The single Cooper-pair box as a charge qubit

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Abstract. We present a series of measurements on nine single Cooper-pair boxes (SCBs), where the charging energy, $E_C$, and the Josephson coupling energy, $E_J$, have been varied. We have investigated both the ground state properties of the SCBs and their quantum coherent properties. The state of the SCBs could be manipulated by an external gate voltage and the charge was measured by coupling it capacitatively to a radio-frequency single-electron-transistor (RF-SET). By ramping the gate voltage and simultaneously measuring the charge of the SCBs using the RF-SET, we could measure the Coulomb staircases of the SCBs. For sufficiently low $E_C$ the SCBs showed a fully 2e periodic Coulomb staircase. For samples with higher $E_C$ the staircase showed a short step for odd number of charges indicating quasi-particle ‘poisoning’. However, if $E_C$ was not too large, the short step could be removed by applying a parallel magnetic field. We attribute this effect to a stronger suppression of the superconducting energy gap in the reservoir than in the box. Using microwave spectroscopy we have determined $E_C$ and $E_J$ for the SCBs. These values agree well with the shape of the Coulomb staircases which we measure. For a limited range of gate voltage, the SCBs were found to behave as model two-level quantum-mechanical systems. A non-adiabatic change in the induced island charge was used to bring two charge states into resonance. The resulting time evolution showed clear charge oscillations between the ground and excited state, with an amplitude above 70% and a frequency given by the energy level separation divided by Planck’s constant. These oscillations had a longest coherence time of $T_2 = 9$ ns, at a point where the pure charge states are degenerate. The coherence time at this point was found to be limited by the relaxation rate. Away from the charge degeneracy point, the coherence time was limited by the pure dephasing rate. The dependence of $T_2$ on gate charge suggested that low frequency fluctuators were the main source of dephasing away from the degeneracy point.
1. Introduction

In the mid-1990s, the research efforts on quantum computation were ignited by a few discoveries. On one hand, it was shown that there were at least two useful quantum algorithms which would be substantially faster on a quantum computer than their classical counterparts. These were the factorization algorithm by Shor [1] and the database search algorithm by Grover [2]. On the other hand, it was also shown that quantum error correction codes could be implemented on a quantum computer [3], indicating that continuous quantum computation could in fact be performed.

These discoveries started a very intense search for suitable hardware to implement qubits. DiVincenzo [4] evaluated the requirements for possible realizations of a general qubit and a large number of systems have been suggested. Two main groups of systems can be identified in the search for suitable qubit systems. On one hand, there are small systems such as ions and molecules where long decoherence times can be achieved relatively easily, but where scaling to a large number of qubits is difficult. On the other hand, there are the solid-state systems where scaling is relatively straightforward, but where decoherence times are typically short.

Among the solid-state qubits, the superconducting qubits have been quite successful. The superconducting qubits come in three main categories: charge qubits [5, 6], flux qubits [7] and phase qubits [8]. The first demonstration of quantum coherence in a solid-state qubit was made by Nakamura et al [9]. They used a single Cooper-pair box (SCB) [5, 6], [9]–[17], which will be the topic of this paper. Since then, quantum coherence in superconducting qubits has been observed by a number of groups [8]–[12], [18]. The coherence properties of the SCB were improved by Vion et al [10] by operating the SCB at a point where, to first order, it is insensitive to charge noise and magnetic noise. Wallraff et al [19] has also demonstrated how an SCB can

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be coupled to photon states in a microwave cavity. More recently, several experiments on coupled superconducting qubits have been reported [20]–[23].

The purpose of the work presented here has been to study the ground state and coherence properties of the single Cooper-pair box, using a radio-frequency single-electron-transistor (RF-SET) as a very sensitive electrometer to read out the charge of the box.

2. The single Cooper-pair box

The single Cooper-pair box consists of a small metallic island which is connected to a reservoir via a tunnel junction. In the superconducting state, Cooper-pairs are free to tunnel to and from the island, whose potential can be controlled by a gate voltage, $V_g$ (see figure 1).

The behaviour of the SCB is governed by two energies, the charging energy $E_C = e^2/2C_\Sigma$, and the Josephson coupling energy, which at low temperature can be written as $E_J = R_Q \Delta / 2R_N$ [24]. Here $C_\Sigma$ is the total capacitance of the island to its environment, $R_N$ is the normal-state tunnel resistance, $\Delta$ the superconducting energy gap and $R_Q = h/4e^2$ is the quantum resistance. If a single junction is replaced by a (small) superconducting loop with two junctions, as in the superconducting quantum interferometer device (SQUID), the effective Josephson coupling energy can be tuned with a magnetic flux penetrating the SQUID loop as $E_J = E_{J_{\text{max}}} \cos(\pi \Phi / \Phi_0)$, where $\Phi_0 = h/2e$ is the flux quantum. In this way, the interplay between charging effects and the Josephson effect can be changed in situ in the experiment.

The energy for adding $n$ Cooper-pairs to the SCB island

$$E_{el} = 4E_C(n - n_g)^2, \quad (1)$$

Figure 1. The setup used in this work. The RF-SET electrometer reads out the charge of the Cooper-pair box. The Cooper-pair box has a SQUID configuration that makes the effective Josephson energy tunable by an external magnetic field. Two voltage sources control the induced charge on the SET and SCB island, respectively. The tank circuit is connected to a microwave circuitry which lets microwaves be reflected off the SET and measured.
where \( n_g = C_g V_g / 2e \) (normalized gate voltage) and terms independent of \( n \) have been omitted. (This we can do since we will only be interested in differences in charging energy of states with different number of charges on the island.) This leads to the charging Hamiltonian

\[
H_{el} = 4E_C \sum_n (n - n_g)^2 |n\rangle \langle n|.
\]  

(2)

The Josephson interaction couples different charge states and favours a well-determined phase, \( \varphi \), across the tunnel barrier. Charging effects, on the other hand, tend to localize charge. The charge and phase operators become conjugated variables and obey a Heisenberg uncertainty relation with a minimum uncertainty \( \Delta \varphi \Delta N \sim 1 \), where \( N \) is the number of Cooper-pairs transferred across the junction. In the charge basis, the mixing of charge states due to the Josephson coupling becomes

\[
H_J = -\frac{E_J}{2} \sum_n (|n+1\rangle \langle n| + |n\rangle \langle n+1|).
\]  

(3)

We limit ourselves to a normalized voltage between \( 0 < n_g < 1 \). In this range, and if \( E_J \) is not too large, we can simplify things by only considering the two lowest energy levels which are comprised of the \( |0\rangle \) and \( |1\rangle \) states. By choosing zero energy to be \( E_0 = 2E_C(1 - 2n_g)^2 \), the Hamiltonian can conveniently be written

\[
H = -\frac{1}{2} \begin{bmatrix} E_{Ch} & E_J \\ E_J & -E_{Ch} \end{bmatrix} = -\frac{1}{2} E_{Ch} \sigma_z - \frac{1}{2} E_J \sigma_x,
\]  

(4)

\[
\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},
\]

where \( E_{Ch} = 4E_C(1 - 2n_g) \) and \( \sigma_x \) and \( \sigma_z \) are Pauli spin matrices. The energy eigenvalues (figure 2(a)) are given by \([6, 15]\)

\[
E_{\pm} = \pm \frac{1}{2} \sqrt{16E_C^2(1 - 2n_g)^2 + E_J^2}.
\]  

(5)

The corresponding eigenstates, apart from an overall phase factor, become

\[
|\Psi_+\rangle = \cos \frac{\theta}{2} |1\rangle - \sin \frac{\theta}{2} |0\rangle, \quad |\Psi_-\rangle = \cos \frac{\theta}{2} |0\rangle + \sin \frac{\theta}{2} |1\rangle,
\]  

