Studying minijets and MPI with rapidity correlations

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Abstract

We propose and carry a detailed study of an observable sensitive to different mechanisms of minijet production. The class of observables measures how the transverse momenta of hadrons produced in association with various trigger objects are balanced as a function of rapidity. It is shown that the observables are sensitive to the model parameters relevant for the minijet production mechanisms: low-$p_T$ cut-off regulating jet cross-section, transverse distribution of partons in protons, and parton distribution functions. We perform our test at different charge-particle multiplicities and collision energies. The MC models, which describe many features of the LHC data, are found to predict quite different results demonstrating high discriminating power of the proposed observables. We also review mechanisms and components of \textsc{Herwig}, \textsc{Pythia}, and \textsc{Sherpa} Monte Carlo models relevant to the minijet production.

1 Introduction

Currently there are a number of the Monte Carlo (MC) generators which successfully describe many features of the inelastic $pp$ collisions at the LHC \cite{1,2,3,4,5}. Since all MC models assume some physics approximations, it is inevitable that they have a number of free parameters which must be fixed by experimental data in the procedure called tuning \cite{6,7,8}. It often happens that the description of experimental data by different MC models is similar, despite the fact that the underlying dynamics in the models differ quite significantly - with hard collisions giving a major contribution in some of the models and significant soft contribution in the other models.

The aim of this paper is to look for the observables which would be especially sensitive to some of the important ingredients of the models. Specifically we will propose observables which are sensitive to two important characteristics of the model: taming of jet production at small $p_T$ and transverse distribution of partons in the colliding protons.
Obviously, the rate of parton-parton scattering has to be tamed at small momentum transfer to avoid an unphysical singular behaviour. The divergence is usually regulated by including a suppression factor, that is quite different in different models. Also, in most of the models the suppression for fixed $p_T$ becomes stronger with increase of collision energy. The relevant details of models used in this study are described in Section 4. The number of parton interactions depends not only on the suppression factor, also on the set of parton distribution functions (PDF) and the overlap of the matter distribution of colliding protons. It often happens that models with very different PDF and model parameters are quite close for some observables to the data and to each other. For instance, Fig. 1 shows $p_T$ distributions of gluons coming from primary and MPI interactions for successful tunes of PYTHIA and HERWIG. One can see that they are very different in the low $p_T$ region, which eventually produces most of the final-state particles in the collision. Nevertheless, the models describe underlying event (UE) observables [10–12] quite satisfactory, as the difference in other mechanisms compensates the discrepancy. This motivates us to propose observables which are sensitive to the underlying dynamics of minijet production and, thus, allows to discriminate models and learn more about underlying dynamics of pp interactions. The correlation between mechanisms and their impact on minijet production will be discussed in Section 4.

![Figure 1: $p_T$ distribution of gluons coming from primary and MPI interactions. The pseudo-rapidity range is $|\eta| < 5$. We show the distribution for the PYTHIA 8 CUETP8M1 [7] and two different settings (tunes) for HERWIG++ 2.7 described in [9].](image)

As mentioned above, the transverse distribution of partons in nucleons is another fitted parameter which is the relevant for jet production. Basically, the rate of the double parton interactions (DPS) is inversely proportional to the transverse area occupied by partons. This parameter of the models can be conveniently coded via so-called sigma effective $\sigma_{\text{eff}}$ defined through: $\sigma_{ij} = \sigma_i \sigma_j / \sigma_{\text{eff}}$, where $\sigma_i$, $\sigma_j$ and $\sigma_{ij}$ are cross sections for single- and double-parton scatters of types $i$ and $j$. Practically in all models it is assumed that transverse distribution of partons does not depend on $x$ of the parton.\footnote{With an exception of the PYTHIA model described in [13]. However, this option is not used in the most...}
the partons are neglected, the inclusive cross section of \( N \) binary collisions is \( \propto \sigma_{1-N}^{\text{eff}} \). Hence the sensitivity to this parameter should grow with the hadron multiplicity, that is usually characterized by charged-particle multiplicity in the experimental measurements.

Note in passing, that the transverse area in which partons are localized found in the fits to the data are at least factor of two smaller than indicated by the HERA data on hard exclusive processes suggesting the one may need to include pQCD effects which lead to decrease of \( \sigma_{\text{eff}} \) with increase of the virtuality of the collision, see the review in [14]. This pattern was implemented in [15][16].

To enhance sensitivity to the low-\( p_T \) suppression mechanism we need observables which minimize soft physics effects and still preserve sensitivity to the presence of the semi-hard collisions. We use here observation that soft mechanisms lead to a short-range correlation in rapidity, while correlation of binary semi-hard collision extends to noticeably larger rapidity intervals. So we suggest to measure how the transverse momenta of hadrons produced in association with a trigger object are balanced as a function of rapidity. The formalism of the proposed observable is given in Section 2. Advantage of such an observable is that the contribution of the \( p-p \) collisions where the trigger and balancing particles belong to the different parton interactions should cancel as long as the parton interactions are independent. Our numerical studies described below demonstrate sensitivity of this variable to the assumed dynamics. A study of the same observable as a function of the multiplicity of final-state particles that in the discussed models originate from fluctuation of the number of hard collisions or a combination of the soft and hard collisions provides an additional discriminating tool. For high multiplicities the discussed observable is sensitive to effects such as screening or formation of quark gluon plasma in collisions of protons. For these reasons we shall study the observable as a function of the charged particle multiplicity. Also, we will investigate the impact of the so-called color-reconnection (CR) mechanism which is in constant development by many Monte Carlo authors [17–25]. The most of mechanisms discussed above are assumed to be dependent on collision energy, therefore we shall perform our tests at two center-of-mass (CM) collision energies, \( \sqrt{s} = 7 \text{ TeV} \) and \( \sqrt{s} = 13 \text{ TeV} \).

