THE COSMOLOGICAL BULK FLOW: CONSISTENCY WITH Λ CDM AND ρ ≈ 0 CONSTRAINTS ON σ8 AND γ

ADI NUSser1 AND MARC DAVIS2

1 Physics Department and the Asher Space Science Institute-Technion, Haifa 32000, Israel; adi@physics.technion.ac.il
2 Departments of Astronomy & Physics, University of California, Berkeley, CA 94720, USA; mdavis@berkeley.edu

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ABSTRACT

We derive estimates for the cosmological bulk flow from the SFI++ Tully–Fisher (TF) catalog. For a sphere of radius 40 h−1 Mpc centered on the Milky Way, we derive a bulk flow of 333 ± 38 km s−1 toward Galactic (l, b) = (276°, 14°) within a 3° 1σ error. Within a radius of 100 h−1 Mpc we get 257 ± 44 km s−1 toward (l, b) = (279°, 10°) within a 6° error. These directions are at 40° to the Supergalactic plane, close to the apex of the motion of the Local Group of galaxies after the Virgocentric infall correction. Our findings are consistent with the Λ CDM model with the latest Wilkinson Microwave Anisotropy Probe (WMAP) best-fit cosmological parameters, but the bulk flow allows independent constraints. For the WMAP-inferred Hubble parameter h = 0.71 and baryonic mean density parameter Ωb = 0.0449, the constraint from the bulk flow on the matter density, Ωm, the normalization of the density fluctuations, σ8, and the growth index, γ, can be expressed as σ8Ωm−0.55/(Ωm/0.266)0.28 = 0.86 ± 0.11 (for Ωm ≈ 0.266). Fixing σ8 = 0.8 and Ωm = 0.266 as favored by WMAP, we get γ = 0.495 ± 0.096. The constraint derived here rules out popular Dvali–Gabadadze–Porrati models at more than the 99% confidence level. Our results are based on the All Space Constrained Estimate (ACSE) model which reconstructs the bulk flow from an all space three-dimensional peculiar velocity field constrained to match the TF data. At large distances, ACSE generates a robust bulk flow from the SFI++ survey that is insensitive to the assumed prior. For comparison, a standard straightforward maximum likelihood estimate leads to very similar results.

Key words: cosmological parameters – large-scale structure of universe

Online-only material: color figures

1 INTRODUCTION

Cosmological bulk flows are the peculiar velocities of whole spherical regions around us. Bulk flows are usually considered for sufficiently large spheres where linear expressions for the velocity and density power spectra are valid. This greatly facilitates the calculation of expected bulk flows in cosmological models, in contrast to analyzing the full field which may involve nonlinear effects on small scales (Feldman et al. 2010; Abate & Erdogdu 2009; Zaroubi et al. 2001; Freudling et al. 1999). In linear theory, the bulk flow of a sphere is solely determined by the gravitational pull of only the dipole component of the external mass distribution. Bulk flows are, therefore, an unmistakable indicator of distant large mass concentrations if they exist. The exact expression of the bulk flow, B(r), of a sphere of radius r is

\[ B(r) = \frac{3}{4\pi} \int_{r} v(x) \, d^3x, \]

where v(x) is the three-dimensional peculiar velocity field as a function of the comoving coordinate x. Beneath this innocuous expression lies a multitude of nuisances. An unbiased estimate of B requires knowledge of v sampled uniformly over the entire volume. However, observational probes of peculiar velocity measurements are available for a few thousand galaxies only with a patchy coverage of the local universe. Further, peculiar velocity probes such as the Tully–Fisher (TF) relation allow us to constrain only the radial component of the peculiar velocities of galaxies.

Recently compiled data on peculiar velocities have triggered renewed interest in the analysis of large-scale flows, including the bulk flow (Davis et al. 2011; Lavaux et al. 2010; Erdogdu et al. 2006). Feldman et al. (2010) report an unusually large bulk flow of 416 ± 78 km s−1 in a sphere of 100 h−1 Mpc which is at odds with the Λ CDM model with the best-fit parameters of the seven-year Wilkinson Microwave Anisotropy Probe (WMAP7; e.g., Jarosik et al. 2011; Larson et al. 2011). Here we provide an alternative estimate of the bulk using a single data set of TF measurements of galaxies, trimmed at faint magnitudes to ensure the linearity of the TF relation. The estimate is based on a method we call ACSE (for the All Space Constrained Estimate). The method computes B(r) using Equation (1) from a three-dimensional v(x) defined everywhere in a large region of space and constrained to match the TF data. For the analysis below, we use the SFI++ survey of spiral galaxies with I-band TF distances (Masters et al. 2006; Springob et al. 2007), which builds on the original Spiral Field I-band Survey (Giovanelli et al. 1994, 1995; Haynes et al. 1999) and Spiral Cluster I-band Survey (Giovanelli et al. 1997b, 1997a). We use the published SFI++ magnitudes and velocity widths, and derive our own peculiar velocities, rather than taking the published distances as given. We shall use the inverse of the Tully–Fisher (ITF) relationship. The main advantage of ITF methods is that samples selected by magnitude, as most are, will be minimally plagued by Malmquist bias effects when analyzed in the inverse direction (Schechter 1980; Aaronson et al. 1982). We assume that the circular velocity parameter, η ≡ log(line width), of a galaxy is, up to a random scatter, related to its absolute magnitude, M, by means of a linear ITF relation, i.e.,

\[ η = s M + η_0. \]

The preparation of the data is done following Davis et al. (2011). We include all field, group, and cluster galaxies. Galaxies in
groups and clusters are treated as individual objects, though the redshifts for template cluster galaxies are replaced by the systematic redshift of the cluster. Galaxies fainter than the estimated magnitude of $-20$ were removed from the sample as those showed significant deviations from a linear TF relation.

