Stable and Metastable Vacua in SQCD

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We study deformations of $N = 1$ supersymmetric QCD that exhibit a rich landscape of supersymmetric and non-supersymmetric vacua.
1. Introduction

In this paper we study the low energy dynamics of supersymmetric QCD (SQCD) in the presence of certain F-term deformations. The starting point of our analysis is an $N = 1$ supersymmetric gauge theory with gauge group $SU(N_c)$ and $N_f > N_c$ flavors of chiral superfields in the fundamental representation of the gauge group, $Q^\alpha_i$, $\tilde{Q}^i_\alpha$ ($\alpha = 1, \cdots, N_c$; $i = 1, \cdots, N_f$). This theory has a global symmetry

$$SU(N_f) \times SU(N_f) \times U(1)_B \times U(1)_R$$

(1.1)

and a non-trivial moduli space of vacua, which has been extensively studied and is rather well understood; see e.g. [1-6] for reviews.

A natural question is what happens when we deform the theory by adding a general superpotential that preserves a particular subgroup of the global symmetry (1.1), such as the non-chiral subgroup $SU(N_f)_{\text{diag}} \times U(1)_B$. A class of superpotentials with this property is

$$W_{\text{el}} = \sum_{n=1}^{n_0} \frac{1}{n!} m_n \text{Tr} M^n$$

(1.2)

where the meson field

$$M^i_j = \tilde{Q}^i Q^j$$

(1.3)

is an $N_f \times N_f$ matrix (the color indices are summed over in (1.3)). Terms with $n > 1$ in (1.2) are non-renormalizable, which is reflected in the fact that the couplings $m_n$ have dimension $[m_n] = 3 - 2n$ at the free fixed point. One can think of the superpotential (1.2) as providing an effective description below a certain energy scale.

In the case $m_n = m_1 \delta_{n,1}$ (1.2) is a mass term for $Q$, $\tilde{Q}$. It has been known for a long time that the resulting theory has $N_c$ supersymmetric vacua, in accordance with the Witten index. More recently, it was found [7] that for $N_c < N_f < \frac{3}{2}N_c$ and small $m_1$ there are metastable non-supersymmetric ground states as well. Such states might be useful for describing supersymmetry breaking in nature.

The purpose of this paper is to study more general superpotentials of the form (1.2). We will mainly discuss the case where only the two lowest terms in (1.2) are non-zero, i.e.

$$W_{\text{el}} = m_1 \text{Tr} M + \frac{1}{2} m_2 \text{Tr} M^2$$

(1.4)
but will also comment on higher order superpotentials. We will see that such superpotentials lead generically to a rich landscape of supersymmetric and non-supersymmetric vacua, and explore some of their properties. Related works include [8-18].

Like in [7], we will find it useful to utilize the Seiberg dual description of SQCD, in which the gauge group is $SU(N_f - N_c)$, and the meson (1.3) becomes a gauge singlet field. We will analyze the dynamics in both the electric and the magnetic descriptions and compare them.

The deformed SQCD with superpotential (1.4) has a simple embedding in string theory, along the lines of [19-30]. In a companion paper [31] we describe the relevant string construction, and in particular its connection to the gauge theory results of this paper. We find that the brane description provides a complementary picture to the gauge theory one.

The plan of the paper is as follows. In section 2 we construct the supersymmetric ground states of the theory (1.4), and verify that the electric and magnetic descriptions give rise to the same vacuum structure once all the relevant quantum effects have been included. In section 3 we describe metastable states in this theory. In section 4 we discuss our results and comment on generalizations.

2. Vacuum structure of deformed SQCD

2.1. A first look at the vacuum structure

The $N = 1$ supersymmetric Yang-Mills theory with gauge group $SU(N_c)$ discussed in the previous section, which we will refer to as the electric theory, is equivalent in the infrared [32] to another gauge theory, which we will refer to as the magnetic theory. The latter has gauge group $SU(N_f - N_c)$ and the following set of chiral superfields: $N_f$ fundamentals of the gauge group, $q^i$, $\tilde{q}_i$, $i = 1, \cdots, N_f$, and a gauge singlet $M_j^i$ which, as suggested by the notation, is identified with the gauge invariant meson field (1.3) in the electric theory.

The magnetic quarks $q$, $\tilde{q}$ and meson $M$ are coupled via the superpotential

$$W_{\text{mag}} = \frac{1}{\Lambda} \tilde{q}_i M_j^i q^j .$$

(2.1)

The scale $\Lambda$ is related to the dynamically generated scales of the electric and magnetic theories, $\Lambda_e$, $\Lambda_m$, by the scale matching relation [1]

$$\Lambda_e^{3N_c - N_f} \Lambda_m^{3(N_f - N_c) - N_f} = (-)^{N_f - N_c} \Lambda^{N_f} .$$

(2.2)
We are interested in adding to the electric Lagrangian the superpotential

\[ W_{\text{el}} = \frac{\alpha}{2} \text{Tr}(\bar{Q}Q)^2 - m \text{Tr}(\bar{Q}Q) = \frac{\alpha}{2} \text{Tr}M^2 - m \text{Tr}M \]  

which has the form (1.4), with \( m_1 = -m \) and \( m_2 = \alpha \). In the magnetic description this corresponds to deforming (2.1) to

\[ W_{\text{mag}} = \frac{1}{\Lambda} \bar{q} M^i q^j + \frac{\alpha}{2} \text{Tr}M^2 - m \text{Tr}M . \]  

(2.4)

Since the superpotential (2.4) is quadratic in \( M \), we can integrate this field out. The resulting superpotential for the magnetic quarks is given by:

\[ W_{\text{mag}} = -\frac{1}{\alpha \Lambda} \left[ \frac{1}{2\Lambda} \text{Tr}(\bar{q}q)^2 - m \text{Tr}(\bar{q}q) \right] . \]  

(2.5)

Comparing to (2.3) we see that the magnetic superpotential has the same qualitative form as the electric one.

Conversely, one can write the electric superpotential (2.3) in a way similar to the magnetic one by integrating in a gauge singlet field \( N \):

\[ W_{\text{el}} = -\frac{1}{\Lambda} \bar{Q}^i N^j Q_j - \frac{\alpha_e}{2} \text{Tr}N^2 + m_e \text{Tr}N . \]  

(2.6)

Requiring that integrating out \( N \) leads back to (2.3) gives

\[ \alpha = \frac{1}{\alpha_e \Lambda^2} , \quad m = \frac{m_e}{\alpha_e \Lambda} . \]  

(2.7)

Comparing (2.6) to (2.4) we see that the two superpotentials have very similar forms. Note also that we chose the normalization of the field \( N \) (which transforms as a singlet under the electric gauge group) such that it is identified in the infrared with the magnetic quark bilinear \( N = \bar{q}q \). This is the dual version of the relation between the singlet meson \( M \) in the magnetic theory and the bilinear in electric quarks, (1.3).

