$H_{TC}\Pi^0$ and $\Pi^+\Pi^-$ pair productions at the planned $e^+e^-$ colliders in the topcolor-assisted technicolor model

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Abstract

The top-pions($\Pi^{0,\pm}$) and the top-Higgs($H_{TC}$) are the typical particles predicted by the topcolor-assisted technicolor (TC2) model and the observation of these particles can be regarded as the direct evidence of the TC2 model. In this paper, we study two pair production processes of these new particles, i.e., $e^+e^- \rightarrow H_{TC}\Pi^0$ and $e^+e^- \rightarrow \Pi^+\Pi^-$. The results show that the production rates are at the level of serval fb for $e^+e^- \rightarrow H_{TC}\Pi^0$ and tens fb for $e^+e^- \rightarrow \Pi^+\Pi^-$. These processes can produce the adequate distinct multi-jet final states and the SM background can be efficiently reduced. So, with the high luminosity at the planned linear colliders, the

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top-pions and top-Higgs can be observable via these two pair production processes without double.

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1 Introduction

Because the Higgs boson predicted in the standard model (SM) has not been found experimentally and the mechanism of the electroweak symmetry breaking (EWSB) is still unknown, one believes that the SM is only an effective theory of the underlying theory at the TeV energy scale. Such underlying theory beyond the SM is called the new physics model. There are several candidates of new physics models (such as: the minimal supersymmetric standard model (MSSM), the technicolor (TC) [1] model, the top sea-saw model [2, 3] and extra dimensions model [4]). One of the most important tasks of the future high energy colliders is to search for the Higgs in the SM or to test the new physics models, furthermore, to reveal the mystery of electroweak symmetry breaking mechanism. In all case, it is likely that the LHC running in 2007 will see first signals of the mechanism at work. However, the complementary information from the linear colliders is needed to understand the underlying theory. Several laboratories in the world have been working on the linear $e^+e^-$ collider projects with an energy from several hundred GeV up to several TeV and the yearly expected luminosity being more like $100fb^{-1}$ or a little more. These are NLC (USA) [5], JLC (Japan) [6], TESLA (Europe) [7]. The running of these high energy and luminosity linear colliders will open an unique window for us to understand the basic theory of particle physics.

