Rate-Region Characterization and Channel Estimation for Cell-Free Symbiotic Radio Communications

Zhuoyin Dai®, Ruoguang Li®, Member, IEEE, Jingran Xu, Yong Zeng®, Senior Member, IEEE, and Shi Jin®, Senior Member, IEEE

Abstract—Cell-free massive MIMO and symbiotic radio communication have been recently proposed as the promising beyond fifth-generation (B5G) networking architecture and transmission technology, respectively. To reap the benefits of both, this paper studies cell-free symbiotic radio communication systems, where a number of cell-free access points (APs) cooperatively send primary information to a receiver, and simultaneously support the passive backscattering communication of the secondary backscatter device (BD). We first derive the achievable communication rates of the active primary user and passive secondary user under the assumption of perfect channel state information (CSI), based on which the transmit beamforming of the cell-free APs is optimized to characterize the achievable rate-region of cell-free symbiotic communication systems. Furthermore, to practically acquire the CSI of the active and passive channels, we propose an efficient channel estimation method based on two-phase uplink-training, and the achievable rate-region taking into account CSI estimation errors is further characterized. Simulation results are provided to show the effectiveness of our proposed beamforming and channel estimation methods.

Index Terms—Cell-free massive MIMO, symbiotic radio, backscattering, channel estimation, active and passive communication.

I. INTRODUCTION

W ith the ongoing commercial deployment of the fifth-generation (5G) mobile communication networks, the academia and industry communities have started the investigation of the key technologies for beyond fifth-generation (B5G) or the sixth-generation (6G) networks [1], [2], [3]. In order to meet the orders-of-magnitude performance improvement in terms of coverage, connectivity density, data rate, reliability, latency, etc., many promising technologies are being investigated, such as extremely large-scale MIMO/surface [4], [5], millimeter wave or TeraHertz communication [6], [7], non-terrestrial networks (NTN) [8], [9], reconfigurable intelligent surface (RIS) [10], [11], and artificial intelligence (AI)-aided wireless communications [13]. On the other hand, cell-free massive MIMO [14] and symbiotic radio communication [15] were recently proposed as the promising B5G networking architecture and transmission technology, respectively, which have received fast-growing attention.

As a radically new potential networking architecture for B5G mobile communication networks, cell-free massive MIMO is significantly different from the classical cellular architecture since it blurs the conventional concepts of cells or cell boundaries [14]. Instead, geographically distributed access points (APs) [16], [17], which are connected to the central processing unit (CPU), cooperatively serve their surrounding users to achieve high macro diversity. Cell-free massive MIMO is expected to mitigate the inter-cell interference issues suffered by small cell systems and provide users with a consistently high quality of service everywhere [18]. Significant research efforts have been devoted recently to the theoretical study and practical design of cell-free massive MIMO systems. For example, the performance of two basic linear precoding schemes, i.e., conjugate beamforming and zero-forcing precoding, was compared for cell-free massive MIMO in [19]. The receiver filter coefficients and power allocation of cell-free massive MIMO were optimized to maximize the minimal user rate or bandwidth efficiency in [20] and [21]. Furthermore, in [22] and [23], the communication resource allocation was optimized to maximize the energy efficiency and spectral efficiency of cell-free massive MIMO systems.

On the other hand, symbiotic radio has been recently proposed as a promising B5G transmission technology [15], which is able to exploit the benefits of both conventional cognitive radio (CR) and the emerging passive ambient backscattering communications (AmBC) to realize spectral- and energy-efficient communications [24]. Specifically, the passive secondary backscatter device (BD) in symbiotic radio...
systems reuses not only the spectrum of the active primary communication as in traditional CR systems, but also its power via passive backscattering technology [25]. According to the relationship between the symbol durations of the primary and secondary signals, symbiotic radio systems can be classified as commensal symbiotic radio (CSR) and parasite symbiotic radio (PSR) [26], [27]. In CSR, the secondary signals have much longer symbol durations than the primary signals, making the secondary backscattering communication contribute additional multipath components to enhance the primary communication. As a result, the primary and secondary communications form a mutualism relationship [24]. On the other hand, for PSR, the primary and secondary signals have equal symbol durations, so that the secondary signals may interfere with the primary signal. However, compared to the CSR case, the secondary communication rate in PSR can be significantly improved. Significant research efforts have been devoted to the study of symbiotic radio systems. For example, in order to maximize the secondary communication rate, an exact penalty beamforming method based on the local optimal solution was proposed in [28]. Besides, the authors in [29] and [30] investigated how to optimize transmit power and reflection coefficients to improve the energy efficiency and achievable rates of symbiotic radio systems.

It is worth remarking that all the aforementioned existing works studied cell-free massive MIMO or symbiotic radio communication systems separately, i.e., cell-free systems with conventional active communication or symbiotic radio transmission in conventional cellular networks or the simplest point-to-point communications. As the promising 5G networking architecture and transmission technology, respectively, it is natural that cell-free networking and symbiotic radio communication would merge into each other to reap the benefits of both. This motivates our current work to investigate cell-free symbiotic radio communication systems, which, to the best of our knowledge, have not been studied in the existing literature. By combining cell-free architecture with symbiotic radio transmission technology, the passive secondary communication in symbiotic radio systems is enhanced by the cooperation gain of distributed APs, thus realizing passive communication with high macro-diversity. In this paper, we study a basic cell-free symbiotic radio system, in which a number of distributed multi-antenna APs cooperatively send primary information to a receiver, and concurrently support the passive backscattering communication of the secondary BD. As such, the distributed cooperation gain by APs can be exploited to enhance both the primary and secondary communication rates. Our specific contributions are summarized as follows:

1. First, we present the mathematical model of the cell-free symbiotic radio communication system, which is a promising system that exploits both the advantages of cell-free networking architecture and symbiotic radio transmission technology. Considering the dependence of the secondary passive communication on the primary active communication, we study the problem of maximizing the secondary communication rate while guaranteeing the performance of the primary communication, which fits well with the characteristics of the cell-free symbiotic radio systems. Under the assumption of perfect channel state information (CSI) of the direct active channels and cascaded passive channels, the achievable rates of both the primary and secondary communications are derived.

2. Next, we relax the assumption of perfect CSI and investigate the practical CSI acquisition method for the considered cell-free symbiotic radio system. Similar to the extensively studied massive MIMO systems, efficient channel estimation for cell-free massive MIMO can be achieved by exploiting the uplink-downlink channel reciprocity [31], [32], [33], i.e., the downlink channels can be efficiently estimated via uplink training. However, different from the existing cell-free massive MIMO systems [14], the channel estimation for cell-free symbiotic radio system requires estimating not only the active direct-link channels, but also the passive cascaded channels. Furthermore, the channel estimation for cell-free symbiotic radio also possesses different properties from that for reconfigurable intelligent surface (RIS)-assisted communications. On the one hand, considering the mutual interference between the primary and secondary transmission in cell-free symbiotic radio systems, the direct link and cascaded backscattering link cannot be treated as a composite equivalent channel for estimation. On the other hand, in cell-free symbiotic radio systems, BD does not need to perform passive beamforming optimization as in RIS-assisted communication. Therefore, we only need to estimate the cascaded AP-BD-receiver channel, rather than estimating its two-hop channels separately, as in RIS-assisted communications [34], [35]. To this end, we propose a two-phase based channel estimation method for cell-free symbiotic radio communication systems. In the first phase, pilot symbols are sent by the receiver while muting the BD, so as to estimate the direct-link channels. In the second phase, pilots are sent through both the direct link and the cascaded backscattering channels, so that, together with the estimation of the direct-link channels, the two hops of the cascaded channel are estimated as a whole. Furthermore, the channel estimation errors in both phases are derived, which are shown to be dependent on the total pilot length and the pilot allocation between the two training phases. The achievable rates under imperfect CSI are derived by taking into account the CSI estimation errors.

