Critical runoff depth estimation for incipient motion of non-cohesive sediment on loose soil slope under heavy rainfall

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ABSTRACT
Incipient motion of non-cohesive sediment, as a branch of water erosion, has been studied for decades, and criteria proposed in predicting incipient motion mainly focus on dimensionless shear stress and shear velocity. The two parameters could provide runoff erosive power, however, are invisible and not easy-measured in experiment. In this paper, critical runoff depth was recognized as another parameter, which is visible, measurable, and closely connected with shear velocity. To get it, a mathematical model coupling surface flow and subsurface flow was established, and they are governed by Navier–Stokes equation and Brinkman–extended Darcy equation, respectively. Velocity derived from the model could be indirectly applied in particle force analysis and finally a new criterion about critical runoff depth was obtained. Accuracy of the criterion was verified by comparison of experimental and theoretical values, e.g. theoretical critical runoff depth ranges from 2.71 \textasciitilde 1.14 mm with a slope angle range of 20 \textdegree \textasciitilde 28 \textdegree and mean particle diameter of 2.5 mm while the corresponding experiment values ranges from 3 \textasciitilde 1 mm and the absolute errors are less than 0.3 mm. Compared with Yang’s formula, this criterion could also be applied in predicting incipient motion of non-cohesive sediment for its significant advantages of high accuracy and concise formulation.

1. Introduction
Soil erosion, as an extremely serious global environment problem, has been of great concern worldwide (Pimentel 2006, Trimble and Crosson 2000). For the triggering mechanisms, it is generally recognized that soil erosion is mainly influenced by exogenic forces, such as wind, freezing-thaw, overland flow and rainfall (Abdulkadir et al. 2019, Geng et al. 2017). Meanwhile, it could also be influenced by materials,
such as organic matter (Liu et al. 2018, Liu et al. 2017). Soil erosion induced by overland flow and rainfall, known as water erosion, is widespread in the world, especially in hilly region. During water erosion process, transport and denudation of soil or other slope sediment would occur under hydraulic conditions, and they could provide provenance for geological hazards, such as landslides and debris flows, seriously threatening local people’s life and property. Therefore, it is necessary to focus on slope failure mechanism from the perspective of water erosion. In nature, soil and other sediment could be classified into cohesive sediment and non-cohesive sediment. Compared with cohesive sediment, non-cohesive sediment would be more easily driven away by water flow as there is no bond between particles.

As water erosion is progressive, some of sediment particles on the surface would be washed away, finally causing integral failure of slope. Thus, as a study branch of water erosion, studies on incipient motion of sediment play a vital role in estimating slope failures. Wiberg and Smith (1987) established a criterion for incipient motion of sediment by analyzing the stability of single soil particle in critical state, and proposed that critical shear velocity or dimensionless shear stress could be regarded as criteria for incipient motion of sediment (Beheshti and Ataie-Ashtiani 2008, Hossein et al. 2016, Wiberg and Smith 1987). Shields (1936) firstly presented an empirical formula for incipient motion criterion on basis of a large number of experiment data, given by Eq. (1), in which the relationship between dimensionless shear stress and shear velocity was constructed.

$$
\Theta_c = \frac{\tau_c}{(\gamma_s - \gamma_w)d_m} = f(\text{Re}^*)
$$

where, $\Theta_c$ is dimensionless shear stress (dimensionless); $\tau_c$ is shear stress (ML$^{-1}$T$^{-2}$); $\gamma_s$ and $\gamma_w$ are unit weight of soil and water, respectively (ML$^{-2}$T$^{-2}$); $d_m$ is mean grain size (L); $\text{Re}^*$ is particle Reynolds number (dimensionless), defined as $\text{Re}^* = \frac{u_f d_m}{v}$, $u_f$ is shear velocity (LT$^{-1}$); $v$ is kinematical viscosity of water (L$^2$T$^{-1}$).

