THE FAR-FIELD HUBBLE CONSTANT

TOD R. LAUER
Kitt Peak National Observatory, National Optical Astronomy Observatories, 2 P. O. Box 26732, Tucson, AZ 85726; lauer@noao.edu

JOHN L. TONRY
Institute for Astronomy, University of Hawaii, 2680 Woodlawn Drive, Honolulu, HI 96822; jt@avidya.ifa.hawaii.edu

MARC POSTMAN
Space Telescope Science Institute, 3700 San Martin Drive, Baltimore, MD 21218; postman@stsci.edu

EDWARD A. AJHAR
Kitt Peak National Observatory, National Optical Astronomy Observatories, 2 P. O. Box 26732, Tucson, AZ 85726; ajhar@noao.edu

AND

JON A. HOLTZMAN
New Mexico State University, Box 30001, Department 4500, Las Cruces, NM 88003; holtz@nmsu.edu

Received 1997 August 26; accepted 1998 January 13

ABSTRACT

We used the Hubble Space Telescope to obtain surface brightness fluctuation (SBF) observations of four nearby brightest cluster galaxies (BCGs) to calibrate the BCG Hubble diagram of Lauer & Postman. This BCG Hubble diagram contains 114 galaxies covering the full celestial sphere and is volume-limited to 15,000 km s\(^{-1}\), providing excellent sampling of the far-field Hubble flow. The SBF zero point is based on the Cepheid calibration of the ground I\(_{KC}\) method as extended to the WFPC2 F814W filter by Ajhar and coworkers. The BCG globular cluster luminosity functions give distances essentially identical to the SBF results. Using the velocities and SBF distances of the four BCGs alone gives \(H_0 = 82 \pm 8\) km s\(^{-1}\) Mpc\(^{-1}\) in the CMB frame, valid on \(\sim 4500\) km s\(^{-1}\) scales; the error includes both systematic and random contributions. Use of BCGs as photometric redshift estimators allows the BCG Hubble diagram to be calibrated independently of recession velocities of the four nearby BCGs, yielding a far-field \(H_0 = 89 \pm 10\) km s\(^{-1}\) Mpc\(^{-1}\) with an effective depth of \(\sim 11,000\) km s\(^{-1}\). The larger error in this case is due to the photometric cosmic scatter in using BCGs as redshift estimators; this \(H_0\) is not significantly different from the more local value. The concordance of the present results with other recent \(H_0\) determinations and a review of theoretical treatments on perturbations in the near-field Hubble flow argue that going to the far field removes an important source of uncertainty, but that there is not a large systematic error to be corrected for to begin with. Further improvements in \(H_0\) depend more on understanding nearby calibrators than on improved sampling of the distant flow.

Subject headings: distance scale — galaxies: clusters: general — galaxies: distances and redshifts — galaxies: photometry

1. INTRODUCTION

Measuring the Hubble constant, \(H_0\), requires the observer to look out far enough so that the bulk velocities of galaxies are trivial compared to the Hubble flow itself. Because of the Virgo Cluster infall pattern, observation of the unbiased Hubble flow can only be contemplated at distances in excess of \(\sim 3000\) km s\(^{-1}\). Furthermore, bulk flows on even larger scales, such as those associated with the Great Attractor, may bias measurement of \(H_0\). Turner, Cen, & Ostriker (1992) and Shi, Widrow, & Dursi (1996), for example, show that under standard theories of structure formation, measurements of \(H_0\) can depart significantly from its true "global" value because of the inhomogeneous distribution of matter in the universe, unless care is taken to sample deeply with large angular coverage. Indeed, a common concern with many recent \(H_0\) determinations is that they are not truly sampling the distant Hubble flow (Bartlett et al. 1995).

Characterizing the far field requires observing large numbers of objects at large distances so that the Hubble diagrams are insensitive to random peculiar velocities or bulk flows. Hubble diagrams at present are largely based on the Tully-Fisher or \(D_n-\sigma\) relationships, the luminosities of supernovae (SNe Ia or SNe II), and brightest cluster galaxies (BCGs). Tully-Fisher distances are available out to \(\sim 9000\) km s\(^{-1}\) and have been recently used to measure a far-field \(H_0\) (Giovanelli et al. 1997), while \(D_n-\sigma\) have full-sky coverage out to only \(\sim 6000\) km s\(^{-1}\). Only a few SNe II have been observed in sufficient detail at large distances (Schmidt et al. 1994), but the SN Ia Hubble diagram is becoming richer with time and provides some sampling of the Hubble flow out to \(\sim 30,000\) km s\(^{-1}\) (Riess, Press, & Kirshner 1996; Hamuy et al. 1996). At present, however, calibration of the SN Ia distance scale remains controversial (see Sandage et al. 1996), and the SN Ia diagrams remain relatively sparse at large distances. In this work we focus on calibrating the BCG Hubble diagram, which is based on a recent characterization of BCGs as relative distance estimators (Postman & Lauer 1995).
In the classic work of Sandage (1972) and Sandage & Hardy (1973), BCGs were used to show that the Hubble flow was linear over a large range in redshift. Lauer & Postman (1994) observed BCGs to define a frame for measuring the peculiar velocity of the Local Group, but as this work was in progress they realized that they could test for $H_0$ variations with distance with greater precision than was previously available in response to the concerns of Turner et al. (1992). Lauer & Postman (1992) presented a Hubble diagram based on the 114 BCGs that defined the volume-limited full-sky sample of Abell clusters within 15,000 km s$^{-1}$, which is shown again here in Figure 1. In brief, the absolute magnitudes of BCGs, $L_m$, measured in apertures of fixed metric size, $r_m$, can be predicted from $z \equiv d \log L_{\text{mm}} / d \log r_m$ (Hoessel 1980). Figure 1 shows the metric luminosities as apparent fluxes, corrected by the $L_{\text{mm}}$ relationship to a standard value of $z = 0.5$.

The BCG Hubble diagram slope is $0.996 \pm 0.030$ of the expected value, consistent with a uniform Hubble flow over 0.01 \leq z \leq 0.05. Lauer & Postman (1992) limit any variation in the apparent or local $H_0$ (the Hubble constant measured over a limited depth, as compared to $H_0$ measured globally over the entire volume) to $\delta H_0 / H_0 < 0.07$. The SN Ia Hubble diagram also shows no evidence for $H_0$ variations with distance; Riess et al. (1996) show its slope (relative to Euclidean) to be $1.005 \pm 0.018$. The full-sky coverage of the Abell cluster sample is crucial, as any dipole pattern caused by large bulk flows (such as that advanced by Lauer & Postman 1994) will integrate out of the Hubble diagram to first order. The linearity of the BCG Hubble diagram shows that an excellent estimate of the far-field $H_0$ can be obtained once the zero point of the diagram is calibrated. We note that BCGs presently provide the only volume-limited sample that explores the Hubble flow at these distances.

A Hubble constant can be obtained from the BCG Hubble diagram once an absolute distance is known to a subset of the galaxies. In essence, one transfers the full sample to a common distance and finds the average absolute luminosity of the BCGs on the assumption that the calibrating set is typical. Random velocities and bulk flows of the BCGs contribute to the “cosmic scatter” in their luminosity distribution but cause no systematic offset (with the caveats discussed in §4.2). We contrast this approach with others that use the apparent distance ratio between the Virgo and Coma clusters, or any other near and single far aggregate of galaxies, to reach the far field. Instead, we are using the BCGs as complete probes of the Hubble flow over a large volume.

We chose to calibrate the BCG Hubble diagram with surface brightness fluctuation (SBF) distance estimates to four of the nearest BCGs. The SBF method (Tonry & Schneider 1988) uses the ratio of the second to first moments of the stellar luminosity function within early-type stellar systems as a distance estimator. The ratio of moments corresponds to an apparent magnitude, $\overline{m}$, that in the near-IR corresponds to the brightness of a typical red giant star. When the images are deep enough so that a star of apparent luminosity $\overline{m}$ contributes more than a single photon to an observation, the random spatial point-to-point SBFs in a galaxy image are dominated by the finite number of stars it comprises rather than photon shot noise. A power spectrum of the SBF pattern provides $\overline{m}$. Use of the SBF method on galaxies with distances known from other methods (see Jacoby et al. 1992 for additional details) provides the zero point $\overline{M}$, allowing absolute distances to be computed from $\overline{m}$.

