The simplified analysis method of nonlinear buckling and displacement of latticed structure

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Abstract. In order to simplify the modeling process of building complex lattice structures by finite element analysis software, a simplified method for nonlinear buckling and deformation analysis of lattice structures based on equivalent inertia moment method is proposed. By comparison with the displacements, the latticed structure is equivalent to a solid structure, and the expression of the equivalent inertia moment of a typical latticed structure is given. The finite element analysis software ANSYS is used to analyze the buckling problem and the deformation of latticed structure and the equivalent solid structure, furthermore, the load-displacement curves of the actual structure and equivalent model are obtained. The results show that the simplified analysis method of nonlinear buckling and deformation of latticed structure is feasible and can satisfy the needs of engineering design.

1. Introduction

Latticed structure is connected by a series of rods, which has the characteristics of light weight, reasonable stress and good wind passing. They are widely used in steel structure buildings, bridges, rocket launching towers, hoisting machinery, and so on. The buckling and nonlinear deformation of complex latticed structure are difficult in design and calculation, so it is necessary to find a simple, practical and effective simplified calculation method.

For structural stability, beams and rods are usually used as the research object to obtain the critical buckling load of structural instability. Xia Chaofan [1] established the relationship between the buckling loads of symmetric and asymmetric compression bars for the high-order modal analysis of the stability of slender compression bars. Humer [2] deduced the buckling load at the bifurcation of the equilibrium path, and used the elliptic integral to solve the post buckling. At present, the buckling and deformation of latticed structure are studied by analytical methods. Usually, latticed structure are abstracted as "beams", and then the deflection differential equations of beams are established for related research. Q.S.Li [3] used the second-order differential equations to give the buckling control equations for non-uniform members under concentrated axial compressive forces.

According to the flexural differential equation method, the overall stability of the crane’s telescopic boom was studied by Simao [4]. Guo Guangjian [5] analyzed the stability of one-way compression and bending lattice member with two-limbed, and verified the stability of lattice members under three different cross-section arrangements. What’s more, many researchers have adopted equivalent methods to study the buckling and deformation of latticed structure. By keeping the axial dimension of the component unchanged, Lu Nianli [6] equivalent latticed structure to solid structure, and gave the
calculation method of its equivalent moment of inertia. [7] used the principle of equivalence to analyze the buckling of the variable cross-section boom structure. China’s code for design of steel structures [8] and code for design of cranes [9] also use the idea of equivalence to express the critical load of the actual structure with the reduced slenderness ratio. The research of the above literature is mainly directed to the linear buckling and deformation of latticed structure, and its analysis object is a single member or a simple combined structure. However, For the nonlinear buckling and deformation analysis of complex structures composed of many members, the finite element method can only be used. Sun Boyong [10] conducted a buckling analysis on the lattice steel column, and the most favorable arrangement and applicable height range are obtained in his study. Xian-Rong Q [11] analyzed the tower crane’s linear and nonlinear buckling process by finite elements method. Qin Y [12] obtained the maximum working range of crane and stability convergence load through eigenvalue buckling and nonlinear buckling analysis. The finite element method can obtain high-precision analysis results by simplifying the structure less, but the modeling is complicated and should be based on the preliminary design.

In order to simplify the modeling process and improve the analysis efficiency, the equivalent inertia moment method is adopted to equivalent the actual latticed structure into a solid structure, and the load-displacement curve tracking of the equivalent structure is established. Then, the finite element software ANSYS is used to establish the actual latticed structure for nonlinear analysis. Finally, the analysis results of the equivalent solid structure are compared with the analysis results of the actual latticed structure to analyze its nonlinear buckling and deformation, which provides a simple and practical calculation method for the nonlinear buckling and deformation analysis of the latticed structure.

2. Equivalent inertia moment method
In view of the stability of lattice structures, many literatures use the equivalent length method to solve the stability of latticed structure, but it is not feasible to perform an overall analysis of the combined structure, [13] uses the energy method to discuss the calculation of the equivalent moment of inertia of latticed structure. Through the internal force analysis of the two-limbed latticed structure, the formula for calculating the lateral displacement is obtained.

\[ \Delta_i = \frac{Q H^3}{3EI_i} \beta \] (1)

where: \( H \) is the height of the two-limbed latticed structure(m);
\( Q \) is the horizontal force subjected by the latticed structure (N);
\( E \) is the elastic modulus of the material (N/m²);
\( I_i \) is the two-limbed latticed structure’s moment of inertia of the limb to the neutral axis (m⁴);
\( \beta \) is the influence coefficient of the mesh of two-limbed latticed structure.

