Generalized geometrical coupling for vector field localization on thick brane in asymptotic Anti-de Sitter spacetime

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It is known that a five-dimensional free vector field $A_M$ cannot be localized on Randall-Sundrum (RS)-like thick branes, namely, the thick branes embedded in asymptotic Anti-de Sitter (AdS) spacetime. In order to localize a vector field on the RS-like thick brane, an extra coupling term should be introduced. In this paper, we generalize the geometrical coupling mechanism by adding two mass terms ($\alpha R g^{MN} A_M A_N + \beta R^{MN} A_M A_N$) into the action. We decompose the fundamental vector field $A_M$ into three parts: transverse vector part $A_\mu$, scalar parts $\phi$ and $A_5$. Then, we find that the transverse vector part $A_\mu$ decouples from the scalar parts. In order to eliminate the tachyonic modes of $A_\mu$, the two coupling parameters $\alpha$ and $\beta$ should satisfy a relation. Combining the restricted condition, we can get a combination parameter as $\gamma = \frac{3}{2} \pm \sqrt{1 + 12\delta}$. Only if $\gamma > 1/2$, the zero mode of $A_\mu$ can be localized on the RS-like thick brane. We also investigate the resonant character of the vector part $A_\mu$ for the general RS-like thick brane with the warp factor $A(z) = -\ln(1 + k^2 z^2)/2$ by choosing the relative probability method. The result shows that, only for $\gamma > 3$, the massive resonant Kaluza-Klein modes can exist. The number of resonant Kaluza-Klein states increases with the combination parameter $\gamma$, and the lifetime of the first resonant state can be long enough as the age of our universe. This indicates that the vector resonances might be considered as one of the candidates of dark matter. Combining the conditions of experimental observations, the constrain shows that the parameter $k$ has a lower limit with $k \gtrsim 10^{-17}$ eV, the combination parameter $\gamma$ should be greater than 57, and accordingly, the mass of the first resonant state should satisfy $m_1 \gtrsim 10^{-15}$ eV.

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I. INTRODUCTION

Over the last decades, brane world theories have received a lot of attention for the success on solving the gauge hierarchy and cosmological constant problems [1, 2]. In the brane world scenario, our universe is a 3-brane embedded in a higher-dimensional bulk. The well-known Randall-Sundrum (RS) models [1] (including RS-1 and RS-2 models) involve one extra dimension with a non-trivial warp factor due to the underlying anti-de Sitter (AdS) geometry.

In the RS thin brane models and their generalizations, the branes have no thickness and there are no dynamical mechanisms responsible for their formation. In order to investigate the dynamical generation of branes and their internal structure, domain wall (or thick brane) models were presented, and for more details on thick brane models, see Refs. [3, 4]. One of the features of thick brane is that usually it is generated by one or more background scalar fields coupled with gravity.

In thick brane models, various fundamental matter fields are living in the higher-dimensional bulk. Therefore, in order to construct a more realistic brane world, on which the four-dimensional gravity and matter fields in the standard model should be localized, it is very necessary and significant to provide an effective localization mechanism for the bulk gravity and matter fields. The results of Refs. [5–11] show that four-dimensional gravity can be localized on the thick branes generated by background scalar field(s) in a five-dimensional asymptotic AdS spacetime. As shown in Refs. [12–17], a free massless scalar field can also be localized on the thick branes. For a Dirac fermion field, without introducing the scalar-fermion coupling (also called the Yukawa coupling) [18–24] or fermion-gravity coupling [25], it has no normalizable zero mode in five-dimensional RS-like brane models. Unfortunately, Ref. [26] gave the essence of “no-go theorem” in the thin brane limit (RS-2 model with an infinite extra dimension) that the localization for a vector field seems to require a richer brane structure, for example, the de Sitter brane [28–31], the brane world with finite extra dimension [27], or a six-dimensional string-like model [32].

