THE CHEMICAL HOMOGENEITY OF OPEN CLUSTERS

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ABSTRACT

Determining the level of chemical homogeneity in open clusters is of fundamental importance in the study of the evolution of star-forming clouds and that of the Galactic disk. Yet limiting the initial abundance spread in clusters has been hampered by difficulties in obtaining consistent spectroscopic abundances for different stellar types. Without reference to any specific model of stellar photospheres, a model for a homogeneous cluster is that it forms a one-dimensional sequence, with any differences between members due to variations in stellar mass and observational uncertainties. I present a novel method for investigating the abundance spread in open clusters that tests this one-dimensional hypothesis at the level of observed stellar spectra, rather than constraining homogeneity using derived abundances as is traditionally done. Using high-resolution APOGEE spectra for 49 giants in M67, NGC 6819, and NGC 2420 I demonstrate that these spectra form one-dimensional sequences for each cluster. With detailed forward modeling of the spectra and Approximate Bayesian Computation, I derive strong limits on the initial abundance spread of 15 elements: \( <0.01 \) (0.02) dex for C and Fe, \( \lesssim 0.015 \) (0.03) dex for N, O, Mg, Si, and Ni, \( \lesssim 0.02 \) (0.03) dex for Al, Ca, and Mn, and \( \lesssim 0.03 \) (0.05) dex for Na, S, K, Ti, and V (at 68% and 95% confidence, respectively). The strong limits on C and O imply that no pollution by massive core-collapse supernovae occurred during star formation in open clusters, which, thus, need to form within \( \lesssim 6 \) Myr. Further development of this and related techniques will bring the power of differential abundances to stars other than solar twins in large spectroscopic surveys and will help unravel the history of star formation and chemical enrichment in the Milky Way through chemical tagging.

Key words: Galaxy: abundances – Galaxy: disk – Galaxy: evolution – Galaxy: formation – Galaxy: fundamental parameters – Galaxy: structure

1. INTRODUCTION

The surface abundances of long-lived stars observed through high-resolution spectroscopy hold the archaeological record of the conditions of their formation. Carefully uncovering this history through analyses of the observed spectroscopic, photometric, and astrometric data has the potential to lead to transformative insights into the nature of star formation, the evolution of massive stars, and the detailed chemical and dynamical evolution of galactic disks. Yet, after many decades of work on the theory of stellar photospheres and orders of magnitude improvements in the quantity, quality, and variety of observed stellar spectra, stellar spectroscopy remains challenging due to incomplete theoretical models and the difficulty of taking into account the many instrumental factors affecting observations. Because of this, abundance uncertainties are still routinely quoted as being “0.1 dex,” a seemingly magic number even though in practice observational setups vary widely.

Measuring stellar abundances for many different elements with uncertainties \( \leq 0.1 \text{ dex} \) opens up a wide range of questions to scientific investigation. Stars are believed to form in groups in molecular clouds (e.g., Shu et al. 1987; Lada & Lada 2003), but exactly how the intracloud medium evolves and mixes and how star formation proceeds in such clouds, especially on timescales of a few Myr (McKee & Tan 2002; Feng & Krumholz 2014), is difficult to study observationally because the young clusters are mostly obscured from view. Determining the spread (or tight limits \( \leq 0.1 \text{ dex} \) on it) in the abundances of elements produced on short timescales by SNe II in surviving clusters would provide strong constraints on analytic and numerical work in this area.

Beyond the individual star clusters, stellar abundances of long-lived stars trace the history of star formation, chemical enrichment, and the interstellar medium. If the majority of stars are born in clusters with tens of thousands of members sharing the same initial abundances, we might be able to chemically tag individual star formation events in the Milky Way by determining abundances for large samples of stars (Freeman & Bland-Hawthorn 2002). If successful, this tagging would provide the chemical and dynamical history of the Milky Way’s disk at a level of detail far surpassing our currently limited, broad-brush picture (e.g., Bland-Hawthorn et al. 2010). To determine whether chemical tagging is possible, three essential questions remain to be answered: (a) What is the level of initial abundance spread in star clusters? (b) Can we measure the variations between the chemical signatures of different clusters to the level determined in (a) in light of observational uncertainties and the effects of stellar evolution on the present-day surface abundances? And (c) do different star-forming clusters have chemical signatures that are sufficiently unique to distinguish each star-forming event, given the “chemical resolution” attained in (b)?

In this paper I present a novel method for addressing question (a) above through observations of the abundance spread in open clusters and use it to determine the most stringent constraints on the chemical homogeneity of open clusters to date. Old open clusters (with ages \( > 1 \text{ Gyr} \)) are those rare star formation remnants that have not been destroyed yet by encounters with molecular clouds. As such, they may constitute a biased sample of the full initial open-cluster population. But it is also likely that an unbiased subset survives the presumably random interactions with gravitational...
inhomogeneities, in which case they can shed light on the properties of all of the clusters that stars form in.

Previous work has established that open clusters are homogeneous at the level of $\approx 0.05$ to 0.1 dex (e.g., De Silva et al. 2006, 2007a, 2007b; Reddy et al. 2012; Ting et al. 2012), although these analyses typically compared the observed scatter to the estimated uncertainties, rather than inferring rigorous limits on the scatter. The advent of large surveys of open clusters has allowed such analyses to be performed for many clusters and many different atomic species, with limits on the dispersion now routinely reaching $\lesssim 0.05$ dex (Blanco-Cuaresma et al. 2015). One of the main limiting factors in these studies is the inability to measure abundances on a consistent scale for different stellar types (e.g., dwarfs, sub-giants, giants), reducing the number of stars available for any analysis.

1.1. Limiting Intrinsic Scatter

Most work on determining or limiting the abundance spread in open clusters does not carefully track the observational uncertainties, even though these are key to establishing whether any measured scatter is real or simply due to measurement errors. To illustrate this, consider a simple experiment where $N$ mock data points $d_i$ with no intrinsic scatter are drawn with Gaussian uncertainties with variance $\delta^2$. The likelihood for the intrinsic scatter $\sigma$ when it is assumed to be Gaussian is

$$\mathcal{L}(\sigma) \propto (\delta^2 + \sigma^2)^{-N/2} \exp \left( -\frac{1}{2} \sum_{i=1}^{N} d_i^2 \right).$$

(1)

This is a $\chi^2$ distribution for the parameter $Q = \sum_{i=1}^{N} d_i^2 / \sigma^2$ with $N + 2$ degrees of freedom. In the large-$N$ limit, this distribution approaches a Gaussian distribution with

$$x = (Q - N - 2) / \sqrt{2(N + 2)} \sim \mathcal{N}(0, 1),$$

(2)

where $\mathcal{N}(0, 1)$ is the unit normal distribution. An upper limit $\sigma_{\text{ul}}$ on $\sigma$ at some confidence level corresponds to a lower limit $Q_{\text{ll}}$ on $Q$; in this case $x$ is equal to some constant $-C$, i.e.,

$$Q_{\text{ll}} - N - 2) / \sqrt{2(N + 2)} = -C.$$

(3)

For large $N$, $\sum_{i=1}^{N} d_i^2 \approx \delta^2 N$ and assuming that $\sigma^2 \ll \delta^2$ we find for the upper limit $\sigma_{\text{ul}}$ on $\sigma$

$$\sigma_{\text{ul}} \propto \delta N^{-1/4}.$$

(4)

In Figure 1, I test this analytic estimate with direct mock-data simulations and inferences. This figure demonstrates that the large-$N$ asymptotic behavior occurs above about 100 data points, with a steeper dependence on $N$ between 10 and 100 data points. The latter is the relevant regime for the data in this paper.

Thus, in the limit of many data points, it is difficult to significantly improve upon the upper limit on the intrinsic scatter by observing more stars, especially given the limited number of stars suitable for high-resolution spectroscopy in all but the nearest clusters. As Figure 1 illustrates, if one were to use the simplistic “0.1 dex” standard abundance uncertainty, about 100,000 stars would be required to limit the intrinsic dispersion to below 0.01 dex. It is therefore of the utmost importance to characterize, understand, and use one’s abundance precision.

1.2. Overview

Motivated by the discussion in the previous section that the abundance precision is of the highest importance in studying abundance scatter, and by the fact that data uncertainties are simplest in the space of the spectra themselves rather than in that of the measured abundances, I propose a novel method here for limiting the abundance scatter in clusters and for evaluating similarity and dissimilarity of abundances in groups of stars more generally. This method evaluates the effect of abundance scatter through forward modeling on the observed stellar spectra where it can be directly compared to the spectral uncertainties. By empirically removing temperature trends in both the observed and simulated spectra, this method is robust to modeling errors related to standard assumptions in stellar spectroscopy and to the main effects of stellar evolution on the surface abundances.

This paper is organized as follows. In Section 2 I propose that a simple empirical model for an open cluster without intrinsic abundance scatter is that all stellar properties are a one-dimensional function of (fundamentally) initial mass and I discuss how we can easily test this model. Section 3 introduces the APOGEE data for the four open clusters that the method in this paper is applied to. In Section 4, I explicitly test the one-dimensional hypothesis for the APOGEE clusters. I demonstrate that all clusters are consistent with this hypothesis and
that this is strongly constraining for the intrinsic scatter in 15 elements that have absorption features in the APOGEE wavelength range. To make the intrinsic-scatter limits precise, Section 5 describes a method using Approximate Bayesian Computation (ABC) that uses detailed simulations of the observed spectra to put robust limits on the intrinsic abundance scatter. I discuss the results, their implications, and future prospects in Section 6.

