A finite-time convergent sliding mode control for rigid underactuated robotic manipulator

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(Received 19 November 2013; final version received 2 May 2014)

Robotic manipulators are widely used in space endeavors nowadays. For space applications, usually lightweight robotic manipulators are preferred. Hence, the system can be made underactuated intentionally. Also, one or more actuators may fail in space. Therefore, the system is highly uncertain and less actuated. The sliding mode control strategy does not depend on exact knowledge of the system model but chattering in states and control is observed. The present work showcases two different control techniques for achieving finite-time convergence and continuous control. The first technique is to choose a zero-error sliding function with a higher order sliding mode control and the second technique is to use a terminal sliding function with appropriate higher order sliding mode control for tracking control of an underactuated manipulator. A comparison is presented for the same.

Keywords: sliding mode control; higher order sliding mode control; terminal sliding mode control; underactuated robotic manipulator; passive joints; locked joints; uncertainties; parameter variations

1. Introduction

Robotic manipulators are positioning devices which have utility in terrestrial and space applications. For space applications, there is always a limit on the weight to be carried. The actuators have motors which are quite heavy. If a few joints are unactuated then the weight can be reduced considerably. Therefore, robotic manipulators for such applications can be deliberately designed to be underactuated. Also, one or more actuators can fail and robotic manipulator may have to complete the task in hand. Handling underactuation in robotic manipulators for space applications becomes a challenging task.

Owing to its utility, various research groups have shown keen interest in studying underactuated robotic manipulators. This paper deals with the problem of controlling the position of end-effector of underactuated robotic manipulator. Many different techniques have been used to control the problem undertaken. A method using holding brakes instead of actuators for the joints is shown in Arai and Tachi (1991). The dynamic coupling between the passive and the active joints is used in Bergerman, Lee, and Xu (1994). In Elangovan and Woo (2004) an adaptive fuzzy sliding control scheme and in Hasan (2012) artificial neural network technique is proposed to control a passive robotic manipulator. Sliding mode control has been used in Neila and Tarak (2011), Kim, Shin, and Lee (2002), Zhihong, Paplinski, and lator. A finite-time convergent sliding mode control for rigid underactuated robotic manipulator is discussed. In Section 2, the dynamics of a general $n$-link underactuated robotic manipulator is discussed. In Section 3, the robust controlling techniques called sliding mode control, higher order sliding mode control and terminal sliding mode control are explained. In Sections 4 and 5, simulation results for
three-link underactuated robotic manipulator and four-link underactuated robotic manipulator using higher order SMC and terminal SMC are discussed respectively. Finally, the conclusions are drawn in Section 7.

2. The system dynamic model

The dynamics of any n-link rigid robotic manipulator can be described in joint space by using Lagrangian approach as inferred by Craig (1989), Jain and Rodriguez (1991) and Spong, Hutchinson, and Vidyasagar (2006):

\[ M(\theta)\ddot{\theta} + C(\theta, \dot{\theta})\dot{\theta} = \tau. \]  (1)

Here, \( \theta = [\theta_1, \theta_2, \ldots, \theta_n]^T \) is the vector of joint angle variables with \( \theta_i \in [0, 2\pi) \) and \( \tau = [\tau_1, \tau_2, \ldots, \tau_n] \) is the controlling torque vector. \( M(\theta) \) is a symmetric and positive-definite inertia matrix and \( C(\theta, \dot{\theta}) \) is a vector of centripetal and coriolis terms. It is assumed that the mass of each link is concentrated at the mid-point of the respective link.

If the robotic manipulator has one or more passive joints, then the dynamics of the underactuated system can be expressed as

\[
\begin{bmatrix}
M_{aa} & M_{ap} \\
M_{pa} & M_{pp}
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_a \\
\ddot{\theta}_p
\end{bmatrix}
+
\begin{bmatrix}
C_{aa} & C_{ap} \\
C_{pa} & C_{pp}
\end{bmatrix}
\begin{bmatrix}
\dot{\theta}_a \\
\dot{\theta}_p
\end{bmatrix}
=
\begin{bmatrix}
\tau_a \\
0
\end{bmatrix}. \]  (2)

Here, \( \theta = [\theta_a, \theta_p]^T \) is the vector of active and passive joint angle variables, respectively, such that \( \theta_p \neq \phi \). The subscript \( p \) denotes passive joints and \( a \) denotes active joints. \( \tau = [\tau_a, 0] \) is the controlling torque vector for active and passive joints, and for all passive joints \( \tau \) is zero.

