Viscosity Mechanisms in Accretion Disks

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The self-sustained turbulence which develops in magnetized accretion disks is suppressed in the weakly-ionized, quiescent disks of close binary stars. Because accretion still proceeds during quiescence, another viscosity mechanism operates in these systems. An anticorrelation of the recurrence times of SU UMa dwarf novae with their mass ratio supports spiral waves or shock-waves tidally induced by the companion star as the main process responsible for accretion in the quiescent disks. Other weakly-ionized gaseous disks in systems lacking a massive companion have to rely on yet another transport mechanism or they could be essentially passive.

Accretion discs\textsuperscript{(1)} are present in a variety of astrophysical objects, such as mass-transfer binaries, young stellar systems and active galactic nuclei\textsuperscript{(2)}. The main uncertainty on the structure of accretion disks is the nature and magnitude of their viscosity\textsuperscript{(3)}, the process by which the gas loses angular momentum to accrete onto the massive central object. Theoretical studies of magnetized accretion disks have shown that magneto-hydrodynamical (MHD) turbulence provides the necessary outward angular momentum transport for accretion to proceed\textsuperscript{(4,5)}. The problem of identifying a viscosity mechanism can also be addressed empirically, by constraining the magnitude of viscosity in unsteady disks, where it directly relates to the observed variability\textsuperscript{(2)}. This is best done in transient close binaries, such as recurrent dwarf novae, with geometries, masses and mass accretion rates that are relatively well known\textsuperscript{(6)}.

The transient nature of accretion in the disks of close binaries is understood as a thermal-viscous limit cycle due to sudden changes of the disk opacity when hydrogen recombines\textsuperscript{(7)}. Global disk evolution models reproduce the main properties of observed outbursts, but only if the efficiency of transport and dissipation is less in quiescence than during outburst\textsuperscript{(8,9)}. This requirement is consistent with the low level of ionization predicted in quiescence. In terms of the magnetic Reynolds number

\[ \text{Re}_M \equiv C_s H / \eta, \]

where \( C_s \) is the sound speed, \( H \) the disk scale height and \( \eta \) the resistivity (inversely proportional to the ionization fraction), local shearing-box MHD simulations\textsuperscript{(10,11)} predict a reduced efficiency of angular momentum transport when \( \text{Re}_M \lesssim 10^4 \), a value comparable to what is expected in quiescent disks\textsuperscript{(12)}. It is unclear, however, whether MHD turbulence is still responsible for transport during this phase, at a reduced level, or is entirely suppressed.

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This question is addressed here by using results from shearing-box simulations as an input for a global numerical model designed to study disk instabilities (13). The model provides reliable values for the temperature, density, and therefore $Re_M$, throughout the disk (thermal ionization and resistive diffusion are the dominant processes in the disks considered here (12)). The viscosity parameter $\alpha$ used in the model (a measure of the disk viscosity in units of $C_s H$) is tabulated as a function of $Re_M$ according to the most recent (zero net flux) MHD simulations that include resistive effects (11). The decay of $\alpha$ is locally limited to an e-folding time-scale corresponding to 4 orbits, to allow for the finite time of field resistive diffusion.

Other model parameters are chosen to represent the disk around the white dwarf of the prototypical dwarf nova SS Cyg (12): mass of the primary $M_1 = 1.2 M_\odot$, disk inner and outer radii $R_{\text{in}} = 5 \times 10^8$ cm and $R_{\text{out}} = 4 \times 10^{10}$ cm, and mass transfer rate of the companion $\dot{M}_T = 10^{-9} M_\odot$ yr$^{-1}$.

The evolution of a disk annulus as it goes into quiescence in a standard model is represented by triangles in Fig. 1. The evolution when physically-motivated values of $\alpha$ are used (circles) shows that the disk does not saturate at a low level of MHD turbulence, but suffers a runaway cooling as dissipation and $Re_M$ become less and less important. Although the evolution was limited to a minimum value $\alpha = 10^{-4}$ and was stopped when $Re_M$ reached 100, the disk was still cooling down at that point. The outcome is independent of the value adopted for the field diffusion e-folding time-scale, as long as it is short compared to the viscous time-scales in the disk. Although the presence of a secondary viscosity mechanism could be responsible for a residual level of MHD turbulence, by maintaining the disk at a temperature ($\sim 2000$ K) sufficient for the field to remain coupled to the gas, the disappearance of self-sustained MHD turbulence is guaranteed.

This is a conservative result, because not using a minimum value of $10^{-4}$ for $\alpha$ would have lead to stronger turbulence decay. Moreover, the enhanced reduction of transport and dissipation shown by higher resolution MHD simulations (10,11) further supports the case for turbulence decay. The runaway decay is found at other radii in the disk as well (the value of $Re_M$ tends to be smaller at smaller radii because of the nearly flat profile of $T_C$ in quiescence and the scaling of $Re_M$ with $H$), and additional models show that the same result holds for the disks of x-ray transients (14,15), around neutron stars and stellar-mass black holes.

Accretion is known to occur during quiescence in transient close binaries (6), so that another transport mechanism operates in the disks. The rapid disk expansion observed during the outbursts of several eclipsing dwarf novae (6) is consistent with MHD-driven accretion because it shows that disk internal stresses dominate transport during this phase. On the other hand, the same disks are observed to shrink between consecutive outbursts, which is a signature that transport is dominated by the tidal torque due to the companion star at least in the outer regions of the disk during quiescence.

