1Gyr in the Life of the Globular Cluster NGC 6397

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ABSTRACT

M4 and NGC 6397 are two very similar galactic globular clusters, which differ mainly in their surface brightness profile. M4 has a classic King-like profile, whereas NGC 6397 has a more concentrated profile, which is often interpreted as that of a post-core collapse cluster. Heggie & Giersz (2008), however, found that M4 is also a post-core collapse cluster, and concluded that the main reason for the difference between the two surface brightness profiles is fluctuations. This conclusion was reached on the basis of Monte Carlo models, however, and in the present Letter we verify that similar fluctuations occur in \(N\)-body models. The models were initialised by generating initial conditions from the Monte Carlo models, however, and in the present Letter we verify that similar fluctuations occur in \(N\)-body models. The models were initialised by generating initial conditions from the Monte Carlo model of NGC 6397 at the simulated age of 12Gyr, and one was followed for 1Gyr. The new models help to clarify the nature of the fluctuations, which have the nature of semi-regular oscillations with a time scale of order 10\(^8\)yr. They are influenced by the dynamical role which is played by primordial binaries in the evolution of the core.

Key words: stellar dynamics – methods: numerical – globular clusters: individual: NGC 6397

1 INTRODUCTION

A long-standing problem in the dynamics of galactic globular clusters is the observed dichotomy in their surface brightness profiles. While most clusters exhibit a profile similar to a classic King profile (King 1966), about one quarter of the clusters exhibit a more cuspy profile (Chernoff & Djorgovski 1989; Trager et al. 1995). The second class of objects are usually described as post-core collapse clusters, as the phenomenon of core collapse leads to an object with high central density and small core radius (Lynden-Bell & Wood 1968; Larson 1970; Lynden-Bell & Eggleton 1980, etc.). By a statistical study of stellar luminosity functions, however, De Marchi, Paresce, & Pulone (2007) suggested that some clusters with a King-like profile might, in fact, be post-core collapse clusters. Independently, Heggie & Giersz (2008) used a Monte Carlo code for star cluster evolution to construct a dynamic, evolutionary model of the King-like globular cluster M4, and were surprised to find that their model was in the post collapse phase of its evolution at the present day. They suggested that the reason it exhibited a finite core radius is that the core was sustained by heating from a population of primordial binary stars. (Many studies had shown that such a population, certainly in the case of idealised models with stars of equal mass, was sufficient to sustain the core in this way after core collapse; see, for example, McMillan, Hut, & Makino (1990); Heggie, Trenti, & Hut (2006).)

The case of the globular cluster NGC 6397 casts serious doubt on this interpretation. This cluster has a very similar mass to M4, and, if anything, a larger population of primordial boundaries, and yet it exhibits a non-King surface brightness profile. If primordial binaries sustain the finite core in M4, then NGC 6397 should exhibit a finite core in the same way.

This conundrum was considered by Giersz & Heggie (2009), who constructed a dynamic evolutionary model for NGC 6397, just as they had previously done for M4. Not surprisingly, this model was also in its post-collapse evolution. Despite having an appropriate primordial binary fraction, however, it exhibited a non-King surface brightness profile that was a fair match to that of NGC 6397. They concluded that primordial binaries and core collapse were insufficient to explain the surface brightness profiles of these two clusters. By considering and also eliminating other alternatives, they concluded that the best explanation for the difference between these clusters was one based on fluctuations. They found that, if they constructed the surface brightness profile from a single model at different times, or from different models started from the same initial conditions, but simply with a different seed for the random number generator, then the variety of surface brightness profiles exhibited differences comparable to the difference between the surface brightness profiles of M4 and NGC 6397. In effect, their conclusion was the following: that a single cluster could sometimes look like M4, and sometimes look like NGC 6397.

Unfortunately it is not clear that a Monte Carlo code, such as the one used for these clusters, simulates fluctuations correctly. Giersz, Heggie, & Hurley (2008) did their best to ensure that the...
Table 1. Data on the initial N-body model (from a Monte Carlo model at 12Gyr)

| Description                  | Value          |
|------------------------------|----------------|
| Number of single stars       | 105615         |
| Number of binaries           | 3277           |
| Total particle number        | 112169         |
| Total mass                   | 62000          |
| Mass of binaries             | 2400           |
| Mass of white dwarfs         | 27000          |
| Mass of neutron stars        | 3100           |
| Tidal radius                 | 22             |
| Half-mass radius             | 3.2            |
| Core radius                  | 0.05           |

Note: masses are given in $M_\odot$, radii in pc. The “core radius” is determined as in the N-body code (Aarseth 2003, pp.265-266).

