Quantum Perturbative Approach to Discrete Redshift.

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Abstract

On the largest scales there is evidence of discrete structure, examples of this are superclusters and voids and also by redshift taking discrete values. In this paper it is proposed that discrete redshift can be explained by using the spherical harmonic integer $l$; this occurs both in the metric or density perturbations and also in the solution of wave equations in Robertson-Walker spacetime. It is argued that the near conservation of energy implies that $l$ varies regularly for wave equations in Robertson-Walker spacetime, whereas for density perturbations $l$ cannot vary regularly. Once this is assumed then perhaps the observed value of discrete redshift provides the only observational or experimental data that directly requires an explanation using both gravitational and quantum theory. In principle a model using this data could predict the scale factor $R$ (or equivalently the deceleration parameter $q$). Solutions of the Klein-Gordon equation in Robertson-Walker spacetimes are used to devise models which have redshift taking discrete values, but they predict a microscopic value for $R$. A model in which the stress of the Klein-Gordon equation induces a metrical perturbation of Robertson-Walker spacetime is devised. Calculations based upon this model predict that the Universe is closed with $2q_0 - 1 = 10^{-4}$.

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1 Introduction

1.1 evidence of discrete structure

There are at least four types of evidence of discrete structure on the largest scales. The first example of discrete structure is given by discrete redshift, the evidence for this will be discussed in the next paragraph. Redshift comes in discrete values with the characteristic velocity \( v_I = 72.2 \pm 0.2 \text{ Kms}^{-1} \), and this leads to a characteristic length \( l_{dr} = v_I H_0^{-1} = 3 \pm 0.8.10^{22} \text{ meters} \), and a characteristic period \( t_{dr} = v_I H_0^{-1} c^{-1} = 3.2 \pm 0.8.10^6 \text{ years} \). Here this result is used in preference to the other examples of discrete structure because of the advantage of having a qualitative result, namely \( v_I = 72.2 \pm 0.2 \text{ Kms}^{-1} \); the actual techniques used might be applicable to the other cases. A second example is that the Universe appears to consist of superclusters and voids Saunders et al (1991) more recent studies of clustering can be found in Cohen (1999) superclusters and voids occur with apparent regularity (Broadhurst et al (1990) they have a characteristic scale of \( l_{sv} = 128 h^{-1} \text{Mpc} = 5.3 \pm 2.6.10^{24} \text{ meters} \), and hence a characteristic period of \( t_{sv} = l_{sv} c = 5.6 \pm 10^8 \text{ years} \). Tytler et al (1993) do not confirm Broadhurst et al’s result: instead of Broadhurst et al’s “apparent regularity with a scale of 128h^{-1} \text{Mpc}.” they find ”There is no significant periodicity on any scale from 10 to 210h^{-1} \text{Mpc.” “ The result was also looked at by Willmer et al (1994) Einasto et al (1997) present evidence for a quasiregular three-dimensional network of rich superclusters and voids, with regions separated by \( \sim 120 \text{Mpc} \); and they say that “if this reflects the distribution of all matter, then there must exist some hitherto unknown process that produces regular structure on large scales”. A third example of discrete structure is that some normal elliptical galaxies have giant shells surrounding them, Malin and Carter (1980). These shells probably consist of stars, the most likely method of their formation is from an intergalactic shock wave or an explosive event in the galaxy. The fourth example is of discrete properties from geological time scales. The characteristic periods \( t_{dr} \) and \( t_{sv} \) are larger than typical geological frequencies. For example Kortenkamp and Dermott (1998) give a periodicity of \( t_d = 10^5 \text{ years} \) for the accretion rate of interplanetary dust. The rate of accretion of dust by the Earth has varied by a factor of 2 or 3. Extraterrestrial helium-3 concentrations in deep sea cores display a similar periodicity but are \( 5.10^4 \text{ years} \) out of phase. The magnetic polarity time sequence (p.672 of Larson and Birkland (1982) gives reversals in the earth’s magnetic field
occurring at intervals \( t_m = 10^3 \) to \( 5.10^3 \) years; and the period of recent glacier advance and retreat is about \( t_g = 2.10^3 \) years, \((p.488\) Larson and Birkland (1982) \[59\]). Naidu and Malmgren (1995) \[71\] give a periodicity of \( t_m = 2,200 \) years for the Asian monsoon system, this is found by looking at fluctuations in upwelling intensity in the western Arabian sea. Clearly it would be good if there was an explanation for these characteristic periods and even better if they could be used to predict presently unknown quantities. The obvious factor to try to predict is the size of the Universe as given by the scale factor \( R \), assuming a Robertson-Walker Universe this is equivalent to deriving a value of the deceleration parameter \( q \). Now the value of the deceleration parameter is assumed in the derivation of \( t_{sv} \), thus models using the characteristic period of discrete redshift are studied.

1.2 observations of discrete red shift

Perhaps the first paper advocating that redshift can only occur in specific discrete values was Cowan (1969) \[24\] who found a periodic clustering of redshifts. This was confirmed by Karlsson (1971) \[22\] who found a number of new peaks in the distribution of quasi-stellar objects; these, together with the peaks at \( z = 1.956 \) and \( z = 0.061 \) formed a geometric series: \( z_1 = 1.96, z_2 = 1.41, z_3 = 0.96, z_4 = 0.060, z_5 = 0.30, z_6 = 0.06 \); this was supported by Arp et al (1990) \[1\]. Green and Richstone (1976) \[41\] did a search for peaks and periodicities in the redshift distribution of a sample of quasars and emission-line galaxies independent of that used in earlier work. In agreement with the results of Burbidge and O’Dell (1972) \[15\], no statistically significant peak was found at a redshift of 1.95, nor any significant periodicity in redshift in either the sample of quasars alone or the sample of quasars and galaxies together. The strong spectral power peak in their distribution of galaxy redshifts, estimated at a confidence level of 97.5 percent, is completely absent in Green and Richstone’s analysis; they conclude that the observed redshift distribution is consistent with a random sample of discrete values from a smooth, aperiodic underlying population. Tifft (1976) \[97\] claimed that well known local galaxies, especially M31, were claimed to consist of two basically opposed streams of outflow material which have an intrinsic difference in red shift of \( 70 - 75 \) Km. \( s^{-1} \). Tiffts’ result was questioned by Monnet and Deharney (1977) \[68\] who say that the two opposing streams of material suggest that the best galaxy candidate for a direct test of Tiffts’ result would be a near face-on galaxy: the Doppler effect due to the rotation is minimized and any expansive motion in the spiral
arms, as connected with the uneven distribution of neutral gas, which can be of the order of \(10 - 25 \text{ Km.s}^{-1}\) at most, gives entirely negligible Doppler effect. On the other hand a real intrinsic redshift would still exhibit its full 70 – 75 Km.s\(^{-1}\) discontinuity. Monnet and Deharneny chose the nearly face-on galaxy NGC628 and find no intrinsic effects as predicted by Tifft and a very smooth velocity field, with a velocity dispersion 12 Km.s\(^{-1}\). In Tifft (1977a) [98] it was claimed that redshift differentials between pairs of galaxies and between galaxies in clusters take preferred values which are various multiples of a basic 72.5 Km. s\(^{-1}\). In Tifft (1977b) [99] the effect was studied for abnormal galaxies. In Tifft (1978a) [100] it was claimed that the asymmetry in galaxy H1 profiles can be related directly to the properties of discrete redshift. In Tifft (1978b) [101] the concept of discrete redshift was applied to dwarf H1 redshifts and line profiles, and a model of redshift based upon the ultimate discrete levels spaced near 12 Km. s\(^{-1}\) was developed. Most optical redshift data are not accurate to show discreteness directly, but it is claimed that 21 cm. data on double galaxies show the effect clearly; Tifft (1980) [102] and Tifft (1982a) [103] using the radio data taken by Peterson (1979) [73] at the NRAO 300’ telescope has claimed that the effect is present with a confidence level of 99.9%. Cocke and Tifft (1983) [22] have claimed that the effect is present in the 21 cm. data on compact groups of galaxies taken by Haynes (1981) [17] and Heleou et al (1982) [47] at the Arecibo telescope with a confidence level of 99.5%. The optical data of Tifft (1982b) [104] also shows the effect strongly. The Fisher and Tully (1981) [38] survey of 21 cm. redshift was found by Tifft and Cocke (1984) [107] to show sharp periodicities at exact multiplies (\(\frac{1}{3}\) and \(\frac{1}{2}\)) of 72.45 Km. s\(^{-1}\); the periodicity at 24.1 Km. s\(^{-1}\) involves galaxies with narrow 21 cm. profiles and the \(\frac{1}{2}\) periodicity at 36.2 Km. s\(^{-1}\) involves galaxies with wide profiles, and there appears to be a progression of periods 24 \(\rightarrow 36 \rightarrow 72\) for galaxies with higher 21 cm. flux levels as the profile width increases. Arp and Sulentic (1985) [7], using data from the Arecibo telescope of over 100 galaxies in more than 40 groups found that: companion galaxies have a higher redshift than the dominant galaxy of the cluster, and that the difference in redshift between the dominant and companion galaxies occurs in multiples of 70 Km. s\(^{-1}\). Sharp (1984) [73] suggested that for double galaxies discrete redshift might be just a statistical effect. Newman et al (1989) [74] suggested that the effect is gaussian random noise and that at least one order of magnitude more data is needed to confirm the effect. Crousdale’s (1989) [20] study supports Tiffts’ work. Mirzoyan and Vardanyan (1991) [65] claim that the values of the redshifts found preferentially in quasistellar objects
essentially coincide with the redshifts for which the strong emission lines of Mg II, C IV, Ly α, in the spectra of these objects fall close to the maximum sensitivity of the U, B, and V light filters. In this case their effect on the conditions for observing quasars is decisive and causes the quasar redshifts to be discretized. Based on a comparison between the observed quasar redshifts and the expected values assuming that this explanation is correct, they conclude that the observed effect of quasar redshift discretization is caused by observational selection. Guthrie and Napier (1991) confirm the effect in near by galaxies, but whereas Tifft finds 24.2, 36.3, or 72.5 Km.s$^{-1}$, they find 37.2 Km.s$^{-1}$. Kruogovenko and Orlov (1992) find a periodic cycle of Seyfert and radio galaxies of about $30 h^{-1}$Mpc between shells. Holba et al (1994) find a non-negligible region where two quasar samples and the galaxy sample are simultaneously fairly periodic. Guthrie and Napier (1996) confirm redshift periodicity in the local supercluster. Khodyachikh’s (1996) findings contradict the explanation of periodicity by selection effects. Tifft (1996) finds 72 and 36 Km.s$^{-1}$ periodicity for galaxy samples from the Virgo cluster, the Perseus and Cancer supercluster regions, and local space. Tifft (1997) studies the redshift of local galaxies for quantization and finds that ordinary spiral galaxies with 21 cm. profile widths near 200 Km.s$^{-1}$ show periodic redshifts. A review of redshift periodicities has been given by Narlikar (1992), he finds that different data sets of extragalactic objects including nearby and distant galaxies and quasars that show statistically significant peaks at periodic intervals of redshift; he says at present the data is not complete in any sense but they are substantial enough to make us worry about the fundamental assumption that the Universe is homogeneous on a large scale. Moreover, he claims evidence of this kind has not only persisted in spite of rigorous statistical analysis but has grown with time so that it cannot be altogether ignored. Recent conference proceedings covering some aspects of discrete redshift, Pitucco et al (1996) and Dwari et al (1996). Newman et al (1994) and Newman and Terzian (1996) analyses the “Power Spectrum Analysis” (PSA) of Yu and Pebbles (1969), this is the statistical analysis used in discrete redshift studies. They find that this method generates a sequence of random numbers from observational data which, it was hoped, is exponentially distributed with unit mean and unit variance. The variable derived from this sequence is approximately exponential over much of its range but the tail of the distribution is far removed from an exponential distribution, so that statistical inference and confidence testing based on the tail of the distribution is unreliable. Newman and Terzian go on to say that
there are six claims of the PSA method which are wrong or involve some hidden assumptions. For purposes of illustration consider the first of these. Let \( N \) points \( x_j \) be distributed in the interval \( 0 \) to \( 2\pi \) and let

\[
z_n = N^{-1/2} \sum_{j=1}^{N} \exp(inx_j).
\]  