The paper is organized as follows. In Section 2 we define the observables in a more formal way, while the justification of kinematic cuts is given in Section 3. In Section 4 a summary of the discussed models is presented. The results of calculations using these models are presented in Section 5. Our conclusions are presented in Section 6.

2 Observables

As mentioned, we will be interested in mechanisms of particle production in hadron-hadron collisions, in particular in finding experimental observables that are sensitive to particular models. As is known, the particle production is driven by the minijets, i.e. semi-hard partons (quark and gluons) produced in a collision of incoming partons (one or many), or in a bremsstrahlung process.

Partons produced in different mechanisms are, in general, correlated in a different way. For example, if we concentrate on rapidity of produced partons, we may expect that bremsstrahlung partons will have short-range correlations, while the partons produced in a hard collision will have a long range tails. 

The use of recent Pythia tunes.
One way to study the correlations is to investigate how the transverse momentum is balanced as a function of rapidity. The practical observable may be constructed as follows (see also Fig. 2). For a given event with \( n \) final state particles, we pick up a particle \( k \) within a fixed rapidity interval and a certain (small) \( p_T \). Let us call this a trigger particle. Then, we define the total transverse momentum of the all remaining final state particles along the trigger particle, contained in a rapidity bin \( \Delta \eta \):

\[
P_{\text{rec}}^{(k)}(\eta) = \sum_{i=1, n, i \neq k}^{N} \left( \hat{O}_k \tilde{p}_{Ti} \right)^y \Theta \left( \left( \eta - \frac{\Delta \eta}{2} \right) < \eta_i < \left( \eta + \frac{\Delta \eta}{2} \right) \right),
\]

where \( \Theta \) is the step function and \( \hat{O}_k \) is the rotation operator which sets the trigger particle \( k \) along the \( y \)-axis, so that in the formula above we add up the \( y \) components of the recoil particles. \( p_{\text{rec}}^{(k)}(\eta) \) can be calculated on the event-by-event basis so that we can define the average \( \langle p_{\text{rec}}^{(k)}(\eta) \rangle \) as

\[
\langle p_{\text{rec}}^{(k)}(\eta) \rangle = \frac{\sum_{k=1}^{N} p_{\text{rec}}^{(k)}(\eta)}{N},
\]

where \( N \) is the total number of events with the required trigger particle present. The total momentum conservation requirement gives, obviously,

\[
\int d\eta \; \langle p_{\text{rec}}^{(k)}(\eta) \rangle = 0.
\]

We can also define similar quantity for the trigger particle, \( \langle p_{\text{trig}}^{(k)}(\eta) \rangle \), to simply counting only the trigger particles.

### 3 Choice of kinematic cuts

This section justifies a choice of the final state objects used to study the mechanisms of the minijet production. We are mostly guided by performance of the LHC general-purpose detectors, ATLAS and CMS. Therefore, usage of charged particles is only option to study minijet production with upper \( p_T \) limit of few GeV. The tracking system of the experiments allows to reliably reconstruct charged particles with \( \eta < 2.5(2.4) \) for ATLAS(CMS) starting from \( p_T \approx 250 \text{ MeV} \). Therefore, we chose \( 2 < \eta < 2.4 \) for trigger in order to maximize possible \( \eta \) distance for recoil particles.

There are two options for choosing the trigger. First is a single charged particle, second is a charged-particle jet. Both approaches have their advantages. The single charged particle is a very simple and stable trigger, which is, in the contrast to the jet trigger, not contaminated by an additional activity from the Underlying Event (UE). The second is expected to be better connected to the initial parton (mainly a gluon). This is illustrated in Fig. 3 where using PYTHIA we investigate to what \( p_T \) of initial gluon the final state trigger corresponds to. These distributions are plotted under assumption that initial gluon, originating in primary scattering or in MPI, can be matched with final state trigger by requirement of maximum distance \( R = \sqrt{(\phi_p - \phi_t)^2 + (\eta_p - \eta_t)^2} \), where \( \phi_p, \eta_p \) and \( \phi_t, \eta_t \) are azimuthal angles (pseudorapidities) of initial gluon and final state trigger object, respectively. As we see in PYTHIA 8 model, we found that for the \( p_T \) range of interest there is a strong spatial correlation between the trigger objects and parent gluons. For \( R < 0.25 \) it is possible to match 80% of them, thus the
value is used to obtain distributions shown in Fig. 3. The $p_T$ windows of trigger are chosen to be sensitive to the suppression of the minijet production. One can see that the distributions are expectedly narrower for charged-particle jets than for single charged particle, even if they correspond to the same gluon $\langle p_T \rangle$. The distribution for single-particle trigger has long tail that is quite noticeable for $p_T > 10$ GeV. While the main disadvantage of usage of charged-particle jet is a contamination by UE. In order to reduce this disadvantage we used small distance parameter of $R = 0.4$ in anti-$k_T$ jet clustering algorithm \[26\]. In this case the UE contribution to the jet is $\sim 0.5$ GeV on average.

