Time-Cost-Quality Trade-off Model for Optimal Pile Type Selection Using Discrete Particle Swarm Optimization Algorithm

Hanaa H. Lateef a, Abbas M. Burhan a*

aCivil Engineering Department, University of Baghdad, Al-Jadriya, Baghdad, Iraq.

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Abstract

The cost of pile foundations is part of the super structure cost, and it became necessary to reduce this cost by studying the pile types then decision-making in the selection of the optimal pile type in terms of cost and time of production and quality. So, the main objective of this study is to solve the time–cost–quality trade-off (TCQT) problem by finding an optimal pile type with the target of "minimizing" cost and time while "maximizing" quality. There are many types in the world of piles but in this paper, the researcher proposed five pile types, one of them is not a traditional, and developed a model for the problem and then employed particle swarm optimization (PSO) algorithm, as one of evolutionary algorithms with the help of (Mat lab software), as a tool for decision making problem about choosing the best alternative of the traded piles, and proposes a multi objective optimization model, which aims to optimize the time, cost and quality of the pile types, and assist in selecting the most appropriate pile types. The researcher selected 10 of senior engineers to conduct interviews with them. And prepared some questions for interviews and open questionnaire. The individuals are selected from private and state sectors each one have 10 years or more experience in pile foundations work. From personal interviews and field survey the research has shown that most of the experts, engineers are not fully aware of new software techniques to help them in choosing alternatives, despite their belief in the usefulness of using modern technology and software. The Problem is multi objective optimization problem, so after running the PSO algorithm it is usual to have more than one optimal solution, for five proposed pile types, finally the researcher evaluated and discussed the output results and found out that pre-high tension spun (PHC) pile type was the optimal pile type.

Keywords: PSO Algorithm; PHC; Optimal Pile Type; Decision Making.

1. Introduction

The foundation cost, of real-world structural systems, can vary from 5% to 20% of the construction cost of the superstructure. And that's not a tiny proportion [1, 2]. And it becomes a necessity to find the best pile foundation type in terms of performance and economy. In the initial stage of large construction residential projects, surveys and studies need to be carried out for its details and stages in order to find appropriate designs and alternatives, which achieve the lowest cost, time and good quality, and This is the first problem that is sought to be solved, so the selection of the right pile type for the foundations is one of these decisions needed to evaluate the performances in terms of time, cost and quality. After this evaluation process, the selection of an optimal type of pile will be carried out the work in the project.

The main types of piles used are driven piles, driven and cast-in-place piles, jacked piles, bored and cast-in-place piles and composite piles. The first three of the above types are also called displacement piles since the soil is displaced. In the case of bored piles, and in some forms of composite piles, the soil is first removed by boring a hole where concrete

*Corresponding author: abbas_alshemosi@yahoo.com

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is placed or various types of precast concrete or other proprietary units are inserted called replacement piles [3]. There are many types of deep foundations, and a classification can be made in various ways. The researcher was reached through the study that (Degree of soil displacement piles type Classification) which consists of (displacement and replacement) piles, is appropriate for multi-story residential complexes [4]. Several researchers made significant efforts to solve (TCQT) problem and the solution methods employed in these studies can be generally categorized into “three” main groups as following:

(1) **Heuristic algorithms:** It is used in the field of optimization to characterize a certain kind of problem-solving methods. There are a great number and variety of difficult problems, which come up in practice and need to be solved efficiently, and this has promoted the development of efficient procedures in an attempt to find good solutions, even if they are not optimal. These methods, in which the process speed is as important as the quality of the solution obtained, are called heuristics or approximate algorithms [5].

