Pulsar emission amplified and resolved by plasma lensing in an eclipsing binary

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Radio pulsars scintillate because their emission travels through the ionized interstellar medium along multiple paths, which interfere with each other. It has long been realized that, independent of their nature, the regions responsible for the scintillation could be used as ‘interstellar lenses’ to localize pulsar emission regions1,2. Most such lenses, however, resolve emission components only marginally, limiting results to statistical inferences and detections of small positional shifts3–5. As lenses situated close to their source offer better resolution, it should be easier to resolve emission regions of pulsars located in high-density environments such as supernova remnants6 or binaries in which the pulsar’s companion has an ionized outflow. Here we report observations of extreme plasma lensing in the ‘black widow’ pulsar, B1957+20, near the phase in its 9.2-hour orbit at which its emission is eclipsed by its companion’s outflow7–9. During the lensing events, the observed radio flux is enhanced by factors of up to 70–80 at specific frequencies. The strongest events clearly resolve the emission regions: they affect the narrow main pulse and parts of the wider interpulse differently. We show that the events arise naturally from density fluctuations in the outer regions of the outflow, and we infer a resolution of our lenses that is comparable to the pulsar’s radius, about 10 kilometres. Furthermore, the distinct frequency structures imparted by the lensing are reminiscent of what is observed for the repeating fast radio burst FRB 121102, providing observational support for the idea that this source is observed through, and thus at times strongly magnified by, plasma lenses10.

On 2014 June 13–16, we took 9.5 h of data of PSR B1957+20 with the 305-m William E. Gordon Telescope at the Arecibo observatory, at observing frequency of 311.25–359.25 MHz (see Methods). These data were previously searched for giant pulses11—sporadically occurring, extremely bright pulses, which are much shorter than regular pulses (around 1 μs compared with tens of microseconds). Although we found many giant pulses at all orbital phases, we noticed that the incidence rate of bright pulses was much higher leading up to and following the radio eclipse.

As can be seen in Fig. 1, most of the pulses near eclipse do not look like giant pulses but rather like brighter regular pulses—they are bright over a large fraction of the pulse profile, and most tellingly, occur in groups spanning several 16-ms pulse rotations, suggesting that the underlying events last for a time of the order of 10 ms. Their properties seem similar to the hitherto mysterious bright pulses associated with the eclipse of PSR J1748−2446A12, suggesting a shared physical mechanism.

High-magnification events are often chromatic, as can be seen from the colours in Fig. 1 (which reflect 16-MHz sub-bands) and is borne out more clearly by the spectra shown in Fig. 2: some show frequency widths comparable to our 48-MHz band, peaking at low or high frequency, whereas others show strong frequency evolution, in some cases tracing out a slope in frequency–time space, in others a double-peaked profile.

For many high-magnification events, the magnification is not uniform across the pulse profile. The bottom panel of Fig. 1 shows this strikingly: at 0.2 s, the main pulse is greatly magnified over five pulses, whereas the entire interpulse is barely affected. In addition, the components of the broad interpulse are often magnified differently from each other, as can be seen most readily in the magnified events in Fig. 3b, c. Thus, the events resolve the pulsar’s various emission regions.

The most strongly magnified events occur within three specific time spans, each lasting for about 5 min, around orbital phase φ = 0.20, φ = 0.30 and φ = 0.32 (see Fig. 1); here, φ = 0.25 corresponds to superior conjunction of the pulsar in its 9.2-h orbit7, and the duration of the eclipses at 350 MHz is about 40 to 60 min (refs. 7–9), or Δφ ≈ 0.1 (a sketch of the system geometry is shown in Extended Data Fig. 1). In each time span, we observe only small overall frequency-dependent time delays, of the order of 10 μs, which indicate modest increases in electron column density, with dispersion measures of the order of 10−4 pc cm−3 (see Fig. 1 as well as Extended Data Fig. 2). Intriguingly, at times when the delays are longer and the dispersion measures thus higher—immediately before and after eclipse, and in between the two post-eclipse periods of strong lensing, at φ ∼ 0.31—the magnifications are less marked, only up to a factor of a few, and correlated over much longer timescales, of the order of 100 ms. After the main lensing periods, up to φ ∼ 0.36, flux variations remain correlated, suggesting that even weaker events are still present (examples of lensing in the aforementioned regions are shown in Extended Data Fig. 3).

