Pseudoscalar glueball and $\eta'$-meson in low-energy QCD expansion

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Abstract

An effective chiral lagrangian of order $p^2$, describing the interaction of light pseudoscalar (PS) mesons with $\eta'$-meson and PS-glueball, has been determined taking into consideration the renorm-group requirements imposed by QCD renormalization. It is shown that the interpolating fields for the lowest singlet quarkic and gluonic states, $\eta^0$ and $\eta^G$, may be involved into the effective theory to be renorm-invariant objects not mixing due to QCD renormalization. It is established that the potential describing the “mass” term of the lagrangian does not depend on $\eta^0$. The dependence on $\eta^G$ is permitted only when there is not direct interaction between $\eta^0$ and $\eta^G$ out of the “mass” term without the octet fields contribution. The peculiarities distinguishing the glueball from excitation over $\eta^0$ have been considered.

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1 Introduction

According to well settled notions, a prediction for glueballs is one of the brightest consequences of QCD. However, so far there is no satisfactory solution for the problem of their quantitative description. A certain reason is lack of clear understanding of the point how one can separate the gluonic contributions from the singlet quark ones in a model-independent way. The correct solution of the problem encounters a set of difficulties, in the long run connected with necessity to observe the local gauge invariance of the theory. One important aspect of the problem is the dependence of the quark-gluon mixing on the scale of the UV renormalization.

The latter problem, as a rule, is not taken into consideration. However, when investigating the wave functions of singlet mesons it exhibits itself inevitably while the quarkic and gluonic composite operators, generating these states, are mixed due to the UV renormalization. For the pseudoscalar (PS) meson channel this phenomenon was first described in [1]. The further investigation of the problem was carried out in [2, 3].

The presence of the nontrivial UV renormalization gives certain difficulties in generalization of PCAC to the case of $\eta'$-meson. Really, Refs. [3, 4] have recently shown that the straightforward generalization of the well-known PCAC formula for $\pi^0 \to \gamma\gamma$ to $\eta' \to \gamma\gamma$ is inconsistent with the renorm-group, and therefore incorrect in principle. The right formula for $\eta' \to \gamma\gamma$ involves a new renorm-invariant constant instead of the decay constant of the axial quark current. Moreover, it is claimed in [3] that one more term should be added to the right formula for $\eta' \to \gamma\gamma$, describing the coupling of the “glue” component of $\eta'$ to photons.

The present paper considers the problem of $\eta'$ together with the problem of PS-glueball, because both states most likely are mixing. The investigation is carried out in the approach of the low-energy expansion of QCD, which is also known as the chiral perturbation theory. Earlier, this very approach allowed one to describe the octet of light PS mesons $\pi, K, \eta$ of the Goldstone nature (see, e.g., [5, 6]). In [7] it was applied to describe the interaction between the lightest PS mesons and heavier meson resonances. Analogously, the $\eta'$-meson and PS-glueball may be involved into the effective chiral theory. But the involving should be performed providing for the renorm-group properties inspired by QCD renormalization.

The next section of the paper presents the short review of the necessary knowledge on the renorm-group properties of the generalization functional of QCD in the presence of the composite operators generating the singlet states of $\eta'$-meson and a PS-glueball. In section 3 the effective chiral lagrangian of order $p^2$ is determined, involving the singlet interpolating field $\eta^0$ of the quarkic nature and some additional singlet field, which may describe a PS-glueball or an excitation over $\eta^0$. Section 4 is devoted to the detailed study of the general properties of the effective theory. The relationship between the currents of the effective theory and the composite operators of QCD is discussed. As well, the restriction on the potential describing the “mass” term of the lagrangian is obtained and the consequences of the restriction are explored. The typical difference was found to be between the contributions of the PS-glueball and the excitation over $\eta^0$ into the effective chiral lagrangian. Section 5 investigates the spectrum of the theory. Section 6 summarizes and discusses the results of the paper.
2 Chiral symmetry and UV renormalization for composite operators in QCD

The effective chiral lagrangian for light PS mesons was most consistently described in the approach of Gasser and Leutwyler \[5, 6\] where its connection with the generalization functional of QCD was preserved. The latter in the presence of the composite operators, generating the octet of mesons and the singlet quarkic and gluonic states, is

\[ e^{iW(V,A,S,P,\Theta)} = \int D[q, \bar{q}, G_\mu] \ e^{i \int d^4x \mathcal{L}_{QCD}(q, \bar{q}, G_\mu; V, A, S, P, \Theta)}. \] (1)

Here \( D[q, \bar{q}, G_\mu] \) is the measure of the functional integral, \( V, A, S, P \) are the sources for the related quarkic composite operators and their chiral partners, \( \Theta \) is the source for the operator of axial gluon anomaly, usually regarded as the generator for the gluonic state. \( \mathcal{L}_{QCD} \) in (1) is the unrenormalized lagrangian of QCD with the sources (the notations are obvious):

\[ \mathcal{L}_{QCD} = \mathcal{L}_{QCD}^0 + \bar{q} \gamma_\mu (V_\mu + \gamma_5 A_\mu) q - \bar{q} (S + i \gamma_5 P) q + \Theta Q, \] (2)

\[ V = \sum_{a=0,1,\ldots,8} (\lambda^a/2) V^a, \quad \ldots \quad (\lambda^0 = \sqrt{2/3} \mathbf{1}), \]

\[ Q = \sqrt{6} \frac{\alpha_s}{8\pi} \frac{1}{2} \epsilon_{\mu\nu\lambda\rho} G^{\mu\nu} G^{\lambda\rho}. \] (3)

Here the nonstandard multiplier \( \sqrt{6} \) \((6 = 2N_f)\) has been introduced in the definition of \( Q \), providing convenient reading for the subsequent formulae. Without sources one should set in (2) \( S = \text{diag}(m_u, m_d, m_s) \), \( P = V = A = \Theta = 0 \).