In order to have a cleaner TF sample, we select only objects with inclination $i > 45^\circ$ to ease problems with inclination corrections. All this leaves us with a sample of 2859 galaxies with redshifts less than $100h^{-1}$ Mpc. The effective depth of the sample defined as the error-weighted mean redshift of galaxies is $\sim 40h^{-1}$ Mpc.

We will refer to the $\Lambda$CDM cosmological model with WMAP7 best-fit parameters as $\Lambda$CDM7 (Larson et al. 2011) for a flat universe, i.e., the total mass density parameter $\Omega_0 = 0.266$, the baryonic density parameter $\Omega_b = 0.0449$, the Hubble constant $h = 0.71$ in units of $100$ km s$^{-1}$ Mpc$^{-1}$, the scalar spectral index $n_s = 0.963$, and $\sigma_8 = 0.8$ for the rms of linear density fluctuations in spheres of $8h^{-1}$ Mpc. Throughout the paper, variants of $\Lambda$CDM7 with different $\Omega_0$ and $\sigma_8$ will be considered. All other parameters will be fixed at their WMAP7 values.

The outline of the paper is as follows. Details of the ASCE method are described in Section 2, while the more standard maximum likelihood estimate (MLE) is outlined in Section 3. Tests of the methods using mock catalogs designed to match the SFI++ catalog are presented in Section 3. Results for the bulk flows from the SFI++ data are given in Section 5 with the subsection Section 5.1 providing a comparison with the $\Lambda$CDM models. Finally, Section 6 discusses the results and some of their cosmological implications.

2. THE ALL SPACE CONSTRAINED ESTIMATE (ASCE)

Observations of distance (peculiar velocity) indicators, such as the SFI++ TF survey, are available for only a small fraction of galaxies in the local universe (out to $\sim 100h^{-1}$ Mpc). The absence of uniformly distributed data prevents a direct application of Equation (1). To circumvent this problem, the ASCE method effectively uses Equation (1) to reconstruct the bulk flow from a three-dimensional field $v(x)$ which satisfies two conditions: (1) at sufficiently large distances from the observed galaxies in the TF data, it has a power spectrum that is dictated by a cosmological model such as the $\Lambda$CDM, and (2) it has radial peculiar velocities at positions of observed galaxies, which are consistent with the TF measurements. The approach is similar to that of constrained realization from noisy data (Hoffman & Ribak 1991; Zaroubi et al. 1995), but it is more general and easier to implement. Assume that the TF catalog contains $i = 1 \ldots N_g$ galaxies with measured redshifts (in km s$^{-1}$), $c z_i$, apparent magnitudes, $m_i$, and line width parameters, $\eta_i$. We write the absolute magnitude of a galaxy as

$$M_i = M_{0i} + P_i ,$$

where

$$M_{0i} = m_i + 5 \log(c z_i) - 15$$

and

$$P_i = -5 \log(1 - u_i/c z_i)$$

with $u_i$ being the radial peculiar velocity of the galaxy. Both $c z_i$ and $u_i$ are defined in the frame of the cosmic microwave background radiation. Assume that an estimate of the underlying cosmological velocity field, $v(x)$, can be written as a linear combination of

$$v(x) = \sum a^\alpha v^\alpha(x) ,$$

where the $N$ fields, $v^\alpha(x)$ ($\alpha = 1 \ldots N_g$), are Gaussian random velocity fields generated using a cosmologically viable power spectrum. In practice, these basis velocity fields will be extracted from a linear cosmological velocity field generated in a very large box using the power spectrum of the $\Lambda$CDM model. We then compute $u^\alpha$, the radial component of $v^\alpha$ at the redshift space positions of the observed galaxies, and define the $P$-basis functions,

$$P^\alpha_i = -5 \log(1 - u^\alpha_i/ c z_i)$$

for the observed galaxies only. The model $P$ is then written as

$$P^M_i = \sum a^\alpha P^\alpha_i .$$

The best-fit mode coefficients, $a^\alpha$, the slope, $s$, and the zero point, $\eta_0$, are found by minimizing the $\chi^2$ statistic

$$\chi^2 = \sum_{\alpha = 1}^{N_g} (s M_{0i} + s P^M_i + \eta_0 - \eta_i)^2 + \sum_{\alpha = 1}^{N_g} (a^\alpha)^2 ,$$

where $\sigma_\eta^2$ is the rms of the intrinsic scatter in $\eta$ about the ITF relation and $N_g$ is the number of galaxies in the sample. The second term of the sum over the squares of $a^\alpha$ is introduced in order to regularize the solution especially in regions of poor data coverage. In the Appendix, we derive this term from a Bayesian formulation. The solution to the equations $\partial \chi^2/\partial a^\alpha = 0$, $\partial \chi^2/d s = 0$, and $\partial \chi^2/d \eta_0 = 0$ is straightforward. The coefficients $a^\alpha$ will be used in Equation (6) to get $v(x)$ everywhere in a region of space large enough to contain the data. For each field $v^\alpha(x)$ we compute its corresponding bulk flow, $B^\alpha(r)$, according to Equation (1) and write our ASCE bulk flow as

$$B_{ASCE}(r) = \sum a^\alpha B^\alpha(r) .$$

3. THE MAXIMUM LIKELIHOOD ESTIMATE (MLE)

For comparison, we will present estimates of the bulk flow obtained with the standard MLE (Kaiser 1988). This method approximates the bulk flow of a sphere of radius $r$ as the vector $B_{MLE}$ which renders a minimum in

$$\chi^2 = \frac{1}{\sigma_\eta^2} \sum_{c z_i < r} \left( s M_{0i} + 2.17s B \cdot \hat{r}_i/c z_i + \eta_0 - \eta_i \right)^2$$

with respect to the three components of $B$. The sum is over galaxies within $r$ and $\hat{r}_i$ is a unit vector in the direction of galaxy $i$. Further, in this expression we have approximated $P = -5 \log(1 - B \cdot \hat{r}_i/c z_i) \approx 2.17B \cdot \hat{r}_i/c z_i$.

