The chiral effective pion–nucleon Lagrangian of order $p^4$ #1

Nadia Fettes$^a$, Ulf-G. Meißner$^a$, Martin Mojžiš$^b$, Sven Steininger$^a$

$^a$Forschungszentrum Jülich, Institut für Ker Physik (Theorie)
D-52425 Jülich, Germany

$^b$Department of Theoretical Physics, Comenius University
SK-28415 Bratislava, Slovakia

Abstract

We construct the minimal effective chiral pion–nucleon SU(2) Lagrangian at fourth order in the chiral expansion. The Lagrangian contains 118 in principle measurable terms. We develop both the relativistic as well as the heavy baryon formulation of the effective field theory. For the latter, we also work out explicitly all $1/m$ corrections at fourth order. We display all relevant relations needed to find the linearly independent terms.

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#2Present address: Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, CA 91125, USA

#3Present address: McKinsey Consulting, Köln, Germany
1 Introduction

Low-energy pion and pion–nucleon physics is described in QCD via chiral perturbation theory (CHPT) [1, 2]. This is an effective field theory designed to solve the Ward identities of the chiral symmetry of QCD order by order. CHPT is based on an effective Lagrangian constructed in a systematic way in terms of the asymptotically observed hadronic fields and consistent with all symmetries of QCD. The effective Lagrangian is organized in the form of a chiral expansion, i.e. an expansion in powers of momenta and light quark masses [3]. At every order, the effective Lagrangian contains free parameters, the so–called low-energy constants (LECs). Various quantities calculated from the effective Lagrangian are related by the same set of LECs entering the results. In addition, there are some rare cases where, to a certain given order in the chiral expansion, predictions are free of LECs. At lowest order, CHPT just reproduces the well–known results of current algebra. However, within CHPT a systematic improvement of current algebra predictions is possible. Furthermore, to restore unitarity perturbatively, one has to calculate (pion) loop corrections because the tree diagrams contributing at lowest order are always real. Every loop increases the chiral order by two, therefore the lowest order results are the tree level ones. The relation between current algebra and the lowest order tree level effective Lagrangian has already been established in the sixties. Loops enter at higher orders, together with tree contributions with insertions from the higher order Lagrangians. In the meson sector, the chiral expansion thus proceeds in steps of two, if one assumes the standard scenario of chiral symmetry breaking with a large quark–antiquark condensate.

In the one–nucleon sector of CHPT the situation is different. Here, due to the fermionic nature of the matter fields, couplings with odd powers in momenta are allowed and the lowest order tree graphs stem from a dimension one chiral Lagrangian. Loops start to enter at third order (in a scheme that respects the power counting, see below). However, for various reasons (convergence, completeness, and so on) calculations are now performed at the complete one–loop level, i.e. at fourth chiral order (for a review and status report, see refs.[4, 5]). The full effective Lagrangian is, however, only known up to third order [2, 3, 7, 8, 9]. The most general fourth order πN chiral Lagrangian is still the missing ingredient in a complete one–loop analysis within pion–nucleon CHPT. It is the purpose of this paper to provide this missing component. We refrain here from discussing the three–flavor case, which certainly deserves more study in the future.

Most CHPT calculations for the πN system (coupled to external fields) have been performed in the framework of Heavy Baryon CHPT (HBCHPT), a specific non-relativistic projection of the theory. This scheme was developed in ref.[10] motivated by the methods used in heavy quark effective field theory and the observation that a relativistic formulation with baryons leads to complications in the power counting when standard dimensional regularization is employed [2]. In HBCHPT the troublesome mass scale, the nucleon mass $m$, appearing in the fermion propagator, is simply transformed in a string of vertex corrections with fixed coefficients and increasing powers in $1/m$. However, as was pointed out recently in ref.[11] and ref.[12], a scheme consistent with the power counting, using the relativistic version of the πN Lagrangian, can also be set up if one performs a different method of regularization. Therefore not only the heavy baryon (HB) projection of the Lagrangian is of interest, but also its fully relativistic version. Note also that if one wants to match the HB approach to the relativistic theory, this is most easily and naturally done starting from the relativistic approach (for a detailed discussion, see e.g. ref.[7]).

The divergent part of the fourth order HBCHPT πN Lagrangian is already known [13]. Also, the
complete but not minimal fourth order Lagrangian with virtual photons has been worked out in ref.\[14\]. In addition, recently an attempt to work out the complete fourth–order isospin–symmetric Lagrangian in the absence of external fields has been presented in ref.\[15\]. However, the complete fourth order heavy baryon as well as relativistic $\pi N$ Lagrangian (including external fields and in particular strong isospin breaking) is constructed here for the first time. The number of LECs in the resulting Lagrangian turns out to be larger than 100. This is similar to the case of the two–loop $O(p^6)$ meson Lagrangian, see refs.\[16, 17\]. In case of such a large number of free parameters in the theory, one might question the predictive power of the whole approach. However, most physical processes are only sensitive to a small number of LECs. As an example, we mention elastic–pion nucleon scattering. At second, third and fourth order, one has four, four and five independent (combinations of) LECs, respectively, and these can be easily determined by a fit to the vast amount of low–energy pion–nucleon scattering data (this argument is spelled out in more detail in ref.\[18\]). In photo–nucleon processes, one usually has even fewer LECs, e.g. the HBCHPT expression of the electric dipole amplitude $E_{0+}$ for neutral pion scattering off nucleons involves only two combinations of LECs up–to–and–including fourth order.

The paper is organized as follows. In Sect. 2, the construction of the Lagrangian is described step by step (choice of fields, construction of invariant monomials, elimination of linearly dependent terms, heavy baryon projection). The result is presented and discussed in Sect. 3. Appendix A contains a more detailed discussion of the elimination of the so-called equation-of-motion terms in the relativistic Lagrangian. Appendix B contains some relevant formulae for the heavy baryon projection.

2 Construction of the Lagrangian

2.1 Choice of fields

Let us briefly collect the basic ingredients for the construction of the effective $\pi N$ Lagrangian. The underlying Lagrangian is that of QCD with massless $u$ and $d$ quarks, coupled to external hermitian $2 \times 2$ matrix–valued fields $v_\mu$, $a_\mu$, $s$ and $p$ (vector, axial–vector, scalar and pseudoscalar, respectively) \[\[\]

$$L = L^0_{\text{QCD}} + \bar{q} \gamma^\mu (v_\mu + \gamma_5 a_\mu) q - \bar{q} (s - i \gamma_5 p) q , \quad q = \begin{pmatrix} u \\ d \end{pmatrix} . \quad (2.1)$$

With suitably transforming external fields, this Lagrangian is locally $SU(2)_L \times SU(2)_R \times U(1)_V$ invariant. The chiral $SU(2)_L \times SU(2)_R$ symmetry of QCD is spontaneously broken down to its vectorial subgroup, $SU(2)_V$. To simplify life further, we disregard the isoscalar axial currents as well as the winding number density.\[\[\]

Explicit chiral symmetry breaking, i.e. the nonvanishing $u$ and $d$ current quark mass, is taken into account by setting $s = M = \text{diag}(m_u, m_d)$.

On the level of the effective field theory, the spontaneously broken chiral symmetry is non-linearly realized in terms of the pion and the nucleon fields \[\[\]. The pion fields $\Phi$, being coordinates of

\[\footnotesize{\#4}\text{We will comment in more detail on that paper below.}

\[\footnotesize{\#5}\text{Note that isoscalar axial currents only play a role in the discussion of the so–called spin content of the nucleon. That topic can only be addressed properly in a three flavor scheme. For a lucid discussion of the $\theta$ term, we refer to ref.\[19\].}
the chiral coset space, are naturally represented by elements $u(\Phi)$ of this coset space. The most convenient choice of fields for the construction of the effective Lagrangian is given by

$$
u = i\{u^\dagger(\partial\nu - ir\nu)u - u(\partial\nu - i\ell\nu)u^\dagger\},$$

$$\chi^\pm = u^\dagger\chi u^\dagger \pm u\chi^\dagger u,$$

$$F_{\mu\nu}^\pm = u^\dagger F_{\mu\nu}^R u \pm uF_{\mu\nu}^L u^\dagger,$$

(2.2)

where

$$\chi = 2B(s + ip),$$

$$F_{\mu\nu}^R = \partial\mu r^\nu - \partial\nu r^\mu - i[r^\mu, r^\nu],$$

$$r^\mu = v^\mu + a^\mu,$$

$$F_{\mu\nu}^L = \partial\mu \ell^\nu - \partial\nu \ell^\mu - i[\ell^\mu, \ell^\nu],$$

$$\ell^\mu = v^\mu - a^\mu,$$

(2.3)

and $B$ is the parameter of the meson Lagrangian of $O(p^2)$ related to the strength of the quark–antiquark condensate [1]. We work here in the standard framework, $B \gg F_\pi$, with $F_\pi$ the pion decay constant. For a discussion of the generalized scenario in the presence of matter fields, see e.g. ref.[21]. The reason why the fields in eq.(2.2) are so convenient is that they all transform in the same way under chiral transformations, namely as

$$X \overset{g}{\rightarrow} h(g, \Phi)Xh^{-1}(g, \Phi),$$

(2.4)

where $g$ is an element of $SU(2)_L \times SU(2)_R$ and the so-called compensator $h(g, \Phi)$ defines a non–linear realization of the chiral symmetry. The compensator $h(g, \Phi)$ depends on $g$ and $\Phi$ in a complicated way, but since the nucleon field $\Psi$ transforms as

$$\Psi \overset{g}{\rightarrow} h(g, \Phi)\Psi, \quad \bar{\Psi} \overset{g}{\rightarrow} \bar{\Psi}h^{-1}(g, \Phi)$$

