THE MUON $g - 2$ REVISITED

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Abstract

I present a short review of the present status of the Standard Model prediction of the anomalous magnetic moment of the muon, with special emphasis on the hadronic contributions.
1 Introduction

The $g$–factor of the muon is the quantity which relates its spin $\vec{s}$ to its magnetic moment $\vec{\mu}$ in appropriate units:

$$\vec{\mu} = g_\mu \frac{e \hbar}{2m_\mu c} \vec{s}, \quad \text{and} \quad g_\mu = 2(1 + a_\mu).$$

In the Dirac theory of a charged spin–1/2 particle, $g = 2$. Quantum Electrodynamics (QED) predicts deviations from the Dirac prediction, because in the presence of an external magnetic field the muon (electron) can emit and reabsorb virtual photons. The correction $a_\mu$ to the Dirac prediction is called the anomalous magnetic moment. It is a quantity directly accessible to experiment \footnote{See e.g. ref. \cite{1} for a simple and lucid review where references to the experimental literature can also be found.}

The experimental world average, at the time of the La Thuile meeting, which included the BNL published result \cite{2} based on the 1999 $\mu^+$ data, was

$$a_\mu(\text{exp.}) = 11,659,202.3(15.1) \times 10^{-10} \ [1.3 \ \text{ppm}]. \quad (2)$$

There is a recent new result from the BNL collaboration \cite{3}, based on $\mu^+$ data collected in the year 2000,

$$a_\mu(\text{exp.}) = 11,659,204.7(5) \times 10^{-10} \ [0.7 \ \text{ppm}]. \quad (3)$$

With this result, the present world average is now

$$a_\mu(\text{exp.}) = 11,659,203.8(8) \times 10^{-10} \ [0.7 \ \text{ppm}]. \quad (4)$$

In this talk, I shall present a review of the various contributions to $a_\mu$ in the Standard Model, with special emphasis in a recent evaluation of the dominant contribution from the hadronic light–by–light scattering \cite{4}, which has the merit to have stopped an avalanche of theoretical speculations, at least temporarily.

2 Some Remarks on the QED Contributions

In QED, the Feynman diagrams which contribute to $a_\mu$ at a given order in the perturbation theory expansion (powers of $\frac{e}{\pi}$), can be classified in four classes:
Diagrams with virtual photons and muon loops

Examples of that are the lowest order contribution in Fig. 1 and the two loop contributions in Fig. 2. In full generality, this is the class of diagrams which includes those with only virtual photons, and the ones where the internal fermionic lines are of the same flavour as the external line. Since $a_\mu$ is a dimensionless quantity, these contributions are purely numerical, and they are the same for the three charged leptons: $l = e, \mu, \tau$. Indeed, $a_l^{(2)}$ from Fig. 1 is the celebrated Schwinger result

$$a_l^{(2)} = \frac{1}{2} \frac{\alpha}{\pi},$$

(5)

![Fig. 1 Lowest Order QED Contribution](image1)

while $a_l^{(4)}$ from the seven diagrams in Fig. 2 gives the result

$$a_l^{(4)} = \left\{ \frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2} \ln 2 + \frac{3}{4} \zeta(3) \right\} \left( \frac{\alpha}{\pi} \right)^2.$$

(6)

At three loops there are 72–Feynman diagrams of this type which contribute. Quite remarkably, they are also known analytically. They bring in transcendental numbers like $\zeta(3)$, the Riemann zeta–function of argument 3, and of higher complexity.

