Direct Observation of Worm-Like Nanochannels and Emergent Magnon Motifs in Artificial Ferromagnetic Quasicrystals

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1. Introduction

Quasicrystalline structures and aperiodic metamaterials find applications ranging from established consumer gadgets to potential high-tech photonic components owing to both complex arrangements of constituents and exotic rotational symmetries. Magnonics is an evolving branch of magnetism research where information is transported via magnetization oscillations (magnons). Their control and manipulation are so far best accomplished in periodic metamaterials which exhibit properties artificially modulated on the nanoscale. They give rise to functional components, such as band stop filters, magnonic transistors and nanograting couplers. Here, spin-wave excitations in artificial ferromagnetic quasicrystals created via aperiodic arrangement of nanoholes are studied experimentally. Their ten-fold rotational symmetry results in multiplexed magnonic nanochannels, suggesting a width down to 50 nm inside a so-called Conway worm. Key elements of design are emergent magnon motifs and the worm-like features which are scale-invariant and not present in the periodic metamaterials. By imaging wavefronts in quasicrystals, insight is gained into how the discovered features materialize as a dense wavelength division multiplexer.
than 100 nm, thereby making any aperiodic interaction driven application challenging to implement. Irregularities in nanobar shape, size, and placement greatly influence the underlying physical phenomena under investigation. The nanoholes offer enhanced domain wall pinning, thereby causing an increased switching field, resulting in an increased stability of a device against magnetic field disturbances. Various shapes of holes have been studied on different periodic lattices, such as square shaped and circular holes on a square lattice. Periodic hole lattices support two prominent types of spin-wave mode patterns: localized (confined) and de-localized modes which can form nanochannels and artificially tailored magnonic minibands. The localized mode patterns occur in the directions perpendicular to the magnetic field and are mostly concentrated at edges of holes (edge modes) or between any two given holes (confined modes). The de-localized modes are connected throughout the ferromagnetic matrix between holes and are perpendicular to the magnetic field direction. These mode patterns appear in a periodic arrangement throughout the lattice. Spin-wave nanochannels exhibit periodically modulated widths. Antidot lattices consisting of periodically arranged holes have hence been investigated thoroughly. Experimental studies on hole arrays with a quasicrystalline arrangement are application challenging to implement. Irregularities in nanobar shape, size, and placement greatly influence the underlying physical phenomena under investigation. The nanoholes offer enhanced domain wall pinning, thereby causing an increased switching field, resulting in an increased stability of a device against magnetic field disturbances. Various shapes of holes have been studied on different periodic lattices, such as square shaped and circular holes on a square lattice. Periodic hole lattices support two prominent types of spin-wave mode patterns: localized (confined) and de-localized modes which can form nanochannels and artificially tailored magnonic minibands. The localized mode patterns occur in the directions perpendicular to the magnetic field and are mostly concentrated at edges of holes (edge modes) or between any two given holes (confined modes). The de-localized modes are connected throughout the ferromagnetic matrix between holes and are perpendicular to the magnetic field direction. These mode patterns appear in a periodic arrangement throughout the lattice. Spin-wave nanochannels exhibit periodically modulated widths. Antidot lattices consisting of periodically arranged holes have hence been investigated thoroughly. Experimental studies on hole arrays with a quasicrystalline arrangement are however missing. Relaxing the condition of periodicity allows one to directly probe the effect of aperiodicity on such type of patterns and explore their possible application (as reported in this article) due to increased spatial and rotational degrees of freedom. In contrast to photonics and plasmonics experimental studies on artificial ferromagnetic quasicrystals (AMQs) which offer unconventional rotational symmetries and a great density of reciprocal vectors are in their infancy. Corresponding dynamic magnetic responses in 2D quasicrystals have neither been classified nor fully exploited in view of manipulation and control of magnons.

