Did Herbert Fröhlich predict or postdict the isotope effect in superconductors?

J E Hirsch

Department of Physics, University of California, San Diego, La Jolla, CA 92093-0319, USA
E-mail: jhirsch@ucsd.edu

Received 28 August 2011
Accepted for publication 9 September 2011
Published 7 October 2011
Online at stacks.iop.org/PhysScr/84/045705

Abstract
Herbert Fröhlich is generally credited with having predicted in 1950 the fundamental role of electron–phonon interactions in superconductivity, and in particular the isotope effect, without any experimental input. Here, we examine the facts on which this belief is based and point out that whether or not the generally accepted view is true depends on the meaning of the word shortly.

PACS number: 74.20.Fg

1. The sequence of events

On 16 May 1950, Herbert Fröhlich’s paper titled ‘Theory of the superconducting state. I. The ground state at the absolute zero of temperature’ was received by the journal Physical Review [1]. The paper (hereafter called the ‘May 16 paper’) proposed that the interaction between electrons and lattice vibrations (phonons) is responsible for superconductivity. The paper made no mention of the fact that in the May issue of the same journal, two experimental papers [2, 3] (both received by the journal on 24 March 1950) were describing measurements of the critical temperature of mercury for different isotopes, reporting that ‘there is a systematic decrease of transition temperature with increasing mass’ [2] and that [3] ‘From these results one may infer that the transition temperature of a superconductor is a function of the nuclear mass, the lighter the mass the higher the transition temperature’.

Three days later, on 19 May 1950, a ‘Letter to the Editor’ by Fröhlich titled ‘Isotope effect in superconductivity’ was received by the journal Proceedings of the Physical Society of London [4]. In that short note (half a page long), Fröhlich stated that the isotope effect experiments ‘have just come to my notice’ and pointed out that the formalism in his May 16 paper in fact predicted the effect. In a note added in proof to the May 16 paper, Fröhlich states: ‘The isotope effect [see Reynolds et al., Phys. Rev. 78, 487 (1950); Maxwell, Phys. Rev. 78, 477 (1950)] which has recently come to my notice follows quantitatively from the proportionality of $S$ with the inverse isotopic mass $1/M$ [see e.g. Eq. (6.9) where $F$ depends on $M$ only and is hence independent of the isotopic mass] as was stated in a recent note [Fröhlich, Proc. Phys. Soc. A 63, 778 (1950)]. This agreement provides a direct check for the fundamental assumptions of the theory.’

A competing explanation of the isotope effect experiments was proposed by John Bardeen in a short paper received by the Physical Review just three days later, on 22 May 1950 [5]. Bardeen had no knowledge of Fröhlich’s work at that time, but had just been made aware of the isotope effect experiments in a telephone call by B Serin on 15 May [6]. In his paper, Bardeen readily acknowledged this, with the words: ‘This approach was stimulated by the recent finding of Reynolds, Serin, Wright, and Nesbitt and of E. Maxwell that the critical temperature, $T_c$, of mercury depends on the isotopic mass.’

About a week after Bardeen had submitted his paper, Fröhlich visited Bell Laboratories (where Bardeen was at the time), where he discussed his work on the electron–phonon interaction and the isotope effect [6]. This is when Bardeen learned, for the first time, about Fröhlich’s work on the subject. According to Bardeen’s recollection [7], Fröhlich stated then that he had developed his theory ‘without knowledge of the isotope effect’. To the author’s knowledge, this occasion was the first recorded time when Fröhlich’s work on the subject became known to anybody other than Fröhlich himself and the journals where Fröhlich had submitted his papers a few days earlier.

Starting with these events (a detailed timeline of these events is given in [8]), the electron–phonon interaction became the focus of theoretical efforts attempting to explain superconductivity after 1950, culminating in the Bardeen–Cooper–Schrieffer (BCS) theory in 1957, generally believed to explain ‘conventional’ superconductivity. Neither
Fröhlich’s nor Bardeen’s 1950 theories stood the test of time. Nevertheless, based on the events of May 1950 recounted above, Fröhlich is generally credited with having predicted the fundamental role of electron–phonon interactions in superconductivity and the isotope effect without any experimental input. In this paper, we analyze the evidence in support of this interpretation of events and suggest the possibility that there could be alternatives.

