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THE PROPERTIES OF HYPERVELOCITY STARS AND S-STARS ORIGINATING FROM AN ECCENTRIC DISK AROUND A SUPERMASSIVE BLACK HOLE

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ABSTRACT

Hypervelocity stars (HVSs), which are observed in the Galactic halo, are believed to be accelerated to large velocities by a process of tidal disruption of binary stars passing close to the supermassive black hole (SMBH) which resides in the center of the Galaxy. It is, however, still unclear where these relatively young stars were born and what dynamical process pushed them to nearly radial orbits around the SMBH. In this paper we investigate the possibility that the young binaries originated from a thin eccentric disk, similar to the one currently observed in the Galactic center. By means of direct N-body simulations, we follow the dynamical evolution of an initially thin and eccentric disk of stars with a 100% binary fraction orbiting around the SMBH. Such a configuration leads to Kozai–Lidov oscillations of orbital elements, bringing a considerable number of binaries to the close vicinity of the black hole. Subsequent tidal disruption of these binaries accelerates one of their components to velocities well above the escape velocity from the SMBH, while the second component becomes tightly bound to the SMBH. We describe the main kinematic properties of the escaping and tightly bound stars within our model, and compare them qualitatively to the properties of the observed HVSs and S-stars, respectively. The most prominent feature is strong anisotropy in the directions of the escaping stars, which is observed for Galactic HVSs but has not yet been explained.

Key words: black hole physics – Galaxy: halo – Galaxy: nucleus – methods: numerical – stars: early-type – stars: kinematics and dynamics

1. INTRODUCTION

Hypervelocity stars (HVSs) are observed in the Galactic halo at distances \( \gtrsim 50 \) kpc from the Galactic center, moving at high velocities \( \gtrsim 300 \) km s\(^{-1}\), i.e., they are not gravitationally bound to the Galaxy. To date, about twenty such stars have been found (Brown et al. 2014), all of spectral type B. The specific location and spectral type of the observed HVSs are, however, limited by the observations themselves. First, in the Galactic halo, the number density of indigenous stars is low, making the survey of HVSs efficient. Second, B-type stars are luminous enough to represent convenient targets at such large distances, and these stars are long-lived enough to be able to travel from the Galaxy to the halo. It is likely that future observations will reveal a number of less luminous HVSs (see Vickers et al. 2015 for one of the first lists of low-mass HVS candidates).

S-stars lie, in a certain sense, on the opposite side of the fiducial Galactic scale from the HVSs—they are the most tightly bound stars to the supermassive black hole (SMBH) residing in the center of the Galaxy (e.g., Ghez et al. 2005; Gillessen et al. 2009; Meyer et al. 2012). The semimajor axes of their orbits range from \( \approx 0.004 \) to \( \approx 0.04 \) pc. S-stars are also classified as main sequence B-type stars and, similarly to HVSs, there are observational limitations which do not allow less luminous main sequence stars to be detected in that region.

It was suggested by Hills (1988), before the first observations of HVSs or S-stars, that both groups (considering their kinematic state, not necessarily their spectral type) should exist as a consequence of tidal break-ups of binary stars in the vicinity of the SMBH. Subsequent works by other authors, particularly after the discovery of the S-stars and the HVSs, have tried to answer the questions of where these binary stars were born and which process drove them into orbits plunging below the tidal break-up radius. Some works have suggested that the binaries originated at distances well above 1 pc from the SMBH and were transported inwards, e.g., within a massive young star cluster (Gould & Quillen 2003) or scattered due to relaxation enhanced by massive perturbers (Perets et al. 2007).

Another possible source of binaries appears to be quite straightforwardly motivated by presence of another group of young stars in the Galactic center, which are observed at radial distances \( 0.04 \) pc \( \lesssim r \lesssim 0.4 \) pc from the SMBH (e.g., Paumard et al. 2006; Bartko et al. 2009, 2010; Do et al. 2013). A subset of these stars apparently orbit around the SMBH in a thin disk (the so-called clockwise disk (CWS); e.g., Levin & Beloborodov 2003). Their binary fraction is currently not well constrained (e.g., Pfuhl et al. 2014) either from observational data or from models of star formation in the vicinity of the SMBH. In spite of the fact that stars from the young stellar disk have moderate values of orbital eccentricities, their pericenters are several orders of magnitude above the tidal break-up radius of binaries that can survive in such a dense environment. Hence, some mechanism has to come into play to excite the orbital eccentricities in order to transport the binaries sufficiently close to the SMBH.

Löckmann et al. (2008) suggested that the mechanism sought may be the Kozai–Lidov resonance, induced by an external, flattened, and axially symmetric source of gravity. In their particular setup this was a second, highly inclined stellar disk which was assumed to be formed at the same time in the Galactic center. Later Madigan et al. (2009), who studied the dynamics of an eccentric disk of stars around a SMBH, noticed that a substantial fraction of stellar orbits undergoes extreme oscillations of their eccentricities due to disk self-gravity. Madigan et al. (2009) interpreted these oscillations as a manifestation of the so-called eccentric instability and claimed that their occurrence relies on the presence of a massive
spherical cluster centered on the SMBH. In Haas & Šubr (2016) we have shown, however, that these oscillations may be interpreted as an imprint of Kozai–Lidov dynamics in the perturbing gravitational field of the stellar disk itself, which does not require presence of the spherical cluster.¹

Both Löckmann et al. (2008) and Madigan et al. (2009) suggested that the oscillations of orbital eccentricities which they observed in their models could contribute to the formation of the HVSs and S-stars. Note, however, that in both works only orbits of single stars were integrated in the full N-body setup. Conclusions about the HVSs and the S-stars formed through the Hills mechanism were based on the numbers of single stars reaching such a vicinity of the SMBH, where a binary would tidally break up. Another class of numerical models of the HVSs and the S-stars is based on integrations of individual binaries injected toward the SMBH with arbitrarily given impact parameters (e.g., Kenyon et al. 2008; Antonini et al. 2010; Zhang et al. 2013). Such experiments allow the evaluation of the statistical properties of the captured and ejected stars. However, the weak point of the models lies in the poorly constrained input parameters of the orbits around the SMBH prior to the tidal break-up.