(2.5)

one can easily construct invariants of the form $\bar{\Psi}O\Psi$ without explicit knowledge of the compensator. Moreover, for the covariant derivative defined by

$$D_\mu = \partial_\mu + \Gamma_\mu, \quad \Gamma_\mu = \frac{1}{2}\{u^\dagger(\partial\mu - ir\mu)u + u(\partial\mu - i\ell\mu)u^\dagger\},$$

(2.6)

it follows that also $[D_\mu, X], [D_\mu, [D_\nu, X]],$ etc. transform according to eq.(2.4) and $D_\mu \Psi, D_\mu D_\nu \Psi,$ etc. according to eq.(2.3). This allows for a simple construction of invariants containing these derivatives. The definition of the covariant derivative eq.(2.6) implies two important relations. The first one is the so-called curvature relation

$$[D_\mu, D_\nu] = \frac{1}{4}[u_\mu, u_\nu] - \frac{i}{2}F_{\mu\nu}^+, \quad [D_\mu, D_\nu] = \frac{1}{4}[u_\mu, u_\nu] - \frac{i}{2}F_{\mu\nu}^-, \quad [D_\mu, D_\nu] = \frac{1}{4}[u_\mu, u_\nu] - \frac{i}{2}F_{\mu\nu}^\pm,$$

(2.7)

which allows one to consider products of covariant derivatives only in the completely symmetrized form (and ignore all the other possibilities). The second one is (be aware that some authors like e.g. in refs.[15, 21], use a different convention which leads to a minus sign in the definition of $F_{\mu\nu}$ as compared to the one used here)

$$[D_\mu, u_\nu] - [D_\nu, u_\mu] = F_{\mu\nu}^-,$$

(2.8)

which in a similar way allows one to consider covariant derivatives of $u_\mu$ only in the explicitly symmetrized form in terms of the tensor $h_{\mu\nu},$

$$h_{\mu\nu} = [D_\mu, u_\nu] + [D_\nu, u_\mu].$$

(2.9)
For the construction of terms, as well as for further phenomenological applications, it is advantageous to treat isosinglet and isotriplet components of the external fields separately. We therefore define

$$\tilde{X} = X - \frac{1}{2} \langle X \rangle,$$

where $\langle \ldots \rangle$ stands for the flavor trace, and we work with the following set of fields: $u_\mu, \tilde{\chi}_\pm, \langle \chi_\pm \rangle, \tilde{F}_\pm, \langle F_+ \rangle$ (the trace $\langle F_- \rangle$ is zero because we have omitted the isoscalar axial current.)

To construct a hermitian effective Lagrangian, which is not only chiral, but also parity (P) and charge conjugation (C) invariant, one needs to know the transformation properties of the fields under space inversion, charge conjugation and hermitian conjugation. For the fields under consideration (and their covariant derivatives), hermitian conjugation is equal to $\pm$itself and charge conjugation amounts to $\pm$transposed, where the signs are given (together with the parity) in Table 1. This table also contains the chiral dimensions of the fields. This is necessary for a systematic construction of the chiral effective Lagrangian, order by order. Covariant derivatives acting on pion or external fields count as quantities of first chiral order.

|                | $u_\mu$ | $\chi_+$ | $\chi_-$ | $F^+_{\mu\nu}$ | $F^-_{\mu\nu}$ | $D_\mu$ |
|----------------|--------|---------|---------|----------------|----------------|--------|
| chiral dimension | 1      | 2       | 2       | 2              | 2              | 1      |
| parity          | -      | +       | -       | +              | -              | +      |
| charge conjugation | +     | +       | -       | -              | +              | +      |
| hermitian conjugation | +   | +       | -       | +              | +              | +      |

Table 1: Chiral dimension and transformation properties of the basic fields and the covariant derivative acting on the pion and the external fields.

In what follows, an analogous information for Clifford algebra elements, the metric $g_{\mu\nu}$ and the totally antisymmetric (Levi-Civita) tensor $\varepsilon_{\lambda\mu\nu\rho}$ (for $d = 4$) together with the covariant derivative acting on the nucleon fields will be needed (for a more detailed discussion, see ref.[9]). In this case, hermitian conjugate equals $\pm \gamma^0$ (itself) $\gamma^0$ and charge conjugation amounts to $\pm$transposed, where the signs are given (together with parity and chiral dimension) in Table 2. The covariant derivative acting on nucleon fields counts as a quantity of zeroth chiral order, since the time component of the derivative gives the nucleon energy, which cannot be considered a small quantity. However, the combination $(i \not{D} - m) \Psi$ is of first chiral order. The minus sign for the charge and hermitian conjugation of $D_\mu \Psi$ as well as the chiral dimension of $\gamma_5$ in Table 2 is formal and requires some comment, which will be given below.

|                | $\gamma_5$ | $\gamma_\mu$ | $\gamma_\mu \gamma_5$ | $\sigma_{\mu\nu}$ | $g_{\mu\nu}$ | $\varepsilon_{\lambda\mu\nu\rho}$ | $D_\mu \Psi$ |
|----------------|----------|-------------|---------------------|---------------------|--------------|---------------------------------|-------------|
| chiral dimension | 1        | 0           | 0                   | 0                   | 0            | 0                               | 0           |
| parity          | -        | +           | -                   | -                   | -            | -                               | -           |
| charge conjugation | +      | -           | +                   | +                   | +            | -                               | -           |
| hermitian conjugation | -    | +           | -                   | +                   | +            | -                               | -           |

Table 2: Transformation properties and chiral dimension of the elements of the Clifford algebra together with the metric and Levi-Civita tensors as well as the covariant derivative acting on the nucleon field.
2.2 Invariant monomials

We are now in the position to combine these building blocks to form invariant monomials.

Any invariant monomial in the effective $\pi N$ Lagrangian is of the generic form

$$\bar{\Psi} A^{\mu\nu...} \Theta_{\mu\nu...} \Psi + \text{h.c.} \ .$$

(2.11)

Here, the quantity $A^{\mu\nu...}$ is a product of pion and/or external fields and their covariant derivatives. $\Theta_{\mu\nu...}$, on the other hand, is a product of a Clifford algebra element $\Gamma_{\mu\nu...}$ and a totally symmetrized product of $n$ covariant derivatives acting on nucleon fields, $D_{\alpha\beta...\omega}^n = \{D_{\alpha}, \{D_{\beta}, \{\ldots, D_{\omega}\}\}\},$

$$\Theta_{\mu\nu...\alpha\beta...} = \Gamma_{\mu\nu...} D_{\alpha\beta...}^n \ .$$

(2.12)

The Clifford algebra elements are understood to be expanded in the standard basis $(1, \gamma_5, \gamma_{\mu}, \gamma_{\mu} \gamma_5, \sigma_{\mu\nu})$ and all the metric and Levi-Civita tensors are included in $\Gamma_{\mu\nu...}$. Note that Levi-Civita tensors may have some upper indices, which are, however, contracted with other indices within $\Theta_{\mu\nu...}$. The structure given in eq.(2.11) is not the most general Lorentz invariant structure, it already obeys some of the restrictions dictated by chiral symmetry. The curvature relation eq.(2.7) manifests itself in the fact that we only consider symmetrized products of covariant derivatives acting on $\Psi$. Another feature of eq.(2.11) is that, except for the $\varepsilon$-tensors, no two indices of $\Theta_{\mu\nu...}$ are contracted with each other. The reason for this lies in the fact that at a given chiral order, $\bar{D} \Psi$ can always be replaced by $-i m \Psi$, since their difference $(\bar{D} + i m) \Psi$ is of higher order. One can therefore ignore $D_{\mu} \Psi$ contracted with $\gamma_{\mu}, \gamma_{5}\gamma_{\mu}$ and also with $\sigma^{\lambda\mu} = i \gamma^{\lambda} \gamma^\mu - ig^{\lambda\mu}$. The last relation explains also why $g^{\lambda\mu} \{D_{\lambda}, D_{\mu}\} \Psi$ can be ignored. Other important restrictions on the structure of $\Theta_{\mu\nu...}$ are discussed in appendix A.

To get the complete list of terms contributing to $A^{\mu\nu...}$, one writes down all possible products of the pionic and external fields and covariant derivatives thereof. Every index of $A^{\mu\nu...}$ increases the chiral order by one, therefore the overall number of indices of $A^{\mu\nu...}$ (as well as lower indices of $\Theta_{\mu\nu...}$) is constrained by the chiral order under consideration. Since the matrix fields do not commute, one has to take all the possible orderings. To get $A^{\mu\nu...}$ with simple transformation properties under charge and hermitian conjugation, all the products are rewritten in terms of commutators and anticommutators. In the SU(2) case, this is equivalent to decomposing each product of any two terms into isoscalar and isovector parts, since the only nontrivial product is that of two traceless matrices, and the isoscalar (isovector) part of this product is given by an anticommutator (commutator) of the matrices. After such a rearrangement of products has been performed, the following relations hold:

$$A^\dagger = (-1)^{h_A} A \ , \quad A^c = (-1)^{c_A} A^T \ ,$$

(2.13)

where $(-1)^{h_A}$ and $(-1)^{c_A}$ are determined by the signs from Table 1 (as products of factors $\pm 1$ for every field and covariant derivative, and an extra factor $-1$ for every commutator). For the Clifford algebra elements one has similar relations,

$$\Gamma^\dagger = (-1)^{h_{\Gamma}} \gamma_0 \Gamma \gamma^0 \ , \quad \Gamma^c = (-1)^{c_{\Gamma}} \Gamma^T \ ,$$

(2.14)

where $h_{\Gamma}$ and $c_{\Gamma}$ are determined by the signs from Table 2.

