One-Particle vs. Two-Particle Crossover in Weakly Coupled Hubbard Chains and Ladders: Perturbative Renormalization Group Approach

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Physical nature of dimensional crossovers in weakly coupled Hubbard chains and ladders has been discussed within the framework of the perturbative renormalization-group approach. The difference between these two cases originates from different universality classes which the corresponding isolated systems belong to.

KEYWORDS: doped Hubbard ladders, dimensional crossover, spin gap, superconductivity, Tomonaga-Luttinger liquid, perturbative renormalization-group

1 Introduction
Physical nature of dimensional crossovers in coupled one-dimensional systems has currently provoked a great deal of controversy. Recent discovery of the superconductivity in the doped spin ladder under pressure has stimulated us to study the dimensional crossover problem in the weakly coupled ladder system. In the present work, based on the perturbative renormalization-group approach (PRG) developed by Bourbonnais and Caron, we discuss the dimensional crossovers induced by the inter-chain/ladder one-particle hopping, $t_{\perp}$, in the weakly coupled chains or ladders, with emphasis on the differences between these systems.

2 One-Particle and Two-Particle Crossovers
The isolated chain or ladder system is specified by the linearized bands and scattering processes generated by the intra-chain/ladder Hubbard repulsion, $U$. For the chain system shown in Fig. 1(a), the scattering processes are specified by dimensionless coupling constants $g^{(1)}$ and $g^{(2)}$ denoting backward and forward scatterings, respectively. For the ladder system shown in Fig. 1(b), there are additional band indices [B (bonding) and A (anti-bonding)] and flavor indices ($\mu = 0, f, t$), specifying the bands and scattering processes, respectively. For the chain and ladder systems, the usual coupling constants with dimension of the interaction energy are $\pi v_F g^{(i)}$ and $2\pi v_F g_{\mu}^{(i)}$, respectively, where $v_F$ is the Fermi velocity at the Fermi points.

When we switch on $U$ and inter-chain/ladder one-particle hopping, $t_{\perp}$, as perturbations to the isolated chains/ladders, dimensional crossovers are governed by the competition between the one-particle and two-particle processes. To study the competition, we set up scaling equations for the inter-chain/ladder one-particle and two-particle hopping amplitudes, which are depicted in Figs. 2(a) and 2(b), respectively. We parametrize the band-width cutoff as $E(l) = E_0 e^{-l}$ with the scaling parameter, $l$.

3 Weakly Coupled Chains
The low-energy asymptotics of the non-half-filled isolated Hubbard chain is characterized by the weak coupling fixed point $g^{(1)*} = 0$, $g^{(2)*} = U/(2\pi v_F)$, which
leads the system to the “Tomonaga-Luttinger liquid” universality class. The scaling equation for the interchain one-particle hopping amplitude [Fig. 2(a)] is written as

\[
\frac{d \ln t_{\perp}(l)}{dl} = 1 - \frac{1}{4} \left[ g^{(1)2} + g^{(2)2} - g^{(1)} g^{(2)} \right],
\]

which gives \(\frac{d \ln t_{\perp}(l)}{dl} \xrightarrow{l \to \infty} 1 - \frac{U^2}{16\pi^2 v_F^2}\). Thus \(t_{\perp}(l)\) becomes relevant and consequently the one-particle crossover dominates the two-particle crossover, at least for weak repulsion, \(U/\pi v_F < 1\), (see Fig. 12 of Ref.[2]) where the PRG scheme is reliable. The one-particle crossover temperature is defined by \(T_{\text{cross}} = E_0 e^{-l_{\text{cross}}/l} = E_0 \frac{t_{\perp}}{E_0} \frac{1}{(1-\theta)}\), where \(l_{\text{cross}}\) is determined through \(t_{\perp}(l_{\text{cross}}) = E_0\). The exact solution tells us the anomalous exponent, \(\theta\), satisfies \(\theta \leq 1/8\), which again indicates \(t_{\perp}\) is always relevant.

In Fig. 3(a), we show the phase diagram of weakly coupled chains, in terms of the intrachain Hubbard repulsion, \(\tilde{U} = U/\pi v_F\), and the temperature, \(\tilde{T} = T/E_0\) for an initial value of the interchain one-particle hopping, \(t_{\perp} = 0.01 E_0\). A similar result has already been given in Ref.[3]. The one-particle crossover dominates the two-particle crossover for a wide region of \(U\). The Tomonaga-Luttinger liquid (TL) phase crosses over to a two-dimensional phase (2D phase) via the one-particle processes. We show for guidance the temperature, \(T_{\text{SDW}}\), at which the amplitude of the interchain two-particle process of the SDW channel diverges. This conclusion, however, has excited lots of controversy. Anderson suggested that an intrachain repulsion of intermediate strength might be sufficient to cause a confinement of the particles within the chain. This subject, however, is beyond the scope of the PRG approach and here we leave the matter open.