Appendix A displays the high signal-to-noise “stacked” red-giant spectra for M67, NGC 6819, and NGC 2420 that I obtain using the method in Section 2. The remaining three appendices discuss relevant technical details of the observed and synthetic APOGEE spectra. Appendix B discusses how I construct an empirical error model for the APOGEE continuum-normalized spectra using repeat observations of stars. Appendix C provides details of the procedure and code to generate synthetic APOGEE spectra tailored to each star for variations of all 15 considered elements. Appendix D presents an investigation of the sensitivity of APOGEE spectra to abundance changes of different elements; these sensitivities are a crucial ingredient in constraining the abundance scatter using forward simulations.

2. AN EMPIRICAL MODEL FOR THE SPECTRA OF OPEN-CLUSTER MEMBERS

The model for open clusters that we are interested in constraining is that they consist of a set of stars born from a well-mixed gas cloud in a negligible amount of time (≤10 Myr). Assuming no scatter in the birth abundances, the most important factor distinguishing different stars is their different initial mass, which spans the range ≈0.1 M⊙ to ≈100 M⊙. Each star’s initial mass together with the common initial abundances determines its subsequent evolution. At the present day, we observe stars in clusters to span a wide range of luminosities, temperatures, surface gravities, etc. due to the range in initial masses. Every photometric and spectroscopic property of cluster stars should then follow a one-dimensional relation as a function of stellar mass. In particular, the spectra of cluster stars near absorption features of different elements should follow a one-dimensional sequence. Without any reference to particular models of stellar photospheres, this is a testable prediction.

An important advantage of this approach to testing the chemical homogeneity of open clusters, is that many of the stellar evolution effects on the surface abundances that normally confound studies of homogeneity by (correctly) showing abundance scatter in the current abundances (e.g., Onenag et al. 2014), are themselves primarily functions of the stellar mass (e.g., gravitational settling or mixing of C, N, and O during dredge-up episodes). Similarly, hydrodynamical effects often parametrized in simplified treatments using micro-and macroturbulence are mostly functions of the current evolutionary state (temperature, gravity) and therefore also functions of stellar mass. Theoretically predicting these functions is difficult, but it is clear that the combination remains one-dimensional and fitting a flexible one-dimensional model allows us to ignore this lack of knowledge.

Besides these deterministic effects, random effects in the initial condition of each star or in its subsequent evolution can break the one-dimensional model. For example, stars are born with a distribution of initial rotation speeds. When these survive to the present time, they will give rise to different line profiles. The effects of magnetic braking likely cause all stars in a cluster to have the same current rotation for the old clusters that we are interested in here (>1 Gyr; Weber & Davis 1967; Skumanich 1972; Kawaler 1988), but a random distribution of inclinations will still give rise to different line profiles. For the giants that I consider in this paper, the line broadening due to rotation can, however, be neglected. Differences in initial rotation might additionally induce variations in the internal mixing that could manifest themselves at the surface today. Interactions between binary stars may also lead to abundance anomalies that would be uncorrelated with mass, or the infall of rocky planets could lead to scatter in the abundances of refractory elements (Meléndez et al. 2009). While this might confound studies of the initial chemical homogeneity of clusters, limits on the abundance scatter obviously constrain the importance of these processes.

Beyond effects intrinsic to the cluster stars, instrumental effects may give rise to scatter at fixed initial stellar mass, even when all stars are observed with the same instrument. Foremost among these are variations in the line-spread function (LSF), which lead to different broadening profiles similar to the case of rotation discussed above. For the APOGEE spectra that I employ in this paper, LSF variations exist, but are small enough that they are only confused with abundance scatter ≲0.01 dex. In the forward modeling approach below, I take the LSF variations among cluster stars fully into account.

The basic model that I will use in this paper is therefore that the spectra of stars in open clusters are a one-dimensional sequence. Because (initial) stellar mass is difficult to observe, I will employ the effective temperature T_eff as a proxy for the mass and use one-dimensional models as a function of temperature. For the red giants that we will consider later, T_eff is a good proxy for the mass and we have photometric T_eff available that are independent of the considered spectra. Each pixel value f_λ^i for different stars i = 1...N in a cluster and wavelengths λ can then be modeled as a function g_λ(T_eff|θ_λ) characterized by parameters θ_λ plus the measurement noise

\[ f_λ^i = g_λ(T_{\text{eff},i}|θ_λ) + \text{noise}. \]  

I model g_λ as a second-order polynomial in T_eff and we can then fit for the parameters θ_λ at each wavelength using the observed f_λ^i and their catalog uncertainties using maximum likelihood; I also include an intrinsic scatter in the fit, but this is always small. This approach is similar to that taken by Ness et al. (2015) for deriving an empirical model of stellar spectra using a calibration sample that can then be applied to determine stellar parameters (T_eff in this case). The approach taken here is different in that I fit the empirical model only as a way of determining whether the spectra of stars in a cluster are the same apart from trends with T_eff.

Once we have determined the best-fitting g_λ(T_{eff,i}|θ_λ) for each pixel λ, we can compute the residuals which are given by

\[ r_λ^i = f_λ^i - g_λ(T_{\text{eff},i}|θ_λ). \] 

If the one-dimensional model provides a good fit, then the distribution of residuals should be consistent with the uncertainty distribution for each pixel.

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1 As we will see, this does not hold exactly because of the presence of both red-giant and red-clump stars at temperatures ≈4750 K. Red-clump stars have slightly different surface gravities at the same T_eff and may also have different C and N abundances due to the effect of convective mixing on the upper giant branch.
This procedure is illustrated in Figure 2. The “data” row of this figure displays APOGEE spectra for stars in M67 (described in more detail below) in the region of an Al feature. Each column sub-panel shows the dependence of a single pixel of the continuum-normalized spectrum on $T_{\text{eff}}$ and a quadratic fit to this dependence. The bottom panels show the residuals from the fit; these are largely consistent with the reported uncertainties. The bottom two sub-figures show the same for two simulations of the data: the middle figure assumes no intrinsic scatter in the Al abundance and the bottom figure uses a scatter of 0.1 dex. For the latter simulation, the residuals are clearly larger than the uncertainties, demonstrating that the scatter around the quadratic fit is strongly constraining for the intrinsic abundance scatter.

Figure 2. The Al spectral line at 16723.5 Å for giants in M67 (top). Each column displays the dependence of a single pixel of the continuum-normalized spectrum on $T_{\text{eff}}$ and a quadratic fit to this dependence. The bottom panels show the residuals from the fit; these are largely consistent with the reported uncertainties. The bottom two sub-figures show the same for two simulations of the data: the middle figure assumes no intrinsic scatter in the Al abundance and the bottom figure uses a scatter of 0.1 dex. For the latter simulation, the residuals are clearly larger than the uncertainties, demonstrating that the scatter around the quadratic fit is strongly constraining for the intrinsic abundance scatter.

3. DATA

The spectroscopic data for open-cluster members that I analyze here comes from the SDSS-III/APOGEE (Majewski et al. 2015), a high-resolution ($R \approx 22,500$) spectroscopic survey that observes in the $H$-band (1.51–1.70 μm) using a 300-fiber spectrograph (Wilson et al. 2010, J. Wilson et al. 2015, in preparation) on the 2.5-m Sloan Foundation telescope (Gunn et al. 2006). I use data from the public Data Release 12 (DR12; Alam et al. 2015; Holtzman et al. 2015) for four open clusters that have a large number of members: M67 ([Fe/H] ≈ 0), NGC 6819 ([Fe/H] ≈ 0), NGC 2158 ([Fe/H] ≈ −0.15), and NGC 2420 ([Fe/H] ≈ −0.2). Members for these clusters are obtained from the catalog in Mészáros et al. (2013) and I use the photometric $T_{\text{eff}}$, surface gravities $\log g$, and overall metallicities [Fe/H] used in that paper. Full details on the APOGEE target selection for these clusters can be found in Zasowski et al. (2013). I only select the stars with $4000 \, \text{K} \leq T_{\text{eff}} \leq 5000 \, \text{K}$, because this range contains most of the stars and spectral modeling is more uncertain at $T_{\text{eff}} < 4000 \, \text{K}$. After this cut, the sample consists of 24 stars, 8 of which are likely red-clump stars, for M67; 30 stars, 13 of which are likely red-clump stars, for NGC 6819; 10 stars in NGC 2158; and 9 stars in NGC 2420. The log $g$ versus $T_{\text{eff}}$ for the giants in these clusters are displayed in Figure 3.
The spectra used here are the apStar spectra that combine data from all APOGEE observations of a given star; I use the version using a “global” weighting of the individual spectra going into the combination. These spectra are sampled on a logarithmic wavelength grid in the restframe of the star (Nidever et al. 2015). In Appendix B, I also use the spectra from the individual hour-long APOGEE observations that are combined to form the apStar spectra; these are also available from the SDSS database on the same wavelength grid. Pixels identified as bad in the APOGEE_PIXMASK bitmask for the reasons BADPIX, CRPIX, SATPIX, UNFIXABLE, BADDARK, BADFLAT, BADDERR, NOSKY, or SIG_SKYLINE (Holtzman et al. 2015) are given very large uncertainties to remove them from further consideration. Uncertainties smaller than 0.5% are set to 0.5%, because of systematic errors at the 0.5% level for APOGEE spectra (Nidever et al. 2015).

