Study of the Flavour Changing Neutral Current Decay mode $B \rightarrow K\gamma\gamma$

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Abstract

We study the neutral current flavour changing rare decay mode $B \rightarrow K\gamma\gamma$ within the framework of Standard Model, including the long distance contributions. It is found that these long distance contributions can be larger than the short distance contribution.

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We have corrected a sign error in one of the terms contributing to the Irreducible (short-distance) contribution in our program. The revised estimates agree well with the ones given in a recent paper [12].

1 Introduction

Radiative decays of the B-meson are very useful indices for testing the underlying theories of flavour changing neutral currents (FCNC) since the amplitudes are very sensitive to QCD contributions as well as to possible contributions from loops with supersymmetric partner particles. Of the radiative
modes, the decay mode $B \to X_s \gamma$ has been measured experimentally and has also been the subject of extensive theoretical investigations. The advent of the B-factories will probably make possible measurement of radiative decay modes with two photons. Hence the basic two photon amplitude $b \to s \gamma \gamma$ has also been studied starting with the work of Lin, Liu and Yao [1], and pursued further in references [2], [3].

The $b \to s \gamma \gamma$ amplitude falls naturally into two categories; an irreducible contribution and a reducible one where the second photon is attached to the external quark lines of the $b \to s \gamma$ amplitude. The irreducible part can be estimated through the basic triangle graph. Evaluating the reducible part at the quark level then presents no problem. For the process $B \to X_s \gamma \gamma$ it is appropriate to consider the amplitude as dominated by the quark level amplitude. However, when we consider an exclusive channel $B \to M \gamma \gamma$ with a specific meson M, it is more appropriate to consider the reducible contributions as arising out of emission of the second photon from the external hadron legs of the amplitude $B \to M \gamma$ rather than restricting oneself to quark level considerations and thereby completely neglecting the spectator contributions in B and M. The amplitude $B \to K \gamma \gamma$ then stands out since the basic one photon amplitude $B \to K \gamma$ vanishes for real photons and we may expect the irreducible part of the amplitude to dominate. There will of course be the usual long distance contributions, the most important being the process $B \to K^* \gamma$ followed by decay of $K^*$, and $B \to K \eta_c$ followed by two photon decay of the $\eta_c$. The corresponding $\eta$ contribution will be small owing to the small $\eta$ coupling to $\bar{c}c$ but we include it also for completeness. The $B \to K \eta'$ amplitude however is anomalously high, probably because of the gluon fusion mechanism. Since $\eta'$ has a sizeable branching ratio into 2 $\gamma$, the $\eta'$ contribution to $B \to K \gamma \gamma$ will prove to be the largest resonance contribution.

This note reports an estimate of the branching ratio for the process $B \to K \gamma \gamma$ taking account of the irreducible contribution as well as all significant resonance contributions. The earliest investigations of this process restricted themselves to the $\eta_c$ contribution [4]; subsequently an estimate was made of a possible contribution from the resonance $\eta'$ with the $\eta'$ arising through gluon fusion from the basic process $b \to sg g$ and subsequently decaying into two photons [5]. The $\eta'$ resonance however is very narrow and its contribution will
show up as a distinct narrow peak in the $\gamma\gamma$ invariant mass spectrum. This can experimentally be easily separated and we could restrict our analysis by excluding this resonance, but we include it for completeness.

There are of course interference terms from each pair of terms. Unfortunately the only interference term whose sign we can determine is that between the irreducible term and the $\eta_c$ resonance contribution. The other signs are unknown. We quote the largest and the smallest branching ratio, and give enough information for the reader to construct the other possibilities.

2 Irreducible triangle contributions

These arise from triangle diagrams with the photon being emitted by the quark loop. The effective Hamiltonian is

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \sum_i C_i(\mu) O_i(\mu),$$

with

$$O_1 = (\bar{s}_i c_j)_{V^-A} (\bar{c}_j b_i)_{V^-A},$$

$$O_2 = (\bar{s}_i c_i)_{V^-A} (\bar{c}_j b_j)_{V^-A},$$

$$O_3 = (\bar{s}_i b_i)_{V^-A} \sum_q (\bar{q}_j q_j)_{V^-A},$$

$$O_4 = (\bar{s}_i b_j)_{V^-A} \sum_q (\bar{q}_j q_i)_{V^-A},$$

$$O_5 = (\bar{s}_i b_i)_{V^-A} \sum_q (\bar{q}_j q_j)_{V^+A},$$

$$O_6 = (\bar{s}_i b_j)_{V^-A} \sum_q (\bar{q}_j q_i)_{V^+A},$$

$$O_7 = \frac{e}{16\pi^2} \bar{s}_i \sigma^{\mu\nu} (m_s P_L + m_b P_R) b_i F_{\mu\nu},$$

$$O_8 = \frac{g}{16\pi^2} \bar{s}_i \sigma^{\mu\nu} (m_s P_L + m_b P_R) T^a_{ij} b_j G^a_{\mu\nu}.$$  

