Commnication via Quantum Neural Network
A. Al- Segher(1) and Nasser Metwally(1,2)
(1) Mathematics Department, Faculty of Science, Aswan Egypt
(2) Math. Dept., College of Science, Bahrain University, 32038 Bahrain

Abstract

In this study, the partially entangled neural networks is used to transfer information between two neurons, where the original teleportation protocol is employed this for this purpose. The effect of the network strength on the fidelity of the transported information is investigated. We show that as the strength of the network increases, the accuracy of the transformed information increases. As a practical application, we consider the spread of swine flu virus between two equivalent tranches of the community. In this treatment two factors are considered, one for humanity and the other for influence factor. The likelihood of infection between different age group is investigated, where we show that the strength of the neural network and the degree of infection plays an important role on transferring infection between different age group. From theoretical point of view, we show that it is possible to control the spread of the virus by controlling the network parameter. Also, by using local rotation, one can decrease the rate of infection between the young.

Keywords: Neural networks, Entanglement, Teleportation.

PACS 03.65.-w, 03.67.-a, 03.65.Yz

1 Introduction

Quantum information, QI is one of the most important events which has considerable progress theoretically and experimentally. To manipulate QI, as sending, coding, etc, one needs entangled quantum channel for these proposes. There are several efforts that has been done to generate entangled channels. These channels have been demonstrated in many different types of systems, such as photons [1], ions [2], atoms [3] and charged qubit [4]. All these types of quantum channels have been used to perform different quantum information tasks as quantum teleportation [5, 6], dense coding, cryptography.

Natural computing is a computational version of the process of extracting ideas from nature to develop computational systems to perform computations [7]. One of the most promising type of quantum channels are the neural networks, NN. This field contains several basic ideas, as concept of processing elements (neurons), which represent the correlated processor units, the transformation performed by these elements and the network dynamics [8, 9]. Due to its potential in describing the brain function, the NN has attracted much attentions, where it represents a model of parallel distributed memory [10, 11]. In fact NN,
can compute any computable functions, so it is classified as neurocomputer. Also, NN is useful for classification and pattern recognition \cite{7}.

Quantum neural network, QNN represents a type of artificial NN, where it has all the features of NN \cite{12}. It is one of a typical approaches to study the human brains, where the information processing in the brain is mediated by the dynamics of a large interconnected neuronal populations. On the other hand, the basic components of neural cytoskeleton are very likely to possess quantum mechanical properties due to their size and structure \cite{13}. As an example the tubuline protein has the ability to flip from one conformation to another. These two conformations act as two basis states of the system. Also the system can lie in a superposition of these two states, which can give a plausible mechanism for creating a coherent state in the brain \cite{14}.

There are different models that have been considered for QNN. As an example, in \cite{15} the authors have proposed a model that is a multilayered neural network and investigated its characteristic features, such as the effect of quantum superposition and probabilistic interpretation. A model of recurrent quantum neural network has been introduced by Behera et al \cite{14}, where they used a linear neural circuit to set up the potential field in which the quantum brain is dynamically excited. Also a theoretical quantum brain has been proposed using a nonlinear schrödinger wave equation \cite{16}. A learning method has been proposed for quantum neural network by Kingo et al. \cite{17}. Shafee \cite{18} has suggested another model by replacing the classical neurons by quantum bits and quantum operators in place of the classical action potentials observed in biological contexts. Pons at. el. \cite{19}, have suggested a model of QNN, where they used the trapped ion chain as quantum neural network. Based on the universality as single qubit rotation gate and two-qubit controlled -Not gate, Panchi and Shiyong have constructed a quantum neuron model \cite{20}.

Communication at long distances in unknown environments often requires combatting noise that may affect or destroy data before it has reached to its intended destination. It is well known that, entanglement provides miracle performances in the transmission of quantum information, where it enables the transmission of quantum state by sending only classical information \cite{5}. Recently, Hayashi \cite{21} investigated the effect of the prior entanglement on quantum network coding. This leads us to our aim; Investigating the effect of non-maximal entangled state (due to the noise the maximum entangled states turn into partially entangled states) on the efficient of sending quantum state via quantum teleportation through a neural quantum network.

