Generic Tool path planning method of T-spline surface CNC milling

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Abstract—T-spline is a new surface modeling technology, which has superior advantages in complex shape representation over traditional NURBS and mesh surfaces. However, the research in CAM field is insufficient, which hinders the further promotion of this technology. To fit the complex topology of T-spline surface, a general tool path planning method of T-spline surface CNC milling is proposed based on the commonly used zig-zag or contour-parallel toolpath pattern. This method utilizes the 2-dimensional scallop height model to calculate side step, the iso-scallop height method to generate tool paths, the minimum energy method to smooth them, and Hermite curves to join them as one tool path, which can realize the milling of T-spline surface model with arbitrary topology. Finally, a T-spline CAM prototype system is successfully developed to implement simulation analysis and machining experiments to verify the algorithms above. And the results of simulations and experiments prove this method can generate a smooth tool path with uniform iso-scallop height distribution, short length, smooth corner transition, and one retraction, which can achieve better finish machining effect.

1.Introduction
T-spline surface is a new mathematical technology that has much superiority over NURBS and mesh surface. It is a generalization of NURBS surface with T-junctions, allowing T-spline surface to significantly reduce the number of superfluous control points and represent the arbitrary topology of free-form surface with one single patch [1]. Nowadays, T-spline is popular in the fields of surface modeling [2] and geometric analysis [3]. Commercial CAD systems like Rhino completely support T-spline as a design utility. But there are rare researches about T-spline in CAM realm. In 2014, Gan et al. [4] first proposed an improved space-filling curve method for the machining of free-form surface represented by T-spline. Moreover, Gan et al. proposed morphing machining strategy method [5] to overcome the pitfalls of conventional iso-height strategy when machining the T-spline artificial bone model. Then Chen and Sui optimized the corner problem of space-filling curve method [6], and applied the gradual rough milling method for T-spline surface rough machining [7].

Although the methods above can be applied to the machining of complex T-spline surface with arbitrary topology, it still cannot get rid of the pitfalls of the frequent changes in cutting direction which makes them unpopular in actual machining. Therefore, this paper proposes a generic method for the arbitrary T-spline surface machining, by generating the iso-scallop height tool paths based on 2D scallop model, smoothing the tool paths based on minimum energy and joining the tool paths with Hermite curves. Based on the widely used zig-zag or contour-offset tool path pattern, this generic
method can generate a short and smooth tool path with one retraction, and promote the further application of T-spline surface technology in CAM realm.

2. Tool Path Planning of T-spline Surface

2.1. T-spline sidestep evaluation based on 2D scallop model

The 2D scallop model simply assumes the corresponding swept sections on adjacent tool paths are coplanar and remaining same curvature. Based on the 2D scallop model, the sidestep can be calculated by (11) when machining with ball-end tool. The $h$ represents the scallop height. The $g$ represents the sidestep. The $r$ represents the radius of the ball-end tool. The $R$ is the radius of the T-spline surface at the cutter contact point. Operator $\text{sign}$ is 0 when the T-spline surface is plane, 1 when the T-spline surface is convex, and -1 when the T-spline surface is concave.

$$g = \sqrt{\frac{8hrR}{R+\text{sign}\cdot r}}$$  \quad (1)

When the tool is a fillet-end tool or a flat-end tool, the $r$ in (1) is equal to the radius of the effective cutting profile on the cutter contact point [8].

T-spline surface can be calculated from (1). The $S$ represents the point on the T-spline surface in Cartesian space. The $u$ and $v$ are the two parameters in the parameter domain. The $A$ is the set of all the control vertices that affect the shape of T-spline surface. The $P_i$ represents the position of a random control vertex $(x_i, y_i, z_i)$ in the Cartesian space. The $W_i$ represents the weight of the control vertex. The $B_{ij}(u, v)$ represents the mixed product of the T-spline.

$$S(u, v) = \frac{\sum_{i \in A} P_i B_{ij}(u, v) W_i}{\sum_{i \in A} B_{ij}(u, v) W_i}$$  \quad (2)

Equation (2) can be simplified as $S = B/W$. Then the $k$-order derivative of T-spline surface is (3).

$$S^{(k)} = \frac{1}{W} \left[ B^{(k)} - \sum_{j=1}^{k} C_{k,j}^{i} W^{(j)} S^{(k-j)} \right]$$  \quad (3)

The first and second fundamental forms of T-spline surface can be calculated by (4). And the $(S_u, S_v, S_{uv})$ represents the mixed product of $S_u$, $S_v$, and $S_{uv}$. 
Given the sidestep, the corresponding two parameter increments in the parameter domain can be inversely solved by (5) and (6).

