Investigating clustering dark energy with 3D weak cosmic shear

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ABSTRACT
As observational evidence increasingly consolidates the case for a cosmological constant Λ being the source of the Universe’s accelerated expansion, the question whether, and if so, how well, future experiments could detect deviations from this standard scenario is raised with urgency. Assuming a dark energy component different from a cosmological constant, the observable effects in general include gravitational clustering described by the fluid’s (rest-frame) speed of sound $c_s$. We employ 3D weak cosmic shear, a proposed method to take advantage of the full 3D information inherent to the cosmic shear field, to explore the capability of future surveys to detect dark energy clustering and the signature of an enhanced amplitude of the matter power spectrum on large scales. For this purpose, we present adequate numerical methods facilitating 3D weak cosmic shear calculations. We find that the possible constraints heavily depend on the dark energy equation of state $w$. If $w$ is not very close to $-1$, constraining the squared sound speed $c_s^2$ within an order of magnitude seems possible with a combination of Euclid and Planck data.

Key words: gravitational lensing: weak – methods: analytical – cosmological parameters – dark energy – large-scale structure of Universe

1 INTRODUCTION
Explaining the accelerated expansion of the Universe is one of the key tasks of cosmology today. If Einstein’s general relativity remains unaltered on cosmological scales, the observed accelerated expansion – if no local effect – is due to a cosmological constant or an unknown cosmological fluid with negative pressure, the dark energy. To this day, all major observations are consistent with a cosmological constant (Bartelmann 2010a; Komatsu et al. 2011). Its unexpectedly tiny value – the cosmological constant problem – and the fact that its energy density is comparable to that of matter just today – the coincidence problem – (see e.g. Carroll 2001) motivate the search for alternative models of dynamically evolving dark energy.

Due to the lack of observational evidence for inhomogeneities in the dark energy, most studies have only investigated the consequences of a perfectly homogeneous dark energy component. Such a fluid is completely determined by its energy density and its equation of state $w$. Its direct effect is restricted to the expansion history, which indirectly causes a scale-independent modification of the growth rate of matter perturbations. In general, however, a cosmological fluid can also leave signatures, possibly scale-dependent, by virtue of its perturbations. This could be, in principle, a means to discriminate between different dark energy models.

Once we include linear perturbations, another characteristic quantity enters the scene, the sound speed $c_s$. This quantity defines a sound horizon such that scales outside and inside this horizon can undergo different evolutions. In general, both $w$ and $c_s$ are necessary to describe the observable effects of a fluid. In order to explore the nature of the dark energy, cosmology has to constrain both $w$ and $c_s$ (Erickson et al. 2002; Hu 2002a; DeDeo, Caldwell & Steinhardt 2003; Hu & Scranton 2004).

The remarkable progress of observational cosmology in determining the fundamental parameters describing our Universe has not yet led to significant constraints on the dark energy sound speed $c_s$ (Bean & Dore 2004; de Putter, Huterer & Linder 2010; Li & Xia 2010). In this work, we study whether next-generation precision observations of the cosmic microwave background (CMB) together with the proposed method of 3D weak cosmic shear (Heavens 2003) have the potential of providing significant progress in this respect.

3D weak cosmic shear is a method to gain precision information about the growth of perturbations (Heavens 2003; Castro, Heavens & Kitching 2005; Kitching, Heavens & Miller 2011). Contrary to ordinary galaxy surveys, it has – like weak lensing in general – the advantage of being independent of galaxy bias models. Only well-understood general relativity is needed from the theoretical side. This is one reason why weak gravitational lensing, since its beginnings (Van Waerbeke et al. 2000), has advanced along with the CMB to one of the cosmological probes with the largest potential (Bartelmann 2010b; Huterer 2010). Weak-lensing methods have in fact proved to be powerful tools to constrain dark energy, i.e. mainly...
its equation of state parameter $w$ (Huterer 2002, 2010; Heavens 2003; Jain & Taylor 2003; Bernstein & Jain 2004; Takada & Jain 2004; Hannestad, Tu & Wong 2006; Huterer, Kitching & Taylor 2006; Amendola, Kunz & Sapon 2008; Hollenstein et al. 2009; Kilbinger et al. 2009).

Most weak-lensing studies consider the case of tomographic measurements where the sample of lensed galaxies is split up into redshift bins on which the standard weak-lensing methods are applied (Hu 1999, 2002b). The advantage of tomography is an enhanced sensitivity due to reduced averaging along a line of sight compared to unbinned cosmic shear spectra, but the shape of the dark matter power spectrum is not measured independently from growth factors and geometry (recent studies about tomography and the relation to 3D weak lensing include Kitching et al. 2011 and Schäfer & Heisenberg 2011).

The 3D version of weak lensing is a complement to standard 2D weak lensing with the aim of retaining the full 3D information contained in the cosmological shear field. The starting point is to not only make use of the angular positions of lensed galaxies on the sky, but to also include their redshifts as a distance measure such that each individual galaxy provides a measure of the tidal shear.

Let us briefly explain our motivation to look specifically into 3D weak lensing as opposed to tomographic methods. Weak-lensing spectra provide an integral measure of the dark matter power spectrum, weighted with the lensing efficiency function. The enhancement of the matter power spectrum due to the clustering of dark energy is restricted to large scales and would thus influence a weak-lensing convergence spectrum only little. A 3D method, however, provides a direct measurement of the amplitude of the dark matter spectrum and would be better suited to distinguish enhanced spectra from unenhanced spectra and therefore to provide constraints on the properties of dark energy and its clustering. This would effectively break the degeneracy between the power spectrum shape and the lensing efficiency, consisting of the growth function and geometrical factors, such that the signature of dark energy induced clustering should be easier to observe.

The organization of this paper is as follows. We first describe clustering dark energy in general, make contact with prominent dark energy models and introduce a parametrization in Section 2. We then explain the 3D weak cosmic shear method in Section 3. A brief description of the Fisher matrix method for forecasting parameter constraints is given in Section 4. We present adequate and efficient numerical tools in Section 5. Our results are shown in Section 6, and we conclude in Section 7.

2 CLUSTERING DARK ENERGY

2.1 The sound speed

The dynamics of the background and the evolution of scalar linear perturbations of a cosmological fluid are fully determined by its equation of state $w = p/\rho$ and its (squared) sound speed $c_s^2 = \delta p/\delta \rho$. If we describe dark energy as a cosmological fluid, coupled to other fluids only by virtue of the gravitational interaction, the natural parameters are $w$ and $c_s^2$.

The sound speed $c_s^2$ defines a characteristic scale $\lambda \propto |c_s^2|$, below which the fluid resists gravitational collapse. In turn, this means that the effects of gravitational clustering are only observable if the scale $\lambda$ lies within the Hubble horizon, $\lambda \lesssim H^{-1}$, where $H = a'/a$ is the conformal Hubble parameter and a prime denotes a derivative with respect to conformal time $\tau$.

In general, the speed of sound is defined by the quotient of the pressure and density perturbations, $c_s^2 = \delta p/\delta \rho$. Both, $\delta p$ and $\delta \rho$, however, are gauge-dependent quantities, whence we shall only consider the gauge-invariant rest-frame speed of sound defined in a frame where the velocity perturbation of the fluid vanishes, $v = 0$.

We can illustrate the role of the sound speed with the help of the evolution equations of linear perturbations, which are obtained from the general energy–momentum conservation equations $T^{\mu\nu} = 0$.

