Quantum Vavilov-Cherenkov radiation from a small neutral particle moving parallel to a transparent dielectric plate

A.I. Volokitin\textsuperscript{1,2,3*} and B.N.I. Persson\textsuperscript{1}

\textsuperscript{1}Peter Grünberg Institut, Forschungszentrum Jülich, D-52425, Germany
\textsuperscript{2}Samara State Technical University, Physical Department, 443100 Samara, Russia and
\textsuperscript{3}Samara State Aerospace University, Physical Department, 443086 Samara, Russia

We study the quantum Vavilov-Cherenkov (QVC) radiation and quantum friction occurring during motion of a small neutral particle parallel to a transparent dielectric plate with the refractive index $n$. This phenomenon occurs above the threshold velocity $v_c = c/n$. The particle acceleration and rate of heating are determined by the friction force and heating power in the rest reference frame of the particle. We calculate these quantities starting from the expressions for the friction force and the radiative energy transfer in the plate-plate configuration, assuming plate at the rest in the lab frame rarefied. Close to the light velocity there is a big difference between the friction force and the radiation power in the rest frame of a particle and in the lab reference frame. This difference is connected with the change of the rest mass of the particle due to absorption of radiation. Close to the threshold velocity the decrease of the kinetic energy of the particle is determined mainly by radiation power in in the lab frame. However, close to the light velocity it is determined also by the heating power for the particle. We establish the connections between the quantities in the different reference frames. For a nanoparticle the QVC radiation intensity can be comparable to classical one. We discuss the possibility to detect QVC radiation.

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I. INTRODUCTION

A remarkable manifestation of the interaction between the electromagnetic field and matter is the emission of light by charged particle moving at a constant superluminally velocity in a medium: the Vavilov-Cherenkov (VC) radiation\textsuperscript{1–6}, which has broad applications in the detection of high-energy charged particles in astrophysics and particle physics. This phenomenon can be explained by classical electrodynamics because moving charge corresponds to a time-dependent current. The problem of the interaction of a point charge with the electromagnetic field in a medium can be reduced to the solution of the equation of the motion for harmonic oscillators under the action of external forces with frequencies

$$\omega = q \cdot v = q\cos \theta = \frac{\omega(q)v\cos \theta}{v_0},$$

where $\theta$ is the angle between the wave vector $q$ and velocity $v$, $v_0 = c/n$ is the phase velocity of the light in the medium with the refractive index $n$, $\omega(q) = v_0 q$ is the eigenfrequency of the electromagnetic wave in the medium. At $\omega = \omega(q)$, there is resonance, and the amplitudes of oscillators grow with time, i.e. radiation occurs. At resonance

$$\cos \theta = \frac{v_0}{v} < 1.$$  \hspace{1cm} (2)

Thus radiation occurs only if the charge velocity $v$ exceeds the phase velocity of light in a medium: $v > v_0$.

A neutral particle moving at a constant velocity can also radiate due to presence of the fluctuating dipole moment. This radiation was described by Frank\textsuperscript{7} and Ginzburg and Frank\textsuperscript{8,9} (see also\textsuperscript{5,6} for review of these work). If a particle has no internal degrees of freedom (e.g., a point charge), then the energy of the radiation is determined by the change of the kinetic energy of the object. However, if a particle has internal degrees of freedom (say, an atom), then two types of radiation are possible. If the frequency of the radiation in the lab reference frame $\omega > 0$, then in the rest frame of a particle, due to the Doppler effect the frequency of the radiation $\omega' = \gamma(\omega - q_x v)$, where $\gamma = \sqrt{1 - \beta^2}$, $\beta = v/c$, $q_x$ is the projection of the wave vector on the direction of the motion. In the normal Doppler effect region, when $\omega' > 0$, the radiated energy is determined by the decrease of the internal energy. For example, for an atom the state changes from the excited state $|1\rangle$ to the ground state $|0\rangle$ in which case $\omega' = \omega_0$ where $\omega_0$ is the transition...
frequency. The region of the anomalous Doppler effect corresponds to $\omega' < 0$ in which case a particle becomes excited when it radiates. For example, an atom could experience the transition from the ground state $|0\rangle$ to the excited state $|1\rangle$ when it radiates in which case $\omega' = -\omega_0$. In such a case energy conservation requires that the energy of the radiation and of the exciton result from a decrease of the kinetic energy of the particle. That is, the self-excitation of a system is accompanied by a slowing down of the motion of the particle as a whole. If the particle is moving relative to the transparent dielectric medium the propagating electromagnetic waves can be excited in the medium in which case $\omega = v_0q$. Thus the condition for the occurrence of the radiation has the form

$$q_x v = v_0 q + \frac{\omega_0}{\gamma} > v_0 q_x + \frac{\omega_0}{\gamma},$$

(3)

It follows that radiation occurs only when $v > v_c = v_0$, which coincide for condition of the radiation from a point charge. Due to the quantum origin, the radiation produced by a neutral particle is denoted as the quantum Vavilov-Cherenkov (QVC) radiation, and the friction associated with this radiation is denoted as quantum friction, which is the limiting case of the Casimir friction produced by thermal and quantum fluctuations of the electromagnetic field; the same fluctuations also give rise to the Casimir – van der Waals forces\textsuperscript{10–12}. The link between QVC radiation and quantum friction in the plate-plate configuration was discussed within framework of a non-relativistic\textsuperscript{13}, semi-relativistic\textsuperscript{14} and fully relativistic theories\textsuperscript{15,16}. The thermal and quantum fluctuation of the current density in one body induces a current density in another body; the interaction between these current densities is the origin of the Casimir interaction. When two bodies are in relative motion, the induced current will lag slightly behind the fluctuating current inducing it, and this is the origin of the Casimir friction. At present the radiation friction is attracting a lot of attention due to the fact that it is one of the mechanisms of noncontact friction between bodies in the absence of direct contact\textsuperscript{16}. The noncontact friction determines the ultimate limit to which the friction force can be reduced and, consequently, also the force fluctuations because they are linked to friction via the fluctuation-dissipation theorem. The force fluctuations (and hence friction) are important for ultrasonic force detection.