3. Furthermore, for both the ideal scenario with perfect CSI and the practical scenario of imperfect CSI with channel estimation errors, we formulate the beamforming optimization problem to characterize the achievable rate-region of the active primary communication and the passive secondary communication. The formulated problems are non-convex in general, making them difficult to solve directly. We show that a closed-form solution can be obtained for the special case when the primary rate target is relatively small. Furthermore, for the general cases, we show that the rate threshold constraint can
be converted into the convex second-order cone (SOC) constraint, and that the nonconcave objective function can be globally lower-bounded by its first-order Taylor expansion. Therefore, efficient algorithms are proposed based on successive convex approximation (SCA) technique [36], [37], [38]. Numerical results are provided to demonstrate the effectiveness of the proposed channel estimation and optimization approaches in cell-free symbiotic radio communication systems.

The rest of this paper is organized as follows. Section II presents the mathematical model of cell-free symbiotic radio communication systems. Under the assumption of perfect CSI, Section III characterizes the achievable rate-region of passive secondary communication and active primary communication by optimizing the transmit beamforming of the APs. In Section IV, a two-phase uplink-training based channel estimation method is proposed, and the achievable primary and secondary communication rates, taking into account the channel estimation errors, are derived. Furthermore, the beamforming optimization problem with imperfect CSI is also investigated in Section IV. Section V presents numerical results to validate our proposed designs. Finally, we conclude the paper in Section VI.

**Notations:** In this paper, scalars are denoted by italic letters. Vectors and matrices are denoted by boldface lower-and uppercase letters, respectively. \( \mathbb{C}^{N \times 1} \) denotes the space of \( N \)-dimensional complex-valued vectors. \( \text{Re} \{ \cdot \} \) and \( \text{Im} \{ \cdot \} \) denote the real and imaginary parts, respectively. \( \mathbb{E}_X [ \cdot ] \) denotes the expectation with respect to the random variable \( X \). \( \text{Ei}(\cdot) \) denotes the exponential integral from \( -\infty \) to \( x \). \( I_N \) denotes an \( N \times N \) identity matrix. For a vector \( \mathbf{a} \), its transpose, Hermitian transpose, and Euclidean norm are respectively denoted as \( \mathbf{a}^\text{T} \), \( \mathbf{a}^\text{H} \), and \( \| \mathbf{a} \| \). Meanwhile, \( \mathbf{a}[m : (m + n)] \) represents the subvector of \( \mathbf{a} \) made up of its \( m \)th to \( (m + n) \)-th elements. \( \log_2(\cdot) \) denotes the logarithm with base 2. Furthermore, \( \mathcal{C}\mathcal{N}(\mu, \sigma^2) \) denotes the circularly symmetric complex Gaussian (CSCG) distribution with mean \( \mu \) and variance \( \sigma^2 \).

II. SYSTEM MODEL

As shown in Fig. 1, we consider a cell-free symbiotic radio communication system, which consists of \( M \) distributed APs, one information receiver, and one BD. The \( M \) APs cooperatively send primary information to the receiver, and simultaneously support the BD for secondary communication via passive backscattering to the same information receiver. The considered system may model a wide range of applications, e.g., with the receiver corresponding to smartphones and the BD being the smart home sensor node. We assume that each AP is equipped with \( N \) antennas, whereas the receiver has one antenna. The BD is equipped with \( L \) antenna elements. Denote by \( \mathbf{g}_m \in \mathbb{C}^{N \times 1} \) and \( \mathbf{F}_m \in \mathbb{C}^{L \times N} \) the multiple-input single-output (MISO) channels and multiple-input multiple-output (MIMO) channels from the \( m \)-th AP to the receiver and BD, respectively, where \( m = 1, \ldots, M \). Further denote by \( \mathbf{q} \in \mathbb{C}^{L \times 1} \) the channel coefficient vector from the BD to the receiver. Thus, the cascaded backscattering channel from the \( m \)-th AP to the receiver via the BD is \( \mathbf{F}_m^H \mathbf{q} \).

In this paper, we focus on the PSR setup [24], where the symbol durations of the primary and secondary signals are equal. Let \( s(n) \sim \mathcal{C}\mathcal{N}(0,1) \) and \( c(n) \sim \mathcal{C}\mathcal{N}(0,1) \) denote the CSCG information-bearing symbols of the primary and secondary signals at the \( n \)-th symbol duration, respectively. Further denote by \( \mathbf{w}_m \in \mathbb{C}^{N \times 1} \) the transmit beamforming vector of the \( m \)-th AP, where its power is \( \| \mathbf{w}_m \|^2 \leq P_m \), with \( P_m \) denoting the maximum allowable transmit power of the \( m \)-th AP. The received signal by the receiver is

\[
\mathbf{r}(n) = \sum_{m=1}^{M} \mathbf{g}_m^H \mathbf{w}_m s(n) + \sqrt{\alpha} \mathbf{q}^H \mathbf{F}_m \mathbf{w}_m s(n) c(n) + z(n),
\]

where \( \alpha \) denotes the power reflection coefficient of the BD, \( z(n) \sim \mathcal{C}\mathcal{N}(0, \sigma^2) \) is the additive white Gaussian noise (AWGN). Based on the received signal \( \mathbf{r}(n) \) in (1), the receiver needs to decode both the primary and secondary signals. Since the backscattering link is typically much weaker than the direct link, the receiver may first decode the primary symbols \( s(n) \), by treating the backscatter interfering signals as noise, whose power is \( \mathbb{E} [ \| \alpha \mathbf{q}^H \mathbf{F}_m \mathbf{w}_m s(n) c(n) \|^2 ] = \alpha \mathbb{E} [ \| \mathbf{F}_m \mathbf{w}_m \|^2 ] \). Therefore, the signal-to-interference-plus-noise ratio (SINR) for decoding the primary information is

\[
\gamma_s = \frac{\alpha \mathbb{E} [ \| \mathbf{g}_m^H \mathbf{w}_m \|^2 ]}{\sigma^2 + \mathbb{E} [ \| \mathbf{F}_m \mathbf{w}_m \|^2 ] + \sigma^2}. \tag{2}
\]

Note that due to the product of \( c(n) \) and \( s(n) \) in the second term of (1), the resulting noise for decoding \( s(n) \) no longer follows Gaussian distribution. However, by using the fact that for any given noise power, Gaussian noise results in the maximum entropy and hence constitutes the worst-case noise [39], [40], the achievable rate of the primary communication in (1) is

\[
R_s = \log_2(1 + \gamma_s). \tag{3}
\]
After decoding the primary information, the first term in (1) can be subtracted from the received signal before decoding the secondary symbols \( c(n) \). The resulting signal is
\[
\hat{r}_c = \sqrt{\alpha} \sum_{m=1}^{M} q^H F_m w_m s(n)c(n) + z. \tag{4}
\]

Note that since \( s(n) \) varies across different secondary symbols \( c(n) \), (4) can be interpreted as a fast-fading channel, whose instantaneous channel gain depends on \( |s(n)|^2 \) [41]. With \( s(n) \sim \mathcal{CN}(0, 1) \), its squared envelope follows an exponential distribution. Therefore, the ergodic rate of the passive backscattering communication (4) can be expressed as [24] and [42]
\[
R_c = \mathbb{E}_{s(n)} \left[ \log_2 \left( 1 + \frac{\alpha |\sum_{m=1}^{M} q^H F_m w_m |^2 |s(n)|^2}{\sigma^2} \right) \right]
= \int_0^\infty \log_2 \left( 1 + \beta_c x \right) e^{-x} dx
= -e^{-\beta_c} \text{Ei}\left( -\frac{1}{\beta_c} \right) \log_2 e, \tag{5}
\]
where \( \text{Ei}(\cdot) \) is the exponential integral, and \( \beta_c = \frac{\alpha |\sum_{m=1}^{M} q^H F_m w_m |^2}{\sigma^2} \) is the average received signal-to-noise ratio (SNR) of the backscattering link.

