Fracture analysis of edge cracked FGM plate under compressive load along crack

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Abstract. Fracture analysis of FGM plate with edge crack under compressive point load along crack is carried out by using extended finite element method (XFEM). In this present work a FGM plate where material properties vary along width with edge crack is considered for the study. The mathematical formulation is carried out in MATLAB and the stress distribution and mixed mode stress intensity factors (SIFs) are obtained for different position of point load. The validation study is carried out for developed MATLAB code with previously published research paper and then fracture analysis of FGM plate with crack under compressive point load along crack is investigated. The present study is carried out to understand the fracture resistance of edge crack FGM plate when the action of load is along crack.

1. Introduction

The functionally graded materials (FGMs) are the types of composite materials where material properties can be varies as per the requirement in required direction. This is firstly utilized by Japanese scientist as thermal resisting material for different applications like aircraft and other engineering applications. During different manufacturing processes and operations, the materials may contain various discontinuities like voids, inclusions and cracks and the presence of these discontinuities strength of material reduces drastically and the stress induces near to the tip of crack is very high. Hence, it is necessary to understand the fractural behavior of material for the reliability and safety of materials.

Many researchers have started using XFEM to solve fracture problem instead of finite element method (FEM), as in FEM every time mesh refinement is required as crack propagates. Where as in XFEM based analysis enrichment functions are utilized near crack tip and crack face, and due to this remeshing is not required. In this direction Belytschko and Black [1] presented crack growth analysis by utilizing FEM with minimal remeshing. This work is carried out by using enrichment functions and the results obtained from 2-D cracked problem shows good accuracy. Duax et al [2] modelled geometries with different discontinuities like geometry with multiple crack, branched crack and crack emerging from hole by using XFEM. In this present work after solving number of problems on material with different discontinuities it is confirmed that this method is very much robust and accurate for solving fracture analysis. Ebrahimi et al [3] investigated the XFEM based fracture analysis of orthotropic material with crack by utilizing Heviside and asymptotic function. The mixed mode SIFs are determined by utilizing interaction integral for calculating J integral and the accuracy and effectiveness of this present method is also checked. Kim and Paulino [4] evaluated the mixed mode SIFs of FGM material with crack by interaction integral method. In this present work mixed mode SIFs are calculated for variety of problem on plate under different loadings. Kunaporn et al [5] presented the analysis of thin walled composite beam with crack along length of the beam analytically.
In this research paper response of damaged beam in terms of safety index and probability of failure is investigated. Sharma [6] presented the XFEM based fracture analysis of 2-D plate with different discontinuities. In this research work it is observed that the SIFs of one crack is greatly affected when it is interacted with other cracks and holes. It is also observed that presence of holes with crack is more severe than presence of inclusion with crack. Lal et al [7-8] presented stochastic mixed mode SIFs of edge cracked composite beam and centre cracked composite plate under tensile, shear and combined loadings by XFEM. In this research work various probabilistic approach are utilized to see the severity of various fracture parameter. Khatri and Lal [9] investigated fracture analysis of emanating crack from hole in isotropic plate by XFEM under in plane loadings. Here, it is observed that reliability of the structure can be enhanced by controlling various fracture parameters. Ahmed [10] presented XFEM based modeling of 1D and 2D domain with crack. In this present work the result is also compared with the analytical and experimental work. Kumar [11] and Oliver [12] presented different types of fracture related problems by theoretical and computational approach respectively.

From the previously presented research papers it is observed that many research articles are available on fracture analysis of FGM plate under different loadings. It is also observed that very less work has been done on the fracture analysis of FGM plate under point compressive load along crack direction.

In the present work, fracture analysis of edge cracked FGM plate under point compressive load is carried out. The mixed mode SIFs, stress distribution and deformation plots are presented for edge cracked FGM plate. The effect of position of point load on cracked FGM plate is also investigated. The mathematical formulation for the present work is carried out in MATLAB.

