Inverse four-wave mixing and self-parametric amplification in optical fibre

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An important group of nonlinear processes in optical fibre involve the mixing of four waves due to the intensity dependence of the refractive index. It is customary to distinguish between nonlinear effects that require external/pumping waves (cross-phase modulation and parametric processes such as four-wave mixing) and those arising from self-action of the propagating optical field (self-phase modulation and modulation instability). Here, we present a new nonlinear self-action effect—self-parametric amplification—which manifests itself as optical spectrum narrowing in normal dispersion fibre, leading to very stable propagation with a distinctive spectral distribution. The narrowing results from inverse four-wave mixing, resembling an effective parametric amplification of the central part of the spectrum by energy transfer from the spectral tails. Self-parametric amplification and the observed stable nonlinear spectral propagation with a random temporal waveform can find applications in optical communications and high-power fibre lasers with nonlinear intracavity dynamics.

Nonlinear fibre optics is a research field at the two-way interface of fundamental nonlinear physics and fibre-optic engineering that encompasses diverse areas of science and technology. Nonlinear effects in optical fibre are critically important for various practical applications ranging from telecommunications to medical fibre lasers (see, for example, refs 1–6 and references therein). However, nonlinear fibre optics is also a remarkable and versatile testbed for the experimental probing of the ideas and concepts of fundamental nonlinear science\textsuperscript{5,7–13}. This, in turn, means that fibre optics is an ideal platform for the invention and development of novel devices based on nonlinear design concepts with functionality not available in linear science engineering. Due to the relatively low threshold for the occurrence of nonlinear effects in fibre, they may adversely impact signal propagation in optical communications or indeed may be positively exploited for the development of all-optical devices for optical networks, fibre lasers, signal-processing components and many other applications.

Typically, fibre nonlinear effects are subdivided into two main categories: phenomena induced by the nonlinearities that arise from scattering (stimulated Brillouin scattering (SBS) and stimulated Raman scattering (SRS)) and those induced by the nonlinear effects due to the Kerr effect, that is, the intensity dependence of the refractive index (self-phase modulation (SPM), cross-phase modulation (XPM), four-wave mixing (FWM), modulation instability (MI) and parametric processes based on FWM\textsuperscript{2,4}). The interaction of two or more waves with different frequencies may lead to power transfer between them according to the corresponding stimulated scattering or parametric processes. A subclass of such phenomena occurs in the degenerate case, when a single wave affects itself through the nonlinear response of the medium. This is called a self-action effect. Although differentiation among the various manifestations of the Kerr nonlinearity is somewhat artificial (as all elementary nonlinear processes resulting from the cubic nonlinearity can be treated formally as a mixing of four waves), from a practical viewpoint it is convenient to distinguish between nonlinear interactions of the optical field under consideration with external fields (for example, pumping waves) co-existing from the very outset (for instance, XPM and parametric processes) and self-action of the propagating light wave (for example, SPM and MI).

In general, optical nonlinear self-action effects may be both spatial (self-focusing, SPM and spatial MI) and temporal (SPM and MI) and...
Experimental set-up

We start with a description of the experiment that initiated this study, the experimental set-up of which is shown in Fig. 1a. In the first set of experiments an RFL was operated at ~1,425 nm, and in the set of cross-check experiments presented at the end of this Article, an RFL operating at 1,276 nm was used. Both Raman lasers operated in the continuous-wave (c.w.) regime with a maximum output power of up to 2 W. The outputs were randomly polarized with a degree of polarization of <5%. It is widely known in fibre optics that nonlinear effects such as FWM and SPM typically manifest themselves as spectrum broadening when relatively high-power c.w. fields propagate in optical fibres. A characteristic feature of the RFL, related to a high in-resonator power and the resulting in-cavity spectral broadening, is that its output spectrum has two peaks with a separation of 0.2–1 nm (Fig. 1b). The double-peak structure of the output spectrum is the result of the FWM-induced spectral broadening inside the Raman converter cavity, which leads to a spectral breadth that exceeds the reflection bandwidth of the fibre Bragg grating (FBG) output coupler and therefore leads to radiation ‘overflowing’ the FBG reflector.