Although the two forms of the magnetic superpotential, (2.4) and (2.5), describe the same long distance physics, there is a physical difference between them, which becomes important for small \( \alpha \). In this limit, the mass of \( M \) in (2.4) is small, and at energies above that mass this field should be included in the dynamics. The description (2.5) does not contain this field; it coincides with (2.4) only at energies well below the mass of \( M \). Similar comments apply to the electric superpotentials (2.3), (2.6) in the limit \( \alpha_e \to 0 \).
To recapitulate, we see that the analysis of the vacuum structure in the electric and magnetic descriptions is essentially identical, up to the replacement \( N_c \leftrightarrow N_f - N_c, \alpha \leftrightarrow \alpha_c, m \leftrightarrow m_e \), etc. We will verify later that the two descriptions lead to the same vacuum structure, in agreement with Seiberg duality.

At first sight, it actually seems that the electric and magnetic theories have different vacuum structures. Consider, for example, the classical magnetic superpotential (2.4). To find the supersymmetric vacua we need to solve the F-term constraints

\[
M^i_j q_j = 0,
\]
\[
\tilde{q}_i M^i_j = 0,
\]
\[
\frac{1}{\Lambda} \tilde{q}_i q_j = m \delta^j_i - \alpha M^j_i.
\]

From (2.8) we learn that \( M \) satisfies the matrix equation

\[
mM = \alpha M^2.
\]

On solutions of the equations of motion one can choose \( M \) to be diagonal. Equation (2.9) implies that its eigenvalues can take only two values, 0 and \( \frac{m}{\alpha} \). Without loss of generality we can take

\[
M = \begin{pmatrix} 0 & 0 \\ 0 & \frac{m}{\alpha} I_{N_f - k} \end{pmatrix}
\]

where \( k = 0, 1, 2, \ldots, N_f \) and \( I_n \) is the \( n \times n \) identity matrix. Plugging (2.10) into the last line of (2.8) implies that

\[
\tilde{q} q = \begin{pmatrix} m \Lambda & 0 \\ 0 & 0 \end{pmatrix}.
\]

The rank of the matrix on the left hand side is at most \( N_f - N_c \). Hence, one must have \( k \leq N_f - N_c \).

Note that the vacua (2.10), (2.11) go off to infinity in field space as the deformation parameter \( \alpha \) (2.4) goes to zero. In particular, for \( \alpha = 0 \) supersymmetry is spontaneously broken. This is consistent with the fact that for \( \alpha = 0 \) the classical superpotential (2.4) has an unbroken \( U(1)_R \) symmetry, with \( R_q = R_{\tilde{q}} = 0, R_M = 2 \). For \( \alpha \neq 0 \) this symmetry is explicitly broken, and one expects [33] that supersymmetric vacua exist.

Expanding around the solution (2.10), (2.11) one finds that the only massless degrees of freedom are gauge fields and fermions associated with pure \( N = 1 \) SYM corresponding to the unbroken gauge group \( SU(N_f - N_c - k) \). As mentioned above, quantum mechanically this theory has \( N_f - N_c - k \) vacua with a mass gap (for \( N_f - N_c - k \geq 2 \)). Thus, the
above analysis implies that the theory with superpotential (2.4) has the following number of vacua (up to Nambu-Goldstone bosons associated with broken global symmetries):
\[
N_{\text{mag}} = 1 + \sum_{k=0}^{N_f - N_c - 1} (N_f - N_c - k) = 1 + \frac{1}{2} (N_f - N_c)(N_f - N_c + 1) .
\] (2.12)

The 1 in (2.12) corresponds to the case \(k = N_f - N_c\) where the expectation values of the baryon fields \(b = q^{N_f - N_c}\) and \(\tilde{b} = \tilde{q}^{N_f - N_c}\) are non-zero, which we will refer to as the baryonic branch.

As mentioned above, the analog of the magnetic superpotential (2.4) in the electric theory is (2.6). This superpotential clearly leads to the same vacuum structure as (2.12) with the replacement \(N_f - N_c \rightarrow N_c\),
\[
N_{\text{el}} = 1 + \sum_{k=0}^{N_c - 1} (N_c - k) = 1 + \frac{1}{2} N_c(N_c + 1) .
\] (2.13)

Thus, the electric and magnetic answers are in general different, whereas Seiberg duality implies that they must agree. The resolution has to do with quantum corrections to the superpotentials of the electric and magnetic theories. We next turn to a discussion of these corrections, first in the special case \(N_f = N_c + 1\), and then in general. We will see that after including quantum effects the number of supersymmetric vacua is given by
\[
N_{\text{vac}} = \max(N_{\text{el}}, N_{\text{mag}})
\] (2.14)
in both the electric and the magnetic theories.

2.2. Quantum corrections for \(N_f = N_c + 1\)

This case is particularly simple, since the magnetic gauge group is empty. At the same time, according to (2.14) the number of vacua is in this case supposed to be given by the electric result (2.13), which is larger than the magnetic one (2.12). The magnetic quarks \(q^i\) are proportional to the electric baryons \(B^i\), and it is convenient to use the latter as the fundamental degrees of freedom. The magnetic superpotential (2.4) is known to receive an important correction proportional to \(\det M\), and takes the form [34]
\[
W_{\text{mag}} = \frac{1}{\Lambda_c^{2N_f-3}} \left( \tilde{B}_i M^i_j B^j - \det M \right) + \frac{\alpha}{2} \text{Tr} M^2 - m \text{Tr} M .
\] (2.15)
The new term in the superpotential leads to a correction to the F-term condition on the third line of (2.8), which now becomes
\[ \tilde{B}_i B^j - (\det M)(M^{-1})^j_i + \Lambda_e^{2N_f-3} (\alpha M^j_i - m \delta^j_i) = 0. \] (2.16)

To analyze the solutions of these equations we need to distinguish between the cases where the meson matrix $M$ is regular and singular. Consider first the case where it is regular, which we will refer to as the mesonic branch. Then, the first two lines of (2.8) (with $q^i \propto B^i$) imply that
\[ B^i = \tilde{B}_j = 0 \] (2.17)
while (2.16) takes the form
\[ \Lambda_e^{2N_f-3} (\alpha M^2 - m M) = (\det M) I_{N_f}. \] (2.18)

As before, on solutions of the equations of motion we can diagonalize $M$. Since the left hand side of (2.18) is proportional to the identity matrix, the $N_f$ eigenvalues of $M$ must take at most two distinct values, which we will denote by $x$ and $y$, and are related as follows:
\[ x + y = \frac{m}{\alpha}. \] (2.19)

The vacua split into classes labeled by an integer $0 \leq k \leq \frac{N_f}{2}$, with $k$ of the eigenvalues of $M$ equal to $y$, and $N_f - k$ equal to $x$. The upper bound on $k$ takes into account the freedom of exchanging $x$ and $y$.

