Rock Breaking Mechanism and Drilling Performance of Harmonic Vibro-Impact Drilling

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Abstract. An improved model of displacement response of rock considering damping is proposed and the energy response model is further presented. Then, the rock breaking mechanism under harmonic vibro-impact is investigated based on the displacement response and energy response. Furthermore, the numerical simulations are conducted to analyze the principal stress of rock and drilling characteristics in axial and torsional directions under harmonic vibro-impact. Finally, the performance of the harmonic vibro-impact drilling technology is verified by the field application. Based on the analysis undertaken, it can be concluded that the periodic harmonic impact force makes the rock subjected to more tensile stress and continuous damage accumulation, which causes the efficient rock breakage. What’s more, the harmonic vibro-impact can also alleviate the stick-slip effect and bounce phenomenon of drill bit to a certain extent. Our investigations confirm that the harmonic vibro-impact can achieve the purpose of rate of penetration (ROP) improvement.

Key Words: Harmonic vibro-impact drilling; Vibration Response; Rock Breaking Mechanism; Numerical Simulation; Field Application.

Nomenclature

\[ L \] \hspace{1cm} \text{Lagrange function}
\[ \dot{q} \] \hspace{1cm} \text{Generalized velocity, m/s}
\[ p \] \hspace{1cm} \text{Generalized momentum, kg·m/s}
\( t \)  
Time, s  

\( H \)  
Hamiltonian function  

\( \omega \)  
Ideal natural frequency of rock, rad/s  

\( \omega_1 \)  
Actual natural frequency of rock, rad/s  

\( \omega_i \)  
Impacting frequency, rad/s  

\( x \)  
Vibration displacement of rock, m  

\( \dot{x} \)  
Vibration velocity of rock, m/s  

\( \ddot{x} \)  
Vibration acceleration of rock, m/s\(^2\)  

\( k \)  
Stiffness of rock, N/m  

\( m \)  
Mass of rock, kg  

\( c \)  
Damping coefficient of rock, N\( \cdot \)s/m  

\( f \)  
Amplitude of impacting force, N  

\( F \)  
Impacting force, N  

ROP  
Rate of Penetration  

PDM  
Positive Displacement Motor  

PDC bit  
Polycrystalline Diamond Compact bit  

WOB  
Weight on Bit  

BHA  
Bottom Hole Assembly  

1. Introduction

Since the idea of utilizing impact energy to drill was put forward, it has been tried in practice for a long time. However, the current technologies of impact energy drilling still cannot meet the demand of drilling at ever-increasing well depth. It is necessary to develop a more efficient technology for rock breaking to enhance penetration rate. Therefore, the harmonic vibro-impact drilling technology is proposed. It is an emerging efficiency rock breaking method which is developed by the Centre for Applied Dynamics Research (CADR) at the University of Aberdeen to improve the rate of penetration in hard formation [1,2]. The technology is realized by applying harmonic vibro-impact to the rock combining the appropriate drilling pressure and rotational speed. In particular, when the impact frequency of the system is equal to the natural frequency of the rock, the rock will resonate, which is so-called the Resonance Enhanced Drilling (RED).

Since the technology was proposed, lots of studies have been done by a number of researchers [3-5]. On the one hand, the research focused on the dynamic characteristics of the impact system [6-9]. A variety of dynamic models of drill string were presented to analyze the motion state of drilling system. On the other hand, a series of experimental tests are conducted indoor [10-13]. By comparing with the rock breaking effect of conventional drilling, the advantages of harmonic vibro-impact drilling technology were verified. In addition, there were a few researches on the fundamental rock-breaking
mechanism of the technology. Aspects of impact characteristic parameters [14-16] and rock mechanics parameters [17-19] were mainly considered in present study.

In fact, the harmonic vibro-impact drilling technology differs from the conventional drilling technology. It is equivalent to applying an extra periodic dynamic load (harmonic impact) to the rock on the basis of static load (drilling pressure). Therefore, the response of the rock under the action of the dynamic load becomes one of the key problems on the rock breaking mechanism of the harmonic vibro-impact drilling technology.

This paper aims to investigate the rock breaking mechanism and drilling performance of the harmonic vibro-impact drilling technology. In Section 2, based on our previous research results [20], the vibration response model of rock under harmonic vibro-impact is further improved by introducing the damping of rock. Meanwhile, the energy response model of rock is presented. In addition, the rock breaking mechanism is concluded from the aspects of displacement response and energy response of rock. Afterwards, numerical simulations and field test are carried out respectively in Section 3 and Section 4 to further prove the drilling effect of the harmonic vibro-impact technology. The research in this article can further promote the on-site and application of harmonic vibro-impact drilling technology, and provide technical support for deep well drilling.

