Tightly Localized Stationary Pulses in Multi-Level Atomic System

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We show the pulse matching phenomenon can be obtained in the general multi-level system with electromagnetically induced transparency (EIT). For this we find a novel way to create tightly localized stationary pulses by using counter-propagating pump fields. The present process is a spatial compression of excitation so that it allows us to shape and further intensify the localized stationary pulses, without using standing waves of pump fields or spatially modulated pump fields.

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Recently, an important progress in electromagnetically induced transparency (EIT) \cite{1, 2, 3, 4, 5, 6} is that experimental realization for the coherent control of stationary pulses was achieved by using standing waves of pump fields in the three-level system \cite{8, 9}. The creation of stationary pulses can enhance the nonlinear couplings between photons or collective excitations corresponding to stored photons, both of which are useful for deterministic logic operations. The key point in the creation of stationary pulses is expressed by the pulse matching phenomenon between the forward (FW) and backward (BW) propagating probe fields \cite{8, 9, 10}. For a three-level system, the technique to generate tightly localized stationary pulses involves the use of standing waves of pump fields with a frequency-comb or spatially modulated pump fields \cite{9}. However, such tight localization cannot be applied directly to applications in quantum nonlinear optics.

On the other hand, coherent manipulation of probe lights has been studied in the four-level double Λ-type system \cite{11, 12} and also in the general multi-level atomic system that interacts with many probe and pump fields \cite{13, 14}. It has also been shown in Ref. \cite{13} that, one can convert different probe lights into each other by manipulating the external pump fields based on such general EIT method, indicating a sort of pulse matching phenomenon between all applied probe fields. This observation also motivates us to probe into a new technique of creating localized stationary pulses based on multi-level atomic system. In this rapid communication, we shall show the tightly localized stationary pulses can be obtained through a spatial compression of excitation the general multi-level EIT system.

We consider the quasi-one dimensional system shown

\begin{figure}
\centering
\includegraphics[width=\textwidth]{figure1}
\caption{(color online) (a) General \(m\)-level atomic system coupled to \(m-2\) quantized probe and classical pump fields which propagate in \(+z\) or \(-z\) directions. (b) No. 1 to No. \(m-3\) pump/probe pulses propagate in the \(+z\) direction, while No. \(m-2\) pump/probe pulse propagates in the \(-z\) direction.}
\end{figure}

in Fig. 1(a) for an ensemble of \(m\)-level atoms interacting with \(m-2\) quantized probe fields which couple the transitions from the ground state \(|b\rangle\) to excited state \(|e_\sigma\rangle\) \((1 \leq \sigma \leq m-2)\) with coupling constants \(g_\sigma\), and \(m-2\) classical pump fields which couple the transitions from the state \(|c\rangle\) to excited ones \(|e_\sigma\rangle\) with Rabi-frequencies \(\Omega_\sigma(z, t)\). All probe and pump fields are co-propagating in the \(+z\) or \(-z\) direction (Fig. 1(b)), and

\begin{equation}
E_\sigma(z, t) = \sqrt{\frac{\hbar \nu_\sigma}{2e_0 V}} \tilde{\xi}_\sigma(z, t) e^{i(k_\sigma z - \nu_\sigma t)},
\end{equation}

\begin{equation}
\Omega_\sigma(z, t) = \Omega_{\sigma0} e^{i(k_\sigma z - \omega_\sigma t)},
\end{equation}

where \(\sigma = 1, 2, ..., m-2\), \(\tilde{\xi}_\sigma\) and \(\Omega_{\sigma0}\) are slowly-varying amplitudes, \(k_\sigma\) and \(k_\sigma\), respectively \(z\)-component wave vectors of probe and pump fields, can be positive or negative. For \(k_\sigma > 0\) and \(k_\sigma > 0\) \((k_\sigma < 0\) and \(k_\sigma < 0\)), it means the \(\sigma\)th pair of probe and pump fields propagate in
the $+z$ ($-z$) direction. We consider all transitions to be at resonance. Under the rotating-wave approximation, the interaction Hamiltonian can be written as:

