Identification of Power Grid Critical Node Based on Voltage Level

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Abstract. With the integration of large clusters of variable renewable energy, plus the fact that modern power system has appeared great varieties of complicated characteristics. Improvement of safe and stable operation of the grid system has become a research focus in the new situation. Such being the case, a critical node identification method based on the theil entropy theory is proposed. By studying the characteristics of power flow and voltage variation, new methods for system safe operation improvement are explored. Firstly, proceeding from the mechanism of load variation on voltage and power flow, the theil entropy index of voltage growth rate is established to describe the equilibrium of voltage variety and the theil entropy index of weighted flow impact rate to describe the uniformity of power flow variation. Secondly, considering both subjective preferences and objective data, binary feature analysis and entropy weight method are combined to get the comprehensive index which can accurately identify the critical nodes for power grid. At last, the simulation results of IEEE30 node system and a southwest certain area power net-work has verified the validity and efficiency of the model. Further more, evaluation results has provided a reference for the construction of security defense system in modern power system.

1. Introduction

The complex characteristics of modern power system are characterized by large scale, complex components, supply and demand balance, and numerous random factors. This will bring high risks and great challenges to the safty and stability of power grid [1]. Critical assessments of large scale complex power system are beneficial to identifying the volunteer part accurately and scientifically, which makes it possible to prevent disasters in the power grid in advance.

Firstly, each branch and node of the power grid is classified by the voltage level. Secondly, vary load capacity of each node. According to the equilibrium degree of voltage amplitude variation and power flow variation, the Theil entropy model of voltage growth rate and weighted flow impact rate are established respectively. Thirdly, apply the binary analysis method and entropy weight method to construct the comprehensive evaluation index of critical nodes. Lastly, verify the scientificity of the physical model through simulation of an IEEE30 node standard system and an actual system in Southwest China.
2. Theil Entropy Theory

Suppose that a discrete information source may send out \( n \) signals \( (A_1, A_2, A_3, \ldots, A_n) \), and the probability of occurrence of each signal is \( x_i \) \( (i = 1, 2, 3, \ldots, n) \). Then these probabilities satisfies:

\[
\begin{align*}
0 & \leq x_i \leq 1 \\
\sum_{i=1}^{n} x_i &= 1
\end{align*}
\]  
(1)

The information entropy of the discrete system is

\[
H(x) = \sum_{i=1}^{n} x_i \log \frac{1}{x_i}
\]  
(2)

According to formula (2), when the probability of occurrence of each event is \( 1/n \), the entropy reaches the maximum. At the same time, a power system shows the most disordered state. The differences between the actual entropy and the maximum entropy is defined as Theil Index, which is the so called Theil Entropy Standard.

\[
T = \log n - H(x) = \sum_{i=1}^{n} x_i \log nx_i
\]  
(3)

The larger the Theil Index, the greater the regional differences [2].

Compared with other regional differential indicators, the Theil Index has a decomposable character, and the decomposition is very simple. The power grid can be partitioned by different voltage levels, and the critical degree of components can be considered jointly with the importance of their locations in the grid. Additionally, after first order decomposition of the theil index, the overall inequalities can be decomposed into intra and inter-class inequalities. We can not only analyze the differences in groups and between groups, but also analyze the proportion of each part, which will provide an accurate location of unbalanced source of the grid. In summary, Theil Index is more suitable for the analysis of uniformity of modern complex power grid under different voltage levels.

3. Theil Entropy Index

3.1. Theil entropy index of voltage growth rate

Assuming that the load capacity of node \( i \) has increased, the variation of the voltage amplitude of the node \( j \) is

\[
\Delta V_j^i = \left| V_j^i - V_j^0 \right|
\]  
(4)

In the formula (4), \( V_j^0 \) stands for voltage amplitude of the node \( j \) in the initial state, \( V_j^i \) stands for the voltage amplitude after variation.

Therefore, the growth rate of the voltage amplitude of the node \( j \) is defined as follows

\[
\gamma_j = \frac{\Delta V_j^i}{V_j^0}
\]  
(5)

In the formula (5), \( V_j^0 \) stands for the rated voltage of the node \( j \).

