Mathematical modeling of dynamic processes of lubricating layers thrust bearing turbochargers

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Abstract. Thrust bearings turbochargers, which include centrifugal and axial compressors partially operate in dynamic mode. These modes can be caused by installation errors, e.g., when installing the hard disk on the rotor with a skew or by external influences, such as rotating stall or surge of the turbocharger. If significant variations occur surge with a frequency of 1÷10 Hz total gas volume filling flow of the compressor and the network. Fluctuations in the gas result in changes of axial load, until the change of sign of the axial force, and hence a displacement of the rotor in the axial and radial directions. Variable turbochargers regimes can sometimes lead to shock loads on the thrust bearing and, consequently, to its destruction.

Thrust bearings (TB) are used for sensing the axial load acting on the rotor, and transmit it to the stator, as well as to lock the rotor axially relative to the housing. When operating in the thrust bearing between the rotating disc 1 (Fig. 1) and pads 2 are formed thin lubricating layer 3, and in the angular direction are separated by aiiters cushion channels 4. These channels provide a supply of liquid lubricant in the layer. When rotating thrust disk in the channel 4 on the disc surface is formed a thin boundary layer 5 with a notional boundary 6 cooperating with lubricating layers adjacent cushions and fresh fluid channel 4. In Fig. 2 shows variations of the basic profiles of the carrier surface flat-wedge pads used in turbochargers [1].

The conducted experimental studies of the thrust bearing shown with fixed pads shown [2] that the surging of the centrifugal compressor (CC) [3] of the lubricating layer pressure waveform is a substantially non-stationary and occurs with changes in time before and during compressor pat (Fig. 3). The sensors D1 and D2 located at the average radius of the opposing pads on the working side of the bearing. The dependence of the output voltage (mV) from the measured pressure was a linear calibration characteristic.

At steady state (Fig. 3, a) on the waveform pressure pulsation observed: sensor D1 they reached 0.14 MPa, and for the sensor D2 - 0.49 MPa. This is due to the inevitable beating due to the skew of thrust disk. Indications sensors D1 and D2 are in antiphase and the rotor rotation frequency. The larger the average pressure values indicative of sensor D2 (1,24 MPa) than the sensor D1 (0,69 MPa) confirms the effect of the skew surface of the bearing pads with respect to the thrust disk mounted without skewing. When surge of the CC (Fig. 3, b) the maximum pressure of the sensor D1 was 0,96 MPa, at the sensor A2 – 2,4 MPa. Surge frequency was 2,5 Hz. On the basis of these experiments, it established the nature of dynamic processes in the lubricating layer of the thrust bearing.
Figure 1. The design scheme of the mathematical model of the TB with the parallel intercushion channel skew. Cross section: a - along the rotational axis and thrust disk; b - A-A; c - B-B

Figure 2. Surface profiler pads of the TB: a - parallel channel intercusion bevel; b - helical surface; c - Raleystep

Figure 3. Waveform pressure lubricating layer D1 and D2 sensors [3]:
   a - steady state (Mach number $M_u = 0.884$);
   b - surge mode of the CC (Mach number $M_u = 0.707$)
Dynamic processes in lubricating boundary layers and the bearing are determined by the behavior of the rotor, disposed in radial and thrust bearings (Fig. 4). Assuming independence of the axial movement from the radial, one can describe the behavior of the rotor in the axial direction. For this purpose, on the basis of the second law of Newton write equation of rotor dynamics in the axial direction:

$$m_r \frac{d^2 y_{d.\text{disp}}}{d\tau^2} = P - F,$$

where $m_r$ - rotor mass is constant and is determined during construction of the compressor; $y_{d.\text{disp}}$ - coordinate disc displacement along the rotor axis; $\tau$ - time; $P$ - carrier bearing ability; $F$ - foreign unsteady gas dynamic force of the CC, acting on the rotor. Here, for the origin of coordinates for the axial motion of the rotor, it takes an arbitrary point beyond its limits.

