Mass Matrix Ansatz for Degenerate Neutrinos Consistent with
Solar and Atmospheric neutrino Data

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Abstract

We suggest mass matrices for neutrinos and charged leptons that can explain solar and atmospheric neutrino data. The resulting flavor mixing matrix $V_\nu$ has a property that $(V_\nu)_{13} = 0$, thus making $\nu_e \leftrightarrow \nu_\mu$ and $\nu_\mu \leftrightarrow \nu_\tau$ oscillations to be effectively a two-channel problem. Phenomenological consequences of the lepton mass matrix ansatze are consistent with the current data on various type of neutrino oscillation experiments except the LSND measurement. Three neutrinos, being almost degenerate with $\sum |m_{\nu_i}| \lesssim 1 \text{ eV}$, can be a part of hot dark matter without any conflict with the constraint from neutrinoless double beta decay experiments. The $\nu_\mu \leftrightarrow \nu_\tau$ oscillation, $\sin^2 2\theta_{\mu\tau}$, is predicted to be 0.86-0.97 with $\Delta m^2_{\mu\tau} \approx 2 \times 10^{-3} \text{ eV}^2$, which is consistent with the atmospheric neutrino data and can be tested further at the planned MINOS and K2K experiments searching for $\nu_\mu \rightarrow \nu_\tau$ oscillation.

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I. INTRODUCTION

In this paper, we suggest a specific form of neutrino and charged lepton mass matrices that follow from permutation symmetry and the quark-charged lepton symmetry, and show that an almost degenerate scenario among three flavor neutrinos [1,2] can explain solar and atmospheric neutrino data following from the standpoint of the mass matrix ansatze. The parameters of the neutrino mass matrix are constrained by the solar and the atmospheric neutrino deficits and the neutrinoless double beta decay experiment. Then the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation, $\sin^2 2\theta_{\mu\tau}$, is predicted to be 0.86–0.97 with $\Delta m^2_{\mu\tau} \simeq 2 \times 10^{-3} \text{eV}^2$, which is consistent with the atmospheric neutrino data [3]. Three neutrinos, being almost degenerate with $\sum |m_{\nu_i}| \lesssim 1 \text{eV}$, can be a part of hot dark matter of the universe.

The flavor mixing, the fermion masses, and their hierarchical patterns still remain to be one of the most fundamental problems in particle physics. As an attempt toward the understanding of the quark mass hierarchy and the flavor mixing, the quark mass matrix ansatz was introduced by Weinberg [4]. The key idea is to make the number of parameters in the mass matrix to be less than the total number of flavor mixing parameters, so that there result relations between mixing parameters and mass eigenvalues. Some call it *calculability*. In particular, the Cabibbo angle is calculable in terms of the quark masses in this scheme. Weinberg’s idea of *calculability* was extended for three and more generations by Fritzsch [5] and Kang et al. [6]. Since then, the Fritzsch-type mass matrix had attracted a great deal of attention until the top quark was discovered. But the Fritzsch texture predicts the top quark mass to be at most about 100 GeV and thus was ruled out [7].

Nevertheless, the Fritzsch type mass matrix is attractive due to its simplicity. Though its original form is phenomenologically ruled out, one may want to generalize the Fritzsch texture by introducing just one more parameter but by maintaining the calculability property. The obvious next move is to increase a nonvanishing entry in the Fritzsch-type mass matrix at the $(2,2)$ element. Recently, a systematic phenomenological study of such generalized mass matrix has been studied by Kang and Kang [8], which is parametrized by

$$M_H = \begin{pmatrix} 0 & A & 0 \\ A & D & B \\ 0 & B & C \end{pmatrix}.$$  \hspace{1cm} (1)

The case of $D = 0$ reduces to the original Fritzsch type. As shown in Ref. [8], this form can be achieved by successive breaking of the maximal permutation symmetry in the mass matrix. Various mass matrix ansätze proposed by others [9] can be identified as special cases of the above form by appropriately relating $D$ to $B$. It has also been shown in [8] that the mass matrix (1) with a finite range of non-zero relation of $D$ to $B$ can be consistent with experimental results including heavy top quark mass, while ruling out several special $D/B$ ratios considered in [9].

