Clearing and Pricing for Coordinated Gas and Electricity Day-ahead Markets Considering Wind Power Uncertainty

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Abstract—Natural gas and electricity systems are becoming increasingly strongly coupled. Gas-fired units (GFUs) are replacing retired coal plants, and the power systems are more dependent on the flexibilities provided by GFUs. The GFUs’ power generation capability relies on the availability of gas resources, which is jointly determined by the capacity of gas suppliers and pipeline networks. However, the gas and electricity markets are operated separately. Consequently, the GFUs are forced to “represent” the entire power system to bid on the gas market; they must make forecasts regarding future gas consumption and bear the risk of improper contracts or being unable to meet generation schedules due to occasional insufficient gas supply. When facing larger shares of renewable energies and more-frequent gas network congestions, the current market framework is particularly unreliable and inefficient, as well as economically unfriendly to the investors of the GFU assets. In this paper, we try to develop a framework that can combine the two markets. By properly pricing the scarce resources, e.g., gas transmission capacity, the joint market can help allocate the resources more efficiently while satisfying the demands. Moreover, a more forward-looking day-ahead market clearing framework is presented by considering the uncertainty brought by renewable energies. The formulation and algorithm of the proposed joint market model will be presented, as are some case studies.

Index Terms—day-ahead market, gas and electricity coordination, wind power, unit commitment, stochastic programming.

NOMENCLATURE

A. Indices, parameters, sets and functions

\( x_{NG}, \lambda^e \) Value of lost natural gas and electric load.

\( \omega, \Omega \) Index and set of the scenarios.

\( \rho \) Weight of the scenarios.

\( \phi^u(\cdot) \) Bus where a unit is located.

\( \phi^w(\cdot) \) Bus where a wind farm is located.

\( \beta(\cdot) \) Gas consumed by a compressor as a function of power output.

\( \Psi(n) \) Set of gas system nodes that are connected to node \( n \).

\( \Xi(n) \) Set of gas suppliers that are located at node \( n \).

\( \Lambda(n) \) Set of gas loads that are located at node \( n \).

\( \Omega(n) \) Set of gas-fired units that are connected to gas node \( n \).

\( \phi(n) \) Set of gas compressors that take gas from node \( n \).

\( Y(g) \) Power system node at which a unit is located.

\( \Gamma \) Generation shift factor matrix.

\( a_{com} \) Parameters of the gas consumption function of a compressor.

\( B^v, B^{GFU} \) Gas node to gas load and gas-fired unit incidence matrix.

\( C^v, C^d \) Startup and shutdown costs of a unit.

\( d, D \) Index and set of the gas loads.

\( f(\cdot) \) Piece-wise generation fuel cost function of the units.

\( f^{fg}(\cdot) \) Piece-wise fuel characteristics of the gas-fired units.

\( G, G, G^{NG} \) Index and set of conventional and natural gas units.

\( \bar{H}, H \) Maximum and minimum power output of a compressor.

\( i, I \) Index and set of the added gas transmission feasibility cuts.

\( j, J \) Index and set of natural gas suppliers.

\( k, W \) Index and set of wind farms.

\( l, L \) Index and set of natural gas loads.

\( L^e, L^g \) Electricity and natural gas load profiles.

\( m, M \) Index and set of power system nodes.

\( n, N \) Index and set of gas system nodes.

\( \bar{p}^{g}, \bar{g} \) Maximum and Minimum output of a unit.

\( q, Q \) Index and set of power transmission lines.

\( \text{ramp} \) Ramping rate limit of a unit.

\( \hat{s} \) Maximum capacity of a gas supplier.

\( t, T \) Index and set of time periods.

\( T_{on}, T_{off} \) Minimum on and off times of the units.

B. Variables

\( \pi \) Gas pressure at a gas network node.

\( \zeta \) Value of natural gas to the power system implied by the locational marginal electricity price.

\( \eta^e \) Locational marginal electricity price.

\( \eta^{ng} \) Gas price.

\( \eta^{ng, cap} \) Locational natural gas capacity price.

\( \varepsilon \) Marginal gas consumption rate of a unit.

\( v^u, v^d \) \{0,1\}, startup and shutdown state variables of a unit.

\( v \) \{0,1\}, on/off state variable of a unit.

\( h \) Power output of a gas transmission compressor.

\( \Delta^{NG} \) Scheduled gas load shifting.

\( \Delta^e \) Scheduled electricity load shifting.

\( p \) Scheduled output of a unit.

\( p^{w} \) Scheduled output of a wind farm.

\( s, s^g \) Scheduled output of a gas supplier.

\( w \) Gas consumed by a GFU.

