RECENT THEORETICAL DEVELOPMENTS IN CP VIOLATION

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We review recent suggestions for testing through $B$ decays the flavor structure of CP violation in the Standard Model. Relative signs of CP asymmetries in U-spin related processes can by themselves test the Kobayashi-Maskawa mechanism in a crude manner. Ratios of charge-averaged decay rates and certain CP asymmetries may constrain tightly the weak phases $\gamma = \phi_3$ and $\alpha = \phi_2$.

1 Introduction

Assuming that a phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix is the dominant source of CP violation, sizable CP asymmetries are predicted in a variety of $B$ decay processes. A major task of present experiments at $e^+e^-$ $B$ factories, following an earlier measurement by the CDF Collaboration at the Fermilab Tevatron, is the search for a time-dependent asymmetry in $B^0(t) \to J/\psi K_S$, which can be clearly interpreted in terms of the weak phase $\beta = \phi_1$. As a consequence of measurements reported at this conference by the BaBar and Belle Collaborations, the present world average value of $\sin 2\beta$ gained the respectable status of a nonzero measurement at three standard deviations, $\sin 2\beta = 0.48 \pm 0.16$, consistent with the CKM prediction. The remarkable success in running the two asymmetric $B$ factories at SLAC and KEK-B, to be soon joined by experiments at the Tevatron, promises a considerable improvement in this important measurement during the coming year.

In addition to time-dependent asymmetries, such as in $B^0(t) \to J/\psi K_S$ or $B^0(t) \to \pi^+\pi^-$, which are related to CKM phases, $B$ decays provide an opportunity of measuring CP rate asymmetries in a large number of two body and multibody $B$ decay modes. In this talk we choose to discuss two classes of decay processes. In the first class, discussed in Section 2, asymmetries can provide crude but very useful tests of the CKM mechanism. While the signs and magnitudes of such direct asymmetries depend in general on unknown strong final state phases, these asymmetries will be shown to be related to each other pairwise in some approximation. The relative signs of pairs of asymmetries in this large class of processes are predicted quite reliably, implying that crude asymmetry measurements can provide simple tests. Measuring “wrong” relative signs would most likely imply new physics.

In a second class of processes studied in Section 3 (which in some cases overlaps with the first class) precise asymmetry measurements are shown to provide tight constraints on CKM phases. In several cases such information may be gained by merely measuring certain ratios of charge-averaged decay rates. Combining all these methods allows for precision tests of the CKM hypothesis of CP violation. Such tests will hopefully provide first clues for physics beyond the Standard Model.

Our brief review will focus on recent work, discussing several central examples which represent a much broader study made during the past decade.

2 A theorem about equal CP rate differences

A subgroup of flavor SU(3), discrete U-spin symmetry interchanging $d$ and $s$ quarks, plays a particularly powerful and important role in charmless $B$ decays. Consider the low energy effective weak Hamiltonian describing $\Delta S = 1$ charmless $B$ decays:

$$H_{\text{eff}}^{(s)} = \frac{G_F}{\sqrt{2}} \left[ V_{ub}^* V_{us} \left( \sum_{i=1}^{10} c_i Q_{i}^{us} + \sum_{3}^{10} c_i Q_{i}^{s} \right) + V_{cb}^* V_{cs} \left( \sum_{i=1}^{10} c_i Q_{i}^{rs} + \sum_{3}^{10} c_i Q_{i}^{s} \right) \right],$$

where $c_i$ are scale-dependent Wilson coefficients. The flavor structure of the various four-quark operators is $Q_{1,2}^{qs} \sim b\bar{q}q\bar{s}, \; Q_{3,6,9}^{s} \sim \bar{b}s \sum q'q', \; Q_{7}^{7,10} \sim \bar{b}s \sum e_q q'q', \; e_q$ are quark charges, $q' = u,d,s,c$. Each of the four-quark operators represents an $s$ component (“down”) of a U-spin doublet, so that one can write in short

$$H_{\text{eff}}^{(s)} = V_{ub}^* V_{us} U^* + V_{cb}^* V_{cs} C^*,$$

where $U$ and $C$ are U-spin doublet operators. Similarly, the effective Hamiltonian responsible for $\Delta S = 0$ decays, in which one replaces $s \to d$, involves $d$ components (“up” in U-spin) of corresponding operators multiplying CKM factors $V_{ub}^* V_{ud}$ and $V_{cb}^* V_{cd}$:

