Deformation and the Nuclear Matrix Elements of the Neutrinoless $\beta\beta$ Decay

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Summary. — In this talk I will review the “state of the art” of the calculations of the nuclear matrix elements (NME) of the neutrinoless double beta decays ($0\nu\beta\beta$) for the nuclei $^{48}$Ca, $^{76}$Ge, $^{82}$Se, $^{124}$Sn, $^{128}$Te, $^{130}$Te and $^{136}$Xe in the framework of the Interacting Shell Model (ISM), and compare them with the NME’s obtained using the Quasiparticle RPA approach (QRPA). I will also discuss the effect of the competition between the pairing and quadrupole correlations in the value of these NME’s. In particular I will show that, as the difference in deformation between parent and grand daughter grows, the NME’s of both the neutrinoless and the two neutrino modes decrease rapidly.

1. Introduction

The discovery of neutrino oscillations in recent experiments at Super-Kamiokande [1], SNO [2] and KamLAND [3] has changed the old conception of neutrinos by proving that they are massive particles. According to the origin of their mass, neutrinos can be...
either Dirac or Majorana particles, the latter case being particularly interesting since it would imply an extension to the standard model of electroweak interactions, and, being neutrinos their own antiparticles in this scenario, lepton number conservation would be broken. Besides, it happens that the best way to detect one of these violating processes and consequently establish the Majorana character of the neutrinos would be detection of the neutrinoless double beta decay ($0\nu\beta\beta$).

Double beta decay is a very slow weak process. It takes place between two even-even isobars when the single beta decay is energetically forbidden or hindered by large spin difference. Two neutrinos beta decay is a second order weak process —the reason of its low rate—, and has been measured in a few nuclei. The $0\nu\beta\beta$ decay is analog but needs neutrinos to be Majorana particles. With the exception of one unconfirmed claim [4, 5], it has never been observed, and currently there is a number of experiments either taking place [6, 7, 8] or expected for the near future —see e.g. ref. [9]— devoted to detect this processes and to set up firmly the nature of neutrinos.

Furthermore, $0\nu\beta\beta$ decay is also sensitive to the absolute scale of neutrino mass, and hence to the mass hierarchy —at present, only the difference between different mass eigenstates is known. Since the half-life of the decay is determined, together with the masses, by the nuclear matrix element (NME) for this process, the knowledge of these NME’s is essential to predict the most favorable decays and, once detection is achieved, to settle the neutrino mass scale and hierarchy.

Two different and complementary methods are mainly used to calculate NME’s for $0\nu\beta\beta$ decays. One is the family of the quasiparticle random-phase approximation (QRPA). This method has been used by different groups and a variety of techniques is employed, with results for most of the possible emitters [10] [11] [12]. This work concerns to the alternative, the interacting shell model (ISM) [13].

In previous works [14] [15], the NME’s for the $0\nu\beta\beta$ decay were calculated taking into account only the dominant terms of the nucleon current. However, in ref. [16] it was noted that the higher order contributions to the current (HOC) are not negligible and it was claimed that they could reduce up to 20%-30% the final NME’s. Subsequently, other QRPA calculations [17] [18] have also taken into account these terms, although resulting in a somewhat smaller correction. These additional nucleon current contributions have been recently included for the first time in the ISM framework [19]. In addition, the short range correlations (SRC), are now modeled either by the Jastrow prescription or by the UCOM method [20].

2. – ISM vs QRPA Nuclear Matrix Elements

The expression for the half-life of the $0\nu\beta\beta$ decay can be written as [21] [22]:

\[
T_{1/2}^{0\nu\beta\beta}(0^+ \to 0^+) = G_{01} |M_0^{0\nu\beta\beta}|^2 \left( \frac{\langle m_\nu \rangle}{m_e} \right)^2,
\]

where $\langle m_\nu \rangle = \sum_k U_{ek}^2 m_k$ is the averaged neutrino mass, a combination of the neutrino
masses $m_k$ due to the neutrino mixing matrix $U$ — as we see, the neutrino mass scale is directly related to the decay rate — and $G_{01}$ is a kinematic factor — dependent on the charge, mass and available energy of the process. $M^{0\nu\beta\beta}$ is the NME object of study in this work.

