Andreev current enhancement and subgap conductance of superconducting hybrid structures in the presence of a small spin-splitting field

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(Dated: May 5, 2014)

We investigate the subgap transport properties of a S-F-N_s structure. Here S (N_s) is a superconducting (normal) electrode, and F is either a ferromagnet or a normal wire in the presence of an exchange or a spin-splitting Zeeman field respectively. By solving the quasiclassical equations we first analyze the behavior of the subgap current, known as the Andreev current, as a function of the field strength for different values of the voltage, temperature and length of the junction. We show that there is a critical value of the bias voltage V∗ above which the Andreev current is enhanced by the spin-splitting field. This unexpected behavior can be explained as the competition between two-particle tunneling processes and decoherence mechanisms originated from the temperature, voltage and exchange field respectively. We also show that at finite temperature the Andreev current has a peak for values of the exchange field close to the superconducting gap. Finally, we compute the differential conductance and show that its measurement can be used as an accurate way of determining the strength of spin-splitting fields smaller than the superconducting gap.

PACS numbers: 74.45.+c,74.25.F-
can be used to determine the strength of a weak exchange or Zeeman-like field in the nanostructure.

**Model and basic equations.** We consider a ferromagnetic wire F. Its length, $L$, is smaller than the inelastic relaxation length. The wire is attached at $x = 0$ to a superconducting (S) and at $x = L$ to a normal (N) electrode. As noticed above, F can also describe a normal wire in a spin-splitting field $B$ (in which case $h = \mu_B B$, where $\mu_B$ is the Bohr magneton) or in proximity with an insulating ferromagnet. We consider the diffusive limit, i.e. we assume that the elastic scattering length is much smaller than the decay length of the superconducting condensate into the F region. In order to describe the transport properties of the system we compute the quasiclassical Green functions [14]. They obey the Usadel equation [15] that in the so-called $\theta-$parametrization reads

$$
\partial_x^2 \theta_{\pm} = 2i \frac{E \pm h}{\mathcal{G}} \sinh \theta_{\pm}.
$$

(1)

Here the upper (lower) index denotes the spin-up (down) component. The normal and anomalous Green functions are given by $g_{\pm} = \cosh \theta_{\pm}$ and $f_{\pm} = \sinh \theta_{\pm}$ respectively. Because of the high transparency of the F/N interface the functions $\theta_{\pm}$ vanish at $x = L$, i.e. superconducting correlations are negligible at the F/N interface. We consider a tunneling barrier at the S/F interface and assume that its tunneling resistance $R_T$ is much larger than the normal resistance $R_F$ of the F layer. Thus, by voltage-biasing the N$\kappa$ the voltage drop takes place at the S/F tunnel interface. To leading order in $R_T / R_F \ll 1$ the Green functions obey the Kupriyanov-Lukichev boundary condition at $x = 0$

$$
\partial_x \theta_{\pm} |_{x=0} = \frac{R_F}{L R_T} \sinh [\theta_{\pm} |_{x=0} - \theta_S],
$$

(2)

where $\theta_S = \arctanh(\Delta/E)$ is the superconducting bulk value of the function $\theta$. Once the functions $\theta_{\pm}$ are obtained one can compute the current through the junction. In particular, we are interested in the Andreev current, i.e. the current for voltages smaller than the superconducting gap due to Andreev processes at the S/F interface. Such current is given by the expression

$$
I_A = \sum_{j=\pm} E \frac{n_{\pm}(E) e^2/2e R_T}{2W \alpha_j(E) - \sqrt{1 - (E/\Delta)^2 \text{Im}^{-1}(\sinh \theta_{\pm} |_{x=0})}}
$$

(3)

where $n_{\pm}(E) = \frac{1}{2} \left(\tanh[(E + eV)/2T] - \tanh[(E - eV)/2T]\right)$ is the quasiparticle distribution function in the N$\kappa$ electrode, $\alpha_j(E) = \int_0^\infty dx \cosh^{-2} |\text{Re} \theta_j(x)|$, $W = \frac{\mathcal{G} R_T}{2 L R_T}$ is the diffusive tunneling parameter, and $\xi = \sqrt{2/G} / \Delta$ is the superconducting coherence length. Eq. (3) is the expression used throughout this article in order to determine the subgap charge transport.

