A simple modification of the maximal mixing scenario for three light neutrinos

Kyungsik Kang
Department of Physics, Brown University
Providence, Rhode Island 02912, USA

Jihn E. Kim
Center for Theoretical Physics and Department of Physics
Seoul National University, Seoul 151-742, Korea

Pyungwon Ko
Department of Physics, Hong-Ik University
Seoul 121-791, Korea

Abstract

We suggest a simple modification of the maximal mixing scenario (with $S_3$ permutation symmetry) for three light neutrinos. Our neutrino mass matrix has smaller permutation symmetry $S_2$ ($\nu_\mu \leftrightarrow \nu_e$), and is consistent with all neutrino experiments except the $^{37}$Cl experiment. The resulting mass eigenvalues for three neutrinos are $m_1 \approx (2.55 - 1.27) \times 10^{-3}$ eV, $m_{2,3} \approx (0.71 - 1.43)$ eV for $\Delta m^2_{LSND} = 0.5 - 2.0$ eV$^2$. Then these light neutrinos can account for $\sim (2.4 - 4.8)\%$ $(6.2 - 12.4)\%$ of the dark matter for $h = 0.8$ $(0.5)$. Our model predicts the $\nu_\mu \rightarrow \nu_\tau$ oscillation probability in the range sensitive to the future experiments such as CHORUS and NOMAD.
The minimal standard model (MSM) has been highly successful in describing interactions among elementary particles from low energy up to \( \sim 100 \text{ GeV} \). The only possible exception may be various types of neutrino oscillation experiments. There have been positive indications from large scale experiments for solar and atmospheric neutrinos that a certain amount of mixing between neutrino species may be present [1]. The recent report from the LSND experiment at the laboratory scale provides us with another hint of such a possible neutrino mixing [2]. Since neutrinos in the MSM are exactly massless, there can be no mixing among them, and it is impossible to accommodate such neutrino mixing data in the framework of the MSM. This situation is rather encouraging, since it is at present the only place where we can grasp a hint of new physics beyond the MSM.

In view of this, it is quite interesting to speculate what type of neutrino mass matrix can fit all the data from the various types of neutrino oscillation experiments. It is our purpose to present one such mass matrix in this work. Most analyses on the neutrino oscillation assume two neutrinos oscillating with one mass difference parameter, \( \Delta m^2 \). However, the LSND experiment and the atmospheric and solar neutrino data hint at least two mass difference parameters, requiring oscillations among at least three neutrinos. For oscillations with three neutrinos, we have two mass differences, three real angles and one phase. In order to simplify the analysis, a certain ansatz for the mass matrix is required. In this vein, we first briefly discuss the maximal mixing scenario for the neutrino sector. We then present our ansatz for the neutrino mass matrix as well as the numerical analyses to fit the atmospheric, LSND, and solar neutrino data from GALLEX and SAGE. In this work, we consider oscillations among three neutrinos only, \( \nu_\alpha \rightarrow \nu_\beta \).

One of the popular ansatz for the neutrino mass matrix is the maximal mixing one (equivalent to a cyclic permutation symmetry among three generations) [3]:

\[
M_{\text{maximal}} = \begin{pmatrix}
 a & b & b^* \\
 b^* & a & b \\
 b & b^* & a \\
\end{pmatrix},
\]

with the mixing matrix \( U \) given by

\[
U_{\text{maximal}} = \frac{1}{\sqrt{3}} \begin{pmatrix}
 \omega_1 & \omega_2 & \omega_3 \\
 \omega_1 & \omega_2 & \omega_3 \\
 \omega_1 & \omega_3 & \omega_2 \\
\end{pmatrix}.
\]

Here, \( \omega_{1,2,3} \) are three complex roots of \( \omega^3 = 1 \) with \( \omega_1 = 1 \). This ansatz was originally proposed in the neutrino sector [3], and extended to the quark sector with a partial success in explaining the quark masses and the Kobayashi-Maskawa matrix elements [4].

