\( NK\pi \) molecular state with \( J^\pi = \frac{3}{2}^- \) and \( I = 1 \)

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Abstract

The structure of the moleculelike state of \( NK\pi \) with spin-parity \( J^\pi = \frac{3}{2}^- \) and isospin \( I = 1 \) is studied within the chiral SU(3) quark model. First we calculate the \( NK \), \( N\pi \), and \( K\pi \) phase shifts in the framework of the resonating group method (RGM), and a qualitative agreement with the experimental data is obtained. Then we perform a rough estimation for the energy of \( (NK\pi)_{J^\pi = \frac{3}{2}^-, I = 1} \), and the effect of the mixing to the configuration \( (\Delta K)_{J^\pi = \frac{3}{2}^-, I = 1} \) is also considered. The calculated energy is very close to the threshold of the \( NK\pi \) system. A detailed investigation is worth doing in the further study.

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For the past few years, many low-energy baryonic resonances have been explained as baryon-meson molecule resonance states within the chiral unitary approach [1]. To understand this kind of mechanism on the quark level and to compare further the similarities and the differences between the results obtained on the hadron level and those on the quark level is very significant. Recently we have extended our chiral SU(3) quark model from the study of baryon-baryon scattering processes to the baryon-meson systems by solving the resonating group method (RGM) equation [2, 3, 4]. We found that some results are similar to those given by the chiral unitary approach study, such as that both the $\Delta K$ system with isospin $I = 1$ and the $\Sigma K$ system with $I = \frac{1}{2}$ have quite strong attractions. We also studied the phase shifts of $\pi N$ and $\pi K$, and got reasonable fit with the experiments in the low energy region [5]. Because our calculation is on the quark level, the parameters can be fixed by the baryon masses and the $NN$ (or $KN$) scattering, and thus the free parameters are largely reduced. These encouraged us to investigate more baryon-meson systems by use of the same group of parameters.

The most interesting case is the system with strangeness $S = +1$, this is because for this five-quark system there is no annihilation to both gluons and vacuum. At the same time, since 2003, several experimental collaborations reported that they observed a new resonance $\Theta^+$, with positive charge and strangeness quantum number $S = +1$ [6]. The mass of this particle is around 1540 MeV and its width is very narrow, $\Gamma_{\Theta} < 25$ MeV. But recently the situation has become more complicated. Several high statistical $\gamma p$ experiments showed their negative results [7], and, conversely, the RHIC-STAR group reported the observation of a resonance state with +2 charges and strangeness $S = +1$, named $\Theta^{++}$, and its mass is around 1530 MeV [8]. Although the experimental result is unclear, the theoretical investigation of the systems with strangeness $S = +1$ from different mechanism to understand if it is possible to have resonance states is still alluring.

In Refs. [2, 3], we studied the $KN$ scattering phase shifts in the chiral SU(3) quark model and further in the extended chiral SU(3) quark model where the vector chiral fields are included by performing a RGM calculation, and we obtained considerably good agreement with the experimental data for the $S$, $P$, $D$, and $F$ partial waves. From these works, we found that there is neither $S$ nor $P$ wave resonance in $KN$ scattering for both isospin $I = 0$ and $I = 1$ cases. This means that the dynamical calculation in the chiral quark model can explain the $KN$ phase shifts, but cannot obtain a resonance state.
We also noticed that though the interactions of $KN S$ wave with $I = 0$ and $I = 1$ are repulsive, the phase shifts of $N\pi P$ wave with $I = \frac{3}{2}$ and of $K\pi S$ and $P$ waves with $I = \frac{1}{2}$ show their interactions are attractive. This means that probably the pion can be a medium to glue a kaon and a nucleon together for some states with suitable quantum numbers.

