Tau Decays Beyond the Standard Model

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Abstract

In a recent paper, 8 semileptonic parameters were defined to specify the most general Lorentz-invariant spin correlation functions for 2-body $\tau$ decays. These parameters can be used to search for anomalous $\Gamma_L$, $\Gamma_T$ polarized-partial widths, for non-CKM-type leptonic CP violation, and for leptonic $\bar{T}_{FS}$ violation. They can also be used to bound the effective-mass scales $\Lambda$ for “new physics” arising from additional Lorentz structures, e.g. from lepton compositeness, tau weak magnetism, weak electricity, or second-class currents. It is emphasized that (i) for these tests “different modes have different merits” and that (ii) the parameters can be measured either by using spin-correlation techniques without polarized beams, or with longitudinally polarized beams.

1 INTRODUCTION

Currently, the bounds are very weak for possible “new physics” in $\tau$ decays. At best, the limits are at the several percent level whereas the errors on the Michel parameters are typically at the per-mill level in $\mu$ decays. During the last five years, impressive high precision electroweak experiments have been performed at the $Z$ boson resonance at LEP and at the SLC. From these experiments, and those by ARGUS, BES, and CLEO, information on $\tau$ decays at the several-percent level has been obtained. Now, the time has come for high precision experiments in $\tau$ decays. Model independent analyses, which do not assume a mixture of $(V \mp A)$ couplings, are now necessary in $\tau$ decays as a means for searching for “new physics” beyond the standard model.

For the purely leptonic decays, $\tau^- \to l^- \bar{\nu}_l \nu_\tau$ and $\tau^+ \to l^+ \nu_l \bar{\nu}_\tau$, there is the classic Michel parametrization. This has been reviewed in Pich’s talk\[2]. Here we will concentrate on

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2-body $\tau$ decays for which the branching ratios are large. In a recent paper \cite{2}, 8 semileptonic parameters were defined to specify the most general Lorentz-invariant spin correlation functions for 2-body $\tau$ decays. They can also be measured at an $e^-e^+$ collider with longitudinally-polarized beams, such as at the SLC or at a future tau/charm factory \cite{3}.

Conclusions of this talk include:

- “Different modes have different merits” in systematic searches for new physics.
- Even without candidates for lepton-number violating decays or for other forbidden modes, new physics can be discovered in on-going and future experiments by exploiting $\tau^-\tau^+$ spin-correlations and/or longitudinal-beam polarization.

Recent papers by theorists on possible “new physics” in $\tau$ decays, include studies of $\Delta L_i \neq 0$ at future colliders \cite{4}, of leptonic $CP$-violation \cite{5,6,7} and of special effects in the third family \cite{8}. Leptoquark and SUSY mechanisms have been proposed for producing observable $CP$-violating dipole moments in $\tau^-\tau^+$ production \cite{9}. The forthcoming BNL experiment has motivated a more precise treatment of higher-order hadronic contributions to the anomalous magnetic moments of the $\mu$ and $\tau$ in the standard model \cite{10}.

2 TESTS FOR “NEW PHYSICS”

By using a general formalism for two-body $\tau$ decays, one can (a) determine the “complete Lorentz structure” of $J_{\text{charged}}^{\text{Lepton}}$ directly from experiment, and (b) test in a model independent manner for the presence of “new physics”. For instance, there are simple tests for non-CKM-type leptonic $CP$ violation and for leptonic $\bar{T}_{FS}$ violation in $\tau$ decays.

2.1 General formalism for two-body $\tau$ decays:

The physical idea is very simple: We introduce 8 parameters to describe the most general spin-correlation function for the decay sequence $Z^0, \gamma^* \rightarrow \tau^-\tau^+ \rightarrow (\rho^-\nu)(\rho^+\bar{\nu})$ followed by $\rho^{ch} \rightarrow \pi^{ch}\pi^o$ including both $\nu_L, \nu_R$ helicities and both $\bar{\nu}_R, \bar{\nu}_L$ helicities. We present the discussion for the $\rho\nu$ channel, but the same formulas hold for the $a_1\nu$ and $K^*\nu$ channels. Thus, by including the $\rho$ polarimetry information that is available from the $\rho^{ch} \rightarrow \pi^{ch}\pi^o$ decay distribution, the polarized-partial-widths for $\tau^- \rightarrow \rho^-\nu$ are directly measurable. For instance, the general angular distribution for polarized $\tau^-_{L,R} \rightarrow \rho^-\nu \rightarrow (\pi^-\pi^o)\nu$ is described by

$$dN/d(cos\theta_1^\tau)d(cos\tilde{\theta}_a)d\tilde{\phi}_a = n_a[1 \pm f_a cos\theta_1^\tau]$$

$$\mp (1/\sqrt{2})sin\theta_1^\tau sin2\tilde{\theta}_a \mathcal{R}_\rho[\omega cos \tilde{\phi}_a + \eta' sin \tilde{\phi}_a]$$