(1)

Yu and Peebles claim that the ensemble average of \( z_n(n \neq 0) \) is

\[
<z_n> = N^{-1/2} \sum <\exp(inx_j)> = N^{-1/2} \sum \int_{0}^{2\pi} \frac{dx_j}{2\pi} \exp(inx_j) = 0
\]  

(2)

Apparently the correct way to approach this is to suppose that the \( x_j \) are identically distributed and independent deviates with distribution \( P(x) \) and to define a characteristic generating function \( F(n) \) by

\[
F(n) \equiv <\exp(inx)> = \int_{-\infty}^{\infty} \exp(inx)dP(x),
\]  

(3)

then

\[
<z_n> = N^{1/2} F(n).
\]  

(4)

If the underlying distribution function \( P(x) \) is normally distributed (gaussian) with a mean \( \mu \) and a variance \( \sigma^2 \) then

\[
<z_n> = N^{1/2} \exp\left(-\frac{n^2\sigma^2}{2}\right).
\]  

(5)

Newman and Terzian also say that astronomers choose too small a frequency class interval (bin with) in their frequency histograms, and that the optimal is \( 1+\log_2(N) \), and this is illustrated by their smoother figures for the larger interval.

1.3 theoretical explanations

Theories to explain discrete redshift have been devised by Cocke and Tifft (1983) \[22\] and Cocke (1985) \[21\], Nieto (1986) \[77\], and Buitrago (1988) \[14\]. These theories depend on the introduction of a redshift quantum mechanical operator; this is essentially equivalent to replacing the emitter’s four-momentum \( P^a \) by an operator. Narlikar and Burbidge (1981) produce an explanation which has a two component model of the Universe with discrete matter. An explanation was devised by Barut et al (1994) \[1\] where:
“It is shown that the energy distribution in this model is periodic and the periods and density decrease with increasing distance, in striking agreement with experimental data.”. Discrete redshift of a quantum energy spectrum in an anisotropic universe was found by Lamb et al (1994) [58]. Discrete redshift might be caused by dislocation solutions to equations, such solutions have been described by Edelen (1994) [32]. Arp (1996) [3] argues that there is a “signal carrier” for inertial mass, which he calls the machion, and this gives rise to periodicity. Greenberger (1983) [12] and Carvalho (1985) [18] generalizes the quantum commutator to \([q,p] = i\hbar + if(q,p)\), where \(f(q,p)\) is a function, Carvalho (1997) [19] uses this to calculate a redshift spectrum with discrete values. Hill et al (1990) [15] constructed three alternative models involving oscillating physics: i) an oscillating gravitational constant, this was also studied by Salgado et al (1996) [85], ii) oscillating atomic electron mass, this was ruled out by Sudarsky (1992) [96] on the basis of the Braginsky-Panov experiment, and iii) oscillating galactic luminosities. A varying Hubble parameter has been used by Morikawa (1991) [69] as an explanation. A Voroni cellular model was used by van de Weygaert (1991) [112] to explain the Broadhurst et al result. Ikeuchi and Turner (1991) [7] also use a three dimensional Voroni tessellation model to explain voids and walls. Williams et al (1991) [117] find that the Voroni foam model predicts a scale length for the galaxy-galaxy correlation function which is too large. Hill et al (1991) [49] suggest a particular coherent sinusoidal peculiar velocity field of amplitude \(\delta/c \approx 3 \times 10^3\) and wavelength \(\lambda \approx 128h^{-1}\)Mpc could explain the result of Broadhurst et al. An oscillating gravitational constant model was constructed by Crittenden and Steinhart (1992) [25] to explain the Broadhurst et al result. Dekel et al [27] argue that the Broadhurst et al result suggests a large scale origin for periodicity. Budinick et al (1995) [1] built another model to explain the Broadhurst et al result. The existence of superclusters and voids is perhaps explained by the cold dark matter theory of White et al (1987) [115]. Redshift periodicity can be used to probe the correctness of general relativity Faroni (1997) [37]. Lui and Hu (1998) [63] suggest an explanation which has the mean free path of heavy elements absorb system varies regularly with cosmic time. Farhi et al (1998) [36] technique might provide an explanation. Some papers on the quantum mechanics of large macroscopic systems are: Greenberger (1983) [12], Agnese (1984) [1], DerSarkissan (1984) [2] and (1985) [24], Carvalho (1985) [18], Silva (1997) [33] Capozziello et al (1998) [16], and Carneiro (1998) [17].
1.4 discrete as opposed to continuous energy spectrum

It is a common error in non relativistic quantum mechanics to suppose that quantization implies that there must be a discrete energy spectrum, see for example Schiff (1949) [87] page 34. For an infinite potential $V(r)$ there are discrete energy levels; however for a potential with $E > V$ there are continuous energy levels when $E > V$. Usually however a discrete spectrum is usually an indication of a system with quantum rather than classical properties. By analogy with non relativistic quantum mechanics a closed gravitational interacting system would be expected to display discrete properties at low energies, i.e. when the gravitational field is weakly interacting, rather than at high energies when the gravitational field is strong. This suggests that cosmology and extra-galactic astronomy - as opposed to particle physics, are the subject areas where a single theory combining quantum mechanics and gravity would be necessary. There are many unusual dynamical properties of large scale systems, such as galaxies, see for example Roberts (1991) [81] and references therein, and it might be that these are directly attributable to quantum corrections to classical theory; however here attention is restricted to systems that are characterized by discrete properties rather than unusual dynamics. Bohr-Sommerfeld quantization rule have been applied to gravitational systems: Wereide (1923) [114] applied them to spherically symmetric spacetimes to find the line element of the electron, and Agnese and Testa (1997) [2] applied them to planetary orbits. There are discrete approaches to quantum gravity Loll (1998) [62], and it might be that discrete structure from near the Planck era is inflated to present day large scale discrete structure. Random walk models suggest a fundamental length of $l \simeq 10^{35}$ cm. Sidharth (1999) [94].

1.5 sectional contents

In section 2 the exact solution for the equation of state $p = (\gamma - 1)\mu$ Robertson-Walker spacetime is derived. This is done because the critical parameter $2q_0$ occurs in latter approximations to the Klein-Gordon equation and these exact solutions clarify when the occurrence of this critical parameter is an artifact of the approximations involved; also the standard approach to redshift in Robertson-Walker spacetime is given. Section 3 consists of general remarks on what properties a theory of discrete redshift would be expected to have and discusses various approaches to constructing a theory. Section 4 is devoted to finding approximate solutions to the
Klein-Gordon equation in Robertson-Walker spacetimes. Section 5 applies these approximate solutions to theories of discrete redshift where the discreteness of radiation originates from the motion of the emitter. Section 6 discusses how discrete redshift might occur via the massive Klein-Gordon equation. Section 7 uses the solutions for the Klein-Gordon equation in the Einstein static universe to induce weak field metric perturbations of Robertson-Walker spacetime, these weak metric perturbations are used to demonstrate discrete redshift.

1.6 conventions

The conventions used are: signature $-+++$, early latin indices $a, b, c \ldots = 0, 1, 2, 3$, middle latin indices $i, j, k \ldots = 1, 2, 3$, Riemann tensor

$$R_{abcd}^a = 2\partial_c\Gamma_{db}^a + 2\Gamma_{[c|j}^a\Gamma_{d]b},$$

(6)

Ricci tensor

$$R_{bd} = R_{b0d},$$

(7)

commutation of covariant derivatives

$$X_{abcda} - X_{abacd} = X_{aecd}R_{bdc} + X_{ebd}R_{a0c}.$$  

(8)

Relativistic units are not used; $c, G,$ and $\hbar$ are put explicitly into equation where appropriate, this is done in order to clarify the relative size of the terms in the approximations used, $c$ is included in the definition of the Hubble constant $H_0$.