At the four loop level, there are 891 Feynman diagrams of this type, and their numerical evaluation is still in progress.
Vacuum Polarization Diagrams from Electron Loops

The simplest example is the Feynman diagram in Fig. 3, which gives a contribution

\[
a_\mu = \left( \left( \frac{2}{3} \right) - \frac{1}{2} \right) \log \frac{m_\mu}{m_e} - \frac{25}{36} + \mathcal{O}\left( \frac{m_\mu}{m_e} \right) \right) \left( \frac{\alpha}{\pi} \right)^2.
\]

These contributions are enhanced by QED short–distance logarithms of the ratio of the muon mass to the electron mass, and are therefore very important. As shown in ref. [10], they are governed by a Callan–Symanzik type equation

\[
\left( m_e \frac{\partial}{\partial m_e} + \beta(\alpha) \alpha \frac{\partial}{\partial \alpha} \right) a_\mu^{(\infty)}(\frac{m_\mu}{m_e}, \alpha) = 0,
\]

where \( \beta(\alpha) \) is the QED–function associated with charge renormalization, and \( a_\mu^{(\infty)}(\frac{m_\mu}{m_e}, \alpha) \) denotes the contribution to \( a_\mu \) from powers of logarithms and constant terms. This renormalization group equation is at the origin of the simplicity of the result in Eq.(7). The factor 2/3 in front of \( \log \frac{m_\mu}{m_e} \) comes from the first term in the \( \beta \)–function and the factor 1/2 is the lowest order coefficient of \( \alpha/\pi \) in Eq.(5), which fixes the boundary condition to solve the differential equation in (8) at the first non–trivial order in perturbation theory i.e., \( \mathcal{O}(\frac{\alpha}{\pi})^2 \).

\[
\begin{tikzpicture}
  \node (x) at (0,0) {$\chi$};
  \node (mu) at (-1,-1) {$\mu$};
  \node (e) at (1,-1) {$e$};

  \draw[thick] (mu) -- (x); \draw[thick] (x) -- (e);
  \draw[thick,decorate,decoration=snake] (mu) -- (x);
  \draw[thick,decorate,decoration=snake] (x) -- (e);
\end{tikzpicture}
\]

Fig. 3 Vacuum Polarization contribution from a Small Internal Mass

Knowing the QED \( \beta \)–function at three loops and \( a_\mu \) (from the universal class of diagrams discussed above) also at three loops, allows one to sum leading, next–to–leading, and next–to–next–to–leading powers of \( \log m_\mu/m_e \) to all orders in perturbation theory. Of course, these logarithms can be reabsorbed in a running fine structure coupling \( \alpha(m_\mu) \). It is often forgotten that the first experimental evidence for the running of a coupling constant in quantum field theory comes in fact from the anomalous magnetic moment of the muon in QED, well before QCD and well before the measurement of \( \alpha(M_Z) \).
Vacuum Polarization Diagrams from Tau Loops

The simplest example is the Feynman diagram in Fig. 4 below,

\[ \alpha(m_\mu) = \left[ \frac{1}{45} \left( \frac{m_\mu}{m_\tau} \right)^2 + \mathcal{O}\left( \frac{m_\mu^4}{m_\tau^4} \log \frac{m_\tau}{m_\mu} \right) \right] \left( \frac{\alpha}{\pi} \right)^2. \] (9)

In full generality, internal heavy masses in the vacuum polarization loops (heavy with respect to the external leptonic line) decouple.

Light–by–Light Scattering Diagrams from Electron Loops

It is well known that the light–by–light diagrams in QED are convergent, (once the full set of gauge invariant combinations is considered). Because of that, it came as a big surprise to find out that the set of diagrams in Fig. 5,

\[ a_\mu^{(3)}|_{\text{l.byl.}} = \left[ \frac{2}{3} \pi^2 \ln \frac{m_\mu}{m_e} + \cdots \right] \left( \frac{\alpha}{\pi} \right)^3 = 20.947 \ldots \left( \frac{\alpha}{\pi} \right)^3. \] (10)
This contribution is now known, analytically, for arbitrary values of the lepton masses \(^{12}\). It can be understood in the framework of effective field theories with different scales of masses. Again, it can be easily shown that internal leptonic loops with a heavy mass compared to the external leptonic line, decouple.

