Sphalerons on Orbifolds

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Abstract

In this work, we study the electroweak sphalerons in a 5D background, where the fifth dimension lies on an interval. We consider two specific cases: flat space-time and the anti-de Sitter space-time compactified on $S^1/Z_2$. In our work, we take the $SU(2)$ gauge-Higgs model, where the gauge fields reside in the 5D bulk; but the Higgs doublet is confined in one brane. We find that the results in this model are close to those of the 4D Standard Model ($SM$). The existence of the warp effect, as well as the heaviness of the gauge Kaluza-Klein modes make the results extremely close to the $SM$ ones.

Keywords: Sphalerons, Kaluza-Klein modes, Warp Factor.

1 Introduction

The Standard Model ($SM$) of the electroweak and strong interactions has been very successful in describing nature at energies around the electroweak scale ($\sim 100$ GeV). However, it fails in answering many fundamental questions in particle physics, like, e.g., the hierarchy problem and the neutrino mass and its smallness, as well other problems related to cosmology like the baryon asymmetry in the universe and dark matter. Therefore a more fundamental theory, which describes nature at higher scales, needs to become known to explain the problems of particle physics and related topics.

It has been realized that the hierarchy problem could be a consequence of the existence of extra dimensions [1]. A popular realization of this concept is the so-called Randall-Sundrum model [2]. There are several variants of this scenario, depending on whether the extra dimension is finite (RS1) or infinite (RS2), and on which of the fields is confined in a brane or lying on the bulk. In the RS1 models, the space-time has the 5D anti-de Sitter ($AdS_5$) geometry

$$ds^2 = g_{MN}dx^M dx^N = a^2(y) \eta_{\mu\nu} dx^\mu dx^\nu - dy^2$$

$$= e^{-2ky} \eta_{\mu\nu} dx^\mu dx^\nu - dy^2,$$

where $y$ is the fifth dimension that has the properties $y \equiv y + 2\pi R$; and $y \equiv -y$; it is compactified on a half-circle $S^1/Z_2$ with two 4D boundaries ($y = 0, \pi R$). The metric $\eta_{\mu\nu} = diag(-1,1,1,1)$ is the usual 4-dimensional one; and $k$ is the $AdS_5$ curvature. In this model, the relation between the Planck and the $TeV$ scales seems to be natural, $TeV \sim \frac{1}{M_{Pl}} = e^{-2\pi kR} M_{Pl}$, where the two fixed points of the fifth dimension $y = 0, \pi R$ represent the Planck and the $TeV$ branes, respectively.
In the first paper \[2\], only gravity resides in the 5D bulk, while the SM fields are confined in the TeV brane. But problems with some of the SM fields are that propagating in the bulk were also considered, like the case of gauge fields \[3, 4\], scalars \[5\], fermions \[6\], the whole SM content \[7\]; and even supersymmetry \[8\].

As mentioned above, the SM fails to explain the origin of matter in the universe \[9\], it does not fulfill the second and the third Sakharov criteria for baryogenesis \[10\]. Although, the first criterion, baryon number violation, is achieved through the $B+L$ anomaly \[11\], where both of the baryon and lepton numbers are violated by 3 units due to the possible transition between two equivalent neighboring vacua of the nontrivial topology of the $SU(2)$ model. It was shown \[11\] that this transition probability is extremely suppressed, $\sim 10^{-162}$, but this is not the case at higher temperatures. The rate of $B$ violating processes is proportional to $T^4$ at the symmetric phase \[12\] and suppressed like $e^{-E_{Sp}/T}$ in the broken phase \[13\], where $E_{Sp}$ is the system’s static energy within the so-called sphaleron configuration \[14, 15\]; a field configuration that corresponds to the top of the barrier between two neighboring vacua. Due to their relevance to the electroweak baryogenesis scenario \[9\], sphalerons were extensively studied in the literature in extended SM variants as in the SM with a singlet \[16, 17\], the Minimal Supersymmetric Standard Model \[18\]; and in the next-to-Minimal Supersymmetric Standard Model \[19\].

