$H_\infty$ generalized dynamic observer for LPV singular systems

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Abstract: The present paper considers the $H_\infty$ observers design for linear parameter varying (LPV) singular systems by introducing a new observer structure named generalized dynamic observer (GDO), which is more general than the proportional observer (PO) and the proportional-integral observer (PIO). The objective is to design a GDO such that the resulting error dynamics is stable and a given level of disturbance attenuation is assured. The stability conditions for the existence of this $H_\infty$ GDO are given in terms of a set of linear matrix inequalities (LMIs). A numerical example is given to show the present approach.

Keywords: Generalized dynamic observer, Singular systems, LPV systems, $H_\infty$, LMI.

1. INTRODUCTION

Real physical processes are often described by nonlinear models. Since design a nonlinear observer is difficult, the linear parameter varying (LPV) approach is a way to deal with this problem, because this approach can be used to approximate nonlinear systems. These classes of systems allow to represent the system as an interpolation of simple local models. The main advantage of LPV systems is that they support the application of linear design tools.

On the other hand, singular systems also known as descriptor systems or differential-algebraic systems can be considered as a generalization of dynamical systems. The singular system representation is a powerful modeling tool since it can describe processes governed by both, differential equations and algebraic equations. This class of systems was introduced by Luenberger (1977) from a control theory point of view and since, great efforts have been made to investigate singular systems and its applications, see Dai (1989); Müller and Hou (1993); Hou and Müller (1995); Darouach and Boutayeb (1995); Müller (2005); Darouach (2012); Alma and Darouach (2014).

When dealing with robust observer problems, there are two basic criteria. The fist one is the minimization of the norm $H_2$, and the second is the minimization of the norm $H_\infty$ of the estimation error performance. In the first case, the $H_2$ observer limits the variance of the error signal in the presence of exogenous inputs and, in the second case the $H_\infty$ observer limits the maximum covariance of the error signal in the presence of bounded exogenous inputs (Borges and Peres, 2006). Furthermore, it has been shown that $H_\infty$ observer technique provides not only a guaranteed noise attenuation level but also robustness against unmodeled dynamics (Nagpal and Khargonekar, 1991).

$H_\infty$ observer design for LPV singular systems has been studied in Yue and Han (2004); Koenig and Marx (2008); Habib et al. (2010), all these results use the proportional observer (PO). In the estimation by a PO there always exists a static error estimation in presence of disturbances. In order to deal with the disadvantage of the PO, proportional-integral observers (PIO) were introduced with an integral gain of the output error in their structure, this change in the structure achieves steady state accuracy in their estimations. Also a new structure of observers was developed by Goodwin and Middleton (1989) and Marquez (2003), known as generalized dynamic observer (GDO). This structure presents an alternative of state estimation which can be considered as more general than PO and PIO.

In this paper, we study the $H_\infty$ GDO design for LPV singular systems. The main contribution of this paper is the introduction of a new observer structure to achieve robust state estimation for LPV singular systems. The stability conditions are given in terms of a set of linear matrix inequalities (LMIs). A numerical example is given to show our approach.

2. PRELIMINARIES

In this section we shall present the notations and some basic results which are used in the sequel of the paper. The symbol ($\ast$) denotes the transpose elements in the
symmetric positions, \( \|f(t)\|_2 = \sqrt{\int_0^\infty [f(t)Tf(t)]dt} \), \( f(t) \in L^2_2(0,\infty) \), where \( L^2_2(0,\infty) \) stands for the space of square integrable functions on \([0,\infty)\). The symbol \( A^\perp \) denotes a maximal row rank matrix such that \( A^\perp A = 0 \), by convention \( A^\perp = 0 \) when \( A \) is of full row rank. The symbol \( A^+ \) denotes any generalized inverse of the matrix \( A \), satisfying \( AA^+ A = A \).

**Lemma 1.** (Skelton et al., 1998) Let matrices \( B, C, D = D^T \) be given, then the following statements are equivalent:

(S1) There exists a matrix \( X \) satisfying
\[
BXC + (BXC)^T + D < 0
\]
(S2) The following statements hold
\[
B^+DB^T < 0 \quad \text{or} \quad BB^T > 0
\]
\[
C^+DC^T + D < 0 \quad \text{or} \quad C^TC > 0
\]
Suppose that the statement (S2) holds. Let \( r_b \) and \( r_c \) be the ranks of \( B \) and \( C \), respectively, and \( (B_1, C_1) \) and \( (C_1, C_r) \) are any full rank factors of \( B \) and \( C \), i.e. \( B = B_1C_r \), \( C = C_1C_r \).

