Chaos synchronizations of chaotic systems via active nonlinear control

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Abstract: This paper not only investigates the chaos synchronization between two LCC chaotic systems, but also discusses the chaos synchronization between LCC system and Genesio system. Some novel active nonlinear controllers are designed to achieve synchronizations between drive and response systems effectively. Moreover, the sufficient conditions of synchronizations are derived by using Lyapunov stability theorem. Numerical simulations are presented to verify the theoretical analysis, which shows that the synchronization schemes are global effective.

1. Introduction

Chaos synchronization is a hot topic that has attracted lots of attentions of scholars since 1990, because chaos synchronization has immense potential applications in secure communications, laser physics, chemical reactor, biomedical and even economy control and so on [1-3]. By now, many approaches have been proposed for synchronizations of chaotic systems such as OGY method, PC method, feedback method, drive-response method, adaptive synchronizations, coupled synchronization, active control method, nonlinear functions synchronization method and so on [4-7]. Most of these methods can synchronize two identical or different chaotic systems. However, the effectiveness of each method is very different. Among these methods, nonlinear function control method has been proved to synchronize some chaotic systems more effectively [8]. Therefore, in this paper, we mainly use active nonlinear synchronization method to investigate the synchronizations of chaotic systems such as LCC system [9] and Genesio system [10, 11].

The rest of this paper is organized as follows. In section 2, we present the synchronization schemes of LCC systems and give some corresponding numerical simulations, followed by the analysis of synchronization between LCC system and Genesio system in Section 3. A conclusion is given in the end.

2. Synchronization of LCC system

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The LCC system, proposed by Lü, Chen and Cheng [9], bridges the gap between the Lorenz and Chen attractors which is described by

\[ \begin{align*}
\dot{x}_1 &= ax_1 - y_1z_1 \\
\dot{y}_1 &= -by_1 + x_1z_1 \\
\dot{z}_1 &= -cz_1 + x_1y_1
\end{align*} \]

where \( x_1, y_1, z_1 \) are state variables and \( a, b, c \) are positive parameters. Differing from other similar chaotic system, system (1) displays not only a two-scroll chaotic attractor when \( a = 4.5, b = 12 \) and \( c = 5 \) (see Fig.1(a)), but also a four-scroll chaotic attractor when \( a = 0.4, b = 12 \) and \( c = 5 \) (see Fig.1(b)). Recently, the linear feedback controlling method was used to synchronize the LCC system successfully [10]. However, this synchronization method is not very effective. Thus, we attempt to find some effective schemes to synchronize LCC systems.

![Fig. 1 The chaotic attractor of system (1): (a) two-scroll chaotic attractor at \( a = 4.5, b = 12 \) and \( c = 5 \), (b) two-scroll chaotic attractor at \( a = 0.4, b = 12 \) and \( c = 5 \).](image)

This section focuses on applying nonlinear function controller method to synchronize LCC system. Assume that system (1) is the drive system and the response system is

\[ \begin{align*}
\dot{x}_2 &= ax_2 - y_2z_2 + u_1 \\
\dot{y}_2 &= -by_2 + x_2z_2 + u_2 \\
\dot{z}_2 &= -cz_2 + x_2y_2 + u_3
\end{align*} \]

where \( u = [u_1, u_2, u_3]^T \) is the controller. Subtracting (1) from (2) leads to the following error system

\[ \begin{align*}
\dot{e}_1 &= ae_1 - y_2z_2 - y_1z_1 + u_1 \\
\dot{e}_2 &= -be_2 + x_2z_2 - x_1z_1 + u_2 \\
\dot{e}_3 &= -ce_2 + x_2y_2 - x_1y_1 + u_3
\end{align*} \]

where \( e_1 = x_2 - x_1, e_2 = y_2 - y_1 \) and \( e_3 = z_2 - z_1 \).

The main result of this section is summarized in Theorem 1.

**Theorem 1.** Systems (1) and (2) can be exponentially and globally synchronized for any initial condition with the following active nonlinear controllers

\[ \begin{align*}
u_1 &= y_2z_2 - y_1z_1 - 2ae_1, \quad u_2 = x_1z_1 - x_2z_2, \quad u_3 = x_1y_1 - x_2y_2
\end{align*} \]

**Proof.** We choose Lyapunov function as follows

\[ V(t) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \]

By using the control law, the time derivative of \( V(t) \) along trajectories (3) can be derived as
\[ \dot{V}(t) = e_i \dot{e}_i + e_2 \dot{e}_2 + e_3 \dot{e}_3 \]
\[ = e_i (ae_i^2 - y_1 z_2 + y_1 z_1 + u_i) + e_2 (-be_2^2 + x_2 z_2 - x_1 z_1 + u_2) + e_3 (-ce_3^2 + x_3 y_2 - x_1 y_1 + u_3) \]
\[ = -ae_i^2 - be_2^2 - ce_3^2 \]
\[ \leq 0 \]

which implies that \( \lim_{t \to +\infty} e_i = 0 \) \( (i = 1, 2, 3) \) and guarantees the exponentially asymptotical stability of the error system (3). Therefore, systems (1) and (2) can achieve exponentially asymptotical synchronization for any initial conditions with nonlinear controllers.

