The Joule Heating Effect on MHD Natural Convective Fluid Flow In A Permeable Medium Over A Semi-Infinite Inclined Vertical Plate In The Presence Of The Chemical Reaction

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Abstract: In the current examination, the impact of the radiation on MHD convective fluid flow stream in a permeable medium over a semi-infinite inclined plate with the impact of the Joule heating. The governing Equation changed into nonlinear ODE’s with the assistance of the similarity transformation. By utilizing the Runge-Kutta fourth order with shooting technique. The effect of the fluid parameters on velocity and temperature along with concentration profiles examined through graphs.

Keywords: RK 4\textsuperscript{th} order, Joule heating effect, Radiation, MHD, Porous medium, ODE’s.

1. INTRODUCTION
For a few issues in engineering and reality, a few movements are brought about by changes in concentration and temperature. There are numerous transportation measures in certain field where heat and mass exchange happen. Magnetohydrodynamics (MHD) is the investigation of the progression of electrically directing fluids magnetic fields. Because of its particular significance in science and innovation, the investigation of MHD affected by radiation transmission of heat and mass has motivated numerous researchers. Sharma and Mishra [1] contemplated the joined impacts of free convection and chemical reaction with heat-mass transition conditions: A semianalytical method. Geetha et al. [2] dissected the free convective heat and mass exchange initiated by a consistent mass motion on an allegorical began a vertical plate with variable temperature. Awasthi et al. [3] explored the impacts of heat and mass motion on MHD free convection course through a permeable medium with radiation and first-request chemical reaction. Rout et al. [4] introduced the chemical reaction impact on MHD free convection stream in a micropolar fluid. Jhankal and Manoj [5] examined the MHD Boundary layer stream past over a contracting sheet with Heat move and Mass attractions. Impacts of thermal radiation on MHD free convection stream past a vertical permeable plate within the sight of chemical reaction-FEM was concentrated by Shankar Goud et al [6]. Thermal radiation impacts on the MHD stagnation point stream over an extending sheet with slip limit conditions were examined by Goud, B.S.Goud.[7]. Isah, B.Y., et al[8] examined the thermal radiation and variable weight impacts on regular convective heat and mass exchange fluid stream in a permeable medium. Mazumdar and Deka [9] dissected the MHD stream past an incautiously begun unending vertical plate within the sight of thermal radiation. Babu et.al [10] considered the radiation impact on MHD heat and mass exchange stream over a contracting sheet with mass pull. Paul et.al [11] examined the chemical reaction impact on transient free convective stream past an endless moving vertical chamber. Shankar Goud et.al [12] researched the stagnation point move through a permeable medium towards extending surface within the sight of heat generation. The impact of thermal radiation on the heat and mass exchange stream of a variable thickness fluid past a vertical permeable plate saturated
by a cross over magnetic field was broke down by Makinde and Ogulu [13]. Odell and Azab[14]
examined the impact of chemical reaction on transient MHD free convection over a moving vertical
plate. Hari Singh Naik et.al[15] inspected the radiation and Hall impact on MHD blended convection
of Casson fluid over an extending sheet. many authors investigated on different field of the plates[17-
30].

In this examination, the consolidated effect for a inclined and semi-vertical plate with MHD
movement of viscous, incompressible fluid in a porous medium within the sight of heat assimilation
and joule heating impact. The resultant arrangement is acquired by RK fourth order with shooting
method and presented through graphs.

2. MATHEMATICAL FORMULATION:
The current examination is essentially on MHD movement of a viscous fluid that is incompressible,
pot a in penetrable medium. A slanted and semi-infinite vertical plate is likewise contemplated in this
investigation. Along the heading of the stream, x*-axis is expected while the normal direction is
assumed as y*. Likewise, a quality B0 magnetic field is applied along the y*-axis. Along these lines,
Viscous dispersal, Joule Dissipation is significant in the energy condition., Cogley radiative heat
transition is additionally distinguished cruciality. In this investigation, all fluid properties are thought
to be consistent except density in the body force term. Under the above assumption, the fluid stream
governing conditions are

\[
\frac{\partial u}{\partial y} = 0 \quad \ldots (1)
\]

\[
v' \frac{\partial u}{\partial y} = u \frac{\partial u}{\partial y} + g\beta(T' - T_\infty) \cos \alpha + g\beta(C' - C_\infty) \cos \alpha - \left( \frac{u}{\rho} + \frac{\sigma B_0^2}{\rho} \right) u' \quad \ldots (2)
\]

\[
v' \frac{\partial V}{\partial y} = -\frac{k}{\rho c_p} \frac{\partial T}{\partial y} + \frac{u}{\rho c_p} \frac{\partial u}{\partial y} \quad \ldots (3)
\]

\[
v' \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} \quad \ldots (4)
\]