(1)
with upper(lower) signs for a L-handed \( \tau^− \)
(R-handed), where
\[
\mathbf{n}_a = \frac{1}{8} (3 + \cos 2\theta_a + \sigma S_p [1 + 3 \cos 2\theta_a])
\]
\[
\mathbf{n}_a f_a = \frac{1}{8} (\xi [1 + 3 \cos 2\theta_a] + \zeta S_p [3 + \cos 2\theta_a])
\]
In this expression, \( \cos \theta_a \) describes the direction of the \( \rho^- \) momentum in the \( \tau^- \) rest frame, and \( \cos \theta_a \) and \( \phi_a \) describe the direction of the \( \pi^- \) in the \( \rho^- \) rest frame. Such formulas for more general spin-correlation functions in terms of the 8 semi-leptonic parameters are given in [2] for unpolarized \( e^-e^+ \) beams, and in [14] for polarized beams.

There are eight \( \tau \) semi-leptonic decay parameters since there are the four \( \rho_{L,T} \rho_{L,R} \)
final states: The first parameter is simply \( \Gamma \equiv \Gamma_L^+ + \Gamma_R^+ \), i.e. the (full) partial width for
\( \tau^- \rightarrow \rho^- \nu \). The subscripts on the Gamma’s denote the polarization of the final \( \rho^- \) (and in the
SM of the intermediate off-shell \( W^- \) boson), either “L=longitudinal” or “T=transverse”; superscripts denote “\( \pm \) for sum/difference of the \( \nu_L \) versus \( \nu_R \) contributions”
\[
\Gamma_L^+ = \left| A(0, -\frac{1}{2}) \right|^2 \pm \left| A(0, \frac{1}{2}) \right|^2
\]
\[
\Gamma_T^+ = \left| A(-1, -\frac{1}{2}) \right|^2 \pm \left| A(1, \frac{1}{2}) \right|^2.
\]
The second is the chirality parameter \( \xi \equiv \frac{1}{2}(\Gamma_L^- + \Gamma_T^-) \). Equivalently,
\[
\xi \equiv (\text{Prob} \ \nu_\tau \text{ is } \nu_L) - (\text{Prob} \ \nu_\tau \text{ is } \nu_R),
\]
\[
\equiv \left| <\nu_L|\nu_\tau > \right|^2 - \left| <\nu_R|\nu_\tau > \right|^2
\]
So a value \( \xi = 1 \) means the coupled \( \nu_\tau \) is pure \( \nu_L \). The remaining two partial-width parameters are defined by
\[
\zeta \equiv (\Gamma_L^- - \Gamma_T^-) / (S_p \Gamma), \quad \sigma \equiv (\Gamma_L^+ - \Gamma_T^+) / (S_p \Gamma).
\]
The definiton for \( \sigma \) implies that
\( \tilde{\sigma} \equiv S_\rho \sigma = (\text{Prob} \ \rho \text{ is } \rho_L) - (\text{Prob} \ \rho \text{ is } \rho_T) \),
is the analogue of the neutrino’s chirality parameter in Eq.(4). Thus, the parameter \( \tilde{\sigma} \) measures the degree of polarization of the emitted \( \rho \). If the exchange is completely via an
off-shell \( W^- \) boson, \( \tilde{\sigma} \) measures the polarization of the \( W^- \) boson.

The interference between these \( \rho/W_L \) and \( \rho/W_R \) amplitudes can be determined by
measuring the four parameters,
\[
\omega \equiv I_L^- / (R^\rho \Gamma), \quad \eta \equiv I_R^+ / (R^\rho \Gamma)
\]
\[
\omega' \equiv I_T^- / (R^\rho \Gamma), \quad \eta' \equiv I_T^+ / (R^\rho \Gamma)
\]
The associated LT-interference intensities are
\[
I_L^\pm = \left| A(0, -\frac{1}{2}) \right| \left| A(-1, -\frac{1}{2}) \right| \cos \beta_a
\]
\[
\pm \left| A(0, \frac{1}{2}) \right| \left| A(1, \frac{1}{2}) \right| \cos \beta_a^R
\]
\[
I_T^\pm = \left| A(0, -\frac{1}{2}) \right| \left| A(-1, -\frac{1}{2}) \right| \sin \beta_a
\]
\[
\pm \left| A(0, \frac{1}{2}) \right| \left| A(1, \frac{1}{2}) \right| \sin \beta_a^R
\]
Here \( \beta_a \equiv \phi_a^0 - \phi_a^\ast \), and \( \beta^R_a \equiv \phi_1^a - \phi_0^{aR} \) are the measurable phase differences of the associated helicity amplitudes \( A(\lambda_\rho, \lambda_\nu) = |A| \exp \imath \phi \).

