Determination of Spectral Characteristics in a Vibration Sensor Microcircuit

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Abstract: This article discusses aspects of the implementation of digital signal processing algorithms in devices with multifunctional diagnostics of bearing assemblies in integrated-circuit form. The possibility of applying the effective implementation of spectral analysis algorithms on a defined basis is considered. The structural flowchart of the filter which performs Goertzel transformation and schemes for implementation of the fast Fourier transform unit is shown. To solve these problems in a Matlab environment were developed mathematical models of Goertzel filter of fast Fourier transform unit was simulated operation of these units for specifying of parameters and evaluation of achieved characteristics. After the evaluation of simulation results, 2 types of spectral analysis units were developed: Goertzel transformation units for accurate calculations and fast Fourier transform units for analysis during the whole operating band. The created units of spectral analysis make it possible to efficiently solve the problems of bearing diagnostics, obtain information on the video spectrum of the vibration signal in the full frequency band and at the same time to determine the exact value or vibration levels at the characteristic frequencies.

Keywords: Vibration diagnostics, Goertzel algorithm, FFT, CORDIC algorithm, custom-designed chips.
Vibrodiagnostic, vibration diagnostic, Goertzel algorithm, CORDIC, FFT, ASIC.

I. INTRODUCTION

Simple characteristic values of the vibration signal, such as the RMS value of vibration velocity, peak value or quantitative average value, do not allow drawing reliable conclusions about the state of the bearing monitored. In particular, for the early identification of failure, these characteristic values are completely unsuitable. Therefore, reliable diagnosis of incipient bearing damages is only possible with an integrated assessment of the high-frequency signals transmitted from the vibration sensor, in the range up to several tens of kilohertz. Evaluation of these high-frequency signals usually carried out with the use of spectral methods of signal analysis [1, 2, 3, 4]. Moreover, one can not only conclude whether the bearing is damaged but also determine the type of damage. Also, such analysis allows us to predict and evaluate the remaining bearing operating life [5].

Spectral analysis is the most important method of processing signals from the vibration sensors. The ability to accurately detect vibrations at a given frequency or, conversely, at a frequency which is not specified by the operating mode, to evaluate signal amplitudes at characteristic frequencies - all this allows improving the analysis process and its quality. The high-frequency (HF) vibration envelope method is one of the main methods of vibration diagnostics of bearings, gearboxes, electric motors and other rotating mechanisms and machines today. This is due to the obvious advantages of the method: interference immunity, information value and, most importantly, the ability to detect defects at an early stage of their development. If any bearing defect is found, a high-frequency (carrier) component of the vibration will be modulated by the low-frequency vibration induced by the defect. Thus, if we select its low-frequency (modulating) component (called the signal "envelope") from the general modulated PF signal, by the frequency location of the amplitude peaks in the envelope spectrum we can unambiguously identify the place of the defect, and by the magnitude of the amplitude - the defect growth. This is the essence of the envelope method. The frequency range of the accelerometer should include all possible resonant frequencies of machine elements - from 1 kHz to 40 kHz or more. [5, 6]. The built-in digital signal processor (DSP) of the chip, which is a part of the multi-functional diagnostics device, allows you:
- to calculate the root mean square value (RMS-value);
- to perform digital filtering with customizable parameters;
- to perform the spectral calculation in accordance with Goertzel’s algorithm and fast Fourier transform (FFT) [6].

By comparing all measured parameters with their admissible limit values, the type and size of the defect in the corresponding bearing are determined.

II. METHODOLOGY

The problem is formulated in such a way that for signals with a low sampling frequency, it is necessary to estimate the amplitudes of the spectral responses with maximum accuracy. The size and power consumption of chips must be minimized. In addition to these requirements, there are also interface restrictions. In the first place capacity of transformation parameters (coefficients) is fixed. It is the same as for the input data is limited as 16 bits. The input data for the units of spectral analysis are real readings. Output data are the amplitude of the signals of certain frequencies and also real readings.

III. RESULTS

A. Goertzel Transform

Goertzel transform is a special implementation of the discrete Fourier transform in the form of a recursive filter. Goertzel algorithm calculates the value of only one frequency response.
At the same time, the implementation of this algorithm is much simpler and requires fewer resources than the fast Fourier transform [6, 9]. The canonical unit diagram implemented in the form of a recursive filter that performs the calculation of the spectral response with a number of \( k \) is shown in Figure 1.