### III. RATE-REGION CHARACTERIZATION
**WITH PERFECT CSI**

In this section, under the assumption that perfect CSI is available at the APs, we aim to characterize the achievable rate-region of the active primary communication and passive secondary communication, by optimizing the transmit beamforming of the \( M \) APs. To this end, the beamforming optimization problem is formulated to maximize the ergodic rate of the backscattering communication in (5), subject to a given communication rate target constraint for the primary communication. By varying the communication rate target, the complete Pareto boundary of the achievable communication rate-region can be obtained. The problem can be formulated as
\[
\begin{align*}
\max_{w, m=1, \ldots, M} & \quad R_c \tag{6a} \\
\text{s.t.} & \quad R_s \geq R_{th}, \tag{6b} \\
& \quad \|w_m\|^2 \leq P_m, \quad m = 1, \ldots, M \tag{6c}
\end{align*}
\]
where \( R_{th} \) denotes the given target threshold for the primary communication rate, and (6c) corresponds to the per-AP power constraint.

It has been shown in [24] that the first-order derivative of the ergodic rate \( R_c \) in (5) with respect to the average received SNR \( \beta_c \) is non-negative. Therefore, \( R_c \) is a monotonically non-decreasing function with respect to \( \beta_c \). Thus, we may replace the objective function of (6) by \( \beta_c \). By further ignoring those constant terms, problem (6) can be equivalently written as
\[
\begin{align*}
\max_{w} & \quad |g^H w|^2 \tag{7a} \\
\text{s.t.} & \quad \log_2 \left( 1 + \frac{|g^H w|^2}{\alpha |h^H w|^2 + \sigma^2} \right) \geq R_{th}, \tag{7b} \\
& \quad \|w[(m-1)N + 1 : mN]\|^2 \leq P_m, \quad m = 1, \ldots, M. \tag{7c}
\end{align*}
\]
where we have defined the cascaded vectors as \( g^T = [g_1^T, g_2^T, \ldots, g_M^T] \), \( h = [F_1^H q; F_2^H q; \ldots; F_M^H q] \) and \( w^T = [w_1^T, w_2^T, \ldots, w_M^T] \).

Before solving problem (7), we first study its feasibility property. Obviously, problem (7) will become infeasible if \( R_{th} \) is too large. It is not difficult to see that problem (7) is feasible if and only if \( R_{th} \leq R_s \), where \( R_s \) is the optimal value for the following optimization problem
\[
\begin{align*}
\max_{w} & \quad \log_2 \left( 1 + \frac{|g^H w|^2}{\alpha |h^H w|^2 + \sigma^2} \right) \tag{8a} \\
\text{s.t.} & \quad \|w[(m-1)N + 1 : mN]\|^2 \leq P_m, \quad m = 1, \ldots, M. \tag{8b}
\end{align*}
\]

Next, we consider solving problem (8) to get the maximum achievable primary communication rate threshold \( R_s \) for problem (7) to be feasible. Note that the objective function of problem (8) is nonconcave with respect to \( w \). Thus, problem (8) cannot be efficiently solved directly with the standard convex optimization technique. Fortunately, the optimal solution can be obtained efficiently by performing convex optimization via the bisection method. To this end, it is not difficult to see that if \( w^* \) is an optimal solution to problem (8), so is \( w^* e^{i\phi} \) for any phase rotation \( \phi \). This is because any arbitrary phase rotation \( \phi \) does not change the objective function in (8a) nor the constraint in (8b) [43], [44]. Therefore, without loss of optimality to problem (8), we may assume that \( g^H w \) is a nonnegative real number, i.e., \( \text{Re}\{g^H w\} \geq 0 \), and \( \text{Im}\{g^H w\} = 0 \). As a result, by further introducing a slack variable \( \mu \), problem (8) can be equivalently written as
\[
\begin{align*}
\max_{w} & \quad \mu \tag{9a} \\
\text{s.t.} & \quad \log_2 \left( 1 + \frac{\text{Re}\{g^H w\}^2}{\alpha |h^H w|^2 + \sigma^2} \right) \geq \mu, \tag{9b} \\
& \quad \|w[(m-1)N + 1 : mN]\|^2 \leq P_m, \quad m = 1, \ldots, M, \tag{9c} \\
& \quad \text{Im}\{g^H w\} = 0. \tag{9d}
\end{align*}
\]

Furthermore, for any given \( \mu \), we may formulate the following feasibility problem
\[
\begin{align*}
\text{Find } & \quad w \tag{10a} \\
\text{s.t.} & \quad \|\sigma, \sqrt{\alpha} h^H w\|_2 \leq \frac{\text{Re}\{g^H w\}}{\sqrt{2\mu - 1}}, \tag{10b} \\
& \quad \|w[(m-1)N + 1 : mN]\|^2 \leq P_m, \quad m = 1, \ldots, M, \tag{10c} \\
& \quad \text{Im}\{g^H w\} = 0. \tag{10d}
\end{align*}
\]

Note that (10b) is equivalent to (9b), which is expressed as an SOC constraint for any given \( \mu \). Thus, problem (10) is an SOC programming (SOCP) problem, which can be efficiently solved by standard convex optimization technique or existing software tools such as CVX [45]. If problem (10) is feasible, then the optimal value \( R_s \) of (8) satisfies \( R_s \geq \mu \); otherwise, \( R_s < \mu \). As a result, the optimal solution to problem (8) can be obtained by solving the SOCP feasibility problem (10),
Theorem 1: When $R_{th} \leq \hat{R}_s$, where
\begin{equation}
\hat{R}_s \triangleq \log_2 \left( 1 + \frac{\left| \sum_{m=1}^{M} \sqrt{P_m} g_m^H h^H_{m,0} q_m^2 \right|^2}{\alpha \left( \sum_{m=1}^{M} \sqrt{P_m} \| F_{m,0}^H q_m \| \right)^2 + \sigma^2} \right),
\end{equation}
the optimal solution and optimal objective value to problem (7) can be obtained in closed-form as
\begin{equation}
w_m^* = \sqrt{P_m} \frac{F_{m,0}^H q_m}{\| F_{m,0}^H q_m \|}, \quad m = 1, \ldots, M,
\end{equation}
\begin{equation}|h^H w_m^*|^2 = \left( \sum_{m=1}^{M} \sqrt{P_m} \| F_{m,0}^H q_m \| \right)^2.
\end{equation}

Proof: Please refer to Appendix.

Under the condition of Theorem 1 and with the closed-form optimal solution to problem (7) given in (12), the resulting primary communication rate $\tilde{R}_c$ is given in (11). Furthermore, by substituting (13) into (5), the secondary communication rate $\tilde{R}_s$ is obtained in closed-form as
\begin{equation}
\tilde{R}_c = -e^{-\frac{h}{\sqrt{\tilde{\beta}_c}}} Ei\left(-\frac{1}{\tilde{\beta}_c}\right) \log_2 e,
\end{equation}
where $\tilde{\beta}_c = \frac{\left| \sum_{m=1}^{M} \sqrt{P_m} \| F_{m,0}^H q_m \|^2 \right|^2}{\sigma^2}$ is the average received SNR of the backscattering link.