\( x \) Booked gas transmission capacity by a GFU.

\( y \) Gas flow in a gas transmission branch.

I. INTRODUCTION

According to the Energy Information Administration [1], the share of gas-fired units (GFUs) in the U.S. electricity supply has increased since 2001, reaching a historically high level of 32.7% in 2015. At the same time, the power sector consumed 35.4% of the annual natural gas supply, compared to 30.7% in 2010 and 22.3% in 2000. The two systems have never
been as strongly coupled as they are today, and this trend is continuing. The Paris Protocol is believed to have accelerated the retirement of coal plants. From both the environmental and economic perspectives, GFUs, which have a lower CO₂ emission rate than coal plants, are among the best substitutes.

Moreover, the deepening penetration of wind and solar power has put increasing pressure on the operational flexibility of power systems. In addition to GFUs, flexibility can also be provided by electric storages and hydro units. However, storage systems are still expensive, and hydro reservoirs are not always available. GFUs have comparative advantages in this context. The International Energy Agency has predicted that GFUs will become more attractive as the share of wind increases [2].

GFUs act as the interface of gas and power systems. The supply, demand and network capacity of one system may have impacts on the other system via GFUs. However, the two systems are largely operated independently at present [3]. Even in the most de-regulated environment, each of the two markets is cleared given the status of the other as boundary conditions [4], which are actually determined by the GFUs. GFUs sign contracts in the gas market based on their forecast of future production, and they also bid in the electricity market within their generation capacity, which is dependent on the availability of gas resources. The coordination efficiency of the two markets largely relies on the GFUs’ performance [5].

However, GFUs have access to only local information; thus, the optimality of the bids is not guaranteed [6]-[7]. Improper bids by GFUs will not only distort the resource allocation but also reduce the GFUs’ own profit. This problem is particularly serious when the gas or power network capacity is scarce and shared by multiple parties. The problem can be resolved by improved coordinated planning of the two systems [8]-[9]. However, a more realistic solution is to modify the current operation framework. Some co-ordinative dispatch and market clearing approaches have been proposed [10]-[15]. In [10]-[11], the authors assemble the gas network constraints into the optimal power flow (OPF) models, which results in complex nonlinear programming problems. The problems are solved using an evolutionary algorithm or the primal-dual interior-point method. In [12], a Benders decomposition approach is applied and the nonlinear gas network constraints are decoupled from the unit commitment (UC) problem. The algorithm shows good efficiency. Similarly, a Lagrangian relaxation method and a fuzzy optimization approach are presented in [13] and [14]. Whereas the above papers only focus on steady-state scheduling, reference [15] analyzes the impacts of gas system dynamics, which should be considered in intra-day scheduling.

In fact, GFUs can learn how to properly bid by trial and error if the status of the systems can be accurately predicted. The need for coordination is not urgent with well-experienced market players. However, the systems’ status becomes highly uncertain and a spontaneous efficiency enhancement becomes infeasible with a significant share of intermittent resources. Due to the underlying risks, investors are less inclined to invest in GFU assets, which in turn impedes the improvement in system flexibility. Researchers have attempted to include the uncertainty into the above coordination approaches to address this problem. A stochastic-programming-based coordinative scheduling tool is presented in [16], in which wind power is described by scenarios. The model can also be formulated as interval [17] or robust optimization problems [18] by modeling the uncertainty using intervals or sets. Moreover, reference [19] shows that a strategic bidding strategy of GFUs can also be beneficial within the current market framework.

Following the notion that scarce resources should be properly priced for more efficient allocation, this paper aims at developing a non-deterministic coordinated gas and electricity market (ND-CGEM) model for day-ahead (DA) clearing. The model is scenario-based and inspired by the studies on stochastic electricity market clearing approaches [20]-[21]. The top priorities include that the GFUs should be free from guessing their future production or bearing the risks from the gas supply, and the value of sufficient and timely gas supply for the power systems should also be reflected in the gas market. Moreover, real-time (RT) scheduling of the gas loads is often impossible or rather costly because the loads are typically less flexible in terms of response time. Therefore, the schedule should be made in advance if some gas loads are to be shifted to reserve capacity for potential needs of the power system in a congested gas network. The above components will be reflected in the proposed model. In summary, the contribution of this paper is multi-fold:

1) Electric power generation and gas supply are coordinated in contrast to the current segregated practice.
2) A coordinated market framework is proposed with effective pricing and market clearing mechanisms for both electricity and natural gas.
3) The uncertainty of renewable energies such as wind and solar is included considering of the inherent heterogeneity of gas and electric supply/networks in terms of timescale and contractual arrangements.

