INTERACTING VECTOR BOSON MODEL ANALYSIS OF THE $^{160}\text{Dy}$ EXCITED STATES SPECTRUM

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Abstract

Analysis of the recently obtained experimental data for collective states of $^{160}\text{Dy}$ is presented. The Interacting Vector Boson Model (IVBM) was applied for the classification of low lying states with positive parity $0^+, 2^+, 4^+, 6^+$ and for description of rotational ground, $S, \gamma$ and two octupole bands. The energies of the bands are reproduced with high accuracy using only one set of the model parameters for all bands.

Independently on that the $^{160}\text{Dy}$ has a very complicated spectrum of excited states, by now this nucleus is widely investigated experimentally. During some last decades a great amount of experiments for the investigation of the excited states spectrum of $^{160}\text{Dy}$ had been performed using different types nuclear reactions [1-18], Coulomb excitation [19-25] and $\beta$ decay $^{160}\text{Tb} \rightarrow ^{160}\text{Dy}$ [26,27], $^{160m,g}\text{Ho} \rightarrow ^{160}\text{Dy}$ and $^{160}\text{Er} \rightarrow ^{160m,g}\text{Ho} \rightarrow ^{160}\text{Dy}$ [28-33]. The results of all the investigations performed to 1996 year were analyzed in detail and presented in Nuclear Data Sheets [34] and Table of Isotopes [35]. During some last years using modern experimental techniques the comprehensive study of the $^{160}\text{Er} \rightarrow ^{160m,g}\text{Ho} \rightarrow ^{160}\text{Dy}$ $\beta$ decay had been repeated with the measuring of $\gamma$ rays, conversional electrons and $\gamma\gamma\gamma$ coincidences spectra [36]. These investigations had supplemented the spectrum of the excited states of $^{160}\text{Dy}$ with more than 10 states and more than 50 $\gamma$ transitions, and also to make agree with the data obtained from nuclear reactions and $\beta$ decay. At the same time a new study of excited states spectrum in $^{160}\text{Dy}$ using $^7\text{Li}$ ions beam with $^{158}\text{Gd}$ as a target had been performed [37]. In these reactions were observed the ground band states with $K^\pi = 0^+$ up to the excitation energy of 7231 keV and $I^\pi = 28^+$, $\gamma$ band with $K^\pi = 2^+$ to energy 6642 keV and $I^\pi = 25^+$, $S$ band to 4875 keV with $I^\pi = 20^+$ and octupole bands $K^\pi = 2^-$ to energy 6967 keV with $I^\pi = 26^-$ and $K^\pi = 1^-$ to energy 4937 keV with $I^\pi = 19^-$. 16 different nature rotational bands in $^{160}\text{Dy}$ nucleus are identified. All this made the
$^{160}$Dy nucleus a very good target for the theoretical nuclear structure models. Recently these bands were analyzed [38] applying Bohr-Mottelson model [39], $Q$ - phonon model [40], variable moment of inertia model with dynamical asymmetry [41], Bohr-Mottelson model with Coriolis interaction [42]. In the same paper the positive parity states were analyzed within the framework of IBM − 1 [43]. Using above approaches a relatively good description for the states energies and transition probabilities for low values of spins was obtained, but in the region of higher values of spins the disagreement with experiment increased noticeably. As a continuation of our theoretical analysis of very rich experimental data for $^{160}$Dy we apply the recently developed Interacting Vector Boson Model (IVBM) [44]. In this model within the framework of the boson representation of the $sp(12, R)$ algebra all possible irreducible representations of the group $SU(3)$ are determined uniquely through all possible sets of the eigenvalues of the Hermitian operators $N$, $T^2$, and $T_0$ or the equivalent $(\lambda, \mu)$ labels in final reduction to the $SO(3)$ representations, which define the angular momentum $L$ and its projection $M$.

This model had illustrated a good description of the energies of different rotational bands for the states with positive and negative parity for instance in $^{226}$Ra [44].

