Flux tubes at Finite Temperature

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The vacuum of QCD is a magnetic (dual superconductor)\(^a\)

\(^a\)G. 'tHooft, Phys. Scripta 25 (1982) 133

The electric field is confined into flux tubes $\rightarrow$ QCD strings

\[ V_{q\bar{q}} \rightarrow \sigma r \]

the dual Meissner effect causes the formation of chromoelectric flux tubes between chromoelectric charges leading to a linear rising potential
Flux tube model

At zero temperature,

\[
\rho_{\mu\nu} = \frac{\langle \text{Tr} \, W \, \Box_{\mu\nu} \rangle}{\langle \text{Tr} \, W \rangle} - \langle \Box_{\mu\nu} \rangle \to a^4 \left( \langle F_{\mu\nu}^2 \rangle_q \bar{q} q - \langle F_{\mu\nu}^2 \rangle_{\text{vac}} \right)
\]

where \( W \) is the Wilson loop and \( \Box_{\mu\nu} \) is the plaquette in the \((\mu, \nu)\) plane,

\[
\Box_{\mu\nu} = 1 - \frac{1}{N_c} \text{Tr} \left[ U_{\mu}(s) U_{\nu}(s + \mu) U_{\mu}^\dagger(s + \nu) U_{\nu}^\dagger(s) \right]
\]

\[
\langle E_i^2 \rangle = -\rho_{i,0} \quad \text{and} \quad \langle B_i^2 \rangle = \rho_{j,k}
\]

and the Lagrangian \((\mathcal{L})\) density is given by

\[
\mathcal{L} = \frac{1}{2} \left( \langle E^2 \rangle - \langle B^2 \rangle \right)
\]
Flux tubes at Zero Temperature

Lagrangian density

Flux tube profile:

Results in lattice spacing units, $a = 0.07261(85) \text{ fm}$ or $a^{-1} = 2718(32) \text{ MeV}$

$24^3 \times 48$ lattice volume with $\beta = 6.2$
Widening in the mediator plane

Square of the width of the flux tube in the mediator plane. As can be seen the tube flux becomes wider as the quark-antiquark distance is increased. We then fit the flux tube width with the leading order one-loop computation in effective string theory\(^a\)

\[
w^2 \left( \frac{R}{2} \right) \text{ (fm}^2\text{)}
\]

The \(B\) parameter can be compared with the theoretical leading order\(^a\) value for the factor of the logarithmic term,

\[
B = \frac{D - 2}{2\pi \sigma} = 0.0640028 \text{ fm}^2
\]

obtained using a string tension of \(\sqrt{\sigma} = 0.44\text{ GeV}\). We find that the width complies, almost within one standard deviation, with the logarithmic widening obtained at leading order in the Nambu-Gotto effective string theory.

\(^a\)F. Gliozzi et al. JHEP 1011, 053 (2010), arXiv:1006.2252.
The central observables that govern the event in the flux tube can be extracted from the correlation of a plaquette with the Polyakov loops,

\[
f_{\mu\nu}(r, x) = \frac{\beta}{a^4} \left[ \frac{\langle L(0) L^\dagger(r) \Box_{\mu\nu}(x) \rangle}{\langle L(0) L^\dagger(r) \rangle} - \langle \Box_{\mu\nu} \rangle \right]
\]

\(x\) denotes the distance of the plaquette from the line connecting quark sources and \(r\) is the quark separation.

or

\[
f_{\mu\nu}(r, x) = \frac{\beta}{a^4} \left[ \frac{\langle L(0) L^\dagger(r) \Box_{\mu\nu}(x_R) \rangle}{\langle L(0) L^\dagger(r) \rangle} - \langle L(0) L^\dagger(r) \Box_{\mu\nu}(x_R) \rangle \right]
\]

where \(x_R\) is the reference point placed far from the quark sources. with

\[
\Box_{\mu\nu}(s) = \frac{1}{N_c} \text{Tr} \left[ U_\mu(s) U_\nu(s + \mu) U^\dagger_\mu(s + \nu) U^\dagger_\nu(s) \right]
\]

the plaquette in the \((\mu, \nu)\) plane and

\[
L(x) = \frac{1}{N_c} \text{Tr} \prod_{t=1}^{N_t} U_4(x, t)
\]

the Polyakov loop.