Lagrangian (2) exhibits a property of the local \( U(3)_L \times U(3)_R \) chiral invariance, provided that the quark transformations

\[ q_L \rightarrow \Omega_L(x) q_L, \quad q_R \rightarrow \Omega_R(x) q_R, \] (4)

\( q_{L,R} = (1 \mp \gamma_5)/2 q \), are accompanied by compensating transformations of the sources:

\[ L_\mu = V_\mu - A_\mu \rightarrow \Omega_L L_\mu \Omega_L^\dagger + i \Omega_L \partial_\mu \Omega_L^\dagger, \]

\[ R_\mu = V_\mu + A_\mu \rightarrow \Omega_R R_\mu \Omega_R^\dagger + i \Omega_R \partial_\mu \Omega_R^\dagger, \]

\[ M = S + iP \rightarrow \Omega_L M \Omega_R^\dagger. \] (5)

It is well known that in quantum theory the anomaly breaks the lagrangian invariance. Nevertheless, the symmetry may be restored if one imposes the additional conditions on the sources. So, one can demand the rotation of \( \Theta \) accompanied to \( U(1)_A \)-rotation of the quark fields:

\[ \Theta \rightarrow \Theta + i \sqrt{1/6} \ln \det(\Omega_L \Omega_R^\dagger) = \Theta - \omega_5^0. \] (6)

Here \( \omega_5^0 = (\omega_R^0 - \omega_L^0)/2 \) is the parameter of \( U(1)_A \)-rotation. Condition (6) compensates completely the effect of the gluon anomaly. To compensate the anomaly depending on the external fields (the sources for the composite operators) one needs an additional term. It is clear that it should equal the Wess-Zumino term with opposite sign, constructed over the
nonet of some auxiliary external PS fields (auxiliary PS sources). Note, owing to the fact that this additional term does not depend on the dynamical fields of the theory, which are the functional integral variables in (1), it does not change the dynamical properties of the theory. In particular, it does not change its UV behaviour.

On the contrary, the insertion of the source-terms in (2) means that the new kinds of interaction are introduced into the theory. They may produce new kinds of UV divergencies. In order to remove the divergencies one needs local counterterms which are at least linear in the sources for the composite operators \[\mathcal{S}\]. To remove the divergencies in all Green functions one needs, in general case, a multitude of counterterms, each containing the certain number of the sources or their derivatives. The only bound at this stage is the requirement of the Lorentz and parity invariance and that the dimension of the counterterms should be equal to the dimension of a lagrangian \[\mathcal{L}\]. In virtue of the chiral invariance the number of the counterterms is highly limited. One can show that only two nontrivial counterterms, involving composite operators \[\mathcal{S}\], are needed. They are

\[
(Z - 1) \left( A^{0\mu} - \partial^\mu \Theta \right) J^0_{\mu5}, \quad (Z_m - 1) \sum_{a=0,1,\ldots,8} (-S^a J^a - P^a J^a_5),
\]

where \(J^0_{\mu5}\) is the singlet axial quark current and \(J^a, J^a_5\) are the scalar and pseudoscalar ones,

\[
J^0_{\mu5} = \bar{q} \gamma_\mu \gamma_5 (\lambda^0/2) q, \quad J^a = \bar{q} (\lambda^a/2) q, \quad J^a_5 = i \bar{q} \gamma_5 (\lambda^a/2) q.
\]

Both counterterms in (7) are chiral-invariant and of dimension four. Note, the first counterterm is chiral-invariant owing to the derivative, acting on \(\Theta\). In fact, it is rather general result that \(\Theta\) may contribute into a chiral-invariant expression only with the derivative operator. Hence, in virtue of dimensional reasons, the first counterterm in (7) is the only one which depends on \(\Theta\) and satisfies the above conditions \[\mathcal{S}\]. The renormalization constant \(Z\) for this counterterm was calculated in \[\mathcal{L}\]. The second counterterm in (7) is like the mass-term one. Its renormalization constant is independent of the quark flavours in the mass-independent scheme.

The rest of counterterms, which are of the contact type (not depending on the operators of the theory), are constructed from invariant combinations of the sources and their derivatives:

\[
(F^R_{\mu\nu})^2, \quad (F^L_{\mu\nu})^2, \quad [\partial^\mu (\partial_\mu \Theta - A^0_\mu)]^2, \\
\partial^\mu (\partial_\mu \Theta - A^0_\mu) \times (S^2 + P^2), \quad (S^2 + P^2)^2.
\]

Here the summation over omitted indexes is implied. Notice, according to the classification in Refs. \[\mathcal{L}, \mathcal{S}\] all these combinations have the common property to belong to order \(p^4\) or higher in the chiral dimension. Therefore, these counterterms play no role in the generalization functional of order \(p^2\).

Usually, the introduction of counterterms (7) is interpreted as a requirement of the multiplicative renormalization for composite operators \(J^0_{\mu5}, J^a, J^a_5\) and of nontrivial renormalization for the operator of the axial gluon anomaly \(Q\):

\[
J^0_{\mu5R} = Z J^0_{\mu5}, \quad J^a_R = Z_m J^a, \quad J^a_5_R = Z_m J^a_5, \\
Q_R = Q - (1 - Z) \partial^\mu J^0_{\mu5}.
\]

\[\text{We neglect here the renormalizations of the fundamental fields of quarks and gluons, which should be performed independently applying the standard technique \[\mathcal{S}\].}\]
Here the subscript $R$ indicates the renormalized operators. (Note, the last formula describes the quark-gluon mixing due to the UV renormalization.) The rest of composite operators introduced in (2) remains invariant. The renormalized lagrangian may be obtained in the approach as a result of substitution into the initial lagrangian (2) of the renormalized operators instead of the bare ones and adding the contact counterterms.

There is also an alternative way to describe counterterms, based on the formal transformation of the sources [9]. This approach, the most convenient in the framework of generating functional, is described as the following substitutions in lagrangian (2):

\[
S^a = Z_m S^a_R, \quad P^a = Z_m P^a_R, \quad \Theta = \Theta_R,
\]
\[
A^0_\mu = Z A^0_\mu_R + (1 - Z) \partial_\mu \Theta_R.
\]

(11)

Notice, due to (11), the expression $\partial_\mu \Theta - A^0_\mu$ is transformed multiplicatively:

\[
\partial_\mu \Theta - A^0_\mu = Z (\partial_\mu \Theta - A^0_\mu)_R.
\]

(12)

In formulae (11) and (12) the quantities, provided with subscript $R$, are interpreted as the renormalized sources. Contact counterterms (9) appear in this approach as a result of non-linear renormalization of the auxiliary source of dimension four which should be added into the initial lagrangian with the unit operator [9]. As a result of substitutions (11) the generation functional $W(S, P, \Theta, A^0, \ldots)$ of the unrenormalized theory becomes the generation functional for renormalized Green functions:

\[
W(S, P, \Theta, A^0, \ldots) = W(Z_m S^a_R, Z_m P^a_R, \Theta_R, Z A^0_\mu_R + (1 - Z) \partial_\mu \Theta_R, \ldots)
\]
\[
\equiv W_R(S_R, P_R, \Theta_R, A^0_\mu_R, \ldots).
\]

(13)

From (13) one can deduce the property of the renorm-invariance of the generation functional written in terms of the renormalized sources.