(6)

where \( \theta \) is the mixing angle defined by \( \cos \theta = E_{Ch} / \sqrt{E_{Ch}^2 + (E_J)^2} \). In the band diagram for the SCB \([25]\) (figure 2(a)), the Josephson coupling opens up a gap at the point \( n_g = 1/2 \), where the \( |0\rangle \) and \( |1\rangle \) states are otherwise degenerate. Far from this charge degeneracy the eigenstates are, for \( 4E_C \gg E_J \), close to pure charge states. At \( n_g = 1/2 \), the ground and excited states are equal weight superpositions of \( |0\rangle \) and \( |1\rangle \). It should be noted that the charge expectation value at this point is equal for both states. Thus, if the read-out is done with charge electrometry, such as with an SET, the two states cannot be discerned. To read out charge, we need to move away from the charge degeneracy point. If the charge is measured as the gate voltage is swept, one obtains a staircase-like structure referred to as the Coulomb staircase \([26]\) (figure 2(b)). For zero temperature and vanishingly small \( E_J \), the staircase is sharp, but for a finite \( E_J \), the resulting staircase will no longer be a step function. It remains rounded even at zero temperature since the ground state is a superposition of two states.
The BCS theory [27] predicts an island with a fixed and even number of charges to have a ground state comprised of only paired electrons. Should the number be odd, the remaining electrons will exist as a quasi-particle excitation above the superconducting energy gap, $\Delta$. In the presence of quasi-particle excitations, the expectation value of the charge can be found by minimizing the free energy of the superconducting box [15] with respect to an even–odd parity term $E_{\text{par}}$, introduced by Averin and Nazarov [28] and investigated by Tuominen et al [29]:

$$E_{\text{par}} = \begin{cases} \hat{\Delta} & \text{for odd number of charges,} \\ 0 & \text{for even number of charges.} \end{cases}$$

Measurements of the odd–even free energy in Cooper-pair boxes have in some cases shown a reduced value compared to the value expected from bulk properties [15]. The reason for this discrepancy is unclear. Other measurements, such as those by Lafarge et al [16] of a box with a normal reservoir showed a value close to the BCS value. The temperature dependence of $\Delta$ can,
for low temperatures \((N_{\text{eff}} e^{-\Delta/k_BT} \ll 1)\), be approximated as

\[
\tilde{\Delta}(T, B = 0) \approx \Delta(B = 0, T = 0) - k_BT \ln N_{\text{eff}},
\]

where \(N_{\text{eff}}\) is the effective number of available quasi-particle excitation states.

In the band diagram of the SCB, the quasi-particle states are lifted up by an amount corresponding to \(\tilde{\Delta}\), and shifted in gate charge by \(e\) (figure 3). For \(\tilde{\Delta} - (E_c - 1/2E_J) \gg k_BT\), the quasi-particle states are not populated and the Coulomb staircase is purely 2\(e\)-periodic (figure 3(a)). For \(\Delta < E_c - 1/2E_J\), the ground state around \(n_g = 1/2\) will have odd parity and the staircase develops a short step around the charge degeneracy (referred to as a long step/short step structure [17], figure 3(b)). For \(\tilde{\Delta} = 0\), the box becomes fully \(e\)-periodic (figure 3(c)).

3. The RF-SET read-out

In the experiments presented here, we have used an SET [30, 31] to measure the charge of the SCB by capacitatively coupling the SCB to the SET. The SET is the most sensitive electrometer
available but, when operated in the conventional way, it has a limited operating speed. Its necessarily high resistance (~50 kΩ) combined with the capacitance of the cryostat cabling (~100 pF) results in an RC cut-off of a few tens of kilohertz, far from the $1/2\pi RC$ intrinsic limit of the SET.

This frequency limitation was overcome by the invention of the radio frequency SET (RF-SET) by Schoelkopf et al [32] and Wahlgren [33] in 1998. The charge sensitivity of the RF-SET was also improved since the bandwidth enables it to measure at higher frequencies where the $1/f$ noise is reduced.

The operation principle of the RF-SET is shown in figure 4. A monochromatic RF carrier wave is sent down into the cryostat. It reaches a directional coupler and continues to a bias-tee, where the SET dc voltage bias is added. The carrier wave is resonant with the inductor and pad capacitor of the tank circuit. The dissipation in the SET, which is controlled by the charge acting on the gate, sets the reflection/absorption ratio of the input power. The reflected component passes back up through the bias-tee and the directional coupler to a cryogenic amplifier followed by room temperature amplifiers. Due to the dissipation in the SET, the carrier becomes amplitude modulated. The properties of the tank circuit can easily be measured by applying a large current to the SET and detecting the shot noise generated in the SET. A typical frequency spectrum from such a measurement is shown in figure 5.

Figure 4. A simplified schematic of the RF-SET set-up. The rf-carrier is launched towards the tank circuit via a directional coupler. The reflected signal which depends on the dissipation in the SET is amplified by a set of amplifiers and then mixed with the original rf-carrier to create an output signal which depends directly on the gate charge of the SET.
Figure 5. The shot noise power spectrum difference between 0 and 100 nA dc current through the SET. A fit to a Lorentzian line shape gives the parameters $\omega_0/2\pi = 339.0$ MHz, bandwidth $\Delta f = 7.4$ MHz and $Q = 23$.

Figure 6. Typical $I$–$V$ characteristics for one of the SETs. The different curves are for different gate voltages. The typical operation point, the DJQP feature and the addition of ac and dc voltages are indicated with arrows.

A typical set of $I$–$V$ characteristics for different gate voltages is shown in figure 6. The transistor is typically operated using a fixed dc-voltage which is centred at some feature which shows a large gate modulation. A smaller RF amplitude then only sweeps a local area of modulation (see figure 6).

For a superconducting SET, the superconducting gap $\Delta$ adds to the Coulomb gap and moves the voltage threshold for sequential quasi-particle tunnelling out between $4\Delta/e$ and $4\Delta/e + 2E_{CS}/e$, where $E_{CS}$ is the charging energy of the SET.

Transistors used in this work are all in the limit $\Delta, E_{CS} \gg E_{JS}, k_B T$, where $E_{JS} = R_Q\Delta/2R_{iS}$ and $R_{iS}$ refers to the source and drain resistances of the SET. Our SETs had a typical charging energy $E_{CS} \sim 1.5$ K. Despite the fact that the Josephson coupling energy in the SET is comparatively small, many higher-order tunnelling processes become possible in the SET [34, 35] and modify the current–voltage characteristics. The Josephson quasi-particle (JQP)
cycle [36] consists of resonant Cooper-pair tunnelling at one junction, followed by tunnelling of two quasi-particles at the other junction. The 3e or double JQP (DJQP) cycle [37] involves resonant tunnelling of a Cooper-pair at one junction followed by a quasi-particle tunnelling at the other junction. This makes a Cooper-pair tunnel resonantly at the second junction before a quasi-particle tunnels at the first junction to return the transistor to its original state and the process repeats itself. The best operating point to optimize the signal and minimize the back-action was close to the DJQP voltage. Unless otherwise noted, the SET was biased at the DJQP feature for all measurements presented here.

4. The measurement setup

The samples were cooled using an Oxford Kelvinox 100 dilution refrigerator with a 3 Tesla persistent current magnet that gave a field parallel to the sample surface.

The sample chip was placed in a vertical position inside the sample. Almost no magnetic field from the 3 Tesla magnet would then penetrate the SQUID loop of the Cooper-pair box. Thus, a separate magnet was fabricated to produce a field perpendicular to the chip (figure 7). This cryostat was not equipped with any magnetic shielding.
A block diagram for the measurement set-up is shown in figure 8, showing cabling and filtering.

