A preference for Dynamical Dark Energy?

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Dynamical Dark Energy (DDE) is a possible solution to the Hubble tension. This work analyses the combination of Baryon Acoustic Oscillation (BAO) data that include 19 points from the range 0.11 \leq z \leq 2.40 and additional points from the Cosmic Microwave Background (CMB) Distant Priors. We test equation of states with a Linear, CPL and a Logarithm dependence on the redshift. DDE seems to be stronger than the standard \Lambda CDM model statistically using different selection criteria. The result calls for new observations and stimulates the investigation of alternative theoretical models beyond the standard model. We also test the same dataset on the \Omega_k CDM model but for it, \Lambda CDM gives better statistical measures.

INTRODUCTION

The Hubble constant $H_0$ measures the current expansion rate of the Universe \cite{1, 2}. An estimation of $H_0$ from direct measurements from the late universe \cite{3–7} yields $H_0 = 73.2 \pm 1.3 \text{km/sec/Mpc}$. Other method uses the measurements of temperature and polarization anisotropies in the Cosmic Microwave Background (CMB), in combination with low-redshift probes of the expansion history, such as Baryon Acoustic Oscillation (BAO) or SNeIa distance measurements \cite{8–15} with $H_0 = 67.27 \pm 0.66 \text{km s}^{-1} \text{Mpc}^{-1}$. The discrepancy between the $H_0$ measurements is one of the fundamental problems in cosmology \cite{16–20}. The question whether the dark energy is a constant energy density or with a dynamical behavior is modeled in different theories \cite{18, 21, 22, 23, 24, 25}. This motivates a host of Dynamical Dark Energy (DDE) parametrisations \cite{26–28} in order to search for deviations from $\Lambda$ in observational data. The DDE also may resolve the Hubble tension, particularly for the Early Dark Energy models \cite{29–32}. This letter combines the CMB (distant priors) and the BAO measurements to present a preference for some DDE model.

THEORY

A Friedmann-Lemaître-Robertson-Walker metric with the scale parameter $a = 1/(1+z)$ is considered, where $z$ is the redshift. The evolution of the universe for it is governed by the Friedmann equation which connects the equation of the state for ACDM background:

$$E(z)^2 = \Omega_r (1+z)^4 + \Omega_m (1+z)^3 + \Omega_k (1+z)^2 + \Omega_A(z),$$  \hspace{1cm} (1)

with the expansion of the universe $E(z)^2 = H(z)/H_0$, where $H(z) := \dot{a}/a$ is the Hubble parameter at redshift $z$ and $H_0$ is the Hubble parameter today. $\Omega_r$, $\Omega_m$, $\Omega_A$ and $\Omega_k$ are the fractional densities of radiation, matter, dark energy and the spatial curvature at redshift $z = 0$. Even though the radiation density at the redshifts probed by BAO is negligible, we include it as $\Omega_r = 1-\Omega_m-\Omega_\Lambda-\Omega_k$. The spatial curvature is expected to be zero for a flat Universe, $\Omega_k = 0$. We can expand this simple model by considering a dark energy component depending on $z$. This can be done with a generalization of the Chevallier-Polarski-Linder (CPL) parametrization \cite{33–35}:

$$\Omega_A(z) = \Omega_A^{(0)} \exp \left[ \int_0^z \frac{3(1+w(z'))dz'}{1+z'} \right]$$  \hspace{1cm} (2)

in which we consider three possible models:

$$w(z) = \begin{cases} 
    w_0 + w_a z & \text{Linear} \\
    w_0 + w_a \frac{z}{\pi+1} & \text{CPL} \\
    w_0 - w_a \log (z+1) & \text{Log}
\end{cases}$$  \hspace{1cm} (3)

which recover $\Lambda$CDM for $w_0 = -1, w_a = 0$.

The distance priors provide effective information of CMB power spectrum in two aspects: the acoustic scale $l_A$ characterizes the CMB temperature power spectrum in the transverse direction, leading to the variation of the peak spacing, and the "shift parameter" $R$ influences the CMB temperature spectrum along the line-of-sight direction, affecting the heights of the peaks. The popular definitions of the distance priors are \cite{36}:

$$l_A = (1+z_s) \frac{\pi D_A(z_s)}{r_s(z_s)} \quad R \equiv (1+z_s) \frac{D_A(z_s) \sqrt{\Omega_m H_0}}{c},$$  \hspace{1cm} (4)