In this paper, we report combined broadband SW spectroscopy, spatially resolved inelastic (Brillouin) light scattering (BLS) and micromagnetic simulations performed on 2D AMQs. They were prepared from ferromagnetic Co$_{20}$Fe$_{60}$B$_{20}$ (CoFeB) thin films by etching out circular nanoholes on the vertices of Penrose P2 and P3 quasicrystalline tilings (Figure S1–S3, Supporting Information). The CoFeB material was amorphous and magnetically isotropic. We prepared lattices of different generations (Experimental Section), that is, lateral extensions, and with mainly two different nanohole diameters $D$ (Table 1). The arrangement of holes followed characteristic geometrical motifs that exhibited a long range order with ten-fold rotational symmetry. In Figure 1a one motif consists of a decagon (highlighted by a white dotted line) for which a central hole is surrounded by ten holes. The variation of the angle $\theta$ of an applied in-plane...
magnetic field $H_0$ induced SW branches of resonant microwave absorption that exhibited ten-fold angular symmetry (Figure 1c,d). This symmetry is not known from periodic lattices with translational invariance and substantiates the quasicrystalline nature of the nanohole lattices. At fixed $\theta$ we observe multiple SW branches in experiments and simulations. We identify spin precession in fundamental motifs (Figure 1e) that do not depend on the generation and are distinctly different from the geometrical ones forming the tileings. Moreover, the aperiodic nanoholes give rise to stripe-like excitations in simulations (Figure 1f) and experiments (Figure 1g). These magnon nanochannels incorporate aperiodic sequences of bends. This aperiodicity is not known from previously investigated periodic hole lattices and might generate magnon band structures that do not depend on the generation and are distinctly different from the geometrical ones forming the tileings. The discovered aperiodic sequences of bends are similar to the Conway worms (white dotted lines in Figure 1b,g) which represent an establish mathematical concept when modeling quasicrystals. These worms form aperiodic Fibonacci chains in the 2D Penrose tileings.\cite{6-8} By BLS we obtain direct images of propagating magnons (Figure 1h) that exhibit irregular wavefronts across neighboring nanochannels which change with magnon frequency. These real-space images taken on a 2D analogue of a natural quasicrystal\cite{32} thereby provide fundamental insight into the formation of wave-like states in quasicrystalline matter. The discovered magnon modes fuel a new class of metamaterials by which irregular worm-like channels transform into a real-world application such as a dense wavelength division multiplexer (DWDM). Going beyond photonics, the magnonic DWDM is ultra-compact, reconfigurable via a rotating field and driven by an electromagnetic wave at a single input frequency.

2. Result and Discussion

2.1. Standing Spin Waves in Artificial Ferromagnetic Quasicrystal

SW spectroscopy was performed by a vector network analyzer connected to a coplanar waveguide (CPW) integrated on top of ten nominally identical AMQs of a given generation (Figure S2, Supporting Information). Penrose P2 and P3 AMQs (Figure 1a,b, and Table 1) both have ten-fold rotational symmetry but exhibit distinct differences concerning the prototiles used to construct them. In a P2 tiling (Figure 1a) the seed tiles are a kite and a dart. In a P3 tiling (Figure 1b) one considers thick and thin rhombi,\cite{6} forming a 2D analogue of a natural icosahedral quasicrystal.\cite{32} Two inter-vertex distances are present in a P2 tiling (we used $d_1 = 810$ nm and $d_2 = 500$ nm) as opposed to only one inter-vertex distance in Penrose P3 tiling (we used $d_2 = 810$ nm). The P2 tiling incorporates a lattice point (nano-hole) density that is larger by 62% compared to P3. For the nanofabrication we kept the inter-vertex distances constant (see above) and thereby created AMQs covering larger and larger areas from generation to generation (Table 1). Figure 1a,b (Figure 1c,d) refer to AMQs P2_SH_3rdGen and P3_SH_3rdGen (P2_SH_5thGen and P3_SH_5thGen), respectively, with nanoholes (H) of small (S) diameter $D = 135$ nm located at the vertices of 3rd (5th) generation (AMQs of 5th generation P2 and P3 tilings are shown in Figure S1, Supporting Information). The angle-dependent SW spectra in Figure 1c,d evidence magnetic anisotropy\cite{33} and a ten-fold rotational symmetry. This rotational symmetry is expected for an infinitely large Penrose lattice. The pronounced variation of resonance frequencies $f$ with $\theta$ reflects the variation of the effective magnetic field $H_{eff}$ which enters the equation of motion of spin precession.\cite{32} We note that according to the Curie

### Table 1. The CoFeB films were 19 nm thick. The overall area of AMQs increases with the generation. Up to 5th generation, the CPW’s signal line width was adjusted to fully cover the AMQs. Errorbars of diameter $D$ of nanoholes indicate the 95% confidence interval of the diameters of ten nanoholes which were randomly chosen from the respective microscopy image.

