On Two-way Communications for Cooperative Multiple Source Pairs Through a Multi-antenna Relay

Chin Choy CHAI*, Chau YUEN†

*Institute for Infocomm Research
1 Fusionopolis Way, #21-01 Connexis, Singapore 138632
E-mail: chaicc@i2r.a-star.edu.sg

†Singapore University of Technology and Design
287 Ghim Moh Road #04-00, Singapore 279623
E-mail: yuenchau@sutd.edu.sg

Abstract—We study amplified-and-forward (AF)-based two-way relaying (TWR) with multiple source pairs, which are exchanging information through the relay. Each source has single antenna and the relay has multi-antenna. The optimal beamforming matrix structure that achieves maximum signal-to-interference-plus-noise ratio (SINR) for TWR with multiple source pairs is derived. We then present two new non-zero-forcing based beamforming schemes for TWR, which take into consideration the tradeoff between preserving the desired signals and suppressing inter-pair interference between different source pairs. Joint grouping and beamforming scheme is proposed to achieve a better signal-to-interference-plus-noise ratio (SINR) when the total number of source pairs is large and the signal-to-noise ratio (SNR) at the relay is low.

Index Terms—Analogue network coding (ANC), two-way relaying (TWR), multiple source pairs, information exchange, analogue relaying, optimal beamforming.

I. INTRODUCTION

By applying the physical-layer network coding [1] or analogue network coding (ANC) [2] in two-way relaying (TWR), only two time slots are required for one complete information exchange using TWR. In the first time slot, both source nodes transmit simultaneously to the relay. In the second time slot, the relay broadcasts the common message which is obtained by combining the received messages. Since both source nodes know their own transmitted signals, each of their self-interference can be completely canceled prior to decoding.

The TWR has been studied in [3] to [10] for the case of single source pair. The beamformer design for AF-based multiple-input-multiple-output (MIMO) TWR is studied in [4][5], in which the receive and transmit beamforming are derived separately and then combined to form the relay beamformer. Furthermore, some strategies to enhance the performance of TWR can be found in [11] to [13] and references therein.

The non-ANC-based TWR for multiple source pairs is studied in [14] to [18]. Unlike the single source pair case, in multiple source pairs scenario, additional inter-pair interference exists between different source pairs, which degrades the TWR performance. In [14], a relay network with multiple source pairs and multiple relay nodes is studied, where all sources and relay stations have multiple antennas. The multiuser TWR is proposed and studied in [15], where multiple source pairs are communicating via multiple relays. In [17] [18], the MIMO TWR where multiple wireless node pairs are communicating via a single decode-and-forward (DF) relay is studied.

Due to the presence of inter-pair interference, previous beamforming solutions for TWR with single source pair is no longer useful and new solutions are required for the case of multiple source pairs. In almost all the above works [14] to [18], the inter-pair interference are canceled using zero-forcing (ZF)-based approach.

In this paper, two new non-ZF-based beamforming schemes or beamformers are proposed for ANC-based TWR. Instead of completely canceling the inter-pair interference for all source pairs by ZF-based methods as was done in previous works, we propose joint grouping and beamforming scheme that divides a given large number of source pairs into smaller subgroups, and then apply the proposed beamformers to each subgroup. To the best of our knowledge, this approach has not been studied in previous works. Simulation results are presented to compare the performance of the proposed schemes.
II. SYSTEM MODEL

We study wireless TWR with a multi-antenna relay and a total number of $K_T$ single-antenna sources, where $K_T$ is an even number. Due to the fact that the inter-pair interference contains the desired signals of all the other sources, these desired signals are also suppressed by any suboptimal beamformer which causes significant loss in the SINR, especially when $K_T$ is large. We propose to overcome this shortcoming by first dividing a large number of $K_T$ source pairs into $N$ subgroups, each with a smaller number of $K$ source pairs, where $K$ is an even number. Then, by using time division approach, the relay performs non-ZF-based beamforming on each subgroup of users at one time, and take turn to serve all source pairs, to achieve a better throughput performance. Next, we consider a given subgroup of $K$ sources, and derive TWR beamformers for these $K$ sources.

