Calculation of force and energy parameters of fracture in the area of the front of the crack in the shell of the reactor of coke chambers

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Abstract. This paper presents a three-dimensional elastic calculation of the stress-strain state of the reactor and the force and energy parameters fracture for an elliptical crack located in a cylindrical shell. It is found that for an elliptic crack, the force and energy parameters of the fracture change non-linearly along the crack front. The qualitative and quantitative relations for the stress intensity factors (SIF and J-integral along the crack front taking into account its size, the shape of the front and the angle of inclination) are obtained.

Reactor (coking chambers) is designed for accumulation of the mixed raw material supplied to the column through the furnace and further coking process and accumulation of the resulting coke in the delayed coking unit of a refinery.

Reactor (Figure 1) is an all-welded hollow cylindrical vessel with an internal diameter of 5500 mm, height 27225 mm, capacity 540 m³, with the upper hemispherical and lower conical bottoms with necks for entering the hydraulic cutting tool and unloading coke.

The reactor is installed on 6 support racks located in the zone of the support belt. The heated raw material of coking enters the reactor through the fitting raw material inputs located in the lower part of the conical bottom. At the top and bottom neck of the reactor there are fitting for the output vapors of the coking and the output vapors of cooling the coke with water and a fitting for filing antispam image additives and fitting for the output vapors during the heating of the chamber and for heating the at start-up.

Defect such as crack in the housing shell are observed due to the cyclic nature of the operation the reactor, which affects the stress-strain state and in general the reliability of the entire reactor. In this paper, the impact of the crack location in the shell area of the device on the force and energy parameters of fracture was assessed.

To assess the danger of such cracks, a three-dimensional numerical experiment was performed, which consists of modeling and subsequent calculation by the finite element method.

Stress intensity factor (SIF) $K_1$, $K_2$, $K_3$ and energy integral $J$ were calculated taking into account the shape, location on the outer surface of the shell, as well as taking into account the actual size and angle of the crack. The stress intensity factors $K_1$, $K_2$, $K_3$ were calculated by the known method according to
the energy release rate using the formula. The energy release rate was calculated by virtual crack propagation [3].

The calculation of the fracture force parameters was performed on a full 3D model of the reactor, the crack was set in the zone of the welded joint of the shell and the conical bottom. The loading was given as an internal pressure of 0.62 MPa. The weight of the device and the hydrostatic pressure of the liquid inside the device and the temperature of the environment were also taken into account.

**Formation of calculation models.** The reactor was made of steel 12HN10T (C-0.12%, Cr<1%, Ni - 10%, Ti<1%) besides; the temperature of the medium 475°C was taken into account in the calculations. The mechanical characteristics of the material used in the calculations given in Table 1.

The following dimensions were taken into account: the thickness of the reactor wall (T) was 21 mm, the diameter of the shell was 5500 mm, and the diameter of the nozzle was 300 mm. The form of the crack in the area of the support wall is shown in Figure 2.

Allowable stress for this material at a temperature of 160°C is equal to 145.5 MPa, at 475°C is equal to 114.0 MPa.

**Table 1. Mechanical characteristics**

| Temperature T, °C | Density, ρ, kg/m³ | Young’s Modulus E, Pa | Poisson’s ratio ν | Yield limit σT, Pa |
|------------------|------------------|----------------------|------------------|------------------|
| 20               | 7900             | 1.98·10¹¹           | 0.36             | 2.25·10⁸ - 3.15·10⁸ |
| 500              | 7690             | 1.66·10¹¹           | 0.36             | 1.35·10⁸ - 2.05·10⁸ |

Mechanical properties in the temperature range from 20°C to 500°C were determined by linear approximation.

**Figure 1. General view of the reactor**

All calculations were performed in a static statement in the ANSYS/Workbench package [2]. All calculations were performed in a static setting in the ANSYS/Workbench software package [2]. One of the variants of the finite element mesh for a full-size the reactor object is shown in Figure 3 (a,c).

The crack in the welded joint of the reactor shell, which has the shape of an ellipse arc with a small radius a, propagated to no more than 0.4 of the sheet wall thickness.
Design parameters of the finite element model. At the first stage, the elastic calculation of the stress-strain state of the reactor by the finite element method was performed. The solid-state reactor model, boundary conditions, and finite element mesh for this calculation are shown in Figure 3. In all the parts of the FE model of the reactor four elements of wall thickness were used. It is found out that the maximum stresses are observed in the zone of welds crossing the shell and branch pipes as well as in the zone of welding support legs of the reactor.