Detailed studies reported that various criteria for incipient motion of sediment are mainly based on dimensionless shear stress (Beheshti and Ataie-Ashtiani 2008, Cao et al. 2006, Cheng 2004) and critical shear velocity (Bong et al. 2016, Mao et al. 2011, Zhao et al. 2013). Various values of dimensionless shear stress were proposed in different literatures, e.g. Chu (1993), Kociuba and Janicki (2014), Komar (1987), Mcnamara and Borden (2004), Milan (2013) and Miller et al. (1977). However, the criteria for incipient motion of non-cohesive sediment are not already unified with different formulations. Furthermore, shear velocity is another criterion frequently employed to predict incipient motion of sediment. Bong et al. (2016) conducted lots of experiments, and analyzed the effect of deposition thickness on incipient motion of sediment according to the criterion for velocity. May (2003) and Zounemat-Kermani et al. (2018) reported different kinds of formulas on basis of shear velocity, describing the incipient motion of sediment under different conditions. Abrahams et al. (1988) and Guy et al. (2009) pointed out that criteria of Shields are not suitable for the shallow water flow on slopes, especially under rainfall conditions.

The parameters of the existing criteria could not be directly observed, e.g. dimensionless shear stress and shear velocity. Although dimensionless shear stress could not
be directly measured, it could be easily calculated by shear velocity according to Eq. (1). Thus, shear velocity is always measured in experiment for evaluation, but there are still some disadvantages of the measurement. On one hand, measurement of flow velocity required complex equipment, and this requirement could not be easily met in the practical engineering application. On the other hand, mean velocity of cross-section could be obtained under rainfall conditions by shallow water equations, but not shear velocity. By comparison, the advantages of critical runoff depth are listed as follows: Firstly, in laboratory tests, runoff depth could be measured just by rulers and recorded by high-speed camera; Besides, the runoff depth could be predicted and measured under different rainfall intensity in the wild. The present study aimed to (a) obtain expression of shear velocity in the model coupling surface flow and subsurface flow, (b) propose a new criterion, of which critical runoff depth is chosen as an estimating parameter, to overcome the disadvantages of measuring shear velocity, and (c) certify the new criterion’s accuracy by experiments and ex-reported criterion.

2. Methods and experiments

In this section, a new criterion for incipient motion of non-cohesive sediment was proposed, of which estimating parameter is critical runoff depth. To get it, a non-linear mathematical model coupling surface flow and subsurface flow in a loose slope was established. In this model, the motion of runoff is governed by the Navier-Stokes equation, and seepage flow in loose soil is subjected to the Brinkman-extended Darcy equation. Through analysis of the results, the flow velocity of fluid in runoff and soil were derived. While the stability of non-cohesive sediment is analyzed, it could be found that lift force and drag force are both associated with shear velocity and runoff depth. Then an explicit expression between shear velocity and runoff depth could be derived, as well as the new criterion of critical runoff depth. Finally, simulation tests were conducted by the authors for flow in the modelled geometry, using a specially designed device in which gravel was used to simulate the loose soil.

2.1. Model establishment

2.1.1. Coupling model of surface flow and subsurface flow

Soil could be considered as a special rock, and transformed from rock while suffering from serious weathering. Loose soil particles pile up on the surface of slope, forming loose soil slope. Loose soil slopes are widespread in mountain regions. Soil particles on these loose slopes are always driven away under heavy rainfall, especially non-cohesive sediment (such as sand and gravel). To investigate the scouring effect of surface runoff, a seepage model considering surface runoff was established (see Figure 1). In the model, \( \theta \) is the slope angle; \( L, b, n \) and \( K \) are the length, thickness, porosity, and permeability of soils, respectively; \( h \) is the runoff depth. The coordinate system was set up, shown in Figure 1.