As a dynamical breaking theory, the technicolor (TC) model was introduced by Weinberg and Susskind in the late 1970’s [11], it can avoid the shortcomings of triviality and unnaturalness [11, 8] arising from the elementary Higgs field. Among the various TC models, the topcolor-assisted technicolor model (TC2) [9, 10, 11] is a more realistic one. One of the most general predictions of the TC2 model is the existence of three isospin-triplet pseudo Goldstone bosons called top-pion ($\Pi^{\pm}, \Pi^{0}$) and an isospin-singlet boson called top-Higgs ($H_{TC}$). These bosons can be regarded as the typical feature of the TC2 model. Thus, study of the production mechanism of these typical particles can provide us the useful information to probe them at the future high-energy colliders. The discovery of these new particles can be regarded as the direct evidence to test the TC2 model.
The new particles predicted by the TC2 model can be probed directly via its decay modes. The decay modes of the neutral top-pion are the tree-level decay processes $\Pi^0 \rightarrow t\bar{t}$ (if this is kinetically allowed), $\Pi^0 \rightarrow t\bar{c}$, $\Pi^0 \rightarrow b\bar{b}$ and the processes $\Pi^0 \rightarrow \gamma\gamma, gg, \gamma Z$ through an internal top quark loop. The branching ratios of these possible decay modes have been calculated in detail\cite{12}. The results show that the neutral top-pion almost decays to $t\bar{t}$ if the mass of neutral top-pion $M_{\Pi}$ is larger than $2m_t$. If $m_t + m_c < M_{\Pi} < 2m_t$, the flavor-changing mode $\Pi^0 \rightarrow t\bar{c}$ will become the dominant decay mode and the branching ratio can be over 60%. Such flavor-changing mode can play an important role in the search of the neutral top-pion due to the clean background. For the top-Higgs, the difference case is that there exist tree-level decay modes $ZZ, W^+W^-$. For the charged top-pions, its decay modes have been studied in reference\cite{13}. The main modes are the tree-level $\Pi^+ \rightarrow t\bar{b}$ and $\Pi^+ \rightarrow c\bar{b}$. The alternative way to probe the top-pions and top-Higgs is to study the production mechanism of these particles. Recently, we have systematically studied some single top-pion production processes in the high energy $e^+e^-, \gamma\gamma$ and $e\gamma$\cite{14,15,16} collisions. For the neutral top-pion, we found that the cross section of $e\gamma \rightarrow e\Pi^0$ is over 10 fb, and the cross sections of the production modes $\Pi^0Z$ in $e^+e^-$ collision and $\Pi^0t\bar{t}, \Pi^0t\bar{c}$ in $\gamma\gamma$ collision are at the level of a few fb. With the high luminosity at the planned linear colliders, the signals should be enough for us to observe the neutral top-pion. Specially, with the large cross section, the process $e\gamma \rightarrow e\Pi^0$ can not only give us more information about the TC2 model but also provide an unique way to distinguish the TC2 model from the SM and other new physics models. At $e^+e^-$ colliders, the main single charged top-pion production processes are $e^+e^- \rightarrow t\bar{b}\Pi^-$ and $e^+e^- \rightarrow W^+\Pi^-$ with the cross section at order of 10 fb for $e^+e^- \rightarrow t\bar{b}\Pi^-$ and a few fb for $e^+e^- \rightarrow W^+\Pi^-$. Because the SM predicts the existence of one neutral Higgs boson, the distinguish of Higgs-like neutral top-pion with the Higgs in the SM need more precise measurement, but any observation of charged Higgs or Higgs-like particles will mean the signal of new physics. Therefore, the probing of charged top-pions is more important to test the TC2 model. Besides the single top-pion production mechanism discussed above, the pair productions $H_{TC}\Pi^0$ and $\Pi^+\Pi^-$ can occur at the tree-level in the $e^+e^-$ collision.
These pair productions might provide more typical information of the TC2 model via the multi-jets. For the heavy top-pions and top-Higgs, such processes can only occur at the planned high energy colliders. In this paper, we study these pair production modes at the planned $e^+e^-$ colliders. We find these two processes can provide enough number of distinct signals and the SM background can be efficiently reduced. These processes can provide a feasible way to test the TC2 model.

As it is known, the $\Pi^0$, $H_{TC}$ and $\Pi^\pm$ look like the heavy $A^0$, $H^0$ and $H^\pm$ in two-Higgs doublet model(2HDM), respectively.\footnote{Many production and decay channels of $A^0$, $H^0$ and $H^\pm$ are very similar to that of $\Pi^0$, $H_{TC}$ and $\Pi^\pm$. For example, like $H_{TC}$, $H^0$ can also be produced via $e^+e^- \rightarrow A^0H^0$, $ZH^0$, $\bar{\nu}\nu H^0$, $e^+e^- H^0$ and decay to $f\bar{f}$, $ZZ$, $W^+W^-$.} To distinguish the scalar in the TC2 model from the Higgs sectors in the 2HDM, we need to point out what different features exist between them. Firstly, the Yukawa couplings of top-pions and top-Higgs to quarks are larger than these of Higgs sectors which can make the signals of the TC2 model become more significant. Secondly, in the TC2 model, the couplings of top-pions to the three family fermions are non-universal and the top-pions and top-Higgs have the large Yukawa couplings to the third family, without the GIM, there exist large flavor-changing couplings, such as: $\Pi^0t\bar{c}$. Therefore, the flavor-changing processes in the TC2 model is very important for us to search for the scalars in the TC2 model and distinguish such scalars from the Higgs sectors. Thirdly, we find that there are some difference in the production between the scalars in the TC2 model and Higgs sectors. For example, there exist flavor-changing production processes $e^+e^- \rightarrow t\bar{c}\Pi^0(H_{TC}), \gamma\gamma \rightarrow t\bar{c}\Pi^0(H_{TC}), pp \rightarrow t\bar{c}\Pi^0(H_{TC})$ which can produce enough signals to observe the $\Pi^0(H_{TC})$. Such flavor-changing production processes for Higgs sectors can be ignored due to the GIM. On the other hand, due to the large Yukawa couplings between $\Pi^0$ and top quark, the loop-level $e^+e^- \rightarrow Z(\gamma)\Pi^0$ become more important than the similar process $e^+e^- \rightarrow Z(\gamma)A^0$. Finally, there are also some difference in the decay modes between them. If $t\bar{t}$ mode is forbidden, with large branching ratio, $t\bar{c}$ become the main decay mode for $\Pi^0$ and $H_{TC}$. But such decay mode does not exist for Higgs sectors. It
should also be noted that $b\bar{b}$ is an important decay mode for Higgs sectors with large $\tan\beta$, but for the scalars in the TC2 model, such decay mode can be ignored due to the small coupling of scalars to $b\bar{b}$. The above discussion can help us to distinguish the scalars in the TC2 model from the Higgs sectors in 2HDM.

