Polarized $Z$ cross sections in Higgsstrahlung for the
determination of anomalous $ZZH$ couplings

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Abstract

The production of a Higgs boson in association with a $Z$ at an electron-positron collider is one of the cleanest methods for the measurement of the couplings of the Higgs boson. In view of the large production cross section at energies a little above the threshold, it seems feasible to make a more detailed study of the process by measuring the cross sections for polarized $Z$ in order to measure possible anomalous $ZZH$ couplings. We show that certain combinations of cross sections in $e^+e^- \rightarrow ZH$ with different $Z$ polarizations help to enhance or isolate the effect of one of the two kinds of anomalous $ZZH$ couplings possible on general grounds of CP and Lorentz invariance. These combinations can be useful to get information on the $ZZH$ coupling in the specific contexts of an effective field theory, two-Higgs-doublet models, and composite Higgs models, in a relatively model-independent fashion. We find in particular that the longitudinal helicity fraction of the $Z$ is expected to be insensitive to anomalous couplings, and would be close to its value in the standard model in the scenarios we consider. We also discuss the sensitivity of the proposed measurements to the anomalous couplings, including longitudinal beam polarizations, which suppress backgrounds, and can improve the sensitivity if appropriately chosen.

Experiments at the Large Hadron Collider (LHC) have studied in great detail the properties of the Higgs boson, especially its couplings to fermions and gauge bosons, with increasing precision. The results seem to be in agreement with the predictions of the standard model (SM) to a good degree of
accuracy. However, there is still a possibility that the Higgs couplings are not precisely those predicted by the SM, and more data from future experiments at the LHC should be able to improve the accuracy of the comparison.

Another prospect for acquiring more information on the Higgs being considered is the construction of an electron-positron collider. At such a collider, the process of associated $Z$ and Higgs production would be an attractive way to study Higgs properties because of a clean environment and a reasonably large cross section at energies not far above threshold. In addition, the Higgs energy-momentum can be reconstructed from the $Z$ decay products regardless of the Higgs decay final state.

In the context of the process $e^+e^- \rightarrow ZH$ alluded to above, deviations from the SM predictions, if seen, would most likely indicate $ZZH$ couplings differing from those in the SM, and perhaps also $\gamma ZH$ couplings. We concentrate here on so-called anomalous $ZZH$ couplings and their effect on the $ZH$ production process. Our aim is to investigate how $Z$ polarization can be used to determine the coefficients of different Lorentz tensors in a general $ZZH$ vertex written in a model-independent way.

In [1] we considered angular asymmetries of charged leptons arising in $Z$ decay which characterize the $Z$ spin density matrix and could be used to determine $ZZH$ couplings. These included polar and azimuthal angle asymmetries of leptons, the azimuthal angular dependence arising from the off-diagonal elements of the density matrix, based on the formalism of [2, 3]. Here we concentrate on the simpler diagonal elements of the density matrix, which are the degrees of polarization of the $Z$, and which can be probed without a detailed study of the azimuthal distribution of leptons. A significant result is that certain combinations of the polarized $Z$ cross sections enable isolation of specific anomalous couplings.

Our study is in the context of a few specific scenarios for $ZZH$ couplings differing from those in the SM. Ref. [1] dealt with a completely model independent set of form factors, only constrained by Lorentz invariance. While one could measure several asymmetries and use them simultaneously to determine or limit the form factors, the process would be fairly complicated, and would be lacking in accuracy because of the many variables involved. However, in the special scenarios considered in this paper, using combinations of polarized cross sections, a certain conceptual and practical simplification results, and the dependence of the observables is only on one, or at most two parameters, leading to better accuracy. Though these scenarios vary in the assumptions that are made in them, they are reasonably model-independent,
within those set of assumptions. The different combinations of polarized cross sections that we suggest have a varied advantage in each scenario.

In all cases, we assume that longitudinally polarized beams would be available, whose signs can be chosen to suppress background and improve the sensitivity of the observables.

Other studies involving $Z$ polarization are [4], in the context of the process $e^+e^- \rightarrow ZH$ and [5, 6] in the context of determination of anomalous $ZZH$ couplings at the LHC.

There have been several studies in the past for the measurement of anomalous $ZZH$ couplings using final state asymmetries in $e^+e^- \rightarrow ZH$ without reference to $Z$ polarization, see [7, 8] and references therein. These do not have the advantage of the kind of observables we are suggesting here.

