Comments on the Hypothesis about Possible Class of Particles Able to Travel Faster then Light: Some Geometrical Models

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Abstract

The hypothesis about possible existence of new class of particles able to travel faster then light as a source of dark matter, recently formulated by L. Gonzalez-Mesters, is analyzed. To this end the general geometrical model for several kinds of matter with different Lorentzian structures coexisting on the same manifold is introduced and the local energy density in cosmological reference frame is calculated in two particular cases. It is shown that the local energy density is positive in both considered cases and hence such models really may describe cosmological dark matter. Nevertheless, some problems may appear during the construction of the cosmological models or the models of the compact objects. Moreover, the simplest generalization of the model lead to some variants of vector-tensor theory of gravitation with preferable reference frame which contradict to observations.
1 Introduction

In this note we discuss the hypothesis about new class of particles able to travel faster than light as a possible source of dark matter, which was proposed recently by L. Gonzalez-Mestres [1]. According to this hypothesis, the apparent Lorentz invariance of classical space-time can be viewed as a symmetry of the equations of motion of observed matter. In this case no reference to absolute properties of space and time is required and besides the usual particles and fields, whose speed could not exceed the speed of light \( c \), another kinds of matter may exists, whose particles have limiting speed \( c_1 >> c \). It was argued, that if there is no direct interaction between such two (or more) kinds of matter then their coexistence does not violate the apparent Lorentz invariance of the laws of physics but may provide most of the cosmic dark matter and produce very high energy cosmic rays compatible with some unexplained discoveries (see for example [1] and references therein). This hypothesis was analyzed in [1] on pure heuristic level without consideration of possible mechanisms of gravitational interaction between different sorts of matter.

In principle, the hypothesis about coexistence of different noninteracting kinds of matter may be associated with some ideas connecting with formalism of quantum gravity. Namely, the space-time in classical gravity has metric with Lorentzian signature while the formalism of continual integration of quantum gravity uses metric with Euclidean signature. The connection between Euclidean and Lorentzian metric is realized by Wick rotation. There are different points of view on this procedure, beginning from the consideration of Wick rotation as a pure formal method until Hawking’s assumption about Euclidean nature of space-time [2]. If this assumption is valid, then it is naturally to suppose, that among the knowing kinds of matter with apparent Lorentzian structure of space-time the other sorts of matter with their own Lorentzian structures may exist.

In another context the assumption about possible coexistence of different sorts of matter on the same manifold was formulated in [3] where the geometrical model for four noninteracting classes of matter with different Lorentzian structures was given. It was pointed out lately, that the existence of matter with nonstandard Lorentzian structure may give contribution into the energy-momentum tensor [4], but the corresponding models were not analyzed in details.

In this note the geometric ideas of papers [3, 4] are used to analyze the hypothesis [1]. To this end in the next section the general geometrical background for different Lorentzian structures which correspond to different sorts of matter on the same Riemannian manifold will be formulated. The resulting energy-momentum tensor is considered in the section 3 for two particular models when the light cone of the usual matter are in the interior of the ”light cone” of the ”superluminal” matter and when the ”light cones” of usual and ”superluminal” matter have empty intersection. Section 4 contains some concluding remarks and discussion.
2 The multiple Lorentziann structures on the Riemannian manifold

It is clear, that in the most general form the hypothesis about coexistence of two or more different Lorentzian structures on the same manifold, which correspond to the different classes of matter, leads to some kind of bi- or multi-metric gravitation theory. To provide the gravitational interaction between particles and fields corresponding to different Lorentziann structures, the metrics, which define these structures, must be connected with each other. In this paper the particular form of such connection is used.

Our approach is based on the well known correspondence between Riemannian structure of manifold, the field of line elements on it and the Lorentziann structures on the same manifolds. Namely, let \( G_{\alpha\beta} \) is a Riemannian metric on manifold, \( u_\alpha \) is a unit vector field \((G_{\alpha\beta} u_\alpha u_\beta = 1)\) representing the field of line elements and \( c = \text{const} > 0 \) is a light speed, then

\[
g_{\alpha\beta} = (c^2 + 1)u_\alpha u_\beta - G_{\alpha\beta}
\]

(1)

is Lorentziann metric on the same manifold. To apply the above equation to the analysis of the correspondence between several Lorentziann structures on the same Riemannian manifold, let’s suppose that space-time has Riemannian metric \( G_{\alpha\beta} \) with signature \((+,-,+,-)\) and the apparent Lorentzian structure, which is associated with observed matter, is defined by equation (1) with the light speed \( c = 1 \) (in geometrical units). Vector field \( u_\alpha \) in (1) may be considered as generator of cosmological reference frame. According to the hypothesis \( [1] \) let’s suppose also that among the usual Lorentziann structure there is additional Lorentziann structure on the same manifold, which correspond to nonobservable (dark) matter and is defined by unit vector field \( v_\beta \neq u_\alpha \) and the ”light speed” \( c_1 > 1 \).