Then, a lot of works have been devoted to find a mechanism for vector field localization, and many literatures show a wide variety of ideas. Kehagias and Tamvakis proposed a dilaton coupling between the vector field and background scalar field [33]. This mechanism has been widely applied in different thick brane models [34–40]. After that, Chumbes, Holf da Silva and Hott proposed a coupling function between the vector field and
the background scalar field [41]. Vaquera-Araujo and Corradini introduced a Yukawa-like coupling, namely, a
Stueckelberg-like action, to realize the localization of the
vector field [42].

Recently, Zhao et al. [43] presented another localization
mechanism of the vector field, i.e., introduced a mass
term $\alpha R g^{MN} A_M A_N$ with $R$ the five-dimensional scalar
curvature. They found that only for a special coupling
parameter $\alpha = -1/16$, the vector part $\hat{A}_\mu$ can be
localized on the thick brane, and there are no tachyonic
modes. Then, Alencar et al. introduced other forms of
the mass term: $\beta R^{MN} A_M A_N$ and $\gamma G^{MN} A_M A_N$ with
$R^{MN}$ and $G^{MN}$ the Ricci tensor and Einstein tensor
[45]. While, in all these mechanisms, in order to eliminate
tachyonic vector modes, the massive parameters $\alpha$
or $\beta$ should be fixed, since there is no more degree of
freedom for the coupling parameter. As a result, the
effective potential of the vector Kaluza-Klein (KK) modes is
fixed and usually there are no resonant vector KK modes
quasi-localized on the brane.

Inspired by the above works, we generalize the mass
term to the following one

$$\frac{1}{2} \left( \alpha R g^{MN} A_M A_N + \beta R^{MN} A_M A_N \right),$$

(1)

since both terms are possible couplings. Then we study
the localization and quasi-localization of the vector field
on the thick brane. For the quasi-localized massive KK
modes, they might be a candidate for dark matter. Note
that, the consistency conditions for this kind of localization
mechanism is just investigated by [46].

We decompose the vector field $A_M$ into three parts:
the transverse component $\hat{A}_\mu$ (transverse vector part),
the longitudinal component $\partial_\mu \phi$ (scalar part), and
the fifth component $A_5$ (scalar part). Here the Latin
indices $(M, N = 0, 1, 2, 3, 5)$ stand for the five-dimensional
coordinate indices, Greek indices $(\mu, \nu = 0, 1, 2, 3)$
correspond to the brane coordinate indices. We find that
the transverse vector part $\hat{A}_\mu$ decouples with the scalar
parts $\phi$ and $A_5$. Besides, in order to eliminate the
tachyonic modes of $A_\mu$, the two parameters in the
coupling term (1), $\alpha$ and $\beta$, should satisfy a relation: $\beta = -1 - 8\alpha \pm \sqrt{1 + 12\alpha}$. With this constraint, we can get
a combination parameter $\gamma = \frac{1}{2} \pm \sqrt{1 + 12\alpha}$, and the
localized condition for the transverse vector part $\hat{A}_\mu$ is
$\gamma > 1/2$. More importantly, we can find the resonant
states under this restrict condition. We investigate the
resonant character of $\hat{A}_\mu$ with the general RS-like thick
brane warp factor $A(z) = -\ln(1 + k^2 z^2)/2$. These res-
onant states can be considered as a candidate for dark
matter.

The remaining parts are organized as follows. In section II, we introduce the generalized model of the vector
field. Then, we calculate the localization of the transverse
part of a five-dimensional vector on the thick brane in section III. After that, we study the resonant character
of the transverse vector part in section IV. Finally, we
conclude our results in the last section.