Altogether, the purely QED contribution to the muon anomalous magnetic moment of the muon, including \(e, \mu, \tau\) lepton loops is known to an accuracy which is certainly good enough for the present comparison between theory and experiment

\[
a_\mu^{(\text{QED})} = \left(11\,658\,470.57 \pm 0.29\right) \times 10^{-10}.
\]  

(11)

This is the number one gets, using the determination of the fine structure constant

\[
\alpha^{-1} = 137.035\,999\,59(52)[3.8\,\text{ppb}],
\]  

(12)

which follows from the comparison between the experimental determination of the electron (positron) anomalous magnetic moments \(^{13}\) and the QED theoretical prediction (see e.g. ref. \(^{14}\) and references therein). The error in Eq.(11) is mostly due to the experimental errors in the determination of the ratios of lepton masses and the error in the numerical integration of some of the four–loop contributions.

The question which naturally arises is whether or not the discrepancy between the experimental numbers in Eqs.(2) and (3) on the one hand and the QED contribution from leptons alone, can be understood in terms of the extra hadronic and electroweak contributions predicted by the Standard Model.

3 Hadronic Vacuum Polarization

This is the contribution illustrated by the diagram in Fig. 6, with the shade in the vacuum polarization indicating hadrons.

\[
\text{Fig. 6 Hadronic Vacuum Polarization Contribution}
\]
All the estimates of this contribution are based on the spectral representation

\[ a_{\mu}^{(h. \ v.p.)} = \frac{\alpha}{\pi} \int_0^\infty dt \frac{1}{t} \frac{\text{Im}\Pi(t)}{\pi} \int_0^1 dx \frac{x^2(1-x)}{x^2 + \frac{t}{m_\mu^2}(1-x)}, \]  

(13)

with

\[ \sigma(t)_{e^+e^- \to \text{hadrons}} = \frac{4\pi^2\alpha}{t} \frac{1}{\pi} \text{Im}\Pi(t). \]  

(14)

The integration kernel in Eq.(13) shows well the underlying physical features.

- The spectral function \( \text{Im}\Pi(t) \), which is positive, is modulated by a known function of \( t \) which is also positive and monotonously decreasing. The integral is therefore positive and dominated by the low–energy region; mostly by the \( \rho \) resonance.

- In QCD, the spectral function at large–\( t \) goes to a constant, which ensures the UV convergence of the integral. Perturbative QCD fails, however, to reproduce the observed hadronic shape of the spectral function below \( t \sim 1.5 \, \text{GeV}^2 \). In fact, perturbative QCD with massless u and d quarks gives an IR–divergent result, pointing out the importance of non–perturbative effects.

- There is a lower bound to the integral in Eq.(13)

\[ a_{\mu}^{(h. \ v.p.)} \geq \frac{\alpha}{\pi} \frac{1}{3} m_\mu^2 \int_0^\infty dt \frac{1}{t^2} \text{Im}\Pi(t), \]  

(15)

which is governed by the slope of the hadronic vacuum polarization at the origin.

I have compiled in Table 1 the most recent evaluations of the integral in Eq.(13) at the time of the La Thuile meeting, and I refer to the original literature for the details of the various evaluations. While I was writing this talk, there appeared a new detailed evaluation [24], which uses the recent \( e^+e^- \) data from the CMD-2 detector at Novosibirsk, as well as the final analysis of hadronic \( \tau \)–decay from the ALEPH and CLEO detectors at LEP. Unfortunately, the results found for \( a_{\mu}^{(h. \ v.p.)} \) from the \( e^+e^- \)–based data and from the \( \tau \)–based data are inconsistent with each other, even after applying radiative corrections and isospin corrections:

\[ a_{\mu}^{(h. \ v.p.)} = \begin{cases} 
(684.7 \pm 6.0_{\text{exp}} \pm 3.6_{\text{rad}}) \times 10^{-10} & [e^+e^- \text{– based}], \\
(701.9 \pm 4.7_{\text{exp}} \pm 1.2_{\text{rad}} \pm 3.8_{\text{SU}(2)}) \times 10^{-10} & [\tau \text{– based}].
\end{cases} \]  