We consider the process $e^+e^- \rightarrow ZH$, where the vertex $Z^*_\mu(k_1) \rightarrow Z\nu(k_2)H$ has the Lorentz structure

$$
\Gamma^V_{\mu\nu} = \frac{g}{\cos \theta_W} m_Z \left[ a_Z g_{\mu\nu} + \frac{b_Z}{m_Z^2} (k_{1\nu}k_{2\mu} - g_{\mu\nu}k_{1\cdot}k_{2\cdot}) + \frac{\tilde{b}_Z}{m_Z^2} \epsilon_{\mu\nu\alpha\beta} k_1^\alpha k_2^\beta \right],
$$

where $g$ is the $SU(2)_L$ coupling and $\theta_W$ is the weak mixing angle. The couplings $a_Z$, $b_Z$ and $\tilde{b}_Z$ are Lorentz scalars, and depending on the framework employed, are either real constants, or complex, momentum-dependent, form factors. The $a_Z$ and $b_Z$ terms are invariant under CP, while the $\tilde{b}_Z$ term corresponds to CP violation. In the SM, at tree level, the coupling $a_Z = 1$, whereas the other two couplings $b_Z$ and $\tilde{b}_Z$ vanish.

We consider here some possibilities for the couplings $a_Z$, $b_Z$ and $\tilde{b}_Z$ in various scenarios.

(a) In an effective field theory (EFT) description of new physics [9, 10] (for applications to $ZH$ production, see the works of Hagiwara and Stong in [8] and of Beneke et al., in [7]), where the SM is the low-energy limit of an extended theory, $a_Z$ would be normalized in the SM to the value of 1 and would get a contribution $\delta a_Z$ of order $1/\Lambda^2$ from dimension-six operators, so that $a_Z = 1 + \delta a_Z$. $b_Z$ and $\tilde{b}_Z$ would get contributions of order $1/\Lambda^2$ from dimension-six operators, and would be suppressed. They would, however, be real, from Hermiticity.

The EFT Lagrangian, including terms up to dimension 6 takes the form

$$
\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}}^{(4)} + \frac{1}{\Lambda^2} \sum_{k=1}^{59} \alpha_k \mathcal{O}_k,
$$

(2)
with a sum of 59 independent terms of dimension 6, of which 11 are relevant for our process. Of these, we can identify \( \delta a_Z = \hat{\alpha}^{(1)}_{ZZ} \) and \( b_Z = \hat{\alpha}_{ZZ} \), where \( \hat{\alpha}^{(1)}_{ZZ} \) and \( \hat{\alpha}_{ZZ} \) in the notation of [11] are combinations of coefficients of dimension-6 operators with a weak coupling factor \( m_Z^2 (\sqrt{2} G_F)^{1/2} \) pulled out.

(b) In two-Higgs-doublet models (for a review, see [12]) at tree level, \( a_Z = \sin(\alpha - \beta) \), where \( \tan \beta = v_2/v_1 \), \( v_{1,2} \) being the vacuum expectation values of the two neutral Higgs fields, and \( \alpha \) is the mixing angle between the physical scalar eigenstates and Lagrangian scalar fields. \( \alpha \) can have a real value different from 1, whereas \( b_Z \) and \( \tilde{b}_Z \) are zero. In these models, we will consider the coupling of the lighter of the two CP-even Higgs particles. In the case of other extensions of the sector with more doublets, singlets or triplets, the situation would be similar.

(c) In composite Higgs models [13, 14, 15] the coupling \( a_Z \) is different from unity, modified by a model-dependent reduction factor. This factor in the so-called minimal composite Higgs models [14, 15] is described by one parameter \( \xi \), and is given by \( \sqrt{1 - \xi} \), where \( \xi = v^2/f^2 \), \( v \) being the scalar vacuum expectation value characterizing the electroweak breaking scale, and \( f \) being the scale of compositeness. For \( f \) of order TeV, \( v/f \ll 1 \), and \( a_Z = \sqrt{1 - \xi} \approx 1 - \frac{1}{2} \xi \). \( b_Z \) in these models is expected to be small, of order \( m_Z^2/f^2 \) [15].