In this paper, we make another step forward by including stellar binaries in N-body models of stellar disks around a SMBH. For the first time, this setup allows us to make predictions about the properties of the HVSs and the S-stars originating from a young stellar disk in a self-consistent way. Similarly to Madigan et al. (2009) and Haas & Šubr (2016), we consider the parent disk of the young stars to be formed by aligned eccentric orbits, i.e., the disk itself is capable of pushing some of its members to extremely oscillating orbits.

1.1. The Kozai–Lidov Mechanism

The key process that we assume drives stellar orbits to highly eccentric states is the Kozai–Lidov resonance. This resonance belongs to a class of nearly Keplerian motions around a dominating central mass which is perturbed by some source of gravity. In the case of the classical Kozai–Lidov mechanism (Kozai 1962; Lidov 1962) the perturbing gravitational potential is assumed to be axially symmetric, which implies a conservation of the projection of the angular momentum of the orbiting body onto the symmetry axis (also called the Kozai–Lidov integral). As the remaining components of the angular momentum vector may change, it is possible for the orbit to undergo oscillations from highly eccentric with a low inclination with respect to the plane of the perturbation to low eccentricity and high inclination. The classical Kozai–Lidov oscillations were considered by Löckmann et al. (2008) who assumed that two mutually highly inclined disks influence each other.

When the axial symmetry of the perturbing potential is lost, none of the components of the angular momentum remains an integral of motion. The specific class of sources leading to such a perturbing potential relevant for celestial mechanics—a body on an elliptical orbit around the central mass—has been studied in the literature (e.g., Ford et al. 2000; Katz et al. 2011; Lithwick & Naoz 2011; Li et al. 2014). If the deviation from the axial symmetry is not strong, the component of the angular momentum vector perpendicular to the plane of the orbit of the perturbing body slowly changes, which leads to modulations of the classical Kozai–Lidov oscillations. The eccentricity of the perturbing body motivates us, as it has previous authors, to call this effect the eccentric Kozai–Lidov mechanism. In our specific setup, with similar orientations of the stellar orbits, the most important variant is the coplanar eccentric Kozai–Lidov mechanism (Li et al. 2014).

2. MODEL AND METHODS

2.1. Model of the Galactic Center

In order to study the role of the Kozai–Lidov mechanism in the production of the HVSs and S-stars, we introduce a model of mutually gravitationally interacting point masses (particles). As there is no explicit characteristic physical scale in the equations of motion of point masses interacting according to Newton’s law of gravity, the model may be arbitrarily rescaled provided the relation $t \equiv r/a G M$ is fulfilled, where $t$, $r$, and $M$ represent the time, length, and mass unit, respectively, and $G$ stands for the gravitational constant. While in the numerical realization it is convenient to set $t = r = G M = 1$, below we describe our model scaled to physical units to make it more accessible. In particular, we consider values motivated by the Galactic center: $M_\odot = 4 \times 10^6 M_\odot$ and $r_0 = 0.004$ pc, which implies $t_a = 1.89$ yr. We numerically integrated two different initial settings, labeled A and B:

(i) The SMBH particle of mass $M_s = 4 \times 10^6 M_\odot$ is initially at rest at the origin of the coordinate system

(ii) A 2000 stars are drawn randomly from a power-law distribution function $n_{M_s} \propto M_s^{-1.5}$ lower and upper boundaries $1 M_\odot$ and $150 M_\odot$, respectively, which implies the total mass of the disk $M_d \approx 24500 M_\odot$. This distribution function is motivated by a recent analysis of the properties of young stars in the Galactic center by Lu et al. (2013).

(ii) The 2000 stars have an equal mass of $M_s = 12.25 M_\odot$, i.e., $M_d = 24500 M_\odot$ as in model A.

(iii) All stars are paired to binaries with a preference for a mass ratio close to unity. This is motivated by the fact that massive stars in the Galactic field tend to have massive companions. Although this is not true for low-mass stars, we used the close to equal-mass pairing for the sake of simplicity. Binary semimajor axes are drawn from the Opik distribution function (Kobulnicky & Fryer 2007), $n_{a_b} \propto a_b^{-1}$ with $a_b \in (0.1, 100)$ au; their initial eccentricity is set to zero and their orbital angular momentum parallel to the angular momentum of the disk.

(iii) All stars are paired to binaries with initial semimajor axis $a_b = 10$ au, zero eccentricity, and orbital angular momentum parallel to the angular momentum of the disk.

(iv) Binaries are placed on elliptic orbits around the SMBH with semimajor axes following a power-law distribution function $n_a \propto a^{-1}$, $a_{in} \equiv 0.04$ pc $\leq a \leq a_{out} \equiv 0.4$ pc. Inclinations with respect to some reference plane are drawn from a distribution function $n_i \propto \sin i$, $i \in (0, 2\pi)$. Motivated by the work of Mapelli et al. (2012), who modeled star formation in the vicinity of the SMBH from an infalling gas cloud, we introduce a non-zero

¹ For further discussion of the eccentric instability see J. Haas & L. Šubr (2016, in preparation).
radial gradient of eccentricities, namely, we initialize them according to $e = 0.9(a - a_{in})/(a_{out} - a_{in})$. Furthermore, in accordance with their results, we construct the orbits around the SMBH such that their eccentricity vectors point to a common direction, i.e., the ellipses are mutually aligned, not randomly oriented.

If not specified otherwise, the results presented below correspond to model A.

