TESTING THE GRAVITATIONAL INSTABILITY HYPOTHESIS?

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ABSTRACT

We challenge a widely accepted assumption of observational cosmology: that successful reconstruction of observed galaxy density fields from measured galaxy velocity fields (or vice versa), using the methods of gravitational instability theory, implies that the observed large-scale structures and large-scale flows were produced by the action of gravity. This assumption is false, in that there exist non-gravitational theories that pass the reconstruction tests and gravitational theories with certain forms of biased galaxy formation that fail them. Gravitational instability theory predicts specific correlations between large-scale velocity and mass density fields, but the same correlations arise in any model where (a) structures in the galaxy distribution grow from homogeneous initial conditions in a way that satisfies the continuity equation, and (b) the present-day velocity field is irrotational and proportional to the time-averaged velocity field. We demonstrate these assertions using analytical arguments and \(N\)-body simulations.

If large-scale structure formed by gravitational instability, then the ratio of the galaxy density contrast to the divergence of the velocity field yields an estimate of the density parameter \(\Omega\) (or, more generally, an estimate of \(\beta = \Omega^{0.6}/b\), where \(b\) is an assumed constant of proportionality between galaxy and mass density fluctuations). In non-gravitational scenarios, the values of \(\Omega\) or \(\beta\) estimated in this way may fail to represent the true cosmological values. However, even if non-gravitational forces initiate and shape the growth of structure, gravitationally induced accelerations can dominate the velocity field at late times, long after the action of any non-gravitational impulses. The estimated \(\beta\) approaches the true value in such cases, and in our numerical simulations the estimated \(\beta\) values are reasonably accurate for both gravitational and non-gravitational models.

Reconstruction tests that show correlations between galaxy density and velocity fields can rule out some physically interesting models of large-scale structure. In particular, successful reconstructions constrain the nature of any bias between the galaxy and mass distributions, since processes that modulate the efficiency of galaxy formation on large scales in a way that violates the continuity equation also produce a mismatch between the observed galaxy density and the density inferred from the peculiar velocity field. We obtain successful reconstructions for a gravitational model with peaks biasing, but we also show examples of gravitational and non-gravitational models that fail reconstruction tests because of more complicated modulations of galaxy formation.

Subject Headings: cosmology: theory — large-scale structure of the Universe — galaxies: clustering
1. Introduction

The fundamental paradigm of modern cosmology postulates a dynamic universe in which the matter distribution, nearly homogeneous at early times, is cast into motion and consequently evolves into a network of swells and troughs corresponding to converging and diverging flows. The general belief is that the motions have been produced by the force of gravity. But do we have direct empirical evidence for this belief? The standard methods for analyzing large-scale velocity and density fields assume that structure formed by gravitational instability, and they have sometimes been thought to test this assumption as well. Here we show that these methods test a rather different combination of assumptions, the most fundamental of which is simply the continuity equation, and that they do not test gravity per se. We demonstrate this point first by considering the relation between velocity and density fields for a broad class of models in the linear approximation, and then by analyzing numerical simulations of a gravitational model and a specific non-gravitational model, the explosion scenario developed by Ostriker & Cowie (1981), Ikeuchi (1981), and others. Throughout this paper, we use explosions only as a convenient example of a non-gravitational model of structure formation; our arguments and results apply to a variety of non-gravitational scenarios.

In the gravitational instability picture of structure formation, gravitational forces amplify small fluctuations, present in the early universe, into the galaxies, clusters, voids, and superclusters observed today. One might think of testing this hypothesis by searching for the expected correlations between large-scale density fluctuations and peculiar velocity flows. “Tests” of this sort take one of two forms. In the density-to-velocity approach, one starts with the observed galaxy density field, assumes some relationship between the galaxy distribution and the underlying mass distribution, predicts the peculiar velocities, and compares them to observations. In the velocity-to-density approach, one starts with the peculiar velocities, predicts the mass density field, and compares it to the galaxy density field.

The first attempts at density-to-velocity analyses were spurred by the discovery of the dipole anisotropy in the cosmic microwave background (CMB) (Smoot, Gorenstein & Muller 1977) and its interpretation as the reflex of the peculiar motion of the Local Group (LG) (e.g. Smoot & Lubin 1979; Lubin & Villela 1986). There have been a number of attempts to account for the origin of this motion in terms of the gravitational acceleration produced by the inhomogeneous matter distribution around the LG. Some of these studies compare the CMB dipole to the dipole of the distribution of optical or IRAS galaxies on the sky (e.g. Meiksin & Davis 1986; Lahav 1987; Harmon, Lahav & Meurs 1987; Lahav, Rowan-Robinson & Lynden-Bell 1988; Lynden-Bell, Lahav & Burstein 1989). More recent studies have utilized the redshift surveys of IRAS galaxies (Rowan-Robinson et al. 1990; Strauss et al. 1992a) and of rich galaxy clusters (Scaramella et al. 1991). The most important recent advances in density-to-velocity analyses rely on systematic surveys of galaxy peculiar velocities, which use the Tully-Fisher or $D_n - \sigma$ relations to obtain redshift-independent distance estimates (e.g. Aaronson et al. 1986; Dressler et al. 1987; see review by Burstein 1990). With these data and a large, homogeneously selected redshift survey, one can attempt to predict peculiar velocities of a large number of galaxies instead of the LG alone (Strauss & Davis 1991; Kaiser et al. 1991; Davis et al., in preparation).

Peculiar velocity surveys also make it possible to reverse the order of analysis and predict the mass density field starting from the observed motions. There are two methods for doing so. In the first method, dubbed POTENT, a 3-dimensional velocity field is constructed from the radial velocity data, under the assumption that the velocity field is a potential flow (Bertschinger & Dekel 1989; Dekel, Bertschinger & Faber 1990, hereafter DBF). The resulting velocity field can be transformed into a density field. In the second method, developed by Kaiser & Stebbins (1991),
the 3-d velocity field (and therefore the density field) is represented by a set of Fourier modes, which are iteratively relaxed until the most probable configuration, given the radial velocity data, is achieved. Recently, Dekel et al. (1993) have compared the POTENT mass density field recovered from peculiar velocity data to the galaxy density field of the 1.936 Jy IRAS redshift survey (Strauss et al. 1992b). A follow-up comparison of more extensive velocity data to the 1.2 Jy redshift survey (Fisher 1992) is underway. Hudson and Dekel (1993) have performed a similar comparison between the POTENT mass density field and the density of optical galaxies.

Results of the multitude of dipole, density-to-velocity, and velocity-to-density studies have been varied, and we will not attempt any detailed comparisons to observations in this paper. It is probably fair to say that, in most of the studies, the observations are consistent with the predictions of gravitational instability theory to within the rather large random and systematic errors in the observational data and the theoretical approximations. Redshift and distance data are accumulating rapidly, so the observational situation should improve significantly over the next few years.

The theoretical underpinning of studies comparing velocity and density fields is usually couched in the language of the gravitational instability hypothesis. Within this framework, the relative amplitudes of density fluctuations and peculiar velocities can be used to estimate the value of the cosmological density parameter $\Omega$ (or, more generally, the value of $\beta \equiv \Omega^{0.6}/b$, where the “bias” factor $b$ is an assumed constant of proportionality between fluctuations in the galaxy density and the underlying mass fluctuations, see equation [1] below.) However, if the velocities are not gravitationally induced, then the derived value of $\beta$ loses its fundamental cosmological significance, since it need not reflect the true value of the density parameter. It is therefore important to ask whether observing the predicted correspondences between velocity and density fields verifies the gravitational instability hypothesis itself (as sometimes implied in the literature, e.g. Yahil 1988). Because the correspondences are usually derived from the equations of gravitational instability, the answer would appear to be “yes.” However, if the primordial mass distribution is homogeneous, then converging flows will produce overdense regions and diverging flows will produce underdense regions, whether or not the peculiar velocities arise from gravitational accelerations. As a result, agreement between predicted and observed density and velocity fields does not necessarily argue against theories in which large-scale structure and peculiar velocities are generated primarily by non-gravitational forces such as cosmological explosions (Ostriker & Cowie 1981; Ikeuchi 1981; Ostriker, Thompson & Witten 1986) or radiation pressure instabilities (Hogan & Kaiser 1983; Hogan & White 1986).

A brief consideration of the equations governing linear velocity and density fields illustrates this point more directly. In the linear theory of gravitational instability, peculiar velocity is proportional to the gradient of the gravitational potential, $\vec{v} \propto -\vec{\nabla} \Phi_g$, and the gravitational potential is related to the density fluctuations $\delta$ through the cosmological generalization of Poisson’s equation. Consequently, $\vec{\nabla} \cdot \vec{v} \propto -\nabla^2 \Phi_g \propto -\delta$. However, if the linearized continuity equation holds ($\vec{\nabla} \cdot \vec{v} \propto -\partial \delta/\partial t$), and the initial galaxy distribution is homogeneous, and the final velocities are proportional to their time-averaged values, then $\vec{\nabla} \cdot \vec{v} \propto -\delta$ regardless of the velocities’ physical origin. Therefore, one would expect that the predicted velocity or density fields will match observations under conditions more general than those of the standard gravitational instability picture.

Conversely, one can anticipate that even if the gravitational instability model is correct, it may fail the density-velocity tests if galaxy formation has been modulated by a non-gravitational process that violates the continuity equation. “Biased” galaxy formation is introduced specifically to allow for such effects in gravitational models (see review by Dekel & Rees 1987). If the efficiency of galaxy
formation is enhanced in some environments and suppressed in others, then the current distribution of galaxies cannot be obtained by taking an initially uniform distribution of galaxy markers and moving them under gravitational forces to their positions at $z = 0$. A linear, scale-independent bias model is often adopted for simplicity,

$$(\delta N/N)_{gal} = b(\delta M/M).$$

In this case the galaxy density contrast $(\delta N/N)$ is proportional to the mass density contrast $(\delta M/M)$, which does obey the continuity equation. However, linear bias is at best a theoretical convenience, and the situation in realistic gravitational instability models is more complex (see, e.g., the detailed numerical study of Cen & Ostriker 1992). We will return to this theme later in the paper.

In the next section, we present the analytic theory of velocity and density fields in greater detail, focusing on the linear regime. In the sections that follow, we apply the usual density-to-velocity and velocity-to-density tests to $N$-body simulations of the explosion scenario and of gravitational models. The simulations are described in §3. In §4 we apply the tests to these simulations, considering both unbiased models, which necessarily obey the continuity equation, and simply biased versions of the gravitational models, in which the galaxy density contrast is closely related to the mass density contrast. In §5 we discuss models that fail the density-velocity tests, with particular attention to models in which non-trivial biasing schemes make the evolution of the galaxy distribution violate the continuity equation. In §6, we apply the POTENT method to some of our simulations to demonstrate that it successfully recovers the 3-d velocity field and the density field from radial velocities alone, regardless of whether the proper velocities are induced by gravity, so long as the continuity equation is satisfied and the velocity field is irrotational and proportional to its time-average. We discuss the implications of our results in §7.

2. Theory

In the standard gravitational instability model, the equations governing the mass distribution are (Peebles 1980):

the continuity equation,

$$\partial_t \delta + \nabla_x \cdot [(1 + \delta) \vec{v}] = 0,$$

the Euler equation,

$$\partial_t \vec{v} + 2H(t) \vec{v} + \left( \vec{v} \cdot \nabla_x \right) \vec{v} = -\nabla_x \Phi_g,$$

and the Poisson equation,

$$\nabla^2_x \Phi_g = 4\pi G \bar{\rho}(t) \delta = \frac{3}{2} H^2(t) \Omega(t) \delta.$$
Together with the boundary conditions $\delta \to 0$ and $\vec{v} \to 0$ as $t \to 0$, equations (2)–(4) may be taken as a definition of the standard gravitational instability model. They can be solved by linear perturbation theory in the limit of small fluctuations. Of particular interest to us is the linear, growing-mode relation between the velocity and density fields:

$$\vec{x} \cdot \vec{v}(\vec{x}, t) = -H f(\Omega) \delta(\vec{x}, t), \quad (5a)$$

$$\vec{v}(\vec{x}, t) = \frac{H f(\Omega)}{4\pi} \int d\vec{y} \, \delta(\vec{y}, t) \frac{(\vec{y} - \vec{x})}{|\vec{y} - \vec{x}|^3}, \quad (5b)$$

with $f(\Omega) \approx \Omega^{0.6}$ (Peebles 1980). Equation (5b) follows from (5a) because the growing-mode velocity field is irrotational (vorticity-free), and it can therefore be derived from a velocity potential, $\vec{v} = -\vec{x} \Phi_v$.

