Excluded Volume Model for Quarkyonic Matter: 3-Flavor Baryon-Quark Mixture

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The single flavor excluded volume model based on the effective size of baryons reproduces the hard-soft density evolution of equation of state (EoS) required by the recent studies of GW170817. This phenomenological model basically realizes the concept of Quarkyonic matter which is introduced from large $N_c$ gauge theory for dense matter. Enhanced nucleon interactions and dynamically generated quark degrees of freedom can reproduce the hard-soft evolution of EoS. In this paper, we extend the excluded volume model to three flavor system by considering electromagnetic charge and possible $\beta$-equilibrium in order to obtain a proper description for the hard-soft behavior of the EoS inferred from the gravitational waves observations.
I. INTRODUCTION

Recent observations of GW170817 and subsequent analyses \[1\] provided important clues for understanding dense nuclear matter. The results strongly imply that the equation of state (EoS) at the highest accessible densities should be hard enough to support $2M_{\odot}$ state \[11, 12\]. At densities somewhat lower than the maximum density, the stiffness should be moderated to satisfy $R_{1.4} \leq 13.5$ km inferred from the tidal deformability observed at the neutron star inspiral \[2\] \[10\]. It seems hard to satisfy both of the constraints. As the nuclear matter density increases, new degrees of freedom are likely to emerge under enhanced energy density of the system. Emergence of new particles usually makes a softer EoS \[13, 14\], as the created quasi-particles are taking low momentum phase space and such massive states, like hyperons and $\Delta$ isobars, have various decay channels which usually lead to lower Fermi level \[12\] \[16\]. Therefore, certain repulsive nuclear interactions are required to support the hard EoS at high density circumstance.

However, the neutron star radius problem cannot be solved only by repulsive interaction. Actually, we need other physical properties to solve the problem. The aforementioned observation confined the possible range of tidal deformability in 90% confidence level \[1\] \[2\]. Subsequent analyses suggested even more compact neutron stars \[3\] \[4\] \[6\] \[7\] \[10\], which require softened EoS beyond some intermediate density regime where the hard EoS is supported. By some unknown physical process, the strong repulsion would be turned off or some kind of phase transition to quark matter can be introduced to explain the observation. However, even if one admits this sudden change, there will still remain debates about the signals such a hypothetical phase transition. As a candidate for solution, it is worth to consider Quarkyonic-like model \[17\] \[20\] which naturally generates hard EoS.

The concept of Quarkyonic matter is based on large $N_c$ quantum chromodynamics (QCD) \[21, 22\]. In the large $N_c$ limit, the Debye screening length diverges $r_{\text{Debye}} \sim O(\sqrt{N_c}) \rightarrow \infty$, as the quark loop is suppressed by $1/N_c$ \[17\]. Thus, if there is a large quark Fermi sphere ($T \rightarrow 0$), the quasi-quark states near the Fermi surface whose momenta are distributed in the range of confinement ($|\vec{k}_{i} - \vec{k}_{j}| < \Lambda_{\text{QCD}}$), $|\vec{k}_{i}| \sim k_{F}^Q$ where $i, j = \{1, \cdots, N_{c}\}$ will be confined in the baryon-like state through similar mechanism to vacuum confinement. In this system, the quark wave functions have two distinguished configurations: almost free quarks filled from the low momentum phase space and the confined waves in the baryon-like state. The confined quarks on the surface are aligned to the momentum direction of baryon-like state ($k_{F}^B \simeq N_{c}k_{F}^Q$). These confined states are understood as Fermi shell distributions of baryon-like states. The transition from the nuclear matter to Quarkyonic matter occurs at few times of nuclear matter density $\rho_{0}$. In the low density regime ($k_{F}^B < \Lambda_{\text{QCD}}$), all the quarks are confined in the nucleon as the quarks can be randomly distributed in the range of $|\vec{k}_{i} - \vec{k}_{j}| < \Lambda_{\text{QCD}}$. Whence $k_{F}^B \sim O(\Lambda_{\text{QCD}})$, quarks begin to fully occupy the low momentum phase space. The saturated states form the Fermi sphere of quarks and, by Pauli’s principle, the momenta of confined quarks should be larger than the saturated momenta, which leads to the shell-like distribution \[19\] \[20\]. In this transition, the total baryon number and energy density are smoothly varying, but the chemical potential of the confined states is suddenly enhanced by large $N_c$ since ($k_{F}^B \sim O(\Lambda_{\text{QCD}}) \rightarrow N_{c}k_{F}^Q$). This is not a first-order phase transition as the chemical potential (intensive variable) is suddenly enhanced during the transition and smooth energy density and density (extensive variable) is expected \[17\]. From this point, most of baryon number increase is taken by the quark degrees of freedom and the shell-like configurations eventually disappear at extreme density limit ($k_{F}^Q \sim O(\sqrt{N_c}\Lambda_{\text{QCD}})$), as the Debye screening begins to block the confinement process ($r_{\text{Debye}} \sim O(N_{c}^{-\frac{1}{2}})$).