\section{The effective chiral lagrangian}

Calculating in (1) the functional integral over the variables corresponding to colour degrees of freedom and heavy hadrons ones, one can obtain the representation for the generalization functional in terms of the effective theory. In case when only the lightest mesons of the Goldstone nature are not integrated out, we get:

\[
e^{iW(V,A,S,P,\Theta)} = \int \mathcal{D}[U] \ e^{i \int d^4 x \mathcal{L}_{\text{eff}}(U;V,A,S,P,\Theta)},
\]

(14)

where $\mathcal{L}_{\text{eff}}$ is the effective chiral lagrangian. The interpolating fields for the mesons are accumulated in (14) in the unitary $3 \times 3$ matrix $U$, satisfying the condition $\det U = 1$. Under the action of the chiral group $SU(3)_L \times SU(3)_R$ matrix $U$ transforms like

\[
U \rightarrow \Omega_L U \Omega_R^T,
\]

(15)

while the flavour-singlet transformations $U(1)_L \times U(1)_R$ do not affect $U$. Usually, $U$ is represented in the exponential parameterization

\[
U = \exp \left( i \sum_{a=1,\ldots,8} \chi^a \eta^a / F \right),
\]

(16)
where $\eta^a$ are the interpolating fields for the mesons.

According to the prescriptions of the chiral perturbation theory [5, 6] lagrangian $\mathcal{L}_{eff}$ in (14) may be represented in the form of the expansion in the derivatives of fields and sources. In the leading order $p^2$ of the expansion $\mathcal{L}_{eff}$ is described by nonlinear $\sigma$-model in the presence of external fields:

$$
\mathcal{L}_{eff}^{(2)} = \frac{F^2}{4} \text{tr}(\nabla_\mu U \nabla^\mu U^\dagger + \chi U^\dagger + \chi^\dagger U) + H (\partial_\mu \Theta - A^0_\mu)^2,
$$

\(17\)

$$
\nabla_\mu U = \partial_\mu U - i\tilde{\Lambda}_\mu U + iU\tilde{R}_\mu, \quad \chi = 2B(S + iP)e^{i\lambda_0\Theta}.
$$

\(18\)

Here the tildes mean that the singlet sources $L^0_\mu$ and $R^0_\mu$ are not taken into account in the definition of $\nabla_\mu U$. Parameter $F$ in (16), (17) stands for the universal decay constant for the octet of mesons. Parameter $B$ is connected with condensate of quarks. $H$ describes the contact term of the singlet sources. One may associate $H$ with the low-energy asymptotic of the propagator for the singlet axial quark current.

Since quantum loops do not contribute into the effective theory in the leading order, the generating functional in the approximation is representable as

$$
W^{(2)}(V, A, S, P, \Theta) = \int d^4x \mathcal{L}_{eff}^{(2)}(U; V, A, S, P, \Theta),
$$

\(19\)

where $U$ is the solution to the classical equations of motion in the presence of the sources. Due to (19), Green functions for composite operators in QCD may be evaluated in terms of the effective theory.

In virtue of (13) and (19), the QCD-inspired renorm-group properties of the effective theory are reduced to the requirement of the renorm-invariance of $\mathcal{L}_{eff}^{(2)}$. In order to provide this property it suffices to demand the renorm-invariance of $U$ and $F$ and the following transformation rules for the constants $B$ and $H$:

$$
B \rightarrow B_R = Z_mB, \quad H \rightarrow H_R = Z^2H.
$$

\(20\)

The property of the renorm-invariance of $F$ and properties (20) for $B$ and $H$ may be verified directly, keeping in mind the above QCD descriptions for these constants. Therefore, the requirement of the renorm-invariance of $\mathcal{L}_{eff}^{(2)}$ is equivalent to that of the renorm-invariance of the interpolating fields $\eta^a$.

Let us now consider the generalization that involves the ninth field $\eta^0$, responsible for the singlet member of the nonet of non-excited quarkic states. (One should retain the $\eta^0$-integration in (14) in this case.) The general way of the including of $\eta^0$ was outlined earlier in [4]. A possibility to determine $\eta^0$ as the very field of the non-excited singlet quarkic state was based on the exclusive property of this field to transform under the action of the full chiral group $U(3)_L \times U(3)_R$ through adding a term only, which is proportional to the parameter for $U(1)_A$-transformation [14]:

$$
\eta^0 \rightarrow \eta^0 + F^0\omega_5.
$$

\(21\)

Thus, $\eta^0 + F^0\Theta$ remains completely chiral-invariant.

The quantity $F^0$ in (21) is a new parameter of the dimension of mass. Its value, obviously, depends on the normalization of $\eta^0$. Usually, it is assumed that $F^0$ may be attributed to
the decay constant for the singlet axial quark current. However, the latter is not renormal-
invariant in view of (10). Therefore, the assumption cannot be combined with the condition
of the renorm-invariance of \( \eta^0 \), which would be highly desirable, especially, taking in mind
the analogy with the previous case of the octet fields. Thus, it is a question whether it is
really possible to introduce \( \eta^0 \) to be renorm-invariant object. In case of the positive answer
the next question is what is the meaning of \( F^0 \) in terms of QCD.

Adjourning, temporarily, the discussion of these questions let us consider, following [9],
the most general form of the chiral-invariant lagrangian involving \( \eta^0 \) up to and including
order \( p^2 \):

\[
\mathcal{L}_{\text{eff}}^{(2)} = \mathcal{L}_0 + v_1 \text{tr} \left( \nabla_\mu U \nabla^\mu U^\dagger \right) + \text{tr} \left( M v_2^* \Sigma^\dagger + M^\dagger v_2 \Sigma \right).
\]  

(22)

Here \( \mathcal{L}_0 \) stands for the lagrangian for \( \eta^0 \), containing no contributions of matrix
\( U \). In the third term, conventionally called as “mass” one, the \( \Sigma \) is the matrix for the nonet of PS
fields:

\[
\Sigma = U e^{i\lambda_0 \eta^0 / F^0}.
\]  

(23)

Notice, \( \Sigma \) involves \( \eta^0 \) divided by \( F^0 \), so, it transforms like \( \Sigma \to \Omega_L \Sigma \Omega_R^\dagger \) when \( \Omega_{L,R} \in U(3)_{L,R} \).

The second, “kinetic”, term in r.h.s. of (22) may be written in terms of \( \Sigma \), too. For this
purpose one has to change the definition of the covariant derivative allowing contributions of
the singlet sources in (18). However, in view of (11), this way does not seem to be reasonable,
because it is hard to define the renorm-group properties of the theory in this case. Therefore,
having in mind that the difference between the two “kinetic” terms does not depend on \( U \)
(and, hence, may be incorporated into \( \mathcal{L}_0 \)), let us keep the proposed above variant, which is
free from the dependence on the dangerous singlet source \( A_{\mu}^0 \).