Monte Carlo model behaves similarly to an N-body model, at least in the range of $N$ where N-body models are commonly carried out, but their investigation was restricted to global quantities, such as the evolution of the total mass, and the half-mass radius; they did not even consider the question of fluctuations. The nature of the fluctuations is also difficult to investigate with a Monte Carlo code, which does not follow the orbital motions of the particles. These considerations motivate the N-body simulations which we report in this letter.

The initial conditions we use are generated from our Monte Carlo model of NGC 6397 evolved to the present day, which we took to be 12Gyr. We placed the model in a tidal field with a tidal radius equal to that of the Monte Carlo model (which used a tidal cut-off), but switched off stellar evolution. One of the runs was continued for a simulation time of one Gyr. Details and results of the runs are given in the following two sections, and the final section of the Letter summarises our conclusions.

2 THE SIMULATIONS

It is unfortunately impossible to specify the initial conditions without access to the complete snapshot of the Monte Carlo model of NGC 6397 at 12 Gyr, but table 1 gives some summary parameters. The Monte Carlo code stores the mass of every star, including binary components. The only positional information on a star (or the barycentre of a binary) which is held by the Monte Carlo code is its radius, and the full position was generated with the assumption of spherical symmetry. For the velocity of each star (or barycentre) the radial and transverse components are available, and the full velocity vector was generated by assuming symmetry about the radial vector. For a binary, the only internal information (apart from the binary masses) is for the semi-major axis and eccentricity. The full relative position and velocity were generated assuming a random value of the mean anomaly, and symmetric distributions of the orbital plane and the line of apsides.

The simulation was run with NBODY6 on a PC equipped with a GPU. The CPU was a quad-core Intel Xeon E5410 at 2.33GHz, and the GPU a GeForce 9800 GTX. As usual, the code uses N-body units (Heggie & Mathieu 1986), but the initial virial radius of the model (the $N$-body unit of length) was 3.43pc, and its crossing time 1.08 Myr ($2\sqrt{2} N$-body time units). The half-mass relaxation time is of order 700Myr.

We actually carried out two runs. One was an exploratory (but scientifically informative) run with dynamically “inert” binaries, i.e. each binary was replaced by a single particle with a mass equal to the combined mass of the components. This was run for an equivalent of almost 260Myr. The main run used dynamically “active” binaries, as described above. We refer to these two runs as “I” and “A” respectively. Both simulations proceeded at a rate of about 1Myr/hr, and the entire run with dynamically active binaries took about 1 month for 1Gyr.

3 RESULTS AND DISCUSSION

3.1 Structural evolution

In the context set out in the Introduction, most of our interest is focused on the inner parts of the models, and information on their spatial structure is given in Figs 1 and 2 for the models with dynamically active and inert binaries, respectively. The core radius is as defined in NBODY6 (Aarseth 2003, pp.265-6), and all radii are referred to the density centre.

We note immediately substantial variations in $\rho$ (defined here to be the 1% Lagrangian radius). Since the crossing time at this radius is of order $10^7$yr, it is clear that these are not the kind of

![Figure 1. Core and Lagrangian radii (mass fractions 0.1 and 1%) for the model with dynamically active binaries (Model A).](image1)

![Figure 2. Core and Lagrangian radius (mass fraction 1%) for the model with dynamically inert binaries (Model I). Data on the 0.1% Lagrangian radius are not available for this model, and the core radius was stored only to 3 decimal places.](image2)
fluctuations caused simply by the motion of stars in and out of the core, but take place on a much longer time scale. By comparing the density within the half-mass radius and \( \bar{r} \), and the above estimate of the half-mass relaxation time, we may estimate that the relaxation time at \( \bar{r} \) is under 1 Myr (the unit of time in these figures), and the variations of largest amplitude (e.g. the rise between 500 and 700 Myr in fig[1] take place on a much longer time scale.