With the above discussions, the remaining task for solving problem (7) is to consider the case $\tilde{R}_s < R_{th} \leq \hat{R}_s$. In this case, due to the non-concave case of (7a) and the nonconvex constraint (7b), problem (7) is non-convex. Thus, it is difficult to find the optimal solution efficiently. Fortunately, an efficient Karush–Kuhn–Tucker (KKT) local optimal solution can be obtained by using the SCA technique. Towards this end, it is first observed that, similar to (10b), without loss of optimality, the rate constraint in (7b) can be written as an SOC constraint. Thus, problem (7) can be equivalently written as
\begin{equation}
\max_w |h^H w|^2
\end{equation}
s.t. $\|\sigma \sqrt{\alpha} h^H w\|_2 \leq \frac{\Re\{g^H w\}}{\sqrt{2R_{th} - 1}},$
\begin{equation}
|w|(m-1)N + 1 : mN| \|^2 \leq P_m, \quad m = 1, \ldots, M,
\end{equation}
\begin{equation}
\text{Im}\{g^H w\} = 0.
\end{equation}

Problem (15) is still non-convex as the objective function (15a) is a convex function with respect to $w$, the maximization of which is a non-convex optimization problem. To address this issue, the SCA technique is applied iteratively to find a KKT local optimal solution [36], [37], [38]. Specifically, consider the current iteration $l$, in which the local point $\{w^{(l)}\}$ is obtained in the previous iteration. Define $F(w) = |h^H w|^2$, and $F(w)$ is a convex differentiable function with respect to $w$. By using the fact that the first-order Taylor expansion of a convex differentiable function provides a global lower bound [46], [47], we have
\begin{equation}
F(w) \geq F(w^{(l)}) + 2 \Re \left\{ w^{(l)} H h^H (w - w^{(l)}) \right\} \triangleq F_{low}(w^{(l)}), \forall w.
\end{equation}

Therefore, by replacing the objective function in (15a) with its global lower bound in (16), we have the following optimization problem
\begin{equation}
\max_w |h^H w^{(l)}|^2 + 2 \Re \left\{ w^{(l)} H h^H (w - w^{(l)}) \right\}
\end{equation}
s.t. $\|\sigma \sqrt{\alpha} h^H w\|_2 \leq \frac{\Re\{g^H w\}}{\sqrt{2R_{th} - 1}},$
\begin{equation}
|w|(m-1)N + 1 : mN| \|^2 \leq P_m, \quad m = 1, \ldots, M,
\end{equation}
\begin{equation}
\text{Im}\{g^H w\} = 0.
\end{equation}

For any given local point $w^{(l)}$, the objective function of (17a) is a concave affine function of the optimization variable $w$, and all constraints are convex. Therefore, problem (17) is a convex optimization problem, which can be efficiently solved with standard convex optimization techniques or readily available software toolboxes, such as CVX [45]. Thanks to the global lower bound in (16), the optimal objective value of the convex optimization problem (17) provides at least a lower bound to that of the non-convex optimization problem (15). By successively updating the local point $w^{(l)}$ and solving (17), a monotonically non-decreasing objective value of (15) can be obtained. The algorithm is summarized in Algorithm 2.

Let $F(w^{*}) = |h^H w^{*}|^2$ denote the objective value of problem (15) with the beamforming vector obtained during the $(l)$-th iteration of Algorithm 2. The value $F(w^{*})$ obtained
Algorithm 2 SCA for Problem (15)

Input: The channel coefficients $g$ and $h$, noise power $\sigma^2$, power reflection coefficient $\alpha$, maximum transmit power $P_m, m = 1, \ldots, M$, rate threshold $R_{th}$ and termination threshold $\kappa_2$.

Output: The beamforming solution $w^*$.

1: Initialization: set the iteration number $l = 0$, and and initialize $w^{(0)}$, so that it is feasible to (15).

2: repeat
   3: For the given local point $w^{(l)}$, solve the convex optimization problem (17) and denote the optimal solution as $w^{*}(l)$. The channel coefficients
   4: Update the local point with $w^{(l+1)} = w^{*}(l)$.
   5: Update $l = l + 1$.
   6: until the fractional increase of the objective value of (15) is below the threshold $\kappa_2$.

during each iteration of Algorithm 2 is monotonically non-decreasing, i.e., $F(w^{(l+1)}) \geq F(w^{*(l)})$, $\forall l$. Furthermore, at the local point $w^{*(l)}$, the lower bound $F_{lm}(w|w^{(l+1)})$ has both identical gradient and value as $F(w)$, therefore, the sequence $\{w^{*(l)}\}, l = 1, 2, \ldots$, converges to a KKT solution to the original non-convex problem (15) [36], [37], [38].

IV. CHANNEL ESTIMATION AND RATE-REGION CHARACTERIZATION WITH IMPERFECT CSI

Note that the above analysis is based on the assumption of perfect CSI on $F_m$, $g_m, m = 1, \ldots, M$, and $q$. In practical wireless communication systems, these channels need to be acquired via e.g., pilot-based channel estimation. In the following, we propose an efficient channel estimation method for cell-free symbiotic radio systems based on two-phase uplink training. Considering the parasitic nature of the secondary transmission on the primary transmission on the one hand, the direct link channel and the cascaded backscattering channel are estimated separately in two different phases. On the other hand, different from [34] and [35], in the estimation phase of the cascaded backscattering channel, the two hops of the cascaded channel are estimated as a whole, since unlike in RIS-assisted communication, the BD does not require the CSI to perform passive beamforming. Furthermore, the achievable rates taking into account the channel estimation errors are derived, and the beamforming optimization problem is revisited with imperfect CSI to characterize the achievable rate-region with imperfect CSI.

In the first phase of the proposed channel estimation method, pilot symbols are sent by the receiver while the BD is muted, so as to estimate the direct-link channels $g_m, m = 1, \ldots, M$. In the second phase, pilots are sent both by the receiver and the BD, so that, together with the estimation of the direct-link channels, the cascaded backscattering channels $F_m^H q, m = 1, \ldots, M$, are estimated. The details are elaborated in the following.

A. Direct-Link Channel Estimation

First, we discuss the uplink training-based estimation of the direct-link channels between the receiver and the $M$ APs. Although muted BD may still play the role of scatterer in the environment, we assume that the strength of the cascaded two-hop channel with muted BD is much weaker than that of the direct link channel, and its impact on the channel estimation is negligible. Denote by $\tau_1$ the length of the uplink
training sequence, and let $P_t$ be the training power. Further denote by $\varphi_1 \in \mathbb{C}^{1 \times 1}$ the pilot sequence, where $\|\varphi_1\|^2 = \tau_1$. The received training signals by $N$ antennas of the $m$th AP over the $\tau_1$ symbol durations, which is denoted as $Y'_m \in \mathbb{C}^{N \times \tau_1}$, can be written as

$$Y'_m = \sqrt{P_t} \tilde{g}_m \varphi_1^H + Z'_m, \quad m = 1, \ldots, M,$$

where $Z'_m$ denotes the i.i.d CSGC noise with zero-mean and power $\sigma^2$.

We consider the general case where the channel coefficient vector $g_m$ may have a non-zero mean, i.e., it consists of the deterministic channel component $\bar{g}_m$ and a random channel component $u_m$ [49] and can be written as

$$g_m = \bar{g}_m + u_m,$$

where the entries in $u_m$ are i.i.d. with zero mean and variance $b_m$. Since $\bar{g}_m$ is deterministic, the main task of estimating $g_m$ is to estimate the random component $u_m$.