The laser radiation generated at 1,424.5 nm was launched into 100-km-long Corning LEAF (large effective area fibre) or single-mode fibre (SMF-28), with a zero-dispersion wavelength (ZDW) around 1,490 nm for LEAF and 1,310 nm for SMF-28. Light thus propagates in the region of normal dispersion for the LEAF fibre and in the region of anomalous dispersion for the SMF-28. The spectra at the input and output of the 100 km lengths of fibre were measured with an optical spectrum analyser (OSA) with a resolution of 0.01 nm.

Narrowing of the optical spectra

The measured optical spectrum at the end of the LEAF fibre shows significant narrowing (Fig. 1b, LEAF), in sharp contrast to the typical nonlinear spectral broadening in SMF-28 (Fig. 1b, SMF) caused by FWM, which has been observed and studied in a number of experimental and theoretical publications. Explaining this atypical narrowing effect is the aim of the present work.

Extensive numerical modelling of light generation in the RFL and its further propagation in the LEAF fibre fully confirms the observed unusual spectral behaviour of a nonlinear wave in a long fibre with normal dispersion. Signal evolution inside the laser cavity was modelled by the set of coupled modified nonlinear Schrödinger equations (NLSEs), taking into account dispersion, Kerr nonlinearity, Raman gain, depletion of the Raman pump wave and fibre losses. All details of the modelling are presented in refs 2 and 33 (see also Methods). Signal evolution in the LEAF fibre was computed using the standard NLSE.

The results of the experiments and numerical simulations presented in Figs 2 and 3 demonstrate spectral narrowing of the c.w. radiation with simultaneous temporal fluctuations. The initial spectrum was converted to a double-scale distribution (a bell-shaped peak in the centre but with exponentially decaying spectral tails introducing a second scaling parameter in the spectral distribution) at the LEAF output both in the experiment and simulation (Fig. 2). A stable spectrum evolution (after ~50 km) along the LEAF fibre is shown in Fig. 3a. We stress that the observed stabilization of the spectrum is a nontrivial nonlinear process. Intensive numerical modelling shows that this happens both in a fibre span with loss and in the corresponding lossless system. The spatiotemporal
dynamics of the signal features highly irregular intensity fluctuations (Fig. 3b). Despite visible irregularities in the temporal field distribution shown in Fig. 3b, in the spectral domain this statistical steady state is very stable and may evolve without major changes over long distances. Due to the fact that multiple modes are involved in building this statistical equilibrium through nonlinear FWM interactions, the process calls for a kinetic description \(^{13,27}\). We believe that this is an interesting and practically important experimental observation of kinetic equilibrium in optical fibre\(^{8,10,12,13,27}\).

We performed a number of experiments with various fibres at different wavelengths, as well as extensive numerical modelling, and conclude that we were able to observe the effect of nonlinear spectral narrowing only in the case of normal fibre dispersion (Supplementary Note 1).

**Figure 3 | Evolution of the signal spectrum and temporal shape along the fibre.** a. Computed power spectrum density evolution along the LEAF fibre, demonstrating a transition to very stable propagation with a distinctive asymptotic spectrum. b. Corresponding spatiotemporal dynamics over an interval of 800 ps. Here, the fluctuating c.w. power \(P(t,z)\) is normalized by the distance-dependent factor \(P_{\text{norm}}(z) = P(0)\exp(\alpha z)\), where \(\alpha = 0.25 \text{ dB km}^{-1}\) is the fibre loss. The two figures illustrate that although the temporal field structure is irregular, the spectrum propagation demonstrates remarkable stability.

**Figure 4 | Signal gain spectra as a function of pump wavelength spacing.** a. Four-wave model, \(P_0(0) = 1.5 \text{ W}, L = 1 \text{ km}. \) The unsaturated single-pass gain for signal \(G_3\) is shown. Black lines show the corresponding wavelengths of the pumps. b. NLSE model, \(P_0(0) = 1.5 \text{ W}, P_3(0) = 300 \text{ mW}, P_4(0) = 0, L = 1 \text{ km}. \) The projection at the top is related to the normal dispersion case: black solid line, deterministic phases of waves; grey circles, averaging over 600 sets of random phases.
Qualitative analysis

