Distributed Interference Management Policies for Heterogeneous Small Cell Networks

Kartik Ahuja, Yuanzhang Xiao and Mihaela van der Schaar
Department of Electrical Engineering, UCLA, Los Angeles, CA, 90095
Email: ahujak@ucla.edu, yxiao@seas.ucla.edu and mihaela@ee.ucla.edu

Abstract

We study the problem of distributed interference management in a network of heterogeneous small cells with different cell sizes, different numbers of user equipments (UEs) served, and different throughput requirements by UEs. We consider the uplink transmission, where each UE determines when and at what power level it should transmit to its serving small cell base station (SBS). We propose a general framework for designing distributed interference management policies, which exploits weak interference among non-neighboring UEs by letting them transmit simultaneously (i.e. spatial reuse), while eliminating strong interference among neighboring UEs by letting them transmit in different time slots. The design of optimal interference management policies has two key steps. Ideally, we need to find all the subsets of non-interfering UEs, i.e. the maximal independent sets (MISs) of the interference graph, but this is NP-hard (non-deterministic polynomial time) even when solved in a centralized manner. Then, in order to maximize some given network performance criterion subject to UEs’ minimum throughput requirements, we need to determine the optimal fraction of time occupied by each MIS, which requires global information (e.g. all the UEs’ throughput requirements and channel gains). In our framework, we first propose a distributed algorithm for the UE-SBS pairs to find a subset of MISs in logarithmic time (with respect to the number of UEs). Then we propose a novel problem reformulation which enables UE-SBS pairs to determine the optimal fraction of time occupied by each MIS with only local message exchange among the neighbors in the interference graph. Despite the fact that our interference management policies are distributed and utilize only local information, we can analytically bound their performance under a wide range of heterogeneous deployment scenarios in terms of the competitive ratio with respect to the optimal network performance, which can only be obtained in a centralized manner with NP complexity. We prove that the competitive ratio is independent of the network size. Through extensive simulations, we show that our proposed policies achieve significant performance improvements (up to 85%) over state-of-the-art policies.
I. INTRODUCTION

Dense deployment of low-cost heterogeneous small cells (e.g. picocells, femtocells) has become one of the most effective solutions to accommodate the exploding demand for wireless spectrum [1] [2] [3]. On one hand, dense deployment of small cells significantly shortens the distances between small cell base stations (SBSs) and their corresponding user equipments (UEs), thereby boosting the network capacity. On the other hand, dense deployment also shortens the distances between neighboring SBSs, thereby potentially increasing the inter-cell interference. Hence, while the solution provided by the dense deployment of small cells is promising, its success depends crucially on interference management by the small cells. Efficient interference management is more challenging in heterogeneous small cell networks, due to the lack of central coordinators, compared to that in traditional cellular networks.

In this paper, we propose a novel framework for designing interference management policies in the uplink of small cell networks, which specify when and at what power level each UE should transmit. Our proposed design framework and the resulting interference management policies fulfill all the following important requirements:

- **Deployment of heterogeneous small cell networks**: Existing deployments of small cell networks exhibit significant heterogeneity such as different types of small cells (picocells and femtocells), different cell sizes, different number of UEs served, different UEs’ throughput requirements etc. Also, the deployment often needs to maximize different network performance criteria.

- **Interference avoidance and spatial reuse**: Effective interference management policies should take into account the strong interference among neighboring UEs, as well as the weak interference among non-neighboring UEs. Hence, the policies should effectively avoid interference among neighboring UEs and use spatial reuse to take advantage of the weak interference among non-neighboring UEs.

- **Distributed implementation with local information and message exchange**: Since there is no central coordinator in small cell networks, interference management policies need to be computed and implemented by the UEs in a distributed manner, by exchanging only local information through local message exchanges among neighboring UE-SBS pairs.

1Although we focus on uplink transmissions in this paper, our framework can be easily applied to downlink transmissions.
• **Scalability to large networks**: Small cells are often deployed over a large scale (e.g. in a city). Effective interference management policies should scale in large networks, namely achieve efficient network performance while maintaining low computational complexity.

• **Ability to optimize different network performance criteria**: Under different deployment scenarios the small cell networks may have different performance criterion, e.g. weighted sum throughput or max-min fairness. The design framework should be general and should prescribe different policies to optimize different network performance criteria.

• **Performance guarantees for individual UEs**: Effective interference management should provide performance guarantees (e.g. minimum throughput guarantees) for individual UEs. As we will discuss in detail in Section II, existing state-of-the-art policies for interference management cannot simultaneously fulfill all of the above requirements.

Next, we describe our key results and major contributions:

1. We propose a general framework for designing distributed interference management policies that maximizes the given network performance criterion subject to each UE’s minimum throughput requirements. The proposed policies schedule maximal independent sets (MISs)\(^2\) of the interference graph to transmit in each time slot. In this way, they avoid strong interference among neighboring UEs (since neighboring UEs cannot be in the same MIS), and efficiently exploit the weak interference among UEs in a MIS by letting them to transmit at the same time.

2. We propose a distributed algorithm for the UEs to determine a subset of MISs. The subset of MISs generated ensures that each UE belongs to at least one MIS in this subset. Moreover, the subset of MISs can be generated in a distributed manner in logarithmic time (logarithmic in the number of UEs in the network) for bounded-degree interference graphs\(^3\). The logarithmic convergence time is significantly faster than the time (linear or quadratic in the number of UEs) required by the distributed algorithms for generating subsets of MISs in [4]–[6].

3. Given the computed subsets of MISs, we propose a distributed algorithm in which each

\(^2\)Consider the interference graph of the network, where each vertex is a UE-SBS pair and each edge indicates strong interference between the two vertices. An independent set (IS) is a set of vertices in which no pair is connected by an edge. An IS is a MIS if it is not a proper subset of another IS.

\(^3\)Bounded degree graphs are the graphs whose maximum degree can be bounded by a constant independent of the size of the graph, i.e. \(\Delta = \mathcal{O}(1)\). As we will show in Theorem 5, for the interference graphs that are not bounded degree graphs, even the centralized solution, given all the MISs, cannot satisfy the minimum throughput requirements.
UE determines the optimal fractions of time occupied by the MISs with only local message exchange. The message is exchanged only among the UE-SBS pairs that strongly interfere with each other, i.e. among neighbors in the interference graph. The distributed algorithm will output the optimal fractions of time for each MIS such that the given network performance criterion is maximized subject to the minimum throughput requirements.

4. Under a wide range of conditions, we analytically characterize the competitive ratio of the proposed distributed policy with respect to the optimal network performance. Importantly, we prove that the competitive ratio is independent of the network size, which demonstrates the scalability of our proposed policy in large networks. Remarkably, our constant competitive ratio is achieved although our proposed policy requires only local information, is distributed, and can be computed in polynomial time, while the optimal network performance can only be obtained in a centralized manner with global information (e.g. all the UEs’ channel gains, maximum transmit power levels, minimum throughput requirements) and NP complexity.

5. Through simulations, we demonstrate significant (up to 85%) performance gains over state-of-the-art policies. Moreover, we show that our proposed policies can be easily adapted to a variety of heterogeneous deployment scenarios, with dynamic entry and exit of UEs.

The rest of the paper is organized as follows. In Section II we discuss the related works and their limitations. We describe the system model in Section III. Then we formulate the interference management problem and give a motivating example in Section IV. We propose the design framework in Section V and demonstrate the performance gain of our proposed policies in Section VI. Finally, we conclude the paper in Section VII.

II. RELATED WORKS

State-of-the-art interference management policies can be divided into three main categories: policies based on power control, policies based on spatial reuse, and policies based on joint spatial reuse and power control. In the following, we discuss their limitations for the considered distributed interference management in heterogeneous small cell networks. We will list some representative references in this section, a detailed list can be found in the online report [7].

A. Distributed Interference Management Based on Power Control

Policies based on distributed power control, with representative references [8]–[10], have been used for interference management in both cellular and ad-hoc networks. In these policies, all the
UEs in the network transmit at a constant power all the time (provided that the system parameters remain the same)\(^4\). The major limitation of policies based on power control is the difficulty in providing minimum throughput guarantees for each UE, especially in the presence of strong interference. Some works \(^[8]–[10]\) use pricing to mitigate the strong interference. However, they \(^[8]–[10]\) cannot strictly guarantee the UEs’ minimum throughput requirements. Indeed, the low throughput experienced by some users, caused by strong interference, is the fundamental limitation of such power control approaches - even the optimal power control policy obtained by a central controller \(^[11]\) can be inefficient\(^5\). Since strong interference is very common in dense small cell deployments (e.g. in offices and apartments where SBSs are installed close to each other \(^[13]\)), more efficient policies are required which can guarantee the individual UEs’ throughput requirements. Also, there exist a different strand of work based on \(^[14]\) which proposes a distributed algorithm to achieve the desired minimum throughput requirement for each UE. However, these works cannot optimize network performance criterion such as weighted sum throughput, max-min fairness etc. and hence are suboptimal.

**B. Distributed Spatial Reuse Based on Maximal Independent Sets**

An efficient solution to mitigate strong interference is spatial reuse, in which only a subset of UEs (which do not significantly interfere with each other) transmit at the same time. Spatial Time reuse based Time Division Multiple Access (STDMA) has been widely used in existing works on broadcast scheduling in multi-hop networks \(^[4]–[6]\)\(^6\). Specifically, these policies construct a cyclic schedule such that in each time slot an MIS of the interference graph is scheduled. The constructed schedule ensures that each UE is scheduled at least once in the cycle.

In terms of performance, STDMA policies \(^[4]–[6]\) cannot guarantee the minimum throughput requirement of each UE, and usually adopt a fixed scheduling (i.e. follow a fixed order in which the MISs are scheduled), which may be very inefficient depending on the given network performance criteria. For example, the policies in \(^[6]\) are inefficient in terms of fairness. In terms

\(^4\) Although some power control policies \(^[8]–[10]\) go through a transient period of adjusting the power levels before the convergence to the optimal power levels, the users maintain constant power levels after the convergence.

\(^5\) In the case of average sum throughput maximization given the minimum average throughput constraints of the UEs, the power control policies are inefficient if the feasible rate region is non-convex \(^[12]\).