The invariant amplitude corresponding to quark level transition $b \rightarrow s\gamma\gamma$ (with an incoming $b$ line and an outgoing $s$ line and the two photons being
emitted by the quark loop) is

\[ M_{b\rightarrow s} = \left[ \frac{16\sqrt{2}\alpha_G F V_{ts}^* V_{tb}}{9\pi} \right] \bar{u}(p_s) \left\{ \sum_q A_q J(m_q^2) \gamma^\rho P_L R_{\mu\nu\rho} \right. \]

\[ + \left. iB(m_s K(m_s^2) P_L + m_b K(m_b^2) P_R) T_{\mu\nu} \right. \]

\[ + \left. C(-m_s L(m_s^2) P_L + m_b L(m_b^2) P_R) \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \right\} u(p_b) \epsilon^{\mu*}(k_1) \epsilon^{\nu*}(k_2), \]

where

\[ R_{\mu\nu\rho} = k_{1\nu}^\rho \delta_{\mu\nu} - k_{2\mu}^\rho \delta_{\mu\nu} + (k_1.k_2) \epsilon_{\mu\nu\rho\sigma}(k_2 - k_1), \]

\[ T_{\mu\nu} = k_{2\mu} k_{1\nu} - (k_1.k_2) g_{\mu\nu}, \]

\[ A_u = 3(C_3 - C_5) + (C_4 - C_6), \]

\[ A_d = \frac{1}{4} [3(C_3 - C_5) + (C_4 - C_6)], \]

\[ A_c = 3(C_1 + C_3 - C_5) + (C_2 + C_4 - C_6), \]

\[ A_s = A_b = \frac{1}{4} [3(C_4 + C_3 - C_5) + (C_3 + C_4 - C_6)], \]

and

\[ B = C = -\frac{1}{4}(3C_6 + C_5). \]

To get to \( M(B \rightarrow K\gamma\gamma) \) from the quark level amplitude \( (M(b \rightarrow s\gamma\gamma)) \), we should replace \( \langle s|\Gamma|b \rangle \) by \( \langle K|\Gamma|B \rangle \) for any Dirac bilinear \( \Gamma \). Recall that \( \langle s|\bar{s}\gamma^\rho b|b \rangle \) gives a factor \( \bar{u}_s \gamma^\rho u_b \) and similarly for \( \bar{s}\gamma^\rho\gamma^5 b, \bar{s}\gamma^5 b, \) and \( \bar{s} b \), and also that \( \langle K|\Gamma|B \rangle \) is zero for \( \Gamma = \gamma^5, \gamma^\rho\gamma^5 \). In this way we get

\[ M_{\text{irred}}(B \rightarrow K\gamma\gamma) = \left( \frac{16\sqrt{2}\alpha_G F V_{ts}^* V_{tb}}{9\pi} \right) \left[ \frac{1}{2} \langle K|\bar{s}\gamma^\rho b|B \rangle \right. \]

\[ + \left. \frac{1}{2} \langle K|\bar{s} b|B \rangle \right\} \left\{ A_u J(m_u^2) R_{\mu\nu\rho} \right. \]

\[ + \left. A_d J(m_d^2) R_{\mu\nu\rho} \right. \]

\[ + \left. C(-m_s L(m_s^2) + m_b L(m_b^2)) \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \right\} \left\} \epsilon^{\mu*}(k_1) \epsilon^{\nu*}(k_2) \right. \]