The rest parts of this paper is organized as follows. In section II, we firstly present a brief review of the TC2 model and then calculate the cross sections of the processes $e^+e^- \rightarrow H_{TC}\Pi^0$ and $e^+e^- \rightarrow \Pi^+\Pi^-$. The numerical results are also discussed in section II. The conclusions are given in Section III.

2 The production cross sections of $e^+e^- \rightarrow H_{TC}\Pi^0$ and $e^+e^- \rightarrow \Pi^+\Pi^-$

2.1 A brief review of the TC2 model

The large top quark mass is suggestive of new dynamics associated with electroweak symmetry breaking. In order to solve the heavy top quark and the EWSB problems, the topcolor model was proposed [17]. In the topcolor model, the top quark participates in a new strong interaction(topcolor) which is spontaneously broken at some high energy scale $\Lambda_t$. The strong dynamics leads to the formation of a condensate $< t\bar{t} >$ and gives rise to a large dynamical mass for top quark. If this top condensate is to be an adequate source of electroweak symmetry breaking, and at the same time, gives a reasonable top quark mass, the scale $\Lambda_t$ must be very high($10^{15}$ GeV). i.e., the topcolor model suffers unnatural problem. Another dynamical breaking theory, technicolor model, can solve the problem related to Higgs field in the SM, but such theory has been unable to provide a natural and plausible understanding of why the top quark mass is so large. The TC2 model[9], incorporating the best features of the technicolor and topcolor, offers a new insight into possible mechanism of EWSB and the origin of heavy top quark mass. At EWSB scale, this model predicts two groups of scalars corresponding to the technicolor condensates and topcolor condensates, respectively. Either of them can be arranged into
a $SU(2)$ doublet, and their roles in TC2 model are quite analogous to the Higgs fields in the model proposed in Ref. which is a special two-Higgs-doublet model in essence. Explicitly speaking, the doublet $\Phi_{TC}$ which corresponds to the topcolor condensates plays a minor role in EWSB and only couples to the third generation quarks, its main task is to generate the large top quark mass. While the doublet $\Phi_{ETC}$ which corresponds to the technicolor condensates is mainly responsible for EWSB and light fermion masses, it also contributes a small portion of top quark mass. The vacuum expectation value (vev) of the top quark pair condensate $f_\pi$ can be given by the Pagel-Stokar formula. For condensation around the EWSB scale of 1 TeV, $f_\pi$ should be near 60 GeV. Once $f_\pi$ is fixed, the vev of the technifermion condensates, $\nu_T$, is uniquely determined by the EWSB requirement