Systematic methods to construct these invariant monomials can also be found in refs.[6, 21].

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The monomial eq.(2.11) can now be written as
\[ \bar{\Psi} A^{\mu \nu \ldots \alpha \beta \ldots} \Gamma_{\mu \nu \ldots} D_n^{\alpha \beta \ldots} \Psi + (-1)^{h_A + h_{\Gamma}} \bar{\Psi} \tilde{D}_n^{\alpha \beta \ldots} \Gamma_{\mu \nu \ldots} A^{\mu \nu \ldots \alpha \beta \ldots} \Psi . \] (2.15)

After elimination of total derivatives (\( \tilde{D}_n \rightarrow (-1)^n D_n \)) and subsequent use of the Leibniz rule, one obtains (modulo higher order terms with derivatives acting on \( A^{\mu \nu \ldots} \))
\[ \bar{\Psi} A^{\mu \nu \ldots \alpha \beta \ldots} \Gamma_{\mu \nu \ldots} D_n^{\alpha \beta \ldots} \Psi + (-1)^{h_A + h_{\Gamma} + n} \bar{\Psi} A^{\mu \nu \ldots \alpha \beta \ldots} \Gamma_{\mu \nu \ldots} D_n^{\alpha \beta \ldots} \Psi . \] (2.16)

We see that to a given chiral order, the second term in eq.(2.15) either doubles or cancels the first one. If the later is true, i.e. if \( h_A + h_{\Gamma} + n \) is odd, this term only contributes at higher orders and can thus be ignored. Consider now charge conjugation acting on the remaining terms of the type given in eq.(2.16) (again modulo higher order terms, with derivatives acting on \( A^{\mu \nu \ldots} \))
\[ 2 \bar{\Psi} A^{\mu \nu \ldots \alpha \beta \ldots} \Gamma_{\mu \nu \ldots} D_n^{\alpha \beta \ldots} \Psi + (-1)^{c_A + c_{\Gamma} + n} 2 \bar{\Psi} A^{\mu \nu \ldots \alpha \beta \ldots} \Gamma_{\mu \nu \ldots} D_n^{\alpha \beta \ldots} \Psi . \] (2.17)

By the same reasoning as before, the terms with odd \( c_A + c_{\Gamma} + n \) are to be discarded. The formal minus sign for the charge and hermitian conjugation of \( D_\mu \Psi \) in Table 2 takes care of this in a simple way. With this convention, for any \( \Theta_{\mu \nu \ldots} \), the two numbers \( h_\Theta \) and \( c_\Theta \) are determined by the entries in Table 2 \( (h_\Theta = h_{\Gamma} + n, c_\Theta = c_{\Gamma} + n) \) and invariant monomials eq.(2.11) of a given chiral order are obtained if and only if
\[ (-1)^{h_A + h_\Theta} = 1 , \quad (-1)^{c_A + c_\Theta} = 1 . \] (2.18)

### 2.3 Elimination of terms

The list of invariant monomials generated by the complete lists of \( A^{\mu \nu \ldots} \) and \( \Theta_{\mu \nu \ldots} \) together with the condition (2.18) is still overcomplete. It contains linearly dependent terms, which can be reduced to a minimal set by use of various identities. In this paragraph, we will discuss the different types of such identities which allow to eliminate the linearly dependent terms.

First of all, there are general identities, like the cyclic property of the trace, implying
\[ \langle a [b,c] \rangle = \langle c [a,b] \rangle , \] (2.19)
or Schouten’s identity
\[ \varepsilon^{\lambda \nu \rho \tau} a^\lambda a^\nu + \varepsilon^{\mu \nu \rho \tau} a^\mu a^\nu + \varepsilon^{\rho \tau \lambda \mu} a^\nu + \varepsilon^{\tau \lambda \mu} a^\rho = 0 . \] (2.20)

Another general identity, frequently used in the construction of chiral Lagrangians, is provided by the Cayley-Hamilton theorem. For 2×2 matrices \( a \) and \( b \), this theorem just implies
\[ \{ a, b \} = a \langle b \rangle + \langle a \rangle b + \langle ab \rangle - \langle a \rangle \langle b \rangle . \] (2.21)

This was already accounted for by the separate treatment of the traces and the traceless matrices. However, there are other nontrivial identities among products of traceless matrices. Let us explicitly mention one such identity, which turns out to reduce the number of independent terms containing four \( u \) fields. It reads
\[ \{ a, \tilde{b} \} [\tilde{a}, \tilde{c}] - \{ \tilde{a}, \tilde{c} \} [\tilde{a}, b] = \{ \tilde{a}, \tilde{a} \} [\tilde{b}, \tilde{c}] - \tilde{a} \{ \tilde{a}, \tilde{b}, \tilde{c} \} \] , (2.22)
where $\tilde{a}, \tilde{b}, \tilde{c}$ are traceless $2 \times 2$ matrices.

Another set of identities is provided by the curvature relation eq.(2.7). In connection with the Bianchi identity for covariant derivatives,

$$[D_{\lambda}, [D_{\mu}, D_{\nu}]] + \text{cyclic} = 0 ,$$  

(2.23)

where “cyclic” stands for cyclic permutations, it entails

$$[D_{\lambda}, F^{\pm}_{\mu\nu}] + \text{cyclic} = \frac{i}{2} [u_{\lambda}, F^{-}_{\mu\nu}] + \text{cyclic} ,$$  

(2.24)

where we have used the Leibniz rule and eq.(2.8) on the right-hand-side. On the other hand, when combined with eq.(2.8), the curvature relation gives

$$[D_{\lambda}, F^{-}_{\mu\nu}] + \text{cyclic} = \frac{i}{2} [u_{\lambda}, F^{\pm}_{\mu\nu}] + \text{cyclic} ,$$  

(2.25)

where we have used the Jacobi identity $[[u_{\lambda}, u_{\mu}], u_{\nu}] + \text{cyclic} = 0$ on the right-hand-side. These relations can be used for the elimination of (some) terms which contain $[D_{\lambda}, F^{\pm}_{\mu\nu}]$.

Yet another set of identities is based on the equations of motion (EOM) deduced from the lowest order $\pi\pi$

$$[D_{\mu}, u^\nu] = \frac{i}{2} \chi^- ,$$  

(2.26)

and $\pi N$ Lagrangians (its explicit form is given in paragraph 3.1)

$$\left( i \not{D} - m + \frac{1}{2} g_A u^\nu \not{\gamma^5} \right) \Psi = 0 ,$$  

(2.27)

$$\Psi \left( i \not{\bar{D}} + m - \frac{1}{2} g_A u^\nu \not{\gamma^5} \right) = 0 .$$  

(2.28)

Strictly speaking, in these equations $m$ and $g_A$ refer to the nucleon mass and the axial-vector coupling constant in the SU(2) chiral limit ($m_u = m_d = 0$, $m_s$ fixed). We will not further specify this but it should be kept in mind. One can directly use these EOM or (equivalently) perform specific field redefinitions — both techniques yield the same result. The pion EOM is used to get rid of all the terms containing $h^\mu_{\nu}$, as well as $[D^\mu, h_{\nu}]$, which can be eliminated using eq.(2.26) together with eqs.(2.7–2.8). The nucleon EOM, the main effect of which is a remarkable restriction of the structure of $\Theta_{\mu\nu\tau...}$, is discussed in full generality in appendix A. Here, we prefer to follow the procedure adopted in ref.[9] and give only the specific relations based on partial integrations and the nucleon EOM used in the reduction from the overcomplete to the minimal set of terms. Some of these relations have already appeared in [9], but for completeness we prefer to show them all. They read:

$$\bar{\Psi} A^\mu i D_\mu \Psi + \text{h.c.} \doteq 2m \bar{\Psi} \gamma_\mu A^\mu \Psi ,$$  

(2.29)

$$\bar{\Psi} A^{\mu\nu} D_\nu D_\mu \Psi + \text{h.c.} \doteq -m \left( \bar{\Psi} \gamma_\mu A^{\mu\nu} i D_\nu \Psi + \text{h.c.} \right) ,$$  

(2.30)

$$\bar{\Psi} A^{\mu\nu\lambda} i D_\lambda D_\nu D_\mu \Psi + \text{h.c.} \doteq m \left( \bar{\Psi} \gamma_\mu A^{\mu\nu\lambda} D_\lambda D_\nu \Psi + \text{h.c.} \right) ,$$  

(2.31)

$$\bar{\Psi} \gamma_5 \gamma_\lambda A^{\mu\lambda} i D_\mu \Psi + \text{h.c.} \doteq 2i m \bar{\Psi} \gamma_5 \sigma_{\mu\lambda} A^{\mu\lambda} \Psi + \left( \bar{\Psi} \gamma_5 \gamma_\mu A^{\mu\lambda} i D_\lambda \Psi + \text{h.c.} \right) ,$$  