4. TESTS

In order to test the performance of ASCE and the MLE reconstructions of the bulk flow from the SFI++ TF data we use 2200 mock catalogs of TF measurements. In each of the catalogs, galaxies with the same positions as in the real SFI++ data are assigned absolute magnitudes, $M_i$, and line width parameters, $\eta_i$, following an artificial ITF relation with slope $s = -0.12$ and intrinsic scatter $\sigma_\eta = 0.057$ (e.g., Davis et al. 2011). The peculiar velocities of galaxies in each mock are taken...
from a linear Gaussian random velocity field in a cubic box of $1454 h^{-1}$ Mpc on the side. Each mock is placed randomly in this large box and the peculiar velocity of each galaxy is then obtained by interpolating the velocity field on the position of the galaxy. A Gaussian random realization of the velocity field is generated for the ASCE-reconstructed bulk, $\Lambda$ field is generated for the galaxy. A Gaussian random realization of the velocity field is then obtained by interpolating the velocity field on the position in this large box and the peculiar velocity of each galaxy is estimated. We emphasize that a basis function $\mathbf{v}(x)$ is to filter out low-frequency modes that would be overfitted by the data, especially at large distances. This smoothing, however, has very little effect on the bulk flows reconstructed by ASCE. We emphasize that a basis function $\mathbf{v}(x)$ is not only defined at the galaxy positions, but also at any point in a sufficiently large volume (radius $10h^{-1}$ Mpc) that contains all the galaxies used in the analysis. Hence, for each basis function, we can measure its corresponding bulk flow, $\mathbf{B}(r)$, and once the coefficients $\alpha$ have been determined from the data by minimization of Equation (8) then the ASCE-reconstructed bulk, $\mathbf{B}_{\text{ASCE}}(r)$, is readily given by Equation (9).

Figure 1 shows scatter plots of the estimated versus true bulk flows of a spherical region of $60h^{-1}$ Mpc in radius. For clarity, results from only 400 randomly selected mock catalogs are shown. Blue dots and red plus signs correspond to $\mathbf{B}_{\text{MLE}}$ and $\mathbf{B}_{\text{ASCE}}$, respectively. Because of the anisotropic distribution of the observed galaxies, the methods may not reconstruct the three Cartesian components equally well. Hence, $x$, $y$, and $z$ bulk flow components in Supergalactic coordinates are shown in the top, middle, and bottom panels, respectively. Blue and red lines in each panel are linear regressions of the estimated true bulk flows. The corresponding mathematical expressions of the regressions are indicated in each panel.

The regularization term in Equation (8) naturally tends to underestimate the coefficients $\alpha$ and subsequently the reconstructed bulk flow. However, the agreement between the ASCE-reconstructed and the true bulk flows seen in Figure 1 clearly demonstrates that the effect is meager. To further explore the quality of the ASCE and MLE reconstructions and to ascertain that the regularization term does not cause a significant reduction in the amplitude of the bulk flow, we apply ASCE and MLE to the mock data but with true velocities amplified by a factor of 1.5. Everything else, including the regularization term in Equation (8), remained the same. The reconstructed $\mathbf{B}_{\text{ASCE}}$ and $\mathbf{B}_{\text{MLE}}$ versus true amplified bulk flows are shown in Figure 2. Both ASCE and MLE perform well even with this amplification of the bulk flow in the mocks.

In both ASCE and MLE, the slopes of the regression lines plotted in Figure 1 are close but not equal to unity. The deviation from unity is significant (compared to the scatter of the points) and persists when the regression is performed using all 2200 mock points. This small but statistically significant bias depends on the radius of the sphere for which bulk flow is computed. The bias can easily be calibrated using the mock catalogs. Hereafter, all reconstructed bulk flows, from ASCE and MLE, are corrected for the systematic bias in the mean of the estimated bulk given the mean of the true value. In practice, we write the corrected estimate of the bulk flow $\mathbf{B}_{\text{corr}}(r)$ from the raw bulk $\mathbf{B}_{\text{raw}}(r)$ (directly reconstructed by either ASCE or MLE) as $\mathbf{B}_{\text{corr}}(r) = C_1 \mathbf{B}_{\text{raw}}(r) + C_2$ where $C_1$ is the ratio of the rms values of the true to raw bulk flows and $C_2$ is a constant term that accounts for the offset between the true and raw bulk flows.

In all panels of Figure 1, the mock $\mathbf{B}_{\text{ASCE}}$ are tightly scattered around their corresponding regression lines. The scatter in $\mathbf{B}_{\text{MLE}}$ appears to be more significant. To quantify the scatter between the reconstructed and true bulk flows, we plot in Figure 3 the cumulative fraction, $P < (\theta)$, of mock catalogs for which the angle between estimated and true bulk flows is less than $\theta$. The solid blue and red dashed curves refer to ASCE and MLE, respectively, while thick and thin curves refer to bulk flows within $40h^{-1}$ Mpc and $100h^{-1}$ Mpc, respectively. The curves are computed after employing the correction to the systematic bias as explained above. The performance of ASCE is excellent. For $40h^{-1}$ Mpc, the direction of $\mathbf{B}_{\text{ASCE}}$ is recovered within $3^\circ$ for about 68% of the mocks. For $100h^{-1}$ Mpc this uncertainty...
Figure 2. Same as Figure 1 but for the peculiar velocities in the mocks amplified by a factor of 1.5.
(A color version of this figure is available in the online journal.)

Figure 3. Cumulative fraction of mock catalogs with estimated bulks directed within an angle smaller than $\theta$ from the direction of the true bulk flow. Blue solid lines and red dashed lines correspond to the ASCE and MLE reconstructions, respectively. Thick and thin lines refer to bulk flows of spheres of radii $40h^{-1}$ Mpc and $100h^{-1}$ Mpc centered on the observer, respectively.
(A color version of this figure is available in the online journal.)

Figure 4. Differential distribution functions of the difference between estimated and true respective Cartesian components. The notation of the lines is the same as in Figure 3.
(A color version of this figure is available in the online journal.)

