Entropy evolution in warm inflation from a 5D vacuum

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Abstract

Using some ideas of Modern Kaluza-Klein theory, we examine the evolution of entropy on a 4D Friedmann-Robertson-Walker (FRW) brane from a 5D vacuum state, which is defined on a 5D background Riemann-flat metric. We found that entropy production is sufficiently important during inflation: \( S > 10^{90} \), for all the initial values of temperature \( T_0 < T_{GU} \).

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I. INTRODUCTION

The standard picture of inflation introduced in 1981\cite{1} relied on a scalar field, called the inflaton, which during inflation was assumed to have no interaction with any other field. The inflationary scenario postulates that the universe underwent a phase of very rapid, accelerated expansion in its distant past. Observations have provided strong support for the paradigm. Standard inflationary scenario is divided into two regimes, slow-rolling expansion and reheating phase. It is assumed that exponential expansion places the universe in a super-cooled phase and subsequently thereafter the universe is reheated. During the inflationary epoch particles are being continually produced but their density is rapidly diminished by the expansion of the universe. In this scenario the radiation energy density $\rho_r$ becomes negligible rapidly since it scales inversely with the fourth power of the scale factor. In such a case, a short time reheating period terminates the inflationary period initiating the radiation dominated epoch. Inflation in presence of non-negligible radiation component is characterized by a non-isentropic expansion. This can be realized in the warm inflation scenarios\cite{2}.

What distinguishes warm inflation is that this particle production is sufficiently strong compared with the effects of the expansion, so that it is possible to produce a non-negligible particle density. Warm inflation occurs when the particle number is large enough to influence the classical inflaton field and produce density fluctuations\cite{3}. Two important facts in warm inflation are that it can provide a graceful exit to the standard inflation dynamics, and can give a physical justification of the slow-rolling conditions, which are incorporated ad hoc in models of standard inflation.

In this paper we examine the evolution of entropy on a 4D brane from a 5D vacuum state, which is defined on a 5D background Riemann-flat metric, using ideas of Modern Kaluza-Klein theory. This theory allow the fifth coordinate to play an important physical role. In the framework of the Induced Matter (or Space-Time-Matter) theory\cite{4}, all classical physical quantities, such as matter density and pressure, are susceptible of a geometrical interpretation. The mathematical basis of it relies in the Campbell’s theorem\cite{5}, which ensures an embedding of 4D general relativity with sources in a 5D theory whose field
equations are apparently empty. That is, the Einstein equations\(^1\) \(G_{\alpha\beta} = -8\pi G T_{\alpha\beta}\) (we use natural units: \(c = \hbar = 1\)), are embedded perfectly in the Ricci-flat equations \(R_{AB} = 0\). Other version of 5D gravity, which is mathematically similar, is the membrane theory, in which gravity propagates freely on the 5D bulk and the interactions of particles are confined to a 4D hypersurface called "brane"\([6]\). Both versions of 5D general relativity are in agreement with observations.

II. 5D VACUUM AND FIELD DYNAMICS

We consider the recently introduced 5D Riemann-flat metric\([7]\)

\[
dS^2 = \psi^2 \frac{\Lambda(t)}{3} dt^2 - \psi^2 e^{2f} \sqrt{\Lambda(t)/3} dt dr^2 - d\psi^2, \tag{1}
\]

where \(\Lambda(t)\) is a time-dependent function, which can be interpreted as a decaying cosmological parameter under certain circumstances\([8]\). Furthermore, \(dr^2 = dx^2 + dy^2 + dz^2\) is the 3D Euclidean metric, \(t\) is the cosmic time and \(\psi\) is the space-like noncompact extra dimension. Since the metric \((1)\) is Riemann-flat (and therefore Ricci-flat), hence it is suitable to describe a 5D vacuum vacuum \((G_{AB} = 0)\) in the framework of Space-Time-Matter (STM) theory of gravity\([9]\). With this aim we shall consider the 5D action