To simplify the model, several assumptions are proposed as follows,

a. Bedrock under loose slope is impermeable, and thickness of loose slope is consistent in \( x \) direction.
b. Surface flow and subsurface flow are steady. Thus, there is no time term and \( \frac{\partial u_x}{\partial t} = \frac{\partial v_x}{\partial t} = 0 \).

c. Flow motion is regarded to be one-dimensional, namely, there is no velocity in \( y \) direction.

d. Slope angle is less than friction angle of sediment to ensure that loose slopes are in stable state without disturbance.

e. Factually, spatial distribution of runoff depth obtained by the shallow water equations is non-uniform. Herein, runoff depth is supposed to be uniform, and the raindrop effect is ignored.

f. Surface flow motion is laminar and could be described by the Navier–Stokes equation, and its expression in \( x \) direction is expressed as

\[
\dot{f}_x - \frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \left( \frac{\partial^2 u_x}{\partial x^2} + \frac{\partial^2 u_x}{\partial y^2} \right) = u_x \frac{\partial u_x}{\partial x} + v_y \frac{\partial u_x}{\partial y} \tag{2}
\]

where, \( u_x \) and \( u_y \) are velocities of surface flow in \( x \) and \( y \) direction respectively (LT\(^{-1}\)), \( P \) is intensity of pressure in \( x \) direction (ML\(^{-1}\)T\(^{-2}\)), \( \rho \) is density of water (ML\(^{-3}\)), \( \dot{f}_x \) represents mass force in \( x \) direction (MLT\(^{-2}\)) and \( \dot{f}_x = g \sin \theta \), \( \theta \) is slope angle (dimensionless) and \( g \) is gravity acceleration (LT\(^{-2}\)).

g. Subsurface flow in loose slopes is described by the Brinkman–extended Darcy equation and its expression in \( x \) direction could be expressed as (Nichele and Teixeira 2015, Sangita and Sharma 2013)

\[
np f_x - n^2 \frac{\partial P}{\partial x} + \eta \left( \frac{\partial^2 v_x}{\partial x^2} + \frac{\partial^2 v_x}{\partial y^2} \right) = \frac{\rho}{n} \left( v_x \frac{\partial v_x}{\partial x} + v_y \frac{\partial v_x}{\partial y} \right) \tag{3}
\]
where, \( v_x \) and \( v_y \) are velocities of subsurface flow in \( x \) and \( y \) direction respectively (LT\(^{-1}\)); \( n \) is porosity of loose slope (dimensionless); \( K \) is permeability of loose slope (L\(^2\)) related to permeability coefficient \( k \), by \( k = K \rho g / \eta \); \( \eta \) is dynamic viscosity of water (ML\(^{-1}\)T\(^{-1}\)) and \( \eta = \rho v \).

### 2.1.2. Derivation of shear velocity

The mass conservation equations of surface flow and subsurface flow in 2D problem are expressed as follows

\[
\begin{align*}
\frac{\partial u_x}{\partial x} + \frac{\partial u_y}{\partial y} &= 0 \quad (4) \\
\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} &= 0 \quad (5)
\end{align*}
\]

As \( u_y = v_y = 0 \), it could be derived, e.g. \( \partial u_x / \partial x = \partial v_x / \partial x = 0 \). Moreover, \( \partial P / \partial x = -\Delta P / L \). Thus, Eq. (2) can be simplified as

\[
\gamma_w \sin \theta + \frac{\Delta P}{L} + \eta \frac{d^2 u_x}{dy^2} = 0 \quad (6)
\]

where, \( L \) is slope length along \( x \) direction (L).

Solving Eq. (6), flow velocity distribution of surface flow could be obtained,

\[
u_x = -\frac{\Delta P + \gamma_w L \sin \theta}{2 \eta L} y^2 + A_1 y + A_2 \quad (7)
\]

Similarly, flow velocity distribution of subsurface flow could be derived as,

\[
u_x = B_1 e^{y \sqrt{n/K}} + B_2 e^{-y \sqrt{n/K}} + \frac{(\Delta P + \gamma_w L \sin \theta)K}{\eta L} \quad (8)
\]

where, \( A_1, A_2, B_1 \) and \( B_2 \) are coefficients determined by boundary conditions.