**Results.** We first compute the Andreev current numerically by solving Eqs. (1.2). In Fig. 1 we show the dependence of the Andreev current on the exchange field $h$ for different values of the bias voltage and temperature for a ferromagnetic F wire of the length $L = 10\xi$. We consider first the zero-temperature limit. For small enough voltages (e.g. $eV = 0.3\Delta$, black solid line in Fig. 1b) the Andreev current decays monotonously with increasing $h$. This behavior is the one expected, since by increasing $h$ the coherence length of the Andreev pairs in the normal region is suppressed, leading to a reduction of the subgap current. For large enough voltages (e.g. $eV = 0.8\Delta$ in Fig. 1b) and keeping the temperature low, the Andreev current first increases by increasing $h$, reaches a maximum at $h \approx eV$, and then drops by further increase of the exchange field, as it is shown for example by the black solid line in Fig. 1b. A common feature of all the low-temperature curves in Fig. 1 is the sharp suppression of the Andreev current at $h \approx eV$.

For large enough temperatures ($T = 0.25\Delta$ in Fig. 1b) one observes a peak at $h \approx \Delta$ (Fig. 1c). The relative height of this peak increases with temperature and voltage as one sees in Figs. 1b and 1c, respectively. In the case of large enough values of $V$ and $T$, one is able to observe both the enhancement of the Andreev current by increasing $h$ and the peak at $h = \Delta$ (see for example blue dashed line in Fig. 1). For values of the exchange field larger than $\Delta$, the Andreev current decreases by increasing $h$ in all cases. In principle all the behaviors of the Andreev current can be observed by measuring the full electric current through the junction as the single particle current is almost independent of $h$.

In order to give a physical interpretation of these results, we first recall the details of the process of two-electron tunneling that gives rise to subgap currents in diffusive systems in the absence of an exchange field. The value of this current is governed by two competing effects. On the one hand, the origin of the subgap current is the tunneling from the normal metal to the superconductor of two electrons with energies $\xi_k$ and $\xi_k$, respectively and momenta $k_1$ and $k_2$, that form a Cooper pair. This process is of the second order in tunneling and therefore involves a virtual state with an excitation on both sides of the tunnel barrier. The relevant virtual state energies are given by the difference $E_k - \xi_{k_1} - \xi_{k_2}$, where $E_k = \sqrt{\Delta^2 + \xi_k^2}$ is the excitation energy of a quasiparticle with the momentum...
electron tunneling. If ever, when these characteristic energies become larger and approaching the value of the gap, the difference $E_k - \xi_{k_1,k_2}$ eventually vanishes. As a result, the amplitude for two-electron tunneling decreases in the presence of a small exchange field (suppression), while in the region above the line the current increases (enhancement). We set $W = 0.007$ in both panels, $T = 0$ in panel (a) and $L = 10 \xi$ in panel (b).

We now turn to the effect of the exchange field $h$ on two-electron tunneling. If $h$ is nonzero, the majority and minority spin electrons at the Fermi level are characterized by different wave vectors $k_{F\pm} = k_F \pm \delta k$, where $\delta k \sim h/v_F$ and $v_F$ is the Fermi velocity. In Fig. 2(a) we show a schematic energy diagram. The wave vectors $k_{F\pm}$ are determined by intersection between the parabolas and the $k$-axis. For a given value of $\epsilon \lesssim \Delta$ and in the absence of an exchange field the relevant excitations with energies $\sim \pm \epsilon$ and wave vectors $k_{1,2}$ are not time-reversed (see Fig. 2a) and therefore their contribution to the current is not coherent. However upon increasing $h$, $|k_1| \rightarrow |k_2|$, i.e. the relevant excitations become more and more coherent, leading to an additional increase of two-electron tunneling. In particular when $h = \epsilon$, $k_1 = -k_2$ (cf. Fig. 2b). If $T \rightarrow 0$ there are no occupied states for $\xi > \epsilon$. Consequently as soon as $h > \epsilon$, the energy window around the Fermi level does not contain time-reversed electrons. This leads to the drop of the Andreev current shown for example in Fig. 1b. In contrast, for finite values of $T$, there are thermally induced quasiparticles with energy $\sim \Delta$, that become exactly time-reversed whenever $h = \Delta$. This leads to the maximum of the current at $h = \Delta$ when the temperature is finite (cf. Fig. 1b). The effects are most clearly seen when plotting the ratio $I_A(h)/I_A(0)$, as the Andreev pair decoherence effects due to temperature or voltage are then divided out.