Let us note here that the models presented above do not involve another important component of the Galactic center—the spherical cluster of old stars. This is due to the numerical complications (stability and CPU time consumption) which would be introduced by adding this cluster into the full N-body setup. By means of additional simple models without binaries, however, we discuss the impact of the spherical cluster on the results of the main models in Section 4.

2.2. Numerical Integrator

We used the NBODY6 code (Aarseth 2003) for the numerical integration of the equations of motion. We added to the original code routines for logging the beginnings and endings of regularizations into a binary file. We further introduced an identification index for the SMBH particle and stored the minimum distance of all other particles to the one representing the SMBH reached during the integration. Finally, we altered the decision making algorithm for adding particles to neighbor lists. Specifically, we weight the standard distance criterion by the mass of the given particle so that the more massive particles are added to the list even when they are at larger distances than the lighter ones. The most prominent target was the SMBH particle which, due to this modification, a member of the neighbor lists of all other (star) particles. The modification of the neighbor list influences the integration and increases its stability. Among many runtime options of the NBODY6 code, let us specifically mention that we switched off the internal evolution of the stars as well as the post-Newtonian corrections to the stellar dynamics.

2.3. Escapers and HVSs

The analysis of the escaping stars in our models is performed as follows. First, we identify all stars that reach a distance of 8 pc from the SMBH with a velocity exceeding the escape velocity from the SMBH at that radius ($\approx 66$ km s$^{-1}$). For each of these stars we search the list of regularization events for the last event, including its ID. This event provides us with information about the ejection time, $t_{ej}$, and radius, $r_{ej}$. Finally, we scan the event list file for all events in time interval $(t_{ej} - \Delta t, t_{ej})$ involving the respective star. We count the number of unique stellar IDs that occur in these events in order to determine the type of ejection mechanism. The time interval $\Delta t$ is taken to be the greater of the orbital periods around the SMBH with a semimajor axis equal to $r_{ej}$ and $1/r_{in}$. We explicitly distinguish the ejection process involving just two stars—a tidal disruption of a binary—which we call the Hills mechanism throughout the rest of the paper. The other ejection cases which involve more than two stars are called multi-body ejections. The method of determination of the ejection event as well as the number of stars involved depend on the details of the regularization techniques implemented in the NBODY6 code, which were not designed specifically for this purpose. Therefore, there is a small fraction (of the order of one per cent) of the escaping stars whose $r_{ej}$, $t_{ej}$, and mechanism of ejection may be misdetected.

Not all of the stars which reach the distance of 8 pc with a velocity larger than the escape velocity from the SMBH will also reach the Galactic halo. In order to present results suitable for comparison to the observed HVSs, we perform a reduction of velocities which accounts for the loss of kinetic energy in the potential of the Galaxy. Specifically, we consider a relatively simple compound spherical potential of the Galaxy and the SMBH (e.g., Kenyon et al. 2008),

$$\Phi(r) = \Phi_{G}(r) + \Phi_{M}(r) = 2\pi G C r_{e}^{2} \left[ \frac{2r_{e}}{r} \arctan \frac{r}{r_{e}} + \ln \left( 1 + \frac{r^{2}}{r_{e}^{2}} \right) \right] - \frac{G M_{*}}{r},$$

where $C$ and $r_{e}$ are free parameters of the model for which we adopt values $C = 1.4 \times 10^{4}$ pc km s$^{-1}$ and $r_{e} = 8$ pc. The specific kinetic energy of the stars then decreases by a value of $\Delta E_{k} \approx 4 \times 10^{6}$ km s$^{-2}$ when traveling from 8 pc to 50 kpc, i.e., from the radius where the Galactic potential starts to dominate to the typical galactocentric distance of the observed HVSs. In the following, we thus distinguish all the stars escaping from the SMBH at the distance of 8 pc (escapers for short) and the HVSs, which are the subset of escapers capable of reaching the galactocentric distance of 50 kpc (i.e., having velocities greater than 894 km s$^{-1}$ at 8 pc).

2.4. S-Stars

Formally, we define the S-stars in our model to be the former companions of the escaping stars originating from the Hills mechanism which stay bound to the SMBH. We will show below that an overwhelming majority of these stars are located below the inner edge of the parent stellar disk, i.e., at the places where the observed S-stars are found in the Galactic center. The orbital elements of the S-stars are evaluated in the rest frame of the SMBH particle.

3. RESULTS

3.1. Properties of the HVSs

We have integrated 150 realizations of model A up to $t = 4 \times 10^{6} t_{\odot}$, which corresponds to $t \approx 7.6$ Myr for scaling $r_{o} = 0.004$ pc and $M_{o} = 4 \times 10^{6}$ M$_{\odot}$. On average, we found 34 escaping stars per realization out of which approximately 5/6 originate from the Hills mechanism, while the remaining 1/6 accounts for other modes of ejection—either a strong interaction of a single star with a binary, or binary–binary scattering. In some rare cases, the event list does not contain any information about regularized interactions with other stars, which may in reality correspond to any of the above mentioned mechanisms, or even a close hyperbolic interaction of two single stars in the tidal field of the SMBH. We excluded these stars from further considerations. After the velocity correction to the Galactic potential, we obtain on average $\approx 12$ HVSs per realization.

Figure 1 shows the distributions of the absolute values of the velocities of all escaping stars at 8 pc and of their subset ejected via the Hills mechanism within model A (thick and thin solid lines, respectively). Comparison of the two distributions clearly shows that the Hills mechanism dominates the ejection process.
for $v \gtrsim 500$ km s$^{-1}$ at $r = 8$ pc. Consequently, the potentially observable HVSs at $r = 50$ kpc practically exclusively originate from the Hills mechanism. The dashed lines in Figure 1 represent the expected velocity distributions of the HVSs at $r = 50$ kpc, i.e., distributions of velocities of the HVSs reduced by motion in the Galactic potential (1).

