Cryptanalysis of dynamic look-up table based chaotic cryptosystems

G. Álvarez *, F. Montoya, M. Romera, G. Pastor

Instituto de Física Aplicada, Consejo Superior de Investigaciones Científicas, Serrano 144—28006 Madrid, Spain

Abstract

In recent years many chaotic cryptosystems based on Baptista’s seminal work have been proposed. We analyze the security of two of the newest and most interesting ones, which use a dynamically updated look-up table and also work as stream ciphers. We provide different attack techniques to recover the keystream used by the algorithms. The knowledge of this keystream provides the attacker with the same information as the key and thus the security is broken. We also show that the dependence on the plaintext, and not on the key, of the look-up table updating mechanism facilitates cryptanalysis.

Key words: Chaotic cryptosystems, Ergodicity, Cryptanalysis, Hash algorithm

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1 Introduction

Since M. S. Baptista proposed in 1998 a new cryptosystem based on the property of ergodicity of chaotic systems [1], a number of new algorithms based on variations of Baptista’s have been published [2,3,4,5,6]. In [7] we analyzed the security and cryptographic robustness of Baptista’s seminal algorithm. The first variation of Wong [2] was cryptanalyzed in [8]. We present in this Letter our results after having thoroughly studied Wong’s second and third algorithms [4,5].

The ergodicity property is exploited in these algorithms by using the logistic map

\[ y_{n+1} = by_n(1 - y_n), \]  

(1)

* Corresponding author: Email: gonzalo@iec.csic.es

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where \( y_n \in [0, 1] \) and the parameter \( b \) is chosen so that Eq. (1) behaves chaotically. In [4], the most interesting addition consists of using a dynamic table for looking up the ciphertext and plaintext which is no longer fixed during the whole encryption and decryption processes. Instead, it depends on the plaintext, being continuously updated during the encryption and decryption processes. This makes cryptanalysis more difficult, but not impossible.

When the \( i \)th message block is encrypted, the look-up table is updated dynamically by exchanging the \( i \)th entry \( l_i \) with another entry \( l_j \). The location of the latter entry, i.e., the value of \( j \), is determined by the current value of \( y \) using the following formula:

\[
v = \left\lfloor \frac{y - y_{\text{min}}}{y_{\text{max}} - y_{\text{min}}} \times N \right\rfloor \tag{2}
\]

\[
 j = i + v \mod N, \tag{3}
\]

where \( y_{\text{min}} \) and \( y_{\text{max}} \) are the end points of the chosen interval \([y_{\text{min}}, y_{\text{max}}]\) and \( N \) is the total number of entries in the table [4].

In [5], the previously described chaotic cryptographic scheme is generalized by allowing the swapping of multiple pairs of entries in the look-up table during the encryption of each input block, and by allowing multiple runs of encryption on the whole message continuously. Starting from the current entry \( i \), \( p \) pairs of entries (\( p \geq 1 \)) are swapped according to the following rule: \( i \leftrightarrow (i + v \mod N) \), \((i + v + 1 \mod N) \leftrightarrow (i + 2v + 1 \mod N) \leftrightarrow (i + 3v + 2 \mod N) \), \( \ldots \), \((i + (p - 1)v + p - 1 \mod N) \leftrightarrow (i + pv + p - 1 \mod N) \). Once the message has been encrypted, the whole process is repeated again \( r \) times, \( r \geq 1 \). The final look-up table is the hash of the message [5].

2 Classical types of attacks

When cryptanalyzing a ciphering algorithm, the general assumption made is that the cryptanalyst knows exactly the design and working of the cryptosystem under study, i.e., he knows everything about the cryptosystem except the secret key. This is an evident requirement in today’s secure communications networks, usually referred to as Kerchoff’s principle [9, p. 24]. According to [9, p. 25], it is possible to differentiate between different levels of attacks on cryptosystems. They are enumerated as follows, ordered from the hardest type of attack to the easiest:

1. Ciphertext only: The opponent possesses a string of ciphertext.
2. Known plaintext: The opponent possesses a string of plaintext, \( p \), and the corresponding ciphertext, \( c \).
(3) Chosen plaintext: The opponent has obtained temporary access to the encryption machinery. Hence he can choose a plaintext string, \( p \), and construct the corresponding ciphertext string, \( c \).