For $k = 0$, $M$ is proportional to the identity matrix, $M = x I_{N_f}$. Plugging this form into (2.18) we find
\[ \Lambda_e^{2N_f-3} (\alpha x - m) = x^{N_f-1}. \] (2.20)

There are $N_f - 1 = N_c$ solutions for $x$, and thus $N_c$ vacua with this form of $M$. For generic $\alpha$, and in particular when $\alpha$ is sufficiently small, the $N_c$ solutions of (2.20) are all distinct and non-vanishing, as expected.

For $k \neq 0$ the two distinct eigenvalues of $M$ satisfy
\[ \Lambda_e^{2N_f-3} (\alpha x - m) = x^{N_f-k-1} y^k, \]
\[ \Lambda_e^{2N_f-3} (\alpha y - m) = x^{N_f-k} y^{k-1}. \] (2.21)

Using (2.19) we can rewrite both lines of (2.21) as
\[ -\alpha \Lambda_e^{2N_f-3} = x^{N_f-k-1} y^{k-1}. \] (2.22)
Since $y$ is linearly related to $x$, (2.22) is a polynomial equation for $x$ of order $N_f - 2$. It has $N_f - 2 = N_c - 1$ distinct solutions with $x, y \neq 0$ (again, for generic $\alpha$). Thus, for odd $N_f$ the number of supersymmetric vacua of the magnetic theory is given by:

$$N_c + \frac{1}{2}(N_f - 1)(N_c - 1) = \frac{1}{2}N_c(N_c + 1).$$

(2.23)

For even $N_f$ we have to analyze separately the case $k = N_f/2$ for which the degeneracies of the two eigenvalues $x$ and $y$ are equal. In this case, due to the symmetry of interchange of $x$ and $y$ we actually have only $\frac{1}{2}(N_f - 2) = \frac{1}{2}(N_c - 1)$ distinct solutions of (2.22). The total number of vacua is in this case

$$N_c + \frac{1}{2}(N_f - 2)(N_c - 1) + \frac{1}{2}(N_f - 2) = \frac{1}{2}N_c(N_c + 1).$$

(2.24)

We see that for both even and odd $N_f$, the total number of vacua in the magnetic description (2.4) agrees with (2.14), except for the contribution of the baryonic branch to which we turn next.

So far we assumed that the meson matrix $M$ is non-degenerate. The analysis needs to be modified when some of its eigenvalues vanish. It turns out that the only non-trivial case is the one in which exactly one eigenvalue of $M$ vanishes. Thus, we take

$$M = \text{diag}(M_1, M_2, \cdots, M_N)$$

and consider the limit $M_1 \to 0$ with the rest of the $M_j$ remaining finite. The first two lines of (2.8) imply that the only non-zero components of $B, \tilde{B}$ are $B^1, \tilde{B}_1$. Eq. (2.16) leads to:

$$M_j = \frac{m}{\alpha}, \quad j = 2, \cdots, N_f,$$

$$B^1 \tilde{B}_1 = \left(\frac{m}{\alpha}\right)^{N_f-1} + m\Lambda^2N_f^{-3}.$$

(2.26)

Including this extra vacuum, which corresponds to the baryonic branch in the classical analysis, brings the total number of vacua into agreement with (2.14).

If $k > 1$ eigenvalues of $M$ go to zero, (2.16) has no solutions. Indeed, in that case the first two lines of (2.8) imply that the non-zero components of $B, \tilde{B}$ lie in the degenerate, $k$ dimensional, subspace. In that subspace, (2.16) implies that

$$B^i \tilde{B}_j \propto \delta^i_j,$$

(2.27)

which is impossible to satisfy since the left hand side has rank 1 while the right hand side has rank $k > 1$.

To summarize, for $N_f = N_c + 1$ we conclude that the number of vacua of the magnetic theory is given by (2.14). The term that goes like $\det M$ in the magnetic superpotential (2.15) is crucial for the analysis. The analog of this term for generic $N_f > N_c + 1$ comes from quantum effects in the magnetic theory. We next turn to a discussion of these effects.
2.3. Quantum corrections for $N_f > N_c + 1$

We would like to extend the analysis of the previous subsection to general $N_f$. Consider a vacuum in which the expectation value of the meson field $M$ has $r$ vanishing eigenvalues and $N_f - r$ non-vanishing ones. As mentioned in subsection 2.1, the F-term constraint on the third line of (2.8) implies that $r \leq N_f - N_c$. Therefore, at least $N_c$ eigenvalues of $M$ must be non-zero. In particular, we can bring it to the form

$$M = \begin{pmatrix} \hat{M} & 0 \\ 0 & M_0 \end{pmatrix}$$  \hspace{1cm} (2.28)

where $M_0$ is a non-degenerate $(N_c - 1) \times (N_c - 1)$ matrix, and $\hat{M}$ is an $(N_f - N_c + 1) \times (N_f - N_c + 1)$ one, which may or may not be degenerate (but whose rank is at least one).

Since $M_0$ is non-degenerate, the flavors of quarks $q, \tilde{q}$ that couple to it via (2.1) are massive and can be integrated out at low energies. This leads to an $SU(N_f - N_c)$ gauge theory with $N_f - N_c + 1$ flavors $q^a, \tilde{q}^\tilde{a}$ ($a, \tilde{a} = 1, \cdots, N_f - N_c + 1$), whose scale is given by

$$\Lambda_L^{2(N_f - N_c) - 1} = \frac{\det M_0}{\Lambda^{N_c - 1}} \Lambda_m^{3(N_f - N_c) - N_f}.$$  \hspace{1cm} (2.29)

This theory is of the sort discussed in the previous subsection. It can be described in terms of the gauge invariant observables

$$N_a^a = \tilde{q}_\tilde{a} \cdot q^a, \quad b_a = q^{N_f - N_c}, \quad \tilde{b}^\tilde{a} = \tilde{q}^{N_f - N_c}.$$  \hspace{1cm} (2.30)

The full magnetic superpotential has the form

$$W_{\text{mag}} = \text{Tr} \left( \frac{1}{\Lambda} \hat{M} N + \frac{\alpha}{2} \hat{M}^2 - m \hat{M} \right) + \text{Tr} \left( \frac{\alpha}{2} M_0^2 - m M_0 \right) + \frac{1}{\Lambda L^{2(N_f - N_c) - 1}} \left( \tilde{b} \cdot N \cdot b - \det N \right).$$  \hspace{1cm} (2.31)

The field $\hat{M}$ appears quadratically in (2.31) and thus can be integrated out. The equation of motion of $\hat{M}$ sets it to

$$\hat{M} = -\frac{1}{\alpha \Lambda} (N - m \Lambda I_{N_f - N_c + 1}).$$  \hspace{1cm} (2.32)

Plugging this into (2.31) (and dropping a constant contribution to the superpotential) leads to

$$W_{\text{mag}} = -\frac{1}{\alpha \Lambda} \text{Tr} \left( \frac{1}{2 \Lambda} N^2 - m N \right) + \text{Tr} \left( \frac{\alpha}{2} M_0^2 - m M_0 \right) + \frac{\Lambda^{N_c - 1}}{\Lambda_m^{3(N_f - N_c) - N_f} \det M_0} \left( \tilde{b} \cdot N \cdot b - \det N \right).$$  \hspace{1cm} (2.33)
The part of the superpotential that depends on $N$, $b$, $\tilde{b}$ is very similar to the one analyzed in the previous subsection, (2.15). The new element is the dependence on $M_0$ which needs to be taken into account.