By such a way we obtain a bimetric-type model, whose metrics \( g_{\alpha\beta} \) and \( q_{\alpha\beta} \), corresponding to the observed and hypothetical dark matter, are defined by

\[
g_{\alpha\beta} = 2u_\alpha u_\beta - G_{\alpha\beta}
\]

(2)

and

\[
q_{\alpha\beta} = (c_1^2 + 1)v_\alpha v_\beta - G_{\alpha\beta}
\]

(3)

Following to \( [1] \), the particles which moves in Lorentziann metric \( q_{\alpha\beta} \) will be called ”superluminal” or ”dark matter”.

It is easy to see that the connection between the determinants of the metrics \( G_{\alpha\beta} \), \( g_{\alpha\beta} \) and \( q_{\alpha\beta} \) are given by (see also )

\[
G = -g = -g/c_1^2
\]

(4)

The correspondence between metrics (2) and (3) are defined by

\[
q_{\alpha\beta} = g_{\alpha\beta} - 2u_\alpha u_\beta + (c_1^2 + 1)v_\alpha v_\beta
\]

(5)

For the following it is necessary to define also the correspondence between the inverse metrics \( g^{\alpha\beta} \) and \( q^{\alpha\beta} \). It is clear that

\[
g^{\alpha\beta} = 2u^\alpha u^\beta - G^{\alpha\beta}
\]

(6)

\(^1\)An analogous equations were used also in \( [3] \) to generate exact solutions of vacuum Einstein equations.
and
\[ q^{\alpha\beta} = \frac{1 + c_1^2}{c_1^2} v_G^\alpha v_G^\beta - G^{\alpha\beta} \]  (7)
where \( u^\alpha = G^{\alpha\beta} u_\beta = g^{\alpha\beta} u_\beta \), \( v_G^\alpha = G^{\alpha\beta} v_\beta \). Using (6) equation (7) may be rewritten as
\[ q^{\alpha\beta} = g^{\alpha\beta} - 2 u^\alpha u^\beta + \frac{1 + c_1^2}{c_1^2} k^{\alpha\beta} \]  (8)
where
\[ k^{\alpha\beta} = 4 u^\alpha u^\beta (u^\rho v_\rho)^2 - 2 (u^\alpha v^\beta + v^\alpha u^\beta) u^\rho v_\rho + v^\alpha v^\beta \]  (9)
with \( v^\alpha = g^{\alpha\beta} v_\beta \).

Using the well known equations of the bimetric formalism it is easy to obtain expressions which connect the covariant derivatives with respect to metrics \( G_{\alpha\beta}, g_{\alpha\beta} \) and \( q_{\alpha\beta} \).

Equations (2)-(9) make possible to analyze the gravitational interaction between of the usual (observable) matter and nonobservable (dark) matter.

3 Energy-momentum tensor of dark matter in the simplest model

For simplicity in this section we consider two massive scalar fields \( \varphi \) and \( \psi \), one of which (\( \varphi \)) corresponds to the usual (observed) matter with metric (2) and the other field (\( \psi \)) corresponds to nonobserved (dark) matter whose own Lorentzian structure are defined by metric (3). Simplest action functional for such model may be written in the form
\[ S = \int (R_g + L_\varphi + c_1 L_\psi) \sqrt{-g} d^4x \]  (10)
where \( R_g \) is the Ricci scalar of the metric (2), \( L_\varphi \) and \( L_\psi \) are the Lagrangians of the fields \( \varphi \) and \( \psi \) correspondingly:
\[ L_\varphi = \frac{1}{2} g^{\alpha\beta} \varphi_{,\alpha} \varphi_{,\beta} + \frac{1}{2} m_\varphi^2 \varphi^2 \]  (11)
and
\[ L_\psi = \frac{1}{2} q^{\alpha\beta} \psi_{,\alpha} \psi_{,\beta} + \frac{1}{2} m_\psi^2 \psi^2 \]  (12)
where \( m_\varphi \) and \( m_\psi \) are the masses of the corresponding fields and the coefficient \( c_1 \) in (10) is present because of (3). The geometrical units with \( c = \kappa = 1 \), where \( \kappa \) is Einsteinian gravitational constant, is used here. The absence of mixed \( \varphi \psi \) terms in (10) provide the Lorentz invariance of the motion equations of the usual matter (\( \varphi \)) and nonobservability of the hypothetical matter (\( \psi \)).