In this paper, we consider a partially entangled quantum neural network, QNN where each neuron entangled with its neighbor. This entangled networks between neurons are used to send information from one neuron to another. This paper is organized as follows: In Sec.2, the properties of the QNN are reviewed. Sending information between neurons is achieved by quantum teleportation is described in Sec.3. The spread of swine flu virous, as
an application of quantum teleportation via neural network, is discussed in Sec.4. Finally Sec.5, is devoted to discuss our results.

2 Description of QNN

A neuron is defined as a two logical state of usual bit ”Yes, No” or ”False, true” or ”of, on” and so on. Also, one can express the state of the neuron as a superposition of the False and true representation as

\[ |\psi\rangle = \alpha |n\rangle + \beta |y\rangle, \]

where with probability \(|\alpha|^2\) the neuron is in state \(|n\rangle\) i.e ”no” has information (unfired), while it is in state \(|y\rangle\) i.e ”Yes” carries information (fired) with probability \(|\beta|^2\) and \(|\alpha|^2 + |\beta|^2 = 1\). In QNN, a neuron receives \(N_i\) real input signals from many other neurons by network channel. Also, the output of a neuron can ”fan-out”, the number of the input signals that can be driven by a single output, to several other neurons. So, we can express on the collective response of all neurons as

\[ |\psi\rangle = \sum_{i=0}^{N} \omega_i |\psi_i\rangle, \]

where \(|\psi_i\rangle\) represent the states of the input neurons and \(\omega_i\) is the corresponding weight. The neural network processing can be implemented by an operator \(\mathcal{U}\), acting on the input states and propagates the information forward to calculate the output state of the neuron as

\[ |\psi(t)\rangle = \mathcal{U} \left( \sum_{i=0}^{N} \omega_i |\psi_i\rangle \right), \]

where, \(\mathcal{U}\) is unknown operator that can be implemented by the gate of the neural network.

On the other hand, in quantum computation the evolutionary operators must be unitary. If we consider the case where \(\mathcal{U} = I\), i.e, identity operator, the output of quantum perceptron is given by

\[ |\psi(t)\rangle = \sum_{i=0}^{N} \omega_i |\psi_i\rangle. \]

This means that quantum preceptron can be defined as a quantum system consists of n-neuron input register and in this case, we have only considered quantum neural network. To achieve quantum computation, the logical function can be performed by applying a group of unitary operations on the neuron states. These operations are called logic gates, which transform the information of the input neuron states to the output ones. The most basic universal gates is the single rotation gate, controlled not gate (cNOT), entangling gate and Hadamard gate. Since we are interested to employ the QNN into the quantum teleportation,
we shall describe the cNOT and hadamard gates in our notations. The effect of the cNOT operation is defined by,

\[ \text{cNOT} |nn\rangle = |nn\rangle, \quad \text{cNOT} |ny\rangle = |ny\rangle, \]
\[ \text{cNOT} |yn\rangle = |yy\rangle, \quad \text{cNOT} |yy\rangle = |yn\rangle, \]

while Hadamard is,

\[ H |n\rangle = \frac{1}{\sqrt{2}} (|n\rangle + |y\rangle), \quad H |y\rangle = \frac{1}{\sqrt{2}} (|n\rangle - |y\rangle) \]