In the standard lepton model in which there is only a \((V-A)\) coupling and \( m_\nu = 0 \), these parameters all equal one except that the two parameters directly sensitive to leptonic \( \bar{T}_{FS} \) violation vanish, \( \omega' = \eta' = 0 \). Note that in the **special case** of a mixture of only \( V \& A \) couplings and \( m_\nu = 0 \), \( \xi \to \xi \). Thus, in this special case \( \zeta \) measures the \( \nu_\tau \) helicity and \( \zeta = \xi \), **but for more general couplings neither property holds** for \( \zeta \) for \( \tau \to \nu_\tau\), \( a_1\nu \), \( K^*\nu \).

Hadronic factors \( S_\rho \) and \( R_\rho \),

\[
S_\rho = \frac{1 - 2 \frac{m_\rho^2}{m_\tau^2}}{1 + 2 \frac{m_\rho^2}{m_\tau^2}}, \quad R_\rho = \frac{\sqrt{2} \frac{m_\rho}{m_\tau}}{1 + 2 \frac{m_\rho^2}{m_\tau^2}} \tag{8}
\]

have been explicitly inserted into the definitions of some of the semi-leptonic decay parameters, so that quantities such as \( q_\rho^2 = m_\rho^2 \) can be smeared over in application due to the finite \( \rho \) width. These factors numerically are \( (S, R)_{a_0, a_1, K^*} = 0.454, 0.445; -0.015, 0.500; 0.330, 0.472 \).

From Table 1 given below for the \( \Gamma_L, \Gamma_T \) polarized-partial-widths, one easily sees that the numerical values of \( \"\xi, \zeta, \sigma, \ldots\" \) are very different for unique Lorentz couplings. This is indicative of the analyzing power of polarization techniques in two-body \( \tau \) decay modes. Both the real and the imaginary parts of the associated helicity amplitudes can be directly measured, c.f. Eqs(3,7).

### 2.2 Tests for non-CKM-CP and \( \bar{T}_{FS} \) violations:

These formulas only assume Lorentz invariance and do not assume any discrete symmetry properties. Therefore, it is easy to use this framework for testing for discrete symmetry properties. In particular, with \( A(\lambda_\rho, \lambda_\nu) \) for \( \tau^- \to \rho^-\nu \) and with \( B(\lambda_\rho, \lambda_\bar{\rho}) \) for \( \tau^+ \to \rho^+\bar{\nu} \) a specific discrete symmetry implies a specific relation among the associated helicity amplitudes:

\[
\begin{array}{ll}
\text{Invariance} & \text{Relation} \\
P & A(-\lambda_\rho, -\lambda_\nu) = A(\lambda_\rho, \lambda_\nu) \\
& B(-\lambda_\rho, -\lambda_\bar{\rho}) = B(\lambda_\rho, \lambda_\bar{\rho}) \\
C & B(\lambda_\rho, \lambda_\bar{\rho}) = A(\lambda_\rho, \lambda_\nu) \\
CP & B(\lambda_\rho, \lambda_\bar{\rho}) = A(-\lambda_\rho, -\lambda_\nu) \\
\bar{T}_{FS} & A^* (\lambda_\rho, \lambda_\nu) = A(\lambda_\rho, \lambda_\nu) \\
& B^* (\lambda_\rho, \lambda_\bar{\rho}) = B(\lambda_\rho, \lambda_\bar{\rho}) \\
CPT\bar{T}_{FS} & B^* (\lambda_\rho, \lambda_\bar{\rho}) = A(-\lambda_\rho, -\lambda_\bar{\rho})
\end{array}
\]

In the \( \tau^- \) rest frame, the matrix element for \( \tau^- \to \rho^-\nu \) is \( \langle \theta^\gamma_1, \phi_1^\gamma, \lambda_\rho, \lambda_\nu | \frac{1}{2}, \lambda_1 \rangle = D_{\rho, \nu}^{\frac{1}{2}}(\phi_1^\gamma, \theta_1^\gamma, 0) A(\lambda_\rho, \lambda_\nu) \) where \( \mu = \lambda_\rho - \lambda_\nu \) and \( \lambda_1 \) is the \( \tau^- \) helicity. For the \( CP \)-conjugate process, \( \tau^+ \to
\( \rho^+ \bar{\nu} \rightarrow (\pi^+ \pi^0) \bar{\nu} \), in the \( \tau^+ \) rest frame \( \langle \theta_2^\tau, \phi_2^\tau, \lambda_\bar{\rho}, \lambda_\bar{\nu} | 1/2, 1 \rangle = D_{\lambda_2, \bar{\mu}}^*(\phi_2^\tau, \theta_2^\tau, 0) B(\lambda_\bar{\rho}, \lambda_\bar{\nu}) \) with \( \bar{\mu} = \lambda_\bar{\rho} - \lambda_\bar{\nu} \).