2. Problem formulation

The displacement field vector in XFEM framework is presented as.

\[
\mathbf{u}(x) = \sum_{i=1}^{N} \mathbf{N}_i(x) \mathbf{u} + \sum_{i=1}^{N} \mathbf{N}_i(x) \mathbf{H}(x) \mathbf{a} + \sum_{i=1}^{N} \mathbf{N}_i(x) \sum_{a=1}^{N_{b+1}} \Phi^a_{b+1}(x) \mathbf{b}^{a+1} + \sum_{i=1}^{N} \mathbf{N}_i(x) \sum_{a=1}^{N_{b+1}} \Phi^a_{b+1}(x) \mathbf{b}^{a+1}
\]  

(1)

where \(\mathbf{u}, \mathbf{a}, \mathbf{b}^{a+1}\) and \(\mathbf{b}^{a+1}\) are the conventional degrees of freedom (dofs), and added dofs for crack face and crack tip. A FGM body with crack is shown whose area is denoted by \(\Omega\) and its outer boundary \(\Gamma\) as in Fig. 1.
Fig. 1 An arbitrary body with crack, under compressive loading

The total dofs is represented as

\[ \text{dofs} = \text{size}(m) + \text{size}(m_{f}) + \text{size}(m_{c}) + \text{size}(m_{t}) \]

Where, \( m = m_{f} + m_{c} \)

Here, \( m_{f}, m_{c} \) and \( m_{t} \) are the nodes for FEM, enrichment for crack face and for crack tip respectively.

\( H(x) \) is the Heaviside function for the enrichment of the crack, and the value is +1 or -1 and \( \Psi \) is the Asymptotic functions for crack tip.

\[ \Psi_{a}^{f} = \sqrt{r} \sin \left( \frac{\theta}{2} \right), \quad \Psi_{c}^{f} = \sqrt{r} \cos \left( \frac{\theta}{2} \right), \quad \Psi_{a}^{c} = \sqrt{r} \sin \theta \cos \left( \frac{\theta}{2} \right), \quad \Psi_{c}^{c} = \sqrt{r} \sin \theta \cos \left( \frac{\theta}{2} \right) \]

The MMSIF and J-integral can be represented as,

\[ J = \frac{K_{I}^{f} + K_{II}^{f}}{E_{f}} \]

Where, \( E_{f} = E \) for plane stress and \( E_{f} = \frac{E}{1-v^{2}} \) for plane strain condition

Equation (4) represents the the \( J \) integral for the body with crack, and Equation 5 represents the summation of two states.

\[ J = \int \left\{ W \delta_{ij} - \sigma_{ij} \frac{\partial u_{j}}{\partial x_{i}} \right\} n_{i} d\Gamma \]

\[ J^{(1-2)} = \int \left[ \frac{1}{2} \left( \sigma_{ij}^{(1)} + \sigma_{ij}^{(2)} \right) (e_{ij}^{(1)} + e_{ij}^{(2)}) \right] \delta_{ij} - \left( \sigma_{ij}^{(1)} + \sigma_{ij}^{(2)} \right) \frac{\partial (u_{i}^{(1)} + u_{i}^{(2)})}{\partial x_{j}} \right\} n_{j} d\Gamma \]

On further solving, get

\[ J^{(1-2)} = J^{(1)} + J^{(2)} + \frac{2}{E_{eff}} (K_{I}^{(1)} K_{I}^{(2)} + K_{II}^{(1)} K_{II}^{(2)}) \]
Now, from equation (3) and (6) get,
\[
I^{(1)} = \frac{2}{E_{\text{eff}}} (K^{(1)}_1 K^{(1)}_\Pi + K^{(1)}_\Pi K^{(1)}_2)
\]  \quad (7)

Where, \(I^{(1, \text{Mode } I)}\) and \(I^{(1, \text{Mode } II)}\) are interaction integrals.

The SIF in XFEM for two states \(K^{(1)}_I\) and \(K^{(1)}_{\Pi}\) are evaluating by putting \(K^{(1)}_I = 1\) and \(K^{(1)}_{\Pi} = 0\) and \(K^{(1)}_I = 0\) and \(K^{(1)}_{\Pi} = 1\) in Equation. 7, we get
\[
K^{(1)}_I = \frac{M^{(1, \text{Mode } I)}}{2 E_{\text{eff}}} \quad \text{and} \quad K^{(1)}_{\Pi} = \frac{M^{(1, \text{Mode } II)}}{2 E_{\text{eff}}}
\]  \quad (8)

The above mathematical formulation is done in MATLAB by utilizing XFEM which is very much efficient for the fracture analysis. In this present approach enrichment functions are used to take care of mesh refinement near crack tip and crack face.

