The Effects of Bose-Condensates on Single Inclusive Spectra and Bose-Einstein Correlations

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Abstract

The implications of the formation of a Bose condensate on one- and two-particle spectra are studied for ultrarelativistic nucleus-nucleus collisions in the framework of a hydrodynamic description. It is found that single particle spectra are considerably enhanced at low momenta. The Bose-Einstein correlation function has an intercept below two. For pion pairs in the central region a two-component structure may appear in the correlation function, which is different from that found in quantum optics. The chaoticity parameter is strongly momentum dependent.

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1 Introduction

In current ultrarelativistic heavy ion collision experiments at the SPS (e.g., $Pb + Pb$ at $E_{beam} = 160$ AGeV) secondary particles are formed at high number densities in rapidity space $^{[1]}$. In future experiments at RHIC and the LHC one expects to obtain even higher multiplicities on the order of a few thousand particles per unit rapidity. If local thermal (but not chemical) equilibrium is established and the number densities are sufficiently large, the pions may accumulate in their ground state and a Bose condensate may be formed $^{[1]}$. A specific scenario for the formation of a Bose condensate, namely, the decay of short-lived resonances, was discussed in Ref. $^{[5]}$ where conditions necessary for the formation of a Bose condensate in a heavy ion collision were investigated. In Ref. $^{[5]}$ it was found that if a pionic Bose condensate is formed at any stage of the collision, it can be expected to survive until pions decouple from the dense matter, and thus it can affect the spectra and correlations of final state pions.

In the present paper we investigate the influence of such a condensate on the single inclusive cross section and on the second order correlation function of identically charged pions (Bose-Einstein correlations BEC) in hadronic reactions for expanding sources.

The proposal to use BEC for the detection of condensates was made a long time ago $^{[6]}$. In that reference the influence of the condensate on the form and the intercept of the second and third order correlation function was studied, with particular emphasis on the possible role of the strong final state interactions, which, contrary to more phenomenological approaches used afterwards, were studied using a quantum statistical Landau-Ginsburg type method.

It was found in $^{[6]}$ that the interaction influences the shape of the correlation function but not the value of its maximum (intercept). On the other hand the amount of condensate influences both the intercept and the form of the correlation function. In particular in analogy with quantum optics, a typical two component structure in the correlation function was found: while for purely

$^{[1]}$Recently a different type of pion condensate, the disordered chiral condensate, has received a lot of attention in the literature (for a review of recent results, see Ref. $^{[2]}$). It has been argued $^{[3]}$ that such a condensate would lead to the creation of squeezed, i.e., two particle coherent states. The possible effects of such squeezed coherent states on Bose-Einstein correlations have been studied in some detail $^{[4]}$.

In what follows, however, we shall restrict ourselves to the case of a conventional Bose condensate which is formed when the pion chemical potential becomes equal to the pion mass and to the associated one-particle coherent states.
chaotic fields the second order correlation function has only one term which depends on the rapidity
difference, for a superposition of coherent and chaotic fields there are two such terms which differ
in a well defined manner and which depend both on the same correlator (cf. eq. (23) below). No
dependence on the total rapidity of the pair appears in this approach because of the assumed boost
invariance and of the fact that in the one dimensional treatment (in rapidity space) used in [6] no
allowance for expansion was made.

The present paper uses a hydrodynamical approach which differs significantly from the previous
approach in that it considers explicitly the expansion of the source and thus the correlation between
position and momenta of produced particles. Such an approach is certainly more adequate for heavy
ion reactions and appears desirable at this stage particularly because of the possibility mentioned
above that a condensate might be produced after the expansion of the system during the freeze-
out process. Furthermore, the momentum dependence of the chaoticity can easily be taken into
account. Last but not least, in the present paper we exploit explicitly the possibility offered by
quantum statistics to treat simultaneously single inclusive cross sections and BEC and to correlate
the possible effects of a condensate in these observables, which is of considerable interest for future
comparisons with experiment.

The remainder of this paper is organized as follows. In section 2 general expressions are derived for
the single particle spectrum and two-particle correlation function of pions emitted from a hydrody-
namically expanding fluid with a superfluid component. In sections 3 and 4 the formalism is applied
to the case of a spherical expansion and a longitudinal scaling expansion, respectively. Numerical
results for both models, obtained for specific parametrizations of the freeze-out hypersurface, are
presented in section 5. Finally, the main results of the paper are summarized in section 6.

