Standard Model with Cosmologically Broken Quantum Scale Invariance

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Abstract: We argue that scale invariance is not anomalous in quantum field theory, provided it is broken cosmologically. We consider a locally scale invariant extension of the Standard Model of particle physics and argue that it fits both the particle and cosmological observations. The model is scale invariant both classically and quantum mechanically. The scale invariance is broken cosmologically producing all the dimensionful parameters. The cosmological constant or dark energy is a prediction of the theory and can be calculated systematically order by order in perturbation theory. It is expected to be finite at all orders. The model does not suffer from the hierarchy problem due to absence of scalar particles, including the Higgs, from the physical spectrum.

1. Introduction

In a recent paper [1] we have considered the possibility that scale invariance may be an exact symmetry in quantum field theory. The basic idea makes nontrivial use of the concept of cosmological symmetry breaking [2]. The possibility that scale invariance may be an exact symmetry in quantum field theory has been considered earlier by many authors [3–14]. An interesting proposal for implementing scale invariance in Standard Model has been introduced by Cheng and collaborators [15, 16]. In Refs. [15, 16] the authors proposed a locally pseudo-scale invariant Standard Model by introducing the Weyl

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vector meson. In Refs. [15, 16] the authors split the scale transformation into a pseudo-scale transformation and general coordinate invariance. The pseudo-scale transformation is similar to the conformal transformations studied earlier [4]. Hence if the action satisfies both pseudo-scale and general coordinate invariance it also obeys scale invariance. The Higgs particle is interpreted as the longitudinal mode of the Weyl meson and hence disappears from the particle spectrum. Phenomenological consequences of this model have also been studied in Refs. [1, 17–19]. Local scale invariance has also been studied in Refs. [20–27].

The phenomenon of cosmological symmetry breaking is inspired by the standard big bang model. Here, at the leading order, the universe is described by a homogeneous and isotropic Friedmann-Robertson-Walker (FRW) metric. The essential point is that the universe is a time dependent solution of the classical equations of motion. All physical phenomena take place in this background. It is not just the metric which might be time dependent. If we add some fields to the action besides gravity, then these fields may also acquire time dependence, as, for example, happens in the case of slow roll models of dark energy [28–36]. Hence to study any process we need to make a quantum expansion around this classical time dependent solution. The important point is that these classical fields need not take values which minimize the potential. It was shown in Ref. [2] that an expansion around such a background time dependent solution breaks some of the symmetries of the action. It was further argued [1, 2] that this is a particularly attractive way to break scale invariance. This phenomenon has most of the attractive features of the well known mechanism of spontaneous symmetry breaking. However the actual mechanism is very different. For example, in contrast to spontaneous symmetry breaking, we do not predict any zero mass Goldstone boson if the symmetry is broken cosmologically. The phenomenon of cosmological symmetry breaking is naturally implemented in the scale invariant Standard Model [15]. In this case, one finds that this model leads to both dark energy and dark matter [1, 19]. Within the framework of global scale invariance the possibility that the background curvature can lead to symmetry breaking has also been considered earlier [6, 8, 12, 37–43].