These equations are valid if there is no coupling, i.e. no energy–momentum exchange, between the fluid and other components such as matter. As usual, we split into background quantities and linear perturbations, $T^{\mu\nu} = T^{\mu\nu}_0 + \delta T^{\mu\nu}$, and we define $T^{\mu\nu}_0 = -p(1 + \delta)$, $T^{\mu\nu}_f = \delta p \delta \mu \delta \nu$ and $T^{\mu\nu}_f = (\delta + \delta + \delta) \delta + \Sigma_j$. We further define a gauge-invariant density perturbation $\Delta = \delta + 3(1 + w)\frac{\dot{a}}{a}(v - B)$ in Fourier space, where $B$ is a metric perturbation defined as in Kodama & Sasaki (1984). Choosing the fluid’s rest frame $v = 0$ and a gauge where $B = 0$, we simply get $\Delta = \delta$. Describing the evolution of perturbations, for a single fluid, in terms of the variable $pa^3\delta$, we find the following second-order differential equation (cf. e.g. Kodama & Sasaki 1984):

\[ (\rho a^3\dot{\delta})^2 + (1 + 3\frac{\dot{\rho}}{\rho}) H(\rho a^3\dot{\delta}) + \left( k^2 c_s^2 - \frac{3}{2} (1 + w) H^2 \right) (\rho a^3\delta) = 0, \]

neglecting anisotropic shear, $\Sigma_j = 0$. A critical scale $k_{crit}^2 = 1/\lambda_{crit}$ is given by the vanishing of the source term $\propto \rho a^3\dot{\delta}$ driving gravitational collapse, i.e.

\[ \lambda_{crit} = \frac{\sqrt{2}}{3} \frac{1}{\sqrt{1 + w}} |c_s^2| H. \]

The perturbation variable $pa^3\delta$ can only grow on subhorizon scales for $\lambda_{crit} \lesssim H^{-1}$, which translates into the approximate relation $c_s^2 \lesssim 1 + w$. Especially for an equation of state $w$ close to $-1$, as preferred by current observations (Komatsu et al. 2011), this only occurs for very small sound speeds $c_s^2 \ll 1$. These effects are restricted to large scales $\lambda \gtrsim \lambda_{crit}$.

In a complete description of the perturbation evolution, we have to cope with the multicomponent fluid of (at least) matter and dark energy. Nonetheless, we can still motivate a corresponding heuristic definition of an effective scale characterizing dark energy clustering (see Section 2.3). We will then also show quantitatively how a clustering dark energy component (with constant $w$ and $c_s^2$) affects the large-scale matter power spectrum $P(k)$.

The (rest-frame) sound speed $c_s^2$ considered here may not be confused with the adiabatic sound speed $c_A^2$, which is only equal to the quotient $\delta p/\delta \rho$ for adiabatic perturbations, i.e. when the entropy perturbation is zero. In general, it is given by $c_s^2 = \bar{\rho}/\bar{p}$. The difference between the two quantities defines a gauge-invariant entropy perturbation $(c_s^2 - c_A^2) \delta/\bar{w}$. For a fluid with constant equation of state $\bar{w}$, the adiabatic sound speed simply reduces to $c_s^2 = \bar{w}$. For a brief introduction to dark energy clustering, see Gordon & Hu (2004).

2.2 Relation to common dark energy models

2.2.1 Quintessence

The most prominent example of dynamical dark energy is standard quintessence (Ratra & Peebles 1988; Wetterich 1988), i.e. a cosmological scalar field $\varphi$ with standard kinetic term and a potential $V(\varphi)$, defining a Lagrangian density $\mathcal{L} = -(1/2)\varphi'\varphi' - \varphi = V(\varphi)$. For suitable choices of the potential $V(\varphi)$, the dynamics of the
background field $\psi$ shows appealing tracker behaviours providing robustness against initial conditions.

The perturbation $\delta\rho$ of the quintessence field usually is of little importance on subhorizon scales, the reason being that the quintessence sound speed $c_s^2$ is unity.

This is easily seen by explicitly writing energy density and pressure perturbations of the scalar field:

$$\delta\rho_\psi = \psi \delta\phi + V_\phi \delta\phi,$$

$$\delta p_\psi = \psi \delta\phi - V_\phi \delta\phi.$$  

(3)

(4)

Since the velocity perturbation $\upsilon$ is proportional to the field perturbation $\delta\phi$, the rest-frame speed of sound (for $\upsilon = \delta\phi = 0$) is $c_s^2 = \delta p_\psi/\delta\rho_\psi = 1$.

We conclude that the detection of a dark energy sound speed $c_s^2 < 1$ would not only challenge the $\Lambda$ cold dark matter ($\Lambda$CDM) model but standard quintessence models as well.

A class of models with very different behaviour, however, is given by coupled quintessence models (Wetterich 1995; Amendola 2000; Amendola, Baldi & Wetterich 2008a). In these models, there is an energy–momentum exchange between the dark energy and other components such as dark matter or neutrinos. Dark energy can then no longer be described as an independent fluid, and the equations of Section 2.1 do not apply. In fact, subhorizon perturbations of the quintessence field can grow in these models. Although not considered in this work, the case of energy–momentum exchange between dark energy and matter has been parametrized and studied in the light of weak lensing (La Vacca & Colombo 2008; Schäfer, Caldera-Cabral & Maartens 2008; Caldera-Cabral, Maartens & Schäfer 2009; De Bernardis et al. 2011).

### 2.2.2 k-essence

Looking at equations (3) and (4), the reason for $c_s^2 = 1$ in standard quintessence is the identical dependence of $\delta\rho_\psi$ and $\delta p_\psi$ on $\delta\phi$. Formally, this could easily be changed by allowing the potential to also depend on $\phi$, $V = V(\phi, \dot{\phi})$. If this dependence can be split into two summands, we could reinterpret the $\dot{\phi}$ dependence as a modification not of the potential but of the kinetic term.

Non-standard kinetic terms are the starting point for k-essence models of dynamical dark energy (Armendariz-Picon, Mukhanov & Steinhardt 2000, 2001). In these models, the Lagrangian $\mathcal{L}$ is a generic function of the standard kinetic term $X = -(1/2)\dot{\phi}^2/\langle \dot{\phi}^2 \rangle$. It is thus possible for the sound speed $c_s^2$ to take any value, without violating causality (Babichev, Mukhanov & Vikman 2008).

The energy density and the pressure are given by the corresponding components of the energy–momentum tensor. They read $\rho = 2\mathcal{L}_X X - \mathcal{L}$ and $p = \mathcal{L}$. The equation of state $w = \rho/p$ and the rest-frame sound speed $c_s^2 = (\delta p/\delta\rho)_\psi$ (Erickson et al. 2002) are then

$$w = \frac{\mathcal{L}}{2\mathcal{L}_X X - \mathcal{L}},$$

(5)

$$c_s^2 = \frac{\mathcal{L}_X}{\mathcal{L}_X + 2\mathcal{L}_XX X}.$$  

(6)

Of course, both $w$ and $c_s^2$ evolve in time and may take very different values at different epochs. The question whether the time evolution of $c_s^2$ could leave characteristic observational imprints was studied by Ansari & Unnikrishnan (2011). In the framework of a specific k-essence type model, 3D cosmic shear has been used to forecast possible constraints on the model parameters (Camera et al. 2010).