The mechanical model of the four-limbed latticed structure with root fixed is shown in figure 1. \( a \) and \( b \) are the distance between two limbs in different planes. When the horizontal force \( F \) is applied, its overall lateral displacement \( \Delta \) is the same as the lateral displacement \( \Delta_i \) of the two-limbed member under the horizontal force of \( F/2 \). That is to say, \( \Delta = \Delta_i \), then:

\[ \Delta_i = \frac{F H^3}{2 3EI_i} \beta \] (2)

For a solid cantilever, let the height as \( H \) and the moment of inertia as \( I \), then the lateral displacement caused by the horizontal force \( F \) at the end is written as:

\[ \delta = \frac{H^3}{3EI} F \] (3)
According to the equal displacement of the ends, the four-limbed latticed structure can be equivalent to a solid structure, let the equivalent moment of inertia is denoted as $I_{eq}$, then the displacement of four-limbed latticed structure can be expressed as:

$$\Delta = \frac{H^3}{3EI_{eq}} F \tag{4}$$

where: $I_{eq}$ is the equivalent moment of inertia of the four-limbed latticed structure (m$^4$).

![Figure 1. The load decomposition model of the four-limbed latticed structure.](image)

According to $\Delta = \Delta_1$, can get

$$F \frac{H^3}{3EI_{eq}} = F \frac{H^3}{2 \cdot 3EI_1} \beta \tag{5}$$

So, the expression of the equivalent moment of inertia is:

$$I_{eq} = \frac{2I_1}{\beta} = \frac{I_2}{\beta} \tag{6}$$

where: $I_2$ is the moment of inertia of the four limbs of the latticed structure to the neutral axis, $I_2 = 4A(a/2)^2$ or $I_2 = 4A(b/2)^2$.

The unfolded drawing of the four-limbed structure which commonly used in engineering is shown in figure 2. In order to facilitate the calculation of the equivalent moment of inertia of the latticed structure, the influence coefficient $\beta$ of the four-limbed structure under different mesh layout is given as shown in table 1.
Figure 2. Different layout of mesh elements in common.

Table 1. Influence coefficient of mesh under horizontal load ($\beta$).

| Types of mesh | A, B | C | D | E, F |
|---------------|------|---|---|------|
| $\beta$       | $1 + \frac{3}{2H^2} \frac{\lambda_2}{\lambda_1}$ | $1 + \frac{3}{2H^2} \frac{\lambda_2 + \lambda_3}{\lambda_1}$ | $1 - \frac{3}{2n} + \frac{3}{4H^2} \frac{\lambda_2 + \lambda_3}{\lambda_1}$ | $1 + \frac{3}{4H^2} \frac{\lambda_2}{\lambda_1}$ |

where: $\lambda_1 = \frac{1}{EA_a a^2}$, $\lambda_2 = \frac{1}{EA_x \cos \alpha \sin^2 \alpha}$, $\lambda_3 = \frac{1}{EA_h} \tan \alpha$, $A_x$, $A_h$, $A_b$ are the cross-sectional area of chord, diagonal mesh, and transverse mesh, respectively.

3. Numerical examples
In this paper, an A-type latticed cantilever is used as the finite element model to do the analysis. The limb bars use 60 mm x 60 mm square steel; the meshes use 50 mm x 50 mm square steel; the height of a single layer is $l = 500$ mm; limb span is $a = b = 500$ mm; the total height is $h$; the elastic modulus of the material is $E = 2.05 \times 10^5$ MPa, the finite element model of the actual latticed structure uses beam188 elements, and the finite element model of the equivalent solid structure uses beam44 elements. Take the slenderness ratio $\lambda = 2h/\sqrt{I/A} = 48, 64, 80, 96, 112, 128$ as examples, based on the method of equivalent moment of inertia, the actual latticed structure is equivalent to a solid structure. The simplified schematic diagram of the actual latticed structure is shown in figure 3, and the inertia moment of the equivalent solid structure is shown in table 2.
Figure 3. Simplified schematic diagram of the latticed structure.