$$f_\pi^2 + v_T^2 = v^2 \simeq (246 GeV)^2.$$ 

For $f_\pi = 60$ GeV, we must have $\nu_T = 239$ GeV. We linearize the theory and rearrange the pions in two orthogonal linear combinations to form the longitudinal degrees of freedom of the weak gauge bosons and a triplet of top-pions, $\Pi^{0,\pm}$, which become physical degrees of freedom. The top-pions are analogous to the neutral CP-odd and charged Higgs scalars of two-Higgs doublet model (2HDM). The theory loosely predicts top-pions to lie in the mass range of 200 GeV. Besides physical top-pions, there are another two CP-even Higgs modes, labeled $H_{TC}$ and $H_{ETC}$, are known as the ”top-Higgs” boson and the ”techni-Higgs” bosons, respectively. Their masses can be estimated in the Nambu-Jona-lasinio (NJL) model in the large $N_c$ approximation. For the top-Higgs boson this is found to be on the order of $M_H \simeq 2m_t$; for the techni-Higgs boson it is much higher. However, it only serves as a rough guide. From the kinetic terms of the effective TC2 Lagrangian in linearized form

$$L_{kin} = (D_\mu \Phi_{TC})^\dagger (D^\mu \Phi_{TC}) + (D_\mu \Phi_{ETC})^\dagger (D^\mu \Phi_{ETC}), \quad (1)$$

we know there exist tree-level couplings $Z^\mu H_{TC} \Pi^0$, $Z^\mu \Pi^+ \Pi^-$ and $A^\mu \Pi^+ \Pi^-$ which can induce the tree-level production processes $e^+ e^- \rightarrow H_{TC} \Pi^0$ and $e^+ e^- \rightarrow \Pi^+ \Pi^-$. These

\footnote{There should be another production $e^+ e^- \rightarrow H_{ETC} \Pi^0$, but the cross section of such process is strongly depressed by the factor $\frac{f_\pi}{v}$ and heavy $H_{ETC}$. On the other hand, techni-Higgs is not the typical particle of the TC2 model, so, we do not study this process in this paper.}
couplings can be written as:

\[
Z^\mu H_{TC}\Pi^0 : -i \frac{g}{2c_w} \frac{v_T}{v} (p_\mu^H - p_\mu^0), \quad Z^\mu \Pi^0 : \frac{g}{c_w} (1 - 2s_w^2)(p_\mu^- - p_\mu^+), \quad A^\mu \Pi^0 : e(p_\mu^- - p_\mu^+). \tag{2}
\]

Where, \(c_w = \cos \theta_w\) (\(\theta_w\) is the Weinberg angle).

### 2.2 The processes \(e^+e^- \rightarrow H_{TC}\Pi^0\)

With the coupling \(Z^\mu H_{TC}\Pi^0\), the process \(e^+e^- \rightarrow H_{TC}\Pi^0\) can be induced at tree-level via \(Z^0\) exchanging. The Feynman diagram of the process is shown in Fig.1(a). The production amplitude of the process can be written directly:

\[
M_{H_{TC}\Pi^0} = \frac{2i\pi\alpha_e v_T}{s^2 c_w^2 v} \bar{v}_{e^+} (p_3 - p_4)(-\frac{1}{2} L + s_w^2) u_{e^-} (p_2) G(p_1 + p_2, M_Z). \tag{3}
\]

Where, \(G(p, M) = \frac{1}{p^2 - M^2}\) denotes the propagator of the particle, \(p_3\) and \(p_4\) denote the momenta of outcoming top-Higgs and neutral top-pion, \(L = \frac{1 - \gamma_5}{2}\). With the above production amplitude, we can directly obtain the production cross section of the process \(e^+e^- \rightarrow H_{TC}\Pi^0\).

To obtain numerical results of the cross section, we fixed the input parameters as: \(M_Z = 91.187\ \text{GeV}, s_w^2 = 0.23, f_\pi = 60\ \text{GeV}\), the mass of top-Higgs \(M_H = 350\ \text{GeV}(\approx 2m_t)\). The electromagnetic fine-structure constant \(\alpha_e\) at a certain energy scale is calculated from the simple QED one-loop evolution with the boundary value \(\alpha_e = \frac{1}{137.04}\). Although the theory predicts top-pions to lie in the mass range of 200 GeV, this can be only regarded as a rough guide. In order to give a general prediction, we expand the mass range to 150-400 GeV. To show the influence of the center of mass energy (\(\sqrt{s}\)) on the cross section, we take \(\sqrt{s} = 800, 1600\ \text{GeV}\), respectively (\(\sqrt{s}=500\) is too small to produce \(H_{TC}\Pi^0\)). The numerical results of the cross section are shown in Fig 2.