### 2.3 Investigating dark energy with 3D weak shear

A frequently employed parametrization of dark energy, which we shall adopt here, is the $w$CDM model, sometimes called $\Lambda$CDM (Turner & White 1997). Contrary to a cosmological constant $\Lambda$ with equation of state $w_\Lambda = -1$, the model allows for an arbitrary dark energy equation of state $w$ which is taken to be constant in time. The model is often extended to a linear evolution of $w$ with respect to the scale factor $a$ (for an attempt to study $w$ as a free function; cf. Hutner & Turner 2001). The simplest generalization for including possible clustering of dark energy is to further introduce a rest-frame sound speed $c_s^2$, also constant in time. In this paper, we completely parametrize the dark energy component by constant numbers $w$ and $c_s^2$.

Dynamical dark energy such as quintessence and k-essence provides a large class of models that cannot be approximated by a simple parametrization such as the $w$CDM model. In fact, the $w$CDM model (for constant $w$) does not resemble very closely any of the prominent dynamical models. Whenever new observational data are published, it is thus not sufficient to study constraints in the $w$CDM model alone; the individual dark energy models must also be studied.

Nonetheless, the $w$CDM model is, in terms of its parameters, a somewhat minimal extension of the standard $\Lambda$CDM model, including the latter as a special case. Hence, it is a useful tool to forecast how strong the deviations from $\Lambda$CDM must be for future observations to detect them.

We now turn to the description of linear perturbations in the presence of a clustering dark energy component parametrized as above. The linear growth of perturbations is described by a growth function $g(k, a)$ that links the Newtonian gravitational potential $\Phi_\text{GR}(a)$ at scale factor $a$ to the one today $\Phi_\text{GR}^0(a) = g(k, a) \Phi_\text{GR}(a)/a$. We shall now introduce the parametrization for $g(k, a)$ that we use for the study of dark energy perturbations in the $w$CDM model.

Dark energy perturbations contribute to the gravitational potential just as matter perturbations via the Poisson equation:

$$k^2 \Phi = -4\pi G \rho_\text{DE} \frac{\Delta(\rho_\text{DE})}{\Delta(\rho_\text{DE})} \equiv -4\pi G \mathcal{Q} \rho_\text{DE} \Delta(\rho_\text{DE}),$$

(7)

where we have used the gauge-invariant density perturbations $\Delta(\rho_\text{DE})$ and $\Delta(\rho_\text{DE})$ and introduced the quantity $Q = Q(k, a)$. It is defined via

$$Q = 1 + \frac{\rho_\text{DE} \Delta(\rho_\text{DE})}{\rho_\text{DE} \Delta(\rho_\text{DE})}.$$  

(8)

An important effect of dark energy perturbations is their influence on the growth of matter perturbations expressed in terms of a modified growth index $\gamma$ defined by $d \ln \Delta(\rho_\text{DE})/d \ln a = \Omega_\text{DE}^{\gamma}$ (Linder & Cahn 2007). As a function of $Q$, we may approximate (cf. Linder & Cahn 2007; Sapone & Kunz 2009; Sapone, Kunz & Amendola 2010)

$$\gamma \approx \frac{3(1 - w - A)}{5 - 6w}, \quad A = \frac{Q - 1}{1 - \Omega_\text{DE}}.$$  

(9)

We follow Sapone et al. (2010), parametrizing $Q(k, a)$ for the $w$CDM model with sound speed $c_s^2$ as

$$Q \approx 1 + \frac{1 - \Omega_\text{DE}^{\gamma} (1 + w) a^{-3w}}{1 - 3w + \gamma^2}, \quad \gamma^2 = \frac{3}{\Omega_\text{DE}^{\gamma}} \mathcal{Q}^2.$$  

(10)

Together, these equations provide a convenient way of obtaining the growth function

$$g(k, a) = \frac{Q(k, a)}{Q^0(k)} \exp \left( \int_1^a \frac{da'}{a'} \Omega_\text{DE}(a')^{\gamma(1 + a')} \right).$$  

(11)

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The explicit appearance of $Q$ in this expression is due to our definition of $g$ describing the growth of the total gravitational potential rather than of the matter perturbations only.

Of course, the growth function could easily be directly obtained by solving the linear perturbation equations numerically. For illustration, we show the linear matter power spectrum $P(k)$ for the $w$CDM model for different sound speeds $c_s^2$ and $w = -0.8$ in Fig. 1. Here, we have used the code CAMB (Code for Anisotropies in the Microwave Background; Lewis, Challinor & Lasenby 2000), which has built-in facilities to work with the $w$CDM model with constant $c_s^2$. We have assumed adiabatic initial conditions.

Perturbations in the dark energy act as an extra source of the gravitational potential in the Poisson equation (7) enhancing the growth of matter perturbations on subhorizon scales. This enhancement, however, is less than a per cent effect for sound speeds $c_s^2 \gtrsim 0.1$ and restricted to large scales. The power spectrum $P(k)$ to have been known with very high precision in order to find significant constraints on $c_s^2$. This becomes even more difficult for $w$ closer to $-1$ (cf. equation 2). Note that the plot also shows superhorizon scales, where the results are gauge-dependent. For our analysis, we will use (subhorizon) scales $k$ between $10^{-3}$ and $10^{-1}$ Mpc$^{-1}$.

With the parametrization of $Q(\kappa, a)$, equation (10), at hand, we can ask above which scale $\lambda_{\text{eff}}$ dark energy clustering could leave observable traces. Let us make the heuristic assumption that the effect of a clustering dark energy component would be observable once roughly $Q(k, a = 1) \gtrsim 1 + \varepsilon$, with $\varepsilon$ for example at the per cent level. This is the case for scales

$$\lambda \equiv \frac{1}{k} \gtrsim \left( \frac{\varepsilon}{1 - \Omega_m^0} \right)^{1/2} \sqrt{\frac{2}{3}} \sqrt{\frac{1}{1 + w} \frac{|c_s|^2}{H_0^2}}.$$  

(12)

For the exemplary values $\varepsilon \approx 1$ per cent and $\Omega_m^0 \approx 0.3$, this defines a critical scale,

$$\lambda_{\text{eff}} \approx 0.1 \frac{|c_s|^2}{\sqrt{1 + w}}.$$  

(13)

with a similar behaviour as the scale given in equation (2). A precise experiment might be able to detect dark energy clustering if the effective scale $\lambda_{\text{eff}}$ lies within the Hubble horizon. In particular, the common choice $w = -1$ refers to $\lambda_{\text{eff}} \rightarrow \infty$. This is already obvious from the parametrization (10) yielding $Q = 1$ on all scales for $w = -1$. In this case, the sound speed becomes irrelevant and dark energy clustering cannot be detected.

3 3D WEAK COSMIC SHEAR

3.1 Convergence

In the presence of a gravitational lens, the observed image points $\theta$ of a galaxy differ from their true positions $\beta$. In a locally linear approximation, the mapping $\theta \mapsto \beta$ is described by a matrix

$$\mathbf{A} = \begin{pmatrix} 1 - \kappa - \gamma_1 & -\gamma_2 \\ -\gamma_2 & 1 - \kappa + \gamma_1 \end{pmatrix}.$$  

(14)

The convergence $\kappa$ determines the magnification of the image, while the shear $(\gamma_1, \gamma_2)$ determines its deformation. Both convergence and shear are calculated from second derivatives of the lensing potential $\phi$, e.g.

$$\kappa = \frac{1}{2} \Delta_{\beta,\gamma} \phi.$$  

(15)

The lensing potential $\phi$ is a projection of the Newtonian gravitational potential $\Phi$. In a flat universe,

$$\phi(\chi) = 2 \int_0^\chi \frac{d\chi'}{\chi'} \frac{\chi - \chi'}{\chi} \Phi(\chi'),$$  

(16)

where $\chi, \chi'$ denote comoving coordinates. The convergence $\kappa$ thus depends on the gravitational potential along the line of sight, which, in turn, is given by the density fluctuations. In this way, gravitational lensing can be used to probe the density field, without relying on galaxy bias models (Jain & Seljak 1997; Hu & White 2001). For general treatments of weak gravitational lensing, see Bartelmann & Schneider (2001) and Bartelmann (2010b).