In this model, the flow velocity distribution of surface flow and subsurface flow should satisfy the following boundary conditions (see Figure 2):

- a. On the surface of the runoff, \( u_x \) is maximal and \( du_x / dy = 0 \) at \( y = h \);
- b. On the interface between loose slope and runoff flow, it was satisfied that the velocities of the surface flow and subsurface flow equal and the shear stress is continuous (Neale and Nader 1974). Thus, \( v_x = u_x \) and \( dv_x / dy = ndu_x / dy \) at \( y = 0 \);
- c. At the bottom of the loose slope layer, \( v_x = 0 \) at \( y = -b \).

Substituting all the boundary conditions into Eqs. (7) and (8), the coefficients \( A_1, A_2, B_1 \) and \( B_2 \) can be calculated as follows:
where, $\Delta P = \gamma_w \Delta H$, $\Delta H/L = i = \tan \theta$, $i$ is hydraulic gradient (dimensionless); $\Delta H$ is water head difference of the slope sides (L); $h$ is runoff depth (L).

Therefore, the ultimate flow velocity formulas of surface flow and subsurface flow are shown as follows (Wei et al. 2018)

$$u_x = -\frac{\gamma_w (\sin \theta + \tan \theta)}{2 \eta} y^2 + \frac{\gamma_w h (\sin \theta + \tan \theta)}{\eta} y$$

$$v_x = \frac{\gamma_w \left(h \sqrt{nK} - Ke^{-b \sqrt{n/K}}\right) (\sin \theta + \tan \theta)}{\eta \left(e^{-2b \sqrt{n/K}} + 1\right)} e^{\gamma \sqrt{n/K}}$$

$$-\frac{\gamma_w \left(h \sqrt{nK} + Ke^{b \sqrt{n/K}}\right) (\sin \theta + \tan \theta)}{\eta \left(e^{2b \sqrt{n/K}} + 1\right)} e^{-\gamma \sqrt{n/K}} + \frac{\gamma_w K (\sin \theta + \tan \theta)}{\eta}$$

The relational expression between shear velocity $u_t$ and drag stress $\tau_0$ on the loose slope surface has been reported as follows (Petit et al. 2015)

$$\tau_0 = \rho u_t^2$$

Moreover, $\tau_0$ could also be derived based on Newton friction law, defined as follows,

$$\tau_0 = \eta \left. \frac{du_x}{dy} \right|_{y=0} = \gamma_w h (\sin \theta + \tan \theta)$$

Therefore, the expression of shear velocity $u_t$ could be obtained as

$$u_t = \sqrt{gh (\sin \theta + \tan \theta)}$$
2.1.3. Force-analyzing model

In fact, sediments are out-of-shape and non-uniform. In this paper, several assumptions are made to simplify the research problem, as follows,

a. Sediment particles are supposed to be circular in 2D problem, with particles numbered in 1, 2, 3 (see Figure 3(a));

b. Exposure degree of the research particle is regarded to be minimal;

c. Diameter of surrounding particles is regarded as a reference diameter, here is the mean diameter $d_m$ of sediment. Thus, in the idealized model, $d_m$ is the diameter of Particle 1 and 2, while $d_i$ is the diameter of Particle 3.

As is shown in Figure 3(b), forces acting on Particle 3 (research particle) include gravity $G$, drag force $F_D$, lift force $F_L$ and normal forces $F_{N1}$, $F_{N2}$. $O$ is the pivot point of Particle 3, $O_1$, $O_2$ and $O_3$ are centres of Particles 1, 2 and 3 respectively, $l_1$, $l_2$ and $l_3$ are force arms of $G$, $F_D$ and $F_L$ respectively, and $l_1 = 0.5d_i\sin(\varphi-\theta)$, $l_2 = 0.5d_i\cos\varphi$, $l_3 = 0.5d_i\sin\varphi$, in which $\varphi = \arcsin\left[\frac{d_m}{d_m+d_i}\right]$.