In this work, we will study the sphaleron configuration for a $SU(2)$ gauge-Higgs model in a 5D background, where the gauge fields propagate in the 5D bulk and the Higgs doublet is confined in a brane. We will focus on the warp effect, by comparing the AdS$_5$ results with the flat geometry case. In the second section, the model is shown, where the equations of motion (EOM) for the Higgs field and the Kaluza-Klein (KK) gauge modes are given. The sphaleron configuration within this model is expressed in section three. In the fourth section, we show the profile functions of the gauge and Higgs fields, as well the values of the sphaleron energy in different cases. These results will be compared by those of the SM. Finally, we give our conclusion.

2 SU(2) Gauge Fields in the Bulk

Let us consider a $SU(2)$ Higgs model in the 5D background \[2\], with a general warp factor $a(y)$. The warp factor $a(y) = 1$ refers to the 5D flat geometry; and $a(y) = e^{-ky}$ refers to the $AdS_5$ one. We have $\mu = 0.3$ and $M = \mu, 5$.

In our model, only the gauge fields propagate in the bulk and the Higgs field is confined in one brane. The action that obeys the symmetry is

$$S = \int d^4xdy\sqrt{G} \{\mathcal{L}_{bulk} + \Delta(y)\mathcal{L}_{brane}\},$$

with $G = \text{det}(g_{MN})$, and $\Delta(y) \equiv 2\delta(y), 2\delta(y - \pi R)$ refers to the Higgs localization in the Planck or TeV branes respectively. The boundary Lagrangian is given by

$$\mathcal{L}_{brane} = g^{\mu\nu}(D_\mu H)^\dagger (D_\nu H) - V(H^\dagger H),$$

with the covariant derivative

$$D_M H = \left(\partial_M - \frac{i}{2}g_5\sigma^aA_M^a\right)H,$$

and $g_5 = g\sqrt{\pi R}$ is the 5D $SU(2)$ dimensionful gauge coupling, where $g$ is the 4D one. The bulk Lagrangian is given by

$$\mathcal{L}_{bulk} = -\frac{1}{4}g^{MN}g^{QW}F_{MQ}^aF_{NW}^a,$$

where the 5D field strength is given by

$$F_{MN}^a = \partial_M A_N^a - \partial_N A_M^a + g_5\epsilon^{abc}A_M^bA_N^c.$$

In what follows, we work in the gauge $(\partial^\mu A_\mu^a = 0, A_5^ a = 0)$. The scalar potential has the usual Mexican hat form

$$V(HH^\dagger) = \lambda (H^\dagger H - v^2/2)^2,$$

where $v$ is the Higgs vev. The equations of motion (EOM) can be obtained by the vanishing of the action variation, $\delta S = 0$, and we get

$$\Delta(y)a^4(y)\left[g^{\mu\nu}D_\mu D_\nu H + \frac{\partial}{\partial H}V(H^\dagger H)\right] = 0,$$

$$\frac{i}{2}g_5\Delta(y)a^4(y)\left[H^\dagger \sigma^aD_\mu H - (D_\mu H)^\dagger \sigma^a H\right] - \partial_5 a^2(y)\partial_5 A_\mu^a + \eta^{\alpha\beta}\partial_\beta F^a_{\alpha\mu} = 0,$$
with the boundary condition $\partial_y A^a_\mu = 0$ at both boundaries, $y=0, \pi R$. The gauge fields have to be factorized using the KK decomposition as

$$A^a_\mu(x, y) = \sum_n A^{a(n)}_\mu(x) \chi^{(n)}(y),$$

with

$$\int_0^{\pi R} \chi^{(n)}(y)\chi^{(n)}(y)dy = \delta_{nn}.$$  \hspace{1cm} (12)

Then, the functions $\chi^{(n)}$ should be the eigenstates of the operator

$$-\partial_y a^2(y) \partial_y \chi^{(n)} = M_n^2 \chi^{(n)},$$

with the condition $\partial_y \chi^{(n)} = 0$ at both boundaries; $M_n$ are the KK masses. The zero mode $\chi^{(0)}(y) = 1/\sqrt{\pi R}$; does not depend on the space-time geometry. In flat space-time, the heavy modes (13) are given by

$$\chi^{(n)}(y) = \sqrt{\frac{2}{\pi R}} \cos \left(\frac{2ny}{R}\right),$$

with the eigenvalues $M_n^2 = 4n^2/R^2$. However, in the AdS$_5$ space-time, they have the form