The remainder of this paper is organized as follows: in Section II, we describe the concepts, assumptions and framework for the ND-CGEM. In Section III, we present the mathematical formulation and solution of the model. Cases studies are presented in Section IV, followed by the conclusions and discussions in Section V.

II. CONCEPTS, ASSUMPTIONS AND FRAMEWORK

In a de-regulation environment, the DA electricity market is typically cleared by solving a security-constrained unit commitment (SCUC) problem. Constrained system balance is the key feature of the SCUC, and the balance is modeled in terms of instantaneous power rather than energy (in DA market, “instantaneous” typically means “hourly”).

In contrast, the upstream natural gas market, is dominated by volume contracts. In other words, the concept of instantaneous balance does not apply. Although long-term contracts are still the most common type of contract, the share of spot and short-term gas trading has been increasing over the past 20 years [22]. Short-term contracts offer the consumers greater flexibility than the traditional long-term take-or-pay contracts, reducing the cost of resource re-allocation and making the gas
market more synchronized to the electricity market.

When gas network congestion is not common or the time criticality of the gas demands is low, the hourly gas balance is indeed not a major concern. However, according to the Federal Energy Regulatory Commission, the competition between heating and electricity companies for gas has caused congestion in the pipelines, leading to delays and spikes in the price of electricity, particularly in winter [23]. A similar issue was reported by ISO New England [24]. When GFUs account for a significant share of power production, timely gas delivery could be crucial to the power system. Therefore, hourly gas balance and network limits should be considered. Network limits can also be resolved by deploying gas storage facilities [25]. However, those facilities are also costly, which motivates us to develop a framework that helps utilize the limited resources in the most efficient manner.

To provide electricity and other ancillary services, GFUs must compete with other gas loads, mainly heating loads, for the resources (gas suppliers and networks). Traditionally, residential loads are assigned higher priority [26], whereas industrial users may choose between firm and interruptible contracts (GFUs typically sign interruptible contracts because of their lower rate). The framework is intended to encourage the provision of flexibility by energy-intensive gas consumers. However, the categorization is rough and static, which is inflexible and occasionally leads to inefficiency. In fact, some gas loads, though with a relatively long response time, are actually well adjustable if informed with adequate lead time. Meanwhile, the flexibility provided by GFUs will be crucial when the power system experiences a large wind power fluctuation; therefore, the power system can be a less elastic gas consumer at particular times. However, such situations cannot be properly addressed by the existing market frameworks because there is no suitable channel for communicating such information dynamically and efficiently. GFUs are definitely not ideal channels because they do not have access to adequate information and the risks of participating in sequentially cleared gas and electricity markets provide false information and cannot encourage them to act as active coordinators.

Instead of allocating resources following a pre-defined priority list, it could be more efficient to price and allocate them dynamically based on the supply and demand. However, a more sophisticated gas market is not sufficient because GFUs do not reflect all market dynamics; instead, they only represent a portion of the generation portfolio. It is also strongly recommended by PJM that operational information be shared between the two systems [27].

All of the above problems naturally lead to the development of a joint electric-gas market framework. In such a framework, (1) gas system resources are allocated considering the condition of the connected power system; (2) the GFUs no longer need to bid in both separate markets and are exposed to far fewer risks; and (3) all of the costs incurred in scheduling gas system resources for the power system are modeled explicitly and do not have to be borne by the GFUs first and then transferred to the end users. Therefore, we propose an ND-CGEM model below.

Before formulating the model, we list some basic assumptions.

**Assumption 1**: The gas loads other than GFUs are categorized into two groups: high-priority gas loads (i.e., residential loads and firm industrial loads) and low-priority gas loads (i.e., interruptible industrial loads). The priority is modeled based on elasticity. Whereas low-priority loads can be curtailed with a moderate amount of compensation, high-priority loads are nearly inelastic.

**Assumption 2**: There is no RT natural gas market. Gas load schedules and network capacity reservation should be made at the DA stage. Moreover, the GFUs can have access to only the gas supply within the reserved capacity.

**Assumption 3**: Similar to the take-or-pay contract, there is a penalty for deviating from the DA gas contracts.

**Assumption 4**: Only the uncertainty of wind power is modeled in this paper. Although uncertainties from other sources can also be described jointly by the scenarios, we suggest that a trade-off be made between modeling accuracy and computational burden.

III. ND-CGEM MODEL FORMULATION AND SOLUTION

A. Overall design of the ND-CGEM model

The ND-CGEM model is modeled as a stochastic programming problem. The problem aims to minimize the expected costs of the entire system and to derive prices for the resources. Fig. 1 shows the overall design of the model. The single DA scenario represents the DA settlement, while the simulated RT scenarios are used to estimate the expected costs and are modeled in a similar manner as the DA one.

