Dense baryonic matter: constraints from recent neutron star observations

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Updated constraints from neutron star masses and radii impose stronger restrictions on the equation of state for baryonic matter at high densities and low temperatures. The existence of two-solar-mass neutron stars rules out many soft equations of state with prominent “exotic” compositions. The present work reviews the conditions required for the pressure as a function of baryon density in order to satisfy these new constraints. Several scenarios for sufficiently stiff equations of state are evaluated. The common starting point is a realistic description of both nuclear and neutron matter based on a chiral effective field theory approach to the nuclear many-body problem. Possible forms of hybrid matter featuring a quark core in the center of the star are discussed using a three-flavor Polyakov–Nambu–Jona-Lasinio (PNJL) model. It is found that a conventional equation of state based on nuclear chiral dynamics meets the astrophysical constraints. Hybrid matter generally turns out to be too soft unless additional strongly repulsive correlations, e.g. through vector current interactions between quarks, are introduced. The extent to which strangeness can accumulate in the equation of state is also discussed.

\section{I. INTRODUCTION}

The investigation of compressed baryonic matter is one of the persistently important themes in the physics of strongly interacting many-body systems. While high-energy heavy-ion collisions probe the transition from the hadronic phase to deconfined quark-gluon matter at high temperatures and relatively low baryon chemical potentials, the access to “cold” and dense baryonic matter comes primarily through observations of neutron stars in which central core densities several times the density of normal nuclear matter can be reached.

Two remarkable examples of massive neutron stars have recently emerged. One of those is the radio pulsar J1614–2230 with a mass $M = (1.97 \pm 0.04) M_{\odot}$ \cite{1}. Even heavier neutron stars were occasionally discussed in the literature (e.g., \cite{2} and references therein), but this one is special because of the high accuracy of its mass determination made possible by the particular edge-on configuration (an inclination angle of almost 90°) of the binary system consisting of the pulsar and a white dwarf. Given this configuration, a pronounced Shapiro delay signal of the neutron star’s pulses could be detected. In the meantime a second neutron star has been found with a comparable, accurately determined mass (J0348+0432 with $M = (2.01 \pm 0.04)M_{\odot}$) \cite{3}, further strengthening the case.

The established existence of two-solar-mass neutron stars rules out many equations of state (EoS) that are too soft to stabilize such stars against gravitational collapse. On the other hand, some selected equations of state based entirely on conventional nuclear degrees of freedom are able to develop sufficient pressure so that the condition to reach $2M_{\odot}$ can be fulfilled.

The present work performs an updated analysis of the constraints on the EoS of strongly interacting baryonic matter provided by these observations. Traditionally, the primary source of information is the mass-radius relation of the star calculated using the Tolman-Oppenheimer-Volkov equations \cite{4} with a given EoS as input. The empirical restrictions on neutron star radii are less severe than those on the mass. Nonetheless, the quest for a stiff EoS at high baryon densities persists as a common theme throughout this investigation.

An essential condition to be fulfilled is the following: the known properties of normal nuclear matter must be considered as a prerequisite for the construction of any realistic EoS, together with the requirement of consistency with the best existing many-body computations of pure neutron matter (including three-body forces etc.). This latter important constraint has often been ignored, e.g. in phenomenological relativistic mean-field model calculations routinely used in supernova simulations.

Neutron star matter interpolates between the extremes of isospin-symmetric nuclear matter and pure neutron matter. The fraction of protons added to the neutron sea is controlled by beta equilibrium. The passage from $N = Z$ matter to neutron-rich matter as it emerges in the core of the star is driven by detailed properties of the isospin-dependent part of the nuclear interaction. These isospin-dependent forces also determine the evolution of the nuclear liquid-gas phase transition from isospin-symmetric matter towards the disappearance of this phase transition around $Z/N \approx 0.05$. Such properties of the phase diagram of highly asymmetric nuclear matter provide further guidance and constraints that we incorporate in our analysis.

At the interface between low-energy quantum chromodynamics (QCD) and nuclear physics, chiral effective field theory (ChEFT) has become the framework for a successful description of the nucleon-nucleon interaction and three-body forces, as well as for the nuclear many-body problem (see Refs. \cite{4,10} for recent reviews). ChEFT is our starting point for a systematic approach to nuclear and neutron matter at densities (and temperatures) well within the hadronic sector of QCD, the one governed by confinement and spontaneous chiral symmetry breaking. The ChEFT approach is used here to set the boundary values, at normal nuclear densities, for the construction of the EoS at higher densities. As will be demonstrated, a sufficiently stiff EoS supporting a two-solar-mass neutron star does indeed result from in-medium ChEFT with “conventional” (nucleon and pion) degrees of freedom plus
three-body forces. Options for a transition to quark matter at very high baryon densities will be examined using a three-flavor Polyakov–Nambu–Jona-Lasinio (PNJL) model. It turns out to be unlikely, however, that such a quark component, even if existent in the deep interior of the star, will be of observable significance. Furthermore, the possible role of hyperons will briefly be discussed, again with the condition in mind that their admixture should not soften the EoS so much that it falls short of supporting a two-solar-mass neutron star.

The aim of the present paper is then twofold: first, to establish boundaries and constraints that any equation of state must fulfill in view of the recent astrophysical observations; secondly, to construct a realistic EoS with a firm foundation in the (chiral) symmetry breaking pattern of low-energy QCD. In Section II the mass constraint together with (less restrictive) constraints on neutron-star radii are summarized in order to impose general limitations for the EoS of neutron star matter. In this context the neutron star crust is briefly discussed. In Section III the equations of state for symmetric and asymmetric nuclear matter and for pure neutron matter are constructed within the framework of in-medium ChEFT. This includes the resummation of short-range interaction ladders to all orders in the large neutron-neutron scattering length. Comparisons with state-of-the-art many-body calculations of neutron matter will be displayed. Section IV is then devoted to astrophysical implications of these EoS results. A summary and conclusions are presented in Section V.

II. EMPIRICAL CONSTRAINTS FROM NEUTRON STARS

Apart from the mass measurements discussed in the introduction, this section briefly reviews and summarizes empirical constraints on neutron star radii and their implications. Thereafter it is shown how the two-solar-mass pulsars (J1614–2230 and J0348–0432), in combination with the radius restrictions, define conditions for acceptable equations of state for neutron star matter.

A. Neutron star radii

In this work we consider constraints on neutron star radii from two independent sources. The first one, Refs. [13,15], following earlier studies in Refs. [13,15], is based on a statistical analysis of the mass-radius curves of four X-ray bursters (EXO 1745–248, 4U 1608–522, 4U 1820–30, KS 1731–260), and four quiescent low-mass X-ray binaries (neutron stars in the globular clusters 47 Tuc, ω Cen, M13 and NGC 6397). Reference [13] amends the previous analyses by considering in addition the low-mass X-ray binaries in the globular clusters NGC 6304 and M28. Analyzing the X-ray spectra of the neutron stars and assuming that all objects have hydrogen atmospheres, one arrives at typical radii for 1.4-solar-mass neutron stars ranging from 10.4 to 12.9 km (95 % confidence level) [12] and 11.4 to 12.8 km (90 % confidence level) [13]. According to Ref. [13] radii of neutron stars having masses between 0.8 $M_\odot$ and 2.0 $M_\odot$ all lie in a band between 10.9 and 12.7 km. An analysis performed in Ref. [16] considering the same objects as in Ref. [13], but assuming a constant radius for all neutron stars, leads to $R = 9.1^{+1.5}_{-1.3}$ km. However, the statistical method used in that analysis results in a radius range that is smaller than the accepted radii assigned to most of the individual neutron stars under consideration.

As a second source we rely on the neutron star radius constraints provided by Fig. 6 of Ref. [17]. This detailed analysis features four independently determined curves of constraints that, together, form a rhombic area in the mass-radius plot. In combination with the two-solar-mass condition a triangular area remains, bounded by radii 11.5 $\lesssim R \lesssim$ 14.5 km. Within the given uncertainties, all acceptable equations of state must generate mass-radius trajectories that pass through this triangle. These radius constraints are deduced from the following specific cases: the light-curve oscillations of the X-ray burster XTE J1814–338 [18]; the thermal spectrum of the radio-quiet isolated neutron star RXJ 1856–3754 as discussed in Ref. [19]; the 90 %-confidence analysis using a hydrogen-atmosphere model to fit the spectra of neutron stars in the globular cluster 47 Tuc [20,21]; and finally, the mass-shedding limit calculated from the spinning period of the fastest known pulsar, J1748–2446ad.