(2.32)
where
\[ \bar{\Psi} \gamma_5 \lambda A^\mu \lambda \alpha D_\alpha D_\mu \Psi + \text{h.c.} \triangleq m \left( \bar{\Psi} \gamma_5 \sigma_{\mu \lambda} A^{\mu \lambda \alpha} D_\alpha \Psi + \text{h.c.} \right) \]
\[ + \left( \bar{\Psi} \gamma_5 \gamma_\mu A^{\mu \lambda \alpha} D_\alpha D_\lambda \Psi + \text{h.c.} \right), \] (2.33)
\[ \bar{\Psi} \gamma_5 \gamma_\lambda A^{\mu \lambda \alpha \beta} i D_\beta D_\alpha D_\mu \Psi + \text{h.c.} \triangleq m \left( i \bar{\Psi} \gamma_5 \sigma_{\mu \lambda} A^{\mu \lambda \alpha \beta} D_\beta D_\alpha \Psi + \text{h.c.} \right) \]
\[ + \left( \bar{\Psi} \gamma_5 \gamma_\mu A^{\mu \lambda \alpha \beta} i D_\beta D_\alpha D_\lambda \Psi + \text{h.c.} \right), \] (2.34)
\[ \bar{\Psi} \sigma_{\alpha \beta} A^{\alpha \beta \mu} D_\mu D_\nu \Psi + \text{h.c.} \triangleq -2m \bar{\Psi} \epsilon_{\alpha \beta \mu \nu} \gamma_5 \gamma^\nu A^{\alpha \beta \mu} \Psi - \left( \bar{\Psi} \sigma_{\beta \mu} A^{\alpha \beta \mu} i D_\alpha \Psi + \text{h.c.} \right) \]
\[ + \left( \bar{\Psi} \sigma_{\alpha \mu} A^{\alpha \beta \mu} D_\beta D_\nu \Psi + \text{h.c.} \right), \] (2.35)
\[ \bar{\Psi} \gamma_5 \sigma_{\alpha \beta} A^{\alpha \beta \mu} D_\mu D_\nu \Psi + \text{h.c.} \triangleq - \left( \bar{\Psi} \gamma_5 \sigma_{\beta \mu} A^{\alpha \beta \mu} D_\alpha \Psi + \text{h.c.} \right) \]
\[ + \left( \bar{\Psi} \gamma_5 \sigma_{\alpha \mu} A^{\alpha \beta \mu} D_\beta D_\nu \Psi + \text{h.c.} \right), \] (2.36)
\[ i \bar{\Psi} \gamma_5 \sigma_{\alpha \beta} A^{\alpha \beta \mu} D_\nu D_\mu \Psi + \text{h.c.} \triangleq - \left( i \bar{\Psi} \gamma_5 \sigma_{\beta \mu} A^{\alpha \beta \mu} D_\nu D_\alpha \Psi + \text{h.c.} \right) \]
\[ + \left( i \bar{\Psi} \gamma_5 \sigma_{\alpha \mu} A^{\alpha \beta \mu} D_\nu D_\beta \Psi + \text{h.c.} \right), \] (2.37)
\[ \bar{\Psi} \gamma_\mu [i D^\mu, A] \Psi \triangleq \frac{g_A}{2} \bar{\Psi} \gamma_\mu \gamma_5 [A, u_\mu] \Psi, \] (2.39)
\[ \bar{\Psi} \gamma_5 \gamma_\mu [i D^\mu, A] \Psi \triangleq -2m \bar{\Psi} \gamma_5 A \Psi - \frac{g_A}{2} \bar{\Psi} \gamma_\mu [A, u_\mu] \Psi, \] (2.40)
\[ \bar{\Psi} \gamma_5 \gamma_\nu [D^\mu, A^\nu] i D_\mu \Psi + \text{h.c.} \triangleq 0, \] (2.41)
\[ \bar{\Psi} \epsilon_{\alpha \beta \mu \lambda} \gamma_5 [D^\nu, A^{\alpha \beta \mu}] i D_\nu \Psi + \text{h.c.} \triangleq 0, \] (2.42)
\[ \bar{\Psi} [D^\mu, A^\nu] D_\nu D_\mu \Psi + \text{h.c.} \triangleq 0, \] (2.43)
\[ \bar{\Psi} \sigma_{\alpha \beta} [D^\mu, A^{\alpha \beta \nu}] D_\nu D_\mu \Psi + \text{h.c.} \triangleq 0. \] (2.44)

Here, the symbol \( \triangleq \) means equal up to terms of higher order.

### 2.4 Heavy baryon projection

The heavy baryon (HB) projection of the relativistic theory is well documented in the literature, see e.g. refs. [10, 7, 8]. Here, we only collect some basic definitions and formulae which are needed in the following. For a more detailed exposition, we refer to the review [4]. The procedure we adopt follows closely the one in ref. [7], therefore we only give some steps for completeness. Basically, one considers the mass of the nucleon large compared to the typical external momenta transferred by pions or external probes and writes the nucleon four–momentum as \( p_\mu = m v_\mu + \ell_\mu, \) \( p^2 = m^2, \)
subject to the condition that \( v \cdot \ell \ll m. \) Here, \( v_\mu \) is the nucleon four–velocity (in the rest–frame, we have \( v_\mu = (1, 0) \)). Note again that strictly speaking, the nucleon mass appearing here should
be taken at its value in the chiral limit. Consequently, one can decompose the wavefunction $\Psi(x)$ into velocity eigenstates

$$\Psi(x) = \exp[-imv \cdot x] \left[ N_v(x) + h_v(x) \right]$$

with

$$\not{v} N_v = N_v, \quad \not{v} h_v = -h_v,$$

or in terms of velocity projection operators $P_v^\pm$

$$P_v^+ N_v = N_v, \quad P_v^- h_v = h_v, \quad P_v^\pm = \frac{1}{2} (1 \pm \not{v}), \quad P_v^+ + P_v^- = 1.$$

One now eliminates the small component $h_v(x)$ either by using the equations of motion or path-integral methods. The Dirac equation for the velocity–dependent nucleon field $N_v$ takes the form

$$iv \cdot \partial N_v = 0$$

to lowest order in $1/m$. Apart from the four-velocity, the covariant spin-operator $S_\mu$ is of crucial importance in the HB projection. Its definition and basic properties are:

$$S_\mu = \frac{i}{2} \gamma_5 \sigma_{\mu \nu} v^\nu, \quad S \cdot v = 0, \quad \{S_\mu, S_\nu\} = \frac{1}{2} (v_\mu v_\nu - g_{\mu \nu}) \cdot \{S_\mu, S_\nu\} = i\epsilon_{\mu \nu \alpha \beta} v^\alpha S^\beta,$$

in the convention $\epsilon^{0123} = -1$. After this projection, the Lagrangian does not contain the nucleon mass term any more and also, all Dirac matrices can be expressed as combinations of $v_\mu$ and $S_\mu$. Therefore, one can translate all terms of the relativistic Lagrangian into their heavy fermion counterparts. In addition, there are $1/m$ corrections to the various operators. These can be worked out along the lines spelled out in appendix A of ref. [7]. The $\pi N$ Lagrangian to fourth order for the “light” components $N_v$ then becomes:

$$\mathcal{L}_{\pi N} = \bar{N}_v \left\{ A^{(1)} + A^{(2)} + \gamma_0 B^{(1)} + \gamma_0 C^{-1(0)} B^{(1)} \right. + A^{(3)} + \gamma_0 B^{(1)} + \gamma_0 C^{-1(1)} B^{(1)} + \gamma_0 B^{(2)} + \gamma_0 C^{-1(0)} B^{(1)} + \gamma_0 B^{(1)} + \gamma_0 C^{-1(0)} B^{(2)} + A^{(4)} + \gamma_0 B^{(1)} + \gamma_0 C^{-1(2)} B^{(1)} + \gamma_0 B^{(2)} + \gamma_0 C^{-1(1)} B^{(1)} + \gamma_0 B^{(1)} + \gamma_0 C^{-1(1)} B^{(2)} + \gamma_0 B^{(2)} + \gamma_0 C^{-1(0)} B^{(2)} + \gamma_0 B^{(3)} + \gamma_0 C^{-1(0)} B^{(1)} + \gamma_0 B^{(1)} + \gamma_0 C^{-1(0)} B^{(3)} \right\} N_v,$$

in the standard notation. Explicit expressions for the operators $A^{(i)}$, $B^{(i)}$, and $C^{-1(i)}$ are collected in appendix B.

### 2.5 Checks

To arrive at the final form of the Lagrangian, which contains more than 100 independent terms, one has to perform rather lengthy algebraic manipulations. Due to the large number of terms, these calculations are prone to errors. To eliminate the possibility of making simple algebraic errors, we went through the whole calculation in two independent ways. We really calculated everything by hand, but simultaneously we have written codes in Mathematica for the construction of the relativistic Lagrangian as well as for the extraction of its HB projection. The Mathematica programs were not used just to check the hand-made algebraic manipulations step by step, we have rather used different approaches in the codes wherever possible. The final coherence of the results obtained in these two different ways is a very nontrivial cross–check.
3 The Lagrangian

The resulting effective Lagrangian is given by a string of terms with increasing chiral dimension,

\[ L_{\pi N} = L^{(1)}_{\pi N} + L^{(2)}_{\pi N} + L^{(3)}_{\pi N} + L^{(4)}_{\pi N} + \ldots, \]  

(3.1)

where the ellipsis denotes terms of chiral dimension five (or higher). We will first briefly summarize the first three terms in this expansion and then display the novel complete and minimal fourth order Lagrangian. We always give the Lagrangian in the relativistic as well as in the heavy baryon formulation.