| Sample name              | Lattice | Generation | Area [$\mu$m$^2$] | Diameter $D$ [nm] | Number of nanoholes |
|-------------------------|---------|------------|------------------|------------------|--------------------|
| P2_SH_3rdGen            | P2      | 3rd        | 7.0 $\times$ 7.4 | 134 $\pm$ 6      | 121                |
| P2_LH_3rdGen            | P2      | 3rd        | 7.0 $\times$ 7.4 | 227 $\pm$ 5      | 121                |
| P2.SH_4thGen            | P2      | 4th        | 11.0 $\times$ 11.6 | 137 $\pm$ 7    | 301                |
| P2_LH_4thGen            | P2      | 4th        | 11.0 $\times$ 11.6 | 216 $\pm$ 3     | 301                |
| P2_SH_5thGen            | P2      | 5th        | 17.5 $\times$ 18.5 | 144 $\pm$ 15   | 761                |
| P2_LH_5thGen            | P2      | 5th        | 17.5 $\times$ 18.5 | 215 $\pm$ 13    | 761                |
| P2_SH_9thGen$^a$        | P2      | 9th        | 117.5 $\times$ 123.6 | 159 $\pm$ 6    | 34 101              |
| P2_LH_9thGen$^a$        | P2      | 9th        | 117.5 $\times$ 123.6 | 242 $\pm$ 6     | 34 101              |
| P3_SH_3rdGen            | P3      | 3rd        | 7.0 $\times$ 7.4 | 134 $\pm$ 6      | 76                 |
| P3_LH_3rdGen            | P3      | 3rd        | 7.0 $\times$ 7.4 | 219 $\pm$ 5      | 76                 |
| P3.SH_4thGen            | P3      | 4th        | 11.0 $\times$ 11.6 | 133 $\pm$ 11    | 191                |
| P3_LH_4thGen            | P3      | 4th        | 11.0 $\times$ 11.6 | 218 $\pm$ 7     | 191                |
| P3_SH_5thGen            | P3      | 5th        | 17.5 $\times$ 18.5 | 137 $\pm$ 9     | 476                |
| P3_LH_5thGen            | P3      | 5th        | 17.5 $\times$ 18.5 | 221 $\pm$ 12    | 476                |
| P3_SH_9thGen$^a$        | P3      | 9th        | 117.5 $\times$ 123.6 | 163 $\pm$ 5     | 21 106              |
| P3_LH_9thGen$^a$        | P3      | 9th        | 117.5 $\times$ 123.6 | 234 $\pm$ 4     | 21 106              |

$^a$The CPW signal line width was two microns and smaller than the lateral extension of the AMQ in case of 9th generation.
principle, we expect a ten-fold rotational symmetry of SW resonances also for small-area five-fold rotationally symmetric AMQs due to the linear polarization of the radio-frequency field in the CPW. This field is composed of, both, left- and right-circularly polarized electromagnetic waves. The two components allow us to excite gyrotropic spin precession for opposing directions of a magnetic field and opposing spin-precessional motion in one and the same segment of the AMQ. Hence, SW resonances in a five-fold rotationally symmetric geometrical motif exhibit a ten-fold rotational symmetry with respect to $\theta$.

Figure 2a,b shows field-dependent SW spectra for the 5th generation AMQ P2_SH_5thGen at $\theta = 0$ deg and 18 deg, respectively. Five prominent SW branches A1, B1, C1, D1, and E1 for $\theta = 0$ deg and A2, B2, C2, D2, and E2 for $\theta = 18$ deg are observed whose frequencies increase with $H_0$ (Figure 2c). We note that similar branches were detected when exploring AMQs of 3rd and 4th generation (Figures S4 and S5, Supporting Information). These branches exist independent of the total lateral size. Comparison of Figure 2a,b reveals that the SW resonances are “separated” more clearly from each other when the field was applied along $\theta = 18$ deg, that is, along an off-symmetry axis of the AMQ. Hence, SW resonances in a five-fold rotationally symmetric geometrical motif exhibit a ten-fold rotational symmetry with respect to $\theta$.

For AMQs with a large (L) hole diameter spectra are shown in Figure 2d,h (and in Figure S5, Supporting Information). Clearer spectra also occurred for large hole diameters in P2 AMQs at 18 deg (Figure 2d). Some resonances are systematically shifted in frequency and are less pronounced in P2_LH (P3_LH) compared to P2_SH (P3_SH) AMQs. We attribute these observations to the larger nanohole diameter (i) modifying $H_{eff}$ via the demagnetization effect and (ii) reducing the area covered by ferromagnetic material, respectively.