Normally, $F_D$, $G$ and $F_L$ are always taken into account in slope stability analysis and they are expressed as follows,

$$G = \alpha_G(\gamma_s-\gamma_w)d_i^3$$  \hspace{1cm} (15)

$$F_D = C_D\frac{\rho \pi d_i^2 u_i^2}{8}$$  \hspace{1cm} (16)

$$F_L = C_L\frac{\rho \pi d_i^2 u_i^2}{8}$$  \hspace{1cm} (17)

where, $\alpha_G$ is volume coefficient (dimensionless) and $\alpha_G = \pi/6$ when Particle 3 is spherical, $C_D$ is drag coefficient (dimensionless), $C_L$ is lift coefficient (dimensionless).
2.2. Derivation of critical runoff depth

For particle failure, rolling failure is the main incipient motion of sediment. When $F_{N2} = 0$, Particle 3 is in critical state, and the runoff depth is named as critical runoff depth. Moreover, critical runoff depth could be more easily measured than shear velocity or dimensionless shear stress.

In Figure 3, the pivot point $O$ is collinear with $F_{N1}$. Therefore, the equilibrium equation of force moments under critical state could be expressed as

$$Gl_1 = F_D l_2 + F_L l_3$$  \hspace{1cm} (18)

Substituting Eqs. (15)–(17) into Eq. (18), it could be derived

$$\frac{\pi}{12} (\gamma_s - \gamma_w) d_i^4 \sin (\varphi - \theta) = \frac{\rho \pi d_i^3 u_i^2}{16} (C_D \cos \varphi + C_L \sin \varphi)$$  \hspace{1cm} (19)

Combining Eqs. (14) and (19), then the explicit expression of critical runoff depth $h_{cr}$ could be derived as

$$h_{cr} = \frac{4(\gamma_s - \gamma_w) d_i \sin (\varphi - \theta)}{3\gamma_w (C_D \cos \varphi + C_L \sin \varphi)(\sin \theta + \tan \theta)}$$  \hspace{1cm} (20)
Apparently, the critical runoff depth $h_{cr}$ is associated with slope angle $\theta$ and particle diameter $d_i$. Eq. (20) could be regarded as a new criterion for incipient motion of non-cohesive sediment.

2.3. Experiments method

To verify above analysis, an experiment device was designed (Figure 4), and a high-speed camera was employed to measure the critical runoff depth $h_{cr}$ while testing. The device mainly consists of spout, water tank, guiding flume, and the main parts are marked in Figure 4.

The gate is used to control the flow flux. When the gate is open, water flows from water tank into guiding flume through water inlet. Reversed filter is set in guiding flume to reduce the flow velocity, ensuring that loose slope would not be directly destroyed by water rush. Loose slope is laid on spout, and slope surface should be kept parallel to spout surface. Slope angle could be changed by adjusting the supporting jack. In order to keep the flow flux constant, water elevation in water tank should be remained constant, with water replenished continuously and excess water drained away through outfall.

As critical runoff depth is associated with the sediment particle size, three kinds of sediment particles were chosen based on particle size, e.g. 0~5mm, 5~10mm and 10~20mm (Figure 5) as Chinese standard GB/T 50123-1999 required (BCS (Bureau of China Standards) 1999). In testing, these materials were respectively laid on the spout. For one size range, the experiments with slope angles of $20^\circ$, $22^\circ$, $24^\circ$, $26^\circ$ and $28^\circ$ were conducted. There are three graduated scales, marked a, b, c respectively, set on upside, middle and downside of the spout to measure critical runoff depth in different parts of the spout (shown in Figure 4(a)). In the test process, the high-speed
camera was working on to catch the moment of incipient motion of sediment. Meanwhile, runoff depth values at scales a, b, c could be recorded and the average could be taken as critical runoff depth.

2.4. Results and analysis

All the recorded results were shown in Table 1, including the recorded runoff depth at a, b, c points, and the average results.

In Table 2, the related parameter values applied in Eq. (20) were listed below.

Through analysis, the experiment values, theoretical values and absolute errors could be calculated, shown in Figure 6, and curves in subgraphs (a), (b) and (c) represent different results with a grain size range of 0~5 mm, 5~10 mm and 10~20 mm.