An IFR-Marconi rf-source with low phase noise was used to send the carrier wave down to 4.2 K, to a 30 dB attenuator and a directional coupler with a $-25$ dB coupling ratio. The signal then continued past an 800 MHz low-pass filter and a dc-block to a superconducting niobium coaxial cable. After passing through a bias-tee, the carrier reached the sample holder with the resonator. The inductors used in the resonant circuit were fabricated on a circuit board (figure 9). Inductances were around 500 nH, which together with the capacitance of the on-chip RF-pad of 400 fF formed the resonant circuit. Resonant frequencies around 300–500 MHz have been used by choosing slightly different inductor values.

The reflected signal passed back up through the directional coupler and was amplified in a cryogenic HEMT amplifier with a 2.5 K nominal noise temperature and 24 dB gain. After two more amplifier stages at room temperature, the signal was mixed with the original rf-carrier in a homodyne way, and the mixer output was recorded on an oscilloscope. For frequency spectrum measurements, the signal from the amplifiers was sent directly to a spectrum analyser.

The high frequency (50 GHz) line for the Cooper-pair box started at room temperature as a copper–beryllium (CuBe) coax. It was thermally anchored at 4.2 and 1.5 K using attenuators. A superconducting niobium coax continued to the mixing chamber where a bias-tee added the SCB gate dc-bias. The signal continued to a V-connector mounted on the sample holder. It launched the signal on to the circuit board from which the signal was carried by bonding wires to the substrate. Continuous microwaves were generated on a 50 GHz microwave source and fast voltage pulses on a fast pulse pattern generator.
Figure 9. A sample holder for RF-SET testing with a printed circuit board and a 7 mm × 7 mm chip. The RF-carrier came up through the bottom of the sample holder and was soldered to the centre of the tank circuit inductor. The other end was gold wire bonded to the RF-pad on chip. The two SMA-contacts were used for gate leads. They too connected first to striplines on the circuit board. The gate lead microstrips were also bonded to the chip. The SCB measurements were done with a similar sample holder but with the SMA-contacts replaced by two 65 GHz V-connectors.

The dc-bias for the SCB gate went down to a 1.9 MHz low-pass filter and a powder filter at 4.2 K through a CuBe coax. From there, a thermocoax continued to a powder filter at the mixing chamber and finally a copper coax connected to the SCB bias-tee. The RF-SET dc-bias is constructed the same way and connected to the RF-SET bias-tee. The gate lead for the RF-SET consisted of a CuBe coax to a 10 dB attenuator at 4.2 K, stainless steel coax to a 20 dB attenuator at 1.6 K and then a stainless steel coax to the mixing chamber and a 15 MHz low-pass filter, mounted on the sample holder. Ramp voltages for sweeping the SCB dc-gate and SET gate were generated with arbitrary waveform generators. Using this setup, all measurement lines were found to be thermally anchored well enough to keep the heat load on the mixing chamber fully acceptable with a base temperature around 20 mK. Details on the filters can be found in [38].

Current–voltage characteristics of the SET was measured through the SET dc-bias line using a modified low-noise current preamplifier. It was configured as a transimpedance amplifier, that biased the sample towards the ground. The bias voltage was amplified using a low-noise voltage preamplifier and connected to a voltmeter (figure 10). An external voltage source was used to bias the transimpedance amplifier.

5. Sample layouts

The samples discussed in this paper have been fabricated using a lift-off technique with two-angle evaporation and oxidation of aluminum [39]. Oxidized silicon was used as the substrate.
Figure 10. A diagram of the $I-V$-measurement set-up. The SET was biased towards ground by a transimpedance amplifier.

Thermally grown wet oxide was used with a thickness varying between 350 and 700 nm. The details about the sample fabrication can be found in [40, 41].

Several different SCB–SET designs have been tested. A few of the issues considered when designing the circuit layout were:

1. Crosstalk between devices: the gate of the box will unavoidably have some coupling to the SET island and vice versa. A compensation voltage of the opposite sign must therefore be applied to the SET gate to cancel the effect but it makes the measurements more cumbersome since a large crosstalk needs to be balanced very carefully.

2. Coupling capacitance: since the coupling strength is proportional to $\kappa = C_c / C_\Sigma$, if the charging energy is halved, the coupling capacitance needs to be doubled somehow to keep the coupling to the SET the same. This can be a problem if a box with low charging energy is wanted while maintaining a good signal-to-noise ratio. The magnitude of the SET back action is also set by $\kappa^2$.

3. Coupling of the box potential to measurement leads: fluctuations in the gate voltage will lead to voltage fluctuations on the island via the gate capacitance. In such a case one might consider reducing the gate capacitance by moving the gate further away from the box. This however will lead to more crosstalk.

4. Coupling to ground: the SCB reservoir should be connected firmly to ground potential to avoid fluctuations in the reservoir potential. This should be valid not just for dc but preferably also up to higher frequencies comparable to the energy splitting in the SCB.
Figure 11. SEM micrographs of different sample designs: (a) version 1.0, no measurements performed; (b) version 2.0, straight design; (c) version 2.1, including large groundplanes; and (d) version 3.0, ‘around-the-corner’ design.

The coupling to ground and to the measurement leads is a difficult problem to quantify. The electrodynamic environment of the sample needs to be calculated by making a full 3D simulation of Maxwell’s equations. Calculating the electrodynamic environment of the sample has not been done in this paper but is an important future step to further improve the SCB.

A number of different layouts have been fabricated in this work. Each layout is characterized by two numbers. The first number defines a completely new layout and the second number represents a variation to the same basic idea. Here, a few different designs are shown.

Version 1.1: this was the first SCB + SET layout that was measured, but the SCB could never be detected. Poor filtering, operation of the SET as a dc-SET, weak coupling to the box and large crosstalk were the main reasons why measurements failed (figure 11(a)).

Version 2.0: this was the first layout where the box could be read out. The box and SET were brought much closer to each other to make the coupling capacitances larger. The gates were also brought closer to the respective islands to decrease the crosstalk. Longer SCB and SET islands also helped with the crosstalk as well as two extended ground planes on either side of the box (figure 11(b)).

Version 2.1: this was the first sample that had microwaves and rectangular dc pulses applied to it. In this way, we successfully observed spectroscopy peaks and coherent charge oscillations. The separation between the two islands and between the islands and gates has now been made even smaller (50 nm) and the ground planes have grown in size. Also, the gates and islands have a ‘dog bone’ shape. All these features help in decreasing the gate crosstalk (figure 11(c)).

Version 3.0: the previous version could only provide a limited amount of coupling capacitance and the coupling to the SET would become too small when the charging energy was decreased. Version 3 solves this problem by putting the box and SET island, at least partially, side-by-side instead of end-to-end. This design also has an extra long box island which helps with the gate crosstalk (figure 11(d)).

Version 3.3: similar to version 3.0 but with a wider box island and a reshaped SET island to improve yield. Although version 3.0 turned out to have a strong box/SET coupling and good signal-to-noise ratio, it had two drawbacks. Firstly, the coherent oscillations never looked as good
in this design as in version 2.x. We suspected poor microwave design to be the reason for this. The ground lead and the gate lead for the box are routed very close to each other. Rectangular dc-pulses on the gate could then couple weakly to the box ground lead thus pulsing not only the gate but the ground potential as well. Secondly, many microwave peaks in the spectroscopy were found to be double peaks. The reason for this was never understood. It could come from the same reason as above, from coupling to a nearby two-level charge fluctuator or possibly from the fact that the SET island actually consists of two islands on top of each other. The tunnel area between the two islands is approximately 0.5 $\mu$m$^2$. By using 45 fF $\mu$m$^{-2}$ as the typical value for the tunnel barrier capacitance [42], the charging energy becomes $E_C/k_B = 40$ mK. This value should be small enough compared to the electron temperature of the SET to remove charging effects between the islands but this design was avoided in future layouts (figure 12(a)).