$$H_{\text{eff}}^{(d)} = V_{ub}^* V_{ud} U^d + V_{cb}^* V_{cd} C^d.$$

This structure of the Hamiltonian implies a general relation between two decay processes, $\Delta S = 1$ and $\Delta S = 0$, in which initial and final states are obtained from each other by a U-spin transformation, $U : d \leftrightarrow s$. Writing the $\Delta S = 1$ amplitude as

$$A(B \to f, \Delta S = 1) = V_{ub}^* V_{us} A_u + V_{cb}^* V_{cs} A_c,$$

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the corresponding $\Delta S = 0$ amplitude is given by

$$A(UB \to Uf, \Delta S = 0) = V_{ub}^* V_{ud} A_u + V_{cb}^* V_{cd} A_c .$$  \hspace{1cm} (5)$$

Here $A_u$ and $A_c$ are complex amplitudes involving CP-conserving phases. The amplitudes of the corresponding charge-conjugate processes are

$$A(\bar{B} \to \bar{f}, \Delta S = -1) = V_{ub} V_{us} A_u + V_{cb} V_{cs} A_c , \hspace{1cm} (6)$$

and

$$A(U\bar{B} \to U\bar{f}, \Delta S = 0) = V_{ub}^* V_{us} A_u + V_{cb}^* V_{cs} A_c . \hspace{1cm} (7)$$

Unitarity of the CKM matrix implies the following relation between CP rate differences

$$\Delta \Gamma(B \to f) \equiv \Gamma(B \to f) - \Gamma(\bar{B} \to \bar{f}) \approx \Delta \Gamma(UB \to Uf) \equiv -[\Gamma(UB \to Uf) - \Gamma(U\bar{B} \to U\bar{f})] .$$  \hspace{1cm} (8)$$

Namely, CP rate differences $(\Delta)$ in decays which go into one another under interchanging $s$ and $d$ quarks have equal magnitudes and opposite signs. This rather powerful result, following from U-spin within the CKM framework, can be demonstrated in numerous decay processes, including two body, quasi-two body, multibody hadronic and radiative $B$ decays. A few examples are

$$\Delta \Gamma(B^0 \to K^+ \pi^-) \simeq -\Delta \Gamma(B_s \to \pi^+ K^-) ,$$

$$\Delta \Gamma(B^0 \to K^{*+} \pi^-) \simeq -\Delta \Gamma(B_s \to \rho^+ K^-) ,$$

$$\Delta \Gamma(B^+ \to K^{*+} \pi^-) \simeq -\Delta \Gamma(B^+ \to \rho^+ K^-) ,$$

$$\Delta \Gamma(B^+ \to K^+ \gamma) \simeq -\Delta \Gamma(B^+ \to \rho^+ \gamma) .$$  \hspace{1cm} (9)$$

U-spin is an approximate symmetry of strong interactions. Naively one would think that the approximation holds up to small terms of order $m_s/m_b$. Considering $B$ decays to two light pseudoscalars, and assuming that the dominant terms in amplitudes factorize, U-spin breaking corrections in these processes are given in terms of ratios of decay constants and form factors involving $s$ and $d$ quarks. This may lead to violations of asymmetry relations, however such violations are not expected to be gross. Furthermore, independently of any assumption, if such asymmetries are large, corresponding to large final state phases in a particular process, it is very unlikely that U-spin breaking can change the sign of these phases. Consequently, the prediction that large CP asymmetries in two U-spin related processes have opposite signs is expected to be robust even in the presence of U-spin breaking effects.

Can one correct for U-spin breaking effects? Since such effects are model-dependent, it would be very useful if they could be directly measured in rates. There exists such a possibility if one assumes that certain rescattering effects can be neglected. Consider the three pairs of U-spin related processes $(B^0 \to K^+ K^-, B_s \to \pi^+ \pi^-)$, $(B^0 \to K^{*+} \pi^-, B_s \to \pi^+ K^-)$ and $(B^0 \to K^+ \pi^-, B_s \to K^{*+} K^-)$. The three $\Delta S = 1$ decays are described by the following SU(3) flavor flow amplitudes

$$A(B_s \to \pi^+ \pi^-) = -PA - E ,$$

$$A(B^0 \to K^+ \pi^-) = -P - T - \frac{2}{3} P_{EW} ,$$

$$A(B_s \to K^+ K^-) = -P - T - \frac{2}{3} P_{EW} - PA - E .$$  \hspace{1cm} (10)$$

The corresponding strangeness conserving decay amplitudes involve other CKM factors but have the same SU(3) structure.

