Irreversibility of a measurement procedure

S V Muravyov
Department of Computer-aided Measurement Systems and Metrology
National Research Tomsk Polytechnic University
Tomsk, 634050, Russia
E-mail: muravyov@camsam.tpu.ru

Abstract – It is shown in the paper that any measurement procedure possesses inherent generic property of irreversibility. The property consists in impossibility of a process where a transition is possible from a finite state of a system to its initial state through the same states but in reverse order. The irreversibility is accompanied by property of sequentiality that means multistage character of a measurement. Awareness of these properties can provide some useful ways and tools for development of new measurement methods.

1. Introduction
It is well known that any measurement process is based on operation of a comparison. This is not a single elementary operation defining the measurement, however its explicit role is generally recognized, see, for example definition of measurement in the 3-rd edition of International Vocabulary of Metrology (VIM):
"process of experimentally obtaining one or more quantity values that can reasonably be attributed to a quantity",
which is provided with the note:
"measurement implies comparison of quantities and includes counting of entities".

When to try to go deeply into the sense of term 'comparison', one may arrive to outcomes appearing to be useful for understanding in more detail a nature of measurement.

Earlier in this research direction, a discrete-mathematical model of measurement procedure had been proposed and investigated by the paper author [1,2]. The formal structure, called PSIQ-model, consists of four elementary operations as follows, see Fig. 1: partitioning of the initial (before measurement) uncertainty interval \( S \) (to that the sought quantity value \( x \) belongs) into subintervals; selecting one of the subintervals and an element \( y_i \in S \) in this subinterval as its representative; inverse mapping the image \( y_i \) of the value \( x \) to the empirical set, i.e. \( x_i = f^{-1}(y_i) \); and questioning whether the result of the inverse mapping \( x_i \) is similar to the element \( x \), fixed in the empirical system, using appropriate measure of similarity. The PSIQ-model allows to describe in concise and formal way different measurement processes implemented in various scales. In this paper we restrict a consideration by only ratio scale measurements.

This paper will extend the treatment of a measurement procedure in form of the PSIQ-model involving the notion of irreversibility.

2. Irreversibility
Reversible is a process where a transition is possible from a finite state of a system to its initial state through the same states but in reverse order. If this transition is not possible, the corresponding process is irreversible [3]. Ice melting and metal fusing can be examples of reversible processes in some extent. Burning, diffusion, thermal conduction and viscous flow are typical examples of irreversible processes.

The concepts of reversibility and irreversibility arise most frequently in thermodynamics, where a reversible process can be characterized as follows: if in forward stroke, the system, at some elementary por-
tion of its state transition, absorbs heat and does work, then, in back stroke, at the same portion, the system releases heat and work is done over it. However, the described situation is ideal one as upon termination of the cycle (that is, the forward and backward strokes) the system returns into the initial state but in its surroundings some changes always take place due to inevitable loss of heat energy. That is why all natural processes are said to be irreversible [3].

In quantum measurement theory [4], one of principal setting is that a dequantization process is irreversible. Dequantization is defined as a transformation of a quantum sensing system input signal in such a way that the signal could be considered to be classical at an instrument output, see also [5-7]. Thus, overall quantum measurement process consisting of dequantization and classical parts is irreversible.

We claim now that any measurement procedure is irreversible and will show this in terms of PSIQ-model using a mental experiment described below.

2.1. Necessary notations and measurement procedure
In the course of the measurement procedure a measurement problem is solving that is

$$\min \{ D(f^{-1}(y), x) \mid y = f(x) \in B, B \subseteq S \},$$

where

- $y = f(x) \in B$ is a form of mapping of an empirical set $A$ to a numerical set $B$ [1],
- $D(f^{-1}(y), x) = |f^{-1}(y) - x| \leq u$ is a distance between preimage of numerical value $y$ and (empirical) measurand $x$, and
- $u$ is a given measurement uncertainty.

Denote the operation of partitioning $B$ into $k$ subintervals $T_i$ through

$$B = \bigcup_{i=1}^{k} T_i,$$

where $k$ can take different values in dependence on particular kind of partitioning.