(2) **Mathematical algorithms:** Mathematical Optimization is often also called Nonlinear Programming, Mathematical Programming or Numerical Optimization. In more general terms Mathematical Optimization may be described as the science of determining the best solutions to mathematically defined problems, which may be models of physical reality or of manufacturing and management systems. It’s divided into three types [6].

  a. **Linear Programming (LP):** is an important concept in optimization techniques in mathematics as it helps to find the most optimized solution to a given problem and helps to find the best solution from a set of parameters or requirements that have a linear relationship [7].

  b. **Integer Programming (IP):** is a mathematical optimization or feasibility program in which some or all of the variables are restricted to be integers. In many settings the term refers to integer linear programming (ILP), in which the objective function and the constraints (other than the integer constraints) are linear [8].

  c. **Dynamic Programming (DP):** refers to simplifying a complicated problem by breaking it down into simpler sub-problems in a recursive manner. While some decision problems cannot be taken apart this way, decisions that span several points in time do often break apart recursively. Likewise, in computer science, if a problem can be solved optimally by breaking it into sub-problems and then recursively finding the optimal solutions to the sub-problems, then it is said to have optimum substructure [9, 10].

(3) **Evolutionary algorithms:** **evolutionary algorithm (EA)** is an artificial intelligence, and a subset of evolutionary computation, a generic population-based meta heuristic optimization algorithm. An EA uses mechanisms inspired by biological evolution, such as reproduction, mutation, recombination, and selection. Candidate solutions to the optimization problem play the role of individuals in a population, and the fitness function determines the quality of the solutions. Evolution of the population then takes place after the repeated application of the above operators [11-14]. Some of evolutionary are as following and as shown in Figure 1:

  a. **Memetic algorithm:** In computer science and operations research, a Memetic Algorithm (MA) is an extension of the traditional genetic algorithm. It uses a local search technique to reduce the likelihood of the premature convergence [15].

  b. **Particle Swarm Optimization (PSO):** is a computational method that optimizes a problem by iteratively trying to improve a solution with regard to a given measure of quality. It solves a problem by having a population of candidate solutions, here dubbed particles, and moving these particles around in the search-space according to simple mathematical formulae over the particle’s position and velocity. Each particle’s movement is influenced by its local best known position, but is also guided toward the best known positions in the search-space, which are updated as better positions are found by other particles. This is expected to move the swarm toward the best solutions [16], [17] and [18].

  c. **Ant Colony Optimization (ACO):** the ant colony optimization algorithm (ACO) is a probabilistic technique for solving computational problems which can be reduced to finding good paths through graphs. Artificial Ants stand for multi-agent methods inspired by the behavior of real ants. The pheromone-based communication of biological ants is often the predominant paradigm used [19], [20].

  d. **Shuffled frog Leaping (SFL):** Shuffled Frog Leaping Algorithm (SFLA) is a recent memetic meta-heuristic algorithm proposed by Eusuff and Lansey in 2003. The SFL algorithm involves a set of frogs that cooperate with each other to achieve a unified behavior for the system as a whole, producing a robust system capable of finding high quality solutions for problems with a large search space such as Economic Dispatch (ED) problem. The algorithm is used to calculate the global optima of many problems and proves to be a very efficient algorithm [21].

  e. **Genetic Algorithm (GA):** genetic algorithm (GA) is a metaheuristic inspired by the process of natural selection that belongs to the larger class of evolutionary algorithms (EA). Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by relying on bio-inspired operators such as mutation, crossover and selection[22],[23].
Among these groups, evolutionary algorithms are the most favourable one because of the fact that they are capable to deal with more than one objective, easily achieve diverse solutions and better in complex problems compared to the other algorithms. The study is limited to use an optimization technique (particle swarm optimization (PSO)) in order to find the optimum pile type of five types of piles [timber, H-steel, cast –in-situ (CIS), precast concrete (PC), pre-high-tension concrete spun pile (PHC)] piles. The Problem is multi objective optimization problem, so after running the PSO algorithm it is usual to have more than one optimal solution [24], and then choosing the best alternative of the traded piles.

This paper aims to apply the PSO algorithm as a tool to assist decision makers for selection of the optimal pile type in terms of cost, time and quality for multi-storey residential buildings in Iraq.

