A BFKL MONTE CARLO APPROACH TO JET PRODUCTION
AT HADRON–HADRON AND LEPTON–HADRON
COLLIDERS

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The production of a pair of jets with large rapidity separation in hadron–hadron
collisions, and of forward jets in deep inelastic scattering, can in principle be used
to test the predictions of the BFKL equation. However in practice kinematic
constraints lead to a strong suppression of BFKL effects for these processes. This
is illustrated using a BFKL Monte Carlo approach.

Perturbative QCD at fixed order in the strong coupling constant \( \alpha_s \)
provides sufficient predictive power for a wide variety of high energy phenomena.
However in some regions of phase space, large logarithms can multiply the
coupling, spoiling the good behaviour of fixed–order expansions. In certain cases
these large logarithms can be resummed, as in the Balitsky, Fadin, Kuraev and
Lipatov (BFKL) equation, and the predictive power of the theory is then in
principle restored.

At hadron colliders the BFKL equation applies to dijet production when
the rapidity separation \( \Delta \) of the two jets (with \( p_{T1} \sim p_{T2} \sim p_T \)) is large.
The emission of (real and virtual) gluons in the rapidity interval between
the two leading jets (see Fig. 1b) generates large logarithmic contributions
\( (\alpha_s \ln(\hat{s}/p_T^2))^n \sim (\alpha_s \Delta)^n \) which when resummed give a dijet subprocess cross
section which increases with \( \Delta \), \( \hat{\sigma}_{jj} \sim \exp(\lambda \Delta) \), with \( \lambda = 12 \ln 2 \alpha_s/\pi \approx 0.5 \), in
contrast to the \( \hat{\sigma}_{jj} \rightarrow \) constant behaviour expected at lowest order. In practice

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Figure 1: Schematic representation of (a) dijet production with large rapidity separation $\Delta$ in hadron–hadron collisions, and (b) forward jet production in deep inelastic scattering. Note that the solid lines at each end of the gluon ladder in (a) and at the bottom of the ladder in (b) represent either quarks or gluons, according to the effective subprocess approximation.

such behaviour can be difficult to observe because $\hat{\sigma}$ gets folded in with parton distribution functions (pdfs), which decrease with $\Delta$ much more rapidly than the subprocess cross section increases:

$$\sigma_{jj} \sim q(x_1) q(x_2) \exp(\lambda \Delta)$$

with $x_{1,2} \sim p_T/\sqrt{s} \exp(\Delta)$. The challenge is to find measurable quantities in dijet production that are insensitive to the pdfs, but that retain the distinctive behaviour characteristic of BFKL resummation. One possibility is the azimuthal decorrelation of the two jets: the multiple emission of soft gluons between the leading jets predicted by BFKL leads to a stronger decorrelation than does fixed–order QCD, and the prediction is relatively insensitive to the pdfs. Another possibility is to look for the increase in $\hat{\sigma}$ with $\Delta$ by considering different collider energies. The idea is to choose $\Delta$'s that correspond to the same parton momentum fractions at different energies so that the pdf dependence is the same for both, thereby allowing the $\Delta$ dependence of $\hat{\sigma}$ to be extracted. This has been studied recently in Ref. see also.

Exactly the same (BFKL) physics applies to the production of ‘forward’ $k_T^2 \sim Q^2$ jets in deep inelastic scattering, where the large rapidity separation is between the current and forward jets, see Fig.

However, a key issue is whether the kinematic regions currently accessible at the Tevatron and at HERA allow the asymptotic BFKL conditions to be fulfilled. In particular, the asymptotic $\exp(\lambda \Delta)$ behaviour is obtained under the assumption that there is no kinematic penalty for emitting arbitrary large
numbers of gluons with \( k_{T_i} \sim p_T \). In practice, there is an overall constraint from \( \sqrt{s} < \sqrt{s} \) and even before this is reached there is a strong suppression of large subprocess energies from the pdfs. To investigate these effects, we have constructed\(^a\) a BFKL Monte Carlo programme (BFKL–MC) which reproduces the analytic predictions by explicitly taking into account the emission of real and virtual gluons to all orders.\(^b\) With such an event generator the effects of kinematic constraints and experimental cuts can readily be taken into account.

