Physics and application of photon number resolving detectors based on superconducting parallel nanowires

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Abstract. The parallel nanowire detector (PND) is a photon number resolving (PNR) detector that uses spatial multiplexing on a subwavelength scale to provide a single electrical output proportional to the photon number. The basic structure of the PND is the parallel connection of several NbN superconducting nanowires (≈100 nm wide, a few nm thick), folded in a meander pattern. PNDs were fabricated on 3–4 nm thick NbN films grown on MgO (T_s = 400 °C) substrates by reactive magnetron sputtering in an Ar/N₂ gas mixture. The device performance was characterized in terms of speed and sensitivity. PNDs showed a counting rate of 80 MHz and a pulse duration as low as 660 ps full-width at half-maximum (FWHM). Building the histograms of the photoresponse peak, no multiplication noise buildup is observable. Electrical and optical equivalent models of the device were developed in order to study its working principle, define design guidelines and develop an algorithm to estimate the photon number statistics of an unknown light. In particular, the modeling provides novel insight into the physical limit to the detection efficiency and to the reset time of these detectors. The PND significantly outperforms existing PNR detectors in terms of simplicity, sensitivity, speed and multiplication noise.

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1. Introduction

Photon number resolving (PNR) detectors are required in the fields of quantum communication, quantum information processing and of quantum optics for two classes of applications. In one case, PNR detectors are needed to reconstruct the incoming photon number statistics by ensemble measurements. This is the case of the characterization of nonclassical light sources such as single photon [1] or $n$-photon [2] state generators or of the detection of pulse splitting attacks in quantum cryptography, where an eavesdropper alters the photon statistics of the pulses [3]. In the second case, PNR detectors are needed to perform a single-shot measurement of the photon number. Applications of this kind are linear-optics quantum computing [4], long distance quantum communication (which requires quantum repeaters [5]) and conditional-state preparation [6].

Among the approaches proposed so far for PNR detection, detectors based on charge-integration or field-effect transistors [7]–[9] are affected by long integration times, leading to bandwidths $< 1$ MHz. Transition edge sensors (TES) [10] show extremely high (95%) detection efficiencies but they operate at 100 mK and show long response times (several hundreds of nanoseconds in the best case). Approaches based on photomultipliers (PMTs) [11] and avalanche photodiodes (APDs), such as the visible light photon counter (VLPC) [2, 12], 2D arrays of APDs [13, 14] and time-multiplexed detectors [15, 16] are not sensitive or are plagued by high dark count rate and long dead times in the telecommunication spectral windows. Arrays of single photon detectors (SPDs) additionally involve complex read-out schemes [14] or separate contacts, amplification and discrimination [17]. We recently demonstrated an alternative approach [18, 19], the parallel nanowire detector (PND), which uses spatial multiplexing on a subwavelength scale to provide a single electrical output proportional to the photon number. The device presented significantly outperforms existing PNR detectors in...
terms of simplicity, sensitivity, speed and multiplication noise. Here, we present the working principle of the device (section 2), a review of fabrication and experimental results (section 3–5), an extensive analysis of the device operation and corresponding design guidelines (section 6) and the first application of a PND to reconstruct unknown incoming photon number statistics (section 7).

2. Photon number resolution principle

The structure of PNDs is the parallel connection of \( N \) superconducting nanowires (N-PND), each of which can be connected in series to a resistor \( R_0 \) (N-PND-R, figure 1(b)). The detecting element is a 4–6 nm thick and 100 nm wide NbN wire folded in a meander pattern. Each section acts as a nanowire superconducting single photon detector (SSPD) \([20]\). In SSPDs, if a superconducting nanowire is biased close to its critical current, the absorption of a photon causes the formation of a normal barrier across its cross section, so almost all the bias current is pushed to the external circuit. In PNDs, the currents from different sections can sum up on the external load, producing an output voltage pulse proportional to the number of photons absorbed.

The time evolution of the device after photon absorption can be simulated using the equivalent circuit of figure 1(a). Each section is modeled as the series connection of a switch that opens on the hotspot resistance \( R_{hs} \) for a time \( t_{hs} \), simulating the absorption of a photon, of an inductance \( L_{kin} \), accounting for kinetic inductance \([21]\) and of a resistor \( R_0 \). The device is connected through a bias \( T \) to the bias voltage source \( V_B \) and to the input resistance of the
preamplifier $R_{\text{out}}$. The $n$ firing sections, in pink, all carry the same current $I_f$ and the $N - n$ still superconducting sections (unfiring), in green, all carry the same current $I_u$. $I_{\text{out}}$ is the current flowing through $R_{\text{out}}$.

Let $I_B$ be the bias current flowing through each section when the device is in the steady state. If a photon reaches the $i$th nanowire, it will cause the superconducting–normal transition with a probability $\eta_i = \eta(I_B/I_{\text{out}}^{(i)})$, where $\eta$ is the current-dependent detection efficiency and $I_{\text{C}}^{(i)}$ is the critical current of the nanowire [20] (the nanowires have different critical currents, being differently constricted [22]). Because of the sudden increase in the resistance of the firing nanowire, its current ($I_f$) is then redistributed between the other $N - 1$ unfiring branches and $R_{\text{out}}$. This argument yields that if $n$ sections fire simultaneously (in a time interval much shorter than the current relaxation time), part of their currents sum up on the external load.