B. Mass-radius relation

Given an equation of state (EoS) relating pressure and energy density, the mass-radius curves for neutron stars are determined by solving the Tolman-Oppenheimer-Volkoff (TOV) equation. This equation describes the structure of a spherically symmetric star composed of isotropic material with corrections from general relativity [4–6]:

$$\frac{dP(r)}{dr} = \frac{\mathcal{G}}{r^2 c^2} [\epsilon(r) + P(r)] \left[ M(r) + 4\pi r^3 \frac{P(r)}{c^2} \right] \times \left[ 1 - 2\mathcal{G}M(r) \frac{1}{c^2 r} \right]^{-1}. \quad (1)$$

Here $\mathcal{G}$ is the gravitational constant, $c$ denotes the speed of light$^1$, $r$ is the radial coordinate, and $\epsilon(r)$ and $P(r)$ are the energy density and pressure, respectively. Moreover, $M(r)$ is the total mass inside a sphere of radius $r$. It is related to the energy density by

$$\frac{dM(r)}{dr} = 4\pi r^2 \frac{\epsilon(r)}{c^2}. \quad (2)$$

Eqs. (1) and (2) supplemented by an EoS, $P = P(\epsilon)$, determine completely the structure of a static (non-rotating), spherical neutron star. The commonly chosen initial boundary conditions for the integration of the TOV equation are the energy density in the core of the neutron star, $\epsilon(0) = \epsilon_c$, and $M(0) = 0$. The radius, $R$, of the neutron star is given by

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1 In all subsequent sections units with $c = 1$ will be used.
the condition \( \epsilon(R) = \epsilon_{\text{Fe}} \), where the energy density on the surface of the star has dropped down to that of atomic iron, \( \epsilon_{\text{Fe}} = 7.9 \text{ g/cm}^3 = 4.4 \cdot 10^{-12} \text{ MeV/fm}^3 \). The neutron star mass is

\[
M \equiv M(R) = \frac{4\pi}{c^2} \int_0^R dr r^2 \epsilon(r) , \tag{3}
\]

the total mass measured by the gravitational field felt by a distant observer.

C. Neutron star equation of state and constraints from observables

The focus in this work is primarily on the equation of state for bulk uniform nuclear and neutron star matter, suitable to describe the core region of the star. Solving the TOV equation requires the knowledge of the EoS in the entire neutron star, including the low-density crust region at its surface. The outer crust is associated with densities \( \varrho \lesssim \varrho_d \) below the neutron-drip point, \( \varrho_d \approx 10^{-3} \varrho_0 \) in units of nuclear saturation density, \( \varrho_0 = 0.16 \text{ fm}^{-3} \). The structure of this outer crust region is quite well established [22, 23]. The inner crust is less well understood. In the transition region to a uniform nuclear medium (in the density range \( 0.1 \varrho_0 \lesssim \varrho \lesssim 0.8 \varrho_0 \)) extended clusters of so-called “pasta” phases [24, 25] might be formed.

In order to describe this multifacet structure of the neutron star’s crust (not covered by our explicit calculations) we use the empirical equation of state as given in Ref. [26] for the low-density region. This EoS is fitted to a Skyrme-Lyon EoS [27] and to experimental data for neutron-rich nuclei according to Refs. [22, 28]. In the following, we refer to this crust EoS as “SLy”.

At a density of about 0.8 \( \varrho_0 \) the nuclei dissolve and turn into a uniform medium of neutrons with a small admixture of protons in the outer core region of the neutron star. From such densities inward to the central core, up to about 2–3 \( \varrho_0 \), chiral effective field theory (ChEFT) is used as an appropriate framework for the description of nuclear and neutron matter (see Sec. [11A]). For the remainder of this section we adopt the ChEFT-based EoS determined in Refs. [7, 29] (FKW) in order to describe the outer core, assuming at this point a (constant) proton fraction of 10%. This FKW EoS is matched to the SLy EoS at their intersection point, \( \varrho_0 \approx 118 \text{ MeV/fm}^3 \) corresponding to a density \( \varrho \approx 0.75 \varrho_0 \).

The extrapolation to the high-density domain of the equation of state is parametrized using three polytropes fitted sequentially to one another (in a way similar to the procedure pursued in Refs. [22, 33]): \( P = K_i \varrho^\gamma_i \), \( i \in \{1, 2, 3\} \). The equation of state for each of the branches is

\[
\epsilon = a_i \left( \frac{P}{K_i} \right)^{1/\gamma_i} + \frac{1}{\Gamma_i - 1} P \quad (i = 1, 2, 3) , \tag{4}
\]

where the \( a_i \) are constants determined by the continuity of \( \epsilon = \epsilon(P) \). It turns out that three polytropes are sufficient [34] in order to represent a large variety of models for dense nuclear matter. We use the FKW EoS up to an energy density \( \epsilon_1 = 153 \text{ MeV/fm}^3 \) corresponding to nuclear saturation density. The polytropes are then introduced in the ranges between \( \epsilon_1 \) and \( \epsilon_2 = 280 \text{ MeV/fm}^3 \), \( \epsilon_2 \) to \( \epsilon_3 = 560 \text{ MeV/fm}^3 \) and at energy densities larger than \( \epsilon_3 \). The parameters \( \Gamma_i \) and \( K_i \) are fixed such that the equation of state is continuous at the matching points. Instead of varying \( \Gamma_1 \) we vary the pressure \( P_2 = P(\epsilon_2) \). Following Ref. [34] the parameters \( P_2 \), \( \Gamma_2 \) and \( \Gamma_3 \) are varied in the following ranges:

\[
\log_{10} \frac{P_2 \text{ fm}}{\text{MeV}} = 0.7 + n_1 \cdot 0.1 \leq 1.6 ,
\Gamma_2 = 1.2 + n_2 \cdot 0.65 \leq 3.8 ,
\Gamma_3 = 1.3 + n_3 \cdot 0.8 \leq 3.7 , \tag{5}
\]

with \( n_1, n_2, n_3 \in \mathbb{N} \). The constraints from neutron star masses and radii then translate into a limited band area of \( P(\epsilon) \). Any acceptable EoS must lie within this belt.

Combining the SLy EoS for \( \epsilon < \epsilon_0 \), the FKW EoS for \( \epsilon_0 \leq \epsilon < \epsilon_1 \) and the three polytropic equations of state for \( \epsilon_1 \leq \epsilon < \epsilon_2 \), \( \epsilon_2 \leq \epsilon < \epsilon_3 \), and \( \epsilon \geq \epsilon_3 \), the TOV equation is solved for each set [5]. We accept a parameter set \( (P_2, \Gamma_2, \Gamma_3) \) if the resulting mass-radius curve reaches or passes beyond the two-solar-mass limit dictated by J1614–2230 and J0348+0432, and if it is within the range of radii suggested by Steiner, Lattimer, Brown [11–13] or, alternatively, passes through the constraining triangle as given by Trümper [17]. For the Steiner-Lattimer-Brown constraints we keep all parameter sets that generate mass-radius curves exceeding the two-solar-mass limit in the radius range 11.0–12.5 km and crossing the \( M = 1.4 M_{\odot} \) line in the radius window 10.5–13.0 km [11, 13]. We ensure that causality is not violated, i.e. the speed of sound, \( c_s \), satisfies the condi-
tion
\[ v_a = \sqrt{\frac{dP}{d\epsilon}} \leq 1 . \] (6)

The result of this analysis is presented in Fig. 1. The bands comprise all polytropes that meet the constraints dictated by the neutron star observables and causality. These emerging “allowed” corridors are consistent with the results reported in Ref. [33].

It is of interest to point out that a state-of-the-art EoS computed using advanced quantum Monte Carlo methods [36], as well as the time-honored EoS resulting from a variational many-body calculation [35] (APR), both pass the test of being within the allowed \( P(\epsilon) \) region. Notably, these equations of state work with “conventional” (baryon and meson) degrees of freedom.

### III. EQUATIONS OF STATE

This section deals with the construction of an EoS for baryonic matter at densities relevant to the description of the neutron star core. The framework is chiral effective field theory (ChEFT), the approach based on the spontaneously broken chiral symmetry of low-energy QCD. ChEFT has been applied successfully to the nuclear many-body problem and its thermodynamics, for symmetric nuclear matter, pure neutron matter and varying proton fractions \( Z/A \) between these extremes (see Ref. [7] for a recent review and references therein). At high baryon densities, the possible appearance of hybrid matter with admixtures of deconfined quark degrees of freedom will also be explored using a Nambu and Jona-Lasinio model including strange quarks. It will be demonstrated, however, that a significant quark matter component is not likely to appear even in the very central region of the neutron star core, given the new observational constraints requiring a sufficiently stiff equation of state.