The solution of Eqn(1) is defined as \( v' = -v_0 = \text{constant} \quad \ldots (5) \)

Here the \( v_0 > 0 \) refers to the constant suction velocity which is normal to the plate. We have the
following Cogley et al[16] as defined below, radiative heat flux: \( \frac{\partial q_r}{\partial y} = 4(T' - T_\infty) \frac{\sigma B_0^2}{\rho} \quad \ldots (6) \)

Where \( I = \int K_A \frac{\partial e_A}{\partial y} d\lambda \), \( K_A \) is the absorption coefficient of the plate and \( e_A \) is Planck’s function.

The appropriate boundary conditions are in this case

\[
u' = 0, \quad \frac{\partial V}{\partial y} = -\frac{q}{k} \frac{\partial C}{\partial y} = -\frac{m}{D} \text{ at } y' = 0 \quad \ldots (7)
\]

\[
u' \rightarrow 0, \quad T' \rightarrow T_\infty, \quad C' \rightarrow C_\infty \quad \text{as } y' \rightarrow \infty \quad \ldots (8)
\]

Eqs(2-4) changed to following form with the help of Eqs(5-6) as following form

\[
-v_0 \frac{\partial u}{\partial y} = u \frac{\partial^2 u}{\partial y^2} + g\beta(T' - T_\infty) \cos \alpha + g\beta(C' - C_\infty) \cos \alpha - \left( \frac{u}{\rho} + \frac{\sigma B_0^2}{\rho} \right) u' \quad \ldots (9)
\]

\[
-v_0 \frac{\partial V}{\partial y} = -\frac{k}{\rho c_p} \frac{\partial T}{\partial y} + \frac{u}{\rho c_p} \frac{\partial u}{\partial y} \quad \ldots (10)
\]

\[
-v_0 \frac{\partial C}{\partial y} = D \frac{\partial^2 C}{\partial y^2} - K_c(C' - C_\infty) \quad \ldots (11)
\]

With the following non-dimensional parameters
\[ f(\eta) = \frac{v_0}{v_0}, \quad \phi = \frac{v_0}{v_0}, \quad Gr = \frac{\beta m q v^2}{\kappa v_0^2}, \quad Gm = \frac{\beta n q v^2}{\kappa v_0^2}, \quad K = \frac{\beta k v_0^2}{v^2} \]

\[
M = \frac{\nu n_0^2}{\nu v_0}, \quad F = \frac{\nu n_0^2}{\nu v_0}, \quad Ec = \frac{\nu n_0^2}{\nu v_0}, \quad Pr = \frac{\nu n_0^2}{\nu v_0}, \quad Sc = \frac{\nu n_0^2}{\nu v_0}, \quad Kr = \frac{\nu n_0^2}{\nu v_0} \]

\[ \phi = \frac{\left( C - C \right) \nu n_0^2}{\nu v_0}, \quad \theta = \frac{\left( \tau - \tau \right) \nu n_0^2}{\nu v_0}, \quad Q = \frac{\nu n_0^2}{\nu v_0} \]

After introducing the nondimensional parameter into Eqn (8-10) becomes as

\[ \dot{f} + f' - \left( \frac{1}{K} + M \right) f' + Gr \theta + Gm \phi = 0 \]

\[ \frac{1}{Pr} \dot{\theta} + \theta' - \left( \frac{\beta m q}{\beta n} v_0^2 - Q \right) \theta + Ec(f')^2 + Me f^2 = 0 \]

\[ \frac{1}{Sc} \dot{\phi} + \phi' - \phi K r = 0 \]

Now the corresponding boundary conditions are transformed to

\[ f = 0, \quad \theta' = -1, \quad \phi' = -1 \quad \text{at} \quad \eta = 0 \]

\[ f \to 0, \quad \theta \to 0, \quad \phi \to 0 \quad \text{as} \quad \eta \to \infty \]

3. SOLUTION OF THE PROBLEM:

The arrangement of nonlinear coupled conventional differential conditions (12-14) trailed by the boundary conditions (15) are unravelled mathematically by the Runge-Kutta fourth-order alongside the shooting method. To start with, the higher-request ODE’s change into synchronous direct ODE’s of first order initial value problems by applying the shooting methods. By utilizing the RK fourth-order to the initial value problem. The progression size \( \Delta \eta = 0.001 \) is utilized to accomplish the mathematical arrangement as the convergence criterion with decimal place accuracy.