4 Monte Carlo models

The general purposes Monte Carlo event generators used in our studies has been reviewed several times, see for example \[27,28\]. Our intention here is not to review them again, but just to provide enough background to set our discussion of the modelling of minijets.

Before we discuss the event generators, let us however start by recalling briefly of the perturbative QCD mechanism of particle production based on the collinear factorization. In fact, it constitutes the skeleton for all MC event generators. We shall also discuss the modification one has to make in the collinear formula to be able to incorporate it into event generators.
4.1 Minijets in perturbative QCD

As mentioned, the particle production mechanism is driven by $2 \rightarrow 2$ perturbative parton production. In the leading order (LO) the cross section for a production of two jets reads (only $gg \rightarrow gg$ channel is included here for simplicity):

$$
\frac{d\sigma_{2\text{jet}}}{dp_T^2dz_1dz_2} = \frac{1}{16\pi} \frac{1}{p_T^4} \frac{z_1z_2}{(z_1 + z_2)^4} f_{g/H}(z_1 + z_2, \mu^2) f_{g/H}\left(\frac{p_T^2}{s} z_1 + z_2, \frac{z_1z_2}{z_1 + z_2}, \mu^2\right) \frac{1}{2} |\mathcal{M}|^2_{gg\rightarrow gg}(z_1, z_2),
$$

where

$$
|\mathcal{M}|^2_{gg\rightarrow gg}(z_1, z_2) = g^4 \frac{9}{2} \left(\frac{z_1^2 + z_2^2}{z_1z_2(z_1 + z_2)^2}\right)\left(\frac{z_1^2 + z_2^2}{z_1z_2(z_1 + z_2)^2}\right)^2,
$$

is the LO matrix element squared and

$$
z_{1,2} = \frac{|\vec{p}_{T1,2}|}{\sqrt{s}} e^{y_{1,2}}.
$$

Above, $f_{g/H}$ are the gluon distributions in a hadron, $\mu^2$ is $\sim p_T^2$, $s$ is the square of CM energy, $\vec{p}_{T1,2}$ are the transverse momenta of the outgoing partons while $y_{1,2}$ are their rapidities. Due to the momentum conservation we have at LO $|\vec{p}_{T1}| = |\vec{p}_{T2}| \equiv p_T$.

There are two related aspects of this mechanism which are relevant at small transverse momenta [29]. First, the dijet cross section is divergent for jet $p_T \rightarrow 0$:

$$
\frac{d\sigma_{2\text{jet}}}{dp_T^2} \sim \frac{\alpha_s^2(p_T^2)}{p_T^2}.
$$

It is however expected that the growth of the spectrum is tamed by some mechanism already in the perturbative domain for $p_T \sim 2-3$ GeV. In phenomenological model of [29] the suppression factor was introduced as follows:

$$
\frac{d\sigma'_{2\text{jet}}}{dp_T^2} = \frac{d\sigma_{2\text{jet}}}{dp_T^2} \frac{p_T^4}{(p_T^2 + p_{T0}^2(s))^2} \frac{\alpha_s^2(p_T^2 + p_{T0}^2(s))}{\alpha_s^2(p_T^2)},
$$

Figure 3: Gluon $p_T$ distribution for various final-state triggers as obtained using Pythia 8 CUETP8M1 model.
Figure 4: Left: Rapidity correlations \( \langle p_T \rangle_{\text{trig}} \) and \( \langle p_T \rangle_{\text{rec}} \) from the basic QCD perturbative model with the \( p_T \) cutoff. The trigger has \( 2.0 < p_T < 3.0 \text{ GeV} \) and rapidity \( 2.0 < y < 2.4 \).
Right: Zoom of the recoil system curves.

where \( p_{T0}(s) \) is a cutoff parameter which depends on the total CM energy of the collision \( s \)

\[
p_{T0}(s) = p_{T0}^{\text{ref}} \left( \frac{s}{s_0} \right)^\lambda ,
\]

where \( p_{T0}^{\text{ref}}, s_0 \) and \( \lambda \) are parameters to be determined from the data.

Second, even with the cutoff, the inclusive dijet cross section can exceed the total inelastic cross section, implying presence of events with multiple hard parton-parton collisions. The average number of the parton collisions is defined to be \( \langle n \rangle \sim \sigma_{2\text{jet}} / \sigma_{\text{ND}} \), where \( \sigma_{\text{ND}} \) is the nondiffractive total cross section, since production of jets in diffraction is strongly suppressed.

In the above model we can simply obtain the expressions for our main observable the average transverse momenta of the trigger and recoil system \( \langle p_T \rangle_{\text{trig}}(y) \) and \( \langle p_T \rangle_{\text{rec}}(y) \). Using the kinematic cuts discussed in Section 3 we show the sample result in Fig. 4. In this case, the trigger has \( 2.0 < p_T < 3.0 \text{ GeV} \) and rapidity \( 2.0 < y < 2.4 \). The result shows a typical pattern of the rapidity correlations encoded in the hard matrix element and will be a useful reference point when discussing realistic mechanisms.

### 4.2 Pythia model

A detailed description of the model is contained in the series of papers [29–32]. Here we give only a very brief summary.

The essential point is that the hard collisions are not completely independent. This is true in three respects. First, the hard collisions are ordered according to their \( p_T \). Second, the correlations are introduced by the proper treatment of the beam remnants. That is, the removal of a parton from the beam affects the remaining multi-parton distribution function in the longitudinal and flavour space. Finally, there are correlations in the transverse momentum space introduced by the so-called primordial \( k_T \).