The plots show that the cross section decreases with \(M_{\Pi}\) due to the phase space depression and falls more sharply for \(\sqrt{s} = 800\ \text{GeV}\). The cross section is not sensitive to \(M_{\Pi}\) when \(\sqrt{s} = 1600\ \text{GeV}\). The change of the cross section with \(\sqrt{s}\) is not monotonous because the influence of \(\sqrt{s}\) on the phase space and \(Z\)-propagator is inverse. In general,
the production rate is at the level of a few fb. We find that $\sqrt{s} = 800$ GeV is an ideal energy to probe light top-pion. For the light top-pion, the production rate can be near 10 fb in the case of $\sqrt{s} = 800$. With yearly expected luminosity about $100 fb^{-1}$, there are $10^3 - 10^4 \Delta_{TC} \Pi^0$ signals can be produced after several running of $e^+e^-$ colliders.

As has been discussed, the possible decay modes of $\Pi^0$ are the tree-level decay modes: $t\bar{t}(if \Pi^0 > 2m_t)$, $t\bar{c}$, $b\bar{b}$ and loop-level decay modes: $gg$, $\gamma\gamma$, $Z\gamma$. As it is known, the couplings of top-pion to the three generation fermions are non-universal and therefore do not possess a Glashow-Iliopoulos-Maiani(GIM) mechanism, this non-universal feature results in a large flavor-changing coupling $\Pi^0 t\bar{c}$. So, in the case of light $\Pi^0$, the main decay mode should be $\Pi^0 \rightarrow t\bar{c}$. In the SM, the cross section of the process with $t\bar{c}$ production is strongly depressed by GIM mechanism. Therefore, $\Pi^0 \rightarrow t\bar{c}$ might provide the typical signals of the TC2 model. With $M_H \approx 2m_t$, the main decay modes of top-Higgs should be tree-level modes $t\bar{c}, Z^0 Z^0$ and $W^+ W^- (H_{TC} \rightarrow t\bar{t}$ might be forbidden or its decay width is very small when $M_H$ is near $2m_t$). With above discussion, we know that the most interesting signals of the $H_{TC} \Pi^0$ production should be the four quark flavor-changing jets $:t\bar{t}t\bar{c}, t\bar{e}t\bar{c}$ which can provide distinct signals to identify the $H_{TC} \Pi^0$ production with the large production rate and clean SM background\(^3\). On the other hand, $e^+e^- \rightarrow H_{TC} \Pi^0$ can produce a large number of $t\bar{c}e\bar{c}$ which can also be produced via $e^+e^- \rightarrow A^0 H^0$ in the 2HDM, but the different pole structure make it possible for us to distinguish them, the pole structure can be easily detected at the $e^+e^-$ colliders. This means that reconstructing the neutral top-pion from $t\bar{c}$ will be important both as a means for measuring the mass of top-pion and also as a means for identifying the neutral scalars via flavor-changing $t\bar{c}$ jet.

In the 2HDM, there exists a similar process $e^+e^- \rightarrow A^0 H^0$. To distinguish the scalars in the TC2 model from the Higgs in the 2HDM via $e^+e^- \rightarrow H_{TC} \Pi^0$, we should compare the cross sections of $e^+e^- \rightarrow H_{TC} \Pi^0$ and $e^+e^- \rightarrow A^0 H^0$. The process $e^+e^- \rightarrow A^0 H^0$ has been studied in reference\(^{[19]}\). We find the the cross section behavior of $e^+e^- \rightarrow A^0 H^0$ is similar to that of $e^+e^- \rightarrow H_{TC} \Pi^0$. The cross sections values of such two processes are