The results of the application of the ASCE and MLE methods to recover the bulk flow from the real SFI++ TF catalog are summarized in Figures 5 and 6. The bulk flows are reconstructed for spheres centered on the Milky Way and of radii from $r = 20h^{-1}$ Mpc to $100h^{-1}$ Mpc in steps of $10h^{-1}$ Mpc. The smallest radius is chosen large enough so that nonlinear effects are not expected to be important (Nusser et al. 1991), facilitating the comparison with cosmological models. The largest radius corresponds to the distance within which the data are used.

Figure 5 shows the Galactic $x$ (blue dotted), $y$ (black solid), and $z$ (red dot-dashed) components of $B_{\text{ASCE}}$ (top panel) and $B_{\text{MLE}}$ (bottom) as a function of $r$. The magnitudes of $B_{\text{ASCE}}$ and $B_{\text{MLE}}$ versus radius are plotted as blue circles and red crosses, respectively, and plotted in Figure 6. The $1\sigma$ error bars in both figures are based on the 2200 mocks. The component $B_y$ is clearly the most significant. This is just a coincidence and has no bearing on the statistical analysis of the results as one can always choose a coordinate system such that the bulk is along a given axis. The solid curve in this figure is the theoretical expectation of $\Lambda$CDM7 but with $\sigma_8 = 0.85$ instead of the default $\sigma_8 = 0.8$. The theoretical curve is computed given the density increased to $7^\circ$. The ASCE method is significantly superior to MLE. The thin blue and thick dashed lines almost overlap, meaning that the performance of ASCE for $100h^{-1}$ Mpc is as good as that of MLE for $40h^{-1}$ Mpc. For $r = 100h^{-1}$ Mpc, MLE recovers the direction only within $27^\circ$ for $68\%$ of the mocks.

Figure 4 plots the differential probability distribution function, $P(\delta B)$, where $\delta B$ refers to the difference in all Cartesian components between estimated and true bulk flows, from the 2200 mocks. The notation of the lines is the same as in Figure 3 and as displayed in Figure 4. The figure also indicates $\sigma$, the rms value of $\delta B$ for the plotted cases. The low values of $\sigma$ corresponding to ASCE demonstrate its excellent ability to recover the true bulk. The performance of MLE is good, but less satisfactory.

5. RESULTS

The results of the application of the ASCE and MLE methods to recover the bulk flow from the real SFI++ TF catalog are summarized in Figures 5 and 6. The bulk flows are reconstructed for spheres centered on the Milky Way and of radii from $r = 20h^{-1}$ Mpc to $100h^{-1}$ Mpc in steps of $10h^{-1}$ Mpc. The smallest radius is chosen large enough so that nonlinear effects are not expected to be important (Nusser et al. 1991), facilitating the comparison with cosmological models. The largest radius corresponds to the distance within which the data are used.

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Three Galactic Cartesian components of the bulk flow as a function of radius. Top and bottom panels correspond to ASCE and MLE estimation, respectively.

(A color version of this figure is available in the online journal.)

Figure 6. Amplitude of the bulk as a function of distance for ASCE and MLE. The solid curve shows the rms value of the bulk flow as expected in a flat universe $\Lambda$CDM model with $\Omega_m=0.266$, $h=0.71$, and $\sigma_8=0.85$.

(A color version of this figure is available in the online journal.)

The power spectrum $p_s(k, \Omega_m, \Omega_b, h, n_s)$ by

$$\sigma_v^2(r) = \frac{H_0^2 f^2}{2\pi^2} \int dk \ p_s(k) W^2(kr),$$

where $f = \Omega_m^{\gamma}$ with a growth index $\gamma \approx 0.55$ for a flat universe (Linder 2005) and $W = W_{TH}(kr)\exp(-k^2 R_s^2/2)$ with $W_{TH}$ is the top-hat window function and the Gaussian window accounts for the fact that the basis functions $\nu^\alpha$ used in ASCE are smoothed with a Gaussian window of $R_s=10h^{-1}$ Mpc in width. The expression (11) is obtained assuming the linear relation $H_0 f \delta = -\nabla \cdot \psi(x)$ between the density contrast, $\delta$, and $\psi(x)$ (Peebles 1980).

The MLE- and ASCE-reconstructed bulks are similar especially at large distances of $r > 40h^{-1}$ Mpc. This is because the data cover space more isotropically at larger distances. Figure 2 clearly demonstrates that ASCE will not cause a significant artificial underestimation of large bulk flows such as in Feldman et al. (2010). To further ascertain if our ASCE-derived bulk flow is robust, the blue circles in Figure 7 show the amplitude of the ASCE bulk flow reconstructed using basis functions generated from a $\Lambda$CDM power spectrum but with a scalar index $n=0.75$ and $\sigma_8=1$. The results are very similar to the ASCE bulk flow shown in Figure 6 despite the significantly enhanced large-scale power. The agreement is particularly striking at large distances.

5.1. Comparison with Cosmological Models

Figure 6 indicates that the estimated bulk flows are consistent with theoretical expectations, but the errors are strongly correlated and a proper statistical analysis must take into account the covariance of the errors. Our large number (2200) of mock catalogs allow a robust determination of the error covariance between the bulk flow estimates at different radii. Since ASCE is significantly superior to MLE, we restrict the comparison with models to ASCE reconstruction. We use all components of $\mathbf{B}_{\text{ASCE}}$ estimated at eight values of $r$ ranging from $r = 30h^{-1}$ Mpc to $100h^{-1}$ Mpc in steps of $10h^{-1}$ Mpc. The reason for not considering smaller radii is that the bulk is most robustly constrained independent of the assumed basis functions at $r > 30h^{-1}$ Mpc. We denote the set of ASCE-reconstructed Cartesian components at these eight values of $r$ with $B_i$ and the corresponding underlying true quantities with $\mathbf{B}_i$. We write the

Figure 7. Same as Figure 6 with ASCE basis functions generated using a $\Lambda$CDM7 power spectrum but with scalar index $n=0.75$ and $\sigma_8=1$, which has more power on large scales compared to our standard choice $n=0.963$.