The event generation goes as follows. After generating a hard interaction with certain \( p_{T \text{max}} \), the following step, i.e. an emission with transverse momentum \( p_T < p_{T \text{max}} \), is described
by the probability distribution

$$
\frac{dP}{dp_T} = \left( \frac{dP_{MPI}}{dp_T} + \sum \frac{dP_{IS}}{dp_T} + \sum \frac{dP_{FS}}{dp_T} \right)
\times \exp \left\{ - \int_{p_T}^{p_{T\text{max}}} dp_T' \left( \frac{dP_{MPI}}{dp_T'} + \sum \frac{dP_{IS}}{dp_T'} + \sum \frac{dP_{FS}}{dp_T'} \right) \right\}, \quad (10)
$$

where the subsequent probabilities in brackets correspond to the probability distribution of another hard collision, the emission from the initial state, and the final state emission, respectively. The exponential ‘Sudakov form factor’ originates from the requirement that no emission took place between $p_T$ and $p_{T\text{max}}$. The initial and final state showers are based on the DGLAP evolution and we do not discuss them here. The MPI probability distribution is impact parameter dependent. Therefore, for the hardest event it reads

$$
\frac{dP_{MPI}}{dp_T d^2 b} = \frac{\mathcal{O}(b)}{\langle \mathcal{O} \rangle} \frac{1}{\sigma_{nd}} \frac{d\sigma}{dp_T} \exp \left\{ - \int_{p_T}^{p_{T\text{max}}} dp_T' \frac{\mathcal{O}(b)}{\langle \mathcal{O} \rangle} \frac{1}{\sigma_{nd}} \frac{d\sigma}{dp_T'} \right\}, \quad (11)
$$

where the cross section $d\sigma/dp_T$ is given by the basic minijet model (8). The matter overlap function $\mathcal{O}(b)$ is

$$
\mathcal{O}(b) \propto \int dt \int d^3r \rho(x,y,z) \rho(x+b,y,z+t), \quad (12)
$$

where $\rho$ is the matter distribution in a single hadron which in recent PYTHIA version is exponential by default. The default tune has no $x$ dependence in the matter distribution. However, it is possible to choose a non-standard setting where the width of the Gaussian does depend on $x$ as $1 + a_1 \log(1/x)$. The average $\langle \mathcal{O} \rangle$ is defined in a special way taking into account that every event has to have at least one collision, see the original papers for details.

The actual number $n$ of binary collisions is determined by the truncation of the iterative procedure when no further emissions can be resolved from Eq. (10).

The effect of the interleaved evolution (10) is most important for the initial state shower and MPI. This is because they compete for the beam energy. The actual correlations are incorporated by modifying the beam remnant in regards to the remaining longitudinal momentum and flavour.

### 4.2.1 Minijet correlations in Pythia

It is instructive to study how the $\langle p_{T\text{rec}} \rangle$ distribution defined in Section 2 depends on the crucial parameters which change the way minijets and MPI are generated. Recall, that in the most simple MPI model, when the hard collisions are completely uncorrelated, contribution to $\langle p_{T\text{rec}} \rangle$ from independent systems cancels out. Hence, only contribution of dijet to which the trigger belongs survives. Since in PYTHIA correlations are present, the distribution will be sensitive to the MPI mechanism. For the purpose of this study, we choose one of the standard PYTHIA tunes (Monash [8]), in which we will play with MPI on/off feature and modify the $p_{T0}$ parameter.

The result with MPI feature on and off is presented in Fig. 5. We also studied the effect of changing the $p_{T0}$ parameter. The simulation was performed with full hadronization. In order to make it possible to connect the present simulations to the experimental measurements and our main results in Section 5, we used only charged particles and required all particles to have $p_T > 0.25$ GeV. Removal of very soft charged particles does not change the conclusions.
Figure 5: $\langle p_T^{\text{rec}} \rangle$ as a function of rapidity in Pythia with hadronization, full beam remnant treatment, initial and final state showers. We study effect of MPI on/off in the simulation. We also change the standard parameter for the $p_{T0}$ (2.4 GeV) cutoff in the MPI model to higher value (4.0 GeV) to observe how this affects the distribution.

First, we observe that the change of $p_{T0}$ for the no-MPI scenario has very little effect. This was already observed in Section 4.1 for the basic perturbative minijet model. Second, for the standard $p_{T0} = 2.4$ GeV the effect of turning on the MPI feature is dramatic. The distribution is scaled down by a factor of about 0.6. If the $p_{T0}$ cutoff is raised to 4 GeV, the scaling factor is only about 0.9.

The above results suggest that: (i) the distribution $\langle p_T^{\text{rec}} \rangle$ is very sensitive to the MPI (at least in the Pythia model), (ii) the $p_{T0}$ cutoff is tightly connected to the number of MPI generated (as one should expect). The second point can be directly illustrated by an explicit calculation. In Fig. 6 we show how the mean value of the MPI number changes when we change $p_{T0}$. We see that for the Monash tune with $p_{T0} = 2.4$ GeV the average number of MPI is more than 10. For $p_{T0} = 4.0$ GeV it narrows down to something between 1 and 2.

