Observational Constraints of Homogeneous Higher Dimensional Cosmology with Modified Chaplygin Gas

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In this work, we have considered the flat FRW model of the universe in \((n + 2)\)-dimensions filled with the dark matter (perfect fluid with negligible pressure) and the modified Chaplygin gas (MCG) type dark energy. We present the Hubble parameter in terms of the observable parameters \(\Omega_m\), \(\Omega_{x0}\) and \(H_0\) with the redshift \(z\) and the other parameters like \(A, B, C, n\) and \(\alpha\). From Stern data set (12 points), we have obtained the bounds of the arbitrary parameters by minimizing the \(\chi^2\) test. The best-fit values of the parameters are obtained by 66%, 90% and 99% confidence levels. Now to find the bounds of the parameters and to draw the statistical confidence contour, we first fixed three parameters \(C, n, \alpha\) and then fixed the three parameters \(A, n, \alpha\). In the first case we find the bounds of \((A, B)\) and draw the contour between them for \(4D(n = 2)\), \(5D(n = 3)\) and \(6D(n = 4)\). In the second case we fixed three different values of \(A\) as 1, 1/3, −1/3 to find the bounds of \((B, C)\) and draw the contour between them. Here the parameter \(n\) determines the higher dimensions and we perform comparative study between three cases : \(4D\) (\(n = 2\)), \(5D\) (\(n = 3\)) and \(6D\) (\(n = 4\)) respectively. Next due to joint analysis with BAO observation, we have also obtained the bounds of the parameters \((A, B)\) by fixing some other parameters \(\alpha\) and \(A\) for \(4D\), \(5D\) and \(6D\).

I. INTRODUCTION

In recent cosmological research work, the theoretical models and range of the cosmological parameters are tested continuously by the combination of different observational astrophysical data. Before achieving Supernova data, it seems that the universe may be occupied by energy density which is very well distributed over large scales \[1\]. In 1992, Cosmic Background Explorer (COBE) \[2, 3\] data suggested that the spectrum of Standard Cold Dark Matter (SCDM) should be modified and proved the necessity of existence of the cosmological constant. The existence of non-zero cosmological constant \(\Lambda\) (which has the equation of state \(w_\Lambda = -1\)) was supported in 1996 \[4\]. The different cosmological observation of SNeIa \[5–8\], large scale redshift surveys \[9, 10\], measurements of the cosmic microwave background (CMB) \[11, 12\] and WMAP \[13, 14\] anticipate that our present universe which is expanding with acceleration, preceded by a period of deceleration. The mysterious observational facts were not explained by the standard big bang Cosmology with perfect fluid. Thus to integrate the recent prediction from observational cosmology, a modification is necessary in the matter sector of the Einstein Gravity. The unknown candidate which is responsible for this accelerating scenario, has the property that the positive energy density and sufficient negative pressure, known as dark energy \[15, 16\].

For \(z > 0.01\), the TONRY data set with the 230 data points \[21\] together with the 23 points from Barris et al \[22\] are valid. Another data set named the “gold” sample (see \[8\]) contains 156 points, which includes the latest points observed by HST and this covers the redshift range \(1 < z < 1.6\). Recently the CMBR data (for recent WMAP results, see \[14\]) strongly support \(\Omega_A + \Omega_m = 1\) for FRW universe in Einstein gravity. Also in 2007, Choudhury et al \[22\] showed that the best-fit value of \(\Omega_m\) for flat model was \(0.31 \pm 0.08\).

One of the most effective candidate of dark energy having positive energy density and negative pressure is Chaplygin gas whose EOS is given by \(p = -B/\rho^\alpha\) \[24\] with \(B > 0\). Later, it has been generalized to the form \(p = -B/\rho^\alpha\) \[25, 26\] and thereafter modified to the form \(p = A\rho - B/\rho^\alpha\) \[27\], which is known as Modified...
Chaplygin Gas (MCG). The Modified form of Chaplygin Gas go with the 3 year WMAP and the SDSS data with the choice of parameters $A = 0.085$ and $\alpha = 1.724$ [28] which are improved constraints than the previous ones ($-0.35 < A < 0.025$) [29].

The drawback of the gravitational force has been successfully explained by proposing the existence of more than three special dimensions [30]. Today, the existence of extra dimensions [31, 32] are supported by large number of promising model and theories. The solutions for MCG in $(n+2)$-dimensional FRW Cosmology are given in section II. Here, the table of $H(z)$ and $\sigma(z)$ is presented for different values of $z$. The $\chi^2$ minimum test for best fit values of parameters are investigated with Stern and Stern+BAO joint data analysis in section III. Finally, some observational conclusions are drawn.

