Wave transformation due to floating thick elastic plate over multiple stepped bottom topography

Praveen K M and D Karmakar
Department of Applied Mechanics and Hydraulics
National Institute of Technology Karnataka, Surathkal, Mangalore – 575025, India

Abstract. The wave transformation due to floating elastic plate is studied over multiple stepped bottom topography. The behaviour of the flexible floating plate is studied considering Timoshenko-Mindlin’s plate thick theory at finite water depth and the edge conditions for the floating plates are considered to be free-free support condition. The mathematical model is developed based on the eigenfunction expansion method to study the hydroelastic behaviour of a floating thick elastic plate at varying water depths, step thickness and plate sizes acted upon by ocean wave. The mode coupling relation is employed along with orthogonality relation and the continuity equations for velocity, pressure, deflection, bending moment, slope and shear force to determine the wave characteristics in reflection and transmission region due to variable bottom topography. The numerical computation is carried out to obtain the hydroelastic behaviour of floating elastic plate acted upon by ocean waves. Further, detail comparison of the numerical results is performed for different step bottom topography for the hydroelastic analysis of floating thick elastic plate. The present study will provide an insight into the effect of seabed profile due to the wave scattering from a large floating thick elastic plate at finite water depth.

1. Introduction
The design and development of VLFS in the coastal regions have increased over the past few decades. Some of the commonly studied large floating structures are floating cities, mobile offshore bases, floating runways, floating parks and floating oil storage base. The study on the scattering of ocean wave due to uneven bottom topography has been a significant topic of research along the coastal regions. The unevenness is the cause for wave refraction, reflection and shoaling along the coastal areas. Hence, the construction of these large floating structures needs further understanding of wave transformation due to floating structures over uneven bottom topography. Due to the huge sizes of these structure as compared to the wavelength of the ocean waves, rigid body motions are negligible and hydroelastic behaviour of the structure becomes significant in the analysis. The major proportion on the study of hydroelastic behaviour along the large floating structures is performed based on thin plate theory assuming the thickness to be small. These large floating structures are substantially thick and hence Timoshenko-Mindlin’s thick plate theory [17] which takes into the effect of rotary inertia and transverse shear deformation is used in the analysis.

The various studies in the analysis of floating thick elastic plates for the hydroelastic characteristics based on Timoshenko-Mindlin’s plate theory were carried out by [2, 9] for the case for sea-ice interaction and [10, 18] for wave-structure. The wave scattering due to the rectangular obstacle was analysed by Mei et al. [16] in a finite depth channel. They employed the variational formulation for the numerical computations. A new approach was adopted by Evans and Linton [8] to solve the wave scattering due to varying bottom topography using 2D linear water wave theory. The varying bottom topography was modelled as a uniform strip to be considered in the variable free surface boundary.
condition. Athanassoulis and Belibassakis [1] employed a consistent coupled mode theory and extension of the theory by [5, 6] was used to analyse the wave propagation over variable bottom topography. The hydroelastic behaviour of a VLFS over varying sea bottom topography was considered by Kyoung et al. [15] for four different cases. They used FEM based on the variational formulation to analyse the influence of bottom topography in the fluid domain. The hydroelastic behaviour of the floating structure was calculated considering the mode superposition method. The wave scattering due to a semi-infinite floating membrane was analysed by Karmakar and Sahoo [12] for a changing bottom topography. The steps are considered to be a finite and infinite step in the analysis of reflection, transmission and deflection of a floating membrane. The flexural gravity wave scattering was analysed by Karmakar et al. [13] due to multiple stepped bottom topography under the action of obliquely incident waves. They used wide spacing approximation to analyse the scattering of waves from multiple steps and submerged blocks. The diffraction pattern due to the obliquely incident wave onto a floating structure with a wall was analysed by Bhattacharjee and Soares [7] over step type bottom topography. They used the eigenfunction expansion method to solve the problem. The effect of varying bottom topography on the wave scattering behaviour of oblique wave interaction with a moored floating membrane was analysed by Karmakar and Soares [14]. The conservation of energy flux was used to derive the energy relation for oblique gravity wave scattering due to floating membrane for various cases of abrupt changes in sea bottom profiles. The wave trapping using porous barrier near a rigid wall along a stepped type bottom topography was studied by Behera et al. [3] under the action of oblique waves. They used the eigenfunction expansion method along with multi-mode approximation for solving the problem. They used a modified mild-slope equation to take into consideration the variations in bottom topography. The efficiency of a dual chamber oscillating water column was analysed by Rezanejad et al. [19] for step type bottom topography. The effect of step type bottom topography on the wave scattering due to the semi-infinite elastic plate was analysed by Guo et al. [11] acted upon by obliquely incident waves. An analytical method using matched eigenfunction expansions method was developed based on potential theory and the plate was modelled considering Euler-Bernoulli beam theory. They analysed the influence of wave angle, plate thickness, edge boundary conditions and bottom topography.

