Consistent relativistic mean-field models: symmetry energy parameter

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Abstract. In this work, we revisit the study published in [Dutra et al., Chinese Physics C 42, 064105 (2018)] where 34 consistent relativistic mean-field models were analyze at the nuclear matter constraints in relation to the role of short-range correlations in the calculation of the symmetry energy and its consequence on the value of the gamma parameter.

1. Introduction

Nuclear matter properties are reasonably well described by different versions of relativistic and non-relativistic models [1, 2, 3, 4]. We use the relativistic mean-field models to analyze the behavior of the symmetry energy in symmetric nuclear matter. The symmetry energy $S(\rho)$ is a very important quantity in nuclear physics and it is related to different nuclear processes [5]. It can be written as the difference between the energy per nucleon of pure neutron matter ($y = 0$) and the energy per nucleon of symmetric matter ($y = 1/2$), where $y = Z/A$ is the proton fraction. This statement can be verified by using the expansion of the energy per nucleon in terms of $\delta = 1 - 2y$, around $\delta = 0$, at a given density as $E(\rho, \delta) \simeq E(\rho, 0) + S(x)\delta^2 + O(\delta^4)$. By taking this parabolic form for $E(\rho, \delta)$ it is possible to use $S(\rho) \simeq S(\delta) = E(\rho, 1) - E(\rho, 0)$ as a good approximation in order to compute the symmetry energy. Many nuclear and astrophysical properties are related to this quantity, as for example, the mass-radius diagram and the cooling process of neutron stars [6, 7]. At finite nuclei, the neutron skin thickness is related to the slope of the symmetry energy [8, 9]. Here, we revisit the study published in [10] to show the behavior of the $\gamma$ parameter, related to the symmetry energy through its potential part when the short range correlations (SRC) are included in the calculations.

2. Relativistic Mean-Field models

The 34 consistent relativistic mean-field models (CRMF) used in this study (see Table 1) were extensively analyzed in the Ref. [11] under the nuclear matter point of view. These models have two different structures: non-linear and density dependent ones. We can find the expressions...
for the symmetry energy to both kinds of models through

\[ S(\rho) = \frac{1}{8} \frac{\partial^2 E(\rho, y)}{\partial y^2} \bigg|_{\rho, y = 1/2} = S^{\text{kin}}(\rho) + S^{\text{pot}}(\rho). \]  

(1)

The expressions are

\[ S_{\text{NL}}(\rho) = \frac{k_F^2}{6E_F^*} + \frac{g_\rho^2}{8m_r^2} \rho^2 - \frac{\left(g_\delta/m_\delta\right)^2}{2E_F^*} \frac{M^2}{\rho} \frac{2E_F^*}{1 + (g_\delta/m_\delta)^2 A(k_F, M^*)}, \]  

(2)

\[ S_{\text{DD}}(\rho) = \frac{k_F^2}{6E_F^*} + \frac{\Gamma_\rho^2}{8m_r^2} \rho^2 - \frac{\left(\Gamma_\delta/m_\delta\right)^2}{2E_F^*} \frac{M^2}{\rho} \frac{2E_F^*}{1 + (\Gamma_\delta/m_\delta)^2 A(k_F, M^*)}, \]  

(3)

with

\[ E_F^* = \left(\frac{k_F^2}{M^2}\right)^{1/2}, \]  

(4)

\[ A(k_F, M^*) = \frac{2}{\pi^2} \int_0^{k_F} \frac{k^4 dk}{(k^2 + M^2)^{3/2}}, \]  

(5)

\[ m_\rho^2 = m_\rho^2 + g_\sigma g_\rho^2 \sigma (2\alpha_2' + \alpha_2 g_\sigma \sigma) + \alpha_3 g_\rho^2 \sigma^2. \]  

(6)

The behavior of the symmetry energy for these models is shown in Fig 1.

![Figure 1. Symmetry energy versus density.](https://example.com/figure1.png)

3. Results

We start by looking at the potential part of the symmetry energy. It can be related to the \( \gamma \) parameter through the following parabolic expression

\[ S^{\text{pot}}(\rho) = S_0^{\text{pot}}(\rho/\rho_0)^\gamma. \]  

(7)
The first one constrains the kinetic part \( (\rho_0) \), binding energy \( (E_0) \), incompressibility \( (K_0) \), effective mass \( (m^*) \), symmetry energy \( (J) \), and slope of symmetry energy \( (L_0) \). For more details and references see [11].