(A color version of this figure is available in the online journal.)
The best-fit \( \sigma_8 \) for eight values of \( \Omega_m \), \( \sigma_8 = (0.236, 0.88) \), while the circle indicates \( (0.266, 0.8) \), corresponding to the best-fit WMAP7 values.

(A color version of this figure is available in the online journal.)

The probability for observing the set \( B_o \) as

\[
P(B_o) = \int dB_i P(B_o|B_i)P(B_i) \tag{12}
\]

where the probability \( P(B_i) \) for the underlying \( B_i \) is computed within the framework of a cosmological model. Here, we adopt the \( \Lambda \)CDM model. For Gaussian velocity fields, the calculation of \( P(B_i) \) is easily performed by integrating standard analytic expressions involving the power spectrum. We assume that the probability \( P(B_o|B_i) \) for \( B_o \), given \( B_i \) is Gaussian with an error covariance matrix computed from the 2200 mocks. Under these assumptions, expression (12) yields

\[
P(B_o) = \frac{1}{\sqrt{(2\pi)^d|\Sigma|}} \exp \left( -\frac{1}{2} B_o^T \Sigma^{-1} B_o \right), \tag{13}
\]

where \( d \) is the number of elements in \( B_o \), i.e., \( d = 24 = 8 \times 3 \), for eight values of \( r \) and three Cartesian components. The \( d \times d \) covariance matrix \( \Sigma = \Sigma_0 + \Sigma_i \), where \( \Sigma_0 \) is the covariance of the errors on \( B_o \) and \( \Sigma_i \) describes the covariance of the underlying quantities \( B_i \). The dependence on the cosmological models comes through \( \Sigma_i \).

5.2. Consistency with \( \Lambda \)CDM7

We begin by assessing how well \( \Lambda \)CDM7 is consistent with the data. To do that we generated \( 10^5 \) sets, \( B_{\text{rnd}} \), each containing \( d = 18 \) numbers selected at random from a Gaussian distribution given by Equation (13) computed with \( \Sigma_i \) for \( \Lambda \)CDM7. For each of those \( 10^7 \) sets of \( B_{\text{rnd}} \) we compute the corresponding \( P(B_{\text{rnd}}) \) using Equation (13) and tabulate the negative of the log of the probability, \( nLP_{\text{rnd}} = -\ln P(B_{\text{rnd}}) \). We also compute \( nLP_o = -\ln P(B_o) \) for the observed \( B_o \) using \( \Lambda \)CDM7. We find that only 26% of the \( 10^7 \) values of \( nLP_{\text{rnd}} \) exceed \( nLP_o \). Therefore, the \( \Lambda \)CDM7 cannot be rejected by the bulk flow results.

5.3. Independent Constraints on \( \sigma_8 \) and \( \gamma \)

The \( \Lambda \)CDM7 expected amplitude of the bulk flow depends separately on the cosmological parameters (see Equation (11) and the parametric form for the power spectrum in Eisenstein & Hu 1998), but the most significant dependence is on \( \sigma_8 \) and \( \Omega_m \) and hence we restrict ourselves here to deriving constraints on these two parameters only. We compute \( nLP_o \) for a grid of values of \( \Omega_m \) and \( \sigma_8 \) used in \( \Sigma_i \), maintaining all other parameters at their default \( \Lambda \)CDM7 values.

Confidence levels (CLs) on \( \Omega_m \) and \( \sigma_8 \) are obtained by inspecting the contours of \( \Delta \chi^2(\Omega_m, \sigma_8) = 2(nLP_o - \min(nLP_o)) \) in the \( (\Omega_m, \sigma_8) \) plane. The minimum of \( nLP_o \) (i.e., \( \Delta \chi^2 = 0 \)) is at \( (\Omega_m, \sigma_8) = (0.236, 0.88) \), marked by the plus sign in the figure. The \( \Lambda \)CDM7 default values \( (\Omega_m, \sigma_8) = (0.266, 0.8) \) are indicated by the circle. The inner and outer contours of \( \Delta \chi^2 \) shown in Figure 8 correspond to 68% and 95% CLs for two degrees of freedom (Press et al. 1992). The \( \Lambda \)CDM7 point is well within the 68% CL. The shape of the contours implies the correlation \( \sigma_8 \sim \Omega_m^{-0.28} \). This reflects the dependence of the shape of the density power spectrum \( p_r \) on \( \Omega_m \), and from the factor \( f(\Omega) \approx \Omega_m^{0.55} \) (see Equation (11)). Only if we neglect the dependence of the shape of \( p_r \) on \( \Omega_m \) we get \( \sigma_8 \sim \Omega_m^{-0.55} \). It is of interest to inspect the constraints when either of the parameters \( \Omega_m \) or \( \sigma_8 \) is fixed at certain values. Figure 9 shows two curves of \( \Delta \chi^2 \) versus \( \sigma_8 \) corresponding to the WMAP7 \( \Omega_m = 0.266 \) and 0.236 giving a minimum of \( \Delta \chi^2 \) in the \( (\Omega_m, \sigma_8) \) plane as seen in Figure 8. Figure 10 plots \( \Delta \chi^2 \) as a function of \( \Omega_m \), for \( \Omega_m \) at the WMAP7 values of 0.8 and 0.88 corresponding to the minimum of \( \Delta \chi^2 \) in Figure 8. In each of the curves in Figures 9 and 10, the value of \( \Delta \chi^2 \) at the minimum of the curve is set to zero. Hence, \( \Delta \chi^2 = 1 \) and 4 correspond to 68% (1\( \sigma \)) and 95% (2\( \sigma \)) CLs, respectively (Press et al. 1992). These curves assume a growth index \( \gamma = 0.55 \) as is appropriate for a \( \Lambda \)CDM model. Hence Figure 9 gives \( \sigma_8 = 0.86 \pm 0.11 \) (1\( \sigma \)) for \( \Omega_m = 0.266 \) and \( \gamma = 0.55 \). However, we see from Equation (11) that by varying \( \gamma \) alone we get the scaling \( \sigma_8(\gamma = 0.55) = \sigma_8(\gamma)\Omega_m^{-0.55} \). We can use this to set a constraint on \( \gamma \) if we adopt \( \sigma_8 = 0.8 \) and \( \Omega_m = 0.266 \) (Larson et al. 2011). Demanding that \( \sigma_8(\gamma) = 0.8 \), the scaling gives \( \gamma = 0.496 \pm 0.096 \). Figure 11 confirms this result. The figure plots \( \Delta \chi^2 \) as a function of \( \gamma \) for the adopted values of \( \sigma_8 \) and \( \Omega_m \) as indicated. The left and right arrows mark the values \( \gamma = 0.42 \) and 11/6. The lower value is expected in \( f(R) \) models (e.g., Gannouji et al. 2009) and
the highest value corresponds to a Dvali–Gabadadze–Porrati (DGP; Dvali et al. 2000; Wei 2008) flat braneworld cosmology. We could also substitute the scaling with $\sigma_8$ and $\Omega_m$ for $f(R)$ and flat DGP models, respectively.