2 Robertson-Walker Spacetime.

2.1 the line element

The Robertson-Walker line element is

$$ds^2 = -c^2N(t)^2dt^2 + R(t)^2d\Sigma_3^2,$$

(9)

where $R$ is the scale factor and

$$d\Sigma_3^2 = d\chi^2 + s(\chi)^2(d\theta^2 + \sin^2\theta d\phi^2),$$

(10)

with

$$s(\chi) = \begin{cases} 
sin(\chi) & \text{for } k = +1 \\
\chi & \text{for } k = 0 \\
\sinh(\chi) & \text{for } k = -1. 
\end{cases}$$
The lapse function $N$ is arbitrary and depends on the time coordinate used. Three common choices for the lapse function are: i) $N = 1$ for which the time coordinate is the same as the proper time of a co-moving observer, ii) $N = R$ for which the line element is conformal to the Einstein static Universe, and the time coordinate referred to as conformal time, and iii) $N = R^3$ for which $\Gamma^t \equiv g^{\alpha \beta} T^t_{\alpha \beta} = 0$, and the time coordinate referred to as harmonic time. For $k = +1$ using the coordinate transformations, Müller (1939) \[70\]

\[
\begin{align*}
\sin(\alpha) &= \sin(\theta)\sin(\chi), \\
\cos(\beta) &= \sqrt{1 + \cos^2(\theta) \tan^2(\chi)}, \\
\cos(\theta) &= \sin(\beta)\sqrt{\sin(\beta) + \tan^2(\alpha)}, \\
\cos(\chi) &= \cos(\alpha)\cos(\beta),
\end{align*}
\]

the three-sphere line element becomes

\[
\begin{align*}
\text{d}^2 \Sigma_{3^+} &= d\alpha^2 + \cos^2(\alpha)d\beta^2 + \sin^2(\alpha)d\phi^2,
\end{align*}
\]

and the three-geodesic distance $\omega$ takes the simple form

\[
\cos(\omega) = \cos(\alpha)\cos(\alpha')\cos(\beta - \beta') + \sin(\alpha)\sin(\alpha')\cos(\phi - \phi').
\]

2.2 Taylor series expansion

The geodesic distance, or world function is known only for a few special cases, Roberts (1993) \[82\]. For $N = 1$, and for times close to an observers time $t_0$ the scale factor can be expanded as a Taylor series

\[
R = R_0 \left[ 1 + H_0 \delta t - \frac{1}{2} q_0 H_0^2 \delta t^2 + \frac{1}{6} j_0 H_0^3 \delta t^3 + O(H_0 \delta t)^4 \right],
\]

where

\[
\delta t = t - t_0, \quad \dot{R} \equiv \partial_t R,
\]

\[
H \equiv \frac{\dot{R}}{R}, \quad q \equiv -\frac{\dot{R} R}{R^2}, \quad j \equiv \frac{\ddot{R} R^2}{R^3}, \quad \text{(14)}
\]

and the subscripted “0” as in $H_0$ refers to the value of the object measured by a observer at $t = t_0$. For arbitrary lapse and $c$ explicit these
become
\[ H \equiv \frac{\dot{R}}{cN R}, \quad q \equiv -\frac{1}{R^2} \left( \frac{\ddot{N}}{N} - \frac{\dot{N} \dot{R}}{N} \right), \]
\[ j \equiv \frac{R^2}{R^3} \left( \ddot{R} - 3 \frac{\dot{R} \dot{N}}{N} - \frac{\dot{R} \ddot{N}}{N} + 3 \frac{R N^2}{N^2} \right). \] (15)

### 2.3 the field equations

For the line element \( l \) the Christoffel symbols are
\[ \{^i_{(u)} \} = \{^i_{(u)} \} = 0, \]
\[ \{^i_{(u)} \} = \frac{\dot{N}}{N}, \quad \{^i_{(u)} \} = \frac{\dot{R}}{R}, \quad \{^i_{(u)} \} = g_{ij} \frac{\dot{R}}{c^2 N^2 R}, \]
\[ \{^\alpha_{\beta} \} = \cos(\alpha) \sin(\alpha), \quad \{^\alpha_{\phi} \} = -\cos(\alpha) \sin(\alpha), \]
\[ \{^\beta_{\alpha} \} = -\tan(\alpha), \quad \{^\phi_{\alpha} \} = \cot(\alpha). \] (16)

The Riemann tensor is
\[ R^{(3)}_{ijkl} = g^{(3)}_{ik} g^{(3)}_{lj} - g^{(3)}_{li} g^{(3)}_{kj}, \]
\[ R_{ijkl} = \left( k \frac{\dot{R}^2}{R^2} + \frac{\dot{R}^2}{c^2 N^2 R^2} \right) (g_{ik} g_{lj} - g_{li} g_{kj}), \]
\[ R_{titj} = \left( -\frac{\dot{R}}{R} + \frac{\dot{N} \dot{R}}{NR} \right) g_{ij}. \] (17)

Using Einstein’s field equations the density \( \mu \) and the pressure \( p \) for a perfect fluid are given by the Friedman equation
\[ \frac{8\pi G}{c^4} \mu = 3 \frac{k}{R^2} + 3 \frac{\dot{R}^2}{c^2 N^2 R^2}, \] (18)
and the pressure equation
\[ \frac{8\pi G}{c^4} p = -\frac{k}{R^2} - \frac{1}{c^2 N^2} \left( 2 \frac{\dot{R}}{R} + \frac{\dot{R}^2}{R^2} - 2 \frac{\dot{N} \dot{R}}{NR} \right). \] (19)

The conservation equation is
\[ \frac{d}{dR} (\mu R^3) = -3pR^2. \] (20)
compare Weinberg [113] equation 15.1.21. For the equation of state

\[ p = (\gamma - 1)\mu, \]  
(21)

\((\gamma - 1)\) times the Friedman equation [18] minus the pressure equation [19] is

\[ 0 = \frac{3\gamma - 2}{R^2} \left( k + \frac{\dot{R}^2}{c^2 N^2} \right) + \frac{2}{c^2 N^2} \left( \frac{\dot{R}}{R} - \frac{\dot{N}\dot{R}}{NR} \right), \]  
(22)

evaluating this at \(t = t_0\) and using [15]

\[ 0 = k(3\gamma - 2) + H_0^2 R_0^2 (3\gamma - 2 - 2q_o), \]  
(23)

which for given \(\gamma\) and \(q\) determines the sign of \(k\) and hence whether the Robertson-Walker line-element is open or closed, in the particular case \(\gamma = 1\) and \(N = 1\), [23] reduces to Weinberg [113] equation 15.2.5. For the equation of state [21] the conservation equation [20] is

\[ \gamma \mu^{1-1/\gamma} \frac{d}{dR} \left( \mu \frac{1}{R} R^3 \right) = \mu_{,R} R^3 + 3\gamma \mu R^2 = 0. \]  
(24)

Integrating

\[ \mu^{1/3} R^3 = a, \]  
(25)

where \(a\) is a constant. Thus \(\mu\) is proportional to \(R^{-3\gamma}\) so that

\[ \frac{\mu}{\mu_0} = \left( \frac{R_0}{R} \right)^{3\gamma}, \]  
(26)

\(\mu_0\) is also given by the Friedman equation [18] at \(t_0\), combining with [26] this gives

\[ \mu = \frac{3c^4}{8\pi G} \left( H_0^2 + \frac{k}{R_0^2} \right) \left( \frac{R_0}{R} \right)^{3\gamma}. \]  
(27)

where a factor of \(c\) is included in our definition of \(H\) [15]. Substituting [25] into [18] the Friedman equation becomes

\[ \left( \frac{\dot{R}}{cN} \right)^2 + k = \left( \frac{R}{R_0} \right)^{2-3\gamma} \left( k + H_0^2 R_0^2 \right). \]  
(28)
2.4 the general solutions for $\gamma$-equation of state

For $3\gamma = 2$ the solutions of the Friedman equation is the generalized Milne universe

$$N = 1, \quad R = R_0(1 + H_0t),$$

(29)

for which the scale factor $R$ is just given by [14] up to first order. For $3\gamma \neq 2$, define

$$R_\rho \equiv R_0 \left( \frac{H_0^2 R_0^2 + k}{\gamma - 2} \right)^{\frac{1}{3\gamma - 2}},$$

(30)

then equation [28] fixes the constant in the solution of Vajk (1967) [111]

$$k = 1, \quad cN = R = R_\rho \left[ \sin \left( \frac{3\gamma - 2}{2} \eta \right) \right]^{\frac{2}{3\gamma - 2}},$$

(31)

$$k = 0, \quad N = 1, \quad R = R_0 \left( \frac{3\gamma cH_0t}{2} \right)^{\frac{2}{3\gamma}},$$

(32)

$$k = -1, \quad cN = R = R_\rho \left[ \sinh \left( \frac{3\gamma - 2}{2} \eta \right) \right]^{\frac{2}{3\gamma - 2}},$$

(33)

where $\eta$ is the time coordinate used when $N = R$. These solutions can be expanded around the origin $t = 0$ of the time coordinate

$$R = R_\rho \left( \frac{3\gamma - 2}{2} \right)^{\frac{2}{3\gamma - 2}} \left[ 1 - \frac{k}{6} \left( \frac{3\gamma - 2}{2} \right) \eta \right]^2 + O \left( \frac{3\gamma - 2}{2} \eta \right)^4.$$

(34)

Using proper time, for $\gamma = \frac{4}{3}$

$$R = R_0 \left[ 1 + kH_0^2 R_0^2 - \frac{kc^2 t^2}{R_0^2} \right]^\frac{\frac{1}{2}}{2},$$

(35)

and for $\gamma = 1$ there is the expansion round the origin

$$R = R_\rho \left[ 1 - \frac{k}{4} \left( \frac{ct}{R_\rho} \right)^2 - \frac{k}{48} \left( \frac{ct}{R_\rho} \right)^4 + O \left( \frac{ct}{R_\rho} \right)^6 \right],$$

(36)

and also an expansion in powers of $t^2$. The substitution $t \to t - t_0$ merely shifts the origin of the time coordinate, i.e. the singularity of the spacetime moves from $t = 0$ to $t = t_0$. Replacing $t$ by

$$t = t_0 + \delta t, \quad \delta t = t - t_0,$$

(37)

in all of the above gives back the Taylor expansion [14].
2.5 the geodesics

The Lagrangian for geodesics is

$$2\mathcal{L} = g_{ab} \frac{dx^a}{ds} \frac{dx^b}{ds},$$  \hspace{1cm} (38)

for timelike, null, and spacelike geodesics $2\mathcal{L} = -1, \quad 0, \quad +1$ respectively. A co-moving geodesic is a timelike geodesic with $\frac{dx^i}{ds} = 0$, the properties and (38) imply that the co-moving velocity vector is always, irrespective of the geometry of the spacetime, given by

$$U^a = \pm \frac{dx^a}{ds} = \pm \left( \frac{1}{cN}, 0 \right).$$  \hspace{1cm} (39)

For null geodesics in Robertson-Walker spacetime

$$0 = -c^2 N^2 \left( \frac{dt}{d\Omega} \right)^2 + g_{ij} \frac{dx^i}{ds} \frac{dx^j}{ds},$$  \hspace{1cm} (40)

using the three-sphere distance (13), (40) can be solved to give

$$\Omega = \int \frac{cN}{R} dt + \omega, \hspace{1cm} (41)$$

thus the radiation vector is

$$k_a = \Omega, a = \left( \frac{cN}{R}, \omega_i \right).$$  \hspace{1cm} (42)

The redshift is given by the equations, Ellis (1971) (33)

$$1 + z = \frac{d\lambda_0}{d\lambda} = \frac{\lambda_0}{\lambda} = \frac{\nu}{\nu_0} = \frac{U^a k_a}{(U^a k_a)_0},$$  \hspace{1cm} (43)

as in (13) the subscript “0” refers to the observer. The quantities at the emitter have no subscript. $\nu$ and $\lambda$ are the frequency and wavelength of the radiation; $s$ is the proper time. Equations (33), (42), and (43) give independently of the choice of $N$,

$$1 + z = \frac{R_0}{R}. \hspace{1cm} (44)$$

For the Taylor series expansion (13) this becomes

$$z = -H_0 (t - t_0) + \left( 1 + \frac{1}{2} q_0 \right) H_0^2 (t - t_0)^2$$

$$- \left( 1 + q + \frac{1}{6} j_0 \right) H_0^3 (t - t_0)^3 + O (H_0 (t - t_0)), \hspace{1cm} (45)$$
the observed values of $H_0$ and $q_0$ are $H_0 = 2.4 \pm 0.8 \cdot 10^{-18}$ sec.$^{-1}$ and $q_0 = 1 \pm 1$, for a recent review see Freedman (1999) [40]. $z$ is found to take the discrete values

$$z_n = \frac{n v_I}{c},$$

where $n$ is an integer and $v_I = 72.2 \pm 0.2$ Km. sec.$^{-1}$, or perhaps a sixth of this value.