3. Robust control for underactuated robotic manipulator

3.1. Sliding mode control

Sliding mode control (SMC) (Utkin, 1977 and Young, Utkin, & Ozguner, 1999) is a technique in which motion of the system trajectory remains along a chosen line/surface/plane of the state space. The system dynamics are governed by the sliding surface and not by the original system parameters and the system behavior becomes invariant (Drazenovic, 1969; Edwards & Spurgeon, 1998; Utkin, 1977) to any disturbance or change in parameters. The simplest form of sliding mode control is first-order sliding mode control, where the control is made to switch between two chosen structures about the sliding surface. This switching causes chattering in system states.

3.1.1. Higher order sliding mode control

In HOSM (Agwan, Janardhanan, & Dewan, 2012; Fridman & Levant, 2002; Janardhanan, 2006; Kamal & Bandyopadhyay, 2012), sliding function \( s \) along with its other higher derivatives becomes zero in finite time. The \( r \)th order sliding mode is defined as a requirement of \( s(t) = \dot{s}(t) = \ddot{s}(t) = \cdots = s^{(r-1)}(t) = 0, \forall t \geq T, T < \infty \) in finite time. Higher order sliding mode forces the system states to slide on the surface in finite time and then remain on it.

3.1.2. Terminal sliding mode control

The design of terminal sliding mode control involves the choice of nonlinear terminal sliding function and determination of a control law, which will lead the system to reach equilibrium in finite time. The terminal sliding function is characterized by

\[ s = [\dot{e} + \beta e^{p/q} \text{sgn}(e)]. \]  (3)

Here, \( \beta > 0 \) and \( p \) and \( q \) are positive odd integers such that \( p > q \).

3.2. Choice of sliding surface and sliding mode

The choice of sliding surface is very important for designing an efficient control. In Cartesian space, the end-effector has to move to the desired position to perform a given job. For this, the unactuated joints are locked at any particular desired angles. By controlling the remaining actuated joints the position of end-effector can be controlled.

The journey of end-effector towards the desired point in Cartesian space is divided into two sections:

3.2.1. Locking of passive joints

The current and desired positions of the end-effector are described by Cartesian coordinates \( x \) and \( y \) and \( x_{\text{desired}} \) and \( y_{\text{desired}} \), respectively. The zero-error sliding surface is chosen as the difference between the current and desired \( x \) and \( y \) coordinates.

\[ s = \begin{bmatrix}
x - x_{\text{desired}} \\
y - y_{\text{desired}}
\end{bmatrix}. \]  (4)

Due to the use of the sliding function shown in Equation (4) for the system considered, the relative degree is 2. Hence, third-order sliding mode control will take the system towards the desired point in finite time. While moving towards the desired position, if the difference between the distances of the actuated and desired end-effector positions from the origin is positive, then the passive joints are locked sequentially by making their velocity and acceleration zero. Thus, sequential locking of the unactuated joints transforms the three-link underactuated and four-link underactuated problems under consideration to two-link fully actuated problems. The degree of freedom reduces to 2 and the reachable workspace also becomes less.

3.2.2. Finite-time convergence

After locking the passive joints, the pictures of the three-link underactuated robotic manipulator and four-link underactuated robotic manipulator are shown in Figure 1(a) and 1(b),
respectively. For a desired position of end-effector in Cartesian space, the unique desired values of active joint angles 1 and 3 are obtained using inverse kinematics. These desired angles are represented by $\theta_{1\text{desired}}$ and $\theta_{3\text{desired}}$, respectively. The actual current values of active joints are indicated by $\theta_1$ and $\theta_3$.

In this section, the aim is to get finite-time convergence to equilibrium with continuous control. This can be achieved using two different control strategies:

1. In the first technique a zero-error sliding function for appropriate higher order sliding mode is taken. The sliding function is taken as the difference between the actual and desired values of joint angles:

$$s = \begin{bmatrix} \theta_1 - \theta_{1\text{desired}} \\ \theta_3 - \theta_{3\text{desired}} \end{bmatrix}.$$  