Fig. 1. Overall design of the ND-CGEM model.

Compared to the SCUC model which is typically a mixed-integer linear programming (MILP) problem, the non-convex gas system constraints make the model more complex. First, even when the integer variables are fixed, the optimization problem is still non-convex and does not have dual variables, without which the prices cannot be defined. Second, solving a mixed-integer programming with non-convex components other than integrality is extremely time consuming, particularly when the non-convex constraints are duplicated for multiple scenarios. This is unacceptable for practical use. Therefore, two modifications are applied to the model. (1) The non-convex gas system constraints are removed from the problem using the general Benders decomposition (GBD) technique. The non-convex constraints, if binding, will be represented by some linearized cuts generated by the sub-problems (as described below). (2) Instead of applying the gas network constraints to each of the RT scenarios, the constraints are used only to limit the network capacity allocated in the DA market (see Assumption 2), and the RT gas consumption is simply constrained by
the booked capacity.

B. Detailed formulation

1) Objective function

The objective function describes the out-of-pocket costs of the entire system, including the UC costs, fuel costs, gas load shedding costs, gas supply deviation fees and electric load shedding costs. The DA scenario is assigned a small weight ($\rho \ll 1$). The objective function is formulated as

$$\min \sum_{\text{gen}} \left(1 + \rho \right) \left[ \sum_{\text{gen}} \left( C^{u}_i \Delta t^{ui} + C^{d}_i \Delta t^{di} \right) + \sum_{\text{node}} \left( L_{\text{g}} \Delta t \right) \right]$$

$$+ \sum_{\text{node}} \rho_{\text{f}} \sum_{\text{gen}} f(p_{\text{gen}}) + \sum_{\text{node}} \Delta t^{\text{me}} + \sum_{\text{node}} \left( \eta + \mu \right) \Delta t^{\text{me}}$$

$$+ \left( \eta - \mu \right) \Delta t^{\text{me}}$$

$$+ \left\{ \sum_{\text{node}} \left( \eta + \mu \right) \Delta t^{\text{me}} + \left( \eta - \mu \right) \Delta t^{\text{me}} \right\}$$

where the weight factors depend on the probability of each scenario satisfying $\sum_{\text{node}} \rho_{\text{f}} = 1$. The objective is subject to the following constraints.

2) Power system constraints

$$v_{\text{in}} - v_{\text{out}} = v_{\text{in}} - v_{\text{out}} \quad \forall g, \forall t \in T / \{1\}$$

$$v_{\text{in}} - v_{\text{out}} \leq \left| v_{\text{in}} - v_{\text{out}} \right| \quad \forall g, \forall t \in T / \{1\}$$

$$v_{\text{in}} - v_{\text{out}} \leq \left| v_{\text{in}} - v_{\text{out}} \right| \quad \forall g, \forall t \in T / \{1\}$$

$$\sum_{\text{gen}} p_{\text{in}} + \sum_{\text{node}} p_{\text{in}} = \left( L_{\text{m}} - \Delta t^{\text{me}} \right) ; \forall g, \forall t$$

$$v_{\text{in}} - v_{\text{out}} \leq \left| v_{\text{in}} - v_{\text{out}} \right| \quad \forall g, \forall t$$

$$p_{\text{in}} - p_{\text{out}} \leq \left| p_{\text{in}} - p_{\text{out}} \right| \quad \forall g, \forall t$$

$$\left| \sum_{\text{gen}} \left( \phi_{\text{in}} \right) p_{\text{in}} + \sum_{\text{k} \in \text{k}} \sum_{\text{node}} \left( \phi_{\text{in}} \right) p_{\text{in}} - \sum_{\text{node}} \sum_{\text{node}} \left( \phi_{\text{in}} \right) p_{\text{in}} - \Delta t^{\text{me}} \right|$$

$$\beta_{\text{me}} \quad \forall q, \forall t$$

$$v_{\text{in}} - v_{\text{out}} \leq \left| v_{\text{in}} - v_{\text{out}} \right| \quad \forall g, \forall t$$

$$p_{\text{in}} - p_{\text{out}} \leq \left| p_{\text{in}} - p_{\text{out}} \right| \quad \forall g, \forall t$$

$$p_{\text{in}} - p_{\text{out}} \leq \left| p_{\text{in}} - p_{\text{out}} \right| \quad \forall g, \forall t$$

$$\sum_{\text{gen}} \left( \phi_{\text{in}} \right) p_{\text{in}} + \sum_{\text{k} \in \text{k}} \sum_{\text{node}} \left( \phi_{\text{in}} \right) p_{\text{in}} - \sum_{\text{node}} \sum_{\text{node}} \left( \phi_{\text{in}} \right) p_{\text{in}} - \Delta t^{\text{me}} \right|$$