This concept was applied to describe the hard-soft density evolution of EoS in the previous literature \[13, 20\]. The shell-like distribution satisfies the aforementioned requirements and leads to a plausible EoS \[19\]. If only the hard-soft density behavior of the Quarkyonic EoS is considered, some strong mean-field potential expressed in polynomials of $\rho_{Q}$ could be introduced. However, such systems have the intrinsic singularity at infinite density and, even if the quark degrees of freedom are allowed, the baryon and quark density simultaneously increases in the physically relevant density regime. This kind of potential cannot lead to soft enough EoS and the thin shell-like distribution. If one considers the hard-core repulsive interaction as suggested and calculated in Refs. \[23\] \[26\] \[31\] \[40\], the effective scale of the hard-core repulsion can be measured by some effective size of the baryon, where spatial overlap is not allowed by intrinsic singularity. The single flavor excluded volume model for the dense baryon state \[20\] dynamically generates the quark degrees of freedom, and the subsequently appearing shell structure provides the hard-soft density evolution of EoS as required in previous studies \[3\] \[13\] \[31\] \[47\].

In this paper, we extend the excluded volume model to a three flavor case and include the effect of electrons and overall charge neutrality within the Quarkyonic framework. The work is organized as follows. In Section II, we first model the baryon-quark mixture under the hard-core repulsive mean-field. By requiring charge neutrality and possible $\beta$-equilibrium, we discuss the conditions which determine the existence of strangeness. Also, in this section, we show that hard-core nature and dynamically generated quark degrees of freedom can reproduce the stiff-moderate evolution of EoS. Finally, in Sec. III, we discuss the possible issues based on the multi-flavor excluded volume model and the expected results from the shell structure, which completes the phenomenological modeling of Quarkyonic concept.
II. THREE FLAVOR EXCLUDED VOLUME MODEL AND QUARK DEGREES OF FREEDOM

A. Hard-core repulsive interaction and excluded volume model for three flavor

The baryon-baryon central potential whose repulsive-core is appearing at the $0.2 - 0.4$ fm scale had been suggested for decades [23–26]. As the distance scale of repulsion was compatible with the physical radius of the nucleons, that scale was just considered as the ideal dense limit where the quark degrees of freedom can appear. However, the subsequent studies reported the developed potential whose repulsive-core is appearing around $0.6$ fm scale [27–29]. This larger scale has been reproduced by the lattice QCD calculation [30, 34–36, 38], which implies that the effective size due to the repulsion can have a significant dynamical role in the dense regime. The lattice QCD calculation suggested the $S$-wave nucleon-nucleon (NN) with short distance singularity in both of spin singlet ($^1S_0$) and triplet ($^3S_1$) channel, as illustrated in Fig. 1. If one simplifies the central potentials by the infinite well-shaped potential with shallow bounding depth, and defines the hard-core radius where the spatial overlap is not allowed by the infinite potential, the mean distance of the hard-core repulsion would be around $0.6$ fm ($n_0 \sim \tau \rho_0$). Similar tendency is found from constituent quark model calculation [39]. The Λ-nucleon (ΛN) interaction can be deduced by SU(3) flavor rotation, which seems more repulsive than NN potential in $^1S_0$ channel and almost same in $^3S_1$ channel, respectively. So ΛN potential can be assumed to have a little bit larger hard-core size than NN potential. On the other hand, the repulsive potential in the physical world could be different from this simple guess because the error band of ΛN potential [34–36, 38] is appearing in non-negligible scale yet. Moreover, if kaon condensation plays a significant role [48–50] and the SU(3) flavor symmetry breaking term becomes important, the ΛN potential can be reduced. In this work, we consider the possible configurations where the hard-core size of ΛN potential can be larger or smaller than the size of NN, assuming the existence of hard-core ΛN potential.

