Dynamically generated $J^P = 1/2^-$ singly charmed and bottom heavy baryons

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Abstract

Approximate heavy-quark spin and flavor symmetry and chiral symmetry plays an important role in our understanding of the nonperturbative regime of the strong interactions. In this work, using the unitarized chiral perturbation theory, we explore the consequences of these symmetries in the description of the interactions between the ground-state singly charmed (bottom) baryons and the pseudo Nambu-Goldstone bosons. In particular, by fixing the only parameter in the theory to reproduce the $\Lambda_b(5912)$ or the $\Lambda_c(2595)$, we predict a number of dynamically generated states, which are contrasted with those of other approaches and available experimental data. In anticipation of future lattice chromodynamics simulations, we calculate the corresponding scattering lengths and compare them with the existing predictions from a $O(p^3)$ chiral perturbation theory study.

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I. INTRODUCTION

In recent years, heavy-flavor hadron physics has yielded many surprising results and attracted a lot of attention due to the intensive worldwide experimental activities, such as BABAR [1], Belle [2, 3], CLEO [4], BES [5], LHCb [6], and CDF [7]. The discoveries and confirmations of the many XYZ particles have established the existence of exotic mesons made of four quarks, such as the \(Z_c(3900)\) [8, 9] and the \(Z(4430)\) [10, 11], and aroused great interest in the theoretical and lattice QCD community to understand their nature, though no consensus has been reached yet (see, e.g., Ref. [12]).

Different from the case of heavy meson states, no similar exotic states have been firmly established in the heavy-flavor baryon sector, partly due to the fact that their production is more difficult. Up to now, there are only a few experimental observations of excited charmed and bottom baryons (see Ref. [13] for a recent and comprehensive review). In the bottom baryon sector, the LHCb collaboration has reported two \(\Lambda_b\) excited states, the \(\Lambda_b(5912)\) and the \(\Lambda_b(5920)\) [14], with the latter being recently confirmed by the CDF Collaboration [15]. In the charmed baryon sector, a number of excited states have been confirmed by various experiments, including the \(\Lambda_c(2595)\), the \(\Xi_c(2790)\), the \(\Lambda_c(2628)\), and the \(\Xi_c(2819)\) [16]. The spin-parity of the first two states and the last two states are assumed to be \(1/2^-\) and \(3/2^-\), respectively, according to quark model predictions.

The conventional picture is that these states are the orbital excitations of the corresponding ground states. There are, however, different interpretations, namely, they are dynamically generated states from the interaction between the ground-state charmed (bottom) baryons with the pseudo Nambu-Goldstone bosons (and other coupled channels) [17–21]. The idea of dynamically generated states is an old one but recently has received a lot of attention. It has been quite successful in solving some long-standing difficulties encountered in hadron spectroscopy, e.g., the nature of the \(\Lambda(1405)\) or the lowest-lying scalar nonet (see, e.g., Ref. [22] for a recent review). In the charmed and bottom baryon sector, various approaches have been adopted to study final state interactions and resulting dynamically generated states, including the so-called unitarized chiral perturbation theory (UChPT) [17], hidden-gauge symmetry inspired approaches [20, 21, 23–27], and heavy-quark symmetry inspired approaches [18, 19, 28–32].

In the present work, we choose the UChPT to study the interaction between the ground-state charmed (bottom) baryons and the pseudo Nambu-Goldstone bosons using the leading order Chiral Lagrangians. In the charmed baryon case, our study differs from that of Ref. [17] in the following...
two respects. First, we adopt different regularization schemes to regularize the loop function in the UChPT. Second, to identify dynamically generated states, we search for poles on the complex plane instead of examining speed plots. Furthermore, we extend the UChPT to study the bottom baryons.

The paper is organized as follows. In Sec. II we briefly recall the UChPT in studies of the interactions between the pseudo Nambu-Goldstone mesons and the ground-state singly charmed (bottom) baryons. Our main results are presented in Sec. III, followed by a short summary and outlook in Sec. IV.

II. THEORETICAL FRAMEWORK

In this section, we briefly recall the essential ingredients of the unitarized chiral perturbation theory (UChPT). There are two building blocks in the UChPT: a kernel provided by chiral Lagrangians up to a certain order and a unitarization procedure. The kernel is standard except in the sector where baryons or heavy hadrons are involved, where non relativistic chiral Lagrangians are frequently used. Common unitarization procedures include the Bethe-Salpeter equation method [33–42], the N/D method [43], as well as the inverse amplitude method (IAM) [44–50]. In the present work, we choose to work with relativistic chiral Lagrangians and in the Bethe-Salpeter equation framework.

The Bethe-Salpeter equation can be written schematically as

$$T = V + VGT,$$

where $T$ is the unitarized amplitude, $V$ the potential, and $G$ the one-loop 2-point scalar function.

In the context of the UChPT, the integral Bethe-Salpeter equation is often simplified and approximated to be an algebraic equation with the use of the on-shell approximation [34, 35]. This approximation works very well. See Ref. [51] for a recent study of off-shell effects in the UChPT and early references on this subject.

The leading order interaction between a singly charmed baryon of the ground-state sexet and anti-triplet and a pseudoscalar meson of the pion octet is provided by the following chiral Lagrangian [17, 52]:

$$\mathcal{L} = \frac{i}{16f_0^2} \text{Tr}(\overline{H}_3(x)\gamma^\mu[H_3(x), [\phi(x), (\partial_\mu\phi(x))]_{-+}]_{++}) + \frac{i}{16f_0^2} \text{Tr}(\overline{H}_{[6]}(x)\gamma^\mu[H_{[6]}(x), [\phi(x), (\partial_\mu\phi(x))]_{-+}]_{++}),$$

(2)
where $f_0$ is the pseudoscalar decay constant in the chiral limit, $\phi$ collects the pseudoscalar octet, $H_{[3]}$ and $H_{[6]}$ collect the charmed (bottom) baryons, respectively:

$$\phi = \sqrt{2} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix},$$ (3)

$$H_{[3]} = \begin{pmatrix} 0 & \Lambda^+_c & \Xi^+_c \\ -\Lambda^+_c & 0 & \Xi^0_c \\ -\Xi^+_c & -\Xi^0_c & 0 \end{pmatrix},$$ (4)

$$H_{[6]} = \begin{pmatrix} \sqrt{2}\Sigma^{++}_c & \Sigma^+_c & \Xi^+_c \\ \Sigma^+_c & \sqrt{2}\Sigma^0_c & \Xi^0_c \\ \Xi^+_c & \Xi^0_c & \Omega^0_c \end{pmatrix}. $$ (5)

The $H$’s for the corresponding ground-state bottom baryons can be obtained straightforwardly by replacing the charm quark content by the bottom counterpart.

