Elastic proton-proton scattering from ISR to LHC energies, focusing on the dip region *

T. Csörgő\textsuperscript{1}, R. J. Glauber\textsuperscript{2}, and F. Nemes\textsuperscript{1,3}

\textsuperscript{1}Wigner Research Centre for Physics
H-1525 Budapest 114, P.O.Box 49, Hungary

\textsuperscript{2}Lyman Laboratory of Physics, Harvard University
17 Oxford St, Cambridge, MA02138, USA

\textsuperscript{3}CERN, CH-1211 Geneva 23, Switzerland

November 12, 2013

Abstract

The differential cross-section of elastic proton-proton collisions is studied at ISR and LHC energies, utilizing a quark-diquark model, that generalizes earlier models of Bialas and Bzdak, and, in addition, a model of Glauber and Velasco. These studies suggest that the increase of the total pp cross-section is mainly due to an increase of the separation of the quark and the diquark with increasing energies. Within the investigated class of models, two simple and model-independent phenomenological relations were found, that connect the total pp scattering cross-section $\sigma_{\text{tot}}$ to the effective quark, diquark size and their average separation, on one hand, and to the position of the dip of the differential cross-section, on the other hand. The latter $t_{\text{dip}} \sigma_{\text{tot}} \simeq C$ relation can be used to predict $t_{\text{dip}}$, the position of the dip of elastic pp scattering for future colliding energies, and for other reactions, where $\sigma_{\text{tot}}$ is either known or can be reliably estimated.

PACS: 13.75.Cs, 13.85.-t, 13.85.Lg, 13.85.Dz

\textsuperscript{*}Presented at the Low x workshop, May 30 - June 4 2013, Rehovot and Eilat, Israel
1 Diffraction - a historical perspective

Diffractive scattering of electrons on various nuclei provided important insights to the charge density distributions of spherical nuclei. The detailed analysis resulted in simple observations by Hofstadter and colleagues, that were summarized in the Nobel lecture of Hofstadter as follows:

- The volume of spherical nuclei is proportional to the mass number $A$.
- The surface thickness is constant, independent of $A$.

These observations revealed structures in atomic nuclei on the femtometer scale. They imply also that the central charge density of large nuclei is approximately constant. For more details, we recommend the Nobel Lecture (1961) by R. Hofstadter [1]. The results summarized there were one of the first observations of images on the femtometer scale, corresponding to nuclear charge density distributions. The more recent historical overview of ref. [2] discusses applications of multiple diffraction theory to high energy particle and nuclear physics. Recently, with 7 and 8 TeV colliding energies of proton-proton reactions at CERN LHC, the resolution of diffractive images in elastic proton-proton scattering reached the sub-femtometer scales, as we demonstrate below.

Our talk at the Low-X 2013 conference discussed two model classes: the Bialas-Bzdak and the Glauber-Velasco models. Our conference contribution follows the lines of that presentation, except for the details of on results from the Bialas-Bzdak model, for which we direct the interested readers to suitable references.

2 Diffraction in quark-diquark models

In a series of papers, Bialas and Bzdak discussed a quark-diquark model of elastic proton proton scattering [3, 4, 5, 6]. Recently, this Bialas-Bzdak or BB model was tested in details on elastic proton-proton scattering data both at the ISR and LHC energies [7] and developed further to obtain a more realistic description of the dip region of elastic pp scattering [8].

In the BB model, the differential cross-section of elastic proton-proton scattering is given by the following formula

$$\frac{d\sigma}{dt} = \frac{1}{4\pi} |T(\Delta)|^2,$$

where $\Delta = |\tilde{\Delta}|$ is the modulus of the transverse component of the momentum transfer. In the high energy limit, $s \to \infty$, $\Delta^2 \approx -t$, where $t$ is the squared four-momentum transfer. The amplitude of the elastic scattering in momentum space, $T(\tilde{\Delta})$ is given by the Fourier-transform of the amplitude in impact parameter space,

$$T(\tilde{\Delta}) = \int_0^{+\infty} \int_{-\infty}^{+\infty} \mathcal{t}_{el}(\vec{b}) e^{i\tilde{\Delta}\vec{b}} d^2b = 2\pi \int_0^{+\infty} \mathcal{t}_{el}(b) J_0(\Delta b) \, db,$$
where the impact parameter is denoted by $\vec{b}$ and $b = |\vec{b}|$. From unitarity conditions one obtains