In a region of the sky covered by a weak-lensing survey, the individual convergences $\kappa$ and shears $\gamma_i$ of the galaxies together allow us to study the 2D fields $\kappa(\theta, \varphi)$ and $\gamma_i(\theta, \varphi)$. If the galaxies’ distances are known (e.g. by a photometric redshift measurement), the fields become 3D, $\kappa(\chi, \theta, \varphi)$ and $\gamma_i(\chi, \theta, \varphi)$. 3D weak cosmic shear is a means to study the statistical properties of these fields (Heavens 2003; Castro et al. 2005).

The importance of the 3D information for weak-lensing precision tests of structure formation has first been studied for tomography (Hu 2002b). Also, the use of spectroscopic redshifts instead of a photometric method has been considered (Ishak & Hirata 2005).

The statistics of the convergence field $\kappa$ are hardly directly observable. But since the statistics of convergence and shear are equivalent, we may use the convergence $\kappa$ instead of $\gamma_i$ in our theoretical calculations.
The first step in a 3D weak cosmic shear calculation is a combined Fourier and spherical harmonic transform, \( \chi \to k, (\theta, \phi) \to (\ell, m) \),

\[
\kappa_{\ell m}(k) = \sqrt{\frac{2}{\pi}} \int \chi^2 d\Omega k(\chi, \theta, \phi) j_\ell(k\chi) Y_{\ell m}^*(\theta, \phi). 
\]  

(17)

By means of the growth factor \( g(k, a) \), we may replace the gravitational potential in equation (16) by the potential of today, \( \Phi_{\ell m}(k) = g(k, a) \Phi_{\ell m}(k)/a \). Note that in clustering dark energy scenarios, the growth factor is scale-dependent. In the transformed variables, equations (15), (16) and (7) take the simple forms

\[
\kappa_{\ell m} = -\frac{(n + 1)}{2} \phi_{\ell m},
\]

(18)

\[
\phi_{\ell m} = \eta_j(k, \nu) \Phi_{\ell m}^0(k'),
\]

(19)

\[
k^2 \Phi_{\ell m} = -4\pi G a^2 \rho_c \Delta_{\ell m}^{(n)},
\]

(20)

where we have, following Heavens (2003), introduced the quantity

\[
\eta_j(k, \nu) = \frac{4}{\pi} \int_0^\infty \chi^2 d\chi j_\ell(k\chi)
\]

\[
\times \int_0^\infty d\chi' \chi - \chi' j_\ell(k'\chi') \frac{g(k', a')}{a'}
\]

(21)

and used the summation convention

\[
A(k, \nu) B(k', \nu) = \int_0^\infty k^2 dk' A(k, \nu) B(k', \nu).
\]

(22)

The appearance of \( Q \) on the right-hand side of the Poisson equation accounts for the direct contribution, \( \alpha_{PDE} \Delta^{(DE)} \), of dark energy perturbations to the gravitational potential. The indirect and dynamical effect of dark energy clustering on the evolution of matter perturbations is neglected.

3.2 Estimator

In Section 3.1, we have seen that the convergence \( \kappa_{\ell m}(k) \) is intimately connected to the density fluctuation field \( \delta_{\ell m}^{(g)}(k) \) by virtue of equations (18)–(20). In other words, we can, for example, use the convergence \( \kappa_{\ell m}(k) \) to probe the matter power spectrum \( P(k) \).

Heavens (2003) has shown how to construct an appropriate estimator for a weak-lensing survey including photometric redshifts. The two main ingredients of this estimator are the following.

(i) The inclusion of the uncertainty of the redshift measurement, for simplicity assumed to be a Gaussian with width \( \sigma_z \), equal for all galaxies:

\[
p(\chi|z)dz = \frac{1}{\sqrt{2\pi}\sigma_z} \exp \left[ -\frac{(z - \bar{z})^2}{2\sigma_z^2} \right] dz.
\]

(23)

We use a typical figure of \( \sigma_z = 0.02 \) (Heavens 2003). An extension of the formalism allowing for individual redshift errors is possible (Kitching et al. 2011).

(ii) The survey’s galaxy distribution encoded in the number density \( n(\chi) \) assumed rotationally symmetric. It constitutes a statistical weight favouring distances \( \chi \) where the density of galaxies is higher. We use the forecasted shape

\[
n(z)dz \propto z^2 \exp \left[ -\left( \frac{z}{\tilde{z}_0} \right)^\beta \right] dz.
\]

(24)

for the Euclid survey. Here, we assume 100 galaxies arcmin\(^{-2}\), \( \tilde{z}_0 = 0.64 \) and \( \beta = 3/2 \), yielding a median redshift of \( \tilde{z}_{\mathrm{med}} = 0.9 \) (Amara & Refregier 2007). For convenience, we consider the idealized case that the full sky is covered. For a realistic sky coverage \( f_{\mathrm{sky}} < 1 \), the errors scale approximately by \( f_{\mathrm{sky}}^{1/2} \).

We may then define the estimator \( \hat{\kappa}_{\ell m} \) for the convergence \( \kappa_{\ell m} \) in terms of the actual convergences \( \kappa_\ell \) of galaxies \( g \) as the harmonic transform

\[
\hat{\kappa}_{\ell m}(k) = \sqrt{\frac{2}{\pi}} \sum g_{\ell m}(k) \kappa_\ell(k\chi) Y_{\ell m}^*(\theta, \phi).
\]

(25)

As explained above, the cosmic shear would be better suited for the analysis of actual observational data. The expectation value of \( \hat{\kappa}_{\ell m} \) is

\[
\hat{\kappa}_{\ell m}(k) = Z_\ell(k, \nu') M_\ell(k', \nu') \kappa_{\ell m}(k'),
\]

(26)

with the summation convention (22) and the quantities

\[
Z_\ell(k, \nu') = \frac{2}{\pi} \int \chi^2 d\chi j_\ell(k\chi) j_\ell(k'\chi) n(\chi),
\]

(27)

\[
M_\ell(k, \nu') = \frac{2}{\pi} \int \chi^2 d\chi j_\ell(k\chi) j_\ell(k'\chi) n(\chi) n(\chi'),
\]

(28)

taking account for the two main ingredients stated above. Abbreviating the product \( B_\ell(k, \nu') \equiv Z_\ell(k, \nu') M_\ell(k', \nu') \eta_j(k', \nu') \), the covariance of \( \hat{\kappa}_{\ell m} \) in terms of the matter power spectrum \( P(k) \) reads

\[
S_{\ell m}(k, \nu') = \langle \hat{\kappa}_{\ell m}(k) \hat{\kappa}_{\ell m}(k') \rangle = A^2 B_\ell(k, \nu') \frac{Q_k^2(k')}{k^{10}} \frac{P(k)}{k_{\mathrm{cld}}^4} B_{k'}(k', \nu'),
\]

(29)

with

\[
A = \frac{(n + 1)}{2} 4\pi G a^2 \rho_m.
\]

The full covariance \( C_{\ell m}(k, \nu') \) is obtained by adding the shot noise \( N_{\ell m}(k, \nu') = (\sigma_z^2/4)M_\ell(k, \nu') \), with \( \sigma_z^2 = 0.1 \) (Heavens 2003). This neglects the non-zero correlation between the ellipticities of neighbouring galaxies due to intrinsic alignments (Heavens, Refregier & Heymans 2000; Schäfer 2009). This small-scale effect, however, does not affect our analysis of the large-scale consequences of a dark energy speed of sound. Further systematic effects have been studied (March et al. 2011) but, in general, do not seem to have a strong impact on parameter estimation (Huterer et al. 2006; Kitching, Taylor & Heymans 2008; Takada & Jain 2009).