To determine whether the magnification events could be due to lensing by inhomogeneities in the companion’s outflow, we first measured the excess delay due to dispersion at a time resolution of 2 s, the shortest at which we can measure it reliably (see Methods). We find that during the periods of strong magnification events, the delay fluctuates by about 1 μs on this timescale. Assuming that the relative velocity between the pulsar and the companion’s outflow is roughly the orbital velocity, 360 km s−1 (ref. 13), the 2-s timescale corresponds to a spatial scale Δx ≈ 720 km. For this scale, the expected geometric delay is Δx/2αc ≈ 0.5 μs (where a = 6.4 light-seconds is the orbital separation and c the speed of light). As this is comparable to the observed dispersive delays, lensing is expected.

To model the magnified pulses, we treat the signal using a standard wave-optics formalism. The electric field received by an observer is the sum of the electric field of the source across the lens plane, with the wave-optics formalism. The electric field received by an observer is the sum of the electric field of the source across the lens plane, with the

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caustics to form, which can lead to a slope in time–frequency space, double-peaked spectra and other interference effects (see Methods). All of these are consistent with the behaviour seen in the measured spectra of the magnified pulses in Fig. 2.

For any lens, different pulse components will be magnified differently if they arise from regions that have projected separations larger than the lens's resolution. We can make a quantitative estimate of the lens resolution by using the magnifications of the events. Continuing to assume the lens is roughly co-located with the companion, that is, at the orbital separation \(a\), the resolution of the lens for given magnification would be \(\approx 1.9R_L/\mu^{1/2}\) for a linear lens, or \(\approx 1.9R_L/\mu^{1/4}\) for a circular one, where \(R_L \equiv \sqrt{\lambda a/\pi}\), or 23 km at our observing wavelength \(\lambda = c/\nu \approx 90\) cm (see Methods for a derivation). A peak magnification of, for instance, \(\mu = 50\) then corresponds to a physical resolution of about 6 km for a linear lens or about 17 km for a circular one.

The inferred resolution is comparable to the radius (approximately 10 km) of the neutron star, and substantially smaller than the light-cylinder radius, \(R_L \equiv cP/2\pi = 76\) km, where the velocity of co-rotating magnetic fields approaches the speed of light, bounding the magnetosphere in which pulsar emission is thought to originate\(^{14}\). Thus, the lensing offers the opportunity to map the emission geometry.

Qualitatively, because the main pulse and interpulse, as well as parts of the wide interpulse beam, are sometimes magnified very differently, the inferred resolution is a lower limit both to the projected separation between the main pulse and the interpulse, and to the size of the interpulse. Perhaps not unexpectedly, the spatial separations do not seem to map directly to rotational phase: we see similar differences between the main pulse and the interpulse, which are separated by half a rotation, and between parts of the interpulse, which are separated by only about 0.1 rotation. This may indicate that the interpulse consists of multiple components, which are not located close together in space.