2. Fröhlich’s version of events

In numerous papers following those cited above, Fröhlich states that he developed his theory independent of the isotope effect experiments. For example:

1951 [9]: ‘The prediction that it is the interaction between electrons and lattice vibrations which is responsible for the establishment of the superconducting state has been verified by the discovery of the isotope effect’; (Ref. 1 is Fröhlich’s May 16 paper.)

1952 [10]: ‘... the situation is best described in terms of a field theory in which the electrons are the sources of the vibrational field. Discussion with the help of perturbation theory led to the introduction of an interaction parameter F. It was found that if F is larger than a critical value F₀ then the electron distribution in momentum space differs in the ground state from the normal distribution. This new state was tentatively identified with the superconductive state which led to a prediction of the isotope effect. Starting from a knowledge of this effect, Bardeen (1950) has developed a theory on similar lines’. (Note how Fröhlich makes a clear distinction between Bardeen’s work that was started with knowledge of the isotope effect, and his own.)

1953 [11]: ‘expression (1.5) predicts that the energy difference S should vary as 1/M. This prediction was confirmed quantitatively with the discovery of the isotope effect... It seems, therefore, that the isotope effect represents a most direct confirmation for the hypothesis advanced above’.

1954 [12]: ‘The conjecture that the interaction between electrons coupled through the field of lattice displacements is responsible for superconductivity (Fröhlich 1950) has been strongly supported by the discovery of the isotope effect (Maxwell 1950; Reynolds, Serin, Wright and Nesbitt 1950; Bar, Mendelssohn, Olsen, Allen and Dawton 1950; Lock, Pippard, Shoenberg, Allen and Dawton 1950)’.

1954 [13]: ‘In fact after the first application of the methods of field theory to electrons in ionic crystals (Fröhlich, Pelzer and Zienau 1950), the use of these methods in metals led to an important step in the theory of superconductivity and to the prediction of the isotope effect (Fröhlich 1950)’.

1961 [14]: ‘While thus a satisfactory solution was long delayed, a most important feature of the interaction was already discovered at the first attempt. It led to the prediction of the isotope effect (Fröhlich 1950) which was discovered soon afterwards (Maxwell 1950, Reynolds et al. 1950, Allen et al. 1950 a, b)’.

1963 [15]: ‘The recognition by the author that the electron–phonon interaction in metals responsible for normal resistivity also induces an electron–electron interaction, in conjunction with the suggestion that this interaction be responsible for superconductivity, transformed the till then exasperatingly vague task of developing the theory of superconductivity into a definite mathematical problem expressed in terms of a characteristic Hamiltonian. Discovery of the isotope effect confirmed this suggestion and, on the basis of a simple model, the mathematical problem found a successful solution in the BCS theory’.

Furthermore, in the 1961 review paper [14], Fröhlich stressed that his proposal was completely independent of any other ideas being considered at the time: ‘A dictum was accepted nearly universally, namely that the ions of the lattice, in view of their large mass (compared with electrons), could play no important role in the establishment of the superconductive state. It was in the face of this opinion that the author conceived the idea (Fröhlich 1950) that just the opposite of the dictum contains the truth.’ And later in this paper he states: ‘the development of the theory of superconductivity is an excellent example of the intuitive approach. My idea was foremost that one should consider the small energy involved in the superconductive transition as the first problem to be answered. Electron–phonon interaction provided this possibility from a purely dimensional point of view.’

3. The community’s version of events

There appears to be universal agreement in the community that Fröhlich’s theory was developed independent of the experiments. This author could not find a single instance in the literature suggesting otherwise.

For example, this version of events was rapidly adopted by the authors of the isotope effect experiments:

E Maxwell, 1952 [16]: ‘Furthermore, theoretical treatments by Fröhlich and Bardeen appeared, which were based on interaction between electrons and lattice vibrations and which specified that Tₐ ∼ M⁻¹/₂. (Fröhlich had developed his theory prior to his knowledge of the isotope effect.)’

B Serin, 1955 [17]: ‘In fact Fröhlich, before hearing of the experimental investigations, had suggested that superconductivity comes about as the result of just such an interaction. (Conversely, the experimentalists were unaware of Fröhlich’s theory.)’