$$\beta_{\text{me}} \quad \forall q, \forall t$$

Except for the logic relationship of the state variables and minimum on/off time limits (constraints (2a)-(2c)), all power system constraints are duplicated for all scenarios. Constraints (2d) and (3a) describe the system balance. Constraints (2e) and (3b) represent the maximum and minimum capacity of the units, respectively. Constraints (2f)-(2g) and (3c)-(3d) limit the units’ upward and downward ramping rates, respectively. Constraints (2h) and (3e) represent the transmission capacity limits. The piece-wise fuel cost functions $f(\cdot)$ are also modeled using constraints but are not explicitly presented here for simplicity.

3) Gas system constraints

The gas system is typically represented by its steady-state and dynamic characteristics [28]. Considering that the market is cleared in an hourly manner, only the steady-state characteristics are considered. However, the gas network models are still complex due to their non-convexity, which cannot be easily linearized as with the power transmission models.

The models of gas suppliers and loads are straightforward. The gas output is limited by the supplier’s capacity. The gas loads have pre-defined profiles. The low-priority loads can be curtailed at low cost but only when informed in the DA market. The gas storage facilities are regarded as suppliers or loads because they do not frequently switch between the charging and discharging modes. The formulations are as follows:

$$0 \leq s_{\mu} \leq S_{\mu}, \quad \forall j, t$$

$$0 \leq \left| \Delta t_{\text{g}} \right| \leq L_{\text{g}} \quad \forall d, t$$

$$0 \leq s_{\mu} \leq S_{\mu}, \quad \forall j, t, \omega$$

$$\Delta t_{\text{g}} = \Delta t_{\text{g}} \quad \forall d, t, \omega$$

The gas transmission system usually consists of two major types of branches: pipelines and compressors. The gas flow in a pipeline is determined by the gas pressure at the two ends, diameter of the pipeline and temperature. While the gas pressure is variable during operation, other factors are regarded as fixed and described by parameters [11]:

$$y_{\text{mn}} = \text{sgn} \left( \pi_{\text{m}} - \pi_{\text{n}} \right) C_{\text{mn}} \sqrt{\pi_{\text{m}} - \pi_{\text{n}}}$$

$$\text{sgn} \left( \pi_{\text{m}} - \pi_{\text{n}} \right) = \begin{cases} 1, & \pi_{\text{m}} - \pi_{\text{n}} \geq 0 \\ -1, & \pi_{\text{m}} - \pi_{\text{n}} < 0 \end{cases}$$

$$y_{\text{mn}} = \frac{\text{sgn} \left( \pi_{\text{m}} - \pi_{\text{n}} \right) h}{k_{\text{c}} - k_{\text{i}}} \left[ \max \left( \pi_{\text{m}}, \pi_{\text{n}} \right) / \min \left( \pi_{\text{m}}, \pi_{\text{n}} \right) \right]$$

The power output of a compressor is limited by its upper and lower bounds, and it is assumed that a compressor consumes gas to generate power, which can be described by a quadratic function.

$$H_{\text{c}} \leq h_{\text{c}} \leq H_{\text{c}}$$

$$\theta \left( h_{\text{c}} \right) = a_{\text{c}} h_{\text{c}}^2 + a_{\text{c}} h_{\text{c}} + a_{\text{c}}$$

As discussed in Section II, the gas transmission constraints are computationally expensive and cannot be included in all scenarios. Hence, constraints (6)-(7) are applied only to constrain the booked capacity of the GFUs. Then, the gas system nodal balance constraints can be modeled as

$$\sum_{\text{node}} y_{\text{mn}} + \sum_{\text{node}} \sum_{\text{node}} \left( \Delta t_{\text{g}} \right) - \sum_{\text{node}} \sum_{\text{node}} \left( \Delta t_{\text{g}} \right)$$

$$- \sum_{\text{node}} \theta \left( h_{\text{c}} \right) = 0, \quad \forall n, t$$

Gas consumption by the GFUs in all DA and RT scenarios should be within the booked capacity, as presented by the following constraints:

$$0 \leq w_{\text{g}} \leq x_{\text{g}}, \quad \forall g \in G^{\text{g}}, t$$
\[
\sum_{j,t} s_j - \sum_{d=G} (L_{dg}^N - \Delta L_{dg}^N) - \sum_{g=D} w_g - \sum_{c \in C} \theta(h_c) = 0 : \eta_{\gamma}(\gamma) \quad \forall n,t \tag{9b}
\]

\[
0 \leq w_{g,t} \leq b_{g,t}^f, \quad \forall g \in G^e, t, \omega \tag{9c}
\]

\[
\sum_{j,t} s_j - \sum_{d=G} (L_{dg}^N - \Delta L_{dg}^N) - \sum_{g=D} w_g - \sum_{c \in C} \theta(h_c) = 0, \quad \forall n,t, \omega \tag{9d}
\]

where, constraints (9b) and (9d) represent the system gas balance for the DA and RT scenarios, respectively, to which the network limits are no longer applied. Considering that the gas consumed by the compressors is minor, the quantity for each scenario is approximated and assumed to be the same for all scenarios.