Substitution of (8) into (12) gives
\[ L_\psi = \frac{1}{2} (g^{\alpha\beta} - 2 u^\alpha u^\beta + \frac{1 + c_1^2}{c_1^2} k^{\alpha\beta}) \psi_{,\alpha} \psi_{,\beta} + \frac{1}{2} m_\psi^2 \psi^2 \]  (13)
where \( u^\alpha = g^{\alpha\beta} u_\beta \) and \( k^{\alpha\beta} \) is defined by (9).
It is clear that variation of (10) with respect to $\varphi$ and $\psi$ gives the usual Klein-Gordon equations for these fields

$$
\frac{1}{\sqrt{-g}} \frac{\partial}{\partial x^\alpha} \left( \sqrt{-g} g^{\alpha \beta} \frac{\partial}{\partial x^\beta} \right) \varphi - m_{\varphi}^2 \varphi = 0 \tag{14}
$$

and

$$
\frac{1}{\sqrt{-q}} \frac{\partial}{\partial x^\alpha} \left( \sqrt{-q} q^{\alpha \beta} \frac{\partial}{\partial x^\beta} \right) \psi - m_{\psi}^2 \psi = 0 \tag{15}
$$

while variation of (10) with respect to $g^{\alpha \beta}$ gives

$$
G_{\alpha \beta} = T_{\alpha \beta} + c_1 \tilde{T}_{\alpha \beta} \tag{16}
$$

Here $G_{\alpha \beta}$ is Einstein tensor corresponding to the metric $g_{\alpha \beta}$, $T_{\alpha \beta}$ is the energy-momentum tensor of classical matter (field $\varphi$) and the additional term $\tilde{T}_{\alpha \beta}$ is the energy-momentum tensor of the field $\psi$ which may be associated with hypothetical "dark" matter moving in metric $g_{\alpha \beta}$. So, the presence of the additional kind of matter with its own Lorentzian structure does not change the local Lorentz invariance of the field (and motion) equations for the usual matter but give additional term in the right-hand side of Einstein equation. In general case this term has the form

$$
-\tilde{T}_{\alpha \beta} = \frac{1}{2} \left\{ \psi_{;\alpha} \psi_{;\beta} - 2 (u_\alpha \psi_{;\beta} + u_\beta \psi_{;\alpha}) u^\rho \psi_{;\rho} \right\} +
$$

$$
2 \frac{c_1^2 + 1}{c_1^2} \left[ (u_\alpha \psi_{;\beta} + u_\beta \psi_{;\alpha}) u^\rho \psi_{;\rho} (u^\sigma v_{\sigma})^2 + (u^\rho \psi_{;\rho})^2 u^\sigma v_{\sigma} (u_\alpha v_\beta + u_\beta v_\alpha) \right] -
$$

$$
\frac{c_1^2 + 1}{c_1^2} \left[ (u_\alpha \psi_{;\beta} + u_\beta \psi_{;\alpha}) u^\rho \psi_{;\rho} u^\sigma v_{\sigma} + (v_\alpha \psi_{;\beta} + v_\beta \psi_{;\alpha}) u^\rho \psi_{;\rho} u^\sigma v_{\sigma} \right] -
$$

$$
\frac{c_1^2 + 1}{c_1^2} \left[ u^\rho \psi_{;\rho} u^\sigma v_{\sigma} (u_\alpha v_\beta + u_\beta v_\alpha) - (v_\alpha \psi_{;\beta} + v_\beta \psi_{;\alpha}) u^\sigma v_{\sigma} \right] -
$$

$$
\frac{1}{4} g_{\alpha \beta} \left( q^{\alpha \beta} \psi_{;\alpha} \psi_{;\beta} + m_{\psi}^2 \psi^2 \right) \tag{17}
$$

The above expression is rather complicated. By this reason it will be considered in two particular cases when the light cones of the usual and "superluminal" matter are collinear ($u_\alpha = v_\alpha$ with $c_1 >> 1$) and orthogonal $u_\alpha \perp v_\alpha$ ($c_1 = 1$) to each other.