The solution of the equation of dynamics (1) can be found in two methods. In the first method specified trajectory equation thrust disk $y_{d.\text{disp}}$, gas dynamic force $F$ and determined bearing capacity $P$ of the bearing. On the basis of the force $P$ further calculated pressure distribution pad according field (the direct problem). In the second method (the inverse problem) are given force $P$, $F$ and the path of movement of the disc is determined $y_{d.\text{disp}}$, entering in equation form a lubricating layer. In the latter case, the equation (1) the initial conditions in the form:

$$y_{d.\text{disp}} = y_{d.\text{disp. st.}} \text{ or } y_{d.\text{disp.}}, \frac{dy_{d.\text{disp.}}}{d\tau} = 0.$$  \hspace{1cm} (2)

The external force $F$ is defined as:

$$F = F_{st} + F_d,$$  \hspace{1cm} (3)

where $F_{st}$ - the constant component, i.e. static force; $F_d$ - the dynamic force.

In the case of rotating stall $F_d$ set harmonically:

$$F_d = F_a \sin \Omega \tau.$$  \hspace{1cm} (4)

where $F_a$, $\Omega$ - the amplitude and frequency of the disturbing force.

In the case of surge $F_d$ it is given in the form:

$$F_d = F_{d1} \exp[-\beta_{F1}(\tau - \tau_{d1})]$$  \hspace{1cm} (5)

where $\beta_{F1}$ - the rate of change of axial force on the interval.

Thrust bearing load-bearing capacity is determined by summing the carrying capacity of its pads:
\[ P_p = \sum_{i=1}^{i=z} P_{pi}. \]  

(6)

where \( Z \) – the number of bearing pads.

The bearing capacity of \( i \)-th pad determined by integrating the field lubricating layer pressure:

\[ P_i = \int_{0}^{R_i} \int_{0}^{r} p_i r d\varphi dr, \]

(7)

where \( p_i \) – pressure distribution in the lubricating layer \( i \)-th pad.

Static and dynamic characteristics of lubricating and boundary layers are using a mathematical model based on the basic laws of conservation of mass, momentum, internal energy. It is described by differential equations of continuity, the Navier-Stokes equations, energy, initial and boundary conditions for them and binding them together geometric shapes of areas, as well as the dependence of the viscosity and density of the lubricant temperature. Numerical implementation of this model provides a rigorous analysis of the bearing characteristics.

A mathematical model of the lubricating boundary layers and non-stationary periodic thermoelastic hydrodynamic (PTUGD) problem [4] the thrust sliding bearing in dimensional form comprises:

1. Reynolds equation that describes the pressure distribution in the lubricating layer \( L_1 \) on the surface of the pad when \( \{R_1 \leq r \leq R_2, 0 \leq \varphi \leq \theta, 0 \leq y \leq h \} \):

\[ r \frac{\partial}{\partial r} \left( r f_0 \frac{\partial p}{\partial r} - f_1 \right) + \frac{\partial}{\partial \varphi} \left( f_0 \frac{\partial p}{\partial \varphi} + \omega r^2 f_2 \right) + Ar^2 = 0 \]

(8)

where \( A = \int_{0}^{h} \rho dy - \int_{0}^{h} \frac{\partial h}{\partial r} \), \( \rho \) - density of the lubricant on the surface of the pad at \( y = h \); \( r, \varphi \) - coordinates; \( p \) – the desired pressure. The pressure drop across the thickness of the lubricating layer is zero \( \frac{\partial p}{\partial y} = 0 \). Included in the equation (8) functions \( f_0, f_1, f_2 \) have the form:

\[ f_0 = \int_{0}^{h} \rho \left( i \frac{m_1}{m_0} \right) dy, \quad f_1 = \int_{0}^{h} \rho \left( j - \frac{n}{m_0} \right) dy, \quad f_2 = \int_{0}^{h} \rho \left( 1 - i \frac{m_0}{m_0} \right) dy, \]

\[ i_K = \int_{0}^{Y} \frac{y}{\mu} dy, \quad m_K = \int_{0}^{Y} \frac{Y}{\mu} dy, \quad k = 0, 1, 2, \]

\[ j = \int_{0}^{Y} \frac{1}{\mu} \rho V_r^2 dydy, \quad n = \int_{0}^{Y} \frac{1}{\mu} \rho V_{\varphi}^2 dydy. \]

(9)

The boundary conditions to equation (8):

at \( r = R_1, r = R_2 \) set pressure \( P_{R_1}, P_{R_2} \),

\( \theta = 0, \varphi = \theta, \varphi = 0 \) pressure gradients are zero \( \frac{\partial p}{\partial \varphi = \theta} = 0, \frac{\partial p}{\partial \varphi = \varphi} = 0 \).