Expanding the Lagrangian of Eq. (2) up to two pseudoscalar fields, one obtains the interaction kernel needed to describe the $\phi(p_2) B(p_1) \rightarrow \phi(p_4) B(p_3)$ process, where $p_i$’s are the four momenta of the respective particles,

$$V = \frac{C_{ij}^{(I,S)}}{4f_0^2} \gamma^\mu (p_2^\mu + p_4^\mu) \approx \frac{C_{ij}^{(I,S)}}{4f_0^2} (E_2 + E_4),$$ (6)

where $C_{ij}^{(I,S)}$ is the Clebsch-Gordan coefficients given in the Appendix. In deriving the final form of $V$, we have assumed that the three momentum of a baryon is small compared to its mass. This is a valid assumption since in the present study we are only interested in the energy region close to the threshold of the respective coupled channels.

The loop function $G$ in the Bethe-Salpeter equation has the following simple form in 4 dimensions:

$$G = i \int \frac{d^4q}{(2\pi)^4} \frac{2M_B}{[(P - q)^2 - m^2_B + i\epsilon][q^2 - M_B^2 + i\epsilon]}.$$ (7)

This loop function is divergent and needs to be properly regularized. In principle, one can either adopt the dimensional regularization scheme or the cutoff scheme. In Ref. [53], a so-called heavy quark symmetry (HQS) inspired regularization scheme has been suggested, which manifestly satisfies both the chiral power counting and the heavy-quark spin and flavor symmetry up to $1/M_H$,.
where $M_H$ is a generic heavy-hadron mass. In the present work, we adopt the HQS regularization scheme, which reads:

$$G_{HQS} = G_{MS} - \frac{2\bar{M}}{16\pi^2} \left( \log \left( \frac{\bar{M}^2}{\mu^2} \right) - 2 \right) + \frac{2m_{sub}}{16\pi^2} \left( \log \left( \frac{m^2}{\mu^2} \right) + a \right), \quad (8)$$

$$G_{MS}(s, M^2, m^2) = \frac{2M}{16\pi^2} \left[ \frac{m^2 - M^2 + s}{2s} \log \left( \frac{m^2}{M^2} \right) - \frac{q}{\sqrt{s}} \left( \log[2q\sqrt{s} + m^2 - M^2 - s] + \log[2q\sqrt{s} - m^2 + M^2 - s] \right. \right.
$$

$$\left. - \log[2q\sqrt{s} + m^2 - M^2 + s] - \log[2q\sqrt{s} - m^2 + M^2 + s] \right) + \left( \log \left( \frac{M^2}{\mu^2} \right) - 2 \right). \quad (9)$$

In the above equations, $m_{sub}$ is a generic pseudoscalar meson mass, which can take the value of $m_\pi$ in the $u, d$ flavor case or an average of the pion, the kaon, and the eta masses in the $u, d, s$ three flavor case. $\bar{M}$ is the chiral limit value of the charmed or bottom baryon masses. In the present study, we use the averaged anti-triplet and sexet charmed or bottom baryon masses given in Table I instead. The difference is of higher chiral order. Clearly the HQS inspired regularization method is a straightforward extension of the minimal subtraction scheme, which, in spirit, is very similar to the extended-on-mass-shell scheme [54]. In our present work, for the sake of comparison, we also present results obtained with the cutoff regularization scheme, where

$$G_{cut} = \int_0^\Lambda q^2 dq \left( \frac{E_M + E_m}{2E_M E_m} \right) \frac{2M}{s - (E_M + E_m)^2 + i\epsilon}, \quad (10)$$

with $E_M = \sqrt{q^2 + M^2}$, and $E_m = \sqrt{q^2 + m^2}$. In the UChPT framework, one usually replaces the underlined $-2$ of Eq. (9) by a subtraction constant to approximate unknown short-range or higher-order interactions. In the following, we refer to this regularization scheme as the $\overline{MS}$ scheme.

In Fig. I the loop function $G$ calculated in different regularization schemes are compared with each other. The subtraction constants or cutoff values have been fixed by reproducing the $\Lambda_b(5912)$ (left panel) or the $\Lambda_c(2595)$ (right panel). In calculating the loop function $G$, the pseudo scalar meson mass is fixed at that of the pion $m = 138$ MeV and the renormalization scale in the dimensional regularization methods is fixed at $\mu = 1$ GeV. It is clear that the loop functions of both the HQS scheme and the cutoff scheme seem to satisfy heavy-quark symmetry to a few percent, while the naive $\overline{MS}$ scheme breaks strongly the symmetry, consistent with the finding of...
FIG. 1. (color online) Loop function $G(M)$ as a function of the heavy-hadron mass $M$ in different regularization schemes: HQS (the heavy quark symmetry inspired scheme), $\overline{\text{MS}}$ (the modified minimal subtraction scheme), CUT (the cutoff scheme), and HH (the exact heavy-quark limit). The subtraction constants or cutoff values have been fixed by reproducing the $\Lambda_b(5912)$ (left panel) or the $\Lambda_c(2595)$ (right panel). In calculating the loop function $G$, the pseudoscalar meson mass is fixed at that of the pion $m = 138$ MeV and the renormalization scale in the dimensional regularization methods is fixed at $\mu = 1$ GeV.

TABLE I. Numerical values of isospin and SU3-multiplet averaged masses, the pion decay constant $f_\pi$, and the SU(3) averaged pseudoscalar meson decay constant $f_0$ (in units of MeV) [16].

| $\bar{3}$ M$^\circ$ | $\Lambda_c$ | $\Xi_c$ | $\bar{6}$ M$^\circ$ | $\Sigma_c$ | $\Xi'_c$ | $\Omega_c$ |
|---------------------|-------------|---------|---------------------|----------|---------|---------|
| $M^\circ$            | 2408.5      | 2469.5  | 2453.5              | 2576.8   | 2695.2  |
| $M_b$                | 2286.5      | 2543.9  | 2453.5              | 2576.8   | 2695.2  |
| $M^\circ$            | 5732.8      | 5893.9  | 5813.4              | 5926     | 6071    |
| $M_b$                | 5619.4      | 5789.5  | 5893.9              | 5926     | 6071    |

$m_\pi$, $m_K$, $m_\eta$, $f_\pi$, $f_0 = 1.17f_\pi$

Ref. [53]. To be conservative, in the following study of dynamically generated charmed (bottom) baryons, we shall present the results calculated in both schemes and view the differences between them as an estimate of inherent theoretical uncertainties.
III. RESULTS AND DISCUSSIONS

The only unknown parameter in the UChPT is related to the regularization of the loop function \( G \), i.e., the subtraction constant \( a \) in the dimensional regularization scheme or the cutoff value \( \Lambda \) in the cutoff scheme. Conventionally, in the latter method one often chooses a cutoff of the order of 1 GeV (the chiral symmetry breaking scale). Requiring the \( G \) function evaluated at threshold to be equal in both methods, one can fix a “natural” value for the subtraction constant. In most cases, the above mentioned prescription allows one to assign some of the dynamically generated states to their experimental counterparts. Once the identification is done, one can slightly fine-tune \( \Lambda \) or \( a \) so that the dynamically generated state coincides with its experimental counterpart and then use the so-obtained \( \Lambda \) or \( a \) to make predictions. We follow the same line of argument in the present work.

