EVOLUTION OF INHOMOGENEOUS CONDENSATES
AFTER PHASE TRANSITIONS

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Abstract

Using the O(4) linear $\sigma$ model, we address the topic of non-equilibrium relaxation of an inhomogeneous initial configuration due to quantum and thermal fluctuations. The space-time evolution of an inhomogeneous fluctuation of the condensate in the isoscalar channel decaying via the emission of pions in the medium is studied within the context of disoriented chiral condensates. We use out of equilibrium closed time path methods in field theory combined with the amplitude expansion. We give explicit expressions for the asymptotic space-time evolution of an initial inhomogeneous configuration including the contribution of thresholds at zero and non-zero temperature. At non-zero temperature we find new relaxational processes due to thermal cuts that have no counterpart in the homogeneous case. Within the one-loop approximation, we find that the space time evolution of such inhomogeneous configuration out of equilibrium is effectively described in terms of a rapidity dependent temperature $T(\vartheta) = T / \cosh[\vartheta]$ as well as a rapidity dependent decay rate $\Gamma(\vartheta, T(\vartheta))$. This rate is to be interpreted as the production minus absorption rate of pions in the medium and approaches the zero temperature value at large rapidities. An initial configuration localized on a bounded region spreads and decays in spherical waves with slower relaxational dynamics at large rapidity.

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I. INTRODUCTION

The dynamics of relaxation of inhomogeneous field configurations out of equilibrium is an important problem and a common theme in cosmology and high energy physics.

In high energy physics the experimental possibility of studying the chiral and quark-gluon phase transition with high luminosity hadron colliders and upcoming heavy-ion colliders makes imperative the understanding of relaxation and transport processes in extreme environments [1,2].

A very exciting possibility [3–7] that can be studied in high energy-high luminosity hadron collisions or relativistic heavy ion collisions within the energy range of the upcoming RHIC and LHC colliders, is that after the chiral phase transition, regions in which the chiral condensate is misaligned with respect to the vacuum state are formed.

In these ultra-high energy heavy ion collisions ($\sqrt{s} \geq 200\text{Gev/nucleon}$) a large energy density (a few Gev/fm$^3$) is deposited in the collision region corresponding to temperatures above the critical value for chiral symmetry restoration $\approx 200\text{Mev}$. In this situation, it is possible that within a volume of a few fm$^3$ an inhomogeneous coherent field configuration is formed corresponding to a region in space in which the chiral symmetry is restored.

The idea [3–7] is that as these regions cool down they relax towards the equilibrium situation emitting a large number of soft pions. Such a configuration has been dubbed a ‘disoriented chiral condensate’ (DCC).

Bjorken, Kowalski and Taylor proposed a ‘baked alaska’ [7] scenario in which this configuration relaxes via the copious emission of pions strongly correlated in isospin. Such a scenario could also explain the Centauro events observed in cosmic rays in which the ratio of charged to neutral pions is different from $1/3$ [2].

A microscopic, field theoretical description of the dynamics of relaxation of these field configurations with typically large amplitudes faces at least two major obstacles. The first is a non-perturbative treatment of inhomogeneous, large amplitude field configurations. The second is a consistent description of non-equilibrium processes that can allow a real time calculation of the evolution.

Recently there has been a surge of interest in the description of non-equilibrium processes both in cosmology and high energy physics. We refer to non-equilibrium processes to those corresponding to the the time evolution of quantum states in field theory which are not Hamiltonian eigenstates as well as the time evolution density matrices that do not commute with the Hamiltonian. In particular the relaxation of strong electric fields [8] and the evolution of homogeneous scalar order parameters during phase transitions in Minkowski and cosmological spacetimes [9].

The non-equilibrium aspects of disoriented chiral condensates have been studied within different approximation schemes by several authors [10–16], but mainly focusing on homogeneous expectation values or correlation functions.

However, to the best of our knowledge, these studies focused on the evolution of a homogeneous but time (or proper time) dependent mean field and fluctuations around it but did not address the relaxational dynamics of inhomogeneous field configurations including quantum and thermal effects.

Moreover, inhomogeneous field configurations appear in cosmology when topological objects, such as textures or cosmic strings involving inhomogeneous field configurations are
considered. Their relaxational dynamics is thought to have a bearing on the spectrum of fluctuations in the cosmic microwave background radiation \[17,18\].

The focus of this article is to study the linear relaxational dynamics of inhomogeneous, coherent field configurations both at zero and finite temperature in the \(O(4)\) linear sigma model. As is known such model is the relevant field theory for the description of chiral condensates.

We use out of equilibrium closed time path methods \[19–23\] combined with the amplitude expansion \[24,25\] to study the time evolution of \(\langle \Phi(\vec{x}, t) \rangle\). We consider small initial field amplitudes such that we can keep just the linear term in the amplitude expansion. In such approximation the field evolution equations linearize.

Whereas the focus is on the description of the dynamics within the setting of the chiral phase transition, the analysis presented below applies to a wide variety of physical situations in which an inhomogeneous scalar order parameter relaxes to the equilibrium situation via the production of lighter scalar fields in a medium.

The main idea is that after a phase transition (within the framework of DCC’s the chiral phase transition) regions are formed within which the scalar order parameter is inhomogeneous. As the zero momentum component rolls towards the equilibrium value, these inhomogeneous configurations will relax in such a way that the spatial gradients of the field configuration become smaller so as to decrease the energy. This relaxation will be accompanied by the production of quanta, such as pions in the case of DCC’s.

Here we study the \(O(4)\) linear sigma model in the broken symmetry state, at temperatures below the chiral phase transition as an effective low energy theory of \(SU(2)_L \times SU(2)_R\) (up and down quarks) which presumably incorporates the effects of strongly interacting QCD on chiral dynamics \[10\].

Wilczek and Rajagopal \[2,10\] have argued that the \(O(4)\) linear sigma model effectively describes the same equilibrium universality class of QCD with two flavors of light quarks and proposed to study it within the context of DCC. Clearly this simple model misses a great number of degrees of freedom, hadrons, vector mesons, etc. and can only be justified as an effective description at temperatures well below the chiral phase transition when the degrees of freedom with much higher masses are not relevant.

The linear sigma model may also be obtained as a Landau-Ginsburg effective theory from a Nambu-Jona-Lasinio model \[26\] which is a popular description of the phenomenology of chiral symmetry at the quark level and which has also been used as an alternative description of DCC’s \[27\].

The \(O(4)\) linear sigma model has also been studied within the context of texture type configurations deemed relevant in certain cosmological models of structure formation \[17\], hence our study may be relevant in these cosmological scenarios as well.

We will restrict our study to the case in which the condensate occurs in the isoscalar channel, without isospin violation. In section II we introduce the model, discuss its range of validity and summarize aspects of non-equilibrium field theory relevant to the calculation. Section III presents the bulk of the results, with explicit analysis of the space-time evolution, including the contribution from thresholds and thermal cuts. We summarize our results in the conclusions section.
II. THE MODEL AND THE TECHNIQUES

We study is the O(4) linear sigma model as an effective low energy description of chiral symmetry aspects of QCD with two (light) flavors of quarks. The Lagrangian density is given by

\[ \mathcal{L} = \frac{1}{2} \partial_{\mu} \vec{\Phi} \cdot \partial_{\mu} \vec{\Phi} - \lambda (\vec{\Phi} \cdot \vec{\Phi} - f_\pi^2)^2 + h \sigma , \]

where \( \vec{\Phi} \) is an \( O(4) \) vector, \( \vec{\Phi} = (\sigma, \vec{\pi}) \). The field \( \sigma \) describes the \( < \bar{q}q > \) condensate while the (isospin) triplet \( \vec{\pi} \) describes the pions. The small magnetic field \( h \approx (120\text{Mev})^3 \) accounts for the explicit symmetry breaking arising from the up and down quark masses and gives the pions a small mass \( m_\pi \approx 130\text{Mev} \). The pion decay constant \( f_\pi \approx 90\text{Mev} \) and \( \lambda \) is fixed by the value of the ‘mass’ of the \( \sigma \) field \( M_\sigma \approx 600\text{Mev} \approx 2\sqrt{2}\lambda f_\pi \). In what follows we will neglect the magnetic field, but explicitly give a mass to the pions and use \( f_\pi \) as the pion decay constant. At this point we recognize that the linear \( \sigma \)-model is a strongly coupled effective theory and any perturbative approximation will be subject to criticism. This important objection notwithstanding, we pursue the study of the non-equilibrium evolution of the condensate with the hope of learning new features of the decay of the condensate that will perhaps be rather generic and persist in higher order calculations or eventually in a non-perturbative treatment.

