Signals classification based on IA-optimal CNN

Yalun Zhang¹ · Wenjing Yu¹ · Lin He¹ · Lilin Cui¹ · Guo Cheng¹

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Abstract

The versatility of the existing A-optimal-based CNN for solving multiple types of signals classification problems has not been verified by different signals datasets. Moreover, the existing A-optimal-based CNN uses a simplified approximate function as the optimization objective function instead of precise analytical function, which affects the signals classification accuracy to a certain extent. In this paper, a classification method called IA-optimal CNN is proposed. To improve the stability of the classifier, the trace of the covariance matrix of the weights of the fully connected layer is used as the optimization objective function, and the parameter optimization model is established without any simplification of the optimization objective function. In addition, to avoid the difficulty of not being able to obtain the analytical expression formula of the partial derivative of the inverse matrix with regard to the networks parameters, a novel dual function is introduced to transform the optimization problem into an equivalent binary function optimization problem. Furthermore, based on the above analytical solution results, the parameters are updated using the alternate iterative optimization method and the accurate weight update formula is deduced in detail. Five signals datasets are used to test the universality of the IA-optimal CNN in signals classification fields. The performance of IA-optimal CNN is showed, and the experimental results are compared with the existing A-optimal-based classification algorithm. Lastly, the following conclusion is proved theoretically: For the A-optimal-based CNN, the trace of the covariance matrix will continue to decrease and approach a convergence value in the iterative process, but it is impossible for the networks to strictly reach the A-optimal state.

Keywords A-optimal · Convolutional neural networks · Signals classification · Alternate iterative optimization · Dual function

1 Introduction

The classification and recognition of signals data can mine the hidden information in the signals, thereby providing the application basis for various fields. For example, mechanical equipment will vibrate due to the operation of parts when working. The equipment vibration signals under negative conditions and the signals under normal operating conditions often have different characteristics. Therefore, vibration signals analysis for power equipment can distinguish different operating conditions of equipment, thereby helping people to diagnose faults and find the cause [1–14]. Different types of sound signals are closely related to the characteristics of different space or objects. The sound quality classification assessment of public places is an important technical background of noise control technology study. In the field of medical applications, in addition to diagnosing heart diseases with the help of electrocardiogram, the acoustic signals of different heart sounds also reflect the health of the heart. The sound signals of human speaking can be used for voice recognition, which is an important research topic in the field of human–computer interaction [15–21]. The human brain is composed of a large number of neurons, and EEG (Electroencephalogram)
is the electrical signals generated by the activities between these neurons. Relevant research and experiments have proved that there is a specific connection between the EEG potential and the type of brain thinking tasks or the external stimuli received. Moreover, the EEG characteristics caused by different brain thinking tasks and external stimuli are also different. Based on above research, brain-computer interface (BCI) technology can be used to identify the thinking intention of the subject by analyzing EEG. Then, this intention can be converted into instruction signals, and mechanical equipment can help people complete a series of tasks by identifying the instruction signals [22–33].

The purpose of signal classification is to identify the real category of signals with unknown labels. Traditional signals classification methods based on feature extraction can be divided into two steps: (1) Classifier training based on training dataset: The signals features of the training dataset will be extracted, and the extracted feature parameters will be input to the initial classifier. The classifier will continuously optimize the parameters according to the error between the classification results and the actual labels, so that the classifier will converge and reach a certain classification accuracy standard; (2) Signals classification based on testing dataset: The signals features of the testing dataset will be extracted, and the extracted feature parameters will be input to the trained classifier. The classifier will recognize the signals according to the input characteristic parameters. The above classification process is shown in Fig. 1.

Both above two steps need to perform feature extraction from the training dataset and the testing dataset. Feature extraction is a preprocessing of the data. For the signals classification problem, the raw data are a vector composed of many vibration amplitudes or electric potential amplitudes. The raw signals without any processing have long data length and large data scale, so it cannot show clear signals characteristics. Therefore, the raw data are generally not directly input to the classifier. Researchers often first extract some feature parameters that can reflect the characteristics of the signals, and then input these feature parameters into the classifier. The classifier will recognize the type of signals based on the different feature parameters. For example, the peak-to-peak represents the difference between the maximum value and minimum value of the signals, which reflects the fluctuation range of the signals. The kurtosis of the signals measures the steepness of the signals waveform. The kurtosis of the waveform which is similar to the normal distribution function is approximately equal to 3, and the kurtosis of a smoother signals is generally less than 3. The RMS (Root Mean Square) of the signals is used to measure the amount of energy contained in the signals.

However, an inconvenience of the traditional method is that there are many features that can be extracted from the signals, such as peak, peak to peak, RMS, kurtosis, skewness, impulse factor, et al. [33–37]. In different application scenarios, the types and quantities of extracted signals features need to be selected based on historical experience. Therefore, for classification problems without prior knowledge, people do not know in advance which features are helpful for signals classification. At this time, it is necessary to manually complete a large number of feature extractions, and the different signals feature combinations will be input into the classifier to test the usability of this combination. Finally, the available feature combinations will be selected according to the discrimination accuracy of the classifier, which greatly increases labor costs and time costs.

Compared with traditional classification methods, CNN (Convolutional Neural Networks) not only has excellent classification and prediction capabilities, but also can automatically extract signals features by special operations such as convolution and pooling. In this way, it is no longer necessary to test the availability of different feature parameters one by one, which greatly reduces the cost of labor and time. CNN first completes the feature extraction of the input data by convolution and pooling operations. Then, it uses the fully connected layer to complete the data classification. When the output results deviate from the real situation, the networks parameters will be continuously adjusted.
updated to make the networks converges adaptively and finally approach the correct classification conclusion. Therefore, since the advent of CNN, people have begun to carry out CNN-based application research in many fields such as Internet behavior detection, image recognition, and signals classification [38–40]. Currently, the world is facing the risk of a Covid-19 pandemic. CNN has even been applied to the diagnosis of Covid-19, showing the strong vitality and adaptability of CNN in solving new problems in new fields [41].

A strategy of the CNN training is that the networks parameters are constantly updated according to the value of loss function. However, due to the different characteristics of different training datasets, networks parameters often be too sensitive to training samples. In particular, the weight of the fully connected layer is likely to have huge differences due to the changes of input dataset. The general existing CNN parameter training models are usually focus on the value of loss function. In this case, the classifier parameters of the fully connected layer will be highly sensitive to samples. However, an excellent classifier should be stable when different training datasets are input to the algorithm [51]. For a classifier optimization model, in the most ideal case, the stability of the networks under various training sample conditions can be maintained when the loss function value is as small as possible.

A-optimal method is a parameter optimization method to improve the stability of the classifier [42–46]. The optimization goal of the A-optimal method is the trace of the covariance matrix of the classification model parameters. The diagonal elements of the covariance matrix of the classifier weights are the variances of the weights. So, the trace of the covariance matrix of the weights is the sum of the weight variances. The variance of the weight measures the dispersion of the weight. The smaller the sum of the weight variances, the smaller the overall volatility of the weights, and the higher the stability of the classifier parameters. He et al. proposed an image classification algorithm based on A-optimal subspace learning [47], which optimizes the parameters by minimizing the trace of the covariance matrix of the classifier parameters, thereby improving the stability of the classifier. Liu et al. proposed the A-optimal NMF model to train the classifier for image recognition. This method introduces nonnegative matrix factorization and uses the A-optimal method to update the parameters of the linear classifier [48]; Li et al. [49] and Yang et al. [50], respectively, introduce the neighborhood regularization and hessian regularization to improve the classification recognition algorithm based on A-optimal. In order to solve the problem that the existing A-optimal classification method can only deal with single-instance and linearly separable problems, Yin introduced the A-optimal method to CNN for the first time to solve the multi-instance classification question. Moreover, compared with existing A-optimal classification methods, the A-optimal-based CNN method is stable for multi-instance learning and has the higher classification accuracy [51].