| Models | \( \rho_0 \) [fm\(^{-3}\)] | \( E_0 \) [MeV] | \( K_0 \) [MeV] | \( m^* \) | \( J \) [MeV] | \( L_0 \) [MeV] |
|--------|----------------|----------------|--------------|--------|--------|----------|
| BKA20  | 0.146          | -15.93         | 237.95       | 0.64   | 32.24  | 75.38    |
| BKA22  | 0.147          | -15.91         | 225.24       | 0.61   | 33.17  | 78.79    |
| BKA24  | 0.147          | -15.95         | 227.06       | 0.60   | 34.19  | 84.80    |
| BSR8   | 0.147          | -16.04         | 230.95       | 0.61   | 31.08  | 60.25    |
| BSR9   | 0.147          | -16.07         | 232.50       | 0.60   | 31.61  | 64.89    |
| BSR10  | 0.147          | -16.06         | 227.41       | 0.60   | 32.72  | 70.83    |
| BSR11  | 0.147          | -16.08         | 226.75       | 0.61   | 33.69  | 78.78    |
| BSR12  | 0.147          | -16.10         | 232.35       | 0.61   | 34.00  | 77.90    |
| BSR15  | 0.146          | -16.03         | 226.82       | 0.61   | 30.97  | 61.79    |
| BSR16  | 0.146          | -16.05         | 224.98       | 0.61   | 31.24  | 62.33    |
| BSR17  | 0.146          | -16.05         | 221.67       | 0.61   | 31.98  | 67.44    |
| BSR18  | 0.146          | -16.05         | 221.13       | 0.61   | 32.74  | 72.65    |
| BSR19  | 0.147          | -16.08         | 220.83       | 0.61   | 33.78  | 79.47    |
| BSR20  | 0.146          | -16.09         | 223.25       | 0.61   | 34.54  | 88.03    |
| FSU-III| 0.148          | -16.28         | 229.54       | 0.61   | 33.89  | 71.72    |
| FSU-IV | 0.148          | -16.28         | 229.54       | 0.61   | 32.56  | 60.44    |
| FSUGold| 0.148          | -16.28         | 229.54       | 0.61   | 31.40  | 51.74    |
| FSUGold4| 0.147         | -16.07         | 232.48       | 0.60   | 31.54  | 64.98    |
| FSUG206| 0.146          | -16.05         | 225.06       | 0.61   | 31.18  | 62.42    |
| G2*    | 0.154          | -16.07         | 214.77       | 0.66   | 30.39  | 69.68    |
| IU-FSU | 0.155          | -16.40         | 231.33       | 0.61   | 31.30  | 47.21    |
| Z271s2 | 0.148          | -16.24         | 271.00       | 0.80   | 34.08  | 76.62    |
| Z271s3 | 0.148          | -16.24         | 271.00       | 0.80   | 33.27  | 67.81    |
| Z271s4 | 0.148          | -16.24         | 271.00       | 0.80   | 32.53  | 60.18    |
| Z271a5 | 0.148          | -16.24         | 271.00       | 0.80   | 31.84  | 53.57    |
| Z271a6 | 0.148          | -16.24         | 271.00       | 0.80   | 31.20  | 47.78    |
| Z271v4 | 0.148          | -16.24         | 271.00       | 0.80   | 34.29  | 77.00    |
| Z271v5 | 0.148          | -16.24         | 271.00       | 0.80   | 34.04  | 73.90    |
| Z271v6 | 0.148          | -16.24         | 271.00       | 0.80   | 33.80  | 70.94    |

| Models | \( \rho_0 \) [fm\(^{-3}\)] | \( E_0 \) [MeV] | \( K_0 \) [MeV] | \( m^* \) | \( J \) [MeV] | \( L_0 \) [MeV] |
|--------|----------------|----------------|--------------|--------|--------|----------|
| DD-F   | 0.147          | -16.04         | 223.32       | 0.56   | 31.63  | 56.00    |
| TW99   | 0.153          | -16.25         | 240.27       | 0.55   | 32.77  | 55.31    |
| DDH3   | 0.153          | -16.25         | 240.18       | 0.55   | 25.34  | 45.33    |
| DD-MEδ | 0.152          | -16.08         | 219.60       | 0.61   | 32.18  | 51.43    |

By writing the slope as

\[
L_0 = 3\rho_0 \left( \frac{\partial S}{\partial \rho} \right)_{\rho=\rho_0} = 3\rho_0 \left[ \left( \frac{\partial S_{\text{kin}}}{\partial \rho} \right)_{\rho=\rho_0} + \frac{\gamma}{\rho_0} S_{\text{pot}}^0 \right] = L_{\text{kin}}(\rho) + L_{\text{pot}}(\rho),
\]

we can find the \( \gamma \) parameter

\[
\gamma = \frac{L_0 - L_{\text{kin}}^0}{3 S_{\text{pot}}^0} = \frac{L_{\text{pot}}^0}{3 S_{\text{pot}}^0},
\]

where the superscript refers to the kinetic part \( (\text{kin}) \) and potential part \( (\text{pot}) \) of these quantities and the subscript 0 means that all these quantities were calculated at the saturation density. For more details see Ref. [10].

