A Strategy for Achieving Smooth Filamentation Cutting of Transparent Materials with Ultrafast Lasers

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Featured Application: High-quality and high-throughput eco-friendly regimes of laser cutting glasses, sapphire and other transparent materials are found based on the model. Possible practical applications are in display manufacturing, the automotive and other industries.

Abstract: A strategy is proposed for achieving a practically important mode of laser processing—a so-called “smooth” laser filamentation cutting (LFC) of transparent materials by a moving beam of a pulse-periodic pico- or subpicosecond laser. With such cutting, the surface of the sidewalls of the cuts have a significantly improved smoothness, and, as a result, the laser-cut plates have increased resistance to damage and cracking due to mechanical impacts during their subsequent use. According to the proposed analytical model, for the case when each filament is formed only by a single laser pulse, the strategy of such cutting is defined by a set of necessary conditions, whose implementation requires, in turn, a certain selection and matching with each other of irradiation parameters (pulse duration and energy, repetition rate, pitch of filaments following) and of material parameters—thermal, optical and mechanical strength constants. The model shows good agreement with experiments on sapphire and tempered glass. Besides, LFC modes are also predicted that provide very high cutting speeds of the order of 1–25 m/s or more, or allow cutting with an extremely low average laser power, but still at a speed acceptable for practical applications.

Keywords: laser cutting; filamentation; transparent materials; smooth cutting; filaments spatial step; pulse repetition rate; pulse energy; pulse duration; analytical model; multi-scale modelling

1. Introduction

Cutting up glass and other optically transparent materials in a desired shape and size is widely used in building construction and architecture, medicine, the automotive industry, mobile phone and tablet manufacture, electronics and many other industries. The development of new high-throughput, energy-efficient and environmentally friendly cutting technologies is important for large industrial-scale production. In comparison with traditional mechanical cutting laser-assisted cutting is promising owing to the prospects of obtaining a higher quality of the surface finish, the possibility of achieving tens or hundreds of times higher speed and environmental friendliness of the overall process. The use of lasers makes it possible to easily cut materials such as tempered and non-tempered glass, hard and superhard transparent materials, for which traditional mechanical methods are inefficient, requiring too much time and labour.

Various methods of laser cutting of transparent materials are known:
- The exposure to continuous-wave (cw) lasers, usually CO2, Nd: YAG, or CO lasers (see, e.g., review [1] and references cited therein, also papers [2,3]), where the laser energy is absorbed on the surface or in the volume of the material. This induces thermal stresses...
that cause the initiation and propagation of material cleaving both into the material depth and also along the laser path on the surface. The separation of parts occurs without applying any auxiliary mechanical stress in the irradiated region, and walls of the cut are smooth and practically do not contain any defects and chips;

Another method is exposure to ultrashort pulses, usually of femtosecond (fs) or picosecond (ps) pulse-periodic lasers when scanning the focused laser beam creates one or several arrays of thin extended filamentary plasma objects (shortly called below as filaments) inside transparent or semi-transparent materials (see the schematic in Figure 1). After such an exposure the material is split along the surface of the arrangement of these arrays when an external mechanical tensile stress is applied there (see, e.g., [4–6]). These elongated filaments can be created by the well-known nonlinear optical phenomenon of self-focusing of a high-intensity laser beam (see, for example, [7,8], as well as reviews [9,10] and references therein). A repeatable traversing of the beam through the material is used to create several arrays in the depth direction and thus to elongate filaments [4,8] (see Figure 1). The method of “multi-focusing”, where the initial beam is split employing a special optical scheme into two or more beams having each of those focused at a certain depth inside the material [4] also enables creating several arrays of filaments differing in the depth of their location (as in Figure 1), but with a single pass of the beam.

Figure 1. Irradiation scheme explaining the main parameters of the model. Each filament is created by a single laser pulse of a focused beam from a pulse-periodic fs, or ps laser, $\lambda$ is its wavelength, $\tau$ is pulse duration, $f$ is pulse repetition rate (PRR), $u$ is the velocity of beam scanning across the material. The focusing system and the laser beam are described by the standard parameters NA and $M^2$. $L$ is the thickness of the irradiated sample, $s_1$ is the distance between neighbouring filaments inside the material, $H$ is the length of the single filament, and at the same time, the width of one array of filaments, $d$ is the vertical distance between adjacent arrays of filaments. The beam scans the material in the $xy$-plane, the $z$-axis is directed into the material depth and coincides with the direction of the filament axis.

Other methods, such as diffraction optics, Bessel non-diffracting beams created with axicons, or intentionally astigmatic lenses (see, e.g., [11–18]), as well as the filament creation by a burst of pulses rather than by a single pulse [17,19], have also opened up a way where the total length of filaments, continuous or discontinuous, may be increased even by a single pass of the beam, thus allowing one to split materials of a greater thickness (up to a few millimetres).

The described methods of laser-induced filamentation cutting (LFC) of transparent materials can be classified as “green technologies”, because such methods use cleaving and hence give a “zero-width” cut. As a result, there is practically no release of the removed material in the form of polluting debris, vapours or droplets. Besides, LFC, if it would be capable of providing high-energy efficiency, a high throughput, and/or high quality of cut sidewalls, could also be important from an economical point of view. As it is well known, just the world market for mobile phones, tablets and wearable electronics
comprises the production of more than 1 billion displays and protective coatings annually for them from thin plates (about 0.1 to 0.70 mm thick) of chemically-tempered, non-tempered glass and sapphire. Besides this, there are also many other applications in the automotive industry, instrumentation, construction, creation of art objects, and other industries, where transparent plates with a relatively great thickness of about 2 to 10 mm or more are required, and the materials used there can include not only glasses, but also other various transparent or semitransparent materials, including superhard and refractory ones, difficult to handle by traditional mechanical processing. Examples are sapphire, diamond, transparent conductors and semiconductors, SiC, GaN, ZnSe, ZnS, CaF$_2$, KCl, KBr, etc.

Despite the aforementioned advantages, a better and deeper level of understanding of the LFC process is still required to loosen constraints acting individually or in various combinations. These are: (i) a low separation rate of only a few mm/s or cm/s in some cases [4,13]; (ii) a lack of smoothness of resulting sidewalls of the cut [4,17,20]; and (iii) the need to use a high average power laser beam for separation—about a few hundreds of Watts—in particular for materials with thicknesses greater than several millimetres. The solution to avoid these shortcomings might be found based on an analytical model of the complex phenomenon of LFC with a pulse-periodic beam scanning over the sample with a high pulse repetition rate of up to several megahertz.

As evidenced by a number of reviews [1,9,10,21–23], the numerous experimental and theoretical papers quoted therein, as well from the number of other references [24–35], a great number of authors in the filamentation physics field have for several decades considered various aspects of individual filament formation, but have not modelled the efficiency, speed and surface quality of cut walls for material separation processes induced with a large array of filaments. This filamentation physics area should be distinguished from the studied here specific field of “LFC physics”, which demands consideration of much higher levels of radiation pumping of filaments used for obtaining a material separation, with correspondingly much higher filament temperatures (as will be shown below, in some cases of the order of $10^5$ K). Besides, although the effects of the repetition rate on the heating of a transparent material by pulsed-periodic radiation of an ultrafast laser have been considered earlier, they were studied not for extended filaments, but almost exclusively for point-like focusing, used for laser direct writing waveguides in a transparent material [36,37]. The effect of a stationary and also a moving pulsed-periodic beam has been studied, but at a beam displacement step much smaller than the laser spot diameter [38]. However, in the given paper on modelling the LFC process, we are going to study the effects of energy release in a transparent material for a moving beam with a high pulse repetition rate (PRR) of up to several megahertz, for a set of long filaments, but not for almost point focusing, and also taking into account the mutual influence of filaments on each other, as well as at a step of their following, comparable or many times exceeding the spot diameter. Such a study has not been performed before.

In practice, the LFC can be completely set for a particular material by four following parameters—the pulse energy inside the material $E_1$, the pulse duration $\tau$, the distance (step of following, or pitch) $s_1$ between neighbour filaments, and the PRR $f$. The optimal values of these parameters, resulting in high-energy efficiency, and/or highly smooth sidewalls of the cut, and/or high-throughput cutting results should be determined in the model depending on the optical, thermophysical and mechanical strength properties of the material, as well as the irradiation parameters. Two of the mentioned parameters, $E_1$ and $\tau$, have been considered as the first stage of modelling in our previous work [39]. In the given paper as the second stage of analytical modelling, two other parameters from above mentioned are modelled and quantitatively calculated—the step of following between neighbour filaments $s_1$ and the PRR $f$.

The focus of the research is on the achievement of an important feature for practical applications—a so-called “smooth” cutting processing mode—where the surface of the
sidewalls of the cut has a better smoothness and does not contain small cracks and roughness of micron size, which may arise when the cut propagates through the material. Obtaining such a result is of great practical importance since cut sidewalls with better smoothness are highly resistant to damage and cracking due to mechanical impacts during further use of the workpiece, for example, as displays and protective plates in mobile phones, and also car windshields, etc. A strategy for obtaining such a better smoothness cutting is formulated. It consists of fulfilling several necessary conditions. Their implementation, in turn, requires, as the model shows, the use of certain irradiation parameters—the energy and duration of the laser pulse, the step of the filaments following in the material, the pulse repetition rate—as well as of certain material parameters and their matching with each other.

Along with this, LFC modes are predicted that are characterized by high throughput, provide very high cutting speeds of the order of 1–25 m/s or more, depending on the material, which are tens or hundreds of times higher than traditional mechanical cutting speeds or allow cutting with an extremely low average laser power, but with an acceptable speed for applications.

2. Methods and Materials

The model considers elongated filaments either created inside the material by a Gaussian beam at its self-focusing point or by an extended in length non-diffracting Bessel beam free of self-focusing, as shown in Figures 1–3. In either case, these objects are designated hereinafter as “filaments”, since their effect on the sample separation is determined, as will be shown below, by their characteristic radius and the volumetric energy density (VED) they acquire, regardless of whether they are due to a self-focusing or Bessel beam. The laser action on the sample surface mentioned in [32,34,35] is not addressed in the paper. Each filament is created by a single pulse.

**Figure 2.** The follow-up to the Figure 1, where other parameters of the model are explained. An individual filament for the sake of simplicity is considered as a cylinder of initial radius \( r_l \) (at the 1/e-level) and a length \( H \gg r_l \), with a volumetric energy density \( w \) uniform along the length, \( E_l \) is the laser pulse energy inside the material, \( E_f \) is the energy of the filament plasma, \( Q \) is the heat released in the surrounding filament material as a result of relaxation of the filament plasma.
Figure 3. Schematic of producing the extended-length focusing inside the transparent material employing axicons.

The main parameters considered in the modelling of a standard scheme of LFC (see, e.g., [4,8,17,39]) are presented in figures 1 and 2. In this scenario, the focused beam of a pulse-periodic ps-, or sub-ps laser is scanned over the material and creates one or several arrays of plasma filaments, with each of these arrays being different from others by its depth inside the sample.

To describe properly the laser heating of the filament a parameter such as a VED of the filament, $w$, is used instead of laser pulse fluence or intensity. This feature $w$, measured in J/cm$^3$, is directly related to the filament temperature. Since many of the processes involved (glass softening, cracking, dissociation, ionisation et al.) have a temperature-activated nature, such an approach enables one to estimate most simply the temperature of the filament in terms of the VED and to tackle the mechanisms and the nature of laser-induced phenomena behind the thus produced laser cleavage. Also, a rectangular temporal shape of the pulse is assumed as a model simplification.

Typical experimental photos of sidewalls of the cuts (see, e.g., [4,8,14,17,20]) suggest that it is a good approximation for modelling to consider an individual filament as a cylinder with a height $H$ (see Figures 1 and 2). Another simplifying approximation is that the filament VED is considered as uniform along the cylinder length, but with initially a Gaussian distribution over the radius, with $r_f$ being the filament radius at the 1/e-level of VED and $r_f << H$. Then a simple integration over the space gives a simple and transparent expression for the filament plasma energy:

$$E_p = (\pi r_f^2 H) w.$$ (1)

The parameter $\pi r_f^2 H$ here corresponds to an effective volume of the mentioned cylindrical filament, and $w$ is a maximal VED value on filament axis. A similar expression $E_p = \pi r_{eff}^2 H w_{max}$ could also be written for a non-Gaussian transverse distribution, if it has an effective concentration around the filament axis rather than being widely distributed across space, in particular, for filaments produced with Bessel beams ($w_{max}$ here is a maximal value of VED in this non-Gaussian distribution, and $r_{eff}$ is its effective radius, which is not so obvious as the mentioned (1/e) Gaussian radius. It can be calculated as $r_{eff}^2 = E_p/(\pi H w_{max}) = \int_0^\infty w(r)2\pi rdr/\pi w_{max}$. However, it requires information on the $w(r)$ dependence, which is not always known.

There are in general two most practical irradiation cases, when (i) filaments do not overlap each other with interspacing $s_1$ several times greater than their characteristic diameter at the 1/2-level of VED decrease, $2r_{1/2}$, that is:

$$s_1 >> 2r_{1/2}$$ (2)
as shown in Figure 1, or (ii) are written inside the sample with a partial overlapping of each other, that is, with a smaller spacing, comparable with a filament diameter, $s_1 \approx 2r_{1/2}$. Only the case (2) with no overlapping is considered in the modelling below. As can be easily shown, for the assumed Gaussian distribution of $w$ the $r_{1/2}$ radius relates with the previously introduced $r_f$ as $r_{1/2} = (\ln2)^{1/2}r_f \approx 0.83r_f$.

The model follows up a concept of an indirect action of laser radiation on the material [40,41] that has been successfully applied to the deep (high-aspect-ratio) drilling of opaque materials. That is, the incident laser energy is at first transformed into the plasma energy (in the given case, the filament plasma) with a conversion coefficient (absorptivity) $A_{LP}$. This is followed by a conversion of the plasma energy into a modification of the material due to the radial transfer and dissipation of the plasma energy from the filament axis over a certain cylindrical volume around the filament. Such an impact zone is formed due to thermal conductivity and/or shock wave. The conversion coefficient of the plasma energy into the released energy is $A_{PH}$. This indirect laser action concept is universal in the sense that it does not depend on whether the material is transparent or opaque.
The thermalisation of the laser-excited electrons, that is, the equalisation of their temperature with the temperature of atoms and ions happens on the timescale of about 1 ps [23,42,43]. For example, the relaxation time for electron-hole plasma in fused silica is $\tau_{e-h} = 0.15$ ps [21,42], and in borosilicate glass BK7 $\tau_{e-h} = 2$ ps [43]. This is comparable with the pulse duration (a few times smaller or greater). That is, an equilibrium temperature maximum of the filament, or at least a relatively small part of it, although great enough in absolute value (few thousands K or more), may be reached for ps and even sub-ps laser pulses already before the pulse termination. Moreover, the filament cools down on a microsecond time scale, and such cooling times are much longer than the mentioned typical time of setting thermal equilibrium.

The material’s exposure to the high laser flux required for cleaving can induce its temperature to rise from room temperature to thousands and even hundreds of thousands of degrees (see below). This drives the material through a chain of transitions, which are not only a softening or melting, but also a dissociation and ionisation, whereas the electronic structure experiences noticeable rebuild even at the pulse timescale, up to the collapse of the bandgap [44,45], or at least to such a decrease of the bandgap $E_g$ from its initial typical for transparent materials value 4–9 eV at room temperature to the value less than laser photon energy $h\nu$ (for example, for the considered below laser wavelengths $\lambda = 1.04$ and 0.52 $\mu$m, $h\nu$ equals to 1.19 and 2.38 eV respectively). Besides, at a high laser excitation, the emission of free-electron plasma created by the laser occurs in a wide spectral range, often with short wavelengths in the UV range, where the material has high linear absorption ($10^4$–$10^5$ cm$^{-1}$ or more). The typical duration of this secondary emission is about 1 $\mu$s to tens of $\mu$s, corresponding to the abovementioned times for filament cooling down [43]. This duration is much longer than that one of the ps-, or sub–ps- laser pulse. As a result, the multiphoton (or tunnelling ionisation by a strong laser field), inverse Bremsstrahlung and the avalanche absorptions, whose role for transparent materials has been stressed out in absorption models developed previously for ultrafast lasers [18,24–28], can be reduced to a usual linear absorption. The availability of the experimental data (see, for example, [46]) show that at relatively low $w$ (correspondingly at low intensities), $A_{L,P}$ first increases with $w$, which indicates the nonlinearity of the absorption process. However, at relatively large $w$, the $A_{L,P}(w)$ dependence reaches saturation at about 0.80 ± 0.05. This practically constant absorptivity is an experimental indication of the transition to linear or quasi-linear absorption.

The use of the mentioned absorption models that presume a permanence of the structure of the electronic levels and also do not take into account absorption of plasma emission becomes thus strongly problematic for laser intensities applied for LFC. Besides, many other processes, such as beam propagation, excitons and defects formation, etc. can also affect the LFC.

It is unlikely that the analytical model could properly take all these processes into account by a calculation “from first principles”—such sophisticated models have not yet been developed—and all input material parameter data required for such calculations are simply unavailable. As the most simple and reliable way of accounting for all these phenomena in the model, we use below a semi-empirical approach, where some crucial parameters required for modelling are not calculated, but rather taken from experiments. For example, these may be the energy conversion coefficients $A_{L,P}$ and $A_{F,H}$ as well as the introduced above filament length $H$. This approach, although looks quite natural in laser materials processing, has not been implemented previously in other papers on laser filaments modelling.

We distinguish between two different types of smoothness when separating: (a) a smoothness between different sets of filaments, as well as between any set of filaments and the free surface of the material, and (b) a smoothness between neighbour filaments of the same array. The former case has been studied previously experimentally [14], and it has been shown that in this case for obtaining sidewalls of improved smoothness some matching a filament length with a thickness of the plate to be cut is required, with this
matching being different for tempered and non-tempered glasses. However, in this paper, we consider the latter case, which has not been previously studied—a smoothness between neighbour filaments of the same array.

3. Model

3.1. Typical Durations of the Processes of Filament Creation and Relaxation of Its Energy

LFC includes the processes of laser excitation and relaxation of the material, whose characteristic durations may differ by orders of magnitude [22]. We will clarify them for our modelling conditions, and also introduce some new time parameters that have not been previously noted.

As mentioned above, the time $t_{eq}$ for equalising the temperature of laser-excited electrons with the temperature of atoms and ions is of the order of 1 ps, and this is comparable to the pulse duration $\tau$, which for LFC is, as we will see below, of the order of 0.15 ÷ 2 ps, that is $\tau \sim t_{eq}$ or $\tau < t_{eq}$ or $\tau > t_{eq}$.