\[
I = \int d^4 x \sqrt{\left(\frac{5}{16\pi G} + \frac{1}{2} g^{AB} \varphi, A \varphi, B\right)}, \tag{2}
\]

where \(\left(\frac{5}{g}\right)\) is the determinant of the covariant metric tensor \(g_{AB}\):

\[
\left(\frac{5}{g}\right) = \psi^8 \left(\frac{\Lambda}{3}\right) e^{6f} \sqrt{\frac{\Lambda}{3}} dt, \tag{3}
\]

and \(\left(\frac{5}{g_0}\right) = \psi_0^8 \left(\frac{\Lambda_0}{3}\right)\) is a constant to make dimensionless the expression \(\left|\frac{\left(\frac{5}{g}\right)}{\left(\frac{5}{g_0}\right)}\right|\).

In order to describe a 5D vacuum on \((1)\), we shall consider \(\varphi\) as a massless test classical scalar field, which is minimally coupled to gravity. For this reason, the Lagrangian related to \(\varphi\) is purely kinetic and free of any interaction on \((1)\). From the mathematical point of view, the second term in the action \((2)\) is constructed by using monogenic fields \(\varphi\), which have null D’Alambertian on the 5D Riemann-flat metric \((1)\)\([10]\).

\(^1\) In this paper we use capital Latin letters that run from 0 to 4 and Greek letters that run from 0 to 3.
The equation of motion for the field $\varphi$ is

$$\ddot{\varphi} + \left[ 3\sqrt{\frac{\Lambda}{3}} - \frac{\dot{\Lambda}}{2\Lambda} \right] \varphi - \frac{\Lambda}{3} e^{-2\int \frac{\Lambda}{3} dt} \nabla^2 \varphi - \frac{\Lambda}{3} \left[ 4\psi \frac{\partial \varphi}{\partial \bar{\psi}} + \psi^2 \frac{\partial^2 \varphi}{\partial \bar{\psi}^2} \right] = 0. \quad (4)$$

Furthermore, since $\varphi$ evolves on a Riemann-flat spacetime and complies with the Einstein equations $G_{AB} = -8\pi G T_{AB}$, ($G$ is the gravitational constant), the energy-momentum tensor $T_{AB}$ must be null on (1).

In the next sections we shall study the interpretation for the dynamics of the scalar field $\varphi$ on an effective 4D FRW brane.

III. DISSIPATIVE DYNAMICS OF $\varphi$ ON A FRW BRANE

We shall assume that the 5D spacetime (1) is dynamically foliated on the fifth coordinate: $\psi = \psi(t)$, such that the effective 4D hypersurface is

$$dS^2 \big|_{\psi=\psi(t)} = d\sigma^2 \big|_{\psi=\psi(t)} - \dot{\psi}^2(t)dt^2$$

$$\equiv \left[ \psi^2(t) \frac{\Lambda(t)}{3} - \dot{\psi}^2(t) \right] dt^2 - \psi^2(t) e^{2\int \frac{\Lambda(t)}{3} dt} dr^2, \quad (5)$$

with the condition $\psi^2(t) \frac{\Lambda(t)}{3} - \dot{\psi}^2(t) > 0$, such that $g_{AB} U^A U^B = 1 \ [U^A = \frac{dX^A}{ds}$ are the components of the penta-velocity]. We can define $\rho = \rho_0 + \Delta \rho$ and $p = p_0 + \Delta p$, the energy density and the pressure on $d\sigma^2 \big|_{\psi=\psi(t)}$ in (5).