$$\chi^{(n)}(y) = \frac{e^{ky}}{a_n} \left[J_1(\alpha_n e^{ky}) - b_n Y_1(\alpha_n e^{ky})\right],$$

$$b_n = J_0(\alpha_n)/Y_0(\alpha_n),$$  \hspace{1cm} (15)

with $\alpha_n = M_n/k$, and $J_i$ and $Y_i$ are the $i$-th order Bessel functions of first and second kind, respectively; and $a_n$ is a normalization factor which is computed using (12):

$$a_n^2 = \frac{e^{2ky}}{2k} \left[J_1(\alpha_n e^{ky}) - b_n Y_1(\alpha_n e^{ky})\right]^2 \bigg|_{y=\pi R}. $$  \hspace{1cm} (17)

The eigenvalues $M_n$ are determined by imposing the boundary condition $\partial_y \chi^{(n)} = 0\big|_{y=\pi R}$, which are the zeros of the quantity

$$Y_0(\alpha_n e^{\pi k R}) J_0(\alpha_n) - J_0(\alpha_n e^{\pi k R}) Y_0(\alpha_n).$$

These eigenvalues can be obtained numerically.

When inserting (11) in (3) and integrating over $y$, we get a 4D Lagrangian $L_{4D}$ as a function of the Higgs doublet and an infinite number of gauge KK modes. The Higgs doublet is coupled to the KK modes through the parameters $\tau_i$. In addition to the quartic couplings between the KK modes, which are characterized by the parameters $\xi_{ijkl}$, there exist also new cubic couplings characterized by $\gamma_{ijk}$. This feature does exist only in non-Abelian theories unlike in the Abelian case [3, 4]. The 4D Lagrangian is given explicitly in the appendix.

There are some geometry-independent properties of these parameters, like the invariance under the permutation between each two indices. Also we have the equalities: $\gamma_{ij} = \xi_{ij0} = \delta_{ij}$. The 4D SM can be recovered by keeping only zero modes in (31), since all the indices of zeroth order in (31) are exactly 1, whatever the nature of space-time.

The physics at the electroweak scale is more sensitive to the first (and maybe the second) KK mode interactions; therefore, we will include in the appendix only the numerical values of the coupling of heavy modes with the first and second KK modes. The existence of the warp factor makes a difference in the masses of the KK modes $(M_i)$ and their couplings $(\gamma_{ijk})$ and $(\xi_{ijkl})$. In what follows, we will investigate the behavior of the sphaleron configuration with respect to these differences.

### 3 Sphaleron Solutions

It was shown that the 5D anomaly is independent of the bulk physics; the cancelation of the 4D anomaly is sufficient to eliminate the 5D one in orbifold theories [24]. Then the problem of fermionic current non-conservation can be treated as in a 4D theory. In the case of a 5D fermion coupled to an external gauge potential $A^{a}_M(x, y)$ on an $S^1/Z_2$ orbifold, the divergent current is given by [21]

$$\partial_M J^M(x, y) = \frac{1}{2} [\delta(y) + \delta(y - \pi R)] F^{\mu\nu} F_{\mu\nu}/16,$$

\hspace{1cm} (19)

\footnote{This result is given in many works, like for e.g. [5, 4] and [20].}
where $J^M$ is the 5D fermionic current and $\tilde{F}^{a\mu} = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} F^{a\alpha\beta}$ is the dual field strength. The last term in (19) represents the usual 4D chiral anomaly for a Dirac fermion in an external gauge potential $A^a_M(x, y)$. Since the fermions in our model are confined in one brane, the expression (19) becomes, after the integration over the fifth coordinate $y$, like the usual 4D formula,
\[
\partial_\mu J^\mu(x) = F^{a\mu(0)} \tilde{F}^{a(0)} / 32,
\]
where the label $(0)$ means that only zero modes are taken into account [21]; and $J^\mu$ is the 4D fermionic current. This means that there is no new contribution to the fermionic currents divergences beside the 4D ones. In our model, the Higgs doublet potential on the brane admits of a minimum, therefore the static energy is bounded from below. In this case, $N_{CS}=1/2$ represents the so-called sphaleron configuration [14, 15].