$$
\hat{V} = - \int \frac{dz}{L} (\hbar N \sum_{\sigma=1}^{m-2} g_{\sigma} \hat{\sigma}_{\sigma b}(z,t) \hat{\mathcal{E}}_{\sigma}(z,t) + hN \sum_{\sigma=1}^{m-2} \Omega_{\sigma 0}(t) \hat{\sigma}_{\sigma c}(z,t) + h.c.),
$$

where $N$ is the total atom number, $L$ is the length of the medium in the $z$ direction, and the continuous atomic variables $\hat{\sigma}_{\mu\nu}(z,t) = \frac{1}{N_z} \sum_{z_j \in N_z} \hat{\sigma}_{\mu\nu}^j(t)$ are defined by a collection of $N_z \gg 1$ atoms in a very small length interval $\Delta z$ [3]. $
\hat{\sigma}_{\sigma b}^j = |\sigma\rangle \langle b| e^{-i(k_{\sigma b} z - \omega_{\sigma b} t)}$ and $\hat{\sigma}_{\sigma c}^j = |\sigma\rangle \langle c| e^{-i(k_{\sigma c} z - \omega_{\sigma c} t)}$ are the slowly-varying parts of the $j$th atomic flip operator. We note that an essential difference between our model and the three-level case is that for the case of multi-frequency optical pulses, the one- and two-photon detunings can be avoided for all optical transitions, and no standing waves of pump fields or spatially modulated pump fields are used.

The evolution of the slowly-varying amplitudes $\hat{\mathcal{E}}_{\sigma}(z,t)$ can be described by the propagation equations

$$
\left( \frac{\partial}{\partial t} + \frac{\nu_{\sigma}}{k_{\sigma\sigma}} \frac{\partial}{\partial z} \right) \hat{\mathcal{E}}_{\sigma}(z,t) = ig_{\sigma} N \hat{\sigma}_{\sigma b}(z,t),
$$

where we note $\nu_{\sigma}/k_{\sigma\sigma} = \pm c$ for the $\pm z$ directional propagation field. Under the condition of low excitation, i.e. $\hat{\sigma}_{bb} \approx 1$, the atomic evolution governed by the Heisenberg-Langevin equations can be obtained by

$$
\hat{\dot{\sigma}}_{\sigma b} = -\gamma_{\sigma b} \hat{\sigma}_{\sigma b} + ig_{\sigma} \hat{\mathcal{E}}_{\sigma} + i\Omega_{\sigma 0} \hat{\sigma}_{\sigma c} + F_{\sigma b},
$$

$$
\dot{\sigma}_{\sigma c} = -\gamma_{\sigma c} \hat{\sigma}_{\sigma c} + i \sum_{\sigma=1}^{m-2} g_{\sigma} \hat{\mathcal{E}}_{\sigma} \hat{\sigma}_{\sigma b} + F_{\sigma c},
$$

where $\gamma_{\mu\nu}$ are the transversal decay rates that will be assumed $\gamma_{\sigma b} = \Gamma$ in the following derivation and $F_{\mu\nu}$ are $\delta$-correlated Langevin noise operators. From the Eq. [1] we find in the lowest order: $\hat{\sigma}_{\sigma c} = (ig_{\sigma} \hat{\mathcal{E}}_{\sigma} + i\Omega_{\sigma 0} \hat{\sigma}_{\sigma b} + F_{\sigma b})/\Gamma$. Substitute this result into Eq. [1] yields $\dot{\sigma}_{\sigma c} = \Gamma^{-1} \Omega_{\sigma 0} \hat{\sigma}_{\sigma c} - \Gamma^{-1} \sum_{\sigma=1}^{m-2} g_{\sigma} \Omega_{\sigma 0} \hat{\mathcal{E}}_{\sigma} - i \sum_{\sigma=1}^{m-2} g_{\sigma} \hat{\mathcal{E}}_{\sigma} \hat{\sigma}_{\sigma b}$, where $\Omega_{\sigma 0} = \sqrt{\sum_{\sigma=1}^{m-2} \Omega_{\sigma 0}^2}$. The Langevin noise terms are neglected in the present results, since under the adiabatic condition the Langevin noise terms have no effect on EIT quantum memory technique.
\[
(\partial_t + c \cos^2 \theta \cos \alpha_{m-2} \partial_z) \hat{\Psi} = -\hat{\theta} \hat{\Phi} + \sum_{j=1}^{m-2} \phi_j \cos \theta \hat{s}_j - \frac{c}{2} \sin 2\theta \cos \alpha_{m-2} \partial_z \hat{\Phi}, +
\]
\[
+ c \cos \theta \sum_{j=1}^{m-3} \prod_{l=j}^{m-3} \cos \phi_l \sin 2\phi_{j-1} \left( \frac{\nu_j}{2c k_p} + \frac{\cos \alpha_{j-1}}{2} \right) \partial_z \hat{s}_j,
\]