II. THE GENERALIZED GEOMETRICAL COUPLING MECHANISM OF VECTOR FIELD

The vector field can be localized on the thick brane
by considering the geometrical coupling term, e.g.,
the coupling between the vector field and the Ricci scalar
(or Ricci tensor), which can be viewed as a mass term
[43–45]. In this paper, we consider the generalized ge-
ometrical coupling (1). Then the full five-dimensional
action for the vector field $A_M$ is given by

$$S = -\frac{1}{4} \int d^5 x \sqrt{-g} \left( F^{MN} F_{MN}
+ 2(\alpha R g^{MN} + \beta R^{MN}) A_M A_N \right),$$

(2)

where $g_{MN}$ is the metric of the five-dimensional bulk
spacetime and $F_{MN} = \partial_M A_N - \partial_N A_M$ is the field
strength. Here, we decompose the vector $A_M$ in the fol-
lowing way:

$$A_M = (\hat{A}_\mu + \partial_\mu \phi, A_5),$$

(3)

where $\hat{A}_\mu$ is the transverse component with the trans-
verse condition $\partial_\mu \hat{A}_\mu = 0$, and $\partial_\mu \phi$ is the longitudinal
component.

We adopt the following metric ansatz to describe the
five-dimensional spacetime:

$$ds^2 = e^{2A(y)} g_{\mu\nu} dx^\mu dx^\nu + dy^2,$$

(4)

where the warp factor $A(y)$ is a function of the extra
dimensional coordinate $y$. So, the Ricci scalar and the
nonvanishing components of the Ricci tensor can be ex-
pressed as

$$R = -4(5A'^2 + 2A''),$$

(5)

$$R^{\mu\nu} = -(4A'^2 + A'') g^{\mu\nu},$$

(6)

$$R^{\mu5} = -4(A'^2 + A''),$$

(7)

where the prime denotes derivative with respect to $y$.

The components of the mass terms in action (2) can the
written as

$$\alpha R g^{\mu\nu} + \beta R^{\mu\nu} = \mathcal{W} g^{\mu\nu},$$

(8)

$$\alpha R g^{55} + \beta R^{55} = \mathcal{G} g^{55},$$

(9)

where

$$\mathcal{W} = -4(5\alpha + \beta) A'^2 - (8\alpha + \beta) A'',$$

(10)

$$\mathcal{G} = -4(5\alpha + \beta) A'^2 - 4(2\alpha + \beta) A''.$$

(11)

By substituting the decomposition (3) into the action
(2), we can split the action into two parts

$$S = S_V(\hat{A}_\mu) + S_S(\phi, A_5),$$

(12)
where

\[ S_V = -\frac{1}{4} \int d^5x \left( \tilde{F}_{\mu\nu} \tilde{F}^{\mu\nu} + 2\partial_5 \tilde{A}_\mu \partial^5 \tilde{A}_\nu \eta^{\mu\nu} e^{2A} \right) + 2W \tilde{A}_\mu \partial^5 \tilde{A}_\nu \eta^{\mu\nu} e^{2A}, \]

and Eq. (18) is rewritten as

\[ -\partial_z \left( e^{A(z)} \partial_z \tilde{\rho}_n \right) + \tilde{\rho}_n e^{3A(z)} \mathcal{W} = e^{A(z)} m_n^2 \tilde{\rho}_n, \]

with \( \mathcal{W} = e^{-2A}(-12\alpha - 3\beta)(\partial_z A)^2 + e^{-2A}(-8\alpha - \beta) \partial_z^2 A. \)

After the field transformation \( \tilde{\rho}_n = e^{-\frac{A(z)}{2}} \rho_n(z) \), Eq. (20) can be rewritten as a Schrödinger-like equation:

\[ \left( -\partial_z^2 + V_v(z) \right) \rho_n = m_n^2 \rho_n, \]

where the explicit expression of the effective potential \( V_v(z) \) is

\[ V_v(z) = \left( \frac{1}{4} - 12\alpha - 3\beta \right)(\partial_z A)^2 + \left( \frac{1}{2} - 8\alpha - \beta \right) \partial_z^2 A. \]

In order to exclude the tachyonic vector modes, the eigenvalues of Schrödinger-like equation (21) should be non-negative, i.e., \( m_n^2 \geq 0 \). So, Eq. (21) should be written in the form of \( Q^2 \rho_n = m_n^2 \rho_n \), with \( Q = -\partial_z + \gamma \partial_A \).