The maximal mixing scenario has many interesting features [5]. For example, the survival probability for a neutrino is independent of its flavor. The \( \nu_e \) survival probability has two plateaus, \( 5/9 \) in the intermediate step, and \( 1/3 \) for \( \frac{L}{E} \gg \frac{1}{(\Delta m^2_{\text{min}})} \) through vacuum oscillations. With vacuum oscillation, one cannot explain solar neutrino data from \( ^{37}\text{Cl} \) and the Ga data simultaneously. Thus one must make a choice between the \( ^{37}\text{Cl} \) and Ga experiments. Here, we choose to interpret the Ga data (GALLEX and SAGE experiments).
through vacuum oscillation and disregard the $^{37}$Cl data, following Ref. [5]. The Ga data requires $\nu_e$ survival probability of $\sim 5/9$, which implies $\frac{1}{E}$ for the solar neutrino is smaller than $1/2\,\Delta m^2_{ij}$ at the minimum. The atmospheric neutrino data and km range laboratory experiments require another $\Delta m^2_{ij}$. Hence, two mass difference scales have been all used, and there is none left for a new scale suggested by the LSND data around $\Delta m^2_{\text{LSND}} \sim O(1) \text{ eV}^2$ with a mixing angle $\sim (\text{a few}) \times 10^{-3}$. The only possibility to explain both the mass shifts at LSND point and at atmospheric data points in the maximal mixing scenario is that there are two thresholds corresponding to a larger $\Delta m^2$ at $\sim O(1) \text{ eV}^2$ and a smaller $\Delta m^2$ at around $10^{-2} \text{ eV}^2$. In this case, the $\nu_e$ survival probability for the solar neutrino problem is $1/3$, and is too small to accommodate the Ga data. Therefore, although the qualitative features of the maximal mixing scenario is encouraging, it is not viable if the LSND data is confirmed in the future. Another way to see this is as follows: the maximal mixing scenario predicts the transition probability for $\nu_\mu \rightarrow \nu_e$ to be $4/9$ in the range of the LSND experiment, which clearly contradicts the reported transition probability, $(\text{a few}) \times 10^{-3}$. Furthermore, the best $\chi^2$ fit to the atmospheric and the solar neutrino data indicates that the masses of three light neutrinos are $m_3 \simeq (85 \pm 10) \text{ meV}$, and $m_{1,2} < 3 \mu\text{eV}$, which are too light to be cosmologically interesting as a hot dark matter component of the missing mass of the universe.

Therefore, we make an ansatz for the neutrino mass matrix which is a simple modification of the maximal mixing one, (1), and study its consequences in this work. We assume that neutrinos are Dirac particles so that the lepton number is to be conserved in our model. Then, each left-handed neutrino ($\nu^L_i$) is accompanied by the right-handed partner ($\nu^R_i$) which is sterile under electroweak interactions.

Note that any $3 \times 3$ matrix $M_{ij}$ can be decomposed as $M = X - iY$ with both $X$ and $Y$ hermitian. Also any hermitian matrix $X$ can be written as $X = S + iA$, where $S$ ($A$) is a real (anti)symmetric matrix. Finally, the symmetric matrix $S$ can be decomposed as the trace part proportional to $\delta_{ij}$ and the traceless symmetric matrix. One can combine the trace part of the symmetric mass matrix $S$ and the real antisymmetric part $A$ in order to get the neutrino mass matrix,

$$M = \mu \begin{pmatrix} 1 & ic & id \\ -ic & 1 & ib \\ -id & -ib & 1 \end{pmatrix},$$

where $\mu$ is the mass scale, $b, c$ and $d$ are all real. We have chosen a basis in which the charged lepton mass matrix is diagonal. Note that the diagonal terms are still universal, and only the off-diagonal elements are modified from the maximal mixing one, (1). Note that this ansatz becomes the maximal mixing one if $b = c = -d$ (i.e., if there is a permutation symmetry among three generations). This form for the mass matrix is sufficiently simple but rich.

