THE KINETIC SUNYAЕV-ZEL’DOVICH EFFECT FROM RADIATIVE TRANSFER SIMULATIONS OF PATCHY REIONIZATION

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ABSTRACT

We present the first calculation of the kinetic Sunyaev-Zel’dovich (kSZ) effect due to the inhomogeneous reionization of the universe based on detailed large-scale radiative transfer simulations of reionization. The resulting sky power spectra peak at $\ell = 2000 - 8000$ with maximum values of $[H(\ell+1)/\ell\ell^2d\ell]_{\text{max}} \sim 1 \times 10^{-12}$. The peak scale is determined by the typical size of the ionized regions and roughly corresponds to the ionized bubble sizes observed in our simulations, $\sim 5 - 20$ Mpc. The kSZ anisotropy signal from reionization dominates the primary CMB signal above $\ell = 3000$. This predicted kSZ signal at arcminute scales is sufficiently strong to be detectable by upcoming experiments, like the Atacama Cosmology Telescope and South Pole Telescope which are expected to have $\sim 1\,$ resolution and $\sim 1\,$K sensitivity. The extended and patchy nature of the reionization process results in a boost of the peak signal in power by approximately one order of magnitude compared to a uniform reionization scenario, while roughly tripling the signal compared with that based upon the assumption of gradual but spatially uniform reionization. At large scales the patchy kSZ signal depends largely on the ionizing source efficiencies and the large-scale velocity fields: sources which produce photons more efficiently yield correspondingly higher signals. The introduction of sub-grid gas clumping in the radiative transfer simulations produces significantly more power at small scales, and more non-Gaussian features, but has little effect at large scales. The patchy nature of the reionization process roughly doubles the total observed kSZ signal for $\ell \sim 3000 - 10^5$ compared to non-patchy scenarios with the same total electron-scattering optical depth.

Subject headings: radiative transfer — cosmology: theory — cosmic microwave background — intergalactic medium — large-scale structure of universe — radio lines

1. INTRODUCTION

The secondary anisotropies of the Cosmic Microwave Background (CMB) are emerging as one of the most powerful tools in cosmology. The small-scale anisotropies are probes of cosmological structures and are thus a valuable tool for studying their formation and properties. The key effect generating small-scale CMB anisotropies is the Sunyaev-Zel’dovich (SZ) effect (Zeldovich & Sunyaev 1969), produced by Compton scattering of the CMB photons on moving free electrons. When this is due to thermal motions the effect is referred to as thermal SZ effect (tSZ), while when it is due to the electrons moving with a net bulk peculiar velocity it is called kinetic SZ effect (kSZ) (Sunyaev & Zeldovich 1980). The former, coming largely from hot gas in low-redshift galaxy clusters, is the dominant effect, but has a different spectrum than the CMB primary anisotropies. The tSZ spectrum has a characteristic zero at $\sim 217$ MHz, so its contribution can in principle be separated. By contrast, kSZ anisotropies have a spectrum identical with that of the primary anisotropies. Separation of its effects is possible only by virtue of its different spatial structure. We consider the kSZ fluctuations as composed of two basic contributions, one coming from inhomogeneous reionization and the other from the fully-ionized gas after reionization. The calculation of the patchy reionization contribution to the kSZ signal based on the first large-scale radiative transfer simulations of reionization is the main focus of this paper. This signal provides a signature of the character of reionization that will complement other approaches like observations of the redshifted 21-cm line of hydrogen and surveys of high-$z$ Ly-α emitters (e.g. Scott & Rees 1994, Tozzi et al. 2000, Iliev et al. 2003, Cardi & Malan 2003, Rhoads et al. 2004, Stanway & et al. 2004, Gnedin & Prada 2004, Furlanetto et al. 2004, Mellema et al. 2006, Malhotra & Rhoads 2006, Shapiro et al. 2006, Bunker et al. 2006, Furlanetto et al. 2006). In addition to better understanding of the physics of reionization, such studies could provide better constraints on the fundamental cosmological parameters, in particular on the primordial power spectrum of density fluctuations at smaller scales than are currently available.

The kSZ from a fully-ionized medium has been studied previously by both analytical and numerical means. When linear perturbation theory is used, the effect is associated with the quadratic nonlinearities in the electron density current and is usually referred to as the Ostriker-Vishniac effect (Ostriker & Vishniac 1986). Calculating the full, nonlinear effect is much more difficult to do analytically, although some models have been proposed (e.g. Jaffe & Komatsu 1998, Hui 2000, Ma & Fry 2002, Zhang et al. 2004). This effect involves coupling of very large-scale to quite small-scale density fluctuation modes and thus requires a very large dynamic range in order to be simulated correctly.
Current simulations are in rough agreement, but have not quite converged yet (e.g. Springel et al. 2001, Ma & Fry 2002, Zhang et al. 2004). All simulations predict significant enhancement of the small-scale anisotropies compared to the analytical theory, due to the nonlinear evolution.

Calculations of the kSZ contribution from patchy reionization are even more challenging. In addition to the above difficulties, such calculations also require detailed modelling of the radiative transfer of ionizing radiation to derive the sizes and distributions of H II regions in space and time (the reionization geometry) and how these correlate with the velocity and density fields. To date this problem has been studied by only a few recent works. Most of these estimates were done by semi-analytical models, which used various simplified approaches to model the inhomogeneous reionization (Gruzinov & Hui 1998, Santos et al. 2003, Zahn et al. 2005, McGinn et al. 2005). While such models are useful since they are cheaper to calculate than full radiative transfer simulations, and so allow for fast exploration of the various scenarios, their results must be checked against full detailed simulations in order to ascertain their reliability. The only two existing numerical studies of this effect, Gnedin & Jaffe (2001) and Salvaterra et al. (2003) used fairly small computational boxes (4h^{-1}Mpc and 20h^{-1}Mpc, respectively). As a consequence, their results significantly underestimate the kSZ signal, as we discuss below.

There are several upcoming experiments which will search for kSZ effect, in particular the Atacama Cosmology Telescope (ACT)4 and South Pole Telescope (SPT)5, both of which are expected to be operational by early 2007. These experiments will have ~ 1' resolution and ~ µK sensitivities, which should be sufficient to detect the kSZ signal.