It should be noted that unlike the heavy meson sector, the present LQCD simulations of charmed [55–59] or bottom [57, 59, 60] baryons still focus on the ground states with the exception of Ref. [61] and Ref. [62], where excited triply charmed and bottom states were studied, respectively. Future LQCD simulations of the excited singly charmed and bottom baryons will be extremely valuable to test the predictions of the present work and those of other studies.

A. Dynamically generated bottom baryons

In Refs. [19, 20], the \( \Lambda_b(5912) \) is found to be dynamically generated. In Ref. [19], the dominant coupled channel is identified as \( \bar{B}N \), while in Ref. [20] it is identified as \( \bar{B}^*N \). In our approach, this state appears naturally as a \( \Sigma_b \pi \) state. It is useful to point out the major differences among the approaches of Ref. [19], Ref. [20], and the present work. The kernel looks similar in all the three cases. However, they differ in the number of coupled channels included and how the transition amplitudes between different channels are obtained. In Ref. [19] the transition amplitudes between different coupled channels are obtained by invoking SU(6) symmetry and heavy-quark spin symmetry, while in Ref. [20], they are obtained through vector meson exchange or pion exchange. The number of coupled channels considered is the largest in Ref. [19], while it is the smallest in our approach. In other words, we only consider the minimum number of channels needed to construct the leading order chiral Lagrangians. In addition, up to the order we are working at, the

\[ \text{We note that preliminary results on the excited state spectroscopy of singly and doubly charmed baryons have recently been presented in conferences [63, 64].} \]
TABLE II. Dynamically generated bottom baryons of $J^P = 1/2^-$. The subtraction constant is fixed in a way such that the $\Lambda_b(5912)$ mass is produced to be 5912 MeV with $\alpha = -16.2$. All energies are in units of MeV and $(S, I)^M$ denotes (strangeness, isospin)$^{SU(3)\text{multiplet}}$.

| Pole position | $(S, I)^M$ | Main channels (threshold) | Exp. [16] |
|---------------|------------|--------------------------|-----------|
| (6082, $-i57$) | $(0, 1)^{[3]}$ | $\Xi_b K(6285.1)$ |           |
| (5922, 0)    | $(0, 0)^{[3]}$ | $\Xi_b K(6285.1)$ |           |
| (5895, 0)    | $(-1, \frac{1}{2})^{[3]}$ | $\Lambda_b K(6115.0)$ |           |
| (6100, $-i32$) | $(-1, \frac{1}{2})^{[3]}$ | $\Xi_b \eta(5928.5)$ |           |
| (6202, 0)    | $(-2, 0)^{[3]}$ | $\Xi_b K(6285.1)$ |           |
| (6201, 0)    | $(1, \frac{1}{2})^{[6]}$ | $\Sigma_b K(6308.6)$ |           |
| (5967, $-i9$) | $(0, 1)^{[6]}$ | $\Sigma_b \eta(5951.0)$ |           |
| (6223, $-i14$) | $(0, 1)^{[6]}$ | $\Sigma_b \eta(6360.9)$ |           |
| (5912, 0)    | $(0, 0)^{[6]}$ | $\Sigma_b \eta(5951.0)$ | $\Lambda_b(5912)$ |
| (6307, $-i12$) | $(0, 0)^{[6]}$ | $\Xi'_b K(6421.6)$ |           |
| (6213, $-i25$) | $(-1, \frac{3}{2})^{[6]}$ | $\Sigma_b K(6308.6)$ |           |
| (5955, 0)    | $(-1, \frac{1}{2})^{[6]}$ | $\Sigma_b K(6308.6)$ |           |
| (6105, $-i17$) | $(-1, \frac{1}{2})^{[6]}$ | $\Xi'_b \pi(6064.0)$ |           |
| (6381, $-i25$) | $(-1, \frac{1}{2})^{[6]}$ | $\Omega_b K(6566.6)$ |           |
| (6377, $-i58$) | $(-2, 1)^{[6]}$ | $\Omega_b \pi(6209.0)$ |           |
| (6180, 0)    | $(-2, 0)^{[6]}$ | $\Xi'_b K(6421.6), \Omega_b \eta(6618.9)$ |           |

The $^{[3]}$ multiplet and the $^{[6]}$ multiplet do not mix. Therefore, one needs to be careful when comparing our predictions with those of Refs. [19, 20].

To study the interaction between the ground-state bottom baryons and the pseudoscalar mesons, we fix the cutoff value or the subtraction constant in such a way that the mass of the $\Lambda_b(5912)$ is produced to be 5912 MeV. Broken SU(3) chiral symmetry then predicts a number of additional resonances or bound states as shown in Tables II and III. It can be seen that in addition to the
TABLE III. The same as Table II, but obtained in the cutoff regularization scheme with $\Lambda = 2.17$ GeV

