Gamma-Ray Burst Afterglow emission with a decaying magnetic field

Elena Rossi & Martin J. Rees
Institute of Astronomy, University of Cambridge, Madingley Road, Cambridge CB3 0HA, England
e-mail:emr,mjr@ast.cam.ac.uk

ABSTRACT

In models for gamma ray burst afterglows, it is normally assumed that the external shock strongly amplifies the magnetic field and that this field maintains a steady value throughout the shocked region. We discuss the effects of modifying this (probably simplistic) assumption by allowing for a short decay time. The observations are incompatible with a post-shock field that decays too rapidly. However if the field pervades only a few percent of the total thickness of the shocked shell (and the electrons undergo only inverse Compton losses in the remainder) the model could be compatible with data. This would suggest a strong dependence of model parameters on the uncertainties of shock physics, therefore calling for independent external density estimates. We claim that afterglow emission should be instead seen as a laboratory where we can understand relativistic shock physics. The model we propose here together with afterglow observations in all wavebands could help to pin down the field structure.

Key words: Gamma–rays: burst — radiation mechanisms: non-thermal—magnetic field

1 INTRODUCTION

It is widely accepted that GRBs afterglow emission is due to non-thermal radiation mechanisms produced in relativistic shocks, as the ejected matter from a highly energetic explosion expands into the surrounding medium. With current facilities we can observe at photon energies up to 10 keV and the modeling of the observed lightcurves shows in general a predominance of synchrotron emission over inverse Compton scattering on the synchrotron photons. (Panaitec & Kumar 2001 (thereafter PK01)). Only in one case, GRB000926, has a Compton component been inferred (Harrison et al. 2001). This fact sets an upper limit on the density of the medium surrounding the explosion, whose properties can help us to unveil GRBs progenitors. In the standard framework the dynamics of the remnant expansion follows an adiabatic law. This behavior is expected when the electrons cannot cool on the expansion timescale or (even when they can) the bulk of the energy remains in relativistic protons and magnetic fields. The post shock magnetic field, necessary for the synchrotron emission, is mainly generated during the shock itself, because the shock compression of the pre-existing field alone would lead to a negligible magnetic energy per particle (e.g. Gruzinov 2001). The magnetic field is normally assumed to be uniform and its lengthscale to be of the order of the remnant scale. The electron energy distribution behind the expanding shock front is given by a power law, down to a minimum injection energy. Modeling eight afterglow lightcurves within this framework has resulted in a wide range of density from $\sim 10^{-3}$ to $\sim 27$ cm$^{-3}$, strongly indicating a low density environment (PK01). This is inconsistent with other independent observations placing GRBs birth in a star forming region: SN bumps in afterglow lightcurves (e.g. Bloom et al. 1999, Lazzati et al 2001, Bloom et al. 2002), metal lines (Antonelli et al 2000, Piro et al 2000, Reeves et al 2002), afterglow localizations close to star forming regions (Bloom, Kulkarni, Djorgovski 2002, Jaunsen et al 2002) and estimations of high density environment from the temporal evolution of external column density (Lazzati & Perna 2002). Another remarkable inference from this model is that the fraction of internal energy given to electrons ($\epsilon_e$) is always much larger then the fraction given to magnetic field ($\epsilon_b$). This implies that the magnetic field is relatively weak and the Compton emission dominates the overall cooling of the electrons.

Recently many publications have investigated the somewhat poorly understood generation of magnetic field in strong relativistic collisionless shocks (e.g. Kazimura et al 1998, Gruzinov and Waxman 1998, Medvedev & Loeb 1999, Gruzinov 2001). Kazimura et al (1998) found generation of a small-scale quasi-static magnetic field; similar results were found by Gruzinov (2001) at small simulation times but his longer run shows that the field quickly decays by Landau damping; as a result synchrotron emission could be insignificant. Medvedev & Loeb (1999) found more reassuring re-
results. Relativistic two-stream magnetic instability can, they claim, generate stable randomly oriented strong magnetic field in the plane of the collisionless shock front, so synchrotron emission is possible. These contradictory results show our ignorance of the possible acceleration process, and of how (or if) a stable magnetic field is created there. In the standard model this ignorance is "parameterized" through $\epsilon_e$ and $\epsilon_B$: the simplistic assumption is made that a persistent magnetic field makes these parameters uniform in space and time. An observation that maybe suggests a time dependence of $\epsilon_e$ and $\epsilon_B$ is the independent determination of these parameters in GRB 970508 afterglow by Wijers and Galama (1999) at 12 days and Frail, Waxman and Kulkarni (2000) at $\sim$ 1 year: while the former found $\epsilon_e = 0.12$ and $\epsilon_B = 0.089$ the latter inferred $\epsilon_e \geq \epsilon_B = 0.5$.