(10)

(11)

2. Lubricant flow rates in the radial \( V_r \) and district \( V_{\varphi} \) directions:
\[ V_r = \frac{\partial p}{\partial r} (i_1 - \frac{m_i}{m_o} i_o) \delta_1 - \frac{1}{r} \left( j - \frac{n}{m_o} i_o \right), \quad (12) \]
\[ V_{\varphi} = \frac{1}{r} \frac{\partial p}{\partial \varphi} \left( i_1 - \frac{m_i}{m_o} i_o \right) \delta_1 + \rho r (1 - \frac{i_o}{m_o}), \quad (13) \]

where \( \delta_1 \) - unit function.

Speed \( V_y \) determined by integration of the equation of continuity in the coordinate given conditions \( V_y = 0 \) at \( y = 0 \):

\[ V_y = -\frac{1}{\rho} \int_0^y \left[ \frac{\partial \rho}{\partial \tau} + \frac{1}{r} \frac{\partial}{\partial r} (\rho V_r) + \frac{1}{r} \frac{\partial}{\partial \varphi} (\rho V_{\varphi}) \right] dy \quad (14) \]

3. The three-dimensional energy equation in the area \( L_2 \) lubricating layer (\( 0 \leq \varphi \leq \theta \)) pad and boundary layer (\( \theta_p < \varphi < \theta \)) intercushion channel when (\( R_1 \leq r \leq R_2 \), \( 0 \leq y \leq h \)):

\[ c \frac{\partial t}{\partial \tau} + \frac{1}{r} \frac{\partial}{\partial r} (c \rho V_r t) + \frac{1}{r} \frac{\partial}{\partial \varphi} \left( \frac{c \rho V_{\varphi} t}{2} - \frac{\lambda_{oil}}{\varphi} \frac{\partial t}{\partial \varphi} \right) + \frac{\partial}{\partial y} \left( c \rho V_y t - \rho \frac{\partial t}{\partial y} \right) = \mu \left\{ \left( \frac{\partial V_{\varphi}}{\partial y} \right)^2 + \left( \frac{\partial V_r}{\partial y} \right)^2 \right\} \quad (15) \]

3.1. Initial conditions:

\[ \tau = 0, \quad t(0, r, \varphi, y) = 0 \quad \text{or} \quad t = t_{stat}(r, \varphi, y). \quad (16) \]

3.2. The coordinate \( r \) put the following boundary condition.

at \( r = r^*(\varphi) \) a function of temperature \( t = t(\tau, r^*, \varphi, y) \) determined by the solution of equation (15) under the conditions:

\[ V_r = 0, \quad \frac{\partial p}{\partial r} = 0 \quad (17) \]

In this case the second term in the left side of the square brackets right side of equation (15) disappears, and the decision depends only on \( \varphi \) coordinate, \( y \). Speed \( V_r \) changes its sign, i.e. it becomes equal to zero at \( r^*(\varphi) \). Meaning \( r^*(\varphi) \) for pads in the form of an annular sector, close to the average radius, and runs exactly to the rectangular pads \( r^*(\varphi) = R_{av} \).

3.3. The coordinate \( \varphi \) on the borders \( \varphi = 0 \) and \( \varphi = \theta \) put periodicity temperature condition [5], i.e. temperature lubricant thickness at the inlet to the lubricating layer is determined by the current pad temperature lubricant outlet channel boundary layer intercushion previous pad:

\[ t_1|_{\varphi=0} = t_{1-1}|_{\varphi=\theta}, \quad t_1|_{\varphi=0} = t_{1-1}|_{\varphi=\theta} \quad (18) \]

3.4. The coordinate \( y \) at \( y = 0 \) the boundary conditions are placed on the surface of the thrust disk:

3.4.1. Lubricating and boundary layers
\[ \frac{\partial t}{\partial y} = 0, \text{ when } y = 0, \quad 0 < \phi < \Theta, \]  

(19)

which means that no heat exchange and lubricating boundary layers with thrust disk, or:

3.4.2. Temperatures and heat fluxes at the boundary «lubrication and boundary layers – the thrust disk» continuous

\[ y = 0, \quad y_d = H_d, \quad \left. \frac{\partial t}{\partial y} \right|_{y=0} = \frac{\lambda_d}{\lambda_{oil}} \left. \frac{\partial T_d}{\partial y} \right|_{y=0}, \]  

(20)

where \( T_d \) – thrust disk temperature; \( H_d \) - thickness of the disc; \( \lambda_{oil} \cdot \lambda_d \) – thermal conductivity of oil and disk.