3) Power and gas coupling constraints

The power and gas systems are coupled by the GFUs. The coupling constraints are modeled as

\[
w_{g,t} = f_g^m (p_g) \quad \forall g \in G^e, t \tag{10a}
\]

\[
w_{g,t} = f_g^m (p_{g,t}) \quad \forall g \in G^e, t, \omega \tag{10b}
\]

The fuel consumption of the GFUs is typically modeled using a quadratic heat rate curve [15] and piece-wise-linearized here.

C. Solution

The above model is a mixed-integer non-convex programming problem with a linear objective function, to which the common commercial MILP solvers are not applicable. Although there are some available non-convex programming solvers, they are designed for general purposes and usually of poor computational performance, which is unacceptable for a market clearing tool. Following the ideas of [12], the GBD method is used in this paper to solve the above problem.

The strategy of the GBD method is divide and conquer. The MILP characteristic of the master problem is retained by isolating the gas network constraints. The removed constraints are compiled as a series of feasibility check sub-problems. Linear cuts are generated if a feasibility check does not pass and then embedded into the master problem, guaranteeing that the market clearing results are at least a feasible solution.

The sub-problem is formulated as follows:

\[
\min \{ \hat{x}^*, \Delta L_{N}^N, s^* \} = \sum_{j,d} \mu_{j,d} + \sum_{g,D} \zeta_{g} \hat{\Delta} L_{dg}^N + \sum_{g \in G^e, t} z_{g} \left( x_{g,t}^* - \hat{x}_{g,t} \right) \tag{11a}
\]

s.t. \( 0 \leq \hat{x}_{g,t} - \hat{x}_{g,t}^* \leq \tilde{x}_{g,t}, \quad \forall f \) \tag{11b}

\( 0 \leq \Delta L_{N}^N + \Delta L_{N}^{D} \leq \Delta L_{N}^{DG} \), \quad \forall d \tag{11c}

\( 0 \leq \hat{x}_{g,t} \leq \hat{x}_{g,t}^* \), \quad \forall g \tag{11d}

\[\sum_{m=0}^{N_{m}} \left( s_{m} + \tilde{s}_{m} \right) - \sum_{c \in C} \theta(h_c) = 0, \quad \forall n \quad \text{and, Constraints (6)-(7)} \]

\[
\sum_{d=0}^{D} \left( L_{dg}^N - \Delta L_{dg}^N - \Delta L_{dg}^{DG} \right) - \sum_{g \in G^e, t} \hat{x}_{g,t} = 0, \quad \forall n \quad \text{and, Constraints (6)-(7)} \]

where, variables marked by "**" are given by the master problem; \( \eta_{\gamma}(\gamma) \) represents the value of natural gas to the power system, as implied by the locational marginal electricity price (LMEP), which is derived by solving the master problem:

\[\zeta_{g} = \eta_{g} \hat{e}_{g} \tag{12a}\]

where, the definition of \( \eta_{g} \) is as presented in Section III-D.

Because the gas system constraints are not temporally coupled, the sub-problems are solved independently for each time period.

The sub-problems are non-convex, and global optimality is not guaranteed. As suggested by [12], a successive linear programming (SLP) approach is effective for solving the problem. Alternatively, the linearized model can be used after an optimum has been found by the interior-point optimizer. Readers may refer to [12] for the detailed algorithm.

If the sub-problem returns a positive optimal objective value, a linear cut is generated as follows:

\[
\hat{\mathcal{S}}(\hat{x}_g, \Delta L_{N}^N, \hat{x}_n) + \sum_{m=0}^{N_{m}} \sum_{\epsilon \in G^e} B_{m,n}^N \left( \Delta L_{dg}^N - \Delta L_{dg}^{DG} \right)
+ \sum_{m=0}^{N_{m}} \sum_{\epsilon \in G^e} B_{m,n}^N \left( \Delta L_{dg}^N - \Delta L_{dg}^{DG} \right) \leq 0 : \beta_{g}^m \tag{13a}
\]

where, \( \mu_{N}^N \) is the dual variable of the correction constraints in the SLP model. To summarize, the market clearing model is solved following the procedures in Fig. 2.