The time of an inertial confinement of pressure in a heated filament $t_{P1}$ and the time of pressure confinement in the space between neighbour filaments $t_{P2}$ can be estimated as the time of passage of a sound wave with a speed $c_s$ of the filament diameter $2r_f$ and the distance between neighbour filaments $s_1$:

$$t_{P1} \approx \frac{2r_f}{c_s}, \quad t_{P2} = \frac{s_1}{c_s},$$  \hspace{1cm} (3)

Until these times have elapsed, it can be assumed that the shock wave and any related destruction do not propagate into the surrounding material from a highly heated filament or from the space between neighbour filaments. When substituting into Equation (3) the characteristic value of the sound velocity in the material $c_s \approx 6 \times 10^5$ cm/s [47], the submicron radius of the filaments used in this method $r_f = 0.25–0.5$ $\mu$m and, for example, $s_1 \approx 2.5–5$ $\mu$m, one gets the estimates: $t_{P1} \approx 0.08–0.17$ ns and $t_{P2} \approx 0.4–0.8$ ns. As seen, $t_{eq} < t_{P1}, t_{P2}$.

The so-called “heat confinement time” in the filament is estimated as [39]:

$$t_{T1} = \frac{r_f^2}{4\chi c_s}. \hspace{1cm} (4)$$

When the radiation energy is deposited into the filament plasma and then is released in the filament in the form of its heating for a time smaller than $t_{T1}$, it can be assumed according to the heat conduction theory that there is practically no radial heat loss beyond the filament radius. Also, with a proper selection of the bulk energy density of the filament $w$, as we will see below, there is a practical absence of plasma energy losses due to thermal radiation in the transparency region of the surrounding material. For example, for a submicron filament radius $r_f = 0.25–0.5$ $\mu$m in a fused silica glass, when $\chi = 5 \times 10^{-3}$ cm$^2$/s, one has $t_{T1} = 30–125$ ns. For sapphire, when $\chi = (1.5–2.4) \times 10^{-2}$ cm$^2$/s, one gets $t_{T1} = 6–40$ ns. It is seen that $t_{T1}, t_{T2} < t_{T1}$.

The time of displacement of the laser beam relative to the material by a distance equal to the filament radius $r_f$ is:

$$t_r = r_f/(4\chi). \hspace{1cm} (5)$$

By analogy with the pressure and the heat confinement times $t_{P1}, t_{P2}$ and $t_{T1}$, the parameter $t_r$ can be called as a beam position retaining time. Here $u = s_f$ is the speed of relative motion of the beam and the material surface. Irradiation of the material with one or several pulses for a time much less than $t_r$ corresponds to the localisation of the energy deposition of these pulses in a space region with a radius that practically does not differ from the filament radius $r_f$. For the submicron radius $r_f = 0.25–0.5$ $\mu$m for tempered glass, when $u = s_f = 100–400$ cm/s, we get $t_r = 125–500$ ns, and for sapphire, when $f = 3$ MHz, $r_f = 0.5$ $\mu$m, $s_f = 4$ $\mu$m, and $u = s_f = 11$ m/s, we get $t_r = 45$ ns (the $u$ values indicated here see in the Section 4 “Discussion” below). It is seen that $t_{T1} < t_r$, or also can be $t_{T1} = t_r$. 

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The effective time \( t_e \) for the release of absorbed radiation energy in the atomic subsystem when it relaxes to a temperature equilibrated state is estimated in order of magnitude by one or another from two parameters \( t_{e-h} \) and \( t_{e-i} \) depending on laser pumping:

\[
t_e = t_{e-h} \text{ or } t_e = t_{e-i}.
\]

Here \( t_{e-h} \) is a characteristic relaxation time of bound electrons in the electron-hole plasma, which is obtained at relatively low fluences or VED, i.e., electrons located at the electronic levels of the material excited by laser photons [21]. Numerically \( t_{e-h} \) ranges from values close to \( t_0 \) (i.e., from 0.15 - 2 ps mentioned above) to few tens of picoseconds, when the formation of laser-induced self-trapped excitons and related defects is taken into account [47,48]. Formation of other long-living states can also contribute.

Another parameter here, \( t_{e-i} \), is an effective relaxation time of free charge carriers of electron-ion plasma obtained at a relatively high VED. It corresponds to the time of transfer of the excitation energy of ions and free electrons of the plasma to heat by their recombination and de-excitation both in the plasma itself and on the sidewalls of the filament. The source of excitation for such plasma can be both laser photons and photons of plasma thermal emission. Their energy can be quite large.

We remind readers here that, according to the Wien displacement law, the wavelength of the maximum of the thermal emission spectrum of a black body varies with the temperature \( T \) as \( \lambda_{max} \ [\text{nm}] = 257/T \ [\text{eV}] \). The photon energy corresponding to this wavelength is \( h\nu_{max} \ [\text{eV}] = 1240/\lambda_{max} \ [\text{nm}] = 4.83\ T \ [\text{eV}] \). As follows from the shape of the broad spectrum of black body thermal emission, photons with the twice greater energy \( h\nu^* \ [\text{eV}] = 2h\nu_{max} = 9.7\ T \ [\text{eV}] \) give although smaller, but a quite comparable contribution to the thermal emission spectrum with respect to the photons \( h\nu_{max} \) in the maximum. For example, the substitution into these expressions of a fairly moderate plasma temperature \( T = 5\ \text{eV} \) gives \( \lambda_{max} = 51\ \text{nm} \), \( h\nu_{max} = 24\ \text{eV} \) and \( h\nu^* = 48\ \text{eV} \). The maximum \( \lambda_{max} \) appears to be located in the region \( \lambda \leq \lambda_{wo} \) of strong absorption (> \( 10^7 – 10^6 \) cm\(^{-1}\)) of the cold material surrounding the filament, where \( \lambda_{wo} \) is a boundary of this high absorption region. For display glass \( \lambda_{wo} = 218\ \text{nm} \) and for fused silica — 140 nm. As seen, the plasma with the indicated temperature provides the presence of thermal emission photons of the high energy of 24–48 eV, which are effectively absorbed by the material. These energies are a few tens of times higher than the laser photon energy, which is 1.19 eV for laser wavelength \( \lambda = 1040\ \text{nm} \), or 2.38 eV for \( \lambda = 520\ \text{nm} \).

The value of \( t_{e-i} \) is not well characterised quantitatively in the literature. The LFC with high VEDs considered in this paper is accompanied by dissociation and disordering of the material. For example, for silica glass, such dissociation starts at a temperature about 2500 K and more [49], which, as we will see below, is relatively low compared to temperatures of few tens of thousands K, that can be reached in LFC with high VEDs. It looks reasonable to expect in such case an intensive formation of defects in the material structure that can make a significant contribution to the formation of long-living electronic states. They may be important for our simulation if an energy relaxation and release through such states could provide a significant contribution to the overall balance of released energy. However, this subject has not been well studied yet. Therefore, it seems highly probable that \( t_{e-i} \) as well as \( t_e \) in (6), may have a value in a rather wide range \( 10^{-15}–3 \times 10^{-8} \) s depending on the material, a level of laser pumping of the filament and the mentioned contribution of long-living states to the energy balance.

Similarly to Equation (4), the time for the formation of the critical impact zone with radius \( R \) around the filament due to thermal conduction is estimated as \( t_{rz} \approx R^2/(4\chi) \). In the LFC conditions, the estimate \( R = s_l/2 \) is valid for \( R \), which gives for \( t_{rz} \):

\[
t_{rz} \approx s_l^2/(16\chi).
\]

For example, for the step \( s_l = 2.5–5\ \mu\text{m} \) in the fused silica glass when \( \chi = 5 \times 10^{-3} \) cm\(^2\)/s, one get \( t_{rz} = 750–3100\ \text{ns} \), and for sapphire, when \( \chi = (1.5–2.4) \times 10^{-2} \) cm\(^2\)/s, for the same step...
range one has $t_{12} = 150–1000$ ns. It is seen that various situations $t_{1} \ll t_{12}$, $t_{1} \approx t_{12}$, or even $t_{1} > t_{12}$ are possible depending on the value of $t_{1}$.

Pulse repetition time (in the discussed here case of creating each filament with a single pulse) $t_{\text{rep}}$ is:

$$t_{\text{rep}} = 1/f.$$  (8)

For typical PRR $f$ in LFC from tens of kHz to several MHz, $t_{\text{rep}}$ is numerically of the order of $0.3–50$ μs = 300–50,000 ns.

Combining the given numerical estimates, we find that for these nine characteristic times of the LFC process at the studied here submicron radii $r_{f} = 0.25–0.5$ μm, for some values of $t_{1}$ a chain of time constants may have in particular the following form:

$$\tau \sim t_{\text{eq}} \ll t_{1n}, \quad t_{1n} \ll t_{T_{1}} \ll t_{T_{2}}, \quad \text{or} \quad t_{T_{2}} \ll t_{\text{rep}}, \quad \text{or} \approx t_{\text{rep}}.$$  (9)

This increasing sequence of time constants shows that the characteristic durations of the processes involved in the LFC sometimes differ very significantly, by orders of magnitude. It makes possible to significantly simplify multi-scale modelling LFC by considering the constituent processes of radiation absorption, energy release, and impact zone formation sequentially and separately, rather than acting altogether at the same time, which would be a too complicated mathematical task both for analytical and numerical computer modelling.

On the other hand, increasing the filament radius several times (from the mentioned $r_{f} = 0.25–0.5$ μm to, for example, $r_{f} = 1.5–2$ μm, which are not considered here), or the use of the material and laser pumping level with another value $t_{E}$ (much greater or much smaller) can change the sequence of characteristic times compared to (9).

As results of modelling depend on the sequence of time constants, we consider below for simplification the situations when:

$$t_{E} < t_{T_{1}}, t_{T_{2}}.$$  (10)

and also when:

$$t_{E} > t_{T_{2}}, t_{T_{1}}.$$  (11)

### 3.2. Strategy for Obtaining Better Smoothness of Cut Walls

A proposed here modification strategy for obtaining a better smoothness of cut sidewalls in LFC consists in the execution of several necessary conditions for this purpose. In turn, these conditions are realised with a certain choice (a certain matching with each other) of the irradiation parameters—the laser pulse energy in the material $E_{1}$, the pulse duration $\tau$, the step of the filaments in the material $s_{1}$ and the PRR $f$, and also of material parameters. Note that the results for $E_{1}$ and $\tau$ in this subsection have been obtained earlier in our paper [39]. They are briefly recalled here, as we will need them in the following sections. But results for two other parameters, $s_{1}$ and $f$, are reported for the first time.

#### 3.2.1. Laser Pulse Energy

To make the LFC process efficient and to impart to it a driving force, it is necessary to eliminate significant losses of filament energy to thermal radiation, which can spread far beyond the limits of the impact zone necessary for separation and thus do not contribute to its formation and propagation around the filament.

To implement this factor, the VED $w$ of the filament should be such that [39]:

$$w \leq w_{1} \text{ or } w \geq w_{2}.$$  (12)

In obtaining this result, we assume that the condition (10) is fulfilled. It means in this case that the filament temperature reaches its maximum vs time at about $t = t_{E}$, when laser excitation of electrons, both in the free-electron plasma and in the electron-hole plasma,
relaxes to a state equilibrated with temperature. Also, as according to (10) \( t_f \) is smaller of the heat confinement time \( t_r \) in the filament, a practically all released heat appears inside the initial filament radius \( r_f \). Numerically, as follows from the given above estimates of \( t_r \) for the submicron radius of the filament \( r_f = 0.25–0.5 \mu m \), this corresponds, for example, to \( t_f < 30–125 \) ns for silica glass and \( t_f < 6–40 \) ns for a sapphire.

The VED bounds \( w_1 \) and \( w_2 \) in (12) correspond to some limits \( T_1 \) and \( T_2 \) for filament plasma temperature, \( T \leq T_1 \) or \( T \geq T_2 \), and:

\[
\begin{align*}
w_1 &= (1/K_{tw})\kappa P(T_1 - T) + w_{dis}, \\
w_2 &= (1/K_{sw})\kappa P(T_2 - T) + w_{dis}. \tag{13}
\end{align*}
\]

Here \( C = C_{sfr} \), and \( C_{sfr} \) is a high-temperature (for \( T > 2500 \) K) specific heat. It is evaluated as \( C_{sfr} = 3RN_m/\mu \) [50,51], where \( R = 8.31 \) J/(mol·K) is the gas constant, \( N_m \) is the number of atoms in the molecule, and \( \mu \) [g/mol] is a molar mass. For example, \( \mu = 60 \) g/mol and \( N_m = 3 \) correspond to both \( \text{SiO}_2 \) and \( \text{SiO}_2 \)-based glasses, and then one gets \( C_{sfr} = 1.25 \) J/(g·K). For sapphire (\( \text{Al}_2\text{O}_3 \)) with \( \mu = 102 \) g/mol and \( N_m = 5 \) one obtains \( C_{sfr} = 1.22 \) J/(g·K), \( \rho \) is material density. \( w_{dis} \) is specific energy for material dissociation. \( T_i \) is initial (room) temperature. The dimensionless coefficient \( K_{sw} \) describes a part of the filament energy \((\pi r_f^2 H)(w - w_{dis})\), which actually goes for material modification through the heat-wave channel (TW is the abbreviation for “thermal wave”). The other part of this energy, \( K_{sw} \), goes for the material modification through a shock-wave channel (SW is the abbreviation for “shock wave”). Based on the law of energy conservation we put:

\[
K_{sw} + K_{sw} = 1. \tag{14}
\]

As \( T_1, T_2 \gg T_r \) they are calculated as:

\[
T_1 = [16(ln2)^{1/2}\beta_{max}\kappa P/\alpha(\sigma r_f)]^{1/3}, \quad T_2 = 2Db/\lambda_{sw}. \tag{15}
\]

The lower temperature limit \( T_1 \), corresponding to “not high” plasma temperatures \( T \leq T_1 \), is defined from the limiting allowed value of the ratio \( \beta \) of contributions of energy flux by thermal radiation (considered as “useless” losses) to the energy flux by heat conduction (considered as “useful” losses for formation and propagation of critical for separation modification zone). These fluxes are generated from the filament plasma during its cooling down. \( \beta_{max} \) denotes this limiting allowed value of the ratio \( \beta \). \( \chi \) is material thermal diffusivity.

On the other hand, \( T_2 \) is a “high enough” temperature, at which and above which the main part of filament energy, although emitted as thermal radiation, but is not lost from the affected zone, as at so high temperature this wide-spectrum thermal radiation appears for more than 90% of its energy to be inside the spectrum of “strong” absorption (\( \alpha > 10^5 \) \( \text{cm}^{-1} \)) of the surrounding cold material. Thus the dominant part of filament energy is effectively absorbed in submicron vicinity of the filament, without escaping from the micron-size affected zone. \( \alpha = 5.67 \times 10^{12} \text{W/(cm}^2 \times \text{K}) \) is Stefan constant. \( D \) is a dimensionless parameter, \( D = 2.5 + 3, b = 2900 \mu m \cdot \text{K} \) is a so-called Wien’s displacement constant. \( \lambda_{sw} \) is a high-absorption spectrum boundary in the short-wavelength range, defined by a condition that for \( \lambda \leq \lambda_{sw} \) the material absorption coefficient \( \alpha(\lambda) \geq 10^5 \text{ cm}^{-1} \). Calculated from (15) values of \( T_1 \) and \( T_2 \) for \( \beta_{max} \) conditionally taken for estimations as \( \beta_{max} = 0.12 \), are equal to several tens of thousands of degrees. For estimations we put that \( K_{sw} = K_{sw} = 0.5 \). VED bounds \( w_1 \) and \( w_2 \) calculated from (13) for such \( K_{sw} \) and with the use of data on \( w_{dis} \) from [52] are given in Table 1. As seen, \( w_1 \) and \( w_2 \) are equal to few hundred kJ/cm\(^3\) depending on the material.

| Table 1. VED bounds \( w_1 \) and \( w_2 \) calculated from (13,15) for \( \beta_{max} = 0.12 \) and several materials at different filament radii \( r_f \) for \( K_{sw} = 0.5 \). |
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| Material        | \( w_1 (\text{kJ/cm}^3) \) | \( w_1 (\text{kJ/cm}^3) \) | \( w_1 (\text{kJ/cm}^3) \) | \( w_1 (\text{kJ/cm}^3) \) | \( w_1 (\text{kJ/cm}^3) \) |
|                 | \( r_f = 0.25 \mu m \) | \( r_f = 0.5 \mu m \) | \( r_f = 1.5 \mu m \) | \( r_f = 0.25 \mu m \) | \( r_f = 0.5 \mu m \) |
| Corning Gorilla Glass 5 | 400             | 336             | 248             | 396             | 480             |
| Eagle XG@Display glass | 396             | 322             | 244             | 460             | 540             |
There is also a “window of unfavourable VED” $w$, defined by an opposite to (12) inequality: $w_1 < w < w_2$. For this range of $w$, the relative part of thermal emission losses $\beta$ becomes very significant and even the dominant and can exceed the allowed limiting level $\beta_{\text{max}}$ by many times (conditionally $\beta_{\text{max}}$ is taken for estimations in Table 1 as $\beta_{\text{max}} = 0.12$). According to (15), a decrease in the filament radius $r_f = r_1/e$ in a range of practical interest from about 1.5 $\mu$m to 0.25 $\mu$m provides an increase of $T_1$. It makes the window of “unfavourable” $w$ to shrink and even to disappear for some materials, in particular, as Table 1 shows, for sapphire and Corning Gorilla Glass 5.

If the filament is formed due to the nonlinear process of self-focusing, the laser beam propagates in its immediate vicinity, as is known [4,21], in a “non-Gaussian” way—mainly along curvilinear trajectories in a so-called “energy reservoir” surrounding the filament. In other words, an effective waveguide results from the refractive index gradient induced by the laser heating and symmetrically distributed across the filament axis. The laser energy drains from such a waveguide into the filament laterally, from different sides, due to multiple reflections and absorptions of the filament for lateral rays coming from the waveguide. In this way, a quite long and homogeneous for most of its length filament is created. The waveguide exists in that region along the length where the filament itself exists—a region of space, both in the filament itself and around it—where concentrated heating of the material and related phenomena such as temperature gradient are most significant. In this lateral energy supply only a part ($A_{L-P}$) of laser pulse energy inside the material $E_1$ transforms into the plasma energy $E_p$, that is:

$$E_p = A_{L-P}E_1. \tag{16}$$

The remaining part, $(1 - A_{L-P})$, of $E_1$ traverses the material through such waveguide without being converted to the filament plasma energy. The lateral energy supply to the filament also takes place when it is formed by Bessel beam, i.e., by laser focusing with a conical lens—axicon [11].

The efficient conversion of the incident laser pulse energy inside the material $E_1$ into the plasma energy of the filament $E_p$ corresponds to the maximisation of the filament absorptivity $A_{L-P}$ in (16). As mentioned in Section 2 above, the absorptivity $A_{L-P}$ is taken for the model based on the proposed semi-empirical approach, i.e., from the available experimental data, where due to its non-linear character $A_{L-P}$ demonstrates a dependence on the VED $w$ (see, e.g., [46]). That is if filaments follow each other having spacing several times (about 3 to 6) greater than their diameter when usually high $w$ are required for LFC (see below), $A_{L-P}$ is taken close to a maximal saturated experimental value about 0.80 ± 0.05 [46]. But in other cases, when a pulse produces about an order of magnitude lower level of VED, a smaller value, $A_{L-P} = 0.3$–0.4 [46] is taken as a better value for the model. This happens if LFC uses filaments that overlap partially, or for a single pulse from a burst of several very close in time pulses when the filament is produced with the burst of pulses.