The reason the $\langle p_T^{\text{rec}} \rangle$ is sensitive to MPI in Pythia can be understood with the help of the following calculation. We switch off the hadronization and use the algorithm that groups the final states with respect to the parent hard process. Then we calculate what is the contribution of subsequent hard collisions to $\langle p_T^{\text{rec}} \rangle$. The results are shown in Fig. 7. We see, that due to $p_T$ ordering of MPI, the $\langle p_T^{\text{rec}} \rangle$ for the subsequent hard collisions is scaled down more and more. Note, that if the trigger belonged to the system with $N_{\text{hard}} = i$, all contributions to $\langle p_T^{\text{rec}} \rangle$ from $N_{\text{hard}} < i$ is zero. Now, the question is how often the trigger belongs to various hard sub-systems. This is answered explicitly by the calculation presented in Fig. 8.

We see that the trigger often originates from the not hardest parton interaction ($N_{\text{hard}} > 1$) that scales down the $\langle p_T^{\text{rec}} \rangle$ distribution. This is a genuine effect of MPI correlations in Pythia caused by ordering of the binary parton collisions. The other correlations have much weaker effect.

In Pythia, in general, the transverse momentum is not conserved in the individual hard parton collisions (but of course it is conserved for the whole event). The idea is schematically illustrated in Fig. 9.

In order to see this explicitly we use again the algorithm to group the final state particles
Figure 6: The distribution of number of parton interactions $N_{\text{MPI}}$ for different settings of $p_{T0}$ cutoff. The events with $N_{\text{MPI}} = 0$ are diffractive events.

Figure 7: $\langle p_{T\text{rec}} \rangle$ decomposed into contributions from subsequent (in $p_T$) hard collisions with $N_{\text{hard}} = 1, 2$ and 3 or more. The grouping into hard subsystems was done without hadronization.
Figure 8: Distribution of a trigger particle among the hard subsystems produced by the MPI mechanism. The subsystems are ordered according to the $p_T$ of the hard process.

Figure 9: Two hard collisions with transverse correlations schematically represented by the transverse momentum exchange between incoming partons. The total transverse momentum does not sum up to zero for each subsystem, but overall is conserved.
before hadronization into groups belonging to different hard process and beam remnants as a separate class. Tracing the final state hadrons back to the hard process is not possible in a unique way. The hard collisions are enumerated $N_{\text{hard}} = 1, 2, \ldots$ from hardest to softest, with $N_{\text{hard}} = 0$ reserved for the beam remnants.

In Fig. [10] we show the $\langle p_T \rangle$ (defined as before but now we do include the trigger) as a function of $N_{\text{hard}}$. We see that indeed there are transverse momentum correlations between MPI. We check that they are generated by the primordial $k_T$ mechanism, that is if the mechanism is switched off, the result for $\langle p_T \rangle (N_{\text{hard}})$ is approximately 0 everywhere.

Note, that our observable $\langle p_T \rangle$ is not sensitive to the direct transverse momentum correlations discussed above.

4.3 Herwig model

The MPI model used in HERWIG has been reviewed several times [1,33–35]. Here we aim just to describe briefly the most important building blocks and the parameters of the model.

The model is formulated in the impact parameter space. At a fixed impact parameter, multiple parton scatterings are assumed to be independent, however, later they are correlated, for example, by imposing energy-momentum conservation or through the colour reconnection mechanism. There are two types of parton-parton scatterings in the model, soft and semi-hard, the both are separated by a transverse momentum scale $p_{\perp}^{\min}$, which is one of the main tuning parameters in the model. We allow the value of $p_{\perp}^{\min}$ to vary with energy and the evolution is govern by a power law, see Eq. [9]. In fact, it is $p_{\perp,0}^{\min} = p_{\perp}^{\min}(7 \text{ TeV})$ and power $\lambda$ that we fit to data. Below $p_{\perp}^{\min}$, scatters are assumed to be non-perturbative, with valence-like longitudinal momentum distribution and “Gaussian” transverse momentum distribution, see right panel of Fig. [9] for two examples how the extrapolation to non-perturbative region can be realized in the model. Above $p_{\perp}^{\min}$, scatters are assumed to be perturbative, and take place according to leading order QCD matrix elements convoluted with inclusive PDFs and an overlap function $A(b)$:
\[ A(b) = \int d^2b_1 G(b_1) \int d^2b_2 G(b_2) \delta^2(b - b_1 + b_2), \tag{13} \]

where

\[ G(b) = \frac{\mu^2}{4\pi} \left( \frac{\mu b}{1-t/\mu^2} \right) K_1(\mu b) \tag{14} \]

is Fourier transform of dipole form factor \(\frac{1}{(1-t/\mu^2)}\) which leads to overlap function

\[ A(b) = \frac{\mu^2}{96\pi} (\mu b)^3 K_3(\mu b), \tag{15} \]

both \(G(b)\), and \(A(b)\) are normalised to unity. \(K_1(x)\) is the modified Bessel function of the \(i\)-th kind and \(\mu\) is the another important parameter of the model which governs the transverse distribution of partons in the proton and can be interpreted as an effective inverse proton radius. A lower values of \(\mu\) lead to the broader matter distribution, therefore higher probability of peripheral collisions. Soft scatters might see a different matter distribution, therefore we allow them to have a different inverse radius \(\mu_{\text{soft}}\) but we keep the functional form of the overlap function from Eq. \(15\). The two soft MPI parameters \(\mu_{\text{soft}}\) and \(\sigma_{\text{soft}}\), the non-perturbative cross section below \(p_{\perp}^{\text{min}}\), are fixed by the inelastic hadron-hadron cross section and the b-inelastic slope parameter, therefore they are not free parameters of the model.

