Simulation of stresses in asphalt-concrete pavement with frost heaving

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\textbf{Abstract.} The article reviews the issue of the stress-strain state of the road surface in the winter period as a result of roadbed soil heaving. The main purpose of the work is to determine the stress fields in the asphalt-concrete pavement, which is necessary for designing of frost-resistant roadbeds in the areas with seasonal freezing of roadbed soils. The following research methods were used: theoretical, laboratory, field. With consideration of different properties of materials and geometric dimensions of the road surface section, stress fields in the asphalt-concrete pavement during freezing were obtained with the use of the software product. Comparison of theoretical studies with the results of experimental and full-scale tests showed that the forms of theoretical solutions describe the stress fields in the freezing asphalt-concrete pavement quite accurately.

\textbf{Introduction}

Strains in the form of cracks, which are often formed during the first three years of highway operation, are widespread in northern districts [1, 2]. Occurrence of cracks on the pavement in winter can be caused by low-temperature cracking [3, 4] or uneven elevation of the pavement surface as a result of frost heaving of soils. In areas with deep seasonal freezing of soils, cracks formed as a result of uneven elevation of the road surface due to frost heaving of soils were most widely spread [5 - 7].

To forecast the future state of highways and minimize operating costs for their maintenance, large-scale studies were performed as described in [8]. In this work, occurrence of cracks on the surface of the asphalt-concrete pavement is described by the mathematical model based on statistical study methods. This allows predicting arising of cracks on the road surface with a certain accuracy, but does not explain the process of crack formation.

It is necessary to study the stress-strain state of roadbed in winter for the purpose to increase frost resistance of road surfaces. Works in this field [9 - 12] are focused on studying the impact of the automobile wheels on the road surface. At present, the topic of the stress-strain state of roadbed as a result of frost heaving remains poorly studied.

To achieve this goal, a mathematical model of stresses of asphalt-concrete pavements caused by frost heaving was built with consideration of elastoplastic strain by the Huber-Mises criterion [13].
Study methods

Soil conditions of the existing highway network were studied and monitoring of strain in asphalt-concrete pavement in winter was performed at the first stage. Observations were performed at specially prepared posts (sections) of the operated road for three calculation schemes for wetting the ground roadbed. Points for monitoring of the pavement strain were fixed with dowels and painted over (Fig. 1). Roadbed was opened at each site to determine the number of layers and their thickness. The material of each layer was also determined. Soils were sampled to determine their characteristics and heaving properties [14]. To ensure the invariability of the reference point during freezing and heaving of soils, freezing reference points were set up, which were used to measure the roadbed surface in winter [15].

Laboratory experimental studies of asphalt-concrete samples (in the form of small beams) were performed at the second stage. Bending tension tests were performed at negative temperatures. As a result, tensile strength characteristics of asphalt-concrete of the A and BI type with 90/130 bitumen were obtained in the range of temperatures from -21°C to -35°C [16].

The third stage is building a mathematical model for determination of stresses in asphalt-concrete pavement as a result of freezing soils heaving in the roadbed based on the available exact solutions of the mechanics of the rigid strained body (elasticity and plasticity theory) [17]. Critical stresses for asphalt-concrete pavement were determined based on the strain strength characteristics of asphalt-concrete of the A and BI type with 90/130 bitumen obtained as a result of the experiments. Critical stresses were compared to the field measurements. Stree fields were built on the example of the 1st area type by the wettening condition of “Teguldet — B. Dorokhovo, km 19+400, in Tomsk region”, where winter monitoring over the pavement strain was performed.

Results

Winter monitoring over the pavement strain as a result of roadbed soils heaving was performed during 2017 to 2018 (Table 1).
Table 1. Results of displacements of pavement points at heaving at highway section “Teguldet – B. Dorokhovo, km 19+400, in Tomsk region”.

| Row No. / section No. | Displacement, mm |
|-----------------------|-------------------|
|                       | 1   | 2   | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  |
| 1                     | 28  | 28  | 27  | 26  | 24  | 25  | 26  | 25  | 21  | 23  | 23  |
| 2                     | 26  | 27  | 24  | 24  | 24  | 20  | 21  | 22  | 22  | 22  | 20  |
| 3                     | 25  | 23  | 20  | 20  | 20  | 18  | 18  | 20  | 19  | 21  |    |
| 4                     | 27  | 24  | 23  | 21  | 24  | 20  | 18  | 19  | 18  | 20  | 18  |
| 5                     | 27  | 20  | 24  | 22  | 19  | 21  | 18  | 21  | 19  | 24  | 28  |
| 6                     | 29  | 26  | 27  | 20  | 19  | 19  | 19  | 21  | 25  | 27  |    |
| 7                     | 33  | 29  | 27  | 25  | 26  | 27  | 25  | 25  | 28  | 29  | 31  |

Note: see distances between points in a row and in a section in Figure 1.