Due to choice of zero error as the sliding function, the relative degree of sliding function is 2. Hence, any order of sliding mode greater than or equal to 2 can be achieved. In second-order sliding mode control the chattering in states is removed but control remains discontinuous. Using third-order sliding mode control (Fridman & Levant, 2002) makes the control torque continuous. Here, the rate of change of torque is modified to obtain the third-order sliding mode control. The control expression is given by

$$\dot{\tau}_{\text{HOSM}} = -\alpha \text{sgn}(\dot{s} + 2e_1 \text{sgn}(e_2)), $$

$$e_1 = (|s|^3 + |s|^2)^{1/6},$$

$$e_2 = \dot{s} + |s|^{2/3} \text{sgn}(s).$$  \hspace{1cm} (6)

After locking passive joints, three-link and four-link underactuated robotic manipulators get converted to two-link fully actuated manipulators. For two-link fully actuated robotic manipulator, the finite-time convergence to the sliding surface for the system under consideration has been shown in Agwan et al. (2012) and Bhave, Janardhanan, and Dewan (2013), using $\eta$ reachability condition.

2. A combination of terminal sliding mode and higher order sliding mode control is used in this methodology. In terminal sliding mode control, a nonlinear sliding function is used. The error in the actual and desired joint angles is denoted by $e$.

$$e = \begin{bmatrix} \theta_1 - \theta_{1\text{desired}} \\ \theta_3 - \theta_{3\text{desired}} \end{bmatrix}.$$  \hspace{1cm} (7)

The first and second derivatives of error are represented by $\dot{e}$ and $\ddot{e}$, respectively. In Equation (3), by taking $p = 1, q = 3$ and $q = 1$, the sliding function takes the form:

$$s = [\dot{e} + e^{1/3} \text{sgn}(e)].$$  \hspace{1cm} (8)

By using the sliding function in Equation (8), the relative degree of sliding function is 1. Hence, any order of sliding mode greater than or equal to 1 can be achieved. Here, the second-order sliding mode control (Fridman & Levant, 2002) will give chatter-free control. For this, the first derivative of $\tau$ has to be changed to shape $\dot{s}$. The control expression for second-order sliding mode control is given by

$$\tau = -\alpha \text{sgn}(\dot{s} + |s|^{1/2} \text{sgn}(s)).$$  \hspace{1cm} (9)

4. Three-link rigid underactuated robotic manipulator with passive second joint

The lengths of links 1, 2 and 3 of manipulator are taken as $L_1$, $L_2$ and $L_3$, respectively. The angles between link 1 and x-axis, between links 1 and 2 and between links 2 and 3 are indicated by $\theta_1$, $\theta_2$ and $\theta_3$, respectively. The coordinates of desired end-effector position are given by $(x_{\text{desired}}, y_{\text{desired}})$. While the end-effector is moving in the direction towards desired position, if the distance between current and desired positions of end-effector is positive, passive joint 2 is locked. After this, the distance between joint 1 and joint 3 becomes effective link 1 and distance between joint 3 and end-effector becomes effective link 2 of the new setup. The diagram of the three link underactuated robotic manipulator after its second joint is locked is shown in Figure 1(a).

To find the length of the new effective link, consider $\Delta OAB$ in Figure 1(a) for three-link manipulator. Applying sine rule to $\Delta OAB$,

$$\frac{L_{1\text{new}}}{\sin(\pi - \theta_2)} = \frac{L_1}{\sin(\theta_2/2)}. $$  \hspace{1cm} (10)

Therefore, the lengths of the new effective links 1 and 2 are given by

$$L_{1\text{new}} = 2L_1 \cos(\theta_2/2),$$

$$L_{2\text{new}} = L_3. $$ \hspace{1cm} (11)

The passive joint 2 is locked if the condition given by Equation (12) is fulfilled.

$$|L_{1\text{new}} - L_{2\text{new}}| < \sqrt{x_{\text{desired}}^2 + y_{\text{desired}}^2},$$

$$\sqrt{x_{\text{desired}}^2 + y_{\text{desired}}^2} > (L_{1\text{new}} + L_{2\text{new}}). $$  \hspace{1cm} (12)

In Figure 1(a), the angle $\theta_{1\text{new}}$ is shown between effective link $L_{1\text{new}}$ and x-axis and $\theta_{2\text{new}}$ is shown between effective links $L_{1\text{new}}$ and $L_{2\text{new}}$. For desired position of end-effector, the desired reference angles between effective link
L_{1\text{new}} \text{ and } x\text{-axis and between } L_{1\text{new}} \text{ and } L_{2\text{new}} \text{ are indicated by } \theta_{1\text{ref}} \text{ and } \theta_{2\text{ref}}, \text{ respectively. Using simple geometry in Figure 1(a), in } \triangle OAB, \angle OBA = \angle CBD = \theta_2/2. \text{ Hence, } 
\theta_{3\text{desired}} = \theta_{2\text{ref}} - \theta_2/2.