where $z_s$ is the redshift at the photon decoupling epoch. Here we use the values of $z_s \approx 1089$ given by the Planck 2018 chains. $r_s$ is the comoving sound horizon. The angular diameter distance $D_A$ is given by

$$D_A = \frac{c}{(1+z)H_0 \sqrt{\Omega_k}} \sin \left[ \Omega_k^{1/2} \int_0^z \frac{dz'}{E(z')} \right],$$  \hspace{1cm} (5)

where $\sin(x) \equiv \sin(x)$, $x$, $\sinh(x)$ for $\Omega_k < 0$, $\Omega_k = 0$, $\Omega_k > 0$ respectively. \cite{37} derives the distance priors in
we use the open-source package Polychord [51] with the GetDist package [52] to present the results. Furthermore, we use mpmath to increase the digits of precision to 32. The prior is a uniform distribution for all the quantities: \( \Omega_m \in [0; 1], \Omega_\Lambda \in [0; 1 - \Omega_m], \Omega_r \in [0; 1 - \Omega_m - \Omega_\Lambda], c/(H_0 r_d) \in [25; 35], w_0 \in [-1.5; -0.5] \) and \( w_a \in [-0.5; 0.5] \). Since the distance prior is defined at the decoupling epoch \((z_s)\) and the BAO - at drag epoch \((z_d)\), we parametrize the difference between \( r_s(z_s) \) and \( r_a(z_d) \) as \( r_s = r_d \times \text{rat} \), where the prior for the ratio is \( \text{rat} \in [0.9; 1.1] \). For testing the \( \Omega_k \) model we use as a prior \( \Omega_k \in [-0.1; 0.1] \). The priors of the other parameters are \( c/(H_0 r_d) \in (25, 35), \Omega_m \in [0.2, 1 - \Omega_k], \Omega_\Lambda \in [0, 1 - \Omega_m - \Omega_k], r_d/r_s \in [0.9, 1.1] \).

To avoid the possible correlations in the BAO dataset, we use the methodology in [53, 54]. This method avoids the use of N-body mocks to find the covariance matrices due to systematic errors and replaces it with an evaluation of the effect of possible small correlation on the final result. The covariance matrix for uncorrelated points is: \( C_{ii} = \sigma_i^2 \). Since we’re trying to simulate correlations, we introduce a number of randomly selected nondiagonal elements in the covariance matrix while keeping it symmetric. The positions of the non-diagonal elements are chosen randomly and the magnitude of those randomly selected covariance matrix element \( C_{ij} \) is set to \( C_{ij} = 0.5\sigma_i\sigma_j \), where \( \sigma_i,\sigma_j \) are the published 1\( \sigma \) errors of the data points \( i,j \). We introduce positive correlations in up to 6 pairs of randomly selected data points (more than 25% of the data). The figures in the appendix show the cornerplots with different randomized points for all the models we employ in this article. The best fit values of the corresponding \( c/(H_0 r_d), \Omega_m \) and \( \Omega_\Lambda \) are shown in the table below. The mean values as well as the errors are not affected significantly from adding a random covariance inside which indicates that we can consider the chosen set of BAO points for effectively uncorrelated. This means that even if there exists a correlation between them in the chosen limit, it wouldn’t change the presented results significantly. Furthermore, we see that while using a model with larger number of inferred parameters increases the errors, the difference in the mean values remains below 10% on average.

\section*{RESULTS}

Table I and Fig. 1 shows the final values obtained by running MCMC on the selected priors. All the models give very similar values for the physical quantities \( c/(H_0 r_d), \Omega_m \) and \( \Omega_\Lambda \), despite the very wide prior imposed on \( \Omega_m \). This comes in contrast with our previous study [54] where the inferred value of \( \Omega_m \) was very sensitive to the result for \( r_d \). Since in the present work we avoid the degeneracy between \( r_d \) and \( H_0 \) by considering the combined quantity \( c/(H_0 r_d) \) this leads to a very strict limit on \( \Omega_m \). From the table it is clear that the

\section*{METHOD}

The dataset we are using is a collection of points from different BAO observations [13, 38–49], to which we add the CMB distant prior [50]. We use a Monte Carlo Markov Chain (MCMC) nested sampler to find the best fit. Concretely, we use the open-source package Planck 2018 TT,TE,EE + lowE which is the latest CMB data from the final full-mission Planck measurement [15].