On the basis of these results, it is possible to determine the components of gradient tensor of displacement \( \frac{\partial u_i}{\partial x_j} \) (\( i = 1 \) — complies with axis \( x \) in the longitudinal direction, \( i = 2 \) — complies with axis \( y \) in the transverse direction, \( i = 3 \) — complies with axis \( z \) in the vertical directions). The antisymmetric part of the displacement gradient tensor

\[
\Phi_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i} \right)
\]

characterizes rotations (deflections). The symmetric part of this tensor

\[
\varepsilon_{ij}^G = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)
\]

characterizes the stretching/compression and shear strain (the Cauchy strain) and can be used in case of small deflections. In case of large deflections, the use of the Cauchy strain measure becomes incorrect. It is necessary to use nonlinear Green’s ratios \([18]\) to describe the stress-strain state at large deflections:

\[
\varepsilon_{ij}^G = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} + \frac{\partial u_k}{\partial x_i} \frac{\partial u_k}{\partial x_j} \right). \tag{1}
\]

If the displacement field is characterized with one component \( u_z \) \( (u_x = 0, \ u_y = 0) \), then Green’s strain tensor components may be expressed as follows:

\[
\varepsilon_{xx}^G = \frac{1}{2} \left( \frac{\partial u_x}{\partial x} \right)^2, \quad \varepsilon_{yy}^G = \frac{1}{2} \left( \frac{\partial u_y}{\partial y} \right)^2, \quad \varepsilon_{xy}^G = \frac{1}{2} \left( \frac{\partial u_y}{\partial x} \right) \left( \frac{\partial u_x}{\partial y} \right), \tag{2}
\]

\[
\varepsilon_{xz}^G = \frac{1}{2} \left( \frac{\partial u_x}{\partial x} \right), \quad \varepsilon_{yz}^G = \frac{1}{2} \left( \frac{\partial u_y}{\partial y} \right), \quad \varepsilon_{zz}^G = \frac{1}{2} \left( \frac{\partial u_x}{\partial z} \right)^2 + \left( \frac{\partial u_y}{\partial z} \right)^2, \tag{3}
\]

Equation (2, 3) was received on the basis of formula (1) on condition that the displacement field is characterized with one component \( u_z \).
The stress field can be determined after determination of strains.

If the strain is elastic, then Hooke’s law is observed, which is as follows in the index form [19]:

\[
\sigma_{xx} = \frac{\mathbf{v}E}{(1+\nu)(1-2\nu)} \left( \varepsilon_{xx}^G + \varepsilon_{yy}^G + \varepsilon_{zz}^G \right) + \frac{\mathbf{v}E}{(1+\nu)} \varepsilon_{xx},
\]

\[
\sigma_{yy} = \frac{\mathbf{v}E}{(1+\nu)(1-2\nu)} \left( \varepsilon_{xx}^G + \varepsilon_{yy}^G + \varepsilon_{zz}^G \right) + \frac{\mathbf{v}E}{(1+\nu)} \varepsilon_{yy},
\]

\[
\sigma_{zz} = \frac{\mathbf{v}E}{(1+\nu)(1-2\nu)} \left( \varepsilon_{xx}^G + \varepsilon_{yy}^G + \varepsilon_{zz}^G \right) + \frac{\mathbf{v}E}{(1+\nu)} \varepsilon_{zz},
\]

\[
\sigma_{xy} = \frac{\mathbf{v}E}{(1+\nu)} \varepsilon_{xy},
\quad \sigma_{xz} = \frac{\mathbf{v}E}{(1+\nu)} \varepsilon_{xz},
\quad \sigma_{yz} = \frac{\mathbf{v}E}{(1+\nu)} \varepsilon_{yz}.
\]

where \( E \) is the pavement elasticity module (assumed to be \( E = 3000 \) MPa in the calculations on the basis of design pavement temperature at section “Teguldet – B. Dorokhovo, km 19+400, in Tomsk region” of -21 °C), \( \nu = 0.30 \) is the Poison’s ratio.

The design pavement temperature was assumed based on the air temperature regime in the area where the monitoring of pavement strain in winter was monitored and was determined by formula [20]:

\[
T_{\text{min}} = 0.859T_{\text{min}} + 1.7, \quad (8)
\]

where \( T_{\text{min}} \) is the 5-day minimum air temperature, °C.