Then the matrix \( X \) in statement (S1) is given by
\[
X = B_1^+KC_r^+ + Z - B_1^+B_rZC_1C_r^+
\]
where \( Z \) is an arbitrary matrix and
\[
K = -R^{-1}B_1^+\partial C_r^T(C_1,\partial C_r^T)^{-1} + S^{1/2}(C_1,\partial C_r^T)^{-1/2}
\]
\[
S = R^{-1} - R^{-1}B_1^+(\partial - \partial C_r^T(C_1,\partial C_r^T)^{-1}C_r)R^{-1}
\]
where \( L \) is a matrix that must satisfy \( \|L\|_2 < 1 \) and \( R \) is a positive definite matrix that satisfies the following inequality
\[
\vartheta = (B_r, R^{-1}B_r^T - D)^{-1} > 0.
\]

3. PROBLEM FORMULATION

Consider the LPV singular system described by
\[
\dot{x}(t) = A(\rho(t))x(t) + B(\rho(t))u(t) + D_1w(t)
\]
\[
y(t) = Cx(t) + D_2w(t)
\]
where \( x(t) \in \mathbb{R}^n \) is the semi-state vector, \( u(t) \in \mathbb{R}^m \) is the input vector, \( w(t) \in \mathbb{R}^r \) is the disturbance vector, and \( y(t) \in \mathbb{R}^p \) is the measurements output vector. Matrices \( E \in \mathbb{R}^{nxn} \), let \( rank(E) = r < n \). Matrices \( A(\rho(t)) \) are known matrices, \( \rho(t) = \{\rho_1(t), \ldots, \rho_j(t)\} \) is the vector of \( j \) variant parameters.

In this paper, the LPV singular systems are considered, which in the parameters of \( r(t) \) vary in a complex polytope of \( \tau \) vertices, where each vertex corresponds to the extreme values of \( \rho(t) \) (Rodrigues et al., 2007). Under this consideration, the structure of the LPV singular system (1) can be written as
\[
\dot{x}(t) = \sum_{i=1}^{\tau} \mu_i(\rho(t))A_i x(t) + B_i u(t) + D_1 w(t)
\]
\[
y(t) = C x(t) + D_2 w(t)
\]
where
\[
\sum_{i=1}^{\tau} \mu_i(\rho(t)) = 1, \quad 0 \leq \mu_i(\rho(t)) \leq 1
\]
\[
\forall i \in \{1, \ldots, \tau\} \quad \text{where} \quad \tau = 2^j, \quad \mu_i(\rho(t)) = \mu(\bar{\rho}_i, \underline{\rho}_i, \rho(t), t)
\]
(\( \bar{\rho}_i \) and \( \underline{\rho}_i \) represent the maximum and the minimum value of \( \rho_i \), respectively).

In the sequel we assume that

**Assumption 1.** \( rank \left[ \begin{array}{c} E \\ C \end{array} \right] = n \).

This condition is generally considered to the impulse observability (Darouach and Boutayeb, 1995).

Now, let us consider the following GDO for system (2)
\[
\dot{\zeta}(t) = \sum_{i=1}^{\tau} \mu_i(\rho(t))(N_i \zeta(t) + H_i v(t) + F_i y(t) + J_i u(t))
\]
\[
\dot{v}(t) = \sum_{i=1}^{\tau} \mu_i(\rho(t))(S_i \zeta(t) + L_i v(t) + M_i y(t))
\]
\[
\dot{x}(t) = \zeta(t) + Q y(t)
\]
where \( \zeta(t) \in \mathbb{R}^{n_0} \) represents the state vector of the observer, \( v(t) \in \mathbb{R}^{q_1} \) is an auxiliary vector and \( \hat{x}(t) \in \mathbb{R}^n \) is the estimate of \( x(t) \). Matrices \( N_i, H_i, F_i, S_i, L_i, M_i \) and \( Q \) are unknown matrices of appropriate dimensions which must be determined such that \( \hat{x}(t) \) converges asymptotically to \( x(t) \) for \( u(t) = 0 \), and for \( u(t) \neq 0 \) we must satisfy \( \sup_{w \in \mathbb{L}_2 \setminus \{0\}} \|\tilde{w}(t)\|_2 < \gamma \), where \( \gamma \) is a given positive scalar.