Combining the lens resolution with the timescale of the lensing events allows us to constrain the projected relative velocity between the lens and the pulsar. A priori, one might expect the outflow velocity to be slow, in which case the relative velocity would just be the orbital motion of about 360 km s\(^{-1}\). Given that strong magnification events typically last about 10 ms, this would then imply a resolution of approximately

Fig. 1 | Strong lensing of the pulsar. a, Magnification distributions for the main pulse, in 10-s time bins, showing periods of lensing near ingress and egress of the eclipse of the pulsar by its companion's outflow. The greyscale is logarithmic, to help individual bright pulses to stand out. b, Pulse profiles as a function of time, averaged over 10 s and in 128 phase bins. Cyan, yellow and magenta represent contiguous 16-MHz sub-bands, from low to high frequency. Near eclipse, plasma dispersion causes frequency-dependent delays. Gaps in the data correspond to calibrator scans. c, d, Enlargements in which each time bin corresponds to an individual 1.6-ms pulse. One sees extreme chromatic lensing in which pulses are magnified by an order of magnitude over tens of milliseconds; the brightest event is magnified by a factor of about 40 across our highest-frequency sub-band. In some events, the main pulse and interpulse are clearly affected differently, indicating that different emission regions corresponding to these are resolved by the lensing structures. e, f, Enlargements for a quiescent period for comparison.
4 km, not dissimilar from what we find above, thus suggesting that
the assumption of a slow outflow velocity is reasonable. By combining
different constraints between the duration and frequency widths of
lensing events, we can set a quantitative limit to the relative velocity
(see Methods for a derivation), of
\[ \nu > 0.5 \frac{R_1}{\Delta \nu_{\text{HWHM}}} \left( \frac{\Delta \nu}{\nu} \right)^{1/2} \text{ HWHM} > 360 \text{ km s}^{-1} \]  

where for the numerical value we use the fact that the tightest con-
straints come from the shortest, least chromatic events, which have
frequency widths \( \Delta \nu / \nu \) \text{ HWHM} \approx 0.1 \text{ and durations } \Delta \nu_{\text{HWHM}} \approx 10 \text{ ms} (\text{measured as half-width at half-maximum, HWHM; see Fig. 2}). This
approximate limit equals the relative orbital velocity, which implies that
the outflow velocity can be (but does not have to be) small in the rest
frame of the companion.

Our results offer many avenues of further research. Ideally, one
would use the lensing to map the pulsar magnetosphere. This requires
better constraints on the lenses, for example from observations over
several eclipses at a range of frequencies. Further observations may
also shed light on the eclipse mechanism. For example, if it is cyc-
lo-synchrotron absorption, large magnetic fields, of about 20 G, are
required\(^1\), which would impart measurable polarization dependence
of the lensing events. Furthermore, combining density and velocity into

Fig. 2 | A range of spectral behaviour in lensing events. Power spectra,
binned to 3 MHz, of the main pulse in consecutive 1.6-ms rotations, for
selected lensing events from eclipse egress. The top panels show the average
magnification in each main pulse, and the side panels the magnification
spectra of the brightest pulse (that is, that of pulse number 0).

Fig. 3 | Profiles of lensing events. a, Frequency-averaged pulse profile
of a 9-min quiescent region, scaled up by a factor of 50 for comparison
purposes; b, c, d, frequency-averaged pulse profiles surrounding bright
events (using 128 phase bins). One sees that the events resolve the
magnetosphere: the main pulse and interpulse are affected differently, as
are parts within the interpulse.
a mass-loss rate of the outflow, one can infer the lifetime, and thus the final fate, of these systems.

Finally, our observations establish that radio pulses can be strongly amplified by lensing in local ionized material. This adds support for the proposal that fast radio bursts (FRBs) can be lensed by host galaxy plasma, leading to variable magnification, narrow frequency structures and clustered arrival times of highly amplified (and thus observable) events. Evidence for high-density environments includes that FRB 110523 is scattered within its host galaxy,\(^\text{[16]}\), and that FRB 121102 has been localized to a star-forming region,\(^\text{[17,18]}\), in an extreme and dynamic magneto-ionic environment.\(^\text{[19]}\) Indeed, the latter’s repeating bursts have spectra\(^\text{[20–22]}\) remarkably similar to those shown in Fig. 2 (for example, compare with Fig. 2 of ref. \textit{[20]}). and, like those bursts, the brightest pulses in PSR B1957+20 are highly clustered in time.