The essential moment, distinguishing (22) from (17), is the presence of invariant functions
\( v_{1,2} \) and \( \mathcal{L}_0 \), containing the dependence of the theory on \( \eta^0 \) and the singlet sources. The only
restriction on these functions is that \( v_1 \) and \( \mathcal{L}_0 \) must be real and even, whereas \( v_2 \) may
be complex and \( v_2^*(\bar{\alpha}) = v_2(-\bar{\alpha}) \), where \( \bar{\alpha} \) stands for the arguments [9]. Since there are
three only invariant combinations of \( \eta^0 \), the singlet sources, and their derivatives, we have
in general case:

\[
\mathcal{L}_0 = \mathcal{L}_0(\eta^0 + F^0 \Theta, \nabla_\mu \eta^0, \nabla_\mu \Theta), \quad v_i = v_i(\eta^0 + F^0 \Theta, \nabla_\mu \eta^0, \nabla_\mu \Theta),
\]  

(24)

where

\[
\nabla_\mu \eta^0 = \partial_\mu (\eta^0 + F^0 \Theta), \quad \nabla_\mu \Theta = h (\partial_\mu \Theta - A_{\mu}^0).
\]  

(25)

Here in the definition of \( \nabla_\mu \Theta \), the multiplier \( h \) is of the dimension of mass. Its role here is
to equate the dimensions of the both covariant derivatives.

Note, that assuming the dependence on the covariant derivatives in \( v_i \) and allowing for
more then quadratic dependence on the derivatives in \( \mathcal{L}_0 \), we have essentially diverged from
[9], where \( v_i \) were considered as derivative-free potentials and \( \mathcal{L}_0 \) did not more then quadratic
in the derivatives. The reason for the assumptions of Ref. [9] was that \( \partial_\mu \eta^0 \), \( \partial_\mu \Theta \) and \( A_{\mu}^0 \)
have the chiral dimension of \( p^1 \), which might be established starting from the equation of motion for \( \eta^0 \) provided that the mass parameter for \( \eta^0 \) is assigned to order \( p^2 \) in the chiral
dimension. The reason for the assignment was that in the chiral limit the mass of \( \eta^0 \) tends to
zero at large \( N_c \) [10], so, it may be made so small as needed. However, because the large-\( N_c \)
argumentation can lead, in fact, to serious consequences for the chiral perturbation theory,
we shall not resort to it in this and next sections, where rather general properties of the effective theory are discussed.

Indeed, thinking $\eta^0$ to be a chiral field (which is the exact result in the limit of large $N_c$), and assuming $L_0$ to involve a derivative-free self-interaction of $\eta^0$, then there are vertices of order $p^0$ in the chiral effective theory. Therefore, defining the generalization functional at order $p^2$ one should take into account the multiloop chiral contributions and, moreover, the contributions of the higher dimensions like $O(p^4)$, etc. This fact follows immediately the formula for the overall chiral dimension for a connected diagram with $L$ chiral loops and $N_d$ vertices of order $p^d$ ($d = 0, 2, \ldots$):

$$D = 2L + 2 + \sum_d N_d(d - 2).$$

The above result frustrates validity of the chiral perturbation theory. However, the situation would not occur if $\eta^0$ had a finite (non-vanish) mass, because in this case evaluating the chiral dimension for a connected diagram one can think the mass of $\eta^0$ as an effectively large parameter suppressing the chiral contributions from $\eta^0$. Note, it does not mean that one should no longer take into account the multiloop contributions of $\eta^0$. It means only that in the case they do not contribute in $D$ and, also, that there is no need take into account the higher-dimensional vertices. That salvages the chiral perturbation theory. The pay for the salvation is loss of the simple representation for the generalization functional, like (19), because in the case the leading order of the chiral expansion does not coincide with quasi-classical approximation.

The analysis of the renorm-group behaviour of the effective theory inspiring by QCD renormalization presents no insuperable problems after the above consideration. The principal moment is to prove the renorm-invariance of the parameter $F^0$. To provide for this property one should require the function $\nu_2$ to transform like $B$ in (20) and the parameter $h$ in such a way to ensure the renorm-invariance of the covariant derivative $\nabla_\mu \Theta$. Owing to (12), this requirement is true if $h^2$ transformed like $H$ in (20), i.e.

$$h \to h_R = Z h.$$  

Since the theory depends on $A_{\mu}^0$ through the covariant derivative $\nabla_\mu \Theta$ only, the parameter $h$ may be attached to the decay constant for the singlet axial quark current. In fact, owing to (10), this observation proves property (27) and, so, the statement that $F^0$ is renorm-invariant. Consequently, the interpolating field $\eta^0$ is renorm-invariant, as well. The meaning of the constant $F^0$ will be discussed below.

The generalization of the results for the case when an additional singlet interpolating field (fields) $\eta^G$ is involved may be performed by analogy. The essential difference between $\eta^G$ and $\eta^0$ is that $\eta^G$ is described as a complete singlet, i.e. it is not affected by any chiral transformation, including $U(1)_A$. Therefore to introduce $\eta^G$ into the effective theory one should simply include the dependence on $\eta^G$ and its derivatives into the invariant functions $v_i$ and extra lagrangian term $L_0$. Then, assuming the renorm-invariance for $\eta^G$, no properties of the theory are changed through the including. The general question now is what is the nature of $\eta^G$. Its possible interpretation is either a glueball or an excitation over $\eta^0$. (We do not consider here the heavy quark and multi-quark contributions). The difference between
both cases may be revealed through the study of the relationship between the quarkic and gluonic composite operators of QCD and currents of the effective theory. Another way is to study the typical features in dependence of $\mathcal{L}_0$ and $v_i$ on $\eta^G$ in both cases. The research of these questions is the aim of the next sections.

4 The currents of the effective theory

The natural way to introduce the currents of the effective theory, related to the composite operators of QCD, is through the variational derivatives of the action of the effective theory on the very sources for the composite operators. For instance, the scalar and pseudoscalar currents are defined as the first derivatives on the sources $S^a$ and $P^a$. So, in the context of lagrangian (22) we have

\begin{align}
\mathcal{J}^a & = -\frac{\delta \mathcal{L}_{\text{eff}}^{(2)}}{\delta S^a} = \text{tr} \left\{ \frac{\chi^a}{2} \left( -v_2 \Sigma - v_2^* \Sigma^\dagger \right) \right\}, \\
\mathcal{J}_5^a & = -\frac{\delta \mathcal{L}_{\text{eff}}^{(2)}}{\delta P^a} = \text{tr} \left\{ \frac{\chi^a}{2} \left( i v_2 \Sigma - i v_2^* \Sigma^\dagger \right) \right\}. 
\end{align}

(For brevity, we use the one and the same symbol for the action and for the lagrangian.)