The invariant measure of strain is intensity of shear deformations:

\[
U = \sqrt{2 \left( \varepsilon_{ij} - \frac{1}{3} \varepsilon_{kk} \delta_{ij} \right) \left( \varepsilon_{ji} - \frac{1}{3} \varepsilon_{kk} \delta_{ij} \right)}.
\]

As the stress invariant, shear stress intensity can be considered [21]:

\[
S = \frac{1}{2} \sqrt{\sigma_{ij} - \frac{1}{3} \sigma_{kk} \delta_{ij} \left( \sigma_{ji} - \frac{1}{3} \sigma_{kk} \delta_{ij} \right)}.
\]

According to the Huber-Mises-Henki condition, the transition from the elastic state to the plastic state occurs when the \( \tau_{\text{Y}} \) = \( S \) condition is met [22]. In this case, stress tensor components may be determined from the following equations [18]:

\[
\sigma_{xx} = \frac{\mathbf{v}E}{(1+\nu)(1-2\nu)} \left( \varepsilon_{xx}^G + \varepsilon_{yy}^G + \varepsilon_{zz}^G \right) + \min \left[ \frac{\mathbf{v}E}{(1+\nu)} \frac{\tau_{\text{Y}}}{U} \varepsilon_{xx} \right],
\]

\[
\sigma_{yy} = \frac{\mathbf{v}E}{(1+\nu)(1-2\nu)} \left( \varepsilon_{xx}^G + \varepsilon_{yy}^G + \varepsilon_{zz}^G \right) + \min \left[ \frac{\mathbf{v}E}{(1+\nu)} \frac{\tau_{\text{Y}}}{U} \varepsilon_{yy} \right],
\]

\[
\sigma_{zz} = \frac{\mathbf{v}E}{(1+\nu)(1-2\nu)} \left( \varepsilon_{xx}^G + \varepsilon_{yy}^G + \varepsilon_{zz}^G \right) + \min \left[ \frac{\mathbf{v}E}{(1+\nu)} \frac{\tau_{\text{Y}}}{U} \varepsilon_{zz} \right],
\]

\[
\sigma_{xy} = \frac{\mathbf{v}E}{(1+\nu)} \varepsilon_{xy},
\quad \sigma_{xz} = \frac{\mathbf{v}E}{(1+\nu)} \varepsilon_{xz},
\quad \sigma_{yz} = \frac{\mathbf{v}E}{(1+\nu)} \varepsilon_{yz}.
\]
The results of calculations of stress fields of the asphalt-concrete pavement are presented in Figures 2 to 7. In Figures 2 and 7, maximum shear displacement arise at the beginning and at the end of pavement section “Teguldet – B. Dorokhovo, km 19+400, in Tomsk region” in the \(xz\) vertical plane. The cracks that appeared on the pavement did not reach the monitoring area (Fig. 1). Perhaps, a greater increase in shear stresses caused formation of a transverse crack on the road surface.

\[
\sigma_{xy} = \min \left[ \frac{vE}{(1+\nu)} \frac{\tau_{xy}}{U} \right] \epsilon_{xy}^G, \quad (14)
\]

\[
\sigma_{xz} = \min \left[ \frac{vE}{(1+\nu)} \frac{\tau_{xz}}{U} \right] \epsilon_{xz}^G, \quad (15)
\]

\[
\sigma_{yz} = \min \left[ \frac{vE}{(1+\nu)} \frac{\tau_{yz}}{U} \right] \epsilon_{yz}^G. \quad (16)
\]

The stresses (Fig. 3) that arose in the \(yz\) plane were caused by irregular freezing of soils under the roadbed and roadside verges. The condition for irregular freezing of the roadbed is large amount of snow on the roadside verges and the absence of snow on the pavement.
Fig. 4. Distribution of tensile stresses on the roadbed surface in the $xy$ plane.

Fig. 5. Distribution of tensile stresses on the roadbed surface along the $xx$ axis.

Fig. 6. Distribution of tensile stresses on the roadbed surface along the $yy$ axis.
Fig. 7. Total intensity of shear stresses on the roadbed surface.

In Figure 4, stresses in the $xy$ plane are insignificant as compared to the obtained value of asphalt-concrete bending strength of 6.8 MPa (at the design negative temperature of -21 °C) for this area. The adequacy of the model is confirmed by the results of field studies of pavements and laboratory experiments (Table 2).

**Table 2.** Comparison of tensile stresses in the proposed mathematical model with the critical stresses obtained in laboratory conditions for asphalt-concrete and field monitoring of the road pavement.

| Section title                  | Section No. | Design tensile stresses according to the proposed mathematical model, MPa | Critical tensile stresses, MPa | Surface condition     |
|-------------------------------|-------------|------------------------------------------------------------------------|-------------------------------|-----------------------|
| Melnikovo – Kozhevnikovo km 22+620 | 5           | 8.0                                                                    | 7.1                           | transverse crack      |
| Parabel – Kargasok km 166+400  | 4           | 10.0                                                                   | 8.2                           | longitudinal crack    |
| Molchanovo - Chazhemo km 231+500 | 1           | 12.0                                                                   | 8.5                           | transverse crack      |
| Teguldet - B. Dorokhovo km 3+844 | 7           | 8.0                                                                    | 7.5                           | diagonal crack        |

It must be noted that there are small areas in Figures 5 and 6 that exceed limit values of the bending strength. Cracks of local nature and not long cracks were not registered. It was very difficult to visually register formation of micro cracks.