Superconductivity textbooks uniformly credit Fröhlich with predicting the isotope effect without experimental input. For example:

Blatt, 1964 [18]: ‘Whereas the distribution of superconductors in the periodic table was known, to some extent, before the work of Fröhlich, the isotope effect was not known to Fröhlich and thus constituted a significant theoretical prediction.’

Schrieffer, 1964 [19]: ‘It was not until 1950 that the basic forces responsible for the condensation were recognized, through the insight of Fröhlich. He suggested that an effective interaction between electrons arising from their interaction with crystal lattice vibrations (phonons) was of primary importance in bringing about the condensation. At this time, independent experiments on the isotope effect in
superconductors were being carried out by Reynolds et al.\textsuperscript{11} and by Maxwell\textsuperscript{12} which gave experimental support to Fröhlich’s point of view.’

Rickayzen, 1965 \textsuperscript{20}: ‘…On the basis of this idea, he predicted the isotope effect independently at the same time as it was being discovered experimentally.’

Tinkham, 1975, 1996 \textsuperscript{21}: ‘Historically, the importance of the electron–lattice interaction in explaining superconductivity was first suggested by Fröhlich\textsuperscript{9} in 1950. This suggestion was confirmed experimentally by the discovery\textsuperscript{6} of the isotope effect, i.e. the proportionality of $T_c$ and $H_c$ to $M^{-1/2}$ for isotopes of the same element.’

In the appendix, we give several more examples of statements by prominent experimentalists and theorists in the field over the years expressing the same view on the issue.

L. Hoddeson, a historian of physics who has written extensively on the early history of superconductivity and on John Bardeen, writes \textsuperscript{22}: ‘As it happened, Bardeen was not the only theorist to connect superconductivity with the electron–lattice interaction. Earlier in 1950, before Maxwell and Serin found the isotope effect experimentally, Herbert Fröhlich had set forth a theory predicting it. When Fröhlich learned of the experimental results a day or two after they appeared in the Physical Review, he sent a letter to the Proceedings of the Royal Society to claim priority for his theory. The competition was on.’

However, upon inquiring with Hoddeson on the meaning of the ‘set forth’ part of the statement above, Hoddeson could not identify a source for it and acknowledged that it is possible that the statement may have been an interpretation of statements by others rather than based on direct evidence (private communication with this author).

4. Bardeen’s early version of events

John Bardeen was consistent in giving Fröhlich full credit for the independent prediction of the isotope effect, both before and after the BCS work. According to the ISI Web of Science database, Bardeen cited Fröhlich’s May 16 paper on 13 different occasions. On the other hand, he never cited Fröhlich’s 19 May note \textsuperscript{4} that explicitly addressed the isotope effect, presumably because in Bardeen’s view it was unnecessary, the prediction being clearly implicit in the May 16 paper. The following are examples of how Bardeen referred to Fröhlich’s work:

1951 \textsuperscript{23}: ‘Prior to his knowledge of the isotope effect, Fröhlich developed a theory of superconductivity based on the self-energy of the electrons arising from interactions with the phonon field.’

1955 \textsuperscript{24}: ‘Fröhlich’s theory, developed without knowledge of the isotope effect, gave a relation between critical temperature, $T_c$, and isotopic mass $\sqrt{M}T_c \sim \text{const}$ which is close to that found empirically.’

1957 (BCS) \textsuperscript{25}: ‘A great breakthrough occurred with the discovery of the isotope effect,\textsuperscript{10} which strongly indicated, as had been suggested independently by Fröhlich,\textsuperscript{11} that electron–phonon interactions are primarily responsible for superconductivity.’

1962 \textsuperscript{26}: ‘As you know, the basis for the current theory was suggested by Fröhlich in 1950 and confirmed by the simultaneous discovery of the isotope effect.’

1969 \textsuperscript{27}: ‘In 1950 Herbert Fröhlich\textsuperscript{8} made a proposal that led eventually to the pairing theory of superconductivity. He based his theory on interactions between electrons and phonons, the quanta of the lattice vibrations. That year also two groups independently discovered that the superconducting transition temperature depends on isotopic mass.’

5. Bardeen’s late version of the events

In the introduction to his 1972 Nobel Prize lecture \textsuperscript{28}, Bardeen wrote, consistent with his earlier statements on the subject:

‘The year 1950 was notable in several respects for superconductivity theory. The experimental discovery of the isotope effect \textsuperscript{4, 5} and the independent prediction of H. Fröhlich \textsuperscript{6} that superconductivity arises from interaction between the electrons and phonons (the quanta of the lattice vibrations) gave the first clear indication of the directions along which a microscopic theory might be sought.’

