3-D Density Imaging Using the Sampling Weighting Function

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Abstract. In order to address the skin effect in 3-D density imaging, a sampling weighting function was introduced, which is similar to the distribution of kernel function. It can describe the attenuation of density cell with depth, and also can describe the attenuation of density cell in horizontal distance. Taking the sampling weighting function as the constraint condition, the single abnormal body model was inversion. Compared with the case without weighting function constraint, the inversion results are in better agreement with the model data. The results show that the sampling weighting function is potential as a constraint in 3D density imaging.

1. Introduction

3-D density imaging technology is widely used in oil, minerals, and other resources survey, regional geological survey, geotectonic research and seismic activity monitoring, and it has attracted widely attention of researchers. Because gravity data can only be observed on the surface, the information obtained is very limited, and the observed gravity data usually contains a lot of noise. When the gravity anomaly with errors is used for density imaging, there will be serious multi solutions. In order to solve the problem about non-uniqueness of the solution and noise, a lot of research has been carried out. There are three main directions. The first direction is joint inversion [1], which combines gravity data with other data and restricts each other, has formed many inversion techniques, such as gravity-magnetic inversion, gravity-magnetic-electric inversion and so on. These inversion techniques have achieved great success. However, the cost of acquiring various physical signals is very high, which limits the application of these imaging technologies. The second direction is to develop advanced optimization algorithms, such as multi-pole algorithm [2], neural network [3], genetic algorithm [4], and so on [5]. Because of the complexity of the algorithm itself, there are relatively many parameters to be optimized and adjusted. At present, they are mostly used in scientific research, and only have better imaging effect for some relatively simple geological bodies. The third direction is to introduce appropriate constraints on the basis of traditional regularization theory, such as density constraints, reference model constraints and so on. By introducing appropriate constraints, the range of solution can be reduced, and the image closer to the real situation can be obtained. This is one of the most extensive research directions of three dimensional density imaging at present.

When introducing constrained model into inversion, it is found that there is a skin effect compared with the real situation [6], that means the distribution of density value tends to be near the surface, rather than reasonable distribution according to the true depth of geological body. The main reason is that for a given observation point, the gravity anomaly weight produced by each density cell is different. The closer density cell contribute the greater the weight to the observation point. With the
distance increases, the weight on the observation point decreases rapidly. However, the results of mathematical inversion usually concentrate on the density cell with larger weight, which leads to the density anomaly concentrate to the surface.

In order to address this problem, Li and Oldenburg introduced a depth weighting function as constraint in the inversion algorithm [7]. The depth weighting function can approximately express the attenuation effect of gravity anomaly with depth, which makes the contribution of density cells of different depths to the objective function approximately the same, and eliminate the skin effect. However, the depth weighting functions only compensate for the weight of density cell decaying with depth, and not fully compensate for the weight decaying with horizontal distance. In order to compensate for the attenuation trend of gravity anomaly with distance more accurately, a weighted function based on sampling function is proposed in this paper. It can not only reflect the attenuation effect of gravity anomaly with depth, but also express the attenuation effect of gravity anomaly with horizontal distance. Using this new weighted function in three-dimensional gravity inversion is expected to obtain better density imaging resolution.

2. Theory of 3-D density imaging

As shown in Figure 1, the underground research space is divided into several closely arranged density cells, whose edges are in the same direction as the coordinate axis, and whose size ranges from:

\[ \xi_1 \leq \xi \leq \xi_2, \quad \eta_1 \leq \eta \leq \eta_2, \quad \zeta_1 \leq \zeta \leq \zeta_2 \]

(1)

![Figure 1. Schematic diagram of 3D density imaging model](image)

Where \( \xi, \eta \) and \( \zeta \) are the coordinate values of three directions in rectangular coordinate system. The gravity anomaly produced by each density cell on the observation point is as follows:

\[
\Delta g(x, y, z) = f \sigma \left[ \xi \ln(\eta + r) + \eta \ln(\xi + r) + \zeta \arctan \left( \frac{\xi r}{\xi \eta} \right) \right] \]

(2)

Where \( \xi, \eta, \zeta \) are the coordinate values of three directions in rectangular coordinate system.