![Fig. 1. Structural diagram of a filter performing Goertzel transform.](image)

Here, the coefficient \( \alpha_k \) is real and determined by the formula (1):

\[
\alpha_k = 2 \cdot \cos \left( \frac{2\pi k}{N} \right)
\]

The coefficient \( W_k \) is complex, determined by the formula (2):

\[
W_k = e \left( j \cdot \frac{2\pi k}{N} \right) = \cos \left( \frac{2\pi k}{N} \right) + j \cdot \sin \left( \frac{2\pi k}{N} \right)
\]

In both formulas: \( k \) is the number of the spectral reading of the transformation; \( N \) is the size of the transformation window.

It should be noted that the multiplication by the coefficient \( W_k \) is performed once - only for the final response. Also note that the real part of the coefficients \( \alpha_k \) and \( W_k \) coincides up to a fixed ratio. The size of the transformation window is determined by the frequency resolution requirements and is chosen to be 4096 readings.

Simulation of the implementation showed that the 16 bits are not enough to represent the coefficients \( \alpha_k \) and \( W_k \). The coefficients for low (and mirror) frequencies coincide with this representation and lead to large distortions of the conversion. This error of the coefficients in the recursive part repeatedly distorts the signal, making spectral analysis impossible for the lower 5..10 frequencies. In figure 2. is presented the Goertzel transform calculation error graph with 16-bit coefficients.

To improve the accuracy of calculations multiplication by harmonic factors \( \alpha_k \) and \( W_k \) was replaced by the operation of a turn signal at a predetermined angle \( \frac{2\pi k}{N} \). The operation of turning a complex reference \( (x, y) \) by an angle \( \varphi \) is generally described as follows (3):

\[
\begin{align*}
X_{\text{rot}} &= x \cdot \cos(\varphi) - y \cdot \sin(\varphi) \\
Y_{\text{rot}} &= x \cdot \sin(\varphi) + y \cdot \cos(\varphi)
\end{align*}
\]

here \( X_{\text{rot}} \) is the actual reference component after the rotation, \( Y_{\text{rot}} \) is the imaginary reference component after the rotation.

Having performed the rotation operation with exclusively real input data and choosing exclusively the real part of the result, we obtain (4):

\[
x_{\text{rot}} = x \cdot \cos(\varphi)
\]

Thus, the operation of multiplication by the harmonic factor in the formula (1) can be replaced by the rotation operation (5):

\[
\alpha_k \cdot x = 2 \cdot \text{real} \left( \text{rotate} \left( (x, 0), \frac{2\pi k}{N} \right) \right)
\]

Rotation operation is efficiently implemented using the CORDIC algorithm [10, 11], and the similarity of \( \alpha_k \) and \( W_k \) coefficients allows us to use one and the same unit for rotation algorithms. The rotation angle is uniquely determined by the frequency harmonic number \( k \). Thus, to configure the conversion with the window 4096, a 12-bit representation of the frequency number is necessary and sufficient.

A similar operation has long been used in FFT implementations [12, 13]. The use of the CORDIC algorithm in relation to Goertzel’s transformation is practically not covered in scientific publications. Figure 4. is presented the Goertzel transform calculation error graph based on the CORDIC algorithm.

![Fig. 4. Goertzel transform calculation error graph based on the CORDIC algorithm.](image)
The values of the worst error estimated of the amplitude of the spectral response for frequencies in typical areas are shown in Table 1. The results of Goertzel transformation options were compared with a similar transformation performed without limiting the bit depth and accuracy of data and coefficients.

Table 1. The worst relative error in typical frequency bands for various Goertzel transform implementations.

| Range (Hz) | Relative error with 16-bit coefficients | Relative error with CORDIC calculations |
|------------|-----------------------------------------|----------------------------------------|
| 0.0001     | 0.499237                                | 0.011048                               |
| 0.001      | 0.383083                                | 0.000199                               |
| 0.004      | 0.222446                                | 0.000015                               |
| 0.007      | 0.004653                                | 0.0000016                              |
| 0.005.15625| 0.000290                                | 0.0000016                              |

The calculation of the amplitude of the output complex result of Goertzel transform can also be performed using the CORDIC algorithm. Figure 5 presents a structural diagram of the Goertzel transform.

The structural difference from the canonical scheme is due to the calculation delay and units operation time synchronization. To ensure the increased accuracy of setting the frequency in the low-frequency region, a customizable decimator is added to the unit structure. Filtering signals outside the target band is carried out by digital filters located in front of the Goertzel converter unit [14, 15, 16]. To enable an analysis of several adjacent or any required frequencies, four Goertzel transducers are installed in the unit.

The structure of the spectral response estimation unit based on Goertzel transform is shown in Figure 6. Here, the decimation value is determined by the signal (Dec_sel) for all transformation units, and the number of the spectral conversion count is determined for each unit separately (K0, K1, K2, K3). Responses are calculated independently and simultaneously.