Radiative friction has deep roots in the foundation of quantum mechanics. The friction force acting on a particle moving relative to the blackbody radiation, which is a limiting case of the fluctuating electromagnetic field, was studied by Einstein and Hopf\textsuperscript{17} and Einstein\textsuperscript{18} in the early days of quantum mechanics. The friction induced by the electromagnetic fluctuations has been studied in the configurations plate-plate\textsuperscript{13,16,19–22}, neutral particle-plate\textsuperscript{16,25–30}, and neutral particle-blackbody radiation\textsuperscript{22,23,35,37,38,41}. While the predictions of the theory for the Casimir forces were verified in many experiments\textsuperscript{12}, the detection of the Casimir friction is still challenging problem for experimentalists. However, the frictional drag between quantum wells\textsuperscript{24,34–36} and graphene sheets\textsuperscript{14,15}, and the current-voltage dependence of nonsuspended graphene on the surface of the polar dielectric SiO\textsubscript{2}\textsuperscript{16}, were accurately described using the theory of the Casimir friction\textsuperscript{24,17,45}.

It was shown in Ref.\textsuperscript{24} that the friction force on a small neutral particle at relativistic motion parallel to flat surface of a dielectric in the \textit{lab} frame can be obtained from the friction force between two plates assuming that the moving plate is sufficiently rarified and consists of particles with the electric polarizability $\alpha$. In this paper, we extend this approach to calculate the friction force in the rest frame of the particle.

II. THEORY

We consider two plates having flat parallel surfaces separated by a distance $d$ and moving with the velocity $v$ relative to each other, see Fig. \textsuperscript{1}. We introduce the two reference frames $K$ and $K'$ which are the rest reference frames for the plates 1 and 2, respectively. According to a fully relativistic theory\textsuperscript{24} the friction force at zero temperature $F_{1x}$ (denoted as quantum friction\textsuperscript{12}) acting on the surface of plate 1, and the radiation power $W_1$ absorbed by plate 1 in the $K$ frame, are determined by formulas

$$\left( \begin{array}{c} F_{1x} \\ W_1 \end{array} \right) = \int_{-\infty}^{\infty} dq_y \int_{0}^{\infty} dq_x \int_{0}^{q_x v} d\omega \int_{0}^{2\pi} \frac{d\gamma}{2\pi} \left( \frac{h}{\hbar} q_x \right) \Gamma_{12}(\omega, q),$$

(4)

where the positive quantity

$$\Gamma_{12}(\omega, q) = -\frac{4}{|\Delta|^2} \left[ (q^2 - \beta k q_x)^2 - \beta^2 k_x^2 q_y^2 \right] \{ \text{Im} R_{1p} [(q^2 - \beta k q_x)^2 \text{Im} R_{2p} |\Delta_{ss}|^2 \\ + \beta^2 k_x^2 q_y^2 \text{Im} R_{2p} |\Delta_{sp}|^2] + (p \leftrightarrow s) \} e^{-2k_x d}$$

(5)
can be identified as a spectrally resolved photon emission rate,
\[
\Delta = (q^2 - \beta k q_x)^2 \Delta_{ss} \Delta_{pp} - \beta^2 k_z q_y^2 \Delta_{sp}, \quad \Delta_{pp} = 1 - e^{-2k_z d} R_{1p} R_{2p}^*, \quad \Delta_{sp} = 1 + e^{-2k_z d} R_{1s} R_{2p}^*
\]

\[
k_z = \sqrt{q^2 - (\omega/c)^2}, \quad k = \omega/c, \quad R_{1p(s)}(s) \text{ is the reflection amplitude for surface 1 in the } K \text{ frame for a } p(s) - \text{polarized electromagnetic wave, } R_{2p(s)}(s) = R_{2p(s)}(\omega', q') \text{ is the reflection amplitude for surface 2 in the } K' \text{ frame for a } p(s) - \text{polarized electromagnetic wave, } \omega' = \gamma(\omega - q_z v), \quad q'_z = \gamma(q_x - \beta k), \quad \Delta_{ss} = \Delta_{pp}(p \leftrightarrow s), \quad \Delta_{sp} = \Delta_{sp}(p \leftrightarrow s). \]

Assuming that the dielectric permittivity of the rarefied plate is close to the unity, i.e. \( \varepsilon - 1 \to 4\pi \alpha N \ll 1 \), where \( N \) is the concentration of particles in a plate in the co-moving reference frame, then to linear order in the concentration \( N \) the reflection amplitudes for the rarefied plate in the co-moving frame are

\[
R_p = \frac{\varepsilon k_z - \sqrt{k_z^2 - (\varepsilon - 1)(\omega/c)^2}}{\varepsilon k_z + \sqrt{k_z^2 - (\varepsilon - 1)(\omega/c)^2}} \approx N \pi q^2 + k_z^2, \quad R_s = \frac{k_z - \sqrt{k_z^2 - (\varepsilon - 1)(\omega/c)^2}}{k_z + \sqrt{k_z^2 - (\varepsilon - 1)(\omega/c)^2}} \approx N \pi q^2 - k_z^2.
\]