For the sake of simplicity the following notation is used
\[
\Phi(\rho(t)) = \Phi(\rho) = \sum_{i=1}^{\tau} \mu_i(\rho(t))\Phi_i, \quad \forall i \in \{1, \ldots, \tau\}
\]
Thus, the observer (4-6) can be rewritten as follows
\[
\dot{\zeta}(t) = N(\rho)\zeta(t) + H(\rho)v(t) + F(\rho)y(t) + J(\rho)u(t)
\]
\[
\dot{v}(t) = S(\rho)\zeta(t) + L(\rho)v(t) + M(\rho)y(t)
\]
\[
\hat{x}(t) = \zeta(t) + Q y(t)
\]
Now, we can give the following lemma.

**Lemma 2.** There exists an observer of the form (8-10) for the system (1) if the matrix \( \left[ \begin{array}{c} N(\rho) \\ H(\rho) \\ S(\rho) \\ L(\rho) \end{array} \right] \) is Hurwitz when \( w(t) = 0 \), and if there exists a matrix \( T \) such that the following conditions are satisfied

(a) \( N(\rho)TE + F(\rho)C - TA(\rho) = 0 \)

(b) \( J(\rho) = TB(\rho) \)

(c) \( S(\rho)TE + M(\rho)C = 0 \)

(d) \( TE + QC = I_n \)

**Proof.** Let \( T \in \mathbb{R}^{q_1 \times m} \) be a parameter matrix and define the transformed error \( \varepsilon(t) = \zeta(t) - TE \zeta(t) \), then its derivative is given by
\[
\dot{\varepsilon}(t) = N(\rho)\varepsilon(t) + H(\rho)v(t) + (J(\rho) - TB(\rho))u(t) + (N(\rho)TE + F(\rho)C - TA(\rho))x(t) + (F(\rho)D_2 - TD_1)w(t)
\]

By using the definition of \( \varepsilon(t) \), equations (9) and (10) can be written as
\[
\dot{v}(t) = S(\rho)\varepsilon(t) + L(\rho)v(t) + (S(\rho)TE + M(\rho)C)x(t) + M(\rho)D_2 w(t)
\]
\[
\hat{x}(t) = \varepsilon(t) + (TE + QC)x(t) + QD_2 w(t)
\]
Now, if conditions (a)-(d) of Lemma 2 are satisfied the following observer error dynamics is obtained from (11) and (12)
\[
\dot{\varphi}(t) = A(\rho)\varphi(t) + B(\rho)w(t)
\]
where
\[
A(\rho) = \begin{bmatrix}
N(\rho) & H(\rho) \\
S(\rho) & L(\rho)
\end{bmatrix}, \\
B(\rho) = \begin{bmatrix}
F(\rho)D_2 - TD_1 \\
M(\rho)D_2
\end{bmatrix}
\]
and \(\varphi(t) = \begin{bmatrix} z(t) \\ v(t) \end{bmatrix} \).

From equation (13) we have
\[
e(t) = \hat{x}(t) - x(t) = \varphi(t) + QD_2w(t)
\]
in this case if \(w(t) = 0\) and matrix \(A(\rho)\) is Hurwitz, then
\[
\lim_{t \to \infty} e(t) = 0.
\]

4. **H\(_\infty\) OBSERVER DESIGN**

4.1 **Parameterization of the observer**

Before giving the solution to the GDO design problem, we shall give the parameterization of all solutions to the algebraic constraints (a)-(d). Condition (d) of Lemma 2 can be written as
\[
[TQ]\Sigma = I_n
\]
where \(\Sigma = \begin{bmatrix} E \\ C \end{bmatrix} \). The necessary and sufficient condition for (16) to have a solution is
\[
\text{rank} \begin{bmatrix} E \\ C \end{bmatrix} = \text{rank} \begin{bmatrix} E \\ I_n \end{bmatrix} = n
\]
(17)

since (17) is satisfied solution to (16) is given by
\[
[TQ] = \Sigma^+
\]
or equivalently
\[
T = \Sigma^+I_0 \\
Q = \Sigma^+0
\]
(19)

where \(\Sigma^+\) is any generalized inverse of \(\Sigma\), such that it verifies \(\Sigma^+\Sigma = \Sigma\).