A. Entropy evolution on the $d\sigma^2$

We shall consider that $\rho_0(t)$ and $p_0(t)$ are the energy density and the pressure on the effective 4D hypersurface $dS^2 \big|_{\psi=\psi(t)}$ in (5) and the system describes an adiabatic evolution on (5), so that (from the thermodynamical point of view) it can be considered as a closed system

$$\frac{d}{dt} \left[ \rho_0 a^3(t) \right] + p_0 \frac{d}{dt} \left[ a^3(t) \right] = 0. \quad (6)$$

Since the universe describes a de Sitter expansion during the inflationary epoch, it is interesting to study the evolution of $\varphi$ on the hypersurface $d\sigma^2 \big|_{\psi=\psi(t)} = g_{\mu\nu}(t, \vec{r}, \psi(t)) \ dx^\mu dx^\nu \neq dS^2 \big|_{\psi=\psi(t)}$, which can be considered as a brane on (5). It is very important to remark that
the equivalence principle is broken on \(d\sigma|_{\psi=\psi(t)}\); \(g_{AB}U^A U^B \neq 1\). Hence, \(\Delta \rho\) and \(\Delta p\) comply with the following equation on \(d\sigma^2|_{\psi=\psi(t)}\):

\[
\Delta \dot{\rho} + 3H \gamma \Delta \rho = \frac{T}{a^3(t)} \dot{S},
\]

(7)

where \(\gamma = 1 + \frac{\Delta p}{\Delta \rho}\), \(a(t) = \psi(t) e^{\int \sqrt{\Lambda} \, dt}\) is the scale factor on \(d\sigma^2|_{\psi=\psi(t)}\), \(H(t) = \dot{a}/a\) is the Hubble parameter, \(T\) is the background temperature of the thermal bath, and \(\dot{S}\) is its entropy. A very interesting case, of particular interest is \(\frac{\Delta p}{\Delta \rho} = 1/3\). In this particular case we can identify \(\Delta \rho\) and \(\Delta p\) with the radiation energy density and its pressure: \(\Delta \rho \equiv \rho_r\) and \(\Delta p \equiv p_r\). In this case \(\gamma = 4/3\) and the system radiates

\[
\dot{\rho}_r + 4H \rho_r = \delta,
\]

(8)

where the interaction \(\delta \equiv \frac{T}{a^3(t)} \dot{\rho} > 0\) is related with the variation of entropy\(^{[11]}\). Notice that the entropy increases with time, so that both sides in (8) are positives. The interesting is that eq. (8) is the same to those of warm inflation, for the evolution of radiation energy density.

**B. Einstein equations on \(d\sigma^2\)**

To understand better the equations (7) and (8), we can start to write the Energy Momentum tensor for a perfect fluid on the 5D Riemann-flat metric (1)

\[
T^A_B = (P + \rho)U^A U_B - P \, g^A_B,
\]

(9)

such that \(U^A U_A = 1\) and \(dS^2 = (\psi^2 \Lambda /3) \, ds^2 - d\psi^2\). Furthermore, we take

\[
\left(\frac{ds}{dS}\right)^2 = \frac{3}{\Lambda \psi^2} \left[1 + U^4 U_4\right].
\]

(10)

In our case, we are choosing \(U^4 = \dot{\psi} U^0\) [see the metric (5)], so that if we take a co-moving frame \(U^i = u^i = 0\), we obtain the following expression:

\[
u^0 u_0 = \frac{1}{1 + \dot{\psi}^2 U^0 U_0} \leq 1,
\]

(11)

where the equality holds when \(\dot{\psi} = 0\).
On the brane $d\sigma^2$, the energy momentum tensor has the form
\[ T_{\alpha\beta}^{\psi(t)} = (P + \rho) \left( \frac{3}{\psi^2 \Lambda} \right) u^\alpha u_\beta - P \delta^\alpha_\beta + (P + \rho) \left[ g^\alpha_\beta - \left( \frac{3}{\psi^2 \Lambda} \right) u^\alpha u_\beta \right] \bigg|_{\psi(t)}. \] (12)

In particular, if we choose $\psi(t) = \sqrt{\frac{3}{\Lambda}}$, we obtain
\[ T_0^0|_{\psi(t)=\sqrt{\frac{3}{\Lambda}}} = \rho, \] (13)
\[ T_x^x|_{\psi(t)=\sqrt{\frac{3}{\Lambda}}} = T_y^y|_{\psi(t)=\sqrt{\frac{3}{\Lambda}}} = T_z^z|_{\psi(t)=\sqrt{\frac{3}{\Lambda}}} = -P, \] (14)

where $P = -\rho$, and
\[ \rho_\varphi = \rho u^0 u_0, \] (15)
\[ \rho_r = \rho \left[ 1 - u^0 u_0 \right]. \] (16)