Without loss of generality, we assume the $k$-th source node $S_k$, $k = \ldots, K$, is to exchange information with another source $S_{\bar{k}}$, where $\bar{k} = 1, \ldots, K$, $k \neq \bar{k}$. Each $k$-th source have single antenna, whereas the relay station $R$ equipped with $M(M \geq K - 1)$ antennas. Let the $M \times 1$ vectors $\mathbf{h}_k$ and $\mathbf{h}_{\bar{k}}$ denote, respectively, the channel response matrices from $S_k$ to $R$, and that from $S_{\bar{k}}$ to $R$. We assume that the elements of $\mathbf{h}_k$ and $\mathbf{h}_{\bar{k}}$ follow the distribution of circularly symmetric complex Gaussian with zero mean and unity variance, which is denoted as $\mathcal{CN}(0, 1)$. Let $s_k(n)$ and $s_{\bar{k}}(n)$ denote respectively, the transmitted signals from $S_k$ to $S_{\bar{k}}$, and from $S_{\bar{k}}$ to $S_k$. We assume the optimal Gaussian codebook is used at each $S_k$, and therefore $s_k(n)$’s are independent random variable each is distributed as $\mathcal{CN}(0, 1)$.

Assume TDD is used and two time slots are needed for information exchange using analogue network coding (ANC)\cite{1}, \cite{2}. In the first time slot, all active source nodes transmit their signals simultaneously, the received baseband signal vector $y_R(n)$ at $R$ is given by,

$$y_R(n) = \mathbf{h}_1 \sqrt{p_1} s_1(n) + \ldots + \mathbf{h}_{K-1} \sqrt{p_{K-1}} s_{K-1}(n) + \mathbf{h}_K \sqrt{p_K} s_K(n) + z_R(n),$$

where $n$ is symbol index, $p_k$ is the transmit power at $S_k$, $z_R(n)$ is the received additive noise vector, and without loss of generality, it is assume that $z_R(n)$ follows the distribution of $\mathcal{CN}(0, \sigma^2)$, $\forall n$, where $\mathbf{I}$ is an identity matrix. Throughout this paper, we assume that the $p_k$’s are given or fixed.

At the relay, we consider AF relaying using linear beamforming which is represented by a $M \times M$ matrix $\mathbf{A}$. The transmit signal $x_R(n)$ at $R$ can be expressed in terms of its inputs $y_R(n)$ as $x_R(n) = \mathbf{Ay}_R(n)$. We assume channel reciprocity for uplink and downlink transmission through the relay. In the second time slot, when $x_R(n)$ is transmitted from $R$, the channels from $R$ to $S_k$ become $\mathbf{h}_k^T$, $k = 1, \ldots, K$. The total transmit power at $R$, denoted as $p_R$, can be shown as,

$$p_R = \sum_{k=1}^{K} \|\mathbf{Ah}_k\|^2 p_k + \text{Tr}(\mathbf{AA}^\dagger)\sigma^2,$$  \hspace{1cm} (2)

where $\text{Tr}(\mathbf{X})$ denotes the trace of $\mathbf{X}$.

The ANC is adopted as follows. We assume that using training and estimation, $\mathbf{h}_k^T \mathbf{Ah}_k$ and $\mathbf{h}_{\bar{k}}^T \mathbf{Ah}_{\bar{k}}$ are perfectly known at $S_k$, $k = 1, \ldots, K$ prior to signal transmission. Each of the $S_k$ can first cancel its self-interference and then coherently demodulate for $s_k$. This yields

$$\tilde{y}_k(n) = \mathbf{h}_k^T \mathbf{Ah}_k \sqrt{\mathbf{p}_k} s_k(n) + \sum_{j \neq k, j \neq \bar{k}} \mathbf{h}_k^T \mathbf{Ah}_j \sqrt{\mathbf{p}_j} s_j(n) + \tilde{z}_k(n), \hspace{1cm} (k = 1, \ldots, K,$$

for $k = 1, \ldots, K$. We assume that the received noise $z_k(n)$ is distributed as $\mathcal{CN}(0, \sigma^2)$, and $z_R(n)$ are independent of $z_k(n)$. $\tilde{z}_k(n) = \mathbf{h}_k^T \mathbf{Az}_R(n) + z_k(n)$, and $\tilde{z}_k(n)$ is distributed as $\mathcal{CN}(0, (\|\mathbf{h}_k^T \mathbf{A}\|^2 + 1)\sigma^2)$. At each $S_k$, coherent signal detection can then be used to recover $s_k$ from $\tilde{y}_k(n)$. The signal-to-interference-plus-noise ratio (SINR) for the $k$-th destination node. $k = 1, \ldots, K$, can be expressed as