(4) Chosen ciphertext: The opponent has obtained temporary access to the decryption machinery. Hence he can choose a ciphertext string, \( c \), and construct the corresponding plaintext string, \( p \).

In each of these four attacks, the objective is to determine the key that was used. The last two attacks, which might seem unreasonable at first sight, are very common when the cryptographic algorithm, whose key is fixed by the manufacturer and unknown to the attacker, is embedded in a device which the attacker can freely manipulate. Daily life examples of such devices are smart-cards, electronic purse cards, GSM phone SIM (Subscriber Identity Module) cards, POST (Point Of Sale Terminals) machines, or web application session token encryption.

### 3 Keystream attacks

Although at first sight the cipher under study might look like a block cipher, in fact it behaves as a stream cipher [9, p. 20], a fundamental weakness as is to be seen. The operation of the algorithm as a stream cipher can be explained as follows. Suppose \( K \) is the key, given by \( y_0 \) and \( b \), and that \( p = p_1 p_2 \ldots \) is the plaintext string. A keystream \( k = k_1 k_2 \ldots \) is generated using Eq. (1). This keystream is used to encrypt the plaintext string according to the rule

\[
c = e_{k_1}(p_1)e_{k_2}(p_2)\ldots = c_1 c_2 \ldots
\]

Decrypting the ciphertext string \( c \) can be accomplished by computing the keystream \( k \) given the knowledge of the key \( K \) and undoing the operations \( e_{k_i} \). In [4] the keystream is the complete orbit followed by iterating Eq. (1) from the initial point \( y_0 \) with parameter near \( b = 4.0 \). The unit interval is divided up into \( N \) equally spaced bins, each corresponding to a symbol of the alphabet in use. However, instead of considering the whole unit interval, only the subinterval \([0.2, 0.8]\) is used. As a consequence of the natural invariant density of Eq. (1), the orbit will visit frequently the forbidden subintervals \([0, 0.2)\) and \([0.8, 1]\).

As an example of how to generate the keystream, let us iterate Eq. (1) starting from \( y_0 = 0.1777 \) and \( b = 3.9999995 \), as in [4]. Let \( s_i \) be the symbol corresponding to each useful subinterval where \( y_i \) lands, and let \( x \) be the iterates which visit the forbidden subintervals. The orbit followed is \( y_i = \{0.5844 \ldots , 0.9714 \ldots , 0.1109 \ldots , 0.3945 \ldots , 0.9555 \ldots , 0.1698 \ldots , 0.5641 \ldots , 0.9835 \ldots , 0.0647 \ldots , 0.2420 \ldots , \ldots \} \), and is transformed into the
keystream \( k = s_{165}xs_{84}xs_{156}xs_{18} \ldots \). Next we show how to recover the keystream using chosen ciphertext, chosen plaintext, and known plaintext attacks. It is important to note that knowing the keystream \( k \) generated by a certain key \( K \) \((y_0 \text{ and } b)\) is entirely equivalent to knowing the key. Therefore, our keystream attacks focus on recovering \( k \).

### 3.1 How to circumvent the look-up table

The election of the look-up table updating method is most unfortunate, since it allows the attacker to easily predict the new positions of the symbols even without the exact knowledge of the value of \( y \). In order to initially simplify our analysis, we use a variable number \( N \) of symbols, \( N = 2^n, n = 1, \ldots, 8 \). First, we assume that the source emits two different symbols, \( S_2 = \{s_1, s_2\} \). When the orbit lands on the first subinterval \([0.2, 0.5)\), the table will be updated following Eqs. (2) and (3). Given that \( 0.2 \leq y < 0.5 \), we have that

\[
0 \leq \nu = \left\lfloor \frac{y - y_{\min}}{y_{\max} - y_{\min}} \times 2 \right\rfloor = \left\lfloor \frac{y - 0.2}{0.6} \times 2 \right\rfloor < 1,
\]

and thus \( \nu = 0 \) and Eq. (3) is equivalent to:

\[
j = i \mod 2.
\]