That is to say, the effective potential should be in the form of

\[ V_v(z) = \gamma^2 (\partial_z A)^2 + \gamma \partial_z^2 A. \]

To this end, the two parameters \( \alpha \) and \( \beta \) should satisfy the following relation:

\[ \beta = -1 - 8\alpha \pm \sqrt{1 + 12\alpha}, \]

and so parameter \( \gamma \) in (23) is given by

\[ \gamma = \frac{3}{2} \pm \sqrt{1 + 12\alpha}. \]

With the relation (24) and the expression (25), the Schrödinger-like equation (21) can be further rewritten as

\[ \left( -\partial_z^2 + \gamma^2 (\partial_z A)^2 + \gamma \partial_z^2 A \right) \rho_n = m_n^2 \rho_n. \]

Now, we investigate the localization of the zero mode of \( \tilde{A}_\mu \), for which \( m_0 = 0 \) and the solution is given by

\[ \rho_0(z) = c_0 e^{\gamma A(z)}, \]

where \( c_0 \) is the normalization constant. According to the orthonormality condition (16), the integration of \( \rho_0^2 \) should be finite, namely,

\[ \int_{-\infty}^{+\infty} \rho_0^2 dz = c_0^2 \int_{-\infty}^{+\infty} e^{2\gamma A(z)} dz = c_0^2 \int_{-\infty}^{+\infty} e^{(2\gamma - 1)A(y)} dy = 1. \]

For the RS-like braneworld scenarios, the warp factor has the following asymptotic behavior as

\[ A(y) \big|_{y \to \pm \infty} \rightarrow -k|y|, \]
where $k$ is the scale parameter of the brane with mass dimension. Plugging it into Eq. (28), we obtain that 
\[ e^{(2\gamma-1)A(y)}|_{y \to \pm \infty} \to e^{-(2\gamma-1)k|y|}. \] (30)

In order to ensure the integration (28) is convergent, the parameter should satisfy $\gamma > 1/2$, i.e., $2 \pm \sqrt{1 + 12\alpha} > 0$. So, the range of the parameter $\alpha$ for different concrete expressions of $\beta$ are:
\[ \alpha > -1/12, \beta = -1 - 8\alpha - \sqrt{1 + 12\alpha}, \] \[ 0 > \alpha > -1/12, \beta = -1 - 8\alpha + \sqrt{1 + 12\alpha}. \] (31) (32)

IV. THE RESONANT CHARACTER OF $\hat{A}_\mu$

In this section, we would like to investigate the massive KK states of the transverse vector part for the vector field. We will mainly look for resonant KK states of the vector field, which are quasi-localized on the brane but will propagate into extra dimension eventually. The resonance spectrum of these KK states is one of the typical characteristics of RS-like brane world models. They can interact with four-dimensional particles, which may lead to the non-conservation of energy and momentum since the KK resonances can escape out of the brane. So, it is possible to probe extra dimensions by detecting resonant states [48]. Besides, some physicists regard those massive KK particles as a candidate of dark matter (see Refs. [49–51] the details). The appearance of these resonances is related to the structure of the brane. Thus, it is important and interesting to study the resonant KK modes on the thick brane with different structures. References [52–57] have considered resonances of graviton and fermion. Besides, Arakawa et al. considered a massive vector field as a candidate of dark matter to explain the strong CP problem [58]. So, we will study the resonances of the five-dimensional vector field.

In order to study the resonant states, Almeida et al. proposed the large peaks of the wave function as the resonant method to study the fermion resonances [52]. Then, Landim et al. researched the resonant states with transfer matrix method [56, 57]. Here, we will choose the relative probability method proposed by Liu et al. [54] to calculate the resonant KK modes of the vector part $\hat{A}_\mu$, since the method is effective for both odd and even KK states. The relative probability is defined as [54]
\[ P = \frac{\int_{z_\min}^{z_\max} |\rho_n(z)|^2 dz}{\int_{z_\min}^{z_\max} |\rho_n(z)|^2 dz}, \] (33)

where $2z_\min$ is approximately the width of the thick brane and $z_{\max} = 10z_\min$. Since the potentials considered in this paper are symmetric, the wave functions are either even or odd. Hence, we can use the following boundary conditions to solve the differential equation (26) numerically:
\[ \rho_n(0) = 0, \rho'_n(0) = 1, \quad \text{for odd KK modes}, \]
\[ \rho_n(0) = 1, \rho'_n(0) = 0, \quad \text{for even KK modes}. \] (34)

