Skyrmion magnetic structure of an ordered FePt monolayer deposited on Pt(111)

S. Polesya, S. Mankovsky, S. Bornemann, J. Minár, and H. Ebert
Department Chemie und Biochemie, Ludwig-Maximilians-Universität München, 81377 München, Germany
(Date: October 22, 2013)

The effect of the Dzyaloshinsky-Moriya interaction on the magnetic structure of an ordered FePt monolayer deposited on Pt (111) surface has been investigated. All exchange interactions are obtained by means of first-principles calculations. The interplay of relativistic exchange coupling interactions leads to a helimagnetic (HM) structure at normal conditions. An applied magnetic field, however, creates a so-called Skyrmion structure, which is formed into a Skyrmion lattice at a certain value of the magnetic field.

PACS numbers: 75.70.Kw, 75.70.Ak, 75.70.Tj, 71.15.-m

The novel topological magnetic structure called Skyrmion crystal (SkX), observed recently in solids, attracts great interest due to various promising physical properties, both from a academical and technological point of view [1-4]. This concerns, in particular, the interaction of Skyrmions (Sk) with an electric current leading to the topological and anomalous Hall effect [5]. The presence of chiral interactions in the system, e.g., the spin-orbit coupling (SOC) induced so-called Dzyaloshinsky-Moriya (DM) interaction intrinsic for systems with lack of inversion symmetry [13]. This interaction can be responsible for a helimagnetic (HM) structure in the absence of an external magnetic field \( \vec{B}_{ext} \), while the vortex-like Sk magnetic structure can be stabilized by \( \vec{B}_{ext} \) [7, 8, 10, 15, 16]. The presence of chiral interactions discerns Skyrmions from the isotropic exchange coupling parameters j [17]. The SkX state was observed experimentally for the first time in the MnSi compound with B20 crystal structure [15, 18]. Since then, the SkX observation in bulk materials was reported also for others systems, e.g., FePt, Fe\(_{1-x}\)Co\(_x\)Si, Mn\(_{1-x}\)Fe\(_x\)Si and Mn\(_{1-x}\)Co\(_x\)Si [19-22]. The conditions for Skyrmion stability in these systems were investigated theoretically by phenomenological considerations based on the Landau-Ginzburg energy functional [10, 15, 16]. To describe properly the itinerant-electron properties of magnetism in these systems in the vicinity of the transition to the paramagnetic (PM) state, Bogdanov and Roessner account also for the energy related to longitudinal spin fluctuations. Interesting is that in 3D materials the temperature window of SkX stability in the phase diagram is rather narrow and located just below the transition temperature to the PM state. In the case of 2D systems, the SkX phase can exist in a larger temperature range approaching the temperature \( T = 0 K \) [20, 23]. Note that SkX observation in 3D systems is rather difficult because of the relatively weak DM interaction, while the formation of a non-collinear magnetic state requires the DM value strong enough to compete with the isotropic exchange interactions. This requirement can be met in deposited monolayers as observed experimentally in particular for Fe/ Ir(111) overlayers [11], Mn/W(110) overlayers [12] having SOC-induced HM structure as well as Sk magnetic structure recently observed in Fe(1ML)/Ir(111) [21] and Pd/Fe(1ML)/Ir(111) [25].

Despite the detailed theoretical investigations based on a phenomenological approach, a complete understanding of the conditions for the SkX appearance cannot be obtained without a detailed analysis of the microscopic origin of the exchange interactions (both, isotropic and anisotropic) and magneto-crystalline anisotropy (MCA), that needs a fully relativistic ab-initio description of the electronic structure of the systems under consideration. Therefore theoretical schemes based on relativistic density functional theory (DFT) [26] are applied to a large extent in computational simulations to support these experimental efforts [11, 13]. In the present work we focus on the first-principles investigation on the formation of the SkX state in a FePt (2 × 1) monolayer deposited on Pt(111). The magnetic properties of this system have been recently studied experimentally [27]. A pronounced non-collinear magnetic structure was predicted [28] due to the strong DM interactions and specific atomic structure of the system consisting of alternating Fe and Pt chains.