I fit for the continuum of each spectrum using the method of Ness et al. (2015). This method identifies a set of continuum pixels by fitting a quadratic model in \((T_{\text{eff}}, \log g, [\text{Fe/H}], [\alpha/\text{Fe}])\) to each pixel for a calibration sample with known \((T_{\text{eff}}, \log g, [\text{Fe/H}], [\alpha/\text{Fe}])\) and selecting those pixels whose values display only a small dependence on \((T_{\text{eff}}, \log g, [\text{Fe/H}], [\alpha/\text{Fe}])\). I use the same calibration sample as used in Ness et al. (2015) and select pixels with linear dependencies less than \((10^{-5}\text{K}, 0.006, 0.012, 0.03)\) in \((T_{\text{eff}}, \log g, [\text{Fe/H}], [\alpha/\text{Fe}])\) (similar to Ness et al. 2015) and additionally limit the pixels to those with intrinsic scatter less than 0.015 to remove pixels with large variations that cannot be attributed to the basic stellar parameters. Using the wavelengths of these continuum pixels, the continuum for each star is determined by fitting a fourth-order polynomial over the wavelength range of each of the three APOGEE detectors to just these wavelengths. As demonstrated in Appendix B using repeat observations, this procedure is highly stable and produces consistent continuum-normalized spectra for different observations of the same star. After continuum normalization, uncertainties smaller than 0.005 are set to 0.005 for the same reason as above. I further remove pixels with errors larger than 0.02 (signal-to-noise ratio <50) from further consideration, because the errors for these low signal-to-noise ratio pixels might not be well-characterized by the reported uncertainty (this includes all of the pixels flagged as bad mentioned earlier). This only removes a few percent of the pixels.

Ideally, the errors in the spectra should be well-characterized by the reported uncertainties in the APOGEE database, which assume uncorrelated errors between different pixels. I test the reported uncertainties in Appendix B using 4143 repeat observations of 1381 stars bright enough that each individual hour-long exposure has high signal-to-noise ratio. These tests demonstrate that the reported uncertainties are typically underestimated by 10%–20%, but ranging up to 100% for significant portions of the wavelength range, especially near the ubiquitous telluric absorption features. Furthermore, these tests show that errors display significant correlations out to dozens of pixels (\(\geq10\text{Å}\)). This is a range that is almost ten times as wide as the reported LSF. This large range over which correlations are significant is most likely due to correlated errors induced by the continuum normalization. In what follows I use the residuals from repeat observations directly as an empirical sampling of the uncertainty in the observed spectra (see further discussion in Appendix B).

Appendix C discusses the details of how I generate synthetic APOGEE spectra for each cluster star individually using its \((T_{\text{eff}}, \log g, [\text{Fe/H}], [\alpha/\text{Fe}])\), the median cluster metallicity, and variations in the abundances of individual elements. Using these synthetic spectra, we can estimate the precision with which we can measure the abundances of individual elements by computing the \(\Delta\chi^2([X/\text{H}])\) from a baseline model where all abundance ratios are solar. I compute this \(\chi^2([X/\text{H}])\) weighting the contribution of each pixel with the pixel-weights that give prominence to clean absorption features of each element (see Appendix D). Assuming perfect knowledge of all other parameters \((T_{\text{eff}}, \log g, [\text{Fe/H}], [\alpha/\text{Fe}])\), but also micro- and macro-turbulence, etc., and that our modeling is perfect, this effectively sets a realistic lower limit on the precision. I compute \(\chi^2([X/\text{H}])\) using the reported uncertainties (i.e., not taking into the account the underestimation of the uncertainties), which also makes the estimated abundance uncertainties a lower limit. These estimated abundance uncertainties are shown in Table 1. From Figure 1, we expect the 95% upper limit on the abundance scatter in each individual cluster to be roughly the precision of an individual abundance measurement (slightly smaller for M67 and NGC 6819, slightly larger for NGC 2420). The precision for some elements (like C, N, and O) sensitively depends on temperatures, leading to a wide range of expected precision. If we combine all clusters, the ~50 stars should give a 95% upper limit that is about 60% of the abundance precision. It is therefore clear that we should be able

![Figure 3](image-url). log \(g\) vs. \(T_{\text{eff}}\) for the four open clusters studied in this paper. Likely red-clump stars have been colored gray for M67 and NGC 6819. These are excluded for the analysis of C and N for M67 and are altogether excluded for the analysis of NGC 6819. PARSEC isochrones (Bressan et al. 2012) for ages of 2.5, 1.6, 2, and 2 Gyr at the cluster’s metallicity are shown as well.
Table 1

| Element | M67 | NGC 6819 | NGC 2420 |
|---------|-----|----------|----------|
| C       | 0.04 (0.03–0.07) | 0.05 (0.02–0.07) | 0.06 (0.02–0.07) |
| N       | 0.07 (0.06–0.09) | 0.06 (0.04–0.08) | 0.05 (0.05–0.08) |
| O       | 0.16 (0.04–0.20) | 0.13 (0.04–0.20) | 0.16 (0.03–0.20) |
| Na      | 0.13 (0.08–0.17) | 0.09 (0.06–0.12) | 0.16 (0.09–0.20) |
| Mg      | 0.04 (0.03–0.05) | 0.03 (0.03–0.04) | 0.03 (0.03–0.04) |
| Al      | 0.09 (0.08–0.10) | 0.04 (0.04–0.04) | 0.04 (0.04–0.05) |
| Si      | 0.06 (0.06–0.06) | 0.04 (0.04–0.05) | 0.04 (0.04–0.05) |
| S       | 0.12 (0.11–0.20) | 0.07 (0.07–0.09) | 0.08 (0.08–0.11) |
| K       | 0.06 (0.06–0.07) | 0.04 (0.03–0.04) | 0.04 (0.04–0.04) |
| Ca      | 0.05 (0.04–0.05) | 0.04 (0.04–0.04) | 0.04 (0.04–0.04) |
| Ti      | 0.07 (0.05–0.10) | 0.06 (0.04–0.09) | 0.10 (0.04–0.12) |
| V       | 0.08 (0.03–0.14) | 0.08 (0.03–0.13) | 0.15 (0.04–0.20) |
| Mn      | 0.06 (0.05–0.07) | 0.03 (0.03–0.04) | 0.06 (0.04–0.06) |
| Fe      | 0.06 (0.05–0.06) | 0.04 (0.04–0.04) | 0.04 (0.04–0.04) |
| Ni      | 0.07 (0.07–0.07) | 0.06 (0.05–0.06) | 0.06 (0.06–0.06) |

Note. Expected abundance uncertainties in the abundance of X (weighted using the pixel weights of Appendix D) of a baseline model with solar abundance ratios for each cluster star. The median precision for each element and each cluster is shown, as well as the full range of all the cluster members in parenthesis. These precisions assume perfect spectral models, perfect knowledge of all other stellar parameters and abundances, and that the APOGEE noise model is correct, but by using the pixel weights they only use parts of the spectrum that are sensitive to each element. Therefore, these represent a realistic estimate of the expected precision that can be hoped to be achieved.

4. ARE OPEN CLUSTERS ONE-DIMENSIONAL SEQUENCES?

To check whether or not the four open clusters described in Section 3 are consistent with forming a one-dimensional temperature (as a proxy for mass) sequence, I fit the quadratic model to the $T_{\text{eff}}$ dependence of each pixel using members in each cluster as discussed in Section 2. M67 and NGC 6819 both have a significant number of both red-clump and first-ascent red-giant stars at similar temperatures (see Figure 3). These lead to two potential issues with the method used in this paper. First, deep mixing along the upper giant branch can change the surface C and N abundances. While this should be a smooth change as a function of temperature, when stars move to the red-clump after the helium flash, as a function of temperature alone this leads to stars in the cluster having a bimodal C and N distribution that cannot be captured with the model from Section 2. Second, red-clump stars have slightly lower log g than first-ascent red-giant stars at the same $T_{\text{eff}}$. This again leads to variations in the spectra at a fixed $T_{\text{eff}}$ that are not included in our model. To avoid these issues, I remove all red-clump stars in M67 when looking at C and N, but not when considering the other elements. The number of red-clump giants is large enough in NGC 6819 that their log g differences lead to substantial scatter and I therefore remove all clump giants altogether in NGC 6819. If we were to fit the spectra as a function of mass rather than $T_{\text{eff}}$, these issues would be avoided, but we currently do not have precise enough masses to do this. In NGC 6819, I also remove the Li-rich giant 2M19411367+4003382, which may not be a cluster member or has anomalously low mass if it is.

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quadratic fit to construct a high signal-to-noise ratio spectrum for the cluster at a given temperature. This combined cluster spectrum at $T_{\text{eff}} = 4750$ K for the red giants in M67, NGC 6819, and NGC 2420 is displayed and discussed in Appendix A.

5. INFERENCE OF THE ABUNDANCE SCATTER

To determine constraints on the abundance scatter of different elements for each cluster, I use ABC to construct an approximation of the posterior probability distribution function (PDF) of the scatter $\sigma_{[X/H]}$ in each element X. ABC is an
inference technique that approximates the PDF without explicitly evaluating the likelihood, but instead making use of forward simulations of the data. To explicitly evaluate the likelihood of $\sigma_{X/H}$ would require an actual model for the noise in the spectra—which is difficult to establish (see Appendix B)—and it would be computationally expensive, because we would need to marginalize over the individual abundances of each cluster member, while properly taking into account the varying LSF. However, it is straightforward to generate simulated data for any $\sigma_{X/H}$ that take into account LSF variations, the noise and its correlations in the spectra, and that are robust against systematics in the abundances due to, e.g., deep mixing or deviations from local thermodynamic equilibrium.