\(^6\) These works \(^[4]–[6]\) do not have the exactly same model as in our setting. However, these works can be adapted to our model. Hence, we also compare with these works to have a comprehensive literature review.
of complexity, for the distributed generation of the subsets of MISs, the STDMA policies in [4]–
[6] require an ordering of all the UEs, and have a computational complexity (in terms of the
number of steps executed by the algorithm) that scales as $O(|V|)$ (in [5], [6]) or $O(|V||E|)$
(in [4]), where $|V|$ and $|E|$ are the number of vertices/UEs and the number of edges in the
interference graph, respectively. Hence, in large-scale dense deployments, the complexity grows
superlinearly with the number of UEs, making the policies difficult to compute. By contrast, our
proposed distributed algorithm for generating subsets of MISs does not require the ordering of
all the UEs, and has a complexity that scales as $O(\log |V|)$, namely sublinearly with the number
of the UEs, for bounded-degree graphs.\footnote{As will be shown in Theorem 5, for graphs which are not bounded degree graphs, even a centralized solution based on all
the MISs cannot satisfy the minimum throughput requirements.}

Finally, the STDMA policies in [4]–[6] are designed for the MAC layer and assume that all
the UEs are homogeneous at the physical layer. In practice, different UEs are heterogeneous due
to their different distances from their SBSs, their different maximum transmit power levels, etc.
This heterogeneity is important, and will be considered in our design framework.

### C. Distributed Power Control and Spatial Reuse For Multi-Cell Networks

As we have discussed, the works in the above two categories either focus on distributed
power control in the physical layer [8]–[10] or focus on distributed spatial reuse in the MAC
layer [4]–[6]. Similar to our paper, some works (representative references [15], [16]) adopted a
cross-layer approach and proposed distributed joint power control and spatial reuse for multi-cell
networks. However, although these works schedule a subset of UEs to transmit at the same time,
the subset is not the MIS of the interference graph [15], [16]. For example, the policies in [15],
[16] schedule one UE from each small cell at the same time, even if some UEs are from small
cells very close to each other. In this case, the UEs will experience strong inter-cell interference.
Hence, the works in [15], [16] cannot perfectly eliminate strong interference from neighboring
cells and exploit weak interference from non-neighboring cells. Moreover, the works in [15],
[16] cannot provide minimum throughput guarantees for the UEs.
III. SYSTEM MODEL

A. Heterogeneous Network of Small Cells

We consider a heterogeneous network of $K$ small cells operating in the same frequency band (see Fig. 1), which represents a common deployment scenario considered in practice [10]. Note that the small cells can be of different types (e.g. picocells, femtocells, etc.) and thereby belong to different tiers in the heterogeneous network. Each small cell $j$ has one SBS, $(SBS-j)$, which serves a set of UEs under a closed access scenario [10]. Denote the set of UEs by $\mathcal{U} = \{1, \ldots, N\}$. We write the association of UEs to SBSs as a mapping $T : \{1, \ldots, N\} \rightarrow \{1, \ldots, K\}$, where each UE-$i$ is served by SBS-$T(i)$. We focus on the uplink transmissions; the extension to downlink transmissions is straightforward when each SBS serves one UE at a time (e.g. TDMA among UEs connected to the same SBS).

Each UE-$i$ chooses its transmit power $p_i$ from a compact set $\mathcal{P}_i \subseteq \mathbb{R}_+$. We assume that $0 \in \mathcal{P}_i, \forall i \in \{1, \ldots, N\}$, namely any UE can choose not to transmit. The joint power profile of all the UEs is denoted by $p = (p_1, \ldots, p_N) \in \mathcal{P} \triangleq \Pi_{i=1}^N \mathcal{P}_i$. Under the joint power profile $p$, the signal to interference and noise ratio (SINR) of UE-$i$’s signal, experienced at its serving SBS-$j = T(i)$, can be calculated as

$$\gamma_i(p) = \frac{g_{ij}p_i}{\sum_{k=1, k \neq i}^N g_{kj}p_k + \sigma_j^2},$$

where $g_{ij}$ is the channel gain from UE-$i$ to SBS-$j$, and $\sigma_j^2$ is the noise power at SBS $j$. The UEs do not cooperate to encode their signals to avoid interference, hence, each UE-SBS pair treats the interference from other UEs as white noise. Hence, each UE-$i$ gets the following throughput [15],

$$r_i(p) = \log_2(1 + \gamma_i(p)).$$

B. Interference Management Policies

The system is time slotted at $t = 0, 1, 2, \ldots$, and the UEs are assumed to be synchronized as in [15], [16]. At the beginning of each time slot $t$, each UE-$i$ decides its transmit power $p^t_i$ and obtains a throughput of $r_i(p^t)$. Each UE $i$’s strategy, denoted by $\pi_i : \mathbb{Z}_+ = \{0, 1, \ldots\} \rightarrow \mathcal{P}_i$, is a mapping from time $t$ to a transmission power level $p_i \in \mathcal{P}_i$. The interference management policy is then the collection of all the UEs’ strategies, denoted by $\pi = (\pi_1, \ldots, \pi_N)$. The average throughput is

$$\bar{r}(\pi) = \frac{1}{T} \sum_{t=0}^{T-1} r_i(p^t).$$

\footnote{Our solutions will be based on spatial time reuse assuming every UE uses the same frequency. Our solutions can be extended to spatial frequency reuse, where we let different MISs operate in non-overlapping frequency bands.}

\footnote{We use the Shannon capacity here. However, our analysis is general and applies to the throughput models that consider the modulation scheme used.}
throughput for UE $i$ is given as $R_i(\pi) = \lim_{T \to \infty} \frac{1}{T+1} \sum_{t=0}^{T} r_i(p^t)$, where $p^t = (\pi_1(t), ..., \pi_N(t))$ is the power profile at time $t$. We assume the channel gain to be fixed over the considered time horizon as in [15]–[20]. However, we will illustrate in Section VI that our framework performs well under dynamic channel conditions (due to fading, time varying channel) as well.

An interference management policy $\pi_{\text{const}}$ is a policy based on power control [8]–[10] if $\pi_{\text{const}}(t) = p$ for all $t$. As we have discussed before, our proposed policy is based on MISs of the interference graph. The interference graph $G$ has $N$ vertices, each of which is one of the $N$ UE-SBS pairs. There is an edge between two pairs/vertices if their cross interference is high (rules for deciding if interference is high will be discussed in Section V) and let there be $M$ edges in the graph. Given an interference graph, we write $I = \{I_1, ..., I_{NMIS}\}$ as the set of all the MISs of the interference graph. Let $p^I_j$ be a power profile in which the UEs in the MIS $I_j$ transmit at their maximum power levels and the other UEs do not transmit, namely $p_k = p_{k}^{\text{max}} \triangleq \max \mathcal{P}_k$ if $k \in I_j$ and $p_k = 0$ otherwise. Let $\mathcal{P}^{MIS} = \{p^I_1, ..., p^{I_{NMIS}}\}$ be the set of all such power profiles. Then $\pi$ is a policy based on MIS if $\pi(t) \in \mathcal{P}^{MIS}$ for all $t$. We denote the set of policies based on MISs by $\Pi^{MIS} = \{\pi : \mathbb{Z}_+ \to \mathcal{P}^{MIS}\}$.

IV. PROBLEM FORMULATION AND A MOTIVATING EXAMPLE

In this section, we formulate the interference management policy design problem and give a motivating example to highlight the advantages of the proposed policy over existing policies.

A. The Interference Management Policy Design Problem

We aim to optimize a chosen network performance criterion $W(R_1(\pi), ..., R_N(\pi))$, defined as a function of the UEs’ average throughput. We can choose any performance criterion that is concave in $R_1(\pi), ..., R_N(\pi)$. For instance, $W$ can be the weighted sum of all the UEs’
throughput, i.e. \( \sum_{i=1}^{N} w_i R_i(\pi) \) with \( \sum_{i=1}^{N} w_i = 1 \) and \( w_i \geq 0 \). Alternatively, the network performance can be max-min fairness (i.e. the worst UE’s throughput) and hence \( W \) can be defined as \( \min_i R_i(\pi) \). The policy design problem can be then formalized as follows:

**Policy Design Problem (PDP)**

\[
\max_{\pi} W(R_1(\pi), ..., R_N(\pi)) \\
\text{subject to } R_i(\pi) \geq R_i^{\text{min}}, \forall i \in \{1, ..., N\}
\]

The above design problem is very challenging to solve even in a centralized manner (it is NP-hard [21] for objective \( W \) as sum throughput, even when we restrict to policies based on power control \( \pi^{\text{const}} \)). Denote the optimal value of the PDP as \( W_{\text{opt}} \). Our goal is to develop distributed, polynomial-time algorithms to construct policies that achieve a constant competitive ratio with respect to \( W_{\text{opt}} \), with the competitive ratio independent of the network size. We achieve our goal by focusing on policies based on MISs \( \Pi^{\text{MIS}} \), among other innovations that will be described in Section V. Next, we provide a motivating example to demonstrate the efficiency of our proposed policy.

**B. Comparisons in a simple example - Motivation**

Consider a network of 2 picocell base stations (PBS) and 2 femtocell base stations (FBS), each serving one UE. The network topology is shown in Fig. 2. We assume a path loss model for channel gains, with path loss exponent 4. The maximum transmit power of each UE is 80 mW, the noise power at each SBS is \( 1.6 \times 10^{-3} \) mW. UEs in different tiers have different minimum throughput requirements: FUE-1 (femtocell UE) and FUE-2 in the femtocells require a minimum throughput 0.4 bits/s/Hz, and PUE-1 (picocell UE) and PUE-2 in the picocells require 0.2 bits/s/Hz. The interference graph is constructed based on a distance based threshold rule similar to [17]. Specifically, an edge exists between two UE-BS pairs if the distance between
any pair of SBSs is less than a threshold, which is set to be 1.2m here. There are two MISs, one (MIS-1) consisting of FUE-1 and FUE-2, and the other (MIS 2) consisting of PUE-1 and PUE-2. Here we consider two performance criteria, namely the max-min fairness and the sum throughput. Next, we describe the existing schemes with which we will compare in this motivating example:

1. **Distributed Constant Power Control Policies** [8]–[10]: In these policies, all the UEs choose constant power levels determined by distributed algorithms utilizing local information (e.g. power levels being used by neighbors) made available through local message exchange.

2. **Optimal Centralized Constant Power Policies** [11]: In these policies, all the UEs choose constant power levels determined by a central controller utilizing global information.