3.1 Dimension one, two and three

At lowest order, the effective \( \pi N \) Lagrangian is given in terms of two parameters, the nucleon mass and the axial–vector coupling constant (in the chiral limit). In its relativistic form, it reads

\[ L^{(1)}_{\pi N} = \bar{\Psi} \left( i \not{D} - m + \frac{g_A}{2} \not{u} \gamma_5 \right) \Psi. \]  

(3.2)

In the HB formulation, the mass term is absent and all Clifford algebra elements can be expressed in terms of the nucleon four–velocity and the spin–vector,

\[ \tilde{L}^{(1)}_{\pi N} = \bar{N}_v \left( iv \cdot D + g_A S \cdot u \right) N_v. \]  

(3.3)

At second order, seven independent terms with LECs appear, so that the relativistic Lagrangian reads (the explicit form of the various operators \( O^{(2)}_i \) is given in Table 3)

\[ L^{(2)}_{\pi N} = \sum_{i=1}^7 c_i \bar{\Psi} O^{(2)}_i \Psi. \]  

(3.4)

The LECs \( c_i \) are, of course, finite. The HB projection is straightforward. We work here with the standard form (see e.g. refs. [7, 4]) and do not transform away the \((v \cdot D)^2\) term from the kinetic energy (as it was done e.g. in ref. [8]),

\[ \tilde{L}^{(2)}_{\pi N} = \frac{1}{2m} \bar{N}_v \left( (v \cdot D)^2 - D^2 - ig_A \left\{ S \cdot D, v \cdot u \right\} \right) N_v + \sum_{i=1}^7 \hat{c}_i \bar{N}_v \hat{O}^{(2)}_i N_v. \]  

(3.5)

Again, the monomials \( \hat{O}^{(2)}_i \) are listed in Table 3 together with the \(1/m\) corrections, which some of these operators receive (this splitting is done mostly to have an easier handle on estimating LECs via resonance saturation [22]).

At third order, one has 23 independent terms. We follow the notation of ref. [4] (see also [8]),

\[ L^{(3)}_{\pi N} = \sum_{i=1}^{23} d_i \bar{\Psi} O^{(3)}_i \Psi. \]  

(3.6)

The LECs \( d_i \) decompose into a renormalized scale–dependent and an infinite (also scale–dependent) part in the standard manner, \( d_i = d^r_i(\lambda) + \kappa_i/F^2 L(\lambda) \), with \( \lambda \) the scale of dimensional regularization,
Table 3: Independent dimension two operators for the relativistic and the HB Lagrangian. The 1/m corrections to these operators in the HB formulation are also displayed.

\[
\sum_{i=1}^{23} \tilde{d}_i \tilde{N}_v \tilde{O}_i^{(3)} N_v + \tilde{N}_v \tilde{O}_i^{\text{fixed}} N_v + \tilde{N}_v \tilde{O}_i^{\text{div}} N_v ,
\]

with the corresponding operators and 1/m corrections collected in Table 3 and \( \tilde{O}_i^{(3)} \) given by \[(3) \]

\[
\tilde{O}_i^{\text{fixed}} = \frac{g_A}{8m^2} [D^\mu, D_\mu, S \cdot u] - \frac{g_A^2}{32m^2} \epsilon^{\mu\nu\alpha\beta} v_\alpha S_\beta \langle F^-_{\mu\nu} v \cdot u \rangle - \frac{1}{4m^2} (v \cdot D)^3
\]

\[
- \frac{g_A}{4m^2} v \cdot D S \cdot u v \cdot D + \frac{1}{8m^2} (i D^2 v \cdot D + \text{h.c.})
\]

\[
- \frac{g_A}{4m^2} \left( (S \cdot D, v \cdot u v \cdot D + \text{h.c.}) + \frac{3g_A^2}{64m^2} (i \langle (v \cdot u)^2 \rangle v \cdot D + \text{h.c.}) \right)
\]

\[
+ \frac{1}{32m^2} \left( \epsilon^{\mu\nu\alpha\beta} v_\alpha S_\beta [u_\mu, u_\nu] v \cdot D + \text{h.c.} \right) - \frac{1}{16m^2} \left( i \epsilon^{\mu\nu\alpha\beta} v_\alpha S_\beta \tilde{F}^+_{\mu\nu} v \cdot D + \text{h.c.} \right)
\]

\[
- \frac{1}{32m^2} \left( i \epsilon^{\mu\nu\alpha\beta} v_\alpha S_\beta \langle F^+_{\mu\nu} v \cdot D + \text{h.c.} \rangle \right)
\]

\[
- \frac{g_A}{8m^2} (S \cdot D^2 + \text{h.c.}) - \frac{g_A}{4m^2} \left( S \cdot D^2 u \cdot D + \text{h.c.} \right)
\]

\[
- \frac{1 + 2c_6}{8m^2} \left( i \epsilon^{\mu\nu\alpha\beta} v_\alpha S_\beta \tilde{F}^+_{\mu\nu} v^\sigma \sigma D_\nu + \text{h.c.} \right) - \frac{1 + 2c_6 + 4c_7}{16m^2} \left( i \epsilon^{\mu\nu\alpha\beta} v_\alpha S_\beta \langle F^+_{\mu\nu} v^\sigma \sigma D_\nu + \text{h.c.} \rangle \right)
\]

\[
+ \frac{1 + 2c_6 + 8m c_4}{16m^2} \left( \epsilon^{\mu\nu\alpha\beta} v_\alpha S_\beta [u_\mu, v \cdot u] D_\nu + \text{h.c.} \right)
\]

\[
+ \frac{g_A}{32m^2} \left( i \epsilon^{\mu\nu\alpha\beta} v_\alpha F^-_{\mu\nu} D_\beta + \text{h.c.} \right) - \frac{g_A^2}{16m^2} (i v \cdot u u \cdot D + \text{h.c.})
\]

\[
+ i \frac{1 + 8m c_4}{32m^2} [v \cdot u, [D^\mu, u_\mu]] + \frac{c_2}{2m} (i \langle v \cdot u u_\mu \rangle D^\mu + \text{h.c.}) .
\]

In addition, there are eight terms just needed for the renormalization (we give these here for
completeness). Their explicit form is:

\[
\tilde{O}_{\text{div}}^{(3)} = \tilde{d}_{24}(\lambda) (i \langle v \cdot D \rangle^3 + \tilde{d}_{25}(\lambda) (v \cdot D + \tilde{d}_{26}(\lambda)(i \langle u \cdot u \rangle v \cdot D + \text{h.c.})) + \tilde{d}_{27}(\lambda) (i \langle (v \cdot u)^2 \rangle v \cdot D + \text{h.c.}) + \tilde{d}_{28}(\lambda) (i \langle \chi_+ \rangle v \cdot D + \text{h.c.}) + \tilde{d}_{29}(\lambda) (S^\mu[v \cdot D, u_\mu] v \cdot D + \text{h.c.}) + \tilde{d}_{30}(\lambda) \left( \epsilon^{\mu\nu\alpha\beta} u_\alpha S_\beta [u_\mu, u_\nu] v \cdot D + \text{h.c.} \right) + \tilde{d}_{31}(\lambda) \left( \epsilon^{\mu\nu\alpha\beta} S_\alpha S_\beta v F_{\mu\nu}^+ v \cdot D + \text{h.c.} \right).
\]

(3.9)

The LECs \( \tilde{d}_i \) have no finite piece \( \tilde{d}_i \).

| \( i \) | \( O^{(3)} \) | \( \tilde{O}^{(3)} \) | \( 4m^2(d_i - \tilde{d}_i) \) |
|-----|----------------|----------------|------------------|
| 1   | \(-\frac{1}{2m}[u_\mu, [D_\nu, u_\mu]]D^\nu + \text{h.c.} \) | \( i[u_\mu, [v \cdot D, u_\mu]] \) | 0 |
| 2   | \(-\frac{1}{2m}[u_\mu, [D^\mu, v_\nu]]D^\nu + \text{h.c.} \) | \( i[u_\mu, [D^\mu, v \cdot u]] \) | \(-\frac{1}{8}(1 + 8mc_4) \) |
| 3   | \( \frac{1}{12m^3}[u_\mu, [D_\nu, u_\mu]]D^{\nu\rho} + \text{h.c.} \) | \( i[v \cdot u, [v \cdot D, v \cdot u]] \) | \( \frac{1}{8}g_A \) |
| 4   | \(-\frac{1}{2m}[i\epsilon^{\mu\nu\alpha\beta} u_\mu u_\nu u_\alpha]D^\beta + \text{h.c.} \) | \( i\epsilon^{\mu\nu\alpha\beta} u_\beta [u_\mu, u_\nu]u_\alpha \) | \( \frac{1}{16}g_A \) |
| 5   | \( \frac{1}{2m}i[\chi_-, u_\mu]D^\mu + \text{h.c.} \) | \( [\chi_-, v \cdot u] \) | 0 |
| 6   | \(-\frac{1}{2m}i[D_\mu, F^+_{\mu\nu}]D^\nu + \text{h.c.} \) | \( v^\mu[D^\mu, F^+_{\mu\nu}] \) | \(-\frac{1}{3}(1 + 2c_6) \) |
| 7   | \(-\frac{1}{2m}i[D_\mu, \langle F^+_{\mu\nu} \rangle]D^\nu + \text{h.c.} \) | \( v^\mu[D^\mu, \langle F^+_{\mu\nu} \rangle] \) | \(-\frac{1}{2}(1 + 2c_6 + 4c_7) \) |
| 8   | \( \begin{aligned} &\frac{1}{12m^3}i\epsilon^{\mu\nu\alpha\beta} (F^+_{\mu\nu} u_\alpha)D^\beta + \text{h.c.} \end{aligned} \) | \( \epsilon^{\mu\nu\alpha\beta} v_\beta (F^+_{\mu\nu} u_\alpha) \) | \( \frac{1}{16}g_A \) |
| 9   | \( \begin{aligned} &\frac{1}{2m}i\epsilon^{\mu\nu\alpha\beta} (F^+_{\mu\nu} u_\alpha)D^\beta + \text{h.c.} \end{aligned} \) | \( \epsilon^{\mu\nu\alpha\beta} v_\beta (F^+_{\mu\nu} u_\alpha) \) | \( \frac{1}{16}g_A \) |
| 10  | \( \frac{i}{16}g_A \) | \( S \cdot u (u_2) \) | 0 |
| 11  | \( \frac{1}{12m}g_A \) | \( S^\mu u^\nu \langle u_\mu, u_\nu \rangle \) | 0 |
| 12  | \( \frac{1}{8m^3}g_A \) | \( S \cdot u \langle [v \cdot u]^2 \rangle \) | \(-\frac{1}{2}g_A (1 + 4mc_4) - \frac{1}{2}g_A^3 \) |
| 13  | \( \frac{1}{8m^2}g_A \) | \( S \cdot u \langle (v \cdot u) \rangle \) | \( \frac{1}{8}g_A (1 + 4mc_4 + \frac{1}{4}g_A^3) \) |
| 14  | \( \frac{1}{8m^3}g_A \) | \( S \cdot u \langle [v \cdot D, u_\mu] \rangle \) | 0 |
| 15  | \( \frac{1}{8m^2}g_A \) | \( -i[S_\mu, S_\nu] \langle [v \cdot D, u_\mu] \rangle u_\nu \) | 0 |
| 16  | \( \frac{1}{8m^2}g_A \) | \( -i[S_\mu, S_\nu] \langle [u_\mu [D_\nu, v \cdot u] \rangle \) | 0 |
| 17  | \( \frac{1}{8m^3}g_A \) | \( S \cdot u \langle \chi_+ \rangle \) | 0 |
| 18  | \( \frac{1}{8m^2}g_A \) | \( (S \cdot u \chi_+) \) | 0 |
| 19  | \( \frac{1}{8m^3}g_A \) | \( i[S_\mu, D_\nu \chi_-] \) | 0 |
| 20  | \( \frac{1}{8m^2}g_A \) | \( -i[S_\mu, D_\nu \chi_-] \) | 0 |
| 21  | \( \frac{1}{8m^3}g_A \) | \( i[S_\mu, D_\nu \chi_-] \) | 0 |
| 22  | \( \frac{1}{8m^2}g_A \) | \( s^{\mu\nu}[F^+_{\mu\nu}, v \cdot u] \) | \( \frac{1}{2}g_A (1 + c_6) \) |
| 23  | \( \frac{1}{8m^3}g_A \) | \( s^{\mu\nu}[F^+_{\mu\nu}, v \cdot u] \) | 0 |