At visual inspection of the welded joint of the shell, after the acoustic emission control, surface defects (cracks) were found. The FE mesh, which was further used for FEM calculations in the fracture weld zone, is shown in Figure 3(b) and Figure 6.

The theory of the method of calculation. The coefficients of intensity of elastic stresses of the first, second and third type $K_1$, $K_2$ and $K_3$, as noted above, were calculated by the rate of energy release determined by the method of virtual crack propagation [2-3]. Figure 4 shows the coordinates of the crack and the integration loop used for the calculations.
Figure 4. Model of the crack

The stresses at the crack tip are described by the following relations:

\[
\sigma_{11}^{(2)} = \frac{1}{\sqrt{2\pi r}} \left[K_I^2 \cos^2 \theta \left(1 - \sin^2 \frac{\theta}{2} \sin \frac{3\theta}{2}\right) - K_{II}^2 \sin^2 \theta \left(2 + \cos^2 \frac{\theta}{2} \cos^2 \frac{3\theta}{2}\right)\right],
\]

\[
\sigma_{22}^{(2)} = \frac{1}{\sqrt{2\pi r}} \left[K_I^2 \cos \theta \left(1 + \sin^2 \frac{\theta}{2} - \sin \frac{3\theta}{2}\right) + K_{II}^2 \sin \theta \cos \theta \cos^2 \frac{3\theta}{2}\right],
\]

\[
\sigma_{12}^{(2)} = \frac{1}{\sqrt{2\pi r}} \left[K_I^2 \cos^2 \frac{\theta}{2} \sin \frac{3\theta}{2} - K_{II}^2 \cos \theta \left(1 - \sin^2 \frac{\theta}{2} \sin \frac{3\theta}{2}\right)\right],
\]

\[
\sigma_{13}^{(2)} = \frac{K_{III}^2}{\sqrt{2\pi r}} \sin \frac{\theta}{2},
\]

\[
\sigma_{23}^{(2)} = \frac{K_{III}^2}{\sqrt{2\pi r}} \cos \frac{\theta}{2},
\]

\[
\sigma_{33}^{(2)} = \begin{cases} 
\nu(s) \left(\sigma_{11}^{(2)} + \sigma_{22}^{(2)}\right) & \text{plane deformed state} \\
0 & \text{plane stress state}
\end{cases}
\]

The force parameters of fracture were defined as:

\[
K_I = \sqrt{JE_s}, \quad K_{II} = \sqrt{JE_s}, \quad K_{III} = \sqrt{JE_s/(1-\nu^2)}.
\]

\(E_s = E/(1-\nu^2)\) for plane deformed state; \(E_s = E\) for plane stress state.

The energy integral was determined by the formula:

\[
J(s) = \frac{K_I^4 + K_{II}^4}{E(s)} + \frac{1 + \nu(s)}{E(s)} K_{III}^2,
\]

\[
J^{(1)}(s) = \frac{1}{E(s)} \left(2K_I^{(1)} + 2K_{II}^{(2)}\right)^2 \left(K_{II}^{(1)} + K_{II}^{(2)}\right)^2 + \frac{1 + \nu(s)}{E(s)} \left(K_{III}^{(1)} + K_{III}^{(2)}\right)^2 = J^{(1)}(s) + J^{(2)}(s) + I(s),
\]

\[
I(s) = \frac{1}{E(s)} \left(2K_I^{(1)} K_{II}^{(2)} + 2K_I^{(2)} K_{II}^{(1)}\right) + \frac{1 + \nu(s)}{E(s)} \left(2K_{III}^{(1)} K_{III}^{(2)}\right),
\]

where, \(K_I^{(1)} = \frac{E_s^{(1)}}{2} I(s)\).
Figure 5. Stress fields in the reactor under the action of operational loads ($\sigma_{eqv} = 120,46$ MPa)

The finite element model (FEM) of the reactor consisted of 956731 twenty-node isoparametric elements and was obtained using the ANSYS/WORKBENCH program [2].

