Letter

Complementarity via error-free measurement in a two-path interferometer

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Abstract

We study both wave-like behavior and particle-like behavior in a general Mach–Zehnder interferometer with an asymmetric beam splitter. An error-free measurement in the detector is used to extract the which-path information. The fringe visibility $V$ and the which-path information $I_{\text{path}}$ are derived, their complementary relation $V + I_{\text{path}} \leq 1$ is found, and the condition for the equality is also presented.

Keywords: complementarity, which-path information, fringe visibility, general Mach–Zehnder interferometer

(Some figures may appear in colour only in the online journal)

1. Introduction

The wave-like nature and particle-like nature of a quantum system are two mutual exclusion properties, and the appearance of these two properties is determined by an experimental instrument, which is known as Bohr’s complementarity principle [1]. The wave–particle duality is a well-known example used to exhibit the curious nature of the complementarity principle. In the recoiling-slit gedanken experiment introduced by Einstein and Bohr, where a particle is sent through a movable slit placed before a double slit, Wootters and Zurek [2] proposed their quantitative formulation of the wave–particle duality. In a two-path interferometer, such as a Mach–Zehnder interferometer (MZI) [3], a complementarity was first found between the\textit{a priori} fringe visibility of the interference pattern and the predictability [4], which were determined by the initial state of the particle. Later on, the wave-like and particle-like nature were characterized by the visibility of the interferometer fringe and the path distinguishability [5], respectively, and a trade-off was found between these quantities. Both the predictability and distinguishability are quantities that measure the which-path knowledge. Since there are many ways to define the measurement of which-path knowledge, the complementarity between the fringe visibility and the which-path knowledge has been studied widely in theory and experiment [4–22].

From the information theory viewpoint, the achievement of knowledge is an information transmission process. It is natural to use measures of information to characterize the particle nature of a quantum system in a two-path interferometer. Here, we employ the mutual information, which is called the which-path information (WPI). The information cannot be transmitted until a measurement is performed, for example, a detector is placed in one path of the MZI in [5], and an error-minimum state distinguishing measurement [23] is performed on the detector after the particle interacts with the detector to acquire path distinguishability. Although such an ambiguous measurement yields a conclusive outcome, the nonvanishing probability of making a wrong guess exists, since errors in the conclusive outcome are unavoidable. The other optimized measurement strategy is error-free discrimination [24] among nonorthogonal states, which allows a nonzero probability of inconclusive outcomes.

In this paper, we study the trade-off between the fringe visibility and WPI in a two-path interferometer made of a
symmetric beam splitter (BS) and an asymmetric BS. The errorfree measurement is used to obtain the WPI. It is found that the magnitudes of the fringe visibility and the WPI are effected by the asymmetric BS and the input state of the particle. A bound between the fringe visibility and the WPI is also found.

The paper is organized as follows. In section 2, we introduce the setup in which a quantum system displays wave-like behavior and particle-like behavior. In section 3, the WPI is defined, and the unambiguous discrimination of the state of the detector is used to obtain the which-path information. In section 4, we make our conclusion.

2. The setups and the state evolution

A general MZI, shown in figure 1, consists of two BSs and phase shifters (PSs). A beam of particles coming from either port a or b is first split into two by beam splitter BS1, and then these beams are recombined by BS2. So, two paths, a and b, are available between BS1 and BS2. A particle taking path a(b) is denoted by the state $|a⟩ = a \hat{a}^\dagger |0⟩ = (|b⟩ = b \hat{b}^\dagger |0⟩)$, where $a\hat{a}^\dagger$ and $b\hat{b}^\dagger$ are the corresponding creation operators in path a and b, and they satisfy $[a, b] = 0$, $[a, a\dagger] = 1, [b, b\dagger] = 1$. States $|a⟩$ and $|b⟩$ support a two-dimensional $H_q$. In this sense, a quantum bit is formed. The state of the particle traveling in this interferometer is characterized by the change of the Bloch vector in the Bloch sphere. Before the particle is incident in the general MZI, its state is described by the density matrix

$$\rho^0 = \frac{1}{2}(1 + S_x \sigma_x + S_y \sigma_y + S_z \sigma_z),$$

where $\sigma_x, \sigma_y, \sigma_z$ are the Pauli matrices and $\sigma_z = |b⟩⟨b| − |a⟩⟨a|$. Here, the initial Bloch vector $S = (S_x, S_y, S_z)$. When $|S| = 1$, the particle is in a pure state, when $|S| < 1$, the particle is in a mixed state.