$$\Delta u = \frac{\pm g(F \frac{du}{dt} + G \frac{dv}{dt})}{\sqrt{EG-F^2} \sqrt{E \left( \frac{du}{dt} \right)^2 + 2F \frac{du}{dt} \frac{dv}{dt} + G \left( \frac{dv}{dt} \right)^2}}$$ (5)

$$\Delta v = \frac{\pm g(E \frac{du}{dt} + F \frac{dv}{dt})}{\sqrt{EG-F^2} \sqrt{E \left( \frac{du}{dt} \right)^2 + 2F \frac{du}{dt} \frac{dv}{dt} + G \left( \frac{dv}{dt} \right)^2}}$$ (6)

### 2.2 Iso-scallop height tool path generation of T-spline surface

By ensuring that the scallop height between adjacent tool paths is just less than the maximum scallop height allowed, the tool path length is shorter and the machining efficiency is significantly improved as well. And the algorithm to generate iso-scallop height tool path for T-spline surface machining are as follows.

- **Step 1**, selecting the initial tool path. The longest edge of T-spline surface contour is selected as the initial tool path for zig-zag tool path pattern. The contour of T-spline surface is selected as the initial tool path for contour-offset tool path pattern.
- **Step 2**, discretizing tool path. In the parameter domain, the tool path is discretized uniformly to get a series of discrete cutter contact points.
- **Step 3**, sidestep evaluating. For each discrete cutter contact point on the tool path, the maximum scallop height allowed is substituted into (1) to calculate the sidestep. And the corresponding position \((u, v)\) in the parameter domain are calculated by (5) and (6).
- **Step 4**, smoothing tool path. In the parameter domain, delete the self-intersecting part of the discrete cutter contact points, and smooth the rest cutter contact points by the simulated annealing algorithm based on minimum energy.
• Step 5, tool path interpolation. In the parameter domain, the B-spline curve interpolation is carried out on the cutter contact points. Then the part outside the boundary of the parameter domain is deleted. The iso-scallop height tool path can be represented by the B-spline curve with effective interval.
• Step 6, joining all tool paths. The iso-scallop height tool paths covering the entire T-spline surface can be generated by repeating steps 2 to 5. Finally, join the tool paths with Hermite curves to get the tool path for T-spline surface machining.

2.3. T-spline tool path smoothing based on minimum energy
When the curvature of T-spline surface changes dramatically, the tool path generated will be distorted, which brings troubles to the actual machining. In this paper, a smoothing algorithm is proposed for the iso-scallop height tool paths based on the minimum energy.

The smaller the energy of the spline, the smoother the curve is. The tool path energy can be changed by adjusting the positions on the T-spline surface of discrete cutter contact points at the expense of sidestep. Equation (7) calculates the energy of discrete tool path. The $j$ and $k_i$ are the coefficients. $P_i$ is the original cutter contact point. $Q_i$ is the adjusted cutter contact point. $l_{i_j} = ||P_i - P_{i-1}||$. Considering the position adjustment of cutter contact point is slight, the influence of their displacement can be ignored. As a result, the $j$ is set as 1 and $k_i$ are all set to be 0.

$$E = j \sum_{i=1}^{n-1} \frac{1}{l_i + l_{i+1}} (\frac{P_{i+1} - P_i}{l_{i+1} + l_i} - \frac{P_{i} - P_{i-1}}{l_{i}}) + \sum_{i=0}^{n} k_i ||P_i - Q_i||^2$$  

(7)

Connect the corresponding cutter contact points on the adjacent tool paths in the parameter domain as a line, which is the sidestep in the parameter domain. Given the maximum adjustable ratio of the sidestep allowed, the simulated annealing algorithm is used to find the new positions of the series of cutter contact points along the sidestep lines, with the minimum energy as the objective function. Through iteratively searching until the temperature drops to the preset value, the iso-scallop height tool path generated by interpolation will be the smoothest, at the expanse of sidestep allowed.

![Figure1. Smoothing T-spline Tool path](image)

2.4. T-spline tool paths joining with Hermite curves
Joining the discrete iso-scallop height tool paths smoothly as one, can avoid frequent tool retraction in the T-spline surface machining process, and improve the machining efficiency further.
In this paper, Hermite curves are applied to join the discrete tool paths, which can avoid tool interference by adjusting the space position of the joining curve conveniently. There are two cases of tool paths joining, which are adjacent tool paths joining and single tool path segments joining. And the start point $CC_0$, the adjusting point $CC_1$ and the end point $CC_2$ are linked with 2 Hermite curves. Moreover, the final tool path for T-spline surface machining is $C1$ continuous without abrupt changes.

The $CC_1 = (CC_0 + CC_2)/2 + D_1 + D_2$, is the adjusting point controlled by vectors $D_1$ parallel to T-spline surface and $D_2$ perpendicular to T-spline surface. The $k$ and $j$ are modulus values. When joining the adjacent paths, $k = |CC_2 – CC_0|/2$. When joining single tool path segments, $k = 0$. The initial value of $j$ is set to be maximum scallop height allowed. Adjust the value of $j$, when tool interference occurs.