Now, by taking \(TE\) from the condition (d) of Lemma 2, and replacing it in condition (a) we obtain
\[
N(\rho) = TA(\rho) + K(\rho)C
\]
(21)

where \(K(\rho) = N(\rho)Q - F(\rho)\).

On the other hand, by replacing \(TE\) in condition (c) of Lemma 2 we get
\[
S(\rho) = Z(\rho)C
\]
(22)

where \(Z(\rho) = S(\rho)Q - M(\rho)\).

**Remark 1.** From the above results, we can see that the determination of matrices of the GDO (4-6) can be done as follows: Matrix \(F(\rho)\) can be deduced as \(F(\rho) = N(\rho)Q - K(\rho),\) matrix \(M(\rho) = S(\rho)Q - Z(\rho),\) and matrix \(J(\rho) = TB(\rho)\). Matrices \(Q, N(\rho)\) and \(S(\rho)\) can be obtained from (20), (21) and (22) respectively. On the other hand parameter matrices \(K(\rho), H(\rho), Z(\rho)\) and \(L(\rho)\) can be obtained from the stability of (14).

By using (21-22) and the definitions of \(F(\rho)\) and \(M(\rho)\), the observer error dynamics (14-15) can be written as
\[
\dot{\varphi}(t) = (A_1(\rho) + \Upsilon(\rho)A_2)\varphi(t) + (\Xi_1(\rho) + \Upsilon(\rho)\Xi_2)w(t)
\]
(23)
\[
e(t) = P\varphi(t) + Qw(t)
\]
(24)

where \(A_1(\rho) = \begin{bmatrix} TA(\rho) \\ 0 \end{bmatrix}, A_2 = \begin{bmatrix} C \\ 0 \end{bmatrix}, \)
\(B_1(\rho) = \begin{bmatrix} TA(\rho)QD_2 - TD_1 \\ 0 \end{bmatrix}, B_2 = \begin{bmatrix} (CQ - I)D_2 \\ 0 \end{bmatrix}, \)
\(P = [I 0], Q = QD_2\) and \(\Upsilon(\rho) = \begin{bmatrix} K(\rho) & H(\rho) \\ Z(\rho) & L(\rho) \end{bmatrix} \).

4.2 **H\(_\infty\) generalized observer design**

In this section we shall present a method for designing an \(H\(_\infty\)\) GDO given by (8-10). This design is obtained from the determination of parameter matrix \(\Upsilon(\rho)\) such that the worst estimation error energy \(\|e(t)\|_2^2\) is minimum for all bounded energy disturbance \(w(t)\). This problem is equivalent to the performance index
\[
J = \int_0^\infty [e(t)^T e(t) - \gamma^2 w(t)^T w(t)]dt
\]
(25)

where \(\gamma\) is a given positive scalar. The solution to this problem is given by the following theorem.

**Theorem 1.** Under Assumption 1, there exists an \(H\(_\infty\)\) GDO (8-10) such that the error dynamics in (23-24) is stable and the performance index (25) is satisfied, if there exists a matrix \(X = X^T = \begin{bmatrix} X_1 & X_2 & X_3 \end{bmatrix} > 0\) such that the following LMIs are satisfied
\[
C^T \begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 - \gamma^2 I_s (QD_2)^T \\ 0 & 0 & -I_n \end{bmatrix} C + T \leq 0
\]
(26)

where
\[
\Pi_1 = (TA_i)^T X_1 + X_1 T A_i \\
\Pi_2 = (TA_i QD_2 - TD_1)^T X_2 \\
\Pi_3 = (TA_i QD_2 - TD_1)^T X_3, \forall i \in [1, \ldots, \tau]
\]
and
\[
\begin{bmatrix} -\gamma^2 I_s (QD_2)^T \\ 0 \end{bmatrix} < 0.
\]
(30)

