A SIMULTANEOUS CONSTRAINT ON THE AMPLITUDE AND GAUSSIANITY OF MASS FLUCTUATIONS IN THE UNIVERSE

JAMES ROBINSON\textsuperscript{1}, ERIC GAWISEND\textsuperscript{2} AND JOSEPH SILK\textsuperscript{1,2,3}

University of California, Berkeley CA 94720-3411

ABSTRACT

We consider constraints on the amplitude of mass fluctuations in the universe, $\sigma_8$, derived from two simple observations: the present number density of clusters and the amplitude of their correlation function. Allowing for the possibility that the primordial fluctuations are non-gaussian introduces a degeneracy in the value of $\sigma_8$ preferred by each of these constraints. However, when the constraints are taken together this degeneracy is broken, yielding a precise determination of $\sigma_8$ and the degree of non-gaussianity for a given background cosmology. For a flat, $\Omega_m = 1$ universe with a power spectrum parameterized by a CDM shape parameter $\Gamma = 0.2$, we find that the perturbations are consistent with a gaussian distribution with $\sigma_8 = 0.49^{+0.08}_{-0.07}$ (95% limits). For some popular choices of background model, including the favored low matter density models, the hypothesis that the primordial fluctuations are gaussian is ruled out with a high degree of confidence.

Subject headings: Cosmology: observations and theory — galaxies: clusters: general — large-scale structure of the universe.

1. INTRODUCTION

Most studies of structure formation in the universe assume that the primordial density perturbations are gaussian. Standard inflationary theories predict gaussian perturbations, and the central limit theorem tells us that any theory involving the superposition of many random processes will give rise to approximately gaussian fluctuations. However, many well motivated theories predict non-gaussian initial conditions, including topological defect theories (Kibble 1976) and certain forms of inflation (Peebles 1983,1997). So far, no convincing observational evidence has been found to confirm or refute the gaussian hypothesis.

Clusters of galaxies, however, provide us with a unique probe of possible non-gaussianity. Being the most massive collapsed structures, they correspond to rare peaks in the primordial density field, so that their statistics respond very sensitively to non-gaussianity in the initial matter distribution. In addition, an analysis of the formation of clusters from given initial conditions requires primarily gravitational physics, and is largely free of complications from star formation and feedback. Finally, although the gravitational evolution which must be modeled is nonlinear, well-tested analytical approximation schemes exist for studying the statistics of the resulting cluster distribution.

In this work we adapt these analytic schemes to the case of non-gaussian fluctuations, and use them to make predictions for two simple observations, the number density of clusters and the amplitude of cluster correlations. We show that the possibility of non-gaussianity introduces a degeneracy in the value of $\sigma_8$ (where $\sigma_R$ is the rms overdensity in a sphere of radius $R\ h^{-1}\text{Mpc}$) preferred by each of these observations. However, combining the two constraints breaks the degeneracy, allowing a precise determination of $\sigma_8$ and the degree of non-gaussianity in the universe.

2. MODEL

We describe the primordial density field by specifying both the power spectrum $P(k)$ and the probability density function (PDF) $p_R(\delta)$ of the fractional over-density $\delta$ after a top-hat smoothing on scale $R$. We make the simplifying assumption that only the $rms$ level of fluctuations changes as a function of scale, that is $p_R(\delta) = P(\delta/\sigma_R)/\sigma_R$ where $P(y)$ is a rescaled PDF, whose distribution has zero mean and an $rms$ value of 1. This assumption should hold reasonably well for topological defect theories, whose perturbations are scale invariant over a range of scales, and more generally, will not affect our results provided the form of the PDF does not change significantly over the narrow range of scales relevant to cluster formation.