\footnote{Y. c. Peng and R. W. Haymaker, SU(2) flux distributions on finite lattices, Phys. Rev. D 47, 5104 (1993)}
Techniques employed to improve the signal

**Multihit**

Replace each temporal link by its thermal average

\[
U_4 \rightarrow \bar{U}_4 = \frac{\int dU_4 U_4 e^{\beta \text{Tr} [U_4 F^\dagger]}\, dU_4 e^{\beta \text{Tr} [U_4 F^\dagger]}}{\int dU_4 e^{\beta \text{Tr} [U_4 F^\dagger]}}
\]

R. Brower et al., Nucl. Phys. B190, 1981.
G. Parisi et al., Phys. Lett. B128, 1983.
**Extended Multihit**

Replace each temporal link by its thermal average with the first $N$ neighbors fixed. Instead of taking the thermal average of a temporal link with the first neighbors, we fix the higher order neighbors, and apply the heat-bath algorithm to all the links inside, averaging the central link.

$$U_4 \rightarrow \bar{U}_4 = \frac{\int [\mathcal{D}U_4] \Omega \, U_4 \, e^{\beta \sum_{\mu,s} \text{Tr} \left[ U_\mu(s) F^\dagger(s) \right]}}{\int [\mathcal{D}U_4] \Omega \, e^{\beta \sum_{\mu,s} \text{Tr} \left[ U_\mu(s) F^\dagger(s) \right]}}$$

N. Cardoso et al., Phys. Rev. D 88, 2013.

By using $N = 2$ we are able to greatly improve the signal, when compared with the error reduction achieved with the simple multihit. Of course, this technique is more computer intensive than simple multihit, while being simpler to implement than multilevel. The only restriction is $R > 2N$ for this technique to be valid.
When computing the fields

\[ f_{\mu\nu}(r, x) = \frac{\beta}{a^4} \left[ \frac{\langle L(0) L^\dagger(r) \Box_{\mu\nu}(x) \rangle - \langle L(0) L^\dagger(r) \Box_{\mu\nu}(x_R) \rangle}{\langle L(0) L^\dagger(r) \rangle} \right] \]

The multihit and extended multihit must only be applied to the Polyakov operator, \( L \). No smearing technique should be applied to the plaquette!
Lattice volume: $48^3 \times 8$

| $\beta$ | $T/T_c$ | $a\sqrt{\sigma^2}$ | # config. |
|--------|---------|-----------------|-----------|
| 5.96   | 0.845   | 0.235023        | 5990      |
| 6.055  | 0.988   | 0.200931        | 5990/4775*|
| 6.1237 | 1.100   | 0.180504        | 3669      |
| 6.2    | 1.233   | 0.161013        | 1868      |
| 6.338  | 1.501   | 0.132287        | 3688      |
| 6.5    | 1.868   | 0.106364        | 1868?     |

where $\sigma$ is the string tension at zero temperature.

All the computations were done in NVIDIA GPUs using CUDA.
Near the phase transition

we can have results from contaminated configurations.
Solution: Remove results from configurations that belong to the second peak!
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\[ F_{\text{avg}} = -T \ln \left( L(0) L^\dagger(r) \right) \]

* \( \rightarrow \) results with extended multihit.
Near the phase transition

we can have results from contaminated configurations.
Solution: Remove results from configurations that belong to the second peak!