First let us try to explain qualitatively the physical mechanism underlying the spectral narrowing of a high-power field containing many longitudinal modes in normal dispersion fibre. The key idea came from the observation that the Raman laser output (with its double-peaked spectrum) being launched into the LEAF fibre resembles a two-pump OPA with two spectrally separated pumps. Of course, there are no separate ‘pumps’ and ‘signal’ in this case. Instead, the input field self-acts by redistributing energy from the peripheral wavelengths (pumps) to the central region (signal). This is obviously only a qualitative picture that helps in understanding the main elementary mechanism of such self-pumping of the central wavelength region at the expense of the tails. Consider those effective pumps to be at frequencies $\omega_1$ and $\omega_2$ at the fibre input. Such effective pumps would amplify the signal and idler with frequencies $\omega_s$ and $\omega_i$, respectively. Consider, $\omega_1 < \omega_2$ and $\omega_3 < \omega_4$. Here, evidently, the signal and idler represent just two spectral components of the same wave packet that, through FWM, obtain energy from the tails of the spectrum (pumps). Note that $\omega_3$ and $\omega_4$ are symmetric with respect to the centre frequency $\omega_c = (\omega_1 + \omega_2)/2 = (\omega_3 + \omega_4)/2$, which is halfway between the two pump frequencies. In fact, the SPA effect does not require a two-pump structure of the input field; similar spectral narrowing can be observed for input waves with a bell-shaped spectrum. This was conclusively confirmed by additional experiments and modelling (see Supplementary Section ‘Gain analysis in the NLSE model’ and the Supplementary materials). The particular example considered here is very useful for understanding the underlying principle; that is, the two spectral peaks in the Raman laser output could be associated with the pumps, whereas the spectral narrowing of the laser output as it propagates in the normal dispersion fibre could be associated with the signal and idler (or just the signal if $\omega_3 = \omega_4$) amplification in an effective fibre OPA.

In this qualitative analysis, consider first the undepleted pump case when signal and idler are small compared to the pumps $P_s \ll P_0 = P_1 + P_2$ (for simplicity, as the analysis in this section is only qualitative, formulae here are given for the case when the idler is absent at $z = 0$). The unsaturated single-pass gain for the signal, $G_s$, may be written as

$$G_s = \frac{P_s(\ell)}{P_s(0)} = 1 + \left[\frac{yP_0}{g} \sinh (gL)\right]^2 \tag{1}$$

where the idler gain is $G_i = G_s - 1$, $g$ is a parametric gain coefficient given by $g^2 = r^2 - (k/2)^2$, $r = yP_0$, and $y$ is a nonlinearity coefficient. The gain coefficient reaches its maximum value when $k = \Delta \beta + \Delta \beta_{NL} = 0$, where $k$ is the total propagation constant, $\Delta \beta = \beta(\omega_s) + \beta(\omega_i) - \beta(\omega_1) - \beta(\omega_2)$ is the propagation constant mismatch and $\Delta \beta_{NL} = yP_0$ is the nonlinear contribution to the wavevector mismatch. The corresponding single pass gain signal is equal to

$$G_{s,\max} = 1 + (\sin (yP_0L))^2 \tag{2}$$

and for the idler

$$G_{i,\max} = (\sin (yP_0L))^2$$

It is convenient to introduce the following notation: $\Delta \omega_0 = \omega_3 - \omega_4$, $\Delta \omega_p = \omega_1 - \omega_c = \omega_3 - \omega_2$. To understand the dispersion, we can expand the propagation constant mismatch in a standard power series in terms of $\Delta \omega_0$ and $\Delta \omega_p$ (refs 15,17): $\Delta \beta = 2 \sum_{m=0}^\infty (\beta_{2m}/(2m)!)(\Delta \omega_0)^{2m} - (\Delta \omega_p)^{2m}$, where $\beta_{2m}$ are even derivatives of $\beta(\omega)$ at $\omega_c$. Consider only the main term in this expansion assuming that $\beta_2 \ll \beta_4$, that is, $\Delta \beta \approx \beta_2[(\Delta \omega_0)^2 - (\Delta \omega_p)^2]$. The shape of the gain spectrum and location of the gain maximum in the spectral domain are given by the condition of phase matching:

$$k = 0 \Rightarrow [(\Delta \omega_0)^2 - (\Delta \omega_p)^2] = -\frac{yP_0}{\beta_2} \tag{3}$$

Equation (3) provides a reasonable qualitative explanation of why the sign of the dispersion matters in the considered experiment. When the fibre dispersion is normal ($\beta_2 > 0$), the gain maximum is located between the pumps and $\Delta \omega_0 > \Delta \omega_p$, but when the fibre dispersion is anomalous ($\beta_2 < 0$), the gain maximum is located outside the area between the pumps, in the frequency domain $\Delta \omega_0 > \Delta \omega_p$ (Fig. 4a).