In the above expressions we introduced the functions

\[ J(m^2) = I_{11}(m^2), \quad K(m^2) = 4I_{11}(m^2) - I_{00}(m^2), \quad L(m^2) = I_{00}(m^2), \]

where

\[ I_{pq}(m^2) = \int_0^1 dx \int_0^{1-x} dy \frac{x^p y^q}{m^2 - 2(k_1.k_2)xy - \iota\epsilon} \quad (6) \]
We use the following parametrization for the matrix elements of the quark vector current:

\[ \langle K | \bar{s} \gamma_{\mu} b | B \rangle = \left( (p_B + p_K)_{\mu} - \frac{m_B^2 - m_K^2}{q^2} q_{\mu} \right) F^{BK}_{1}(q^2) \]

\[ + \left( \frac{m_B^2 - m_K^2}{q^2} \right) q_{\mu} F^{BK}_{0}(q^2), \]

with \( q = p_b - p_K = k_1 + k_2 \). It then follows that

\[ \langle K | \bar{s} b | B \rangle = \left( m_b - m_s \right)^{-1} q_{\mu} \langle K | \bar{s} \gamma^{\mu} b | B \rangle \]

\[ = \left( m_b - m_s \right)^{-1} (m_B^2 - m_K^2) F^{BK}_{0}(q^2). \]

Also observe that

\[ q^{\rho} R_{\mu \nu \rho} = (k_1 + k_2)^{\rho} R_{\mu \nu \rho} \]

\[ = (k_1 \cdot k_2) \epsilon_{\mu \nu \rho \sigma} (k_1^\rho k_2^\sigma - k_2^\rho k_1^\sigma). \]

For the form factors appearing above, we use the explicit numerical dependence of \( F(q^2) \) on \( q^2 \) given by Cheng et al [6].

### 3 Resonance contributions

#### 3.1 The \( \eta_c \) resonance

The \( \eta_c \) contribution to the decay process comes via the t-channel decay \( B \to K \eta_c \), with then \( \eta_c \) decaying into two photons. Since this has to be added to other contributions, we need to be careful about the sign of the term.

The \( T \)-matrix element for this process can be written as

\[ \iota \langle K \gamma \gamma | T | B \rangle = \langle K \eta_c | - \iota \mathcal{H} | B \rangle \left( \frac{q^2}{q^2 - m_{\eta_c}^2 + i m_{\eta_c} \Gamma_{\eta_c}^{\gamma \gamma} + \Gamma_{\eta_c}^{\gamma \gamma} \text{total}} \langle \gamma \gamma | - \iota \mathcal{H} | \eta_c \rangle. \]

Noting that in the lowest order, for any states \( X, Y \)

\[ \langle X | T | Y \rangle = - \langle X | \mathcal{H} | Y \rangle, \]

we get

\[ \langle K \gamma \gamma | T | B \rangle = - \frac{\langle K \eta_c | T | B \rangle \langle \gamma \gamma | T | \eta_c \rangle}{q^2 - m_{\eta_c}^2 + i m_{\eta_c} \Gamma_{\eta_c}^{\gamma \gamma} + \Gamma_{\eta_c}^{\gamma \gamma} \text{total}}, \]
The amplitude \( \langle \gamma \gamma | T | \eta_c \rangle \) is parametrized as
\[
\langle \gamma \gamma | T | \eta_c \rangle = 2i B_{\eta_c} \epsilon^{\mu \nu \alpha \beta} \epsilon_{1 \mu}^* \epsilon_{2 \nu}^* k_1^\alpha k_2^\beta.
\] (12)

We can determine \( B_{\eta_c} \) from the \( \eta_c \rightarrow \gamma \gamma \) decay rate:
\[
\Gamma(\eta_c \rightarrow \gamma \gamma) = \left( \frac{1}{2} \right) \frac{1}{2m_{\eta_c}} \int \frac{d^3 k_1}{(2\pi)^3 2k_1^0} \frac{d^3 k_2}{(2\pi)^3 2k_2^0} \left( 2\pi \right)^2 \delta^4(k_\eta - k_1 - k_2)|\langle \gamma \gamma | T | \eta_c \rangle|^2.
\] (13)

We have
\[
\sum_{\text{spins}} |\langle \gamma \gamma | T | \eta_c \rangle|^2 = 2 |B_{\eta_c}|^2 m_{\eta_c}^2,
\]
and so
\[
\Gamma(\eta_c \rightarrow \gamma \gamma) = \frac{|B_{\eta_c}|^2 m_{\eta_c}^2}{16\pi}.
\]

Next we need the \( B \rightarrow K \eta_c \) amplitude
\[
\langle K \eta_c | T | B \rangle = -\langle K \eta_c | \mathcal{H}_{\text{eff}} | B \rangle.
\]