In Ref. [9], the possibility of the $\Theta$ particle to be a $NK\pi$ state with $J^\pi = 1/2^+$ and $I = 0$ has been studied. But we notice that the $N\pi P$-wave state with isospin $\frac{3}{2}$ and total angular momentum $\frac{3}{2}$ has a strong attraction, and the isospin $\frac{1}{2} K\pi S$- and $P$-wave scattering phase shifts also show that their interactions are attractive. These encourage us to study the possibility that there will be a $NK\pi$ bound state near its threshold with special quantum numbers because of the comparatively strong attractive interaction of $N\pi$ and $K\pi$. In this work, we suggest that the state of $NK\pi$ with $J^\pi = \frac{3}{2}^-$ and $I = 1$ could be a very interesting state. We estimate the energy of the $NK\pi$ three-body system, where the interactions of $KN$, $K\pi$ and $N\pi$ are obtained from the chiral SU(3) quark model. Further the configuration mixing between $(NK\pi)_{J^\pi=\frac{3}{2}^-, I=1}$ and $(\Delta K)_{J^\pi=\frac{3}{2}^-, I=1}$ is also considered. Although the calculation is only a rough estimate, the result is quite interesting. It shows that when the configuration mixing is included, the energy of the system is quite near the threshold of $NK\pi$ (1572 MeV). This gives us a hint that the system of $(NK\pi)_{J^\pi=\frac{3}{2}^-, I=1}$ could be a moleculelike bound state. We also notice that if the system of $(NK\pi)_{J^\pi=\frac{3}{2}^-, I=1}$ is really bound, its width must be very narrow, because when and only when the pion of the system is absorbed by the nucleon, can the decay process to a $K$ and a $N$ occur. All these features show that the $(NK\pi)_{J^\pi=\frac{3}{2}^-, I=1}$ state is an interesting state, and the possibility of the $\Theta$ particle to be the $(NK\pi)_{J^\pi=\frac{3}{2}^-, I=1}$ state cannot be excluded if the $\Theta$ particle does really exist.

Because the state $(N\pi)_{l=1,s=\frac{1}{2},j=\frac{3}{2},t=\frac{1}{2}}$ has a quite strong attraction, we thus start from the coupling of $(N\pi)_{l=1,s=\frac{1}{2},j=\frac{3}{2},t=\frac{1}{2}}$ with a kaon to construct the wave function of the $NK\pi$ system. When the relative motion wave function between the $(N\pi)_{l=1,s=\frac{1}{2},j=\frac{3}{2},t=\frac{1}{2}}$ and the kaon is taken to be $S$ wave, there are only two possible states: isospin $I = 1$ and 2. A simple analysis shows that the state with $I = 1$ is more favorable. Therefore we suppose the $NK\pi$ system with $J^\pi = \frac{3}{2}^-$ and isospin $I = 1$ could be a molecular state.

The wave function of the $NK\pi$ system with $J^\pi = \frac{3}{2}^-$ and $I = 1$ can be written simply
as follows:

\[ \Psi_{NK\pi} = \left( (N\pi)_{l=1,s=\frac{1}{2},j=\frac{1}{2},t=\frac{3}{2},K} \right)_{L=1,S=\frac{1}{2},J=\frac{3}{2},I=1}, \]  

(1)

where \( l \) is the orbital angular momentum of \( N\pi \) relative motion, and \( s \) and \( t \) represent the spin and isospin of \( N\pi \), respectively. \( L \) and \( I \) are the total orbital angular momentum and isospin of the \( NK\pi \) system. When the orbital wave function is taken to be the harmonic oscillator wave function, \( \Psi_{NK\pi} \) can be expressed as follows:

\[ \Psi_{NK\pi} = \left\{ \begin{array}{ll}
\sqrt{M_N} \phi_s(\vec{r}_N)\phi_p(\vec{r}_\pi) - \sqrt{M_\pi} \phi_p(\vec{r}_N)\phi_s(\vec{r}_\pi) \\
\phi_s(\vec{r}_K)
\end{array} \right\}_{l=1,s=\frac{1}{2},t=\frac{3}{2},J=\frac{3}{2},I=1}. \]  

(2)

Performing a re-coupling expression of \( \Psi_{NK\pi} \), one obtains

\[ \Psi_{NK\pi} = \left\{ \begin{array}{ll}
N \sqrt{M_N M_{NK}} \left( \sqrt{\frac{8}{9}} (K\pi)_{l=0,t=\frac{1}{2}} + \sqrt{\frac{1}{9}} (K\pi)_{l=0,t=\frac{3}{2}} \right) \\
+ \sqrt{M_N M_K} \sqrt{M_{NK}} \left( \sqrt{\frac{8}{9}} (K\pi)_{l=1,t=\frac{1}{2}} + \sqrt{\frac{1}{9}} (K\pi)_{l=1,t=\frac{3}{2}} \right) \\
\right\}_{J=\frac{3}{2},I=1}, \]  