In virtue of lagrangian (22) depends linearly on $S^a$ and $P^a$, the very currents $\mathcal{J}^a$ and $\mathcal{J}_5^a$ possess the property of independence on the sources themselves. Since this property and equality $W_{QCD}^{(2)} = W_{\text{eff}}^{(2)}$, we have the following relations between the matrix elements in QCD and ones in the effective theory made of the identical sets of operators $\mathcal{J}^a$, $\mathcal{J}_5^a$, $\mathcal{J}^a_5$:

\begin{equation}
< a | J^{a_1} \ldots J^{a_n} \ldots J_5^{a_{n+m}} | b >_{QCD} = < a | J^{a_1} \ldots \mathcal{J}^a \ldots \mathcal{J}_5^{a_{n+m}} | b >_{\text{eff}}. 
\end{equation}

Here $< a |$ and $| b >$ stands for the vacuum state or any other states described by the effective theory ($\pi, K, \eta, \eta' \ldots$).

Relation (30) means that in $p^2$-approximation the composite operators $J^a$, $J_5^a$, operating in QCD (in the indicated above space of states), act identically to the operators $\mathcal{J}^a$, $\mathcal{J}_5^a$, operating in the effective theory. The direct consequence from this observation is the requirement that both sets of operators should have identical chiral-symmetry properties at fixed sources. In case of the transformations $SU(3)_L \times SU(3)_R$ and $U(1)_V$ this requirement is fulfilled automatically in view of the transformation rule (15) and the property that $\eta^0$ is the exact singlet under these transformations. In case of the axial-singlet transformation $U(1)_A$ the requirement leads to the nontrivial consequence. Indeed, as directly follows from exact expressions (28), (29), and due to $U(1)_A$-transformation properties for $\Sigma$, the transformation rules required for $\mathcal{J}^a$, $\mathcal{J}_5^a$, are only fulfilled when $v_2$ is not changed under the transformation. At the fixed sources the latter only can take place when $v_2$ has no dependence on the field $\eta^0$ and its derivatives. In view of (24), this means that $v_2$ does not depend on its allowing arguments $F^0 \Theta$ and $\nabla_\mu \eta^0$. Thus, $v_2$ may depend on $\eta^G$, $\partial_\mu \eta^G$ and $\nabla_\mu \Theta$ only. The equivalent proof of this result, based on the analysis of permutation relations between the currents and the generator of $U(1)_A$-transformation, is given in Appendix.

The restriction obtained on the function $v_2$ allows one to establish the important corollary concerning the properties of the singlet-field lagrangian $\mathcal{L}_0$. The idea is to inspect if the
dependence on $\eta^0$ will appear in $\nu_2$ when $\eta^G$ is integrated out from the theory. It is easy to show that in the case when $\nu_2$ involves $\eta^G$ or its derivatives, the dependence cannot appear if $\mathcal{L}_0$ admits of no interaction between $\eta^0$ and $\eta^G$. (In this case $\eta^0$ does not contribute into the equation of motion for $\eta^G$ in the leading order $p^0$ of the lagrangian (22). The contributions into $\nu_2$, going through the dependence on $\eta^0$ and $\eta^G$ in the $p^2$-terms of the lagrangian, are irrelevant here, because in the end they contribute beyond the order $p^2$ of the lagrangian.) In this case $\mathcal{L}_0$ is representable as the sum of two independent lagrangians, one for $\eta^0$ and another for $\eta^G$:

$$\mathcal{L}_0 = \mathcal{L}_{\eta^0}(\eta^0 + F^0 \Theta, \nabla_\mu \eta^0, \nabla_\mu \Theta) + \mathcal{L}_{\eta^G}(\eta^G, \partial_\mu \eta^G, \nabla_\mu \Theta).$$  \hspace{1cm} (31)

The second case is when $\nu_2$ involves neither $\eta^G$ nor $\partial_\mu \eta^G$. In this case $\mathcal{L}_0$ may well contain an additional term describing the interaction between $\eta^0$ and $\eta^G$.

The first case considered above ($\nu_2$ depends on $\eta^G$ or its derivatives) means that $\eta^0$ and $\eta^G$ can interact with each other without the octet fields contribution through the “mass” term only. When the quark masses vanish and the sources are turned-off they cannot interact at all without the octet fields which become the Goldstone bosons in the limit. Such behaviour is typical for objects one of which is an excited state, because the latter in the chiral limit seems cannot enter into the strong interaction with any object without the emission of the Goldstone bosons. On this ground one may conclude that the most probable interpretation for $\eta^G$ in this case is an excitation over $\eta^0$, i.e. the quark excitation or a hybrid state, in dependence of the type of the degrees of freedom being excited in $\eta^0$. In the second case ($\nu_2$ does not depend on $\eta^G$ and its derivatives) both fields, $\eta^0$ and $\eta^G$, can well interact with each other without the Goldstones when quark masses and sources are turned-off. This picture of interaction is typical for non-excited states. Since there is not another state neighboring in energy, one may consider $\eta^G$ as a PS-glueball in this case.

Let us now consider the singlet currents $Q$ and $\mathcal{J}^0_{\mu 5}$ related to the QCD composite operators $Q$ and $J^0_{\mu 5}$:

$$Q = \delta \mathcal{L}^{(2)}_{\text{eff}} / \delta \Theta, \quad \mathcal{J}^0_{\mu 5} = \delta \mathcal{L}^{(2)}_{\text{eff}} / \delta A^0_\mu. \hspace{1cm} (32)$$

In contrast to the above case, these currents act not identically to the operators $Q$ and $J^0_{\mu 5}$ determined in the space of states of the effective theory. (The same property may be established as well for the octet-vector and octet-axial quark currents.) That fact follows immediately from the nonlinear character of the dependence of lagrangian (22) on the corresponding sources. However, any relation linear in the composite operators should take place as well in the effective theory. For instance, it is easy to show by the straightforward calculation that following the QCD renormalization the singlet currents $Q$ and $\mathcal{J}^0_{\mu 5}$, although being made of the renorm-invariant interpolating fields, transform like:

$$\mathcal{J}^0_{\mu 5} \to Z \mathcal{J}^0_{\mu 5}, \quad Q_R = Q - (1 - Z) \partial^\mu \mathcal{J}^0_{\mu 5}. \hspace{1cm} (33)$$