Figure 10. Curves of $\Delta \chi^2$ as a function of $\Omega_m$ for $\sigma_8 = 0.8$ (blue solid line) and $\sigma_8 = 0.88$ (red dot-dashed line).

Figure 11. Curves of $\Delta \chi^2$ as a function of the growth index $\gamma$ given $\sigma_8 = 0.8$ and $\Omega_m = 0.266$. The left and right arrows indicate $\gamma$ values obtained in $f(R)$ and flat DGP models, respectively.

The analysis presented here uses a trimmed version of the SFI++ in which galaxies fainter than $M = -20$ are removed. This ensures the linearity of the TF relation. Further, to avoid dealing with selection effects imposed on the magnitudes we use the inverse TF relation (see Strauss & Willick 1995 for a thorough review of this issue). Additionally, to minimize inhomogeneous Malmquist bias (Lynden-Bell et al. 1988), we do not place galaxies at their TF-inferred distances, but at their measured redshifts, which have significantly smaller observational errors. We also collapse the main known galaxy clusters.

The bulk flows estimated here are remarkably featureless and do not seem to reflect the gravitational effects of any of the individual main nearby clusters. Bulk flow estimates from TF-like relations are traditionally featureless (e.g., Dekel 1994), in contrast to velocity dipoles estimated from the distribution of galaxies in redshift surveys (e.g., Nusser & Davis 1994). In the analysis here, we have collapsed clusters, and therefore, signatures of individual clusters in our estimated bulk flows could be smeared out. It is instructive to explore how much we are missing by collapsing clusters and whether the signature of infall on clusters of nearby galaxies can actually be clearly seen in SFI++ or similar data. As an illustrative representative case we plot in Figure 12 individual peculiar velocities of 54 SFI++ galaxies contained in a cylinder of 6° in radius and 2600 km s$^{-1}$ centered on the Virgo cluster.

6. DISCUSSION

The bulk flows estimated here are remarkably featureless and do not seem to reflect the gravitational effects of any of the individual main nearby clusters. Bulk flow estimates from TF-like relations are traditionally featureless (e.g., Dekel 1994), in contrast to velocity dipoles estimated from the distribution of galaxies in redshift surveys (e.g., Nusser & Davis 1994). In the analysis here, we have collapsed clusters, and therefore, signatures of individual clusters in our estimated bulk flows could be smeared out. It is instructive to explore how much we are missing by collapsing clusters and whether the signature of infall on clusters of nearby galaxies can actually be clearly seen in SFI++ or similar data. As an illustrative representative case we plot in Figure 12 individual peculiar velocities of 54 SFI++ galaxies contained in a cylinder of 6° in radius and 2600 km s$^{-1}$ centered on the Virgo cluster.

Figure 12. Individual peculiar velocities, $V_{TF}$, of galaxies in the line of sight to Virgo, plotted against the redshift (red circles) and the estimated distance $d_{TF}$ (blue plus signs). The centroid of these galaxies is at $cz \sim 0$ and $V_{TF} \sim 0$.

(A color version of this figure is available in the online journal.)
investigate the full information in the peculiar velocity measurements. This could be done by analysis of power spectra and correlation functions by maximum likelihood techniques (e.g., Gorski et al. 1989; Jaffe & Kaiser 1995; Zaroubi et al. 1997; Juszkiewicz et al. 2000; Bridle et al. 2001; Abate & Erdogdu 2009). However, the bulk flow is particularly appealing because of its simplicity and the fact that it is entirely linear for sufficiently large spheres. The constraints from peculiar velocities, including the bulk flow, are unique since they are local at redshifts very close to zero and they directly probe the growth index \( \gamma = \frac{\ln f}{\ln a} \), where \( f \) is the linear growth factor (Peebles 1980; Linder 2005). Adopting the \( \Lambda \)CDM cosmological parameters (Larson et al. 2011), we derive a local constraint of \( \gamma = 0.495 \pm 0.096 \). This constraint is completely independent of the biasing relation between galaxies and mass. Further, it is essentially a constraint at \( z = 0 \). By contrast, the lowest redshift constraint obtained from a study of redshift distortions in the Two-Degree Field galaxy redshift survey is at \( z \approx 0.15 \) (Hawkins et al. 2003). Our constraint significantly improves upon previous constraints on \( \gamma \) (Dosset et al. 2010; Wei 2008) derived at higher redshifts. This result could help us distinguish between alternative theories for structure formation (e.g., Amendola et al. 2005; Guzzo et al. 2008; Keselman et al. 2010).