The most recent calibration of the SBF method is presented by Tonry et al. (1997). Major components of this work are: (1) understanding how $\overline{M}$ varies with stellar population, (2) determining the zero point of the method, and (3) establishing the universality of the calibration. Tonry et al. (1997) observe in the $I_{\text{Kc}}$ band, which minimizes variations in $I_{\text{Kc}}$ with stellar population ab initio. They also show that variations in $I_{\text{Kc}}$ are fully characterized by the $(V-I)$ colors of the stellar systems. Based on 149 nearby galaxies, they find

$$I_{\text{Kc}} = (-1.74 \pm 0.07) + (4.5 \pm 0.25)[(V-I)_0 - 1.15] .$$

(1)

This relationship has a scatter of only 0.05 mag and agrees well with the theoretical calculations of Worthey (1993a, 1993b), both in slope and in zero point. Tammann (1992) was concerned that an earlier SBF calibration based on $(V-I)$ was incomplete and that $I_{\text{Kc}}$ additionally depended on the galaxies’ Mg$_2$ indices. In response, Tonry et al. (1997) use their extensive sample to show that there is no correlation between the residuals of equation (1) and Mg$_2$. The zero point of equation (1) is based on Cepheid distances to seven spiral galaxies with bulge SBF observations. Tonry et al. present numerous comparisons of SBFs to planetary nebula luminosity function, Tully-Fisher, $D_{\sigma}$, SN Ia, and SN II distances, finding no evidence for any systematic offset between SBF bulge and elliptical galaxy measurements, nor any other systematic effect that challenges the calibration.

Although the nearest BCGs are too far away for the SBF method to work from the ground, the high spatial resolution of the Hubble Space Telescope (HST) allows SBFs to be used beyond the 15,000 km s$^{-1}$ depth of the Lauer & Postman (1994) sample. An important caveat is that there is no direct match to the $I_{\text{Kc}}$ filter among the WFPC2 filter set. The F814W filter is a close analog to $I_{\text{Kc}}$. 

Fig. 1.—BCG Hubble diagram. $R$-band metric luminosities of the BCGs, corrected by the $L_{\text{mm}}$ relationship, plotted as a function of velocity in the Local Group frame. The line is the mean Hubble relation fitted.
(see Holtzman et al. 1995a) but requires additional calibration to tie it to the Tonry et al. (1997) zero point. Ajhar et al. (1997) accomplished this task in preparation for the present work by comparing HST F814W SBF observations to the $I_{KC}$ results for 16 galaxies in the Tonry et al. sample. For the WFPC2 CCDs and F814W filter, Ajhar et al. find

$$M_{1814} = (-1.73 \pm 0.07) + (6.5 \pm 0.7)(V-I) - 1.15,$$

with scatter similar to that about equation (1). A key difference between equations (1) and (2) is the steeper relationship between $M_{1814}$ and $(V-I)$, which Ajhar et al. (1997) show is consistent with the differences between the F814W and $I_{KC}$ filters. Calibration of HST for SBF work is thus crucial for the present work.

2. OBSERVATIONS AND REDUCTIONS

2.1. BCG Sample Selection

We selected the BCGs in four Abell clusters, A262, A3560, A3565, and A3742, for observation. These are among the nearest of the Postman sample (1994) and were chosen to minimize HST exposure time. We also wanted to minimize the effects of bulk flows on placement of the calibrating BCGs within the Hubble diagram, so we selected BCGs positioned in such a way that their mean photometric offset about the Hubble line in Figure 1 is largely insensitive to whether the Hubble diagram is constructed from velocities referenced to the cosmic microwave background (CMB), Local Group (LG), or the Abell cluster (AC) frame solution of Lauer & Postman (1994). Figure 2 shows how the positions of the BCGs in the Hubble diagram change with changing velocity system. Complete frame invariance of the results is difficult to achieve with only four BCGs, however; we discuss this issue later where our results are affected by it.

The BCG properties are given in Table 1. Photometry, details of BCG identification, and so on are discussed by Postman & Lauer (1995). Velocities are given in the CMB, LG, and AC frames (see Lauer & Postman 1994). The velocities are a weighted average of all galaxies selected to be within the given Abell cluster; the estimated velocity error is 184 km s$^{-1}$ (see Postman & Lauer 1995). Extinctions are from Burstein & Heiles (1984).

2.2. HST Observations and Reductions

Images were obtained in WFPC2 using the F814W filter. The galaxies were centered in the high-resolution PC1 chip. While suitable data were obtained in the flanking Wide Field Camera (WFC) CCDs, we chose to analyze the PC data only, given its superior resolution and the greater brightness of the central portions of the BCGs with respect to the sky. At the BCG distances, the apparent luminosity of an F814W SBF “star,” $I_{B14}$, is extremely faint, and so long exposures are required; the total exposures are given in Table 1. Although our ideal criterion is to obtain at least five photons per $I_{B14}$ star, our data contained only 2.3–3.8 photons per $I_{B14}$. The reasons for the shortfall were (1) that HST was 5% less sensitive through F814W than prelaunch numbers suggested, (2) that the galaxies were about 5% more distant than we had guessed from their redshifts, and (3) that $M_{I_{B14}}$ was significantly fainter (0.65 mag in the case of A262) because the galaxies were redder than anticipated, and $I_{B14}$ was more sensitive to color than we assumed. Nevertheless, all four galaxies yielded a strong SBF signal that could be accurately determined.

Because compact artifacts can strongly affect the SBF power spectrum, we built the total exposures from sets of “dithered” half-orbit images to eliminate hot pixels, CCD defects, as well as cosmic-ray hits. Each individual exposure was typically 1200 s long, with the actual exposure time set to maximize the total exposure obtained with two roughly equal exposures per orbit. The dither pattern consisted of moving the telescope between exposures in a skewed-

![Fig. 2.—BCG Hubble diagram with frame variations. This figure shows how the BCGs move within the Hubble diagram as the velocity frame changes. The lines start at the positions of the BCGs in the CMB frame and move through the Local Group frame, ending with the solid points in the Abell cluster frame. The four BCGs with SBF distances are labeled.](image)

| Abell Cluster | R.A.   | Decl.   | $V_{CMB}$ (km s$^{-1}$) | $V_{LG}$ (km s$^{-1}$) | $V_{AC}$ (km s$^{-1}$) | $E_{B-V}$ | Date       | Time (s) |
|--------------|--------|---------|-------------------------|------------------------|------------------------|-----------|------------|-----------|
| 262          | 01 52 46.3 | + 36 09 05 | 4650 | 5130 | 5310 | 0.060 | 1996 Feb 11 | 16400     |
| 3560         | 13 31 53.3 | + 33 14 04 | 4020 | 3510 | 3360 | 0.038 | 1996 Jan 16 | 9200      |
| 3565         | 13 36 39.1 | + 33 57 57 | 4110 | 3630 | 3450 | 0.030 | 1996 Jan 19 | 11600     |
| 3742         | 21 07 52.3 | + 47 10 43 | 4680 | 4800 | 4740 | 0.018 | 1996 Apr 22 | 16500     |

*Note.* Units of right ascension are hours, minutes, and seconds, and units of declination are degrees, arcminutes, and arcseconds. Velocities are weighted averages of all available cluster data given in the Cosmic Microwave Background (CMB), Local Group at rest (LG), or Abell Cluster (AC) frames (see Lauer & Postman 1994; Postman & Lauer 1995). Values of $E_{B-V}$ are from Burstein & Heiles (1984). The last two columns are the date of the HST observations and total exposure time.
square-spiral pattern, designed to achieve integral pixel shifts in the WFC CCDs. The pattern optimized removal of fixed-pattern CCD defects, while preserving the exact shape of any SBF pattern from exposure to exposure. While one might argue for an approach that would instead optimize information recovered with subpixel stepping, we did not have the exposure time available to obtain an equal number of the required 2 × 2 half-pixel steps and were concerned with the effects on the SBF pattern from any ad hoc interpolation scheme that might be required to assemble the completed image from a random set of offsets. Ironically, we did not use the WFC images in the present analysis, given the excellent quality and superior resolution of the PC1 images; however, this was a decision made after the data were in hand.