2 Basic formalism

Applying the current formalism [7, 8, 9, 10] we write the amplitude for the emission of a pion of
four-momentum \( k = (E, \vec{k}) \) as a superposition of contributions from the thermal excitations and
from the condensate,

\[
J(k) = J_{th}(k) + J_{co}(k) ,
\] (1)
with

\[ J_{th}(k) = \sum_l J_{th}^l(k) = \sum_l e^{i\phi_l^{th}} e^{ikx_l} j_l^{th}(k), \tag{2} \]

\[ J_{co}(k) = \sum_n J_{co}^n(k) = \sum_n e^{i\phi_n^{co}} e^{ikx_n} j_n^{co}(k), \tag{3} \]

where the labels “th” and “co” indicate the thermal and the condensate component of the distribution. The indices \( l \) and \( n \) label source elements centered at space-time point \( x_l \) and \( x_n \), respectively, and \( \exp(i\phi_l^{th}) \) and \( \exp(i\phi_n^{co}) \) are the corresponding fluctuating phase factors.

For the thermal component, two phases that characterize the emission from two different source elements, \( \phi_l^{th} \) and \( \phi_l^{th}' \) with \( l \neq l' \), are taken to be uncorrelated. Likewise, we assume that there exists no correlation between the phases related to emission from the thermal component and from the condensate, \( \phi_l^{th} \) and \( \phi_n^{co} \). If, and to what degree, the phase factors for emission from the condensate are correlated for two different source elements depends on the details of the formation and evolution of the condensate. There are two extreme cases:

(a) The phases \( \phi_n^{co} \) are completely correlated, i.e., the differences \( \phi_n^{co} - \phi_n^{co} \) do not fluctuate. In the case of an expanding source this would imply that the condensate must have been formed at an early stage when the hadronic matter was concentrated in a space-time volume sufficiently small for this kind of global phase coherence to be established. This corresponds to the presence of an expanding superfluid with a globally coherent phase.

(b) The phases \( \phi_n^{co} \) are completely uncorrelated. This is what one would expect if the condensate is formed at a late stage when the process of condensate formation (caused, e.g., by the decay of short-lived resonances) occurs independently in different fluid cells.

In the following we shall consider only the case (b), i.e., we assume that the Bose condensate is not phase correlated in different fluid cells. This situation appears as more realistic than case (a) although the formation of a global superfluid as envisaged in (a) is in itself also of interest and might deserve further study.

The single and double inclusive momentum distribution are

\[ E \frac{d^3 N}{d^3 k} = \langle J^*(k)J(k) \rangle, \tag{4} \]
The two-particle correlation function then takes the form
\[ E_1 E_2 \frac{d^6 N}{d^3 k_1 d^3 k_2} = \langle J^*(k_1) J^*(k_2) J(k_2) J(k_1) \rangle , \]
where the averaging is performed over the space-time positions \( x_l, x_n \) and the phases \( \phi_l^{th}, \phi_n^{co} \).

It is useful to introduce the following correlators:
\[ D^{th}(k_1, k_2) \equiv \left\langle \sum_l \left( J_l^{th}(k_1) \right)^* J_l^{th}(k_2) \right\rangle , \]
\[ D^{co}(k_1, k_2) \equiv \left\langle \sum_n \left( J_n^{co}(k_1) \right)^* J_n^{co}(k_2) \right\rangle , \]
\[ G^{th}(k_1, k_2) \equiv \left\langle \sum_l |J_l^{th}(k_1)|^2 |J_l^{th}(k_2)|^2 \right\rangle , \]
\[ G^{co}(k_1, k_2) \equiv \left\langle \sum_n |J_n^{co}(k_1)|^2 |J_n^{co}(k_2)|^2 \right\rangle . \]
The particle spectrum may then be expressed as the sum of a thermal and a condensate contribution,
\[ E \frac{d^3 N}{d^3 k} = E \frac{d^3 N}{d^3 k} \bigg|_{th} + E \frac{d^3 N}{d^3 k} \bigg|_{co} , \]
with
\[ E \frac{d^3 N}{d^3 k} \bigg|_{th} = D^{th}(k, k) , \]
and
\[ E \frac{d^3 N}{d^3 k} \bigg|_{co} = D^{co}(k, k) , \]
and the two-particle inclusive distributions may be written as
\[ \langle J^*(k_1) J^*(k_2) J(k_2) J(k_1) \rangle = \left| D^{th}(k_1, k_2) + D^{co}(k_1, k_2) \right|^2 - G^{th}(k_1, k_2) - G^{co}(k_1, k_2) . \]
The two-particle correlation function then takes the form
\[ C_2(k_1, k_2) = \frac{\langle J^*(k_1) J^*(k_2) J(k_2) J(k_1) \rangle}{\langle J^*(k_1) J(k_1) \rangle \langle J^*(k_2) J(k_2) \rangle} \]
\[ = 1 + \frac{\left| D^{th}(k_1, k_2) + D^{co}(k_1, k_2) \right|^2 - G^{th}(k_1, k_2) - G^{co}(k_1, k_2)}{(D^{th}(k_1, k_1) + D^{co}(k_1, k_1))(D^{th}(k_2, k_2) + D^{co}(k_2, k_2))} . \]
We first consider the thermal correlators \( D^{th}(k_1, k_2) \) and \( G^{th}(k_1, k_2) \) to further evaluate these expressions. Let \( L \) be the number of source elements that contribute to particles of momenta.
In the limit of large $L$ the term $G^{th}(k_1, k_2) \propto L$ can be neglected compared to the term $|D^{th}(k_1, k_2)|^2 \propto L^2$ in eq. (13). In the case of hydrodynamics, the sum over source elements translates into an integral over the freeze-out hypersurface. Introducing the average and the relative four-momentum of the pair, $K \equiv \frac{1}{2}(k_1 + k_2)$ and $q \equiv k_1 - k_2$, respectively, one has