\(^3\)In the SM, the production rates of $t\bar{t}t\bar{c}, t\bar{e}t\bar{c}$ are very small due to the GIM mechanism.
not significantly different for the same parameters. For example, \( \sigma(e^+e^- \rightarrow H_{TC}\Pi^0) = 4.08, 3.95 \text{fb} \) for \( \sqrt{s} = 800, 1600 \text{ GeV} \) and \( M_{\Pi} = 300 \text{ GeV} \), \( \sigma(e^+e^- \rightarrow A^0H^0) = 5, 3.9 \text{fb} \) for \( \sqrt{s} = 800, 1600 \text{ GeV} \) and \( M_A = 300 \text{ GeV} \). We should distinguish the neutral \( H_{TC}, \Pi^0 \) depending on their different feature of decay modes and pole structure. \( \Pi^0(H_{TC}) \rightarrow t\bar{c} \) can provide the typical signals of the TC2 model. For neutral Higgs boson, \( b\bar{b}, \tau^+\tau^- \) are important decay modes, specially for large \( \tan\beta \), but the decay modes \( b\bar{b} \) can be ignored and \( \tau^+\tau^- \) does not exist for \( H_{TC}, \Pi^0 \). On the other hand, the neutral Higgs boson can decay to supersymmetric particles and there are not similar decay modes for \( H_{TC}, \Pi^0 \).

2.3 The process \( e^+e^- \rightarrow \Pi^+\Pi^- \)

The charged top-pion pair \( \Pi^+\Pi^- \) can be produced in \( e^+e^- \) annihilation via virtual \( \gamma \) or \( Z^0 \) exchanging as shown in Figs.1(b). Such process is very important in searching for the charged top-pions because it can produce the distinct signals of the charged top-pions. The production amplitudes are:

\[
M_{\Pi^+\Pi^-}^\gamma = 4\pi \alpha_e \bar{v}_{e^+}(p_1)(\not{p_4} - \not{p_3})u_{e^-}(p_2)G(p_1 + p_2, 0) \tag{4}
\]

\[
M_{\Pi^+\Pi^-}^Z = \frac{-4\pi \alpha_e}{s^2_w c^2_w}(1 - 2s^2_w)\bar{v}_{e^+}(p_1)(\not{p_4} - \not{p_3})(s^2_w - \frac{L}{2})u_{e^-}(p_2)G(p_1 + p_2, M_Z) \tag{5}
\]

We ignore the electroweak contribution to the masses of charged top-pions and take \( M_{\Pi^+} = M_{\Pi^-} = M_{\Pi} \). The cross section of \( e^+e^- \rightarrow \Pi^+\Pi^- \) is shown in Fig.3.

It is shown that the behavior of the cross section plots of \( e^+e^- \rightarrow \Pi^+\Pi^- \) versus \( M_{\Pi} \) is similar to that of \( e^+e^- \rightarrow H_{TC}\Pi^0 \). For \( \sqrt{s} = 500 \text{ GeV} \), the cross section falls sharply to a very small rate with \( M_{\Pi} \) increasing. So, the energy 500 GeV is not suitable to search for the heavy charged top-pion pair. In the most case, the cross section is at the order of tens fb which is significantly larger than that of \( e^+e^- \rightarrow H_{TC}\Pi^0 \). The process \( e^+e^- \rightarrow \Pi^+\Pi^- \) includes an extra \( \gamma - \text{propagator contribution} \). Such production rate corresponds about \( 10^3 \Pi^+\Pi^- \) pairs with the luminosity \( 100^{-1} \text{fb} \). This abundant production allows to enforce tight requirements on the event pre-selection and the mass reconstruction. The most
promising decay modes to search for the charged top-pions are $\Pi^+ \to t\bar{b}$ and the flavor-changing mode $\Pi^+ \to c\bar{b}$. In the case of $\Pi^+ \to t\bar{b}$, the signals of the charged top-pion pair production is $t\bar{t}b\bar{b}$. In order to efficiently distinguish the signals from the underlying backgrounds and to measure the top-pion mass, it is important to obtain a clean charged top-pion signal in the mass distribution of the multi-jet final states. To identify the production mode $t\bar{t}b\bar{b}$, we insist on 8 jets or 1 lepton plus 6 jets (in particular, fewer than 10 visible lepton/jets so as to discriminate from the 4t final states) and possibly require that one W and the associated t be reconstructed. In particular, since final states contain at least four b jets, in order to eliminate any residual QCD background, we need one or two b-tags without incurring significant penalty. Such b-tagging should have efficiency of 60% or better. For the light charged top-pions, the branching ratio of $\Pi^+ \to c\bar{b}$ can be comparative to that of $\Pi^+ \to t\bar{b}$. In this case, $\Pi^+ \to c\bar{b}$ is also an important mode which induces the signals $c\bar{b}\bar{c}b$. Although $\Pi^+ \to c\bar{b}$ is a flavor-changing decay mode, $c\bar{b}\bar{c}b$ production is not the flavor-changing process. Therefore, the SM background can not be ignored. The major irreducible background should come from $e^+e^- \to Z^0Z^0 \to c\bar{c}b\bar{b}$. The mistagging of b-quark and s-quark will make the $e^+e^- \to W^+W^-$ become important which significantly enhances the background. So, the efficient b tagging and mass reconstruction of the charged top-pion is very necessary to reduce the background.

