Light Quark Mass Difference and Isospin Breaking In Electromagnetic Pion Production

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Abstract

It is demonstrated that there is a dynamic isospin breaking effect in the near threshold $\gamma^*N \to \pi N$ reaction due to the mass difference of the up and down quarks, which also causes isospin breaking in the $\pi N$ system. The photopion reaction is affected through final state $\pi N$ interactions (formally implemented by unitarity and time reversal invariance). It is also demonstrated that the near threshold $\gamma \bar{N} \to \pi N$ reaction is a practical reaction to measure isospin breaking in the $\pi N$ system, which was first predicted by Weinberg about 20 years ago but has never been experimentally tested.

Since the discovery of a large mass difference between the up and down quarks ($m_u \simeq 5 MeV, m_d \simeq 9 MeV$) there has been considerable interest in the possibility of observing dynamical isospin breaking in the pion-nucleon system [1, 2, 3, 4, 5]. The theoretical consensus is that the magnitude of isospin breaking is not $(m_d - m_u)/(m_d + m_u) \simeq 27\%$, but instead is $\simeq (m_d - m_u)/\Lambda_{qcd} \simeq 2\%$ [3], where $\Lambda_{qcd} \simeq 0.2$ GeV. In this paper it will be shown for the first time that the $\gamma p \to \pi^0 p$ reaction is an excellent candidate to measure dynamic isospin violations due to the up, down quark mass difference.

Weinberg first showed that there is an isospin violating effect in the s wave $\pi N$ scattering length $a(\pi N)$ due to the up, down quark mass difference [3].

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This predicted effect, which occurs for $\pi^0 N$ scattering or charge exchange but not for $\pi^\pm N$ scattering, has more recently been calculated by Chiral Perturbation Theory (ChPT) \cite{4,5}. The predicted magnitude of this effect is the same (to within a factor of $\sqrt{2}$) in $\pi^0 N$ scattering and charge exchange reactions. However, since the magnitude of $a(\pi^0 N)$ is small, the relative magnitude of the isospin violating term is $\simeq 30\%$ \cite{1,4}. By contrast, for charge exchange where the isospin conserving amplitude is larger, the relative isospin violation is estimated to be $\simeq 2$ to $3\%$ \cite{1,4}.

In this paper the (dynamic) isospin breaking effect of the up and down quark mass difference is shown to be present in the near threshold $\gamma N \rightarrow \pi N$ reaction. This is in addition to the well known (static) isospin breaking effect due to the threshold difference between the $\gamma p \rightarrow \pi^0 p$ and $\gamma p \rightarrow \pi^+ n$ channels. Of this $\simeq 2/3$ is due to the Coulomb effect and $\simeq 1/3$ is caused by the up, down quark mass difference. The separation of the two threshold energies leads to the prediction of a unitary cusp in the $\gamma p \rightarrow \pi^0 p$ reaction near the $\pi^+ n$ threshold due to the two step $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ reaction \cite{6,7,8,9}. The magnitude of this unitary cusp is due to the large ratio of the electric dipole multipole amplitudes, $R = E_{0+}(\gamma p \rightarrow \pi^+ n)/E_{0+}(\gamma p \rightarrow \pi^0 p) \simeq -20$ \cite{6}. The first derivations of the unitary cusp \cite{7} used a K matrix approach to calculate the effect of charge exchange in the final state. Due to their single scattering approximation, these calculations violate unitarity as will be discussed below. The chiral perturbation theory (ChPT) calculations are basically isospin conserving but the biggest isospin non-conserving effect due to the pion mass difference (which is almost purely electromagnetic in origin) is inserted by hand \cite{6}. These calculations also violate unitarity due to their truncation at the one loop (single rescattering) level. There has been recent progress in performing ChPT calculations in $\pi N$ scattering which take the isospin violations due to the Coulomb interaction into account \cite{5}.