**Online content**

Any Methods, including any statements of data availability and Nature Research reporting summaries, along with any additional references and Source Data files, are available in the online version of the paper at https://doi.org/10.1038/s41586-018-0133-z.

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Author contributions R.M. discovered the magnified pulses; wrote the majority of the manuscript; and created the figures. I.-S.Y. developed and wrote the sections on wave-optics formalism in the Methods; M.H.v.K. and U.-L.P. guided the analysis and interpretation of the results; M.H.v.K. also helped to write the manuscript and influenced the content and presentation of the figures; F.X.L., V.C. and K.V. quantified the excess dispersion measure associated with the eclipse. N.M. wrote the code used to produce the coherently de-dispersed single pulse profiles used in this analysis, and derived an improved orbital ephemeris relative to which time delays are measured. D.L. worked on criteria to systematically separate giant pulses from pulses magnified by plasma lensing. All authors contributed to the interpretation of results.

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METHODS
Calculating magnifications. The data were taken in four 2.4-h sessions on 2014 June 13–16 and were recorded as part of a European VLBI network programme (GP 052). The data span 311.25–359.25 MHz, in three contiguous 16-MHz sub-bands, and we read, de-dispersed (with a dispersion measure of 29.1162 pc cm$^{-3}$) and reduced the data as described in ref. 11. As one extra step, we accounted for the wander in orbital period of PSR B1957+20 by adjusting the time of ascending node in the ephemeris, such that it minimized the scatter of the arrival times of giant pulses associated with the main pulse across all four days of observation.

We define the magnification of a pulse as the ratio of its flux to the mean flux in a quiescent region far from eclipse. Specifically, we construct an intensity profile of each pulse using 128 phase gates, subtract the mean off-pulse flux in each 16-MHz sub-band separately, measure the flux in an eight-gate (approximately 100 $\mu$s) window around the peak location (which we find from folded profiles, averaged in 2-s bins to obtain sufficient signal-to-noise), and divide by the average flux in the same pulse window measured in a 9-min section far from eclipse.

To construct the spectra of lensing events, we start by binning power spectra in 3-MHz bins. We correct these approximately for the bandpass and the effects of interstellar scintillation (which still has a small amount of power on 3-MHz scales) by dividing them by the average spectra in the 15 s before and after (excluding the lensing event itself). This time span is chosen to be safely less than the timescale of about 84 s on which the interstellar scintillation pattern varies$^{11}$, ensuring the dynamic spectrum is stable on these scales. With 30 s of data, it is also very well measured: each 3-MHz channel has signal-to-noise ratio S/N$\approx$ 150.

**Evidence that strong plasma lensing must occur.** To obtain the properties of the lensed images, and in particular to determine whether we are in the ‘strong’ or ‘weak’ lensing regime—that is, whether or not multiple images are formed—we use evidence that strong plasma lensing must occur. When a source is separated from the focal point in direction by $\Delta R = 720$ km. Using equation (3), this corresponds to $\Delta t = 300$ cycles at 2-s bins to obtain sufficient signal-to-noise, and divide by the average flux in the same pulse window measured in a 9-min section far from eclipse.

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Before continuing, we note that at first glance it might seem puzzling for the magnification of the full two-dimensional lens plane at a resolution better than the Fresnel scale, whereas our time-delay measurements are limited to the one-dimensional trajectory of the pulsar, and to an order of magnitude larger scales. Hence, for our analysis we use a simplified model, amenable to wave-optics analysis, in which each strong lensing event is associated with an elliptical perfect lens that, is, there is a single focal point to which all paths within the lens contribute perfectly coherently, and all paths outside the lens do not contribute at all. As shown in Extended Data Fig. 4, our model is parametrized by the location of the focal point relative to the centre of the lens, $(X, Y)$, the semi-major and semi-minor axes of the lens, $(\Delta x_{\text{lens}}, \Delta y_{\text{lens}})$ and the orientation relative to the projected pulsar trajectory.