In the present study, the scattering of waves in the presence of multiple stepped bottom topography below the floating elastic plate is analysed considering Timoshenko-Mindlin’s plate theory. In order to obtain the solution for the wave interaction with a freely floating thick elastic plate, the eigenfunction expansion method is applied which is related along with orthogonal mode coupling equation. The boundary condition at the edges of the floating thick elastic plate is considered to be free edge support condition. The continuity equations for velocity, pressure, deflection, bending moment, slope and shear force are considered in the analysis along the step interfaces. The effect of stepped bottom topography is analysed by varying the water depth for various steps in plate covered region. The numerical analysis is carried out for the wave behaviour along the reflection and transmission region. A hydroelastic analysis is carried out for large floating thick elastic plate over multiple stepped bottom topography acted upon by ocean waves.

2. Mathematical Formulation

The scattering of waves due to freely floating thick elastic plate over multiple stepped bottom topography is formulated considering linearized wave theory. The floating elastic plate is acted upon by a monochromatic wave along the positive x – direction. A 2D plate is modelled with x – axis along the plate and the y – axis across the plate vertically downward as presented in Fig. 1. The fluid domain in finite water depth is divided into upstream open water region at \( I_i \equiv (-a_i < x < \infty) \) with \( 0 < y < h_i \), the finite thick floating elastic plate covered region \( I_j \equiv (-a_j < x < -a_{j+1}) \) with \( 0 < y < h_j \) for \( j = 2, 3, ..., N \) and downstream open water region \( I_{N+1} \equiv (-\infty < x < -a_N) \) with \( 0 < y < h_{N+1} \). The plate edges at \( x = -a_i \) and \( x = -a_N \) satisfies the free edge support condition. The floating thick elastic plate has a considerable thickness and is modelled based on Timoshenko-Mindlin plate theory.
is the Poisson’s

\[
\mu = \frac{E}{2(1+\nu)}
\]

at \( x=0 \),

\[
EI d p j N = \text{plate thickness},
\]

\[
\int E I d p j N = \text{plate density},
\]

and

\[
GE
\]

for \( \text{atm} \) and \( \text{at} \), \( \text{at} \). \( \text{at} \) and \( \text{at} \).

\[
\rho \Phi_{ij} - \rho g \zeta_j = p_{\text{atm}} \quad \text{at} \quad y = 0,
\]

\[
\Phi_{ij} = 0, \quad \text{at} \quad y = h_j, \quad j = 1, 2, ..., N + 1.
\]

where \( p_{\text{atm}} \) is the atmospheric pressure. The plate is assumed to satisfy the Timoshenko-Mindlin’s equation [9], which includes the rotary inertia and transverse shear deformation terms in the form

\[
\left( \partial_x^2 - \frac{\rho \mu G}{\mu G} \partial_y^2 \right) \left( \partial_t^2 - \frac{\rho \nu d}{12} \partial_y^2 \right) \zeta_j + \rho \nu d \partial_t^2 \zeta_j = \left( 1 - \frac{EI}{\mu Gd} \partial_x^2 + \frac{\rho \nu d^2}{12\mu G} \partial_t^2 \right) p, \quad j = 2, 3, ..., N.
\]

where, \( p \) is the pressure, \( d \) is the plate thickness, \( \rho \mu \) is the plate density. \( \mu \) is the transverse deformation of the plate, \( E \) is the Young’s Modulus, \( EI = Ed^3/12(1-\nu^2) \) is the plate rigidity, \( \nu \) is the Poisson’s ratio and \( G = E/2(1+\nu) \) is the shear modulus of the plate. The surface elevation is assumed to be simple harmonic motion in time with frequency. The surface deflection \( \zeta_j(x,t) \) and velocity potential \( \Phi_j(x,y,t) \) are rewritten as \( \zeta_j(x,t) = \text{Re} \{ \eta_j(x) \} e^{i\omega t} \) and \( \Phi_j(x,y,t) = \text{Re} \{ \phi_j(x,y) \} e^{i\omega t} \) where ‘Re’ denotes the real part. The linearized free surface boundary condition in open water region is given by

\[
\partial_x \phi_j - \frac{\omega^2}{g} \phi_j = 0, \quad j = 1, N + 1. \quad \text{for} \quad x > -a_1 \quad \text{and} \quad x < -a_N.
\]