| Pole positions $(S, I)^M$ | Main channels | Exp. [16] |
|--------------------------|--------------|----------|
| $(6083, -i72)$ $(0, 1)^3$ | $\Xi_b K(6285.1)$ |          |
| $(5903, 0)$ $(0, 0)^3$ | $\Xi_b K(6285.1)$ |          |
| $(5895, 0)$ $(-1, 1/2)^3$ | $\Lambda_b \bar{K}(6115.0)$ |          |
| $(6103, -i29)$ $(-1, 1/2)^7$ | $\Xi_b \pi(5927.5), \Xi_b \eta(6337.4), \Lambda_b \bar{K}(6115.0)$ |          |
| $(6198, 0)$ $(-2, 0)^3$ | $\Xi_b \bar{K}(6285.1)$ |          |
| $(6221, 0)$ $(1, 1/2)^6$ | $\Sigma_b K(6308.6)$ |          |
| $(5966, -i9)$ $(0, 1)^6$ | $\Sigma_b \pi(5951.0)$ |          |
| $(6234, -i19)$ $(0, 1)^6$ | $\Sigma_b \eta(6360.9)$ |          |
| $(5912, 0)$ $(0, 0)^6$ | $\Sigma_b \pi(5951)$ | $\Lambda_b (5912)$ |
| $(6305, -i16)$ $(0, 0)^6$ | $\Xi'_b K(6421.6)$ |          |
| $(6227, -i27)$ $(-1, 3/2)^6$ | $\Sigma_b \bar{K}(6308.6)$ |          |
| $(5951, 0)$ $(-1, 1/2)^6$ | $\Sigma_b \bar{K}(6308.6)$ |          |
| $(6094, -i12)$ $(-1, 1/2)^6$ | $\Xi'_b \pi(6064.0)$ |          |
| $(6356, -i33)$ $(-1, 1/2)^6$ | $\Omega_b K(6566.6)$ |          |
| $(6350, -i52)$ $(-2, 1)^6$ | $\Omega_b \pi(6209.0)$ |          |
| $(6148, 0)$ $(-2, 0)^6$ | $\Xi'_b \bar{K}(6421.6)$ |          |

$\Lambda_b(5912)$, both regularization schemes generate a number of other states, whose experimental counterparts cannot be identified. Future experiments are strongly encouraged to search for these states.

In Ref. [19] only $(S, I) = (0, 0)$ and $(-1, 1/2)$ sectors are studied. For $J = 1/2$, three $\Lambda_b$ states and three $\Xi_b$ states are identified. From the couplings of those dynamically generated states to the corresponding coupled channels (Table III and IV of Ref. [19]), it is clear that none of those states couple dominantly to the coupled channels considered in the present study. For instance, their $\Lambda_b(5912)$ and the bound state with $M = 6009.3$ MeV couple only moderately to $\Sigma_b \pi$ and
\( \Lambda_\eta \), respectively. The same is true for the two \( \Xi_\eta \) states with \( M = 6035.4 \text{ MeV} \) and \( M = 6072.8 \text{ MeV} \).

In Ref. \[20\], 10 states are found in the \( J^P = 1/2^- \) sector. Among them, one \( I = 0 \) state with \( M = 5969.5 \) and one \( I = 1 \) state with \( M = 6002.8 \text{ MeV} \) couple strongly to \( \Sigma_\eta \pi \). Because of the different coupled channels considered in both works and the fact that the \([3]\) and \([6]\) multiplet do not mix at leading order ChPT, we must refuse the temptation to associate them to our dynamically generated states. One needs to keep in mind that in our present work only the smallest number of coupled channels are taken into account which are dictated by approximate SU(3) chiral symmetry. Introduction of additional coupled channels inevitably requires further less justified assumptions.

TABLE IV. Dynamically generated charmed baryons of \( J^P = 1/2^- \). The subtraction constant is fixed in a way such that the \( \Lambda_c(2595) \) mass is produced to be 2591 MeV with \( a = -9.5 \). All energies are in units of MeV and \((S, I)^M\) denotes (strangeness, isospin)\textsuperscript{SU(3) multiplet}.

| Pole position | \((S, I)^M\) | Main channels (threshold) | Exp. [16] |
|---------------|--------------|---------------------------|----------|
| (2723, 0)     | \((0, 0)^[3]\) | \( \Xi_c K(2965.1) \)    |          |
| (2655, \(-i26\)) | \((-1, \frac{1}{2})^[3]\) | \( \Lambda_c \bar{K}(2782.1), \Xi_c \pi(2607.5) \) |          |
| (2965, 0)     | \((-2, 0)^[3]\) | \( \Xi_c \bar{K}(2965.1) \) |          |
| (2948, 0)     | \((1, \frac{1}{2})^6\) | \( \Sigma_c K(2949.1) \)    |          |
| (2674, \(-i51\)) | \((0, 1)^6\) | \( \Sigma_c \pi(2591.5) \)    |          |
| (2999, \(-i16\)) | \((0, 1)^6\) | \( \Sigma_c \eta(3001.4), \Xi_c \bar{K}(3072.4) \) |          |
| (2591, 0)     | \((0, 0)^6\) | \( \Sigma_c \pi(2591.5) \)    | \( \Lambda_c(2595) \) |
| (3070, \(-i12\)) | \((0, 0)^6\) | \( \Xi_c \bar{K}(3072.4) \)    | \( \Lambda_c(2940) \)? |
| (2949, \(-i35\)) | \((-1, \frac{3}{2})^6\) | \( \Sigma_c \bar{K}(2949.1) \)    |          |
| (2694, 0)     | \((-1, \frac{1}{2})^6\) | \( \Sigma_c \bar{K}(2949.1) \)    |          |
| (2827, \(-i55\)) | \((-1, \frac{1}{2})^6\) | \( \Xi_c \pi(2714.7) \)    | \( \Xi_c(2790) \)? |
| (3124, \(-i43\)) | \((-1, \frac{1}{2})^6\) | \( \Omega_c \bar{K}(3190.8) \)    |          |
| (2947, 0)     | \((-2, 0)^6\) | \( \Xi_c \bar{K}(3072.4), \Omega_c \eta(3243.1) \) |          |
TABLE V. The same as Table IV but obtained in the cutoff regularization scheme with $\Lambda = 1.35$ GeV.

| Pole positions ($S,I$) | Main channels (threshold) | Exp. [16] |
|------------------------|---------------------------|-----------|
| (2707, 0)              | (0, 0)$^35$ $\Xi_cK(2965.1)$ |           |
| (2655, $-i28$)         | (−1, $\frac{1}{2}$)$^35$ $\Lambda_c\overline{K}(2782.1),\Xi_c\pi(2607.5)$ |           |
| (2965, 0)              | (−2, 0)$^33$ $\Xi_c\overline{K}(2965.1)$ |           |
| (2950, $-i1$)          | (1, $\frac{3}{2}$)$^6$ $\Sigma_cK(2949.1)$ |           |
| (2672, $-i3$)          | (0, 1)$^6$ $\Sigma_c\pi(2591.5)$ |           |
| (3001, $-i18$)         | (0, 1)$^6$ $\Sigma_c\eta(3001.4),\Xi_c\overline{K}(3072.4)$ |           |
| (2591, 0)              | (0, 0)$^6$ $\Sigma_c\pi(2591.5)$ | $\Lambda_c(2959)$ |
| (3072, $-i15$)         | (0, 0)$^6$ $\Xi_c\overline{K}(3072.4)$ | $\Lambda_c(2940)$? |
| (2949, $-i34$)         | (−1, $\frac{3}{2}$)$^6$ $\Sigma_c\overline{K}(2949.1)$ |           |
| (2683, 0)              | (−1, $\frac{1}{2}$)$^6$ $\Sigma_c\overline{K}(2949.1)$ |           |
| (2813, $-i4$)          | (−1, $\frac{1}{2}$)$^6$ $\Xi_c\pi(2714.7)$ | $\Xi_c(2790)$? |
| (3121, $-i61$)         | (−1, $\frac{1}{2}$)$^6$ $\Omega_cK(3190.8)$ |           |
| (2909, 0)              | (−2, 0)$^6$ $\Xi_c\overline{K}(3072.4)$ |           |