Model A appears to give a velocity distribution that is considerably broader and shifted to higher velocities with respect to the observed HVSs (see Figure 2). Figure 2, however, also shows that the velocity distribution of the HVSs strongly depends on the initial properties of the stellar disk. Model B, which starts with all binaries initially relatively weakly bound ($a_b = 10$ au), gives considerably fewer HVSs with velocities exceeding 1000 km s$^{-1}$ at 50 kpc. This can be quite naturally explained by the fact that the more tightly bound binaries from model A become tidally disrupted closer to the SMBH, passing it with a larger velocity which is then inherited by the escaping star. Let us further note that the distribution function of the binary semimajor axes in model B evolve (broaden) quite rapidly due to two-body relaxation. The typical semimajor axis of tidally broken-up binaries which led to HVSs in model B is about 1 au.

Figure 2 also demonstrates that the distribution of velocities of the HVSs is sensitive to the shape of the Galactic potential. The two distributions plotted for model B (dashed lines) differ by the amount of reduced kinetic energy—the thick line corresponds to $\Delta E_K = 4 \times 10^5$ (km s$^{-1}$)$^2$, according to formula (1), while the thin line corresponds to $\Delta E_K = 5 \times 10^5$ (km s$^{-1}$)$^2$. Note that the simple model of Galactic potential (1) may have a similar level of inaccuracy. For example, while its parameters are such that it reasonably well fits the mass of the Galactic nuclear star cluster within the distance of $\approx 8$ pc from the SMBH, it very likely does not reproduce well the density distribution in that region and, consequently, it underestimates the depth of the Galactic potential. The way in which the reduction of kinetic energy translates to changes in velocities directly implies that stars with lower velocities are affected more, as can also be inferred from Figure 2.

Considering the large uncertainty in the initial properties of the stellar disk, together with the uncertainty of the depth of the Galactic potential, we are not able to provide a unique prediction of the velocity spectrum of the HVSs which may be produced by the mechanism described in this paper. Some level of uncertainty also has to be considered on the observational side when comparing models to observational data.

In Figure 3, we plot the distribution function of the ejection times of the escapers within model A. Again, we distinguish the distributions of all escapers and the HVSs. We see an initial peak at $t = 0$, which is particularly apparent for the overall distribution, i.e., it is dominated by the low-velocity escapers. These originate from those primordial binaries that are immediately broken up in the tidal field of the SMBH. More interesting is the relatively narrow peak of ejection of the HVSs, which rises at $t \approx 1$ Myr and slowly decays during the subsequent few million years. This feature is a consequence of the coplanar eccentric Kozai–Lidov mechanism, which is
responsible for the production of the majority of the HVSs in our setup. Our results show that most of the oscillating orbits are located close to the inner edge of the disk, thus having similar characteristic timescales of their eccentric Kozai–Lidov cycles. Hence, a large fraction of the affected binaries reach high eccentricity at a common time. A typical representative of a tidally disrupted binary is shown in Figure 4. Initially, the lines represent orbital elements of the binary barycenter. At $t = 1.15$ Myr, the eccentricity of its orbit around the SMBH reaches a value $e > 0.99$ and the binary breaks up. From that time on, the lines in Figure 4 represent orbital elements of the component which remains bound to the SMBH, while the other one escapes. The oscillations of orbital elements of the bound star show a pattern typical for coplanar eccentric Kozai–Lidov cycles (Li et al. 2014; see also Haas & Šubr 2016) with flips from co-rotation to counter-rotation, and vice versa. As time proceeds, the pattern of oscillations changes, which is likely due to evolution of the source of the perturbing potential (the stellar disk) in the N-body environment. The semimajor axis of the bound star just after the tidal break-up is $a \approx 0.015$ pc, which is approximately one half of that of the binary before the event. The escaping star becomes unbound to the SMBH, having a velocity of $\approx 140$ km s$^{-1}$ at a distance of 8 pc from the SMBH, i.e., this particular star does not contribute to the population of HVSs.

Figure 5 demonstrates that tidal break-ups of binaries within model A occur in a relatively wide range of radii. This is caused by the initially wide range of (intrinsic) binary semimajor axes considered in our model, which determine how deep into the potential well of the SMBH the binary can penetrate before being tidally broken up. We also see that HVSs practically exclusively originate from radii $\lesssim 0.0005$ pc.

Figure 6 shows the positions of the escaping stars on the sphere in the sinusoidal projection. There is a clear anisotropy in the distribution, with the majority of stars escaping along the plane of the stellar disk. This is a natural consequence of the coplanar eccentric Kozai–Lidov mechanism, which is the main process that brings the binaries from the disk to extremely eccentric orbits in our models. In spite of the fact that the orbits change their inclination during the secular evolution, this only happens hand-in-hand with evolution of eccentricities. In particular, in the high-inclination phase ($i \approx 90^\circ$), the eccentricity is so large and the argument of pericenter is close to 0 or $\pi$ that the orbit is embedded in the disk and the velocity vector of the star is nearly parallel with the disk plane for the major part of the orbit. It is then natural to expect that the unbound component of a tidally broken-up binary follows a trajectory nearly parallel to the stellar disk. In addition to the strong tendency of escaping at low latitudes with respect to the disk, we also observe a significant clustering of the ejected stars around a certain value of the azimuthal angle in the plane of the disk. We attribute this to the nature of the eccentric Kozai–Lidov mechanism, which relies on specific mutual orientation of the orbit and the eccentric perturbation of the central potential. Although current observations are incomplete in sky-coverage, they indicate a statistically significant anisotropy of the distribution of HVSs on the sky (Brown et al. 2014), which is in qualitative agreement with the outcome of our model.