Since we want to understand the relaxation of inhomogeneous perturbations of the condensate around the equilibrium value \( f_\pi \), we write

\[ \sigma(\vec{x}, t) = f_\pi + \phi(\vec{x}, t) + \chi(\vec{x}, t) , \]

\[ \phi(\vec{x}, t) = \langle (\sigma(\vec{x}, t) - f_\pi) \rangle , \]

\[ \langle \chi(\vec{x}, t) \rangle = 0 , \]

where the average above stands for the expectation value in the non-equilibrium state discussed below, and the fluctuation field \( \chi \) describes the \( \sigma \) mesons in the broken symmetry state. The quantum state that leads to (2.3) is an inhomogeneous coherent state of \( \sigma \) mesons.

A. Out of Equilibrium Techniques

The field theoretical methods to describe processes out of equilibrium are known and described at length in the literature [19–23]. The basic ingredient is the time evolution of an initially prepared density matrix, which leads to the generating functional of non-equilibrium Green’s functions in terms of a path integral representation along a complex contour in time. The contour involves a forward time branch, a backward time branch and a third branch down the imaginary time axis to time \( \tau = -i\beta \) if the initial density matrix describes an ensemble at initial temperature \( 1/\beta \). It can be proven (see references above) that the third, imaginary branch only determines the boundary conditions on the Green’s functions, but does not enter in the calculation of real-time correlation functions.

The fields living on the forward and backward branches will be labelled with + and −, respectively, and the effective Lagrangian that enters in the path integral representation of the non-equilibrium generating functional is given by
\[ \mathcal{L}_{\text{noneq}} = \mathcal{L}[\bar{\Phi}^+] - \mathcal{L}[\bar{\Phi}^-]. \]  

(2.5)

From this path integral representation it is possible to construct a perturbative expansion of the non-equilibrium Green’s functions in terms of modified Feynman rules:

1. The number of vertices is doubled. Those in which all the fields are on the + branch are the usual interaction vertices, while the vertices in which the fields are on the − branch have the opposite sign.

2. The combinatoric factors are the same as in usual field theory.

3. The spatial Fourier transform of the (bosonic) propagators are

\[ G_{k^+}(t, t') = G_{k^+}^>(t, t')\Theta(t - t') + G_{k^-}^>(t, t')\Theta(t' - t), \]  

(2.6)

\[ G_{k^-}(t, t') = G_{k^-}^>(t, t')\Theta(t' - t) + G_{k^-}^<(t, t')\Theta(t - t'), \]  

(2.7)

\[ G_{k^+}^+(t, t') = -G_{k^-}^<(t, t'), \]  

(2.8)

\[ G_{k^-}^+(t, t') = -G_{k^-}^>(t, t'), \]  

(2.9)

\[ G_{k}^x(t, t') = i \int d^3x e^{-i\vec{k} \cdot \vec{x}} \langle \Phi(\vec{x}, t)\Phi(\vec{0}, t') \rangle, \]  

(2.10)

\[ G_{k}^y(t, t') = i \int d^3x e^{-i\vec{k} \cdot \vec{x}} \langle \Phi(\vec{0}, t)\Phi(\vec{x}, t) \rangle, \]  

(2.11)

where \( \Phi \) is a generic boson field. That is \( \Phi = \sigma \) or \( \Phi = \vec{\pi} \).

Now we have to specify the properties of the initial state. A particularly convenient choice is that of a thermal initial state at temperature \( T \). Then the density matrix of this initial state is

\[ \rho = e^{-H_0/T}, \]  

(2.12)

where \( H_0 \) is the Hamiltonian for times \( t < 0 \). This choice of the initial state determines the boundary conditions on the Green’s functions; these are the usual periodicity conditions in imaginary time (KMS conditions):

\[ G^<(\vec{x}, t; \vec{x}', t') = G^>(\vec{x}, t - i\beta; \vec{x}', t'). \]  

(2.13)

Finally, the free-field Green’s functions for generic boson fields are constructed from the following ingredients:

\[ G_{k}^x(t, t') = \frac{i}{2\omega_k} \left\{ [1 + n_b(\omega_k)]e^{-i\omega_k(t-t')} + n_b(\omega_k)e^{i\omega_k(t-t')} \right\}, \]  

(2.14)

\[ G_{k}^y(t, t') = \frac{i}{2\omega_k} \left\{ [1 + n_b(\omega_k)]e^{i\omega_k(t-t')} + n_b(\omega_k)e^{-i\omega_k(t-t')} \right\}, \]  

(2.15)

\[ \omega_k = \sqrt{\vec{k}^2 + m^2}, \quad n_b(\omega_k) = \frac{1}{e^{\beta\omega_k} - 1}, \]  

(2.16)

where \( m \) is the mass of the boson. An important property that will be used in the calculations that follow is the relation

\[ G_{k}^x(t, t') = G_{k}^x(t', t). \]  

(2.17)

Our goal is to study the dynamics of the expectation value of the chiral order parameter \( \phi(\vec{x}, t) = \langle (\sigma(\vec{x}, t) - f_\pi) \rangle \). We obtain the equation of motion for \( \phi \) using the tadpole method [28, 24, 25].

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To implement this method in the non-equilibrium formulation, set
\[ \sigma^\pm(x, t) = f_\pi + \phi(x, t) + \chi^\pm(x, t) \] (2.15)
in the Lagrangian \[ \mathcal{L}_3 \], and consider \[ \phi(x, t) \] as a c-number background field, keeping the linear, cubic and quartic terms as interactions. The expectation value of the fluctuation \( \chi^\pm \) is computed in presence of the background of fields \( \phi \) and the condition \( \langle \chi^\pm \rangle = 0 \) is imposed to all orders in perturbation theory. We restrict our study to the case of linear relaxation, so that we obtain the equation of motion linearized in the amplitude of \( \phi(x, t) \), an approximation that is valid for small amplitude fluctuations from the minimum of the tree level potential. This equation can be obtained in a systematic loop expansion using the usual Feynman rules with the doubled interaction vertices and the non-equilibrium propagators. To one loop order there are two tadpole contributions to the equation of motion: one from the quartic vertex that is absorbed in a mass renormalization and the other from the cubic vertex that is absorbed in a renormalization of \( f_\pi \). The remaining one loop Feynman diagrams that contribute to the equation of motion are shown in Figure 1. The details have already been presented elsewhere [24,25]. The equation \( \langle \chi^+(x, t) \rangle = 0 \) leads to
\[ \int d^3x' dt' \left\{ \left( \langle \chi^+(x, t) \chi^+(x', t') \rangle - \langle \chi^+(x, t) \chi^-(x', t') \rangle \right) \right\} = 0 \] (2.16)
\[ \left[ \frac{\partial}{\partial^2} \phi(x, t') + M_\sigma^2(T) \phi(x, t') + \int d^3x'' dt'' \Sigma_{\text{ret}}(x' - x'', t' - t'') \phi(x'', t'') \right] = 0 . \]

We have absorbed a momentum independent but temperature dependent tadpole in a renormalization of \( M_\sigma \), and used the property (2.14). \( \Sigma_{\text{ret}} \) is the retarded self-energy.

We are interested in the decay of an inhomogeneous chiral condensate into pions. In the linear \( \sigma \) model the chiral condensate is represented by the \( \sigma \) field. The dynamics of the decay process of such configuration will be determined by the imaginary part of the self-energy on mass shell.

In a medium, different kind of processes contribute to the imaginary part of the on-shell self-energy of the external particle [29,31]. In particular, collisional processes are always present and are responsible for a collisional lifetime of the particle in the medium. Decay (and recombination) processes are only present if the kinematics of decay is allowed; these contribute to an imaginary part of the self-energy on shell only when the lowest multiparticle threshold is below the single particle particle pole in the spectral density of the one-particle propagator. Since the ‘mass’ of the \( \sigma \) field (\( \approx 600 \) Mev) is larger than twice the pion mass (\( \approx 130 \) Mev), the one-loop diagram with pions in the internal loop will contribute to an imaginary part on mass shell of the sigma particle through processes in which the \( \sigma \) decays into two pions in the medium, as well as the inverse process of pion recombination into \( \sigma \).

Decay and recombination processes will also be present in higher order contributions to the self-energy, but we will only consider the contribution from the one-loop diagrams shown in figure 1.

