Some remarks about mass and Zitterbewegung substructure of electron

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Abstract - It is shown that the electron Zitterbewegung, that is, the high-frequency microscopic oscillatory motion of electron about its centre of mass, originates a spatial distribution of charge. This allows the point-like electron behave like a particle of definite size whose self-energy, that is, energy of its electromagnetic field, owns a finite value. This has implications for the electron mass, which, in principle, might be derived from Zitterbewegung physics.

Key words - Elementary particle masses, Zitterbewegung theory.

Introduction

The nature of mass of elementary particles is one of the most intriguing topics in modern physics. Electron deserves a special attention because, differently from quarks which are bound in composite objects (hadrons), it is a free particle. Moreover, differently from neutrinos whose vanishingly small mass is still controversial, its mass is a well-known quantity.

At the beginning of the past century, the question of the nature of electron mass was debated on the ground of the electromagnetic theory by various authors (Abraham, Lorentz, Poincaré). Actually, the electron charge \( e \) originates in the surrounding space an electromagnetic field \( \vec{E}, \vec{H} \), associated with energy density \( (E^2 + H^2) / 8\pi \) and momentum density \( (\vec{E} \times \vec{H}) / 4\pi c \). These densities, when integrated over the space, account for electron self-energy and inertial mass. However, peculiar models must be introduced in order to avoid divergent results. The simplest assumes that charge \( e \) is spread on the surface of a sphere of radius \( r_0 \). Consequently, an electron at rest originates the electric field \( \vec{E} = e \vec{r} / r^3 \) for \( r \geq r_0 \) and \( \vec{E} = 0 \) for \( r < r_0 \). So, electron self-energy is found to be

\[
\omega_e = \frac{1}{2} \int_{r_0}^{\infty} \frac{e^2}{r^2} dr = \frac{e^2}{2r_0}.
\]

By putting: \( \omega_e = m_e c^2 \) (\( m_e \) standing for the electron rest mass: \( 9.1 \cdot 10^{-28} \text{ g} \)) and \( 2r_0 = r_e \), we get: \( r_e = e^2 / m_e c^2 = 2.8 \cdot 10^{-13} \text{ cm} \). This quantity is referred to as the classical electron radius [1].

The advent of quantum physics made obsolete these models. In quantum electrodynamics, electrons are considered structureless point-like particles. Therefore, in order to evade the consequent divergence problem, renormalization methods were introduced which regard the divergent electron mass as an unobservable quantity. On that account, empirical mass is utilized in applications.

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Nowadays, keeping this state of affairs in mind, most theoretical physicists ascribe particle masses to interactions with the Higgs field, that is, a hypothetical field associated with zero-spin bosons, which permeates the universe. Accordingly, electron mass is assumed proportional to the Higgs vacuum expectation value \( v \), that is,
\[
m_e = g_e v / \sqrt{2},
\]  
(2)
g\(_e\) standing for a coupling constant specific for the electron [2]. This picture provides a physical basis for particle masses, but, owing to the unknown values of coupling constants, actual masses remain empirical.

The electron Zitterbewegung

Electron is rightly described by the relativistic Dirac equation. Besides the spin moment \( \vec{S} \), which is represented by a rank-3 four-dimensional antisymmetric tensor, electron shows an electromagnetic moment whose density-components are represented by an antisymmetric rank-2 four-dimensional antisymmetric tensor [3]. In the reference frame in which electron is at rest the three magnetic components, when integrated over space, yield a moment \( \vec{M} \) related to spin by
\[
\vec{M} = -\frac{e}{m_e c} \vec{S},
\]  
(3)
which, by putting \( \mathcal{M} = |\vec{M}| \), gets
\[
\mathcal{M} = \frac{e\hbar}{4\pi m_e c} = \mu_B,
\]  
(4)
\( \mu_B \) standing for the Bohr magneton. Performing a Lorentz transformation, owing to the different tensorial characters, \( \vec{M} \) and \( \vec{S} \) are no longer parallel and \( \mathcal{M} \) is only approximately equal to the Bohr magneton. So, an electron, in its ground state moving in the field of a charge \( Ze \), shows the magnetic moment
\[
\mathcal{M} = \mu_B \left(1 + 2\sqrt{1 - \alpha^2 Z^2}\right) / 3,
\]  
(5)
\( \alpha \) standing for the fine structure constant [4]. The three electric components of moment tensor yield in turn a dipole moment \( \vec{P} \) related to magnetic moment by
\[
\vec{P} = \frac{\vec{M} \times \vec{r}}{c}
\]  
(6)
[5] [2]. These features of the electromagnetic tensor hardly are compatible with a point-like electron. They fit better with an electron showing an internal structure.