The detail description of this algebraic model one may find in [45]. Here we present only necessary for our purposes expressions for energy spectrum and the decomposition rules for the considering in the model chains (1).

\[
sp(12, R) \supset sp(4, R) \otimes so(3) \quad \cup \\
\quad \cup \\
u(6) \supset u(2) \otimes su(3)
\]

Written in terms of the $(\lambda, \mu)$ labels facilitates together with the decomposition rules the energy spectrum produced by the IVBM Hamiltonian are as follow:

\[
E((\lambda, \mu); L; T_0) = \alpha N + \alpha_1 N(N + 5) + \beta_3 L(L + 1) + \\
\alpha_3(\lambda^2 + \mu^2 + \lambda \mu + 3\lambda + 3\mu) + cT_0^2
\]

$N$ − even $\longrightarrow 0, 2, 4, 6......$

\[
T = \frac{N}{2}, \frac{N}{2} - 1, \frac{N}{2} - 2, ..., 0 \text{ or } 1
\]

\[
T_0 = -T, -T + 1,...T
\]

\[
\lambda = 2T
\]

\[
\mu = \frac{N}{2} - T
\]

\[
K = \text{min}(\lambda, \mu), \text{min}(\lambda, \mu) - 2, ... , 0 \text{ or } 1
\]
\[ K = 0 \implies L = \max(\lambda, \mu), \quad L = \max(\lambda, \mu) - 2, \ldots, 0, 1 \]

\[ K \neq 0 \implies L = \max(\lambda, \mu), \quad L = \max(\lambda, \mu) - 1, \ldots, 0, 1 \]

The parity of the states is defined as \( \pi = (-1)^T \). The index \( K \) appearing in this reduction is related to the projection of \( L \) in the body fixed frame and is used with the parity to label the different bands in the energy spectra of the nuclei. This allows us to describe both positive and negative parity bands. Further we use the connection:

\[ N = 4L \]

Thus, taking into account the reducing rules \( \Box \) the energy can be rewritten only in terms of the pseudospin \( T \) and angular momentum \( L \):

\[ E(L, T) = 4\alpha L + 4\alpha_1 L (5 + 4L) + \beta_3 L(L + 1) + \alpha_3 \left[ 6L + 4L^2 + 3T^2 + 3T \right] + cT_0^2. \] (5)

In our investigations of the experimental excited states in \( ^{160}\text{Dy} \) we start with the study of the low lying \( 0^+ \) states within the framework of simplified pairing vibrational model. With other words we make the classification of \( 0^+ \) states basing on phenomenological monopole part of collective Hamiltonian for single level approach written in terms of boson creation and annihilation operators \( R_+ \), \( R_- \) and \( R_0 \)

\[ H = \alpha R_+ R_- + \beta R_0 R_0 + \frac{\beta\Omega}{2} R_0, \] (6)

constructed with the pairs of fermion operators \( a^\dagger \) and \( a \) of the fermions placed at subshell \( j \) and model parameters \( \alpha, \beta \) and \( \Omega = \frac{2j + 1}{2} \):

\[ R_+ = \frac{1}{\sqrt{2}} \sum_{m} (-1)^j m \alpha_{jm} \alpha_{j-m}^\dagger, \]
\[ R_- = \frac{1}{\sqrt{2}} \sum_{m} (-1)^j m \alpha_{j-m} \alpha_{jm}, \]
\[ R_0 = \frac{1}{\sqrt{2}} \sum_{m} (\alpha_{jm} \alpha_{jm} - \alpha_{j-m} \alpha_{j-m}^\dagger). \] (7)

\[ [R_0, R_\pm] = \pm R_\pm, \quad [R_+, R_-] = 2R_0 \]

Applying the Holstein-Primakoff [47] transformation to the operators \( R_+ \), \( R_- \) and \( R_0 \)

\[ R_- = \sqrt{2\Omega - b^\dagger b} b; \quad R_+ = b^\dagger \sqrt{2\Omega - b^\dagger b}; \quad R_0 = b^\dagger b - \Omega. \] (8)

where \( b^\dagger, b \) are new (ideal) boson creation and annihilation operators with commutation rules:

\[ [b, b^\dagger] = 1, \quad [b, b] = [b^\dagger, b^\dagger] = 0. \] (9)
Now, the initial Hamiltonian (6) written in terms of ideal bosons has the form:

\[
H = Ab^\dagger b - Bb^\dagger bb^\dagger b.
\]  