3 Quantum teleportation

In this section, the entangled neural network is used to perform the original quantum teleportation protocol \cite{5}. To achieve this task we assume that the neural network is built on the postulates given by Shiafee \cite{18}. In this consideration the neuron is represented by qubit, such that the state of the neuron is described by equation (1) . Let us assume that the neural network is partially entangled. This means that the state of each two neighboring neurons is non maximal entangled states. Assume that the state of any two connected neurons is defined by the density operator,

\[ \rho = \frac{1+q}{2} |nn\rangle\langle nn| + \frac{p}{2} (|nn\rangle\langle yy| + |yy\rangle\langle nn|) + \frac{1-q}{2} |yy\rangle\langle yy|, \]

where \( q^2 + p^2 = 1 \) and \( 0 \leq p \leq 1 \) and \( 0 \leq q \leq 1 \) (see \cite{22}). If, we put \( p = 1 \), one gets the maximum Bell state \( |\phi^+\rangle = |\phi^+\rangle \) \cite{23}. The following describes the steps to implement the Bennett Protocol \cite{5}.

1. **Step one:** A neuron \( B \) has got information from another neuron \( A \) (which can be order, computing, decision, etc.). The aim of neuron \( B \) is transferring these information to another neuron \( C \), which is a member in the network. Let us assume that the unknown information coded in the neuron state \( |\psi_A\rangle = \alpha |n\rangle + \beta |y\rangle \) and the neurons \( B \) and \( C \) share an entangled neuron state defined by \( \rho_{BC} \), then the total state of the system is given by \( \rho_S = \rho_A \otimes \rho_{BC} \), where \( \rho_A = |\psi_A\rangle\langle \psi_A| \).

2. **Step two:** Neuron \( B \) performs local operations on his state and the neuron \( \rho_A \). These local operations include the cNOT operation which is a two neurons gate as defined by \( \text{cNOT} \). After this operation one gets

\[ \rho_S^{(1)} = (\text{cNOT} \otimes I)\rho_S(I \otimes \text{cNOT}), \]

where \( I \) is a unitary operator effects on the neuron \( C \). The second operation is the Hadamard operation \( \text{H} \), which transform the state \( \rho_S^{(1)} \) to

\[ \rho_S^{(2)} = (\text{H} \otimes I \otimes I)\rho_S^{(1)}(I \otimes I \otimes \text{H}). \]
Alice makes measurements on the basis $|nn\rangle, |ny\rangle, |yn\rangle$ and $|yy\rangle$ randomly on her own neuron and the state which has got from the neuron $A$. Alice gets a two classical bit of information and sends them to Bob.

4. **Step four**: As soon as Bob gets the classical data from Alice, he performs suitable rotations on his neuron to obtain the final information as shown in Table(1).

In Fig.1, we plot the fidelity, $F$ of the teleported state, where we assume the case where Alice measures $|yy\rangle\langle yy|$. From this figure, it is clear that when $p = 0$, then the quantum network is classically correlated and given by $\rho = |nn\rangle\langle nn|$. As one increases the value of the parameter $p$ on the expance of $q$, where $p = \sqrt{1 - q^2}$, the quantum network becomes a partially entangled. The corresponding fidelity, $F$ increases as $p$ increases. For $p = 1$, the network is maximally entangled and given by $\rho = \frac{1}{2}(|nn\rangle\langle yn| + |yn\rangle\langle nn|)$ and the fidelity of the teleported state $F = 1$ (maximum value). On the other hand, there is no effect on the type of the teleported state on the degree of the fidelity. So, one can teleporte classical

