The salient features of Mobile WiMAX (Worldwide interoperability for Microwave Access) are the higher data rate, mobility, scalability and Quality of service. Smart antenna technologies, multicast and broadcast service, fractional frequency reuse are certain advanced features. One of the main real time applications of Mobile WiMAX is the Internet Protocol Television. Video, voice and data are all IP data services, but each has its own Quality of Service (QoS) requirements. The use of structural and functional self-organization principles is one of effective ways to increase performance and improve the main indices of the quality of service (Quality of Service, QoS) by the systems using the WiMAX technology, based on the IEEE 802.16 standard [1, 2]. The use of self-organization solutions makes it possible to respond effectively to variations of the state and conditions of the wireless network operation, which can be dictated. For example, failure or overload of the network elements, oscillations of traffic, arriving into the network, by dynamics of change in the signal-to-noise conditions.

A high level of self-organization can be reached through refinement of the network protocols and mechanisms responsible for scheduling of accessible network resources. It should be noted, that the IEEE 802.16 standard does not define mechanisms of the network planning and resource scheduling, leaving the choice with the operators of link and equipment manufacturers (vendors). Such resources include: the network traffic (information resource); the communication channels capacity (channel resource); queues (buffer resource); and also frequency subcarriers (frequency resource), that is especially important for wireless networks. Frequency subcarrier is the primary structural unit of the OFDM, logical association forms a unit of frequency resource called a sub-channel. The group of sub-channels in turn forms a frequency channel [3].

The widely known solutions of allocation of frequency resource are aimed at solving the task of subcarriers scheduling. Thus, the number of subcarriers forming one frequency channel can be different and it is defined with the width of the frequency channel. The choice of this or that width of a frequency channel makes it possible to define the number of sub-channels formed by equal sets of subcarriers. As a result, the task of the frequency of resource scheduling, concerning the fixed subcarrier to the sub-channels, should be reduced to the task of the sub-channels scheduling between subscribers stations (SS) of the network.

Thereupon, the mathematical model of the sub-channels scheduling is used in the networks with application of a scalable version of the OFDMA. It gives us opportunity to simplify the width of the frequency channel which is necessary. Also, it will make it possible to use the given model in the IEEE 802.16e standard. In its turn, the task of structural self-organization is considered in the proposed model as the task of sub-channels scheduling, which allows making an account of the technological features of the wireless network [3, 4].
2. Analysis of known mechanisms

The methods for scheduling of the accessible resources using the Round Robin Scheduler algorithm were analyzed. It’s one of the algorithms employed by process and network schedulers in computing. As the term is generally used, time slices are assigned to each process in equal portions and in a circular order, handling all processes without priority (also known as cyclic executive). Round-robin scheduling is simple, easy to implement, and starvation-free. Round-robin scheduling can also be applied to other scheduling problems, such as data packet scheduling in computer networks. It is the Operating System concept. The use of the Round Robin Scheduler algorithm assumes the selection of equally accessible resource for every US. In its base implementation, the Round Robin Scheduler selects a time interval for every US, within the framework of which this US gains an exclusive access to the channel. In this case, the data transmission rate of the US is limited by the actual value of a signal-to-noise ratio (SNR) [5, 6].

The methods where the Max C/I Ratio algorithm had been used were also analyzed; this algorithm presents an accessible resource of the user’s station with the best values of the signal-to-noise ratio (SNR), not providing a validity of this resource scheduling between the US. Moreover, the methods of accessible resources scheduling, using the algorithm of proportional fair scheduling of service (Proportional Fair Scheduling) are analyzed, it’s a compromise-based scheduling algorithm. It is based upon maintaining a balance between two competing interests: trying to maximize total [wired/wireless network] throughput while at the same time allowing all users at least a minimal level of service. This is done by assigning each data flow a data rate or a scheduling priority (depending on the implementation) that is inversely proportional to its anticipated resource consumption. The performed analysis has shown that the Proportional Fair Scheduling algorithm promotes the US, which has a high SNR value, simultaneously providing a sufficient number of accessible resources for US with the worst SNR value [7].