In Figure 6, the theoretical values and experiment values of $h_{cr}$ are both negatively correlated to slope angle $\theta$, and positively correlated to grain size $d_i$. Moreover, the theoretical values could match the experiment values very well, and the absolute errors are kept in an allowable range, less than 3.5 mm. Especially, the prediction errors are kept in the range less than 0.5 mm, 1 mm, 3.5 mm with the grain size range of 0~5 mm, 5~10 mm, 10~20 mm, respectively.

3. Comparison with other formulas in the form of shear velocity

Most formulas describing the incipient motion of non-cohesive sediment are based on shear velocity or dimensionless shear stress. Eq. (20) should be converted into the formula of shear velocity to compare with other formulas.

Due to the mathematical model established in this paper, relationship between shear velocity and runoff depth are obtained (seeing Eq. (14)). Thus, shear velocity $u_f$ could also be expressed as follows:

$$u_f = \sqrt{\frac{4(\gamma_s - \gamma_w)d_i \sin (\varphi - \theta)}{3\rho(C_D \cos \varphi + C_L \sin \varphi)}} \quad (21)$$

Another formula about shear velocity (named as Yang’s formula) proposed by Yang et al. (2004) is given by
Table 1. Experiment values obtained in scouring tests with different grain sizes and slope angles.

| Grain size (mm) | Slope angle (°) | Runoff depth measured by graduated scales (mm) | Averaged results (mm) |
|----------------|----------------|---------------------------------------------|----------------------|
| 0 < d_i ≤ 5   | 20             | 3.00                                        | 3.00                 |
| 2.5            | 2.50           | 2.50                                        | 2.50                 |
| 24             | 1.80           | 1.80                                        | 1.60                 |
| 26             | 1.50           | 1.20                                        | 1.20                 |
| 28             | 1.00           | 1.00                                        | 1.00                 |
| 5 < d_i ≤ 10   | 20             | 5.00                                        | 5.00                 |
| 7.5            | 5.50           | 5.50                                        | 5.50                 |
| 22             | 4.80           | 4.80                                        | 4.80                 |
| 24             | 3.50           | 3.50                                        | 3.20                 |
| 26             | 2.80           | 2.50                                        | 2.50                 |
| 28             | 1.50           | 1.50                                        | 1.50                 |
| 10 < d_i ≤ 20  | 20             | 8.50                                        | 8.00                 |
| 15             | 6.80           | 6.80                                        | 6.50                 |
| 22             | 5.20           | 5.00                                        | 5.00                 |
| 24             | 4.00           | 4.00                                        | 3.80                 |
| 26             | 2.50           | 2.50                                        | 2.50                 |

Table 2. Parameter values used in evaluating the critical runoff depth h_{cr}.

| Unit weight of water γ_w (kN/m^3) | Unit weight of soil γ_s (kN/m^3) | Drag coefficient C_D | Lift coefficient C_L |
|-----------------------------------|----------------------------------|-----------------------|----------------------|
| 18                                | 10                               | 0.4^*                 | 0.1^*                |

^*Values of drag coefficient and lift coefficient are referred to Zhou et al. (2016).

Figure 6. Comparisons between theoretical and experiment values of h_{cr} with different slope angles. (a) 0 < d_i ≤ 5mm, d_m = 2.5mm; (b) 5 < d_i ≤ 10mm, d_m = 7.5mm; (c) 10 < d_i ≤ 20mm, d_m = 15mm.

\[ u_i = \frac{0.917}{d_{90}} \left( \frac{h}{d_{90}} \right)^{\frac{1}{2}} \left[ \frac{3.33 \gamma_s - \gamma_w}{\rho} d_i + \phi(\xi_i, \theta) \left( \frac{\gamma_s' \gamma_s c - \gamma_w}{\gamma_s' c} \right)^{10} \frac{c}{\rho d_i} \sqrt{\frac{d_m}{d_i}} \right]^{\frac{1}{2}} \]  