The theory of tidally induced spiral waves or shock-waves predicts a reduced effective torque for smaller
ratios of the companion mass to the accretor mass (16, 17). Because the recurrence times of dwarf novae represent the time-scales for mass and angular momentum redistribution in the quiescent disks, an anticorrelation of the recurrence times with mass ratios is expected if tidal torques dominate transport in the quiescent disks (18). An anticorrelation exists for a sample of SU UMa dwarf novae (Fig. 2). This sample represents all SU UMa stars with well known masses and recurrence times (19). The anticorrelation, which is significant for normal and super outbursts, supports tidal effects due to the mass-transferring star as the dominant transport mechanism during quiescence in transient close binaries. In the same six systems, no apparent correlation of \( t_{\text{rec}} \) or \( T_{\text{rec}} \) with the orbital period is found. For U Gem type dwarf novae, another subclass for which there are enough systems with well known masses and recurrence times to repeat the exercise, no correlation of the recurrence times with mass ratio or orbital period is found.

A possible reason why the relation is apparent only for SU UMa stars is that an anticorrelation of recurrence times with the mass ratio \( q \) can easily be masked by differences in the rates at which mass is transferred by the companion, because these also influence the frequency of outbursts. SU UMa stars may constitute an ideal subsample of dwarf novae to test for such a relation because they mostly lie in a small range of orbital periods, below the period gap (6). Mass transfer is thought to be driven by gravitational radiation in these systems (20), which means that it should primarily depend on the geometry and masses of the system. The case of mass transfer driven by magnetic braking (when the binary orbital angular momentum is lost via the wind of the mass-transferring companion), which applies to most U Gem type dwarf novae above the period gap, may be more complex. The much wider range of orbital periods covered by U Gem stars may also contribute to the absence of a relation for these systems.

Differences in the mass transfer rates of SU UMa stars themselves could explain the relation (Fig. 2). However, in the limit \( q << 1 \) and under the assumption that the secondary star fills its Roche lobe, predictions for the mass transfer rate due to gravitational radiation give \( \dot{M}_T \propto q^2 M_1^{8/3} \), where \( M_1 \) is the accretor mass (21). There is no evidence for a dependence of the recurrence times with \( M_1 \) in the data set considered here, as expected if the primary reason for the anticorrelation is a reduced efficiency of transport for smaller values of \( q \).

The anticorrelation is consistent with a more general, qualitative trend among transient close binaries for longer recurrence times in systems with smaller mass ratios (omitting possible complications due to evolutionary issues and differences in the mass transfer rates or the accretion flow structures). This is the case of WZ Sge type dwarf novae, which have longer recurrence times and lower mass ratios than the rest.

\(^3\) SU UMa type dwarf novae show characteristic, long superoutbursts in addition to the more frequent normal outbursts of U Gem type dwarf novae.
of the dwarf nova population (6). Similarly, soft x-ray transients containing accreting black holes, as a class, have longer recurrence times than similar systems containing less massive, accreting neutron stars (14).

Theoretically, it has been argued that the tidal perturbations due to the companion star probably do not lead to significant transport in the inner regions of the disks of close binaries because of their relatively large Mach numbers. This would be particularly true of the cold, quiescent disks considered here. However, no study to date has considered a realistic, steeply increasing profile of surface density with radius, as predicted for the quiescent disks by disk instability models (9). Because such a profile favors tidally-induced transport in the inner regions of the disk (22), this question remains open.

The concept of accretion driven by MHD turbulence during outburst and by tidal perturbations during quiescence has several interesting implications. It complicates the modeling of the disks of transient close binaries, because, in quiescence, α is plausibly non-uniform with radius, and becomes time-variable as the disk shrinks and couples less and less with time to the tidal perturbation (a minimum value for the disk outer radius is set by the specific angular momentum of the accreted gas). It also allows for an efficiency of transport in quiescent disks which differs from system to system because it depends on the binary star parameters.4 Finally, it implies that, in the absence of a massive companion, weakly-ionized disks must rely on yet another transport mechanism or they could be unable to accrete. Self-gravity is a plausible candidate for angular momentum transport in the disks of T-Tauri stars and active galactic nuclei, which are more massive than the disks considered here. However, in systems with disks similar to those of close binaries, such as supernova fallback disks or the disk which lead to the planetary system around the pulsar PSR 1257+12, the absence of a massive companion could result in essentially passive disks.

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Fig. 1. The evolution into quiescence of a disk annulus located at $2 \times 10^{10}$ cm from a central white dwarf is shown in a surface density ($\Sigma$) vs. central temperature ($T_c$) diagram. The solid line represents the disk thermal equilibria. The middle section, which corresponds to partially ionized gas, is unstable and forces the annulus to a cyclic behavior. The triangles represent the evolution of the annulus, from top to bottom, in a standard model with arbitrarily chosen values of the viscosity parameter $\alpha$ on the stable branches ($\alpha_{\text{hot}} = 0.1, \alpha_{\text{cold}} = 0.01$). The circles represent the evolution in a model where the value of $\alpha$ is consistently taken as a function of $RE_M$ in the low ionization regime. In this case, the annulus experiences a runaway cooling, down to values of $RE_M < 100$. At such a low level of ionization, MHD turbulence dies away.
Fig. 2. The recurrence times of normal and super outbursts of a sample of well-studied SU UMa stars are shown as a function of their mass ratio. There is an indication for systems with longer recurrence times having lower mass ratios. From left to right, the six systems are OY Car, CU Vel, Z Cha, VW Hyi, WX Hyi and YZ Cnc, with white dwarf masses $M_1/M_\odot = 0.68, 1.23, 0.84, 0.63, 0.9$ and $0.82$, respectively. The data are taken from the Ritter & Kolb catalogue\cite{ritter_kolb}. Masses are usually determined via time-resolved photometric and spectroscopic methods.