Optimal Structural Design of a Circular Cylindrical Ridge Wave Ultrasonic Motor

T H Yu¹
Department of Electronic Engineering, National United University, 2, Lien Da Road, Nan Shih Li, Miaoli 36063, Taiwan
E-mail: yth@nuu.edu.tw

Abstract. Exact solutions for flexural ridge wave propagation around a circular cylindrical tube have not previously been possible. Rather, such problem used to be solved numerically or using an empirical formula. This study investigates the dispersive properties of ridge waves traveling circumferentially around piezoelectric tubes, as well as their resonant modes. Based on the separation of variables, the displacements of ridge waves are represented as the product of a cross-sectional coordinate-dependent function and the propagator along the circumference of a tube. The dispersion equation of ridge waves is formulated by using Hamilton’s principle and the so-called bi-dimensional finite element method (Bi-d FEM). Furthermore, the dispersion curves of traveling waves and resonant frequencies corresponding to standing waves are solved numerically. Several applications, such as an optimal design for circular cylindrical ridge wave ultrasonic motors, are illustrated.

1. Introduction
Research on wedge acoustic waveguides began in the early 1970s, after Lagasse [1] discovered the waveguide behavior of wedge tips and sparked an increasing interest in research on guided wave propagation. That study revealed that, when a wedge is in a truncation-free state, energy will concentrate along the anti-symmetrical flexural waves propagated by the wedge tip and approximately within a one wavelength range from said wedge tip, and that this phenomenon is dispersion free. McKenna et al [2] used the flexible waves propagated by flat edges to approximate wedge-guided waves and showed that, under conditions in which the wedge has truncated corners, the flexible wave’s phase velocity will change when its frequency increases.

In 1972, Lagasse [1] and Maradudin et al [3] utilized numerical calculation to prove that a wedge tip possesses waveguide functions within a specific range of angles, and that the energy of an anti-symmetrical flexural guided wave being propagated along a wedge tip will mostly be limited to an approximately one wavelength range from the wedge tip. In this case, the wave's phase velocity will be lower compared to that of a Rayleigh wave with the same wavelength. When the wedge is in a truncation-free state, this guided wave will be dispersion free.

In 1973, Lagasse et al [4] crafted a simple empirical formula from the numerical analysis results of a study, and this formula allows for the wave propagation speed of an antisymmetrical flexural guided wave (when the integer multiple of the vertex angle is smaller than or equal to 90°) to be expressed as

¹ To whom any correspondence should be addressed.
\[ V = V_R \sin(n\theta), \] where \( V_R \) is the Rayleigh wave speed, \( \theta \) is the wedge’s vertex angle, and \( n \) is the guided wave’s mode number.

In 1973, Lagasse \[5\] applied the variation principle and finite element method to examine the wave propagation behavior of linear acoustic waveguides. Given its applicability to a variety of different geometric cross-sections, this approach became an important tool for studying acoustic waveguides.

In 1974, Mckenna \textit{et al} \[2\] approximated an infinitely long wedge with an extremely small vertex angle as a plate, and applied the plate theory to derive the relationship between the phase velocity and wave number of truncation-free and truncated (vertex angle) wedge-guided waves. It was revealed that the dispersion phenomenon cannot be prevented in an actual wedge since it will become more pronounced when the vertex angle becomes more truncated.

In 1992, Jia and Billy \[6\] utilized the pulsed laser excitation source Nd:YAG to cause instantaneous heating and expansion at a wedge tip and generate elastic waves, used an optical interferometer to receive the produced signals, obtained two groups of waveform signals from different detection points, and finally obtained the phase velocities of the two groups of waveforms using their phase difference.