#### A. Chiral effective field theory

In-medium ChEFT incorporates the essentials of low-energy pion-nucleon and pion-pion interactions together with the Pauli principle and a systematically structured hierarchy of nucleon-nucleon forces that include one- and two-pion exchange dynamics plus important three-body correlations. In the present work we use an equation of state for neutron star matter (neutron matter with an admixture of protons) based on three-loop in-medium ChEFT calculations of nuclear and neutron matter [17, 29, 30].

The starting point is the chiral meson-baryon effective Lagrangian in its isospin SU(2) sector, with pions as the “light” (Goldstone boson) degrees of freedom coupled to nucleons as “heavy” sources. This Lagrangian is organized as an expansion in powers of pion momentum (derivatives of the pion field) and pion mass (the measure of explicit chiral symmetry breaking by the small non-zero \( u \)- and \( d \)-quark masses):

\[ L_{\pi N} = L_{\pi N}^{(1)} + L_{\pi N}^{(2)} + \ldots . \] (7)

At leading order we have

\[ L_{\pi N}^{(1)} = \bar{\Psi} \left[ i \gamma_\mu (\partial^\mu + \Gamma^\mu) - M_0 + g_A \gamma_\mu \gamma_5 u^\mu \right] \Psi , \] (8)

with the isospin doublet Dirac field of the nucleon, \( \Psi = (u, d)^T \). The vector and axial vector quantities

\[ \Gamma^\mu = \frac{1}{2} [\xi^\dagger, \partial^\mu \xi] = \frac{i}{4f_{\pi}} \vec{\pi} : (\vec{\pi} \times \partial^\mu \vec{\pi}) + \ldots , \] (9)

\[ u^\mu = \frac{i}{2} [\xi^\dagger, \partial^\mu \xi] = - \frac{1}{2f_{\pi}} \vec{\pi} : \partial^\mu \vec{\pi} + \ldots , \] (10)

involve the isovector pion field \( \vec{\pi} \) via \( \xi = \exp[(i/2f_{\pi}) \vec{p} \cdot \vec{\pi}] \).

The last steps in the preceding equations result when expanding \( \Gamma^\mu \) and \( u^\mu \) to leading order in the pion field. Up to this point the only parameters that enter are the nucleon mass \( M_0 \), the nucleon axial vector coupling constant \( g_A \) and the pion decay constant \( f_{\pi} \), all to be taken at first in the chiral limit. The pion decay constant plays the role of an order parameter for spontaneous chiral symmetry breaking. It sets a characteristic scale, \( 4\pi f_{\pi} \sim 1 \text{ GeV} \). The effective field theory is designed to work at excitation energies and momenta small compared to that scale.

At next-to-leading order, \( L_{\pi N}^{(2)} \), the chiral symmetry breaking quark mass term enters. It has the effect of shifting the nucleon mass from its value in the chiral limit to the physical mass. The nucleon sigma term

\[ \sigma_N = m_q \frac{\partial M_N}{\partial m_q} = \langle N| m_q (\bar{u}u + \bar{d}d)|N \rangle \] (11)

measures the contribution of the non-vanishing quark mass, \( m_q = \frac{1}{2}(m_u + m_d) \), to the nucleon mass \( M_N \). Its empirical value is in the range \( \sigma_N \approx (45 \pm 8) \text{ MeV} \) and has been deduced [27] by extrapolation of low-energy pion-nucleon data using dispersion relation techniques. Up to this point, the \( \pi N \) effective Lagrangian, expanded to second order in the pion field, has the form

\[ L_{\text{eff}}^N = \bar{\Psi} \left( i \gamma_\mu \partial^\mu - M_N \right) \Psi - \frac{g_A}{2f_{\pi}} \bar{\Psi} \gamma_\mu \gamma_5 \vec{\pi} \cdot \partial^\mu \vec{\pi} \\
- \frac{1}{4f_{\pi}^2} \bar{\Psi} \gamma_\mu \gamma_5 \vec{\pi} \cdot (\vec{\pi} \times \partial^\mu \vec{\pi}) \\
+ \frac{\sigma_N}{2f_{\pi}^2} \bar{\Psi} \vec{\pi}^2 + \ldots \] (12)

where we have not shown a series of additional terms involving \( (\partial^\mu \vec{\pi})^2 \) that appear in the complete Lagrangian \( L_{\pi N}^{(2)} \). These terms come with low-energy constants \( c_{3,4} \), encoding physics at smaller distances or higher energies. These constants need to be fitted to experimental data, e.g. from pion-nucleon scattering.

The “effectiveness” of such an effective field theory relies on the proper identification of the active low-energy degrees of freedom. Pion-nucleon scattering is known to be dominated by the \( p \)-wave \( \Delta(1232) \) resonance with spin and isospin \( 3/2 \). The excitation energy of this resonance, given by the mass difference \( \Delta = M_\Delta - M_N \sim 293 \text{ MeV} \) is small, just slightly larger than twice the pion mass. If the physics of the
\( \Delta(1232) \) is absorbed in low-energy constants such as \( c_{3,4} \) of an effective theory that works with pions and nucleons only, the limit of applicability of such a theory is narrowed down to an energy-momentum range small compared to \( \Delta \). The effective Lagrangian is therefore often extended by incorporating the \( \Delta(1232) \) isobar as an explicit degree of freedom, and this is the version of ChEFT that we use here to construct an EoS for neutron star matter.

The pion-nucleon vertices entering Eq. (12) generate a systematically organized hierarchy of pion exchange mechanisms in the nucleon-nucleon interaction: one-pion exchange at leading order (LO), two-pion exchange processes at next-to-leading order (NLO) and so forth. These explicitly calculated long- and intermediate-range parts are supplemented by NN contact terms that encode short distance dynamics not resolved in detail at small momenta far below the chiral symmetry breaking scale \( 4\pi f_{\pi} \) of order 1 GeV. The constants associated with these contact terms are parameters to be fixed and fine-tuned by comparison with empirical data.

In the standard version of ChEFT, terms involving important p-wave pion-nucleon scattering information through the low-energy constants \( c_{3,4} \) appear at next-to-next-to-leading order (N3LO). Three-body NNN forces also emerge for the first time at N2LO. As mentioned, the version we use in this work is the one with \( \Delta(1232) \) degrees of freedom treated explicitly. In this case, two-pion exchange processes involving intermediate \( \Delta \) excitations are promoted from N3LO to NLO, rescaling the constants \( c_{3,4} \) and improving the convergence of the approach. The importance of the \( N \rightarrow \Delta \) transition in generating the very large spin-isospin polarizability of the nucleon is underlined in this way. This also emphasizes the significance of virtual \( \Delta \) excitations in providing a prominent part of the central attraction in the \( 2\pi \) exchange NN force at intermediate distances, as well as an important piece of the three-body interaction.

This scheme has been applied successfully to the description of symmetric and asymmetric nuclear matter as well as pure neutron matter. In particular, nuclear thermodynamics, the liquid-gas phase transition, its evolution as a function of the proton fraction \( Z/A \) and its disappearence in neutron matter, are well reproduced. The isospin dependence of explicit two-pion exchange processes in the nuclear medium plays an important role in this context. In-medium ChEFT provides a systematic way to handle such mechanisms, including the action of the Pauli principle in the presence of filled Fermi seas of neutrons and protons with varying proportions. The Pauli principle is implemented through the in-medium nucleon propagator,

\[
G(E, \vec{p}) = \frac{i}{E - \frac{\vec{p}^2}{2M_N} + i\epsilon} - 2\pi\delta \left( E - \frac{\vec{p}^2}{2M_N} \right) \Theta(p),
\]

where

\[
\Theta(p) = \frac{1 + \tau_3}{2} \theta(k^p_F - |\vec{p}|) + \frac{1 - \tau_3}{2} \theta(k^n_F - |\vec{p}|),
\]

and \( k^p_F \) and \( k^n_F \) are the proton and neutron Fermi momenta, respectively. Intermediate and long-range pion exchange dynamics (Fock terms from one-pion exchange and all explicit two-pion exchange processes in the presence of the medium) are computed up to three-loop order in the energy density. Contact terms (subject to resummations as described in Ref. [41]) are adjusted to properties of symmetric nuclear matter (the empirical binding energy per nucleon and the equilibrium density) and to the symmetry energy at \( k^0_F = 1.36 \text{ fm}^{-1} \).