In the method proposed by Yin, the trace of the covariance matrix of the weight of the fully connected layer is used as the optimization objective function, and the gradient calculation of objective function with regard to the classifier parameters is based on the above objective function [51]. In this part, Yin simplified the formula of the trace of the covariance matrix and replaced the original optimization objective function with the simplified formula. Therefore, the calculation of the gradient is not based on the precise optimization objective function but on an approximate function, which may affect the final accuracy of the model. In addition, Yin’s literature uses only one signals dataset to test the performance of different algorithm, but the conclusions based on a single type of sample dataset may not be comprehensive and accurate. Finally, Yin introduced the A-optimal method to CNN and gave the implementation steps of the A-optimal CNN method, but did not discuss whether the A-optimal-based CNN can finally reach the A-optimal state when the algorithm training ends.

This paper proposes an Improved A-optimal CNN method. Hereinafter, the “Improved A-optimal CNN” proposed in this paper will be referred to as “IA-optimal CNN” for short. In the following, the IA-optimal CNN architecture including the convolutional layer, the pooling layer, the activation layer, and the fully connected layer will be introduced, and its data processing to complete the multi-instance classification will be described. Thereafter, to maintain the stability of networks under the condition that the loss function value is as small as possible, the trace of the covariance matrix of the weight of the fully connected layer is used as the optimization goal and the goal of networks parameter update is to minimize the value of the trace. In this part, the calculation expression formula of the trace of the covariance matrix will not be simplified in any way, and the precise optimization objective function is used to derive the weight update formula in detail to eliminate the possible classification error caused by the simplified objective function. To avoid the problem that it is difficult to give an analytical solution to the partial derivative of the inverse matrix with regard to convolution kernel, a novel binary dual function is introduced and the optimization of the trace is transformed into an optimization problem. Then, the alternate iterative optimization method is adopted as the weight update strategy. In addition, different types of activation functions are test their ability to express nonlinear characteristics of signals. To test the algorithm performance and the universality in more signals classification fields, five signals datasets are used as
samples datasets. The algorithm result is compared with existing A-optimal classification algorithms. Finally, the following question is discussed: In the iterative process of the A-optimal-based CNN algorithm, whether the networks can finally reach the optimal A-optimal state.

The main improvements of the model proposed in this paper and the corresponding work focus are shown in Fig. 2.

2 IA-optimal CNN method

The proposed IA-optimal CNN method will be introduced in detail in this chapter, which includes three parts: (1) Architecture of IA-optimal CNN: In this section, the architecture of IA-optimal CNN will be showed. IA-optimal CNN includes the input layer, the convolutional layer, the activation layer, the pooling layer, the fully connected layer, and the output layer. The data processing and calculation rules of different layer will be introduced, respectively; (2) Optimization goal: In this section, the trace of the covariance matrix of the weight of the fully connected layer will be used as the optimization goal to train network parameters. To ensure the mathematical rigor of the algorithm, the subsequent formulas will be derived based on the precise optimization objective function without simplification. In addition, a novel dual function is introduced as the optimization strategy to avoid calculation difficulties during the formula derivation, which provide a theoretical basis for network parameter training; (3) Training strategy: In this section, based on the conclusions of the previous section, the original optimization problem is transformed into a binary matrix function optimization problem. The alternating iteration method is used to train the network parameters, and the algorithm steps for parameter updating are explained.

2.1 Architecture of IA-optimal CNN

The IA-optimal CNN includes the input layer, the convolutional layer, the activation layer, the pooling layer, the fully connected layer, and the output layer. The networks is oriented to multi-instance learning and recognition. The input of the networks is a dataset including a number of signals (bag), and the output of the networks is a sequence of classification results of the input signals dataset. In a bag, if the classification result of any instance (a sub-sequence signals) is positive, the classification result of this bag is positive; if the classification result of all the instances is negative, the classification result of this bag is negative. The architecture of IA-optimal CNN is shown in Fig. 2.

Input Layer: The input layer is used to accept input signals. Suppose that the dataset input to the networks

Fig. 2 The main improvements of the model proposed in this paper and the corresponding work focus.
contains \( n \) signals whose sampling points are all \( D \), and each signals \( S_i \) is regarded as a bag. Each signals will be divided into \( l \) equal parts by a sliding window, each sub-sequence signals is an instance, and the number of sampling points in each instance is \( d \). Then, the \( j \)-th sub-sequence signals of the \( i \)-th sample signals can be denoted as \( x_{ij} \). For the convenience of subsequent calculations, the \( l \) sub-sequences are combined by column into a matrix, then each matrix \( X_i \) can represent a bag, denoted as \( X_i = [x_{i1}, x_{i2}, \ldots, x_{il}] \in \mathbb{R}^{d \times l}, \, i = 1, 2, \ldots, n \).

Convolution Layer: The convolution layer performs convolution operations on the input data. The output of convolution layer is a feature map. Suppose that the convolutional layer contains \( m \) convolution kernels, and each convolution kernel \( w_k \) is a column vector, denoted as \( w_k \in \mathbb{R}^{d \times 1}, \, k = 1, 2, \ldots, m \). For the convenience of subsequent calculations, the \( m \) convolution kernels are combined by column into a matrix, then the kernels of the convolution layer can be expressed as a convolution kernel matrix, denoted as \( W = [w_1, w_2, \ldots, w_m] \in \mathbb{R}^{d \times m} \). After the convolution kernels of \( m \) channels are, respectively, convolved with \( X_i \), the \( m \) feature row vectors \( c_i \in \mathbb{R}^{1 \times l} \) \((i = 1, 2, \ldots, n)\) will be obtained. The above row vectors are combined by row into a matrix, then the output of \( X_i \) after the convolution operation can be expressed as an matrix \( C_i \), denoted as:

\[
C_i = \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_m \end{bmatrix} = \begin{bmatrix} w_{1}^{T}x_{i1} & w_{2}^{T}x_{i1} & \cdots & w_{m}^{T}x_{i1} \\ w_{1}^{T}x_{i2} & w_{2}^{T}x_{i2} & \cdots & w_{m}^{T}x_{i2} \\ \vdots & \vdots & \ddots & \vdots \\ w_{1}^{T}x_{il} & w_{2}^{T}x_{il} & \cdots & w_{m}^{T}x_{il} \end{bmatrix} = W^{T}X_i
\]  

(1)

Activation Layer: An activation function \( \sigma(\cdot) \) is used to process the matrix \( C_i \) so that the networks can express nonlinear characteristics of the signals. After using the activation function \( \sigma(\cdot) \) for each element in the matrix \( C_i \), the output of the activation layer is obtained, denoted as \( D_i = \sigma(C_i) \in \mathbb{R}^{m \times l} \).

Pooling Layer: The pooling layer performs down-sampling operations on the feature maps output by the convolutional layer to obtain feature vectors that can be used for classification and discrimination. The pooling size is \( 1 \times 1 \), the row stride is 1, and the maximum pooling is selected as the pooling method. A column feature vector \( z_i \) of size \( m \times 1 \) can be obtained:

\[
z_i = \begin{bmatrix} \max_j \{\sigma(c_1)\} \\ \max_j \{\sigma(c_2)\} \\ \vdots \\ \max_j \{\sigma(c_m)\} \end{bmatrix} = \begin{bmatrix} \max_j \{\sigma(w_1x_{ij})\} \\ \max_j \{\sigma(w_2x_{ij})\} \\ \vdots \\ \max_j \{\sigma(w_mx_{ij})\} \end{bmatrix}
\]  

(2)

Input \( n \) sample signals (\( n \) bags) in a batch, and then \( n \) column feature vectors \( z_i \) can be obtained. The above vectors are combined by column into a matrix, the output of \( n \) sample signals through the pooling layer can be expressed as an output matrix \( Z = [z_1, z_2, \ldots, z_n] \in \mathbb{R}^{m \times n} \).