However, in the next section he wrote:

‘The isotope effect was discovered in the spring of 1950 by Reynolds, Serin, et al. [4] at Rutgers University and by E. Maxwell [5] at the U.S. National Bureau of Standards. Both groups measured the transition temperatures of separated mercury isotopes and found a positive result that could be interpreted as $T_c M^{1/2} \sim \text{constant}$, where $M$ is the isotopic mass. If the mass of the ions is important, their motion and thus the lattice vibrations must be involved. Independently, Fröhlich, [6] who was then spending the spring term at Purdue University, attempted to develop a theory of superconductivity based on the self-energy of the electrons in the field of phonons. He heard about the isotope effect in mid-May, shortly before he submitted his paper for publication and was delighted to find very strong experimental confirmation of his ideas.’

This is a very significant departure from the previous version of events. For the first time, it is stated that Fröhlich knew about the isotope effect experiments before he submitted his May 16 paper!

This new version of events is further supported by the paper written by Bardeen for Fröhlich’s Festschrift on occasion of his retirement \textsuperscript{29} in 1973. On the one hand, Bardeen wrote: ‘Fröhlich did his original work on electron–phonon interactions and superconductivity while he was visiting at Purdue University in the spring semester of 1950. Independently, without his knowledge, two groups had been measuring the transition temperatures of separated mercury isotopes and found a positive result that could be interpreted as $T_c M^{1/2} = \text{constant}$, where $M$ is the isotopic mass.’

In the next paragraph, Bardeen wrote that the experimental papers reporting the isotope effect “appeared in mid-May. It was not until this time, just before he was ready to submit his
Thus, Bardeen stated twice in the literature unequivocally that Fröhlich knew about the isotope effect experiments before he submitted his May 16 paper that did not mention the isotope effect experiments.

In the same paper [29], Bardeen recounts that the experimental results on the isotope effect had been reported at a small meeting almost two months earlier:

“At that time, a great deal of research in low temperature physics in the United States was sponsored by the US Office of Naval Research, and periodic meetings were held to exchange information. The isotope effect was first announced by the two groups at an ONR sponsored meeting held at the Georgia Institute of Technology in March, 1950, but was not publicized outside of the low temperature community. Papers were submitted for publication shortly after the meeting, and appeared in mid-May.’

According to a report on that meeting by W T Ziegler published in Science magazine in May 1950 [30], the meeting was held on 20–21 March and was attended by more than 60 scientists from 24 institutions. On this topic, the Ziegler report states: ‘Among the interesting experimental results reported in the field of superconductivity were those on the mercury isotopes. Experiments on these isotopes indicated that the normal transition temperature decreased by 0.01◦ per unit increase in mass number.’

6. Fröhlich’s own recount of events

In 1982, Fröhlich wrote in the proceedings of a NATO Advanced Study Institute on Advances in Superconductivity [31]:

‘The isotope effect in superconductors was in fact discovered during the same period. At that time I was at a 2–3 month visit to Purdue University, and submitted my paper on leaving on the 16.5.1950. I then spent a couple of days at Princeton and there, at my breakfast table, found the Physical Review with the two letters reporting the isotope effect [12, 14]. On checking I found my M-dependence confirmed and on the 19.5.1950 sent a letter [15] to claim confirmation of the basic idea, electron–phonon interaction.’

Clearly, the statement is intended to imply that Fröhlich did not know about the isotope effect experiments before he read the Physical Review at his breakfast table in Princeton. However, note that it does not say so explicitly. If it did, it would be in direct contradiction to the statements of John Bardeen recounted in the previous section, since Fröhlich went to Princeton right after submitting his May 16 paper. (This statement also suggests there may be some confusion in Fröhlich’s mind between submission and receipt dates, but this is irrelevant to the issue at hand.)

To the author’s knowledge, this statement is the only one ever made by Fröhlich in the literature on how he learned about the isotope effect experiments. The statement leaves us completely in the dark as to whether, and if yes where, when and how, he might have ‘heard about the isotope effect in mid-May, shortly before he submitted his paper for publication’, as affirmed by Bardeen in his Nobel lecture.