The first pair of U-spin processes involve only quark amplitudes $PA + E$ in which, in the absence of large rescattering effects, the spectator quark in the $B$ or $B_s$ meson participates in the interaction. These decays can be used to test the smallness of rescattering corrections. Unless amplified by rescattering, these amplitudes are expected to be suppressed by $f_B/m_B$ relative to the dominant amplitudes occurring in the other two pairs of processes. Thus, the branching ratio of $B^0 \to K^+ K^-$ is expected to be of the order of $10^{-7}$ or smaller, compared to $10^{-5}$ characterizing the branching ratios of the other four processes. In order to test the assumption of small rescattering effects, the present experimental upper limit $B(B^0 \to K^+ K^-) < 1.9 \times 10^{-6}$, should be improved by at least one order of magnitude.

Assuming that such a stringent bound is obtained, one can then neglect corresponding $PA + E$ terms in $B_s \to K^+ K^-$ and $B^0 \to \pi^+ \pi^-$. This implies in the limit of U-spin symmetry

$$A(B_s \to K^+ K^-) \simeq A(B^0 \to K^+ \pi^-) ,$$

$$A(B_s \to \pi^+ K^-) \simeq A(B^0 \to \pi^+ \pi^-) .$$  \hspace{1cm} (11)$$

The rates of these four processes can be used to measure U-spin corrections. For instance, assuming factorization these corrections are given by ratios of form factors

$$\frac{A(B_s \to K^+ K^-)}{A(B^0 \to K^+ \pi^-)} = \frac{F_{B_s,K}(m^2_K)}{F_{B\pi}(m^2_K)} ,$$

$$\frac{A(B_s \to \pi^+ K^-)}{A(B^0 \to \pi^+ \pi^-)} = \frac{F_{B_s,K}(m^2_K)}{F_{B\pi}(m^2_{\pi})} .$$  \hspace{1cm} (12)$$

The two ratios of form factors are expected to be equal within about 1%, since the variation of the two form factors from $q^2 = m^2_{\pi}$ to $q^2 = m^2_K$ is tiny for a relevant scale of order $m^2_B$. We conclude that (once the smallness of rescattering has been established) the rates of these four processes can be used not only to determine the U-spin breaking factor in the ratio of amplitudes, but also to check the factorization assumption by finding equal ratios of amplitudes in the two cases.
3 Stringent constraints on weak phases

In the present section we discuss recent developments in suggestions for determining the weak phase $\gamma$. We will also comment briefly on an old idea for resolving penguin uncertainties in the determination of $\sin 2\alpha$ from $B^0(t) \rightarrow \pi^+\pi^-$. 

3.1 $\phi_3 = \gamma$ from $B^0, B_s \rightarrow K^{\pm}\pi^\mp$

The processes in (11) play a useful role in determining $\gamma$. Here we describe a scheme based on $K\pi$ decays of $B^0$ and $B_s$ mesons [11]. We will briefly comment on a complementary method using the other two processes.

Writing the amplitudes for $B^0 \rightarrow K^{+}\pi^-$ and $B_s \rightarrow K^{-}\pi^+$ as in Eqs. (1) and (2), respectively, we note that the rates for these processes and their charge-conjugates depend on four quantities, $|V_{ub}V_{us}A_u|$, $|V_{cb}V_{cs}A_c|$, $\delta_{K\pi} \equiv \text{Arg}(A_u^*A_c^*)$ and $\gamma \equiv \text{Arg}(-V_{ub}^*V_{us}V_{cb})$. Because of the equality of CP rate-differences in the two processes, a determination of $\gamma$ requires another input. This input is provided by $|A(B^+ \rightarrow K^0\pi^+)| = |V_{ub}^*V_{us}A_u|$, where small rescattering corrections are neglected as argued above.