Because \( R_{p(s)} \ll 1 \) for the rarefied plate, it is possible to neglect the multiple-scattering of the electromagnetic waves between the surfaces. In this approximation \( \Delta_{pp} \approx \Delta_{ss} \approx \Delta_{sp} \approx \Delta_{sp} \approx 1 \),

\[
\Delta \approx (q^2 - \beta k q_x)^2 - \beta^2 k_z q_y^2 = \frac{(qq')^2}{q^2},
\]

\[
(q^2 - \beta k q_x)^2 \text{Im} R_{2p} |\Delta_{ss}|^2 + \beta^2 k_z q_y^2 \text{Im} R_{2s} |\Delta_{sp}|^2 \approx \frac{(qq')^2}{q^2} \text{Im} R_{2p} + \beta^2 k_z q_y^2 \text{Im} (R_{2p} + R_{2s}'),
\]

\[
\Gamma_{12} = -4 \left[ (\text{Im} R_{1p} \text{Im} R_{2p} + \text{Im} R_{1s} \text{Im} R_{2s}) \left( 1 + \gamma^2 \beta^2 k_z q_y^2 \right) \right] + \gamma^2 \beta^2 k_z q_y^2 \left( \text{Im} R_{1p} \text{Im} R_{2s} + \text{Im} R_{1s} \text{Im} R_{2p}' \right) \left( \frac{q^2}{q^2} \right)^2.
\]

The friction force \( f_z \) acting on a particle, and the radiation power \( w \) absorbed by it, can be obtained in the \( K \) frame from Eq. (10) assuming the plate 2 as sufficiently rarefied \( \ll \) (see Fig. 1). In this case the friction force acting on the surface 2, \( F_{2z} \), and the radiation power absorbed by it, \( W_2 \), are

\[
\begin{pmatrix} F_{2x} \\ W_2 \end{pmatrix} = \begin{pmatrix} -F_{1x} \\ -W_1 \end{pmatrix} = N' \int_d^\infty dz \begin{pmatrix} f_z(z) \\ w(z) \end{pmatrix},
\]

FIG. 1: The schemes of the configurations (a) plate-plate, (b) particle-plate in the \( \text{lab} \) frame and (c) particle-plate in the rest frame of a particle. The friction force in the particle-plate configuration can be obtained from the friction force in the plate-plate configuration assuming rarefied the plate 2.
where \( N' = \gamma N \) is the concentration of particles in the plate 2 in the \( K' \) frame,

\[
\left( \frac{f_x(z)}{w(z)} \right) = \frac{1}{\gamma^2} \int d^2q \int_{0}^{vq_z} d\omega \left( \frac{hq_x}{\hbar \omega} \right) e^{-2k_zz} \frac{k_z^2}{k_z^2} \left[ \text{Im} R_{\nu p}(\omega) \phi_p + \text{Im} R_{\nu s}(\omega) \phi_s \right] \text{Im} \alpha(\omega'),
\]

\[
\phi_p = (\omega'/c)^2 + 2\gamma^2 (q^2 - \beta^2 q_x^2) \frac{k_z^2}{q_x^2}, \quad \phi_s = (\omega'/c)^2 + 2\gamma^2 \beta^2 q_y^2 \frac{k_z^2}{q_z^2}.
\]

In the rest reference frame of an object the radiation power absorbed by it is equal to the heating power for the object. Thus \(-w\) is equal to the heating power for the plate 1. Eqs. (12) agree with the results obtained in\(^{28,34,35}\). However, as shown in Refs.\(^{33,41}\), the acceleration and heating of the particle are determined by the friction force \( f_x' \) and the radiation power \( w' \) absorbed by the particle in the rest reference frame of a particle (the \( K' \) frame)

\[
m_0 \gamma^2 \frac{dv}{dt} = m_0 \frac{dv'}{dt'} = f_x',
\]

\[
w' = \frac{dm_0}{dt'} c^2
\]

where \( m_0 \) is the rest mass of particle, \( v' \ll v \) and \( t' \) are the velocity and time in the \( K' \), respectively. These quantities can be also obtained assuming the plate 2 as sufficiently rarefied (see Fig. [1]). In this case in the \( K' \) frame the friction force acting on the surface 2, \( F_{2x}' \), and the radiation power absorbed by it, \( W_{2}' \), are

\[
\left( \begin{array}{c} F_{2x}' \\ W_{2}' \end{array} \right) = \left( \begin{array}{c} -\bar{F}_{1x} \\ \bar{W}_1 \end{array} \right) = N \int_{-\infty}^{\infty} dz \left( \begin{array}{c} f_x'(z) \\ w'(z) \end{array} \right),
\]

where \( \bar{F}_{1x} \) and \( \bar{W}_1 \) are obtained from \( F_{1x} \) and \( W_1 \) after the replacement of the indexes 1 \( \leftrightarrow \) 2,

\[
\left( \begin{array}{c} f_x' \\ w' \end{array} \right) = \frac{1}{\pi^2} \int_{0}^{\infty} dq_z \int_{-\infty}^{\infty} dq_y \int_{0}^{vq_y} d\omega \left( \frac{hq_x}{-\hbar \omega} \right) e^{-2k_zz} \frac{k_z^2}{k_z^2} \left[ \text{Im} R_{\nu p}(\omega') \phi_p' + \text{Im} R_{\nu s}(\omega') \phi_s' \right] \text{Im} \alpha(\omega),
\]

\[
\phi_p' = (\omega/c)^2 + 2\gamma^2 (q^2 - \beta^2 q_x^2) \frac{k_z^2}{q_x^2}, \quad \phi_s' = (\omega/c)^2 + 2\gamma^2 \beta^2 q_y^2 \frac{k_z^2}{q_z^2}.
\]

The relation between the different quantities in the \( K \) and \( K' \) frames can be found using the Lorentz transformations for the energy-momentum tensor for a plate 2 according to which

\[
F_{2x} = \gamma \left( F_{2x} + v \frac{W_{2}'}{c^2} \right), \quad W_2 = \gamma(W_2' + \nu F_{2x}'),
\]

Using Eqs. (11) and (15) gives

\[
f_x = f_x' + \frac{w'}{c^2}, \quad w = w' + v f_x'.
\]