3. **Distributed MIS STDMA-1** [6] and **STDMA-2** [4]: These policies construct a subset of the MISs of the interference graph in a distributed manner and schedule the MISs according to a fixed procedure. Different works adopt different schedules, and we differentiate them by referring to them as MIS STDMA-1 and STDMA-2.

4. **Distributed Joint Power Control and Spatial Reuse** [15], [16]: These policies schedule UEs in different cells in the network based on their channel gain to maximize the sum throughput. The schedule of subsets of UEs is determined by power matched scheduling (PMS).

We first propose a distributed, fast algorithm, which allows the UE-SBS pairs to compute a subset of MISs within a number of steps/computations growing logarithmically $O(\log N)$ in the number of UEs. Next, we propose a distributed optimization based framework which allows the UEs to determine the fractions of time occupied by the MISs in a distributed manner. The schedule ensures minimum throughput guarantees to all the UEs in the network and maximizes the given network performance criterion. Our framework only requires message passing among the neighboring UEs in the interference graph. Also, we will show in Section VI that our framework is robust to the dynamic entry and exit of UEs.

We compare the performance of our proposed policy and existing policies for the set-up depicted in Fig. 2 in Table 1. We can see that our proposed policy outperforms centralized and distributed constant power policies and distributed PMS by 375% and 32.8%, in terms of max-min fairness and sum throughput, respectively. Our proposed policy also outperforms distributed STDMA policies by 30%-40%. As we will see in Section VI, the performance gain can be even higher (up to 85% improvement over these policies). Also in this motivating example, the proposed policy achieves the same performance as the benchmark problem (see Section VI).
### Table I
COMPARISONS IN TERMS OF MAX-MIN FAIRNESS & SUM THROUGHPUT CRITERION

| Policies                              | Max-min Performance (bits/s/Hz) | Performance Gain % | Sum Performance (bits/s/Hz) | Performance Gain % |
|---------------------------------------|---------------------------------|--------------------|-----------------------------|--------------------|
| Distributed constant power [8]–[10]   | 0.28 (Upper bound)              | 375 %              | 6.1 (Upper bound)           | 32.8 %             |
| Optimal centralized constant power [11]| 0.28                            | 375%               | 6.1                         | 32.8 %             |
| Distributed MIS STDMA-2/1 [4], [6]    | 0.96                            | 38.5%              | 6.25                        | 30.0%              |
| Distributed PMS [15], [16]            | 0.28 (Upper Bound)              | 375%               | 6.1 (Upper bound)           | 32.8 %             |
| Proposed (Section-V)                  | 1.33                            | -                  | 8.12                        | -                  |
| Benchmark Problem (BP) (Section-VI)   | 1.33                            | -                  | 8.12                        | -                  |

Figure 3. Steps in the Design Framework.

### V. DESIGN FRAMEWORK FOR DISTRIBUTED INTERFERENCE MANAGEMENT

#### A. Proposed Design Framework

Our proposed design framework (See Fig. 3) consists of four steps. In Step 1, each UE-SBS pair identifies strongly interfering UE-SBS pairs, in order to get a local view (i.e. its neighbors) of the interference graph. In Step 2, the UE-SBS pairs determine a subset of MISs, such that each UE-SBS pair belongs to at least one MIS in the subset, in a distributed fashion. In Step 3, each UE-SBS pair determines the optimal fraction of time allocated to each MIS found in Step 2 in a distributed fashion (The optimal fraction depends on the performance criterion.). In Step 4, UE-SBSs determine the cycle length and the number of slots taken by each MIS, based on the optimal fractions of time computed in Step 3. Next, we describe the four steps in detail.

**Step 1. Identification of the interfering neighbors:** In Step 1, each UE-SBS pair identifies the UE-SBS pairs that strongly interfere with it. Essentially, each pair obtains a local view (i.e. its neighbors) of the interference graph. Note that an edge exists between two pairs if at least one of them identifies the other as strongly interfering.

Specifically, each UE-SBS pair is first informed of other pairs in the geographical proximity by managing servers (e.g. femtocell controllers/gateways) [22] [23] [18] [19]. Then each pair can...
Figure 4. Illustration of the distributed generation of MISs in Step 2.

decision whether another pair is strongly interfering based on various rules, such as rules based on Received Signal Strength (RSS) in the Physical Interference Model [22] [18] [19], and rules based on the locations in the Protocol Model [17]. If one pair identifies another pair as strongly interfering, its decision can be relayed by the managing servers to that pair, such that any two pairs can reach consensus of whether there exists an edge between them.

Step 2. Distributed generation of MISs that span all the UEs: In Step 2, the UE-SBS pairs generate a subset of MISs in a distributed fashion. It is important that the generated subset spans all the UEs, namely every UE is contained in at least one MIS in the subset. Otherwise, some UEs will never be scheduled. The distributed algorithm is comprised of two phases: first, distributed coloring of the interference graph based on [24], and second, extension of the color classes to the MISs. We assume that all the UEs are synchronized and carry out their computation simultaneously (i.e. not sequentially). We now explain the algorithm in detail. The pseudo-code can be found in Table II and III in the Appendix given at the end.

Phase 1. Distributed coloring of the interference graph: The goal of this phase is to let each UE-SBS pair- \( i \) choose a color from \( C^0_i = \{1, ..., H\} \cap \{1, ..., d_i + 1\} \), such that no neighbors choose the same color. Here \( H \) is the maximum number of colors given to all SBSs at the installation and \( d_i \) is the degree (number of neighbors in the interference graph) of \( i^{th} \) pair.

The distributed coloring works as follows. At the beginning of each time slot \( t \), each UE- \( i \)

\(^{10}\) The maximum number of colors \( H \) should be set to be larger than the maximum number of UE-SBS pairs interfering with any UE-SBS pair. The SBSs can determine \( H \) according to the deployment scenario. \( H \) in general will also include the number of UEs that use the same SBS who interfere with each other along with the other neighboring UEs. For example, \( H \) can be 10-15 in an office building with dense deployment of SBSs, and can be 3-5 in a residential area.
chooses a color from the set of remaining colors $C^t_i$ randomly, and informs its neighbors of its tentative choice. This information can be transmitted using the back-haul network/X2 interface, which are used for inter-cell interference coordination (ICI) \cite{23}. If the tentative choice of the UE does not conflict with any of its neighbor, then it fixes and confirms the color choice, informs the neighbors of the confirmation and will not contend for colors any further in Phase 1. The neighbors delete the color chosen by $i$ from their lists $C^{t+1}_j$, $\forall j \in N(i)$, where $N(i)$ is the set of $i$’s neighbors. On the other hand, if there is a conflict then the UE does not choose that color and repeats the same procedure in the next time slot. There are $\lceil c_1 \log_{\frac{4}{3}} N \rceil + 1$ time slots in Phase 1, known to the SBSs at installation where $c_1$ is the parameter given by the protocol. Phase 1 is successful if all the UEs acquire a color which implies that the set of color classes (i.e. the set of UE-SBS pairs with the same color) obtained span all the UEs.

**Phase 2. Extending color classes to the MISs:** Each color class obtained at the end of Phase 1 is an independent set (IS) of the graph. In Phase 2, we extend each of these ISs to MISs and generate additional MISs in this step. Note that after Phase 1, each UE-SBS pair has chosen one color and deleted some colors from its list. But there may still be remaining colors in its list that are not acquired by any of its neighbors. The goal of Phase 2 is to utilize these remaining colors. At each time slot in Phase 2, UE-$i$ chooses each color from the remaining colors in its list independently with probability $c$. Each UE-$i$ then sends this set of its tentative choices to its neighboring UEs, and receives their neighbors’ choices. For any tentative choice of color, if there is a conflict with at least one neighbor, then that color is not fixed; otherwise, it is fixed. At the end of each time slot, each UE deletes its set of fixed colors from its list, and transmits this set of fixed colors to its neighbors, who will delete these fixed colors from their lists as well. Hence, a UE deletes a particular color, if and only if this UE itself or some of its neighbors have chosen this color. Based on this key observation, we can see that if a color is not in any UE’s list, the set of UEs who have chosen that color is a MIS. If all the UEs have an empty list, then for any color in the set $\{1, ..., H\}$, the set of UEs with this color is a MIS. There are $\lceil c_2 \log_x N \rceil + 1$ time slots in Phase 2, known to SBSs at installation, where $x = \frac{1}{1-(c)^H(1-c)^H}$, and $c_2$ is the parameter given by the protocol. We say that Phase 2 is successful, if it finds $H$ MISs, equivalently, if all the UEs have an empty list after $\lceil c_2 \log_x N \rceil + 1$ time slots.

**Example:** Fig. 4 shows the interference graph of a network of 5 UE-SBS pairs. At the start, each UE-SBS pair has a list of 3 colors, $\{\text{Red, Yellow, Green}\}$ to choose from. At the end of
Phase 1, which runs for $P_1 = \lceil c_1 \log_4 5 \rceil$ time slots, UEs 2,3 acquire Yellow, UEs 4,5 acquire Red and UE 1 acquires Green. UEs 2,3 delete all the colors from their list of remaining colors, as Yellow is acquired by them and Red, Green by their neighbors. UE 1 also deletes all the colors, since its neighbors use Yellow and Red, but UE 4(UE 5) has Yellow(Green) color in its list. At the end of Phase 1, Red color class is MIS, while Yellow and Green are not. At the end of Phase 2, which runs for $P_2 = \lceil c_2 \log_x 5 \rceil + 1$ time slots, UE 4 (UE 5) acquires the remaining color Yellow (Green). At the end of Phase 2, the color classes Green and Yellow are MISs. Next, theorem shows that the Phase 1 and 2 are successful with a high probability.

Theorem 1. For any interference graph with the maximum degree $\Delta \leq H - 1$, the proposed algorithm in Table II and III outputs a set of $H$ MISs that span all the UEs in $(\lceil c_1 \log_4 N \rceil + \lceil c_2 \log_x N \rceil + 2)$ time slots with a probability no smaller than $(1 - \frac{1}{N^{c_1-1}})(1 - \frac{1}{N^{c_2-1}})$, where $c_1$ and $c_2$ are design parameters that trade-off the run time and the success probability.

See Appendix at the end for all proof sketches, and see online report [7] for detailed proofs.