Still, another important property, which was named Zitterbewegung by Schrödinger [3], derives from Dirac equation when dealing with expected electron

\[2\) Really, the electron dipole moment was first derived by J. Frenkel on the ground of mere relativistic considerations.
\[3\) Zitterbewegung can be translated: “Jitterbehaviour”.

2
velocity \[6\]. Different treatments have been given on this subject \[7, 8, 9\]. We consider here the treatment, given by De Broglie, which is especially convenient for our purposes \[3\]. In Dirac theory, operators for \(x, y, z\) velocity components are \(-c\alpha_1, -c\alpha_2, -c\alpha_3\), respectively. So, expected velocity on \(x\) axis is

\[
\langle \dot{x} (t) \rangle = \iiint \sum_{k=1}^{k=4} \psi_k^* (r, t) (-c\alpha_1) \psi_k (r, t) \, d^3 r, \tag{7}
\]

\(\psi_k\) standing for the spinor components. These are given in general by the Fourier’s expansion

\[
\psi_k (r, t) = \iiint \{a_k (\vec{p}) \exp [(2\pi i/\hbar) (Wt - \vec{p} \cdot \vec{r})] + b_k (\vec{p}) \exp [(2\pi i/\hbar) (-Wt - \vec{p} \cdot \vec{r})] \} \, d^3 \vec{p}, \tag{8}
\]

where \(a_k\) and \(b_k\) mean the amplitudes of positive and negative energy states, respectively. The latter cannot be omitted owing to the completeness theorem of Fourier’s expansion. Fermi actually showed that without the contribution of negative energy states a free electron cannot originate Thomson’s scattering of light. This scattering appears as a sort of resonance of the quantum jump of energy \(2me^2/c^2\) between positive and negative energy states \[10\]. Substituting eq. (8) into eq. (7), a sum of conjugated terms is found, representing an oscillation of angular frequency \(4\pi W/\hbar\), that is,

\[
-c \sum_{k=1}^{k=4} \{a_k^* \alpha_1 b_k \exp [(4\pi i/\hbar) Wt] + b_k^* \alpha_1 a_k \exp [(4\pi i/\hbar) Wt] \} = cA_1 \cos \left( \frac{4\pi W}{\hbar} t + \varphi_1 \right), \tag{9}
\]

where amplitude \(A_1\) and phase \(\varphi_1\) depend on moment \(\vec{p}\). The electron Zitterbewegung consists just of this oscillatory behaviour of velocity. It is due to the beat between positive and negative energy states. This result entails that the Ehrenfest theorem is not right in relativistic quantum mechanics. Performing integration with respect to time, straightforward transformations lead to

\[
\langle x (t) \rangle = \langle v_x \rangle t + \sum_{\sigma} \frac{\sigma_e^2}{4\pi W} A_1 \sin \left( \frac{4\pi W}{\hbar} t + \varphi_1 \right), \tag{10}
\]

where

\[
\langle v_x \rangle = \sum_{\sigma} \frac{\sigma_e^2}{W} \sum_{k=1}^{k=4} (a_k^* a_k - b_k^* b_k) \tag{11}
\]

is the time-independent part of \(x\)-component of velocity. In previous equations \(\sigma\) stands for the element \(dp_x dp_y dp_z\) of momentum space. Amplitude \(A_1\) and phase
\(\varphi_1\), like coefficients \(a_k\) and \(b_k\), vary from one element \(\sigma\) to the other. Analogous results are found for \(y\) and \(z\) components.

The electron substructure

We are dealing here with the electron rest mass. We assume, therefore, that energy \(W\) barely exceeds \(m_e c^2\). So, terms dependent on moment \(-\vec{p}\) can be disregarded in comparison with \(m_e c^2\). By putting \(W = m_e c^2\) and omitting time-independent velocity components, previous results allow us to write

\[
\langle x (t) \rangle = \frac{\lambda_e}{4\pi} \sum_\sigma \sigma A_1 \sin \left( \frac{4\pi}{T_Z} t + \varphi_1 \right),
\]

\[
\langle y (t) \rangle = \frac{\lambda_e}{4\pi} \sum_\sigma \sigma A_2 \sin \left( \frac{4\pi}{T_Z} t + \varphi_2 \right),
\]

\[
\langle z (t) \rangle = \frac{\lambda_e}{4\pi} \sum_\sigma \sigma A_3 \sin \left( \frac{4\pi}{T_Z} t + \varphi_3 \right),
\]

(12)

where \(\lambda_e = h/m_e c\) is the Compton wavelength and

\[
T_Z = \frac{h}{m_e c^2} = 8.1 \cdot 10^{-21} \text{s}
\]