As before, the F-term equations of motion of $N$, $b$, $\tilde{b}$ have two kinds of solutions: one in which $N$ is non-degenerate, and another in which it has exactly one vanishing eigenvalue.

Consider first the mesonic branch, where $N$ is non-degenerate. In this case it is convenient to go back to (2.31) and integrate out $N$, $b$, $\tilde{b}$. This amounts to setting $b = \tilde{b} = 0$ in (2.31) and replacing $N$ by the solution of its equation of motion,

$$\hat{M} N = \frac{\Lambda^{N_c}}{\Lambda_{m}^{3(N_f-N_c)-N_f}} \frac{\det N}{\det M_0} I_{N_f-N_c+1} .$$

(2.34)

Solving for $N$ and substituting in (2.31) (using (2.2)) we get

$$W_{\text{mag}} = \text{Tr} \left( \frac{\alpha}{2} M^2 - m M \right) - (N_f - N_c) \left( \frac{\det M}{\Lambda_{e}^{3N_c-N_f}} \right) \frac{1}{N_f-N_c} I_{N_f} .$$

(2.35)

where $M$ is the full meson matrix (2.28), which is non-degenerate in this case. Of course, the determinant term in (2.35) is nothing but the well known non-perturbative superpotential [1-6] for the meson field in SQCD. We could have gotten it directly from (2.4) by assuming that $M$ is a non-degenerate matrix and integrating out the massive quarks $q$, $\tilde{q}$.

The fact that we got it with the correct coefficient is a check on the algebra.

We can now generalize the discussion of the previous subsection and look for (non-singular) solutions of the F-term equations corresponding to the superpotential (2.35):

$$\alpha M^2 - m M = \left( \frac{\det M}{\Lambda_{e}^{3N_c-N_f}} \right)^{\frac{1}{N_f-N_c}} I_{N_f} .$$

(2.36)

Again, the matrix $M$ has only two distinct eigenvalues $x$, $y$ satisfying the relation (2.19), and vacua are labeled by an integer $k$ which keeps track of the number of times the eigenvalue $y$ (say) appears.

Considerations similar to those that led to (2.22) give

$$x^{N_c-k} y^{k-N_f+N_c} = (-\alpha)^{N_f-N_c} \Lambda_{e}^{3N_c-N_f} .$$

(2.37)

Together with (2.19) this gives a polynomial equation for $x$, whose order depends on $N_f$, $N_c$ and $k$. 9
Consider first the case $N_f \leq 2N_c$. For $k \leq N_f - N_c$, (2.37) is a polynomial of degree $N_c - k$. Thus it has $N_c - k$ solutions. For $N_f - N_c \leq k \leq \frac{N_f}{2}$ the degree of the polynomial and the number of solutions are given by $2N_c - N_f$. Summing over $k$ one finds that the number of vacua (up to global symmetries) is $\frac{1}{2}N_c(N_c + 1)$, as before (2.23).

For $N_f \geq 2N_c$ the picture is slightly different. For $0 \leq k \leq N_c$ one finds $N_f - N_c - k$ solutions, and for $N_c \leq k \leq \frac{N_f}{2}$, $N_f - 2N_c$ solutions. The total in this case is $\frac{1}{2}(N_f - N_c)(N_f - N_c + 1)$, again in agreement with (2.14).

It is not surprising that for $N_f > 2N_c$ the number of vacua agrees with the magnetic answer, while for $N_f < 2N_c$ it does not. The non-perturbative superpotential in (2.33) gives a correction to the classical superpotential for $M$, (2.4), that goes like $M^{\frac{N_f}{N_f - N_c}}$. For $N_f > 2N_c$ this correction is subleading at large $M$ relative to the leading, $M^2$, term, and one does not expect it to change the number of vacua. On the other hand, for $N_f < 2N_c$ it changes the behavior of the potential at infinity, and it is natural that the number of vacua changes.

So far we assumed that the matrix $N$ (2.30) is regular. As we saw in the previous subsection, the only other case we need to consider is the baryonic branch, in which the rank of $N$ is $N_f - N_c$ (i.e. one of the eigenvalues of $N$ goes to zero). We can choose this eigenvalue to be $N_1$. The F-term equations of the superpotential (2.31) take in this case the form:

$$
\tilde{M}_1^1 = \frac{\Lambda}{\Lambda^2(N_f - N_c) - 1} \left[ \det N \right]_{11} - b_1 \tilde{b}^1 = \frac{m}{\alpha};
$$

$$
\tilde{M}_{i}^{j} = 0; \quad N_i^j = m \Lambda \delta_i^j, \quad i, j > 1;
$$

$$
M_0 = \frac{m}{\alpha} I_{N_c - 1},
$$

(2.38)

where $[\det N]_{11}$ is the determinant of $N$ with the first row and column discarded (i.e. the (11) minor of the matrix $N$).

Thus, the meson matrix $M$ takes the form

$$
M = \begin{pmatrix} M^{(1)} & 0 \\ 0 & 0 \end{pmatrix}
$$

(2.39)

where $M^{(1)}$ is proportional to the $N_c \times N_c$ identity matrix,

$$
M^{(1)} = \frac{m}{\alpha} I_{N_c}.
$$

(2.40)

The $(N_f - N_c + 1) \times (N_f - N_c + 1)$ matrix $N$ has a block proportional to the $(N_f - N_c)$ dimensional identity matrix (2.38). Overall, we find that the number of vacua in the magnetic gauge theory with the superpotential (2.4) agrees precisely with (2.14).
2.4. Matching the electric and magnetic descriptions

In the previous two subsections we performed a detailed analysis of the supersymmetric vacua of the magnetic gauge theory (2.4), and in particular reproduced (2.14) in that description. It is interesting to check this result in the electric description. In fact, this does not require any additional work. As mentioned above, the description of the electric gauge theory via the superpotential (2.6) is identical to the magnetic one (2.4) with the substitutions

\[
\begin{align*}
N_c & \rightarrow N_f - N_c , \\
\Lambda_m & \rightarrow \Lambda_e , \\
\Lambda & \rightarrow -\Lambda , \\
(m, \alpha) & \rightarrow (m_e, \alpha_e) , \\
(q, \bar{q}) & \rightarrow (Q, \bar{Q}) , \\
M & \rightarrow N .
\end{align*}
\]

(2.41)

As a check, note that the transformation on the first three lines of (2.41) is a symmetry of the scale matching condition (2.2).

