Gravitational Radiation From Pulsar Creation

Leonard S. Kisslinger¹, Bijit Singha¹, Zhou Li-juan²
1) Department of Physics, Carnegie Mellon University, Pittsburgh, PA
   kissling@andrew.cmu.edu       bsingha@andrew.cmu.edu
2) School of Science, Guangxi University of Science and Technology, Guangxi
   zhoulijuan05@hotmail.com

Abstract

We estimate the gravitational wave amplitude as a function of frequency produced during the creation of pulsars from the gravitational collapse of a massive star. The three main quantities needed are the magnitude of the magnetic field producing pulsar kicks, the temperature which determines the velocity of the pulsar and the duration time for the gravitational radiation.

Keywords: Gravitational wave, pulsars, pulsar kick

1 Introduction

Shortly after the collapse of a massive star often a neutron star is created. Because of the strong magnetic field and high temperature the neutron star has a large velocity and emits electromagnetic radiation, which is why it is called a pulsar. The origin of a pulsars velocity is called a pulsar kick. In research on pulsar kicks[1, 2] the magnitude of B, the magnetic field, and T, the temperature of the neutron star at the time of the pulsar kick, were estimated. We use these parameters in the present estimate of gravitational wave creation from pulsar creation, for which there are no published previous estimates.

In a study of gravitational waves (GW) generated by magnetic fields[6] it was found that the GW created during the EWPT could be detected by Lisa Interferometer Space Antenna (LISA) while the GW created during the QCDPT cannot be detected by LISA. There is also an article[7] on the circular polarization of GW created during the EWPT and QCDPT.

Previous research on gravitational radiation from neutron stars[3, 4] was based in part on gravitational radiation from cosmological turbulence[5].

There have been studies of gravity waves generated by cosmological phase transitions, the Electroweak Phase Transition (EWPT) and the Quantum Chromodynamic Phase Transition (QCDPT). In a study of gravity waves generated by magnetic fields[6] it was found that the gravitational waves generated during the EWPT might be detected by the Laser Interferometer Space Antenna (LISA) while gravitational waves generated during the QCDPT cannot be detected by LISA.

With current theory the EWPT and QCDPT are first order phase transitions, so bubbles of the new universe form within the older universe during the phase transition. A study of gravitational waves created by bubble collisions during the EWPT and QCDPT estimated the degree of circular polarization of the gravitational waves[7].

The formalism used Refs[6, 7] is not suitable for estimates of the creation of gravitational radiation from pulsar creation. However, the research in Refs[3, 4, 5] was based in part on gravitational radiation from cosmological turbulence which we use in our present research.
2 Theory of gravitational radiation from magnetic fields

Einstein’s General Theory of Relativity is based on the equation (with no cosmological constant)

\[ R^{\alpha\beta} - \frac{1}{2} R g^{\alpha\beta} = 8\pi G T^{\alpha\beta}, \]

(1)

where \( R^{\alpha\beta}, R \) the Ricci tensor, Ricci scalar, \( g^{\alpha\beta} \) is the metric tensor, \( T^{\alpha\beta} \) is the energy-momentum tensor and \( G \) is Newton’s gravitational constant.

In Ref.[5] \( T_{ij}(x) \), with \( i, j = 1, 2, 3 \) is

\[ T_{ij}(x) = w u_i(x) u_j(x), \]

(2)

with \( w = \rho \), the mass density, and \( u \) the velocity of the material emitting the gravitational radiation. In Fourier space at time \( t \)

\[ T_{ij}(k, t) = \frac{V}{(2\pi)^3} \int dq (B_i(q, t) B_j(k - q, t) - \delta_{ij} B_l(q, t) B_l(k - q, t)), \]

(3)

where \( V \) is a volume factor.

The energy density in gravitational radiation scales like \( a^{-4} \), where \( a \) when the temperature of the universe was \( T \), with \( a_o \) the scale factor now, is

\[ \frac{a}{a_o} = 8.0 \times 10^{-16}(100/g)^{1/3}(100\text{GeV}/T), \]

(4)

where \( g \) is the number of degrees of freedom at temperature \( T \).

The energy momentum tensor from a magnetic field \( \vec{B} \) is

\[ T^{(B)}_{ij}(k, t) = \frac{V}{(2\pi)^3} \int dq [B_i(q, t) B_j(k - q, t) - \delta_{ij} B_l(q, t) B_l(k - q, t)], \]

(5)

where \( t \) is the time the magnetic field \( B \) is generated.