We will encode the numerical value $y = f(x)$ in the following way:

$$y = f(x) = < q_1, q_2, \ldots, q_m >,$$

where $m$ takes values in dependence on particular kind of partitioning and

$$q_i = \begin{cases} 0 \text{ if } y \in T_i \\ 1 \text{ if } y \notin T_i \end{cases}.$$
In order to validate expression (4) we suppose that there is a possibility to judge of a membership of \( y \) in \( T \) by result of comparing \( x \) and \( g \), where \( g = f^{-1}(y) \) is a current preimage of \( y \). That is if \( x \geq g \) then \( y \notin T \) and if \( x < g \) then \( y \in T \). It means that expression (4) is equivalent to

\[
q_i = \begin{cases} 0 & \text{if } x < g \\ 1 & \text{if } x \geq g \end{cases}.
\]

Finally, we need a method of implementing transformation of the current numerical value \( y \) into its current preimage \( g \), i.e. \( g = f^{-1}(y) \). This can be done accounting the length (or cardinality) \( |T_i| \) of the current interval \( T_i \), that is,

\[
g_i = \begin{cases} g_{i-1} - |T_i| & \text{if } x < g \\ g_{i-1} + |T_i| & \text{if } x \geq g \end{cases}.
\]

Measurement procedure:

\[
y \leftarrow <0,0,\ldots,0> \quad T_0 \leftarrow \{0, 1, \ldots, y_n\} \quad // \text{initialization}
\]

DICHOTOMY (1, 0)

\[
\text{procedure DICHOTOMY}(i, g):
\]

P: \( T_{i-1} \leftarrow T_i^1 \cup T_i^2, |T_i^1| = |T_i^2| \);

S: \( T_i \leftarrow T_i^1 \)

I: \( g \leftarrow g + |T_i| \)

Q: \( \text{if } x \geq g \text{ then } q_i \leftarrow 1, T_i \leftarrow T_i^2 \)
\text{else } g \leftarrow g - |T_i| \)
\text{if } |T_i| > 1 \text{ then DICHOTOMY}(i + 1, g)

Figure 2. Formal view of the measurement procedure in terms of PSIQ-model.

2.2. Mental experiments with the measurement procedure

Let the empirical system is modeled by set \( A \subseteq \mathbb{N}_0 \), \( A = \{0, 1, 2, \ldots, 7\} \), and an element \( x \in A \), \( x = 4 \), is fixed (see Fig. 3). Measurement procedure shown in Fig. 2 should identify the element \( x = 4 \) among other elements of \( A \) and assign for it an appropriate numerical value \( y = <q_1, q_2, q_3> \).

Particular steps and corresponding current solutions of the procedure are reduced to Table 1 and additionally illustrated by Fig. 3. One can observe that the procedure correctly identifies the empirical element \( x = 4 \) and determines the right numerical value \( y = <1, 0, 0> \).

In order to make sure that the considered procedure is reversible one can fulfill a backward transition from its last step to its initial step. In fact, it is possible to move in inverse order through the same current
solutions as shown in Table 1 taking into account that expression (6) should be transformed to its inverse form

\[
g_{i-1} = \begin{cases} 
  g_i + |T_i| & \text{if } x < g_i \\
  g_i - |T_i| & \text{if } x \geq g_i 
\end{cases}
\]  

(7)

| Table 1 |
| --- |
| **Ideal measurement procedure steps** |

| i-th step | \( T_i \) | \( y \notin T_i \) | \( q_i \) | Current solutions |
| --- | --- | --- | --- | --- |
| 0 | \{0, 1, 2, 3, 4, 5, 6, 7\} | initial interval \( B = T_0; g = 0 \) |
| 1 | \{0, 1, 2, 3\} \cup \{4, 5, 6, 7\} | \( \varepsilon \) | 1 | partition and selection of \( T_i^1 = \{1, 2, 3, 4\} \); \( g = 4; x = g; q_1 = 1 \Rightarrow \) selection of \( T_i^2 = \{4, 5, 6, 7\} \); |
| 2 | \{4, 5\} \cup \{6, 7\} | \( \varepsilon \) | 0 | partition and selection of \( T_i^1 = \{4, 5\} \); \( g = 4 + 2 = 6; x < g; q_2 = 0 \Rightarrow g = 6 - 2 = 4 \) |
| 3 | \{4\} \cup \{5\} | \( \varepsilon \) | 0 | partition and selection of \( T_i^1 = \{4\} \); \( g = 4 + 1 = 5; x < g; q_3 = 0 \Rightarrow g = 5 - 1 = 4 \) \( y = <1,0,0> \) |

![Figure 3](image)

Figure 3. Current partitions and solutions of the measurement procedure.