(16)
Table 1: *Compilation of recent estimates from Hadronic Vacuum Polarization*

| Authors                     | Contribution to $a_\mu \times 10^{10}$ |
|-----------------------------|----------------------------------------|
| Davier–Höcker [16]         | 692.4 ± 6.2                            |
| Jegerlehner [17]           | 697.40 ± 10.45                         |
| Narison [18]               | 702.06 ± 7.56                          |
| de Trocóniz–Ynduráin [19]  | 695.2 ± 6.4                            |

Higher order hadronic vacuum polarization contributions were first estimated in ref. [21]. The most recent evaluation in [22] gives the result

$$a_{\mu}^{(h.o.-h. v.p.)} = -10.0 (0.6) \times 10^{-10}. \quad (17)$$

Concerning this evaluation, one should realize that there may be here a potential problem of double counting with some of the lowest order estimates. This is because *part* of the radiative hadronic corrections of the type indicated by the diagram in Fig. 7 below have already been included in the experimental cross section used to evaluate the *lowest order* contribution in Fig. 6.

**Fig. 7** *Higher Order Hadronic Vacuum Polarization Contribution*

This issue of double counting is under investigation at present [23].

4 Hadronic Light–by–Light Scattering

These are the contributions illustrated by the diagrams in Fig. 8 below
Hadronic Light–by–Light Contributions

All the estimates of these contributions made so far are model dependent. There has been progress, however, in identifying the dominant regions of virtual momenta, and in using models which incorporate some of the required features of the underlying QCD theory. The combined frameworks of QCD in the 1/$N_c$–expansion and of chiral perturbation theory have been very useful in providing a guiding line to possible estimates.

Recent progress in this domain has come from the observation that, in large–$N_c$ QCD and to leading order in the chiral expansion, the dominant contribution to the muon $g – 2$ from hadronic light–by–light scattering comes from the contribution of the diagrams which are one particle (Goldstone–like) reducible; these are the diagrams in Fig. 9 below. The first of these diagrams (Fig. 9a) produces a $\log^2(\mu/m)$–term with a coefficient which is an an exact QCD result:

$$a_{\mu}^{(\pi^0)} = \left(\frac{\alpha}{\pi}\right)^3 \left\{ \frac{N_c^2}{48\pi^2} \left[ \frac{m^2}{F_{\pi}^2} \log^2 \left(\frac{\mu}{m}\right) + \mathcal{O} \left[ \log \left(\frac{\mu}{m}\right) + \kappa(\mu) \right] \right] \right\}. \quad (18)$$

Here, $F_{\pi}$ denotes the pion coupling constant in the chiral limit ($F_{\pi} \sim 90$ MeV); the $\mu$–scale in the logarithm is an arbitrary UV–scale, and $m$ an infrared mass (either $m_\mu$ or $m_{\pi}$).

The dependence on the $\mu$–scale in Eq.(18) would be removed, if one knew the terms linear in $\log \mu$ from Fig. 9b, as well as the constant $\kappa(\mu)$ from the local counterterms generated by Fig. 9c. Unfortunately, the determination of some of the coefficients of the $\log \mu$–terms cannot be made in a completely model independent way; neither the determination of the constant $\kappa(\mu)$. Nevertheless, Eq.(18) plays a fundamental role in fixing the overall sign of the hadronic light–by–light scattering contribution to the muon $g – 2$. In the various hadronic model calculations of this contribution, there