From equations (1) and (16) a simple relationship follows for the laser pulse energy inside the material as a function of $w$, as well as the length $H$ and the radius $r_f$ of the filament:

$$E_1 = \pi r_f^2 H w / A_{L-P}, \tag{17}$$

where $w$ satisfies to the condition (12).

On the other hand, if $r_f$, $H$ and $E_1$ are known from experiment, the VED can be estimated from (17) as:

$$w = A_{L-P}E_1 / (\pi r_f^2 H). \tag{18}$$
For the correct comparison of model estimations with experimental results, it is worth noticing that Gaussian radius at the (1/e²)-level for VED profile, \( r/(1/e) \), relates to the \( r/(1/e) \) radius of our model as \( r/(1/e) = (1/\sqrt{2})r/(1/e) \).

We also assume that for self-focusing \( r/(1/e) \) can be given in terms of the waist radius \( r_0 \) for the classical focusing of the Gaussian beam in a transparent material, i.e., \( r_0 = qr_0 \). Here a coefficient \( q \) can be attributed to a plasma defocusing and beam propagation effects and in general, may depend both on the VED and pulse duration, \( q = q(w, \tau) \). But still within indicated below a range of interest of \( w \) one can put \( q = 1 \).

As for the parameter \( r_0 (1/e) \), it is given by the ratio \( r_0 = (\lambda M^2)/(\sqrt{2} \pi \tau \lambda) \) [32]. Thus, one can put for the filament formed by self-focusing:

\[
\frac{r_0}{r} = q(w, \tau) \frac{\lambda M^2}{\sqrt{2} \pi \tau \lambda}.
\]

An important consequence of this ratio for further modelling is that, as \( q = 1 \), the filament radius \( r_\ell \) like \( r_0 \), decreases with shortening the laser wavelength \( \lambda \) and with an increase of the numerical aperture of beam focusing NA.

To confirm that \( q \) is very close to 1, one can estimate the diffusion length of electrons \( L_e \approx 2(D_e \tau)^{1/2} \) excited by an intense laser for the pulse duration \( \tau \). Here \( D_e \) is the electron diffusion coefficient, which for crystalline materials has a characteristic value of the order of 1 cm²/s, and for amorphous materials—less than this value by about an order of magnitude or more. For a considered below characteristic value for LFC \( \tau = 0.2–0.4 \) ps and \( D_e = 1 \) cm²/s, one get \( L_e = (0.9–1.3)10^{-2} \mu m \). Then the inequality \( L_e \ll r_\ell \) is fulfilled even for smallest filament radii considered in this paper, around \( r_\ell = 0.24 \mu m \) (which is given by the above formula for \( \lambda = 0.52 \mu m \)) and for the largest value \( D_e = 1 \) cm²/s. This inequality corresponds to a negligible effect of a diffusive spread of electrons from the focus zone radius \( r_0 \) for the pulse duration, and hence \( r_\ell \approx r_0 \). It means that \( q \approx 1 \).

When the filament is made by an extended in focal length Bessel beam using axicon (see Figure 3), the focusing radius \( r_\ell \) for the filament is expressed by the ratio functionally similar to the given above Gaussian beam parameter \( r_0 (1/e) \): \( r_\ell = 1.20\lambda/(\sqrt{2} \pi \sin \delta_0) \) [11], where \( \delta_0 = (n - 1)(\pi - \gamma)/2 \), and \( \gamma \) is an angle at the axicon apex. As seen in this paper, \( r_\ell \) is defined by the wavelength and the apex angle \( \gamma \). But the length of the focus area (i.e., of the filament) is regulated, as seen from Figure 3, by another parameter—the width of the laser beam incident on the axicon. This means that \( r_\ell \) and \( H \) for Bessel beam focusing can vary independently. It is assumed that a similar to (19) ratio \( r_\ell = q(r_\ell \tau) r_0 \) is fulfilled here also, where \( q(r_\ell \tau) \) is a defocusing coefficient analogous to the introduced above parameter \( q \). In a similar way, one can show that \( q(r_\ell \tau) = 1 \). Following to the given above \( r_\ell (\lambda) \) dependence, the filament radius \( r_\ell \) is then also expected to decrease with \( \lambda \) shortening as \( r_\ell \approx \lambda \), similar to the considered above case of self-focusing.

The relationship (17) leads to a simple dependence: \( E_1 \propto r_\ell^2, H \). As we have seen, \( r_\ell \propto r_0 \). Due to the mentioned dependence \( r_\ell^2 \propto \lambda^2 \), one gets then:

\[
E_1 \propto \lambda^2, H.
\]

Therefore, if the laser wavelength \( \lambda \) is taken shorter, for example, twice, as \( \lambda = 0.52 \mu m \) instead of 1.04 \mu m, the pulse energy \( E_1 \) can decrease \( \propto \lambda^2 \) by 4 times only due to the reduction of \( r_\ell \) but can also decrease several times more, if the filament length \( H \) is made shorter.

### 3.2.2. Laser Pulse Duration

An important necessary condition for obtaining after irradiation a separation with a better smoothness of cut side walls is to ensure an increased uniformity of the diameter of each formed filament along its length. As our analysis of experiments [13] shows, it is necessary for this that the power of a single laser pulse \( P_1 = E_1/\tau \) (under the made above model assumption of a rectangular temporal shape of the pulse) is close in order of mag-
mitude to the critical power \( P_c \) for self-focusing, i.e., \( E_1/\tau = \Gamma P_c \), where the empirical parameter \( \Gamma \) is chosen in the range 1 to 2. The given ratio is equivalent to a matching the duration and the energy of laser pulse with one another in such a way that \( \tau = E_1/(\Gamma P_c) \). When substituting here \( E_1 \) from (17), one finds for pulse duration [39]:

\[
\tau = \frac{\pi r_f^2 Hw}{(A_{L-\rho} \Gamma P_c)}, (21)
\]

where, as in (17), the VED \( w \) satisfies the condition (12).

For comparison, in the opposite case, when \( \Gamma \) is large (e.g., about 10 or more), a known in the literature multifilament formation occurs, i.e., a filament branching into smaller and more curved filaments, but such a situation would be completely undesirable from the point of view of obtaining a better smoothness of cut walls. Therefore, so high \( \Gamma \) is not used for a choice of \( \tau \) by (21).

As follows from the obtained relations (17) and (21), that the required for LFC pulse duration \( \tau \) and energy \( E_1 \) in the material demonstrate a size effect – they increase with the length \( H \) and radius \( r_f \) of the created filament, as well as with VED \( w \). These relations can be applied to quantify the pulse energy and duration for a great variety of filaments sizes: for radii \( r_f \) in the range of practical interest 0.2 \( \mu \)m to 2 \( \mu \)m or more and lengths \( H \)—about 10 \( \mu \)m to 10–20 mm, which corresponds to the separation of both very thin and very thick materials. A practical range of \( w \) for LFC, as we shall see below, can be from about 1 kJ/cm\(^3\) to 300–600 kJ/cm\(^3\) and more.

### 3.2.3. Step of Filaments Following

Another necessary condition for obtaining a separation with better smoothness is obviously a formation of continuous long modification zone extended along a path of laser beam motion relative to the material. This requires a suitable step \( s_1 \) of filaments following in the material. Indeed, let \( R \) be the radius of the cylindrical impact zone around each filament, where the material modification reaches a certain critical level for obtaining a smooth separation at the subsequent splitting stage. Such a zone occurs each time after termination of each next laser pulse when the plasma energy of the aroused filament is relaxed in the form of heat and/or shock wave radially propagating in the material from the filament axis (see Figure 4). Then, based on the value \( R \), the step of the filaments in the material \( s_1 \) favourable for creating the mentioned continuous modification zone along the path of the beam should be limited from above according to the ratio:

\[
s_1 \leq s_{1\text{max}} = 2R. (22)
\]

Here \( s_{1\text{max}} \) is the maximum allowable step for a given \( R \), which ensures that mentioned critical modification zones from two neighbour filaments in the material touch each other, and a continuous path of modified material along the laser beam track begins to appear (see Figure 4c). At a smaller step \( s_1 < s_{1\text{max}} \) (that is, as follows from (22), at \( s_1/R < 2 \)) these zones overlap with each other (see Figure 4d). As a result, a continuous long modification zone is formed along the path of the laser beam movement relative to the material, which is necessary for obtaining a smooth separation. However, if the smoothness of the separation is not important, then \( s_1 \) can be chosen even so that the condition (22) is not met, i.e., \( s_1 > 2R \) (as in Figure 4b).
Figure 4. Schematic of the formation of impact zones in the material around filaments created sequentially in time, when the pulsed-periodic laser beam moves relative to the material. The possibilities of merging or not merging these zones are shown. (a) $\pi$ is the virtual cross-section plane of a set of filaments in the material thickness. $u$ is a direction of the laser beam movement relative to the material; (b–d) show cross-sections of a set of filaments in the plane $\pi$ at different steps of the filaments in the material $s_i$ with respect to the radius $R$ of the cylindrical impact zone with a critical for obtaining a smooth separation modification of the material around each filament; (b) $s_i > 2R$—there is no joining of impact zones; (c) $s_i = 2R$—the impact zones are adjacent to each other, and a continuous path of modified material along the laser beam track begins to appear; (d) $s_i < 2R$ (shown $s_i = 1.8R$)—the impact zones partially overlap with each other, forming a continuous path of modified material along the laser beam track.

To calculate $R$ and according to (22) also $s_{\text{max}}$, the key parameters are the already mentioned $w$ and a minimal VED $w_{\text{mod}}$ (in $[J/cm^3]$) for obtaining a smooth separation within mentioned critical zones. The $w_{\text{mod}}$ parameter, as we will see below, is from one to several $kJ/cm^3$ depending on the material. Under the condition (2) such value is almost two orders of magnitude less than the characteristic values $w_1$, $w_2$ and $w$. We consider three possible mechanisms of material modification, which can be conventionally designated as a thermal mechanism, a thermally induced deformation mechanism, and a shock-wave damage mechanism. Depending on the implementation of one or another mechanism, one or another parameter is critical for obtaining a separation: an achievement in the critical modification zone of a temperature not below a certain threshold, or an achievement there of a critical thermally-induced deformation, also not below some threshold, or an achievement in this area of a mechanical stress not below a threshold strength and fracture of material under the action of the shock wave passage. These material modification thresholds, sufficient for further material separation, determine the $w_{\text{mod}}$ value in each case.

In our geometry, the radial energy transfer in the material from the filament axis at the plasma relaxation stage is common for all three of these mechanisms. Therefore, the dependence of $R$ on $w$ and $w_{\text{mod}}$ for an instantaneous (short) time of energy release in the filament is functionally universal: $w_{\text{mod}}(\pi R^2 H) \propto KA_{\text{mod}}(w - w_{\text{min}})(\pi r f^2 H)$, where $\pi R^2 H$ is the volume of the modification zone, $\pi r f^2 H$ is the volume of the filament, and $KA_{\text{mod}}(w - w_{\text{min}})(\pi r f^2 H)$ is the energy spent to modify the surrounding material. This ratio gives for the modification radius $R$: 
Here \((w - w_{\text{dis}})\) is the filament VED, \(w\), used to modify the surrounding material, with the subtraction of irreversible losses spent for material dissociation, \(w_{\text{dis}}\), which are not replenished when energy is released from the filament plasma in the surrounding material. A dimensionless conversion coefficient \(A_{P-H}\) is an analogue of the surrounding material absorptivity, which describes the fraction of the filament plasma energy that actually passed into the energy for modification of material surrounding this filament for the relaxation time \(t_E\) of the excited energy levels of the material. We put here that \(A_{P-H} \approx 0.9\) taking into account that under the described above condition (12) of optimal choice of \(w\) only an insignificant fraction of energy (about \(1 - A_{P-H} \approx 0.1\)) can be lost to the plasma emission in the mentioned above transparency region of the material surrounding the filament. This thermal emission leaves the modification zone and propagates for long distances (even can leave the material at all), thus not participating in the formation and expansion of this zone. The dimensionless coefficient \(K (< 1)\) defines a fraction of the indicated above energy \(A_{P-H}(w - w_{\text{dis}})(\pi r_f^2 H)\), going, as was already mentioned above, to the material modification on one or another channel: on a channel of modification by the thermal wave (in this case \(K = K_{\text{tw}}\)), or on a channel of modification due to the shock wave (in this case \(K = K_{\text{sw}}\)). \(\gamma (< 1)\) is a dimensionless numerical coefficient that also may depend on the mechanism of energy transfer—by a heatwave due to thermal conductivity, or by a shock wave, that is \(\gamma = \gamma_{\text{tw}}\) or \(\gamma = \gamma_{\text{sw}}\). We show below the applicability of the Formula (23) for each of these mechanisms.

### 3.2.4. Threshold Modification Temperature for the Thermal Mechanism

In the case of a thermal mechanism, the zone of modification critical for separation is defined as a region of space of a radius denoted further as \(R_T\) around the filament, in which (i.e., at \(r \leq R_T\)) a temperature is reached not lower a certain threshold, which is hereinafter referred to as \(T_{\text{mod}}\).

For amorphous transparent materials (for example, glasses), the temperature of intense volumetric expansion \(T_{\text{exp}}\) can be chosen as \(T_{\text{mod}}\), which, in turn, as follows from the data [53,54], is located near a so-called glass transition temperature (strain point) \(T_{\text{strain}}\), i.e.,

\[
T_{\text{mod}} \approx T_{\text{exp}} \approx T_{\text{strain}}.
\]  

(24)

For example, \(T_{\text{strain}} = 571 \, ^{\circ}\text{C}\) for chemically-tempered glass [55]. Such a definition of \(T_{\text{mod}}\) in the indicated zone inside the radius \(R_T\) is justified by the fact that in this case the maximum vs time temperatures reached at each point inside this zone (i.e., at \(r \leq R_T\)) exceed \(T_{\text{exp}}\), i.e., \(T \geq T_{\text{exp}}\). This means that in this zone there is a known and characteristic for such temperatures, namely a sharp acceleration of the temperature expansion of the heated volume due to a sharp increase (almost by an order of magnitude) of the volume expansion coefficient when the point \(T = T_{\text{strain}}\) is exceeded. As a result, a significant compressive pressure develops in the indicated zone and an immediate vicinity outside it, while simultaneously initiating mobility, softening and deformability of micrograins of the material, which altogether leads to their compression, sintering and greater material homogenisation, and also a formation of higher density phases. Therefore, upon cooling down after the pulse indicated compressive stresses in this area may be replaced with tensile ones. The merger of these impacts from many filaments, established when driving the laser beam, into a single continuous extended modified layer in the bulk material, extending on the trail of the beam movement on the material (with a proper choice of the step of the filaments in the material \(s_i\), leads to a selected dominant common orientation of such tensile stress—perpendicular to the direction of movement of the beam in the material front plane. At the following stage of mechanical splitting after irradiation, all these
factors together appear favourable for obtaining a plate separation that does not randomly change direction (which would lead to the appearance of rough sidewalls of separation), but has the same distinct direction over a long distance, and has a more homogenised (i.e., non-grained) side walls profile, having as a result a better smoothness due to the softening, deformability and mobility of the material grains in the $T > T_{\text{strain}}$ affected zone.

Crystalline transparent materials, unlike amorphous ones, do not have a glass transition point, but, as a rule, are characterised by a relatively high and well-defined melting point $T_{\text{melt}}$ (as shown in Table 2 for sapphire). The crossing the point $T_{\text{melt}}$ at the heating stage is often also characterized by a sharp increase in the material volume, as with the transition through the point $T_{\text{strain}}$ in the case of amorphous materials, and with an analogous development of the compressive stress. This compressive stress is replaced by a tensile one at the material cooling down stage after the pulse. Therefore, for crystalline materials, the threshold temperature for obtaining material separation $T_{\text{mod}}$ as a temperature of intense volumetric expansion $T_{\text{exp}}$ is taken equal to the melting point $T_{\text{melt}}$:

$$T_{\text{mod}} = T_{\text{exp}} = T_{\text{melt}}.$$  

Table 2. Parameters of amorphous and crystalline materials [55–57] used in the modelling.

| Parameter                        | Fused Silica | Corning EAGLE XG™ | Corning Gorilla Glass 5 | Sapphire |
|----------------------------------|--------------|-------------------|-------------------------|-----------|
| Mean specific heat $C_2$, J/(g·K) | 1.09         | 1.07              | 1.0                     | 1.2       |
| Mean density $\rho_2$, g/cm³     | 2.2          | 2.38              | 2.43                    | 3.97      |
| Strain point $T_{\text{strain}}$, °C | 670–900     | 669               | 571                     | -         |
| Melting point $T_{\text{melt}}$, °C | -            | -                 | -                       | 2050      |
| Thermal expansion coefficient $\alpha_T$, $10^{-6}$ K⁻¹ | 1.3          | 3.55              | 7.9                     | 5.5       |
| Young modulus $Y$, GPa            | 73.6         | 73.6              | 76.7                    | 345–470   |
| Compression strength $\sigma_c$, GPa | 0.59         | 0.6               | 1.8 ÷ 2.4              | 2.4–3.0   |

3.2.5. The Threshold Modification Temperature for the Mechanism of Thermally-Induced Deformation

In the mechanism of thermally-induced strain, the temperature not below some critical point, $T_c$, can be considered as a threshold for the formation of the critical for separation modification zone with a radius denoted, respectively, as $R_{TD}$ (TD stands for “thermal deformation”). A property of this value $T_c$ is that when a region of material is heated locally inside the sample volume to this point and is accompanied with a thermal expansion $(-\Delta\rho/\rho)$, there is a mechanical stress increase in the material up to the compressive strength $\sigma_c$, followed by irreversible destruction of the material structure and, as a consequence, a mechanical weakening of the material for its subsequent cleaving. It means that in this case, one can put for critical modification point $T_{\text{mod}}$ that $T_{\text{mod}} = T_c$. For estimates, we can assume that the critical expansion value $(-\Delta\rho/\rho)$ and $\sigma_c$ are related by the relation known from the elasticity law: $\sigma_c = Y(-\Delta\rho/\rho)_c$, where $Y$ is Young’s modulus. Substituting here the density change in the form $(-\Delta\rho/\rho) = \alpha_T(T_c - T_i)$, where $\alpha_T$ is the coefficient of thermal expansion of the material, we get an estimate for $T_c$:
\[ T_{\text{mod}} \approx T_e \approx T_i + \frac{\sigma_e}{\alpha T} . \]  

(26)

3.2.6. The Radius of the Modification Zone under the Action of Heatwave

The estimates of \( R_T \) and \( R_{TD} \) for the separately considered mentioned above thermal and thermally induced deformation mechanisms are based on the radial propagation of heat due to its diffusion in the material from the filament axis at the cooling stage of the material after the termination of the energy release from the absorbed laser pulse. We assume that the heat release in the material from a single filament occurs for a finite time \( t_e \) and that the power of the pulse of the heat release \( q(t) \) for the time \( 0 \leq t \leq t_e \) is constant, \( q(t) = \text{const} = q_0 \) [W]. The filament radius \( r_f \) is considered as negligibly small compared to \( R \), i.e.:

\[ r_f \ll R. \]  

(27)

We also consider times \( t \) that exceed the introduced above in (4) time \( t_T \) for heat confinement inside the filament radius:

\[ t >> t_T = r_f^2/(4 \chi), \]  

(28)

at which the propagation of heat beyond the filament radius \( r_f \) becomes really significant due to the above definition of the parameter \( t_T \), as the time of heat confinement inside the filament radius.