$$t_{el}(\vec{b}) = 1 - \sqrt{1 - \sigma(\vec{b})}. \quad (3)$$

The inelastic proton-proton cross-section in the impact parameter space for a fixed impact parameter $\vec{b}$ is given by the following integral

$$\sigma(\vec{b}) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} d^2s_q d^2s_q' d^2s_d d^2s_d' D(s_q, s_q') D(s_d, s_d') \sigma(s_q, s_q', s_d, s_d'; \vec{b}), \quad (4)$$

where the integral is taken over the two-dimensional transverse position vectors of the quarks $s_q, s_q'$ and diquarks $s_d, s_d'$.  

3 Bialas - Bzdak model of elastic pp scattering

The BB model approximates the quark-diquark distribution inside the proton with a Gaussian form \([3, 4, 5, 6]\)

$$D(s_q, s_d) = \frac{1 + \lambda^2}{\pi R^2_{q,d}} e^{-(s^2_q + s^2_d)/R^2_{q,d} + \lambda s^2_q}, \quad \lambda = m_q/m_d, \quad (5)$$

and, in order to define a model that can be analytically integrated and compared to data in a straightforward way, the model is formulated in simple and if possible Gaussian terms. The BB model also supposes that protons are scattered elastically if and only if all of their constituents are scattered elastically

$$\sigma(s_q, s_d; s_q', s_d'; \vec{b}) = 1 - \prod_{a,b \in \{q,d\}} \left[1 - \sigma_{ab}(\vec{b} + s_a' - s_b')\right], \quad (6)$$

where the inelastic differential cross-sections of the constituents are parametrized with Gaussian distributions as well

$$\sigma_{ab}(s) = A_{ab} e^{-s^2/R^2_{ab}}, \quad R^2_{ab} = R^2_a + R^2_b. \quad (7)$$

Here $A_{ab}$ are suitably chosen normalization factors, $a, b$ stand for $q,d$, denoting quarks and/or diquarks, while $R_q$ and $R_d$ stand for the Gaussian radii, that characterize in the BB model the quark and diquark inelastic scattering cross-sections, respectively.
to have a quark-diquark structure. The diquark is assumed to be scattered as a loosely bound state of two correlated quarks. These models with three uncorrelated quarks in the proton, both for the protons [9]. In its original form, the BB model has been integrated analytically, as noted by Bialas and Bzdak, models with three uncorrelated quarks in the proton, labelled as \( p = (q, q, q) \) were tested before at ISR energies and they are known to fail, with other words, we know that the quarks are correlated inside the protons [9]. In its original form, the BB model has been integrated analytically, both for the \( p = (q, d) \) and the \( p = (q, (q, q)) \) scenarios, assuming that the real part of forward scattering is negligible.

Figure 1: Scheme of the scattering of two protons, when the proton is assumed to have a quark-diquark structure. The diquark is assumed to be scattered as a single entity (left) or as composition of two quarks (right). This figure is a snapshot and all the model parameters follow a Gaussian distribution. Note, that a center of mass energy dependent Lorentz-contraction determines the longitudinal scale parameters.
Figure 2: (Color online.) Results of Minuit fits of both versions of the Bialas-
Bzak model at ISR energies. Left panel indicates the scenario $p = (q, d)$, where
the diquark is assumed to scatter as a single entity while the right panel indicates
the scenario $p = (q, (q, q))$, where the diquark inside the proton is considered to
be a scattering object consisting of two quarks.