(3)

and

\[ \Psi_{NK\pi} = \left\{ \begin{array}{ll}
\pi \sqrt{M_N M_{NK}} \left( \sqrt{\frac{2}{3}} (NK)_{l=0,t=0} + \sqrt{\frac{1}{3}} (NK)_{l=0,t=1} \right) \\
+ \sqrt{M_N M_K} \sqrt{M_{NK}} \left( \sqrt{\frac{2}{3}} (NK)_{l=1,t=0} + \sqrt{\frac{1}{3}} (NK)_{l=1,t=1} \right) \\
\right\}_{J=\frac{3}{2},I=1}. \]  

(4)

Here

\[ M_{N\pi} = M_N + M_\pi, \quad M_{NK} = M_K + M_\pi, \]
\[ M_{NK} = M_N + M_K, \quad M = M_N + M_K + M_\pi. \]  

(5)

Once the interactions of \( NK \), \( N\pi \) and \( K\pi \) are obtained, the energy of \( (NK\pi)_{J^\pi=\frac{3}{2},I=1} \) can be estimated by using the above wave functions.

To get the interactions of \( NK \), \( N\pi \) and \( K\pi \), we draw support from the chiral SU(3) quark model that has been quite successful in explaining the \( NN \), \(YN \) and \( KN \) scattering data \[2, 3, 10\]. The effective potentials of \( NK \), \( N\pi \) and \( K\pi \) can be extracted from the phase shift fitting calculations. Here we briefly introduce the chiral SU(3) quark model and show the phase shifts of \( NK \), \( N\pi \), and \( K\pi \) in the low energy region, and then we give a rough estimate for the energy of the system \( (NK\pi)_{J^\pi=\frac{3}{2},I=1} \).
In the chiral SU(3) quark model, we introduce the coupling between quarks and chiral fields to describe the low-momentum medium-range nonperturbative quantum chromodynamics (QCD) effect. The interacting Lagrangian $\mathcal{L}_I$ can be written as follows:

$$\mathcal{L}_I = -g_{ch}\bar{\psi}(\sum_{a=0}^{8}\sigma_a\lambda_a + i\sum_{a=0}^{8}\pi_a\lambda_a\gamma_5)\psi.$$  \hfill(6)

Here scalar nonet fields $\sigma_a$ and pseudoscalar nonet fields $\pi_a$ are all included. $g_{ch}$ is the coupling constant of chiral fields, its value is determined by the relation

$$\frac{g_{ch}^2}{4\pi} = \frac{9}{25}\frac{m_u^2 g_{NN\pi}^2}{M_N^2 4\pi},$$  \hfill(7)

and $g_{NN\pi}$ is taken to be the experimental value.

In this model, we also employ an effective OGE interaction to govern the short range behavior and a confinement potential to provide the nonperturbative QCD effect in the long distance. Therefore the total Hamiltonian of the system can be written as follows:

$$H = \sum_i T_i - T_G + \sum_{i<j}[V_{qq}(ij) + V_{q\bar{q}}(ij)].$$  \hfill(8)

The interaction between two quarks is expressed as follows:

$$V_{qq}(ij) = V_{ij}^{OGE} + V_{ij}^{conf} + V_{ij}^{ch},$$  \hfill(9)

and the interaction between quark and antiquark has two parts: direct interaction and annihilation parts,

$$V_{q\bar{q}}(ij) = V_{q\bar{q}}^{dir} + V_{q\bar{q}}^{ann},$$  \hfill(10)

with

$$V_{q\bar{q}}^{dir} = V_{q\bar{q}}^{conf} + V_{q\bar{q}}^{OGE} + V_{q\bar{q}}^{ch},$$  \hfill(11)

and

$$V_{q\bar{q}}^{ch} = \sum_j (-1)^{G_j}V_{q\bar{q}}^{ch,j}.$$  \hfill(12)

Here $(-1)^{G_j}$ represents the G parity of the $j$th meson. The confinement potential is taken as the quadratic form. The chiral field coupling interaction includes two parts: scalar part and pseudoscalar part. Their expressions and corresponding parameters can be found in Refs. [2, 3].
The annihilation part is complicated. The \( NK \) system is the simplest one, because \( u(d)\bar{s} \) can annihilate neither to gluons nor to vacuum. From the \( NK \) scattering calculation where the annihilation of \( u(d)\bar{s} \) into a kaon meson has been considered [2, 3], we found that the annihilation interaction is unimportant in the low energy scattering processes. For the system \( NK\pi \), when we treat it as a molecule-like state, the distance between two particles is comparatively long, thus the relative momenta of \( NK, N\pi \) and \( K\pi \) would be in the low energy region. In this sense, we can neglect the annihilation part to get the approximate effective interaction of \( NK, N\pi \) and \( K\pi \) by fitting their low energy scattering phase shifts.