When the orbit lands on the second subinterval \([0.5, 0.8)\), then \( 0.5 \leq y < 0.8 \), and thus \( \nu = 1 \) and Eq. (3) becomes:

\[
j = (i + 1) \mod 2.
\]

If the source emits four different symbols, \( S_4 = \{s_1, s_2, s_3, s_4\} \), then \( \nu = 0 \) if the orbit lands on \([0.2, 0.35)\), \( \nu = 1 \) if the orbit lands on \([0.35, 0.5)\), \( \nu = 2 \) if the orbit lands on \([0.5, 0.65)\), and \( \nu = 3 \) if the orbit lands on \([0.65, 0.8)\).

The generalization for higher order sources is immediate. Even when multiple pairs are swapped at each encryption run, as in [5], the look-up table evolution is easily predicted. As a consequence, the look-up table plays no significant security role during the encryption process. It is not necessary to know the exact value of \( y \) to predict the next update. It suffices to know the subinterval where \( y \) lands. Therefore, the updated table depends solely on the plaintext, and not on the key \((y_0 \text{ and } b)\), to the advantage of the attacker. When encrypting the same plaintext using different keys, the same updating sequence will take place for the look-up table. Likewise, the same message will always yield the same hash value regardless of the key used.
3.2 How to obtain the keystream

In this subsection different attacks are described. Each of them aims at the recovery of the keystream.

3.2.1 Chosen ciphertext attack

The chosen ciphertext attack is straightforward. Simply request the plaintext of the one-block ciphertexts $c = 1$, $c = 2$, $c = 3$, ..., until the desired length of the keystream is reached. Either the correct symbol (when the iterate lands on a site) or an error (when it lands outside the boundaries) is obtained, one by one. Once the desired length of keystream has been recovered in this way, any message encrypted with the same values of $y_0$ and $b$ can easily be decrypted. Under this attack, the dynamically updated look-up table has no effect at all. This attack requires as many one-block ciphertexts as the length of the keystream that is to be recovered. This attack does not work on [5], because the attacker does not know the encrypted values of $p$ and $r$, the first two blocks of the ciphertext.

3.2.2 Chosen plaintext attack

Let us deal with the chosen plain text attack next. To make things even simpler at first, we assume the fourth order symbol source $S_4$ and that $r_{\text{max}} = 1$ (see [4]). Although unknown to the cryptanalyst, the system key $K$ is given by $y_0 = 0.1777$ and $b = 3.9999995$, using the interval $[0.2, 0.8)$, as in [4].

Our goal is to find out the exact position of all occurrences of $s_1, s_2, s_3,$ and $s_4$ in the keystream. But this task is not as easy as requesting the ciphertext of $p = s_1 s_1 s_1 \ldots$, then of $p = s_2 s_2 s_2 \ldots$, etc., as in [7]. The dynamic look-up table prevents knowing whether a certain symbol corresponds to its original position in the table or has been already changed. But given that the changes produced in the table are known even when the exact value of $y$ is unknown, the following attack can be designed.

First, in order to know the exact position of all symbols $s_1$ in the keystream $k$, we need to construct an adequate plaintext $p$. Table 1 represents the final result of the process followed to compute the correct value of $p$. We start constructing Table 1 by filling in columns $i$ and $k_i$, already known in advance. Columns 0, 1, 2, and 3 reflect the current state of the look-up table, i.e., which symbol is at which position at any given moment. Next, we proceed row by row, in the following way:

(1) Assign to $p_i$ the value of the symbol ($s_1, s_2, s_3,$ or $s_4$) in the previous row...
in the subinterval corresponding to \( k_i \). At start, the look-up table is not yet altered.

(2) Calculate \( j = (i + k_i) \mod 4 \).