Equations (5a) and (5b) are the basic equations for velocity-to-density and density-to-velocity reconstructions, respectively, in the linear regime. Such reconstructions directly test a combination of two hypotheses: (1) that $\vec{x} \cdot \vec{v} \propto -\delta$, and (2) that the velocity field is irrotational. In a density-to-velocity reconstruction, the irrotational assumption is used to compute the full velocity field from its divergence. A velocity-to-density reconstruction that utilized the 3-dimensional velocity field would not require this assumption, but observations provide only the radial component of the velocity field, and one must assume a potential flow in order to build the full, 3-dimensional field.

If structure formed by gravitational instability, then the constant of proportionality between $\vec{x} \cdot \vec{v}$ and $-\delta$ provides a measure of $f(\Omega)$; by working in velocity units, one can avoid any dependence on the Hubble constant $H$.

It has often been argued that successful velocity-to-density and density-to-velocity reconstructions provide confirmation of the gravitational instability scenario (e.g. Yahil 1988; similar arguments appear in much of the literature on cosmological dipoles and velocity fields). However, the two conditions tested by these methods are by no means unique to gravitational instability models. Whatever the source of peculiar velocities, the linearized form of the continuity equation (2) implies that

$$\partial_t \delta = -\vec{x} \cdot \vec{v}. \quad (6)$$

If the galaxy distribution is smooth at high redshift, one can integrate both sides of this equation from $t = 0$ (and $\delta = 0$) to $t = t_0$ to obtain

$$\delta = -t_0 \vec{x} \cdot \langle \vec{v} \rangle, \quad (7)$$

where $\langle \vec{v} \rangle_t \equiv \int_0^{t_0} \vec{v} dt/t_0$ is the time-averaged velocity field. Therefore, any model that has homogeneous initial conditions and obeys the continuity equation will yield successful reconstructions, provided that its present-day velocity field is irrotational and is proportional to the time-averaged velocity field.

In fact we require only that the present-day velocity field be proportional to the irrotational part of the time-averaged field, because a divergence-free component that might have existed in the past but decayed since would not leave behind any associated density perturbations. With this qualification, the proportionality condition is necessary as well as sufficient; an initially homogeneous model that obeys the continuity equation and satisfies $\vec{x} \cdot \vec{v} \propto -\delta$ today must, by equation (7), have a present-day velocity divergence that is proportional to its time-averaged value. Irrotationality does not follow from the continuity equation alone, however; it must be adopted as a separate condition. Expansion of the universe always erodes vorticity, so an irrotational velocity field is likely to arise in many scenarios, both gravitational and non-gravitational, at least on scales
in the linear regime. Irrotationality is not in itself a sufficient condition for successful reconstructions, since it does not guarantee proportionality between the present and time-averaged velocity fields.

In the sections that follow, we will present several examples of non-gravitational models that pass the usual density-velocity tests. We start by analyzing the behavior of some idealized models in the linear regime, then proceed in §4 to numerical simulations of more realistic, non-linear models.

2.1 A Simple Example: A One-component Universe with No Gravity ($\Omega = 0$)

As our first example, we consider the simple but extreme case of an $\Omega = 0$ universe filled with massless test particles. In such a universe, there are no gravitational forces. We assume that the test particles are uniformly distributed at very early times, and that they acquire peculiar velocities from non-gravitational forces that produce an impulsive perturbation, $\vec{V}(\vec{x})\delta_{\text{DIRAC}}(t - t_e)$, acting at time $t = t_e$. Apart from this initial impulse, we ignore any pressure forces.

On scales where density inhomogeneities and the associated velocities are small, the governing equations are the linearized continuity equation (6) and the linearized Euler equation,

$$\partial_t \vec{v} + 2H(t)\vec{v} = \vec{V}(\vec{x})\delta_{\text{DIRAC}}(t - t_e).$$

(8)

By combining these, we find that for $t > t_e$ the density perturbations evolve according to

$$\partial_t^2 \delta + 2H(t)\partial_t \delta = 0,$$

(9)

where the effects of the initial perturbations are absorbed into the initial conditions at time $t = t_e$: $\delta(\vec{x}, t_e) = 0$, and $\partial_t \delta(\vec{x}, t_e) = \zeta(\vec{x}) \equiv -\vec{\nabla}_x \cdot \vec{V}(\vec{x})$. Noting that $H(t) = 1/t$ and $a = t/t_e$ (the expansion scale factor is normalized so that $a_e = 1$), we find that

$$\delta(\vec{x}, t) = \zeta(\vec{x})t_e(a - 1)/a,$$

$$t\partial_t \delta(\vec{x}, t) = \zeta(\vec{x})t_e/a.$$  

(10)

At any finite time, $\partial_t \delta$ and $\delta$ are clearly proportional to each other, i.e. the ratio $\delta/\partial_t \delta$ is independent of spatial position.

We can combine equations (6) and (10) to obtain the continuity equation in terms of the density contrast:

$$\vec{\nabla}_x \cdot \vec{v} = \frac{-\delta}{(a - 1)t}.$$  

(11)

The essential point of this paper is made by comparing equation (11), which is derived assuming that gravity is non-existent, to equation (5a), which is derived from the usual linear theory for the growth of perturbations under the influence of gravity. Both equations indicate the same relationship between $\vec{\nabla}_x \cdot \vec{v}$ and $\delta$, differing only by the constant of proportionality. Furthermore, for $a \sim$ a few (in the non-gravitational case) and $\Omega \sim 1$ (in the gravitational case), the predicted velocities are of comparable magnitudes. If the velocity impulse in the non-gravitational model is irrotational, then the velocity field remains irrotational at later times, and equation (5b) also holds, apart from the constant of proportionality.

It is equation (5a) or (5b) that is traditionally used to estimate $\Omega$, given the large-scale velocity and density fields. In the present case, examination of equation (11) shows that an observer (or
theorist) erroneously applying (5a) or (5b) in an empty universe would infer an effective value of \( \Omega \) determined by

\[
 f(\Omega_{\text{eff}}) = \left( \frac{1}{a-1} \right). \tag{12}
\]

Even though the true value of the cosmological density parameter is \( \Omega = 0 \) (i.e. there is no gravitating mass present), one always infers a non-vanishing value for \( \Omega_{\text{eff}} \). The solid line in Figure 1 shows the evolution of \( \Omega_{\text{eff}} \); it is initially infinite, but it declines and asymptotically approaches zero. In this model the ratio of the inferred \( \Omega \) to the true \( \Omega \) is always infinite.

To summarize, the example of an \( \Omega = 0 \) universe demonstrates that a proportionality between the density field and the divergence of the velocity field is not solely a property of the gravitational instability hypothesis. Analyzing the density and the velocity fields with equations derived within the framework of this hypothesis will yield an estimate of \( \Omega \), but unless the motions are gravitational in origin the inferred value of \( \Omega \) is not a measure of the cosmological density parameter.

### 2.2 A Two-component Universe

Now let us consider a two-component universe consisting of dark matter and baryons. We want to study the evolution of the two fluids under the combined influence of gravitational and non-gravitational forces. We characterize the mean densities of the dark matter and the baryons by the parameters \( \Omega_D \) and \( \Omega_B \), respectively. For algebraic simplicity, we shall assume that the total mass density is such that \( \Omega_D + \Omega_B = 1 \). We again assume that non-gravitational forces produce an impulsive perturbation in the velocity field of the baryonic component at time \( t = t_e \), and that they are negligible at later times. In the linear regime, the equations governing the evolution of the baryons and the dark matter are:

the continuity equations,

\[
 \partial_t \delta_i + \vec{\nabla}_x \cdot \vec{v}_i = 0, \quad i = D, B, \tag{13}
\]

the Euler equations,

\[
 \begin{align*}
 \partial_t \vec{v}_D + 2H(t)\vec{v}_D &= -\vec{\nabla}_x \Phi_g(\vec{x}, t), \\
 \partial_t \vec{v}_B + 2H(t)\vec{v}_B &= -\vec{\nabla}_x \Phi_g(\vec{x}, t) + \vec{V}_B(\vec{x})\delta_{\text{DIRAC}}(t - t_e),
\end{align*} \tag{14}
\]

and the Poisson equation,

\[
 \nabla^2 x \Phi_g = 4\pi G \sum_{i=D,B} \bar{\rho}_i(t)\delta_i(\vec{x}, t) = \frac{3}{2}H^2(t) \sum_{i=D,B} \Omega_i \delta_i(\vec{x}, t). \tag{15}
\]
The comoving gravitational potential $\Phi_g$ mediates the gravitational interaction between the two fluids. In these equations, the subscripts “D” and “B” denote dark matter and baryons, respectively.

By combining the above equations and absorbing the effects of the impulse perturbation into the initial conditions, we find that the density perturbations, at time $t > t_e$, evolve according to

$$\partial_t^2 \delta_i + 2H(t)\partial_t \delta_i = \frac{3}{2}H^2(t) \sum_{\nu=D,B} \Omega_i \delta_i. \quad (16)$$

The general solution to this pair of differential equations can be expressed as (Hogan & Kaiser 1989)

$$\begin{bmatrix} \delta_B \\ t\partial_t \delta_B \\ \delta_D \\ t\partial_t \delta_D \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2/3 & -1 & -1/3 & 0 \\ 1 & 1 & -\Omega_B & -\Omega_B \\ 2/3 & -1 & -\Omega_B/3 & 0 \end{bmatrix} \begin{bmatrix} \Delta_1(\vec{x})a(t) \\ \Delta_2(\vec{x})a(t)^{-3/2} \\ \Delta_3(\vec{x})a(t)^{-1/2} \\ \Delta_4(\vec{\zeta}) \end{bmatrix}. \quad (17)$$

The spatial functions $\Delta_i(\vec{x})$ can be determined by imposing the appropriate initial conditions and inverting the equation. Assuming $\Omega_B \ll 1$, it is simpler to expand the functions $\Delta_i$ as a series: $\Delta_i = \Delta_{i0} + \Omega_B \Delta_{i1} + \Omega_B^2 \Delta_{i2} + \cdots$ and iteratively solve for $\Delta_i$ to the desired order.

Our scenario is constructed along the lines of theories that invoke non-gravitational forces (such as cosmological explosions or radiation pressure instabilities) to initiate the formation of large-scale structure. We assume that the baryon and dark matter distributions are smooth at early times, and that the non-gravitational forces disturb the baryonic component impulsively at $t = t_e$. The appropriate initial conditions for this scenario are: $\delta_{D,B} = 0$, $\partial_t \delta_B = 0$, and $\partial_t \delta_B = \zeta(\vec{x})$, where $\zeta(\vec{x}) \equiv -\vec{\nabla}_x \cdot \vec{V}_B(\vec{x})$. At any subsequent time, the baryonic density perturbations are given, to first order in $\Omega_B$, by:

$$\begin{align*}
\delta_B(\vec{x},t) &= 3\zeta(\vec{x})t_e \left[ 1 - a^{-1/2} + \frac{1}{5}\Omega_B \left( a - a^{-3/2} + 5a^{-1/2} - 5 \right) \right], \\
t\partial_t \delta_B(\vec{x},t) &= \zeta(\vec{x})t_e \left[ a^{-1/2} + \frac{1}{5}\Omega_B \left( 2a + 3a^{-3/2} - 5a^{-1/2} \right) \right]. \quad (18)
\end{align*}$$

In the limit of $\Omega_B \rightarrow 0$, the initial velocities simply decay as $a^{-2}$ (in comoving units), and the density contrast $\delta_B$ asymptotes to $3\zeta(\vec{x})t_e$. When $\Omega_B$ is not zero, the dark matter eventually begins to fall into the baryon perturbations, and the perturbed baryons and dark matter produce gravitational accelerations parallel to the initial velocities. These accelerations sustain the velocities and the growth of the density fluctuations.