Version 4.0: the purpose of this design was to combine the best features of the previous versions. The SET and box islands were put side-by-side for good coupling. All leads were separated far from each other to minimize parasitic coupling and eliminate the ground-to-gate coupling in the above design. The SET had to be flipped to the other side in order not to have overlapping SET islands. The SET island will then be on top of the electrodes. The drawback with this is that the tunnel area increases slightly and lowers the charging energy and sensitivity. All ground planes were also removed in order to have a very clean design with no complications, such as an SET island that consists of two islands (figure 12(b)).

Version 4.2: quasi-particle traps made from gold were added for the first time. The traps were not fabricated separately but the box was simply fabricated close to one of the photolithography gold pads (figure 12(c)).

Version 4.4: this time, normal metal quasi-particle traps in gold were fabricated in a separate e-beam step and added to both the SET and the box (figure 12(d)).

Table 1 shows the list of samples presented in this paper. Coherence data are presented in section 6.3. No spectroscopy could be done on sample 8 to determine $E_{J}^{\text{max}}$. 

![Figure 12. SEM micrographs of (a) version 3.3; (b) version 4.0; (c) version 4.2, the high brightness area (lower-left) is a gold photolithography pad that acts as a quasi-particle trap; and (d) version 4.4, gold quasi-particle traps have here been fabricated on both the SCB and SET in a separate step (bright areas).](http://www.njp.org/)

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Table 1. List of fabrication parameters and sample parity properties for the measured samples. Sample 10 was measured at Yale University, see [52]. Spectroscopy could not be used to determine $E_{J}^{\text{max}}$ on sample 8 since it was close to being completely $e$-periodic. Samples 6 and 7 had, for maximum $E_{J}$, a ground state composed of more than two charge states. However, a magnetic field was used in experiments to reduce $E_{J}$ to values where ground and excited state has a charge difference close to $2e$. $B_{2e}$ is the magnetic field where the box was purely $2e$ periodic.

| Sample | Layout | $E_{C}$ (K) | $E_{J}^{\text{max}}$ (K) | $C_{F}/C_{C}$ % | QP traps | $B_{2e}$ (mT) | $t_{i}/t_{r}$ (nm) | $B_{||}$ (mT) | $E_{C} - \frac{1}{2}E_{J}$ (K) |
|--------|--------|-------------|----------------|----------------|-----------|--------------|----------------|--------------|----------------|
| 1      |        | 2.1         | 1.65           | 0.69           | no        | L/S          | 25/65          | 710          | 1.3            |
| 2      |        | 3.0         | 1.25           | 1.05           | no        | 370–490      | 25/65          | 820          | 0.73           |
| 3      |        | 3.0         | 1.40           | 0.60           | no        | L/S          | 25/65          | 710          | 1.1            |
| 4      |        | 3.3         | 0.74           | 0.53           | no        | $2e$         | 25/65          | 1100         | 0.48           |
| 5      |        | 3.3         | 0.83           | 0.73           | no        | $2e$         | 25/65          | n/m          | 0.47           |
| 6      |        | 2.2         | 0.5            | 1.0            | no        | $2e$         | 25/65          | n/m          | 0.47           |
| 7      |        | 4.0         | 0.43           | 0.97           | no        | $2e$         | 25/65          | n/m          | -0.06          |
| 8      |        |             | 0.58           | n/m            | box       | ‘$e$’        | 40/65          | n/m          | n/m            |
| 9      |        | 4.4         | 0.67           | 0.58           | box/set   | $2e$         | 25/65          | 680          | 0.38           |
| 10     |        | 3.3         | 1.79           | 0.63           | no        | L/S          | 25/65          | n/m          | 1.5            |

6. Results

6.1. The Coulomb staircase

As the SCB gate charge was ramped and the charge of the qubit was measured with the RF-SET, a Coulomb staircase [26] was observed. During this ramp a compensating voltage was applied to the SET gate to cancel out the direct cross capacitance between the SCB gate and the SET. In figure 13, a typical staircase is shown. For reference, the corresponding staircase in the normal state when the box behaves as a single electron box [43, 44] is also shown.

Suppression of $\Delta$ and thus also of $\tilde{\Delta}$ was in our experiments done by applying a strong magnetic field to the sample in a direction parallel to the chip. The average box charge read out by the RF-SET as a function of parallel magnetic field for sample 9 can be seen in figure 14(a). A field ranging between 500 and 680 mT was applied. For fields up to $B = 500$ mT, the staircase was fully $2e$-periodic. A further increase in field suppressed $\tilde{\Delta}$ enough to produce a short step in the $2e$-periodic staircase. At $B = 680$ mT, $\tilde{\Delta}$ has been reduced to zero and the staircase becomes $e$-periodic. The direction of the parallel field is indicated in figure 14(b).

However, not all samples showed a fully $2e$-periodic staircase at zero magnetic field. The behaviour shown in figure 14(a) represents the ideal magnetic field dependence for the Cooper-pair box, which was observed in samples 4–7 and 9. However, in other measured samples, three other behaviours of the parity as a function of the applied field has been seen. In samples 1 and 3, a long-step/short-step staircase as in figure 3(b) could be observed even at zero field. However, the length $S$ of the short step decreases with increasing field up to some intermediate field value (figure 14(a)), contrary to what is expected from further suppression of $\tilde{\Delta}$. Above this intermediate value, it starts to increase again until the short and long steps are of equal length, i.e.
Figure 13. A typical Coulomb staircase for a single Cooper-pair box (blue curve). For reference, the corresponding curve when the box is in the normal state is shown (red curve). Note that both the period and the step height increases by a factor of two when going from the normal to the superconducting state. Data from sample 2.

Figure 14. (a) A parallel magnetic field ranging between 500 and 680 mT has been applied to sample 9 to continuously reduce $\Delta$ to zero, changing a $2e$-periodic staircase into an $e$-periodic staircase. Below 500 mT the staircase is all $2e$-periodic. Each trace has been offset proportionally to the applied magnetic field. (b) The white arrows indicate the direction of the in-plane, parallel magnetic field.

The staircase is fully $e$-periodic. In sample 2, the same behaviour is observed except for that the short step decreases further in length and disappears completely for a finite field range between 370 and 490 mT (figure 15(a)). For sample 8, the box is completely $e$-periodic except for a small field range where there was a small difference in length between the steps (table 1).

The applied magnetic field in all these samples was applied in a direction parallel to the evaporated aluminum film (figure 14(b)). For thin aluminum films in a magnetic field, the critical
Figure 15. (a) The length of the short step plotted as a function of the normalized critical field for samples 1–4 and 9. The length of the short and long steps is determined such that $L + S = 2e$ (see inset). Samples 4 and 9 are $2e$-periodic up to a normalized field of $B/B_{c\parallel} \approx 0.75$. Sample 2 has a range of $2e$-periodicity between $B/B_{c\parallel} = 0.52 - 0.60$. Only samples 1 and 3 show a long step/short step structure. (b) The $\tilde{\Delta}/k_B$ deduced from equation (9). The dashed line is a linear extrapolation to $\tilde{\Delta}/k_B \approx 2.6$ K corresponding to a $T_c$ of 1.5 K (note that the $x$-axis starts at 0.5).

parallel field is expected to be inversely proportional to the film thickness $d$

$$H_{c\parallel} = 2\sqrt{6} \frac{H_c \lambda}{d},$$

where $H_c$ is the thermodynamic critical field and $\lambda$ the penetration depth [45]. In all samples, except for sample 8, the thickness of the SCB island was $t_i = 25$ nm and the thickness of the reservoir electrodes $t_r = 65$ nm (table 1). Thus, given the thicknesses of the SCB island and
reservoir, the superconducting gap for an increasing magnetic field will be suppressed faster in the reservoir compared to the island.