2. Rock Breaking Mechanism of Harmonic Vibro-Impact

2.1. Physical Modeling

The rock vibration model is shown in figure 1, where the action between drill bit and rock is simplified to that between indenter and rock. Here, the indenter is simulated as a series of springs and the coupling force between the springs is ignored. In order to simplify the analysis, the area under the indenter is divided infinitely. Since the force exerted by the indenter on the surface of the rock is the same, the relationship between one point of the indenter and the micro-element of the rock can replace the action between the entire indenter and the rock. Therefore, one small micro-element is extracted for analysis. Assume that the rock is an isotropic homogeneous medium, has a mass of \( m \), a natural frequency of \( \omega \) and a damping of \( c \). The harmonic force exerted by the indenter is \( F \) with an impact frequency of \( \omega_i \).

The influence of harmonic force on the drilling efficiency of drill bit is focused in this paper. Although other factors such as pore pressure and liquid column pressure also affect on the drilling efficiency, they don’t have an influence on the law of change. Therefore, these factors are not considered in the simplified model.
2.2. Mathematical Modeling

Under the action of high-frequency harmonic force, the rock produces a different response from the static load, which directly determines the drilling efficiency of the drill bit. Therefore, this paper analyzes the rock-breaking mechanism under harmonic dynamic load from two perspectives of displacement response and energy response of rock.

2.2.1 Displacement Response [20]

When an external force applies on the rock, the vibration equation of rock can be given by,

\[ m\ddot{x} + c\dot{x} + kx = F(t) \]  

(1)

The excitation applied by the indenter is periodic and the impact force is defined as,

\[ F = f \cos(\omega t + \beta) \]  

(2)

Substituting equation (2) into equation (1), the general solution can be gotten,

\[ x = ae^{-\lambda t} \cos(\omega t + \sigma) + b_1 \cos(\omega t + \beta + \delta) \]  

(3)

Where, \( \lambda = \frac{c}{2m}, \ b_1 = \frac{f}{m\sqrt{\left(\omega^2 - \omega_i^2\right)^2 + 4\lambda^2 \omega_i^2}}, \ \omega_1 = \sqrt{\omega^2 - \lambda^2}, \ \text{and} \ \tan \delta = \frac{2\lambda \omega_i}{\omega_i^2 - \lambda^2}. \)

Equation (3) is the displacement response equation of rock with damping under harmonic vibro-impact.

2.2.2 Energy Response [21]

It can be known from the Hamiltonian energy function that the response energy of the rock is only a function of time and has nothing to do with the position of impact system under harmonic vibro-impact.

For the one-dimensional vibration model of rock, the Lagrangian function can be expressed as,

\[ L = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2 - \int Fdx \]  

(4)
\[
P = \frac{\partial L}{\partial \dot{q}} = \frac{\partial L}{\partial \dot{x}} = m\ddot{x}
\]

Therefore, the Hamiltonian energy equation is obtained as follows,

\[
H = pq - L = m\ddot{x} \cdot \dot{x} - \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2 + \int F dx = \frac{1}{2} m\dot{x}^2 + \frac{1}{2} kx^2 + \int F dx
\]

As the rock is in a state of micro-vibration, based on the vibration response equation of the rock obtained in section 2.2.2, we can have,

\[
\frac{1}{2} m\ddot{x}^2 = \frac{1}{2} m\left[a^2 \lambda^2 e^{-2i\lambda t} \cos^2 (\omega_1 t + \alpha) + a^2 \omega_1^2 e^{-2i\omega_1 t} \sin^2 (\omega_1 t + \alpha) + b_1^2 \omega_1^2 \sin^2 (\omega_1 t + \beta + \delta) + 2a^2 \lambda \omega_1 e^{-2i\lambda t} \cos(\omega_1 t + \alpha) \sin(\omega_1 t + \alpha) + 2ab_1 \lambda \omega_1 e^{-2i\lambda t} \cos(\omega_1 t + \alpha) \sin(\omega_1 t + \beta + \delta) + 2ab_1 \omega_1 \omega_1 e^{-2i\omega_1 t} \sin(\omega_1 t + \alpha) \sin(\omega_1 t + \beta + \delta) \right]
\]

\[
\frac{1}{2} kx^2 = \frac{1}{2} k\left[a^2 e^{-2i\lambda t} \cos^2 (\omega_1 t + \alpha) + b_1^2 \cos^2 (\omega_1 t + \beta + \delta) + 2ab_1 \omega_1 \omega_1 e^{-2i\omega_1 t} \cos(\omega_1 t + \alpha) \cos(\omega_1 t + \beta + \delta) \right]
\]

\[
\int F dx \approx Fx = afe^{-2i\lambda t} \cos(\omega_1 t + \alpha) \cos(\omega_1 t + \beta) + ab_1 \cos(\omega_1 t + \beta + \delta) \cos(\omega_1 t + \beta)
\]