Figure 6. Finite element model of the crack

Boundary conditions. The boundary conditions and the finite element mesh are shown in Figure 3(a, b). In the support racks of the device, vertical displacement restrictions were set, in the zone support of the mobility rack contact conditions were set, taking into account the slippage of the support surface of the rack.

Loads in the form of axial forces and moments from strapping by technological pipelines were applied to the fittings. The temperature of the environment inside the reactor was set to 475°C.

The model of a crack. The area with an elliptical crack was described by isoparametric elements of the second order and set separately from the main model using the CRACK tools in the Static Structural module [2]. As a result, a hybrid FEM of the reactor with an elliptical crack was formed. The area with a crack with a radial-ring structure of the FE is shown in Figure 3 and Figure 6 with different magnification. Six elements were located along front of the crack, eight elements were divided in the radial direction around the tip of the crack to simulate the fracture and the singularity type stress near the fracture tip [3]. The crack was modeled in the form of an ellipse with parameters $C=2.0-28.0$ mm, $\alpha=0.4-6.0$ mm, located in the zone of the radius transition from the branch pipe to the shell of the reactor. The orientation of the crack was chosen so that the X-axis was located along the short radius of the ellipse, and the Z-axis along the long side of the front of the crack.

Results of calculation of force and energy parameters of fracture. The dependence of the force parameters fracture $K_1$, $K_2$ and $K_3$ and the $J$-integral on the crack front is shown in Figure 8-9, where $C$ is the length of the crack front measured from the crack end located on the reactor surface. For the analyzed crack depth and length it was found that the stress intensity factors $K_1$, $K_2$ and $K_3$ change nonlinearly along the crack front. For Figure 7(a) stress fields in the crack tip region are shown.
The effect of depth and crack length. Non-destructive testing methods are often revealed cracks of different geometry located in the weld zone of the shell and the bottom of the reactor. The crack can be formed and have different shapes, which leads to a change in the values of the force parameters of fracture and, accordingly, affect the operational survivability of such a connection. The shape of the crack can change when the depth of the crack changes at a constant length, which should also be taken into account when analyzing the structural strength. In this calculation, the crack was located in the weld zone of the reactor shell, as shown in Figure 3(c). Figure 7(b) shows the region of the crack opening and isopoly movements. The results of calculation force of the fracture parameters $K_1$, $K_2$ and $K_3$ and $J$-integral are shown in Figure 9-10, the calculation is performed on the FE model of the reactor with an elliptical crack located horizontally taking into account its size. The length of the crack $C$ varied from 10 mm to 28 mm at the maximum depth of the crack $a=2.4$ mm. The crack was given in the form of an ellipse.

It is established that the fracture force parameters $K_1$, $K_2$ reach a maximum at the peak depth along the crack front (figure 8). With the growth of the crack length $2C$ from 10 mm to 28 mm, there is a monotonic increase in the values of SIF ($K_1$, $K_2$, $K_3$) and the energy integral. With a change in the crack length from 10 mm to 28 mm at a maximum depth along the crack front $a=2.0$ mm, the first-kind SIF ($K_1$) grew more than 2.0 times.

Effect of the angle of inclination of the crack. The crack can is located at a certain angle, which must also be considered in the analysis of structural strength.

Figure 7. Von Mises stress at the crack tip (a) and crack opening (b)
Therefore, a numerical experiment was performed to assess the effect of the crack shape and its location and angle on the force parameters of the fracture.

The results of the calculation of force parameters fracture for the elliptical shape of the crack at an angle of 45° to the reactor axis are shown in Figure 11. The geometry of the cracks is ranged in length \( C = 24 \) mm with a depth of \( a = 8.0 \) mm. It is found out that the dependences of the intensity factor of the first kind and the \( J \)-integral on the crack length along its front, calculated for the crack located on the reactor shell, vary along the crack front and depend on the size, shape and angle of its inclination. Thus, for an elliptical crack, the SIF of the first kind \( (K_1) \) reaches a maximum at the most distant point along the front of the crack front, and at the exit to the surface of the \( K_1 \) decreases. For example, for a crack with parameters \( C = 24 \) mm and \( a = 8 \) mm, and located at an angle of 45° to the axis of the reactor, the increase in SIF \( K_1 \) along the crack front reaches values of \( 398.67 \) MPa·mm\(^{1/2}\).