Our system has a 5D $SU(2)$ gauge symmetry; it is invariant under the gauge transformation
\[
H \rightarrow UH, \quad i \frac{g}{2} \sigma^a A^a_M \rightarrow i \frac{g}{2} \sigma^a A^a_M + \partial_M U U^\dagger,
\]
where $U$ is a $SU(2)$ element. In the gauge $A^a_5 = 0$, the matrix $U$ should be independent of the fifth dimension; and only the zero mode may ensure the $SU(2)$ gauge invariance. This means that the sphaleron configuration can be defined for the system $(H, A^{a(0)})$ using the 4D transformation matrix $U(\mu, x)$ [15],
\[
U(\mu, x) = \begin{pmatrix}
   e^{i \tau_\mu} (\cos \mu - i \sin \mu \cos \theta) & e^{i \varphi} \sin \mu \sin \theta \\
   -e^{-i \tau_\mu} \sin \mu \sin \theta & e^{-i \varphi} (\cos \mu + i \sin \mu \cos \theta)
\end{pmatrix};
\]
but this system $(H, A^{a(0)})$ is coupled to the heavy $KK$ modes $A^{a(n \neq 0)}$; this effect will be investigated in this work. The sphaleron configuration can be obtained by making $\mu = \pi / 2$.

For reasons of simplicity, we will not use the sphaleron configuration [15], but another, equivalent, representation [22]:
\[
H(x) = \frac{v}{\sqrt{2}} L(r) \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad A^a_0 = 0, \quad A^a_5(x, y) = 2 \frac{\epsilon_{akl} x^i}{g r^2} \sum_n \left[ 1 - f^{(i)}(r) \right] \chi^{(i)}(y).
\]
Here the heavy modes are represented by a similar form as the zero one in order to make the generalization of the orthogonal gauge $x, A^a_5 = 0$ consistent for all the $KK$ modes.

Then, when inserting (23) in (9) and (10), we get the differential equations governing the $f^{(i)}(r)$ modes and $L(r)$. The field’s profile functions $L$ and $f^{(i)}$ are given by the solutions of the system
\[
\frac{\partial}{\partial \zeta} \zeta^2 \frac{\partial}{\partial \zeta} L = 2L \sum_n \sum_m \tau_n \tau_m \left( 1 - f^{(n)} \right) \left( 1 - f^{(m)} \right) + \frac{\lambda}{2 g^2} \zeta^2 L (L^2 - 1),
\]
\[
\zeta^2 \frac{\partial^2}{\partial \zeta^2} f^{(i)} = -\frac{\lambda}{2} L^2 \sum_n \sum_m \tau_m \left( 1 - f^{(m)} \right) - 2 \left( 1 - f^{(i)} \right) - \frac{\lambda}{2} \frac{M^2}{g^2} \left( 1 - f^{(i)} \right) + 6 \sum_m \sum_n \gamma_{imk} \left( 1 - f^{(m)} \right) \left( 1 - f^{(k)} \right)
- 4 \sum_m \sum_k \sum_l \xi_{imkl} \left( 1 - f^{(m)} \right) \left( 1 - f^{(k)} \right) \left( 1 - f^{(l)} \right),
\]
where $\zeta = g v r$ is the dimensionless radial coordinate, $M_\ell$ are the $KK$ modes eigenmasses; and the $\tau$ parameters, $\gamma_{ijk}$ and $\xi_{ijkl}$ are given in the appendix. Here, one needs to mention that the equations (23), (24) and (25) are referring to both cases where the Higgs doublet is localized on the Planck or TeV branes. Here one needs to mention that in the TeV brane case, the Higgs doublet as well the 4D brane parameters needs to be redefined (for e.g. $a(\pi R) H \rightarrow H$) in order to be canonically normalized.