![FIG. 1. The 2D contour plot of \( c/(H_0 r_d), \Omega_m \) and \( \Omega_\Lambda \) with different parametrization of DDE. The upper panel shows the results for \( \Omega_m \) vs. \( c/(H_0 r_d) \) and the lower panel shows the results for \( w \) and \( w_a \). \( \Lambda\text{CDM} \) corresponds to \( w_0 = -1 \) and \( w_a = 0 \) which is outside the distributions and give strong sign for DDE. The grey lines show the 1\( \sigma \) and 2\( \sigma \) of \( \Omega_m \) and \( c/(H_0 r_d) \) as measured by Planck 2018, while on the bottom plot the gray cross shows where we recover \( \Lambda\text{CDM} \).](image-url)
TABLE I. The posterior distribution for $c/(H_0 r_d)$, $\Omega_m$, $\Omega_\Lambda$, $\Omega_K$, $w_0$, $w_a$ for different parametrization of DDE.

| Model   | $c/(H_0 r_d)$ | $\Omega_m$ | $\Omega_\Lambda$ | $\Omega_K$ | $w_0$ | $w_a$ |
|---------|---------------|------------|-------------------|------------|-------|-------|
| ACDE    | 29.6 ± 0.3    | 0.331 ± 0.02 | 0.668 ± 0.019   | 0.000      | -1.000 | 0     |
| Linear  | 29.1 ± 0.8    | 0.344 ± 0.019 | 0.655 ± 0.02   | 0.000      | -1.125 ± 0.231 | -0.237 ± 0.166 |
| CPL     | 29.3 ± 0.8    | 0.328 ± 0.022 | 0.671 ± 0.025  | 0.000      | -1.087 ± 0.186 | 0.079 ± 0.322   |
| Log     | 29.2 ± 0.8    | 0.326 ± 0.025 | 0.673 ± 0.026  | 0.000      | -1.096 ± 0.241 | -0.02 ± 0.244   |
| $\Omega_k$ CDM | 29.1 ± 0.3 | 0.349 ± 0.018 | 0.745 ± 0.018  | -0.094 ± 0.003 | -1.000 | 0     |

TABLE II. DIC and BF selection criteria for different models compared to the default $\Lambda$CDM model.

| Model   | DIC | $\Delta$DIC | BF  |
|---------|-----|-------------|-----|
| ACDE    | 23.4|             |     |
| Linear  | 23.1| 0.35        | 1.1 |
| CPL     | 23.5| -0.05       | 0.3 |
| Log     | 23.6| -0.11       | 0.4 |
| $\Omega_k$ CDM | 23.2| 0.21        | 121 |

TABLE III. The tension between the models and the combined observations.

| Model   | Planck 2018 | R20 + H0LiCOW |
|---------|-------------|----------------|
| ACDE    | 2.18        | 0.58           |
| Linear  | 1.34        | 0.26           |
| CPL     | 1.27        | 0.36           |
| Log     | 1.23        | 0.32           |
| $\Omega_k$ CDM | 4.03 | 0.31 |

The current dataset favors $w \leq -1$ and also $\Omega_k < 0$. In all cases the ratio $r_d \sim 0.915$ placing the drag horizon at about $r_d = 161 Mpc$ if one assumes $r_0 = 146.8 Mpc$ which is about 5% higher than the expected value.

As for $\Omega_k$ CDM we see that the error on $\Omega_k$ seems very small. According to our observations, if one uses a smaller prior still bracketing the zero (i.e. $[-0.05, 0.005]$), the inferred value for $\Omega_k$ will tend to the left (negative) limit. We didn’t use that smaller prior as it results in larger reduced $\chi^2$ and also, it implies the assumption of a flat universe.

To compare the different models, we use well-known statistical measures. The results can be seen in Table 2. When comparing the models, we do not use the standard AIC and BIC measures, since for small datasets, they are dominated by the number of parameters in the model (which are 4 for ACDE, 6 for the DE models and 5 for $\Omega_k$ CDM). For this reason, we use deviance information criterion (DIC) and the Bayes Factor (BF) which rely on the numerically evaluated likelihood and evidence, making them more unbiased. The DIC criterion, just like the AIC, selects the best model to be the one with the minimal value of the DIC measurement. The reference table we use for DIC is: $\Delta$DIC > 10 shows strong support for the model with lower DIC, $\Delta$DIC = 5 – 10 shows substantial support for the model with lower DIC, $\Delta$DIC < 5 gives ambiguous support for the model with lower DIC. In our case, we have 2 DDE models with DIC smaller than that of ACDE, but all of the $|\Delta$DIC| < 5 (or even 2, which is the limit for AIC), showing that the evidence distinguishing any of the models is very weak. The same applies for $\Omega_k$ CDM.