In table. 2 generalized values of force and energy parameters of fracture for the entire range of considered crack lengths located in the most loaded cylindrical part of the reactor vessel are presented. The observed change in \( K_1 \) along the crack front is qualitatively consistent with a number of published similar elastic calculations for comparable geometric shapes of the welded joint and crack sizes. The results of similar three-dimensional elastic calculations by the finite element method are given in the works of the authors [4-6] for different crack lengths and coincide well with the given data.

![Figure 10. Dependences SIF (K1) and J-integral for the elliptical crack shape located at an angle of 45° to the reactor axis 1-2C=24 mm; a=8 mm](image)

It should be admitted that the comparisons above with published solutions are approximate due to incomplete information about the geometry of the object and the loading conditions.

**Conclusion**

The elastic calculation of the power and energy parameters of fracture for an elliptical crack located in the zone of the reactor shell is carried out. The dependences of the stress intensity factor of the first, second and third kind and the energy integral are obtained taking into account the shape of the crack and its size and angle of inclination. It is shown that for a crack located in the cylindrical shell of the reactor, the stress intensity factor of the first kind can vary (increase) along the crack front by 2.0 times or more and can reach maximum values at the most remote point of the crack front. In this case, the greatest danger is represented by cracks located in the welded joint zone of the cylindrical shell of the reactor vessel.
Table 2. Values of force and energy parameters of fracture

| a/C, mm | $K_1$, MPa× $mm^{1/2}$ | $K_2$, MPa× $mm^{1/2}$ | $K_3$, MPa× $mm^{1/2}$ | J, mJ/m² |
|--------|------------------------|------------------------|------------------------|----------|
| 1/2    | 112.87                 | 17.19                  | -14.6 ±20.0            | 0.061    |
| 1/4    | 85.26                  | 19.59                  | -10.6 ±14.1            | 0.035    |
| 4/8    | 189.19                 | 41.35                  | -13.3 ±17.5            | 0.17     |
| 4/12   | 192.51                 | 43.53                  | -13.4 ±18.0            | 0.18     |
| 4/28   | 228.32                 | 51.75                  | -14.3 ±16.9            | 0.25     |
| 4/28   | 429.63                 | 3.69 ±30.47            | 13.06                  | 0.06     |
| 4/34   | 425.30                 | 3.47 ±33.44            | 13.01                  | 0.06     |
| 4/44   | 334.08                 | 2.58 ±25.53            | 5.88 ±22.9             | 0.06     |
| 4/8    | 287.35                 | 71.62 ±86.52           | 74.85                  | 0.41     |
| 4/32   | 341.07                 | 17.72 ±80.44           | 97.06                  | 0.58     |
| 4/24   | 327.87                 | 22.64 ±84.73           | 94.05                  | 0.54     |
| 4/28   | 328.31                 | 20.78 ±80.26           | 94.18                  | 0.54     |
| 8/24   | 398.67                 | 68.20 ±113.85          | 114.13                 | 0.81     |
| 8/32   | 417.04                 | 68.99 ±120.80          | 123.14                 | 0.88     |
| 8/64   | 424.33                 | 44.36 ±122.24          | 123.96                 | 0.92     |
| 12/24  | 417.99                 | 98.80 ±146.05          | 126.4                  | 0.89     |
| 12/32  | 484.25                 | 104.11 ±151.99         | 140.67                 | 1.19     |
| 12/64  | 499.40                 | 36.65 ±154.41          | 123.61                 | 1.29     |

References

[1] Vessels and apparatus. Norms and methods of strength calculation. Calculation of shells and heads from influence of support loads, Federal standart 52857.5-2007, Moscow, Standatrinform, 2007.

[2] Ansys Release 15.0. Dokumentation. Canonsburg: Ansys Inc.

[3] E. M. Morozov, G. P. Nikishkov, Metod finite element in fracture mechanics (LKI Publishing, URSS, 2008)

[4] Y. Murakami, Handbook of stress intensity factors (Mir, Moskow, 1990)

[5] M. Bergman 1995 Stress Intensity Factors for Cir-cumferential Surface Cracks in Pipes, Fatigue and Fracture of Engineering Materials and Structures, v. 18, pp. 1155-1172.

[6] Yuh J. Chao, X. K. Zhu 2000 Constraint-modified J-R curves and its application to ductile crack growth, International Journal of Fracture, v.106, pp. 135-160.