(10)

\[
A = \alpha(2\Omega + 1) - \beta\Omega, \quad B = \alpha - \beta.
\]

|$n\rangle$ - boson state is determined as:

\[
|n\rangle = \frac{1}{\sqrt{n!}}(b^\dagger)^n|0\rangle, \text{ where } b|0\rangle = 0
\]

(11)

Thus the energy spectrum produced by Hamiltonian (10) is the parabolic function of the number of ideal monopole bosons $n$:

\[
E_n = An - Bn^2
\]

(12)

In Figure 1 we show the new representation of available experimental data for $0^+$ states in $^{160}$Dy as distributed by number of ideal monopole bosons $n$. The average energy deviations $<|E_{\text{expt}} - E_{\text{calc}}|>$ are 10.7 KeV. The parameters $A$ and $B$ of (12) are evaluated by fitting the experimental energies of the different $0^+$ states of a given nucleus to the theoretical ones applying all possible permutations of the classification numbers $n$ and extracting the distribution corresponding to the minimal value of $\chi^2$ - square. In Figure 2 as an additional example we present our results for description of $0^+$ excited states in new experimental data for $^{158}$Gd nucleus. Even in this case we obtain a very good agreement between our distribution and experiment - the average deviation less than 13 KeV per point. With nice accuracy the experimental energies for low lying collective states follow the parabolic distribution function of number of collective excitations. Now we can label every $K^\pi = 0^+$ state by an additional characteristic $n$ number of monopole bosons determining it’s collective structure. It is interesting to point out that the ordering of the states in respect to their number of phonons does not necessarily correspond to increase of excitation energy. For some nuclei the lowest excited $K^\pi = 0^+$ states have more collective structure ( larger $n$ ) than the states with higher excitation energies.

Of course it is straightforward now to see whether the low lying excited states having different from zero spin can be also represented in the same form of the energies distributed by parabolic type function and can we consider the new classification parameter as a measure of collectivity determining each low lying state.

Using the $(\lambda, \mu)$ labels facilitates and choosing for instance $(\lambda, 0)$ multiplet together with the reducing rules (9) after simple regrouping of the terms in (2) for any fixed value of angular momentum $L$ the energy spectrum corresponding to this $(\lambda, 0)$ multiplet is:

\[
E(\lambda) = A\lambda - B\lambda^2 + C
\]

(13)
here $A, B$ and $C$ are the combinations of free model parameters of $2\alpha, \alpha_3, \beta_3$ and $\alpha_1$. Hence choosing any permitted by (3) $(\lambda, \mu)$ multiplet we again may classify the low lying excited states energies in even even nuclei applying the parabolic type distribution function and considering label $C$ as a measure of collectivity of the corresponding excited states. In Figure 3 we show the distributions of the $2^+, 4^+, 6^+$ excited states in $^{160}\text{Dy}$. In these figures we use the notion $n = \lambda$. And in this case we have a good description with the average energy deviations $<|E_{\text{expt}} - E_{\text{calc}}| > 39.4, 21.9$, and $50.4$ keV for the levels with $I^* = 2^+, 4^+$, and $6^+$ respectively.

From the energy spectrum written in terms of $\lambda$ and $\mu$ labels (4) we can determine the expressions for the rotational bands energies as follow:

For $K^* = 0^+$ ground-state band $\{\lambda = 0, \mu = 2L\}$ and $T = 0, T_0 = 0$

$$E_{gr} = 4\alpha L + \beta_3 L(L + 1) + 4\alpha_1 L(5 + 4L) + 2\alpha_3(6L + 4L^2)$$

(14)

$K^* = 0^+ S$ band $\{\lambda = 4, \mu = 2L - 2\}$ and $T = 2, T_0 = 1$

$$E_s = c_1 + 4\alpha L + \beta_3 L(L + 1) + 4\alpha_1 L(5 + 4L) + 2\alpha_3(6L + 4L^2)$$

(15)