(13)

is the time light requires to cover a space equal to Compton wavelength. It is twice the Zitterbewegung period. According to eqs. (12), electron motion consists of a superposition of oscillations of different amplitudes and phases. If oscillations on \(x, y, z\) axes have the same phase, electron moves along a \(\vec{s}\) axis passing through the origin. If also amplitudes have a common value \(A\), electron moves along the \(\vec{s}\) axis performing angle 54.7° with respect to axes, that is,

\[
s = \sqrt{3}\Lambda \sin \left( \frac{4\pi}{T_Z} t \right).
\]

(14)

where \(\Lambda = \lambda_e \sigma A/4\pi\). Owing to factor \(\sigma A/4\pi\), length \(\Lambda\) is expected to be small in comparison with Compton wavelength. If amplitudes are equal but different phases: \(\varphi_1 = \varphi_3 = \pi/2, \varphi_2 = 0\) are considered, electron performs a turn in \(x, y\) plane accompanied by an oscillation along \(z\) axis, that is, utilizing cylindrical coordinates \(r, \vartheta, z\),

\[
r = \Lambda, \quad \vartheta = 4\pi \frac{t}{T_Z}, \quad z = \Lambda \cos \left( \frac{4\pi}{T_Z} t \right).
\]

(15)

The electron moves on a trajectory wrapped round a cylinder of radius \(\Lambda\) and height \(2\Lambda\).

Previous equations show that the oscillating electron owns a dynamical substructure. But, the uncertainty principle sets restrictions to electron oscillations. In fact, by dividing relation \(\delta w \cdot \delta t \simeq h\) by \(m_e c^2\), we obtain

\[
\frac{\delta w}{m_e c^2} \cdot \frac{\delta t}{T_Z} \simeq 1.
\]

(16)
Owing to this reciprocity law, when electron energy \( w \) is determined with a precision such that \( \delta w << m_e c^2 \), indetermination on time is \( \delta t >> T_Z \). In this case the oscillating motion of electron cannot be observed. This occurs, for instance, when dealing with atomic spectroscopy. In fact, the most important quantity which controls precision in atomic spectroscopy is the Rydberg constant \( R_H = 13.6056981 \text{ eV} \). It is known with seven digits, corresponding to an indetermination \( \delta w \) smaller than \( 10^{-7} \text{ eV} \). Taking into account that \( m_e c^2 = 0.5 \text{ MeV} \), we have \( \delta w/m_e c^2 \leq 2 \cdot 10^{-13} \), so that \( \delta t/T_Z \geq 5 \cdot 10^{12} \). This large indetermination in time forces us to eliminate \( t \) in previous equations. Let consider, for instance, the oscillation given in eq. (14). In a half-period electron moves from \( s = -\sqrt{3} \Lambda \) to \( s = +\sqrt{3} \Lambda \). Therefore, probability \( dP \) that electron is found in the space between \( s \) and \( s + ds \) is: \( dP = 4 \cdot \frac{dt}{T_Z} \), \( dt \) standing for the time required to cross space \( ds \). We have thus: \( dP/ds = 4 / \left( sT_Z \right) \). Calculating \( \frac{\cdot s}{2} \) and eliminating \( t \), we get

\[
\frac{dP}{ds} = \frac{1}{\pi \sqrt{3\Lambda^2 - s^2}}.
\] (17)

Analogous considerations apply to trajectories (15). In this case, a complete period is required to cover the trajectory, which leads to: \( dP/ds = 2 / \left( sT_Z \right) \). We have from eqs. (15)

\[
ds^2 = \Lambda^2 d\theta^2 + dz^2 = \Lambda^2 \left[ 1 + \sin^2 \left( \frac{4\pi}{T_Z} \right) \right] dt^2,
\] (18)

which leads to

\[
\frac{dP}{ds} = \frac{1}{2\pi \sqrt{2\Lambda^2 - z^2}}.
\] (19)

The linear oscillation of eq. (17) differs from the cylindrical oscillation of eq. (19) because the former allows charge to be found at the axis origin, while the second places charge only on the cylinder surface. Taking into account that in eqs. (12) a number of oscillations are added up, this is tantamount to say that the oscillating electron originates a spatial distribution of charge. Linear oscillations originate a central distribution of charge, while cylindrical oscillations on \( x, y, \) and \( z \) axes originate an empty shell of charge. This is a very interesting feature, because such shell mimics the distribution of charge on the spherical surface postulated in eq. (1).

It is to be emphasized that the foregoing results are not in conflict with the point-like nature of electron observed in high-energy experiments, such as electron-positron collisions. Late experiments of this kind have shown that electron radius does not exceed \( 10^{-16} \text{ cm} \), that is, at least three orders of magnitude less than the classical radius. In these experiments the collision time \( \tau \), as determined in the reference frame of the electron, is given by the ratio between the impact parameter, that is, the electron size \( 10^{-16} \text{ cm} \), and the velocity of light \( c \), that is, \( \tau = 3 \cdot 10^{-27} \text{ s} \). This time is small with respect to \( T_Z \). In addition,
Lorentz contraction along the direction of collision makes sharper the electric field pulse originated by the colliding positron so further reducing $\tau$. It follows that eqs. (17, 19) cannot be applied.