Table 4: Independent dimension three operators for the relativistic and the HB Lagrangian. The 1/m corrections to these operators in the HB formulation are also displayed. Note that we have corrected for two typographical errors which appeared in \( \tilde{O}_{20}^{(3)} \) in the 1/m corrections to the operators \( \tilde{O}_4^{(3)} \) and \( \tilde{O}_{20}^{(3)} \).
This paragraph constitutes the main result of our work. We have found that there are 118 independent dimension four operators, which are in principle measurable. The last four ($i = 115, \ldots, 118$), however, are special in the sense that they are pure contact interactions of the nucleons with the external sources that have no pion matrix elements. For example, the operators 115 and 116 contribute to the scalar nucleon form factor but not to pion–nucleon scattering. The relativistic dimension four Lagrangian takes the form

$$\mathcal{L}_{\pi N}^{(4)} = \sum_{i=1}^{118} e_i \bar{\Psi} O_i^{(4)} \Psi ,$$

(3.10)

and the monomials $O_i^{(4)}$ are tabulated in Table 5. Also given in that table are the processes and observables with the least number of particles (including nucleons, pions, photons and W bosons) to which each operator can contribute. As pointed out in the introduction, many of the operators contribute only to very exotic processes, like three or four pion production induced by photons or pions. Moreover, for a given process, it often happens that some of these operators appear in certain linear combinations. Note also that operators with a separate $\langle \chi^+ \rangle$ simply amount to a quark mass renormalization of the corresponding dimension two operator. The HB projection leads to a much more complicated Lagrangian,

$$\hat{\mathcal{L}}_{\pi N}^{(4)} = \bar{N}_v \left( \sum_{i=1}^{118} \hat{e}_i \hat{O}_i^{(4)} + \sum_{i=1}^{23} e'_i W_i + \sum_{i=1}^{67} \left( e''_i X^\lambda_{ij} D_{ij} + \text{h.c.} \right) \right) + \sum_{i=1}^{23} \left( e'''_{ij} Y_{ij}^{\mu\nu} D_{ij} + \text{h.c.} \right) + \sum_{i=1}^{4} \left( e''''_{ij} Z_{ij}^{\mu\nu} D_{ij} + \text{h.c.} \right) + \frac{1}{8m^3} \left( v \cdot D D_{ij} + \text{h.c.} \right) N_v ,$$

(3.11)

where the first sum contains the 118 low–energy constants in the basis of the heavy nucleon fields. Various additional terms appear: First, there are the leading $1/m$ corrections to (most of) the 118 dimension four operators. These contribute to the difference in the LECs $e_i$ and $\hat{e}_i$; the $\hat{e}_i$ are tabulated in Table 5. Furthermore we have additional terms with fixed coefficients. They can be most compactly represented by counting the number of covariant derivatives acting on the nucleon fields. The corresponding (tensor) structures $W_i$, $X^\lambda_{ij}$, $Y_{ij}^{\mu\nu}$, and $Z_{ij}^{\mu\nu}$ are collected in Tables 7, 8, 9, 10 together with the pertinent coefficients $e'_i, \ldots, e''''_i$. There are two other fixed coefficient terms which are listed in the last line of eq.(3.11).

| $i$ | $O_i^{(4)}$ | $\hat{O}_i^{(4)}$ |
|-----|-------------|-----------------|
| 1   | $\langle u \cdot u \rangle \langle u \cdot u \rangle$ | $\langle u \cdot u \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi \pi \pi N$ |
| 2   | $\langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle$ | $\langle u_\mu u_\nu \rangle \langle u^\mu u^\nu \rangle$ | $\pi N \rightarrow \pi \pi \pi N$ |

Table 5:

#7 We note that the mastertable in ref.[13] contains more terms, 199 to be precise. In that work, however, no attempt was made to find the minimal basis. This is similar to what was done for the dimension three Lagrangian, compare e.g. refs.[23] and [3].
| $i$ | $O_i^{(4)}$ | $\tilde{O}_i^{(4)}$ | $\pi N \rightarrow \pi \pi \pi \pi N$ |
|-----|-------------|----------------|---------------------------------|
| 3   | $-\frac{1}{4m^2} \langle u \cdot u \rangle \langle u_\mu u_\nu \rangle D^{\mu \nu} + \text{h.c.}$ | $\langle u \cdot u \rangle \langle (v \cdot u)^2 \rangle$ | $\pi N \rightarrow \pi \pi \pi \pi N$ |
| 4   | $-\frac{1}{4m^2} \langle u_\mu u_\nu \rangle \langle u^\lambda u^\rho \rangle D^{\mu \nu} + \text{h.c.}$ | $\langle u_\mu v_\nu \rangle \langle u^\lambda v^\rho \rangle \langle (v \cdot u)^2 \rangle$ | $\pi N \rightarrow \pi \pi \pi \pi N$ |
| 5   | $\frac{1}{8m^2} \langle u_\mu u_\nu \rangle \langle u_\mu u_\nu \rangle D^{\lambda \mu \rho} + \text{h.c.}$ | $\langle u_\mu u_\nu \rangle \langle v \cdot u \rangle \langle (v \cdot u)^2 \rangle$ | $\pi N \rightarrow \pi \pi \pi \pi N$ |
| 6   | $-\frac{1}{8m^2} \left[ \sigma^{\mu \nu} [u_\mu, u_\nu] \langle u \cdot u \rangle \sigma^{\mu \nu} \right]$ | $[S^\mu, S^\nu] [u_\mu, u_\nu] \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi \pi \pi \pi N$ |
| 7   | $-\frac{1}{8m^2} \left[ u_\lambda, u_\nu \right] \langle u_\mu u_\rho \rangle \sigma^{\lambda \mu \rho} D^{\nu} + \text{h.c.}$ | $[S^\mu, S^\nu] [u_\mu, u_\nu] \langle u \cdot u \rangle (v \cdot u)^2$ | $\pi N \rightarrow \pi \pi \pi \pi N$ |
| 8   | $\frac{1}{8m^2} \left[ [u_\mu, u_\nu] \mu \right] u^\lambda \sigma^{\mu \nu}$ | $[S^\mu, S^\nu] \langle [u_\mu, u_\nu] \mu \rangle u^\lambda \langle u \cdot u \rangle \sigma^{\mu \nu}$ | $\pi N \rightarrow \pi \pi \pi \pi N$ |
| 9   | $-\frac{1}{8m^2} \langle [u_\lambda, u_\nu \rangle \langle u_\mu u_\rho \rangle \sigma^{\lambda \mu \rho} D^{\nu} + \text{h.c.}$ | $[S^\mu, S^\nu] \langle [u_\mu, u_\nu] \mu \rangle u^\lambda \langle u \cdot u \rangle \sigma^{\mu \nu}$ | $\pi N \rightarrow \pi \pi \pi \pi N$ |
| 10  | $-\frac{1}{4m^2} \langle h_\mu h^\nu \rangle$ | $\langle h_\mu h^\nu \rangle$ | $\pi N \rightarrow \pi N$ |
| 11  | $-\frac{1}{4m^2} \langle h_\mu h^\nu \rangle$ | $\langle h_\mu h^\nu \rangle$ | $\pi N \rightarrow \pi N$ |
| 12  | $\frac{1}{4m^2} \langle h_\mu h^\nu \rangle$ | $\langle h_\mu h^\nu \rangle$ | $\pi N \rightarrow \pi N$ |
| 13  | $\frac{1}{4m^2} \langle h_\mu h^\nu \rangle$ | $\langle h_\mu h^\nu \rangle$ | $\pi N \rightarrow \pi N$ |
| 14  | $\frac{1}{4m^2} \langle h_\mu h^\nu \rangle$ | $\langle h_\mu h^\nu \rangle$ | $\pi N \rightarrow \pi N$ |
| 15  | $\frac{1}{4m^2} \langle h_\mu h^\nu \rangle$ | $\langle h_\mu h^\nu \rangle$ | $\pi N \rightarrow \pi N$ |
| 16  | $\frac{1}{4m^2} \langle h_\mu h^\nu \rangle$ | $\langle h_\mu h^\nu \rangle$ | $\pi N \rightarrow \pi N$ |
| 17  | $\frac{1}{4m^2} \langle h_\mu h^\nu \rangle$ | $\langle h_\mu h^\nu \rangle$ | $\pi N \rightarrow \pi N$ |
| 18  | $\frac{1}{4m^2} \langle h_\mu h^\nu \rangle$ | $\langle h_\mu h^\nu \rangle$ | $\pi N \rightarrow \pi N$ |
| 19  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 20  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle (v \cdot u)^2 \rangle$ | $\pi N \rightarrow \pi N$ |
| 21  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle (v \cdot u)^2 \rangle$ | $\pi N \rightarrow \pi N$ |
| 22  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle (v \cdot u)^2 \rangle$ | $\pi N \rightarrow \pi N$ |
| 23  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 24  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 25  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 26  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 27  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 28  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 29  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 30  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 31  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 32  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 33  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 34  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |
| 35  | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\langle \chi_+ \rangle \langle u \cdot u \rangle$ | $\pi N \rightarrow \pi N$ |