Theorem 1 characterizes the performance of our proposed algorithm, in terms of the run time of the algorithm and the lower bound of the success probability. When the parameters $c_1$ and $c_2$ are chosen to be large, the lower bound of the success probability increases at the expense of a longer run time. When the maximum degree of the interference graph is larger, we need to set a higher $H$, which results in a longer run time. This is reasonable, because it is harder to find coloring and MISs when the number of interfering neighbors is higher. Finally, we can see that the lower bound of the successful probability is very high even under smaller $c_1$ and $c_2$, especially if the number of UEs is large. Note that the exact successful probability should depend on the probability $c$, while the lower bound in Theorem 1 does not. Also, note that the interference graph here is a bounded-degree graph since the maximum degree is bounded by a given constant, $H - 1$. The algorithms in [4], [6] (require ordering of the vertices, work sequentially and have a higher complexity) can be used to output the MISs spanning all the UEs for arbitrary graphs. However, we will show in Theorem 5, that the restriction to bounded-degree graphs is a must to ensure that the minimum throughput requirement of each UE is satisfied for any MIS based policy.

Step 3. Distributed computation of the optimal fractions of time for each MIS: Let the set of MISs generated in Step 2 be $\{I'_1, ..., I'_H\}$. Here, the UE-SBS pairs compute the fractions of time allocated to each MIS in a distributed manner.
Note that when an MIS is scheduled, the UEs in this MIS transmit at their maximum power levels, and the other UEs do not transmit. Define $R^k_i$ as the instantaneous throughput obtained by UE $i$ in the MIS $I'_k$, which can be calculated as: \[
 \log_2(1 + \frac{g_{T(i)I'_k}^i}{\sum_{r=1,r \neq i}^N g_{T(r)I'_k}^i p_{I'_k}^r + \sigma^2_{T(i)}}), \]
 where $p_{I'_k}^i = p_{\text{max}}^i$ if $i \in I'_k$ and $p_{I'_k}^i = 0$ otherwise. To determine $R^k_i$, the UE needs to know the accumulative interference it experiences when transmitting in $I'_k$. This can be done by having an initial cycle of transmissions by the MISs, i.e. UEs in MISs, in the order of the indices (i.e. the order of their colors), in which the SBSs of the UEs can measure the received SINR.

From now on, we assume that the network performance criterion $W(y)$ is concave in $y$ and is separable, namely $W(y_1, \ldots y_N) = \sum_{i=1}^N W_i(y_i)$. Examples of separable criteria include weighted sum throughput and proportional fairness. Note also that our framework can be extended to deal with max-min fairness, although it is not separable (see the discussion in the online report [7]). Then the problem of computing the optimal fractions of time for the MISs is given as follows:

**Coupled Problem (CP)**

$$\text{max}_{\alpha} \sum_{i=1}^N \left( \sum_{k=1}^H \alpha_k R^k_i \right)$$

subject to

$$\sum_{k=1}^H \alpha_k R^k_i \geq R^\text{min}_i, \forall i \in \{1, \ldots N\}$$

$$\sum_{k=1}^H \alpha_k = 1, \alpha_k \geq 0, \forall k \in \{1, \ldots, H\}$$

Since each UE $i$ knows only its own utility function $W_i$ and its minimum throughput requirement $R^\text{min}_i$, it cannot solve the above problem by itself directly. To get around this difficulty, we let each UE-$i$ have a local estimate of the fractions of time allocated to all the MISs (including those that do not include UE-$i$). Denote UE-$i$’s local estimate of the fraction of time allocated to MIS $I'_k$ by $\beta^k_i$. We impose an additional constraint that all the UEs’ local estimates are the same, such that they reach to a consensus. Such a constraint is still global, because any two UEs, even when they are not neighbors and far away from each other, need to have the same local estimate. Hence, global message exchange among any pair of UEs is needed to solve CP$^{[1]}$.

$^{[1]}$If the UEs could exchange messages globally, i.e. broadcast messages to all the UEs in the network, and if the network performance criterion is strictly concave, we could use standard dual decomposition with augmented Lagrangian in [25] to derive a distributed algorithm. However, in large networks, the UEs cannot exchange messages globally with other UEs, and the network performance criterion may not be strictly concave (e.g. the weighted sum throughput is linear).
Now we reformulate the CP into a decoupled problem (DP) that involves only local coupling among the neighbors and that can be solved by Alternating Direction Method of Multipliers (ADMM) [26]. If UE $i$ and $l$ are connected by an edge $(i,l)$ then for each set $I'_k$ define $\theta^{k}_{(i,l)i} = \beta^{k}_i$ and $\theta^{k}_{(i,l)l} = -\beta^{k}_l$, note that these auxiliary variables are introduced to formulate the problem into the ADMM framework [26]. Define a polyhedron for each $i$, $\mathcal{T}_i = \{\beta_i| s.t. \ 1^T\beta_i = 1, \beta_i \geq 0, R'_i \beta_i \geq R'^{\text{min}}_i\}$, where $\beta_i = (\beta^{1}_i, \ldots, \beta^{H}_i)$ and $R_i = (R^{1}_i, \ldots, R^{H}_i)$ and $(\cdot)'$ corresponds to the transpose. Let $\beta = (\beta_1, \ldots, \beta_N) \in \mathcal{T}$, where $\mathcal{T} = \prod_{i=1}^{N} \mathcal{T}_i$ and $\prod$ corresponds to the Cartesian product of the sets. Also, let $\beta^k_i = (\beta^{1}_i, \ldots, \beta^{N}_i), \forall k \in \{1, \ldots, H\}$. Define another polyhedron $\Theta^k_{(i,l)} = \{(\theta^{k}_{(i,l)i}, \theta^{k}_{(i,l)i}') : \theta^{k}_{(i,l)i} + \theta^{k}_{(i,l)i}' = 0, -1 \leq \theta^{k}_{(i,l)i} \leq 1, \forall s \in \{i, l\}\}$, $\Theta^k = \prod_{(i,l) \in E} \Theta^k_{(i,l)}$ here $E = (e_1, \ldots, e_M)$ is the set of all the $M$ edges in the interference graph. A vector $\theta^k \in \Theta^k$ is written as $\theta^k = (\theta^{k}_{e_1, z(e_1)}, \theta^{k}_{e_1, t(e_1)}, \ldots, \theta^{k}_{e_M, z(e_M)}, \theta^{k}_{e_M, t(e_M)})$, here $z(e_i), t(e_i)$ correspond to the vertices in the edge, $e_i$. Similarly define, $\theta = (\theta^1, \ldots, \theta^H) \in \Theta$, where $\Theta = \prod_{k=1}^{H} \Theta^k$.

**Decoupled Problem (DP)**

\[
\min_{\beta \in \mathcal{T}, \theta \in \Theta} - \sum_{i=1}^{N} W_i (R'_i \beta_i)
\]

subject to $D^k \beta^k - \theta^k = 0, \forall k \in \{1, \ldots, H\}$

Here, $D^k \in \mathbb{R}^{2M \times N}$, is a matrix in which each row has exactly one non-zero element which is 1 or $-1$. Each element of the matrix, $D^k_{ij}$ is evaluated as follows, the index $v$ can be uniquely expressed in terms of quotient $q$ and the remainder $w$ as $v = 2q + w$, and if $j \neq z(e_{q+1}), j \neq t(e_{q+1})$ then $D^k_{ij} = 0$. If $w = 1, j = z(e_{q+1})$, then $D^k_{ij} = 1$ else if $w = 0, j = z(e_{q+1})$ then $D^k_{ij} = 0$. Also, if $w = 0, j = t(e_{q+1})$, then $D^k_{ij} = -1$ else if $w = 1, j = t(e_{q+1})$ then $D^k_{ij} = 0$.

**Theorem 2:** For any connected interference graph [12], the coupled problem is equivalent to the decoupled problem.

The above theorem is important, because it shows that the CP, which cannot be solved without global information and global message exchange, is now transformed into an equivalent problem, DP which can be solved by ADMM with local message exchange.

We denote the optimal solution to the DP by $W^{G}_{\text{distributed}}$. We solve the DP by ADMM briefly described next (more detail in Table IV in the Appendix), and prove the rate of convergence, namely how fast the error from the optimal solution $W^{G}_{\text{distributed}}$ decreases as the number of iterations increase.

---

12 A graph is connected, if there exists a path of edges connecting any two nodes.
We associate with each constraint $D^k \beta^k - \theta^k = 0$ a price vector which we denote as follows, $\lambda^k = (\lambda^k_{e_1z(e_1)}, \lambda^k_{e_1t(e_1)}, \ldots, \lambda^k_{e_Mz(e_M)}, \lambda^k_{e_Mt(e_M)})$. We can write the augmented Lagrangian for DP as follows, $L_y(\beta, \theta, \lambda) = -\sum_{i=1}^{N} W_i (R_i, \beta_i) + \sum_{k=1}^{H} (\lambda^k)' (D^k \beta^k - \theta^k) + \frac{1}{2} \sum_{k=1}^{H} ((D^k \beta^k - \theta^k))^2$, here $\lambda = (\lambda^1, \ldots, \lambda^H)$. The ADMM procedure relies on computing the optimal vector $\beta_i(t) = (\beta_1^i(t), \ldots, \beta_H^i(t))$, by each UE-$i$ in the current time slot $t$ given the price variables and auxiliary variables at time $t-1$, i.e. $\lambda^k_{e_i}(t-1), \theta^k_{e_i}(t-1) \forall k \in \{1, \ldots, H\}, \forall e \in E(i)$, here $E(i)$ is the set of edges with $i$ as a vertex. Then, the price variable $\lambda^k_{e_i}(t-1)$ and auxiliary variable $\theta^k_{e_i}(t-1)$ is updated parallelly by each UE-$i$ based on the $\beta^k_i(t)$ and the neighbor $j$’s $\beta^k_j(t)$, here $e = (i, j), \forall e \in \tilde{E}(i), \forall k \in \{1, \ldots, H\}$. This iteration of updating $\beta_i(t)$ and price, auxiliary variables is repeated $P$ times.

**Theorem 3:** If DP is feasible, then the ADMM algorithm in Table IV converges to the optimal value $W^G_{\text{distributed}}$ with a rate of convergence $O(\frac{1}{P})$.

DP is feasible refers to the feasible region resulting from the constraints in DP to be non-empty.