In simulations, the fourth order Runge-Kutta integration method is used to solve the systems of differential equations with time step size 0.01. We select the parameters as \( a = 4.5 \), \( b = 12 \) and \( c = 5 \). The initial values for the drive and response systems are \( x_i(0) = -1, y_i(0) = 1, z_i(0) = 1 \) and \( x_2(0) = 2, y_2(0) = -3, z_2(0) = 5 \) respectively, while the initial values of error systems (3) are \( e_i(0), e_2(0), e_3(0) = (3, -4, 4) \). Fig. 2 (a – c) shows the time response of states \( x_i, y_i, z_i \) for the drive system (1) and the states \( x_2, y_2, z_2 \) for the response system (2). Fig. 2(d) displays the time response of the error system (3). Obviously, the synchronization errors coverage to zero with exponentially asymptotical speed and two systems with different initial values are achieved chaos synchronization very quickly.

![Graphs showing time response of variables](image)

**Fig. 2.** The time response of variables \( x_i, y_i, z_i \) for drive system, \( x_2, y_2, z_2 \) for response system and error system (3) with the control law (4): (a) signals \( x_1 \) and \( x_2 \); (b) signals \( y_1 \) and \( y_2 \); (c) signals \( z_1 \) and \( z_2 \); (d) dynamics of error states \( e_1, e_2, e_3 \).

3. **3. Synchronization between LCC system and Genesio system**
In this section, we pay our attentions on the synchronization between LCC system and Genesio system. The Genesio system, proposed by Genesio and Tesi, is one of paradigms of chaos because it captures many characteristics of chaotic systems [10]. Recently, backstepping approach was used to synchronize two Genesio systems [11]. However, the chaos synchronization between LCC system and Genesio system has not been investigated.

The dynamics equations of Genesio chaotic system is described as

\[
\begin{align*}
\dot{x} &= y \\
\dot{y} &= z \\
\dot{z} &= kx + gy + hz + x^3
\end{align*}
\]

where \( k, g, h \) are parameters. When \( k = -6, g = -2.92, h = -1.2 \), system (5) is chaotic.

In this section, we choose Genesio system as the drive system and LCC system as the response system. Therefore, Subtracting (5) from (2) leads to the following error system

\[
\begin{align*}
\dot{e}_1 &= anx_2 - y_2z_2 - y + u_1 \\
\dot{e}_2 &= -by_2 + x_2z_2 - z + u_2 \\
\dot{e}_3 &= x_2y_2 - kx - gy - x^3 - cz_2 - hz + u_3
\end{align*}
\]

where \( e_1 = x_2 - x, e_2 = y_2 - y \) and \( e_3 = z_2 - z \).

In Theorem 2, we show that these two different chaotic systems can be effectively synchronized by nonlinear function controllers.

**Theorem 2.** Systems (1) and (5) can be synchronized with exponentially asymptotical effectiveness for any initial condition with the following active nonlinear controllers

\[
\begin{align*}
u_1 &= y_2z_2 + y + ax - 2ax_2 \\
u_2 &= -x_2z_2 + z + by \\
u_3 &= -x_2y_2 + kx + gy + hz + cz_2 + x^3 - e_3
\end{align*}
\]

The proof is similar to that of Theorem 1, omitting it here.

In the following, numerical simulations are carried out to verify the method proposed in this section. The fourth order Runge-Kutta integration method is used to solve the systems of differential equations (2) and (5) with time step size 0.001. We select the parameters as \( a = 4.5, b = 12, c = 5, k = -6, g = -2.92 \) and \( h = -1.2 \). The initial values for the drive and response systems are \( x(0) = -1, y(0) = 0, z(0) = 1 \) and \( x_2(0) = 1, y_2(0) = -2, z_2(0) = 3 \) respectively, while the initial values of error system (3) are \((e_1(0), e_2(0), e_3(0)) = (2, -2, 2)\). Fig. 5 (a-c) shows the time response of states \( x, y, z \) for the drive system (1) and the states \( x_2, y_2, z_2 \) for the response system (2) and Fig. 6 displays the time response of the error system (3).
Fig. 5. The time response of states for variables $x, y, z$ in drive system (5), $x, y, z$ in response system (2) and error system (6): (a) signals $x$ and $x_1$; (b) signals $y$ and $y_1$; (c) signals $z$ and $z_1$; (d) dynamics of errors states $e_x, e_y, e_z$.

4. Conclusion

In this paper, by using nonlinear function controllers, we not only achieve the synchronizations of two LCC chaotic systems, but also realize the synchronization between LCC system and Genesio system. Lyapunov stability methods are used to theoretically give the synchronization conditions, which are verified by some numerical simulations. The analysis shows that the nonlinear function controllers are exponentially asymptotical effective to synchronize these chaotic systems.

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