Regarding the phenomenological form of the mass matrix Ramond et al. [10] narrowed down a few years ago possible forms of mass matrices having texture zeros at the supersymmetric unification scale. Eq.(1) was, of course, one of the mass matrix patterns considered in Ref. [10]. However, they constructed the different patterns of mass matrices for the up- and down-quark sectors, whereas Ref. [8] assumed the same form of mass matrices for both sectors. In this paper, we assume the same form of mass matrix for the charged lepton sector because the charged leptons exhibit a similar hierarchy in mass.
On the other hand, all neutrino masses are zero and lepton numbers are exactly conserved in the context of the standard model (SM). However, the current experimental anomalies of solar \cite{11-14} and atmospheric \cite{15-18} neutrinos lead us to speculate nearly degenerate but non-zero neutrino masses and mixing among the three flavors, as they can be interpreted as originating from the neutrino oscillations. The deficit of the solar neutrino flux is sometimes explained economically by the Mikheyev-Smirnov-Wolfenstein (MSW) effect \cite{19}. The “atmospheric neutrino anomaly” can be interpreted by the muon neutrino oscillation into other neutrino, possibly, of tau flavor. The recent CHOOZ experiment \cite{20} which is a long baseline experiment indicates that one has to invoke a large mixing between the $\nu_\mu \leftrightarrow \nu_\tau$, which is supported by the more recent result from the Super-Kamiokande Collaborations \cite{3}. It has been suggested by several authors \cite{1,2} that almost degenerate neutrinos are needed to accommodate the solar and atmospheric neutrino observations as well as the cosmological constraint that arises when we regard neutrinos as candidates for the hot dark matter within the three-flavor framework.

II. MASS MATRIX FOR CHARGED LEPTON

Let us start with the new class of mass matrix Eq.(1) for charged lepton. Since the matrix $M_H$ contains four independent parameters, one might think that the “calculability” \cite{21} might have been lost. However, one can make additional ansatz to relate $B$ and $D$ via $B = wD$ with the same ratio parameter $w$ for both the up- and down-quark sectors, to maintain the “calculability” \cite{21}.

In this paper, we assume the quark-charged lepton symmetry for the mass matrix so that the matrix form of charged lepton sector is exactly the same as the new type of quark mass matrix Eq.(1). Let us diagonalize the mass matrix of the charged lepton sector. In general, a hermitian matrix $M_H$ can be diagonalized by a single unitary transformation $U_L M_H U_L^\dagger$, while a mass matrix needs a biunitary transformation $U_L M_H U_R^\dagger = \text{diag}[m_1, m_2, m_3]$ in general. Then, we can write $U_L M_H U_L^\dagger = K \cdot \text{diag}[m_1, m_2, m_3]$ where $K = U_L U_R^\dagger$ is a diagonal matrix having the diagonal elements $\pm 1$ or a phase factor $e^{i\phi}$ in general. Since we deal with the real mass matrix in our problem, $U_L$ is a real matrix and $K$ is real too. As discussed in Ref. \cite{8}, because of the empirical mass hierarchy $m_1 \ll m_2 \ll m_3$, $K = \text{diag}[1, -1, 1]$ irrespective of the sign of $D$ and $K = \text{diag}[-1, 1, 1]$ only for positive $D$. In view of the hierarchical pattern of the charged lepton masses, it is also natural to expect that $A < |D| \ll C$, and the case of $K = \text{diag}[-1, 1, 1]$ for positive $D$ can be excluded if the same ratio parameter $w$ as that for the quark mass matrices is required. Then, the parameters $A, B, C$ and $D$ can be expressed in terms of the charged lepton masses and $w$. For the other case $K = \text{diag}[1, 1, -1]$, the characteristic equation for the mass matrix does not admit any solution.

The Case I with $K = \text{diag}[-1, 1, 1]$: From the characteristic equation for the $M_H$, the mass matrix $M_H$ can be written by

$$M_H = \begin{pmatrix}
0 & \sqrt{m_1 m_3 - \epsilon} & 0 \\
\frac{m_1 m_3 - \epsilon}{m_3 - \epsilon} & m_2 - m_1 + \epsilon & w(m_2 - m_1 + \epsilon) \\
0 & w(m_2 - m_1 + \epsilon) & m_3 - \epsilon \\
\end{pmatrix}, \quad (2)$$
in which the small parameter \( \epsilon \) is related to \( w \), i.e., \( w \approx \pm \sqrt{\frac{m_2}{m_3}} \left( 1 + \frac{m_1}{m_2} - \frac{m_2}{2m_3} \right) \), whose range is determined from the experiments. Note the sign of \( B \) is undetermined from the characteristic equation but the KM matrix elements are independent of the sign of \( B \).

