The emerging field of quantum information theory is a fruitful combination of quantum theory and classical information theory, leading to surprising new insights into both fields. One of the most important goals in the development of a quantum information theory is to provide analogs of the central theorems of classical information theory, most notable Shannon’s noiseless and noisy coding theorem [1]. For noiseless quantum channels such an analog has been found by Schumacher [2]. The analog theorem known for noisy quantum channels [3] exploiting coherent information does not provide a closed analytical formula for the capacity of a given noisy quantum channel. Finding the latter is, in general, a difficult problem because it involves maximization over all possible coding and decoding procedures.

One of the major open problems in this context is the question whether quantum channel capacities are additive. Following pioneering papers connecting channel capacities with entanglement distillation [4] and identifying bound entanglement [5], it has been conjectured [6] that channels capacities are not additive, i.e. that there exist quantum channels \( \mathcal{E}_j, j = 1, 2 \) such that their capacities \( Q(\mathcal{E}_j) \) satisfy a superadditivity relation,

\[
Q(\mathcal{E}_1 \otimes \mathcal{E}_2) > Q(\mathcal{E}_1) + Q(\mathcal{E}_2),
\]

Obtaining equality in this expression for all channels \( \mathcal{E}_j \) would on the other hand imply additivity of the quantum channel capacity. Despite of considerable effort, this question remains unanswered so far.

One of the candidates for nonadditivity [7] in the bipartite scenario (one sender and one receiver) are so called binding entanglement (BE) channels introduced in [7]. i.e. channels that can produce only bound entanglement [8] after sending one of any given two entangled subsystems. BE channels have all capacities zero [9] and their possible nonadditivity is connected to the conjectured existence of NPT bound entanglement [8].

In this paper, we consider multiparty communication scenarios where quantum information is sent from several senders to several receivers through a noisy quantum channel. For such multiparty communication channels, natural generalizations of the definition of quantum channel capacities are possible. We establish a connection of different kinds of quantum channel capacities to the capability of the channel to create (distillable) multipartite entangled states. We show for certain scenarios that the ability of a channel to faithfully transmit quantum information is equivalent to its capability to generate a certain kind of (distillable) multipartite entangled states. This connection allows us to show that all these multipartite quantum channel capacities are, in general, not additive. The considered channels are multipartite versions of binding entanglement channels. We give an example of several such channels all of which have zero capacity (and cannot produce pure entangled states), while simultaneous availability of all these channels allows to faithfully transmit quantum information, i.e. the new channel has non-zero capacity.

1. Quantum channel capacity

We consider a quantum channel \( \mathcal{E} \) which is described by a completely positive trace preserving linear map \( \mathcal{E} : B(\mathcal{H}_{e}) \to B(\mathcal{H}_{o}) \) from the space \( B(\mathcal{H}_{e}) \) of bounded linear operators on the input Hilbert space \( \mathcal{H}_{e} \) to the space \( B(\mathcal{H}_{o}) \) of bounded linear operators on the output Hilbert space \( \mathcal{H}_{o} \). We consider isomorphic system Hilbert spaces \( \mathcal{H}_{s_{in}} \) and \( \mathcal{H}_{s_{out}} \) and coding (C) and decoding (D) operations, where \( C : B(\mathcal{H}_{s_{in}}) \to B(\mathcal{H}_{o}^{\otimes n}) \) and \( D : B(\mathcal{H}_{o}^{\otimes n}) \to B(\mathcal{H}_{s_{out}}) \). These coding and decoding operations define a \( (n, \epsilon) \) code if

\[
\min_{\langle \phi \rangle \in \mathcal{H}_{s_{in}}} F(\langle \phi \rangle, (D \circ E^{\otimes n} \circ C)\langle \phi \rangle) \geq 1 - \epsilon. \tag{2}
\]

