the initial magnetization $M^{(0)}$ is defined by the equation

$$M^{(0)} = \frac{1}{2}\tanh\left(\frac{\beta_i(H + 2dJM^{(0)})}{2}\right).$$

Therefore, the problem is reduced to one spin in the effective field $\tilde{H} = H^z + 2dJM^{(0)}$. The formula (25) with the substitution $H^z \rightarrow H^z + 2dJM^{(0)}$ describes the time-dependence of the magnetization of the Heisenberg model after the magnetic field flipping. The time-dependence of the magnetization for the spin system at different values of $dJ$ is presented in Fig. 5. As it follows from this Figure, the time-dependence of the magnetization strongly depends on the relation $|H|/dJ$ keeping the other parameters fixed. Let us analyse the expression (25) at quite long times, when the relation (26) can be applied. Either the first or the second terms in the curled brackets contribute to the integral in (25), when the sign of $\tilde{H} = -|H| + 2dJM^{(0)}$ is negative or positive, respectively. This leads to different, negative or positive, corrections to the magnetization due to the magnetic field flipping. Therefore, the equation $\tilde{H} = 0$ defines the critical magnetic field value $H_{cr}$, below which the magnetic field flipping does not lead to a change of the magnetization sign. Equation $\tilde{H} = 0$ together with (36) gives the expression, which connects $H_{cr}$ with $J$ and $T_i$:

$$\frac{H_{cr}}{dJ} = \tanh\left(\frac{H_{cr}}{T_i}\right).$$

The initial temperature dependence of $H_{cr}$ is presented in Fig. 6. As it follows from this Figure, at low temperatures, the critical value of magnetic field is $dJ$, i.e. it is of order of the ferromagnetic coupling energy, which should be overcome to change the sign of the magnetization of
Mean-field critical temperature of ferromagnetic transition is $T_c = 0.5 dJ$.

Figure 6. Temperature-dependence of $H_{cr}$. The time-dependence of the magnetization for the spin system at different values of $H$ is presented in Fig.7. As it follows from this Figure, the magnetization does not flip when the magnetic field is lower than the critical value $H_{cr} \simeq 0.1915 \omega_c$ at given temperature $T_i = 0.1 \omega_c$ (see Fig.6). In this case the magnetization oscillates with a reasonably large amplitude, similarly to the case of the $\phi^4$-theory \cite{32,33} and does not relax to the overturned magnetization. In the $\phi^4$-theory a vector self-interacting theory is considered. In our case the interaction part is due to the spin interactions.

Finally, in Fig.8 we present the normalized magnetization deviation $\frac{M(0) - M(t)}{[(t - t_i) \omega_c]}$. The rate of change of the magnetization corresponds to the Fermi golden rule for the scattering cross section. After an initial rapid increase the rate converges to a constant valid for intermediate times, as well known. For very large times the perturbation theory is no longer valid. In this regime we have to consider higher order corrections which complicates the solution. This will be considered in a future publication.

5. CONCLUSION

The problem of the time-dependence of the magnetization of a spin system in a time-dependent magnetic field is interesting both from the point of view of its practical applications, and from the theoretical point of view. In this paper we have studied the behavior of the Heisenberg ferromagnet coupled to a heat bath in a time-dependent external magnetic field. In particular, we considered the case when its direction is suddenly changed. As expected the behavior of the system strongly depends on the phonon bath properties.

It has also been shown that there is a critical
value of the magnetic field below which the magnetization does not relax to the equilibrium value after the magnetic field flipping. This situation is analogous to the $\phi^4$-theory case, where also exists a critical value of the external source $A$. In this work we considered a microscopic description for the vector spin system directly instead of using an effective bosonic theory from the start. Representing the spin operators by Majorana fermions we constructed the path integral representation for the spin system. Also, we used a physical way to introduce the relaxation to equilibrium coupling the system to a heat bath. For simplicity we used a phonon heat bath. The spin-phonon coupling can be integrated exactly introducing an effective time-dependent interaction between the spins. Afterwards we bosonized the model via a Hubbard-Stratonovich transformation leading to an effective bosonic theory (after integrating out the Majorana fermions) which has some resemblances with the $\phi^4$-theory but with significant differences like time-dependent coefficients and the introduction of cubic terms (like $\phi^3$) due to the presence of a three field vertex which originates in the three-spin correlation function which does not vanish in general due to the nature of the spin commutation relations.