(3) Update the look-up table by interchanging the symbols in the subintervals \( i, j \).

(4) Proceed to the next row.

After proceeding in this way, the plaintext that corresponds to all symbols \( s_1 \) in the keystream is worked out: \( p = 0 \ 0 \ 0 \ 0 \ 0 \ldots \) This plaintext is always periodic. The corresponding ciphertext is requested, obtaining: \( c = 10 \ 9 \ 6 \ 6 \ 7 \ldots \). Hence, it is known for sure that there is a true symbol \( s_1 \) at the 10th position, and at the 19th, etc. The partial keystream already obtained is \( k = xxxxxxxxs_1 xxxxxxxxs_1 xxxxxxxxs_1 xxxxxxxxs_1 \ldots \)

Next, we are to obtain the exact position of all symbols \( s_2 \) by constructing Table 2. This table informs us that we have to request the ciphertext of \( p = 1 \ 0 \ 2 \ 2 \ 2 \ 0 \ 3 \ 3 \ 3 \ 0 \ 1 \ 1 \ldots \), obtaining \( c = 4 \ 11 \ 14 \ 5 \ldots \). The improved partial keystream already obtained is \( k = xxxs_2 xxxxs_1 xxxxs_2 xxxxs_1 xxxxs_2 xxxs_1 \ldots \)

If we repeat this process, generating Table 3 and 4 for symbols \( s_3 \) and \( s_4 \), and requesting the corresponding ciphertexts, we would obtain the following complete keystream:

\[
\begin{align*}
k &= s_3 xxss_2 xxss_3 xxss_1 xxss_4 xxss_2 xxss_1 xxss_4 xxss_3 xxss_1 xxss_4 xxss_2 xxss_1 \ldots \\
&\quad (4)
\end{align*}
\]

The generalization for higher order sources is immediate. This attack is very inexpensive too, since it only requires \( N = 2^n \) plaintexts, \( 1 \leq n \leq 8 \).

When \( r_{\text{max}} > 1 \), the same procedure must be followed. However, there will be blanks in the recovered keystream, because many valid iterations will be skipped. In many cases, though, it is possible to narrow down the number of possible symbols. Once the partial keystream has been worked out, while trying to decrypt a ciphertext following the decryption method, iterates will land on an \( x \). The only possible symbols for those \( x \) are those lying before the \( x \) at a distance smaller than \( r_{\text{max}} \). In this way, the possibilities are greatly reduced, in many cases to only one possible value (the correct one). When more than one value is possible, there are two courses of action. If the plaintext is not of random nature, then the gaps can easily be filled selecting by the context one symbol amongst the possible ones. On the other hand, if the plaintext is random, then a new plaintext must be requested, made by all the previous correct symbols plus one of the candidates. If the ciphertext is equal to the expected one, then the guess was correct. Otherwise, new plaintexts must be encrypted until the correct guess is used.

There is still another possible modification presented in [4]. A new parameter,
called threshold \(y_{\text{threshold}}\), can be introduced along with the key. The current value of \(y\) can be checked against this threshold, so that the table is updated only when \(y > y_{\text{threshold}}\). However, although it is assumed that this addition improves security, it is very easy to deduce which is the subinterval to which \(y_{\text{threshold}}\) belongs. It can be observed in Table 1 that when the plaintext symbol to be encrypted is \(s_1\), the table is never effectively updated. After the \(N = 2^n\) tables are constructed, if \(y_{\text{threshold}} > y_{\min}\), then there must occur repeated values in the recovered keystream. Given that the position of \(s_1\) is always correct in the keystream, we know for sure that symbols \(s_t\) for \(t > 1\) which coincide with a previous \(s_1\) symbol must be incorrect. The greatest value of \(t\) indicates which is the symbol \(y_{\text{threshold}}\) belongs to. This attack works on [5] too. The attacker can still predict the evolution of the look-up table with the only knowledge of the plaintext.