We remark that we implicitly made use of the isomorphy between \( \mathcal{H}_{s_{in}} \) and \( \mathcal{H}_{s_{out}} \) to define the fidelity \( F \) between two states on different Hilbert spaces, that is for some arbitrary but fixed bases \( \{ |\phi_i \rangle \} \) and \( \{ |\chi_i \rangle \} \) with \( |\phi_i \rangle \in \mathcal{H}_{s_{in}} \) and \( |\chi_i \rangle \in \mathcal{H}_{s_{out}} \), we identify \( |\phi_i \rangle \equiv |\chi_i \rangle \). The rate \( R \equiv 1/n \log \dim \mathcal{H}_{s_{in}} \) of the code is called achievable if for all \( \epsilon, \delta > 0 \) and sufficiently large \( n \) there exists a code of rate \( R - \delta \). The quantum capacity \( Q(\mathcal{E}) \) of a bipartite quantum channel \( \mathcal{E} \) is defined as the supremum of all achievable rates \( R \) (see Ref. [11] for a rigorous definition).

Coding and decoding operations may be assisted by forward classical communication \( (\rightarrow) \) or two–way classical communication \( (\leftrightarrow) \) which gives rise to quantum capacities \( Q^{\rightarrow} \) and \( Q^{\leftrightarrow} \) respectively. We remark that a minimal pure state fidelity \( F = 1 - \epsilon \) for all \( |\phi \rangle \in \mathcal{H}_{s_{in}} \) implies an entanglement fidelity \( F_e \geq 1 - 3/2\epsilon \) for all density operators \( \rho \) whose support lies entirely in that subspace [11].
That is, when transmitting part of an entangled state $|\Psi\rangle$ which is a purification of $\rho$, we have that the resulting state has fidelity $F \geq 1 - 3/2\epsilon$ with respect to $|\Psi\rangle$.

One can generalize the definition of channel capacity to a multipartite scenario with $N$ spatially separated senders $A_1, \ldots, A_N$ and $M$ spatially separated receivers $B_1, \ldots, B_M$. In this case, the input Hilbert space is given by the tensor product of the Hilbert spaces of the senders, $\mathcal{H}_{\text{in}} = \mathcal{H}_{A_1} \otimes \cdots \otimes \mathcal{H}_{A_N}$, and the output Hilbert space is a tensor product of the Hilbert spaces of the receivers, $\mathcal{H}_{\text{out}} = \mathcal{H}_{B_1} \otimes \cdots \otimes \mathcal{H}_{B_M}$, while the quantum channel $E : \mathcal{B}(\mathcal{H}_{\text{in}}) \to \mathcal{B}(\mathcal{H}_{\text{out}})$ as before. In such a scenario the allowed operations (in particular $\mathcal{C}$ and $\mathcal{D}$) are restricted to local operations $A_1, \ldots, A_N - B_1, \ldots, -B_M$. We denote by LOCC local operations assisted some kind $C$ of classical communication between senders and receivers (e.g. $C = \leftrightarrow$ denotes two way classical communication between each sender $A_i$ and all receivers $B_j$). For any subset of sender $\tilde{A}$ and any subset of receivers $\tilde{B}$ one can define a channel capacity $Q_{\tilde{A} \to \tilde{B}}$ which measures the amount of quantum information that can be sent from the set of senders $\tilde{A}$ to the set of receivers $\tilde{B}$. In that case, we have that $\mathcal{H}_{\text{in}} = \mathcal{H}_{\tilde{A}} \cong \mathcal{H}_{\text{out}} = \mathcal{H}_{\tilde{B}}$ and all operations are LOCC, where $\mathcal{H}_{\tilde{A}} = \otimes_{A_i \in \tilde{A}} \mathcal{H}_{A_i}$ is some (arbitrary) Hilbert space of parties $A_j \in \tilde{A}$ and similarly for $\mathcal{H}_{\tilde{B}}$ (see Fig. 1).