These uncertainties, coupled with the apparent problems with the standard model, motivate us to explore the consequences for afterglow emission of a different scenario, where the propagation of the magnetic field to large scale is disfavored. In this contest we particularly focus on the possibility that external density could be larger, keeping the total energy to a reasonable value, without loosing a general agreement with data. In fact, at this stage, we deal with sparse set of measurements, almost never simultaneous at all wavelengths. These data can be modeled in different frameworks, (e.g. GRB 970508 (Frail et al 2000, Chevalier & Li 2000), GRB 980519 (Frail et al 2000b, Chevalier & Li 1999), GRB 000301C (Berger et al 2000, Li & Chevalier 2001), GRB 991208 (Galama et al 2000, Li & Chevalier 2001 and GRB 000418 (Berger et al 2001)). This analysis is meant to investigate how robust are the current estimation of shock and density parameters.

In this paper we present this more general theory for the GRB afterglow emission (§2); our conclusion (§3) is that a qualitative comparison of this model with data suggests the possibility that a shorter magnetic lengthscale can be involved, affecting the estimates of parameters, but only a quantitative modeling of data can give the conclusive answer.

2 SHORT MAGNETIC LENGTHSCALE THEORY

The assumption of a post shock magnetic field persisting during all the expansion time has three main consequences for the synchrotron (S) emission and spectrum. First the emitting region linear dimension is $R \simeq c \times t_{\text{exp}}$, where $t_{\text{exp}}$ is the expansion time in the lab frame; this enters in the calculation of the observed peak flux $F_p$. Then the observed cooling frequency $\nu_c$ is computed using the Lorenz factor, $\gamma_c$, of the electrons that cool radiatively on a timescale equal to the remnant age. Finally the lengthscale for synchrotron self-absorption has to be compared with the dimension of the fireball in order to compute the observed absorption frequency $\nu_a$. The only break frequency which is not affected by any assumption about the extension of the magnetic field, is the observed peak frequency $\nu_m$, corresponding to the minimum random Lorentz factor $\gamma_m$: it depends crucially only on the fractions of internal energy given to electrons, $\epsilon_e$, and given to the magnetic field $\epsilon_B$. Throughout the entire emission volume electrons cool also via inverse Compton (IC) on the synchrotron photons (Self Synchrotron Compton: SSC). This cooling process dominates the electrons cooling if the Compton parameter $Y = L_{\text{IC}}/L_S$ (IC and S luminosity ratio) is greater than one. This happens if $\epsilon_b \ll \epsilon_e$ and current estimates find in most objects $\epsilon_b/\epsilon_e \sim 10^{-1} - 10^{-2}$, (Wijers & Galama 1999, Granot et al 1999, PK01).

If the magnetic field behind the shock persists for an average timescale of $t_p < t_{\text{exp}}$, its lengthscale is $\lambda \simeq c \times t_p < R$. In principle $\lambda$ could depend on time, however we do not have any consolidated understanding of this process, therefore for simplicity we will assume throughout this paper that $\lambda/R = \delta$ is constant during the afterglow phase. At each time the emitting region is then divided in two zones (see Fig 1). The first zone (region A) is the layer with dimension $\lambda$ just behind the shock front, where there are newly accelerated electrons and magnetic field: here electrons emit via S and SSC; the second zone (region B) of linear dimension $R - \lambda$, is filled by electrons that have just cooled by S and SSC emissions and that now Compton scatter the radiation coming from the first layer. We suppose that the newly accelerated electrons are advected with the flow; they produce synchrotron radiation only for a short time (with a cooling break therefore only at very high energies) in region A near the shock front and then they emit most of the IC radiation while downstreaming through region B. On the other hand if the electrons diffused freely, so that they could move back into the magnetized region after having lost most of their energy via IC, then the synchrotron spectrum would have a lower-energy break. This is probably less likely therefore we concentrate on the first case. The total observed spectrum then results from the sum of region A and region B spectra.

2.1 region A: S and SSC emission

The consequences of $\lambda < R$ on the S spectrum in region A can be derived easily from what we mentioned before. The observed peak frequency $\nu_m$ remains unchanged

$$\nu_m = \nu_m,$$

(1)
and
\[ \tilde{\gamma}_m \propto \tilde{\nu}_m^{1/2} = \gamma_m. \] (2)

In our generalized model with \( \delta < 1 \) we denote quantities by the symbol ‘\( q \)’, where ‘\( q \)’ is defined for \( \delta = 1 \), (for the explicit definitions of quantities in the standard model we refer to Panaitescu & Kumar 2000).