The amplitude of the gravitational wave created by magnetic fields, \( h_c^B \), as a function of \( f \), the frequency of the gravitational radiation today, and \( f_B = \tau_B^{-1} \), where \( \tau_B \) is the duration time of the source of the \( B \) field from Ref[5] Eq(75) is

\[ h_c^B(f, \eta_o) = \frac{3^{3/2}}{25/3\pi^{2/3}} \frac{G\epsilon^{2/3} f_B^{-1/3} a^{14/3}}{H_o \sqrt{\Omega_{rad}}} f^{-4/3} \xi(f \eta_o, f \eta_{end}) \]

\[ \epsilon \approx \frac{27}{8} k_B^2 \nu^3 \]

\[ \xi(f \eta_o, f \eta_{end}) = \xi(f) = \int_{\eta_o}^{\eta_{end}} dq' \sin(f \eta_{end} - f \eta') \eta' \]

\[ H_o \sqrt{\Omega_{rad}} = \frac{a_o}{\eta_{end}}, \]

\[ w = B^2/(4\pi u_B^2) \]
with \( k_D \) the wave number corresponding to the smallest-scale motions, \( \nu \) is the kinematic viscosity of the source, \( \eta_0, \eta_{\text{end}} \) are the conformal time at the beginning and end of the process creating the gravitational wave, and \( u_B \) is the velocity associated with the creation of the \( B \) field.

Therefore, to determine the gravitational wave amplitude produced by magnetic field generation one needs the magnitude of the magnetic field, \( B \), the temperature, \( T \), and the duration time, \( \tau_B \). These quantities are also needed for our estimate of the gravitational wave amplitude produced by pulsar creation, which is reviewed in the next section.

3 Pulsar Kicks With Magnetic Field \( B \) and Temperature \( T \)

Neutron stars, often called pulsars, are created by the gravitational collapse of a massive star. The pulsars move with much greater velocities than other stars in our galaxy, called the pulsar kick. Neutrinos produced by the URCA process dominate the emission of energy during the first 10-20 seconds after the collapse of a heavy star\(^1\), with the URCA process which dominates neutrino production in neutron stars\(^8\).

\[
n + n \rightarrow n + p + e^- + \bar{\nu}^e ,
\]

with \( \bar{\nu}^e \) an anti-electron neutrino. The electrons are in Landau levels due to the strong magnetic field \( B \). With a magnetic field \( B \approx 10^{16} \) Gauss, which is created before the URCA process\(^1\), the momentum given to the pulsar with the duration time \( \tau_T = 1/f_B \approx 10s \),

\[
p_{\text{ns}} \simeq 0.43 \times 10^{27} (T/10^9 K)^7 (R_{\text{ns}}^3 - R_{\nu}^3) \text{gm cm s}^{-1} ,
\]

with \( R_{\text{ns}} \), \( R_{\nu} \) the radius of the protoneutron star and the radius of the neutrinosphere.

In Ref.\(^1\) \( R_{\nu} \) was estimated using a standard value for \( R_{\text{ns}} \), with the result

\[
R_{\text{ns}} = 10 \text{ km} \quad R_{\nu} \approx 9.96 \text{ km} .
\]

From Eqs(8,9)

\[
p_{\text{ns}} \simeq 5.14 \times 10^{-4} (T/10^{10})^7 M_{\text{ns}} v_{\text{ns}} ,
\]

where \( M_{\text{ns}} \) is the mass of the neutron star and \( v_{\text{ns}} \) is the velocity of the neutron star.

Using \( M_{\text{ns}} = 2 \times 10^{33} \) gm \( \approx \) the mass of the sun, and including a factor of 0.4, from Eq(10) the velocity of the neutron star \( v_{\text{ns}} \) is

\[
v_{\text{ns}} \simeq 1.03 \times 10^{-4} (T/10^{10})^7 \text{ km/s} .
\]
From Eq(11) one obtains the pulsar velocity as a function of temperature, $v_{ns}(T)$, as shown in Figure 1 below.

Figure 1: The pulsar velocity as a function of $T$
4 Gravitational Radiation From Pulsar Creation

From sections Theory of gravitational radiation from magnetic fields and Pulsar Kicks With Magnetic Field $B$ and Temperature $T$, we now estimate the gravitational wave amplitude as a function of the linear frequency, $h_c(f)$, from the creation of pulsars.

Note: In order to calculate $h_c(f)$ and plot it as a function of $f$ as in Ref[6] we must determine the quantities $\epsilon, \xi(f_{\eta_o}, f_{\eta_{\text{end}}}), H_o\sqrt{\Omega_{\text{rad}}}$, and $w$ in Eq(6), using Refs[1], [2] and other articles related to pulsar creation.