In this paper we present the first calculations of the kSZ effect from patchy reionization based on large-scale radiative transfer simulations. We simulate reionization using a (100 h^{-1}Mpc)^3 simulation volume, which is sufficient to capture the relevant large-scale density and velocity perturbations, an important improvement over previous efforts. We use the ray-tracing code C^2-Ray (Mellemma et al. 2006) to follow the radiation from all ionizing sources in that volume identified with the resolved halos, which are of dwarf galaxy size or larger. The halos and underlying density field are provided by a very large N-body simulation with the code PMFAST (Merz et al. 2003). The results of these simulations on the reionization character, geometry and observability of the redshifted 21-cm line of hydrogen were discussed in detail in Iliev et al. (2006), Paper I and Mellemma et al. (2006), Paper II). A preliminary version of our current results was presented in Iliev et al. (2006).}

2. SIMULATIONS

Our simulations follow the evolution of a comoving simulation volume of (100 h^{-1}Mpc)^3, corresponding to an angular size ~ 1 deg on the sky at the relevant redshift range. Our methodology and simulation parameters were described in detail in Papers I and II. Here we provide just a brief summary. We start with performing a very large pure dark matter simulation of early structure formation, with 1624^3 ~ 4.3 billion particles and 3248^3 grid cells using the code PMFAST (Merz et al. 2005). This allows us to reliably identify (with 100 particles or more per halo) all halos with masses 2.5 \times 10^9 M_\odot or larger. We find and save the halo catalogues, which contain the halo positions, masses and detailed properties, in up to 100 time slices starting from high redshift (z ~ 30) to the observed end of reionization at z ~ 6. We also save the corresponding density and bulk peculiar velocity fields at the resolution of the radiative transfer grid. Unfortunately, radiative transfer simulations at the full grid size of our N-body computations are impractical on current computer hardware, thus we solve for the radiative transfer on coarser grids, of sizes 203^3 = (3248/10)^3 or 406^3. These grid resolutions allow us to derive reliably the angular sky power spectra for l ~ 430 – 90,000 for 203^3, and up to l ~ 180, 000 for 406^3. Throughout this study we assume a flat ΛCDM cosmology with parameters ($\Omega_m, \Omega_\Lambda, \Omega_b, h, \sigma_8, n) = (0.27, 0.73, 0.044, 0.7, 0.9, 1$)5 (Spergel et al. 2003), where $\Omega_m$, $\Omega_\Lambda$, and $\Omega_b$ are the total matter, vacuum, and baryonic densities in units of the critical density, h is the Hubble constant in units of 100 km s^{-1} Mpc^{-1}, $\sigma_8$ is the standard deviation of linear density fluctuations at present on the scale of 8h^{-1}Mpc, and n is the index of the primordial power spectrum. We use the CMBfast transfer function (Seljak & Zaldarriaga 1996). All calculations are done in flat-sky approximation, which is appropriate for our relatively small angular sizes.

All identified halos are assumed to be sources of ionizing radiation and each is assigned a photon emissivity proportional to its total mass, M, according to

$$\dot{N}_\gamma = f_* \frac{M\Omega_b}{\mu m_p t_s\Omega_0},$$

where $t_s$ is the source lifetime, $m_p$ is the proton mass, $\mu$ is the mean molecular weight and $f_*$ is a photon production efficiency which includes the number of photons produced per stellar atom, the star formation efficiency (i.e. what fraction of the baryons are converted into stars) and the escape fraction (i.e. how many of the produced ionizing photons escape the halos and are available to ionize the IGM).

The radiative transfer is followed using our fast and accurate ray-tracing photoionization and non-equilibrium chemistry code C^2-Ray. The code has been tested in detail for correctness and accuracy against available analytical solutions and a number of other cosmological radiative transfer codes (Mellemma et al. 2006a, Iliev et al. 2006). The radiation is traced from every source on the grid to every cell.

We have performed four radiative transfer simulations. These share the source lists and density fields given by the underlying N-body simulation, but adopt different assumptions about the source efficiencies and the sub-grid density fluctuations. The runs and notation are the same as in Paper II: runs f2000 and f250 assume 6 3248 = N_{nodes} \times (512 \times 2 \times 24), where N_{nodes} = 7 (with 4 processors each), 512 cells is the Fourier transform size and 24}
$f_\gamma = 2000$ and 250, respectively, and no sub-grid gas clumping, while $f_{2000 \text{C}}$ and $f_{250 \text{C}}$ adopt the same respective efficiencies, $f_\gamma = 2000$ and 250, but also add a sub-grid gas clumping, $C(z) = \langle n^2 \rangle / \langle n \rangle^2$, which evolves with redshift according to

$$C_{\text{subgrid}}(z) = 27.466 e^{-0.114z + 0.001328z^2}.$$  \hspace{1cm} (2)

The last fit was obtained from another high-resolution PMFAST N-body simulation, with box size $(3.5 \, h^{-1} \, \text{Mpc})^3$ and a computational mesh and number of particles of $3248^3$ and $1624^3$, respectively. These parameters correspond to a particle mass of $10^3 M_\odot$ and minimum resolved halo mass of $10^8 M_\odot$. This box size was chosen so as to resolve the scales most relevant to the gas clumping - on scales smaller than these the gas fluctuations would be below the Jeans scale, while on larger scales the density fluctuations are already present in our computational density fields and should not be doubly-counted. The expression in equation (2) excludes the matter inside collapsed minihalos (halos which are too small to cool atomically, and thus have inefficient star formation) since these are shielded, unlike the generally optically-thin IGM. This self-shielding results in a lower contribution of the minihalos to the total number of recombinations than one would infer from a simple gas clumping argument [Shapiro et al. 2004, Iliev et al. 2005a,b]. The effect of minihalos could be included as sub-grid physics as well, see [Ciardi et al. 2006]. This results in slower propagation of the ionization fronts and further delay of the final overlap. The halos that can cool atomically are assumed here to be ionizing sources and their recombinations are thus implicitly included in the photon production efficiency $f_\gamma$ through the corresponding escape fraction.

3. **kSZ FROM PATCHY REIONIZATION**

The kSZ effect is the CMB temperature anisotropy along a line-of-sight (LOS) defined by a unit vector $\mathbf{n}$ induced by Thomson scattering from flowing electrons:

$$\frac{\Delta T}{T_{\text{CMB}}} = \int d\eta e^{-\tau_{\text{es}}(\eta)} a_n \sigma_T \mathbf{n} \cdot \mathbf{v},$$  \hspace{1cm} (3)

where $\eta = \int_0^t dt'/a(t')$ is the conformal time, $a$ is the scale factor, $\sigma_T = 6.65 \times 10^{-25} \, \text{cm}^{-2}$ is the Thomson scattering cross-section, and $\tau_{\text{es}}$ is the corresponding optical depth.