The approximations made in the K matrix \cite{7} and published ChPT papers \cite{6} can be overcome by using a 3 channel S matrix approach in which unitarity and time reversal invariance are satisfied (a preliminary version of this work has been presented previously \cite{8}). This has the advantage that both static and dynamic isospin breaking, and final state multiple scattering (to all orders), are taken into account. The S matrix for the 3 open channels ($\gamma p, \pi^0 p, \pi^+ n$) can be written as:
The off diagonal matrix elements for the photo-production of the $\pi^0p$ and $\pi^+n$ channels are written as $iM'_0$ and $iM'_c$, where $M'_0$ and $M'_c$ are proportional to the multipole amplitudes. For the important case of production of s wave pions these are the $E_{0^+}$, $(L_{0^+})$ transverse (longitudinal) multipoles. Although not explicitly written here, the S matrix elements are for a fixed value of $W$, the total CM energy, and the quantum numbers $l$ and $j$, the $\pi N$ orbital and total angular momenta. The notation of using 0 (c) for the neutral (charged) channel is conveniently generalized to neutron targets where the three channels are $\gamma n$, $\pi^0n$, and $\pi^-p$. Time reversal invariance requires that the S matrix be symmetric ($S_{ij} = S_{ji}$) and unitarity requires that $S^+S = SS^+ = 1$. The form of the 2x2 $\pi N$ portion of the S matrix has been chosen to be separately unitary and time reversal invariant. Eq. (1) is symmetric by construction. Applying the unitary constraint and assuming the weakness of the electromagnetic interaction by dropping terms of order $e^2$, one obtains:

$$
M'_0 = e^{i(\delta_0 + \delta)} [A'_0 \cos \frac{\phi}{2} + iA'_c \sin \frac{\phi}{2}]
$$

$$
M'_c = e^{i(\delta_0 + \delta_c)} [A'_c \cos \frac{\phi}{2} + iA'_0 \sin \frac{\phi}{2}]
$$

where $A'_0$ and $A'_c$ are smoothly varying, real functions, of the CM energy, and can be identified as proportional to the multipole matrix elements $M_0$ (for $\gamma p \rightarrow \pi^0p$) and $M_c$ (for $\gamma p \rightarrow \pi^+n$) in the absence of the final state $\pi N$ interaction. These equations show the connection between electromagnetic pion production and $\pi N$ interactions. Note that the phase shifts $\delta_0$, $\delta$ appear as $e^{i\phi}$ in photopion reactions compared to $e^{2i\delta}$ in elastic scattering because the $\pi N(\gamma N)$ interactions take place in the final (initial) state only. Eqs. (1) and (2) are valid for both photo- and electro-production, i.e. for both real and virtual photons. In general $M'_0$, $M'_c$, $A'_0$, and $A'_c$ are functions of $q^2$ (the invariant four momentum transfer) as well as $W$, $l$, and $j$. To order $e^2$ the $\pi N$ sector parameters $\delta_0$, $\delta_c$, and $\phi$ are functions of $W$, $l$, and $j$ only.
In this treatment terms of order $e^2$ have been neglected with the exception of $\delta$, which is small compared to both $\delta_0$ and $\delta_c$ (except at the $\pi^0$ threshold), and the mass difference between the charged and neutral pions which is put in by hand. This is the largest $O(e^2)$ effect found in $\pi N$ scattering [3] and in photoproduction [3]. The effect of the $O(e^2)$ terms have been worked out, although not presented here. In general they produce a small coupling between the electromagnetic and $\pi N$ sectors, so that e.g. the 2x2 $\pi N$ sector of the $S$ matrix is no longer separately unitary. When the experimental situation reaches a sufficient level of accuracy these corrections should then be implemented.

