Measurement error in small area estimation: a literature review

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Measurement error in small area estimation: a literature review

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Abstract. Recent development in small area estimation has been related to the measurement errors in the auxiliary variables. The development has been driven by the fact that in standard small area estimation the auxiliary variables in the model are assumed to be measured without error. This type of auxiliary variables is not easy to find in practice. In fact, the auxiliary variables cannot be provided by survey data since the data is subject to sampling errors. Hence, it is of interest to the researchers to understand how to use the auxiliary variables which are based on survey data in small area estimation. Several studies on small area estimation models with measurement error have been conducted recently and they usually are based on types of measurement error assumptions namely functional and structural measurement errors. This paper reviews recent development of measurement errors in small area estimation. Illustrative examples are also provided using real data.

1. Introduction
Sample survey is method of data collection with several advantages, such as saving time, money, and energy. Sample survey are generally designed to produce reliable direct estimate of mean or total of variable of interest for large areas or domain. In recent years, demand for reliable estimates for small domain or area has increased from both the public and private sectors. The term small domain or area typically refers to a population for which reliable statistics of interest cannot be produced due to certain limitations of the available data. Small domain includes a geographical area such as provinces, municipalities, district, or sub-district, and demographic group such as age, race, or gender. In sample survey, estimation of small domain could arise simply due to the sampling design that aims to provide reliable data for large areas and pay little attention to the small domain of interest. It is seldom possible to produce a large enough overall sample size to support direct estimates for all domains of interest. Indirect estimate are used to increase the effective sample size by borrowing strength from related areas that through linking model using census and administrative data and other auxiliary data associates with the small areas. Small area estimation is indirect estimation method to solve this problem. Model based estimation of small area estimation has been receiving a lot of attention in statistical literature where the goals is to extract information from other source. Depends on the type of data available, small area models are classified into two types: (i) area-level models that relate small area direct estimates to area specific auxiliary variables: such models are used if unit level data are not available; (ii) unit-level models that relate the unit values of a study variable to associated unit-level auxiliary variables with known area means and area specific auxiliary variables. A comprehensive
account of model based small area estimation under area level and unit level model is given by Jiang & Lahiri [1], Preffermann [2] [3], and Rao & Molina [4].

Properties of the predictors of small area means, such as bias and mean square error (MSE) are derived conditionally on the auxiliary information and under the assumption that auxiliary data are measured without error. However, when the auxiliary information used in the models are measured with error, small area estimators that ignore such error may be worse than direct estimators. In regression models, the presence of measurement error in auxiliary variables is known to cause bias in model parameter estimation and lead to loss of power to detect interesting relationship among variables [5]. In small area models, using survey data for auxiliary variable means that the data can be measured with error. Error in survey data came from due lack of memory, rounding, and other obvious reason. Within the classical measurement error framework, there are two major classes of models, the classical functional model and the classical structural model [6]. Functional model assumed that the true value of the covariates or auxiliary variables are unknown but fixed and constant; and structural model assumed that the true value of the covariates or auxiliary variables are unknown but random. In next chapter we will present a review of the existing literature related to the use of auxiliary information measured with error in small area statistical model.

2. Small area estimation with functional measurement error models

2.1. Area level

Functional measurement error models in area-level small area estimation was begin developed by Ybarra & Lohr [7]. Area-level model small area estimation is broadly use when the auxiliary variables is only at the area level. The model is defined as

\[ y_i = x_i^T \beta + u_i + e_i; \quad i = 1, \ldots, m \]  

(1)

where \( e_i \) represent the sampling error, assumed to have zero mean and known design variance, \( \text{Var}(e_i) = \sigma^2_e \) or \( e_i \sim N(0, \sigma^2_e) \). The random effects \( u_i \) are assumed to be independent with zero mean and variance \( \sigma^2_u \) or \( u_i \sim N(0, \sigma^2_u) \). If the parameter \( \beta, \sigma^2_u, \sigma^2_e \) are known, the best linear unbiased predictor (BLUP) of \( \theta_i \) under this model is

\[ \hat{\theta}^{FU}_i = y_i + (1 - y_i)x_i^T \beta \]  