The special case of a centred circular lens with radius $R$, that is, $\Delta x_{\text{lens}} = \Delta y_{\text{lens}} = \mu$ and $X = Y = 0$, the set-up would produce an Airy disk. By comparing this to the actual path-integral of an un lensed image (that is, with $\phi_{\text{DM}} = 0$), we can identify the un lensed case with a circular lens with radius $R_{\mu} = R_{\text{F}}/\sqrt{\pi}$. Integrating the electric field over an elliptical lens, the magnification (defined as the increase in intensity $\mu = H(\ell)$, where $I = E^2/I_{\text{ref}}$, will be proportional to the area squared:

$$\mu = \frac{\Delta x_{\text{lens}}}{R_{\mu}} \left( \frac{\Delta y_{\text{lens}}}{R_{\mu}} \right)^2$$

Before continuing, we note that at first glance it might seem puzzling for the magnification to be proportional to image area squared, as that seems to violate energy (flux) conservation. However, the image is also more highly beamed; what we calculated is the magnification in the centre of the beam. Viewed from an angle, a linear phase term is induced across the originally coherent region. Thus, when forming images, we find intrinsic variability corresponding to variations in delay of $\Delta \phi_{\text{DM}} \approx 1 \mu$s, which suggests that the inhomogeneities correspond to differences in the dispersion phase of $\Delta \phi_{\text{DM}} = \nu \Delta \phi_{\text{DM}} \approx 300$ cycles at 2-s timescales.

To compare this to the geometric phase, we need to translate the timescale of 2 s to a length scale. Assuming the relative motion between the pulsar and the lensing material is dominated by the orbital motion, of 360 km s$^{-1}$, the spatial scale is 720 km. Using equation (3), this corresponds to $\Delta x_{\text{F}}$ (720 km) $\approx 200$ cycles.

The fact that $\Delta \phi_{\text{DM}}$ is comparable to and slightly larger than $\Delta \phi_{\text{F}}$ at the distance scale of 720 km guarantees that somewhere around this scale, slopes in $\phi_{\text{DM}}$ and $\phi_{\text{F}}$ will sometimes cancel. It also implies that multiple stationary points with $\nabla \phi = 0$ are likely. Because the phases result from a continuous function of $x$, $y$, and $\nu$, the stationary points must emerge or annihilate in pairs, leading to so-called caustics, where one has not just $\nabla \phi = 0$ but also $\det(\partial \phi/\partial \nu) = 0$ (ref. 25).
We can write the above in terms of the magnification for a specific lens model.
We will consider two extremes, the first a very anisotropic lens, effectively one-
dimensional, for which $\Delta y_{\text{ens}} = R_1$. For this case,
\[ x_{\text{ens}} \approx \frac{1.9 R_1}{\mu^{1/4}} \]  
(10)

In the opposite direction, we consider a circular lens, with $\Delta x_{\text{ens}} = \Delta y_{\text{ens}}$. For
this case,
\[ x_{\text{ens}} = y_{\text{ens}} \approx \frac{1.9 R_1}{\mu^{1/4}} \]  
(11)

Here, dividing by the distance $a$ and writing in terms of diameter $D = 2\Delta x_{\text{ens}}$, one
recovers the usual relation for the angular resolution of a circular lens, $1.22\lambda/D$.

A similar result can be derived in frequency space. Given the different frequency
dependencies of the geometric and dispersive phases, a lens cannot be fully in focus
across all frequencies, but will have some characteristic frequency width. To show
this, we first assume that the focal point is co-located with the centre of the elliptical
lens, that is, $(X, Y) = (0, 0)$, and that perfect coherence occurs at some frequency $\nu_c$, that is, that at $\nu_c$ one has
\[ \phi_{\text{GM}}(X, Y, \nu) = -\phi_{\text{DM}}(X, Y, \nu) = \frac{\nu_c(x^2 + y^2)}{2ac} \]  
(12)