Eq. 2 is a generalization of the final state interaction theorem of Fermi and Watson [11] who first pointed out the connection between photo-pion production and $\pi N$ scattering. In their derivation, as in this one, time reversal and unitarity (to order $e^2$) were assumed. However they made the additional assumption that if one could neglect $e^2$ (and higher order terms) isospin would be conserved (this was before the time of quarks). This reduces the dimensionality of the $S$ matrix to 2x2 which gives the simple solution $S(\gamma p \rightarrow \pi N : I) = \pm |S| e^{i \delta(\pi N : I)}$ where $I=1/2,3/2$ is the isospin of the $\pi N$ system. If isospin were conserved, so that the two thresholds are degenerate, Eq. 2 reduces to the Fermi-Watson theorem. For the near threshold region in which the CM pion momentum $q \rightarrow 0$, the s wave phase shifts are $\delta_0 = [2a_3 + a_1]q/3 = a_{\pi^0p}q$, $\delta_c = [a_3 + 2a_1]q/3 = a_{\pi^+n}q$, and $\phi/2 = [a_1 - a_3]q/3 = a_c q$ where $a_i$ is the s wave $\pi N$ scattering lengths in the isospin $2I = 1, 3,$ or designated charge states. However isospin conservation is badly violated in the threshold region due to the threshold energy difference between the $\pi^+n$ and $\pi^0p$ channels and the additional dynamic isospin violating effect due to the up and down quark mass difference. This generalization of the Fermi-Watson theorem removes the approximation of isospin conservation. Note the interesting feature that below the $\pi^+n$ threshold there are only two open channels and Eq. 4 reduces to the form of the Fermi-Watson theorem, i.e. $M'_0 = \text{real number} \cdot e^{i \delta_0}$ or, equivalently, that $\tan(\delta_0) = \text{Im}E_{0^+}/\text{Re}E_{0^+}$.

It is of interest to show the connection between the $S$ matrix formulation which takes the final state scattering into account to all orders and the original $K$ matrix derivations [4] which only took a single final state scattering into account. This single scattering limit can be taken by expanding $e^{i \delta_0} \approx 1 + i \delta_0$, and neglecting second order terms like $\delta_0 \delta_c$, thus obtaining $M_0 \approx A_0 + iA_0 \delta_0 + iA_c a_{cex} q_c$. By examining the region below the $\pi^+n$
that causes the isospin breaking in $\pi N$ the s wave combined with the prediction of isospin breaking for the s wave scattering enters into $\beta$. Combining the two experimental (theoretical) numbers as:

\[
\beta = \frac{iA_c f_{\text{ex}}(\pi^+ n \rightarrow \pi^0 p)}{\cos(\phi/2)} \simeq E_{0^+}(\gamma p \rightarrow \pi^+ n) a_{\text{exx}}(\pi^+ n \rightarrow \pi^0 p)
\]

The value of $\beta$ can be calculated from the experimental value of $a_{\text{exx}}(\pi^- p \rightarrow \pi^0 n)$ [13]. Assuming isospin is conserved $a(\pi^+ n \leftrightarrow \pi^0 p) = -a(\pi^- p \leftrightarrow \pi^0 n)$ [13]. For $E_{0^+}(\gamma p \rightarrow \pi^+ n)$ the ChPT prediction of $28.2 \pm 0.6$ [14] is used. This is in agreement with the preliminary value of $27.6 \pm 0.3$ from a recent experiment [16]. Combining the two experimental (theoretical) numbers gives $\beta = 3.59 \pm 0.17$ ($\beta = 3.51 \pm 0.22$) [14].
lengths for charge exchange reactions $\delta a_{cex}$, that there is an isospin violating contribution to $\beta$,

$$\delta \beta = E_{0+}(\gamma p \rightarrow \pi^+ n)\delta a_{cex}(\pi^+ n \rightarrow \pi^0 p)$$

$$\delta \beta/\beta = \delta a_{cex}(\pi^+ n \rightarrow \pi^0 p)/a_{cex}(\pi^+ n \rightarrow \pi^0 p)$$  \hspace{1cm} (5)

In the second line it has been assumed that there is no quark mass effect on $E_{0+}(\gamma p \rightarrow \pi^+ n)$. It is estimated that $\delta a_{cex}/a_{cex} \simeq 2$ to 3\% \cite{1,4}. A better theoretical calculation of this effect should be performed.

The electric dipole amplitude $E_{0+}(\gamma p \rightarrow \pi^0 p)$ with its unitary cusp is presented in Figs. 1 and 2. The recent Mainz/TAPS \cite{17} and Saskatoon \cite{18} results for $\text{Re}E_{0+}$ are presented in Fig. 4, where only the statistical errors are shown. The small deviations between the data sets suggest the magnitude of the systematic errors. Considering this, the agreement between the calculated curves and the data is satisfactory. The curves are the ChPT calculation with three empirical low energy parameters used in fitting the data \cite{2}, and a unitary fit to the data \cite{17} using Eq. 3 which has $\beta = 3.67$ \cite{14} and a linear function of photon energy for $A_0$ with two parameters which were fit to the data. For this case $R = E_{0+}(\gamma p \rightarrow \pi^+ n)/E_{0+}(\gamma p \rightarrow \pi^0 p) \simeq -20$ \cite{3} and the effect of the two step charge exchange reaction is dramatic in the real (imaginary) part below(above) the $\pi^+ n$ threshold. Approximately 1 MeV above the $\pi^+ n$ threshold the $|\text{Im}E_{0+}| = |\text{Re}E_{0+}|$. The negative sign in $R$ makes $\text{Im}E_{0+}$ have the opposite sign from $\text{Re}E_{0+}$.