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1 The matter oscillation effect, i.e., the MSW mechanism, has also been suggested to interpretate this Homestake experiment in Ref. [3]. In this case, the relevant $\Delta m^2$ is around $10^{-4} \text{ eV}^2$. Since we have only two $\Delta m^2$ around $\sim O(1) \text{ eV}^2$ and $\sim 10^{-2} \text{ eV}^2$, the MSW mechanism is irrelvant to our study in this work.
enough to give nontrivial analytic formulae for the survival and transition probabilities for three neutrinos. If there is a solution when two of the off-diagonal elements are the same (say, $b = d$ for example), then the permutation symmetry among three generations ($S_3$) in the original maximal mixing case (1) breaks down to $S_2$. This would imply that our mass matrix ansatz depends on three real parameters and, thus it is one of the simplest modifications to the maximal mixing ansatz, which can accommodate LSND, atmospheric and solar neutrino data. In fact, this is the case (with $b^2 = d^2$) as discussed in the following. It breaks the original $S_3$ possessed by (1) into $S_2$ in a particular way. There may be several other ways to break this permutation symmetry, which will be considered elsewhere. There may be some underlying dynamical reasons for the above form of the mass matrix, but we take it as a simple phenomenological ansatz for the moment.

Three eigenvalues of the mass matrix (3) are

$$m_1 = \mu, \quad m_{2,3} = \mu(1 \pm N),$$

where $N = (b^2 + c^2 + d^2)^{1/2}$. The corresponding eigenvectors form the mixing matrix $U$ which relates the weak eigenstate $\nu_\alpha$ to the mass eigenstates $\nu_i$ as $\nu_\alpha = \Sigma_i U_{\alpha i} \nu_i$. The indices $\alpha = e, \nu, \tau$ label the flavor eigenstates, and $i = 1, 2, 3$ label the mass eigenstates of three neutrinos.

Then one can easily verify that

$$P(\nu_e \to \nu_e) = 1 - \frac{1}{N^4} \left[ (\Delta_{21} + \Delta_{31}) b^2(N^2 - b^2) + \frac{1}{2} \Delta_{32} (N^2 - b^2)^2 \right],$$

$$P(\nu_\mu \to \nu_e) = \frac{1}{N^4} \left[ (\Delta_{21} + \Delta_{31}) b^2 d^2 + \frac{1}{2} \Delta_{32} (c^2 N^2 - b^2 d^2) \right],$$

$$P(\nu_\mu \to \nu_\mu) = 1 - \frac{1}{N^4} \left[ (\Delta_{21} + \Delta_{31}) d^2(N^2 - d^2) + \Delta_{32} \left\{ \frac{2(c^2 N^2 - b^2 d^2)}{c^2 + d^2} + \frac{2(c^2 N^2 + b^2 d^2) - c^2 d^2}{2(c^2 + d^2)^2} \right\} \right],$$

where

$$P(\nu_\tau \to \nu_\mu) = \frac{1}{N^4} \left[ (\Delta_{21} + \Delta_{31}) c^2 d^2 + \Delta_{32} \left( \frac{2(c^2 N^2 + b^2 d^2)(c^2 + d^2) - c^2 d^2}{2(c^2 + d^2)^2} \right) \right],$$

and

$$\Delta_{ij} = 2 \sin^2 \left( \frac{1.27 L \Delta m^2_{ij}}{E} \right),$$

with $\Delta m^2_{ij}$ as $m_i^2 - m_j^2$ in eV$^2$ and $L/E$ in km/GeV. Since $\sum \Delta m^2_{ij} = 0$, there exist only two independent mass difference parameters.

Note that the heights of the plateaus for the $\nu_e$ survival probability are functions of $b^2/N^2$ only, even in the presence of nonvanishing $c$ and $d$. Since $P(\nu_\mu \to \nu_e)$ depends on two mass differences, one can identify the mass difference $\Delta m^2_{LSND} \sim O(1)$ eV$^2$ either as $\Delta m^2_{31}$ or as $\Delta m^2_{32}$. The other mass difference is taken to be $0.72 \times 10^{-2}$ eV$^2$ in order to solve the atmospheric neutrino problem. Thus, there is a reasonable hierarchy between two mass differences. In the following, we discuss two possibilities separately.