2.2. Emergent Magnonic Motifs and Worm-Like Nanochannels Explored by Micromagnetic Simulation

To get a microscopic understanding of the experimental observations, we performed micromagnetic simulations. The simulated power spectrum for AMQ P2_SH_3rdGen at $\theta = 0$ deg suggests five distinct modes (Figure 3a). The number and frequencies agree well with SW resonance frequencies obtained experimentally (Figure S6, Supporting Information). Due to the inhomogeneous magnetic field created by the demagnetization effect (Figure S7, Supporting Information), several spin-wave modes of different eigenfrequencies are excited: Local phase and power maps indicate that branch A1 reflects edge modes (not shown) with spin precession close to the edges of nanoholes. Branch B1 (Figure 3b) contains stripe-like extended modes which remind one of modes seen in periodic nanohole lattices. However, in case of a P2 AMQ with a high density of vertices the stripe-like modes are multi-segmented as nanoholes selectively block spin precession at some places. Branch
C1 (Figure 3c) contains patches of pronounced excitations that appear to be confined between, for example, four holes (indicated by rhombi with dotted white lines). Note that the overall mode pattern exhibits a strict mirror symmetry with respect to the horizontal central axis of the AMQ, which is parallel to the applied field. Branches D1 and E1 are attributed to higher order confined modes (not shown). In the case of a P3_SH_3rdGen AMQ, we extract five salient modes (Figure 3d). Based on the simulations we classify them as follows: F1 (edge mode), G1 (extended modes shown in Figure 3e), H1 (patches of confined modes shown in Figure 3f), I1 and J1 (higher order confined modes, not shown). Differences between P2_SH and P3_SH AMQs for a field along $\theta = 0$ deg are as follows: (1) stripe-like modes in P2 are blocked in many locations, whereas in P3 nanochannels extend more through the structure. (2) Patches highlighted with white dotted lines in Figure 3c,f represent a mode motif of magnons confined in thin rhombi. Importantly, AMQs with enlarged nanohole diameters repeat this rhombus-like mode motif confined between only four holes: For both P2 in Figure 3g and P3 in Figure 3h they are clearly visible; the additional holes in P2 compared to P3 suppress spin-precessional motion in large parts and reduce the magnon mode pattern to the rhombus-like motif almost completely (Figure 3g). We emphasize that the rhombus is a geometrical motif for P3, but not for P2. The observation of an identical magnon motif in P2 and P3 AMQs hints towards the emergence of magnon mode motifs different from the underlying geometrical motifs. (3) Simulated resonances in P2 (Figure 3a) are slightly better resolved as compared to P3 (Figure 3d) consistent with experiments.

For AMQ P2_SH_3rdGen (Figure 4a) and AMQ P3_SH_3rdGen (Figure 4e) with $H_0$ applied at $\theta = 18$ deg we identify five modes in the corresponding simulations. The resonance frequencies are in good agreement with the experimentally observed branches at 90 mT (Figure S6, Supporting Information). The signal strengths of SW resonances B2 to D2 are comparable to one another in contrast to signals obtained for $\theta = 0$ deg on P2 (Figure 3a). In the experiments corresponding branches at $\theta = 18$ deg were indeed better resolved compared to $\theta = 0$ deg. Branch B2 (Figure 4b) reflects stripe-like modes extending in a direction transverse to the applied field direction. In Figures 4b–d we indicate the decagon-like geometrical motif enclosed by ten nanoholes which is highlighted by a white dotted line in Figure 1a. Thereby a one-to-one
comparison between geometrical (structural) and mode motifs can be performed. In Figure 4b, close to the center of the geometrical motifs the modes exhibit bends. The bends imply that the aperiodically arranged nanoholes make propagation in SW nanochannels highly sensitive to the nanohole diameter (Figure S8, Supporting Information). For P2 AMQs, we find the SW nanochannels to be multi-segmented. This is different for P3 AMQs: In Figure 1f (and P3_LH in Figure S8 in the Supporting Information), long 1D nanochannels are identified that exhibit bends separated by different distances. Their appearances resemble worms whose bends vary from channel to channel. The aperiodicity prohibits identical sequences of holes within neighboring nanochannels suggesting channel-dependent magnon states. The minimum width of a nanochannel analyzed along a Conway worm (highlighted by white dots in Figure 1b) amounts to 50 nm. Due to this confinement spin waves in the nanochannels are in the exchange-dominated regime, and therefore the anisotropy of spin waves induced by the dipolar interaction in the long wavelength limit is not responsible for the existence of the worm-like nanochannels. We expect narrower nanochannels for further decreased inter-vertex distances in the Penrose tiling as the spin-wave dispersion relation is isotropic in the exchange regime. Furthermore, dissecting a Conway worm (Experimental Section) creates another Conway worm. Interestingly there is always at least one Conway worm in the Penrose tiling produced by the matching rule, which can propagate through the entire lattice. In case of low-damping material such nanochannels are expected to transmit spin-wave signals from edge to edge. Our experiment shows that inside the bulk of the AMQ there are many more aperiodic nanochannels guiding spin waves than these very long edge-connecting worms. We attribute our observation to the infinite set of worms of short and long ties that are expected for a regular Penrose tiling. The worms are distributed according to the Fibonacci sequence.