The general expression for the temperature field in a material under these conditions, as is known from the heat conduction theory, at the heating stage has the form:

\[ T(r, t) - T_{i} = \frac{q_0}{4\pi C \rho \chi H} B \left( \frac{r^2}{4\chi t} \right) \text{ when } 0 \leq t \leq t_e \]  

(29)

and at the cooling stage:

\[ T(r, t) - T_{i} = \frac{q_0}{4\pi C \rho \chi H} \left[ B \left( \frac{r^2}{4\chi t} \right) - B \left( \frac{r^2}{4\chi(t-t_e)} \right) \right] \text{ when } t > t_e. \]  

(30)

In these expressions, \( T_i (= 20 \degree C) \) is the initial (room) temperature, \( r \) is the radius of a position of the point in the material from the filament axis. We consider \( r >> r_f \), \( t \) is the time, \( C, \rho \) and \( \chi \) are, respectively, the specific heat capacity, density and thermal diffusivity of the material, \( H \) is the length of the filament. Function \( B \) is defined as:

\[ B(z) = \int_z^{\infty} e^{-\xi} d\xi = -E_{1}(-z), \]  

(31)

where \( E_{1}(-z) \) is a special function, the so-called integral exponential function, given as

\[ E_{1}(-z) = -\int_z^{\infty} e^{-\xi} \frac{d\xi}{\xi}. \]  

Its numerical values are known only in a tabulated form [58].

In the limiting case of a short energy release time \( t_e \), the difference in square brackets of (30) can be expressed in terms of derivatives: \( T(r, t) - T_{i} = \frac{q_0}{4\pi C \rho \chi H} \left( \frac{\partial B}{\partial t} \right) \). This, as it is easy to show, gives a simpler expression for the temperature at the cooling stage of interest:

\[ T(r, t) - T_{i} = \frac{Q}{\pi H(4\chi t) C \rho \chi} \text{exp} \left( -\frac{r^2}{4\chi t} \right). \]  

(32)

Here \( Q = q_0 t_e \) is an amount of heat released in the filament [J]. The obtained solution (32) for temperature has the same form as the well-known from textbooks so-called fundamental solution of the heat conduction equation, i.e., the solution for the temperature field created by a heat release source in a material in the form of an infinitely thin thread with an instantaneous heat release. The small duration of heat release \( t_e \), as the analysis shows, is determined by the fulfilment of the condition:
\[ t_2 \ll t_2 = \frac{R^2}{(4\chi)}, \]  

(33)

where \( t_2 \) is the introduced above in (7) time for the formation of the critical impact zone with radius \( R \) \((R_T \text{ or } R_{TD})\) around the filament due to the heat conduction.

Note that when using the above assumption about the Gaussian distribution of \( w \) in the filament, a slightly different description of the temperature is also possible, which, in contrast to (32), is more physically correct, since it is formally limited at \( t \to 0 \) [30]:

\[
T(r, t) - T_i = \frac{q}{4\pi \rho c \chi} \exp\left(-\frac{r^2}{\xi^2(t)}\right), \quad \xi(t) = (r_f^2 + 4\chi t)^{1/2}.
\]

Here \( r_f(t) \) is a Gaussian \((1/e)\)-radius of the temperature profile around filament after pulse termination. As seen, \( \xi(t) \) increases with time due to radial heat conduction. However, for the times defined by the condition (28) \( t \gg t_1 = r_f^2/(4\chi) \) that are of interest to us according to the condition (2), we can neglect the term \( r_f^2 \) in comparison with \( 4\chi t \) \((r_f^2 \ll 4\chi t)\) in the expression for \( r_f(t) \). Then this solution for \( T \) practically coincides with the more convenient for performing calculations and simpler solution (32), which is therefore used further for the necessary calculations.

Under conditions (27) and (33), we first consider a relatively simple case where the pulse repetition time \( t_{\text{rep}} \) is much greater than the cooling time of the shown in Figure 5a point \( r = R \). This means that the next pulse coming in the time \( t_{\text{rep}} \) and creating the next filament \((Y_2 \text{ in Figure 5a})\), from which the specified point \( r = R \) is at the same distance \( R \) (since when condition (22) is fulfilled and \( s = s_{\text{max}} = 2R \) this point is in the middle between the filaments \( Y_1 \text{ and } Y_2 \)), induces heating in the point \( r = R \) that has already completely cooled down from the first pulse. Reaching the threshold value \( T = T_{\text{mod}} \) is then determined by the action of the heat release from only one filament. In the opposite case, when the pulse repetition time \( t_{\text{rep}} \) is sufficiently short compared the time for cooling down from the first pulse, the heating at the point \( r = R \) is formed by heat release from two adjacent filaments, as shown in Figure 5b, which will also be discussed below.

![Figure 5. Schematic of heating of the space between two neighbour filaments due to the propagation of energy in the material by the thermal diffusion. The condition \( r_f \ll R \) is met for the filament radius \( r_f \). (a) The point \( r = R = 0.5s_1 \) (in the plane \( \pi \) of the cross-section of a set of filaments shown in Figure 4a) is placed in the middle between two neighbour filaments separated one from another by the step \( s_1 \) of filaments following. (b) 1 is the heating curve for the point \( r = R \) by the filament created by the pulse \( Y_1 \) coming to the point \( x = 0 \) at the time \( t = 0 \). \( \Delta T_1 \) is the maximum of this curve reached at the time \( t = t_{\text{max}}. \) 2 is the similar heating curve for the same point \( r = R \), but by the filament created by the next pulse \( Y_2 \) coming to the point \( x = s_1 \) at a distance \( s_1 \) of the step of filaments following from the previous point \( x = 0 \) and with a delay \( t_{\text{rep}} \) relative to the pulse \( Y_1 \). The same maximum \( \Delta T_1 \) of this curve is reached at \( t = t_{\text{max}} + t_{\text{rep}}. \) At this time, the heating by the first filament


decreases due to cooling down to \( \eta \Delta T_1 \), where \( \eta (<1) \) is a coefficient of heat preservation. \( 3 \) is the curve of the resulting heating of the point \( r = R \) by both these neighbour filaments. Its maximum \( = (1+\eta)\Delta T_1 \) is reached at the time \( = (t_{\text{max}}+t_E) \).

The radius of the modification zone critical for separation under these conditions is defined as the maximum radial distance of penetration into the material of the isotherm \( T = T_{\text{mod}} \) from the filament axis. Verification of compliance with conditions (27) and (33) is possible if data on the value of \( t_E \) for this material is available only after performing calculations of \( R \) and \( s_1 \).

To calculate the radius of the modification zone around the filament we take the fixed \( r = r_1 \) in the expression (32) for the temperature. Considering then \( T(r_1, t) \) as a function of time only, from the condition \( dT(r_1, t)/dt = 0 \), we can easily find the position in time \( t_{\text{max}} \) of the temperature maximum \( T = T_{\text{max}} \) at the radius \( r_1 \):

\[
t_{\text{max}} = r_1^2 / (4\chi).
\]  

(34)

At \( t > t_{\text{max}} \), the cooling of the point \( r = r_1 \) starts, that is, its temperature decreases. It follows from (34) that the relation \( r_1 = 2(\chi t_{\text{max}})^{1/2} \) is fulfilled. Therefore, reaching the maximum temperature at the radius \( r_1 \) can be interpreted as an arrival to this point from the heated filament of thermal diffusion wave, which has a length of \( 2(\chi t_{\text{max}})^{1/2} \). The substitution of \( r = r_1 \) and \( t = t_{\text{max}} \) with the use of (34) into (32) gives the maximum temperature \( T_{\text{max}}(r_1) \):

\[
T_{\text{max}}(r_1) - T_i = \frac{q}{e \pi r_1^2 H}.
\]  

(35)

It is seen from here that the heating \( [T_{\text{max}}(r_1) - T_i] \) decreases \( \propto 1/r_1^2 \) with increasing the distance \( r_1 \). For \( r_1 = R \) and \( T_{\text{max}} = T_{\text{mod}} \), we find from (35) the relation between \( R \) and \( T_{\text{mod}} \):

\[
T_{\text{mod}} - T_i = \frac{q}{e \pi R^2 H}.
\]  

(36)

From here, the modification radius is calculated, i.e., the distance of the maximum penetration of the isotherm \( T = T_{\text{mod}} \) from the filament axis into the surrounding material:

\[
R = \left[ \frac{Q}{e \pi C \rho(T_{\text{mod}} - T_i)H} \right]^{1/2}.
\]  

(37)

The time \( t_{\text{mod}} \) for formation of the critical modification zone in these new terms according to (34) is:

\[
t_{\text{mod}} = R^2 / (4\chi).
\]  

(38)

As seen now, \( t_{\text{mod}} \) coincides with the introduced above in (7) time \( t : \) for the formation of the critical impact zone with radius \( R \) around the filament due to thermal conduction.

We introduce now a new term: the energy \( Q_{\text{tw}} \) released from the relaxation of the filament energy in the material and spent to the material modification through the thermal wave channel. It can be represented in general form by the following expression:

\[
Q_{\text{tw}} = K_{\text{tw}} A_{\text{P-H}} (w - w_{\text{dis}})(\pi r_1^2 H).
\]  

(39)

Here \( \pi r_1^2 H \) is, as previously in (1), the effective volume of the filament. \( (w - w_{\text{dis}}) \) is a VED of the filament \( (w) \) with subtraction of irreversible losses spent on the material dissociation \( (w_{\text{dis}}) \), not restored during the further energy release from the plasma filament to an ambient material. The term \( (w - w_{\text{dis}})(\pi r_1^2 H) \) thus describes the filament energy, which propagating radially from the filament in the material, goes to modify the material surrounding the filament. The dimensionless conversion coefficient \( A_{\text{P-H}} (\approx 0.9) \) is an analogue of the absorptivity and describes the fraction of energy release from the filament plasma that actually converted to the heat during the relaxation time \( t_E \) (P-H is the abbreviation for “plasma-to-heat”) and is thus able to modify the material. It is taken into account that under the conditions (12) of the optimal choice of \( w \) described above, the value
of $A_{P-H}$ is large, i.e., it reaches values of about 0.9. In such case, only a small fraction of energy (about $1 - A_{P-H} \approx 0.1$) can be lost to the above-mentioned plasma thermal radiation in the transparency region of the surrounding filament material, leaving the modification zone for rather long distances and thus not taking part in this modification.

For comparison, the energy $Q_{sw}$ released from the relaxation of the filament energy in the material which goes for material modification through the shock wave channel can be similar to (39) represented as:

$$Q_{sw} = K_{sw}A_{P-H}(w - w_{dis})(\pi r_f^2 H).$$

(40)

Substitution of (39) into (37) gives the radius $R$ of the critical modification zone in the form:

$$R = \left[ \gamma_{tw} K_{tw} A_{P-H} \frac{(w - w_{dis})}{C\rho(T_{mod} - T_i)} \right]^{1/2} r_f.$$

(41)

Here $\gamma_{tw} = 1/e = 0.37$ is the value of the coefficient $\gamma$ in the considered case of heat-conduction propagation of energy in the material. Functionally, this relation really coincides with the introduced above general expression (23), if to put in it $\gamma = \gamma_{tw}$, $K = K_{tw}$, and $w_{mod}$ is taken as:

$$w_{mod} = C\rho(T_{mod} - T_i).$$

(42)

In this formula, for the specific heat capacity $C$ and the density $\rho$ of the material in the presence of their changes with temperature, their average in the range of $T_i = 20^\circ C$ to $T_{mod}$ values are taken. These are denoted further as $C_2$ and $\rho_2$ to distinguish them from the above-mentioned values $C_1$ and $\rho_1$ at the initial temperature $T_i$ and also from the introduced above in the discussion of Equation (13) values of $C = C_{HT}$ at high temperatures $> 2500$–$3000$ K.

For the mentioned thermal mechanism we put in formulas (41) and (42) for amorphous materials according to (24) $T_{mod} = T_{strain}$, $R = R_T$ and $w_{mod} = w_{T_mod}^T$. Then one get from (41) for $R_T$:

$$R_T = \left[ \gamma_{tw} K_{tw} A_{P-H} \frac{(w - w_{dis})}{C\rho(T_{mod} - T_i)} \right]^{1/2} r_f,$$

where $w_{T_mod}^T = C_2\rho_2(T_{strain} - T_i)$.

(43)

Similarly, for crystalline materials according to (25), we put $T_{mod} = T_{melt}$, which gives:

$$R_T = \left[ \gamma_{tw} K_{tw} A_{P-H} \frac{(w - w_{dis})}{C\rho(T_{mod} - T_i)} \right]^{1/2} r_f,$$

where $w_{mod}^T = C_2\rho_2(T_{melt} - T_i)$.

(44)

On the other hand, for the mechanism of the thermally induced deformation (TD), respectively, it is necessary to put in (41) according to (26) $T_{mod} = T_c$ and also $R = R_{TD}$, which gives:

$$R_{TD} = \left[ \gamma_{tw} K_{tw} A_{P-H} \frac{(w - w_{dis})}{C\rho(T_{mod} - T_i)} \right]^{1/2} r_f,$$

where $w_{mod}^{TD} = C_2\rho_2(T_c - T_i) = \frac{C_2\rho_2}{\sigma_T} \sigma_c$.

(45)

3.2.7. Radius of Shock-Wave Modification

In the mechanism of the modification zone formation by a shock wave, the plasma of each filament creates a so-called thermobaric effect: due to the development of the indicated above a high temperature of several tens of thousands of degrees, a high sub-megabar (or even megabar) pressure is achieved in the filament. Therefore, each such filament acts as a source of a cylindrical micro-explosion inside the material. The radial propagation of the shock wave changes the structure of the surrounding material, inducing its destruction. A certain modification zone is formed around the filament in the form of a
cylinder with a radius $R_{sw}$, about the same length $H$, as that one of the filaments itself, and with a corresponding volume $V = \pi R_{sw}^2 H$. At the same time, we put also that in whole this modified zone, a threshold of mechanical stresses is reached not lower than a mechanical strength $\sigma_c$. Thus, there is a destruction of the material and its corresponding mechanical weakening, sufficient to provide for a mechanical cleaving in the material after irradiation along the path of the laser beam movement. Let $\gamma_{sw} Q_{sw}$ be the energy expended on the destruction of the material in the mentioned volume $V$. Then, from the known approximate relation $\sigma_c V = \gamma_{sw} Q_{sw}$ [27] and using the relation (40) for $Q_{sw}$, the radius of critical modification by shock wave $R_{SW}$ can be found as:

$$R_{SW} = \left(\frac{\gamma_{SW} K_{SW} (w - w_{dis})}{w_{mod}}\right)^{1/2} r_f,$$

where $w_{mod} = \sigma_c$. (46)

This expression is functionally the same as the general expression (23) introduced above, if we put in (23) $\gamma = \gamma_{sw}$, $K = K_{sw}$, and $w_{mod} = \sigma_c$. The conversion coefficient $A_{P-H}$ is taken as previously in expressions (43–45) as $A_{P-H} = 0.9$. Irreversible energy expenditures on the dissociation $w_{dis}$ are also expected the same as previously. There is no precise information on the magnitude of the compressive strength $\sigma_c$, for example, for chemically tempered glass Corning Gorilla Glass 5, only an estimate is known: $\sigma_c \geq 0.85$ GPa [55], which in kJ/cm$^3$ is $\sigma_c \geq 0.85$ kJ/cm$^3$. For greater certainty, we take into account, that, as is well known from the literature, for tempered glasses $\sigma_c$ is 3–4 times higher than for non-tempered glasses, for which, in turn, the value of $\sigma_c$ is known and according to reference data is about 0.59–0.6 GPa. Therefore, it can be assumed that for tempered glasses $\sigma_c$ is about 1.8–2.4 GPa, i.e., 1.8–2.4 kJ/cm$^3$, and for sapphire $\sigma_c$ about 2.4–3 GPa (=2.4–3 kJ/cm$^3$). These values are given in Table 2.

For crystalline materials, the threshold parameter $\sigma_c$ in (46) may depend on the orientation of the filament axis relative to the crystallographic axes of the material. In particular, not the compressive strength, but the tensile strength, which is known to be 1–2 orders of magnitude lower than the indicated compressive strength, may be important. In this case, the radius $R_{SW}$ can increase several times according to (46).

Equation (46) assumes that the shock wave is initiated by the de-excitation for a sufficiently short time $t_E$, i.e.:

$$t_E < t_{P1}. (47)$$

In numbers according to the $t_{P1}$ estimates in Section 3.1, this means that for the filament radius $r_f = 0.25–0.5 \mu$m $t_E < 80–170$ ps. However, if the de-excitation is much slower:

$$t_E \gg t_{P2}, (48)$$

that is, according to the $t_{P2}$ estimates in Section 3.1 for $r_f = 0.25–0.5 \mu$m, at $t_E > 0.4–0.8$ ns, a significant decrease in the shock wave amplitude and a corresponding decrease in $R_{SW}$ should be expected. $R_{SW}$ estimates in this case are not included in this model.

3.2.8. The Pulse Repetition Rate

Another factor in obtaining a better smoothness of the cut walls is that the filaments in each of their arrays, despite their possible mutual influence on each other due to the temperature fields and gradients created by them, must, nevertheless, be straight. Otherwise, when the filaments are curved, after the material separation passing through these filaments, it is unlikely to expect flat smooth sidewalls of separation—most likely they will be rough. From this point of view, it looks reasonable and useful to minimise the influence of neighbour filaments in the array on each other, i.e., to minimise the influence of the temperature gradient and heating from the previous filament at the location and at the time of occurrence of the next new filament in time and space.

We will show in this section that in the framework of the thermal or thermal deformation mechanism when $R = R_1$ or $R = R_{1D}$ and the impact zone is formed due to the energy...
transfer in the material by thermal diffusion, this requires the choice of the PRR $f$ in some specific ranges.