B. Dynamically generated charmed baryons

Once the subtraction constant is fixed in the HQS approach, one can use the same constant to predict the counterparts of the dynamically generated bottom baryons. We have performed such a calculation and found that the $\Lambda_c(2595)$ can indeed be identified as a $\Sigma_c\pi$ state, as first pointed out in Refs. [17, 65]. To account for moderate heavy-quark flavor symmetry breaking corrections, we slightly fine-tune the subtraction constant in the dimensional regularization scheme or the cutoff value in the cutoff regularization scheme so that the mass of the $\Lambda_c(2595)$ is reproduced to be 2591 MeV$^2$. The predictions are then tabulated in Tables IV and V.

A comparison with the predictions of Refs. [18, 21] is again complicated by the same factors as mentioned previously. For instance, four ($S = 0, I = 0$) states and five ($S = 0, I = 1$) states

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2 Experimentally, the $\Lambda_c(2595)$ is found at $2592.25 \pm 0.28$ MeV with a width of $2.6 \pm 0.6$ MeV [16]. We need to slightly increase $f_0$ to put the $\Lambda_c(2595)$ exactly at this position because of the closeness of the $\Sigma_c\pi$ threshold.
are predicted in Ref. [21]. The number of dynamically generated states in Ref. [21] is even larger. Somehow, it seems that the number of states generated is proportional to the number of coupled channels considered.

In addition, our $\Lambda_c(2595)$ is predominantly a $\Sigma_c\pi$ state, where it is more of a $DN$ state in Ref. [21] and a $D^*N$ state in Ref. [18]. Despite of the different dominant components, it is clear that coupled channel effects or multi quark components may not be negligible in the wave function of the $\Lambda_c(2595)$. The same can be said about the $\Lambda_b(5912)$.

In Tables IV and V, we have temporarily identified the states appearing at $\sqrt{s} = (3070 - i12)$ MeV and $\sqrt{s} = (2827 - i55)$ MeV as the $\Lambda_c(2940)$ and the $\Xi_c(2790)$. The identification is based on the mass of these states and their main decay channels [16].

C. Further Discussions

Superficially, exact heavy quark flavor symmetry would dictate that the number of dynamically states in the bottom sector and in the charm sector is the same. A comparison of Table III and Table IV (or Table III and Table V) shows that this is almost the case, but not exactly. For instance, some counterparts of the dynamically generated bottom baryons in the charm sector are missing, such as the counterparts of the [3] states at $\sqrt{s} = (6082 - i57)$ MeV and $\sqrt{s} = (6100 - i32)$ MeV. A closer look at these channels reveal that they simply become too broad and develop a width of $200 \sim 300$ MeV. It should be noted that we have not considered any states broader than 200 MeV in our study.

The broadening of these states can be traced back partially to the weakening of the corresponding potentials and partially to the calibration of our framework to reproduce the $\Lambda_b(5912)$ in the bottom sector and to reproduce the $\Lambda_c(2595)$ in the charm sector. Since the $\Lambda_c(2593)$ is much closer to the threshold of its main coupled channel than the $\Lambda_b(5912)$, the calibration implies a weaker potential in the charm sector than the bottom sector. Due to this weakening, the dynamical generation of some charmed baryons require a slight readjustment of the potential by changing either $f_0$ and $a$ slightly within a few percent. Otherwise, they will show up as cusps. The pole positions of these states have been underlined to denote such a fine-tuning.

One should note that we have used an averaged pseudoscalar decay constant, $f_0 = 1.17 f_\pi$, in our calculations. Using the pion decay constant, $f_0 = f_\pi$, will not change qualitatively our results and conclusions, but can shift the predicted baryon masses by a few tens of MeV depending on
the particular channel. We have not given explicitly such uncertainties in Tables II, III, IV, and V but one should keep in mind the existence of such uncertainties (or freedom) in our approach. In addition, the differences between the results obtained in the dimensional regularization scheme and those obtained in the cutoff scheme also indicate inherent theoretical uncertainties of the UChPT method.

D. Scattering Lengths

Scattering lengths provide vital information on the strong interactions. Although direct experimental measurements of the scattering lengths between a charmed (bottom) baryon with a

| (S, I)^M | Channel | a | (S, I)^M | Channel | a |
|-----------|---------|---|-----------|---------|---|
| (1, 1/2)^3 | Λ_bK(6115.0) | −0.112 | (1, 3/2)^6 | Σ_bK(6308.6) | −0.138 |
| (0, 1)^7 | Ξ_bK(6285.1) | −0.240 − i0.040 | (1, 1/2)^6 | Σ_bK(6308.6) | −0.419 |
| (0, 1)^7 | Λ_bπ(5757.4) | 0.003 | (0, 2)^6 | Σ_bπ(5951.0) | −0.102 |
| (0, 0)^7 | Ξ_bK(6285.1) | −0.205 − i0.003 | (0, 1)^6 | Ξ'_bK(6421.6) | −0.211 − i0.007 |
| (0, 0)^7 | Λ_bη(6167.3) | −0.150 | (0, 1)^6 | Σ_bη(6360.9) | −0.273 − i0.014 |
| (−1, 3/2)^7 | Π_bπ(5927.5) | −0.067 | (0, 1)^6 | Σ_bπ(5951.0) | 1.162 |
| (−1, 1/2)^7 | Π_bπ(5927.5) | −0.417 | (0, 0)^6 | Ξ'_bK(6421.6) | −0.398 − i0.019 |
| (−1, 1/2)^7 | Π_bη(6337.4) | −0.163 − i0.025 | (0, 0)^6 | Σ_bπ(5951.0) | −0.598 |
| (−1, 1)^7 | Λ_bK(6115.0) | −0.379 − i0.224 | (−1, 3/2)^6 | Ξ'_bπ(6064.0) | 0.012 |
| (−2, 1)^7 | Ξ_bK(6285.1) | −0.118 | (−1, 3/2)^6 | Σ_bK(6308.6) | −0.350 − i0.061 |
| (−2, 0)^7 | Ξ_bK(6285.1) | −0.507 | (−1, 1/2)^6 | Ξ'_bπ(6064.0) | 0.467 |
| (−1, 1)^6 | Ξ'_bη(6473.9) | −0.233 − i0.024 | (−2, 1)^6 | Ω_bπ(6209.0) | 0.099 |
| (−1, 1/2)^6 | Ω_bK(6566.6) | −0.282 − i0.015 | (−2, 0)^6 | Ξ'_bK(6421.6) | −0.217 |
| (−1, 1/2)^6 | Σ_bK(6308.6) | −0.187 − i0.005 | (−2, 0)^6 | Ω_bη(6618.9) | −0.189 − i0.003 |
| (−2, 1)^6 | Ξ'_bK(6421.6) | −0.239 − i0.140 | (−3, 1/2)^6 | Ω_bK(6566.6) | −0.155 |
pseudoscalar meson cannot be foreseen in the near future, rapid developments in lattice chromodynamics may soon fill the gap. In Tables VII and VI we tabulate the scattering lengths calculated in the dimensional regularization scheme, defined as