Defining two charge-averaged ratios of rates

$$R = \frac{\Gamma(B^0 \rightarrow K^+\pi^-)}{\Gamma(B^0 \rightarrow K^0\pi^-)} , \quad R_s = \frac{\Gamma(B_s \rightarrow K^{+}\pi^-)}{\Gamma(B_s \rightarrow K^{-}\pi^+)} ,$$

and CP violating pseudo-asymmetries

$$A_0 \equiv \frac{\Delta(B^0 \rightarrow K^+\pi^-)}{\Gamma(B^0 \rightarrow K^0\pi^-)} , \quad A_s \equiv \frac{\Delta(B_s \rightarrow K^{+}\pi^-)}{\Gamma(B_s \rightarrow K^{-}\pi^+)} ,$$

one finds

$$R = 1 + r^2 + 2r \cos \theta_c \cos \gamma , \quad (15)$$

$$R_s = \tan \theta_c^2 + (r / \tan \theta_c)^2 - 2r \cos \theta_c \cos \gamma , \quad (16)$$

$$A_0 = -A_s = -2r \sin \delta_{K\pi} \sin \gamma , \quad (17)$$

where $r \equiv |V_{ub}^*V_{us}A_u| / |V_{cb}^*V_{cs}A_c|$. SU(3) breaking can be checked in (12) and used for improving the precision in $\gamma$ obtained from these four quantities. It is estimated [12] that a precision of 10$^\circ$ in $\gamma$ can be achieved in experiments to be performed at the Fermilab Tevatron Run II program [3].

Alternatively, one may compare time-dependence in the $U$-spin related decays $B^0(t) \rightarrow \pi^+\pi^-$ and $B_s(t) \rightarrow K^+K^-$ [4]. Here one is measuring in the two processes CP asymmetries of the form

$$\text{Asym}(t) = A_{\text{mix}} \sin(\Delta m t) + A_{\text{dir}} \cos(\Delta m t) . \quad (18)$$

The four measurable, $A_{\text{mix}}$ and $A_{\text{dir}}$ in the two processes, can be expressed in terms of $\beta$, $\gamma$, the ratio of penguin and tree amplitudes in $B^0 \rightarrow \pi^+\pi^-$ and their relative strong phase. This allows a determination of $\gamma$ with a precision comparable to that achieved when studying $B, B_s \rightarrow K\pi$.

3.2 $\phi_3 = \gamma$ from $B^\pm \rightarrow K\pi$

A large number of charmless $B$ and $B_s$ decays to two light pseudoscalars can be related to each other under approximate flavor SU(3) symmetry. It was noted a long time ago [4] that hadronic weak amplitudes can be classified in SU(3) in terms of quark diagrams. Starting with the papers [13] this framework has been applied to the $\Delta B = 1$, $\Delta C = 0$ low energy effective Hamiltonian [14] and its $\Delta S = 0$ counterpart for the purpose of determining weak phases. A large number of proposals of this kind were made in the past seven years.

Here we will focus on a particular recent application. Although SU(3) is only an approximate symmetry it will be applied to subleading terms in decay amplitudes so that SU(3) breaking corrections will be second order. We will make use of an SU(3) proportionality relation [15] between electroweak penguin operators ($Q_9, 10$) and the current-current operators ($Q_{1, 2}$) transforming as given SU(3) representations [3, 6 and 15], in which the proportionality constant is given purely in terms of ratios of Wilson coefficients and CKM factors. For instance

$$H_{\text{EW}L}^{(s)}(T_5) = -\frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} \frac{V_{ub}^*V_{us}}{V_{cb}^*V_{cs}} H_{\text{CC}}^{(s)}(T_5) , \quad (19)$$

where $(c_9 + c_{10})/(c_1 + c_2) \approx -1.12\alpha$, $\delta_{\text{EW}} \approx 0.65 \pm 0.15$. This SU(3) equality, implying a relation between hadronic amplitudes, simplifies the study of processes governed by $T_5$ transitions.

Where does only $T_5$ contribute? The answer to this question is simple [16]. In $B \rightarrow (K\pi)_{I=3/2}$ and in $B^+ \rightarrow \pi^+\pi^0$ where the final states are “exotic” and belong to a 27 representation. The amplitude of the first process can be written in terms of SU(3) graphical contributions

$$A(B^+ \rightarrow K^0\pi^+) + \sqrt{2} A(B^+ \rightarrow K^+\pi^0) = \quad (20)$$

$$- (T + C + P_{\text{EW}} + P_{\text{EW}}^*) = -(T + C)(1 - \delta_{\text{EW}} e^{-i\gamma}) .$$

Note that the $\Delta I = 0$ penguin contributions dominating the two amplitudes in the left-hand-side are equal and cancel by isospin alone. Hence the resulting SU(3) relation (21) applies to the subdominant current-current and electroweak contributions.