“Measurement of a non-real helicity amplitude implies a violation of \( \tilde{T}_{FS} \) invariance”. This true since by exact \( T \) invariance the amplitudes for the process of interest and the time reversed process must be equal:

\[
\mathcal{M}(\tau^- \rightarrow \rho^- \nu^-_\tau) = \mathcal{N}(\tau^- \leftarrow \rho^- \nu^-_\tau)
\]

(9)

This can be written as

\[
\langle \lambda_\rho \lambda_\nu | H_{eff} | \lambda_\tau \rangle = \langle \lambda_\tau | H_{eff} | \lambda_\rho \lambda_\nu \rangle
\]

(10)

in terms of the “effective Hamiltonian” \( H_{eff} \) which describes the transition. In quantum mechanics, the right-hand-side equals the complex conjugate matrix element

\[
\langle \lambda_\tau | H_{eff} | \lambda_\rho \lambda_\nu \rangle = \overline{\langle \lambda_\rho \lambda_\nu | H_{eff}^\dagger | \lambda_\tau \rangle}
\]

(11)

but with the initial and final states interchanged and Hermitian-adjoint \( H_{eff}^\dagger \). If there are no “final state interactions”, the effective Hamiltonian will be Hermitian \( H_{eff} = H_{eff}^\dagger \), so

\[
\langle \lambda_\rho \lambda_\nu | H_{eff} | \lambda_\tau \rangle = \overline{\langle \lambda_\rho \lambda_\nu | H_{eff}^\dagger | \lambda_\tau \rangle}
\]

(12)

or

\[
\mathcal{M}(\tau^- \rightarrow \rho^- \nu^-_\tau) = \mathcal{M}(\tau^- \rightarrow \rho^- \nu^-_\tau)^* \]

(13)

Therefore, for an exclusive tau decay mode, the associated transition amplitude will be real by canonical \( T \)-invariance if there are no “final state interactions.” We refer to this as \( \tilde{T}_{FS} \)-invariance.

A violation of \( \tilde{T}_{FS} \)-invariance could occur because of the exchange of a \( Z' \) boson between the final \( \rho^- \) and the final \( \nu^-_\tau \) in which the \( Z' \) couples differently to the \( \rho_L \) versus the \( \rho_T \). Or \( \tilde{T}_{FS} \)-violation could occur because of a fundamental violation of canonical \( T \)-invariance. Whatever the cause might turn out to be, the experimental discovery of a violation of \( \tilde{T}_{FS} \)-invariance in a tau two body decay mode would be very significant.

In this formalism,

- If the primed parameters \( \omega' \neq 0 \) and/or \( \eta' \neq 0 \) \( \Rightarrow \tilde{T}_{FS} \) is violated:

In this formalism there are four parameters \( \eta, \eta', \omega, \omega' \) which can be used to test for leptonic \( \tilde{T}_{FS} \) violation. This occurs because the trigonometric structure of Eqs.(7) implies the two constraints

\[
(\bar{\eta} \pm \bar{\omega})^2 + (\bar{\eta}' \pm \omega')^2 = \frac{1}{4} [(1 \pm \xi)^2 - (\bar{\sigma} \pm \bar{\zeta})^2].
\]

(14)
among these four parameters.

To test for leptonic $T_{FS}$ violation, besides the $\omega$ parameter which can be measured from $I_4$ in both the $\rho$ and $a_1$ modes, there is the $\eta'$ parameter which can be obtained from $I_5$ in both the $\rho$ and $a_1$ modes. Also there are the $\eta$ and $\omega'$ parameters which only appear in S2SC distributions for the $a_1$ modes. The $a_1$ mode parameters are more sensitive [2] for searching for leptonic $T_{FS}$ violation than the simple $I_4$ distribution considered in Ref.[11].

Canonical CPT invariance implies only equal total widths between a particle and its antiparticle. Canonical CPT invariance does not imply equal partial widths between CP-conjugate decay modes of a particle and its antiparticle. Indeed, in nature in the kaon system the partial widths of the neutral kaons do differ for the particle and the antiparticle.

The barred parameters $\bar{\xi}, \bar{\zeta}, \ldots$ have the analogous definitions [2] for the CP conjugate modes, $\tau^+ \to \rho^+ \bar{\nu}, \ldots$. For instance,

$$\bar{\xi} = (\text{Prob } \bar{\nu}_\tau \text{ is } \bar{\nu}_R) - (\text{Prob } \bar{\nu}_\tau \text{ is } \nu_L),$$

$$\bar{\Gamma}_{\pm}^L = |B(0, \frac{1}{2})|^2 \pm |B(0, -\frac{1}{2})|^2.$$ (15)

Therefore, in this formalism

- If any $\xi \neq \bar{\xi}, \zeta \neq \bar{\zeta}, \ldots \implies \text{CP is violated:}$

As was shown in [5], if only $\nu_L$ and $\bar{\nu}_R$ exist, there are two simple tests for “non-CKM-type” leptonic CP violation in $\tau \to \rho \nu$ decay. Normally a CKM leptonic phase will contribute equally at tree level to both the $\tau^{-}$ decay amplitudes and so will cancel out in the ratio of their moduli and in their relative phase (for exceptions see footnotes 14, 15 in [5]).