The bilinear and trilinear interaction vertices needed for the one loop calculation are obtained from the following part of the interaction Lagrangian density
\[ \mathcal{L}_3 = g \left\{ \phi(\chi^+)^2 + (\chi^+)^3 + \phi(\pi^+)^2 + \chi^+(\pi^+)^2 - (\to -) \right\} , \] (2.17)
\[ g = 4\lambda f_\pi = \frac{M_\sigma^2}{2f_\pi} . \] (2.18)
The retarded self-energy to this order is found to be:

\[
\begin{align*}
\Sigma_{\text{ret}}(\vec{x} - \vec{x}', t - t') &= \Sigma_{\text{ret},\sigma}(\vec{x} - \vec{x}', t - t') + \Sigma_{\text{ret},\pi}(\vec{x} - \vec{x}', t - t') , \\
\Sigma_{\text{ret},\sigma}(\vec{x} - \vec{x}', t - t') &= 18 i g^2 \{ [G^<_{\sigma}(\vec{x} - \vec{x}', t - t')]^2 - [G^<_{\pi}(\vec{x} - \vec{x}', t - t')]^2 \} \Theta(t - t') , \\
\Sigma_{\text{ret},\pi}(\vec{x} - \vec{x}', t - t') &= 6 i g^2 \{ [G^<_{\pi}(\vec{x} - \vec{x}', t - t')]^2 - [G^<_{\pi}(\vec{x} - \vec{x}', t - t')]^2 \} \Theta(t - t') .
\end{align*}
\]  

(2.19)

The Green’s functions in (2.19) are given by (2.13) appropriate for the quanta of the fields \( \chi \) and \( \vec{\pi} \) respectively. We have also used the fact that the pion Green’s functions are diagonal in isospin.

Since we are dealing with a real field, the retarded self-energy \( \Sigma_{\text{ret}} \) is given by \[22,23\]

\[
\Sigma_{\text{ret}}(\vec{x}, t; \vec{x}', t') = 2 \text{ Re} \Sigma^>(\vec{x}, t; \vec{x}', t') .
\]  

(2.20)

Translational invariance of the self-energy makes it convenient to write the equation of motion for the spatial Fourier transform of \( \phi \). Introduce

\[
\phi(\vec{x}, t) = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot \vec{x}} \delta(\vec{p}, t) ,
\]  

(2.21)

\[
\Sigma_{\text{ret}}(\vec{x} - \vec{x}', t - t') = \int \frac{d^3p}{(2\pi)^3} e^{i\vec{p} \cdot (\vec{x} - \vec{x}')} \Sigma(\vec{p}, t - t') .
\]  

(2.22)

In order to solve the integro-differential equation we will impose the initial condition \( \dot{\phi}(\vec{x}, t < 0) = 0 \) and that this configuration is ‘released’ at time \( t = 0 \) \[32\]. Under these conditions the evolution equation becomes

\[
\begin{align*}
\dot{\delta}(\vec{p}, t) + \omega_p^2 \delta(\vec{p}, t) + \int_0^t dt' \Sigma(\vec{p}, t - t') \delta(\vec{p}, t') &= 0 , \\
\omega_p^2 &= |\vec{p}|^2 + M^2(T) .
\end{align*}
\]  

(2.23)

(2.24)

Using the free-field Green’s functions given above for the \( \chi \); \( \vec{\pi} \) fields we find

\[
\begin{align*}
\Sigma(\vec{p}, t - t') &= \Sigma_{\sigma}(\vec{p}, t - t') + \Sigma_{\pi}(\vec{p}, t - t') \\
\Sigma_{\sigma}(\vec{p}, t - t') &= -18 g^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{k,\sigma}\omega_{k+p,\sigma}} \{(1 + 2 n_{k,\sigma}) \sin[(\omega_{k+p,\sigma} + \omega_{k,\sigma})(t - t')] \\
&\quad - 2 n_{k,\sigma} \sin[(\omega_{k+p,\sigma} - \omega_{k,\sigma})(t - t')] \} \\
\Sigma_{\pi}(\vec{p}, t - t') &= -6 g^2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega_{k,\pi}\omega_{k+p,\pi}} \{(1 + 2 n_{k,\pi}) \sin[(\omega_{k+p,\pi} + \omega_{k,\pi})(t - t')] \\
&\quad - 2 n_{k,\pi} \sin[(\omega_{k+p,\pi} - \omega_{k,\pi})(t - t')] \}
\end{align*}
\]  

(2.25)

where

\[
\begin{align*}
\omega_{k,\sigma;\pi} &= \sqrt{k^2 + M_{\sigma;\pi}^2} , \\
n_{k,\sigma;\pi} &= \frac{1}{e^{\beta \omega_{k,\sigma;\pi}} - 1} .
\end{align*}
\]  

(2.27)

(2.28)

The equation of motion (2.23) can now be solved via a Laplace transform. With the boundary conditions \( \delta(\vec{p}, t = 0) = \delta_1(\vec{p}) ; \dot{\delta}(\vec{p}, t = 0) = 0 \) and denoting the Laplace transforms
of $\delta(p, t)$, $\Sigma(p, t)$ by $\delta(p, s)$, $\Sigma(p, s)$ respectively (here $s$ is the Laplace transform variable) we find

$$\delta(p, s) = \frac{\delta_i(p)}{s^2 + \omega_p^2 + \Sigma(p, s)},$$

(2.29)

with $\omega_p^2$ given by (2.24). The time evolution is found by integration in the complex $s$-plane along the Bromwich contour $s = i\omega + \epsilon$; $-\infty \leq \omega \leq \infty$ with $\epsilon$ chosen so that the contour lies to the right of the real part of all the singularities of $\delta(p, s)$.

The processes that contribute to the imaginary part of the kernel that deserve to be studied in detail, for $\omega > 0$ these give the following contribution: (for $\omega < 0$ changes sign)

$$\begin{align*}
\text{Im}K(p, i\omega + 0^+) &= \pm \pi \text{sign}(\omega) \left[ \rho(p, |\omega|) - \rho(p, -|\omega|) \right],
\end{align*}$$

(2.33)

where the sign($\omega$) reflects the retarded nature of the kernel [23,30].

There are two different processes that contribute to the imaginary part of the kernel that deserve to be studied in detail, for $\omega > 0$ these give the following contribution: (for $\omega < 0$ changes sign)

$$\begin{align*}
\text{Im}K(p, i\omega + 0^+) &= \text{Im}K^{(1)}(p, i\omega + 0^+) + \text{Im}K^{(2)}(p, i\omega + 0^+),
\end{align*}$$

(2.34)

$$\begin{align*}
\text{Im}K^{(1)}(p, i\omega + 0^+) &= \frac{1}{16\pi^2} \int \left[ \frac{d^3k}{\omega_p\omega_{k+p}} \right] (1 + 2n_k) \delta(\omega - \omega_p - \omega_{k+p}),
\end{align*}$$

(2.35)

$$\begin{align*}
\text{Im}K^{(2)}(p, i\omega + 0^+) &= \frac{1}{16\pi^2} \int \left[ \frac{d^3k}{\omega_p\omega_{k+p}} \right] 2n_k \left[ \delta(\omega + \omega_p - \omega_{k+p}) - \delta(\omega_p - \omega - \omega_{k+p}) \right].
\end{align*}$$

(2.36)

The processes that contribute to $\text{Im}K^{(1)}$ [29,31] are the decay $\sigma \rightarrow \pi\pi$ with Boltzmann weight $(1 + n_k)(1 + n_{k+p})$ minus the recombination process $\pi\pi \rightarrow \sigma$ with Boltzmann weight
when the particles in the loop are pions, or the process $\sigma \to 2\sigma$ and its inverse $2\sigma \to \sigma$ with similar weights when the particles in the loop are $\sigma$. Clearly these processes will only contribute above the two-particle threshold.

The processes that contribute to $\text{Im} K^{(2)}$ are the $\sigma\pi \to \pi$ with weight $n_\vec{k}(1+n_\vec{k})\bar{\vec{k}} + \vec{p}$ when the particles in the loop are pions, or the process $\sigma\sigma \to \sigma\sigma$ with the corresponding weights when the particles in the loop are the $\sigma$ mesons. Clearly these latter process can only occur in the heat bath for non-zero momentum transfer and will lead to new thresholds and discontinuities. These are identified with the Landau damping processes in the medium \cite{30} as they only occur for space-like momenta. An important point to consider is that these processes do not contribute to the relaxation of a homogeneous condensate (zero momentum transfer) and are, therefore, a new feature of the inhomogeneous situation.