| Alice Deuity | Bob state $\rho_b =$ | Bob Deuity |
|--------------|-------------------|-----------|
| $|nn\rangle\langle nn|$ | $|\frac{b_2^2+\lambda_2}{2}nn\rangle\langle nn| + \frac{a_2^2+\lambda_2}{2}n\rangle\langle y| + \frac{b_2^2+\lambda_2}{2}y\rangle\langle n| + \frac{a_2^2+\lambda_2}{2}y\rangle\langle y|$ | $I_{\rho_b}I$ |
| $|ny\rangle\langle ny|$ | $|\frac{b_2^2+\lambda_2}{2}n\rangle\langle n| - \frac{a_2^2+\lambda_2}{2}n\rangle\langle y| - \frac{b_2^2+\lambda_2}{2}y\rangle\langle n| + \frac{a_2^2+\lambda_2}{2}y\rangle\langle y|$ | $\sigma_x\rho_b\sigma_x$ |
| $|yn\rangle\langle yn|$ | $|\frac{a_2^2+\lambda_2}{2}n\rangle\langle y| + \frac{a_2^2+\lambda_2}{2}y\rangle\langle n| + \frac{a_2^2+\lambda_2}{2}y\rangle\langle y|$ | $\sigma_y\rho_b\sigma_y$ |
| $|yy\rangle\langle yy|$ | $|\frac{a_2^2+\lambda_2}{2}n\rangle\langle n| - \frac{a_2^2+\lambda_2}{2}n\rangle\langle y| - \frac{a_2^2+\lambda_2}{2}y\rangle\langle n| + \frac{a_2^2+\lambda_2}{2}y\rangle\langle y|$ | $\sigma_y\rho_b\sigma_y$ |

Table 1: The possible measurements (Alice side), obtained result (Bob side) and the local operations which done by Bob to get the desired coded information, where $\lambda_1 = \frac{1+q}{2}$, $\lambda_2 = \frac{1-q}{2}$ and $\lambda_3 = \frac{3}{2}$.
information as well as quantum information with the same efficiency. Then by controlling the
devises which generate entangled network, one can send the information with high fidelity.

4 Application: The spread of swine flu virus

The prevalence of swine virus is one of the most difficult challenges facing the world these
days. In this contribution, we use the QNN, which has been described in Sec.(2) to investigate
the possibility of controlling the spread of this virus. In the present protocol, we consider
two categories of the populations with the same social and economic circumstances, and
each of them contains different age groups. Let's assume that each one of these category is
described by a pair of information: the first part of which represents human characteristics
such as young, old, rich, poor, have a medical culture and no medical culture. The second
part represents information on the affecting factors such as: non-infected, infected, awareness
and non awareness, has culture of health and has no culture of health). In fact, there are a
lots of human and influence factors, so we consider some of them to illustrate the basic idea
of this protocol.

Let us assume that the human properties are coded in the phase, while the influence
factors coded in the amplitude. So, if the neuron is described by \(\psi = \alpha|1\rangle + \beta|0\rangle\), then with
probability \(|\alpha|^2\), the neuron represents the pair (young, non-infected), while with probability
\(|\beta|^2\), it represents the pair (young, infected). In this description, we set the positive phase
(+) for the young and (−) for the old and the amplitude 1 for non-infected and 0 for infected.
Table(2), gives a complete description to some of these factors.

| Human factor | phase | Influence factors | amplitude |
|--------------|-------|------------------|-----------|
| young        | +     | non-infected     | 1         |
| old          | −     | infect           | 0         |
| rich         | +     | awareness        | 1         |
| poor         | −     | non-awareness    | 0         |

Table 2: This table is an example of coding the information in a neuron.

Assume that we have a class of human cases of pathological, described by the neuron
\(\psi_n = \alpha|0\rangle + \beta|1\rangle\). According to the definitions given in Table (2), this state describes a
young infected with probability \(|\alpha|^2\) and it is non-infected with probability \(|\beta|^2\). In view of the
friction between the two entangled tranches of the community, the information which coded
in the neuron \(\psi_n\), where the virus which can be transmitted through it, is transformed to
the other segment of the community. Also, the virus can be transmitted to another person in
the same category. Since there is an environmental pollutions, the transformed information
| Age group | Healthy case | Probability | Measurements |
|-----------|--------------|-------------|--------------|
| old       | Infected     | $|B_1|^2$    | $|yy\rangle\langle yy|$ |
| old       | non-infected | $|A_1|^2$    |              |
| young     | Infected     | $|B_2|^2$    |              |
| young     | non-infected | $|A_2|^2$    |              |
| young     | Infected     | $|B_3|^2$    |              |
| young     | non-infected | $|A_3|^2$    |              |
| old       | Infected     | $|B_4|^2$    | $|yy\rangle\langle yy|$ |
| old       | non-infected | $|A_4|^2$    |              |

Table 3: This table shows the age group and the possibility of non-infected or infected.