### 3 Remarks on Explanations of Discrete Redshift.

#### 3.1 three distinctions categorizing discrete redshift

There are THREE distinctions that should be made to categorize any theory of discrete redshift. The FIRST is whether the discrete properties enter via the radiation connecting the emitter and observer, or by the motion of either (or both) the emitter and observer. The SECOND is whether the emitter has real discrete motion, or only apparent discrete motion, or neither. The THIRD is whether the effect is due to quantizing the whole system, or part of it, or is not due to quantum mechanics at all.

#### 3.2 via connecting radiation

To elaborate on the FIRST distinction, from equation 43 it is apparent that discrete redshift must come from discrete differences in the ratio of the observer’s and emitters proper time $\frac{d\text{obs}}{d\text{em}}$. The underlying structure of spacetime may be so unusual as to forbid the introduction of a time-like four-vector $U^a$ or a null tangent vector $k_a$; however if this can be done then there are two choices for the origin of discrete properties: either the particles time-like four-vector or the connecting radiation appears to take discrete values. If the metric itself takes discrete values then both of these would presumably occur.

#### 3.3 real discrete motion

To elaborate on the SECOND distinction. Real discrete motion means that the emitter’s motion is discrete no matter how it is measured. Apparent discrete motion means that the motion of the emitter merely appears to be discrete to the observer; to put this another way if the observer chooses to observe from a different vantage point the observer might not measure the same discrete motion of the emitter. Real discrete motion has the disadvantage
that there would have to be boundaries at the edges of where the emitter’s four-velocity jumps, these boundaries would lead to other effects, such as refraction and reflection on the boundary surface. Such effects have not been observed in association with measurements of discrete redshift. Monet and Deharveny (1977) [68] do not observe what would be expected from real discrete motion, see subsection 1.2. The existence of superclusters and voids, Sanders et al (1991) [86], however might be an example of an effect of real discrete motion. The quantum perturbations of Grishchuk (1997) [43], Yamamoto et al (1996) [119], and Modanese (2000) [67] might give real discrete motion.

3.4 whole system quantization

To elaborate on the THIRD distinction. The idea of the whole system being treated quantum mechanically is essentially that of quantum cosmology. If part of the system is quantized it could be either the connecting radiation or the emitting matter. Of course there is the possibility that discrete redshift may not have a quantum origin at all, for example, it might be of fluid dynamical origin.

3.5 quantum emitter

In this paper the idea that the discrete properties originate in the quantum treatment of the emitting matter will be pursued. Before proceeding with this some remarks will be made on the possibility of discrete redshift originating in the properties of the connecting radiation from quantum cosmology. There are several possibilities which might give discrete connecting radiation, here two will be mentioned. The FIRST is that the connecting radiation could undergo a scattering process which makes it discrete, the scattering process would have to occur in a wide variety of circumstances in order to explain the many scales and wavelengths over which discrete redshift occurs. The SECOND is that solutions to Maxwell’s equations in Robertson-Walker spacetime involve discrete properties from spherical harmonics. This will be discussed further in Section 5.

3.6 quantum cosmology

Quantum cosmology consists of a large amount of theory in which the whole Universe is considered quantum mechanically (see Tipler (1986) [108] and Kiefer (1999) [54] for reviews); and it is the obvious framework in which to
start looking for an explanation of discrete redshift. There might be a quantum analog of the Friedman equation which possess solutions in which the metric is discrete. Developing quantum cosmology as it is found in the literature produced nothing along these lines, therefore four more simplistic approaches where attempted: as it would be anticipated that even a simple model of a quantum expanding gas would produce a qualitative result which involved discrete redshift. The dynamics of Robertson-Walker spacetimes are determined by the Friedman equation and the conservation equation. The first approach consisted of applying naïve operator substitutions to the Friedman equation; by this is meant replacing each symbol in the equation by an operator of the form $-ia\partial_b$ where $a$ is chosen to be the most general combination of $\hbar, c, G, R, R_0$ which provides a dimensionally correct substitution for the symbol, and $b$ can be variously considered to be a four-vector index or a time index etc… . This approach failed because it produced inconsistencies when applied to the conservation equation and it was impossible to eliminate $R$. A method similar to this which works has been constructed by Rosen (1993), see also Capozziello et al (1998). The second approach was to apply naïve operator substitutions to the Newtonian analog of the Friedman equation (see for example Weinberg (1972)). It might be anticipated that this would produce a well-defined theory because it involves only a quantum generalization of Newtonian cosmology, however in common with much of Newtonian cosmology $\dot{R}$ and $c$ occur in places that lead to inconsistencies. The third approach is to note that there are special relativistic gravitational theories, some of which are discussed in North (1965). It would be hoped that they would lead to equations with $\dot{R}$ and $c$ in consistent places, however this approach also failed. The fourth approach is that Schrödinger’s derivation of Robertson-Walker redshift by thermodynamic analogy might be extendable to include quantum mechanics and thence discrete redshift.

3.7 constituent quantization

Assuming that the Robertson-Walker spacetime remains correct, it is necessary to identify what constituents of its contents needs to be quantized and by what mechanism. In general relativity the co-moving emitter travels on geodesics for which

$$2\mathcal{L} = U_a U^a = \frac{1}{m} p_a p^a. \quad (47)$$
This equation can be naïvely quantized by using the operator substitution
\[ p_a \rightarrow i\hbar \nabla_a, \quad (48) \]
giving the Klein-Gordon equation
\[ \left( \Box - \frac{m^2c^2}{\hbar^2} \right) \psi = 0, \quad (49) \]
but it is not clear what the interpretation of \((49)\) is in the present context. For example, should the mass in the Klein-Gordon equation be interpreted as the mass of the emitting galaxy or the emitting atom or something in between, and has the single particle theory described by \((47)\) become a many particle theory described by \((49)\)? Here what the Klein-Gordon field describes will be discussed later. The Universe will be taken to have closed Robertson-Walker geometry, because it is for closed systems that discrete properties usually occur.

3.8 references for the KG equation in RW spacetime

The Klein-Gordon equation in Robertson-Walker spacetimes has been studied for a variety of purposes by: Schrödinger (1939) [89], (1956) [91], Müller (1940) [70], Lifshitz (1946) [60], Lifshitz and Khalatnikov (1963) [61], Ford (1976) [39], Barrow and Matzner (1980) [8], and Klainerman and Sarnak (1981) [55]. Maxwell’s equation in Robertson-Walker spacetimes has been studied by Schrödinger (1940) [90], and Mashhoon (1973) [65]. Dirac’s equation in Robertson-Walker spacetime has been studied by Schrödinger (1938) [88], (1940) [90], and Barut and Duru (1987) [10].

4 The Klein-Gordon Equation in Robertson-Walker Spacetime.

4.1 spherical harmonics

In Robertson-Walker spacetime the Klein-Gordon equation is
\[ 0 = -\frac{1}{NR^3} \left( \frac{R^3 \dot{\phi}}{N} \right) \cdot + \frac{c^2}{R^2} K(\phi) - \frac{c^4 m^2}{\hbar} \phi, \quad (50) \]
In the coordinates $12$, $K(\phi)$ takes the form

$$K(\phi) = \sec(\alpha)\csc(\alpha) (\cos(\alpha)\sin(\alpha)\phi_\alpha)_\alpha + \sec^2(\alpha)\phi_{\beta\beta} + \cosec^2(\beta)\phi_{\alpha\alpha}. \quad (51)$$

Define

$$Y \equiv \sin^{\lvert n \rvert\alpha} \cos^{\lvert p \rvert\alpha} (\alpha) \exp i(\lvert n \rvert\alpha + \lvert p \rvert\beta), \quad (52)$$

then

$$\frac{K(\phi)}{Y} = - (\lvert n \rvert + \lvert p \rvert) (\lvert n \rvert + \lvert p \rvert + 2). \quad (53)$$

The coordinate ranges $0 < \alpha < \frac{1}{2}\alpha$ and $0 < \beta, \phi < 2\pi$ imply that $n$ and $p$ are integers, thus defining

$$l \equiv \lvert n \rvert + \lvert p \rvert, \quad (54)$$

gives

$$K(\phi) = Y^i_{\lvert i} = -l(l + 2)Y, \quad (55)$$

and

$$Y^i_i = -l^2\frac{Y}{R^2}. \quad (56)$$

The complex conjugate to equation 55 and 56 also holds, furthermore

$$Y^iY^\dagger_i = \left[ n^2(\cot^2(\alpha) + \cosec^2(\alpha)) + p^2(\tan^2(\alpha) + \sec^2(\alpha)) - 2np \right] Y Y^\dagger, \quad (57)$$

and

$$(Y Y^\dagger)^i_{\lvert i} = 4 \sin^{\lvert 2n \rvert \alpha} \cos^{\lvert 2p \rvert \alpha} \left( n^2 \cot^2(\alpha) + p^2 \tan^2(\alpha) - 2np - p - n \right), \quad (58)$$

also

$$(Y Y^\dagger)_{\lvert i} = Y^i_i, \quad i \neq j, \quad (59)$$

$$i(Y Y^\dagger_{\alpha} - Y^\dagger Y_{\alpha}) = 0,$$

$$i(Y Y^\dagger_{\beta} - Y^\dagger Y_{\beta}) = 2pY Y^\dagger,$$

$$i(Y Y^\dagger_{\beta} - Y^\dagger Y_{\beta}) = 2nY Y^\dagger, \quad (60)$$

where the dagger "$\dagger$" denotes complex conjugate. Equations 57, 58, 59, and 60 cause difficulties when considering the stress of a Klein-Gordon field. Defining the dimensionless scalar field

$$\phi Y = \left( \frac{R}{R_0} \right)^{-\frac{3}{2}} \left( \frac{N}{N_0} \right)^{\frac{1}{2}} \psi, \quad (61)$$
the Klein-Gordon equation for $\psi \neq 0$ becomes
\begin{equation}
0 = \frac{\dddot{\psi}}{\psi} + X + c^2 l(l + 2) \frac{N^2}{R^2} + \frac{c^4 m^2 N^2}{\hbar}, \tag{62}
\end{equation}
where
\begin{equation}
X \equiv \frac{1}{2} \left( \frac{\dddot{N}}{N} - 3 \frac{\dddot{R}}{R} \right) - \frac{3}{4} \left( \frac{\dddot{R}}{R} - \frac{\dddot{N}}{N} \right), \tag{63}
\end{equation}

4.2 Hill’s equation

For $R$ and $N$ consisting of trigonometric functions this equation is similar to Hill’s equation, see for example p.406 Whittaker and Watson (1927) \[116\].