(22)

where, \( \phi(\xi_i, \theta) \) and \( \psi(\xi_i, \theta) \) are functions of relative exposure degree \( \xi_i \) and slope angle \( \theta \), determined by Eqs. (23) and (24); \( \gamma_s' \) and \( \gamma_s' c \) are stable dry unit weight and dry unit weight of sand.

\[ \phi(\xi_i, \theta) = \left( \frac{17}{12} - \xi_i \right) \left( \sqrt{2\xi_i - \xi_i^2 \cos \theta - (1 - \xi_i) \sin \theta} + \frac{1}{4} \right)^{-\frac{1}{2}} \]  

(23)
Table 3. Data list of incipient velocity of non-cohesive sands under different slope angles (m/s).

| slope angle | data category | < 0.5 mm | 0.5 ~ 1.0 mm | 1.0 ~ 2.0 mm | 2.0 ~ 5.0 mm |
|-------------|---------------|----------|--------------|--------------|--------------|
| 5°          | A*            | 0.0959   | 0.2030       | 0.3262       | 0.4193       |
|             | B*            | 0.1537   | 0.2332       | 0.3347       | 0.4437       |
|             | C*            | 0.1324   | 0.1660       | 0.1917       | 0.3793       |
| 10°         | A             | 0.0911   | 0.1685       | 0.2922       | 0.3919       |
|             | B             | 0.1418   | 0.2152       | 0.3087       | 0.4093       |
|             | C             | 0.1241   | 0.1525       | 0.2157       | 0.3499       |
| 15°         | A             | 0.0844   | 0.1359       | 0.2601       | 0.3802       |
|             | B             | 0.1278   | 0.1940       | 0.2787       | 0.3690       |
|             | C             | 0.1148   | 0.1371       | 0.1939       | 0.3164       |
| 20°         | A             | 0.0785   | 0.1162       | 0.2425       | 0.3566       |
|             | B             | 0.1110   | 0.1685       | 0.2418       | 0.3206       |
|             | C             | 0.1042   | 0.1191       | 0.1684       | 0.2776       |
| 25°         | A             | 0.0741   | 0.1052       | 0.2044       | 0.3055       |
|             | B             | 0.0900   | 0.1366       | 0.1961       | 0.2599       |
|             | C             | 0.0920   | 0.0973       | 0.1376       | 0.2311       |
| 30°         | A             | 0.0696   | 0.0966       | 0.1457       | 0.2201       |
|             | B             | 0.0606   | 0.0920       | 0.1321       | 0.1751       |
|             | C             | 0.0774   | 0.0684       | 0.0967       | 0.1712       |

\[
\psi(\xi_i, \theta) = \frac{1}{\cos \beta - \sin \beta \sqrt{1 - \xi_i \xi_i}}
\] (24)

The results calculated by the two formulas are listed in Table 3, employing the experiment material parameters complemented by Chen et al. (1996). And the experiment data are also listed in Table 3. In Table 3, it was easily found that theoretical values about shear velocity by Eq. (21) could fit well with the experiment values, e.g. when grain size is less than 0.5 mm, theoretical values by Eq. (21) ranges from 0.1324 to 0.0774 m/s with a slope angle range of 5° ~ 30° while the experiment values are ranging from 0.0959 to 0.0696 m/s. Meanwhile, the theoretical values of critical runoff depth by Eq. (21) under different slope angles and grain sizes were listed in Table 4. It is showing that critical runoff depth ranges from 1.81 mm to 0.08 mm with the grain size less than 0.5 mm. As the grain size is increasing, the critical runoff depth obtained by Eq. (21) is increasing.

Figure 7 compared the experiment and theoretical values by the two formulas with different grain sizes respectively. In subgraphs (a) and (b), the relative errors of Eq. (21) are mostly less than that of Yang’s formula, and the theoretical values match well with the experiment data. In subgraphs (c) and (d), although the relative errors of Eq. (21) are greater than that of Yang’s formula, the theoretical values can also match well with the experiment values.

Compared with Yang’s formula, Eq. (21) has a concise form and only two independent variables, but it could match well with the experiment values. However, the relative errors are increasing as the grain size increases.