The two tests for leptonic CP violation are:

$$\beta_a = \beta_b,$$

and

$$r_a = r_b,$$

where $r_a = \frac{|A(0, \frac{1}{2})|}{|A(0, -\frac{1}{2})|}$, $\beta_a = \phi_a^L - \phi_a^R$, $\beta_b = \phi_b^L - \phi_b^R$, and $r_a = r_b$, where $r_a = \frac{|A(-1, -\frac{1}{2})|}{|A(0, -\frac{1}{2})|}$, $r_b = \frac{|B(1, \frac{1}{2})|}{|B(0, \frac{1}{2})|}$. Sensitivity levels for $\tau \to \rho \nu$ and $\tau \to a_1 \nu$ decays are to about 0.05 to 0.1% for $r_a = r_b$, and to about 1° to 3° for $\beta_a = \beta_b$ at 10GeV and at 4GeV without using polarized $e^-e^+$ beams [5, 2].

2.3 Tests for anomalous $\Gamma_L, \Gamma_T$ polarized-partial-widths:

The contribution of the longitudinal($L$) and transverse($T$) $\rho/W$ amplitudes in the decay process is projected out by the simple formulas:

$$I_{\rho}^{\nu L,\nu R} \equiv \frac{1}{2}(I_{\rho}^L \pm I_{\rho}^R) = |A(0, \frac{1}{2})^2 A(\mp 1, \frac{1}{2}) | \cos \beta_a^{L,R}$$

$$= \frac{\Gamma}{2} (\bar{\eta} \pm \bar{\omega})$$

$$I_{\omega}^{\nu L,\nu R} \equiv \frac{1}{2}(I_{\omega}^L \pm I_{\omega}^R) = |A(0, \frac{1}{2})^2 A(\mp 1, \frac{1}{2}) | \sin \beta_a^{L,R}$$

$$= \frac{\Gamma}{2} (\bar{\eta}' \pm \bar{\omega}')$$
Table 1: Tests for anomalous $\Gamma_L, \Gamma_T$ polarized-partial-widths for $\rho_{L,T} \nu_{L,R}$ final states:

|                      | $V \mp A$ | $S \pm P$ | $f_M + f_E$ | $f_M - f_E$ |
|----------------------|----------|----------|------------|------------|
| **Analytic**         |          |          |            |            |
| $\Gamma^-_L/\Gamma$  | $\pm \frac{1}{2}(1 + S_\rho)$ | $\pm 1$ | $\frac{2r^2+\rho^2}{2r^2+\rho^2}$ | $-\frac{1}{3}$ |
| $\Gamma^+_L/\Gamma$  | $\pm \frac{1}{2}(1 - S_\rho)$ | $0$ | $\frac{2r^2+\rho^2}{2r^2+\rho^2}$ | $-\frac{2}{3}$ |
| $\Gamma^-_L/\Gamma$  | $\frac{1}{2}(1 + S_\rho)$ | $1$ | $\frac{2r^2+\rho^2}{2r^2+\rho^2}$ | $+\frac{1}{3}$ |
| $\Gamma^+_T/\Gamma$  | $\frac{1}{2}(1 - S_\rho)$ | $0$ | $\frac{2r^2+\rho^2}{2r^2+\rho^2}$ | $+\frac{2}{3}$ |
| **Numerical**        |          |          |            |            |
| $\Gamma^-_L/\Gamma$  | $\pm 0.7(\pm 0.5)$ | $\pm 1$ | $0.0(0.2)$ | $-0.3$ |
| $\Gamma^-_T/\Gamma$  | $\pm 0.3(\pm 0.5)$ | $0$ | $1.0(0.8)$ | $-0.7$ |
| $\Gamma^+_L/\Gamma$  | $0.7(0.5)$ | $1$ | $0.0(0.2)$ | $+0.3$ |
| $\Gamma^+_T/\Gamma$  | $0.3(0.5)$ | $0$ | $1.0(0.8)$ | $+0.7$ |

Values for unique Lorentz couplings. $S_\rho$ is defined in Eq.(8). Entries are for $\rho^-$ ($a_1^-$, if value differs).

$$\Gamma^\nu_{L^{\nu \nu R}} = \frac{1}{2} (I_L^+ \pm I_L^-) = |A(0, \mp \frac{1}{2}|^2$$

$$= \frac{\Gamma}{4}(1 + \bar{\sigma} \pm \xi \pm \bar{\zeta})$$

$$\Gamma^\nu_{T^{\nu \nu R}} = \frac{1}{2} (I_T^+ \pm I_T^-) = |A(\mp 1, \mp \frac{1}{2}|^2$$

$$= \frac{\Gamma}{4}(1 - \bar{\sigma} \pm \xi \mp \bar{\zeta})$$

In the first line, $\beta^L_a = \beta_a$ but we normally suppress such L superscripts, e.g. Eq.(7). Unitarity, requires the two right- triangle relations

$$(I_R^\nu_L)^2 + (I_R^\nu_T)^2 = \Gamma^\nu_L \Gamma^\nu_T$$

$$(I_R^\nu_R)^2 + (I_R^\nu_R)^2 = \Gamma^\nu_R \Gamma^\nu_R.$$

Table 1 gives the analytic forms and numerical values of the $\Gamma_L, \Gamma_T$ partial-widths for unique Lorentz couplings. An important experimental goal is to determine whether or not these partial widths are anomalous in nature versus the standard lepton model’s $(V - A)$ predictions (single Higgs doublet) because the $W_L$ versus $W_T$ partial widths might have distinct dynamical differences if electroweak dynamical symmetry breaking occurs in nature.