where we have defined
\[
\cos \alpha_\sigma = \frac{\sum_{j=1}^\sigma \frac{\nu_j}{2c k_p}}{\sum_{j=1}^{m-3} \Omega_{0j}^2 \prod_{l=1, l \neq j}^{m-3} g_l^2}, \quad \sigma = 1, 2, ..., m - 3
\]
and \(\hat{s}_j = \partial_{\phi_j} \hat{\Psi} / \cos \theta\). It then follows that
\[
\hat{s}_1 = \prod_{j=2}^{m-3} \cos \phi_j (-\sin \phi_1 \mathcal{E}_1 + \cos \phi_1 \mathcal{E}_2), \quad \hat{s}_2 = \prod_{j=2}^{m-3} \cos \phi_j (-\sin \phi_2 (\cos \phi_1 \mathcal{E}_1 + \sin \phi_1 \mathcal{E}_2) + \cos \phi_2 \mathcal{E}_3),
\]
and generally
\[
\hat{s}_k = \prod_{j=k+1}^{m-3} \cos \phi_j \hat{s}_k, \quad k = 1, 2, ..., m - 3.
\]

By comparing Eqs. (10) and (13) with the corresponding DSP and BSP fields in the three-level system, one can see a key difference is the appearance of \(\hat{\Psi}\) and \(\sigma\) fields in the three-level system, one can verify that the field \(\hat{s}_j(z, t)\) from the probe fields in our model. The adiabatic condition in the present case can be fulfilled only if \(\hat{s}_j(z, t) = 0\) for all \(j\). However, the input probe pulses are generally independent of each other so that the fields \(\hat{s}_j\) need not be zero. To study the dynamics of the DSP field, we should therefore investigate first the pulse matching between all the probe fields needed for adiabatic condition. Bearing

this idea in mind, we next explore the evolution of a set of normal fields by introducing

\[
\hat{G}_{j,j+1} = -\sin \phi_{j,j+1} \hat{\mathcal{E}}_j(z, t) + \cos \phi_{j,j+1} \hat{\mathcal{E}}_{j+1}(z, t),
\]

where \(j = 1, 2, ..., m - 3\) and \(\tan \phi_{j,j+1} = g_j \Omega_{j+1,0} / g_{j+1} \Omega_{j,0}\). From the Eq. (14) and together with the results of \(\sigma_{bc}\), one can verify that the field \(\hat{G}_{j,j+1}\) satisfies the equation

\[
(\partial_t - c \cos^2 \beta \cos 2\phi_{j,j+1} \partial_z) \hat{G}_{j,j+1} = -\frac{(g_j^2 \Omega_{j+1}^2 + g_{j+1}^2 \Omega_j^2)}{\Gamma} \frac{\cos \beta \gamma^2}{\Omega_0^2} \hat{G}_{j,j+1} -
- \frac{1}{2} g_j g_{j+1} \sqrt{N} \sin 2\beta \partial_t \hat{\mathcal{E}}_{j,j+1} + c \cos^2 \beta \sin 2\phi_{j,j+1} \partial_z \hat{\mathcal{E}}_{j,j+1} + F(\hat{\mathcal{E}}_\sigma, \sigma \neq j, j + 1)
\]