4 PARAMETER ESTIMATION

We apply a standard Fisher information matrix method to investigate possible future parameter constraints from upcoming weak-lensing surveys. The Fisher information matrix \( F_{\mu\nu} \) is a square matrix whose indices label (cosmological) parameters \( p_{\mu} \). We choose the parameters \( p_{\mu} \in \{\Omega_m, A_s, h, n_s, \theta, \log_10c_s^2\} \), assuming flatness: \( \Omega_\Lambda = 1 - \Omega_m \). The Fisher matrix determines stringent bounds on how precise a parameter \( p_{\mu} \) can be constrained. If all parameters are estimated from the experimental data, the individual uncertainty \( \Delta p_{\mu} \) does not go below the Cramér–Rao bound, \( \Delta p_{\mu} \geq \sqrt{\langle F^{-1} \rangle_{\mu\mu}} \) (for an introduction, see Tegmark, Taylor & Heymans 1997). The Cramér–Rao bound not only applies to individual parameters, but also determines optimal confidence regions for a set of parameters. For two parameters \( p_{\mu} \) and \( p_{\nu} \), the corresponding coefficients of \( F^{-1} \) are a quadratic form defining an error ellipse.

Formally, the Fisher matrix is defined via the likelihood \( L \),

\[
F_{\mu\nu} = \langle -\partial_\mu \partial_\nu \ln L \rangle.
\]

(30)
The likelihood \( L \equiv L(\kappa_{\ell m} | p) \) is the probability for an experiment to measure the value \( \kappa_{\ell m} \) for the estimator given cosmological parameters \( p \).

The cosmological parameters enter the likelihood in two ways. First, they predict a power spectrum \( P(k) \) and a growth function \( g(k, a) \), which, by equation (29), are decisive quantities for the covariance of the estimator. Secondly, they define the background evolution and hence the distance measures entering the quantities \( Z_i, M_i \) and \( \eta_i \).

If the likelihood \( L \) is a multivariate Gaussian in the data with covariance matrix \( C \), the Fisher matrix is given by

\[
F_{\ell \nu} = \frac{1}{2} \text{tr} \left[ C^{-1} (\partial_{\ell} C) C^{-1} (\partial_{\nu} C) \right],
\]

sensitive to the derivatives of the covariance \( C \) with respect to the cosmological parameters.

For our estimator \( \kappa_{\ell m}(k) \), the covariance matrix carries the indices \((\ell, m, k, k')\). Since different modes \( \ell \) and \( m \) are uncorrelated, the covariance matrix \( C \) splits into blocks. Further, the covariance \( C_{\ell}(k, k') \) from Section 3.2 is assumed to be independent of \( m \), whereby all \( 2\ell + 1 \) blocks for a given \( \ell \) are identical. In terms of the covariance \( C_{\ell}(k, k') \), we may reformulate equation (31) to

\[
F_{\ell \nu} = \sum_{\ell=\min}^{\ell=\max} \frac{2\ell + 1}{2} \text{tr} \left[ C_{\ell}^{-1} (\partial_{\ell} C_{\ell}) C_{\ell}^{-1} (\partial_{\nu} C_{\ell}) \right].
\]

It should be kept in mind that the Cramér–Rao bounds are realistic estimates of the actual constraints only if the likelihood \( L \) as a function of the parameters \( p \) is a Gaussian. This is often violated in the case of parameters that are difficult to measure and therefore weakly constrained, such as the sound speed parameter \( c_s^2 \). The broad likelihood extends to regions where the dependence of the matter power spectrum \( P(k) \) on \( c_s^2 \) (cf. Fig. 1) cannot be approximated linearly (Ballesteros & Lesgourgues 2010). This also affects the weak-lensing convergence spectrum considered in this work. Fig. 1 suggests that the logarithm \( \log P(k) \) is a more natural parameter to describe the reaction of the model to variations in the dark energy speed of sound. We thus choose \( \log \Theta \) as a model parameter in our analysis but emphasize that the Cramér–Rao bounds we calculate are only rough estimates of the actual future constraints.

A very practical feature of the Fisher matrix is its additivity. Given Fisher matrices \( F^{(A)} \) and \( F^{(B)} \) for two independent parameters \( A \) and \( B \), the joint Fisher matrix providing the combined parameter constraints is simply \( F^{(A+B)} = F^{(A)} + F^{(B)} \). This follows directly from the multiplication of the corresponding likelihoods and the definition of the Fisher matrix (equation 30). In our case, we can use this formalism to include prior information from other experiments than weak gravitational lensing.

As prior information, we use a Fisher matrix \( F^{(\text{MB})}_{\mu \nu} \) for the CMB based on forecasts for the Planck satellite. We include temperature \((TT)\), polarization \((EE)\) and the cross-correlation spectrum \((TE)\). We calculate the Fisher matrix \( F^{(\text{MB})} \) following Perotto et al. (2006). The predicted noisy spectra \( C_{\ell}^{TT} \) (temperature only), \( C_{\ell}^{TE} \) (E-mode polarization) and \( C_{\ell}^{TT} \) (cross-correlation) are encoded in a \( 3 \times 3 \) matrix,

\[
\mathbf{A}_{\ell} = \frac{2}{(2\ell + 1) f_{\text{sky}}} \left( \begin{array}{ccc}
\left( C_{\ell}^{TT} \right)^2 & \left( C_{\ell}^{TE} \right)^2 & \left( C_{\ell}^{TT} \right) \left( C_{\ell}^{EE} \right) \\
\left( C_{\ell}^{EE} \right)^2 & \left( C_{\ell}^{TT} \right)^2 & \left( C_{\ell}^{EE} \right) \left( C_{\ell}^{EE} \right) \\
\left( C_{\ell}^{TE} \right)^2 & \left( C_{\ell}^{TE} \right)^2 & \frac{1}{2} \left( \left( C_{\ell}^{TT} \right)^2 + \left( C_{\ell}^{TE} \right)^2 \right)
\end{array} \right),
\]

with a fraction \( f_{\text{sky}} \) of the CMB covered. From this, we evaluate the Fisher matrix,

\[
F^{(\text{MB})}_{\mu \nu} = \sum_{\mu=TT,EE,TE} \sum_{\nu=TT,EE,TE} \partial_{\mu} C^{PP} \left( A^{-1} \right)_{\mu \nu} \partial_{\nu} C^{QQ},
\]

with the indices \( PP', QQ \in \{TT, EE, TE\} \).