Fully Connected Layer: The fully connected layer reduces the dimensionality of the output vector of the pooling layer and initially completes the signals classification. The classification of each input sample \( S_i \) is a binary discrimination, and the output is a Boolean scalar \( b_i \). The calculation method of the output \( b_i \) of the fully connected layer is \( b_i = \omega^{T}z_i \), \( \omega \) is the fully connected layer weight. \( \text{'}b_i = 1\text{'}\) means ‘the sample \( S_i \) is judged as positive’, and \( \text{'}b_i = 0\text{'}\) means ‘the sample \( S_i \) is judged as negative.’ In different questions, the meaning of positive or negative is different.

Output Layer: The real category label of sample \( S_i \) is \( y_i \). \( \text{'}y_i = 1\text{'}\) means ‘sample \( S_i \) is a positive signals’ and \( \text{'}y_i = 0\text{'}\) means ‘sample \( S_i \) is a negative signals.’ Then, the loss function for evaluating the classification accuracy of sample \( S_i \) is:

\[
\text{error}(S_i) = (y_i - \omega^{T}z_i)^2
\]  

(3)

If \( n \) samples (\( n \) bags) are input in a batch, the classification label of each sample can be obtained. The \( n \) Boolean scalars are combined into a row vector \( b \), the total loss function is:

\[
\text{error}(S) = \sum_{i=1}^{n} (y_i - \omega^{T}z_i)^2 = \sum_{i=1}^{n} (y_i - b_i)^2
\]  

(4)

In order to avoid the over-fitting, an L2 regularization term is added to the total loss function to limit the weight. Finally, the loss function used in the IA-optimal CNN method is:

\[
\text{loss}(S) = \sum_{i=1}^{n} (y_i - \omega^{T}z_i)^2 + \lambda \|\omega\|^2_2
\]  

\[
= \|y - \omega^{T}Z\|^2_2 + \lambda \|\omega\|^2_2
\]  

(5)

where, \( \|\cdot\|^2_2 \) is the 2-norm, \( \lambda \) is the L2 regularization coefficient, and \( y = (y_1, y_2, \ldots, y_n) \) is the label row vector of \( n \) samples.

IA-optimal CNN needs to be continuously trained to minimize the total loss function value of \( n \) samples input in a batch, so the algorithm parameters need to be
In order to continuously update the weight \( \omega_0 \) that minimizes loss(\( S \)):

\[
\omega_0 = \arg \min_{\omega} \left\{ \text{loss}(S) = \|y - \omega^T Z\|_2^2 + \alpha \|\omega\|_2^2 \right\}
\] (6)

It should be noted that although the architecture of the networks, the form of the loss function, and the selection of the regularization method can be changed, these changes may make it difficult to obtain the analytical expression of the partial derivative of the optimization objective function with regard to the networks’ parameter in the subsequent derivation process. The analytical expression formula of the partial derivative is necessary to achieve A-optimal state, so the networks design has its unique characteristics.

### 2.2 Optimization goal

In order to continuously update the weight \( \omega_0 \) of the fully connected layer and the convolution kernel matrix \( W \), it is necessary to derive the mathematical relationship between loss(\( S \)), \( W \) and \( \omega_0 \). In this section, the weight \( \omega_0 \) will be derived when loss(\( S \)) takes the minimum value. According to the A-optimal criterion, the optimization goal of the convolution kernel matrix \( W \) is to take the minimum value of the trace of the weight covariance matrix \( \text{Tr}(\text{cov}(\omega_0)) \). In addition, a novel dual function is introduced to transform this optimization problem into a problem of finding the minimum value of a binary function. This approach will provide important support for the gradient calculation in Sect. 2.3.

#### 2.2.1 Optimization goal: trace of the covariance matrix of the weight

The loss function can be written as follows:

\[
\text{loss}(S) = \|y - \omega^T Z\|_2^2 + \alpha \|\omega\|_2^2
\]

\[
= (y - \omega^T Z)(y - \omega^T Z)^T + \alpha \omega^T \omega
\]

\[
= y^T y - 2\omega^T Z y + \omega^T Z Z^T \omega + \alpha \omega^T \omega
\] (7)

In order to obtain the weight \( \omega_0 \) when loss(S) takes the minimum value, the derivative of loss(\( S \)) with regard to \( \omega \) is set to 0.

\[
\frac{\partial \text{loss}(S)}{\partial \omega} = -2Zy^T + 2Z Z^T \omega + 2\alpha \omega = 0
\] (8)

The analytical expression of \( \omega_0 \) is

\[
\omega_0 = (Z Z^T + \alpha I)^{-1} Z y^T
\] (9)

For a certain sample set \( \{S\} \), \( y \) is an invariant, so only \( Z \) in formula (9) is a variable. \( Z \) is a function of the convolution kernel matrix \( W \), so \( \omega_0 \) is also a function of the convolution kernel matrix \( W \).

The trace of the covariance matrix of the weight \( \omega_0 \) can be written as follows:

\[
\text{cov}(\omega_0) = \text{cov}\left((Z Z^T + \alpha I)^{-1} Z y^T\right)
\]

\[
= (Z Z^T + \alpha I)^{-1} Z \text{cov}(y^T) Z^T (Z Z^T + \alpha I)^{-1}
\]

\[
= \sigma^2 (Z Z^T + \alpha I)^{-1} Z Z^T (Z Z^T + \alpha I)^{-1}
\]

\[
= \sigma^2 (Z Z^T + \alpha I)^{-1} (Z Z^T + \alpha I)^{-1}
\]

\[
= \sigma^2 \left[(Z Z^T + \alpha I)^{-1} - \alpha (Z Z^T + \alpha I)^{-2}\right]
\] (10)

where \( \sigma^2 I = \text{cov}(y^T) \).

In the literature [51] published by Yin, in order to simplify the calculation, the above formula (10) is simplified, and the following conclusions are used:

\[
\text{cov}(\omega_0) = \sigma^2 (Z Z^T + \alpha I)^{-1}
\] (11)

Moreover, Yin took function (11) as the optimization objective function to complete the subsequent formula derivation. This approach reduces the complexity of formula calculation to a certain extent, but may have a certain impact on the rigor of the conclusion.

In order to ensure the accuracy, this article does not simplify the formula, but complete the subsequent formula derivation based on the expression formula (10). Therefore, the trace of the covariance matrix of the weight \( \omega_0 \) is:

\[
\text{Tr}(\text{cov}(\omega_0)) = \sigma^2 \text{Tr}\left((Z Z^T + \alpha I)^{-1} - \alpha (Z Z^T + \alpha I)^{-2}\right)
\] (12)

In the process of networks training, we hope to find the convolution kernel matrix \( W \) that minimizes \( \text{Tr}(\text{cov}(\omega_0)) \):

\[
W_0 = \arg \min_{W} \{\text{Tr}(\text{cov}(\omega_0))\}
\] (13)

In the following section, the detailed steps to find \( W_0 \) and the formula derivation process will be given.