The linearized kinematic condition at the surface is combined with Timoshenko-Mindlin’s equation for thick plates, we obtain the plate covered boundary condition as shown below

\[
\left\{ \frac{EI}{\rho g - m_0 \omega^2} \partial_x^4 + \left( \frac{m_0 \omega^2 I}{\rho g - m_0 \omega^2} - S \right) \partial_x^2 + \left( 1 - \frac{m_0 \omega^2 IS}{EI} \right) \right\} \partial_x \phi_j + \frac{\rho \omega^2}{\rho g - m_0 \omega^2} \left\{ 1 - \frac{m_0 \omega^2 IS}{EI} - S \partial_x^2 \right\} \phi_j = 0, \quad j = 2, 3, ..., N.
\]
where \( \rho \) is the density of water, \( m_s = \rho \rho_d \) is the mass of the plate, \( I = \rho d^2 / 12 \) is the rotary inertia and \( S = EI / \mu Gd \) is the shear deformation. The far-field radiation condition is considered as shown below

\[
\phi_j (x) = \begin{cases} 
(e^{-ik_0x} + R_0 e^{ik_0x}) f_{10} (0) & \text{as } x \to \infty, \\
T_0 e^{-ik_{j+1}y} f_{(N+1)0} (y) & \text{as } x \to -\infty,
\end{cases}
\]

(8)

with \( R_0 \) and \( T_0 \) are the complex wave amplitudes for reflection and transmission region with \( k_{j0} \) at \( j=1,N+1 \) being the positive real roots which satisfies the open water dispersion relation given by

\[
k_{j0} \tan k_{j0} h_j - \omega^2 / g = 0.
\]

(9)

For a freely floating elastic plate, the bending moment and the shear force at the edges \( x = -a_j \) and \( x = -a_{N} \) are as given by the relation

\[
\partial_y^3 \phi_j (x,y) = 0 \quad \text{and} \quad \partial_y^4 \phi_j (x,y) = \omega^2 \partial_y^2 \phi_j (x,y), \quad j = 2, N \text{ at } x = -a_j \text{, and } x = -a_N, \quad y = 0
\]

(10)

with \( \omega = ma_0^2 (S + I) / EI \). The continuity equations for velocity and pressure at the interface \( x = -a_j, j = 1,2,...,N, 0 < y < h_{j+1} \) for \( j = 1,2,...,N \) is given by

\[
\phi_j = \phi_{(j+1)} \text{ and } \phi_j = \phi_{(j+1)} \text{ at } x = -a_j, 0 < y < h_{j+1} \text{ for } j = 1,2,...,N.
\]

(11)

The continuity of deflection, bending moment, slope and shear force at the step edges \( x = -a_j, 0 < y < h_{j+1} \) for \( j = 2,...,N - 1 \) given by (Karmakar, et al. [12]) is applied as follows

\[
\partial_y \phi_j (x,y) = \partial_y \phi_{(j+1)} (x,y), \quad \partial_y^3 \phi_j (x,y) = \partial_y^3 \phi_{(j+1)} (x,y), \quad \partial_y^4 \phi_j (x,y) = \partial_y^4 \phi_{(j+1)} (x,y), \quad \partial_y^2 \phi_j (x,y) = \partial_y^2 \phi_{(j+1)} (x,y)
\]

and

\[
\partial_y^4 \phi_j (x,y) = \partial_y^4 \phi_{(j+1)} (x,y)
\]

(12)

for \( j = 2,3,...,N - 1 \).

3. Method of Solution

The scattering of waves due to the presence of multiple stepped bottom topography below finite floating thick elastic plate is analysed based on Timoshenko-Mindlin plate theory. The boundary value problem is formulated considering free-free edge condition. The velocity potentials \( \phi_j (x,y), j = 1,2,...,N+1 \) satisfies Eq. (1) in the fluid domain and boundary condition (4), (6), (7) and (8) as explained in the above section. The velocity potentials \( \phi_j (x,y), j = 1,2,...,N+1 \), based on eigenfunction expansion formulae for the respective regions are given by

\[
\phi_1 (x,y) = \left( I_0 e^{-ik_0x} + R_0 e^{ik_0x} \right) f_{10} (0) + \sum_{n=1}^{N} R_n e^{-ik_nx} f_{in} (y) \quad \text{for } x > -a_1, 0 < y < h_1,
\]

\[
\phi_j (x,y) = \left[ \sum_{n=0,j}^{N} \left( A_n e^{ik_nx} + B_n e^{-ik_nx} \right) f_{jn} (y) + \sum_{n=1}^{N} \left( A_n e^{ik_nx} + B_n e^{-ik_nx} \right) f_{jn} (y) \right], \quad \text{for } x \in I_j,
\]