$$D^{th}(k_1, k_2) = \int d^4x \ g^{th}(x, K) \ e^{iq \cdot x}, \ (15)$$

with the thermal source function for pions

$$g^{th}(x, k) = \frac{g_\pi}{(2\pi)^3} \int_\Sigma \frac{k \ d\sigma(x') \ \delta^4(x - x')}{\exp\left[\frac{k \ u(x') - \mu_\pi(x')}{T_f(x')}\right] - 1}. \ (16)$$

In eq. (16) $d\sigma(x)$, $u(x)$ and $T_f(x)$ are the volume element of the freeze-out hypersurface $\Sigma$, the 4-velocity of the fluid and the freeze-out temperature at space-time point $x$, respectively. The factor $g_\pi$ denotes the degeneracy factor of the pions while $\mu_\pi(x)$ is the pionic chemical potential at space-time point $x$. For simplicity, in the following sections we shall assume that $\mu_\pi(x) = \mu_\pi = \text{const.}$, i.e., that a pionic Bose condensate is formed at each space-time point on the freeze-out hypersurface.

Having discussed the terms $D^{th}(k_1, k_2)$ and $G^{th}(k_1, k_2)$ which are due to the thermal part, we now proceed to consider the corresponding expressions related to the condensate, $D^{co}(k_1, k_2)$ and $G^{co}(k_1, k_2)$. In what follows each fluid element will be treated as a macroscopic system in so far as it will be assumed that the condensate in each fluid cell is identified with the lowest momentum state in the rest frame of that fluid cell. This is in line with the conventional meaning of the concept of a condensate which refers to a phase of matter. That is to say, for a fluid cell centered around $x_l$ moving with velocity $\vec{u}_l$ we have

$$|J_l^{co}(k)|^2 \propto \delta^3(\vec{k} - m_\pi \vec{u}_l), \ (17)$$

i.e., as emphasized in Ref. [3], particles emitted from the condensate move with the collective velocity of the fluid. In the models of expanding sources which will be discussed in Sections 3 and 4 there is a one-to-one relation between the position $x_l$ of the source element and its velocity $u_l$. As far as particles emitted from the condensate are concerned, eq. (17) then implies that for each momentum $\vec{k}$ only one single source element contributes to the spectrum. Note that nevertheless this contribution will be comparable in magnitude to the contributions of the thermal excitations since the condensate constitutes a macroscopically occupied quantum state.
From eqs. (7), (9) and (17) it follows that

$$D^{\text{co}}(k_1, k_2) = G^{\text{co}}(k_1, k_2) = 0 \quad \text{for} \ k_1 \neq k_2. \quad (18)$$

The contribution of the condensate to the single inclusive spectrum can then be written as

$$\left. \frac{d^3 N}{d^3 k} \right|_{\text{co}} = D^{\text{co}}(k, k) = \int d^4 x \ g^{\text{co}}(x, k) \quad (19)$$

with

$$g^{\text{co}}(x, k) = \int_\Sigma d\sigma(x') u(x') \ n_{\text{co}}(x') \ E \delta^3(\vec{k} - m_\pi \vec{u}) \delta^4(x - x'). \quad (20)$$