In 1994, Krylov \[7\] utilized geometrical acoustics to calculate the wave propagation speed of the guided waves of wedges in water, and revealed that guided waves are affected by water, with their wave velocities being slower (compared to a vacuum environment), and that a nonlinear relationship exists between the reduction in wave velocity and the vertex angle of a wedge. That study also predicted that low-speed wedge waveguides can be used to develop a propeller that mimics the motions of a stingray exercising in water. In 2007, Krylov and Pritchard \[8,9\] were finally able to develop a wedge propeller model similar to those for twin hull vessels.

In 1996, Hladky-Hennion \[10\] applied a bi-dimensional finite element method to analyze the wave propagation behavior of linear and cylindrical wedge acoustic waveguides, and verified this method by comparing the empirical formula from the study by Lagasse \textit{et al} \[4\] with the experimental results from the study by Jia and Billy \[6\].

In 1998, Yang and Liaw \[11\] utilized experimental techniques that combined 3D finite element analysis and laser-ultrasonics to analyze and measure linear and disk-type wedges. Wave signals measured in the time domain were subjected to the 2D fast Fourier transform (2D-FFT) procedure in order to obtain the phase velocity dispersion curve of guided waves. That study indicated that the propagation of guided waves by disk-typed wedges was affected by curvature, which caused the dispersion phenomenon to occur. Furthermore, the phase velocity dispersion curve began to undergo negative slope change when the frequency increased.

In 1999, Krylov \[12\] utilized geometrical acoustics to derive a simpler analytical solution for describing the wave propagation behavior of the guide waves of conical wedges and cone-like wedges. That study revealed that, when the radius of curvature was greater than the wavelength of the guided waves, the results were similar to those from the finite element study by Hladky-Hennion \[7\], with an excellent level of consistency being achieved.

In 2005, Tominaga \textit{et al} \[13\] proposed a linear ultrasonic motor that had a rectangular structure. The definition provided by Lagasse \textit{et al} \[4\] holds that single mode flexible vibrations are limited by the characteristics of ridges and produce traveling waves that propagate along the ridges, and in this case, the elliptical motion curve of the flexible waves will cause the sliding block above the rectangular structure to move in the opposite direction of the propagating waves.

Yin and Yu \[14\] performed a numerical simulation which revealed that the elliptical motion of guided waves caused the circumferential displacement of the wedge tip to become overly small; thus, even though the energy of flexible waves is limited to the wedge tip, ultrasonic motors are still unable to provide sufficient torque to rotate the rotor. In response, that study proposed a new ultrasonic motor with a rotor based where the circumferential displacement was greatest. This study incorporated the new linear ultrasonic motor design proposed by Tominaga \textit{et al} \[13\] that utilized ridge waveguides, and the new rotor design proposed by Yin and Yu \[14\]. The wave propagation behaviors of piezoelectric tube ridges were examined, and a comparison involving single geometric shapes and
different geometric parameters was carried out so as to develop an optimized reference for designing ultrasonic motors driven by ridge waveguides.

2. Basic Theoretical Analysis

2.1. Constitutive Equation Of Piezoelectric Material

This study utilized the piezoelectric ceramic material PZT-4, which is a transversely isotropic material. Piezoelectric materials produce direct and inverse piezoelectric effects, and therefore, the application of positive pressure on piezoelectric materials will cause the piezoelectric body to generate voltage to maintain its original state, so as to resist the shortening and compression of the electric dipole moment. This phenomenon is called the direct piezoelectric effect of converting mechanical energy to electrical energy. When piezoelectric materials are subjected to electric field effects, their electric dipole moment will become elongated and the piezoelectric body will change according to the direction of the electric field. This phenomenon is called the inverse piezoelectric effect of converting electrical energy to mechanical energy. When a piezoelectric structure is subjected to electric fields or mechanical forces, its constitutive equation is as follows:

\[
\begin{align*}
T &= eS - eE \\
D &= eS - e^T E
\end{align*}
\]  

where \(T\) and \(S\) are the stress tensor and strain tensor, respectively, \(e\) is the elastic stiffness matrix, \(e\) is the tensor of the piezoelectric constants, \(D\) is the electric displacement, \(E\) is the electric field strength, and \(e^T\) is the piezoelectric constant tensor of the constant strain. Furthermore, \(u\) and \(\Phi\) are defined as the displacement vector and potential vector, respectively.

