Where are the beryllium neutrinos?

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Abstract

We show that present experiments imply that neutrinos are nonstandard at the 87% C.L., independently of solar or nuclear physics. Moreover, if neutrinos are standard, the $^7$Be flux must be almost zero. Even if we arbitrarily disregard one of the experiments, the neutrino flux must still be less than half of the value predicted by standard solar models.

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It is a widespread statement that the Solar Neutrino Problem (SNP) is already at the level of $^7$Be neutrinos, and it does not concern anymore just the rare, hard to predict $^8$B neutrinos.

In this letter we intend to show clearly what is the essence of the problem, by relating directly the results of solar neutrino experiments to the fluxes of the $^7$Be, $^8$B and CNO neutrinos ($\Phi_{^7\text{Be}}$, $\Phi_{^8\text{B}}$ and $\Phi_{\text{CNO}}$) under the hypothesis that neutrinos are standard (i.e. no mass, no mixing, no magnetic moment, ...).

We shall employ methods similar to the ones we introduced in Ref. [1], which have also been considered by other authors [2–7], and keep into account the most recent experimental results [8–11]. Specifically, we shall address the following questions: (1) What can be concluded about $\Phi_{^7\text{Be}}$ from the present experimental data for standard neutrinos? (2) At which confidence level (C.L.) we can claim that either neutrinos are nonstandard or experimental results are incorrect? (3) If neutrinos are standard and experimental results are correct, what is the predicted range for $\Phi_{^7\text{Be}}$? (4) Can future experiments aimed at the measurement of the beryllium flux distinguish standard neutrinos from the Mikheyev-Smirnov-Wolfenstein (MSW) solution?

We have to qualify in some detail the expression “experimental results are correct”. In the rest of the paper, we discuss separately the case when we assume all experimental results to be correct, and when one of them is disregarded. We use the standard procedure of quadratically combining systematical and statistical errors for estimating confidence limits. Alternative procedures are not obviously superior, and give similar results as long as one does not go in the tails of the distributions. Thus the sentence “correct experimental result” includes the statements that errors are correctly evaluated by experimentalists, and correctly treated by us.

Our discussion is practically independent of solar models, both standard and non-standard ones. Our main (very reasonable) assumption about the Sun is that the present neutrino flux can be derived from the present value of the solar constant (stationary Sun). We also use the (very weak) assumption that the ratio $x \equiv \Phi_{\text{pep}}/\Phi_{\text{pp+pep}}$ is the same as in the Standard Solar Model (SSM), i.e. $x_{\text{SSM}} = 2.38 \times 10^{-3}$. In fact, we made solar models with input parameters that vary wildly out of the range defining the SSM, and found that this ratio is practically constant. Similarly, we take $y = \Phi_{N}/\Phi_{\text{CN}} = y_{\text{SSM}} = 0.54$, and we neglect the hep and fluorine neutrinos.

Consequently, for standard neutrinos, i.e. neutrinos whose differential flux is conserved independently of their energy from production to detection, there remain only four unknowns:

$$\Phi_{^7\text{Be}}, \quad \Phi_{^8\text{B}}, \quad \Phi_{\text{pp+pep}} \quad \text{and} \quad \Phi_{\text{CN}}.$$  \hfill (1)

These four unknowns are constrained by exactly four equations, if all three experimental data are correct and the Sun is stationary [12].

Firstly, we have the luminosity equation

$$K = \sum_i \left( \frac{Q}{2} - \langle E \rangle_i \right) \Phi_i,$$  \hfill (2)

where $K$ is the solar constant, $Q$ is the energy released in the fusion reaction $4p + 2e \rightarrow \alpha + 2\nu$, and $\langle E \rangle_i$ is the average neutrino energy of the $i$th flux $\Phi_i$. Neglecting the contribution of
boron neutrinos, which is expected to be even smaller than the error on $K$ (of the order of 0.2%), and substituting the numerical constants, Eq. (2) becomes:

$$65.531 = 0.980 \times \Phi_{pp+pep} + 0.939 \times \Phi_{Be} + 0.937 \times \Phi_{CN},$$

where all fluxes, here and in the following, are in units of $10^9 \text{cm}^{-2} \text{s}^{-1}$. We use this equation to express $\Phi_{pp+pep}$ as function of the other unknown fluxes.

Therefore, the gallium experiments measure the combination:

$$S^{Ga} = (79.75 + 2.43 \cdot 10^3 \times \Phi_B + 6.14 \times \Phi_{Be} + 7.49 \times \Phi_{CN}) \text{ SNU},$$

where SNU denotes solar neutrino units. The weighting factors are given by the cross sections for neutrino capture on gallium nuclei. The uncertainties as a function of energy of these cross sections give an overall uncertainty of $\pm 3\% [13]$. We use as gallium experimental data the average between GALLEX and SAGE data:

$$S^{Ga}_{exp} = (74.4 \pm 9) \text{ SNU}.$$  

Since this error is definitely larger than the one on the neutrino-capture cross section, we ignore this latter uncertainty.