**Step 4. Determining the cycle length and transmission times:** At the end of Step 3, all the UEs have a consensus about optimal fractions, i.e. $\beta^*_i = \gamma^* = (\gamma^*_1, \ldots, \gamma^*_H), \forall i \in \{1, \ldots, N\}$ allocated to the MISs. The MISs transmit in the order of their indices (i.e. $\{1, \ldots, H\}$) in cycles. In each cycle of transmission, MIS $I_k$ transmits for $\lceil \gamma^*_k \frac{10^{\text{tol}}}{\gamma^*_k} \rceil$ slots, where $\gamma^*_k$ is the smallest non-zero fraction assigned to any MIS and $10^{\text{tol}}$ characterizes the difference the UEs can tolerate in the exact fractions $\gamma^*$ and the one yielded by the cycle computed through this procedure.

**B. Performance Guarantees for Large Networks and Properties of Interference Graphs**

In this subsection, we provide performance guarantees for our proposed framework described in Subsection V-A. Specifically, we prove that the network performance, $W^G_{\text{distributed}}$, achieved by the proposed distributed algorithms has a constant competitive ratio with respect to the optimal value, $W_{\text{opt}}$, of the PDP. Moreover, we prove that the competitive ratio does not depend on the network size. Our result is strong, because the PDP is NP-hard and requires global information to solve, while our proposed framework requires the UEs to have only local information based message exchange, and converges in polynomial time.

Before characterizing the competitive ratio analytically, we define some auxiliary variables. Define the upper and lower bounds on the UEs’ maximum transmit power levels and throughput requirements as, $0 < p_{\text{ub}}^{\text{max}} \leq p_i^{\text{max}} \leq p_{\text{ub}}^{\text{max}}, \forall i \in \{1, \ldots, N\}$ and, $0 < R_{\text{ub}}^{\text{min}} \leq R_i^{\text{min}} \leq R_{\text{ub}}^{\text{min}}, \forall i \in \{1, \ldots, N\}$.
\{1, \ldots, N\} \) respectively. Let \( D_{ij} \) is the distance between UE \( i \) and SBS \( j \). Define upper and lower bounds on the distance between any UE and its serving SBS and the noise power at the SBSs as, \( 0 < D_{lb} \leq D_{ij} \leq D_{ub}, \forall i \in \{1, \ldots, N\} \) and, \( \sigma^2_{lb} \leq \sigma^2_j \leq \sigma^2_{ub}, \forall j \in \{1, \ldots, K\} \) respectively. We assume that the channel gain is \( g_{ij} = \frac{1}{(D_{ij})^{np}} \), where \( np \) is the path loss exponent.

**Definition 1 (Weak Non-neighboring Interference):** The interference graph \( G \) exhibits \( \zeta \)-Weak Non-neighboring Interference (\( \zeta \)-WNI) if for each UE \( i \) the maximum interference from its non-neighbors is bounded, namely \( \sum_{j \notin N(i), j \neq i} g_{jT(i)}p^\max_j \leq (2^\zeta - 1)\sigma^2_{ub}, \forall i \in \{1, \ldots, N\} \).

Define \( \Delta^\max = \frac{\log_2(1+ \frac{p^\max_{ub}}{(D_{lb})^{np}} \sigma^2_{ub})}{R_{ub}} - 1 \). Then we have the following theorem for the network performance criterion, sum throughput:

**Theorem 4:** For any interference graph, if the maximum degree \( \Delta \leq \Delta^\max \) and it exhibits \( \zeta \)-WNI then, our proposed framework of interference management described in Subsection V-A achieves a performance \( W^G_{\text{distributed}} \geq \Gamma \cdot W^\text{opt} \), where \( \Gamma = \frac{R_{ub}}{\log_2(1+ \frac{p^\max_{ub}}{(D_{lb})^{np}} \sigma^2_{ub})} \) is the constant competitive ratio, which is independent of the network size.

Note that the analytical expression of competitive ratio, \( \Gamma = \frac{R_{ub}}{\log_2(1+ \frac{p^\max_{ub}}{(D_{lb})^{np}} \sigma^2_{ub})} \) does not depend on the size of the network. Our results are derived under the conditions that the interference graph has a maximum degree bounded by \( \Delta^\max \), and that the interference from non-neighbors is bounded (i.e. \( \zeta \)-WNI). Note that these do not restrict the size of the network (for more detail, see the online report [7]). Also, see that the competitive ratio, \( \Gamma \) holds as long as conditions in the theorem are satisfied and it does not explicitly depend on the rule used for constructing the interference graph.

Both Theorem 1 and 4 required the maximum degree of the interference graph to be bounded by a given constant. Here, we show that constraint on the degree is natural and is a must to ensure feasibility, i.e. to satisfy the minimum throughput requirements of every UE. Specifically, we prove that if the maximum degree exceeds some threshold, then no policy based on scheduling MIs in \( \Pi^{MIS} \) (a large space of policies, see Section III) is feasible. Let the construction of interference graph be based on a distance based threshold rule similar to [17]. An edge exists between two UE-SBS pairs if and only if, the distance between two SBS is no greater than \( D_{th} \). We define the threshold of the maximum degree as \( \Delta^* \) (See the Appendix for the expression).

---

13We can extend this result for weighted sum throughput, with weights \( w_i = \Theta(\frac{1}{N}) \), it is not done to avoid complex notations.
Theorem 5: If the maximum degree of the interference graph $\Delta \geq \Delta^*$, then any policy based on scheduling MISs in $\Pi^{MIS}$ fails to satisfy the minimum throughput requirements of the UEs.

The intuition behind Theorem 5 is that, if the degree of the interference graph is large then there must be a large number of UE-SBS pairs which interfere with each other strongly (mutually connected) which makes it impossible to allocate each UE enough transmission time to satisfy its minimum throughput requirement.

C. Self-Adjusting Mechanism for Dynamic Entry/Exit of UEs

We now describe how the proposed framework can adjust to dynamic entry/exit by the UEs in the network without recomputing all the four steps. We allow the UEs to enter and exit, but number of SBSs is fixed. We only allow let one UE enter or leave the network in any time slot.

1. UE leaves the network: Suppose a UE $i$ which was transmitting to SBS $T(i)$ leaves the network. If the UE $i$ was transmitting in a set of colors $C_i$, then as soon as it leaves, these colors can be potentially used by some neighbors, $\mathcal{N}(i)$. The SBS $T(i)$ which was serving the UE $i$ can have other UEs which are still in the network and transmitting to it. Then for each color $c' \in C_i$ it first searches among the UEs which it serves that are not already transmitting in $c'$ and who also do not have a neighboring UE-SBS pair which is already transmitting in $c'$. Let the set of such UEs be $UE^{c'}_{i,left}$. SBS $T(i)$ allocates color $c'$ to the UE whose index is $\arg \max_{j \in UE^{c'}_{i,left}} R^{c'}_{ij}$. In case $UE^{c'}_{i,left}$ is empty then that color, $c'$ is left unused.

2. UE enters the network: Suppose a UE $i$ registered with SBS $T(i)$ enters the network. SBS $T(i)$ creates the list of colors $C^{valid}_{i,enter}$, which are either unused or the UEs transmitting in the colors are transmitting at more than their minimum throughput requirement. SBS $T(i)$ allocates some portions from the fractions of time allocated to the colors in $C^{valid}_{i,enter}$, to satisfy UE-$i$'s throughput requirement to the best possible extent, making sure that the minimum throughput requirements of UEs transmitting to SBS $T(i)$ in $C^{valid}_{i,enter}$ are not violated. If the requirement of UE-$i$ is not satisfied then, SBS $T(i)$ requests the neighboring UE-SBSs to announce the set of colors which are either not being used or in which their corresponding UEs are operating at more than the throughput requirement. From the list of colors received, $T(i)$ chooses those in which UE $i$ can transmit without conflicting with neighbors and for each color in the list it sends the request (portion of time needed) to the neighbors. SBS $T(i)$ and the neighbors go through a phase of communication (more detail in online report [7]), based on which SBS $T(i)$ can decide how much time UE-$i$ can transmit in the colors allocated to the neighbors.
VI. ILLUSTRATIVE RESULTS

In this section, we evaluate our proposed policy under a variety of scenarios which include a) varying interference strength, b) large networks, c) different performance criteria, d) time-varying channel conditions and e) dynamic entry and exit of UEs.

We compare our policy with the optimal centralized constant power control policy [11], the distributed MIS STDMA-1 [6] and STDMA-2 [4], distributed PMS [15], [16], in terms of sum throughput and max-min fairness. We do not compare with distributed constant power control policies [8]–[10], because their performance is upper bounded by the optimal centralized power control. Since it is difficult to compute the solution of PDP, we now define another centralized optimization problem which solves for the optimal policy among a class of policies defined as

\[ \Pi_{BC} = \{ \pi = (\pi_1, ..., \pi_N) : \pi_i : \mathbb{Z}_+ \rightarrow \{0, p_{i}^{\text{max}}\} \forall i \in \{1, ..., N\} \}. \]

Any policy in \( \Pi_{BC} \) schedules a subset of UEs in each time slot, and let the UEs in the subset transmit at the maximum power levels and the others remain silent. The optimal policy in \( \Pi_{BC} \) schedule subsets of UEs in the optimal way at each time slot. We formally state the benchmark problem (BP) as follows:

**Benchmark Problem (BP)**

\[
\max_{\pi \in \Pi_{BC}} W(R_1(\pi), ..., R_N(\pi))
\]

subject to \( R_i(\pi) \geq R_{i}^{\text{min}}, \forall i \in \{1, ..., N\} \)

Note that the above problem is very general and admits optimal solutions that are very efficient, because we allow the policy to schedule any subset of UEs. Also, note that this problem is NP-hard (detailed explanation in the online report [7]).