2. Research Methodology

Steps of the research methodology were as the following:

- **Processing Data:** The researcher conducted open questioners, and personal interviews with several experts, specialized in design and execution pile foundation fields, for obtaining data of the pile types criteria, by choosing three criteria (cost, time and quality) from a number of criteria as shown in the Table 1, which is supposed to be present in the optimal pile type. These criteria values were obtained from expert respondents by Specific ranges as following:

  A- Very High [1-0.8] av. =0.9,  
  B- High, [0.8-0.6] av. = 0.7,  
  C- Medium [0.6-0.4] av. = 0.5,  
  D- Low [0.4-0.2] av. = 0.3,  
  E- Very Low [0.2-0] av. = 0.1.

By Excel Software 2013, output of Respondents data are processed and analyzed to get preliminary results before optimization as shown in the Figure 2.

- **Define Initial Parameters:** The definitions of the parameters are represented in the Equation 1: (w) is the inertia weight; \( V_{ij} \) is velocity of the particle i, the position of the particle i is denoted as \( X_{ij} \); \( C_1 \) and \( C_2 \) are two acceleration coefficients, which denote for cognitive and social parameters respectively and are set as 2; \( r_1 \) and \( r_2 \) are two uniformly distributed random numbers that are generated within the range of \([0, 1] \); \( g_{best} \) is the best position of the particle; \( \rho_{best} \) is the global best position of the entire swarm; and, \( t \) is the iteration number.

  The new position of the particle is updated by using Equation 2; here the new position of the particle is denoted by \( X_{ij}^{t+1} \), \( X_{ij} \) is the current position, and \( V_{ij}^{t+1} \) is the updated velocity of the particle [25-27].

- **Evaluate Responses:** The process of evaluating the results of respondents’ data is done by using the (Excel and PSO) tools with the assistance of MATLAB (R 2017a).
- Evaluate selected O.F: The process of analyzing and evaluating alternatives applies evaluation criteria to alternatives or options in a way that facilitates decision making. The evaluation process was by refining alternatives to develop the final alternative. PSO as a Multi-level evaluation to provides an opportunity to modeling and Check for convergence, then updating parameters to refine alternatives to meet the desired goals or outcomes more effectively with a greater understanding of the alternative’s strengths and weaknesses in each criterion.

Table 1. Expert responded(input data) about cost, time and quality of proposed pile type alternatives

| Pile Type Criteria | Timber | Steel | Pored | PC | PHC |
|-------------------|--------|-------|-------|----|-----|
| Production time   | 0.5    | 0.2   | 0.9   | 0.3| 0.1 |
| Quality control   | 0.3    | 0.8   | 0.5   | 0.7| 0.5 |
| Cost              | 0.3    | 0.8   | 0.3   | 0.7| 0.8 |

Table 1 shows a project have 5 alternatives. Let ti, ci and qi denote the (production time, cost of the pile type and the quality attained by performing pile type (i)). The nature of each pile type determines the (time, cost and quality) level (0-1) assigned to them, the objective is to construct the complete and efficient time, cost and quality profile to offer decision support in choosing pile type for project. There would be a constraint to ensure that one and only one pile type is assigned.

2.1. Preliminary Results

The results of the data analysis obtained by using the Excel program, which in the researcher view is a preliminary results, as shown in Figure 2, the lowest value of the cost is (0.5), time is (0.1) and the highest value for quality is (0.8).

As for quality, it is considered an important criterion, so the steel and PHC piles are the best but the cost of Steel is high. Therefore, PHC piles are the best one.

3. Particle Swarm Optimization (PSO)

Particle swarm optimization (PSO) is a population-based stochastic optimization method widely used to search for an ideal solution in the search space. Eberhart and Kennedy (1995) first created it. This optimization method is a kind of swarm intelligence inspired by the social behavior and dynamic motion of flocks of birds and fish [25, 28]. Basically, the PSO algorithm integrates the particles’ selfexperiences with their social experiences in search of globally optimal alternatives. PSO utilizes particles to move around in the search space to find the best alternative, and these particles also constitute a population called swarm.[26, 27]. In this algorithm, each particle has a memory (i.e., ability to remember) to store its flying experience especially for identifying its best position. The algorithm aims to move particles gradually towards better areas of the solution space for obtaining optimal solutions. The direction of the movement of each particle is adjusted according to the function of algorithm.