The T-B phase diagram of FePt/Pt(111) has been obtained performing Monte Carlo simulations using a standard Metropolis algorithm [29] and based on the extended Heisenberg model accounting for the SOC induced anisotropy of the exchange interactions. The corresponding Hamiltonian is given by:

\[
H = - \sum_{i,j(i\neq j)} J_{ij} \hat{e}_i \cdot \hat{e}_j - \sum_{i,j(i\neq j)} D_{ij} \cdot [\hat{e}_i \times \hat{e}_j] + \sum_i K_i (\hat{e}_i) \cdot \hat{e}_i \]

(1)

with the isotropic exchange coupling parameters \( J_{ij} \), the DM vector \( D_{ij} \) and the anisotropy constants \( K_i (\hat{e}_i) \) ac-
counting for the on-site magnetocrystalline anisotropy (MCA) energy associated with each individual moment oriented along \( \hat{e}_i \).

The main results presented here are obtained taking into account only the exchange interactions between magnetic Fe atoms, neglecting the contribution from the Pt atoms having a small induced magnetic moment which gives a small correction to the critical temperature.

A fully relativistic approach for the calculation of the exchange interaction tensor \( J_{ij} \) based on the magnetic force theorem was used [30]. This gives access to the isotropic and DM exchange coupling parameters used in an extended Heisenberg model, Eq. (1).

The electronic structure calculations for a monolayer of FePt on Pt(111), denoted FePt/Pt(111), have been performed within the local density approximation for DFT [31], using the spin-polarized relativistic Korringa-Kohn-Rostoker multiple scattering formalism [32]. In this scheme, the Dirac Green’s function was calculated self-consistently for FePt/Pt(111) assuming pseudomorphic deposition on a Pt slab consisting of 37 atomic layers and having the experimental lattice constant of bulk Pt (\( a = 3.924 \) Å). All calculations have been performed within the atomic sphere approximation (ASA) to the potentials and lattice relaxations near the surface have been neglected (see Ref. [33] for more details).

The structure of the \((2 \times 1)\) FePt/Pt(111) system is depicted in Fig. 1 (a) consisting of a monolayer of alternating Fe and Pt atomic chains deposited on a Pt(111) surface. Figure 1 (b) represents the isotropic exchange coupling parameters \( J_{ij} \) and \( D_{x}^{ij}, D_{y}^{ij} \) and \( D_{z}^{ij} \) components of the DM interaction vector. Strong \( D_{ij} \) interactions between the first neighboring Fe atoms indicate the favorite conditions for the appearance of a HM structure in the system. The properties of \( D_{ij} \) along different directions \( R_{ij} \) (Fig. 1 (b) and Table I) are clearly governed by the system symmetry [34, 35]. The symmetry plane between the in-chain Fe atoms 1 and 2 (Fig. 1 (a)) forces the \( D_{12} \) component along the chain to be equal to 0. The symmetry plane crossing the Fe atoms at positions 1 and 3 allows a non-zero component of \( D_{13} \) interaction along the direction parallel to Fe and Pt chains. Summarizing the results for the DM interaction for other distances one can conclude that they have pronounced in-plane components (see Fig. 1) responsible for a rotation of the magnetic moments within a corresponding plane orthogonal to the film. As it follows from the model consideration by Fert and Levy [36] giving the expression for DM interactions in the form \( D_{ij} \sim \frac{(R_{ij} \times R_{mi})(R_{ij} \times R_{mj})}{R_{ij} R_{mi} R_{mj}} \) (\( j, i \) correspond to the sites of magnetic atoms while \( n \) corresponds to the sites of atoms mediating exchange interaction), the DM interactions are mediated by the Pt atoms and their big magnitude is essentially determined by strong SOC of the Pt atoms. This allows also to draw conclusions about the main responsibility of the Pt atoms within the FePt monolayer for the \( D_z \) component of the DM vector, which is much smaller than \( D_x \) and \( D_y \) at short distances \( R_{ij} \). On the other hand, the large magnitude of the \( D_x \) and \( D_y \) components is fully determined by the substrate Pt atoms, giving evidence to the crucial role of their strong SOC values for the in-plane components of the \( D_{ij} \) interactions between the Fe atoms. At large distances \( R_{ij} \) all three components have the same order of magnitude, due to contributions of many Pt atoms involved into the mediation of Fe-Fe DM exchange interactions. Thus, one can clearly see a pronounced SOC induced effect in the present system leading to DM couplings strong enough to compete with the isotropic exchange coupling and to create a pronounced non-collinear magnetic structure.