Also, the algorithm of max-min fairness had been used in communication networks, multiplexing and division of scarce resources, max-min fairness is said to be achieved by an allocation if and only if the allocation is feasible and an attempt to increase the allocation of any participant necessarily results in the decrease in the allocation of some other participant with an equal or smaller allocation.

In best-effort statistical multiplexing, a first-come first-served (FCFS) scheduling policy is often used. The advantage with max-min fairness over FCFS is that it results in traffic shaping, meaning that an ill-behaved flow, consisting of large data packets or bursts of many packets, will only punish itself and not other flows. Network congestion is consequently to some extent avoided [8].

Fair queuing is an example of a max-min fair packet scheduling algorithm for statistical multiplexing and best effort packet-switched networks, since it gives scheduling priority to users that have achieved the lowest data rate since they became active. In case of equally sized data packets, round-robin scheduling is max-min fair [8].

The analysis has shown that the most reasonable mechanism for scheduling of the access to radio resources of the WiMAX technology would be the mechanism including features of the Round Robin and Max C/I Ratio algorithms. The choice of the algorithm depends on the load category and value. The exact choice of the algorithm for the access scheduling is especially important at a great load.

Also, as a result of the analysis, it was found, that the use of the Proportional Fair Scheduling is aimed at application to the interactive “best effort” class of data to prevent a situation, at which some US will never receive an access to the radio resource. The use of the indicated class of service (CoS) provides delivery of the user’s stations data in accordance with the possibilities without the transmission rate warrants. The improvement of the quality of service, when planning the radio resource of each US, should be aimed at support of the guaranteed speed of transmission with the possibility of access to an additional (not guaranteed) bandwidth. But none of the mechanisms, which have been analyzed, is capable to provide a similar CoS.

As a result of the performed analysis, the decision has been made on the necessity of developing a mathematical model for the frequency resource planning in the downward communication channel of the WiMAX technology, formulated as the sub-channels scheduling tasks to support the guaranteed transmission speed of user's stations [9, 10].

3. The purpose and objectives of the study

WiMAX networks were expected to be the main Broadband Wireless Access (BWA) technology that provided several services such as data, voice, and video services which in turn were defined by the IEEE 802.16 standard. Scheduling in WiMAX became one of the most challenging issues, since it was responsible for distributing available resources of the network among all subscribers’ stations; this has led to the demand of constructing and designing high efficient scheduling algorithms in order to improve the network utilization.

Taking into consideration the detected disadvantages of the known mechanisms of the frequency resources scheduling in the downward channel, the problem which is raised in this paper is assuring QoS by available resources which directly influence the quality of the equipment design. The WiMAX resource allocation algorithms determine which users to schedule, how to allocate subcarriers to them, and how to determine the appropriate power levels for each user on each subcarrier. We present a brief survey of recent scheduling research. The purposes of scheduling are to achieve the optimal usage of resources, to assure the QoS guarantees. The mathematical model, introduced by a number of linear and nonlinear conditions-limitations is analyzed in this research. Therefore, the model that consists in the statement of the sub-channels scheduling task is proposed. This model gives a solution of tasks of rescheduling of the accessible capacity of the download channel of the WiMAX technology for the information transmission in the direction of users stations, taking into account their territorial remoteness.

4. Mathematical model for sub-channel scheduling in the wireless network of the IEEE 802.16 standard

In the IEEE 802.16a and IEEE 802.16d standards, the OFDMA scheme with a fixed “window” of fast Fourier transform (FFT) of 2048 subcarriers size is utilized with an operating bandwidth of the channel of 20 MHz. In the IEEE 802.16e standard, the scalable OFDMA version is
used, which is realized at the expense of the FFT “window” change, that makes it possible to vary an operating bandwidth of the channel in the limits from 1.25 MHz up to 20 MHz [8]. But it should be noted that the choice of the frequency channel width is performed by an operator of the link, when designing the wireless network, and cannot be changed during its functioning.