For any $\sigma_{X/H}$, I draw a set of $N$ abundances [X/Fe] for the $N$ cluster members, generate synthetic spectra using the procedure described in Appendix C, and then fit the $T_{\text{eff}}$ dependence of each pixel using the quadratic model described in Section 2 in the same way as for the data. An example of this is displayed for one of the Al lines in Figure 2. The top panel of that figure shows the data, while the middle and bottom panels show simulated data with $\sigma_{X/H} = 0.0$ dex and $\sigma_{X/H} = 0.1$ dex, respectively. When running ABC, we retain those $\sigma_{X/H}$ that lead to similar residuals from the quadratic fit as found in the data. By only considering a match between the data and the simulated data in terms of their residuals, we focus the comparison on the abundance scatter, rather than on whether the simulations produce the exact same continuum levels, the same line strengths (which may be affected by such effects as deviations from local thermodynamic equilibrium or hyperfine structure), the correct behavior of weak and strong lines of a given element, and whether evolutionary changes in the surface abundances are included.

ABC produces an approximation to the PDF by (a) simulating $\sigma_{X/H}$ from its prior (which I take to be uniform between 0 and 0.1 dex), and (b) only storing those $\sigma_{X/H}$ that lead to simulated data that are “the same” as the real data (Tavaré et al. 1997; Pritchard et al. 1999). By considering the meaning of the PDF (“the probability distribution of $\sigma_{X/H}$ given the data”), it is clear that this procedure works, because it generates a Monte Carlo sampling of $\sigma_{X/H}$ that are constrained to be the same as the data and are therefore a sampling of the PDF. See, for example, Marin et al. (2012) for a recent review of ABC.

ABC requires one to specify what it means for simulated data $D'$ to be same as the actual data $D$. This is done by defining a metric $\rho(D', D)$ that expresses how close the simulated data are to the real data. The PDF for $\sigma_{X/H}$ is constructed using the $\sigma_{X/H}$ that generate $D'$ that satisfy $\rho(D', D) \leq \epsilon$ and ABC produces an exact sampling of the PDF in the limit $\epsilon \rightarrow 0$. Of course, it is difficult to generate simulated data that are exactly like the actual data, especially in the presence of random noise. If the data and model have a lower-dimensional sufficient statistic $\mu(D)$ that encapsulates all of the information about $\sigma_{X/H}$ that is contained in the data, this situation is significantly ameliorated. While a rigorous sufficient statistic does not exist for the problem considered in this paper, as I argue below there are summary statistics that can be used to significantly reduce the dimensionality of the data and make this problem tractable for ABC.

In principle, we need to constrain all 15 $\sigma_{X/H}$ simultaneously, because all 15 elements affect the spectra and disentangling their effects is difficult, especially for C, N, and O. Because I only derive upper limits on $\sigma_{C/H}$, however, I can consider each element separately. That is, the lack of scatter in the spectra near, e.g., CN features implies a limit on both $\sigma_{C/H}$ and $\sigma_{N/H}$ that can be established by varying $\sigma_{C/H}$ and $\sigma_{N/H}$ separately. Stronger, covariant limits on $\sigma_{C/H}$ and $\sigma_{N/H}$ could be determined by considering them simultaneously, but I do not attempt this here as it significantly increases the computational complexity. Similarly, we need not worry too much about whether the limit $\epsilon$ used to decide which simulated data are close to the actual data is a good limit, because setting it too high will only weaken the upper limits on $\sigma_{X/H}$.

I consider two summary statistics when running the ABC simulations. The first is the Kolmogorov–Smirnov distance $D_a$ between the cumulative distribution of normalized residuals of the data and the simulated data, that is, the maximum difference between these distributions. For each element, these cumulative distributions are computed by weighting the residuals by the weights for that element (see Appendix D). For the data these cumulative distributions are shown in Figure 4. This figure also displays the median and interquartile range of the cumulative distributions of simulated data computed for $\sigma_{X/H} = 0.1$ dex.

The second summary statistic that I employ is based on the covariance matrix of the normalized residuals of different pixels. For the data and the simulated data, I compute the covariance matrix $\text{Cov}_{ij}$ between pixels $i$ and $j$. For each simulated data set, I calculate the difference

$$\Delta\text{Cov}_{ij} = \sqrt{\sum_j (w_i w_j)^{1/2} (\text{Cov}_{ij}^{\text{data}} - \text{Cov}_{ij}^{\text{sim}})^2},$$

where $w_i$ and $w_j$ are the weights for a given element. The covariance matrix $\text{Cov}_{ij}$ for the data in M67 and for the two simulated data sets from Figure 2 are displayed in Figure 5. It is clear that $|\Delta\text{Cov}_{ij}|$ is a good sufficient statistic, especially for elements with many absorption features, because any abundance scatter in a given element will give rise to correlated residuals at the positions of absorption features of that element. For example, in Figure 5, the simulated data with $\sigma_{\text{Al/H}} = 0.1$ dex has both large scatter at the positions of the Al lines, and large off-diagonal correlations between pixels in different lines. The fact that these are absent for the data puts a strong constraint on the Al scatter in M67.

As discussed in Section 2, variations in the LSF for different stars can arise to scatter in the residuals. While the mock-data simulations take any LSF variations for APOGEE into account, neither of the two summary statistics corrects for the effect of LSF variations. Doing so would require a metric function $\rho(D', D)$ that distinguishes between stars with different LSFs. Alternatively, a procedure to homogenize the LSF could be applied to both the data and the simulated data (in its crudest form, this would consist of convolving all data to the worst LSF). The effect of LSF variations can be seen in the zero-scatter simulation in the middle panel of Figure 5 as the regions of negative correlation surrounding the line centers and the positive correlation between the central pixels of the two Al lines shown. These features are absent in simulations using the same LSF for all stars.

The distribution of these two summary statistics in the ABC simulations for M67 are shown in Figure 6 for Al and Fe. It is clear that $|\Delta\text{Cov}_{ij}|$ is a good summary statistic, especially at
large intrinsic scatter, as it strongly correlates with $\sigma_{[X/H]}$. The statistic $D_n$ distinguishes between different $\sigma_{[X/H]}$ to a lesser extent, but is important for identifying those simulations that are most like the data, as $D_n$ captures some of the information in the shape of the distribution of residuals that is not captured by $|\Delta \text{Cov}_{ij}|$.

To construct the PDF for $\sigma_{[X/H]}$ for each element $X$ and each cluster, I then run ABC simulations as described above. A flowchart of how these simulations are run is given in Figure 7. Before the start of each simulation, I compute a fine grid in $[X/H]$ from $-0.20$ dex to $+0.20$ dex with respect to the standard cluster abundances (median [Fe/H] of all cluster members with solar abundance ratios) with a spacing of $0.01$ dex; this grid is computed for each star individually. Subsequent simulations that require $[X/H]$ within the grid use linear interpolation to generate the spectra; spectra for $[X/H]$ outside of the grid are computed on-the-fly and added to the grid; interpolation is only ever performed for $[X/H]$ located between grid points within $0.01$ dex from the nearest grid points. Each simulation then proceeds by drawing a $\sigma_{[X/H]}$ from the prior (uniformly between 0 and 0.1 dex), generating simulated spectra, performing the quadratic $T_{\text{eff}}$ fit for each pixel, and computing the two summary statistics $D_n$ and $|\Delta \text{Cov}_{ij}|$.

Good limits on $D_n$ and $|\Delta \text{Cov}_{ij}|$ are determined by inspecting their distribution by eye and I run ABC simulations until the distributions appear to have converged (that is, small changes in the limits on $D_n$ and $|\Delta \text{Cov}_{ij}|$ stop mattering) and until about 1000 $\sigma_{[X/H]}$ samples from the PDF have accrued. The limits on $D_n$ and $|\Delta \text{Cov}_{ij}|$ for Al and Fe in M67 are displayed as dashed lines in Figure 6.

I have performed tests of the code verifying that no constraints on the abundance scatter are possible in the following limiting cases: (a) When only using three stars in a cluster, because then the quadratic fit is always perfect; and (b) when calculating the $D_n$ and $|\Delta \text{Cov}_{ij}|$ summary statistics using the weights of a different element than the one whose intrinsic scatter is being constrained (choosing two elements with no overlapping weights, like Al and C or Ca and Mn), because then the statistics are not sensitive to abundance variations. All such tests passed.

The cumulative distributions of the PDF for each element and each cluster are displayed in Figure 8. The 68% and 95% upper limits for each individual cluster are given in Table 2. The cumulative PDFs in Figure 8 demonstrate that we obtain strong limits on the abundance scatter, especially in M67 and NGC 6819, where we have the most cluster members. All elements are consistent with having no scatter; only for Ca in M67 and Al and Mn in NGC 6819 do the cumulative PDFs have a mild preference for a scatter of $\approx 0.03$ dex, but with a tail toward zero scatter. Multiple elements for NGC 2420 have an almost flat PDF, especially O, Na, and K; this is simply due to the fact that these features are very weak in this more metal-poor cluster and that only a single low-$T_{\text{eff}}$ star is included (for which we can get a precise O abundance).