We solve the Schrödinger-like equation (25) with the general RS-like warp factor $A(z) = -\ln(1 + k^2z^2)/2$. According to the supersymmetric quantum mechanics, the supersymmetric partner potentials will share the same spectrum of massive excited states. So, we can judge whether there are resonances by analyzing the shape of the supersymmetric partner potential (we call it the dual potential). For our case, the dual potential corresponding to (23) is $V^\text{(dual)}_\nu(z) = \gamma^2(\partial_z A)^2 - \gamma \partial_z^2 A$. If there is no well or quasi-well in the dual potential, then there is no resonances. Thus, only for $\gamma > 3$, there might exist resonances. We solve the KK states numerically. The result shows that both the parameters $k$ and $\gamma$ will affect the properties of the resonant states.

Figure 1 shows the influence of the combination parameter $\gamma$ on the effective potential $V_\nu(z)$ and the resonant KK modes of the vector field $\hat{A}_\mu$. Figure 1(a) shows that the height of the potential barrier increases with the combination parameter $\gamma$, which indicates there are more resonant KK modes for larger $\gamma$, and this can be confirmed from Figs. 1(b), 1(c), 1(d). Combining Figs. 1(b), 1(c), 1(d), we can see that the mass of the first resonant KK modes, the number of resonant states, and the mass gap of the resonant KK modes increase with the parameter $\gamma$.

The effect of the scale parameter $k$ is shown in Fig. 2. From Fig. 2(a), we can see that the scale parameter $k$ can influence not only the width of the potential well, but also its height. With the increasing of the scale parameter $k$, the potential well becomes narrower and higher. From Figs. 2(b), 2(c), 2(d), we can see that the mass of the first resonant KK mode and the mass gap of the resonant KK modes increase with the parameter $k$. However, the number of the resonances does not change with $k$ for a fixed $\gamma$.

![Figure 1](image1.png)

**Fig. 1:** The influence of the combination parameter $\gamma$ on the effective potential $V_\nu$ and the probabilities $P$ (as a function of $m^2$) for the odd-parity (blue dashed lines) and even-parity (red lines) massive KK modes. The scale parameter is set as $k = 1$. 
TABLE I and II are the specific values of the mass $m_n$, the relative probability $P$, the width $\Gamma$, and the lifetime $\tau$ for different parameters. Here, we define the width $\Gamma = \Delta m_n$ at half maximum of the peak and $\tau = 1/\Gamma$. Table I shows that with the increasing of the parameter $\gamma$, the relative probability $P$ of the corresponding $n$-th resonant state becomes larger, and the lifetime of the resonant state becomes longer. From Tab. II, we can see that when the parameter $\gamma$ is fixed, all of the mass $m_n$, the width $\Gamma$, and the lifetime $\tau$ for the corresponding $n$-th resonant state are influenced by the parameter $k$. However, the values of $m_n/k$ are basically the same for different values of $k$, so do the relative probability $P$, the relative width $\Gamma/k$, and the relative lifetime $\tau \ast k$. So we can make a coordinate transformation as $\bar{z} = k z$ to offset the effect of $k$. Combining these two tables, we can see that the lifetime $\tau$ increases with $\gamma$, while decreases with $k$, which means that if the parameter $\gamma$ is large enough or the parameter $k$ is small enough, the lifetime of the resonant states can be long enough as the age of our universe. So, in this case, we can consider the resonant states as one of the candidates of dark matter.

Then, we calculate the lifetime of the first resonant state in order to check whether it can be a candidate of dark matter or not. For convenience, we make a coordinate transformation $\bar{z} = k z$, and define the scaled mass $\bar{m}_1 = m_1/k$ and the scaled lifetime $\bar{\tau} = 1/\Gamma$ for the first resonant state in the Natural System of Units, where $\Gamma = \Delta \bar{m}_1$ is the half maximum of the peak for the first resonant state. Note that, both $\bar{m}_1$ and $\bar{\tau}$ are dimensionless.

