Numerical modelling of guided waves dispersion curves in an aluminium flat plate by finite element analysis

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Abstract. This work presents a practical approach to obtain the dispersion curves for a plate via numerical modelling using the commercial software ANSYS. The method is based on the dynamic analysis of the waveguide to extract the mode shapes (eigenvectors) for each resonant frequency (eigenvalue) from the natural frequency response without damping. Each eigenvalue provides a frequency and the dimensions of the model (wavelength), which are the basis of the dispersion curves. Then, an algorithm is developed in MATLAB to stack the different mode solutions with its respective frequencies using the modal assurance criterion. The data obtained is compared with the dispersion curves provided by the GUIGW software.

1. Introduction

Engineering structures require suitable and effective methods for condition monitoring that avoids any possible accident in the operating personnel and keeps the industrial infrastructure safe [1]. Therefore, structural health monitoring (SHM) becomes a necessity in order to guarantee a good structural performance, avoiding any type of functioning setback and providing feedback and even predictive prognosis about the condition of the system. Due to this, the guided waves technique is getting recognition from the industrial and academic communities [2, 3]. The technique is a non-destructive test (NDT) based on the analysis of propagated elastic waves (Lamb waves in the case of plates) that, as mentioned by Majda Bakhcha et al. [4], have the capacity of propagation over long distances through the element under inspection, given by the resonance between the guided wave frequency and the natural frequency of the waveguide, which in turn, allows screening bigger areas as the wave is launched in one specific spot or side of the material.

In [5], it is pointed out that the ultrasonic waves travel through the structure intercepting each other producing stationary interference in the cross-section in the direction of propagation. Thus, it is very convenient to examine the interference patterns and modes, which exist in infinite numbers, and which only propagate for certain combinations of frequency and wavenumber, according to the elastodynamic properties of the element under examination.

This paper presents a practical approach to obtain the guided waves dispersion curves of a plate via a finite element (FE) method. Here, the mass and stiffness matrices are used for a modal analysis using commercial software (ANSYS) where the propagation of time-harmonic
disturbances in the plate (isotropic and homogeneous) are obtained by applying periodicity conditions. Additionally, the boundary conditions in the plate are considered stress-free. A good agreement is achieved between the obtained results of the dynamic analysis by ANSYS (eigenvalues and eigenvectors) organized in dispersion curves and the dispersion curves obtained by the software GUIGW [6].

2. Modal analysis using FEM

There are four different approaches to determine the dispersion curves: The first one based on the superposition of bulk waves (SPBW), as Lowe particularises in [7], which includes transfer-matrix and global matrix methods. The second one, the semi-analytical finite element methods (SAFE) or waveguide finite element method (WFE), which resolves the dynamics of the guided wave propagation using a FEM framework. As Barki [8] states, it is relatively simple to use in comparison with other methods that were previously used. For the third one, the dispersion curves are obtained in an experimental way, requiring a great experimental effort. Finally, a numerical approach is proposed by Sorohan in [5], in which a dynamic analysis (natural frequencies of the waveguide and its shape modes) provides the wavenumber - frequency relation, which is the foundation of the dispersion curves, highlighting the advantages of the dynamic analysis of FE models in providing a numerical tool able to determine the dispersion curves of complex structures.

The finite element method (FEM) proposes an approximation of the solution by discretisation of the domain, dividing a complex problem into small finite elements [9, 10]. Within each finite element (FE), a polynomial formulation is assumed and the element contribution is assembled into a larger system of equations that models the elastic response. In this case, once the waveguide is discretised by a mesh to compute natural frequencies, the FE environment will not consider applied loads, i.e., a “free vibration” model is considered. The dynamic analysis to determine the natural frequencies and mode shapes is based on the unforced motion governed by the equation of motion written in reduced matrix form in Equation (1):

\[
M \ddot{u} + Ku = 0, \quad (1)
\]

which can be considered as generalised Newton’s second law. Here, \( M \) is the mass matrix, \( K \) is the stiffness matrix, \( \ddot{u} \) is the acceleration vector and \( u \) is the displacement vector. A free-vibration (\( \ddot{u} \approx u\lambda \)) solution of harmonic motion is assumed, such that, Equation (1) can be simplified in Equation (2):

\[
(K - \omega^2 M) = 0, \quad (2)
\]

ending in the basic equation solved in a typical undamped modal analysis as the classical eigenvalue problem.