![Fig. 2. Procedures for solving the ND-CGEM model.](image)

D. Definition of the prices

Consistent with the traditional locational marginal pricing method, the prices are defined based on the dual variables of the master problem, with the integer variables fixed at their optimized values:

\[
\eta_{m}^e = \eta_{m}^{e,0} + \sum_{q \in Q} \Gamma_m^p \hat{p}_q, \quad \forall m \tag{14a}
\]

It is assumed that the gas suppliers in a system share the same gas cost and do not bid strategically. This assumption is made to simplify the spot market model and is a fair assumption because the gas prices are typically uniform within a specific range of area. Alternatively, arbitrage will occur to rebuild the equilibrium. Moreover, the gas network constraints are imposed only on the allocation of booked gas demand capacity. Thus, the gas prices are solely determined by the dual variables of constraints (9b):

\[
\eta_{m}^e = \eta_{m}^{e,0} \tag{15a}
\]

The power system as a whole must pay for the obtained gas capacity of the GFUs based on the capacity prices. The capacity prices are positive only when some low-priority loads are curtailed to fulfill the power system (potential) demand. Otherwise, they are zero. Though the gas prices are uniform, the capacity prices are locationally discrepant, which is determined by the dual variables of the added cuts:

\[
\eta_{g}^{e,\text{cap}} = \sum_{\epsilon \in G^e} B_{g,\epsilon}^\text{FGU} \mu_{g,\epsilon}^N, \quad \forall \epsilon \in G^e \tag{16a}
\]
It could be controversial to price the gas capacity with the dual variables of the linearized cuts. However, it is a good approximation for the value of gas capacity. More sophisticated convexification approaches for the gas transmission network model might be helpful to improve the approximation.

IV. NUMERICAL CASES

A modified IEEE Reliability Test System [29] with 18 units (total capacity: 2,090 MW) and a 14-node gas transmission system with 3 gas suppliers (see Fig. 3) are used in this paper to demonstrate the performance of the proposed ND-CGEM framework. The model data of the two systems are provided in [30] and [31]. It is assumed that G3 and G13 are GFUs located at node 12 and 5 in the gas system, respectively. Without loss of generality, all gas loads are assumed to be 80% high-priority and 20% low priority.

A wind farm with a capacity of 400 MW is located at bus 8. The wind power is assumed to follow a beta distribution, with the parameters estimated based on the historical data provided by [32] and scaled down to fit the test system. To simulate the RT scenarios, 5,000 samples are generated following the estimated distribution and are reduced to 30 using the fast forward selection method [33]. Fig. 4 presents the wind power forecast and the associated scenarios of a specific day.

The electricity market is traditionally cleared without considering the gas system. With the conventional SCUC tool, we can examine how the uncertainty of wind power impacts the schedules of the power system. Fig. 5 shows that for a specific day, the GFUs are committed for more periods when uncertainty is considered, which implies that the power system relies on the GFUs’ capacity to firm the variability of wind power.

However, as noted above, there are two underlying risks. First, the quantity of gas consumed by the GFUs is uncertain. Unless the GFUs are informed of all the market information, including the wind power forecasts and distributions, they are incapable of making efficient gas purchase contracts to minimize the deviation penalties. Taking G3 as an example, Fig. 6 demonstrates the total gas consumption and profiles of G3 during the day in different scenarios. As indicated by the results, the GFUs are exposed to significant risks of deviation penalties even if the gas contracts are signed daily; needless to say, the hourly profile is even more volatile.

Furthermore, from Hour 17 to Hour 22, during which the GFU is heavily used is also a gas load peak on that day (see Fig. 7). If the two systems are not coordinated, the gas loads will not be rescheduled to reserve gas supply for GFUs to generate power and the GFU will suffer from insufficient gas supply (see Fig. 8), which means that the power system might have to commit more expensive resources or even curtail loads during RT operation. However, there could be potential overall social welfare improvement if such a shortage in gas supply for power
generation is predicted by electric-gas market coordination and the gas load is properly compensated for being curtailed. Therefore, running a joint optimization-based coordinated market is technically more effective than the non-coordinated market.

The results indicate that the gas prices are $2/kcf at the gas suppliers and that the costs of curtailing the low-priority and high-priority loads are $3/kcf and $1,000/kcf respectively. The option of shifting the loads across the periods is not considered but can be easily modeled within the framework. The ND-CGEM model is tested based on the above assumptions.