![FIG. 1: Multiparty communication scenario with $\tilde{A} = A_1, A_2, A_3$, $\tilde{B} = B_1, B_2$. Note that party $A_4 \notin \tilde{A}$ may prepare its system in some suitable state, while party $B_3 \notin \tilde{B}$ can e.g. perform suitable measurements to assist the communication.](image)

We point out that although $Q_{\tilde{A} \to \tilde{B}} > 0$, one might have the situations that (i) the parties $A_k \in \tilde{A}$ cannot transmit information if they prepare the states of their systems separately and (ii) the information is distributed among the parties $B_j \in \tilde{B}$ and cannot be accessed locally. Situation (i) might appear when a channel is only capable to transmit entangled states, while (ii) might occur when the output states of a channel are entangled. If some senders $A_k$ and some receivers $B_j$ are not involved in the transmission, and are excluded from classical communication with the remaining system, one can equivalently consider the reduced channel $\tilde{E}$ which corresponds to the initial channel $E$ followed by trace over all parties $A_k, B_j$.

Given the variety of possible choices of $\tilde{A}, \tilde{B}$ and the allowed classical communication $C$, it is not hard to imagine that such a multipartity communication system is very rich and displays many new and interesting features.

2. Distillability in multipartite systems

In the following, we establish a connection of the different channel capacities to different kinds of multipartite distillability. Consider again a multi-local scenario with spatially separated parties $\{A_k\}$ and $\{B_j\}$ and subsets $\tilde{A}, \tilde{B}$ and some kind of classical communication $C$. We have that $\rho_{AB}$ is distillable in the sense $\tilde{A} \tilde{B}$ if an entangled state shared between the systems $\tilde{A}$ and $\tilde{B}$ can be created from several copies of $\rho_{AB}$ by means of LOCC. That is, a state $\rho_{AB}$ has $D_{\tilde{A} \tilde{B}}(\rho) > 0$ if and only if there exists a (multilocal) transformation $E : \mathcal{B}(\mathcal{H}_{\tilde{A} \tilde{B}}) \to \mathcal{B}(\mathcal{H}_{\tilde{A} \tilde{B}})$ such that $\rho_{AB}^N \to \mathcal{C}(\rho_{AB})$ with $\langle \Phi | \sigma | \Phi \rangle > 0$ for all entangled states $|\Phi\rangle$. We remark that the states $|\chi\rangle, |\psi\rangle$ might itself be entangled, which can imply that $\rho_{AB}$ cannot be used to accomplish certain tasks (e.g. teleportation) by means of only local operations.

We proceed by pointing out some non–trivial relations between different kinds of distillability. First we have that $\exists B_j \in \tilde{B}, D_{\tilde{A} \tilde{B}}(\rho_{AB}) > 0 \Rightarrow D_{\tilde{A} \tilde{B}}(\rho_{AB}) > 0$. The converse is, however, not generally true. Consider for instance a tripartite GHZ state, $|\Psi\rangle_{AB1} = 1/\sqrt{2}(|000\rangle + |111\rangle)$ and LOCC. Clearly, $D_{A(B1)(B2)}(|\Psi\rangle) > 0$, however $D_{A\tilde{B}}(|\Psi\rangle) = 0\forall j$. Note that e.g. the state of the system $AB1$ is described by the reduced density operator $\rho_{AB1} = 1/2(|00\rangle\langle 00| + |11\rangle\langle 11|)$ regardless of the operation performed at $B2$, since the results of possible measurements in $B2$ cannot be communicated to $A$ or $B1$. We have that $\rho_{AB1}$ is separable and hence $D_{A\tilde{B}}(|\Psi\rangle) = 0$.

On the other hand for LOCC and $A_k \in \tilde{A}$, $B_j \in \tilde{B}$ one can show that

$$\exists (A_k, B_j), D_{A_k B_j}(\rho_{AB}) > 0 \iff D_{\tilde{A} \tilde{B}}(\rho_{AB}) > 0, \tag{3}$$

that is the possibility to create entanglement between the composed systems $\tilde{A}$ and $\tilde{B}$ is equivalent to the possibility to create entanglement between at least one of the individual parties $A_k$ and $B_j$. It remains to show that from $|\Phi\rangle = 1/\sqrt{2}(|\chi\rangle_{A_1} |\psi\rangle_{B_1} + |\chi\rangle_{A_2} |\psi\rangle_{B_2})$ with $|\chi\rangle \neq |\chi\rangle$, $|\psi\rangle \neq |\psi\rangle$ one can create a maximally entangled state shared between $A_k$ and some $B_j$ by means of LOCC.