These relation also can be found using the Lorentz transformation for the energy-momentum for a particle according to which

\[
p_x = \gamma (p_x' + v m_0), \quad \varepsilon = \gamma (m_0 c^2 + v p_x'),
\]

where \( p_x \) and \( \varepsilon \) are the momentum and energy of a particle in the \( K \) frame, respectively, and \( p_x' \) and \( m_0 c^2 \) are the same quantities in the \( K' \) frame. When the derivative of 4-momentum is taken with respect to \( \text{lab} \) time, then the factor \( \gamma \) disappears in the right place because \( dt = \gamma dt' \) and the relations (18) are obtained\(^{11}\). From the inverse transformations

\[
F_{2x}' = \gamma \left( F_{2x} - v \frac{W_2}{c^2} \right), \quad W_2' = \gamma(W_2' - \nu F_{2x}'),
\]
follows
\[ f'_x = \gamma^2 (f_x - \frac{w}{c}), \quad w' = \gamma^2 (w - vf_x). \] (21)

These relations also can be obtained as above using the Lorentz transformation for the energy-momentum for a particle. We note that in the contrast with the relations (18) on the right side of the relations (21) there is an extra factor \( \gamma^2 \). This is because the friction force and radiation power are not the 4-vectors. The kinetic energy of a particle in the lab frame \( \epsilon_K = \epsilon - mc^2 \) where the total energy of a particle in the \( K \) frame \( \epsilon = \gamma (mc^2 + p'_x v) \). The rate of change of the kinetic energy
\[ \frac{d\epsilon_K}{dt} = w' + f'_x v - \frac{w'}{\gamma} = w - \frac{w'}{\gamma} = vf_x - \frac{(\gamma - 1)w'}{\gamma^2}. \] (22)
Thus the rate of change of the kinetic energy in the \( K \) frame is equal to the friction force power in this frame only when \( w' = 0 \).

### III. RESULTS

For the transparent dielectrics the reflection amplitudes are given by the Fresnel’s formulas
\[ R_p = \frac{in^2 k_z - \sqrt{n^2(\omega/c)^2 - q^2}}{in^2 k_z + \sqrt{n^2(\omega/c)^2 - q^2}}, \quad R_s = \frac{ik_z + \sqrt{n^2(\omega/c)^2 - q^2}}{ik_z - \sqrt{n^2(\omega/c)^2 - q^2}}. \] (23)

There is no restriction on the imaginary part of the particle polarizability in the integration range \( 0 < \omega < v_n v \). Thus the integrand in Eq. (12) is nonzero only in the range \( \omega_0 q_e < \omega < q_e v \), where the imaginary part of the reflection amplitude is nonzero, thus the critical velocity \( v_c = v_0 = c/n \).

Assuming that the imaginary part of the particle polarizability is determined by the formula
\[ \text{Im} \alpha = R^3 \omega_0^2 \frac{\omega/\tau}{(\omega^2 - \omega_0^2)^2 + (\omega/\tau)^2}, \]
where \( R \) is the radius of the particle, \( \omega_0 \) is the plasmon frequency for the particle and \( \tau \) is the damping constant, then close to the resonance at \( \omega \approx \omega_0 \)
\[ \text{Im} \alpha = R^3 \omega_0 \frac{\pi}{2} \delta (\omega - \omega_0), \]
and from Eq. (16) the resonant contributions to the friction force and the heating power close to the threshold velocity in the \( K' \) frame are dominated by the contributions from \( p \)-polarized waves, and are given by (see Appendix A)
\[ f_{x p}^{res} = -\frac{\hbar R^3 \omega_0}{4d^4(n^2 - 1)} \frac{v - v_0}{v_0}[3 + 4q_0d + 2(q_0d)^2]e^{-2q_0d}, \] (24)
\[ w_{p}^{res} = \frac{\hbar R^3 \omega_0^3}{2d^3(n^2 - 1)} \frac{v - v_0}{v_0}(1 + q_0d)e^{-2q_0d} \] (25)
where \( q_0 = (n^2 - 1)\omega_0/((v - v_0)n^2) \). In the off-resonant region \( \omega \ll \omega_0 \)
\[ \text{Im} \alpha = R^3 \frac{\omega}{\omega_0 \tau}, \]
and again the dominant contributions are given by the \( p \)-polarized waves (see Appendix B):
\[ f_{x p}^{off res} \approx -\frac{5}{4\pi} \frac{\hbar R^3 v_0^2}{\omega_0^2 \tau} \frac{n^4}{(n^2 - 1)^3} \left( \frac{v - v_0}{v_0} \right)^3, \] (26)
\[ w_{p}^{off res} \approx \frac{35}{64\pi} \frac{\hbar R^3 v_0^3}{\omega_0^2 \tau} \frac{n^6}{(n^2 - 1)^4} \left( \frac{v - v_0}{v_0} \right)^4. \] (27)
From Eqs. (24)–(27) follows that close to the threshold velocity \( w' \ll f'x/e \). Thus, from Eq. (18) follows that \( f_x \approx f'_x \), which agrees with the results obtained in Ref. 36, and \( w \approx f_x v \). The change of the kinetic energy in this limit is determined by the radiation power from a particle in the \( K' \) frame, but the change of the internal energy, and consequently the change of the rest mass of the particle, is small. The off-resonant contribution to friction force from the frequency range \( \omega \ll \omega_p \) is only important close to the threshold velocity \( ((v - v_0)/v_0 \ll 1) \), while far from the threshold velocity the friction force is dominated by the resonant contribution from \( \omega \approx \omega_0 \), as was already noted in Ref. 36.

