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Abstract

We present a model where all fermions are contained in a single irreducible representation of an SU(19) gauge symmetry group. If there is only one scalar field, Yukawa interactions are controlled by a single number rather than by one or more $3 \times 3$ matrices of couplings. The low-energy concept of flavor emerges entirely from the scalar-sector parameters; more specifically, entries of the Standard Model Yukawa matrices are controlled by several vacuum expectation values.

1. Introduction

The three-family fermion structure of the Standard Model is inherited by SU(5) [1] and SO(10) [2, 3] grand unified theories. Therefore these models cannot explain the intricate pattern of masses and mixing angles observed at low energies, much less why there are three families in the first place. Yet if all fermions are part of a single irreducible representation $\Psi$ of a gauge group, then, given an appropriate scalar field $\Phi$, the Yukawa interactions

$$\mathcal{L}_Y = y\Psi\Phi$$

would be controlled by a single number $y$. If feasible, such a setup would shed light on the origin of flavor.

But a viable model of this type also needs to reproduce numerous low-energy flavor observables (fermion masses and mixing parameters), which at first glance seems difficult. Yet this worry is unwarranted because low-energy flavor parameters are not functions of $y$ alone. Indeed, the scalar $\Phi$ contains several singlets $(1,1,0)$ and doublets $(1,2,\pm1/2)$ of $G_{\text{SM}} \equiv \text{SU}(3)_C \times \text{SU}(2)_L \times U(1)_Y$, and the low-energy-Yukawa interactions depend on their vacuum expectation values (VEVs).
If Nature behaves in this way, flavor, superficially looking like a property of fermions, is actually an emergent phenomenon originating exclusively from the scalar potential. We are therefore describing a mechanism of flavorgenesis.

Unification of all Standard Model fermion families within one [4, 5], or several [6–8], irreducible representation of a unified group has been considered before, but none of these proposals correctly accommodate three—and only three—families of light fermions.

Recently it has been pointed out [9, 10] that the three-family chiral structure of the Standard Model fits adequately into the 171-dimensional representation of SU(19), and that this gauge theory might be uniquely suited for flavor unification. In this letter we present a grand-unified model with the aforementioned structure that is capable of reproducing low-energy data. Explicit relations between entries of the Standard Model Yukawa matrices and the vacuum expectation values of the fundamental theory are constructed.

2. The 171-dimensional representation of SU(19)

The fermion content of grand-unified theories is severely restricted by the requirement that only the Standard Model fermions are light. If there are no confining interactions [5], additional fermions must be vector-like under G_{SM} in order to obtain a large mass prior to electroweak symmetry breaking. That is, any new fermion X must be matched with an X^c having the same chirality and opposite quantum numbers. Using this requirement, a systematic study [9] reveals that it is difficult to unify all fermions in a single irreducible representation. Only one non-trivial case is known: unification of all fermions in the 171-dimensional irreducible representation of SU(19). This uniqueness justifies a serious study of such a gauge theory.

Under SO(10), a subgroup of SU(19), the 171 transforms as

\[
\begin{align*}
\text{SU(19)} & \to 3 \times \text{16} + \text{120} + 3 \times \text{1} \, .
\end{align*}
\]

(2)

The three 16s are known to contain all Standard Model fermions plus three right-handed neutrinos. This accounts for 48 out of the 171 components in equation (2). The remaining degrees of freedom transform as real SO(10) representations (120 or singlets) and are, therefore, vector-like under G_{SM}. All in all, only three Standard Model families of fermions are chiral. Yet this is only a counting argument; it does not imply that all the light fermions are fully contained in the three 16s. In fact, the 120 contains two copies of the vector

1We only consider 4-dimensional space-time. Family unification has also been discussed in higher dimensions. In particular, reference [10] discusses some features of SU(19) family unification in 6 dimensions. See also reference [11].

2Since they do not interact with the Standard Model gauge bosons, right-handed neutrinos, if they exist, might also be light.

3The Standard Model contains 45 fermions (distributed over 3 families), so it is trivially possible to embed these fields in the fundamental representation of the SU(45) group, or SU(45 + n) if we account for n right-handed neutrinos.
fermions \((d^c, d)\) and \((L, L^c)\), together with one copy of \((Q, Q^c)\), \((u^c, u)\) and \((e^c, e)\); these states can mix with the ones in the three \(16\)s in equation (2).