For consideration of density-velocity tests, the crucial feature of equation (18) is that $\partial_t \delta_B(\vec{x})$ is proportional to $\delta_B(\vec{x})$ at all times. We can therefore substitute $\delta_B$ for $\partial_t \delta_B$ in the continuity equation (13) and find that $\vec{\nabla}_x \cdot \vec{v}_B \propto -\delta_B$, just as in the gravitational instability scenario. At all times $\vec{\nabla}_x \cdot \vec{v}_B \propto \vec{\nabla}_x \cdot \vec{\nabla}_B$, and if $\vec{v}_B$ is irrotational then we can recover the full velocity field from its divergence. The results for this scenario are thus identical to those for gravitational instability (equations 5a and 5b), except that the quantity $f(\Omega)$ is replaced by

$$f(\Omega_{eff}) = \frac{1}{2} \left[ \frac{a^{-1/2} + \frac{1}{5}\Omega_B \left( 2a + 3a^{-3/2} - 5a^{-1/2} \right)}{1 - a^{-1/2} + \frac{1}{5}\Omega_B \left( a - a^{-3/2} + 5a^{-1/2} - 5 \right)} \right]. \quad (19)$$

The dotted curve in Figure 1 shows the evolution of $\Omega_{eff}(a)$ for a case where $\Omega_B = 0.1$. The value of $\Omega_{eff}$ is initially very large, but it drops to its minimum of 0.24 by $a \approx 10$, then asymptotically rises.
to unity. The initial drop in $\Omega_{\text{eff}}$ reflects the decaying influence of the impulse perturbations, and the return to $\Omega_{\text{eff}} \approx 1$ occurs as the dark matter catches up to the perturbed baryons. We draw attention to the fact that, with the exception of a relatively brief period following the impulse, the value of $\Omega_{\text{eff}}$ ranges between 0.24 and 1. Hence, measurements yielding $\Omega_{\text{eff}} < 1$ do not necessarily imply an open universe. In the same vein, measurements of $\Omega_{\text{eff}}$ that yield values of $O(1)$ do not preclude the possibility of a non-gravitational influence having been the original cause of structures in the universe.

Linear theory solutions are more complicated in the case of a universe with $0 < \Omega < 1$, but on physical grounds we expect the qualitative results to be similar. In particular, the same correlations between density and velocity fields should appear, just with different constants of proportionality. In the non-gravitational scenario, $\Omega_{\text{eff}}$ will asymptotically approach the true value of $\Omega$, though in a two-component model with small $\Omega_B$ it is possible that the dark matter will not catch up by the time the universe enters free expansion, and that $\Omega_{\text{eff}}$ will get stuck between $\Omega_B$ and $\Omega$.

### 2.3 Non-linear Effects

The relations in §2.1 and §2.2 are derived from linearized equations, so they are valid only in the linear regime. We can extend the relation between the velocity and the density fields into the non-linear regime by defining a comoving displacement field:

$$\vec{\xi}(\vec{x}, t) \equiv \vec{x}(\vec{q}, t) - \vec{q} = t \langle \vec{v}(\vec{x}, t) \rangle_t.$$  \hspace{1cm} (20)

Here $\vec{x}(\vec{q}, t)$ is the comoving position at time $t$ of a mass element originally at comoving position $\vec{q}$, and $\langle \vec{v}(\vec{x}, t) \rangle_t$ is the time-averaged, comoving peculiar velocity of this mass element. Mass conservation requires that $\rho_x(\vec{x}) d\vec{x} = \rho_q d\vec{q}$, so

$$\rho(\vec{x}, t) / \bar{\rho}(t) = \left\| I - \frac{\partial \vec{\xi}}{\partial \vec{x}} \right\|,$$  \hspace{1cm} (21)

where $I$ is the identity matrix and vertical bars denote the Jacobian determinant (Nusser et al. 1991). Equation (21) remains valid until orbit crossing occurs. After this time $\vec{\xi}(\vec{x}, t)$ is no longer single-valued, but we might still expect equation (21) to apply on large scales, after smoothing over regions of orbit crossing. In a two-component universe, equation (21) holds separately for the density and displacement fields of the two components.

If the present-day peculiar velocities are proportional to the time-averaged peculiar velocities, that is $\vec{v}(\vec{x}, t_0) = \alpha \langle \vec{v}(\vec{x}, t_0) \rangle_t$, then equation (21) determines the relation between the present-day velocity and density fields:

$$\rho(\vec{x}, t_0) / \bar{\rho}(t_0) = \left\| I - \frac{t_0}{\alpha} \frac{\partial \vec{v}}{\partial \vec{x}} \right\|.$$  \hspace{1cm} (22)

On scales where orbit crossing is unimportant, this equation applies equally well to gravitational and non-gravitational models, provided only that the initial matter distribution is uniform and that present and time-averaged velocities are indeed proportional. Confirming relation (22) observationally can therefore provide evidence only for this combination of assumptions, not for the more specific assumption that the displacements and velocities were generated by gravitational instability. Furthermore, comparison of the observed velocity and density fields only yields the constant of proportionality relating the present and time-averaged velocities, which in general need not be related to $\Omega$. 

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For the case of gravitational instability, Nusser et al. (1991) propose using the Zel’dovich (1970) quasi-linear approximation for the displacements and the velocities:

\[ \vec{\xi}(\vec{x}, t) = D(t) \vec{\psi}(\vec{q}) \implies \vec{v}(\vec{x}, t) = \dot{D}(t) \vec{\psi}(\vec{q}), \]

where \( D(t) \) is the growth rate of linear perturbations and \( \dot{D}/D = Hf(\Omega) \). Inserting this approximation into the mass conservation equation (21) yields

\[ \frac{\rho(\vec{x}, t)}{\bar{\rho}(t_0)} = \left| I - \frac{D}{\dot{D}} \frac{\partial \vec{v}}{\partial \vec{x}} \right|. \]  

In the limit of small \( D \), this yields the linear relation \( \delta = -(Hf)^{-1} \vec{\nabla}_x \cdot \vec{v} \). The approximation (24) does not satisfy the Euler and Poisson equations of gravitational instability theory. Instead, it is a solution of the continuity equation under the Zel’dovich ansatz that the velocity is proportional to the displacement. Still, Nusser et al. (1991) find, based on N-body simulations, that it provides an excellent approximation to the true velocity-density relation in a mildly non-linear gravitating system, i.e. in the range \(-0.7 \lesssim \delta \lesssim 4.5\).

One can also devise quasi-linear approximations for computing the smoothed peculiar velocity field given observed density contrasts (e.g., Nusser et al. 1991). However, non-linear corrections have a smaller impact when computing velocities from densities, and we will not consider them in this paper.

The basic message of this section is that even mildly non-linear effects do not distinguish gravitational models from non-gravitational models, because the most successful approximation in this regime is derived from the continuity equation and the proportionality of velocity and displacement, independently of gravity. Dynamical non-linearities in gravitational systems eventually break the proportionality between velocities and displacements. Such deviations from the Zel’dovich approximation could, in principle, be used to differentiate gravitational and non-gravitational theories. However, the observational data on large-scale velocity flows are unlikely to permit detection of such subtle effects in the foreseeable future, and they would in any case be difficult to distinguish from non-linearities in the relation between galaxy and mass density. In the highly non-linear regime, the existence of “fingers-of-god” indicates that gravity plays a major role in the dynamics of galaxy clusters, but it does not tell us that clusters formed by gravitational processes alone.

### 2.4 Biased Galaxy Formation

Observations do not probe the baryon distribution directly; rather, they map out the galaxy distribution. It is possible that the galaxy distribution evenly traces the underlying distribution of baryonic matter, but it is also possible that galaxies form more efficiently in some regions and less efficiently in others. The observed morphology-density relation demonstrates that such efficiency variations occur at least for individual galaxy types. The idea of “biased” galaxy formation — preferential formation of galaxies in high density regions — became popular when it was realized that such a scheme could reconcile the assumption of an \( \Omega = 1 \) universe with observations suggesting lower \( \Omega \) (see review by Dekel & Rees 1987). Numerical simulations of the cold dark matter model indicate that some degree of biasing is a natural prediction of this theory (White et al. 1987; Gelb 1992; Cen & Ostriker 1992; Katz, Hernquist & Weinberg 1992) and, indeed, that physical processes are likely to produce bias in a wide range of theoretical scenarios.

The simplest mathematical prescription that describes biasing is the “linear bias” model, in which one assumes that the galaxy and baryon density contrasts are proportional to each other and
that the galaxy and baryon velocity fields are the same, i.e. $\delta_G = b_B \delta_B$, where $b_B$ is the baryonic “bias parameter”, and $\vec{v}_G = \vec{v}_B$. It is clear that our earlier results for the relations between linear density and velocity fields continue to hold with this biasing model, except that $f(\Omega_{\text{eff}})$ is replaced by the parameter $\beta \equiv f(\Omega_{\text{eff}})/b_B$. For the gravitational instability scenario, equations (5) become

$$
\vec{\nabla}_x \cdot \vec{v}_G = -H \beta \delta_G(\vec{x},t),
$$

(25a)

$$
\vec{v}_G(\vec{x},t) = \frac{(H \beta)}{4\pi} \int d\vec{y} \, \delta_G(\vec{y},t) \frac{(\vec{y} - \vec{x})}{|\vec{y} - \vec{x}|^3}.
$$

(25b)

Similar relations will hold for the sort of non-gravitational model discussed in §2.2, except that the constant of proportionality will no longer reflect the true cosmological value of $\beta$.

It makes no sense to apply the linear bias model once baryon fluctuations grow to amplitudes $\delta_B \sim 1/b_B$, since it can yield the absurd prediction that $\delta_G < -1$. We must therefore adopt a different strategy to incorporate biasing into the quasi-linear relation discussed in §2.3. A simple approach is to put the bias factor inside the determinant of equation (24), yielding

$$
\delta_G(\vec{x}) = \left| I - (H \beta)^{-1} \frac{\partial \vec{v}_G}{\partial \vec{x}} \right| - 1.
$$

(26)

In the linear limit, this yields equation (25a). Some of our numerical simulations of gravitational models (see §3.1) employ a biasing scheme in which galaxies are identified with high peaks of the initial density field, and for these simulations we find that putting the bias factor inside the determinant yields better correlations than a simple linear bias model. There is no guarantee, however, that this prescription will work for other biasing schemes.

The cosmic mass distribution must obey the continuity equation, but the galaxy distribution need not. In a biased model, structure in the galaxy distribution is not created by displacing an initially uniform population of objects; it also reflects variations in the efficiency of galaxy formation from one place to another. The continuity argument presented at the beginning of this section does not apply directly to the galaxy distribution in such a model. Nonetheless, to the extent that the relation between galaxies and mass is described by linear bias, the standard relation between velocity and density will continue to hold, with the $1/b_B$ change in the constant of proportionality. Mild non-linearities in the bias relation may be difficult to distinguish from kinematic or dynamical non-linearity in the velocity-density relation. However, if the connection between galaxies and mass is non-local, then the resulting model can fail density-velocity tests in a drastic way, even if the mass distribution and the velocity field grow entirely by gravitational instability. We return to this issue in §5.

3. Simulations

The arguments in the previous section are somewhat idealized, as they are based largely on linear theory and the linear biasing model. In the following three sections, we illustrate these arguments with concrete examples derived from $N$-body simulations of gravitational and non-gravitational models. Although the density-to-velocity and velocity-to-density techniques discussed in §1 have been tested on $N$-body simulations of gravitational models in other papers (Nusser et al. 1991; Davis et al. 1991; Dekel et al. 1993), we repeat the exercise here because we want to compare our results for explosion simulations to results for gravitational simulations analyzed with identical techniques on the same scales. We intend to demonstrate that the correlations between velocity and density fields discussed in §2 can be found in realistically non-linear models of the
### Table 1a: Parameters of Gravitational Simulations

| Model     | $L$  | $a_f/a_i$ | $N_t$ | $N_p$  | $N_m$  |
|-----------|------|-----------|-------|--------|--------|
| CDM       | 96   | 24        | 48    | $64^3$ | $128^3$|
| Biased    | 96   | 12        | 24    | $64^3$ | $128^3$|

Notes to Table 1a:
- Column 3: size of simulation cube, in $h^{-1}$ Mpc
- Column 4: ratio of final to initial expansion factor
- Column 5: number of timesteps (equal intervals in $\Delta a$)
- Column 6: number of particles
- Column 7: number of cells in density/potential mesh

#### FIGURE 2

Figure 2 — Slices from our $N$-body simulations of the cold dark matter (CDM) model (see Table 1a for defining parameters). Each column shows three contiguous slices from a single simulation. Each slice is 1/6 the thickness of the simulation cube, so that one half of the simulation volume is shown in total. The scale is marked in $h^{-1}$ Mpc. The left-hand column shows slices from an unbiased simulation. The middle column shows slices through the galaxy distribution of a biased simulation. The right-hand column shows slices through the mass distribution of the same biased simulation. This mass distribution is sampled to the same mean density as the galaxy distribution, so that visual differences are not dominated by differences in particle density.

universe, regardless of the mechanism that caused the velocity flow. We also want to investigate the impact of biased galaxy formation, which has not been well studied in previous work.