Figure 2: Figs. (a&c), represent a case of old where the probability $|A_1|^2$ of non-infected while the probability $|B_1|^2$ for old infected is shown in Figs. (b&d). In this case the reserver measures $|yy\rangle\langle yy|$.
interacts with these unhealthy environment and there may be a transfer of swine flu virus. For this example, we find with probability 25%, the virus can be transformed as shown in Table(3). The values of $A_i, B_i$ and $i = 1, 2, 3, 4$ are given by

$$A_1 = \frac{p \sqrt{1 - \alpha^2}}{\sqrt{(1 - q)^2 \alpha^2 + p^2 (1 - \alpha^2)}}, \quad B_1 = \frac{(1 - q)\alpha}{\sqrt{(1 - q)^2 \alpha^2 + p^2 (1 - \alpha^2)}}$$

$$A_2 = \frac{p \alpha \sqrt{1 - \alpha^2}}{\sqrt{(1 + q)^2 (1 - \alpha^2) + p^2 \alpha^2 (1 - \alpha^2)}}, \quad B_2 = \frac{(1 + q)(1 - \alpha^2)}{\sqrt{(1 + q)^2 (1 - \alpha^2) + p^2 \alpha^2 (1 - \alpha^2)}}$$

$$A_3 = \frac{(1 + q) \alpha}{\sqrt{(1 + q)^2 \alpha^2 + p^2 (1 - \alpha^2)}}, \quad B_3 = \frac{p \sqrt{1 - \alpha^2}}{\sqrt{(1 + q)^2 \alpha^2 + p^2 (1 - \alpha^2)}}$$

$$A_4 = \frac{p \sqrt{1 - \alpha^2}}{\sqrt{(1 + q)^2 \alpha^2 + p^2 (1 - \alpha^2)}}, \quad B_4 = \frac{(1 + q) \alpha}{\sqrt{(1 + q)^2 \alpha^2 + p^2 (1 - \alpha^2)}}$$

(10)

Table(3), describes the output as a result of two types of measurements that can be done by the person who receives the transmitted information. In the upper part of Table(3),
we assume that the recipient has measured $|yy\rangle\langle yy|$. As a result of these measurements there are four possibilities: the first is infected old, the second is non-infected old, the third is infected young and the fourth is non-infected young. The bottom part of Table(3), represents the output of the second type of measurements, where in this case the receiver measures $|yn\rangle\langle yn|$. Also, one gets a similar possibilities with different probabilities.

It is clear that, the probabilities $|A_i|^2$, $|B_i|^2$ for $i = 1, \ldots, 4$ depend on the strength of the channel $p$, where $p = \sqrt{1-q^2}$ and the initial state setting which is described by the parameters $\alpha$ and $\beta$. On the other hand, the fidelity of transmitted information increases as the channel parameter $p$ increases, this is clear from Fig.(1). So if we were able to control the laboratory devices to generate entangled channel to minimize the fidelity, one can reduce the rate of infections. Also, from theoretical point of view, if the receiver makes some kind of rotation, one can reduce the rate of infection between young.

In Fig.(2), we plot the probabilities $A_1, B_1$ which represent the non-infected and infected probabilities respectively. Fig.(2a), shows the behavior of the probability of an old non-infected person, $|A_1|^2$. From this figure it is clear that the person is completely non-infected i.e $|A_1|^2 = 1$ for large values of the network strength. But as $p$ increases the probability $|A_1|^2$ decreases and it becomes zero for $p \geq 0.75$. This means that for larger values of $p$ the symptoms of the disease can appear and its rate increases for $p \geq 0.75$. This is clear from Fig.(1b), for $p < 0.75$ and the amplitude, $|B_1|^2 = 0$. From these figures we can see that the strength of the channel and the rate of infection playing a central role in the behavior of the spreading the virous.