The total phase at a nearby frequency is then given by
\[ \phi(x, \nu + \Delta \nu) = \frac{\nu_c + \Delta \nu}{\nu_c + \Delta \nu} \frac{\nu_c(x^2 + y^2)}{2ac} \]  
(13)

the dominant term of which is linear in $\Delta \nu$. If this term is of order 1 within the
elliptical region, the image will no longer be magnified. Perfect cancellation does
not happen in this case, but one can define characteristic width. Using half-width
at half-maximum, one finds
\[ \frac{\Delta \nu}{\nu_c} \approx \frac{2.9 R_1^2}{3 \Delta x_{\text{ens}}^4 - 2 \Delta x_{\text{ens}}^2 \Delta y_{\text{ens}}^2 + 3 \Delta y_{\text{ens}}^4} \]  
(14)

In terms of the magnification, we find for the special case of an effectively
one-dimensional lens
\[ \frac{\Delta \nu}{\nu_c} \approx \frac{1.7}{\mu} \]  
(15)

whereas for a circular lens
\[ \frac{\Delta \nu}{\nu_c} \approx \frac{1.5}{\mu^{1/2}} \]  
(16)

When the focal point is not in the centre of the elliptical region, there will be
linear terms proportional to $\Delta X \nu$ and $\Delta Y \nu$ that also contribute to the phase. They
make the dependence on $\Delta \nu$ even steeper, which means that equation (14) is an
upper bound on the frequency width of magnified images.

It is possible for the frequency and position shifts to cancel each other, so that
after moving the source by some distance, it is still strongly magnified at a nearby
frequency. That leads to a slope of strongest magnification in time–frequency space.
Following a derivation similar to that outlined above for the behaviour with frequency
and position separately, we find that the slope is related only to the offset of the source
from the focal point. For instance, for the case that the source is travelling along the
semi-major axis of the lens, that is, along a trajectory $(x(t), y(t)) = (x(t), 0)$, we find
\[ \frac{d\nu}{dx} = \frac{\nu_c}{2x_c} \]  
(17)

Finally, it is possible that the pulsar is close to the focus of more than one lens
at the same time. As the pulsar moves, these multiple focal points lead to multiple
images which change intensity and interfere with each other. This can lead to a wide
variety of spectral behaviour, and might well be responsible for the multi-peaked
structures seen in some panels of Fig. 2.

A lower bound on the relative velocity. Within the perfect lens approximation,
one can derive a lower bound on the relative, transverse velocity between the lens and
the pulsar from the durations and frequency widths of the lensing events.

The duration of strongly magnified events is predicted to be
\[ \Delta t_{\text{HWHM}} \approx \frac{0.7 R_1^2}{\sqrt{\Delta x_{\text{ens}}^2 \nu_c^2 + \Delta y_{\text{ens}}^2 \nu_c^2}} > \frac{0.7 R_1^2}{\nu_c \Delta x_{\text{ens}}} \]  
(18)

where the inequality corresponds to making the most conservative estimate that
the lens is effectively one-dimensional, extended in the $x$ direction, and that the
full velocity $\nu_c$ is directed along it.

In the frequency space, the same event will have its width bounded from above
by equation (14):
\[ \frac{\Delta \nu}{\nu_c} \text{ HWHM} < \frac{1.8 R_1^2}{\Delta x_{\text{ens}}^2} \]  
(19)

Combining the two limits to eliminate $\Delta x_{\text{ens}}/R_1$, we find:
\[ \nu_c > 0.5 R_1 \frac{\Delta \nu}{\nu_c} \text{ HWHM}^{1/2} \]  
(20)

Lensing consistency checks. Our analysis in terms of lensing was motivated by
the facts that the bright pulses before and after eclipse do not look like giant pulses,
that they occur at specific orbital phases where dispersion measure fluctuations
are large, and that, in most events, the entire profile increases in flux (although
not always by the same factor across the profile), with the enhancements lasting
for several pulses.