4. Discussions

Water erosion is a serious environment problem around the world, mainly influenced by rainfall, runoff and seepage flow. One of slope failure modes is from partial failure...
to integral failure, thus it is necessary to study incipient motion of sediment. In this paper, a new criterion for critical runoff depth is proposed, and it should be noted that the criterion could be transformed as the formula based on shear velocity. Meanwhile, accuracy of the new criterion was certified by experiments and ex-reported formulas, e.g. Yang's formula. It is showed that the new criterion, where there are two independent variables, has a concise form compared with Yang's formula, and could be used to predict the incipient motion of non-cohesive sediment.

Certified by experiments, the theoretical results are reliable but are slightly less than experiment values. Several possible reasons about errors were listed: (a) The sediment materials used in experiments are out-of-shape, and have non-uniform
distribution of grain size; (b) The flow flux is difficult to control and the runoff depth
could not be kept as a constant; (c) As the surface of loose slope is rugged, the flow
pattern near the surface is turbulence which could not meet the assumption that the
flow motion is laminar. This phenomenon is highlighted as the grain size increases,
and absolute errors are positively correlated to grain size $d_i$ shown in Figure 6.

Moreover, there are several limitations of the new criterion as follows:

Firstly, the new criterion is just suitable for non-cohesive sediment. Cohesive sedi-
ment is widespread worldwide and strength criteria rather than the new criterion
could be used in the stability of cohesive sediment. As particles were bonded together
by cohesion, stability analysis of single particle is not suitable for cohesive sediment
any more. Thus, in further studies, we hope to propose a criterion predicting critical
runoff depth when incipient motion of cohesive sediment is observed;

Secondly, runoff depth is supposed to be a constant under rainfall conditions.
Factually, runoff induced by rainfall could be expressed by the shallow water equations,
and runoff depth is various in space and time. Therefore, we also hope to propose a criter-
ion combining rainfall conditions to estimate incipient motion of sediment;

Thirdly, materials used in experiments are out-of-shape, obeying the assumption
that particles in 2D problem is circular. However, it could simulate the factual engin-
eering environment;

Finally, in the new criterion, it is obvious that critical runoff depth is only influ-
enced by slope angle and granular size. Through analysis, critical runoff depth is posi-
tively corelated with grain size and negatively corelated with slope angle. Virtually,
incipient motion of sediment could also be affected by exposure degree, particle
shape, different materials (e.g. organic matter) and so on. In further study of incipient
motion of sediment, these factors could be considered in the criterion.

5. Conclusions

To find a convenient and easy-measured method predicting incipient motion of non-
cohesive sediment, a new criterion based on critical runoff depth $h_{cr}$ was proposed.
In the new criterion, $h_{cr}$ is negatively correlated to slope angle $\theta$ and positively corre-
lated to grain size $d_i$.

Meanwhile, in order to certify the accuracy of the new criterion, simulation experi-
ments were conducted, with different grain size and slope angle. Grain size ranges
include 0 ~ 5 mm, 5 ~ 10 mm and 10 ~ 20 mm, and slope angle changes from 20° to
28°. The results show that theoretical values from Eq. (20) could fit well with experi-
ment values. For instance, when grain size range is 0 ~ 5 mm, the theoretical value of
critical runoff depth is 2.71 mm with a slope angle of 20° and the experiment value is
3 mm; the theoretical value decreased to 1.14 mm with a slope angle of 28° while the
experiment value is 1 mm. The results proved that the critical runoff depth could be
applied as an estimating parameter in predicting incipient motion of non-cohesive
sediment. The new criterion could be transformed to formulas based on shear vel-
ocity, and was compared with another formula. It has significant advantage of high
predicting accuracy and concise formulation.

In conclusion, critical runoff depth could be greatly used to predict incipient
motion of non-cohesive sediment. By experiment and ex-reported literature, critical
runoff depth was certified and it could be applied in the field of predicting incipient motion of non-cohesive sediment.

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