The Press-Schechter (1974) formalism (hereafter PS) allows us to compute the number density of clusters within a given background cosmology for gaussian fluctuations. The derivation suggests a simple way to adapt the PS formula for theories with a general PDF (as done by Chiu, Ostriker & Strauss 1997): The probability that a given region of space lies inside a collapsed region with pre-collapse radius larger than $R$ is taken to be $\int_{\delta_C}^\infty p_R(\delta)d\delta$, where $\delta_C$ is a critical threshold for collapse. Differentiating with respect to $R$ and dividing by the pre-collapse volume we obtain the number density $n(R)$ of collapsed objects with pre-collapse radius $R$, where

$$n(R)dR = \frac{3f}{4\pi R^2} \left[ \frac{d}{dR} \left( \int_{\delta_C}^\infty p_R(\delta)d\delta \right) \right] dR. \quad (1)$$

As in the gaussian case (Bond et al. 1991), we have mul-
tially by a correction factor $f = 1/\int_0^\infty P(y)dy$, to ensure that the entire mass of the universe is accounted for. We integrate the above expression to obtain cumulative number counts $N_{> R}$ for all objects with a pre-collapse radius greater than $R$:

$$N_{> R} = \int_R^\infty dR' \frac{3f}{4\pi R'^3} P(y_c(R')) \frac{dy_c(R')}{dR'}$$

where $y_c(R) = \delta_c/\sigma_R$. We convert to cumulative number counts as a function of mass and temperature using the method of Eke, Cole & Frenk (1996).

The peak-background split, first suggested by Kaiser (1984), provides a simple scheme for estimating the bias, and hence the correlation amplitude, of collapsed objects. Consider the manner in which the addition of a long wavelength background fluctuation $\delta_b(x)$ modulates the local number density of collapsed objects. The threshold for critical collapse due to short wavelength fluctuations is modified locally from $\delta_c$ to $\delta_c - \delta_b(x)$, so the number density of collapsed objects is modified from its background value $N_{> R}$ to $N_{> R}(x) = (1 + \delta_b(x))N_{> R}/\delta_c$. The factor $1 + \delta_b(x)$ comes from the linear growth of perturbations. Taylor expanding to first order in $\delta_b$ we obtain

$$N_{> R}(x) = (1 + \delta_b(x)) \frac{dN_{> R}}{d\delta_c}.$$

The large scale cluster-cluster correlation function is given by $\xi_{CC}(x) = (\delta_c(x)\delta_c(0))$, where $x = |x|$ and $\delta_c(x) = (N_{> R}(x) - N_{> R})/N_{> R}$ is the fractional overdensity in the cluster distribution. Defining bias $b$ via $\xi_{CC}(x) = b^2 \xi(x)$ where $\xi(x) = (\delta_b(x)\delta_b(0))$ is the correlation function of the dark matter on large scales, we find $b = 1 - (dN_{> R}/d\delta_c)/N_{> R}$.

Substituting using equation 3 yields the bias:

$$b = 1 + \frac{P[y_c(R')]/(R^3 \sigma_R)}{\int_R^\infty dR' P[y_c(R')]/(R^3 \sigma_R)} - 3 \frac{\int_R^\infty dR' P[y_c(R')]/(R^3 \sigma_R)}{\int_R^\infty dR' P[y_c(R')]/(R^3 \sigma_R)}.$$

For each of our models we specify the background cosmology in terms of $\Omega_m$ and $\Omega_\Lambda$. We then parameterize the power spectrum $P(k)$ of matter fluctuations as a CDM spectrum (Bardeen et al. 1986, eq. G3) with shape parameter $\Gamma$ (where $\Gamma \approx \Omega h$) and normalization specified by $\sigma_8$. Over the range of scales relevant to cluster formation this form gives a useful fit to power spectra in a variety of structure formation scenarios. We choose the PDF $P(y)$ to be either a $\chi^2$ or a log-normal distribution, translated and renormalized to have mean zero and standard deviation 1. In each case there is one free parameter which allows us to dial the amount of deviation from gaussianity, which we quantify in terms of a new parameter $T = \sqrt{2\pi} \int_{-\infty}^\infty P(y)dy/\int_{-\infty}^\infty e^{-y^2/2}dy$, where $T$ is the probability of obtaining a three or higher standard deviation peak for the PDF in question relative to that for a gaussian PDF. To obtain log-normal and $\chi^2$ distributions with $T$ less than 1, we reflect the PDF by making the transformation $P(y) \rightarrow P(-y)$. Some sample PDFs, along with the value of $T$, are illustrated in Fig. 1. Our models are thus specified by five parameters: $\Omega_m$, $\Omega_\Lambda$, $\Gamma$, $\sigma_8$ and $T$.