From the results in Table 2, we see that for M67 and NGC 6819 we obtain strong constraints on the scatter in C, N, O, Mg, Al, Si, Mn, Fe, and Ni. We obtain weaker constraints for Na, S, K, Ca, Ti, and V and also obtain weaker results overall for NGC 2420. Given that I find no evidence for any abundance scatter in any of the clusters, it is reasonable to combine the PDFs into joint constraints on the abundance scatter (assuming all clusters have the same intrinsic scatter and placing a limit on this). The PDF obtained from combining the PDFs of all three clusters and the 95% upper limits on $\sigma_{[X/H]}$ from this combined PDF are indicated in Figure 8; the combined constraints are also included in Table 2. We see that the combined constraints are strong. With the exception of Ti, all elements have a scatter constrained to be less than 0.05 dex at 95% confidence.

The constraints on Fe and C are particularly strong: any intrinsic scatter has to be $< 0.007$ dex and $< 0.009$ dex in Fe and C, respectively, at 68% confidence ($< 0.013$ dex and $< 0.019$ dex at 95% confidence). We also obtain strong constraints on the intrinsic scatter in N and O, which has to be $< 0.013$ dex and $< 0.010$ dex, respectively, at 68% confidence ($< 0.022$ dex and $< 0.025$ dex at 95% confidence). That I find the strongest limits on C, N, O, and Fe is not surprising, because these elements have by far the most abundant absorption features in the near-infrared wavelength region used here, but the fact that I have been able to extract these constraints from the complicated molecular features for C, N, and O and in the light of deep mixing along the giant branch, demonstrates the power of the method developed here.
I also obtain strong limits on the scatter in Mg, Si, and Ni; these are all roughly \( \lesssim 0.015 \) dex and \( \lesssim 0.03 \) dex at 68% and 95% confidence, respectively. The limits on the scatter in Al, Ca, and Mn are less strong, but are nevertheless \( \lesssim 0.02 \) dex and \( \lesssim 0.03 \) dex at 68% and 95% confidence. The weaker features of Na, S, K, Ti, and V give weaker limits that are about 0.025 to 0.03 dex at 68% and about 0.05 dex at 95% confidence.

6. DISCUSSION AND CONCLUSIONS

6.1. Prospects for Chemical Tagging

The novel technique introduced here for constraining the abundance scatter in open clusters has several advantages over traditional techniques that determine each individual star’s abundances and constrain the scatter in these abundances. First, the new technique is robust to systematic uncertainties in the abundances stemming from the fact that obtaining consistent abundances over wide ranges of stellar types is difficult. The systematics introduced by, e.g., deviations from the assumptions of local thermodynamic equilibrium, one-dimensional radiative transfer, and uncertainties in the line list will cause offsets in the abundances that are smooth functions of stellar mass. By remaining agnostic about the overall trend of the spectra of the cluster members analyzed here, I avoid all of these issues directly. Second, many of the stellar-evolutionary effects on the surface abundances due to, e.g., deep mixing or gravitational settling, also change the abundances in a manner that is perhaps not well understood, but that is largely a deterministic function of the stellar mass. Therefore, I was able to constrain the initial abundance scatter rather than the present-day scatter and in particular constrain the scatter in C and N. Third, the use of forward simulations and ABC makes it straightforward to include a large variety of real-world complications in the observed spectra such as non-Gaussian and variable LSFs, correlated noise in the spectra, and uncertainties coming from the applied continuum normalization.
It is easy to think of factors that should lead to a break-down of the one-dimensional assumption that I propose here for clusters. The large spread in initial rotation velocities will give rise to differences in the spectra of young-cluster members that are largely orthogonal to those from differences in the initial mass. While the initial differences in rotation speeds will have largely disappeared due to magnetic braking for older open clusters (Kawaler 1988), the effect of the different viewing angles would still give rise to scatter in the spectra and for the technique used in this paper to work, the $v \sin i$ of each star probably needs to be inferred prior to the forward simulations. For the giants that I studied here, rotational velocities are small and I was therefore able to ignore this complication. Beyond the effects of the current velocity, initial velocity differences may still lead to spectral scatter today if they led to different mixing histories, changing the surface abundances of, e.g., C, N, and Li (Pinsonneault et al. 1990; Meynet & Maeder 2002). Effects of binarity and, in particular, of mass transfer between

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**Figure 8.** Cumulative posterior distribution functions for the intrinsic abundance scatter in 15 elements obtained from the ABC simulations. The color-coding of the three clusters is the same as in Figure 4: M67 is in red, NGC 6819 is in yellow, and NGC 2420 is in blue. The black curve gives the cumulative distribution function from the combination of all three clusters; the 95% upper limit from this combined PDF is given in each panel. The 68% and 95% upper limits for all three clusters and the combined PDF are given in Table 2.
Limits on Intrinsic Abundance Scatter

|        | M67     | NGC 6819 | NGC 2420 | Combined |
|--------|---------|----------|----------|----------|
| 68%    | 95%     | 68%      | 95%      | 68%      | 95%      | 68%      | 95%      |
| C      | 0.016   | 0.030    | 0.013    | 0.025    | 0.027    | 0.058    | 0.009    | 0.019    |
| N      | 0.015   | 0.031    | 0.023    | 0.043    | 0.034    | 0.069    | 0.013    | 0.022    |
| O      | 0.022   | 0.041    | 0.017    | 0.039    | 0.055    | 0.088    | 0.010    | 0.025    |
| Na     | 0.035   | 0.069    | 0.037    | 0.070    | 0.065    | 0.094    | 0.025    | 0.049    |
| Mg     | 0.019   | 0.036    | 0.034    | 0.059    | 0.021    | 0.045    | 0.014    | 0.027    |
| Al     | 0.019   | 0.035    | 0.045    | 0.069    | 0.020    | 0.041    | 0.018    | 0.030    |
| Si     | 0.015   | 0.030    | 0.026    | 0.048    | 0.045    | 0.077    | 0.014    | 0.027    |
| S      | 0.028   | 0.058    | 0.052    | 0.085    | 0.050    | 0.084    | 0.024    | 0.046    |
| K      | 0.030   | 0.053    | 0.030    | 0.059    | 0.061    | 0.091    | 0.025    | 0.043    |
| Ca     | 0.033   | 0.049    | 0.026    | 0.047    | 0.020    | 0.042    | 0.019    | 0.029    |
| Ti     | 0.051   | 0.083    | 0.039    | 0.070    | 0.046    | 0.084    | 0.031    | 0.054    |
| V      | 0.038   | 0.066    | 0.030    | 0.061    | 0.053    | 0.089    | 0.022    | 0.041    |
| Mn     | 0.019   | 0.034    | 0.038    | 0.059    | 0.030    | 0.063    | 0.021    | 0.031    |
| Fe     | 0.010   | 0.019    | 0.012    | 0.025    | 0.024    | 0.048    | 0.007    | 0.013    |
| Ni     | 0.022   | 0.045    | 0.025    | 0.049    | 0.035    | 0.068    | 0.014    | 0.028    |

Note. The 68% and 95% upper limits on the intrinsic abundance scatter in 15 elements. Those obtained from each individual cluster are given as well as those from combining all clusters.

The tight constraints on the initial abundance scatter in open clusters place strong limits on how star formation in molecular clouds—the progenitors of open clusters—proceeds. This is especially the case because of the strong limits on the scatter in C, N, and O. C and O are produced in large quantities in core-collapse supernovae (CCSNe). Using the yields at solar metallicity from Chieffi & Limongi (2004) and Limongi & Chieffi (2006), we find that \( \approx 0.9 M_\odot \) and \( 3.5 M_\odot \) of C is produced in a single CCSN of a \( 35 M_\odot \) and a \( 60 M_\odot \) star, respectively; for O the yield is even higher: \( \approx 5 M_\odot \) and \( 8.5 M_\odot \) for the same two masses. Assuming that this amount of C or O is mixed in with \( \approx 20,000 M_\odot \) of gas with solar abundance ratios from Asplund et al. (2005), subsequent stars would have C abundances higher by \( \approx 0.02 \) dex and 0.04 dex and O abundances higher by \( \approx 0.02 \) dex and 0.03 dex for a single CCSN of a \( 35 M_\odot \) and \( 60 M_\odot \) star. For comparison, the amount of Fe produced in these CCSNe is only about 0.1–0.2 \( M_\odot \) and raises the Fe abundances of new stars by \( \approx 0.004 \) dex.

The fact that the initial scatter in C and O is constrained to be \( \lesssim 0.025 \) dex at 95% confidence implies that no pollution by massive CCSNe occurred before most of the stars formed. The initial masses of M67 and NGC 6819 were likely in the range 10,000 \( M_\odot \) to 20,000 \( M_\odot \) (Hurley et al. 2005; Yang et al. 2013). Using the IMF from either Kroupa (2001) or Chabrier (2003), we would expect \( \approx 11 \) and 5 stars with masses greater than \( 35 M_\odot \) and \( 60 M_\odot \), respectively, in a cluster with a mass of 20,000 \( M_\odot \). Thus, we would expect CCSNe of \( 35 M_\odot \) to \( 60 M_\odot \) stars to occur in M67 and NGC 6819 and potentially even higher mass CCSNe, which would lead to even larger abundance scatter. For star formation lasting for a time \( \tau_{\text{SF}} \), the lack of a CCSN when \( K \) massive stars with lifetimes \( \tau_{\text{CCSN}} \) are expected to form gives the following PDF for \( \tau_{\text{SF}} \) (assuming a flat prior):

\[
p(\tau_{\text{SF}}|\text{no CCSN}) \propto \begin{cases} 1 & \text{if } \tau_{\text{SF}} < \tau_{\text{CCSN}} \\ \frac{\tau_{\text{CCSN}}}{\tau_{\text{SF}}}^K & \text{otherwise.} \end{cases}
\]