We define the fraction of thermally produced particles of four-momentum \(k\), i.e., the momentum dependent chaoticity \(p(k)\), as

$$p(k) \equiv \frac{E \frac{d^3 N}{d^3 k}}{E \frac{d^3 N}{d^3 k}} = \frac{D^{\text{th}}(k, k)}{D^{\text{th}}(k, k) + D^{\text{co}}(k, k)} \quad (21)$$

and the normalized correlator of two thermal currents

$$d^{\text{th}}(k_1, k_2) = \frac{D^{\text{th}}(k_1, k_2)}{[D^{\text{th}}(k_1, k_1) D^{\text{th}}(k_2, k_2)]^{1/2}}. \quad (22)$$

Substituting the expressions (15), (16) and (19) – (22) into (14) we find for the two-particle Bose-Einstein correlation function

$$C_2(\vec{k}_1, \vec{k}_2) = 1 + 2 \sqrt{p(k_1) p(k_2)} (1 - p(k_1))(1 - p(k_2)) \ \hat{\Theta}(|\vec{k}_1 - \vec{k}_2|) \ d^{\text{th}}(k_1, k_2)$$

$$+ p(k_1) p(k_2) |d^{\text{th}}(k_1, k_2)|^2, \quad (23)$$

where

$$\hat{\Theta}(x) = \begin{cases} 
1 & : \ x \leq 0 \\
0 & : \ x > 0
\end{cases} \quad (24)$$

i.e., one has

$$C_2(\vec{k}_1, \vec{k}_2) = 1 + p(k_1) p(k_2) |d^{\text{th}}(k_1, k_2)|^2 \quad \text{for} \ k_1 \neq k_2. \quad (25)$$

Note that the “true” intercept – obtained by evaluating (23) at \(\vec{k}_1 = \vec{k}_2\) – is given by

$$C_2(\vec{k}, \vec{k}) = 1 + 2p(k) - p^2(k), \quad (26)$$

whereas the value obtained by extrapolating (23) to \(\vec{k}_1 = \vec{k}_2\) is

$$C_2^{\text{extrapol}}(\vec{k}, \vec{k}) = 1 + p^2(k). \quad (27)$$
The reason for the difference between (26) and (27) is that in the idealized case considered here $D^{co}(\vec{k}_1, \vec{k}_2)$ and $G^{co}(\vec{k}_1, \vec{k}_2)$ vanish for $\vec{k}_1 \neq \vec{k}_2$ (cf. Eq. (18)), while

$$G^{co}(\vec{k}, \vec{k}) = |D^{co}(\vec{k}, \vec{k})|^2 = |J_l(\vec{k})|^4,$$  

(28)

where $l$ labels the single source elements which contributes to the emission of pions of momentum $\vec{k}$ from the condensate (cf. eqs. (23,24)).

3 Spherically expanding source

Let us assume a space-like freeze-out hypersurface parametrized as $t = t(r)$ where $t = x^0$ is the time coordinate and $r = |\vec{x}|$ the radial coordinate. We also assume a radial velocity field $u = u(r) \vec{e}_r$ with non-vanishing gradient, i.e., $(\partial u/\partial r) \neq 0$. The pionic freeze-out temperature is taken to be $T_f = m_\pi = \text{const.}$ The volume element of the 3-dimensional freeze-out hypersurface and the 4-velocity of the spherically expanding relativistic fluid are ($\nu = 0, 1, 2, 3$)

$$d\sigma^\nu(x) = \left(1, \frac{\partial t}{\partial r} \vec{e}_r\right) r^2 \sin \theta \, d\theta \, d\phi,$$

(29)

$$u^\nu(x) = (u^0(r), u(r)\vec{e}_r), \quad u^0(r) = \sqrt{1 + [u(r)]^2}.$$

(30)

3.1 Single inclusive momentum distributions

With the 4-momentum $k^\nu = (E, \vec{k}) = (E, k \cdot \vec{e}_k)$ and $\vec{e}_r \cdot \vec{e}_k = \cos \theta \equiv z$, the thermal part of the single inclusive momentum distribution is given through

$$E \frac{d^3 N}{d^3 k} \bigg|_{th} = \frac{g_\pi}{(2\pi)^3} \int_{\Sigma} d\sigma^\nu k^\nu \exp \left[\left(\frac{u^\nu k^\nu - \mu_\pi}{T_f}\right) - 1\right]$$

$$= \frac{g_\pi}{(2\pi)^3} \int_{R^\perp} r^2 dr \int_{-1}^{1} dz \frac{E - k z \partial t/\partial r}{\exp \left[\left(\frac{Eu^0(r) - k z u(r) - \mu_\pi}{T_f}\right) - 1\right]}.$$

(31)

In eq. (31) $E = \sqrt{m_\pi^2 + \vec{k}^2}$ and $R^\perp$ is the radial extension of the pion source.

