Wormhole Shadows

Takayuki Ohgami† and Nobuyuki Sakai†
Graduate School of Science and Engineering, Yamaguchi University, Yamaguchi 753-8512, Japan

We propose a new method of detecting Ellis wormholes by use of the images of wormholes surrounded by optically thin dust. First, we derive steady solutions of dust and more general medium surrounding the wormhole by solving relativistic Euler equations. We find two types of dust solutions: one is a static solution with arbitrary density profile, and the other is a solution of dust which passes into the wormhole and escapes into the other side with constant velocity. Next, solving null geodesic equations and radiation transfer equations, we investigate the images of the wormhole surrounded by dust for the above steady solutions. Because the wormhole spacetime possesses unstable circular orbits of photons, a bright ring appears in the image, just as in Schwarzschild spacetime. This indicates that the appearance of a bright ring solely confirms neither a black hole nor a wormhole. However, we find that the intensity contrast between the inside and the outside of the ring are quite different. Therefore, we could tell the difference between an Ellis wormhole and a black hole with high-resolution very-long-baseline-interferometry observations in the near future.

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I. INTRODUCTION

General relativity and other extended gravitational theories admit a spacetime with nontrivial topology such as a wormhole. A wormhole is a tunnel-like structure which connects two distant or disconnected regions. The original wormhole solution, which is called the Einstein-Rosen bridge, was discovered by Einstein and Rosen [1]. Because this wormhole is not traversable, it was regarded as nothing but a mathematical product. Many years later, Ellis [2] obtained a new wormhole solution: a spherically symmetric solution of Einstein equations with a ghost massless scalar field. Morris and Thorne [3] proved that these Ellis wormholes are traversable: instantaneous space movement and time travel could be achieved by passing through the wormhole. These wormholes have neither singularity nor horizon, and the tidal force is so weak that people can withstand. If such a wormhole exists, it could become a fascinating tool for voyaging to far galaxies or engaging in time travel.

The stability of traversable wormholes has been studied by several researchers. Shinkai and Hayward [4] showed by numerical simulations that Ellis wormholes are unstable. González et al. [5] considered more general wormhole solutions with a ghost scalar field and found that they are also unstable. These results indicate that Ellis wormholes and other traversable wormholes with a ghost scalar field are practically nonexistent. However, Das and Kar [6] pointed out that another matter could contribute to supporting the Ellis geometry. Furthermore, if we adopt modified gravitational theories, a matter like a ghost scalar field, which makes spacetimes unstable, may not be required. Therefore, a traversable wormhole is still a viable subject not only in theoretical physics, but also in observational astrophysics.

A possible method for probing wormholes, gravitational lensing of Ellis wormholes has been intensively studied in the literature. Basic properties of their gravitational lensing were investigated theoretically in Ref. [7]. Since Cramer et al. [8] pointed out anomalous features of the light curve of a distant star lensed by a wormhole, observational research to find wormholes by using the microlensing effect has proceeded [9]. In addition to the light curve, the lensed images [10] and the lensed spectra [11] of Ellis wormholes have also been discussed as observable quantities.

As for black holes, another method of probing them by electromagnetic observations is the use of shadows, which are the images of optical or radio sources around a black hole. Black hole shadows were originally discussed by Bardeen [12] and have recently attracted much attention [13]. Black hole shadows have not only been studied theoretically but also applied to probing black holes by very-long-baseline-interferometry (VLBI) observations [14]. Therefore, we expect that shadows could also be used to probe wormholes by VLBI observations. The shadows of a rotating wormhole were studied by Nedkova, Tinchev, and Yazadjiev [15]. They calculated the outline of the shadow in the presence of an extended source behind the wormhole.

In this paper we consider the images of wormholes surrounded by optically thin dust. Recently it was shown that Ellis wormholes possess unstable circular orbits of photons [16], as we discuss in Sec. IV. Because we expect a bright ring to appear in the image, just as in Schwarzschild spacetime, it is important to explore whether it is possible to identify wormholes by observing shadows.

This paper is organized as follows. In Sec. II, we introduce the Ellis wormhole and discuss its spacetime structure briefly. In Sec. III, to set up dust models used in our shadow analysis, we derive steady solutions of dust and a more general medium surrounding the wormhole...
by solving relativistic Euler equations. In Sec. IV, we derive null geodesic equations and discuss photon trajectories around the wormhole. In Sec. V, we investigate—by solving the radiative transfer equation as well as the null geodesic equations—the images of the wormhole surrounded by dust for the models obtained in Sec. II. Section VI is devoted to concluding remarks.