$K^* = 1^-$ octupole vibrational band $\{\lambda = 2, \mu = 2L - 1\}$ and $T = 1, T_0 = 1, L \geq 1$

$$E_{\text{oct }1^-} = c_1 - 4\alpha L + \beta_3 L(L + 1) + 4\alpha_1 L(5 + 4L) + \alpha_3\{10 + 5(2L - 1) + (2L - 1)^2\}$$

(16)

$K^* = 2^+ \gamma$ vibrational band $\{\lambda = 4, \mu = 2L - 2\}$ and $T = 2, T_0 = 1, L \geq 2$

$$E_\gamma = c_2 + 4\alpha L + \beta_3 L(L + 1) + 4\alpha_1 L(5 + 4L) + \alpha_3\{28 + 7(2L - 2) + (2L - 2)^2\}$$

(17)

$K^* = 2^-$ octupole vibrational band $\{\lambda = 2, \mu = 2L - 1\}$ and $T = 1, T_0 = 1, L \geq 2$

$$E_{\text{oct }2^-} = c_2 - 4\alpha L + \beta_3 L(L + 1) + 4\alpha_1 L(5 + 4L) + \alpha_3\{10 + 5(2L - 1) + (2L - 1)^2\}$$

(18)

Our previous calculations of rotational bands energies with different forms of nuclear density shapes [46] had shown that the moment of inertia depends on number of monopole bosons $n$ approximately as:

$$I(n) \approx I(0)(1 + xn)$$

(19)

where $x$ is connected with the diffuseness parameter $s$, compressibility coefficient $C_0$, one-phonon energy $E_0$ and nuclear half-radius $R$ as:

$$x = \frac{E_0 R^2 \left((-3 + 20\pi) R^4 + 30 (-1 + 4\pi) R^2 s^2 + 45 (-1 + 4\pi) s^4\right)}{8 C_0 \pi^2 \left(R^6 + 13 R^4 s^2 + 45 R^2 s^4 + 45 s^6\right)}$$

(20)
That is why in our calculations we choose parameter $\beta_3$ to be

$$\beta_3 = \frac{1}{2\Gamma(n)} = \frac{\beta_0}{1 + nx} \quad (21)$$

We apply this approximation in our calculations of the energies of rotational bands determined by \text{(21)}. In Figures 4 - 5. is shown the comparison of our calculations with experiment. It is important to point out that all these bands are calculated with the same set of parameters:

| $\alpha$   | $\alpha_1$ | $\alpha_3$ | $x$      | $\beta_0$ |
|------------|------------|------------|----------|-----------|
| 0.00568396 | -0.0274276 | 0.05485516 | 0.0020997| 0.23      |

In these figures are also presented the corresponding values of number of monopole bosons $n$ building the collective excited 0$^+$ state (entering (21)) which mainly determines the moment of inertia of each band. The agreement between calculated and experimental energies is very good and average energy deviation $\langle |E_{\text{expt}} - E_{\text{calc}}| \rangle$ for all bands under consideration is less than 9 KeV per point.

For vindication of the right positions of the bands with different parities and check the parity splitting we had calculated the staggering functions of the fifth order for experimental points and calculated data:

$$\Delta^5 E(L) = 6(E(L) - E(L - 1)) - 4(E(L - 1) - E(L - 2)) - 4(E(L + 1) - E(L)) + E(L + 2) - E(L + 1) + E(L - 2) - E(L - 3) \quad (23)$$