Therefore, the OFDMA scalable version, used in the IEEE 802.16e standard, will be considered further in the given paper with the aim to develop solutions by preliminary selecting of the frequency channel width.

In view of the above mentioned, the following input data are assumed to be known in the proposed model:
1) \( N \) – the total number of the SS in the network;
2) \( L \) – the number of sub-channels used depending on the selected width of the frequency channel. For the mode of the full usage of the subcarriers (Full Usage of Subcarriers, DL FUSC), the amount of sub-channels can accept values 2, 8, 16, 32, and for the “partial” mode of subcarriers usage (Partial Usage of Subcarriers, DL PUSC) – 3, 15, 30, 60;
3) \( R_{nk} \) – the required transmission rate for service of the \( n \)-th SS (Mbps);
4) \( R^L \) – capacity of one sub-channel scheduled by the \( n \)-th SS.

In the WiMAX technology, the duration of a frame can vary and accept the values equal to 2; 2.5; 4; 5; 8; 10; 12.5; 20 ms. Recognizing that the useful part of the character has a fixed duration \( T_0 = 89.6 \text{ ms} \), the number of characters in the frame will accept the values 19, 24, 39, 49, 79, 99, 124, 198, according to the indicated durations of the frame. Moreover, between the characters there is a guard interval, which can accept four values concerning the duration of the useful part of the character ms; ms; 2 ms; ms.

The capacity of the sub-channel of the \( n \)-th SS \( R^L \) represents the number of the transmitted bit per a time unit (second) and can be calculated according to the formula [9]:

\[
R^L = \frac{K^\text{c}}{T_0 + T^g + T_{\text{RTG}} + T},
\]

where \( R^c \) – is the speed of the code used at coding of a signal of the \( n \)-th SS (for example, for modulation 16-QAM 1/2 parameters \( R^c = 1/2 \)) [5]; \( K^\text{c} \) – is the bit load of the character of the \( n \)-th SS (for example, for modulation 16-QAM the parameter \( K^\text{c} = 4 \)) [5]; \( K \) – is the number of subcarriers for the data transmission on one sub-channel (for the DL FUSC sub-mode \( K = 48 \), and for DL PUSC \( K = 24 \)); \( T_{\text{RTG}} = 105 \mu\text{s} \) – is the duration of the interval of switching from reception to transmission (receive/transmit transition gap, RTG); \( \mu \) – is the duration of an interval of switching from transmission to reception (transmit/receive transition gap, TRG); BLER – is the probability of the block error obtained at the expense of the HARQ mechanism (Hybrid automatic repeat request) [10].

In the course of solving the task of sub-channels scheduling within the framework of the proposed model, it is necessary to provide the calculation of the control variable \( X_{nk} \), defining the order of the sub-channels scheduling. According to the solved task physics, the following limitation is superimposed on the control variable (2):

\[
X_{nk} \in \{0,1\},(n=1,N,l=1). \tag{2}
\]

The total number of the control variables depends on an amount of the user’s servers in the network, and used sub-channels accordingly, defined by the expression \( N^*L \) when calculating the required variables \( X_{nk} \) is necessary to meet a number of the important conditions – limitations:

1) Condition of fixing of one sub-channel only for one subscriber station is defined according to the expression (3):

\[
\sum_{n=1}^{N} X_{nk} \leq 1(l=1,L). \tag{3}
\]

2) Condition of scheduling the transmission speed for the \( n \)-th subscriber’s station on the \( l \)-th sub-channel not exceeding the capacity of the sub-channel is defined according to the expression (4):

\[
R^L \sum_{k=1}^{L} X_{nk} \geq R. \tag{4}
\]

The required variables (2) calculation according to conditions – limitations (3, 4) is expedient for realizing during the solution of the optimization task, providing a minimum or maximum previously selected criterion of the quality of the sub-channels scheduling task solution.