Therefore, the relevant low-energy components are

\[
\text{SU(19)} \rightarrow \underbrace{4Q + 4u^c + 5d^c + 5L + 4e^c + Q^c + u + 2d + 2L^c + e + (\text{more vector fermions})}_{\text{SU(3)_C \times SU(2)_L \times U(1)_Y}}.
\]

(3)

From this discussion we conclude that, regardless of other details, this SU(19) fermion automatically reproduces the number of light fermions in the Standard Model.

It is worth noting that SU(19) contains a large number of inequivalent G_{SM} subgroups,\(^4\) which should not be surprising given the sizeable difference between the rank of the two groups. Yet we are only interested in a particular embedding, which can be understood as follows. One can embed G_{SM} in SO(10) such that the spinor representation decomposes into a family of SM fermions plus a right-handed neutrino, as usual. Since SO(10) is a special maximal subgroup of SU(16) \([12]\), the fundamental representation of SU(16) is also irreducible under SO(10).\(^5\) Furthermore, the special unitary group SU(19) contains SU(16) \(\times\) SU(3)_F \(\times\) U(1) as a maximal subgroup \([10]\). We may therefore write the following symmetry breaking chain

\[
\text{SU(19)} \rightarrow \underbrace{(16, 1) + (1, 3)}_{\text{SO(10) \times SU(3)_F}} \rightarrow \underbrace{Q + u^c + d^c + L + e^c + N^c + N_k^c}_{\text{G_{SM} \times SU(3)_F}} (k = 1, 2, 3)
\]

(4)

The SU(3)_F group is important—even though it must be broken—because it commutes with G_{SM} and, as such, plays the role of a flavor group. For this reason it is worth keeping track of the transformation properties of the various fields under SU(3)_F. The first six terms on the right hand side of branching rule (4) stand for the well known quantum numbers of the SM fermions plus sterile neutrinos, which are all singlets under SU(3)_F. On the other hand, the irreducible representation \(N_k^c\) is invariant under G_{SM}, but unlike \(N^c\) it is a triplet of SU(3)_F.

However, there is an even bigger SU(4)_F that contains SU(3)_F and commutes with G_{SM}. Indeed, SU(5) \(\times\) SU(4)_F is also a subgroup of SU(19), under which

\[
\text{SU(19)} \rightarrow \underbrace{(5, 1) + (10, 1) + (1, 4)}_{\text{SU(5) \times SU(4)_F}} \rightarrow \underbrace{Q + u^c + d^c + L + e^c + N^c}_{\text{G_{SM} \times SU(4)_F}} (i = 1, \ldots, 4),
\]

(5)

where \(N_i^c\) is a quadruplet under SU(4)_F. We use lower Latin indices for it, reserving upper Latin indices for an anti-quadruplet. This SU(4)_F is noteworthy for being the biggest simple subgroup of SU(19) which commutes with G_{SM} \(\subset\) SU(19). There is a reason why we have been writing \(Q, u^c, \ldots\) (group representations) instead of \(Q, u^c, \ldots\) (actual fields): while the SU(19) model does not contain a 19-dimensional representation, the 19’s decomposition is crucial to the identification of the various fermion and scalar components used.

\(^4\)There are thousands of such subgroups \([9]\).

\(^5\)There are also models embedding a single Standard Model family, with or without right-handed neutrinos, into the fundamental representations of SU(15) or SU(16) \([13–16]\).

3
The 171-dimensional fermion representation corresponds to the anti-symmetric product of two fundamental SU(19) representations. We can therefore view all fermions of equation (3) as belonging to a $19 \times 19$ anti-symmetric matrix with the following block form

$$171 = \begin{pmatrix}
\begin{array}{cccc}
\phi^e & \phi^L & \phi^Q & \phi^e \\
\phi^e & u & Q^c & L^c_1 \\
\phi^e & \times & e^c & d_1 \\
\phi^e & \times & \times & L_5 \\
\phi^e & \times & \times & \times \\
\phi^e & \times & \times & \times \\
\phi^e & \times & \times & \times \\
\phi^e & \times & \times & \times \\
\phi^e & \times & \times & \times \\
N^c_i & \times & \times & \times \times \times \times \\
\end{array}
\end{pmatrix}
\frac{1}{\sqrt{2}}. \tag{6}
$$

For example, the block $171_{LN^c_i}$ has the G\textsubscript{SM} quantum numbers $L \times N^c_i = (1, 2, -1/2)$, hence the name $L_i$. It is also a quadruplet of SU(4)\textsubscript{F}, unlike $L_5 \equiv 171_{Qu^c}$ which is a singlet of the flavor group. In total there are 4+1 L fields in the 171. Equation (6) encodes this information for all field components relevant at low energies.\textsuperscript{6} Also note that the right-handed neutrinos $N^c_{ij} \equiv 171_{N^c_iN^c_j}$ correspond to a sextet of SU(4)\textsubscript{F} (with $ij$ anti-symmetrized).