After analyzing the kinematical regions, we find:

$$\text{Im} K^{(1)}(\vec{p}, i\omega + 0^+, T) = \left\{ \frac{1}{8\pi} \sqrt{1 - \frac{4m^2}{\omega^2 - p^2}} + \frac{T}{4\pi p} \ln \left[ \frac{1 - e^{-\beta\omega^+_p}}{1 - e^{-\beta\omega^-_p}} \right] \right\} \times \Theta(\omega^2 - p^2 - 4m^2), \quad (2.37)$$

$$\text{Im} K^{(2)}(\vec{p}, i\omega + 0^+, T) = \frac{T}{4\pi p} \ln \left[ \frac{1 - e^{-\beta\omega^+_p}}{1 - e^{-\beta\omega^-_p}} \right] \Theta(p^2 - \omega^2), \quad (2.38)$$

$$\omega^\pm_p = \frac{\omega}{2} \pm \frac{p}{2} \sqrt{1 - \frac{4m^2}{\omega^2 - p^2}}, \quad (2.39)$$

where $m$ is the mass of the particles in the loop, $m = M_\pi$ for $\Sigma_\pi$ or $m = M_\sigma$ for $\Sigma_\sigma$. In the expression for $\text{Im} K^{(1)}$ we have explicitly separated out the $T = 0$ and $T \neq 0$ contributions. Note that whereas $\text{Im} K^{(1)}$ is non-zero above the two particle threshold and has a contribution that survives in the zero temperature limit, $\text{Im} K^{(2)}$ is non-zero only for $T \neq 0$, and for space-like momenta and does not contribute directly to the imaginary part on shell but does contribute to the relaxational dynamics as discussed below. The explicit form of the imaginary part at finite temperature is one of the novel results of this study. For small momentum $p/T \ll 1$ we find

$$\text{Im} K^{(1)}(\vec{p}, i\omega + 0^+, T) \approx \text{Im} K^{(1)}(\vec{p}, i\omega + 0^+, T = 0) \left[ 1 + 2n(\omega^-) \right], \quad (2.40)$$

$$\text{Im} K^{(2)}(\vec{p}, i\omega + 0^+, T) \approx \frac{1}{8\pi} \sqrt{1 - \frac{4m^2}{\omega^2 - p^2}} 2n(\omega^-) \Theta(p^2 - \omega^2), \quad (2.41)$$

and we recognize \cite{25} as the imaginary part obtained in the homogeneous case \cite{25}.

The $\pi:\sigma$ self-energies are now given in terms of the above kernel as:

$$\Sigma_\pi(\vec{p}, \omega, T) = 3g^2 K(\vec{p}, \omega ; m = M_\pi, T), \quad (2.42)$$

$$\Sigma_\sigma(\vec{p}, \omega, T) = 9g^2 K(\vec{p}, \omega ; m = M_\sigma, T). \quad (2.43)$$
B. Zero Temperature Limit

The real part of the kernel can be obtained from the imaginary part from the dispersion integral (2.31), but is very difficult to compute in the general case. Its zero temperature limit can be found analytically. The integral in (2.31) has a logarithmic divergence which is subtracted at \( s = 0 \), \( \vec{p} = 0 \). This subtraction can be absorbed in a further renormalization of \( M_\sigma \) and we find that for \( T = 0 \):

\[
K(\vec{p}, s) = \frac{1}{4\pi^2} \left[ \sqrt{1 + \frac{4m^2}{s^2 + p^2}} \text{Argtanh} \left( \frac{1}{\sqrt{1 + \frac{4m^2}{s^2 + p^2}}} \right) - 1 \right],
\]

(2.44)

where again, \( m \) refers to the mass of the particle in the loop. Along the imaginary axis and below the two particle cut, the kernel is real and given by

\[
K(\vec{p}, s = i\omega) = \frac{1}{4\pi^2} \left[ \sqrt{\frac{4m^2}{\omega^2 - p^2}} - 1 \right],
\]

(2.45)

whereas above the two-particle cut \( \omega^2 > 4m^2 + |\vec{p}|^2 \) the kernel has an imaginary part, \( K(\vec{p}, i\omega \pm 0^+) = K_R(\vec{p}, \omega) \pm iK_I(\vec{p}, \omega) \) with

\[
K_R(\vec{p}, \omega) = \frac{1}{4\pi^2} \left[ \sqrt{1 - \frac{4m^2}{\omega^2 - p^2}} \text{Argtanh} \left( \frac{1}{\sqrt{1 - \frac{4m^2}{\omega^2 - p^2}}} \right) - 1 \right],
\]

(2.46)

\[
K_I(\vec{p}, \omega) = \frac{1}{8\pi} \sqrt{1 - \frac{4m^2}{\omega^2 - p^2}}.
\]

(2.47)

Using the parameters of the linear \( \sigma \) model, we find that

\[
Re\Sigma_\pi(\vec{p}, \omega, T = 0) + Re\Sigma_\sigma(\vec{p}, \omega, T = 0)|_{\omega = \sqrt{p^2 + M_\sigma^2}} = 0.045M_\sigma^2,
\]

(2.49)

\[
Im\Sigma_\pi(\vec{p}, \omega, T = 0)|_{\omega = \sqrt{p^2 + M_\sigma^2}} = 1.195M_\sigma^2.
\]

(2.50)

Thus we see that although the linear \( \sigma \) model is strongly coupled, the one-loop correction to \( M_\sigma \) is very small. However the imaginary part on shell (related to the width of the particle, see below) is very large.

C. Analytic structure

To perform the inverse Laplace transform we need to understand the analytic structure of the Laplace transform in the complex s-plane.

From the analysis presented above, we find that for arbitrary temperature, \( \delta(\vec{p}; s) \) is analytic except for

1. isolated single particle poles at \( s = \pm i\Omega(\vec{p}, T) \) with \( \Omega(\vec{p}, T) \) being the solutions of

\[
-\Omega^2(\vec{p}, T) + |\vec{p}|^2 + M_\sigma^2(T) + \Sigma(s = \pm i\Omega, \vec{p}, T) = 0
\]

(2.51)

and
2. discontinuities across the imaginary axis determined by the imaginary part of the self-energy obtained above:

\[\delta(\vec{p}, s = i\omega + 0^+) - \delta(\vec{p}, s = i\omega - 0^+) = i \delta(\vec{p}) S(\omega, \vec{p}, T), \quad (2.52)\]

where we have introduced the spectral density \(S(\omega, \vec{p}, T)\) given by

\[S(\omega, \vec{p}, T) = \frac{2\omega \Sigma_I(\omega, \vec{p}, T)}{[\omega^2 - |\vec{p}|^2 - M_0^2 - \Sigma_R(\omega, \vec{p}, T)]^2 + \Sigma_I(\omega, \vec{p}, T)^2}.
\] \( (2.53)\)

### III. TIME EVOLUTION OF THE ORDER PARAMETER

The real-time evolution of the spatial Fourier transform of the condensate is obtained via the inverse Laplace transform

\[\delta(\vec{p}, t) = \int_{-i\infty+\epsilon}^{i\infty+\epsilon} ds \frac{\delta(\vec{p}) s}{2\pi i e^{st}} \frac{\delta(\vec{p}) s}{s^2 + M_0^2(T) + |\vec{p}|^2 + \Sigma(\vec{p}, s)}. \quad (3.1)\]

The integral is performed by deformation of the contour, wrapping around the single particle poles which are solutions of the equation (2.51) and slightly to the right and left of the multiparticle cuts along the imaginary axis. Thus the inverse Laplace transform will have two contributions: from the poles \(\delta_{\text{pole}}(\vec{p}, t)\) and from the cuts \(\delta_{\text{cut}}(\vec{p}, t)\) and we can write in general

\[\delta(\vec{p}, t) = \delta_{\text{pole}}(\vec{p}, t) + \delta_{\text{cut}}(\vec{p}, t). \quad (3.2)\]

There are several different cases that we can study.

#### A. Stable Case (\(M_\sigma < 2M_\pi\))

Although this is not a realistic case for the chiral phase transition described by the linear \(\sigma\) model, this case is still worth studying in order to compare it to the unstable case \((M_\sigma > 2M_\pi)\) to be described later. Furthermore, we will see that many results from this particular case do apply in the unstable case which is of interest for the chiral phase transition.

1. **Zero Temperature**

At zero temperature the self-energy is manifestly Lorentz covariant as can be seen from the expressions (2.46, 2.47).

We first analyze the one-particle pole contribution, which is a straightforward generalization of the homogeneous \(\vec{p} = 0\) case [25]. The condition (2.51) is satisfied for

\[\Omega^2(\vec{p}, T = 0) = |\vec{p}|^2 + M_0^2, \quad (3.3)\]

\[M_0^2 - M_\sigma^2 - \Sigma(s = iM_0, \vec{p} = 0, T = 0) = 0. \quad (3.4)\]
Then the pole contribution to the inverse Laplace transform is

\[ \delta_{\text{pole}}(\vec{p}, t) = \delta_i(\vec{p}) Z \cos(\sqrt{p^2 + M_0^2} t), \tag{3.5} \]

\[ Z = \left[ 1 - \frac{\partial \Sigma(iM, p = 0)}{\partial M^2} \right]^{-1} \Big|_{M=M_0}, \tag{3.6} \]

where \( Z \) is the (finite) wave-function renormalization constant, defined on shell. We will now consider the case in which \( \delta_i(\vec{p}) = \delta_i \) (independent of \( \vec{p} \)) and later study the general case by convolution.