3.5. The coordinate \( y \) at \( y = h \) posed boundary conditions:

3.5.1. Within \( y = h, \quad 0 < \phi < \Theta_p \) on the surface of pad temperature and heat flux at the interface «lubricant layer – pad» continuous

\[ y = h, \quad y_p = 0, \quad t \left|_{y=h} = T_p \right|_{y=0} = \lambda_{oil} \left. \frac{\partial T_p}{\partial y} \right|_{y=h} = \lambda_p \left. \frac{\partial T_p}{\partial y} \right|_{y_p=0}, \]  

(21)

where \( T_p \) – pad temperature; \( \lambda_{oil} \cdot \lambda_p \) – thermal conductivity of oil and pad;

3.5.2. Within \( y = h, \quad \Theta_p < \phi < \Theta \) conditional on the boundary of the boundary layer 6 (see Fig. 1) are considered:

3.5.2.1. temperature is known and equal to the temperature of the intercushion channel

\[ t = t_o \]  

(22)

or:

3.5.2.2. Heat transfer occurs only by convection

\[ \left. \frac{\partial t}{\partial y} \right|_{y=h} = 0. \]  

(23)

As seen from the boundary conditions (20) and (21) to be used needs to know the temperature distribution \( T_d, T_p \) in the thrust disk and pad. This temperature distribution can be obtained by solving the differential equation of heat conduction for the thrust disk and pad.

4. Three-dimensional equation describing the temperature distribution in the thickness of pad in the area \( L_A(0 \leq y_p \leq H_p, R_1 \leq r \leq R_2, 0 \leq \phi \leq \Theta_p) \):

\[ c_p \rho_p \frac{\partial T_p}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda_p \frac{\partial T_p}{\partial r} \right) + \frac{1}{r^2} \frac{\partial}{\partial \phi} \left( \lambda_p \frac{\partial T_p}{\partial \phi} \right) + \frac{\partial}{\partial y_p} \left( \lambda_p \frac{\partial T_p}{\partial y_p} \right) \]  

(24)

where \( c_p, \rho_p, \lambda_p \) – coefficient of heat capacity, density and coefficient of thermal conductivity of the pad material.

4.1. Initial conditions:

\[ \tau = 0, \quad T_p(0, r, \phi, y_p = 0) = 0 \quad \text{or} \quad T_p = T_{p, \text{stat}}(r, \phi, y_p). \]  

(25)
4.2. For equation (24) at the interface «lubricant layer - pad» has been adopted continuity of temperatures and heat flows from the conditions (21).

4.3. On the boundary with the \( y_p = H_p \), the boundary condition of the third kind:

\[
\lambda_p \left. \frac{\partial T_p}{\partial y_p} \right|_{y_p = H_p} = \alpha_{T_p H_p} \left( T_{a4} - T_p \left( r, r, \varphi, H_p \right) \right),
\]

(26)

where \( \alpha_{T_p H_p}, T_{a4} \) - heat transfer coefficient and the ambient temperature on the back side pad.

4.4. On the boundary with the \( \varphi = 0 \) and \( \varphi = \Theta_n \) the boundary condition of the third kind:

\[
\lambda_p \left. \frac{\partial T_p}{\partial \varphi} \right|_{\varphi = 0} = -\alpha_{T_p \varphi} \left( T_{a1} - T_p \left( r, r, 0, y_p \right) \right),
\]

(27)

\[
\lambda_p \left. \frac{\partial T_p}{\partial \varphi} \right|_{\varphi = \Theta_n} = \alpha_{T_p \Theta} \left( T_{a1} - T_p \left( r, r, \Theta_n, y_p \right) \right),
\]

(28)

where \( \alpha_{T_p \varphi}, \alpha_{T_p \Theta} \) - pad with heat transfer coefficients \( \varphi = 0 \) and \( \varphi = \Theta_n \); \( T_{a1} \) - temperature intercushion channel environment.

