Slowing the Decay of an Excited Atom by the Presence of a Detector

D. R. Fredkin

Department of Physics, University of California, San Diego, La Jolla, CA 92093

Amiram Ron

Department of Physics, Technion—Israel Institute of Technology, Haifa, ISRAEL 32000

(Dated: November 6, 2018)

The decay of an excited atom is shown to slow down in the presence of a photo detector. This is similar to the behavior of an atom in a mistuned cavity, and under the conditions of the quantum Zeno effect. No external perturbing field is applied to measure the survival probability of the unstable state.

PACS numbers: 03.65.Xp, 03.65.Yz, 06.20.Dk

INTRODUCTION

It has long been known that the rate of decay of an excited atom can be decreased, or increased, relative to its vacuum value, by changing the environment of the atom, e.g., placing it between two mirrors [1]. The rate of decay can also be changed by measuring the state of the system sufficiently frequently [2], the so-called quantum Zeno effect. In the present paper we will show that, furthermore, an excited atom may be prevented from returning to its ground state merely by being watched by a suitable detector.

As early as 1946 Purcell [3] made the observation that the spontaneous emission rate of an atom is increased when placed in a cavity tuned to its transition frequency. It was later argued [4] that this rate can be decreased when the cavity is mistuned, or in principle even may completely inhibited [5]. This effect was first observed [6] for dye molecules near a conducting plane, and later on for Rydberg atoms [7], and in the visible range [8].

In a theoretical paper, Misra and Sudarshan [9] pointed out that, assuming that measurements are described by von Neumann’s state reduction postulate, frequently measuring an unstable state of a system inhibits its decay. It was first demonstrated experimentally by Itano et al. [10] that a quantum transition of an atomic system is restrained by frequently measuring the state of the atom. Their analysis of the experiments in terms of the collapse of the wave function following the measurements agrees remarkably well with the experimental results. It was later shown [11] that analysis of this experiment does not require the projection postulate; standard quantum dynamical analysis can account for the time evolution of the atom under the influence of the external field, which “measures” the state of the system. A decade later Fischer et al. [12] observed that the escape rate of trapped cold atoms by tunneling is either decelerated or accelerated by frequently measuring the number of atoms which remained trapped. The quantum Zeno effect was understood as due to an active intervention of a measuring entity in the early evolution of the system; the survival probability of the quantum state is $1 - a t^2$, where $a$ is a constant, for sufficiently short time, $t$ (see e.g. [13, 14, 15] for a detailed analysis).

In the present paper we investigate the time evolution of an excited atom in the presence of a detector, without active intervention of an external field with the atom, and without restricting ourselves to the early time dynamics. We demonstrate that the mere presence of a detector slows down the transition of the atom to the ground state. We start with a model Hamiltonian of the system composed of an atom, the radiation field, and a detector made of one or many atoms, which can be ionized by the radiation emitted by the excited atom. The radiation field simply mediates between the excited atom and the atoms of the detector, which are initially in their ground state. We investigate the evolution of the entire system, generalizing the standard procedure of Weisskopf and Wigner [16].

MODEL HAMILTONIAN

We consider a two level atom at the origin of the coordinate system, and a collection of detector atoms, perhaps constituting a spherical shell detector near radius $R$. The Hamiltonian of the atom, with ground state $|l\rangle$, of energy $\epsilon_l = 0$, and the excited state $|u\rangle$, of energy $\epsilon_u = h\omega_0$, is expressed as $H_a = h\omega_0 |u\rangle \langle u|$. The atom is coupled to the electromagnetic field, whose normal modes, described in terms of the creation and annihilation operators $b^\dagger_{k,\lambda}$ and $b_{k,\lambda}$, have wave vector $k$, polarization unit vector $\hat{\epsilon}_{k,\lambda}$, with $\hat{k} \cdot \hat{\epsilon}_{k,\lambda} = 0$, and frequencies $\omega_k = ck$. The Hamiltonian of the field is $H_r = h \sum_{k,\lambda} \omega_k b_{k,\lambda}^\dagger b_{k,\lambda}$, and its coupling to the atom is given by $H_{ra} = -d_a \cdot \mathbf{E}(r,t)$. Here $\mathbf{E}(r,t)$ is the radiation electric field, and $d_a$ is the atom’s dipole moment. In the rotating wave approximation the coupling Hamiltonian is

$$H_{wa} = h \sum_{k,\lambda} \left( \alpha_{k,\lambda} |u\rangle \langle l| b_{k,\lambda} + \alpha^*_{k,\lambda} b_{k,\lambda}^\dagger \langle l| |u\rangle \right),$$

