Reply on RC3
Adnane Osmane et al.

Author comment on "Quantifying the non-linear dependence of energetic electron fluxes in the Earth's radiation belts with radial diffusion drivers" by Adnane Osmane et al., Ann. Geophys. Discuss., https://doi.org/10.5194/angeo-2021-47-AC3, 2021

We thank the reviewer for carefully reading the manuscript and providing us with constructive feedback. Our answer to the specific comments (in bold and italic) can be found below.

- **Line 19, “Their study are...” --> “Their studies are...”**
  It is now corrected.

- **Line 61, the Balikhin et al. citation is missing the publication year**
  It is now corrected.

- **Lines 99-100, it is easy to think of scenarios where one can gain information about the likelihood of event X, given Y. However, it is not so easy to think of scenarios where one can lose information. If X and Y are unrelated then no information is gained about X given Y. Can the authors elaborate on this?**
  Indeed, if X and Y are not dependent on one another, we have not lost information. But if a variable X (e.g., ULF wave power) and Y (MeV electron fluxes) are dependent on one another under some conditions (e.g., large solar wind speed), the removal of the conditions upon which the dependence is strong can result in a loss of information (reduction of mutual information), and thus a loss of knowledge. We have added this clarification to the text.

- **Lines 168-169, the sentence is a bit awkward. The authors probably want to say ...mutual information and Pearson correlation is an indication that the correlation should not be interpreted linearly (or something like that).**
It is now corrected.

- **Line 210**, there should be a comma between “radial diffusion” and “is a leading”.

  It is now corrected.

- **Line 238**, there should be a description that SOPA is an instrument on board of Los Alamos National Laboratory (LANL) spacecraft.

  We have added this description.

- **Line 256**, “... and positive vice-versa”. Should this be “...negative vice-versa”?

  Indeed, good catch.

- **Line 261**, should the value above the shaded area represents a mutual information that has least three (not six) sigma significance?

  Yes, it is three sigma. We corrected it.

- **Lines 260-270 and Figures 4 and 5**. One of the main differences between mutual information and correlation in Figures 4 and 5 is that mutual informations consistently have very pronounced secondary peaks at time offset around -100 h whereas the secondary peaks in the Pearson correlations appear to be less pronounced or less significant. Can the authors discuss this?

  This is a good observation. Overall, we show that the Pearson correlation is missing out about 20-30% of the statistical dependence due to its inability to capture nonlinearities. We would need to look at each of these peaks in isolation to quantify this, but we can postulate that the difference we see in secondary peaks has a similar explanation.

  1- In mutual information plots, Figures 4a, 4c, 5a, and 5c, the secondary peaks probably correspond to negative correlations, as inferred from their Pearson correlation counterparts. The anti-correlations can also be seen in Figures 6 and 7. Can the authors explain this anti-correlation between F1.2 and Sgr and Sgeo at time offset -100 h? The anti-correlations between F130 and Sgr and Sge can also be seen in Figure 11 at about the same time offset.
In Figure 4, the secondary peak around -100h corresponds indeed to a small negative correlation. This negative correlation can be explained by a loss of relativistic electrons associated to an enhancement in ULF wave power. ULF wave power locally increase the magnitude of the mean magnetic field sampled by the particles. So one possible consequence is that locally cyclotron resonances can take place for particles with higher parallel velocity, and result in enhanced scattering and losses. This is beyond the scope of the paper and it is speculative, but it might be interesting for future study to search for ULF wave modulation of local wave-particle interactions in terms of mutual information.

- Lines 345-346, the authors claim that their results show that quantitatively the dependence is modest. This claim is repeated on line 386 and elsewhere in the manuscript. Table 1 shows adjusted correlations of 0.6 to 0.8. In many studies of space science, correlations of 0.7-0.8 would be considered strong or very good. “modest” is probably a subjective term. Can the authors comment on what they would quantitatively consider modest or strong or weak correlations?

This statement certainly needs clarification. The statistical dependence between 130 keV and ULF wave power is NOT modest since a Pearson correlation of 0.78 and a mutual information of 0.67 is large. However, the Pearson correlation for the relativistic electron and ULF wave power, ranging between 0.4 and 0.59 is modest (significant, but not large) when compared to the statistical dependence between 130 keV and ULF wave power. When compared to previous results, e.g., Simms et al. find correlations ranging between 0.15 and 0.65 between the maximum electron fluxes 48-120 hours after the beginning of a storm and average ULF wave power for a given phase storm. In our case, we correlate the maximum ULF wave power with the maximum electron fluxes measured over an hour interval. We have added this clarification to the revised manuscript and provided more details in our discussion of the results of Simms et al.