The “small” parameters, in addition to pion mass and momentum, now include the Fermi momenta, \( k^p_F/4\pi f_{\pi} \ll 1 \). The energy density is derived as an expansion in powers of Fermi momenta and generally written as

\[
\epsilon(k^p_F, k^n_F) = \epsilon_0(k_F) + \delta^2 A_2(k_F) + \ldots,
\]

introducing the asymmetry parameter \( \delta = (\varrho_n - \varrho_p)/\varrho \) with the neutron and proton densities,

\[
\varrho_{n,p} = \frac{(k^{n,p}_F)^3}{3\pi^2},
\]

and the total baryon density, \( \varrho = \varrho_p + \varrho_n \). For symmetric nuclear matter, \( \varrho = 2k^p_F/(3\pi^2) \). Symmetric nuclear matter and pure neutron matter correspond to the limiting cases \( \delta = 0 \) and \( \delta = 1 \), respectively. A good approximation for \( \delta \lesssim 1 \) relevant for neutron star matter, with a small admixture of protons controlled by beta equilibrium, is given by extrapolating around the neutron matter limit, \( \delta = 1 \), using the \( \delta^2 \) term.

![FIG. 2: Energy per particle, \( E/N = \epsilon/\rho - M_N \), for pure neutron matter as a function of density. Solid curve: ChEFT result used in the present work. Long-dashed curve: variational many-body calculation of Akmal, Pandharipande and Ravenhall (APR) Short-dashed curve: fit to results of Quantum Monte Carlo computation by Armani et al. (QMC). The baryon density \( \varrho \) is given in units of normal nuclear matter density, \( \varrho_0 = 0.16 \text{ fm}^{-3} \).](image)
at densities as high as 3.5 times the density of normal nuclear matter. At such densities the neutron Fermi momentum, \(k_F^n \sim 2.5 \text{ fm}^{-1}\), continues to be appreciably smaller than the chiral symmetry breaking scale of order \(4\pi f_\pi \sim 1 \text{ GeV}\), rendering the ChEFT expansion in powers of \(k_F^p/4\pi f_\pi\) still meaningful.

B. Quark matter: PNJL model with vector interaction

At very high baryon densities the principal possibility exists that nucleons dissolve into a sea of quarks. In this subsection, quark matter is described using the Polyakov-loop-extended Nambu and Jona-Lasinio (PNJL) model with \(N_f = 2 + 1\) quark flavors, taking into account two degenerate light (up and down) quarks with masses \(m_u = m_d\) and a heavier strange quark with mass \(m_s\). The PNJL approach has been developed and discussed extensively in the literature [42–48].

Neutron stars are “cold” systems, with temperatures \(T\) typically below a few MeV. Given the \(u\) and \(d\) current-quark masses of the same order, it is useful to prepare the EoS of quark matter at finite \(T\) and then take the limit \(T \to 0\) (done here also in view of neutron star cooling issues that are, however, not part of the present work).

The starting point is the (Euclidean) action of the (local) PNJL model:

\[
\mathcal{S}_{\text{PNJL}} = \int_0^\beta d\tau \int d^3 x \bar{q}(x)(-i\gamma^\nu D^\nu + \gamma^0\mu + \bar{\mu})q(x) + \int_0^\beta d\tau \int d^3 x \mathcal{L}_{\text{int}} + \beta V \mathcal{U}(\Phi[A], \bar{\Phi}[A]; T),
\]

where \(q(x) = (u(x), d(x), s(x))^T\) is the three-flavor quark field and \(\bar{\mu} = \text{diag}(\mu_u, \mu_d, \mu_s)\) denotes the (current) quark mass matrix. We work in the isospin limit with \(m_u = m_d\). Quark chemical potentials are incorporated in the matrix \(\mu = \text{diag}(\mu_u, \mu_d, \mu_s)\).

The interaction part of the Lagrangian, \(\mathcal{L}_{\text{int}}\), is given as:

\[
\mathcal{L}_{\text{int}} = \frac{1}{2} G \sum_{a=0}^8 \left[(\bar{q}\lambda^a q)^2 + (\bar{q}i\gamma_5\lambda^a q)^2\right] + \mathcal{L}_v
\]

\[
- K \left[\det(\bar{q}(1 + \gamma_5)q) + \det(\bar{q}(1 - \gamma_5)q)\right].
\]

The first term in the first line describes the chirally invariant combination of scalar and pseudoscalar interactions between quarks, with coupling strength \(G\) of dimension (length)\(^2\). The flavor SU(3) Gell-Mann matrices \(\lambda_i\) \((i = 1, \ldots, 8)\) are supplemented by \(\lambda^0 = \lambda_0 = \sqrt{2/3}\) times the 3 \(\times\) 3 unit matrix. The second term in the first line describes additional vector and axial-vector interactions. Their general form, invariant under chiral SU(3)\(_L\) \(\times\) SU(3)\(_R\) symmetry, is [51,52]:

\[
\mathcal{L}_v = -\frac{1}{2} g \sum_{a=1}^8 (\bar{q}v^\mu \lambda^a q)^2 - \frac{1}{2} g_{v,0} (\bar{q}v^\mu \lambda_0 q)^2
\]

\[- \frac{1}{2} g \sum_{a=1}^8 (\bar{q}v^\mu \gamma_5 \lambda^a q)^2 - \frac{1}{2} g_{a,0} (\bar{q}v^\mu \gamma_5 \lambda_0 q)^2.
\]

Using vector dominance and the small difference between the masses of \(\rho\) and \(\omega\) mesons, one can choose [52,53] \(g_{v,0} = g_{a,0} \equiv g\). In the following we work with a simplified ansatz keeping only the single term,

\[
\mathcal{L}_v \to -\frac{1}{2} G_v (\bar{q}v^\mu q)^2,
\]

with vector-coupling strength \(G_v = \frac{2}{3} g\). If a color current-current interaction is chosen to start with, a Fierz transformation would relate the vector and scalar couplings as \(G_v = \frac{5}{2} G\).

The term in the second line of Eq. (18) is the Kobayashi-Maskawa-'tHooft determinant [54,55] that describes the (anomalous) breaking of the axial U(1)\(_A\) symmetry and gives rise to the large mass of the \(\eta'\) meson.

The PNJL model is non-renormalizable. It operates with a characteristic three-momentum cutoff scale \(\Lambda\), such that the effective interaction between quarks is “turned off” for momenta \(|\vec{p}| > \Lambda\). No additional divergences appear at finite temperature and density. We adopt the cutoff prescription given in Ref. [56] and use the following parameters [53]: \(m_u = m_d = 3.6\text{ MeV}\), \(m_s = 87.0\text{ MeV}\), \(\Lambda = 750\text{ MeV}\), \(G = 3.64/\Lambda^2\), \(K = 8.9/\Lambda^4\). With this parameter setting the empirical meson spectrum and the measured pseudoscalar decay constants in vacuum are well reproduced. The value of the vector coupling strength, \(G_v\), is varied in order to investigate the impact of the repulsive vector interaction on the equation of state. A study comparing various parameter sets within a similar framework is presented in [57].

In Eq. (17) the color gauge covariant derivative \(D^\nu = \partial^\nu + iA^\nu\) involves the SU(3)_{\text{flavor}} matrices \(\lambda_a, a \in \{1, \ldots, 8\}\). The gauge coupling is absorbed in the definition of \(A^0\). The temporal gauge field \(A^0\) is treated as a constant Euclidean background field in the form \(A_\tau = iA^0 = A_3^0 = A_2^0 + A_8^0\). The last term in Eq. (17) is the Polyakov-loop effective potential \(U\), multiplied by the volume \(V\) and the inverse temperature \(\beta = T^{-1}\), and constructed as follows:

\[
\frac{U(\Phi, \bar{\Phi}; T)}{T^4} = -\frac{1}{2} b_2(T) \Phi \bar{\Phi}
\]

\[+ b_4(T) \ln \left[1 - 6\Phi \bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi \bar{\Phi})^2\right]
\]

with \(\Phi\) and \(\bar{\Phi}\) as follows:

\[
\Phi = \frac{1}{3} \left[e^{\frac{\lambda_3}{2} + \frac{\lambda_8}{2}} + e^{-\frac{\lambda_3 - \lambda_8}{2}} + e^{\frac{\lambda_3 - \lambda_8}{2}}\right]
\]

The coefficients \(b_2(T)\) and \(b_4(T)\) are parametrized to reproduce pure-gauge lattice QCD results (cf. Refs. [45,47,49]). The temperature \(T_0\) appearing in \(b_2(T)\) and \(b_4(T)\) is set to the transition temperature for the confinement-deconfinement crossover in the presence of two light and one heavy quark, as discussed in Ref. [50].