The comoving peak intensity is \( I' \propto \tilde{n}' B' \lambda' \), where \( \tilde{n}' \), \( B' \) and \( \lambda' \) are respectively the external density, the magnetic field and the emitting region linear dimension in the comoving frame; therefore the observed peak flux \( \tilde{F}_p \) is smaller than \( F_p \) by a factor \( \delta \):
\[ \tilde{F}_p \simeq \Gamma^3 (\tilde{n}' B' \lambda') \frac{R^2}{\Gamma^2} = \delta \times F_p, \] (3)
where \( \Gamma \) is the bulk Lorentz factor. Since the magnetic field persists for a shorter time, a smaller fraction of electrons cool significantly, before the magnetic field disappears in that region. The Lorentz factor of the electrons, whose cooling time is equal to the typical timescale of the system, \( \tilde{\gamma}_c \) is therefore higher:
\[ \tilde{\gamma}_c \propto \frac{1}{\nu_c B^2 (1 + Y)} = \gamma_c \times Y \delta^{-1}, \] (4)
where \( Y = (1 + \tilde{Y})/(1 + \tilde{\gamma}) \). The observed cooling frequency \( \tilde{\nu}_c \propto \Gamma^2 \tilde{\gamma}_c \) is then
\[ \tilde{\nu}_c = \nu_c \times Y^2 \delta^{-2}. \] (5)

The synchrotron self-absorption optical thickness can be approximated as
\[ \tilde{\tau}_a(\nu) \propto \left( \frac{\tilde{n} A}{\nu_c B^2 \nu_p} \right) (\nu/\tilde{\nu}_p)^{-5/3} = \tilde{\tau}_a(\nu/\tilde{\nu}_p)^{-5/3}, \] (6)
for \( \nu < \tilde{\nu}_p \), where \( p = c \) when the electrons are radiative (\( \tilde{\gamma}_c < \tilde{\gamma}_m \)) and \( p = m \) when they are adiabatic (\( \tilde{\gamma}_c \approx \tilde{\gamma}_m \)). We will refer to the first case as fast-cooling regime and to the second case as slow-cooling regime. The absorption frequency corresponds to \( \tilde{\tau}_a(\nu_a) = 1 \) and in slow cooling regime is given by
\[ \tilde{\nu}_a = \nu_a \times \delta^{3/5}, \] (7)

where we use Eq. 4 and Eq. 5 in fast cooling regime Eqs 4 and 5 \( (Y = 1) \) give
\[ \tilde{\nu}_a = \nu_a \times \delta^{8/5}. \] (8)

Eqs 4, 5 and 8 are calculated in the case \( \tilde{\nu}_a < \tilde{\nu}_p \) (see Granot & Sari 2001 for the different possible S spectra).

Nevertheless \( \tilde{\nu}_a \geq \tilde{\nu}_p \) for high densities
\[ \tilde{n} \geq 7 \times 10^2 \delta^{-1/2} E_{53}^{-12/10} (1 + \tilde{Y})^{-3-e_{b_2}}^{-27/10} \] (9)
in fast cooling and in slow cooling
\[ \tilde{n} \geq 8.2 \times 10^5 \delta^{1/2} E_{53}^{-1/2} \epsilon_{b_2} \epsilon_{e_2}^{-2} (1 + z) \tilde{\gamma}_{E_{day}}^{1/2} \] (10)
where \( E \) is the isotropic equivalent energy and \( \tilde{\gamma}_{E_{day}} \) is the observed time in days. The inferred external density is reasonable for \( \delta = 0.1 \) (\( \tilde{n} \geq 3.6 \times 10^9 \tilde{\nu}_m^{-3/2} \)) and \( \tilde{n} \geq 8.5 \times 10^6 \tilde{\gamma}_{E_{day}}^{1/2} \) respectively and for \( \delta = 0.01 \) (\( \tilde{n} \geq 2 \times 10^6 \tilde{\gamma}_{E_{day}} \) and \( \tilde{n} \geq 8.5 \times 10^7 \tilde{\gamma}_{E_{day}}^{-3/2} \)) but for lower \( \delta \) the density lower limits expressed in Eq. 4 and Eq. 5 are perhaps too high for the afterglow environment. For simplicity we focus thereafter on \( \tilde{\nu}_a < \tilde{\nu}_c \) but the reader should keep in mind that these cases hold as long as Eq. 4 and Eq. 5 are not satisfied.

The magnetic energy \( B^2 \propto n \Gamma^2 \) and the Compton parameter are higher at the beginning of the afterglow evolution and so the electrons are more likely then to be radiative. Since the minimum injected \( \gamma_m \propto \Gamma^2 \) decreases with time while \( \tilde{\gamma}_c \) increases as the efficiency of radiative loss decreases, the electrons undergo a transition to the adiabatic regime. The transition time between the fast cooling and the slow cooling regime is
\[ \tilde{T}_{fs} = T_{fs} \times \delta^2; \] (11)
for
\[ \tilde{n} < 40 \delta^{-2} E_{53}^{-1} (Y_F + 1) - \epsilon_{B_2}^{-2} \epsilon_{e_2}^{-2} (1 + z)^{-1} \tilde{\gamma}_{E_{day}}^{-1} \] (12)
the slow-cooling regime dominates the fireball evolution during times when the afterglow is observed \( (1 \leq 1 \) day).