It is interesting to note that both the power counting rules of ChPT and the constraints of unitarity lead to pion rescattering in the final state as a critical dynamical ingredient. Although the unitary and ChPT curves both agree with the data for $\text{Re}E_{0+}$ there is an important difference between them. The value of $\beta = 2.78$ \cite{1,4} calculated for the $\gamma p \rightarrow \pi^0 p$ reaction is smaller then the one calculated using the separately predicted values of $E_{0+}(\gamma p \rightarrow \pi^+ n)$ and $a_{cex}(\pi^+ n \rightarrow \pi^0 p)$ of $3.51 \pm 0.22$. This difference can be clearly seen in Fig. 2. The reason for this discrepancy, which was discussed by the ChPT authors \cite{3}, is due to the fact that the ChPT calculation is carried out to one loop which is not sufficient for $\text{Im}E_{0+}$. This is a general feature of chiral perturbation theory in which the imaginary part of the amplitude is not calculated as accurately as the real part, and thus unitarity is only approximately satisfied at a given order \cite{19}. As will be discussed below the difference between the unitary and one loop ChPT values of $\beta$ can be observed in future experiments in which $\text{Im}E_{0+}$ will be measured directly.

To accurately measure the magnitude of the unitary cusp and to exploit
Figure 1: $\text{Re}E_{0^+}$ (in units of $10^{-3}/m_\pi$) for the $\gamma p \rightarrow \pi^0 p$ reaction versus photon energy $k$. The dashed dot curve is the ChPT fit [6] and the solid curve is the unitary fit (Eq. 1) [17]. The solid diamonds (circles) are the Mainz (Saskatoon) points [17, 18]. The errors are statistical only.
Figure 2: $\text{Im}E_{0+}$ (in units of $10^{-3}/m_\pi$) for the $\gamma p \rightarrow \pi^0p$ reaction versus photon energy $k$. The dashed dot curve is the ChPT calculation [6] and the solid curve is the unitary calculation (Eq. [7]).
the connection between electromagnetic pion production and low energy $\pi N$ interactions, one must measure $\text{Im}E_{0+}$. In photoproduction this requires experiments with polarized beams and/or targets [8]. For the sake of brevity the formulas connecting the cross sections and polarization observables to the multipoles [20] will not be quoted here. To briefly demonstrate the power of polarized photo-pion experiments, two asymmetries are shown in Fig. 3: $\Sigma$ for linearly polarized photons with an unpolarized target; and $T$ for unpolarized photons but with a target polarized normal to the reaction plane. The results presented in Fig. 3 use the p wave predictions of chiral perturbation theory [6] and the unitary fit to $E_{0+}$ discussed above [17]. $\Sigma$ is primarily sensitive to the p wave multipoles and since the unitary fit has essentially the same p wave multipoles as ChPT, the curves for ChPT and the unitary fit are almost identical. By contrast, $T$ is sensitive to a linear combination of p wave multipoles times $\text{Im}E_{0+}$, and shows its rapid rise above the $\pi^+n$ threshold. For $T$, the large difference in $\text{Im}E_{0+}$ between the unitary fit [17] and ChPT to one loop [8] should be straightforward to distinguish experimentally. A proposed experiment with tagged photons using an active, polarized proton target at Mainz estimates that $\beta$ can be measured to $\simeq 1$ to 2% [21]. Similar results could be obtained using a laser backscattering source [22, 23] or small angle electron scattering with polarized, internal, targets in a storage ring facility [24].