The PS formula is based on analytic results for the collapse of spherically symmetric perturbations. In the case of gaussian fluctuations, it provides remarkably good fits to the multiplicity function of cluster scale objects (Gross et al. 1997). For non-gaussian fluctuations, one may worry that the spherical collapse picture is less applicable, particularly in the case $T \ll 1$, where the highest density objects will tend to form from matter swept out of low density regions. As for cluster correlations, our result for the cluster bias in the gaussian case is identical to that of Mo & White (1996), which has been shown by Mo, Jing & White (1996) to give a good fit to the amplitude of cluster correlations in N-body simulations. For highly non-gaussian fluctuations, problems could arise in the separation of the long and short wavelength fluctuations, which are not necessarily uncorrelated. However, if most clusters form from rare peaks in the initial density field, and if the long and short wavelength modes are nearly uncorrelated, then there is no reason to expect the modified PS or peak-background split formalisms to lead to errors. Our predictions seem to be born out qualitatively by the results of published simulations of non-linear gravitational clustering from non-gaussian initial conditions (Park, Spergel & Turok 1991; Weinberg & Cole 1992; Borgani et al. 1994). However, a detailed quantitative comparison with numerical simulations remains to be carried out.

3. Results

We compare our predictions with two sets of observational data. Firstly, we compare predictions for the cluster temperature function at $z = 0.05$ with observations computed from the catalogue of nearby clusters compiled by Henry & Arnaud (1991). We compute the cumulative number counts at two values of the temperature, and estimate the covariance matrix by bootstrap resampling. For each resampling, we add gaussian random errors to the temperature consistent with the quoted observational uncertainty, and an additional 10% systematic error to allow for uncertainties in the $M$ vs. $T$ normalization (Eke, Navarro & Frenk 1998). The observational data, along with 2$\sigma$-errorbars for our two chosen temperatures, are shown in the top panel of Fig. 2. Secondly, we compare with the amplitude of the cluster-cluster correlation function, quantified by $r_0$, the value of $r$ for which $\xi_{CC}(r) = 1$. Croft et al. (1997) (hereafter C97) have computed $r_0$ as a function of mean cluster separation $d$ for subsamples of the APM cluster survey with different lower bounds on the cluster richness. Their results are shown in the bottom panel of Fig. 2. We compare model predictions with just the $d = 30 h^{-1}$Mpc data point, since this point is derived from the largest number of clusters, and the points are highly correlated. We use $r_0 = 13.2 \pm 0.91 \pm 1.0h^{-1}$Mpc, where we have corrected the correlation amplitude for redshift space distortions by subtracting 1$h^{-1}$Mpc, as suggested by the simulations in C97. The first errors are their 2$\sigma$ statistical errors and the second are 2$\sigma$ errors. We have added to account for systematic uncertainties in the redshift space correction. For each model we use the techniques described above to compute the cluster correlation length $r_0$ at $z = 0$ for clusters with pre-collapse radius determined by their mean separation $d = N_{> R}^{-1/3} = 30 h^{-1}$Mpc.

In Fig. 2 we compare observations and predictions for a set of four models, each with $\Omega_m = 1$, $\Omega_\Lambda = 0$ and
\( \Gamma = 0.2 \). Models 1, 2 and 3 have a normalization given by \( \sigma_g = 0.5 \). Model 1 has gaussian fluctuations, model 2 has \( T = 0.0348 \) (a reflected \( \chi^2 \) distribution with 50 degrees of freedom), and model 3 has \( T = 16.3 \) (a \( \chi^2 \) with 1 degree of freedom). Model 4 has the same PDF as model 3, but a lower normalization, \( \sigma_g = 0.2 \).