The next obvious observation is that the long-term fluctuations have a comparable amplitude for the two runs. But the maximum and minimum radii are lower in run \( \bar{r} \). In particular, the deep minima in run \( \bar{r} \) are not found in run \( \bar{r} \) A. It is natural to attribute these facts to the presence of primordial binaries: run \( \bar{r} \) has to make its own binaries, which requires higher density than the burning of an existing binary. The amplitude of the fluctuations in the inner Lagrangian radii is smaller than that of the core radius, but not inconsiderable, resulting in variations in the mean density within the 1% Lagrangian radius by a factor of around 3:1.

Though we have not shown data on the outer Lagrangian radii, we find that the variations we observe are largely confined to the inner few percent of the mass. In fact the 10% Lagrangian radius fluctuates with a relative amplitude of order 7%, and the oscillations are approximately 180° out of phase with those of the core. The relative amplitude of oscillations drops to less than 1% at the half-mass radius.

The initial decrease in both runs is an interesting feature. One might suspect that it is due to some property of the Monte Carlo code, which perhaps maintains thermal equilibrium in a structure which would not be in thermal equilibrium in an \( N \)-body model. Another possibility is a response of the system to the cessation of mass loss by stellar evolution. And yet, in the overall range of data in these figures, the structure at the start is not unusual. Furthermore, it might be remembered that this model was selected because, at 12 Gyr, it yielded a surface brightness profile resembling that of NGC6397, and this happens only intermittently, even in the Monte Carlo model.

Fig[3] when considered in conjunction with Figs[1] and [2] allows a direct qualitative comparison between the Monte Carlo and \( N \)-body models in respect of the fluctuations in the inner radii, though the definition of the core radius adopted in the Monte Carlo model is different (see caption). This figure shows oscillations of a similar amplitude and range of time scales to those exhibited by Run A (Fig[1]).

In an attempt to make this statement more quantitative we show in Fig[4] the autocorrelation of the core radius for Run A and the Monte Carlo model, though it should be borne in mind that these correspond to different time intervals. In both cases we have restricted the offset \( \tau \) in the autocorrelation (defined to be \( \langle x(t)x(t+\tau) \rangle \) for a zero-mean normalised signal \( x(t) \)) to be less than half the duration of the measurements. The result for the \( N \)-body model is striking, and confirms the visual impression from Fig[1] of fairly regular oscillations with a period of about 400Myr. There is a faint suggestion of similar structure in the autocorrelation function of the Monte Carlo model.

Another structural quantity of interest is, of course, the total mass, and the \( N \)-body run provides a check on this aspect of the Monte Carlo model. In the \( N \)-body model the rate of mass loss increases for the first few tens of Myr, presumably because escapers take some time to reach the boundary (at 2 tidal radii) where escapers are removed. For the period after 200 to 400Myr, however, the rate of mass loss is virtually constant, and yields \( d\ln M/dt = -1.35 \times 10^{-9} \pm 2 \times 10^{-7} \), where the unit of time is 1Myr. For the Monte Carlo model discussed above, we do not have data extending beyond 12Gyr. For another model differing only in the initial seed, however, in the same period the corresponding result is \( d\ln M/dt = -1.546 \times 10^{-9} \pm 5 \times 10^{-7} \). This difference of 13% should be corrected for the fact that there is no mass loss through stellar evolution in the \( N \)-body model. In the Monte Carlo model this contributes only about 3% of the total, but the direct loss of this mass also induces further loss of stars (by tidal overflow) because it make the potential well more shallow. It is difficult to quantify this induced mass loss, but it appears that the discrepancy in the total rate of mass loss between the two models is less than 10%.
Since so much of the mass at small radii is in the form of degenerate remnants, the influence of the fluctuations shown in Fig. 1 on the surface brightness profile is not obvious. Insufficient data were collected from Run I, but Fig. 5 provides a useful comparison of two snapshots from Run A. These are taken at times of 200 and 340 Myr, which correspond, respectively, to low and high values of the radii plotted in Fig. 1 (though not the extreme values). As expected, the central surface densities are lower at the time when the Lagrangian and core radii are larger.

Let us consider these profiles in more detail. Neutron stars and white dwarfs are the dominant components at the centre, each contributing about half of the projected number density. (Note that the surface density of neutron stars in Fig. 5 is divided by 10, to avoid overlap with the white dwarfs.) Beyond the core, however, the projected density of neutron stars decreases more strongly, as is expected from their greater mass. Indeed they are actually the dominant central component in terms of spatial mass density. The component which contributes least to the projected central density, by a factor of order 10, are the non-degenerate stars, though they begin to dominate at radii beyond those shown in the figure.

