Towards Quantum Enigma Cipher III
-Communication performance-

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Abstract—Cloud computing system based on data centers has recently attracted considerable attention. In that system, all data are communicated via a high speed optical network between a customer and data center or between data centers. There is a serious threat so called “Eavesdropper data center business”, which means the eavesdropper can get all data from the transmission line and sell specific data selected by the protocol analyzer to malicious people who want to get the secret data. So we need to consider cyber attack against Layer-1 (physical layer). Quantum cryptography has been developed to protect such an attack. In order to apply such a new security technologies, the communication performance is very important as well as its security, because the data speed is more than several Gbit/sec. This research note III will discuss communication performances of quantum key distribution (QKD) and quantum enigma cipher, and explains that QKD based on single photon signals cannot realize appropriate data speed, but quantum enigma cipher can.

I. INTRODUCTION

It has been claimed by many research institutes that quantum key distribution allows two remote users to generate a secure random key for practical networks. However, in reality, this is not true, because communication performance of QKD is extremely poor. The reason is that they have to employ single photon or very weak light as the transmission signal. QKD researchers would like to simultaneously reach long distances and high rates, but it is impossible by the law of physics. In order to avoid this problem, they employed a theory of a rate-loss tradeoff in which they restrict the meaning of “rate” as the term. That is, their rate means bit per pulse or mode, not bit per sec that is the most essential for communication performance. In general, such an unit has to be discussed under the non vanishing situation of signals in the communication process. Let us describe a confusion in the following:

[To achieve QKD with high rates is demanded if we compete with the the conventional infrastructure for fiber based classical communication where the general IT network sets rate as high as several Gbit/sec for distance up to several hundred km. So there has been an effort towards high-rate QKD.]

Team of “high rate” in the above QKD and Gbit/sec of fiber link has no relation in this case. QKD by single photon signal cannot realize a secure communication for Gbit/sec as data. Despite the fact, QKD group uses ambiguous term. An example of the typical trick is a paper entitled “Quantum secured gigabit optical access networks”[1] by A.Shields et al (Toshiba group).” It is clear that there is a mismatch between title and contents.

Recently MIT groups have pointed out that such a description on QKD is not fair. They clearly describe the communication performance as follows [2,3,4]:

[QKD would enable full IT security by means of one time pad, but its secret key rate has to equal to the desired communication rate (bit/sec). Unfortunately demonstrated key rates fall far short of what is needed for widespread use of one time pad. At present, therefore, QKD’s principal application is to rekey classical cryptosystems, making overall security that of the classical system. In fact, numerous QKD protocols have demonstrated robust extraction of secret key over 200 km with key rates up to the order of 10 bit/sec.]

As general users pointed out many times, when we consider secure communication systems, we have to take into account the following requirements on the encryption system to protect the data of gigabit per sec:

Requirement of specifications:
(1) Data-speed: 1 Gbit/sec ∼ 100 Gbit/sec
(2) Distance: 1000 km ∼ 10000 km
(3) Encryption scheme: Symmetric Key Cipher
(4) Security: Provable security, Secure against Brute force attack (exhaustive search trial for secret key) by means of computer and also physical devices.

In this note, we will reaffirm that QKD cannot provide such performance, but quantum enigma cipher [5] can.

II. BASIS OF COMMUNICATION PERFORMANCE

A. Single photon communication

Let us describe the energy loss channel for photon number. The input-output relation for the photon channel of transmissivity κ is given as follows:

\[ P(n|k) = nC_k(\kappa)^k(1-\kappa)^{n-k} \]  (1)
where \( n \) and \( k \) are photon number at input and output, respectively. For the single photon signal, the arrival probability of the photon at the channel output is 

\[
P(k = 1) = \kappa
\]

Thus, the rate of speed becomes 

\[
R_S = \kappa
\]

The loss property of an optical fiber with low loss is 0.2 dB/km for the wavelength 1.55 \( \mu \)m. This means \( \kappa = 0.01 \) for the fiber length 100km. Before we start QKD protocol, the rate of the channel is already \( \kappa = 0.01 \). So, when the input rate of this channel is 1 Gbit/sec, the output rate is 10 Mbit/sec.