The accelerated electrons that upscatter synchrotron photons give rise to a SSC component in the spectrum (for the SSC in the standard model see Sari & Esin 2000). During the afterglow phase the medium has a Thomson optical depth
\[ \tilde{\tau} = \frac{1}{3} \tilde{n} \sigma_T R \delta \] (13)
of the order of \( \tilde{\tau} \simeq 10^{-5} \delta \) for \( \tilde{n} = 10^2 \text{cm}^{-3} (\sigma_T \equiv \text{Thomson cross section}, \text{therefore only a minor fraction of the emission is upscattered, and the synchrotron flux is not significantly altered. Nevertheless if } \tilde{Y} > 1 \text{ the electron cooling is Compton dominated and SSC upscatters low energy photons, that might be detected as a Compton component in x-ray and, in the future, in higher energy band spectra (Sari, Narayan & Piran 1996). The parameter } \tilde{Y} \text{ can be defined as the mean number of scatterings times the average fractional energy change per scattering, gained by a photon traversing a finite medium (Rybicki & Lihtman 1979). For a Thomson thin, relativistic medium } \tilde{Y} \text{ is}
\[ \tilde{Y} \simeq \frac{4}{3} \int \tilde{\gamma}^2 N(\tilde{\gamma}) d\tilde{\gamma}; \] (14)

where \( N(\tilde{\gamma}) \) is the normalized \( \tilde{\gamma} \) distribution (see e.g. PK00 for explicit formulae for \( \gamma_a < \gamma_p \)). For \( \tilde{Y} > 1 \) one may need to consider more than one scattering per photon; however the energy of a photon in the rest frame of the second scattering electron is a factor \( \gamma^3 \) higher than the original energy, generally exceeding the Thompson regime limit (511 keV); the scattering cross section is then substantially reduced (Klein-Nishina (KN) regime) and further scatterings inhibited. As a consequence we will consider only single scattering of S photons. The KN effect limits also the electrons that mostly contribute to the SSC luminosity. They have a Lorentz factor
\[ \tilde{\gamma} \leq \tilde{\gamma}_{KN} \simeq 511 \Gamma/\tilde{n} \tilde{\gamma}_{E_{fp}}; \] (15)
where \( \tilde{\gamma}_{E_{fp}} \) is the S energy peak frequency (\( \tilde{\gamma}_c \) in slow cooling and \( \tilde{\nu}_m \) in fast cooling). In zone A the S spectrum can have a very high \( \tilde{\gamma}_c \) (see Eq. 5); consequently in slow cooling \( \tilde{\gamma}_c \) could be greater than \( \tilde{\gamma}_{KN} \) and the Klein-Nishina effect substantially decreases \( \tilde{Y} \) (we will refer to this situation as “zone A in KN regime”). Moreover if \( \tilde{Y} \simeq 1 \) (which is the case in most simulations) we can omit the contribution from \( \tilde{\gamma} > \tilde{\gamma}_{KN} \) Eq. 14 for \( 2 < p < 3 \) gives then
\[
Y_S (1 + Y_S) = Y_S (1 + Y_S) \times \delta^{(p-2)},
\]
for \( \gamma_c \leq \gamma_{KN} \) and
\[
Y_S (1 + Y_S)^{2(p-3)} = Y_S (1 + Y_S)^{2(p-3)} \times \delta^{(7-3p)},
\]
for \( \gamma_c \leq \gamma_{KN} \). If the electrons are in the fast cooling regime it is very unlikely that \( \gamma_m > \gamma_{KN} \) and Eq. 14 gives
\[
\tilde{Y}_F = \frac{3}{4} \tilde{\gamma}_m \tilde{\gamma}_c = Y_F;
\]
note that \( \tilde{Y}_F \) does not depend on \( \delta \), because most of the electrons cool before the field disappears. Given a S seed spectrum \( \tilde{F}_S(\nu) \), the resulting single scattering IC spectrum is (Rybicki & Lightman, 1979)
\[
\tilde{F}_p^{IC} = A \int_0^1 \tilde{F}_S \left( \frac{\nu}{4\gamma^2 X} \right) f(X) dX \int_{\tilde{\gamma}_1}^{\tilde{\gamma}_2} N(\tilde{\gamma}) d\tilde{\gamma},
\]
where \( A = 3\sigma_T R \delta \) (in this model), \( X = \nu / 4\gamma^2 \nu_a \) and \( f(X) = 2X \ln X + X - 2X + 1 \) includes the exact cross section angular dependence in the limit \( \gamma \gg 1 \) (Blumenthal and Gold, 1970). For the cases we are treating it is possible to use for the first integral the analytic expressions given by Sari & Esin (2000) in their Appendix A. The IC spectral peak flux and break frequencies for \( \delta < 1 \) are related to the IC spectrum for \( \delta = 1 \) in the following way
\[
\tilde{F}_p^{IC} \approx \tilde{\tau}_B \tilde{F}_p = F_p^{IC} \times \delta^2
\]
using Eq. 13 and Eq. 14
\[
\tilde{\nu}_m^{IC} \approx 2\tilde{\gamma}_m^{-2} \nu_m = \nu_m^{IC},
\]
using Eq. 13
\[
\tilde{\nu}_c^{IC} \approx 2\tilde{\gamma}_c^{-2} \nu_c = \nu_c^{IC} \times \left( \frac{Y \delta^{-1}}{4} \right)^{\frac{1}{3}},
\]
using Eq. 13
\[
\tilde{\nu}_a^{IC} \approx 2\tilde{\gamma}_a^{-2} \nu_a = \nu_a^{IC} \times \delta^{3/5},
\]
in the slow cooling regime using Eq. 13 and
\[
\tilde{\nu}_a^{IC} \approx 2\tilde{\gamma}_c^{-2} \nu_a = \nu_a^{IC} \times \delta^{-2/5}
\]
in the fast cooling regime, using Eq. 13