### 2.4 Tests for additional Lorentz structures:

Besides model independence, a major open issue is whether or not there is an additional chiral coupling in the tau’s charged-current. A chiral classification of additional structure
is a natural phenomenological extension of the symmetries of the standard $SU(2)_L \times U(1)$ electroweak lepton model. For $\tau^- \rightarrow \rho^- \nu_{L,R}$, the most general Lorentz coupling is

$$\rho^*_\mu \bar{u}_\nu \left( p \right) \Gamma^\mu u_\tau \left( k \right)$$ (16)

where $k_\tau = q_\rho + p_\nu$. It is convenient to treat the vector and axial vector matrix elements separately. In Eq.(16)

$$\Gamma^\mu_V = g_V \gamma^\mu + \frac{f_M}{2\Lambda} i\sigma^{\mu\nu}(k-p)_\nu + \frac{g_S^-}{2\Lambda} (k-p)^\mu$$

$$+ \frac{g_S^+}{2\Lambda} (k+p)^\mu + \frac{g_{T^+}}{2\Lambda} i\sigma^{\mu\nu}(k+p)_\nu$$

$$\Gamma^\mu_A = g_A \gamma^\mu \gamma_5 + \frac{f_E}{2\Lambda} i\sigma^{\mu\nu}(k-p)_\nu \gamma_5 + \frac{g_P^-}{2\Lambda} (k-p)^\mu \gamma_5$$

$$+ \frac{g_{P^+}}{2\Lambda} (k+p)^\mu \gamma_5 + \frac{g_{T^5}}{2\Lambda} i\sigma^{\mu\nu}(k+p)_\nu \gamma_5$$

The parameter $\Lambda = \text{"the effective-mass scale of new physics"}$. In effective field theory this is the scale at which new particle thresholds are expected to occur or where the theory becomes non-perturbatively strongly-interacting so as to overcome perturbative inconsistencies. It can also be interpreted as a measure of a new compositeness scale. In old-fashioned renormalization theory $\Lambda$ is the scale at which the calculational methods and/or the principles of “renormalization” breakdown.

Without additional theoretical or experimental inputs, it is not possible to select what is the “best” minimal set of couplings for analyzing the structure of the tau’s charged current. For instance, by Lorentz invariance, for the $\rho$, $a_1$, $K^*$ modes there are the equivalence theorems that for the vector (axial-vector) current

$$S \approx V + f_M, \; T^+ \approx -V + S^-$$ (17)

$$P \approx -A + f_E, \; T^5_5 \approx A + P^-$$ (18)

There are similar but different equivalences for the $\pi$, $K$ modes, see Eq.(21). Therefore, from the perspective of searching for the fundamental dynamics, it is important to investigate what limits can be set for a variety of Lorentz structures (including $S^\pm$, $P^\pm$, $T^\pm$, and $T^5_5^\pm$) and not just for a kinematically minimal, but theoretically prejudiced, set.

Ref.[11] gives the limits on $\Lambda$ in GeV for real $g_i$’s from the $\rho$ and $a_1$ modes: Effective mass scales of $\Lambda \sim 1 - 2 TeV$ can be probed at 10GeV, and at 4GeV for the $(S + P)$ and the $f_M + f_E$ couplings. For determination of ideal statistical errors, we assume $10^7 (\tau^- \tau^+)$ pairs at 10GeV and separately at 4GeV; at $M_Z$ we assume $10^7 Z^0$’s with $\text{BR}(Z^0 \rightarrow \tau^- \tau^+) = 0.03355$; $BR_\rho = 24.6\%$, $BR_{a_1} = 18\%$ for the sum of neutral/charged $a_1$ modes, and $BR_{\pi} = 11.9\%$.

We listed the ideal statistical error for the presence of an additional $V + A$ coupling as an error $\delta(\xi_A)$ on the chirality parameter $\xi_A$ for $\tau^- \rightarrow A^- \nu$. Equivalently, if one ignores
possible different L and R leptonic CKM factors, the effective lower bound on an additional $W^\pm_R$ boson (which couples only to right-handed currents) is

$$M_R = \{\delta(\xi_A)/2\}^{-1/4}M_L$$

For the $\{\rho^-, \rho^+\}\{\{a_1^-, a_1^+\}$ mode, from $\delta(\xi_{\rho}) = 0.0012(0.0018)$ this gives equivalently $M_R > 514 GeV (464 GeV)$. Probably, $10^8(\tau^-\tau^+)$ pairs will be accumulated by a $\tau$/charm factory at 4GeV, so all the potential 4GeV statistical-error bounds might be improved by a factor of 3.2.