3. Methodology

The approach presented here starts with the selection of the Structural option as a context filter in ANSYS APDL. In this work, the dispersion curves for a thin aluminium plate are obtained, hence the PLANE182 element is used to define the topology of the mesh. The linear element has four nodes, with two degrees of freedom per node: translations in the nodal \( x \) and \( y \) directions [11]. Due to the fact that the waves are modeled along the thickness, thus, the \( z \) direction is not relevant, the plane strain condition is selected for the elastic model. The material properties for the aluminium are a Young’s modulus \( E = 70 \) GPa, Poisson’s ratio \( \nu = 0.33 \), and mass density \( \rho = 2700 \) Kg/m\(^3\). For the geometry, a thickness \( h \) of 5 mm and a length \( L \) of 90 mm are defined,
considering that the wavenumber increment is $\Delta k = 2\pi/L$ where $L$ must be between 10 and 20 times $h$.

In [12], it is recommended to use between 10–20 nodes per wavelength, which can be expressed as in Equation (3), where $l_e$ is the element length and $\lambda_{\text{min}}$ is the shortest wavelength of interest and reckons that, if highly accurate numerical results are needed, then, a finer mesh might be required.

$$l_e \leq \frac{\lambda_{\text{min}}}{10 - 20} \quad (3)$$

The element size was chosen so that the propagating waves can be accurately described for a frequency range of interest considered (0 KHz – 1000 KHz). According to the procedure proposed in [5], a mechanical coupling between the left end and right end cross-section areas is required to determine the dynamic behaviour of the waveguide, see Figure 1. Thus, the degrees of freedom associated with the nodes on boundary 1 are constrained to the nodes on boundary 2, Equation (4).

$$U_2 = U_1 \quad (4)$$

Figure 1. Finite element mesh representation used in plate analysis for Lamb mode computation (plotted with an arbitrary scale).

Subsequently, the modal analysis is selected to evaluate the eigenvalues and eigenvectors. The Block Lanczos algorithm is especially powerful when searching for eigenfrequencies in a given part of the eigenvalue spectrum of a given system, as expressed in [11]. Finally, once the model is submitted to analysis and after the computation stage, the results are displayed as a list of natural frequencies (eigenfrequencies) and mode shapes (eigenvectors). Data is saved in an array to be transferred to the Matlab Workspace, where the dispersion curves are post-processed. Once the data obtained from the finite element analysis has been obtained, a propagation modes discrimination and grouping stage is performed via wave structures using the eigenvectors derived for each eigenfrequency. For this purpose, the $x$-axis displacements of a random vertical line of nodes belonging to the cross-section of the plate are evaluated [13].

4. Results

The displacement vectors for each eigenfrequency are used to build the modal assurance criterion (MAC) matrix which is the basis to compare the different mode shapes and stack them in the different propagation modes such as $S_0$, $A_0$, etc., as shown in Table 1. Then, another array is created with the terms which satisfy a MAC threshold, in our case 0.9, where each column represents a group of frequencies and every element in one column belongs to the same model. However, it is possible that different modes of the same type (symmetric or antisymmetric)
interfere in the columns, therefore it is recommended to choose a high MAC criterion value,
where only the frequencies that belong to the same mode are related. Due to the fact that each
mode propagates in positive and negative directions, repeated values are debugged. Finally, the
different modes can be labelled according to the wave structure in symmetric or antisymmetric.

It should be noted that once the list of frequencies by mode type has been found, as explained
in [5], the eigenvectors are orthogonal to the mass and stiffness matrix, therefore, an ascending
order of frequencies is expected. Thus, under the assumption of \( kL = \pm 2p\pi \), the frequencies are
shown in an order of submultiples of the main length \( \lambda_{\text{max}} \) (\( p = 0, 1, 2, 3 \)).