We extend the excluded volume model for the 3-flavor ($n$, $p$, and $Λ$) circumstance, and each baryon is assumed to have its own effective hard-core size $n_0$ from the repulsive interaction [34,40]. This hard core size can be understood as arising from a repulsive mean-field singular at short distance $\tau \rho_0$. Then, one can redefine the Fermi momentum and number density to include this repulsive nature as follows:

$$n_{B_i}^{ex} = \frac{n_{B_i}}{1 - n_{B_i}/n_0} = \frac{2}{(2\pi)^3} \int_0^{K_F^{B_i}} d^3k,$$

(1)

$$n_{\bar{B}} = n_n + n_p + (1 + \alpha) n_{\Lambda},$$

(2)

where $\alpha$ determines the strength of hard core repulsive interaction between surrounding baryon and Λ hyperon. The strength is determined in range of $-0.2 < \alpha < 0.2$. The $K_F^{B_i}$ denoted in capital letter represents the enhanced baryon Fermi momentum in the reduced available space. Following Ref. [20], the energy density of the 3-flavor baryonic...
system can be written as

\[ \varepsilon_B = \left( 1 - \frac{n_B}{n_0} \right) \frac{1}{\pi^2} \sum_{i}^{\{n,p,\Lambda\}} \int_{0}^{K_{F,i}^2} dkk^2 \left( m_{B_i}^2 + k^2 \right)^{\frac{\alpha}{2}} + \frac{(3\pi^2)^{\frac{\alpha}{2}}}{4\pi^2} n_e \]

where the electron mass is suppressed and non-relativistic limit is taken in last line. The chemical potential for each baryon can be expressed as

\[ \mu_i = \frac{\partial \varepsilon_B}{\partial n_i} = \left( 1 - \frac{n_B}{n_0} \right) \left\{ \frac{K_{F,i}^2}{\pi^2} \left( K_{F,i}^2 + m_{B_i}^2 \right)^{\frac{\alpha}{2}} \frac{\partial K_{F,i}^2}{\partial n_{B_i}} + \sum_{j \neq i}^{\{n,p,\Lambda\}} \frac{K_{F,j}^2}{\pi^2} \left( K_{F,j}^2 + m_{B_j}^2 \right)^{\frac{\alpha}{2}} \frac{\partial K_{F,j}^2}{\partial n_{B_j}} \right\} \]

\[ - \frac{\omega_i}{n_0} \sum_{i}^{\{n,p,\Lambda\}} \int_{0}^{K_{F,i}^2} dkk^2 \left( m_{B_i}^2 + k^2 \right)^{\frac{\alpha}{2}} \]

\[ \simeq m_i + \frac{(3\pi^2)^{\frac{\alpha}{2}}}{10\pi^2 m_i} \left( \frac{5}{3} n_{B_i}^{ex} \right)^{\frac{\alpha}{2}} + \omega_i \sum_{j}^{\{n,p,\Lambda\}} \frac{(3\pi^2)^{\frac{\alpha}{2}}}{10\pi^2 m_j} \left( \frac{2}{3} n_{B_j}^{ex} \right)^{\frac{\alpha}{2}} + \cdots, \]

where the non-relativistic limit is taken in the last line and \( \omega_i = \partial n_B/\partial n_i \) (\( \omega_{n,p} = 1, \omega_\Lambda = 1 + \alpha \)) The partial derivatives are calculated as

\[ \frac{\partial K_{F,i}^2}{\partial n_{B_i}} = \pi^2 \left( 3\pi^2 n_{B_i}^{ex} + \left( K_{F,i}^2 \right)^3 \right)^{\frac{\alpha}{2}} \left( 1 - \frac{n_B}{n_0} \right)^{-2} \left( 1 - \frac{(n_B - \omega_i n_{B_j})}{n_0} \right) \]

\[ \frac{\partial K_{F,j}^2}{\partial n_{B_j}} = \pi^2 \left( 3\pi^2 n_{B_j}^{ex} + \left( K_{F,j}^2 \right)^3 \right)^{\frac{\alpha}{2}} \left( 1 - \frac{n_B}{n_0} \right)^{-2} \left( \frac{\omega_i n_{B_j}}{n_0} \right)^{-2} \]