(1)
where
\[ \alpha_{k,\lambda} = -i\mu_e \sqrt{2\pi\omega_k/hV} \langle \mathbf{p}_d \cdot \mathbf{e}_{k,\lambda} \rangle, \]  
(2)

\( V \) is the quantization volume, and \( \mu_e = \langle u|d_A|l \rangle \). The detector is made of identical atoms, located at positions \( r_i \), whose ionization energy \( E_I = h\omega_I \), is much smaller than \( h\omega_0 \). The ground state of a detector atom is \( |g\rangle \), with energy \( \epsilon_g = 0 \), the ionized states are \( |c\rangle \), with energies \( \epsilon_c = h\omega_c \), and the detector’s Hamiltonian is
\[ H_d = h \sum_{i,c} \omega_c |c\rangle_i \langle c|_i. \]  
(3)
The dipole coupling of the \( i^{th} \) detector atom to the field is \( H_{wd,i} = -d_i \cdot \mathbf{E}(r_i, t) \), where \( d_i = \mathbf{p}_d d_D \) is the dipole moment operator of the detector’s atom. The detector-field coupling Hamiltonian in rotating wave approximation is
\[ H_{wd} = h \sum_{i,c,k,\lambda} \left\{ g_{k,\lambda,c} e^{i\mathbf{k} \cdot \mathbf{r}_i} |c\rangle_i \langle g|_i b_{k,\lambda} \right. \]
\[ + \left. g_{k,\lambda,c}^* e^{-i\mathbf{k} \cdot \mathbf{r}_i} b_{k,\lambda}^\dagger |g\rangle_i \langle c|_i \right\}, \]  
(4)
and \( \mu_c = \langle c|d_D|g\rangle \).

**SINGLE ATOM DETECTOR**

We first consider a single atom detector at \( \mathbf{r} \). The initial state of the system is assumed to be: the atom is excited, while the field modes are empty, and the detector is in its ground state. We are interested in the evolution in time of the system from the initial state. The states available to the system are

1. \( \text{the initial state } |\phi_0\rangle = |u, \{n_{k,\lambda} = 0\}, g\rangle \), of amplitude \( A_0 \),

2. \( \text{single photon states } |\phi_k,\lambda\rangle = |l; \{n_{k,\lambda} = 1, n_{k',\lambda'\neq k,\lambda} = 0\}, g\rangle \), of amplitude \( A_k,\lambda \), and

3. ionization states \( |\phi_c\rangle = |l; \{n_{k,\lambda} = 0\}; c\rangle \), of amplitude \( A_c \).