The currently observed HVSs are exclusively late B-type stars (e.g., Brown et al. 2014), which is, nevertheless, just a selection effect and the mass spectrum of the HVSs has to be considered unknown at this time. The numerical model,
however, enables us to predict the mass spectrum of the HVSs. Figure 7 shows the mass function of the escaping stars produced during the whole integration of model A. We see that it somewhat deviates from the initial mass function, in particular, it is flatter at the low-mass end. This feature is more prominent for the HVSs, i.e., it is primarily these stars that are responsible for the deviation from the initial mass function. We see two possible reasons for such a flattening of the mass function which are not mutually exclusive. First, more massive binaries have on average a larger binding energy than lighter ones, i.e., they are able to sink deeper into the potential well of the SMBH before tidally breaking up. Consequently, they achieve higher ejection velocities, i.e., they have a larger probability of being detected as HVSs. A second, and probably more important, reason is that it is preferably the more massive stars (binaries) which sink toward the SMBH within the parent disk due to two-body relaxation. In spite of the fact that this process is not as pronounced as in self-gravitating systems without a dominating central mass, our simulations show that the mass function at the inner edge of the disk already becomes somewhat flatter at $t \approx 1$ Myr. As the eccentric Kozai–Lidov cycles have a shorter period at smaller radii in our settings, they preferentially push binaries from this region to highly eccentric orbits on which they can tidally break up.

### 3.2. S-stars

Let us now analyze the properties of the S-stars in our model as they were defined in Section 2.4. On average we obtain $\approx 28$ S-stars per realization of model A up to $t = 7.6$ Myr. In the subsequent paragraphs, we will also qualitatively discuss the comparison of the properties of the S-stars in our model with those of S-stars observed in the Galactic center. Note, however, that such a comparison has to be handled with caution for two primary reasons. First, the number of observed S-stars is quite small ($\approx 20$), which makes any statistical analysis only marginally reliable. Second, we have a strict definition of S-stars in this paper based on their dynamical history. Such a definition cannot be applied to the stars observed in the Galactic center. In fact, to our knowledge there is no widely accepted definition of the observed S-stars. Therefore, it is very difficult to compare the outcomes of numerical models with the observational data.

Figure 8 shows the main statistical properties of the captured S-stars. The left panel presents the distribution of their inclinations within model A. We see two dominant peaks around $i = 0$ and $\pi$, i.e., co-rotating and counter-rotating with respect to the parent disk. Similarly to the distribution of the velocity vectors of the ejected stars, the high anisotropy of the orientations of the orbits of the S-stars is a consequence of the Kozai–Lidov mechanism. Despite the vague definition of the observed S-stars, it is quite safe to state that the young stars observed within the projected distance of $\approx 1'' \approx 0.04$ pc from the SMBH have relatively randomly oriented orbital planes (at least in comparison to the coherent orientations of orbits of the stars forming the young stellar disk above this radius). Hence, there has to be some process which randomizes the orbital orientations in order to smear out the high anisotropy introduced by the process of formation discussed in this paper. Indeed, it has been suggested by various authors that the orientations of the orbits of the S-stars may have undergone considerable evolution due to stellar dynamics. In particular, resonant relaxation processes (Rauch & Tremaine 1996) within the nuclear star cluster were discussed, e.g., by Hopman & Alexander (2006). Another process capable of changing the orientations of the orbits is Kozai–Lidov oscillations due to a secondary, arbitrarily inclined stellar disk (Lockmann et al. 2008). A certain combination of the two processes was considered by Chen & Amaro-Seoane (2014). Our model does not allow us to verify the resonant relaxation of the orbital parameters of the S-stars as it does not include the spherical component of the nuclear star cluster for numerical reasons. We tried to overcome this limitation by means of a follow-up model. It consists of the S-stars whose initial kinematic state was taken from the main model A at $t = 2 \times 10^6 t_0 \approx 3.8$ Myr, regardless of the time of their formation. The motivation for this choice of time is two-fold. On one hand, at $t \approx 3.8$ Myr, on average a considerable number of S-stars ($\approx 21$) are already formed in the main model. On the other hand, we wish to test whether the orbits of a substantial fraction of the S-stars can be randomized within a few millions of years, which is the estimated life-time of the currently observed young stellar disk in the Galactic center. The S-stars were embedded in a spherical cluster modeled by 500 stars of equal mass (1 $M_\odot$) with random orientations of their orbits, a thermal distribution of eccentricities, and a distribution of the semimajor axes $a \propto a^{1/2}$ in (0.002, 0.02 pc), i.e., within the domain of the S-stars. The distributions of the orbital elements of the S-stars after 3.8 Myr of the follow-up dynamical evolution are plotted in Figure 8 with dashed lines. We observe a substantial evolution of the orientations of the orbits, which tend to become randomized. Hence, we may state that our model of production of the S-stars via the Hills mechanism is compatible with the orientations of the observed S-stars.

The distribution of eccentricities of the S-stars in our model A appears to be close to the thermal one, $n_\varepsilon \propto e$ (Figure 8, middle panel) and it does not evolve considerably in time when embedded in the spherical cluster. The distribution is roughly compatible with that of early-type stars with determined orbital elements observed in the Galactic center (thick dashed–dotted line, data taken from Gillessen et al. 2009). The situation
changes if only stars with a semimajor axis smaller than 1″ are considered (thin dashed–dotted line in Figure 8). Gillessen et al. (2009) reported a super-thermal eccentricity distribution \( n_e \propto e^{2.6 \pm 0.9} \) for this subset of stars in the Galactic center, which is steeper than the slope we obtained within the considered models A and B. The apparent differences between the outcomes of our two models suggest, however, that further variations of the initial setup may lead to an even more super-thermal distribution of eccentricities. Furthermore, in order to obtain more realistic results, the numerical model would have to include all resonant and relaxational processes for the whole integration time. Finally, the comparison with the observational data needs to account for the problem of the definition of S-stars mentioned above.

