ON THE EFFECT OF BINDING IN DEEP INELASTIC SCATTERING

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Abstract

The phenomenon of scaling in deep inelastic lepton-nucleon scattering is usually explained in terms of the Feynman parton model, and the logarithmic corrections to scaling are explained in the framework of perturbative QCD. For testing the validity of the parton model, we consider the deep inelastic electron scattering in a model in which the system electromagnetic current operator explicitly satisfies relativistic invariance and current conservation. Let the struck particle have the fraction $\xi$ of the total momentum in the infinite momentum frame. Then it is shown that, due to binding of particles in the system under consideration, the Bjorken variable $x$ no longer can be interpreted as $\xi$, even in the Bjorken limit and in zero order of the perturbation theory. We argue that, as a result, the data on deep inelastic scattering alone do not make it possible to determine the $\xi$ distribution of quarks in the nucleon.

1 Introduction

The phenomenon of scaling in deep inelastic scattering (DIS) was first explained in the framework of approach developed by Bjorken [1]. Another explanation was proposed by Feynman [2] in the framework of the parton model. According to this model, the process of absorption of a virtual photon with the 4-momentum $q$ such that $|q^2|$ is very large, can be described assuming that the nucleon consists of point-like partons which do not interact with each other at small distances. Let $P'$ be the 4-momentum of the nucleon in the infinite momentum frame (IMF) where the nucleon moves along the
positive direction of the $z$ axis with the velocity close to the velocity of light. Let also the struck parton have the fraction $\xi$ of the nucleon’s momentum. Then the 4-momentum of this parton in the final state is equal to $\xi P' + q$.

We consider the process in the Bjorken limit when $|q^2|$ and $2(P'q)$ are very large, but the quantity $x = |q^2|/2(P'q)$ is not too close to 0 or 1. Then assuming that $(\xi P' + q)^2$ does not exceed the square of the nucleon’s mass, we conclude that in the Bjorken limit $\xi = x$. For this reason the authors of some textbooks and papers even do not distinguish the quantities $\xi$ and $x$.

In QCD the partons are naturally identified with quarks, and the fact that they do not interact at small distances is treated as the consequence of the asymptotic freedom. More exactly, using some assumptions, it can be shown that in the Bjorken limit $\xi = x$ and Bjorken Scaling take place in zero order in $\alpha_s$ (where $\alpha_s$ is the QCD running coupling constant), while the interaction between quarks and gluons can be taken into account perturbatively (leading to logarithmic breaking of scaling and the relation $\xi = x$). Note however that the technique of the operator product expansion (OPE) developed by Wilson and others (see, for example, Refs. [3]) formally does not use any relation between $\xi$ and $x$, and in principle one cannot exclude the possibility that, even in the Bjorken limit and in zero order of the perturbation theory, scaling takes place while $\xi \neq x$. The relation $\xi = x$ in the framework of the OPE can be obtained only at some additional assumptions—see Refs. [4]. Moreover, it is not even clear whether the OPE series is convergent or asymptotic [5].

The problem arises how to take into account the effect that initially quarks are in the bound state — in the nucleon. It is clear that this effect cannot be considered in the framework of perturbative QCD.

Our experience in nonrelativistic quantum mechanics and in nuclear physics tells that the effect of binding is not important at large momentum transfer. For example, the results of calculations of the electron scattering from nuclei show that at small momentum transfer there is a coherent process on the nucleus as a whole (and therefore the scattering amplitude is proportional to $Z$ and the cross-section is proportional to $Z^2$), while at large momentum transfer the cross-section is an incoherent sum of the cross-sections on each nucleon in the nucleus (therefore, this cross-section is proportional to $Z$).

This picture was questioned by many authors after the discovery of the EMC effect [6]. The central point of the extensive discussion in the literature was whether the EMC effect can be explained in the framework of conven-
tional nuclear physics. We shall not discuss this problem but note that the large group of authors stated that this can be done if the effects of the interaction between the nucleons and relativistic effects are taken into account (see, for example the calculations in Refs. [7, 8, 9]).

It is important to note that the above calculations in conventional nuclear physics (not taking into account the EMC effect) and the analogous calculations in atomic physics have been carried out in the framework of the impulse approximation, where it is assumed that the electromagnetic current operator (ECO) of the system under consideration can be represented as a sum of the ECO’s for the constituents comprising this system. Such an approximation is reasonable in nonrelativistic quantum mechanics, but it is well-known that in the relativistic case the ECO should necessarily contain the terms depending on the interaction between the constituents, since otherwise the ECO satisfies neither relativistic invariance nor current conservation.

In the parton model it is assumed that the partons in the IMF are free to the extent that the impulse approximation is valid. On the other hand, the interaction between them cannot be eliminated at all since in this case the nucleon will not be bound. Are these assumptions compatible with each other? The answer to this question can be given only in the framework of explicitly solvable models.

In the present paper the effect of binding in DIS is investigated in the framework of the model in which the ECO explicitly satisfies relativistic invariance, current conservation, cluster separability, and the condition that the interaction terms in the ECO do not renormalize the total system electric charge. However these conditions are not sufficient for choosing a unique solution. We choose a special solution considered in Ref. [10]. The essence of our results becomes clear already in the case of $N = 2$ particles, and then these results are generalized to the case when $N$ is arbitrary (including $N = \infty$).

The major objection against such an approach maybe that the ECO $\hat{J}^\mu(x)$ ($\mu = 0, 1, 2, 3$) obtained in such a way is nonlocal in the sense that it is not derived from a local Lagrangian (we use $x$ to denote a point in Minkowski space as well as the Bjorken variable, but this should not lead to misunderstanding). In particular, it is not clear whether the commutator $[\hat{J}^\mu(x), \hat{J}^\mu(0)]$ necessarily vanishes when $x$ is a space-like vector. Let us note however that if a theory is nonlocal in the above sense, this does not necessarily imply that it is unphysical. Indeed, as it has become clear already in 30th, in relativistic
quantum theory there is no operator possessing all the properties of the position operator. In particular, the quantity $x$ in the Lagrangian density $L(x)$ is not the coordinate, but some parameter which becomes the coordinate only in the classical limit. Therefore the physical condition is that the above commutator should vanish when $|x^2| \to \infty$ but $x^2 < 0$. We shall see in Sec. 4 that this condition is indeed satisfied.

Anyway, the relation between $\xi$ and $x$ derived in Sec. 3 is in fact kinematical. This relation shows that $\xi \neq x$ even in the Bjorken limit and in zero order of the perturbation theory. At the same time, in our approach the scaling and the Callan-Gross relation \cite{11} remain.

The paper is organized as follows. In Sec. 2 we give a detailed calculation of the hadronic tensor for systems of two particles in the impulse approximation, and in Sec. 3 the same is done for systems of $N$ particles. The corresponding results are well-known, and the analogous calculations were carried out elsewhere (see, for example, Refs. \cite{12, 13, 14} and references cited therein), but the major purpose of these sections is to prepare the reader for the consideration of the case when the ECO contains the interaction. In Sec. 4 we briefly describe the results of Ref. \cite{10} needed in Sec. 5 where these results are used for the explicit calculation of the hadronic tensor for systems of $N$ particles. We hope that the main part of the paper is self-contained, and even the unexperienced reader can follow our calculations.

2 Impulse approximation for the system of two particles

Let us consider a system of two particles with the masses $m_i$ and the electric charges $e_i$ ($i = 1, 2$). If $p$ is the 4-momentum of some particle then $p \perp$ means the projection of $p$ onto the plane $xy$, and, instead of the temporal and the $z$ components of $p$, we use the $\pm$ components defined as $p^\pm = (p^0 \pm p^z)/\sqrt{2}$. We also use $\sigma_i$ to denote the projection of the spin of particle $i$ on the $z$ axis.

The Hilbert space $H$ for the system under consideration is the space of functions $\varphi(p_{1\perp}, p_{1\perp}^\pm, \sigma_1, p_{2\perp}, p_{2\perp}^\pm, \sigma_2)$ such that

$$\sum_{\sigma_1, \sigma_2} \int \|\varphi(p_{1\perp}, p_{1\perp}^\pm, \sigma_1, p_{2\perp}, p_{2\perp}^\pm, \sigma_2)\|^2 d\rho(p_{1\perp}, p_{1\perp}^\pm) d\rho(p_{2\perp}, p_{2\perp}^\pm) < \infty \quad (1)$$
where
\[ d\rho(p_\perp, p^+) = \frac{d^2p_\perp dp^+}{2(2\pi)^3p^+} \]  
(2)

Instead of the individual particle variables we introduce the total momentum variables and the internal momentum variables. The former are the \( \perp \) and \(+\) components of the 4-vector \( P = p_1 + p_2 \), and, following Ref. [15], the latter can be defined as

\[ \xi = \frac{p_1^+}{P^+}, \quad k_\perp = p_\perp - \xi P^+ \]  
(3)

As shown in this reference, it is also possible to choose as the internal variables the set \( k = (k_\perp, k^z) \) where \( k^z \) is defined from the conditions

\[ \xi = \frac{\omega_1(k) + k^z}{M(k)}, \]  
(4)

\( \omega_i(k) = (m_i^2 + k^2)^{1/2} \), and \( M(k) = \omega_1(k) + \omega_2(k) \). It is easy to show that \( P^2 = M(k)^2 \), and therefore the mass operator of the two-particle system is the operator of multiplication by \( M(k) \).