7. Discussion of the facts

Note that the fact that Fröhlich was visiting Purdue University while developing his theory of superconductivity, as opposed to being in his home base in Liverpool, makes absolutely no difference within the generally accepted version of events. Instead, if Fröhlich learned about the isotope effect experiments through an informal channel rather than by reading the Physical Review, this would have been presumably facilitated by being in the US rather than in England during that time.

The author sees no reason to doubt that Bardeen’s statements that Fröhlich learned about the isotope effect experiments before he submitted his May 16 paper for publication were factually accurate. Recall that one of these statements was in Bardeen’s 1972 Nobel lecture, an unlikely place to make such a remark lightly. If those statements had been factually inaccurate, it is reasonable to expect that Fröhlich would have explicitly challenged them. No such challenge exists in the literature to the author’s knowledge. Fröhlich passed away in 1991, 18 years after Bardeen put forth this version of events, and it is inconceivable that Fröhlich would not have been aware of these statements, one of them made by Bardeen on Fröhlich’s own Festschrift.

How Bardeen found out that Fröhlich knew about the experiments before submitting his paper, and why Bardeen chose to disclose this information in his 1972 and 1973 papers, are interesting questions that probably have interesting answers.

Assuming then that indeed Fröhlich was aware of the experiments ‘just before he was ready to submit his own paper for publication’, it is rather peculiar that he made no mention of them in the paper, nor of the fact that he was ‘delighted to find very strong experimental confirmation of his ideas’, as stated by Bardeen.

There is no source given in Bardeen’s statement to his assertion that Fröhlich learned about the experiments only shortly before submitting his May 16 paper. The fact that a meeting was held two months earlier attended by over 60 scientists where the experimental results were reported makes it at least plausible that Fröhlich could have heard about the experiments quite a bit earlier than ‘shortly’ before submitting his paper. Note that Bardeen produced a theory proposing an explanation for the experiments in a period of less than seven days following the telephone call from Serin. (15 May was the date of the telephone call, and 22 May the receipt date of Bardeen’s paper by the journal.)

We conclude that Bardeen’s source of information that Fröhlich learned about the isotope effect experiments only shortly before submitting his paper, after Fröhlich’s theory had been completed, is likely to have been statements by Fröhlich himself to Bardeen. If Fröhlich was truthful, indeed he predicted the isotope effect and the fundamental role of electron–phonon interactions in superconductivity independent of the isotope effect experiments. If Fröhlich was not truthful, he did not.

If Fröhlich was truthful, the question remains: why didn’t he mention the isotope effect experiments in any way, shape or form, in his May 16 paper?

If Fröhlich was not truthful, many questions remain: what did Fröhlich know, and when did he know it? From what
source(s)? How did this knowledge influence his work? And how (if at all) did the distortion of the real facts influence the development of the field?

From a purely probabilistic point of view, the probability that the two events, Fröhlich submitting his May 16 paper and Fröhlich learning about the isotope effect experiments, would happen within a short time interval of each other, if the two events are truly independent of each other, becomes increasingly unlikely the shorter the time interval between the two events relative to the total time from the discovery of superconductivity on 8 April 1911 to 16 May 1950, namely 14 283 days.

This author believes that the true version of events is likely to have been very different from the generally accepted version. In a future publication, I will discuss an alternative version and arguments in support of it, as well as argue that the fact that the accepted version may not have corresponded to the true facts had a very significant detrimental effect on the course of development of the field.

Acknowledgments

The author is grateful to F Marsiglio for a discussion that stimulated his interest in this subject, to L J Sham and S K Sinha for discussions and feedback and to L Hoddeson for a private communication.

Appendix

The following are some examples of statements by prominent experimentalists and theorists in the field over the years expressing the view that Fröhlich’s work was completely independent of the isotope effect experiments. We have not found a single instance in the literature with a differing viewpoint.

Tisza, 1950 [32]: ‘The present manuscript was essentially completed when Fröhlich’s theory of superconductivity came to the author’s attention. (Compare also the recent note of Bardeen.) This theory is based on the interaction of free electrons with the lattice vibrations and led to a brilliant prediction of the isotope effect’.