Figure 4. Micromagnetic simulations for an applied field of 90 mT along $\theta = 18$ deg (off-symmetry axis) of P2 and P3 AMQs and comparison of nanohole diameters. a) Simulated SW spectrum ($\text{Im}(m_z)$) of Penrose P2_SH_3rdGen with nanoholes of diameter $D = 135$ nm. Circles and letters are consistent with labels introduced for experimental data. Power maps ($m_z^2$) overlaid on corresponding local phase maps indicating regions of spin-precessional motion in P2 for branches b) B2, c) D2, d) E2. e) Spectrum simulated for an AMQ P3_SH_3rdGen and spin-precessional motion of f) $H_2$ at $f = 11.13$ GHz in a 3rd generation AMQ. g) Spin-precessional motion at the same frequency in P3_SH_4thGen, that is, a 4th generation AMQ. A mirror symmetry becomes apparent for the localized mode patches (highlighted white dotted lines). The yellow semi-transparent decagons and stars revisit underlying geometrical motifs for P2 and P3, respectively.
results: The patches appear within (Figure 1e and Figure 4c) or at the edges of the geometrical motifs (Figure 4d). The well-defined patches can be understood in analogy to periodic antidot lattices\cite{25,29} in which standing magnon modes reflected confinement and localization due to edges of nanoholes and inhomogeneities of $H_{\text{eff}}$\cite{35} respectively. We note that the emergent magnonic motif is seen in Figures 1e and 4d–d all exhibit a mirror symmetry axis being perpendicular to the applied field. A consistent mirror symmetry axis is found for mode patches of P3 at $\theta = 18$ deg (Figures 1f and 4f,g). Axes of mirror symmetries thus depend on the orientation of $H_0$ consistent with a report about nanobar-based quasicrystals\cite{36}. Figure 4f,g substantiates that a SW motif identified in generation three of AMQ P3 (dashed white rhombus) occurs in generation four at structurally similar positions within the geometrical motif (highlighted by the yellow star). Emergent magnon motifs identified throughout this work thus reflect self-similar motifs reappearing in different generations of quasicrystals.