It can be shown also that $Q$ and $\mathcal{J}^0_{\mu 5}$ satisfy the Ward identity (on the equations of motion for the interpolating fields) which coincides with the anomalous Ward identity in QCD for $Q$ and $J^0_{\mu 5}$. The relations of the kind of (30) are also true provided the operators $Q$ or $J^0_{\mu 5}$ and $Q$ or $\mathcal{J}^0_{\mu 5}$ were inserted only once. In particular, the following relations take place

$$< 0|Q|\eta^{0,G} > = < 0|Q|\eta^{0,G} >, \quad < 0|J^0_{\mu 5}|\eta^{0,G} > = < 0|J^0_{\mu 5}|\eta^{0,G} >. \hspace{1cm} (34)$$
From the above properties it is natural to regard $Q$ and $J_{\mu 5}^0$ as the effective-theory analogs to the gluonic operator $Q$ and axial singlet quark current $J_{\mu 5}^0$ of QCD. The analogy would be strong when only the linear correlations were considered. Notice, equalities (34) allow one to consider the properties of $\eta^0$ and $\eta^G$, initially introduced in the effective theory, to be dependent on the behaviour of the matrix elements $\langle 0 | Q | \eta^{0,G} \rangle$ and $\langle 0 | J_{\mu 5}^0 | \eta^{0,G} \rangle$ defined in QCD. This property will be exploited below for further discussion of the differences between the glueball and excitation over $\eta^0$.

5 The spectrum of the effective theory

To investigate the spectrum of the effective theory it is reasonable to combine the chiral expansion with quasi-classical approximation. Then, lagrangian (22) may be regarded as the effective action for the mesons in the presence of the external fields, which are the sources for the QCD composite operators, too. Applying the combined approximation we are able also to make use the large-$N_c$ approximation, which can assist us to recognize the nature of the extra singlet field $\eta^G$.

At first, let us study the lagrangian $L_0$ in the quadratic approximation on the fields and sources. Starting from the most general expression for $L_0$ provided with the symmetry required, we have

$$ L_0 = \frac{1}{2} \alpha_1 (\nabla_\mu \eta^0)(\nabla^\mu \eta^0) + \frac{1}{2} \alpha_2 (\partial_\mu \eta^G)(\partial^\mu \eta^G) + \frac{1}{2} \alpha_3 (\nabla_\mu \Theta)(\nabla^\mu \Theta) + \alpha_4 (\nabla_\mu \eta^0)(\partial^\mu \eta^G) + \alpha_5 (\nabla_\mu \eta^0)(\nabla^\mu \Theta) + \alpha_6 (\partial_\mu \eta^G)(\nabla^\mu \Theta) - \frac{1}{2} \beta_1 (\eta^0 + F^0 \Theta)^2 - \frac{1}{2} \beta_2 (\eta^G)^2 - \beta_3 (\eta^0 + F^0 \Theta)\eta^G. \quad (35) $$

Here $\alpha_i$ and $\beta_i$ are the constants. Some of them may be removed or fixed through more special consideration. In this way, the constant $\alpha_1$ may be absorbed by the normalization for the field $\eta^0$ and parameter $F^0$. So, without loss of generality we may set $\alpha_1 = 1$ providing for the canonical normalization for $\eta^0$. The fourth term in (35) may be removed out by the linear transformation

$$ \eta^0 \rightarrow \eta^0 - \alpha_4 \eta^G, \quad (36) $$

diagonalizing the kinetic terms in (35). Owing to the transformation (36) does not run counter to (21), it is allowed for $\eta^0$. Generally, any transformation of the kind describes the uncertainty in definition of $\eta^0$, which may be associated with the indeterminate contributions of gluons and other types of contributions into the singlet state of quarkic nature. The very transformation (36) fixes the uncertainty in the context of the effective theory, and then $\eta^0$ becomes the interpolating field for observable state.

Performing transformation (36) and fixing the canonical normalization for $\eta^G$, let us rewrite $L_0$ in the form

$$ L_0 = \frac{1}{2} (\nabla_\mu \eta^0)(\nabla^\mu \eta^0) + \frac{1}{2} (\partial_\mu \eta^G)(\partial^\mu \eta^G) + \frac{1}{2} \alpha_3 (\nabla_\mu \Theta)(\nabla^\mu \Theta) + \alpha_0 (\nabla_\mu \eta^0)(\nabla^\mu \Theta) + \alpha_4 (\partial_\mu \eta^G)(\nabla^\mu \Theta) - \frac{1}{2} M_0^2 (\eta^0 + F^0 \Theta)^2 - \frac{1}{2} M_G^2 (\eta^G)^2 - q(\eta^0 + F^0 \Theta)\eta^G. \quad (37) $$
Here $M_0$, $M_G$ are the mass parameters for the singlet interpolating fields and $q$ is the parameter describing their mixing.

Due to (32) and (37), the currents $J^0_{\mu_5}$ and $Q$ in the linear approximation on the fields and turned-off sources may be represented as

$$J^0_{\mu_5} = -h \partial_\mu (\alpha_0 \eta^0 + \alpha_G \eta^G),$$

$$Q = -F^0(M_0^2 + \partial^2)\eta^0 - F^0 q \eta^G - h \partial^2 (\alpha_0 \eta^0 + \alpha_G \eta^G) \simeq h(\alpha_G M_G^2 + \alpha_0 q) \eta^G + h(\alpha_0 M_0^2 + \alpha_G q) \eta^0 + \text{(mass)}. \tag{38}$$

Here symbol ‘$\simeq$’ means that the equations of motion were used. The last term in (39), designated like ‘(mass)’, stands for the contributions which are proportional to the current quark masses. Owing to (34), $h \alpha_0$ in (38) is equal to the decay constant for the singlet axial quark current. Assuming $\alpha_0$ to be absorbed by $h$ we may set $\alpha_0 = 1$.

Using (34) and (38), (39), one can determine the large-$N_c$ behaviour of the parameters of lagrangian $\mathcal{L}_0$ depending on the nature of $\eta^G$. To this end, let us put the quark masse to be vanish (the chiral limit) and consider the following well-known formulae [10]:

$$< 0| J^0_{\mu_5} | \text{glueb} > \sim 1, \quad < 0| J^0_{\mu_5} | \text{quark} > \sim N_c^{1/2}, \quad (40)$$

$$< 0| Q | \text{glueb} > \sim 1, \quad < 0| Q | \text{quark} > \sim N_c^{-1/2}. \quad (41)$$

From here and (38), (39), taking into consideration the quark nature of $\eta^0$, and the property $M_G \sim 1$ postulated independently on the nature of $\eta^G$, one can deduce that $M_0^2 \sim N_c^{-1}$ and $h \sim N_c^{1/2}$, and also that

$$q \sim N_c^{-1/2}, \quad \alpha_G \sim N_c^{-1/2} \quad (42)$$

for $\eta^G$ is a glueball, and

$$q \sim N_c^{-1}, \quad \alpha_G \sim N_c^{-1} \quad (43)$$

for $\eta^G$ is an excitation over $\eta^0$.