The main technical characteristics of the unit are presented in Table 2. The above-mentioned estimates of the unit area were performed taking into account the technological process and the digital library XFAB 180 nm. Table 2. The main technical characteristics of the unit for evaluating the spectral responses based on Goertzel transform.

| Characteristic                          | Decimation Units | Value     |
|-----------------------------------------|------------------|-----------|
| Sampling rate (input data rate)         | Hz               | 62500     |
| Clock frequency                         | Hz               | 16000000  |
| The bit depth of the input data         | bit              | 16        |
| The bit depth of the output data        | bit              | 16        |
| Window size                             | counts           | 4096      |
| Upper analysis frequency limit          | Hz               | 15625     |
|                                         | 4 Hz             | 7812.5    |
|                                         | 8 Hz             | 3906.3    |
|                                         | 16 Hz            | 1953.1    |
|                                         | 32 Hz            | 976.6     |
|                                         | 64 Hz            | 488.3     |
|                                         | 128 Hz           | 244.1     |
|                                         | 256 Hz           | 122.1     |
| Frequency resolution                    | Hz               | 7.63      |
|                                         | 4 Hz             | 3.81      |
|                                         | 8 Hz             | 1.91      |
|                                         | 16 Hz            | 0.95      |
|                                         | 32 Hz            | 0.48      |
|                                         | 64 Hz            | 0.24      |
|                                         | 128 Hz           | 0.12      |
|                                         | 256 Hz           | 0.06      |
| Duration of calculation without taking into account the time of data arrival | sec | 0.00001 |
|                                         | 2 sec            | 0.13      |
|                                         | 4 sec            | 0.26      |
|                                         | 8 sec            | 0.52      |
|                                         | 16 sec           | 1.05      |
|                                         | 32 sec           | 2.10      |
|                                         | 64 sec           | 4.19      |
|                                         | 128 sec          | 8.39      |
|                                         | 256 sec          | 16.78     |
Table 3. Relative error estimates for low and characteristic frequencies

| Harmonic number | Frequency (Hz) | Maximum relative error | Relative error |
|-----------------|----------------|------------------------|---------------|
| 0               | 0              | 0.023698               | 0.021651      |
| 1               | 6.25           | 0.004014               | 0.002692      |
| 2               | 12.5           | 0.001052               | 0.000625      |
| 3               | 17.75          | 0.000547               | 0.000315      |
| 5               | 31.25          | 0.000256               | 0.000115      |
| 10              | 62.5           | 0.000083               | 0.000029      |
| 14              | 87.5           | 0.000045               | 0.000015      |
| 19              | 118.75         | 0.000044               | 0.000016      |
| 38              | 237.4          | 0.000046               | 0.000014      |
| 77              | 481.25         | 0.000051               | 0.000016      |
| 115             | 718.75         | 0.000042               | 0.000016      |
| 154             | 962.5          | 0.000047               | 0.000015      |

It should be noted that the error at the 0th frequency is caused by rounding in integer calculations. The maximum use of sequential computing in the implementation of the CORDIC algorithm allowed us to obtain the required Goertzel transform unit of a small area. The use of customizable decimation provided increased resolution in the low-frequency region - the target region.

B. Fast Fourier Transform

Fast Fourier Transform is an algorithm for accelerated calculation of the discrete Fourier transform. A feature of the requirements for this unit was the use of memory with an 8-bit capacity of input data: 8192×8 bits. To store the intermediate results of the calculation, 64 bits were used for each complex sample: 4 bytes per real and imaginary parts. The FFT window size was determined by the requirements for frequency resolution and the number of available RAM units. The selected window size is 1024 readings. When implementing the transformation, a sequential organization of calculations was used. A generalized unit diagram is shown in Figure 9.

C. Window overlay

To improve the quality of spectral analysis in a limited time interval, a window overlay is used. The Hann window was chosen as a compromise between the frequency resolution and the level of suppression of the side lobes of the signal [17]. The characteristics of this window in the frequency and time
domain are presented in Figure 10. In addition to direct multiplication by the window coefficients, the unit writes data in memory, buffers the input data.

![Window Viewer](image)

**Fig. 10.** Hann window characteristics in time and frequency domain.

D. **FFT core**

The basis of the FFT calculation algorithm is the “butterfly” – the operation of selecting and updating complex samples [18, 19], figure 11.