Here, the total energy density on the brane $d\sigma^2$ is given by matter and radiation energy densities ($\rho_\varphi, \rho_r$): $\rho = \rho_\varphi + \rho_r$ and the total pressure $P$ is given by matter $P_\varphi$ and radiation $P_r$, such that $P_r = \rho_r/3$: $P = P_\varphi + \rho_r/3$.

The Friedmann equations on the metric $d\sigma^2$ are
\[ 3H^2 = -8\pi G \left[ \rho_\varphi + \rho_r \right], \] (17)
\[ 3H^2 + 2\dot{H} = 8\pi G \left[ P_\varphi + \rho_r/3 \right], \] (18)

which are exactly the same as those of warm inflation.

**IV. AN EXAMPLE**

Now we consider the foliation where $\psi^2(t) = 3/\Lambda(t)$. In this case the effective 4D metric \((5)\) is
\[ dS^2|_{\psi(t)=\sqrt{\frac{3}{\Lambda(t)}}} = d\sigma^2|_{\psi(t)=\sqrt{\frac{3}{\Lambda(t)}}} - \frac{3}{\Lambda(t)} \left( \frac{\dot{\Lambda}(t)}{2\Lambda(t)} \right)^2 dt^2, \] (19)

where
\[ d\sigma^2|_{\psi(t)=\sqrt{\frac{3}{\Lambda(t)}}} = dt^2 - \frac{3}{\Lambda(t)} e^{2\sqrt{\frac{3}{\Lambda(t)}}} dr^2 \equiv dt^2 - a^2(t) dr^2. \] (20)

From the thermodynamical point of view, the hypersurface described by the brane $d\sigma^2|_{\psi(t)=\sqrt{\frac{3}{\Lambda(t)}}}$ can be considered as an open system with respect to the closed one $dS^2|_{\psi(t)=\sqrt{\frac{3}{\Lambda(t)}}}$ in \((19)\). We shall consider that \((20)\) is our physical spacetime.
The equation of motion for the field $\varphi(t, \vec{r})$ on the metric (20) [induced from the 5D vacuum defined by the action (5), on the metric (1)], is

$$\ddot{\varphi} + \frac{3}{a} \dot{\varphi} - \frac{1}{a^2} \nabla^2 \varphi - \frac{\Lambda(t)}{3} \left[ 4 \psi \frac{\partial \varphi}{\partial \psi} + \psi^2 \frac{\partial^2 \varphi}{\partial \psi^2} \right] = \frac{-\delta}{\dot{\varphi}}, \quad (21)$$

where the interaction on the right-hand side of (21) has its origin in the temporal dependence of the fifth coordinate on the brane (20).

In order to study the evolution of entropy $S$ during warm inflation, we shall consider the case where

$$\gamma = \frac{4}{3} - \frac{1}{3} \left( t - t_0 \right), \quad (22)$$

where $t \geq t_0$, $t_0$ being the time when inflation starts. Notice that at the beginning of inflation (i.e., for $t \simeq t_0$ the parameter $\gamma \simeq 4/3$ and $\Delta P/\Delta \rho \simeq 1/3$. However, after a few Planckian times one recovers $\Delta P/\Delta \rho \simeq 0$, so that $\gamma \simeq 1$, for a matter dominated universe. In other words, the evolution of $\rho$ is governed by the equation

$$\dot{\rho} + \gamma(t) H \rho = \delta, \quad (23)$$

where $\gamma(t)$ is given by eq. (22). We consider $a(t) = a_0 \left( t/t_0 \right)^{[n(t)+1]}$, so that $n(t) = (t_0/t)$. In this case the Hubble parameter is

$$H = n \ln(t/t_0) + \left( \frac{n + 1}{t} \right) = \frac{t_0}{t^2} \left[ 1 - \ln(t/t_0) \right] + \frac{1}{t}. \quad (24)$$

In the figure (1) we have plotted the evolution of $\Gamma/(3H)$, as a function of time, such that $\Gamma(t) = \delta/\dot{\varphi}^2$. Notice that dissipation $\Gamma$ is of the order of the expansion $3H$ during all the inflationary stage, so that the production of entropy remains high in this epoch.