The kinematic factor $G_{01}$ depends on the value of the coupling constant $g_A$. Therefore, we have to take this into account when comparing the values of NME’s obtained with different $g_A$ values. In these cases we will use a NME modified as:

$$M'_{0\nu\beta\beta} = \left(\frac{g_A}{1.25}\right)^2 M^{0\nu\beta\beta}$$

These $M'_{0\nu\beta\beta}$s are directly comparable between them no matter which was the value of $g_A$ employed in their calculation, since they share a common $G_{01}$ factor — that of $g_A = 1.25$. In this sense, the translation of $M'_{0\nu\beta\beta}$s into half-lives is transparent. The QRPA results obtained with different $g_A$ values are already expressed in this way by the authors of refs. [23, 12] while the results of refs. [17, 18, 24] will be translated by us into the above form when compared with other results.

We have calculated the ISM NME’s both taking the UCOM and Jastrow ansatzs for the short range correlations. The former use the correlator of the $ST=01$ channel [25], throughout the calculation. The correlator of the other important — even — channel is very similar to this one, and it should not make much difference on this result.

In figure 1 the ISM and QRPA results for the NME’s are compared within this UCOM treatment of the SRC. The same figure but considering Jastrow type SRC was shown in ref. [19], but, inadvertently, the QRPA results from refs. [17, 18, 24] obtained with $g_A = 1.0$ where not transformed properly. This is corrected in figure 2. By comparing both figures, it is confirmed that there is a common trend: when the nuclei that participate in the decay have a low level of quadrupole correlations, as in the decays of $^{124}$Sn and $^{136}$Xe, both approaches agree. The QRPA in a spherical basis seems not to be able to capture the totality of the quadrupole correlations when they are strong. As these correlations tend to reduce the NME’s, the QRPA produces NME’s that are too large in $^{76}$Ge, $^{82}$Se, $^{128}$Te, and $^{130}$Te. For both ISM and QRPA the only net effect of UCOM is an increase of the Jastrow results of about 20%.

3. – The Influence of Deformation in the NME’s

An important issue regarding $0\nu\beta\beta$ decay is the role of pairing and deformation. It has recently been discussed in ref. [19] that the pairing interaction favors the $0\nu\beta\beta$ decay and that, consequently, truncations in seniority, not including the anti-pairing-like effect of the missing uncoupled pairs, tend to overestimate the value of the NME’s. On the other hand, the NME is also reduced when the parent and grand-daughter nuclei have different deformations [26]. Thus, we have studied the interplay between both pairing and deformation, this is, to which extent a wave function in the laboratory frame, truncated in seniority, can capture the correlations induced by the quadrupole-quadrupole part of
the nuclear interaction, and its eventual influence in 0νββ NME’s. There is an extra motivation to pursue this study; the possibility to carry on an experiment with 150Nd, which is a well deformed nuclei, decaying into 150Sm which is a much less deformed one.

To study the interplay between pairing, seniority truncations, and quadrupole correlations we need first to decide how to measure the quadrupole correlations of the ground state. Our choice is to refer to non energy-weighted sum rule:

\[
\langle Q^2 \rangle = \sum_i |\langle 2_1^+ | Q | 0^+ \rangle|^2
\]

(3)