It is easy to see that under the interchange of particles 1 and 2, \( k \rightarrow -k \) and \( \xi \rightarrow 1 - \xi \). For this reason it is sometimes convenient to use the notations \( k_1 = -k_2 = k \) and \( \xi_1 = 1 - \xi_2 = \xi \).

Let us introduce the 4-vectors \( k_i = (\omega_i(k_i), k_i) \) and \( G = P/M(k) \). Since \( G^2 = 1 \), only three components of \( G \) are independent, for example \( G_\perp \) and \( G^+ \). Let \( \beta(G) \equiv \beta(G_\perp, G^+) \in SL(2, C) \) be the matrix with the components

\[ \beta_{11} = \beta_{22}^{-1} = 2^{1/4}(G^+)^{1/2}, \quad \beta_{12} = 0, \quad \beta_{21} = (G^x + iG^y)\beta_{22} \]  
(5)

We use \( L(l) \) to denote the Lorentz transformation corresponding to \( l \in SL(2, C) \). Then a direct calculation shows that

\[ p_i = L[\beta(G)]k_i \quad (i = 1, 2) \]  
(6)

Therefore \( L[\beta(G)] \) has the meaning of the boost, and \( k \) is the momentum in the c.m. frame. We note that these quantities are the same as the "canonical" ones if \( G_\perp = 0 \).
A direct calculation gives

$$d\rho(p_1, p_1^+)d\rho(p_2, p_2^+) = d\rho(P_1, P_1^+)d\rho(int),$$

$$d\rho(int) = \frac{d^3k d\xi}{2(2\pi)^3\xi(1-\xi)} = \frac{M(k)d^3k}{2(2\pi)^3\omega_1(k)\omega_2(k)}$$  (7)

We introduce the Hilbert space $H_{int}$ as the space of functions $\chi(k, \sigma_1, \sigma_2)$ such that

$$||\chi||^2 = \sum_{\sigma_1\sigma_2} \int |\chi(k, \sigma_1, \sigma_2)|^2 d\rho(int) < \infty$$  (8)

(note that $\xi \in [0, 1]$). We shall write the function $\chi$ in the form $\chi(k^\perp, \xi, \sigma_1, \sigma_2)$ if $(k^\perp, \xi)$ are chosen as the independent variables, and $(k_1^\perp, \xi_1)$ are connected with them as explained above.

In the scattering theory the state in which particle 1 has the momentum $p_1'$ and the spin projection $\sigma_1'$, and particle 2 has the momentum $p_2'$ and the spin projection $\sigma_2'$, is the product $|p_1', \sigma_1'\rangle|p_2', \sigma_2'\rangle$ where

$$|p_i', \sigma_i'\rangle = 2(2\pi)^3 p_i'^{+\delta(2)}(p_i^\perp - p_i'^\perp)\delta(p_i^\perp - p_i'^\perp)\delta_{\sigma_i\sigma_i'}$$  (9)

($\delta_{\sigma_i\sigma_i'}$ is the Cronecker symbol). If the particles are in the bound state described by the wave function $\chi' \in H_{int}$, and the system as the whole has the 4-momentum $P'$ then, as shown by several authors (see, for example, Ref. [10]), the above choice of the variables makes it possible to write the wave function of such a system by analogy with Eq. (9):

$$|P', \chi'\rangle = 2(2\pi)^3 P'^{+\delta(2)}(P_-^\perp - P_-'^\perp)\delta(P_-^+ - P_-'^+)|\chi'\rangle$$  (10)

where $\chi'$ is normalized as $||\chi'||^2 = 1$.

We shall always assume that all particles having the electric charge are structureless and their spin is equal to $1/2$. Then the one-particle ECO for particle $i$ acts over the variables of this particle as

$$J_i^\nu \varphi(p_i^\perp, p_i^+, \sigma_i) = r_i \sum_{\sigma_i'} \int [\bar{w}_i(p_i, \sigma_i)\gamma^\mu w_i(p_i', \sigma_i')] \cdot \varphi(p_i^\perp, p_i'^+, \sigma_i')d\rho(p_i', p_i'^+)$$  (11)

and over the variables of other particle it acts as the identity operator. Here $r_i = e_i/e_0$ is the ratio of the particle electric charge to the unit electric
charge, \( w_i(p_i, \sigma_i) \) is the Dirac light cone spinor, \( \gamma^\mu \) is the Dirac \( \gamma \)-matrix, and \( \bar{w} = \gamma^+ \gamma^0 \). The form of \( w_i(p_i, \sigma_i) \) in the spinor representation of the Dirac \( \gamma \)-matrices is

\[
 w_i(p_i, \sigma_i) = \sqrt{m_i} \begin{pmatrix} \beta(p_i \gamma^0 / m_i) \chi(\sigma) \\ \beta(p_i \gamma^+ / m_i)^{-1+} \chi(\sigma) \end{pmatrix}
\]

(12)

where \( \chi(\sigma) \) is the ordinary spinor describing the state with the spin projection on the \( z \) axis equal to \( \sigma \).

The impulse approximation implies that the ECO for the system as a whole is a sum of the ECO’s for the constituents. In particular, for a system of two particles \( J^\mu(0) = J^\mu_1(0) + J^\mu_2(0) \). This relation together with Eqs. (9-11) makes it possible to calculate the matrix element of the operator \( J^\mu(0) \) between any two-particle states.

When the bound state of two particles absorbs a virtual photon with the large momentum, we should expect that, since the relative momentum in the final state is large, the interaction between the particles in this state can be neglected. Therefore the inclusive cross-section is fully defined by the tensor

\[
 W^{\mu\nu} = \frac{1}{4\pi} \sum_{\sigma_1,\sigma_2} \int (2\pi)^4 \delta(4) (P' + q - p_1'' - p_2'') \langle P', \chi'| J^\mu(0) | p_1'', \sigma_1'', p_2'', \sigma_2'' \rangle d\rho(p_{1\perp}'', p_{1+}'') d\rho(p_{2\perp}'', p_{2+}'')
\]

(13)

It is well-known that the average value of this tensor over the all initial spin states is equal to

\[
 W^{\mu\nu}(P', q) = \frac{(q^\mu q^\nu - g^{\mu\nu})}{q^2} F_1(x, q^2) +
\]

\[
 \frac{1}{P'(q)} (P'^\mu - \frac{q^\mu (P'q)}{q^2}) (P'^\nu - \frac{q^\nu (P'q)}{q^2}) F_2(x, q^2),
\]

(14)

and our goal is to calculate the structure functions \( F_1(x, q^2) \) and \( F_2(x, q^2) \).

Knowing the momenta \( p_i'' \) in the final state and using Eq. (3) we can calculate the relative momentum \( k'' \) in the final state. Let \( M'' = \omega_1(k'') + \omega_2(k'') \) be the mass of the final state. Then a standard calculation gives

\[
 (2\pi)^4 \delta(4) (P' + q - p_1'' - p_2'') d\rho(p_{1\perp}'', p_{1+}'') d\rho(p_{2\perp}'', p_{2+}'') = \frac{k'' do''}{16\pi^2 M''}
\]

(15)
where $k'' = |k''|$, and $do''$ is the element of the solid angle for the unit vector $k''/k''$. In the Bjorken limit $k'' = M''/2$ and

$$M'' = \left[ \frac{|q^2|(1-x)}{x} \right]^{1/2} \quad (16)$$

since $M''^2 = (P' + q)^2$.

As follows from Eqs. (3), (4), (6) and (11)

$$\langle p_1'', \sigma_1'', p_2'', \sigma_2'' | J''(0) | P', \chi' \rangle = \sum_{i=1}^{2} \sum_{\sigma_i} \frac{r_i}{\xi_i} [\tilde{w}_i(p_i'', \sigma_i'') \gamma_{\nu} w_i(p_i', \sigma_i')] \chi'(d_i, \sigma_i', \tilde{\sigma}_i'') \quad (17)$$

where $\tilde{\sigma}_1'' = \sigma_2'', \tilde{\sigma}_2'' = \sigma_1'', p_i' = L[\beta(P'_\perp/M(d_i), P'^+/M(d_i))]d_i, d_i = (\omega_i(d_i), d_i)$, and the vectors $d_i$ are defined by the conditions

$$L[\beta(P'_\perp/M(d_i), P'^+/M(d_i))] (\tilde{\omega}_i(d_i), -d_i) = L[\beta(P''_\perp/M '', P'^+/M'')] (\tilde{\omega}_i(k_i''), -k_i'') \quad (18)$$

where $P'' = p_1'' + p_2''$, $\tilde{\omega}_1 = \omega_2$, and $\tilde{\omega}_2 = \omega_1$.