$$a_{jj} = -\frac{M_j}{4\pi(M_j + m_j)} T_{jj}^{(S,I)}$$

for channel $j$ with strangeness $S$ and isospin $I$, where $M_j$ and $m_j$ are the respective baryon and meson masses of that channel. For the sake of comparison, we list the ChPT results of Ref. [52]. One should note, however, Ref. [52] calculated the scattering lengths up to $O(p^3)$ and considered the virtual contributions of the spin-3/2 baryons as well. While in our study only the leading order
($\mathcal{O}(p)$) ChPT kernel is used and in addition we work with the UChPT. Therefore, the differences shown in Table VII and VI can be easily understood.

Examining the scattering lengths in the charm sector, we notice that because of the existence of a bound state just below their respective threshold, the scattering lengths for the $\Sigma_c K$ channel with $(S, I)^M = (1, 1/2)^6$ and for the $\Sigma_c \pi$ channel with $(S, I)^M = (0, 0)^6$ are quite large and negative, i.e., $a_{\Sigma_c K} = -7.851$ and $a_{\Sigma_c \pi} = -25.327$. Therefore, future Lattice QCD study of these two channels may be able to test to what extent that the scenario of these states being dynamically generated is true.

IV. SUMMARY AND OUTLOOK

We have studied the interaction between a singly charmed (bottom) baryon and a pseudoscalar meson in the unitarized chiral perturbation theory using leading order chiral Lagrangians. It is shown that the interactions are strong enough to generate a number of dynamically generated states. Some of them can be naturally assigned to their experimental counterparts, such as the $\Lambda_b(5912)$ and the $\Lambda_c(2595)$. By slightly fine-tune the subtraction constant in the dimensional regularization scheme or the cutoff value in the cutoff regularization scheme so that the $\Lambda_b(5912)$ and the $\Lambda_c(2595)$ masses are produced, we predict a number of additional states, whose experimental counterparts remain unknown. We strongly encourage future experiments to search for these states.

In anticipation of future lattice chromodynamics simulation of scattering lengths, as already happened in the light baryon sector or the heavy meson sector, we have calculated the scattering lengths between the charmed (bottom) baryons and the pseudoscalar mesons. A comparison between our results and those of the $\mathcal{O}(p^3)$ ChPT study confirmed that there are indeed strong attraction in some of the coupled channels, which hint at the possible existence of dynamically generated states.

Approximate heavy-quark spin symmetry implies that the interaction between a ground state spin-1/2 baryon and a pseudoscalar meson and that between a ground-state spin-3/2 baryon with a pseudoscalar meson is the same in the limit of infinite heavy quark mass. Therefore, a detailed study of the latter interaction will be a natural extension of the present study. In addition, one may wish to study the effects of higher-order kernels in the unitarized chiral perturbation theory. Such studies are in progress and will be reported somewhere else.
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VI. APPENDIX

In this section, we tabulate the Clebsch-Gordan coefficients appearing in Eq. (6) for the anti-triplet (Tables VIII to XIV) and sexet (Tables XV to XXIV) ground-state charmed or bottom baryons interacting with the pseudoscalar mesons.

**TABLE VIII.** \((S = 1, I = 1/2)\)

| \(\Lambda_c K\) | \(\Lambda_c K\) |
|----------------|----------------|
|                 | 1              |

**TABLE IX.** \((S = 0, I = 1)\)

| \(\Xi_c K\) | \(\Lambda_c \pi\) |
|--------------|-------------------|
| \(\Xi_c K\) | 0 \(\ -1\)       |
| \(\Lambda_c \pi\) | \(-1\) \(\ 0\) |

**TABLE X.** \((S = 0, I = 0)\)

| \(\Xi_c K\) | \(\Lambda_c \eta\) |
|--------------|-------------------|
| \(\Xi_c K\) | \(-2\) \(\sqrt{3}\) |
| \(\Lambda_c \eta\) | \(\sqrt{3}\) \(\ 0\) |

[1] D. Bernard [BaBar Collaboration], PoS DIS 2013, 179 (2013) [arXiv:1311.0968 [hep-ex]].
TABLE XI. \((S = -1, I = 3/2)\)

| \(\Xi_c\pi\) | \(\Xi_c\pi\) |
|-------------|-------------|
| \(\Xi_c\pi\) | 1           |

TABLE XII. \((S = -1, I = 1/2)\)

| \(\Xi_c\pi\) | \(\Xi_c\eta\) | \(\Lambda_c\bar{K}\) |
|-------------|--------------|---------------------|
| \(\Xi_c\pi\) | -2           | 0                   |
| \(\Xi_c\pi\) | 0            | \(-\sqrt{3}/2\)    |
| \(\Xi_c\eta\) | 0            | \(\sqrt{3}/2\)     |
| \(\Lambda_c\bar{K}\) | \(-\sqrt{3}/2\) | \(\sqrt{3}/2\) |
| \(\Lambda_c\bar{K}\) | -1           |                     |

TABLE XIII. \((S = -2, I = 1)\)

| \(\Xi_c\bar{K}\) |
|-------------------|
| \(\Xi_c\bar{K}\) | 1           |

TABLE XIV. \((S = -2, I = 0)\)

| \(\Xi_c\bar{K}\) |
|-------------------|
| \(\Xi_c\bar{K}\) | -1       |

TABLE XV. \((S = 1, I = 3/2)\)

| \(\Sigma_c\bar{K}\) |
|----------------------|
| \(\Sigma_c\bar{K}\) | 2         |