From the above expression (32) for the spatio-temporal temperature change in the material due to the energy release of the filament, we estimate the material heating from the previous filament at the location and at the time of occurrence of the next new filament in time and space, i.e., at time $t = t_{\text{rep}}$ ($t = 0$ is taken as the time of occurrence of the previous filament) and at a distance $r = s_1$ from this previous filament, as shown in Figure 5a:

$$T(r = s_1, t = t_{\text{rep}}) - T_i = \frac{q}{\pi \rho_0 c_0 \chi_{\text{rep}}} \exp \left( - \frac{s_1^2}{4 \chi_{\text{rep}}} \right).$$

(49)

Considering $s_1$ to be fixed, we search for such $t_{\text{rep}}$ at which this heating (49) is small, i.e., for example, is no more than 10% of the characteristic heating value $(T_{\text{mod}} - T_i)$ corresponding to the threshold point $T_{\text{mod}}$ in the considered case. In the limiting case, we use for estimates not an inequality, but a strict equality:

$$T(r = s_1, t = t_{\text{rep}}) - T_i = 0.1(T_{\text{mod}} - T_i).$$

(50)

To find solutions of this equation for $t_{\text{rep}}$, a new dimensionless variable $a$ is introduced, such that:

$$t_{\text{rep}} = \frac{s_1^2}{4 a \chi}, \quad a(4 \chi t_{\text{rep}}) = s_1^2.$$

(51)

Substituting (51) into (49), then into the condition (50) and using also the expression (36) for $T_{\text{mod}}$ results in a transcendental equation for $a$: $ae^{-a} = (0.1/e)(s_1/R)^2$. For each given ratio $(s_1/R)$, the solution of this equation has two roots $a_1$ and $a_2$, which can be easily found numerically. They are shown in Table 3. According to (51), $a_1$ and $a_2$ correspond to two solutions for $t_{\text{rep}}$:

$$t_{\text{rep}} = s_1^2/(4 a_1 \chi), \quad t_{\text{rep}} = s_1^2/(4 a_2 \chi),$$

and, respectively, two solutions for the PRR $f = 1/t_{\text{rep}}$:

$$f_1 = 4a_1\chi/s_1^2, \quad f_2 = 4a_2\chi/s_1^2.$$

(53)

Table 3. Calculation of parameters $a_1$ and $a_2$ as a function of the ratio $(s_1/R)$, characterising the degree of overlapping of critical impact zones from neighbour filaments.

| $s_1/R$ | $a_1$ | $a_2$ |
|---------|-------|-------|
| 2       | 3.02  | 0.175 |
| 1.8     | 3.33  | 0.137 |
| 1.6     | 3.66  | 0.105 |
| 1.4     | 4.02  | 0.0779|
| 1.2     | 4.43  | 0.0560|

Thus, the condition $T(r = s_1, t = t_{\text{rep}}) - T_i \leq 0.1(T_{\text{mod}} - T_i)$ is fulfilled in two ranges of pulse repetition frequency: in a so-called “high-frequency” range:

$$f \geq f_1,$$

(54)

where $f_1$ is the lower limit of this high-frequency range, and in the so-called “low-frequency” range:

$$f \leq f_2,$$

(55)

where $f_2$ is the upper limit of the low-frequency range, with $f_2 \leq f_1$.

For glasses, when irradiated in the low-frequency range of repetition frequencies $f \leq f_2$, the characteristic heating value $(T_{\text{mod}} - T_i)$ for the middle between two neighbour filaments in space and time, as shown in Figure 5a, is formed, as we will see below, mainly as the heating from only one filament and according to (24) is $(T_{\text{strain}} - T_i)$, which, for example, for Corning Gorilla Glass 5 according to the Table 2 is 551 °C. However, for the
processing in the high-frequency range \( f \geq f_i \), the same characteristic heating for obtaining separation \( (T_{\text{strain}} - T) \) is formed, as we will see below, under the effect of heating from two neighbour filaments, since in this case, the pulse repetition time is so short that the indicated middle point (or more correctly—line) does not have time to cool down after the action of the \( Y_1 \) filament to the arrival of heating from the next \( Y_2 \) filament. With this in mind, in such case, the almost half smaller heating from a single filament is sufficient: 

\[
T_1 - T_i = (0.5-0.57)(T_{\text{strain}} - T_i),
\]

i.e., numerically, for example, again for Corning Gorilla Glass 5—only \( 276-314 \) °C. The value of the disturbing heating \( T(r = s_i, t = t_{\text{rep}}) - T_i \leq 0.1(T_{\text{mod}} - T_i) \) from the previous filament at the time and place of the appearance of the next new filament is then numerically equal to \( \leq 55 \) °C and \( \leq (28-31) \) °C, respectively, for these two frequency ranges, and this heating has indeed a fairly small value.

From Table 3 it is seen that depending on the above-selected ratio \( (s_i/R) \), which characterises the degree of overlap of the above-mentioned critical affected zones from neighbour filaments, the high-frequency limit \( f_i \) according to (53) changes proportionally to \( a_1 \) in the range:

\[
f_i = (12-18)\chi/s_i^2.
\]

The coefficient 12 here corresponds to \( s_i/R = 2 \) when the impact zones only touch each other, as in Figure 4c. And the coefficient 18 corresponds to \( s_i/R = 1.2 \) when the impact zones overlap significantly, as shown in Figure 4d. The low-frequency limit \( f_2 \) changes according to (53) and \( a_2 \) value given in Table 3 from \( 0.22\chi/s_i^2 \) for \( s_i/R = 1.2 \) to \( 0.70\chi/s_i^2 \) for \( s_i/R = 2 \):

\[
f_2 = (0.22-0.70)\chi/s_i^2.
\]

As we will see below, high-frequency range (54) provides for achieving high, including the highest, cutting speeds of the order of \( 1-25 \) m/s or more, depending on the material. Such high values exceed by tens and hundreds of times the speed of other known laser and non-laser methods of cutting transparent materials. On the other hand, the low-frequency range provides the possibility of LFC at a very low average laser power of the order of only one to several hundred mW.

3.2.9. \( T_{\text{mod}} \) and \( s_i \) at High PRR

Having in hands expressions (52) for \( t_{\text{rep}} \) in two practically interesting ranges of pulse repetition frequencies (54) and (55), we can now estimate the modification radius \( R \) taking into account a finite \( t_{\text{rep}} \), when the resulting heating from two neighbour filaments becomes important for reaching the critical for separation temperature at the point \( r = R \), as shown in Figure 5. If we assume that the pulse \( Y_1 \) falls on the material at time \( t = 0 \) and creates a filament at a point \( x = 0 \), then the next pulse \( Y_2 \) creates its filament, respectively, at time \( t = t_{\text{rep}} \) (\( t_{\text{rep}} \) is the pulse repetition time) and at a point \( x = s_i \). Here \( s_i \) is, as previously, the step of the filaments following in the material, and the \( x \)-axis is directed along the irradiated surface of the material in the direction of the beam movement relative to it—see Figure 4a. We estimate the contributions of these pulses to the heating of the point \( x = R \), taking into account the obtained above finite time delay \( t_{\text{rep}} \) between them. We take for estimates according to (22) \( s_i = s_{\text{max}} = 2R \), i.e., \( R = 0.5s_i \). Thus, the location of this point is at the same distance \( R = 0.5s_i \) from both these filaments. As can be seen from Figure 5b, the filament created by the pulse \( Y_1 \) causes at this point \( x = R \) the maximum heating, denoted as \( \Delta T_i \), at the time, which according to (34) is:

\[
t_{\text{max}1} = \frac{s_i^2}{16\chi} = 0.0625 \frac{s_i^2}{\chi}.
\]

The second filament created by the pulse \( Y_2 \) falling on the material with a delay of \( t_{\text{rep}} \) to the first pulse causes the maximum heating at the same point \( R \), as shown by curve 2, of the same value \( \Delta T_i \) (since this point is located at the same distance \( R \) from this second filament), but, accordingly, at the time delayed by \( t_{\text{rep}} \):
The resulting heating under the effect of these two neighbour filaments is represented by curve 3 (Figure 5b). It is seen that this curve has a maximum value of:

$$\Delta T = \Delta T_1 + \eta \Delta T_1 = (1 + \eta)\Delta T_1$$  \hspace{1cm} (60)$$

at the time:

$$t_{max2} \approx t_{max1} + t_{rep}.$$ \hspace{1cm} (61)$$

Here $\eta \Delta T_1$ is the value of the heating from the first pulse at the stage of the cooling down at the indicated time $t = t_{max2}$. As can be seen from Equation (60) and Figure 5b, taking into account the action of two neighbour filaments gives rise to an increase in the maximum heating by $(1 + \eta)$ times compared to the above-considered heat diffusion heating from only one filament. To estimate the coefficient of heat preservation $\eta$ for the above practically interesting ranges of pulse repetition frequency $f \geq f_1$ and $f \leq f_2$, a relation obvious from Figure 5b is used:

$$\eta = \frac{T(t=0.5s_1, t=t_{max1}+t_{rep}) - T_i}{\Delta T_1},$$ \hspace{1cm} (62)$$

in which the temperatures in the numerator and the denominator are calculated by Equation (32). Using (52), we substitute into (32) and (62) the parameter $t_{rep}$, whose value according to Table 3 at $s_1 = 2R$ is $t_{rep} = s_1^2/(12.1\chi) = 0.0826s_1^2/\chi$, as well as $t_{max1}$ from (58) and the expression (35) for $\Delta T_1$. It gives: $\eta(t_{opt1}) = 0.76$. Thus, for $f = f_1$ one has $\eta = 0.76$. For $f >> f_1$, when $t_{rep} << t_{opt1}$, i.e., the delay between pulses is practically absent, we have obviously $\eta = 1$. Therefore in the high-frequency range (54) $f \geq f_1$, an estimate for $\eta$ is valid:

$$0.76 \leq \eta \leq 1.$$ \hspace{1cm} (63)$$

Similarly, for the low-frequency range (55), when $f \leq f_2$, and according to (51), as well as Table 3 $t_{opt}(f = f_2) = t_{opt2} = 1.427s_1^2/\chi$ for $R = 0.5s_1$, we find an estimate for $\eta$:

$$0 < \eta \leq 0.11.$$ \hspace{1cm} (64)$$

Taking into account the correction factor $(1 + \eta)$ in (60), which, as we see, depends on the pulse repetition frequency $f$ and actually takes into account the contributions to the heating from two neighbour filaments, the required heating from one filament $\Delta T_1$ to achieve the same critical temperature for separation $T_{exp}$ according to (60) becomes less:

$$\Delta T_1' = \frac{(T_{exp} - T_i)}{(1+\eta)}.$$ \hspace{1cm} (65)$$

However, such a reduction in the heating leads to a corresponding decrease by $(1 + \eta)$ times of the threshold $w_{mod}$ in one pulse. Therefore, taking into account the obtained estimates (63) and (64) for $\eta$ instead of the previous expressions (43–45) for $R_T$ and $R_{TD}$, we obtain new, corrected formulas for these high-frequency and low-frequency ranges. For thermal mechanism and amorphous materials:

$$s_1 \leq s_{1max} = 2R_T,$$ \hspace{1cm} where $R_T = \left[\gamma_T K_T W_{A-P-H} \frac{(W - w_{dis})}{w_{mod}}\right]^{1/2} \gamma_T,$$$$

$$w_{mod}^T = \frac{c_2p_2(T_{sof} - T_i)}{1 + \eta},$$ \hspace{1cm} $\eta = 0.76$–1 for $f \geq f_1,$ \hspace{1cm} or $\eta = 0$–0.11 for $f \leq f_2.$ \hspace{1cm} (66)$$

Similarly for the thermal mechanism and crystalline materials:

$$s_1 \leq s_{1max} = 2R_T,$$ \hspace{1cm} where $R_T = \left[\gamma_T K_T W_{A-P-H} \frac{(W - w_{dis})}{w_{mod}}\right]^{1/2} \gamma_T,$$$$

$$w_{mod}^T = \frac{c_2p_2(T_{sof} - T_i)}{1 + \eta},$$ \hspace{1cm} $\eta = 0.76$–1 for $f \geq f_1,$ \hspace{1cm} or $\eta = 0$–0.11 for $f \leq f_2.$ \hspace{1cm} (67)$$
\[ w_{mod}^{T} = \frac{c_{p}T_{0}(T_{text}-T_{i})}{1+\eta}, \quad \eta = 0.76-1 \text{ for } f \geq f_i, \text{ or } \eta = 0-0.11 \text{ for } f \leq f_i. \]

For the mechanism of thermally-induced deformation, respectively:

\[ s_{1} \leq s_{1\text{max}} = 2R_{TD}, \quad \text{where } R_{TD} = \left[ \gamma_{f}K_{Tw}A_{P-H} \left( \frac{w-w_{\text{dis}}}{w_{\text{mod}}} \right) \right]^{1/2} r_{f}, \]

\[ w_{mod}^{T} = \frac{c_{p}T_{0}(T_{text}-T_{i})}{1+\eta}, \quad \eta = 0.76-1 \text{ for } f \geq f_i, \text{ or } \eta = 0-0.11 \text{ for } f \leq f_i. \] (68)

We recall that after the calculation of \( R \) using obtained corrected expressions (66)–(68), it is necessary to verify the validity of the made above approximation (33) of the short duration of the energy deposition in the material from the filament: \( t_{e} \ll R^{2} / (4\chi) \). For example, as we will see in the “Discussion” section below, for glasses \( \chi \approx 5 \times 10^{-3} \text{ cm}^{2} / \text{s} \) and \( s_{\text{max}} \approx 5-6 \mu \text{m} \). Then this inequality holds for \( t_{e} < 1.5 \mu \text{s} \), which seems practically feasible.

On the other hand, if it turns out that for other \( s_{\text{max}} \) (for example, for glasses at \( s_{\text{max}} \approx 2.5 \mu \text{m} \)) or other materials, the condition opposite to (33) is met, the calculation of \( s_{\text{max}} \) and \( \text{PRR} \) requires then the use of other formulas given in the following section.

3.2.10. Step of Filaments following and \( \text{PRR} \) in the Approximation of “Long Time” of Energy Release

In the approximation of a “long time” of energy release \( t_{e} \), when the condition (11) opposite to (10) and (33) is fulfilled:

\[ t_{e} > R^{2} / (4\chi), \] (69)

the spatio-temporal dependence of the temperature around the filament has the form (30). But for calculation of the maximal in time temperature at a radius \( r \), it is sufficient to consider times \( t \leq t_{e} \), described by (29). For a fixed \( r \), this function monotonically increases with \( t \), so the maximum is reached at \( t = t_{e} \):

\[ T_{\text{max}}(r) - T_{i} = T(r, t = t_{e}) - T_{i} = \frac{q_{0}}{4\pi C \rho \chi H} B \left( \frac{r^{2}}{4\chi t_{e}} \right). \] (70)

We recall that since \( r \leq R \), in this expression according to (69) the parameter \( z = r^{2} / (4\chi t_{e}) \leq 1 \), or \( z = 1 \). To find the radius of modification \( R \) from the expression (70), we use the method as in Ref. [59], when for any special function (in the given case for \( B(z) \)), the values of which are known only in a tabulated form, its approximation is used in the form of a relatively simple analytical exponential function \( B(z) \). In this case, as can be easily shown by a numerical computer interpolation of tabulated values, \( B(z) \approx B(z) \), where the approximation is \( B(z) = B_{0} \exp(-Az^{1/2}) \) with \( B_{0} = 5.47 \) and \( A = 3.3 \). Calculations of \( B(z) \) using this simple analytical formula in comparison with tabulated values of \( B(z) \) show that this approximation is valid in the range \( 0.01 \leq z \leq 2.6 \) (covering the interesting for us range \( 0.01 \leq z \leq 1 \)) with a relative accuracy of at least 12%. With the use of this approximation, the ratio (70) takes the following simple form:

\[ T_{\text{max}}(r) - T_{i} = \frac{q_{0}}{4\pi C \rho \chi H} B_{0} \exp \left( -A \frac{r}{2\sqrt{\chi t_{e}}} \right). \] (71)

Substituting here \( r = R \), \( T_{\text{max}} = T_{\text{mod}} \) and \( q_{0} = Q_{tw} / t_{e} \), we now can easily solve this equation with respect to \( R \):

\[ R_{TD} = \frac{1}{A} 2 \sqrt{\chi t_{e}} \ln \left( \frac{Q_{tw}}{Q_{th}} \right) \quad \text{where } Q_{th} = 4\pi C t_{e} H C \rho (T_{\text{mod}} - T_{i}) / B_{0}. \] (72)
By substituting here the general expression (39) for \( Q_{tw} \) and taking also into account the relations (24) and (25), we obtain the dependence of \( R_s, s_{1 max} = 2R \) and \( s_1 \) on the filament radius \( r_f \) and the VED \( w \). For the thermal mechanism, when \( R = R_T \), we obtain:

\[
s_1 \leq s_{1 max} = 2R_T, \quad \text{where} \quad R_T = \frac{1}{A} 2\sqrt{\frac{\chi t_E^2}{4R}} \ln \left[ \frac{\frac{K_{tw} \sigma - \mu R_0 (w-w_{dis})(T)}{w_{mod}}}{4R} \right],
\]

(73)

\[
w_{mod}^T = C_2 \rho_2 (T_{strain} - T_i) \quad \text{for amorphous materials,}
\]

(74)

\[
w_{mod}^T = C_2 \rho_2 (T_{melt} - T_i) \quad \text{for crystalline materials.}
\]

For the thermally-induced deformation when \( R = R_{TD} \), respectively:

\[
s_1 \leq s_{1 max} = 2R_{TD}, \quad \text{where} \quad R_{TD} = \frac{1}{A} 2\sqrt{\frac{\chi t_E^2}{4R}} \ln \left[ \frac{\frac{K_{tw} \sigma - \mu R_0 (w-w_{dis})(T)}{w_{mod}^{TD}}}{4R} \right],
\]

(75)

\[
w_{mod}^{TD} = C_2 \rho_2 (T_c - T_i) = \frac{C_2 \rho_2}{\rho_T} \sigma_c.
\]

(76)

The condition for the applicability of equations (72), (73) and (75) is the obvious relation \( R_T, R_{TD} > 0 \), i.e., the fulfilment of the condition \( Q_{tw}/Q_{th} \geq 1 \), as well as the fulfilment of the inequality (69), rewritten as \( R_T^2, R_{TD}^2 < 4\chi t_E \). From these limitations, we find the range of acceptable \( Q_{tw} \):

\[
Q_{tw} = (1 \div \exp A) Q_{th} = (1 \div 27) Q_{th}.
\]

(77)

Similar to the above calculations for the frequency limits \( f_1, f_2 \), we find the repetition frequencies in the considered mode (69), which allow minimising the mentioned above influence of the temperature gradient and heating from the previous filament at the location and at the time of occurrence of the next, new filament. For the temperature field around the filament, in this case, we use the expression (71), as well as the same condition of smallness (50) of heating from the previous filament at the location and at the time of occurrence of the next new filament at time \( t = t_{rep} \). The solution for \( f \) is then found in the form:

\[
f \geq f^*, \quad f^* = \frac{1}{k_1} \left( \frac{k_1}{2} \right)^2 \left[ 1 + \frac{2(\chi t_E)^{1/2}}{AR} \ln 10 \right]^2,
\]

(78)

where \( f^* \) is a lower limit of this range.