Fig. 9 shows the capacity that is scheduled to the GFUs in the joint market compared to the maximum gas demand among the scenarios when the gas system is not considered. The increase in supply capacity for G3 from Hour 22 to Hour 23 is due to a re-balance of power output between G3 and G13. Although not all demands are satisfied, the ND-CGEM model does choose to curtail some of the low-priority loads (see Table I) to reduce the overall system costs. The prices of gas supply capacity for the two GFUs are also presented in Fig. 10. The results indicate that the gas capacity price is zero when there is no congestion in the gas networks.

The joint cleared market can also help determine the optimal volume of DA gas supply contracts. As shown in Fig. 10, the deviations are distributed around zero, which helps the GFUs, i.e., the entire power system, reduce deviation penalties.

The optimized expected overall cost of the gas and electricity system is $1,612,614, which is 0.19% higher than the overall cost when the two systems are not coordinated ($1,612,614). Although at a lower cost, the non-coordinated schedules are actually infeasible. To compare, an additional 1,000 wind power scenarios are generated to simulate the RT operation of the system under coordinated and non-coordinated DA schedules. Fig. 11 shows the average system-wide RT LMEP among the simulated scenarios. The results indicate that the re-dispatch costs can be higher than expected without coordinating the DA schedules of the two systems.

| Cases | Expected cost (non-coordinated) | Expected cost (ND-CGEM) | Simulated cost (non-coordinated) | Simulated cost (ND-CGEM) | Simulated cost reduction |
|-------|---------------------------------|-------------------------|----------------------------------|-------------------------|-------------------------|
| 1     | 1,672,320                       | 1,674,809               | 1,714,020                        | 1,676,839               | 2.2%                    |
| 2     | 1,533,726                       | 1,533,726               | 1,546,916                        | 1,547,040               | 0.0%                    |
| 3     | 1,537,054                       | 1,558,423               | 1,575,033                        | 1,566,549               | 0.5%                    |
| 4     | 1,661,755                       | 1,664,132               | 1,686,451                        | 1,666,506               | 1.2%                    |
| 5     | 1,516,533                       | 1,516,533               | 1,522,264                        | 1,522,292               | 0.0%                    |
| 6     | 1,672,320                       | 1,674,809               | 1,714,020                        | 1,676,839               | 0.7%                    |
| 7     | 1,533,726                       | 1,533,726               | 1,546,916                        | 1,547,040               | 1.3%                    |
| 8     | 1,557,054                       | 1,558,423               | 1,575,033                        | 1,566,549               | 0.0%                    |
| 9     | 1,661,755                       | 1,664,132               | 1,686,451                        | 1,666,506               | 0.6%                    |
| 10    | 1,516,533                       | 1,516,533               | 1,522,264                        | 1,522,292               | 1.5%                    |

The simulation results for more days are presented in Table II. As expected, the non-coordinated approach always tends to underestimate the actual costs, whereas the ND-CGEM framework provides a better estimation. Moreover, the actual cost with the ND-CGEM approach can be 2% lower than the non-coordinated approach by making better DA schedules. The difference is minor when the gas network is not congested (e.g., cases 2, 5, 7 and 10).

All of the cases are run on a laptop with a Xeon E3-1535M CPU and 16 GB of RAM. The optimization problems are solved using the commercial solver Gurobi 6.5. When the relative gap for the MILP problems is set at $10^{-4}$, the average solving time for the above cases is approximately 231 s, with 1-5 outer-loop iterations. When the number of RT scenarios increases to 80, the time increases to approximately 411 s.
Table III shows the iteration process of a specific case. The number of sub-problem iterations indicates the overall number of SLP problems calculated for the 24-hour gas transmission feasibility problems. Further improvements in the algorithms might be helpful in applying the proposed framework to larger systems.

V. CONCLUSIONS AND DISCUSSION

This paper proposes and examines a framework of joint gas and electricity markets considering wind power uncertainty. With the ND-CGEM model, the GFUs are no longer responsible for making forecasts regarding future gas consumption to bid in the gas market considering natural gas-fired generation in the power system operation or bearing the risks of improper gas contracts and insufficient gas supply. Moreover, the gas transmission resources are better allocated. When the gas network capacity is urgently demanded by the power system, the joint market can help curtail some of the low-priority gas loads by compensating for the lost utility. The case studies indicate that better DA market clearing results can be achieved.

The solution algorithm shows acceptable computational performance, although further improvements might be needed when applying the framework to larger systems. The proposed stochastic marketing clearing framework can also be applied in a deterministic setting using only one scenario in the real-time market simulation.

For future works, a convexification of the gas transmission network model might be helpful to overcome the local optimum of nonlinear sub-problems. Moreover, decomposition algorithms can also be applied to the stochastic optimization problem to increase the computational efficiency.

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