The operator \( Q \) represents the mass quadrupole. No effective “nuclear” charges are included. Using \(^{82}\text{Kr}\) as our test bench, we proceed to compute \( \langle Q^2 \rangle \), first with our standing effective interaction and different seniority truncations \( (s_m \text{ means the maximum seniority allowed in the wave functions of parent and grand daughter nuclei}) \). The results
are drawn in Figure 3 as the black circles labeled $\lambda = 0$. We can see that at $s_m = 4$ —roughly, the implicit level of seniority truncation in the spherical QRPA— some 70% of the full quadrupole correlations are incorporated in the wave function. We would like to know how this behavior evolves when more correlations are enforced in the system. For this we recalculate the ground state of $^{82}\text{Kr}$ with a new hamiltonian that consists of the standing one plus a quadrupole-quadrupole term $\lambda Q \cdot Q$, whose effect will be gauged by its influence in the sum rule. To have an idea of the relevant range of values of $\langle Q^2 \rangle$ in this nucleus and valence space, we have gone to the limit of pure quadrupole-quadrupole interaction with degenerate single particle energies, getting $\langle Q^2 \rangle \approx 4500 \text{ fm}^4$. The results for $\lambda = 1$ and $\lambda = 2$ are also shown in Figure 3 ($\lambda = 1$ corresponds to $\lambda_{qq} = 0.025$ in Figs. 6-9 and $\lambda_{qq} = 1$ in Fig. 10 to $\lambda_{qq} = 0.1$. It is evident in the figure that, as we try to increase the correlations, the $s_m = 4$ truncation becomes more and more ineffective. For $\lambda = 1$, only 57% of the full correlations are present, and for $\lambda = 2$ only 50%. The situation is different for $^{82}\text{Se}$: while for $\lambda = 0$ the values of $\langle Q^2 \rangle$ as a function of seniority are similar, albeit a bit smaller than the $^{82}\text{Kr}$ ones, for $\lambda = 1$ and $\lambda = 2$ there is scarcely
any increase of the ground state correlations. This means also that, as we increase $\lambda$, the “deformation” of $^{82}$Kr grows, whereas that of $^{82}$Se remains constant.

This behavior offers us the opportunity of exploring the effect of the difference in deformation between parent and grand daughter in the $0\nu\beta\beta$ NME’s. To this goal, we have computed the Gamow-Teller matrix element for different values of $\lambda$—the amount of extra quadrupole-quadrupole interaction—and $s_m$—the maximum seniority allowed in the wave function—. The results are gathered in Figure 4. For $s_m = 0$, we observe that the Gamow-Teller matrix element grows as a function of $\lambda$. This may seem paradoxical, but is not, because at this seniority truncation, the only effect of adding more quadrupole-quadrupole interaction is to augment the pairing content of the wave functions, thus increasing $M_{GT}$. At $s_m = 4$, $M_{GT}$ remains constant as a function of $\lambda$, meaning that the minor increase of the correlations of $^{82}$Kr, that we have shown in Figure 3, is barely enough to compensate the increase of $M_{GT}$ at $s_m = 0$. On the contrary, the full space results are sensitive to the difference in deformation—or, to be more precise, to the difference in the level of quadrupole correlations in the ground state—between parent and grand daughter. The effect goes in the direction of reducing the value of $M_{GT}$. In the $A = 82$ decay, doubling the quadrupole correlations in $^{82}$Kr, roughly halves $M_{GT}$. 

Fig. 3. – Quadrupole correlations in the ground state of $^{82}$Kr as a function of the amount of quadrupole-quadrupole interaction $\lambda Q \cdot Q$ added to the hamiltonian and of the maximum seniority $s_m$ permitted in the wave functions.
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We can go much further in the exploration of the deformation effects, calculating the NME of the decay for initial and final states computed with different amounts of supplementary quadrupole-quadrupole interaction. As before, we measure the quadrupole correlations by means of the mass quadrupole sum rule. The results of this search are plotted in Fig. 5. We observe that the NME decreases almost linearly as the difference of the sum rules for the final and initial states increases. In fact, the maximum values of the NME are reached when this difference is close to zero. On the contrary for large differences the NME can be extremely quenched. Notice that the values in the figure range between 0.07 and 2.7, a factor of 40 span. The $\lambda=0$ value is 2.18, indicating that the difference in quadrupole correlations between $^{82}$Kr and $^{82}$Se is not very large.

4. – $0\nu$ (Unphysical) Mirror Decays: A Case Study

We have also studied the transitions between mirror nuclei in order to have a clearer view of the role of deformations in the NME's. These transitions have the peculiarity that the wave functions of the initial and final nuclei are identical (provided Coulomb effects are neglected) and consequently the interplay of the $0\nu\beta\beta$ operator and of the nuclear wave functions in the NME may be easier to understand.

We have studied four parent nuclei with six valence protons and four valence neutrons,
Fig. 5. – $^{82}$Se $\rightarrow$ $^{82}$Kr NME, $M^{0\nu}$, as a function of the difference between the mass quadrupole sum rule between $^{82}$Kr and $^{82}$Se for a large number of different values of the strength of the added quadrupole quadrupole interaction.