\subsection{Region B: IC emission}

Zone B extends over a distance \((R - \lambda)\); since we are mainly interested in \( \delta \leq 10^{-1} \), we approximate \((R - \lambda) \approx R \) throughout this paper. This region is populated by electrons that have radiatively cooled via S and SSC for a time \( t_b \). If they were originally injected at the front shock with a power-law energy distribution \( N(\tilde{\gamma}) \propto \tilde{\gamma}^{-p} \) then the steady state distribution in region B depends on whether the electrons were adiabatic or radiative in region A. The \( \tilde{\gamma}_{cB} \) for electrons that cool in a dynamical timescale \( t_{exp} \) is in fact
\[
\tilde{\gamma}_{cB} = \frac{m_e c}{4\sigma_T R_{rad}^2 \nu_{rad} \delta t_{exp}},
\]
where \( U_{rad} \) is the S radiation energy density and the factor 1/2 takes into account that only half of the S photons travel from region A towards region B in the electrons rest frame. The calculation of \( U_{rad} \) in the two different regimes leads to
\[
\tilde{\gamma}_{cB} \propto \begin{cases}
(1) \, \frac{n^2 e^2 \Gamma_p \tilde{\gamma}_m^{-p-1} R^2 \delta^{-1}}{\tilde{\gamma}_m < \tilde{\gamma}_c} \\
(2) \, \frac{n^2 e^2 \Gamma_p \tilde{\gamma}_m^{-2} R^2 \delta^{-1}}{\tilde{\gamma}_m > \tilde{\gamma}_c}
\end{cases}
\]
If electrons in region A are in the slow cooling regime, the new steady state configuration is
\[
N(\tilde{\gamma}) \propto \begin{cases}
\tilde{\gamma}^{-(p+1)} \, \tilde{\gamma}_m < \tilde{\gamma} < \tilde{\gamma}_{cB} \\
\tilde{\gamma}^{-(p+2)} \, \tilde{\gamma}_{cB} < \tilde{\gamma} < \tilde{\gamma}_c
\end{cases}
\]
for slow cooling \( \tilde{\gamma}_m < \tilde{\gamma}_{cB} \) or
\[
N(\tilde{\gamma}) \propto \begin{cases}
\tilde{\gamma}^{-(p+1)} \, \tilde{\gamma}_m < \tilde{\gamma} < \tilde{\gamma}_{cB} \\
\tilde{\gamma}^{-(p+2)} \, \tilde{\gamma}_{cB} < \tilde{\gamma} < \tilde{\gamma}_c \\
\tilde{\gamma}^{-(p+2)} \, \tilde{\gamma}_c < \tilde{\gamma} < \tilde{\gamma}_m
\end{cases}
\]
for fast cooling \( \tilde{\gamma}_m > \tilde{\gamma}_{cB} \). Since most of the electrons in region A have two sources of energy loss (S and SSC), because \( t_b < t_{exp} \) and SSC can be inhibited further for KN effect. If electrons in region A undergo fast cooling, the most common case is \( \tilde{\gamma}_{cB} < \tilde{\gamma}_c \) and the new steady state configuration is
\[
N(\tilde{\gamma}) \propto \begin{cases}
\tilde{\gamma}^{-(p+2)} \, \tilde{\gamma}_m < \tilde{\gamma} < \tilde{\gamma}_{cB} \\
\tilde{\gamma}^{-(p+3)} \, \tilde{\gamma}_{cB} < \tilde{\gamma} < \tilde{\gamma}_c
\end{cases}
\]
The “external Compton” spectrum in zone B is calculated by means of Eq. 13 with \( A = \frac{3\sigma_T R}{\delta} \) and the Compton parameter \( \tilde{\tau}_B \) using Eq. 13 and Eq. 25 with \( \tilde{\tau}_B = \tilde{\tau}_B \). Even if zone B has a higher number of IC emitting electrons, the Compton parameter can be lower than in region A: a higher fraction of the electrons in region B cool efficiently and they are in average less energetic than in region A, therefore increasing the density parameter \( \tilde{\tau}_B \) decreases while \( \tilde{Y} \) increases because in zone A \( \tilde{\gamma}_c \) moves towards low values and the KN effect is less and less important (until the electrons become radiative and \( \tilde{Y} \) does not depend on density). In this situation the SSC radiation can actually account for most of the energy that goes into IC luminosity, (this is the case in Figs 13 and 14 right panels); anyway the SSC usually dominates the spectrum at high energy because it has a higher energy break. The peak flux is