Figure 3 shows that both the scaled mass $\bar{m}_1$ and the scaled lifetime $\log(\bar{\tau})$ linearly depend on the parameter $\gamma$, and the fit functions can be expressed as

$$\bar{m}_1 = -3.2 + 2.0 \gamma,$$

(35)

$$\log(\bar{\tau}) = 4.7 + 0.2 \gamma.$$

(36)

FIG. 3: The influence of the combination parameter $\gamma$ on the scaled mass $\bar{m}_1$ and the scaled lifetime $\log(\bar{\tau})$ of the first resonant state. The black dots are numerical results, the red solid line is the fit function for $\bar{m}_1$ with $a_m = -3.2$ and $b_m = 2.0$, and the blue solid line is the fit function for $\log(\bar{\tau})$ with $a_\tau = 4.7, b_\tau = 0.2$.

It is known that the age of our universe is of about 13.8 billion years, i.e., $4.35 \times 10^{17}$s. So, if we consider the first resonant state as one of candidates of dark matter, its lifetime should be larger than the age of universe, i.e., $\tau \gtrsim 4.35 \times 10^{17}$s, or in the Natural System of Units, $\tau = 1/(k \bar{\Gamma}) = \bar{\tau}/k \gtrsim 6.6 \times 10^{32}$eV$^{-1}$.

(37)

Thus, the restriction of the scale parameter $k$ can be expressed as

$$k \lesssim 1.5 \times 10^{-33} \text{ eV} \simeq 7.5 \times 10^{-29 + 0.2 \gamma} \text{ eV}.$$

(38)

In addition, in the brane world theory considered in this paper, the relation between the four-dimensional effective Planck scale $M_{Pl}$ and the five-dimensional fundamental scale $M_*$ is given by [11]:

$$M_{Pl}^2 = M_*^3 \int_{-\infty}^{\infty} dz e^{3A(z)} = 2 M_*^3/k.$$

(39)

Theoretically, if the energy scale reaches the five-dimensional fundamental scale $M_*$, the quantum effect of gravity cannot be ignored. Experimentally, in the recent experiment of the Large Hadron Collider (LHC), the collision energy is 13 TeV and the result shows that the quantum effect of gravity can be ignored, which means that the five-dimensional fundamental scale $M_* > 13$ TeV. Thus, the constrain on the parameter $k$ is

$$k > 4.4 \times 10^{-17} \text{ eV}.$$

(40)

By combining the two conditions (38), (40) and the fit function (35), we can get the restricted expressions of the mass of the first resonant state $m_1$ with the combination parameter $\gamma$ as

$$m_1 > (8.8 \gamma - 14.1) \times 10^{-17} \text{eV},$$

(41)

$$m_1 \lesssim (1.5 \gamma - 2.4) \times 10^{-28 + 0.2 \gamma} \text{eV}.$$

(42)
The shadow regions of Fig. 4 show the available ranges of the parameters $k$ and $m_1$, respectively. From Fig. 4(a), we can see that only if $\gamma > 57$, the two restricted conditions (38) and (40) of $k$ could be satisfied, which means that the parameter $\gamma$ has a lower limit. Correspondingly, Fig. 4(b) shows there is a lower limit for the first resonant state mass $m_1$, i.e., $m_1 \gtrsim 10^{-15}$eV.