Then, matrix \(Y_i\) is parameterized as
\[
Y_i = X^{-1}(B_i^+ C_i^+ + Z - B_i^+ B_r Z C_i^+ )
\]
(31)

where
\[
K_i = -R_i^{-1} B_i^T \vartheta_i C_i^+ (C_r, \vartheta_i C_i^+ )^{-1} + S_i^{1/2} L(C_r, \vartheta_i C_i^+ )^{-1/2}
\]
(32)
\[
\vartheta_i = (B_i R_i^{-1} B_i^T - D_i) > 0
\]
(33)
\[
S_i = R_i^{-1} - R_i^{-1} B_i^T [\vartheta_i - \vartheta_i C_i^+ (C_r, \vartheta_i C_i^+ )^{-1} C_r, \vartheta_i ] B_i R_i^{-1}
\]
(34)

with \(D_i = \begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 - \gamma^2 I_s (QD_2)^T \\ 0 & 0 & -I_n \end{bmatrix} \), matrices \(\Pi_1, \Pi_2\) and \(\Pi_3\) are defined in equations (27-29),
Consider the following Lyapunov function

\[ V(\varphi(t)) = \varphi(t)^T X \varphi(t) \]  

(35)

where \( X = \begin{bmatrix} X_1 & X_2 \\ X_3 & X_4 \end{bmatrix} > 0 \), then the derivative of (35) along the trajectory of system (14) is

\[ \dot{V}(\varphi(t)) = \varphi(t)^T (A(\rho)X + XA(\rho))\varphi(t) + \varphi(t)^T \dot{X} \]  

(36)

From performance index (25) we get

\[ J < \int_0^\infty |e(t)^T e(t) - \gamma^2 w(t)^T w(t) + \dot{V}(\varphi(t))| dt \]  

(37)

so, a sufficient condition for \( J < 0 \) is that

\[ e(t)^T e(t) - \gamma^2 w(t)^T w(t) + \dot{V}(\varphi(t)) < 0, \quad \forall t \in [0, \infty) \]  

(38)

Replacing \( e(t) \) from (24) and \( \dot{V}(\varphi(t)) \) from (36), we have that inequality (38) is equivalent to

\[ \begin{bmatrix} \varphi(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} A(\rho)X + XA(\rho) + \dot{X} \mathbb{P} \\ \mathbb{B}(\rho)X + Q^T \mathbb{P} \end{bmatrix} \begin{bmatrix} \varphi(t) \\ w(t) \end{bmatrix} < 0 \]  

(39)

The sufficient condition to satisfy inequality (39) is to get

\[ \begin{bmatrix} A(\rho)X + XA(\rho) + \dot{X} \mathbb{P} \\ \mathbb{B}(\rho)X + Q^T \mathbb{P} \end{bmatrix} < 0 \]  

(40)

negative definite. So, applying the Schur complement (40) and replacing matrices \( A(\rho) \) and \( \mathbb{B}(\rho) \) from (23) we get

\[ \begin{bmatrix} (A_1(\rho) + \mathbb{Y}(\rho)A_2)^T X + X (A_1(\rho) + \mathbb{Y}(\rho)A_2) \\ (B_1(\rho) + \mathbb{Y}(\rho)B_2)^T X \end{bmatrix} < 0 \]  

(41)

which can be written as

\[ B \mathcal{X}(\rho) C + (B \mathcal{X}(\rho) C)^T + D(\rho) < 0 \]  

(42)

where \( \mathcal{X}(\rho) = XY(\rho) \), \( B = \begin{bmatrix} I_{q_0+q_1} \\ 0 \end{bmatrix} \), \( C = [A_2 \ B_2 \ 0] \) and

\[ D(\rho) = \begin{bmatrix} A_1(\rho)^T X + X A_1(\rho) \\ B_1(\rho)^T X \end{bmatrix} < 0 \]  

(43)

According to Lemma 1, the inequality (42) is equivalent to

\[ B^T D(\rho) B^T < 0 \quad \text{or} \quad B \mathbb{S} B^T > 0 \]  

(44)

with \( B^T = \begin{bmatrix} 0 & I_n \\ 0 & 0 \end{bmatrix} \) and \( C^T = \begin{bmatrix} A_1^T \\ B_1^T \end{bmatrix} \). By using the definition of matrices \( C^T \) and \( D(\rho) \) and taking into account the definition of (7), the inequality (44) is equivalent to (26), and by using matrices \( B^T \) and \( D(\rho) \) the inequality (43) is equivalent to (30). From Lemma 1, if conditions (43) and (44) are satisfied, the parameter matrix \( Y(\rho) \) is obtained as in (31-34).