At this point it is convenient to introduce the proper time and (radial) spatial rapidity variables as these exhibit the Lorentz properties more clearly:

\[ \vartheta_r = \frac{1}{2} \ln \left[ \frac{t + r}{t - r} \right], \]

\[ \tau = \sqrt{t^2 - r^2}, \quad r = \tau \sinh[\vartheta_r], \quad t = \tau \cosh[\vartheta_r]. \tag{3.7} \]

The spatial Fourier transform leads to the pole contribution

\[ \phi_{\text{pole}}(\vec{x}, t) = -\frac{Z \delta_i M_0^2 \cosh[\vartheta_r]}{2\pi \tau} J_2(M_0 \tau) \theta(\tau^2), \tag{3.8} \]

where \( J_2 \) is a Bessel function. Recalling that \( \delta_i = \int d^3x \phi(\vec{x}, t = 0) \) it is clear that this solution is Lorentz invariant, since the product \( \delta_i \cosh[\vartheta_r] \) is Lorentz invariant. The pole contribution then gives fluctuations which propagate inside the light-cone as a massive relativistic wave as expected. For large times and distances eq.(3.8) gives

\[ \phi_{\text{pole}}(\vec{x}, t) \overset{r, t \to \infty}{=} \delta_i \cosh[\vartheta_r] \tau \frac{3}{2} \left[ C_1 M_\sigma^2 \sin(2M_\sigma \tau) + C_2 M_\pi^2 \sin(2M_\pi \tau) \right], \tag{3.9} \]

where \( C_1, C_2 \) are constants determined by the spectral densities at the respective two-particle thresholds. When \( M_\sigma^2 \tau^2 ; M_\pi^2 \tau^2 \gg 1 \), the amplitude of \( \phi_{\text{cut}} \) is smaller than the one of \( \phi_{\text{pole}} \) by a factor \( \tau^3/2 \) and is clearly subleading at long distances. This is analogous to the factor \( t^{3/2} \) for the homogeneous case \[25\].
2. Non-Zero Temperature

Unlike the $T = 0$ case, we cannot make use of manifest Lorentz covariance, since this property is not available in the rest frame of the heat bath. However the situation is qualitatively similar to the zero temperature case. The positions of the one-particle pole are now given by

\[
s = \pm i \Omega(p, T) ,
\]

\[
\Omega(p, T) = \sqrt{M^2 + p^2 + \Sigma(i\Omega(p, T), p, T)} .
\]

The pole contribution is

\[
d_{\text{pole}}(\vec{p}, t, T) = \delta_i Z(p, T) \cos(\Omega(p, T) t) ,
\]

where the wave-function renormalization (defined on shell) is now

\[
Z(p, T) = \left[ 1 - \frac{\partial \Sigma(iM, p, T)}{\partial M^2} \bigg|_{M=\Omega(p,T)} \right]^{-1} .
\]

The lack of Lorentz invariance introduces a complicated dependence on the momentum $p$, and the inverse Fourier transform cannot be performed analytically as it was the case for $T = 0$.

However, for $t > r \gg M^{-1}, M^{-1}$ (i.e., $t > r \gg \mathcal{O}(1\text{fm})$) one can compute $\phi_{\text{pole}}(\vec{x}, t, T)$ by the stationary phase approximation. We have

\[
\phi_{\text{pole}}(\vec{x}, t, T) = \frac{\delta_i}{2\pi^2 r} \int_0^\infty dp p \sin(pr) \ Z(p, T) \cos \left[ t \Omega(p, T) \right] .
\]

This integral has stationary points at

\[
\pm p_0 = \pm \frac{M_\sigma r}{\sqrt{t^2 - r^2}} + \mathcal{O}(g^2) ,
\]

for large $t$ and $r$ and time-like intervals, where by $\mathcal{O}(g^2)$ we refer to terms that are small in the formal weak coupling expansion or in the strongly coupled $\sigma$ model numerically small as given by equation 2.49.

These saddle point values have an illuminating physical interpretation which is better displayed in terms of (radial) rapidity (3.19) and momentum rapidity variables defined by:

\[
\vartheta_p = \frac{1}{2} \ln \left[ \frac{\omega + p}{\omega - p} \right] ,
\]

\[
p = M_\sigma \sinh[\vartheta_p] , \quad \omega = M_\sigma \cosh[\vartheta_p] .
\]

Using these variables, the saddle point condition becomes

\[
\vartheta_p = \pm \vartheta_r ,
\]

and we will just refer to $\vartheta$ as the rapidity variable irrespective of coordinate or momentum. The contribution from the saddle points yields
\[
\phi_{\text{pole}}(\vec{x}, t, T) \overset{r,t \to \infty}{=} -\frac{\delta_i}{\sqrt{2}} Z(p = M_\sigma \sinh[\vartheta], T) \left[ \frac{M^{3/2} \cosh[\vartheta]}{\tau^{3/2}} + O(g^2) \right] \\
\times \cos \left\{ M_\sigma \tau \left[ 1 + \frac{1}{2M_\sigma^2} \Sigma(iM, p = M_\sigma \sinh[\vartheta], T) + O(g^4) \right] - \frac{\pi}{4} \right\} \\
\times \left\{ 1 + O\left( \frac{1}{t^2}, \frac{1}{r^2} \right) \right\}.
\]
(3.22)

The fluctuations for \( T \neq 0 \) propagate as spherical waves, similarly to the zero temperature case displayed in eq.(3.9). However the phase and amplitude depend on \( \tau \) and \( \vartheta \) through the temperature dependence of \( \Sigma \) as well as of \( Z \), which breaks manifest Lorentz covariance.

The cut contribution is
\[
\delta_{\text{cut}}(\vec{p}, t, T) = \delta_{\text{cut}, 1}(\vec{p}, t, T) + \delta_{\text{cut}, 2}(\vec{p}, t, T),
\]
(3.23)
where we have separated the ‘normal’ (zero temperature) two-particle cut
\[
\delta_{\text{cut}, 1}(\vec{p}, t, T) = \frac{\delta_i}{\pi} \int_0^{\infty} S(\omega, \vec{p}, T) \cos(\omega t) d\omega
\]
(3.24)
from the thermal contribution with support below the light-cone
\[
\delta_{\text{cut}, 2}(\vec{p}, t, T) = \frac{\delta_i}{\pi} \int_0^{\infty} S(\omega, \vec{p}, T) \cos(\omega t) d\omega.
\]
(3.25)

Again, as in the zero temperature case \( M \) is the smallest mass.

Let us first study \( \delta_{\text{cut}, 1} \). At large \( t \) the most important contribution to the integral will be from the region near the thresholds and the largest contribution will be from the smallest threshold (two pion). From the expression for the imaginary part given by eq. (2.37,2.39) we see that near the thresholds \( \omega_+ \to \omega_- \) and
\[
\Sigma_I(\omega, p, T) \approx \Sigma_I(\omega, p, T = 0) \left[ 1 + 2n \left( \frac{\omega}{2} \right) \right],
\]
(3.26)
with \( n \) the Bose occupation number. Performing the change of variables \( \omega \to \sqrt{\omega^2 + p^2} \) the integral over \( \omega \) is similar to (3.11) but with the finite temperature factor \( \left[ 1 + 2n \left( \frac{\sqrt{\omega^2 + p^2}}{2} \right) \right] \) in the integrand. For large distances, the spatial Fourier transform can be done by the stationary phase approximation. The values of the stationary phase are
\[
p_s = \pm \omega \sinh[\vartheta].
\]
(3.27)

Finally, at large times the integral over \( \omega \) obtains the largest contribution near the thresholds \( \omega = 2M_\pi \); \( \omega = 2M_\sigma \) and we find the leading behavior for large times and distances (for time-like intervals)
\[
\phi_{\text{cut}, 1}(\vec{x}, t, T) \overset{r,t \to \infty}{=} \delta_i \frac{\cosh[\vartheta]}{\tau^{3/2}} \left\{ C_1(T) \left[ 1 + 2n \left( M_\sigma \cosh[\vartheta] \right) \right] M_\sigma^2 \sin(2M_\sigma \tau) + \\
C_2(T) \left[ 1 + 2n \left( M_\pi \cosh[\vartheta] \right) \right] M_\pi^2 \sin(2M_\pi \tau) \right\},
\]
(3.28)
with $C_1(T) ; C_2(T)$ the finite temperature counterpart of the zero temperature constants and the Bose enhancement factors are a result of the enhancement factors near thresholds as given by eq. (3.20). The temperature dependence of the constants $C_1 ; C_2$ is rather weak through the temperature dependence of the pole which can be neglected to the lowest order. Then to this order we see that the most important effects of temperature are through the Bose-enhancement factors which are the same as in the homogeneous case but with an effective, rapidity dependent temperature $T(\vartheta) = T / \cosh[\vartheta]$.