$^{26}$Mg, $^{50}$Cr, $^{66}$Ge and $^{110}$Xe, decaying into the four grand daughter nuclei with four valence protons and six valence neutrons, $^{26}$Si, $^{50}$Fe, $^{66}$Se and $^{110}$Ba. The valence spaces considered are the $sd$-shell, the $pf$-shell, $r_{3g} (1p_{3/2}, 1p_{1/2}, 0f_{5/2}, 0g_{9/2})$ and $r_{4h} (0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2})$. The starting interactions are USD [27], KB3 [28], GCN28.50 and GCN50.82 [29]. The deformation of the nuclei is modified as in the previous section, and it is quantified also in the same way.

The results for $A = 66$ in the case of equally deformed initial and final nuclei, are shown on Fig. 6. There we see that, as the nuclei become more deformed, the NME and the pairing content of the wave function get smaller, while the quadrupole sum rule grows. All these changes are nearly linear for reasonable deformations and then the saturation is approached more smoothly. Note that the purely quadrupole interaction ($\lambda_{qq} \rightarrow \infty$ limit) gives a NME which is about a half of the value obtained with no additional quadrupole.

Fig. 7 shows the same quantities than Fig. 6 but now only the final nucleus has been artificially deformed by adding an extra quadrupole-quadrupole term. In addition, the overlap between initial and final wave functions has been included. We see that now, the reduction of the NME is more pronounced and, what is more interesting, that it follows closely the overlap between wave functions. This means that, if we write the final wave function as: $|\Psi\rangle = a |\Psi_0\rangle + b |\Psi_{qq}\rangle$, the $0\nu\beta\beta$ operator only connects the parts of the
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Fig. 6. $^{66}\text{Ge} \rightarrow ^{66}\text{Se}$ NME, $M_{0\nu}^0$, as a function of the strength of the added quadrupole quadrupole interaction. Equally deformed case; the same amount of extra QQ interaction is added to $^{66}\text{Ge}$ and $^{66}\text{Se}$. On the right hand y axis the pairing and quadrupole sum rules are represented, normalized so that their maximum value is 1. Note the change of scale in the x axis at $\lambda = 0.2$.

The behavior of the NME’s with respect to the difference of deformation between parent and grand daughter is common to all the other transitions between mirror nuclei that we have studied. Therefore we can submit that this is a robust result. However, when we consider the transitions between equally deformed nuclei, the evolution of the NME’s with the deformation that we have found in A=66 is only shared by the A=110 case. When the valence space is a full major oscillator shell —A = 26 and A = 50— the situation is quite different. Indeed, what is observed is that the NME does not decrease for moderate values of $\lambda$ but remains rather constant until a point —with large deformation— where its value increases significantly —up to 50%—. This is due to the fact that, at this point, the major contribution to the NME ceases to come only form the decay of pairs coupled to $J = 0$, since other values like $J = 2, 4, 6$, which usually have a contribution to the NME contrary to that of $J = 0$, reverse sign and grow until being
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Fig. 7. – Same as previous figure, but now the additional quadrupole interaction is only added to $^{66}$Se. The normalized overlap between the initial and final states is also included.

comparable with this contribution, thus resulting in this notorious rise of the NME. In the $sd$-shell and $pf$-shell cases, the $\lambda_{qq}\rightarrow\infty$ limit is equivalent to Elliott’s SU(3) limit, and, the fact that both the initial and final nuclei should belong to the same irrep of SU(3) may be the reason of the increase of the NME, but, for the moment we have not found a formal explanation.

5. – 2$\nu$ (Unphysical) Mirror Decays: A Case Study

Very similar conclusions may be reached for the effect of deformation in $2\nu\beta\beta$ decay. For instance, the mirror nuclei diagonal and non-diagonal NME’s are represented as in the $0\nu\beta\beta$ case in Figs. 8 and 9 for the $A=66$ transition. We see that the figures resemble very much that of the previous section, with the only exception that, in the equally deformed case, the lowering of the NME due to deformation is more pronounced.