For \( \sigma_8 = 0.8 \), the constraint on \( \gamma \) disfavors DGP models at \( \sim 2\sigma \) level, but it is consistent with \( f(R) \) gravity models (e.g., Starobinsky 2007; Gannouji et al. 2009; Wu et al. 2009; Fu et al. 2010). However, \( \sigma_8 \) in these models should be computed self-consistently, assuming the same normalization at the recombination epoch. Based on WMAP7, this implies \( \sigma_8 = 0.63 \) and 0.855 for DGP and \( f(R) \) models, respectively. Adopting these \( \sigma_8 \) values for these models, we get \( \gamma \approx 0.315 \pm 0.091 \) and 0.55 \( \pm 0.098 \) for the south Galactic pole and \( f(r) \), respectively (C. Di Porto 2010, private communication). In the DGP model, the expected value for \( \gamma \) at \( z = 0 \) is 0.664 (Wu et al. 2009), which is ruled by our constraint at more than the \( 3\sigma \) level. The \( f(R) \) model cannot be ruled out at a high CL by the constraint derived here.

Our results are in agreement with the analysis of Sandage et al. (2010). Peculiar velocities from supernovae, although very sparse, yield bulk flows that are consistent with WMAP7 (Dai et al. 2011; Colin et al. 2011). The results are in agreement with WMAP7. The direction of the bulk flow is robust and agrees with the direction of the motion of the Local Group after correcting for the Virgo-centric infall (Sandage et al. 2010), but we strongly disagree with the bulk flows of Feldman et al. (2010) who find a significant large bulk flow at \( r = 100h^{-1} \) Mpc using the untrimmed SFI++ survey, other individual data sets, and a composite catalog. We have opted to use a single uniformly calibrated catalog, namely the SFI++, excluding faint galaxies that spoil the linearity of the ITF (Davis et al. 2011). We have also refrained from using composite data since minor miscalibration errors between different catalogs could lead to large artificial flows when these catalogs are combined. Further, we place galaxies at their measured redshifts rather than estimated distances from the TF relation. This greatly suppresses inhomogeneous Malmquist bias which is known to lead to significant spurious signal especially at large distances. We also refrained from using a Gaussian window so that the bulk flow within a certain radius is completely unaffected by the increasing uncertainties at large distances.

We have seen that the MLE and the ASCE methods give very similar results. Further, the ASCE bulk flow at \( r > 30h^{-1} \) Mpc is almost completely independent of the cosmological model used in generating the basis functions. This is clearly demonstrated by the comparison of Figures 6 and 7. However, in ASCE, even if the results turned out to be sensitive to the assumed model used in generating the basis function, the validity of the model can still be confidently assessed. The reason is that the sensitivity would imply that the data are insufficient for constraining the bulk flow within the framework of the assumed model used in generating the basis function. Fortunately, this ambiguity is irrelevant for the SFI++ used here since the corresponding bulk flow is extremely insensitive to the model used in generating the basis functions.

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**APPENDIX**

**THE REGULARIZATION TERM**

The motivation for the second term on the right-hand side of Equation (8) can be found in a Bayesian statistical formulation. Our model for the three-dimensional velocity field defined at any point in space \( x \) is given in terms of an expansion, \( \mathbf{v}^M(x) = \sum_{\alpha} a^\alpha \mathbf{v}^\alpha(x) \), over \( N_\alpha \) basis functions \( \mathbf{v}^\alpha(x) \), each corresponding to a realization of a random Gaussian field with a cosmologically motivated power spectrum. Given the data, the probability \( P(\mathbf{v}^M|\text{data}) \) for \( \mathbf{v}^M(x) \) is

\[
P(\mathbf{v}^M|\text{data}) \propto P(\text{data}|\mathbf{v}^M)P(\mathbf{v}^M),
\]

where \( P(\text{data}|\mathbf{v}^M) \) is assumed to be Gaussian with \(-\ln P(\text{data}|\mathbf{v}^M) = \sum_i (s M_\alpha + P^M + \eta_\alpha - \eta_j)^2/\sigma_\alpha^2 \) with \( P^M \) being related to the radial component of \( \mathbf{v}^M \) by Equation (5). This probability function gives rise to the first term on the right-hand side in Equation (8). The prior function \( P(\mathbf{v}^M) \) is the probability for the realization of the particular velocity field model \( \mathbf{v}^M \) independent of the data. For a Gaussian random field

\[
-\ln P(\mathbf{v}^M) = \sum_{x,x',J,J'} v^M_{x,J}(x) \Xi^{-1} v^M_{x',J}(x'),
\]

where the indices \( J \) and \( J' \) refer to the three velocity components and \( \Xi(x,x',J,J') \) is the velocity correlation function. Substituting the expansion \( \mathbf{v}^M = \sum_{\alpha=1}^{N_\alpha} a^\alpha \mathbf{v}^\alpha \) yields

\[
-\ln P(\mathbf{v}^M) = \sum_{\alpha,\beta} a^\alpha a^{\beta} \sum_{x,x',J,J'} v^\alpha_{x,J}(x) \Xi^{-1} v^\beta_{x',J}(x').
\]

Since \( \mathbf{v}^\alpha \) are all space-independent fields, the terms with \( \alpha \neq \beta \) will be negligibly small. Since all of \( \mathbf{v}^\alpha \) are generated with the same power spectrum, the term \( S = \sum_{x,x',J,J'} v^\alpha_{x,J}(x) \Xi^{-1} v^\beta_{x',J}(x') \) will be independent of \( \alpha \). Therefore, \(-\ln P(\mathbf{v}^M) = S \sum_{\alpha} (a^\alpha)^2 \). If the model is also to represent a random realization of the same power spectrum as each of the basis functions \( \mathbf{v}^\alpha \) then the sum in Equation (A2) must also be equal to \( S \). Hence

\[
P(\mathbf{v}^M) \propto P(|a^\alpha|) \propto \exp(-\sum_{\alpha} (a^\alpha)^2/2) \]
REFERENCES