A. Performance under time-varying channel conditions

Consider a 3x3 square grid of 9 SBSs with the minimum distance between any two SBSs being \( d = 4.7 \text{m} \). Each SBS serves one UE, who has a maximum power of 1000 mW and a minimum throughput requirement of 0.45 bits/s/Hz. The UEs and the SBSs are in two parallel horizontal hyperplanes, and each SBS is vertically above its UE with a distance of \( \sqrt{10} \text{m} \). Then the distance from UE \( i \) to another SBS \( j \) is \( D_{ij} = \sqrt{10 + (D_{ij}^{BS})^2} \), where \( D_{ij}^{BS} \) is the distance between SBSs \( i \) and \( j \). The channel gain from UE \( i \) to SBS \( j \) is a product of path loss and Rayleigh fading \( f_{ij} \sim \text{Rayleigh}(\beta) \), namely \( g_{ij} = \frac{1}{(D_{ij})^2} f_{ij} \). The SBSs identify neighbors using a distance based rule with the threshold distance as in Subsection V-B with \( D^{th} = 7 \text{m} \). Note that different thresholds lead to different interference graphs, and hence different performance, which will be discussed next. Although, we use a distance based threshold rule, our framework
is general and does not rely on a particular rule. The resulting interference graph for this setting is graph 3 shown in Fig. 6.

At the beginning, the UE-SBS pairs generate the set of MISs (Step 2 of the design framework in Section V), and compute the optimal fractions of time allocated to each MIS (Step 3). In our simulation, we assume a block fading model [27] and the fading changes every 100 time slots independently. To reduce complexity, the UEs do not recompute the interference graph and the MISs, but will recompute the optimal fractions of time under the new channel gains every 100 time slots. In Fig. 5, we compare the performance of the proposed policy with state of the art policies under different variances $\beta$ of Rayleigh fading. We do not plot the performance of distributed PMS for this scenario since it is upper bounded by optimal centralized constant power control (when there is one user per cell). We do not plot the distributed MIS STDMA -1 either, when the performance criterion is sum throughput, because it cannot satisfy the minimum throughput constraints. From Fig. 5, we can see that in terms of both sum throughput and max-min fairness, our proposed policy achieves large performance gain (up to 86%) over existing policies, and achieves performance close to the benchmark (as close as 9%).

Selecting the Optimal Interference Graph: For different values of $d$, there can be five possible interference graphs, which are shown in Fig. 6. In Fig. 7 a) we show that as the grid size $d$ decreases ($d = 4.7m$, $d = 3.7m$ and $d = 2.5m$), the levels of interference from the adjacent UEs increases, and as a result, the interference graph with higher degrees perform better (as $d$ decreases, the optimal graph changes from graph 3 to graph 1).
Figure 6. Different interference graphs for the 3 x 3 BS grid.

Graph 1 is optimal for $d=4.74$ m
Graph 2 is optimal for $d=3.70$ m
Graph 3 is optimal for $d=2.52$ m

Minimum throughput achieved by any user (bits/s/Hz)

Fig. 7 a). Comparison of max-min fairness under different grid sizes, b) Sample paths of sum throughput under dynamic entry/exit of UEs in the network

B. Performance scaling in large networks

Consider the uplink of a femtocell network in a building with 12 rooms adjacent to each other. Fig. 8 illustrates 3 of the 12 rooms with 5 UEs in each room. For simplicity, we consider a 2-dimensional geometry. Each room has a length of 6 meters. In each room, there are $P$ uniformly spaced UEs, and one SBS installed on the left wall of the room at a height of 2m. The distance from the left wall to the first UE, as well as the distance between two adjacent UEs in a room,

Figure 8. Illustration of the setup with 3 rooms.
Figure 9. Comparison of max-min fairness and average throughput per UE against state of the art for large networks.

is \( \frac{6}{(1+P)} \) meters. Based on the path loss model in [28], the channel gain from each SBS \( i \) to a UE \( j \) is \( \frac{1}{(D_{ij})^{2\Delta_n}} \), where \( \Delta = 10^{0.25} \) is the coefficient representing the loss from the wall, and \( n_{ij} \) is the number of walls between UE \( i \) and SBS \( j \). Each UE has a maximum transmit power level of 1500 mW and a minimum throughput requirement of \( R_{min} = 0.025 \) bits/s/Hz. Here, we consider that the UEs use a distance based threshold rule as in Section V-B with \( D_{th} = 10 \) m. This results in interference graphs which connects all the UE-SBS pairs within the room and in the adjacent rooms. We vary the number \( P \) of UEs in each room from 5 to 15 and compare the performance in Fig. 9. Note that the optimal centralized constant power policy cannot satisfy the feasibility conditions for any number of UEs in each room. Hence, only the performance of distributed MIS STDMA-1,2 and distributed PMS is shown in Fig. 9. We can see that under both criteria, the performance gain of our proposed policy varies between 22% and 57%. Note that since the number of UEs is large a comparison with the BP (which is NP-hard) is not possible.

C. Self-adjusting mechanism for dynamic entry/exit of the UEs

The self-adjusting mechanism proposed in Subsection V-C is aimed to provide incoming UEs with the maximum possible throughput without affecting the incumbent UEs, and to reuse the time slots left vacant by exiting UEs efficiently. Consider the same setup as in Section VI-B with 3 rooms and a maximum of \( P = 3 \) UEs in each room. Each UE has a maximum transmit power of 1000 mW and a minimum throughput requirement of 0.25 bits/s/Hz.

We assume that at a given time only one UE either enters or leaves the network. In Fig. 7 b) we show the sample paths showing the sum throughput achieved when the number of UEs are changed over time. \( R_{min_{tol}} \) shows the minimum throughput achieved at any point
in the sample path. We can see that the self-adjusting mechanism works well by guaranteeing the minimum throughput requirement of 0.23 bits/s/Hz, which is just 0.02 bits/s/Hz below the guarantee obtained in the scenarios without dynamic entry and exit.

VII. CONCLUSION

We proposed a design framework for distributed interference management in large-scale, heterogeneous networks, which are composed of different types of cells (e.g. femtocell, picocell), have different number of UEs in each cell, and have UEs with different minimum throughput requirements and channel conditions. Our framework allows each UE to have only local knowledge about the network and communicate only with its interfering neighbors. There are two key steps in our framework. First, we propose a novel distributed algorithm for the UEs to generate a set of MISs that span all the UEs. The distributed algorithm for generating MISs requires $O(\log N)$ steps (which is much faster than state-of-the-art) before it converges to the set of MISs with a high probability. Second, we reformulate the problem of determining the optimal fractions of time allocated to the MISs in a novel manner such that the optimal solution can be determined by a distributed algorithm based on ADMM. Importantly, we prove that under wide range of conditions, the proposed policy can achieve a constant competitive ratio with respect to the policy design problem which is NP-hard. Moreover, we show that our framework can adjust to UEs entering or leaving the network. Our simulation results show that the proposed policy can achieve large performance gains (up to 85%).

APPENDIX

Proof of Theorem 1: The success probability of Phase 1 is high, $(1 - \frac{1}{N^{\epsilon_1}})$ (lower bound), (see [24] for detail), here we analyze Phase 2. We first show that, if the list of remaining colors given as, $C_1^n$ is empty at $n \geq \lceil c_1 \log_4 N \rceil + \lceil c_2 \log_x N \rceil + 2$ and if this holds $\forall i \in \{1, ..., N\}$ then the Phase 2 has converged to a set of $H$ MISs which span all the UEs. Let us assume otherwise, i.e. $C_1^n$ is empty $\forall i \in \{1, ..., N\}$ however, the set corresponding to some color $h \in \{1, ..., H\}$, $I'_h$ is not a MIS. $I'_h$ has to be an IS (see [7]). Since $I'_h$ is not maximal then $\exists$ at least one UE-$j \not\in I'_k$ which can be added to this set without violating independence. From the assumption, we have $C_1^n = \phi$ which implies that the color $h$ was deleted at some stage from the original list of all the colors either in Phase 1 or 2. The deletion of $h$ was a result of that color being acquired finally by at least one of the neighbors $k \in \mathcal{N}(j)$ since $j \not\in I'_k$. In that case, $j$ cannot acquire $h$ as it will violate the independence property. Next, we show that indeed the
Table II
GENERATING MISs in a distributed manner, algorithm for UE $i$

| Phase 1 - Initialization: $T_{\text{tent}}^i = \phi$, $T_{\text{final}}^i = \phi$, tentative and final choice of UE $i$, $R_{\text{tent}}^{N(i)} = \phi$, $R_{\text{final}}^{N(i)} = \phi$ tentative and final choice made by the neighbors, $C_i^0 = \{1, \ldots, H\} \cap \{1, \ldots, d_i + 1\}$ the current list of subset of available colors, $C_i = \phi$, list of colors used by $i$, $F_{\text{colored}}^i = \phi$, $C_{1i}^0 = \{1, \ldots, H\}$, the current list of all available colors |
| for $n = 0$ to $\lceil c_1 \log_2 N \rceil$ |
| $T_{\text{tent}}^i = \phi$, $T_{\text{final}}^i = \phi$ |
| if($F_{\text{colored}}^i = \phi$) |
| $T_{\text{tent}}^i = \text{rand}(C_i^0)$, rand represents randomly selecting a color and informing the neighbors about it. |
| $R_{\text{tent}}^{N(i)} = \{T_{\text{tent}}^k, \forall k \in N(i)\}$ |
| If($T_{\text{tent}}^i \neq R_{\text{tent}}^{N(i)}(j)$, $\forall j \in N(i)$), here UE-$i$ checks if there is a conflict with any of the neighbor’s choice |
| $T_{\text{final}}^i = T_{\text{tent}}^i$, $C_i = \{T_{\text{final}}^i\}$ if there is no conflict then UE-$i$ transmits its final color choice to the neighbors, |
| else |
| $T_{\text{final}}^i = \phi$ |
| end |
| $R_{\text{final}}^{N(i)} = \{T_{\text{final}}^k, \forall k \in N(i)\}$ |
| $C_{i+1}^n = C_i^n \cap \{R_{\text{final}}^{N(i)} \cup T_{\text{final}}\}$, $C_1^n = C_1^n \cap \{R_{\text{final}}^{N(i)} \cup T_{\text{final}}\}$ |
| if($T_{\text{final}}^i \neq \phi$) |
| $F_{\text{colored}}^i = 1$ |
| end |
| end |

list of all colors available $C_{1i}^0$ is empty at the end of Phase 2 with a high probability. Let $U^n$ correspond to the number of UEs which have a non-empty list at the beginning of time slot $n$ and, let $T_n(U^n)$ correspond to the total time needed before all the UEs have an empty list. The probability that a UE at time slot $n$ with a non-empty list will have an empty list in next time slot is always greater than $c^H(1-c)^{H^2}$. This can be explained as, if the UE chooses all the colors in the list assuming (worst case $H$ number of colors remain) and all the neighbors (worst case $H$ neighbors) do not choose any color, then all the colors in the UE’s list will be deleted. From this, we get $E(U^{n+1}) \leq (1 - c^H(1-c)^{H^2})U^n = \frac{1}{x}U^n$ and $T_n(U^n) = 1 + T_n(U^{n+1})$. Assuming that the Phase 2 will start with $N$ UEs whose list are non-empty (worst case) and from [29] we get $P(T_n(N) \geq \lceil c_2 \log_x N \rceil) \leq \frac{1}{N^2 - \tau}$. This gives the lower bound on success probability of Phase 2 and thereby the result in the Theorem. (Q.E.D)