3) Condition of balancing the capacity degree between subscribers’ stations is defined according to the expression (5):

\[
\frac{R^L \sum_{k=1}^{L} X_{nk} \geq \frac{n}{N},(n=1,N)}{R} \geq 1. \tag{5}
\]

\( b \) – the lower threshold is exceeded, bandwidth is allocated to the \( n \)-th subscriber’s station, with respect to the desired value. The optimality criterion takes the form.

The required variables (2) calculation according to conditions – limitations (3, 4) is expedient for realizing during the solution of the optimization task, providing a minimum or maximum previously selected criterion of the quality of the sub-channels scheduling task solution.

5. Research solution of scheduling sub-channel in WiMAX

To analyze the solutions on the sub-channels scheduling in the download communication channels obtained with the known methods and also with the offered model (2)–(5), an example should be considered, where the following were used as the input data for the sub-channels scheduling in the download communication channel:

- the number of subscriber’s stations (SS) \( N = 3 \);
- the number of sub-channels formed in the download communication channel \( L = 16 \).

As an example, the optimization task, set in this work, was solved using the MatLab system. In this case, the program “fmincon” of the optimization package – Optimization Toolbox – was used. Fig. 1, a-c shows how the overall performance of the download communication channel varies depending on the required transmission speed, for example, in the case when all subscribers’ stations required guaranty for providing the throughput.
Fig. 1. The download communication channel performance depends on the required transmission speed in view of the used way of the sub-channels scheduling: 
a – first SS; b – second SS; c – third SS

As the simulation results have shown (Fig. 1, a–c) the overall performance of the download communication channel, the requirement throughput is shown in blue color and allocations throughput is shown in red color. The proposed method shows that allocation throughput is more or equal to the requirement throughput, but in this case we can see that there is no balancing for the allotment capacity of sub-channels between subscribers’ stations.

Fig. 2 demonstrates the simulation results representing the dynamics of change in the degree of balancing of the download communication channel capacity between the subscribers’ servers as defined in the expression (5). Regardless of the little difference between the thresholds, as shown in Fig. 2, the different capacity of sub-channels, the thresholds for all subscribers stations are approximately on the same level, which is proved in this research.

Fig. 3 demonstrates the maximum threshold that can be allocated to n-th of subscribers’ station and dependence on requirement throughput as defined according to the expression (6). Regardless of the little difference between the thresholds, as shown in Fig. 2, the different capacity of sub-channels, the thresholds for all subscribers stations are approximately on the same level, which is proved in this research.

6. Conclusion

One of the main tasks in the wireless network operating with the use of the WiMAX technology is the task of supporting the required quality of service, which includes the need to schedule the required transmission speed in the WiMAX technology can be reached by solving the task of the sub-channels scheduling in the downward communication channel.

In this connection, the available mechanisms of the sub-channels scheduling between users’ stations in the download communication channel of the wireless network operating with the use of the WiMAX technology have been analyzed. Based on the detected disadvantages of the known mechanisms of the frequency resources scheduling in the
downward channel, the mathematical model, introduced by a number of linear and nonlinear conditions-limitations is proposed. The novelty of this model consists in the statement of the sub-channels scheduling task as the tasks of rescheduling of the accessible capacity of the downward channel of the WiMAX technology for the information transmission in the direction of users stations, taking into account their territorial remoteness (type of the system of modulation and coding).

It has been marked, that the formulated task on the sub-channels scheduling of the downward channel from the point of view of physics of processes, taking place in the wireless network, belongs to the class of the tasks of frequency resources balancing, i.e. the number of sub-channels scheduled to the users' stations, and from the mathematical point of view it is a task of nonlinear programming. The conducted analysis has also shown that known methods were effective only under conditions of the low-level requirements to the transmission speed. The model presented in this paper works under conditions of high level requirements to the transmission speed of the users' servers, providing a guaranteed speed of transmission to every user's station with the possibility of access to an additional transmission bandwidth.

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