The desired angles for active joints 1 and 3 are given by

\begin{align}
\theta_{1\text{desired}} &= \theta_{1\text{ref}} - \frac{\theta_2}{2}, \\
\theta_{3\text{desired}} &= \theta_{2\text{ref}} - \frac{\theta_2}{2}.
\end{align}

### 4.1. Simulation results and discussion

The parameters of three-link rigid underactuated robotic manipulator considered for simulation in MATLAB are shown in Table 1.

The initial joint angles and velocities for joints 1, 2 and 3 are assumed to be zero.

| Link parameter | Three-link | Four-link |
|----------------|------------|----------|
| \(m_1\)       | 1 kg ± 0.2 kg | 1 kg ± 0.2 kg |
| \(m_2\)       | 1 kg ± 0.2 kg | 1 kg ± 0.2 kg |
| \(m_3\)       | 1 kg ± 0.2 kg | 1 kg ± 0.2 kg |
| \(m_4\)       | –          | 1 kg ± 0.2 kg |
| \(L_1\)       | 0.5 m      | 0.4 m     |
| \(L_2\)       | 0.5 m      | 0.6 m     |
| \(L_3\)       | 0.5 m      | 0.3 m     |
| \(L_4\)       | –          | 0.7 m     |
| \((x_{\text{desired}}, y_{\text{desired}})\) | (0.4332, 1.0409) m | (0.9309, -0.6667) m |

### 4.1.1. Choice of zero error as a sliding function for third-order sliding mode control

The third-order sliding mode control forces the end-effector to move towards its desired position in smooth manner. Due to the use of zero error as sliding function, the distance
between actual and desired end-effector position reaches zero in finite time along with the sliding function as seen in Figure 2(a). Fifty simulations were performed to cover the joint space and the average convergence time was found to be 3.6 s.

It can be seen in Figure 2(b), that by using third-order sliding mode control for the system under consideration, the control torques are continuous. Thus, by properly choosing the sliding function and sequentially locking the unactuated joints, finite-time convergence is achieved.

4.1.2. Choice of terminal sliding function for the second-order sliding mode control

The use of nonlinear terminal sliding function with appropriate higher order sliding mode forces the system to converge to equilibrium very fast as seen in Figure 3(a). For 50 simulations spread over the entire joint space, the average time taken by the end-effector to reach its desired position is 2.7 s.

The control signal is continuous but less smooth as compared to the technique using zero-error sliding function with higher order sliding mode as seen in Figure 3(b).

5. Four-link robotic manipulator with passive joints 2 and 4

The lengths of links 1, 2, 3 and 4 of manipulator are taken as \( L_1, L_2, L_3 \) and \( L_4 \), respectively. In Figure 1(b), the angles between link 1 and x-axis, between links 1 and 2, between links 2 and 3 and between links 3 and 4 are indicated by \( \theta_1, \theta_2, \theta_3 \) and \( \theta_4 \), respectively. While moving towards the desired end-effector position, when the distance between actual and desired end-effector positions becomes positive, passive joints 2 and 4 are locked sequentially. The distance between joints 1 and 3 is effective link 1 and distance between joint 3 and end-effector is effective link 2.

After locking passive joints 2 and 4, the picture of robotic manipulator is shown in Figure 1(b).

Applying cosine rule to \( \Delta OAB \) and \( \Delta BCD \), the new effective link length can be found as

\[
L_{1\text{new}} = \sqrt{L_1^2 + L_2^2 + 2L_1L_2 \cos(\theta_2)},
\]

\[
L_{2\text{new}} = \sqrt{L_3^2 + L_4^2 + 2L_3L_4 \cos(\theta_4)}.
\]