#### 2.2.2 Optimization strategy: dual function

Since \( \sigma^2 \) is a constant value, so the following conclusion is established:

\[
W_0 = \arg \min_{W} \left\{ Q(Z) = \text{Tr}\left((Z Z^T + \alpha I)^{-1} - \alpha (Z Z^T + \alpha I)^{-2}\right)\right\}
\] (14)

In order to find the convolution kernel matrix \( W_0 \) that minimizes \( \text{Tr}(\text{cov}(\omega_0)) \), a novel dual function \( H(\varphi, Z) \) is introduced, and the expression is as follows:

\[
H(\varphi, Z) = \text{Tr}\left(-\varphi (Z Z^T + \alpha I)^2 \varphi^T + 2 Z Z^T \varphi^T\right)
\] (15)

**Theorem** \( Q(Z) \) is the minimum value of the dual function \( H(\varphi, Z) \) with regard to the matrix \( \varphi \in \mathbb{R}^{n \times m} \).
\[ Q(Z) = \min_{\varphi \in \mathbb{R}^{m \times n}} \left\{ H(\varphi, Z) = \text{Tr} \left( -\varphi(ZZ^T + xI)^2 \varphi^T + 2Z^T \varphi^T \right) \right\} \] (16)

**Proof** In order to obtain the weight matrix \( \varphi_0 \in \mathbb{R}^{n \times m} \) when \( H(\varphi, Z) \) takes the minimum value, the derivative of \( H(\varphi, Z) \) with regard to \( \varphi \) is set to 0.

\[ \frac{\partial H(\varphi, Z)}{\partial \varphi} = -2 \varphi(ZZ^T + xI)^2 + 2Z^T = 0 \] (17)

The analytical expression of \( \varphi_0 \) is:

\[ \varphi_0 = Z^T(ZZ^T + xI)^{-2} \] (18)

Substitute \( \varphi_0 \) into \( H(\varphi, Z) \), the minimum value of \( H(\varphi, Z) \) can be calculated as follows:

\[
H(\varphi_0, Z) = \text{Tr} \left( -\varphi_0(ZZ^T + xI)^2 \varphi_0^T + 2Z^T \varphi_0^T \right) \\
= \text{Tr} \left( -Z^T(ZZ^T + xI)^{-2} Z + 2Z^T(ZZ^T + xI)^{-2} Z \right) \\
= \text{Tr} \left( (ZZ^T + xI)^{-2} ZZ^T \right) \\
= \text{Tr} \left( (ZZ^T + xI)^{-2}(ZZ^T + xI - xI) \right) \\
= \text{Tr} \left( (ZZ^T + xI)^{-1} - x(ZZ^T + xI)^{-2} \right) \\
= Q(Z)
\] (19)

Thus, \( Q(Z) \) is the minimum value of the dual function \( H(\varphi, Z) \) with regard to the matrix \( \varphi \in \mathbb{R}^{n \times m} \). The optimization problem can be equivalently converted to the following problem:

\[
W_0 = \arg \min_{W \in \mathbb{R}^{n \times m}} \left\{ \min_{\varphi \in \mathbb{R}^{n \times m}} H(\varphi, Z) \right\} \\
= \arg \min_{W \in \mathbb{R}^{n \times m}} \left\{ \min_{\varphi \in \mathbb{R}^{n \times m}} \text{Tr} \left( -\varphi(ZZ^T + xI)^2 \varphi^T + 2Z^T \varphi^T \right) \right\}
\] (20)

The optimization problem can be regarded as the optimization of a binary matrix function.

\[
W_0, \varphi_0 = \arg \min_{W \in \mathbb{R}^{n \times m}, \varphi \in \mathbb{R}^{n \times m}} \left\{ H(\varphi, Z) \right\} \\
= \arg \min_{W \in \mathbb{R}^{n \times m}, \varphi \in \mathbb{R}^{n \times m}} \left\{ \text{Tr} \left( -\varphi(ZZ^T + xI)^2 \varphi^T + 2Z^T \varphi^T \right) \right\}
\] (21)

The significance of this step is that the inverse matrix \((ZZ^T + xI)^{-1}\) is included in the analytical solution of \( Tr(\text{cov}(\omega_0)) \). In order to calculate \( \partial Tr(\text{cov}(\omega_0))/\partial w_k \) to obtain the update strategy, the derivative of \((ZZ^T + xI)^{-1}\) with regard to \( w_k \) must be calculated. Due to the existence of the inverse matrix, it is very difficult to directly obtain the analytical solution of \( \partial Tr(\text{cov}(\omega_0))/\partial w_k \). Therefore, the problem of finding the minimum value of \( Tr(\text{cov}(\omega_0)) \) needs to be transformed into the problem of finding the minimum value of its dual function \( H(\varphi, Z) \). By doing so, the inconvenience of obtaining the derivative of the inverse matrix can be avoided.

Based on the conclusions of this section, the training process of the IA-optimal CNN method will be given in the next section, including the update strategy of the network parameter and the detailed derivation process.

### 3 Training strategy

In this section, based on the dual function and its properties introduced in the previous section, the analytical formula of the gradient of the \( H(\varphi, Z) \) with regard to the convolution kernel \( w_k \) will be derived, the minimum value of \( H(\varphi, Z) \) will be found by the alternate iteration method, and the detailed steps of parameter updating based on the gradient descent method will be given.

#### 3.1 Alternate iterative optimization

CNN can continuously adjust the networks parameters according to the loss function value during the training process, so the loss function value of CNN will continuously decrease and gradually converge. The commonly used gradient descent method is to make the networks parameters change in the direction of the gradient. Therefore, the IA-optimal CNN method also uses the gradient descent method to complete the parameter updating.

According to Sect. 2.2.1, the analytical expression of the weight \( \omega_0 \) when \( \text{loss}(S) \) takes the minimum value is \( \omega_0 = (ZZ^T + xI)^{-1}Zy \). In the iterative process, the weight of the fully connected layer can be updated according to this formula. The weight \( \omega_0 \) is a function of the convolution kernel matrix \( W \), so the next round of convolution kernel matrix \( W \) can be adjusted according to the \( \omega_0 \). The optimization goal of \( W \) is to make \( Tr(\text{cov}(\omega_0)) \) as small as possible.

According to Sect. 2.2.2, the problem of finding the minimum value of \( Tr(\text{cov}(\omega_0)) \) has been transformed into the problem of finding the minimum value of \( H(\varphi, Z) \). \( H(\varphi, Z) \) is a binary matrix function, so the alternate iteration method is used for optimization in parameter update. The main steps are as follows:

1. After the end of the \( t \)-th iteration, calculate \( \text{loss}(S) \) and update the weight \( \omega_{t+1} \) of the fully connected layer according to formula (9);
2. Keep the convolution kernel matrix \( W \) unchanged, and update \( \varphi_{t+1} \) according to formula (18).
3. Keep $\phi_{t+1}$ unchanged and update $w_k$ according to the gradient descent method:

$$w_k(t + 1) = w_k(t) - \beta \frac{\partial H(\phi_{t+1}, Z)}{\partial w_k(t)}$$ (22)

where, $\beta$ is the learning rate coefficient, and $W_{t+1}$ can be updated by combining all the $w_k(t + 1)$. In the next section, the calculation process of the partial derivative of the dual function with regard to the convolution kernel $w_k$ will be showed.

### 3.2 Derivative of the dual function with regard to the convolution kernel

The $H(\phi, Z)$ can be written as follows:

$$H(\phi, Z) = Tr\left(-\phi(ZZ^T + z^T \phi^T + ZZ^T \phi^T)\right)$$

$$= Tr\left(-\phi(ZZ^T \phi^T - 2\phi ZZ^T \phi^T - \phi Z^T \phi^T + 2Z^T \phi^T)\right)$$

$$= -Tr(ZZ^T \phi^T \phi) - 2Tr(ZZ^T \phi^T \phi) - \phi Z^T \phi^T$$

$$= -2Tr(ZZ^T \phi^T \phi)$$ (23)

In order to facilitate writing, the following provisions are made:

$$Tr(1) \triangleq Tr(ZZ^T \phi^T \phi)$$

$$Tr(2) \triangleq Tr(ZZ^T \phi)$$

$$Tr(3) \triangleq Tr(Z^T \phi)$$ (24)

Then, formula (23) can be rewritten as:

$$H(\phi, Z) = -Tr(1) - 2Tr(2) + 2Tr(3) - z^2 Tr(\phi \phi^T)$$ (25)

In the following, $\partial Tr(1)/\partial w_k$, $\partial Tr(2)/\partial w_k$ and $\partial Tr(3)/\partial w_k$ will be calculated, respectively.