The device shows PNR capability if the height of the current pulse through $R_{\text{out}}$ for $n$ firing stripes $\bar{I}_{\text{out}}^{(n)}$ is $n$ times higher than the pulse for one $\bar{I}_{\text{out}}^{(1)}$, i.e. if the leakage current drained by each of the unfiring nanowires $\delta I_{\text{lk}} = I_u - I_B$ is negligible with respect to $I_B$. The leakage current is also undesirable because it lowers the signal available for amplification and temporarily increases the current flowing through the still superconducting (unfiring) sections, eventually driving them normal. Consequently, $\delta I_{\text{lk}}$ limits the maximum bias current allowed for the stable operation of the device and then the detection efficiencies of the sections. The leakage current depends on the ratio between the impedance of a section $Z_S$ and $R_{\text{out}}$ and it can be reduced by engineering the dimensions of the nanowire (thus its kinetic inductance) and of the series resistor (see section 6). The design without series resistors simplifies the fabrication process, but, as $Z_S$ is lower, $\delta I_{\text{lk}}$ significantly limits the detection efficiency of the device.

3. Fabrication

NbN films 3–4 nm thick were grown on MgO (100) substrates (substrate temperature 400 °C [23]) by reactive magnetron sputtering in an argon–nitrogen gas mixture. Using an optimized sputtering technique, our NbN samples exhibited a superconducting transition temperature of $T_C = 10.5$ K for 40 Å thick films. The superconducting transition width was equal to $\Delta T_C = 0.3$ K.

Both the designs with and without the integrated series resistors were implemented. A SEM picture of a 6-PND-R fabricated on MgO is shown in figure 1(b). The size of the detector active area ($A_d$) ranges from $5 \times 5$ to $10 \times 10 \mu m^2$ with the number of parallel branches ($N$) varying from 4 to 14. The nanowires are 100 nm wide and the filling factor ($f$) of the meander is 40%. The length of each nanowire ranges from 25 to 100 µm.

The three nanolithography steps needed to fabricate the structure have been carried out by using an electron beam lithography (EBL) system equipped with a field emission gun (acceleration voltage 100 kV, 20 nm resolution). In the first step, e-beam lithography is used to define pads (patterned as a 50 Ω coplanar transmission line) and alignment markers on a 450 nm thick polymethyl methacrylate (PMMA; a positive tone electronic resist) layer. The sample is then coated with a Ti–Au film (60 nm Au on 10 nm Ti) deposited by e-gun evaporation, which is selectively removed by lift-off from un-patterned areas. In the second step, a 160 nm thick hydrogen silsesquioxane (HSQ FOX-14, a negative tone electronic resist) mask is defined reproducing the meander pattern. The alignment between the different layers is performed using the markers deposited in the first lithography step. All the unwanted material, i.e. the material
not covered by the HSQ mask and the Ti/Au film, is removed by using a fluorine-based (CHF$_3$ + SF$_6$ + Ar) reactive ion etching (RIE). Finally, with the third step the series resistors (85 nm AuPd alloy, 50% each in weight), aligned with the two previous layers, are fabricated by lift off via a PMMA stencil mask. Our process is optimized to obtain both an excellent alignment between the different e-beam nanolithography steps (error of the order of 100 nm) and a nanowire with high width uniformity ($<10\%$ [24]).

4. Measurement set-up

Electrical and optical characterizations have been performed in a cryogenic probe station with an optical window and in a cryogenic dipstick.

In the cryogenic probe station (Janis), the devices were tested at a temperature $T = 5$ K. Electrical contact was realized by a cooled 50 $\Omega$ microwave probe attached to a micromanipulator, and connected by a coaxial line to the room-temperature circuitry. The light was fed to the PNDs through a single-mode optical fiber coupled to a long working distance objective, allowing the illumination of a single detector.

In the cryogenic dipstick, the devices were tested at 4.2 K. The light was sent through a single-mode optical fiber coupled to a short focal length lens, placed far from the plane of the chip in order to ensure uniform illumination. The number of incident photons per device area was estimated with an error of 5%.

The bias current was supplied through the dc port of a 10 MHz–4 GHz bandwidth bias-$T$ connected to a low noise voltage source in series with a bias resistor. The ac port of the bias-$T$ was connected to the room temperature, low-noise amplifiers. The amplified signal was fed either to a 1 GHz bandwidth single shot oscilloscope or to a 40 GHz bandwidth sampling oscilloscope for time-resolved measurements and statistical analysis. The devices were optically tested using a fiber-pigtailed, gain-switched laser diode at 1.3 $\mu$m wavelength (100 ps long pulses, repetition rate 26 MHz) and a mode-locked Ti:sapphire laser at 700 nm wavelength (40 ps long pulses, repetition rate 80 MHz).

Throughout the paper, the single photon detection efficiency of an $N$-PND ($\tilde{\eta}$) or of one of its sections ($\eta$) are defined with respect to the photon flux incident on the area covered by the device (active area $A_d$, typically $10 \times 10 \mu$m$^2$) or by one section ($A_d/N$), respectively.

5. Device characterization

Figure 2(a) shows a single-shot oscilloscope trace of the photoresponse of a 5-PND under laser illumination ($\lambda = 700$ nm, 80 MHz repetition rate). Pulses with five different amplitudes can be observed, corresponding to the transition of one to five sections. The measured 80 MHz counting rate represents an improvement of three orders of magnitude over most of the PNR detectors at telecom wavelength [7, 14, 25], with the only exception of the SSPD array [17].