**Conclusion**

Development of the pavement stress prediction model allows considering material characteristics and determining the most critical conditions of the pavement during operation of roads in winter. Based on the results of the study, the model was built that adequately
predicted stresses in pavement. Application of the model will ensure the quality of designing frost-resistant roadbeds in areas with deep seasonal freezing. However, the model requires further calibration and continued monitoring of strains in road pavement in winter. To ensure that the model conforms to the actual conditions of pavement operation, more initial information on the pavement material is needed. For example, the effect of aging of bitumen on the strain strength characteristics of asphalt-concrete. Development of a model for predicting behavior of materials of the underlying layers of pavement as a result of frosty soils heaving in the roadbed is not less important.

References

1. M. Sarady, Licentiate thesis, Uppsala University, Uppsala 2014.
2. Premature asphalt concrete pavement cracking. Final report (Oregon Department of Transportation, 2015)
3. B.B. Teltaev, Reports of the National Academy of the Republic of Kazakhstan 302, 40–65 (2015). (in Russian)
4. Wu, Zhong, and Danny X. Xiao, Development of DARWin-ME Design Guideline for Louisiana Pavement Design. No. FHWA/LA. 11/551. 2016
5. S.V. Bel'kovskij, Proektirovanie gruntových osnovani j svoovershensstvovannykh pokryti j s uchjotom ih raboty v zimnih uslovijah [Design of soil bases for improved coatings, taking into account their work in winter conditions], 53 – 114 (1953). (in Russian)
6. F. Rengmark, Highway Research Board 33, 137-157 (1963).
7. Summary report of a cooperative analysis by teams from Iowa, Kansas, Nebraska North Dakota and Oklahoma. Transverse cracking of asphalt pavements. Final report (U.S. Department of Transportation, 1982).
8. N.G. Göransson, Uppföljning av vägars tekniska tillstånd: lägesrapport för observationssträckor ingående i LTPP-projektet till och med december 2017, 1-2018, (Swedish National Road and Transport Research Institute, Linköping, 2018).
9. B.S. Radovskiy, A.S. Suprun, Proektirovanie dorozhnyh odezhdy dlja dvizhenija bol'shegruzhnyh avtomobilej [Designing of road clothes for the heavy vehicles (Budivennik, 1989), (in Russian)
10. B.B. Teltaev, Doctoral thesis, Institute of Mechanics and Engineering, National Academy of Sciences of the Republic of Kazakhstan (Almaty, 1998). (in Russian)
11. A.V. Smirnov, Raschjot dorozhnyh i aerodromnych konstrukcij na dinamicheskie vozdejstvia [Calculation of road and airfield structures for dynamic impacts] (SibADI, Omsk, 2008). (in Russian)
12. M. Li, H. Wang, American society of civil engineers, Vol. 144(2), 04018014 (2018).
13. J. Lubliner, Plasticity theory (Courier Corporation, 2008).
14. S. Efimenko, V. Efimenko, A. Sukhorukov, AIP Conference Proceedings, 1698, 1 (2016).
15. V.S. Churilin, A.V. Sukhorukov, M.B. Badina, Youth, science, technology: new ideas and perspectives (YSIP-2017), Proceedings of the IV International scientific conference, 334 – 336 (Tomsk, 2017). (in Russian)
16. V.S. Churilin, Vestnik of TSUAB, 5 (2015). (in Russian)
17. O.V. Matvienko, V.P. Bazuev, V.S. Churilin, Dorogi i mostyi [Roads and Bridges], 36 (2017). (in Russian)
18. V.V. Novozhilov, Osnovy nelinejnoj teorii uprugosti [Fundamentals of the nonlinear theory of elasticity] (Moscow, 2003). (in Russian)
19. A.G. Gorshkov, E.I. Starovojtov, D.V Tarlakovskiy, Teorija uprugosti i plasticnosti [Theory of elasticity and plasticity] (Fizmatlit, Moscow, 2002). (in Russian)
20. Superpave Performance Graded Asphalt Binder Specification and Testing. Asphalt Institute Superpave. Series 1 (1997).
21. O.V. Matvienko, V.P. Bazuev, V.N. Venik, Vestnik of TSUAB, 4, 165 - 176 (2017). (in Russian)
22. D.M. Klimov, A.G. Petrov, D.V. Georgievskiy, Vjazkoplasticheskie techenija. Dinamicheskij haos, ustojchivost', peremeshivanie [Viscous plastic flows. Dynamic chaos, stability, mixing] (Nauka, Moscow, 2005). (in Russian)