The Bayes factor, calculated always as the ratio between the evidence of ACDE over one of the other models, definitely shows negative support for the ACDE model (having BF<1) compared to the CPL and the Log models. Following Jeffrey’s scale [55], the evidence in support of the DDE models is on the limit of substantial (this is the conservative limit, some runs gave strong to very strong support for DDE). The support for $\Omega_k$ CDM is very weak, signs of which have been seen also elsewhere [54, 56, 57]. Interestingly, the $\Omega_k$ value excludes zero at the 95% confidence interval (again due to the very small error on the inferred $\Omega_k$).

The dataset combines the $H_0$ and the $r_d$ in one quantity. Therefore we estimate the new variable $c/(H_0 r_d) \sim 30$. Fig 2 shows the values of the $c/(H_0 r_d)$ for different models vs. the result from Planck 30.24 ± 0.08 and the combined result of 28.67 ± 1.47. The tension between these measurements is 1.065σ. Table shows the tensions rate (in number of sigmas) between our fit and these of the early Planck 2018 estimation and the late time esti-
mation from R20 + H0LicoW + BBN. The DDE models reduce the tension with the tension from the ΛCDM model both from the Planck 2018 data and the late time estimation. However we cannot state that a complete solution to the cosmic tension is achieved.

**DISCUSSION**

This paper is raising the possibility for DDE preference over ΛCDM, using the DIC and the BF comparison seems that there is a strong support for CPL or Log DDE over ΛCDM. This is particularly interesting considering the wide priors we use, the fact the DDE models have more parameters and that we have one additional parameter – the ratio between the two sound horizons. DDE models are particularly interesting in connection to modified gravity theories.

Papers have shown possible extensions to the standard model: [58] gives a strong preference for DDE from some earlier BAO dataset. [59] shows that a combined analysis of CMB anisotropy and the luminosity distance data excludes a flat universe. [60, 61] do not detect any significant deviation from GR based on DES and BOSS combined measurements. Combined with our results, there is a possibility for DDE as a viable extension of the standard model and future experiments should shed more light whether it is the best model.

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Complete Cornerplots
FIG. 3. The covariance test plot for the $\Lambda$CDM model.
FIG. 4. The covariance test plot for the Linear model.
FIG. 5. The covariance test plot for the CPL model.

| CPL | $c/(H_0 r_d)$ | $\Omega_m$ | $\Omega_\Lambda$ | $w_0$ | $w_a$ | $r_d/r_s$ |
|-----|---------------|------------|----------------|------|------|----------|
| $n = 0$ | 29.4 ± 0.8 | 0.316 ± 0.034 | 0.683 ± 0.035 | -1.040 ± 0.256 | 0.076 ± 0.330 | 0.923 ± 0.019 |
| $n = 4$ | 28.9 ± 0.9 | 0.325 ± 0.024 | 0.674 ± 0.028 | -1.180 ± 0.229 | 0.108 ± 0.325 | 0.916 ± 0.014 |
| $n = 6$ | 28.9 ± 0.7 | 0.329 ± 0.022 | 0.670 ± 0.022 | -1.197 ± 0.205 | 0.108 ± 0.350 | 0.914 ± 0.011 |
FIG. 6. The covariance test plot for the Log model.
\[ \Omega_k_{\Lambda CDM}, n=0 \]
\[ \Omega_k_{\Lambda CDM}, n=4 \]
\[ \Omega_k_{\Lambda CDM}, n=6 \]

\[ \Omega \]
\[ k_{CDM} \]
\[ c/(H_0 r_d) \]
\[ \Omega_m \]
\[ \Omega_\Lambda \]
\[ r_d/r_s \]

\[ n = 0 \quad 29.3 \pm 0.3 \quad -0.094 \pm 0.004 \quad 0.359 \pm 0.022 \quad 0.735 \pm 0.020 \quad 0.917 \pm 0.01 \]
\[ n = 4 \quad 29.4 \pm 0.3 \quad -0.095 \pm 0.004 \quad 0.363 \pm 0.023 \quad 0.731 \pm 0.025 \quad 0.916 \pm 0.013 \]
\[ n = 6 \quad 29.2 \pm 0.3 \quad -0.094 \pm 0.004 \quad 0.351 \pm 0.019 \quad 0.743 \pm 0.017 \quad 0.921 \pm 0.016 \]

**FIG. 7.** The covariance test plot for the \( \Omega_k \Lambda CDM \) model.