### 3.2.3 Known plaintext attack

Under this attack, each plaintext/ciphertext pair allows for the recovery of a portion of the keystream. For the sake of simplicity and without loss of generality, let us use once again the \(S_4\) source, \(r_{\max} = 1\), and \(y_{\text{threshold}} = y_{\min}\). Let us set \(p = 1\ 0\ 3\ 0\ 0\ldots\), whose ciphertext is \(c = 4\ 1\ 1\ 5\ 9\ 5\ldots\). Table 5 can be easily constructed, which allows to recover a correct portion of the keystream: \(k = xxxs_2xxxxxxxxxs_2xxxxs_4xxxxxxxxxs_2xxxxs_2\ldots\). This process should be repeated with as many known plaintexts as possible, to recover as big a portion of the keystream as possible. Therefore, this attack does not guarantee total recovery of the keystream. It would work in a similar way for [5].

### 3.3 How to decrypt using the keystream

In the previous subsection, different methods to obtain the complete keystream where introduced. Next, we explain how to recover the plaintext from a ciphertext when the keystream is known.

For simplicity, the fourth order symbol source \(S_4\) is used. We assume the attacker already knows the keystream given by Eq. (4) and possesses the following ciphertext: \(c = 1\ 0\ 3\ 0\ 4\ 4\ldots\). In order to decrypt it, Table 6 is constructed in the following way. First, fill all the values of \(k_i\) with the symbol found in the keystream at the positions indicated by the ciphertext. Next, according to the current state of the look-up table, assign the correct value to \(p_i\) which corresponds to each \(k_i\). Calculate the value of \(j\) and update the look-up table accordingly. Move to the next row and repeat the process until the ciphertext has been exhausted.
4 Security of the hash

We have tested that breaking the hash algorithm is possible when \( p = 1 \) and \( r = 1 \), even without the knowledge of the key \((y_0, b)\). Let us consider for example the following message: “Transfer $10000 to Alvarez’s account.”. If encoded using 4-bit symbols, its hash is \( h = 1E825BC0A36974FD \), expressed in hexadecimal. Changing the message into “Transfer $30005 to Alvarez’s account.” produces exactly the same hash. In order to avoid attacks on the hashing scheme, it is all important that \( r > 1 \) and \( p > 1 \). In effect, in [5] two runs and a small value of \( p \) are suggested. Greater values would increase security, but penalize on speed.

Although in [5] it is claimed that this hash can be treated as a message authentication code (MAC), in fact it can not. A MAC is a key-dependent one-way hash function. However, as already proved, the look-up table, and hence the hash, does not depend on the key \((y_0, b)\). Therefore, this scheme does not behave as a MAC but as a one-way hash function, even though the knowledge of the key is necessary to verify the hash when both authenticity and secrecy are to be provided.

5 Conclusions

In spite of dynamically updating the look-up table, the same fundamental weakness present in Baptista’s algorithm [1] is reproduced in the chaotic cryptosystems proposed in [4,5], as proved by our different attacks. As a consequence of our attacks, an important conclusion is that implementations of these algorithms can never reuse the same key because if so, they are easily broken. Furthermore, the look-up table does not depend on the key, but only on the plaintext, thus facilitating cryptanalysis. After these attacks, we conclude that the lack of security, along with the low encryption speed, discourage the use of these algorithms for secure applications. We are to investigate how the weaknesses outlined in this Letter might affect the security of other Wong’s variants [6].

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Table 1
Plaintext \( (p_i) \) needed to find out the exact position of symbols \( s_1 \) \((k_i = s_1 = 0)\).

| \(i\) | \(j\) | 0 | 1 | 2 | 3 | \(k_i\) | \(p_i\) |
|------|------|---|---|---|---|--------|--------|
| \(-\) | \(-\) | \(-\) | \(-\) | \(-\) | \(-\) | \(-\) | \(-\) |
| 0 | 0 | \(s_1\) | \(s_2\) | \(s_3\) | \(s_4\) | \(0\) | \(s_1\) |
| 1 | 1 | \(s_1\) | \(s_2\) | \(s_3\) | \(s_4\) | \(0\) | \(s_1\) |
| 2 | 2 | \(s_1\) | \(s_2\) | \(s_3\) | \(s_4\) | \(0\) | \(s_1\) |
| 3 | 3 | \(s_1\) | \(s_2\) | \(s_3\) | \(s_4\) | \(0\) | \(s_1\) |