Then, the diagonalizing matrix \( U_L' \) can be written as

\[
U_L' = U_{23}(\theta_{23}) \cdot U_{12}(\theta_{12})
\]

where

\[
U_{12} = \begin{pmatrix} \cos \theta_{12} & \sin \theta_{12} & 0 \\ -\sin \theta_{12} & \cos \theta_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{23} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \theta_{23} & \sin \theta_{23} \\ 0 & -\sin \theta_{23} & \cos \theta_{23} \end{pmatrix}
\]

Since (1,1), (1,3) and (3,1) elements of \( M_H \) are zero, we may put \( U_{13}(\theta_{13}) = 1 \) without loss of generality. The mixing angles \( \theta_{12} \) and \( \theta_{23} \) can be written to a very good approximation as

\[
\tan \theta_{12} = \sqrt{\frac{m_1}{m_2}}
\]

and

\[
\tan \theta_{23} = \frac{1}{2w} \left[ \left( 1 + \frac{m_1 - m_2}{m_3} \right) - \sqrt{\left( 1 + \frac{m_1 - m_2}{m_3} \right)^2 + 4w^2 \left( \frac{m_1 - m_2}{m_3} \right)} \right]
\]

The Case II with \( K = \text{diag}[1, -1, 1] \): For a negative \( D \), the real symmetric matrix \( M_H \) can be diagonalized as \( U_L'M_HU_L^\dagger = \text{diag}[m_1, -m_2, m_3] \), thus reversing the signs of both \( m_1 \) and \( m_2 \) in Eqs. (2),(5) and (6). As we noted, a positive \( D \) in this case is excluded for the reasons of naturalness due to the charged lepton mass hierarchy and calculability.

In both cases discussed above, it turns out that the experimentally allowed range of \( w \) in the quark mass matrices is \( 0.97 \lesssim |w| \lesssim 1.87 \) in the leading approximation \[8\]. Thus, we will assign the value of \( w \) for the charged lepton sector to the above range. However, physical observables such as survival and transition probabilities for \( \nu_\alpha \)'s are insensitive to the precise value of \( w \) in the allowed range, as discussed in the following.

III. MASS MATRIX FOR NEUTRINOS

It is likely that the mass matrix of the charged lepton sector is not appropriate for the neutrino sector, since the neutrino oscillation experiments do not seem to support such hierarchical pattern for neutrino masses as that of quark or charged lepton masses but rather nearly degenerate neutrinos within the three-flavor framework \[1,2\]. We will show that such an almost degenerate neutrino scenario can follow from a neutrino mass matrix, which is clearly different from the approach used by others \[3\]. We assume that three light neutrinos are Majorana particles. This can be partly motivated by the fact that there is a dimension-5 operator which can generate the Majorana masses for SM neutrinos after electroweak symmetry is spontaneously broken, if one considers the SM as an effective field theory of more fundamental theories \[23\].
In order to construct a neutrino mass matrix so as to be consistent with the experiments, we consider three observations for neutrinos which may be accounted for by assuming massive neutrinos:

- solar neutrino data from four different experiments, the HOMESTAKE [11], GALLEX [12], SAGE [13], and the KAMIOKANDE II-III [14].
- atmospheric neutrino data measured by four experiments, the KAMIOKANDE [15], Super-Kamiokande [16], Soudan2 [17] and IMB [18].
- constraint from the neutrinoless double beta decay experiments [24]

\[
\langle m_{\nu_e} \rangle \equiv \left| \sum_{i=1}^{3} \eta_i V_{ei}^2 m_i \right| \leq 0.45 \text{ eV} \tag{7}
\]

where \( \eta_i = \pm 1 \) depending on the CP property of \( \nu_i \).

- the likely need for neutrinos as a candidate of hot dark matter [25].

As is well known, the solar neutrino deficit can be explained through the MSW mechanism if \( \Delta m_{\text{solar}}^2 \approx 6 \times 10^{-6} \text{ eV}^2 \) and \( \sin^2 2\theta_{\text{solar}} \approx 7 \times 10^{-3} \) (small angle case), or \( \Delta m_{\text{solar}}^2 \approx 9 \times 10^{-6} \text{ eV}^2 \) and \( \sin^2 2\theta_{\text{solar}} \approx 0.6 \) (large angle case) and through the just-so vacuum oscillations if \( \Delta m_{\text{solar}}^2 \approx 10^{-10} \text{ eV}^2 \) and \( \sin^2 2\theta_{\text{solar}} \approx 0.9 \) [26]. The atmospheric neutrino problem can be accommodated if \( \Delta m_{\text{atmos}}^2 \approx 2 \times 10^{-3} \text{ eV}^2 \) and \( \sin^2 2\theta_{\text{atmos}} \approx 1.0 \). Especially the recent results from CHOOZ and Super-Kamiokande seem to disfavor the \( \nu_\mu \rightarrow \nu_e \) oscillation as a possible solution to the atmospheric neutrino problem [20]. So one has to invoke for large mixing between \( \nu_\mu \leftrightarrow \nu_\tau \). If the light neutrinos account for the hot dark matter of the universe, one has to require [25]

\[
\sum_{i=1,2,3} |m_{\nu_i}| \lesssim 6 \text{ eV} \tag{8}
\]