(2)

where \( y_i = \sigma^2_u/(\sigma^2_u + \sigma^2_e) \). In practice, \( \beta \) and \( \sigma^2_e \) are unknown, these parameters must be estimated from the data. Suppose an estimator \( \hat{x}_i \) with \( \text{MSE}(\hat{x}_i) = C_i \). Since \( x_i \) may be measured with error, and \( x \) is non stochastic, we substitute an estimator \( \hat{x}_i \) for \( x_i \) and used instead

\[ y_i = \frac{\sigma^2_u}{\sigma^2_u + \sigma^2_e} \hat{x}_i^T \beta + u_i + e_i \]  

(3)

Then Ybarra-Lohr model for measurement error denote \( \hat{\theta}^{ME}_i = y_i + (1 - y_i)\hat{x}_i^T \beta \) where

\[ y_i = \frac{\sigma^2_u}{\sigma^2_u + \beta^T C_i \beta + \sigma^2_e} \]  

(4)

Let \( w_1, \ldots, w_m \) be a set of finite weight bounded away from 0. The estimated regression parameters \( \hat{\beta}_w \) is define

\[ \hat{\beta}_w = \left( \sum_{i=1}^{m} w_i \hat{x}_i \hat{x}_i^T - C_i \right)^{-1} \sum_{i=1}^{m} w_i \hat{x}_i y_i \]  

(5)

where \( w_i = 1/(\sigma^2_u + \beta^T C_i \beta + \sigma^2_e) \) for \( i = 1, \ldots, m \).

To estimate consistent estimator of \( \sigma^2_e \), Ybarra-Lohr suggested

\[ \hat{\sigma}^2_e(w) = (m - p)^{-1} \sum_{i=1}^{m} \left\{ \left( y_i - \hat{x}_i^T \hat{\beta}_w \right)^2 - \hat{\sigma}^2_e - \hat{\beta}_w^T C_i \hat{\beta}_w \right\} \]  

(6)

Singh [8] showed that the Ybarra-Lohr estimator of coefficient regression is slightly inefficient and that applying methods of bias correction like SIMEX and MCCS increases the efficiency of the small area estimates.
Arima et al. [9], henceforth abbreviated ADL, used a hierarchical Bayes for the measurement error problem for the area-level small area setup. It is assumed that measurement errors occur at random. This implies that the observed measurement $\hat{x}_i$ is modelled as random variable whose mean is equal to the true auxiliary variable $x_i$ and its variance is fixed equal to $C_i$. Then the measurement error model can be written as

- Stage 1: $y_i | \theta^i = N(\theta_i, \sigma^2_i)$, $i = 1, ..., m$
- Stage 2: $x_i, \beta, \delta, \sigma^2_u \sim N(x_i^T \beta + z_i^T \delta, \sigma^2_u)$, $i = 1, ..., m$
- Stage 3: $\hat{x}_i \sim N(x_i, C_i)$, $i = 1, ..., m$
- Stage 4: a uniform prior on $x_i$, $i = 1, ..., m$ and $\pi(x_1, ..., x_m, \beta, \delta, \sigma^2_u) \propto 1$

We set $z = (x_1, ..., x_m)^T$ as $m \times q$ matrix of the $q$ precisely observed auxiliary variables for $m$ small areas, $x = (x_1, ..., x_m)^T$ and $\hat{x} = (\hat{x}_1, ..., \hat{x}_m)^T$ and assuming that $x$ and $z$ is full column rank matrix, the posterior distribution is

$$\pi(\theta_1, ..., \theta_m, x_1, ..., x_m, \beta, \delta, \sigma^2_u | y_1, ..., y_m, w_1, ..., w_m) \propto \frac{1}{\sigma^2_m} \prod_{i=1}^{m} \left[ \exp \left( - \frac{(y_i - \theta_i)^2}{2\sigma^2_i} + \frac{(\theta_i - x_i^T \beta - z_i^T \delta)^2}{2\sigma^2_u} + \frac{\hat{x}_i - x_i)^T C_i^{-1}(\hat{x}_i - x_i)}{2} \right) \right]$$

(7)

ADL showed that the above posterior density is proper under the mild condition that $m > p + q + 2$. Also when $m > p + q + 6$ the model parameters $\beta, \delta, \sigma^2_u$ have finite posterior variances.