As mentioned earlier, we aim to constrain the above anomalous couplings with specific combinations of polarized cross sections. A hermitian, traceless, \( 3 \times 3 \) density matrix gives a complete description of the polarization states of spin one, characterized by 8 polarization parameters [17]. The diagonal elements of the density matrix correspond to pure polarization states. With the final-state phase space appropriately put in, these diagonal elements would give us production cross sections with definite Z polarization. We are interested in constructing combinations of these polarized Z cross sections, such that each combination would be dominantly sensitive to one of the two couplings.

On the experimental side, polarization of weak gauge bosons has been measured at the LHC in \( W + \text{jet} \) production [18, 19], \( Z + \text{jet} \) production [20, 21], \( W \) produced in the decay of top quarks [22] and more recently in \( WZ \) production [23] and same-sign \( WW \) production [24]. The gauge-boson polarizations and helicity fractions are inferred from the angular distributions of the fermions to which the gauge bosons decay [25]. It should be possible to determine the Z polarization in the \( e^+e^- \rightarrow ZH \) in the same way. See
for a recent suggestion for measurement of $W$ polarization.

For the following, we assume for simplicity of some of our expressions that there is no CP violation, $\tilde{b}_Z = 0$. However, there is no loss of generality because a nonzero $\tilde{b}_Z$ would not contribute to the observables we consider here. The implications on including loop-level contributions from, say, triple-Higgs couplings can be different and important.

The density matrix elements and the helicity amplitudes from which they are constructed were obtained in [1]. The relevant diagonal elements of the density matrix integrated over phase space are given by

\[
\sigma(\pm, \pm) = \frac{F(P_L, P_L)g^4m_Z^2|\vec{k}_Z|^2}{96\pi\sqrt{s}\cos^4\theta_W(s - m_Z^2)^2} \left[ |a_Z|^2 - 2\text{Re}(a_Zb_Z^*) \frac{E_Z\sqrt{s}}{m_Z^2} + |b_Z|^2 \frac{E_Z^2s}{m_Z^4} \right],
\]

\[
\sigma(0, 0) = \frac{F(P_L, P_L)g^4E_Z^2|\vec{k}_Z|^2}{96\pi\sqrt{s}\cos^4\theta_W(s - m_Z^2)^2} \left[ |a_Z|^2 - 2\text{Re}(a_Zb_Z^*) \frac{\sqrt{s}}{E_Z} + |b_Z|^2 \frac{s}{E_Z^2} \right].
\]

Here, $E_Z = (s - m_{H}^2 + m_Z^2)/(2\sqrt{s})$ is the $Z$ energy in the c.m. frame, and $|\vec{k}_Z| = \sqrt{E_Z^2 - m_Z^2}$ is the magnitude of the $Z$ three-momentum. $c_V$ and $c_A$ are respectively the vector and axial-vector couplings of the $Z$ to the electron, given by $c_V = \frac{1}{2}(-1 + 4\sin^2\theta_W)$ and $c_A = -\frac{1}{2}$. The beam polarization dependence enters through the combination

\[
F(P_L, P_L) = (1 - P_L\bar{P}_L)(c_V^2 + c_A^2 - 2P_L^{\text{eff}}c_Vc_A),
\]

where $P_L$ and $\bar{P}_L$ are respectively the degrees of electron and positron beam longitudinal polarization, and $P_L^{\text{eff}} = (P_L - \bar{P}_L)/(1 - P_L\bar{P}_L)$. The total cross section is then $\sigma = \sigma(\pm, \pm) + \sigma(0, 0) + \sigma(-, -)$.

We now take up three combinations constructed out of the polarized cross sections, and discuss their features and advantages.

1. The quantity $\sigma_T - 2\sigma_L$, where $\sigma_T \equiv \sigma(\pm, \pm) + \sigma(-, -)$ and $\sigma_L \equiv \sigma(0, 0)$ are respectively the cross sections for the production of transverse and longitudinally polarized $Z$, is independent of $b_Z$ to first order, and depends only quadratically on $b_Z$.