We come now at the question of electron mass. The opinion that the Zitterbewegung substructure has implications for electron self-energy, that is, for electron rest mass, is not new. It was first advanced by A. O. Barut and A. J. Bracken [8]. These workers, however, left out the possibility that this dynamical substructure originates a spatial distribution of charge. It, in principle, allows evaluation of electron field and consequently of its self-energy, like in eq. (11). For clarity sake, let consider in succession the following steps.

1°) Amplitudes and phases in eqs. (12) are found on the ground of the Zitterbewegung theory. Then, by proceeding as in eqs. (17, 19), electron oscillations are transformed in a spatial distribution of charge.

2°) Evaluation of field and self-energy: $w_e = w_Z(m_e, h, c, e)$ of this distribution.

3°) After substitution of $m_e$ with a variable "trial mass" $\mu$, electron self-energy is evaluated again as a function of $\mu$. Then, function $w_Z(\mu, h, c, e)$ is compared with trial energy $\mu c^2$. If equation

$$w_Z(\mu, h, c, e) = \mu c^2$$

has a solution $\mu = \mu_0$ different from zero, it follows that $m_e = \mu_0$ is the expected electron mass given as a quantity dependent on the fundamental constants $h, c, e$. It is worth to point out in this connection that possible further solutions of eq. (20) might be interpreted as the high-mass flavours, $mu$ and $tau$, of electron.

Discussion and conclusions

Unfortunately a thorough Zitterbewegung theory has not yet been expressed. Apart from amplitudes and phases, different points would require a more deep analysis. Perhaps, it might be necessary to interpret Dirac’s equation as a quantized field equation, rather than at the one-particle level [8]. This poor state of affairs is due to the fact that most theorists relegate Zitterbewegung to the position of an unobservable mathematical curiosity. Consequently, evaluation of actual electron mass is left out for now. Nevertheless, the previous qualitative arguments are interesting because they show that in low energy experiments the point-like electron takes a spatial substructure. Moreover, they show that it is possible, in principle, to deal with the problem of particle masses without introducing new physics as occurs with the Higgs-field theory. The last, on the other hand, is somewhat disappointing because the assignment of the coupling constants to particles is arbitrary. This omits explaining why masses of particles are so different. On the contrary, according to Zitterbewegung theory, particle masses come from interaction with their own fields, so that neutrinos, which show only the short-range weak field, are the less massive. Mass of electrons which show both weak and electromagnetic interactions follows. Quarks, which combine all kinds of interactions, are the most massive particles.

As for the range of weak forces, Higgs-field theory ascribes such very short range to the high masses of weak bosons $W^+, W^-$ and $Z^0$. This poses the
problem of the nature of these masses. In the Higgs-field theory, when the boson system is in its ground state, massless weak bosons acquire mass through a mechanism referred to as spontaneous symmetry breaking. Unlike this interpretation, we proposed in a previous paper that weak bosons remain massless and the short range of weak interactions is due to the screening properties of the negative-kinetic-energy distribution of neutrinos that, according to Dirac’s equation, permeates the universe [9]. The general conclusion ensues: all field-bosons are massless and all fermions acquire mass owing to the fields themselves originate.

References
[1] See for instance: L. Page and N. I. Adams - Electrodynamics (Dover, 1965) Ch. 4.
[2] G. Kane - Modern Elementary Particle Physics (Addison-Wesley, 1987) Ch. 8.
[3] L. De Broglie - L’Electron Magnetique (Hermann, Paris, 1934) Ch. XXI.
[4] G. Breit - Nature 122, 649 (1928).
[5] J. Frenkel - Zeits. f. Phys., 37, 243 (1926).
[6] E. Schrödinger, Sitzungsbl. Preuss. Akad. Wiss. Phys.-Math. Kl. 24, 418 (1930); 3, 1 (1931).
[7] P. A. M. Dirac - The Principles of Quantum Mechanics (Oxford, 1958) § 69.
[8] A. O. Barut and A. J. Bracken, Phys. Rev. D 23, 2454 (1981).
[9] B. R. Holstein - Topics in Advanced Quantum Mechanics (Addison-Wesley, 1992) Ch. VI.
[10] E. Fermi - Rev. Mod. Phys. 4, 120 (1932).
[11] P. Brovetto, V. Maxia and M. Salis - Il Nuovo Cimento A 112, 531 (1999).