\[
\tilde{F}_p^{IC}(\nu) \approx \tilde{\tau}_B \tilde{F}_p,
\]
which is a factor \( \delta^{-1/2} \) higher than \( \tilde{F}_p^{IC} \) in region A; while the spectral breaks are related to the S seed spectrum in slow cooling as follows
\[
\tilde{\nu}_a^{IC} \approx 2\tilde{\gamma}_B^2 \nu_a,
\]
\[
\tilde{\nu}_m^{IC} \approx 2\tilde{\gamma}_m^2 \nu_m,
\]
\[
\tilde{\nu}_c^{IC} \approx 2\tilde{\gamma}_c^2 \nu_c
\]
if region B is in the slow cooling regime as well, and
\[
\tilde{\nu}_a^{IC} \approx 2\tilde{\gamma}_B^2 \nu_a,
\]
\[
\tilde{\nu}_m^{IC} \approx 2\tilde{\gamma}_m^2 \nu_m,
\]
\[
\tilde{\nu}_c^{IC} \approx 2\tilde{\gamma}_c^2 \nu_c
\]
if B is in the fast cooling regime. If S seed spectrum is in the fast cooling regime and \( \tilde{\gamma}_{cB} < \tilde{\gamma}_c < \tilde{\gamma}_m \)
\[
\tilde{\nu}_a^{IC} \approx 2\tilde{\gamma}_B^2 \nu_a,
\]
\[
\tilde{\nu}_m^{IC} \approx 2\tilde{\gamma}_m^2 \nu_m,
\]
\[
\tilde{\nu}_c^{IC} \approx 2\tilde{\gamma}_c^2 \nu_c
\]
3 DISCUSSION AND CONCLUSIONS

In Fig 2 and Fig. 3 left panels, we show as examples how the spectrum (respectively for fast and slow cooling) modifies if a short magnetic lengthscale is taken into account. Note that S and IC peak fluxes are lower (see also Eq. 5, Eq. 14, Eq. 15), a lower energy break corresponding to the absorption frequency is present (see Eqs. 3 and 5) and the cooling frequency is at higher energies (see Eq. 3). The SSC component in the slow cooling spectra of region A (Fig 2) is depleted because the KN effect is important. In the right panels we raise the density until \( F_\nu \propto \dot{n}^{1/2} \delta = F_\nu \propto n^{1/2} \), thus \( \dot{n} = \frac{\dot{n}}{\delta} \). We obtain \( \nu_c = \nu_e \) and the \( \delta < 1 \) spectrum fails to match the standard spectrum only at radio wave-lengths. In this band such high densities, in fact, overcompensate the shorter magnetic lengthscale and \( \nu_c \) becomes greater then \( \nu_e \). Another important feature is that the total (region A plus region B) \( Y \) parameter is never much higher then \( Y \), even when \( Y \) corresponds to a density \( \delta^2 \) lower; in fact when \( F_\nu = F_\nu^* \), the main contribution to IC luminosity comes from region A and Eq \( \ref{5} \) and Eq \( \ref{6} \) hold: only \( Y_S \) depends on density and \( \delta \) but the dependence are very weak (\( Y_S (1+Y_S) (3-p) \propto \dot{n} (\nu_e-2)/2 \times \delta (\nu_e - 2) \)). These illustrative examples of how the model works should now be followed by a modeling of real data. In fact the general agreement between the two spectral shapes suggests that data could also be well reproduced in this framework, with possibly different values for the break frequencies and peak flux respect to the standard model. As regard the corresponding estimated density compared to the standard model, there are two competitive facts: the peak flux is brought to observed ranges by a higher density but the absorption frequency cannot be too high (see Fig. 2 and 3) (note that unlike the standard model a high density, \( \dot{n} \sim 10^{45} \text{cm}^{-3} \) does not produce an extreme inverse Compton component, which is not observed); Therefore, even if there is the possibility that a higher density is required, the suitable value can be derived within the errors only by a broadband modeling of data, with a consistent variations of all the parameters.