with
\[
\tan^2 \beta = \frac{N \Omega_j^2 \Omega_{j+1}^2}{g_j^4 \Omega_{j+1}^2 + g_{j+1}^4 \Omega_j^2} \frac{(g_j^2 - g_{j+1}^2)^2}{\Omega_0^4}.
\]
and \(\hat{\mathcal{E}}_{j,j+1} = \cos \phi_{j,j+1} \hat{\mathcal{E}}_j(z, t) + \sin \phi_{j,j+1} \hat{\mathcal{E}}_{j+1}(z, t)\). \(F(\hat{\mathcal{E}}_\sigma)\) includes no \(\hat{\mathcal{E}}_j\) or \(\hat{\mathcal{E}}_{j+1}\). The first term in the right hand side of Eq. (15) reveals a very large ab-
sorption of $\hat{G}_{j,j+1}$, which results in a large decay in
the field $\hat{G}_{j,j+1}$ and then the system satisfies the pulse
matching condition $\{13, 15, 16\}: \hat{E}_{j+1} = \tan \phi_{j,j+1} \hat{E}_j$.
For an explicit numerical estimation, we set some typical
values $[2, 4]: g_j \approx g_{j+1} \sim 10^3 \text{s}^{-1}, N \approx 10^5,$
$\Gamma \approx 10^8 \text{s}^{-1}$, so that the life time of field $\hat{G}_{j,j+1}(z,t)$
is about $\Delta t < 10^{-8}$ which is much smaller than the
storage time $[4]$. Furthermore, by introducing the adiabatic parameter
$\tau \equiv (\sum_j (1/g_j)^2)^{1/2}/\sqrt{N}T$ where $T$ is the
characteristic time scale, we can calculate the lowest
order in Eq. $[17]$ and obtain $\Phi \approx 0, \hat{G}_{j,j+1} \approx 0$. On the
other hand, under the condition of pulse matching, one
can verify that $\hat{s}_j(z,t) \propto \hat{G}_{j,j+1} = 0$. Thus equation $[10]$
is reduced to the shape- and state-preserving case

$$\left(\partial_t + c \cos^2 \theta \cos \alpha_m - 2 \partial_z\right) \Psi(z,t) = 0. \quad (16)$$

The formula $[10]$ is the main result of the present work. The
group velocity of the DSP field is

$$V_g = \cos^2 \theta \sum_{j=1}^{m-2} \frac{\Omega_j^2}{g_j} \prod_{l=1,l \neq j}^{m-2} \frac{g_l^2}{g_j} \frac{1}{\sum_{j=1}^{m-2} \frac{\Omega_j^2}{g_j} \prod_{l=1,l \neq j}^{m-2} \frac{g_l^2}{g_j}} \quad (17)$$

One should bear in mind that the wave vectors $k_{pj}$ can be
positive (in the $+z$ direction) or negative (in the $-z$
direction). So, by adjusting the Rabi-frequencies for exter-
pal pump fields properly under the adiabatic condition
so that $\cos \alpha_{m-2} = 0$, we can obtain a zero velocity for the
DSP field. In particular, one may set No. 1 to No. $m-3$ pump/probe pulses in the $+z$ direction, while No.
m - 2 pump/probe pulse in the $-z$ direction (Fig. 1(b))

and $\Omega_{m-2,0} = \sum_{j=1}^{m-3} \frac{\Omega_j^2}{g_j}$, in an experiment so that
the group velocity $V_g = 0$. In this way, we create the
multi-frequency stationary pulses with each component:

$$\hat{E}_1 = \cos \theta \prod_{j=1}^{m-3} \cos \phi_j \Psi(z,t),$$

$$\hat{E}_l = \cos \theta \sin \phi_{l-1} \prod_{j=l}^{m-3} \cos \phi_j \Psi(z,t), \quad (18)$$