We calculate the kSZ anisotropy signal from our simulation data as follows. We first calculate the line-of-sight integral in equation (3) for each individual output slice of the radiative transfer simulation and for all LOS along each of the three box axes. The contribution to the total LOS integral from each light-crossing time of the box is then obtained by linear interpolation between the nearest results from the simulation output times. The individual light-crossing time contributions are then all added together to obtain the full LOS integral given in equation (3). All these integrals are done for each LOS through the box along the direction of light propagation, which allows us to produce kSZ maps, in addition to the statistical signals. In order to avoid artificial amplification of the fluctuations resulting from repeating the same structures along the line of sight, after each light-crossing time we randomly shift the box in the directions perpendicular to the LOS and rotate the box, so that the LOS cycles the directions along the x, y and z axes of the simulation volume.

For comparison we also consider two simplified cases in addition to our simulations, the cases of instantaneous and uniform reionization. We define instant reionization as a sharp transition from completely neutral to fully-ionized IGM at redshift $z_{\text{instant}}$. We pick $z_{\text{instant}} = 13$, which yields the same integrated electron scattering optical depth as our simulation $f_{250}$. We also define a “uniform reionization” scenario to be one that has the same time-dependent reionization history (and hence, also the same $\tau_{\text{es}}$) as simulation $f_{250}$, but spatially uniform (i.e. not patchy). We then derive the kSZ temperature fluctuations for these two scenarios using the same density and velocity data and same procedures as for the actual simulations. We consider these simplified models in order to demonstrate the effects of reionization being extended and patchy in nature.

4. **RESULTS**

4.1. **kSZ maps**

We show the kSZ maps of temperature fluctuations, $\delta T / T_{\text{CMB}}$, yielded by all of our cases in Figure 1. These have a total angular size of approximately $50' \times 50'$, corresponding to our computational box size. The resolution of the maps is that of the full radiative transfer grid ($203 \times 203$ pixels), corresponding to pixel resolution of $\sim 0.25'$. All maps utilize the same color map in order to facilitate their direct comparison. The maps derived from our simulations of inhomogeneous reionization (Figure 1, top and middle) all show fairly strong fluctuations, both positive and negative, of order $\delta T \sim 10 \, \mu K$ at angular scales of a few arcminutes ($\sim 10 \, h^{-1} \, \text{Mpc}$) and up to $\delta T \sim 20 \, \mu K$ at the full map resolution. The fluctuations are at somewhat smaller scales when the sources are less efficient photon producers ($f_{250}$ and $f_{250 \text{C}}$), compared to the high-efficiency cases ($f_{2000}$ and $f_{2000 \text{C}}$). The variations are noticeably enhanced when sub-grid gas clumping is included in the reionization model ($f_{2000 \text{C}}$ and $f_{250 \text{C}}$) and there are a number of regions with very strong features. In comparison, the artificial models of instant and uniform reionization (Figure 1, bottom) show fluctuations with much lower amplitudes and with a less well-defined typical scale. Since these simple scenarios were constructed to produce the same total electron scattering optical depth as simulation $f_{250}$ and they use the same density and velocity fields as the simulations, any observed differences are due to the patchiness of reionization. The lower signal in these last two cases is expected since the kSZ effect in uniformly-ionized gas exactly cancels in the first order of the linear theory [Kaiser 1984a]. This cancellation is broken when patchiness is present, however, which enhances the anisotropies.

The mean and rms of the temperature fluctuations for all cases are summarized in Table 1. Based on these, we see that, indeed, the uniform and instant reionization cases have means very close to zero, as expected, while the realistic, patchy reionization simulations yield mean temperature fluctuations that are low, of order $\sim 10^{-7}$, but not zero. The rms values are $\sim 10^{-6}$ for all patchy reionization cases and lower than that by factor of $\sim 2$ ($\sim 3$) for the instant (uniform) reionization cases.

These observations based on the maps are confirmed by their corresponding pixel PDF distributions, shown...
Fig. 1.— kSZ maps from simulations: f2000 (top left), f250 (top right), f2000C (middle left), f250C (middle right) instant reionization at $z_{\text{instant}} = 13$ which gives the same total electron scattering optical depth as simulation f250 (bottom left), and spatially-uniform reionization with the same reionization history and thus same total electron scattering optical depth as simulation f250 (bottom right). (Images produced using the Ifrit visualization package of N. Gnedin).

in Figures 2 and 3. On each panel we also plot the Gaussian distribution with the same mean and standard deviation. All distributions are surprisingly close to Gaussian around their mean values given the maps. However noticeable departures from Gaussianity do occur in the wings of the PDFs. In particular, the PDFs for patchy reionization with sub-grid clumping (f2000C, and f250C) are significantly non-Gaussian. For the realization being considered here there is an over-abundance of bright regions by up to an order of magnitude compared to the
corresponding Gaussian distributions. The reionization scenarios without sub-grid clumping show much weaker non-Gaussian features there. This indicates that the observed PDF of kSZ from patchy reionization may give us important information on the level of small-scale gas clumpiness during reionization. The PDFs derived from the uniform and instant reionization scenarios are significantly less wide than the simulated ones (which was also shown by their lower rms values, as was discussed above), and are also much closer to Gaussian.

### 4.2. Sky power spectra

In Figure 4 we show the 2D sky power spectra derived from our simulation data. These were obtained by calculating the sky power spectra for each box light-crossing time and adding these, which yields a smoother final result than would calculating the power spectrum directly from the final map. The kSZ anisotropy signal from inhomogeneous reionization dominates the primary CMB anisotropy above $\ell = 3000$. The sky power spectra peak strongly at $\ell = 3000 - 5000$, to a maximum of $|\ell(\ell+1)C_\ell/(2\pi)|_{\text{max}} \sim 7 \times 10^{-13}$ when the ionizing sources are highly-efficient (cases f2000 and f2000C).

The peak values for the simulations with lower-efficiency sources (f250 and f250C) are only slightly lower, at $|\ell(\ell+1)C_\ell/(2\pi)|_{\text{max}} \sim 4 \times 10^{-13}$, and the peaks are somewhat broader and moved to smaller scales, $\ell \sim 3000 - 7000$, than for the high-efficiency cases. The introduction of sub-grid gas clumping in the radiative transfer simulations produces significantly more power at small scales and slightly broader peaks when compared to the similar cases without sub-grid clumping. The scales at which the power spectra peak roughly correspond to the typical ionized bubble sizes observed in our simulations, namely $\sim 5 - 20$ Mpc comoving. These typical sizes depend on the assumed source efficiencies and sub-grid clumping. At large scales the patchy kSZ signal depends only on the source efficiencies, being higher for more efficient photon emitters, since these tend to produce larger ionized regions on average.