In the dither pattern, each exposure would be shifted from the previous one by ~10 WFC pixels (~1") in either the row or column direction, spiraling around the original pointing, with the exact shift adjusted by ±1 WFC pixel to avoid having any object land on the same row or column that it may have fallen on in a previous exposure. The latter criterion also meant that the exposure was simultaneously stepped ±1 pixel in a direction perpendicular to the major step, resulting in the spiral being skewed from perfect alignment with WFC row and column axes. The detailed shifts in all cases reflected the slight misalignment of the WFC CCDs with respect to perfect quadrature with the HST sky axes (see Holtzman et al. 1995b).

Integral pixel offsets for the WFC produce nonintegral steps in PC1, but for F814W, the PC1 images are nearly Nyquist-sampled, so shifting the images to a common center may be done with little error. As it is, however, we chose to stack the PC1 images with centering only to the nearest pixel to avoid the complexity of patching in the defects prior to interpolation. While this produces a slight blurring at the 1 pixel scale, this has little effect on measurement of the SBF signal, because the final composite image remains photon shot-noise limited at the highest spatial frequencies. This can be simply understood by considering the enclosed energy curve of the point-spread function (PSF). Holtzman et al. (1995b) show that only 32% of the light within the F814W PSF falls within its central core, corresponding to only a single photon for the typical exposure time in the present work.

The final composite PC1 images were assembled with an algorithm that looks at the statistical properties of the data set at any pixel location and rejects extreme values that might be caused by cosmic-ray hits or hot pixels. The BCG images are shown in Figures 3–6 (Plates 7–10), with the galaxies themselves largely subtracted to emphasize fine detail. Globular clusters are readily visible into the centers of all images. This is the strength of using HST for SBF work—globulars, background galaxies, and dust clouds are easily recognized and excluded when measuring the SBF power spectrum. Dust clouds are also visible in three galaxies, A262, A3560, and A3565; indeed, their centers are completely or nearly obscured by dust.

Although SBF are measured from the PC1 images, we do need the WFC images to measure the sky levels, which are required as part of the analysis. Sky levels were measured from a 20' × 20' patch extracted from the far corner of WF3. This is the corner of WF3 diagonally opposite the WFPC2 field, and is thus the portion of the WFPC2 field most distant from the BCG centers; the typical displacement of the sky patch from the galaxy centers is ~125". The galaxies still contribute light to the sky measurements at this modest distance; however, their contribution to the sky is readily estimated and corrected for, using ground-based surface photometry extending to much larger radii. The sky values given in Table 2 have been corrected for galaxy light contributions.

### 2.3. (V−I) Colors

As equation (2) shows, the $M_{\text{I}}$ value for a given BCG depends strongly on its $(V−I)$ color. We measure $(V−I)$ from ground-based photometry over an annulus between 5" and 15" in radius from the galaxy centers to match the area of the PC1 field used for the SBF measurements. The photometry for the three southern BCGs was obtained under excellent conditions at the Cerro Tololo Inter-American Observatory 1.5 m telescope. For A262, the $I_{\text{KC}}$ image was obtained at the KPNO 4 m, and John Blakeslee kindly obtained the $V$ image at the MDM Observatory 2.4 m. For the small redshifts of the BCGs, the $k$-corrections are $k_V \approx 2.0e$, and $k_I \approx 1.1e$; Galactic absorption corrections are $A_V: A_I = 3.04: 1.88: 1.00$. The observed $(V−I)$ and reduced $(V−I)_0$ colors are both given in Table 2. In passing, we note that ground $(V−I_{\text{KC}})$ and WFPC2 $(V_{\text{F814W}}−I_{\text{F814W}})$ are essentially identical (Holtzman et al. 1995a).

While we did not obtain WFPC2 F555W data, we did compare the F814W photometry to the ground $I_{\text{KC}}$ data. For $(V−I) \sim 1.25$, Holtzman et al. (1995a) find $I_{\text{KC}}−I_{\text{F814W}} \approx −0.04$; unfortunately, even with this transformation, the WFPC2 $I_{\text{F814W}}$ fluxes are still brighter than the ground values by 0.04 mag, with a spread of 0.05 mag. Ajhar et al. (1997) found excellent agreement between WFPC2 $I_{\text{F814W}}$ and ground $I_{\text{KC}}$, transformed to $I_{\text{F814W}}$ for their sample; so the present mismatch is disappointing. Since to first order we might expect any deviations in ground $V$ to correlate with those in $I_{\text{KC}}$, we chose to base

| Abell Cluster | $I_{\text{F814}}$ | SBF | Sky | $(V−I)$ | $(V−I)_0$ | $M_{\text{I}}$ | $(m−M)$ | ± | GCLF |
|--------------|-----------------|-----|-----|---------|------------|-------------|-----------|---|------|
| 262 .......... | 33.35 ± 0.15    | 2.3 | 21.40 | 1.401 | 1.317 | −0.642 | 33.77 | 0.19 | 25.85 |
| 3560 .......... | 32.33 ± 0.10 | 3.4 | 21.61 | 1.289 | 1.234 | −1.183 | 33.35 | 0.14 | 25.12 |
| 3565 .......... | 32.47 ± 0.08 | 3.8 | 21.70 | 1.286 | 1.239 | −1.151 | 33.47 | 0.13 | 25.72 |
| 3742 .......... | 33.08 ± 0.08 | 3.0 | 21.73 | 1.305 | 1.270 | −0.950 | 33.88 | 0.12 | 25.77 |

Note.—SBF amplitudes are given as $I_{\text{F814}}$ magnitudes, with no reddening or $k$-corrections, or as number of $e^−$ detected. Sky is $I_{\text{F814}}$ magnitudes per square arcsecond after correction for galaxy-light contamination. Colors are $(V−I)$ observed and $(V−I)_0$ after reddening and $k$-corrections, $M_{\text{I}}$ is the predicted SBF absolute luminosity calculated from eq. (2), and GCLF is the $I_{\text{F814}}$ turnover magnitude of the globular cluster luminosity function.
PSF is appropriate to use for the fluctuation analysis, with mismatch and normalization error amounting to about 0.05 mag.

An observed power spectrum always has excess power at wavenumbers below 10 or so (wavelengths longer than ~30 pixels) arising from poor flattening, poor galaxy subtraction, dust, and so on. With the exception of A262, we found very good agreement with the PSF beyond these contaminated wavenumbers. We normally use the rms variation in $P_0$ as a function of where we begin the power-spectrum fit, as an additional error component. In the case of A262, there is a range of $P_0$ values that are allowed by the power spectrum; this range is reflected in the larger error budget for this galaxy. PSF fits to the power spectra of the BCGs are shown in Figure 7.

The contribution from the residual point sources was small, always less than a 0.20 mag correction to $P_0$ and more typically 0.10 mag. Since we think we know this variance contribution quite well, at least to 25%, the contributed error is small. As a test of this, we routinely analyze different annuli independently, and (except for A262, wherein the central annulus was obscured by dust) we used 4 annuli at 1", 2", 4", and 8" mean radius. If we have an error in the residual point-source correction (or an unknown source of variance which does not scale with galaxy brightness), it will show up as a radial gradient in $m$. A3560 had a gradient of about 0.1 mag in each of the outer annuli, but the other three had no gradient at the 0.05 mag level, giving us confidence that we have modeled the residual variance well and have no unforeseen source of variance. The gradient in A3560 may very well be real; there is a 0.04 mag color gradient in $(B-R)$ from the center to 20" in radius, which would produce a gradient in $m$ consistent with what we see.