**B. Coherent state communication**

Let us consider the transmission of coherent state signal. There is an important theorem as follows:

**Theorem [6]:** Only coherent state can maintain the pure state property against energy loss channel.

Thus, the arrival probability of signal itself is 

\[
P(\text{pulse}) = 1 - |<0|\alpha >_{\text{out}}|^2 = 1 - \kappa |\alpha|^2
\]

\[
\approx 1 - \kappa |\alpha|^2 >> 1
\]

where \( |\alpha >_{\text{out}} \) is the coherent state at the output of the channel, and \( \alpha \) is the input amplitude of coherent state signal. In the conventional optical communications, \( |\alpha|^2 = 10^6 \) at 1 Gbit/sec. Thus, when the input rate is 1 Gbit/sec, the output is also 1 Gbit/sec.

**C. PPM communication**

The pulse position modulation (PPM) is one of modulation schemes in communication systems. A typical structure used in quantum optical communication is

\[
|\Psi_i >= |0 >_1 |0 >_2 \ldots |\alpha >_i |0 >_{i+1} \ldots |0 >_N
\]

where \( i \) and \( N \) mean ith signal and slot length, respectively. That is, the information \( i \) as the data corresponds to the position of the coherent state with non zero amplitude. The total energy per symbol is \( <n> = |\alpha|^2 \). So, the energy per pulse becomes \( |\alpha|^2/N \). Thus, one symbol in this scheme can convey \( \log_2 N \) bits under the low average energy. The arrival probability of symbol is

\[
P(\text{symbol}) = 1 - |<0|\alpha >_{\text{out}}|^2 = 1 - \kappa |\alpha|^2
\]

It looks very efficient in the sense of the energy constraint when we use \( N >> 1 \). However, it is not efficient in the sense of frequency bandwidth, because this requires \( N \) times of the bandwidth of the conventional communication systems. So far, many ideas have published to improve the bandwidth explosion, keeping a good energy efficiency. The most famous method is to employ Reed-Solomon code. Even though such coding theories can improve the bandwidth problem, still there are many serious inefficiencies in the total performance of communications.

**III. Communication performance of QKD based on single photon or weak light**

**A. Rate theory of QKD**

The rate theory is one of the most important subjects in the theoretical QKD. So far, the theoretical analysis of rate for QKD has been dealt with the unit of bit per pulse or mode. Recently, the incomplete bound [7] of the rate has been replaced by the rigorous and beautiful analysis [8] as follows:

\[
R = \log \frac{1}{1 - \kappa} \sim 1.44 \kappa \text{ bit/pulse}
\]

This means that when the input rate is 1 Gbit/sec, the output under the ultimate efficiency is

(i) 100 Kbit/sec for 100 km,
(ii) 10 bit/sec for 200 km.

This fact supports the comment of MIT paper [2-4] refered in the introduction of this paper. Thus, QKD has a serious defect in the sense of communication performance. To date, nobody is interested in such an inefficient communication system. Thus, the author cannot understand how the QKD system can provide the security for “gigabit access network [1]”. The real situation is that QKD experiment has been demonstrated in the conventional fiber transmission system of gigabit data communication. Thus their title is misleading. In fact, the headquarter of Toshiba believed that real gigabits data has been encrypted by quantum system.

**B. Cryptosystem based on QKD**

(1) AES+QKD

It is clear that the QKD is the most inefficient scheme in modern communication systems, so we have to use the QKD for providing the secret key of a mathematical encryption such as AES (Advanced Encryption Standard). AES has in general the secret key of 256 bits. QKD can provide 256 bits with the delay time of 26 seconds, because the bit rate is 10 bits/sec when the communication distance is 200 km. Thus we need 26 seconds for one round of refresh key. AES operating at 1 Gbit/sec sends 26 Gbits during each 26 seconds based on the previous key. The eavesdropper can get the correct ciphertext of 26 Gbits from the transmission line. This is sufficient to launch the crypto-analysis. In addition, AES can be decrypted by the Brute
force attack under the correct cipher text and plaintext of 256 bits. This means that the plaintext (data) of 26 Gbits -256 bits can be decrypted by only 256 bits, in principle.