4.5. On the boundary with the \( r = R_1 \) and \( r = R_2 \) the boundary condition of the third kind:

\[
\lambda_p r \left. \frac{\partial T_p}{\partial r} \right|_{r = R_1} = -\alpha_{T_p R_1} \left( T_{a2} - T_p \left( r, R_1, \varphi, y_p \right) \right),
\]

(29)

\[
\lambda_p r \left. \frac{\partial T_p}{\partial r} \right|_{r = R_2} = -\alpha_{T_p R_2} \left( T_{a3} - T_p \left( r, R_2, \varphi, y_p \right) \right),
\]

(30)

where \( \alpha_{T_p R_1}, \alpha_{T_p R_2} \) - pad with heat transfer coefficients \( r = R_1 \) and \( r = R_2 \); \( T_{a2}, T_{a3} \) - ambient temperature with \( r = R_1 \) and \( r = R_2 \).

5. The two-dimensional equation describing the temperature distribution in the thickness of thrust disk in the area \( L_d (R_1 \leq r \leq R_2, 0 \leq y_d \leq H_d) \):

\[
c_d \rho_d \frac{\partial T_d}{\partial \tau} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \lambda_d \frac{\partial T_d}{\partial r} \right) + \frac{\partial}{\partial y_d} \left( \lambda_d \frac{\partial T_d}{\partial y_d} \right)
\]

(31)

where \( c_d, \rho_d, \lambda_d \) - specific heat ratio, density ratio and warm-resistant disc material. Temperature gradient \( \partial T_d / \partial \varphi = 0 \).

5.1. Initial conditions:

\[
\tau = 0, \quad T_d \left( 0, r, y_d \right) = 0 \quad \text{or} \quad T_d = T_{d, \text{stat}} \left( r, y_d \right).
\]

(32)

5.2. For equation (31) at the interface «thrust disk - the lubricating layer and boundary layer - intercushion channel» has been adopted continuity of temperatures and heat flows from the conditions (20).

5.3. At \( y_d = 0 \) in the case of one-sided bearing on the rear side of the boundary condition of the third kind:

\[
\lambda_d r \left. \frac{\partial T_d}{\partial r} \right|_{r = R_1} = -\alpha_{T_d R_1} \left( T_a - T_d \left( r, R_1, y_d \right) \right),
\]

(33)
\[ \lambda_d \frac{\partial T_d}{\partial r} \bigg|_{r=R_2} = \alpha_{T_d} R_2 \left( T_a - T_d(r, R_2, y_d) \right), \]

where \( \alpha_{T_d} R_1, \alpha_{T_d} R_2 \) — disc heat transfer coefficients of \( r = R_1 \) and \( r = R_2 \); \( T_a \) — at ambient temperature \( r = R_1 \) and \( r = R_2 \).

Communication between the differential equations (8), (15), (24), (31) and their boundary conditions are set through such physical properties of fluids such as viscosity and density, forms the lubricant and the boundary layers, depending on the lubricant temperature, the working geometry the surface of the pad and some mode parameters. Therefore, depending on the further attachable to PTUGD mathematical model.

6. The dependence of the viscosity and density of the lubricant temperature:

\[ \mu = \mu_0 \exp \left( -\beta(t-t_0) + \beta_1(t-t_0)^2 \right), \]

\[ \rho = \rho_0 \exp \left( -\alpha_{oil}(t-t_0) \right), \]

where \( t_0, \mu_0, \rho_0 \) — temperature, viscosity, oil density on giving a intercushion channel; \( t \) - oil temperature; \( \beta, \beta_1 \) - temperature coefficient of viscosity, depending on the grade of oil and the temperature range under consideration; \( \alpha_{oil} \) - temperature coefficient of volumetric expansion of the oil.