In contrast, the uniform reionization scenario (with the mean reionization history of simulation f250) yields a kSZ signature which is much lower than the non-uniform reionization scenarios, indicating a very large boost of the signal due to the effects of patchiness. The boost is largest, approximately one order of magnitude, at and above the typical scale of the patches ($\ell < 7000$), but it still exists at smaller scales, where it is a factor of two or more. The other simplified scenario, of instant reionization with the same integrated optical depth as f250, produces larger kSZ anisotropy than a uniform reionization does. However, it is still well below the realistic patchy reionization signals, by factor of $\sim 3$ for $\ell < 7000$. At smaller scales the reionization signal from the instant reionization scenario becomes similar to the ones from

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**TABLE 1**

| simulation | f2000 | f2000C | f250 | f250C | uniform | instant |
|------------|-------|-------|------|-------|---------|---------|
| mean       | $8.86 \times 10^{-8}$ | $9.88 \times 10^{-8}$ | $-2.17 \times 10^{-7}$ | $5.57 \times 10^{-8}$ | $-3.29 \times 10^{-12}$ | $-3.37 \times 10^{-12}$ |
| rms        | $1.13 \times 10^{-6}$ | $1.22 \times 10^{-6}$ | $1.06 \times 10^{-6}$ | $1.09 \times 10^{-6}$ | $4.24 \times 10^{-7}$ | $6.67 \times 10^{-7}$ |
simulated non-uniform reionization. The distribution of
the power in these two simplified scenarios is much flatter,
with less indication of a characteristic scale. This implies
that the sharp peaks yielded by the inhomogeneous
reionization scenarios are dictated by the size of
the ionized patches, since both the density and the ve-
clocity fields are shared among our models. This point is
discussed further in §4.3.

This behavior can be understood further by consider-
ing the contributions from different redshift intervals to
the integrated signal. In Figure 4 we show the contribu-
tions from the total signal to different redshift intervals
of the IGM is already ionized, the patchiness still plays
an important role in assembling the kSZ signal.

In the homogeneous (uniform and instant) reioniza-
tion cases the shape of the power spectra is essentially the
same for all redshift intervals, quite flat, with a broad
peak at \( \ell \sim 5000 \). The signal from the uniform reio-

In linear theory the velocity and density perturbations
at redshift \( z \) are related through the continuity equa-
tion:

\[
\bar{v}(k,t) = \frac{i a(t) \dot{D}(t)}{k^2} \ddot{D}(t) \delta(k,t),
\]

where \( a = (1 + z)^{-1} \) is the scale factor, and \( D(t) \) is the
growth factor of linear perturbations, which for a flat
\( \Lambda \)CDM background cosmology is given by

\[
D(z) = \frac{5 \Omega_0 E(z)}{2} \int_z^\infty \frac{1 + z'}{[E(z')]^3} dz',
\]

with \( E(z) = H(z)/H_0 = \Omega_0 (1 + z)^3 + \Omega_\Lambda^{1/2} \), where
\( H(z) \) and \( H_0 \) are the values of the Hubble constant at
redshift \( z \) and at present, respectively. In terms of the
power spectra of the density, \( P_\delta \), and the velocity, \( P_{vv} \),
equation (4) can be re-written as

\[
P_{vv}(k,t) = \frac{a^2(t)}{k^2} \left[ \frac{\dot{D}(t)}{D(t)} \right]^2 P_\delta(k,t),
\]

or

\[
\Delta_v^2(k,t) = \frac{a^2(t)}{k^2} \left[ \frac{\dot{D}(t)}{D(t)} \right]^2 \Delta_\delta^2(k,t),
\]

where we defined

\[
\Delta_\delta^2 \equiv \frac{k^3}{2 \pi^2} P_\delta
\]

and

\[
\Delta_v^2 \equiv \frac{k^3}{2 \pi^2} P_{vv}.
\]
Substituting equation (10) into equation (7) we finally of the density field (dotted, black) and velocity field (in km s\(^{-1}\)). Shown are the signals for every three box light crossing times, roughly corresponding to (bottom to top on the left) redshifts \(z > 15\) (green), \(15 > z > 11\) (red), \(z < 11\) (magenta). In the last two cases the \(z > 15\) contribution is very low (uniform) or zero (instant) and thus not show. For comparison, on the last panel we also show the corresponding redshift-binned results from the model of Zhang et al. (2004) (see § 18).

Using equation (5) it is straightforward to show that

\[
\frac{\dot{D}}{D} = \frac{\ddot{a}}{a} - \frac{\dot{a}}{a} + \frac{5\Omega_0}{2} \left( \frac{1 + z}{E(z)} \right)^2 \frac{4\dot{E}(z)}{E(z)} D(z)
\]

\[
= -\frac{3H_0\Omega_0(1 + z)^2}{E(z)} \left[ 1 - \frac{5}{3(1 + z)D(z)} \right]. \tag{10}
\]

(\frac{\Delta^2_{\delta}(k, z)}{k^2}) = \frac{9H_0^2\Omega_0^2(1 + z)^4}{4E^2(z)} \left[ 1 - \frac{5}{3(1 + z)D(z)} \right]^2. \tag{11}

In Figure 6 we show the CMBfast density power spectrum at redshift \(z = 13.6\), and the corresponding linear-theory velocity power spectrum given by equation (11), along with the density and velocity power spectra obtained from our N-body simulation data (re-gridded to our radiative transfer grid of \(203^3\)). Comparison of the two sets of data shows a good agreement between them and indicates that at these scales and redshifts our density and velocity fields are clearly still in the linear regime. Note, however, that the bulk velocities derived from the simulation data do include the non-linear effects at smaller scales, up to the full resolution of the underlying N-body simulation, at \(3248^3\) cells, or \(\sim 30\ h^{-1}\) kpc comoving per cell, averaged at the radiative transfer grid resolution. The velocity power spectra were derived from the simulated data by first calculating the power spectrum of each of the three velocity components and then averaging these, which minimizes the variance at large scales.