First of all, the analytic formulas of the elements of the matrices $Z$, $ZZ^T$ and $ZZ^T ZZ^T$ need to calculate. The elements of matrix $Z$ can be written directly:

$$Z(k, i) = \max_{j=1}^{n} \{\sigma(w_k^T x_{ij})\} = \sigma(w_k^T x_{ij}^*)$$

$$\delta_k^* = \arg \max_j \{\sigma(w_k^T x_{ij})\}$$ (26)

where, $Z(k, i)$ represents the element in the $k$-th row and $i$-th column of the matrix $Z$, and other matrix elements are expressed in the same way in the following.

According to (26), the expressions of the row vector of the $k$-th row of the matrix $Z$ and the column vector of the $k'$-th column of the matrix $Z^T$ can be obtained:

$$\begin{align*}
Z(k,:) &= [\sigma(w_k^T x_{1j}^*) \cdots \sigma(w_k^T x_{nj}^*)] \\
Z^T(:,k') &= [\sigma(w_k^T x_{1j}^*) \cdots \sigma(w_k^T x_{nj}^*)]'
\end{align*}$$ (27)

The analytic formula for the elements in the $k$-th row and $k'$-th column of matrix $ZZ^T$ can be given:

$$ZZ^T(k,k') = Z(k,:) \cdot Z^T(:,k') = \sum_{i=1}^{n} \sigma(w_k^T x_{ij}^*) \sigma(w_k^T x_{ij}^*)$$ (28)

the expressions of the row vector of the $k$-th row and the column vector of the $k'$-th column of the matrix $ZZ^T$ can be obtained:

$$\begin{align*}
ZZ^T(k,:) &= \sum_{i=1}^{n} \sigma(w_k^T x_{ij}^*) \sigma(w_k^T x_{ij}^*) \\
ZZ^T(:,k') &= \sum_{i=1}^{n} \sigma(w_k^T x_{ij}^*) \sigma(w_k^T x_{ij}^*)'
\end{align*}$$ (29)

Based on (29), the analytic formula for the elements in the $k$-th row and $k'$-th column of matrix $ZZ^T ZZ^T$ can be given:

$$ZZ^T ZZ^T(k,k') = \sum_{i=1}^{m} \sum_{j=1}^{n} \sigma(w_k^T x_{ij}^*) \sigma(w_k^T x_{ij}^*) \sigma(w_k^T x_{ij}^*) \sigma(w_k^T x_{ij}^*)$$ (30)

So, the analytical expressions of $Tr(1)$, $Tr(2)$, and $Tr(3)$ can be written as following:

$$\begin{align*}
Tr(1) &= Tr(ZZ^T \phi^T \phi) \\
&= Tr\left([ZZ^T ZZ^T](\phi^T \phi)\right) \\
&= \sum_{k=1}^{m} \sum_{k'=1}^{m} ZZ^T(k,k') \cdot \phi^T \phi(k',k)
\end{align*}$$ (31)

$$\begin{align*}
Tr(2) &= Tr(ZZ^T \phi^T \phi) \\
&= Tr\left([ZZ^T](\phi^T \phi)\right) \\
&= \sum_{k=1}^{m} \sum_{k'=1}^{m} ZZ^T(k,k') \cdot \phi^T \phi(k',k)
\end{align*}$$ (32)
\[ Tr(3) = Tr(Z^T \phi^T) \]
\[ = \sum_{k=1}^{m} \sum_{i=1}^{n} Z(k, i) \cdot \phi(i, k) \]
\[ = \sum_{k=1}^{m} \sum_{i=1}^{n} \sigma(w_{k,i}^T x_{i:d}) \cdot \phi(i, k) \]  \hspace{1cm} (33)

In order to calculate the derivative of (31)–(33) with regard to \( w_k \), write the terms containing \( w_k \) and the terms not containing \( w_k \) separately:

\[ Tr(1) = \sum_{k' \neq k} \sum_{k'' \neq k} \left( \sum_{i=1}^{n} \sigma(w_{k,i}^T x_{i:d}) \sigma(w_{k',i}^T x_{i:d}) \cdot \sigma(w_{k'',i}^T x_{i:d}) \right) \cdot \phi^T \phi(k'', k') \]
\[ + \sum_{k' \neq k} \left[ \sum_{i=1}^{n} \sigma(w_{k,i}^T x_{i:d}) \sigma(w_{k',i}^T x_{i:d}) \right] \cdot \phi^T \phi(k', k) \]
\[ + \sum_{k'' \neq k} \left[ \sum_{i=1}^{n} \sigma(w_{k,i}^T x_{i:d}) \sigma(w_{k'',i}^T x_{i:d}) \right] \cdot \phi^T \phi(k'', k) \]
\[ + \sum_{i=1}^{n} \sigma(w_{k,i}^T x_{i:d}) \cdot \phi^T \phi(k, k) \]  \hspace{1cm} (34)

\[ Tr(2) = \sum_{k' \neq k} \sum_{k'' \neq k} \left[ \sum_{i=1}^{n} \sigma(w_{k,i}^T x_{i:d}) \sigma(w_{k',i}^T x_{i:d}) \right] \cdot \phi^T \phi(k'', k') \]
\[ + \sum_{k' \neq k} \left[ \sum_{i=1}^{n} \sigma(w_{k,i}^T x_{i:d}) \sigma(w_{k',i}^T x_{i:d}) \right] \cdot \phi^T \phi(k', k) \]
\[ + \sum_{k'' \neq k} \left[ \sum_{i=1}^{n} \sigma(w_{k,i}^T x_{i:d}) \sigma(w_{k'',i}^T x_{i:d}) \right] \cdot \phi^T \phi(k'', k) \]
\[ + \sum_{i=1}^{n} \sigma(w_{k,i}^T x_{i:d}) \cdot \phi^T \phi(k, k) \]  \hspace{1cm} (35)

\[ Tr(3) = \sum_{k' \neq k} \sum_{i=1}^{n} \sigma(w_{k',i}^T x_{i:d}) \cdot \phi(i, k') + \sum_{i=1}^{n} \sigma(w_{k,i}^T x_{i:d}) \cdot \phi(i, k) \]  \hspace{1cm} (36)

According to the above conclusion, the partial derivative of the dual function \( H(\phi, Z) \) with regard to \( w_k \) can be obtained. For the convenience of writing, the analytical formulas of \( \partial Tr(1)/\partial w_k \), \( \partial Tr(2)/\partial w_k \), and \( \partial Tr(3)/\partial w_k \) will be derived, respectively:

\[ \frac{\partial H(\phi, Z)}{\partial w_k} = -\frac{\partial Tr(1)}{\partial w_k} - 2\sigma \frac{\partial Tr(2)}{\partial w_k} + 2 \frac{\partial Tr(3)}{\partial w_k} \]  \hspace{1cm} (40)

Substitute (37) \sim (39) into (40), the analytical expression of \( \partial H(\phi, Z)/\partial w_k \) can be obtained. Substitute \( \partial H(\phi, Z)/\partial w_k \) into (22), the update formula of the convolution kernel can be obtained.
3.3 Algorithm steps for updating

In Sect. 2.3.1 and 2.3.2, the optimization goal of networks parameter and calculation process of parameter updating is studied. Summarize the training process of IA-optimal CNN as follows: Initialize the networks structure and networks parameters. Input the training dataset and output the classification result. If the number of iterations $p$ is less than the upper limit $P$, the convolution kernel matrix and weight will be, respectively, updated according to the parameter update strategy proposed in Sect. 2.3.2; otherwise, the algorithm ends and the final convolution kernel matrix and weight are output.