2.2. Hamilton’s Principle

Under boundary conditions in which the surface traction of the cylindrical acoustic waveguide is zero, Hamilton's principle dictates that the first variation of total potential will be zero when the elastomer reaches dynamic equilibrium,

\[
\int_{t_1}^{t_2} \delta(T - H) dt = 0
\]

and the first variation of all the field variables of the timepoints \(t_1\) and \(t_2\) will be zero, with \(T\) and \(H\) being the elastomer’s total kinetic energy and enthalpy.

Applying Hamilton's principle, the variation when time is integrated upon using the Lagrangian function is designated as zero, making it possible to obtain the equation of motion for the cylindrical acoustic waveguide.

\[
\int_{t_1}^{t_2} \delta L dt = 0
\]

2.3. Bi-dimensional finite element analysis

The finite element method is applied to discretize the cross section of the cylindrical acoustic waveguide into numerous continuous small elements, and the displacement and potential at any point within an element are expressed as the product of the interpolation function and the node displacement and potential. The elements used in this study are isoparametric elements such as 2D four-node Q4 elements.

Considering that the flexible waves of a cylindrical acoustic waveguide are propagated circumferentially along the circumference, a state of resonance is maintained by the wave motion at the cross section perpendicular to the circumference. Based on coordinate system in Figure 1, the displacement vector and potential vector are respectively assumed to be

\[
u = N_d d
\]
\[ \Phi = N_N \varphi \]

where \( d \) is the displacement vector of a node, and \( \varphi \) is the potential vector of a node. The strain \( S \) and electrical potential energy \( \Phi \) are then expressed as the following matrix

\[ S = B_d d \]

\[ E = -B_{\varphi} \varphi \]

2.4. Dispersion Equation

The expressions of the displacement \( u \), strain \( S \), electrical potential energy \( \Phi \), electric field \( E \), and interpolation function of any point within a unit element in Equations (6) and (7) are substituted into Equations (2) and (3).

By combining the discretized elements to form a global matrix, the equation of motion for the entire system can be obtained,

\[
\begin{bmatrix}
M & 0 & D
\end{bmatrix}
\begin{bmatrix}
\dot{\Phi}
\end{bmatrix}
+
\begin{bmatrix}
K_{uu} & K_{u\varphi}

K_{u\varphi} & K_{\varphi\varphi}
\end{bmatrix}
\begin{bmatrix}
\Phi
\end{bmatrix}
=
\begin{bmatrix}
0
\end{bmatrix}
\]

where \( m, k_{uu}, k_{u\varphi}, k_{\varphi\varphi} \) form the real number symmetric matrix. Equation (25) is an eigenvalue problem, and the sufficient condition for establishing the system of equations’ non-trivial solution is

\[
\det\left[
\begin{bmatrix}
K_{uu} & K_{u\varphi}

K_{u\varphi} & K_{\varphi\varphi}
\end{bmatrix}
-\omega^2
\begin{bmatrix}
M & 0

0 & 0
\end{bmatrix}
\right] = 0
\]

Using a tube made from general materials as an example, Figure 2 illustrates the tube cross-section deformations that correspond to axial modes \( m = 1 \sim 3 \), while Figure 3 shows the dispersion curves for the calculated wave number \( k \) and frequency \( \omega \). The 3D ANSYS business suite was used to compare the simulation results, and it was revealed that the two sets of results were consistent with each other.
2.5. Correction of Ridge Displacement