### 2.3. Wavelength Division Multiplexing Utilizing Worm-Like Nanochannels

Using BLS microscopy (Figure S3, Supporting Information) we evidenced spin-precessional motion in the aperiodically bent nanochannels (Figure 1g). Data were taken near a CPW by which we excited the magnons. In the experiment, we observed a minimum width of 300 nm which is larger than the predicted value of 50 nm. The discrepancy is attributed to the diffraction limit of micro-focus BLS as explained in ref. [36]. Regions of large spin-precessional motion varied as a function of frequency and field orientation as will be discussed in the following (see also Figures S9 and S10, Supporting Information). Large segments of worm-like nanochannels were resolved best at relatively low frequency consistent with simulations. BLS with phase resolution allows us to directly image the phase evolution of waves in the quasicrystalline structure when excited by the integrated microstructured CPW. We evidence irregular wavefronts of propagating SWs (Figures 1h and 5a–c) indicating that magnon states are different from nanochannel to nanochannel. The wavefronts vary significantly with excitation frequency. The lengths over which the phase $k(x, y)^{\prime}$ $y$ varies by $2\pi$ in nanochannels vary with frequency (vertical bars in Figure 5a–c). $k(x, y)$ is the wave vector of spin waves which is not constant along a nanochannel as it depends on the aperiodically modulated effective field $H_{\text{eff}}(x, y)^{[35]}$. The irregular phase evolution (wavefront) is in contrast to 2D periodic antidot lattice magnonic crystals\cite{25} for which spin waves in neighboring nanochannels are in phase and wavefronts are parallel to the CPW if the applied magnetic field direction is collinear with the CPW’s signal line. The SW band structures for each nanochannel are identical due to the translational invariance of the magnonic crystals. In AMQs each SW nanochannel is modulated by a different sequence of nanoholes thanks to the quasicrystalline nature. Note that the arrangement of nanoholes is related to a 1D Fibonacci sequence, which suggests the formation of a specific spin-wave band structure.\cite{37} For the same spin-precessional frequency differently propagating magnon states are thereby created on a single chip in neighboring nanochannels. They enable dense wavelength division multiplexing (Figure S11, Supporting Information). In photonics, a DWDM allows one to exploit numerous wavelengths of light on a single optical fiber. For photons wavelength and frequency are coupled via the speed of light and hence a corresponding number of different input frequencies is required. In the quasicrystal-based DWDM only a single input frequency is needed to generate different wavelengths. Going beyond photonics the configuration of the magnonic DWDM can be altered ("gated") by an applied magnetic field. Another gating device such as a phase shifter using local magnetic fields\cite{38} could be used to tune wavefronts of spin waves in the quasicrystal, and in turn, the phase differences between neighboring nanochannels (Figure S11, Supporting Information) would be modified. This would create a wave-based logic device with a large number of both processing units and outputs which offers an areal density much larger than the recently proposed (de)multiplexers\cite{39,40}. Thanks to the scale invariance of demagnetization fields, further downscaling of the size of the magnonic DWDM is possible. In Figure 5d we demonstrate the rotation of the extended modes in nanochannels experimentally by choosing $\theta = 90$ deg for BLS performed at an excitation frequency of 11.2 GHz. The observed rotation of channels goes beyond the photonic channels (waveguides) created by tailored defects in hyperuniform disordered solids which are fixed in space.\cite{41} The extended magnon modes transform into localized excitations in the experiment when we increase the frequency to 12.9 GHz (Figure 5e). The transformation is predicted by simulations. Localized modes were found in theoretical studies also in bi-component magnonic AMQs.\cite{28}

### 3. Conclusion

In our study, spin waves in 2D AMQs are thoroughly investigated using broadband all-electrical spectroscopy, spatially resolved inelastic light scattering and micromagnetic simulations. Scale invariant spin-wave spectra indicate the emergence of peculiar magnon mode patterns in 2D AMQs which we characterize by the simulations. They exhibit specific mirror symmetries different from the underlying quasicrystalline lattices. The mirror symmetry axis varies with the direction of the applied field $H_0$ and takes either zero or 90 deg with respect to $H_0$. The unconventional rotational symmetries of AMQs is reflected in the angular dependent SW spectra. In analogy to their plasmonic counterparts,\cite{10} the aperiodically arranged nanoholes could be exploited in gating couplers, that is, microwave-to-magnon transducers. Their unconventional rotational symmetries are advantageous in order to optimize multi-directional magnon emission relevant for integrated magnonic circuits.\cite{17,18} Particularly intriguing are the aperiodically bent nanochannels which we observed. For them we find a width down to 50 nm in simulations. Due to the quasiperiodic arrangement of the nanoholes, spin-wave states are expected to be different from nanochannel to nanochannel. Consistently we detect irregular wavefronts of spin waves in the AMQs using spatially resolved BLS. They are created due to an inhomogeneous internal field $H_{\text{eff}}$ which is not active in
photonics and plasmonics. The materials-by-design approach and imaging by a state-of-the-art microscopy technique allowed us to explore the implication of Conway worms defined for the theoretical description of quasicrystals concerning a real-world application. We propose a quasicrystal-based DWDM that can be operated at a single microwave frequency and is ultra-compact because of the significant wavelength reduction inherent to magnonics.

4. Experimental Section

Creation of Penrose P2 and P3 AMQs: Figure S12a, Supporting Information shows the methodology behind constructing Penrose P2 tilings of different generation (and overall size). The authors started with two prototiles, called kites and darts. A kite is a quadrilateral with four interior angles of 72°, 72°, 72°, and 144°; a dart is a quadrilateral with four internal angles of 36°, 72°, 36°, and 216°. In this study, the authors began with the configuration formed by joining five kites at