From the top panel of Fig. 2 we see that increasing \( T \) increases the number density of clusters of a given temperature. The reason for this is clear from Fig. 1: for the value of \( \sigma_g \) considered here, only peaks of height roughly 3\( \sigma \) and higher (shown by the heavily shaded region under each curve) will collapse, since the ratio of the critical overdensity to the standard deviation \( \sigma_g \) in a sphere which will collapse to form a typical mass cluster is \( y_c = \delta_c/\sigma_g = 1.69/0.5 \simeq 3 \). So in this “standard” case, the parameter \( T \) directly counts the expected number of clusters.

Now we consider the effect of changes in the PDF on the amplitude of the cluster correlation function. From the bottom panel of Fig. 2 we see that increasing \( T \) decreases the correlation amplitude, for reasons which are also illustrated in Fig. 1. Adding a small background fluctuation modifies the local value of the critical overdensity for collapse, leading to a spatial modulation in the number density of collapsed objects. The lightly shaded regions under the curves in Fig. 1 show the effect for each PDF of modifying \( y_c \) from 3.0 to 2.5. For the \( T > 1 \) PDF which has a long flat tail, this modification produces only a small enhancement in the number of objects capable of collapse, and hence there is only a small enhancement in the clustering amplitude of those collapsed objects. For the \( T < 1 \) PDF where the positive tail falls off very sharply, this modification greatly enhances the local number of collapsed objects, and consequently there is large enhancement in the clustering amplitude of those objects.

Finally, we consider the effect on each of the above results of changing \( \sigma_g \) for a fixed PDF. Model 4 illustrates the effect of decreasing \( \sigma_g \) for the case \( T > 1 \). As \( \sigma_g \) is decreased, the number density of clusters decreases, the reason being that the number of peaks with high enough over-density to collapse is decreased. The top panel of Fig. 2 shows that for \( T = 16.3, \sigma_g = 0.2 \) provides a good fit to the temperature function data. In the bottom panel of Fig. 2, we see that as \( \sigma_g \) is decreased, the amplitude of the cluster-cluster correlation function also decreases. This is due to the fact that decreasing \( \sigma_g \) decreases the amplitude of the underlying dark matter correlation function, but leaves the bias roughly unchanged. Model 4 shows that for \( T = 16.3, \sigma_g = 0.2 \) provides a worse fit to the correlation function data than \( \sigma_g = 0.5 \).

Thus we see that the two datasets we have chosen are complementary: variations in the parameter \( T \) introduce a degeneracy in the preferred value of \( \sigma_g \) for each dataset, but the degeneracies are “orthogonal”. For the temperature function, increasing \( T \) requires lower \( \sigma_g \), while for the correlation amplitude, increasing \( T \) requires higher \( \sigma_g \). So taken in combination, these data allow a precise determination of both \( \sigma_g \) and \( T \). We quantify this by computing values of the \( \chi^2 \) statistic for each model relative to the two datapoints in the top panel of Fig. 2 and the \( d = 30h^{-1}\text{Mpc} \) data point in the bottom panel. In Fig. 3 we show goodness of fit contours in the \( \sigma_g \) vs. \( T \) plane for log-normal PDF models with a range of background cosmologies and values of the parameter \( \Gamma \). (We use three values of \( \Gamma \) which span the range allowed by measurements of the APM cluster power spectrum by Tadros, Efstathiou & Dalton 1997.)

The dotted blue line (\( T = 1 \)) shows the location in the \( T \) vs. \( \sigma_g \) plane of a gaussian PDF. The constraints on \( \sigma_g \) from present day cluster number abundances have been extensively investigated in this restricted case (White, Efstathiou & Frenk 1993; Eke et al. 1996; Viana & Liddle 1996) and the results obtained here are in close agreement. In addition, Mo et al. (1996) have investigated the combined constraints of cluster correlation amplitude and number abundance measurements for the case of gaussian fluctuations. Our results along this “line of gaussianity” can be considered an update of their analysis, as we use more recent cluster correlation data (C97) with smaller observational errors.