5. ILLUSTRATIVE EXAMPLE

Consider the following singular system

\[ E = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad A(\rho) = \begin{bmatrix} -1 & 0 & 3 \\ 3 & \rho(t) & -2 \end{bmatrix}, \quad B(\rho) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \]  

\[ D_1 = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 \end{bmatrix} \]  

and \( D_2 = 0.1 \)

The parameters \( \mu_i(\rho(t)) \) are:

\[ \mu_1(\rho(t)) = \frac{\rho(t) - \rho(t)}{\rho(t)} = \frac{8 - \rho(t)}{3} \]  

(45)

\[ \mu_2(\rho(t)) = \frac{\rho(t) - \rho(t)}{\rho(t) - \rho(t)} = \frac{\rho(t) - 5}{3} \]  

(46)

Matrix \( T \) can be determined from (19), so \( T = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \).

By following Theorem 1 for the \( H_\infty \) GDO design matrices \( \mathcal{L}, \mathcal{Z} \) and \( \mathcal{R} \) have been chosen as:

\[ \mathcal{L} = \begin{bmatrix} 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 0.1 \end{bmatrix}, \quad \mathcal{Z} = \begin{bmatrix} 8 & 2 & 9 & 7 \\ 9 & 3 & 7 & 5 \\ 9 & 3 & 8 & 5 \\ 8 & 3 & 3 & 9 \end{bmatrix} \]  

and \( \mathcal{R} = I_6 \times 0.01 \).

The election of matrix \( \mathcal{Z} \) is totally arbitrary, matrix \( \mathcal{L} \) must satisfy \( \|\mathcal{L}\|_2 < 1 \) and matrix \( \mathcal{R} \) is positive definite and must provided matrices \( \tilde{v}_i \) positive definite.

Now, by fixing \( \gamma = 5.92 \) and solving inequalities (26) and (30) matrix \( X \) is obtained

\[ X = \begin{bmatrix} 34.06 & 2.93 & 0 & 0 & 0 & 0 \\ 2.93 & 2.16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 26.23 & 0 & 0 & 0 \\ 0 & 0 & 0 & 26.23 & 0 & 0 \\ 0 & 0 & 0 & 0 & 26.23 & 0 \\ 0 & 0 & 0 & 0 & 0 & 26.23 \end{bmatrix} \]  

Parameter matrix \( \mathcal{Y}_i \) is parameterized as in (31-34)

\[ \mathcal{Y}_1 = \begin{bmatrix} -2.09 & 0.05 & 0.05 \\ 0.97 & 1.08 & 1.08 \\ -3.60 & 0.38 & 0.38 \\ 0.21 & -3.43 & 0.38 \\ 0.21 & 0.38 & -3.43 \\ 0.21 & 0.38 & 0.38 & -3.43 \end{bmatrix} \]  

(47)

\[ \mathcal{Y}_2 = \begin{bmatrix} -2.24 & 0.03 & 0.03 \\ -1.58 & 0.52 & 0.52 \\ -3.64 & 0.38 & 0.38 \\ 0.17 & 3.43 & 0.38 \\ 0.17 & 3.43 & 0.38 \\ 0.17 & 3.43 & 0.38 \\ 0.17 & 3.43 & 0.38 \end{bmatrix} \]  

(48)

Now, we can obtain all the observer matrices as

\[ N_1 = \begin{bmatrix} -1 & 0 & 0.91 \\ 5 & -2 & -1.03 \end{bmatrix}, \quad N_2 = \begin{bmatrix} -1 & 0 & 0.76 \\ 8 & -2 & -3.58 \end{bmatrix} \]  

(49)

(50)
$F_1 = F_2 = \begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$, $J_1 = \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$, $J_2 = \begin{bmatrix} 8 \\ 1 \\ 0 \end{bmatrix}$, $H_1 = \begin{bmatrix} 0.05 & 0.05 & 0.05 \\ 1.08 & 1.08 & 1.08 \\ 0.38 & 0.38 & 0.38 \end{bmatrix}$, $H_2 = \begin{bmatrix} 0.03 & 0.03 & 0.03 \\ 0.52 & 0.52 & 0.52 \\ 0.38 & 0.38 & 0.38 \end{bmatrix}$.