7. The clearance in the thrust bearing with fixed pads.

7.1. The surface with parallel bevel intercushion channel (Fig. 2a)

\[ h = h_{2o} - y_{d, \text{disp.}} + \delta_{\text{tilt}} \left( 1 - \frac{r \sin \varphi}{\eta_k} \right) \delta_k - \frac{\delta_d}{2R_2} r \cos (\varphi - \omega \tau) + \delta_{\text{step}} \left( 1 - \frac{\varphi}{\theta_k} \right) \delta_k - \frac{\delta_d}{2R_2} r \cos (\varphi - \omega \tau) + \delta_{\text{tilt}} \right) \]

7.2. Helical bevel wedge surface (Fig. 2b)

\[ h = h_{2o} - y_{d, \text{disp.}} + \delta_{\text{tilt}} \left( 1 - \frac{\varphi}{\theta_k} \right) \delta_k - \frac{\delta_d}{2R_2} r \cos (\varphi - \omega \tau) + \delta_{\text{step}} \left( 1 - \frac{\varphi}{\theta_k} \right) \delta_k - \frac{\delta_d}{2R_2} r \cos (\varphi - \omega \tau) + \delta_{\text{tilt}} \right) \]

7.3. Raley stepped surface (Fig. 2c)

\[ h = h_{2o} - y_{d, \text{disp.}} + \delta_{\text{step}} \left( 1 - \frac{\varphi}{\theta_k} \right) \delta_k - \frac{\delta_d}{2R_2} r \cos (\varphi - \omega \tau) + \delta_{\text{tilt}} \right) \]

where \( h_{2o} \) — the thickness of the lubricant layer at the position of the disc at the origin; \( y_{d, \text{disp.}} \) — thrust disk location coordinate is given its own equation or determined by solving the equations of the dynamics of the rotor (1); \( \delta_{\text{tilt}} = (h_1 - h_2) \) - the depth of the bevel at the front edge of the pad;
\( \delta_{\text{step}} = \text{const} \) – depth of Raley steps; \( \alpha_p \) – coefficient of linear expansion cushion material; 

\( T_p(\theta_p, r, y_p) \) – temperature distribution at the trailing edge pad; \( \delta_r \) – unit function; \( \delta_d \) – beating (double amplitude) of the disk to the outer radius \( R_2 \); \( \omega \) – disk angular velocity; \( \tau \) – time.

8. Conditional boundary layer thickness when \( \theta_p \leq \varphi \leq \theta \):

\[
h = \left[ h_{\varphi=\theta_p} \left( \frac{(\theta - \theta_p)^2}{(\theta - \theta_p)^2} + \frac{(\varphi - \theta_p)^2}{(\varphi - \theta_p)^2} \right) \right] + \left[ \frac{h_{\varphi=\theta_p} - h_{\varphi=0}}{\theta^2} \right] + \varepsilon_1 (h_{20} - y_{d,\text{disp}}) \left( \frac{(\varphi - \theta_p)}{\theta} \right) + \varepsilon_2 \left( h_{20} - y_{d,\text{disp}} \right) \left( \frac{(\varphi - \theta_p)^2}{\theta^3} \right),
\]

where \( \varepsilon_1, \varepsilon_2 \) – parameters of the concavities and convexities, allowing investigate the effect on the bearing characteristics of the conventional shapes of the boundary layer thickness.

Reduced system of mathematical equations PTUGD model is valid for the case of laminar flow in the lubricating grease and boundary layers. This mode may occur when the Reynolds number, defined by the characteristic width of the channel, to \( 10^5 \), i.e. \( \text{Re}_{cr} < 10^5 \), the turbulent regime above [6]. In practice, in most cases, this condition is satisfied.

As a result of numerical implementation composed PTUGD mathematical model of the thrust bearing with different profiles of surfaces of bearing pads must be prepared depending on the time proportion distribution of pressure, temperature and velocity of lubrication, forms a gap, the maximum temperature values lubricant in the clearance, pad and thrust disk, and the minimum value of the clearance and such integral characteristics of the bearing as the bearing capacity, power loss due to friction and lubrication flow rate. It should also be determined coefficients of stiffness and damping of the bearing lubricating layer when the thrust disc axially in dynamic conditions, including depending on the geometric dimensions and operating parameters.

The obtained data will be used in developing thrust bearing design with fixed pads with an increased service life under dynamic conditions.

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