The friction force acting on the elementary charge \( e \), due to the classical Vavilov-Cherenkov radiation, is determined by the well-known formula 39

\[
f^{\text{class}}_x = \frac{e^2}{2\beta d^2} \frac{1}{n^2 - 1} \left[ \frac{n^2 \gamma}{\sqrt{n^2 - 1} - \beta} \right].
\]

(28)

In the ultra relativistic limit \( (\gamma \gg 1) \), Eq. (23) is significantly simplified taking the form \( f^{\text{class}}_x = -\frac{e^2}{2d^2} \). In Fig. 2 it is compared with the friction force due to the quantum Vavilov-Cherenkov radiation on a neutral silver nano particle with the radius \( R = 4\)nm and with losses (red, \( \tau^{-1} = 2.4 \cdot 10^{14} \text{s}^{-1} \)) and no losses (green, \( \tau^{-1} = +0 \)), the surface plasmon frequency for a particle \( \omega_0 = \omega_p/\sqrt{3} \), where \( \omega_p = 9.01\)eV is the bulk plasmon frequency for silver, at \( d = 10\)nm. Far from the threshold velocity these friction forces are of the same order of the magnitude.

In the ultra relativistic limit \( (\gamma \gg 1) \)

\[
\text{Im} \alpha(\omega') = -\frac{\pi}{2} \frac{\omega_0 R^3}{v} \delta(\omega' + \omega_0) = -\frac{\pi}{2} \frac{\omega_0 R^3}{v} \delta \left( \frac{q}{\gamma v} - \frac{\omega_0}{\gamma v} \right)
\]

(29)

The negative sign of the imaginary part of the particle polarizability means that a particle behaves as an object with negative absorption amplifying certain incident waves. This phenomenon is closely connected to superradiance first introduced by Zel’dovich 36. He argued that a rotating object amplifies certain incident waves and speculated that this would lead to spontaneous emission when quantum mechanics is considered. The contributions to the friction force, and the radiation power, from the \( s- \) and \( p- \) polarised waves in the \( K \) frame are given by (see Appendix A)

\[
f_{xx} = -\frac{3\hbar \omega_0 R^3}{2d^4(n + 1)} \left[ 1 + \left( \frac{\omega_0 d}{c \gamma} \right) C \right], \quad p_s = -\frac{3\hbar \omega_0 R^3}{2d^4(n + 1)}
\]

(30)

\[
f_{px} = -\frac{3\hbar \omega_0 R^3}{2d^4} \frac{n}{n + 1} \left[ 1 + \left( \frac{\omega_0 d}{c \gamma} \right) C' \right], \quad p_p = -\frac{3\hbar \omega_0 R^3}{2d^4} \frac{n}{n + 1}.
\]

(31)
where

\[
C = \frac{2}{3\pi} \sqrt{\frac{n+1}{n-1}} \left[ \frac{n}{\sqrt{n^2-1}} \tanh^{-1} \frac{\sqrt{n^2-1}}{n} - 1 \right],
\]

\[
C' = \frac{2}{3\pi} n \sqrt{\frac{n+1}{n-1}} \left[ \frac{n^2}{\sqrt{n^2-1}} \tanh^{-1} \frac{\sqrt{n^2-1}}{n} - \frac{n}{\sqrt{n^2-1}} \tanh^{-1} \frac{\sqrt{n^2-1}}{n} \right].
\]

In the \(K'\) frame the friction force and the heat absorbed by a particle can be obtained from the corresponding quantities in the \(K\) frame using the Lorentz transformations \(21\). For example, for the contributions from the \(s\)-polarized waves

\[
f'_s = \gamma^2 (f_s - \beta w_s) \approx - \frac{3\hbar \omega_0 R^3}{2d^4(n+1)} \left[ 1 + C' \frac{\omega_0 d}{c} \right], \quad (32)
\]

and

\[
w'_s = \gamma^2 (w_s - v f_s) \approx \frac{3\hbar \nu_0 R^3}{2d^4(n+1)} C' \frac{\omega_0 d}{c}. \quad (33)
\]

Thus, contrary to the \(K\) frame where the friction force and the power of photon emission are finite, in the \(K'\) frame the friction force and the radiation power both diverge as \(\sim \gamma\). These results also can be obtained by the direct calculations in the \(K'\) frame. The radiation power \(w\) is determined mostly by the friction force power \(f_s v\) in the \(K\) frame (see Eq. \(21\)). From Eq. \(22\) it follows that the particle heating power also contributes significantly in this limit to the change of the kinetic energy.

\[\text{IV. DISCUSSION}\]

A silver particle with \(R = 4\) nm has mass \(m_0 \approx 2.68 \cdot 10^{-21}\) kg. Close to the threshold velocity \((v \approx v_0 = c/n)\) at \(n = 2\) it has the kinetic energy \(E_K = m_0 c^2 (\gamma - 1) \approx 2 \cdot 10^2\) TeV, which is larger than the energy of proton in Large Hadron Collider 7 TeV. However the energy of a particle can be decreased by decreasing its radius. According to Eqs. \(28\) and \(24\) the intensity of the QVC radiation exceeds intensity CVC radiation when

\[
\hbar \omega_0 \left( \frac{R}{d} \right)^3 > \frac{e^2}{2d} \quad (34)
\]

The point dipole approximation is valid for \(d > R\). Assuming \(d = 2R\) the condition \(34\) gives \(d > 4\alpha d_0\) where the fine structure constant \(\alpha \approx 1/137\) and \(d_0 = c/\omega_0\). At typical value of \(\omega_0\) in the UV range \(d_0 \sim 10\) nm and thus \(d > 0.3\) nm. For a particle with \(R = 0.4\) nm and \(d = 1\) nm the reduction factor for the particle mass \(\sim 10^{-3}\) and the energy \(\sim 0.1\) TeV. The characteristic frequency of radiation \(\omega \sim v/d\). The threshold velocity \(v_0\) depends on the refractive index \(n\). Recently the transparent metamaterials were developed with very high reflective index in the UV range\(34\). At \(n = 20\) the threshold velocity \(v_0 \sim 10^7\) m/s and at \(d = 1\) nm the frequency of the radiation \(\sim 10^{16}\) s\(^{-1}\) i.e. in the near UV range. The energy of a particle in this case is \(\sim 1\) GeV. Thus, in principle the QVC radiation can be detected with the present experimental setups but only in the region where relativistic effects are small.