Based on this, the training process of IA-optimal CNN can be written as follows:

- **Algorithm:** Training of IA-optimal CNN

- **Input:** Signals samples dataset, $\{S\}$
  - Label vector, $y$
  - L2 regularization coefficient, $\alpha$
  - Learning rate, $\beta$
  - Maximum number of iterations, $P$

- **Step 1:** Initialization
  - Convolution kernel matrix, $W$
  - Weight, $\omega$
  - Iteration counter, $p=1$

- **Step 2:** Input data and obtain the classification result;

- **Step 3:** If $p<P$
  - $p$ times:
    - Update $\omega$ base on formula (9);
    - Update $\phi$ base on formula (18);
    - Update $W$ base on formula (22);
    - To Step 2
  - Else
    - To Step 4

- **Step 4:** Output: $W$ and $\omega$

The output convolution kernel matrix $W$ and weight $\omega$ can be used as the final parameters of IA-optimal CNN, and the trained networks can be used to classify the signals from the testing dataset.

4 Experiments

In this chapter, experiments are carried out with different types of signals data to test the classification ability of the IA-optimal CNN. The classification results are shown, and the algorithm performance are compared with the existing A-optimal-based classification algorithm.

4.1 Experimental data

In order to verify the ability of IA-optimal CNN on classifying different types of signals data, this article uses five datasets: P300 EEG signals dataset, bearing fault signals dataset, English spoken digit signals dataset, arrhythmia signals dataset, gearbox signals dataset. The above five datasets cover a wide range of engineering application fields, and the signals generation mechanisms are diverse, which can be used to test the classification universality of the different method for different signals datasets.

P300 EEG signals is an evoked EEG signals, which is generated when the human brain is stimulated by a small probability event. The EEG data are collected by the character matrix flashing experiment. During the experiment, a character matrix is displayed on the computer screen, and a “target character” is displayed at the same time to let the subjects pay attention to this character. Then, each row or column of the character matrix flashes in a random order. When the row or column contains the target character is flashed, the P300 potential will appear in the EEG signals. The experimental dataset contains 20 channels of collected data from the subjects. The IA-optimal CNN method will recognize the input EEG signals sample and determine whether it contains P300 [22]. The bearing fault signals of mechanical equipment are collected by the sensors installed on the bearing failure test stand, and the sampling frequency is 12 kHz. In order to distinguish the fault signals from the normal signals, the dataset also contains the normal bearing signals waveform at the same sampling frequency. The selected fault type is that outer ring fault magnitude is 0.021 mil, and the fault location is 6 o’clock. The IA-optimal CNN method will be used to recognize the input signals samples and determine whether the signals are a fault signals and then determine whether the bearing has this type of fault [5]. The English spoken digit dataset consists of the audio of 6 speakers, each of whom repeats different digits 50 times, and the speakers all use English pronunciation when speaking. In this experiment, digital 1 audio data are used, and all audio will be converted into acoustic signals, and then input into the networks for signals classification. The IA-optimal CNN method will recognize the input acoustic signals samples and determine whether the signals is the audio signals of the target digit [52]. The arrhythmia signals dataset contains 48 dual-channel Holter signals records, and each record contains the ECG waveform signals with a duration of more than 30 min and expert diagnosis results. The records came from 47 subjects in the arrhythmia laboratory. The IA-optimal CNN method will use the dataset to distinguish between normal beat signals and arrhythmia signals [53]. The gearbox signals dataset collects signals data under different working conditions based on drivetrain dynamic simulator. The gearbox is driven by a motor, and the experiment is carried out on a simulated failure test bench. The rotating speed-system load is set to be 20 Hz–0 V. The IA-optimal CNN method will distinguish the signals under normal operating conditions and the fault
signals under the damage conditions that the gear miss one of feet [54].

The computer brand used for calculation is DELL, and the processor model is Intel(R)Core(TM)i5-8250U CPU @ 1.60 GHz 1.80 GHz. The experiment uses cross-validation. Each rotation takes 80% of the sample set as training samples dataset and the remaining 20% as testing samples dataset. The final metrics is the average of all test results. L2 regularization coefficient \(\beta=10^{-4}\), maximum number of iterations \(P=1000\).

4.2 Performance comparison

The performance comparison section is divided into two parts: In the first part, the performance difference of the IA-optimal CNN method using different activation functions will be compared, and the method with the best performance will be selected as the object for subsequent comparison with other algorithms; In the second part, the IA-optimal CNN will be compared with the existing A-optimal-based classification algorithm to show the performance difference.

4.2.1 Performance comparison of IA-optimal CNN using different activation functions

Based on the same dataset, the accuracy of the IA-optimal CNN using six different activation functions will be compared with each other. The six activation functions are Sigmoid function, Tanh function, ReLu function, Leaky-ReLu function, ELU function, and Cube function used by Yin [51]. The activation functions \(\sigma(x)\) and the corresponding partial derivative \(\frac{\partial \sigma(w^T x_{id})}{\partial w_k}\) can be calculated by the following formula [55, 56]:

\[
\sigma_1(x) = \text{Sigmoid}(x) = \frac{1}{1 + \exp(-x)}
\]

\[
\frac{\partial \sigma_1(w^T x_{id})}{\partial w_k} = \left(1 - \sigma_1(w^T x_{id})\right) \cdot \sigma_1(w^T x_{id}) \cdot x_{id}
\]

\[
\sigma_2(x) = \tanh(x) = \frac{\exp(x) - \exp(-x)}{\exp(x) + \exp(-x)}
\]

\[
\frac{\partial \sigma_2(w^T x_{id})}{\partial w_k} = \left(1 - \sigma_2^2(w^T x_{id})\right) \cdot x_{id}
\]

\[
\sigma_3(x) = \text{ReLu}(x) = \begin{cases} 0(x < 0) \\ x(x \geq 0) \end{cases}
\]

\[
\frac{\partial \sigma_3(w^T x_{id})}{\partial w_k} = \begin{cases} 0(w^T x_{id} < 0) \\ x_{id}(w^T x_{id} \geq 0) \end{cases}
\]

\[
\sigma_4(x) = \text{Leaky ReLu}(x) = \begin{cases} \gamma x(x < 0) \\ x(x \geq 0) \end{cases}
\]

\[
\frac{\partial \sigma_4(w^T x_{id})}{\partial w_k} = \begin{cases} \gamma x_{id}(w^T x_{id} < 0) \\ x_{id}(w^T x_{id} \geq 0) \end{cases}
\]

\[
\sigma_5(x) = \text{ELU}(x) = \begin{cases} e \cdot (e^x - 1)(x < 0) \\ x(x \geq 0) \end{cases}
\]

\[
\frac{\partial \sigma_5(w^T x_{id})}{\partial w_k} = \begin{cases} -e \cdot \exp(-w^T x_{id}) \cdot x_{id}(w^T x_{id} < 0) \\ x_{id}(w^T x_{id} \geq 0) \end{cases}
\]

\[
\sigma_6(x) = \text{Cube}(x) = x^3
\]

\[
\frac{\partial \sigma_6(w^T x_{id})}{\partial w_k} = 3(x^2) \cdot x_{id}
\]

The accuracy comparison results are shown in the following table:

According to the results in the table, the following conclusions can be obtained: (1) When using IA-optimal CNN for signals recognition, arrange the recognition accuracy in descending order, the rank is as follows: arrhythmia signals, English spoken digit signals, bearing failure signals, gearbox signals, P300 signals; (2) When classifying P300 signals, arrhythmia signals and bearing failure signals, the networks using sigmoid activation function have the highest accuracy. When classifying English spoken digit signals and gearbox signals, the networks using tanh activation function have the highest accuracy; (3) When using ReLu or Leaky-ReLu activation functions for classification and recognition, the accuracy rate is generally low. The possible reason is that the ability to extract nonlinear features of the signals is insufficient when ReLu or Leaky-ReLu is used as activation functions based on the IA-optimal CNN structure.