A more quantitative understanding of the effects discussed above can be get by analyzing some limiting cases in which simple analytical expressions for the current can be derived. We first focus our analysis on the zero-temperature limit. Due to the tunneling barrier at the S/F interface the proximity effect is weak and hence one can linearize Eqs. (12) with respect to $R_F/R_T \ll 1$. After a straightforward calculation one obtains the Andreev current in this limit,

$$I_A = \frac{W \Delta_0^2}{2eR_T} \sum_{j=\pm} \int_0^\infty \frac{dE}{\Delta_0^2 - E^2} \times \text{Re} \left[ \frac{i\Delta_0}{E + j\hbar} \frac{\tanh \left( \frac{E + jh}{2\Delta_0} \xi \right)}{\xi} \right].$$

For a large exchange field, $h \gg \Delta_0 > \epsilon$ one can evaluate this expression obtaining

$$I_A \approx \frac{R_F \Delta_0}{8eL R_T^2} \sqrt{\frac{2}{\hbar}} \log \left( \frac{\Delta_0 + e\epsilon}{\Delta_0 - e\epsilon} \right).$$

Thus, the Andreev current decays as $h^{-1/2}$ for large values of $h$ in accordance with our numerical results (see Fig. 1).

In the case of small values of $h$, $\epsilon \lesssim e\epsilon < \Delta_0$, one can evaluate Eq. (4) in the long-junction limit, i.e. when $L \gg \sqrt{D}/\hbar$. In this case the Andreev current reads

$$I_A = \frac{\Delta_0 \xi R_F}{eL R_T^2} \sum_{j=\pm} \arctanh \left( \frac{e\epsilon + jh}{\Delta_0 + jh} \right) + \arctan \left( \frac{e\epsilon - jh}{\Delta_0 + jh} \right).$$

This expression describes the two different behaviors obtained in Fig. 1 for $h \leq e\epsilon$. For small voltages $I_A$ decreases by increasing the field $h$. However, for large enough values of the voltage $I_A$ is enhanced by the presence of the field. From Eq. (6) we can determine the voltage $e\epsilon$, at which the...
crossover between these two behaviors takes place, by expanding the expression for the current up to second order in $h/eV \ll 1$, i.e. up to the first non-vanishing correction to $I_A$ due to the exchange field. This expansion leads to the following transcendental expression which determine the voltage $V^*$ at which the crossover takes place,

$$\left(\frac{\Delta_0}{eV^*}\right)^{3/2} = \frac{3}{\pi} \left( \text{arctanh} \sqrt{\frac{eV^*/\Delta_0}{\Delta_0}} + \text{arctan} \sqrt{\frac{eV^*/\Delta_0}{\Delta_0}} \right).$$

From here we get $eV^* \approx 0.56\Delta_0$. For $V < V^*$ the Andreev current decays monotonically with $h$ while for $V > V^*$ it increases up to a maximum value at $h \leq eV$. This is in agreement with our numerical results in Fig. 1.

For an arbitrary length $L$ and finite temperature we have computed the value of $V^*$ numerically. In Fig. 3 we show the results. The solid black line gives the values of $V^*$ as a function of $L$ and $T$ [the (a) and (b) panels of Fig. 3 respectively]. The area below the black line corresponds to the range of parameters for which the Andreev current is suppressed by the presence of a spin-splitting field, while the area above the solid line corresponds to the range of parameters for which the unexpected enhancement of the subgap current takes place.

According to Fig. 3(a) at $T = 0$ the value of $V^*$ first decreases as $L$ increases, reach a minimum and then grows again up to the asymptotic value $eV^*/\Delta_0 \approx 0.56\Delta_0$. Also the dependence of $V^*$ on the temperature is non-monotonic having a maximum value at $T \sim 0.2\Delta_0$.