The velocity power spectrum has a broad peak at fairly large scales, \(k \sim 0.01 - 0.1\ h\ \text{Mpc}^{-1}\), corresponding to scales \(\sim 60 - 600\ \text{Mpc} h^{-1}\). Thus, our simulation volume is sufficiently large to reach this velocity peak, but is still missing some velocity power from the largest scales. We estimate the missing power as follows. The rms of the velocity field is given by

\[
v_{\text{rms}}^2 = \int \Delta^2_v(k) d\ln(k). \tag{12}
\]

Thus, integrating over the full linear power spectrum yields the total power in the velocity field, \(v_{\text{rms}, \text{tot}}^2\) in the linear theory. A finite simulation box of size \(L_{\text{box}}\) would not include the modes with wave-numbers below \(k_{\text{min}} = 2\pi / L_{\text{box}}\). Integrating \(\Delta^2_v\) over the wave-numbers \(k > k_{\text{min}}\), we obtain the total velocity power for a given box size. The results are shown in Figure 7 where we plot the fraction of total linear-theory velocity power \(v_{\text{rms}, \text{box}}^2 / v_{\text{rms}, \text{tot}}^2\) present in a simulation box vs. the box size \(L_{\text{box}}\). A simulation of volume \((100\ h^{-1}\text{Mpc})^3\) would thus be missing about \(\sim 50\%\) of the total \(v_{\text{rms}}^2\) velocity...
power as given by the linear theory. Simulations with smaller volumes would miss much larger fraction of the velocity power, $\sim 70\%$ for 50 $h^{-1}\text{Mpc}$ box, $\sim 90\%$ for 20 $h^{-1}\text{Mpc}$ box, and over 99% for 4 $h^{-1}\text{Mpc}$ box.

The large-scale bulk motions not included in our simulation occur on scales of $\sim 100$ Mpc or larger, well above the characteristic size of the ionized patches ($\sim 10$ Mpc or less) (Iliev et al. 2006b, see also Figure 9), thus on such scales the reionization patchiness averages out and the ionization fraction approaches the mean for the universe. We can approximately account for the missing large-scale velocity power as follows. At each light-crossing time we can assume that the whole simulation volume is moving with some random velocity, $v_{\text{box}}$. In order to account for the missing velocity power, the value of the component of this random velocity at redshift $z$ along the line-of-sight is given by

$$v_{\text{box}}(z) = v_{\text{rms,missing}} \sqrt{-2 \ln q} \cos(2\pi\theta),$$

where $v_{\text{rms,missing}} \equiv \left[ v_{\text{rms, tot}}(z)^2 - v_{\text{rms, box}}(z) \right]^{1/2}$, and $q$ and $\theta$ are uniformly-distributed random variables between 0 and 1. The form of equation (13) ensures that $v_{\text{box}}(z)$ is a Gaussian-distributed random variable with a zero mean and rms of $v_{\text{rms,missing}}$ (Box & Muller 1958). In terms of the contribution from this redshift to the temperature anisotropy this yields

$$\left( \frac{\Delta T}{T_{\text{CMB}}} \right)_\text{tot}(z) = \left( \frac{\Delta T}{T_{\text{CMB}}} \right)_\text{box}(z) + \tau_{\text{es}}(z) \frac{v_{\text{box}}(z)}{c},$$

where $\tau_{\text{es}}(z)$ is the corresponding contribution to the total electron-scattering optical depth. Note that for uniform reionization the effects from large-scale velocity fields would exactly cancel, since the flow is potential (Kaiser 1984). However, the patchiness breaks this cancellation and the large-scale velocities increase the signal by increasing the ionized bubble velocities along the LOS. The ionized regions are at much smaller scale than these large-scale velocities and uncorrelated with them, thus avoiding the usual cancellation.

The power spectra with large-scale velocity corrections included are shown in Figure 8. For comparison, we also show three representative calculations of the post-reionization contribution from fully-ionized gas after reionization: the quadratic-order Ostriker-Vishniac effect (Vishniac 1987) expressed in terms of a product of the linear power spectra; the same expression, but with the nonlinear density power spectrum substituted for one of the linear ones, which partially accounts for the nonlinear effects; and the recent detailed nonlinear model of Zhang et al. (2004). The last three models are rescaled to our adopted cosmology using $\ell(\ell + 1)C_\ell \propto \sigma_8^2$ (Zhang et al. 2004), and assume reionization overlap at $z_{\text{ov}} = 8$, to allow for direct comparison. Confronting first the three post-reionization models, we see that they agree fairly well at large, linear scales, but strongly diverge, by up to one order of magnitude at small scales, where the nonlinearities become very important. Including the corrections due to the nonlinear density power spectrum yields modestly higher kSZ signal than the linear OV calculation, and similar power spectrum shape overall. Both peak at roughly the same scale, $\ell \sim 2000$. In contrast, the Zhang et al. (2004) result finds much larger signal, especially at small scales, which peaks at $\ell \sim 20,000$. Compared to our patchy reionization kSZ predictions, all
uniform-ionization power spectra are much less sharply peaked, i.e. they lack a well-established characteristic scale. In terms of their peak values, the OV and OV with nonlinear corrections models yield values lower than any of our patchy reionization results, while [Zhang et al. (2004)] find a similar signal around the patchy power spectra peaks. At small scales ($\ell > 10^4$) the patchy reionization kSZ signals are similar to or lower than the OV result and are much lower than the expected full nonlinear post-reionization contribution. Thus, the best range to aim for when attempting to detect the patchy reionization signal is $\ell = 3000 - 10^4$.

4.4. Characteristic scales and LOS spectra
The characteristic scale corresponding to the peak of the power spectra in Figure 8 is largely dictated by the typical scales of the ionized bubbles, since, as we discussed in §4.3, the velocity and density are in the linear regime and do not have characteristic scales within the range corresponding to our simulation volume. However, the ionized patches do have a typical scale. In Figure 9 we show $\Delta_x$, the power spectrum of the ionized fraction (which is the same as the one of the neutral fraction) for all of our simulation cases at the redshifts when the volume ionized fractions are $x_v = 0.3, 0.5$ and $0.8$. In all cases the power spectra peak around wave-numbers $k \sim 1 \text{ h Mpc}^{-1}$, which scale corresponds to $\ell \sim 7000$. As reionization progresses, the peak moves gradually to slightly larger scales, resulting in a moderate widening of the peak of the kSZ temperature anisotropy sky power spectra and slightly less well-defined typical scale than seen in the single-redshift 3D power spectra. In order to quantify the power spectra evolution better, in Figure 10 we show the evolution vs. redshift of the peak amplitude, $\Delta_{x, max}$, the scale at which it occurs, $k_{max}$, and the effective width of the peak, given by

$$\text{width} = \bar{k} \sinh \left\{ \frac{\delta (\ln k)^2}{x_{\text{rms}}^2} \right\}^{1/2}$$