On similar devices, the single-photon detection efficiency ($\tilde{\eta}$) at $\lambda = 1.3 \mu$m and the dark-count rate DK were measured as a function of the bias current at $T = 2.2$ K [18]. The lowest DK value measured was 0.15 Hz for $\tilde{\eta} \sim 2\%$, yielding a noise equivalent power (NEP) = $4.2 \times 10^{-18}$ W Hz$^{-1/2}$ [26]. This sensitivity outperforms most of the other approaches by one–two orders of magnitude (with the only exception of transition-edge sensors [25], which require a much lower operating temperature).
We investigated the temporal response of a $10 \times 10 \, \mu m^2$ 4-PND-R probed with light at 1.3 $\mu m$ wavelength using a 40 GHz sampling oscilloscope (figure 2(b)). All four possible amplitudes can be observed. The pulses show a full-width at half-maximum (FWHM) as low as 660 ps. In a traditional $10 \times 10 \, \mu m^2$ SSPD, the pulse width would be of the order of 10 ns FWHM, so the recovery of the output current $I_{out}$ through the amplifier input resistance is a factor $\sim 4^2$ faster (see section 6.2), which agrees with results reported by other groups [27, 28]. As shown in section 6.2, the very attractive $N^2$ scaling rule for the output pulse duration unfortunately does not apply to the device recovery time.

6. PND design

We aim at providing a detailed understanding of the device operation and guidelines for the design of PNDs with optimized performance in terms of efficiency, speed and sensitivity.

The first step is to define the relevant parameter space. The width of the nanowire ($w = 100 \, \text{nm}$) and the filling factor ($f = 50\%$) of the meander are fixed by technology, the thickness of the superconducting film ($t = 4 \, \text{nm}$) is the optimum value yielding the maximum device efficiency and the active area ($A_d = 10 \times 10 \, \mu m^2$) is fixed by the size of the core of single mode fibers to which the device must be coupled. We consider single-pass geometries (no optical cavity), but the same guidelines can be applied to cavity devices with optimized absorption [29]. The parameters of the PND-R that can be used as free design variables are: the number of sections in parallel $N$, the value of the series resistor $R_0$ and the value of the inductance of each section $L_0$. The number of sections in parallel $N$ can be chosen within a discrete set of values ($N = 2, 3, 4, 6, 7, 10$ and $17$), which satisfy the constraints of $w$, $f$, size of the pixel and that the number of stripes in each section is to be odd (we consider the geometry...
Table 1. Inductance ($L_0$) and number of squares (SQ) of each section for all possible values of $N$. The width of the nanowires is $w = 100$ nm, the thickness is $t = 4$ nm. The kinetic inductance per square was estimated ($L_{\text{kin}}/\Box = 90$ pH) from the time constant of the exponential decay of the output current ($\tau_{\text{out}} = \tau_f = L_{\text{kin}}/R_{\text{out}}$, see section 6.2) for a standard $5 \times 5 \mu$m$^2$ SSPD [23].

| $N$ | $L_0$ (nH) | SQ |
|-----|------------|----|
| 2   | 225        | 2500 |
| 3   | 153        | 1700 |
| 4   | 117        | 1300 |
| 6   | 81         | 900  |
| 7   | 63         | 700  |
| 10  | 45         | 500  |
| 17  | 27         | 300  |

...of figure 1(b)). The value of $L_0$ is the sum of the kinetic inductance of each meander $L_{\text{kin}}$ and of a series inductance that can be eventually added. $L_{\text{kin}}$ is not a design parameter, as it is fixed by $w$, $t$, $f$, $A_d$ and $N$. If no series inductors are added (bare devices, $L_0 = L_{\text{kin}}$), the value of $L_0$ for each $N$ is listed in table 1.

An additional free parameter, relative to the read-out, is the impedance seen by the device on the RF section of the circuit $R_{\text{out}}$, which is 50 $\Omega$ (of the matched transmission line) in the actual measurement set-up (see section 4), but which can be varied in principle from zero to infinite introducing a cold preamplifier stage.

The target performance specifications are the single-photon detection efficiency ($\eta$), the signal to noise ratio (SNR) and the maximum repetition rate (speed), which must be optimized under the constraints that the operation of the device is stable and that it is possible to detect a certain maximum number of photons ($n_{\text{max}}$) dependent on the specific application.

A comprehensive description of PND operation should combine thermal and electrical modeling of the nanowires [30]. In this work, a purely electrical model (see section 2 and figure 1(a)) has been used to make a reliable guess on how the performance of the device varies when moving in the parameter space.

In this model, the dependence of $L_{\text{kin}}$ on the current flowing through the nanowire was disregarded, and it was assumed constant. Furthermore, it has been shown [30] that changing the values of the kinetic inductance of an SSPD or of a resistor connected in series to it results in a change of the hotspot resistance and of its lifetime, eventually causing the device to latch to the normal state. The simplified analysis presented here does not take into account these effects, and considers both $R_{\text{hs}}$ and $T_{\text{hs}}$ as constant ($R_{\text{hs}} = 5.5$ k$\Omega$, $T_{\text{hs}} = 250$ ps), and that the device cannot latch. However, the results of this approach can still quantitatively predict the behavior of the device in the limit where the fastest time constant of the circuit $\tau_f$ (see section 6.2) is much higher than the hotspot lifetime ($\tau_f \gg T_{\text{hs}}$), and give a reasonable qualitative understanding of the main trends of variation of the performance of faster devices ($\tau_f \sim T_{\text{hs}}$).

In order to gain a better insight into the circuit dynamics (see section 6.2) and to reduce the calculation time, the $N + 1$ mesh circuit of figure 1(a) can be simplified to the three mesh circuit of figure 3(a) applying the Thévenin theorem on the $n$ firing sections and on the remaining $N - n$ still superconducting (unfiring) sections, separately.
Figure 3. (a) Simplified circuit of an \( N \)-PND-R, where the two sets of \( n \) firing and \( N - n \) unfiring sections have been substituted by their Thévenin equivalents. (b–d) Simulated time evolution of \( I_u \) (b), \( I_{\text{out}} \) (c) and \( I_f \) (d) for a 6-PND-R as \( n \) increases from 1 to 6. The parameters of the circuit are: \( L_0 = L_{\text{kin}} = 81 \) nH, \( R_0 = 50 \) \( \Omega \), \( R_{\text{out}} = 50 \) \( \Omega \), \( R_{\text{hs}} = 5.5 \) k\( \Omega \) and \( t_{\text{hs}} = 250 \) ps.