If we look to a non mirror —but again fictitious— transition, for instance that of $^{48}\text{Ti} \rightarrow ^{48}\text{Cr}$, we get the results shown in Fig. 10. If we compare with those of ref. 26,
which are again the equivalent ones for the $0\nu\beta\beta$ transition, we see that there are not substantial changes. In this sense, deformation seems to affect similarly to $0\nu\beta\beta$ and $2\nu\beta\beta$ decays.

6. Summary

After a brief discussion of the “state of the art” results for the nuclear matrix elements of the neutrinoless double beta decay in the context of the Interacting Shell Model and of the Quasiparticle Random Phase Approximation, we have analyzed the role of the pairing correlations and the deformation in the NME’s, concluding that seniority truncations are less reliable when the quadrupole correlations are large. Since the NME’s are reduced when the deformation of parent and grand daughter is different, a bad treatment of the quadrupole correlations can lead to an artificial enhancement of the NME’s in the transitions among nuclei with unequal deformations, or in the cases of nuclei with different –and large- amounts of quadrupole correlations.

Fig. 8. – Equally deformed $^{66}\text{Ge} \rightarrow ^{66}\text{Se}$ $2\nu$NME. Note the change of scale in the x axis at $\lambda = 0.2$. 

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Fig. 9. – The same as the previous figure, but now the only nuclei calculated with additional quadrupole interaction is the final one. The normalized overlap between initial and final states is also included.

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REFERENCES

[1] Fukuda Y., Hayakawa T., Ichihara E., Inoue K., Ishihara K., Ishino H., Itow Y., Kajita T., Kameda J., Kasuga S., Kobayashi K., Kobayashi Y., Koshow Y., Miura M., Nakahara M., Nakayama S., Okada A., Okumura K., Sakurai N., Shiozawa M., Suzuki Y., Takeuchi Y., Totsuka Y., Yamada S., Earl M., Habig A. and Kearns E., *Phys. Rev. Lett.*, **81** (1998) 1562.
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Fig. 10. – Influence of deformation in the $^{48}$Ti $\rightarrow$ $^{48}$Cr decay.

[2] Ahmad Q. R., Allen R. C., Andersen T. C., Anglin J., Barton J. C., Beier E. W., Bercovitch M., Bigu J., Biller S. D., Black R. A., Blevis I., Boardman R. J., Boger J., Bonvin E., Boulay M. G., Bowler M. G., Bowles T. J., Brice S. J., Browne M. C., Bullard T. V., Bühler G., Cameron J., Chan Y. D., Chen H. H., Chen M., Chen X. and Cleveland B. T., Phys. Rev. Lett., 89 (2002) 011301.

[3] Eguchi K., Enomoto S., Furuno K., Goldman J., Hanada H., Ikeda H., Ikeda K., Inoue K., Ishihara K., Itoh W., Iwamoto T., Kawaguchi T., Kawashima T., Kinoshita H., Kishimoto Y., Koga M., Koseki Y., Maeda T., Mitsu T., Motoki M., Nakajima K., Nakajima M., Nakajima T., Ogawa H., Owada K., Sakabe T. and Shimizu I., Phys. Rev. Lett., 90 (2003) 021802.

[4] Klapdor-Kleingrothaus H. V., Dietz A., Harney H. L. and Krivosheina I. V., Mod. Phys. Lett. A, 16 (2001) 2409.

[5] Klapdor-Kleingrothaus H. V., Krivosheina I. V., Dietz A. and Chkvorets O., Phys. Lett. B, 586 (2004) 198.

