Summary: We prove that, for any finite set $A \subset \mathbb{Q}$ with $|AA| \leq K|A|$ and any positive integer $k$, the $k$-fold product set of the shift $A + 1$ satisfies the bound

$$|\{(a_1 + 1)(a_2 + 1) \cdots (a_k + 1) : a_i \in A\}| \geq \frac{|A|^k}{(8k^4)^k K}.$$ 

This result is essentially optimal when $K$ is of the order $c \log |A|$, for a sufficiently small constant $c = c(k)$. Our main tool is a multiplicative variant of the $\Lambda$-constants used in harmonic analysis, applied to Dirichlet polynomials.

MSC:

- 11P70 Inverse problems of additive number theory, including sumsets
- 11B30 Arithmetic combinatorics; higher degree uniformity

Keywords:

- Dirichlet polynomials
- product sets

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