II. ELLIS WORMHOLE SPACETIME

The Ellis wormhole spacetime is one of the traversable ones, and it is expressed by the line element

$$ds^2 = -dt^2 + dr^2 + (r^2 + a^2)(d\theta^2 + \sin^2 \theta \, d\varphi^2), \quad (2.1)$$

where $a$ is the throat radius of the wormhole. This line element indicates that there is no singularity in this spacetime. To understand the geometry of the spacetime intuitively, we draw an embedding diagram as follows. Introducing a new radial coordinate as

$$r^* = \sqrt{r^2 + a^2}, \quad (2.2)$$

Comparing (2.3) and (2.2), we obtain the relation

$$z = \pm a \cdot \text{arccosh} \frac{r^*}{a}. \quad (2.4)$$

Figure 1 is a graph of (2.4) with the $\varphi$ direction, which indicates the visual image of the spacetime structure of the Ellis wormhole. We see that two separated spaces are connected by the throat like a tunnel.

III. MOTION OF INTERSTELLAR MEDIUM

In this section, we examine the general relativistic motion of the interstellar medium around the Ellis wormhole under the assumption that its self-gravity is negligible. The equations of motion of the perfect fluid in Schwarzschild spacetime and their solutions were presented in a textbook by Shapiro and Teukolsky [18]. Here we apply their method to the Ellis wormhole.

A. Thermodynamics

Suppose a local Lorentz frame comoving with fluid particles. Let $n$, $\rho$ and $P$ be, respectively, the number density, the total energy density, and the pressure of the fluid particles, which are measured in the reference frame. Then the first law of thermodynamics is written as

$$dQ = d\left(\frac{\rho}{n}\right) + Pd\left(\frac{1}{n}\right), \quad (3.1)$$

where $dQ$ is the heat gained per particle. $\rho/n$ and $1/n$ represent, respectively, the energy and the volume per particle. If the process is quasistatic (i.e., in thermal equilibrium at all times) and adiabatic, then $dQ = Tds = 0$, where $s$ is the entropy per particle and $T$ is the temperature. It follows that

$$0 = d\left(\frac{\rho}{n}\right) + Pd\left(\frac{1}{n}\right). \quad (3.2)$$

This equation displays energy conservation and is rewritten in the following, simpler form:

$$\frac{d\rho}{dn} = \frac{\rho + P}{n}. \quad (3.3)$$

B. Fluid dynamics

Assuming that the particle number is conserved, we have the general relativistic continuity equation,

$$(nu^\mu)_{;\mu} = 0, \quad (3.4)$$

where $u^\mu$ is the four-velocity of the particle fluid. Here the semicolon denotes a covariant derivative: $A^\mu_{\beta\gamma} = \partial_\beta A^\mu_{\gamma} + \Gamma^\mu_{\beta\gamma} A^\gamma$. We also assume that the particle fluid is a perfect fluid whose energy-momentum tensor is written as

$$T^{\mu\nu} = (\rho + P)u^\mu u^\nu + Pg^{\mu\nu}. \quad (3.5)$$

Applying the conservation law of energy-momentum,

$$T^\nu_{\mu;\nu} = 0, \quad (3.6)$$

to (3.5) with (3.3), we obtain the general relativistic Euler equation,

$$(\rho + P)u_{\mu;\nu} u^\nu = -P_{\mu} - u_{\mu} P_{\nu} u^\nu. \quad (3.7)$$

We have two basic equations (3.4) and (3.7), for the general perfect fluid.
Here we suppose spherically symmetric flow without angular momentum. This assumption is just for the simplicity of obtaining analytic solutions for dust flow. Then the four-velocity of the fluid takes the form
\[ u^α = \frac{dx^α}{dτ} = (u^t, u^r, 0, 0), \quad (3.8) \]
where \( τ \) is the proper time of the fluid. By use of the relation, the four-velocity of the fluid takes the form
\[ \frac{∂u}{∂t} + \frac{1}{r} \frac{∂u}{∂r} [u^r (u^t + 1) + 2ru^r] = 0, \quad (3.9) \]
\[ (\rho + P) \left( \frac{∂u}{∂t} u^t + \frac{∂u^r}{∂r} u^r \right) + \frac{P}{r} [1 + (u^r)^2] + \frac{∂P}{∂t} u^r u^t = 0. \quad (3.10) \]

We also assume that the flow is stationary, that is, \( ∂/∂t = 0 \). By use of the relation, \( -1 + u^t u^α = -(u^t)^2 + (u^r)^2 \), we rewrite (3.9) and (3.10) as
\[ \frac{u'}{n} + \frac{u'}{u} + \frac{2r}{r^2 + a^2} = 0 \quad (3.11) \]
\[ uu' = -\frac{1}{\rho + P} \frac{dP}{dr} (1 + u^2), \quad (3.12) \]
where \( u' ≡ d/dr, u ≡ |u^r| \).