3.2.11. \( R_T \) and \( R_{TD} \) for High Pulse Repetition Rate

The high pulse repetition rate \( f \geq f^* \) indicated in (78) provides a fairly short pulse repetition time \( t_{rep} \), similar to the situation discussed above when the condition (33) was met. Therefore, as follows from Figure 5b, the selected point \( r = R \) at the time of the maximum heating from the second filament created by the second pulse \( Y_2 \), cools down only slightly from the maximum heating created by the first filament and by the first pulse \( Y_1 \). To achieve the same critical temperature for separation \( T_{strain} \) (or \( T_{melt} \) for crystalline materials) as a result of the beam passing through the material, it is hence sufficient, as above, to have about two times less heating from a single filament \( \Delta T_i = T_i - T_i = (T_{strain} - T_i)/2 \).

This reduction in heating for the required modification of the material according to equations (74) and (76) leads to a decrease in the thresholds at high frequencies \( f \geq f^* \) by about two times:

\[
w_{mod}^T \approx 0.5C_2 \rho_2 (T_{strain} - T_i) \quad \text{for amorphous materials,}
\]

(79)

\[
w_{mod}^T \approx 0.5C_2 \rho_2 (T_{melt} - T_i) \quad \text{for crystalline materials,}
\]

(80)
This decrease of the thresholds according to Formulas (73) and (75) leads to an increase in $R_T$ and $R_{TD}$, as well as to an increase in the corresponding step $s_{1max}$ at the same $Q_{sw}$.

3.2.12. Modification Mechanism Preferable for Smooth Separation

For the convenience of performing the necessary estimates in the proposed model, it is important to determine the predominant of the three mentioned above competing mechanisms of the modification zone formation. To simplify the modeling, we consider them as acting separately, not altogether. In such case, a mechanism which gives the maximum radius of the modification zone from the above radii $R_T, R_{TD}$, $R_{SW}$ should be considered as a dominant mechanism. It is also obvious that the smooth separation of interest can be realized in the case when the thermal (T) mechanism is dominant. Thus, to obtain a smooth separation, the condition must be met:

$$R_T > R_{TD}, R_{SW}. \quad (82)$$

Indeed, in this case, we can hope for the formation of the impact zone without destruction in the form of scattered cracking in the solid material as a result of irradiation. As can be seen from Figure 6a, although the thermal deformation mechanism formally leads to cracking of the material directly inside the zone $r \leq R_{TD}$, the boundary of which is conventionally designated by a zigzag circular contour, however, this zone completely and with excess is covered by the zone of action of the thermal mechanism $r \leq R_T$. Therefore, although there is a cracking in the impact zone of the thermal deformation mechanism $r \leq R_{TD}$, as a result of annealing in the larger zone $r \leq R_T$ this cracking has no chance to occur when separation is completed. In the case of mechanical splitting following irradiation, a straight and smooth separation line is possible in this case, indicated as 1 in Figure 6a.
for these mechanisms. (b) The dominance of the thermal deformation mechanism when \( R_{TD} > R_t \). As can be seen, at radii \( r \) such that \( R_t < r \leq R_{TD} \), there is a characteristic area of damage and cracking of the material as a result of irradiation. (2) is a zigzag-like line (surface) of separation, randomly changing direction, obtained in this case by mechanical splitting after irradiation since it passes along random microcracks and boundaries between pieces of material formed as a result of cracking in the indicated damage area of the material. Such a surface 2 corresponds to a rough, non-smooth separation of the material.

However, the situation changes, and such annealing of the cracked material in the entire \( r \leq R_{TD} \) zone does not occur if the thermal deformation (TD) or the shock wave (SW) mechanisms are dominant. In this case (see Figure 6b) \( R_t < R_{TD} \) (or \( R_t < R_{SW} \)) and at radii \( r \) such that \( R_t < r \leq R_{TD} \) (or \( R_t < r \leq R_{SW} \)), there is a relatively large area of material cracking, characteristic for the thermal mechanism (TD) or the shock wave (SW) mechanisms. This cracked area is not annealed, since it is not covered by a relatively small impact zone of the thermal mechanism \( r \leq R_t \). The surface of the mechanical splitting obtained in this case after irradiation (designated on the cross-section in Figure 6b as the line 2) passes through this cracking area. It has a zigzag-like, randomly changing direction since it passes through random numerous microcracks and boundaries between pieces of damaged material formed as a result of cracking in the indicated not annealed damage area. Such shape of the separation surface is rough and non-smooth. Moreover, on both halves of the sample obtained during the subsequent separation after irradiation, there will be a broken cracked (or microcracked) superficial layer corresponding to the area of damage on both sides of this rough separation surface 2 in this figure. The presence of such a cracked superficial layer has an extremely unfavourable effect on the mechanical strength of the material during its further operation after separation, since it serves as the nuclei of cracks that can occur and propagate with accidental or non-accidental mechanical impacts during subsequent use of the product.

A comparison of situations (a) and (b) in Figure 6 shows that smoother separation becomes possible when the thermal mechanism dominates in the formation of the impact zone.

Material parameters for fulfillment of the condition (82) for obtaining a smoother separation can be easily found, as all three modification radii are defined by formulas of the general type (23): (43) or (44) for \( R_t \), (45) for \( R_{TD} \) and (46) for \( R_{SW} \). If we assume for the estimations in these formulas that \( \gamma_{sw} \approx \gamma_{tw} = 1/e = 0.368 \), the condition of separation with a better smoothness (82) is satisfied when simple relations between the corresponding thresholds of these 3 mechanisms are fulfilled:

\[
\begin{align*}
 w^{T}_{mod} &< w^{TD}_{mod} \quad \text{and} \quad w^{T}_{mod} < w^{SW}_{mod}. \\
\end{align*}
\]  

(83)

Given the expressions (66) or (67) for \( w^{T}_{mod} \) and (66) for \( w^{TD}_{mod} \), the first inequality in (81) corresponds to the following condition:

\[
T_{strain} < T_c
\]  

(84)

for amorphous materials, and:

\[
T_{melt} < T_c
\]  

(85)

for crystalline materials. These conditions indicate the simple physical fact that for obtaining a separation with a better smoothness, it is important that the point of intense volumetric expansion, estimated as \( T_{exp} = T_{strain} \) for amorphous materials (or, respectively, as \( T_{exp} = T_{melt} \) for crystalline materials), is reached earlier (i.e., at a lower VED \( w \)) than the critical for material cracking point \( T_c \) in the mechanism of the thermally-induced deformation.

By substituting expression (26) for \( T_c \) into (84) and (85), one then obtains a dimensionless criterion for obtaining a separation with a better smoothness for amorphous materials.
\[
\frac{\alpha r Y (T_{\text{strain}} - T_i)}{\sigma_c} < 1, \quad (86)
\]

and, correspondingly, a similar criterion for crystalline materials:

\[
\frac{\alpha r Y (T_{\text{melt}} - T_i)}{\sigma_c} < 1. \quad (87)
\]

Besides, from the second inequality in (83), \( w_{\text{mod}}^r < w_{\text{mod}}^{SW} \), similarly, when substituting the expression (66) or (67) for \( w_{\text{mod}}^r \) and the expression (46) for \( w_{\text{mod}}^{SW} \), we find the following dimensionless criterion for obtaining a separation with a better smoothness:

\[
\frac{C_2 \rho_2 (T_{\text{strain}} - T_i)}{(1 + \eta)\sigma_c} < 1 \quad (88)
\]

for amorphous materials, and:

\[
\frac{C_2 \rho_2 (T_{\text{melt}} - T_i)}{(1 + \eta)\sigma_c} < 1 \quad (89)
\]

for crystalline materials. The physical meaning of fulfilling conditions (88) and (89) is that to obtain a separation with a better smoothness, it is important that the temperature point of intense volumetric expansion, estimated in the model as \( T_{\text{exp}} = T_{\text{strain}} \) for amorphous materials (or, respectively, as \( T_{\text{exp}} = T_{\text{melt}} \) for crystalline materials), is reached earlier (i.e., at a lower VED \( w \)) than the material strength limit \( \sigma_c \) under the action of a shock wave.

The simultaneous fulfilment of the criteria (86,88) for amorphous materials and the criteria (87,89) for crystalline materials thus determines the materials preferred for obtaining a separation with a better smoothness. These, as can be seen from the obtained ratios (86–89), are materials with small \( \alpha r, \ Y, \ T_{\text{strain}} \) or \( T_{\text{melt}} \), but with a large \( \sigma_c \), which, when heated to the point \( T_{\text{strain}} \) (or \( T_{\text{melt}} \)), are not subjected to destruction and cracking under the action of thermally-induced deformations, as well as destruction and cracking under the action of a shock wave propagating from the heated filament. Also, it is important to use the high-frequency range of repetition frequencies (54) \( f \geq f_1 \) compared to the low-frequency range (55) \( f \leq f_2 \), since with an increase in \( f \), as we saw from (63) and (64), the heat preservation factor \( \eta \) increases, and, thus, more favourable conditions are created for fulfilling the criteria (88) or (89).

The examples given in Table 4 for calculation of the fulfilment of the indicated criteria (86–89) for several materials show that such a material is the chemically tempered Corning Gorilla Glass 5 (as well as, possibly, other chemically tempered Corning Gorilla Glass 3 and Corning Gorilla Glass 2, which have similar parameters \( \alpha r, \ Y, \ T_{\text{strain}}, \sigma_c, C_2 \) and \( \rho_2 \)). However, these criteria are not met for non-tempered glass Corning EAGLE XGTM, as well as for the crystalline material—sapphire. Therefore, in the considered LFC with each filament created by a single pulse, a smooth separation for these materials is not expected.

Table 4. Verification of criteria (86,88) for amorphous materials and (87,89) for crystalline materials, the simultaneous fulfilment of which is preferable for obtaining a separation with better smoothness. The parameters \( \alpha r, \ Y, \ T_{\text{strain}}, \ T_{\text{melt}}, \sigma_c, C_2 \) and \( \rho_2 \), required for calculations, are taken from Table 2. Initial temperature is \( T_i = 20 \, ^\circ\text{C} \), heat preservation factor is \( \eta = 0.76 \).

| Amorphous material | Quartz Glass | Corning EAGLE XGTM | Corning Gorilla Glass 5 | Sapphire |
|-------------------|--------------|---------------------|------------------------|---------|
| \( \frac{\alpha r Y (T_{\text{strain}} - T_i)}{\sigma_c} \) | 0.11–0.14 | 0.28 | 0.14–0.19 | - |
| \( \frac{C_2 \rho_2 (T_{\text{strain}} - T_i)}{(1 + \eta)\sigma_c} \) | 1.5–2.0 | 1.6 | 0.32–0.42 | - |
Crystalline material
\[ \frac{\alpha \gamma (T_{\text{melt}} - T_i)}{\sigma_c} \]
\[ \frac{c_2 \rho (T_{\text{melt}} - T_i)}{(1 + \eta) \sigma_c} \]
- - - 1.3–2.2
- - - 1.5–1.9

The obtained criteria (86,88) and (86,89), based on the data on the materials parameters, predict explicitly ability to a separation with a better smoothness depending on the material. The given example with tempered glass makes it possible to indicate the directions of material science construction of the properties of new materials to achieve the values of the mentioned parameters \( \alpha \), \( \gamma \), \( \sigma_c \), \( c_2 \), \( \rho \), \( T_{\text{strain}} \) or \( T_{\text{melt}} \) required for obtaining a separation with a better smoothness.

### 3.3. Speed and Average Power of the Beam

The speed \( u \) of the relative movement of the laser beam and the material (and, correspondingly, the processing speed) is defined by the known formula:

\[ u = s_1 f, \]  
(90)

where the choice of \( s_1 \) and \( f \) is defined above depending on material and radiation parameters. For example, for the thermal modification mechanism, if the condition (33) of a short duration \( t_e \) of heat release is fulfilled, we obtain the following estimate for \( f = f_1 \), or \( f = f_2 \):

\[ u \approx a \chi / s_1, \]  
(91)

where \( a = a_1 \) or \( a = a_2 \) (and \( a_1 \gg a_2 \)) correspond according to (53) to the frequency limits \( f_1 \) and \( f_2 \), respectively.

It is seen from (89) that a high speed \( u \) (and processing speed, respectively) can be obtained by using the high-frequency range, where \( a \) is higher. Also, \( u \) increases with a decrease in the step of the filaments following \( s_1 \), which according to (66) is achieved by reducing \( w \), as well as by reducing the filament radius \( r_f \) if, as follows from the discussion of the Formula (19) above, a shorter wavelength \( \lambda \) is used. But at the same time, according to the ratios (56,57), where \( f_{1,2} \propto 1/s_1^2 \), a higher pulse repetition rate \( f \) is also required. Besides, as follows from (89), \( u \) increases for materials with a higher thermal diffusivity \( \chi \).

The necessary average power \( P \) of the beam is found in terms of \( E_1 \) and \( f \) obtained above, according to the obvious relation \( P = E_1 f \). For example, for the thermal mechanism of the impact zone formation, when the condition (33) of short duration of heat dissipation is fulfilled, at \( f = f_1 \) or at \( f = f_2 \), with taking into account the foregoing ratio \( f_{1,2} \propto a \chi / s_1^2 \), where \( a = a_1 \) for \( f = f_1 \) and \( a = a_2 \) for \( f = f_2 \), expressions (17) for \( E_1 \) and (66) for \( s_{\text{max}} \) and with choosing \( s_1 \) as \( s_1 = 0.9 s_{\text{max}} \) (when according to Table 3 \( a = a_1 = 13.3 \) or \( a = a_2 = 0.547 \)), we obtain the following estimate of \( P \):

\[ P \cong \frac{0.399 \pi}{T_{\text{cw}} K_{\text{cw}} A_{P-H} A_{L-P} (w - w_{\text{dia}})} a H_{\chi} w_{\text{mod}}, \]  
(92)

It can be seen that \( P \) decreases many times for the low-frequency range, where \( a \) is less than in the high-frequency range, as well as for filaments with shorter length \( H \), for materials with a lower thermal diffusivity \( \chi \) and having a smaller threshold \( w_{\text{mod}} \). An increase in the conversion coefficients \( A \), \( A_{P-H} \) and \( K_{\text{cw}} \) also leads to \( P \) decrease. It is also seen that \( P \) does not depend on \( r_f \) i.e., on the wavelength, in contrast to the discussed above estimate (91) for \( u \).

### 3.4. LFC Throughput

The LFC throughput \( \Omega \) is defined as the ratio of the product \( uH \) to the average power of the laser beam in the material \( P \), i.e., \( \Omega = uH/P \). The higher the \( uH \) at the same \( P \), the higher the throughput \( \Omega \). Substituting the expressions \( P = E_1 f \) and \( u = s_1 f \) from (90) gives
Thus, $\Omega$ can be equivalently defined as the ratio of the cut area $s_1H$ per pulse to the pulse energy. Using here the expression (17) for $E_1$, as well as choosing the step of filaments following as $s_1 = 0.9s_{1\max}$, where $s_{1\max}$ is given by the expressions (22,23), leads to the relation:

$$\Omega = \frac{s_1H}{E_1}. \quad (93)$$

It shows that $\Omega$ does not depend on the filament length $H$ and increases when:

(i) The ratio $(w - w_{dis})^{1/2}/w$ is maximal. This takes place, as can be easily shown, at $w = 2w_{dis}$;

(ii) The filament radius $r_f$ is minimal, which is the case, as follows from the discussion of the expression (19) above, when using a shorter wavelength of laser radiation $\lambda$;

(iii) The modification threshold $w_{mod}$ is minimised. When the energy transfer mechanism by heat conduction rather than by the shock wave dominates in the formation of the impact zone around the filament, it is preferable for this, as we have seen above, to use repetition frequencies in the high-frequency range ($54 \leq f \leq f_1$, but not in the low-frequency range ($55 \leq f \leq f_2$). In this case, the increase of the heat preservation factor $\eta$ in the range $f \geq f_1$ following the estimates (63) and (64) leads according to the equations (66)–(68) to a decrease in the $w_{mod}$ by almost two times compared to the low-frequency range $f \leq f_2$;

(iv) The energy conversion coefficients $A_{L-P}$ and $A_{P-H}$ are maximal, which emphasizes the importance of using filaments with the VED $w$ that satisfies the conditions (12) for eliminating significant losses of filament energy to thermal radiation.

4. Discussion and Comparison with Experiments

The equations obtained above (2,4,6,7,12–15,17,19,21,22,53–57,66–68,73–81,86–93) completely solve the problem of choosing the irradiation parameters for creating filaments by single pulses with taking into account the numerous factors listed above for obtaining LFC with a better smoothness. The key parameters are the energy of a single laser pulse in the material $E_1$, the pulse duration $\tau$, the step of filaments following in the material $s_1$ and the PRR $f$. Besides, based on these parameters, the LFC throughput $\Omega$, the relative velocity of the beam and material $u$ and the average power of the pulse-periodic beam $P$ are also calculated. The dependences of these parameters on the filament VED $w$, as well as the radius $r_f$ and the length of the filament $H$ are found. The allowed ranges of $w$ variation are given by limitations (12), which eliminate significant losses of the filament energy to thermal radiation. The bounds $w_1$ and $w_2$ of these ranges of $w$, as well as the VED threshold $w_{mod}$ for creating the critical for separation modification zone in the material around the filament, depend on the optical, thermophysical and strength properties of the material.

A schematic of the algorithm for a sequential calculation of the parameters $E_1$, $\tau$, $s_1$ and $f$ in the model is shown in Figure 7 when the energy transfer mechanism by heat conduction dominates in the formation of the impact zone around the filament. For an alternative shock-wave modification mechanism, smooth separation according to the model is not achieved, but this scheme, if we put in it $\eta = 0$ and $f_1 = f_2$, can be used to calculate the parameters $E_1$, $\tau$, $s_1$ and $f$, which in this case provide for a separation with more straight and more uniform in diameter filaments. The algorithm scheme shown contains 45 parameters in total, their physical meaning is explained in the model above. This number includes the input parameters: of radiation, of material and size of filaments, as well as the actual unknown parameters to be found—these are the irradiation parameters $E_1$, $\tau$, $s_1$ and $f$ (shown in Figure 7 in yellow), which determine the smooth separation. The search for the unknown $E_1$, $\tau$, $s_1$ and $f$ is performed in five consecutive stages. To illustrate
the use of this algorithm, we calculate in the sections below these parameters and compare them with the experiments.

**Figure 7.** Schematic of the algorithm for calculating the main irradiation parameters $E_1$, $\tau$, $s_1$ and $f$ (highlighted in yellow), which are preferred for the LFC with a better smoothness of separation for the heat diffusion modification mechanism. For the alternative shock-wave mechanism, the smooth separation is not achieved, but this scheme, if formally to put in it $\eta = 0$ and $f_1 = f_2$, can be used in this case as well—to create a separation with filaments which are more straight and more uniform in diameter along the length. 5 consecutive stages are shown for calculation of unknowns $E_1$, $\tau$, $s_1$ and $f$.