The estimates and errors for $m$ are derived from the averages of the values determined for each annulus, and the formal uncertainty in the $P_0$ fitting procedure added in quadrature to 0.05 mag for the PSF uncertainty. With the exception of A3560, where we think we see a real gradient in $m$, the scatter between the different annuli is consistent with the error estimates. We list the apparent SBF $I_{814}$ fluxes in Table 2 uncorrected for extinction and prior to $k$-correction. The SBF signal strength in electrons is also shown.

2.5. The Turnover of the Globular Cluster Luminosity Function

Measuring $m$ requires characterization of the galaxy's globular cluster luminosity function (GCLF) to estimate the residual variance contributed by the undetected (faint) portion of the GCLF as well as by the undetected faint galaxies. Figure 8 shows the luminosity functions of the objects found in the images of our sample along with the GCLF, background galaxy, and combined luminosity function fits. The fitting procedure naturally produces an estimate of the GCLF turnover magnitude in the $I$ band for an assumed Gaussian $\sigma$ width of 1.4 mag. The $m_I^o$ GCLF turnover magnitudes are listed in Table 2. The estimated turnovers are all near the $I \sim 25$ completeness limit and are uncertain by 0.2–0.3 mag.

Based on the SBF distance moduli, the mean absolute magnitude of the four BCG GCLF turnovers in the $I$ band is $\langle M_I^o \rangle = -8.29 \pm 0.18$; the error is consistent with the...
measurement errors and an estimated intrinsic scatter in the GCLF technique of \( \sim 0.25 \) mag. Few deep \( I \)-band GCLFs of giant elliptical galaxies have been published for comparison; however, \( \langle M_V^0 \rangle \) is consistent with the M87 measurements of Whitmore et al. (1995), who found \( m_V^{0,\text{M87}} = 22.67 \). Using the SBF distance modulus to the Virgo Cluster of \( 31.03 \pm 0.05 \) (Tonry et al. 1997) yields \( M_V^{0,\text{M87}} = -8.36 \). For additional comparison, a crude estimate of \( M_V^0 \) can be based on the following assumptions: (1) Combining the distance modulus to Virgo with the average observed apparent magnitude for the GCLF turnover in Virgo in the \( V \) band of \( m_V = 23.75 \pm 0.05 \) (Blakeslee \\& Tonry 1996) yields \( M_V^0 = -7.28 \pm 0.07 \). (2) Taking the mean color \( (V-I)_0 = 1.10 \pm 0.1 \), based on the average of the mean \( (V-I)_0 \) found in Coma's IC 4051 of \( (V-I)_0 = 1.08 \) (Baum et al. 1997) and that found in M87 of \( (V-I)_0 = 1.12 \) (Whitmore et al. 1995), and applying an uncertainty of 0.1 mag yields an estimate of \( M_V^0 = -7.28 - 1.10 = -8.38 \pm 0.12 \), also in good agreement with the BCG value.

3. MEASUREMENT OF \( H_0 \)

The present data set permits two approaches to measuring \( H_0 \). The first and most obvious approach is simply to form Hubble ratios for the four BCGs and average them in an optimal way. The BCGs all have velocities in excess of \( 4000 \) km s\(^{-1} \) (in the CMB frame) and may be far enough away so that their average Hubble ratio might be close to the true value of \( H_0 \). At the same time, this approach does not make use of the BCG Hubble diagram, nor does it transfer the results to the 15,000 km s\(^{-1} \) far field. The BCGs instead are simply treated as test particles, without reference to their photometric properties (although we do use cluster averages for the velocities).

The second approach is to use the BCGs as redshift estimators to avoid any use of the Doppler velocities of the four SBF BCGs themselves, in an effort to skip over the near velocity field—this is the approach promised by the title of our paper. The frame independence of both the BCG
Hubble diagram and the $L_m$-$z$ relationship argues that we can successfully reduce the BCGs to a common distance. The SBF distances then permit calibration of the BCGs as absolute rather than relative distance estimators.

In either approach, the mathematical formalism used to derive a Hubble ratio is the same. For each SBF observation, we compute a distance measurement $D$ in megaparsecs from the expression

$$D = \text{dex} \left[ 0.2 (I_{814} - M_{814} - 25) \right],$$

where $I_{814}$ and $M_{814}$ are corrected for extinction and $k$-dimming. The Hubble ratio is then just

$$H_0 = v / D,$$

where $v$ is either the observed velocity of the BCG in the appropriate reference frame (for the first approach) or the estimated velocity of the BCGs using the prescription in Postman & Lauer (1995) and summarized in § 3.2 (for the second approach).

3.1. $H_0$ from the SBF BCG Hubble Ratios Alone

The BCG $(V - I)_0$ colors and the F814W SBF calibration of equation (2) yield the absolute fluctuation magnitude $M_{814}$. Using a $I_{814}$ $k$-correction of $\approx 7z$ (Tonry et al. 1997) and the measured values for the apparent $I_{814}$, we derive the distance moduli and errors listed in Table 2. Converting these to distances in megaparsecs (eq. [4]) and using the velocities in the CMB, LG, and AC frames, we compute Hubble ratios in Table 3. The errors listed here include the estimated velocity error of $184 \text{ km s}^{-1}$, plus a nominal $100 \text{ km s}^{-1}$ allowance for peculiar velocity with respect to the local velocity field (added in quadrature), although this term could plausibly be as large as $5\%$. There is no allowance for bulk flow since we are explicitly trying to remove this by examining different reference frames. The average Hubble ratios are the weighted logarithmic averages, and the errors are those expected given the individual distance errors.

While this sample is too small to solve for a preferred reference frame at $4000 \text{ km s}^{-1}$, we do see the insensitivity of the average Hubble ratio among the three frames because of the sampling over the sky. It is also apparent that $\chi^2$ prefers the CMB frame to the Local Group or Abell cluster frame, but again the numbers are small. Because these SBF distances appear to be extremely consistent and accurate, a larger sample observed by HST throughout the sky could
after adopting a linear Hubble flow and an ad hoc The indeed based on comparing the physical properties of BCGs Postman and Lauer (1992, 1994) Postman (1995),

\[ L_{\text{m}} \text{ of physical radius for a given value of } \Omega. \]

finding the angular aperture at which the enclosed surface

\[ 17\% \text{ accuracy from a surface brightness curve of growth by } \]

nocity within the metric radius & Oke The (Gunn 1975).

metric radius also implies an error in the apparent lumi-

\[ D = 0.24 \text{ mag, which translates into a typical distance error of } \]

on a \[ a \]

\[ L \]

\[ \text{BCG in the } \eta \text{ Postman sample at } 15,000 \text{ km} \]

s\[ -1 \]

\[ \Omega \]

\[ \text{SBF BCGs to } \eta \text{ the nearby } \]Coma Cluster by finding the ratio of the Virgo to the Virgo Cluster but adopting the velocity of the much more distant Coma Cluster by finding the ratio of the Virgo to Coma distance from some form of relative distance estimator, such as the \[ D_{\text{r-\sigma}} \]method.

In the present case, the transference to the far field is somewhat less obvious, since we are not making an explicit comparison of the four SBF BCGs to, say, the most distant BCG in the Lauer & Postman (1994) sample at 15,000 km \[ s^{-1} \]. Instead, we will implicitly compare the SBF BCGs to the entire Lauer & Postman sample on the presumption that all BCGs can be transferred to a common distance based on a simple linear Hubble flow model. One can add a bulk flow to this model, but in the end this makes little difference, as we discuss below. The method works as a far-field measure because the vast majority of the BCGs are near the outer portions of the volume-limited sample.