To compare the cross section of $e^+e^- \to \Pi^+\Pi^-$ with that of similar process $e^+e^- \to H^+H^-$, we find that the former is significantly larger than the latter \(^4\) which provide some useful information to distinguish the charged top-pions from charged Higgs. The $t\bar{b}$ is the main decay mode for both charged top-pions and charged Higgs, such mode is not suitable to distinguish these particles. To obtain the identified signals of the charged top-pions, we should probe charged top-pions via the flavor-changing decay mode $\Pi^+ \to c\bar{b}$. $\tau\nu_\tau$ can also provide the identified signals of charged Higgs which does not exist for the charged top-pions.

The production rates of the processes $e^+e^- \to H_{TC}\Pi^0$ and $e^+e^- \to \Pi^+\Pi^-$ are studied \(^4\)For example, $\sigma(e^+e^- \to \Pi^+\Pi^-)=21.51, 15.09$ fb with $\sqrt{s} = 800, 1600$ GeV and $M_{\Pi}=300$ GeV, $\sigma(e^+e^- \to H^+H^-)=10.02,8.1$ fb with $\sqrt{s}=800,1600$ GeV and $M_H=300$ GeV.
in this section. In order to give a believable prediction of the yearly event rate, we also need to consider the overall efficiency factor for detector coverage and for experimentally isolating and detecting these modes. Such efficiency can be reasonable estimated as 40%. On the other hand, the mass reconstruction and b-tagging will also reduce the efficiency to detect the signals. Considering these efficiencies, we can safely estimate that there should be abundant signals can be identified with the high luminosity at the planned linear colliders, and furthermore, the masses of these new particles might be measured with high accuracy.

3 The conclusions

In the framework of the TC2 model, $H_{TC}\Pi^0$ and $\Pi^+\Pi^-$ pair productions at the planned $e^+e^-$ colliders are studied in this paper. We find that the production rates are at the level of a few fb for $H_{TC}\Pi^0$ production and tens fb for $\Pi^+\Pi^-$ production. These pair productions can produce multi-jet final states and the SM background can be efficiently reduced. We conclude that the top-pions and top-Higgs predicted by the TC2 model should be experimentally observable via these processes at the planned colliders with high luminosity. Therefore, $H_{TC}\Pi^0$ and $\Pi^+\Pi^-$ pair productions at $e^+e^-$ colliders are very promising production mechanism of the top-pions and top-Higgs.

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Fig. 1. The Feynman diagrams of the processes $e^+e^- \rightarrow H_T\Pi^0$ and $e^+e^- \rightarrow \Pi^+\Pi^-$. 

Fig. 2. The cross section of $e^+e^- \rightarrow H_T\Pi^0$ versus top-pion mass $M_{\Pi}(150-400 \text{ GeV})$ for $S^{1/2}=800\text{ GeV}$ and $S^{1/2}=1600\text{ GeV}$, respectively.
Fig. 3. The cross section of $e^+e^- \rightarrow \Pi^+\Pi^-$ versus top-pion mass $M_\Pi(150-400 \text{ GeV})$ with $\sqrt{s}=500 \text{ GeV}$ (solid line), 800 GeV (dot line), 1600 GeV (dash line), respectively.