Clear that the considered procedure is ideal one as it was implicitly supposed that comparison of values \( x \) and \( g \) in expressions (5), (6) and (7) is fulfilled without errors. Now let us suppose \( g \) have an additive error \( \varepsilon \) that accidentally takes values \( \pm 1 \). Table 2 shows that presence of \( \varepsilon \) catastrophically changes operation of the measurement procedure which leads, in this case, to the wrong result \( y = <0,1,1> \).

It is evident that the presence of \( \varepsilon \) makes the considered measurement procedure to be irreversible as, in this case, both forward and backward transitions between PSIQ-model steps are unpredictable.

Generally speaking, irreversibility of this kind of measurement procedure immediately follows from the fact that inverse mapping \( g' = f^{-1}(y) \) never coincides with preimage \( g \) since if \( g \) is erroneous there are no empirical conditions able to guarantee validity of the hypothesis \( g' = g \) [8].
Table 2
Steps of the measurement procedure with erroneous $g$

| $i$-th step | $T_i$ | $y_{\notin T_i}$ | $q_i$ | $\varepsilon$ | Current solutions |
|-------------|-------|-----------------|------|-------------|------------------|
| 0           | $\{0, 1, 2, 3, 4, 5, 6, 7\}$ |               |      |             | initial interval $B = T_0$ |
| 1           | $\{0, 1, 2, 3\} \cup \{4, 5, 6, 7\}$ | $\varepsilon$ | 1    | +1          | partition and selection of $T_1^1 = \{0, 1, 2, 3\}$; $g = 5$; $x < g$; $q_1 = 0 \Rightarrow g = 5 - 4 = 1$ |
| 2           | $\{0, 1\} \cup \{2, 3\}$ | $\varepsilon$ | 0    |             | partition and selection of $T_2^1 = \{0, 1\}$; $g = 1 + 2 = 3$; $x \geq g$; $q_2 = 1 \Rightarrow$ selection of $T_2^2 = \{2, 3\}$ |
| 3           | $\{2\} \cup \{3\}$ | $\varepsilon$ | 0    | -1          | partition and selection of $T_3^1 = \{2\}$, $g = 3$; $x \geq g$; $q_3 = 1 \Rightarrow$ selection of $T_3^2 = \{3\}$, $y = <0,1,1>$ |

3. Sequentiality

Sequentiality of measurement procedure means its multistage character which can become apparent as iterativity or recursivity.

This follows from presence of the necessary elementary operation of partition in a measurement procedure and is evident from consideration of corresponding PSIQ-model.

The number of partitioning intervals can be infinite if a measurement accuracy is infinite. However, under infinite accuracy of some particular measurement result, general description of the empirical system becomes rougher, see, e.g. [9].

4. Conclusion

Irreversibility and sequentiality are deserving attention properties of any kind of measurement procedures. Awareness of them can provide some useful ways and tools for development of new measurement methods, see, e.g. [10].

Acknowledgment

This work was partly supported by the Russian Science Foundation, Project # 14-19-00926.

References

[1] Muravyov S V and Savolainen V 1996 Discrete-mathematical approach to formal description of measurement procedure Measurement 18 71-80.
[2] Muravyov S V and Savolainen V 1996 Some structural properties of formal model of measurement procedure Measurement 18 81-7.
[3] Atkins P W 1997 The Second Law W.H. Freeman & Co. New York.
[4] Vorontsov Yu I 1989 Theory and methods of macroscopic measurements Nauka Publishers Moscow
[5] The Measurement Problem. Retrieved February 15, 2014, from Information Philosopher Web site http://www.informationphilosopher.com/problems/measurement.
[6] Kadomtsev B B 2003 Classical and quantum irreversibility Physics-Uspekhi 46 1183-201.
[7] Antonov V and Kondratyev B 2011 Entropy and irreversibility in classical and quantum mechanics Journal of Modern Physics 2 519-32.
[8] Muravyov S V 2012 Chaotic results of multidimensional ordinal measurements *Proc XX IMEKO World Congress (September 9-14, 2012, Busan, Republic of Korea)* 3 2071-74.

[9] Knorring V G 2013 *Istoriya i metodologiya nauki i tekhiki* Saint-Petersburg Polytechnic University Press (in Russian).

[10] Srivastav A, Ray A and Gupta S 2007 Irreversibility-based measure of slowly evolving anomalies *American Control Conference (9-13 July 2007, New York)* 3228-33.