It is convenient to consider the process in the reference frame where $P'_\perp = q_\perp = 0$, and $P'^z$ is positive and very large. By analogy with the Breit frame for elastic processes we choose the reference frame in which $P' + P'' = 0$. It is easy to show that in this reference frame

$$q^0 = 2|P'|(1-x), \quad P'^+ = \sqrt{2}|P'|, \quad q^+ = -\sqrt{2}|P'|x, \quad P''^+ = \sqrt{2}|P'|(1-x) \quad (19)$$

Then, as follows from Eq. (18)

$$d_{1\perp} = k_{1\perp}'', \quad d_{2\perp} = k_{2\perp}'', \quad 1 - \xi_1 = \frac{1}{2}(1-x)(1-cos\theta), \quad 1 - \xi_2 = \frac{1}{2}(1-x)(1+cos\theta) \quad (20)$$

where $k''^z = k'' cos\theta$, and the quantities $\xi_i$ are expressed in terms of $d_i$ according to Eq. (4).

We assume that the internal wave function $\chi'(d)$ effectively cuts the contribution of large momenta, and therefore the contribution to the integrals containing $\chi'(d)$ is given only by the momenta with $|d| \leq m_0$, where $m_0$ is
some parameter satisfying the condition \( m_0^2 \ll |q|^2 \). Then, as follows from Eqs. (15-17) and the first pair of expressions in Eq. (20), only those \( \theta \)'s contribute to Eq. (13) for which \( 1 - |\cos \theta| \leq \frac{m_0^2}{|q|^2} \). Using additionally Eq. (14), the second pair of expressions in Eq. (20), and the condition \(|dz| \leq m_0\), we conclude that in the right-hand side of Eq. (17) the term with \( i = 1 \) is not negligible only if \( \cos \theta \) is close to -1 \((1 + \cos \theta \leq \frac{m_0^2}{|q|^2})\), while the term with \( i = 2 \) is not negligible only if \( \cos \theta \) is close to +1 \((1 - \cos \theta \leq \frac{m_0^2}{|q|^2})\).

Therefore, as follows from Eq. (20), \( \xi_1 = x \) in the first term, and \( \xi_2 = x \) in the second one, in agreement with the interpretation of the quantity \( x \) in the parton model. We also see that both particles absorb the virtual photon incoherently.

It is easy to see that in both regions of \( \cos \theta \) we can write \( d\sigma'' = d^2k_{\perp}'' / k^{''2} \). Therefore, as follows from Eqs. (5-7), (12), (13), (15-17), (19) and (20)

\[
W^{\mu\nu} = \sum_{i=1}^{2} i^2 \int \langle \chi'(k_{\perp}, \xi_i = x) | S^{\mu\nu}_i | \chi'(k_{\perp}, \xi_i = x) \rangle \frac{d^2k_{\perp}}{4(2\pi)^3 x(1-x)} \quad (21)
\]

where we do not write the spin variables in the arguments of the function \( \chi' \), the scalar product is taken over these variables, and the tensor operator \( S^{\mu\nu}_i \) is as follows. It is equal to zero if either \( \mu \) or \( \nu \) is equal to \( \pm \), while if \( j, l = x, y \) then \( S^{jl}_i = \delta_{jl} + 2i\epsilon_{jl}s^{z}_i \), where \( \epsilon_{jl} \) is the antisymmetric tensor with \( \epsilon_{xy} = -\epsilon_{yx} = 1, \epsilon_{11} = \epsilon_{22} = 0 \), and \( s^{z}_i \) is the \( z \) component of the spin operator for particle \( i \).

3 Impulse approximation for the system of \( N \) particles

If the bound state consists of \( N \) particles, we can choose any pair of particles, say the pair of particles \( i_1 \) and \( i_2 \), and construct the external and internal variables for this pair as described above. Let \( P_{i_1i_2} = p_{i_1} + p_{i_2} \) be the total 4-momentum of the pair \( i_1i_2 \). Using the analogous procedure, we can construct from \( P_{i_1i_2} \) and \( p_{i_3} \) the total momentum of the system \( i_1i_2i_3 \) and the relative variables describing the motion of the system \( i_1i_2 \) relative particle \( i_3 \). Then the internal variables in the system \( i_1i_2i_3 \) are these variables and the internal variables for the system \( i_1i_2 \). It is obvious that in such a way we can construct the external and internal variables for a system with any number of particles, but the choice of the internal variables is not unique.
Let $P_i$ be the total 4-momentum of the system consisting of particles $1, 2, \ldots, i-1, i+1, \ldots, N$, $\tilde{i}nt$ be a set of the internal variables for this system, and $dp(\tilde{i}nt)$ be the volume element in the set $\tilde{i}nt$. Then $P = p_i + P_i$ is the total 4-momentum of the system consisting of all the particles $1, 2, \ldots, N$, and we use $k_i, \xi_i$ to denote the variables describing the motion of particle $i$ relative the system $1, 2, \ldots, i-1, i+1, \ldots, N$. By analogy with Eq. (3)

$$\xi_i = \frac{p_i^+}{P^+}, \quad k_i = p_{i\perp} - \xi_i P^+$$

(22)

Let $M_i$ be the free mass operator of the system $1, 2, \ldots, i-1, i+1, \ldots, N$ as a function of $\tilde{i}nt$. Then, by analogy with Eq. (4), we can introduce the vector $k_i = (k_i, k_i^z)$ such that

$$\xi_i = \omega_i(k_i) + k_i^z = \omega_i(k_i) + \left(M_i^2 + k_i^2\right)^{1/2}$$

(23)

where $M(k_i, M_i) = \omega_i(k_i) + (M_i^2 + k_i^2)^{1/2}$.

By analogy with the two-particle case, we can introduce the internal space $H_{int}$ as the space of functions $\chi(k_i, \xi_i, \sigma_i, \tilde{i}nt)$ such that

$$||\chi||^2 \equiv \sum_{\sigma_i} \int |\chi(k_i, \xi_i, \sigma_i, \tilde{i}nt)|^2 \frac{d^2k_{i\perp}d\xi_i}{2(2\pi)^3(1 - \xi_i)} dp(\tilde{i}nt) < \infty$$

(24)

Let the initial state of the system of $N$ particles be a bound state with the total 4-momentum $P'$ and the internal wave function $\chi'(k_i, \xi_i, \sigma_i, \tilde{i}nt)$ the norm of which in the space $H_{int}$ is equal to unity. Then the wave function of such a system can be written in the form of Eq. (10) (see, for example, Ref. [16]). After absorbing the virtual photon with large momentum, particle $i$ becomes free, but the rest of the system can consist of some number of free particles and of some number of subsystems in the bound states. If $\chi_i''(\tilde{i}nt)$ is the internal wave function of the system $1, 2, \ldots, i-1, i+1, \ldots, N$ in the final state, then the wave function of the system of $N$ particles in the final state can be written as

$$|p_i'', \sigma_i'', P_i'', \chi_i''\rangle = 2(2\pi)^3p_i'' + \delta(2)(P_i - P_i''\rangle \delta(p_i'' - P_i')\delta\sigma_i'' \cdot \frac{1}{4\pi} \sum_{\sigma_i'' \tilde{i}nt} \int (2\pi)^4 \delta(4)(P' + q - p_i'' - P_i''\rangle \langle P', \chi'\rangle J^\mu(0)p_i''', \sigma_i''$$

(25)

Instead of Eq. (13) we should write the hadronic tensor in the form

$$W^{\mu\nu} = \frac{1}{4\pi} \sum_{\sigma_i'' \tilde{i}nt} \int (2\pi)^4 \delta(4)(P' + q - p_i'' - P_i''\rangle \langle P', \chi'\rangle J^\mu(0)p_i''', \sigma_i''$$

10
where a sum is taken over all possible spin states of particle \(i\) and all possible internal states of the system \(1, \ldots, i-1, i+1, \ldots, N\).

Let \(k''_i\) be the relative momentum of particle \(i\) and the system \(1, \ldots, i-1, i+1, \ldots, N\), and \(E_i(d_i) = (M_i^2 + d_i^2)^{1/2}\). Then, as follows from Eqs. (10), (11) and (25)

\[
\langle p''_i, \sigma''_i, P''_i, \chi''_i(\tilde{\text{int}}) | J''_i(0) | P'_i, \chi'_i(k_i, \sigma_i, \tilde{\text{int}}) \rangle = r_i \xi_i \sum_{\sigma'_i} \int \chi''_i(\tilde{\text{int}})^* \cdot \left[ \bar{w}(p''_i, \sigma''_i) \gamma_i w(p'_i, \sigma'_i) \chi'_i(d_i, \sigma'_i, \tilde{\text{int}}) \right] d\rho(\tilde{\text{int}}) \tag{27}
\]

where \(p'_i = L[\beta(P'_i/M(d_i, M_i), P''_i/M(d_i, M_i))] d_i\), and \(d_i\) is defined by the condition, which can be written as Eq. (18) if \(M(d_i)\) is replaced by \(M(d_i, M_i)\), \(\tilde{\omega}_i\) is replaced by \(E_i\), and \(M'' = \omega(k'') + E_i(k'')\).