2.2. Unit level

We consider the case of nested error model with single area level auxiliary variables $x_i$ subject to measurement error. Ghosh & Sinha [10] studied the unit level regression model for small area estimation when the area level auxiliary variables is subject to functional measurement error. Ghosh-Sinha proposed a nested error model with an area-level auxiliary variables, $x$, subject to measurement error. It given by

$$y_{ij} = b_0 + b_1 x_i + u_i + e_{ij}, \quad j = 1, ..., N_i, i = 1, ..., m$$

(8)

$$X_{ij} = x_i + u_{ij}, \quad j = 1, ..., N_i, i = 1, ..., m$$

(9)

where $N_i$ is the known population size of the $i$-th area, $y_{ij}$ is the value of the study variable associated with the $j$-th unit in the $i$-th area and $x_i$ is the unknown true-area specific auxiliary variables associated with $y_{ij}$. Further, the random errors $e_{ij}$, measurement errors $u_{ij}$, and the area-level random effect $u_i$ are assumed to be mutually independent with $e_{ij} \sim N(0, \sigma^2_e)$, $u_{ij} \sim N(0, \sigma^2_u)$, and $u_i \sim N(0, \sigma^2_u)$. Ghosh-Sinha considered the case of a functional measurement error model (9), where $x_i$ is a fixed and unknown parameter and focused on EB estimation. The vector of model parameters is denoted by $\phi = (b_0, b_1, \sigma^2_e, \sigma^2_u, \sigma^2_u)^T$. A sample of size $n_i$ is selected from the $i$-th area and the sample data, without loss of generality is denoted by $\{y_{ij}, X_{ij}; j = 1, ..., N_i; i = 1, ..., m\}$. We write $y_i = (y_{i1}, ..., y_{iN_i})^T$ as $y_i = (y_i^{(1)}, y_i^{(2)})^T$ with $y_i^{(1)} = (y_{i1}, ..., y_{iN_i})^T$ and $y_i^{(2)} = (y_{iN_{i+1}}, ..., y_{iN_{i+i}})^T$ and $X_i^{(1)} = (X_{i1}, ..., X_{iN_i})^T$. We assume that the small area model given (8) and (9) hold for the sample data $(y_i^{(1)}, X_i^{(1)}; i = 1, ..., m)$ that is, no sample selection bias and the sampling design is not informative. The goal is to estimate (or more appropriately predict) the small area means from the sample data and given by

$$\hat{y}_i = \frac{1}{N_i} \sum_{j=1}^{N_i} y_{ij} \quad (i = 1, ..., m)$$

Ghosh-Sinha obtained the Bayes predictor of $y_i$ given by

$$\hat{y}_i^B = (1 - f_i B_i) \hat{y}_i + f_i B_i (b_0 + b_1 x_i)$$

(10)
where \( f_i = (N_i - n_i)/N_i, i = 1, \ldots, m \) is the finite population corrected factor, \( \bar{y}_i = n_i^{-1} \sum_{j=1}^{n_i} y_{ij} \), and \( B_i = \sigma_e^2 / (\sigma_e^2 + n_i \sigma_y^2) \). Since \( \hat{y}_i^{IB} \) is a function of unknown \( x_i \), pseudo-Bayes predictor of \( y_i \) as

\[
\hat{y}_i^{IB} = (1 - f_B) \bar{y}_i + f_B(b_0 + b_1 \bar{x}_i)
\]

where \( \bar{x}_i = \bar{x} = n_i^{-1} \sum_{j=1}^{n_i} x_{ij} \) and \( \phi \) is method of moments estimators of the model parameters. Pseudo-Bayes predictor in (11) is not efficient because used only the sample means of the observed auxiliary variables value to estimate the true auxiliary variables values.