Although the quantitative description remains to be worked out, it is clear that for samples 1–4 and 9 in figure 15(a), the larger gap in the island creates an energy barrier, \( \tilde{\Delta}_{\text{island}} - \tilde{\Delta}_{\text{reservoir}} \), for tunnelling of quasi-particles onto the island. This graded gap effect becomes larger for increasing field and, in the case of samples 2, 4 and 9, makes the box completely 2\( e \)-periodic for a finite range in magnetic field. For a further increase in magnetic field, the box starts to show a short step again when \( \tilde{\Delta} \) on the island is reduced below \( E_C - 1/2E_J \). The short step then continues to grow until it becomes \( S = 1e \) at \( B_{c1} \). A similar method was used by Aumentado et al [46], who created a graded gap effect by introducing oxygen during evaporation of the aluminum island. This increased the gap on the island and significantly improved the parity of Cooper-pair transistors.

The critical temperature of our films with respect to oxygen content and film thickness was investigated by Lindqvist [47]. It was found that for the thicknesses used in this work, the critical temperature was approximately 1.5 K. This temperature could be increased up to \( \sim 2K \) by evaporating aluminum in a partial pressure of oxygen.

The odd–even free energy can be estimated from the widths of the short (S) and the long (L) steps of the Coulomb staircase using the relation [16]

\[
\tilde{\Delta}(B) = E_C \frac{L - S}{L + S},
\]

provided that \( \tilde{\Delta} \) has been reduced by a magnetic field to a value smaller than \( E_C \). The \( \tilde{\Delta}(B) \) deduced from this relation for samples 1–4 and 9 is shown in figure 15(b). The dashed line extrapolates back to \( \tilde{\Delta} = 2.6K \) which corresponds to \( T_c = 1.5K \) assuming the BCS relation \( \tilde{\Delta} = 1.76k_BT_c \).

Sample 8 has been evaporated with a thicker SCB island (40 nm) compared to the leads (65 nm) which significantly reduces the graded gap effect. This sample was close to being completely \( e \)-periodic, despite the fact that sample 8 had normal metal quasi-particle traps implemented at a distance <1.5 \( \mu m \) from the island.

Thus, the conclusion is that graded gaps are in our system more effective than normal metal traps at reducing quasi-particle tunnelling to the island. Single Cooper-pair boxes with \( 2e \)-periodicity can reliably be fabricated by applying the graded gap method to samples with a limited charging energy. The measurements indicate that this limit for \( E_C/k_B \) is somewhere between 0.83 K and 1.25 K, or if we consider \( (E_C - E_J/2)/k_B \) to be the important parameter the range is rather 0.48–0.73 K (table 1).

### 6.2. Spectroscopy

Applying the rotating wave approximation, the case of a two level system in a transverse time-dependent potential can be solved analytically when the potential is an oscillating sinusoid. In the SCB eigenbasis, the problem is defined as (cf equations (5) and (6))

\[
H = E_\pm |\Psi_\pm\rangle \langle \Psi_\pm| + E_+ |\Psi_+\rangle \langle \Psi_+| + V(t),
\]

\[
V(t) = \gamma e^{i\omega t} |\Psi_-\rangle \langle \Psi_+| + \gamma e^{-i\omega t} |\Psi_-\rangle \langle \Psi_+|,
\]

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where $\gamma$ is the strength of the field and $\omega/2\pi$ the frequency, both are real and positive. If the field is switched on at $t = 0$ and the SCB state starts in the ground state, $|c_-|^2 = 1$ and $|c_+|^2 = 0$, the time evolution, as first derived by Rabi [48], becomes

$$|c_+|^2 = \frac{\sin^2 \left( \sqrt{1 + \left( \frac{\hbar(\omega - \omega_\pm)}{2\gamma} \right)^2} \gamma t \right)}{1 + \left( \frac{\hbar(\omega - \omega_\pm)}{2\gamma} \right)^2}, \quad |c_-|^2 = 1 - |c_+|^2, \quad (12)$$

where $\omega_\pm$ is the level splitting between the upper and lower states. If the system is driven at resonance, $\omega = \omega_\pm$, the state oscillates with its maximum amplitude and a frequency $\nu = \gamma/\pi \hbar$. Thus, the oscillation frequency is proportional to the applied field strength. Furthermore, the average charge for $\hbar \omega_\pm \gg E_J$ and $n_g < 1/2$ becomes

$$\langle Q \rangle = 2e \langle |c_+|^2 \rangle = e \frac{1}{1 + \left( \frac{\hbar(\omega - \omega_\pm)}{2\gamma} \right)^2}. \quad (13)$$

Thus, the half-width at half-maximum (HWHM) of the Lorentzian frequency spectrum is equal to $2\gamma/\hbar$. The resulting resonance peaks in the Coulomb staircase therefore become narrower for weaker drive amplitude. Charge fluctuations, back action from the RF-SET and temperature can broaden the peaks during the time it takes to complete a measurement. Thus, the HWHM line width of a resonant peak extrapolated to zero power can only be used to calculate a worst case decoherence time $T_2$ of the system.

Mixing of the states in the Cooper-pair box was in experiments done by applying a high-frequency voltage to the box gate, and continuously measuring the box charge with the RF-SET. The resulting staircase has a series of peaks and dips where the field frequency matches the level splitting (figure 16(a)). The position of these peaks, together with the level splitting given by the resonant frequency, allows for the determination of the box charging energy and Josephson energy by fitting to equation (5). The peak separation around the charge degeneracy is measured for maximum $E_J$ and for the smallest $E_J$ where the peak position can still be resolved (which is done at a considerably higher microwave power). For sample 9, a fit to the energy eigenvalues determines the charging and maximum Josephson energy to $E_C/k_B = 0.67$ K and $E_J^{\text{max}}/k_B = 0.58$ K, respectively (figure 16(b)). The charging and Josephson energies of all samples (except sample 8) was determined in this way, and the values for all samples can be found in table 1.

From the measured values of $E_C$ and $E_J$ we could calculate the expected staircase and compare that to the staircase measured on the same sample. This comparison is shown for sample 2 in figure 17, as can be seen we get an excellent agreement between the measured and the calculated staircase. This kind of comparison could be done over a wide range of $E_J$, as the external magnetic field was changed. These results have been presented in [49].

### 6.3. Coherent oscillations by fast pulses

The way we have probed the coherence properties of the SCB was to apply fast rectangular pulses in a way similar to that of Nakamura et al [9]. The SCB Hamiltonian (4) can be rewritten...
Figure 16. (a) An applied microwave field results in a series of peaks and dips from ground and excited state mixing due to an oscillating potential. For higher powers, two-photon peaks occur (data not shown). (b) The resonant peak position above can be fitted to the level separation of the box, which determines the charging and Josephson energy. Data from sample 9.

to the equivalent description of a spin-1/2 particle in a magnetic field

\[ H = -\frac{1}{2} E_C \sigma_z - \frac{1}{2} E_J \sigma_x = -\hat{\sigma} \cdot \mathbf{h}, \]  

\[ \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \sigma_y = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}, \]  

where \( \mathbf{h} \) is an effective magnetic field with components \( \mathbf{h} = [E_J/2, 0, 2E_C(1 - 2n_e)] \). A general spin-1/2 system can be described as points on the Bloch sphere where spin-up corresponds to

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the $|0\rangle$ state and spin-down corresponds to the $|1\rangle$ state. A general state can be written as

$$|\Psi\rangle = \cos\left(\frac{\theta}{2}\right)e^{-i\phi/2}|0\rangle + \sin\left(\frac{\theta}{2}\right)e^{-i\phi/2}|1\rangle,$$

where $\theta$ is defined as the angle between $[0, 0, h_z]$ and $|\Psi\rangle$ and $\phi$ as the rotation in the $xy$-plane from $[h_x, 0, 0]$. The time evolution of the system is calculated by applying the time evolution operator

$$U(t_2, t_1) = e^{-iH(t_2-t_1)/\hbar},$$

to the initial state $|\Psi_0\rangle$, which is prepared by putting the box at $n_{g0}$ far away from the degeneracy point ($n_g = 1/2$). At this point, the SCB is in an eigenstate, parallel to the field axis. At a time $t_1$, a gate voltage pulse is applied to the gate with an amplitude $\Delta n_g$ such that $n_{g0} + \Delta n_g = 1/2$, which points the field in the direction $[h_x, 0, 0]$. This abrupt change makes the state of the SCB