Similarly to the standard model there are univocal correlations between the estimated spectral parameters and the derived fireball energy, external density and shock parameters. They are

\[
\tilde{E} \propto F_\nu^{3/2} \nu_e^{-5/6} \nu_c^{1/4} \nu_a^{-5/6} \delta^{-1/2} T^{-0.5} \quad (40)
\]

\[
\tilde{n} \propto F_\nu^{-3/2} \nu_a^{25/12} \nu_e^{-3/4} \nu_a^{25/6} \delta^{1/2} T^{-7/2} \quad (41)
\]

\[
\tilde{\nu}_c \propto F_\nu^{-1/2} \nu_c^{11/12} \nu_e^{-1/4} \nu_a^{5/6} \delta^{1/2} T^{-3/2} \quad (42)
\]

\[
\tilde{\nu}_B \propto F_\nu^{1/2} \nu_c^{-5/4} \nu_a^{-5/4} \nu_a^{-5/2} \delta^{-3/2} T^{5/2} \quad (43)
\]

in slow cooling regime while for a radiative regime

\[
\tilde{E} \propto F_\nu^{3/2} \nu_c^{-1/6} \nu_a^{-5/6} \delta^{-1/2} T^{-0.5} \quad (44)
\]

\[
\tilde{n} \propto F_\nu^{-3/2} \nu_c^{17/6} \nu_a^{25/6} \delta^{1/2} T^{7/2} \quad (45)
\]

\[
\tilde{\nu}_c \propto F_\nu^{-1/2} \nu_a^{1/2} \nu_c^{3/2} \nu_a^{-5/6} \delta^{1/2} T^{-3/2} \quad (46)
\]

\[
\tilde{\nu}_B \propto F_\nu^{1/2} \nu_c^{-5/4} \nu_a^{-5/2} \delta^{-3/2} T^{-5/2} \quad (47)
\]

Some constraints must be added to this set of equations. Because \( \epsilon_e + \epsilon_B + \epsilon_p = 1 \), where \( \epsilon_p \) is the fractional energy that goes to protons, we most likely expect \( \epsilon_e + \epsilon_B < 0.5 \).

The kinetic energy in the afterglow should not be much lower then the energy released in the \( \gamma \)-ray phase, otherwise very large efficiencies of radiation, incompatible with internal shock models, would be required (Lazzati, Ghisellini & Celotti 1999). Finally the apparent source size, the rate of expansion and the energy in each afterglow should be consistent with observations in the radio wavebands of interstellar scintillation and its quenching (Waxman, Kulkarni & Frail 1998). In principle these requirements together with observations of the spectral parameters at a particular time would set limits on \( \delta \) and therefore on the structure of the magnetic field generated in shocks. Unfortunately the current data do not allow a model independent estimation of spectral parameters. Actually the spectral shape, and in particular the break frequencies and the peak flux, are not unequivocally constrained by data in even the best studied afterglows (e.g. GRB 970508, GRB 000418). These spectral features are derived by modeling the sparse measurement collected in various wavebands at different times: very often different versions of the standard model can fit the data, consequently finding different spectral parameters (see for example Berger et al 2000, for GRB 000418). Moreover these model dependent quantities (above all \( \nu_c \)) enter with higher dependences than \( \delta \) in the estimation of the fireball parameters (Eq. \( \ref{5} \) and \( \ref{6} \)); therefore the break frequencies and peak flux, derived under the assumption that \( \delta = 1 \), cannot obviously be used to figure out the value of \( \delta \). These equations and the physical limits on the fireball parameters should be used instead to constrain the overall modeling of data. The results would actually lead to very interesting conclusions.