This converges to a flat distribution between zero and \( \tau_{\text{CCSN}} \) for large \( K \), because of the increasing probability that a massive star is formed at the onset of star formation and because we cannot distinguish \( \tau_{\text{SF}} \) that are smaller than \( \tau_{\text{CCSN}} \). For 11 expected \( 35 M_\odot \) stars (with lifetimes \( \tau_{\text{CCSN}} \approx 5.7 \) Myr) as well as for 5 expected \( 60 M_\odot \) stars (\( \tau_{\text{CCSN}} \approx 4 \) Myr; Bressan et al. 2012), this gives an upper limit on \( \tau_{\text{SF}} \) of \( \approx 0.6 \) Myr at 95% confidence. This limit would obviously weaken if massive stars preferentially form after low-mass stars—although this is not expected to be the case (McKee & Tan 2002) and they may even form the earliest (Maschberger et al. 2010)—or if a significant portion of the SNe ejecta are introduced into a warm environment.
The fact that I find no scatter in the abundances of light elements also directly demonstrates that the type of pollution that occurs in globular clusters does not happen for open clusters. Globular clusters display significant abundance scatter and anti-correlations in the abundances of light elements (C, N, O, Na, Mg, Al; Gratton et al. 2004), believed to stem from pollution of the intracluster medium by intermediate-mass asymptotic giant branch stars (e.g., Ventura et al. 2001), fast-rotating massive stars (Decressin et al. 2007), or massive binaries (de Mink et al. 2009). The abundance scatter in all of the light elements commonly studied in globular clusters is \(<0.03\) dex at 95% confidence in the open clusters analyzed here (except for Na, for which the limit is slightly weaker).

That the initial abundance scatter in open clusters is as small as 0.01 to 0.02 dex as found here challenges our understanding of the structure of molecular clouds. To attain this level of homogeneity, the gas and dust in star-forming clouds has to be very well mixed (Feng & Krumholz 2014; Hopkins & Lee 2015) and, as argued above, star formation has to proceed within about 6 Myr. This is an important new constraint on the timescale of star formation in molecular clouds (e.g., Elmegreen 2000; Tan et al. 2006; Matzner 2007). The timescale constraint derived here is limited not by the constraint on the abundance spread, but instead by whether a CCSN of a massive star is likely to have occurred and to have polluted the star-forming gas. Therefore, the kind of limits derived here will not be able to be improved much further.

6.3. Final Remarks

For many of the questions relating to the formation and evolution of star clusters and galactic disks that we may answer using detailed measurements of stellar abundances, the precision in the abundances is of much higher importance than their overall accuracy. However, much of the modeling effort currently going into the analysis of large spectroscopic surveys is focused on improving the theoretical modeling of stellar photospheres (e.g., Magic et al. 2013) or line formation beyond the simplest models (e.g., Bergemann et al. 2012). While more realistic modeling of the stellar photospheres, radiative transfer, and line formation will provide a significant improvement for any spectroscopic analysis, it is unlikely that all systematic effects in the abundances will be removed through these efforts in the near future. This is especially the case for the effects of, e.g., deep mixing or atomic diffusion that actually change the surface abundances in a manner that is not entirely well understood (e.g., Onehag et al. 2014).

The method introduced here is wholly focused on obtaining the highest possible abundance precision given the observational limitations. It does this at the expense of some of the information in the spectra, which instead of being used to constrain the abundance scatter, is used to build an empirical model of the spectra. That the new technique leads to some of the tightest constraints on the intrinsic abundance scatter in open clusters and that it does this based on the complex infrared APOGEE spectra of giants that likely have intrinsic variations in C and N, is a testament to the strength of this new technique. I expect that extensions of this technique to other groupings of stars and to the whole Galactic disk population will lead to fundamentally new insights into the formation and evolution of stellar populations in the Milky Way.

It is with great pleasure that I thank the APOGEE ASPCAP team for many valuable discussions regarding infrared spectroscopy and APOGEE. In particular, I thank Carlos Allende Prieto, Katia Cunha, Anibal García-Herranz, Jon Holtzman, Szabolcs Mészáros, Matthew Shetrone, and Olga Zamora for help with the APOGEE ATLAS9 model atmospheres, Turbospectrum and MOOG, the APOGEE line list, the APOGEE LSF, and the APOGEE pipeline. I further acknowledge insightful conversations with and comments from the anonymous referee, Ewan Cameron, David Hogg, Chris McKee, Melissa Ness, Garrett Somers, and Yuan-Sen Ting. I also thank the Natural Sciences and Engineering Research Council of Canada for financial support of this project.

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APPENDIX A

HIGH SIGNAL-TO-NOISE RATIO MEAN H-BAND SPECTRA FOR M67, NGC 6819, AND NGC 2420

In this appendix, I display the combined cluster spectra for M67, NGC 6819, and NGC 2420. These are obtained from the quadratic fits to each pixel using the modeling of Section 2. Because the spectra of members of these clusters are all consistent with being a function of a single variable, we can use the quadratic fit to their \(T_{\text{eff}}\) dependence to construct a very high signal-to-noise-ratio spectrum for each cluster. I compute the cluster spectrum at \(T_{\text{eff}} = 4750\) K, which is close to the median \(T_{\text{eff}}\) of all of the considered cluster members. To avoid any confusion due to the inclusion of red-clump stars in M67 and NGC 6819, I only include first-ascent red giants in the quadratic fit and the cluster spectrum is therefore that of a red giant in these clusters at \(T_{\text{eff}} = 4750\) K.

The cluster spectra are shown at high resolution in Figures 9 and 10. The three spectra are clearly very similar, especially those for M67 and NGC 6819 which are close in age and metallicity. Almost every single wiggle in the spectra is repeated in all three spectra, demonstrating the extremely high signal-to-noise ratio of these combined spectra. They could be used to obtain precise abundances for these clusters, but this is not done here as it is beyond the scope of this paper.

APPENDIX B

EMPIRICAL INVESTIGATION OF THE APOGEE SPECTRAL ERRORS USING REPEAT OBSERVATIONS

As discussed in Section 3, the APOGEE spectra have associated pixel-level uncertainties in the standard DR12 data products. These are obtained from a noise model that tracks the Poisson photon-counting noise through the APOGEE pipeline. These uncertainties are for single pixels only; correlations between the errors of neighboring pixels are not tracked. Nidever et al. (2015) tested these uncertainties using the scatter in the repeat observations of stars with six individual hour-long exposures, demonstrating that the uncertainties overall track the scatter well, but also finding a systematic error floor at the 0.5% level (see discussion in Section 3). This test did not distinguish
between pixels at different wavelengths and regions with significant sky-emission or telluric-absorption lines were avoided.

I perform a similar, but more detailed, test here and furthermore determine an empirical model for the spectral errors using repeat observations. I select giants from the APOGEE DR12 data set with $T_{\text{eff}} \geq 4500$ K and $\log g < 3.5$. Of these giants, I consider those with $10 < H < 11$ with three hour-long exposures that on average have an overall signal-to-noise ratio per half-resolution element larger than 100 (thus, their combined spectrum has signal-to-noise ratio larger than 300). These characteristics are similar to the majority of the open-cluster members considered in this paper. This sample consists of 1381 stars with 3 repeat observations each for a total of 4143 individual spectra. All of the individual spectra have an overall signal-to-noise ratio larger than 80.

I then continuum-normalize each individual-exposure spectrum as well as the combined APOGEE spectrum for each of these stars in the manner described in Section 3 and compute

![Figure 9. Combined continuum-normalized spectra between $\lambda \approx 15,150$ and 16,150 Å for a first-ascent red giant at $T_{\text{eff}} = 4750$ K in M67, NGC 6819, and NGC 2420. These are determined from the quadratic fit to the $T_{\text{eff}}$ dependence of each pixel using the red-giant members in each cluster (excluding red-clump stars). Strong, clean atomic lines from the compilation of Smith et al. (2013) for most of the elements considered in this paper (and some others) are indicated. The spectra for M67 and NGC 2420 are offset by $-0.3$ and $0.3$, respectively. Almost every feature, including weak ones, are reproduced in all three spectra, demonstrating that these spectra contain very little noise.](image)

Figure 9. Combined continuum-normalized spectra between $\lambda \approx 15,150$ and 16,150 Å for a first-ascent red giant at $T_{\text{eff}} = 4750$ K in M67, NGC 6819, and NGC 2420. These are determined from the quadratic fit to the $T_{\text{eff}}$ dependence of each pixel using the red-giant members in each cluster (excluding red-clump stars). Strong, clean atomic lines from the compilation of Smith et al. (2013) for most of the elements considered in this paper (and some others) are indicated. The spectra for M67 and NGC 2420 are offset by $-0.3$ and $0.3$, respectively. Almost every feature, including weak ones, are reproduced in all three spectra, demonstrating that these spectra contain very little noise.
the normalized residuals

$$\frac{\Delta f_i^j}{\delta \lambda} = \frac{f_i^j - f_i^{combined}}{\delta \lambda}$$  \hspace{1cm} (8)$$

for each star $i$, where $\delta \lambda$ is the pipeline uncertainty for each pixel $\lambda$. Pixels with signal-to-noise ratio less than 50 or with any of the bad pixel flags discussed in Section 3 are removed from further consideration.