Datta et al. [11], henceforth abbreviated DRT, used both response and auxiliary variable values to estimate true auxiliary variable values. In particular, DRT gives an efficient estimator of \( x_i \) given by

\[
\hat{x}_i(\phi) = \bar{x}_i + \frac{b_1 \sigma_x^2 (y_i - b_0 - b_1 \bar{x}_i)}{\phi}
\]

Then correspond pseudo-Bayes predictor of \( y_i \) is given by

\[
\hat{y}_i^{PEB} = (1 - f_A) \bar{y}_i + f_A(b_0 + b_1 \bar{x}_i)
\]

where \( A_i = \sigma_x^2 / (\sigma_x^2 + n_i \sigma_u^2 + b_1^2 \sigma_y^2) \).

The PEB (pseudo-empirical Bayes) predictor, \( \hat{y}_i^{PEB} \), of \( y_i \) is obtaining by replacing \( \phi \) in the (13) by a consistent estimators \( \phi \) with method-of-moments estimator proposed by Ghosh-Sinha. Let

\[
SSW_X = \sum_{i=1}^{m} \sum_{j=1}^{n_i} (x_{ij} - \bar{x}_i)^2, \quad SSW_y = \sum_{i=1}^{m} \sum_{j=1}^{n_i} (y_{ij} - \bar{y}_i)^2
\]

\[
MSW_X = SSW_X/(n_T - m), \quad MSW_y = SSW_y/(n_T - m)
\]

where \( n_T = \sum_{i=1}^{m} n_i \) is the total sample size. Then Ghosh-Sinha have show consistent estimator of \( \sigma_u^2 \) and \( \sigma_x^2 \) by

\[
\hat{\sigma}_u^2 = MSW_X \quad \text{and} \quad \hat{\sigma}_x^2 = MSW_y
\]

Further, consistent estimator of \( b_1 \) and \( b_0 \) by

\[
\hat{b}_1 = (MSB_X/MSB_Y - MSW_X) \bar{y}_i \quad \hat{b}_0 = \bar{y} - \hat{b}_1 \bar{x}_i
\]

where \( \bar{x} = n^{-1} \sum_{i=1}^{m} n_i \bar{x}_i, \bar{y} = n^{-1} \sum_{i=1}^{m} n_i \bar{y}_i \), \( MSB_X = (m - 1) \sum_{i=1}^{m} n_i (\bar{x}_i - \bar{x})^2 \) and \( \hat{b}_1 = \sum_{i=1}^{m} n_i \bar{y}_i (\bar{X}_i - \bar{x}) / (m - 1) \). The remaining \( \sigma_u^2 \) is consistently estimated by

\[
\hat{\sigma}_u^2 = \max \left[0, \left\{ MSB_y - MSW_y - \hat{b}_1^2 (MSB_X - MSW_X) / g_m \right\} \right]
\]

where \( MSB_Y = (m - 1) \sum_{i=1}^{m} n_i (\bar{y}_i - \bar{y})^2 \) and \( g_m = n_T - \sum_{i=1}^{m} n_i \hat{\sigma}_u^2 / n_T \).

The PEB predictor of is given by

\[
\hat{y}_i^{PEB} = (1 - f_A) \bar{y}_i + f_A(b_0 + b_1 \bar{x}_i)
\]

where \( A_i \) is obtained from \( A_1 \) by replacing \( \phi \) with \( \hat{\phi} \).

Torabi [12] suggest a PEB predictor of \( y_i \) from DRT that depends on the survey weights and satisfies the design consistency property. Torkashvand et al. [13] suggest a PEB predictors of small area means based on the James-Stein estimator of true auxiliary variable subject to the functional measurement error. The result is the PEB predictor based on the James-Stein estimator perform better than maximum likelihood method and the method of moments. The new PEB predictor is also asymptotically optimal.