Figures 3(b)–(d) show the simulation results for the time evolution of the currents flowing through \( R_{\text{out}} \) and through the unfiring \( (I_u) \) and firing \( (I_f) \) sections of a PND with six sections and integrated resistors (6-PND-R) and for the number of firing sections \( n \) ranging from 1 to 6. As \( n \) increases, the peak values of the output current \((I_{\text{out}}, \text{figure 3(b)})\) and of the current through the unfiring sections \((I_u, \text{figure 3(c)})\) increase. The firing sections experience a large drop in their current \((I_f, \text{figure 3(d)})\), which is roughly independent of \( n \). The observed temporal dynamics will be examined in the following sections.

6.1. Current redistribution and efficiency

Let \( \delta I_{1k}^{(n)} \) be the peak value of the leakage current drained by each of the still superconducting (unfiring) nanowires when \( n \) sections fire simultaneously. The stability requirement translates into the condition that for each unfiring section \( I_B + \delta I_{1k}^{(n_{\text{max}})} \leq I_C \) (as the leakage current increases with \( n, \delta I_{1k}^{(n_{\text{max}})} \) represents the worst case). This limits the bias current and therefore the
Figure 4. Peak value of the leakage current $\delta I_{1k}$ drained by each of the still superconducting (unfiring) nanowires (a) and of the output current $I_{out}$ (b) when only one section fires plotted as a function of the number of sections in parallel $N$ and of the value of the inductance of each section $L_0$. The value of the series resistor $R_0$ and of the output resistor $R_{out}$ is 50 $\Omega$. The orange line highlights bare devices, the colored bars correspond to devices that respect the constraints on the geometry of the structure, whereas the gray bars refer to purely theoretical devices that just show the general trend. The leakage current and the output current are expressed in % of the bias current $I_B$ because they are proportional to it.

The leakage current for $n = 1$ is first investigated and its dependence on $n$ is then presented for some particular combinations of design parameters. The dependence of $\delta I_{1k}^{(1)}$ on $N$ and $L_0$ at fixed $R_0$ and $R_{out}$ (both equal to 50 $\Omega$) is shown in figure 4(a): an orange line highlights bare devices ($L_0 = L_{kin}$, see table 1) and the colored bars are relative to devices that respect the constraints on the geometry of the structure ($L_0 > L_{kin}$), while the gray bars refer to purely theoretical devices that just show the general trend. For any $N$, the current redistribution increases with decreasing $L_0$, as the impedance of each section decreases. Keeping $L_0$ constant, $\delta I_{1k}^{(1)}$ decreases with increasing $N$, as the current to be redistributed is fixed and the number of channels draining current increases. For this reason also the increase of redistribution with decreasing $L_0$ becomes weaker for high $N$.

The dependence of $\delta I_{1k}^{(1)}$ on $R_0$ is shown in figure 5(a) for some bare devices and $R_{out} = 50 \, \Omega$. As expected, the redistribution decreases as $R_0$ increases because the impedance of each section increases with respect to the output resistance. For the same reason, $\delta I_{1k}^{(1)}$ is strongly
Figure 5. (a) Variation of the peak value of the leakage current per unfiring section of some bare devices for $n = 1 \left(\delta I_{1k}\right)$ as the resistance of the series resistor $R_0$ varies from 10 to 400 Ω. (b) Peak value of the output current for $n = 1 \left(\overline{I}_{\text{out}}^{(1)}\right)$ as a function of $R_0$ for some bare devices.

Figure 6. Variation of the peak value of the leakage current per unfiring section (a) and of the output current (b) of the set of bare devices for $n = 1 \left(\delta I_{1k}^{(1)}\right)$ and $I_{\text{out}}^{(1)}$, respectively) as the resistance of the output resistor $R_{\text{out}}$ decreases by one order of magnitude from 50 to 5 Ω (in blue and orange, respectively), while $R_0 = 50 \Omega$.

The variation of the leakage current with the number of firing stripes $n = 1 \left(\delta I_{1k}^{(n)}\right)$ for the set of bare devices is presented in figure 7(a). The dependence is superlinear ($\delta I_{1k}^{(n)} > n \delta I_{1k}^{(1)}$), as the current to be redistributed per firing stripe is always the same (see section 6.3), but the number of channels draining current decreases. Furthermore, as expected, the curves for different design parameter sets never cross, which means that all the design guidelines presented in figures 5(a), 6(a), and 7(a) for $n = 1$ still apply for higher $n$.

In conclusion, the result of this simplified analysis is that, in order to minimize the leakage current and thus maximize the efficiency, $N$, $L_0$ and $R_0$ must be made as high as possible and reduced (to $\sim 3\%$ of $I_0$) when $R_{\text{out}}$ is decreased by one order of magnitude from 50 to 5 Ω, keeping $R_0$ constant (figure 6(a)).
\[ R_{\text{out}} \] as low as possible. We note, however, that \( R_0 \) cannot be increased indefinitely to avoid the nanowire latches to the hotspot plateau before \( I_B \) reaches \( I_C \) [23].

6.2. Transient response and speed

Before proceeding to the analysis of the SNR and speed performances of the device, it is necessary to discuss the characteristic recovery times of the currents in the circuit.