The effect of the BS on the state of the incoming particles is described by the operator $B(\beta) = \exp[i\beta(a\hat{a}^\dagger b + b\hat{b}^\dagger a)]$, which preserves the total number of particles in this MZI. In the subspace spanned by the basis $\{|a⟩, |b⟩\}$, the BS performs a rotation around the $y$ axis by angle $\beta$, which is denoted by

$$U_{B}(\beta) = \exp(-i\frac{\beta}{2} \sigma_y) = \left(\begin{array}{cc} \sqrt{T} & -\sqrt{R} \\ \sqrt{R} & \sqrt{T} \end{array}\right),$$

Here, $r$ and $t$ respectively represent the reflection coefficient and transmission coefficient of the beam splitter.

$$r = \sin^2\frac{\beta}{2}, \quad t = \cos^2\frac{\beta}{2}.$$

The PS in path $d \in \{a, b\}$ is described by $P(\phi_d) = \exp(i \phi_d \hat{d}^\dagger \hat{d})$. If the parameters $\phi_a$ and $\phi_b$ have the same magnitude but a different sign, i.e. $\phi_a = -\phi_b = \phi$, a rotation around the $z$ axis by angle $\phi$ is realized by PS1 and PS2,

$$U_{P}(\phi) = \exp(-i\frac{\phi}{2} \sigma_z).$$

To acquire the WPI, a detector is usually placed in one of the paths (e.g. the a path). As long as the particle goes through the general MZI, the operator

$$M = 1 + \sigma_z I + \frac{1}{2} - \sigma_z U,$$

is performed on the detector, where $U$ is unitary. The final state of the particle and the detector reads

$$\rho_f = U_{B}(\beta)MU_{P}(\phi)U_{B}(\beta)^\dagger \rho^0 D_{\text{in}} U_{P}(\phi)^\dagger U_{B}(\beta)^\dagger \rho^0 M^\dagger,$$

where $\rho^0$ is the initial state of the detector, and BS1 is assumed to be symmetric.

The probability that we find the particle at the output port a reads

$$p(\phi) = |\text{tr}_d [\frac{1}{2}(1 - \sigma_z) \rho_f]|$$

$$= \frac{1}{2}(1 + S_z \cos \beta) + \frac{1}{2} \sqrt{S_x^2 + S_y^2} \sin \beta |\text{tr}_d(U_{P}(\rho^0)M^\dagger U^\dagger)| \cos(\alpha + \gamma + \phi),$$

where $\alpha$ and $\gamma$ are defined as

$$\alpha = \arctan \frac{S_y}{S_x}, \quad \gamma = -i \ln \frac{|\text{tr}_d(U_{P}(\rho^0))|}{|\text{tr}_d(U_{P}(\rho^0)M)|}. \quad (8)$$

The fringe visibility, which documents the wave-like property of the particle, is defined via the probability in equation (7) as

$$V = \frac{\max P(\phi) − \min P(\phi)}{\max P(\phi) + \min P(\phi)}, \quad (9)$$

where the maximum and minimum is achieved by adjusting $\phi$, and one can easily obtain that

$$V = \frac{\sin^2 \frac{\beta}{2}}{1 + S_z \cos \beta \sqrt{S_x^2 + S_y^2} |\text{tr}_d(U_{P}(\rho^0))|}. \quad (10)$$
To show the dependence and when $\rho_\omega \rho U(1) = \rho_\omega |\rangle \langle \rho| U(1)$. That the fringe visibility first increases and (b) the cross section of (a) when $S_x = -0.5, 0, 0.5$; (c) the cross section of (a) when $\beta = \pi/4, \pi/2, 3\pi/4$.

![Figure 2](image1.png)

(a) The fringe visibility as a function of the $S_x$ and $\beta$ with $\text{tr}(\rho(U^\rho D)) = 1/3$ and $S_x^2 + S_y^2 + S_z^2 = 1$; (b) the cross section of (a) when $S_x = -0.5, 0, 0.5$; (c) the cross section of (a) when $\beta = \pi/4, \pi/2, 3\pi/4$.

Figure 3. A schematic representation of the setup with four input and output ports.