3. Small area estimation with structural measurement error models

Early small area estimation with structural measurement error in the auxiliary variables was development by Ghosh et al. [14]. Ghosh et al., henceforth abbreviated GSK, develop empirical Bayes (EB) and hierarchical Bayes (HB) procedures for simultaneous estimation of finite population strata means as well as the super population parameter when the auxiliary variables, say \( x \), are measured with error. In this model, we assume that \( x \) is stochastic. In the common terminology, this is the so-called structural measurement error model. This is in contrast to the functional measurement error model where \( x \) is non stochastic. GSK proposed a nested error linear regression population model with an area-level auxiliary variables, \( x \), subject to measurement error. The model is given by

\[
y_{ij} = b_0 + b_1 x_i + u_i + e_{ij}, \quad (j = 1, \ldots, n_i, i = 1, \ldots, m)
\]

\[\text{doi:10.1088/1755-1315/187/1/012034}\]
\[ X_{ij} = x_i + \eta_{ij}, \quad j = 1, \ldots, N_j, i = 1, \ldots, m \] 

where, as before, \( N_i \) is the known population size of the \( i \)-th area, \( y_{ij} \) is the value of the study variable associated with the \( j \)-th unit in the \( i \)-th area and \( x_i \) is the unknown true-area specific auxiliary variables associated with \( y_{ij} \). In the structural approach, in addition to the random errors \( e_{ij}, \eta_{ij} \) and random effect \( u_{ij} \), unknown auxiliary variables \( x_i \)'s are also considered random. We assume independence among all these components, with corresponding distributions specified by \( e_{ij} \sim N(0, \sigma_e^2), \eta_{ij} \sim N(0, \sigma_\eta^2), u_{ij} \sim N(0, \sigma_u^2), x_i \sim N(0, \sigma_x^2) \), \( j = 1, \ldots, N_i, i = 1, \ldots, m \). The vector of model parameters is denoted by \( \phi = (b_0, b_1, \mu_x, \sigma_u^2, \sigma_e^2, \sigma_\eta^2, \sigma_x^2)^T \). In their EB approach, GSK first derived a predictor for non sampled response variable \( N_i - n_i \) units, conditional on \( \phi \), and the observed sample, which they denote \( \hat{y}_{i1} \). The goal is to estimate the small area mean \( y_i = \frac{1}{N_i} \sum_{j=1}^{N_i} y_{ij} \). In particular, for any non sampled \( y_{ij}, j = n_{i+1}, \ldots, N_i \), they obtained

\[ \hat{y}_i^B = E\left( y_i | y_i^{(1)} \right) = (1 - f_l D_l) \bar{y}_i + f_l D_l (b_0 + b_1 \mu_x) \] 

where \( D_l = \frac{\sigma_\eta^2}{\sigma_e^2 + \eta_1 (\sigma_u^2 + \beta \sigma_x^2)}, \bar{y}_i = n_i^{-1} \sum_{j=1}^{n_i} y_{ij}, \) and \( f_l = (N_i - n_i) / N_i, i = 1, \ldots, m \). To pursue the EB approach, GSK replace unknown model parameters in (19) by their estimators to derive an EB predictor of an non sampled unit and finally of \( y_i \). GSK used method of moment for estimation of the parameters \( \phi \).

Torabi et al. [15], henceforth abbreviated TDR, consider the same model and focused on the EB estimation. They noticed that the above Bayes predictor for an un sampled \( y_{ij} \) does not involve sample information on the auxiliary variables value, which is present in the actual data and it should be adequately used in the predictor, by conditioning on the observed values, say \( X_i^{(1)} \), as well. The expression of new Bayes predictor for each of the \( N_i - n_i \) non sampled units is the obtained as

\[ \hat{y}_i^B = (1 - f_l G_l) \bar{y}_i + f_l G_l (b_0 + b_1 \mu_x) + f_l G_l \left( \frac{n_i \sigma_x^2}{\sigma_u^2 + n_i \sigma_x^2} \right) b_1 (\bar{X}_i + \mu_x) \] 