### 4.1. LFC of Sapphire

Cutting 300 µm-thick sapphire by the laser with wavelength $\lambda = 1.04$ µm has been performed experimentally [20]. As it can be read directly from the picture of the cut walls there with the scale shown, the step of filaments in the material is $s_1 \approx 4$ µm and the length of each filament is about $H \approx 45$ µm. The level of the average power and the scan speed of the laser beam reported in [20] are respectively $P < 2$ W and $u = 30$ cm/s. The values of $f$ and $E_1$ are not reported in this paper. However, they can be evaluated based on the reported $s_1$, $P$ and $u$. Indeed, the pulse repetition rate can be found from (90) as $f = u/s_1$, which gives $f = 75$ kHz. Then according to the relation $E_1 = P/f$ for the indicated $P$ and $f$ we also find $E_1 < 26.7$ µJ.

To test the adequacy of the model, it is interesting to find out whether these LFC parameters $E_1$, $s_1$, $f$, $P$ and $u$ can be explained based on the proposed model when using for calculations the model algorithm shown in Figure 7?

For this $w_{\text{thin}}$ for sapphire is taken as 78 kJ/cm³ [57], also $A_{\text{L-P}} = 0.8$ and $A_{\text{P-H}} = 0.9$ as has been mentioned above.

LFC of sapphire according to our model is not smooth, since the parameters of this crystalline material, as can be seen from Table 4, do not satisfy the necessary conditions (87) and (89) for smooth separation The predominant mechanism in this case is not a thermal mechanism, but rather a shock-wave mechanism of the impact zone formation around the filament. However, this is not favourable for obtaining smooth separation. Photos of
the sidewalls of separation surfaces, which are given in the paper [20], confirm this conclusion about a non-smooth separation. For such a mechanism, we take $\gamma_{ww}$ the same as $\gamma_{ww}$. I.e., $\gamma_{ww} = 1/e = 0.37$. Also, we put that in the relation (14) $K_{ww} = K_{ww}$, and hence, $K_{ww} = 0.5$. We also use $P$: ($\lambda = 1.04 \mu m$) $= 2.8$ MW [60] and $\Gamma = 1$–2.

The filament radius is not reported in the considered experiment [20], but its calculation with Equation (19) at $M^2 = 1.25$ for the given wavelength $\lambda = 1.04 \mu m$, NA = 0.6 and $q = 1$ gives an estimation $r_f = 0.49 \mu m$. According to the algorithm diagram in Figure 7, we use these parameters as input data.

Further, on the stage 1, VED bounds are chosen as $w_1 = 860$ kJ/cm$^3$ and $w_2 = 1020$–1230 kJ/cm$^3$ according to calculations given in Table 1 for a very close value of $r_f = 0.5 \mu m$. The choice of the frequency range, low-frequency or high-frequency one, for calculations within the framework of the indicated dominant shock-wave modification mechanism, is not essential.

In stage 2, the VED $w$ is taken according to the condition (12) for the elimination of thermal emission losses as $w < w_1 = 860$ kJ/cm$^3$, namely as $w = 460$ kJ/cm$^3$. For the shock-wave mechanism of modification, in this case, we put $\eta = 0$ and according to (46) use for this material the mechanical strength indicated in Table 2 $\sigma_m = 3.0$ GPa as $w_{mod}^{sw} \approx 3.0$ kJ/cm$^3$.

In stage 3, we calculate the laser pulse energy in the material $E_1$, which is not reported directly in the considered experiment [20]. For the above input parameters $A_L, r_f, \gamma_{ww}$ and $H$, as well as the $w$ chosen on the previous stage, Equation (17) gives then $E_1 = 19.5 \mu J$. Also, according to (46), we calculate the modification radius $R_{SW}$ for the shock-wave mechanism—for the above values of the input parameters $\gamma_{ww}, K_{ww}, \eta_{ww}, r_f, \eta_{dis}$ and the VED $w$ chosen on the previous stage, this gives $R_{SW} = 2.25 \mu m$. If smooth separation were possible in this case, then calculates the maximum step of filaments following $s_{max}$ to obtain it would give according to (22) $s_{max} = 2 R_{SW} = 4.5 \mu m$. The choice of the step value as $s_1 = 0.9 s_{max} = 1.8 R_{SW}$, when the impact zones from individual filaments overlap with each other, would lead to an estimate $s_1 = 4.1 \mu m$.

In stage 4, we find the pulse duration $\tau$ for the case when the filament is formed by only one pulse. In Equation (21) we use the value $E_1 = 19.5 \mu J$ found on the previous stage, and also use the input parameters indicated above: $P_0 (\lambda = 1.04 \mu m) = 2.8$ MW [60] and $\Gamma = 1$–2. This gives $\tau = 3.5$–7.0 ps. For the LFC throughput using (93) for the found above $s_1 = 4.1 \mu m$, $H = 45 \mu m$ and $E_1 = 19.5 \mu J$, we get: $\Omega = 9.5 \mu m^2/\mu J$.

In stage 5 for the PRR $f = 75$ kHz mentioned above and extracted from experimental values and also for the calculated $E_1 = 19.5 \mu J$ we find the average laser power in the material as $P = E_1 f = 1.5$ W. Also, for the above $s_1$ and the same $f$, we find from (90): $u = s_1 f = 31$ cm/s. This estimate is in good agreement with the scan speed $u = 30$ cm/s reported in the experiment under discussion.

In the absence of a smooth separation in LFC for the given material, when it is only a matter of mechanical weakening this material in the irradiation zone regardless of the quality of the mechanical separation obtained after irradiation (i.e., not necessarily smooth), it seems that lower pulse energies and average beam powers can be chosen, but probably not more than about 1.5–2 times lower, compared to the $E_1$ and $P$ indicated here above and calculated as for smooth separation. Also, the parameter $\Gamma$, which describes the multiplicity of exceeding for the power of a single pulse $P_1$ over the critical power for self-focusing $P_c$ and is necessary for estimating the pulse duration according to the Formula (21), can be 2–3 times lower or higher, i.e., $\Gamma$ instead of 1–2 can be of the order of $\Gamma = 0.25$–5. If still the indicated above, calculated $E_1$ and $P$ are used, then the step of filaments following can also be chosen more than the found value $s_1 = 4.1 \mu m$ up to several tens of percent.

For the given model calculations, the shock-wave mechanism of the impact zone formation around each filament has been considered as the dominant. However, an alternative thermo-deformation mechanism gives a practically comparable contribution, as this mechanism has a very close modification threshold: according to (68) $w_{mod}^{TD} \approx 2.4$–3.6
kJ/cm², while \( w_{\text{min}}^{w} = 2.4-3.0 \) kJ/cm². Therefore, the radial transfer of energy from the filament to the surrounding material due to thermal diffusion, which is characteristic of this alternative mechanism, is also important for obtaining separation. In this relation, when choosing the pulse repetition frequency \( f \) to obtain more regular and more straight filaments, it seems preferable, as in the case of the thermal mechanism, to use frequencies in the two found frequency ranges (54) and (55): \( f \geq f_i \) and \( f \leq f_2 \), or according to (78) \( f \geq f^* \).

In this case, when \( s_i/R = 1.8 \), according to Table 2 the frequency limits are \( f_1 = 13.3\chi/s_i^2 \) and \( f_2 = 0.547\chi/s_i^2 \). For the thermal diffusivity of sapphire at high temperatures \( \chi = (1.5-2.4) \times 10^{-2} \) cm²/s [57] and the above step \( s_i \), we find: \( f_1 = 1.2-1.9 \) MHz and \( f_2 = 49-78 \) kHz. From this, it can be seen that the repetition rate \( f = 75 \) kHz extracted above from the experimental data is close to the \( f_1 \) limit of the low-frequency range (53), i.e., \( f = f_1 \).

The results of calculations in comparison with the experimental data are summarised in Table 5. It can be seen that the calculated \( E_i, s_i, P \) and \( u \) are in good agreement with the values obtained from the experiment when \( f \) is chosen as 75 kHz, i.e., close to the calculated limit \( f_1 \) of the low-frequency range.

### Table 5. Comparison of experimental [20] and theoretical values of the \( E_i, s_i, P \) and \( u \) parameters for the LFC sapphire.

|         | Pulse Energy \( E_1, \mu \) | Pulse Duration \( \tau, \text{ps} \) | Scanning Step \( s_i, \mu \) | Repetition Rate Limit \( f_2, \text{kHz} \) | Repetition Rate \( f, \text{kHz} \) | Average Power \( P, \text{W} \) | Scanning Speed \( u, \text{m/s} \) |
|---------|-----------------------------|-------------------------------|-----------------------------|---------------------------------|---------------------------------|-----------------------------|-----------------------------|
| Experiment [20] | 26.7 | - | 4 | 75 | <2 | - | 2.4–3.0 kJ/cm² |
| Model   | 19.5 | 3.5–7.0 | 4.1 | \( f_2 = 49–78 \) kHz | 75 | 1.5 (\( f = 75 \) kHz) | 0.3 |

However, the model also predicts, in this case, another, high-frequency range (54), \( f \geq f_i \), which has not been studied in the experiment [20]. If we choose \( f \) in this range, for example, as \( f = 2.5 \) MHz (i.e., \( f \geq f_i = 1.2–1.9 \) MHz), then according to the expression \( P = Ef \), using the above value \( E_1 = 19.5 \) \( \mu \), we find a much higher required average laser beam power \( P = 49 \) W than \( P = 1.5 \) W estimated above in the low-frequency range. The scanning speed of the beam relative to the material according to (90) at such \( f \) and \( s_i \) is very large: \( u = s_i f = 10.3 \) m/s. If we choose \( f \) even more, for example, \( f = 6 \) MHz, then with the same \( s_i \) we find \( u = s_i f = 24.6 \) m/s and \( P = 117 \) W.

These particular numbers show that the high-frequency range \( f \geq f_i \) provides for high, in particular, for a sapphire—very high, cutting speeds of 25 m/s and more, which by tens and hundreds of times exceed cutting speeds of glass, sapphire or other transparent materials with known mechanical methods and by tens times exceeds the characteristic rate of separation with the known methods of thermo-cleaving by irradiation with continuous-wave lasers. However, for such high-speed LFC modes, a relatively high average laser power of tens of W is required. At the same time, it can be seen that in the low-frequency mode, the laser separation requires a much lower average power of the laser beam (for example, as we estimated above and as confirmed by the experiment, for sapphire—at the level of 1.5 W), at which, nevertheless, the processing separation speed is provided, although not a very high, but acceptable for practical applications, specifically—about 0.3 m/s. Due to the correct choice of the repetition frequency for this material, if not a smooth separation, but, as already mentioned above, increased uniformity and straightness of the filaments is obtained.

#### 4.2. LFC of Tempered Glass

In the experiment [5,6], a smooth laser cutting of chemically-tempered Corning® Gorilla® Glass 5 with a thickness of 550 \( \mu \)m has been observed at speeds up to 4 m/s. The depth of the hardened layer (DOL, depth of layer) on the front and back of the plate was 20 \( \mu \)m. According to the reports of these authors, the laser source (Spirl® HE laser, from Spectra Physics, Rankweil, Austria) had pulse energies of >120 \( \mu \)J and average output powers of > 16 W. The laser offers also process flexibility at the wavelengths of 520 and
1040 nm with programmable pulse energy, repetition rate, and pulse width between 340 fs and 10 ps. The femtosecond pulses can be also supplied in a burst train with a controlled number of pulses up to 20 and with the time spacing between the pulses in the burst around 13 ns. Besides chipping free edges, the quality of a laser cut (defined by the average roughness of the cross-section) of < 0.1 μm has been achieved using the developed at Spectra-Physics process ClearShape™. This process is not disclosed in the papers [5,6], but authors report that at a wavelength of 1040 nm it allows glass cutting by using relatively low average power of < 4 W.

We will try now to explain these experimental parameters of radiation, which actually provide for this material a smooth separation, based on the proposed model. Note that according to the model, chemically tempered glass is preferred for obtaining smooth separation by its properties, since, as the criteria (86) and (88) necessary for obtaining a separation with better smoothness for amorphous material are met (see Table 2). It means that the predominant mechanism of the impact zone formation around the filament for this material is the thermal, rather than the shock-wave one. When calculating the irradiation parameters \( E_s, \tau, s_i, f, P \) and \( u \) necessary for smooth separation, we follow below the algorithm scheme shown in Figure 7.

The radius of the filament in this experiment is not reported. However, as in the case of sapphire in Section 4.1, it can be estimated theoretically: for \( M_f = 1.25, \lambda = 1.04 \mu m \) and the numerical focusing aperture \( NA = 0.6 \) according to (19), with \( q = 1 \), as discussed, we obtain \( \eta = 0.49 \mu m \). In the absence of data on the length of the filament in the experiment under consideration, we take \( H = 45 \mu m \), as for the LFC of sapphire in Section 4.1 above. For the tempered glass, we take \( w_{dis} = 64 \text{kJ/cm}^3 \) [52], \( P_f (\lambda = 1040 \text{nm}) = 5 \text{MW} \) [19] and \( I_f = 1–2 \). We also put \( A_{L-P} = 0.8 \) and \( A_{F-H} = 0.9 \), as discussed earlier. For the thermal mechanism in this case, \( \gamma_{tw} = 1/e = 0.37 \). It is also assumed for estimates, as above for sapphire, that \( K_{sw} \approx K_{os} \), and hence, taking into account the relation (14), \( K_{sw} \approx 0.5 \). Reference data from Table 2 are used in the model: \( \rho = 2.43 \text{g/cm}^3, T_{strain} = 571 °C \) [55], \( T_i = 20 °C \) and \( C_2 = 0.94–1.0 \text{ J/(g×K)} \). According to Figure 7, all these data are used as input parameters.

Next, in stage 1, we select VED bounds as \( w_1 = 336 \text{kJ/cm}^3 \) and \( w_2 = 396–480 \text{kJ/cm}^3 \) according to calculations given in Table 1 for this material and for a very close value of \( \eta = 0.5 \mu m \). For the thermal modification mechanism, which is the dominant one, in this case, we choose for a certainty the high-frequency range of pulse repetition frequencies, i.e., \( f \geq f_i \). The exact value of the limiting frequency \( f_i \) will be found below.

In stage 2, according to Table 1, we choose the VED of the filament plasma as \( w = 270 \text{kJ/cm}^3 \), which corresponds to the fulfilment of the condition (12) \( w \leq w_1 = 336 \text{kJ/cm}^3 \), at which, according to our model, there is a significant suppression of the filament energy losses for thermal emission, thus providing for the driving force and efficiency of the LFC process. To calculate \( w_{mod}^T \), we use the heat preservation factor \( \eta = 0.76 \), which, as noted above, corresponds to the choice of the repetition frequency \( f = f_i \) in the high-frequency range. The value of the “low-temperature” heat capacity is taken as \( C = C_2 = 1.0 \text{ J/(g×K)} \), which in the indicated high-frequency range according to (60) corresponds to a heating by the value \( \Delta T_i = (T_{strain} - T_i)/(1 + \eta) = 313 °C \), i.e., from \( T_i = 20 °C \) to 333 °C. When substituting the data \( C, T_i, T_{strain} \) and \( \rho \) into (66), we get \( w_{mod}^T = 0.76 \text{kJ/cm}^3 \).

In stage 3, we calculate the energy of the laser pulse in the material \( E_1 \), which is not directly reported in this experiment [5,6]. From (17), we get \( E_1 = 11.5 \mu J \), and from (66), \( S_{max} = 6.6 \mu m \). To ensure partial overlap of the impact zones, we further select for a certainty the step of filaments following, as noted earlier, \( s_i = 0.9 S_{max} \), which gives \( s_i = 5.9 \mu m \).

In stage 4, the calculation of the pulse duration according to (21) (for a filament created by only one pulse) at the above \( E_1, T_f \) and \( P \) gives \( \tau = 1.15–2.3 \text{ps} \). But if the filament is created by a burst of pulses, for example, from \( N = 6 \) pulses, then the energy of a single pulse from the burst \( E_0 \) can be estimated approximately as \( E_0 = E_1/N \), which gives \( E_0 = 1.92 \mu J \), and the duration of a single pulse from the burst according to (21) will then be \( \tau_0 = \tau/N \), i.e., \( \tau_0 = 192–384 \text{ fs} \). This estimate is consistent with the experiment under discussion, where, as the authors indicate, pulses with \( \tau < 400 \text{ fs} \) have been used. For such a ratio \( s_i \)}
The calculated parameters are summarised in Table 6. As can be seen, they are in good agreement with the experimental values for the smooth LFC of chemically tempered Corning® Gorilla® Glass 5.

In stage 5, we choose the PRR as \( f = f_1 \), i.e., \( f = 153–191 \) kHz. The speed of the relative motion of the beam and the material \( u \) is calculated according to (90), which, when substituting the indicated \( s_i \) and \( f_1 \), gives \( u = 0.90–1.13 \) m/s. The average power of pulse-periodic laser radiation in the material \( P = E_1 f \) when using the pulse energy found above in the material \( E_1 \) and the indicated PRR \( f \) for one set of filaments will be \( P = 1.76–2.20 \) W. With a higher PRR \( f = 700 \) kHz and the same \( s_i \) and \( E_1 \), respectively, we get \( u = 4.1 \) m/s and \( P = 8.1 \) W. The obtained range \( u = 0.90–4.1 \) m/s is consistent with the speeds reported by the authors of the experiment: \( < 4 \) m/s.

However, a real separation of chemically tempered glass requires the creation of two sets of filaments—one usually is placed near the backside of the plate, and the other closer to the front side. Moreover, to obtain the smoothness of the separation as a whole, and not only between filaments of one array, it is important that at least one of these arrays of filaments covers both the hardened layer itself and also enters the area of the non-hardened remaining bulk material [14]. With the width of the set of filaments \( H = 45 \) \( \mu \)m considered here, which exceeds the thickness of the hardened surface layer of 20 \( \mu \)m, this is possible. Therefore, when creating not one, but simultaneously two these arrays of filaments in the material, for example, using the multi-focusing method, the required total pulse energy and average power are, respectively, twice as much as those indicated above, i.e., \( E_1 = 23 \) \( \mu \)J and \( P = 3.5–4.4 \) W. This estimate closely corresponds to the average power reported by the authors of the experiment for the separation of this plate: \( P < 4 \) W. Besides, for the higher PRR \( f = 700 \) kHz above mentioned, when \( u = 4.1 \) m/s, for the same \( E_1 = 23 \) \( \mu \)J, we find for the average laser power \( P = E_1 f \) the value 16.1 W, which is also close to the value \( > 16 \) W mentioned in Refs. [5,6].

If each filament is created by a burst of pulses, then for the burst heating to be effective, its duration \( T_{\text{burst}} \) should not exceed the time \( f_1 \) for heat confinement in the filament with the radius \( r_f \) mentioned above in (4), i.e., \( T_{\text{burst}} < r_f^2/(4\chi) \) [39]. For the \( r_f \) and \( \chi \) indicated here, we obtain then an estimate: \( T_{\text{burst}} < 120–150 \) ns. Given that for \( N \) pulses in the burst, the distance between them in time is estimated by the obvious ratio as \( \Delta t = T_{\text{burst}}/(N - 1) \), for the number of pulses in the burst \( N = 6 \), we then find \( \Delta t < 24–30 \) ns, and for \( N = 10 \), we find respectively \( \Delta t < 13.3–16.7 \) ns. Both of these estimates of \( \Delta t \) are in a good agreement with the intervals between pulses \( \Delta t = 13 \) ns in the burst used for this laser source in the discussed experiment. The calculated parameters are summarised in Table 6. As can be seen, they are in a good agreement with the experimental values \( \tau, P, u \) and \( \Delta t \).