Taking equations (7) ~ (9) into equation (6), the energy response equation of the rock is therefore given by,

\[
H = \frac{1}{2} m\left[a^2 \lambda^2 e^{-2i\lambda t} \cos^2 (\omega_1 t + \alpha) + a^2 \omega_1^2 e^{-2i\omega_1 t} \sin^2 (\omega_1 t + \alpha) + b_1^2 \omega_1^2 \sin^2 (\omega_1 t + \beta + \delta) + 2a^2 \lambda \omega_1 e^{-2i\lambda t} \cos(\omega_1 t + \alpha) \sin(\omega_1 t + \alpha) + 2ab_1 \lambda \omega_1 e^{-2i\lambda t} \cos(\omega_1 t + \alpha) \sin(\omega_1 t + \beta + \delta) + 2ab_1 \omega_1 \omega_1 e^{-2i\omega_1 t} \sin(\omega_1 t + \alpha) \sin(\omega_1 t + \beta + \delta) \right] + \frac{1}{2} k\left[a^2 e^{-2i\lambda t} \cos^2 (\omega_1 t + \alpha) + b_1^2 \cos^2 (\omega_1 t + \beta + \delta) + 2ab_1 \omega_1 \omega_1 e^{-2i\omega_1 t} \cos(\omega_1 t + \alpha) \cos(\omega_1 t + \beta + \delta) \right] + afe^{-2i\lambda t} \cos(\omega_1 t + \alpha) \cos(\omega_1 t + \beta) + ab_1 \cos(\omega_1 t + \beta + \delta) \cos(\omega_1 t + \beta)
\]

2.3. Rock Breaking Mechanism

Figure 2 shows the displacement response of rock which is got from its vibration response equation.
Figure 2. The displacement response of rock with different impact frequencies

As can be seen from figure 2, the displacement response of rock is in the form of harmonic motion under the vibro-impact, which indicates that the rock oscillates up and down regularly. When the indenter impacts the rock at a non-resonant frequency (corresponding to the low frequency region and the high frequency region in figure 2), the displacement response of the rock is weak. However, when the impact frequency of the indenter approaches or reaches the resonance frequency of the rock (corresponding to the resonant region in figure 2), it is obvious that the response displacement of the rock reaches a peak.

According to the energy response equation of the rock, the energy response under the harmonic vibro-impact can be depicted as shown in figure 3.

Figure 3. The history curves of the energy response of rock under different drilling conditions

The time histories plotted in figure 3 suggest that the response energy of rock under harmonic vibro-impact is also regularly attenuated in a harmonic form. When the indenter impacts the rock with the harmonic force at a non-resonant frequency, the response energy of rock fluctuates strongly. When the indenter impacts the rock at a resonant frequency, the response energy of rock reaches a steady state. At this time, the vibration of rock keeps the same pace with the impact load, and the impact energy of the indenter is utilized to the greatest extent to change the motion state of rock, which means the impact system efficiency is optimized.

In summary, the vibration response of rock under harmonic vibro-impact is regular and periodic, whether it is displacement response or energy response, which indicates that the harmonic impact force
actually applies a continuous and stable alternating stress on the rock. The stress makes the rock not only subjected to compressive stress, but also subjected to tensile stress. As is well known, the tensile strength of rock is much smaller than the compressive strength of rock, thus the tensile stress caused by harmonic force makes it easier for rock to reach its tensile fracture limit and break, which finally improve the rock breaking efficiency. Besides, since the harmonic force applied is high frequency, the harmonic vibro-response of the rock under the dynamic load causes damage accumulation continuously. Eventually, when the total damage of rock reaches its fatigue failure limit, the fracture will occur even the strain of rock does not reach its fracture limit. This is also one of the reasons for the improvement of drilling efficiency. In particular, when the impact frequency of the harmonic force is the same as the natural frequency of the rock, the rock reaches the state of resonance. At this time, the response of the rock is the most violent, and volume breakage is more likely to occur.