All three components show a lower surface density and a larger core at the later epoch (340 Myr) compared to the earlier epoch (200 Myr). It is interesting to attempt to quantify this in terms of spatial fluctuations, i.e. weighted by luminosity. In particular, the most luminous stars are centrally concentrated relative to most non-degenerate stars. Because stellar evolution is absent from the N-body simulation, however, it is not an entirely appropriate model for considering how fluctuations affect the surface brightness. The discussion of our Monte Carlo model in Giersz & Heggie (2009) is more complete in this respect.

3.2 Surface brightness profiles

Figure 5. Surface densities of three groups of stars in Run A at two times. Only the central 30 arcsec (approximately) is shown. For neutron stars the surface density is reduced by a factor 10, to avoid confusion with the surface density of white dwarfs.

Observational description, which is usually expressed in terms of surface brightness, i.e. weighted by luminosity. In particular, the most luminous stars are centrally concentrated relative to most non-degenerate stars. Because stellar evolution is absent from the N-body simulation, however, it is not an entirely appropriate model for considering how fluctuations affect the surface brightness. The discussion of our Monte Carlo model in Giersz & Heggie (2009) is more complete in this respect.

3.3 Binaries

Run A contains over 3000 binaries, and it is best to begin with run I, where the effects of individual binaries are clearer (Fig. 6). By comparing with Fig. 6, we can surmise that one or two binaries were responsible for terminating the core collapse at about 70 Myr and initiating the subsequent expansion. For a period beginning at 125 Myr, however, no binaries were present, suggesting that the continuing modest expansion was powered gravothermally (Sugimoto & Betttwieser 1983). A second phase of core collapse and binary formation appears to have started at about 225 Myr, but is incomplete by the end of the run.

Figure 6 shows similar data for run A. The numbers are not included, but the binaries are separated into two groups. Those labelled “new” exclude binaries which have evolved from primordial binaries by exchange. If, however, two binaries collide, resulting in a hierarchical triple system, the outer motion of the hierarchy is regarded as a “new” binary. In any event we see that both types of binary have an active role to play. The correlation with the evolution of the radii (Fig. 1) is less clear than for run I, though the initial contraction seems to be associated with a period of relatively sluggish binary activity.

While neither run shows the kind of gravitational oscillations which are so evident in simulations of systems with equal masses, it should not be surprising to find evidence of gravothermal effects in a multi-mass system of the kind which we are studying (see the discussion in Giersz & Heggie (2009)).

The rate of change of the binary fraction in run A is $+2.05 \times 10^{-7} \pm 2.5 \times 10^{-8}$, where the unit of time is 1 Myr. For the Monte Carlo model the corresponding value is $+1.56 \times 10^{-7} \pm 1.5 \times 10^{-8}$, and these results are pretty consistent within the errors.
4 CONCLUSIONS

We have carried out an N-body simulation of the globular cluster NGC6397, in order to study the evolution of its central structure over a period of 1Gyr starting at the present day. The simulation was initialised using the results of a Monte Carlo model (Giersz & Heggie, 2009) which approximately fits the present-day profiles of surface brightness and velocity dispersion, and the mass function at two radii. The main limitation of the N-body model is that there is no stellar evolution. This apart, it is in many respects the most realistic N-body simulation of a specific globular cluster of which we are aware.

The model provides a new check on the reliability of the Monte Carlo code, and suggests that the evolution of the binary fraction is satisfactory, while the rate of escape of mass appears to be in agreement to better than 10%.

The results show that the population of primordial binaries in the cluster (about 3%) suppresses the deepest collapses of the core, which nevertheless still exhibits substantial fluctuations on a time scale of many core relaxation times. Their amplitude is sufficient to change the mean density of the innermost 1% of the mass by a factor of order 3:1. These changes are reflected in variations in the core radius, whether measured in terms of the spatial density, as in an N-body model, or by the radius at which the surface density decreases to half its central value. These changes manifest themselves in all components that we have studied: neutron stars (the dominant component in the central density), white dwarfs, and non-degenerate stars, which are the least dominant.

In view of our results, it is interesting to think of globular clusters as variable stellar systems, in much the same way that many stars are variable. The time scales and mechanisms are vastly different, and the variations in globular clusters are confined to the vicinity of the core. But it is another demonstration of the deep physical resemblance between these two types of thermal, self-gravitating objects.

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