Numerical study of energy loss rate of fast proton in
cold dense electron gas in strong magnetic field

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Abstract. Energy loss of a proton propagating through a low-temperature gas of magnetized
electrons is studied by molecular dynamics method for the case when proton velocity is much
more than electrons thermal velocity. Strong uniform magnetic field is considered when
electron Larmor radius is much smaller than distance of closest approach (Coulomb radius).
High electron density, and hence small values of the Coulomb logarithm, is considered. The
dependence of energy loss on angle between the proton velocity and the magnetic field direction
is studied. Results of calculations are compared to Derbenev and Skrinskii theoretical approach
to average friction force.

1. Introduction
The problem of determining of energy exchange rate between protons and magnetized electrons
first arose in connection with the idea of electron cooling. The idea of effective cooling of
ion beams by closely propagating electron beams was proposed by Budker in 1966. An original
approach to finding the average friction force acting on a proton by electrons in a strong magnetic
field was developed by Derbenev and Skrinskii in [1]. In this approach the friction force was
estimated in linear response approximation for the case when electrons motion in the direction
transverse to magnetic field is completely frozen.

Interest to the problem of energy exchange of magnetized charged particles has grown due
to experiments on antihydrogen, see, e.g., [2]. In these experiments antiprotons were injected
into the cloud of positrons to produce antihydrogen. The temperature of the positrons cloud
was made extremely low \( \sim 10 \) K. Such a low temperature leads to small value of the Coulomb
logarithm that arose from the divergence of Coulomb cross-section at small deflection angles.
This fact can break the validity of Derbenev and Skrinskii approach since it was derived in
approximation of high values of the Coulomb logarithm. Moreover unlike the typical experiments
on electron cooling in experiments on antihydrogen positrons are far more magnetized since the
Larmor radius is much smaller than Coulomb distance of minimal approach.

In the present paper, the molecular dynamics method is used for direct numerical calculation
of the energy loss rate for a proton propagating through a cold and dense electron gas in a
strong magnetic field. Density and temperature typical to experiments on antihydrogen were
considered. Calculation results were used to check the validity of the energy loss rate which
follows from [1] in these conditions.
2. Derbenev and Skriniskii approach

In Derbenev and Skriniskii approach [1] the average friction force exerted on a proton moving in electron gas in strong magnetic field is presented as a sum of a non-magnetized force \( F_0 \) and a magnetized one \( F_A \):

\[
F = F_0 + F_A.
\]  

(1)

The non-magnetized component corresponds to collisions which are not disturbed by magnetic field and is written in a usual way:

\[
F_0 = -\frac{4\pi n e^4}{m_e} \int L^0(u) \frac{u}{u^3} f_0(v_e) d^3 v_e,
\]  

(2)

where \( n \) is electron density, \( e \) is an elementary charge, \( m_e \) is electron mass, \( v_e \) is velocity of the electron, \( u = v - v_e \) is relative velocity of the proton and the electron, \( f_0(v_e) \) is electron velocity distribution.

The non-magnetized Coulomb logarithm \( L^0(u) \) is

\[
L^0(u) = \ln \frac{u A / \Omega}{e^2 / \left(m_e u^2 \right)},
\]  

(3)

where \( \Omega = eB / (m_e c) \) is electron Larmor frequency and \( u_A = v - v_{e||} \) is the relative velocity of the proton and electron Larmor circle (electron motion in transverse to magnetic field direction is frozen).

In [1], magnetized force component is derived in linear response theory approach based on perturbative treatment of collective plasma response. In result the force was written as one dimensional integral:

\[
F_A = -\frac{2\pi n e^4}{m_e} \frac{\partial}{\partial v} \int \left( \frac{v^2}{u^3 A} L^A(u_A) + 2 \right) f(v_{e||}) dv_{e||},
\]  

(4)

where \( v_\perp \) is the proton velocity in transverse to magnetic field direction. Magnetized Coulomb logarithm \( L^A \) is

\[
L^A(u_A) = \ln \frac{u_A / \omega_p}{e^2 / \left(m_e u^2 A \right)},
\]  

(5)

where \( \omega_p = (4\pi n e^2 / m_e)^{1/2} \) is the plasma frequency for electrons.

For the case when \( v \gg v_e \) considered in the present paper, (4) has asymptotes:

\[
F_{A||} = -v_{e||} \frac{2\pi n e^4}{m_e v^3} \left( \frac{3v^2}{v^2 A} L^A + 2 \right),
\]  

(6)

\[
F_{A\perp} = -v_{e\perp} \frac{2\pi n e^4}{m_e v^3} \left( \frac{v^2}{v^2 A} L^A \right).
\]  

(7)

In [3], the magnetized force was treated in binary collision approximation by Parkhomchuk, and different expressions for asymptotic behavior of friction force for \( v \gg v_e \) were obtained:

\[
F_{A||} = -v_{e||} \frac{2\pi n e^4}{m_e v^3} \left( \frac{2v^2}{v^2 A} L^A \right),
\]  

(8)

\[
F_{A\perp} = -v_{e\perp} \frac{2\pi n e^4}{m_e v^3} \left( \frac{v^2 - v_{e\perp}^2}{v^2} L^A \right).
\]  

(9)

To calculate the energy loss rate the following expression is used:

\[
\nu_i = \frac{2 F_i}{m_p v_i},
\]  

(10)

where \( i \) is || or \( \perp \) and \( m_p \) is mass of a proton.