(13)

\[
\phi_{N+1} (x,y) = T_0 e^{-ik_{N+1}x} f_{(N+1)0} (y) + \sum_{n=1}^{N} T_n e^{ik_{N+1}x} f_{(N+1)n} (y) \quad \text{for } x < -a_N, 0 < y < h_{N+1}.
\]

where eigenfunctions \( f_{jn} (y) = \frac{\cos k_{jn} (h_j - y)}{\cos k_{jn} h_j} \) for \( n = 0, I, II \) and \( f_{jn} (y) = \frac{\cos k_{jn} (h_j - y)}{\cos k_{jn} h_j} \) for \( n = 1,2,... \)
\( k_{j0} \) for \( j = 1, N+1 \) are the eigenvalues. The eigenvalues correspond to the roots for the dispersion relation in open water region given by Eq. (9) with \( k_{j0} = \imath \kappa_{j0} \) for \( n = 1,2,... \). The values of \( R_n, T_n \) at \( n = 0,1,2,..., A_n \) and \( B_{j0} \) for \( j = 2,3,...,N \) for \( n = 0, I, II, 1,2,... \) are the unknown constants. The eigenvalues \( k_{j0} \) for \( j = 2,3,..,N \) are the roots for the plate covered dispersion relation as given by

\[
(\alpha_0 - \alpha k_{j0}^2 + \alpha k_{j0}^4) k_{j0} \tanh k_{j0} h_j - \left( \beta_0 - \beta k_{j0}^2 \right) = 0.
\]

where \( \alpha_0 = \left\{ 1 - m\omega^2 \left( \frac{IS}{EI} \right) \right\}, \quad \alpha_i = \left\{ \frac{m_i\omega^2 I}{(p}\right\} - S \right\}, \quad \alpha_2 = \frac{EI}{(p}\right\}, \quad \beta_i = -\frac{\rho \omega^2 S}{(p}\right\),

\[
\beta_0 = \frac{\rho \omega^2}{(p}\right\} \left( 1 - m_i\omega^2 \left( \frac{IS}{EI} \right) \right) \]

and \( h_j \) is the water depth at respective regions. The plate covered dispersion relation has one real root \( k_{j0} \) and four complex conjugate roots \( k_{j0} \) form \( \pm \alpha \pm \imath \beta \) along with an infinite purely imaginary roots \( \kappa_{j0} \) for \( n = 1,2,... \).

The orthogonality relation as given by Eq. (15) is applied for the eigenfunctions \( f_{jn}(y) \) based on the orthogonal mode-coupling relation as given by Eq. (16a) and (16b).

\[
\left\{ f_{jm}, f_{jn} \right\}_{j=1,N+1} = \left\{ \begin{array}{ll}
0 & \text{for } m \neq n, \\
C_n^* & \text{for } m = n,
\end{array} \right.
\]

\[
\left\{ f_{jm}, f_{jn} \right\}_{j=2,3,...N} = \left\{ \begin{array}{ll}
0 & \text{for } m \neq n, \\
C_n^* & \text{for } m = n,
\end{array} \right.
\]

\[
\left\{ f_{jm}, f_{jn} \right\}_{j=1,N+1} = \int_0^{h_i} f_{jm}(y)f_{jn}(y)dy,
\]

\[
\left\{ f_{jm}, f_{jn} \right\}_{j=1,N+1} = \int_0^{h_i} f_{jm}(y)f_{jn}(y)dy - \frac{\alpha_0}{Q(k_{n0})} \left( f_{jm'}(0)f_{jn}(0) \right)
\]

\[
+ \frac{\frac{1}{\alpha_0}}{\left( Q(k_{n0}) \right)} \left( f_{jm'}(0)f_{jn}(0) + f_{jm}(0)f_{jn'}(0) \right) + \frac{\frac{1}{\beta_0}}{P(k_{n0})} \left( f_{jm}(0)f_{jn}(0) \right).
\]