### 3.1 Gravitational Models

For our gravitational simulations, we assume an $\Omega = 1$ universe and adopt the cold dark matter (CDM) power spectrum (Bardeen et al. 1986) for the initial fluctuations in the mass distribution. The adopted CDM power spectrum is an adequate representation of reality for the purposes of this paper, even though, in detail, it may fail rigorous tests of its applicability to the observed universe (c.f. Ostriker 1993 and references therein). We use the Zel’dovich approximation (Zel’Dovich 1970) to transform the initial conditions into positions and growing mode velocities of the particles, and evolve the resulting distribution using a particle-mesh (PM) $N$-body code written by C. Park (Park 1990). This code uses a staggered-mesh technique (Melott 1986) to achieve higher force resolution (by about a factor of two) than a conventional PM code. Tests against analytical solutions and other $N$-body codes show that the PM code produces reliable results down to the limits of its force resolution, $\sim 1 - 2$ mesh cells (Park 1990; Weinberg et al., in preparation). The simulations of gravitational models employ $64^3$ particles on a $128^3$ mesh representing a periodic cube of length $96h^{-1}$ Mpc, for a Hubble constant of $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$. Table 1 lists more details of the simulations.

Since the $N$-body simulations only follow the evolution of the mass, the identification of galaxies in the simulations requires further assumptions. The usual schemes for identifying galaxies either
assume that galaxies trace the matter (the unbiased scenario, corresponding to bias parameter $b = 1$) or implement an ad hoc prescription to model biased galaxy formation, such as flagging high peaks in the initial density field as sites for galaxy formation (Bardeen et al. 1986). We explore both schemes. For our unbiased models we adopt the standard normalization condition that the rms fluctuation of the initial density field linearly extrapolated to $z = 0$ be unity in spheres of radius $8h^{-1}$ Mpc. We create a galaxy catalog by randomly sampling the particle distribution to a density of $0.01h^3$ Mpc$^{-3}$, the observed number density of bright ($L^*$-ish) galaxies. We normalize our biased models so that the rms linear mass fluctuation is 0.5 at $8h^{-1}$ Mpc, and we use the peak-background split scheme (Bardeen et al. 1986; Park 1991) to identify biased particles corresponding to $\nu > 1.6\sigma$ peaks of the initial conditions on a scale of $0.6h^{-1}$ Mpc. With this choice of peak parameters, the mean galaxy density is again $0.01h^3$ Mpc$^{-3}$, and the bias factor is $b = 2$. The rms galaxy fluctuation in $8h^{-1}$ Mpc spheres is, therefore, unity, as in the unbiased model. Strictly speaking, the galaxy distribution in the peak-biased scenario does not satisfy the continuity equation because the galaxies are “born” clustered. However, the fluctuations in the final galaxy density field are closely coupled to the fluctuations in the mass density, which does obey the continuity equation.

Figure 2 shows slices through the galaxy distributions of the gravitational models. The three panels in each column display three contiguous slices from a single simulation. Each slice is $1/6$ the thickness of the simulation cube, so that one half of the simulation is shown in total. The left-hand column shows slices from one of the unbiased simulations. The middle column shows slices from the biased simulation with the same initial fluctuations. The galaxy distributions of the two models look similar, although structure in the unbiased model is clumpier, while the biased model has slightly larger voids and a somewhat more filamentary appearance. Of course, peculiar velocities are lower in the biased model (by a factor of two in the linear regime) because the amplitude of mass fluctuations is lower. The right-hand column of Figure 2 shows the mass distribution of the biased model. We have sampled the mass distribution to $0.01h^3$ Mpc$^{-3}$ so that the visual differences between the mass and galaxy distributions are not dominated by differences in the particle densities.

For the purposes of this paper, it is not terribly important whether the peaks biasing scheme gives an accurate description of galaxy formation in the CDM model, or whether the CDM models or the explosion models reproduce all features of the observed galaxy distribution. We are interested in broader questions about scales that are mildly non-linear, and it is sufficient that our simulations yield qualitatively realistic structure while illustrating the basic effects of gravitational instability, biased galaxy formation, and non-gravitational perturbations.

### 3.2 Explosion Models

For our non-gravitational models, we use numerical simulations of the explosion scenario. These simulations are akin to those described by Saarinen, Dekel & Carr (1987), but we use different numerical techniques, many more particles, larger physical volumes, and a wider range of initial conditions. A detailed discussion of the simulations and statistical analysis of the structure that they produce will appear elsewhere (Weinberg, Dekel & Ostriker, in preparation); here we provide a summary of the models and the numerical methods. The explosion scenario serves here as a representative example of a non-gravitational theory of structure formation — a convenient example because we happen to have simulations of it. However, our goal is to learn about the relation between velocity and density fields, not to defend or refute the explosion model, so we will not attempt any comprehensive evaluation of the model’s observational successes or failures.

The detailed physics of the explosions is not particularly important for our purposes. We simply assume that some explosive process sweeps baryonic material onto expanding, spherical shells. We assume that, in the absence of interactions, the shells would have a power-law distribution of radii up
to some maximum, \( n(R) \propto R^{-4} \) for \( R \leq R_{\text{max}} \). Ostriker & Strassler (1989) and Weinberg, Ostriker & Dekel (1989) have argued that a model with an \( R^{-4} \) distribution can produce a reasonable match to the distribution of void sizes in the CfA slice and to the distribution of galaxy cluster masses, respectively. The physical scale corresponding to \( R_{\text{max}} \) is treated as a free parameter, to be determined \textit{a posteriori} by fitting the observed level of galaxy clustering. We have also examined an explosion model in which all shells have equal radii; results are similar to those reported here for the power-law model.

We consider three different cosmological scenarios. In the first, we set \( \Omega_S = \Omega = 1 \), where \( \Omega \) is the density parameter and \( \Omega_S \) represents the cosmological density of the material that can be swept up onto shells. One would normally expect blast waves to collect only baryonic matter, and setting \( \Omega_B = \Omega_S = 1 \) would violate nucleosynthesis constraints. However, \( \Omega_S = 1 \) is possible if explosions occur early, so that dark matter has time to catch up, or if the “explosions” have a gravitational origin, with expanding shells developing around deep negative density perturbations (c.f. Bertschinger 1985; Weinberg & Cole 1992). Our second scenario assumes an open, baryon-dominated universe, with \( \Omega_S = \Omega = 0.17 \) at redshift \( z = 0 \). Finally, we consider a scenario with \( \Omega = 1 \) and \( \Omega_S = 0.1 \), \textit{i.e.} 90% of the mass is assumed to be in a collisionless dark matter component that is not directly disturbed by the explosions, though it can respond gravitationally to the perturbed baryon distribution. For brevity, we will refer to these three scenarios as “flat” (\( \Omega_S = \Omega = 1 \)), “open” (\( \Omega_S = \Omega = 0.17 \)), and “dark” (\( \Omega_S = 0.1, \Omega = 1 \)), respectively.

We evolve the explosion models in two phases, a “hydrodynamic” phase and a “gravitational” phase. Our idealization is that gravitational effects can be ignored during the early evolution of the blast waves, the hydrodynamic phase, but that after a certain point (physically speaking, the point at which the shells begin to fragment), the evolution enters a gravitational phase during which matter is effectively collisionless and hydrodynamics can be ignored. Because the large-scale growth and interactions of cosmological shells are similar regardless of whether they are collisional or collisionless, our results should not be sensitive to this idealization. Of course the fragmentation process must involve gravity, but it does so on scales much smaller than those that we are considering, where the density field is strongly non-linear.

The hydrodynamic phase would be simple to compute (for our rather simplified purposes) if shells did not overlap: we would simply choose random locations for the shell centers (the “explosions”), project particles lying within the shell radius out to the surface, and assign them radial peculiar velocities. However, overlap regions are unavoidable, and they are tricky to manage “by hand.” Therefore, we turn to a technique based on Burgers’ equation (Burgers 1974), which describes the flow of a fluid with bulk viscosity. Burgers’ equation was introduced into cosmology by Gurbatov, Saichev & Shandarin (1985, 1989), who used it as a clever extension of the Zel’dovich approximation for gravitational instability. Weinberg & Gunn (1990) describe an efficient technique for integrating the equation in three dimensions. We employ this technique here, but instead of regarding Burgers’ equation as an approximation for gravitational instability, we return to its original, hydrodynamic interpretation and treat it as a solution for the purely hydrodynamical interactions of a system of blast waves. Given an initial velocity field that is the gradient of a potential, we can compute the velocity field at any later time and integrate particle orbits along this evolving velocity field. A spike in the initial velocity potential drives a spherical outflow, which, because of the viscosity term in Burgers’ equation, sweeps all particles in its path onto a thin shell, whose radius grows with time. Where two shells emanating from neighboring spikes collide, viscosity prevents the shells from interpenetrating, and it redirects particle velocities in a momentum-conserving way along the wall that separates the two bubbles. Burgers’ equation thus automatically handles blast wave interactions in a physically plausible manner.
Table 1b: Parameters of Explosion Simulations

| Model  | $\Omega$ | $\Omega_S$ | $L$  | $ff(a_h)$ | $a_f/a_h$ | $N_t$ | $N_p$  | $N_m$  |
|--------|---------|-----------|-----|---------|---------|------|-------|-------|
| Flat   | 1.0     | 1.0       | 108 | 3.0     | 2       | 16   | $64^3$| $128^3$|
| Open   | 0.17    | 0.17      | 98  | 3.0     | 4       | 32   | $64^3$| $128^3$|
| Dark   | 1.0     | 0.1       | 98  | 3.0     | 6       | 48   | $2 \times 64^3$| $128^3$|

Notes to Table 1b:
Column 2: density parameter
Column 3: density parameter of “shell” component
Column 4: size of simulation cube, in $h^{-1}$ Mpc
Column 5: formal filling factor at end of hydrodynamic phase
Column 6: ratio of final expansion factor to expansion factor at end of hydrodynamic phase
Column 7: number of timesteps (equal intervals in $\Delta a$)
Column 8: number of particles; in dark simulations there are $64^3$ particles of each species
Column 9: number of cells in density/potential mesh

We place the potential spikes (“explosion” sites) at random locations in a periodic, cubical simulation volume $V$, which contains $64^3$ particles representing the baryonic component (or, more generally, the “shell” component). The height of a potential spike determines the radius of the associated shell, in cube units. We distribute the spike heights so that, in the absence of shell collisions, the distribution of shell sizes would be $n(R) = n_0 R^{-4}$ for $R_{\text{max}} \geq R \geq R_{\text{max}}/16$. By the end of the hydrodynamic phase, many of the smaller bubbles have been swept up or crushed between larger shells. The constant $n_0$ is chosen so that the formal filling factor is

$$ff = \frac{1}{V} \int_{R_{\text{max}}/16}^{R_{\text{max}}} \frac{4\pi}{3} R^3 n(R) dR = 3.$$  \hspace{1cm} (27)

The formal filling factor of large shells ($R_{\text{max}} \geq R \geq R_{\text{max}}/2$) is 0.75. The choice of filling factor defines the end of the hydrodynamic phase — shells grow hydrodynamically until they fill the specified volume. Before beginning the gravitational phase, we multiply the Burgers’ equation velocity field by a constant factor, chosen so that the peculiar expansion velocity of an isolated, undisturbed shell would be 20% of its Hubble expansion velocity, $V_p = 0.2 H \sqrt{\Lambda}$. In principle, the expansion velocity is another parameter of the model; we choose the value implied by the self-similar solution in an $\Omega_S = \Omega = 1$ scenario (Bertschinger 1985), which also seems reasonable for our other cases.

For the gravitational phase of the simulations, we take the particle positions and velocities at the end of the hydrodynamic phase as initial conditions and evolve them forward using Park’s PM code. We again use $64^3$ particles on a $128^3$ mesh. For the dark models we use an additional $64^3$ particles to represent the dark component. These particles are uniformly distributed and unperturbed at the beginning of the gravitational phase, but they quickly start to chase after the baryon perturbations.