We integrate the differential equations (3.11) and (3.12) and obtain the form of conservation equations
\[ \dot{M} = 4πnmu(r^2 + a^2) = \text{const}, \quad (3.13) \]
\[ \left( \frac{\rho + P}{n} \right)^2 (1 + u^2) = \left( \frac{\rho_∞ + P_∞}{n_∞} \right)^2 (1 + u_∞^2), \quad (3.14) \]
where the subscript \( ∞ \) denotes quantities at infinity and \( \dot{M} \) is the rest-mass accretion rate. Because Eqs. (3.13) and (3.14) include four unknown variables—the particle number density \( n \), the energy density \( \rho \), the pressure \( P \), and the velocity \( u \)—we need additional conditions to obtain explicit solutions.

Hereafter, we assume that the Ellis wormhole is surrounded by dust, which is characterized by
\[ \rho = nm, \quad P = 0. \quad (3.15) \]

With this condition, we obtain the steady flow solutions
\[ u = u_∞ = \text{const.}, \quad (3.16) \]
\[ \dot{M} = 4πρu_∞ (r^2 + a^2) = \text{const}. \quad (3.17) \]

We find two types of solutions, as follows:
- \( u = u_∞ = 0, \rho = \text{arbitrary function of } r. \)
- \( u = u_∞ \neq 0, \rho \propto (r^2 + a^2)^{-1}. \)

The second type implies the solution that dust passes into the wormhole and escapes into the other side with constant velocity, and its reverse. For this case we show the profile of \( ρ(r) \) in Fig. 2. Note that the region \( r < 0 \) corresponds to the opposite side of the wormhole.

**IV. PHOTON TRAJECTORIES**

In preparation for the investigation of wormhole shadows in the next section, we derive the null geodesic equations and discuss qualitative properties of photon trajectories.

**A. Null geodesic equations and effective potential**

Null geodesic equations are generally written as
\[ \frac{dk^μ}{dλ} + Γ^μ_{ρμ}k^ρ k^ρ = 0, \quad \text{with} \quad k_μ k^μ = 0, \quad (4.1) \]
where \( λ \) and \( k^μ ≡ dx^μ/dλ \) are the affine parameter and the null vector, respectively. The geodesics in the \( \theta = \pi/2 \) plane are given by
\[ \frac{d}{dλ} k^t = 0, \quad \frac{d}{dλ} [(r^2 + a^2)k^r] = 0, \quad (4.2) \]
\[ \frac{d}{dλ} k^r - r(k^φ)^2 = 0, \quad (4.3) \]
\[ -(k^t)^2 + (k^r)^2 + (r^2 + a^2)(k^φ)^2 = 0. \quad (4.4) \]
Because Eq. (4.3) is also derived by (4.2) and (4.4), we do not have to solve it. By integrating (4.2), we obtain two
conserved quantities,

\[ E = k^t, \quad L = (r^2 + a^2)k^\varphi \] (4.5)

Then (4.4) becomes

\[ k^t r^2 + V_{\text{eff}}(r) = E^2, \quad V_{\text{eff}}(r) \equiv \frac{L^2}{r^2 + a^2}. \] (4.6)

We show the effective potential \( V_{\text{eff}} \) in Fig. 3, which will be discussed in the next subsection.

![Fig. 3: Effective potential of the Ellis wormhole. The maximum point at \( r = 0 \) corresponds to the unstable circular orbits of photons.](image)

It follows from (4.5) and (4.6) that

\[ \frac{dr}{d\varphi} = \frac{k^r}{k^\varphi} = \pm \frac{r^2 + a^2}{L} \sqrt{E^2 - V_{\text{eff}}}, \] (4.7)

which gives spatial trajectories of photons, which is discussed in the next subsection. For use in Sec. V, we derive the relation from (4.7),

\[ \lim_{r \to \infty} \frac{1}{r^2} \frac{dr}{d\varphi} = \pm \frac{E}{L}. \] (4.8)

**B. Photon trajectories**

Photon trajectories in the Ellis geometry were studied in Ref. [16]. Here we reanalyze them and discuss their characteristics.