In showing the distribution of the eccentricities of the captured stars, let us mention that works by other authors (e.g., Perets et al. 2009; Madigan et al. 2011; Antonini & Merritt 2013) discussing the origin of S-stars often expect their eccentricities coming from the Hills mechanism to be exclusively greater than 0.9. Our results indicate that this assumption may not be valid in the case when the orbit around the SMBH evolves secularly, i.e., the binary becomes gradually more perturbed when passing closer and closer to the SMBH during subsequent revolutions around it. Let us also remark that Hills (1991) obtained extreme values of eccentricities of captured stars only for highly radial orbits, while larger impact parameters led to mean values of eccentricity considerably smaller than 0.9.

The S-stars produced by the Hills mechanism in model A occupy a rather wide range of semimajor axes: 0.001 pc \( \lesssim a \lesssim 0.01 \) pc (see the right panel of Figure 8). This is in agreement with the observational data, which currently provide us with the smallest semimajor axis of \( \approx 0.004 \) pc (S0-102; Meyer et al. 2012) while the outermost S-stars are located close to the inner edge of the young stellar disk (\( \approx 0.04 \) pc). Comparison of the semimajor axis distribution function of all stars and that of the S-stars in the right panel of Figure 8 indicates that the region below 0.01 pc is exclusively occupied by the S-stars in our model, which justifies their formal definition in Section 2.4. As expected, the distribution of the semimajor axes did not change as their two-body relaxation time is several orders of magnitude longer than the integration time.

Finally, we also evaluated the mass function of the captured S-stars in model A. We found it to be similar to that of their former companions, i.e., the escaping stars ejected via the Hills mechanism (see Figure 7 and its description in the previous section). This result is not surprising for the initial setup of our model in which the pairing of stars into binaries was biased toward equal masses.

### 3.3. Disk Structure Evolution

On a timescale of the order of ten million years, both secular and relaxational processes influence the structure of the parent stellar disk. Due to the relatively coherent motions of the stars and, therefore, the small initial velocity dispersion in the disk, two-body relaxation is capable of altering its radial density profile, making it flatter and broadening its extent both below and above its initial inner and outer edges, respectively (see the right panel of Figure 8). Except for the population of S-stars, which were transported below 0.01 pc due to the Hills mechanism, the radial density profile does not differ significantly from that of models consisting of only single stars presented in our previous works (Šubr & Haas 2014; Haas & Šubr 2016).

Figure 9 shows that the two-body relaxation is also mainly responsible for heating up the disk during the initial phase of evolution (\( \lesssim 1 \) Myr) which manifests itself by a power-law growth of the root mean square inclination of the orbits, \( \langle i_{\text{rms}} \rangle \propto t^{1/4} \) (e.g., Stewart & Ida 2000). At \( t \approx 1 \) Myr,
accelerated growth of $i_{\text{rms}}$ is observed, which is a consequence of relatively coherent Kozai–Lidov oscillations of a subset of orbits which flip to counter-rotation, thus pushing $i_{\text{rms}}$ to high values. The evolution of $i_{\text{rms}}$ is qualitatively the same as that of model M8 from Haas & Šubr (2016) which has initial global characteristics of the disk (distribution of orbital elements and total mass) identical to the current model A. We attribute the offset of the two models at $t \lesssim 1$ Myr to faster relaxation of model A which, unlike the former model M8, has a broad mass spectrum. Another additional source of heating of the disk in model A could be the binaries, which are not present in model M8. Recent semi-analytical estimates of Mikhaloff & Perets (2016) show, however, that binaries contribute only marginally to heating of Keplerian stellar disks around SMBHs in comparison to the process of two-body relaxation. Another difference between the evolution of $i_{\text{rms}}$ of the two models can be observed at $t \gtrsim 1$ Myr. Model M8 exhibits damped oscillations of $i_{\text{rms}}$, which are a consequence of the longer lasting coherence of the Kozai–Lidov oscillations of individual orbits. There are at least two reasons why this feature is not observed in model A. First, tidal disruptions of primordial binaries lead to abrupt changes of orbital elements which affect the Kozai–Lidov cycles. Second, as argued above, model A undergoes faster two-body relaxation due to the mass spectrum, which gradually demotes the eccentric perturbation. Both these arguments can also be used to explain the higher value of $i_{\text{rms}}$ of model A at $t \gtrsim 3$ Myr, which is likely due to orbits that stay frozen in the counter-rotating phase after interruption of their Kozai–Lidov cycles.

In Haas & Šubr (2016), we already pointed out that $i_{\text{rms}}$ is not straightforwardly related to the disk geometrical thickness. The argument for this statement is basically identical to that in Section 3.1 of the current paper: undergoing the coplanar Kozai–Lidov oscillations, the orbits reach a high-inclination state with such values of eccentricity and argument of pericenter that they are still embedded in a relatively thin disk structure. The azimuthal projection of a snapshot of one realization of model A at $t = 7.6$ Myr presented in Figure 10 confirms that, except for several outliers, the stellar disk is well confined within a cone with a half-opening angle $\approx 30^\circ$.

4. DISCUSSION

We showed by means of a relatively simple model that both the S-stars and the HVSs may have a common origin in an eccentric stellar disk formed by fragmentation of a gaseous cloud falling onto the SMBH. Provided they were born as binaries, the growth of the eccentricity of their orbits around the SMBH due to the Kozai–Lidov mechanism induced by the disk itself may have brought them to the tidal break-up radius at which they broke up, leaving one component tightly bound to the SMBH with the other ejected away at a high velocity. The kinematic properties of the HVSs and the S-stars in our model qualitatively agree with the properties of observed stars. We have to keep in mind, however, that our knowledge of these properties is incomplete due to the observational limits and, therefore, a model matching such incomplete observational data may actually be wrong. Only the improved observational data that are expected to become available in the future will enable us to test our model. Until then, there is still room to improve the model to make it more realistic.