$$\gamma_k = \frac{\|\mathbf{h}_k^T \mathbf{Ah}_k\|^2 p_k}{\sum_{j \neq k, j \neq \bar{k}} \|\mathbf{h}_k^T \mathbf{Ah}_j\|^2 p_j + (\|\mathbf{h}_k^T \mathbf{A}\|^2 + 1)\sigma^2}, \hspace{1cm} (3)$$

III. PROPOSED SCHEMES

A. Optimal beamformer

We define the uplink (UL) channel gain matrix $\mathbf{H}_{UL} = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_K]$ and denote the singular value decomposition (SVD) of $\mathbf{H}_{UL}$ as $\mathbf{H}_{UL} = \mathbf{U} \Sigma \mathbf{V}^H$, where the $M \times K$ matrix $\mathbf{U}$, and the $K \times K$ matrix $\mathbf{V}$, are with orthogonal column vectors, and $\Sigma$ is singular value matrix with dimension $K \times K$ where $[\Sigma]_{\ell, \ell} = \sigma_\ell$ for $\ell = 1, \ldots, K$. We have the following new result.

Proposition: The optimal beamforming matrix to achieve maximum SINR in (3) has the following structure:

$$\mathbf{A}^{\text{opt}} = \mathbf{U}^* \mathbf{B}^{\text{opt}} \mathbf{U}^H, \hspace{1cm} (4)$$
where $B^{opt}$ is a $K \times K$ matrix.

**Proof:** The above new result can be proven by extending the proof of [3][10], which considers the TWR for the case of single source pair who are exchanging information. For the case of multiple source pairs, each source is also subject to interference from the other source pairs. This so-called inter-pair interference term also depends only on $B$ as \( \sum_{j \neq k,j \neq k} |h_{kj}^T A h_j|^2 = \sum_{j \neq k,j \neq k} |h_{kj}^T U^* B U^H h_j|^2 \), which also spans the total signal subspace of $B$. Therefore, we obtain $B^{opt}$. The beamforming matrix $B$ can be solved as shown next.

Let $\hat{h}_k = U^H h_k$, $k = 1, \ldots, K$, represent the effective channel from $S_k$ to $R$, with $A$ given in [4]. Similarly, let $\hat{h}_k^T$ represent the effective channel from $R$ to $S_k$. The SINR formula in [3] can be written in terms of $B$. Throughout this paper, the optimization problems are formulated in terms of the beamforming matrix $B$ and the effective channels $\hat{h}_k$.

The minimum power (MP) beamformer is derived by minimizing the total relay transmit power with respect to the relay beamforming matrix $B$ (or equivalently, $A$), subject to SINR constraints $\gamma_k \geq \gamma^*_k, k = 1, \ldots, K$,

$$\min_B \sum_{k=1}^K ||B\hat{h}_k||^2 p_k + \text{Tr}(BB^H)\sigma^2,$$

subject to $\gamma_k \geq \gamma^*_k, k = 1, \ldots, K$.

For given transmit powers $p_k, k = 1, \ldots, K$, we can use the same approach as in [10], to develop an efficient algorithm based on the second-order cone programming (SOCP) [19] to solve the problem in (5).

**B. Suboptimal beamformer**

We derive the suboptimal minimum interference (MI) beamformer that does not require computations via optimization technique. The tradeoff between the desired signals and interference is taken into account by minimizing the sum of inter-pair interference plus AF noise at the output of the beamformer, subject to the constraints that the desired signal gain for each $k$-th receiver is equal to a constant $\beta_k$. The additive Gaussian noise $\hat{z}_k(n)$ at each $k$-th source, which is not affected by the beamforming matrix $B$, and is neglected in the optimization. Again in this case, the joint design of both receive and transmit beamforming is considered. The interference minimization problem is formulated as,

$$\min_B \sum_{k=1}^K I_k(B),$$

subject to $|h_{kj}^T B h_k|^2 p_k = \beta_k, k = 1, \ldots, K$.

where

$$I_k(B) = \sum_{j \neq k,j \neq k} |h_{kj}^T B h_j|^2 p_j + ||h_{kj}^T B||^2 \sigma^2, k = 1, \ldots, K.$$

The above constraints in (6) are introduced to preserve the desired signal components at each source, so as to minimize the inter-pair interference plus AF noise component that point into any undesired direction. In (7), $I_k(B)$ represents the sum of inter-pair interference (from other source pairs) plus AF noise that is imposed on $S_k$. To solve for $B$, the UL and DL channel response of each $k$-th source is assumed to be known at the relay.