The spectrum of the effective theory in the chiral limit can be determined by diagonalizing the mass terms in (37). The final result at large $N_c$ looks like

$$M^2_{\eta^0} = (M_0^2 M_G^2 - q^2)/M_G^2, \quad M^2_{\eta^0} = M_G^2. \quad (44)$$

Here the symbols $\eta'$ and $\eta''$ represent the observable states having a certain value of mass (the subscript zero means that the states are considered in the chiral limit). From the first equality in (44) and (42), (43) one may deduce that $M^2_{\eta^0} \sim N_c^{-1}$, and that in the leading order in $N_c^{-1}$ the mixing parameter $q$ can contribute into $M^2_{\eta^0}$ if $\eta^G$ is the glueball only. On the contrary, when $\eta^G$ is the excitation over $\eta^0$, then $q$ can contribute in the next-to-leading order only (if it does not equal identically zero). Therefore it must be eliminated from the theory in the case in $p^2$-approximation when one equates the order of magnitude of $O(N_c^{-1})$ to $O(p^2)$. Note, the similar result has been obtained in the preceding section on the ground of rather general consideration.

Now let us introduce the “mass” term of the effective lagrangian. Putting

$$\nu_2 = \frac{1}{2} BF^2 \left(1 + ib \eta^G / F^0 + \ldots \right), \quad (45)$$
where \( b \) is a new constant responsible for \( \eta^G \)-dependence in \( \nu_2 \), we get from (29) in the linear approximation:

\[
\mathcal{J}^a_{s=0} = -BF^2 \left( \eta^a / F \right),
\]

(46)

\[
\mathcal{J}^0_s = -BF^2 \left( \eta^0 + b\eta^G \right) / F^0.
\]

(47)

Owing to the similar to (40) formula for the large-\( N_c \) behaviour of the PS quark currents, and since \( F \sim N_c^{1/2} \), it follows from (47) that \( F^0 \sim N_c^{1/2} \). Then, depending on the nature of \( \eta^G \), the parameter \( b \) in (47) behaves at large \( N_c \) as

\[
b \sim \begin{cases} 
N_c^{-1/2}, & \text{for } \eta^G \text{ is a glueball} \\
1, & \text{for } \eta^G \text{ is an excitation over } \eta^0.
\end{cases}
\]

(48)

We see that in the first case the parameter \( b \) is suppressed by large \( N_c \). Therefore, when determining the spectrum in \( p^2 \)-approximation combined with large \( N_c \), one should eliminate \( b \), because it contributes through the “mass” term which is already suppressed. In the second case of (48) parameter \( b \) may well contribute in \( p^2 \)-approximation.

The consequence from (47) and (34), which is of the great importance, is the formula for the parameter \( F^0 \), representing it in terms of QCD variables:

\[
F^0 = \frac{< \bar{u}u >_0}{< 0|J^0_s|^0_0>}. \tag{49}
\]

Here the equality \( BF^2 = -< \bar{u}u >_0 \) has been exploited where \( < \bar{u}u >_0 = \) the chiral quark condensate \( < \bar{u}u >_0 = < \bar{d}d >_0 = < s \bar{s} >_0 \). Note, to within designations, (49) is equivalent to the result obtained earlier in the framework of PCAC [3].

Turning-on the quark masses one may obtain the mass matrix for the observable states. When \( m_u = m_d \neq m_s \) it describes the \( \eta^8 - \eta^0 - \eta^G \) mixing. In this very basis the squared mass matrix is

\[
\mathcal{M}^2 = \begin{pmatrix}
\frac{1}{3} (4M_K^2 - M_\pi^2) & \frac{2\sqrt{2}}{3} \xi (M_\pi^2 - M_K^2) & \frac{2\sqrt{2}}{3} b \xi (M_\pi^2 - M_K^2) \\
\frac{M_\eta^2}{3} + \frac{1}{3} \xi^2 (2M_\pi^2 + M_\eta^2) & q + \frac{1}{3} b \xi^2 (2M_\pi^2 + M_\eta^2) & M_\omega^2 \\
\end{pmatrix}
\]

(50)

Here \( M_\pi \) and \( M_K \) are the pion and kaon masses, \( \xi = F/F^0 \). If \( \eta^G \) is an excitation over \( \eta^0 \), then one should put \( q = 0 \) in (50). When \( \eta^G \) is a glueball, then \( b = 0 \). Notice, in the second case with \( \xi = 1 \) the matrix \( \mathcal{M}^2 \) is equivalent to the Kawai matrix [4] with the two parameters (not counting \( M_\pi^2 \)) instead of three ones in [4].

The eigenvalues of the squared mass matrix (50), which are the squared masses of eigenstates \( \eta, \eta', \eta'' \) resulting from the mixing of \( \eta^8, \eta^0, \eta^G \), may be evaluated by fitting the data for the radiative decays \( P \to \gamma \gamma, P \to V \gamma \) and \( V \to P \gamma \), where \( P = \eta, \eta', V = \omega, \rho \).

In the effective chiral theory these decays, violating the internal parity, are described by Wess-Zumino term (see, e.g., [12, 13]). In the approach considered here this term should be constructed over the field matrix \( \Sigma \), involving \( \eta^0 \) divided by \( F^0 \). Omitting the tedious and rather standard calculation, let us present the final result for the masses of \( \eta, \eta', \eta'' \) in the case when \( \eta^G \) is the glueball:

\[
M_\eta^2 = (0.52 \pm 0.02 \text{ GeV})^2, \quad M_{\eta'}^2 = (0.99 \pm 0.13 \text{ GeV})^2, \quad M_{\eta''}^2 = (0.00 \pm 3.74 \text{ GeV}^2) \leq (1.94 \text{ GeV})^2.
\]

(51)
The errors in (51) are the consequences of the 20%-errors, assumed in the mass formulae for $\eta, \eta', \eta''$, and the errors of the experimental data, which were put into the fitting procedure. On definition, the quantity $M_{\eta''}^2$ was assumed to be non-negative through the fitting.