Magnetic torque calculations show a rather strong in-plane MCA of 1.1 meV per Fe atom with the mag-

![FIG. 1. (a) Geometry of the system and directions of in-plane components of DM exchange interactions between Fe1 and Fe2 atoms, Fe1 and Fe3 atoms, etc. (b) Calculated exchange coupling parameters: isotropic, \( J_{ij} \) (top panel) and \( D_{x}^{ij}, D_{y}^{ij} \) and \( D_{z}^{ij} \) components of DM interactions between Fe atoms for FePt/Pt(111).](image)
netic easy axis being perpendicular to the Fe and Pt chains. The MCA energy was taken into account in all present MC simulations based on Heisenberg Hamiltonian Eq. (1). First, the calculations have been performed in the absence of an external magnetic field. The resulting non-collinear magnetic structure obtained at \( T = 1 \) K is presented in Fig. 2. The period of the helicity in the continual model represented by Landau-Ginsburg energy functional is determined by the ratio of exchange stiffness and Dzyaloshinsky-Moriya constants, \( A/D \) [8]. In general, these values are represented by tensors of first rank, \( D^α = |\sum_j \vec{D}_{ij} R^2_{ij}| \), and second rank, \( A^αβ = \sum_j J_{ij} R^2_{ij} \), characterizing a spatial anisotropy of the system, that is important for the present case having \( C_{nv} \) symmetry. In particular, these values give the anisotropy of the energy of spin-wave excitations with wave vector \( \vec{q} = q\hat{n}_α \) along different directions, \( \hat{n}_α = \hat{x}, \hat{y} \): \( ω(q^α) = A^αα(q_α)^2 + D^α q_α + ω_0 \), with spin-wave coefficients \( A^αα = \frac{2μ_B^2}{M} A^αα \) and \( D^α = \frac{2μ_B^2}{M} D^α \), and \( ω_0 \) the spin-wave gap created by magnetocrystalline anisotropy.

The strong anisotropy of the exchange stiffness tensor is well recognized considering the first-neighbor isotropic exchange coupling parameters (see Table 1) within and between the Fe chains. This leads to a complex HM structure with a different period along different directions, i.e. the period along the chains is essentially longer than perpendicular to the Fe chains, as can be clearly seen in Fig. 2. Raising the temperature from \( T = 0 \) K, the magnetization of the system exhibits two phase transition: from HM to FM state at \( T_H = 40 \) K, and from FM to the paramagnetic (PM) at \( T_C = 90 \) K.

The presence of an external magnetic field \( \vec{B}_{ext} \) perpendicular to the surface plane, exceeding a certain critical value, gives raise to the formation of Skyrmions in the system under consideration. The distribution of the magnetic moments within the single Skyrmion obtained using the MC simulation is shown in Fig. 3. One can clearly see the rather small Skyrmion size (compared with the period of the HM structure) due to a small \( A/D \) ratio. Its different size in two orthogonal directions within the plane is determined by the spatial anisotropy of the exchange coupling parameters, \( J_{ij} \) and \( \vec{D}_{ij} \). The in-plane tangential components of the magnetic moments governed by \( D^2_{ij} \) component of DM interactions are close to 0, in line with the discussions by Roessler et al. [16] on the magnetic moment distribution within the Skyrmions in FM system with \( C_{nv} \) symmetry.

Analyzing the requirements for the Skyrmion formation for FePt/Pt(111), one can use rather simple qualitative arguments. In this consideration the effect of a demagnetizing field can be neglected being small in the case of a magnetic monoatomic overlayer. A competition of isotropic exchange and DM interactions determine the period of the HM structure without magnetic field and anisotropy by minimizing the energy of the system. In the presence of an out-of-plane magnetic field the minimization of the Zeeman energy is realized by minimizing the 2D area with magnetic moments opposite to the direction of the magnetic field (the Zeeman energy gain is shown by yellow color, while the energy loss is shown by blue in Fig. 3 (a)). However, this costs the energy orig-

---

**TABLE I.** Components of the DM vector \( \vec{D}_{ij} \) and the isotropic exchange constant \( J_{ij} \) (in meV) for the FePt/Pt(111).