Here \( n_{k,\lambda} \) is the number of photons in the mode. Simplifying the notation by \( k,\lambda \rightarrow k \) and \( g_{k,c} \rightarrow g_{k,\lambda,c} e^{i\mathbf{k} \cdot \mathbf{r}} \), the state of the system is then
\[ |\Psi(t)\rangle = A_0(t) |\phi_0\rangle + \sum_k A_k(t) |\phi_k\rangle + \sum_c A_c(t) |\phi_c\rangle, \]  
(5)
and the equations of motion for the amplitudes are:
\[ \left( \frac{d}{dt} + i\omega_0 \right) A_0(t) = -i \sum_k \alpha_k A_k(t), \]  
(6)
\[ \left( \frac{d}{dt} + i\omega_c \right) A_k(t) = -i \alpha_k^* A_0(t) - i \sum_c \bar{g}_{kc} A_c(t), \]  
(7)
\[ \left( \frac{d}{dt} + i\omega_c \right) A_c(t) = -i \sum_k \bar{g}_{kc} A_k(t). \]  
(8)
The Laplace transform of these equations, with \( F(s) = \int_0^\infty dt \ e^{-st} F(t) \), is
\[ (s + i\omega_0) A_0(s) = 1 - i \sum_k \alpha_k A_k(s), \]  
(9)
\[ (s + i\omega_c) A_k(s) = -i \alpha_k^* A_0(s) - i \sum_c \bar{g}_{kc} A_c(s), \]  
(10)
\[ (s + i\omega_c) A_c(s) = -i \sum_k \bar{g}_{kc} A_k(s). \]  
(11)
We use Eq.(11) to eliminate the field amplitudes, i.e. we solve for \( A_k(s) \):
\[ A_k(s) = \frac{-i \alpha_k^*}{(s + i\omega_k)} A_0(s) - i \sum_{c'} \frac{\bar{g}_{kc'}}{(s + i\omega_k)} A_{c'}(s), \]  
(12)
substitute it into Eq.(9), and Eq.(11) to obtain
\[ (s + i\omega_c + K(s)) A_0(s) = 1 - \sum_c M_{ac}(s) A_c(s), \]  
(13)
\[ (s + i\omega_c) A_c(s) + \sum_{c'} N_{cc'}(s) A_{c'}(s) \]
\[ = -M_{ca}(s) A_0(s), \]  
(14)
where the Laplace transforms of the propagators are
\[ K(s) = \sum_k \frac{|\alpha_k|^2}{(s + i\omega_k)}, \]  
(15a)
\[ M_{ac}(s) = \sum_k \frac{\alpha_k \bar{g}_{kc}}{(s + i\omega_k)} \]  
(15b)
\[ M_{ca}(s) = \sum_k \frac{\alpha_k^* \bar{g}_{kc}}{(s + i\omega_k)} \]  
(15c)
\[ N_{cc'}(s) = \sum_k \frac{\bar{g}_{kc} \bar{g}_{kc'}}{(s + i\omega_k)}, \]  
(15d)
Using Eq.(2), and Eq.(4), and replacing
\[ \sum_k \rightarrow \frac{V}{(2\pi)^3} \int d^3k \sum_{\lambda}, \]  
(16)
Eq. [13] and Eq. [14] become
\[
(s + i\omega_0 + |\mu_a|^2 I(s)) A_0(s) = 1 - \mu_a J(s, r) \sum_c \mu_c^+ A_c(s)  
\]  
and
\[
(s + i\omega_c) A_c(s) + \mu_c I(s) \sum_{c'} \mu_{c'}^+ A_{c'}(s)  
\]
where
\[
J(s, r) = \frac{1}{(2\pi)^2} \hbar \int \frac{d^3 k}{\lambda} \sum \frac{\omega_k}{(s + i\omega_k)} (\hat{P}_a \cdot \hat{e}_{k,\lambda}) (\hat{P}_a \cdot \hat{e}_{k,\lambda})  
\]
\[
J(s, r) = \frac{1}{(2\pi)^2} \hbar \int \frac{d^3 k}{\lambda} \sum \frac{\omega_k e^{-ik \cdot r}}{(s + i\omega_k)} (\hat{P}_d \cdot \hat{e}_{k,\lambda}) (\hat{P}_a \cdot \hat{e}_{k,\lambda})  
\]
We now introduce the sum over the ionization states,
\[
L(s) = \sum_c |\mu_c|^2 / (s + i\omega_c)  
\]
and obtain
\[
A_0(s) = \frac{1}{s + i\omega_0 + |\mu_a|^2 I(s)} U(s, r)  
\]
where
\[
U(s, r) = 1 - \frac{L(s) J^2(s, r)}{I(s) [1 + L(s) J(s)]}  
\]
Remember that the probability of survival of the atom in its excited state is given by
\[
P(t) = |A_0(t)|^2,  
\]
where
\[
A_0(t) = \int e^{s t} A_0(s) \, ds  
\]
and C is the standard contour of integration for the inverse Laplace transform.