Using the completeness of the states in the internal space of the system \(1, \ldots, i-1, i+1, \ldots, N\) and Eqs. (15), (16), (19), (26) and (27) we get (compare with Eq. (21))

\[
W^{\mu\nu} = \sum_{i=1}^{N} \int \langle \chi'(k_{i\perp}, \xi_i = x, \tilde{\text{int}}) | S_i^{\mu\nu} | \chi'(k_{i\perp}, \xi_i = x, \tilde{\text{int}}) \rangle \frac{d^2k_{i\perp} d\rho(\tilde{\text{int}})}{4(2\pi)^3 x(1-x)} \tag{28}
\]

Let us introduce the notation

\[
\rho_i(x) = \int \langle \chi'(k_{i\perp}, \xi_i = x, \tilde{\text{int}}) | \chi'(k_{i\perp}, \xi_i = x, \tilde{\text{int}}) \rangle \frac{d^2k_{i\perp} d\rho(\tilde{\text{int}})}{2(2\pi)^3 x(1-x)} \tag{29}
\]

Then, as follows from Eqs. (7), (8) and (24), \(\rho_i(x)dx\) is the probability of the event that particle \(i\) in the bound state has the value of \(\xi_i\) in the interval \((x, x+dx)\).

As follows from Eqs. (14) and (28), the structure functions \(F_1\) and \(F_2\) depend only on \(x\):

\[
F_1(x) = \frac{1}{2} \sum_{i=1}^{N} r_i^2 \rho_i(x), \quad F_2(x) = 2xF_1(x) \tag{30}
\]

(the last equality is known as the Callan-Gross relation [11]). These expressions for the structure functions were derived by many authors in the
framework of the parton model (see also Sec. 6). Equation (28) also makes it possible to write the expression for the polarized structure functions, but we shall not dwell on this question.

One might think that the above results are natural since they fully agree with the parton model. However the following question arises. Since the ECO in the impulse approximation does not satisfy relativistic invariance and current conservation (see the next section for more details), the results for the structure functions depend on the reference frame in which these functions are calculated. An argument in favor of choosing the IMF is that in this reference frame the tensor $W^{\mu\nu}$ given by Eq. (28) satisfies the continuity equation $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$. Another well-known arguments are based on the approach proposed by Weinberg [17] and developed by several authors (see, for example, Refs. [18, 19]). Let us note however that though quantum field theory in the IMF seems natural and has some advantages, it also has some serious difficulties which are not present in the usual formulation [20].

In our opinion, a rather strange feature of the above results is as follows. By looking through the derivation of these results one can easily see that the initial state is treated in fact not as the bound state but as the free state of noninteracting particles. Indeed, we have never used the fact that the initial state is the eigenstate of the mass operator $\hat{M}$ with the eigenvalue $M'$: $\hat{M}\chi' = M'\chi'$. In the impulse approximation the relation between the quantities $d_i$ and $k_i''$ (see Eq. (18)) is derived from the condition that the 4-vectors $(\tilde{\omega}(d_i), -d_i)$ and $(\tilde{\omega}(k_i''), -k_i'')$ are connected by the Lorentz boosts in the initial and final states. It is natural that particle $i$ does not interact with the other particles in the final state, but it is strange that we neglect the interaction in the initial state and write the free mass $M(d_i)$ instead of the real mass $M'$ which has the initial state.

The effect of binding can be explicitly taken into account in models where the ECO satisfies relativistic invariance and current conservation. This problem is considered in the subsequent sections.
4 Electromagnetic current operator for systems of interacting particles

In the following we use $\hat{J}^\mu(x)$ to denote the ECO for a system of interacting particles, while $J^\mu(x)$ is used to denote the ECO in the impulse approximation.

Let $\hat{U}(a) = \exp(i\hat{P}_\mu a^\mu)$ be the representation operator corresponding to the displacement of the origin in spacetime translation of Minkowski space by the 4-vector $a$. Here $\hat{P} = (\hat{P}^0, \hat{\mathbf{P}})$ is the operator of the 4-momentum, $\hat{P}^0 = \hat{E}$ is the Hamiltonian, and $\hat{\mathbf{P}}$ is the operator of ordinary momentum. Let also $\hat{U}(l)$ be the representation operator corresponding to $l \in SL(2, C)$. Then $\hat{J}^\mu(x)$ must be the selfadjoint relativistic vector operator such that

$$\hat{U}(a)^{-1}\hat{J}^\mu(x)\hat{U}(a) = \hat{J}^\mu(x-a)$$ (31)

$$\hat{U}(l)^{-1}\hat{J}^\mu(x)\hat{U}(l) = L(l)^\mu_\nu \hat{J}^\nu(L(l)^{-1}x)$$ (32)

where a sum over repeated indices $\mu, \nu = 0, 1, 2, 3$ is assumed. As follows from Eq. (31), the continuity equation $\partial\hat{J}^\mu(x)/\partial x^\mu = 0$ can be written in the form

$$[\hat{J}^\mu(x), \hat{P}_\mu] = 0$$ (33)

Since at least some of the operators $\hat{U}(a)$ and $\hat{U}(l)$ depend on interactions in the system under consideration, the immediate consequence of Eqs. (31-33) is that $\hat{J}^\mu(x)$ also depends on these interactions and thus $\hat{J}^\mu(x)$ cannot be written only as a sum of the constituent ECO’s. This fact was first pointed out by Siegert [21].

Let

$$\hat{Q} = \int \hat{J}^\mu(x)d\sigma_\mu(x)$$ (34)

be the system electric charge operator where $d\sigma_\mu(x) = \lambda_\mu \delta(\lambda x - \tau)d^4x$ is the volume element of the space-like hypersurface defined by the time-like vector $\lambda$ ($\lambda^2 = 1$) and the evolution parameter $\tau$. Then the important physical condition is that the interactions do not renormalize the electric charge, i.e. $\hat{Q}$ does not depend on the choice of $\lambda$ and $\tau$ and has only one eigenvalue equal to the sum of electric charges of constituents. It is well-known that Eq. (33) ensures that $\hat{Q}$ does not depend on $\tau$ and $\lambda$ but this condition does not ensure that $\hat{Q}$ has the same value as for noninteracting particles.
In addition, the operator $\hat{J}^\mu(x)$ should satisfy the cluster separability condition. Briefly speaking, this condition implies that if the interaction between any subsystems $\alpha_1, \ldots, \alpha_n$ comprising the system under consideration is turned off then $\hat{J}^\mu(x)$ must become a sum of the ECO’s $\hat{J}_{\alpha_i}^\mu(x)$ for the subsystems.

To explicitly construct the ECO satisfying the above properties it is necessary to choose first the explicit realization of the representation of the Poincare group for the system under consideration. Dirac was the first who singled out three forms of relativistic dynamics: instant, front and point ones [22]. As proved by Sokolov and Shatny [23], these forms are unitarily equivalent to each other. In Ref. [14] the problem of constructing the ECO was first explicitly solved in the point form, and then, using the unitary operators constructed in Ref. [23], the ECO was constructed in the instant and front forms. For this reason, in the present paper we use the solution in the point form. By definition, the description in the point form implies that the operators $\hat{U}(l)$ are the same as for noninteracting particles, i.e. $\hat{U}(l) = U(l)$, and thus interaction terms can be present only in the 4-momentum operators $\hat{P}$ (i.e. in the general case $\hat{P}^\mu \neq P^\mu$ for all $\mu$).

In the point form it is convenient to use 4-velocities (instead of 4-momenta) as the external variables while the internal variables can be chosen as above (see Eqs. (3) and (6)). If $g_i = p_i/m_i$, then it is easy to see that, by analogy with Eq. (7),

$$d\rho(g_1^+, g_2^+) = d\rho(G^+, G^-)d\rho(int)$$

but now

$$d\rho(int) = \frac{M(k)^2d^2k_1d\xi}{2(2\pi)^3\xi(1-\xi)} = \frac{M(k)^3d^3k}{2(2\pi)^3\omega_1(k)\omega_2(k)}$$

The internal two-particle Hilbert space $H_{int}$ can be formally defined as the space of functions satisfying Eq. (5), but with $d\rho(int)$ given by Eq. (36).