3. Numerical Simulation of Harmonic Vibro-Impact Drilling

3.1. Modeling and Solution

In this study, the ABAQUS software is used to simulate the drill string-bit-rock system for analyzing the rock breaking effect and drilling performance of the harmonic vibro-impact drilling. In order to match the actual drilling situation, the finite element model includes rotary table, drill pipe, bottom-hole assembly, drill bit and rock, as shown in figure 4.

Figure 4. The finite element model of drilling system

As show in figure 4, the drill bit is a PDC bit with five blades and the rock is limestone. In the model, a constant angular velocity is applied to the rotary table, and a WOB and harmonic dynamic load are
exerted on the BHA and drill bit, respectively. In addition, the whole model is constrained in the X direction and the bottom of the rock is fixed with all directions. To make the simulation more accurate, the teeth of the PDC bit and parts of the rock which contact with the drill bit are meshed more finely than other parts. Thus, the fineness of the meshing has no effect on the results. The specific drilling parameters and rock characteristics parameters in the simulation are given in Table 1 and Table 2.

### Table 1. Drilling parameters

| Parameters       | Angular Velocity rad/s | WOB kN | Impact Force kN | Impact Frequency Hz |
|------------------|-------------------------|--------|-----------------|--------------------|
| Values           | 4.7                     | 15     | 7.5             | 100                |

### Table 2. Characteristics parameters of limestone

| Parameters       | Density t/mm³ | Elastic Modulus MPa | Poisson's Ratio | Yield Strength MPa | Friction Angle ° | Fracture Strain |
|------------------|---------------|---------------------|-----------------|--------------------|-----------------|-----------------|
| Values           | 2.75×10⁹      | 51500               | 0.33            | 120                | 30.16           | 0.075           |

### 3.2. Results and Discussion

#### 3.2.1 Principal Stress of Rock

![Figure 5](image)

**Figure 5.** The maximum principal stress distribution of rock: (a) conventional drilling; (b) harmonic vibro-impact

Figure 5 shows the maximum principal stress distribution of rock element under the conventional drilling and harmonic vibro-impact drilling, where the positive sign represents the tensile stress and the negative sign represents the compressive stress. Comparing Fig. 5a and Figure 5b, it can be seen that more rock elements under the drill bit are subjected to tensile stress under harmonic vibro-impact compared with the conventional drilling, which is the result of the regular harmonic vibration response of rock caused by harmonic vibro-impact. As the tensile strength of rock is much less than the compressive strength, the rock is more likely to be broken under the impact of harmonic vibration.

#### 3.2.2 Axial Motion

In figure 6 and figure 7, the axial drilling characteristics of conventional drilling and harmonic vibro-impact drilling are presented. As shown in figure 6, in both drilling methods, the progression of the drill bit increases in a stepwise form with the increasing of time due to the stick-slip effect.
Figure 6. Time history curves of progression under different drilling conditions

Figure 7. Time history curves of ROP under different drilling conditions

It can be obtained from the simulation results that under the same drilling conditions, the maximum rate of penetration (ROP) is 39.59 mm/s and the average ROP is 5.58 mm/s in the conventional drilling, which are corresponding to 54.77 mm/s and 8.56 mm/s in the harmonic vibro-impact drilling. It is worth noting that the average ROP is increased by 46.3%, which obviously reveals that the harmonic vibro-impact significantly improves the axial drilling efficiency.

In addition, it can also be gotten from figure 7 that the maximum reverse velocity is 15.94 mm/s in the conventional drilling, while that is 12.17 mm/s in the harmonic vibro-impact drilling. What’s more, the stability of axial ROP of harmonic vibro-impact drilling is better than that of conventional drilling. It is intuitive that the harmonic vibro-impact alleviates the phenomenon of bit bounce, which further demonstrates that harmonic vibro-impact drilling can improve the axial drilling performance and achieve the purpose of increasing ROP.

3.2.3 Torsional Motion
Figure 8. Time history curves of angular velocity under different drilling conditions

Figure 9. Time history curves of angular acceleration under different drilling conditions

Figure 8 and figure 9 show the torsional drilling characteristics of conventional drilling and harmonic vibro-impact drilling. As it can be seen from figure 8, the rotational angular velocity of the drill bit in the harmonic vibro-impact drilling decreases slightly compared with the conventional drilling. However, from the perspective of the stability of angular velocity, the standard deviation of angular velocity in conventional drilling is 6.2 rad/s, and that is 5.7 rad/s in harmonic vibro-impact drilling, which confirms that the angular velocity of drill bit under harmonic vibro-impact is more stable. Besides, it is important to note that the stability is mainly reflected in the stickiness stage of the drill bit. Therefore, we can conclude that not only the stickiness stage of drill bit is shortened, but also the phenomenon of reverse rotation of drill bit is alleviated.