The transient response of the simplified equivalent electrical circuit of the N-PND (figure 3(a)) to an excitation produced in the firing branch can be easily found analytically. Therefore, the transient response of the current through the firing sections \( I_f \), through the unfiring sections \( I_u \) and through the output \( I_{\text{out}} \) after the nanowires become superconducting again \( (t \geq t_{\text{hs}}) \) can be written as

\[
\begin{align*}
I_t &\propto \frac{N-n}{N} \exp(-t/\tau_s) + \frac{n}{N} \exp(-t/\tau_f), \\
I_u &\propto \exp(-t/\tau_s) - \exp(-t/\tau_f), \\
I_{\text{out}} &\propto \exp(-t/\tau_f),
\end{align*}
\]

(1)

where \( \tau_s = L_0/R_0 \) and \( \tau_f = L_0/(R_0 + NR_{\text{out}}) \) are the ‘slow’ and the ‘fast’ time constants of the circuit, respectively.

This set of equations describes quantitatively the time evolution of the currents after the healing of the hotspot in the case \( \tau_f \gg t_{\text{hs}} \), and it provides a qualitative understanding of the recovery dynamics of the circuit for shorter \( \tau_f \).

The recovery transients \( (t \geq t_{\text{hs}}) \) of \( I_{\text{out}} \), \( \delta I_{\text{lk}} \) and \( I_f \) for a 4-PND-R simulated with the circuit of figure 3(a) are shown in figures 8(a)–(c), respectively (in blue) for different numbers of firing sections \( (n=1–4) \). As \( n \) increases from 1 to 4, the recoveries of \( I_{\text{out}} \) and \( \delta I_{\text{lk}} \) change only by a scale factor. On the other hand, the transient of \( I_f \) depends on \( n \) and becomes faster with increasing \( n \), as qualitatively predicted by the first of equations (1). Indeed, \( I_f \) consists of the sum of a slow and a fast contribution, whose balance is controlled by the number of firing sections \( n \). To prove the quantitative agreement with the analytical model in the limit \( \tau_f \gg t_{\text{hs}} \), the simulated transients of \( I_{\text{out}} \), \( \delta I_{\text{lk}} \) and \( I_f \) have been fitted (figures 8(a)–(c), respectively, in

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Figure 8. Recovery transients ($t \geq t_{hs}$) of $I_{out}$ (a), $\delta I_{lk}$ (b), and $I_f$ (c) for a 4-PND-R as $n$ increases from 1 to 4. The simulated transients are in blue, the fitted curves are in orange. The parameters of the circuit used for the simulations are: $L_0 = L_{kin} = 117 \text{nH}$, $R_0 = 50 \Omega$, $R_{out} = 50 \Omega$, $R_{hs} = 5.5 \text{k}\Omega$ and $t_{hs} = 250 \text{ps}$. The three sets of curves are fitted by equations (1) (multiplied by $K$ and shifted by $t_0$), where the values of $\tau_s$ and $\tau_f$ are shown in the insets.

In order to quantify the speed of the device, we take $f_0 = (t_{reset})^{-1}$ as the maximum repetition frequency, where $t_{reset}$ is the time that $I_f$ needs to recover to 95% of the bias current after a detection event.

According to the results presented above, which are in good agreement with experimental data (figure 2(b)), $I_{out}$ decays exponentially with the same time constant for any $n$ ($\tau_{out} = \tau_f$), which, for a bare $N$-PND, is $N^2$ times shorter than a normal SSPD of the same surface [27, 28]. This, however, does not relate to the speed of the device. Indeed, $t_{reset}$ is the time that the current through the firing sections $I_f$ needs to rise back to its steady-state value ($I_f \sim I_B$). In the best case of $n = N$, $I_f$ rises with the fast time constant $\tau_f$, but in all other cases the slow
Figure 9. Dependence of $f_0$ on $L_0$ and $R_0$. No data are presented for $f_0 > 4$ GHz, where no reliable predictions can be made using this simplified model.

The speed performance of the device is then limited by the slow time constant ($\tau_s \approx 3 \cdot \tau_s$), which means that an $N$-PND is only $N$ times faster than a normal SSPD of the same surface, being as fast as a normal SSPD whose kinetic inductance is the same as one of the $N$ section of the $N$-PND.

Figure 9 shows the dependence of $f_0$ on $L_0$ and $R_0$. For $\tau_s < t_{hs}$ (i.e. $f_0 > 4$ GHz in our model) the speed of the device may be limited by the hotspot temporal dynamics, and so no reliable predictions can be made using our simplified model.

6.3. SNR

The peak value and the duration of the output current pulse are a function of the design parameters (see below and section 6.2, respectively). As the output pulse becomes faster, amplifiers with larger bandwidth are required and thus electrical noise becomes more important. In order to assess the possibility to discriminate the output pulse from the noise, we define the SNR as the ratio between the maximum of the output current $I_{\text{out}}$ and the rms value of the noise-current at the preamplifier input $I_{\text{n}}$, $\text{SNR} = I_{\text{out}} / I_{\text{n}}$.

The peak value of the output current when $n$ sections fire simultaneously (see figure 3(b), relative to a 6-PND-R) can be written as

$$I_{\text{out}}^{(n)} = n \left( I_B - I_{t}^{(n)*} \right) - (N - n) \delta I_{1k}^{(n)*},$$

where the starred values refer to the time $t = t^*$, when the output current peaks.

As $n = 1$ represents the worst case, in order to evaluate the performance of the device in terms of the SNR, the dependence of $\overline{T_{\text{out}}}^{(1)}$ from the design parameters is investigated:
\( T_{\text{out}}^{(1)}(N, L_0, R_0, R_{\text{out}}) \). The dependence of \( T_{\text{out}}^{(1)} \) on \( N \) and \( L_0 \) at fixed \( R_0 \) and \( R_{\text{out}} \) (both equal to 50 \( \Omega \)) is shown in figure 4(b). Inspecting the values of \( T_{\text{out}}^{(1)} \) and \( \delta I_{1k}^{(1)} \) for the same device in figure 4, it becomes clear that they add up to a value well above \( I_B \), which is due to the fact that the output current and of the leakage current peak at two different times, \( t^* \) and \( t_{1k} \), respectively (figure 3). Furthermore, as \( t_{1k} > t^* \), the output current is not significantly affected by redistribution, because \( I_{\text{out}} \) is maximum when \( \delta I_{1k} \) is still beginning to rise.