To produce the database for the dispersion curves generation, some calculations are
performed, e.g., the wavenumber \( k \) is obtained from the wavelength which is related to the
waveguide length, see Equation (5). On the other hand, the angular velocity is related to the
period of oscillation \( T \) via Equation (6), considering that the wave propagates parallel to the
largest distance of the plate, a longitudinal wave is analytically described in time by Equation
(7).

\[
\begin{align*}
  k &= \frac{2\pi}{\lambda}; \\
  \omega &= \frac{2\pi}{T};
\end{align*}
\]  

The phase velocity \( C_{\text{ph}} \) can easily be obtained using Equation (8) since the values of frequency
and wavelength are available. Mathematically, the group velocity \( C_{\text{G}} \) is defined as the derivative
of the frequency of a partial wave with respect to wavenumber, Equation (9).

\[
\begin{align*}
  \psi(x, t) &= \sum A_i \cdot \cos(k_i x - \omega_i t) \quad (7) \\
  C_{\text{ph}} &= \lambda \cdot v = \frac{\omega}{k}; \\
  C_{\text{G}} &= \frac{d\omega}{dk};
\end{align*}
\]  

| Table 1. List of modes frequencies (Hz) obtained in the range 0-1 MHz. |
|-----------------------------|-----------------------------|-----------------------------|-----------------------------|
| \( \lambda \) [mm] | A0 | A0' | A1 | A1' | S0 | S0' |
|-----------------|----|----|----|----|----|----|
| 0.090 | 5908.76 | 5883.14 | 312400.72 | 312400.72 | 59858.41 | 59726.02 |
| 0.045 | 22301.03 | 22209.35 | 337736.56 | 337631.63 | 119255.91 | 118994.30 |
| 0.030 | 46282.84 | 46104.78 | 337736.56 | 365159.79 | 177639.74 | 177256.30 |
| 0.023 | 75104.84 | 74834.38 | 365366.36 | 398710.64 | 234247.62 | 233756.45 |
| 0.018 | 106824.43 | 106462.14 | 399028.66 | 436092.47 | 287947.66 | 287374.00 |
| 0.015 | 140222.63 | 139771.35 | 436522.92 | 475769.70 | 337090.03 | 336475.30 |
| 0.013 | 174564.49 | 174027.45 | 476308.56 | 516630.04 | 379672.16 | 379070.74 |
| 0.011 | 209410.87 | 208791.04 | 517269.66 | 557802.71 | 414351.73 | 413805.53 |
| 0.010 | 244499.61 | 243799.51 | 558532.11 | - | 441801.34 | 441310.58 |
| 0.009 | 279674.83 | 278866.48 | - | - | - | 463962.93 |
| 0.008 | 314845.01 | 313990.03 | - | - | - | 484278.38 |
| 0.007 | 349958.04 | 349027.70 | - | - | - | 503995.70 |
| 0.006 | 384986.04 | 383981.31 | - | - | - | 524090.31 |
The wave frequencies \( f \) are obtained only for a limited series of wavelengths \( k \), due to this the derivative cannot be accurately obtained. In [14], to overcome this limitation, it is proposed that two further analyses must be performed for new sets of lengths \( L \), with the value of \( \Delta L \) between 0.5% and –2.5% of \( L \). In order to achieve an accurate shift in the frequencies with respect to the two sets of wavelength when a finite difference formulation is used, the \( \Delta L \) must not be very small. Hence, the central difference theorem was used to estimate each coordinate (group velocity, frequency). Figure 2 shows the resulting dispersion curves for the isotropic plate. The figure reports the wavenumber, wavelength, phase velocity and group velocity vs. frequency, obtained by modal analysis. The discrete points were numerically obtained using FEA and the dashed lines behind them were obtained using the software GUIGW. Notice the good agreement between the FE results and the reference solution for all curves.

Figure 2. Dispersion curves for an aluminium plate. The discrete points were numerically obtained using FEA and the dashed lines behind them were obtained by GUIGW.

5. Conclusions

In this paper, the dispersion curves (phase velocity and group velocity versus frequency) for a stress-free, isotropic and homogeneous plate using a modal analysis via a commercial finite element package were obtained. The results proved the studied approach to be accurate in the generation of the dispersion curves, comparing the good agreement of the obtained results with the values provided by the GUIGW software. It is found that to amplifying the resolution and range of the curves more points can be found as the procedure is repeated for different lengths. Also, notice that the discretisation by MAC number presents certain sensibility problems since in some cases there are high numbers above 0.9 for different types of mode.

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