Proof of Theorem 2: The two problems which are introduced to transit from CP to DP are,
### Table III

| Phase 2-Initialization: $Tx_{\text{tent},i}^\text{set} = \phi, Tx_{\text{final},i}^\text{set} = \phi$, the set of tentative and final colors chosen by $i$, $Rx_{\text{tent},i}^\text{set} = \phi$, the set of tentative and final colors chosen that are received from the neighbors, $x = \frac{1}{1 - (c)^2 (1 - c)^2}$ | 
|---|---|
| for $n = \lfloor c_1 \log_2 N \rfloor + 1$ to $\lceil c_2 \log_2 N \rceil + 1$ | 
| $Tx_{\text{tent},i}^\text{set} = \phi, Tx_{\text{final},i}^\text{set} = \phi$, for $m = 1$ to $|C1_i^+|$ | 
| with probability $c$, $Tx_{\text{tent},i}^\text{set}(m) = C1_i^+(m)$, randomly selecting and informing the neighbors about tentative choice | 
| with probability $1 - c$, $Tx_{\text{tent},i}^\text{set}(m) = \phi$ | 
| end | 
| $Rx_{\text{tent},i}^\text{set} = \cup_{k \in N(i)} Tx_{\text{tent},k}^\text{set}$, set of tentative color choices of the neighbors of $i$ | 
| for $r = 1$ to $|Tx_{\text{tent},i}^\text{set}|$ | 
| If($Tx_{\text{tent},i}^\text{set}(r) \neq Rx_{\text{tent},i}^\text{set}(j) \forall j \in N(i)$) | 
| $Tx_{\text{final},i}^\text{set}(r) = Tx_{\text{tent},i}^\text{set}(r)$ | 
| else | 
| $Tx_{\text{final},i}^\text{set}(r) = \phi$ | 
| end | 
| $C_i = C_i \cup Tx_{\text{final},i}^\text{set}$ | 
| $Rx_{\text{final},i}^\text{set} = \cup_{k \in N(i)} Tx_{\text{final},k}^\text{set}$, set of final color choices of the neighbors of $i$ | 
| $C1_i^{n+1} = C1_i^n \cap \{Rx_{\text{final},i}^\text{set} \cup Tx_{\text{final},i}^\text{set}\}$ | 
| end | 

### Table IV

ADMM Update Algorithm for UE $i$

| Initialization: $\beta_i(0) = \beta_1^\text{init}, \beta_i^\text{init} \in \mathcal{T}_i$, and it can be chosen arbitrarily and $\theta_i^k(0) = \theta_i^\text{init}, \forall k \in \{1, \ldots, H\}, \forall e \in \tilde{E}(i)$, $\theta_i^\text{init} \in \Theta_i^k$ and $\tilde{E}(i)$ is the set of edges which have $i$ as an end point and set $\lambda_{e,i}^k(0) = 0, \forall e \in \tilde{E}(i), \forall k \in \{1, \ldots, H\}$ | 
| --- | 
| For $t=0$ to P-1 | 
| $\beta_i(t + 1) = \arg\min_{\beta_i \in \mathcal{T}_i} - (W_i(R_i^t, \beta_i)) - \sum_{k=1}^H \sum_{e \in E(i)} \lambda_e^k(t)D_{ei}\beta_i^k + \frac{\rho}{2} \sum_{k=1}^H \sum_{e \in E(i)} (D_{ei}\beta_i^k - \theta_i^k(t))^2$ | 
| $\beta_i(t + 1)$ is transmitted to all the interfering neighbors, $N(i)$ | 
| $\lambda_{e,i}^k(t)$ is transmitted to the neighbor with the edge $e$, $\forall k \in \{1, \ldots, H\}$ and $\forall e \in \tilde{E}(i)$ | 
| Update $\forall k \in \{1, \ldots, H\}$ and $\forall e \in \tilde{E}(i)$ | 
| $\lambda_{e,i}^k(t + 1) = \frac{1}{2}(\lambda_{e,i}^k(t) + \lambda_{j,i}^k(t)) - \frac{1}{2} (D_{ei}\beta_i^k(t + 1) + D_{ej}\beta_j^k(t + 1))$, here $j$ is the other end point of $e$. | 
| $\theta_i^k(t + 1) = \frac{1}{2} (\lambda_{e,i}^k(t + 1) - \lambda_{e,i}^k(t)) + D_{ei}\beta_i^k(t + 1)$ | 
| end |
Global Primal Problem (GPP) \[ \text{max} \sum_{k=1}^{H} \sum_{i=1}^{N} \beta_{i}^{k} W_{i}(\sum_{i=1}^{N} \beta_{i}^{k} R_{i}^{k}) \]
subject to \[ \sum_{k=1}^{H} \beta_{i}^{k} R_{i}^{k} \geq R_{i}^{\text{min}}, \sum_{k=1}^{H} \beta_{i}^{k} = 1, \forall i \in \{1, ..., N\} \]
\[ \beta_{i}^{k} = \beta_{i}^{l}, \forall i \neq l, \forall k \in \{1, ..., H\}, \beta_{i}^{k} \geq 0, \forall i \in \{1, ..., N\}, \forall k \in \{1, ...H\} \]

The second problem, Local Primal Problem (LPP) is the same as GPP except we choose a subset of the constraints from the above problem. Basically, instead of an equality constraint between the UE’s estimate and every other UE in the network, we only keep the equality constraints between the UE and its neighbors, i.e. \( \beta_{i}^{k} = \beta_{i}^{l}, \forall k \in \{1, ..., H\}, \forall l \in N(i) \). To show that problems GPP and CP are equivalent, we need to show that from \( \beta^{*} = (\beta_{1}^{*}, ..., \beta_{N}^{*}) \), an optimal argument of GPP, we can obtain an optimal argument of CP, i.e. \( \alpha^{*} \) and vice versa. Since \( \beta^{*} \) is the optimal value (assuming feasibility) we know that \( \beta_{i}^{*} = \beta_{j}^{*} \) (component-wise) holds \( \forall i, j \in \{1, ..., N\} \). Let \( \alpha^{'} = \beta_{i}^{*} \). \( \alpha^{'} \) satisfies the constraints in CP. The objective of CP at \( \alpha^{'} \) attains the optimal value of GPP. We need to establish that \( \alpha^{'} \) is indeed the optimal argument of CP. Assume that \( \alpha^{'} \) is not the optimal value, then there exists another \( \alpha^{*} \) which is indeed the optimal. Next, using \( \alpha^{*} \), we can obtain another \( \beta^{'} \) as follows, \( \beta_{1}^{*} = \alpha^{*} \text{ and } \beta_{i}^{*} = \beta_{i}^{1}, \forall i \in \{1, ..., N\} \). The objective of GPP at \( \beta^{'} \) should be higher than \( \beta^{*} \) which contradicts \( \beta^{*} \) being the optimal argument. Note that if either of CP or GPP is infeasible then the other problem can be shown to be infeasible as well. On the same lines we can show that from an \( \alpha^{*} \) we can obtain \( \beta^{*} \) as well. To show that GPP and LPP are equivalent, we use the following fact, since LPP consists of a subset of the constraints then the solution of LPP is upper bound to the solution to GPP. We need to show that the gap between the solution of LPP and GPP is always 0. Note that for an optimal solution of LPP, \( \gamma^{*} = (\gamma_{i}^{*}, ..., \gamma_{N}^{*}) \) we know that \( \gamma_{i}^{*} = \gamma_{j}^{*} \) (component-wise). If we can show that \( \gamma_{i}^{*} = \gamma_{j}^{*} \) \( \forall j \in \{1, ..., N\} \) then LPP and GPP will be equivalent. Assume that this does not hold then \( \exists i, j \) such that \( \gamma_{i}^{*} \neq \gamma_{j}^{*} \). Since, the interference graph is connected \( \exists \) a path \( i \rightarrow j = \{i_{1}, ..., i_{s}\} \) which implies, \( \gamma_{i_{1}}^{*} = \gamma_{i_{2}}^{*} ... = \gamma_{j}^{*} \). This leads to a contradiction, thereby establishing the claim. Lastly, we have to show that DP is equivalent LPP. Given \( \gamma^{*} \) the optimal solution of LPP, define \( \kappa = \gamma^{*} \). Let \( \theta = (\theta^{1}, ..., \theta^{H}) \) which satisfies \( D^{k} \kappa^{k} - \theta^{k} = 0, \forall k \in \{1, ..., H\} \), where \( \kappa^{k} = (\gamma_{i_{1}}^{k}, ..., \gamma_{N}^{k}) \). It can be shown using the same approach as we did for GPP and CP that \( (\kappa, \theta) \) is indeed optimal argument for DP. (Q.E.D)
**Proof of Theorem 3:** We only need to show that the assumptions in [26], namely the feasibility of DP, along with compactness of $\mathcal{T}, \Theta$ to ensure convergence at rate $O(1/T)$. Both $\mathcal{T}, \Theta$ are closed and bounded polyhedron implying that they are compact. (Q.E.D)