Figure 17. (a) The Coulomb staircase when continuous microwaves are applied to sample 2. (b) The filled circles indicate the energy separation $\Delta E$ versus $\Delta n_g$ for the applied frequencies $\nu_1 = 22.8$ GHz, $\nu_2 = 28.4$ GHz, $\nu_3 = 32$ GHz, $\nu_4 = 37.1$ GHz and $\nu_5 = 44$ GHz. These points are fitted to $\Delta E$ with $E_C/k_B = 1.25$ K and $E_J/k_B = 0.94$ K (solid line). (c) The Coulomb staircase obtained at the same applied $B$-field as above (solid line) together with the simulated Coulomb staircase for $E_C/k_B = 1.25$ K and $E_J/k_B = 0.94$ K (dotted line). (d) Same as (c), but with the derivative of the Coulomb staircase.
precess around the new field axis with the Larmor frequency
\[ \nu = \frac{1}{\hbar} (E_+ - E_-) = \frac{E_J}{\hbar}. \] (18)

The change in gate voltage must be non-adiabatic to bring the box into oscillation. The pulse rise
time should thus be fast compared to the \( \hbar/E_J \) timescale. At \( t = t_2 \), the field is switched back
to its original state. In the case of vanishing \( E_J/4E_C \) ratio, the state vector begins to rotate in a
plane parallel to the \( xy \)-plane. The probability coefficients, \( |c_0|^2 \) and \( |c_1|^2 \), of the resulting wave
dunction

\[ |\Psi_2\rangle = c_0(t)|0\rangle + c_1(t)|1\rangle, \] (19)

become fixed and the final state hence becomes a superposition of the \( |0\rangle \) and \( |1\rangle \) states.

Measurements of time-resolved coherent oscillations were done by slowly ramping the box
gate voltage (and the compensation voltage on the SET gate) while at the same time adding
to the box gate a pulse train of fast rectangular voltage pulses with a fixed repetition rate. No
compensation for these pulses was done on the SET gate. The pulse train induced an excited
state component to the charge \( \Delta Q_{\text{box}} = Q_{\text{box}}^m - Q_{\text{box}}^{\text{off}} \) which was time averaged over the pulse
train by the continuous measurement of the RF-SET. Typically, each staircase was obtained
from an average of 1000 traces. The evolution of the oscillations were measured by successively
increasing the pulse duration time, \( \Delta t \).

Such oscillations can be seen in figure 18, which shows the pulse-induced charge, \( \Delta Q_{\text{box}} = \Delta Q_{\text{box}}^m - \Delta Q_{\text{box}}^{\text{off}} \) (z-axis colour mapping), as a function of the ramped box gate voltage and the
pulse duration time. A pulse-amplitude \( \Delta n_g = 0.25 \) and a repetition time of \( T_R = 150 \text{ ns} \) has

**Figure 18.** The application of non-adiabatic voltage pulses creates charge
oscillations between the ground and excited states when the pulse amplitude
takes the system close to the charge degeneracy. The oscillations are measured
on sample 4. The shift at 3.8 ns is due to a background charge jump.
been used (data from sample 4). Clear oscillations of charge are evident in the cross section at a distance equal to the $\Delta n_g = 0.25$ pulse height from the charge degeneracy points. On either side of this point, the oscillation signal decays rapidly. At $t = 3.8$ ns a single, strong background charge fluctuator close to the SCB offsets the pattern by $0.2e$. Figure 19(a) shows the cross section corresponding to the state evolution at the charge degeneracy (charge jump corrected). Figure 19(b) shows the measurement data and a fit to a sinusoid with $E_J/h = 6.9$ GHz and an exponential decay constant of $T_2 = 2.7$ ns. Data from sample 4.

The measured amplitude (or visibility of the oscillations) can be split into two parts, the state preparation fidelity and the readout fidelity. The state preparation can be reduced in two ways, either if the initial state at $n_g = 0$ is not a pure charge state, or if the rise time of the pulse is not sufficiently fast compared to $\hbar/E_J$. We find that the calculated state preparation fidelity in most cases was very similar to the measured amplitude. The reduced state preparation fidelity can be accounted for by combination of the finite rise time of the pulse, and by the initial state not being a pure charge state. Thus we can conclude that our measurement fidelity was close to 100% [11].

As argued above, the amplitude of the oscillations depends on the value of $E_J$. If the rise time of the pulse is not sufficiently fast compared to $\hbar/E_J$, the probability of reaching the exited state (i.e. the state preparation fidelity) will not be 100%. Since the rise time of our pulses were constant, we can improve the amplitude of the oscillations by reducing $E_J$ with the external magnetic filed. A smaller $E_J$ makes the state evolution slower and thus the state preparation fidelity higher. A small value of $E_J$ also helps in making the initial state very close to a pure charge state. However, this is done at the expense of decoherence since a smaller $E_J$ also gives a shorter decoherence time. This increased decoherence could be understood as increased relaxation if the spectral density probed by the relaxation is increasing for decreasing frequency.

Figure 19. (a) A cross-section of figure 18 at $n_g = 0.5$ (charge jump in figure 18 corrected). (b) The data above with a fit to a sinusoid with $E_J/h = 6.9$ GHz and an exponential decay constant of $T_2 = 2.7$ ns. Data from sample 4.
Another possibility is that for lower $E_J$, the curvature of the energy band is getting stronger, and thus pure dephasing may start to play a role due to the second order (curvature) coupling to charge fluctuations. The origin of this spectral density is however still not known. This reduction of $T_2$ can be seen in figure 20, where $E_J$ was suppressed such that the amplitude became quite large but the decoherence time became quite short. The amplitude extrapolated back to zero $\Delta t$ was close to 100%. Note that, due to the averaging, the maximum signal is $1e$. Typically, amplitudes above 70% could be achieved by going to low values of $E_J$.

6.4. Measuring the relaxation and decoherence times

As is clear from the above section, the $T_2$ decoherence time at the degeneracy point can be inferred directly from the decay of the free evolution (figure 19(b)). In figure 21, the same fitting procedure has been used to determine $T_2$ in the vicinity of the degeneracy point. The decoherence time has a clear maximum at the degeneracy point and it decays rapidly as we go away from the degeneracy point. The longest $T_2$ was 9 ns.

Using a double pulse procedure we could also determine the decoherence rate further away from the degeneracy point [50]. Similar measurements have been performed by the NEC group [51]. Figure 22, shows the decoherence rate for sample 9 measured with both methods: double-pulse measurements and decay of the free precession oscillations. The decoherence rates from both methods agree well in the charge range where they overlap.

Furthermore, these decoherence rates are also consistent with the decoherence rates estimated from the HWHM width (cf equation (13)) of the spectroscopy peaks, extrapolated to zero microwave power. These measurements typically give values of the order of a few hundred ps. Measurements on sample 10 done at Yale University [52] resulted in a worst case estimate of $T_2 = 325$ ps.

The relaxation time $T_1$ away from the degeneracy point can be measured by studying the time-averaged decay of the excited state probability as a function of the pulse repetition rate. For
Figure 21. The decoherence time $T_2$ extracted from fitting the experimental data to an exponential decay, for gate voltages around the charge degeneracy. Data from sample 9.