Using the effective potential shown in Fig. 3, we can discuss qualitative properties of photon trajectories in the wormhole spacetime. The trajectories are classified into three types: A, B, and C. The maximum at the throat \( r = 0 \) corresponds to the unstable circular orbit of photons. Type C represents the photon which passes by the throat. Type A represents the photon which passes into the throat and escapes into the other side. Type B represents the photon which approaches and winds many times in the vicinity of the unstable circular orbit.

Figure 3(a) shows photon trajectories around the wormhole. On the \( \theta = \pi/2 \) plane we define the rectangle coordinates \((x, y)\) as

\[ x = r^* \cos \varphi, \quad y = r^* \sin \varphi. \] (4.9)

These trajectories end up at the observer at \( r = 300a, \varphi = 0 \). Labels A, B, and C correspond to those on the effective potential in Fig. 3. The dashed line of A represents the trajectory in the other side of the wormhole \((r < 0)\).

For reference, we also show trajectories around a Schwarzschild black hole in Fig. 4(b). The trajectory \( B' \) in Fig. 4(b) represents the photon which approaches and winds many times in the vicinity of the unstable circular orbit, which is analogous to the trajectory B in Fig. 4(a). Accordingly, in both spacetimes one could observe brightening there when gas falls into the wormhole or the blackhole. This similarity is important, but we are also interested in the observational difference between the two.

**V. WORMHOLE SHADOWS**

In this section, we investigate the optical images of the wormhole surrounded by dust, using the solutions obtained in Sec. III.

**A. Apparent position of optical sources**

We consider the rectangle coordinates \((x, y)\) defined by (4.9) on the \( \theta = \pi/2 \) plane, where the center of the wormhole is located at the origin and the observer at \((x_o, 0)\) \((\varphi = 0)\), as shown in Fig. 5. We denote the intersection of the \( y \) axis with the tangent to the ray at the observer by \( y = \alpha \). Therefore, we can regard \( \alpha \) as the apparent length from the center.

The equation of this tangent line is

\[ \frac{x}{x_o} + \frac{y}{\alpha} = 1, \quad \text{i.e.,} \quad r^* \cos \varphi + \frac{r^*}{\alpha} \sin \varphi = 1. \] (5.1)

Differentiating (5.1) and taking the limit of \( r^* \to x_o, \varphi \to 0 \), we obtain

\[ \alpha = -x_o \frac{d\varphi}{dr^*}. \] (5.2)

Furthermore, taking the limit of \( x_o \to \infty \) and using (4.8), we find

\[ \alpha \to \frac{L}{E}. \] (5.3)

As Fig. 5 shows, \( L/E \) represents the apparent positions of optical sources; therefore, it is interpreted as an impact parameter.
We numerically integrate the null geodesic equations from the observer to the sources. Once we specify the initial point and the orbit plane, the remaining parameter of the geodesic equations is the impact parameter $L/E$ only.

### B. Radiation intensity

To calculate the observed intensity of radiation emitted from optically thin gas, we solve the general relativistic radiative transfer equation, which is generally expressed as \[ \frac{d\mathcal{J}}{d\lambda} = \frac{\eta(\nu)}{\nu^2} - \nu \chi(\nu) \mathcal{J}, \quad \mathcal{J} \equiv \frac{I(\nu)}{\nu^3} \] (5.4)

where $\nu$ is the photon frequency, $I(\nu)$ is the specific intensity, $\mathcal{J}$ is the invariant intensity, $\eta(\nu)$ is the emission coefficient and $\chi(\nu)$ is the absorption coefficient. Because Eq. (5.4) is the differential equation along null geodesics, we should solve the null geodesic equations simultaneously.

Here we make the following assumptions, for simplicity:

- The dust does not absorb radiation, i.e., $\chi(\nu) = 0$.
- $\eta(\nu)$ is proportional to the dust density which is measured along the null geodesics, i.e., $\eta(\nu)d\lambda \propto \rho u_\mu dx^\mu$.

Introducing a positive factor $H(\nu)$, which is proportional to the spectrum of the dust sources, we express $\eta(\nu)$ as

$$\eta(\nu)d\lambda = -H(\nu)\rho u_\mu dx^\mu.$$ (5.5)

With these assumptions we can integrate (5.4) as

$$\mathcal{J} = -\int \frac{H(\nu)}{\nu^3} \rho u_\mu dx^\mu.$$ (5.6)

The integration in (5.6) should be performed alongside the null geodesics. The frequency measured by observers comoving with the dust particles is given by

$$\nu = -u_\mu k^\mu.$$ (5.7)

Generally we should fix the spectrum of the dust sources, i.e., $H(\nu)$. Here, for simplicity, we assume a flat spectrum, $H(\nu) = \text{const.}$
C. Numerical analysis

We numerically calculate the intensity distribution as follows:

(i). Put the observer at \( x_o = 300a \).