As one can see from (51) the estimate for $\eta''$ is very rough to make any conclusion about for what real state the $\eta''$ stands. To solve this problem one needs study in detail the decays of $\eta''$, which is beyond the framework of the present work. Notice only, that for the study it is important to know [14] whether the parameter $b$ in (45) is really equal to zero when $\eta^G$ is the glueball. According to section 4 it should exactly equal zero if the mixing parameter $q$ does not. However, for the glueball there is not strong restriction that $q$ should differ zero at any price. Therefore, it is desirable to get the quantitative estimate. Unfortunately, the result of the above fitting, which is $q = 0.1 \pm 1.0$, permits no certain conclusion. Note, that the another result $\xi = 0.92 \pm 0.12$ allows one to conclude that the normalization constant $F_0$ for the singlet field $\eta^0$ is fitted with high accuracy and that it coincides within errors with the universal decay constant $F$ for the octet of mesons.

6 Summary and discussion

The present paper has shown that the UV renormalization, mixing in QCD the quarkic and gluonic composite operators (which generate the $\eta'$-meson and PS-glueball), in the effective theory does not affect the mutual configuration of the interpolating fields for quarkic and gluonic states. Nevertheless, the QCD renormalization of the composite operators may well be reproduced as the renormalization of the related currents of the effective theory. The interpolating field $\eta^0$ for the lowest singlet quarkic state may be introduced into the effective theory being normalized on the very special renorm-invariant constant. (It is determined as the ratio of the chiral quark condensat to the normalization constant for the singlet pseudoscalar quark current. However, its value coincides within errors with the universal octet decay constant $F_\pi$.) The interpolating field for the lowest gluonic state may be involved so that it saturates at large $N_c$ the gluonic current of the effective theory.

The general way to involve singlet fields into the effective chiral lagrangian is through the potentials describing the “kinetic” and “mass” terms of the lagrangian, and through some extra terms which are the kinetic and mass terms of the singlet fields themselves and their mutual- and self-interaction. The present paper investigation has shown that the interpolating field $\eta^0$ makes no contribution into the potential of the “mass” term. Consequently, another singlet interpolating field, $\eta^G$, may only contribute into the “mass” term when there is not direct interaction between $\eta^G$ and $\eta^0$ out of the “mass” term without the octet fields contribution. The latter property is shown to be peculiar for an excitation over $\eta^0$. On the contrary, PS-glueball may well enter the interaction. In particular, it can mix with $\eta^0$ in the chiral limit and, then, it makes no contribution into the “mass” term of the effective chiral lagrangian.

The latter property results in serious consequences for the decay modes of the PS-glueball. So, Ref. [14] has shown that with non-vanish parameter $b$, describing the contribution of $\eta^G$ into the “mass” term of the lagrangian (in Ref. [14] this state is unreasonably identified with the PS-glueball), the principal decay mode of $\eta''$ is predominantly $K\bar{K}\pi$. In the case when $b$ vanishes, this mode occurs through the mixing of $\eta^G$ with $\eta^0$ and $\eta^8$ only. If the mixing
is large there may be the copious decay. However, to get to know more on the question one needs the additional study. The results obtained above may serve as the first step in this trend.

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**Appendix**

The $U_A(1)$-transformation properties for the scalar and pseudoscalar currents are determined by the permutation relations

$$
\left[ \mathcal{O}_5^0, \mathcal{J}^a \right] = i \, d^{0ab} J^b_5, \quad \left[ \mathcal{O}_5^0, \mathcal{J}^a_5 \right] = -i \, d^{0ab} J^b. \tag{A.1}
$$

Here $d^{0ab} = \sqrt{2/3} \, \delta^{ab}$, $\mathcal{O}_5^0$ is the generator for $U_A(1)$-transformations. According to the standard construction it equals the spatial integral over the temporal component of the Noether current $\mathcal{I}^{\mu}_0$. Owing to (21), the latter may be represented as

$$
\mathcal{I}^{\mu}_0 = \frac{1}{F_0} \frac{\partial L_{\text{eff}}^{(2)}}{\partial (\partial_\mu \eta^0)}. \tag{A.2}
$$

It is not difficult to show that, due to (28) and (29), both relations in (A.1) are equivalent to the single relation

$$
\left[ \mathcal{O}_5^0, \nu_2 e^{i\lambda_\eta^0/F_0} \right] = \lambda_0 \nu_2 e^{i\lambda_\eta^0/F_0}. \tag{A.3}
$$

Let us now make use the fact that in view of (A.2) the temporal component of $\mathcal{I}^{\mu}_0$ coincides, up to the factor $F_0$, with the canonical momentum, conjugated to $\eta^0$. From here and in view of the canonical permutation relations for $\eta^0$, one can deduce the following permutation relations:

$$
\left[ \mathcal{O}_5^0(n), \eta^0(y) \right] = -i F_0 \delta(x - y), \quad \left[ \mathcal{O}_5^0(n), \partial_n \eta^0(y) \right] = i F_0 \partial_n \delta(x - y). \tag{A.4}
$$

Here $n$ runs over the spatial values $n = 1, 2, 3$. Thanks to (A.4), one can write the permutation relation for any operator $\Phi$, admitting a power decomposition:

$$
\left[ \mathcal{O}_5^0, \Phi \right] = -i F_0 \left[ \frac{\partial \Phi}{\partial \eta^0} - \partial_n \frac{\partial \Phi}{\partial (\partial_n \eta^0)} - \mathcal{F}_0 \left( \frac{\partial \Phi}{\partial (\partial_0 \eta^0)} \right) \right]. \tag{A.5}
$$

Here $\mathcal{F}_0$ is a functional satisfying the condition $\mathcal{F}_0(0) = 0$. (It is possible that $\mathcal{F}_0$ identically equals zero. In general case, $\mathcal{F}_0$ arises in (A.5) because of the commutator of $\mathcal{O}_5^0$ with $\partial_0 \eta^0$ from $\Phi$.) Applying (A.5) to l.h.s of (A.3) one gets the equation on $\nu_2$:

$$
\frac{\partial \nu_2}{\partial \eta^0} - \partial_n \frac{\partial \nu_2}{\partial (\partial_n \eta^0)} - \mathcal{F}_0 \left( \frac{\partial \nu_2}{\partial (\partial_0 \eta^0)} \right) = 0. \tag{A.6}
$$

Due to Lorentz-invariance, (A.6) means that $\partial \nu_2/\partial (\partial_\mu \eta^0) = 0$ and $\partial \nu_2/\partial \eta^0 = 0$, q.e.d. If second and third terms in (A.6) taken together form a Lorentz-invariant combination $\partial_\mu \partial \nu_2/\partial (\partial_\mu \eta^0)$, then (A.6) becomes simply $\delta \nu_2/\delta \eta^0 = 0$. From here, owing to arbitrariness of $\eta^0$, the same result follows.

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