An interesting point of this multi-flavor hard-core model is the third contribution in the chemical potential. Even if there only few numbers of a specific particle, the chemical potential can be enhanced if the space is taken by the other finite size particles. This nature can be understood as follows: the excluded volume by some particles makes the system cost more energy to create a fast particle wave function. The enhanced energy by the strong repulsive mean-field at high density is realized in terms of hard-core repulsion with the effective size of the particle. For the most simplified example, one can use the mean-field theory of the hard-core limit with \( \alpha = 0 \) and \( m_i = m_B \) where \( \mu_n \simeq \mu_p \simeq \mu_\Lambda \) are appearing in the hard-core density limit \( n_B \simeq n_0 \).

For the asymmetric realization, one should consider electromagnetic charge and related \( \beta \)-decay channels \( n \rightarrow p + e, n \rightarrow \Lambda \) with \( m_n = m_p \simeq 1 \text{ GeV}, m_\Lambda \simeq 1.2 \text{ GeV} \). Considering physical constraints,

\[ n_p = n_e, \quad \mu_n = n_p + n_e, \quad \mu_\Lambda = n_\Lambda \]

\[ n_\Lambda \simeq n_n \simeq n_\Lambda \]

each baryon density can be calculated in \( n_B \) variation as plotted in Fig. [2]. As one can find in the density profile plotted in Fig. [2a], the large scale of \( \alpha \) (stronger repulsion) compared to the other baryons expels \( \Lambda \) hyperon. In this case \( n_\Lambda \) vanishes around \( n_B \sim n_0 \). One can guess the reason for this from the non-relativistic expansion of Eq. [4], where the chemical potential has singular contributions near the hard-core limit. For the \( \mu_\Lambda \) with \( \alpha = 0.2 \), the contribution of third term dominates the other contribution but this contribution is weaker in the \( \alpha = 0 \) case. The less singular behavior allows \( n_\Lambda \simeq n_\Lambda \) by [4].

In Fig. [2b], the hyperon density \( n_\Lambda \) is presented for three different values of \( \alpha \). In the symmetric hard-core input, \( n_\Lambda \simeq n_n \) around hard-core limit, although the rest mass of \( \Lambda \) hyperon is higher than nucleon \( (m_\Lambda = m_p < m_\Lambda) \). A similar density configuration appears when a weaker repulsion (negative \( \alpha \)) is assigned. The corresponding EoS is plotted in Fig. [3] for three different values of \( \alpha \). We observe that decreasing \( \alpha \) or even using a negative value leads to the very stiff EoS, due to the singular hard-core interaction. As one can find in Eq. [4], all chemical potentials have a singularity around the hard-core limit, which leads to causality violating \( n^2 \gg 1 \). At this stage, one can recall the argument of Hagedorn model and construct an analogous argument. According to Hagedorn model, extensive values such as entropy has singularity around the critical temperature \( S/S_0 \sim (T_0 - T)^{-\alpha} \) if the energy spectrum grows.

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\[ \partial K_{F,i}^2 = \frac{\pi^2}{\left( 3\pi^2 n_{B_i}^{ex} + \left( K_{F,i}^2 \right)^3 \right)^{\frac{\alpha}{2}}} \left( 1 - \frac{n_B}{n_0} \right)^{-2} \left( 1 - \frac{(n_B - \omega_i n_{B_j})}{n_0} \right) \]

\[ \partial K_{F,j}^2 = \frac{\pi^2}{\left( 3\pi^2 n_{B_j}^{ex} + \left( K_{F,j}^2 \right)^3 \right)^{\frac{\alpha}{2}}} \left( 1 - \frac{n_B}{n_0} \right)^{-2} \left( \frac{\omega_i n_{B_j}}{n_0} \right)^{-2} \]
as $\rho(E) \sim e^{E/T_0}$. When the temperature exceeds $T_0$, the Hagedorn system prefers to create new degrees of freedom rather than keep the exponentially populating high energy states. In our system, the EoS has intrinsic singularity around the hard-core density. If the total baryon number density goes beyond the hard-core density, the system would prefer to generate new degrees of freedom, the quarks.\footnote{By definition \footnote{1}, the baryon density has upper limit $n_0$ in our excluded volume model. The argument is given in context of the repulsive singular potential, without assuming the hard-core size.}