The BCG \[ L_{\text{m-\sigma}} \]distance estimator, presented in Lauer & Postman (1992, 1994) and Postman & Lauer (1995), is indeed based on comparing the physical properties of BCGs after adopting a linear Hubble flow and an ad hoc \[ H_{\Omega} \]. The \[ L_{\text{m-\sigma}} \]relation works by allowing \[ L_{\text{m}} \]to be predicted based on \[ \sigma \]as measured at the metric radius \[ r_{\text{m}} \]. The scatter in \[ L_{\text{m}} \]is 0.24 mag, which translates into a typical distance error of \[ \sim 17\% \]; this is larger than the error expected for a pure inverse-square distance estimator, because an error in the metric radius also implies an error in the apparent luminosity within the metric radius (Gunn & Oke 1975). The \[ L_{\text{m-\sigma}} \]relation is equivalent to assuming that BCGs all have the same average-enclosed surface brightness as a function of physical radius for a given value of \[ \sigma \]. Postman & Lauer (1995) showed that they could estimate BCG redshifts to 17% accuracy from a surface brightness curve of growth by finding the angular aperture at which the enclosed surface brightness and \[ \sigma \]were consistent with the \[ L_{\text{m-\sigma}} \]relationship, then using the aperture size as a metric distance estimator.

Figure 9 shows the plot of estimated versus observed redshift presented by Postman & Lauer (1995). The estimated redshift results from calculating the velocity required to bring a given BCG onto the ridgeline of the \[ L_{\text{m-\sigma}} \]relation. The scatter in \[ z_{\text{a}} \]thus reflects the scatter of the BCGs about the ridgeline. Lauer & Postman (1994) showed that this scatter is strongly dominated by random photometric differences between the BCGs rather than random peculiar velocities or velocity errors—photometric scatter is constant with redshift, as are the residuals about the \[ L_{\text{m-\sigma}} \]relationship, while the decreasing relative importance of random velocities with increasing Hubble velocity would cause the scatter to decline with redshift. In other words, the \[ L_{\text{m-\sigma}} \]relationship itself has little, if any, dependence on the random peculiar velocities of the nearer BCGs.

The estimated redshifts for the four SBF BCGs (given in Table 3) are thus the best estimates of their true Hubble velocities, as based only on their photometric properties. Hubble ratios can then be calculated, given the SBF dis-

| ABELL | \( D \) (Mpc) | \( z_{\text{r}} \) | \( H_{\Omega} \) |
|-------|--------------|-------------|-----------|
| 262   | 57 ± 5       | 0.0250      | 82 ± 8    |
| 3560  | 47 ± 3       | 0.0126      | 86 ± 7    |
| 3565  | 49 ± 3       | 0.0108      | 83 ± 6    |
| 3742  | 60 ± 3       | 0.0187      | 78 ± 6    |
| Averages |            |             | 82 ± 8    |
| \( \chi^2/N \) |          |             | 0.3       |

\[ \text{Note—The third column is the redshift estimated from the BCG } L_{\text{m-\sigma}} \text{ relationship (see Postman & Lauer 1995). Hubble ratios are in } \text{km s}^{-1} \text{ Mpc}^{-1}. \] The row denoted “Averages” gives the logarithmic average of the individual ratios, with error bars reflecting all systematic errors (see text). \( \chi^2/N \) is calculated using errors without common, systematic contributions and gives an indication of the internal consistency of the ratios.
ances; the implied far-field Hubble ratios are given in Table 3. Again, the average value is best estimated with a error-weighted logarithmic average of the ratios. The far-field Hubble constant implied is $H_0 = 89 \pm 10$ km s$^{-1}$Mpc$^{-1}$. The error will be discussed in detail in the next section, but we note here that it is dominated by the BCG photometric scatter about the $L_{wz}$ relationship. The large random scatter, in fact, gives the far-field $H_0$, a larger error than that in the more local “SBF-alone” $H_0$ derived in the previous section.

In this context, it is worth noting that A262 is among the most deviant BCGs with respect to its position in the $L_{wz}$ relationship, a conclusion echoed in the highly deviant Hubble ratio that it yields using its photometric redshift as the velocity (see the last column in Table 3). The $\chi^2$ value for the far-field $H_0$ is also large. Deleting A262 yields a lower far-field $H_0 = 79 \pm 10$ km s$^{-1}$Mpc$^{-1}$. This value is consistent with the former value, but it does suggest that a more complete sampling of the nearby BCGs may be an attractive way to improve the accuracy of the BCG far-field $H_0$.

As the far-field $H_0$ does not explicitly depend on the velocities of the individual SBF BCGs, it is explicitly independent of the velocity frame. There may be implicit dependences on frame, since different choices of Hubble velocity will affect the particular placement of any given BCG within the $L_{wz}$ relationship; however, such effects are tiny. For example, at $z = 0.5$, the ridgeline value of $L_{wz}$ varies by only 1% among the three frames. More to the point, the $z_w$ varies by only 2% for A262 and less than 1% for the other BCGs over the various frames. This result is not surprising, since any dipole pattern caused by a large bulk flow superimposed on the BCG recession velocities will integrate out of the $L_{wz}$ relationship because it is defined from the full sky.

A more relevant question is whether the nearby SBF BCG set may be offset from the more distant BCGs by some global perturbation of the Hubble flow that is limited to, say, 5000 km s$^{-1}$ or some other small fraction of the 15,000 km s$^{-1}$ limit of the Abell cluster sample. This was the sort of problem posed by Turner, Cen, & Ostriker (1992) and addressed by the Lauer & Postman (1992) BCG Hubble diagram. The linearity of the BCG Hubble diagram argued that $|d_H| < 0.07$ between 3000 and 15,000 km s$^{-1}$. Further, Lauer & Postman (1992) showed that beyond 9000 km s$^{-1}$, which includes the bulk of the BCG sample, $|d_H| \leq 0.02$ on radial shells of 3000 km s$^{-1}$. In other words, there is no evidence that scatter in the $L_{wz}$ relationship or its ridgeline is affected by any global variations in $H_0$ within 15,000 km s$^{-1}$. The $L_{wz}$ relationship is defined by the full BCG sample, unweighted by distance; the effective scale of the far-field BCG Hubble constant is just the average BCG recession velocity, or $\sim 11,000$ km s$^{-1}$.

3.3. Errors in $H_0$

Both the “SBF-alone” and far-field $H_0$ values are affected by several random errors. The strong dependence of $\dot{M}_{814}$ on $(V-I)$ puts a premium on accurate colors. We do not have multiple ground observations of the BCGs, but previous experience plus the quality of the standard star solutions argues that the error in $(V-I)$ is likely to be close to 0.012 mag. We also include a 10% uncertainty in $E_{B-V}$, and, $\sim 20\%$ errors in the $(V-I)$ and $I_{814}$ $k$-corrections. The final net color error gives a typical $0.09$ mag error for estimating $\dot{M}_{814}$ from equation (2); this is added in quadrature to the measurement error in $I_{814}$ given in Table 2 to give the total random distance error. Calculation of the Hubble ratios also includes a 184 km s$^{-1}$ velocity error and 100 km s$^{-1}$ peculiar velocity term for the SBF-alone measures and a 17% redshift error for the Hubble diagram values, as noted above. The errors in the individual Hubble ratios given in Table 3 reflect the random error contributions only.

Important systematic errors include uncertainties in the F814W SBF calibration, the absolute zero point of the $I_{814}$ relationship on which it is based, and in the $HST$ PSF. Scatter about the $HST$ $M_{814}$ versus $(V-I)$ relationship argues that the uncertainty in predicting $\dot{M}_{814}$, given perfect $(V-I)$, is $\sim 0.10$ mag (Ajhar et al. 1997). Tonry et al. (1997) give $\sim 0.07$ mag as the error in the $I_{814}$ SBF zero point, which is itself a composite of the statistical errors in the ground $I_{814}$ versus $(V-I)$ relationship plus uncertainties in the Cepheid calibration of the relationship. The systematic error associated with the PSF is 0.05 mag, as discussed in §2.4. If we add these errors in quadrature, we get a total systematic uncertainty of 0.13 mag in predicting distances.