At the end of the hydrodynamic phase, the two-point correlation functions of the particle distributions are roughly $r^{-1}$ power-laws on small scales, the $-1$ power-law index reflecting the two-dimensional nature of the dominant structures, the shells. Gravitational clustering causes the correlation functions to steepen on small scales. We identify the present epoch as the time when...
the correlation function most closely matches the observed $r^{-1.8}$ power-law. This occurs after an expansion factor of two (following the start of the gravitational phase) for the flat model, four for the open model, and six for the dark model. We fix the size of the simulation box (in $h^{-1}$ Mpc) by requiring that the rms fluctuation of the baryon particle distribution in spheres of radius $8h^{-1}$ Mpc match the observed galaxy clustering amplitude $\sigma_8 \approx 1$ (Davis & Peebles 1983). The simulation boxes turn out to be $108h^{-1}$ Mpc for the flat model and $\sim 98h^{-1}$ Mpc for the open and dark models. If the shells in the simulations had evolved without interacting with their neighbors, the largest shells would have reached $R_{\text{max}} \sim 35 - 40 h^{-1}$ Mpc. However, the volume filling factor of the explosions is sufficiently high that all the shells are likely to have their growth slowed by collisions with other shells. Finally, to create “galaxy” distributions, we can adopt either of two approaches. We can assume that galaxies evenly trace the baryonic mass, in which case we just randomly sample the baryon particle distributions to a density of $0.01 h^3$ Mpc$^{-3}$, or we can assume that the high gas density of a blast wave shock is a necessary prerequisite for galaxy formation (see Ostriker & Cowie 1981; Vishniac 1983), in which case we randomly sample only those baryon particles that have been swept onto shells before the end of the hydrodynamic phase. With our assumed power-law distribution, the explosions completely fill space (except for a tiny fraction left undisturbed because of our arbitrary lower cutoff $R_{\text{min}}$), so all baryons are “on-shell”, and the two approaches are equivalent. If the shells do not fill space, then the two assumptions yield different results, as we discuss in §5. Additional information about the explosion N-body simulations appears in Table 1b.

An interesting feature of Burgers’ equation is that, if the velocity field is irrotational at the initial time, then it remains irrotational at all later times. At the end of the hydrodynamic phase, therefore, the velocity fields of our simulations are irrotational. Since gravity does not generate large-scale vorticity, we can guess that the model velocity fields will remain approximately irrotational on large scales even during the gravitational phase, and we find this to be the case in our simulations. To the extent that Burgers’ equation gives a reasonable description of the interactions of hydrodynamic blast waves, we expect this low vorticity to be a generic feature of the explosion scenario.

Figure 3 shows slices through the galaxy distributions of the explosion models, in a format similar to Figure 2. The first column displays the flat model, the second column the open model, and the third column the dark model. The three galaxy distributions look quite similar because we choose the same sites for the “explosions” in each case and because our normalization procedure ensures that the small scale clustering as measured by the two-point correlation function is similar in the three models. The fourth column shows the dark matter distribution in the dark model. The dark matter has begun to reflect the pattern in the baryon distribution, but a substantial fraction of it remains in the voids. The range of shell sizes in these models gives the structure an irregular
appearance, with qualitative variations from slice to slice, in rough agreement with the observed galaxy distribution.

4. Density-Velocity Analysis

To analyze the simulations, we must transform the final galaxy positions and velocities into smooth density and velocity fields. Smoothing removes gross non-linearities, and in any practical situation it is also needed to reduce shot noise errors. Smoothing is part of the POTENT velocity-to-density procedures from start to finish. It is only partly implemented in existing observational applications of density-to-velocity analysis, where velocities predicted from the smoothed density field are usually compared to estimated velocities of individual galaxies, unsmoothed. In this paper we will adopt the more physically appropriate procedure of comparing the predicted velocity field to the “observed” velocity field smoothed at the same scale, though we recognize that this approach is trickier to implement in the real universe.

Creating a smoothed density field from the galaxy positions in our periodic simulations is straightforward: we use cloud-in-cell interpolation to compute a density field on a grid and smooth it by Fourier convolution with a Gaussian. Creating a smoothed velocity field is more problematic because in low density regions there may be no galaxy particles available to trace the local flow, leaving the velocity field undefined. The galaxy momentum field is well defined everywhere, however; where there are no galaxies the momentum is zero. We therefore define the smoothed velocity field to be the ratio of the smoothed momentum field to the smoothed density field. Contributions to the velocity field are thus weighted by galaxy density. Other smoothing schemes can be used to achieve something close to volume-weighted smoothing (§6 below, also Nusser et al. 1991). In the linear regime, the various velocity smoothing schemes should be equivalent if galaxies trace mass. In general, however, different schemes will yield somewhat different results.

We should note that in the non-linear regime, galaxy-weighted smoothing can introduce artificial vorticity into the smoothed velocity field because (1) smoothing tends to mix streamlines, and (2) the galaxies may trace an underlying curl-free field unevenly. In the models to be examined, however, we find that the vorticity of the smoothed fields is small, and, to a good approximation, the velocity field traced by the galaxies can be assumed to be irrotational.

In this paper we present results for Gaussian smoothing lengths of 1200 km s\(^{-1}\) and 600 km s\(^{-1}\). The larger scale is typical of the smoothing now used for the POTENT velocity-to-density analyses (e.g. Dekel et al. 1993). As more and better peculiar velocity data become available, increasing the sampling density, it may be possible to reduce the smoothing scales to \(\sim 600\) km s\(^{-1}\), at least in selected nearby regions.

4.1 Recovering the Density Field

We begin by comparing the true density fields with those reconstructed from the corresponding velocity fields. We reconstruct the density fields according to one of the following prescriptions:

\[
\delta_{G1} = - (H\beta)^{-1} \vec{\nabla}_x \cdot \vec{v}_G, \\
\delta_{G2} = \left\| I - (H\beta)^{-1} \frac{\partial \vec{v}_G}{\partial \vec{x}} \right\| - 1. 
\]  

(28)

For the linear theory reconstruction \(\delta_{G1}\) (c.f. equation 25a), we first set the ratio \(\beta \equiv f(\Omega)/b\) to unity and take the divergence of the smoothed velocity field to obtain \(\delta_{G1}\). Dividing the rms
FIGURE 4

Figure 4 — Velocity-to-density reconstruction of an unbiased gravitational simulation. The left panel shows a slice through the smoothed galaxy density field of an unbiased CDM simulation, with a Gaussian smoothing length $R_s = 1200 \, \text{km} \, \text{s}^{-1}$. The heavy solid line marks the mean density contour; positive (solid) and negative (dotted) contours are separated by 0.2 in $\delta$. The central panel shows a slice through the density field recovered from the smoothed velocity field using the linear approximation $\delta_{G1}$ of equation (28). The right-hand panel shows the density field reconstructed via the quasi-linear approximation $\delta_{G2}$. Both methods recover the true density field quite accurately.

FIGURE 5

Figure 5 — (a) Scatterplots of the velocity-to-density reconstructions of unbiased gravitational models. Each panel plots the reconstructed density contrast against the true density contrast $\delta_G$ for a set of 4096 pixels uniformly spaced throughout the simulation volume. Left hand panels show a smoothing length of 1200 km s$^{-1}$, right-hand panels a smoothing length of 600 km s$^{-1}$. Upper panels show the linear theory reconstruction $\delta_{G1}$, lower panels the quasi-linear reconstruction $\delta_{G2}$ (equation 28). Fluctuations are normalized by the rms fluctuation $\sigma$ of the corresponding density field. Diagonal lines mark the $y = x$ relation expected for a perfect reconstruction. The points follow this relation with modest scatter and some curvature; the latter is most noticeable in the linear reconstructions and at the smaller smoothing length. Values of the linear correlation coefficient $r$ are listed in each panel. (b) Same as (a), but for biased gravitational models.

fluctuation of the reconstructed density field by that of the simulation density field then yields the estimated value of $\beta$. The reconstruction $\delta_{G2}$ (c.f. equation 26 and the accompanying discussion) uses the quasi-linear relation derived from the Zel’dovich approximation by Nusser et al. (1991). In computing $\delta_{G2}$, we determine the value of $\beta$ by iterating until the rms density contrast of the reconstructed field matches that of the simulation field.

4.1.1 Gravitational Models

Figure 4 juxtaposes a contour plot of a single slice through the smoothed galaxy density field of one of the unbiased gravitational simulations against corresponding slices from the two reconstructed density fields, derived from the smoothed galaxy velocity field by equation (28). The smoothing scale is $R_s = 1200 \, \text{km} \, \text{s}^{-1}$. The reconstructed density fields have been scaled so that their rms fluctuations match that of the simulation density field. The heavy contour traces the mean density, while lighter solid (dotted) contours map out regions of positive (negative) density contrasts. The contour interval is 0.2 (in $\delta$). The location of the slice is in the middle of the range covered by the upper left panel of Figure 2.

A comparison of the contour maps shows that the linear theory reconstruction $\delta_{G1}$ tends to underestimate the magnitude of the density contrasts in high density regions. The quasi-linear reconstruction $\delta_{G2}$ fares somewhat better, as it should. However, the rms density contrast of the fields is only 0.22, so the differences between the two reconstructions are not large.
The scatterplots in Figure 5a present a more quantitative comparison between the true and reconstructed density fields of this simulation. For a set of 4096 pixels uniformly spaced throughout the simulation volume, we plot the pixel’s density contrast in the reconstructed field against its density contrast in the true smoothed density field. We normalize the density contrasts by the rms fluctuation of the corresponding field; these rms values are listed in each panel. The left-hand panels show results for a smoothing length of 1200 km s\(^{-1}\), and the right-hand panels show results for 600 km s\(^{-1}\) smoothing. Upper panels show the linear theory reconstruction \(\delta_{G1}\), lower panels the quasi-linear reconstruction \(\delta_{G2}\). A perfect reconstruction would yield equal rms fluctuations in the true and reconstructed fields and a set of points lying on the \(y = x\) diagonal, marked in each panel by a solid line. While all of the plots show scatter and some show curvature, they also display clear correlations, regardless of the reconstruction scheme or the smoothing scale. As a quantitative measure, we list in each panel the linear correlation coefficient,

\[
\rho = \frac{\sum_i (x_i - \bar{x})(y_i - \bar{y})}{\sigma_x \sigma_y},
\]

where \(x\) and \(y\) refer to the quantities plotted on the \(x\)- and \(y\)-axes, respectively. A value \(\rho = 1\) corresponds to a perfect correlation, \(\rho = 0\) to no correlation, and \(\rho = -1\) to perfect anti-correlation (see Press et al. 1992 for further discussion). The values in Figure 5a range from 0.84 to 0.96.

Focusing first on the left-hand panels, we see that the linear theory reconstruction (top panel) overestimates the amplitude of deep negative density contrasts and underestimates the amplitude of high positive density contrasts, so that the scatterplot exhibits significant curvature. Furthermore, the rms density contrast of the reconstructed density field, \(\sigma_{G1}\), is lower than that of the true density field, \(\sigma_G\). The quasi-linear method (lower panel) removes the curvature in the scatterplot, yielding a fairly tight, linear correlation between the reconstructed and true density contrasts. The rms contrast \(\sigma_{G2}\) equals \(\sigma_G\) by construction, since we iterate the assumed value of \(\beta\) until these two values match. Results for the 600 km s\(^{-1}\) smoothing length (right-hand panels) are generally similar, although there is greater scatter in the reconstructed densities, the curvature in the linear theory scatterplot is more severe, and even the quasi-linear scheme tends to underestimate the highest densities in the simulation. The plots in Figures 4a and 5a are all derived from a single run; results for the other two gravitational runs are similar.

Table 2a lists the estimates of \(\beta\) derived from these reconstructions (averaged over the three independent simulations), as well as the true values. The linear theory estimate \(\beta_1\), in column 4, is simply the ratio \(\sigma_{G1}/\sigma_{G}\). The quasi-linear estimate \(\beta_2\), in column 5, is determined by iterating the value of \(\beta\) in equation (28) for \(\delta_{G2}\) until the rms fluctuation of the reconstructed density field matches the measured value. The inferred values of \(\beta\) are always smaller than the true value of \(\beta = 1\), and the linear theory estimates are lower than the quasi-linear estimates. These trends hold separately in each of the three simulations, as well as in the average results. At 1200 km s\(^{-1}\) smoothing, linear theory yields \(\beta_1 = 0.87\), and the quasi-linear estimate yields \(\beta_2 = 0.90\); the latter value indicates that even the Zel’dovich approximation cannot completely remove the effects of non-linearity. Non-linear effects are more severe at the 600 km s\(^{-1}\) smoothing length, and the resulting \(\beta\) estimates are lower, \(\beta_1 = 0.74\) and \(\beta_2 = 0.85\). Despite the systematic tendency to underestimate \(\beta\), all of the inferred values correspond reasonably well to the true value.