The probability distribution of number of scatters is Poissonian at a given impact parameter, but the distribution over impact parameter is a considerably broader than Poissonian. The number of soft and hard scatters is chosen according to this distribution and generated according to their respective distributions. Each hard scatter is evolved back to the incoming hadron according to the standard parton shower algorithm, therefore the evolution is not interleaved like in Pythia model and additional scatterings are not ordered in any kinematic.
variable. Energy-momentum conservation is imposed by rejecting any scatters that take the total energy extracted from the hadron above its total energy. As mentioned before the individual scatters might be colour correlated using a colour reconnection model, described in detail in Ref. [24], in that model a reconnection probability $p_{\text{reco}}$ is applied. To summarize there are four main parameters of the model: $p_{\perp,0}^\text{\ min}$, $\Lambda$, $\mu^2$ and $p_{\text{reco}}$, which are fitted to the experimental data. Unlike Pythia, Herwig does not have a large family of tunes, usually no more than one tune is released with a new version of the program. Therefore, in order to study in a meaningful way effect of the parameters variation in HERWIG we decided to use the two tunes prepared for the same version of the program [9]. The both tunes, which we label by Var1 (the default tune of HERWIG++ 2.7) and Var2, provide good description of the UE data over the collision energy range from 300 GeV to 7 TeV. This is visualized in the left panel of Fig. 11 (see [9] for details), where we show the $\chi^2/N.d.f.$ value of the fit as a function of $p_{\perp,0}^\text{\ min}$ and $\mu^2$, the both tunes are marked by black (Var1) and white (Var2) dots. A visible strong correlation between $p_{\perp,0}^\text{\ min}$ and $\mu^2$ (a long thin blue valley in Fig. 11) reflects the fact that a smaller hadron radius means more likely central collisions and as a consequence more multiple scattering, which can be compensated to give a similar amount of underlying-event activity by having fewer perturbative MPIs, i.e. a larger value of $p_{\perp,0}^\text{\ min}$. The best fit value is $p_{\perp,0}^\text{\ min} = 2.80 \text{GeV}$, $\mu^2 = 1.65 \text{GeV}^2$ (Var2), but one can obtain good fits for higher $p_{\perp,0}^\text{\ min} = 3.91 \text{GeV}$, $\mu^2 = 2.3 \text{GeV}^2$ (Var1). As one can see from Fig. 11 (right-panel) the $p_\perp$ spectra looks significantly different for these two tunes$^2$ therefore they are well suited for our studies. It is worth to mention the default HERWIG++ 2.7 tune gives value of $\sigma_{\text{\ eff}} = 14.8 \text{ mb}$, which is closer then $\sigma_{\text{\ eff}} = 20.6 \text{ mb}$ from Var2, to $\sigma_{\text{\ eff}}$ obtained from the combination of the two most precise results for this observable from CDF and D0 measurements $\sigma_{\text{\ eff}} = (13.9 \pm 1.5) \text{ mb}^3$.

Finally, we will also show results of HERWIG 7 which has new model for soft interactions including diffractive final states and multiple particle production in multiperipheral kinematics, see [38] for the details.

### 4.4 Sherpa model

Minimum bias events in Sherpa [5] are simulated using the Shrimps package [39, 40] which is based on Khoze-Martin-Ryskin (KMR) model [41]. The KMR model is a multi-channel eikonal model in which the incoming hadrons are described as a superposition of Good-Walker states, which are diffractive eigenstates that diagonalize the T-matrix. Each combination of colliding Good-Walker states gives rise to a single-channel eikonal. The final eikonal is the superposition of the single-channel eikonals. The number of Good-Walker states is 2 in Shrimps (the original KMR model includes 3 states). Each single-channel eikonal can be seen as the product of two parton densities, one from each of the colliding Good-Walker states. The evolution of the parton densities in rapidity due to extra emissions and absorption on either of the two hadrons is described by a set of coupled differential equations. The parameter $\Delta$, which can be interpreted as the Pomeron intercept, is the probability for emitting an extra parton per unit of rapidity. The strength of absorptive corrections is quantified by the parameter $\Lambda$, which can be related to the triple-Pomeron coupling. A small region of size $\Delta Y$ around the beams is excluded from the evolution due to the finite longitudinal size of the parton densities.

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$^2$For all other parameters of the tunes see the Appendix.

$^3$The most of the $\sigma_{\text{\ eff}}$ agrees with this value, however it is worth noting that for example analysis of the exclusive photoproduction of $J/\psi$ [37] suggests larger values of $\sigma_{\text{\ eff}}$. 

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The boundary conditions for the parton densities are form factors, which have a dipole form characterized by the parameters $\Lambda^2$, $\beta_0^2$, $\kappa$ and $\xi$. In this framework the eikonals and the cross sections for the different modes (elastic, inelastic, single- and double-diffractive) are calculated.

Inelastic events are generated by explicitly simulating the exchange and re-scattering of gluon ladders. The number of primary ladders is given by a Poisson distribution whose parameter is the single-channel eikonal. The decomposition of the incoming hadrons into partons proceeds via suitably infrared continued PDFs.

