Linear magnetosonic waves in solar wind flow tubes

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Abstract: Nakariakov et al. (1996) investigated the linear magnetosonic waves trapped within solar wind flow tubes, where they accounted for a slab having boundaries at \( x = \pm d \) and extended up to infinity in the \( y \) and \( z \) directions. Srivastava and Dwivedi (2006) claimed to extend that work by considering a two-dimensional slab. We find that the work of Srivastava and Dwivedi (2006) is not for a two-dimensional slab and has a number of discrepancies. Further, their results for body waves are not reliable.

Keywords: Solar wind - Magnetosonic waves

1 Introduction

HELIOS spacecraft observations (Thieme et al., 1990) supported Parker’s assumption (1963) that the solar wind could be fine-structured in the form of flow tubes. In these flow tubes, the magnetosonic waves may be excited assuming the Alfvén speed to be less inside the tube than that outside. Nakariakov et al. (1996) (hereinafter referred to as NRM) investigated one-dimensional problem by considering a slab having boundaries at \( x = \pm d \) and extended up to infinity in the \( y \) and \( z \) directions. Srivastava and Dwivedi (2006) (hereinafter referred to as SD) claimed to extend the work of NRM by accounting for a two-dimensional slab with a symmetric expansion (\( \delta \)) in the edges of the slab. They obtained expressions for surface as well as body waves.

NRM considered a slab having boundaries at \( x = \pm d \) and extended up to infinity in the \( y \) and \( z \) directions. For one-dimensional case, the linearized equations of ideal MHD are (NRM)

\[
\frac{d^2 V_{xi}}{dx^2} - m_i^2 V_{xi} = 0 \tag{1}
\]

where \( i \) is either \( o \) (for inside) or \( e \) (for outside) the slab. The transversal plasma velocity is \( V_{xi} \) exp \( i(\omega t - kx) \) and

\[
m_i^2 = \left[ \frac{a_{Ai}^2 - (a - M_i)^2}{a_{T_i}^2} \right] \left[ \frac{a_{Si}^2 - (a - M_i)^2}{a_{S_i}^2} \right] k^2 \tag{2}
\]

where

\[
a = \frac{\omega}{k C_{Ao}} \quad a_{Ai}^2 = \frac{C_{Ai}^2}{C_{Ao}^2} \quad a_{Si}^2 = \frac{C_{Si}^2}{C_{Ao}^2}
\]
Here, all the variables are normalized with $C_{Ao}$. \( M_i \) is Alfvén Mach number and \( a \) the phase speed in the units of Alfvén speed $C_{Ao}$. So, NRM accounted for one-dimensional problem where the thickness 2d of the slab along the $x$-direction is not changing. When the speed of sound is much larger than all other velocities (the case of incompressible plasma), the dispersion relation is

$$\frac{\rho e [a_{Ac}^2 - (a - M_e)^2]}{\rho_o [1 - (a - M_o)^2]^2} = \begin{cases} -\tanh(kd) & \text{kink surface waves} \\ -\coth(kd) & \text{sausage surface waves} \end{cases}$$

and there are only surface waves.

SD accounted for a two-dimensional slab having boundaries at $x = \pm d$, $y = \pm d$ at the base and at $x = \pm (d + \delta)$, $y = \pm (d + \delta)$ at the top. Later on they converted the expression $\pm (d + \delta)$ into $\pm d \pm \delta$ without any reason. The later expression carries some other values in addition to the previous ones and those values are irrelevant. For the symmetric expansion in the edges of the slab (\( \delta \)), the conservation of magnetic flux gives

$$B_{0z} (2d)^2 = B_z (2d + 2\delta)^2$$

where $B_z$ and $B_{0z}$ are the magnetic field strengths at the top and the base of the slab, respectively. Thus, we have

$$\delta = \left( \frac{B_{0z} - B_z}{B_z} \right) \frac{d}{2}$$

This expression differs from equation (2) of SD and is derived for the situation that $\delta << d$. Equation (2) of SD is not even dimensionally correct. It is further interesting to find a plus-minus sign in equation (2) of SD, as the sign of $\delta$ is decided by the relative values of $B_z$ and $B_{0z}$. Hence, $\delta > 0$, when we have $B_{0z} > B_z$ and for $\delta < 0$, we have $B_{0z} < B_z$. We have taken $B_{0z} > B_z$, so that $\delta$ is positive. Though SD claimed a two-dimensional treatment of the problem, but they also used the MHD equation (1) which is for a one-dimensional case only. Further, SD considered the boundary conditions

$$\frac{V_{xo}(x = \pm d)}{\omega - kU_o} = \frac{V_{xe}(x = \pm d)}{\omega - kU_e} \quad (4)$$

$$pT_o(x = \pm d) = pT_e(x = \pm d) \quad (5)$$

$$pT_i = \frac{iC_{Ao}\rho_i a_{fi}^2 \left[ a_{Ti}^2 - (a - M_i)^2 \right]}{k(a - M_i) \left[ a_{Si}^2 - (a - M_i)^2 \right]} \quad (6)$$

where $\rho_i$ is the gas density, which have been used by NRM for one-dimensional case. SD did not mention anything about the $y$ coordinate. It categorically shows that except giving a figure and conservation of magnetic flux, SD did not do anything with the two-dimensional case. Their treatment appears as one-dimensional case.
Let us now look into the equations of SD. The equations (4), (5) and (6) give boundary conditions at \( x = \pm d \) and nothing is said even about the top at \( x = \pm (d + \delta) \). Let us assume similar boundary conditions at the top also.