Based on the analysis of the angular acceleration in figure 9, we can also come to similar conclusions that the average angular acceleration is higher and the standard deviation of angular acceleration is lower in harmonic vibro-impact drilling. The simulation results demonstrate that harmonic vibro-impact not only improve the cutting effect of the drill bit, but also mitigates its stick-slip problem to some extent.

4. Applications

In order to realize the field application of the harmonic vibro-impact drilling technology and evaluate its drilling performance, Northeast Petroleum University independently develops the axial oscillation impactor. The working principle of the tool is that the flow of the drilling fluid is converted into a periodic vibro-impact transmitted to the drill bit, and then the drill bit exerts the harmonic force on the rock at the bottom of the well. The axial oscillation impactor with stable performance is applied to downhole drilling operations of S well for field test, as shown in figure 10. Also, the working
performance parameters of the tool are shown in Table 3.

![Figure 10. The field operation of axial oscillating impactor](image)

**Table 3. Working performance parameters of axial oscillating impactor**

| Parameters              | Wellbore Diameter | WOB | Rotation Velocity | Flow | Density of Drilling Fluid |
|-------------------------|-------------------|-----|-------------------|------|--------------------------|
| Value                   | 9-1/2”            | 8-16| 50-70             | 28-32| ≤1.4                     |

The test interval of S well is 3970m–4220m, of which the BHA are PDM+PDC in the 3970m–4140m section and PDM + axial oscillating impactor + PDC in the 4140m–4220m section. The lithology encountered in the test interval includes conglomerate, mudstone and siltstone. The instantaneous ROP of the PDC bit at different depths and the average ROP of different lithology are drawn in figure 11.

![Figure 11. Curves of ROP in the test interval of S well with depth variation](image)

It can seen from figure 11 that, before the axial oscillating impactor was used, the PDC bit encountered the conglomerate once with an average ROP of 1.42m/h, and the mudstone four times with an average ROP of 2.22m/h, 1.94 m/h, 2.31 m/h, and 1.29 m/h. The siltstone formation was drilled three times and the average ROP are 1.68m /h, 1.72 m/h and 1.83 m/h, respectively. After using the axial oscillation impactor, the PDC bit drilled the siltstone once with an average ROP of 1.95 m/h, the mudstone once with an average ROP of 2.04 m/h, and the conglomerate twice with an average ROP of 1.52m /h and 1.35m/h, respectively. By comparing the ROP of the same lithology before and after using the axial oscillation impactor, it can be found that the average ROP of conglomerate is increased by 1.44%, the average ROP of mudstone is increased by 5.2%, and the average ROP of siltstone is increased by 12.1%.
As we know, as the drilling depth increases, the drill bit's ROP decreases due to the formation pressure, the abrasion of drill bit and other problems. However, in this application example, the ROP of the drill bit is improved to different degrees for different lithology with the increasing of the depth after using the axial oscillation impactor, which is enough to prove that the harmonic vibro-impact can play a role in improving the rate of penetration.

5. Conclusions
In this paper, an improved vibration response model of rock considering damping has been investigated and the energy response model of rock has been further proposed. Then, the rock breaking mechanism of the harmonic vibro-impact drilling has been analyzed basing on the displacement response and energy response. The harmonic vibro-impact drilling technology applies the periodic harmonic impact force on the rock, so that the rock subjects to more tensile stress and continuous damage accumulation, which makes the breaking efficiency is improved. In particular, the rock is most likely to be broken when resonance drilling occurs.

The numerical simulations have been carried out to analyze the performance of the harmonic vibro-impact drilling. The results demonstrate that more rock elements under the drill bit are subjected to tensile stress under harmonic vibro-impact. Furthermore, the harmonic vibro-impact can not only improve the ROP and alleviate bit bounce in the axial direction, but also shorten the sticking time and slow down the reversal phenomenon of the drill bit in the torsional cutting direction, which will help to improve the drilling efficiency of the drill bit.

Eventually, the axial oscillating impactor is utilized in the field test of S well for further verifying its improvement effect of ROP. The results of the preliminary test drilling show that the application of harmonic vibro-impact to the drilling process brings an increase in the ROP even the depth of the well is increasing. In a word, it can be concluded that the harmonic vibro-impact drilling can be a competitive technique in the drilling industry.

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