Our forecast bases on expected properties of the Planck satellite (Knox 1995; Hollenstein et al. 2009). We adopt the expected instrument properties as listed in table 1 of Hollenstein et al. (2009), namely a sky coverage \( f_{\text{sky}} = 0.65 \), a beam width \( \theta_{\text{FWHM}} = 7 \) arcmin, temperature noise \( \Delta_T = 28 \mu \text{K arcmin} \) and polarization noise \( \Delta_\nu = 57 \mu \text{K arcmin} \). For the numerical calculation of the theoretically predicted multipoles, we employ \( \text{CAMB} \).

5 METHOD

In principle, we have already collected the ingredients for our 3D weak-lensing calculations, namely the covariance of the estimator (cf. equation 29) and the Fisher information matrix (equation 32). Due to the presence of multiple nested integrals, the actual calculation is involved and motivates the choice of adequate numerical approaches and techniques. We present our strategies in this section.

5.1 The quantities \( Z_i, M_i \) and \( \eta_i \)

The expectation value \( \kappa_{\ell m} \) of the 3D convergence estimator (equation 26) mainly is the application of \( Z_i \) (27), \( M_i \) (28) and \( \eta_i \) (21) on today’s gravitational potential \( \Phi_1^{(0)}(k) \):

\[
\kappa_{\ell m} \propto Z_i(k, k') M_i(k', k'') \eta_i(k'', k''') \Phi_1^{(0)}(k'''),
\]

where each multiplication corresponds to a k integration according to the convention (equation 22). We have introduced the shorthand \( B_\ell(k, k'') \equiv Z_i(k, k') M_i(k', k'') \eta_i(k'', k''') \) for the product.

Before we explain an elaborate way to calculate \( B_\ell \) with high precision, we first turn to a simplified approximate approach. Recalling that the sequence of functions

\[
f_\ell(x) = \sqrt{\frac{2}{\pi}} y \sqrt{x} j_\ell(x), \quad \xi \equiv y(x + 1), \quad y \equiv \ell + 1 \quad (36)
\]

approaches the Dirac delta function \( \delta_D \) for \( \ell \rightarrow \infty \), we may, for sufficiently large \( \ell \), use the approximation

\[
j_\ell(k x) \approx \sqrt{\frac{\pi}{2 k}} \frac{1}{k} \delta_D \left( x - \frac{k}{k} \right).
\]

In this approximation, the quantities \( Z_i, M_i \) and \( \eta_i \) take simple forms, namely

\[
Z_i(k, k') \approx \frac{y}{k^2} p \left( \frac{y}{k} \right), \quad M_i(k, k') \approx \frac{1}{k^2} n \left( \frac{y}{k} \right) \delta_D (k - k'), \quad \eta_i(k, k') \approx \frac{2}{k^2} \frac{k}{k'} g(k, a(y/k)) \quad \text{for } k \leq k', \quad 0 \text{ else.}
\]

Calculating the final product \( B_\ell \) now does no longer pose difficulties:

\[
B_\ell(k, k') \approx \frac{y}{k^2} \frac{2g(k', a(y/k))}{a(y/k)} \times \int_0^\infty d\chi p \left( \frac{y}{k} \right) \frac{n(\chi) - \frac{\chi - \frac{k}{k'}}{k'}}{k^2}.
\]
The matrix $B_\ell(k, k') = Z_\ell(k, k') M_\ell(k', k'') \eta_\ell(k'', k')$ for $\ell = 10$ (upper surface) with the difference between the full integration and the approximation given by equation (41) (lower surface and contours), which shows a small oscillatory feature close to the steep edge of $B_\ell(k, k')$ amounting to less than 10 per cent of the amplitude.

We compare this approximate result with the full expression in Fig. 2.

Although useful for a first impression, these approximate results do not allow for a precise calculation of the covariance $C_\ell(k, k')$. We thus develop a more sophisticated strategy.

5.2 Covariance

The signal and noise parts of the covariance matrix are given in Section 3.2. While the noise part $\alpha M_\ell$ is uncomplicated, the direct evaluation of the signal $S_\ell$ (29) would, in a first step, require the calculation of $Z_\ell$, $M_\ell$, and $\eta_\ell$, which contain highly oscillating integrands [cf. equations (27), (28) and (21)]. A second step, the product $B_\ell(k, k') = Z_\ell(k, k') M_\ell(k', k'') \eta_\ell(k'', k')$ has to be calculated. Taken together, these are seven nested integrals. Calculating the signal covariance $S_\ell$, then requires two further integrations.

Fortunately, the orthogonality relation for spherical Bessel functions,

$$\int_0^\infty k^2 dk \ j_1(k\chi) j_1(k\chi') = \frac{\pi}{2\chi^2} \delta_0(\chi - \chi'), \quad (42)$$

can be used to solve several $k$ integrals analytically. The remaining expression for $B_\ell$ reads

$$B_\ell(k, k') = \frac{4}{\pi} \int \chi' d\chi' j_1(k\chi) \int d\chi \ p(\chi|\chi') n(\chi) f_\ell(k', \chi), \quad (43)$$

where

$$f_\ell(k, \chi) = \int d\chi' j_1(k\chi') \frac{\chi - \chi'}{\chi \chi'} \frac{g(k, a')}{a'} \ , \quad (44)$$

The number of nested integrals in the calculation of $B_\ell$ is reduced to three.

We will show that an efficient evaluation of the inner integral in equation (43) is possible using a fast Fourier transform (FFT). We therefore have to sample $f_\ell$ at discrete coordinates $\{\chi_i\}$. For each $\chi_i$, we need not calculate the full integral (44) but only an integral from $\chi_{i-1}$ to $\chi_i$. This is possible once we write the integral in a way that makes the integrand independent of the integral bound:

$$f_\ell(k, \chi) = \int_0^\infty \frac{d\chi'}{a'} j_1(k\chi') \frac{g(k, a')}{a'} = \frac{1}{\chi} \int_0^\infty d\chi' j_1(k\chi') \frac{g(k, a')}{a'}. \quad (45)$$

In redshift space, the conditional probability $p(z'|z)$ is a Gaussian (cf. equation 23). Inserting this property, we reformulate the inner integral in equation (43) as a convolution:

$$\int_0^\infty d\chi \ p(\chi|\chi') n(\chi) f_\ell(k', \chi) = \frac{dz'}{dz} \int_0^\infty \frac{d\chi}{dz} p(z' - z) f_\ell(k', z) \ . \quad (46)$$

For convolution integrals, fast solving methods exist. This is due to the convolution theorem stating that the Fourier coefficients of the individual functions can be multiplied to give the Fourier coefficients of the convolution. The Fourier transform of the Gaussian is again a Gaussian and thus analytically known. In the last factor, we use the sampled values of $f_\ell$ to perform a FFT.

5.3 Fisher matrix

Once the covariance $C_\ell(k, k')$ is known, the Fisher matrix $F_{\ell\ell'}$ can, in principle, be calculated according to equation (32). In terms of linear algebra, the Fisher matrix is given by a trace, which is a basis-independent operation. This opens the possibility of calculating the covariance in another basis, allowing for a more efficient numerical calculation.

Working with the tools of linear algebra, we find it more transparent to abandon the summation convention (22) for a moment and to work with standard notation instead. All earlier expressions can easily be reproduced if quantities of the type $A_\ell(k, k')$ are replaced by ordinary matrices

$$A_{\ell k} = \sqrt{k^2 / \Delta k} A_\ell(k, k') \sqrt{k^2 / \Delta k}, \quad (47)$$

with a discrete step size $\Delta k$. The additional factors automatically reproduce the summation convention once a matrix multiplication is performed, $\sum_k k^2 \Delta k \rightarrow \int k^2 dk$.