B. Dynamically generated quark degrees of freedom

Quark degrees of freedom can be generated via various physical scenarios. If some type of phase transition occurs, the entire nuclear matter becomes quark phase after some critical density. However, the quarkyonic-like model discussed in this paper does not have such kind of nature. In $N_c \to \infty$ limit, quark wave functions near the Fermi surface are confined in baryon-like states. In the low density regime, the matter behaves as normal nuclear matter.
but the quark wave functions become saturated from the low momentum state when the matter density reaches few times of $\rho_0$ where $k_F^B \sim O(\Lambda_{QCD})$. Whence the saturated quarks are forming their own Fermi sphere, the momenta of confined wave functions are enhanced by Pauli’s exclusion principle as they should take larger momentum than the fully occupied lower phases. Around the onset moment, the pressure is continuous and stiffly increasing as the chemical potential should show stiffness ($k_F^B \simeq N_c k_F^Q$), different from the first-order phase transition. Thus, if one models this Quarkyonic picture within the hard-core repulsion between the baryons, the appearance of quark degrees of freedom should not accompany any sign of discontinuity of EoS. As a first step, we allow the quark degree of freedom pretending that Pauli’s exclusion principle does not exist. The energy density can be written as

$$\varepsilon_{\text{mix}} = \left(1 - \frac{n_B}{\rho_0}\right) \frac{1}{\pi^2} \sum_i \int_0^{k_F^B} dk k^2 \left(m^2_{B_k} + k^2\right)^{\frac{3}{2}} + \frac{N_c}{\pi^2} \sum_j \int_0^{k_F^Q} dk k^2 \left(m^2_{Q_k} + k^2\right)^{\frac{3}{2}} + \frac{(3\pi^2)^{\frac{1}{3}}}{4\pi^2} n_e^4. \quad (10)$$

Recall the isospin symmetric configuration in Ref. [20]. If electromagnetic charge is not concerned, the quark degrees of freedom is naturally appearing by dynamical requirement $\mu_B = N_c \mu_Q$. In three-flavor extension, this simple relation needs modification. Our system described by Eq. (10) is mixture of baryons and possible quarks with electron clouds. For a given total baryon number density, we have two constraints for the baryon and lepton number. The first constraint is obtained from the baryon number conservation:

$$n_B = n_B + n_Q, \quad (11)$$
$$dn_B = dn_B + dn_Q$$
$$= dn_n + dn_p + dn_A + dn_\bar{a} + dn_\bar{q} + dn_\bar{\bar{q}} = 0, \quad (12)$$

where the tilde denotes the unit of baryon number. The second constraint is the constraint of electromagnetic charge neutrality [17]:

$$n_e = n_p + 2n_\bar{q} - n_\bar{q} - n_\bar{s}, \quad (13)$$
$$dn_e = dn_p + 2dn_\bar{a} - dn_\bar{a} - dn_\bar{s}. \quad (14)$$

Also, we have the additional constraints from the $\beta$-equilibrium:

$$\mu_\bar{q} = \mu_\bar{q} + 3\mu_e, \quad (15)$$
$$\mu_\bar{d} = \mu_\bar{s} \quad \text{when} \ n_\bar{s} \neq 0, \ n_\bar{s} = 0 \ if \ \mu_\bar{s} < N_c m_s, \quad (16)$$

where $\mu_Q = N_c \mu_Q$ denotes the quark chemical potential in the unit of baryon number. At the minimum, the deviation of energy density is zero:

$$d\varepsilon_{\text{mix}} = \mu_n dn_n + \mu_p dn_p + \mu_\Lambda dn_\Lambda + \mu_\bar{a} dn_\bar{a} + \mu_\bar{q} dn_\bar{q} + \mu_\bar{d} dn_\bar{d} + \mu_e dn_e$$
$$= \mu_n (dn_n + dn_p) + \mu_\Lambda dn_\Lambda + (\mu_\bar{q} - \mu_e) (dn_\bar{q} + dn_\bar{d}) + (\mu_\bar{s} - \mu_e) dn_\bar{s} = 0, \quad (17)$$

where the aforementioned constraints are used. By using the constraint (12), one can obtain following relation:

$$\mu_n = N_c \mu_\bar{q} - \mu_e, \quad (18)$$

which corresponds to the three-flavor modification of the constraint $\mu_N = N_c \mu_Q$ in Ref. [20]. This constraint has following conditions in each different configuration:

$$\begin{align*}
& \text{if} \ n_\Lambda = 0, \ n_\bar{s} = 0, \ \mu_n = N_c \mu_\bar{d} - \mu_e = \mu_p + \mu_e, \quad (19) \\
& \text{if} \ n_\Lambda \neq 0, \ n_\bar{s} \neq 0, \ \mu_n = N_c \mu_\bar{d} - \mu_e = \mu_\Lambda = \mu_p + \mu_e = N_c \mu_s - \mu_e, \quad (20) \\
& \text{if} \ n_\Lambda \neq 0, \ n_\bar{s} = 0, \ \mu_n = N_c \mu_\bar{d} - \mu_e = \mu_\Lambda = \mu_p + \mu_e, \quad (21) \\
& \text{if} \ n_\Lambda \neq 0, \ n_\bar{s} \neq 0, \ \mu_n = N_c \mu_\bar{d} - \mu_e = \mu_p + \mu_e = N_c \mu_s - \mu_e. \quad (22)
\end{align*}$$

Now one can calculate the number density of each particle which satisfies Eq. (18), with the baryon number chemical potential and the stiffness of the EoS determined from the density configuration. In Fig. 3(a) we show the density profile for the mean field mixture, for $\alpha = 0.2$. The sum of baryon number density is saturated near $\rho_0$ and $\Lambda$ appears around $n_B \simeq 6\rho_0$ with very small scale. For the hadron sector, it is almost a 2-flavor system in the physically relevant regime. On the other hand, in both the symmetric configuration ($\alpha = 0$) and negative $\alpha, \Lambda$ appear in a little bit lower density regime $n_B \simeq 4.8\rho_0$ and $n_B \simeq 4.2\rho_0$ respectively with non-negligible amount, as can be seen in Fig. 3(b). The
The quark density profile is barely changed but the hadron sector is changed: strong (and positive) $\alpha$ increases the isospin density of the system. The $d$ quark degree of freedom appears from the $n_B \simeq 0$ as $\beta$-equilibrium conditions are satisfied in the low density regime. Comparing with the plots of Fig. 2, $d$ quark plays some part of the proton role even at low density regime. The $\Lambda$ degree of freedom appears in the high density regime, contrary to the early appearance of $d$ quark (Eq. (9) is satisfied at beyond hard-core density as strong $\alpha = 0.2$ is assigned). Because this model has only hard-core repulsive interaction at high density, and does not have any attractive interaction in the low density regime. The density behavior is similar to the 3-flavor free baryon gas, where $\mu_\Lambda > \mu_{n,p}$ in the density regime.

One can check whether the EoS is hard or soft via sound velocity:

\[ v_s^2 = \left. \frac{\partial p}{\partial \varepsilon_{\text{mix}}} \right|_{\text{mix}} = \frac{n_B}{\mu_B} \frac{\partial n_B}{\partial \mu_B}. \]  

As one can find in Fig. 4(a), the baryon number chemical potential shows stiff increment whence $u$ quark degrees of freedom appear. At the moment, $n_B$ is already reached around the hard-core density $n_0$. The stiffness becomes moderated at onset of $s$ quark degrees of freedom. The corresponding $v_s^2$ is plotted in Fig. 4(b) and the EoS is shown in Fig. 6. The sound velocity increases to $v_s^2 \simeq 0.4$ around the hard-core regime and seems converge to the ideal limit $1/3$ at high density limit. This analysis is valid for three values of $\alpha$ used in this work. The scale of the sound velocity

FIG. 4. (Color online) Density profile for $\alpha = 0.2$ (a) and strange density profile for different values of $\alpha$ (b). In the latter plot, thick(thin) curves are correspondent to $n_\Lambda(n_s)$.

FIG. 5. (Color online) Baryon number chemical potential (a) and sound velocity square (b) in total baryon number variation.
\[ \alpha = -0.2 \]
\[ \alpha = 0 \]
\[ \alpha = 0.2 \]

\( \alpha = 0 \) seems not huge enough to accommodate the recent analyses based on GW observation. The plausible scale can be found if we consider Pauli's exclusion principle which leads to the shell structure.