Peierls, 1959 [33]: ‘In 1950, Fröhlich suggested that superconductivity might be due to an interaction between different electrons in the metal which was transmitted through the vibrations of the crystal lattice. . . . Fröhlich’s view was strikingly confirmed by the discovery about the same time of the isotope effect’.

Ginsberg, 1962 [34]: ‘It is interesting to note that slightly prior to the publication of the experimental indications of the isotope effect, it had been suggested that the electron-phonon interaction might be responsible for superconductivity. (Ref 20 is to Fröhlich’s May 16 paper.)

Matthias, 1964 [35]: ‘Fröhlich predicted a gap in the electron spectrum on the top of the Fermi surface in the superconducting state for a one-dimensional model. This gap was indeed detected optically, by microwave spectroscopy (6) as well as by specific heat measurements (7). He also predicted the isotope effect, according to which the transition temperature \( T_c \) varies with the inverse square root of the atomic mass, thus clearly indicating a correlation with lattice vibrations.’

Cleason and Lundqvist, 1974 [36]: ‘An important step towards a microscopic theory was taken by the theoretical prediction of the isotope effect by Fröhlich and the independent experimental discovery of Maxwell and Reynolds et al. [12, 14].’

Pippard, 1987 [37]: ‘It was a notable triumph for Fröhlich that he predicted the isotope effect which was so speedily confirmed by four different groups.’

Mott, 1991 [38]: ‘As the phonon frequency enters into the expression, Fröhlich’s theory showed that the transition temperature should depend on the isotopic mass of the metal concerned, as was found to be valid experimentally by several workers at about the same time. Fröhlich immediately saw that these results were predicted by his work’.

Geballe, 1993 [39]: ‘The discovery of the inverse square-root dependence of \( T_c \) upon mass for the different isotopes of tin, lead and mercury in 1950, by four different groups in the US and Britain, restimulated Bardeen’s intense interest in the electron–phonon interaction. Evidence for this isotope effect, predicted in a model proposed in that same year by Herbert Fröhlich, had actually been sought experimentally at Leiden in the 1920s, when Pb isotopes first became available’.

Schrieffer, 1992 [40]: ‘Independently and without knowledge of the experimental results on the isotope effect, Herbert Fröhlich attempted to deduce the Meissner effect by proposing a theory of superconductivity based on the electron–phonon interaction.’

Schrieffer and Tinkham, 1999 [41]: ‘The discovery of the isotope effect by Maxwell (1950) and Reynolds et al (1950), namely that \( T_c \sim M^{-\alpha} \) where \( M \) is the atomic mass and \( \alpha \sim 1/2 \), gave strong support to the view that superconductivity is the result of the electron–phonon interaction. Prior to this discovery, Fröhlich (1950) had worked out a model based on this interaction but ran into formal difficulties and the approach did not describe the properties of a superconductor.’

Goodstein and Goodstein, 2000 [42]: ‘The foundations that would eventually lead to a microscopic theory began to be laid down, also around 1950, when Herbert Fröhlich realized that the observation that good conductors (copper, gold) tend not to become superconductors might mean that superconductivity is produced by a relatively strong interaction between the conduction electrons and the lattice vibrations, or phonons, in those metals that were not good normal conductors. The Hamiltonian he produced to study the question the Fröhlich Hamiltonian implicitly contained the isotope effect, the prediction, later confirmed, that the superconducting transition temperature is inversely proportional to the square root of the mass of the ions in the lattice (Fröhlich didn’t realize he had made this prediction until he read about the experiments that demonstrated the effect).’
References