Consistent with the expectations of plasma lensing, in which lenses focus light
but arrive with associated ‘shadows’ where light is scattered away from our direct
line of sight, we find that in regions with many strongly lensed pulses, the average
flux received is unchanged (see Extended Data Fig. 3). Also consistent with plasma
lensing is that the strongly magnified pulses are highly chromatic, often peaking at
high or low frequencies, and sometimes showing slopes in frequency time space
or interference patterns characteristic of caustics.

Finally, as a concrete example, the peak magnifications of $\mu \approx 70$ are comparable
to the magnifications expected for plasma lensing. The measured dispersive time
delay changes provide direct evidence that the geometric and dispersive phases
$\phi_{\text{GM}}$ and $\phi_{\text{DM}}$ can cancel over distances of about 720 km. The cancellation is likely
to happen mostly in one spatial direction; setting $\Delta y_{\text{ens}} = 720 \text{ km}$ and $\Delta x_{\text{ens}} = R_1$, our perfect lens model predicts a magnification of the order of 100, comparable
to the observed value.

Data availability. The data underlying the figures are available in text files at
https://github.com/ramain/B1957/LensingData

Code availability. The raw data were read using the baseband package: https://
github.com/mhvk/baseband

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Extended Data Fig. 1 | Shock geometry. The brown dwarf companion is irradiated by the pulsar wind, causing it to be hotter on the side facing the pulsar, and inflated, nearly filling its Roche lobe. Outflowing material is shocked by the pulsar wind, leaving a cometary-like tail of material. This tail is asymmetric because of the companion's orbital motion, which leads to eclipse egress lasting substantially longer than ingress. The companion and separations are drawn roughly to scale, whereas the pulsar is not: the light cylinder radius of 76 km would be indistinguishable on this figure. The inclination of the system is conservatively constrained to $50^\circ < i < 85^\circ$ (ref. 13). $v_p$ and $v_c$ are the velocities of the pulsar and companion, $a$ is the semi-major axis of the orbit, and $R_{\text{Roche}}$ is the Roche lobe of the companion.
Extended Data Fig. 2 | Dispersion measure near the radio eclipse. Shown is the excess dispersion measure relative to the interstellar dispersion, with insets focusing on regions of strong lensing. The excess dispersion is estimated from delays in pulse arrival times (see Methods); at our observing frequency of 330 MHz, $\Delta DM = 0.001$ pc cm$^{-3}$ corresponds to a delay of 38 $\mu$s. The scatter around the curves is intrinsic. During periods of strong lensing, it is at a level of $2.6 \times 10^{-5}$ pc cm$^{-3}$, corresponding to variations in delay time of 1 $\mu$s.
Extended Data Fig. 3 | Pulse profiles and magnification distributions.

a–e, Each row shows pulse profiles for 10-s segments (left, in 128 phase bins), and the corresponding magnification distributions. The segments are taken from: a, a quiescent region, showing a log-normal magnification distribution (repeated using a dotted line in b–e for comparison); b, eclipse ingress, showing some lensing events, reflected in the tail to high magnification; although hard to see, the average flux is reduced to about 70% by absorption in eclipsing material; c, the first post-eclipse lensing period, showing extreme magnifications but little change in average flux; d, in between the two post-eclipse strong lensing periods, showing only weak lensing on relatively long, approximately 100-ms timescales; e, the second post-eclipse period of strong lensing.
Extended Data Fig. 4 | Geometry of a lensing region. An almost edge-on (left) and a face-on (right) view of the lensing geometry. We assume that the source is at a separation from the pulsar equal to the semi-major axis $a$ of the binary system and moves on a trajectory parallel to the lens plane. A source at the focal point $(X, Y)$ will illuminate the entire elliptical lens coherently, leading to strong magnification. In general, the focal point may not be at the centre of the ellipse (although it is likely to be within it), and the source trajectory $[x_s(t), y_s(t)]$ may not intersect the focal point or the elliptical region.