Mathematical study for an infectious disease with awareness-based SIS-M model

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Abstract. In this article, we discuss a mathematical model of Susceptible-Infected-Susceptible-Media (SIS-M), which considers the level of human awareness on certain presence of disease. To construct the model, we divided the population based on their health status into susceptible individuals unaware and aware of the disease and the infectious individuals. The level of awareness included private awareness associated with direct contacts between unaware and aware populations, global awareness due to reported cases of infection, and regular awareness campaigns from media or policy makers. The dynamical behaviour of the model was analysed rigorously. The disease-free equilibrium point, the endemic equilibrium point, and the basic reproduction number were shown in this model analytically and numerically. We found that the disease-free equilibrium point was locally asymptotically stable if $R_0 < 1$, and unstable if $R_0 > 1$. From the sensitivity analysis of $R_0$, it was found that there was a minimum intensity for the awareness campaign so that the level of awareness manifested in the efforts of individuals to protect themselves from disease successfully eradicate the disease from the community. This result indicates that the efforts of individuals in protecting themselves can have a massive effect on the existence of the disease in the community.

1. Introduction

Infectious diseases are caused by pathogenic microorganisms, such as bacteria, viruses, parasite or fungi. The diseases can spread directly or indirectly, from one individual to another [1]. From the 2018 Indonesian Basic Health Research data (Riskesdas) [2], the prevalence of acute respiratory infections (ARI) according to the diagnosis of health workers (specialist doctors, general practitioners, midwives, and nurses) in Indonesia in 2018 decreased by 4.4% compared to 2013 which reached around 14%. For the prevalence of pulmonary TB and malaria, both occupied 0.4%. Besides, Indonesia also still has challenges from HIV cases, which are increasing every year. In 2015 there were 30,935 cases, 2016 recorded 41,250 cases and 2017 recorded 48,300 cases. Infectious diseases can also cause a pandemic, including SARS, poliomyelitis, H1N1, Ebola, MERS-CoV, diphtheria, and drug-resistant TB [2].

The level of awareness plays a vital role in disease control. The result of the 2018 Riskesdas by the Health Research and Development Agency showed that the level of health awareness of the Indonesian people only reached 20% of the total population in Indonesia. This lack of public health awareness is due to the lack of knowledge about health and these diseases. This will make the community vulnerable to the danger of diseases such as infectious diseases. In this globalization era, various efforts to raise awareness of infectious diseases in the community include using media such as posters, television...
advertisements, newspapers, and campaigns and through education such as counseling by health service providers.

To model the spread of diseases, various efforts have been made both in terms of health, social, economics, and even mathematics. Mathematical models are used to explain a system, study the influence of system components, and make prediction about system behavior. In 2017, a deterministic SIS model that involves human awareness in the prevention of disease transmission was introduced [3]. The level of human consciousness is involved in the model as a variable that has its own dynamics. However, demographic factors (birth and death) were not included in the model. Meanwhile, various literature states that the number of births and deaths plays a role in how quickly a disease spread. Another awareness-based model has also been studied [4, 5, 6, 7, 8, 9].

Based on the above description, we modified the introduced model [3] by considering human demography into the model. The structure of this article is as follows. In Section 2, the construction of the model is given, mathematical analysis regarding the existence and local stability of equilibrium points, and how basic reproduction number becomes the threshold number of the model are explained in Section 3. Some numerical simulations are given in Section 4 and followed with conclusion in the last section.

2. Model construction

In this section, we construct a mathematical model using the SIS model. To develop the model, the human population is assumed to be constant. The population is divided into three classes, these are, the susceptible with unaware of the disease, the susceptible with aware of the disease and the infected humans. The proportion of them in the total population are denoted by $S_n(t), S_a(t), I(t)$ respectively. It is also assumed that the disease spreads due to the direct contact between the susceptible and the infected individuals only. The unaware susceptible become aware because of getting knowledge about the disease from some sources, such as media campaign, the number of reported cases, and from aware susceptible individuals. Furthermore, let $M(t)$ be the level of awareness in the population at time $t$. The level of awareness is dynamics and depends on the situation. As time passes, some awareness will lose its impact on people.