With the same notations, the chlorine experiment measures the combination:

$$S^{Cl} = (0.247 + 1.09 \cdot 10^3 \times \Phi_B + 0.236 \times \Phi_{Be} + 0.396 \times \Phi_{CN}) \text{ SNU},$$

where the cross section for $^8\text{B}$ neutrino capture on Cl has been taken from Ref. [14]. This time there is an overall uncertainty of $\pm 1\%$. The corresponding experimental result is:

$$S^{Cl}_{exp} = (2.32 \pm 0.23) \text{ SNU}.$$  

Again we note that the error on the neutrino-capture cross section can be neglected.

Finally, the Kamiokande experiment measures the $^8\text{B}$ neutrino flux:

$$\Phi_B = (2.9 \pm 0.42) \times 10^{-3}$$

We can substitute the experimental data, Eqs. (3), (5) and (8), into the two equations (4) and (7), and solve them:

$$\Phi_{Be} = 4.9 \pm 3.4 \quad \text{and} \quad \Phi_{CN} = -5.7 \pm 5.6.$$  

On the other hand, fluxes cannot be negative, so we stick to the most favourable case $\Phi_{CN} = 0$, i.e. the one that allows the highest $^7\text{Be}$ flux.

Figure 1 shows the areas allowed at $1\sigma$ in the $(^8\text{B}, ^7\text{Be})$ plane for $\Phi_{CN} = 0$. We note that if all experiments are correct, the allowed areas intersect in the unphysical region ($\Phi_{Be} \leq 0$). Even if we disregard one of the three experimental results the physical region, $\Phi_{Be} > 0$, looks disfavored. At this point, we must point out that we are just demanding the fluxes to be positive to say that the region is physical. The introduction of any additional physical information will further constrain the allowed region, e.g. the knowledge about the nuclear reaction producing the $^8\text{B}$ neutrinos limits the values of the $^8\text{B}$ flux given the $^7\text{Be}$ flux.
Thus we conclude that it appears unlikely that neutrinos are standard and at least two out of three experimental results are correct.

In an attempt to quantify this assertion, we make a chi-square analysis. We take $\Phi_B$ and $\Phi_{Be}$ as free parameters, whereas we assume $\Phi_{CN} = 0$, the case most favorable for standard neutrinos. We evaluate the iso-$\chi^2$ curves both for the case when all experiments are taken into account, and for the cases when one of them is disregarded in turn (see Fig. 2). Statistics tells us, under the standard hypothesis, the probability that the fluxes be in the region defined by $\chi^2 < \chi^2_{\text{max}}$. By requiring $\Phi_{Be} \geq 0$, we find that, when all experiments are taken into account, there is at least a 87% probability that neutrinos are non standard. Again we say at least, since these are just the C.L. at which we can exclude standard neutrinos at once without using any information about the nuclear physics or about the Sun. The analogous probabilities for the cases when one of the experiments is disregarded are reported in Table I.

The most severe constraint is given, as expected, by the combination of the chlorine and Kamiokande data, that excludes standard neutrinos at the 88% C.L. On the contrary, the less severe constraint is given, as already noticed in Refs. [12,15], by the combination of the chlorine and gallium data: we can exclude standard neutrinos without further hypothesis only at the 45% C.L.

Let us take a somehow different attitude, and make some model independent predictions testable in future experiments. Assuming that experimental results are correct and neutrinos are standard, we determine the regions of the $(^8\text{B}, ^7\text{Be})$ plane that contains the true values of $\Phi_B$ and $\Phi_{Be}$ at the 90%, 95% and 98% C.L. These regions are also shown in Fig. 4 and the corresponding upper limits on the $\Phi_{Be}$ are reported in Table I. The combination of all three experiments limit the $\Phi_{Be}$ to 0.6 at the 95% C.L., and to 1.1 at the 98% C.L. Even if we consider the least restrictive case, and exclude the Kamiokande result, the $\Phi_{Be}$ must still be less than 2.1 at the 95% C.L., and less than 2.7 at the 98% C.L.