The relevant piece of \( \mathcal{H}_{\text{eff}} \) for this matrix element is
\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left[ C_2 (\bar{c}b) V_{-A} (\bar{s}c) V_{-A} + C_1 (\bar{c}_i b_j) V_{-A} (\bar{s}_j c_i) V_{-A} \right] \quad (14)
\]
\[
= -\frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* \left[ C_1 (\bar{c}c) V_{-A} (\bar{s}b) V_{-A} + C_2 (\bar{c}_i c_j) V_{-A} (\bar{s}_j b_i) V_{-A} \right].
\]

The second expression is obtained by Fierz transforming the first one. Thus
\[
\mathcal{H}_{\text{eff}} = - \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (C_1 + \frac{C_2}{3})(\bar{c}c) V_{-A} (\bar{s}b) V_{-A}.
\] (15)

Between \( |B\rangle \) and \( |K\rangle \) only the V part of \( (\bar{s}b) V_{-A} \) contributes whereas for the c-loop with two photons only the A part of \( (\bar{c}c) V_{-A} \) is relevant. Hence,
\[
\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (C_1 + \frac{C_2}{3})(\bar{c} \gamma_\mu \gamma_5 c)(\bar{s} \gamma^\mu b).
\] (16)

Using factorization we can write
\[
\langle K \eta_c | T | B \rangle = \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^* (C_1 + \frac{C_2}{3}) \langle K | \bar{s} \gamma^\mu b | B \rangle \langle \eta_c | \bar{c} \gamma_\mu \gamma_5 c | 0 \rangle.
\] (17)

Define
\[
\langle 0 | A^\mu_\rho | \eta_c \rangle \equiv i f_{\eta_c} q_\mu; \quad A^\mu_\rho = \bar{c} \gamma_\mu \gamma_5 c
\]
\[ q^\mu \langle K | \bar{s} \gamma_\mu b | B \rangle = F_0(m_{\eta_c}^2)(m_B^2 - m_K^2) \]

We thus get

\[ \langle K | \eta_c | T | B \rangle = -i \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* f_{\eta_c} F_0(m_{\eta_c}^2)(m_B^2 - m_K^2)(C_1 + \frac{C_2}{3}) \] (18)

where a dipole form of the form factor \( F_0(m_{\eta_c}^2) \) is used.

Now we fix the relative sign between \( B_{\eta_c} \) and \( f_{\eta_c} \). This can be easily done through the anomaly equation [7]

\[ \langle \gamma \gamma | T | \eta_c \rangle = \epsilon_1^{*\mu}(k_1) \epsilon_2^{*\nu}(k_2) \Gamma_{\mu\nu} \]

\[ = \epsilon_1^{*\mu}(k_1) \epsilon_2^{*\nu}(k_2) \frac{ie^2D}{2\pi^2 f_{\eta_c}} \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \] (19)

Comparing with our definition of \( B_{\eta_c} \) in eq. (12), we identify

\[ B_{\eta_c} = \frac{e^2D}{4\pi^2 f_{\eta_c}}, \]

and see that the relative sign is positive. However, we emphasise that we do not use this theoretical result to determine the numerical value of either \( B_{\eta_c} \) or \( f_{\eta_c} \), but merely use it to fix the relative sign.

The total contribution due to \( \eta_c \) resonances is thus

\[ \mathcal{M}_{\eta_c} = 2B_{\eta_c} f_{\eta_c} \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* (C_1 + \frac{C_2}{3}) F_0(m_{\eta_c}^2)(m_B^2 - m_K^2) \]

\[ \epsilon_1^{*\mu}(k_1) \epsilon_2^{*\nu}(k_2) \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \frac{1}{q^2 - m_{\eta_c}^2 + i m_{\eta_c} \Gamma_{\text{total}}^{\eta_c}}. \] (20)

### 3.2 The \( \eta \) contribution

Analogous to \( \eta_c \), the \( \eta \)-resonance will also contribute to the decay amplitude because of its coupling to \( c\bar{c} \) channel. This contribution, \( \mathcal{M}_\eta \), has exactly the same form as eq. (20) with the parameters \( B_{\eta_c}, f_{\eta_c}, F_0(m_{\eta_c}^2), m_{\eta_c} \) and \( \Gamma_{\text{total}}^{\eta_c} \) being replaced by their \( \eta \)-counterparts. However in this case we cannot define the relative sign of the \( \eta \) and any of the other components of the amplitude.
3.3 The $\eta'$ contribution