The position and velocity of the each particle is adjusted according to the best position visited by itself (i.e., pbest) and the best position visited by the entire swarm (i.e., gbest) at each step. The PSO algorithm is initialized randomly. In a D-dimensional space position of particle i^th is represented with \( X_i(t) = (X_{i1}(t), X_{i2}(t), ..., X_{iD}(t)) \), and its
velocity is presented with $V_i(t) = V_{i1}(t), V_{i2}(t), ..., V_{iD}(t))$, and as well as its best position is shown by $P_{i}^{best} = (P_{i1}, P_{i2}, ..., P_{iD})$ [23, 25, 29].

3.1. Steps of Basic PSO Algorithm

Steps of basic PSO algorithm are as follows:

1. Initialize swarm (i.e., initialize particles position and velocity randomly);
2. Evaluate each particle position based on the objective (fitness) function;
3. Update particles (pbest) (if the current position is better than its previous position);
4. Determine the best particle (gbest) (choose the particle with best fitness value as the gbest from entire swarm);
5. Update particles velocity (Using Eqn. 1)
6. Move particles to their new positions (Using Eqn. 2)
7. Go to step 2 until stopping criteria is not satisfied;

\begin{align*}
V_{ij}^{t+1} &= wV_{ij}^t + C_1 r_1 (P_{i}^{best} - X_{ij}^t) + C_2 r_2 (g_{best} - X_{ij}^t) \tag{1} \\
X_{ij}^{t+1} &= X_{ij}^t + V_{ij}^{t+1} \tag{2}
\end{align*}

3.2. Problem Formulation

Time-cost-quality trade-off problem is a multi-objective optimization problem [30]. In this study, on three conflictive objectives are optimized simultaneously. In other words, while time (T) and cost (C) are minimized, quality (Q) is maximized. Equation 3 represents the objective (fitness) function of the multi objective pile types selection problem; the first objective of the trade-off problem is the minimizing driven time of the pile (causes minimizing total project duration). This objective is expressed with Equation 4; where T is the total production time of the pile type (i), and (n) is the number of pile type option. Minimizing total cost of the project is the second objective. This objective is calculated by Equation 5; the third objective is maximizing the quality by Equation 6 [31].

\begin{align*}
&f \rightarrow Minimization \left( f_T, f_C, \frac{1}{f_Q} \right) \tag{3} \\
&Min f_1 = \sum_{i=1}^{n} t_i \tag{4} \\
&Min f_2 = \sum_{i=1}^{n} c_i \tag{5} \\
&Max f_3 = \frac{1}{n} \sum_{i=1}^{n} Q_i \tag{6}
\end{align*}

4. Implementation of the Developed Multi Objective Optimization Model

In this study, the PSO technique was developed using the MATLAB (R 2017a) software. The data obtained by the questionnaires is used to run the algorithm.

In the first step, the duration of the pile types’ production was calculated by taking the sum of the durations of pile types. In the second step, the cost of the pile types also was calculated by taking the sum of the costs. The overall quality of the pile types was evaluated in the third step by taking the invers of the sum of the qualities (because it was maximization not minimization like time and quality). The parameters and boundaries of the objectives are presented in Table 2.

| Parameters of the model | Values | Parameters of the model | Values |
|-------------------------|--------|-------------------------|--------|
| Swarm Size              | 5      | Time (up–low )          | (0.9 - 0.1 ) |
| Archive size            | 20     | Cost (up–low )          | (0.9 - 0.1 ) |
| W (min–max)             | (0-0.2)| Quality (up–low )       | (0.9 – 0.1 ) |
| (C1, C2)                | (0.02-0.012) | Maximum iteration number | 50    |
5. Results and Discussion of PSO

Since the discussed problem is multi objective optimization problem, so after running the PSO algorithm it is usual to have more than one optimal solution as shown in Figures 4 to 6, and Table 3. This research aims to find the optimal pile type to the decision makers which they can select the optimal solution, which fulfils their targets. Among the obtained solutions, five optimal solutions are presented.