In the next section we will discuss the numerical method used for direct calculations of the loss rates.
3. Molecular dynamics model

To simulate the cooling process we placed a proton and a number of electrons in a cubic molecular dynamics simulation cell and solve classical equation of motion for the particles. Unchanged Coulomb potential was used in the calculations. To deal with the Coulomb potential divergence a variable time-step leap-frog integration scheme was used. The step value was chosen so that the total energy of the system was conserved within the predefined limits (better than 1%). Positions of large time-step particles were changed during small time-step particles movements using linear force extrapolation. Lorentz force was included in the calculations according to the Boris method [4]. Continuity of the system was simulated using periodic boundary conditions. The calculations were carried out within the framework of the microcanonical ensemble.

In all the calculations initial energy of the proton was \( E_p(0)/\left(3/2k_B\right) = 150\,000 \) K, the temperature of the electrons was \( T_e = 15 \) K. Magnetic field value was \( B = 30\,000 \) Gauss, number of electrons was \( N_e = 300 \). Initial positions of the particles were chosen randomly.

As seen from the formulas in the previous section the energy loss depends on the angle of the proton velocity relative to the magnetic field direction. To study this dependence the calculations were performed for different values of axial to transverse proton velocity components ratio \( r \):

\[
r = \frac{v^2_\parallel}{v^2_\perp}.
\]

In all the calculations initial value of the proton velocity

\[
v = \sqrt{\frac{2E_p}{m_e}} = \sqrt{v^2_\parallel + v^2_\perp}
\]

was fixed, initial values of \( v_\parallel \) and \( v_\perp \) were chosen according to the given \( r \) and directions of axial and transverse velocities were chosen randomly. For every set of initial conditions 128 molecular dynamics runs were performed and time dependencies of the proton velocity components were averaged over all the runs.

In the next section we will discuss the results of our calculations.

4. Results and discussion

In figure 1, typical calculated proton velocity components dependencies on time are presented. The calculations were made for \( n = 2 \times 10^9 \) cm\(^{-3} \). From (7) and (9) it follows that for small
Figure 2. Dependence of the energy exchange rate upon the axial to transverse ratio $r$ for $n = 2 \times 10^9$ cm$^{-3}$: circles—calculated axial rate; squares—calculated transverse rate; solid lines—linear response approach (6) and (7); dashed lines—binary collisions approach (8) and (9).

Figure 3. Dependence of the energy exchange rate upon the axial to transverse ratio $r$ for $n = 2 \times 10^8$ cm$^{-3}$: points and lines—the same as in figure 2.

$v_{\perp}$ the transverse component of the friction force is directed along the corresponding velocity component. Transverse velocity growth for this case is shown in figure 1(b).

To estimate the energy loss rate the calculated time dependencies of the proton velocity components were fitted by the exponent function $\exp(-\nu_{i}t)$. The results for $\nu_{i}$ for electron density $n = 2 \times 10^9$ cm$^{-3}$ are presented in figure 2. The negative values of $\nu$ correspond to transverse velocity growth. Also in figure 2 theoretical curves are shown. Coulomb logarithm values for these conditions are small: $L^{0} \approx 0.6$ and $L^{A} \approx 6$. It seems from the figure that...
linear response theory [1] agrees surprisingly well with the axial energy loss rate for a such electron density, but for transverse direction it significantly overestimates the friction force. The approximation of binary collisions, on the contrary, is better consistent with the calculated data for the transverse direction. The similar result for binary approximation for transverse direction was obtained in numerical simulation [5], although in that work much more hot plasma was studied and electron-electron interaction was neglected in the simulation.

In figure 3, results of calculations for lower electron density \( n = 2 \times 10^8 \text{ cm}^{-3} \) are shown. There is slightly more agreement between the theory and the calculations results since the the Coulomb logarithm is larger and again linear response approach works better for axial direction and binary collisions model for transverse direction.

5. Conclusion
We performed simulation of energy loss process of a proton propagating through the cold and dense electron gas in strong magnetic field. Calculations results agree reasonable well with theoretical formulas despite the fact that the formulas have logarithmic accuracy and strictly speaking are valid only for large values of the Coulomb logarithm \( L \gg 1 \) while in considered conditions the Coulomb logarithm values are about unity. Calculations show that the linear response approximation works well for estimation of axial energy loss rate but overestimate the transverse rate. The binary collision approach is more consistent with the results of the calculation of the transverse velocity.

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