(1) : $\Delta m^2_{31} = \Delta m^2_{LSND} \sim O(1)$ eV$^2$ and $\Delta m^2_{32} = 0.72 \times 10^{-2}$ eV$^2$.
In this case, the heights of the intermediate plateaus for the \( \nu_e \) and \( \nu_\mu \) survival probabilities are given by

\[
P(\nu_e \rightarrow \nu_e) = 1 - 2 \frac{b^2}{N^2} \left( 1 - \frac{b^2}{N^2} \right),
\]

(10)

\[
P(\nu_\mu \rightarrow \nu_\mu) = 1 - 2 \frac{d^2}{N^2} \left( 1 - \frac{d^2}{N^2} \right),
\]

(11)

and the transition probabilities can be approximated as

\[
P(\nu_\mu \rightarrow \nu_e) = 4 \frac{b^2 d^2}{N^4} \sin^2 \left( \frac{1.27L \Delta m^2_{31}}{E} \right),
\]

(12)

\[
P(\nu_\mu \rightarrow \nu_\tau) = 4 \frac{c^2 d^2}{N^4} \sin^2 \left( \frac{1.27L \Delta m^2_{31}}{E} \right).
\]

(13)

The transition probability is often described in terms of two parameters, \( \Delta m^2 \) and \( \theta_{\alpha\beta} \) for which

\[
P(\nu_\alpha \rightarrow \nu_\beta) = \sin^2 2\theta_{\alpha\beta} \sin^2 \left( \frac{1.27L \Delta m^2_{31}}{E} \right).
\]

(14)

Therefore, we can make the following identifications:

\[
\sin^2 2\theta_{e\mu} = \frac{4b^2 d^2}{N^4},
\]

(15)

\[
\sin^2 2\theta_{\mu\tau} = \frac{4c^2 d^2}{N^4},
\]

(16)

in our model (for \( \Delta m^2_{31} \gg \Delta m^2_{32} \)).

Experimental results for the neutrino oscillations are shown in the \( (\sin^2 2\theta, \Delta m^2) \) plane. In the plot presented by the LSND group, there are small regions in this plane which indicates a possible transition of \( \bar{\nu}_\mu \rightarrow \bar{\nu}_e \). This region is not ruled out by other laboratory searches such as BNL E776 [7], KARMEN [8], BUGEY [9] and others [10]-[11]. For each possible \( \Delta m^2_{LSND} \), we show the possible value(s) of \( \sin^2 2\theta_{e\mu} \) in Table 1. For the same \( \Delta m^2_{LSND} \), there is an upper bound on \( \sin^2 \theta_{\mu\tau} \) from FNAL E531 [12], CHARM II [13] and CDHSW [14], and we also list these numbers in Table 1.

For each \( \Delta m^2_{31} = \Delta m^2_{LSND} \) given in Table 1, one can solve Eq. (4) to get the neutrino masses. For example, \( \Delta m^2_{31} = 6 \text{ eV}^2 \) leads to

\[
m_1 = 7.35 \times 10^{-4} \text{ eV},
\]

\[
m_2 \approx -m_3 \approx 2.45 \text{ eV},
\]

(17)

with \( \Sigma_i |m_{\nu_i}| = 4.9 \text{ eV} \). (The negative \( m_3 \) can be remedied by a chiral transformation of \( \nu_3 \) field.) For other values of \( \Delta m^2_{31} \), we show the resulting neutrino masses from our mass matrix ansatz (3) in the fourth column of Table 1.

These light neutrinos can contribute to the missing mass of the universe (the hot dark matter) in amount of [15]
\[ \Omega h^2 = 7.83 \times 10^{-2} \frac{g_{\text{eff}}}{g_{\ast \ast}(T_D)} \left( \frac{m_\nu}{eV} \right), \]

where \( g_{\ast \ast}(T_D) = 10.75 \) and \( g_{\text{eff}} = (3g)/4 = 3/2 \) are the effective degrees of freedom contributing to the entropy density \( s \) and to the ratio \( Y = n/s \), \( n \) being the number density, respectively. The parameter \( h \) is related to the Hubble constant \( H_0 \) as \( H_0 = 100h \text{ km sec}^{-1} \text{ Mpc}^{-1} \). So, for the solution \( \Delta m^2 \), three light neutrinos can constitute 8.3% (21.4%) of the missing mass of the universe for \( h = 0.8 \) (0.5), which is again cosmologically interesting. (Here, we have assumed that three sterile right-handed neutrinos decouple much earlier than the left-handed neutrinos, and that they don’t affect the results of the standard cosmology.) The results for other values of \( \Delta m^2_{31} \) are listed in the last column of Table 1.