The velocity associated with the creation of the $B$ field, $u_B$, has been estimated by measuring the radial velocities of OBN (enhanced nitrogen) stars[9] for a period of 27 days, with the result

$$u_B \approx 90 \text{ km/s}.$$  \hspace{1cm} (12)

From Eq(12) and using $B \approx 10^{16} \text{ Gauss or } B \approx 10^{19} M_W^2$, with $M_W \approx 80 \text{ GeV}$

$$w = B^2/(4\pi u_B^2) \approx 6.4 \times 10^{30} \text{GeV}^2\text{s}^2/\text{km}^2.$$  \hspace{1cm} (13)

From Ref[1] the duration time $\tau_B \approx 1 \text{ s}$, so

$$f_B \approx 1.0 \text{ s}^{-1}.$$  \hspace{1cm} (14)

From Ref[5] Eq(14) and Eq(6)

$$k_B^4 = \frac{8\kappa \rho_{\text{vac}}}{27u^3\tau_s w}, \hspace{1cm} (15)$$

with the source time interval $\tau_s \approx 1.0 \text{ s}$ and the efficiency $\kappa \approx 1.0$. From Eqs(6,15)

$$h_c^B(f, \eta_o) = \frac{3^{3/2}}{2^{5/2}3^{2/3}} G_w^{1/3}(\rho_{\text{vac}}/\tau_s)^{2/3} f_B^{-1/3} a_{14/3}^{-1/3} f^{-4/3} \xi(f_{\eta_o}, f_{\eta_{\text{end}}})$$

$$= 0.76 G_w^{1/3}(\rho_{\text{vac}}/\tau_s)^{2/3} f_B^{-1/3} (a/a_o)^{14/3} a_{11/3}^{1/3} \eta_{\text{end}} f^{-4/3} \xi(f),$$  \hspace{1cm} (16)

where $\eta_o, \eta_{\text{end}}=(1\text{s},2\text{s})$ are the conformal time at the beginning and end of the process creating the gravitational wave.
Defining $A$ by

$$h_c^B (f, \eta_0) = A f^{-4/3} \xi (f) ,$$

(17)

using $c = c_h = 1.0$, Newton’s gravitational constant $G = 6.71 \times 10^{-39} \text{GeV}^2$, the temperature $T = 10^{10} \text{ K}$ so $(a/a_0)^{14/3} \approx 105.0$, free energy density $\rho_{\text{vac}} = 9.9 \times 10^{-30} \text{ gm} / (\text{cm}^3)$, and the other parameters in Eq (16) given above, one finds

$$A \approx 5.27 \times 10^{-21} \text{ cm} .$$

(18)

Our results for the gravitational wave amplitude produced via pulsar creation are shown in Figure 2.

![Figure 2: $h_c^B (f) = A f^{-4/3} \xi (f)$](image)

5 Conclusions

Our conclusion, based on our results shown in Figure 2, is that the gravitational wave amplitude produced via pulsar creation is smaller than that produced via the Cosmological Electroweak Phase Transition[6] and is too small to be measured by the Lisa Interferometer Space Antenna (LISA)[10] or any other gravitational wave detector in the near future.
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References

[1] Ernest M. Henley, Mikkel B. Johnson and Leonard S. Kisslinger, Phys. Rev. D 76, 125007 (2007)
[2] Leonard S. Kisslinger and Mikkel B. Johnson, Mod. Phys. Lett. A 27, 1250215 (2012)
[3] A. Melatos and C Peralta, Astrophys. J. 709, 77 (2010)
[4] Paul D. Lansky, Mark, F. Bennett and Andrew. Melatos, Phys. Rev. D 87, 063004 (2013)
[5] Authur Kosowsky, Andrew Mack and Tina Kahniashvili, Phys. Rev. D 66, 024030 (2002)
[6] Tina Kahniashvili, Leonard Kisslinger and Trevor Stevens, Phys. Rev. D 81, 023004 (2010)
[7] Leonard Kisslinger and Tina Kahniashvili, Phys. Rev. D 92, 043006 (2015)
[8] John N. Bahcall and Richard A Wolf, Phys. Rev. 140, B1452 (1965)
[9] Hugo Levato, Nidia Morrell, Beatriz Garcia and Stella Malaroda, Astrophysical j. Supplement Series 68, 319 (1988)
[10] Neil J. Cornish, Phys. Rev. D 65, 022004 (2001)