For locking passive joint 2, the conditions

\[
|L_{1\text{new}} - (L_3 + L_4)| < \sqrt{x_{\text{desired}}^2 + y_{\text{desired}}^2},
\]

\[
\sqrt{x_{\text{desired}}^2 + y_{\text{desired}}^2} < (L_{1\text{new}} + L_3 + L_4)
\]

have to be fulfilled. After joint 2 is locked, if the conditions

\[
|L_{1\text{new}} - L_{2\text{new}}| < \sqrt{x_{\text{desired}}^2 + y_{\text{desired}}^2},
\]

\[
\sqrt{x_{\text{desired}}^2 + y_{\text{desired}}^2} < (L_{1\text{new}} + L_{2\text{new}})
\]

are satisfied then passive joint 4 is locked. The angle \( \theta_{1\text{new}} \) is shown between effective link \( L_{1\text{new}} \) and x-axis and \( \theta_{2\text{new}} \) is shown between effective links \( L_{1\text{new}} \) and \( L_{2\text{new}} \). For desired position of end-effector, the desired reference angles between effective link \( L_{1\text{new}} \) and x-axis and between \( L_{1\text{new}} \) and \( L_{2\text{new}} \) are indicated by \( \theta_{1\text{ref}} \) and \( \theta_{2\text{ref}} \), respectively. Using simple geometry in Figure 1(b), the desired angles for active joints 1 and 3 are given by

\[
\theta_{1\text{desired}} = \theta_{1\text{ref}} + \alpha,
\]

\[
\theta_{3\text{desired}} = \theta_{3\text{ref}} + \theta_{2\text{ref}} + \beta - \theta_1 - \theta_2,
\]

where \( \alpha \) and \( \beta \) are found by applying sine rule to the triangles formed by sides \( L_1, L_3 \) and \( L_{1\text{new}} \) and sides \( L_3, L_4 \) and \( L_{2\text{new}} \), respectively, in Figure 1(b).

\[
\alpha = \arcsin \left( \frac{-L_2 \sin(\theta_2)}{L_{1\text{new}}} \right),
\]

\[
\beta = \arcsin \left( \frac{-L_4 \sin(\theta_4)}{L_{2\text{new}}} \right).
\]
5.1. Simulation results and discussion

The parameters of four-link rigid underactuated robotic manipulator considered for simulation in MATLAB are shown in Table 1. The initial joint angles and velocities for joints 1, 2, 3 and 4 are assumed to be zero.

5.1.1. Choice of zero error as the sliding function for third-order sliding mode control

Higher order sliding mode pushes the system to its equilibrium position in a smooth way in finite time. In Figure 4(a), it is seen that the distance between actual and desired end-effector position in Cartesian space reduces to zero in finite time. The average convergence time for 50 simulations in joint space in this case was found to be 4 s. Higher order sliding mode control gives continuous torques for active joints 1 and 3 as shown in Figure 4(b).

5.1.2. Choice of terminal sliding function for second-order sliding mode control

Terminal sliding function forces the distance between actual and desired end-effector position to reach zero in finite time. For 50 simulations spread over entire joint space the average convergence time was observed to be 3.9 s, as shown in Figure 5(a). The control signals are continuous and observed to be non-smooth as shown in Figure 5(b). The terminal sliding mode control makes the system reach its equilibrium point faster as compared to HOSM but experiences some oscillations.

6. Zero-error sliding function with HOSM vs terminal sliding function with HOSM

For both three-link and four-link underactuated robotic manipulators, simulations were performed for 50 cartesian points covering all the four quadrants. In Table 2, for
three-link and four-link underactuated robotic manipulator, the mean time taken by end-effector to converge using the above shown two techniques is presented. It is seen that the technique of using zero-error sliding function with HOSM takes approximately 20% more time in case of three-link and 10% more in case of four-link underactuated robotic manipulator as compared to the technique of using terminal sliding function with HOSM. It is observed that the control with only HOSM is smoother as compared to technique with TSM and HOSM but takes more time for convergence to equilibrium.

7. Conclusion

In the present work, we have designed sliding surfaces and control methods for finite time and robust position control of a planar articulated, underactuated manipulator. The concepts of terminal sliding mode and higher order sliding mode were used for this purpose. Two different control strategies are suggested: zero-error sliding function with appropriate higher order sliding mode and combination of terminal sliding mode control with higher order sliding mode control. These strategies are implemented to control the movement of end-effector of a three-link and four-link underactuated manipulator. Both the techniques are efficient for tracking a Cartesian point by an \( n \)-link rigid underactuated robotic manipulator by locking its passive joints. However, it is seen that the technique using terminal sliding mode control with HOSM gives faster convergence to equilibrium as compared to the technique using zero-error sliding function with higher order sliding mode control. Both the techniques make the control torque continuous.

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