### 3. Results and Discussions

In FGM material properties can be varied as per the requirement. The validation study is carried out to check the effectiveness of the present method. Table 1 represents the mixed mode SIFs of edge cracked FGM plate under uniform tensile load. The present method is based on XFEM and the results of the present method are very much close to the previously published result as represented in Table 1. In this validation study material properties vary along x direction and loading is perpendicular to the crack. In this validation study dimension of plate is considered as \((1 \times 8)\) unit and material properties of FGM plate vary along \(W=1\) unit. The variation of first mode SIF \(K_1\) with respect to \(a/W\) ratio for different value of modulus ratio \((E_2/E_1)\) is presented in Table 1. The present results obtained from XFEM is validated from the results of previous published research paper [4].

| Methods                  | \(E_2/E_1\) | \(a/W\) |
|--------------------------|-------------|---------|
|                          | 0.2        | 0.3     | 0.4     | 0.5     | 0.6     |
| Kim and Paulino [4]      | 0.1        | 1.284   | 1.847   | 2.554   | 3.496   | 4.962   |
|                          | 0.2        | 1.390   | 1.831   | 2.431   | 3.292   | 4.669   |
|                          | 1          | 1.358   | 1.658   | 2.110   | 2.822   | 4.030   |
|                          | 5          | 1.132   | 1.370   | 1.794   | 2.366   | 3.448   |
|                          | 10         | 1.003   | 1.228   | 1.588   | 2.175   | 3.212   |
| Present simulation XFEM  | 0.1        | 1.3191  | 1.8537  | 2.4601  | 3.2401  | 4.7243  |
|                          | 0.2        | 1.3970  | 1.8423  | 2.4415  | 3.2104  | 4.4662  |
|                          | 1          | 1.3698  | 1.7240  | 2.3459  | 3.0556  | 4.1763  |
|                          | 5          | 1.2581  | 1.4243  | 2.0927  | 2.6398  | 3.4428  |
|                          | 10         | 1.1775  | 1.3079  | 1.9040  | 2.3285  | 2.9254  |

Table 1 Variation of first mode SIF for edge cracked FGM plate under tensile loading.
The above mathematical formulation which is done in MATLAB is utilized for further study. In this present work material properties vary in Y direction and loading is along the crack. Fig. 2 (a-b) represent the dimension and loading of edge cracked FGM plate and enrichment for crack face ad crack tip.

![Geometry of FGM plate with loadings](image1)
![Crack enrichment](image2)

Fig. 2 (a) Geometry of FGM plate with loadings (b) Crack enrichment

In FGM plate material property varies in such a way that lower side of FGM plate is 100% alloy whereas upper side is 100% ceramic. Dimension of FGM plate is L=80 mm and H= 20 mm. The material properties of and $E_a = 70$ GPa, $v_a = 0.33$, $E_c = 300$ GPa and $v_c = 0.21$ represent elastic modulus of alloy, poisson’s ratio of alloy, elastic modulus of ceramic and poisson’s ration of ceramic. The applied point load is 100 MPa. The elastic modulus of FGM plate is represented as

$$E(y) = E_a e^{\beta y}$$

where

$$\beta = \frac{1}{H} \left( \frac{E_c}{E_a} \right)$$

The volume fraction of ceramic ($V_c(y)$) and alloy ($V_a(y)$) in FGM plate along y direction are represented as

$$V_c(y) = \frac{E_a E_c - E_a e^{\beta y} - E_c}{E_c - E_a}$$

(10a)

$$V_a(y) = 1 - V_c(y)$$

(10b)

The poisson’s ratio of FGM plate along x direction is represented as

$$v(x) = \frac{\nu_a V_a(y) E_a + \nu_c V_c(y) E_c}{V_a(y) E_a + V_c(y) E_c}$$

(11)

FGM plate is modeled and the Elastic Modulus, Volume fraction and Poisson’s ratio is plotted along x direction as shown in Fig. 3 and Fig. 4 respectively.
Table (2-3) and represent the variation of mixed mode SIFs with respect to crack length for position of point load as shown in Fig. 2(a) for isotropic and FGM plate. The elastic modulus and poisson’s ratio of Isotropic plate are 70 GPa and 0.33 respectively. The dimension and loading of Isotropic plate are considered as same as for FGM plate.