The tangent vectors of the three linking points are $V_0$, $V_1$ and $V_2$. The $V_0$ and $V_2$ are in the same direction as the tangent vector of the tool path at the cutter contact points, and the modulus value is equal to $|CC_2 – CC_0|$.

The $f$ in forward direction along the joining curve is the unit tangent vector of Hermite curve, and the normal vector $n$ is calculated by (11). The $n_0$ and $n_2$ are the normal vectors of T-spline surface at $CC_0$ and $CC_2$ respectively. The $l$ represents the chord length of the joining curve. The $L_c$ represents the chord length from the current point to the start point of the curve.

$$ f = (CC_2 – CC_0) \times (V_0 - V_2) \times (CC_2 – CC_0)$$

$$ D_1 = \frac{k(CC_2 – CC_0) \times (V_0 - V_2) \times (CC_2 – CC_0)}{|CC_2 – CC_0||V_0 - V_2||CC_2 – CC_0|}$$

$$ D_2 = \frac{j(CC_2 – CC_0) \times (V_0 - V_2)}{|CC_2 – CC_0||V_0 - V_2|}$$

The tangent vectors of the three linking points are $V_0$, $V_1$ and $V_2$. The $V_0$ and $V_2$ are in the same direction as the tangent vector of the tool path at the cutter contact points, and the modulus value is equal to $|CC_2 – CC_0|$.

$$ V_1 = \frac{(CL_2 – CL_0) \times (V_2 + V_0)}{|CL_2 – CL_0|^2}$$

The $f$ in forward direction along the joining curve is the unit tangent vector of Hermite curve, and the normal vector $n$ is calculated by (11). The $n_0$ and $n_2$ are the normal vectors of T-spline surface at $CC_0$ and $CC_2$ respectively. The $l$ represents the chord length of the joining curve. The $L_c$ represents the chord length from the current point to the start point of the curve.

$$ n = f \times (\lambda \cdot n_0 + (1-\lambda) \cdot n_2) \times f, \quad \lambda = L_c / L$$

3. SIMULATION AND ANALYSIS

In this paper, the T-spline mouse surface and T-spline face surface with complex topology are used for simulation experiment. The T-spline mouse surface is made up by joining 5 NURBS surfaces with T-spline technology, and the T-mesh pre-image in parameter domain is concave. T-spline face surface is generated by fitting mesh surface with T-spline technology. The T-mesh pre-image in parameter domain is square with two rectangular holes inside.
In order to show the tool path and cutting simulation result effect clearly, the T-spline mouse surface is milled with a Φ10 mm ball-end tool. The maximum scallop height allowed is set to be 0.1mm. And the maximum adjustable ratio of the sidestep allowed is 0.2. The simulation results between the space-filling curve method and the generic method in this paper is shown in Fig. 3.

The T-spline face surface is milled with a Φ8 mm ball-end tool. The maximum scallop height allowed is set to be 0.1mm. And the maximum adjustable ratio of the sidestep allowed is 0.2. The simulation results between the space-filling curve method and the generic method in this paper is shown in Fig. 4.

The comparison of the two methods are shown in Table 1.
### TABLE 1. SIMULATION RESULTS COMPARISON

| T-spline Model | ISFC method | The Generic method |
|----------------|-------------|---------------------|
|                | Path length | Corners             | Path length | Corners |
| Mouse model    | 3585.89mm   | 288                 | 3224.51mm   | 38      |
| Face model     | 5135.78mm   | 472                 | 4143.74mm   | 52      |

According to the simulation results, both methods can generate short tool path with one retraction for T-spline surface machining. However, there are enormous turning corners of the tool path generated by space filling curve, which change the tool direction frequently. Moreover, the length of iso-scallop height tool path is shorter. And the whole iso-scallop height tool path is smooth. The scallop height after machining with iso-scallop height tool path is just less than the maximum scallop height allowed and uniformly distributed on the T-spline surface, which is more popular in the actual machining.

### 4. Conclusions

The research about T-spline technology in the CAM realm is insufficient at present. And the existing space-filling curve method for T-spline surface machining will lead to the frequent changes of the tool direction, which is not popular in the actual machining. To solve the problems above and adapt to the complex topology of T-spline surface, this paper proposes a generic method for the arbitrary T-spline surface machining, by generating the iso-scallop height tool paths based on 2D scallop model, smoothing the tool paths based on minimum energy and joining the tool paths with Hermite curves. According to the results of simulation and analysis, this generic method can generate a shorter and smoother tool path with one retraction, much fewer turning corners and uniformly distribution of scallop height based on the widely used zig-zag or contour-offset tool path pattern, which can achieve better finish machining effect and help the promotion and application of T-spline technology in the CAM realm.

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