The second cut gives the following contribution to the amplitude

$$
\phi_{\text{cut}, 2}(\vec{x}, t, T) = \frac{\delta_\tau}{4\pi^3 r} \int_0^\infty dp \rho \sin pr \int_{-p}^p S(\omega, \vec{p}, T) \cos(\omega t) d\omega .
$$

(3.29)

For large time and distances it is dominated by the point $\omega = p = 0$. Defining $\rho = \frac{\omega}{p}$, we obtain

$$
\text{Im} \Sigma_\pi(\rho p, p, T) \omega \rho \rightarrow 0 3g^2 F_\pi(\rho, T) + \mathcal{O}(p^2) ,
$$

(3.30)

$$
\text{Re} \Sigma_\pi(\rho p, p, T) \omega \rho \rightarrow 0 3g^2 G_\pi(\rho, T) + \mathcal{O}(p^2) ,
$$

(3.31)

where the function $G_\pi(\rho, T)$ can be found from the appropriate self-energy after some algebra but will not be relevant for our purposes, and the function $F_\pi(\rho, T)$ is given by

$$
F_\pi(\rho, T) = \frac{\rho}{4\pi} \frac{1}{e^{\frac{M_\pi}{T} - 1}} .
$$

(3.32)

$\Sigma_\sigma$ has an expression similar to $\Sigma_\pi$, but with the factor 3 replaced by 9 and $M_\pi$ by $M_\sigma$.

We will approximate $M_\sigma^2 + 3g^2 G_\pi(\rho, T)$ by the ‘pole mass’ which at low temperature (compared to $M_\sigma$) can be approximated by $M_\sigma$ as discussed above. It is convenient to change integration variables in eq. (3.29) setting $y = pr$. For large $r$ and $t$ and $T \ll M_\sigma$ the contribution from the loop with $\sigma$ mesons is negligible, since only the lowest mass intermediate states contribute a long distances and the $\sigma$ mesons are not thermally excited. In this case we find

$$
\phi_{\text{cut}, 2}(\vec{x}, t, T) \approx \frac{\delta_\tau 3g^2}{2\pi^3 M_\sigma^4 r^3} \int_0^\infty y^3 dy \sin y \int_{-1}^{+1} \rho d\rho \frac{F_\pi(\rho, T)}{1 + \frac{3g^2 F_\pi(\rho, T)}{M_\sigma^2} \sin[pr]} \cos(ty) .
$$

(3.33)

Using the symmetry of the integrand we can recast this result as

$$
\phi_{\text{cut}, 2}(\vec{x}, t, T) = \frac{\delta_\tau 3g^2}{2\pi^3 M_\sigma^4 r} \int_0^\infty p^3 dp \int_{-1}^{+1} \rho d\rho \frac{F_\pi(\rho, T)}{1 + \frac{3g^2 F_\pi(\rho, T)}{M_\sigma^2} \sin[p(r - t\rho)]} .
$$

(3.34)

The integral over $p$ yields the third derivative of the Dirac delta and we finally obtain

$$
\phi_{\text{cut}, 2}(\vec{x}, t, T) \mid_{r,t \rightarrow \infty} = \frac{\delta_\tau 3g^2}{2\pi^2 M_\sigma^4 \sinh[\vartheta] \cosh^4[\vartheta]} \frac{\theta(\tau^2)}{\tau^5} \frac{d^3}{d\rho^3} \left[ \frac{\rho F_\pi(\rho, T)}{1 + \frac{3g^2 F_\pi(\rho, T)}{M_\sigma^2} \sin[pr]} \right] \bigg|_{\rho = \tanh[\vartheta]} .
$$

(3.35)
In the general case, there is a similar contribution from the $\sigma$ meson loop, but with $F_\pi$ replaced by $F_\sigma$ as mentioned above.

Finally the large distance, large time behavior is given by

$$\phi(\vec{x}, t) = \phi_{\text{pole}}(\vec{x}, t) + \phi_{\text{cut}, 1}(\vec{x}, t) + \phi_{\text{cut}, 2}(\vec{x}, t),$$

with the asymptotic results given by (3.22, 3.28, 3.35).

We emphasize that the contribution from the thermal cut yields a non-oscillatory contribution decaying as $t^{-5}$ or $r^{-5}$ and is subleading with respect to the contributions from the pole and from the ‘normal’ two-particle cut given by eq. (3.28) at long distances. Thus we obtain one of the important results of this work: the contributions from processes such as $\sigma\pi \to \pi$ (and the inverse process), that can occur only in a plasma of excitations, are asymptotically subleading compared to the one-particle pole and the contributions from processes such as $\sigma \to \pi\pi$ (and its inverse).

### 3. Time evolution for an initial Gaussian wave packet

In this subsection we will compute the time evolution in the case in which the initial state is a Gaussian wave packet. That is, we will assume

$$\phi_\xi(\vec{x}, t = 0) = \left(\frac{1}{2\pi \xi^2}\right)^{3/2} e^{-\frac{|\vec{x}|^2}{2\xi^2}},$$

where we included a convenient normalization factor. We have to convolute this initial condition with the results obtained in the previous subsection. Namely

$$\phi_\xi(\vec{x}, t, T) = \int d^3 y \phi_\xi(\vec{x} - \vec{y}, t = 0) \phi(\vec{y}, t, T),$$

where $\phi(\vec{y}, t, T)$ is the amplitude for the evolution with the initial condition corresponding to a $\phi(\vec{x}, t = 0) = \delta_3(\vec{x})$ as studied in the previous sections.

Within the setting of the chiral phase transition and the possibility of the formation and evolution of DCC’s, we are interested in initial packets of spatial sizes $\xi > 1\text{fm}$ (therefore $\xi \gg M_\sigma^{-1} \approx 0.2\text{fm}$), and the evolution for distances and times $r; t \gg \xi$.

We first analyze the pole contribution. By using (3.17) in (3.38) and performing the simple integral over $d^3 y$, we get the result

$$\phi_{\xi, \text{pole}}(\vec{x}, t, T) = \frac{1}{2\pi^2 r} \int_0^\infty dp \, p \sin(pr) \, e^{-\frac{1}{2}p^2}\xi^2 Z(p, T) \cos \left[ t\sqrt{p^2 + M_\sigma^2 + \Sigma(i\Omega(p, t), p, T)} \right].$$

We see that the only difference with (3.17) is the Gaussian factor in the integrand.

For $r, t \gg \xi \gg M_\sigma^{-1}$ the stationary phase points are

$$\pm p_0 = \pm M_\sigma \sinh[\vartheta] \left[ 1 + \mathcal{O} \left( \frac{M_\sigma \xi^2}{\tau} \right) \right] + \mathcal{O}(g^2).$$

(3.40)
This analysis holds for all other contributions (from both cuts), with the final conclusion that asymptotically for large distances and times compared with the size of the initial packet, the results obtained for the $\delta$-function initial condition can be used with the simple modification of the additional Gaussian pre-factor

$$ e^{-\frac{1}{2}p^2\xi^2} \approx e^{-\frac{1}{2}M_\sigma^2\xi^2 \sinh^2[\vartheta]} . $$

Thus at time and distances much larger than $\xi$ we obtain

$$ \phi_\xi(x, t, T) = e^{-\frac{1}{2}M_\sigma^2\xi^2 \sinh^2[\vartheta]} \phi_{\xi=0}(x, t, T) . $$

(3.42)

Only for distances comparable to the size of the initial packet or very close to the light cone do the corrections to the above results arising from the initial size of the packet become important. It is clear that the asymptotic space-time evolution is completely determined by the lowest energy thresholds and the fact that higher mass degrees of freedom are not described by the linear $\sigma$ model should not modify the asymptotic behavior for distances larger than a few fm.

A remarkable consequence of the saddle point conditions which determine the long time-long distance behavior of the contributions from the multiparticle cuts is that the temperature effects enter in terms of an effective temperature that depends on rapidity given by

$$ T_{\text{eff}}(\vartheta) = \frac{T}{\cosh[\vartheta]} $$

(3.43)

in the sense that the Boltzmann factors that enter into the expressions have this effective temperature dependence. Thus the space-time evolution can be interpreted as that the inhomogeneous, non-equilibrium configuration is relaxing with a rapidity dependent temperature that becomes smaller at large rapidities. This is a one-loop result and may be modified by higher order corrections, a possibility worthy of further study.