The coherent part of the single inclusive momentum distribution is

$$E \frac{d^3 N}{d^3 k} \bigg|_{co} = n_{co} \int_{\Sigma} d\sigma^\nu u_\nu E \delta^3(\vec{k} - m_\pi \vec{u})$$
\[
\frac{1}{\pi} \int_0^\pi \sin \theta \, d\theta \int_0^{2\pi} \sin \theta \, d\phi \left[ u^0(r) - u(r) \frac{\partial}{\partial r} \right] = n_\text{co} \int_0^{R_\perp} r^2 \, dr \int_0^{\pi} \sin \theta \, d\theta \int_0^{2\pi} \sin \theta \, d\phi \left[ u^0(r) - u(r) \frac{\partial}{\partial r} \right] E \delta^3 \left( k_\perp - m_\pi u(r) \bar{e}_r \right)
\]

\[
= \left\{ \begin{array}{ll}
\frac{E}{m_\pi} \left( E - \frac{k_\perp}{m_\pi} \frac{\partial}{\partial r} (r_0) \right) \left( \frac{\partial}{\partial r} (r_0) \right)^{-1} & : k \leq m_\pi u_{\text{max}} \\
0 & : k > m_\pi u_{\text{max}}
\end{array} \right., \quad (32)
\]

where \( r_0 = r \left( u = \frac{k_\perp}{m_\pi} \right) \) and \( u_{\text{max}} \) is the maximal value of the velocity at freeze-out. The total momentum distribution is obtained by inserting eqs. (31) and (32) into eq. (10).

3.2 Bose-Einstein correlation functions

For illustration we restrict ourselves to the central momentum region. To be specific, we consider the case \( \vec{K} \equiv \frac{1}{2}(\vec{k}_1 + \vec{k}_2) = 0 \), i.e., \( \vec{k}_1 = -\vec{k}_2 \). The quantity that remains to be calculated in order to construct the BEC function is the thermal correlator \( D_{\text{th}}(k_1, k_2) \). With \( E_1 = E_2 \) one has \( q^0 \equiv E_1 - E_2 = 0 \). Using the notation \( \vec{q} \equiv \vec{k}_1 - \vec{k}_2 \) we obtain \( q^\mu x_\mu = -\vec{q} \cdot \vec{x} = -qr \cos \theta \) and

\[
D_{\text{th}}(k_1, k_2) \bigg|_{\vec{K} = 0} = \frac{g_{\pi}}{(2\pi)^3} \int_\Sigma \exp \left[ \left( K^\mu u_\mu - \mu_\pi / T_f \right) / T_f \right] - 1
= \frac{g_{\pi}}{(2\pi)^3} \int_0^{2\pi} d\phi \int_0^{\pi} \sin \theta \, d\theta \int_0^{R_\perp} r^2 \, dr \frac{E_K e^{-iqr \cos \theta}}{\exp \left[ (E_K u^0(r) - \mu_\pi) / T_f \right] - 1}
= \frac{g_{\pi}}{(2\pi)^3} \int_0^{R_\perp} r^2 \, dr \frac{2E_K \sin(qr)/q}{\exp \left[ (E_K u^0(r) - \mu_\pi) / T_f \right] - 1}. \quad (33)
\]

In section 5 explicit parametrizations for \( t(r) \) and \( u(r) \) will be used to evaluate numerically the above expressions.

4 Longitudinally expanding source

Here we assume a space-like freeze-out hypersurface parametrized by the radius-dependent longitudinal proper-time \( \tau(r) \). For the longitudinal component of the velocity we take the scaling ansatz \( v_\| = x_\| / t \). To be specific, the 4-volume element of the freeze-out hypersurface and the 4-velocity
of the relativistic fluid are written as
\[ d\sigma^\nu = ( \cosh \eta, \sinh \eta, \frac{\partial \tau}{\partial r} \vec{e}_r(r) ) \tau(r) \ d\eta \ dr \ d\phi, \]
(34)

\[ u^\nu = ( U(r) \cosh \eta, U(r) \sinh \eta, u_\perp(r) \vec{e}_r(r) ), \quad U(r) = \sqrt{1 + [u_\perp(r)]^2}, \]
(35)

where \( u_\perp(r) \) and \( \eta \) are the transverse (radial) component of the fluid 4-velocity and the space-time rapidity of the fluid, respectively.