![Scheme of one operation of selecting, calculating and updating FFT readings](image)

**Fig. 11.** The scheme of one operation of selecting, calculating and updating FFT readings is a “butterfly”.

The main computing core is just engaged in the organization of the data sample, performing arithmetic operations of the “butterfly” and writing the data in the memory [18, 19]. Instead of multiplying by the complex harmonic factor, the complex signal rotation operation based on the CORDIC algorithm is also used. This was done to exclude a sufficiently large ROM of harmonic factors and increase the accuracy of calculations [12, 13]. This FFT implementation uses a parallel-serial computation circuit. Sampling and writing in memory, the rotation operation is performed sequentially. At the same time, delays are synchronized so that new data is selected and processed before the rotation operation is completed. This allowed us to accelerate the transformation process, without additional hardware costs.

E. **Calculation of response amplitudes**

The calculation of amplitudes is also implemented by the CORDIC algorithm with sequential implementation. In addition to calculating the absolute value of the unit also performs descaling of the data (for the window size) and writes to the memory in a convenient form for later processing. The main technical characteristics of the FFT unit are presented in table 4. The given estimates of the unit area are made taking into account the technological process and the digital library XFAB 180 nm.

| Characteristic                                  | Units | Value  |
|------------------------------------------------|-------|--------|
| Sampling rate (input data rate)                | Hz    | 62500  |
| Clock frequency                                | Hz    | 16000000 |
| The bit depth of input data                    | bit   | 16     |
| Bit depth output                               | bit   | 16     |
| Window size                                    | counts | 1024   |
| Frequency resolution                           | Hz    | 61.04  |
| Duration of the window overlay operation       | sec.  | 0.0164 |
| (including data accumulation)                  |       |        |
| FFT calculation operation duration             | sec.  | 0.0205 |
| Duration of calculation of modules (amplitudes)| sec.  | 0.0030 |
| of spectral responses                          |       |        |
| FFT calculation duration (total)               | sec.  | 0.04   |
| Unit area (without memory unit)                | mkm²  | 107147 |
| The area of the memory unit                    | mkm²  | 864302 |

The graph of results transformation for the signal
Determination of Spectral Characteristics in a Vibration Sensor Microcircuit

from the sensor connected to a working rolling bearing is shown in Figure 12. The data are calculated on a model that is completely identical to the hardware implementation.

![FFT calculation results in a signal from a serviceable rolling bearing sensor.](image)

The assessment of the relative error is given as in Table 5. Comparison of the results of calculations carried out with the results of similar calculations, but are not limited to the bit (floating point) on one and the same data set.

| Error          | Value     |
|----------------|-----------|
| Maximum relative error | 0.0000061 |
| Relative error   | 0.000025  |

### IV. DISCUSSIONS

#### A. Comparison of implementations of spectral response methods

In the current implementation of the unit and transform Goertzel and FFT solving the same problem of obtaining the amplitudes of the spectral responses have different characteristics. FFT allows you to get the full spectral picture of the digitized signal, Goertzel transformation, however, allows one to achieve increased accuracy in the choice of frequencies for analysis. The characteristics of the realized Goertzel and Fourier transform units are given in table 6.

### Table 6. Goertzel and FFT Conversion Unit

| Characteristic                        | Units | Goertzel Transformation | FFT |
|---------------------------------------|-------|-------------------------|-----|
| Sampling rate (input data rate)       | Hz    | 62500                   | 62500 |
| Clock frequency                       | Hz    | 160000000               | 16000000 |
| The bit depth of input data           | bit   | 16                      | 16 |
| Bit depth output                      | bit   | 16                      | 16 |
| Frequency resolution                  | Hz    | from 7.63 to 0.06       | 61.04 |
| Upper bound of available analysis band| Hz    | from 15625 to 122.1     | 32125 |
| Number of simultaneously calculated samples | pcs  | 4                      | 512 |
| Duration of calculation, excluding data arrival time | sec  | 0.00001                 | 0.0235 |

The FFT unit allows you to simultaneously receive spectral responses in the entire frequency band but has a low-frequency resolution. Goertzel’s transform unit allows to simultaneously receive spectral responses for only 4 frequencies, but the ability to set the frequency is significant, from 8 times to 100 times (in the low-frequency region) more accurate than that of the FFT unit. The arithmetic accuracy of calculating the spectral response amplitudes is the same for both of these implementations.

### V. CONCLUSION

The maximum use of sequential calculations made it possible to fit the implementation of the units for calculating the spectral characteristics in a small area. The use of complex number rotation operations instead of multiplying by harmonic factors allowed increasing the accuracy of the calculation. The implementation of the rotation functions and finding the amplitude of the complex number according to the CORDIC algorithm also ensured the effective embodiment of the unit in the target microcircuit.

Implementation of parallel Goertzel and Fourier transform units in a microcircuit will allow you to constantly have information about the spectrum in the full frequency band and at the same time determine the exact value or levels of characteristic frequencies.

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