For the Einstein static universe there is the solution
\begin{equation}
\phi = C_+ \exp(i\nu t) + C_- \exp(-i\nu t), \quad \nu^2 = \frac{c^2 l(l + 2)}{R_0^2} + \frac{m^2 c^4}{\hbar^2}, \tag{64}
\end{equation}
where $C_+$ and $C_-$ are constants. For the generalized Milne universe there is the massless solution, Schrödinger (1939) \[89\],
\begin{equation}
\phi = C_+ \frac{\tau}{\tau} + C_- \frac{\tau}{\tau}, \tag{65}
\end{equation}
with
\begin{equation}
\tau = -(2H_0 R_0 R^2)^{-1},
\end{equation}
and
\begin{equation}
rt^2 = 1 - \frac{c^2 l(l + 2)}{H_0^2 R_0^2}.
\end{equation}

For the closed $\gamma = \frac{4}{3}$ spacetime \[26\] there is the massless solution, Lifshitz (1946) \[60\],
\begin{equation}
\phi = \cosec(\eta) \left( C_+ \exp(+i(l + 1)\eta) + C_- \exp(-i(l + 1)\eta) \right). \tag{66}
\end{equation}
In general \[62\] is intractable and it is necessary to use approximate WKB solutions to it.

4.3 WKB approximation

The WKB approximation is derived as follows, Alvarez (1989) \[4\]. Assume the differential equation can be put in the form
\begin{equation}
y'' + \frac{f(x)y}{\hbar^2} = 0, \tag{67}
\end{equation}

where $y' = \frac{dy}{dx}$. Let

$$y = \exp\left(\frac{iz}{\hbar}\right),$$

(68)

then (67) becomes

$$i\hbar z'' - z'^2 + f(x) = 0,$$

taking $\hbar$ to be small

$$z' = \pm \sqrt{f},$$

(70)

which gives the first order approximation

$$y = \exp\left(\frac{i}{\hbar} \int \sqrt{f} dx\right).$$

(71)

substituting the derivative of (70), $z''$, into (69)

$$z'^2 = f \left(1 \pm \frac{i\hbar f'}{2\sqrt{f}}\right),$$

(72)

taking the square root and disregarding terms in $\hbar^2$,

$$z' = \pm \sqrt{f} + \frac{i\hbar f'}{4\sqrt{f}},$$

(73)

where all combinations of sign are possible, choosing the sign in front of the second term to be positive, using equation (59), and integrating

$$y = f^{-\frac{1}{4}} \exp \left(\pm \frac{i}{\hbar} \int \sqrt{f} dx\right).$$

(74)

The second order approximation is then given by the linear combination

$$y = f^{-\frac{1}{4}} \left\{ C_+ \exp\left(\frac{i}{\hbar} \int \sqrt{f} dx\right) + C_- \exp\left(-\frac{i}{\hbar} \int \sqrt{f} dx\right) \right\}.$$

(75)

Substituting back into (67) this approximation holds if

$$\frac{f}{\hbar^2} > \frac{5}{16} \frac{f''}{f^2}.$$

(76)
4.4 WKB applied to the KG equation

Applying the WKB approximation to the Klein-Gordon equation implies that $\phi$ is of the form

$$
\phi = A(t) Y(\alpha, \beta, \gamma) \{ C_+ \exp(+i\bar{\nu}(t)) + C_- \exp(-i\bar{\nu}(t)) \},
$$

(77)

where $C_+$ and $C_-$ are dimensionless constants. There are two functions to be determined. The first is the dimensionless frequency $\nu$,

$$
\nu t = \bar{\nu} = \int \left( X + c^2 l(l+2) N^2 R^2 + \frac{c^4 m^2 N^2}{\hbar^2} \right)^{\frac{1}{2}} dt,
$$

(78)

where the constant of integration is a phase factor which is taken to vanish here. When $t$ is proper time $\nu$ is referred to as the proper frequency. The second is the dimensionless amplitude

$$
A = \left( \frac{R}{R_0} \right)^{-\frac{3}{4}} \left( \frac{N}{N_0} \right)^{\frac{1}{2}} A_\psi
$$

$$
= D \left( \frac{R}{R_0} \right)^{-\frac{3}{4}} \left( \frac{N}{N_0} \right)^{\frac{1}{2}} \left( X + c^2 l(l+2) N^2 R^2 + \frac{c^4 m^2 N}{\hbar^2} \right)^{-\frac{1}{4}},
$$

(79)

where $D$ is a dimensional constant added in order to keep $A$ dimensionless. Both the frequency $\nu$ and the amplitude $A$ are sensitive to the choice of time coordinate; for example in proper, conformal, and harmonic times respectively

$$
N = 1 : X = -\frac{3}{4} \left( \frac{2\dot{R}}{R} + \frac{\dot{R}^2}{R^2} \right) = \frac{3}{4} \left( \frac{8\pi G p}{c^4} + \frac{c^2}{R^2} \right),
$$

$$
N = R : X = -\frac{R_{\text{eff}}}{R},
$$

$$
N = R^3 : X = 0.
$$

(80)

The amplitude $A$ is computed from straightforward substitution; however the frequency integral usually has to be approximated.

4.5 the Taylor series approximation for the frequency

For the Taylor series approximation

$$
X = \frac{3}{4} \left\{ (2q_0 - 1)H_0^2 + 2(j_0 - 1)H_0^3(t-t_0) + O(t-t_0)^2 \right\}.
$$

(81)
Up to second order in the Taylor series expansion \(14\) this expression is time
dependent, also it depends on the quantity \((2q_0 - 1)\) from \(23\) this is just the
critical number which determines whether a pressure free \(\gamma = 1\) spacetime is
open or closed. Thus any prediction of \((2q_0 - 1)\) using \(81\) is just an artifact
of the approximations involved rather than a prediction based on \(\gamma = 1\)
spacetime, this is the reason that it is necessary to work with exact solutions
to Einstein’s equations rather than use Taylor series approximations. From
\(81\) and \(78\)
\[
\nu = \left\{ \frac{3}{4}(2q_0 - 1)H_0^2 + \frac{c^2l(l+2)}{R_0^2} + \frac{c^4m^2}{h^2} \right\}^{\frac{1}{2}}
+ \frac{1}{4} \left\{ \frac{3}{4}(2q_0 - 1)H_0^2 + \frac{c^2l(l+2)}{R_0^2} + \frac{c^4m^2}{h^2} \right\}^{-\frac{1}{2}} \cdot \left\{ 2(j_0 - 1)H_0^2 - \frac{2c^2l(l+2)}{R_0^2} \right\} H_0(t - t_0)
+ \mathcal{O}(t - t_0)^2. \tag{82}
\]

4.6 the dimensionless frequency for the \(\gamma\)-solutions

For perfect fluids with \(N = R\) the solution to Einstein’s field equations with
\(k = 1\) gives
\[
X = \frac{3\gamma - 2}{2} + \frac{3\gamma - 4}{2} \cot^2 \left( \frac{3\gamma - 2}{2} \eta \right). \tag{83}
\]

For \(m = 0\) the frequency \(\tilde{\nu}\) is
\[
\tilde{\nu} = \int \left\{ \frac{3\gamma - 2}{2} + \frac{3\gamma - 4}{2} \cot^2 \left( \frac{3\gamma - 2}{2} \eta \right) + l(l+2) \right\} \frac{1}{2} d\eta. \tag{84}
\]

Assuming that \(l\) is large and expanding
\[
\nu = \sqrt{l(l+2)} \left\{ \eta + \frac{1}{2l(l+2)} \left( \eta + \frac{3\gamma - 4}{2 - 3\gamma} \cot \left( \frac{3\gamma - 2}{2} \eta \right) \right) \right\} + \mathcal{O}(l^{-4}). \tag{85}
\]

For \(m \neq 0\) it is assumed that \(\hbar\) is small in line with the assumption needed
for the WKB approximation. Equivalently, expressions for the wavelength
are of the form
\[
\frac{1}{\lambda_{\text{total}}^2} = \frac{1}{\lambda_{\text{compton}}^2} \pm \frac{1}{\lambda_{\text{geometry}}^2}, \tag{86}
\]
where \(\lambda_{\text{compton}} = \frac{\hbar}{mc}\) is the Compton wavelength. The assumption that \(\hbar\) is
small implies that the total wavelength \(\lambda_{\text{total}}\) is dominated by the Compton
term. Usually, as can be seen from the sign in front of the geometric contribution is positive, thus $\lambda_{\text{total}}$ is shorter than the Compton wavelength. Expanding in $\hbar$ the frequency is

\[ \tilde{\nu} = \int \left\{ \frac{3\gamma - 2}{2} + \frac{3\gamma - 2}{2} \cot^2 \left( \frac{3\gamma - 2}{2} \eta \right) + l(l+2) + \frac{m^2 c^2 R^2}{\hbar^2} \right\}^{\frac{1}{2}} d\eta \]

\[ = \frac{mc}{\hbar} I_1 + \frac{\hbar}{2mc} \left\{ \frac{3\gamma - 2}{2} + l(l+2)I_2 + \frac{3\gamma - 4}{2} I_3 \right\} + O(\hbar^3), \quad (87) \]

where

\[ I_1 \equiv \int R \, d\eta, \quad I_2 \equiv \int \frac{d\eta}{R}, \quad I_3 \equiv \int \cot^2 \left( \frac{3\gamma - 2}{2} \eta \right) \frac{d\eta}{R}. \quad (88) \]

Evaluating these integrals for $\gamma = \frac{4}{3}$ and $\gamma = 1$ gives

\[ \tilde{\nu} = \frac{cmR_0}{\hbar} \left( H_0^2 R_0^2 + 1 \right)^{\frac{1}{2}} \cos(\eta) \]

\[ + \frac{\hbar(l+1)^2}{2cmR_0} \left( H_0^2 R_0^2 + 1 \right)^{-\frac{1}{2}} \ln \left| \tan \frac{\eta}{2} \right| \]

\[ + O(\hbar^3), \quad (89) \]

and

\[ \tilde{\nu} = \frac{cmR_0}{\hbar} \left( H_0^2 R_0^2 + 1 \right) \left\{ \frac{\eta}{2} - \cos \left( \frac{\eta}{2} \right) \sin \left( \frac{\eta}{2} \right) \right\} \]

\[ + \frac{\hbar}{2cmR_0} \left( H_0^2 R_0^2 + 1 \right) \cot(\eta) \left\{ \frac{1}{3} \cot^2 \left( \frac{\eta}{2} \right) - \frac{1}{2} (l^2 + 2l + \frac{1}{2}) \right\} \]

\[ + O(\hbar^3), \quad (90) \]

respectively.