Fig. 2 shows the dijet cross section as a function of \( \Delta \) for the naive BFKL and improved BFKL–MC cases at \( \sqrt{s} = 630, 1800 \) GeV, with \( p_{T1}, p_{T2} > 20 \) GeV. Asymptotic QCD LO is also shown for reference. The naive BFKL cross section (dashed curve) is always largest, because it includes the analytic subprocess cross section \( \hat{\sigma}_{jj} \sim \exp(\lambda \Delta) \), which corresponds to the emission of any number of gluons with arbitrarily large energies. The prediction falls off rather than increases because \( \hat{\sigma} \) is multiplied by the pdfs, but even those incorporate only lowest order kinematics in this case. When exact kinematics for entire events are included in both the subprocess cross section and the pdfs, as in the BFKL–MC (solid curve), there is a dramatic suppression of the total cross section.\(^b\) In fact the suppression is so strong that it drives the BFKL–MC cross section below that for asymptotic QCD LO. The reason is due to simple kinematics: the QCD LO cross section contains only two jets, but the BFKL–MC cross section also includes additional jets, each of which increases the subprocess centre–of–mass energy and elicits a corresponding price in parton densities. In the naive BFKL calculation, the contribution to the subprocess energy from additional jets is ignored and their net effect is to combine with the virtual gluons to increase the subprocess cross section.

Finally we consider forward jet production in deep inelastic scattering at HERA, Fig. 1b. It is relatively straightforward to adapt the dijet formalism to calculate the cross section for the production of a forward jet with a given \( k_T \) and longitudinal momentum fraction \( x_j \gg x_{Bj} \). In fact there is a direct correspondence between the variables: \( p_{T2} \leftrightarrow k_T \) and \( \Delta \leftrightarrow \ln(x_j/x_{Bj}) \). In the DIS case the variable \( p_{T1} \) corresponds to the transverse momentum of the \( q\bar{q} \) pair in the upper ‘quark box’ part of the diagram. In practice this variable is integrated with the off–shell \( \gamma^* g^* \rightarrow q\bar{q} \) amplitude such that \( p_{T1}^2 \sim Q^2 \). As a result, it is appropriate to consider values of \( k_T^2 \) of the same order, and to consider the (formal) kinematic limit \( x_j/x_{Bj} \rightarrow \infty \), \( Q^2 \) fixed. In this limit we obtain the ‘naive BFKL’ prediction \( \hat{\sigma}_{jet} \sim (x_j/x_{Bj})^\lambda \), the analogue of \( \hat{\sigma}_{jj} \sim \exp(\lambda \Delta) \).

\(^a\)The technical details can be found in Ref.\(^5\). A similar technique has been used in Ref.\(^4\).
\(^b\)The running of \( \alpha_s \), which we include, also contributes to the suppression, but it has a much smaller effect than kinematics.
Fig. 3 shows the differential structure function $\frac{\partial^2 F_2}{\partial x_j \partial k_T^2}$ as a function of $x_{Bj}$ at HERA, with

$$x_j = 0.1, \quad Q^2 = 50 \text{ GeV}^2, \quad Q^2/2 < k_T^2 < 4Q^2. \quad (2)$$

The lower dashed curve is the leading–order prediction from the process $\gamma^* \mathcal{G} \to q\bar{q}\mathcal{G}$, with $\mathcal{G} = g, q$, with no overall energy–momentum constraints. This is the analogue of the $\hat{\sigma}_{jj} \to$ constant prediction for dijet production. Note that here the parton distribution function at the lower end of the ladder is evaluated at $x = x_j$, independent of $x_{Bj}$. In practice, when $x_{Bj}$ is not small we have $x > x_j$ and the cross section is suppressed, as indicated by the lower solid curve in Fig. 3. The upper dashed curve is the asymptotic BFKL prediction.
with the characteristic \((x_j/x_{Bj})^\lambda\) behaviour. Finally the upper solid line is the prediction of the full BFKL Monte Carlo, including kinematic constraints and pdf dependence. There is evidently a significant suppression of the cross section. We emphasise that Fig. 3 corresponds to ‘illustrative’ cuts and should not be directly compared to the experimental data. Nevertheless, the BFKL–MC predictions do appear to follow the general trend of the H1 and ZEUS measurements\(^{10}\). A more complete study, including realistic experimental cuts and an assessment of the uncertainty in the theoretical predictions, is under way and will be reported elsewhere\(^9\).

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Figure 3: Differential structure function for forward jet production in $ep$ collisions at HERA. The curves are described in the text.