Table 2
Plaintext \( (p_i) \) needed to find out the exact position of symbols \( s_2 \) \((k_i = s_2 = 1)\).

| \(i\) | \(j\) | 0 | 1 | 2 | 3 | \(k_i\) | \(p_i\) |
|------|------|---|---|---|---|--------|--------|
| \(-\) | \(-\) | \(-\) | \(-\) | \(-\) | \(-\) | \(-\) | \(-\) |
| 0 | \(s_2\) | \(s_1\) | \(s_3\) | \(s_4\) | \(1\) | \(s_2\) | \(s_2\) |
| 1 | \(s_2\) | \(s_3\) | \(s_1\) | \(s_4\) | \(1\) | \(s_1\) | \(s_3\) |
| 2 | \(s_2\) | \(s_3\) | \(s_4\) | \(s_1\) | \(1\) | \(s_3\) | \(s_1\) |
| 3 | \(s_1\) | \(s_3\) | \(s_4\) | \(s_2\) | \(1\) | \(s_3\) | \(s_2\) |
| 4 \(\equiv 0\) | \(s_3\) | \(s_1\) | \(s_4\) | \(s_2\) | \(1\) | \(s_3\) | \(s_2\) |
| 5 \(\equiv 1\) | \(s_3\) | \(s_4\) | \(s_1\) | \(s_2\) | \(1\) | \(s_1\) | \(s_2\) |
| 6 \(\equiv 2\) | \(s_3\) | \(s_4\) | \(s_2\) | \(s_1\) | \(1\) | \(s_4\) | \(s_3\) |
| 7 \(\equiv 3\) | \(s_1\) | \(s_4\) | \(s_2\) | \(s_3\) | \(1\) | \(s_4\) | \(s_3\) |
| 8 \(\equiv 0\) | \(s_4\) | \(s_1\) | \(s_2\) | \(s_3\) | \(1\) | \(s_4\) | \(s_3\) |
| 9 \(\equiv 1\) | \(s_4\) | \(s_2\) | \(s_1\) | \(s_3\) | \(1\) | \(s_1\) | \(s_3\) |
| 10 \(\equiv 2\) | \(s_4\) | \(s_2\) | \(s_3\) | \(s_1\) | \(1\) | \(s_2\) | \(s_3\) |
| 11 \(\equiv 0\) | \(s_1\) | \(s_2\) | \(s_3\) | \(s_4\) | \(1\) | \(s_2\) | \(s_2\) |

Table 3
Plaintext \( (p_i) \) needed to find out the exact position of symbols \( s_3 \) \((k_i = s_3 = 2)\).

| \(i\) | \(j\) | 0 | 1 | 2 | 3 | \(k_i\) | \(p_i\) |
|------|------|---|---|---|---|--------|--------|
| \(-\) | \(-\) | \(-\) | \(-\) | \(-\) | \(-\) | \(-\) | \(-\) |
| 0 | \(s_3\) | \(s_2\) | \(s_1\) | \(s_4\) | \(2\) | \(s_3\) | \(s_3\) |
| 1 | \(s_3\) | \(s_4\) | \(s_1\) | \(s_2\) | \(2\) | \(s_3\) | \(s_3\) |
| 2 | \(s_1\) | \(s_4\) | \(s_3\) | \(s_2\) | \(2\) | \(s_1\) | \(s_3\) |
| 3 | \(s_1\) | \(s_2\) | \(s_3\) | \(s_4\) | \(2\) | \(s_3\) | \(s_3\) |
Table 4
Plaintext \((p_i)\) needed to find out the exact position of symbols \(s_4\) \((k_i = s_4 = 3)\).