We perform sum over polarizations and the angular part of the integrations of Eq. [19], and Eq. [20].
\[
I(s) = \frac{2}{3\pi \hbar c^3} \int_0^\infty dw' \frac{\omega^{\prime 3}}{(s + i\omega')}  
\]
and
\[
J(s, r) = \frac{1}{2\pi \hbar c^3} \int_0^\infty d\omega' \frac{\omega^{\prime 3}}{(s + i\omega')} D(\omega' r / c),  
\]
where
\[
D(z) = \bar{p}_d \cdot \bar{p}_a (S(z) + T(z)) + (\bar{r} \cdot \bar{p}_d) (\bar{r} \cdot \bar{p}_a) (S(z) - 3T(z)),  
\]
with
\[
S(z) = \frac{1}{2} \int_0^1 d\xi e^{-iz \xi} = \frac{\sin z}{z},  
\]
\[
T(z) = \int_0^1 d\xi e^{-iz \xi} \xi^2 = -S''(z).  
\]
Also we replace in Eq. [24],
\[
\sum_c \int_{\omega_c}^\infty d\omega_c \rho(\omega_c) = \int_{\omega_0}^\infty d\omega' \rho(\omega'),  
\]
where $\rho(\omega_c)$ is the density of states for ionization of the detector’s atom, and write
\[
L(s) = \int_{\omega_0}^\infty d\omega' \rho(\omega') \frac{|\mu_c|^2}{(s + i\omega')}  
\]
We can now estimate the orders of magnitude of the functions in Eq. [26]. By inspection we have $J \sim J \sim \omega_0^3 / \hbar c^3$, and $L \sim |\mu_c|^2 \rho(\omega) \sim e^2 a^2 / \omega_0$, where $e$ is the electron’s charge, and $a$ is the order of Bohr radius. Further since $e^2 / a \sim \hbar \omega_0$, and $c / \omega_0 \sim 2\pi / \lambda$, the radiation wavelength, $LI \sim (e^2 / a) / \hbar \omega_0 (a\omega_0 / c)^3 \sim (2\pi a / \lambda)^3 \ll 1$, and we can neglect $LI$ with respect to one in the denominator of Eq. [28]. Also we notice that $|\mu_a|^2 I / \omega_0 \sim (2\pi a / \lambda)^3 \ll 1$.

It is instructive at this point to investigate Eq. [25] in the absence of the detector. We write explicitly
\[
A_0^{(0)}(t) = \int \frac{ds}{2\pi i} e^{s t} \frac{1}{s + i\omega_0 + |\mu_a|^2 I(s)},  
\]
where $I(s)$ is given by Eq. [20]. This is the familiar Weisskopf–Wigner expression for the survival amplitude of an excited atom in vacuum radiation field. Except for very short times, which is the main interest of previous studies of the quantum Zeno effect, and very long times [13], the Weisskopf–Wigner approximation for $A_0(t)$ is valid: one can safely substitute $s = -i\omega_0 + \gamma$, with $\gamma \ll \omega_0$, in Eq. [20], and evaluate the integral of Eq. [30]. Discarding the imaginary part of the integral in Eq. [26], which just contributes an energy shift, we obtain
\[
I(s) = \frac{2\omega^3}{3\hbar c^3},  
\]
which yields
\[
A_0^{(0)}(t) = e^{-\Gamma t / 2 - i\omega_0 t},  
\]
and the survival probability
\[
P^{(0)}(t) = e^{-\Gamma t}.  
\]
Here $\Gamma = 4\omega_0^3 |\mu_a|^2 /3\hbar c^3$ is the Weisskopf–Wigner relaxation rate, or Einstein $A$ coefficient, for the excited state of the atom.

We return now to Eq. (24), reinstate the contribution of the detector, and write

$$A_0(t) = \int_C \frac{ds}{2\pi i} e^{it} \frac{1}{s + i\omega_0 + |\mu_a|^2 I(s, r)}.$$  

In the spirit of the Weisskopf–Wigner procedure we substitute $s = -i\omega_0 + \gamma$, with $\gamma \ll \omega_0$, in Eq. (27), and Eq. (29), and obtain

$$J(s, r) = \frac{\omega_0^3}{4\pi l c^3} D(\omega_0 r/c),$$  

and

$$L(s) = \pi |\mu_e|^2 \rho(\omega_0).$$  

Again with Eq. (21) the probability of the survival of the excited state is then

$$P(t) = e^{-iU(r)t},$$  

where

$$U(r) = 1 - \frac{9}{64\pi^2} \beta D^2 (\omega_0 r/c),$$

is the reduction factor due to a single detector atom at position $r$, and

$$\beta = 2\pi\omega_0^3 |\mu_e|^2 \rho(\omega_0) /3\hbar c^3 \sim (2\pi a/\lambda)^3.$$  

When the detector is in the far field region, we obtain

$$U(r) = 1 - \frac{9}{4} \beta \frac{c^2}{\omega_0^2} R^2 \sin^2 \omega_0 r/c,$$

where $l = \hat{p}_d \cdot \hat{p}_a - (\hat{r} \cdot \hat{p}_d) (\hat{r} \cdot \hat{p}_a)$ is the dipole-dipole geometric factor, while for a detector at $r \to 0$, we have $U(r) = 1 - \beta (\hat{p}_d \cdot \hat{p}_a)^2$.