Like $\eta_c$ and $\eta$, the $\eta'$ can also contribute via the effective Hamiltonian, eq.(15). However, unlike $\eta_c$ and $\eta$, the $\eta'$ has a very strong coupling to a two gluon state. Theoretical models for $\eta'$ production in B-decay via $gg$-states exist both when the gluons arise from a basic $b \to sgg$ process \[8\] and also when the spectator quark in B-meson emits a gluon to combine with a basic $b \to sg$ process \[4, 5\]. These estimates have considerable theoretical uncertainties. Fortunately, experimental data on $B \to K\eta'$ exists, which is sufficient for our purpose since the invariant mass of the two photons will be strongly peaked at $m_{\eta'}^2$. Calling the coupling of $B^i \to K^i\eta'$ ($i = +, 0$) as $F(B^i K^i \eta')$, the $\eta'$ contribution to our amplitude can be written as

$$M_{\eta'} = 2 B_{\eta'} F(B^i K^i \eta') \epsilon_1^{*\mu}(k_1) \epsilon_2^{*\nu}(k_2) \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \frac{1}{q^2 - m_{\eta'}^2 + i m_{\eta'} \Gamma_{\eta'}^{total}}$$  (21)

where $B_{\eta'}$ is defined as in eq.(12) with $\eta_c \to \eta'$. Also, $F(B^i K^i \eta')$ is related to the decay rate $\Gamma(B^i \to K^i \eta')$ as:

$$\Gamma(B^i \to K^i \eta') = \frac{1}{16 \pi m_B} |F(B^i K^i \eta')|^2 \lambda^\perp(1, \frac{m_K^2}{m_B^2}, \frac{m_{\eta'}^2}{m_B^2})$$  (22)

Once again the relative sign of the contribution is unknown.

3.4 $K^*$ resonance contribution

In this case we have the following situation:

$$B^i(p_B) \longrightarrow K^{*i} + \gamma(k_1), \quad i = +, 0$$

followed by

$$K^{*i} \longrightarrow K^i + \gamma(k_2),$$

and the process with $k_1 \leftrightarrow k_2$.

The effective Hamiltonian for the first vertex is

$$H_{\text{eff}} = 4 \frac{G_F}{\sqrt{2}} V_{cb} V_{cs}^{*} C_7 \left( \frac{e m_b}{16 \pi^2} \right) \bar{s}_L \sigma_{\mu\nu} b_R F^{\mu\nu}$$  (23)

and the $T$-matrix element $T^i$ for the process $K^{*i} \to K^i + \gamma$ is

$$T^i = g_{K^*K^\gamma} \epsilon_\delta(p_B - k_1) \epsilon_\beta^{*}(k_2) k_{2\alpha}(p_B - k_1) \lambda^\perp(1, \lambda, \lambda, \lambda)$$  (24)
with $|g^i_{K^* K \gamma}|$ to be determined from the decay rate of $K^{*i} \to K^i + \gamma$. Summing over the final spins and averaging over the initial spins we get

$$\frac{1}{3} \sum |T^i|^2 = \frac{1}{3} |g^i|^2 \left( \frac{m_{K^*}^2 - m_K^2}{2} \right)^2,$$

so $|g_i|$ is determined from

$$\Gamma^i = \frac{|g^i|^2}{96\pi} \left( \frac{m_{K^*}^2 - m_K^2}{m_{K^*}} \right)^3. \quad (26)$$

The matrix element of

$$O_7 = \left( \frac{e m_b}{16\pi^2} \right) \bar{s} L \sigma_{\mu \nu} b_R F^{\mu \nu}$$

is

$$\langle K^{*i}(q)|O_7|B^i(p + q)\rangle = -i\left( \frac{e m_b}{32\pi^2} \right) F^i \left[ \epsilon^\alpha(p) \epsilon^\beta(p') - \epsilon^\beta(p) \epsilon^\alpha(p') \right]$$

$$\epsilon^\sigma(q) \left[ i\epsilon_{\alpha\beta\sigma\tau} q^\tau - 2g_{\sigma\alpha} q_{\beta'} \right] \quad (27)$$

with $F^i$ to be determined from the radiative decay $B^i \to K^{*i} \gamma$. With these definitions the complete $T$-matrix element can be written as