4 Weakly Coupled Ladders

Dimensional crossovers in the weakly coupled ladder are very different from those in the weakly coupled ladders, since the low-energy asymptotics of the isolated Hubbard ladder is characterized by the strong coupling fixed point \(g^{(1)*} = -1\), \(g^{(2)*} = 0\), \(g^{(1)*} = 1\), \(g^{(2)*} = -3/4 + U/8\pi v_F\), \(g^{(2)*} = 1/4 + U/8\pi v_F\), \(g^{(2)*} = 1\), which leads the system to the “spin gap metal” (SGM) phase. The scaling equation for the interladder one-particle hopping amplitude [Fig. 2(a)] is written as

\[
\frac{d \ln t_{\perp}(l)}{dl} = 1 - \left[ g^{(1)2} + g^{(2)2} + g^{(1)} g^{(2)} \right],
\]

which gives \(\frac{d \ln t_{\perp}(l)}{dl} \xrightarrow{l \to \infty} -U^2/32\pi^2 v_F^2 - 7/8\). Thus \(t_{\perp}(l)\) becomes always irrelevant. However it can grow at an early stage of scaling before the intraladder couplings grow sufficiently. Then the competition between the one-particle and two-particle crossovers takes place.

In the weakly coupled ladders, the two-particle process is dominated by the interladder hopping (Josephson tunneling) of \(d\)-wave like Cooper pairs. The lowest order scaling equation for the hopping amplitude, \(V^{\text{SCd}}\), which is depicted in
Fig. 2(b), is written as

$$\frac{dV^{\text{SCd}}(l)}{dl} = - \left[ \frac{t_\perp(l)g^{\text{SCd}}(l)}{E_0} \right]^2 + 2g^{\text{SCd}}(l)V^{\text{SCd}}(l) - \frac{1}{2} \left[ V^{\text{SCd}}(l) \right]^2,$$

where $g^{\text{SCd}} = \frac{1}{2}(g^{(1)}_t + g^{(2)}_t - g^{(1)}_0 + g^{(2)}_0)$ denotes the coupling for the SCd pair field. We have solved (3) with the initial condition, $V^{\text{SCd}}(0) = 0$. The third term of the r.h.s of (3) causes divergence of $V^{\text{SCd}}$ at a critical scaling parameter, $l_c$, determined by $V^{\text{SCd}}(l_c) = -\infty$, which gives the $d$-wave superconducting transition temperature, $T_c = E_0 e^{-l_c}$.

In Fig.3 (b), we show the phase diagram of weakly coupled ladders under the same conditions as in Fig.3(a). In this case, the one-particle crossover is strongly suppressed due to the intraladder correlation effects and $T_{\text{cross}}$ does not exist for larger $\tilde{U}$ than some crossover value, $\tilde{U}_c \sim 0.8$. For $U < U_c$, the SGM phase crosses over to the two-dimensional phase (2D phase) via the one-particle processes. Then the interladder coherent band motion takes places. For $U > U_c$, the SGM phase transits to the $d$-wave superconducting phase (SCd phase) via the two-particle processes. At $T < T_c$, coherent Josephson tunneling of the Cooper pairs in the interladder transverse direction occurs, while the interladder one-particle motion is incoherent. In both of coupled chains and ladders, in the temperature region, $T < T_{\text{cross}}$, the physical properties of the system would strongly depend on the shape of the 2D Fermi surface.

5 Conclusion
In the present work, we discussed nature of the dimensional crossovers in the weakly coupled chains and ladders, with emphasis on the difference between the two cases within the framework of the PRG approach. The difference of the universality class of the isolated chain and ladder profoundly affects the relevance or irrelevance of the inter-chain/ladder one-particle hopping. The strong coupling phase of the isolated ladder makes the one-particle process irrelevant so that the $d$-wave superconducting transition can be induced via the two-particle crossover in the weakly coupled ladders. The weak coupling phase of the isolated chain makes the one-particle process relevant so that the two-particle crossover can hardly be realized in the coupled chains.

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Figure 1: Linearized bands and scattering processes in the isolated chain (a) and ladder (b). Solid and broken lines represent the propagators of right-moving (R) and left-moving (L) electrons, respectively. In (b), $m$ and $\bar{m}$ denote different bands.

Figure 2: Diagrammatic representations of the scaling equations for the inter-chain/ladder one-particle hopping amplitude (a), and inter-chain/ladder two-particle hopping amplitude (b). In (b), only the superconducting channel is shown. A black circle and a shaded square represent an intra-chain/ladder scattering process and an inter-chain/ladder two-particle hopping amplitude, respectively.

Figure 3: Phase diagram of the weakly coupled Hubbard chains (a) and ladders (b), in terms of the intra-chain/ladder Hubbard repulsion, $\tilde{U} = U/\pi v_F$, and the temperature, $\tilde{T} = T/E_0$ for $t_\perp/E_0 = 0.001$. 

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