One defines a charge-averaged ratio of rates

$$R_s^{-1} = \frac{2[B(B^+ \rightarrow K^+\pi^0) + B(B^+ \rightarrow K^-\pi^0)]}{B(B^+ \rightarrow K^0\pi^+) + B(B^- \rightarrow K^0\pi^-)} . \quad (21)$$
The amplitudes in the numerator and denominator involve a common dominant penguin amplitude and current-current and electroweak contributions which are related by (20). Expanding in subdominant contributions one derives the following inequality, to leading order in small quantities

$$| \cos \gamma - \delta_{EW} | \geq \frac{|1 - R_-^a|}{2\epsilon} ,$$  

(22)

where

$$\epsilon = \frac{|V_{ub}^* V_{ts}|}{|V_{tb} V_{ts}|} \left| \frac{|T + C|}{|P + EW|} \right| = \frac{\sqrt{2} \text{Re} \left( \frac{f_K}{f_\pi} \right) |A(B^+ \to 0^0_3)|}{|V_{ua}^* F_\pi| \left| A(B^+ \to K^0_3) \right|} ,$$  

(23)

SU(3) breaking in subdominant terms is introduced through $f_K/f_\pi$.

A useful constraint on $\gamma$ follows for $R_-^a \neq 1$. The error of the present average value $R_-^a = 1.45 \pm 0.46$, ought to be reduced before drawing firm conclusions about allowed values of $\gamma$. Further information about $\gamma$, applying also to the case $R_-^a = 1$, can be obtained by measuring separately $B^+$ and $B^-$ decay rates. The solution obtained for $\gamma$ involves uncertainties due to SU(3) breaking in subdominant amplitudes and an uncertainty in $|V_{ub}/V_{tb}|$, both of which affect the value of $\delta_{EW}$. Combined with errors in $\epsilon \propto |A(B^+ \to 0^0_3)|/A(B^+ \to K^0_3)$, and in rescattering effects, one may expect to reach a precision in $\gamma$ as small as 10 or 20 degrees.

3.3 $\phi_3 = \gamma$ from $B \to DK$

In $B^+ \to DK^+$ two amplitudes interfere due to color-favored $b \to c u s$ and color-suppressed $b \to u c s$ transitions. The relative weak phase between the two amplitudes is $\gamma$. We will describe two variants based in this useful property which permits a measurement of $\gamma$. A brief discussion is included of recent progress made in studying relevant amplitudes and strong phases.

(a) $B$ decay to $K$ and flavor specific $D^0$ modes

The three-body decay $B^+ \to (K^- + \pi^+) D K^+$ involves an interference between the two cascade amplitudes,

$$A a_{K\pi} \equiv A(B^+ \to D^0 K^+ ) A(D^0 \to K^- \pi^+) ,$$  

(24)

and

$$\bar{A} a_{K\pi} \equiv A(B^+ \to \bar{D}^0 K^+ ) A(\bar{D}^0 \to K^- \pi^+) .$$  

(25)

The first amplitude $A$ is color-suppressed and subsequently the $D^0$ decays into a Cabibbo-favored mode with amplitude $a_{K\pi}$. The second amplitude $\bar{A}$ is color-favored, and subsequently $\bar{D}^0$ decays with a doubly Cabibbo-suppressed (DCS) amplitude $\bar{a}_{K\pi}$. The relative weak phase between $A$ and $\bar{A}$ is $\gamma$, their strong phase-difference will be denoted $\delta$, $\text{Arg}(A/\bar{A}) = \delta + \gamma$, and the relative phase between $a_{K\pi}$ and $\bar{a}_{K\pi}$ (including a relative weak phase $\pi$) will be denoted $\Delta_{K\pi} = \text{Arg}(a_{K\pi}/\bar{a}_{K\pi})$. Omitting a common phase space factor,

$$\Gamma(B^+ \to (K^- + \pi^+) D K^+) = |Aa|^2 + |\bar{A} \bar{a}|^2 + 2|A\bar{a}a\bar{A}| \cos(\delta + \Delta + \gamma) ,$$  

(26)

where $a \equiv a_{K\pi}$, $\bar{a} \equiv \bar{a}_{K\pi}$, $\Delta \equiv \Delta_{K\pi}$.