With the pilot sequence $\varphi_1$ and the deterministic channel component $\bar{g}_m$ known to the APs, we may project $Y'_m$ to $\varphi_1$, and remove the term caused by the deterministic channel component $\bar{g}_m$, which yields

$$\hat{y}'_m = \frac{1}{\sqrt{P_t}} Y'_m \varphi_1 - \tau_1 \tilde{g}_m = \tau_1 u_m + \frac{1}{\sqrt{P_t}} \hat{z}'_m,$$

where $\hat{z}'_m \triangleq Z'_m \varphi_1$ is the resulting noise vector. It can be shown that $\hat{z}'_m$ is i.i.d. CSGC noise with power $\tau_1 \sigma^2$, i.e., $\hat{z}'_m \sim \mathcal{CN}(0, \tau_1 \sigma^2 I_N)$.

With $u_m$ being a zero-mean random vector, its linear minimum mean square error estimation (LMMSE), denote by $u_m \in \mathbb{C}^{N \times 1}$, is [50]

$$\hat{u}_m = \mathbb{E}[u_m y_m^H] (\mathbb{E}[y_m y_m^H])^{-1} y'_m = R_{u_m} \tau_1 R_{u_m} + \sigma^2 I_N)^{-1} y'_m,$$

where $R_{u_m} = \mathbb{E}[u_m u_m^H] = b_m I_N$ denotes the covariance matrix of $u_m$, but without restricting to any specific distribution. In this case, we can simplify (25) and derive the estimation of $\tilde{g}_m$ as

$$\hat{u}_m = \frac{P_t b_m}{P_t \tau_1 b_m + \sigma^2} Y'_m,$$

$$\tilde{g}_m = \tilde{g}_m + \hat{u}_m.$$

It is observed from (26) that the mean and covariance matrix of $\hat{u}_m$ are

$$\mathbb{E}[\hat{u}_m] = 0, \quad \mathbb{E}[\hat{u}_m] = \frac{\epsilon_1 b_m^2}{1 + \epsilon_1 b_m} I_N,$$

where we have defined the transmit training energy-to-noise ratio (ENR) as $\epsilon_1 \triangleq \frac{\tau_1}{\sigma^2}$.

Let $\tilde{g}_m$ denote the direct-link channel estimation error of the $m$th AP, i.e., $\tilde{g}_m = g_m - \tilde{g}_m = \hat{u}_m$. Its mean and covariance matrix are

$$\mathbb{E}[\tilde{g}_m] = \mathbb{E}[\hat{u}_m] = 0,$$

$$\mathbb{E}[\tilde{g}_m] = \mathbb{E}[\hat{u}_m] = \frac{\epsilon_1 b_m^2}{1 + \epsilon_1 b_m} I_N.$$

It is observed from (31) that as the transmit training ENR $\epsilon_1$ increases, the variance of the channel estimation error reduces, as expected.

### B. Backscattering Channel Estimation

With the estimation $\tilde{g}_m$ for the direct-link channels obtained in the first phase, in the second phase, pilot symbols are sent from both the receiver and the BD to estimate the cascaded passive backscattering channels $F_m q_m, m = 1, \ldots, M$. In typical scenarios such as the Internet of Things (IoT), BD is often deployed close to the receiver [51]. Therefore, the channel $q_m$ between the BD and the receiver may have a non-zero mean, denoted as $q$, while we assume that the channel $F_m$ between the $m$th AP and BD has a zero mean for simplicity. Therefore, the cascaded backscattering channel $h_m$ can be written as

$$h_m = F_m q = F_m q + F_m v,$$

where $v$ denotes the random channel component of $q$ whose entries are i.i.d with zero-mean and variance $\epsilon_v$.

Note that for the cascaded channel $h_m$, $\mathbb{E}[h_m] = \mathbb{E}[F_m q + F_m v] = 0$. Furthermore, the corresponding covariance matrix $R_{h_m}$ of $h_m$ is

$$R_{h_m} = \mathbb{E}[(F_m q + F_m v)(F_m q + F_m v)^H] = \epsilon_m I_N,$$

where $\epsilon_m = (L \epsilon_v + ||q||^2) \epsilon_{F_m}$, with $\epsilon_{F_m}$ denoting the variance of the entries in $F_m$.

Let $\tau_2$ denote the length of the training sequence in the second phase and $\varphi_2 \in \mathbb{C}^{\tau_2 \times 1}$ be the pilot sequence sent by the receiver with $||\varphi_2||^2 = \tau_2$. The received training signal by the $m$th AP can be written as

$$Y''_m = \sqrt{P_t} \alpha_m h_m \varphi_2^H + \sqrt{P_t} (\tilde{g}_m + \tilde{g}_m) \varphi_2^H + Z''_m,$$

where $Z''_m$ denotes the i.i.d. CSGC noise with power $\sigma^2$. Note that without loss of generality, we assume that the pilot symbols backscattered by the BD are all equal to 1. After subtracting the terms related to the estimation $\tilde{g}_m$ of the direct-link channels from (34), we have

$$Y''_m = \sqrt{P_t} \alpha_m h_m \varphi_2^H + \sqrt{P_t} (\tilde{g}_m + \tilde{g}_m) \varphi_2^H + Z''_m,$$

With the pilot sequence $\varphi_2$ known to the APs, we may scale $Y''_m$ by $\frac{1}{\sqrt{P_t} \alpha_m}$ and projecting $Y''_m$ to $\varphi_2$, yielding

$$\hat{y}'_m = \frac{1}{\sqrt{P_t} \alpha_m} Y''_m \varphi_2 = \tau_2 h_m + \frac{\tau_2}{\sqrt{\epsilon_1}} \tilde{g}_m + \frac{1}{\sqrt{P_t} \alpha_m} \tilde{z}'_m,$$

where the noise vector is defined as $\tilde{z}'_m \triangleq Z''_m \varphi_2$. It can be shown that $\tilde{z}'_m \sim \mathcal{CN}(0, \tau_2 \sigma^2 I_N)$. 


Based on the (36), the LMMSE estimation \( \hat{h}_m \) of the cascaded channel \( h_m \) is derived as
\[
\hat{h}_m = \mathbb{E}[h_m y'_m^{-1}] = \mathbb{E}[y'_m y'_m^{-1}]^{-1} y'_m = R_{h,m}(\mathbf{\bar{R}}_{x,m} + \frac{\sigma^2}{\alpha P_1} \mathbf{I}_N)^{-1} y'_m.
\] (37)

where \( R_{x,m} = \mathbb{E}[x_m x'_m] \) is the covariance matrix of \( x_m \).

According to the covariance matrix \( R_{x,m} \) and \( R_{h,m} \) that have been derived in (31) and (33), respectively, (37) can be simplified as
\[
\hat{h}_m = \frac{\alpha P_1 \epsilon_m}{\alpha P_1 \tau_2 \epsilon_m + \frac{\epsilon_m}{\epsilon_m} + \frac{\sigma^2}{\alpha^2 \epsilon_m} + \frac{\sigma^2}{\epsilon_m} + \frac{1}{\epsilon_m}} y'_m.
\] (38)

Define the transmit training ENR in the second phase as \( e_2 = \frac{\epsilon_m \alpha P_1}{\alpha^2 \epsilon_m} \). Then with (31) and (38), we have
\[
R_{h,m} = \mathbb{E}[\hat{h}_m \hat{h}_m^H] = \frac{\alpha P_1 \epsilon_m}{\alpha P_1 \tau_2 \epsilon_m + \frac{\epsilon_m}{\epsilon_m} + \frac{\sigma^2}{\alpha^2 \epsilon_m} + \frac{\sigma^2}{\epsilon_m} + 1} \mathbf{I}_N.
\] (39)

Let \( \hat{h}_m = h_m - \hat{h}_m \) denote the cascaded channel estimation error. Then the covariance matrix of \( h_m \) is derived as
\[
R_{h,m} = \mathbb{E}[\hat{h}_m \hat{h}_m^H] = \frac{\epsilon_m}{\alpha P_1 \tau_2 \epsilon_m + \frac{\epsilon_m}{\epsilon_m} + \frac{\sigma^2}{\alpha^2 \epsilon_m} + \frac{\sigma^2}{\epsilon_m} + 1} \mathbf{I}_N.
\] (40)

It follows from (31) that if \( e_1 \to \infty \), in which case the direct-link channel \( g_m \) is perfectly estimated without any error, the variance of the estimation error in (40) reduces to the same form as in (31).