### 4.7 further approximation

Using proper time $30$ and $80$ give for the expansion around the origin

\[ X = \frac{3(3\gamma - 2)}{4R_0^2} + O(t). \quad (91) \]
Defining
\[ l'^2 \equiv l^2 + 2l + \frac{3(3\eta - 2)}{4}, \tag{92} \]
the proper frequency is
\[ \nu = \nu_\rho + O(t), \tag{93} \]
where
\[ \nu_\rho^2 = \frac{c^2l'^2}{R_\rho^2} + \frac{c^4m^2}{\hbar^2}. \tag{94} \]
For \( \gamma = 1 \), using 80 and 36 the integral for the proper frequency is
\[ (\bar{\nu}, t)^2 = \nu_\rho^2 + \frac{c^2l'^2}{2R_\rho^2} \left( \frac{ct}{R_\rho} \right)^2 + \frac{11c^2l'^2}{48R_\rho^2} \left( \frac{ct}{R_\rho} \right)^4 + O \left( \frac{ct}{R_\rho} \right)^6, \tag{95} \]
expanding the square root and integrating gives the proper frequency
\[ \nu = \nu_\rho \left\{ 1 + \frac{c^2l'^2}{12\nu_\rho R_\rho^2} \left( \frac{ct}{R_\rho} \right)^2 + \left( \frac{11}{3} \nu_\rho - \frac{c^2l'^2}{R_\rho^2} \right) \frac{c^2l'^2}{160\nu_\rho^2 R_\rho^2} \left( \frac{ct}{R_\rho} \right)^4 + O \left( \frac{ct}{R_\rho} \right)^6 \right\}. \tag{96} \]

5 Discrete Redshift via the Connecting Radiation.

5.1 explanation using the frequency of the connecting radiation

The simplest way to produce a theory of discrete redshift is to note that in 13 \( z \) has an expansion in terms of the frequency \( \nu \) and that the solutions to the massive Klein-Gordon equation also involve a frequency. From 80 this frequency depends on the choice of time coordinate, however the \( l \) dependent term is usually larger than the \( X \) term so that this choice only makes a small difference; because of this it is sufficient to use the proper frequency. Choosing the massless proper frequency 93, the equation for the redshift 43 becomes
\[ \nu_0 (1 + z) = \frac{cl'}{R_\rho} + O(t), \tag{97} \]
which implies that
\[ l' \simeq (1 + z) \frac{\nu_0}{H_0} \left( \frac{3\gamma - 2}{2q_0 + 2 - 3\gamma} \right)^\frac{3\gamma - 2}{3\gamma - 2} \left( \frac{2q_0}{2q_0 + 2 - 3\gamma} \right)^\frac{1}{3\gamma - 2}, \tag{98} \]
as \( \frac{\nu_0}{H_0} \sim 10^{26} - 10^{32} \), \( l \) must be a very large number, as noticed by Schrödinger (1939) 89.
5.2 preservation of proper frequency

Equation 97 depends on $z, 1, \nu_0, c, H_0, q_0$ and $\gamma$. $z$ varies and $\nu_0, H_0$ and $q_0$ are constants by definition, thus at least one of $l, c$ or $\gamma$ must also vary. $c$ and $\gamma$ cannot vary enough to explain $z$, therefore suggesting that $l$ must vary. By the variables separable assumption 61 $l$ is independent of time, however here it is taken that $l$ varies slowly so that 7 and 8 holds in approximation only. In this section $\nu$ is assumed to be the frequency of the electromagnetic connecting radiation and the nature of the variation in $l$ is taken to be such that $\nu$ maintains the measured value of $z$; and this property is a particular example of a property here called the preservation of proper frequency. In general at the most fundamental level a co-moving quantum system would be expected to have basic states dependent upon a proper frequency $\nu$ proportional to $\frac{c^2(l+2)}{R^2} + \frac{\nu^2m^2}{h^2}$. The scale factor $R$ is time dependent, in order for the proper frequency to be nearly conserved (or perhaps fixed by some other considerations), it is necessary for $l$ to vary. This requirement is here call the preservation of proper frequency. This principle might imply that some quantity of matter on microscopic scales depends on $R$ and $l$. Precisely what this quantity may be is unclear. The value of the fundamental constants may vary with time, Dirac (1937) [30], and it could be that these depend on $R$ and $l$. An alternative way of viewing the preservation of proper frequency is by using the Planck equation; because $\nu$ remains nearly a constant the energy $E = \hbar \nu$ will remain nearly constant. From 45 and 46 the characteristic time interval corresponding to one unit of discrete redshift is

$$t_{\text{char}} = \frac{v_I}{cH_0} = 3 \pm 1 \times 10^6 \text{ years},$$

(99)

thus making direct measurements depending on $l$ unlikely. Suppose that the value of discrete redshift 16 corresponds to $l$ varying by a factor of one

$$\delta z = \frac{v_I}{c} = z_{i+1} - z_i = \frac{\nu_{i+1} - \nu_i}{\nu_0}.$$  

(100)

Using the value of $\nu_0$ from 97

$$\frac{v_I}{c(1 + z)} = \left( \frac{1 + 4/l + \left(\frac{9\rho}{4} + \frac{3}{2}\right)/l^2}{1 + 2/l + \left(\frac{9\rho}{4} - \frac{3}{2}\right)/l^2} \right)^{\frac{1}{2}} - 1$$

(101)

expanding for large $l$

$$\frac{v_I}{c(1 + z)} = \frac{1}{l} + O(l^{-2}).$$

(102)
From \( \text{[45]} \) this gives \( l \sim 10^7 \). As \( l \) is large \( l \simeq l' \) and substituting for \( l' \) from \( \text{[18]} \)

\[
\frac{\nu_I \nu_0 \sqrt{3\gamma - 2}}{cH_0} = (2q_0)^{\frac{1}{2-3\gamma}} (2q_0 + 2 - 3\gamma)^{\frac{\gamma}{2(3\gamma - 2)}}.
\]

This gives a very large \( q_0 \sim 10^{38} - 10^{50} \), well outside the observational limits and comparable in size to \( 10^{42} \) the Dirac (1937) \( \text{[30]} \) dimensionless constant.

### 5.3 refinements

The above model has scope for refinements: for example by using Maxwell’s equations instead of the Klein-Gordon equation, and more importantly choosing that the field \( \phi \) depends on both \( t \) and \( \chi \) so that the radiation connects observer and emitter; however this is not pursued as the model has serious problems which it is unlikely that these refinements would overcome. The most important of these is that it predicts a value for \( q_0 \) many orders of magnitude larger than allowed for by observation. It does not give consistent values for \( l \), if \( \nu \) is taken to be given by the value given by \( \text{[48]} \) and \( \text{[42]} \) then \( l \sim 1 \), \( \text{[48]} \) gives \( l > 10^{26} \), and \( \text{[42]} \) gives \( l \sim 10^7 \). It predicts that \( \nu_I \) should depend on frequency and this is not observed, \( \nu_I \) has the same value using either optical or radio data. The model is not quantum mechanical because the massless Klein-Gordon equation does not depend explicitly on \( \hbar \). The positive aspects of the model are that it is simple and predicts apparent discrete motion.

### 6 Discrete Redshift via the Massive Klein-Gordon Equation.

#### 6.1 associate Klein-Gordon momenta with the comoving velocity

The scalar field solutions of section \( \text{[4]} \) can be interpreted as being the wave function for an element of quantized matter in Robertson-Walker spacetime. The stress of the scalar field can be calculated by substitution. Associated with this stress is the momentum density

\[
P_a = T_{ab} U^b,
\]

where \( U^b \) is a time-like vector-field tangential to an observers world-line. In the present case \( U^b \) can be taken to be the co-moving vector \( \text{[29]} \). Corre-
sponding to the momentum density \( P_a \) there is the velocity vector

\[
V_a = \frac{4\pi R^3}{3m} P_a.
\]  

(105)

In the present case this vector has only the component \( V_t \). This velocity vector can be taken to be almost co-moving

\[
V_a \approx U_a,
\]  

(106)

where \( U_a \) is the co-moving vector. Two immediate consequences of this assumption are that: the redshift constructed using \( V_a \) is almost the same as that constructed from \( U_a \), and that the redshift is discrete. The almost co-moving stress assumption can be used to calculate the size of the scale factor by methods similar to those of the previous section. It predicts a scale factor of microscopic size. A variant of this approach is to take the vector

\[
W_a = \frac{\phi_a}{\sqrt{-\phi_c\phi^c}},
\]  

(107)

to be almost co-moving. This variant is similar to the co-moving stress approach, but it also predicts angular terms because of non-vanishing of \( W_i \). The reason that the moving stress approach gives the wrong result is that it is unrealistic to consider the stress given by a single scalar field. It might be hoped that a statistical ensemble of such wave functions would produce the correct co-moving stress. It is not immediately apparent how to construct such ensembles. For a review of statistical mechanics in curved spacetime see Ehlers (1971) [33]. It can be assumed that such statistical ensembles give back the solutions of section 4 and that the single Klein-Gordon equation produces a quantum perturbation of this. This approach is pursued in section 7.

6.2 other massive Klein-Gordon approaches

There are several other approaches based on solutions to the massive Klein-Gordon equation. In non-relativistic quantum mechanics there is the equation

\[
\langle p_a \rangle = -i\hbar \int d^3x \phi^\dagger \partial_a \phi,
\]  

(108)

see for example Schiff (1949) [87] equation 7.8. This equation would give non-vanishing \( p_i \) components, so that the requirement that \( \frac{p_a}{m} \) is approximately co-moving again would not hold; also it is not clear what the interpretation of (108) is in relativistic quantum mechanics. Another approach is
to use the group velocity $c_{\text{group}}$ of $\phi$, as defined by Schrödinger (1939) [89], to define an effective energy and hence $p_i$ component; however the existence of a non-negligible group velocity $c_{\text{group}}$ would again imply non-negligible $p_i$ components. There is an entirely different approach which consists of investigating how the hydrogen atom behaves in a Robertson-Walker universe, see for example Trees (1956) [109]; the problem with this approach is that it requires the value of $l$ to be small and is thus unsuitable for the problem in hand.