The researcher discovered that there is no important distinction between the outcomes acquired from the Excel program and the outcomes of the PSO program due to fewer data and alternatives used in the search.

![Flowchart of the PSO algorithm methodology](image)

**Figure 3. Flowchart of the PSO algorithm methodology**

![MATLAB (R 2017a) output result](image)

**Figure 4. MATLAB (R 2017a) output result for Relationship between pile types and (Time in %) objective**
The main contribution of this study is to consider the pile type selection problem as a multi objective optimization problem. The data was used to run the developed PSO algorithm. For engineers, the "quality" is the dominant criteria for comparison so the indication after ranking the values as shown in Tables 3, 4 refers that, the optimal pile type was (0.1, 0.5, 0.8), and this presented in PHC pile type, so that easy for the decision maker to choose this pile type. The developed algorithm was quite satisfactory and the result was pointed out that PHC was the best choice.

Table 3. PSO and MATLAB (R 2017a) software output about optimized cost, time and quality of proposed pile type alternatives

| Criteria | TIME Minute/ML | COST ID/ML | QUALITY (%) |
|----------|----------------|------------|-------------|
| Timber   | 0.5            | 0.5        | 0.3         |
| Steel    | 0.2            | 0.8        | 0.8         |
| Pored    | 0.9            | 0.5        | 0.3         |
| PC       | 0.3            | 0.7        | 0.7         |
| PHC      | 0.1            | 0.5        | 0.8         |
| Rang     | 0.1-0.9        | 0.1-0.9    | 0.1-0.9     |

Figure 5. MATLAB (R 2017a) result for Relationship between pile types and (Cost in %) objective

Figure 6. PSO by MATLAB (R 2017a) result for Relationship between pile types and (Quality in %) objective
5.1. The Evaluation of PSO Algorithm

The evaluation process of PSO Algorithm should take into account operating the software before any discussion conducted with the specialist persons concerned with the subject of pile foundations management. Ten expert were selected to perform the evaluation process of algorithm. All of them were from (National Center for Engineering Consultancy, Baghdad Buildings Department, AL Rasheed Company, AL Farooq Company) as shown in Table 5. The evaluation form consists of seven questions is used to collect the opinions of evaluators about the ability PSO Algorithm. Table 6 contains the answers of evaluators and the evaluation degree for each question.

Table 5. Description for the Sample of PSO Algorithm Evaluation.

| Work place                        | Academic degree | Experience years | Position address                        | Current position                                      |
|-----------------------------------|-----------------|------------------|----------------------------------------|------------------------------------------------------|
| Baghdad Buildings Department      | B.Sc. Civil     | 32               | Chief senior Engineer                   | Head of technical division                           |
| Baghdad Buildings Department      | B.Sc. Civil     | 34               | Chief senior Engineer                   | Resident engineer of ministry of planning building project |
| AL Farooq company                 | B.Sc. Surveying | 32               | Chief senior Engineer                   | Head of roads survey and design division              |
| AL Farooq company                 | B.Sc. Civil     | 32               | Chief senior Engineer                   | Head of supervision and monitoring department         |
| AL Farooq company                 | M.Sc. Civil     | 31               | Engineer expert                         | Head of design and planning department                |
| National Center for Engineering Consultancy | M.Sc. Civil | 31 | Chief senior Engineer                   | Head of roads design division                         |
| National Center for Engineering Consultancy | B.Sc. Civil | 25 | Chief Engineer                          | Head of roads maintenance department                  |
| AL Rasheed company                | M.Sc. Civil     | 13               | Assistant Chief Engineer                | Director of world bank loan projects                  |
| AL Rasheed company                | B.Sc. Civil     | 12               | Senior Engineer                         | Supervision and monitoring engineer                   |
| AL Rasheed company                | B.Sc. Civil     | 11               | Senior Engineer                         | Head of studies division                               |

Table 6. The Evaluation Questions and the Answers.