Suppose one power system has \( N \) nodes, \( L \) branches, \( S \) voltage levels, and the voltage level \( q \) has \( N_q \) nodes. Then, according to the first order decomposition of the Theil Index, the theil entropy of voltage growth rate can be defined as follows:

\[
T_{vi} = \sum_{q} \frac{\gamma_q}{\gamma} T_v + \sum_{q} \frac{\gamma_q}{\gamma} \ln \frac{N_q/N}{N} = T_{iv} + T_{ivq}
\]  
(6)

\[
T_v = \sum_{q} \frac{\gamma_q}{\gamma_q} \ln \left( \frac{\gamma_q/N_q}{1/N_q} \right)
\]  
(7)
In the formula, \( T_{V_0} \) stands for the whole equalization of the voltage growth rate, \( T_{V_1} \) stands for the equalization of voltage growth rate in the same voltage level, \( T_{V_2} \) stands for the equalization of voltage growth rate between different voltage levels, \( T_{V_3} \) stands for the equalization of voltage growth rate of voltage level \( q \), \( \gamma_{V_4} \) stands for the voltage growth rate of all nodes in voltage level \( q \), \( \gamma_{V_5} \) stands for the voltage growth rate of all nodes in the grid, \( \gamma_{V_6} \) stands for the voltage growth rate of node \( n \) in voltage level \( q \).

3.2. Theil entropy index of weighted flow impact rate

Load change of one node not only causes the fluctuation of voltage amplitude, but also causes the variation of transmission current. The uniformity and aggregation of transmission current distribution play an important role in cascading failures [3].

Suppose a system is in a balanced state which works under normal conditions. When node \( i \) increases an unit load, the power flow variation of branch \( k \) is

\[
\Delta p_i^k = p_i^k - p_0^k
\]

The initial power flow value of the branch \( k \) is \( p_0^k \) the power flow after change is \( p_i^k \). At the same time, the flow impact rate of branch \( k \) is

\[
\eta_i^k = \frac{\Delta p_i^k}{\sum_{k=1}^{L} \Delta p_i^k}
\]

\( L \) stands for the total number of branches in system.

Considering the capacity margin of the residual branch itself, even the distribution of power flow is balanced, the variation of system may also cause the power flow of some heavy-load branches off-limit. Therefore, the weighted entropy model is applied in this paper as a safety constraint. The load rate of each branch is used in each voltage level and the average load rate is used between voltage levels. Then the improved theil model of weighted flow impact rate is as follows:

\[
T_{wp} = \frac{\sum_q \eta_q T_r}{\eta} + \sum_q \mu_q \frac{n_q}{N_q} \ln \left( \frac{n_q}{\eta_q} \right) + \frac{1}{N} = T_{wp} + T_{wp} \]

\[
T_{wp} = \sum_q \mu_q \eta_q \ln \left( \frac{n_q}{\eta_q} \right) \left( \frac{1}{N_q} \right)
\]

In the formula, \( \mu_q \) stands for the average load rate of all branches in voltage level \( q \), \( \mu_{wp} \) stands for the load rate of branch \( l \) in the voltage level \( q \). \( T_{wp} \) stands for the whole equalization of the power flow distribution, \( T_{wp} \) stands for the equalization of power flow distribution in the same voltage level, \( T_{wp} \) stands for the equalization of power flow distribution between different voltage levels, \( T_{wp} \) stands for the equalization of power flow distribution of voltage level \( q \). \( \eta_q \) stands for the flow impact rate of all branches in voltage level \( q \), \( \eta_q \) stands for the overall flow impact rate of the system, \( \eta_{wp} \) stands for the flow impact rate of branch \( l \) in the voltage level \( q \).

The larger the \( T_{wp} \) is, power flow are more easily to transfer to these branches with little margin, and power flow distribution will be more uneven. Power flow off-limit has a larger probability to occur. When a load is disturbed, the safety and stability of the system will bear a serious impact, that is to say, this node proves to be a critical node in power grid.

3.3. Critical node comprehensive evaluation index

When node voltage or power flow is step beyond the boundary, it will greatly compromises the overall security of the system, causing large-scale cascading failures, and bring serious threat to power grid [4].
Therefore, the critical nodes assessment should consider the influence of active and reactive power and the build a comprehensive evaluation index from a global perspective. Define comprehensive evaluation index of critical node as follows:

$$C_i = \lambda_1 T_{vi} + \lambda_2 T_{wp} \quad (12)$$

In the formula, $\lambda_1$ and $\lambda_2$ are the comprehensive weights of attributes [5]. The larger $C_i$ is, the system voltage increase and power flow increase are more aggregated in the system. In all, the position of the node in the power grid is more critical.

4. Critical Node Identification Process
The identification process of critical node in power grid is shown in Figure 1.

5. Example Simulation
5.1. Simulation and analysis of critical nodes in IEEE30 node system
In this paper, the IEEE30 node system is used as a simulation test system. The specific parameters are shown in document [6]. Nodes and branches are numbered as shown in Figure 2. This system consists of 24 PQ nodes, 5 PV nodes, and node 1 is a balanced node. In addition, it is divided into two voltage levels: the high voltage level is 132 kV, the low voltage level is 33 kV. The comprehensive weight indexes calculated by experiment are $\lambda_1=0.685$, $\lambda_2=0.315$. The calculated theil entropy indexes are shown in figure 3 and figure 4.
odes of the system and electrical connection with