7 Discrete Redshift via Quantum Perturbation of Classical Solutions.

7.1 general weak metric perturbation

The metric is taken to be of the form

$$g_{ab} = \bar{g}_{ab} + h_{ab}, \quad g^{ab} = \bar{g}^{ab} - h^{ab},$$

(109)

where $\bar{g}_{ab}$ is a given background field, in the present case this is the Robertson-Walker metric [3], and $h_{ab}$ is a small perturbative term. The connection is

$$\Gamma^a_{bc} \equiv \{a}_{(bc)} + \frac{1}{2} K^a_{bc},$$

(110)

where the contorsion is

$$K_{abc} = h_{ba;c} + h_{ca;b} - h_{bc;a},$$

(111)

and $\{a\}_{bc}$ is the Christoffel symbol of the background field $\bar{g}_{ab}$, ";'" is the covariant derivative with respect to the background field $\bar{g}_{ab}$. For any connection which is a sum of the Christoffel connection and a contorsion tensor, the Riemann tensor is

$$R^a_{bcd} = \bar{R}^a_{bcd} + K^a_{[d|b|c]} + K^a_{ce} S^e_{cd} + \frac{1}{2} K^a_{[c|e]} K^e_{d]b},$$

(112)

In the present case the torsion $S^a_{bc}$ and the cross terms in $h_{ab}$ are taken to vanish, then after using [3] for the commutation of covariant derivatives, the Riemann tensor becomes

$$R^a_{bcd} = \bar{R}^a_{bcd} - \frac{1}{2} h^a_{e} \bar{R}^{e}_{bcd} - \frac{1}{2} h_{be} \bar{R}^{ea}_{cd} + \frac{1}{2} (h^a_{d} ; b c - h^a_{db} c - h^a_{d} ; bd + h^a_{eb} d),$$

(113)
Contracting and again using 8,  
\[ R_{bd} = \tilde{R}_{bd} - h^{fe} \tilde{R}_{efbd} + \frac{1}{2} h_{be} \tilde{R}_{bd} + \frac{1}{2} h_{de} \tilde{R}_{b}^{c} + \frac{1}{2} (h_{d,cb} + h_{b,cd} - \Box h_{db} - h_{bd}), \]  
(114)

where \( h = h_{a}^{a}. \) Equations 112, 113 and 114 differ from (I4) and (I5) of Lifshitz and Khalatnikov (1963) [61] as they leave out all the terms involving products of \( h_{ab} \) and \( \bar{R}_{cdef}. \)

7.2 perturbing the stress of a perfect fluid

Perturbations of the stress usually (see for example Lifshitz and Khalatnikov (1963) [61] and Sacks and Wolfe (1967) [84]) are of a perfect fluid which obeys

\[ R_{ab} = (\mu + p)U_{a}U_{b} + \frac{1}{2}(\mu - p)g_{ab}, \]  
(115)

the first conservation equation

\[ \mu_{a}U^{a} + (\mu + p)U_{a} = 0, \]  
(116)

and the second conservation equation

\[ (\mu + p)\dot{U}_{a} + (g_{a}^{b} + U_{a}U_{b})p_{,b} = 0, \]  
(117)

where

\[ \dot{U}_{a} = U_{a,b}U^{b}. \]  
(118)

The perfect fluid is linearly perturbed thus

\[ \mu = \bar{\mu} + \delta \mu, \quad p = \bar{p} + \delta p, \quad U_{a} = \bar{U}_{a} + \delta U_{a}, \quad g_{ab} = \bar{g}_{ab} + \delta g_{ab}. \]  
(119)

Identifying

\[ \delta \mu = -\phi_{c}\phi^{c} + V(\phi), \quad \delta p = -\phi_{c}\phi^{c} - V(\phi), \quad \delta U_{a} = \frac{\phi_{a}}{\sqrt{-\phi_{c}\phi^{c}}}, \quad \delta g_{ab} = h_{ab}, \]  
(120)

the Klein-Gordon equation and the scalar field stress are recovered at the second and third orders. The solution to the Klein-Gordon equation given in Section 8 are not compatible with this linearization because the first order perturbation produces cross equations in the perfect fluid and scalar field which are not obeyed. This can be readily verified by investigating the
time component of the first order perturbation of the second conservation equation \[117\] for the Einstein static universe this gives

\[
\left( \frac{\phi_t}{\sqrt{-\phi_c \phi^c}} \right)_t = 0,
\]

which is incompatible with the solution \[64\]. Usually perturbation theory fixes the values of $\delta \mu$ and $\delta \rho$, thus not giving the freedom necessary to replace them with scalar fields.

### 7.3 perturbing the stress of a scalar field

Here perturbation are taken to given by the scalar field stress \[77\], thus

\[
R_{ab} = \bar{R}_{ab} + \phi_a \phi_b^\dagger + \phi_a^\dagger \phi_b + g_{ab} \frac{m^2 c^2}{\hbar^2} \phi \phi^\dagger,
\]

(122)

The components of the Ricci tensor follow immediately after noting

\[
\phi \phi^\dagger = A^2 Y Y^\dagger \left( C_+^2 + C_-^2 + 2C_+ C_- \cos(2\bar{\nu}) \right)
\]

\[
\phi_t \phi_t^\dagger = A^2 Y Y^\dagger \left( C_+^2 + C_-^2 + 2C_+ C_- \cos(2\bar{\nu}) \right)
\]

\[-4AA_t Y Y^\dagger \sin(2\bar{\nu})
\]

\[+ A^2 Y Y^\dagger \left( C_+^2 + C_-^2 - 2C_+ C_- \cos(2\bar{\nu}) \right)
\]

\[
\phi_i \phi_i^\dagger + \phi_i^\dagger \phi_i = AA_t (Y Y_i^\dagger + Y_i Y^\dagger) \left( C_+^2 + C_-^2 + 2C_+ C_- \cos(2\bar{\nu}) \right)
\]

\[+ iA^2 (Y Y_i^\dagger - Y_i Y^\dagger) (C_+^2 - C_-^2) \bar{\nu}_t
\]

\[-2(Y Y_i^\dagger + Y_i Y^\dagger) C_+ C_- \bar{\nu}_t \sin(2\bar{\nu}),
\]

\[
\phi_i \phi_j^\dagger + \phi_j^\dagger \phi_i = A^2 (Y_i Y_j^\dagger + Y_j^\dagger Y_i) \left( C_+^2 + C_-^2 + 2C_+ C_- \cos(2\bar{\nu}) \right).
\]

(123)

### 7.4 the $R_{tt}$ component

In general the equations resulting from [122] are intractable and therefore attention is restricted to the Einstein static universe. The perturbations are taken to be in the harmonic gauge

\[
h_{a;b}^b = \frac{1}{2} h_{a;},
\]

(124)
the $R_{tt}$ component of $122$ is

$$R_{tt} - \bar{R}_{tt} = -\frac{1}{2} \Box h_{tt}$$

$$= YY^\dagger \left\{ (C_+^2 + C_-^2)(2\nu^2 - \frac{c^2 m^2}{\hbar^2}(c^2 - h_{tt})) + 2C_+ C_- \cos(2\nu t)(-2\nu^2 - \frac{c^2 m^2}{\hbar^2}(c^2 - h_{tt})) \right\}. \quad (125)$$

due to the presence of $YY^\dagger$, which obeys $58$, this equation appears to be intractable. Now $YY^\dagger = \sin^{2n}(\alpha) \cos^{2n}(\alpha)$, expanding the trigonometrical functions and taking $n = 0$ (implying $p = l$) gives $YY^\dagger$ is approximately one for small $\alpha$. Taking the cross term $m^2 h_{tt}$ to be negligible and assuming that $h_{tt}$ is only a function of $t$, $125$ reduces to

$$h_{tt,tt} = 2C^{-2}(C_+^2 + C_-^2) \left(2\nu^2 - \frac{c^4 m^2}{\hbar^2}\right)$$

$$-4C^{-2}C_+ C_- \left(2\nu^2 + \frac{c^4 m^2}{\hbar^2}\right) \cos(2\nu t), \quad (126)$$

which has solution

$$h_{tt} = C_- (C_+^2 + C_-^2) \left(2\nu^2 - \frac{c^4 m^2}{\hbar^2}\right) t^2$$

$$+C_+ C_- \frac{2\nu^2 + c^4 m^2 \hbar^{-2}}{c^2 \nu^2} \cos(2\nu t) + B_1 + B_2, \quad (127)$$

where $B_1$ and $B_2$ are constants.

### 7.5 the $R_{ti}$ component

The $R_{ti}$ component of $122$ is

$$R_{ti} - \bar{R}_{ti} = h_{ti} - \frac{1}{2} \Box h_{ti}$$

$$= -2(Y Y_i^\dagger + Y_i Y_i^\dagger) C_+ C_- \nu \sin(2\nu t)$$

$$+i(Y Y_i^\dagger - Y_i Y_i^\dagger)(C_+^2 - C_-^2) \nu$$

$$+\frac{c^2 m^2}{\hbar^2} h_{tt} YY^\dagger (C_+^2 + C_-^2 + 2C_+ C_- \cos(2\nu t)). \quad (128)$$
The \( i = \beta \) and \( i = \phi \) components of the first term vanish, however the \( i = \alpha \) term does not as \( YY_\alpha^\dagger + Y_\alpha Y^\dagger = 2(n \cot(\alpha) - p \tan(\alpha))YY^\dagger \); again taking \( n = 0 \) and expanding the trigonometrical functions gives that this term vanishes for small \( \alpha \). By equations 50 the angular part of the second term is non-vanishing and remains so after expanding for small \( \alpha \), by assuming that \( C_2^+ \) is of the same magnitude as \( C_2^- \) this term can be taken to vanish. Similarly to the \( R_{tt} \) component the \( m^2 h_{ij} \) term can be taken to be negligible. Hence all of the left hand side of 128 can be taken to vanish, thus \( h_{ti} = 0 \) is an approximate solution to 128.

7.6 the \( R_{ij} \) component

The \( R_{ij} \) component of 122 is

\[
R_{ij} - \bar{R}_{ij} = -g_{ij} R_0^2 + 3 h_{ij} - \frac{1}{2} h_{ij} \nonumber
\]

\[
= \left( Y_i Y_j^\dagger + Y_j Y_i^\dagger + (g_{ij} + h_{ij}) \frac{m^2 c^2}{h^2} YY^\dagger \right) \nonumber
\]

\[
\times \left( C_2^+ + C_2^- + 2 C_+ C_- \cos(2 \nu t) \right). \tag{129}
\]

Now \( Y_i Y_j^\dagger + Y_j Y_i^\dagger \) is non-vanishing for \( i = j = \alpha, \beta, \phi \) and \( i = \beta, j = \phi \) or \( i = \phi, j = \beta \). Taking \( n = 0 \) and expanding there remains just the \( i = j = \phi \) component and it is of size \( l^2 \); the spatial axes can be rotated so that

\[
Y_i Y_j^\dagger + Y_j Y_i^\dagger = l^2 \bar{g}_{ij}^{(3)}. \tag{130}
\]

Similarly to the \( R_{tt} \) component \( YY^\dagger \) is taken to be approximately one and \( m^2 h_{ij} \) is taken to be negligible. Subject to the ansatz

\[
h_{ij} \equiv \sigma(t) \bar{g}_{ij}^{(3)}, \tag{131}
\]

129 reduces to

\[
\sigma_{,tt} = 2c^2 \left( l^2 + \frac{m^2 c^2 R_0^2}{h^2} \right) \left( C_2^+ + C_2^- + 2 C_+ C_- \cos(2 \nu t) \right), \tag{132}
\]

which has solution

\[
\sigma = +c^2 (C_2^+ + C_2^-) \left( l^2 + \frac{c^2 m^2 R_0^2}{h^2} \right) l^2
\]

\[
-2c^2 C_+ C_- \frac{l^2 + c^2 m^2 R_0^2 h^{-2}}{\nu^2} \cos(2 \nu t) + B_3 t + B_4, \tag{133}
\]

35
where $B_3$ and $B_4$ are constants.