Thus we see that all three neutrinos may be almost degenerate in their masses, with \( m_{\nu_e} \lesssim O(1) \) eV, rather than \( m_{\nu_1} \ll m_{\nu_2} \ll m_{\nu_3} \), as sometimes assumed in the three-neutrino mixing scenarios [1].

In this paper, we would like to account for the solar and atmospheric neutrino deficits as \( \nu_e - \nu_\mu \) and \( \nu_\mu - \nu_\tau \) oscillations, respectively. Once \( \nu_\mu \) and \( \nu_\tau \) mixing is taken to be maximal, the corresponding \( 2 \times 2 \) mass matrix can be given by

\[
\begin{pmatrix}
A & B \\
B & A
\end{pmatrix} .
\tag{9}
\]

\[\text{1} \text{The recent LSND data [27], if confirmed, indicates } \Delta m_{\text{LSND}}^2 \sim 1 \text{ eV}^2 \text{ and } \sin^2 \theta_{\text{LSND}} \sim 10^{-3}. \text{ Since the conclusions of two different analyses [27,28] do not agree with each other, we do not consider the possibility alluded by the LSND data [27] in this work. See, however, Ref. [24] for a discussion when the LSND data is included.}\]
After diagonalizing, we get the eigenvalues $m_{\nu_i} = A \pm B$. Note that if the parameter $B$ is taken to be small, the atmospheric data of $\Delta m^2_{\text{atmos}}$ can be accommodated. In addition, $\Delta m^2_{\text{solar}}$ can be accommodated by allowing non-zero (1,1) element of the $3 \times 3$ mass matrix as follows

\[
\begin{pmatrix}
C & 0 & 0 \\
0 & A & B \\
0 & B & A
\end{pmatrix}
\] (10)

which can be diagonalized by

\[
U_\nu = \begin{pmatrix}
1 & 0 & 0 \\
0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\
0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}
\end{pmatrix}.
\] (11)

One can solve for $A$, $B$ and $C$ by requiring three conditions, $\Delta m^2_{\text{solar}} = 10^{-5} \text{eV}^2$, $\Delta m^2_{\text{atmos}} = 2 \times 10^{-3} \text{eV}^2$ and Eq. (7). Then, the set of parameters $(A, B, C)$ is given by:

\[
(A, B, C) = (0.33383, 0.001498, 0.3323) \text{(eV)}
\] (12)

for which three light neutrinos are almost degenerate with mass around 0.34 eV.

**IV. NEUTRINO MIXING MATRIX AND PREDICTIONS**

Combining the $U_L^\dagger$ given by Eq. (3) with $U_\nu$ of Eq. (11), we get the neutrino mixing matrix,

\[
V_\nu \equiv U_\nu^\dagger U_L^\dagger = \frac{1}{\sqrt{2}} \begin{pmatrix}
\sqrt{2}c_{12} & \sqrt{2}s_{12} & 0 \\
\frac{s_{12}(-c_{23} + s_{23})}{s_{12}(c_{23} + s_{23})} & c_{12}(c_{23} - s_{23}) & s_{23} + c_{23} \\
\frac{-c_{12}(c_{23} + s_{23})}{s_{12}(c_{23} + s_{23})} & -c_{12}(s_{23} - c_{23}) & -s_{23} + c_{23}
\end{pmatrix}.
\] (13)

where we have abbreviated $\cos \theta_{ij}$ and $\sin \theta_{ij}$ as $c_{ij}$ and $s_{ij}$ respectively. We note that the mixing matrix is independent of neutrino masses, although it depends on the charged lepton masses. For the whole range of $0.97 \lesssim |w| \lesssim 1.87$, the neutrino mixing matrix is given by

\[
|V_\nu| = \begin{pmatrix}
0.9952 & 0.0692 & 0.0 \\
0.0453 & 0.6520 & 0.7531 \\
0.052 & 0.7947 & 0.6551
\end{pmatrix} \sim \begin{pmatrix}
0.9952 & 0.0692 & 0.0 \\
0.0440 & 0.6326 & 0.7307 \\
0.0506 & 0.7271 & 0.6356
\end{pmatrix}.
\] (14)

Note that our lepton mixing matrix predicts zero for $(V_\nu)_{13}$ element, i.e., the $\nu_e$-tau coupling is forbidden, which makes $\nu_e \leftrightarrow \nu_\mu$ and $\nu_\mu \leftrightarrow \nu_\tau$ oscillations to be effectively a two-channel problem.