These results are shown clearly in Fig.(1c) and Fig.(1d), where we plot the contour diagram for the probabilities $|A_1|^2$ and $|B_1|^2$. This description gives a clear vision for the effect of the rate of infection and the network strength. From these figures, we can notice that there is gradual change from completely dark regions to shine ones. This means that the probability increases gradually from almost zero at (completely dark region) to a maximum value for the completely shine regions. As an example, in Fig.(1c) the brightness decreases as one increases the strength of the network, but the effect of the infection rate appears much earlier. This means that for large values of $p$, the degree of entanglement of the neural network increases and then the possibility of the infection increases. So at $p = 1$, the probability $|A_1|^2$ is almost zero, therefore there is an infection. Conversely, in Fig.(1D), the shine regions increases as one increases the network strength, $p$ and the infection rate $\alpha$. This confirm what has been displayed in Fig.(1b), where the probability of infection $|B_1|^2 = 1$ at $p = 1$.

The other two possibilities are plotted in Fig.3, where the dynamics of this amplitude represents a case of non-infected young with probability $|A_2|^2$ while he is infected with probability $|B_2|^2$. One can see that for small values of $\alpha$ and $p$, the young is not infected, but the symptoms of disease appears as one increases the strength $p$ only. As a remark, for
large values of $\alpha$ and small values of $p$, the degree of infection is maximum (see Fig.(3a)). Also for the probability of infection is plotted in Fig.(3a), where both parameters play an important role on the behavior of $|B_2|^2$. This behavior can be seen clearly from the contour graph for each of $|A_2|^2$ and $|B_2|^2$.

The probabilities $|A_2|^2$ and $|B_2|^2$ are displayed as contour diagram in Fig.(3c) and Fig.(3D) respectively. In Fig.(3C), the shining region increases for both parameters, but the dark region is larger than the bright one. This means that the probability of non-infected is smaller than the probability of infected. The converse behavior is seen in Fig.(3d), where the dark region (non-infected) is very small compared with the shine regions which represents (infection).

Finally, the behavior of the probabilities $|A_3|^2$ and $|B_3|^2$ are shown in Fig.(4), where the receiver measures $|nn\rangle\langle nn|$. In Fig.(4a), we plot the probability of a non-infected young, $|A_3|^2$. This figure shows that the probability increases the network strength increases and it is almost zero for small values. This means that for small values of $p$, the probability that the young is infected is increases. This phenomena is shown in Fig.(4b), where $|B_2|^2 = 1$ as one decreases the network’s strength. The confirmation of these results is shown in Figs.(4c&4d),

Figure 4: The same as in Fig.(2), but for non-infected young (Figs.(a&c)) and the infected in (Figs.(b&d)) and the receiver measures $|nn\rangle\langle nn|$. 
where the dark regions appear for a small range of $\alpha$ and $p$ as shown in Fig.(4c), while it increases for a large range of these parameters. Finally concerning the last possibility which for old non-infected is represented by the probability $|A_4|^2$ and infected old with probability $|B_4|^2$, we can see that they have the same behavior of $|B_3|^2$ and $|A_3|^2$. In other words, the probability for non-infected old equals to the probability of infected young and the probability that old infected equals the non-infected young. So, we obtain the same behavior and there is no need to re-plot them.

5 Conclusion

In this contribution, we employ the quantum neural network to achieve the quantum teleportation protocol. In our treatments we assume that the neurons are connected with unperfect channels. The sensitivity of the fidelity of the transmitted information is investigated for the network’s parameter as well as for the structure of the input information. For large values of the channel parameter, the channel becomes a maximum entangled and consequently the fidelity is maximum. The idea of quantum teleportation is applied on a practical example, where we investigate the possibility of controlling on the spread of swine flu virus. In this context, we show that the virus can be transformed to a similar segment of society with different probabilities. Also, it has been shown, that the case of patients has no effect on the spread of the virus for some possibility. For other possibility the degree of infection plays the central role, while for some other cases both parameters paly an equivalent role. On the other hand, by local rotations young people can avoid HIN1 infection.