When, for example, considering the stars as finite-size objects rather than point masses, we may expect them to physically collide. It was suggested by various authors that binary stars orbiting a SMBH may merge due to the secular evolution of their (binary) eccentricity (e.g., Antonini et al. 2010, 2011; Prodan et al. 2015; Stephan et al. 2016). This event may prevent the production of the HVSs, provided the binary components merge before they reach the tidal break-up radius. Whether or not stellar collisions occur depend on a complicated mixture of poorly constrained parameters that determine the timescales of the binary Kozai–Lidov oscillations and the Kozai–Lidov cycles of the orbit around the SMBH. The fraction of binaries that merge due to oscillations of the binary eccentricities varies from a few per cent (Prodan et al. 2015) up to ~10% (Stephan et al. 2016). It is, however, not clear whether the binaries that produce HVSs in our model would be affected on a similar level. In other words, the fraction of merging binaries is likely to vary across the whole parameter space and we do not know its value in the parts that contribute to production of HVSs.

The production rate of the HVSs and the S-stars could also change when other sources of gravity capable of modifying the Kozai–Lidov dynamics are involved. One of the observed and theoretically expected constituents of the Galactic nucleus, which was neglected in our main models, is the old component of the nuclear star cluster (which consists of late-type stars and very likely also compact stellar remnants). Considering it to be spherically symmetric to the first order of approximation, it generally has a tendency to damp the Kozai–Lidov oscillations. This effect for the case of an axisymmetric perturbation causing these oscillations was discussed, e.g., in Ivanov et al. (2005), Šubr et al. (2007), and Karas & Šubr (2007); further extension to the case of an eccentric perturbation, i.e., for the higher order Kozai–Lidov resonances, can be found in Haas & Šubr (2016). In the latter paper, we demonstrated that with an increasing mass of the spherical cluster, the higher order effects of the Kozai–Lidov mechanism are damped first, while the classical (quadrupole) cycles can survive larger masses. The limiting mass of the cluster beyond which the Kozai–Lidov mechanism is unable to push the stellar orbits to extreme eccentricities depends on the mass distribution in both the cluster and the disk, and also on the eccentricities of the stellar orbits. In particular, we found that a moderate to high initial eccentricity
of the disk is a key feature, as it may lead to a global angular momentum flow through the disk which is capable of pushing a subset of orbits to higher eccentricities. Consequently, they can reach the resonant configuration which leads to extreme Kozai–Lidov oscillations. Our numerical model in the current paper is not suitable for direct testing of the influence of the stellar cluster on the production of the HVSs, as the addition of several tens of thousands of particles would require much more computational time and, at the same time, would lead to lower numerical stability. An alternative approach, which is commonly used in similar studies, namely, modeling the cluster by a smooth static potential, would require modeling the SMBH as an external potential as well. This, however, again leads to a lower numerical stability of our integrations when the binary tidal break-ups occur.

Here we make an attempt to estimate the rate of production of HVSs and the S-stars based on numerical integrations of the models without primordial binaries, which is a numerically less complicated setup. The key idea lies in the empirical fact that the majority of tidal break-up events occurred below 0.0005 pc in our model A (see Figure 5). Models without the primordial binaries can then be used to determine the fraction of stars that reach the SMBH at a distance smaller than the given value. Such a fraction can be determined for systems embedded in the spherical potential as well as for isolated systems. Specifically, we integrated several models, similar to model A introduced in Section 2.1, with the following three key differences:

(i) The SMBH is modeled as a fixed Keplerian potential.
(ii) The gravity of the spherical cluster is modeled by a fixed potential \( \Phi_c \propto r^{3/2} \) which corresponds to a mass distribution \( \rho_c(r) \propto r^{-1/2} \). It has a single free parameter, \( M_c \), which determines the mass of the cluster enclosed within a radius \( a_{\text{out}} \) (the maximum initial value of the semimajor axis of the stellar orbits in the disk).
(iii) All stars are initially single.

We let these models with \( M_c = 0, \ 1 M_\odot, \ 4 M_\odot \) and \( 10 M_\odot \) evolve for \( \approx 10 \) Myr and monitor the minimal radial distance, \( p_{\text{min}} \), of the stars from the SMBH reached during this time interval. Figure 11 shows the cumulative distribution functions of \( p_{\text{min}} \) for the above described models with different values of \( M_\odot \). We see that the spherical cluster of a mass equal to the mass of the disk leads to only a small decrease of \( N_{p_{\text{min}}} \) at \( p_{\text{min}} \approx 0.0005 \) pc with respect to the isolated system. Therefore, we also do not suppose the number of HVSs to be altered considerably in this case. For \( M_c = 4 M_\odot \), the number of stars plunging below 0.0005 pc is decreased by a factor of \( \approx 5 \) with respect to the case without the spherical cluster, and we expect the number of HVSs to be decreased by a similar factor, i.e., to \( \approx 2 \) in contrast to the \( \approx 12 \) obtained for the isolated model (see Section 3.1). Finally, the spherical cluster of mass \( M_c = 10 M_\odot \) nearly completely damps the Kozai–Lidov oscillations and we do not expect any HVSs or S-stars to be formed via the Hills mechanism within the models described in this paper in such a case (see, however, Haas & Šubr 2016 for discussion of other initial setups in which a considerable amount of oscillating orbits occurred even for \( M_c = 10 M_\odot \)). Note that the distribution of \( p_{\text{min}} \) does not provide us directly with the expected number of HVSs, as not all particles reaching the critical value would still be in the form of binaries, even in the case of a 100% initial binary fraction, and the argumentation above relies on the comparison of isolated models with and without the primordial binaries.