The rate for the charge-conjugate process, $B^- \to (K^+ - \pi^-) D K^-$, has a similar expression in which $\gamma$ occurs with an opposite sign, while strong phases are invariant under charge-conjugation. The CP asymmetry in this process, involving an interference of $Aa$ and $\bar{A}\bar{a}$, is proportional to $\sin(\delta + \Delta) \sin \gamma$, becoming maximal for $|Aa/\bar{A}\bar{a}| = 1$, $\delta + \Delta = \pi/2$.

Let us summarize the present updated information on the parameters appearing in Eqs. (26). The DCS amplitude $\bar{a}$ was measured recently by CLEO and by FOCUS, resulting in an average value $|\bar{a}_{K\pi}/a_{K\pi}| = (1.23 \pm 0.10) \tan^2 \theta_\epsilon = 0.063 \pm 0.005$. SU(3) symmetry predicts a value of $\tan^2 \theta_\epsilon$ indicating some amount of SU(3) breaking in $a/\bar{a}$. Model-dependent studies of $\Delta$ suggest that this phase, which vanishes in the SU(3) limit, can be as large as about 20 degrees or be even larger. Recently a method was suggested for measuring $\Delta$ at a charm factory. This phase plays an important role in studies of $D^0 - \bar{D}^0$ mixing. Finally, the ratio $A/\bar{A}$ is estimated, $|A/\bar{A}| \sim 0.1$, using a CKM factor $|V_{us}^* V_{cb}|/|V_{ub}^* V_{ts}| \approx 0.4$ and a color-suppression factor of about 0.25 measured in $B \to D\pi$ decays. The latter measurements also indicate a small value for $\delta$.

We conclude that the two amplitudes interfering in Eqs. (26) are anticipated to be comparable in magnitude, $|Aa/\bar{A}\bar{a}| \sim 0.6$ and to involve a possibly large relative strong phase $\delta + \Delta$. This is crucial for a feasible determination of $\gamma$ from the rate (26) and its charge-conjugate. To solve for $\gamma$ requires observing another doubly Cabibbo-suppressed $D^0$ decay mode. Such a study in the $K^+ - \pi^-$ channel is reported at this conference. This method requires a large number of $B$’s, at least of order $10^8 - 10^9$ since $B(B^+ \to D^0 K^+) B(\bar{D}^0 \to K^- \pi^+) = (4.2 \pm 1.4) \times 10^{-8}$.

(b) $B$ decay to $K$ and $D^0$ CP-eigenstate modes

Neglecting very small CP violation in $D^0 - \bar{D}^0$ mixing, one can write neutral $D$ meson even/odd CP states (decaying, for instance, to $K^+ K^-$ or $K S^0$) as $D^0_{\pm} = (D^0 \pm \bar{D}^0)/\sqrt{2}$. Consequently, one has up to an overall phase

$$\sqrt{2} A(B^+ \to D^0_{\pm} K^+) = \pm |A| + |A| \exp[i(\delta + \gamma)] .$$  

(27)

Let us define charge-averaged ratios of rates for positive and negative CP states relative to rates corresponding to
color-favored neutral $D$ flavor states

$$R_\pm = \frac{2[\Gamma(B^+ \rightarrow D\pi^0) + \Gamma(B^- \rightarrow D\bar{\pi}^0)]}{\Gamma(B^+ \rightarrow D\pi^0) + \Gamma(B^- \rightarrow D\bar{\pi}^0)} ,$$  

and two corresponding pseudo-asymmetries

$$A_\pm = \frac{\Gamma(B^+ \rightarrow D\pi^0) - \Gamma(B^- \rightarrow D\bar{\pi}^0)}{\Gamma(B^+ \rightarrow D\pi^0) + \Gamma(B^- \rightarrow D\bar{\pi}^0)} .$$

These quantities do not require measuring the color-suppressed rate $\Gamma(B^+ \rightarrow D\pi^0)$ and its charge-conjugate. One finds

$$R_\pm = 1 + |A/\bar{A}|^2 \pm 2|A/\bar{A}| \cos \delta \cos \gamma ,$$

$$A_- = -A_+ = |A/\bar{A}| \sin \delta \sin \gamma .$$

In principle, Eqs. (31) provide sufficient information to determine the three parameters $|A/\bar{A}|, \delta$ and $\gamma$, up to certain discrete ambiguities. However, a value $|A/\bar{A}| \sim 0.1$ would be too small to be measured with good precision. One still obtains two interesting bounds

$$\sin^2 \gamma \leq R_\pm ,$$

implying new constraints on $\gamma$. Assuming, for instance, $|A/\bar{A}| = 0.1$, $\delta = 0$, $\gamma = 40^\circ$, one finds $R_- = 0.85$. With $10^8 \ B^+B^-$ pairs, using measured $B$ and $D$ decay branching ratios, one estimates an error $R_- = 0.85 \pm 0.05$. In this case, Eq. (31) excludes the range $73^\circ < \gamma < 107^\circ$ with 90% confidence level. Including measurements of the CP asymmetries $A_\pm$ could further constrain $\gamma$.