According to our calculations for the energies of the bands (Figure 4 - Figure 5) we had calculated the staggering functions with replaced states $I^\pi = 18^+, 4.181, MeV, I^\pi = 20^+, 4.875 MeV$ from $S$ band to the ground band while the states $I^\pi = 18^+, 3.67 MeV, I^\pi = 20^+, 4.279, I^\pi = 22^+, 4.936 MeV, I^\pi = 24^+, 5.648 MeV, I^\pi = 26^+, 6.413 MeV, I^\pi = 28^+, 7.231 MeV$ from the ground to $S$ band (which really produces much better agreement with experiment than the calculations with previous straightening [37]. Hence we have proposed that the sequence of states $I^\pi = 18^+, 4.181, MeV, I^\pi = 20^+, 4.875 MeV$ belongs to the ground band while the states $I^\pi = 18^+, 3.67 MeV, I^\pi = 20^+, 4.279 MeV, I^\pi = 22^+, 4.936 MeV, I^\pi = 24^+, 5.648 MeV, I^\pi = 26^+, 6.413 MeV, I^\pi = 28^+, 7.231 MeV$ must be related to the $S$ band, moreover that for simultaneous description of the bands with previous straightening [37] the additional parameter is required. In Figure 7 are presented odd-even staggering functions for $S \ (K^\pi = 0^+)$ and octupole $\ (K^\pi = 2^-)$ bands, $S \ (K^\pi = 0^+)$ and $\gamma \ (K^\pi = 2^-)$ bands, for $S \ (K^\pi = 0^+)$and octupole $S \ (K^\pi = 1^-)$ bands, ground $\ (K^\pi = 0^+)$ and $\gamma \ (K^\pi = 2^+)$ bands calculated for experimental and theoretical data. The agreement between theory and experiment is good. In Figure 8 we show the comparison of the staggering functions for octupole band $\ (K^\pi = 1^-)$ and the band $\ (K^\pi = 0^+)$, so far determined as a ground band.
in [37]. In the case of straightening [37] the agreement of the calculated and experimental data is sensitively worse. The same situation one can find in Figure 9, where are compared staggering functions for octupole \( K^π = 2^- \) and ground bands. To prove that this rearrangement of some of states between \( S \) and ground bands is not a sort of mere assertion we must analyze the behavior of the \( B(E2) \) transitions in the region of crossing ground and \( S \) bands states. Indeed, the transition probability even in simple rigid rotor model depends on intrinsic quadrupole momentum \( Q_0 \) that in our consideration is a function of number of monopole bosons and increases with increase of number of monopole bosons \( n \) [44]. Thus it should be good to make detail theoretical analysis of \( B(E2) \) transitions within the framework of our model. This work is in progress.

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References

[1] G.D.Burke, G.Lovhoiden, T.F.Thorsteinsen, Nucl.Phys. A483 (1988) 221.
[2] C.Wesselborg, P.Von Brentano, K.O.Zell, R.D.Heil, H.H.Pitz, U.E.P.Berg, U.Kneissl, S.L.Lindenstruth, U.Seemann, R.Stock, Phys.Lett. 207B (1988) 22.
[3] W.N.Shelton, Phys.Lett. 20 (1966) 651.
[4] T.Grotdal, K.Nybo, T.Thorsteinsen, B.Elbek, Nucl.Phys. A110 (1968) 385.
[5] T.Ramsoy, J.Rekstad, M.Guttormsen, A.Henriquez, F.Ingebretsen, T.Rodland, T.F.Thorsteinsen, G.Lovhoiden, Nucl. Phys. A438 (1985) 301.
[6] G.M.Jin, J.D.Garrett, G.Lovhoiden, T.F.Thorsteinsen, J.C.Waddington, J.Rekstad, Phys.Rev.Lett. 46 (1981) 222.
[7] J.V.Maher, J.J.Kolata, R.W.Miller, Phys.Rev. C6 (1972) 358.
[8] H.J.Riezebos, M.J.A.De Voigt, C.A.Fields, X.W.Cheng, R.J.Peterson, G.B.Hagemann, A.Stolk, Nucl.Phys. A465 (1987) 1.
[9] A.Johnson, H.Ryde, S.A.Hjorth, Nucl.Phys. A179 (1972) 753.
[10] S.M.Ferguson, H.Ejiri, I.Halpern - Nucl.Phys. A188 (1972) 1.
[11] M.Fenzl, J.Imazato, Z.Phys. 266 (1974) 135.
[12] A.J.Gresswell, P.A.Butler, D.Cline, R.A.Cunningham, M.Devlin, F.Hammachi, R.Ibbotson, G.D.Jones, P.M.Jones, M.Simon, J.Simpson, J.F.Smith, C.Y.Wu, Phys.Rev. C52 (1995) 1934.
[13] E.K.Meintyre, T.J.Hallman, K.S.Kang, C.W.Kim, Y.K.Lee, L.Madansky, G.R.Mason, Phys.Lett. 137B (1984) 339.
[14] M.J.A. De Voigt, W.J. Ockels, Z. Sujkowski, A. Zglinski, J. Mooibroek, Nucl. Phys. A323 (1979) 317.