The expression for \( t_{1k} \) can be derived from equation (1): \( t_{1k} = L_0/(N \cdot R_{\text{out}}) \ln(1 + N \cdot R_{\text{out}}/R_0) \), which means that increasing the device speed (decreasing \( L_0 \) or \( R_0 \)), \( N \) or \( R_{\text{out}} \) makes the redistribution faster and then \( T_{\text{out}}^{(1)} \) lower.

So, for any given \( N \), \( T_{\text{out}}^{(1)} \) decreases (figure 4(b)) with decreasing \( L_0 \), both because \( \delta I_{1k}^{(1)} \) is higher and because \( t_{1k} \) is lower. Keeping \( L_0 \) constant, \( T_{\text{out}}^{(1)} \) decreases with increasing \( N \) because even though \( \delta I_{1k}^{(1)} \) decreases, the redistribution peaks earlier and the number of channels draining current increases.

The dependence of \( T_{\text{out}}^{(1)} \) on \( R_0 \) is shown in figure 5(b) for some bare devices and \( R_{\text{out}} = 50 \Omega \). Even though \( \delta I_{1k}^{(1)} \) decreases as \( R_0 \) increases (figure 5(a)), the output current decreases due to the redistribution speed-up (decrease of \( t_{1k} \)): \( \delta I_{1k}^{(1)} \) increases despite of the decrease of the peak value of the leakage current. On the other hand, a decrease in \( R_{\text{out}} \) makes the redistribution much less effective, as \( t_{1k} \) decreases slower with decreasing \( R_{\text{out}} \) than with increasing \( R_0 \). Indeed, as shown in figure 6(b) for bare devices, \( T_{\text{out}}^{(1)} \) significantly increases when \( R_{\text{out}} \) is decreased by one order of magnitude from 50 to 5 \( \Omega \), keeping \( R_0 \) constant.

In conclusion, in order to maximize the output current, \( N \), \( R_0 \) and \( R_{\text{out}} \) must be minimized, while \( L_0 \) must be made as high as possible.

The rms value of the noise-current at the preamplifier input \( I_n \) can be written as \( I_n = \sqrt{S_n \Delta f} \), where \( S_n \) is the noise spectral power density of the preamplifier and \( \Delta f \) is the bandwidth of the output current \( I_{\text{out}} \), which is estimated as \( \Delta f = 1/\tau_{\text{out}} \), where \( \tau_{\text{out}} = \tau_t = L_0/(R_0 + NR_{\text{out}}) \) is the time constant of the exponential decay of \( I_{\text{out}} \) (see section 6.2). \( I_n \) is then a function of the parameters of the device and of the read-out through \( S_n \) and \( \tau_t \). As \( S_n \) varies consistently with the type of preamplifier used, in our analysis of \( I_n \) we take into account only the variation of \( \Delta f \). Assuming that \( S_n \) is constant, \( I_n \) is thus minimized, minimizing \( N \), \( R_0 \) and \( R_{\text{out}} \) and maximizing \( L_0 \).

The same optimization criteria apply then naturally to the SNR. The dependence of \( T_{\text{out}}^{(1)} \sqrt{\Delta f} \) on \( N \) and \( L_0 \) for an input resistance of \( R_{\text{out}} = 50 \Omega \) is shown in figure 10.

The main design guidelines that can be deduced from the analysis of sections 6.1–6.3 are summarized in table 2. The type of dependence of \( \delta I_{1k} \), \( f_0 \), \( T_{\text{out}} \) and \( \Delta f \) on the design parameters \((L_0, R_0, R_{\text{out}}, N)\) is indicated.

7. Application to the measurement of photon number statistics

We wish to determine whether the PND can be used to measure an unknown photon number probability distribution. Indeed, the light statistics measured with a PND differ from the original due to non-idealities such as the limited number of sections and limited and non-uniform efficiencies \((\eta_i)\) of the different sections. As a PND can be modeled as a balanced lossless \( N\)-port beam splitter, every channel terminating with a SPD [19], the input–output transformation can be formalized as follows.
Figure 10. $I_{\text{out}}^2 \sqrt{\Delta f}$ as a function of $N$ and $L_0$.

Table 2. Dependence of $\delta I_{1k}$, $f_0$, $I_{\text{out}}$ and $\Delta f$ on the design parameters: increasing with increasing the parameter (↗), decreasing with increasing the parameter (↘), independent (–).

| $L_0$ | $R_0$ | $R_{\text{out}}$ | $N$ |
|-------|-------|------------------|-----|
| $\delta I_{1k}$ | ↘ | ↘ | ↗ |
| $f_0$ | ↘ | – | – |
| $I_{\text{out}}$ | ↘ | ↘ | ↘ |
| $\Delta f$ | ↘ | ↗ | ↗ |

Let an $N$-PND be probed with a light whose photon number probability distribution is $S = [S(m)] = [s_m]$, and its output be sampled $H$ times. The result of the observation can be of $N + 1$ different types (i.e. 0, . . . , $N$ stripes firing), so a histogram of the $H$ events can be constructed, which can be represented by a $(N + 1)$-dimensional vector $r = [r_i]$, where $r_i$ is the number of runs in which the outcome was of the $i$th type. The expectation value of the statistics obtained from the histogram is $E[Q_{\text{ex}} = r/H] = Q$, where $Q = [Q(n)] = [q_n]$ is the probability distribution of the number of measured photons.