C. Achievable Rate Analysis

In this subsection, we derive the achievable primary and secondary rates based on the channel estimation \( \hat{g}_m \) and \( \hat{h}_m \), \( m = 1, \ldots, M \), by taking into account the channel estimation errors. By substituting \( g_m = \hat{g}_m + g_m \) and \( F_m^H q = h_m + \hat{h}_m \) into (1), the received signal can be written as
\[
r(n) = \sum_{m=1}^{M} \left[ (\hat{g}_m + g_m) w_m s(n) + \sqrt{\alpha} (h_m + \hat{h}_m) w_m s(n)c(n) \right] + z(n).
\] (41)

In addition to the interference from the backscatter symbols \( c(n) \), the term caused by the channel estimation error \( \hat{g}_m \) is also treated as noise for the decoding of the primary signals \( s(n) \) [40], [52]. Therefore, (41) can be decomposed as
\[
r_s(n) = DS' \cdot s(n) + ER + ST + z(n),
\] (42)
where \( DS' \), ER, and ST denote the desired signal, estimation errors and the secondary transmission signal, respectively, which are given by
\[
DS' = \sum_{m=1}^{M} \hat{g}_m^H w_m,
\] (43)
\[
ER = \sum_{m=1}^{M} (\hat{g}_m + \sqrt{\alpha} h_m c(n)) w_m s(n),
\] (44)
\[
ST = \sum_{m=1}^{M} \sqrt{\alpha} h_m w_m s(n)c(n).
\] (45)

Therefore, the resulting SINR can be expressed as (46), shown at the bottom of the page, and the corresponding achievable rate is \( R_s = \log_2 (1 + \gamma_s) \).

Note that for any given channel estimation \( \hat{g}_m \) and \( \hat{h}_m \), since the channel estimation errors \( \hat{g}_m \) and \( \hat{h}_m \) are random, the SINR in (46) and hence its rate \( R_s \) is random. By taking the expected achievable rate with respect to the random estimation errors \( \hat{g}_m \) and \( \hat{h}_m \), we obtain the result (47), shown at the bottom of the page, where the average channel estimation-error-plus-noise power \( E \) is denoted as
\[
E = \sum_{m=1}^{M} P_m \left[ \frac{b_m}{1 + e_1 b_m} + \frac{\epsilon_m (1 + e_1 b_m + 1)}{\alpha^2 \epsilon_m + \sigma^2} \right] \alpha^2,
\] (48)
In (47), the lower bound of the expected primary communication rate $E[R_s]$ is denoted by $\tilde{R}_{s,\text{LB}}$. Note that the inequality in (47) follows from Jensen’s inequality, and the fact that $\log_2(1 + C/x)$ is a convex function for $x > 0$.

Next, we derive the achievable rate of the secondary signals $c(n)$. After decoding $s(n)$, the primary signals $s(n)$ can be subtracted from (41) based on the estimated channel $\hat{g}_m$. The resulting signal is

$$r_c = \sum_{m=1}^{M} \left[ \sqrt{\alpha_m} (\hat{h}_m + \tilde{h}_m) w_m s(n) c(n) + \hat{g}_m^H w_m s(n) \right] + z(n).$$

(49)

By treating the terms caused by the channel estimation error $\tilde{g}_m$ and $\hat{h}_m$ as noise, (49) can be decomposed as

$$r_c = DS'' \cdot c(n) + ER + z(n),$$

(50)

where $ER$ accounts for the estimation errors given in (44), and $DS''$ denotes the desired signal in the decoding of the secondary signals $c(n)$, which is given by

$$DS'' = \sum_{m=1}^{M} \sqrt{\alpha_m} h_m^H w_m s(n).$$

(51)

The resulting SINR is

$$\gamma_c = \frac{[DS'']^2}{E_{s,c}[|ER|^2] + \sigma^2} = \frac{\alpha \sum_{m=1}^{M} h_m^H w_m |s(n)|^2}{\sum_{m=1}^{M} \sum_{l=1}^{M} w_m^H (g_m g_m^H + \alpha h_m h_m^H) w_l + \sigma^2},$$

(52)

and the corresponding achievable rate is $R_c = \log_2(1 + \gamma_c)$.

Note that different from (43), as the desired channel $DS''$ also depends on the primary symbols $s(n)$, the SINR in (52) is a random variable that depends on both $|s(n)|^2$ and the channel estimation errors. The expectation of $R_c$, which is taken with respect to both $|s(n)|^2$ and the channel estimation errors. Therefore, we can obtain $E[R_c]$ given in (53), shown at the bottom of the page. In (53) the inequality is obtained by applying Jensen’s inequality to the convex function $\log_2(1 + C/x)$, and $\beta_c' = \frac{\alpha \sum_{m=1}^{M} h_m^H w_m}{E}$ represents the average SINR for the secondary signals taking into account the channel estimation errors. Similarly, we denote the lower bound of the expectation rate $E[R_c]$ with $\tilde{R}_{c,\text{LB}}$.

$$E[R_c] = E_{g_m, h_m, s(n)} \left[ \log_2(1 + \gamma_c) \right] \geq E_{s(n)} \left[ \log_2(1 + \frac{\alpha \sum_{m=1}^{M} h_m^H w_m |s(n)|^2}{E}) \right]$$

$$= E_{s(n)} \left[ \log_2 \left( 1 + \frac{\alpha \sum_{m=1}^{M} h_m^H w_m |s(n)|^2}{E} \right) \right]$$

$$= -e^{-E} Ei\left( -\frac{1}{\beta_c'} \right) \log_2 e$$

$$\leq \tilde{R}_{c,\text{LB}}.$$
their locations correspond to the $4 \times 4$ grid points, with the x- and y-coordinates chosen from the set \{-375m, -125m, 125m, 375m\}. The small-scale fading coefficients for the channels of different communication links follow the i.i.d. CSCG distribution with zero mean and unit variance. Furthermore, the large-scale channel gains of AP-to-BD and AP-to-receiver links are modeled as $b_m = \beta_0 d_m^{-\gamma}$, where $\beta_0 = \left( \frac{1}{\lambda} \right)^2$ is the reference channel gain, with $\lambda = 0.0857$m denoting the wavelength, $d_m$ represents the corresponding channel link distance of AP-to-BD or AP-to-receiver, and $\gamma$ denotes the path loss exponent. We set $\gamma = 2.7$ for the AP-to-BD and AP-to-receiver channels. For convenience, the channels are assumed to have zero mean, i.e., $\mathbf{g}_m = \mathbf{F}_m^{H} \mathbf{q} = 0_{N \times 1}$, and the large-scale channel coefficients $\epsilon_m$ of the cascaded AP-BD-receiver channels are modeled as $\epsilon_m = L\xi_m \xi_q = 0.0001 L\xi_m$, where $\xi_m$ and $\xi_q$ represent the large-scale coefficients of AP-to-BD and BD-receiver channels, respectively. The power reflection coefficient is $\alpha = 1$, and the transmitter-side SNR for data and pilot transmission are set as $\frac{E_m}{\sigma_m^2} = \frac{E_q}{\sigma_q^2} = 130 \text{ dB}$, $m = 1, \ldots, M$, which may correspond to $P_m = P_l = 20 \text{ dBm}$ and $\sigma^2 = -110 \text{ dBm}$. The termination threshold for Algorithm 1 is set as $\kappa_1 = 0.5\%$. We further denote $\tau_{\text{total}} = \tau_1 + \tau_2$ as the total pilot length used for channel estimation in phase 1 and phase 2, and $l_1 = \frac{\tau_1}{\tau_{\text{total}}}$ and $l_2 = \frac{\tau_2}{\tau_{\text{total}}}$ denote the ratios of the total pilot length allocated for the first and second training phases, respectively.