### 4.1 Single inclusive momentum distributions

Since our ansatz is invariant under boosts in the longitudinal (beam) direction we may without loss of generality restrict ourselves to the case of rapidity \( y = 0 \). With the 4-momentum \( k^\nu = (E, \vec{k}) = (m_\perp \cosh y, m_\perp \sinh y, k_\perp \vec{e}_k) \) and \( \vec{e}_r \cdot \vec{e}_k \equiv \cos \psi \) the thermal part of the single inclusive momentum distribution is given by
\[
E \frac{d^3 N}{d^3 k} \bigg|_{\text{th}} = \left[ \frac{g_\pi}{(2\pi)^3} \int_\Sigma d\sigma^\nu k^\nu \frac{1}{\exp[(u^\nu k^\nu - \mu_\pi)/T_f] - 1} \right]_{y=0}
\]
\[
= \frac{g_\pi}{(2\pi)^3} \int_\infty^{+\infty} d\eta \int_0^{R_\perp} \tau(r)rdr \int_0^{2\pi} d\phi \left[ m_\perp \cosh \eta - k_\perp \cos \psi \left( \frac{\partial \tau}{\partial r} \right) \right]
\]
\[
\times \frac{1}{\exp[(m_\perp \cosh \eta U(r) - k_\perp \cos \psi u_\perp(r) - \mu_\pi)/T_f] - 1}. \quad (36)
\]

In eq. (36) \( R_\perp \) is the radial extension of the pion source and \( m_\perp = \sqrt{m_\pi^2 + k_\perp^2} \).

The coherent part of the single inclusive momentum distribution is
\[
E \frac{d^3 N}{d^3 k} \bigg|_{\text{co}} = \left[ n_{\text{co}} \int_\Sigma d\sigma^\nu u^\nu E \delta^3(\vec{k} - m_\pi \vec{u}) \right]_{y=0}
\]
\[
= n_{\text{co}} \int_\infty^{+\infty} d\eta \int_0^{R_\perp} \tau(r)rdr \int_0^{2\pi} d\phi \left( U(r) - u_\perp(r) \frac{\partial \tau}{\partial r} \right)
\]
\[
\times m_\perp \delta^3(\vec{k} - m_\pi \vec{u}) \bigg|_{y=0}
\]
\[
= \left\{ \begin{array}{l}
\frac{n_{\text{co}} r_0}{m_\pi} \frac{\tau(r_0) (m_\perp - \frac{k_\perp}{m_\pi} \frac{\partial \tau}{\partial r}(r_0)) \left( \frac{\partial u_\perp}{\partial r}(r_0) \right)}{k_\perp} : \quad k_\perp \leq m_\pi u_{\text{max}}, \\
0 : \quad k_\perp > m_\pi u_{\text{max}},
\end{array} \right. \quad (37)
\]

where \( r_0 = r(u_\perp = \frac{k_\perp}{m_\pi}) \) and \( u_{\text{max}} \) is the maximal value of the transverse velocity field at freeze-out.

The total momentum distribution is obtained by inserting (36) and (37) into eq. (10).
4.2 Bose-Einstein correlation functions

As in the preceding section we restrict ourselves to the case \( \vec{K} = \frac{1}{2}(\vec{k}_1 + \vec{k}_2) \equiv 0 \), i.e., \( \vec{k}_1 = -\vec{k}_2 \). Since we have assumed the source to be invariant under boosts in longitudinal direction, the generalization to average pair momenta \( K_\perp = 0, K_\parallel \neq 0 \) is straightforward. We shall consider the correlation function in the transverse direction \( C_2(q_\perp, \Delta y = 0) \), i.e., the case \( K_\parallel = k_\parallel = y_i = 0 \) and \( E_K = E_i = m_{\perp K} = \sqrt{m_i^2 + k_{i\perp}^2} \) \((i = 1, 2)\). Then for construction of the BEC function the only remaining term to specify is \( D(k_1, k_2) \).