The gravity anomaly of the \( j \)-th density cell on the \( i \)-th observation point can be expressed as follows:

\[
\Delta g_{ij} = G_{ij} \sigma_j
\]

(3)
When the coordinates of observation points and density cells have been obtained, the size of $G_{ij}$ can be calculated by formula (2). Finally, the relationship between all observation points and density cells can be written in matrix form.

$$
\begin{bmatrix}
\Delta g_1 \\
\Delta g_2 \\
\vdots \\
\Delta g_M
\end{bmatrix} = 
\begin{bmatrix}
G_{11} & G_{12} & \cdots & G_{1M} \\
G_{21} & G_{22} & \cdots & \vdots \\
\vdots & \vdots & \ddots & \vdots \\
G_{N1} & \cdots & \cdots & G_{NM}
\end{bmatrix}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\vdots \\
\sigma_M
\end{bmatrix}
$$

(4)

On the basis of forward modeling, the gravity anomaly is taken as a known condition, and according to formula (4), the problem of three-dimensional density imaging can be reduced to solving linear equations.

$$
\Delta g = G\sigma
$$

(5)

Because the number of gravity anomaly data is usually much smaller than the number of density cells, it is unrealistic to solve directly (5). In this study, the regularization inversion method is used to transform the density imaging problem into the problem of finding the minimum value of the objective function. The objective function is as follows:

$$
\Phi(m) = \Phi_1(m) + \lambda\Phi_2(m) = \|\Delta g - G\sigma\|^2 + \lambda\|W_m(\sigma - \sigma_0)\|^2
$$

(6)

Where $\Phi$ is the total objective function, $\Phi_1$ is the objective function of observation data, When $\Phi_1$ is the minimum value, it shows that the gravity anomaly obtained by density imaging has the best fitting degree with the known gravity anomaly. $\Phi_2$ is the objective function of a priori constraint. The priori information is represented by the weighted matrix $W_m$. $\lambda$ is the regularization factor, which is used to control the weight between $\Phi_1$ and $\Phi_2$.

3. The results of 3-D density imaging

![Figure 2](image-url)

**Figure 2.** (a) the distribution of the kernel function. (b) the distribution of the sampling weighting function.

Assuming that all density cells have the same density, the gravity anomaly generated by these density cells at different positions on a certain observation point are shown in Figure 2 (a), that is the distribution of the kernel function $G_{ij}$. It can be seen from the Figure2 (a) that the contribution of
different density cells to the same observation point varies greatly. In order to describe the difference of contribution accurately, a sampling weighting function was introduced in this paper, whose expression is as follows:

\[ W_s = \frac{\sin(j\pi/N)}{j\pi/N} \]  

Where \( j \) is the number of density cell, \( Z \) is the number of density cell layers, and \( N \) is the total number of density cells. The distribution of the sampling weighting function is shown in Figure 2 (b). Compared with figure 2 (a), they are similar and can simulate the attenuation trend of gravity anomaly contribution with distance.

\[ \text{Figure 3} \quad \text{(a) The initial gravity anomaly calculated according to the model. (b) The cross section of the abnormal body at x = 50km.} \]

In order to verify the effect of sampling weighting function, we assume a uniform space, with a cross-sectional area of 100km * 100km and a depth of 50km. There is a anomaly source with a density of 1. The vertical and horizontal position of the abnormal source is 30-70km, and the depth is 12.5-25km. The cross section of the abnormal body at x = 50km is shown in Figure 3 (b), and the initial gravity anomaly calculated according to equation (5) is shown in Figure 3 (a).

\[ \text{Figure 4} \quad \text{(a) The gravity anomaly calculated according to the inversion result. (b) The inversion result of density distribution without weighting function constraint.} \]

(a)  

(b)
If the influence of density cell location is not considered, the direct inversion results are shown in Figure 4. Figure 4(a) is the gravity anomaly calculated according to the inversion result, and Figure 4(b) is the inversion density distribution result. It can be seen from the figure4 that the inversion gravity anomaly is in good agreement with the forward modeling results, but the distribution of anomaly field sources is concentrated to the surface.

Figure 5 is the result of inversion with sampling weighting function, Figure 5(a) is the distribution of gravity anomaly based on inversion result, and Figure 5(b) is the inversion density distribution result. The inversion results of gravity anomalies are in good agreement with the forward modeling results. Compared with Fig. 4 (b), the distribution of anomalous field sources is more consistent with the model.

4. Conclusion
In order to address the skin effect in 3-D density imaging, a sampling weighting function was introduced, which is similar to the distribution of kernel function. It can describe the attenuation of density cell with depth, and also can describe the attenuation of density cell in horizontal distance. In order to verify the effect of sampling weighting function, a single abnormal body model was inversion, which with a density of 1, and the vertical and horizontal position is 30-70km, and the depth is 12.5-25km. Compared with the case without weighting function constraint, the inversion results are in better agreement with the model data. The results show that the sampling weighting function is effective as a constraint in 3D density imaging.

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