A fundamental problem with imposing scale invariance is that it may be anomalous [44–46]. Hence it is not clear whether the locally pseudo-scale invariant model proposed in Ref. [15] is meaningful quantum mechanically. In Ref. [3] it was conjectured that conformal invariance may not be anomalous. In Ref. [4], the authors showed that conformal invariance is not anomalous if it is suitably extended to arbitrary number of dimensions, provided the symmetry is broken spontaneously. We have also argued in Ref. [1] that pseudo-scale invariance need not be anomalous in theories which are cosmologically broken. Here we study scale invariance in the context of cosmological symmetry breaking. We also apply this to the Standard Model of particle physics with local scale invariance [15]. An interesting prediction of pseudo-scale invariance is that it does not admit a cosmological constant term in the action [1, 28, 43, 54]. Hence this symmetry might potentially solve the cosmological constant problem [47–53]. However cosmological constant is gen-
erated due to the phenomenon of cosmological symmetry breaking [1, 2]. If pseudo-scale invariance indeed holds at the quantum level then we expect that we should find finite results for cosmological constant at one loop. In a recent paper [54] we have explicitly demonstrated this result and computed the finite value of the one loop contribution to the cosmological constant. The calculation in Ref. [54] was performed in the adiabatic limit, where we assume that the background classical solution is very slowly varying with time. There also exist several other approaches to solving the cosmological constant problem [47,55–61,63]. The possibility that scale or conformal invariance might provide a solution to the cosmological constant problem has also been discussed in [28,43]. Here the author works within the framework of anomalous conformal invariance. The cosmological constant problem is solved by assuming a fixed point in the beta function. The model we consider in the present paper is very different since we impose exact local conformal invariance. We demonstrate that a locally conformal invariant extension of the Standard Model fits both the cosmological and high energy physics observations if the conformal invariance is broken cosmologically.

The essential idea can be captured by considering a simple model where we include only one real scalar field besides gravity. This simple model displays global scale invariance. The action for this model may be written as,

\[ S = \int d^4x \sqrt{-g} \left\{ \frac{1}{2} g^{\mu\nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{\lambda}{4} \Phi^4 - \frac{\beta}{8} \Phi^2 R \right\}, \]  

where \( R \) is the Ricci scalar and \( \Phi \) is a real scalar field. The model has no dimensionful parameter. In four dimensions the pseudo-scale or conformal transformation can be written as follows:

\[ x \rightarrow x, \]
\[ \Phi \rightarrow \Phi/\Lambda, \]
\[ g^{\mu\nu} \rightarrow g^{\mu\nu}/\Lambda^2. \]  

In Refs. [1, 2] the authors assumed an FRW background metric with scale factor \( a(t) \) and the curvature parameter \( k = 0 \). The model admits a classical solution [1],

\[ a(t) = a_0 \exp(H_0 t) \]  

with the scalar field,

\[ \Phi_0 = \sqrt{\frac{3\beta}{\lambda}} H_0, \]  

where \( H_0 \) is the Hubble constant, which is found to be independent of time in this case. Similar solutions have also been considered earlier [8,12,28,37,42,43]

We expand the scalar field \( \Phi \) around this classical solution,

\[ \Phi(x) = \Phi_0 + \phi(x), \]  

where \( \phi(x) \) represent the quantum fluctuations. Similarly the metric is expanded around its classical solution.
2. Quantum Scale Invariance

In this section we demonstrate that we can extend scale invariance as an exact symmetry in quantum field theory. Using dimensional regularization, we write a regulated action which is exactly scale invariant. Here scale transformation is extended to arbitrary dimensions using the earlier works [1, 4]. However our precise proposal for the regulated action is different. We propose the following regulated action in \( d = 4 - \epsilon \) dimensions,

\[
S = \int \! d^d x \sqrt{-\bar{g}} \left( \frac{1}{2} \bar{g}^{\mu \nu} \partial_\mu \Phi \partial_\nu \Phi - \frac{\lambda}{4} \Phi^4 (\bar{R}^2)^{\epsilon/4} - \frac{\beta}{8} \Phi^2 \bar{R} \right).
\]

Here we are using the notation of Ref. [64] and denote all quantum gravity variables, such as \( g_{\mu \nu}, R \) etc, with a bar. This action is invariant under the generalized pseudo-scale or conformal transformation,

\[
\begin{align*}
x & \to x, \\
\Phi & \to \Phi / \Lambda, \\
\bar{g}^{\mu \nu} & \to \bar{g}^{\mu \nu} / \Lambda^{b(d)}, \\
\bar{g}_{\mu \nu} & \to \bar{g}_{\mu \nu} \Lambda^{b(d)}, \\
A_\mu & \to A_\mu, \\
\Psi & \to \Psi / \Lambda^{c(d)},
\end{align*}
\]