\[\text{V. CONCLUSION}\]

A small neutral particle moving parallel to transparent dielectric plate emits quantum Vavilov-Cherenkov (QVC) radiation when the velocity exceeds a threshold velocity. This radiation is responsible for quantum friction, which we have studied in the particle rest reference frame and in the \(lab\) frame, using a fully relativistic theory. The friction forces in the particle-plate configuration in the different reference frames were calculated from the corresponding results in the plate-plate configuration considering one of the plates as sufficiently rarefied. We have shown that in the realistic situation the friction force acting on a neutral nanoparticle due to QVC radiation can be comparable in the magnitude with the friction force acting on a charged particle due to the classical Vavilov-Cherenkov (CVC) radiation. Thus, in principle QVC radiation can be detected using the same experimental setup as for CVC radiation. The challenges for future experiments are to accelerate a particle, having sufficiently large fluctuating dipole moment and to the velocities close to the light velocity at small separation from a transparent dielectric surface. Non relativistic QVC radiation can be observed using transparent dielectric with refractive index \(\sim 10\) in the near UV region.
VI. ACKNOWLEDGEMENT

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Appendix A: Particle with no losses

1. Close to the threshold velocity

The resonant contribution to the friction force comes from $\omega$ in the range

$$\omega_0 < \omega < \frac{v - v_0}{1 - v_0/v^2}q_x.$$ 

Near resonance the particle polarizability can be approximated by the formula

$$\text{Im} \alpha = R^3\omega_0^2 \frac{\omega \gamma}{(\omega_0^2 - \omega_0^2)^2 + \omega^2 \gamma^2} \approx R^3\omega_0^2 \frac{\pi}{2} \left[ \delta(\omega - \omega_0) - \delta(\omega + \omega_0) \right]$$ (A1)

Close to the threshold velocity ($(v - v_0)/v_0 \ll 1$) and at the resonance

$$k_{n z}^2 = -\frac{1}{v_0} \left[ \frac{n^2}{(n^2 - 1)}(v - v_0)q_x - \omega_0 \right] 2q_x - q_y^2.$$ (A2)

Introducing new variables $q_x = q_0 x$, $q_y = q_0 y_{max}$ where $q_0 = \omega_0(n^2 - 1)/(v - v_0)n^2)$,

$$y_{max}^2 = \frac{2n^2}{n^2 - 1}x(x - 1)\frac{v - v_0}{v_0} \ll x^2$$

the imaginary part of the reflection amplitudes can be written in the form

$$\text{Im} R'_w \approx \frac{2}{\pi} q_{\text{max}} \sqrt{1 - y^2}$$

The $p$-wave resonant contribution to the friction force in the $K'$ frame is given by

$$f'_x \approx -\frac{2^4h^4q^4_0}{\pi^2(n^2 - 1)} \left( \frac{\pi}{2} R^3\omega_0 \right) \frac{v - v_0}{v_0} \int_1^\infty dx x^2(x - 1)e^{-2q_0dx} \int_0^1 dy \sqrt{1 - y^2}$$

$$= -\frac{\hbar R^3\omega_0}{4d^2(n^2 - 1)} \frac{v - v_0}{v_0} [3 + 4q_0d + 2(q_0d)^2]e^{-2q_0d}$$ (A3)

$$w' \approx \frac{2^4h\omega_0q^3_0}{\pi^2(n^2 - 1)} \left( \frac{\pi}{2} R^3\omega_0 \right) \frac{v - v_0}{v_0} \int_1^\infty dx x(x - 1) \int_0^1 dy \sqrt{1 - y^2}$$

$$= \frac{\hbar R^3\omega_0^2}{2d^3(n^2 - 1)} \frac{v - v_0}{v_0} (1 + q_0d)e^{-2q_0d}$$ (A4)

2. Limiting case $v \to c$

In the ultra relativistic limit ($\gamma \gg 1$)

$$\text{Im} \alpha(\omega') = -\frac{\pi}{2} \omega_0 R^3 \delta(\omega' + \omega_0) = -\frac{\pi}{2\gamma} \frac{\omega_0 R^3}{v} \delta \left( q_x - \frac{\omega - \omega_0}{\gamma v} \right)$$ (A5)
After the integration in Eq. (12) over \( q \),

\[
\begin{align*}
\frac{k_z^2}{k^2} &= \left(\frac{\omega}{\gamma v}\right)^2 + \frac{2\omega_0}{\gamma v} + \left(\frac{\omega_0}{\gamma v}\right)^2 + q_y^2 \\
\frac{k_{nz}^2}{k^2} &= \left[\frac{\omega}{v}(n\beta - 1) - \frac{\omega_0}{\gamma v}\right] \left[\frac{\omega}{v}(n\beta + 1) + \frac{\omega_0}{\gamma v}\right] - q_y^2
\end{align*}
\]  

(A6)

(A7)

For \( \omega < C\gamma v/d \) where \( C < 1 \): \( k_z \approx q_y \),

\[
\begin{align*}
k_{nz}^2 &\approx (n^2 - 1) \left(\frac{\omega}{v}\right)^2 - q_y^2
\end{align*}
\]  

(A8)

Introducing the new variable

\[
q_y = \frac{\omega}{v} y \sqrt{n^2 - 1}
\]

where \( 0 \leq y \leq 1 \), the imaginary part of the reflection amplitudes can be written in the form

\[
\text{Im} R_s = \frac{2k_z k_{nz}}{k_z^2 + k_{nz}^2} = 2y \sqrt{1 - y^2},
\]

(A9)

\[
\text{Im} R_p = \frac{2n^2 k_z k_{nz}}{n^2 k_z^2 + k_{nz}^2} = \frac{2n^2 y \sqrt{1 - y^2}}{1 + (n^2 - 1)y^2},
\]