With \( q^0 \equiv E_1 - E_2 = 0 \) we obtain \( q^\nu x_\nu = -\vec{q} \cdot \vec{x} \equiv -q_\perp r \cos \psi \) (since \( q_\parallel = 0 \)) and

\[
D(k_1, k_2) \bigg|_{K=0} (\Delta y = 0) = \frac{g_\pi}{(2\pi)^3} \int_\Sigma \frac{d\sigma_{\nu\epsilon} e^{iq^\nu x_\nu}}{\exp}\frac{K_\nu d\sigma_{\nu\epsilon} e^{iq^\nu x_\nu}}{\exp} \int \frac{d\eta}{\cosh \eta} \int_0^{2\pi} d\psi \int_0^{R_\perp} r dr \tau(r) \frac{m_{\perp K}}{\sqrt{m_{\perp K}^2 + q_\perp^2}} \exp \left[ \frac{(E_K U(r) - \mu_\pi)}{T_f} - 1 \right]
\]

\[\text{(38)}\]

with the Bessel function \( J_0 \).

In the next section explicit parametrizations for \( \tau(r) \) and \( u_\perp(r) \) will be given and the results for eqs. (36),(37) and (38) will be discussed after their numerical evaluation.

5 Parametrizations and Results

For the sake of illustration of the above results, we shall consider the case of a specific heavy-ion reaction, namely \( S + S \) at 200 AGeV. Application of the relativistic \((3+1)\)-dimensional hydrodynamic code HYLANDER \cite{12, 13} yielded a successful description \cite{13} of rapidity and transverse momentum spectra of mesons and baryons and a quantitatively correct prediction of BEC of pions \cite{14, 15}.

In order to get reasonable descriptions for \( t(r) \), \( u(r) \), \( \tau(r) \) and \( u_\perp(r) \) we have chosen the following
parametrizations and parameters from the hydrodynamical solution mentioned above:

\[
t(r) = \tau(r) = \begin{cases} 
\tau_0 - \frac{1}{\alpha_0} r^2 & : r \leq R_\perp \\
0 & : r > R_\perp 
\end{cases}
\]
with \( \tau_0 \, c/fm = \alpha_0 / fm \, c = 4.5 \), \( (39) \)

\[
u(r) = \upsilon_\perp(r) = \begin{cases} 
u_0 r & : r \leq R_\perp \\
0 & : r > R_\perp 
\end{cases}
\]
with \( \upsilon_0 \, fm/c = 0.13 \), \( (40) \)

The pionic chemical potential was taken to be \( \mu_\pi = 0.139 \, GeV \), since we consider the formation of a pionic Bose-condensate. The transverse radius has been fixed to \( R_\perp = 3.8 \, fm \).

The remaining parameter is the condensate number density \( n_{co} \) which we write in fractions of the thermal number density

\[
n_{th} = \frac{g_\pi}{(2\pi)^2} \int_0^{2\pi} k^2 dk \frac{1}{\exp \left[ \frac{E - \mu_\pi}{T_f} \right] - 1}.
\]
(41)

In Fig. 1 the results of the numerical evaluations of the single inclusive momentum distributions \( E dN/d^3k \), the momentum-dependent chaoticities \( p \) and the Bose-Einstein correlation functions \( C_2 \) are shown for the spherically and for the longitudinally expanding source. The quantity \( n_{co} \) here takes four different values ranging from \( 0.0 \, n_{th} \) to \( 1.0 \, n_{th} \).

It can be seen that the results for the spherical and the longitudinal expansion scenario look quite similar although there are some quantitative differences according to the momentum dependence. The Bose-condensate affects the plotted functions only over a limited momentum range. This is due to the fact that there exists a maximum velocity \( u_{max} = u(R_\perp) = \upsilon_\perp(R_\perp) = 0.494 \, c \) that enters in all condensate contributions. Thus the maximum momentum where the Bose-condensate contributes to the single inclusive momentum distributions is \( k_{max} = m_\pi u_{max} = 68.7 \, MeV/c \) and the maximum momentum difference over which the BEC can have a reduction is twice this value, namely \( q_{max} = 137.4 \, MeV/c \).

Of course, theoretically the largest value \( u_{max} \) could assume is the speed of light \( c = 1 \). Therefore, there exists an absolute maximum value for the momenta of particles which originate from a Bose-condensate: in case of a pionic Bose-condensate it would be \( k_{max} = m_\pi c = 139.6 \, MeV/c \). Bose-Einstein correlation functions would be then affected over a momentum difference range with
$q_{\text{max}} = 279.2 \text{ MeV/c}$, which would result in a disappearance of the sharp peak in the BEC for $S + S$, because this peak would be shifted to unobservably large momentum differences. It should be mentioned that from such a structure in the double inclusive signal one could in principle determine the maximum flow velocity of the fluid.