The static energy of the system is given by
\[
E = \frac{4 \pi a}{g} \int_0^\infty d\zeta \left[ \frac{\dot{L}^2}{g^2} (L^2 - 1)^2 + \frac{\dot{L}^2}{g^2} (L^2 - 1)^2 + L^2 \sum_n \sum_m \tau_n \tau_m \left( 1 - f^{(n)} \right) \left( 1 - f^{(m)} \right) \right]
+ 4 \sum_n \left( \frac{\partial^2}{\partial \zeta^2} f^{(n)} \right)^2 + \frac{\dot{L}^2}{g^2} (L^2 - 1)^2 + 4 \sum_n \sum_k \sum m \gamma_{nmk} \gamma_{nmk} \left( 1 - f^{(m)} \right) \left( 1 - f^{(k)} \right) \left( 1 - f^{(n)} \right)
\]
\[
+ \frac{\lambda}{g} \sum_n \sum m \sum k \sum l \xi_{nmkl} \left( 1 - f^{(n)} \right) \left( 1 - f^{(m)} \right) \left( 1 - f^{(k)} \right) \left( 1 - f^{(l)} \right).
\]
When comparing equations (23), (24) and (25) with their corresponding equations in [15]; we find that instead of the gauge profile function $f$, we have a summation over an infinite number of $f^{(i)}$; and also the Higgs-gauge, cubic and quartic gauge-gauge couplings get modified as
\[ \begin{align*}
L^2 (1 - f)^2 & \to \sum_m \tau_n \tau_m L^2 (1 - f^{(n)}) (1 - f^{(m)}), \\
(1 - f)^3 & \to \sum_m \sum_{n \neq m} \gamma_{nmk} (1 - f^{(n)}) (1 - f^{(m)}) (1 - f^{(k)}), \\
(1 - f)^4 & \to \sum_m \sum_k \sum_l \xi_{nmkl} (1 - f^{(n)}) (1 - f^{(m)}) (1 - f^{(k)}) (1 - f^{(l)}),
\end{align*} \]

(27)
in addition to the presence of mass terms for non-zero gauge KK modes. Indeed, when neglecting the massive gauge KK modes, the EOM (24) and (25) tend to (11); and (26) tends to (10) in [15].

The convergence of the energy functional (26) implies the following boundary conditions on the profiles functions \( L \) and \( f^{(i)} \):

\[
\text{For } \zeta \to 0: \quad L \sim \zeta; \quad f^{(0)} \sim \zeta^2; \quad f^{(i)} \sim 1,
\]

(28)
and \( \zeta \to \infty: \quad L \sim 1; \quad f^{(0)} \sim 1; \quad f^{(i)} \sim 1.
\]

(29)

We use the relaxation method to integrate this system of differential equations. The infinite summations in (24), (25) and (26) over the gauge KK modes are practically impossible analytically as well as numerically. We expect that the contributions of the heavy gauge KK modes \( (n \geq 1) \) are just corrections to the energy of the system \( (H, A^{a(0)}) \); we will consider only a finite number \( N \) of the KK modes and then examine the variation of the energy (26), as well as the profile functions \( L \) and \( f^{(n)} \) with respect to this number \( N \) for both cases of flat and warped geometries, with different values of the warp factor and the first KK mass.

### 4 Numerical Results and Discussion

In our computations, we will take the Higgs mass to be around 120 GeV, i.e., \( \lambda \simeq 0.12 \). For a rigorous comparison between the flat and warped cases, we fix the mass of the first heavy KK mode, which represents in a way the scale of the new physics beyond SM, and we will consider the values 600 GeV, 2 TeV and 10 TeV. In general, the warp factor \( w = e^{8kR_w} \) value is chosen in a way as to represent the hierarchy between the Planck and TeV scales, i.e. \( w \sim 10^{16} \). But since we are interested also to investigate its effect on the sphaleron configuration, we will vary the size of the extra dimension to give it different values for the warp factor: \( w = 10^4, 10^8 \) and the desired one, \( 10^{16} \).

![Figure 1: The masses of the gauge KK modes for the cases of flat and warped geometry with different values of the warp factor w.](image)

In Fig. 1 the masses of KK modes are shown for both flat and warped backgrounds, where the first KK heavy mode mass is chosen to be 1 TeV. It is clear that the flat modes are just multipliers of the first heavy one, while the existence of the warp factor makes the warped mode masses increasing with respect to the warp factor \( w \).