![Graph](image1)

**FIG. 1:** \( KN \) \( S \)-wave phase shifts as a function of the laboratory momentum of kaon meson. The first subscript denotes the isospin quantum number and the second one twice of the total angular momentum of the \( KN \) system. The hole circles and the triangles correspond respectively to the phase shifts analysis of Hyslop et al. [11] and Hashimoto [12].

![Graph](image2)

**FIG. 2:** \( K\pi \) phase shifts as a function of the energy of center of mass motion. The subscript denotes the orbit angular momentum of the \( K\pi \) relative motion and the superscript the isospin of the \( K\pi \) system. The hole circles and the triangles correspond respectively to the phase shifts analysis of Mercer et al. [13] and Estabrooks et al. [14].
Figs. 1-3 show the phase shifts of $KN$, $K\pi$, and $N\pi$. One can see that in the low energy region the calculated phase shifts are almost consistent with the experimental data.

Although the interactions of $KN$ $S$ wave are repulsive in both isospin $I = 0$ and $I = 1$ channels, we notice that in the system $(NK\pi)_{J^P = \frac{3}{2}^-, I = 1}$, there are 1 pair of $N\pi$ in the $P_{33}$ channel, 0.60 pairs of $K\pi$ in $P$ wave with $I = \frac{1}{2}$, and 0.28 pairs of $K\pi$ in $S$ wave with $I = \frac{1}{2}$. The calculated phase shifts indicate that all these interactions are attractive. This shows that the $\pi$ plays a very important role to glue the $K$ and the $N$ together, and as a consequence, the energy of the state $(NK\pi)_{J^P = \frac{3}{2}^-, I = 1}$ can be very close to its threshold.

We perform a rough estimate of the energy of the $(NK\pi)_{J^P = \frac{3}{2}^-, I = 1}$ system. The radial part of the wave function is taken to be the harmonic oscillator form. In this framework, the kinetic energy of the system is $2\omega$, where $\omega$ is the harmonic oscillator frequency of the system. In our calculation, we treat it as a parameter. Substituting $M_N$, $M_K$, and $M_\pi$ by their experimental values in Eqs. (1)-(4), we can obtain that in the $(NK\pi)_{J^P = \frac{3}{2}^-, I = 1}$ system there are:

1 pair of $(N\pi)_{l=1,j=\frac{3}{2},t=\frac{1}{2}}$, attraction
0.28 pairs of $(K\pi)_{l=0,j=0,t=\frac{1}{2}}$, attraction
0.60 pairs of $(K\pi)_{l=1,j=1,t=\frac{1}{2}}$, attraction
0.64 pairs of $(NK)_{l=0,j=\frac{1}{2},t=0}$, repulsion
0.32 pairs of $(NK)_{l=0,j=\frac{1}{2},t=1}$, repulsion (13)
Where it can be seen from the phase shifts whether the interaction is attractive or repulsive. When the frequency of the harmonic oscillator, $\omega$, is taken to be around several tens MeV, the calculated energy of $(NK\pi)_{J^*=\frac{3}{2}^-,I=1}$ is about $70-80$ MeV higher than the threshold of $NK\pi$. We emphasize again that this is just a very rough estimation, because the harmonic oscillator wave function cannot offer an exact description of this three-body system. And as is well known that the energy of the system obtained from more accurate solution must be lower than that from the estimation for a same Hamiltonian.

Because for the $(N\pi)_{J^T=\frac{3}{2}^+}$ system the effect from the $\Delta$ resonance state is very important, we would now like to consider the coupling of the configuration $(\Delta K)_{J^*=\frac{3}{2}^-,I=1}$ to the system $(NK\pi)_{J^*=\frac{3}{2}^-,I=1}$. First, we notice that the $\Delta K$ interaction is attractive in the isospin one channel $\left[\frac{3}{2},\frac{1}{2}\right]$, and it’s energy can be taken to be the value near the $\Delta K$ threshold $\left[\frac{3}{2},\frac{1}{2}\right]$. Then we calculate the matrix element of the $\Delta$ and $N\pi$ interacting vertex

$$\sqrt{4\pi}\frac{f_{\Delta N\pi}}{m_{\pi}}(\bar{\sigma}_{\Delta N} \cdot \vec{q})(\bar{\tau}_{\Delta N} \cdot \vec{\phi}). \quad (14)$$