with \( G_l = \frac{\sigma_\eta^2}{n_i b_0^2 \sigma_u^2 + (n_i \sigma_u^2 + \beta \sigma_x^2)(\sigma_x^2 + n_i \sigma_x^2)} \) and \( \bar{X}_i = n_i^{-1} \sum_{j=1}^{n_i} X_{ij} \). Notice that both \( D_l \) and \( G_l \) depend on the area index only through the corresponding sample sizes. Using method of moments estimators of the model parameters, suggested in GSK, the latter author also provided their EB predictors of \( y_i \). Both of authors established asymptotic optimality of their respective EB predictors by showing that as the number of areas \( m \to \infty \), the empirical mean of the mean squared of differences of the EB predictors and the Bayes predictors goes to zero. Torabi [16] suggest a PEB predictor from the model of TDR that depends on the survey weights. The result show that the estimator borrows strength across areas through the model and make use of the survey weights to preserve the design consistency as the area sample size increases.

GSK also propose hierarchical Bayes (HB) procedures for simultaneous estimation of finite population strata means as well as the super population parameter when the auxiliary variables are measured with error. The measurement error model can be written as

- **Stage 1:** \( y_{ij} = \theta_i + e_{ij}, \quad j = 1, \ldots, n_i, i = 1, \ldots, m \) with \( e_{ij} \sim N(0, \sigma_e^2) \)
- **Stage 2:** \( \theta_i = b_0 + b_1 x_i + u_i, \quad i = 1, \ldots, m \) with \( u_i \sim N(0, \sigma_u^2) \) and \( X_{ij} = x_i + \eta_{ij} \) with \( \eta_{ij} \sim N(0, \sigma_\eta^2) \)
- **Stage 3:** \( x_i \sim N(\mu_x, \sigma_x^2), \quad i = 1, \ldots, m \)
- **Stage 4:** \( b_0, b_1, \mu_x, \sigma_u^2, \sigma_e^2, \sigma_\eta^2, \sigma_x^2 \) are mutually independent with \( b_0, b_1, \mu_x \) iid Uniform\((-\infty, \infty); \)

\( \sigma_e^2 \sim IG(a_e/2, b_e/2), \sigma_\eta^2 \sim IG(a_\eta/2, b_\eta/2), \sigma_u^2 \sim IG(a_u/2, b_u/2), \sigma_x^2 \sim IG(a_x/2, b_x/2), \) where \( IG(\alpha, \beta) \) denote an inverse gamma distribution with pdf \( f_{\alpha,\beta}(z) \propto \alpha \exp(-\alpha/z)z^{-\beta-1} \) for \( z > 0 \).

Bayesian computations with this prior distributions are straight forward via Gibbs sampling. GSK proved that the posterior distribution based on their improper prior is proper under some mild
condition. While the use of conjugate priors by GSK simplifies the computation, the shape and the scale parameters of the inverse gamma distribution were selected by them arbitrarily in a subjective manner. Arima et al. [17], henceforth abbreviated ADL, derive the Jeffreys prior for model GSK. The reason using Jeffreys prior is reasonable objective prior when the entire parameter vector is the parameter of interest. ADL prove that if the number of areas is at least seven, the corresponding posterior distribution of the parameter vector is proper. For this prior, while the computing is less that straightforward, samples from the posterior distribution can be obtained via Adaptive Rejection Metropolis Sampling.

4. Case study
In this case study used data National Socio-Economic Survey (Survei Sosial Ekonomi Nasional, Susenas) 2015 and Labour Force Survey (Survei Angkatan Kerja Nasional, Sakernas) 2015 in regency of Jawa Timur. The parameter used in this case is the average expenditure per capita per month for district in regency of Jawa Timur. In Jawa Timur there are 605 district in 29 regency. There are 29 non sampled district. Auxiliary variable obtained from Sakernas 2015. Only one auxiliary variable used in this model is proportion of the population work in the agricultural sector. Auxiliary variable assume that measurement with error because come from data survey.