When we determine \( b^2 \) and \( d^2 \), it is important to satisfy all the constraints shown in Table 1. One might try to perform the \( \chi^2 \) fit to the available data on the neutrino oscillations. Instead, we choose to scan \( d^2/N^2 \) for each \( \Delta m^2 \) in the first column of Table 1. For each \( d^2/N^2 \), the parameter \( b^2 \) is determined by the mixing angle given by the LSND experiments, and \( c^2/N^2 = (1 - b^2/N^2 - d^2/N^2) \). Then, we require that the resulting \( \sin^2 2\theta_{\mu\tau} \) satisfy the upper limit given in the third column of Table 1. We also calculate the survival probabilities for \( \nu_e \) and \( \nu_\mu \) at the intermediate level (the laboratory and the km range scale) and require them to be larger than 0.95 in order to satisfy the null results in various types of disappearance experiments for the \( \nu_\mu \) and \( \bar{\nu}_e \) beams. For \( \Delta m^2 = 6 \text{ eV}^2 \) or larger (the first and the second rows), the resulting \( d^2/N^2 \approx 0.995 - 1.00 \), which corresponds to almost no disappearance of \( \nu_\mu \) for all ranges of \( L \) and \( E \). Thus, we reject \( d^2/N^2 \) around 1. For \( \Delta m^2_{31} \leq 2 \text{ eV}^2 \), the allowed ranges for \( d^2/N^2 \) are typically around 0.010–0.020. The corresponding \( b^2/N^2 \)'s are also in the same range as \( d^2/N^2 \). Thus, as discussed in the following, we can accommodate the laboratory scale and the large scale neutrino experiments, by choosing small (but nonvanishing) \( b^2 \) and \( d^2 \).

In particular, there is a small region in which \( b^2 = d^2 \) gives acceptable fits to all available data on neutrino oscillation experiments except for the \( ^{37}\text{Cl} \) data. This is quite interesting, since it corresponds to residual permutation symmetry \( (S_2) \) between \( e \) and \( \mu \) in (3) with three real parameters. In other words, we have \( |M_{e\mu}| = |M_{\mu\tau}|. \) Our matrix with \( b^2 = d^2 \) breaks the original symmetry of the maximal mixing one \( (S_3) \) into \( S_2 \), and thus may be regarded as one of the simplest modifications to the maximal mixing one, (1). In the following, we demonstrate that the ansatz (3) can reasonably fit all the data with \( b^2/N^2 = d^2/N^2 = 0.015 \) for \( \Delta m^2_{31} = 2 \text{ eV}^2 \), except for the HOMESTAKE data, with a reasonable accuracy at present.

In Figure 1, we show the resulting survival probability for \( \nu_e \) in the solid curve along with various types of neutrino oscillation data, the \( \nu_e \) disappearance experiments at reactors \[8]- [11] and the solar neutrino experiments \[12]- [15]. Using three neutrino mixing with four parameters, we get two step survival probability for \( \nu_e \to \nu_e \). The plateau for the large \( L \) is about 0.49, a little bit lower than 5/9 used before in the maximal mixing case. Thus, the solar neutrino deficit is solved in terms of vacuum oscillations. The intermediate plateau of

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2 In order to average the oscillation probabilities, we adopt the prescription by Harrison et al. [3], which amounts to replacing \( \cos(x/2) \) by \( \sin x/x \).
\(\sim 0.97\) for \(\nu_e\) is the prediction of our specific form of the mass matrix, and is consistent with all the existing data. The KRASNOYARSK data has a relatively large error bar, and our curve is within two \(\sigma\) of the data.