By analogy with the method proposed by Bakamdjian and Thomas in the instant form [24], it is possible to introduce the interaction into the two-particle system as follows (see, for example, Refs. [27, 26]). First, we can express all the representation generators of the Poincare group as functions of the operators $M$, $G$, and the two-body spin operator $S$. In this case $\hat{P} = MG$, where $P$ is the free two-body momentum operator, and the representation generators of the Lorentz group are functions of only $G$ and $S$. Then we
replace $M$ by the two-body mass operator $\hat{M}$ which acts only in $H_{\text{int}}$. If $\hat{M}$ commutes with $S$ then the commutation relations for the Poincare group generators will not be broken. After this procedure the generators of the Lorentz group remain the same as for the noninteracting particles, but the 4-momentum operator $\hat{P} = \hat{M}G$. In the general case the two-body generators obtained in such a way should be subject to some unitary transformation $A$, but we shall not discuss this question and assume that $A = 1$ (see Refs. [27, 10] for more details).

As follows from Eq. (31)

$$\hat{J}_\mu(x) = \exp(i\hat{P}x)\hat{J}_\mu(0)\exp(-i\hat{P}x) \quad (37)$$

Therefore, if the operators $\hat{P}$ are known, it is sufficient to construct only the operators $\hat{J}_\mu(0)$ with the correct properties.

Let $\varphi(G)$ be a function of $G$ with the range in $H_{\text{int}}$. The action of $\hat{J}_\mu(0)$ in $H$ can be defined as

$$\hat{J}_\mu(0)\varphi(G) = 2\int \hat{M}^{3/2} \hat{J}_\mu(G, G')\hat{M}^{3/2} \varphi(G')d\rho(G'_{\perp}, G'^+) \quad (38)$$

where the kernel $\hat{J}_\mu(G, G')$ is an operator in $H_{\text{int}}$ for any fixed values of $G$ and $G'$.

We use $\beta(G, G')$ to denote $\beta((G + G')/(G + G')) \in \text{SL}(2, \mathbb{C})$ and $L(G, G')$ to denote $L[\beta(G, G')]$. We also introduce the 4-vectors

$$f = L(G, G')^{-1}G, \quad f' = L(G, G')^{-1}G' \quad (39)$$

These 4-vectors are constructed as the c.m. frame 4-velocities of two particles with unit masses and the 4-velocities $G$ and $G'$ (compare with Eq. (3)). Let us note that this is only a formal construction since $G$ and $G'$ in Eq. (39) have the sense of the 4-velocities of one and the same system in the final and initial states. Nevertheless, as follows from Eq. (39), the 4-vectors $f$ and $f'$ are such that

$$f^2 = f'^2 = 1, \quad f + f' = 0, \quad f^0 = f'^0 = (1 + f^2)^{1/2} \quad (40)$$

Therefore the 4-vectors $f$ and $f'$ are fully determined by the spatial part $f$ of the 4-vector $f$.

It can be shown (see Ref. [10]) that as follows from Lorentz invariance

$$\hat{J}_\mu(G, G') = L(G, G')_{\nu}^\mu \hat{J}_\nu(f) \quad (41)$$
where we use $\hat{\jmath}^\nu(f)$ to denote $\hat{J}^\nu(f, f')$. We see that the kernel of the operator $\hat{J}^\mu$ is fully determined by an operator the action of which in $H_{int}$ depends only on $f$.

The continuity equation (33) in terms of $\hat{\jmath}^\nu(f)$ reads

$$f^0[\hat{M}, \hat{\jmath}^0(f)] = f\{\hat{M}, \hat{\jmath}(f)\} \quad (42)$$

where we use curly brackets to denote the anticommutator. As shown in Ref. [10], the condition that the operator $\hat{\jmath}^0(f)$ is the same as for noninteracting particles will be satisfied if $\hat{\jmath}^0(0) = \jmath^0(0)$, i.e. the operator $\hat{\jmath}^0(0)$ does not depend on the interaction. Let us choose the coordinate axes in such a way that $f_\perp = 0$. Then, as follows from Eq. (42), the continuity equation does not impose any constraint on the operator $\hat{\jmath}^\perp(f)$. In addition, as shown in Ref. [10], Eq. (42) makes it possible to find $\hat{\jmath}^z(f)$ if $\hat{\jmath}^0(f)$ is known. We conclude that one of the possibilities to construct the ECO satisfying all the above properties is to choose the operators $\hat{\jmath}^0(f)$ and $\hat{\jmath}^\perp(f)$ in the same form as they have in the case of noninteracting particles:

$$\hat{\jmath}^\nu(f) = \jmath^\nu(f) \quad \text{if} \quad \nu \neq z \quad (43)$$

We shall assume that this condition is satisfied.

As noted in the Introduction, there is also a problem, whether the ECO satisfies the locality conditions, and in particular, the equal time commutation relations. Generally speaking, we expect that these conditions are not satisfied. However, as follows from Eqs. (37) and (38),

$$\hat{J}^\mu(0, x)\hat{\jmath}^\nu(0)\varphi(G) = 4e^{\exp(-i\hat{M}(Gx))} \int \hat{M}^{3/2} \hat{J}^\mu(G, G')\hat{\jmath}^\nu(G', G')d\rho(G_{\perp'}, G_{\perp}). \exp\left(i\hat{M}(G'x)\right)$$

$$\int \hat{M}^{3/2} \hat{J}^\mu(G', G')d\rho(G_{\perp}, G'_{\perp})(G'_{\perp}) (44)$$

If the kernel of the ECO is sufficiently smooth, then it follows from Eq. (44) that the strong limit of $\hat{J}^\mu(0, x)\hat{\jmath}^\nu(0)$ is equal to zero if $|x| \to \infty$. Analogously it is easy to see that the same is valid for the strong limit of $\hat{\jmath}^\nu(0)\hat{J}^\mu(0, x)$. As noted in the Introduction, just these conditions should be necessarily satisfied.

The explicit expression for the action of the free two-particle operator $\hat{\jmath}^\nu(f)$ in $H_{int}$ can be found from Eqs. (38-40), since in the case of noninteracting particles the ECO is a sum of the ECO’s for particles 1 and 2 (see
Ref. [10] for more details). The result is (compare with Eqs. (17) and (18))

\[ j^\nu(f)\psi(k, \sigma_1, \sigma_2) = \sum_{i=1}^{2} \sum_{\sigma_i'} \frac{r_i}{2\omega(d_i)} \left[ \frac{M(d_i)}{M(k_i)} \right]^{3/2} \cdot [\tilde{\omega}_i(h_i, \sigma_i)\gamma^\nu w_i(h'_i, \sigma'_i)]\psi(d_i, \sigma'_i, \tilde{\sigma}_i) \]

where

\[ h'_i = L[\beta(f'_+, f'^+)\omega_i(d_i, d_i), \quad h_i = L[\beta(f_+, f^+)\omega_i(k_i, k_i), \quad \tilde{\omega}_i = L[\beta(f'_+, f'^+)\omega_i(k_i, -k_i)] \]

and instead of Eq. (18), the vectors \( d_i \) are defined by the conditions

\[ L[\beta(f'_+, f'^+)\omega_i(d_i) - d_i] = \tilde{L}[\beta(f'_+, f'^+)\omega_i(k_i)] \]

As follows from Eqs. (3) and (47)

\[ f^+ [\tilde{\omega}_i(d_i) - d_i^z] = f^+ [\tilde{\omega}_i(k_i) - k_i^z] \]

and, if \( f_+ = 0 \),

\[ d_\perp = k_\perp, \quad d_i^z = (1 + 2|f|^2)k_i^z - 2f_0 f^z \tilde{\omega}_i(k_i) \]

The problem of constructing the ECO for systems with \( N > 2 \) particles is much more complicated than in the case of two particles since cluster separability imposes considerable restrictions on the choice of the solution (see Ref. [10]). For this reason we shall consider the case when (in the spirit of the parton model) only the N-particle interaction is present while there are no interactions in the subsystems of the system under consideration. In this case the action of the operator \( j^\nu(f) \) in the N-particle internal space \( H_{int} \) can be determined by analogy with Eq. (45):

\[ j^\nu(f)\psi(k_i, \sigma_i, \tilde{i}nt) = \sum_{i=1}^{N} \sum_{\sigma_i'} \frac{r_i}{2\omega(d_i)} \left[ \frac{M(d_i)}{M(k_i, M_i)} \right]^{3/2} \cdot [\tilde{\omega}_i(h_i, \sigma_i)\gamma^\nu w_i(h'_i, \sigma'_i)]\psi(d_i, \sigma'_i, \tilde{i}nt) \]

where \( h_i \) and \( h'_i \) are defined by Eq. (16), \( M_i \) is the free mass of the system 1,...,\( i - 1, i + 1,...,N \), \( d_i \) is defined by Eq. (17) with \( \tilde{\omega}_i(d_i) \) replaced by \( (M_i^2 + d_i^2)^{1/2} \), and for the index \( i \) in the argument of the wave function in the left-hand side we can take any integer from 1 to \( N \). By analogy with the two-particle case, we also assume that the N-particle operator \( j^\nu(f) \) satisfies Eq. (43).
5 Hadronic tensor for the system of \( N \) particles

