Dephasing of coupled qubit system during gate operations due to background charge fluctuations

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Abstract

A quantum computer that can be constructed based on a superconducting nanocircuits has previously been proposed. We examine the effect of background charge fluctuations on a coupled Josephson charge qubit system used in such a computer. In previous work, the background charge fluctuations were found to be an important dephasing channel for a single Josephson qubit. We investigate the effect of fluctuations in the bias at the charge degeneracy point of a Josephson charge qubit system. Evaluated quantities are gate fidelity and diagonal elements of the qubit’s density matrix. The fluctuation leads to gate error, however quantum gate operation becomes more accurate with increasing interaction between qubit systems.

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I. INTRODUCTION

Among the various proposals for quantum computation, quantum bits (qubits) in solid state materials, such as superconducting Josephson junctions and quantum dots, have the advantage of scalability. Proposals to implement a quantum computer using superconducting nanocircuits are proving to be very promising, and several experiments have already highlighted the quantum properties of these devices. Such a coherent-two-level system constitutes a qubit and the quantum computation can be carried out as the unitary operation functioning on the multiple qubit system. Essentially, this quantum coherence must be maintained during computation. However, it is difficult to avoid dephasing due to the system’s interaction with its external environment. The dephasing is characterized by a dephasing time of $T_2$. Various environments can cause dephasing.

Background charge fluctuations (BCFs) have been observed in diverse kinds of systems. In nanoscale systems, BCFs are electrostatic potential fluctuations arising due to the dynamics of an electron, or a hole, on a charge trap. In particular, the charges in charge traps fluctuate with the Lorentzian spectrum form, which is called random telegraph noise in the time domain. The random distribution of the positions of such dynamical charge traps and their time constants leads to BCFs or $1/f$ noise. In solid-state charge qubits, these BCFs result in a dynamical electrostatic disturbance and hence, dephasing. It should be noted that this dephasing process does not mean the qubit being entangled with the environment, but rather, that the stochastic evolution of an external classical field is suppressing the density matrix elements of the qubit after averaging out over statistically distributed samples.

It has been shown that BCFs are important dephasing channels for a single Josephson charge qubit system. In the present study, we investigate the effect of BCFs on the two-qubit-gate operation. To construct a controllable quantum computer, one requires the suppression of dephasing and accurate universal quantum gate which consists of single qubit operations and two-qubit operations is required. Therefore, to address these manipulations, we examine the dephasing of a coupled qubit system, which is an experimentally current topic. There is a lot of interest in understanding what causes dephasing and its role in these systems. In Sec. II, we discuss the dephasing in a Josephson charge...
qubit system. Sec III is a brief conclusion. Similar subjects are also discussed in terms of decoherence-free subspace in Ref. [24].

II. COUPLED CHARGE QUBIT SYSTEM

The system under consideration is a pair of Cooper pair boxes [5]. Under appropriate conditions (charging energy $E_{C_1,2}$ and the Josephson coupling $E_{J_1,2}$ are much larger than and temperatures $k_BT \ll E_{J_1,2}, E_{C_1,2}$) only two charge states in each box are important, and the Hamiltonian of the pair of qubits $H_{qb}$ reads

$$H_{qb} = \frac{E_{J_1}}{2}\sigma_x^1 + \frac{E_{J_2}}{2}\sigma_x^2 + \frac{\delta E_{C_1}}{2}\sigma_z^1 + \frac{\delta E_{C_2}}{2}\sigma_z^2 + \frac{E_m}{4}\sigma_z^1\sigma_z^2,$$

where the charge bases $\{|0\rangle, |1\rangle\}$ are expressed using Pauli matrices. We chose the charge degeneracy point $\delta E_{C_1,2} = 0$ except for our last result, where $\delta E_{C_i} = E_{C_i}(1 - C_i^i V_i^i / e)$, $(i = 1, 2)$ and $C_i^i$ and $V_i^i$ are capacitance and gate bias of i-th Cooper pair box. The environment is a set of BCFs electrostatistically coupled to the qubits $H_{qb}$ reads

$$H_{qb} = \frac{\hbar J_2}{2}(\sigma_z^1 + \sigma_z^2)(d^d d - \frac{1}{2}),$$

where $d^d$ and $d$ are the electron creation and annihilation operators of a charge trap, and the coupling with the qubit is such that each BCF produces a bistable extra bias $\hbar J$. Because the qubit Hamiltonian consists of $E_{J_1,2}$ and $E_m$, the dephasing is accompanied by dissipation.

It should be noted that we evaluated a collective environment. The two Cooper pair boxes feel the same fluctuations in our model.

