GENERALISED VDM AND $F_2$ DATA AT LOW $Q^2$ *

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The generalised vector meson dominance model (GVDM) gives a good description of $F_2$ data at very low $Q^2$. At intermediate $Q^2$ a GVDM component avoids problems when applying the large-$Q^2$ DIS formalism, such as a negative gluon distribution in the proton. The fluctuations of the exchanged photon into vector mesons is also a natural part of a model with hadronic fluctuations of the target proton, which gives a good description of the non-perturbative $x$-shape of the proton’s parton density functions at the starting scale $Q_0^2$ for DGLAP evolution.

1 Introduction

Data on the proton structure function $F_2$ from $ep$ (or $\mu p$) scattering are usually interpreted in terms of the formalism for deep inelastic scattering (DIS) where $d\sigma/dx dQ^2 \sim F_2(x, Q^2) = \sum q^2 (xq(x, Q^2) + x\bar{q}(x, Q^2))$. $F_2$ is, therefore, interpreted in terms of the quark density functions $xq(x, Q^2)$ in the proton and the gluon density enters via the DGLAP equations for evolution in $Q^2$. This formalism has also been applied to $F_2$ data at low $Q^2$ where the exchanged photon is not far from being on-shell. Parametrising such $F_2$ data in terms of quark and gluon density functions results in gluon distributions that tend to be negative at small $Q^2$ [1], since otherwise the strong DGLAP evolution, driven primarily by the gluon at small $x$, gives too large parton densities and thereby a poor fit to $F_2$ in the genuine DIS region at large $Q^2$. Although one may argue that the gluon density is not a directly observable quantity and hence might be negative, it certainly is in conflict with the normal interpretation in terms of the probability for a gluon with momentum fraction $x$ in the proton. In particular, such a gluon distribution may not be universal and applying it in other interactions may, therefore, give incorrect results.

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The mistake is here to apply the formalism for DIS also in the low-$Q^2$ region, where the momentum transfer is not large enough that the parton structure of the proton is clearly resolved, as illustrated in Fig. 1a. For $Q^2 \lesssim 1\text{GeV}^2$, there is no hard scale involved and the interaction is a soft photon-proton interaction. The dominant cross-section is here given by the photon fluctuating into a vector meson state which then interacts with the proton in a strong interaction (Fig. 1b), i.e. the vector meson dominance model.

2 Generalised vector meson dominance model in $ep$ at very low $Q^2$

Due to quantum fluctuations a photon may appear as a vector meson, i.e. $|\gamma\rangle = C_0 |\gamma_0\rangle + \sum_V \frac{e}{f_V} |V\rangle + \int m_V dm_V (\cdots)$. The sum is over the vector meson states $V = \rho, \omega, \phi \ldots$ and the integral is for the generalisation to include a continuous mass spectrum [2]. This hadronic state then interacts with the target proton, resulting in transverse and longitudinal cross-sections $\sigma_{T,L}^{\text{GVDM}}(\gamma p \to X) = \sum_V P_{T,L}(\gamma \to V) \sigma_{T,L}(V p \to X)$, where the fluctuation probability includes the vector meson propagator and is given by GVDM phenomenology [3]. The soft hadronic cross-section can be taken as the standard parametrisation $\sigma(V p \to X) = A_V s^\epsilon + B_V s^{-\eta}$, with $\epsilon \approx 0.08$ in the pomeron exchange term which dominates over the reggeon exchange term at high energies.

In $ep$ scattering, $F_2(x, Q^2) = \frac{(1-x)Q^2}{4\pi^2\alpha} (\sigma_T + \sigma_L)$ and $s_{\gamma p} = Q^2 \frac{1-x}{x} + m_p^2 \approx Q^2/x$ at small-$x$. Inserting the full GVDM expressions for $\sigma_{T,L}$ results in [3]