**B. Unstable Case ($M_{\sigma} > 2M_{\pi}$)**

As is well known (see [23][33]), if the $\sigma$ particle is unstable (resonance), namely if $M_{\sigma} > 2M_{\pi}$, the pole is above the two-particle threshold and moves off into the second (unphysical) Riemann sheet at a distance $O(g^2)$ from the imaginary axis in the $s$-plane. Thus in this case we have no pole contribution. However, if the theory is weakly coupled, the pole is very close to the cut, and will give the dominant contribution to the integral over the cut itself. In this case we have to consider only the cut contribution since, as explained in detail for the homogeneous case [23], the poles of $\varphi_\mu(s)$ get a real part and move off into the second Riemann sheet in the $s$-plane. Thus

$$ \delta_{\text{pole}}(\vec{p}, t) = 0 . $$

(3.44)

Thus in this case we find

$$ \delta(\vec{p}, t) = \frac{2}{\pi} \delta_i(\vec{p}) \int_0^\infty \frac{\omega \Sigma_I(\omega, \vec{p}, T) \cos(\omega t)}{[\omega^2 - |\vec{p}|^2 - M_{\sigma}^2 - \Sigma_R(\omega, \vec{p}, T)]^2 + \Sigma_I(\omega, \vec{p}, T)^2} . $$

(3.45)
This expression leads to the following sum rule obtained previously by Pisarski \[34\] in a different manner:

\[
\frac{2}{\pi} \int_0^\infty \frac{\omega \Sigma_I(\omega, \vec{p}, T)}{\omega^2 - |\vec{p}|^2 - M_0^2 - \Sigma_R(\omega, \vec{p}, T) + \Sigma_I(\omega, \vec{p}, T)^2} = 1 .
\] (3.46)

In this unstable case, this sum rule illuminates the relationship between the Landau damping contribution to the absorptive part of the self-energy and the on-shell contribution. Notice that the Landau damping contribution to the absorptive part of the self-energy \(\text{Im} K^{(2)}\) given by eq.(2.38) has the same form as the finite temperature contribution to \(\text{Im} K^{(1)}\) \[2.37\]. Thus the sum rule above determines that the contribution to the spectral density below the light cone, that is the Landau damping term given by eq.(2.38) is borrowed from the spectral density above the physical two-particle cut. Therefore, although \(\text{Im} K^{(2)}\) does not contribute directly to the width of the meson and its decay rate in the medium, it does so indirectly through the sum rule.

In weakly coupled theories such that \(g^2 \ll 1\) the spectral density features a narrow resonance and has the Breit-Wigner form. Thus the integral over the discontinuity feels the pole and can be approximated by a Breit-Wigner resonance in the second Riemann sheet. When the position of the resonance is far away from the two particle thresholds (many widths) the integral is dominated by the resonance for long times but eventually the large time behavior will be determined by the behavior of the spectral density at threshold \[25\].

In what follows we will focus on the weakly coupled case and address the strongly coupled linear \(\sigma\) model afterwards.

1. **Zero Temperature**

In the case \(g^2 \ll 1\), and after the change of variables \(\omega \rightarrow \sqrt{\omega^2 + |\vec{p}|^2}\) the spectral density can be approximated by the Breit-Wigner form

\[
S(\omega, \vec{p}, T = 0) \approx \frac{2\omega \Sigma_I(\omega = M_0, \vec{p} = 0, T = 0)}{(\omega^2 - M_0^2)^2 + \Sigma_I^2(\omega = M_0, \vec{p} = 0, T = 0)} ,
\] (3.47)

\[
M_0^2 = M_0^2 + \Sigma_R(\omega = M_0, \vec{p} = 0, T = 0) .
\] (3.48)

The integral over the discontinuity of the self-energy, given by \[3.10\], can be approximated by

\[
\delta_{\text{cut}}(\vec{p}, t) \simeq \delta_i \frac{Z}{e^{-\Gamma(p)t}} \cos(t \sqrt{M_0^2 + p^2 + \alpha}) , \quad \Gamma(p) \ll M_0 ,
\] (3.49)

where the wave function renormalization \(Z\) and \(\alpha\) \(p\)-independent and given by

\[
Z = \left[ 1 - \frac{\partial \Sigma_R(iM_0, p = 0)}{\partial M_0^2} \right]^{-1} , \quad \alpha = -Z \frac{\partial \Sigma_I(iM_0, p = 0)}{\partial M_0^2} ,
\] (3.50)

and the damping rate is given by

\[
\Gamma(p) = \frac{Z \Sigma_I(M, p = 0)}{2 \sqrt{M_0^2 + p^2}} .
\] (3.51)
At one loop order we find from (2.47) and (2.42)
\[
\Gamma(p) = \frac{3g^2}{16\pi\sqrt{M^2 + p^2}} \sqrt{1 - \frac{4M^2}{M^2}} = \frac{M_r \Gamma(0)}{\sqrt{M^2 + p^2}}.
\] (3.52)

The expression (3.52) displays the Lorentz contraction of the width (dilation of the lifetime) with respect to that in the rest frame.

The Fourier transform of \(\delta_{\text{cut}}(\vec{p}, t)\) can be evaluated for large \(t\) and \(r\) with the stationary phase method. We find
\[
\phi(\vec{x}, t) \overset{r, t \to \infty}{\approx} \frac{\delta_i Z M^2}{4\pi^2} \frac{\cosh[\vartheta]}{\tau} e^{-\Gamma(0)\tau}
\times \sin \left\{ M_r \tau \left[ 1 + \frac{1}{2} \left( \frac{2\Gamma(0)}{M_r} \tanh[\vartheta] \right)^2 + \mathcal{O}\left( \frac{2\Gamma(0)}{M} \tanh[\vartheta] \right)^4 \right] + \alpha \right\}.
\] (3.53)

There is now an exponential damping in the amplitude. However for proper times longer than \(\approx \frac{1}{\Gamma(0)} \ln[\Gamma(0)/M_r]\), the most important contribution to the integral arises from the region near the two-pion threshold leading to a long time tail given by the pion contribution to \(\phi_{\text{cut}}(\vec{x}, t)\) in equation (3.12).

For an initial Gaussian wave packet (3.37) the field \(\phi_{\xi}(\vec{x}, t)\) is given for large \(t\) and \(r\) by
\[
\phi_{\xi}(\vec{x}, t, T) = e^{-\frac{1}{2}M^2 \xi^2 \sinh^2[\vartheta]} \phi_{\xi=0}(\vec{x}, t, T)
\] (3.54)
just as in the stable case because the stationary phase condition is the same as in that case.

2. Non-Zero Temperature

As shown in equations (3.23), (3.24), (3.25), at finite temperature the are several different cuts which contribute to \(\delta_{\text{cut}}(\vec{p}, t, T)\). The most important contribution arises from the two-particle cut starting at \(\omega^2 = 4M^2 + p^2\) corresponding to the process of \(\sigma\) decay into two pions and pion recombination. This is clearly seen because the resonance will be at a position \(4M^2 + p^2 < \Omega^2(p) < 4M^2 + p^2\) because the \(\sigma\) meson can now kinematically decay ‘on-shell’ in two pions. For weak coupling the sharp resonance dominates the integral and one obtains a result for \(\delta_{\text{cut,1}}(\vec{p}, t, T)\) analogous to (3.49), with the finite temperature self-energy. The two-particle cut starting at \(4M^2 + p^2\) and the thermal cut below the light-cone will give a contribution similar to the stable case studied in the previous section. Thus we concentrate on studying the contribution from the two-pion cut. Since manifest Lorentz covariance is lost (in the rest frame of the thermal bath), in the \(T \neq 0\) case all the parameters will depend on \(p\)

\[
Z(p, T) = \left[ 1 - \frac{\partial \Sigma_R(iM, p, T)}{\partial M^2} \bigg|_{M=\Omega(p, T)} \right]^{-1},
\] (3.55)

\[
\Gamma(p, T) = Z(p, T) \frac{\Sigma_I(i\Omega(p, T), p, T)}{2\Omega(p, T)},
\] (3.56)

\[
\alpha(p, T) = -Z(p, T) \frac{\partial \Sigma_I(iM, p, T)}{\partial M^2} \bigg|_{M=\Omega(p, T)},
\] (3.57)
where $\Omega(p, T)$ is given by equation (3.14).

The inverse Fourier transform of $\delta_{\text{cut},1}(\vec{p}, t, T)$ can be evaluated for large times and distances with the saddle point method to obtain, in the Breit-Wigner approximation

$$
\phi_{\text{BW}}(\vec{x}, t, T) = \frac{\delta_{\text{cut}} M_{\sigma}^2 Z(h(\vartheta), T) \cosh[\vartheta]}{4\pi^2} e^{-\Gamma(\vartheta, T)\tau} \sin[M_{\sigma} \tau + \alpha(h, T)] \theta(\tau),
$$

(3.58)

where $h(r, t) = M_{\sigma} \sinh[\vartheta]$. More explicitly for weak coupling, $\Gamma(\vartheta, T)$ takes the form

$$
\Gamma(\vartheta, T) = \frac{3 g^2 T}{8 \pi M_{\sigma}^2 \sinh[\vartheta] \cosh[\vartheta]} \log \frac{\sinh A_+ (\vartheta)}{\sinh A_- (\vartheta)}.
$$

(3.59)

with

$$
A_{\pm} (r, t) = \frac{M_{\sigma} \cosh[\vartheta]}{4 T} \left( 1 \pm \tanh[\vartheta] \sqrt{1 - \frac{4 M_{\pi}^2}{M_{\sigma}^2}} \right).
$$

(3.60)

For high temperatures and $M_{\pi}^2 \ll M_{\sigma}^2$ one finds the remarkable result (valid in the weak coupling case)

$$
\Gamma(\vartheta, T) = \frac{3 g^2 T \vartheta}{2 \pi M_{\sigma}^2 \sinh[2\vartheta]}.
$$

(3.61)

Notice again that the Boltzmann factors depend on the effective temperature $T(\vartheta) = T / \cosh[\vartheta]$ and that $\Gamma$ is a function of $\vartheta$ and $T(\vartheta)$. In particular, notice that (3.59) approaches the zero temperature limit at very large rapidities.