This can be seen using the following lemma (see also [13]): For all states $|\Psi_0\rangle_{A_1 \ldots A_N} \neq |\Psi_1\rangle_{A_1 \ldots A_N}$ which are not of the form (i) $|\Psi_0\rangle = |\psi_0\rangle_{A_1} \otimes |\Phi_0\rangle_{A_2 \ldots A_N}$ and $|\Psi_1\rangle = |\psi_1\rangle_{A_1} \otimes |\Phi_1\rangle_{A_2 \ldots A_N}$ with $|\psi_0\rangle \neq |\psi_1\rangle$, there exists a projector $P_{A_1} = \langle \psi_1 | A_1 \langle \Phi_1 |$ such that the resulting states $|\Psi_j\rangle = P_{A_1} |\Psi_j\rangle / ||P_{A_1} |\Psi_j\rangle ||$ fulfill $|\Psi_0\rangle \neq |\Psi_1\rangle$. The proof is by contradiction: Assume on the opposite that $|\Psi_0\rangle = |\Psi_1\rangle$ for all projectors $P_{B1}$... One readily convinces oneself that this implies that either $|\Psi_0\rangle = |\Psi_1\rangle$ or (i) is fulfilled, from which the lemma follows.

We sequentially apply the lemma to systems $A_1, A_2, \ldots, A_{M-1}$ and stop if (i) applies at some point. In step one we have e.g. that either $|\chi_{A_1}\rangle, |\chi_{A_2}\rangle$ fulfills (i), which leaves parties $A_1, \tilde{B}$ with a state of the form $(|\psi_1\rangle_{A_1} |\psi\rangle_{B_1} + |\psi_2\rangle_{A_i} |\psi\rangle_{B_2})$ while the other parties are
factored out. If (i) does not apply, then from the lemma follows that there exists a projective measurement in $A_1$ such that the resulting state of systems $A_2, \ldots, A_M \otimes B$ is of the same form as the initial state $|\Phi\rangle$, but the number of parties $A_k$ is decreased by one (and party $A_1$ is factored out). Doing the same in system $B$, we have that one ends up with a state of the form $(|\varphi_1\rangle_{A_k}|\gamma_1\rangle_{B_j} + |\varphi_2\rangle_{A_k}|\gamma_2\rangle_{B_j})$ shared between two parties $A_k$ and $B_j$ for some $(k, j)$, while all other parties are factored out. This state is distillable, e.g. by means of filtering measurements in $A_k$ and $B_j$ which ends the proof of Eq. 3.

3. Relation between $Q_{A \rightarrow B}$ and $D_{A \rightarrow B}$

We will now show the qualitative equivalence of non–zero multipartite channel capacities and the capability of the channel to create distillable entanglement. We will restrict ourselves to channels with only a single sender $A \equiv A_1$ and several receivers $B_1, \ldots, B_M$ and classical communication $C$ which contains at least forward communication from $A$ to all receivers $B_j \in B$. Our results are a generalization of the results found in Ref. [10] for bipartite communication channels to the multipartite case.