III. DISCUSSION

In this paper, we made a 3-flavor extension of the single flavor excluded volume model for the baryon-quark mixture as the first step. Considering electromagnetic charge and possible \( \beta \)-decay channels, the density behavior of each degrees of freedom was calculated. The hard-core size of \( \Lambda \) hyperon was parametrized by \( \alpha \) whose value would be around \( \alpha = 0.2 \), that leads to almost 2-flavor system which reproduces the hard-soft behavior EoS which is required by the gravitational wave observation.

The EoS is almost not affected by the variation of \( \alpha \) for the physically relevant density regime, since the strange particles are still missing on this regime (this behavior clearly appears in Figs. 4(b) and 5(b)). Also, the EoS for the mean field mixture is barely affected by the variation of \( \alpha \), comparing with the nucleons system (see Figs. 3 and 6), but for intermediate to large densities \( \alpha = 0.2 \) has a stiffer pressure compared to the other values. One can guess the reason as follows; if Pauli's exclusion principle is not accounted for, the quark degrees of freedom are appearing from the very low density regime \((n_B \approx 0)\). The early onset of the quarks leads to relatively hard EoS in the low density regime \((n_B < \rho_0)\). As the constituent quark mass is assumed in this model, the rest mass contribution to the energy density is of similar order of the nucleon contribution while the kinetic contribution is relatively small. Around the hard-core density, the system effectively contains four degrees of freedom, which leads to the relatively soft EoS in comparing with previously reported studies.

Although the hard-core repulsion can enhance the energy density and allows the quark degrees of freedom, one should consider Pauli’s exclusion principle which dynamically generates the baryon-like shell structure to describe the correct Quarkyonic matter concept. The nucleons are having enhanced momenta by Pauli’s exclusion principle \((k_P^\beta \simeq N_c k_Q^Q)\) in the dynamically generated shell-like momentum distribution, which can leads to the hard enough EoS to support 2\( M_{\odot} \) state. It should be considered for the momentum conservation constraint at the onset of quark Fermi sea and subsequent matching constraints between the perturbative and confined quark momenta, which accompany the complexity in the numerical calculation. Model construction is now in progress.

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[39] A. Park, S. H. Lee, T. Inoue and T. Hatsuda, arXiv:1907.06351 [hep-ph].
[40] K. Sasaki et al., arXiv:1912.08630 [hep-lat].
[41] K. Masuda, T. Hatsuda and T. Takatsuka, PTEP 2013, no. 7, 073D01 (2013) doi:10.1093/ptep/ptt045 arXiv:1212.6803 [nucl-th].
[42] T. Kojo, P. D. Powell, Y. Song and G. Baym, Phys. Rev. D 91, no. 4, 045003 (2015) doi:10.1103/PhysRevD.91.045003 arXiv:1412.1108 [hep-ph].
[43] P. Bedaque and A. W. Steiner, Phys. Rev. Lett. 114, no. 3, 031103 (2015) doi:10.1103/PhysRevLett.114.031103 arXiv:1408.5116 [nucl-th].
[44] Y. L. Ma and M. Rho, arXiv:1811.07071 [nucl-th].
[45] I. Tews, J. Carlson, S. Gandolfi and S. Reddy, Astrophys. J. 860, no. 2, 149 (2018) doi:10.3847/1538-4357/aac267 arXiv:1801.01923 [nucl-th].
[46] T. Kojo, AIP Conf. Proc. 2127, no. 1, 020023 (2019) doi:10.1063/1.5117813 arXiv:1904.05080 [astro-ph.HE].
[47] Y. Fujimoto, K. Fukushima and K. Murase, arXiv:1903.03400 [nucl-th].
[48] D. B. Kaplan and A. E. Nelson, Phys. Lett. B 175, 57 (1986). doi:10.1016/0370-2693(86)90331-X
[49] M. J. Savage and M. B. Wise, Phys. Rev. D 53, 349 (1996) doi:10.1103/PhysRevD.53.349 [hep-ph/9507288].
[50] K. S. Jeong, G. Gye and S. H. Lee, Phys. Rev. C 94, no. 6, 065201 (2016) doi:10.1103/PhysRevC.94.065201 arXiv:1606.00594 [nucl-th].
[51] R. Hagedorn, Nuovo Cim. Suppl. 3, 147 (1965).