Table 5:
| $i$ | $O_i^{(4)}$ | $\tilde{O}_i^{(4)}$ | $\pi N \rightarrow \pi N$ |
|-----|-------------|-----------------|-----------------|
| 36  | $i \langle u\mu [D^\mu, \bar{\chi}_-] \rangle$ | $i \langle u\mu [D^\mu, \bar{\chi}_-] \rangle$ | $\pi N \rightarrow \pi N$ |
| 37  | $-\frac{i}{2} [u\mu, [D_\nu, \bar{\chi}_-]] \sigma^{\mu\nu}$ | $i[S^\mu, S^\nu] [u\mu, [D_\nu, \bar{\chi}_-]]$ | $\pi N \rightarrow \pi N$ |
| 38  | $\langle \chi_+ \rangle \langle \chi_+ \rangle$ | $\langle \chi_+ \rangle \langle \chi_+ \rangle$ | $m, \pi N \rightarrow \pi N$ |
| 39  | $\bar{\chi}_+ \langle \chi_+ \rangle$ | $\bar{\chi}_+ \langle \chi_+ \rangle$ | $m, \pi N \rightarrow \pi N$ |
| 40  | $\langle \bar{\chi}_+ \bar{\chi}_+ \rangle$ | $\langle \bar{\chi}_+ \bar{\chi}_+ \rangle$ | $m, \pi N \rightarrow \pi N$ |
| 41  | $\bar{\chi}_- \langle \chi_- \rangle$ | $\bar{\chi}_- \langle \chi_- \rangle$ | $\pi^0 N \rightarrow \pi^0 N$ |
| 42  | $i \langle F^+_{\mu\nu} \rangle [u^\mu, u^\nu]$ | $i \langle F^+_{\mu\nu} \rangle [u^\mu, u^\nu]$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 43  | $-\frac{i}{4m_\pi} \langle F^+_{\mu\nu} \rangle [u^\lambda, u_\nu] D^{\mu\nu} + \text{h.c.}$ | $i\mu < F^+_{\mu\nu} > [u^\lambda, v \cdot u]$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 44  | $-\frac{i}{2} \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu}$ | $i[S^\mu, S^\nu] \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 45  | $\frac{1}{2} \langle \gamma^+_{\nu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu}$ | $i[S^\mu, S^\nu] \langle F^+_{\mu\nu} \rangle \langle u^\lambda u_\nu \rangle$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 46  | $\frac{1}{8m_\pi^2} \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu} \sigma^{\mu\nu}$ | $i[S^\mu, S^\nu] \langle F^+_{\mu\nu} \rangle \langle v \cdot u \rangle^2$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 47  | $\frac{1}{8m_\pi^2} \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu} \sigma^{\mu\nu} \sigma^{\mu\nu}$ | $i[S^\mu, S^\nu] \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 48  | $\frac{i}{4m_\pi} \langle F^+_{\lambda\mu} \rangle h_{\nu} \gamma^5 \gamma^\mu \gamma^\nu D^{\nu} + \text{h.c.}$ | $S^\mu v^\nu \langle F^+_{\lambda\mu} \rangle h_{\nu}$ | $\gamma^* N \rightarrow \pi N$ |
| 49  | $\frac{1}{23m_\pi} \langle F^+_{\lambda\mu} \rangle h_{\nu} \gamma^5 \gamma^\mu \gamma^\nu D^{\nu} + \text{h.c.}$ | $S^\mu v^\nu \langle F^+_{\lambda\mu} \rangle h_{\nu}$ | $\gamma^* N \rightarrow \pi N$ |
| 50  | $\frac{i}{4m_\pi} \langle F^+_{\lambda\mu} \rangle h_{\nu} \gamma^5 \gamma^\mu \gamma^\nu D^{\nu} + \text{h.c.}$ | $S^\lambda \nu v^\nu v^\rho \langle F^+_{\lambda\mu} \rangle h_{\nu \rho}$ | $\gamma^* N \rightarrow \pi N$ |
| 51  | $\frac{i}{4m_\pi} \langle F^+_{\lambda\mu} \rangle h_{\nu} \gamma^5 \gamma^\mu \gamma^\nu D^{\nu} + \text{h.c.}$ | $S^\mu v^\nu u^\lambda \langle D_\lambda, \langle F^+_{\lambda\mu} \rangle \rangle$ | $\gamma^* N \rightarrow \pi N$ |
| 52  | $\frac{i}{4m_\pi} \langle F^+_{\lambda\mu} \rangle h_{\nu} \gamma^5 \gamma^\mu \gamma^\nu D^{\nu} + \text{h.c.}$ | $S^\mu v^\nu u^\lambda \langle D_\lambda, \langle F^+_{\lambda\mu} \rangle \rangle$ | $\gamma^* N \rightarrow \pi N$ |
| 53  | $\frac{i}{4m_\pi} \langle F^+_{\lambda\mu} \rangle h_{\nu} \gamma^5 \gamma^\mu \gamma^\nu D^{\nu} + \text{h.c.}$ | $S^\mu v^\nu u^\lambda \langle D_\lambda, \langle F^+_{\lambda\mu} \rangle \rangle$ | $\gamma^* N \rightarrow \pi N$ |
| 54  | $\frac{i}{4m_\pi} \langle F^+_{\lambda\mu} \rangle h_{\nu} \gamma^5 \gamma^\mu \gamma^\nu D^{\nu} + \text{h.c.}$ | $i[S^\mu, S^\nu] [D_\lambda, \langle D_\lambda, \langle F^+_{\lambda\mu} \rangle \rangle]$ | $\text{em. ff.}$ |
| 55  | $i \langle F^+_{\mu\nu} \rangle [u^\mu, u^\nu]$ | $i \langle F^+_{\mu\nu} \rangle [u^\mu, u^\nu]$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 56  | $-\frac{i}{4m_\pi} \langle F^+_{\mu\nu} \rangle [u^\lambda, u^\nu] D^{\mu\nu} + \text{h.c.}$ | $i\mu < F^+_{\mu\nu} > [u^\lambda, v \cdot u]$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 57  | $-\frac{i}{2} \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu}$ | $i[S^\mu, S^\nu] \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 58  | $-\frac{i}{2} \langle F^+_{\mu\nu} \rangle [u^\lambda u^\mu] \sigma^{\mu\nu}$ | $i[S^\mu, S^\nu] \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 59  | $\frac{1}{8m_\pi^2} \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu} \sigma^{\mu\nu} + \text{h.c.}$ | $i[S^\mu, S^\nu] \langle F^+_{\mu\nu} \rangle \langle v \cdot u \rangle^2$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 60  | $\frac{1}{8m_\pi^2} \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu} \sigma^{\mu\nu} \sigma^{\mu\nu} + \text{h.c.}$ | $i[S^\mu, S^\nu] \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 61  | $\frac{i}{2} \langle F^+_{\lambda\mu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu}$ | $i[S^\mu, S^\nu] \langle F^+_{\lambda\mu} \rangle \langle u \cdot u \rangle$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 62  | $\frac{i}{2} \langle F^+_{\lambda\mu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu}$ | $i[S^\mu, S^\nu] \langle F^+_{\lambda\mu} \rangle \langle u \cdot u \rangle$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 63  | $\frac{i}{2} \langle F^+_{\lambda\mu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu}$ | $i[S^\mu, S^\nu] \langle F^+_{\lambda\mu} \rangle \langle u \cdot u \rangle$ | $\gamma^* N \rightarrow \pi \pi N$ |
| 64  | $\frac{i}{8m_\pi^2} \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle \sigma^{\mu\nu} \sigma^{\mu\nu} \sigma^{\mu\nu} + \text{h.c.}$ | $i[S^\mu, S^\nu] \langle F^+_{\mu\nu} \rangle \langle u \cdot u \rangle$ | $\gamma^* N \rightarrow \pi \pi N$ |