There are several possible strategies to test isospin conservation from measurements in the near threshold $\gamma N \rightarrow \pi N$ reaction. A definitive demonstration of this effect would involve precision measurements of the real and imaginary parts of $E_{0+}$ for the unitary cusp contributions to the $\gamma n \rightarrow \pi^0n$ and $\gamma p \rightarrow \pi^0p$ reactions. Although the predicted effect of $\simeq 2$ to 3% in $\beta$ is larger than the usual $\simeq 1\%$ effect expected on the basis of electromagnetic effects, it represents a serious experimental challenge to accomplish this goal, particularly since free neutron targets are not available to make this measurement. A simpler strategy would be to perform a precision measurement of the unitary cusp for the $\gamma p \rightarrow \pi^0p$ reaction, extract the value of $\beta$, and compare the result of with the value obtained using unitarity and isospin conservation quoted above. An equivalent strategy is to obtain $a_{\text{ex}}(\pi^+n \rightarrow \pi^0p)$ from the measured value of $\beta$ and compare it to the accurately measured value of $a_{\text{ex}}(\pi^-p \rightarrow \pi^0n)$ [13], obtained from the width of the 1s state in pionic hydrogen, to see if there is any deviation from the pure isospin prediction that these values are equal and opposite [15]. It is important to note that there is a PSI proposal [13] to improve the pionic hydrogen measurement so
Figure 3: The polarized photon (Σ) and polarized target (T) asymmetries (in %) at a $\pi^0$ center of mass angle of 90° for the $\gamma p \rightarrow \pi^0 p$ reaction versus photon energy k. The lower curve is the ChPT calculation \cite{6} for Σ. The two curves for T which rise rapidly at the $\pi^+ n$ threshold at 151.4 MeV are the ChPT calculation (dashed curve) \cite{6} and the unitary calculation (solid curve).
that the error in $a_{cex}(\pi^- p \to \pi^0 n)$ will be reduced to $\sim 1\%$.

It is therefore of interest to discuss the information that might be obtained from measurements of the $s$ wave $\pi N$ scattering lengths, $a(\pi N)$, from photopion reactions with the values that have been obtained using conventional pion beams. The values for $a(\pi N)$ for several physical channels are presented in Table 1 (the appropriate isospin relations [25] were used when required). The most accurate and direct measurements come from pionic hydrogen experiments [13]. In order to test isospin symmetry the PSI group has also measured pionic deuterium in order to get at the $\pi^- n$ scattering length [13]. Unfortunately the two body corrections in deuterium are large and add significantly to the error which makes the isospin conservation test uncertain at the required level of accuracy, so that these values are not quoted here.

The results of the two most recent empirical analyses of the $\pi N$ data for pion kinetic energies greater than $\sim 30$ MeV, extrapolated to threshold, are also presented [27, 26] in Table 1. In the SAID study the scattering lengths come from a dispersive analysis, the errors are hard to ascertain and are not quoted [26]. The chiral perturbation theory results [4] are consistent with experiment. If isospin symmetry is exact there are two independent scattering lengths in the $I=1/2, 3/2$ states, and six possible physical $\pi N$ elastic scattering and charge exchange reactions, so that there are many possible tests if three or more channels can be measured precisely. This is precisely the gap that can be filled by measurement of the final state $\pi N$ interactions in photopion production, e.g. $\pi^0 p$ or $\pi^+ n \to \pi^0 p$ in the $\gamma p \to \pi^0 p$ reaction.

Another test of isospin conservation has been made for medium energy $\pi N$ scattering (pion kinetic energy from 30 to 100 MeV) [27, 28]. In both cases the scattering amplitudes in the $I=1/2$ and $3/2$ states were obtained from the data for $\pi^{\pm} p$ elastic scattering. From these, the prediction for the $\pi^- p \to \pi^0 n$ charge exchange amplitude made on the assumption of isospin conservation was compared to the empirical charge exchange amplitude. An isospin violation at the $\sim 7\%$ level has been found by both analyses [27, 28], primarily in the $s$ wave amplitude. These analyses depend on the quality of the data and on the accuracy of the Coulomb corrections, which have been criticized as being inconsistent with the strong interaction calculations that were employed [4]. If isospin is indeed violated at the $7\%$ level, it is not clear how to relate this to the isospin breaking predictions for the $s$ wave scattering length [1, 4] which is applicable at lower energies. What is required is an extension of the isospin breaking calculations to medium energies. A test of the existing predictions requires experiments at lower energies which would
Table 1: S-wave $\pi N$ scattering lengths $a(\pi N)$ for several channels in units of $10^{-2}/m_\pi$ ($N = n$ or $p$)