We note that in this protocol we used a quantum neural network consists of two neurons. We believe that there are many factors affecting the spread of swine flu virus. Therefore, we can develop this protocol to include many of the human factors and external factors, and then strategies can be developed to understand the process of transmission. In fact, this protocol was an attempt to understand the theory of the spread of the virus as an application of quantum neural networks. And we hope that this study contributes to a vision to reduce the spread of the epidemic.

Acknowledgements: The designated project has been fulfilled by financial support from the unite research of IT-House, Aswan, Egypt.

References

[1] A. Aspect, Nature bf 398 189 (1999); B. Weber, H. P. Specht, T. Mueller, J. Bochmann, M. Muecke, D. L. Moehring and G. Rempe, Phys. Rev. Lett. 102, 030501 (2009).
[2] R. Blatt and D. Wineland, Nature **453** 1008 (2008).

[3] J. León and C. Sab, Phys. Rev. A**79** 012301 (2009).

[4] N. Matwally and A. El-Amin, Physica E **41** 718 (2009).

[5] C. H. Bennett, G. Brassard, C. Crepeau, R. Jozsa, A. Perese and W. K. Wootters, Phys. rev. Lett. **70** 1895 (1993).

[6] S. Olmschenk, D. N. Matsukevich, P. Maunz, D. Hayes, L.-M. Duan and C. Monroe, Science **323**, 486 (2009); L. Slodicka, M. Jezek and J. Fiurasek Physical Review A **79**, 050304(R) (2009).

[7] L. N. de Castro, Phys. of Life Rev., **4** 1-36 (2007).

[8] N. Kouda, N. Matsui, H. Nishimura and F. Peper, Neural Processing Letters Vol 16 No 1 67-80 (2002); Neural Computing and applications, Vol 14 No 2 114-121 (2005).

[9] P. Gralewicz, quant-ph/0401127 (2004).

[10] D. J. Amit, "modling brain function, (cambridge University Press, Cambridge 1989).

[11] M. Mézard, G. Parisi and M.A. Virasoro " Spin Glass Theory and Beyond (World Scientific, Singapore 1987).

[12] M. V. Altaisky, quant-ph/0107012 (2001); F. Shafee quant-ph/0202016 (2004).

[13] R. Penrose,"Shodows of the Mind (Oxford University, 1994)".

[14] L. Behera, I. Kar and A. Elitzur, quant-ph/0407001 (2004).

[15] N. Kouda, N. Matsui and H. Nishimura” Systems and computers in japan, Vol 45 no 13, 34-55 (2004); K. Mori, T. Isokawa, N. Kouda, N. Matsui and H. Nishimura; " international joint conference on Neural networks” july 16-21 (2006).

[16] L. Behera, B. Sundaram. G. Singhal and M. Agarawal, IEEE Trans. Neural network (2003).

[17] M. Kinjo, S. Sato and K. Nakajima, international joint conference on Neural networks” july 16-21 (2006).

[18] F. Shafee, quant-ph/0202016 (2004); F. Shafee, Engineering Application of Artificial intelligence 20 429 (2007).

[19] M. Pons, V. Ahufinger, C. Wunderlich and A. Sanpera, quant-ph/060701 (2007).
[20] Li Panchi and Li Shiyong, J. of Systems Engineering and Electronics, **19** No 1, 167-174 (2008).

[21] M. Hayashi, Phys. Rev. A **76** 4 04030(R) (2007).

[22] B.-G. Englert, N. Metwally, J. Mod. Opt. **47** (2000) 221; B.-G. Englert, N. Metwally, in: R. Brylinski, G. Chen (Eds.), Mathematics of Quantum Computation, CRC Press, Boca Raton, FL, 2002, p. 25.

[23] R.F. Werner, Phys. Rev. A **40** (1989) 4277;