The dispersion equation of the cylindrical acoustic waveguide is an eigenvalue problem, and the eigenvectors that are obtained are all normalized to a characteristic length of 1. If one intends to normalize these eigenvectors via other methods, this will have to be carried out separately. Assuming that the corrected displacement is the product of the displacement obtained via the eigenvector corresponding to the eigenvalue and the constant of proportionality $A$, its equation may be expressed as follows

\begin{align}
U_r &= AU_r \\
U_z &= AU_z \\
U_\theta &= AU_\theta
\end{align}

\( m = 1 \quad \text{(2)} \quad m = 2 \quad \text{(3)} \quad m = 3 \)

**Figure 2.** Tube cross-section deformations corresponding to axial modes $m = 1$–3.

**Figure 3.** Dispersion curves for the frequency and wave number of the tube ridges' flexible waves.
where, \( \overline{U}_r \), \( \overline{U}_z \), and \( \overline{U}_\theta \) are the displacement components prior to correction, and \( U_r \), \( U_z \), and \( U_\theta \) are the displacement components after correction.

The kinetic energy and strain per unit of time inside the tubal wall are summed up and expressed as 1 unit of energy. Given that an elastic wave's strain energy and kinetic energy are equal within one cycle, the sum of the two is expressed as

\[
2(\pi A^2 \omega^2 \mathbf{U}^T \mathbf{M} \mathbf{U}) = 1
\]

with the constant of proportionality \( A \) being

\[
A = \frac{1}{\omega} \sqrt{\frac{1}{2\pi \mathbf{U}^T \mathbf{M} \mathbf{U}}}
\]

Thus, the real sizes of the \( U_r \), \( U_z \), and \( U_\theta \) which correspond to the unit time and energy can be obtained. Using a tube made from stainless steel materials as an example, Figures 4 and 5 illustrate the distribution of the radial displacement \( U_r \) and the circumferential displacement \( U_\theta \) prior to the correction, with the horizontal axis \( \Lambda \) in the figures representing the tube's height-to-diameter ratio. Figures 6 and 7 illustrate the distribution of the radial displacement \( U_r \) and the circumferential displacement \( U_\theta \) after the correction, revealing a significant difference prior to and after the correction, as well as a more reasonable post-correction displacement distribution. For this study \( n=4 \) was selected for the electrodes’ arrangement. Figure 6 shows that maximum radial displacement \( U_r \) is achieved when \( \Lambda=1.334 \), while Figure 7 shows that the maximum circumferential displacement \( U_\theta \) is achieved when \( \Lambda=1.231 \); thus, the optimal structure for the cylindrical ridge wave ultrasonic motor can be achieved by using these \( \Lambda \) ratios.

![Figure 4. Distribution of the radial displacement of the tube ridges' flexible waves prior to the correction.](image-url)
Figure 5. Distribution of the circumferential displacement of the tube ridges' flexible waves prior to the correction.

Figure 6. Distribution of the radial displacement of the tube ridges' flexible waves after the correction.
3. Simulation and Experimental Measurement Results

3.1. Production of Ultrasonic Motor

The ultrasonic motor prototype developed for this study utilized a PZT-4 piezoelectric tube manufactured by Eleceram Inc., Taiwan. The piezoelectric tube's height, outer radius, and inner radius were 12.5mm, 12.5mm, and 10.5mm, respectively; hence, its thickness was 2mm. Nitric acid was first used to wash off the metal electrodes from the inner and outer section of the piezoelectric tube, after which electrodes were then coated onto the tube surface based on the flexible wave mode that the design intended to excite. The arrangement of the electrodes was primarily carried out via screen printing, which involved the printing of silver paste onto the piezoelectric tube, where it served as the driving electrodes for the transducer. Before the electrodes were coated, the screen had to be tightly affixed to the outer surface of the piezoelectric tube, and a scraping knife was then used to extrude the silver paste, so as to allow it to pass through the electrode section of the screen. Lastly, the piezoelectric tube, on which the electrodes had been properly placed, was then placed into the oven and baked at 120° C to solidify the silver paste and complete the electrode placement process.