The emissions from the ladders are then generated in a Markov chain. The pseudo-Sudakov form factor contains several factors: an ordinary gluon emission term, a factor accounting for the Reggeisation of the gluons and a recombination weight taking absorptive corrections into account. The emission term has the perturbative form $\alpha_s(k_T^2)/k_T^2$, that, as we have already seen in PYTHIA and HERWIG models needs to be continued into the infrared region. In SHERPA in the case of $\alpha_s$ the transition into the infrared region happens at $Q^2$ while in the case of $1/k_T^2$ the transition scale is generated dynamically and depends on the parton densities and is scaled by $Q_0^2$.

The propagators of the filled ladder can be either in a colour singlet or octet state, the probabilities are again given through the parton densities. The probability for a singlet can also be regulated by hand through the parameter $\chi_S$. A singlet propagator is the result of an implicit re-scattering.

After all emissions have been generated and the colours assigned, further radiation is generated by the parton shower. The strength of radiation from the parton shower can be regulated with $K_{T\text{-Factor}}$, which multiplies the shower starting scale. After parton showering partons emitted from the ladder or the parton shower are subject to explicit re-scattering, i.e. they can exchange secondary ladders. The probability for the exchange of a re-scattering ladder is characterised by $\text{RescProb}$. The probability for re-scattering over a singlet propagator receives an extra factor $\text{RescProb}_1$. After all ladder exchanges and re-scatterings but before hadronization colour can be rearranged in the event. Finally, the event is hadronized using the standard Sherpa cluster hadronization. In our studies we used the default settings of Multiple Interaction Models in Sherpa 2.2.2, for completeness we list the Shrimps parameters in the Appendix B.

5 Results

In this Section we show comparisons of distributions for the observables defined in Sec. 2 for few recent versions and tunes of PYTHIA, HERWIG, and SHERPA. Since some model parameters depend on $\sqrt{s}$, including the suppression of jet cross-section at very low $p_T$, results are presented for 7 and 13 TeV. For the analysis we use non-diffractive inelastic events. Two approaches to the trigger object are studied, first using a single charged particle as a trigger, second using a charged-particle jet as a trigger. The former is more model dependent, but is less affected by UE contribution. It is worth noting, that the latter also provides a better connection to the parent parton.

Fig. 12 shows $p_T^{\text{rec}}$ as a function of pseudorapidity at $\sqrt{s} = 13$ TeV for two techniques. In the trigger region (near $2 < \eta < 2.4$), in the case of charged particle trigger (left panel) we see a peak that is mostly caused by particles strongly correlated to a triggered particle (which would be clustered to the same jet), in the case of jet trigger the particles clustered in to the trigger jet are excluded from the calculation of $p_T^{\text{rec}}$, see Eq. 1, therefore we see a dale. In the regions
Figure 12: Rapidity correlation of recoiled system with respect to single-charged particle with $1.5 \leq p_T < 2$ GeV (a) and with respect to charged-particle jet with $3 \leq p_T < 3.5$ GeV (b) at $\sqrt{s} = 13$ TeV.

distant ($\eta < 0$) from the trigger object, one can see that for the Pythia model the difference between approaches is modest. This is expected since $p_T$ ranges of different trigger objects correspond on average to the same $p_T$ of the parent gluon (see Fig. 1). However, differences between techniques are more pronounceable for the other models. This is probably due to the different fragmentation and hadronization. A peculiar feature of Herwig 7 is that it generates a recoil peak that lies at opposite $\eta$ region with respect to the trigger object, this seems to be a feature of new Soft MPI model [38], which we plan to investigate more in the future. Finally, it is worth to notice that Sherpa produces the least amount of long-range correlations.

Let us turn to the discussion of dependence of the rapidity correlation on a collision energy. Fig. 13 shows a comparison of the observable at $\sqrt{s} = 7$ TeV and $\sqrt{s} = 13$ TeV. All presented models show increased yield of particle associated with the trigger jet with collision energy, while recoil is reduced in the most of studied $\eta$ range. However, the $p_T^{\text{rec}}$ distribution for $\sqrt{s} = 7$ TeV and $\sqrt{s} = 13$ TeV converge in $\eta$ regions distant from the trigger.

The correlation studied at different $N_{\text{ch}}$ reflects transverse structure of colliding protons. The $N_{\text{ch}}$ is here defined as a number of stable charged particles with $p_T > 250$ MeV and $|\eta| < 2.4$. Here, we choose a single charged particle as the trigger. This is motivated by increasing with $N_{\text{ch}}$ UE contribution into a jet cone. For instance, the average $p_T$ density of charged particles is roughly 3 GeV per square unit at $N_{\text{ch}} = 100$. That can contribute as half of trigger jet momentum. Fig. 14 shows comparison of the $p_T^{\text{rec}}$ distribution in different $N_{\text{ch}}$ domains at $\sqrt{s} = 13$ TeV for various MC models. Pythia and Herwig models exhibit fast increase up to $N_{\text{ch}} \approx 60$ and then a saturation. Such behaviour mimics the transverse $p_T$ density in the UE analyses [10][12]. At the same time Sherpa model shows continuous increase of peak along the trigger direction and recoil. The differences between two tunes of Herwig++ 2.7 could be explained by differences in the transverse proton structure in the tunes, which is conventionally characterized by $\sigma_{\text{eff}}$. The peak along the trigger particle is expected to be higher for lower
σ_{eff} and indeed this is confirmed by the Herwig++ results in Fig. 14 where tune Var1 has smaller σ_{eff} (14.8 mb) compared to tune Var2 (20.6 mb). For the Pythia CUETP8M1 σ_{eff} is 26 mb as obtained in the inclusive 4-jet production [7], and indeed the Pythia tune shows results that are close to Herwig Var2.