II. BASIC EQUATIONS AND SOLUTIONS FOR MCG IN HIGHER DIMENSIONAL FRW COSMOLOGY

We consider the $(n+2)$ dimensional flat ($k = 0$), homogeneous and isotropic universe described by FRW metric is given by [33, 34]

$$ds^2 = dt^2 - a^2(t)[dr^2 + r^2 dx_n^2]$$ (1)

where $a(t)$ is the scale factor and

$$dx_n^2 = d\theta_1^2 + sin^2\theta_1 d\theta_2^2 + \ldots + sin^2\theta_1 sin^2\theta_2 \ldots sin^2\theta_{n-1} d\theta_n^2$$ (2)

The modified Einstein’s field equations in higher dimension are given by

$$\frac{n(n+1)}{2} \left(\frac{\dot{a}}{a}\right)^2 = \rho_m + \rho_x$$ (3)

and

$$\frac{n}{a} \ddot{a} + \frac{n(n-1)}{2} \left(\frac{\dot{a}}{a}\right)^2 = -p_x$$ (4)

where $\rho_m$ is the energy density of the dark matter (pressureless fluid) and $\rho_x, p_x$ are the energy density and pressure of dark energy (choosing $8\pi G = c = 1$).

Now we consider the Universe is filled with Modified Chaplygin Gas (MCG) whose equation of state (EOS) is given by [27]

$$p_x = A\rho_x - \frac{B}{\rho_x^\alpha}, \quad A > 0, \quad B > 0, \quad 0 \leq \alpha \leq 1$$ (5)

We also assume that the dark matter and dark energy are separately conserved. So the energy conservation equations in homogeneous higher dimensional cosmology are

$$\dot{\rho}_m + (n+1)H\rho_m = 0$$ (6)

and

$$\dot{\rho}_x + (n+1)H(\rho_x + p_x) = 0$$ (7)

where $H$ is the Hubble parameter defined as $H = \frac{\dot{a}}{a}$. From the first conservation equation (6) we have the solution of $\rho_m$ as

$$\rho_m = \rho_{m0}(1 + z)^{n+1}$$ (8)
where $\rho_{m0}$ is the present value of the density of matter and $z = \frac{1}{a} - 1$ is the cosmological redshift. From the second conservation equation (7) we have the solution of the energy density $\rho_x$ as

$$\rho_x = \left[ \frac{B}{A+1} + \frac{C(1+z)^{(n+1)(\alpha+1)(A+1)}}{(A+1)} \right]^\frac{1}{\alpha+1}$$

(9)

where $C$ is the integration constant which can be interpreted as one of the contribution of dark energy. Now the equation (9) can be written as

$$\rho_x = \rho_{x0} \left[ \frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} (1+z)^{(n+1)(\alpha+1)(A+1)} \right]^\frac{1}{\alpha+1}$$

(10)

where $\rho_{x0}$ is the present value of the dark energy density.

### III. OBSERVATIONAL DATA ANALYSIS MECHANISM

We now investigate the expected bounds of the theoretical parameters by $\chi^2$ statistical best fit test with the basis of $H(z)$-z (Stern) \[35\] and Stern+BAO \[36–41\] joint data analysis. We also determine the statistical confidence contours between any two parameters of of MCG in FRW higher dimensional cosmology. To investigate the bounds of model parameters here we consider Stern ($H(z)$-z) data set with 12 data of $H(z)$-z (Stern) given by \[35\]

| $z$ | $H(z)$ | $\sigma(z)$ |
|-----|--------|------------|
| 0   | 73     | ± 8        |
| 0.1 | 69     | ± 12       |
| 0.17| 83     | ± 8        |
| 0.27| 77     | ± 14       |
| 0.4 | 95     | ± 17.4     |
| 0.48| 90     | ± 60       |
| 0.88| 97     | ± 40.4     |
| 0.9 | 117    | ± 23       |
| 1.3 | 168    | ± 17.4     |
| 1.43| 177    | ± 18.2     |
| 1.53| 140    | ± 14       |
| 1.75| 202    | ± 40.4     |

Table 1: $H(z)$ and $\sigma(z)$ for different values of $z$.