At the same time there are several questions which remain open like the interpolation between the short time and the long time behaviors. Also, in the problem of quantum spinodal decomposition, the possibility of the formation of a magnetic bubble of the stable phase immersed in a background of the unstable phase and the determination of its radius time-dependence is an interesting open problem. These and some other questions are planned to be studied in the nearest future where a detailed comparison of the Heisenberg model with the $\phi^4$-theory will be carried out.

REFERENCES

1. R.P. Feynman and F.L. Vernon, Ann. Phys. (N.Y.) 24 (1963) 118.
2. A.O. Caldeira and A.J. Leggett, Physica 121A (1983) 587.
3. H.G. Schuster and V.R. Vieira, Phys. Rev. B 34 (1986) 189.
4. J. Zanella and E. Calzetta, cond-mat/0203566.
5. L. Dolan and R. Jackiw, Phys. Rev. D 9 (1974) 3320; A.J. Niemi and G.F. Semenoff, Ann. Phys. (N.Y.) 152 (1984) 105; Y. Fujimoto, R. Grijanis, and H. Nishino, Phys. Lett. 141B (1984) 83; F. Cooper, S. Habib, Yu. Kluger et al, Phys. Rev. D 50 (1994) 2848.
6. A.J. Leggett, S. Chakravarty, A.T. Dorsey et al, Rev. Mod. Phys. 59 (1987) 1; A. Zawadowski, Phys. Rev. Lett. 45 (1980) 211; K. Vlárvar and A. Zawadowski, Phys. Rev. B 28 (1983) 1564; ibid 28 (1983) 1582; ibid 28 (1983) 1596; A. Muramatsu and F. Guinea, Phys. Rev. Lett. 57 (1986) 2337; P.D. Sacramento and P. Schlottmann, Phys. Rev. B 43 (1991) 13294.
7. V.R. Vieira, Phys. Rev. B 39 (1989) 7174.
8. V.R. Vieira and I.R. Pimentel, Phys. Rev. B 39 (1989) 7196.
9. J. Villain, F. Hartmann-Bourton, R. Sessoli, and A. Rettori, Europhys. Lett. 27 (1995) 159.
10. P. Politi, A. Rettori, F. Hartmann-Bourton, J. Villain, Phys. Rev. Lett. 75 (1995) 537.
11. D.A. Garanin, Phys. Rev. B 55 (1997) 3050.
12. N.V. Prokof'ev and P.C.E. Stamp, Phys. Rev. Lett. 80 (1998) 5794.
13. N.V. Prokof'ev and P.C.E. Stamp, Rep. Prog. Phys. 63 (2000) 669.
14. E. Abrahams and T. Tsuneto, Phys. Rev. 152 (1966) 416.
15. H.T.C. Stoof, Phys. Rev. B 47 (1992) 7979.
16. I.J.R. Aitchison and D.J. Lee, Phys. Rev. B 56 (1997) 8303.
17. I.J.R. Aitchison, G. Meticas and D.J. Lee, Phys. Rev. B 62 (2000) 6638.
18. S.G. Sharapov, H. Beck and V.M. Loktev, Phys. Rev. B 64 (2001) 134519.
19. S.G. Sharapov and H. Beck, Phys. Rev. B 65 (2002) 134516.
20. S.M. Alamoudi, D. Boyanovsky, and S.-Yu. Wang, cond-mat/0204232.
21. J. Schwinger, J. Math. Phys. 2 (1961) 407.
22. K.V. Keldysh, Sov. Phys. JETP 20 (1965) 1018.
23. S. Doniach, Phys. Rev. 144 (1966) 382.
24. F.A. Berezin and M.S. Marinov, Ann. of Phys. 104 (1977) 336.
25. V.R. Vieira, Phys. Rev. B 23 (1981) 6043.
26. P.D.S. Sacramento and V.R. Vieira, J. Phys. C 21 (1988) 3099.
27. O. Kahn, “Molecular Magnetism”, VCH, New York, 1993.
28. D. Gatteschi, A. Caneschi, L. Pardi, and R. Sessoli, Science 265 (1994) 1054.
29. A. Caneschi, A. Gatteschi, J. Laugier, R. Ray et al, J. Am. Chem. Soc. 110 (1988) 2795.
30. R. Sessoli, D. Gatteschi, A. Caneschi, and M. Novak, Nature (London) 365 (1993) 141.
31. C. Paulsen and J. G. Park, to be published.
32. F.J. Cao and H.J. de Vega, Phys. Rev. D 63 (2001) 045021.
33. F.J. Cao and H.J. de Vega, Phys. Rev. D 65 (2002) 045012.
34. R. Kubo, J. Phys. Soc. Jpn. 12 (1957) 570; P.C. Martin and J. Schwinger, Phys. Rev. 115 (1959) 1342.