4.2.2 Performance comparison of different A-optimal-based classification algorithms

In this section, the algorithm performance of IA-optimal CNN will be compared with five existing A-optimal-based classification method. Algorithms used for comparison are A-optimal subspace learning method proposed by He [36], the nonnegative matrix factorization method proposed by Liu [37], the method based on neighborhood regularization proposed by Li [38], the method based on Hessian energy Regularization proposed by Yang [50], and the A-optimal CNN method proposed by Yin [51]. The five metrics to evaluate networks performance are accuracy, precision, recall, G-score, and F1-score. The above metrics can be calculated using following equations:
\[
\text{Accuracy} = \frac{TP + TN}{TP + FP + FN + TN} 
\]
\[
\text{Precision} = \frac{TP}{TP + FP} 
\]
\[
\text{Recall} = \frac{TP}{TP + FN} 
\]
\[
G - \text{score} = \sqrt{\text{Precision} \cdot \text{Recall}} 
\]
\[
F1 - \text{score} = \frac{2 \cdot \text{Precision} \cdot \text{Recall}}{\text{Precision} + \text{Recall}} 
\]

where, ‘TP’ means ‘True-Positive’, ‘TN’ means ‘True-Negative’, ‘FP’ means ‘False-Positive’, ‘FN’ means ‘False-Negative’. The experimental results are shown in the following table:

In order to more intuitively show the performance difference between the algorithms, the above data are drawn into the following picture:

According to the results in Tables 2-6 and Figs. 3-7, the following conclusions can be obtained: (1) The five metrics of IA-optimal CNN are superior to other A-optimal classification algorithms. The five metrics measure the performance of different classification methods from different aspects to ensure that the evaluation metric system is as comprehensive as possible. It can be seen that IA-optimal CNN has advantages in all metrics and shows stronger recognition capabilities; (2) IA-optimal CNN has the best classification performance on all signals datasets, which proves that IA-optimal CNN algorithm has good versatility for a variety of signals classification problems. The five different signals datasets used in this paper involve different application fields, and the generation mechanism of signals are also different. Among the existing classification algorithms based on A-optimal, using five types of datasets, IA-optimal CNN ranks first in all the classification metrics, which can show the universality of IA-optimal CNN; (3) From Table 2-6, it can be found that if the datasets are arranged according to the classification accuracy of IA-optimal CNN from largest to smallest, the order is: arrhythmia signals dataset, English spoken digit signals dataset, bearing failure signals dataset, gearbox signals dataset, P300 EEG signals dataset. The accuracy rankings of other A-optimal-based classification algorithms are roughly the same. This sort can show the difficulty degree of distinguishing different types of signals in different datasets. The quality of the data in the arrhythmia signals dataset and the English spoken digit signals dataset are relatively better. For most classification algorithms, different types of signals in these two datasets are easier to distinguish. The signals in the bearing failure signals dataset and gearbox signals dataset are more complex and are easily interfered by noise in the engineering environment, so the classification accuracy is relatively low.

5 Discussion: can the A-optimal-based CNN finally converge strictly to the A-optimal state?

In the above, the IA-optimal CNN framework is given, the update strategy of the networks is studied in detail, and the performance of algorithms is compared with different methods based on different types of datasets.

In the IA-optimal CNN method, the alternate iterative optimization method is used to update the convolution kernel matrix \( W \). The convolution kernel matrix is continuously updated along the gradient direction, so that \( \text{Tr}(\text{cov}(\omega)) \) continues to decrease until it converges. In this case, the new problem will arise: Can this method make the trained networks reach a strict A-optimal state? Can \( \text{Tr}(\text{cov}(\omega)) \) converge to its theoretical minimum in the end?

According to Sect. 2.2.1, when \( \partial(\text{loss}(S))/\partial\omega = 0 \), the expression of \( \omega_0 \) is \( \omega_0 = (ZZ^T + xI)^{-1}ZY^T \). In this formula, only \( Z \) is a variable. Set \( B = (ZZ^T + xI)^{-1} \), then formula (12) can be written as:

\[
\text{Tr}(\text{cov}(\omega)) = \sigma^2(\text{Tr}(B) - z\text{Tr}(B^2)) 
\]
In order to obtain the matrix $B$ when $\text{Tr}(\text{cov}(\omega))$ takes the minimum value, the derivative of $\text{Tr}(\text{cov}(\omega))$ with regard to $B$ is set to 0:

$$
\frac{\partial \text{Tr}(\text{cov}(\omega))}{\partial B} = \sigma^2 (I - 2\sigma B^T) = 0 \tag{53}
$$

The analytical expression of $B_0 \in \mathbb{R}^{m \times m}$ is:

$$
B_0 = \frac{1}{2\sigma} I \tag{54}
$$

The matrix $Z_0Z_0^T \in \mathbb{R}^{m \times m}$ when $\text{Tr}(\text{cov}(\omega))$ takes the minimum value can be calculated:

$$
Z_0Z_0^T = \begin{bmatrix}
\sigma & 0 & \cdots & 0 \\
0 & \sigma & \cdots & 0 \\
\vdots & \vdots & \ddots & 0 \\
0 & 0 & \cdots & \sigma
\end{bmatrix} \tag{55}
$$

### Table 1 Accuracy comparison—six activation functions

| $\sigma(x)$ | P300 EEG signals | Bearing failure signals | English spoken digit signals | Arrhythmia signals | Gearbox signals |
|-------------|------------------|-------------------------|-----------------------------|-------------------|----------------|
| Sigmoid     | 0.778            | 0.851                   | 0.869                       | 0.899             | 0.836          |
| Tanh        | 0.765            | 0.823                   | 0.889                       | 0.879             | 0.837          |
| ReLu        | 0.661            | 0.725                   | 0.811                       | 0.814             | 0.717          |
| Leaky-ReLu  | 0.643            | 0.722                   | 0.784                       | 0.813             | 0.702          |
| ELU         | 0.738            | 0.787                   | 0.830                       | 0.851             | 0.759          |
| Cube [51]   | 0.741            | 0.827                   | 0.844                       | 0.856             | 0.788          |

### Table 2 Algorithm performance comparison/P300 EEG signals

| Method                                    | Accuracy | Precision | Recall | G-score | F1-score |
|-------------------------------------------|----------|-----------|--------|---------|----------|
| A-optimal subspace learning [49:He]       | 0.661    | 0.515     | 0.634  | 0.571   | 0.568    |
| nonnegative matrix factorization [50:Liu] | 0.685    | 0.535     | 0.628  | 0.580   | 0.578    |
| neighborhood regularization [51:Li]       | 0.703    | 0.549     | 0.649  | 0.597   | 0.595    |
| Hessian energy Regularization [52:Yang]   | 0.673    | 0.524     | 0.618  | 0.569   | 0.567    |
| A-optimal CNN [53:Yin]                    | 0.741    | 0.563     | 0.692  | 0.624   | 0.621    |
| IA-optimal CNN                            | 0.778    | 0.587     | 0.720  | 0.650   | 0.647    |

### Table 3 Algorithm performance comparison / bearing failure signals

| Method                                    | Accuracy | Precision | Recall | G-score | F1-score |
|-------------------------------------------|----------|-----------|--------|---------|----------|
| A-optimal subspace learning [49:He]       | 0.701    | 0.489     | 0.665  | 0.570   | 0.564    |
| nonnegative matrix factorization [50:Liu] | 0.796    | 0.627     | 0.748  | 0.685   | 0.682    |
| neighborhood regularization [51:Li]       | 0.753    | 0.525     | 0.750  | 0.628   | 0.618    |
| Hessian energy Regularization [52:Yang]   | 0.724    | 0.506     | 0.769  | 0.624   | 0.610    |
| A-optimal CNN [53:Yin]                    | 0.827    | 0.703     | 0.846  | 0.771   | 0.768    |
| IA-optimal CNN                            | 0.851    | 0.789     | 0.871  | 0.829   | 0.828    |