Given this input, the grand-canonical potential \(\Omega = -\ln Z\) is calculated in mean-field approximation with the partition function.
function $Z$ constructed from the action $S_{\text{PNJL}}$ of Eq. (17). Details are relegated to the Appendix. The result is the thermodynamic potential $\Omega_{\text{MF}}$ given in Eq. (46). It involves the expectation values of the scalar fields, $\bar{\sigma}_i = -G \langle \bar{q}_i q_i \rangle$ ($i \in \{u, d, s\}$) representing the chiral condensates for each quark species, and of the vector field, $\bar{v} = G_v \langle q^I q \rangle$, related to the baryon number density of the quarks.

Minimization of $\Omega_{\text{MF}}$ determines the fields $\bar{\sigma}_i$, $\bar{v}$, $A^3_i$, and $A^8_i$ from the set of equations

$$\frac{\partial \Omega_{\text{MF}}}{\partial \bar{\sigma}_i} = \frac{\partial \Omega_{\text{MF}}}{\partial \bar{v}} = \frac{\partial \Omega_{\text{MF}}}{\partial A^3_i} = \frac{\partial \Omega_{\text{MF}}}{\partial A^8_i} = 0. \quad (22)$$

In particular, dynamical quark masses emerge from the gap equations (47). In mean-field approximation it follows that $\Phi = \bar{\Phi}$ and consequently $A^8_i = 0$ as shown in Refs. [45,46]. In the limit $T \to 0$ one actually has $\Phi = \bar{\Phi} = 0$.

With the aim of describing charge-neutral matter in chemical equilibrium inside neutron stars, the equations (22) have to be supplemented by the following conditions for the densities and chemical potentials of the quarks and leptons involved:

$$\begin{align*}
\frac{2}{3} \varrho_u - \frac{1}{3} \varrho_d - \frac{1}{3} \varrho_s - \varrho_e - \varrho_\mu &= 0, \\
\mu_d &= \mu_u + \mu_e, \quad \mu_d = \mu_s, \quad \mu_e = \mu_\mu.
\end{align*} \quad (23,24)$$

Eq. (23) expresses charge neutrality when both electrons and muons participate in establishing chemical (beta) equilibrium. The particle densities are calculated from

$$\varrho_i = - \left( \frac{\partial \Omega}{\partial \mu_i} \right)_{T, V, \{\mu_j\}_i \neq i}. \quad (25)$$

For the particle densities of the leptons $e, \mu$ we simply use those derived from the thermodynamic potential, $\Omega_{\text{lepton}}$, of a free gas of electrons and muons. Beta equilibrium in terms of the processes

$$
d \leftrightarrow u + e^- + \bar{\nu}_e, \quad s \leftrightarrow u + e^- + \bar{\nu}_e, \\
d \leftrightarrow u + \mu^- + \bar{\nu}_\mu, \quad s \leftrightarrow u + \mu^- + \bar{\nu}_\mu,
$$

is expressed by Eqs. (24) (neglecting chemical potentials for neutrinos).

Consider now the EoS for beta-equilibrated quark matter. The gap equations (22) are solved simultaneously under the constraints of charge neutrality (23) and beta equilibrium (24). Only one of the chemical potentials remains as a free parameter. With the mean-field thermodynamic potential $\Omega_{\text{MF}}$, the pressure of the system is

$$P = -\Omega_{\text{MF}} - \Omega_{\text{lepton}}. \quad (26)$$

The energy density is calculated using the Gibbs-Duhem relation,

$$\epsilon = TS - P + \sum_i \mu_i \varrho_i, \quad (27)$$

where the particle densities, $\varrho_i$, are given in Eq. (25), and the entropy density $s$ is determined as

$$s = - \left( \frac{\partial \Omega}{\partial T} \right)_{V, \{\mu_j\}}. \quad (28)$$

Resulting equations of state at $T = 0$ are shown in Fig. 3 for different values of the vector coupling strength $G_v$. It actually turns out that low temperatures $T \lesssim 10$ MeV do not affect the EoS for $q = \frac{1}{3}(\varrho_u + \varrho_d + \varrho_s) \gtrsim 0$. In what follows we use $T = 0$ throughout. Fig. 3 displays a qualitative change in the properties of the EoS, depending sensitively on the vector coupling strength. For $G_v = 0$ the low-temperature EoS features a first-order chiral phase transition leading to an EoS that is far too soft and fails to satisfy the neutron star constraints. This first-order transition disappears and turns into a continuous crossover once the repulsive vector interaction strength exceeds a critical value, $G_v^{\text{crit}} \approx 0.9 G$. The constraints from neutron star observables would require a further strengthening of the vector repulsion between quarks, up to $G_v \approx 1.5 G$ as demonstrated in Fig. 3.

![Fig. 3: Equations of state at $T = 0$ derived from the 3-flavor PNJL model with inclusion of charge neutrality and beta equilibrium conditions. The blue dashed and solid lines show results for different vector coupling strengths $G_v$, as indicated in the figure. The black solid line displays the EoS derived from in-medium chiral effective field theory as described in the previous section III A and discussed further in the next section. The grey bands show the constraints from neutron star observables (see Fig. 1).](image-url)

It is instructive to study the particle ratios,

$$\frac{\varrho_i}{\varrho_u + \varrho_d + \varrho_s} \quad (i = u, d, s, e),$$

as they emerge from this (P)NJL model, as a function of the baryon density

$$\varrho = \frac{1}{3}(\varrho_u + \varrho_d + \varrho_s).$$

These particle ratios turn out to be universal: they do not depend on the strength of the vector interaction. This is because the vector field, $\bar{v}$, appears only in the combination $\mu_i - \bar{v}$ with the chemical potentials $\mu_i$. The result is shown in Fig. 4.

The muon fraction is always zero because the muon chemical potential never exceeds the muon mass. At low baryon densities, $\varrho \lesssim 3 \varrho_0$ with $\varrho_0 = 0.16$ fm$^{-3}$, the relative proportion of
with the energy per nucleon, \( \tilde{E} = E/A \), given as a function of the density \( \varrho = \varrho_n + \varrho_p \) and the proton fraction, \( x_p = \varrho_p/\varrho \). The expansion of \( \tilde{E} \) provided by in-medium chiral effective field theory is actually in powers of the Fermi momentum, i.e. in fractional powers of the density \( \varrho \). The nucleon mass is taken as the average of neutron and proton masses, \( M_N = \frac{1}{2} (M_n + M_p) \). As mentioned previously it is useful to write the energy per nucleon as an expression to second order in the asymmetry parameter, \( \delta = (\varrho_n - \varrho_p)/\varrho \), given the small proton fraction \( x_p \) encountered in the neutron star interior. With the calculated energies per nucleon for symmetric nuclear matter, \( \tilde{E}_{SM} \), and pure neutron matter, \( \tilde{E}_{NM} \), and the symmetry energy, \( S(\varrho) = \tilde{E}_{NM}(\varrho) - \tilde{E}_{SM}(\varrho) \):

\[
\tilde{E} = \tilde{E}_{SM}(\varrho) + S(\varrho)(1 - 2x_p)^2 \\
= (1 - 2x_p)^2 \tilde{E}_{NM}(\varrho) + 4x_p(1 - x_p) \tilde{E}_{SM}(\varrho).
\]

(30)

The ChEFT calculation of the symmetry energy at nuclear saturation density, \( \varrho_0 = 0.16 \text{ fm}^{-3} \), gives

\[
S_{\text{ChEFT}}(\varrho_0) = 33.5 \text{ MeV},
\]

(31)

compatible with empirically deduced values that range between 26 and 44 MeV [65]. It is common to expand the symmetry energy around nuclear saturation density,

\[
S(\varrho) = S(\varrho_0) + \frac{L}{3} \left( \frac{\varrho - \varrho_0}{\varrho_0} \right) + \ldots
\]

(32)

The \( L \) value,

\[
L = 3\varrho_0 \frac{\partial S}{\partial \varrho} \bigg|_{\varrho = \varrho_0},
\]

(33)

is poorly known and supposed to be in the range 50 MeV \( \lesssim L \lesssim 140 \) MeV (see [65] and references therein). Our calculation gives

\[
L_{\text{ChEFT}} = 48 \text{ MeV},
\]

(34)

at the lower side of the empirical bandwidth. The significance of the \( L \) value is that it scales linearly with the neutron-skin thickness (i.e., the difference between the root-mean-square radii of neutron and proton distributions) of heavy nuclei [67]. Implications of the symmetry energy for neutron stars are discussed in Ref. [68].