If the modeling succeeded in reproducing the data with \( \delta < 1 \), a different value for the external density, energy and shock parameters would possibly be derived. If it indicates a denser environment, this could be more compatible with the strong observational evidence that would place GRBs explosions in a star forming region. However a very short scale magnetic field (say \( \dot{n} \sim 10^{34} \)) would be probably ruled out, since extremely high densities would be required in order to get fluxes in the observed ranges and the radio flux (\( \nu < \nu_e \)) would probably be too heavily absorbed: \( F_\nu \propto \dot{n}^{-1/2} \) (in adiabatic) and \( F_\nu \propto \delta^{-1} \dot{n}^{-5/2} \) (in radiative regime). An other implication of the possibility to fit the very same data with models based on different shock physics assumptions is that parameter estimations greatly suffer from shock theory uncertainties. In this work we have only explored the effect of a short-scale magnetic field, but other reasonable variations of the standard model (such as time dependent \( \epsilon_B, \epsilon_e \) and \( p \)) may affect the parameter estimates as well. This would call for alternative model independent external density and energy estimates in order to constrain, together with the afterglow modeling, the shock physics.

On the other hand if this model can reproduce data only with \( \delta = 1 \), it could be either evidence in favor of an extremely stable field or the indication that some other modifications of the theory must be taken into account. For example, as already mentioned, \( \delta \) can vary with time: for instance if the Rayleigh-Taylor instability occurs at the contact discontinuity, the lengthscale would grow as the fireball evolves and consequently late multibands observations would be more consistent with \( \delta = 1 \), while early measurements would require \( \delta < 1 \). Observational tests of this possibility can be performed using the theory developed in this
work; in fact we can look for a coherent changing in time of $\delta$, modeling instantaneous spectra at different times, with all the remaining parameters kept constant. Of, course, with a very good dataset, this approach can be extended to all the other shock parameters, including $\epsilon_B, \epsilon_e$ and $p$. Such a procedure would yield unbiased estimates of shock parameters variability, and the GRB afterglow would become a laboratory where shock physic can be tested. The best case up to now for constraining $\delta$ and its temporal behavior is probably GRB 000926, but in a year SWIFT, REM and possibly the VLA will provide us with more simultaneous data (also at early times) in X-ray, optical, infrared and longer wavelengths and therefore more constraining fits will be performed. This work actually highlights the importance of observing afterglows at all wavelengths, from radio to hard x-ray, but emphasizes as well that current estimations from this data of the external density surrounding $\gamma$-ray bursts may depend on uncertain details of shock physics.

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Figure 2. An example of how the spectrum modifies if $\delta < 1$. Total (solid line) and SSC+IC (dot line) spectra for $\delta = 0.01$ and total (3 dots-dash line) and SSC (dot-dash line) spectra for $\delta = 1$ at 1 day. In the left panel the two total spectra have all parameters, except for $\delta$, the same: $n=1.35 \text{ cm}^{-3}$, $E=3 \times 10^{52} \text{ erg}$, $\epsilon_e = 6 \times 10^{-2}$, $\epsilon_B = 4 \times 10^{-3}$, $p=2.2$ at a redshift $z=1$. These parameters are typical of broadband afterglow lightcurves modeled in the standard framework (PK01). Both spectra are in the slow cooling regime (also zone B with $\gamma_cB < \gamma_c$). The SSC in zone A is highly inhibited by the KN effect; therefore most of the IC component is due to “external Compton” in zone B. The resulting Compton parameters are $\tilde{Y} = 0.40$ and $Y = 1.05$. In the right panel we raise the density in the $\delta = 0.01$ spectrum until $\tilde{F}_p = F_p$, $\tilde{n} = 1.35 \times 10^4 \text{ cm}^{-3}$. Zone B is now in the fast cooling regime therefore $\gamma_cB$ moves towards low values but since in zone A the electrons are almost all in Thompson regime the total $\tilde{Y}$ increases, $\tilde{Y} \simeq 1.10$; region A now dominates the IC emission. It is worth noticing that the two total spectra are distinguishable only in the radio waveband where the $\delta = 0.01$ flux is more absorbed.

Figure 3. Same as Fig. 2 with parameters $n = 27 \text{ cm}^{-3}$, $E = 1.8 \times 10^{53} \text{ erg}$, $\epsilon_e = 0.3$ and $\epsilon_B = 8 \times 10^{-3}$ and $p=2.43$ and $\delta = 0.1$. This set of parameters results from the best fit of 000926 (Harrison et al. 2001). The redshift of this burst is $z = 2.0639$. In the left panel both spectra are in fast cooling (also zone B with $\gamma_cB < \gamma_c$). The Compton parameters are $\tilde{Y} = 2.59$ and $YF = 2.34$. In the right panel we raise the value of density in the $\delta = 0.1$ spectrum until $\tilde{F}_p = F_p$, $\tilde{n} = 2.7 \times 10^3 \text{ cm}^{-3}$. The spectrum is obviously still in fast cooling with a lower $\gamma_cB$; therefore the total $\tilde{Y}$ is lower, $\tilde{Y} \simeq 2.37$ and the main contribution to the IC luminosity comes from region A. Again the two total spectra are distinguishable only in the radio waveband.