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2Ref. [24] provides a discussion of this point using a renormalization group approach. Essentially the same arguments have been recently emphasized in ref. [26].
appear indeed hadronic scales (usually the $\rho$–mass), which act as an UV–regulator, and play the role of $\mu$ in Eq. (18). Therefore, letting the hadronic scale become large, and provided that the model incorporates correctly the basic chiral properties of the underlying QCD theory, must reproduce the characteristic universal $\log^2(\mu)$ behaviour of Eq. (18), with the same coefficient. This test, when applied to the most recent existing calculations \cite{27,28} (prior to the Knecht–Nyffeler calculation in ref. 4) failed to reproduce the sign of the coefficient of the $\log^2(\mu)$–term in Eq. (18), though the results from the calculations, when extrapolated to large UV–scales, agreed in absolute value with the coefficient of the $\log^2(\mu)$–term. The authors of refs. \cite{27,28} have later found mistakes in their calculations which, when corrected, reproduce the effective field theory test. Their results, now, agree with the Knecht–Nyffeler calculation \cite{4} which we report on next.

The Knecht–Nyffeler Calculation

In full generality, the pion pole contribution to the muon anomaly has hadronic structure, as represented by the shaded blobs in Fig. 10 below. The authors of ref. \cite{4} have shown that, for a large class of off–shell $\pi^0\gamma\gamma$ form factors (which includes the large–$N_c$ QCD class), the contribution from these diagrams has an integral representation over two euclidean invariants $Q_1^2$ and $Q_2^2$ associated with the two loops in Fig. 10:

$$a_{\mu}^{\pi^0(\text{by 1})} = \int_0^\infty dQ_1^2 \int_0^\infty dQ_2^2 \mathcal{W}(Q_1^2, Q_2^2) \mathcal{H}(Q_1^2, Q_2^2),$$

where $\mathcal{W}(Q_1^2, Q_2^2)$ is a skeleton kernel which they calculate explicitly, and $\mathcal{H}(Q_1^2, Q_2^2)$ is a convolution of two generic $\mathcal{F}_{\pi^0\gamma^*\gamma^*}(k_1^2, k_2^2)$ form factors. In Large–$N_c$ QCD,

$$\mathcal{F}_{\pi^0\gamma^*\gamma^*}(k_1^2, k_2^2)|_{N_c \to \infty} = \sum_{ij} \frac{c_{ij}(k_1^2, k_2^2)}{(k_1^2 - M_i^2)(k_2^2 - M_j^2)},$$

with the sum extended to an infinite set of narrow states.

Fig. 10 Hadronic Light–by–Light from a $\pi^0$ State
In practice, the calculation in 4) has been done by restricting the sum in Eq.(20) to one and two vector states, and fixing the polynomial $c_{ij}(k_1^2; k_2^2)$ from general short–distances and long–distances QCD properties. This way, they obtain the result

$$a_{\mu}^{(\pi^0, 1\text{ by } 1)} = (5.8 \pm 1.0) \times 10^{-10}, \quad (21)$$

where the error also includes an estimate of the hadronic approximation. Further inclusion of the $\eta$ and $\eta'$ states results in a final estimate

$$a_{\mu}^{(\pi^0 + \eta + \eta', 1\text{ by } 1)} = (8.3 \pm 1.2) \times 10^{-10}. \quad (22)$$

**A Remark on the Constituent Quark Model (CQM)**

This is perhaps a good place to comment on an argument which is often used in favor of the constituent quark model as a simple way to estimate the hadronic light–by–light scattering contribution to the muon $g - 2$. Since the argument has even appeared in print in some recent papers, I feel obliged to abrogate it here, so as to stop further confusion.