$$\langle K \gamma \gamma | T | B \rangle = \mathcal{M}_{K^*} \quad (28)$$

$$= \left[ T^{\mu \nu}(k_1, k_2) + (\mu \leftrightarrow \nu, k_1 \leftrightarrow k_2) \right] \epsilon^{* \mu}(k_1) \epsilon^{* \nu}(k_2)$$

with

$$T^{\mu \nu}(k_1, k_2) = -\left( \frac{e m_b g^i F^i}{16\pi^2} \right) G_F \sqrt{2} V_{cb} V_{cs}^* C_7 \epsilon^{\alpha \nu \gamma \delta} k_{2\alpha} (p_B - k_1)_{\gamma} k_{1\beta'}$$

$$\left[ \left( g_{\delta \sigma'} - \frac{(p_B - k_1)\delta(p_B - k_1)_{\sigma'}}{m_{K^*}^2} \right) \left( p_B - k_1 \right)^2 - m_{K^*}^2 + m_{K^*} \Gamma_{K^*}^{total} \right]$$

$$\left[ \epsilon^{\mu \beta' \sigma' \tau'} (p_B - k_1)_{\tau'} - (g^{\mu \sigma'} (p_B - k_1)_{\beta'} - g^{\beta' \sigma'} (p_B - k_1)_{\mu}) \right]. \quad (29)$$

In the last expression we have used the antisymmetry of $F^{\mu \nu}$ to write the terms in this particular form.

Also note that the sign of this contribution can not be unambiguously fixed as was done for the $\eta_c$ contribution.
4 Results

The total contribution to the process $B \rightarrow K\gamma\gamma$ is

$$M_{\text{tot}} = M_{\text{irred}} + M_{\eta c} + \xi_{K^*}M_{K^*} + \xi_{\eta}M_{\eta} + \xi_{\eta'}M_{\eta'}$$  \hspace{1cm} (30)

where $\xi_{\alpha} = \pm 1$ are sign parameters, introduced because the signs of these terms are unknown. The total decay rate is given by

$$\frac{d\Gamma}{d\sqrt{s_{\gamma\gamma}}} = \left(\frac{1}{512m_B^3}\right)\sqrt{s_{\gamma\gamma}}\left[1 - \frac{s_{\gamma\gamma}}{m_B^2} + \frac{m_K^2}{m_B^2}\right]^2 - \frac{4m_K^2}{m_B^2}\int_0^\pi d\theta \sin \theta |M_{\text{tot}}|^2$$  \hspace{1cm} (31)

where $\sqrt{s_{\gamma\gamma}}$ is the C.M. energy of the two photons while $\theta$ is the angle which the decaying B-meson makes with one of the two photons in the $\gamma\gamma$ C.M. frame. The numerical values of various parameters used in our calculation are listed in Appendix.

Figure 1 shows the spectrum of our results given as a function of the invariant mass of the two photons, for the case of neutral $B$ meson decay. The figure shows the expected resonance peaks in the $\gamma\gamma$ spectrum at the positions of the $\eta$, $\eta'$, and $\eta_c$, and the $K^*$ resonance contribution is spread out across this projection of the Dalitz plot. We have chosen the sign parameters $\xi_{\alpha} = 1$ in all cases in the graph. Figure 2 contrasts the irreducible and the total contribution to the branching ratio as a function of momentum transferred squared over the whole kinematic range.

The various contributions to the branching ratio are given in Table 1, with the maximum branching ratio ($\xi_{\alpha} = +1$ for all $\alpha$), and the minimum branching ratio ($\xi_{\alpha} = -1$ for all $\alpha$). We see that we predict the branching ratio in the range

$$1.439 \times 10^{-6} \leq \text{Br}(B \rightarrow K\gamma\gamma) \leq 1.485 \times 10^{-6}$$  \hspace{1cm} (32)

and that the largest single contribution to the branching ratio comes from the $\eta'$ resonance term. In fact the resonance terms (neglecting interference) are far far greater than the irreducible term.