### Table 6. Comparison of experimental [5,6] and theoretical values of the parameters \( \tau, P, u \) and \( \Delta t \) for the smooth LFC of chemically tempered Corning® Gorilla® Glass 5.

|        | Pulse Energy \( E_1, \mu J \) | Pulse Duration \( \tau, \) fs | Scan Step \( s_1, \mu m \) | Repetition Rate \( f, \) kHz | Average Power \( P, \) W | Average Speed \( u, \) m/s | Interval between Pulses in the Burst, \( \Delta t, \) ns |
|--------|-------------------------------|-------------------|-------------------|-----------------|-----------------|-----------------|-----------------|
| Experiment [5,6] | - | 400 | - | - | 4 | 4 | 13 |
| Model | 23 | 192–384 | 5.9 | 153–191 | 3.5–4.4 | 0.90–1.13 | 700 | 16.1 | 4.1 | <13–17 |

For comparison, we calculate for this material the throughput of the LFC \( \Omega \) for the most favourable conditions of maximising \( \Omega \) indicated above in Section 3.4. To do this, as mentioned above, we also apply a twice shorter wavelength \( \lambda = 0.52 \) \( \mu \)m and assume that according to (19), the filament radius at the same \( NA = 0.6 \) becomes \( r_f = 0.244 \) \( \mu \)m, i.e., also decreases by half compared to the previously used value \( r_f = 0.49 \) \( \mu \)m for \( \lambda = 1.04 \) \( \mu \)m. We use VED bounds \( \omega_1 = 400 \) \( \text{kJ/cm}^3 \) and \( \omega_2 = 396–480 \) \( \text{kJ/cm}^3 \) according to calculations given
in Table 1 for a very close value of $r_f = 0.25 \mu m$. We use the value of $w$ based on the condition indicated above in Section 3.4 $w = 2u_{	ext{dis}}$, i.e., $w = 128 \text{kJ/cm}^3$. In this case, as can be seen, the condition (12) $w \leq \omega \nu$ is fulfilled, under which, according to our model, there are no significant energy losses from the filament to thermal emission, providing thus for the driving force and efficiency of the LFC process. Also, following the condition (iii) in Section 3.4, we choose $f$ in the high-frequency range (54), for example, as $f = 3f_i$. For so high $f$, $f >> f_i$, we put according to (61) $\eta \approx 1$. Such $\eta$, as follows from (60), corresponds to a single pulse heating at the edge of the modification zone by the value $\Delta T_i = (T_{\text{strain}} - T_i)/2 = 275 ^\circ C$, i.e., from $T_i = 20 ^\circ C$ to $295 ^\circ C$. For such a temperature range, we take the value $C_2 = 0.98 J/(g*K)$ as the “low-temperature” heat capacity $C = C_2$. Then the modification threshold when substituting these $C_s$, $\rho$ and $\Delta T_i$ into (66) is $w_{\text{mod}}^T = 0.655 \text{kJ/cm}^3$.

According to (17) for the indicated $w$, $r_f$, $A_s$ and the filament length, as above, $H = 45 \mu m$, the pulse energy in the material is $E_1 = 1.35 \mu J$. Using the same $w_{\text{dis}}$, $\gamma_r$, $K_{\text{ae}}$ and $A_{\text{f}}$, $h_2$, as above in this section, according to (66) we find the step as $s_i = 0.99_{\text{max}} = 7.24 \mu m$. From (93) for such $s_i$, $E_1$ and $H$, we find $\Omega = 59 \mu m^2/\mu J$. For comparison, we recall that for the above-given example of the LFC of tempered glass at $\lambda = 1.04 \mu m$, the LFC throughput was only $\approx 23 \mu m^2/\mu J$, which is 2.6 times less.

In this case, the calculation of the pulse duration according to (21), if each filament is created by only one pulse with the indicated energy $E_1 = 1.35 \mu J$ at $f = 1-2$ and $P_1(\lambda = 520 \text{nm}) = 1.3 \text{MW}$, gives $\tau = 0.52-1.04 \text{ps}$. But if the filament is created by a burst of pulses, for example, from $N = 6$ pulses, as in the above example, then the duration of a single pulse from the burst is $\tau_0 = \tau/N$, i.e., $\tau_0 = 86-172 \text{fs}$.

For the indicated above step $s_i = 0.99_{\text{max}} = 1.77 \mu m$, which corresponds to the ratio $(s_i/R) = 1.8$, we find the limit $f_i$ for this ratio according to Table 3 as $f_i = 13.3 \chi/s_i^2$. For the above-mentioned thermal diffusivity $\chi$ of tempered glass and $s_i = 1.77 \mu m$ this relation gives $f_i = 1.70-2.12 \text{MHz}$.

The PRR is chosen for certainty as $f = 3f_i$, i.e., $f = 3 \times 2.12 = 6.36 \text{MHz}$. For the average laser power in the material $P = Ef_i$ with indicated $E_1$ and $f$, we get $P = 8.6 \text{W}$. When creating in the material not one, but, as noted above, two arrays of filaments simultaneously, for example, by the multi-focusing method, the resulting pulse energy and average power are required, respectively, 2 times greater, i.e., $E_1 = 2.70 \mu J$ and $P = 17.2 \text{W}$. The relative speed of the beam and material $u = sf_i$ for indicated $s_i$ and $f$ is $u = 11.3 \text{m/s}$.

As we can see, the indicated irradiation mode with almost half the $w$, twice the shorter wavelength $\lambda$ and PRR in the high-frequency range, in addition to maximising throughput for a smooth LFC of tempered glass, provides also for a very high cutting speed, 11.3 m/s, which exceeds the speeds of previously known laser and non-laser glass cutting methods by tens and hundreds of times.

### 4.3. LFC at very Low Average Laser Power

It is also of interest to find such LFC modes that at extremely low average laser power and low pulse energy allow, nevertheless, obtaining a cutting speed which is although not very high, but still acceptable for practical applications.

We search for such a mode in the low-frequency range PRR, in which, according to the ratio $P = Ef$, the average laser power in the material $P$ is small due to a significantly lower repetition frequency $f$ compared to the high-frequency range considered above. In this case, to obtain the maximum speed $u$, we use according to (90) the maximum PRR in this range, i.e., as follows from (55), $f = f_s$. As follows from Equation (91), a further increase in $u$ is possible by reducing the step of following $s_i$. To do this, according to (66), we use, as in the example above, a smaller filament radius $r_f$ (for example, as follows from (19), $r_f = 0.244 \mu m$ due to the use of a twice shorter wavelength $\lambda = 520 \text{nm}$). And we also use a smaller filament VED $w$, for example, $w = (0.9-1)2w_{\text{dis}}$, i.e., $w = (115-128) \text{kJ/cm}^3$, instead of $w = 270 \text{kJ/cm}^3$ considered above in the interpretation of the experiment with the LFC of tempered glass at the wavelength 1040 nm.
To calculate particular parameters of such LFC, we follow, as above, the calculation algorithm shown in Figure 7. As input, we use the parameters of tempered glass indicated in the example above. A stage-by-stage (1, 2, ..., 5) calculation of unknown irradiation parameters according to the scheme gives the following results:

1) We take VED bounds as \( w_1 = 400 \text{ kJ/cm}^3 \) and \( w_2 = 396-480 \text{ kJ/cm}^3 \) according to calculations given in Table 1 for a very close value of \( \eta = 0.25 \mu \text{m} \).

2) The VED \( w \) is taken as \( w = 1.88 \text{ kJ/cm}^2 = 115 \text{ kJ/cm}^3 \). In this case the condition (12) \( w \leq w_1 \) is evidently fulfilled, corresponding to a significant elimination of filament energy losses for thermal emission and thus providing for a driving force and an efficiency for LFC. We put \( \eta = 0.1 \) for the considered low-frequency range, and also take \( C = C_2 = 1.0 \text{ J/(g*K)} \). The modification threshold according to (64) is then \( w_{mod} = 1.22 \text{ kJ/cm}^3 \).

3) The pulse energy in the material for the same filament length \( H = 45 \mu \text{m} \) as in Section 4.2 is from (17) \( E_1 = 1.21 \mu \text{J} \), and the step of filaments following is from (66) \( s_1 = 0.9 s_{lim} = 4.74 r_1 = 1.16 \mu \text{m} \).

4) LFC throughput from (93) is \( \Omega = 4.3 \mu \text{m}^2/\mu \text{J} \), which is noticeably lower than the value \( \Omega = 59 \mu \text{m}^2/\mu \text{J} \) obtained above in the high-frequency mode. From Table 3, for the limit \( f_s \), we find \( f_s = 0.547 x/s_i^2 \), which gives \( f_s = 162-203 \text{ kHz} \) for the above thermal diffusivity \( \chi \) of this material.

5) We take \( f = 162 \text{ kHz}, \) i.e., \( f \leq f_s \). We then finally find the average laser power in the material and the relative velocity of the beam and target: \( P = 196 \text{ mW} \) and \( u = 0.19 \text{ m/s} \).

As we can see, a proper choice of irradiation parameters allows LFC with an acceptable (though not very high) speed of smooth separation, reaching about 0.2 m/s, when using a fairly small average laser power of about 200 mW and a very small pulse energy in the material of the order of 1 \( \mu \text{J} \). For the LFC of tempered glass, when, as we saw above, it is required to simultaneously create two arrays of filaments in the thickness of the material, these figures will double, respectively, to about 400 mW and 2 \( \mu \text{J} \).

4.4. LFC of Thick Plates

For LFC thick materials (i.e., with a thickness \( L \) of several millimetres or even centimetres), it may be necessary to create correspondingly many times longer filaments. As we saw above in the examples for LFC sapphire and tempered glass, the total length of the filaments \( H_L \) for separation was from \( 0.13 \text{ L} \) to almost \( L \). If this ratio is maintained for the LFC of thick materials, for example, with a thickness of \( L = 10 \text{ mm} \), this corresponds to the total length of about \( H_L = 1.3-10 \text{ mm} \) of filaments with one or more of their arrays in the material depth. The methods discussed above in the Introduction, which use axicon focusing, Bessel beams, and multi-focusing, make it possible to obtain such long filaments of several millimetres—see, for example, [4,17,18]. Then scaling of the results obtained for \( \lambda = 0.52 \mu \text{m} \) calculations with an increase in the filament length from 45 \( \mu \text{m} \) to, for example, \( H_L = 10 \text{ mm} \) for thick plates, i.e., by 222 times, according to the obtained formulas, if we assume that the filament has the same radius \( r_1 = 0.244 \mu \text{m} \), leads to an increase in the same number of times of the pulse energy in the material, i.e., from 1.21 \( \mu \text{J} \) to \( E_s = 269 \mu \text{J} \), and the average power in the material from the above value of 196 mW to \( P = 43.5 \text{ W} \) at the same \( s_1 = 1.16 \mu \text{m}, f = 162 \text{ kHz} \) and \( u = 0.19 \text{ m/s} \).

However, if we use the parameters calculated above for irradiation at the wavelength \( \lambda = 1.04 \mu \text{m} \), which were obtained when comparing the model with the experiment [5,6] for a smooth LFC of chemically tempered Corning® Gorilla® Glass 5 (see Table 6), then with the same increase in the filament length to 10 mm (by 222 times), we obtain for the same \( w = 270 \text{ kJ/cm}^3, s_1 = 5.9 \mu \text{m}, f = 153-191 \text{ kHz} \) and \( u = 0.90-1.13 \text{ m/s} \), that the pulse energy in the material increases by the same number of times from the previously obtained value of 11.5 \( \mu \text{J} \) to \( E_s = 2550 \mu \text{J} \). The average laser power in the material increases from the above value of 1.76–2.20 W to \( P = 390-490 \text{ W} \), if we assume that the filament has the same radius \( r_1 = 0.49 \mu \text{m} \) as in the experiment [5,6]. This average laser power for \( \lambda =
1.04 μm is by the order of magnitude greater than $P = 43.5$ W obtained above for $\lambda = 0.52$ μm.

5. Conclusions

In this paper, an analytical model of laser filamentation cutting (LFC) of transparent materials is proposed, which predicts the process parameters for achieving high cutting speeds (of several m/s), as well as a mode important for practical applications—the so-called “smooth” laser cutting, when the surface of the sidewalls of the cut has a better smoothness and practically does not contain roughness with periods of less than a few microns. As a result, laser-cut plates have an increased resistance to damage and cracking due to mechanical impacts during further use, which is important in applications, for example, as mobile phone displays, car windshields, etc.

Such parameters quantified in the model are the radiation parameters—the step of filaments following in the material $s_1$, the pulse repetition frequency $f$ of the laser (in addition to the other two necessary parameters analytically calculated in our previous work—the pulse energy $E_1$ in the material and the pulse duration $\tau$). As the model shows, $E_1$, $\tau$, $s_1$, and $f$ are a function of the radius $r_f$ and the length $H$ of the filament, the volumetric energy density $w$ of the filament plasma, the bounds $w_1$ and $w_2$, which determine the ranges of $w$ for an elimination of the energy losses of the filament to thermal radiation, as well as the threshold $w_{\text{mod}}$ for the modification critical for obtaining separation of the material.

According to the model, obtaining a cut with a better smoothness is determined by several necessary conditions, the implementation of each of them requires a certain choice of radiation and material parameters. These necessary conditions are:

- Obtaining high energy efficiency of the separation process by eliminating the energy losses of the filament due to thermal radiation. To do this, the pulse energy $E_1$ in the material according to the model is chosen in such a way that the plasma of the filament is pumped to the VED $w$, which satisfies the condition $w \leq w_1$, or $w \geq w_2$. The limits of $w_1$ and $w_2$, for which usually $w_1 < w_2$, are determined in the model by the thermal and optical properties of the material and are numerically of the order of several hundred kJ/cm³;
- Ensuring increased uniformity of the diameter of each emerging filament along its length. For this purpose, the pulse duration $\tau$ is matched with the pulse energy $E_1$ so that the power of a single laser pulse $P_i = E_1/\tau$ is approximately equal to the critical power $P_c$ for self-focusing, i.e., $P_i = \Gamma P_c$, where $\Gamma = 1–2$;
- Creation in the volume of the material of a continuous modified layer from the filament to the filament with the value of the volumetric energy density released in it (in [J/cm³]) not lower than the minimum (or threshold) value $w_{\text{mod}}$ necessary to obtain a separation. For this purpose, the step $s_1$ of filaments following in the material is selected in such a way that the zones of modification critical for separation from neighbour filaments join or even overlap with each other;
- Formation of straight filaments of a regular shape in each of their arrays by minimising their influence on each other, i.e., minimising the heating and temperature gradient from the previous filament at the location and at the time of occurrence of the next, new filament. To do this, the PRR $f$ of laser pulses should be selected in one of two ranges found in the model, conventionally called as high-frequency range, $f \geq f_1$, and low-frequency range, $f \leq f_2$, where the limits $f_1$ and $f_2$ differ by several tens of times with $f_1 > f_2$;
- Formation of the impact zone from the filament without destruction in the form of scattered cracking in the solid material as a result of laser irradiation. This requires the dominance of the thermal mechanism in the formation of the impact zone radius, but not of the alternative shockwave or thermal deformation mechanisms, which just lead to scattered cracking formation in the solid material, excluding thus a smooth
separation after irradiation. As the model shows, the dominance of the thermal mechanism occurs when two dimensionless criteria for material and radiation parameters are met simultaneously. It requires the use of a specific class of materials—tempered glasses, which corresponds to experimental observations of smooth cutting for these materials. On the other hand, these criteria are not met for sapphire and non-tempered glasses. In experiments, a smooth separation in LFC for these materials really is not observed.

Materials with a fairly long de-excitation time, \( t_E \gg t_2 \) (that is, \( t_E \gg 0.4–0.8 \) ns according to the estimates of the pressure confinement time \( t_P \) made in Section 3.1 for \( r_j = 0.25–0.5 \) μm) may also provide the dominance of the thermal mechanism—see the inequality (48). For such \( t_E \), a significant decrease in the amplitude of the shock wave and a corresponding decrease in the radius \( R_{SW} \) of modification zone due to the shock-wave mechanism is expected, which is preferable for obtaining a smoother separation.

The model searches for a set of four indicated parameters (\( E_1, \tau, s_1 \) and \( f \)) that allow us to fulfil all these necessary conditions for obtaining a smoother separation. In practice, these four parameters are sufficient to define completely the LFC process.

The LFC process is very complicated to model, as it is an effect of in a total of about 40 radiation and material parameters. We have proposed an algorithm for the sequential calculation of \( E_1, \tau, s_1 \) and \( f \) as a function of these numerous parameters.

A comparison of the calculation according to this algorithm with the LFC experiments for sapphire and tempered glass shows a good agreement for the entire set of \( E_1, \tau, s_1 \) and \( f \), which indicates the reliability and adequacy of the constructed model. Besides, from this comparison, it is found that the experimental values of VED \( w \) both for sapphire and tempered glass satisfy the inequality \( w < w_1 \). According to the condition (12) obtained, it corresponds to the practical elimination of the filament energy losses for thermal emission.

As a next step, the model is used to predict new modes of LFC that have a positive property in addition to better smoothness.

For example, it is shown that the choice of the PRR in the high-frequency range, \( f \geq f_1 \), and the use of twice shorter laser radiation wavelength as 1.04 μm, allow obtaining very high cutting speeds—about 1–25 m/s or more, depending on the material. These speeds can be tens or hundreds of times higher than the characteristic cutting speeds of glass, sapphire and other transparent materials by known mechanical methods, and tens of times higher than the characteristic separation speeds by known controlled thermal splitting methods using continuous-wave lasers.

On the other hand, it is found that the choice of the PRR in the low-frequency range, \( f \leq f_2 \), and the use of the shorter radiation wavelength (0.52 μm) make possible LFC with an extremely low average laser power—only about one or several hundred mW, which provides a cutting speed of about 0.2 m/s. This value, although not a very high, but nevertheless is acceptable for applications.

Besides, LFC modes that are characterised by high throughput are found, i.e., a high ratio of the cut-off area \( s_i H \) per pulse to the pulse energy \( E_1 \). It is shown that here, too, the reduction of the filament radius by almost half due to the use of a shorter wavelength than 1.04 μm, for example, \( \lambda = 0.52 \) μm, provides significant advantages in obtaining a higher LFC throughput. This also allows reducing by many times the average laser power for the LFC of thick plates with a thickness of several millimetres. Thus, it enables reducing the depth of the heat-affected zone along the sidewalls of the separation.

It is worth to note that the considered process of LFC of transparent materials is environmentally friendly since it occurs without the use of any technical liquids, abrasives, liquid or gaseous chemical etchants. Besides, when the filaments created in the material do not reach the front and/or back surfaces of the plate to be processed, there is practically no release of the material into the surrounding space in the form of vapours, droplets or solid fragments during the process.
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Conflicts of Interest: Page: 41

The authors declare no conflicts of interest.

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