From the solutions (8) and (10) we now express the Hubble parameter $H$ in terms of redshift parameter $z$ and two dimensionless density parameters $\Omega_{m0} = \frac{2\rho_{m0}}{n(n+1)H_0^2}$ and $\Omega_{x0} = \frac{2\rho_{x0}}{n(n+1)H_0^2}$ as follows:

$$H(z) = H_0 \left[ \Omega_{x0} \left[ \frac{B}{(1+A)C+B} + \frac{(1+A)C}{(1+A)C+B} (1+z)^{(n+1)(\alpha+1)(A+1)} \right]^\frac{1}{\alpha+1} + \Omega_{m0}(1+z)^{n+1} \right]^\frac{1}{2}$$

(11)

This equation can be written in the form $H(z) = H_0 E(z)$, where $E(z)$ known as normalized Hubble parameter contains five model parameters $A, B, C, n, \alpha$ beside the redshift parameter $z$. Now to find the bounds of the parameters and to draw the statistical confidence contour (66%, 90% and 99% confidence levels) we first fixed three parameters $C, n, \alpha$ and then fixed the three parameters $A, n, \alpha$. In the first case we find the bounds of $A, B$ and draw the contour between them. In the second case we fixed three different values of $A$ as 1, 1/3, $-1/3$ to find the bounds of $B, C$ and draw the contour between them. Here the parameter $n$ determines the higher dimensions and we perform comparative study between three cases: 4D ($n = 2$), 5D ($n = 3$) and 6D ($n = 4$) respectively.
Figs. 1 - 3 show that the variation of $B$ against $A$ for $\alpha = 0.0001$ and $C = 1.5$ in 4D, 5D and 6D respectively for different confidence levels. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted in these figures for the $H(z)$-z (Stern) analysis.

A. Stern ($H(z)$-z) Data Analysis

Here we analyze the model parameters using twelve data [35] of Hubble parameter for different redshift given by table 1. The corresponding error $\sigma(z)$ also available in the table. Here we perform $\chi^2$ test as a tool to find the minimum values of the parameters and draw the contours with three particular confidence limit. For this purpose we first define the $\chi^2$ statistic with 11 degree of freedom as

$$\chi^2_{\text{Stern}} = \sum \frac{(H_E(H_0, A, B, C, n, \alpha, z) - H_{\text{obs}})^2}{\sigma(z)^2}$$

(12)

where $H_E$ and $H_{\text{obs}}$ are theoretical and observational values of Hubble parameter at different redshifts respectively and $\sigma(z)$ is the corresponding error as per Table 1. Since we are interested to determine model parameters, $H_0$ is considered as a nuisance parameter and can be safely marginalized. We consider the observed parameters $\Omega_{m0} = 0.28$, $\Omega_{x0} = 0.72$, $H_0 = 72 \pm 8 \text{ Kms}^{-1} \text{ Mpc}^{-1}$ and a fixed prior distribution. Here we shall determine the model parameters $A, B, C$ by minimizing the $\chi^2$ statistic. The probability distribution can be written as

$$L = \int e^{-\frac{1}{2}\chi^2_{\text{Stern}}} P(H_0) dH_0$$

(13)

where $P(H_0)$ is the prior distribution function for $H_0$. As per our theoretical model of MCG the two parameters should satisfy the two inequalities $A \leq 1$ and $B > 0$. We now plot the graphs for different confidence levels i.e., 66%, 90% and 99% confidence levels and for three different dimensions (4D, 5D and 6D). Now our best fit analysis with Stern observational data support the theoretical range of the parameters. When we fix the two parameters $C = 1.5$ and $\alpha = 0.0001$, the 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours for $(A, B)$ are plotted in figures 1, 2 and 3 for 4D ($n = 2$), 5D ($n = 3$) and 6D ($n = 4$) respectively and we see that $A$ becomes negative in this case. If we fix the parameter $A$ and $\alpha = 0.0001$, the 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours for $(B, C)$ are plotted in (i) figures 4-6 for 4D, 5D and 6D respectively with $A = 1$, (ii) figures 7-9 for 4D, 5D and 6D respectively with $A = 1/3$ and (iii) figures 10-12 for 4D, 5D and 6D respectively with $A = -1/3$. The best fit values of $(B, C)$ and minimum values of $\chi^2$ for different values of $A = 1$, $1/3$, $-1/3$ in different dimensions are tabulated in Table 2. For each dimension, we compare the model parameters through the values of the parameters and by the statistical contours. From this comparative study, one can understand the convergence of theoretical values of the parameters to the values of the parameters obtained from the observational data set and how it changes from normal four dimension to higher dimension (6D).
Figs. 4 - 12 show that the variation of $B$ against $C$ for $\alpha = 0.0001$ in 4D, 5D and 6D respectively for different confidence levels for $A = 1$ (figs. 4-6), $A = 1/3$ (figs. 7-9), $A = -1/3$ (figs. 10-12). The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted in these figures for the $H(z)$-$z$ (Stern) analysis.
Table 2: $H(z)$-z (Stern): The best fit values of $A$, $B$ and the minimum values of $\chi^2$ in three different dimensions and for $\alpha = 0.0001$ and $C = 1.5$.