In an attempt to enhance the relative contribution of the irreducible term we have calculated the partial branching ratio with a cut on $\sqrt{s_{\gamma\gamma}} > m_{\eta_c} + 2\Gamma_{\eta_c} \approx 3.02$ GeV. In this partial branching ratio the $\eta_c$ term does dominate and is about four the $K^*$ contribution. We could try to further reduce the non-irreducible background by introducing a further cut in the Dalitz plot to stay
Table 1: Contributions to the $B^0 \rightarrow K^0 \gamma \gamma$ branching ratio — the total branching ratio and with a cut on $\sqrt{s_{\gamma \gamma}}$

| Contribution  | Branching ratio | With cut |
|---------------|-----------------|----------|
|               | $\times 10^{-7}$| $\times 10^{-7}$|
| Resonance     |                 |          |
| $\eta_c$      | 3.86            | 1.94     |
| $\eta$        | 0.004           | 0        |
| $\eta'$       | 9.69            | 0.005    |
| $K^*$         | 1.07            | 0.56     |
| Irreducible   | 0.0195          | 0.0193   |
| Interference  |                 |          |
| $\eta_c - I$  | -0.025          | 0.076    |
| $\eta' - I$   | $\mp 0.13$      | 0        |
| $\eta_c - K^*$| $\mp 0.048$    | $\mp 0.099$ |
| $\eta' - K^*$ | $\pm 0.054$    | $\pm 0.018$ |
| $K^* - I$     | $\mp 1.5 \times 10^{-4}$ | $\mp 1.43 \times 10^{-4}$ |
| Maximum BR    | 14.85           | 2.72     |
| Minimum BR    | 14.39           | 2.48     |

above the $K^*$ peak in $\sqrt{s_{K\gamma}}$, but because of the already small branching ratio, we do not suggest this additional cut.

For completeness, we also quote the analogous individual contributions to the branching ratio for the corresponding charged decay mode ($B^+ \rightarrow K^+ \gamma \gamma$) in Table 2.

Comparing the the two branching ratios for the neutral and charged deacy modes, one finds that although the $\eta'$ contribution is larger in the case of the charged mode, the overall branching fractions are almost identical, whether one considers the complete branching ratio, or just the part above the cut. This is because of the fact that the larger branching fraction of the charged mode $B^+ \rightarrow K^{*+} \gamma$ gets compensated by the smaller value of the branching ratio for the mode $K^{**+} \rightarrow K^+ \gamma$.

From the tables above, it is clear that above the imposed cut, the $\eta$ and $\eta'$ do not contribute significantly. But the same is not found to be true for
Table 2: Contributions to the $B^+ \to K^+\gamma\gamma$ branching ratio — the total branching ratio and with a cut on $\sqrt{s_{\gamma\gamma}}$

| Contribution     | Branching ratio with cut | With cut |
|------------------|--------------------------|----------|
|                  | $\times 10^{-7}$         | $\times 10^{-7}$ |
| Resonance        |                          |          |
| $\eta_c$         | 3.85                     | 1.94     |
| $\eta$           | 0.0037                   | 0        |
| $\eta'$          | 12.61                    | 0.005    |
| $K^*$            | 0.9                      | 0.46     |
| Irreducible      | 0.0196                   | 0.0193   |
| Interference     |                          |          |
| $\eta_c - I$     | $-0.025$                 | 0.076    |
| $\eta' - I$      | $\mp 0.13$              | 0        |
| $\eta_c - K^*$   | $\mp 0.043$             | $\mp 0.082$ |
| $\eta' - K^*$    | $\pm 0.067$             | $\pm 0.018$ |
| $K^* - I$        | $\mp 1.6 \times 10^{-4}$| $\pm 1.47 \times 10^{-4}$ |
| Maximum BR       | 17.6                     | 3.61     |
| Minimum BR       | 17.12                    | 3.41     |

$\eta_c$. In particular, the interference terms are practically unimportant, with or without the cut. This is essentially due to small contribution from the irreducible sector while for interference between two resonance contributions, the fact that the cut is practically far from the resonances results in a negligible contribution.

The value obtained for $Br(B \to K\gamma\gamma)$, is much much higher than the corresponding branching ratio for the inclusive process $B \to X\gamma\gamma$, where the total branching ratio is quoted to be $(1.7 - 3.37) \times 10^{-7}$ [3]. Even the exclusive branching ratio with much of the long distance resonance effects eliminated by the cut in $\sqrt{s_{\gamma\gamma}}$ we have suggested, could exceed these values (or at least be of similar size). However, it is known [9] that the inclusive rate calculation based on quark transition picture is not valid when $M_X^2$ becomes low, which is true in our case. Thus we should not be too surprised by our larger result for just one part of the inclusive branching ratio.