The evolution of entropy is given by

$$\dot{S} = \frac{a^4}{2\pi T_0 a_0} \frac{\dot{H}^2}{HG}, \quad (25)$$

so that

$$S(t) = -\frac{1}{2\pi T_0 a_0 G} \int_{t_0}^{t} \frac{a^4(\tau)}{\dot{H}^2} \frac{\dot{H}^2}{H} \, d\tau. \quad (26)$$

In the next table we endorse the entropy produced during warm inflation for $k = 1.5$ and $p = 1$, taking into account different values of initial temperature $T_0$


\begin{table}
\begin{tabular}{|c|c|c|c|}
\hline
\( k = 1.5 \ ; \ p = 1 \) & \( S \sim 10^{90} \) & \( S \sim 10^{94} \) & \( S \sim 10^{98} \) \\
\hline
\( T_0 = 10^{-19} \ G^{-1/2} \) & \( t_f = 10^9 \ G^{1/2} \) & \( t_f = 10^{9.5} \ G^{1/2} \) & \( t_f = 10^{10} \ G^{1/2} \) \\
\( T_0 = 10^{-15} \ G^{-1/2} \) & \( t_f = 10^{9.5} \ G^{1/2} \) & \( t_f = 10^{10} \ G^{1/2} \) & \( t_f = 10^{10.5} \ G^{1/2} \) \\
\( T_0 = 10^{-11} \ G^{-1/2} \) & \( t_f = 10^{10} \ G^{1/2} \) & \( t_f = 10^{10.5} \ G^{1/2} \) & \( t_f = 10^{11} \ G^{1/2} \) \\
\( T_0 = 10^{-7} \ G^{-1/2} \) & \( t_f = 10^{10.5} \ G^{1/2} \) & \( t_f = 10^{11} \ G^{1/2} \) & \( t_f = 10^{11.5} \ G^{1/2} \) \\
\hline
\end{tabular}
\end{table}

where we are using natural units \( h = c = 1 \), so that the Planckian mass is given by \( M_p = G^{-1/2} \) and \( G \) is the gravitational constant. The interesting here is that models with smaller initial temperature generate more entropy in shortest times.

V. FINAL COMMENTS

In this letter we have studied the evolution of entropy on a 4D FRW metric, which can be considered as a brane on a 5D Riemann-flat metric on which we define a vacuum. The generation of entropy is a consequence of dissipative dynamics of the inflaton field, which can be induced from a 5D vacuum state. Furthermore, the equivalence principle is broken on the FRW brane on which the ”extra force” that acts on the inflaton field must be interpreted as non-conservative.

In the example here studied we have examined different possibilities of evolution with different initial conditions for warm inflation. Our results show how the rate of production of entropy of the universe increases during inflation when \( T_0 \) is smallest. In this sense, models of warm inflation with smallest initial temperature are favored because they produce more entropy in shortest times. Anyway, we found that for all \( T_0 \) which are below \( T_{GU} \approx 10^{15} \ G^{-1/2} \), warm inflation produces a sufficiently high amount of entropy to assure sufficiently flatness of the universe: \( S > 10^{90} \).

Acknowledgments

The authors acknowledge CONICET and UNMdP (Argentina) for financial support.

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FIG. 1: Evolution of $\Gamma/(3H) = -\dot{\Lambda}/[3\Lambda H]$ as a function of cosmic time $t$ [expressed in Planckian times $G^{-1/2}$]. Notice that dissipation is of the order of expansion during all the inflationary phase.