| \( i-j \) | \( R_{ij} \) | \( D^x_{ij} \) | \( D^y_{ij} \) | \( D^z_{ij} \) | \( J_{ij} \) |
|--------|-------|-------|-------|-------|-------|
| 1-2    | 0.707 | 0.00  | 2.44  | 0.39  | 23.90 |
| 1-3    | 1.225 | -2.47 | 0.00  | 0.00  | 4.59  |
| 1-4    | 1.414 | -0.31 | 0.50  | -0.69 | -0.56 |
| 1-5    | 1.414 | 0.00  | 0.39  | -0.17 | -1.07 |

---

**FIG. 2.** Helimagnetic structure in FePt/Pt(111). The arrows represent the magnetic moments of Fe atoms.

**FIG. 3.** Magnetic moment distribution within the Skyrmion. Yellow and blue colors in (a) represent schematically the region giving gain and loss of Zeeman energy in the presence of a magnetic field; blue color in (b) shows the region giving loss of the exchange energy contributed by in-plane components of magnetic moments. (c) and (d): Structure of single Sk obtained with the contributions of the Fe-Pt exchange interactions taken into account (the same color-code as in (a) and (b)). Long arrows show the spontaneous magnetic moments on Fe atoms, short arrows indicate the induced magnetic moments on Pt atoms.
inating from the inter-atomic isotropic exchange, due to a non-collinear orientation of the in-plane components of the magnetic moments within the Skyrmion (see Fig. 3 (b), where the exchange energy loss is shown by blue color), assuming that the Skyrmion size is close to the period of the HM structure. The competition of these two factors determine the conditions for the formation of Skyrmions at some critical value of the external magnetic field $\vec{B}_{\text{ext}}$. Despite the strong simplification neglecting the role of other contributions (due to exchange interactions and MCA) to the SkX energetics, this qualitative consideration visualizes the role of the magnetic field for the SkX formation as a ground state ($T = 0$ K), relevant in the case 2D system.

As was mentioned above, the 2D anisotropy of the system leads to a corresponding shape of the Skyrmions obtained via calculations accounting for Fe-Fe exchange interactions only. One calculation has been performed to investigate the effect of Fe-Pt interactions on the shape of the Skyrmion, accounting for the induced character of the Pt magnetic moment. The result, obtained at $T = 1$ K, is presented in Fig. 3 (c). One can see that the Skyrmion shape in this case is more symmetric. This occurs due to an additional Fe-Pt exchange contribution competing with DM interactions and leading that way to the increase of the Skyrmion size along direction perpendicular to the Fe chains.

The magnetic structure of FePt/Pt(111) and its behavior at different temperature and external magnetic field, perpendicular to the surface, has also been investigated via Monte Carlo simulations. For this, only the interactions within the Fe subsystem with well defined local magnetic moments have been taken into account. The MC unit cell containing 2500 Fe atoms was extended using periodic boundary conditions. The in-plane magnetic anisotropy was taken into account with MCA direction perpendicular to Fe chains and $E_{MCA} = 1.1$ meV (obtained in present calculations). The important role of the magnetic anisotropy for the stabilization of SkX state in some systems was discussed in the literature. Therefore, a set of additional MC calculations has been performed for the out-of-plane MCA direction (with the MCA energy equal to 1.1 meV) as well as for the MCA energy set to 0. All these results exhibit rather small differences indicating a weak effect of the MCA for the SkX stabilization in the system under consideration. Therefore, below we present only the results obtained for in-plane MCA.

More results of the MC simulation can be seen in Fig. 4 representing the low-temperature part of the $T$-$B$ phase diagram. Without magnetic field the system exhibits a rather pronounced SOC-induced HM structure (Fig. 4 (b)). An external magnetic field, $\vec{B}_{\text{ext}}$, leads at low temperature to the formation of Skyrmions coexisting with the HM structure (Fig. 4 (c)). Raising of the magnetic field at low temperature leads to the formation of a Skyrmionic lattice (Fig. 4 (d)). The area corresponding to this state is represented in red color in the phase diagram. At high magnetic fields and low temperature the concentration of Skyrmions is reduced resulting in the mixed FM + Sk state (Fig. 4 (e)) until the transition to the FM state. At these values of the magnetic field the transition to the FM + HM + Sk state does not occur. On the other hand, at high temperature and low magnetic field, as it was discussed above, a mixed FM + HM + Sk state is observed (Fig. 4(f)), while an increasing of the magnetic field leads to a mixed FM + Sk state (Fig. 4 (g)). At all values of the magnetic field the temperature increase leads to a transition from the non-collinear magnetic state to the FM state.