The constituent quark model contribution from the diagram in Fig. 11 below,

![Fig. 11 Hadronic Light–by–Light in the Constituent Quark Model](image)

can be easily extracted from the work of Laporta and Remiddi in ref. [12], with the result

$$a_{\mu}^{(\text{CQM})} = \left( \frac{\alpha}{\pi} \right)^3 N_c \frac{2}{9} \left\{ \frac{3}{2} \zeta(3) \left[ \frac{M_Q}{m_\mu} \right]^2 + \mathcal{O} \left[ \left( \frac{m_\mu}{M_Q} \right)^4 \log^2 \left( \frac{M_Q}{m_\mu} \right) \right] \right\}. \quad (23)$$

Seen from a low energy effective field theory point of view, the constituent quark mass $M_Q$ in the CQM should provide the UV–regulating scale. However, the model is not a good effective theory of QCD and, therefore, it fails to
reproduce the characteristic QCD $\log^2 M_Q$ behaviour when $M_Q$ is allowed to become arbitrarily large; in fact the CQM result above, decouples in the large $M_Q$–limit. The argument of a positive contribution based on the CQM is certainly a simple argument, but unfortunately it is wrong.

Notice however that, contrary to the naive CQM, the constituent chiral quark model of Georgi and Manohar \cite{29} (see also ref. \cite{30}) does indeed reproduce the correct $\log^2 M_Q$ behaviour in the $M_Q \to \infty$ limit. This is because, in this model, the Goldstone particles couple with the constituent quarks in a way which respects chiral symmetry, and the pion pole diagram appears then explicitly. (The same happens in the Nambu–Jona-Lasinio model as well as in its extended version, the ENJL–model \cite{31}). These models, however, suffer from other diseases\footnote{See e.g. the discussion in ref. \cite{32}}, and therefore are not fully reliable to compute the hadronic light–by–light scattering contribution. Hopefully, they will be progressively amended so as to incorporate further and further QCD features, in particular the short–distance constraints, following the line discussed in refs. \cite{33,34}, as already applied to the evaluation of the one particle (Goldstone–like) reducible diagrams in ref. \cite{4} reported above. This is why, at the moment, one can only claim to know the hadronic light–by–light scattering contribution with a cautious generous error, which takes into account these uncertainties. While awaiting for further improvement, the value quoted (at present) by our group in Marseille, based on the combined work of refs. \cite{27,28} (appropriately corrected) and ref. \cite{4}, is

$$a_{\text{hadronic}}^{(\text{light by light})} = (8 \pm 4) \times 10^{-10}. \tag{24}$$

5 Electroweak Contributions

The contribution to the anomalous magnetic moment of the muon from the electroweak Lagrangian of the Standard Model, at the one loop level, originates in the three diagrams of Fig. 12 below,

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig12.png}
\caption{Weak Interactions at the one loop level}
\end{figure}

\footnote{See e.g. the discussion in ref. \cite{32}}
where we also indicate the size of their respective contributions. Their analytic evaluation gives the result 35)

\[
a^{(W)}_\mu = \frac{G_F m^2_\mu}{\sqrt{2} 8\pi^2} \left[ \frac{5}{3} + \frac{1}{3} (1 - 4 \sin^2 \theta_W) + \mathcal{O} \left( \frac{m^2_\mu}{M^2_Z} \log \frac{M^2_Z}{m^2_\mu} \right) \right] + \frac{m^2_\mu}{M^2_H} \int_0^1 dx \frac{2x^2(2 - x)}{1 - x + \frac{m^2_\mu}{M^2_H} x^2} \]  

= 19.48 \times 10^{-10}. \tag{25}
\]

Notice that the contribution from the Higgs decouples and is very small.

Let us recall that the present world average experimental error in the determination of the muon anomaly is 3\( \Delta a^{\mu}_{\text{Exp}} = \pm 8 \times 10^{-10} \), and, hoping for a continuation of the BNL experiment, it is expected to be further reduced. A theoretical effort on the evaluation of the two–loop electroweak corrections is therefore justified. It is convenient to separate the two–loop electroweak contributions into two sets of Feynman graphs: those which contain closed fermion loops, which we denote by \( a^{EW(2)}_\mu (\text{ferm}) \), and the others which we denote by \( a^{EW(2)}_\mu (\text{bos}) \). In this notation, the electroweak contribution to the muon anomalous magnetic moment is

\[
a^{EW}_\mu = a^{W(1)}_\mu + a^{EW(2)}_\mu (\text{bos}) + a^{EW(2)}_\mu (\text{ferm}). \tag{26}\n\]

We shall review the calculation of the two–loop contributions separately.