The results depend both on the background cosmology and the power spectrum shape parameter \( \Gamma \). For low values of \( \Omega_m \), the value of \( \sigma_g \) preferred by the temperature function data increases. This is just as observed in the gaussian case, and is due to the fact that for low values of \( \Omega_m \), clusters of typical mass form from spheres with a larger pre-collapse radius \( R \), for which \( \sigma_R \) is less than \( \sigma_g \). The value of \( \sigma_g \) preferred by the correlation amplitude data, however, is almost independent of \( \Omega_m \) and \( \Omega_\Lambda \). This is because the correlation amplitude is measured for clusters with a fixed value of \( d \), and consequently for clusters with the same pre-collapse radius (not the same mass). Finally, as \( \Gamma \) increases, the value of \( \sigma_g \) preferred by the correlation amplitude also increases. This is because the corresponding change in the shape of the power spectrum decreases the amount of power on scales of order the cluster correlation length for a given value of \( \sigma_g \).

Results for the best fit values of \( \sigma_g \) and \( T \) from the combined datasets for the family of log-normal PDFs in the various background cosmologies are summarized in Table 1. We have performed the same calculation for \( \chi^2 \)-parameterized PDFs and find very similar results. Thus we expect that we can infer results for other models provided we work out the appropriate value of the parameter \( T \) (for instance \( T \simeq 14 \) for textures – Park et al. 1991). In future work we will expand on the analysis presented here by including observations of the evolution of cluster abundance and making use of updated datasets at low redshift.

4. CONCLUSIONS

We have modified the Press-Schechter and the peak-background split formalisms to compute the cluster number density and cluster correlation amplitude in non-gaussian models. The best-fit value of \( \sigma_g \) and level of non-gaussianity depend on the choice of background cosmology, but we have demonstrated how these two quantities can be constrained simultaneously. For a flat, \( \Omega_m = 0, \Gamma = 0.2 \) universe we find that the fluctuations are consistent with gaussian, with \( \sigma_g = 0.49^{+0.08}_{-0.07} \) (95% limits). For low-\( \Omega_m \) models with \( \Gamma = 0.2 \), gaussian fluctuations are ruled out with a high degree of confidence, while non-gaussian fluctuations (\( T > 1 \)) are strongly preferred. Taken in conjunction with accurate knowledge of the background pa-
rameters (which we are rapidly gaining from supernovae and CMB observations), this method can provide a powerful constraint on the nature of density fluctuations in the universe.

Some concerns remain regarding the validity of our modification of the PS and peak-background split formalisms to the case of non-gaussian fluctuations, and some systematic errors may not yet be accounted for. This issue should be settled by an analysis of n-body simulations with non-gaussian initial conditions. However, our formulae have been well tested for the gaussian case, so the result that gaussianity is ruled out for certain choices of background model is not subject to these uncertainties. At the very least, we have demonstrated the power of two simple and well measured datasets, the cluster temperature function and the amplitude of cluster correlations, to simultaneously constrain the amplitude of mass fluctuations and the level of non-gaussianity in the universe.

We would like to thank P. Ferreira, M. Davis and M. Markevitch for helpful discussions. This work has been supported in part by a grant from the NSF, and E.G. acknowledges the support of an NSF Graduate Fellowship.