where \( b(d) = 4 / (d - 2) \) and \( c(d) = (d - 1) / (d - 2) \). Here we have also included the transformation law for vector and spinor fields, which we will need later. The term \( (\bar{R}^2)^{\epsilon/4} \) which multiplies the \( \Phi^4 \) term is treated by expanding around the classical solution. We have [64],

\[
\begin{align*}
\bar{g}_{\mu \nu} & = g_{\mu \nu} + h_{\mu \nu}, \\
\bar{R} & = R + h_\beta^{\beta \alpha} - h_\alpha^{\beta \alpha} - h_\alpha^{\nu} R_\nu^{\alpha} + \ldots,
\end{align*}
\]

where \( g_{\mu \nu} \) is the classical metric and \( R \) the classical curvature scalar. Here we shall assume the classical metric to be the FRW metric. In this case we find that \( R = -12 H_0^2 \). In Eq. \( \text{S} \), "\( \cdot \)" denotes covariant derivatives.

The transformation, Eq. \( \text{7} \) is similar to what was also proposed in Ref. [4]. However our regulated action differs from earlier proposals [1, 4] since we use the Ricci scalar, rather than the scalar field, raised to a fractional power to regulate the quartic coupling term. The advantage of regulating in this manner is that as long as we neglect quantum gravity effects, the action is manifestly renormalizable. When we expand \( \bar{R} \) around its classical solution, then at the leading order we simply recover the standard \( \Phi^4 \) interaction. At higher orders, however, the term \( (\bar{R}^2)^{\epsilon/4} \) leads to additional interaction vertices, involving scalar fields and all possible powers of the gravitational field. This is an expansion in powers of \( \epsilon \) and hence the corresponding couplings go to zero in four dimensions. These additional vertices make the higher loop analysis of quantum gravity somewhat more complicated. However these contributions are suppressed and as long as we are not
interested in quantum gravity corrections, they can be ignored. For the action given in Eq. theses corrections are suppressed by powers of $1/\beta$. In the regularization proposed in [1, 4], however, we would pick up additional powers of the scalar field in the action. These would generate additional interaction vertices which make the loop analysis of the theory more complicated. Furthermore renormalizability of the theory is not obvious even if we ignore quantum gravity contributions.

The one loop analysis of the model, Eq. 6 has been performed in Ref. [54]. In Ref. [54], the authors used a different regularization and hence in the present case we might expect some additional terms at one loop. These will arise due to the expansion of the term $(\bar{R}^2)^{\epsilon/4}$ which generates additional vertices. Since the additional vertices that get generated in the next to leading order term are proportional to $\epsilon$, it is clear that all additional contributions are finite at one loop. These can contribute only to the cosmological constant. For all other quantities these give contributions suppressed by powers of $1/\beta$. The cosmological constant in the model, Eq. 6 has been explicitly shown to be finite at one loop [54], as expected from pseudo-scale invariance.

Finally we define the path integral measure. The scalar field measure may be written as

$$\Pi_x D \left[ (-g)^{1/4} (R^2)^{1/4} \Phi(x) \right]$$

Here we have used the classical curvature scalar in order to scale the measure proposed in [46] such that it is invariant under the generalized pseudo-scale transformations. It is clear that this is possible only if we are expanding around a curved background. Alternatively we could use the classical field $\Phi_0$ to scale the measure, as long as it acquires a non-zero value. It is clear that both the action and the measure are exactly invariant under the generalized pseudo-scale transformations. Here we specialize only to the scalar field measure. The full quantum gravity measure may also be constructed following Refs. [65, 66].

We emphasize that here we have shown that the theory is exactly invariant under a generalized pseudo-scale transformation, displayed in Eq. Hence this symmetry transformation is not anomalous. The regulated action, Eq. makes sense as long as we are expanding around a non-trivial gravitational background. If we expand around a flat background, $R = 0$, then the regulator is ill-defined. Alternatively we may use the regularization proposed in [1, 4]. In this case the regulator is well defined, as long as we expand around a non-zero value of the classical scalar field.