To see how the vacua we found in the previous subsections arise in the electric description (2.6), consider for example the case \(N_c < N_f < 2N_c\) (the regime \(N_f > 2N_c\) is very similar). In the magnetic description we found vacua in which the meson matrix \(M\) (1.3) takes the form (up to global symmetries)

\[
M = \text{diag}(x^{N_f-k}, y^k)
\]

(2.42)

with \(x\) and \(y\) satisfying the relations (2.19), (2.37). The number of solutions as a function of \(k\) is \(N_c - k\) for \(0 \leq k \leq N_f - N_c\), and \(2N_c - N_f\) for \(N_f - N_c \leq k \leq \frac{N_f}{2}\).

To analyze this case in the electric variables we need to use the map (2.41). Due to the transformation of the number of colors, the electric theory is actually in the opposite regime of the analysis of subsection 2.3, \(N_f > 2\tilde{N}_c = 2(N_f - N_c)\). According to that analysis, the electric meson matrix \(N\) takes in this case the form

\[
N = \text{diag}(x_e^{N_f-k}, y_e^k)
\]

(2.43)

where now for \(0 \leq k \leq \tilde{N}_c = N_f - N_c\) there are \(N_f - \tilde{N}_c - k = N_c - k\) solutions, and for \(N_f - N_c = \tilde{N}_c \leq k \leq \frac{N_f}{2}\) there are \(N_f - 2\tilde{N}_c = 2N_c - N_f\) solutions. Comparing to
the magnetic analysis, we see that the degeneracies are exactly the same for all \(k\), and we should identify the electric and magnetic vacua for each value of \(k\) separately.

We next show that the detailed form of the meson matrix one finds in the electric and magnetic descriptions is indeed the same for each \(k\). To facilitate the comparison, we need to translate the results of the electric analysis, which give the auxiliary gauge singlet meson matrix \(N\) to those for the meson matrix \(M\) \([1.3]\). The equation of motion of \(N\) arising from the superpotential \([2.6]\) gives the relation between the two:

\[
\frac{M}{Λ} = m_e I_{Nf} - α_e N .
\] (2.44)

Plugging \([2.42]\), \([2.43]\) into \([2.44]\) we find the following relations between the eigenvalues:

\[
\begin{align*}
x &= Λ(m_e - α_e x_e) , \\
y &= Λ(m_e - α_e y_e) .
\end{align*}
\] (2.45)

These relations can be simplified by recalling that \(x_e\) and \(y_e\) satisfy an analog of \([2.19]\) obtained by making the replacements \([2.41]\),

\[
x + y = \frac{m_e}{α_e}
\] (2.46)

using which we can rewrite \([2.45]\) as

\[
\begin{align*}
x &= α_e Λ y_e , \\
y &= α_e Λ x_e .
\end{align*}
\] (2.47)

The non-trivial check is that the polynomial equation satisfied by \(x\), \(y\), \([2.37]\), and the corresponding equation for \(x_e\), \(y_e\),

\[
x_e^{N_f - N_c - k} y_e^{k - N_c} = (± α_e) N_c Λ_{m}^{3(N_f - N_c) - N_f}
\] (2.48)

are compatible for all \(k\). Plugging \([2.47]\) into \([2.37]\) and using the scale matching condition \([2.2]\) as well as \([2.7]\) one finds that this is indeed the case.

2.5. The classical limit

In the previous subsections we found that the number of supersymmetric vacua of the deformed SQCD system described by \([2.3]\) \(-\) \([2.6]\) is given by \([2.14]\). For \(N_f ≥ 2N_c\) it agrees with the classical analysis of the magnetic theory, while in the electric description
some of the vacua exist only in the quantum theory and go off to infinity in the classical limit. For $N_f \leq 2N_c$ it is the other way around.

To see how this happens in detail, we can take the classical limit of our general results. Consider, for example, the limit in which the magnetic theory becomes classical. In this limit (which is equivalent via (2.2) to $\Lambda^{3N_c-N_f}_{\text{mag}} \to \infty$) the quantum corrections to the classical superpotential vanish (see e.g. (2.34)). To see what happens to the quantum vacua, we need to analyze the behavior of the eigenvalues $(x, y)$ of subsection 2.3 in this limit.

Consider, for example, the case $N_f \leq 2N_c$. The equations for the eigenvalues, (2.19), (2.37) take different forms for different $k$. For $0 \leq k \leq N_f - N_c$ one has

$$x^{N_c-k} = (-\alpha)^{N_f-N_c} \Lambda_e^{3N_c-N_f} \left( \frac{m}{\alpha} - x \right)^{N_f-N_c-k}.$$  

(2.49)

This is a polynomial equation of degree $N_c-k$. The solutions exhibit two types of behavior in the classical limit. There are $N_f - N_c - k$ solutions of the form

$$x = \frac{m}{\alpha} - \epsilon , \quad y = \epsilon ,$$

(2.50)

with

$$\epsilon^{N_f-N_c-k} \simeq (-\alpha)^{N_c-N_f} \left( \frac{m}{\alpha} \right)^{N_c-k} \Lambda_e^{N_f-3N_c}.$$  

(2.51)

Note that $\epsilon \to 0$ in the classical limit. Thus, these solutions are small deformations of the classical solutions found in subsection 2.1.

In addition, (2.49) has $2N_c - N_f$ solutions in which

$$x^{2N_c-N_f} \simeq (-)^k \alpha^{N_f-N_c} \Lambda_e^{3N_c-N_f}.$$  

(2.52)

These solutions go to $x = \infty$ in the classical limit.

For $N_f - N_c \leq k \leq \frac{N_f}{2}$ (2.49) is a polynomial equation of degree $2N_c-N_f$. Its solutions have the form (2.52), so they too go to infinity in the classical limit. The baryonic vacuum (2.38) is a small deformation of the classical one.

For $N_f \geq 2N_c$ one can perform a similar analysis and verify that all the solutions of (2.37) are small deformations of the classical ones, as expected.

---

1 The analysis of the classical limit in the electric theory is very similar.
3. Metastable vacua

In the previous section we discussed the supersymmetric ground states of deformed SQCD (1.4). In this section, we will see that this theory has many non-supersymmetric metastable ground states as well. As in [7], we will restrict attention to the regime \( N_f < \frac{3}{2} N_c \), where the magnetic theory is free in the infrared, and can be thought of as the effective low energy description of the asymptotically free electric gauge theory. A similar analysis can be performed in the free electric phase.