REFERENCES

Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 304, 15
Bond, J. R., Cole, S., Efstathiou, G., & Kaiser, N. 1991, ApJ, 379, 440
Borgani, S., Coles, P., Moscardini, L., & Manidhis, P. 1994, MNRAS, 266, 524
Chiu, W. A., Ostriker, J. P., & Strauss, M. A. 1997, astro-ph/9708250
Croft, R. A. C., Dalton, G. B., Efstathiou, G., Sutherland, W. J., & Maddox, S. J. 1997, astro-ph/9701049
Eke, V. R., Cole, S., & Frenk, C. S. 1996, MNRAS, 282, 263
Eke, V. R., Navarro, J. F., & Frenk, C. S. 1998, ApJ, in press, astro-ph/9708070
Gross, M. A. X., Somerville, R. S., Primack, J. R., Holtzman, J., & Klypin, A. 1998, astro-ph/9712142
Henry, J. P., & Arnaud, K. A. 1991, ApJ, 372, 410
Kaiser, N. 1984, ApJ, 284, L9
Kibble, T. W. B. 1976, J. Phys., A9, 1387
Mo, H. J., Jing, Y. P., & White, S. D. M. 1996, MNRAS, 284, 189
Mo, H. J., & White, S. D. M. 1996, MNRAS, 282, 347
Park, C., Spergel, D. N., & Turok, N. 1991, ApJ, 372, L53
Peebles, P. J. E., 1983, ApJ, 274, 1
Park, C., Spergel, D. N., & Turok, N. 1991, ApJ, 372, L53
Peebles, P. J. E., 1997, ApJ, L1
Press, W. H., & Schechter, P. 1974, ApJ, 187, 425
Tadros, H., Efstathiou G., & Dalton, D. 1997, astro-ph/9708259
Viana, P. T. P., & Liddle, A. R. 1996, MNRAS, 281, 551
White, S. D. M., Efstathiou, G., & Frenk, C. S. 1993, MNRAS, 262, 1023
Weinberg, D. H., & Cole, S. 1992, MNRAS, 259, 652
Fig. 1.— Three PDFs. The heavily shaded region shows the area contributing to peaks of height $3\sigma$ and above, while the lightly shaded region shows the additional contribution to peaks of height $2.5\sigma$ and above.

| $\Omega_m$ | $\Omega_\Lambda$ | $\Gamma$ | $\sigma_8$    | $T$    |
|----------|----------------|--------|-------------|-------|
| 1.0      | 0.0            | 0.1    | $0.42^{+0.08}_{-0.05}$ | $3.8^{+1.7}_{-1.2}$ |
| 1.0      | 0.0            | 0.2    | $0.49^{+0.07}_{-0.08}$ | $2.0^{+1.7}_{-1.1}$ |
| 1.0      | 0.0            | 0.3    | $0.56^{+0.10}_{-0.08}$ | $0.9^{+1.3}_{-0.7}$ |
| 0.3      | 0.0            | 0.1    | $0.59^{+0.15}_{-0.09}$ | $5.8^{+2.8}_{-1.8}$ |
| 0.3      | 0.0            | 0.2    | $0.71^{+0.17}_{-0.13}$ | $3.8^{+3.2}_{-2.1}$ |
| 0.3      | 0.0            | 0.3    | $0.83^{+0.21}_{-0.16}$ | $2.1^{+3.2}_{-1.6}$ |
| 0.3      | 0.7            | 0.1    | $0.61^{+0.15}_{-0.10}$ | $6.3^{+3.1}_{-2.1}$ |
| 0.3      | 0.7            | 0.2    | $0.73^{+0.12}_{-0.10}$ | $4.0^{+3.6}_{-2.1}$ |
| 0.3      | 0.7            | 0.3    | $0.87^{+0.21}_{-0.17}$ | $2.3^{+3.2}_{-1.7}$ |

Table 1

Best fit values of $\sigma_8$ and $T$, with 95% confidence limits.
Fig. 2.— The cluster temperature function (black jagged line, top panel), and the cluster correlation length $r_0$ (datapoints, lower panel), with predictions for four models ($\Omega_m = 1.0, \Omega_\Lambda = 0.0, \text{and } \Gamma = 0.2, \sigma_8$ and $T$ labeled in lower panel).

Fig. 3.— Confidence intervals (68%—dark band, 95%—light band) in the $T$ vs. $\sigma_8$ plane for the cluster temperature function (green band, top left to bottom right) and the cluster correlation length (red band, bottom left to top right). The blue dotted line corresponds to gaussianity ($T = 1$), and the panels are labeled with the values of $\Omega_m$, $\Omega_\Lambda$, and $\Gamma$. 