**Bosonic Contributions**

The leading logarithmic terms of the two–loop electroweak bosonic corrections have been extracted using asymptotic expansion techniques. In fact, these contributions have now been evaluated analytically, in a systematic expansion in powers of \( \sin^2 \theta_W \), up to \( \mathcal{O} (\sin^2 \theta_W)^3 \), where log \( \frac{M^2_W}{m^2_\mu} \) terms, log \( \frac{M^2_H}{M^2_W} \) terms, \( \frac{M^2_H}{M^2_W} \log \frac{M^2_H}{M^2_W} \) terms and constant terms are kept 36). Using \( \sin^2 \theta_W = 0.224 \) and \( M_H = 250 \text{ GeV} \), the authors of ref. 36 find

\[
a^{EW(2)}_\mu (\text{bos}) = \frac{G_F m^2_\mu}{\sqrt{2} 8\pi^2} \frac{\alpha}{\pi} \times \left[ -5.96 \log \frac{M^2_W}{m^2_\mu} + 0.19 \right] = \frac{G_F m^2_\mu}{\sqrt{2} 8\pi^2} \left( \frac{\alpha}{\pi} \right) \times (-79.3). \tag{27}\n\]

**Fermionic Contributions**

The discussion of the two–loop electroweak fermionic corrections is more delicate. Because of the cancellation between lepton loops and quark loops in
the electroweak $U(1)$ anomaly, in the diagrams in Fig. 13, one cannot separate hadronic from leptonic effects any longer. In fact, as discussed in refs. [37, 38], it is this cancellation which eliminates some of the large logarithms which were incorrectly kept in a previous calculation in ref. [39]. It is therefore appropriate to separate the two–loop electroweak fermionic corrections into two classes: one is the class arising from Feynman diagrams like in Fig. 13, with both leptons and quarks in the VVA–triangle, including the graphs where the $Z$ lines are replaced by $\Phi^0$ lines, if the calculation is done in the $\xi_Z$–gauge. We denote this class by $a_{\mu}^{\text{EW}(2)}(l, q)$. The other class is defined by the rest of the diagrams, where quark loops and lepton loops can be treated separately, which we call $a_{\mu}^{\text{EW}(2)}(\text{ferm-rest})$ i.e.,

$$a_{\mu}^{\text{EW}(2)}(\text{ferm-rest}) = a_{\mu}^{\text{EW}(2)}(l, q) + a_{\mu}^{\text{EW}(2)}(\text{ferm-rest}) .$$

The contribution from $a_{\mu}^{\text{EW}(2)}(\text{ferm-rest})$ brings in $m^2_t/M^2_W$ factors. It has been estimated, to a very good approximation, in ref. [38] with the result,

$$a_{\mu}^{\text{EW}(2)}(\text{ferm-rest}) = \frac{G_F}{\sqrt{2}} \frac{m^2_\mu}{8\pi^2} \frac{\alpha}{\pi} \times (-21 \pm 4) . \quad (28)$$

Concerning the contributions to $a_{\mu}^{\text{EW}(2)}(l, q)$, it is convenient to treat the contributions from the three generations separately. The contribution from the third generation can be calculated in a straightforward way, with the result [37, 38]