Let us first consider the problem of calculating the hadronic tensor for the system of two particles. Let \( g_i'' \) (\( i = 1, 2 \)) be the 4-velocities of the particles in the final state. Since the normalization of the states should be the same as in Eq. (9), the wave function of the final state is

\[
|g_1'', \sigma_1'', g_2'', \sigma_2''\rangle = \frac{2}{m_i} \int (2\pi)^3 g_i'' + \delta(2)(g_i'' - g_i') \delta(g_i^+ - g_i'^+) \delta(\sigma_i, \sigma_i') (51)
\]

Let \( \psi' \in H_{\text{int}} \) be the internal wave function of the initial bound state such that \(|\psi'||^2 = 1\), and \( M'\) be the mass of this state. Then \( \hat{M}\psi' = M'\psi' \). If \( P' \) is the 4-momentum of the initial state and \( G' \) is its 4-velocity then \( P' = M'G' \), and, since the normalization should be the same as for the wave function in Eq. (10), we write

\[
|G', \psi'\rangle = \frac{2}{M'} \int (2\pi)^3 G' + \delta(2)(G' - G') \delta(G^+ - G'^+) \psi' (52)
\]

Now the hadronic tensor should be written in the form

\[
W^{\mu\nu} = \frac{1}{4\pi} \sum_{\sigma_1'\sigma_2'} \int (2\pi)^4 \delta(4)(P' + q - p_1'' - p_2'') \langle G', \psi' | \hat{J}^{\mu}(0) | g_1'', \sigma_1''; g_2'', \sigma_2'' \rangle \langle g_1'', \sigma_1'', g_2'', \sigma_2'' | \hat{J}^{\nu}(0) | G', \psi' \rangle d\rho(p_1'', p_1'^+) d\rho(p_2'', p_2'^+) (53)
\]

We again use Eqs. (6), (15) and (16), and define the 4-velocity of the final state as \( G'' = P''/M'' \). As follows from Eqs. (35) and (36), the wave function given by Eq. (52) can be rewritten in the form

\[
|g_1'', \sigma_1'', g_2'', \sigma_2''\rangle = (2\pi)^3 G'' + \delta(2)(G'' - G'') \delta(G^+ - G'^+) \delta(\sigma_1, \sigma_1') \delta(\sigma_2, \sigma_2') \cdot \frac{2}{M(k'')^3} \omega_1(k'') \omega_2(k'') \delta^3(k - k'') (54)
\]

Therefore, as follows from Eqs. (35), (36), (38), (52) and (54),

\[
\langle g_1'', \sigma_1'', g_2'', \sigma_2'' | \hat{J}^{\nu}(0) | G', \psi'(k', \sigma_1, \sigma_2) \rangle = \frac{2}{M' m_1 m_2} (M'' M')^{3/2} \cdot \hat{J}^{\nu}(G'', G') \psi'(k'', \sigma_1'', \sigma_2'') (55)
\]
Since the ECO satisfies relativistic invariance and current conservation, the results for the structure functions do not depend on the reference frame in which $W^{\mu\nu}$ is calculated, and in any frame $q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0$. As follows from Eqs. (39-41), it is convenient to choose the reference frame in which $G'' + G' = 0$, since in this case the Lorentz transformation $L(G'', G')$ is the identity operator, and thus $J^{\nu}(G, G') = j^{\nu}(f)$ with $f = G''$. Let us note that the condition $G'' + G' = 0$ is not the same as the condition $P'' + P' = 0$ defining the Breit frame, since the masses of the initial and final states are different.

We again suppose that $P'_\perp = q_\perp = 0$ and $P'^z > 0$. Therefore $G'_\perp = G''_\perp = 0$, and $G'^z > 0$. If $G'' + G' = 0$ then $f^z = G'^z < 0$, and, since $G'' = (M'G' + q)/M''$, we find that

$$q^0 = (M'' - M')G'^0, \quad q = -(M'' + M')G'$$

As follows from these expressions

$$G'^2 = |f|^2 = \frac{(M'' - M')^2 - q^2}{4M''M'}$$

In the Bjorken limit $M'' \gg M'$, and, since $M'^2 = (P' + q)^2$, we get from Eqs. (16) and (57)

$$|f|^2 = \frac{|q^2|^{1/2}}{4M'[x(1 - x)]^{1/2}}$$

Therefore $|f| \gg 1$ in the Bjorken limit.

Using Eqs. (13), (15), (16) and (59), we obtain that if $\nu \neq z$, then in the reference frame under consideration

$$\langle g_1^\nu, \sigma_1^\nu, g_2^\nu, \sigma_2^\nu | \hat{J}^{\nu}(0)|G', \psi'(k, \sigma_1, \sigma_2) \rangle \sum_{i=1}^{2} \sum_{\sigma'_i} r_i \frac{M(d_i)^{3/2}(M')^{1/2}}{2\omega(d_i)m_1 m_2}$$

$$[\bar{w}_i(h_i^\nu, \sigma_i^\nu)\gamma^\nu w_i(h'_i, \sigma'_i)]\psi(d_i, \sigma'_i, \tilde{\sigma}_i')$$

where $h'_i, h_i^\nu = h_i$ and $d_i$ are defined by Eqs. (16) and (17) with $f'_\perp = f_\perp = 0$, $k_{i\perp} = k_{i\perp}''$.

As follows from Eq. (19), in the reference frame under consideration

$$d_i^z = (1 + 2|f|^2)[k_i^z - \bar{w}_i(k_i^\nu)] - \frac{\bar{w}_i(k_i^\nu)}{4|f|^2}$$

(60)
Since \( \tilde{\omega}_i(k_{i''})/4|f^z|^2 = M'(1-x)/2 \) (see Eqs. (14) and (58)), we conclude that the condition \(|d| \leq m_0\) (see Sec. 2) can be again satisfied only if \(1 - |\cos \theta| \leq m_0^2/|q^2|\) for \(i = 1\) and \(1 + |\cos \theta| \leq m_0^2/|q^2|\) for \(i = 2\). Therefore, the presence of the interaction in the ECO does not change the conclusion that (at least if \(\mu, \nu \neq z\)) the constituents absorb the virtual photon incoherently.

Let us now consider Eq. (48). As follows from Eq. (4), \(\tilde{\omega}_i(d_i) - d_i^z = M(d_i)(1 - \xi_i)\). Taking into account the fact that the free mass operator can also be written as a function of \(d_i^\perp\) and \(\xi_i\), and using Eq. (49), we can write \(M(d_i) = M(k_{i''}, \xi_i)\). We also take into account that in the reference frame under consideration

\[
f^+ = \sqrt{2}|f^z|, \quad f^+ = \frac{1}{2\sqrt{2}|f^z|}
\]

Therefore, using Eqs. (14) and (58) we get the final result

\[
M(k_{i''}, \xi_i)(1 - \xi_i) = M'(1-x)
\]  

(62)

where, as can be shown from Eq. (4), the explicit expression for \(M(k_{i''}, \xi_i)\) is

\[
M(k_{i''}, \xi_i) = \left[\frac{m_1^2}{\xi_1} + \frac{m_2^2}{\xi_2} + \frac{k_{i''}^2}{\xi_1\xi_2}\right]^{1/2}
\]  

(63)

We see that the equality \(\xi_i = x\) takes place only if one neglects the difference between the free mass and the mass of the bound state. This equality was obtained from Eq. (18) while the relation (62) was obtained from Eq. (47) at \(k_{i''} = k_i\). Since in the reference frame under consideration \(P' = M'f^+\), \(P'' = M''f\), we can rewrite Eq. (47) in the form

\[
L[\beta(P', M')]\tilde{\omega}_i(d_i, -d_i) = L[\beta(P'', M'')\tilde{\omega}_i(k_{i''}, -k_{i''})]
\]  

(64)

Therefore, instead of the free mass \(M(d_i)\) in Eq. (18), the mass of the initial bound state \(M'\) enters into Eq. (64). This explains the result given by Eq. (12).

An analogous effect (called \(x\)-rescaling) was observed by the authors investigating the original EMC effect [3], and in Sec. 6 we discuss the difference between our results and those in Refs. [7, 8, 9].

Equation (64) has the clear physical meaning. Indeed, the Lorentz boost in the left-hand side is the real physical boost since \(P'\) and \(M'\) are the real
4-momentum and the mass which has the initial state. Let the virtual photon be absorbed by particle 1. Then the left-hand side of Eq. (64) \( h'_2 \) has the meaning of the momentum of particle 2 in the initial state since the 4-momentum of this particle in the c.m. frame of the initial state is \((\omega_2(d_1), -d_1)\). Analogously, the right-hand side of Eq. (64) \( h''_2 \) has the meaning of the momentum of particle 2 in the final state. Thus Eq. (64) tells that the momentum of particle 2 does not change. As follows from Eqs. (6) and (47), \( h'_1 = p'_1 \) and \( h''_2 = p''_2 \) since in the final state the two-particle system is free. At the same time, the quantities \( h'_1 \) and \( h''_2 \) are not equal to the free momenta \( p'_1 \) and \( p''_2 \) in the initial state since \( h'_1 \) and \( h''_2 \) are defined by the Lorentz boost depending on the physical mass \( M' \) while \( p'_1 \) and \( p''_2 \) are defined by the Lorentz boost depending on the free mass \( M(d_1) \). Meanwhile, Eq. (18) just tells that \( p'_2 = p''_2 \) as it should be from the definition of the impulse approximation. We conclude that since in the presence of the interaction the quantity \( h'_2 \) can be interpreted as the physical 4-momentum of particle 2 in the initial state, while \( p'_2 \) no longer can be interpreted in such a way, Eq. (64) is reasonable while Eq. (18) is not.