A nonzero channel capacity $Q_{A \rightarrow B}^C$ implies —by definition— that there exists a LOCC$^C$ implementable coding and decoding procedure such that a subspace $\mathcal{H}$ with dim $\mathcal{H} \geq 2$ can be reliable transmitted. Since the entanglement fidelity is $F_e = 1 - 3/2e$ with $e$ arbitrarily small, one can use this coding and decoding procedure to success fully transmit one half of a maximally entangled state of two qubits $|\Phi\rangle = 1/\sqrt{2}(0_A|\Psi_0\rangle + 1|\Psi_1\rangle)$ with $\langle \Psi_0 | \Psi_1 \rangle = 0$ such that the resulting state $\rho_{AB}$ fulfills $F = \langle \Phi | \rho_{AB} | \Phi \rangle \geq 1 - 3/2e$. This already shows that a maximally entangled state shared between $A$ and the joint system $B$ can be created by means of LOCC$^C$ if $Q_{A \rightarrow B} > 0$. In turn we have that whenever such a state $|\Phi\rangle_{AB}$ is available it can be used to faithfully transmit quantum information by means of teleportation (which only involves forward classical communication) [3]. Note, however, that the operations required in the original teleportation scheme to achieve deterministic teleportation for all possible measurement outcomes may not be locally implementable since the states $|\Psi_j\rangle$ are not necessarily of product form. As we did not fix a basis in $\mathcal{H}_{sni}$ in the definition of $Q_{A \rightarrow B}$, we have that $Q_{A \rightarrow B} > 0$, however the quantum information is not in all cases locally accessible.

The capability of a channel $\mathcal{E}$ to generate (distillable) pure state entanglement under LOCC$^C$ is completely determined by the entanglement properties of the state $E_{AB}$ [14] corresponding to $\mathcal{E}$ via the isomorphism

$$E \equiv 1_{A} \otimes E_{B}|\Phi\rangle\langle\Phi|,$$

and $|\Phi\rangle = 1/\sqrt{\mathcal{d}}\sum_{k=1}^{\mathcal{d}} |k\rangle_A|k\rangle_B$. We have that the channel can generate distillable pure state entanglement in the sense $A \otimes B$ iff $E$ is distillable (assuming LOCC$^C$ in both cases). To see this, assume on the one hand that $E$ is distillable. Since $E$ can be created by sending one half of a maximally entangled state through the channel $\mathcal{E}$, the channel can generate states which are distillable and thus pure entangled states. Assume on the other hand that by sending a certain state, say $\tau_A$, through the channel $\mathcal{E}$ one can generate distillable states $\sigma_{AB}$. Since $E$ can be used to implement $\mathcal{E}$ probabilistically by using only local operations and forward classical communication [14], one can generate $\sigma_{AB}$ from $\tau_A$ and $E$. Since $\sigma_{AB}$ is by assumption distillable, so is $E$.

Thus for both forward one way and two way classical communication and any subset of parties $B \in B$ we have the following result [13]:

$$Q_{A \rightarrow B}(\mathcal{E}) > 0 \iff D_{A \rightarrow B}(E) > 0$$

Note that for two–way classical communication Eq. (5) together with Eq. (3) implies that if $D_{A \rightarrow B}(E) = 0$ for all $B_j \in B$ then all quantum capacities $Q_{A \rightarrow B}$ are zero.

4. Example for non–additivity of $Q_{A \rightarrow B}$

We now use this fact to provide an example which shows that multipartite quantum channel capacities are not additive. In the following we restrict ourselves to two–way classical communication and thus omit the symbol $\leftrightarrow$.

We consider a three party communication scenario and introduce three quantum channels $E_{a}$, $a = 1, 2, 3$ from a sender $A$ to two receivers $B$ and $C$ with $\mathcal{H}_B = \mathcal{H}_A = \mathcal{H}_0 = \mathcal{H}_B \otimes \mathcal{H}_C = \mathcal{Q}^2 \otimes \mathcal{Q}^2$. From now on we use the shorthand notation $Q$ to refer to any of the capacities $Q_{A \rightarrow B_1}, Q_{A \rightarrow B_2}, Q_{A \rightarrow B_1B_2}$, since the results we obtain hold for any of these capacities. On the one hand, we show that each of the channels $E_a$ is not capable to create pure state entanglement shared between any two of the parties $A, B, C$. It follows that the quantum capacities $Q$ of each channel $E_a$ are zero. On the other hand, we show that $Q$ of a channel $\mathcal{E}$ created by randomly choosing one of the channels $E_a$ with equal probability is non–zero which implies non–additivity of quantum channel capacities in this multi–party communication scenario. Note that quite remarkably the entanglement capability of the channels is enhanced by classical mixing, a procedure usually believed to diminish the entanglement properties of states and operations.