The two panels of Figure 2 indicate CERN Minuit fit results of the BB model
to differential cross-section data on elastic proton-proton scattering at the ISR
energy of $\sqrt{s} = 23.5$ GeV. Left plots correspond to the scenario $p = (q, d)$ while
the right panel stands for the scenario $p = (q, (q, q))$. The top parts show the
data points and the result of the best fit, while the lower parts indicate the
relative deviation of the model from data in units of measured error bars. As
the original BB model is singular around the dip, 3 data points were left out
from the optimization, that are located closest to the diffractive minimum,
and indicated with filled (red) circles in Figure 2. The fit range was restricted
to $0.36 \leq -t \leq 2.5$ GeV$^2$, so that a fair comparison could subsequently be
made with the first TOTEM results on proton-proton elastic scattering at LHC
energy of 7 TeV of ref. [10]. The best fits are shown with a solid (black) line
in the fitted range, while their extrapolation to lower $t$ values are also shown,
with dashed (green) lines. The confidence levels, after fixing the values of $\lambda$ and
$A_{qq}$ to 0.5 and 1, respectively, come very close to 0.1%, indicating that the fit
quality is similar, statistically acceptable in both scenarios. Similar fit qualities
were reported at each ISR energies of 30.7 GeV, 52.8 GeV and 62.5 GeV, see
ref. [7] for details. Figure 3 shows the comparison of the BB model to TOTEM
data on elastic pp scattering at 7 TeV LHC energies, indicating a qualitative
change, as compared to the fit results at ISR energies: the quality of this fit is
statistically not acceptable, CL is significantly below 0.1%, and the fit deviates
from the data in particular in the dip region. The bottom parts indicate, that
the shape of the differential cross-section in the dip region, around the first diffractive minimum is not reproduced correctly by the original BB model at 7 TeV LHC energies, and, as also can be seen on this Figure 3 this shortcoming cannot be fixed by leaving out a few data points around the diffractive minimum from the optimization procedure. The details of these BB fits are described in ref. [7].

Figure 3: (Color online.) The result of the fit of the original version of the Bialas-Bzdak model at 7 TeV LHC energies in two different scenarios: left panel stands for the \( p = (q, d) \) scenario, when the diquark is assumed to scatter as a single entity, while the right panel stands for the \( p = (q, (q, q)) \) case, when the internal structure of the diquark is resolved as a correlated system of two quarks.

Recently, two of us generalized the Bialas-Bzdak model by adding a small real part to the forward scattering amplitude, to investigate, if the description of the dip region can be improved can be made statistically acceptable in this way. The results of this scenario are described in detail in ref. [8]. A small real part was added to the forward scattering amplitude by using an analogy of with the Glauber-Velasco model, and assuming that even if all the parton level scatterings are elastic, the proton-proton scattering can, with a small probability, become inelastic. In this manner, a parton level \( \rho \) parameter was introduced. The results, detailed in ref. [8], indicate that a small real part indeed improves the agreement of the BB model with data in the dip region, and the fits become statistically acceptable in the whole \( t \) region, including all the data points from dip region, if the energy of the collisions is limited to the ISR energy range of \( \sqrt{s} = 23.5 - 62.5 \) GeV. At the LHC energy of \( \sqrt{s} = 7 \) TeV, the generalized Bialas-Bzdak or the \( \alpha \)BB model resulted in an improvement, that reduced the
disagreement between the BB model and the data substantially and filled the dip region rather dramatically. However, even this improvement did not result in a statistically acceptable fit quality to the differential cross-section of elastic proton-proton collisions at this LHC energy, although good quality fits were obtained if the fit range was limited to the dip region. As a consequence, we kept on searching for a model that is able to describe elastic pp scattering data at LHC energies, and investigated the performance of the Glauber-Velasco model [11]. Before reporting the results, let us summarize what we have learned till now from the detailed fits using the original Bialas-Bzdak or BB model, and its generalized version when a small real part is added to its forward scattering amplitude, see refs. [7, 8] for further details.