(ii) Transmission systems can send 1 Gbits/sec

To encrypt 1 Gbits, we have to store the generated key sequence of 1 Gbits in the Hard-Disk by spending $10^8$ seconds (3 years). Then 1 Gbits is sent at 1 second. Again we have to wait $10^8$ seconds (3 years) to send the next 1 Gbits.

IV. COMMUNICATION PERFORMANCE OF QUANTUM ENIGMA CIPHER

A. Rate of quantum enigma cipher by coherent state

As discussed in the previous section, the signal does not disappear in the coherent state communication with large energy like typical classical communication system. Since the quantum enigma cipher (QEC) can employ such large energy as the transmission signal, the rate of speed is

$$R_S \sim 1$$  \hspace{1cm} (10)

So, in general, the speed at input of the channel can be kept at the output. This performance is very important to apply physical cipher to the real network. To derive the real rate of quantum enigma cipher in such cases, we will need the information theoretical rate theory. It depends on noise, modulation scheme, error correcting code, scheme of mathematical encryption box, and randomization scheme. However, IEEE standard for the optical network requires error free in point to point link, for example,

$$P_e(Bob) \sim 10^{-9}$$  \hspace{1cm} (11)

with no time delay. It means that the real application of physical cipher to IEEE standard link requires the following rate:

$$R_E = 1$$  \hspace{1cm} (12)

The quantum enigma cipher will satisfy the requirements described in the introduction of this paper and the above requirement. So,

$$R_E(QEC) \sim 1$$  \hspace{1cm} (13)

though it depends on the implementation method.

B. Cryptosystem by quantum enigma cipher

The quantum enigma cipher consists of an integration of mathematical encryption box and physical randomization for ciphertext of mathematical encryption box [5]. The mathematical encryption box has a secret key of the length $|K_e|$ bits and PRNG for expansion of the secret key. The physical encryption box has a mechanism to create ciphertext as signal and it has a function to induces an error when the eavesdropper receives the ciphertext as signal. Consequently the different ciphertext sequences are observed in the legitimate’s receiver and the eavesdropper’s receiver, respectively. The quantum mechanics plays a fundamental role to create the following situation.

$$P_e(Eve) \gg P_e(Bob \text{ or } Alice) \sim 10^{-9}$$  \hspace{1cm} (14)

There are many implementation methods to realize quantum enigma cipher. The communication performances may especially depend on the structures of both mathematical encryption box and physical randomization. But since we can employ a high power coherent state signals for the quantum enigma cipher, we have no signal arrival probability problem. In addition, it may have error free communication. As a result, in general the quantum enigma cipher does not degrade any input rate. Thus, we can simply replace the tranceiver in the conventional optical communication infrastructure (fiber or space) by the quantum enigma cipher tranceiver. This is also very important in the real world network.

Let us denote some examples. Schemes [9,10] based on difference of quantum detection performance and quantum illumination [11,3] do not degrade the rate, because the rate is kept in the physical randomization in the such simplest cascade schemes.

However, a scheme by PPM signal structure may have a tradeoff for security-rate [12]. In the case of quantum noise randomized stream cipher $\alpha/\eta$[13,14] or Y-00 [15,16], basically the rate is not degraded through the encryption process. However, when the masking effect to randomize the ciphertext is very small in the real setting, we have to employ additional randomization methods or a new technique. Although well known randomizations [17,18] do not degrade the input rate, some methods such as deliberate randomization in the data stream may degrade it. In this case, we have to design the randomization carefully.

V. CONCLUSION

This note has introduced the communication performance of QKD and quantum enigma cipher which is essential for application of all quantum cryptography to the real world. Consequently, it has been explained that the communication performance of QKD by single photon has no prospect. This fact had been confirmed already in 2003 by several governments. However, the development of such QKDs was tolerated by such governments under the several reasons. Now time has come to change the
direction of the development of a quantum cryptography with a new concept, because the ciber attack against physical layer of networks become a real possibility at present.

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