[6] Arnold R., Augier C., Baker J., Barabash A., Boudin G., Brudanin V., Caffrey A. J., Caurier E., Egorov V., Errahmane K., Etienne L., Guyonnet J. L., Hubert F., Hubert P., Jouillet C., Jullian S., Kochetov O., Kovalenko V., Konovalov S., Lalanne D., Leccia F., Longuemare C., Lutter G., Marquet C., Mauger F., Nowacki F., Ohsumi H., Piquemal F., Reyss J. L., Saakyan R., Sarazin X., Simard L., Simovic F., Shitov Y., Smolnikov A., Stekl L., Suhonen J., Sutton C. S., Szklarz G., Thomas J., Timkin V., Tretyak V., Umato V., Vala L., Vanushin I., Vasilyev V., Vorobev V. and Vylov T., Phys. Rev. Lett., 95 (2005) 182302.
[7] Arnaboldi C., Artusa D. R., Avignone F. T., Balata M., Bandac I., Barucci M., Beeman J. W., Brofferio C., Buccci C., Ceppellini S., Carbone L., Cebrian S., Cremone O., Cresswick R. J., de Waard A., Fiorini E., Frossati G., Guardincerri E., Giuliani A., Gorla P., Haller E. E., McDonald R. J., Morales A., Norman E. B., Nucciotti A., Olivieri E., Pallavicini M., Palmieri E., Pasca E., Pavan M., Pedretti M., Pessina G., Pirro S., Previtali E., Risegari L., Rosenfeld C., Sangiorgio S., Sisti M., Smith A. R., Torres L. and Ventura G., *Phys. Rev. Lett.*, 95 (2005) 142501.

[8] Bloxham T., Boston A., Dawson J., Dobos D., Fox S. P., Freer M., Fulton B. R., Gössl C., Harrison P. F., Junker M., Kiel H., McGrath J., Morgan B., Münstermann D., Nolan P., Oehl S., Ramachers Y., Reeve C., Stewart D., Wadsworth R., Wilson J. R. and Zuber K., *Phys. Rev. C*, 76 (2007) 025501.

[9] Avignone F. T., Elliott S. R. and Engel J., *Rev. Mod. Phys.*, 80 (2008) 481.

[10] Suonen J. and Civitarese O., *Phys. Rept.*, 300 (1998) 123.

[11] Rodin V. A., Faessler A., Simkovic F. and Vogel P., *Nucl. Phys. A*, 766 (2006) 107.

[12] Rodin V. A., Faessler A., Simkovic F. and Vogel P., *Nucl. Phys. A*, 793 (2007) 213.

[13] Caurier E., Martinez-Pinedo G., Nowacki F., Poves A. and Zuker A. P., *Rev. Mod. Phys.*, 77 (2005) 427.

[14] Retamosa J., Caurier E. and Nowacki F., *Phys. Rev. C*, 51 (1995) 371.

[15] Caurier E., Nowacki F., Poves A. and Retamosa J., *Phys. Rev. Lett.*, 77 (1996) 1954.

[16] Simkovic F., Pantis G., Vergados J. D. and Faessler A., *Phys. Rev. C*, 60 (1999) 055502.

[17] Kortelainen M. and Suonen J., *Phys. Rev. C*, 75 (2007) 051303.

[18] Kortelainen M. and Suonen J., *Phys. Rev. C*, 76 (2007) 024315.

[19] Caurier E., Menendez J., Nowacki F. and Poves A., *Phys. Rev. Lett.*, 100 (2008) 052503.

[20] Feldmeier H., Neff T., Roth R. and Schnack J., *Nucl. Phys. A*, 632 (1998) 61.

[21] Doi M., Kotani T., Nishura H., Okuda K. and Takasugi E., *Phys. Lett. B*, 103 (1981) 219.

[22] Doi M., Kotani T. and Takasugi E., *Prog. Theor. Phys. Suppl.*, 83 (1985) 1.

[23] Simkovic F., Faessler A., Rodin V. A., Vogel P. and Engel J., *Phys. Rev. C*, 77 (2008) 045503.

[24] Suonen J. and Kortelainen M., *Int. J. Mod. Phys. E*, 17 (2008) 1.

[25] Roth R., Hergert H., Papakonstantinou P., Neff T. and Feldmeier H., *Phys. Rev. C*, 72 (2005) 034002.

[26] Caurier E., Nowacki F. and Poves A., *Eur. Phys. J. A*, 36 (2008) 195.

[27] Wildenthal B. H., *Prog. Part. Nucl. Phys.*, 11 (1984) 5.

[28] Poves A. and Zuker A., *Phys. Rep.*, 70 (1981) 235.

[29] Gniady A., Caurier E. and Nowacki F., to be published.