We observe that the irreversibility of the decay process can be understood as due to the continuum of the ionization states of the detector’s atom. This could also explain why there would not be coherent reflection of the excitation to the radiated atom. This allows us to avoid including an explicit irreversibility in the model, just as the continuum of photon states enabled Weisskopf and Wigner to get away with a discussion based entirely on wave functions.

Notice that since $\beta$ is very small, the reduction due to one atom in the detector is quite small, however when the detector is comprised of many atoms the reduction can be significant.

**DISCUSSION**

Our Hamiltonian Eq. (33) for a many atom detector allows coherent transfer of ionization from one detector atom to another. In practice, we do not expect such coherent transfer to occur because the electron produced by ionization is removed in the course of measurement and replaced by some other externally supplied electron. If we neglect the coherent transfer and assume all the atoms of the detector to be identical, we may conjecture that we can simply replace Eq. (39) by

$$U = 1 - \frac{9}{4} \beta N_a \frac{c^2}{\omega_0^2} R^2 \sin^2 \omega_0 R/c.$$  

The efficiency of the detector is hidden within $\beta$. If $U \approx 1$, the coupling between the detector atoms should not have been neglected. A more detailed calculation would have to include the transfer of electrons into and out of the detector array and would have to be described by an equation of motion, containing non-Hamiltonian terms, for a density matrix.

In conclusion, we observe that to inhibit the decay of an unstable state, the latter need be just constantly watched by appropriate detectors. No active measurements, perturbing the atom, are needed, and neither is the early time evolution of the atom crucial for decelerating (or accelerating) the decay rate. Although it may not be true that “A watched pot never boils”[17], we have shown that at least a watched pot boils more slowly.

One of us (A.R.) would like to acknowledge very helpful discussions with A. Ben-Kish, O. Firstenberg, A. Fisher, and M. Shuker.

[1] See recent paper by J. Eschner, Ch. Raab, F. Schmidt-Kater, and R. Blatt, Nature 413, 495, (2001), and references cited therein.
[2] See recent review by K. Koshino, and A. Shimizu, Physics Report 412, 191, (2005), and references cited therein.
[3] E. M. Purcell, Phys. Rev. 69, 681, (1946).
[4] D. Kleppner, in Atomic Physics and Astrophysics, edited by M. Chretien and E. Lipworth (Gordon and Breach, NY, 1971), Sec. 6.3, p. 5.
[5] Daniel Kleppner, Phys. Rev. Lett. 47, 233, (1981).
[6] K. H. Drexhage, in Progress in Optics, edited by E. Wolf (North-Holland, Amsterdam, 1974), Vol. 12, p. 165.
[7] R. G. Hulet, E.S. Hilfer, and D. Kleppner, Phys. Rev. Lett. 55, 2137, (1985).
[8] D. J. Heinzen, J. J. Childs, J. E. Thomas, and M. S. Feld, Phys. Rev. Lett. 58, 1320, (1987).
[9] B. Misra ans E.C.G. Sudarshan, J. Math. Phys. 18, 756 (1977).
[10] W.M. Itano, D.J. Heinzen, J.J. Bollinger, and D.J. Wineland, Phys. Rev. A 41, 2295 (1990).
[11] See e.g., V. Frerichs and A. Schenzle, Phys Rev. A 44, 1962 (1991), and E. Block and P.R. Berman, Phys. Rev. A 44, 1466 (1991), and M.J. Gagen and G.J. Milburn, Phys. Rev. A 47, 1467 (1992).
[12] M.C. Fischer, B. Gutiérrez-Medina, and M.G. Raizen, Phys. Rev. Lett. 87, 040402 (2001).
[13] L.A. Khalfin, Zh. Eksp. Theo. Fiz. 33, 1371 (1958) [Sov. Phys. JETP 6, 1503 (1958)]
[14] A. Peres, Ann. Phys. NY 129, 33,(1980).
[15] P. Facchi, H. Nakazato, and S. Pascazio, Phys. Rev. Lett. 86, 2699 (2001), and P. Facchi, and S. Pascazio, in Progress in Optics, Ed. E. Wolf (Elsevier, Amsterdam, 2001) 42, 147.
[16] V. Weisskopf and E. Wigner, Z. Phys. 63, 54 (1930).
[17] Elizabeth C. Gaskell, Mary Barton, a tale of Manchester life xxxi (1848): “What’s the use of watching? A watched pot never boils.”