[15] S.J. Feenstra, W.J. Ockels, J. Van Klinken, M.J.A. De Voigt, Z. Sujkowski, Phys. Lett. 69B (1977) 403.

[16] A. Johnson, H. Ryde, J. Sztarkier, Phys. Lett. 34B (1971) 605.

[17] H. Morinaga, P.C. Gugelot, Nucl. Phys. 46 (1963) 210.

[18] G.T. Ewan, G.I. Andersson, Proc. Roy. Soc. Edinburgh Sect. A 70 (1972) 155.

[19] F.K. McGowan, W.T. Milner, Phys. Rev. C23 (1981) 1926.

[20] F. Kearns, G. Varley, G.D. Dracoulis, T. Inamura, J.C. Lisle, J.C. Willmott, Nucl. Phys. A278 (1977) 109.

[21] R.O. Sayer, E. Eichler, N.R. Johnson, D.C. Hensley, L.L. Riedinger, Phys. Rev. C9 (1974) 1103.

[22] R.N. Oehlberg, L.L. Riedinger, A.E. Rainis, A.G. Schmidt, E.G. Funk, J.W. Mihelich, Nucl. Phys. A219 (1974) 543.

[23] B. Elbek, M.C. Olesen, O. Skilbreid, Nucl. Phys. 19 (1960) 523.

[24] F. Kearns, G.D. Dracoulis, T. Inamura, J.C. Lisle, J.C. Willmott, J. Phys. (London). A7 (1974) L11.

[25] Y. Yoshizawa, B. Elbek, B. Herskind, M.C. Olesen, Nucl. Phys. 73 (1965) 273.

[26] M.A. Ludington, J.J. Reidy, M.L. Wiedenbeck, D.J. McMillan, J.H. Hamilton, J.J. Pinajian, Nucl. Phys. A119 (1968) 398; Erratum Nucl. Phys. A127 (1969) 693.

[27] R.E. McAdams, O.H. Ottoesn, Z. Phys. 250 (1972) 359.

[28] M.P. Avotina, E.P. Grigoriev, B.S. Dzhelepov, A.V. Zolotavin, V.O. Sergeev, BAS USSR. Phys. 30 (1966) 530.

[29] N.A. Bonch-Omslovskaya, J. Vrzal, E.P. Grigoriev, J. Liptak, G. Pfrepper, J. Urbanets, D. Christov, BAS USSR. Phys. 32 (1968) 98.

[30] E.P. Grigoriev, K.Y. Gromov, Z.T. Zhelev, T.A. Islamov, V.G. Kalinnikov, U.K. Nazarov, S.S. Sabirov, BAS USSR. Phys. 33 (1969) 635.

[31] E.P. Grigoriev, J. Zvolski, N.A. Tikhonov, V.I. Fominykh, BAS USSR. Phys. 34 (1970) 2059.

[32] A.A. Aleksanrov, V.S. Buttsev, T. Vylov, E.P. Grigoriev, K.Y. Gromov, V.G. Kalinnikov, N.A. Lebedev, BAS USSR. Phys. 38 (1974) 2096.

[33] A.A. Aleksanrov, V.S. Buttsev, T. Vylov, E.P. Grigoriev, K.Y. Gromov, V.G. Kalinnikov, N.A. Lebedev, BAS USSR. Phys. 38 (1974) 2103.
[34] C.W.Reich, Nucl. Data Sheets. 78 (1996) 547.

[35] R.B.Firestone, Table of Isotopes. 8th Edit. New York. 1996. V. 2.

[36] J.Adam, Yu.A.Vaganov, V.Vagner, V.P.Volnykh, V.Zvolska, J.Zvolski, Y.S.Ibraheem, T.A.Islamov, V.G.Kalinnikov, B.Krack, N.A.Lebedev, A.F.Novgorodov, A.A.Solnyshkin, V.I.Stegailov, Zh.Sereeter, M.Fisher, P.Caloun, BRAS. Phys. 66 No 10 (2002) 1384.