We note that the fringe visibility can also be defined by the probability at the output port $b$. However, the fringe visibility measured in either output port $a$ or $b$ is expected to be different if the BS2 is symmetric (i.e. $\beta = \pi/2$).

Equation (10) shows that both the BS2 and the initial state have an influence on the fringe visibility. It is found that for a given $\beta$, more of the wave’s nature appears when the particle is in a pure state ($|\beta|$ = 1). To show the dependence of the wave nature of the BS2 and the initial state, we have plotted the fringe visibility $V$ as a function of the $\beta$ and $S_x$ for $|\text{tr}(\rho(U^\rho D))| = 1/3$ and $|\beta| = 1$ in figure 2. It can be observed for a given $S_x(\beta)$ that the fringe visibility first increases and then decreases as $\beta(S_x)$ increases, and the fringe visibility obtains the maximum $C \equiv |\text{tr}(\rho(U^\rho D))|$ when $\cos \beta = -S_x$. This is the reason the quantified wave-particle duality in [5] is presented by choosing $S_x = 0$ when $\beta = \pi/2$. The value of the fringe visibility is zero in the following situations. (1) The effect of the BS2 for the particle is full transmission or full reflection, corresponding to $\beta = 0$ or $\pi$. (2) The particle only travels along the a path or $b$ path, corresponding to $S_x = 1$ or $-1$.

It is well-known that the wave-like property characterized by the fringe visibility is complementary to the particle-like property which gives rise to the WPI. A decrease of the fringe visibility predicts an increase of the WPI. From equation (10), one finds that, except when it is in its initial state, BS2 affects the WPI, which indicates that the asymmetric BS2 introduces an additional WPI [19]. Then, the setup which measures the particle-like property is different from the one which is obtained simply by removing BS2 in figure 1. Actually, the particle-like property is measured by the setup with four input and output ports, as shown in figure 3. Since two paths $c$ and $d$ have been introduced, the initial density matrix for the total system reads

$$\rho_{in}^{CD} = \rho_{in}^{D} \otimes |00\rangle_{cd} \langle 00| \otimes \rho_{in}^{P}. \quad (11)$$

where $|00\rangle_{cd}$ is the vacuum state of the input ports $c$ and $d$. Before the particle meets BS2 and BS3, the state of the particle and the detector is the same as the state before BS2 in figure 1. BS2 acts on paths $a$ and $c$ and BS3 acts on paths $b$ and $d$. The performance of BS2 and BS3 is denoted by $B_2 = \exp((-\pi/2)(a^d c - c^d a))$ and $B_3 = \exp(-\pi/2)(b^d \hat{a} - \hat{a}^d b)$ respectively. After the particle goes through BS2 and BS3, the state for the particle appearing in either output $a$ or $d$ reads

$$\rho_{f}^{OD} = \omega_d |d\rangle \langle d| \rho_{in}^{D} + \omega_a |a\rangle \langle a| \rho_{in}^{D} U^{\dagger} + \frac{\sqrt{r}}{1 + S_x(t-r)} e^{i\omega(S_x + iS_y)} |a\rangle \langle d| \rho_{in}^{D} \rho_{in}^{P}\rho_{in}^{D} U^{\dagger} + \frac{\sqrt{r}}{1 + S_x(t-r)} e^{-i\omega(S_x - iS_y)} |d\rangle \langle a| \rho_{in}^{D} U^{\dagger}. \quad (12)$$

Here are the prior probabilities

$$\omega_a = \frac{t(1 + S_x)}{1 + S_x(t-r)} , \quad \omega_b = \frac{r(1 + S_x)}{1 + S_x(t-r)}. \quad (13)$$

for finding the particle in outputs $a$ and $d$ respectively. Since the particle detected at output port $a(d)$ is obtained by
transmission (reflection) from path \(a(b)\), we change the letter \(d\) in equation (12) to \(b\), which is rewritten as
\[
\rho_D^b = \omega_b|b\rangle\langle b|^D + \omega_i|a\rangle\langle a|U\rho_D^aU^\dagger
+ \frac{\sqrt{r}}{1 + S_i(t - r)}e^{i\phi(S_i + iS_f)}|a\rangle\langle b|U\rho_D^bU^\dagger
+ \frac{\sqrt{r}}{1 + S_i(t - r)}e^{-i\phi(S_f - iS_i)}|b\rangle\langle a|\rho_D^aU^\dagger.
\]
(14)