3 DIFFERENT MODES HAVE DIFFERENT MERITS

In contrast to the purely leptonic modes[1, 12], the tau semi-leptonic modes are qualitatively distinct since they enable a second-stage spin-correlation. From existing results, a quantitative comparison with the ideal sensitivity in the purely leptonic case is possible if we assume an arbitrary mixture of $V$ and $A$ couplings with $m_\nu = 0$. Then the semi-leptonic chirality parameter $\xi_\rho$ and the chiral polarization parameter $\xi_{Lepton}$ can be compared since then they both equal $(|g_L|^2 - |g_R|^2)/(|g_L|^2 + |g_R|^2)$. By using $I_4$ to obtain $\xi_\rho$ from $\{\rho^-, \rho^+\}$, the statistical error [2] is $\delta(\xi_{\rho}) = 0.006$ at $M_Z$. This is a factor of 8 better than the pure leptonic mode's $\delta(\xi_{Lepton}) = 0.05$ error[13] from averaging over the $\mu$ and $e$ modes and using $I_3(E_1, E_2, \cos \psi_{12})$ where $\psi_{12}$ is the opening angle between the two final charged leptons in the cm-frame. A complete determination of the purely leptonic parameters for $\tau^- \to \mu^- \nu_\mu \nu_\tau$ will require a difficult measurement of the $\mu$ polarization, see Fetscher[12].

3.1 $\rho, K^*$ modes:

For the $\rho^-$ mode, the errors for $(\xi, \zeta, \sigma, \omega)$ based on simple four-variable spin-correlation function $I_4$ are slightly less than 1%: For $10^7(\tau^-\tau^+)$ pairs at 10 GeV: from the $\{\rho^-, \rho^+\}$ mode and using the four-variable distribution $I_4$, the ideal statistical percentage errors are for $\xi$, 0.6%; for $\zeta$, 0.7%; for $\sigma$, 1.3%; and for $\omega$, 0.6%. The CP tests for these semileptonic parameters are about $\sqrt{2}$ worse. Typically the $a_1$ values for these parameters are about 3 times worse than the $\rho$ values.

In analogy with the Pauli anomalous magnetic moment, an obvious signature for lepton compositeness would be an additional tensorial coupling. In this regard, it is useful to first test for the presence of only $\nu_L$ couplings which would exclude a significant contribution from the $g_- = f_M - f_E$ tensorial coupling:

- For the $a_1$ and $\rho$ modes there are 3 logically independent tests for only $\nu_L$ couplings: $\xi = 1$, $\zeta = \sigma$, and $\omega = \eta$.

In addition, if $T_{FS}$ violation occurred then the non-zero parameters $\omega' = \eta'$ if there are only $\nu_L$ couplings. An additional tensorial $g_+ = f_M + f_E$ coupling would preserve these 3
signatures for only $\nu_L$ couplings. But, such a tensorial $g_+$ coupling would give them non-(V − A)-values: $\zeta = \sigma \neq 1$ and $\omega = \eta \neq 1$. Second, for a $g_+$ coupling there is the prediction that for $\Lambda$ large

$$ (\zeta - 1) = (1 - \omega) \frac{g}{I} $$

(19)

where the ratio “$g/l$” is a known function \cite{2} of $m_\rho$ and $m_\tau$. Numerically $(g/l)_\rho = 0.079$.

These $\nu_L$ signatures and Eq.(19) also occur for an additional $(S + P)$ coupling but with the ratio $(g/l)$ replaced by $(a/d)$, which varies from 5.07 to 12.1 across $(m_\rho \pm \Gamma/2)$. Fortunately, here the $\pi$ mode can again be used to limit the presence of an additional $(S + P)$ coupling versus a $g_+ = f_M + f_E$ coupling since the latter does not contribute to the $\pi$ mode.

3.2 $a_1$ mode:

For the kinematic description of $\tau^- \rightarrow a_1^- \nu \rightarrow (\pi_1^- \pi_2^- \pi_3^+ )\nu$, the normal to the $(\pi_1^- \pi_2^- \pi_3^+ )$ decay triangle is used in place of the $\pi^-$ momentum direction of the $\tau^- \rightarrow \rho^- \nu \rightarrow (\pi^- \pi^+ \nu$ sequential decay.

