Is there maximal mixing in the lepton sector?

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We discuss the potential of long-baseline neutrino oscillation experiments to determine deviations from maximal $\nu_\mu$-$\nu_\tau$ mixing. We compare the obtainable sensitivities to predictions from neutrino mass models and to the size of quantum corrections. We find that the theoretical expectations for deviations are typically well within experimental reach.

One of the most interesting results in recent particle physics is the evidence for large generation mixing in the lepton sector, which has been established by neutrino oscillation experiments. Two of the three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ commonly used to parameterize the lepton mixing matrix are large: the “solar” mixing angle $\theta_{12}$ is approximately $33^\circ$, where maximal mixing is excluded at more than $5\sigma$ [1]. The best-fit value of the “atmospheric” mixing angle $\theta_{23}$ is very close to maximal mixing [2], where current data are still consistent with rather large deviations from maximality: at $3\sigma$ the allowed range is $0.31 \leq \sin^2 \theta_{23} \leq 0.72$ [1]. These results are in sharp contrast to the quark sector, where generation mixing is small. Maximal mixing is very interesting from the theoretical point of view, since it corresponds to a very particular flavour structure indicating an underlying symmetry. On the other hand, if significant deviations from maximality were established, the value of $\theta_{23}$ could just be a numerical coincidence.

A precise measurement for the leading atmospheric neutrino oscillation parameters will be mainly obtained from the $\nu_\mu$ survival probability determined by future long-baseline experiments. In addition to this disappearance channel, we include all appearance channels available for a given experiment in the analysis, which in some cases slightly increases the sensitivity to $\theta_{23}$. For quantitative evaluations, we discuss the next generation of conventional beam experiments, MINOS [3], ICARUS [4], and OPERA [5]. We show their combined results after five years of running time each. In addition, we investigate the potential of the first-generation superbeams JPARC-SK [6] and NuMI off-axis [7]. To estimate the potential after ten years from now, we combine the conventional beams and first-generation superbeams [8]. Eventually, we consider also the JPARC-HK superbeam upgrade [6] and a representative setup for a neutrino factory (labeled NuFact-II). The analysis techniques and precise definitions for the discussed experiments can be found in [8–10]. The most important parameter values are also given in the caption of Fig. 1.

In this figure, we show the potential of the discussed experiments to exclude maximal $\theta_{23}$. We simulate data for fixed “true values” of $\theta_{23}$ and $\Delta m^2_{31}$ and test if they can be fitted by $\theta_{23} = \pi/4$. For a fixed set $(\theta_{23}, \Delta m^2_{31})$ realized by nature, one can exclude maximal mixing at a certain confidence level (CL) if these values are within the corresponding shaded region. Thus, one can easily read off how far $\theta_{23}$ has to be from $\pi/4$ in order to be distinguished from it. For the current best-fit value of $\Delta m^2_{31}$, these results are summarized in Tab. I. From Fig. 1 and Tab. I, one can read off that the best sensitivity to maximal mixing is obtained by JPARC-HK: deviations as small as $4\%$ of $\sin^2 \theta_{23}$ from maximal mixing could be established at $90\%$ CL. The neutrino factory is not as good as one may expect, since it measures far away from the oscillation maximum. In fact, one can show that the sensitivity can be improved by about a factor of two for baselines much longer than 3000km. For all experiments the sensitivity strongly decreases for low values of $\Delta m^2_{31} \lesssim 2 \cdot 10^{-8}$ eV$^2$, which is well within the current $3\sigma$ range. In particular, because of the sharp energy spectrum the NuMI superbeam could provide excellent results only in a rather narrow region of $\Delta m^2_{31}$ around $3 \cdot 10^{-3}$ eV$^2$. Eventually, if $\Delta m^2_{31}$ is not too low, the combination of conventional beams, JPARC-SK, and NuMI will provide a rather good measurement at the $10\%$-level after about ten years from now.