![Figure 1. Transmission Diagram for SIS epidemiology model with level of awareness](image)

We use transmission diagram in Figure 1 for our model. All the born individual will be classified into unaware susceptible, with a constant per capita recruitment rate $A$. A disease is characterized by a transmission rate $\beta$, which is reduced by the factor $0 \leq \sigma_s \leq 1$ that represents the decrease of infection in susceptibility due to being aware. The aware individuals will do some prevention such as use of face mask, vaccine, medicine, etc. because of the awareness that they get. The infection is assumed does not
confer the immunity, it means after being infected, individual recovered at a rate $r$ and a proportion $p$ will return into susceptible aware whereas the remaining proportion of $q$ will return into susceptible unaware. The construction of the model is as follows.

$$\frac{dS_n}{dt} = A - \beta IS_n - \eta MS_n + \gamma S_a + rqI - \mu S_n,$$

$$\frac{dS_a}{dt} = -\sigma S_n + \eta MS_n - \gamma S_a + rpl - \mu S_a,$$

$$\frac{dI}{dt} = \beta IS_n + \sigma S_n S_a - rI - \mu I,$$

$$\frac{dM}{dt} = \omega_0 + \alpha_0 I + \alpha S_a - \gamma_0 M,$$

with initial condition

$$S_n(0) = S_{n_0} \geq 0, \quad S_a(0) = S_{a_0} \geq 0, \quad I(0) = I_0 \geq 0, \quad M(0) = M_0 \geq 0,$$

where $S_{n_0} + S_{a_0} + I_0 = 1$ and $p + q = 1$.

Before we proceed to the model analysis, we need to state the following theorem about the positiveness and boundedness of the solution of model (1).

**Theorem 1.** The solutions $S_n(t), S_a(t), I(t), M(t)$ of the model (1) with the initial condition (2) are non-negative and bounded for every $t \geq 0$.

Based on the above discussion, we define

$$\Omega = \{(S_n, S_a, I, M) \in \mathbb{R}^4_+ : S_n + S_a + I = 1; \ 0 \leq M \leq M_0 e^{-\gamma_0 t} + \frac{\omega_0 + \alpha_0 \alpha + \gamma_0}{\gamma_0} (1 - e^{-\gamma_0 t}) \}.$$

**3. Model Analysis**

In this section, mathematical analysis is performed to determine the equilibrium points, which then the existence and stability criteria of these points will be further studied. The basic reproduction number as an endemic threshold will also be shown in this section.

The system (1) has a disease-free equilibrium point which is given by

$$DFE = (S_n^0, S_a^0, I^0, M^0),$$

where

$$S_n^0 = 1 - z_0, \quad S_a^0 = z_0, \quad M^0 = \frac{\alpha S_a^0 + \omega_0}{\gamma_0},$$

and

$$z_0 = \frac{1}{2} \left(1 - \frac{\gamma \omega_0 + \eta \omega_0 + \mu \gamma_0}{\eta \alpha} \right) + \frac{1}{4} \left(1 - \frac{\gamma \omega_0 + \eta \omega_0 + \mu \gamma_0}{\eta \alpha} \right)^2 + \frac{\omega_0}{\alpha}.$$

Since $0 \leq z_0 \leq 1$, for any values of parameter, the disease-free equilibrium point is biologically feasible.

The endemic equilibrium point for system (1) is given by

$$EE = (S_n^+, S_a^+, I^+, M^+),$$

where

$$S_n^+ = \frac{A + \gamma S_n^+ + rq I^+}{\beta I^+ + \eta M^+ + \mu}.$$
\[ S_a^+ = \frac{\eta M^+ S_n^+ + r p I^+}{\sigma_s B I^+ + \gamma + \mu}, \]
\[ M^+ = \frac{\omega_0 + a_0 I^+ + a S_a^+}{\gamma_0}, \]

and \( I^+ \) satisfying the equation
\[ a I^+^2 + b I^+ + c = 0, \]
where
\[ a = \beta^2 \mu^2 \sigma_s (\gamma_0 \beta + \eta \alpha_0) (k_2 - 1), \]
\[ b = \beta \mu [(\eta \mu^2 \sigma_s + \eta \mu \sigma_s) (\alpha + \alpha_0) + A \beta \eta \alpha_0 (1 - \sigma_s) + \beta \mu q r \gamma_0] (1 - k_1), \]
\[ c = [(\beta \eta \mu^3 \omega_0 + \beta \eta \mu^2 r \omega_0 + \beta \gamma \gamma_0 \mu^2 r + \beta \mu^2 \gamma_0 + \beta \mu^2 r + \gamma_0) (1 - \sigma_s) + (A \alpha \beta \eta \mu + A \alpha \beta \eta \mu r)] (k_0 - 1). \]