We assume that the $p_k$’s are given, and write $|h_{kj}^T B h_k|^2 p_k = |f_k^T b|^2$. \( \sum_{j \neq k,j \neq k} |h_{kj}^T B h_j|^2 p_j = \sum_{j \neq k,j \neq k} |d_{kj}^T b|^2 \), and $||h_{kj}^T B||^2 = ||G_k b||^2$, where $k \neq \tilde{k}$, and $b$ denotes a $K^2 \times 1$ equivalent beamforming weights vector which is generated by the rule of [8] as,

$$V(Q) = \begin{bmatrix} q_1 \\ \vdots \\ q_K \end{bmatrix}.$$ (8)

The problem (6) can be written in terms of $b$ as,

$$\min_b \sum_{k=1}^K \left( \sum_{j \neq k,j \neq k} |d_{kj}^T b|^2 + ||G_k b||^2 \sigma^2 \right)$$

subject to $|f_k^T b| = \beta_k, k = 1, \ldots, K$.

We further define $R_k = \sum_{j \neq k} d_{kj}^T d_{kj}$, $N_k = G_k^H G_k$, where both $R_k$, and $N_k$ are $K^2 \times K^2$ matrices. By writing $||G_k b||^2 = b^H N_k b$, $\sum_{j \neq k,j \neq k} |d_{kj}^T b|^2 = b^H R_k b$, and $\Phi = \sum_{k=1}^K (R_k + N_k \sigma^2)$, the problem (9) can be written as,

$$\min_b b^H \Phi b,$$

subject to $C^H b = g$.

where $C = [f_1^T, f_2^T, \ldots, f_K^T]$ is the $K^2 \times K$ constraint matrix, and $f_k$ has been defined in the paragraph after Eq.(8). Each of the column of $C$ imposes a constraint on the equivalent beamforming weight vector $b$. The $K \times 1$ response vector $g = [\beta_1 \ \beta_2 \ \cdots \ \beta_K]^T$ contains the K scalar constraints values $\beta_k, k = 1, \ldots, K$, which are chosen to satisfy the SINR requirements of each source.
The beamforming vector solution, denoted as $b_{MI}$, that satisfies (10) is solved as [20],

$$b_{MI} = \Phi^{-1}C(C^H\Phi^{-1}C)^{-1}g.$$  \hspace{1cm} (11)

The MI relay beamformer, denoted as $B_{MI}$, can then be obtained from $b_{MI}$ as $B_{MI} = \mathcal{V}^{-1}(\alpha b_{MI})$, where $\mathcal{V}^{-1}$ denotes the inverse operation of $\mathcal{V}$ defined in (6). It can be shown that the solution $B_{MI}$ maximizes the SINR of each source, where the constant $\alpha$ is used to control the total relay power $p_R$ in (3). The iterative steps to search for $\alpha$ are presented next.

C. Joint grouping and beamforming scheme

We propose the following joint grouping and beamforming scheme that divides a given large number of source pairs into smaller subgroups, and then apply the above beamformers to each subgroup. For simplicity, here we consider that the grouping is done arbitrarily. By doing so we can reduce the feedback of channel state information and beamforming calculation during each relaying.