For the Higgs-gauge and gauge-gauge couplings, they are given in unit of the \( SU(2) \) coupling \( g \); by the parameters \( \tau, \gamma \) and \( \xi \). All these parameters are of order \( O(1) \) in the flat geometry. In warped geometry, the situation is different, the \( \tau \) parameters; that represent the couplings of the Higgs with gauge KK modes, depend on which boundary the Higgs filed is located in. If the Higgs field is located in the Planck brane, these parameters are negative and their
modulus is less than unity and decaying with respect to the $KK$ masses, and also with respect to the warp effect. If the Higgs doublet is located in the TeV brane, the values of the $\tau$ parameters are of the order $O(1)$ but positive for odd modes and negative for the even ones; and their modules are almost stable with respect to the $KK$ masses. The previous difference between the two cases will not change significantly the profile functions of $L$ and $f^{(i)}$ or the sphaleron energy (26). The difference between the sphaleron energy in both cases is less than $0.004\%$ for $w = 10^{16}$ and $M_1 = 1 \text{ TeV}$.

The $\gamma$ parameters that describe the cubic couplings between the gauge $KK$ modes are also small in the AdS$_5$ background and decaying with respect to the $KK$ masses. However, the $\xi$ parameters that represent the quartic couplings between the gauge $KK$ modes are large (for e.g. $\xi_{1,1,1,1} \sim 46$) and decaying with respect to the $KK$ masses but still remaining large (for e.g. $\xi_{30,30,30,30} \sim 27$).

The profile functions $L$ and $f^{(0)}$ are given in Fig. 2. They are very close to the SM ones to a very high precision for both the cases of flat and warped geometries. This feature does not depend on $N$, the number of the heavy modes taken into account to solve (24) and (25). However, the profile functions of the heavy modes $f^{(i)}$, as shown in Fig. 3, are just deviations from 1; and these deviations decrease with respect to the $KK$ masses.

![Figure 2: The profile functions $L$ (upper curve) and $f^{(0)}$ (lower curve) for the SM case, flat geometry and the warped geometry as a function of the dimensionless radial coordinate $\zeta$. Each profile function is almost identical for the different cases. This plot was performed taking into account the first 10 heavy $KK$ modes for both flat and warped geometries for $M_1 = 1 \text{ TeV}$ and $w = 10^{16}$.](image)

![Figure 3: From up to down, here are the profile functions $f^{(i)}$, of the first five heavy modes for the flat geometry case (up) and warped geometry (down) for the same values of $M_1$ and $w$ taken in Fig. 2 as a function of the dimensionless radial coordinate $\zeta$.](image)

We remark that the profile functions of the heavy modes $f^{(i)}$, are more suppressed in the case of warped geometry than in the flat one. However, the suppression effect decreases if we decrease the warp factor; for, e.g., when taking the warp factor to be $w = 10^4$ instead of $10^{16}$, the maximum of $f^{(1)}$ (the upper curve in the right side of Fig. 3) increases from 1.00037 to 1.00075. This suppression increases also if we increase the first $KK$ mode mass.
Due to the fact that the profile functions of $L$ and $f^{(0)}$ practically do not change with respect to the SM results, and in addition to the suppression of the heavy modes profile functions, one expects that the sphaleron energy should not be very different from the SM value, but this is not guaranteed due to the infinite number of terms in Eq. (26), as well the increasing $KK$ mass values, unless confirmed numerically.

To check this, we compute the sphaleron energy (26) taking into account a finite number $N$ of $KK$ modes for the different values of the first heavy $KK$ mode mass and the warped factor mentioned above. The sphaleron energy dependence on the index $N$ is shown in Fig. 4.

![Figure 4: The dependence of the sphaleron energy on the number of heavy KK modes that are taken into account to estimate (26); for different values of the first KK mode mass.](image)