The quantum channels $E_a$, $a = 1, 2, 3$ are described by trace preserving completely positive maps which we write in the Kraus representation $E_{\rho} \equiv \sum_{k} A_k\rho A_k^\dagger$. The map $E_1$ is specified by the Kraus operators $A_k \in \{1/2(\sigma_0 \sigma_0 + \sigma_3 \sigma_3), 1/\sqrt{2}(\sigma_1 \sigma_1 + \sigma_2 \sigma_2), 1/\sqrt{2}(\sigma_2 \sigma_0 - \sigma_0 \sigma_2), 1/\sqrt{2}(\sigma_3 \sigma_3 - \sigma_0 \sigma_0 + \sigma_1 \sigma_1 - \sigma_2 \sigma_2), 1/\sqrt{2}(\sigma_0 \sigma_2 + \sigma_2 \sigma_0), 1/2(\sigma_3 \sigma_1), 1/2(\sigma_3 \sigma_2), 1/2(\sigma_3 \sigma_3), 1/2(\sigma_0 \sigma_1), 1/2(\sigma_0 \sigma_2), 1/2(\sigma_0 \sigma_0), 1/2(\sigma_2 \sigma_0), 1/2(\sigma_1 \sigma_2), 1/2(\sigma_2 \sigma_2), 1/2(\sigma_1 \sigma_1), 1/2(\sigma_1 \sigma_2), 1/2(\sigma_1 \sigma_3), 1/2(\sigma_3 \sigma_1), 1/2(\sigma_3 \sigma_2), 1/2(\sigma_3 \sigma_3)\}$ where $\sigma_j$ are the Pauli matrixes with $\sigma_0 \equiv 1_2$. One readily checks that $\sum_k A_k^\dagger A_k = 1$ which ensures that $E_1$ is trace preserving.

Similarly, the map $E_2$ is is determined by Kraus operators $A_k \in \{1/2(\sigma_0 \sigma_0 + \sigma_3 \sigma_3), 1/2(\sigma^+ (\sigma_0 \sigma_3) + \sigma^- (\sigma_0 \sigma_3)), 1/2(\sigma_0 \sigma_0), 1/2(\sigma_3 \sigma_3), 1/2(\sigma_0 \sigma_0), 1/2(\sigma_3 \sigma_3), 1/2(\sigma_0 \sigma_0), 1/2(\sigma_3 \sigma_3), 1/2(\sigma_0 \sigma_0), 1/2(\sigma_3 \sigma_3)\}$. Again one can easily check that $E_2$ is trace preserving. Finally, the map $E_3$ is obtained from $E_2$ by permuting systems 1 and 2.

To determine the (distillability) properties of the maps
\( E_n \) we make use of the isomorphism Eq. (1). The mixed state \( E_n \) associated to \( E_n \) is obtained by applying \( E_n \) to particles \( B,C \) of the state \( \mathcal{P}_{\Phi}^{(A_1A_2B)} \equiv \langle \Phi | \Phi \rangle \) where 

\[
\langle \Phi | \equiv \langle \Phi^+ | A_1B \otimes \langle \Phi^+ | A_2C \rangle \text{ with } \langle \Phi^+ | = 1/\sqrt{2}(|00\rangle + |11\rangle).
\]