3. The genesis of flavor

There are two possible ways to couple two 171-dimensional representations to a single scalar field $\Phi$,

$$\mathcal{L}_Y = y_{171}^{ab} 171_{cd} \Phi^{abcd}, \tag{7}$$

corresponding to either a completely antisymmetric 3876 or a mixed-symmetric 10830 irreducible representation of SU(19). In what follows we consider the former case, ignoring for the moment the scalar potential and possible issues with breaking the SU(19) gauge symmetry group down to G\textsubscript{SM}.

To shed light on flavor one must establish a connection between the Standard Model Yukawa couplings and the parameters of the SU(19)-phase Lagrangian. For that sake we only need to consider those components of $\Phi$ which can influence the light fermion spectrum: those that transform as $(1, 1, 0)$, $(1, 2, +1/2)$, and $(1, 2, -1/2)$ under G\textsubscript{SM}. We shall denote these types of fields by generic symbols S, H and $\tilde{H}$, respectively.

There are 15 S-type singlets, 11 H’s, and 17 $\tilde{H}$’s in $\Phi = 3876$. Using the block notation introduced above it is straightforward to identify their location; for example, 3876$^{d^c\bar{d}^c\bar{u}^c}$ contains an H doublet. All S’s, H’s, and $\tilde{H}$’s are found in the blocks

$$S : \begin{pmatrix}
S_L & S_{DL} & S_D & S_{UD} & S_{QDL} & S_{EL} & S_N \\
\end{pmatrix}
\begin{pmatrix}
d^cQu^c, d^cQ\bar{u}^c, d^c\bar{u}^c\bar{u}^c, d^c\bar{d}^c\bar{u}^cN^c_i, \bar{d}^c\bar{d}^c\bar{u}^cN^c_i, d^c\bar{L}Q\bar{N}^c_i, d^c\bar{L}e^cN^c_i, \bar{L}L^c_i, \bar{L}L^c_i, N^c_iN^c_jN^c_kN^c_l
\end{pmatrix}. \tag{8}$$

\textsuperscript{6}Some blocks in equation (6) contain fields beyond the ones shown here. For example 171_{Qu^c} contains both the $(1, 2, +1/2)$ state, which we called $L_5$, and also $(8, 2, +1/2)$ which is vector-like and, thus, irrelevant at low energies.
For later convenience, specific names were given to most of these fields. Note that, just like fermions, some of these scalars are singlets of the SU(4)_F flavor group (for example S_L), while others are anti-quadruplets with an upper index (such as H_{DE}) and others yet are anti-sextets (H_{QN} is one such case). The only scalar which transforms as a quadruplet of SU(4)_F is H_{N,i} \equiv \epsilon_{ijkl} \mathbf{3876}^{L_i N_j N_k N_l}.  

All the fermion and scalar components that are important for the low-energy phenomenology have been identified in equations (6) and (8)–(10), but not all of them are light. In particular, the VEVs of the electroweak singlets—the S’s—determine which combinations of the Q, u^c, d^c, L, and e^c fields become super-heavy. Furthermore, it is necessary to consider some fine tuning of scalar parameters, so that a linear combination of all the H’s and \tilde{H}’s forms the light Standard Model scalar doublet.

Expanding the SU(19)-invariant Yukawa interaction in expression (7), we find

\[ \mathcal{L}_Y \supset \frac{1}{3} Q_i Q^c S^{\dagger}_{i QDL} + \sqrt{\frac{2}{3}} u^c_i u S_{UD} + \sqrt{\frac{2}{3}} e^c_i e S_{EL} + \frac{1}{4} \sqrt{\frac{2}{3}} \epsilon_{ijkl} N_{ij}^c N_{kl}^c S_N \]

\[ + \frac{\sqrt{2}}{3} \left( d^c_i \right)^T \begin{pmatrix} -S^i_{QDL} \sqrt{2S^i_{UD}} \\ -S^i_{DL} \sqrt{2S^i_{SD}} \end{pmatrix} \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} + \frac{1}{\sqrt{3}} \begin{pmatrix} L^i \end{pmatrix}^T \begin{pmatrix} \sqrt{2H_{DE}^i} \\ \sqrt{2H_{QN}^i} \end{pmatrix} \begin{pmatrix} L^c \end{pmatrix} \]

