Plasma effects in harmonic emission in relativistic laser interactions with overdense plasma targets examined from single particle radiation models

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Abstract. Intense femtosecond laser pulses incident on supercritical plasmas can generate reflected high-order harmonics of the fundamental frequency that extend over the xuv spectral region, characterized by a power decay $P_m \sim m^{-p}$, where $m$ is the harmonic number. It has been shown in earlier reports that plasma effects can modify the emission spectrum producing decay indexes $p$ significantly weaker than the universal value $8/3$ predicted by the similarity theory, these being within the range $4/3 \leq p \leq 8/3$. In this paper we employ two single particle radiation models to demonstrate that plasma density effects substantially modify the emission radiated by a relativistic electron accelerated by the electric field of the laser pulse, and show that the harmonic power decay is commonly characterized by the indexes $4/3$ and $5/3$, with remarkable similitudes to those particle-in-cell results recently reported in ultrarelativistic laser-plasma interactions.

1. Introduction

Multiple harmonic generation at relativistic energies from laser irradiated solid targets has become a subject of enduring interest for the diversity of potential technological applications in nonlinear optics, and more recently as sources that are promissory for generating intense attosecond light pulses at short wavelengths [1]. High harmonic generation could provide extremely short pulses at the attosecond scale in the xuv regime, much shorter that the conventional x-ray sources, and appropriate for exploring the ultrafast nature of the dynamical processes in matter.

The spectral power decay of laser-generated harmonics in relativistic interactions with overdense plasmas has been characterised by various authors [2,3] according to laser-plasma parameters. In the mildly relativistic regime Gibbon [3] proposed an empirical relation from PIC simulations for the harmonic efficiency $\eta$, and showed this to be scaled as $\eta \sim 17.2 a_0^4 m^{-p}$, where the decay index $p = 5$, $a_0$ is the normalized quiver momentum $a_0 \sim 0.85 (I_{18} \lambda_L^2)^{1/2}$, with $I_{18}$ denoting the laser intensity in units of $10^{18}$ W/cm$^2$, and $\lambda_L$ the wavelength of the incident light measured in microns. Relativistic effects arise for $a_0 \gtrsim 1$. The spectral power density was estimated in experiments of laser pulses of a few ps duration to span over decreasing values of $p$ for increasing laser intensities, from instance $p = 5.5$ at $a_0 \sim 0.64$ to $p = 3.38$ at $a_0 \sim 2.85$ [4].
More recently, Baeva et al. [5] proposed a model on the grounds of the similarity model, that asserts that the interaction physics remains similar solely for constant values of the parameter $S = (n_e/n_c)/a_0$, with $n_e/n_c$ being the ratio of the ambient electron plasma density to the critical density. This restriction lead to associate a “universal” value $8/3$ to the harmonic power decay. Furthermore, another feature predicted by the $S$-similarity analysis states a cut-off in the harmonic spectra at $m = \sqrt{8\alpha\gamma_s^3}$, where $\gamma_s$ is the relativistic factor of the plasma surface and $\alpha$ is of order one. No experimental support was found in experiments of the spectrum cut-off, though the spectral power decay was showed to be fitted by indexes in the range $2 < p < 2.7$, with the upper limit in consonance with the predicted “universal” value.

![Figure 1](image-url)

**Figure 1.** (a) PIC simulation results for the harmonic spectra from $s$-polarized laser pulses incident on a plane target plasma and (b) for $p$-polarized light; $a_0 = 20, n_e/n_c = 40$, pulse length $\tau_p = 17$ fs, $\lambda_L = 1054$ nm, angle of incidence $\theta = 23^\circ$, $m = \omega_m/\omega_L$; (c),(d) show the incident $E_i$ and reflected fields $E_{ry}$ for these two polarizations, (e) shows the peak Langmuir condensate density, and (f) the contours of the corresponding Langmuir electric field, $E_x$.