Figure 5. Phase-resolved magnon propagation in a quasicrystal and control of nanochannels. Spatially resolved phase evolution of spin waves propagating along k (white arrow in (a)) in the P3 quasicrystal of Figure 1h at a) 4.5 GHz, b) 7.6 GHz, and c) 8.2 GHz for 10 mT. The black vertical bars indicate a phase difference of 2π. The length of the bar can be attributed to the wavelength in the selected channel. d) Worm-like nanochannels (bright) imaged by BLS without phase resolution at an excitation frequency of 11.2 GHz for θ = 90 deg and 90 mT. This angle is consistent with the channel configuration shown in Figure 1f. e) BLS imaging at an increased frequency of 12.9 GHz. The spin-precessional motion is found to be confined to narrow patches. The scale bars are 1 μm. The asterisk marks a position where there was a slight stitching error in the x, y positioning system.
a common vertex; this configuration called “0th generation”. Next, each kite was dissected into two smaller kites and a dart, and rejoined according to the matching rule\(^6\) to obtain the 1st generation tiling. Each application of such dissection rule generated a tiling of a higher generation containing a correspondingly larger number of prototypical elements. The 1\(^{st}\) generation tiling was rescaled to make it the same size as the 0th generation tiling; kites and darts of the 1st generation were smaller in size, compared to those of the 0th generation. Continued dissection and rescaling produced higher-generation Penrose P2 tilings, and the deflation process was then terminated at desired finite generation.

In the case of Penrose P3 tilings (Figure S12b, Supporting Information), it was started with two rhombi that have equal edges but different angles: (1) Thick rhombus with interior angles of 72°, 108°, 72°, and 108°. (2) Thin rhombus with interior angles of 36°, 144°, 36°, and 144°. Here, the authors started with the configuration formed by joining ten acute Robinson triangles (half of thin rhombi) at a common vertex; this configuration we termed “0th generation”. Two neighboring acute Robinson triangles were bisected into one thin and one thick rhombi that leads to “1st generation” P3 tiling. Subsequently, each thick rhombus was dissected into two thick and one thin rhombi; whereas thin rhombus got divided into one thick and thin rhombi, and were attached together according to matching rules,\(^9\) producing next generation of Penrose P3 tiling. As in the case of P2 tiling, we rescaled the thick and thin rhombi so that the subsequent generation thick and thin rhombi were of smaller sizes as compared to ones in 0th generation.

Finally, the edges of kites and darts were taken out and inserted nanoholes on vertices of kites and darts in P2 tiling. In the case of P3 tiling edges of rhombi were taken out and inserted nanoholes. The next step was to convert the final P2 and P3 tilings into a GDS file format that was imported into commercial electron beam lithography (EBL) software.

**Sample Fabrication:** A 19 nm thick amorphous CoFeB (Co\(_{90.8}\)Fe\(_{7}\)B\(_{2}\)) thin film was sputtered\(^{40}\) on a Si substrate. The aperiodic nanohole masks were patterned on hydrogen silsesquioxane negative electron beam resists using EBL. Ion beam etching was then performed to form a decagonal mesa and open the nanoholes in the magnetic thin film to form different AMQs as summarized in Table 1. Consecutively, CPW were patterned via EBL, electron beam evaporation of Ti/Au, and lift-off processing. They covered ten nominally identical AMQs. A bitmap containing the required P2 and P3 geometry was imported into OOMMF and discretized on a grid of 10 nm (100 nm). At each step we extracted the full width at half maximum of the local amplitude distribution in the corresponding line was recorded as a function of the spin precessional amplitude \(\theta\) of the applied field \(H\). A 2-port VNA allowed us to generate a microwave magnetic field with frequencies ranging from 10 MHz to 26.5 GHz. The applied microwave current generated an in-plane rf-magnetic field perpendicular to the long axis of the CPW. The microwave with a power of 0 dBm was applied at the port 1 of the CPW in order to excite magnetization precession. The precession-induced voltage was detected at port 2 via reading the scattering parameter \(S_{21}\) where the numbers 2 and 1 in the subscript denote the detection and excitation port. An external magnetic field \(\mu_0 H_0\) of up to 90 mT was applied under an angle \(\theta\) between the external field \(H_0\) and the CPW’s long axis. In order to increase signal to noise ratio, \(\Delta S_{21} = S_{21}(H_0) - S_{21}(\text{Ref})\) was evaluated where \(S_{21}(H_0)\) and \(S_{21}(\text{Ref})\) represent scattering parameters measured at a given field \(H_0\) and at 90 mT along \(\theta = 90^\circ\), respectively.