Table 1. Direct estimation, GSK, and TDR for average expenditure per capita per month

| Area | N_i | n_i | Average expenditure per capita per month |
|------|-----|-----|-----------------------------------------|
|      |     |     | Direct Estimate | GSK | TDR |
| 1    | 12  | 12  | 616.128,76      | 631.377,09 | 631.377,09 |
| 2    | 21  | 20  | 594.304,51      | 649.289,46 | 642.940,50 |
| 3    | 14  | 14  | 666.477,78      | 667.306,61 | 667.306,61 |
| 4    | 19  | 19  | 745.368,13      | 774.112,86 | 774.112,86 |
| 5    | 22  | 24  | 710.892,37      | 765.211,14 | 758.295,43 |
| 6    | 26  | 24  | 571.700,59      | 627.087,29 | 618.325,75 |
| 7    | 33  | 32  | 730.821,77      | 753.174,53 | 749.893,32 |
| 8    | 21  | 21  | 570.695,96      | 609.288,43 | 609.288,43 |
| 9    | 31  | 28  | 583.872,34      | 622.444,88 | 612.430,75 |
| 10   | 24  | 22  | 696.293,95      | 746.823,73 | 734.829,10 |
| 11   | 23  | 22  | 559.891,01      | 558.060,12 | 553.383,81 |
| 12   | 17  | 17  | 587.515,17      | 560.505,06 | 560.505,06 |
| 13   | 24  | 24  | 563.818,57      | 560.323,35 | 560.323,35 |
| 14   | 24  | 23  | 702.022,23      | 697.098,82 | 691.760,27 |
| 15   | 18  | 18  | 1.269.040,05    | 1.292.475,85 | 1.292.475,85 |
| 16   | 18  | 18  | 851.190,80      | 899.081,39 | 899.081,39 |
| 17   | 21  | 20  | 645.927,20      | 659.769,87 | 653.318,43 |
| 18   | 20  | 18  | 580.295,04      | 591.657,73 | 578.651,15 |
| 19   | 15  | 15  | 725.347,92      | 798.224,67 | 798.224,67 |
| 20   | 18  | 17  | 675.531,44      | 706.527,46 | 697.581,05 |
| 21   | 19  | 19  | 604.664,98      | 692.327,12 | 692.327,12 |
| 22   | 28  | 24  | 607.873,13      | 612.970,85 | 597.065,70 |
| 23   | 20  | 20  | 713.063,98      | 756.035,98 | 756.035,98 |
| 24   | 27  | 23  | 788.582,24      | 813.062,65 | 789.564,23 |
| 25   | 18  | 18  | 923.785,93      | 913.879,78 | 913.879,78 |
| 26   | 18  | 17  | 526.329,28      | 590.126,07 | 582.652,61 |
| 27   | 14  | 12  | 537.886,53      | 561.126,52 | 538.637,41 |
| 28   | 13  | 13  | 510.135,09      | 511.139,89 | 511.139,89 |
| 29   | 27  | 25  | 708.535,89      | 685.242,30 | 676.279,97 |
We use model GSK and TDR in (17) and (18) that assume $x$ is stochastic. Table 1 shows the result of estimation of average expenditure per capita by regency in Jawa Timur. It show that estimates using the Bayes method with unit level (district) measurement error model to estimate regency level.

5. **Concluding Remarks**

Small area estimation methods typically combine direct estimates from a survey with prediction from the model in order to obtain accurate estimate of population quantities, auxiliary variables used in the model strongly affect the accuracy of the estimation. However, it is not possible to measure the auxiliary variables without any error. We have presented a review of the existing literature, considering unit-level and area-level models from a frequentist and a Bayesian approach. The topic of measurement error are deeply investigated in small area literature, but some aspect may be developed because in official statistics, measurement error is crucial problem. Some studies that can be develop related to small area estimation model with the measurement error are:

- To evaluate the prediction model
- To develop alternative model to be compared
- To compare functional and structural model
- To construct a model when there is a violation of mutual assumptions between the auxiliary variables within area.

6. **References**

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