2. Plasma effects

In mildly relativistic interactions, Boyd and Ondarza-Rovira (BOR) found evidence of emission at the plasma line and at its harmonics [6]. In the UR regime, the ponderomotive force of the Langmuir field is strong enough to govern plasma behavior over time scales $t > \omega_p^{-1}$ and highly localized electrostatic fields are generated in correspondence with density structures inside the plasma. These structures were identified as sources of emission at the plasma frequency and at its harmonics.
Particle-in-cell simulations where used to explore the harmonic power decay in ultrarelativistic laser-plasma interactions by BOR [7-9] and spectral decays characterized by prevalent indexes $p = 5/3$ were found and, less commonly, $p = 4/3$ in $p$-polarized light incident on dense plasma targets. When the incident laser pulse is normal to the plasma surface, the spectrum is invariantly characterized by a power law described by the index $8/3$, regardless of the combination of parameters $(a_0, n_e/n_c)$, as shown in figure 1(a) [9]. Contrasting this, plasma effects are evident in the power harmonic decay for oblique incidence, since the excitation of plasma oscillations enable the enhancement of emission with lower harmonic power decay indexes, resulting in much flatter spectra, as shown in figure 1(b). The cases here shown correspond to the combination $a_0 = 20$ and $n_e/n_c = 40$, for a pulse length $\tau_p = 17$ fs, angle of incidence of $\theta = 23^\circ$, and $m = \omega_m/\omega_L$. For $p$-polarization field amplification is also observed (figure 1(d)), being an order of magnitude greater than the $s$–polarized case, figure 1(c), with an estimated width of about 10 as. These reflected attosecond spikes correlate with the electron bunching observed within the plasma (figure 1(e)), and were identified as the sources of the emission.

Beside this, the generation of localized high electrostatic fields are found to be generated at sites of electron density bunching, as depicted in figure 1(f). Additionally, as figure 2 shows, the electron trajectories were followed around the regions in the plasma where these intense electrostatic fields were observed, and for which the emission of a reflected attosecond spike was correlated.

3. Single particle radiation models
In what follows we show that from simple dynamical models, the harmonic radiation emission from a charged particle accelerated by a strong electromagnetic pulse and perturbed by an external electrostatic field or by the plasma restoring force, is characterized by power decay indexes $5/3$ or $4/3$, in similitude with PIC results, as discussed in the previous section.

3.1. Plasma and harmonic emission from relativistic electrons perturbed by a soliton field
Radiation from electron bunching in large-amplitude Langmuir waves excited in electron beam-plasma interactions has been studied in the past. Weatherall and Hobbs [10] showed that electrostatic bunching was an effective means of generating plasma harmonics. Subsequently, Weatherall and Benford (WB) [11] reported a spectrum showing strong emission at the plasma line and its harmonics with a power spectrum below a cut-off at $\omega \sim \omega_c$. 

![Figure 2. PIC simulation results for the electron trajectories around the region in the plasma where the emission source is correlated from the $E_x$ contours and the reflected field.](image-url)
\[ P(\omega) \sim \omega^{-5/3}; \quad \omega \lesssim \omega_c = 2 \gamma_b^2 c/D, \quad (1) \]

where \( \gamma_b \) is the relativistic factor for the electron beam and \( D \) denotes the scale length of the localised electrostatic structures described by the Zakharov model of turbulence, [12]. Carrying this over to our PIC spectra, the cut-off frequency identified in (1) corresponds to \( m_c = (\gamma_b^2/\pi)(\lambda_L/D) \). For \( \gamma_b \sim 6, (\lambda_L/D) \sim 20, m_c \sim 200 \). We note in passing that all three signatures of this spectrum, namely plasma emission at \( \omega_p \) and its harmonics, spectral decay with \( p = 5/3 \) and a cut-off in a range typically \( 100 \lesssim m_c \lesssim 300 \) are consistent with the BOR spectra reported in references [6] and [9]. Furthermore, the localized electrostatic structures in their strongly turbulent state, the so-called soliton, is consistent with the trajectory crossings, as shown in figure 2.