The median of the normalized residuals $\Delta f_i^j / \delta \lambda$ is displayed in Figure 11 and it measures the bias in the spectra taking into account the effects of continuum normalization. It is clear that the bias is small for all wavelengths, although minor polynomial trends especially in the green detector (middle panel) and the red detector (bottom panel) remain; these are due to the polynomial continuum fitting not being quite reproducible between different observations of the same star. I have also performed the same test using the standard APOGEE continuum-normalization method (García Pérez et al. 2015) and found significantly larger biases (up to about 0.5) that could negatively affect parameter and abundance determinations from these spectra. Similarly, I found larger biases when using only a second-order polynomial as in Ness et al. (2015).

The standard deviation of the normalized residuals is also shown in Figure 11 and is typically about 1.1 to 1.2, but with large wavelength ranges where the standard deviation is larger than 1.5. Because the three individual exposures are compared to their combined value, the distribution of the sum of the

Figure 10. Same as Figure 9, but for the wavelength range $\lambda \approx 16,150$–$16,950$ Å.
squares of the normalized residuals should follow a $\chi^2$ distribution with two degrees of freedom if the pipeline uncertainties are correct and the standard deviation of all residuals should be approximately $2/3$. Therefore, the fact that the standard deviation is larger than $2/3$ demonstrates that the pipeline uncertainties are underestimated. Figure 11 also contains the median sky and telluric spectra that were used to correct the individual exposures used here. As expected because I remove any pixels near sky lines, the location of sky lines does not appear to be correlated with large values of the standard deviation of the residuals. However, these large values do appear to coincide with regions with significant telluric absorption. It is therefore likely that the underestimated uncertainties are due to issues with the telluric correction.

We can use the same normalized residuals to investigate the correlations between the errors of neighboring pixels. Figure 12 displays the correlation for five pixels chosen to represent a range of detectors and of regions with low and high scatter in the normalized residuals. This figure clearly demonstrates that there typically are significant correlations out to dozens of pixel offsets, corresponding to $\gtrsim 10\,\text{Å}$. This is much wider than the wavelength region over which the line spread function is significant and these correlated errors are most likely due to correlations induced by the continuum normalization, although they may also have some contribution from scattered light.

When relating the differences in the spectra of stars to those expected from scatter in the abundances, it is essential to have a good understanding of the random errors and their correlations that affect these differences. Rather than using the repeat observations to build an empirical noise model incorporating the correlations between pixels, I directly use the normalized residuals determined from the repeat observations as an empirical sampling of the noise. This has the advantage of being incredibly straightforward. When looking at the residuals of the spectra in a given cluster from the one-dimensional model using the method of Section 2 as applied in Section 4, I simply compare to the distribution determined from the repeat observations. Similarly, in the forward simulations described in Section 5 and in Appendix C, I simply draw from the set of normalized residuals in the process of determining a mock error. However, because the residuals come from a comparison between an individual exposure and a combined spectrum that contains information from this individual exposure, the residuals are slightly smaller than the true error (because the

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**Figure 11.** The median (bias; black line near zero) and standard deviation (black line above one) of the normalized residuals of individual exposures vs. combined continuum-normalized spectra of 1381 stars with three exposures that individually have high signal-to-noise ratio. Each panel displays one of the three APOGEE detectors, which span three different wavelength ranges. The median is near zero, demonstrating that the continuum-normalized spectra are unbiased, with only minor effects due to the polynomial continuum-normalization. The standard deviation should be $<1$ if the reported APOGEE spectral uncertainties were correct (see text), but is typically 1.1 to 1.2, with large wavelength regions where it is $>1.5$. The blue and red lines display the average sky and telluric spectra used to correct the individual spectra for the effect of sky-emission and telluric-absorption lines. Regions of high scatter in the residuals appear to largely coincide with those with significant telluric absorption.

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three residuals for a given star only have two degrees of freedom). For the purposes of this paper, this is a conservative mistake, because it means that we are slightly underestimating the uncertainties in the spectra. This will slightly inflate any limit on the abundance scatter of the open clusters studied here. This could be fixed in the future by using the residuals determined from repeat observations to infer a noise model or it could be ameliorated by increasing the number of repeat exposures to \( \gg 3 \). Another problem is that if the underestimation of the pipeline uncertainties is truly due to the telluric-absorption correction, a model for the uncertainties in the spectra. This will slightly in

\[ \lambda \approx 15490 \, \AA \]
\[ \lambda \approx 15725 \, \AA \]
\[ \lambda \approx 16025 \, \AA \]
\[ \lambda \approx 16660 \, \AA \]
\[ \lambda \approx 16825 \, \AA \]

**Figure 12.** Correlations between the errors in neighboring pixels determined from repeat observations (see Figure 11) for five different central pixels. Significant correlations exist out to dozens of pixel offsets, both in regions of high and low scatter in the repeats (see Figure 11). This is much wider than the line-spread function and is most likely due to the continuum normalization.

APPENDIX C
SYNTHETIC APOGEE SPECTRA

In Sections 4 and 5, I employ synthetic APOGEE spectra varying the abundances of 15 elements with absorption lines in the APOGEE wavelength range. This appendix explains how I generate these synthetic spectra. For each individual star, I generate a model atmosphere at the median metallicity of the cluster and at the \((T_{\text{eff}}, \log g)\) of the star, using solar abundance ratios for all elements. The model atmosphere is obtained using linear interpolation of the grid of atmospheres computed using the ATLAS9 code by Mészáros et al. (2012). These atmospheres and the synthetic spectra computed using them all use the solar abundances from Asplund et al. (2005). I compute synthetic spectra varying the abundances of individual elements using Turbospectrum (Alvarez & Plez 1998), adopting the microturbulence prescription as a function of \( \log g \) used in APOGEE DR12, an isotopic ratio \( ^{12}C/^{13}C = 15 \) appropriate for giants, and using a Gaussian macroturbulence with a full-width-at-half-maximum of \( 6 \, \text{km} \, \text{s}^{-1} \). This macroturbulence is at the high end of what is expected for the giants in this sample, but oversmoothing the spectrum is conservative in that it would weaken any result on abundance variations from spectral scatter. I employ the same line list as used in APOGEE’s DR12 (Shetrone et al. 2015), with astrophysical \( gfs \) determined by fitting the \( H \)-band spectra of the Sun and Arcturus, but fitting to the center-of-disk solar flux (M. Shetrone, private communication), i.e., with the total-flux versus center-of-disk bug in the APOGEE DR12 line list fixed (see Shetrone et al. 2015). Synthetic spectra are computed in air wavelengths over the wavelength range 15,000 to 17,000 \( \AA \) with a wavelength step of 0.05 \( \AA \).

Each of the 300 APOGEE fibers has a different LSF. Variations in the width of the LSF between different fibers are typically 10 to 20%, but the LSF of individual fibers are stable at the 1% level (Nidever et al. 2015). The APOGEE LSF is non-Gaussian and detailed forward modeling of the spectra needs to take the non-Gaussian, variable nature of the LSF into account. I compute the LSF of each fiber using the Gauss-Hermite-expansion fit to the LSF of each fiber (which additionally includes a wide Gaussian for the wings of the LSF) and the wavelength calibrations for all three detectors, which are publicly available (Nidever et al. 2015). For each cluster star I average the LSFs of the fibers used for the hour-long individual exposures. The raw synthetic spectra from Turbospectrum are interpolated onto a wavelength grid in vacuum wavelengths using the transformations from Ciddor (1996). They are then convolved with the LSF and brought onto the same wavelength grid as the observed spectra. These spectra are then continuum-normalized using the procedure described in Section 3.

To add errors to the continuum-normalized synthetic spectra, I draw from the set of 4143 normalized residuals from repeat observations (see Appendix B). This normalized error is then “de-normalized” by multiplying it with the uncertainty array of the observed spectrum and the result is added to the synthetic spectrum. Any bad pixels in the observed spectrum of a cluster star are also labeled as bad in the synthetic spectra for that star. In drawing from the normalized residuals, only those residuals with a smaller number of bad pixels than the observed spectrum are used. In generating synthetic spectra for changes in different elements, this number of bad pixels is computed by weighting with the sensitivity weights for each element that are discussed in Appendix D. That is, only bad pixels in the spectral regions with absorption features for the given element are taken into account. This procedure creates synthetic spectra that are very similar to the observed spectra in LSF, errors, and distribution of bad pixels.

All of the code to generate these synthetic spectra is available online as part of a general-use APOGEE data-analysis Python package called apogee, available at http://github.com/jobovy/apogee. This package allows one to download and open the necessary data files containing the APOGEE catalog and spectra. Tools for reading the ATLAS9 APOGEE model atmosphere grid\(^2\), interpolating within the grid using linear interpolation, and outputting the model atmospheres in a format suitable for MOOG (Sneden 1973) or Turbospectrum are included in apogee.modelatm. Synthetic spectra can be calculated using MOOG or Turbospectrum with a similar Python interface using functions in apogee.modelspec.

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\(^2\) Available here: http://www.iac.es/proyecto/ATLAS-APOGEE/
moog and apogee.modelspec.turbospec. These include functions to simply compute the high-resolution theoretical spectra or to generate full synthetic spectra including LSF and macroturbulence convolution, re-sampling onto the observed wavelength grid, and continuum normalization.

Tools for computing the LSF and convolving with it efficiently using sparse-matrix algebra are contained in apogee.spec.lsf. Continuum-normalization using the standard APOGEE method (García Pérez et al. 2015) or the method of Ness et al. (2015) (as used in this paper) are implemented in apogee.spec.cont. Various tools for handling the sensitivity windows described in Appendix D are included in apogee.spec.windows. The code to fit a linear or quadratic model in stellar labels ($T_{\text{eff}}$ in this paper, see Section 2 and Ness et al. 2015) is included in apogee.spec.cannon. Aside from these tools used in the analysis in this paper, the apogee package also contains a full implementation of the standard APOGEE stellar-parameters and elemental-abundances pipeline using FERRE.3 This is included in apogee.modelspec.ferre, which allows for interpolation of model spectra from the standard APOGEE grids (Zamora et al. 2015) and for performing the APOGEE stellar-parameter and abundance fits for any APOGEE spectrum.