The Kahler potential of the singlet meson superfield \( M \) in (2.4) in the free magnetic phase is expected to take the form

\[
K = \frac{1}{a|\Lambda_e|^2} M^\dagger M + \cdots
\]

near the origin \( M = 0 \). The positive real constant \( a \) is not easy to calculate (see [7] for further discussion). It is convenient to define a superfield \( \Phi \) via

\[
M = \sqrt{a} \Lambda_e \Phi
\]

such that the Kahler potential for \( \Phi \) and the magnetic quarks is canonical near the origin of field space,

\[
K = \text{Tr}q^\dagger q + \text{Tr}\tilde{q}^\dagger \tilde{q} + \text{Tr}\Phi^\dagger \Phi + \cdots.
\]

Corrections to the Kahler potential (3.3) are due to physics at or above the scale \( \Lambda_m \) where the magnetic gauge theory breaks down and is replaced by the asymptotically free dual electric theory discussed above. The leading corrections are expected to be quartic in the fields and suppressed by two powers of \( \Lambda_m \),

\[
\delta K \sim \frac{1}{|\Lambda_m|^2} \text{Tr}(\Phi^\dagger \Phi)^2 + \cdots.
\]

In order to be able to ignore them we will require the expectation values of the fields \( q, \tilde{q}, \Phi \) to be much smaller than \( |\Lambda_m| \).

The magnetic superpotential (2.4) is given by

\[
W_{\text{mag}} = h \tilde{q}_i \Phi^i q^j - \text{Tr} \left( h \mu^2 \Phi - \frac{1}{2} h^2 \mu_\phi \Phi^2 \right) = \frac{1}{\Lambda} \tilde{q}_i M^i_j q^j + \text{Tr} \left( \frac{1}{2} \alpha M^2 - m M \right).
\]
By equating the two expressions in (3.3) we can relate the parameters as follows:

\[ h = \sqrt{a \frac{\Lambda_c}{\Lambda}} , \quad \mu^2 = m \Lambda , \quad \mu_\phi = a \Lambda^2 . \] 

(3.6)

The classical supersymmetric vacua (2.10), (2.11) take in these variables the form

\[ h\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \mu^2 I_{N_f-k} \end{pmatrix} , \] 

(3.7)

\[ \tilde{q}q = \begin{pmatrix} \mu^2 I_k & 0 \\ 0 & 0 \end{pmatrix} . \] 

(3.8)

To construct the metastable states, it is convenient to further split the \((N_f-k) \times (N_f-k)\) block at the lower right corner of (3.7), (3.8) into blocks of size \(n\) and \(N_f-k-n\) as follows:

\[ h\Phi = \begin{pmatrix} 0 & 0 & 0 \\ 0 & h\Phi_n & 0 \\ 0 & 0 & \mu^2 I_{N_f-k-n} \end{pmatrix} , \] 

(3.9)

and

\[ \tilde{q}q = \begin{pmatrix} \mu^2 I_k & 0 & 0 \\ 0 & \tilde{\varphi}\varphi & 0 \\ 0 & 0 & 0 \end{pmatrix} . \] 

(3.10)

\(\varphi\) and \(\tilde{\varphi}\) are \(n \times (N_f-N_c-k)\) dimensional matrices. They correspond to \(n\) flavors of fundamentals of the gauge group \(SU(N_f-N_c-k)\) which is unbroken by the non-zero expectation value of \(q, \tilde{q}\) in (3.10). \(\Phi_n\) and \(\tilde{\varphi}\varphi\) are \(n \times n\) matrices. The supersymmetric ground state (3.7), (3.8) corresponds to \(h\Phi_n = \frac{\mu^2}{\mu_\phi} I_n, \varphi = \tilde{\varphi} = 0\).

As we will see next, there are metastable vacua near the origin as well. We will restrict to the regime

\[ \Lambda_m \gg \mu \gg \mu_\phi \] 

(3.11)

where the first inequality is as in [7], and the second implies that the term proportional to \(\mu_\phi\) is a small perturbation of the superpotential considered in [7]. Since in general the expectation value of \(\Phi\) (3.3) can be large in the regime (3.11), it is convenient to discuss separately the cases \(n = N_f-k\) and \(n < N_f-k\).
3.1. Metastable vacua with $n = N_f - k$

In this case, the dynamics of the fields $\Phi_n, \varphi, \tilde{\varphi}$ near the origin of field space is obtained from the underlying $SU(N_f - N_c)$ gauge theory by giving an expectation value $\mu$ to $k$ flavors of magnetic quarks $q, \tilde{q}$, (3.10). The low energy theory is an $SU(N_f - N_c - k)$ gauge theory with $N_f - k = n$ light flavors, which is infrared free in the regime $N_f < \frac{3}{2}N_c$. Its scale, $\Lambda_l$, is related to that of the underlying theory, $\Lambda_m$, via the scale matching relation

$$\Lambda_m^{3(N_f-N_c)-N_f} = \mu^{2k} \Lambda_l^{3(N_f-N_c-k)-n}.$$  \hspace{1cm} (3.12)

A useful way of writing (3.12) is

$$\left(\frac{\mu}{\Lambda_m}\right)^{2k} = \left(\frac{\Lambda_m}{\Lambda_l}\right)^{3(N_f-N_c-k)-n}.$$  \hspace{1cm} (3.13)

In the regime (3.11), the left hand side of (3.13) is very small. Since the power on the right hand side is negative, we conclude that $\Lambda_m \gg \Lambda_l$. Using the fact that (3.12) also implies that

$$\left(\frac{\mu}{\Lambda_l}\right)^{2k} = \left(\frac{\Lambda_m}{\Lambda_l}\right)^{3(N_f-N_c)-N_f}$$  \hspace{1cm} (3.14)

we conclude that the hierarchy of scales is

$$\mu \ll \Lambda_l \ll \Lambda_m.$$  \hspace{1cm} (3.15)

Thus, we see that as long as $\mu \ll \Lambda_m$, the gauge dynamics is weakly coupled for all energies well below $\Lambda_m$. At energies above $\mu$, the full magnetic gauge group is restored. At an energy of order $\mu$ the theory crosses over to the one with gauge group $SU(N_f - N_c - k)$, which is also infrared free and due to (3.15) is weakly coupled. Therefore, we can mostly neglect the gauge dynamics in what follows.