$Q(n)$ is related to the incoming distribution $S(m)$ by the relation:

$$Q(n) \sum_m P^N(n|m) \cdot S(m),$$

where $P^N(n|m)$ is the probability that $n$ photons are detected when $m$ are sent to the device. Equation (2) may be rewritten in a matrix form as $Q = P^N \cdot S$, where $P^N = [P^N(n|m)] = [p_{nm}^N]$ is the matrix of the conditional probabilities.

It has been shown [31, 32] that an unknown incoming photon number distribution $S$ can be recovered if $Q$ and $P^N$ are known.

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Figure 11. Conditional probability matrix for a $8.6 \times 8 \mu m^2$ 5-PND (with no integrated series resistors), calculated from the vector $\eta$ of the five single-photon detection efficiencies (relative to $T = 4.2 \, K$, $\lambda = 700 \, nm$) of the different sections of the device (inset).

7.1. Matrix of conditional probabilities

The matrix of the conditional probabilities of an $N$-PND depends only on the vector of the $N$ single-photon detection efficiencies of the different sections of the device $\eta = [\eta_i]$ through the relations presented in [18, 19]. The vector $\eta$ can be then determined from the statistics $Q_{ex}$ measured when probing the device with a light of known statistics $S$.

For example, using a laser light source with Poissonian photon number probability distribution, the probability distribution of the number of measured photons $Q$ (expressed by (2)) was fitted to the experimentally measured distribution $Q_{ex}$ using $\eta$ as a free parameter. The resulting $\eta$ and matrix of the conditional probabilities are shown in figure 11 for a 5-PND at $\lambda = 700 \, nm$.

The $P^N$ matrix provides a full description of the detector. Once $P^N$ is known, several approaches can be used to reconstruct $S$ from the histogram $r$. In the case no assumptions on the form of $S$ are made, the maximum likelihood (ML) method is the most suitable, as it is the most efficient in solving this class of problems [33].

7.2. ML method

Let $R = R_0, \ldots, R_N$ be the random vector of the populations of the $(N+1)$ different bins of the histogram after $H$ observations. The joint probability density function $L(r|Q)$ for the occurrence of the particular configuration $r = r_0, \ldots, r_N$ of $R$ is called the likelihood function of $r$ and it is
given by [33]:

\[
L \left( \mathbf{r} | \mathbf{Q} \right) = H! \prod_{i=0}^{N} \frac{q_i^{r_i}}{r_i!},
\]

where \( \mathbf{Q} = [q_i] \) is the probability distribution of the number of measured photons, i.e. the vector of the probabilities to have an outcome in the bin \( i (i = 0, \ldots, N) \) in a single trial.

Considering \( \mathbf{Q} \) as a function of \( \mathbf{S} \) through equation (2), we can rewrite the likelihood function of the vector \( \mathbf{r} \), depending on the parameter \( \mathbf{S} \):

\[
L \left( \mathbf{r} | \mathbf{S} \right) = H! \prod_{i=0}^{N} \frac{\left( \sum_{m} p_{i,m}^N s_m \right)^{r_i}}{r_i!},
\]

which is then the probability of the occurrence of the particular histogram \( \mathbf{r} \) when the incoming light has a certain statistics \( \mathbf{S} \).

As \( \mathbf{r} \) is measured and then it is known, \( L \left( \mathbf{r} | \mathbf{S} \right) \) can be regarded as a function of \( \mathbf{S} \) only, i.e. \( L \left( \mathbf{r} | \mathbf{S} \right) \) is the probability that a certain vector \( \mathbf{S} \) is the incoming probability distribution when the histogram \( \mathbf{r} \) is measured. The best estimate of the incoming statistics that produced the histogram \( \mathbf{r} \) according to the ML method is the vector \( \mathbf{S}_e \), which maximizes \( L \left( \mathbf{r} | \mathbf{S} \right) \), where \( \mathbf{r} \) is treated as fixed. So, the estimation problem can in the end be reduced to a maximization problem.

For numerical calculations, it is necessary to limit the maximum number of incoming photons to \( m_{\text{max}} \) (in the following calculations, \( m_{\text{max}} = 21 \)). As \( \mathbf{S} \) is a vector of probabilities, the maximization must be carried out under the constraints that the \( s_n \) are positive and that they add up to one. The positivity constraint can be satisfied changing variables: \( s_n = \sigma_n^2 \). Instead of \( L \), we maximize the logarithm of \( L \):

\[
l(\Sigma) = \ln(L(\Sigma)) = \ln(C) + \sum_{i=0}^{N} r_i \ln \left( \sum_{m=0}^{m_{\text{max}}} p_{i,m}^N \sigma_m^2 \right),
\]

where \( \Sigma = [\sigma_n] \) and \( C \) is a constant.

The condition that the \( s_n \) add up to one can be taken into account using the Lagrange multipliers method:

\[
F(\Sigma, \alpha) = l(\Sigma) - \alpha \left[ \sum_{m=0}^{m_{\text{max}}} \sigma_m^2 - 1 \right].
\]

After developing [34] the set of \( m_{\text{max}} + 2 \) gradient equations \( \nabla F(\Sigma, \alpha) = 0 \), we obtain that \( \alpha = H \) and that the set of \( m_{\text{max}} + 1 \) nonlinear equations to be solved with respect to \( \Sigma \) is

\[
\sigma_l \left[ \sum_{i=0}^{N} \frac{r_i p_{i,l}^N}{\sum_{m=0}^{m_{\text{max}}} p_{i,m}^N \sigma_m^2} - H \right] = 0,
\]

for \( l = 0, \ldots, m_{\text{max}} \). The set of equations (6) can be solved by standard numerical methods.
7.3. ML reconstruction

To test the effectiveness of the reconstruction algorithm, a $8.6 \times 8 \mu m^2$ 5-PND was tested with the coherent emission from a mode-locked Ti:sapphire laser, whose photon number probability distribution is approximately Poissonian ($S(m) = \mu^m \cdot \exp(-\mu)/m!$). Therefore, $S$ could be determined by measuring the mean photon number per pulse $\mu$ with a power meter. To determine $Q_{ex}$, histograms of the photoresponse voltage peak were built for varying $\mu$. The signal from the device was sent to the 1 GHz oscilloscope, which was triggered by the synchronization generated by the laser unit. The photoresponse was sampled for a bin time of 5 ps, making the effect of dark counts negligible.