The results in Fig. 1 are calculated for the true value $\theta_{13} = 0$. For $\theta_{13}$ close to the current bound, none of the shown results changes drastically. Only the neutrino factory potential is slightly improved, since the $\nu_e \rightarrow \nu_\mu$

| Experiment(s) | $[0.5 - \sin^2 \theta_{23}]$ |
|--------------|-----------------|
|               | 90% CL | $3\sigma$ |
| Conventional beams | 0.100 20% | 0.148 30% |
| JPARC-SK      | 0.057 11% | 0.078 16% |
| NuMI          | 0.079 16% | 0.126 25% |
| After ten years | 0.050 10% | 0.069 14% |
| JPARC-HK      | 0.020 4%  | 0.024 5%  |
| NuFact-II     | 0.055 11% | 0.075 15% |

TABLE I: Minimal values of $[0.5 - \sin^2 \theta_{23}]$ required to exclude maximal mixing at 90% CL and $3\sigma$ (absolute and relative values). For the oscillation parameters, we use the same values as in Fig. 1 and $\Delta m^2_{31} = 2.5 \cdot 10^{-3}$ eV$^2$. 

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channel contributes somewhat to the measurement of $\theta_{23}$. For all experiments, the results are basically independent of the mass hierarchy. Note that although the sensitivity to $|\Delta m_{23}^2|\sim 0.5 - \sin^2 \theta_{23}$ is rather good, in general it is very difficult to determine the sign ("$\theta_{23}$-degeneracy" [12, 13]). Finally, we remark that irrespective of the true value of $\sin^2 \theta_{23}$, the achievable accuracy is very similar to the sensitivities shown in Fig. 1 and Tab. I.

Let us now analyze theoretical expectations for the deviation from maximal atmospheric mixing. It could either be a feature of a mass model itself, or it could stem from quantum corrections due to the running of $\theta_{23}$ between high energy, where the model is defined, and low energy, where the experiments are performed. As to the first possibility, there exists a large variety of models aiming to explain the observed neutrino properties, utilizing various approaches such as Grand Unification, flavour symmetries, sequential right-handed neutrino dominance, textures, or combinations of these. Many of them are based on a version of the see-saw mechanism. There are models where the predicted $\theta_{23}$ lies in a range that does not include maximal mixing at all [14, 30, 35]. In many other cases a large atmospheric angle can be explained, while almost maximal mixing would require some tuning, see e.g. [18, 39–45].

Other works, for instance [15–17, 46], predict a value of $\theta_{23}$ rather close to $\pi/4$ at leading order, but various sources cause deviations that are typically still within the reach of future experiments.

In many cases, these deviations are related to small parameters, such as mass ratios. For example, even if we assume that maximal $\theta_{23}$ is predicted from proper-
ties of the neutrino mass matrix, corrections can stem from the charged lepton sector, with a typical order of magnitude of \( |0.5 - \sin^2 \theta_{23}| = O(m_\mu/m_\tau) \sim 0.06 \). Analogously, assuming that maximal \( \theta_{23} \) is predicted from the charged lepton mass matrix, a hierarchical neutrino mass matrix might induce \( |0.5 - \sin^2 \theta_{23}| = O(m_2/m_3) \sim 0.17 \) [47]. Deviations of this order of magnitude are also typical in models based on sequential right-handed neutrino dominance, where maximal \( \theta_{23} \) in leading order can originate from the dominant right-handed neutrino and the subdominant contribution leads to corrections (see e.g. [28–32]).

Thus, the described classes of models, summarized in Tab. II, favor deviations from maximal atmospheric mixing that will be measurable unless \( \Delta m_{31}^2 \) is very small. If they are not found experimentally, some new ingredients will be necessary. A value of \( \theta_{23} \) very close to \( \pi/4 \) corresponds to a rather particular configuration of lepton mixing parameters, which is clearly not compatible with the assumption of a neutrino mass matrix without any structure [48] and would require some theoretical reason. One option is employing flavour symmetries that enforce virtually maximal atmospheric mixing, see e.g. [19–25]. On the other hand, if maximal mixing is excluded experimentally by a broad margin, this will favor either a numerical coincidence without an underlying symmetry or models which can accommodate or even predict significant deviations. Either way, precise measurements of \( \theta_{23} \) will provide crucial information on the flavour structure of lepton mass models.