$$l = 2, ..., m-2.$$  

These components interfere to create a sharp spatial en-
velope. It is helpful to present a comparison of our results
with those obtained for a three-level system $[8, 9]$: i) In the present system, all optical pulses can couple in resonance to the corresponding atomic transitions, thus all the applied probe fields with different frequencies contribute to the generation of stationary pulses. This means the present process is a spatial compression of excitation, which allows us to shape and intensify the lo-
calized stationary pulses. Our technique can therefore be
expected to enhance further nonlinear couplings and be
applied in the most straightforward manner, e.g. to ap-
lications in quantum nonlinear optics $[17]$. Numerical

results in Fig. 2 show how tight localization of stationary
pulses can be readily obtained when the multi-level system is applied. In contrast, for a three-level system,

![FIG. 2: (color online) Localization of created stationary pulses for 5-level (red solid line) cases, where three input probe lights are used and the parameters are set as $\omega_{e_2e_1} = \omega_{e_3e_1} \approx \omega_2/100$ (a). As a comparison, blue dashed line shows the stationary pulses created in 3-level system (b) by employing one standing wave of pump fields. The probe lights are used with the envelop $e^{-z^2}$.

a frequency-comb is used to create a localized pulse, fil-
tering the off-resonant input probe pulses and retaining
only the resonant one for creation $[8]$. Generally, the total number of probe photons created by a frequency-
comb in a three-level system is much less than in the
current model; ii) The present technique can be freely
controlled. For example, based on our results, we see that
the pulse matching in the present case is between all of the probe pulses with different frequencies, say,

$$\hat{E}_1 = \prod_{j=1}^{m-1} \tan \phi_{j,j+1} \hat{E}_j \quad (l \geq 1, \sigma \leq m-2).$$

Thus, in principle, one can use just one pump field to achieve the stationary pulse by adjusting its intensity to match those of the other pump fields; iii) It requires no standing waves in the pump fields or spatially modulated pump fields to create localized stationary pulses.

Experimentally, the simplest multi-level system is an
ensemble of four-level double $\Lambda$-type $^{87}\text{Rb}$ atoms. The
schematic diagram for experimental realization is shown
in Fig. 3. All the atoms first are trapped in state $|b\rangle$
($5^2S_{1/2}$) and only the $\pm z$ directional propagation pump
fields ($\Omega_1$ and $\Omega_2$) are applied to couple the transi-
tions from $|c\rangle$ ($5^2S_{1/2}$) to $|e_1\rangle$ ($5^2P_{1/2}(F = 1)$) and $|e_2\rangle$
($5^2P_{3/2}(F = 1)$) respectively. We then input the probe
pulses ($\hat{E}_{1,2}$) and allow the system to achieve adiabatic
condition. Finally, by adjusting $\Omega_1$ or $\Omega_2$ so that $g_1\Omega_{20} = g_2\Omega_{10}$, we can create the stationary pulses for probe fields

$$\hat{E}_1(z,t) = \cos \theta \cos \phi \hat{\Psi}, \hat{E}_2(z,t) = \cos \theta \sin \phi \hat{\Psi},$$

where $\hat{\Psi}$ is determined by the Eq. [8] with $m = 4$. It can be ex-
expected that when the level number $m$ becomes larger, the more tightly localized stationary pulses can be created. According to the numerical results in Fig. 2, the effect becomes substantial when $m \geq 5$.

In conclusion, we have shown the tightly localized stationary pulses can be obtained in the general multi-level EIT system. We have examined the dynamics of DSPs in detail and found that, all the applied probe pulses with different frequencies contribute to the stationary pulses. The present process is therefore a spatial compression of excitation, which may be able to enhance further nonlinear optical couplings and will have interesting applications in quantum nonlinear optics [17]. In particular, our technique may open up a novel way towards the spatial compression of many probe photons with small losses. According to the results in [12], if initially input is a non-classical probe pulse, e.g. a quantum superposition of coherent states, we may also generate entangled stationary pulses within our model.

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