In [1] we showed that a decay-lepton angular asymmetry $A_{\pm} = \frac{3}{16}(\sigma_T - 2\sigma_L)/\sigma$ can be used to put a limit on $\text{Re} b_Z$. However, the numerator of this asymmetry, which is proportional to

\[
\Delta\sigma_1 \equiv \sigma_T - 2\sigma_L = \frac{F(P_L, P_L)g^4|\vec{k}_Z|^3}{48\pi\sqrt{s}\cos^4\theta_W(s - m_Z^2)^2} \left[ -|a_Z|^2 + \frac{s}{m_Z^2} |b_Z|^2 \right],
\]
is actually independent of $b_Z$, to first order in $b_Z$. The limit on $\text{Re } b_Z$ from the asymmetry used in $[1]$ actually comes from the $b_Z$ dependence of the denominator, which is the cross section. Thus, in models where $b_Z$ arises only as a small effect, possibly from loops, the quantity $\sigma_T - 2\sigma_L$ can be used to determine $a_Z$ (assumed non-SM) independently of $b_Z$.

(a) In EFT, with $a_Z = 1 + \delta a_Z$, and keeping only first order in higher-dimensional couplings,

$$\Delta \sigma_1 = \frac{F(P_L, \bar{P}_L)g^4 |\vec{k}_Z|^3}{48\pi \sqrt{s} \cos^4 \theta_W(s - m_Z^2)^2} \left[-1 - 2\delta a_Z \right], \quad (7)$$

and can be used to determine $\delta a_Z$.

(b) In 2HDM like models, there is no tree-level higher derivative $b_Z$-type coupling (which is non-renormalizable). $b_Z$ may arise only at loop level and is therefore small. Hence the quadratic term in $b_Z$ can be neglected. So $\Delta \sigma_1$ can be used to determine $a_Z$, which is $\sin(\alpha - \beta)$, as mentioned earlier.

(c) Again, in the case of composite models, $b_Z$ is expected to be small, and hence the quadratic term can be neglected. Hence $\Delta \sigma_1$ can again be used to determine $a_Z$, and hence the parameter $\xi$.

2. Another useful combination of polarized cross sections is $\Delta \sigma_2 \equiv \sigma_T - (2m_Z^2/E_Z^2)\sigma_L$, which evaluates to

$$\Delta \sigma_2 = \frac{F(P_L, \bar{P}_L)g^4 |\vec{k}_Z|^3}{48\pi \sqrt{s} \cos^4 \theta_W(s - m_Z^2)^2} \left[\frac{2\text{Re}(a_Z b_Z^*) \sqrt{s}}{E_Z} + |b_Z|^2 \left(\frac{E_Z^2 + 2m_Z^2}{E_Z^2 m_Z^2} s \right) \right]. \quad (8)$$

This is independent of $|a_Z|^2$, and hence proportional to $b_Z$.

(a) To linear order in the EFT couplings,

$$\Delta \sigma_2 = \frac{F(P_L, \bar{P}_L)g^4 |\vec{k}_Z|^3}{48\pi E_Z \cos^4 \theta_W(s - m_Z^2)^2} \left(-2\text{Re } b_Z \right). \quad (9)$$

$\Delta \sigma_2$ can thus be used to determine $b_Z$, thus giving information complementary to that obtained from $\Delta \sigma_1$.

(b) In SM and models like 2HDM, $\Delta \sigma_2$ will be zero. Of course, at the loop level, the answer will be non-zero, and this would get dominant contribution from say triple-Higgs couplings, since tree-level contributions are eliminated.

(c) In composite models $\Delta \sigma_2$ will also be zero at tree level, or in some models, suppressed by $m_Z^2/f^2$ $[15]$. 

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3. The longitudinal helicity fraction $F_0 \equiv \sigma_L/\sigma$ is given by

$$
\frac{\sigma_L}{\sigma} = \frac{E_Z^2|aZ|^2 - 2\text{Re}(aZb_Z)E_Z\sqrt{s} + s|bZ|^2}{(2m_Z^2 + E_Z^2)|aZ|^2 - 6\text{Re}(aZb_Z)E_Z\sqrt{s} + s|bZ|^2(1 + 2E_Z^2/m_Z^2)}.
$$

(10)

The value of the helicity fraction is independent of beam polarization. To first order in $b_Z/a_Z$,

$$
F_0 = \frac{E_Z^2}{2m_Z^2 + E_Z^2} \left(1 + \frac{\text{Re}(a_Z b_Z)}{|a_Z|^2} \frac{4\sqrt{s}|\bar{k}_Z|^2}{E_Z(2m_Z^2 + E_Z^2)}\right).
$$

(11)

(a) To first order in dimension-six EFT couplings,

$$
F_0 \approx \frac{E_Z^2}{2m_Z^2 + E_Z^2} \left(1 + \text{Re}(b_Z) \frac{4\sqrt{s}|\bar{k}_Z|^2}{E_Z(2m_Z^2 + E_Z^2)}\right).
$$

(12)

We see that the SM contribution dominates. There is a correction from the $b_Z$ coupling, but no dependence on the $\delta a_Z$ type of couplings. A measurement of the helicity fraction would thus be mainly model independent, with only a mild dependence on the dimension-six couplings entering $b_Z$.