### 7.7 the harmonic gauge condition

The solutions for $h_{ab}$ given by (127), (131), and (133) do not obey the harmonic gauge condition (124). This is a result of the approximations made for $Y$. From (130) $h_{,t}$ will depend on $l$ but from (127) $h_{t,t}$ will not, thus violating the time component of the harmonic gauge condition. No approximation for $Y$ which allows the harmonic gauge condition to be preserved are known. (127), (131), and (133) give the weak field metric perturbations

$$N^2 = \bar{N}^2 - c^{-2}h_{tt}, \quad R^2 = \bar{R}^2 + \sigma. \quad (134)$$

Note that the equations (125), (128) and (129) are not equivalent to the differential equations that arise if the substitutions

$$N = \bar{N} + \epsilon_1, \quad R = \bar{R} + \epsilon_2, \quad (135)$$

are used in (18) and (19), because for example, there are no second derivatives of $N$ in (18) and (19).

### 7.8 the change in redshift

In general weak metric perturbations induce a complicated change in the redshift. This has been calculated for the conformally flat ($k = 0$) case by Sacks and Wolfe (1967) [84]. The present case is much simplified because the metric perturbations are of the form (134), the new values of $N$ and $R$ can be used for the vectors (39) and (42) to give the redshift of the form (44), thus

$$1 + z = \frac{R_0}{R} = \frac{R_0}{R} \left(1 + \frac{\sigma}{R^2}\right) \simeq \frac{R_0}{R} \left(1 - \frac{\sigma}{2R^2}\right). \quad (136)$$

Appealing to the principle of the preservation of proper frequency, introduced in section 5, a change in the value of $l$ by a factor of one is taken to correspond to one unit of discrete redshift

$$\frac{v_l}{c} = |\delta z|, \quad \delta z \equiv z_{l+1} - z_l \simeq \frac{1 + z}{2R^2} (\sigma_l - \sigma_{l+1}). \quad (137)$$

This equation allows rough estimates to be made of the size of $R_0$; as such an estimate can only be made of the order of magnitude of $R$ it can be assumed that $\frac{1 + z}{2R^2} \simeq R_0^2$, this still leaves $C_+, C_-, l, t$ and $R$ of unknown size.
The frequency $\nu$ is large compared to the time scales involved in \(133\), as for example the Compton frequency of the electron $\nu_e = \frac{m_e c^2}{\hbar} \approx 10^{21}$ sec$^{-1}$; thus the $t^2$ term in \(133\) is larger than the cos term and \(137\) becomes

$$R_0^2 = (C_+^2 + C_-^2)(2l + 1)t^2 c^3 v_l^{-1}. \quad (138)$$

Using the $T'_t$ component of the scalar field’s stress

$$C_+^2 + C_-^2 = \frac{8\pi G}{c^2} \left( \frac{2l}{R_0^2} + \frac{c^2 m^2}{\hbar^2} \right)^{-1}, \quad (139)$$

where $\mu_S$ is the density of the scalar field, and the cos term is taken to be negligible and $YY^\dagger \sim 1$, thus \(138\) becomes

$$R_0^2 = 8\pi G \mu_S \left( \frac{2l}{R_0^2} + \frac{c^2 m^2}{\hbar^2} \right)(2l + 1)t^2 \frac{c}{v_l}. \quad (140)$$

It has been assumed that $C_+^2 + C_-^2$ is a constant and that $\mu_S$ is $l$ dependent, this implies that the substitution for $C_+^2 + C_-^2$ takes place after equation \(137\) has been applied.

### 7.9 some incompatible conditions

Rather than deriving a value of $R_0$ from \(140\), it is shown what values of $R_0, l, t$ and $s$ are compatible with this equation. First it is proved that the following conditions are incompatible:

i) \(140\) holds,

ii) the ”$t$” in \(140\) is less than $H_0^{-1}$,

iii) $R > 10^{22}$ meters (this is a typical distance between galaxies),

iv) $\mu_s < 10^5$ Kg m$^{-3}$ (this is an extremely high density compared with a typical stellar interior density)

$\mu_c = 10^{-26}$ Kg m$^{-3}$ the critical density for a Robertson-Walker universe,

$\mu = 10^{-5}$ Kg m$^{-3}$ a typical photosphere density (see p.163 Allen (1973)),

$\mu = 10^{+3}$ Kg m$^{-3}$ an average stellar density; it would be expected that $\mu_s < \mu_c$ for the weak metric approximations used in deriving \(140\) to work,

v) $\nu_{\text{geometry}} \ll \nu_e$ (this is necessary if the scalar field $\phi$ is chosen to represent a known field).

Proof: v) implies that the $\frac{l}{R}$ term in \(140\) can be neglected thus

$$R_0^2 = \frac{8\pi G}{c^2} \frac{\hbar^2}{m^2 c^2}(2l + 1)t^2 \frac{c^2}{v_l^2}. \quad (141)$$
using ii)

\[ R_0^2 < \frac{8\pi G\mu_S h^2(2l + 1)}{m^2 c v_l H_0^2}, \]  

(142)

again using v)

\[ R_0 < \frac{16\pi G\mu_S h}{mv_l H_0^2}, \]  

(143)

in SI units this is \( R_0 < 10^{18} \mu_S \), from which conditions ii) and iii) can be seen to be incompatible.

### 7.10 some compatible conditions

The most realistic compatible conditions are:

i) \( \Box \) holds,

ii) the ”t” in \( \Box \) equals \( H_0^{-1} \),

iii) \( R_0 = 10^{28} \) meters \(^1\) (this implies that \( 2q_0 - 1 = 10^{-4} \)),

iv) \( \mu_S = 10^{-13} \text{Kg m}^{-3} \) (this is well above the critical density but below a typical photosphere density),

v) \( \nu = t_p^{-1} \), where \( t_p^{-1} = h^\frac{4}{v} G^{-\frac{1}{2}} c^{-\frac{3}{2}} \simeq 10^{-44} \text{sec.} \) is the Planck time,

\(^1\) (this forces the scalar field \( \phi \) to be a hypothetical field rather than a known field, together with the above value of \( R_0 \) it implies that \( l = 10^{63} \) which is the Dirac dimensionless constant to the power of \( \frac{3}{2} \)).

Proof: Re-arranging \( \Box \)

\[ \frac{c^2 m^2 R_0^2}{\hbar^2} = 2l(8\pi G\mu_S t^2 c v_l^{-1} - 1) + 8\pi G\mu_S t^2 c v_l^{-1}, \]  

(144)

using ii) for the value of \( t \) and using SI units

\[ 10^{25} R_0^2 \simeq 2l(10^{31} - 1) + 10^{31}. \]

(145)

Now v) implies that \( \nu_{\text{geometry}} \gg \nu_{\text{compton}} \) giving

\[ l \simeq 10^{35} R_0 \simeq 10^{-6} \mu_S^{-1} R_0, \]

(146)

and the values of \( R_0, \mu_S \) and \( l \) given above can be shown to obey \( \Box \) by substitution.
7.11 summary

To summarize some of the deficiencies of the above model. There are at least three technical deficiencies: there is no proof that the dynamical equations are consistent; the Einstein static universe has been assumed in order to solve the perturbation equations, but the background metric is time dependent; and various approximations have been made for the angular terms, in particular the approximations result in the loss of the initially assumed harmonic gauge condition. There are at least three physical deficiencies: the equation for discrete redshift in the form depends on contrary to observation; the most realistic compatible conditions for equation require a hypothetical field with the unusual property of a frequency of the order of the inverse Planck time; and the result requires a scalar field density above the critical density of a Robertson-Walker-Friedman universe.

8 Conclusion.

Properties of Robertson-Walker spacetimes can be discrete if they depend on the spherical harmonic integer \( l \), in particular redshift is discrete if there is a mechanism to connect it to this integer. Density perturbations have been known for a long time to depend on this integer and thus integer dependent redshift could have been predicted before it was observed. The problems with introducing discrete redshift via density perturbations include: density perturbations are irregular whereas the value of discrete redshift is constant irrespective of other conditions, and more importantly density perturbations provide no mechanism which will alter \( l \). Solutions to the Klein-Gordon equation and other field equations also depend on the spherical harmonic integer \( l \). The requirement that the energy of these fields is almost conserved implies that, as discussed in section the proper frequency is preserved; this in turn implies that the value of \( l \) changes in a regular manner proportional to the increase in the scale factor \( R \). In principle all quantum fields have the Universe as an ultimate boundary condition and are thus presumably \( l \) dependent. Here it was found that theories using only solutions to the Klein-Gordon equation predict a microscopic value for the scale factor \( R \). It was suggested that the large scale behaviour of Robertson-Walker spacetime is governed, as it is classically, by the Friedmann equation and that the Klein-Gordon solutions in this background induce weak metric perturbations of the spacetime. It might be that solutions for fields involve other integers, apart from the spherical harmonic integer, and this could
also lead to discrete redshift via induced metric perturbations. From \[20\] it is not clear that this coupling is well-defined as the interactions between the Klein-Gordon field and the background fluid may be non-negligible. To produce a realistic prediction of the size of the scale factor using induced metric perturbations, it was necessary to make some very coarse technical and physical assumptions, including the requirement that the scalar field has a frequency approximately equal to the inverse Planck time, this precludes the scalar field representing a known particle. Any theory based upon metric perturbations will predict what in section 3 is called real discrete motion; this implies that there should be boundary effects where the value of the discrete redshift jumps. It is hoped that using the theory of quantum fields on curved spacetimes will remove, or reduce the bounds on, the free parameters such as \( C_+ \) and \( C_- \) in the induced weak metric perturbations, and will give a more rigorously defined theory. \[\]

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\(^1\)The referee suggests:’- take away ’\(2q_0 - 1 = 10^{-4}\)’ from the Abstract and add in the Conclusions that the value \(q_0 = 1/2\) is obtained in the absence of the cosmological constant. Taking it into account, the value of \(q_0\) would be in agreement with the recent observations on the Ia type supernovae.’
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