- Divide the total number of $K_T$ source pairs into $N$ subgroups, $N = 1, 2, \ldots$, and let $K = K_T/N$ denote the number of source in each subgroup.
- For each $n$-th subgroup, $n = 1, \ldots, N$, compute the MI beamforming matrix as follows:
  - Given $\alpha \in [0, \alpha_{\text{max}}]$, $\mathbf{p} = [p_1 \ p_2 \ \ldots \ p_K]^T$.
  - Initialize $\alpha_{\text{lower}} = 0$, $\alpha_{\text{upper}} = \alpha_{\text{max}}$.
  - Compute the SVD of the UL channel matrix $\mathbf{H}_{UL} = \mathbf{U}\Sigma\mathbf{V}^H$.
  - Estimate the correlation matrix $\Phi$.
  - Compute the constraint matrix $C$ and the response vector $g$.
  - Compute the beamforming weights using $b_{MI} = \Phi^{-1}C(C^H\Phi^{-1}C)^{-1}g$.
  - Obtain the MI relay beamforming matrix by arranging the beamforming weights of $b_{MI}$ into the matrix form, $B_{MI} = \mathcal{V}^{-1}(\alpha b_{MI})$.
  - Repeat
    1. Set $\alpha \leftarrow \frac{1}{2}(\alpha_{\text{lower}} + \alpha_{\text{upper}})$.
    2. Update $\alpha$ by the bisection method [19]. If $p_R$ is less than a given power constraint, set $\alpha_{\text{lower}} \leftarrow \alpha$; otherwise, $\alpha_{\text{upper}} \leftarrow \alpha$.
    - Until $\alpha_{\text{upper}} - \alpha_{\text{lower}} \leq \delta_\alpha$, where the small positive constant $\delta_\alpha$ is chosen to ensure sufficient accuracy.

- Compute the MI relay beamforming matrix as $A_{MI} = U^Hb_{MI}U^H$ for each $n$-th subgroup, $n = 1, \ldots, N$.
- Select the number of subgroup $N$ and its corresponding $A_{MI}$ which results in the largest achievable sum-rate.

IV. SIMULATION RESULTS AND DISCUSSIONS

In this section, the sum rate performance of the proposed TWR beamformers are presented. For simplicity, the channel correlations between the $j$-th and the $k$-th sources are set to be equal, that is, $|h_{j}^Hh_k|^2 = \rho$, $\forall j \neq k$.

In Fig. 1 two pairs of single-antenna source ($K = 4$) and a multi-antenna relay ($M = 8$) are considered in the simulations. The transmit power at each source $S_1, S_2, S_3$ and $S_4$ are fixed as $p_1 = p_2 = p_3 = p_4 = 10\text{dB}$, and the relay transmit power is set to be $p_R = 10\text{dB}$. Both sources within each pair are set to have identical SINR requirement. With no channel correlation between different sources, the proposed MI beamformer achieves the same achievable rate region as the optimal MP beamforming scheme. However, when the channel correlations between different source pairs, and within each source pair, are set to be higher, the proposed MI beamformer achieves a smaller rate region (not shown here due to space constraint). This is because when the channels of different source pairs are correlated, the desired signal components that point in the direction of the inter-pair interference will also be suppressed by the MI beamformer, which in turn reduces the achievable rate for each source.

![Fig. 1. Comparison on the achievable rate region for the proposed MP and the MI relay beamformers with $M = 8$, $K = 4$, fixed $p_1 = p_2 = p_3 = p_4 = 10$, $P_R = 10$, and $\rho = 0$.](image)

In Fig. 2 we present the achievable sum-rate versus SNR at the relay, which is defined as $p_R/\sigma^2$, for the proposed MI
beamformer with various number of subgroups. Four pairs of single-antenna source (\(K = 8\)) and a multi-antenna relay (\(M = 8\)) are considered, and the channels between different sources are uncorrelated. Time division is used to serve different subgroup. We observe that either the smaller subgroups or large group transmission does not always perform the best in TWR. With low SNR at the relay, the case of four groups (each with one source pair) performs the best, whereas for large SNR, the case of a single group (with four source pairs) performs the best. This is because with either large number of source pairs or small relay transmit power, the MI beamformer tends to suppress the interference more for the case of single group (with more interfering users), which results in SINR loss.

To overcome this shortcoming, joint grouping and beamforming scheme can be used to reduce SINR loss as follows. For small SNR, we should apply the proposed beamformers to four different subgroups, and for large SNR, we should apply the proposed beamformers to a single group (with four source pairs). The improved sum-rate performance by joint grouping and MI beamforming scheme is highlighted by the solid line in Fig. 2. The optimal grouping and selection of the SNR thresholds correspond to different number of subgroups are interesting subjects for further study.