Thus, the behavior of the magnetic structure of FePt/Pt(111) in the presence of a finite external field shows striking differences when compared with the properties of 3D systems, e.g. FeSi, MnSi, MnGe, etc. This is, first of all, due to the well defined Fe local magnetic moment indicating a small effect of longitudinal fluctuations on the formation of Skyrmionic magnetic properties. A Skyrmion lattice state in the phase diagram appears at low temperature, in contrast to 3D systems, where the Skyrmion lattice formation occurs close to the temperature of the transition to the disordered conditions.
magnetic state. Because the Curie temperature in the system is rather high, the temperature increase leads first to the transition from the Skyrmionic to the FM state and then from the FM to the PM state, in contrast to the SkX-PM transition observed in the 3D bulk compounds FeSi, MnSi and MnGe. Finally, we would like to stress that strong in-plane components of $\vec{D}_{ij}$ interactions, governed by the substrate Pt atoms in FePt/Pt(111), lead to a small Sk size (see above), that makes this system attractive for technological applications. Note however, that too strong DM interactions can lead to a stabilization of the SkX as a ground state \([24, 30]\), even without external magnetic field. One can expect different behavior of this state when compared to those discussed above, although detailed investigations of its phase diagram so far have not been done.

Financial support by the DFG via SFB 689 (Spinphänomene in reduzierten Dimensionen) is thankfully acknowledged.