Small spin-splitting fields, as those studied in the present work, can be created by applying an external magnetic field $B$, in which case $h = \mu_B B$ or by the proximity of a ferromagnetic insulator as discussed in Ref. 13. It may be also an intrinsic exchange field of weak ferromagnetic alloys (see, for example, Ref. 21). Such small exchange fields are in principle difficult to detect. However, as we show in Fig. 4 by measuring the subgap differential conductance $G = dI/dV$ at low temperatures, one can accurately determine the value of $h$. At $T = 0$ the conductance shows two well defined peaks, one at $eV = h$ and the other at $eV = \Delta$. These are related to a sudden increase of the coherence length between the electron-hole pairs in the ferromagnet and of the two-particle tunneling amplitude respectively. As we have seen above, at small voltages $eV < h$ electrons with majority spins do not find time-reversed partners in the narrow energy window around the Fermi energy, i.e. such pairs show weak coherence. By increasing the voltage $eV \lesssim h$, the contribution of time-reversed electrons to the current gradually increases and consequently the differential conductance increases, reaching a maximum at $eV = h$. Further increase of the voltage, $eV > h$, leads to an increasing contribution to the current from non time-reversed electron-hole pairs and therefore to a suppression of the coherent contribution to $G$. At $h < eV \lesssim \Delta$ the two-electron tunneling amplitude increases as $(eV - \Delta)^{-1}$ due to virtual state contributions with energies $eV$ close to the gap; as a result the conductance shows a sharp increase. For $h \rightarrow 0$ (normal metal) the peak moves toward $eV \rightarrow 0$ (not shown here), which corresponds to the zero bias peak discussed, for example, in Ref. 22.

In conclusion, we present an exhaustive study of the subgap charge current through S-F-N$_2$ hybrid structures in the presence of a spin-splitting field. We have demonstrated the existence of a threshold bias voltage $V^*$ above which the Andreev current can be enhanced by an exchange field. We also have shown that at finite temperatures the Andreev current has a peak for values of the exchange field close to the superconducting gap $\Delta$. Finally, we have proposed a way to determine the strength of small exchange fields by measuring the differential conductance. Beyond the fundamental interest, our results can also be useful for the implementation of a recent and interesting proposal[13] which suggests a way to detect the odd-triplet component[21] of the superconducting condensate induced in a normal metal in contact with a superconductor and a ferromagnetic insulator. The latter induces an effective exchange field in the normal region. The amplitude of such induced exchange fields is smaller than the superconducting gap[22]. Therefore the proposed ferromagnet proximity system in Ref. 13 is a candidate to observe the phenomena described in the present work.

The authors thank E.I. Kats for useful discussions, F.S.B. and A.O. acknowledge the Spanish Ministry Economy and Competitiveness under Project No. FIS2011-316 28851-C02-02. The work of A.O. was supported by the CSIC and the European Social Fund under the JAE-Predoc program.

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6 Throughout this article $\Delta$ denotes the temperature dependent BCS gap, while $\Delta_0$ its value at $T = 0$. 

FIG. 4. (Color online) The bias voltage dependence of differential conductance at $T = 0$ for fields: $h = 0.3$ (black solid) and $h = 0.8$ (blue dashed). Here $G_A = 4R_T G_L$, $W = 0.007$ and $L = 10\xi$. 

\[\Delta \approx 3/2 \left( \text{arctanh} \sqrt{eV^*/\Delta_0} + \text{arctan} \sqrt{eV^*/\Delta_0} \right).\]
We neglect here the contribution to $I_A$ from partial Andreev reflection processes at energies above the gap. In the case of tunneling barriers, this contribution leads to small corrections and can be neglected. We neglect the contribution to $I_A$ from partial Andreev reflection processes at energies above the gap. In the case of tunneling barriers, this contribution leads to small corrections and can be neglected. We neglect the contribution to $I_A$ from partial Andreev reflection processes at energies above the gap. In the case of tunneling barriers, this contribution leads to small corrections and can be neglected. We neglect the contribution to $I_A$ from partial Andreev reflection processes at energies above the gap. In the case of tunneling barriers, this contribution leads to small corrections and can be neglected.