4 What have we learnt so far?

The original version of the Bialas-Bzdak model gave a statistically acceptable description of elastic pp scattering data at ISR energies, if the data points close to the diffractive minimum were left out from the fit. If these data points were included and also a small real part was added to the model, as detailed in ref. [8], the fits at the ISR energies from $\sqrt{s} = 23.5$ GeV to 62.5 GeV become statistically acceptable, good quality fits, in the fit range of $0.36 \leq -t \leq 2.5$ GeV$^2$. Two model parameters could be fixed at all energies ($A_{qq} = 1$ and $\lambda = 1$) while maintaining the statistically acceptable fit quality. The parameter $\alpha$, that was introduced as a parton level ratio of the real to imaginary part of the forward scattering amplitude, remained indeed in the region of very small values, $\alpha = 0.01 \pm 0.01$ except at 52.8 GeV, where $\alpha = 0.02 \pm 0.01$ value was found. Although these $\alpha$ parameters are within errors consistent with zero, a small but non-vanishing value provided qualitatively better fits in the dip region, as detailed in ref. [8]. The best fit parameters, that described the quark structure of the protons geometrically, took also rather interesting values. For example, the quark radius $R_q$ within 2 standard deviations was consistent with an energy independent value of $R_q = 0.27 \pm 0.01$ fm. The diquark size indicated a nearly constant value, varying between $R_d = 0.71 \pm 0.01$ to $0.77 \pm 0.01$ fm, slightly increasing with increasing $\sqrt{s}$. Although the fit to the TOTEM data 7 TeV were not statistically acceptable, the best parameter values for the quark and diquark radii were in the same range, except a slight decrease of the diquark size in the $p = (q, (q, q))$ model at 7 TeV. We observed that the biggest variation, when the energy is increased to 7 TeV, is observable in the scale that measures the typical quark-diquark distance, $R_{qd}$. This value was in the range of $R_{qd} = 0.23 \pm 0.01$ fm at ISR energies in the $p = (q, (q, q))$ model, while it increased to the value of $0.73 \pm 0.01$ fm at 7 TeV. Similar trend of increasing quark-diquark separation is seen in the $p = (q, d)$ scenario. Graphically, the evolution of the proton elastic scattering structure is illustrated on Figure 3 where the best fit parameters are also indicated on the sub-plots, generated for the case of the $p = (q, (q, q))$ scenario. The same qualitative behaviour of increasing quark-diquark distance is observed also in the $p = (q, d)$ picture, see ref. [8] for further details.
Figure 4: Visualization of the fit results of Bialas-Bzdak models, extended to a small real part, for the case of \( p = (q, (q, q)) \), when the diquark is assumed to be resolvable as a weekly bound state of two quarks. The main effect of increasing \( \sqrt{s} \) is apparently the increasing value of \( R_{qd} \), the typical quark-diquark distance.

As discussed both in refs. [7] and [8], the \( p = (q, d) \) and the \( p = (q, (q, q)) \) models provide similar quality of data description both at ISR energies (where they are both statistically acceptable) and at 7 TeV LHC energy (where both fail to describe TOTEM data in a statistically acceptable manner). Nevertheless we compare the best fit values of the different energies, to try to get a qualitative insight assuming, that the missing element of the model will not modify dras-
tically the best fit parameters at LHC energies. Given that the $p = (q, d)$ and the $p = (q, (q, q))$ Bialas-Bzdak models correspond to two different assumptions about the internal structure of the protons, it was a kind of surprise for us, that the measured total pp cross-section $\sigma_{tot}$ was phenomenologically related to the parameters of the BB model in a model-independent way, i.e. the following relation is approximately valid for both scenarios:

$$\sigma_{tot} \approx 2\pi R_{eff}^2 = 2\pi (R_q^2 + R_d^2 + R_{qd}^2).$$  \hspace{1cm} (8)$$

This approximation was found to be valid within a relative error of about 9% at ISR energies, while at the LHC energies it yields only an ball-park value, order of magnitude estimation ($\frac{\sigma_{tot}}{2\pi R_{eff}^2} = 1.42$). We also have observed an interesting scaling property of the differential and the total proton-proton elastic scattering cross-section, namely the product of the total cross-section times the $t$ of the dip is within errors a constant:

$$t_{dip}\sigma_{tot} \approx C$$  \hspace{1cm} (9)$$

where $C = 54.8 \pm 0.7$ mb GeV$^2$ from a fit. We find that this relation is valid within 5% relative error at each ISR and also at 7 TeV LHC energies. A similar relation holds for a light scattering from a black disc, however, with a significantly different constant value, $C_{blackdisc} \approx 35.9$ mb GeV$^2$.