| \(i\) | \(j\) | 0 | 1 | 2 | 3 | \(k_i\) | \(p_i\) |
|------|------|---|---|---|---|--------|--------|
| -    | -    | \(s_1\) | \(s_2\) | \(s_3\) | \(s_4\) | -      | -      |
| 0    | 3    | \(s_4\) | \(s_2\) | \(s_3\) | \(s_1\) | 3      | \(s_4\) |
| 1    | 0    | \(s_2\) | \(s_4\) | \(s_3\) | \(s_1\) | 3      | \(s_1\) |
| 2    | 1    | \(s_2\) | \(s_3\) | \(s_4\) | \(s_1\) | 3      | \(s_1\) |
| 3    | 2    | \(s_2\) | \(s_3\) | \(s_1\) | \(s_4\) | 3      | \(s_1\) |
| 4    | 0    | \(s_4\) | \(s_3\) | \(s_1\) | \(s_2\) | 3      | \(s_4\) |
| 5    | 1    | \(s_3\) | \(s_4\) | \(s_1\) | \(s_2\) | 3      | \(s_2\) |
| 6    | 2    | \(s_3\) | \(s_1\) | \(s_4\) | \(s_2\) | 3      | \(s_2\) |
| 7    | 3    | \(s_3\) | \(s_1\) | \(s_2\) | \(s_4\) | 3      | \(s_2\) |
| 8    | 0    | \(s_4\) | \(s_1\) | \(s_2\) | \(s_3\) | 3      | \(s_4\) |
| 9    | 1    | \(s_1\) | \(s_4\) | \(s_2\) | \(s_3\) | 3      | \(s_3\) |
| 10   | 2    | \(s_1\) | \(s_2\) | \(s_4\) | \(s_3\) | 3      | \(s_3\) |
| 11   | 3    | \(s_1\) | \(s_2\) | \(s_3\) | \(s_4\) | 3      | \(s_3\) |

Table 5
Partial keystream \((k_i)\) recovered when both plaintext \((p_i)\) and ciphertext \((c_i)\) are known.

| \(i\) | \(j\) | 0 | 1 | 2 | 3 | \(k_i\) | \(p_i\) | \(c_i\) |
|------|------|---|---|---|---|--------|--------|--------|
| -    | -    | \(s_1\) | \(s_2\) | \(s_3\) | \(s_4\) | -      | -      | -      |
| 0    | 1    | \(s_2\) | \(s_1\) | \(s_3\) | \(s_4\) | \(s_2 = 1\) | \(s_2\) | 4      |
| 1    | 2    | \(s_2\) | \(s_3\) | \(s_1\) | \(s_4\) | \(s_2 = 1\) | \(s_1\) | 11     |
| 2    | 1    | \(s_2\) | \(s_1\) | \(s_3\) | \(s_4\) | \(s_4 = 3\) | \(s_4\) | 5      |
| 3    | 0    | \(s_4\) | \(s_1\) | \(s_3\) | \(s_2\) | \(s_2 = 1\) | \(s_1\) | 9      |
| 4    | 0    | \(s_1\) | \(s_4\) | \(s_3\) | \(s_2\) | \(s_2 = 1\) | \(s_1\) | 5      |
Table 6
Plaintext \((p_i)\) recovered when the keystream \((k_i)\) is known.

| \(i\) | \(j\) | 0  | 1  | 2  | 3  | \(k_i\) | \(p_i\) |
|------|------|----|----|----|----|---------|--------|
| -    | -    | s₁ | s₂ | s₃ | s₄ | -       | -      |
| 0    | 2    | s₃ | s₂ | s₁ | s₄ | \(s₃ = 2\) | s₃     |
| 1    | 0    | s₂ | s₃ | s₁ | s₄ | \(s₄ = 3\) | s₄     |
| 2    | 3    | s₂ | s₃ | s₄ | s₁ | \(s₂ = 1\) | s₃     |
| 3    | 3    | s₂ | s₃ | s₄ | s₁ | \(s₁ = 0\) | s₂     |
| 4    | 0    | s₄ | s₃ | s₂ | s₁ | \(s₃ = 2\) | s₄     |