\[ - \frac{2}{3} Q_i H_{QN}^{ij} u^c_j + \frac{\sqrt{2}}{3} \left( d^c_i \right)^T \begin{pmatrix} \sqrt{2H_{DE}^i} \\ \sqrt{2H_{QN}^i} \end{pmatrix} Q_j + \frac{1}{\sqrt{3}} \left( L^i \right)^T \begin{pmatrix} -\epsilon_{ijkl} H_{N,i.l} \\ \sqrt{2H_{QN}^k} \end{pmatrix} \begin{pmatrix} N^c \end{pmatrix} \]

\[ + \frac{\sqrt{2}}{3} \left( L^i \right)^T \begin{pmatrix} -\sqrt{2H_{DE}^i} \\ \sqrt{2H_{QN}^i} \end{pmatrix} e^c_j \]  

(11)

plus other terms which are unimportant. Flavorgenesis is manifest in this expression: family-replication emerges from a fundamental theory which had none, and the flavor structure seen at low energies is a result of an alignment of VEVs.

As an example, consider the up-quarks. The four-component VEV S_{i QDL} defines the heavy combination of the four Q’s, while the VEV S_{UD} plays an analogous role for the u^c’s. Therefore, these VEVs define 3 \times 4 semi-unitary matrices U_Q and U_{u^c} that project out the light fields. As for the scalar doublets, their projection onto the Standard Model Higgs (H_X \equiv \Lambda_X H_{SM} + heavy components, H_Y \equiv \Lambda_Y H_{SM}^2 + heavy components with |\Lambda_{X,Y}| \leq 1) can be determined from the scalar potential. Note that in the corresponding relations for SU(4)_F sextets like H_{QN} = \Lambda_{QN} H_{SM}^2 + \cdots the proportionality coefficients \Lambda_{QN} form a 4 \times 4 anti-symmetric matrix; these coefficients are in general complicated functions of the scalar potential parameters.

With all this at hand, the Standard Model up-quark Yukawa matrix Y_U at the unifica-
tion scale is computed readily:

\[ Y_U = -\frac{2}{3} U Q \Lambda Q N U_T^T. \]  

(12)

A similar but somewhat more complicated analysis can be made for down quarks, charged leptons and neutrinos.

As for the neutrinos, note that there are five different types of VEVs (those of \( S_{QDL}^i \), \( S_{EL}^i \), \( S_L \), \( S_{DL} \) and \( S_N \)) contributing to the light neutrino masses through a rather elaborate seesaw mechanism. One finds that if \( \langle S_N \rangle \) is significantly smaller than the remaining VEVs—assumed to trigger the SU(19) symmetry breaking—then one of the light neutrinos is practically massless.

Besides this observation, we have checked that it is possible to reproduce any pattern of the Standard Model fermion masses and mixing angles, if the VEVs of the scalars (S, H and \( \tilde{H} \)) are treated as free. This means that expression (11) is not ruled out by data, but at the same time, none of the Standard Model flavor parameters can be predicted without looking carefully into the scalar potential.

4. Symmetry Breaking

It is tempting to consider that the very same scalar representation \( \Phi \) that interacts with fermions is also responsible for breaking SU(19) to G_{SM}; if possible, this would be unlike, for example, SO(10) and E\(_6\) grand unified theories where multiple scalar irreducible representations are typically needed. And in our model the required group-rank reduction is even bigger—from 18 to 4—so it seems hard to do so with a single irreducible representation. However, the current scalar is a four-index-antisymmetric tensor, for which many breaking patterns are possible [17–21].

Indeed, our analysis shows that \( \Phi = 3876 \) has the right components to break SU(19) into G_{SM}, and the VEVs that achieve this breaking are solutions of the tadpole equations. Yet as far as we could tell, for all of these VEVs there are tachyonic scalars in the spectrum. The rest of this section expands on these statements.

It is a direct consequence of the branching rule (5) that \( F \equiv SU(4)_F \times U(1)^4 \) is the biggest subgroup of SU(19) which commutes with G_{SM}, so at the very least F must be completely broken. In order to break SU(4)_F, three scalars in the fundamental representation, with linearly independent VEVs, are needed [22]. Moreover, on very general grounds, one needs a different VEV (which does not need to be charged under SU(4)_F) to break each of the remaining U(1)s. This suggests that \( \Phi \) should contain at least 3 S’s that are quadruplets of the flavour group, and 4 S’s that are singlets under it. It turns out that this corresponds exactly to the set of available G_{SM} preserving directions in \( \Phi \), see expression (3).