Beta equilibrium involving electrons and muons, \( n \leftrightarrow p + e^- + \bar{\nu}_e \) and \( n \leftrightarrow p + \mu^- + \bar{\nu}_\mu \), together with charge neutrality imply:

\[
\varrho_n = \varrho_e + \varrho_\mu, \quad \mu_n = \mu_p + \mu_e, \quad \mu_e = \mu_\mu.
\]

(35)

(36)

where the neutron and proton chemical potentials are given by

\[
\mu_{n,p} = \left( \frac{\partial \epsilon}{\partial \varrho_{n,p}} \right)_V.
\]

(37)

The lepton charge densities, \( \varrho_e, \varrho_\mu \), and the corresponding chemical potentials, \( \mu_e, \mu_\mu \), are again assumed to be those of a free Fermi gas of electrons and muons.
Incorporating the conditions (35) and (36) the equation of state \( P(\epsilon) \), applicable for neutron star matter in beta equilibrium at zero temperature, is derived using

\[
P = -\epsilon + \sum_i \mu_i \rho_i \quad (i = n, p; e, \mu).
\]

(38)

At very low densities this EoS based on ChEFT is matched again to the “SLy” EoS as in Fig. 1. The complete result is shown by the solid black curve in Fig. 3. Evidently the ChEFT equation of state satisfies the astrophysical constraints over the whole range of relevant energy densities. The exact microscopic treatment of the Pauli principle acting on the in-medium pion-exchange processes and the repulsive three-nucleon correlations provide the required stiffness of the EoS in the dense medium to support two-solar-mass neutron stars.

The proton fraction \( x_p \) in neutron star matter follows from the ChEFT equation of state is shown in Fig. 5. The smallness of the proton admixture (which stays systematically below a maximum of less than 7\% reached at about twice the density of normal nuclear matter) justifies the ansatz quadratic in \( x_p \) as written in Eq. (30).

Given the pressure as a function of energy density the TOV equations (1) and (2) are solved. The resulting ChEFT mass-radius relation for neutron stars is shown in Fig. 6. It turns out that there is only a marginal difference between the results for pure neutron matter and matter in beta equilibrium with its small proton admixture. In either case the equation of state is sufficiently stiff to pass beyond the two-solar-mass threshold.

Next, consider the calculated density profile of a neutron star with a mass \( M = 2 M_\odot \) displayed in Fig. 7. As a general feature of a stiff EoS, the density \( \rho_c \) reached in the center of the star is by far lower than the values characteristically associated with many previous neutron star models which worked with softer equations of state. In the present example, the central density does not exceed \( \rho_c \approx 4.8 \rho_0 \).

One might of course raise concerns about how far ChEFT calculations can be extrapolated into the high density regime. Such calculations are expected to work reliably up to densities \( \rho \lesssim 2 \rho_0 \) at which the neutron Fermi momenta, \( k_F^n \lesssim 0.4 \) GeV, are small compared to the chiral symmetry breaking scale, \( 4\pi f_\pi \sim 1 \) GeV. Issues of convergence in the chiral in-medium expansion, beyond NLO and three-loop order in the energy density \( \epsilon \), remain open at this point (including e.g. questions about the role of genuine four-nucleon correlations). On the other hand, a look back at Fig. 2 nonetheless demonstrates that the ChEFT EoS reproduces quantitatively the available realistic many-body computations of neutron matter even up to about four times \( \rho_0 \). The Fermi momentum, as a small parameter of the chiral in-medium expansion, increases only with the third root of the density. At \( \rho_\pi \sim 4 \rho_0 \), the corresponding Fermi momentum \( k_F^\pi \sim 0.5 \) GeV still re-
mains sufficiently small to justify an expansion in powers of $k_F^2/4\pi f$, though with increasing uncertainty.

Models based on the chiral symmetry of QCD describe the nucleon [70] as a compact valence quark core with a radius of about 1/2 fm, surrounded by a pionic cloud. The meson cloud determines most of the empirical proton rms charge radius of 0.88 fm. For the neutron the picture of core and cloud is analogous except that the electric charges of quark core and meson cloud now add up to form the overall neutral object. Even at $\varrho \sim 5\varrho_0$ the average distance between two neutrons is still about 1.1 fm, hence the baryonic cores do not yet overlap at that density. The pionic field surrounding the baryonic sources is of course expected to be highly inhomogeneous and polarized in compressed matter, but this effect is properly dealt with in chiral EFT. It is therefore perhaps not so surprising that an EoS based entirely on nucleons (plus $\Delta$ isobars) and pionic degrees of freedom works well for neutron stars, if only the mechanisms for generating stiffness and high pressure are properly incorporated. A similar reasoning is found e.g. in Ref. [59].

B. Hybrid stars

This subsection deals with the possibility that the inner core of the neutron star is composed of quark matter. It is obviously not realistic to think of a quark matter EoS for the entire core region. But a combination of a suitable quark matter equation of state for the inner core with the ChEFT EoS from the previous section, describing the outer core, is still an option. In the following we discuss two scenarios: first an ansatz featuring quark-hadron continuity, and secondly a first-order phase transition involving a coexistence region of hadronic and quark matter.

1. Quark-hadron continuity

The quark-hadron continuity picture has been discussed previously in Refs. [43] [57] [60] [62]. It is based on the assumption that the outer and inner core regions of the hybrid neutron star are characterized by a smooth, continuous transition between the nucleonic and quark matter regions. We follow an ansatz introduced in Ref. [57] and combine the ChEFT EoS representative of hadronic (nucleonic plus pionic) matter, $P_H(\varrho)$, with the quark matter EoS derived from the PNJL model, $P_Q(\varrho)$:

$$P(\varrho) = P_H(\varrho) f_H(\varrho) + P_Q(\varrho) f_Q(\varrho) ,$$

with interpolating functions:

$$f_H(\varrho) = \frac{1}{2} \left[ 1 - \tanh \left( \frac{\varrho - \varrho_0}{\Gamma} \right) \right] ,$$

$$f_Q(\varrho) = \frac{1}{2} \left[ 1 + \tanh \left( \frac{\varrho - \varrho_0}{\Gamma} \right) \right] .$$

The parameters $\varrho_0$ and $\Gamma$ determine the location and the width of the transition region between the nucleonic and quark matter sectors. The functions $P_H(\varrho)$ and $P_Q(\varrho)$ are matched continuously. The density, $\varrho = \varrho(\varrho)$, can be determined from the EoS [59] by integrating

$$\frac{d\varrho}{\varrho} = \frac{d\varrho}{P(\varrho) + \varrho} .$$

In Fig. 8 we show the EoS derived from Eq. (39) for different values of the NJL vector coupling strength $G_v$. We have chosen $\Gamma = 300$ MeV/fm$^3$ and $\varrho = 800$ MeV/fm$^3$, representing a transition region $3.0 \varrho_0 \lesssim \varrho \lesssim 5.5 \varrho_0$. As in Sec. IV A of Ref. [59] the “SLy” EoS has been matched smoothly to the ChEFT EoS at $\varrho = 100$ MeV/fm$^3$. It is evident from the figure that a hadron-quark hybrid scenario meets the constraints from neutron star observables only if the repulsive vector coupling between quarks is sufficiently large, $G_v > G$. This is confirmed by the mass-radius plot shown in Fig. 9. At this point our results are roughly consistent with those of Ref. [57] despite their use of a different hadronic EoS.

![FIG. 8: Equations of state representing the quark-hadron continuity scenario using different quark vector couplings. Quark matter (PNJL) and nuclear matter (ChEFT) equations of state are matched continuously at $\varrho = \varrho_0 = 800$ MeV/fm$^3$. Solid curve: $G_v = 1.5G$; dashed curve: $G_v = 0$. The grey areas are those of Fig. 1 representing constraints from neutron star observables.]

2. Hadron-quark first-order phase transition

In the previous section the transition region from the hadronic to the quark phase was chosen by means of the parameters $(\Gamma, \varrho)$ of the interpolating functions (40). In this section a different approach is taken assuming a first-order phase transition from hadronic to quark matter with an extended coexistence region of the two phases. The system is characterized by two conserved quantities: electric charge and baryon number. For such systems with more than one conserved charge, the Maxwell construction is generalized and replaced by the Gibbs condition [63]. In the present case, this condition describing mechanical and chemical equilibrium is:

$$P_H(\mu_n, \mu_e) = P_Q(\mu_n, \mu_e) ,$$
The choice of the chemical potentials \( \mu \) is expressed as the pressure balance between hadronic and quark components in terms of the neutron and electron chemical potentials. For the nucleonic phase, the proton chemical potential is \( \mu_p = \mu_n - \mu_e \). The muon chemical potential is \( \mu_\mu = \mu_e \).