### 3. Standard Model with Local Scale Invariance

We now consider the locally scale invariant extension of the Standard Model [15]. The action in four dimensions may be written as,

$$S = \int d^4x \sqrt{-g} \left[ -\frac{\beta}{4} \mathcal{H}^4 \mathcal{H}' + g^{\mu\nu} (D_\mu \mathcal{H})^\dagger (D_\nu \mathcal{H}) - \frac{1}{4} g^{\mu\nu} g^{\alpha\beta} (A_\mu A_\nu) \right]$$
\[
B_{\mu \nu}B_{\nu \rho} - \frac{1}{4} g^{\mu \rho} g^{\nu \sigma} \varepsilon_{\mu \nu \rho \sigma} - \lambda (H^\dagger H)^2.
\]

Here \(H\) is the standard Higgs field doublet,
\[
H(x) = \begin{pmatrix}
h_1(x) \\
h_2(x)
\end{pmatrix}.
\]

The covariant derivative,
\[
D_\mu = \partial_\mu - ig\mathbf{T} \cdot A_\mu - ig'Y^2 B_\mu - f CS_\mu,
\]

where \(B_\mu\) is the \(U(1)\) gauge field, \(A_\mu = \tau^a A^a_\mu\) is the \(SU(2)\) gauge field multiplet and \(S_\mu\) is the Weyl vector meson. As usual \(\mathbf{T}\) denotes the \(SU(2)\) generators, \(Y\) the \(U(1)\) hypercharge and \(C\) the scaling or conformal charge. For the Higgs field \(C = 1\). The field tensors \(A_{\mu \nu}\), \(B_{\mu \nu}\) and \(E_{\mu \nu}\) are the field strength tensors for the gauge fields \(A_\mu\), \(B_\mu\) and \(S_\mu\) respectively. The scalar, \(R'\), is related to the Ricci scalar, \(R\), by the relationship,
\[
R' = R - 6f S^\kappa \varepsilon_{\kappa} - 6f^2 g^{\mu \nu} S_\mu S_\nu.
\]

In the action, Eq. 10, we have not included the spinor fields. These can be easily added [15].

The classical equations of motion admit a solution with the FRW scale parameter \(a(t)\) given by Eq. 3 and the Higgs field,
\[
H_0^\dagger H_0 = \frac{3\beta^2}{2\lambda} H_0^2
\]

and all the remaining fields equal to zero. As before the Hubble parameter \(H_0\) is constant in this case. The Weyl vector field may also be nonzero depending on the initial conditions. In this case also one may choose a gauge such that \(H_0^\dagger H_0\) is constant [19]. In general, however, the Hubble parameter depends on time. As long as it varies slowly we can use the adiabatic approximation and ignore its time dependence while computing the Feynman diagrams. Similarly we can treat the time dependence of the scale factor \(a(t)\) in the adiabatic approximation [54].

The classical solution, Eq. 13, breaks pseudo-scale invariance. As we expand around this classical solution we find that the gravitational constant is generated. The electroweak symmetry is broken and the \(W\) and \(Z\) bosons acquire masses [15]. The \(W\) boson mass is found to be
\[
M_W^2 = g^2 (H_0^\dagger H_0).
\]

The Weyl meson also acquires a mass,
\[
M_S^2 = 2f^2 (H_0^\dagger H_0),
\]

due to the breakdown of pseudo-scale invariance. The Higgs boson disappears from the particle spectrum and acts like the longitudinal mode of the Weyl vector meson [15]. This phenomenon was illustrated in the context of cosmological symmetry breaking in Ref. [1]. The fermions acquire masses by their Yukawa interactions. The model predicts a cosmological constant whose value can be adjusted by fixing \(\lambda\) to fit the current cosmological
observations. Furthermore the Weyl meson acts like a dark matter candidate \cite{15, 19}. Hence the model fits all the particle and cosmological observations. The classical solution for the Higgs field is related to the Planck Mass by the formula,

\[ \beta \left( \mathcal{H}_0^\dagger \mathcal{H}_0 \right) = \frac{M^2_{\text{Pl}}}{4\pi}. \]  