3.4 $\phi_2 = \alpha$ from $B \rightarrow \pi\pi$

The phase $\alpha = \pi - \beta - \gamma$ occurs in the time-dependent rate of $B^0(t) \rightarrow \pi^+\pi^-$ and would dominate its asymmetry if only a “tree” amplitude $T$ contributes. A smaller penguin amplitude $P$, which carries a different weak phase, implies a more general form of the time-dependent asymmetry, which includes in addition to the $\sin(\Delta mt)$ term a cos($\Delta mt$) term due to direct CP violation

$$A(t) = a_{dir} \cos(\Delta mt) + \sqrt{1 - a_{dir}^2} \sin 2(\alpha + \theta) \sin(\Delta mt) .$$

This provides two equations for three unknowns, $a_{dir}$, $\theta$ and $\alpha$, which is insufficient for measuring $\alpha$. $a_{dir}$ and $\theta$ can be expressed in terms of $|P/T|$, $\text{Arg}(P/T)$ and $\alpha$. Consequently, knowledge of $|P/T|$ or $\theta$ could provide very useful information about $\alpha$. Applying flavor SU(3) to measured $B \rightarrow \pi\pi$ and $B \rightarrow K\pi$ decay rates one finds $|P/T| = 0.3 \pm 0.1$. QCD based studies of $|P/T|$ obtain a small value around 0.1, however these calculations involve systematic theoretical uncertainties.

A clean way of eliminating the penguin effect is by measuring also the time-integrated rates of $B^0 \rightarrow \pi^0\pi^0$,

$B^+ \rightarrow \pi^+\pi^0$ and their charge-conjugates. One constructs the isospin triangle

$$A(B^0 \rightarrow \pi^+\pi^-)/\sqrt{2} + A(B^0 \rightarrow \pi^0\pi^0) = A(B^+ \rightarrow \pi^+\pi^0)$$

and its charge-conjugate in which $A(B^+ \rightarrow \pi^+\pi^0)$ is a common base. The correction 20 in (32) is given by the angle between $A(B^0 \rightarrow \pi^+\pi^-)$ and $A(B^0 \rightarrow \pi^+\pi^-)$. A tiny electroweak penguin term, forming a very small angle between $A(B^+ \rightarrow \pi^+\pi^0)$ and $A(B^- \rightarrow \pi^-\pi^0)$, can be taken into account analytically.

A small $B \rightarrow \pi^0\pi^0$ branching ratio (probably around $10^{-6}$ but hard to estimate) may be a potential difficulty for separating $B^0$ and $B^0$ decays into $\pi^0\pi^0$. A combined rate measurement for $B^0$ and $B^0$, avoiding the need for flavor tagging, is considerably easier. Such a measurement was shown to imply useful upper bounds on $\theta$ if the combined rate is sufficiently small. An interesting question is whether it can also lead to a lower bound on $\theta$, thereby constraining this angle tightly to a narrow range (and consequently fixing $\alpha$) for some values of the combined $\pi^0\pi^0$ rate and the other measurables.

4 Conclusion

In our brief conclusion we wish to make a few general recommendations for experimentalists. The experimental task becomes harder as we go down the list.

- Measure 

  - crudely as many as possible CP asymmetries in hadronic and radiative $B$ decays. Relative signs of U-spin related asymmetries, which are relatively easily measured, test the CKM picture.
  - Certain asymmetries are predicted by CKM to be very small. Measuring sizable asymmetries in these channels would be signals of new physics.
  - Measure precisely certain charge-averaged rates, which imply interesting constraints on $\phi_3 = \gamma$.
  - Measure precisely those CP asymmetries from which $\phi_1 = \beta, \phi_2 = \alpha$ and $\phi_3 = \gamma$ can be determined in an accurate manner.

The final goal of measuring CP asymmetries in different processes is to test and overconstrain the CKM parameters in a manifold and critical manner, thereby opening a window into new physics.

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