The main structure of the ultrasonic motor consisted of a stator and a rotor, and the stainless steel base of the motor stator was made using the lathe processing method. Based on the details discussed above, the optimal structural design was used to divide the stator base into an upper and lower section, with the upper section and lower section having a height of 12.5 mm and 10 mm, respectively, resulting in $\Lambda \approx 1.3$. The piezoelectric tube is firmly affixed to the stator base using epoxy resin AB glue. Figure 8 shows the completed motor stator prototype.
3.2. Motor Stator Modal Analysis

The 3D ANSYS business suite's finite element software package includes a structural dynamics analysis program consisting of three components, namely, the pre-processing, analysis, and post-processing components. For the pre-processing component, a geometric model of the motor stator was first established, with the PZT-4 piezoelectric tube forming the upper section and the base forming the lower section, and stainless steel (steel use stainless, SUS) being used. The height H of the motor stator's base was determined based on the optimal structure design analysis. With respect to element type, the SOLID5 element was used. This element is a 3D coupled-field solid element with eight nodes, and each of these nodes has a displacement and potential difference for the x, y, and z directions. An appropriate number of elements was used to mesh the model, and as shown in Figure 9, the number of elements and the number of nodes were 67,200 and 83,040, respectively.

The resonant modes of the ultrasonic motor's stator structure comprised a longitudinal mode, torsional mode, and flexural mode. The longitudinal mode of motion moved along the axial and radial directions. In this study, the resonant response of the flexural mode was used as the ultrasonic motor's primary driving force. Two integers \((m, n)\) were used to indicate the numbering and characteristics of the resonant modes, with \(n\) representing the number of circumferential modes and \(m\) indicating the number of axial modes. Figure 10 shows that, when \(m = 1\), the free end of a motor stator's ridge will generate 1–6 waveforms, and the number of waveforms will represent the number of circumferential modes \(n\). As the bottom section of the motor stator examined in this study was fixed, \(m\) was thus greater than or equal to 1.
3.3. Measurement of Motor Stator's Resonance Frequency

In recent studies, the laser doppler interferometer has been widely used as an instrument for measuring the vibration velocity and displacement of structures. It works by interfering with the reflected light and reference light that is projected onto the material being measured, and using the difference in their optical distances to calculate the speed or displacement of the object on the surface. The measuring instruments and devices used in this study are illustrated in Figure 11. A 1V sinusoidal signal was output by the HP 8751A (Agilent Technologies, Santa Clara, CA, USA) network analyzer and amplified 40 times using the NF-4051 power amplifier. Next, the voltage signal was then transmitted to one of the motor stator's actuator groups. The doppler interferometer was used to measure the out-of-plane vibration displacement of the piezoelectric tube ridge's outer surface, and the measured displacement signals were then input into the network analyzer. By dividing the structural vibration displacement's frequency response function by the input load's frequency spectrum, the frequency response of the system can then be obtained.

Figure 12 shows the frequency response of the motor stator's out-of-plane displacement when the stator had a \( n=4 \) electrode arrangement. The figure revealed that the maximum out-of-plane vibration displacement was achieved when the drive frequency was 33.675 kHz, indicating that this frequency was the motor stator's resonance frequency.
3.4. Measurement of ultrasonic motor’s performance