Models employing flavour symmetries, GUT relations or textures typically operate at a very high energy scale. Consequently, their predictions are modified by radiative corrections, i.e. the renormalization group (RG) running to low energy, where experiments take place. This means that even for a model predicting exactly maximal atmospheric mixing, one expects to measure deviations of the order of magnitude of the running effects [49]. Of course, the combination of deviations from \( \pi/4 \) at high energy and quantum corrections could, in principle, produce nearly maximal mixing at low energy. However, this possibility appears unnatural, since it requires a conspiracy between two effects that are not related in general.

One can easily estimate the size of the RG effects using the differential equation for \( \theta_{23} \) derived in [49]. It immediately follows that the effects are negligible in the Standard Model due to the smallness of the charged lepton Yukawa couplings. In the MSSM, these are enhanced by \( \tan \beta \), the ratio of the two Higgs vevs, so that the situation can change. In addition to the oscillation parameters, the running depends on the mass of the lightest neutrino, the value of the Majorana CP phases in the lepton mixing matrix, and \( \tan \beta \). The MSSM results are shown in Fig. 2. For a considerable parameter range, one finds corrections to \( \theta_{23} \) comparable to the precision of future experiments. Note that this is a conservative estimate, as we have neglected additional contributions coming from neutrino Yukawa couplings above the seesaw scale [50–52], which can cause sizable effects even in the Standard Model. Physics above the GUT scale could also contribute [53]. This provides a further argument why precision experiments have a good chance of measuring deviations from maximal atmospheric mixing.

In summary, we have discussed the potential of future long-baseline experiments to test maximal atmospheric mixing. The comparison with fermion mass models has shown that the deviations from maximal mixing predicted by many of them are large enough to be experimentally accessible. We have furthermore discussed the effects of renormalization group running, which connects the models built at very high energy scales with the measurements at low energies. These effects are also likely to cause deviations from maximality accessible by planned experiments. We conclude that if no deviation from maximal mixing can be established, the models will be severely constrained. This result will point towards a symmetry for maximal \( \theta_{23} \) and indicate small quantum corrections. Finally, compared to experiments involving quarks, measurements in the leptonic sector do not suffer from the limitation by hadronic uncertainties. Thus, in the long term, the combination of precision measurements of the atmospheric angle and other neutrino parameters, such as \( \theta_{13} \), has the potential to play an important role for exploring GUT-scale physics.

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| Model(s) | Refs. | \(|0.5 - \sin^2 \theta_{23}|) |
|----------|-------|--------------------------|
| Minimal SO(10) | [14] | > 0.16 |
| SO(10) + flavour symmetry | [15–17] | \( \lesssim 0.05 \) |
| SO(10) + texture | [18] | \( \lesssim 0.11 \) |
| Flavour symmetries | [19–25] | 0 |
| | [26] | 0.02 |
| | [27] | 0.04 |
| Sequential RH neutrino dominance | [28, 29] | 0.1 |
| + Flavour symmetries | [30–32] | 0.1 |
| + Type II see-saw upgrade | [33] | 0.01 .. 0.1 |
| Texture zeros | [34] | 0.07 |
| | [35] | > 0.1 |
| Perturbations of textures | [36] | \( \lesssim 0.16 \) |
| | [37, 38] | 0.005 .. 0.1 |

TABLE II: Selection of theoretical expectations for \(|0.5 - \sin^2 \theta_{23}|\) at tree level. The numbers should be considered as order of magnitude statements.
FIG. 2: Deviations of $\sin^2 \theta_{23}$ from 0.5 due to the running in the MSSM between high energy $M_U \approx 2 \cdot 10^{16}$ GeV, where maximal $\theta_{23}$ has been taken as initial condition, and low energy $M_{EW} \approx 10^2$ GeV. The contour lines correspond to deviations roughly equal to the 90% CL sensitivities listed in Tab. I, $\Delta \sin^2 \theta_{23} = 0.02, 0.05, 0.08$ and 0.1, respectively. In the left figure, the corrections are shown as a function of $\tan \beta$ and $m_1$, the mass of the lightest neutrino for a normal mass scheme. The right plot illustrates the dependence on the Majorana CP phases $\varphi_1$ and $\varphi_2$ (as defined in [49]) for $m_1 = 0.075$ eV. We have used $\Delta m^2_{21} = 2.5 \cdot 10^{-3}$ eV$^2$ and the same values as in Fig. 1 for the other parameters as further boundary conditions.

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