| Channel | Pionic Matsinos [13] | Matsinos (Sp98) [27] | SAID [26] | ChPT [28] | Estimated isospin breaking [1, 4] |
|---------|---------------------|----------------------|-----------|-----------|-----------------------------|
| $\pi^- p$ | 8.83(0.08)     | 8.14(0.10)     | 8.83     | 8.70(0.86) | $\simeq 0$        |
| $\pi^- p \rightarrow \pi^0 n$ | $-13.01(0.59)$ | $-10.93(0.08)$ | $-12.5$  | $-12.45(0.75)$ | $\simeq 2$ to $3\%$ |
| $\pi^0 N$ | 0.41(0.09)     | 0.0     | 0.0     | $-0.1(0.7)$ | $\simeq 30\%$ |

measure the s wave scattering lengths.

In addition to the measurement of $\text{Im}E_{0^+}$ above the $\pi^+ n$ threshold discussed above, one could contemplate the more difficult measurement when only the $\pi^0 p$ channel is open. As was discussed previously, from a measurement of both the Re and Im parts of $E_{0^+}$ (or $L_{0^+}$) one obtains $\delta_0$, for which there is a predicted $\simeq 30\%$ isospin breaking effect. However, because of the small expected size of $\delta_0 \simeq 1^\circ$ in the energy region below the $\pi^+ n$ threshold, this is a very difficult task. It may be at the limits of feasibility using either a laser backscattering source, where an intense photon flux is concentrated in a small energy interval [22, 23], or with internal target, small angle scattering with polarized internal targets [24].

In conclusion, the connection between the $\gamma^* p \rightarrow \pi^0 p$ reaction and $\pi N$ scattering has been presented in a rigorous, model-independent way, which is unitary and time reversal invariant, and where the isospin breaking due to the threshold difference between the $\pi^0 p$ and $\pi^+ n$ channels, and the mass difference between the up and down quarks, is taken into account. This leads to a predicted unitary cusp due to the two step $\gamma p \rightarrow \pi^+ n \rightarrow \pi^0 p$ charge exchange reaction. The magnitude of the unitary cusp is given by $\beta \equiv E_{0^+}(\gamma p \rightarrow \pi^+ n) \cdot a_{\text{ex}}(\pi^+ n \rightarrow \pi^0 p)$. This unitary cusp has been recently observed in photoproduction experiments [17, 18]. It has been shown for the first time that there is a dynamical isospin breaking effect in the value of $\beta$, due to the mass difference of the up and down quarks, in electromagnetic pion
production. This is linked by unitarity and time reversal invariance to a predicted quark mass effect in $\pi^0 N$ scattering and pion charge exchange \([1, 3, 4]\). At present there are accurate measurements for $a_{\pi^- p}$ and $a_{\text{ceex}}(\pi^- p \rightarrow \pi^0 n)$ from pionic atoms performed at PSI \([13]\). To check isospin conservation requires at least one more precision measurement in another charge channel. This is more readily performed in electromagnetic meson production for two reasons: 1) in order to accurately measure the s wave scattering length, it is important to work at very low energies (e.g. $\leq 10$ MeV) at which $\pi N$ experiments are hard to perform since the low energy charged pions decay and also since $\pi^0$ beams can not be made at any energy; and 2) in electromagnetic pion production one can access charge states that cannot be reached with conventional pion beams (this is important for isospin checks). It is shown that photoproduction experiments with polarized targets can lead to a precise measurement of the small but interesting isospin violating effects. In addition to the polarization observables in photoproduction discussed above, similar information can be obtained in threshold $\pi^0$ electroproduction from the combination of the transverse- longitudinal (TL and TL') structure functions \([20]\). This possibility will be discussed in a future publication. Finally we stress that the important program of precisely measuring the $\pi N$ and photopion reactions that are required to test the predicted isospin violation, is difficult but feasible.

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