**Table 2a: Inferred \(\beta\) values for Gravitational Models**

| Model | \(R_s\) | \(\beta_{true}\) | \(\beta_1\) velocity-to-density | \(\beta_2\) density-to-velocity |
|-------|--------|------------------|---------------------------------|------------------------------|
| CDM   |        |                  |                                 |                              |
| Unbiased | 1200   | 1.0              | 0.87                            | 0.90                         |
|        | 600    | 1.0              | 0.74                            | 0.85                         |
| Biased |        |                  |                                 |                              |
|        | 1200   | 0.5              | 0.46                            | 0.47                         |
|        | 600    | 0.5              | 0.42                            | 0.48                         |

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FIGURE 6

Figure 6 — Velocity-to-density reconstruction of a simulation of the flat ($\Omega_S = \Omega = 1$) explosion model, with a smoothing length of $1200 \text{ km s}^{-1}$. The format is the same as Figure 4: true density contrast on the left, linear theory reconstruction in the middle, quasi-linear reconstruction on the right, contour spacing of 0.2 in $\delta$. Non-gravitational forces play a major role in this model, but the reconstruction is still quite successful because the galaxy distribution obeys the continuity equation.

Figure 5b shows scatterplots of true and reconstructed densities for a biased CDM simulation, with the same format as Figure 5a. Results are similar to those for the unbiased model, and therefore noteworthy. As mentioned previously, the galaxy distribution in the peak-biased scenario does not satisfy the continuity equation, since the galaxies are “born” clustered. However, the fluctuations in the final galaxy density field are roughly proportional to the fluctuations in the underlying mass density, which does obey continuity equation, hence the correlations between density and velocity. The rms density contrasts of the linear theory reconstructions, $\sigma_{G1}$, are lower than the corresponding values of $\sigma_G$ by roughly a factor of two, since the divergence of the velocity field yields an estimate of the mass density contrast rather than the (biased) galaxy density contrast. The linear theory scatterplots also show significant curvature, especially at $R_s = 600 \text{ km s}^{-1}$. The quasi-linear reconstructions are iterated to yield $\sigma_{G2} = \sigma_G$, and this procedure removes the curvature in the correlations. This result is encouraging, since it suggests that the quasi-linear reconstruction scheme, designed to account for non-linear effects of gravitational evolution, can also correct for departures from linear biasing. However, we have tested this procedure only for our adopted prescription of peaks biasing, and it is not clear how well the result will generalize to other biasing prescriptions.

The last two rows of Table 2a list the values of $\beta$ estimated from the biased CDM simulations. Since $\Omega = 1$ and $b = 2$, the true value in the simulations is $\beta \equiv f(\Omega)/b = 0.5$. The derived values are slightly lower than this ($0.42 \leq \beta \leq 0.48$), and again the non-linear estimates are more accurate.

4.1.2 Explosion Models

Figure 6 illustrates velocity-to-density reconstructions of an explosion simulation. The left-hand contour plot shows a slice through the smoothed density field of one of the flat ($\Omega_S = \Omega = 1$) model simulations, and the right-hand plots show slices through the two reconstructed density fields. The location of the slices corresponds to the middle of the range covered by the upper left panel of Figure 3. The smoothing scale is $R_s = 1200 \text{ km s}^{-1}$, and the contour interval is 0.2 (in $\delta$). The reconstructed density fields have been scaled so that their rms fluctuations match that of the simulation density field.

As in the gravitational reconstructions, linear theory tends to underestimate the density contrasts in high density regions. More seriously, however, linear theory reconstruction overestimates the magnitude of density contrasts in low density regions, producing excessively deep negative perturbations. The quasi-linear reconstruction fares better in both the high and low density regions, though it is still not perfect. Part of the discrepancy reflects the non-linear nature of the density contrasts in the voids, but another problem arises from noise in the smoothed simulation velocity field used to reconstruct the density field. Because there are very few tracer particles (galaxies) in the voids, the smoothed velocity field is not well defined in these regions.
**FIGURE 7**

Figure 7 — Scatterplots of the velocity-to-density reconstructions of the flat explosion model. Format is the same as Figure 5. Results are similar to those for gravitational instability models, though there are differences of detail that reflect the greater prominence of large, empty voids in the explosion model. Scatterplots for the open ($\Omega_S = \Omega = 0.17$) and dark ($\Omega_S = 0.1, \Omega = 1$) explosion models are nearly identical, except for differences in normalization.

Figure 7 presents scatterplots of the density fields contoured in Figure 6. The format is the same as that used for the gravitational models in Figure 5. The scatterplots of the explosion model look somewhat different because the 1-point probability distribution is different: relative to the gravitational models, the very low density regions in the explosion model occupy a larger fraction of the volume, while the very high density regions occupy a smaller fraction. In spite of these differences, and in spite of the fact that the structure in this simulation has been sculpted primarily by the explosions, there is a good overall correlation between the simulation density field and the reconstructions, as expected from the contour plots of Figure 6. The demonstration of these correlations is one of our major results. Figures 6 and 7 reinforce the point made analytically in §2, that non-gravitational models can exhibit the same sorts of velocity-density correlations that are predicted by gravitational instability theories. Section 2 based this argument primarily on linear theory, but here we show that the result holds even for a realistically clustered, non-linear model.

At both the 1200 km s$^{-1}$ and 600 km s$^{-1}$ smoothing lengths, the scatter in the reconstructed densities of the explosion model is somewhat larger than it is in the gravitational models, particularly at the lowest densities. The non-linear Zel’dovich reconstruction removes most of the curvature in the density correlations, as expected based on the discussion in §2.3, but it does not do much to reduce the scatter. The increased scatter reveals itself in the values of the correlation coefficient $r$, which are lower than those in Figure 5, differing mostly at the 600 km s$^{-1}$ smoothing length. Greater scatter arises partly from the more non-linear nature of the voids in the explosion model — they are larger and deeper than those in the gravitational models — and partly from the lack of velocity tracers in the large, empty voids. In an observational analysis, many of the points at lower densities would be eliminated from these scatterplots or given little weight because of the poor knowledge of the velocity field, e.g. they would be flagged by the “$R_1$” criterion of DBF. Observational errors would also add to the scatter in the gravitational reconstructions. It seems unlikely, therefore, that the subtle differences in scatter and curvature that can be seen between Figures 5 and 7 would be discernible in a realistic case.

Density contour plots and scatterplots for the open and dark explosion models are very similar to those shown for the flat model in Figures 6 and 7, except for differences in normalization that reflect the lower velocities in these low-$\Omega_S$ models. Furthermore, in the dark models we obtain the same correlations, up to a constant factor, regardless of whether we compare the reconstructed density to the baryonic mass density or to the total mass density; gravity pulls the dark matter into the pre-existing baryon perturbations, so on large scales there is a simple, linear bias between the baryon fluctuations and the total mass fluctuations. Table 2b lists the values of $\beta$ inferred from the explosion reconstructions, along with the true $\beta$ values. The inferred $\beta$’s are $\sim 0.7$ for the flat model, where the true $\beta = 1$, and $\sim 0.3$ for the open model, where the true $\beta = 0.17^{0.6} = 0.34$. For the $\Omega = 1$, dark model we measure the ratio of rms galaxy fluctuations to rms mass fluctuations directly.
Table 2b: Inferred $\beta$ values for Explosion Models

| Model | $R_s$ | $\beta_{true}$ | $\beta_1$ velocity-to-density | $\beta_2$ density-to-velocity |
|-------|-------|----------------|-------------------------------|-----------------------------|
| Flat  | 1200  | 1.00           | 0.76                          | 0.71                        |
|       | 600   | 1.00           | 0.72                          | 0.69                        |
| Open  | 1200  | 0.34           | 0.30                          | 0.28                        |
|       | 600   | 0.34           | 0.29                          | 0.28                        |
| Dark  | 1200  | 0.25           | 0.21                          | 0.20                        |
|       | 600   | 0.25           | 0.23                          | 0.23                        |

on the 600 km s$^{-1}$ and 1200 km s$^{-1}$ smoothing scales, obtaining $\beta_{true} = 0.25 - 0.26$. Alternatively, we could define $b$ by the ratio of galaxy fluctuations to baryonic mass fluctuations, as we did in §2, and use $\Omega_b$ instead of $\Omega_{tot}$ in the definition of $\beta$. Since galaxies trace the baryon distribution in the dark explosion model, this definition yields $\beta_{true} = 0.1^{0.6/1} = 0.25$, the same value as before. The former definition of $\beta$ will behave more sensibly with time, increasing as the dark matter catches up to the baryons, but at this epoch we get the same answer from either method. The inferred $\beta$ values for the dark model range from 0.20 to 0.23.

The first thing to note about Table 2b is that the inferred values of $\beta$ are in the range 0.2 – 1, and hence are not unlike those derived from observations (e.g. Dekel et al. 1993). More remarkable is the fact that the inferred values of $\beta$ correspond quite closely to the true values; only for the flat model do the fractional errors exceed 20%. In the absence of gravity, the velocities produced by explosions (or by other non-gravitational perturbations) would decay kinematically in a few expansion times, once the non-gravitational forces themselves turned off. However, gravity does play a role if $\Omega > 0$, and the velocities only decay until they reach the level that can be sustained by the most rapidly growing gravitational mode. If the velocities lie below those that gravity can sustain, then gravitational accelerations raise them to the growing-mode amplitude in roughly a Hubble time. In either case, the inferred $\beta$ approaches the true value at late times, i.e. several expansion factors after the non-gravitational effects become unimportant. The gravitational phase of our flat model lasts only an expansion factor of 2, and in this model the inferred $\beta$’s deviate significantly from the correct value. Our other models expand by factors of 4 or 6 in the gravitational phase, and they yield more accurate $\beta$ estimates.

The match between the inferred and true $\beta$’s in Table 2b suggests that the velocities in most of our explosion models are in fact sustained by gravity at the final epoch. It may seem churlish to describe a model as non-gravitational if the final velocity field is dominated by gravitational accelerations. However, the distinction is far more than semantic. First, one can construct models in which the non-gravitational perturbations produce most of the displacements, and hence most of the spatial structure, even if the final velocities are sustained mainly by gravity. Second, the physical basis of the “seed” fluctuations is completely different in gravitational and non-gravitational models. Most non-gravitational theories locate the source of structure in “mundane” astrophysics after recombination, while most gravitational instability models assume that the seeds of structure arise from “exotic” processes in the very early universe.

We should emphasize that the density-velocity correlations in Figure 7 do not arise simply because gravitational accelerations have come to dominate over the original velocities — on large
scales these gravitational accelerations are simply proportional to the original velocities, so they affect the velocity divergence field only by a constant overall factor. In fact, velocity-to-density reconstructions of the explosion models at the end of the hydrodynamic phase, before any $N$-body evolution, yield similar correlations, and we showed in §2.1 that these correlations would arise even in a model with no gravity at all. The correlation between density and velocity divergence reflects the physics of the continuity equation, not gravity per se. We should also note that the agreement between true and inferred $\beta$ values in non-gravitational models generally develops only after several expansion factors, and that the value of “several” depends on the amplitude of the original velocities and on the values of $\Omega$ and $b$.

### 4.2 Recovering the Velocity Field

Having compared the density fields reconstructed from velocities to the actual simulation density fields, we now turn our attention to the reverse task of reconstructing velocity fields from smoothed density fields. We adopt the linear theory prescription

$$\vec{v}_G = -\vec{\nabla} \Phi_{v,G_1}, \quad \nabla^2 \Phi_{v,G_1} = (H\beta)\delta_G,$$

where $\delta_G$ is the smoothed galaxy density contrast (c.f. equation 25a). We first determine the velocity field assuming $\beta = 1$; the actual value of $\beta$ can then be inferred by demanding that the rms value of the reconstructed velocity field equal the rms value of the simulation velocity field.