$L_1 = L_2 = \begin{bmatrix} -3.43 & 0.38 & 0.38 \\ 0.38 & -3.43 & 0.38 \\ 0.38 & 0.38 & -3.43 \end{bmatrix}$, $Q = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$, $S_1 = \begin{bmatrix} 0 & 0.21 \\ 0 & 0.21 \\ 0 & 0.21 \end{bmatrix}$, $S_2 = \begin{bmatrix} 0 & 0.17 \\ 0 & 0.17 \\ 0 & 0.17 \end{bmatrix}$, and $M_1 = M_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

The initial conditions for the system are $x(0) = [0.1, 0.1, 0]^T$, for the GDO are $\zeta(0) = [0, 0, 0]^T$ and $v(0) = [0, 0, 0]^T$. The results of the simulation are depicted in Figures 1-5. Figure 1 shows the input $u(t)$ and the disturbance $w(t)$. Figure 2 shows the variation of the parameter $\rho(t)$ and the weighting states of each model. Figures 3-5 show the system states and their estimations by the GDO.

6. CONCLUSION

In this paper an $H_\infty$ GDO design for LPV singular systems with disturbances is treated. The conditions for the existence and stability of the observer are given in terms of a set of LMIs. The obtained observer guarantees the disturbance attenuation in a prescribed level. A numerical example is presented to show the performance of our approach.

REFERENCES

Alma, M. and Darouach, M. (2014). Adaptive observers design for a class of linear descriptor systems. Automatica, 50, 578–583.
Fig. 5. Estimation of $x_3$.

Borges, R.A. and Peres, P.L.D. (2006). $H_\infty$ LPV filtering for linear systems with arbitrarily time-varying parameters in polytopic domains. In 45th IEEE Conference on Decision and Control. USA.

Dai, L. (1989). *Singular control systems*. New York: Springer-Verlag.

Darouach, M. (2012). On the functional observer for linear descriptor systems. *Systems & Control Letters*, 61, 427–434.

Darouach, M. and Boutayeb, M. (1995). Design of observers for descriptor systems. *IEEE Transactions on Automatic Control*, 40, 1323–1327.

Goodwin, G.C. and Middleton, R.H. (1989). The class of all stable unbiased state estimators. *Systems & Control Letters*, 13, 161–163.

Habib, H., Rodrigues, M., Mechmeche, C., and Braiek, N.B. (2010). Robust $H_\infty$ fault diagnosis for multimodel descriptor systems: A multi-objective approach. In 18th Mediterranean Conference on Control and Automation.

Hou, M. and Müller, P.C. (1993). Design of a class of Luenberger observers for descriptor systems. *IEEE Transactions on Automatic Control*, 38, 1666–1671.

Koenig, D. and Marx, B. (2008). $H_\infty$-filtering and state feedback control for discrete-time switched descriptor systems. *IET Control Theory and Applications*, 3, 661–670.

Luenberger, D.G. (1977). Dynamic equations in descriptor form. *IEEE Transactions on Automatic Control*, AC-22, 312–321.

Marquez, H.J. (2003). A frequency domain approach to state estimation. *Journal of the Franklin Institute*, 340, 147–157.

Müller, P.C. (2005). Modelling and control of mechatronic systems by the descriptor approach. *Journal of Theoretical and Applied Mechanics*, 43, 593–607.

Müller, P.C. and Hou, M. (1993). On the observer design for descriptor systems. *IEEE Transactions on Automatic Control*, 38, 1666–1671.

Nagpal, K.M. and Khargonekar, P.P. (1991). Filtering and smoothing in an $H_\infty$ setting. *IEEE Transactions on Automatic Control*, 36, 152–166.

Rodrigues, M., Theilliol, D., Aberkane, S., and Sauter, D. (2007). Fault tolerant control design for politopic LPV systems. *International Journal of Applied Mathematics and Computer Science*, 17, 27–37.

Skelton, R.E., Iwasaki, T., and Grigoriadis, K. (1998). *A unified algebraic approach to linear control design*. Taylor & Francis.

Yue, D. and Han, Q.L. (2004). Robust $h_\infty$ filter design of uncertain descriptor systems with discrete and distributed delays. *IEEE Transactions on Automatic Control*, 52, 3200–3212.