The first remark on the results in Fig. 4 is that the sphaleron energy does differ significantly from the SM value; its largest deviation is in the case of a small mass of the first $KK$ heavy mode with flat geometry (first plot in Fig. 4), which is $-0.06\%$, i.e. much less than $1\%$. Also, the existence and largeness of the warp factor makes the sphaleron energy practically identical to the SM value. However, this feature is due to the sphaleron configuration itself, i.e. $(H, A_{\mu}^{(0)})$, rather than the decoupling effect of the heavy $KK$ modes, because if we consider an extreme case of a flat geometry with a small mass for the first $KK$ mode (for e.g. $300 \text{ GeV}$, and then $100 \text{ GeV}$), the sphaleron energy decreases only by $-0.9\%$ and $-6\%$, respectively. This can be explained by the fact that most of the sphaleron energy is coming from the contributions of the gauge zero mode and the Higgs fields; and the profile functions of these fields are determined by self-interactions as well as interactions with each other rather than their interactions with the heavy $KK$ modes. Then one can say that the heavy $KK$ modes are just compensating fields in the EOM (24) and (25), as in the case of the singlet in the model of SM+singlet [17]. This could explain the fact that the contributions of the $KK$ modes to the sphaleron energy (26) are very small even though their cubic ($\gamma$) and quartic ($\xi$) coupling are (very) large. Indeed, the sphaleron energy (26) is more sensitive to the first $KK$ eigenmass rather than to the couplings $\tau, \gamma$ and $\xi$.

At finite temperature, we do not expect to have a deviation in sphaleron field’s profile functions as well as in the values of sphaleron energy from the results of the SM [13]; and the $B+L$ anomaly is almost the same as in the standard theory. Then the criterion for a strongly first-order phase transition remains the known one, $v_{c}/T_{c} \geq 1$. 

5 Conclusion

In this work, the sphaleron configuration for a Higgs model in a 5D space-time is studied, where the Higgs is confined in a brane and the gauge field resides in the 5D bulk. When we made the KK decomposition of the gauge field, we found that possible interactions (cubic and quartic) between different KK modes are possible due to the non-Abelian nature of the symmetry group unlike the Abelian case [3,4]. The strength of these interactions depends on the space-time nature. The strength of the interaction with the Higgs doublet depends on where it is located in.

We defined the sphaleron configuration in this case, where we got the equations like the SM case, but corrected by the existence of the KK heavy modes. Practically the profile functions of the Higgs and zero mode gauge fields do not change when comparing with the SM results; and the heavy mode profile functions are just little deviations from 1. The suppression of this deviation from unity is proportional to the KK order. Also the existence of a strong warp factor (like $w = 10^{16}$) suppresses these deviations by one order of magnitude.

We checked also that the sphaleron energy has the same value as the SM one. The heavy KK modes do not practically contribute to the sphaleron energy; and their presence decreases the value of sphaleron energy by $-0.25\%$ for a light mass of the first KK heavy mode ($600\: GeV$) in a flat geometry. The existence of a warp factor; or the increasing of the mass of the first KK heavy mode, which represents somehow the new physics scale, suppresses the deviation from the SM results.

This allows us to suppose that at finite temperature, the previous results should differ from those of the SM. In addition to the fact that the 5D $B+L$ anomaly is identical to the 4D one, the criterion of a strong first-order phase transition, $v_c/T_c \geq 1$, is still valid for these models.

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A Explicit 4D Lagrangian

The 4D theory can be obtained by integrating over the fifth dimension. Here we explicitly give the 4D Lagrangian with its different parameters that describe the couplings of the gauge KK modes with themselves as well as with the Higgs doublet. It is given by

$$L_{4D} = \eta^\mu \eta^\nu \partial_\mu H^\dagger \partial_\nu H - V H^\dagger H - \frac{1}{2} g m_H \sum_n \tau_n A_\mu^{(n)} H^\dagger \sigma^\mu H + \frac{1}{2} g m_H \sum_n \tau_n \sum_n m_n^2 H^\dagger H$$

$$+ \delta_{nm} M_n^2 A_\mu^{(n)} A_\nu^{(m)} - \frac{1}{2} \eta^\mu \eta^\nu \sum_n \left[ \partial_\mu A_\alpha^{(n)} \partial_\nu A_\alpha^{(n)} - \partial_\nu A_\alpha^{(n)} \partial_\mu A_\alpha^{(n)} \right] - g m_H \sum_n \sum_n \xi_{nmk} \sum_{m,k} A_\alpha^{(n)} A_\alpha^{(m)} A_\beta^{(m)} A_\beta^{(k)}.$$  

(30)