[1] A. N. Bogdanov and U. K. Rößler, Phys. Rev. Lett. 87, 037203 (2001).
[2] M. Lee, W. Kang, Y. Onose, Y. Tokura, and N. P. Ong, Phys. Rev. Lett. 102, 186601 (2009).
[3] T. Schulz, R. Ritz, A. Bauer, M. Halder, M. Wagner, C. Franz, C. Pfleiderer, K. Everschor, M. Garst, and A. Rosch, Nature Physics 8, 301 (2012).
[4] F. Jonietz, S. Mihlbauer, C. Pfleiderer, A. Neubauer, W. Münzer, A. Bauer, T. Adams, R. Georgii, P. Bni, R. A. Duine, K. Everschor, M. Garst, and A. Rosch, Science 330, 1648 (2010).
[5] A. Neubauer, C. Pfleiderer, B. Binz, A. Rosch, R. Ritz, P. G. Niklowitz, and P. Böni, Phys. Rev. Lett. 102, 186602 (2009).
[6] Y. Li, N. Kanazawa, X. Z. Yu, A. Tsukazaki, M. Kawasaki, M. Ichikawa, X. F. Jin, F. Kagawa, and Y. Tokura, Phys. Rev. Lett. 110, 117202 (2013).
[7] A. N. Bogdanov and D. A. Yablonov, Sov. Phys. JETP 68, 101 (1989).
[8] A. Bogdanov and A. Hubert, J. Magn. Magn. Materials 138, 255 (1994).
[9] U. K. Rößler, A. N. Bogdanov, and C. Pfleiderer, Nature 442, 797 (2006).
[10] A. B. Butenko, A. A. Leonov, U. K. Rößler, and A. N. Bogdanov, Phys. Rev. B 82, 052403 (2010).
[11] K. von Bergmann, S. Heinze, M. Bode, E. Y. Vedmedenko, G. Bihlmayer, S. Blügel, and R. Wiesendanger, Phys. Rev. Lett. 96, 176203 (2006).
[12] M. Bode, M. Heide, K. von Bergmann, P. Ferriani, S. Heinze, G. Bihlmayer, A. Kubetzka, O. Pietzsch, S. Blügel, and R. Wiesendanger, Nature 447, 190 (2007).
[13] M. Heide, G. Bihlmayer, and S. Blügel, Phys. Rev. B 78, 140403 (2008).
[14] I. E. Dzyaloshinskii, Sov. Phys. JETP 19, 960 (1964).
[15] C. Pappas, E. Lelièvre-Berna, P. Falus, P. M. Bentley, E. Moskvin, S. Grigoriev, P. Fouquet, and B. Faraday, Phys. Rev. Lett. 102, 197202 (2009).
[16] U. K. Roessler, A. A. Leonov, and A. N. Bogdanov, J. Phys. Chem. Solids 200, 022029 (2010).
[17] N. S. Kiselev, A. N. Bogdanov, R. Schäfer, and U. K. Rößler, (2011).
[18] S. Mihlbauer, B. Binz, F. Jonietz, C. Pfleiderer, A. Rosch, A. Neubauer, R. Georgii, and P. Bni, Science 323, 915 (2009).
[19] X. Z. Yu, Y. Onose, N. Kanazawa, J. H. Park, J. H. Han, Y. Matsui, N. Nagaosa, and Y. Tokura, Nature 465, 901 (2010).
[20] X. Z. Yu, N. Kanazawa, Y. Onose, K. Kimoto, W. Z. Zhang, S. Ishiwata, Y. Matsui, and Y. Tokura, Nature Materials 10, 106 (2011).
[21] A. Bauer, A. Neubauer, C. Franz, M. Münzer, M. Garst, and C. Pfleiderer, Phys. Rev. B 82, 064404 (2010).
[22] W. Münzer, A. Neubauer, T. Adams, S. Mihlbauer, C. Franz, F. Jonietz, R. Georgii, P. Böni, B. Pedersen, M. Schmidt, A. Rosch, and C. Pfleiderer, Phys. Rev. B 81, 041203 (2010).
[23] J. H. Han, J. Zhang, Z. Yang, J.-H. Park, and N. Nagaosa, Phys. Rev. B 82, 094429 (2010).
[24] S. Heinze, K. von Bergmann, M. Menzel, J. Brede, A. Kubetzka, R. Wiesendanger, G. Bihlmayer, and S. Blügel, 7, 713 (2011).
[25] N. Romming, C. Hanneken, M. Menzel, J. E. Bickel, B. Wolter, K. von Bergmann, A. Kubetzka, and R. Wiesendanger, Science 9, 636 (2013).
[26] A. H. MacDonald and S. H. Vosko, J. Phys. C: Solid State Phys. 12, 2977 (1979).
[27] J. Honolta, T. Y. Lee, K. Kuhnle, A. Enders, R. Skomski, S. Bornemann, S. Manovskiy, J. Minár, J. Staunton, H. Ebert, M. Hessler, K. Fauth, G. Schütz, A. Buchbaum, M. Schmid, P. Varga, and K. Kern, Phys. Rev. Lett. 102, 067207 (2009).
[28] S. Manovskiy, S. Bornemann, J. Minár, S. Polevaya, H. Ebert, J. B. Staunton, and A. I. Lichtenstein, Phys. Rev. B 80, 014422 (2009).
[29] K. Binder, Rep. Prog. Phys. 60, 487 (1997).
[30] L. Udvardi, L. Szunyogh, K. Paloútás, and P. Weinerberger, Phys. Rev. B 68, 104436 (2003).
[31] S. H. Vosko, L. Wilk, and M. Nusair, Can. J. Phys. 58, 1200 (1980).
[32] H. Ebert, in Electronic Structure and Physical Properties of Solids, Vol. 535 of Lecture Notes in Physics, edited by H. Dreyssé (Springer, Berlin, 2000), p. 191.
[33] S. Bornemann, O. Šípr, S. Manovskiy, S. Polevaya, J. B. Staunton, W. Wurth, H. Ebert, and J. Minár, Phys. Rev. B 86, 104436 (2012).
[34] T. Moriya, Phys. Rev. 120, 91 (1960).
[35] A. Crpieux and C. Lacroix, J. Magn. Magn. Materials 182, 341 (1998).
[36] A. Fert and P. M. Levy, Phys. Rev. Lett. 44, 1538 (1980).
[37] A. I. Lichtenstein, M. I. Katsnelson, and V. A. Guabanov, J. Phys. F: Met. Phys. 14, L125 (1984).
[38] S. Polevaya, S. Manovskiy, O. Šípr, W. Meindl, C. Strunk, and H. Ebert, Phys. Rev. B 82, 214409 (2010).
[39] U. K. Roßler, A. A. Leonov, and A. N. Bogdanov, J. Phys.: Condens. Matter 303, 012105 (2011).
[40] Y.-Q. Li, Y.-H. Liu, and Y. Zhou, Phys. Rev. B 84, 205123 (2011).