(ii). For a given value of \( \alpha = L/E \), we solve the null geodesic equations from the observer. We can choose a value of the initial (observed) frequency \( \nu_o \) arbitrarily because the ratio of \( \nu_o \) to the emitted frequency \( \nu_e \) does not depend on \( \nu_o \).

(iii). With the values of \( \nu \) at each point, which is determined by the null geodesic equations, we integrate (5.6) to obtain the intensity \( I \). We adopt the fourth-order Runge-Kutta method for all integrations.

(iv). We continue the integrations until \( r = 300a \) again, where the gas density is sufficiently small.

(v). Iterate (ii) \( \sim \) (iv) by changing the value of \( \alpha \).

As for the dust distribution, we consider two cases.

- Case 1: static top-hat distribution.
  \[
  u = 0, \quad \rho = \begin{cases} 
  \text{const} > 0 & (\ -10 < r/a < 10) \\
  0 & (\text{otherwise})
  \end{cases}
  \]

- Case 2: nonstatic dust that flows into the wormhole and escapes into the other side.
  \[
  u = 10^{-7}, \quad \rho \propto \{(r/a)^2 + 1\}^{-1}.
  \]

Figures 6 and 7 show the numerical results for radiation intensity for cases 1 and 2, respectively. Figures 6(a) and 7(a) are the graphs of \( I_{\nu}(\alpha)/I_{\nu}(0) \), which does not depend on \( \nu \) for the flat spectrum, \( H(\nu) =\text{const} \). \( \alpha_{\text{max}} \) denotes the value of \( \alpha \) where \( I_{\nu}(\alpha) \) is the maximum; it corresponds to the photon which winds around the throat infinite times. Figures 6(b) and 7(b) display the optical images. In both cases a bright ring appears, in accord with the peak in Figs. 6(a) and 7(a).

We also investigate case 2 for various values of the fluid velocity \( u \). As long as the fluid is nonrelativistic (i.e., \(|u| \ll 1\)), the result does not change essentially even if the flow is in the reverse direction (i.e., \( u < 0 \)). We may therefore conclude that the profile and appearance in Fig. 7 is the general feature for nonstatic dust that flows into or out of the wormhole.

As we discussed in Sec. IV.B, the appearance of a bright ring is a characteristic common to black holes. This indicates that the appearance of a bright ring solely confirms neither a black hole nor a wormhole. Therefore, we closely reinvestigate the image of the Schwarzschild black hole surrounded by optically thin dust. The stationary solution of dust flow in the Schwarzschild spacetime is expressed as \[18\]
\[
  u \propto r^{-\frac{1}{2}}, \quad \rho \propto r^{-\frac{1}{2}}.
\]

VI. CONCLUDING REMARKS

We have proposed a new method for detecting Ellis wormholes by use of the images of wormholes surrounded by optically thin dust. First, we derived steady solutions of dust and a more general medium surrounding
the wormhole by solving relativistic Euler equations. We
found two types of dust solutions: one is a static solution
with an arbitrary density profile, and the other is a solu-
tion for dust which passes into the wormhole and escapes
into the other side with constant velocity. Next, solv-
ing null geodesic equations and radiation transfer equa-
tions, we investigated the images of the wormhole sur-
rrounded by dust for the above steady solutions. Because
the wormhole spacetime possesses unstable circular or-
bits of photons, a bright ring appears in the image, just
as in Schwarzschild spacetime.

We have assumed that the dust distribution is spher-
ically symmetric and steady state. This is just for sim-
plicity to obtain analytic solutions of dust flow; if we
consider more realistic situations, we should obtain nu-
merical solutions of rotating dust flow. On the other
hand, the effect of the angular velocity on radiation flux
is negligible as long as the velocity is nonrelativistic.

Our results indicate that the appearance of a bright
ring solely confirms neither a black hole nor a worm-
hole. This could be a serious problem for identifying
black holes by optical/radio observations. However, we
found that the intensity contrast between the inside and
the outside of the ring are quite different. Therefore,
we could be able to tell the difference between an El-
is wormhole and a black hole with high-resolution very-
long-baseline-interferometry observations in the near fu-
ture.

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