In Figure 2, the survival probability for \(\nu_\mu\) using our ansatz is shown in the solid curve, along with the \(\nu_\mu\) disappearance experiment data \[20\] \[21\]. The survival probability for \(\nu_\mu\) at large \(L\) is 0.48, and the intermediate plateau has a height of 0.97. The curve agree with the available data quite well.

For the atmospheric neutrino data, we show the data point for the so-called \(R\) defined by

\[
R = \frac{(N_\mu/N_e)_{\text{Data}}}{(N_\mu/N_e)_{\text{MC}}},
\]

along with our prediction

\[
R = \frac{P_{\mu\mu} + P_{e\mu}/r}{P_{ee} + rP_{\mu e}}
\]

in Table 2, where \(r\) is the incident \((\mu/e)\) ratio. From Table 2, we observe that most of the predicted \(R\) values are consistent with all the atmospheric neutrino data from KAMIOKANDE, IMB, and others \[22\] - \[26\], considering some data points have large errors.

For these numbers with \(\Delta m^2_{31} = 2\ eV^2\), we predict \(\sin^2 2\theta_{\mu\tau} \approx 6 \times 10^{-2}\), which is just below the current upper limit, \(8 \times 10^{-2}\). This range of \(\sin^2 2\theta_{\mu\tau}\) may be probed at CHORUS, NOMAD, FNAL P803, CERN/ICARUS and FNAL/SOUDAN2 \[27\]. It would be interesting to test our predictions for the \(\nu_\mu \rightarrow \nu_\tau\) oscillation in the future. Similar results can be drawn for other values of \(\Delta m^2_{31}\). Thus, our mass matrix ansatz \(3\) not only describes all the available data on neutrino oscillations, but also predicts the mixing angle for \(\nu_\mu \rightarrow \nu_\tau\) in an interesting range which lies within sensitivity of the near-future experiments.

Let us briefly discuss the second case : (II) \(\Delta m^2_{31} = 0.72 \times 10^{-2}\ eV^2\), and \(\Delta m^2_{32} = \Delta m^2_{LSND}\). In this case, it is easy to verify that there is no solution for \(b^2/N^2\) and \(d^2/N^2\) which satisfy the constraints from the laboratory scale experiments from BUGEY, BNL E766, and those in the second and the third columns of Table 1. So, our mass matrix ansatz prefer the solution (I) for which \(m_1\) is smallest around \(10^{-3} - 10^{-4}\ eV\), and the other two are nearly degenerate with \(m_2 \approx m_3 \approx O(1)\ eV\).

In summary, the neutrino mass matrix ansatz \(3\) with four real parameters is one of the simple modifications to the maximal mixing ansatz (that fails to fit the new LSND data) which can fit various types of neutrino experiments except for the \(^{37}\text{Cl}\) solar neutrino data \[17\]. In particular, there are solutions with \(b^2 = d^2\) with residual permutation symmetry among two generations \((\nu_\mu \leftrightarrow \nu_e)\). In this sense, our mass matrix ansatz could be regarded as one of the simplest modifications to the maximal mixing ansatz. The resulting light neutrinos have masses \(17\), and thus they can constitute about \(2.4\% - 4.8\%\) of the missing mass of the universe for \(h = 0.8(0.5)\), and thus cosmologically interesting unlike the maximal mixing scenario. In our model, the transition probability for \(\nu_\mu \rightarrow \nu_\tau\) is close to the current upper limit, depending on \(\Delta m^2_{LSND}\) as shown in the third column of Table 1. Since it lies within the reach of various future experiments such as CHORUS, NOMAD, etc., the observation of \(\nu_\mu - \nu_\tau\) oscillation would constitute a definite test of our mass matrix ansatz along with the confirmation of the LSND data.
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Note Added in Proof

In Eqs. (5)-(8), we did not show terms involving $s_{ij} \equiv \sin \left(\frac{1.27L\Delta m_{ij}^2}{E}\right)$, since these terms vanish under taking averages, or they are irrelevant to our study. See Ref. [5] for more details.
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FIGURES

FIG. 1. The survival probabilities $P(\nu_e \rightarrow \nu_e)$ using our ansatz (3) for the case (I), along with the reactor experiment data from KARMEN, ILL/GOSGEN, BUGEY, KRASNOYARSK, and the solar neutrino data from KAMIOKA, HOMESTAKE, SAGE and GALLEX.