Figure 22. Gate charge dependence of the decoherence rate $\Gamma_2 = T_2^{-1}$ measured from the decay of the free precession oscillations for $E_J = 9.4$ GHz (circles) and $E_J = 3.6$ GHz (squares), and using twin pulses for $E_J = 9.4$ GHz (diamonds). The lines show the expected $\Gamma_2$ assuming the low-frequency noise couples to charge and including an incoherent contribution from relaxation $E_J = 9.4$ GHz (dashed line) and $E_J = 3.6$ GHz (solid line). The dotted line represents the pure dephasing for $E_J = 9.4$ GHz. Data from sample 9.

A certain gate voltage, the pulse amplitude and pulse duration are set to values which maximize the probability of ending up in the excited state, i.e. one spin flip produced by a $\pi$-pulse (or three spin flips produced by a $3\pi$-pulse in the case where the spin flip time is shorter than the experimental minimum pulse duration time, $\sim$80 ps). When the pulse repetition rate is swept, the resulting decay of the excited state probability will contain two components. One from the finite repetition rate and one from relaxation into the ground state. A simple calculation gives...
the oscillation amplitude decay for different repetition rates as

$$\Delta Q_{\text{box}}(T_R) = n_0 \frac{2T_1}{T_R} \tanh \left( \frac{T_R}{2T_1} \right),$$  \hspace{1cm} (20)$$

where $n_0$ is the observed initial peak amplitude that accounts, e.g., the finite $E_J/E_C$ ratio and the finite rise time of the voltage pulses ($\sim 30$ ps) as discussed earlier. In the case of perfectly rectangular voltage pulses and a vanishing $E_J/E_C$ ratio, the maximum observable contrast for short $T_R$ is $1/e$ due to population mixing. Figure 23 shows the measured decay of the oscillation amplitude for sample 7 and fitted to equation (20). Measurements at Yale University on sample 10 resulted in a substantially longer relaxation time $T_1 = 1.3 \, \mu s$ [52]. They used a different technique based on microwaves rather than rectangular pulses.

The coherence properties of the samples investigated in this paper are summarized in table 2. From table 2 it is clear that measured $T_1$ times range between 50 ns to $\sim 1 \, \mu s$. In order to compare relaxation times $T_1$ between samples, it should be taken into account that they are measured at different gate voltages. Assuming noise couples to the charge degree of freedom, in the Bloch sphere picture this $\sigma_z$ fluctuation can be divided into two components. One parallel to the effective field direction ($\cos \theta$) and one perpendicular component ($\sin \theta$). The transverse fluctuation causes mixing between states for fluctuations with a frequency equal to the level spacing in the box, which can be estimated [6] using Fermi’s golden rule

$$\Gamma_1 = \frac{1}{T_1} = \left( \frac{e}{\hbar} \right)^2 \sin^2 \theta S_{V_{\text{box}}} (\omega \pm),$$  \hspace{1cm} (21)$$

where $S_{V_{\text{box}}} (\omega)$ is the spectral density of voltage fluctuations on the box island. If we assume $S_{V_{\text{box}}} (\omega)$ to be constant over the relevant frequencies we can multiply $T_1$ by $\sin^2 \theta$, to compare different samples.
Table 2. List of sample properties extracted from both staircase measurements, spectroscopy and coherent oscillations. Sample 8 has no $E_j^{\text{max}}$ data since the staircase was close to being completely $e$-periodic. The coherent oscillations in version 3 never looked as good as in the other versions. Hence, no $T_2$ values are listed for samples 2 and 4 since a fit to a sinusoid could not be made. No coherence times are listed for sample 3 due to a broken high frequency coax cable.

| Sample | $E_C$ (K) | $E_j^{\text{max}}$ (K) | QP | $B_{2e}$ (mT) | $t_i/t_e$ (nm) | $T_1$ (ns) | $n_{g0}$ | $T_1 \sin^2 \theta$ (2e) (ns) | $T_2$ (ns) | $4E_C/E_j^{\text{max}}$ |
|--------|----------|------------------------|----|-------------|---------------|-------------|--------|-------------------------------|----------|------------------------|
| 1      | 1.65     | 0.69                   | no | L/S         | 25/65         | 92          | 0.18  | 0.3                           | 3        | 9.6                     |
| 2      | 1.25     | 1.05                   | no | 370–490     | 25/65         | 50          | 0.24  | 6.3                           | –        | 4.8                     |
| 3      | 1.40     | 0.6                    | no | L/S         | 25/65         | n/m         | n/m    | n/m                           | n/m      | 9.3                     |
| 4      | 0.74     | 0.53                   | no | $2e$        | 25/65         | 140         | 0.23  | 3.6                           | –        | 5.6                     |
| 5      | 0.83     | 0.73                   | no | $2e$        | 25/65         | 160         | 0.26  | 3.6                           | 2        | 4.6                     |
| 6      | 0.5      | 1.0                    | no | $2e$        | 25/65         | 60          | 0.24  | 1.6                           | 2        | 2.0                     |
| 7      | 0.43     | 0.97                   | no | $2e$        | 25/65         | 110         | 0.25  | 10                            | 9        | 1.8                     |
| 8      | 0.58     | box                    | ‘e’ | 40/65       | 930          | 0.13        | 40    | 2 n/m                         | n/m      | 4.6                     |
| 9      | 0.67     | box/set                | 2e | 25/65       | 150          | 0.25        | 4.7   | n/m                           | n/m      | 11.8                    |
| 10     | 1.79     | 0.63                   | no | L/S         | 25/65         | 1300        | 0.17  | 23                            | n/m      | 11.8                    |

Slow fluctuations parallel to the effective field change the precession frequency of the state and lead to a dephasing rate

$$\Gamma_\phi = \frac{1}{T_\phi} = \left( \frac{e}{\hbar} \right)^2 \cos^2 \theta S_{vbox}(\omega \to 0).$$  \hspace{1cm} (22)

Figure 21 shows how $T_2$ varies close to the degeneracy point for samples 7 and 9 respectively. The coherence time has a maximum at the degeneracy point $n_g = 1/2$ but quickly decreases to the sides. This suggests that low-frequency charge noise quickly dephases the box when the gate voltage is moved away from $n_g = 1/2$. Assuming that the decoherence is dominated by $1/f$ charge fluctuations we can fit our data to equation (22). As can be seen in figure 22, the data agrees well with this assumption away from the degeneracy point, and we can extract a value for the strength of the $1/f$ noise

$$S_q(\omega) = \frac{\alpha}{|\omega|}.$$  \hspace{1cm} (23)

We get a value of $\sqrt{\alpha} = 4 \times 10^{-3}e$ which is substantially higher than what is normally measured in SETs [53]–[55]. A possible reason for this high value could be that our fast rectangular pulses, which have a considerable amplitude and contain a wide-frequency spectrum, may shake up the background charges in the environment of the qubit. According to (equation (22)), the pure dephasing rate goes to zero at the degeneracy point (in the leading order) equation (22). At that point the decoherence will either be limited by the relaxation or by the second-order term of the charge fluctuations. As can be seen in table 2 the relaxation rates extrapolated to the degeneracy point come quite close to the maximum $T_2$ that we measure in most of the samples. Thus we can conclude that the coherence times in these samples are
relaxation-limited at the charge degeneracy, but limited by dephasing and due to $1/f$ noise as soon as we move away from the degeneracy point.