Table 2. Equivalent moment of inertia of equivalent solid structure.

| Layer number | Slenderness ratio | $I_{e}$ ($10^4$mm$^4$) |
|--------------|-------------------|------------------------|
| 12           | 48                | 8.6381                 |
| 16           | 64                | 8.7927                 |
| 20           | 80                | 8.8662                 |
| 24           | 96                | 8.9067                 |
| 28           | 112               | 8.9312                 |
| 32           | 128               | 8.9481                 |

3.1. Linear buckling analysis

Linear buckling is also called the first type of instability. In this type of instability, the critical buckling load $P_c$ of the structure is obtained by solving the equilibrium equation (7),

$$\{[K] + \mu [K(\sigma_0)]\} \{\Delta u\} = 0$$

(7)

where: $[K]$ is the linear stiffness matrix;
$\mu$ is the eigenvalue value;
$K(\sigma_0)$ is the geometric stiffness matrix;
$\Delta u$ is the displacement increment matrix.

Buckling analysis was performed on the actual latticed structure and the relative equivalent solid structure, the comparison results of the critical buckling load are shown in figure 4.
Figure 4. The comparison of critical load between latticed structure and the equivalent solid structure.

It can be seen from figure 4 that the errors between the critical load of the actual latticed structure and the equivalent solid structure are within 2%, and the error gradually decreases as the slenderness ratio increases. Therefore, the proposed equivalent analysis method can be used for the analysis of the eigenvalue buckling of the actual latticed structure in engineering.

3.2 Geometrical nonlinear buckling analysis

Econd type of instability occurs in most structures in actual engineering. In the second type of instability, the entire load-displacement path can be obtained by solving the nonlinear equation (8),

\[
[K][u] = \{P\}
\]

where: \([K]\) is the stiffness matrix; 
\([u]\) is the displacement matrix; 
\([P]\) is the external force.

The nonlinear buckling analysis of the actual lattice structure and the equivalent solid mesh structure is carried out, and the arc length method is used to track the whole process of the loaddisplacement curve of the structure. First, the load displacement curves of structures with different slenderness ratios are obtained, and the critical loads of structures are determined by the load deformation curves. Then, the displacement load curves of the equivalent solid structure is compared with that of the actual latticed structure. The load-displacement curves are shown in figure 5.

As can be seen from figure 5, the Y value at point A is the critical load for nonlinear buckling analysis of the actual latticed structure, the Y value at point B is the critical load for nonlinear buckling analysis of the equivalent solid structure, and the Y value at point C is the critical load for linear buckling analysis of the actual latticed structure, and the Y value at point D is the critical load of the linear buckling analysis of the equivalent solid structure. From the above calculation and analysis results, it can be seen that the linear analysis has a higher structural stability bearing capacity than the nonlinear analysis. When the slenderness ratio is the same, the trends of load-displacement curve of the actual latticed structure and the equivalent solid structure are basically the same. The structural deformation gradually increases as the load increases, and the structure maintains the load-bearing capacity before the first
The comparative analysis results indicate that the simplified analysis method based on the equivalent moment of inertia can be used for the nonlinear buckling analysis of latticed structure.

Figure 5. Nonlinear buckling analysis of structural load-displacement curves.
3.3. Geometrically nonlinear full process analysis
The whole process of geometric nonlinearity analysis is carried out for the actual latticed structure and the equivalent solid structure. The structure is subjected to both lateral loads $Q$ and axial loads $P$. The arc length method was used to obtain the load-displacement curves of the structure. Dimensionless coefficients are used in calculation, where $K = \frac{Qh^2}{EI}$, $m = \frac{P}{P_{cr}}$. Figure 6 shows the load-displacement curves of actual latticed structure and equivalent solid latticed structure with different slenderness ratios at $m = 0.5$, $K = 0.3$, 0.6, 0.9, 1.2, and 1.5.
It can be seen from figure 6 that, when the slenderness ratio $\lambda$ and the coefficient $k$ are unchanged, the trends of load-displacement curve of the actual latticed structure and the equivalent solid latticed structure are basically the same. Therefore, the comparative analysis shows that the proposed equivalent analysis method can also be adopted to analyze the whole process of geometric nonlinearity of the latticed structure in engineering.

4. Conclusion
In this paper, in order to simplify the modeling process of building complex lattice structures by finite element analysis software, a simplified analysis method for nonlinear buckling and deformation of latticed structure based on the equivalent inertia moment method is proposed. The stability calculation of...
the actual lattice structure is transformed into the equivalent calculation of the solid mesh structure, and the theoretical formula of the equivalent moment of inertia is derived. The load-displacement curve of the actual latticed structure is compared with the load-displacement curve of the equivalent solid structure. The trends of load-displacement curve of the actual latticed structure and the equivalent solid structure are basically the same and the errors between the critical load of the actual latticed structure and the equivalent solid structure are within 2%. The results show that the proposed equivalent analysis method can satisfy the analysis of nonlinear buckling and deformation of lattice structures and can satisfy the needs of engineering design.

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