A. Channel Estimation Error and Convergence of SCA Algorithm

As can be inferred from (47) and (53), with imperfect CSI, not only the additive noise but also the channel estimation error will impair the performance of cell-free symbiotic radio systems. Therefore, Fig. 2 shows the normalized channel estimation-error-plus-noise power, i.e., $E/\sigma^2$, where $E$ is given in equation (48) and the normalization is taken with respect to the noise power $\sigma^2$. The ratio $l_1$ of training phase 1 varies from 0.05 to 0.95. The total pilot length $\tau_{\text{total}}$ is set as 50 and 100, respectively. Note that with perfect CSI, we have $E/\sigma^2 = 1$. It is observed from Fig. 2 that the curve with $\tau_{\text{total}} = 100$ always stays below the curve with $\tau_{\text{total}} = 50$. This is expected since the accuracy of channel estimation can be improved with a longer pilot length. Furthermore, for both $\tau_{\text{total}} = 100$ and $\tau_{\text{total}} = 50$, as ratio $l_1$ increases, $E/\sigma^2$ first decreases, and then increases slowly after reaching its lowest point. This is expected since for a given total training length, there exists a trade-off between the estimation error in phase 1 and phase 2, as can be inferred from (48). Specifically, the increase of $l_1$ increases the training ENR $\epsilon_1$, which effectively reduces the primary channel estimation error in (31), while the secondary cascaded channels suffer from more serious estimation errors. Besides, it can be inferred from Fig. 2 that the average channel estimation-error-plus-noise power $E$ is more sensitive to the pilot length of phase 1 than that of phase 2. Specifically, when a short pilot was allocated to phase 1 (say $l_1 = 0.1$), the corresponding $E/\sigma^2$ is much higher than its counterpart with $l_1 = 0.9$, which corresponds to a low pilot allocation to phase 2. This is expected since the estimation error of the primary channels in phase 1 also affects the channel estimation accuracy of the secondary cascaded link in phase 2, as can be inferred from (40). As a result, with a limited pilot length, higher priority should be given to phase 1 in order to decrease the estimation error of the whole system.

Fig. 3 shows the optimal pilot allocation for which $E/\sigma^2$ achieves the minimum value. It is observed that around the optimal point, the curves are rather flat, indicating that the impact of the channel estimation error would be comparable for a wide range of pilot length allocations, say for $0.3 < l_1 < 0.9$. Therefore, in the following, we choose $l_1 = 0.1, 0.5, 0.9$ as the representative values to show the impact of different pilot length allocations on system performance.

Fig. 3 and Fig. 4 show the convergence of the proposed SCA-based algorithm in Algorithm 2 with total pilot lengths of $\tau_{\text{total}} = 50$ and 100, respectively. The iterations start with randomly generated initial local points, with a rate threshold of $R_{\text{th}} = 12 \text{ bps/Hz}$ and an iteration terminating threshold of $\kappa_2 = 0.5\%$. Note that different curves in
Fig. 4. Convergence of the SCA iteration process with different primary pilot length ratio $l_1$, where $R_{th} = 12 \text{bps/Hz}$, $\tau_{total} = 100$.

Fig. 6. Achievable rate-region of cell-free symbiotic radio system, where $\tau_{total} = 100$.

Fig. 5. Achievable rate-region of cell-free symbiotic radio system, where $\tau_{total} = 50$.

Fig. 7. Achievable rate-region of the proposed SCA-based algorithm and the benchmark weighted-MRT, where $\tau_{total} = 50$.

Fig. 3 and Fig. 4 show the convergence with perfect CSI and different primary pilot length ratio $l_1$, respectively. As shown in the figure, the passive secondary communication rate, which is the optimization objective of problem (7), increases monotonically during iterations, which is in accordance with Lemma 1. Furthermore, Fig. 3 and Fig. 4 show that only a few iterations are needed for Algorithm 2 to converge.

**B. Achievable Rate-Region**

Fig. 5 shows the achievable rate-region of the primary and secondary communication rates with perfect and imperfect CSI, respectively. In the case of imperfect CSI, the primary pilot length ratios are selected as $l_1 = 0.1, 0.5, 0.9$, and the simulation results are obtained by running 500 experiments at each primary pilot length ratio based on the random cell-free symbiotic radio system channel realization. The total pilot length $\tau_{total}$ is set to 50. Note that each point of the curve corresponds to a primary-secondary rate pair, by varying the primary communication threshold $R_{th}$ from $\hat{R}_s$ to $\bar{R}_s$ with step size 1. When $\hat{R}_s < R_{th} \leq \bar{R}_s$, we use the solid and dashed lines to denote the SCA-based achievable rate-region and the SDR-based outer bound, respectively. It is observed that the SCA-based achievable rate-region is very close to that of the SDR-based outer bound for both perfect CSI and estimated CSI with different primary pilot length ratios. This indicates that our proposed SCA-based algorithm can well characterize the achievable rate-region of the cell-free symbiotic radio system. Meanwhile, the dashed line also shows the portion with $R_{th} \leq \hat{R}_s$, where the optimization problem has a closed-form solution given in Theorem 1, and the primary and secondary communication rate can be obtained in (11) and (14). If $\hat{R}_s \leq R_{th} \leq \bar{R}_s$, the achievable rate of the secondary backscattering communication decreases monotonically as the primary communication threshold $R_{th}$ increases, and eventually approaches zero. It is observed from Fig. 5 that with the total pilot length $\tau_{total}$ fixed, the achievable
rate-region, taking into account imperfect CSI estimation, is highly dependent on the allocation of the pilot $\tau_l$. Out of the three pilot allocations considered, $\tau_l = 0.5$ gives the best performance. This is consistent with Fig. 2, where $\tau_l = 0.5$ gives smaller channel estimation error than $\tau_l = 0.1$ and $\tau_l = 0.9$. Furthermore, Fig. 5 also shows that $\tau_l = 0.9$ outperforms $\tau_l = 0.1$. This is also consistent with Fig. 2, by comparing their respective channel estimation errors.

Fig. 6 shows the achievable rate-region with a fixed total pilot length $\tau_{total} = 100$. Similar to Fig. 5, a higher primary communication threshold $R_{th}$ brings a lower secondary communication rate. Besides, when comparing Fig. 6 to Fig. 5, it is observed that a larger rate-region is achieved for any given pilot allocation $\tau_l$ in Fig. 6 than its counterpart in Fig. 5. This is expected since a longer pilot length enhances the estimation accuracy of both the active primary and passive secondary channels, thus enlarging the achievable rate-region significantly. Meanwhile, it is observed that the performance gap between $\tau_l = 0.5$ and $\tau_l = 0.9$ is smaller in Fig. 5 than that in Fig. 6, while the reverse is true for the gap between $\tau_l = 0.5$ and $\tau_l = 0.1$. This implies that the performance improvement to increase the priority for primary channel estimation is more significant when $\tau_{total}$ is relatively small. Thus, when the pilot length $\tau_{total}$ is severely limited, higher priority should be given to the training phase. This is expected since the estimation of the direct-link channels in the first phase affects not only the primary communication rate, but also the quality of the channel estimation of the backscattering channels.