(15)

where

$$\delta (\ln k)^2 = \frac{1}{x_{\text{rms}}^2} \int ((\ln k)^2 - (\ln k)^2) \Delta_x^2 d\ln k.$$ 

(16)

Here

$$\bar{k} = \exp (\ln k) = \exp \left\{ \frac{1}{x_{\text{rms}}^2} \int (\ln k) \Delta_x^2 d\ln k \right\}$$

(17)

and

$$x_{\text{rms}}^2 = \int \Delta_x^2 d\ln (k),$$

(18)
which gives the total integrated power per logarithmic interval. We also show $k$ (in $h^{-1}$Mpc) and $x_{\text{rms}}^2$. The amplitude of the peak initially rises, peaking at the point of maximum patchiness, $x_m \approx 0.5$, and slightly decreases thereafter. The scale of the peak, $k_{\text{max}}$, (in $h^{-1}$Mpc) steadily increases in time, as the typical ionized regions grow. Approximate linear fits (in the form $a + bz$) to $k_{\text{max}}(z)$ are given by $a = -0.528 (-0.329, -1.135, -0.847)$ and $b = 0.085 (0.089, 0.17, 0.19)$ for simulation f2000 (f2000C, f250, f250C). The effective width of the power spectra and $\bar{k}$ are largely constant and roughly equal to each other, except for simulation f250C at early times. Finally, the rms initially strongly rises, peaking at slightly later times than the patchiness itself peaks (at $x_m \approx 0.6$).

During the later stages of reionization ($x_m > 0.3$), which contribute most of the kSZ signal the ionized density fluctuations at large scales are proportional to the density fluctuations with a bias coefficient $b(z)$ [Iliev et al. 2006b; see also Figure 11]. Note however that...
this is not the case at earlier times, when the ionized regions are relatively small and isolated and do not follow the large-scale density perturbations.). Based on this fact we can derive a handy approximate expression for the power spectrum of the ionized gas density, $\Delta_{\delta, H II}$, as follows. Let

$$\Delta_{\delta, H II}(k, z) = b(z)W(k, z)\Delta_{\delta}(k, z),$$

(19)

where $W(k, z)$ is some window function asymptoting to 1 at large scales. We recognize that because of the zero or one nature of reionization (i.e. a region is either ionized or neutral when ionization fronts are unresolved, as is the case here), the random field $\delta n_e/\bar{n}_e$ is more complex than this relation between the power spectra indicates. However, it is reasonable expect that for low $k$, the electron density structure will have such a proportionality, with the window function being related to a quadratic superposition of form factors for the various H II patches. If the reionization contribution from dense regions is lessened because of negative feedback from earlier ionizers on nearby galaxies, a situation we have ignored, the relation could be considerably modified. In spite of such complications, a simple relationship characterized by a characteristic scale related to the average size of the H II regions turns out to prevail as we now show.

The bias $b(z)$ is readily calculated from the simulation data (at the largest scale available from our simulation volume) and is shown in Figure 11 (values shown on the right axis). The bias starts close to zero since very little of the gas is ionized, and has a peak value of 2-3, reached again around the time of maximum patchiness. For redshifts around the time of maximum patchiness (for mass-weighted ionized fraction $0.3 < x_m < 0.8$) and at large scales ($k \geq 1 - 2$) the window function $W(k, z)$ is reasonably well-fit (within factor of ~ 2, and generally much better) by the following functional form:

$$W(k, z) = \frac{1}{1 + k^2/k_0^2(z)}$$

(20)

where $k_0(z)$ is a characteristic scale parameter which gives the best fit at the turnover. Samples of this window function, $b(z)W(k, z)$, for simulation f250 are shown in Figure 11. At small scales this function turns over and the above fit is not applicable. The values of $k_0(z)/2$ at $x_m = 0.3, 0.5$ and 0.8 (in h$^{-1}$Mpc) are shown in Figure 11 (squares). They are very close to the scale at which the ionized fraction power spectra peak, $k_{\text{max}}$.

The addition of the missing large-scale velocity power outlined in the previous section boosts the sky power spectra at all scales, but especially at the largest ones, which further widens the peak. Nonetheless, the characteristic ionized bubble scale remains imprinted on the final results through their peak at $\ell \sim 2000 - 9000$.

To gain some further insight in the complex nature of the derived kSZ signal, in Figures 12 and 13 we show some sample line-of-sight (LOS) cuts through the simulation volume (simulation f250) of the density field in units of the mean, $n_H/\bar{n}_H$, ionized fraction, $x_H$, electron number density in units of the mean gas density, $n_e/\bar{n}_H = x_H n_H/\bar{n}_H$, and the velocity component along that LOS, $v_x$, and the accumulation of the kSZ temperature anisotropy integral (from left to right), $\Delta T_{\text{ksz}}(< x)$. Early in the evolution ($z = 13.62$; Figure 12), the H II regions tend to be associated with high-density peaks, where the first halos (ionizing sources) form (inside-out reionization scenario). The ionized fraction distribution acts as a filter, picking the density field portions that would contribute to the CMB temperature anisotropies. The ionized regions are correlated with the high-density peaks (inside-out reionization), but the correlation is not perfect, there are density peaks which still remain neutral, while at the same time there are ionized regions which have a low overdensity, or are even underdense. The velocity field fluctuations are dominated by the largest scales and are generally not closely correlated with the density field or the ionized fraction. The velocity field determines the sign of the contribution of a given ionized region to the total temperature anisotropy. A number of different situations can be observed in our data. For example, in Figure 12 (left) the ionized regions are relatively closely spaced together, separated by

Fig. 11.— Ratio of the power spectra of the ionized density and the total density, $\Delta_{\delta, H II}/\Delta_{\delta}$ for simulation f250 at volume ionized fractions $x_v = 0.3$ (left), $x_v = 0.5$ (middle), and $x_v = 0.8$ (right). The corresponding redshifts are indicated on each panel.
Fig. 12.— LOS cuts through the simulation volume (simulation f250) at $z = 13.622$ at which time the volume-weighted ionized fraction is $x_v = 0.095$, and the mass-weighted one is $x_m = 0.12$.