6.5. SET back action

The interaction between the RF-SET and the SCB necessarily leads to dephasing during read-out of the coherent state. Furthermore, the SET can cause mixing of the SCB levels which, given enough time ($T_1$), destroys the information about the quantum state. The back action from the RF-SET on the SCB occurs due to voltage fluctuations $S_V(\omega)$, on the SET island. The mixing time can be expressed as [6, 56]

$$\Gamma_1 = \frac{1}{T_1} = \frac{e^2}{\hbar^2} \kappa^2 \sin^2 \theta S_V(\Delta E/\hbar),$$

(24)

where $\kappa = C_c/C_\Sigma$ is the coupling coefficient, $\Delta E$ the level splitting in the box and $\sin \theta = E_J/\Delta E$. Thus the mixing time is dependent on the spectral density of the voltage fluctuations at the frequency $\omega \pm = \Delta E/\hbar$ corresponding to the SCB level splitting. These voltage fluctuations are of two origins. The SET shot noise dominates at low frequencies. At higher frequencies, the noise from the quantum fluctuations of the SET will dominate. At those high frequencies, the SET can be considered as a resistor in parallel with the junction capacitances, giving rise to a spectral density $S_V(\omega) = 2\hbar \omega \Re[Z_{set}(\omega)]$. To obtain the total spectrum, it is necessary to solve the full quantum problem, which was done by Johansson et al [57, 58].

We have found that when the SCB is read out with the SET biased at the JQP resonance and at higher voltages, this introduces non-equilibrium effects visible in the Coulomb staircase [59]. For high enough voltages, a staircase would become $e$-periodic although being completely $2e$-periodic for lower bias. The DJQP resonance was found to introduce significantly less back action than the other two for an acceptable decrease in charge sensitivity [60]. Thus, measurements of the SCB were in this work, unless otherwise specified, done by biasing the SET at the DJQP.

Investigations of how the continuous measurement of the RF-SET affects the box have been limited to two measurements. Firstly, the shape of the Coulomb staircase was in sample 9 found to be independent of bias current for currents below $\sim 200$ pA (figure 24). Generally, for most samples, it was possible to find a bias condition on the DJQP resonance that did not seem to influence the shape of the Coulomb staircase.

On the same sample, no systematic influence on the $T_1$ coherence time was found for bias voltages on the DJQP resonance, or at the JQP resonance for SET currents below $\sim 200$ pA. All times were of the order of 125 ns. For currents at the JQP resonance higher than $\sim 300$ pA, $T_1$ coherence times would only drop to $\sim 80$ ns. This must be considered a small reduction since the Coulomb staircase started to show significant back action effects at these currents. These facts do not exclude the presence of back action but rather indicates that the amount of back action, if present, does not change when the bias current through the RF-SET is smaller than $\sim 200$ pA.

7. Discussion

7.1. Ground state properties

One of the initial goals of this project was the fabrication of a $2e$-periodic single Cooper-pair box. This was achieved by choosing a low $E_C$ and by using the graded gap effect. The thickness of
Average charge <n><sub>2e</sub>

Normalized gate voltage n<sub>g</sub>=C<sub>g</sub>V<sub>g</sub>/2e

Figure 24. Coulomb staircases measured on sample 9 for different bias currents through the SET. Curves are offset proportionally to the RF-SET bias current. The three lowest traces are measured on the DJQP feature. The other traces are taken on the JQP feature and for higher bias voltages.

the SCB island was fabricated substantially thinner than the reservoir electrodes. In this way, the superconducting gap in the SCB reservoir was suppressed more quickly by an applied magnetic field compared to the SCB island. For charging energies below \( \sim 1 \text{ K} \), this led to Cooper-pair boxes which were 2\( e \)-periodic for all magnetic fields up to a field where superconducting gap of the SCB island was suppressed below the ground state energy for Cooper-pairs, \( \Delta - (E_C - 1/2 E_J) < k_B T \).

This graded gap effect appeared to have a stronger effect than the addition of normal metal quasi-particle traps.

7.2. Relaxation and coherence measurements

From table 2 we see that relaxation times measured far away from the charge degeneracy point, except for samples 8 and 10, range between 50 and 160 ns. The relaxation times are lower by factors of 3–30 than what we calculate from the environment of the SCB, including the SET read-out device [6, 57, 61]. Measurements of the coherence time \( T_2 \) in the vicinity of \( n_g = 1/2 \) (especially on sample 7) indicate that the relaxation time \( T_1 \) sets the limit on the coherence time at this point. The behaviour of \( T_2 \) around the charge degeneracy suggests that low frequency charge fluctuators dephase the box quickly as soon as the gate voltage is moved away from \( n_g = 1/2 \). An improvement of the \( T_1 \) relaxation time should lead to an improved coherence time at \( n_g = 1/2 \) since the dephasing rate is expected at first order to go to zero at this point. The exception to this conclusion is sample 8 which showed a relaxation time close to 1 \( \mu \text{s} \). This was initially believed to be the result of the quasi-particle traps added, but attempts to reproduce this result (sample 9) failed. This leaves some room for future work, e.g. by cooling down one more such sample which is nominally identical.

Relaxation time measurements of an SCB with RF-SET read out done at Jet Propulsion Labs (JPL) showed a 63 ns relaxation time [12] measured at \( n_{g0} = 0.34 \). This extrapolates to a relaxation time at degeneracy of 2.4 ns, in agreement with the lower end of the samples in

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this paper. Furthermore, measurements on a Cooper-pair box by the NEC group [62], where the probe junction read out was used, proposed a relaxation time $T_1 \approx 10\text{ ns}$ at the charge degeneracy. Further work by this group [63], where the relaxation rate could be measured directly at the charge degeneracy, again resulted in a relaxation time $T_1 \approx 5\text{ ns}$. The coherence time was also found to be limited by dephasing as soon as the gate voltage was moved away from $n_g = 1/2$. However, the results from sample 10 obtained at Yale University showed a $T_1$ well above $1\mu s$ by manipulating the box with microwaves instead of fast pulses. One conclusion from these measurements is that results very similar to most samples presented in this work have been obtained also at NEC and JPL. These results were measured using two different read out techniques (RF-SET and probe junction) which supports our belief that the short coherence times are not limited by the continuous measurement of the RF-SET. The long coherence time in sample 10 further supports this. One notable difference between the above measurements is that consequently shorter coherence times have been found using fast pulses. Clearly, what is needed in the future is a coherence time measurement using both fast pulses and RF-manipulation on the same sample to settle this question.

Longer relaxation rates compared to the above results have been demonstrated by Vion et al [10]. In this case, a Cooper-pair transistor was integrated in a loop interrupted by a large Josephson junction with the switching probability of the large junction used for read out. The parameters of the box (or transistor) were such that $4E_C/E_J = 0.8$, which means that it was no longer in the pure charging limit but in an intermediate charge/phase limit. Measurements at the charge degeneracy yielded $T_2 = 0.5\mu s$ and $T_1 = 1.8\mu s$. Another experiment at Yale University [19] on a Cooper-pair box in a microwave resonator, resulted at the charge degeneracy of the order of $0.5\mu s$ and a very long relaxation time $T_1 \approx 10\mu s$ [64]. This box was again in the charging limit with $4E_C/E_J = 3.5$. Notably, none of these two experiments used fast voltage pulses for the manipulation of the box.

8. Conclusions

We have fabricated a large number of single Cooper-pair boxes with varying parameters, and tested their properties as potential qubits. We find that the Coulomb staircase can be made $2e$-periodic by designing the charging energy to be sufficiently low. For intermediate values of $E_C$ we also demonstrate that the parity of the SCB can be tuned by a parallel magnetic field if the box electrode is made substantially thinner than the reservoir. The energy scales of the qubit can be extracted by microwave spectroscopy and the extracted values agree very well with those extracted from the staircases. This agreement is valid over a wide range of magnetic fields. Coherent oscillations were observed in seven of the samples and we find a high visibility of the oscillations. The relaxation and decoherence times were investigated as a function of both gate charge and Josephson coupling energy. We find relatively short decoherence times which are the longest at the charge degeneracy point. At this point the decoherence seems to be limited by relaxation, but as we move away from the degeneracy point the decoherence is limited by pure dephasing caused by charge fluctuations. The relaxation rates are larger by factors of 3–30 than what we calculate from the environment of the SCB including the SET read-out device.
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