(b) In models like SM and 2HDM, where $b_Z = 0$, the helicity fraction is independent of $a_Z$ and therefore a model-independent quantity, $E_Z^2/(2m_Z^2 + E_Z^2)$, which approaches 1 at high energies [27]. This high-energy behaviour is as expected because the longitudinally polarized $Z$ bosons give a dominant contribution, which can be seen in eq. (4). The model-independence of $F_0$ seems interesting to verify experimentally. Corrections from loops would be interesting to check.

(c) In composite Higgs models, since $b_Z$ is expected to be small, the value of $F_0$ is approximately the same as in the SM, or as that in the 2HDM.

We now discuss how sensitive the measurements of these three quantities would be in the determination of the relevant couplings.

The statistical sensitivity is determined by comparing the expected number of new-physics events $\Delta N$ with the statistical fluctuation $\sqrt{N_{SM}}$ in the number of events in the SM. Thus, the 1-$\sigma$ limit on a coupling can be obtained by equating these two quantities. To linear order,

$$
\Delta N_i = L(\Delta \sigma_i - \Delta \sigma_{i,SM}^L) = L \frac{\partial \Delta \sigma_i}{\partial c_j} c_j = \sqrt{L \sigma_{i,SM}^L},
$$

(13)
where $c_j$ ($j = 1, 2$) is the anomalous coupling ($\delta a_Z$ or $b_Z$) and $L$ is the integrated luminosity. Then, the 1-$\sigma$ limit on $c_j$ from a measurement of $\Delta_i$ is given by

$$c_j^{\text{lim}} = \frac{\sqrt{\sigma_i^{\text{SM}}}}{\sqrt{L|\partial \Delta \sigma_i / \partial c_j|}}.$$  \hspace{1cm} (14)

At the LHC, a measurement of $W$ polarization in top decay with c.m. energy of 8 TeV and an integrated luminosity of 20 fb$^{-1}$, an accuracy of about 2\% for $F_0$ and about 3\% for the left-handed helicity fraction $F_L$ \cite{22}. This implies an overall accuracy of 2-3\% for our observable $F_0$, though it is likely to be better for $Z$ polarization at an $e^+e^-$ collider. However, for the measurement of our first two observables, which are combinations of polarized cross sections and not ratios, the efficiency would be expected to be lower. The efficiency would also depend on the actual final states into which $Z$ and $H$ decay. The corresponding backgrounds and the kinematic cuts needed to suppress the backgrounds would determine the efficiency. A discussion of backgrounds for earlier proposals for linear colliders may be found in \cite{28} and references therein, and for more recently planned colliders can be found in \cite{29} for the CEPC and \cite{30} for the FCC-ee and the CEPC. Here, we have not incorporated effects of cuts and assumed an ideal efficiency. We reiterate that our main motive is to show the usefulness of certain polarization combinations which have not been considered before.

To get an idea of the event rates, we have first plotted in Fig. 1 the

![Figure 1](image_url)

Figure 1: The SM cross section $\sigma_{\text{SM}}$ as a function of $\sqrt{s}$ for different electron and positron polarization combinations.
We estimate the 1-σ limits possible on $\delta a_Z$ using $\Delta \sigma_1$ and on $b_Z$ using $\Delta \sigma_2$ for various configurations of $e^+e^-$ colliders, for both polarized and unpolarized beams. We observe that there is improvement in the limits in the presence of negative electron polarization and/or positive positron polarization as compared to the unpolarized case. Furthermore, inclusion of (negative) positron polarization results in an improvement as compared to the case when only the electron beam is polarized.