A motor's performance is usually determined by its torque and revolution speed. An example is the automatic measurement system shown in Figure 13. The function generator first output a sinusoidal signal with the same frequency as the resonance frequency of the motor stator's specific circumferential mode. Using a phase shifter and piezo amplifier (EPA-104), two sets of signals with a phase difference of ±90° are then output to the motor stator's A and B actuators, so as to generate traveling waves. When the rotor is moved by the friction force, the keyway will drive the motor shaft. Since the coupling connects the shaft to the encoder (HRT-3A/1000 ppr, Hontko Inc.), the motor's revolution speed can thus be calculated. Furthermore, Futek's load cell (LSM400) and weights are used to measure the motor's torque. For the automated measurement system used in this experiment, NI's PCI -7344 motion controller was used to read and decode the encoder's digital signals, and its PCI-6035E data capture card was used to read the analog signals of the load cell. The LabVIEW graphics control software was utilized to display the revolution speed and torque values of the ultrasonic motor. As shown in Figure 14, an encoder was used in conjunction with a load cell to measure the motor's revolution speed at different torque levels. As the rotor and cotton thread were slidable, a range of different weights could be tangentially hung at the other end of the rotor for the purpose of measuring the motor's torque. Figure 15 shows the speed-voltage curve for the ultrasonic motor's measured revolution speed and drive voltage, while Figure 16 shows the motor's torque-speed curve. These figures indicate that, when the F(1, 4) mode was used, a voltage level in excess of 100 Vpp was necessary to overcome the static friction and cause the rotor to rotate. Thus the maximum available torque and maximum revolution speed were 11.6 mN-m and 334 rpm, respectively.
Figure 13. Measurement architecture for motor drive performance.

Figure 14. Schematic diagram showing the measurement of the ultrasonic motor's maximum torque.

Figure 15. Ultrasonic motor's revolution speed and drive voltage curve.
4. Conclusion
The tube’s resonant modes included the number of circumferential modes $n$ and the number of axial modes $m$. With regard to the elastic waves, when $n$ is an integer, the tube ridge's flexible waves that are propagating in the clockwise and counter-clockwise direction will combine to form standing waves; and when $n$ is not an integer, progressive waves will be bi-directionally transmitted instead. The basic concepts examined in this study are based on the above. The data from a bi-dimensional finite element wave propagation analysis and an ANSYS-based 3D vibration value analysis were compared, and the results revealed were considerably good for both the isotropic and piezoelectric tubes.

A bi-dimensional finite element analysis allowed for the dispersion curve to be used as a 2D map, which provided a reference for determining an ultrasonic motor's optimal structure. Furthermore, an examination of the circumferential displacement of the motor rotor and stator's contact points could also be performed to identify the optimal contact points for the two components. In this study, the flexible waves of low-order circumferential modes were used to drive an ultrasonic motor, and the numerical simulation and experimental results exhibited a high level of consistency.

5. References
[1] Lagasse P E 1972 *Electrics Letters* **8**(4) 372-373
[2] Mckenna J, Boyd G D and Thurston R N 1974 *IEEE Transactions on Sonics and Ultrasonics* **21**(3) 178-186
[3] Maradudin A A, Walls R F and Ballard R L 1972 *Physics Rev. B* **6** 1106-1111
[4] Lagasse P E, Mason I M and Ash E A 1973 *IEEE Transactions on Microwave Theory and Techniques* **21**(4) 225-236
[5] Lagasse P E 1973 *Journal of Acoustical Society of America* **53** 1116-1122
[6] Jia X and Billy M D 1992 *Applied Physics Letters* **61** 2970-2972
[7] Krylov V V 1994 *IEEE Ultrasonics Symposium* 793-796
[8] Krylov V V and Pritchard G V 2007 *Journal of Fluids and Structures* **23**(2) 297-307
[9] Krylov V V and Pritchard G V 2007 *Applied Acoustics* **68**(1) 97-113
[10] Haldky-Hennion A C 1996 *Journal of Sound and Vibration* **194**(2) 119-136
[11] Yang C H and Liaw J S 1998 *IEEE Ultrasonics Symposium* 1139-1142
[12] Krylov V V 1999 *Journal of Sound and Vibration* **227**(1) 215-221
[13] Tominaga M, Kaminaga R, Friend J R, Nakamura and Ueha S 2005 *IEEE Transactions on Ultrasonics, Ferroelectrics and Frequency Control* **52**(10) 1735-1742

[14] Yin C C and Yu T H 2006 *IEEE Ultrasonics Symposium* 156-159

**Acknowledgments**

We would like to sincerely thank the Ministry of Science and Technology in Taiwan (MOST 106-2221-E-239-011) for funding this research.