Figure 4a depicts the signal parametric gain $G_s$ (in dB) as a function of distance $\Delta \ell$ between the pumps and signal wavelength. Black lines show the wavelengths of the pumps. In the normal dispersion regime the gain spectrum is bell-shaped, with the maximum amplification in its central part. When the dispersion is anomalous, symmetric gain maxima are always located outside the area between the pumps, which effectively leads to significant spectrum broadening.

Note that, for a central frequency gain, when the signal coincides with the idler ($\Delta \omega_0 = 0$), we have degenerate OPA and signal amplification should be considered in the framework of the degenerate OPA model. The qualitative analysis presented in this section explains the difference between normal and anomalous dispersion propagation regimes. However, this simple model does not take into account pump depletion and the generation of additional waves through FWM. This changes the quantitative characteristics.

![Figure 5 | Estimate of FWM product during signal amplification. Top: dependence of signal gain $G$ (in dB) on relative phase difference $\theta(0)$ and wavelength spacing between two pumps $\Delta \lambda$. Bottom: dependence of $F = \Delta P_s/\Delta P_{FWM}$ on relative phase difference $\theta(0)$ and wavelength spacing between two pumps $\Delta \lambda$.](image-url)
of the SPA process and requires a more accurate analysis (presented in the following section).

**Gain analysis in the NLSE model**

The four-wave model described above gives only a qualitative picture. Many other frequency components are generated and interact with each other due to FWM in the process under consideration. Some of these waves may be well phase-matched and so may reach levels comparable with the signal. To give a more realistic evaluation of the SPA we now use the NLSE to obtain the signal amplification spectrum in the presence of parasitic FWM components depleting the effective gain. Figure 4b shows the signal gain spectrum corresponding to the two-pump fibre OPA with varying distance between the pumps. Here, the total pump power is 1.5 W, the signal c.w. power at the fibre input is 300 mW and the idler is absent at z = 0. The gain spectrum is still bell-shaped if the distance between the pumps is properly chosen (Fig. 4b, top, solid lines), and maximum amplification is achieved in the frequency band between the pumps.

Note that we previously considered an ideal case—a phase-insensitive parametric amplifier. The real laser output has the form of a multimode light field without phase locking, so the SPA of such a field is phase-sensitive. To study the impact of the effect of random initial phases we performed additional simulations. We considered a signal and idler with equal powers (300 mW total power) at the fibre input, and the relative phase difference between the four involved light waves at the fibre input \( \theta(0) = \theta_1(0) + \theta_2(0) - \theta_3(0) - \theta_4(0) \) was assumed to be a random value with a uniform probability distribution bounded between \( -\pi \) and \( \pi \). Statistical analysis was performed with 600 different sets of random phases. A statistical signal gain spectrum averaged over the 600 sets is shown by grey circles in Fig. 4b, top. It has a characteristic high peak at the central frequency, similar to the simplified four-wave model.

Although we have shown that the maximum signal amplification can be achieved near the central frequency, new spectral components could still be amplified simultaneously with the signal and lead to broadening of the laser spectrum. To investigate pump energy transfer along the fibre, we consider signal amplification at the central frequency (where the signal coincides with the idler, \( \omega_1 = \omega_3 = \omega_4 \)). To estimate the value of the FWM product during signal amplification we introduce the dimensionless function \( F(\Delta \lambda, \theta(0)) \), defined as the ratio between the pump energy transferred to the signal at frequency \( \omega = \omega_3 \) and the pump energy transferred to other frequencies due to FWM: \( F = \Delta P_s / P_{\text{FWM}} \), where \( \Delta P_s = P_s(L) - P_s(0) \) and \( P_{\text{FWM}} = P_p(0) + P_p(0) - P_s(L) - P_s(L) \). \( F(\Delta \lambda, \theta(0)) \) is a figure of merit indicating how effectively pump energy transfers to the signal, that is, the efficiency of signal...
amplification. \( F = 0 \) corresponds to \( G_3 = 0 \) (that is, no amplification for the \( \omega_3 \) signal), whereas \( F \) tends to infinity when the only spectral component amplified is the \( \omega_3 \) signal.