3. Information gain via the error-free measurement

After tracing over the degree of the particle in equation (14), we obtain the final state of the detector
\[
\rho_D^a = \omega_b\rho_D^b + \omega_i\rho_D^a.
\]
(15)

To obtain the WPI, we have to discriminate the states \(\rho_D^a\) and \(\rho_D^b\) with the prior probabilities \(\omega_b\) and \(\omega_i\) in an optimal way. Here, we perform the error-free measurement on the detector. This kind of measurement gives two results: a conclusive one without any error and an inconclusive one. The conclusive result means the which-path that the particle takes is definitely known. Mathematically, to calculate the overlap 
\[
\rho_D^a \equiv U\rho_D^bU^\dagger
\]
with the prior probabilities \(\omega_b\) and \(\omega_i\), is always unambiguously detectable. Therefore, the optimal way. Here, we perform the error-free measurement requires
\[
\rho_D^a \equiv \omega_b\rho_D^b + \omega_i\rho_D^a = 0.
\]
(16)
The joint probability that a particle travels along path \(d\) \(\in \{a, b\}\) and the which-path result \(k\), indicated by the measurement of the detector reads
\[
Q(\mu, k) = Tr_D(\mu|\Pi_k\rho_D^{|k|}).
\]
(17)
Then, the amount of WPI [25] obtained from the error-free measurement is given by
\[
\mathcal{I}_{\text{path}} = \sum_{\mu \in a,b} \sum_{k \in a,b} Q(\mu, k) \log\left[\frac{Q(\mu, k)}{Q(\mu)Q(k)}\right].
\]
(18)

For the sake of simplicity, we assume that the detector is initially in a pure state \(\rho_D^{|r\rangle} = |r\rangle\langle r|\). Since the unitary operator \(U\) is arbitrary, the states \(|r\rangle\) and \(|s\rangle\) \(\equiv U|r\rangle\) can be assumed to be linearly independent, and the POVM is constructed as
\[
\Pi_a = \alpha|r\rangle\langle r|,
\]
\[
\Pi_b = \beta|s\rangle\langle s|,
\]
\[
\Pi_0 = (1 - \beta S^2)|r\rangle\langle r| + \beta SC|r\rangle\langle r| r\rangle\langle r| + \beta SC|r\rangle\langle r| + (1 - \beta S^2 - \alpha)|r\rangle\langle r|,
\]
(19)
where the states \(|r\rangle\) and \(|s\rangle\) are orthogonal to
\[
|r\rangle = \frac{1}{S}(|s\rangle - C|r\rangle),
\]
\[
|s\rangle = \frac{1}{S}(|r\rangle - C|s\rangle),
\]
(20)
respectively. The capital letters \(S = \sqrt{1 - C^2}\) and \(C = \langle r|s\rangle\), where the maximum value of the fringe visibility becomes an overlap between two linearly independent states. In equation (19), parameters \(\alpha\) and \(\beta\) are chosen to minimize the probability of failure
\[
Q = \omega_bTr(\rho_D^a\Pi_0) + \omega_iTr(\rho_D^b\Pi_0).
\]
(21)
By the Cauchy inequality and the resolution of the identity, we derive the lower bound on the probability of failure [26]
\[
Q \geq 2\sqrt{\omega_b\omega_i}F(\rho_D^a\rho_D^b),
\]
(22)
where the fidelity [27] is defined as
\[
F = Tr\left[\sqrt{\rho_D^a\rho_D^b}\right] = C.
\]
(23)
The lower bound of the failure probability is achieved if and only if
\[
\omega_bTr(\rho_D^a\Pi_0) = \omega_iTr(\rho_D^b\Pi_0) = \sqrt{\omega_b\omega_i}F(\rho_D^a\rho_D^b).
\]
(24)

The probability of the \(k\)th outcome, \(tr(\Pi_k\rho_D^{|k|})\), is always real and non-negative, and requires \(0 \leq \alpha, \beta \leq 1\). The choice of the measurement that discriminates \(|r\rangle\) and \(|s\rangle\) unambiguously depends on the relation between the ratio \(\sqrt{\omega_b/\omega_i}\) and the overlap \(C\):