For the quark matter phase, the quark chemical potentials are expressed in terms of \( \mu_n \) and \( \mu_e \) according to

\[
\begin{align*}
\mu_u &= \frac{1}{3}(2\mu_p - \mu_n) = \frac{1}{3}(\mu_n - 2\mu_e), \\
\mu_d &= \mu_s = \frac{1}{3}(2\mu_n - \mu_p) = \frac{1}{3}(\mu_n + \mu_e).
\end{align*}
\]

The choice of the chemical potentials \( \mu_n, \mu_e \) is arbitrary. Note that \( P_Q \) also depends on the mean fields \( \bar{\sigma}, \bar{v} \) which are in turn dependent on \( \mu_n \) and \( \mu_e \). The total baryon density in the coexistence region is \([63]\):

\[
\rho = \chi \rho_Q + (1 - \chi) \rho_H,
\]

where \( \chi \) (with \( 0 \leq \chi \leq 1 \)) denotes the proportion of quark matter in the hadron-quark mixed system. The combinations \( \chi \rho_Q \) and \( (1 - \chi) \rho_H \) are the densities of deconfined quarks and confined baryons, respectively, in the coexistence region. Global charge neutrality implies:

\[
\chi \sum_{i=u,d,s} q_i \rho_i + (1 - \chi) (\rho_p - \rho_e - \rho_\mu) = 0,
\]

where the \( q_i \) denote the quark charges.

The Gibbs condition \([42]\) together with Eq. \([45]\) allows to eliminate two of the three quantities \( \mu_n, \mu_e, \chi \). The pressure is a function of the remaining (free) parameter. The resulting equation of state is shown in Fig. \(10\). The corresponding particle densities (for the case \( G_v = 0 \)) are displayed in Fig. \(11\). The coexistence region in the case without vector interaction, \( G_v = 0 \), extends over the baryon density interval \( 4 \rho_0 \lesssim \rho \lesssim 9 \rho_0 \). For \( G_v = 0.5 G \) the coexistence region is \( 6 \rho_0 \lesssim \rho \lesssim 10 \rho_0 \). Hence the phase transition takes place over a broad density range and moves toward higher densities as the vector repulsion is increased. An interesting feature observed in Fig. \(10\) is the increase of the proton fraction to about 10% in the coexistence region. This is primarily to compensate the increasing supply of negative charges from the emergent \( d \) and \( s \) quarks.

![FIG. 9: Solutions of the TOV equations (1) and (2) (mass-radius relation) for neutron stars using the EoS given in (39). The lines correspond to different vector coupling strengths, as indicated in the figure. The shaded areas are as in Fig. 6](image)

![FIG. 10: Equations of state including a first-order phase transition between hadronic and quark matter. The transition region itself is characterized by the flat parts of the curves. The (upper) solid curve includes a vector repulsion of \( G_v = 0.5 G \) between quarks, while the (lower) dashed curve is found using \( G_v = 0 \). The grey areas are as in Fig. 9](image)

![FIG. 11: Particle ratios as a function of the (normalized) baryon density for the particles as indicated in the figure. The first-order coexistence region is marked by the rapid decrease of neutrons and the steep rise of quarks. The case without vector interaction (\( G_v = 0 \)) is shown.](image)
the $M(R)$ curve downward causing instability of the neutron star. Stability is recovered only if the repulsive vector interaction between quarks is introduced (with $G_v = 0.5 \, G$ in our example). However, in this case the first-order phase transition moves to densities $\rho \gtrsim 6 \, \rho_0$, exceeding the maximal central density that can be realized in the inner core of the star.

In order to elaborate further on this point, it is instructive to have a look at the density profile of a neutron star with $M = 1.95 \, M_{\odot}$, calculated using $G_v = 0$ in the hybrid sector. In this case which just barely satisfies the empirical constraints, a possible quark-hadron coexistence domain is restricted to a small part of the inner core within a radius of about 2 km. The central density, $\rho_c \approx 5 \, \rho_0$, is only slightly larger than $\rho_c \approx 4.8 \, \rho_0$ of the two-solar-mass neutron star reached with the “conventional” EoS based on chiral EFT.

We have emphasized repeatedly that the required stiffness of the equation of state keeps the central density of a two-solar-mass neutron star within limits not exceeding typically five times $\rho_0$. The actual bulk baryon densities relevant for the determination of the star mass are significantly lower. Recalling Eq. (6) and approximating the energy density roughly as $\epsilon \sim M_N \, \rho$, one notes that $r^2 \, \rho(r)$ rather than the density profile itself matters in the integration of the mass up to the star radius $R$. For illustration we plot the dimensionless, scaled quantity $(r/R)^2 \rho(r)/\rho_0$ in Fig. 14 and observe that the characteristic bulk densities stay around 2-3 $\rho_0$ and hence in a density range where nuclear chiral EFT can be applied. In this plot the difference between a “conventional” ChEFT scenario and an EoS including hadron-quark coexistence is almost invisible.

Qualitatively similar results concerning the possibility of hadron-quark coexistence are found in Ref. [64] in a model combining a relativistic mean field (RMF) equation of state for the hadronic sector with a non-local PNJL model for quark matter. While the non-local effective interaction between quarks does not make much of a difference compared to the local couplings used in the present work, it should be noted that RMF-based equations of state usually fail to satisfy at least one of the EoS criteria, namely the requirement of consistency with the most advanced many-body calculations of neutron matter [71][72].
C. Comments on hyperon admixtures to the EoS

Admixtures of Λ and Σ hyperons to the EoS of dense baryonic matter in neutron stars have been under discussion for a long time. While Σ hyperons are not likely to appear since the absence of Σ hypernuclei suggests a weakly repulsive ΣN interaction, the low-energy Λ-nuclear interaction is attractive. From hypernuclear phenomenology it is known that the Λ-nuclear mean field is about half as strong as the Hartree-Fock potential experienced by a nucleon in the nuclear medium.

In neutron star matter, Λ hyperons can take over the role of the neutrons when this becomes energetically favourable at baryon densities exceeding 2-3 times $\rho_0$. Recent examples of calculations including hyperons in the EoS can be found in Refs. [73, 74]. From these and similar calculations it is now widely accepted that the softening of the equation of state produced by Λ admixtures, in the absence of additional repulsive interactions, reduces the maximum mass of a neutron star to values way below two solar masses. Such additional repulsive forces acting on the hyperons in matter are required in order to maintain a sufficiently steep slope of the pressure $P(\epsilon)$ at high densities.

Our present work features an equation of state for the hadronic sector based on in-medium chiral SU(2) effective field theory. A fully consistent chiral SU(3) approach to baryonic matter including hyperons and the complete pseudoscalar meson octet is not yet available. However, we can present a rough estimate of the admixture of Λ hyperon admixtures to the previously derived EoS that combines chiral EFT in the hadronic sector with the three-flavor NJL model for quark matter (see Figs. 10, 11), by simply adding a Λ contribution to the energy density, using an attractive mean-field (Hartree) potential adjusted to reproduce hypernuclear data.

![FIG. 15: Particle ratios as a function of baryon density $\rho$ (in units of $\rho_0 = 0.16$ fm$^{-3}$) for the particles indicated, as in Fig. 11 but with inclusion of Λ hyperons.](image)

The result, Fig. 15, can be considered as typical and representative for a large class of similar model calculations. The onset of hadron-quark coexistence at $\rho \simeq 3.5 \rho_0$ takes place for a system in which a substantial fraction of neutrons is now substituted by Λ hyperons (implemented here according to the RMF treatment of Ref. [75]). However, the corresponding EoS has now become far too soft. It does not satisfy the pertinent constraints and fails to support a two-solar-mass neutron star. The only possibility to preserve stability of the star within the “allowed” regions of Fig. 1 is through extra repulsive interactions of the hyperon with the surrounding baryonic medium. To clarify the origin of such repulsive hyperon-nuclear interactions at high baryon density is a key question for the near future. Short-distance three-body forces with a hyperon involved could be part of this discussion [76], as well as repulsive momentum dependent components in the $\Lambda N$ interaction as they emerge in a chiral SU(3) approach at next-to-leading order [77]. Such interactions would play only a minor role in Λ hypernuclei but would be increasingly important at the higher baryon densities encountered in the center of a neutron star.

V. SUMMARY AND CONCLUSIONS

The present work updates the equation of state for dense baryonic matter in view of the by now well established existence of two-solar-mass neutron stars. This study consists of two parts with the following aims: first, to set the constraints for the pressure as a function of energy density from the new mass determinations together with (less accurate) limits on neutron star radii; secondly, to construct equations of state that are compatible with these observational constraints, while at the same time satisfying the conditions provided by nuclear physics and known properties of nuclear and neutron matter.