We have plotted in Fig. 2, for the case of $L = 2ab^{-1}$, the limits on $\delta a_Z$ and $b_Z$ as functions of the c.m. energy for various combinations of beam polarization. It is clear from these plots that the optimal limits in either case are obtained for a c.m. energy in the region of 350 GeV. We also see that longitudinal beam polarization we have chosen helps to improve the sensitivity to a great extent. In addition to the results shown in the plots we find that further increase in positron polarization, if feasible, would improve the sensitivity further. As for example, for $\sqrt{s} = 500$ GeV, in going from $P_{e^-} = -0.8$ and $P_{e^+} = +0.3$ to $P_{e^-} = -0.8$ and $P_{e^+} = +0.6$, the limit on $\delta a_Z$ improves from $9.1 \times 10^{-4}$ to $8.2 \times 10^{-4}$ and on $b_Z$ from $4.4 \times 10^{-4}$ to $4.0 \times 10^{-4}$.

We may compare the limits we consider possible here using polarized
cross section combinations with the limits estimated in [1] with the use of asymmetries. The limits on \( b_Z \) we find here are better by an order of magnitude, even though in [1] \( a_Z \) was assumed to be exactly equal to 1, whereas here we only make a linear approximation. It should of course be borne in mind that in either case, in practice, the limits would be somewhat lower and dependent on a more detailed analysis including the effects of cuts and detector efficiency, which we have not attempted here.

To conclude, we have studied how \( Z \) polarization in \( e^+e^- \to HZ \) can be used to measure general Lorentz-invariant \( HZZ \) interaction described by three couplings, \( a_Z, b_Z \) and \( \tilde{b}_Z \), of which the first two are CP-conserving.

Instead of the full \( Z \) spin density matrix which would require complicated distributions or asymmetries of the \( Z \) decay products, we suggest looking at only the diagonal elements of this matrix. These correspond to cross sections for the production of \( Z \) with definite polarizations, which are more easily accessible. We find that certain combinations of \( Z \) production cross sections with definite \( Z \) polarization can help to enhance or isolate the effect of one of the two CP-even couplings.

The specific combinations of longitudinally polarized (\( \sigma_L \)) and transversely polarized (\( \sigma_T \)) cross sections we consider are

\[
\Delta \sigma_1 \equiv \sigma_T - 2\sigma_L, \quad \Delta \sigma_2 \equiv \sigma_T - (2m_Z^2/E_Z^2)\sigma_L
\]

and the longitudinal helicity fraction \( F_0 \) given by

\[
F_0 \equiv \frac{\sigma_L}{\sigma_L + \sigma_T}
\]

We show that \( \Delta \sigma_1 \) is independent of \( b_Z \) to first order, and can therefore be used to determine \( a_Z \). On the other hand, \( \Delta \sigma_2 \) is proportional to \( b_Z \) and has no leading \( |a_Z|^2 \) dependence, so that it can be used to determine \( b_Z \). To leading order in \( b_Z/a_Z \), \( F_0 \) takes its SM value, viz., \( E_Z^2/(2m_Z^2 + E_Z^2) \), regardless of the value of \( a_Z \).

We consider in a relatively model-independent fashion three scenarios, viz., EFT, 2HDM and composite Higgs, and analyze what role the measurement of the three cross section combinations \( \Delta \sigma_1, \Delta \sigma_2 \) and \( F_0 \) can play in these scenarios. We also estimate the 1-\( \sigma \) limits on the couplings that could be placed using \( \Delta \sigma_1 \) and \( \Delta \sigma_2 \).

We have also considered the effect of longitudinal beam polarization. While the limits on the couplings from \( \Delta \sigma_1 \) and \( \Delta \sigma_2 \) are improved by a choice of electron and positron beam polarizations \(-0.8\) and \(+0.3\), \( F_0 \) is independent of beam polarization.

We find that limits of the order of a few times \( 10^{-4} \) to \( 10^{-3} \) can be obtained for ranges of \( \sqrt{s} \) from 250 to 500 GeV and \( L \) values from 2 to 30 ab\(^{-1}\) envisaged at various electron-positron colliders.
esting to measure at future colliders. A more detailed analysis using event
generators and detector simulation would be necessary to check how well the
estimates made here survive after realistic kinematic cuts and detection effi-
ciencies. Further, it would be interesting to investigate if such cross section
combinations yield useful results in a $pp$ environment, like at the LHC.

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