We include no systematic error in velocity for $H_0$ estimated from the Hubble diagram, given the large random error in the estimated velocities. For the SBF-alone $H_0$ estimate, however, we do include an error for its relation to the true far-field value. Since the four SBF BCGs are among the nearest of the Lauer & Postman (1994) sample, we take the net uncertainty in $H_0$ measured at $\sim 4500$ km s$^{-1}$ scales as $7\%$, given the variation of $\delta H \sim 0.07$, allowed out to 15,000 km s$^{-1}$ by the Lauer & Postman (1992) BCG Hubble diagram. In terms of velocity, this error corresponds to $\sim 300$ km s$^{-1}$ at the SBF BCG distances, similar to plausible bulk flows on this scale regardless of how the BCGs sample the Lauer & Postman (1994) flow. The errors in the final average $H_0$ values given in Table 3 reflect the distance error and, except for the Hubble diagram solution, the 7% far-field error added in quadrature to the statistical error.

There are a number of paths for improving the present results. The Hubble diagram $H_0$ can best be refined by obtaining SBF distances to more nearby BCGs. Refining the $H_0$ estimated solely from SBF distances, however, requires observation of more distant galaxies, as the present use of near-field velocities is an important uncertainty. Improving the F814W SBF calibration will also be useful. Fortunately, $HST$ SBF observations for the nearest galaxies can generally be done within an orbit and in many cases may be made from images obtained for other purposes, as was the case for the Ajhar et al. (1997) sample. Because this calibration may improve, we have attempted to make the path from $I_{814}$ to distances readily visible. Revised values of $H_0$ can thus be quickly derived when better data become available.

4. DISCUSSION

4.1. Comparison of Recent $H_0$ Measurements

The present $H_0$ values are somewhat larger than many recent $H_0$ measurements based on calibration of Hubble diagrams of a variety of distance estimators, but they are consistent with the higher of the comparisons. Giovanelli et al. (1997) find $H_0 = 69 \pm 5$ km s$^{-1}$ Mpc$^{-1}$, using Cepheid distances to 12 galaxies, which were used to calibrate a composite Tully-Fisher relationship based on 24 clusters of galaxies within 9000 km s$^{-1}$. As do we, Giovanelli et al. emphasize their independence from the classic Virgo/Coma
distance ratio approach. Giovanelli et al. did not show a Hubble diagram or list their clusters, but from their description of the cluster distribution we estimate that the effective depth of their $H_0$ determination is $\sim 6000$ km s$^{-1}$. Calibration of the Tully-Fisher relationship has been a major goal of the HST Cepheid key project as well (Freedman et al. 1994). Mould et al. (1995) find $H_0 = 82 \pm 11$ km s$^{-1}$ Mpc$^{-1}$, based on a Tully-Fisher calibration that reaches beyond 4000 km s$^{-1}$ but that also depends heavily on the Virgo Cluster sample. Freedman (1997) gives $H_0 = 73 \pm 8$ km s$^{-1}$ Mpc$^{-1}$, as a provisional summary of the key project work to date.

Supernovae also are providing excellent probes of the Hubble flow. Riess, Press, & Kirshner (1996) present a Hubble diagram based on their light-curve-shape method applied to 20 SNe Ia; using three Cepheid calibrators, they conclude $H_0 = 64 \pm 6$ km s$^{-1}$ Mpc$^{-1}$ on $\sim 7000$ km s$^{-1}$ scales. Hamuy et al. (1996) find $H_0 = 63 \pm 4$ km s$^{-1}$ Mpc$^{-1}$, an essentially identical result, using their $\Delta m_{15}$ decay rate estimator applied to 29 SNe Ia with an effective depth of $\sim 12,000$ km s$^{-1}$. Proper use of SNe Ia as distance estimators remains controversial, however, with significant disagreement on how the light-curve decay rate relates to the SN Ia peak luminosity. Sandage et al. (1996), for example, emphasize a Hubble diagram approach as well, using SNe Ia, and conclude $H_0 = 57 \pm 4$ km s$^{-1}$ Mpc$^{-1}$.

This value, while consistent with those of Riess et al. (1996) and Hamuy et al. (1996), argues for $H_0$ near the lower ends of their error bars rather than the upper ends, as would be more consistent with the Tully-Fisher results cited above and the present BCG results.

There is an abundance of other recent measurements of $H_0$ that we could cite, but as many of them are based heavily or exclusively on the Virgo or Coma Cluster, we find them less attractive than methods featuring rich sampling of the Hubble flow. It is also worth noting that while we are comparing $H_0$ estimates that all depend on the HST Cepheid calibration work, different methods make use of different calibrators, which may account for a portion of the variance in $H_0$ among authors. At this writing, Feast & Whitelock (1998) are arguing that the Cepheid scale itself should be revised based on Hipparcos parallax measurements. If so, then the Tonry et al. (1997) and Ajhar et al. (1997) SBF zero points will need revision.

Two promising alternative approaches for measuring $H_0$, which are completely independent of local calibration, are gravitational lens-induced time-delay observations and measurement of the Sunyaev-Zeldovich (SZ) effect for $z \geq 0.05$ clusters. As Freedman (1997) summarizes, however, measurements of the SZ effect are neither accurate nor consistent enough at this time to provide an $H_0$ that challenges the more local measures. Gravitational lenses also have the potential to produce a far-field $H_0$ that steps over all the distance-ladder problems that bedevil more traditional methods; however, detailed mass distributions of the lenses are required, which presently limits their accuracy. Kundic et al. (1997) find $H_0 = 64 \pm 13$ km s$^{-1}$ Mpc$^{-1}$, based on the observed time delay between the two QSO images in the classic 0957+561 lens at $z = 0.36$. In contrast, Schechter et al. (1997) present time delays and mass models for the lens PG 1115+080 that favor $H_0 = 42$, although they also present a model that gives $H_0 = 64$. It is thus difficult to make a strong case at present that gravitational lenses, which probe the Hubble flow on extremely large scales, are yielding consistent $H_0$ values significantly smaller than the $z \leq 0.05$ measurements.

4.2. Is the BCG Far Field Far Enough?

Observational evidence suggests that the far field has been reached. While the sampling of the Hubble flow remains sparse beyond 15,000 km s$^{-1}$, the limiting radius of Lauer & Postman (1992) and the SN Ia Hubble diagrams of Riess et al. (1996) and Hamuy et al. (1996) are consistent with linear flows out to $\sim 30,000$ km s$^{-1}$. Going to cosmological distances, Kim et al. (1997) use SNe Ia to constrain $\delta_H$ by comparing 28 SNe with $0.35 < z < 0.65$ to 18 SNe of the Hamuy et al. (1996) sample. The motivation for developing such SN Hubble diagrams is to measure $\Omega_M$ and $\Lambda$; such cosmological tests presume an unbiased local Hubble flow as a point of departure. On the other hand, with assumed $\Omega_M$ and $\Lambda$, one can test for significant deviation over extremely large scales. For $\Omega_M \leq 1$, Kim et al. find $\delta_H < 0.05$ (1σ), or $\delta_H < 0.1$ (95% confidence). For $\Omega_M \leq 1$ with $\Lambda = 0$, or $\Omega + \Lambda = 1$, one can get $\delta_H \leq -0.1$, corresponding to global $H_0$ actually larger than the local value.

Going out far enough to measure the unbiased $H_0$ means reaching the scale on which mass density fluctuations no longer generate significant velocity perturbations of the Hubble flow. The most important bias to consider for a full-sky determination of $H_0$ is the global radial retardation or acceleration of the Hubble flow that occurs within significant mass overdensities or underdensities. The possibility that we are within a large bubble of lower than cosmic density, for example, has been proposed as a way of reconciling apparently high $H_0$ values measured nearby with the estimated age of the universe and the often-cited concern (e.g., Bartlett et al. 1995) that extremely far-field $H_0$ measures (such as those from the SZ effect or lenses) are lower than the more local measures (although, as noted above, the case for this is weak). Bertschinger (1985) and Ryden (1994) present analytic treatments of how voids grow with time. The voids effectively expand faster than the cosmic scale factor; observers well inside the voids would see linear, if spuriously rapid, Hubble flows. Shi et al. (1996) in general find $\delta_H \sim -0.6 \delta M/M$, where $\delta M/M$ is the relative mass deficit of the void.