There are two recent predictions for the \( \gamma \) parameters. The first one constrains the kinetic part of \( S \) at saturation obtained from free proton-to-neutron ratios measured at intermediate energy nucleus-nucleus collisions with SRC included and predicts the value 0.25 ± 0.05 [12]. The second one constrains \( S \) at saturation without SRC. This constraint was taken from ASY-EOS.
at GSI, where the elliptic flows of neutron and light-charged particles in an Au-Au reaction resulted in $0.72 \pm 0.19$ [13].

To obtain the $\gamma$ value we use Equation (9) and calculate its value for three cases:

(i) complete kinetic term for the different models:

$$S^\text{kin}_i(\rho) = \frac{k^2_F}{6E^*_i}$$

where $i = \text{NL, DD}$, with $E^*_i = (k^2_F + M^*_i)^{1/2}$ and

$$M^*_\text{NL} = M - g\sigma, \quad M^*_\text{DD} = M - \Gamma(\rho)\sigma,$$

for symmetric matter. The Fermi momentum is written in terms of density as

$$k^F = \left(\frac{3\pi^2}{2}\rho\right)^{1/3}.$$  

The potential part for the symmetry energy, in this case, is given by the remaining terms in Eq. (2) for the NL model, and Eq. (3) for the density dependent one. This analysis shows that only 7 parametrizations have the $\gamma$ value inside the interval $\gamma = 0.72 \pm 0.19$. They are: BKA20, BKA22, BKA24, BSR11, BSR19, BSR20, and G2*.

(ii) separation of the really kinetic term, the one without any dependence of the interaction with the mesons, from the rest of the symmetry energy:

$$S^\text{kin}_\text{NL}(\rho) = S^\text{kin}_\text{DD}(\rho) = \frac{k^2_F}{6E^*},$$

with $E^* = (k^2_F + M^2)^{1/2}$, for the kinetic part, and

$$S^\text{pot}_\text{NL}(\rho) = S^\text{pot}_\text{DD}(\rho) = \frac{k^2_F}{6E^*_\text{NL}} - \frac{k^2_F}{6E^*_\text{DD}} + \frac{g^2F}{8m^*_F}\rho,$$

for the potential one.

Here we found 20 parametrizations with $\gamma$ coefficients in agreement with the range $\gamma = 0.72 \pm 0.19$. They are: BKA20, BKA22, BKA24, BSR8, BSR9, BSR10, BSR11, BSR12, BSR15, BSR16, BSR17, BSR18, BSR19, FSU-III, FSUGZ03, FSUGZ06, G2*, Z271s2, Z271s3, and Z271s4.

(iii) replace the kinetic part of the symmetry energy by the one proposed in Ref. [12]:

$$S^\text{pol}_i(\rho) = S_i(\rho) - S^\text{kin}_\text{SRC}(\rho),$$

where $i = \text{NL, DD}$. The expressions for the total symmetry energy $S_i(\rho)$ are given by Eq. (2), or Eq. (3) for the nonlinear or density dependent models respectively.

$$S^\text{kin}_\text{SRC}(\rho) = \left(2^{2/3} - 1\right) \frac{3k^2_F}{10M} - \Delta S^\text{kin}(\rho),$$

with

$$\Delta S^\text{kin}(\rho) = \frac{c_0k^2F}{2M\pi^2} \left[\lambda \left(\frac{\rho}{\rho_0}\right)^{1/3} - \frac{8}{5} \left(\frac{\rho}{\rho_0}\right)^{2/3} + 3\rho + 5\lambda\rho_0\right],$$

where the parameters $c_0 = 4.48$ and $\lambda = 2.75$ are also taken from Ref. [12].

In this case, only 6 parametrizations agree with the range $\gamma = 0.25 \pm 0.05$. 

4
4. Summary
In this work, we reviewed the subject already analyzed in reference [10]. The behavior of the total symmetry energy was presented in Fig. 1. For the cases where the kinetic term does not contain the short-range correlations the CRMF predicts $\gamma$ values in the interval $0.72 \pm 0.19$. More precisely, in cases 1 and 2, there are respectively 7 and 20 models within this interval. In case 3, where SRC was included, a decrease in the value of the range was observed and 6 parameterizations were able to describe the $\gamma$ values within the intervals $0.25 \pm 0.05$.

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