1. Concerning the first part, the observational constraints determine a band of acceptable neutron star equations of state that are characterized by their pronounced stiffness: at baryon densities $\rho \simeq 0.8$ fm$^{-3}$, about five times the density of normal nuclear matter in equilibrium, the pressure must be at least $P \gtrsim 150$ MeV fm$^{-3}$ in order to support $2M_\odot$ neutron stars. This conclusion does not depend on the detailed composition of the matter forming the core of the star. Our results at this point are compatible with related studies reported in Refs. [11, 13, 32, 33].

By analysing the systematics of mass-radius trajectories within the acceptable limits, the stiffness condition on the equation of state has an important implication: the maximum density in the center of the star cannot exceed much the margin of five times nuclear matter density, corresponding to neutron Fermi momenta less than 0.6 GeV and average kinetic energies of less than 100 MeV.

2. The modeling of the equation of state in the second part of this work has been governed by the following criteria. The theory used to construct this equation of state should accurately reproduce:

a) nuclear phenomenology and the thermodynamics of symmetric nuclear matter;

b) advanced many-body calculations, such as recent Monte Carlo computations, of pure neutron matter;
the symmetry breaking pattern of low-energy QCD and its implications for the nuclear many-body problem.

In-medium chiral effective field theory is a systematic framework that satisfies these three criteria. The energy density and pressure resulting from this approach at three-loop order does generate the required stiffness of the neutron star equation of state, based on the explicit treatment of two-pion exchange processes, three-body forces, and their in-medium behaviour with proper inclusion of Pauli principle effects. In-medium ChEFT is expected to work reliably up to baryon densities of about 2 ϱ0, twice the density of normal nuclear matter. Extrapolations to much higher densities are promising but still subject to uncertainties about convergence properties. Open issues include, for example, the role of four-body correlations as they are encountered in the hierarchy of chiral effective interactions. Nonetheless, ChEFT at three-loop reproduces quantitatively recent Monte Carlo computations of pure neutron matter even up to about four times ϱ0. At the same time, the pertinent baryon densities reached in neutron stars, given the stiffness condition on the EoS, are not extremely high. As pointed out in the present work, the bulk properties of the star rest primarily on radial regions where the density does not exceed about 1.5 times ϱ0. The physics at such densities is considered to be well accessible by ChEFT methods.

3. Possible scenarios for the appearance of hybrid hadron-quark matter in the deep interior of neutron stars have been explored in this work, combining the ChEFT equation of state in the hadronic phase with either continuous or first-order transitions to quark matter. The quark matter component is described schematically in terms of a three-flavor (P)NJL model. Hybrid stars built with such a model are found unable to pass beyond the two-solar-mass line unless an additional repulsive vector-current interaction between quarks is introduced in order to generate a sufficiently stiff equation of state. At the same time, such strong repulsion in the quark sector eliminates the first-order chiral phase transition that is characteristic of more basic versions of the NJL model, in favor of a smooth chiral crossover at low temperatures and high densities.

The resulting hybrid equation of state, compatible with the criteria under the previous points 1 and 2, does not feature an extended region of quark matter in the inner core of a neutron star. Likewise, the admixture of strangeness (in the form of hyperons or deconfined strange quarks) is not substantial in such a constrained scenario. The presence of Λ hyperons would have again to be accompanied by strongly repulsive correlations in order to sustain the necessary pressure. In summary, the present work supports the idea that neutron stars are indeed predominantly composed of neutrons rather than more exotic forms of matter.

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A. APPENDIX: Some details of the PNJL model

The PNJL grand-canonical potential Ω = −ln Z, with Z derived from the action S_{PNJL} of Eq. (17) in mean-field approximation, is:

\[ Ω_{MF} = − \ln Z_{MF} = \frac{1}{β V} S_{PNJL,MF} \]

\[ = \sum_{a} \sum_{i\in\{u,d,s\}} \sum_{n\in\mathbb{Z}} \int_{\Lambda} \frac{d^3 p}{(2\pi)^3} \left[ \frac{3}{4G} \ln \left( \frac{m^2_i + \vec{p}^2 + M^2_{ai}}{2Gv} + \frac{\bar{σ}_u \bar{σ}_d \bar{σ}_s}{4G} \right) + \frac{4G}{2G} \bar{σ}_u \bar{σ}_d \bar{σ}_s \right] \]

\[ + U(Φ, \bar{Φ}; T) \]  

This result is found by standard bosonization of Eq. (17), introducing expectation values of the scalar fields, \( \bar{σ}_i = -G \langle q_i q_i \rangle \), and of the vector field, \( v = G_v \langle q' q \rangle \). The dynamically generated (constituent) quark masses are determined by the gap equations

\[ M_u = m_u + \bar{σ}_u + \frac{K}{2G^2} \bar{σ}_d \bar{σ}_s, \]

\[ M_d = m_d + \bar{σ}_d + \frac{K}{2G^2} \bar{σ}_u \bar{σ}_s, \]

\[ M_s = m_s + \bar{σ}_s + \frac{K}{2G^2} \bar{σ}_u \bar{σ}_d. \]

These masses (or, equivalently, the scalar mean fields \( \bar{σ}_i \)) serve as order parameters for the chiral transition. The shifted Matsubara frequencies \( ω^a;i \) with \( a \in \{0, ±\} \) are given by:

\[ ω^{0;i}_a = \omega_n - i μ_i + \frac{A^2_i}{2\sqrt{3}}, \]

\[ ω^{1;i}_a = \omega_n - i μ_i + \frac{A^2_i}{2\sqrt{3}}, \]

where \( ω_n = (2n + 1)πT \), \( n \in \mathbb{Z} \), denote the fermionic Matsubara frequencies and the \( μ_i \) are the chemical potentials for each quark species. The thermodynamic potential is written

\[ Ω_{MF} = Ω_Λ + Ω_{free} + Ω_{bos} + U(Φ, \bar{Φ}; T) \]  

\[ Ω_Λ \] is the fermionic part of Eq. (46) with quark momenta cut off at \( |\vec{p}| = Λ \); after performing the Matsubara summation one finds:

\[ Ω_A = -6 \sum_{i\in\{u,d,s\}} \int_{|\vec{p}| \leq Λ} \frac{d^3 p}{(2\pi)^3} E^{(i)}(\vec{p}) \]

\[ - 2T \sum_{i\in\{u,d,s\}} \int_{|\vec{p}| \leq Λ} \frac{d^3 p}{(2\pi)^3} \left\{ \ln \left[ 1 + 3(Φ + \bar{Φ} e^{-βE^{(i)}(\vec{p})}) e^{-βE^{(i)}(\vec{p})} + e^{-3βE^{(i)}(\vec{p})} \right] \right\} \]

\[ \ln \left[ 1 + 3(Φ + \bar{Φ} e^{-βE^{(i)}(\vec{p})}) e^{-βE^{(i)}(\vec{p})} + e^{-3βE^{(i)}(\vec{p})} \right] \]  

\[ \int_{\Lambda} \frac{d^3 p}{(2\pi)^3} \]

\[ \int_{\Lambda} \frac{d^3 p}{(2\pi)^3} \]

\[ \int_{\Lambda} \frac{d^3 p}{(2π)^3} \}

\[ \{ \ln \left[ 1 + 3(Φ + \bar{Φ} e^{-βE^{(i)}(\vec{p})}) e^{-βE^{(i)}(\vec{p})} + e^{-3βE^{(i)}(\vec{p})} \right] \]  

\[ \ln \left[ 1 + 3(Φ + \bar{Φ} e^{-βE^{(i)}(\vec{p})}) e^{-βE^{(i)}(\vec{p})} + e^{-3βE^{(i)}(\vec{p})} \right] \]  

\[ (50) \]
\[
E_{\pm}^{(i)}(\vec{p}) = \sqrt{\vec{p}^2 + M_i^2} \pm (\mu_i - \bar{v}). \tag{51}
\]

The potential \(\Omega_{\text{free}}\) is the contribution of a gas of quarks with momenta above the cutoff \(\Lambda\). These high-momentum quarks have their current-quark masses and do not interact. This added contribution makes sure that recover the correct Stefan-Boltzmann limit is recovered for the pressure and the energy density. The last two pieces are

\[
\Omega_{\text{free}} = \frac{\tilde{\sigma}_u^2 + \tilde{\sigma}_d^2 + \tilde{\sigma}_s^2}{4G} - \frac{\bar{v}^2}{2G_v} \tilde{\sigma}_u \tilde{\sigma}_d \tilde{\sigma}_s \tag{52}
\]

and the Polyakov effective potential \([20]\).

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