It is worth mentioning at this point that if instead of using the explicit numerical dependence of $B \to K$ form factors, following Chen etal[10] we assume
\[ F_0^{BK}(0) = F_1^{BK}(0) = 0.341 \text{ and } F_0^{BK}(q^2) = \frac{F_0^{BK}(0)}{1 - \frac{q^2}{M_{pole}^2}} \text{ with } M_{pole} = 6.65 \text{ GeV} \]

From [11], we get an even higher value for the branching ratio.

Observation of this decay experimentally thus is expected to provide an interesting test of the underlying theory leading to this value. In particular the branching ratio above a cut which removes the \( \eta, \eta' \) and \( \eta_c \) contributions will, ideally speaking, be a good test of the magnitude of the irreducible and will thus be sensitive to the effects of high mass intermediate states, even to physics beyond the standard model.

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**Appendix**

We give the input parameters used in the numerical calculations.

|   | \( C_1 \) | \( C_2 \) | \( C_3 \) | \( C_4 \) | \( C_5 \) | \( C_6 \) | \( C_7 \) | \( C_8 \) |
|---|---|---|---|---|---|---|---|---|
|   | -0.222 | 1.09 | 0.010 | -0.023 | 0.007 | -0.028 | -0.301 | -0.144 |

Table 3: The approximate values of \( C_i \)s at \( \mu = m_b \)

\[ m_b = 4.8 \text{ GeV} \quad m_c = 1.5 \text{ GeV} \quad m_t = 175 \text{ GeV} \]
\[ m_s = 0.15 \text{ GeV} \quad m_u = m_d = 0 \]
\[ m_{B^0} = 5.2792 \text{ GeV} \quad \Gamma_{B^0_{total}} = 4.22 \times 10^{-13} \text{ GeV} \]
\[ m_{B^+} = 5.2789 \text{ GeV} \quad \Gamma_{B^+}^{\text{total}} = 4.21 \times 10^{-13} \text{ GeV} \]
\[ m_{K^0} = 0.497672 \text{ GeV} \quad m_{K^+} = 0.493677 \text{ GeV} \]
\[ m_{\eta_c} = 3 \text{ GeV} \quad B_{\eta_c} = 2.74 \times 10^{-3} \text{ GeV}^{-1} \]
\[ \Gamma_{\eta_c}^{\text{total}} = 1.3 \times 10^{-2} \text{ GeV} \quad f_{\eta_c} = 0.35 \text{ GeV} \]
\[ m_\eta = 0.547 \text{ GeV} \quad \Gamma_\eta^{\text{total}} = 1.18 \times 10^{-6} \text{ GeV} \]
\[ B_\eta = 13.254 \times 10^{-3} \text{ GeV}^{-1} \quad f_\eta = -2.4 \times 10^{-3} \text{ GeV} \]
\[ m_{\eta'} = 0.95778 \text{ GeV} \quad \Gamma_{\eta'}^{\text{total}} = 0.203 \times 10^{-3} \text{ GeV} \]
\[ Br(B^0 \to K^0\eta') = 4.7 \times 10^{-5} \quad Br(B^+ \to K^+\eta') = 6.5 \times 10^{-5} \]
\[ m_{K^{*0}} = 0.896 \text{ GeV} \quad g_{K^{*0}K\gamma} = 0.384916 \text{ GeV}^{-1} \quad \Gamma_{K^{*0}}^{\text{total}} = 50.5 \text{ MeV} \quad F_i = 0.52 - 0.72 \]
\[ m_{K^{*+}} = 0.8916 \text{ GeV} \quad g_{K^{*+}K\gamma} = 0.355 \text{ GeV}^{-1} \quad \Gamma_{K^{*+}}^{\text{total}} = 50.8 \text{ MeV} \]

We follow the Wolfenstein parametrization of the CKM matrix with

\[ A = 0.8 \quad \lambda = 0.22 \quad \eta = 0.34 \]
\[ V_{tb} \sim 1 \quad V_{ts} = -A\lambda^2 \]
\[ V_{cb} = A\lambda^2 \quad V_{cs} = 1 - \frac{\lambda^2}{2} \]

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Figure 1: Our result for the spectrum of $B^0 \rightarrow K^0 \gamma \gamma$ (above). The same results plotted with logscale on $y$-axis (below). The parameters used are listed in the appendix.
Figure 2: The spectrum of $B^0 \rightarrow K^0 \gamma \gamma$ with and without resonances. We plot the differential $BR$ as a function of momentum transferred squared over the whole kinematic regime.