[37] A.Jungclaus, B.Binder, A.Dietrich, T.Hartlein, H.Bauer, Ch.Gund, D.Pansegrau, D.Schwalm, J.L.Egido, Y.Sun, D.Bazzacco, G.de Angelis, E.Farnea, A.Gadea, S.Lunardi, D.R.Napoli, C.Rossi-Alvarez, C.Ur, G.B.Hagemann, Phys. Rev. C 66 (2002) 014312.

[38] J.Adam, V.P.Garistov, M.Homsek, J.Dobes, J.Zvolski, J.Mrazek, A.A.Solnyshkin, nucl-th/0408

[39] Bohr A., Mottelson B.R. Nucl.Structure, Benjamin. N.Y. 1975. V. 4.

[40] Jolos R.V. Yadernaya Fizika. 64. No 3 (2001) 520.

[41] Begzhanov R.B., Bilenkii V.M., Dubro V.G. “Structure of deformed nuclei.” M: Energoatomizdat. 1983.

[42] Peker L.K., Pearlstein S., Rasmussen J.O., Hamilton J.H. Phys. Rev. Lett. 50 (1983) 1749.

[43] Arima A., Iachello F. Ann. Phys. N.Y. 99 (1976) 253 and 111 (1978) 201.

[44] H.Ganev, V.P.Garistov, A.Georgieva, Phys. Rev. C69 (2003) 0143305; Vladimir P. Garistov "Phenomenological Description of the Yrast Lines”, nucl-th/0201008 4, 2002; V. P. Garistov, nucl-th/0008067 V. P. Garistov, nucl-th/0309058 V. P. Garistov, Nuclear Theory, Proceedings of the XXI Workshop on Nuclear Theory, edited by V. Nikolaev, Heron Press Science Series, 77-85; V. P. Garistov, Nuclear Theory, Proceedings of the XXII Workshop on Nuclear Theory, edited by V. Nikolaev, Heron Press Science Series, 305-311.

[45] A.Georgieva, P.Raychev, R.Roussev Phys G: Nucl. Phys. 8 (1982) 1377; A.Georgieva, P.Raychev, R.Roussev Phys G: Nucl. Phys. 9 (1983) 521; A.Georgieva, M.Ivanov, P.Raychev, R.Roussev Int. J. Theor. Phys. 28 (1989) 769.

[46] T.Holstein, H.Prinnakoff, Phys. Rev. 58, (1940) 1098; A.O. Barut, Phys. Rev. 139, (1965) 1433; R. Marshalek Phys. Lett. B 97 (1980) 337; C. C. Gerry, J. Phys. A 16, (1983) 11.
Figure 1. Distribution of $0^+$ states by number of monopole bosons in $^{160}$Dy.

Figure 2. Distribution of $0^+$ states by number of monopole bosons in $^{158}$Gd.

$E(n) = 0.60095n - 0.02945n^2$

$E(n) = 0.51051n - 0.03071n^2$
Figure 3. Distribution of $2^+$, $4^+$ and $6^+$ states in $^{160}$Dy (IVBM, $n=\lambda/4$).

For $2^+$ states:

$$E(n) = 0.75293n - 0.02184n^2 - 3.64795$$

For $4^+$ states:

$$E(n) = 0.36113n - 0.01309n^2 - 0.07936$$

For $6^+$ states:

$$E(n) = 0.85994n - 0.0521n^2 - 0.16602$$
Figure 4. Comparison of IVBM energies with experiment for the ground and S bands in $^{160}$Dy.

Figure 5. Comparison of IVBM energies with experiment for $\gamma$ band in $^{160}$Dy.
Figure 6. Comparison of IVBM energies with experiment for the octupole bands in $^{160}$Dy.
Figure 7. Comparison of theoretical and experimental staggering functions $\Delta^5 E(L)$ for different bands of $^{160}$Dy.
Figure 8. Comparison of theoretical and experimental staggering functions $\Delta^5(L)$ for octupole $K^z=1^-$ band of $^{160}$Dy.
Figure 9. Comparison of theoretical and experimental staggering functions $\Delta^5 E(L)$ for octupole $K^\pi=2^-$ band of $^{160}$Dy.