Fig. 7 and Fig. 8 plot the achievable rate-region obtained by the proposed SCA-based algorithm with $\tau_{total} = 50$ and $\tau_{total} = 100$, respectively. The benchmark scheme with weighted maximum ratio transmission (weighted-MRT) is also considered. In weighted-MRT, the beamforming of each AP consists of a weighted sum of the normalized direct link channel and the cascaded channel. The weight of the secondary communication channels varies from 0 to 1 with a step size of 0.1. It is observed that the proposed SCA-based achievable rate-region characterization is much larger than the weighted-MRT scheme. Furthermore, in the achievable rate-region characterization, we can obtain the ultimate performance $\tilde{R}_s$ of the primary transmission by the bisection method proposed in the previous section, which cannot be achieved by the weighted-MRT scheme. This can be explained by the fact that weighted-MRT is a simple weighted summation of channels and cannot effectively achieve trade-offs between the primary and secondary communications.

VI. Conclusion

In this paper, a novel cell-free symbiotic radio system was investigated, in which a number of distributed APs cooperatively send primary information to the receiver, while concurrently supporting the secondary backscattering communication. The achievable rates of both the active primary and passive secondary communications were first derived under the assumption of perfect CSI. Furthermore, a two-phase uplink-training based channel estimation method was proposed to effectively estimate the direct-link channel and cascaded backscattering channel, and the achievable rates were revisited with channel estimation errors. In order to characterize the achievable rate-region, a beamforming optimization problem was formulated to maximize the passive secondary communication rate with a given active primary communication rate constraint, for both perfect CSI and imperfect CSI. Efficient algorithms were proposed to solve the formulated optimization problem. The performance of the cell-free symbiotic radio communication systems was validated with extensive simulation results.

APPENDIX

Proof of Theorem 1

To prove Theorem 1, we first consider a relaxed problem of (7) by omitting the primary communication rate target constraint (7b). The problem is formulated as

$$\max_{w_{m}, m = 1, \ldots, M} \sum_{m=1}^{M} q_j^H F_m w_m^2$$

subject to

$$\|w_m\|^2 \leq P_m, \quad m = 1, \ldots, M.$$  \hfill (56b)

It is not difficult to see that the optimal solution to problem (56) is the per-AP maximum ratio transmission (MRT) beamforming with maximum transmit power, which is given by (12) in Theorem 1. In this case, the corresponding primary communication rate on the left hand side of (7b) is given in $\tilde{R}_s$ shown in (11).

Therefore, when the condition $R_{th} \leq \tilde{R}_s$ given in Theorem 1 is satisfied, the solution in (12) satisfies both constraints in (7b) and (7c), and thus it is also feasible to the original problem (7). Furthermore, since the beamforming vectors in (12) is the optimal solution to the relaxed problem (56) and is also feasible to (7), it must also be the optimal solution to the original problem (7). This conclusion is obvious since the former has a larger feasibility region than the latter.

This completes the proof of Theorem 1.
[49] Ö. Özdogan, E. Björnson, and J. Zhang, “Performance of cell-free massive MIMO with Rician fading and phase shifts,” IEEE Trans. Wireless Commun., vol. 18, no. 11, pp. 5299–5315, Nov. 2019.

[50] S. K. Sengijpta, Fundamentals of Statistical Signal Processing: Estimation Theory. Oxford, U.K.: Taylor & Francis, 1995.

[51] J. Yuan, Y. C. Liang, J. Joung, G. Feng, and E. G. Larsson, “Intelligent reflecting surface-assisted cognitive radio system,” IEEE Trans. Commun., vol. 69, no. 1, pp. 675–687, Jan. 2021.

[52] Y. Zeng, R. Zhang, and Z. N. Chen, “Electromagnetic lens-focusing antenna enabled massive MIMO: Performance improvement and cost reduction,” IEEE J. Sel. Areas Commun., vol. 32, no. 6, pp. 1194–1206, Jun. 2014.

Zhuoyin Dai received the B.S. degree in electronic and information engineering from the University of Electronic and Science Technology of China, Chengdu, China, in 2020. He is currently pursuing the Ph.D. degree with the Southeast University, Nanjing, China. His current research interests include cell-free massive MIMO, symbiotic radio, and reconfigurable intelligent surfaces.

Ruoguang Li (Member, IEEE) received the Ph.D. degree from the School of Electrical Engineering, Beijing University of Posts and Telecommunications, in 2020. From 2017 to 2019, he was a Visiting Ph.D. Student with the Department of Electrical and Computer Engineering, University of Houston, Houston, TX, USA. He is currently a Post-Doctoral Fellow with the National Mobile Communications Research Laboratory, Southeast University, Nanjing, China. His research interests include integrated sensing and communication, mobile edge computing, and resource allocation.

Jingran Xu received the B.S. degree in communication engineering from the Nanjing University of Posts and Telecommunications, Nanjing, China, in 2020. She is currently pursuing the Ph.D. degree with Southeast University, Nanjing. Her research interests include symbiotic radio and cell-free massive MIMO.

Yong Zeng (Senior Member, IEEE) received the Bachelor of Engineering (Hons.) and Ph.D. degrees from Nanyang Technological University, Singapore, in 2009 and 2014, respectively. From 2013 to 2018, he was a Research Fellow and Senior Research Fellow at the Department of Electrical and Computer Engineering, National University of Singapore. From 2018 to 2019, he was a Lecturer at the School of Electrical and Information Engineering, The University of Sydney, Australia. He is currently a Full Professor with the National Mobile Communications Research Laboratory, Southeast University, China, and also with Purple Mountain Laboratories, Nanjing, China. He was listed as a Highly Cited Researcher by Clarivate Analytics for four consecutive years (2019–2022). He was a recipient of the Australia Research Council (ARC) Discovery Early Career Researcher Award (DECRA), 2020 IEEE Marconi Prize Paper Award in Wireless Communications, 2018 IEEE Communications Society Asia-Pacific Outstanding Young Researcher Award, 2020 & 2017 IEEE Communications Society Heinrich Hertz Prize Paper Award, 2021 IEEE ICC Best Paper Award, and 2021 China Communications Best Paper Award. He is the Symposium Chair of IEEE GLOBECOM 2021 Track on Aerial Communications, the Workshop Co-Chair of ICC 2018–2022 Workshop on UAV Communications, and the Tutorial Speaker of GLOBECOM 2018/2019 and ICC 2019 Tutorials on UAV Communications. He serves as an Associate Editor for IEEE COMMUNICATIONS LETTERS and IEEE OPEN JOURNAL OF VEHICULAR TECHNOLOGY, a Leading Guest Editor for IEEE WIRELESS COMMUNICATIONS on “Integrating UAVs into 5G and Beyond” and China Communications on “Network-Connected UAV Communications.”

Shi Jin (Senior Member, IEEE) received the B.S. degree in communications engineering from the Guilin University of Electronic Technology, Guilin, China, in 1996, the M.S. degree from the Nanjing University of Posts and Telecommunications, Nanjing, China, in 2003, and the Ph.D. degree in communications and information systems from Southeast University, Nanjing, in 2007. From 2007 to 2009, he was a Research Fellow with the Adastral Park Research Campus, University College London, London, U.K. He is currently with the Faculty of the National Mobile Communications Research Laboratory, Southeast University. His research interests include space time wireless communications, random matrix theory, and information theory. He and his coauthors received the 2011 IEEE Communications Society Stephen O. Rice Prize Paper Award in the field of communication theory and the 2010 Young Author Best Paper Award by the IEEE Signal Processing Society. He serves as an Associate Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, the IEEE COMMUNICATIONS LETTERS, and the IET Communications.