This corresponds to sending one part of a maximally entangled four level system through the quantum channel \( E \) from \( A \) to \( B \) and \( C \) such that qubits \( A_1, A_2 \) remain at site \( A \). One finds that

\[
E_1^{(A_1A_2B)} = \frac{(2|\Psi^+_0\rangle \langle \Psi^+_0| + 1 - P_{1010} - P_{0101})}{16}
\]

\[
E_2^{(A_1A_2B)} = 2(2|\Psi^+_0\rangle \langle \Psi^+_0| + 1 - P_{1010} - P_{0101})/16, \quad (6)
\]

where \( P_{ijkl} \equiv |ijkl\rangle\langle ijkl| \) and \( |\Psi^+_0\rangle \equiv 1/\sqrt{2}(|00\rangle + |11\rangle) \) and \( E_3 \) is obtained from \( E_2 \) by exchanging \((A_1B)\) with \((A_2C)\). One can readily determine the entanglement properties of these density operators. We have that

\[
E_{1B} = 0 \neq E_{1C} \geq 0 \text{ and } E_{2B} \sim 0 \geq E_{2C} = 0 \text{ and similarly for } E_3, \text{ where we denote by } E_{iC} \text{ the partial transposition of the density operator } E_i \text{ with respect to party } \beta \text{.}
\]

Note that \( E_n \) belongs to a class of states which has been completely characterized with respect to its entanglement properties in Ref. [17], for states optimality of partial transposition with respect to a certain system already implies separability of this system from the remaining ones, e.g. \( E_2 = \mathcal{S}_k(\mathcal{S}_C(\mathcal{S}_C(\mathcal{S}_C(\mathcal{S}_C|\Psi^+_0\rangle \langle \Psi^+_0| + 1 - P_{1010} - P_{0101})/16)\rangle) \). It follows that the channel \( E_1 \) can only create states where particle \( B \) (and also particle \( C \)) is separable and thus no pure state entanglement shared between any of the groups \((A_1A_2)B \) is separable, i.e. \( D^{|\Psi^+_0\rangle}_E(A_1A_2) = D^{|\Psi^+_0\rangle}_E(C) = 0 \) [17]. We thus have that the quantum capacity of \( E_1 \) is zero, \( Q(E_1) = 0 \). Similarly, we have that \( E_2, E_3 \) are separable with respect to system \((A_1A_2)B \) which leads to \( Q(E_2) = Q(E_3) = 0 \). Note that the considered channels are multipartite binding entanglement channels since they are isomorphic to bound entangled states \( E_n \).

We consider now the situation where all three channels \( E_n \) are available and show that \( Q(E_1, E_2, E_3) > 0 \). In particular, we consider a channel \( E \) which is obtained by a classical mixture of the three channels \( E_n \), i.e. randomly choosing one of the channels. The new set of Kraus operators for the channel \( E \) is given by the Kraus operators of the channels \( E_n \) with a pre-factor \( 1/\sqrt{3} \). The density operator \( E \) corresponding to \( E \) is given by \( E = (E_1 + E_2 + E_3)/3 \) and one finds

\[
E^{(A_1A_2B)} = 1/16(2|\Psi^+_0\rangle \langle \Psi^+_0| + 1 - (7)
\]

\[
-1/3(P_{1010} + P_{0101} + P_{1011} + P_{0101} + P_{0001} + P_{1110})
\]

We have \( E_{1B} \sim 0 \neq E_{1C} \equiv 0 \) which implies that \( E \) is distillable entangled, i.e. maximally entangled pure states shared between \((A_1A_2) - B \) and \((A_1A_2) - C \) can be created. This is due to the fact that \( E \) belongs to a class of states \( \rho \) for which non–positive partial transposition with respect to all relevant partitions is a sufficient condition for distillability [17]. Note that the distillation procedure requires two–way classical communication. From Eq. (4) follows that also the quantum capacity \( Q(E) > 0 \), which implies that the quantum capacity of multiparty communication channels is not additive.

In this paper, we have considered multiparty communication scenarios where information is sent from a single sender to several receivers. We have introduced (natural) definitions of quantum channel capacities in the multipartite setting and established a connection of these quantum capacities to the capability of the channel to create distillable entangled states. This connection allowed us to show that all these quantum capacities are –in the case of two way classical communication– not additive.

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