The electron dynamics was studied from the numerical integration of the equations of energy and motion

\[
\frac{d(m v)}{dt} = e \left( E + \frac{v}{c} \times B \right) - \frac{2}{3} \pi e D \rho_0 e^{-i \omega_p t} \hat{p}, \quad (2)
\]

where \( e \) and \( m \) are the electric charge and mass of the electron, respectively. The density oscillation of amplitude \( \rho_0 \) is localized within a scale length \( D \), oscillates at the plasma frequency \( \omega_p \) and has a dipole moment in the direction of the unit vector \( \hat{p} \). In figure 3(a) the beam radiation power obtained from the WB model is shown, characterized by a strong feature in the spectrum at the plasma frequency. The harmonic power decay is governed by the 5/3 index.

In figure 3(b) the single particle radiation spectrum obtained from (2) is shown for the case \((a_0, n_e/n_c) = (30, 100)\) exhibiting a power spectrum decay \( p \sim 5/3 \). For other two combinations of parameters, figures 4(a) and 4(b) show that the power decay is described by \( p \sim 5/3 \) for \((30, 200)\) and by \( p \sim 4/3 \) for \((40, 100)\). We performed single particle simulations for a large number of combinations \((a_0, n_e/n_c)\) and found that indexes 5/3 and 4/3 remained predominant in the observed spectra.

**Figure 3.** (a) Beam radiation power spectrum from WB model [11], and (b) single particle radiation from a laser-accelerated electron perturbed by a soliton field (2), for \( a_0 = 30, n_e/n_c = 100 \), both having power decays \( p \sim 5/3 \).
Figure 4. Harmonic spectra from (2) for (a) $a_0 = 30$, $n_e/n_c = 200$, and (b) $a_0 = 40$, $n_e/n_c = 100$, having respectively power decays $p \sim 5/3$, and $4/3$.

3.2. Plasma and harmonic emission from relativistic electrons perturbed by plasma potential

The radiation from electrons accelerated by an intense laser pulse and perturbed by the restoring force of the plasma was studied by solving numerically the equations of energy and motion

$$\frac{d(mv^\gamma)}{dt} = e \left( E + \frac{v}{c} \times B \right) - m\omega_p^2 r,$$

where the second term on the right represents the restoring force that oscillates at $\omega_p$. Different aspects of the particle dynamics were studied for a number of combinations of the laser amplitude and plasma density. In particular, the electron energy gain during and after the action of the laser pulse was explored. It was found that a similarity effect for the electron energy exists as a function of the parameter $S$. In figure 5(a) the similarity effect for the electron energy is shown for fixed amplitudes $a_0$ and varying densities, while figure 5(b) shows this effect for fixed plasma densities and varying laser intensities.

Figure 5. Similarity effects: maximum electron energy during the pulse vs $S = (n_e/n_c)/a_0$, for (a) $a_0=10$, 20, 40 & 60 and (b) $n_e/n_c = 30, 60, ..., 240$.

In figures 6(a-d) the radiation spectra for various combinations of the parameter $S$ are shown, and where the power decay indexes $5/3$ are observed for these cases. Figure 6(c) shows for comparison a PIC harmonic spectrum from an ultrarelativistic laser-plasma interaction, characterized by a $\sim 4/3$ power decay index, reported by other authors [13].
Figure 6. Harmonic spectra for (a) \((a_0, n_e/n_c; S) = (4.70; 4.25)\), (b) for \((20, 100; 5)\), (c) PIC harmonic spectrum from [13] showing a 5/3-4/3 decay for \((60, 95; 1.6)\), and (d) for an \(s\)-polarization -like case condition, \((15, 450; 30)\), showing an 8/3 decay.

4. Conclusions
In this paper we demonstrate the influence of plasma effects on harmonic spectra for relativistic laser-plasma interactions, from both PIC simulations and single particle models. Two possible particle radiation mechanisms were explored consistent with the highly localized electrostatic fields generated inside the plasma during the interaction and with the electron trajectory crossings that correlate with both the electron density bunching and the reflected attosecond electric field spikes observed from PIC simulations. Radiation from perturbed plasma electrons either by a soliton field or by a plasma potential is reminiscent of the 5/3 UR power law decay observed in PIC emission spectra.

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