![FIG. 4: The limit range of the scale parameter $k$ and the mass of the first resonant state $m_1$. The blue lines are restrictions from the five-dimensional fundamental scale $M$, should larger than 13 TeV and the black lines are restrictions from the lifetime of the first resonant state should be on the magnitude of the universe life.](image)

V. CONCLUSION

We generalized the geometrical coupling mechanism in order to localize a five-dimensional vector field on RS-like thick branes. The key feature of the mechanism is to introduce two mass terms of the vector field, which are proportional to the five-dimensional Ricci scalar and the Ricci tensor, respectively. We decomposed the vector field $A_{\mu}$ into three parts: the vector part $\hat{A}_{\mu}$, the scalar part $\phi$ and $A_5$. With the transverse condition $\partial_{\mu} \hat{A}^\mu = 0$, we got a decoupled action of $\hat{A}_{\mu}$. We find that when the two parameters $\alpha$ and $\beta$ in the action (2) satisfy the relation $\beta = -1 - 8\alpha \pm \sqrt{1 + 12\alpha}$, the effective potential $V_\gamma(z)$ of the vector KK modes can be expressed as $V_\gamma(z) = \gamma^2 (\partial_{\gamma} A)^2 + \gamma \partial^2 A$ with $\gamma = \frac{1}{2} \pm \sqrt{1 + 12\alpha}$ and the tachyonic KK modes of $\hat{A}_\mu$ can be excluded. For $\gamma > 1/2$, the zero mode of $\hat{A}_\mu$ can be localized on the brane.

Then, we investigated the resonances of the vector field by using the relative probability method and considered the possibility of these resonances as one of the candidates of dark matter. We analyzed the influence of the parameters $k$ and $\gamma$ on the resonant behavior. We found that, only for $\gamma > 3$, the massive resonant KK modes could exist. Both the two parameters affect the height of the potential and hence the vector resonances. The number of the resonant states only increases with the parameter $\gamma$. We also considered scaled lifetime $\tau$ and the scaled mass $m_1$ of the first resonant state. We found that both the scaled mass $m_1$ and the scaled lifetime $\log(\tau)$ can be fitted by a linear function of $\gamma$ approximately. In order to view the first resonant vector KK state as dark matter, its lifetime should be long enough as the age of our universe. This would introduce some constrains on the parameters $k$ and $\gamma$ as well as the mass of the first resonance, i.e., $k \gtrsim 10^{-17}$eV, $\gamma > 57$, and $m_1 \gtrsim 10^{-15}$eV.

Note: When we finish this work, we find another work [59] that also considered the same localization mechanism (2) for the vector filed.

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| $\gamma$ | $n$ | $m_n$ | $P$ | $\Gamma$ | $\tau$ |
|---|---|---|---|---|---|
| 10 | 1 | 4.0917 | 0.9474 | 1.0998 \times 10^{-2} | 9.0924 \times 10^{-4} |
| 2 | 5.1461 | 0.3027 | 2.911 \times 10^{-2} | 34.3351 |
| 15 | 1 | 5.1822 | 0.9973 | 1.5778 \times 10^{-4} | 8.6372 \times 10^{-4} |
| 2 | 6.8525 | 0.9032 | 1.4593 \times 10^{-4} | 6.8523 \times 10^{-4} |
| 3 | 7.6902 | 0.2839 | 6.5012 \times 10^{-4} | 1.5382 \times 10^{-4} |
| 25 | 1 | 6.8457 | 0.9997 | 1.09557 \times 10^{-4} | 9.1277 \times 10^{-4} |
| 2 | 9.3884 | 0.9468 | 1.598 \times 10^{-4} | 6.2589 \times 10^{-4} |
| 3 | 10.9936 | 0.9189 | 9.0939 \times 10^{-4} | 1.0996 \times 10^{-4} |
| 4 | 12.1288 | 0.814 | 8.2448 \times 10^{-4} | 1.2129 \times 10^{-4} |
| 5 | 12.7800 | 0.2546 | 1.9736 \times 10^{-4} | 5.0668 \times 10^{-4} |

**TABLE I:** The influence of combination parameter $\gamma$ on the mass spectrum $m_n$, the relative probability $P$, the width of mass $\Gamma$, and the lifetime $\tau$ of the KK resonances. The scale parameter $k$ is set as $k = 1$.