**Proof of Theorem 4:** Here, we need to show three things, 1. if $\Delta \leq \Delta^{max}$ then the distributed policy yields a feasible solution, 2. the size of any MIS is $\geq \frac{N}{\Delta+1}$, thereby using this to show that the distributed policy, if feasible will yield a network performance of at least $\frac{N}{\Delta+1} \log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})$ and 3. the upper bound on the network performance, sum throughput here is $N \log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})$. In the Phase I of the algorithm the maximum number of colors used is $\Delta+1$, since each UE selects colors from subset of $\{1, ..., H\} \cap \{1, ..., d_i + 1\}$. The first $\Delta+1$ output MISs, $\{I'_1, ..., I'_{\Delta+1}\}$ span all the UEs in the network. If the fraction of time assigned to each of these $\Delta+1$ MISs is, $\frac{R_{ub}^{min}}{\log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})}$ then such an assignment is feasible since $\Delta \leq \Delta^{max} \Rightarrow (\Delta + 1) \frac{R_{ub}^{min}}{\log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})} \leq 1$. Using the fact that network exhibits $\zeta-$WNI we get the minimum instantaneous throughput that can be obtained by UE-$i$ as, $\log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})$, and minimum instantaneous throughput of any UE as, $\log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})$. Thus, each UE $i$’s throughput requirement is satisfied, $\frac{R_{ub}^{min}}{\log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})} \geq R_{ub}^{min}$. Assume that $\exists$ an MIS whose size is $S < \frac{N}{\Delta+1}$. Each UE in the MIS can exclude a maximum of $\Delta$ UEs from being included in the MIS. This implies that $S(\Delta+1)$, represents the total number of UEs excluded and the UEs in the MIS which put together should exceed $N$. Since this is not the case here, this leads to contradiction, hence $S \geq \frac{N}{\Delta+1}$. This combined with minimum instantaneous throughput of any UE, we get the lower bound $\frac{N}{\Delta+1} \log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})$ for our policy. The upper bound on the optimal network performance is obtained by summing maximum instantaneous throughput of any UE $\log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})$ for all UEs, $N \log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})$. Computing the ratio of the lower bound of proposed scheme $\frac{\log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})}{\log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})}$ which is no less than, $\frac{R_{ub}^{min}}{\log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})}$ since $\Delta \leq \Delta^{max}$. (Q.E.D)

**Proof of Theorem 5:** Let $\Delta^* = 6\eta$ with $\eta = \frac{\log_2(1 + \frac{1}{(D_{ub})^{np_2}p_2\sigma^2}) R_{ub}^{min}}{p_{ub}^{max}}$. We assume that the interference graph is constructed using a distance threshold rule (Subsection V-B). Note that each UE’s minimum throughput requirement is at least $R_{ub}^{min}$, this combined with maximum instantaneous throughput of any UE $\log_2(1 + \frac{p_{ub}^{max}}{(D_{ub})^{np_2}p_2\sigma^2})$, yields that each UE needs at least
fraction of time slots. First, we need to show that if there exists a clique (subset of UEs which are mutually connected) in the interference graph of size, \( X \) greater than \( \eta \) then the minimum throughput constraints cannot be satisfied. Assume that there does exist such a clique, then any MIS based scheduling policy will allocate separate time slots to each UE in the clique. This implies that \( X \frac{R_{\text{min}}}{\log_2(1 + \frac{p_{\text{ub}}}{(D_{lb}^\delta)^{\beta_{\text{SBS}}}})} \) is the total fraction separate time slots needed which has to be less than 1. But as \( X > \eta \), this leads to infeasibility. Next, if \( \Delta \geq \Delta^* \), we will have at least one clique satisfying this condition. If \( \Delta \geq \Delta^* \exists \) UE-\( i \) with a degree \( d_i \geq 6\eta \), this implies that within a radius of \( D^{th} \) around SBS-\( T(i) \) \( 6\eta \) SBSs. This circle around SBS-\( T(i) \) can be partitioned into 6 sectors subtending \( \pi/3 \) at the center. The distance between any two points located in the sector is \( \leq D^{th} \) (See [7]), thus any two points in the sector are connected. If we have more than \( 6\eta \) SBSs in the circle then at least one sector has to have more than \( \eta \) SBSs (Pigeonhole principle), which implies that a clique of size \( X > \eta \) will exist. (Q.E.D)

REFERENCES

[1] E. Hossain, L. B. Le, and D. Niyato, “Self-organizing small cell networks,” in Radio Resource Management in Multi-Tier Cellular Wireless Networks. Hoboken, NJ, USA: John Wiley Sons, Inc., 2013.

[2] A. Ghosh, N. Mangalvedhe, R. Ratasuk, B. Mondal, M. Cudak, E. Visotsky, T. A. Thomas, J. G. Andrews, P. Xia, H. S. Jo et al., “Heterogeneous cellular networks: From theory to practice,” Communications Magazine, IEEE, vol. 50, no. 6, pp. 54–64, 2012.

[3] J. G. Andrews, H. Claussen, M. Dohler, S. Rangan, and M. C. Reed, “Femtocells: Past, present, and future,” Selected Areas in Communications, IEEE Journal on, vol. 30, no. 3, pp. 497–508, 2012.

[4] R. Ramaswami and K. K. Parhi, “Distributed scheduling of broadcasts in a radio network,” in INFOCOM’89. Proceedings of the Eighth Annual Joint Conference of the IEEE Computer and Communications Societies. Technology: Emerging or Converging, IEEE. IEEE, 1989, pp. 497–504.

[5] I. Cidon and M. Sidi, “Distributed assignment algorithms for multihop packet radio networks,” Computers, IEEE Transactions on, vol. 38, no. 10, pp. 1353–1361, 1989.

[6] A. Ephremides and T. V. Truong, “Scheduling broadcasts in multihop radio networks,” Communications, IEEE Transactions on, vol. 38, no. 4, pp. 456–460, 1990.

[7] K. Ahuja, Y. Xiao, and M. van der Schaar, “Online report for distributed interference management in smallcell networks,” 2014. [Online]. Available: [http://medianetlab.ee.ucla.edu/papers/report_smallcell.pdf]

[8] J. Huang, R. A. Berry, and M. L. Honig, “Distributed interference compensation for wireless networks,” Selected Areas in Communications, IEEE Journal on, vol. 24, no. 5, pp. 1074–1084, 2006.

[9] C. U. Saraydar, N. B. Mandayam, and D. Goodman, “Pricing and power control in a multicell wireless data network,” Selected Areas in Communications, IEEE Journal on, vol. 19, no. 10, pp. 1883–1892, 2001.

[10] E. J. Hong, S. Y. Yun, and D.-H. Cho, “Decentralized power control scheme in femtocell networks: A game theoretic approach,” in Personal, Indoor and Mobile Radio Communications, 2009 IEEE 20th International Symposium on, Sept 2009, pp. 415–419.
[11] M. Chiang, C. W. Tan, D. P. Palomar, D. O’Neill, and D. Julian, “Power control by geometric programming,” Wireless Communications, IEEE Transactions on, vol. 6, no. 7, pp. 2640–2651, 2007.

[12] S. Stanczak, M. Wiczanowski, and H. Boche, Fundamentals of resource allocation in wireless networks: theory and algorithms. Springer, 2009, vol. 3.

[13] J. Ling, D. Chizhik, and R. Valenzuela, “On resource allocation in dense femto-deployments,” in Microwaves, Communications, Antennas and Electronics Systems, 2009. COMCAS 2009. IEEE International Conference on. IEEE, 2009, pp. 1–6.

[14] G. J. Foschini and Z. Miljanic, “A simple distributed autonomous power control algorithm and its convergence,” Vehicular Technology, IEEE Transactions on, vol. 42, no. 4, pp. 641–646, 1993.

[15] S. G. Kiani and D. Gesbert, “Optimal and distributed scheduling for multicell capacity maximization,” Wireless Communications, IEEE Transactions on, vol. 7, no. 1, pp. 288–297, 2008.

[16] D. Gesbert, S. G. Kiani, A. Gjendemsjo et al., “Adaptation, coordination, and distributed resource allocation in interference-limited wireless networks,” Proceedings of the IEEE, vol. 95, no. 12, pp. 2393–2409, 2007.

[17] K. Jain, J. Padhye, V. N. Padmanabhan, and L. Qiu, “Impact of interference on multi-hop wireless network performance,” Wireless networks, vol. 11, no. 4, pp. 471–487, 2005.

[18] H. Li, X. Xu, D. Hu, X. Qu, X. Tao, and P. Zhang, “Graph method based clustering strategy for femtocell interference management and spectrum efficiency improvement,” in Wireless Communications Networking and Mobile Computing (WiCOM), 2010 6th International Conference on. IEEE, 2010, pp. 1–5.

[19] S. Uygungelen, G. Auer, and Z. Bharucha, “Graph-based dynamic frequency reuse in femtocell networks,” in Vehicular Technology Conference (VTC Spring), 2011 IEEE 73rd. IEEE, 2011, pp. 1–6.

[20] P. Lee, T. Lee, J. Jeong, and J. Shin, “Interference management in lte femtocell systems using fractional frequency reuse,” in Advanced Communication Technology (ICACT), 2010 The 12th International Conference on, vol. 2. IEEE, 2010, pp. 1047–1051.

[21] C. W. Tan, S. Friedland, and S. H. Low, “Spectrum management in multiuser cognitive wireless networks: Optimality and algorithm,” Selected Areas in Communications, IEEE Journal on, vol. 29, no. 2, pp. 421–430, 2011.

[22] K. Han, S. Woo, D. Kang, and S. Choi, “Automatic neighboring bs list generation scheme for femtocell network,” in Ubiquitous and Future Networks (ICUFN), 2010 6th International Conference on. IEEE, 2010, pp. 251–255.

[23] D. Lopez-Perez, I. Guvenc, G. De La Roche, M. Kountouris, T. Q. Quek, and J. Zhang, “Enhanced intercell interference coordination challenges in heterogeneous networks,” Wireless Communications, IEEE, vol. 18, no. 3, pp. 22–30, 2011.

[24] Ö. Johansson, “Simple distributed δ < i > δ/δ+i 1-coloring of graphs,” Information Processing Letters, vol. 70, no. 5, pp. 229–232, 1999.

[25] D. P. Bertsekas and J. N. Tsitsiklis, Parallel and distributed computation: numerical methods. Prentice-Hall, Inc., 1989.

[26] E. Wei and A. Ozdaglar, “On the o (1/k) convergence of asynchronous distributed alternating direction method of multipliers,” arXiv preprint arXiv:1307.8254, 2013.

[27] A. Goldsmith, Wireless communications. Cambridge university press, 2005.

[28] S. Y. Seidel and T. S. Rappaport, “914 mhz path loss prediction models for indoor wireless communications in multifloored buildings,” Antennas and Propagation, IEEE Transactions on, vol. 40, no. 2, pp. 207–217, 1992.

[29] R. M. Karp, “Probabilistic recurrence relations,” Journal of the ACM (JACM), vol. 41, no. 6, pp. 1136–1150, 1994.