![Figure 5: The $|t_{dip}|\sigma_{tot,exp}$ ratio, directly obtained from experimental data. The dashed line indicates 1, which value within errors is consistent with all the data from $\sqrt{s} = 23.5$ GeV to 7 TeV.](image)

Given that there are theoretically well established formulas for the description of the rise of the total pp scattering cross-section with increasing energies,
the above formula can be well used to predict the position of the (first) minimum or the dip in the differential cross-section of pp collisions and also can be extrapolated, or, predicted for pA and AB collisions [15]. Given that we could not find a statistically acceptable quality fit with the Bialas-Bzdak model to 7 TeV TOTEM data on elastic pp scattering at LHC, neither in the original form, nor when a small real part is added to the forward scattering amplitude of this model, we started to look for alternative interpretations and derivations of \( \frac{d\sigma}{dt} \). One possibility is to allow for not only small values of the real part of the forward scattering amplitude, but still keep the basic structure of the Bialas-Bzdak model. The studies in this direction will be reported elsewhere. In the next section we report about the other natural direction, that we investigated in detail. In particular, when we added a small real part to the forward scattering amplitude to the Bialas-Bzdak model in ref. [8], we were introducing a parton level \( \rho \) parameter inspired by the Glauber-Velasco model of refs. [11, 12]. In the next section, we summarize this model and report about its first comparisons to TOTEM data.

5 Overview of the model of Glauber and Velasco

In this section, we follow the lines of the presentation of the Glauber-Velasco model, as described in refs. [11, 12]. The Glauber diffractive multiple scattering theory is utilized to describe elastic collisions of two nucleons, which are pictured as clusters of partons. The parton distributions are assumed to have form factors given by the experimentally measured electric charge form factors. Differential cross sections calculated in this way showed good agreement with the experimentally measured ones over a broad range of \( pp \) and \( p\bar{p} \) energies, when the parton-parton scattering amplitude is given a suitable parametrization [11, 12]. The range of the parton-parton interaction derived from these data is found to increase steadily with energy. The absorption processes that take place are localized in the overall nucleon-nucleon interaction by calculating the shadow profile function. The emerging picture corresponded to an opaque region of interaction that grows in radius with increasing energy. The surface region of the interaction seems however to maintain a remarkably fixed shape as the radius grows. In the multiple diffraction theory of Glauber and Velasco, the elastic scattering amplitude for diffractive collisions can be written as an impact parameter integral

\[
F(t) = i \int_0^\infty J_0 \left( b \sqrt{-t} \right) \left\{ 1 - \exp \left[ -\Omega(b) \right] \right\} b \, db.
\] (10)

Any particular model is characterized by the opacity function \( \Omega(b) \), which in general may be a complex valued function. If we picture the two colliding nucleons as clusters of partons that scatter one another with the averaged scattering amplitude \( f(t) \), then the opacity function can be written in the form of an integral over momentum transfers \( q \).
\[ \Omega(b) = \frac{\kappa}{4\pi} (1 - i\alpha) \int_0^\infty J_0(q b) G_{p,E}^2(-t) \frac{f(t)}{f(0)} q dq, \]  

(11)

where \( q = \sqrt{-t} \). The constants \( \kappa \) and \( \alpha \) in this expression are real-valued and need to be determined empirically. The function \( G_{p,E}(t) \) is the form factor for the parton density in the proton. Following ref. \[12\], we shall assume it to be the same as the observed electric form factor for the proton. One choice of parametrization we have investigated is

\[
\frac{f(t)}{f(0)} = \frac{e^{i(b_1|t| + b_2 t^2)}}{\sqrt{1 + a |t|}}.
\]

(12)

The BSWW form factor, corresponding to the distribution of electric charge in the proton, is described with a four-pole parametrization \[14\]

\[
G_{p,E}(q^2) = \sum_{i=1}^n a_i^E \left( \frac{m_i^E}{q^2 + m_i^E} \right)^2, \quad \sum_{i=1}^n a_i^E = 1, \quad G_{p,E}(0) = 1.
\]

(13)

The differential cross-section for elastic pp collisions is evaluated as

\[
\frac{d\sigma_{el}}{d|t|} = \pi |F(t)|^2.
\]

(14)

The parameters of the BSWW form factor are given by the following table:

| \( a_i^E \) | \( (m_i^E)^2 \) (fm\(^{-2}\)) |
|------------|-----------------|
| 0.219      | 3.53            |
| 1.371      | 15.02           |
| -0.634     | 44.08           |
| 0.044      | 154.20          |

Table 1: Best fit parameters \[14\] of the four-pole fit of Eq. (13).
Figure 6: Glauber-Velasco model fit to the differential cross-section of proton-proton elastic scattering data at 7 TeV, in the range of $0.36 \leq -t \leq 2.5$ GeV$^2$.