(A10)

and

\[
\phi_p \approx \phi_s \approx 2\gamma^2 \left(\frac{\omega}{v}\right)^2 \frac{(n^2 - 1)y^4}{1 + (n^2 - 1)y^2}.
\]

(A11)

The contributions from the \( s \)- and \( p \)- polarised waves in the \( K \) frame are given by

\[
\begin{align*}
f_{sx} &= -\frac{4\hbar \omega_0 R^3}{\pi} \int_0^1 dy \frac{(n^2 - 1)^2 y^4 \sqrt{1 - y^2}}{1 + (n^2 - 1)y^2} \int_0^\infty d\left(\frac{\omega}{v}\right) \left(\frac{\omega}{v} + \frac{\omega_0}{\gamma v}\right)^3 e^{-2(\omega/v)y/d\sqrt{n^2 - 1}} \\
&= -\frac{3\hbar \omega_0 R^3}{2d^2(n + 1)} \left[1 + \left(\frac{\omega_0 d}{c\gamma}\right) C\right]
\end{align*}
\]

(A12)

where

\[
C = \frac{2}{3\pi} \sqrt{\frac{n + 1}{n - 1}} \left[\frac{n}{\sqrt{n^2 - 1}} \tanh^{-1} \frac{\sqrt{n^2 - 1}}{n} - 1\right].
\]

\[
w_s = -\frac{4\hbar v \omega_0 R^3}{\pi} \int_0^1 dy \frac{(n^2 - 1)^2 y^4 \sqrt{1 - y^2}}{1 + (n^2 - 1)y^2} \int_0^\infty d\left(\frac{\omega}{v}\right) \left(\frac{\omega}{v}\right)^3 e^{-2(\omega/v)y/d\sqrt{n^2 - 1}}
\]

(A13)

\[
f_{px} = -\frac{4\hbar \omega_0 R^3}{\pi} \int_0^1 dy \frac{n^2(n^2 - 1)^2 y^4 \sqrt{1 - y^2}}{[1 + (n^2 - 1)y^2][1 + (n^4 - 1)y^2]} \int_0^\infty d\left(\frac{\omega}{v}\right) \left(\frac{\omega}{v} + \frac{\omega_0}{\gamma v}\right)^2 e^{-2(\omega/v)y/d\sqrt{n^2 - 1}}
\]

(A14)
where

\[ C' = \frac{2}{3\pi} n \sqrt{\frac{n+1}{n-1}} \left[ \frac{n^2}{\sqrt{n^2-1}} \tanh^{-1} \frac{\sqrt{n^2-1}}{n} - \frac{n}{\sqrt{n^2-1}} \tanh^{-1} \frac{\sqrt{n^2-1}}{n} \right], \]

\[ w_p = \frac{4\hbar}{\pi \omega_0 R^3} \int_0^1 dy \frac{y \sqrt{1-y^2}}{1+(n^4-1)y^2[1+(n^4-1)y^2]} \int_0^\infty d\left(\frac{\omega}{v}\right) \left(\frac{\omega}{v}\right)^3 e^{-2(\omega/v)y d} \sqrt{n^2-1}. \]

In the deriving of the formulas (A12–A15) the values of the following integrals were used

\[ \int_0^1 dy \frac{\sqrt{1-y^2}}{1+(n^2-1)y^2} = \frac{\pi}{n+1} \]

\[ \int_0^1 dy \frac{y \sqrt{1-y^2}}{1+(n^2-1)y^2} = \frac{1}{n^2-1} \left[ \frac{n}{\sqrt{n^2-1}} \tanh^{-1} \frac{\sqrt{n^2-1}}{n} - 1 \right] \]

\[ \int_0^1 dy \frac{\sqrt{1-y^2}}{1+(n^4-1)y^2[1+(n^4-1)y^2]} = \frac{\pi}{n(n+1)} \]

\[ \int_0^1 dy \frac{y \sqrt{1-y^2}}{1+(n^4-1)y^2[1+(n^4-1)y^2]} = \frac{1}{n^2(n-1)} \left[ \frac{n^2}{\sqrt{n^4-1}} \tanh^{-1} \frac{\sqrt{n^4-1}}{n^2} - \frac{n}{\sqrt{n^2-1}} \tanh^{-1} \frac{\sqrt{n^2-1}}{n} \right]. \]

In the \( K' \) frame the friction force and the heat absorbed by a particle can be obtained from the corresponding quantities in the \( K \) frame using the Lorentz transformations (21). For example for the contributions from the \( s \)-polarized waves

\[ f_s' = \gamma^2 (f_s - \beta w_s) \approx -\frac{3\hbar \omega_0 R^3}{2d^4(n+1)} \left[ 1 + C' \gamma \frac{\omega_0 d}{c} \right], \]

and

\[ w_s' = \gamma^2 (w_s - \omega f_s) \approx \frac{3\hbar \omega_0 R^3}{2d^4(n+1)} C' \gamma \frac{\omega_0 d}{c}. \]

Thus, contrary to the \( K \) frame where the friction force and the power of photon emission are finite, in the \( K' \) frame the friction force and the radiation power both diverge as \((1-\beta)^{-1/2}\). These results can be confirmed by the direct calculations in the \( K' \) where

\[ \text{Im} \omega = -\frac{\pi}{2} \omega_0 R^3 \delta(\omega - \omega_0) \]

(A18)

After the integration in Eq. (16) over \( \omega \)

\[ k_z^2 = q_x^2 + q_y^2 - \left(\frac{\omega_0}{c}\right)^2 \]

\[ k_{nz}^2 \approx \gamma^2 \left[ n^2 \beta^2 \left(\frac{q_x - \omega_0}{v}\right)^2 - \left(\frac{q_x - \omega_0}{c}\right)^2 \right] - q_y^2 \]

\[ \approx \gamma^2 (n^2 - 1) \left(\frac{q_x - \omega_0}{v}\right)^2 - q_y^2 \]