where

\[
P(k_{n0}) = \left( \alpha_0 - \alpha k_{j0}^2 + \alpha k_{j0}^4 \right) Q(k_{n0}) = \left( \beta_0 - \beta k_{j0}^2 \right), \quad C_n^* = \frac{2 k_{jn} h_j + \sinh 2 k_{jn} h_j}{4 k_{jn}\cosh^2 k_{jn} h_j}, \quad j = 1, N + 1
\]

\[
C_n^* = \frac{\left( \alpha_0 - \alpha k_{j0}^2 + \alpha k_{j0}^4 \right) 2 k_{jn} h_j + \left( \alpha_0 - 3\alpha k_{j0}^2 + 5\alpha k_{j0}^4 \right) \sinh 2 k_{jn} h_j + (4 \beta_0 k_{jn} \cosh^2 k_{jn} h_j)}{\left( 4 k_{jn}\cosh^2 k_{jn} h_j \right)} (\alpha_0 - \alpha k_{j0}^2 + \alpha k_{j0}^4), \quad j = 2,3,...,N.
\]

The constant term \( C_n^*, C_n^*, P(k_{jn}) \) and \( Q(k_{jn}) \) for \( n = 1,2,... \) are obtained by substituting \( k_{jn} = \imath \kappa_{jn} \) for \( j = 1,2,3,... \).

The unknown constants are calculated by applying the mode-coupling relation for the velocity potentials \( \phi_{j+1}(x) \), \( j = 1,2,... \) at the interface \( x = -a_1, -a_2, \ldots, -a_N \) with eigenfunctions \( f_{jn}(y) \). The system of linear equations is obtained by using continuity equations across the vertical interface and the edge support conditions. We obtain infinite series of a sum of the algebraic equations which are curtailed up to a finite number of \( M \) steps to formulate a system of \( 2\left((M + 1) + N(M + 3)\right) \) equations for \( N \) number of multiple stepped bottom topography. Solving the above equations, the unknown constants \( R_n, T_n, n = 0,1,2,... \) and \( A_{jn}, B_{jn}, n = 0, I, II, 1,2,..., j = 2,3,..., N \) are obtained. The coefficients in the reflection and transmission region are given by Eq. (17), which are checked to satisfy the energy balance relation \( K_2^2 + \chi K_3^2 = 1 \).
and (b)

10. 2

10

+ higher water depths. The unity in the

6

=  

( =  

=  

6  

along non

k k h

4  

are

(2

(2

8

(8

=  

sinh 2 2 sinh 2

=  

sinh 2 2 sinh 2

k k h

+ + +

+ + +

+=  +

4.2

Surface deflection and strain

The hydroelastic behaviour in terms of surface deflection and strain along the plate length at changing plate thickness and water depths are shown in Fig 3(a, b) and Fig 4(a, b) along the plate covered region over a three-step bottom topography. The hydroelastic behaviour is noticed to reduce as the waves progressed towards the transmission region due to the difference in the bottom topography along the plate covered region. From Fig. 3(a) and Fig. 4(a), the hydroelastic behaviour is noticed to decrease with the increasing plate thickness which may be due to the increase in plate rigidity. Further, the hydroelastic behaviour is also noticed to reduce with the increasing water depth which may be to decrease in wave
height at higher water depths as presented in Fig. 3(b) and Fig. 4(b). The variations in the hydroelastic behaviour at regular intervals of step length are observed due to the obstruction caused at the step edge.

![Figure 3](image1.png)

**Figure 3.** The deflection along the plate length at varying non-dimensional values of (a) plate thickness and (b) water depth for 03 step bottom topography at $k_0 h = 3$.

![Figure 4](image2.png)

**Figure 4.** Wave included strain along the plate length at varying non-dimensional values of (a) plate thickness and (b) water depth for 03 step bottom topography at $k_0 h = 3$.