| The Questions                                           | Excellent 100-80 | V. Good 80-60 | Good 60-40 | Middle 40-20 | Acceptable 20-0 | The degree | The evaluation |
|---------------------------------------------------------|------------------|---------------|------------|--------------|-----------------|------------|----------------|
| What is your opinion about the importance of program for project planners? | 2                | 6             | 1          | 1            | 0               | 68         | V. Good        |
| What do you think about the concept of PSO for the decision makers? | 4                | 5             | 1          | 0            | 0               | 76         | V. Good        |
| What do you think about the ability of program in assisting decision-makers, projects managers, planners, and contractors in their work? | 7                | 3             | 0          | 0            | 0               | 84         | Excellent      |
| What do you think about the role of program in developing the project feasible studies in Iraq? | 8                | 2             | 0          | 0            | 0               | 86         | Excellent      |
| What do you think about the importance of PSO in contracts management and selection of the best tenders in Iraqi companies? | 5                | 3             | 2          | 0            | 0               | 76         | V. Good        |
| What is the importance of program for Competition of contractors in bidding | 6                | 2             | 2          | 0            | 0               | 78         | V. Good        |
| What is the importance of program for your company? | 8                | 1             | 1          | 0            | 0               | 84         | Excellent      |

Note: Degree=\[\frac{\sum (fi*av.)/sum fi}{\sum fi}\]
fi=frequency

Table 4. Ranking of(time, cost,quality)Values for pile types

| Time valu | Cost valu | Quality valu |
|-----------|-----------|-------------|
| 0.1       | 0.5       | 0.8         |
| 0.2       | 0.5       | 0.8         |
| 0.3       | 0.5       | 0.7         |
| 0.5       | 0.7       | 0.3         |
| 0.9       | 0.8       | 0.3         |
According to the answers of evaluators as shown in Table 6, the following results can be summarized as follows:

- Most respondents confirmed that PSO is very good (in degree 68%) for planners, so it can be used to achieve a workable budget for complex projects. PSO is theoretically very easy and very good for decision-makers.
- 76% of respondents pointed that PSO concept has a very good ability to assist decision-makers in their works.
- 84% of respondents pointed that PSO has Excellent ability to assist decision-makers, projects managers, planners, and contractors in their work of selecting the most feasible alternatives.
- Most of respondents (86%) confirmed that this program has an excellent role in developing the feasible studies for complex projects.
- 76% of respondents pointed that PSO is very good at managing agreements and selecting the best tenders in Iraqi companies.
- 78% of respondents pointed that PSO is very good in Competition of contractors in bidding.
- Most evaluators (84%) confirmed that such program has an excellent importance for Iraqi companies if it is seriously applied.

6. Conclusions

- No previous studies about PHC piles carried out in Iraq neither PSO as a decision making.
- The research showed that PHC piles are the best option in time, cost and quality, and this is reflected in the duration, cost and quality of the project as a whole, and this is regarded a solution to Iraq's housing problem.
- PHC piles are distinguished ability bearing high weights, it is excellent in foundations of multi-story residential complexes.
- If the PHC product available in the market, there will be competition and promote the Market situation in addition to the quick profits of the plant because of the good features of the production.
- Using contemporary technology promotes job, decreases energy, and improves output as pointed by the researcher.
- Noticed from the case study that the majority of the cost savings was arrived by improving the design efficiency and changing the traditional methods of pile production.
- It is evidently proves that by promoting the new system (innovation) for the execution of piling construction, cost saving and environmental protection can also be achieved.
- From personal interviews and field survey the research has shown that most of the experts, engineers are not fully aware of new soft wear techniques to helps them in choosing alternatives, despite their belief in the usefulness of using modern technology and software.
- Most of Iraqi projects do not have an adequate database.
- The general focus of this study is to recommend PHC pile as the best option linked to price, time and quality for housing construction in Iraq.
- In the complex case of many data and the criteria with sub-criteria, the researcher believes that PSO method faster and easier In the case of Availability of a good programmer.
- One of the most significant factors to disrupt the decision-making, the obvious weakness in determining the relevant standards, is the lack of Iraqi companies to an integrated information system contributes to support and rationalize decision-making.

7. Conflicts of Interest

The authors declare no conflicts of interest.

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