Fig. 13.— LOS cuts through the simulation volume (simulation f250) at $z = 11.310$ ($x_v = 0.66$, $x_m = 0.70$).

\[ \sim 10 \text{ Mpc}, \] falling within the same large-scale velocity fluctuation. All H II regions except one are moving in the positive direction, resulting in a relatively large integrated temperature anisotropy of $\sim 4.4 \mu K$. The other sample case from the same redshift (Figure 12 right) includes three ionized regions along the LOS. These are separated by 40-50 comoving Mpc, and thus correspond to completely uncorrelated density fluctuations. The velocity field does not appear to correlate them, either, but nonetheless the temperature anisotropies they produce happen to almost exactly cancel each other, resulting in $\Delta T_{\text{ksz}} \approx 0$ along that LOS. At later times (Figure 13) most of the volume is already ionized and each LOS encounters a number of ionized regions, but ultimately almost all of the total signal is contributed by the two highest-density peaks along the LOS, both of which here happen to move in the negative direction, while the contributions of the multiple other H II regions largely cancel each other. Once again, the density and velocity fields are not correlated. The ionized fraction is less correlated with the high-density regions than at early times. These examples demonstrate some of the complexities one has to deal with when trying to derive the kSZ signal from patchy reionization analytically, underscoring the need for large-scale simulations of this effect.

5. COMPARISONS TO PREVIOUS WORK

Previous simulations of the kSZ effect from patchy reionization (Gnedin & Jaffe 2001; Salvaterra et al. 2005) predicted significantly lower kSZ signals than the ones we find. Their power spectra reach maximum values of $[\langle \ell (\ell +1) C_\ell / (2 \pi) \rangle_{\text{max}} \sim 2 \times 10^{-14}$ and $\sim 1.6 \times 10^{-13}$, respectively, compared to $[\langle \ell (\ell +1) C_\ell / (2 \pi) \rangle_{\text{max}} \sim 1 \times 10^{-12}$ for our simulations. This discrepancy is due to the small volumes used in these simulations. As we showed in § 4.3 considering small volumes significantly reduces the power in the velocity field fluctuations. At the scale of the simulation box the bulk velocities are zero by definition, and the density is the mean one for the universe, thus any larger-scale fluctuations are not included, and the ones close to the box size are underestimated. The reionization patchiness imprints its characteristic scale on the density and velocity fluctuations, effectively smoothing the small-scale fluctuations below the typical
bubble sizes. On the other hand, the large-scale fluctuations still should contribute to the kSZ signal, since the H II regions are moving with the large scale bulk motions.

In addition to the missing large-scale power, these earlier reionization simulations also did not follow sufficient volumes to properly sample the size distribution of the ionized bubbles. The typical sizes of these ionized patches are of order 5-20 comoving Mpc, and become even larger at later times. These large characteristic bubble scales are result of the strong clustering of ionizing sources at high redshifts. Capturing the full bubble size distribution requires simulation volumes of order $(100 \, h^{-1}\text{Mpc})^3$ \cite{Iliev+06}, and as a result the kSZ power spectra found by the smaller-box reionization simulations were largely flat, with no clear characteristic scale.

Several semi-analytical models for calculating the kSZ signature of patchy reionization have been proposed in recent years \cite{Gruzinov+98, Santos+03, McQuinn+05}. Gruzinov & Hu (1998) proposed a very simple model, whereby the ionized patches are randomly distributed and have a given characteristic size $R$. Furthermore, they assumed that the density, velocity and ionization fraction fields are all uncorrelated with each other. The ionized fraction auto-correlation function is approximated as a Gaussian with rms given by \cite{Iliev+06}. Under these assumptions the kSZ power spectrum can be calculated analytically. The quantitative details of the signal predicted by this model depend on the assumptions made about the typical ionized patch size, and the time and duration of reionization, but generically it predicts a distribution which is very strongly peaked, more so than our simulations find. This is related to the assumed Gaussian distribution around a fixed characteristic size of the ionized patches. The actual simulations also find a characteristic size for the ionized bubbles, but one that is also evolving with redshift, and different size distributions around it, which yields somewhat less sharp peaks than this analytical model predicts.

\cite{Santos+03} proposed another simple model for the reionization patchiness, which assumes that the correlation function of the ionized patches is proportional to the one for the density field, with a bias factor which is time-dependent, but not scale-dependent. The power is filtered at small scales, below the typical size of the ionized bubbles using a Gaussian filter in k-space. The time-dependence of the typical size of the ionized patches is assumed to be proportional to $[1-x_m(t)]^{-1/3}$, where $x_m(t)$ is the mean mass-weighted ionized fraction at time $t$. The last function is based on the semi-analytical models of reionization of \cite{Haiman+03}. The resulting kSZ power spectra are fairly flat, with much less well-defined characteristic scale than our simulation results, as should be expected based on the quickly-evolving typical patch size assumed in this model. The peak amplitudes of the results is somewhat higher than the ones we find, by factors of $\sim 2-3$, probably due to their assumed values of the effective bias.

More recently, \cite{McQuinn+07} applied the semi-analytical reionization model of \cite{Furlanetto+04} to estimate the kSZ signal. They found lower fluctuations in their “single reionization episode” models (by factor of $\sim 3$), although these models have similar mean reionization histories (i.e. evolution of the mean volume-weighted ionized fraction) to our simulations. Their results also predict a peak at somewhat larger scales ($\ell \sim 2000$) than ours ($\ell \sim 2000-9000$). Their “extended reionization” scenarios peak to similar values to the ones we find ($\ell (\ell+1)C_{\ell}/(2\pi)_{\text{max}} \sim 10^{-12}$) again at $\ell \sim 2000$, but have a very different assumed reionization histories than any of our simulations. They found that the kSZ post-reionization contribution dominates the patchy signal at the scales of interest, while we find that the two signals are comparable in magnitude.