Here, \( k_2, k_1, \) and \( k_0 \) are functions of some parameters which cannot be shown in this article because it has a very long expression. Since the variables \( S_n, S_a^+ \), and \( M \) dependent on the variable \( I \), the endemic equilibrium will exist if \( I^+ > 0 \). Since the positive roots of \( I \) depends on the quadratic polynomial, then the system (1) will have:

a. One endemic equilibrium point if \( \frac{c}{a} < 0 \)

b. Two positive endemic equilibrium points if \( \frac{b}{a} < 0, \frac{c}{a} > 0, \) and \( b^2 - 4ac \geq 0; \)

c. Two identical endemic equilibrium points if \( \frac{b}{a} < 0, \frac{c}{a} > 0, \) and \( b^2 - 4ac = 0; \)

d. No endemic equilibrium point otherwise.

After we have the disease-free equilibrium point, we define the basic reproduction number as the average number of secondary infections produced by a typical case of an infection in a population where everyone is susceptible [5]. Using the next generation matrix approach [6], the basic reproduction number is given by
\[ R_0 = \frac{\beta ((1 - z_0) + \sigma_s z_0)}{\mu + r}. \]

Readers can see [9-15] for more examples about the application of the next-generation for calculating the basic reproduction number in some epidemiological models.

Next, for the stability of the two equilibrium points above, we form the Jacobian Matrix by linearization of the system (1) near the equilibrium point. The Jacobian Matrix is given as follows:
\[ J = \begin{bmatrix} -\beta I - \eta M - \mu & \gamma & -\beta S_n + qr & -n S_n \\ \eta M & -\sigma_s B I - \gamma - \mu & -\sigma_s B S_a + (1 - q) r & \eta S_n \\ \beta I & \sigma_s B I & \sigma_s B S_a + \beta S_n - \mu - r & 0 \\ 0 & \alpha & \alpha_0 & -\gamma_0 \end{bmatrix} \]

**Theorem 2.** The disease-free equilibrium DFE is locally asymptotically stable if \( R_0 < 1 \)

**Proof.** By substituting the disease-free equilibrium to the Jacobian matrix (6), the characteristic equation is
\[ (\alpha z_0 \eta \lambda + 2 \alpha z_0 \eta \gamma_0 - \alpha \eta \gamma_0 - \eta \lambda \omega_0 + \eta \gamma_0 \omega_0 + \lambda \gamma \gamma_0 + \gamma \gamma_0^2 + \lambda^2 \gamma_0 + \lambda \mu \gamma_0 + \lambda \gamma_0^2 + \mu \gamma_0^2) \]
\[ (\lambda + \mu)(\sigma_s \beta z_0 - \beta z_0 + \beta - \lambda - \mu - r) = 0. \]

We get \( \lambda_1 = -\mu < 0, \lambda_2 = \beta z_0 (\sigma_s - 1) + \beta - \mu - r \), and the rest are determined by the roots of the quadratic equation
\[ \lambda^2 \gamma_0 + (\alpha z_0 \eta + \eta \omega_0 + \gamma \gamma_0 + \mu \gamma_0 + \gamma_0^2) \lambda + 2 \alpha z_0 \eta \gamma_0 - \alpha \eta \gamma_0 - \eta \gamma_0 \omega_0 + \gamma \gamma_0^2 + \mu \gamma_0^2 = 0. \]

So \( \lambda_2 < 0 \) when \( R_0 < 1 \). Thus, the disease-free equilibrium is locally asymptotically stable if \( R_0 < 1 \). The other roots will be negative if only if
\[ 2\alpha z_0 \eta + \gamma \gamma_0 + \mu \gamma_0 > \alpha \eta + \eta \omega_0 \Rightarrow z_0 > \frac{1}{2} \left( 1 - \frac{\gamma \gamma_0 + \eta \omega_0 + \mu \gamma_0}{\eta \alpha} \right) \]

which always holds by Eq. (3), then Eq. (7) always has negative real part. □

Next, for the stability of the endemic equilibrium, the characteristic polynomial of Jacobian matrix at the endemic equilibrium point \( EE \) is written in the form

\[ P(\lambda) = \lambda^3 + a_2 \lambda^2 + a_1 \lambda + a_0, \]

where \( a_i \) are functions of the parameters that have a very long expression. By the Routh-Hurwitz criterion, the endemic equilibrium point will be locally asymptotically stable if the following conditions are satisfied

\[ a_0 > 0, \quad a_1 > 0, \quad a_2 > 0, \quad a_2 a_1 > a_0. \]