The potential for $\Phi_n, \varphi, \tilde{\varphi}$ near the origin of field space has two relevant contributions. One is the tree level potential that follows from (3.3), (3.5). The second is the one loop potential, which to leading order in $h$, $\frac{\mu^2}{\mu}$ is identical to that computed in [7], and is given by

$$V_{1\text{-loop}} = b|h|^2 \mu^2 \text{Tr} \Phi_n^\dagger \Phi_n$$  \hspace{1cm} (3.16)

$^3$ In the regime (3.11), the expectation value of $q$ is such that the corrections to the Kahler potential discussed around eq. (3.4) can be neglected, as in [7].
with \( b \) a numerical constant,
\[
b = \frac{\ln 4 - 1}{8\pi^2} (N_f - N_c) .
\] (3.17)

The full one loop potential for \( \Phi_n, \varphi, \tilde{\varphi} \) has the form
\[
\frac{V}{|h|^2} = |\Phi_n\varphi|^2 + |\tilde{\varphi}\Phi_n|^2 + |\varphi - \mu^2 I_n + h\mu\Phi_n|^2 + b|h\mu|^2 \text{Tr} \Phi_n^\dagger \Phi_n .
\] (3.18)

We are looking for a minimum of the potential at \( \varphi = \tilde{\varphi} = 0 \). Differentiating (3.18) w.r.t. \( \Phi_n \) we find that if there is one, it is located at
\[
h\Phi_n = \frac{\mu^2 \mu^*_\phi}{|\mu^\phi|^2 + b|\mu|^2} I_n \simeq \frac{\mu^2 \mu^*_\phi}{b|\mu|^2} I_n ,
\] (3.19)

and its vacuum energy is given to leading order in \( h, \frac{\mu^\phi}{\mu} \) by
\[
V \simeq n|h\mu|^2 .
\] (3.20)

Expanding around this solution one finds that the mass matrix of \( \varphi, \tilde{\varphi} \) has eigenvalues
\[
m^2_{\pm} = \frac{|\mu|^4}{(|\mu^\phi|^2 + b|\mu|^2)^2} [|\mu^\phi|^2 \pm b|h|^2(|\mu^\phi|^2 + b|\mu|^2)] \simeq \frac{1}{b^2} (|\mu^\phi|^2 \pm |b|\mu|^2) .
\] (3.21)

Equation (3.21) implies that for sufficiently small \( \mu^\phi \) some of the fluctuations of \( \varphi, \tilde{\varphi} \) are tachyonic. To avoid tachyons one must have
\[
\frac{|\mu^\phi|^2}{|\mu|^2} > \frac{|bh|^2}{1 - b|h|^2} \simeq |bh|^2 .
\] (3.22)

The condition (3.22) is compatible with \( |h|, \frac{|\mu^\phi|}{|\mu|} \) being arbitrarily small. If it is satisfied, the quarks \( \varphi, \tilde{\varphi} \) are massive, and the vacuum (3.19) is locally stable.

For \( k < N_f - N_c - 1 \), this vacuum contains an unbroken \( SU(N_f - N_c - k) \) gauge theory, which is weakly coupled at energies above the scale of the masses of \( \varphi, \tilde{\varphi} \) (3.21). For energies well below these masses this gauge theory confines and has, as before, \( N_f - N_c - k \) vacua.

For \( k = N_f - N_c - 1 \) there are no unbroken gauge fields and the theory is weakly coupled at long distances.

For \( k = N_f - N_c \) (the largest value it can take), the fields \( \varphi, \tilde{\varphi} \) in (3.10) do not exist. Hence, in this case the constraint (3.22) is absent and the resulting metastable vacuum is present for arbitrarily small \( \mu^\phi \). In the limit \( \mu^\phi \to 0 \) it corresponds to the metastable state discussed in [7]. Even when (3.22) is satisfied, the vacuum with \( k = N_f - N_c \) is more long-lived than those with \( k < N_f - N_c \) since the mode of instability towards condensation of \( \varphi, \tilde{\varphi} \) is absent in this case.
3.2. Metastable vacua with \( n < N_f - k \)

In order to trust an analysis based on the Kahler potential (3.3) we have to demand that all components of \( \Phi \) (3.9) are much smaller than \( \Lambda_m \). In the present case this implies

\[
\frac{\mu^2}{\mu_\Phi} \ll h \Lambda_m .
\]  

(3.23)

A useful way of thinking about (3.23) is as the requirement

\[
\frac{\mu}{\Lambda_m} \ll h \frac{\mu_\Phi}{\mu},
\]  

(3.24)

which is compatible with all three couplings \( \frac{\mu}{\Lambda_m}, h \) and \( \frac{\mu_\Phi}{\mu} \) being small. Furthermore, this requirement is very natural in the brane realization of the theory, [31].

Like in the previous subsection, we would like the gauge dynamics to be weak near the origin of the field space of \( \varphi, \tilde{\varphi}, \Phi_n, (3.9), (3.10) \). The dynamics of these fields is obtained from the magnetic \( SU(N_f - N_c) \) gauge theory by giving masses \( \frac{\mu^2}{\mu_\Phi} \) to \( N_f - k - n \) flavors (see (3.9)) and vacuum expectation values \( \mu \) to \( k \) flavors (see (3.10)). In the regime (3.11), the masses are much larger than the expectation values. Thus we can analyze the reduction from the \( SU(N_f - N_c) \) magnetic gauge theory to the low energy \( SU(N_f - N_c - k) \) one in two steps, by first incorporating the masses, and then the expectation values. We would like the theory to remain weakly coupled throughout this process.

In the first step, we go from an \( SU(N_f - N_c) \) SYM theory with \( N_f \) flavors and scale \( \Lambda_m \) to one with the same gauge group, \( n + k \) flavors and scale \( \Lambda_1 \). The scale matching relation between the two theories is

\[
\Lambda_m^{3(N_f - N_c) - N_f} \left( \frac{\mu^2}{\mu_\Phi} \right)^{N_f - k - n} = \Lambda_1^{3(N_f - N_c) - (k + n)}. 
\]  

(3.25)

The fact that the magnetic gauge theory is not asymptotically free is reflected in the power of \( \Lambda_m \) on the left hand side of (3.25) being negative. The theory with \( n + k \) flavors can be either asymptotically free or not, which is reflected in the fact that the power of \( \Lambda_1 \) on the right hand side can be positive or negative.

Consider first the case\[4\]

\[
k + n > 3(N_f - N_c)
\]  

(3.26)

\[4\] In the free magnetic phase \( N_f > 3(N_f - N_c) \) there are solutions to this constraint. For large generic \( N_f, N_c \), the number of such solutions is large as well.
in which it is not asymptotically free. Rewriting (3.23) in the form
\[
\left( \frac{\mu^2}{\Lambda_m} \right)^{N_f-k-n} = \left( \frac{\Lambda_1}{\Lambda_m} \right)^{3(N_f-N_c)-(k-n)}
\]
(3.27)
and using the fact that the left hand side is very small due to (3.23), we conclude that
\[
\Lambda_1 \gg \Lambda_m .
\]
(3.28)
Hence the $SU(N_f - N_c)$ gauge dynamics is weakly coupled both for energies larger than $\frac{\mu^2}{\mu_\phi}$, for which the number of flavors is $N_f$ and for lower energies, where it is $n + k$.