For long proper times, $\tau \approx \Gamma^{-1} \ln(\Gamma/M_{\sigma})$, the threshold will dominate the long time-large distance behavior with a contribution which is given by eq. (3.28). The contribution from the thermal cut is the same as in the stable case and given by eq. (3.35). In the weak coupling limit the contribution from the Breit-Wigner resonance and that of the threshold can just be added to yield the total contribution from the cut, and since the contribution of the thermal cut is added to that of the ‘normal’ cut, we finally find in the unstable case:

$$
\phi_{\text{unst}}(\vec{x}, t, T) = \phi_{\text{BW}}(\vec{x}, t) + \phi_{\text{thresh}}(\vec{x}, t) + \phi_{\text{cut},2}(\vec{x}, t),
$$

(3.62)

with the asymptotic results given by (3.58,3.28,3.35)

It is now straightforward to show as in the previous section for the stable case, that for an initial Gaussian wave packet the final result is just multiplied by the factor $e^{-\frac{1}{2} M_{\sigma}^2 \xi^2 \sinh^2[\vartheta]}$ because of the saddle point condition.

Thus at time and distances much larger than $\xi$ we obtain

$$
\phi_{\text{unst}, \xi}(\vec{x}, t, T) = e^{-\frac{1}{2} M_{\sigma}^2 \xi^2 \sinh^2[\vartheta]} \phi_{\text{unst}, \xi=0}(\vec{x}, t, T).
$$

(3.63)

The remarkable result of this section is that the space-time description of the evolution of a wave packet that decays, is that the effective decay rate is a function of rapidity which itself depends on the rapidity dependent effective temperature $T(\vartheta)$. We recall that in a medium this decay rate does not give the rate of production of pions through the decay of the sigma meson, but the difference between the production and annihilation rate as discussed previously in section II.
Let us now discuss the strongly coupled case. The approximations and conclusions obtained above are within the framework of a weakly coupled theory. As was discussed in section II, the imaginary part on shell for the linear sigma model description of pion physics is very large (see equation 2.50), thus the validity of the Breit-Wigner approximation to describe the space-time evolution must be called into question. Figure 2 shows $S(y, \vec{p})/M_\sigma$ as a function of $y = \sqrt{\omega^2 - p^2}/M_\sigma$ for $T = 0.1M_\sigma$; $|\vec{p}| = 10M_\sigma$ compared to the Breit-Wigner approximation. We see that in most of the range, but in a small region near the lowest threshold the Breit-Wigner approximation is excellent. We find the same result for temperatures all the way up to the pion mass and for a large range of momenta from very small to very large compared to $M_\sigma$. Clearly what is less accurate within the Breit-Wigner approximation is the extension of the integration region in $\omega$ to $-\infty$ because the resonance is rather broad. However one can extend the integration region and subtract the contribution from threshold, which is the dominant one at large rapidities and proper times. This is precisely what was done to arrive to the result given by eq. (3.62) above. Thus we conclude that even in the strong coupling case the asymptotic behavior is well approximated by eq.(3.63) for the case of a Gaussian initial packet.

IV. CONCLUSIONS

We have studied the relaxational dynamics of an inhomogeneous condensate fluctuation in the O(4) linear sigma model near the broken symmetry state, both at zero and non-zero temperature.

Explicit expressions are obtained for the self-energies at zero and finite temperature and we point out that at finite temperature there are new relaxational processes with origin in thermal cuts and that are only present in the inhomogeneous case with no counterpart in the relaxation of an homogeneous condensate.

For initial Gaussian fluctuations we have given explicit expressions for the asymptotic space-time evolution of the inhomogeneous fluctuation including the effect of thresholds both at zero and finite temperature. At finite temperature we obtained the decay rate of this inhomogeneous non-equilibrium configuration, in the medium this quantity is the rate of production minus the rate of absorption of pions. The space-time evolution is described in terms of an effective temperature and “decay rate” that depend on rapidity. We find to one loop that relaxational processes are described in terms of $T(\vartheta) = T/\cosh[\vartheta]$ and $\Gamma(\vartheta, T(\vartheta))$, and for large rapidities the finite temperature decay rate approaches the zero temperature limit.

We systematically compute the field behaviour for large times and distances compared with the inverse of the typical mass scale ($M$) in the model. This means $t \gg 1/M$, $r \gg 1/M$ and $\tau = \sqrt{t^2 - r^2} \gg 1/M$. For $t = r \gg 1/M$ the behaviour will be quantitatively different.

For large $\tau$, the field $\phi(\vec{x}, t)$ propagates as spherical waves for an initial wave packet of arbitrary shape concentrated around the origin.

Our asymptotic results can be summarized as follows.

• **Stable Case: $M_\sigma < 2M_\pi$**

$T = 0$. The pole contribution $\phi_{\text{pole}}(\vec{x}, t)$ dominates, giving an amplitude that decays as $\tau^{-3/2}$ and oscillates as a function of $\tau$ with frequency $M_0$ (physical $\sigma$ mass).
The cut gives contributions smaller than \( \phi_{\text{pole}} \) by a factor \( \tau^{-3/2} \) and oscillating with frequencies equal to the threshold positions [eqs. (3.9-3.12)].

\( T \neq 0 \). The pole contribution dominates for large \( \tau \). \( \phi_{\text{pole}} \) decays with the same power \( \tau^{-3/2} \) as for \( T = 0 \), but the amplitude and frequency change quantitatively [see eq. (3.22)].

A new cut running from \(-ip\) to \(+ip\) appears at non-zero temperature. It gives a non-oscillating contribution for large \( \tau \) that decays as \( \tau^{-5} \) [see eq. (3.35)].

**Unstable Case:** \( M_\sigma > 2M_\pi \)

\( T = 0 \). \( \phi \) is now a resonance (a pole in the second Riemann sheet) yielding an exponentially damped amplitude that oscillates with frequency \( M_0 \). The damping being the width of the resonance [see eq. (3.53)].

Eventually, for very large \( \tau \), the power-like tail coming from the cut contribution will dominate over the exponentially damped contribution of the resonance.

\( T \neq 0 \). It is similar as for \( T = 0 \), except that the damping rate becomes a non-trivial function of \( t, r, T \) and \( M_0 \) given by eq. (3.53).

The thermal cut with its non-oscillating contribution to \( \phi(\vec{x}, t) \) is also present here and dominates for \( \tau \to \infty \).

All the asymptotic results hold to first order in the field amplitude. However, since the field vanishes for \( \tau \to \infty \), our results are true for any initial amplitude provided \( \tau \) is large enough.

The relaxation of an initially Gaussian inhomogeneous fluctuation is described in terms of the spreading of the packet and decay in spherical waves, and we found that the time scale for relaxation, production and absorption of pions is a function of rapidity such that for larger rapidities the relaxational and decay processes are slower. We are currently extending these studies to the case of non-equilibrium fluctuations produced during the stage of parametric amplification as the \( \sigma \) rolls to the ground state [13].

We believe that the techniques developed in this work and the non-equilibrium aspects found here will be of use in a variety of physical contexts. One that comes to mind concerns the emission of Goldstone bosons from an axion string [35,36]. This emission is important in determining the exact upper bound on the axion decay constant, and thus on whether axion models for solving the strong CP problem are still viable. This problem will entail some modifications of the tools described here, the most notable one having to do with the existence of zero modes for the string configuration.

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**Figure Captions**

Fig. 1: One-loop Feynman diagrams contributing to the equation of motion. The dashed line corresponds to the insertion of the background field \( \phi \). Thin lines correspond to pion propagators, thick lines to \( \sigma \) propagators. The tadpoles had been absorbed in mass and \( f_\sigma \) renormalization.

Fig. 2: Comparison between \( S(y, \vec{p}, T)/M_\sigma \) and the Breit-Wigner approximation for \( T = 0.1M_\sigma \); \( |\vec{p}| = 10M_\sigma \) vs \( y = \sqrt{\omega^2 - p^2}/M_\sigma \). Solid line is the Breit-Wigner approximation, dotted line the full spectral density.
This figure "fig1-1.png" is available in "png" format from:

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