$$F_2 = \frac{(1-x)Q^2}{4\pi^2\alpha} \left\{ \sum_V r_V \left( \frac{m_V^2}{Q^2 + m_V^2} \right)^2 \left( 1 + \xi \frac{Q^2}{m_V^2} \right) + r_C \frac{m_0^2}{Q^2 + m_0^2} \right\} A \frac{Q^{2\epsilon}}{x^\epsilon}$$
Here, the last factor originating from $\sigma(Vp \rightarrow X)$ only includes the pomeron term, since the reggeon term is negligible in the small-$x$ region relevant here. An overall normalisation constant $A$ is introduced giving ratios $A_{V,C}/A$ included in the parameters $r_{V,C}$. In the curly bracket, conventional VDM gives the sum over vector mesons with the characteristic vector meson propagators and the fluctuation constants $r_V = \frac{4\pi\alpha}{f_V^2}A_V$ involving the vector meson decay constant $f_V$. Besides the dominating contribution from transverse photons, the VDM sum contains a longitudinal contribution through the $\xi$-term. The term with $r_C = 1 - \sum_V r_V$ originates from the integral over the continuous mass spectrum with a lower limit $m_0$ (only the transverse contribution is here included since the longitudinal one is small). Altogether, GVDM gives a more complex $Q^2$ dependence than the simple VDM for transverse photons. The parameters involved are known from GVDM as $r_{V=\rho,\omega,\phi,C} = 0.67, 0.062, 0.059, 0.21; \xi \approx 0.6$ and $m_0 = 0.9 \text{ GeV}$ [3].

3 Comparison to $F_2$ data

The above expression for $F_2$ compares very well with the HERA $F_2$ data at low $Q^2$, as shown in Fig. 2. The fit gives $\chi^2 = 89/(70 - 3) = 1.3$ with values as expected for the three free parameters used in the fit, namely $\epsilon = 0.09$, $A = 71 \mu b$, $\xi = 0.6$ [3]. This demonstrates that at $Q^2$ clearly below 1 GeV$^2$ the HERA $ep$ cross-section can be fully accounted for by GVDM using parameter values as determined from old investigations related to fixed target data.

At larger $Q^2$ GVDM does not give the correct $Q^2$ dependence since the resulting $F_2$ increases with $Q^2$. This may be interpreted physically as a need for a form factor suppression and we introduce the factor $(Q_0^2/Q^2)^a$ for $Q^2 > Q_0^2$ to phase out GVDM. Instead, the parton model should become applicable in the DIS region. As shown in Fig. 3, a good description of HERA $F_2$ data at intermediate $Q^2$ can be obtained by combining GVDM, with fitted values $a = 1.8$ and $Q_0^2 = 1.26$ in the form factor, and parton density functions that fit HERA $F_2$ data at larger $Q^2$. As can be seen, GVDM gives a negligible contribution for $Q^2 \gtrsim 3 \text{ GeV}^2$. 
Figure 2: $F_2$ at low $Q^2$: GVDM compared to HERA $ep$ data from ZEUS [4].

Figure 3: $F_2$ at intermediate $Q^2$: GVDM contribution to complete model including DIS parton density functions compared to H1 data [5].
4 Model for $x$-shape of parton distributions at $Q_0^2$

The parton distributions used in Fig. 3 are not just parametrisations, but are obtained from a model [6] where valence quarks and gluons are derived from momentum fluctuations according to a gaussian distribution with a width given by the uncertainty relation and the proton size. Sea quarks and gluons are obtained from similar momentum fluctuations in hadronic fluctuations of the proton, i.e. $|p⟩ = α_0|p_0⟩ + α_{pπ}|p_0π^0⟩ + α_{nπ}|nπ^+⟩ + ... α_{ΛK}|ΛK^+⟩ + ...$

This model gives a good description of available $F_2$ data with only a few fitted parameters [6]. Furthermore, it gives $u_v(x) \neq d_v(x)$ and $\bar{u}(x) \neq \bar{d}(x)$ in qualitative agreement with data, as well as $s(x) \neq \bar{s}(x)$ of interest for the NuTeV anomaly [3]. This model for parton distributions via hadronic fluctuations, fits very naturally together with GVDM based on hadronic fluctuations of the photon.

5 Conclusions

The full generalised vector meson dominance model, including contributions from a continuous mass spectrum and longitudinal polarisation states, reproduces HERA $F_2$ data at very low $Q^2$ using parameter values in agreement with old analyses of GVDM. Introducing a form factor damping at larger $Q^2$ gives a smooth transition into the deep inelastic region where a description of $F_2$ in terms of parton distribution functions become appropriate.

References

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