Let us search for an orthogonal transformation $T^\ell$ of the covariance matrix $C_{\ell k} = \sqrt{k^2 / \Delta k} C_\ell(k, k') \sqrt{k^2 / \Delta k}$,

$$C^\ell = (T^\ell)^\dagger C T^\ell. \quad (48)$$

A good choice would, when applied on $C^\ell$, produce the orthonormality relation for spherical Bessel functions. Such a choice is given by

$$T_{\ell k'} = \sqrt{\frac{2}{\pi}} \sqrt{k^2 / \Delta k} j_\ell(k\rho) \sqrt{\rho^2 / \Delta \rho} \ . \quad (49)$$

The noise part $N^\ell \propto M^\ell$ becomes particularly simple,

$$M^\ell_{\rho\rho'} = \sum_{k, k'} T_{\ell k} M_{\ell k} T_{\ell k'} = n(\rho) \delta_{\rho\rho'} \ . \quad (50)$$

For the transformed signal part, $S^\ell = (T^\ell)^\dagger S^\ell T^\ell$, the product $B^\ell$ is transformed from the left-hand side only,

$$B^\ell_{\rho\rho'} = \sum_k T_{\ell k} B_{\ell k} \ . \quad (51)$$
In fact, this transformation further simplifies $\mathbf{B}^\ell$ by virtue of the orthogonality relation,

$$
\mathbf{B}^\ell_{\kappa k} = 2\sqrt{2 \pi} \rho^2 \Delta \rho \int_0^\infty d\chi \, p(\rho | x) n(x) \times \int_0^\chi d\chi' \, j_\ell(k' \chi') \frac{X - X'}{X X'} g(k', a') \sqrt{k^2 \Delta k}.
$$

(52)

Applying the matrix $\mathbf{T}^\ell_{\kappa k}$ introduced above on a quantity $\mathbf{A}_{\kappa k}$ can be understood as undoing the transformation $\chi \rightarrow k$ in the harmonic transform (cf. equation 17). This means that the Fourier mode $k$ is replaced by a comoving distance, now labelled by $\rho$. Hence, the application of $\mathbf{T}^\ell_{\kappa k}$ avoids unnecessary integrations originating from the harmonic transform.

Finally, we have all the necessary tools for an efficient calculation of the Fisher matrix at our disposal.

6 RESULTS

The Fisher matrix formalism (cf. Section 4) and our numerical methods, explained in Section 5, enable us to estimate which constraints on the dark energy sound speed will be possible with the weak-lensing data of Euclid. The constraints depend, however, on the assumed fiducial parameters since the Fisher matrix is defined by derivatives at these points (31). Unfortunately, the dependence of sound speed constraints on the fiducial values of both the sound speed $c_s^2$ itself and the equation of state $w$ is very strong. This is illustrated by the scale $\lambda_{\text{eff}}$ introduced in Section 2.3 (equation 13), below which dark energy clustering is not expected to be observable. This scale is a function of both $c_s^2$ and $w$; it exceeds the Hubble horizon for $c_s^2 \gg 1 + w$. In particular, the most natural fiducial value for $w$ mimicking the standard $\Lambda$CDM model, i.e. $w = -1$, is a singular choice, $\lambda_{\text{eff}} \rightarrow \infty$. The question of how well the sound speed $c_s^2$ can be constrained crucially depends on how close the equation of state $w$ is to the value $-1$.

In order to explore this behaviour quantitatively, we apply the Fisher matrix formalism to estimate the uncertainties of the dark energy sound speed and equation of state as functions of the fiducial values $c_s^2$ and $w$. In Section 4, we argued that a natural parameter to constrain is the order of magnitude $\log_{10} c_s^2$ rather than $c_s^2$ itself. The relative error on the sound speed approximately is $\Delta c_s^2 / c_s^2 \approx \ln(10) \Delta \log_{10} c_s^2$. This becomes imprecise for large uncertainties. In the case of the equation of state, we estimate $\Delta w / |w|$. For simplicity, we assume all other cosmological parameters to be exactly known, fixed to the 7-year Wilkinson Microwave Anisotropy Probe (WMAP7) recommended $\Lambda$CDM parameters (Komatsu et al. 2011).

The Fisher matrix is then a $2 \times 2$ matrix, and the uncertainties are estimated as explained in Section 4. We combine CMB and 3D weak-lensing constraints. In our numerical calculation, the multipoles $\ell$ run from $\ell_{\text{min}} = 2$ to $\ell_{\text{max}} = 50$, the mode $k$ from $k_{\text{min}} = 10^{-3}$ Mpc$^{-1}$ to $k_{\text{max}} = 10^{-1}$ Mpc$^{-1}$ in $N_k = 200$ equidistant steps. The included redshift range is $z_{\text{min}} = 10^{-4}$ to $z_{\text{max}} = 10$ in $N_z = 1000$ steps. For the CMB Fisher matrix, we include, as in all subsequent calculations, multipoles from $\ell = 2$ to 2250. The uncertainties are shown in Fig. 3. These results should be taken as a first approximation due to the limitations of the Fisher formalism when applied to weakly constrained parameters (cf. Section 4).

The lower figure, which shows the relative error $\Delta w / |w|$ on the dark energy equation of state $w$, is easily interpreted. The constraints on the equation of state parameter $w$ are largely independent of the assumed sound speed $c_s^2$. Therefore, at least, the uncertainty in the sound speed $c_s^2$ does not worsen the accuracy with which $w$ can be known, nor will a wrong assumption on $c_s^2$ introduce a significant bias on the estimate of $w$.

In the upper figure, we see that, conversely, the sound speed constraints heavily depend on the fiducial values, as explained above. For $w \geq -0.95$ and sufficiently small $c_s^2$, the estimated error $\Delta \log_{10} c_s^2$ is smaller than 1. We may thus hope that the combination of 3D weak cosmic shear and the CMB will determine the order of magnitude of $c_s^2$.

For subsequent calculations, we choose the exemplary fiducial value $c_s^2 = 10^{-2}$. For $w$, the most natural choice, $w = -1$, is not adequate. If we still chose $w$ close to $-1$, e.g. $w = -0.99$ or $-0.9$, all results would strongly depend on the exact value chosen. Instead, we decide to go further away from the observationally preferred value and use $w = -0.8$ for illustration.

Adopting these choices for $c_s^2$ and $w$ as the fiducial values, together with the $\Lambda$CDM WMAP7 recommended parameters (Komatsu et al. 2011), we now calculate the full Fisher matrices for our six cosmological parameters: fractional matter density $\Omega_m$, scalar initial perturbation amplitude $A_s$, Hubble parameter $h$, scalar spectral index $n_s$, equation of state $w$ and sound speed $c_s^2$. We choose higher numerical precision, $\ell_{\text{max}} = 300$ and $N_k = 500$, and...
avoid non-linear scales. The resulting confidence regions for 3D weak cosmic shear alone and for the combined constraints with the CMB are seen in Fig. 4.

3D weak cosmic shear obviously provides interesting constraints on all the six cosmological parameters included in our analysis. Some constraints considerably improve when the CMB Fisher matrix is added. This is not true for the sound speed. We emphasize, of course, that our choice of fiducial parameters of $w$ and $c_s^2$ is only illustrative. In more realistic cases with $w \approx -1$; the constraints will be much weaker (cf. Fig. 3).