Figure 5 shows the value of the signal gain \( G = 10 \log_{10}(P(L)/P(0)) \) and dimensionless function \( F \) (bottom row) in the plane of parameters \( \Delta L - \theta(0) \). It can be seen that the FWM product can be neglected if the spectral distance between the pumps is properly chosen (undesirable FWM product is minimized and all the pump energy transfers to the signal).

Discussion

An interesting question is 'What are the conditions for spectral compression and broadening in normal dispersion fibre?' In Fig. 6 we considered the broadening factor \( \Delta L_{\text{out}}/\Delta L_{\text{in}} \) as a function of initial spectral width, power, fibre dispersion and cavity length. Figure 6a depicts the evolution of the broadening factor \( \Delta L_{\text{out}}/\Delta L_{\text{in}} \) with an initial spectral width \( \Delta \lambda_0 \) in numerical simulations. When the spectral width at \( z = 0 \) is less than 0.5 nm, we still observe spectrum broadening in a 100-km-long fibre. However, if \( \Delta \lambda_0 \) exceeds 0.5 nm, spectrum narrowing takes place. We verified that this effect is observed both in lossy and lossless cases (for details see Supplementary Note 2). We also studied the impact of dispersion (Supplementary Note 3) and the statistics of the compressing signal (Supplementary Note 4). In Fig. 6b–e we consider in more detail two different points along the line, corresponding to spectrum broadening and narrowing (marked 1 and 2 in Fig. 6a). For \( \Delta \lambda_0 = 0.23 \) nm, the broadening factor first monotonically increases with fibre length \( L \) as long as it is shorter than 5 km (Fig. 6d). With a further increase of the propagation distance, the initial rise is followed by a decrease. This kind of evolution of a multimode c.w. field was previously observed in ref. 32. Scaling of \( L_f/L_{\text{NL}} \) along the propagation distance is shown in Fig. 6e. Spectral broadening occurs when the dispersion length at the fibre input is much greater than the nonlinear length. On the other hand, when the dispersion length becomes comparable with the nonlinear length, we observe nonlinear spectral narrowing. The observed nonlinear spectral broadening depends on the ratio of the dispersive and nonlinear lengths \( L_f/L_{\text{NL}} \). Light with a broader bandwidth can therefore also be compressed provided that after rescaling of the parameters the factor \( L_f/L_{\text{NL}} \) is the same as in the studied examples.

We also experimentally verified that the RFL spectrum narrows when transmitted through other fibres with normal dispersion (Fig. 6f,g). Laser radiation generated at 1.276 nm was launched into the same 100 km length of SMF-28, which has normal dispersion at this wavelength (\( \beta_2 = 3.1 \) ps\(^2\) km\(^{-1}\), \( \gamma = 1.8 \) W\(^{-1}\) km\(^{-1}\)). We observed spectral narrowing both in the experiment and simulation. In the experiment, a narrow and low-power (0.7 W) RFL spectrum is broader after propagation in SMF (Fig. 6f). However, when the initial width and power increase (1.3 W), one can observe compression (Fig. 6g) and the formation of a stable spectrum.

The remarkable spectral stability of the evolved state (in normal dispersion fibre), despite a random temporal behaviour, indicates that we observe an asymptotic kinetic regime resulting from the optical wave turbulence of a multitude of elementary waves in the nonlinear system considered\(^{8,10,12,13,34}\). It is also worth pointing out that this surprising finding that the evolved spectral distribution does not change appreciably with propagation may potentially be exploited in fibre lasers and optical telecommunications. In fibre lasers, nonlinear compression may lead to increased spectral brightness compared to systems using direct spectral filtering, thereby avoiding the additional losses inevitable with filters. In optical communications, spectrally stable nonlinear propagation regimes may lead to new techniques for mitigation of the nonlinear transmission impairments that are a major challenge in modern high-capacity systems.

Conclusion

We have presented a new self-action effect—self-parametric amplification or inverse FWM, that may occur during high-power wave propagation in normal dispersion fibre and that manifests itself as a spectral compression of light. This is different from the compression of pre-chirped coherent pulses.\(^{35-40}\). The observed effect is the result of a nonlinear energy redistribution from the tails of the signal spectrum to the central region. This can be considered as an effective SPA of the central part of the wave packet spectrum by the peripheral pumps. The presented simple theory of SPA explains all the key features observed in the experiments and full numerical modeling. We believe that the remarkable stability of the observed spectral field distribution may offer new interesting applications in high-power fibre lasers and optical telecommunications.