| $k$ | $n$ | $m_n$ | $m_n/k$ | $P$ | $\Gamma$ | $\Gamma/k$ | $\tau$ | $\tau \times k$ |
|---|---|---|---|---|---|---|---|---|
| 0.2 | 1 | 1.3681 | 6.8405 | 0.9999 | 2.1928 \times 10^{-1} | 1.0964 \times 10^{-4} | 4.5602 \times 10^{-1} | 9.1204 \times 10^{-1} |
| 2 | 2 | 1.8852 | 9.4262 | 0.9987 | 2.6522 \times 10^{-1} | 1.3261 \times 10^{-4} | 5.7704 \times 10^{-1} | 7.5408 \times 10^{-1} |
| 3 | 1 | 1.9626 | 10.9813 | 0.9872 | 1.8213 \times 10^{-1} | 9.1065 \times 10^{-4} | 5.4904 \times 10^{-1} | 1.0981 \times 10^{-1} |
| 2 | 2 | 2.4306 | 12.1529 | 0.8212 | 1.6546 \times 10^{-1} | 8.2731 \times 10^{-4} | 6.0767 \times 10^{-1} | 1.2153 \times 10^{-1} |
| 3 | 5 | 2.5470 | 12.7350 | 0.3192 | 3.5534 \times 10^{-1} | 1.7672 \times 10^{-4} | 2.8294 \times 10^{-1} | 5.6588 \times 10^{-1} |
| 2 | 1 | 1.3689 | 6.8403 | 0.9999 | 2.1902 \times 10^{-1} | 1.0951 \times 10^{-4} | 4.5657 \times 10^{-1} | 9.1314 \times 10^{-1} |
| 2 | 2 | 1.8761 | 9.3680 | 0.9923 | 2.6686 \times 10^{-1} | 1.3343 \times 10^{-4} | 5.7472 \times 10^{-1} | 7.4944 \times 10^{-1} |
| 3 | 2 | 2.2077 | 11.0039 | 0.9361 | 1.8175 \times 10^{-1} | 9.0875 \times 10^{-4} | 5.5021 \times 10^{-1} | 1.1004 \times 10^{-1} |
| 3 | 4 | 2.24305 | 12.1414 | 0.8164 | 1.6507 \times 10^{-1} | 8.2535 \times 10^{-4} | 6.0578 \times 10^{-1} | 1.2116 \times 10^{-1} |
| 3 | 5 | 2.6334 | 12.8167 | 0.3082 | 3.5109 \times 10^{-1} | 1.7554 \times 10^{-4} | 2.8482 \times 10^{-1} | 5.6964 \times 10^{-1} |
| 5 | 1 | 1.4195 | 6.6391 | 0.9991 | 5.4417 \times 10^{-1} | 1.0883 \times 10^{-4} | 1.8377 \times 10^{-1} | 9.1885 \times 10^{-1} |
| 2 | 2 | 1.7203 | 9.4014 | 0.9986 | 1.4616 \times 10^{-1} | 1.4833 \times 10^{-4} | 1.3484 \times 10^{-1} | 6.7419 \times 10^{-1} |
| 3 | 3 | 1.48917 | 10.9784 | 0.9822 | 4.3535 \times 10^{-1} | 8.7471 \times 10^{-4} | 2.8654 \times 10^{-1} | 1.4327 \times 10^{-1} |
| 3 | 4 | 0.8046 | 12.1616 | 0.8203 | 4.1115 \times 10^{-1} | 8.2234 \times 10^{-4} | 2.43 \times 10^{-1} | 1.2166 \times 10^{-1} |
| 3 | 5 | 0.63192 | 12.7383 | 0.3209 | 7.8591 \times 10^{-1} | 1.5718 \times 10^{-4} | 1.2724 \times 10^{-1} | 6.3227 \times 10^{-1} |

**TABLE II:** The influence of scale parameter $k$ on the mass spectrum $m_n$, the relative value $m_n/k$, the relative probability $P$, the width of mass $\Gamma$, the relative width of $\Gamma/k$, the lifetime $\tau$, and the relative $\tau \times k$ of the KK resonances. The combination parameter $\gamma$ is set as $\gamma = 25$.
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