(15)

Furthermore the model predicts dark energy,

\[ \rho_\Lambda = \lambda \left( \mathcal{H}_0^\dagger \mathcal{H}_0 \right)^2. \]  

(16)

The important issue that we need to settle is whether the model is consistent quantum mechanically. This would be true as long as scale invariance is not anomalous. Based on earlier works \cite{1, 4, 54} and the arguments presented in the previous section, we expect this to be true. We next explicitly write down the regulated action in \( d \) dimensions. The action in \( d \) dimensions may be written as,

\[ S = \int d^dx \sqrt{-\bar{g}} \left[ -\frac{\beta}{4} \mathcal{H}^\dagger \mathcal{H} \mathcal{R}^l + g^{\mu\nu}(D_\mu \mathcal{H})^\dagger (D_\nu \mathcal{H}) - \frac{4}{1} \bar{g}^{\mu\nu} \bar{g}^{\alpha\beta} (A_{\mu\alpha} A_{\nu\beta} - \mathcal{H}^\dagger \mathcal{H}^2 \mathcal{R}^l - \frac{\lambda}{8}) \right], \]  

(17)

where, as before, we denote all quantum gravity variables with a \( \bar{\text{bar}} \). Here we have used the scalar \( \mathcal{R}^l \), which transforms covariantly under pseudo-scale transformations, in order to regulate the action. The regulated fermionic action, corresponding to the kinetic energy terms, can also be written easily in \( d \) dimensions as,

\[ S_{\text{fermions}} = \int d^dx \sqrt{-\bar{g}} \left( \bar{\Psi} \gamma^\nu e_{\nu}^a \left( D_\mu - \frac{1}{2} \sigma_{\mu\nu} \sigma_{\nu} e_{\rho}^a \right) + \right) \Psi, \]  

(18)

where \( e_{\nu}^a \) is the tetrad. The scaling charge for fermion and the tetrad \( e_{\nu}^a \) are \( c(d) \) and \( b(d)/2 \), respectively. In the connection \( \Gamma^\rho_{\mu\nu} \) the derivatives of the metric are to be replaced by the suitable pseudo-scale covariant derivatives with the suitable charge which can be obtained from Eq. \cite{7}. The Yukawa interaction terms may be written in \( d \) dimensions as,

\[ S_{\text{Yukawa}} = \int d^dx \sqrt{-\bar{g}} X_{ab} \bar{\Psi}_a \mathcal{H} \Psi_b \left( \mathcal{R}^l \right)^{\epsilon/8} + \text{h.c.} \]  

(19)

for two fermion species \( a \) and \( b \). Here \( X_{ab} \) are the Yukawa couplings.

The model is renormalizable as long as we ignore quantum gravity contributions. The gravitational sector of the model is expected to be non-renormalizable. It is very economical since it only introduces only one vector field, the Weyl meson, besides the Standard Model fields. The model solves the hierarchy problem since it contains no physical scalar fields. By hierarchy problem we refer to the stability of the electroweak symmetry breaking scale to loop corrections. The problem arises primarily due to the presence of physical scalar fields in the particle spectrum \cite{67}. In the present model, all the scalar fields get eliminated and do not appear as a physical particle, thereby solving
the hierarchy problem. The model predicts absence of the Higgs particle from the physical spectrum. The Higgs particle essentially acts as the longitudinal mode of the Weyl vector meson. As in the case of the Standard Model, the particle content of the model is best seen by choosing a particular gauge, as discussed in [1, 15]. This is similar to the unitary gauge in the Standard Model and all unphysical degrees of freedom do not appear in this gauge. In this gauge the Higgs field does not appear in the Lagrangian and we recover the standard Einstein action besides the matter terms. However for loop calculations it is convenient to use a different gauge where the scalar fields appear as internal lines but never appear as physical particles. Hence for loop calculations it may be better to work in a general gauge and then specialize to the unitary gauge at the end of the calculation.