Methods

Methods and any associated references are available in the online version of the paper.

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Author contributions
S.B.P. initiated the study and carried out the experiments. A.E.B. designed and conducted the numerical modelling. S.K.T., A.E.B. and M.P.F. guided the theoretical and numerical studies. S.K.T., S.B.P., A.E.B., W.R.L.C and M.P.F. analysed the data. S.K.T., A.E.B., S.B.P. and W.R.L.C. wrote the paper.

Additional information
Supplementary information is available in the online version of the paper. Reprints and permissions information is available online at www.nature.com/reprints. Correspondence and requests for materials should be addressed to S.K.T.

Competing financial interests
The authors declare no competing financial interests.
Methods

Propagation in LEAF. Signal propagation down the LEAF fibre was modelled using the NLSE with losses:

\[
\frac{\partial A}{\partial z} = -i \beta_2 \frac{\partial^2 A}{\partial t^2} + i\gamma |A|^2 A - \frac{\alpha}{2} A
\]

where \(A(z,t)\) is the electric field envelope, \(\beta_2\) is the second-order dispersion coefficient at the central frequency \(\omega_0\), \(\gamma = n_2 \omega_0 / (cA_{\text{eff}})\) is the Kerr nonlinearity coefficient with nonlinear refractive index \(n_2\) and effective fibre cross-section area \(A_{\text{eff}}\) for the fundamental mode, and \(\alpha\) is the fibre attenuation coefficient. The equation was solved using the split-step Fourier transform method. The following fibre parameters are used in the simulations: \(\beta_2 = 4.3 \, \text{ps}^2 \, \text{km}^{-1}\), \(\gamma = 2.16 \, \text{W}^{-1} \, \text{km}^{-1}\), \(\alpha = 0.25 \, \text{dB} \, \text{km}^{-1}\).

Raman fibre laser. To model the laser generation we used a NLSE-based model previously reported to be efficient for modelling RFLs:

\[
\frac{\partial A^\pm_p}{\partial z} = -i \frac{\beta_2}{2} \frac{\partial^2 A^\pm_p}{\partial t^2} + i\gamma_p \left[ |A^\pm_s|^2 + (2 - f_R) |A^\pm_p|^2 \right] A^\pm_p - \frac{\alpha_p}{2} A^\pm_p \frac{\partial A^\pm_s}{\partial z} - \frac{1}{\nu_s - \nu_p} \frac{\partial A^\pm_s}{\partial t} - \frac{i}{2} \beta_2 s \frac{\partial^2 A^\pm_p}{\partial t^2} + i\gamma_s \left[ |A^\pm_s|^2 + (2 - f_R) |A^\pm_p|^2 \right] A^\pm_s + \frac{\delta_p}{2} \left[ |A^\pm_s|^2 + |A^\pm_p|^2 \right] A^\pm_s - \frac{\alpha_s}{2} A^\pm_s
\]

The boundary conditions describe the pump input and the reflection of the optical field from the fibre Bragg gratings:

\[
A^+(0, t) = A_{\text{in}}, \quad A^{-}(L, \omega) = \sqrt{R_{\text{in}}(\omega)} A^+_{\text{in}}(L, \omega)
\]

\[
A^-(0, \omega) = \sqrt{R_{\text{in}}(\omega)} A^-_{\text{in}}(0, \omega), \quad A^+(L, \omega) = \sqrt{R_{\text{out}}(\omega)} A^+_{\text{in}}(L, \omega)
\]

where \(R_{\text{in}}(\omega)\) and \(R_{\text{out}}(\omega)\) are the reflectivities (with respect to power) at the left and right cavity ends, respectively.

The model describes spectral broadening during one roundtrip and the building of a steady state over many roundtrips. We integrated equations (4) along \(z\) using the split-step Fourier transform method and an iterative procedure similar to that used for the modelling of Brillouin fibre lasers. For example, to integrate the equation for \(A^+_p(z, t)\) we substituted into the equation \(A^+_p(z, t)\) obtained from a previous iteration, and so on. The generation becomes stable after \(10^5 - 10^6\) roundtrips, depending on the power.

References

41. Preda, C. E., Fotiadi, A. A. & Mégret, P. Numerical approximation for Brillouin fiber ring resonator. Opt. Express 20, 5783–5788 (2012).