4. Numerical Simulation

In this section, numerical simulation will be conducted to strengthen our previous analytical result. Since our model shows the importance of \( R_0 \), it is relevant to show how \( R_0 \) behaved when parameters varying. This will be discussed in section 4.1. Lastly, numerical simulation about the autonomous model (1) was conducted for some scenarios to see our model's behaviour.

4.1. Sensitivity Analysis of \( R_0 \)

In this section, we discuss how \( R_0 \) behave on the change of \( \omega_0 \) and \( \sigma_s \). Using \( A = 0.05, \beta = 0.3, \eta = 0.01, \gamma = 0.1, \gamma_0 = 0.3, \mu = 0.05, r = 0.2, p = 0.9, q = 0.1, \alpha = 0.3, \alpha_0 = 30, \) and substituting them into the \( R_0 \) gave us Figure 2.

![Figure 2](image)

**Figure 2.** Sensitivity of \( R_0 \) with respect to \( \omega_0 \) and \( \sigma_s \)

If \( \sigma_s \) is assumed constant, the greater the value \( \omega_0 \), the smaller the value of \( R_0 \). The opposite condition occurs in the value of the parameter \( \sigma_s \) which is a parameter for reducing the spread of disease due to the presence of susceptible individuals who care for infectious diseases. However, Figure 2 shows that, when \( \omega_0 < \omega_0^{\text{min}} = 0.8499 \), for each value of \( \sigma_s \) it only produces a value of \( R_0 > 1 \) which is indicated by a red curve. Therefore, the intensity of regular campaigns carried out by the government plays an important role in increasing individual awareness of the disease, because no matter how an individual's efforts in protecting themselves from the disease if not balanced with more knowledge about infectious diseases obtained from regular campaigns, the disease will remain spread.
4.2. Autonomous Simulation

This simulation involves the variation in the values of $\omega_0$, i.e $\omega_0 = 0.2$, $\omega_0 = 0.8932948159$, and $\omega_0 = 1.5$, which gives varying $R_0$ values of 1.1478, 1, and 0.9 respectively. To conduct this simulation, we use the following parameters value: $A = 0.05$, $\beta = 0.3$, $\eta = 0.01$, $\gamma = 0.1$, $\gamma_0 = 0.3$, $\mu = 0.05$, $r = 0.2$, $p = 0.9$, $q = 0.1$, $\sigma_x = 0.04$, $\alpha = 0.3$, $\alpha_0 = 30$.

The stable disease-free equilibrium points for each case of $\omega_0$ are given in Table 1.

| $\omega_0$ | $R_0$ | $(S_n, S_d, I, M)$ |
|------------|-------|------------------|
| 0.2        | 1.14  | (0.8270, 0.1563, 0.0165, 2.4791) |
| 0.9        | 1.0   | (0.8263, 0.1736, 4.842 × 10^{−11}, 3.1512) |
| 1.5        | 0.9   | (0.7403, 0.2596, 0, 5.3596) |

**Figure 3.** Autonomous simulation of model 1 for various of $\omega_0$

From the graphs in Figure 3, it can be seen the greater the value of the parameter $\omega_0$, the greater the level of human concern for infectious diseases. By increasing the level of concern for the disease, it has succeeded in reducing the spread of the disease, which is marked by the reduction in the infected individual.

5. Conclusion

In this article, a modification of the SIS-M model by authors in [3] has been conducted by considering the demography effect on the model. Mathematical analysis of the equilibrium points and the basic reproduction number has been shown analytically.

The role of the policy makers in regular campaigns has positive implications for controlling the spread of infectious diseases. Also, it was found there was a minimum intensity for a regular campaign so that the efforts of individuals to protect themselves from disease succeeded in reducing the spread of infection from the community. Therefore, we hope that the government can maximize the intensity of regular campaigns so that the public is better educated and more alert in protecting themselves from the danger of infectious diseases.
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