#### 4.2.1 Gravitational Models

Figure 8 shows a slice through the smoothed velocity field of one of the unbiased gravitational simulations and a corresponding slice from the reconstructed velocity field. The smoothing length is $R_s = 600 \text{ km s}^{-1}$. The location of the slice is the same as that of the slice through the density field in Figure 4. Arrows mark the projection of the smoothed velocity onto the $x-y$ plane; multiplying the lengths of the arrows by $100h$ yields velocities in km s$^{-1}$. The reconstructed velocity field matches the simulation velocity field accurately in both direction and amplitude. At 1200 km s$^{-1}$ smoothing (not shown), the agreement between the two velocity fields is even better.

The scatterplots of Figure 9a compare the true and reconstructed velocity fields. At each of 512 uniformly spaced pixels, we plot the $x$-, $y$-, and $z$-components of the reconstructed velocity field against the corresponding components of the simulation velocity field. We normalize the velocities by the rms (1-dimensional) value of the corresponding velocity field; these rms values are listed in each panel. These plots demonstrate close agreement between the true and reconstructed velocity fields at both smoothing lengths. Non-linear effects are clearly much smaller here than in the density reconstructions considered above. This difference is expected because at a given smoothing length peculiar velocities are more strongly affected by larger scale perturbations, which are closer to the linear regime. In addition, the constraint that densities remain non-negative necessarily imposes an asymmetry between the evolution of positive and negative density fluctuations once density contrasts approach unity. No such constraint applies to velocities; indeed, isotropy implies that there must be no systematic differences between positive and negative velocities. As a result, non-linear evolution of velocity fields cannot produce the sort of curvature that appears in the density scatterplots; only anti-symmetric distortions that treat positive and negative velocities identically are consistent with isotropy. A subtle distortion of this sort can be seen in the right-hand panel of Figure 9a.

Figure 9b shows velocity scatterplots for the biased gravitational model. Results are similar to those in Figure 9a: linear correlations with modest scatter. Column 6 of Table 2a lists the values of
FIGURE 8

Figure 8 — Density-to-velocity reconstruction of an unbiased gravitational model. The left panel shows a slice through the smoothed velocity field of an unbiased CDM simulation, with a 600 km s\(^{-1}\) smoothing length. Vectors indicate the smoothed velocity projected onto the \(x - y\) plane. The right panel shows the same slice through the velocity field that is reconstructed from the smoothed density field using the linear approximation \(\vec{v}_{G1}\) of equation (29). The reconstruction recovers the direction and amplitude of the true velocity field.

FIGURE 9

Figure 9 — (a) Scatterplots of the density-to-velocity reconstructions of unbiased gravitational models. Each panel plots the three components of the reconstructed velocity field against the three components of the true velocity field \(\vec{v}_{G}\) for a set of 512 pixels uniformly spaced throughout the simulation volume. Left-hand panels show a smoothing length of 1200 km s\(^{-1}\), right-hand panels a smoothing length of 600 km s\(^{-1}\). Velocities are normalized by the rms 1-d value of the corresponding field; each panel lists these values in km s\(^{-1}\). Diagonal lines mark the \(y = x\) relation expected for a perfect reconstruction. The points follow this relation with small scatter. (b) Same as (a), but for biased gravitational models.

\(\beta\) derived from these reconstructions (and averaged over the three independent runs). For unbiased models, the inferred \(\beta\)'s range from 0.87 to 0.92, and for biased models they range from 0.46 to 0.48. The inferred \(\beta\) values are systematically low, but they are more accurate than the values derived from the velocity-to-density reconstructions discussed earlier. This difference again reflects the fact that velocity fields are more linear than density fields at the same smoothing scale.

Since we derive our reconstructed velocity fields from velocity potentials, they are irrotational by construction. The success of the reconstructions demonstrates that the smoothed velocity fields of the simulations are themselves nearly irrotational. We can quantify this result by decomposing each simulation velocity field into a component that is curl-free (irrotational) and a residual component that is divergence-free, and which therefore cannot be derived from a scalar velocity potential. The rms amplitude of the divergence-free velocity component is typically smaller than the rms amplitude of the curl-free component by about a factor of five.

4.2.2 Explosion Models

Figure 10 shows slices through smoothed velocity fields of an explosion simulation and the corresponding reconstruction. The format is the same as that of Figure 8, and the smoothing length is \(R_s = 600\) km s\(^{-1}\). The location of the slices is the same as that of the density slices in Figure 6. The true and reconstructed velocity fields agree very well.

Figure 11 plots the reconstructed velocities against the true velocities, using the same format as Figure 9. At both smoothing lengths we find linear correlations with relatively little scatter. Scatterplots for the open and dark explosion models (not shown) are nearly identical. Column 6 of Table 2b lists the values of \(\beta\) derived from these reconstructions. The inferred \(\beta\)'s are similar
to those found from the velocity-to-density reconstructions, and the remarks at the end of §4.1.2 apply here as well.

The success of the density-to-velocity reconstructions indicates that the velocity fields as traced by the galaxies in these explosion simulations are nearly irrotational. If we decompose the velocity fields into curl-free and divergence-free components, we find that the rms amplitude of the curl-free component is larger by about a factor of five for $R_s = 1200 \, \text{km s}^{-1}$ and four for $R_s = 600 \, \text{km s}^{-1}$. The explosion velocity fields thus have about the same amount of vorticity as the velocity fields of our gravitational models. This is an important result because it indicates that the POTENT method should recover the correct 3-dimensional velocity field from radial velocity data even for the explosion models considered here. We address this point more directly in §6.

5. Models That Fail

The analytic arguments in §2 and the numerical examples in §4 demonstrate that some non-gravitational models can produce correlations between density and velocity fields that are the same as those predicted by the standard gravitational instability scenario, though they may sometimes yield incorrect estimates of $\beta$ if analyzed in the standard fashion. The presence of such correlations does not, therefore, provide evidence exclusively for the gravitational instability hypothesis. However, the success of the models considered in §4 does not imply that all plausible models will pass the velocity-density tests. In this section we discuss some models that fail these tests, in the sense that they yield only weak correlations between the galaxy density field and the galaxy flow field. We focus our attention on two models in which the physical processes that influence galaxy formation yield a distribution where the galaxy density and the galaxy flow are not related to each other as prescribed by the continuity equation. Galaxies in these models are biased with respect to the baryonic mass in a non-trivial way, so regions of high and low galaxy density are not necessarily associated with converging and diverging flows, respectively.

We first consider a model in which peculiar velocities and the evolution of the mass distribution are driven by gravitational instability, but where the formation of galaxies is modulated by a non-gravitational process. Specifically, we examine the scenario proposed by Babul & White (1991),
who suggested that radiation from high-redshift quasars might suppress nearby galaxy formation, giving rise to apparent voids in the galaxy distribution that are not truly empty of matter. Babul & White proposed this effect as a possible mechanism for reconciling the standard CDM model with observations like the APM angular correlation function (Maddox et al. 1990) and the QDOT counts-in-cells (Efstathiou et al. 1990; Saunders et al. 1991), which imply strong galaxy clustering on large scales. They noted, however, that the model might have difficulty explaining the observed correspondence between galaxy density and velocity fields, and we show here that this concern is justified. In order to simulate this “voided” CDM model, we start with the mass distribution of the unbiased CDM simulations analyzed in §4, but we generate the catalog of galaxy particles by first excising all particles lying inside randomly placed spheres of radius $15h^{-1}$ Mpc (the quasar “exclusion” zones), then randomly sampling the remaining particle distribution to a density of $0.01h^{3}$ Mpc$^{-3}$. The number of exclusion zones is chosen so that they occupy 60% of the volume in total. We ignore any effects that quasars might have on the true or inferred (via Tully-Fisher, $D_n - \sigma$, etc.) velocity field of the galaxies.

Figure 12 shows scatterplots of the velocity-to-density reconstructions of the voided CDM model, using the quasi-linear method and smoothing lengths of 600 and 1200 km s$^{-1}$. Figure 13 shows scatterplots of the density-to-velocity reconstructions of this model. Although the plots display some correlations, the scatter is much larger than in our earlier reconstructions of either gravitational or explosion models (compare Figure 12 to Figures 5 and 7; Figure 13 to Figures 9 and 11). Values of the correlation coefficient confirm the visual impression of increased scatter; for example, at 1200 km s$^{-1}$ the correlation coefficient is $r = 0.63$ in both reconstructions of voided CDM, while for the CDM and explosion models analyzed earlier the correlation coefficients at this smoothing length always exceed 0.9. Residual correlations are present in the voided model because the galaxy distribution satisfies the continuity equation in some regions of the simulation volume. Regions of high galaxy density are associated with genuine high matter density, and therefore with converging velocity flows. These high density regions are consequently recovered in the reconstructed density maps, while convergent flows are recovered in the reconstructed velocity maps. Violation of the continuity equation, and therefore the failure of the reconstruction scheme,
FIGURE 14

Figure 14 — Comparison of the reconstructed density contrast to the mass density contrast $\delta_\rho$ of the voided CDM model. The velocity-to-density technique recovers the mass distribution of this model with reasonable but not high accuracy.

FIGURE 15

Figure 15 — Velocity-to-density reconstruction of an explosion model in which shells do not completely fill space, for comparison to Figure 7. We assume that all shells have the same radius, and we assume that galaxies form only on the shells. This model leaves false voids in the regions between shells, which are empty of galaxies but not of mass. The velocity-to-density reconstructions fail drastically because the evolution of the galaxy distribution does not obey the continuity equation.

FIGURE 16

Figure 16 — Density-to-velocity reconstruction of the shell-galaxy explosion model, for comparison to Figure 11. Only weak correlations between the true and reconstructed velocity fields exist for this model.

is more likely to occur in regions of low galaxy density, as such regions do not necessarily represent low matter density. A high mass density fluctuation located in a region of suppressed galaxy formation will not appear in the galaxy map, but through its gravity it will make its presence felt in velocity space. Consequently, a density map reconstructed from the velocity field will show a high density region where none exists in the galaxy map. Similarly, the “voids” in the galaxy map make spurious contributions to the predicted velocity flow pattern.

Although it does not recover the galaxy map accurately, the velocity-to-density reconstruction of the “voided” CDM model yields a reasonable map of the underlying mass distribution. This is not surprising, since the galaxy velocity field traces the mass velocity field, and the mass distribution necessarily obeys the continuity equation. Figure 14 plots the reconstructed density against the mass density, and the correlations are much tighter than in Figure 12. Nonetheless, there is more scatter here than in the reconstruction of the standard CDM model (Figure 5) because the velocity field is not well sampled inside the voids, so that the smoothed (galaxy-weighted) velocity field is afflicted by “sampling-gradient bias” (see DBF).

As our second model, we consider an explosion scenario in which the explosion blast waves do not completely fill space. We further assume that the high gas densities of a blast wave shock are a necessary prerequisite for galaxy formation, so that galaxies form only on shells (see Ostriker & Cowie 1981; Vishniac 1983). The simulations are similar to those described in §3.2, but instead of
using a power-law distribution of shell radii, we place 16 shells of equal radius at random locations in the simulation cube. We choose the shell radius so that the formal filling factor is \( ff = 0.75 \) at the end of the hydrodynamic phase, and we evolve for an expansion factor of four in the gravitational phase, adopting \( \Omega_S = \Omega = 1 \). We identify as galaxies only those particles that were swept up onto shells before the end of the hydrodynamic phase. The resulting galaxy distribution contains a combination of true voids in the baryon distribution, created by the explosions, and false voids, which contain undisturbed baryons at the mean density but no galaxies. The evolution of this distribution violates the continuity equation, in the sense that some of the voids are created by suppressing galaxy formation instead of by moving galaxies out; these voids do not have corresponding outflow signatures in the peculiar velocity field. Figures 15 and 16 show scatterplots of the velocity-to-density and density-to-velocity reconstructions, respectively, for this version of the explosion model. Some correlations are present, but correlation coefficients are low, and the scatter is very large, much larger than it is for the model in which galaxies faithfully trace the baryonic mass \((c.f. \text{Figures 7 and 11})\). Correlations arise in regions where shells have collided and eliminated the false voids between them. Consequently, we expect correlations in this model to grow with time as the shells expand to fill all of space. Qualitatively, results for this modified explosion model and results for the voided CDM model are quite similar. An important common feature is that the scatter in the velocity-density correlations does not drop significantly as one goes from \( 600 \text{ km s}^{-1} \) to \( 1200 \text{ km s}^{-1} \) smoothing length; in some cases it even goes up. This is in stark contrast to our results in §4, where scatter arises from non-linear effects, and correlations are always tighter at the larger smoothing length.