| $n$  | $A$     | $B$     | $\chi^2_{\text{min}}$ |
|------|---------|---------|-----------------------|
| 2(4D)| -0.330  | 0.419   | 7.065                 |
| 3(5D)| -0.334  | 0.511   | 7.154                 |
| 4(6D)| -0.381  | 0.625   | 7.464                 |

Table 3: $H(z)$-z (Stern): The best fit values of $B$, $C$ and the minimum values of $\chi^2$ for different values of $A$ in three different dimensions and for $\alpha = 0.0001$.

| $A$  | $n$  | $B$     | $C$     | $\chi^2_{\text{min}}$ |
|------|------|---------|---------|-----------------------|
| 1    | 2(4D)| 0.694   | 0.774   | 11.484                |
|      | 3(5D)| 0.731   | 0.887   | 15.114                |
|      | 4(6D)| 0.505   | 0.754   | 19.191                |
| $\frac{1}{3}$ | 2(4D)| 0.396   | 0.420   | 8.382                 |
|      | 3(5D)| 0.449   | 0.240   | 10.378                |
|      | 4(6D)| 0.634   | 0.466   | 12.769                |
| $-\frac{1}{3}$ | 2(4D)| 0.411   | 0.300   | 7.066                 |
|      | 3(5D)| 0.254   | 0.302   | 7.152                 |
|      | 4(6D)| 0.574   | 0.668   | 7.632                 |

**B. Stern + BAO Joint Data Analysis**

The Baryon Acoustic Oscillations (BAO) in the primordial baryon-photon fluid, leave a characteristic signal on the galaxy correlation function, a bump at a scale $\sim 100$ Mpc, as observed by Eisenstein et al [42]. The peaks and troughs seen in the angular power spectrum arise from gravity-driven acoustic oscillations of the coupled photon-baryon fluid in the early Universe. The interpretation of BAO measurements are the effects of non-linear gravitational evolution, of scale-dependent differences between the clustering of galaxies and of dark matter and for spectroscopic surveys, redshift distortions of the clustering, which can shift the BAO features. Sloan Digital Sky Survey (SDSS) survey is one of the first redshift survey (46748 luminous red galaxies spectroscopic sample, over 3816 square-degrees of sky approximately five billion light years in diameter) by which the BAO signal has been directly detected at a scale $\sim 100$ Mpc (SDSS confirmed the WMAP results that the sound horizon in the today’s universe). The SDSS catalog provides a picture of the distribution of matter such that one can search for a BAO signal by seeing if there is a larger number of galaxies separated at the sound horizon. We shall investigate the two parameters $A$ and $B$ for our model using the BAO peak joint analysis for low redshift (with range $0 < z < 0.35$) using standard $\chi^2$ distribution. The BAO peak parameter may be defined by

$$A = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}} \left( \frac{1}{z_1} \int_0^{z_1} \frac{dz}{E(z)} \right)^{2/3}$$

where

$$\Omega_m = \Omega_{m0}(1 + z_1)^3 E(z_1)^{-2}$$

Here, $E(z)$ is the normalized Hubble parameter and $z_1 = 0.35$ is the typical redshift of the SDSS data sample. This quantity can be used even for more general models which do not present a large contribution of dark energy at early times. Now the $\chi^2$ function for the BAO measurement can be written as in the following form

$$\chi^2_{\text{BAO}} = \frac{(A - 0.469)^2}{0.017^2}$$
where the value of the parameter $A$ for the flat model ($k = 0$) of the FRW universe is obtained by $A = 0.469 \pm 0.017$ using SDSS data set [42] from luminous red galaxies survey. Now the total joint data analysis (Stern+BAO) for the $\chi^2$ function defined by

$$\chi^2_{\text{tot}} = \chi^2_{\text{Stern}} + \chi^2_{\text{BAO}}$$

(17)

Now our best fit analysis with Stern+BAO observational data support the theoretical range of the parameters. In figures 13, 14 and 15, we plot the graphs of $(A, B)$ for different confidence levels 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours for 4D, 5D and 6D respectively and by fixing the other parameters $C = 1.5$ and $\alpha = 0.0001$.

| $n$ | $A$  | $B$  | $\chi^2_{\text{min}}$ |
|-----|------|------|-----------------------|
| 2(4D) | −0.323 | 0.440 | 767.456 |
| 3(5D) | −0.327 | 0.517 | 767.508 |
| 4(6D) | −0.373 | 0.632 | 767.787 |

Table 4: $H(z)$-$z$ (Stern+BAO): The best fit values of $A$, $B$ and the minimum values of $\chi^2$ in three different dimensions and for $\alpha = 0.0001$ and $C = 1.5$.