6. SUMMARY

We have derived the kSZ CMB anisotropies due to the inhomogeneous reionization of the universe. This is the first such calculation based on detailed, large-scale radiative transfer simulations of this epoch. As we have shown above, these simulations follow large enough volume to capture the full range of scales relevant to the large-scale reionization geometry. They also include most of the velocity fluctuations power, which is not the case for smaller-box simulations. We have also approximately corrected for the velocity power still missing from the box. The resulting sky power spectra peak at $\ell = 2000-8000$ with maximum values of $\ell (\ell+1)C_{\ell}/(2\pi)_{\text{max}} \sim 1 \times 10^{-12}$. The angular scale of the peak roughly corresponds to the typical ionized bubble sizes observed in our simulations, which is $\sim 5-20$ Mpc, depending on the assumed source efficiencies and the gas clumping at small scales. The kSZ anisotropy signal from reionization dominates the primary CMB signal above $\ell = 3000$. At large scales the patchy kSZ signal depends largely on the source efficiencies and the large-scale velocity fields. It is higher when sources are more efficient at producing ionizing photons, since such sources produce large ionized regions, on average, than less efficient sources. The introduction of sub-grid gas clumping in the radiative transfer simulations produce significantly more power at small scales, but has little effect at large scales. The sub-grid gas clumping also significantly enhances the non-Gaussianity of the kSZ maps, resulting in up to one order of magnitude more of the brightest kSZ regions than a Gaussian would predict. The integrated kSZ signal is strong enough to be detected by upcoming experiments, like ACT and SPT. However, the separation of the patchy reionization signal from the contribution by the fully-ionized gas after reionization seems difficult.

Our current simulations do not include the smallest atomically-cooling ionizing sources, these with total mass $10^9 \, M_\odot \lesssim M_{\text{tot}} < 2.5 \times 10^9 \, M_\odot$, as these are not resolved in our base N-body simulations. These smaller sources have important effects on the early global reionization history, but are also strongly suppressed due to Jeans-mass filtering in the ionized regions \cite{Iliev+06}. During most of the evolution the reionization process is thus dominated by the larger sources, which are resolved here, and which dictate the large-scale reionization geometry. Consequently, we do not expect that the presence of the low-mass ionizing sources would change our current conclusions significantly.
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REFERENCES

Box, G. E. P. & Muller, M. E. 1958, Ann. Math. Stat., 29, 610
Bunker, A., Stanway, E., Ellis, R., McMahon, R., Eyles, L., & Lacy, M. 2006, New Astronomy Review, 50, 94
Ciardi, B. & Madau, P. 2003, ApJ, 596, 1
Ciardi, B., Scannapieco, E., Stoehr, F., Ferrara, A., Iliev, I. T., & Shapiro, P. R. 2006, MNRAS, 366, 689
Furlanetto, S., Oh, S. P., & Briggs, F. 2006, ArXiv Astrophysics e-prints (astro-ph/0608032)
Furlanetto, S. R., Sokasian, A., & Hernquist, L. 2004a, MNRAS, 347, 187
Furlanetto, S. R., Zaldarriaga, M., & Hernquist, L. 2004b, ApJ, 613, 1
Gnedin, N. Y. & Jaffe, A. H. 2001, ApJ, 551, 3
Gnedin, N. Y. & Prada, F. 2004, ApJL, 608, L77
Gruzinov, A. & Hu, W. 1998, ApJ, 508, 435
Haiman, Z. & Holder, G. P. 2003, ApJ, 595, 1
Hu, W. 2000, ApJ, 529, 12
Iliev, I. T., et al. 2006a, MNRAS, 371, 1057
Iliev, I. T., Mellema, G., Pen, U.-L., Merz, H., Shapiro, P. R., & Alvarez, M. A. 2006b, MNRAS, 369, 1625
Iliev, I. T., Mellema, G., Shapiro, P. R., & Pen, U. L. 2006c, MNRAS, submitted (astro-ph/0607517)
Iliev, I. T., Pen, U.-L., Bond, J. R., Mellema, G., & Shapiro, P. R. 2006d, New Astronomy Reviews, in press (astro-ph/0607209)
Iliev, I. T., Scannapieco, E., Martel, H., & Shapiro, P. R. 2003, MNRAS, 341, 81
Iliev, I. T., Scannapieco, E., & Shapiro, P. R. 2005a, ApJ, 624, 491
Iliev, I. T., Shapiro, P. R., Ferrara, A., & Martel, H. 2002, ApJL, 572, L123
Iliev, I. T., Shapiro, P. R., & Raga, A. C. 2005b, MNRAS, 361, 405
Jaffe, A. H. & Kámionkowskí, M. 1998, Phys. Rev. D, 58, 043001
Kaiser, N. 1984, ApJ, 282, 374
Ma, C.-P. & Fry, J. N. 2002, Physical Review Letters, 88, 211301
Malhotra, S. & Rhoads, J. 2006, ApJ, 647, 95L
McQuinn, M., Furlanetto, S. R., Hernquist, L., Zahn, O., & Zaldarriaga, M. 2005, ApJ, 630, 643
Mellema, G., Iliev, I. T., Alvarez, M. A., & Shapiro, P. R. 2006a, New Astronomy, 11, 374
Mellema, G., Iliev, I. T., Pen, U. L., & Shapiro, P. R. 2006b, MNRAS, in press (astro-ph/0603518)
Merz, H., Pen, U.-L., & Trac, H. 2005, New Astronomy, 10, 393
Ostriker, J. P. & Vishniac, E. T. 1986, ApJL, 306, L51
Rhoads, J. E., et al. 2003, AJ, 125, 1006
Salvaterra, R., Ciardi, B., Ferrara, A., & Baccigalupi, C. 2005, MNRAS, 360, 1063
Santos, M. G., Cooray, A., Haiman, Z., Knox, L., & Ma, C.-P. 2003, ApJ, 598, 756
Scott, D. & Rees, M. J. 1990, MNRAS, 247, 510
Seljak, U. & Zaldarriaga, M. 1996, ApJ, 469, 437
Shapiro, P. R., Ahn, K., Alvarez, M. A., Iliev, I. T., Martel, H., & Ryu, D. 2006, ApJ, 646, 681
Shapiro, P. R., Iliev, I. T., & Raga, A. C. 2004, MNRAS, 348, 753
Spergel, D. N. et al. 2003, ApJS, 148, 175
Springel, V., White, M., & Hernquist, L. 2001, ApJ, 549, 681
Stanway, E. R. et al. 2004, ApJL, 604, L13
Sunyaev, R. A. & Zeldovich, I. B. 1980, MNRAS, 190, 413
Tozzi, P., Madau, P., Meiksin, A., & Rees, M. J. 2000, ApJ, 528, 597
Vishniac, E. T. 1987, ApJ, 322, 597
Zahn, O., Zaldarriaga, M., Hernquist, L., & McQuinn, M. 2005, ApJ, 630, 657
Zeldovich, Y. B. & Sunyaev, R. A. 1969, Ap&SS, 4, 301
Zhang, P., Pen, U.-L., & Trac, H. 2004, MNRAS, 347, 1224