Including both $\nu_L$ and $\nu_R$ helicities, the composite decay density matrix for $\tau^- \rightarrow a_1^- \nu \rightarrow (\pi_1^- \pi_2^- \pi_3^+ )\nu$ is

$$ R' = S_1^+ R^+ + S_1^- R^- $$

(20)

where the sequential decay density matrices $R^\pm$ describing $\tau^- \rightarrow a_1^- \nu \rightarrow (\pi_1^- \pi_2^- \pi_3^+ )\nu$ are given in Ref.[2]. The $S_1^\pm$ factors do depend on the strong-interaction form-factors used to describe the decay $a_1^- \rightarrow \pi_1^- \pi_2^- \pi_3^+$. However, when the 3-body Dalitz plot is integrated over, only the $S_1^+$ term remains, so it can be absorbed into the overall normalization factor which removes any arbitrary form-factor dependence. Similarly, by asymmetric integration, the $S_1^-$ factor can be absorbed into an overall normalization factor.

To test for leptonic $\tilde{T}_{ES}$-violation, besides the $\omega$ parameter which can be measured from $I_4$ in both the $\rho$ and $a_1$ modes, there are the following from the \{a_1^-, a_1^+\} mode: using $I_5^-$ the errors are for $\eta$, 0.6%; using $I_7$ for $\eta'$, 0.013; and using $I_7^-$ for $\omega'$, 0.002. See Sec.(2.2) above.

3.3 $\pi, K$ modes:

The $\tau^- \rightarrow \pi^- \nu, K^- \nu$ modes each generally provide less information since here only two of the semi-leptonic parameters can be measured, i.e. the partial widths $\Gamma_{\pi,K}$ and the chirality parameter $\xi_{\pi,K} = \frac{|A(-\frac{1}{2})|^2 - |A(\frac{1}{2})|^2}{|A(-\frac{1}{2})|^2 + |A(\frac{1}{2})|^2}$. Second, the important weak-magnetism and weak-electricity couplings, $f_M$ and $f_E$, do not contribute to these modes; but, they can be precisely measured by the $\rho$ and $a_1$ modes. Third, by Lorentz invariance, there are the strong equivalence theorems

$$ S^- \approx S \approx T^+ \approx V, P^- \approx P \approx T_5^+ \approx A $$

(21)

because there are only two independent decay amplitudes.
Nevertheless, from the π mode there is good separation (> 127GeV from CLEO II data) of $V - A$ from a $T^+ + T_5^+$ coupling, whereas these couplings cannot be separated in the $\rho$ and $a_1$ modes. Second, the $S + P$ coupling is also excluded to $\Lambda > 127GeV$. Third, there is direct measurement of the chirality parameter $\xi_\pi$, i.e. of the probability that the emitted $\nu_\tau$ is L-handed. Unfortunately, the fundamental $S^-$ and $P^-$ couplings which do not contribute to $\tau \rightarrow \rho \nu, a_1 \nu, K^* \nu$ are suppressed in $\tau^- \rightarrow \pi^- \nu, K^- \nu$ decay since $q \cdot V \sim \frac{m_\pi^2}{2 \Lambda} g_{S^-}$ and $q \cdot A \sim \frac{m_\pi^2}{2 \Lambda} g_{P^-}$.

It is also important to note what cannot be precisely measured by two-body $\tau$ decay modes:

(i) The present and potential experimental bounds on $(S^- \pm P^-)$ couplings are exceptionally poor or non-existent from measurements of the $\pi$, $\rho$ and $a_1$ modes[11, 2]. (ii) The S2SC functions do not enable a measurement of any relative phase between the $\nu_L$ and $\nu_R$ helicity amplitudes[2].

4 TESTS WITH LONGITUDINALLY-POLARIZED BEAMS

For the case of longitudinally polarized beams, we assume 100 % polarization and study the 3-variable distribution $I_3^P(\theta_{beam}, E_{\rho^-}, \hat{\theta}_{\pi^-})$. In the center-of-mass frame, $\theta_{beam}$ is the angle between the final tau momentum and the initial $e^-$ beam, and $E_{\rho^-}$ is the energy of the final $\rho^-$. The angle $\hat{\theta}_{\pi^-}$ is the direction of the final $\pi^-$ momentum in the $\rho^-$ rest frame [when boost is directly from the center-of-mass frame]. We call this the “$P_L$ method”. Instead of the angle between the $e^-$ and $\tau^-$ momenta, one could use the angle between the $e^-$ and the final $\rho^-$ momenta; work on this alternative 3-variable distribution in in progress[14].

See Table 2 for the errors for measurement of the $(\xi, \zeta, \sigma, \omega)$ parameters based on $I_3^P$ and on $I_4$. In general, by using longitudinally-polarized beams the errors for the $\rho^-$ mode are slightly less than 0.4% and about a factor of 7 better than by using the S2SC function $I_4$. The CP tests for these semileptonic parameters are $\sqrt{2}$ worse by the $P_L$ method, or by the S2SC method. Typically the $a_1$ values are 2-4 times worse than the $\rho$ values. However, for $\xi$, the error for the $a_1$ mode by the $P_L$ method is about 3 times better than that for the $\rho$ mode.

Both methods are comparable for the two tests for non-CKM-type leptonic $CP$ violation. Table 3 shows the sensitivities of the $\rho$ and $a_1$ modes.

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