IV. DISCUSSIONS

In this work, we have considered the flat FRW model of the universe in $(n + 2)$-dimensions filled with the dark matter (perfect fluid with negligible pressure) and the modified Chaplygin gas (MCG) type dark energy. We present the Hubble parameter in terms of the observable parameters $\Omega_{m0}$, $\Omega_{x0}$ and $H_0$ with the redshift $z$ and the other parameters like $A$, $B$, $C$, $n$ and $\alpha$. We have chosen the observed values of $\Omega_{m0} = 0.28$, $\Omega_{x0} = 0.72$ and $H_0 = 72 \text{ Kms}^{-1} \text{ Mpc}^{-1}$. From Stern data set (12 points), we have obtained the bounds of the arbitrary parameters by minimizing the $\chi^2$ test. The best-fit values of the parameters are obtained by 66%, 90% and 99% confidence levels. Now to find the bounds of the parameters and to draw the statistical confidence contour, we first fixed three parameters $C, n, \alpha$ and then fixed the three parameters $A, n, \alpha$. In the first case we find the bounds of $(A, B)$ and draw the contour between them for 4D($n = 2$), 5D($n = 3$) and 6D($n = 4$). In the second case we fixed three different values of $A$ as 1, 1/3, −1/3 to find the bounds of $(B, C)$ and draw the contour between them. Here the parameter $n$ determines the higher dimensions and we perform comparative study between three cases: 4D ($n = 2$), 5D ($n = 3$) and 6D ($n = 4$) respectively. Finally due to joint analysis with Stern+BAO observational data, we find the bounds of $(A, B)$ and draw the contour between them for 4D($n = 2$), 5D($n = 3$) and 6D($n = 4$).
We have plotted the graphs for different confidence levels i.e., 66%, 90% and 99% confidence levels and for three different dimensions (4D, 5D and 6D). Now our best fit analysis with Stern observational data support the theoretical range of the parameters. When we fix the two parameters \( C = 1.5 \) and \( \alpha = 0.0001 \), the 66\% (solid, blue), 90\% (dashed, red) and 99\% (dashed, black) contours for \((A, B)\) are plotted in figures 1, 2 and 3 for 4D \((n = 2)\), 5D \((n = 3)\) and 6D \((n = 4)\) respectively. The best fit values of \((A, B)\) and minimum values of \(\chi^2\) in different dimensions (4D, 5D and 6D) are tabulated in Table 2 and we see that \(A\) becomes negative in this case. If we fix the parameter \(A\) and \(\alpha = 0.0001\), the 66\% (solid, blue), 90\% (dashed, red) and 99\% (dashed, black) contours for \((B, C)\) are plotted in (i) figures 4-6 for 4D, 5D and 6D respectively with \(A = 1\), (ii) figures 7-9 for 4D, 5D and 6D respectively with \(A = 1/3\) and (iii) figures 10-12 for 4D, 5D and 6D respectively with \(A = −1/3\). The best fit values of \((B, C)\) and minimum values of \(\chi^2\) for different values of \(A = 1, 1/3, −1/3\) in different dimensions are tabulated in Table 3. For each dimension, we compare the model parameters through the values of the parameters and by the statistical contours. From this comparative study, one can understand the convergence of theoretical values of the parameters to the values of the parameters obtained from the observational data set and how it changes from normal four dimension to higher dimension (6D). Next due to joint analysis with Stern+BAO observational data, we have also obtained the bounds of the parameters \((A, B)\) by fixing some other parameters \(\alpha\) and \(C\) for 4D, 5D and 6D. In figures 13, 14 and 15, we have plotted the graphs of \((A, B)\) for different confidence levels 66\% (solid, blue), 90\% (dashed, red) and 99\% (dashed, black) contours for 4D, 5D and 6D respectively and by fixing the other parameters \(C = 1.5\) and \(\alpha = 0.0001\). The best fit values of \((A, B)\) and minimum values of \(\chi^2\) for Stern and BAO data in different dimensions (4D, 5D and 6D) are tabulated in Table 4. In summary, the conclusion of this discussion suggests that for different dimension (4D, 5D and 6D) cosmological observation can put upper bounds on the magnitude of the correction coming from quantum gravity that may be closer to the theoretical expectation than what one would expect.

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