Aaronson, M., Huchra, J., Mould, J., Schechter, P. L., & Tully, R. B. 1982, ApJ, 258, 64
Abate, A., & Erdogdu, P. 2009, MNRAS, 400, 1541
Amendola, L., Quercellini, C., & Giallongo, E. 2005, MNRAS, 357, 429
Bridle, S. L., Zehavi, I., Dekel, A., Lahav, O., Hobson, M. P., & Lasenby, A. N. 2001, MNRAS, 321, 333
Colin, J., Mohayee, R., Sarkar, S., & Shaﬁeloo, A. 2011, MNRAS, 414, 264
Dai, D.-C., Kinney, W. H., & Stojkovic, D. 2011, J. Cosmol. Astropart. Phys., JCAP04(2011)015
Davis, M., Nusser, A., Masters, K., Springob, C., Huchra, J. P., & Lemson, G. 2011, MNRAS, 413, 2906
Davis, M., & Nusser, A. 1994, ApJ, 421, L1
Dekel, A. 1994, ARA&A, 32, 371
Dossett, J., Ishak, M., Moldenhauer, J., Gong, Y., & Wang, A. 2010, J. Cosmol. Astropart. Phys., JCAP04(2010)022
Dvali, G., Gabadadze, G., & Porrati, M. 2000, Phys. Lett. B, 485, 208
Eisenstein, D. J., & Hu, W. 1998, ApJ, 496, 605
Erdogdu, P., et al. 2006, MNRAS, 373, 45
Feldman, H. A., Watkins, R., & Hudson, M. J. 2010, MNRAS, 407, 2328
Freudling, W., et al. 1999, ApJ, 523, 1
Fu, X., Wu, P., & Yu, H. 2010, Eur. Phys. J. C, 68, 271
Gannouji, R., Moraes, B., & Polarski, D. 2009, J. Cosmol. Astropart. Phys., JCAP02(2009)034
Giovanelli, R., Haynes, M. P., Herter, T., Vogt, N. P., da Costa, L. N., Freudling, W., Salzer, J. J., & Wegner, G. 1997a, AJ, 113, 53
Giovanelli, R., Haynes, M. P., Herter, T., Vogt, N. P., Wegner, G., Salzer, J. J., da Costa, L. N., & Freudling, W. 1997b, AJ, 113, 22
Giovanelli, R., Haynes, M. P., Salzer, J. J., Wegner, G., da Costa, L. N., & Freudling, W. 1994, AJ, 107, 2036
Giovanelli, R., Haynes, M. P., Salzer, J. J., Wegner, G., da Costa, L. N., & Freudling, W. 1995, AJ, 110, 1059
Gorski, K. M., Davis, M., Strauss, M. A., White, S. D. M., & Yahil, A. 1989, ApJ, 344, 1
Guzzo, L., et al. 2008, Nature, 451, 541
Hawkins, E., et al. 2003, MNRAS, 346, 78
Haynes, M. P., Giovanelli, R., Chamaaux, P., da Costa, L. N., Freudling, W., Salzer, J. J., & Wegner, G. 1999, AJ, 117, 2039
Hoffman, Y., & Ribak, E. 1991, ApJ, 380, L5
Jaffe, A. H., & Kaiser, N. 1995, ApJ, 455, 26
Jurisic, N., et al. 2011, ApJS, 192, 14
Juszkiewicz, R., Ferreira, P. G., Feldman, H. A., Jaffe, A. H., & Davis, M. 2000, Science, 287, 109
Kaiser, N. 1988, MNRAS, 231, 149
Karachentsev, I. D., & Nasonova, O. G. 2010, MNRAS, 405, 1075
Keselman, J. A., Nusser, A., & Peebles, P. J. E. 2010, Phys. Rev. D, 81, 063521
Larson, D., et al. 2011, ApJS, 192, 16
Lavaux, G., Tully, R. B., Mohayee, R., & Colombi, S. 2010, ApJ, 709, 483
Linder, E. V. 2005, Phys. Rev. D, 72, 043529
Lynden-Bell, D., Faber, S. M., Burstein, D., Davies, R. L., Dressler, A., Terlevich, R. J., & Wegner, G. 1988, ApJ, 326, 19
Ma, C., & Bertschinger, E. 1995, ApJ, 455, 7
Masters, K. L., Springob, C. M., Haynes, M. P., & Giovanelli, R. 2006, ApJ, 653, 861
Nusser, A., & Davis, M. 1994, ApJ, 421, L1
Nusser, A., Dekel, A., Bertschinger, E., & Blumenthal, G. R. 1991, ApJ, 379, 6
Peebles, P. J. E. 1980, The Large-Scale Structure of the Universe (Princeton, NJ: Princeton Univ. Press)
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in FORTRAN. The Art of Scientific Computing (2nd ed.; Cambridge: Cambridge Univ. Press)
Sandage, A., Reindl, B., & Tammann, G. A. 2010, ApJ, 714, 1441
Schechter, P. L. 1980, AJ, 85, 801
Springob, C. M., Masters, K. L., Haynes, M. P., Giovaneli, R., & Marinoni, C. 2007, ApJS, 172, 599
Starobinsky, A. A. 2007, Sov. J. Exp. Theor. Phys. Lett., 86, 157
Strauss, M. A., & Willick, J. A. 1995, Phys. Rep., 261, 271
Wei, H. 2008, Phys. Lett. B, 664, 1
Wu, P., Yu, H., & Fu, X. 2009, J. Cosmol. Astropart. Phys., JCAP06(2009)019
Zaroubi, S., Bernardi, M., da Costa, L. N., Hoffman, Y., Alonso, M. V., Wegner, G., Willmer, C. N. A., & Pellegrini, P. S. 2001, MNRAS, 326, 375
Zaroubi, S., Hoffman, Y., Fisher, K. B., & Lahav, O. 1995, ApJ, 440, 446
Zaroubi, S., Zehavi, I., Dekel, A., Hoffman, Y., & Kolatt, T. 1997, ApJ, 486, 21