Figure 6 indicates, that the Glauber-Velasco model is able to describe successfully the differential scattering cross-section of elastic pp collisions at the 7 TeV LHC energies: the fit quality is statistically acceptable, with CL $> 0.1\%$. We have tested the model at the ISR energy range of 23.5 GeV - 62.5 GeV too, where the similarly good quality fits were found. The detailed results will be reported in a manuscript that is currently under preparation.

6 Summary

In summary, we have analyzed elastic proton-proton scattering data from the 23.5 GeV ISR energies to 7 TeV LHC energies, using various forms of the Bialas-Bzdak model. We found that the scenario when the proton is considered to be a quark-diquark state provides a fit quality that is similar to the case when the diquark is resolved as a correlated quark-quark system within the framework of the same model. Adding a small real part to the forward scattering amplitude of the original Bialas-Bzdak model provides a statistically acceptable description of elastic pp scattering data at the ISR energies, however, even this generalized Bialas-Bzdak model fails to describe TOTEM data on elastic pp scattering at 7 TeV. Given that the generalization of the Bialas-Bzdak model followed the lines of the Glauber-Velasco model, we tested also the performance of the Glauber-Velasco model in its original form, and found that it was describing elastic proton-proton scattering both at ISR and at LHC energies when the fit range was restricted to $0.36 \leq -t \leq 2.5$ GeV$^2$. 
7 Acknowledgments

T. Cs. would like to thank for professor Glauber for his kind hospitality at Harvard University as well as for his inspiring and fruitful visits to Hungary and CERN, that made the completion of these results possible. He also would like to express his gratitude to the organizers of the Low-X 2013 conference for an invitation and for creating an inspiring and useful meeting. This research was supported by the Hungarian OTKA grant NK 101428 and by a Ch. Simonyi fund, as well as by the Hungarian Academy of Sciences, by a HAESF Senior Leaders and Scholars fellowship and by the US DOE.

References

[1] R. Hofstadter, Nobel Lecture, 1961, from http://www.nobelprize.org/
[2] R. J. Glauber, Nucl. Phys. A 774 (2006) 3 [nucl-th/0604021].
[3] A. Bialas and A. Bzdak, Acta Phys. Polon. B 38 (2007) 159 [hep-ph/0612038].
[4] A. Bzdak, Acta Phys. Polon. B 38 (2007) 2665 [hep-ph/0701028].
[5] A. Bialas and A. Bzdak, Phys. Lett. B 649 (2007) 263 [nucl-th/0611021].
[6] A. Bialas and A. Bzdak, Phys. Rev. C 77 (2008) 034908 [arXiv:0707.3720 [hep-ph]].
[7] F. Nemes and Csörgő, Int. J. Mod. Phys. A 27 (2012) 1250175 [arXiv:1204.5617 [hep-ph]].
[8] T. Csörgő and F. Nemes, [arXiv:1306.4217 [hep-ph]].
[9] W. Czyz and L. C. Maximon, Annals Phys. 52 (1969) 59.
[10] G. Antchev et al. [TOTEM Collaboration], Europhys. Lett. 95, 41001 (2011) [arXiv:1110.1385 [hep-ex]].
[11] R. J. Glauber and J. Velasco, Phys. Lett. B 147 (1984) 380.
[12] R. J. Glauber and J. Velasco, “Multiple Diffraction Theory of anti-p p and p p Elastic Scattering,” Print-88-0334 (HARVARD).
[13] G. Antchev et al. TOTEM Collaboration, Europhys. Lett. 96 (2011) 21002 [arXiv:1110.1395 [hep-ex]].
[14] F. Borkowski, G. G. Simon, V. H. Walther, and R. D. Wendling, Nucl. Phys. B93 (1975) 461.
[15] M. Csanád, T. Csörgő, F. Nemes and T. Novák, in preparation.