TABLE XVI. \((S = 1, I = 1/2)\)

| \(\Sigma_c\bar{K}\) |
|----------------------|
| \(\Sigma_c\bar{K}\) | -1       |

[2] D. Santel [Belle Collaboration], PoS DIS 2013, 162 (2013).
**TABLE XVII.** $(S = 0, I = 2)$

| $\Sigma_c \pi$ | $\Sigma_c \pi$ |
|----------------|----------------|
| 2              | 2              |

**TABLE XVIII.** $(S = 0, I = 1)$

| $\Xi'_c K$ | $\Sigma_c \eta$ | $\Sigma_c \pi$ |
|------------|-----------------|----------------|
| $\Xi'_c K$ | 0               | $-\sqrt{3} - \sqrt{2}$ |
| $\Sigma_c \eta$ | $-\sqrt{3}$ | 0 | 0 |
| $\Sigma_c \pi$ | $-\sqrt{2}$ | 0 | $-2$ |

**TABLE XIX.** $(S = 0, I = 0)$

| $\Xi'_c K$ | $\Sigma_c \pi$ |
|------------|----------------|
| $\Xi'_c K$ | $-2 - \sqrt{3}$ |
| $\Sigma_c \pi$ | $-\sqrt{3}$ | $-4$ |

**TABLE XX.** $(S = -1, I = 3/2)$

| $\Xi'_c \pi$ | $\Sigma_c K$ |
|---------------|---------------|
| $\Xi'_c \pi$ | 1 | $\sqrt{2}$ |
| $\Sigma_c K$ | $\sqrt{2}$ | 0 |

**TABLE XXI.** $(S = -1, I = 1/2)$

| $\Xi'_c \pi$ | $\Xi'_c \eta$ | $\Omega_c K$ | $\Sigma_c \bar{K}$ |
|---------------|----------------|--------------|-------------------|
| $\Xi'_c \pi$ | $-2$ | 0 | $-\sqrt{3}$ | $1/\sqrt{2}$ |
| $\Xi'_c \eta$ | 0 | 0 | $-\sqrt{3}$ | $3/\sqrt{2}$ |
| $\Omega_c K$ | $-\sqrt{3}$ | $-\sqrt{3}$ | $-2$ | 0 |
| $\Sigma_c \bar{K}$ | $1/\sqrt{2}$ | $3/\sqrt{2}$ | 0 | $-3$ |
[3] Y. Kato [Belle Collaboration], PoS Hadron 2013, 053 (2014).
[4] K. K. Seth, arXiv:1107.2641 [hep-ex].
[5] H. -B. Li [BESIII Collaboration], EPJ Web Conf. 72, 00011 (2014).
[6] P. d. Simone [LHCb Collaboration], EPJ Web Conf. 73, 03007 (2014).
[7] P. Palni [CDF Collaboration], Nucl. Phys. Proc. Suppl. 233, 151 (2012).
[8] M. Ablikim et al. [BESIII Collaboration], Phys. Rev. Lett. 110, no. 25, 252001 (2013) [arXiv:1303.5949 [hep-ex]].
[9] Z. Q. Liu et al. [Belle Collaboration], Phys. Rev. Lett. 110, 252002 (2013) [arXiv:1304.0121 [hep-ex]].
[10] R. Aaij et al. [LHCb Collaboration], Phys. Rev. Lett. 112, 222002 (2014) [arXiv:1404.1903 [hep-ex]].
[11] S. K. Choi et al. [BELLE Collaboration], Phys. Rev. Lett. 100, 142001 (2008) [arXiv:0708.1790 [hep-ex]].
[12] N. Brambilla, S. Eidelman, B. K. Heltsley, R. Vogt, G. T. Bodwin, E. Eichten, A. D. Frawley and A. B. Meyer et al., Eur. Phys. J. C 71, 1534 (2011) [arXiv:1010.5827 [hep-ph]].
[13] V. Crede and W. Roberts, Rept. Prog. Phys. 76, 076301 (2013) [arXiv:1302.7299 [nucl-ex]].

TABLE XXII. \((S = -2, I = 1)\)

| \(\bar{\Xi}_c^' K\) | \(\Omega_c\pi\) |
|-----------------|-----------------|
| \(\bar{\Xi}_c^' K\) |  1  | \(\sqrt{2}\) |
| \(\Omega_c\pi\)  |  \(\sqrt{2}\)  |  0  |

TABLE XXIII. \((S = -2, I = 0)\)

| \(\bar{\Xi}_c^' K\) | \(\Omega_c\eta\) |
|-----------------|-----------------|
| \(\bar{\Xi}_c^' K\) |  -1  | \(\sqrt{6}\) |
| \(\Omega_c\eta\)  |  \(\sqrt{6}\)  |  0  |

TABLE XXIV. \((S = -3, I = 1/2)\)