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\(^2\) If $\Delta m^2_{\text{atmos}} = 10^{-2} \text{eV}^2$ is used instead, the three neutrinos are almost degenerate with mass around 1 eV [30].
Now, we check if the solutions of three neutrino mass eigenvalues satisfy the constraint coming from the neutrinoless double $\beta$–decay, as well as other data from neutrino oscillation experiments. The neutrino mixing matrix Eq. (13) and neutrino mass eigenvalues lead to

$$\langle m_{\nu_e} \rangle \simeq 0.33 \text{ eV}$$

for $w = 0.97 - 1.87$. All of these solutions are well below the current upper limit given in Eq. (7). If we begin to increase the neutrino masses in order to make it dominant hot dark matter candidates, we cease to satisfy the $(\beta\beta)_{00}$ constraint, Eq. (7).

Next, we study the transition and survival probabilities of the neutrinos. In order to calculate the transition probabilities, the mass differences $\Delta m^2_{ij} = m^2_{\nu_i} - m^2_{\nu_j}$ should be identified with $\Delta m^2_{\text{solar}}$ or $\Delta m^2_{\text{atmos}}$. Among the possibilities, it turns out that only the case for $\Delta m^2_{\text{solar}} = \Delta m^2_{12}$ and $\Delta m^2_{\text{atmos}} = \Delta m^2_{23}$ can fit the available data quite well, and thus we will consider henceforth only this case. In particular, we find that the probability $P(\nu_e \to \nu_\mu)$ and $P(\nu_\mu \to \nu_\tau)$ is changed up to about 10% with the value of $w$.

Further test of our ansatz is provided with the long baseline experiments searching for $\nu_\mu \to \nu_\tau$ oscillation in the range of $\Delta m^2_{\mu\tau} \simeq 10^{-3} \text{ eV}^2$. The MINOS [31] and K2K [32] sensitivities to $\Delta m^2$ at 90% CL can go down to $\Delta m^2 = 1.2 \times 10^{-3} \text{ eV}^2$ and $2.0 \times 10^{-3} \text{ eV}^2$, respectively, while the ICARUS [33] sensitivity is achieved at $\Delta m^2 = 3.0 \times 10^{-3} \text{ eV}^2$. Our prediction is that

$$\sin^2 2\theta_{\mu\tau} \simeq 0.86 - 0.97$$

with $\Delta m^2_{\mu\tau} = 2 \times 10^{-3} \text{ eV}^2$ for the allowed range of $w$. This can be tested at the MINOS and K2K experiments searching for the $\nu_\mu \to \nu_\tau$ oscillations in the foreseeable future, but is beyond the sensitivity to $\Delta m^2$ at 90% CL being achieved at ICARUS. Future experiment on the $\nu_\mu \leftrightarrow \nu_\tau$ oscillation from the MINOS and K2K will exclude our model for charged lepton and neutrino mass matrices.

V. CONCLUSION

In conclusion, we investigated in this paper phenomenological consequences of the lepton mass matrix ansätze with the minimal number of parameters, three each in the charged lepton and Majorana neutrino mass matrices. We find the ansätze Eqs. (1) and (10) lead to a lepton mixing matrix which is consistent with the current data on various types of neutrino oscillation experiments except the controversial LSND data. Three light Majorana neutrinos can constitute a part of hot dark matter, with $\Sigma |m_{\nu_i}| \sim 1 \text{ eV}$ without contradicting the constraint from neutrinoless beta decay experiments. The resulting amplitude of $\nu_\mu \leftrightarrow \nu_\tau$ oscillation $\sin^2 2\theta_{\mu\tau}$ is 0.86 – 0.97 with $\Delta m^2_{\mu\tau} = 2 \times 10^{-3} \text{ eV}^2$ for the range of $w$ under consideration, which is consistent with the atmospheric neutrino oscillation and will be tested at the MINOS and K2K experiments. Finally, three neutrinos being almost degenerate, we expect that the lepton family number breaking effects in $\mu \to e\gamma$ and $\mu \to 3e$ and analogous tau decays will be very small.
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