For simplicity, we also adopt the harmonic oscillator wave function in the calculation. When the size parameter $b$ is chosen to be around $0.8-1.0$ fm and the coupling constant $f_{\Delta N\pi}^2$ is taken to be 0.29, we get

$$<\Delta | \sqrt{4\pi}\frac{f_{\Delta N\pi}}{m_{\pi}}(\bar{\sigma}_{\Delta N} \cdot \vec{q})(\bar{\tau}_{\Delta N} \cdot \vec{\phi}) | (N\pi)_{IJ=\frac{3}{2}^+,\frac{1}{2}^-}> \approx 30-50 \text{ MeV}. \quad (15)$$

As a consequence, the energy of the $NK\pi$ system will be $10-20$ MeV reduced by the mixing with the configuration $(\Delta K)_{J^*=\frac{3}{2}^-,I=1}$.

Here we mention that in the case of $(K\pi)_{J^T=\frac{1}{2}^+}$, the effect of the $\kappa$ resonance should also be considered. Because the mass of $\kappa$ is about $345$ MeV higher than $m_K + m_\pi$, which is much larger than the mass difference between $N\pi$ and $\Delta$, thus the effect of the $\kappa$ resonance must be smaller than that of the $\Delta$ resonance. As an approximation, we neglect it in this work.

In this article, we have estimated the energy of the $NK\pi$ system with $J^\pi = \frac{3}{2}^-$ and $I = 1$. By fitting the corresponding phase shifts in the low energy region, the $KN$, $N\pi$ and $K\pi$ interactions are obtained from the chiral SU(3) quark model. The estimated energy of $(NK\pi)_{J^*=\frac{3}{2}^-,I=1}$ is about $70-80$ MeV higher than the threshold of the $NK\pi$ system. After considering the mixing with the configuration $(\Delta K)_{J^*=\frac{3}{2}^-,I=1}$, the energy of this system can go down about $10-20$ MeV. The present calculations are rough and yet do not produce a
bound state of the three body system. However, further refinements in the interaction and
the method of calculation could lead to such a state and it is worth calling the attention to
such a possibility. In fact recent, also qualitative, calculations with a different quark model
also suggest that the Θ+ could be a \( \frac{3}{2}^- \) state \[18\].

As mentioned, a new resonance Θ+ with \( M_\Theta = 1540 \text{ MeV} \) and \( \Gamma_\Theta < 25 \text{ MeV} \) has been
observed by several labs since 2003 \[6\]. Although there are also some experimental groups
who have reported negative results \[6, 7\], research is ongoing. This is because the Θ+ has
strangeness quantum number \( S = +1 \), so that if it does exist, it must be at least a 5-quark
system, and if it can be explained as a pentaquark it will be the first multi-quark state people
found. At the same time there are many theoretical works trying to explain the structures
and the properties of the Θ particle with various quark models or other approaches \[19\].

Because the mass of Θ, \( M_\Theta \), is higher than the threshold of nucleon-kaon system, \( M_N + M_K \),
it is not easy to understand why its width is so narrow, unless it has very special quantum
numbers. As to the mass of Θ, although it is predicted by the original chiral soliton model
\[20\] quite well, it is difficult to generate the experimental Θ’s mass and understand its narrow
width by the reasonably dynamical calculation based on the constituent quark model when
it is treated as a pentaquark state \[21, 22\]. In Ref. \[22\], we calculated the energies of 17
lowest five-quark configurations in the framework of the chiral quark model, and found that
for both cases of \( J^\pi = \frac{1}{2}^- \) and \( J^\pi = \frac{1}{2}^+ \), their energies are always about 250 – 350 MeV
higher than the experimental value. It seems that either the Θ does not exist, which is
consistent with the new high statistical experiments \[7\], or the structure of this particle has
to be understood in some other mechanisms. If the report from RIHC-STAR group is true,
i.e., there is a Θ++ with \( S = +1 \) and \( M_\Theta \simeq 1530 \text{ MeV} \) \[8\], we suppose that this Θ++ particle
might be explained as a three-body molecule-like state of \( NK\pi \) with \( J^\pi = \frac{3}{2}^- \) and \( I = 1 \).
This is because: (1) its energy is close to the threshold of \( NK\pi \), thus it can be expected to
be a bound state; (2) its decay width must be quite narrow, because when and only when
the pion of the system is absorbed by the nucleon, it can decay to \( K \) and \( N \); and (3) it is
not easy to be formed in the \( K + N \) process. Though the above opinion is only a qualitative
discussion, we think more accurate study of the structure and the properties of this system
is worth doing in the future work.

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