\[
\frac{V_{xo}[x = \pm (d + \delta)]}{\omega - kU_o} = \frac{V_{xe}[x = \pm (d + \delta)]}{\omega - kU_e}
\]

(7)

\[
pT_o[x = \pm (d + \delta)] = pT_e[x = \pm (d + \delta)]
\]

(8)

For the solutions outside the slab, equation (7) of SD should be as the following.

\[
V_{xe}(x) = \begin{cases} 
A_1 \exp[-m_e\{x - (d + \delta)\}] & x > (d + \delta) \\
A_2 \exp[+m_e\{x + (d + \delta)\}] & x < -(d + \delta)
\end{cases}
\]

(9)

which correspond to the top of the slab. Here, \( A_1 \) and \( A_2 \) are constants. For the solutions inside the slab, equation (8) of SD should be as the following.

\[
V_{xo}(x) = \begin{cases} 
A \sinh(m_o x) & \text{for sausage surface modes} \\
A \cosh(m_o x) & \text{for kink surface modes} \\
A \sin(n_o x) & \text{for sausage body modes} \\
A \cos(n_o x) & \text{for kink body modes}
\end{cases}
\]

(10)

where \( n_o^2 = -m_o^2 \) and \( A \) is a constant. These expressions are the same at the base as well as at the top of the slab. On applying boundary conditions (7) and (8) along with (6), we get for surface waves as

\[
\frac{\rho_e m_o [a_{Ae}^2 - (a - M_o)^2]}{\rho_o m_e [1 - (a - M_o)^2]} = \begin{cases} 
-\tanh[m_o(d + \delta)] & \text{kink surface waves} \\
-\coth[m_o(d + \delta)] & \text{sausage surface waves}
\end{cases}
\]

(11)

The upper case corresponds to the kink waves whereas the lower to the sausage waves. For the body waves, we have

\[
\frac{\rho_e n_o [a_{Ae}^2 - (a - M_e)^2]}{\rho_o m_e [1 - (a - M_e)^2]} = \begin{cases} 
-\tan[n_o(d + \delta)] & \text{kink body waves} \\
\cot[n_o(d + \delta)] & \text{sausage body waves}
\end{cases}
\]

(12)

Here, also the upper case corresponds to the kink waves whereas the lower to the sausage waves.

2 Dispersion relations

Equation (2) shows that \( m_i \) tends to \( k \) in two situations: (i) when \( a = M_o = M_e \), (ii) when the speed of sound is much larger than all other velocities (the case of incompressible plasma).

(i) For \( a = M_o = M_e \), the steady shear flows are equal inside as well as outside the slab. Under such situation, equation. (11) reduces to

\[
\frac{\rho_e C_{Ae}^2}{\rho_o C_{Ao}^2} = \begin{cases} 
-\tanh\{k(d + \delta)\} & \text{kink surface waves} \\
-\coth\{k(d + \delta)\} & \text{sausage surface waves}
\end{cases}
\]

giving no dispersion relation which relates \( a \) and \( k \).
(ii) When the speed of sound is much larger than all other velocities (the case of incompressible plasma), the dispersion relation is

\[
\frac{\rho_e}{\rho_o} \left[ a_{2e}^2 - (a - M_e)^2 \right] = \left\{ \begin{array}{ll}
-\tanh\{k(d + \delta)\} & \text{kink surface waves} \\
-\coth\{k(d + \delta)\} & \text{sausage surface waves}
\end{array} \right. \quad (13)
\]

For \( \delta = 0 \), these expressions are same as those obtained by Nakariakov et al. (1996) for one-dimensional case. Moreover, there are only surface waves and the body waves do not exist.

We could not see any way to replace \( n_o \) by \( k \) in equation (12). Aforesaid expressions show that \( n_o \) can be replaced by \( ik \), leading to non-existence of body waves. But SD have replaced \( n_o \) by \( k \) in their equations (11) and (12) and obtained the expressions

\[
\frac{\rho_o}{\rho_e} \left[ a_{2e}^2 - a^2 \right] = \left\{ \begin{array}{ll}
-\tan\{k(d + \delta)\} & \text{kink body waves} \\
\cot\{k(d + \delta)\} & \text{sausage body waves}
\end{array} \right. \quad (14)
\]

where they taken \( M_e = 0 \) and \( M_o = M \). This expression is not correct as \( n_o \) cannot be replaced by \( k \).

3 Conclusions

The above discussion categorically shows that all the equations of SD are objectionable and full of discrepancies. In particular, their expressions for When the equations used in the calculations are not correct, the results obtained from them cannot be reliable.

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