The total number of HVSs and S-stars formed by the Hills mechanism induced by the Kozai–Lidov oscillations of binary stars orbits around the SMBH is not the only factor that could be affected by presence of the spherical stellar cluster. The strong anisotropy of the ejected stars, as well as the temporal burst of their production described in Section 3.1 are features of the coplanar eccentric Kozai–Lidov mechanism. The increasing mass of the stellar cluster first damps this mode (Haas & Šubr 2016) and, therefore, we assume its effects on the HVS velocity directions and times of ejection to be suppressed as well. The classical (quadrupole) Kozai–Lidov oscillations are also likely to bring stellar orbits to highly eccentric states with specific orientations of the orbital planes, in this case, nearly parallel to the plane of the disk. Nevertheless, there is no reason to expect a preferred direction of the eccentric vector in this plane. In Haas & Šubr (2016) we showed that these classical oscillations occur from the very beginning of the disk evolution until the end of our integrations, thus they are assumed to suppress the dominant peak in the distribution of ejection times of the HVSs. The single-star models introduced above are not suitable for determining the properties of escaping stars. Therefore, complex models including a large fraction of primordial binaries in the disk as well as the gravity of the spherical cluster deserve to be the subject of future investigations.

There is growing evidence for tidal disruptions of stars in the vicinity of SMBHs in distant galaxies (see e.g., Saxton et al. 2012; Komossa 2015). An interesting side-effect of the tidal break-up of binaries on orbits undergoing Kozai–Lidov oscillations, is that it may prevent the individual stars from being tidally disrupted. The Kozai–Lidov oscillations change the orbital elements of orbits around the SMBH regardless of the multiplicity of the orbiting system. At the maximum of eccentricity, the pericenter may lie below the stellar tidal radius, i.e., if the system were formed by a single star, it would be disrupted. However, a binary tidally breaks up before it reaches the stellar tidal radius. Due to that, one star is ejected away, thus being saved from tidal disruption, while the other one remains parked on a tightly bound orbit whose pericenter is, however, typically well above the stellar tidal radius. The orbit of the bound star may not further undergo Kozai–Lidov oscillations, as it has abruptly changed its orbital elements.
during the binary tidal disruption event. Hence, it may also be saved from being tidally disrupted.

Finally, let us recall that the process discussed in this work led to a substantial number of stars being ejected from the stellar disk with velocities only marginally exceeding the escape velocity from the SMBH. These stars should be found in the Galactic bulge. Interestingly, several young stars at distances of the order of tens of parsecs from the SgrA* are observed. Their origin is unclear. Some of them may have been ejected from the young star clusters Arches and Quintuplet. However, it has been discussed by Habibi et al. (2014) that these two clusters cannot be the birth places of all of these stars. Let us close our discussion with a suggestion that the young stellar disk may also have contributed to this population of young stars.

5. CONCLUSIONS

We have investigated, by means of direct N-body integrations, the production of HVSs and S-stars from an eccentric stellar disk around the SMBH, through tidal break-ups of binaries brought to the vicinity of the SMBH via the coplanar eccentric Kozai–Lidov mechanism induced by the stellar disk itself. In agreement with the principle of Occam’s razor, this model of the origin of the HVSs and the S-stars observed in the Galactic halo and nucleus, respectively, relies on as small a number of constituents as possible. At the same time, however, it assumes that recurrent star formation episodes occur in the Galactic center. The currently observed HVSs required at least 50 Myr to travel from the Galactic center to the halo, i.e., they cannot originate from the young stellar disk currently observed in the Galactic center which is \( \lesssim 7 \) Myr old (e.g., Paumard et al. 2006; Lu et al. 2013).

The velocity distribution of the HVSs in our model, as well as their total number, strongly depends on the properties of the initial binary population. Unfortunately, this piece of information is unavailable for the case of the young stellar disk observed in the Galactic center, which otherwise is considered to be a template configuration. Therefore, any strong statement on the viability of our model based on observational data cannot be made at this time. The most prominent “smoking gun” of our hypothesis appears to be the strong anisotropy of the distribution of the HVSs on the sky, which is a robust feature of the model and is in agreement with the current observational data. Another characteristic feature of the HVSs in our model is a burst-like mode of their formation. This feature is neither confirmed, nor excluded for the currently observed HVSs (Brown et al. 2014). Future observations may bring stronger constraints in this point. Note, however, that (i) despite our model predicting one dominant peak of ejection, a large fraction of the HVSs are produced in a wide time range and (ii) a more sophisticated model including the spherical cluster is likely to lower the dominant peak.

Hand-in-hand with production of the HVSs, our model of tidal break-ups of binary stars originating from an eccentric disk also leads to the transportation of stars to orbits tightly bound to the SMBH within a few millions of years. These could be then observed as S-stars. The strong initial anisotropy of the normal vectors of their orbital planes is very quickly (within a few few millions of years) smeared out due to resonant relaxation within an embedding stellar cluster. Hence, unlike in the case of the HVSs, we do not observe any characteristic feature of the distribution functions of the orbital elements of the S-stars formed in our model, which could serve as a strong test bed of its relevance. Finally, let us note that the quite rapid formation of the population of S-stars from an eccentric disk implies that, according to our model, some of the currently observed S-stars may have originated from the young stellar disk currently observed in the Galactic center. On the other hand, the life-time of the observed S-stars allows them to be original companions of the HVSs currently observed in the Galactic halo, i.e., originating from some previous star formation episode. The two scenarios are not mutually exclusive. In fact, we estimate the number of S-stars originating from the currently observed young stellar disk in the Galactic center to be small, when considering damping role of the spherical component of the nuclear stellar cluster.

In addition to the properties of the HVSs and the S-stars, we also evaluated the evolution of the parent stellar disk. We found that it evolves in a similar way to the models without primordial binaries. In particular, the binaries do not contribute significantly to heating of the disk, i.e., the model of the isolated disk is not able to reproduce the spatial distribution of all the young stars in the Galactic center, a substantial fraction of which are observed well above the plane of the young stellar disk (the CWS). The hypothesis of a common origin for all the young stars observed in the Galactic center from a single parent disk may be kept viable if we assume some other perturbing source(s) of gravity to be present (see, e.g., Subr et al. 2009; Haas et al. 2011a, 2011b for discussions of influence of an outer massive gaseous torus on the evolution of the stellar disk).

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