### 5 Conclusion

The wave transformation due to floating elastic plate over multiple stepped bottom topography is analysed considering Timoshenko Mindlin’s thick plate theory at finite water depth. A three-stepped bottom topography is assumed to analyse the impact of multiple stepped bottom topography. At lower non-dimensional wavenumber, no significant variations are observed due to three step bottom topography. Whereas, the transmission of waves is noticed to decrease with the reduction in water depths at higher non-dimensional wavenumber which must be carefully considered for the design of floating structures over a steeped bottom topography. The hydroelastic behaviour is observed to decrease along the plate length towards the transmission region due to the steps at regular intervals, which restrained the propagation of waves. The increase in plate rigidity and plate thickness are noticed to reduce the hydroelastic characteristics of the freely floating elastic structures. The significant changes in the hydroelastic behaviour of freely floating thick elastic plate above multiple stepped bottom topography are observed due to the variations in water depth, which is mainly due to increasing wavelength with the reduced wave height. A steep increase in hydroelastic behaviour is observed in plate covered region may be mainly due to the higher difference in water depth between the mediums of interaction.
Acknowledgement
The authors are thankful to National Institute of Technology Karnataka Surathkal and MHRD for providing necessary support. The authors also acknowledge Science and Engineering Research Board (SERB), Department of Science & Technology (DST), Government of India for supporting financially under the Young Scientist research grant no. YSS/2014/000812 and DST for India-Portugal Bilateral Scientific Technological Cooperation Project grant no. DST/INT/Portugal/P-13/2017.

References
[1] Athanassoulis G A and Belibassakis K A 1999 A consistent coupled-mode theory for the propagation of small-amplitude water waves over variable bathymetry regions J. of Fluid Mechanics 389 p 275–301.
[2] Balmforth N J and Craster R V 1999 Ocean waves and ice sheets J. of Fluid Mechanics 395 p 89-124.
[3] Behera H Kaligatla R B and Sahoo T 2015 Wave trapping by porous barrier in the presence of step type bottom Wave Motion 57 p 219-30.
[4] Belibassakis K A and Athanassoulis G A 2004 Hydroelastic responses of Very Large Floating Structures lying over variable bathymetry regions. The 14th International Offshore and Polar Engineering Conference, International Society of Offshore and Polar Engineers p 584–591.
[5] Belibassakis K A and Athanassoulis G A 2005 A coupled-mode model for the hydroelastic analysis of large floating bodies over variable bathymetry regions J of Fluid Mechanics 531 p 221–249.
[6] Belibassakis K A, Athanassoulis G A and Gerostathis Th. 2013 Hydroelastic analysis of very large floating structures in variable bathymetry regions. Proc. 10th HSTAM Intern. Congress on Mechanics. Chania, Crete, Greece, 25–27 May, 2013.
[7] Bhattacharjee J and Guedes Soares C 2011 Oblique wave interaction with a floating structure near a wall with stepped bottom Ocean Engineering 38(13) p 1528–44.
[8] Evans D V and Linton C M 1994 On step approximations for water-wave problems. J. of Fluid Mechanics 278(1) p 229–49.
[9] Fox C and Squire V A 1991 Coupling between the ocean and an ice shelf. Annals of Glaciology p 101-8.
[10] Gao R P Tay Z Y Wang C M and Koh C G 2011 Hydroelastic response of very large floating structure with a flexible line connection Ocean Engineering 38 p 1957-66.
[11] Guo Y Liu Y and Meng X 2016 Oblique wave scattering by a semi-infinite elastic plate with finite draft floating on a step topography Acta Oceanologica Sinica 2016 35(7) p 113-21.
[12] Karmakar D and Sahoo T 2008 Gravity wave interaction with floating membrane due to abrupt change in water depth Ocean Engineering 35(7) p 598–615
[13] Karmakar D, Bhattacharjee J and Sahoo T 2010 Oblique flexural gravity-wave scattering due to changes in bottom topography Ocean Engng Mathematics 66(4) p 325–341.
[14] Karmakar D and Guedes Soares, C 2012 Oblique scattering of gravity waves by moored floating membrane with changes in bottom topography Ocean Engineering 54 p 87–100.
[15] Kyoung J H, Hong S Y, Kim, B W and Cho S K 2005 Hydroelastic response of a very large floating structure over a variable bottom topography Ocean Engineering 32 p 2040-52.
[16] Mei C C and Black J L 1969 Scattering of surface waves by rectangular obstacles in waters of finite depth. J. of Fluid Mechanics 38(3) p 499-511.
[17] Mindlin R D 1951 Influence of rotary inertia and shear on flexural motion of isotropic elastic plates J of Applied Mechanics (ASME) 18 p 31-38.
[18] Praveen K M, Karmakar D and Nasar T 2016 Hydroelastic analysis of floating elastic thick plate in shallow water depth Perspectives in Science 8 p 770-772.
[19] Rezanejad K, Bhattacharjee J and Guedes Soares C 2015 Analytical and numerical study of dual-chamber oscillating water columns on stepped bottom Renewable Energy 75 p 272–282.