Since we wish to compare the results with those obtained in the impulse approximation, we note that, as follows from Eqs. (6) and (36), \( \psi'(d, \sigma_1, \sigma_2) = \frac{m_1 m_2}{M(d)} \chi'(d, \sigma_1, \sigma_2) \) (65)

Then a simple calculation using Eqs. (4), (15), (16), (46), (53), (58-60) shows that in the reference frame under consideration the components of the tensor \( W^{\mu \nu} \) with \( \mu, \nu \neq z \) are given by

\[
W^{\mu \nu} = \sum_{i=1}^{2} r_i^2 \int \langle \chi'(k_{i \perp}, \xi_i)|S_{i \mu}^{\nu}|\chi'(k_{i \perp}, \xi_i)\rangle \left[ 1 + \frac{k_i^2}{\omega_i(k)} \right]^2 \frac{d^2 k_{i \perp}}{(2\pi)^3 \xi_i (1 - x)} \] (66)

where, as follows from Eqs. (62) and (63), \( \xi_i \) is a function of \( k_{i \perp} \) and \( x \) which should be defined from the condition

\[
\left[ \frac{m_i^2}{\xi_i} + \frac{\tilde{m}_i^2}{1 - \xi_i} + \frac{k_i^2}{\xi_i (1 - \xi_i)} \right]^{1/2} (1 - \xi_i) = M'(1 - x) \] (67)

By analogy with the calculations in Sec. 3, we can easily generalize the above calculations to the case of \( N \) particles in the model considered in Sec.
where $\hat{j}(\mathbf{f})$ is given by Eqs. (34) and (33). In the reference frame where $\mathbf{P}_\perp = \mathbf{P}_\perp = 0$, $P'_z > 0$ and $\mathbf{G'} + \mathbf{G}'' = 0$,

$$W^{\mu\nu} = \sum_{i=1}^{N} r_i^2 \int \langle \chi'(|k_{i \perp}, \xi_i, \tilde{\mathbf{y}})|S^{\mu\nu}_i|\chi'(|k_{i \perp}, \xi_i, \tilde{\mathbf{y}})\rangle \cdot$$

$$[1 + \frac{k_i^z}{\omega_i(k)}]^2 \frac{d^2k_{i \perp} d\rho(\tilde{\mathbf{y}})}{4(2\pi)^3 \xi_i (1 - x)}$$

(68)

where $\xi_i$ is a function of $k_{i \perp}, M_i, x$ defined by the equations

$$M(k_{i \perp}, \xi_i, M_i)(1 - \xi_i) = M'(1 - x),$$

$$M(k_{i \perp}, \xi_i, M_i) = [\frac{m_i^2}{\xi_i} + \frac{M_i^2}{1 - \xi_i} + \frac{k_i^z}{\xi_i (1 - \xi_i)}]^{1/2}$$

(69)

and $k_i^z$ is a function of $k_{i \perp}, M_i, x$ defined by Eq. (23).

It is easy to see that the explicit expression for $\xi_i$ is

$$\xi_i = \frac{m_i^2 + k_i^z}{m_i^2 + k_i^z + M_i^2 (1 - x)^2} \quad \text{if} \quad m_i = M_i,$$

$$\xi_i = \frac{1}{2} \left(1 - \alpha_i - \beta_i + [(1 - \alpha_i - \beta_i)^2 + 4\alpha_i]^{1/2}\right) \quad \text{if} \quad M_i > m_i,$$

$$\xi_i = \frac{1}{2} \left(1 - \alpha_i - \beta_i - [(1 - \alpha_i - \beta_i)^2 + 4\alpha_i]^{1/2}\right) \quad \text{if} \quad M_i < m_i$$

(70)

where

$$\alpha_i = \frac{m_i^2 + k_i^z}{M_i^2 - m_i^2}, \quad \beta_i = \frac{M_i^2 (1 - x)^2}{M_i^2 - m_i^2}$$

(71)

Since $x \in [0, 1]$, it follows from Eq. (70) that $\xi_i \in [\xi_i^{\min}, 1]$ where $\xi_i^{\min} = \xi_i^{\min}(k_{i \perp}, M_i)$ is a function of $k_{i \perp}, M_i$ which can be defined from Eq. (71) at $x = 0$. It is easy to see that $0 < \xi_i^{\min} < 1$.

As follows from Eqs. (14) and (68), the scaling and the Callan-Gross relation [11] also take place if the interaction in the ECO is taken into account since

$$F_1(x) = \sum_{i=1}^{N} r_i^2 \sum_{\sigma_i} \int \langle \chi'(|k_{i \perp}, \xi_i, \sigma_i, \tilde{\mathbf{y}})|S^{\mu\nu}_i|\chi'(|k_{i \perp}, \xi_i, \sigma_i, \tilde{\mathbf{y}})\rangle \cdot$$

$$\frac{k_i^z}{\omega_i(k)} \frac{d^2k_{i \perp} d\rho(\tilde{\mathbf{y}})}{4(2\pi)^3 \xi_i (1 - x)},$$

$$F_2(x) = 2xF_1(x)$$

(72)
6 Discussion

Though there exist a vast literature devoted to the parton model, only a few authors investigated the problem, what explicit physical conditions should be satisfied for the validity of this model (see, for example, Refs. [12, 13, 14, 28]). The results of Secs. 2 and 3 show that the impulse approximation is the sufficient condition for ensuring the validity of the parton model in the Bjorken limit (in agreement with the above references). It is obvious from Eqs. (3) and (22) that the quantity $\xi_i$ is indeed the fraction of the total momentum in the IMF which has the particle interacting with the virtual photon, and the results show that indeed $\xi_i = x$ in the Bjorken limit.

It is easy to show that if the point-like particle $i$ with the spin 1/2 and the initial 4-momentum $P'_i$ absorbs the virtual photon with the 4-momentum $q$, then $F_{2i}(x/\xi_i) = \delta(x/\xi_i - 1)$, and therefore, as follows from Eq. (30),

$$F_{2i}(x) = \sum_{i=1}^N r_i^2 \int_0^1 \frac{F_{2i}(x/\xi_i)}{\xi_i} \rho_i(x/\xi_i) d\xi_i$$

in full agreement with the interpretation of the function $\rho_i(\xi_i)$ (see Sec. 3) and with the expression used by many authors.

The general expression for the hadronic tensor is

$$W^{\mu\nu} = \frac{1}{4\pi} \sum_n (2\pi)^4 \delta^4(P' + q - P_n) \langle P', \chi'| J^\mu(0) |n\rangle \langle n| J^\nu(0) |P', \chi'\rangle$$

(74)

where a sum is taken over all possible intermediate states $|n\rangle$, and $P_n$ is the 4-momentum of the state $|n\rangle$. It is well-known that using Eq. (37) and the completeness of the states $|n\rangle$, it is easy to transform Eq. (74) to the form

$$W^{\mu\nu} = \frac{1}{4\pi} \int e^{qx} \langle P', \chi'| J^\mu(x) J^\nu(0) |P', \chi'\rangle d^4x$$

(75)

The product of the ECO’s in this expression can also be replaced by the commutator since the second term in the commutator does not contribute to the integral.

The usual argument in favor of the impulse approximation is that since at large $q$ only the region of small $x$ contributes to the integral (see, for example, Ref. [29]), the asymptotic freedom guarantees that $J^\mu(x)$ can be replaced by the free ECO $J^\mu(x)$ with a good accuracy. Then using again the
completeness of the states $|n\rangle$, we obtain that the hadronic tensor will be replaced by the following expression

$$W^{\mu\nu} \to \frac{1}{4\pi} \sum_n (2\pi)^4 \delta^{(4)}(P'(0) + q - P_n^{(0)}) \langle P', \chi' | J^{\mu}(0) | n \rangle \langle n | J^{\nu}(0) | P', \chi' \rangle$$  \hspace{1cm} (76)$$

Here $P'(0)$ and $P_n^{(0)}$ are not the real 4-momenta in the initial state and in the state $|n\rangle$, but the total 4-momenta of the free constituents comprising these states.