At this point, it is mandatory to recall that these upper values we found for $\Phi_{Be}$ are very much lower than the values predicted by solar models, both standard and non standard. For reference, in our solar model we find that the beryllium flux is $(4.79 \pm 0.24) \times 10^9 \text{cm}^{-2} \text{sec}^{-1}$, and the quoted upper values are only 13% and 45% of this value at the 95% C.L., respectively for all experiments and for the best case, the one that excludes Kamiokande. Up to our knowledge, it is not possible construct solar models with such a low $^7\text{Be}$ flux without choices of input parameters and/or nuclear cross sections that cannot be believed (a part for the case of an hypothetical low energy $^3\text{He} + ^3\text{He}$ resonance, see [1]). For the sake of discussion, we can use the approximate power-law dependence of the fluxes on the central temperature $T_c$ [12], $\Phi_{Be} \sim T_c^{10}$ and $\Phi_B \sim T_c^{20}$, and find that we need a 7% central temperature reduction to reduce the beryllium flux to half of its standard value. Correspondingly, the boron flux is reduced to about 1/5 of its standard value. These reduced values barely fall inside the 98% C.L. curve for the combination of the chlorine and gallium data (such a $^8\text{B}$ low flux is clearly incompatible with Kamiokande). However, we can produce solar models with such a large central temperature reduction only at the price of large and unreasonable changes of the solar parameters [12].

If we combine this analysis in the context of standard neutrinos with corresponding analyses in the context of the MSW solution of the SNP, which also predicts almost independently of the SSM a strong reduction of the $^7\text{Be}$ flux [14], we have nowadays a very robust prediction that the future experiments aimed at the detection of $^7\text{Be}$ neutrinos should
measure a $^7$Be flux considerably lower than the one predicted by SSM’s.

It will be extremely interesting to see this low flux confirmed by future experiments. The discrimination between different mechanisms that achieve such low fluxes will, however, necessitate results from other experiments.

In conclusion, the combination of all present experiments tell us that neutrinos are non-standard at the 87% C.L., even if we knew nothing about the solar reactions that power our Sun. If we insist that neutrinos are standard, the $^7$Be flux must, however, be less than 13% of the SSM value at the 95% C.L.

If we are willing of throwing away one of the experimental data (no reason to do that), the highest value of the $^7$Be flux compatible with standard neutrino and two experimental data is $2.7 \times 10^9$ cm$^{-2}$ sec$^{-1}$ at the 98%, corresponding to the combination of the chlorine and gallium data.
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FIGURES

FIG. 1. We present in the ($^8$B, $^7$Be) plane the regions allowed by the present experimental results. Dashed lines correspond to central values, solid lines denote 1σ limits. We use the following experimental data: (2.32±0.23) SNU, (74.4±9.0) SNU, and (2.9±0.4)×10^6 cm^{-2}s^{-1} for the chlorine (Cl), gallium (Ga), and Kamiokande (Ka) results, respectively. We show the least stringent limits (see text), i.e. when we assume a null CNO neutrino flux. The diamond shows the fluxes predicted by our SSM with 1σ error bars.

FIG. 2. C.L. curves in the ($^8$B, $^7$Be) plane for: (a) chlorine + gallium + Kamiokande data, (b) chlorine + gallium data, (c) chlorine + Kamiokande data and (d) gallium + Kamiokande data. Solid curves correspond to 90%, 95% and 98% C.L., while the dashed curve corresponds to the C.L. at which the $^7$Be is negative (see Table). The diamonds show the fluxes predicted by our SSM with 1σ error bars.
TABLE I. The second column shows at which C.L. four different combinations of present experimental data imply a negative $^7$Be flux, i.e. nonstandard neutrinos (see text). The combinations are (a) chlorine + gallium + Kamiokande, (b) chlorine + gallium, (c) chlorine + Kamiokande and (d) gallium + Kamiokande. The other columns show the maximum beryllium flux allowed by the same combinations of data at the 90%, 95% and 98% C.L. The data correspond to the ones shown in Fig. 2.

| C.L.    | Maximum $^7$Be flux [$10^9$cm$^{-2}$s$^{-1}$] |
|---------|---------------------------------------------|
|         | $^7$Be $\leq$ 0                             | 90% C.L. | 95% C.L. | 98% C.L. |
| (a) Cl+Ga+Ka | 87%                                      | 0.2      | 0.6      | 1.1      |
| (b) Cl+Ga    | 45%                                      | 1.7      | 2.1      | 2.7      |
| (c) Cl+Ka    | 88%                                      | 0.1      | 0.9      | 1.6      |
| (d) Ga+Ka    | 59%                                      | 1.1      | 1.6      | 2.1      |
This figure "fig1-1.png" is available in "png" format from:

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This figure "fig1-2.png" is available in "png" format from:

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Figure 2

\[ \Phi_{\text{Be}} \left[ 10^9 \, \text{cm}^{-2} \, \text{s}^{-1} \right] \]

(c) Cl+Ka

(d) Ga+Ka

(a) Cl+Ga+Ka

(b) Cl+Ga