The color reconnection effect which is one of the least-understood elements of MPI models. Therefore, it is naturally to test whether the proposed observable is sensitive to the CR. Fig. 15 shows results of switching the CR off in Pythia 8 and Herwig models. The former shows most significant differences near the trigger, while the rapidity correlations almost converges at η = −1. For the Herwig++ 2.7 the effect is qualitatively similar, however the versions with CR and CR-off do not converge within studied η-range. Such behaviour can be qualitatively explained by the fact that the trigger jet “absorbs” softer jets during the CR procedure.

6 Summary and conclusions

We have introduced a new observable which probes interplay between soft and hard physics at moderate \( p_T \) via probing long and short range rapidity correlations of transverse momenta of charged particles/minijets. The basic idea is to study how the transverse momenta of hadrons produced in association with a trigger object are balanced as a function of rapidity (the precise definition is given in Section 2). It is shown that the observable is sensitive to basic mechanisms and components used in the present MC models, such as a suppression of low-\( p_T \) jet production, parton distribution functions, a transverse geometry of proton, a color reconnection mechanism, and their evolution with collision energy. We demonstrated that predictions of different MC models which described well many characteristics of the hadron production at LHC differ significantly for suggested observable. The most prominent discrepancy between models appears when the correlation is studied as a function of charged-particle multiplicity. It is important to stress that changing of parameters within a single model results in the expected changes for the measured distribution. Therefore, proposed measurements can help to disentangle various mechanisms relevant for minijet production. It is worth mentioning that our tests have revealed quite peculiar features of Herwig 7 and Sherpa 2.2.2 models.

Moreover, it would be highly useful to perform the proposed tests with MC models that use new NNPDF 3.1 [42], especially NNLO PDFs. These PDFs are measured down to very low \( x \) and with a much better precision. Such tunes of Pythia 8 model are under intensive development by CMS collaboration [43]. Therefore, we will update our study as soon as the tune will become publicly available.

We performed our tests taking into account performance of general purpose detectors at LHC such as ATLAS and CMS. Hence, one may hope that prompt experimental studies of the quantities we calculated will be possible. The data necessary for proposed study are available from low pileup LHC runs. Standard amounts of minimum bias data, that are usually few ten million events, are enough for the measurement, however special trigger is desired.

Obviously, the discussed correlations are sensitive to the various collective effects. Hence, it would be also interesting to study such correlations also in \( pA \) and \( AA \) scatterings.

The proposed measurements can be extended by using as trigger particles two hadrons with azimuthal angle difference \( \phi_1 - \phi_2 \sim \pi/2 \). One would measure \( \langle k(\phi_1, \Delta y_1) \rangle \) and \( \langle k(\phi_2, \Delta y_2) \rangle \) and compare the results with the measurements of the same quantities with one trigger particle which we studied in this paper. We expect that such an observable would have an enhanced sensitivity to the contribution of the multiparton interactions and collective effects.
Figure 13: Comparison of rapidity correlation of recoiled system with respect to charged-particle jet with $3 \leq p_T < 3.5$ GeV at $\sqrt{s} = 7$ TeV and $\sqrt{s} = 13$ TeV for different models.
Figure 14: Comparison of rapidity correlation of recoiled system with respect to single charged particle with $1.5 \leq p_T < 2$ GeV at $\sqrt{s} = 13$ TeV in different $N_{ch}$ domains for various MC models.
Figure 15: Effect of color reconnection in PYTHIA 8 and HERWIG++ 2.7 models.

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A Herwig++ Parameters

| parameter          | Only UE data in fit | UE data and $\sigma_{\text{eff}}$ in fit |
|--------------------|---------------------|------------------------------------------|
| $\mu^2 \ [GeV^2]$  | 1.65                | 2.30                                      |
| $p_{\text{disrupt}}$ | 0.22                | 0.80                                      |
| $p_{\text{reco}}$  | 0.60                | 0.49                                      |
| $p_{\perp}^{\text{min}} \ [GeV]$ | 2.80                | 3.91                                      |
| $b$                | 0.29                | 0.33                                      |

Table 1: Parameters of the underlying event tunes. The last two parameters describe the running of $p_{\perp}^{\text{min}}$ according to Eq. [9].

B SHERPA Parameters

| parameter          | value               |
|--------------------|---------------------|
| $SOFT\_COLLISIONS$ | Shrimps             |
| $Shrimps\_Mode$    | Inelastic           |
| $\Delta Y$         | 1.50                |
| $\Lambda^2$        | 1.376               |
| $\beta_0^2$        | 18.76               |
| $\kappa$           | 0.6                 |
| $\xi$              | 0.2                 |
| $\lambda$          | 0.2151              |
| $\Delta$           | 0.3052              |
| $Q_0^2$            | 2.25                |
| $\chi_S$           | 1.0                 |
| $Shower\_Min.K_T^2$| 4.0                 |
| $Diff\_Factor$     | 4.0                 |
| $K_T^2\_Factor$    | 4.0                 |
| $RescProb$         | 2.0                 |
| $RescProb1$        | 0.5                 |
| $Q_{\text{RC}}^2$  | 0.9                 |
| $ReconnProb$       | -25                 |
| $Resc.K_T,\text{Min}$ | off              |
| $Misha$            | 0                   |

Table 2: Parameters of the SHERPA 2.2.2 model for the production of minimum bias events.
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