We have also analyzed the single-shell-radius explosion models assuming that galaxies form with equal efficiency everywhere, so that they trace the baryonic mass distribution. In this case the model does obey the continuity equation, and the velocity-density correlations are similar to those obtained for the power-law explosion model studied in §4. As mentioned in §3.2, it is natural to assume that galaxies trace the baryonic mass in the power-law model, since the explosion blast waves completely fill space.

We argued in §2 that a gravitational or non-gravitational model would pass the velocity-density tests, at least on scales in the linear regime, provided that (a) the galaxy distribution obeys the continuity equation, in the sense that the distribution is initially uniform and fluctuations are created only by moving galaxies from one place to another, and (b) the present-day galaxy velocity field is irrotational and proportional to the time-averaged galaxy velocity field, and hence to galaxy displacements. The biased CDM model of §4 actually violates condition (a) because the galaxies are “born clustered,” but the final galaxy fluctuations are approximately proportional to fluctuations in the mass distribution, which does obey the continuity equation. The two models that we have considered in this section fail the velocity-density tests because the processes that modulate galaxy formation violate condition (a) in more radical ways, yielding a non-trivial bias between the galaxy distribution and the underlying distribution of baryonic matter.

One can also imagine plausible variations of the explosion model that violate condition (b). We have so far considered explosion models in which all of the blasts are coeval: the ratio of expansion velocity to Hubble velocity is the same for every unperturbed shell. If explosions occur over a wide range of epochs, on the other hand, then the velocities associated with different blasts will decay by different factors, and no single constant of proportionality will relate present-day velocities to time-averaged velocities. While the relation \( \nabla \cdot \vec{v} \propto -\delta \) will hold in local regions, the constant of proportionality (and hence the inferred value of \( \Omega \)) will vary from place to place, adding scatter to any global correlations between the velocity and density fields. However, this spatial variation will die out as gravitationally induced velocities come to dominate over residual velocities from the explosions, and for reasonable parameter choices it may well be unimportant.
Condition (b) is also violated to some degree in the hybrid model discussed by Thompson & Park (1992), where explosions perturb the baryon distribution but there are independent fluctuations in a gravitationally dominant, collisionless dark matter component. As the explosion velocities decay and gravitational accelerations induced by the dark matter grow, velocities of the baryon fluid will change in both amplitude and direction, destroying the link between velocity and displacement. This hybrid model behaves quite differently from the explosion models with dark matter that we considered in §2 and §4; there the dark matter perturbations develop in response to the explosion-induced baryon perturbations, so they produce gravitational accelerations that are parallel and proportional to the explosion velocities. One would expect weaker velocity-density correlations in a hybrid model, but quantitative predictions are difficult without detailed modelling, since the baryons and dark matter affect each other in ways that depend on Ω_b and on the relative perturbation strengths.

6. POTENT Reconstructions

In sections 4 and 5, we reconstructed the density maps for various models using the full 3-d velocity fields traced by the galaxies in the simulations. However, real observations provide only the radial component of peculiar velocities. If the velocity field is a potential flow, then one can recover the 3-d velocity field from the radial velocity field using the POTENT method of Bertschinger & Dekel (1989). In this scheme, one integrates the radial velocity field along radial rays to construct the velocity potential; the gradient of this potential yields the 3-d velocity field. The associated density field can be computed by the prescriptions discussed in §2.

We have already commented that the vorticity of the smoothed velocity fields in our models is small. We expect, therefore, that the POTENT procedure should derive fairly accurate 3-d velocity fields given the radial velocity fields of these models, and that results for velocity-to-density reconstructions should be similar to those obtained using the full 3-d fields. In this section, we demonstrate these points explicitly by analyzing three of our simulations using the full POTENT machinery.

From each simulation, we create a data set containing the positions and radial peculiar velocities of galaxies within 8,000 km s\(^{-1}\) of an “observer” at the center of the cube (using periodic replicas of the simulation volume where necessary). We do not introduce errors in the positions and velocities of the “galaxies.” We analyze these simulated data sets using the techniques developed by DBF for analysis of observational data. We smooth the discrete radial velocity data onto a spherical grid using a tensor window with a spherical Gaussian weighting function of radius 1000 km s\(^{-1}\). We also weight the contribution of each galaxy by the volume enclosing it and its four nearest neighbors, in order to minimize the “sampling-gradient bias” that results from inhomogeneous sampling of the radial velocity field. The velocity potential at each spherical grid-point is computed by integrating the smoothed radial velocity field along rays originating from observer’s position in the center of the catalog volume. We interpolate the velocity potential onto a cubic grid with a spacing of 500 km s\(^{-1}\) using a cloud-in-cell scheme, and differentiate it to obtain the estimated 3-d velocity field \(\vec{v}_p\) in Cartesian coordinates. The corresponding density field \(\delta_p\) is reconstructed using the quasi-linear procedure outlined in §2.3. For a more detailed exposition of the smoothing and weighting scheme, we refer the reader to DBF.

Figure 17 shows scatterplots of the true and the POTENT-reconstructed density and velocity fields for three simulations: the unbiased CDM model (top panels), the flat explosion model (middle panels), and the modified explosion model discussed in §5, where galaxies form only on shells (bottom panels). We compare the fields inside a sphere of radius 6000 km s\(^{-1}\) instead of the full
Figure 17 — POTENT reconstructions of simulated data sets drawn from three of our models. The input data sets contain positions and radial velocities of particles within 8,000 km s\(^{-1}\) of a central “observer.” The 3-d velocity field, smoothed with a Gaussian window of radius 1,000 km s\(^{-1}\), is recovered by integrating the smoothed radial field along radial rays and differentiating the resulting potential. The density field is reconstructed from the divergence of the 3-d velocity field by the quasi-linear method discussed in §2.3. Left hand panels plot the smoothed, 3-d velocities recovered by POTENT against the true velocities smoothed directly in 3-d. Right hand panels plot the POTENT density contrast against the model galaxy density contrast. Reconstructions of the velocity and density fields of the unbiased CDM model (top panels) and the explosion model (middle panel) are quite successful because the velocity fields are nearly irrotational and the galaxy distributions obey the continuity equation. Recovered velocities of the shell-galaxy explosion model (lower left panel) show more scatter because of the poor sampling of the velocity field in this heavily voided model. Density reconstruction of this model (lower right panel) fails completely because the galaxy distribution is biased in a way that violates the continuity equation; this result is similar to that obtained directly from the 3-d velocity field (Figure 15).

8000 km s\(^{-1}\) volume used for the reconstruction, in order to avoid edge effects. We smooth the true density and velocity fields using a 1000 km s\(^{-1}\) Gaussian window, and to ensure a fair comparison we also weight the true velocity field by the 4th-neighbor volume.

From the upper panels, we see that POTENT is quite successful at recovering the velocity and density fields of the unbiased CDM simulation. These results are unsurprising, since the velocity field is very nearly a potential flow on these scales, and since the model satisfies the other conditions required for successful velocity-to-density reconstructions. The results from the full POTENT procedure are similar to those obtained directly from 3-d velocity field at a smoothing length of 1200 km s\(^{-1}\) (lower left panel of Figure 5a), with a slightly larger scatter that reflects both the shorter smoothing length and the imperfect recovery of the 3-d velocity field in the POTENT analysis.

From the middle panels, we see that POTENT successfully recovers the velocity and density fields of the explosion model. There is some scatter in the recovered velocity field, presumably reflecting the presence of a small amount of vorticity in the flow and the absence of velocity tracers in the large voids, which induces sampling-gradient bias. Nonetheless, the plot of reconstructed density versus true density is quite similar to that obtained directly from the 3-d field at 1200 km s\(^{-1}\) smoothing (lower left panel of Figure 7). This result is important, as it reinforces our claim that non-gravitational models, analyzed by POTENT, can yield the same correlations between velocity and density that are found in standard gravitational models, and in observations.

POTENT is less successful at recovering the velocity field of the shell-galaxy explosion model — the correlation coefficient between reconstructed and true velocities is only \(r = 0.64\), compared to \(r = 0.84\) and \(r = 0.87\) for the other two models. We should emphasize that this reconstruction of the 3-d velocity field has nothing to do with the density-to-velocity reconstructions discussed in previous sections; the input is the radial velocity field, not the galaxy density field, and the essential assumption is that of potential flow, not the continuity equation. The relatively poor recovery of the velocity field reflects the dominance of large voids in the galaxy distribution. Galaxies in this model provide poor spatial tracers of the velocity field, so the smoothed radial velocity field suffers from sampling-gradient bias. The reference “true” velocity field suffers from this bias as well, and the bias may artificially induce vorticity that cannot be recovered by POTENT (see the discussion at the beginning of §4).
While the velocity reconstruction in this model is less than perfect, the failure of the density reconstruction is catastrophic, yielding a correlation coefficient of only 0.24 (see the lower right panel of Figure 17). This failure is unsurprising, and it reflects the fact that the galaxy distribution in the shell-galaxy explosion model does not evolve according to the continuity equation. The results are quite similar to those obtained directly from the 3-d velocity field (Figure 15); even if POTENT recovered the velocity field perfectly, the velocity-to-density analysis would not recover the galaxy density field. In application to observations, comparisons of POTENT density fields to the density fields of IRAS or optical galaxies typically yield correlation coefficients higher than 0.5, so this model is probably excluded by existing data.

The POTENT analysis reinforces our previous findings regarding velocity-to-density reconstructions. POTENT densities should correlate with galaxy densities so long as the galaxy distribution satisfies the continuity equation and the velocity field is irrotational, whether or not the velocities are gravitationally induced. If physical processes modulate galaxy formation in a way that violates continuity, then the reconstructed density field will not match the observed galaxy distribution.

7. Conclusions

The gravitational instability hypothesis predicts specific correlations between large-scale density and velocity fields. We have argued that finding these correlations does not provide confirmation of the gravitational instability hypothesis because the same results arise in a wider class of models. Specifically, one expects to find these correlations in any model where (a) structure in the galaxy distribution grows from homogeneous initial conditions in a way that obeys the continuity equation, and (b) the present-day velocity field is irrotational and proportional to the time-averaged velocity field. The explosion models analyzed in §4 provide particular examples of non-gravitational models that pass the usual density-velocity tests.

In gravitational models, the ratio of the galaxy density contrast to the divergence of the peculiar velocity field yields an estimate of \( \Omega \) (or of \( \beta = \Omega^{0.6}/b \) in the case of linear biasing). In non-gravitational models, the peculiar velocity field may contain a mixture of growing and decaying modes, so the value of \( \beta \) estimated from this ratio may not correspond to the true cosmological value. However, if non-gravitational forces shut off at some epoch, then modes of the velocity field that are not sustained by gravitational accelerations decay thereafter, on the cosmic expansion timescale. As a result, one can find models in which non-gravitational forces initiate the formation of spatial structure but gravity dominates the present-day velocity field, and in such models the estimated \( \beta \) should coincide with the true value.

One can adopt either a pessimistic or an optimistic reading of our results. The pessimist would emphasize our conclusion that density-velocity comparisons do not fundamentally test gravitational instability. The value of \( \beta \) estimated from these comparisons is guaranteed to reflect the true cosmological value only if the gravitational hypothesis holds, and the large-scale fields used for these estimates do not provide the means to test this assumption. Even agreement between the \( \beta \) estimated from large-scale fields and values estimated by other techniques would not imply that spatial structure formed by gravitational instability, only that gravitational accelerations sustain the present-day velocities.

The optimist would note, first of all, that there are physically interesting models that fail the density-velocity tests. These include models like voided CDM (c.f. §5 and Babul & White 1991) in which the mass fluctuations and peculiar velocities are generated by gravity but where galaxy formation is modulated on large scales by non-gravitational processes, which bias the galaxy
distribution in a way that violates the continuity equation. Also included are models in which hydrodynamic forces sculpt the spatial distribution of the baryons but independent fluctuations in a collisionless dark matter component produce gravitational accelerations that dominate the present velocity field, since these independent accelerations break the link between present and time-averaged velocities. The optimist would also note that in the broad class of models for which present and time-averaged velocities are proportional, differentiating the velocity field does yield a scaled map of the true mass distribution (with some unknown scaling factor), even if the velocities have a non-gravitational origin, and even if most of the mass is non-luminous. Thus, comparisons of large-scale, galaxy density and velocity fields offer a fairly general way to study the relation between the distributions of galaxies and mass in the universe, although they do not tell us whether gravity created these distributions.

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