### Table 4 Algorithm performance comparison/English spoken digit signals

| Method                                    | Accuracy | Precision | Recall | G-score | F1-score |
|-------------------------------------------|----------|-----------|--------|---------|----------|
| A-optimal subspace learning [49:He]       | 0.711    | 0.571     | 0.703  | 0.634   | 0.630    |
| nonnegative matrix factorization [50:Liu] | 0.778    | 0.594     | 0.725  | 0.656   | 0.653    |
| neighborhood regularization [51:Li]       | 0.803    | 0.652     | 0.784  | 0.715   | 0.712    |
| Hessian energy Regularization [52:Yang]   | 0.731    | 0.568     | 0.691  | 0.627   | 0.624    |
| A-optimal CNN [53:Yin]                    | 0.844    | 0.747     | 0.850  | 0.797   | 0.795    |
| IA-optimal CNN                            | 0.889    | 0.833     | 0.900  | 0.866   | 0.865    |
To satisfy this condition, the matrix $Z$ must have the following form:

$$Z_{0}(k, i) = \begin{cases} \sqrt{x(k = i)} \\ 0(k \neq i) \end{cases}$$

(56)

According to Sect. 2.3.2, the elements in $Z_0$ are obtained by maximum pooling, namely:

$$Z_0(k, i) = \max_{j=1}^{n} \{\sigma(w^T_kx_j)\}$$

(57)

Obviously, for any input sample $X_i$, it is almost impossible to find a convolution kernel matrix $W$ that meets the formula (57).

Therefore, the answer to the question in this chapter is: For all the classification algorithms based on A-optimal including IA-optimal CNN and A-optimal CNN proposed by Yin [51]. $Tr(cov(\omega))$ can continuously reduce in the process of algorithm iterations, but the theoretical minimum cannot be achieved, and the networks cannot strictly achieve the best A-optimal state. Nevertheless, the classification algorithm based on A-optimal still makes the value of $Tr(cov(\omega))$ as small as possible when conditions permit, which can ensure the stability of the optimization model to a greater extent.

### 6 Conclusions

This paper proposes an novel IA-optimal CNN method and test it universality to solve different signals classification problems. The main contributions of our works include five aspects: (1) The existing A-optimal-based CNN simplifies the trace of the covariance matrix of the fully connected layer weights and uses the simplified calculation formula as parameter optimization objective function, which affect the classification accuracy to a certain extent. IA-optimal CNN did not perform any simplification on the trace of the covariance matrix and directly used the precise analytical expression of optimization objective function to complete the subsequent formula derivation. This approach makes the results more precise; (2) The A-optimal-based CNN framework uses gradient descent method for network training, so it is necessary to calculate the analytical expression of the partial derivative of the optimization objective function with regard to the network parameters. The precise expression of the trace of the covariance matrix contains the inverse matrix, so it is difficult to directly write the analytical solution of the partial derivative with regard to the network parameter. For this reason, a novel dual function is proposed and the following conclusions are proved mathematically: the minimum value of this dual function is exactly equal to the optimization objective function. According to this conclusion, the original

#### Table 5 Algorithm performance comparison/arrhythmia signals

| Method                                | Accuracy | Precision | Recall | G-score | F1-score |
|---------------------------------------|----------|-----------|--------|---------|----------|
| A-optimal subspace learning [49:He]   | 0.704    | 0.685     | 0.698  | 0.692   | 0.691    |
| nonnegative matrix factorization [50:Liu] | 0.786    | 0.726     | 0.754  | 0.740   | 0.740    |
| neighborhood regularization [51:Li]   | 0.793    | 0.743     | 0.806  | 0.774   | 0.773    |
| Hessian energy Regularization [52:Yang] | 0.774    | 0.700     | 0.780  | 0.739   | 0.738    |
| A-optimal CNN [53:Yin]                | 0.855    | 0.813     | 0.871  | 0.842   | 0.841    |
| IA-optimal CNN                        | 0.899    | 0.880     | 0.884  | 0.882   | 0.882    |

#### Table 6 Algorithm performance comparison/gearbox signals

| Method                                | Accuracy | Precision | Recall | G-score | F1-score |
|---------------------------------------|----------|-----------|--------|---------|----------|
| A-optimal subspace learning [49:He]   | 0.695    | 0.524     | 0.681  | 0.597   | 0.592    |
| nonnegative matrix factorization [50:Liu] | 0.733    | 0.665     | 0.716  | 0.690   | 0.690    |
| neighborhood regularization [51:Li]   | 0.708    | 0.596     | 0.740  | 0.664   | 0.660    |
| Hessian energy Regularization [52:Yang] | 0.717    | 0.632     | 0.702  | 0.666   | 0.665    |
| A-optimal CNN [53:Yin]                | 0.807    | 0.726     | 0.822  | 0.773   | 0.771    |
| IA-optimal CNN                        | 0.837    | 0.787     | 0.847  | 0.816   | 0.816    |
optimization problem is transformed into an equivalent binary matrix function optimization problem, avoiding the difficulty of not being able to obtain the analytical solution of the derivative of the inverse matrix with regard to network parameter; (3) To improve the nonlinear expression ability of the network, the Tanh function and Sigmoid function are used as the activation function instead of Cube function which is used in A-optimal CNN; (4) To further verify the versatility of the A-optimal-based CNN algorithm on signals classification problems, five different signals datasets are used as samples to test the algorithm performance; (5) The following conclusion is proved mathematically: In the iterative process of A-optimal-based CNN, the trace of the covariance matrix will continuously shrink and approach a convergence value, but the theoretical minimum cannot be achieved.

The results of algorithm performance test show that compared with the existing A-optimal classification algorithm, the IA-optimal CNN has the best performance in classifying five datasets. In addition, the accuracy distribution of IA-optimal CNN is relatively more stable and the degree of dispersion is low, which verifies the accuracy and stability of the IA-optimal CNN.
7 Future works

At present, the research on the A-optimal-based CNN network architecture and mathematical principles is still in its infancy, and there are few related theories and application results. The A-optimal-based CNN has specific operating rules, so the further expansion of the network depth is restricted. According to the principle of A-optimal CNN, the dimensionality of the data will reduce once after each round of convolution and pooling. The signals are a one-dimensional vector. If the depth of the convolutional layer and the pooling layer increase, the structure of the input signals must be changed continuously according to the network depth. We have not yet found a data input structure with clear mathematical meaning to ensure the feasibility of this approach. So, further research is necessary for the issues such as “how to expand the network structure to a deeper network,” “how to adjust the data input format or network architecture to adapt to the deeper network,” and “how the training time, algorithm complexity and classification accuracy will change as the network depth increases.” Therefore, the depth expansion of the IA-optimal CNN network, the time-consuming of deep network iteration, the computational complexity, and the accuracy of algorithm will be the focus of the future research.

In addition, based on the specific application background, the structure of IA-optimal CNN can be improved to make it more targeted to solve different types of signals classification problems. For example, when using vibration signals for fault diagnosis, based on full research for the
structural mechanism and mechanical characteristics of the equipment, the IA-optimal-CNN-based fault signals diagnosis networks for different types of power equipment can be designed [57, 58]. When classifying signals based on multiple sensor channels, the signals data from different sensors can be used as the input of different channels of CNN to complete the signals classification based on data fusion[41, 59]. Taking into account the noise interference and data loss that may occur in the process of sensor signals transmission, the model for loss data recovery can also be considered when designing the algorithm. In this case, multiple structures such as SNN, RNN, GRNN, LSTM and SGTM can be combined with A-optimal-based CNN [60–66].

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**Compliance with ethical standards**

**Conflict of interest** All authors declare that they have no conflict of interest.

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![Fig. 11 Accuracy box plots/arrhythmia signals](image1)

![Fig. 12 Accuracy box plots/gearbox signals](image2)
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