The Fisher matrix $F_{\mu \nu}$ for 3D weak lensing is obtained from a summation of all multipoles $\ell$ (cf. equation 32). It is instructive to examine which multipoles most contribute to the parameter constraints. We therefore plot the uncertainties of all the parameters as functions of the maximum multipole $\ell_{\text{max}}$ in equation (32); see Fig. 5.

Let us first consider the parameters other than $c_s^2$. These show two distinct behaviours. The constraints on the parameters $w$, $\Omega_m$, and $h$ are strongly improved by going to larger multipoles. The two parameters $A_s$ and $n_s$ characterizing the primordial scalar perturbation spectrum are already tightly constrained for low multipoles. This is linked to the different sensitivities of the two independent observations, 3D weak lensing and the CMB, on these parameters. Looking again at the error ellipses in Fig. 4, we see that the constraints of 3D weak lensing alone on $w$, $\Omega_m$, and $h$ are not much weaker than the combined ones. Here, 3D weak lensing can establish strong constraints with increasing $\ell_{\text{max}}$. On the other hand, the CMB is more sensitive to $A_s$ and $n_s$, whereby 3D weak lensing, regardless of $\ell_{\text{max}}$, cannot contribute very much to the constraints.

The case of the dark energy sound speed $c_s^2$ is different. The fact that the uncertainty does not decrease significantly with increasing $\ell_{\text{max}} \gtrsim 20$ is mainly the consequence of clustering dark energy being a large-scale phenomenon (cf. Section 2.3 and Fig. 1). Plotting the covariances $C_{\ell}(k, k')$ of the estimator (see Section 3.2) for increasing multipoles $\ell$, we see how the maximal sensitivity moves to smaller scales (Fig. 6). In fact, for low multipoles $\ell$, 3D weak shear probes the scales of interest where dark energy clustering mainly occurs. The maxima seen in Fig. 6 are related to the fact that the galaxy distribution $n(\chi)$ (equation 24) peaks at a comoving distance $\chi_*$ characterizing the survey. Approximating Bessel functions by Dirac deltas (cf. equation 37), this distance roughly corresponds to the scale $k \approx \ell / \chi_*$. This explains the shift of the maximum for varying $\ell$ observed in Fig. 6.

Another way to study the $\ell$ dependence for the parameter constraints is to look at the direct contribution of a multipole $\ell$ to the diagonal elements $F_{\mu \mu}$ of the Fisher matrix (equation 32). These
quantities can be interpreted as a (squared) sensitivity $s^2_{\mu}$ per $\ell$ mode,

$$s^2_{\mu} = \text{tr} \left( C^{-1}_{\ell} \partial_{\mu} C \right)^2$$

(53)

Another way of interpreting equation (53) is that $s^2_{\mu}$ describes the derivative of the measurement with respect to a cosmological parameter normalized by the noise of the measurement, such that it assumes large values for strong dependences of the signal on the cosmological model and small noise contributions. At the same time, $s^2_{\mu}$ is the contribution to the Fisher matrix entry for the parameter $p_\mu$ from each mode $k_{\ell m}$ of the convergence field. The number of modes for each $\ell$ is given by $2\ell + 1$.

We show the sensitivity $s^2_{\mu}$ for the cosmological parameters in Fig. 7. Again, we observe that the sensitivity of 3D weak cosmic shear on the dark energy sound speed $c_s^2$ mostly comes from the first multipoles. This confirms the impression already obtained from the covariances $C_{\ell}(k, k')$ shown in Fig. 6, and emphasizes the fact that the influence of a non-trivial sound speed is a large-scale phenomenon. At the same time, the plot explains the two orders of magnitude difference in constraints on the dark energy sound speed compared to the other cosmological parameters.

Figure 6. Qualitative results for the covariance $C_{\ell}(k, k')$ for $\ell = 5$, 10 and 20, and $w = -0.8$. Brighter regions mark larger values. The dashed lines mark the scales $k_{\text{eff}} = 1/h_{\text{eff}}$ in the case $c_s^2 = 1$.

7 SUMMARY

We have studied the potential of 3D weak cosmic shear to constrain a possible clustering of dark energy with the data of next-generation surveys. We have parametrized the clustering dark energy component by two parameters characteristic for a generic cosmological fluid, its equation of state $w$ and its (rest-frame) sound speed $c_s^2$.

(i) For the 3D weak cosmic shear analysis, we have developed adequate numerical tools allowing for an efficient calculation of the covariance and Fisher matrices. These tools were shown to be numerically very efficient, which ultimately allowed us to sweep through the dark energy parameter space while retaining sufficient numerical accuracy.

(ii) The capability of future observations to constrain $c_s^2$, and thereby the clustering of dark energy, strongly depends on the dark energy equation of state $w$. If $w$ is close to $-1$, dark energy perturbations are mainly present at very large scales possibly outside the Hubble horizon. The effects of clustering dark energy would hardly be observable at all if $c_s^2 \gg 1 + w$. As, indeed, current observations prefer values of $w$ very close to $-1$ (Komatsu et al. 2011), this seems to be the decisive caveat.

(iii) Due to the sensitivity of sound speed constraints to the assumed exact value of $w$, we have estimated the uncertainties $\Delta \log_{10} c_s^2$ and $\Delta w/|w|$ as functions of the fiducial parameters ($w, c_s^2$) (cf. Fig. 3). The numbers are combined constraints based on assumed properties of Euclid (Heavens 2003) and the Planck satellite (Perotto et al. 2006; Knox 1995; Hollenstein et al. 2009).

(iv) For the considered range of fiducial parameters ($-0.99 \lesssim w \lesssim -0.6$, $10^{-1} \lesssim c_s^2 \lesssim 1$), the estimated constraints on the sound speed $\Delta \log_{10} c_s^2$ vary between the extreme cases of 0.1 and 3. If $w \gtrsim -0.95$, the combination of Euclid and Planck is promising to constrain the order of magnitude of $c_s^2$ provided that the true sound speed is small enough. This would be considerable progress compared not only to constraints possible with current observational data (de Putter et al. 2010; Li & Xia 2010) but also, for most of the parameter space, to the constraints expected from weak-lensing tomography and galaxy surveys alone (Sapone et al. 2010). Constraining $c_s^2$ within one or two orders of magnitude could also be possible with Planck and next-generation galaxy surveys (Takada 2006; Ballesteros & Lesgourgues 2010) or for neutral hydrogen surveys (Torres-Rodriguez & Cress 2007; Torres-Rodriguez, Cress & Moodley 2008). Although not our focus here, the constraints of 3D weak cosmic shear together with the Planck satellite on the dark energy equation of state $w$ are worth mentioning: in fact, according to Figs 3 and 4, $w$ can be constrained below the per cent level (for other constraints from 3D weak cosmic shear, see Heavens 2003;
Heavens et al. 2006). Additionally, the true value of $w$ largely determines the accuracy on the sound speed $c_s$.

There are two very different conclusions one could draw from these constraints on $c_s^2$. First, we may regard $c_s^2 = 1$ and $w \approx -1$ as the natural values as they refer to unclustered dark energy such as a cosmological constant. Then, small deviations from $c_s^2 = 1$ are interesting; but these seem hardly observable in next-generation experiments. Secondly, however, one may argue that $c_s^2$ is a completely unknown parameter with a natural range from 0 to 1; then, Euclid and 3D weak lensing could single out an order of magnitude in which $c_s^2$ lies. This could be a decisive step for discriminating between different dark energy models.

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