[1] Fröhlich H 1950 Theory of the superconducting state. I. The ground state at the absolute zero of temperature Phys. Rev. 79 845–56
[2] Reynolds C A, Serin B, Wright W H and Nesbitt L B 1950 Superconductivity of isotopes of mercury Phys. Rev. 79 487
[3] Maxwell E 1950 Isotope effect in the superconductivity of mercury Phys. Rev. 79 477
[4] Fröhlich H 1950 Isotope effect in superconductivity Proc. Phys. Soc. A 63 778
[5] Bardeen J 1950 Zero-point vibrations and superconductivity Phys. Rev. 79 167
[6] Hoddeson L and Daitch V 2002 True Genius: The Life and Science of John Bardeen (Washington, DC: Joseph Henri Press) p 156
[7] Bardeen J 1995 Afterword: background leading to the microscopic theory of superconductivity, in Fritz London: A Scientific Biography by K Gavroglou (Cambridge: Cambridge University Press) pp 267–72
[8] Hoddeson L, Schubert H, Heims S J and Baym G 1992 Out of the Crystal Maze: Chapters from the History of Solid-State Physics ed L Hoddeson, H Schubert, S J Heims and G Baym (New York: Oxford University Press) pp 548–50
[9] Fröhlich H 1951 Crystal structure and superconductivity Nature 168 280
[10] Fröhlich H 1952 Interaction of electrons with lattice vibrations Proc. R. Soc. A 215 291
[11] Fröhlich H 1953 Superconductivity and lattice vibrations Physica 19 755
[12] Fröhlich H 1954 On the theory of superconductivity: the one-dimensional case Proc. R. Soc. A 223 296
[13] Fröhlich H 1954 Electrons in lattice fields Advances in Physics 3 325–61
[14] Fröhlich H 1961 The theory of the superconductive state Rep. Prog. Phys. 24 1
[15] Fröhlich H 1963 Phys. Lett. 7 346
[16] Maxwell E 1952 Superconductivity of the isotopes of tin Phys. Rev. 86 235
[17] Serin B 1955 The magnetic threshold curve of superconductors Prog. Low. Temp. Phys. 1 138
[18] Blatt J M 1964 Theory of Superconductivity (New York: Academic)
[19] Schrieffer J R 1964 Theory of Superconductivity (Redwood City, CA: Addison-Wesley)
[20] Rickayzen G 1965 Theory of Superconductivity (New York: Wiley)
[21] Tinkham M 1996 Introduction to Superconductivity 2nd edn (New York: McGraw-Hill)
[22] Hoddeson L 2001 John Bardeen and the theory of superconductivity: a late revision of a homework assignment for J. M. Luttinger J. Stat. Phys. 103 625
[23] Bardeen J 1951 Electron-vibration interactions and superconductivity Rev. Mod. Phys. 23 261
[24] Bardeen J 1956 Theory of superconductivity Handbuch der Physik vol 15 (Berlin: Springer) p 274
[25] Bardeen J, Cooper L N and Schrieffer J R 1957 Theory of superconductivity Phys. Rev. 108 1175
[26] Bardeen J 1962 Review of the present status of the theory of superconductivity IBM J. Res. Dev. 6 3
[27] Bardeen J 1969 Advances in superconductivity Phys. Today 22 40
[28] Bardeen J 1992 Electron–phonon interactions and superconductivity (Nobel Lecture) Nobel Lectures, Physics 1971–1980 ed S Lundqvist (Singapore: World Scientific)
[29] Bardeen J 1973 Electron–phonon interactions and superconductivity, in Cooperative Phenomena ed H Haken and M Wagner (Heidelberg: Springer) pp 63–78
[30] Ziegler W T 1950 ONR cryogenics conference Science 111 525
[31] Fröhlich H 1983 History of the theory of superconductivity Advances in Superconductivity ed B Deaver and J Ruvalds (New York: Plenum) p 1
[32] Tisza L 1950 Theory of superconductivity Phys. Rev. 80 717
[33] Peierls R E 1959 Superconductivity: a step in the solution of a puzzle The New Scientist 6 953
[34] Ginsberg D M 1962 Experimental foundations of the BCS theory of superconductivity Am. J. Phys. 30 433
[35] Matthias B T 1964 Superconductivity: II. The facts Science 144 373
[36] Cleason T and Lundqvist S 1974 The microscopic theory of superconductivity—verifications and extensions Phys. Scr. 10 5
[37] Pippard A B 1987 Early superconductivity research (except Leiden) IEEE Trans. Magn. 23 371
[38] Mott N 1992 Herbert Fröhlich. 9 December 1905–23 January 1991 Biographical Mem. Fellows R. Soc. 1991
[39] Geballe T H 1993 Superconductivity: from physics to technology Phys. Today 46 52
[40] Schrieffer J R 1992 John Bardeen and the theory of superconductivity Phys. Today 45 46
[41] Schrieffer J R and Tinkham M 1999 Superconductivity Rev. Mod. Phys. 71 S313
[42] Goodstein D and Goodstein J 2000 Richard Feynman and the history of superconductivity Phys. Perspect. 2 30