1. Colinear light cones. In this case, which is closely connected with the ideas of paper [1], $u_\alpha = v_\alpha$, $c_1 >> 1$, tensor $k^{\alpha \beta} = u^\alpha u^\beta$ and (13) takes the form

$$
L_\psi = \frac{1}{2} (g^{\alpha \beta} - \frac{c_1^2 - 1}{c_1^2} u^\alpha u^\beta) \psi_{;\alpha} \psi_{;\beta} + \frac{1}{2} m_{\psi}^2 \psi^2 \tag{18}
$$

For the energy-momentum tensor of this field we have the following expression

$$
-\tilde{T}_{\alpha \beta} = \frac{1}{2} \left\{ \psi_{;\alpha} \psi_{;\beta} - \frac{c_1^2 - 1}{c_1^2} (u_\alpha \psi_{;\beta} + u_\beta \psi_{;\alpha}) u^\rho \psi_{;\rho} \right\} - \frac{1}{2} g_{\alpha \beta} L_\psi \tag{19}
$$
The local energy density in the reference frame which is defined by vector field $u_\alpha$ is equal to

$$\tilde{T}_{\alpha\beta}u^\alpha u^\beta = (u^\rho \psi,_{\rho})^2 \frac{c_1^2 - 2}{2c_1^2} + \frac{1}{4}m_\psi^2 \left( c_1^2 + \psi^2 \right)$$

(20)

Here we use well known equation $q^{\alpha\beta} \psi,_{\alpha} \psi,_{\beta} = m_\psi^2 c_1^2$. It is clear, that the local energy density in this case is positive if $c_1^2 > 2$, non-negative if $c_1^2 = 2$ and may be both positive and negative if $1 < c_1^2 < 2$.

2. Orthogonal light cones. In this case $c_1 = c = 1$, $u^\alpha v_\alpha = 0$, $k^{\alpha\beta} = \psi,_{\alpha} \psi,_{\beta}$ and (13) reads

$$L_\psi = \frac{1}{2} \left( g^{\alpha\beta} - 2u^\alpha u^\beta + 2u^\alpha v^\beta \right) \psi,_{\alpha} \psi,_{\beta} + \frac{1}{2}m_\psi^2 \psi^2$$

(21)

Direct calculation gives the following expression for energy momentum tensor of the field $\psi$ in this case

$$-\tilde{T}_{\alpha\beta} = \frac{1}{2} \left\{ \psi,_{\alpha} \psi,_{\beta} - 2 \left( u_\alpha \psi,_{\beta} + u_\beta \psi,_{\alpha} \right) u^\rho \psi,_{\rho} \right\} +$$

$$\left( v_\alpha \psi,_{\beta} + v_\beta \psi,_{\alpha} \right) v^\rho \psi,_{\rho} - \frac{1}{2}g_{\alpha\beta} L_\psi$$

(22)

The local energy density in the reference frame which is defined by vector field $u_\alpha$ (in cosmological reference frame) is equal to

$$\tilde{T}_{\alpha\beta}u^\alpha u^\beta = \frac{3}{2} \left( u^\rho \psi,_{\rho} \right)^2 + \frac{1}{4}m_\psi^2 \left( 1 + \psi^2 \right)$$

(23)

It is easy to see, that the local energy density in this case is positive because $u^\rho \psi,_{\rho}$ and $m_\psi^2$ cannot become zero simultaneously.

The last model may be generalized as follows [3]. Let vector field $u_\alpha$ generate Lorentzian structure which corresponds to the observed matter and let the fields $v^i_\alpha$ ($i = 1, 2, 3$) form an orthonormal frame in orthogonal space sections, i.e. $G^{\alpha\beta} v^i_\alpha v^j_\beta = \delta^{ij}$ and $u^\alpha v^i_\alpha = 0$. Analogously to the above, it may be supposed that every field $v^i_\alpha$ corresponds to some kinds of non-observable (“dark”) matter with its own Lorentziann structure

$$g^{i\alpha} = 2v^j_\alpha v^j_\beta - G_{\alpha\beta} = g_{\alpha\beta} - 2u_\alpha u_\beta + 2v^i_\alpha v^j_\beta$$

In this case the full energy-momentum tensor $T_{\alpha\beta}$ will consist of four terms: the term corresponding to the usual matter and three additional terms which correspond to the nonobserved matter. In the simplest case when hypothetical kinds of matter are represented by massive scalar fields these additional terms will have the form (22).

4 Concluding remarks

In the modern classical general relativity metric of space-time depends on the matter distribution. However the signature of the metric, which defines the local Lorentziann structure of space-time, is supposed to be fixed. It may be supposed that the local Lorentziann structure of space-time is also defined by the features of matter. If such supposition is true then the
question about uniqueness of such structure is naturally appears. This question may be reformulated also as the question about possible coexistence on the same manifold several different classes of matter which generate different non-equivalent Lorentzian structures.