The cosmological constant generated in this model is expected to be finite at all orders in perturbation theory due to scale invariance. In Ref. [54] we have explicitly demonstrated this at one loop order in a toy model with a real scalar field. In order to fit the small value of observed dark energy, we need to choose a very small value of the coupling $\lambda$. The choice of small value of the coupling is a shortcoming of the model. However once this is chosen we do not expect fine-tuning at loop orders. The contribution due to the Higgs field at higher orders in perturbation theory is extremely small since the coupling is so small. Furthermore we also need to include loop corrections due to the fermion and vector fields. These are likely to be constrained due to scale invariance. We see this as follows. If a field is minimally coupled to gravity then its contribution to the Einstein equations arise through the energy-momentum tensor, $T_{\mu \nu}$. This is related to the conformal current by the relationship

$$ (J^\mu)_{,\mu} = T^\mu_\mu = 0. \quad (20) $$

The fact that the trace of the energy momentum is zero as an operator identity clearly imposes constraints on the size of the vacuum energy that any field might contribute. Let us assume an isotropic and homogeneous fluid such that the expectation value of the trace, $< T^\mu_\mu > = \text{diag}(\rho, -p, -p, -p)$, where $\rho$ is the total energy density and $p$ the total pressure. We first consider a field which contributes only to relativistic energy density, with the equation of state $p_R = \rho_R/3$, and to the vacuum energy density, with $p_V = -\rho_V$. Here $p_R$ is the pressure due to the relativistic gas and $p_V$ due to vacuum. Similarly $\rho_R$ is the energy density due to the relativistic gas and $\rho_V$ due to vacuum. From Eq. (20) we obtain the constraint, $\rho - 3p = 0$, where $\rho = \rho_R + \rho_V$ and $p = p_R + p_V$. In this case the constraint implies that $\rho_V = 0$. Similarly if we assume that a field contributes only to non-relativistic and vacuum energy density, we find that $\rho_V = -\rho_{NR}/4$, where $\rho_{NR}$ denotes the non-relativistic energy density.

Hence we find that for a minimally coupled field the contribution to vacuum energy is constrained and hence requires no fine tuning. Such a constraint does not apply to the Higgs field since it does not couple minimally. The loop contribution due to Higgs is small as already explained. We shall explore the contribution due to Weyl meson in a separate publication. We expect it also to be constrained by scale invariance. Hence we
expect that no fine tuning may be required at loop orders.

The main shortcoming of the model is that the parameters $\lambda$ and $1/\beta$ are found to be very small compared to unity. The model provides no explanation for their extreme values. Furthermore it is not clear how inflation arises in this model, although an inflationary solution to this model has also been suggested [68]. The model describes both the high energy physics as well as the cosmological observations. Hence it is an attractive generalization of the Standard Model of particle physics.

4. Conclusions

In this paper, following earlier works [1, 4, 54], we have argued that scale invariance can be implemented exactly in quantum field theory. We considered a locally scale invariant extension of the Standard Model [15] and argued that it fits both the particle physics and cosmological data. Furthermore the model does not suffer from fine tuning problems due to absence of scalar particles in the particle spectrum and due to scale invariance. The cosmological constant is expected to be finite at all orders in perturbation theory and hence a prediction of the model. At one loop this was explicitly demonstrated in Ref. [54].

The model is particularly attractive since it proposes only one extra field, namely the Weyl vector meson, besides the Standard Model particles. This additional field also serves as a dark matter candidate. The model does contain some parameters which take extreme values. Both the parameters $\lambda$ and $1/\beta$ are very small compared to unity. So far we have no explanation for why they are so small. Furthermore it is not clear how inflation arises in this model. The model may be extended to include supersymmetry although it is not necessary for the solution of the hierarchy problem. It may also be interesting to explore grand unified models with local scale invariance.

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