| \(\Omega_c\bar{K}\) |
|-----------------|
| \(\Omega_c\bar{K}\) | 2  |
[14] RAaij et al. [LHCb Collaboration], Phys. Rev. Lett. 109, 172003 (2012) [arXiv:1205.3452 [hep-ex]].
[15] T. A. Aaltonen et al. [CDF Collaboration], Phys. Rev. D 88, no. 7, 071101 (2013) [arXiv:1308.1760 [hep-ex]].
[16] J. Beringer et al. [Particle Data Group Collaboration], Phys. Rev. D 86, 010001 (2012).
[17] M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A 730, 110 (2004) [hep-ph/0307233].
[18] C. Garcia-Recio, V. K. Magas, T. Mizutani, J. Nieves, A. Ramos, L. L. Salcedo and L. Tolos, Phys. Rev. D 79, 054004 (2009) [arXiv:0807.2969 [hep-ph]].
[19] C. Garcia-Recio, J. Nieves, O. Romanets, L. L. Salcedo and L. Tolos, Phys. Rev. D 87, 034032 (2013) [arXiv:1210.4755 [hep-ph]].
[20] W. H. Liang, C. W. Xiao and E. Oset, Phys. Rev. D 89, 054023 (2014) [arXiv:1401.1441 [hep-ph]].
[21] W. H. Liang, T. Uchino, C. W. Xiao and E. Oset, arXiv:1402.5293 [hep-ph].
[22] T. Hyodo and D. Jido, Prog. Part. Nucl. Phys. 67, 55 (2012) [arXiv:1104.4474 [nucl-th]].
[23] J. -J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. Lett. 105, 232001 (2010) [arXiv:1007.0573 [nucl-th]].
[24] J. -J. Wu, R. Molina, E. Oset and B. S. Zou, Phys. Rev. C 84, 015202 (2011) [arXiv:1011.2399 [nucl-th]].
[25] J. -J. Wu and B. S. Zou, Phys. Lett. B 709, 70 (2012) [arXiv:1011.5743 [hep-ph]].
[26] C. W. Xiao, J. Nieves and E. Oset, Phys. Rev. D 88, 056012 (2013) [arXiv:1304.5368 [hep-ph]].
[27] C. W. Xiao and E. Oset, Eur. Phys. J. A 49, 139 (2013) [arXiv:1305.0786 [hep-ph]].
[28] J. M. Flynn, E. Hernandez and J. Nieves, Phys. Rev. D 85, 014012 (2012) [arXiv:1110.2962 [hep-ph]].
[29] O. Romanets, L. Tolos, C. Garcia-Recio, J. Nieves, L. L. Salcedo and R. G. E. Timmermans, Phys. Rev. D 85, 114032 (2012) [arXiv:1202.2239 [hep-ph]].
[30] O. Romanets, L. Tolos, C. Garca-Recio, J. Nieves, L. L. Salcedo and R. Timmermans, Nucl. Phys. A 914, 488 (2013) [arXiv:1212.3943 [hep-ph]].
[31] C. Garcia-Recio, J. Nieves, O. Romanets, L. L. Salcedo and L. Tolos, Phys. Rev. D 87, 074034 (2013) [arXiv:1302.6938 [hep-ph]].
[32] F. -K. Guo, C. Hidalgo-Duque, J. Nieves and M. P. Valderrama, Phys. Rev. D 88, no. 5, 054014 (2013) [arXiv:1305.4052 [hep-ph]].
[33] N. Kaiser, P. B. Siegel and W. Weise, Nucl. Phys. A 594, 325 (1995) [nucl-th/9505043].
[34] J. A. Oller and E. Oset, Nucl. Phys. A 620, 438 (1997) [Erratum-ibid. A 652, 407 (1999)] [hep-ph/9702314].
[35] E. Oset and A. Ramos, Nucl. Phys. A 635, 99 (1998) [nucl-th/9711022].
[36] B. Krippa, Phys. Rev. C 58, 1333 (1998) [hep-ph/9803332].
[37] J. Nieves and E. Ruiz Arriola, Nucl. Phys. A 679, 57 (2000) [hep-ph/9907469].
[38] U. G. Meissner and J. A. Oller, Nucl. Phys. A 673, 311 (2000) [nucl-th/9912026].
[39] M. F. M. Lutz and E. E. Kolomeitsev, Nucl. Phys. A 700, 193 (2002) [nucl-th/0105042].
[40] C. Garcia-Recio, J. Nieves, E. Ruiz Arriola and M. J. Vicente Vacas, Phys. Rev. D 67, 076009 (2003) [hep-ph/0210311].
[41] T. Hyodo, S. I. Nam, D. Jido and A. Hosaka, Phys. Rev. C 68, 018201 (2003) [nucl-th/0212026].
[42] B. Borasoy, U.-G. Meissner and R. Nissler, Phys. Rev. C 74, 055201 (2006) [hep-ph/0606108].
[43] J. A. Oller and E. Oset, Phys. Rev. D 60, 074023 (1999) [hep-ph/9809337].
[44] T. N. Truong, Phys. Rev. Lett. 61, 2526 (1988).
[45] A. Dobado, M. J. Herrero and T. N. Truong, Phys. Lett. B 235, 134 (1990).
[46] A. Dobado and J. R. Pelaez, Phys. Rev. D 47, 4883 (1993) [hep-ph/9301276].
[47] A. Dobado and J. R. Pelaez, Phys. Rev. D 56, 3057 (1997) [hep-ph/9604416].
[48] F. Guerrero and J. A. Oller, Nucl. Phys. B 537, 459 (1999) [Erratum-ibid. B 602, 641 (2001)] [hep-ph/9805334].
[49] A. Gomez Nicola and J. R. Pelaez, Phys. Rev. D 65, 054009 (2002) [hep-ph/0109056].
[50] J. R. Pelaez, Mod. Phys. Lett. A 19, 2879 (2004) [hep-ph/0411107].
[51] M. Altenbuchinger and L. -S. Geng, Phys. Rev. D 89, 054008 (2014) [arXiv:1310.5224 [hep-ph]].
[52] Z. -W. Liu and S. -L. Zhu, Phys. Rev. D 86, 034009 (2012) [arXiv:1205.0467 [hep-ph]].
[53] M. Altenbuchinger, L. -S. Geng and W. Weise, Phys. Rev. D 89, 014026 (2014) [arXiv:1309.4743 [hep-ph]].
[54] T. Fuchs, J. Gegelia, G. Japaridze and S. Scherer, Phys. Rev. D 68, 056005 (2003) [hep-ph/0302117].
[55] R. A. Briceno, H. -W. Lin and D. R. Bolton, Phys. Rev. D 86, 094504 (2012) [arXiv:1207.3536 [hep-lat]].
[56] C. Alexandrou, V . Drach, K. Jansen, C. Kallidonis and G. Koutsou, arXiv:1406.4310 [hep-lat].
[57] L. Liu, H. -W. Lin, K. Orginos and A. Walker-Loud, Phys. Rev. D 81, 094505 (2010) [arXiv:0909.3294 [hep-lat]].
[58] Y. Namekawa et al. [PACS-CS Collaboration], Phys. Rev. D 87, no. 9, 094512 (2013) [arXiv:1301.4743 [hep-lat]].
[59] Z. S. Brown, W. Detmold, S. Meinel and K. Orginos, arXiv:1409.0497 [hep-lat].
[60] R. Lewis and R. M. Woloshyn, Phys. Rev. D \textbf{79}, 014502 (2009) [arXiv:0806.4783 [hep-lat]].

[61] M. Padmanath, R. G. Edwards, N. Mathur and M. Peardon, arXiv:1307.7022 [hep-lat].

[62] S. Meinel, Phys. Rev. D \textbf{85}, 114510 (2012) [arXiv:1202.1312 [hep-lat]].

[63] M. Padmanath, R. G. Edwards, N. Mathur and M. Peardon, arXiv:1311.4806 [hep-lat].

[64] M. Padmanath, R. G. Edwards, N. Mathur and M. Peardon, arXiv:1311.4354 [hep-lat].

[65] J. Hofmann and M. F. M. Lutz, Nucl. Phys. A \textbf{763}, 90 (2005) [hep-ph/0507071].