Using the environment variable $X(t) = \langle d^d(t)d(t)\rangle_r - 1/2$, where $\langle A(t)\rangle_r$ is a trace of the operator $A(t)$ about the electron reservoir of the charge trap, we rewrite the perturbation Hamiltonian in terms of the Pauli matrix as

$$\mathcal{H}_1 = \frac{\hbar J}{2}(\sigma_z^1 + \sigma_z^2)X(t) = JX(t)V_1,$$

where we denote that the charge trap is strongly coupled with its charge reservoir and the time evolution of $X(t)$ is a Poisson process (BCF). We assume $\langle X(t_1)X(t_2)\rangle = e^{-|t_1 - t_2|/\tau}$, which corresponds to dephasing due to a single trap where $\langle \rangle$ denotes the ensemble average and $\tau$ is the time constant of a BCF. In the interaction representation, $U_0(t) = e^{-\frac{\hbar J}{\tau}H_{qb}t}$, $U_1(t) = e^{\frac{i}{\hbar} \int_0^t \mathcal{H}_1(t')dt'}$ and $V_1(t) = U_0(t)V_1U_0^\dagger(t)$. The ensemble averaged density matrix
\[ \rho(t) \] at time \( t \) in a second-order perturbation approximation is, 
\[ \langle \rho(t) \rangle = U_0(t)\rho(0)U_0^\dagger(t) - J^2 \int_0^t dt_1 \int_0^{t_1} dt_2 U_0(t)e^{-\frac{t_{1-t_2}}{\tau}}[\mathbf{V}_1(t_1),[\mathbf{V}_1(t_2),\rho(0)]]U_0^\dagger(t). \]

The gate fidelity is defined as \( \mathcal{F} = Tr(\rho_0(t)\langle \rho(t) \rangle) \), where \( \rho_0(t) = U_0(t)\rho(0)U_0^\dagger(t) \).

In Fig. 1, we show the \( E_J t \) dependence of \( -\ln(\mathcal{F}(t)) \), where the initial density matrix is \( |00\rangle \langle 00 | \). For simplicity we set \( E_{J_1} = E_{J_2} \). The solid lines denote \( E_m/E_J = 20 \) and the dotted lines denote \( E_m = 0 \). We choose parameters \( J/E_J = 0.5 \) and \( E_J \tau = 10^{-1}, 10^{-2}, 10^{-3} \). At \( t = 0 \), fidelity is 1, and it decreases with time. In the short time regime \( t \ll \frac{\hbar}{E_J} \), the fidelities show \( -\ln(\mathcal{F}(t)) \propto t^4 \). The lowest-order Gaussian behavior \( (t^2) \) originating from the term like \( [\sigma_z^1 + \sigma_z^2, [\sigma_z^1 + \sigma_z^2, \rho(0)]] \) is absent, since we started from a diagonal qubits' density matrix. As \( E_J \tau \) increases, fidelity worsens 

The fidelity of \( E_m \neq 0 \) is larger than that of \( E_m = 0 (> 0) \). The reason is as follows. The fluctuation \( V_1 \) only induces transitions between the ground state and the 2nd excited state of \( H_{qb} \), and between the 2nd excited and the 3rd excited state. When \( E_m = 0 \), the sum of the dephasing rates for these two transitions is given by \( 2J^2 \tau \). As \( E_m \) increases, the dephasing rate decreases down to \( J^2 \tau \), since the dephasing by the transition between the ground state and the 2nd excited state is gradually suppressed 

Thus, the dephasing rate of \( E_m = 0 \) is larger than that of \( E_m \neq 0 \). Therefore, it is expected that interaction between the qubits leads to more reliable quantum gate operations.

Figure 2 shows the Positive Operator-Value Measurement (POVM) results where \( I_1 \) is the sum of the density matrix elements of \( \langle 10|\langle \rho(t)\rangle|10 \rangle \) and \( \langle 11|\langle \rho(t)\rangle|11 \rangle \), with parameters \( J_C = 1 \text{ GHz}, \tau = 0.1 \text{ ns}, E_{J_1} = 13.4 \text{ GHz}, E_{J_2} = 9.1 \text{ GHz} \) and \( E_m = 11.6 \text{ GHz} \), which are the same as those in the experiment in Ref. 27. We compared the Fourier spectrum for \( J = 0 \) and that of \( J = 1 \text{GHz} \). There are two peaks in Fourier spectrum with finite width which correspond to different \( E_{J_1,2} \) of qubit system. The spectrums also show that the peak widths of the qubit system are larger for \( |J| \neq 0 \). This signals that the dephasing due to BCF occurs, as we have shown in the analysis of gate fidelity. For fidelity, \( \mathcal{F}(t) = 1 \) when \( J = 0 \) and \( \mathcal{F}(t) < 1 \) when \( |J| \neq 0 \).

We also examined the time evolution of coherent transition between \( |00 \rangle \leftrightarrow |10 \rangle \). We choose \( \delta E_{C_1} = 0 \), \( \delta E_{C_2} = 152 \text{ GHz} \) and \( E_m = 11.6 \text{ GHz} \). This corresponds to single qubit coherence oscillation. The coherent oscillation is robust against BCF compared with the case of two-qubit operations. This behavior is consistent with the experiment Ref. 27. We speculate that this result depends on the way of the environment coupling to qubits.
III. CONCLUSION

We examined the effect of BCF on the coupled qubit system during gate operation. The fluctuation leads to gate error, however quantum gate operation becomes more accurate with increasing interaction between qubit systems when one start from \(|00\rangle \langle 00|\).

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FIG. 1: Time dependence of gate fidelity of two-qubit operation.
FIG. 2: Time dependence of the diagonal element of the qubit density matrix.