4. RESULTS AND DISCUSSION

Figures 2 to 13 explain the arrangements in the above examination. In the current study, the boundaries taken over are three dependent fluid dynamic factors (\( f, 0, \phi \)), one independent variable (\( \eta \)), body power control boundaries and thermophysical boundaries. The parametric defaults are \( M = 1, Gr = 5, Gm = 5, K = 0.5, Ec = 0.001, Pr = 0.71, Sc = 0.22, Kr = 2.5, F = 1 \). During calculations, characteristic parameter esteems are taken as \( M = 1, K = 0.5, Gr = Gm = 5, Ec = 0.001, Sc = 0.22, Kr = 0.71, F = 1 \) For each graph, numerical qualities are derived for all boundaries.

Figures 2 to 7 depict velocity profiles for changing boundaries. The impact of porous media that expanding \( Gr \) prompts higher speeds. Consequently, as \( Gr \) builds, the thermal lightness power increments. The proportion of solutal lightness power to thick power is \( Gr \). An expansion in fluid speed is portrayed with expanded estimations of \( Gr \).}

Figure 6 portrays the impact of the heat age boundary on the speed profile. Structure the figure shows that the speed bends increment with an expansion of the \( Q \) esteems.
Figure 7 purposeful the effect of the edge of tendency on the speed profile. It is noticed that the speed diminishes with the expansion in $\alpha$.

Temperature profiles are outlined in Figures 8-11. From Figures 8 and 9, an expansion in $Ec$ the outcome in temperature increments, and $Pr$ depicts a lower temperature esteem. $Pr$ is the proportion of energy diffusivity to thermal diffusivity. Consequently, it is sure that thermal diffusivity diminishes with an expansion in $Pr$ subsequently initiating fluid temperature all through the limit layer area.

From figure 10, an expansion in $F$ portrays a drop in temperature. In figure 11, the conduct of the heat age boundary on the temperature profile. It is discovered that the temperature increments with an expansion of $Q$ esteems. Figures 12 and 13 spotlight on fixation profiles for the impact of $Sc$ and $Kr$ separately. On expanding estimations of $Sc$ and $Kr$, fixation is found to decrease.

Because of the chemical reaction in the limit layer locale, the speed of the fluid diminishes. It very well may be perceived from this that the fixation field diminish is because of chemical species utilization. Additionally, lightness impacts will in general decrease, prompting a diminished speed of fluid stream accordingly. From figure 11, an expansion in $Gr$ prompts lesser skin erosion. From figure 12, with an expansion in $Ec$, $Nu$ is found to increment. $Nu$ is the distinction in the pace of heat move. So also, from figure 13, on expanding $Sc$, Sherwood number $Sh$ is found to increment. This shows that on expanding $Sc$, fluid fixation diminishes. Plainly, mass diffusivity is found to cause an expansion in the concentration within the boundary layer region.
**Fig 3.** Velocity $v/s$ Gr

**Fig 4.** Velocity $v/s$ Gm.
Fig. 5. velocity $v$ vs $Q$

Fig. 6. velocity $v$ vs $\alpha$
5. CONCLUSIONS
The RK fourth order and shooting method gives the closed-form answers for the nonlinear ODEs. The impacts of the different physical boundaries associated with the stream issue are frequently examined and graphically tended to. The ends are as per the following, dependent on the perceptions and conversations above.

- The velocity fluid stream improves by expanding Gr, K, Q, and Gm.
- It has been indicated that the fluid velocity diminishes with the $M$ and $\alpha$ rise.
- Also, the temperature increments with an expansion of Ec, and the converse impact happens with $F$, $Pr$, and $Q$ increment.
- It has it was discovered that the fluid focus diminishes ascending in Sc and Kr.
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