It is clear from the considerations in Secs. 2 and 5, that the replacement $P_n \to P_n^{(0)}$ is reasonable at least in some cases. The argument in favor of the replacement $P' \to P'^{(0)}$ is also well-known: the quantum theory on the light cone is such that the $\perp$ and $+$ components of the vectors $P'$ and $P'^{(0)}$ are equal to each other and only the "minus" components differ, but these components are equal to zero in the IMF. Therefore, as far as the $x$ dependence of $\hat{J}^{\mu}(x)$ is concerned, the effect of binding is indeed negligible. At the same time we do not see the reason why $\hat{J}^{\mu}(0)$ can be replaced by $\hat{J}^{\mu}(0)$, and this is just the impulse approximation. If we expand $\hat{J}^{\mu}(0)$ in powers of $\alpha_s$, the same should be done with the initial state, but the perturbation theory cannot be used in this case.

In our opinion, the crucial point in understanding the situation is that not only the 4-momentum of the initial state, but also the mass of this state enter into the calculations. Therefore we cannot confine ourselves to the consideration of only the quark absorbing the virtual photon, and the large distances necessarily come into play. The impulse approximation unambiguously leads to the prescription that for the mass of the initial state we should take not the physical mass $M'$, but the mass of the system of free constituents, i.e. the nonphysical quantity. In contrast with the case of the "minus" components of the vectors $P'$ and $P'^{(0)}$ in the IMF, we cannot make the difference between the above masses negligible.

In our calculations in Sec. 5 we used the ECO which satisfies relativistic invariance and current conservation, but, as noted above, these conditions are not sufficient for choosing a unique solution. Nevertheless the ECO under consideration unambiguously leads to the prescription that for the mass of the initial state we should take its physical value $M'$. Therefore, we believe, that though our result (68) for the hadronic tensor is model-dependent, the relation between $\xi_i$ and $x$ given by Eqs. (67), (69-71) does not depend on the choice of the solution for the ECO if the quarks absorb the virtual photon.
incoherently. Indeed, as explained in Sec. 3, these expressions are only the consequence of relativistic kinematics. For this reason we expect that in the general case the relations (69-71) will be valid if \( M_i \) is replaced by the mass operator \( \hat{M}_i \) of the subsystem \( 1,\ldots,i-1,i+1,\ldots,N \). However, to prove this statement it is necessary to construct the ECO in the case when the interactions in the subsystems of the system under consideration are present.

In the literature the impulse approximation is often associated with the Feynman diagrams in which the virtual photon interacts with only one constituent, while the other constituents are spectators. Let us note however that each Feynman diagram can be unambiguously calculated only if the underlying dynamics is known (for both the interaction between the constituents and the ECO satisfying Eqs. (31-33)). Meanwhile usually this is not the case, and the Feynman diagrams are calculated using some prescriptions. Our solution for the ECO unambiguously leads to Eq. (47) which shows that for the particle which does not interact with the virtual photon \( h_{\text{initial}} = p_{\text{final}} \) (see Sec. 3). This looks like the impulse approximation. However, as explained in Sec. 3, the quantity \( h_{\text{initial}} \) is not equal to the free 4-momentum \( p_{\text{initial}} \). The difference between these quantities cannot be described in the perturbation theory. So it is not clear what is the interpretation of our result on the language of Feynman diagrams.

As follows from Eq. (69), the relation \( \xi_i = x \) takes place only in the nonrelativistic approximation. Therefore we should expect that in the real nucleon \( \xi_i \) considerably differs from \( x \). Thus, in contrast with Eq. (30), for determining the structure functions it is necessary to know not only the functions \( \rho_i(\xi_i) \), but also the dependence of the internal wave function on the transverse momenta. As the result, the DIS data do not make it possible to determine the \( \xi_i \) distribution of quarks in the nucleon if there are no additional experimental information. We can also expect that the sum rules which are based only on the parton model are not reliable.

Let us consider, for example, the Gottfried sum rule (30), according to which the quantity

\[
S_G = \int [F_{2p}(x) - F_{2n}(x)] \frac{dx}{x}
\]

is equal to 1/3. Here \( F_{2p}(x) \) and \( F_{2n}(x) \) are the structure functions for the proton and neutron respectively. This sum rule easily follows from Eqs. (24), (29), (30) and the condition \( ||\chi'|| = 1 \) if we assume that the neutron wave function can be obtained from the proton one if one of the \( u \) quarks in the
proton is replaced by the $d$ quark (though some authors argue that this is not the case). We suppose that particle 1 in the proton is the $u$ quark, particle 1 in the neutron is the $d$ quark and all other particles are the same. Then, as follows from Eq. (72)

$$S_G = \frac{1}{3} \sum \sigma \int \frac{|\chi'(k_{1\perp}, \xi_1, \sigma, \tilde{t})|^2}{\omega_1(k_1)} \left[ 1 + \frac{k_{1\perp}^2}{\omega_1(k_1)} \right] \frac{d^2 k_{1\perp} \, dp(\tilde{t})}{2(2\pi)^3 \xi_1(1-x)}$$  \hspace{1cm} (78)

where $\xi_1$ is a function of $k_{1\perp}, M_1, x$ defined by Eq. (70). Now using Eqs. (23) and (69-71) we change the integration variable from $x$ to $\xi_1$. Then it is easy to show that

$$S_G = \frac{1}{3} \sum \sigma \int \frac{d^2 k_{1\perp} \, dp(\tilde{t})}{(2\pi)^3} \int_{\xi_{1\min}}^1 \frac{d\xi_1}{2\xi_1(1-\xi_1)} \cdot |\chi'(k_{1\perp}, \xi_1, \sigma, \tilde{t})|^2 \left[ 1 + \frac{k_{1\perp}^2}{\omega_1(k_1)} \right]$$  \hspace{1cm} (79)

It is not clear what is the effect of the last multiplier in the integrand since both $k_{1\perp}^x > 0$ and $k_{1\perp}^z < 0$ are possible. However in the general case $\xi_{1\min}$ can considerably differ from zero. Therefore, comparing Eq. (79) with the normalization integral (24), it is natural to expect that $S_G < 1/3$. Recently the quantity $S_G$ was calculated in Ref. [31] using the data of Ref. [32], and the result was $S_G = 0.235 \pm 0.026$.

Analogously, the DIS data only do not make it possible to determine the contributions of the $u$, $d$ and $s$ quarks to the nucleon spin, and the well-known problem of the "spin crisis" does not arise (the present status of this problem is described, for example, in Refs. [33, 34]). Indeed, these contributions (usually denoted as $\Delta q = (\Delta u, \Delta d, \Delta s)$) are given by some integrals over $\xi_i \in [0, 1]$. Meanwhile, the DIS data make it possible to calculate some integrals over $x \in [0, 1]$. Since the integrals over $x$ can be transformed to the integrals over $\xi_i \in [\xi_{i\min}, 1]$, we see that the DIS data do not make it possible to determine the contributions of $\xi_i \in [0, \xi_{i\min}]$. Thus it is natural to expect that the parton model underestimates the quantities $\Delta q$.

At the same time, the DIS experiments make it possible to check the well-known results which are not based on the parton model (for example, the Bjorken sum rules [35]).

In conclusion we compare our results with those obtained by several authors investigating the original EMC effect [4]. This is possible in the formal case when the nucleons are point-like.
The first calculations of the EMC effect were carried out in Refs. [7] and others. However, as shown in Ref. [8], the above works did not take into account the "flux factor" (see also Refs. [9] and many others). The flux factor used in these references is equal to 

$$z_i = A(\omega_i(k_i) - k_i^z) / M_A$$

where \(A\) is the mass number of the nucleus under consideration and \(M_A\) is its mass (equal to \(M'\) in our notations).

If we work in the impulse approximation then Eq. (73) is the convolution formula for \(F_2(x)\). Since \(x/\xi_i = Ax/A\xi_i\), we conclude that the result for the flux parameter in the impulse approximation is \(A\xi_i\) where \(\xi_i\) is given by Eq. (23). Note however, that since the ECO in the impulse approximation is not relativistically invariant, the result depends on the reference frame and on the form of dynamics (see Ref. [36] for more details). We derived our result in the IMF while the EMC effect is usually considered in the reference frame in which the initial nucleus is at rest.

Let us now consider the case when the interaction in the ECO is taken into account. Then if \(N = 2\), our result which follows from Eq. (63) is \(\tilde{\omega}_i(k_i) - k_i^z = M'(1 - x)\) while in the approach of Refs. [8, 9] the relation \(\omega_i(k_i) - k_i^z = M'x\) takes place. If \(N\) is arbitrary, then our result (69) can be written as \((M_i^2 + k_i^2)^{1/2} - k_i^z = M'(1 - x)\) while the result of the above references is again \(\omega_i(k_i) - k_i^z = M'x\).

Above we have argued that Eqs. (62) and (69) are in fact kinematical. In addition, it is easy to see that the expression for \(F_2(x)\) in Eq. (72) cannot be written as the one-dimensional convolution formula. This can be expected in any model in which the interaction in the ECO is taken into account (see also Refs. [4, 77]).

These considerations show that the interpretation of the original EMC effect [6] has to be revisited. We suppose to consider this problem elsewhere.

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