Some remarks on geometric scaling

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In order to explain a striking symmetry of the HERA data for virtual photon-proton cross section, I propose a simple model based on the elementary 2-gluon exchange dipole-dipole cross section, and which exhibits geometric scaling. I also suggest that geometric scaling should manifest itself in exclusive processes. A preliminary search for this property in the HERA data is presented.

1 Geometric scaling

Geometric scaling [1] refers to the property that the total virtual photon-proton cross section $\sigma(x, Q)$ at small-$x$ depends on the combined variable $Q/Q_s(x)$ only, where

$$Q_s(x) = 1 \text{ GeV} \cdot (x/x_0)^{-\lambda_0/2}, \text{ with } x_0 \sim 3 \times 10^{-4} \text{ and } \lambda \sim 0.3 \quad (1)$$

is the so-called saturation scale. This scale is related to the mean transverse momentum of the partons inside the proton. Its growth with $1/x$ stems from gluon recombination (for a review, see [2]). The fact that geometric scaling holds is often considered a hint for saturation effects in deep-inelastic scattering.

Our first remark concerns a point that has not been much commented so far: the almost perfect symmetry of $Q/Q_s \times \sigma(Q/Q_s)$ under the interchange of $Q$ and $Q_s$ [1]. We point out that this symmetry is already present in the 2-gluon exchange dipole-dipole cross section and starting from this observation, we propose a simple model for $\sigma$. Our second remark is that the saturation scale should be understood as an impact parameter-dependent quantity $Q_s(x, b)$, and that exclusive amplitudes exhibit local geometric scaling, in the sense that they should depend on $Q/Q_s(x, b)$ only.

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2 A simple model

The elementary dipole-dipole cross section of two dipoles of respective size $1/Q$ and $1/Q_s$ reads

$$\sigma_{dd}(1/Q, 1/Q_s) = 2\pi \alpha_s^2 \min(1/Q, 1/Q_s)^2 (1 + |\log(Q/Q_s)|).$$

The fundamental observation is that the magnitude of the cross section is given by the squared size of the smallest interacting dipole.

Let’s try a simple model for $\sigma$ [3]. Assume that the proton is a collection of independent dipoles of size $1/Q_s$ uniformly spread over a surface $1/\Lambda^2_{QCD}$. The photon is itself represented by $q\bar{q}$ dipoles of size $1/Q$ whose flux we approximate by a constant $N$. The interaction proceeds through the exchange of two gluons between the photon dipole and one of the dipoles in the proton. With these simple assumptions, the $\gamma^* p$ cross section reads

$$\sigma = \{\text{flux of dipoles in the photon}\} \times \{\text{number of dipoles in the proton}\} \times \sigma_{dd}$$

$$= N \frac{1}{\Lambda^2_{QCD}} 2\pi \alpha_s^2 \min(Q_s^2/Q^2, 1)(1 + |\log(Q/Q_s)|).$$

It is easy to see that $Q/Q_s \times \sigma(Q/Q_s)$ is symmetric under the replacement $Q \leftrightarrow Q_s$. For $Q \gg Q_s$, this model is equivalent to the BFKL evolution with appropriate boundary conditions, where the QCD dynamics determine the energy evolution of $Q_s(x)$. For $Q \ll Q_s$, it turns out that this model can be qualitatively understood as an effective picture for color glass condensate [4].

However, this model has to be supplemented by multiple gluon-pair exchanges when $Q \sim Q_s$, which we implemented in a Glauber-Mueller inspired way. Indeed, in this kinematical region, the probability for dipole-dipole interaction can become quite large even for moderate values of $\alpha_s$. The resulting model, depending on 5 free parameters, gives a good fit to all recent ZEUS data [5] for $F_2 (\chi^2 = 1.15/d.o.f)$, as can be seen on Fig.1 (for details, see [3]).

The success of the obtained parametrization is also a first step toward the quantitative estimate of the statistical relevance of geometric scaling.

3 Local geometric scaling

So far, most theoretical and phenomenological discussions on saturation have assumed a unique saturation scale $Q_s(x)$. However, the value of the saturation scale is related to the quark and gluon density, thus for a realistic proton, it is
dependent on the impact parameter $b$. A deeper understanding of saturation effects thus requires to take into account the transverse profile of the nucleus. Some progress has been made very recently on the theoretical side (see for example [6, 7], and [8] for a phenomenological model), and as pointed out in Ref.[9], it is possible to extract the $b$-profile from experimental data.

We have shown in Ref.[10] that amplitudes $A(x, Q, b)$ which are explicitly dependent on $b$, like elastic diffraction (in practice: diffractive production of vector mesons of mass $M$) exhibit geometric scaling, i.e.

$$A(x, Q, b) = A((Q^2+M^2)/Q_s^2(x, b)).$$  

(4)
We have checked that this statement, which can be derived from QCD, is supported by experimental measurements. On Fig.2, we represent the amplitude $\mathcal{A}$ that we deduced from the present data on elastic $\rho$ meson production, as a function of the scaling variable $(Q^2 + M^2)/Q_s^2(x, b)$, for 3 different values of $b$ (for each $b$, the form (1) was assumed for $Q_s(x, b)$). One sees that the result is a good hint for local geometric scaling.

![Local geometric scaling](image)

Figure 2: Local geometric scaling. The data were taken from Ref.[11]. Note however that the displayed points are not true measurements, but are only derived from experimental data through the procedure explained in Ref.[10]. The bands give an (upper) estimate of the uncertainties.
4 Conclusion and outlook

Geometric scaling may be considered as an indirect manifestation of saturation effects, and has been understood within saturation models. In this talk, we have essentially advocated the study of local geometric scaling, which would give deeper insight into saturation physics. It requires the experimental extraction of scattering amplitudes at fixed impact parameter, which we believe is doable with the present experimental data on diffraction.

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