(1) When \(\sqrt{\omega_b/\omega_i} \leq C\), the minimum probability of failure \(Q = \omega_b + C\omega_i\) is achieved by selecting the following measurement operators: \(\Pi_a = 0, \Pi_b = |s\rangle \langle s| \) and \(\Pi_0 = |s\rangle \langle s|\). Here, state \(|s\rangle\) is never detected, and the optimal POVM becomes a von Neumann projective measurement. Actually, in this case, \(\omega_b > \omega_i\), the state of the detector is more likely to be in state \(\rho_D^a\), so we make the failure direction along \(\rho_D^a\) to obtain the minimum probability of failure. The joint probability is obtained as
\[
Q(b, b) = (1 - C^2)\frac{\sin^2 \frac{\beta}{2}(1 - S_i)}{1 + S_i \cos \beta},
\]
(25)

\[
Q(a, b) = 0,
\]
(26)

\[
Q(a, 0) = \frac{\cos^2 \frac{\beta}{2}(1 - S_i)}{1 + S_i \cos \beta},
\]
(27)

\[
Q(b, 0) = C^2\frac{\sin^2 \frac{\beta}{2}(1 - S_i)}{1 + S_i \cos \beta}.
\]
(28)

Then, the amount of WPI via the von Neumann projective measurement reads
and $r$. Is achieved by selecting the measurement $C$, the minimum probability of failure $Q = 2C \sqrt{\omega_a/\omega_b}$ is achieved by choosing the following measurement operators

$$
\Pi_a = \frac{1}{S^2} (1 - C \cot \frac{\beta}{2} \sqrt{1 + S_x}) |r\rangle \langle r|,
$$

$$
\Pi_b = \frac{1}{S^2} (1 - C \cot \frac{\beta}{2} \sqrt{1 + S_x}) |s\rangle \langle s|,
$$

$$
\Pi_0 = C \cot \frac{\beta}{2} \sqrt{1 + S_x} |r\rangle \langle r| + C \sqrt{1 - S_x} |s\rangle \langle s| + C (1 - C \cot \frac{\beta}{2} \sqrt{1 + S_x}) |s\rangle \langle r| + \Pi_a + \Pi_b
$$

This measurement is more general than the von Neumann projective measurement. Via equation (17), the joint probability reads

$$
Q(a, a) = \frac{\cos^2 \frac{\beta}{2} (1 + S_x)}{1 + S_x \cos \beta} (1 - C \cot \frac{\beta}{2} \sqrt{1 + S_x}),
$$

$$
Q(b, b) = \frac{\sin^2 \frac{\beta}{2} (1 - S_x)}{1 + S_x \cos \beta} (1 - C \cot \frac{\beta}{2} \sqrt{1 + S_x}),
$$

$$
Q(a, b) = Q(b, a) = 0,
$$

$$
Q(a, 0) = C \frac{\cos^2 \frac{\beta}{2} (1 + S_x)}{1 + S_x \cos \beta} \tan \frac{\beta}{2} \sqrt{1 + S_x},
$$

$$
Q(b, 0) = C \frac{\sin^2 \frac{\beta}{2} (1 - S_x)}{1 + S_x \cos \beta} \cot \frac{\beta}{2} \sqrt{1 + S_x}.
$$

Then the amount of WPI obtained from the POVM measurement is calculated as

$$
I_{\text{path}} = \frac{\cos^2 \frac{\beta}{2} (1 + S_x)}{1 + S_x \cos \beta} (1 - C \cot \frac{\beta}{2} \sqrt{1 + S_x})
$$

$$
\times \log \frac{1 + S_x \cos \beta}{\cos^2 \frac{\beta}{2} (1 + S_x)}
$$

$$
+ \frac{\sin^2 \frac{\beta}{2} (1 - S_x)}{1 + S_x \cos \beta} \cot \frac{\beta}{2} \sqrt{1 + S_x}.
$$

According to its definition given by equation (18).

(3) When $\sqrt{\omega_a/\omega_b} \geq 1/C$, the minimum probability of failure $Q = \omega_b + C \omega_a$ is achieved by selecting the measurement operators $\Pi_a = |r\rangle \langle r|$, $\Pi_b = 0$ and $\Pi_0 = |r\rangle \langle r|$. Here, the state $|r\rangle$ is never detected, and the optimal POVM becomes a von Neumann projective measurement. In this case, $\omega_b > \omega_a$.

