Heavy-light decay constants from relativistic $N_f=2,0$ QCD

H.P. Shanahan for the CP-PACS Collaboration

DAMTP, University of Cambridge, Cambridge, CB3 9EW, England, UK

Abstract

We present results on an analysis of the decay constants $f_{B_d}$ and $f_{B_s}$ with two flavours of sea quark. The calculation has been carried out on 3 different bare gauge couplings and 4 sea quark masses at each gauge coupling, with $m_{\pi}/m_{\rho}$ ranging from 0.8 to 0.6. We employ the Fermilab formalism to perform calculations with heavy quarks whose mass is in the range of the b-quark. A comparison with a quenched calculation using the same action is made to elucidate the effects due to the sea quarks.

1. Introduction

An accurate determination of the parameters $f_{B_d}\sqrt{B_{d}}$ and $\xi = f_{B_s}\sqrt{B_{s}}/f_{B_d}\sqrt{B_{d}}$, in conjunction with the (future) experimental data on $\Delta m_d$ (and $\Delta m_s$) will provide excellent constraints on $|V_{td}|$ and $|V_{ts}|/|V_{td}|$. Their calculation in the quenched approximation, using the plaquette gluon action, has been carried out using a number of different formulations for heavy quarks and the results are converging.

A major uncertainty in these results, however, is that they may be susceptible to large corrections due to the effect of sea quarks. The penultimate step in eliminating this uncertainty is to consider the effect of two flavours of sea quark ($N_f = 2$). Here we present the status of such a study, using the clover action for heavy quark, which we started last year.

As the lattice spacings available to us are relatively coarse, we deal with the large mass of $b$ quark $m_{ba} > 1$ in the formalism of Ref. [6], which has been previously applied in the quenched calculation of B meson decay constants.

In order to reduce discretisation effects, we have employed an RG-improved action in the gluon sector. Since this action was not considered in previous studies of $f_{B}$, we repeat the calculation in quenched QCD to compare with full QCD results.

A comparison is also made with preliminary NRQCD results for the decay constants obtained on the same full QCD configurations.

2. Results

The computational details of this calculation are described in [10]. In Fig. 1 we plot results for $f_{B}$, for $N_f = 2$ and in the quenched approximation ($N_f = 0$); sea quark mass is extrapolated to the chiral limit for $N_f = 2$. A clear increase of 10–20% is seen from two flavours of sea quark.

We also include the preliminary results from NRQCD in Fig. 1 for two values of finite sea quark mass at $\beta = 1.95$ in full QCD (filled squares), and one value in quenched QCD (open squares). In Fig. 2 we present a similar comparison of the Fermilab and NRQCD approaches for $f_{B_d}$. In both figures we find good consistency of results between the two approaches.

Finally, we plot the ratio $f_{B_s}/f_{B_d}$ in Fig. 3 using
the $K$ meson mass to set the strange quark mass. We observe only mild variation of the ratio with respect to $a$ for even the coarsest of our lattice spacings.

As a preliminary result we quote

\begin{align}
\frac{f_{B_s}^{n_f=2}}{f_{B}^{n_f=2}} &= 251 \pm 3 \pm 4(m_s) \pm 10 \text{ (fit) MeV, (1)} \\
\frac{f_{B_s}^{n_f=2}}{f_{B}^{n_f=2}} &= 210 \pm 7 \pm 14 \text{ (fit) MeV, (2)} \\
\frac{f_{B_s}^{n_f=2}}{f_{B}^{n_f=2}} &= 1.20 \pm 4 \pm 2(m_s) \pm 3 \text{ (fit). (3)}
\end{align}

The first errors are statistical. The error labelled $(m_s)$ is due to the ambiguity of using the mass of the $\phi$ or $K$ to set the strange quark mass.

The central values are determined by assuming the results independent of $a$ for the two finer lattice spacings. The resulting systematic error (fit) is derived by taking the difference between a constant and a linear fit in $a$ for all three points. We should also add an uncertainty due to the choice of scale; our preliminary estimate is 15–20 MeV from comparison of results using the scale determined from $m_H$.

3. Conclusions

At this point it seems clear that there exists a systematic difference between the $N_f = 0$ and 2 data. Encouragingly enough, the preliminary NRQCD results are also in agreement with the relativistic results in both cases as well. One worrying point is that the quenched results for $f_{B_s}$ is approximately 10% larger than the quenched results using the Wilson action\[2]. Clearly this effect needs further examination. The ratio $f_{B_s}/f_{Bd}$ appears to be less affected by discretisation effects and is not substantially different from previous quenched calculations. It should be noted that even a systematic error of 10% for this ratio would be of substantial use for phenomenologists. It seems plausible then that a calculation of $f_{B_s}/f_{Bd}$ could be carried out for three flavors of dynamical quarks on a comparatively coarse lattice.

This work is supported in part by the Grants-in-Aid of Ministry of Education, Science and Culture (Nos. 09304029, 10640246, 10640248, 10740107, 11640250, 11640294, 11740162). HPS is supported by Research for the Future Program of JSPS, and also by the Leverhulme foundation.

References

[1] For a recent review, see e.g., A.J. Buras, proceedings of the Lake Louise Winter Institute, Feb. 14-20, 1999, TUM-HEP-349/99, [hep-ph/9905437](http://arxiv.org/abs/hep-ph/9905437).
[2] S. Hashimoto to appear in the Proceedings of Lattice '99, 28 June-3 July 1999, Pisa, Italy.
[3] M.J. Booth, Phys. Rev. D51, 2338, (1995).
[4] S.R. Sharpe and Y. Zhang, Phys. Rev. D53, 5125 (1996).
[5] H. P. Shanahan et al., CP-PACS Collaboration, Nucl. Phys. Proc. Suppl. 73 (1999) 375.
[6] A. X. El-Khadra, A. S. Kronfeld and P. B. Mackenzie, Phys. Rev. D55 (1997) 3933.
[7] A.X. El-Khadra, et al., Phys. Rev. D58 (1998) 014506.
[8] S. Aoki et al. (JLQCD Collaboration), Phys. Rev. Lett. 80 (1998) 5711.
[9] A. Ali Khan et al., CP-PACS collaboration, to appear in the Proceedings of Lattice '99.
[10] H.P. Shanahan et al., CP-PACS collaboration, [hep-lat/9909053](http://arxiv.org/abs/hep-lat/9909053).