FIG. 2. The survival probabilities of $\nu_\mu$ using our ansatz (3) for the case (I), along with the accelerator experiments from CDHS-SPS and CHARM-PS.
TABLES

TABLE I. The allowed regions for $\Delta m^2$ and $\sin^2 \theta_{e\mu}$ consistent with the LSND as well as BNL E766, and the corresponding upper limit for $\sin^2 \theta_{\mu\tau}$ from FNAL E531 and CDHSW. The fourth column is the predicted neutrino masses by our mass matrix ansatz (3). The last column shows contributions of three light neutrinos to the missing mass of the universe for $h = 0.8$ (0.5). See the text for details.

| $\Delta m^2$ (eV$^2$) | $\sin^2 2\theta_{e\mu}$ | $\sin^2 2\theta_{\mu\tau}$ | $(m_1, m_2, m_3)$ in eV | $\Omega$ (%) |
|------------------------|--------------------------|-----------------------------|------------------------|------------|
| 20                     | $\sim 3 \times 10^{-3}$  | $< 4 \times 10^{-3}$        | $(4.02 \times 10^{-4}, 4.47, -4.47)$ | 15.1% (38.9%) |
| 6                      | $2 \times 10^{-3}$       | $< 2 \times 10^{-2}$        | $(7.35 \times 10^{-4}, 2.45, -2.45)$ | 8.3% (21.4%) |
| 2                      | $(1 \sim 2) \times 10^{-3}$ | $< 8 \times 10^{-2}$        | $(1.27 \times 10^{-3}, 1.43, -1.43)$ | 4.8% (12.4%) |
| 1                      | $(2 \sim 6) \times 10^{-3}$ | $< 0.1$                     | $(1.80 \times 10^{-3}, 1.0, -1.0)$ | 3.4% (8.8%) |
| 0.5                    | $(0.7 \sim 2) \times 10^{-3}$ | $< 0.2$                     | $(2.55 \times 10^{-3}, 0.71, -0.71)$ | 2.4% (6.2%) |

TABLE II. The atmospheric neutrino data $R$ for various $L/E$ along with our predictions for $\Delta m_{31}^2 = 2$ eV$^2$, $\Delta m_{32}^2 = 0.72 \times 10^{-2}$ eV$^2$ and $b^2/N^2 = d^2/N^2 = 0.015$. We show the $r = (\mu/e)_{\text{incident}}$ values for each data point also.

| Experiments | $r$ | $L/E$ (km/GeV) | Measured  | Prediction |
|-------------|-----|----------------|-----------|------------|
| KAMIOKA \[22\] | 4.5/1 | 5 | $1.27^{+0.61}_{-0.38}$ | 0.99 |
| (Multi-GeV) | 3.2/1 | 10 | $0.63^{+0.21}_{-0.16}$ | 0.97 |
| | 2.2/1 | 100 | $0.51^{+0.15}_{-0.12}$ | 0.41 |
| | 3.2/1 | 1000 | $0.46^{+0.18}_{-0.12}$ | 0.31 |
| | 4.5/1 | 2000 | $0.28^{+0.10}_{-0.07}$ | 0.22 |
| KAMIOKA \[22\] | 2.1/1 | 80 | $0.59 \pm 0.10$ | 0.50 |
| (Sub-GeV) | | | | |
| IMB \[23\] | 2.1/1 | 12800 | $0.62 \pm 0.10$ | 0.48 |
| FREJUS \[24\] | 2.1/1 | 1000 | $0.54 \pm 0.13$ | 0.47 |
| NUSEX \[25\] | 2.1/1 | 500 | $0.87 \pm 0.18$ | 0.47 |
| SOUDAN \[26\] | 2.1/1 | 1000 | $0.69 \pm 0.21$ | 0.47 |
This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9503436v4
Figure 1

Figure 2
This figure "fig1-2.png" is available in "png" format from:

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