In the second step of the reduction we give an expectation value $\mu$ to $k$ flavors of quarks and go down to an $SU(N_f - N_c - k)$ gauge theory with $n$ light flavors and scale $\Lambda_l$. The scale matching relation is (compare to (3.12))
\[
\Lambda_1^{3(N_f-N_c)-(k+n)} = \mu^{2k} \Lambda_l^{3(N_f-N_c)-n} .
\]
(3.29)
In the regime (3.26) the power of $\Lambda_l$ in (3.29) is negative. Thus, the low energy $SU(N_f - N_c - k)$ gauge theory with $n$ flavors is weakly coupled below the scale $\Lambda_l$. Using the fact that $\mu \ll \Lambda_1$, one can see from (3.29) that $\Lambda_l$ is in the range
\[
\mu \ll \Lambda_l \ll \Lambda_1 .
\]
(3.30)
We see that the theory is weakly coupled at all scales. At energies much above $\mu$ it is governed by the intermediate theory with scale $\Lambda_1$, which as we saw before is weakly coupled. At the energy $\mu$ it crosses over to the low energy theory with the scale $\Lambda_l$, which due to (3.30) is weakly coupled as well. Consequently, the gauge dynamics can be neglected, as in the previous subsection, and one can proceed as there, with similar conclusions.

To summarize, we find that deformed SQCD in the regime (3.11), (3.24) in coupling space has metastable vacua of the form (3.9), (3.10), (3.19), with $k$ and $n$ satisfying the constraint (3.26). The states with $k < N_f - N_c$ further require (3.22) for their existence. Those with $k = N_f - N_c$ exist throughout this regime.

The above analysis can be repeated for the case where the constraint (3.26) is not satisfied. One finds that there are regions in parameter space in which the gauge dynamics is weakly coupled and metastable states exist. We will not describe the details here.

Finally, it is interesting to ask what happens to the states with $k = N_f - N_c$, $n < N_c$ as $\mu_\phi \to 0$. As $\mu_\phi$ decreases, at some point we leave the weak coupling regime (3.23) and the analysis presented above becomes unreliable. The limit $\mu_\phi \to 0$ involves strongly coupled dynamics. The brane construction of [31] seems to suggest that these vacua survive in the limit, and thus in massive SQCD there are additional metastable states to those studied in [7]. More work is required to resolve this issue conclusively.
4. Discussion

In this paper we discussed supersymmetric QCD in the presence of the superpotential \ref{2.3}. We described the supersymmetric vacua of the theory, and generalized the discussion of metastable supersymmetry breaking states in \cite{7} to this case. We saw that by tuning the parameters of the theory one can make the non-supersymmetric states arbitrarily long-lived. In other regions of parameter space, which can be studied using weakly coupled field theory, many of these states become unstable and disappear.

The mechanism for the emergence of metastable states in the theory with $\alpha \neq 0$ \ref{2.3} is slightly different than that in the theory with $\alpha = 0$ studied in \cite{7}. There, the classical theory spontaneously broke supersymmetry and had a pseudo-moduli space of non-supersymmetric states. One loop effects lifted the pseudo-moduli space and replaced it by an isolated metastable state. In our case, classically there are supersymmetric ground states and no non-supersymmetric metastable states. The latter are due to a competition between the classical and one loop contributions to the potential. The fact that one can ignore higher loop contributions to the potential in studying these states is analogous to what happens in the $\epsilon$ expansion in quantum field theory.

The theory with quartic superpotential in \ref{2.3} requires a UV completion. One possibility is to introduce the singlet meson $N$, and rewrite it as \ref{2.6}, which is renormalizable. Alternatively, one can proceed as follows. Consider first SQCD with vanishing superpotential and $N_f < 3N_c$. Due to asymptotic freedom, it approaches a free fixed point in the UV, while at long distances it is governed by a non-trivial fixed point, at which the scaling dimension of the meson operator \ref{1.3} is smaller than its free field value. For $N_f < 2N_c$, the IR dimension is sufficiently small that the quartic perturbation in \ref{2.3} becomes a relevant perturbation of this fixed point. Thus, in this regime the theory with superpotential \ref{2.3} is well defined in the UV. Our analysis of metastable states was performed (following \cite{7}) in the region $N_f < \frac{3}{2}N_c$, in which the magnetic theory is infrared free and the electric theory is UV complete.

A natural generalization of the problem studied here is to higher order superpotentials of the form \ref{1.2}. The supersymmetric ground states can be analyzed as in section 2. For example, in the mesonic branch, in which the matrix $M$ is non-degenerate, the F-term equation \ref{2.36} takes in the general case the form

$$
\sum_{n=1}^{n_0} \frac{1}{(n-1)!} m_n \text{Tr} M^n = \left( \frac{\det M}{\Lambda_c^{3N_c-N_f}} \right)^{\frac{1}{N_f-N_c}} I_{N_f} . \quad (4.1)
$$
Since the left hand side is a polynomial of degree $n_0$, the matrix $M$ has at most $n_0$ distinct eigenvalues, $(x_1, x_2, \cdots, x_{n_0})$. In a general vacuum, it takes the form

$$M = \text{diag}(x_1^{l_1}, x_2^{l_2}, \cdots, x_{n_0}^{l_{n_0}})$$

(4.2)

with $l_j = 0, 1, \cdots, N_f$ and $\sum_j l_j = N_f$. It seems clear that for large $N$ the number of vacua grows like $N^{n_0}$, where $N$ denotes collectively $N_f$, $N_c$ and any linear combinations of the two. The growth of metastable states with $N$ is even faster since we can follow the construction of section 3 in each block of $M$ (1.2) separately.

From the perspective of effective field theory it is natural to consider superpotentials with $n_0$ of order $N_f$, $N_c$. The number of stable and metastable vacua grows in this case at least as fast as $N!$. To ensure that these vacua are all long-lived one needs to fine tune the couplings $m_n$ in (1.1) accordingly. Conversely, if the couplings are generic, one expects a distribution of lifetimes, so that at least some of the non-supersymmetric vacua are long-lived.

An interesting generalization of the analysis of this paper is to a system in which the $SU(N_f)_{\text{diag}}$ symmetry of (1.2) is gauged. This leads to adjoint SQCD, which was studied using a generalization of Seiberg duality in [35-37]. One can use these results as well as those of this paper to analyze the metastable states in this system.

As mentioned in the introduction, SQCD with the superpotential (2.3) has a simple embedding in string theory, as a low energy theory on a system of intersecting D-branes and NS5-branes. In a companion paper [31] we describe this embedding and show that much of the structure found in the gauge theory description in this paper is nicely realized in the brane system.

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5 E.g. for applications to phenomenology.
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