As it was argued in [1], the coexisting of such different classes of matter without direct interaction between them does not contradict to the apparent Lorentz invariance of the laws of physics. On the other hand it may provide most of the cosmic dark matter and produce very high energy cosmic rays compatible with some unexplained discoveries (see for example [1] and references therein).

It is shown that such different classes of matter, which is mentioned in [1], may be naturally described using the well known correspondence (1) between Riemannian structure of manifold (Riemannian metric $G_{\alpha\beta}$), the field of line elements on it (represented by the unit vector field $u_{\alpha}$) and the Lorentzian structures (Lorentzian metric $g_{\alpha\beta}$) on the same manifold. Using this correspondence, two Lorentzian structures $g_{\alpha\beta}$ and $q_{\alpha\beta}$ on the same manifold $(M^4, G)$ may be connected by equation (2), which may be used for description of the gravitational interaction between the usual matter, which moves in metric $g_{\alpha\beta}$, and hypothetical matter moving in metric $q_{\alpha\beta}$. Using the simplest model it was shown that the presence of the additional kind of matter with its own Lorentzian structure does not change the local Lorentz invariance of the field (and motion) equations for the usual matter but gives additional term in the right-hand side of Einstein equation. This additional term may be considered as the energy-momentum tensor of dark matter.

In the case then hypothetical dark matter is represented by massive scalar field $\psi$ its energy-momentum tensor is given by equation (19) for colinear light cones or (22) for orthogonal light cones. The corresponding local energy density in cosmological reference frame connected with vector field $u_{\alpha}$ is given by equations (20) and (23) respectively. In both cases the local energy density is positive, so both models may describe cosmological dark matter. It is clear, that in the case of colinear light cones the homogeneous isotropic cosmological solutions for metric $g_{\alpha\beta}$ may be easily constructed as well as in the standard general relativity while in the case of orthogonal light cones metric $g_{\alpha\beta}$ may be spatially homogeneous in this case only if the metrics $q_{\alpha\beta}$ are static. On the other hand, there are no obvious problems with construction of the models of the compact objects for the usual matter with the metric $g_{\alpha\beta}$ in the case with orthogonal light cones, while the case of colinear light cones may lead to the apparent violation of the equivalence principle because any concentration of the usual (observable) matter is an attractor for dark matter and vice versa.

The action (10), which where considered in the previous section, is non-symmetric with respect to metrics $g_{\alpha\beta}$ and $q_{\alpha\beta}$: action (10) may be varied with respect to metric $g_{\alpha\beta}$ or with respect to metric $q_{\alpha\beta}$ but not both. Moreover, the fields $u^\alpha$ and $v^\alpha$ remain undefined and their nature is unclear. The introduction of these fields in Lagrangian as usual massive or massless vector fields leads to some variant of vector-tensor gravitation theory with preferable reference frame, whereas it is known that the existence of such frames contradicts to observations [3].

Thus, the hypothesis that the cosmological dark matter is generated by the hypothetical particles which able to travel faster then light has rather simple geometrical realization, but its consistency with observations is problematic.
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References

[1] L. Gonzalez-Mestres, Physical and cosmological implications of a possible class of particles able to travel faster than light, Paper hep-ph/9610474 of electronic library; Cosmological Implications of a Possible Class of Particles Able to Travel Faster than Light, Nucl. Phys. Proc. Suppl., (1996), 48, 131, (Paper astro-ph/9601090 of electronic library).

[2] S. W. Hawking, Euclidean Quantum Gravity, in: Recent Developments in Gravitation, ed. S. Deser, Plenum Press, 1978.

[3] M. Yu. Konstantinov, Topological transitions in classical gravity: the scalar-tensor formalism, in: The Problems of Gravitation Theory and Elementary Particles. v. 16, eds. K.P. Stanyukovich and V.N. Melnikov, Moscow, Energoatom, 1985, pp.148-157.

[4] M. Yu. Konstantinov, Super-Light Speed Travel and Causality, in: “Astrophysics and Cosmology after Gamov. The conference devoted to the 90th anniversary G.A. Gamov, September 5-10, 1994, Odessa, Ukraine”, Moscow, Cosmoinform, 1994, p. 16.

[5] M.M. Beilinson, Izvestya vuzov. Fizika, (1982), N 1, 50-53. (in Russian)

[6] C.M. Will, Theory and experiment in gravitational physics, Cambridge U.P., 1981.