The parameters $\tau_n$, $\gamma_{nmk}$ and $\xi_{nmkl}$ are given by

$$\tau_n = \sqrt{\pi R} \int_0^{\pi R} \sqrt{C} \Delta(y) \chi^{(n)}(y) \, dy, \quad \gamma_{nmk} = \sqrt{\pi R} \int_0^{\pi R} dy \chi^{(n)}(y) \chi^{(m)}(y) \chi^{(k)}(y),$$

$$\xi_{nmkl} = \pi R \int_0^{\pi R} dy \chi^{(m)}(y) \chi^{(n)}(y) \chi^{(k)}(y) \chi^{(l)}(y).$$  

(31)

In a flat space-time, these parameters can be reduced to

$$\tau_n = 1/\sqrt{2}, \quad \gamma_{nmk} = \{ \delta_{0,m+k-n} + \delta_{0,m-k-n} + \delta_{0,m+k+n} \} / \sqrt{2},$$

$$\xi_{nmkl} = \{ \delta_{0,n+m+k-l} + \delta_{0,n+m-k+l} + \delta_{0,n+m+k+l} + \delta_{0,n-m+k+l} \} / 2.$$  

(32)

In the $AdS_5$ space-time, the formulae of the $\tau_i$ parameters are given in both the cases where Higgs field is confined in the Planck (Pl) and $TeV$ branes by

$$\tau_n^{(Pl)} = \sqrt{\pi R} \chi^{(n)}(0), \quad \tau_n^{(TeV)} = \sqrt{\pi R} \chi^{(n)}(\pi R).$$  

(33)
Table 1: Different values of the parameters $\tau_i$ for different values of the warp factor in both the cases where the Higgs doublet is confined in the Planck brane (left) or TeV brane (right).

| i   | $w = 10^4$ | $w = 10^8$ | $w = 10^{16}$ | $w = 10^4$ | $w = 10^8$ | $w = 10^{16}$ |
|-----|------------|------------|---------------|------------|------------|---------------|
| 1   | -0.1955  | -0.1352   | -0.0945      | 2.1549    | 3.0379    | 4.2930       |
| 2   | -0.1453  | -0.0950   | -0.0645      | -2.1509   | -3.0363   | -4.2924      |
| 3   | -0.1236  | -0.0782   | -0.0523      | 2.1495    | 3.0359    | 4.2923       |
| 4   | -0.1107  | -0.0683   | -0.0453      | -2.1488   | -3.0356   | -4.2922      |
| 5   | -0.1018  | -0.0617   | -0.0405      | 2.1484    | 3.0355    | 4.2921       |
| 6   | -0.0952  | -0.0568   | -0.0371      | -2.1481   | -3.0354   | -4.2921      |
| 7   | -0.0900  | -0.0530   | -0.0344      | 2.1479    | 3.0353    | 4.2921       |
| 8   | -0.0858  | -0.0500   | -0.0322      | -2.1477   | -3.0353   | -4.2921      |
| 9   | -0.0823  | -0.0473   | -0.0304      | 2.1475    | 3.0352    | 4.2920       |
| 10  | -0.07936 | -0.0452   | -0.0289      | -2.1474   | -3.0352   | -4.2920      |

In the following table, we give the first 10 values of the $\tau_i$ parameters for different values of the warp factor. For the parameters $\gamma$ and $\xi$, it is easy to check that they depend only on the warp factor $w$, and not on the first $KK$ mass $M_1$. Their formulae are complicated; and therefore they could be computed numerically.

As stated above in section 2, it is important to estimate the couplings of the heavy modes with the zero and first one (and maybe the second one). Here we give the numerical values of $\gamma_{1,i}$, which represents the cubic coupling of two one modes with a heavier one ($i \geq 2$), or equivalently, the quartic coupling of a zero mode, two one modes and a heavier one. We give also the value of $\xi_{1,i,1}$, which represents the quartic coupling of three one modes and a heavier one, taking the value of the warp factor to be $w = 10^4$, $10^8$, $10^{16}$.

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Table 2: Different values of the cubic $\gamma_{1,i,i}$ and $\gamma_{1,2,i}$ and quartic $\xi_{1,1,1,i}$ and $\xi_{1,1,2,i}$ gauge-gauge couplings for $w = 10^4$, $10^8$, $10^{16}$.

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