The single inclusive momentum distributions are quite sensitive to the presence of a Bose-condensate; for the BEC this effect is even more pronounced. The presence of a Bose-condensate of about only 1% (i.e. $\kappa \equiv n_{co} = 0.01 n_{th}$) results in a decrease of the intercept of the shown two-particle correlation functions to 1.83 for the spherically expanding source and 1.86 for the longitudinally expanding source. In quantum optics as well as in the approach used in [6] the sensitivity of the intercept on the amount of coherence is much weaker. Thus to get a decrease of the (“true”) intercept from 2 to 1.8, a value of $\kappa = 0.5$ is necessary. Furthermore, due to a limited value of $q_{\text{max}}$ a part of the tail of the two-particle correlation functions will not be affected by the pionic bose-condensate and a peak will therefore come into existence. To what extent such peaks due to an existing pionic Bose-condensate can be observed in experimental data depends on several factors:

(i) the size of the source, which determines the inverse width of the correlation functions;
(ii) the maximum of accessible velocity at freeze-out of the pionic source which determines the position of the peak;
(iii) deformation of the correlation function due to further contributions from resonance decay;
(iv) the averaging over acceptance regions in the experiment;
(v) the width of the momentum distribution in the bosonic ground state.

A study of the influence of these factors on the effects reported here is in preparation [16].

6 Summary

We have shown that the formation of a pionic Bose condensate can influence single inclusive spectra as well as Bose-Einstein correlation functions quite strongly. Among other things we find an enhancement of single inclusive momentum spectra at low momenta, and a reduction of the two-particle correlation function on a limited momentum range resulting in a bumpy structure of the BEC. The structure depends on the maximum velocity of the fluid. Within this treatment an
explicitly momentum-dependent chaoticity is presented for the first time due to a maximum freeze-out velocity rather than to geometrical effects of the source. Whether these effects will survive in more realistic treatments has yet to be proven. However the results obtained so far confirm the interest and need to take into account coherence in theoretical approaches to BEC.

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References

[1] S. Margetis and the NA49 Collaboration, Nucl. Phys. A590 (1995) 355c.

[2] S. Gavin, Nucl.Phys. A590 (1995) 163c.

[3] I.J. Kogan, JETP Lett. 59 (1994) 307; R.D. Amado and I.J. Kogan, Phys. Rev. D51 (1995) 190.

[4] I.V. Andreev and R.M. Weiner, Phys. Lett. B373 (1996) 159.

[5] U. Ornik, M. Plümer, D. Strottman, Phys. Lett. B314 (1993) 401.

[6] G.N. Fowler, N. Stelte, and R.M. Weiner, Nucl. Phys. A319 (1979) 349.

[7] E.V. Shuryak, Sov.J.Nucl.Phys. 18, 667 (1974).

[8] G.I. Kopylov and M.J. Podgoretsky, Sov.J.Nucl.Phys. 18, 336 (1974).

[9] M. Gyulassy, S.K. Kauffmann, and L.W. Wilson, Phys. Rev. C20 (1979) 2267.

[10] I.V. Andreev, M. Plümer, and R.M. Weiner, Phys. Rev. Lett. 67 (1991) 3475; Int. J. Mod. Phys. A8 (1993) 4577.

[11] B.R. Schlei, U. Ornik, M. Plümer, R.M. Weiner, Phys. Lett. B293 (1992) 275.

[12] U. Ornik, F. Pottag, R.M. Weiner, Phys. Rev. Lett. 63 (1989) 2641.

[13] J. Bolz, U. Ornik, R.M. Weiner, Phys. Rev. C46 (1992) 2047.

[14] J. Bolz, U. Ornik, M. Plümer, B.R. Schlei, R.M. Weiner, Phys. Lett. B300 (1993) 404; Phys. Rev. D47 (1993) 3860.

[15] Th. Alber et al., Phys. Rev. Lett. 74 (1995) 1303; Z. Phys. C66 (1995) 77; Th. Alber for the Collaborations NA35 and NA49, Nucl. Phys. A590 (1995) 453c.

[16] B.R. Schlei et al., in preparation.
Figure Captions

**Fig. 1** Single inclusive spectra, momentum-dependent chaoticities and BE correlation functions for a spherically and a longitudinally expanding source, respectively. The different line styles correspond to different condensate densities $n_{co}$ compared to the thermal number densities $n_{th}$. The left column of the figure shows the results for the spherically expanding source, whereas the right column shows the results for the longitudinally expanding source.
Figure 1