When limited to popular initial power spectra, however, one predicts only small $\delta_H$ over the large volumes sampled by BCGs and SNe Ia. Turner et al. (1992), for example, confined their analysis to very modest scales compared to the Hubble diagrams now available. The large variations in $\delta_H$ that they observed under CDM and PIB power spectra occurred for volumes limited to 3000 km s$^{-1}$. For volumes limited to 6000 km s$^{-1}$, they found $\langle \delta_H \rangle < 0.05$, depending on the power spectrum. Shi et al. (1996) likewise conclude that $\langle \delta_H \rangle \sim 0.05$ on 15,000 km s$^{-1}$ scales with “reasonable” models of galaxy and structure formation.

The question of whether or not we have gone out far enough in measuring the Hubble flow thus remains an issue only if there is significantly more power in mass fluctuations on large scales than would be expected under standard theories. In this context, we can posit that the 689 km s$^{-1}$ bulk flow observed by Lauer & Postman (1994) in the volume limited to 15,000 km s$^{-1}$ is indeed evidence for such power on large scales. Strauss et al. (1995) and Feldman & Watkins (1994) both find the Abell cluster bulk flow to be incompatible at $\sim 95\%$ confidence with all standard models of galaxy formation considered. Tegmark, Bunn, & Hu
argue further that the Lauer-Postman bulk flow is not compatible with degree-scale measurements of the CMB anisotropy power spectrum. Even so, however, Shi et al. (1996), taking the Harrison-Zeldovich power-spectrum shape and normalization parameters that best fit the Lauer-Postman flow (see Jaffe & Kaiser 1995), still find that only modest $\delta_H = 0.05$ would be expected. At the same time, Shi et al. (1996) show that the range of power spectra considered by Jaffe & Kaiser (1995) to fit the Lauer-Postman bulk flow would admit $\delta_H$ as large as 0.12 in the limiting extreme case.

Of course, power spectra give only a statistical expectation for the local distribution of matter. One remains free to argue that we are within a local density anomaly that exceeds the $\pm 1 \sigma$ fluctuations at some level. In this context, Shi et al. (1996) discuss the CMB dipole and quadrupole anisotropies that would be observed within the anomaly and conclude that they would put strong constraints on the allowed geometry of the local density fluctuation (see also Tomita 1996). We conclude that, while one can construct models of the universe for which the far field remains at distances well in excess of those explored here, they are extremely unfavored by what we know of the power spectrum of initial mass fluctuations in the universe.

4.3. The Abell Cluster Bulk Flow

The accuracy of these SBF distances offers the means to test the validity of different reference frames, although we cannot legitimately solve for an independent, best-fit reference frame with only four points. As described in section § 3.1, we consider three reference frames: CMB (for obvious reasons), LG (since for smooth flows this has zero dipole locally), and AC frame, following the Lauer & Postman (1994) bulk flow. Although we tried to remove all systematic, common contributors to the errors in the SBF distances when calculating $\chi^2$ for our $H_0$ estimates, we find that $\chi^2/N$ in Table 3 ($N = 3$) has a value of 0.3 for the CMB frame. This may indicate that we have overestimated our errors, since such a value or lower will occur only 18% of the time by chance, so we bear in mind that all the true $\chi^2$ values may be larger than what is listed in Table 3. At the same time, it seems unlikely that our random errors could be overestimated by a factor of almost 2—what is necessary to raise $\chi^2/N$ in the CMB frame to unity—so at least some of the concordance of results in this frame surely is coincidental. Taking the numbers at face value, the probability that $\chi^2/N$ is at least as large as 1.2 (the LG value) is 0.31, and the probability that $\chi^2/N$ is at least as large as 2.4 (the AC frame value) is 0.07. Thus, our observations offer the most support for these four clusters being at rest in the CMB frame.

The very small number of SBF distances does not permit us to say whether this rejects the AC frame or not, since the AC frame was chosen to minimize the scatter of the $L_m-\alpha$ relationship for 119 clusters at distances much greater than our sample here. Additionally, we are guessing at a random velocity component of 209 km s$^{-1}$ (100 and 184 in quadrature). If the random velocity is as large as 500 km s$^{-1}$, the AC frame will have unity $\chi^2/N$ (0.1 in the CMB frame).

It is clear that A262 is problematic for $L_m-\alpha$, as seen in Figures 1, 2, and 9; it has a very low surface brightness, and hence its distance is estimated to be large. Likewise, the SBF and $L_m-\alpha$ distances are consistent within the errors for A3560, A3565, and A3742, but there is a 3$\sigma$ inconsistency with A262. The cosmic scatter in the $L_m-\alpha$ is clearly related to intrinsic variations in metric surface brightness among the BCGs. Lauer & Postman (1994) note that this scatter greatly dominates any variation in $L_m$ that is due to peculiar velocity, but they rely on the assumption that intrinsic variations in BCG surface brightness are not correlated over large scales and will average out of a large sample. In the case of A262, however, the AC frame does partially compensate for the discrepancy between its $L_m-\alpha$ distance and its redshift. The rms scatter in the $L_m-\alpha$ relationship for just these four clusters is 0.41 mag in the CMB frame but only 0.27 mag in the Abell cluster frame. If we delete A262, the rms $L_m-\alpha$ residuals drop to 0.29 mag (CMB) and 0.20 mag (AC). In their analysis Lauer & Postman tried deleting outliers (such as A262) and found the AC frame to be quite stable, so this is not the whole story. Nevertheless, it seems clear that a larger sample of SBF distances could not only provide a very accurate reference frame and random velocity amplitude but also allow us to understand why the Lauer & Postman sample and the $L_m-\alpha$ relation point to the AC frame.

4.4. The Distance to the Virgo Cluster

Lauer & Postman (1992) originally attempted to calibrate the BCG Hubble diagram on the presumption that NGC 4472, the Virgo Cluster BCG, was typical. Adopting a 14.4 Mpc distance to NGC 4472, they found $H_0 = 77 \pm 8$ km s$^{-1}$ Mpc$^{-1}$; however, if they adopted the Sandage & Tammann (1990) 21.9 Mpc Virgo distance for NGC 4472, $H_0 = 51 \pm 5$ would be implied. Our present results for $H_0$ are essentially identical to that obtained with the short distance to NGC 4472 and are significantly different from that obtained with the long distance, arguing that the distance to the Virgo core is indeed close to the shorter value. If NGC 4472 were at the distance of 21.9 Mpc, then it would be among the brightest of the Lauer & Postman (1994) sample, deviating from the $L_m-\alpha$ ridgeline by more than 2$\sigma$. Of course, one could question this conclusion by challenging the SBF calibration, since it is already known that SBF distances imply a short distance to Virgo (Tonry et al. 1997); however, this still remains as an important consistency check. The relative distances inferred from BCGs are consistent with SBF distances.

5. CONCLUSION

We have used HST to obtain SBF distances to four BCGs beyond 4000 km s$^{-1}$ to calibrate the Lauer & Postman (1992) BCG Hubble diagram, producing an estimate of the global value of $H_0$ valid on $\sim 11,000$ km s$^{-1}$ scales. This method gives $H_0 = 89 \pm 10$ km s$^{-1}$ Mpc$^{-1}$ and is based on the full Lauer & Postman (1994) 15,000 km s$^{-1}$ volume-limited BCG sample. As such, the result is independent of Virgo or Coma Cluster distances and membership issues, as well as the recession velocities of the four BCGs studied. The large error reflects the photometric scatter about the $L_m-\alpha$ ridgeline, which was used to transfer the BCG Hubble diagram to the SBF distance scale. As more BCGs are observed with HST, the formal errors in this far-field $H_0$ should decrease.