V. CONCLUSIONS

New optimal and suboptimal beamformers for ANC-based TWR with multiple source pairs are derived by taking into account the tradeoff between the desired signals and the inter-pair interference. For low SNR, a better sum-rate performance can be achieved by first dividing a large number of source pairs into smaller subgroups, and then apply beamforming to each subgroup using time division.

ACKNOWLEDGMENT

This research is partly supported by the Singapore University Technology and Design (grant no. SRG-EPD-2010-005).

REFERENCES

[1] S. Zhang, S.-C. Liew, and P. P. Lam, “Physical-layer network coding”, Proc. of MobiComm 2006, pp. 358-365, 2006.
[2] S. Katti, S. Gollakota, and D. Katabi, “Embracing wireless interference: Analog network coding”, Computer Science and Artificial Intelligence Laboratory Technical Report, MIT-CSAIL-TR-2007-012, Feb. 23, 2007.
[3] I. Hammerström, M. Kuhn, C. Esli, J. Zhao, A. Wittneben, and G. Bauch, “MIMO two-way relaying with transmit CSI at the relay”, in Proc. IEEE Signal Proc. Adv. Wireless Comm. (SPAWC), Jun. 2007.
[4] T. Unger and A. Klein, “Linear transceive filters for relay stations with multiple antennas in the two-way relay channel”, 16th IST Mobile and Wireless Communications Summit, Budapest, Hungary, Jul. 2007.
[5] T. Unger and A. Klein, “Maximum sum rate of non-regenerative two-way relaying in systems with different complexities”, in Proc. IEEE ICC, May 2008.
[6] T. Cui, F. F. Gao, T. Ho, and A. Nallanathan, “Distributed space-time coding for two-way wireless relay networks”, in Proc. IEEE ICC, Beijing, China, May 2008.
[7] R. F. Wyrembelski, T. J. Oechtering, I. Bjelaković, C. Schnurr, and H. Boche, “Capacity of Gaussian MIMO bidirectional broadcast channels”, in Proc. IEEE Int. Symp. Inf. Theory (ISIT), Jul. 2008.
[8] Y. C. Liang and R. Zhang, “Optimal analogue relaying with multi-antennas for physical layer network coding”, in Proc. IEEE ICC, pp. 3893-3897, May 2008.
[9] M. Eslamifar, W. H. Chin, C. Yuen, and Y. L. Guan, “Performance Analysis of Two-Way Multiple-Antenna Relaying with Network Coding”, Proc. VTC-Fall 2009.
[10] R. Zhang, Y. C. Liang, C. C. Chai, and S. Cui, “Optimal beamforming for two-way multi-antenna relay channel with analogue network coding”, IEEE Journal on Selected Areas in Communication, vol. 27, no. 5, October 2009, pp. 3988-3999.
[11] M. Eslamifar, C. Yuen, W. H. Chin, and Y. L. Guan, “Max-Min Antenna Selection for Bi-Directional Multi-Antenna Relaying”, Proc. VTC-Spring 2010.
[12] L. Song, G. Hong, B. Jiao, and M. Debbah, “Joint relay selection and analog network coding using differential modulation in two-way relay channels”, IEEE Transactions on Vehicle Technology, Vol. 59, No. 6, Jun. 2010.
[13] T. Abe, H. Shi, T. Asai and H. Yoshino, “Relay techniques for MIMO wireless networks with multiple source and destination pairs”, in EURASIP Journal on Wireless Communications and Networking, pp. 1-9, 2006.
[14] B. Rankov and A. Wittneben, “Spectral efficient protocols for half-duplex fading relay channels”, IEEE Journal on Selected Areas in Communications, vol 25, no. 2, pp. 379-389, 2007.
[15] M. Chen and A. Yener, “Multiuser two-way relaying for interference limited systems”, in Proc. IEEE ICC, May 2008.
[16] C. Esli and A. Wittneben, “Multiuser MIMO two-way relaying for cellular communications”, in Proc. IEEE ICC, May 2008.
[17] C. Esli and A. Wittneben, “One- and two-way decode-and-forward relaying for wireless multiuser MIMO networks”, in Proc. IEEE GLOBECOM, September 2008.
[18] O. L. Frost, III, “An algorithm for linearly constrained adaptive array processing”, Proc. IEEE, Vol. 60, No. 8, pp. 926-935, August 1972.