**Brillouin Light Scattering Microscopy:** Spin-wave eigenmodes were imaged via Brillouin light scattering (BLS) microscopy with and without phase resolution at room temperature using a setup similar to refs. [43,44]. Figure S3, Supporting Information shows the experimental setup. The end of one CPW was electrically bonded to a printed-circuit board, which was connected to a signal generator applying a microwave current. The corresponding magnetic microwave field excited spin precession in the AMQ close to the CPW at a fixed frequency. The power was such that spin precession was excited in the linear regime. A magnetic field of 90 mT was applied under an angle \(\theta\) via a permanent magnet for BLS measurement without phase resolution. For phase-resolved BLS measurement, a magnetic field of 10 mT was applied after that a field of 90 mT was applied in order to first saturate the AMQ. A lens with a numerical aperture of 0.85 was used to focus a 473 nm wavelength laser power of 300 nm onto the AMQ. Laser power was set to 1 and 0.85 mW for BLS measurement without and with phase resolution, respectively. Energy shifts of reflected laser light due to the inelastic magnon–photon scattering were detected by a triple-tandem Fabry–Prot interferometer. The AMQ was positioned under the laser with a spot using \(x, y\) piezo-positioning system. The step size to acquire the images of Figures 1g,h and 4, Figures S9 and S10, Supporting Information was 100 nm. We exploited phase-resolved inelastic light scattering while exciting spin waves phase-coherently at the straight CPW. As the CPW was microstructured the excitation was inhomogeneous and generated propagating spin waves. Each spin wave was taken by a wave vector \(k\). The acquisition time of inelastically scattered photons outside the CPW was chosen to be 2.14 times longer than in regions between signal and ground lines to compensate for the different excitation strength near a CPW. The 120 nm thick Au layer of the CPW did not allow us to monitor a signal from the magnonic channels underneath the ground and signal lines. The nanohole positions depicted in Figure 1g were reconstructed in that we overlaid (i) atomic force microscopy (AFM) images taken on the studied AMQ with (ii) the optical image taken in the BLS microscope, (iii) the BLS data, and (iv) the exposure masks for CPWs and electron beam lithography of nanoholes. Characteristic notches and dust particles identified in AFM and BLS microscopy data were used for a pre-alignment of the nanohole lattices. Finally we adjusted precisely the lattice of nanoholes given by the exposure mask relative to BLS data. Still we shifted this mask only within about 200 nm in lateral directions (i.e., by less than the diameter of the laser spot) to avoid that a maximum BLS intensity peak resided inside a hole with a diameter of about 200 nm.

**Simulations:** Micromagnetic simulations using OOMMF\(^{[5]}\) were performed to obtain a microscopic insight into SW excitations in AMQs. A bitmap containing the required P2 and P3 geometry was imported into OOMMF and discretized on a grid of 10 nm \(\times\) 10 nm \(\times\) 19 nm. A global in-plane 90 mT DC magnetic field was applied along the \(x\) direction, and equilibrium magnetization configuration was determined. Subsequently, a spatially homogeneous Gaussian pulse of 20 mT amplitude and 2.5 ps duration was applied along the \(z\)-axis (out of plane). In the simulation we explored the standing spin-wave modes due to uniform excitation. The perpendicular component of magnetization \(m_z\) was recorded as a function of \(x, y\) and the time step. A fast Fourier transformation (FFT) was performed on the magnetization of each pixel along the time axis to obtain the resonance spectrum. Then sums of power and phase of \(m_z\) were calculated to display the SW spatial profile for relevant frequencies. \(\text{Im}(m_z)\) was then integrated over the whole geometry, and plotted as a function of frequency \(f\) in order to observe the microwave absorption. \(\text{Im}(m_z)\) corresponds to the \(S_{21}\) observed in experiment. The input parameters used in the simulations were as follows: saturation magnetization \(\mu_0 M_{\text{s}} = 1.8\) T, exchange constant \(A = 13 \times 10^{-12}\) J m\(^{-1}\), and damping constant 0.007. In simulations, the global magnetic field was fixed along \(x\)-axis and the relevant bitmap containing the geometry was oriented at 0° and 18°. To illustrate the spatial distribution of spin-precessional motion under uniform excitation we show maps of the local phase and power (square of the spin precessional amplitude \(m_z\)). The widths of nanochannels in simulations (experiments) were extracted in that we followed the amplitudes of spin-precessional motion along a Conway worm with a step size of 10 nm (100 nm). At each step we extracted the full width at half maximum of the local amplitude distribution in the corresponding spin-wave nanochannel.
Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Keywords

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