APPENDIX D
SENSITIVITY OF APOGEE SPECTRA TO ABUNDANCE VARIATIONS OF DIFFERENT ELEMENTS

APOGEE’s abundance pipeline uses a set of numbers as a function of wavelength for each element to weight the contribution of different pixels to the $\Delta \chi^2$ when fitting for the abundance of that element. These numbers give high weight to pixels that are highly sensitive to the abundance of the element in question and not that sensitive to the abundance of other elements. They are computed from the derivatives of model spectra at $T_{\text{eff}} = 4000$ K, log $g = 1$, and overall metallicity of $-2.0$, $-1.0$, and $0.0$ and they also take into account how well a model for the spectrum of Arcturus reproduces the high resolution, high signal-to-noise ratio observed spectrum of Arcturus of Hinkle et al. (1995), how well the whole APOGEE sample is fit at each pixel, and how well the stars analyzed in detail by Smith et al. (2013) are modeled. Full details on this procedure are given in García Pérez et al. (2015).

In this paper, I make use of these weights (also referred to as “windows”) to analyze the spectral variations induced by abundance changes of different elements. That is, when determining, for example, the impact of Al variations, the APOGEE weights are used to only consider wavelengths that are sensitive to the Al abundance. This appendix describes some further analysis of the sensitivity of the standard APOGEE DR12 windows to changes in the abundances of different elements. The main purposes of this analysis are to specifically focus on abundance changes near solar metallicity and to create a subset of the standard windows that is less sensitive to the abundances of other elements, in particular C and N.

To do this, I compute a baseline synthetic APOGEE spectrum using the procedure described in Appendix C for a star with $T_{\text{eff}} = 4750$ K, log $g = 2.5$, solar abundances, micro-turbulence of $2$ km s$^{-1}$, and convolving with the average LSF of all APOGEE fibers. I then compute a set of spectra that vary the abundances of all 15 elements considered in this paper separately by ±0.1 dex. The variations around the baseline spectrum are displayed for a few example elements in Figure 13. I then compute the rms deviation for this ±0.1 dex change in all elements for each element’s windows, weighting by the APOGEE weights. For example, for Al I use the Al weights and then compute the rms variation in the spectrum for all 15 elements. This then returns the effect of each individual element’s abundance changes on the spectrum near the Al absorption features. In Figure 13, the element whose windows we are interested in is always displayed in blue and other elements are ranked by the relative contribution to the spectral scatter near the absorption features of that element.

Figure 13 demonstrates that the standard APOGEE weights for a given element (e.g., Mg) include many wavelength ranges with significant contributions from other elements (e.g., the two reddest Mg windows). To clean the list of windows for each element, I compute the variation induced by other elements for each individual window for that element (roughly, an individual absorption feature, but for this can be quite extended for the molecular features). For example, for each of the two Al windows in Figure 13, I compute the variation induced by Al and all other elements. I then remove any window that produces less than a 0.01 change in the continuum-normalized spectrum when varying the abundance of that window’s element by ±0.1 dex or if any of the other elements induce a variation greater than 34% of that induced by the window’s corresponding element. For example, if the variation due to Al in the first Al window is 0.005, the window would be removed (this is not the case). Or if another element, say Mg, creates a spectral variation larger than 34% of that of Al in this window, the window would be removed (also not the case). Many of the elements require some fine tuning of these cuts to not remove too many individual windows: K is kept, because there is only a single line in the spectrum, other elements are allowed to contribute up to 100% or 200% for C and N, respectively (basically, because of the ubiquitous CN features), O is kept down to changes as small as 0.005 and up to contributions of other elements of 500% (because most O features are weak). Additionally, for Na, Ti, V, Mn, and Ni I only consider the contribution of other elements and use a cut-off of 34%, 30%, 40%, 25%, and 50%, respectively. While these cuts are somewhat arbitrary, they have been chosen to keep a reasonable number of windows for each element that are not too affected by variations in other elements.

The weighted variations induced in the spectrum by ±0.1 dex abundance changes for the final set of weights are given in Table 3. Along the diagonal, this table gives the magnitude of spectral changes for each element weighted by the weights for that element. The off-diagonal entries show how much variation abundances changes in other elements induce, given as a percentage of the main element’s variation. This demonstrates that we end up with a relatively clean set of weights for each element. Most ±0.1 dex abundance changes induce weighted spectral variations of about 0.01 to 0.025, which is larger than the typical noise of each pixel. This weighted spectral variation, however, does not show the number of pixels at which such large variations exist. Elements

3 http://www.as.utexas.edu/~hebe/ferre/
Figure 13. Variations in the spectrum of a $T_{\text{eff}} = 4750$ K, $\log g = 2.5$ star with solar abundances induced by $\pm 0.1$ dex changes in the abundance of different elements. As examples, this figure shows those spectral variations in regions of the spectrum with C (top panel), Mg (bottom left panel), and Al (bottom right panel) absorption features; all variations are exaggerated by a factor of ten. The whole $H$-band wavelength range is broken into small regions and the separation in x-tickmarks is always 2 Å; the wavelength of a single tickmark in each region is indicated. Variations due to elements other than C in the top panels and other than Mg or Al in the bottom panels are ordered by their weighted rms variation, computed using the standard APOGEE weights for C, Mg, and Al, respectively. These weights are displayed as the gray lines. Only half of the features for C are displayed here. C has ubiquitous absorption features, but disentangling them from those of N and O is difficult. Some of the Mg APOGEE weights cover wavelength regions where Mg does not have absorption features (such as the two reddest regions) and I remove these from consideration here.

Table 3
Spectral Variations for 0.1 Dex Abundance Changes

| Window | C  | N  | O  | Na | Mg | Al | Si | S  | K  | Ca | Ti | V  | Mn | Fe | Ni |
|--------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
|        | 0.022 | 30 | 31 | 1  | 17 | 2  | 8  | 1  | 9  | 1  | 0  | 1  | 1  | 30 | 2  |
| N      | 113 | 0.013 | 65 | 1  | 23 | 4  | 12 | 1  | 0  | 2  | 0  | 0  | 0  | 23 | 2  |
| O      | 123 | 82 | 0.007 | 2  | 40 | 46 | 17 | 3  | 0  | 6  | 8  | 1  | 3  | 118 | 11 |
| Na     | 29 | 21 | 20 | 0.010 | 5  | 0  | 3  | 0  | 0  | 0  | 1  | 0  | 0  | 12 | 2  |
| Mg     | 9  | 5  | 3  | 0  | 0.030 | 1  | 5  | 0  | 0  | 0  | 0  | 1  | 1  | 34 | 0  |
| Al     | 16 | 11 | 7  | 0  | 0.025 | 7  | 0  | 0  | 1  | 0  | 0  | 0  | 14 | 1  |
| Si     | 12 | 7  | 6  | 0  | 12  | 1  | 0.023 | 0  | 0  | 1  | 2  | 0  | 0  | 17 | 1  |
| S      | 19 | 13 | 21 | 0  | 13  | 1  | 4  | 0.015 | 0  | 1  | 0  | 0  | 0  | 14 | 1  |
| K      | 58 | 45 | 36 | 1  | 12  | 1  | 6  | 0  | 0.026 | 1  | 0  | 0  | 2  | 12 | 1  |
| Ca     | 13 | 6  | 1  | 0  | 3  | 0  | 4  | 0  | 0  | 0.027 | 1  | 0  | 0  | 34 | 0  |
| Ti     | 7  | 9  | 12 | 0  | 3  | 0  | 2  | 0  | 0  | 0  | 0.015 | 0  | 0  | 0  | 12 | 1  |
| V      | 36 | 27 | 23 | 2  | 4  | 1  | 11 | 0  | 0  | 3  | 1  | 0.011 | 2  | 30 | 1  |
| Mn     | 3  | 4  | 23 | 0  | 5  | 1  | 3  | 0  | 0  | 0  | 0  | 0.029 | 5  | 2  | 2  |
| Fe     | 12 | 9  | 8  | 0  | 8  | 1  | 7  | 1  | 0  | 1  | 6  | 0  | 1  | 0.027 | 2  |
| Ni     | 22 | 7  | 8  | 0  | 10 | 1  | 6  | 1  | 0  | 1  | 0  | 0  | 46 | 0.019 | 1  |

Note. This table gives the changes in the spectrum of a $T_{\text{eff}} = 4750$ K, $\log g = 2.5$ star with solar abundances due to abundance changes of $\pm 0.1$ dex for the 15 elements considered in this paper. Each row weights the spectral changes due to each individual element using the weights for this row’s element that I derived from the standard APOGEE weights in Appendix D. Thus, the first line uses the weights for C to compute the weighted spectral variations due to different elements. The diagonal gives the actual weighted variation in the continuum-normalized spectrum, while the off-diagonal entries list the changes as a percentage of the diagonal entry. For example, N induces changes that are 34% of those induced by the same change in C when weighting using C weights. Using the set of weights from Appendix D, the weighted spectral regions for all elements except for C, N, and O have only minor contributions from other elements.
with a large number of pixels with non-zero weights will lead to stronger constraints on abundance variations.

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