However, this counting is only indicative as, in some cases, the symmetry-breaking power of the S’s may be further enhanced: arranging, for instance, the 3 quadruplet VEVs \( \langle S_{QDL}^i \rangle, \langle S_{UP}^i \rangle \) and \( \langle S_{EL}^i \rangle \) to be linearly independent but not perpendicular to each other \( (\sum_i \langle S_X^i \rangle \langle S_Y^i \rangle \neq 0) \), it is possible to break the SU(4)_F flavor group completely along with
two extra U(1)'s. As a consequence, the VEVs of up to two of the SU(4)_F-invariant S's can even be null and, yet, the entire SU(4)_F × U(1)^4 factor together with any residual symmetry between SU(19) and G_SM (such as SU(5) in equation (5)) may still be broken appropriately.

As an example consider the following vacuum direction

\[
\langle S^i_{QDL} \rangle = (v^1_{QDL}, v^2_{QDL}, 0, 0), \quad \langle S^i_{UD} \rangle = (v^1_{UD}, v^2_{UD}, 0, 0), \quad \langle S^i_{EL} \rangle = (0, 0, v_{EL}, 0),
\]

(13)

\[
\langle S_{DL} \rangle = 0, \quad \langle S_D \rangle = v_D, \quad \langle S_L \rangle = v_L, \quad \langle S_N \rangle = v_N.
\]

It can be shown that this configuration breaks the SU(19) gauge symmetry down to G_SM and that it is also a solution of the tadpole equations for the renormalizable potential

\[
V = -m^2 (\Phi \Phi^*) + \lambda_1 (\Phi \Phi^*)^2 + \lambda_2 (\Phi \Phi^*)^{a} (\Phi \Phi^*)^{b} + \lambda_3 (\Phi \Phi^*)^{cd} (\Phi \Phi^*)^{ab}.
\]

(14)

Here we used the abbreviations \( (\Phi \Phi^*) \equiv \Phi^{abcd} \Phi^{*}_{abcd} \), \( (\Phi \Phi^*)^{x} \equiv \Phi^{xbcd} \Phi^{*}_{ybcd} \) and \( (\Phi \Phi^*)^{xx'} \equiv \Phi^{xx'cd} \Phi^{*}_{yxx'cd} \).

Note that this potential has a U(19) symmetry [17], which includes the gauge group and a global U(1) associated to a rephasing of the entire \( \Phi \) scalar.

For the VEV direction (13) we have obtained the correct number of would-be Goldstone bosons corresponding to the SU(19)/G_SM coset.\(^7\) However, for all such solutions there are always some tachyonic scalars in the spectrum. This issue can possibly be overcome if we admit all S’s to take non-zero VEVs. But such a case is arduous to analyse so we leave this question open. It could also be worth considering radiative corrections [23].

5. Final thoughts

In this paper we have presented a potentially realistic unified model where all fermions are contained in a single irreducible representation of the SU(19) gauge group; thus, a single number controls all Yukawa interactions at the fundamental level. The Standard Model family replication is an emergent phenomenon only manifest at low energies. We have shown explicitly that the Standard Model fermion masses and mixing parameters can depend exclusively on the VEVs of a single scalar field; in this sense the concept of flavor originates entirely from the scalar sector.

Furthermore, we found that the model is compatible with the low-energy data if all VEVs are taken as free parameters. Such a consistency check does not provide any further insight into the fermion masses and mixing parameters, but this could change with a dedicated analysis of the scalar potential.

Remarkably, the same scalar which governs all the Yukawa interactions is capable of breaking the original gauge symmetry all the way down to G_SM. An initial study of the tree-level potential suggests that tachyonic scalars are a problem, but a more thorough analysis

\(^7\)The global U(1) rephasing symmetry is also broken, so one would think that there is a truly massless Goldstone boson. However, this is not the case because a mixture of this rephasing U(1) and the gauge symmetry generators is unbroken.
might still reveal viable points in the parameter space. If that is not the case, the situation can be amended by adding another scalar or perhaps by considering radiative corrections.

Due to its fermion content, the model has a gauge anomaly. This shortcoming may be resolved in various ways without affecting its main features (but perhaps making the whole setup less elegant); for example, only part of the symmetry group might be gauged, or further fermionic representations can be added within a confining sector.

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