$$a_{\mu}^{\text{EW}(2)}(\tau, t, b) = \frac{G_F}{\sqrt{2}} \frac{m^2_\mu}{8\pi^2} \frac{\alpha}{\pi} \times \left[ -3 \log \frac{M^2_Z}{m^2_\tau} - \log \frac{M^2_Z}{m^2_b} - \frac{1}{3} \log \frac{m^2_t}{M^2_Z} + \frac{8}{3} \right] + \mathcal{O} \left( \frac{M^2_Z}{m^2_t} \log \frac{m^2_t}{M^2_Z} \right) = \frac{G_F}{\sqrt{2}} \frac{m^2_\mu}{8\pi^2} \frac{\alpha}{\pi} \times (-30.6) . \quad (29)$$
As emphasized in ref. [37], an appropriate QCD calculation when the quark in the loop of Fig. 1 is a light quark should take into account the dominant effects of spontaneous chiral symmetry breaking. Since this involves the $u$ and $d$ quarks, as well as the second generation $s$ quark, it is convenient to lump together the contributions from the first and second generation. A recent evaluation of these contributions [40], which incorporates the QCD long–distance chiral realization as well as short–distance constraints, gives the result

$$a_{\mu}^{EW(2)}(e, \mu, u, d, s, c) = \frac{G_F}{\sqrt{2}} \frac{m_{\mu}^2}{8\pi^2} \frac{\alpha}{\pi} \times (-28.5 \pm 1.8).$$  \tag{30}

Putting together the numerical results in Eqs. (27), (28), (29) with the new result in Eq. (30), we finally obtain the value

$$a_{\mu}^{EW} = \frac{G_F}{\sqrt{2}} \frac{m_{\mu}^2}{8\pi^2} \left[ \frac{5}{3} + \frac{1}{3} \left(1-4\sin^2 \theta_W \right)^2 - \frac{\alpha}{\pi} \left(159 \pm 4\right) \right] = (15.2 \pm 0.1) \times 10^{-10},$$  \tag{31}

which shows that the two–loop correction represents a sizeable reduction of the one–loop result by an amount of 22%.

6 Summary of the Standard Model Contributions

The situation, at present, concerning the evaluation of the anomalous magnetic moment of the muon in the Standard Model, can be summarized as follows:

- **Leptonic QED contributions**

  $$a_{QED}(\mu) = 11\,658.470.57 \pm 0.29 \times 10^{-10}$$

- **Hadronic Contributions**

  - **Hadronic Vacuum Polarization**

    It is clear that, given the present experimental accuracy in Eq. (31), we need now a better understanding of the hadronic vacuum polarization contributions. Issues like the possible double counting already mentioned, and the improvement in the treatment of radiative corrections and isospin corrections [41, 42] have now become extremely important. For reference, I shall choose the two results from the most recent determination in Eq. (16), combined with the higher order vacuum polarization estimate in Eq. (17).
– Hadronic Light–by–Light Scattering

\[ a_{\text{hadronic}}^{(\text{light by light})} = \left(8 \pm 4\right) \times 10^{-10} \]

Work in progress

• Electroweak Contributions

\[ a_{\text{EW}} = (15.2 \pm 0.1) \times 10^{-10} \]

The sum of these contributions, adding experimental and theoretical errors in quadrature, gives then a total

\[ a_{\mu}^{\text{SM}} = \begin{cases} 
(11 659 168.5 \pm 8.1) \times 10^{-10} & \text{[e}^+e^- \text{– based]}, \\
(11 659 185.7 \pm 7.4) \times 10^{-10} & \text{[}\tau\text{– based]}.
\end{cases} \]

(32)

to be compared to the experimental world average in Eq. (4)

\[ a_{\mu}^{\exp} = (11 659 203 \pm 8) \times 10^{-10}. \]

Therefore, with the input for the Standard Model contributions discussed above, one finds:

\[ a_{\mu}^{\exp} - a_{\mu}^{\text{SM}} = \begin{cases} 
(34.5 \pm 11.4) \times 10^{-10} & 3.0\sigma \text{ discrepancy [e}^+e^- \text{– based]}, \\
(17.3 \pm 10.9) \times 10^{-10} & 1.6\sigma \text{ discrepancy [}\tau\text{– based]}.
\end{cases} \]

We should be prepared for a new avalanche of theoretical speculations!

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