The state of the detector is more likely to be in state $\rho^D_{\text{out}}$, so the failure direction is chosen along $\rho^D_{\text{out}}$ to obtain the minimum probability of failure. The joint probability is obtained as

$$
Q(a, a) = (1 - C^2) \frac{\cos^2 \frac{\beta}{2} (1 + S_x)}{1 + S_x \cos \beta}.
$$
The amount of WPI is given by

\[ \beta = -Q \frac{1}{C} \log \frac{1 + S_x \cos \beta}{\cos^2 \frac{\beta}{2}(1 + S_x)} \]  

(43)

Equations (29), (38) and (43) show that the WPI is a piecewise function of the parameters \( S_x, \beta, \) and \( C \). Although all the components of the Bloch vector determine the fringe visibility, only \( S_x \) occurs in the expression of the WPI, indicating that \( I_{\text{path}} \) is independent of the initial state of the quantum particle. In figure 4, we plot the WPI as a function of \( S_x \) and \( \beta \) with the overlap \( C = 1/3 \). The \( I_{\text{path}} \) under the conditions \( \sqrt{\omega_1/\omega_0} \in (0, C), (C, C^{-1}) \) and \( (C^{-1}, +\infty) \) is shown in the ranges in purple, brown and cyan in figure 4(a), respectively. The white range in figure 4(a) indicates that \( I_{\text{path}} \) is a discontinuous function of \( S_x \) and \( \beta \). It can be observed in figure 4 that \( I_{\text{path}} \leq 1 - C \) for any \( S_x \) and \( \beta \), and the position along the \( S_x(\beta) \) axis where \( I_{\text{path}} = 1 - C \) occurs varies with the different given \( \beta(S_x) \). In figure 5(a),

\[ Q(b, a) = 0, \]

\[ Q(a, 0) = C^2 \cos^2 \frac{\beta}{2}(1 + S_x) \]

\[ Q(b, 0) = \sin^2 \frac{\beta}{2}(1 - S_x) \]  

(40)

(41)

(42)

\[ I_{\text{path}} = (1 - C^2) \frac{\cos \frac{\beta}{2}(1 + S_x)}{1 + S_x \cos \beta} \log \frac{1 + S_x \cos \beta}{\cos^2 \frac{\beta}{2}(1 + S_x)} \]

(43)
we have plotted $I_{\text{path}}$ as a function of the parameters $\beta(S_x)$ and $C$ for a given $S_x(\beta)$. One also finds that $I_{\text{path}}$ is less than or equal to $1 - C$, $I_{\text{path}}$ decreases as $C$ increases, the position where the peak occurs is fixed for a different overlap $C$. \text{—i.e.} the peak appears at $\beta = 2\pi/3$ when $S_x = 1/2$ in figure 5(c)\text{—}and $S_x = -1/2$ when $\beta = \pi/3$ in figure 6(c).

From equation (38), we find that the maximum of $I_{\text{path}}$ can be achieved once $\cos \beta = -S_x$.

The wave-like and the particle-like property are quantitatively described by the fringe visibility $V$ in equation (10) and the WPI $I_{\text{path}}$ in equation (18). Since $V \leq C$ and $I_{\text{path}} \leq 1 - C$, we obtain the complementary relation

$$V + I_{\text{path}} \leq 1,$$

and the equals sign holds in equation (44) when $\cos \beta = -S_x$.

4. Conclusion

We have investigated the complementarity of the fringe visibility and the WPI in an MZI with one asymmetric BS. Although the fringe visibility measured in either two output ports is different, there exists an upper limit, i.e. $V \leq C$. The upper bound $C = |\text{tr}(U \rho_{\text{in}}^B)|$ is determined by the initial state of the detector and the unitary operator performed on it. To describe the maximum value of the fringe visibility $C$ we can achieve when the quantum system is initially in a pure state with $\cos \beta = -S_x$. To observe the particle-like behavior of this quantum system, a four-path interferometer must be introduced due to the asymmetrical BS2. The WPI is characterized by the WPI $I_{\text{path}}$, which is obtained via unambiguous discrimination of the state of the detector. Although $I_{\text{path}}$ is dependent on the asymmetric BS and the initial state of the quantum system, the WPI is bounded by the following inequality, $I_{\text{path}} \leq 1 - C$. The maximum $I_{\text{path}}$ is achieved when $\cos \beta = -S_x$. It is also found that $V + I_{\text{path}} \leq 1$.

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