Table 3 represents the variation of mixed mode SIFs with respect to crack length for position of point load as shown in Fig. 2(a). From this present study it is observed that as crack length increases magnitude of MMSIF also increases. The maximum value of $K_I$ and $K_{II}$ are observed when loading is at centre of the plate, where as minimum value of $K_I$ and $K_{II}$ are observed when loading is at right of the plate. The maximum reduction of $K_I$ and $K_{II}$ for loading at B as compared to loading at A are 37.79 % and 58.57% at crack length 15 mm and 10 mm respectively, whereas the maximum reduction of $K_I$ and $K_{II}$ for loading at C as compared to loading at A are 75.83 % and 15.95% at crack length 15 mm and 10 mm respectively. It is observed that for same crack length and loading FGM plate has less value of first mode SIF ($K_I$) than Isotropic plate because in FGM elastic modulus increases along crack length direction.

Table 2 Normalized mixed mode SIFs of edge crack Isotropic plate

| Crack length | Loading at A | Loading at B | Loading at C |
|--------------|--------------|--------------|--------------|
|              | $K_I$        | $K_{II}$     | $K_I$        | $K_{II}$     | $K_I$        | $K_{II}$     |
| 10           | 2.0073       | 0.0814       | 1.2788       | -0.1428      | 0.8484       | 0.0941       |
| 11           | 2.1035       | 0.0836       | 1.3340       | -0.1462      | 0.8853       | 0.0961       |
| 12           | 2.2603       | 0.0912       | 1.4249       | -0.1531      | 0.9473       | 0.1032       |
| 13           | 2.4165       | 0.0914       | 1.5171       | -0.1599      | 1.0084       | 0.1036       |
| 14           | 2.6320       | 0.1087       | 1.6387       | -0.1679      | 1.0842       | 0.1140       |
| 15           | 2.7456       | 0.1083       | 1.7256       | -0.1778      | 1.1430       | 0.1137       |
Table 3 Normalized mixed mode SIFs of edge crack FGM plate

| Crack length | Loading at A |       |       | Loading at B |       |       | Loading at C |       |
|--------------|--------------|-------|-------|--------------|-------|-------|--------------|-------|
|              | $K_I$        | $K_{II}$ | $K_I$  | $K_{II}$ | $K_I$ | $K_{II}$ | $K_I$ | $K_{II}$ |
| 10           | 2.0047       | 0.0815 | 1.2771 | -0.1433 | 0.8473 | 0.0945 |
| 11           | 2.1012       | 0.0837 | 1.3323 | -0.1469 | 0.8843 | 0.0965 |
| 12           | 2.2564       | 0.0911 | 1.4223 | -0.1540 | 0.9456 | 0.1035 |
| 13           | 2.4103       | 0.0916 | 1.5130 | -0.1606 | 1.0058 | 0.1041 |
| 14           | 2.5993       | 0.1089 | 1.6215 | -0.1689 | 1.0801 | 0.1145 |
| 15           | 2.7309       | 0.1082 | 1.6987 | -0.1788 | 1.1312 | 0.1141 |

Fig. 5(a) represents the deformation plot of cracked FGM plate under point load at A for crack length 10 mm. In Fig. 5(b-d) represent the stress distribution plot of FGM plate with edge crack under point load of 100 MPa at A, B and C respectively for crack length 10 mm. From the above distribution it is observed that maximum stress is observed when point load is at centre (A), whereas minimum stress is observed when point load is at right (C).

![Fig. 5(a) Deformed shape for load at A](image)
![Fig. 5(b) Stress distribution for load at A](image)
![Fig. 5(c) Stress distribution for load at B](image)
![Fig. 5(d) Stress distribution for load at C](image)
4. Conclusions

Mixed mode SIFs, deformation and stress plots are investigated for FGM plate with edge crack under under point load at different locations. The XFEM based mathematical formulation is done in MATLAB. The observations from the present study are given as:

- From this present study it is observed that as crack length increases magnitude of MMSIF also increases.
- The maximum value of $K_I$ and $K_{II}$ are observed when loading is at centre of the plate, where as minimum value of $K_I$ and $K_{II}$ are observed when loading is at right of the plate.
- The maximum stress is observed when point load is at centre (A), whereas minimum stress is observed when point load is at right (C).
- In FGM material fracture resistance can be improved as compared to Isotropic material.

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