The device was characterized in terms of its conditional probability matrix $P^S$ ([18, 19], figure 11), so it was possible to carry out the ML estimation of the different incoming distributions with which the device was probed. Because of the bound on the number of incoming photons which can be represented in our algorithm ($m_{\text{max}} = 21$) and as, for a coherent state, losses simply reduce the mean of the distribution, the ML estimation was performed considering $\mu^* = \mu/10$ and $\eta^* = 10\eta$ (the efficiency of each section being < 10%).

Figure 12 shows the experimental probability distribution of the number of measured photons $Q_{ex}$ obtained from the histograms measured when the incoming mean photon number is $\mu = 1.5, 2.8$ and 4.3 photons pulse$^{-1}$ (figures 12(a)–(c), respectively, in orange), from which the incoming photon number distribution is reconstructed. The ML estimate of the incoming probability distribution $S_e$ is plotted in figures 12(d)–(f) (light blue), where it is compared with the real incoming probability distribution $S$ (green). The estimation is successful only for low photon fluxes ($\mu = 1.5$ and 2.8; figures 12(d) and (e)) and it fails already for $\mu = 4.3$ (figure 12(f)). In figures 12(a)–(c), $Q_{ex}$ (orange) is compared with the ones obtained from $S$ and $S_e$ through relation (2) ($Q_e$, $Q_{ex}$ in green and light blue, respectively).

The main reasons why the reconstruction fails are not the low efficiencies of the sections of the PND or the spread in their values, but rather the limited counting capability ($N = 5$) and a poor calibration of the detector, i.e. an imperfect knowledge of its real matrix of conditional probabilities. This assessment is supported by the following argument. If we generate $Q_{ex}$ with a Monte-Carlo simulation [19] using the same $\eta$ vector of figure 11 and a Poissonian or thermal incoming photon number distribution and then we run the ML reconstruction algorithm (using the same $P^S$, which this time describes the detector perfectly), $S$ can be estimated up to much higher mean photon numbers. However, to alleviate this problem, a self-referencing measurement technique might be used [35].

8. Conclusion

A new PNR detector, the PND, has recently proven to significantly outperform existing approaches in terms of sensitivity (NEP = $4.2 \times 10^{-18}$ W Hz$^{-1/2}$), speed (80 MHz count rate) and multiplication noise [18, 19] in the telecommunication wavelength range.

An electrical equivalent circuit of the device was developed in order to study its operation and to perform its design. In particular, we found that the leakage current significantly affects only the PND detection efficiency, whereas it has a marginal effect on its SNR. Furthermore, in order to gain a better insight into the device dynamics, the $(N+1)$-mesh equivalent circuit of the $N$-PND was simplified and reduced to a three-mesh circuit, so that the analytical expression of its transient response could be easily found. With this approach, we could predict a physical limit.
Figure 12. (a)–(c) Probability distribution of the number of measured photons obtained from experimental data $Q_{ex}$ (orange), from $S$ ($Q$, in green) and $S_e$ ($Q_e$, in light blue) through relation (2) for $\mu = 1.5, 2.8$ and 4.3 photons pulse$^{-1}$, respectively. (d)–(f) Real incoming probability distribution $S$ (green) and its ML estimate $S_e$ (light blue) for $\mu = 1.5, 2.8$ and 4.3 photons pulse$^{-1}$, respectively. The 8.6 $\times$ 8 $\mu$m$^2$ 5-PND was tested under uniform illumination in a cryogenic dipstick dipped in a liquid He bath at 4.2 K. The light pulses at 700 nm from a mode-locked Ti:sapphire laser were 40 ps wide (after propagation in the optical fiber) and the repetition rate was 80 MHz. The average input photon number per pulse $\mu$ was set with a free space variable optical attenuator.
to the recovery time of the PND, which is slower than that previously estimated. Additionally, the figures of merit of the device performance in terms of efficiency, speed and sensitivity ($\delta I_{1k}$, $f_0$, SNR) were defined and their dependence on the design parameters ($L_0$, $R_0$, $R_{out}$, $N$) was analyzed.

In order to prove the suitability of the PND to reconstruct unknown light statistics by ensemble measurements, we developed an ML estimation algorithm. Testing a 5-PND with Poissonian light, we found the reconstruction of the incoming photon number probability distribution to be successful only for low photon fluxes, most likely due to the limited counting capability ($N = 5$) and the poor calibration (i.e. the imperfect knowledge of the real matrix of conditional probabilities) of the detector used, and not to its low detection efficiency ($\eta \sim 3\%$). Additional simulations will be needed to evaluate the performance of our detector for the measurement of other, nonclassical photon number distributions. Finally, despite the high sensitivity and speed of PNDs, their present performance in terms of detection efficiency ($\eta = 2\%$ at $\lambda = 1.3\ \mu m$) does not allow their application to single shot measurements, as required for linear-optics quantum computing [4], quantum repeaters [5] and conditional-state preparation [6]. Nevertheless, the $\eta$ of SPDs based on the same detection mechanism can be increased to $\sim 60\%$ [29], and could potentially exceed 90% using optimized optical cavities.

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