Our review of the present understanding of the formation of large-scale structure argues that we are likely to have fairly sampled the far field. Even theories with enough power on large spatial scales to generate bulk flows as large
as those observed by Lauer & Postman (1994) are unlikely to have deviations outside of $|\delta H| \lesssim 0.05$ for the volume sampled by the BCG Hubble diagram. In contrast, the compatibility of our results with those based on more nearby objects argues that there is little effect on $H_0$ and the depth of the measurements. Going to the far field most likely removes a source of uncertainty rather than correcting for a systematic error. Indeed, we find $H_0 = 82 \pm 8$ km s$^{-1}$ Mpc$^{-1}$ just from Hubble ratios based on the SBF distances and observed recession velocities to the four SBF-calibrated BCGs at $\sim 4000$ km s$^{-1}$ alone, a result consistent with our far-field result. We note that, even including systematic effects, this $H_0$ estimate actually has smaller errors bars than the far-field $H_0$, given the latter's large random error from using BCGs as redshift estimators.

Both estimates of $H_0$ rest on calibration of the SBF method and an understanding of its systematic effects. At the fundamental level, we are tied to the nearby Cepheid calibrators. Changes in the Cepheid scale will propagate through the present results through the Tonry et al. (1997) and Ajhar et al. (1997) calibrations. As noted in § 1, Tonry et al.'s (1997) SBF calibration is tied to seven spiral galaxies with Cepheid distances. Further, Tonry et al. have observed enough galaxies to perform an exhaustive series of tests, finding no systematic offsets between SBF observations of bulges and elliptical galaxies. A weaker link is transferring the ground $I_{BC}$ method to the WFPC2 F814W filter, a task accomplished by Ajhar et al. (1997); we will attempt to refine this calibration as more nearby systems are observed with $HST$. We conclude that the major uncertainties in the distance scale are those close to home rather than far away.

We thank Guy Worthey, Barbara Ryden, and Robert Kennicutt for helpful discussions, and John Blakeslee for the photometry of A262. This research was supported in part by $HST$ GO analysis funds provided through STScI grant GO-05910.03-94A.

REFERENCES

Ajhar, E. A., Lauer, T. R., Tonry, J. L., Blakeslee, J. B., Dressler, A., Holtzman, J. A., & Postman, M. 1997, AJ, 114, 626
Bartlett, J. G., Blanchard, A., Silk, J., & Turner, M. S. 1995, Science, 267, 980
Baum, W. A, Hammergren, M., Thomsen, B., Groth, E. J., Faber, S. M., Grillmair, C. J., & Ajhar, E. A. 1997, AJ, 113, 1483
Bertschinger, E. 1984, ApJS, 58, 1
Blakeslee, J. P., & Tonry, J. L. 1996, ApJ, 465, L19
Burstein, D., & Heiles, C. 1984, ApJS, 54, 33
Feast, M. W. 1998, MNRAS, 293, L27
Feldman, H. A., & Watkins, R. 1994, ApJ, 430, L17
Freedman, W. L. 1997, in Proc. 18th Texas Symp. on Relativistic Astrophysics, ed. A. Olinto, J. Frieman, & D. Schramm (Singapore: World Scientific), in press
Freedman, W. L., et al. 1994, Nature, 371, 757
Giovanelli, R., Haynes, M. P., da Costa, L. N., Freudling, W., Salzer, J. J., & Wegner, G. 1997, ApJ, 477, L1
Gunn, J. E., & Oke, J. B. 1975, ApJ, 195, 255
Hamuy, M., Phillips, M. M., Suntzeff, N. B., Schommer, R. A., Maza, J., & Aviles, R. 1996, AJ, 112, 2398
Floessel, J. G. 1980, ApJ, 241, 493
Holtzman, J. A., Burrows, C. J., Casertano, S., Hester, J. J., Trauger, J. T., Watson, A. M., & Worthey, G. 1995a, PASP, 107, 1065
Holtzman, J. A., et al. 1995b, PASP, 107, 156
Jacoby, G. H., et al. 1992, PASP, 104, 599
Jaffe, A. H., & Kaiser, N. 1995, 455, 26
Kim, A. G., et al. 1997, ApJ, 476, L63
Kundic, T., et al. 1997, ApJ, 482, 75
Lauer, T. R., & Postman, M. 1992, ApJ, 400, L47
---. 1994, ApJ, 425, 418
Mould, J., et al. 1995, ApJ, 449, 413
Postman, M., & Lauer, T. R. 1995, ApJ, 440, 28
Riess, A. G., Press, W. H., & Kirshner, R. P. 1996, ApJ, 475, 88
Ryden, B. S. 1994, ApJ, 423, 534
Sandage, A. 1972, ApJ, 178, 1
Sandage, A., & Hardy, E. 1973, ApJ, 183, 743
Sandage, A., Saha, A., Tamman, G. A., Labhardt, L., Panagia, N., & Macchetto, F. D. 1996, ApJ, 460, L15
Sandage, A., & Tammann, G. A. 1990, ApJ, 365, 1
Schechter, P. L., Mateo, M., & Saha, A. 1993, PASP, 105, 1342
Schechter, P. L., et al. 1997, ApJ, 475, L85
Schmidt, B. P., Kirshner, R. P., Eastman, R. G., Phillips, M. M., Suntzeff, N. B., Hamuy, M., Maza, J., & Aviles, R. 1994, ApJ, 432, 42
Shi, X., Widrow, L. M., & Dursi, L. J. 1996, MNRAS, 281, 565
Strass, M. A., Cen, R., Ostriker, J. P., Lauer, T. R., & Postman, M. 1995, ApJ, 444, 507
Tammann, G. A. 1992, Phys. Scr., T43, 31
Tegmark, M., Bunn, E. F., & Hu, W. 1994, ApJ, 434, 1
Tonry, J. L., Blakeslee, J. B., Ajhar, E. A., & Dressler, A. 1997, ApJ, 475, 399
Tonry, J. L., & Schneider, D. P. 1988, AJ, 96, 807
Turner, E. L., Cen, R., & Ostriker, J. P. 1992, AJ, 103, 1427
Whitmore, B. C., Sparks, W. B., Lucas, R. A., Macchetto, F. D., & Biretta, J. A. 1995, ApJ, 454, L73
Worthey, G. 1993a, ApJ, 409, 530
---. 1993b, ApJ, 418, 947
Fig. 3.—The 710 × 711 pixel (≈ 32") portion of the PC1 image centered on the BCGs in Abell 262. The central portion of the montage shows the galaxy and its central dust clouds; this region is masked from the SBF analysis. The remainder shows the residual after the galaxy model is subtracted (with 8 × deeper stretch); most of the objects seen are globular clusters. Some dust is still visible, and this is also masked for the SBF analysis.

Lauer et al. (see 499, 580)
Fig. 4.—The 771 × 781 pixel (≈ 34") portion of the PC1 image centered on the BCGs in Abell 3560. The central portion of the montage shows the galaxy and its central dust ring; this region is masked from the SBF analysis. The remainder shows the residual after the galaxy model is subtracted (with 40 × deeper stretch); most of the objects seen are globular clusters.

LAUER et al. (see 499, 580)
FIG. 5.—The 791 × 742 pixel (≈ 33") portion of the PC1 image centered on the BCGs in Abell 3565. The central portion of the montage shows the galaxy and its central dust lane; this region is masked from the SBF analysis. The remainder shows the residual after the galaxy model is subtracted (with 60 × deeper stretch); most of the objects seen are globular clusters.

LAUER et al. (see 499, 580)
Fig. 6.—The $766 \times 766$ pixel ($\approx 33''$) portion of the PC1 image centered on the BCGs in Abell 3742. No dust was seen in this image, so only the very central portions were excluded from SBF analysis. The concentration of globular clusters around the galaxy is evident.

Lauer et al. (see 499, 580)