(A20)
Introducing the new variable

\[ q_y = \gamma \sqrt{n^2 - 1} \left( q_x - \frac{\omega_0}{v} \right) y \]

where \( 0 \leq y \leq 1 \) and \( q_x \geq \omega_0/v \), the same equations (A9) and (A10) are obtained for \( \text{Im} R_{p(s)} \) and

\[ \phi_p \approx \phi_s \approx 2 \gamma^4 \left( q_x - \frac{\omega_0}{v} \right)^2 \frac{(n^2 - 1)^2 y^4}{1 + (n^2 - 1)y^2}. \]  

(A21)

The contributions from the \( s- \) and \( p- \) polarised waves in the \( K' \) frame are given by

\[
f'_{xx} = -\frac{4\hbar}{\pi} R_0^3 \gamma^4 \int_0^1 dy \frac{(n^2 - 1)^2 y^4 \sqrt{1 - y^2}}{1 + (n^2 - 1)y^2} \int_0^\infty dq_x q_x \left( q_x - \frac{\omega_0}{v} \right)^2 e^{-2(q_x - \omega_0/v)y \sqrt{n^2 - 1}} \]

\[= -\frac{3\hbar \omega_0 R_0^3}{2d^4(n+1)} \left[ 1 + C \gamma \frac{\omega_0 d}{c} \right], \]

(A22)

\[
w'_s = \frac{4\hbar}{\pi} \omega_0 R_0^3 \gamma^4 \int_0^1 dy \frac{(n^2 - 1)^2 y^4 \sqrt{1 - y^2}}{1 + (n^2 - 1)y^2} \int_0^\infty dq_x \left( q_x - \frac{\omega_0}{v} \right)^2 e^{-2(q_x - \omega_0/v)y \sqrt{n^2 - 1}} \]

\[= \frac{3\hbar \omega_0 R_0^3}{2d^4(n+1)} C \gamma \frac{\omega_0 d}{c}. \]

(A23)

**Appendix B: The off-resonant contribution for a particle with losses close to the threshold velocity**

The off-resonant contribution to the friction force comes from \( \omega \) in the range

\[ 0 < \omega < \frac{v - v_0}{1 - vv_0/c^2} q_x \ll \omega_0. \]

In this frequency range the low-frequency approximation for the particle polarisability can be used

\[ \text{Im} \alpha = R_0^3 \omega_0^2 \frac{\omega^\gamma}{(\omega^2 - \omega_0^2)^2 + \omega^2 \gamma^2} \approx R_0^3 \frac{\omega^\gamma}{\omega_0^2} \]

Close to the threshold velocity \(((v - v_0)/v_0 \ll 1) \omega \) is small and to lowest order in \((v - v_0)/v_0 \ll 1 \)

\[ k_{nz}^2 = \left( \frac{\omega}{v_0} \right)^2 - q^2 \]

\[= \gamma^2 \left[ q_x (v - v_0) - \omega (1 - vv_0/c^2) \right] q_x (v + v_0) - \omega (1 + vv_0/c^2) - q_y^2 \]

\[= \frac{1}{v_0} \left[ \frac{n^2}{n^2 - 1} (v - v_0) q_x - \omega \right] 2q_x - q_y^2. \]  

(B1)

The integration over \( q_y \) is restricted by the range 0 < \( q_y < q_x y_{\text{max}} \), where

\[ y_{\text{max}}^2 = 2 \frac{n^2}{n^2 - 1} \frac{v - v_0}{v_0} \ll 1. \]

Introducing new variables \( q_y = q_x y_{\text{max}} y \)

\[ \omega = q_x v_0 y_{\text{max}}^2 \frac{1 - y^2}{2}. \]
the imaginary part of the reflection amplitudes can be written in the form

\[
\text{Im} R_s = \frac{2k_x k_{2n}}{k_x^2 + k_{2n}^2} \approx \frac{2k_{2n}}{k_x} \approx \frac{2}{q_x} \sqrt{\frac{1}{v_0} \left( \frac{n^2}{(n^2 - 1)}(v - v_0) - \omega \right)} 2q_x - q_y^2
\]

\[
= 2y_{max} \sqrt{1 - y^2} \sqrt{1 - z} \sim \sqrt{\frac{v - v_0}{v_0}},
\]

and \(\text{Im} R_s = \frac{\text{Im} R_s'}{n^2}\),

\[
\phi_s' \approx 2q_x^2, \quad \phi_s' \sim q_x^2 y_{max} \ll q_x^2.
\]

To lowest order in \((v - v_0)/v_0\) the friction force is determined only by the contribution from the \(p\)-polarized waves which is given by

\[
f'_x \approx \frac{2\hbar y_{max}^2 v_0^2 R_x^3}{n^2 \pi^2} \int_0^\infty dq_x q_x^5 e^{-2q_x d} \int_0^1 dy (1 - y^2)^{5/2} \int_0^1 dz \sqrt{1 - z}
\]

\[
= -\frac{5}{4\pi} \frac{\hbar R^4 v_0^2}{d^5 \omega_0^5} \frac{n^4}{(n^2 - 1)^3} \left( \frac{v - v_0}{v_0} \right)^3,
\]

and the heat absorbed by the particle in the \(K'\) frame is given by

\[
w'_x \approx \frac{\hbar y_{max}^3 v_0 R_x^3}{n^2 \pi^2} \int_0^\infty dq_x q_x^5 e^{-2q_x d} \int_0^1 dy (1 - y^2)^{7/2} \int_0^1 dz \sqrt{1 - z}
\]

\[
= \frac{35}{64\pi} \frac{\hbar R^4 v_0^2}{d^5 \omega_0^5} \frac{n^6}{(n^2 - 1)^4} \left( \frac{v - v_0}{v_0} \right)^4,
\]

\[
= \text{Im} R'_s/n^2.
\]
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