N. Cardoso and P. Bicudo, Phys. Rev. D 85, 077501 (2012).
Results for $Q\bar{Q}$

Results for $\beta = 6.055, \ T = 0.988T_c$ With Multihit:

The results from extended multihit are only valid for $R > 4$. 
Results for $\bar{Q}Q$

Results for $\beta = 6.055, \, T = 0.988 \, T_c$

$\langle E^2 \rangle$, $-\langle B^2 \rangle$, $\mathcal{L}$

$y\sqrt{\sigma}$

$R = 4$  $R = 6$  $R = 8$  $R = 10$  $R = 12$

$\langle E^2 \rangle$, $-\langle B^2 \rangle$, $\mathcal{L}$

$y\sqrt{\sigma}$

$R = 4$  $R = 6$  $R = 8$  $R = 10$  $R = 12$
Results for $\bar{Q}Q$

$\beta = 5.96, \ T = 0.845\ T_c$

$\beta = 6.055, \ T = 0.988\ T_c$, without contaminated configurations

$\beta = 6.1237, \ T = 1.100\ T_c$
Results for $Q\bar{Q}$

$\beta = 6.2, \ T = 1.233 T_c$

$\beta = 6.338, \ T = 1.501 T_c$

$\beta = 6.5, \ T = 1.868 T_c$
Results for $Q\bar{Q}$: in Middle of the Sources

\[
\begin{align*}
\langle E^2 \rangle & \quad -\langle B^2 \rangle & \quad L \\
O/\sigma^2 & \quad r/\sqrt{\sigma} & \quad r/\sqrt{\sigma} & \quad r/\sqrt{\sigma}
\end{align*}
\]
Results for \( QQ \)

\[
f_{\mu\nu}(r, x) = \frac{\beta}{a^4} \left[ \frac{\langle L(0) L(r) \Box_{\mu\nu}(x) \rangle - \langle L(0) L(r) \Box_{\mu\nu}(x_R) \rangle}{\langle L(0) L(r) \rangle} \right]
\]
Results for QQ

Below the critical temperature

\[ \beta = 5.96, \ T = 0.845 \ T_c \]

\[ \beta = 6.055, \ T = 0.988 \ T_c, \text{ with contaminated configurations} \]

\[ \beta = 6.055, \ T = 0.988 \ T_c, \text{ without contaminated configurations} \]
Above the critical temperature

\( \beta = 6.2, \ T = 1.233 T_c \)

\( \beta = 6.5, \ T = 1.868 T_c \)
**Results**

**Q̅Q**

\[
\begin{array}{ccc}
\langle E^2 \rangle & -\langle B^2 \rangle & L \\
\end{array}
\]

\[
\begin{array}{ccc}
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 \\
0.5 & 0.0 & 0.5 & 1.0 & 1.5 & 2.0 & 2.5 & 3.0 & 3.5 \\
0.5 & 0.0 & 0.5 & 1.0 & 1.5 & 2.0 & 2.5 & 3.0 & 3.5 \\
\end{array}
\]

\[O/\sigma^2\]

\[<E^2>\]

\[<B^2>\]

\[L\]

\[T = 1.1 T_c \quad R = 1.083 \sqrt{\sigma}\]

\[T = 1.501 T_c \quad R = 1.058 \sqrt{\sigma}\]

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**QQ**

\[
\begin{array}{ccc}
\langle E^2 \rangle & -\langle B^2 \rangle & L \\
\end{array}
\]

\[
\begin{array}{ccc}
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 \\
0.0 & 0.2 & 0.4 & 0.6 & 0.8 & 1.0 & 1.2 & 1.4 & 1.6 & 1.8 \\
0.5 & 0.0 & 0.5 & 1.0 & 1.5 & 2.0 & 2.5 & 3.0 & 3.5 \\
0.5 & 0.0 & 0.5 & 1.0 & 1.5 & 2.0 & 2.5 & 3.0 & 3.5 \\
\end{array}
\]

\[O/\sigma^2\]

\[<E^2>\]

\[<B^2>\]

\[L\]

\[T = 1.1 T_c \quad R = 1.083 \sqrt{\sigma}\]

\[T = 1.501 T_c \quad R = 1.058 \sqrt{\sigma}\]