Figure 2. Connection diagram of IEEE 30-bus System

Figure 3 shows the distribution of the voltage growth rate of each node, Generators can maintain the stability of voltage level because of the powerful reactive support. Therefore, the voltage growth rate of the load 3, 4, 6, 7 and other nodes near generators are low, and the voltage fluctuation is little. The node 16, 17, 18 lie in the load center, far from generators, lacking reactive power support. Once the node load increases, the voltage variation are obvious. Although generators have sufficient reactive compensation to maintain the stability of the voltage, when the load fluctuates obviously, the generators need to increase its output immediately to maintain the balance of power flow. At this time, the connection nodes like node 3, 4, 6, 7 are easy to be affected by the variation of the network power flow. Nodes 27, 28 and 29 belong to the terminal nodes of the system and electrical connection with the balanced node are weak. So, they are hard to be affected by the external fluctuation. The theil entropy indexes of weighted flow rate are low.

Table 1. Vulnerable nodes of IEEE 30-bus system

| rating | theil entropy of voltage growth rate | theil entropy index of weighted flow rate | comprehensive evaluation index | Node number |
|--------|-------------------------------------|-----------------------------------------|--------------------------------|-------------|
| 1      | 0.8277                              | 0.7581                                  | 0.7800                         | 15          |
| 2      | 0.9246                              | 0.7111                                  | 0.7783                         | 16          |
| 3      | 0.6929                              | 0.8008                                  | 0.7668                         | 14          |
| 4      | 0.1838                              | 1.0000                                  | 0.7429                         | 3           |
| 5      | 0.5163                              | 0.8399                                  | 0.7380                         | 12          |
| 6      | 0.1236                              | 0.9860                                  | 0.7143                         | 6           |
| 7      | 0.1544                              | 0.9456                                  | 0.6964                         | 4           |
As can be seen from table 1, the first three nodes like 15, 16 and 14 are mainly concentrated in the central area of the power grid. It shows this region is more critical in the system and needs more focus of operators. The first 10 nodes are basically in the load center or near the generators, bearing the main tasks of transmitting. If these nodes are disturbed, the power flow and voltage growth will gather together, leading to the gradual deterioration of the network. Take node 6 as an example. The node 6 is a main transformer branch node, responsible for injecting power flow into the system. Although the voltage growth rate is 0.1236, the weighted flow rate is high. If its load is disturbed, the whole network will have a large-scale flow transferation, which will aggravate the burden of the remaining nodes and branches. Therefore, the node 6 is more critical.

5.2. Simulation and analysis of practical model system
In order to verify the practicability of critical node identification model in this paper, a certain Southwest Power Grid during the dry season is simulated and analyzed. The reginal network topology is shown in Figure 5, transmission network is divided into 110 kV and 35 kV, with a total of 17 nodes, 3 nodes, and node 1 is a balanced node.

![Figure 5. Topological map of real power grid structure in a southwest certain area](image)

Table 2. Critical nodes identification of a southwest certain area power network

| Rating | Node Number | Comprehensive evaluation index |
|--------|-------------|--------------------------------|
| 1      | 2           | 0.9836                         |
| 2      | 4           | 0.9620                         |
| 3      | 8           | 0.9398                         |
| 4      | 3           | 0.8736                         |
| 5      | 6           | 0.6137                         |
| 6      | 9           | 0.5720                         |
| 7      | 14          | 0.5323                         |
| 8      | 10          | 0.5275                         |
| 9      | 15          | 0.5026                         |
| 10     | 18          | 0.4802                         |

Using the comprehensive evaluation index proposed in the second section, all the nodes in the system are sorted, and the most critical 10 nodes of the power grid are identified. The ranking result is shown in Table 2.

From table 2, it can be seen that the first 4 nodes of the power grid are node2 (110 kV), node 4 (110 kV), node 8 (110 kV) and node 3 (110 kV), which constitute the 110 kV backbone transmission.
network of this region, connecting the power generation centre and load centre, bearing the important
tasks of the power transmission and interconnection. If load disturbances happened to these critical
nodes, whether it is voltage growth or power flow transferation, it will produce a large degree of
aggregation in power grid, seriously, nodes or branches will exit running and the power grid will
become an isolated network. The node 6 (110 kV), node 9 (35 kV), node 14 (35 kV) and node 10
(35 kV) are all near-transformer nodes. The position of these nodes are very important in power grid.

On one hand, the node is closely connected with the distribution network. Compared with the
transmission network, the operation condition and the component distribution of distribution network
are more complex, so these nodes more vulnerable to load disturbance; on the other hand, these nodes
bear heavy load and are far away from main network, lacking reactive power compensation and
voltage support. Voltage growth rate is high, so operators need to pay more attention to it.

In summary, the simulation of the local power grid proves the practicability of the model
established in this paper.

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