Table 5:
| $i$ | $O_i^{(4)}$ | $\tilde{O}_i^{(4)}$ |
|-----|-------------|-----------------|
| 65  | $\frac{1}{8m^2}u_\rho \langle \widetilde{F}_{\mu \lambda}^+ u_\nu \rangle \sigma^{\mu \nu} D^{\lambda \rho} + \text{h.c.}$ | $i[S^\mu, S^\nu] v^\lambda v \cdot u \langle \widetilde{F}_{\mu \lambda}^+ u_\nu \rangle$ | $\gamma N \to \pi \pi N$ |
| 66  | $\frac{1}{8m^2}u_\nu \langle \widetilde{F}_{\mu \lambda}^+ u_\rho \rangle \sigma^{\mu \nu} D^{\lambda \rho} + \text{h.c.}$ | $i[S^\mu, S^\nu] v^\lambda u_\nu \langle \widetilde{F}_{\mu \lambda}^+ v \cdot u \rangle$ | $\gamma N \to \pi \pi N$ |
| 67  | $-\frac{i}{4m^2} \langle \widetilde{F}_{\mu \lambda}^+ h_\nu \rangle \gamma_5 \gamma_\mu D^\nu$ | $S^{\mu \nu} \langle \widetilde{F}_{\mu \lambda}^+ h_\nu \rangle$ | $\gamma N \to \pi N$ |
| 68  | $-\frac{i}{4m^2} \langle \widetilde{F}_{\mu \lambda}^+ h_\nu \rangle \gamma_5 \gamma_\nu D^\mu$ | $S^{\nu \mu} \langle \widetilde{F}_{\mu \lambda}^+ h_\nu \rangle$ | $\gamma N \to \pi N$ |
| 69  | $-\frac{i}{24m^2} \langle \widetilde{F}_{\mu \lambda}^+ h_\rho \rangle \gamma_5 \gamma_\lambda D^{\mu \rho} + \text{h.c.}$ | $S^{\lambda \nu \mu \nu} v^{\rho} \langle \widetilde{F}_{\mu \lambda}^+ h_\rho \rangle$ | $\gamma N \to \pi N$ |
| 70  | $-\frac{i}{4m^2} \langle \widetilde{F}_{\mu \lambda}^+ h_\nu \rangle \varepsilon^{\lambda \mu \nu \rho} D^{\rho \sigma} + \text{h.c.}$ | $i \varepsilon^{\lambda \mu \nu \rho} v^{\tau} v^{\rho} \langle \widetilde{F}_{\mu \lambda}^+ h_\rho \rangle$ | $\gamma N \to \pi N$ |
| 71  | $-\frac{i}{4m^2} \langle \mu^{\nu} D_{\lambda \mu}, \widetilde{F}_{\mu \nu}^+ \rangle \gamma_5 \gamma_\mu D^\nu$ | $S^{\mu \nu} \langle \mu^{\lambda} D_{\lambda \mu}, \widetilde{F}_{\mu \nu}^+ \rangle$ | $\gamma N \to \pi N$ |
| 72  | $-\frac{i}{4m^2} \langle \mu^{\nu} D_{\lambda \mu}, \widetilde{F}_{\mu \nu}^+ \rangle \gamma_5 \gamma_\nu D^\mu$ | $v^{\nu} \langle \mu^{\lambda} S \cdot u D_{\lambda \mu}, \widetilde{F}_{\mu \nu}^+ \rangle$ | $\gamma N \to \pi N$ |
| 73  | $-\frac{i}{4m^2} \langle \mu^{\nu} D_{\lambda \mu}, \widetilde{F}_{\mu \nu}^+ \rangle \gamma_5 \gamma^\nu D^\mu$ | $S^{\nu \nu} \langle \nu^{\nu} v^{\nu} D_{\lambda \mu}, \widetilde{F}_{\mu \nu}^+ \rangle$ | $\gamma N \to \pi N$ |
| 74  | $-\frac{i}{2} \langle D^\lambda, D_{\lambda \mu}, \widetilde{F}_{\mu \nu}^+ \rangle \sigma^{\mu \nu}$ | $i [S^\mu, S^\nu] \langle D^\lambda, D_{\lambda \mu}, \widetilde{F}_{\mu \nu}^+ \rangle$ | em. ff. |
| 75  | $u_\lambda \langle F_{\mu \nu}^+ h_\rho \rangle \varepsilon^{\mu \nu \rho}$ | $\varepsilon^{\mu \nu \rho \nu} u_\lambda \langle F_{\mu \nu}^- h_\rho \rangle$ | $WN \to \pi \pi N$ |
| 76  | $-\frac{i}{4m^2} u_\lambda \langle F_{\mu \nu}^+ h_\rho \rangle \varepsilon^{\mu \nu \rho \nu} D^\lambda + \text{h.c.}$ | $\varepsilon^{\mu \nu \rho \nu} v^\tau u \langle F_{\mu \nu}^- h_\rho \rangle$ | $WN \to \pi \pi N$ |
| 77  | $-\frac{i}{4m^2} u_\lambda \langle F_{\mu \nu}^+ h_\rho \rangle \varepsilon^{\mu \nu \tau} D^{\rho \tau} + \text{h.c.}$ | $\varepsilon^{\mu \nu \rho \nu} v^\tau u \langle F_{\mu \nu}^- h_\rho \rangle$ | $WN \to \pi \pi N$ |
| 78  | $-\frac{i}{4m^2} u_\lambda \langle F_{\mu \nu}^+ h_\rho \rangle \varepsilon^{\mu \nu \tau} D^{\rho \tau}$ | $\varepsilon^{\mu \nu \rho \nu} v^\tau u \langle F_{\mu \nu}^- h_\rho \rangle$ | $WN \to \pi \pi N$ |
| 79  | $\frac{1}{4m^2} \langle F_{\mu \nu}^- [u^\lambda, v^\lambda] \rangle \gamma_5 \gamma^\nu D^\mu + \text{h.c.}$ | $i S^{\mu} \langle F_{\mu \nu}^- [u^\lambda, v^\lambda] \rangle$ | $WN \to \pi \pi N$ |
| 80  | $\frac{1}{4m^2} \langle F_{\mu \nu}^- [u^\lambda, v^\lambda] \rangle \gamma_5 \gamma^\nu D^\mu + \text{h.c.}$ | $i v^{\nu} \langle F_{\mu \nu}^- [u^\lambda, S \cdot u] \rangle$ | $WN \to \pi \pi N$ |
| 81  | $-\frac{i}{4m^2} \langle F_{\mu \nu}^- h_\lambda \rangle \gamma_5 D^{\mu \nu} + \text{h.c.}$ | $v^{\mu} v^{\nu} \langle F_{\mu \nu}^- h_\lambda \rangle$ | $WN \to \pi N$ |
| 82  | $\frac{i}{2} \langle F_{\mu \nu}^- h_\lambda \rangle \sigma^{\mu \nu}$ | $[S^\mu, S^\nu] \langle F_{\mu \nu}^- h_\lambda \rangle$ | $WN \to \pi N$ |
| 83  | $-\frac{i}{8m^2} \langle F_{\mu \nu}^- h_\rho \rangle \gamma^{\mu \nu} D^{\rho} + \text{h.c.}$ | $[S^\mu, S^\nu] \varepsilon^{\mu \nu \rho \nu} \langle F_{\mu \nu}^- h_\rho \rangle$ | $WN \to \pi N$ |
| 84  | $-\frac{i}{8m^2} \langle F_{\mu \nu}^- h_\rho \rangle \sigma^{\mu \nu} D^{\rho}$ | $[S^\mu, S^\nu] \sigma^{\mu \nu} \langle F_{\mu \nu}^- h_\rho \rangle$ | $WN \to \pi N$ |
| 85  | $\langle u^{\mu} [D^\nu, F_{\mu \nu}^+] \rangle$ | $\langle u^{\mu} [D^\nu, F_{\mu \nu}^+] \rangle$ | $WN \to \pi N$ |
| 86  | $-\frac{i}{4m^2} \langle u^{\mu} [D^\nu, F_{\mu \nu}^-] \rangle D^{\mu \nu}$ | $v^{\nu} \langle v \cdot u [D^\lambda, F_{\mu \nu}^-] \rangle$ | $WN \to \pi N$ |
| 87  | $\frac{i}{2} \langle u^{\mu} [D^\nu, F_{\mu \nu}^+] \rangle \sigma^{\mu \nu}$ | $[S^\mu, S^\nu] \langle u^{\lambda} [D^\lambda, F_{\mu \nu}^-] \rangle$ | $WN \to \pi N$ |
| 88  | $\frac{i}{2} \langle u^{\mu} [D^\nu, F_{\mu \nu}^+] \rangle \sigma^{\mu \nu}$ | $[S^\mu, S^\nu] \langle u^{\mu} [D^\lambda, F_{\mu \nu}^-] \rangle$ | $WN \to \pi N$ |
| 89  | $\langle F_{\mu \nu}^+ \rangle \langle F^{+\mu \nu} \rangle$ | $\langle F_{\mu \nu}^+ \rangle \langle F^{+\mu \nu} \rangle$ | $\gamma N \to \gamma N$ |
| 90  | $-\frac{i}{4m^2} \langle F_{\mu \nu}^+ \rangle \langle F^{+\mu \nu} \rangle$ | $v^{\mu} v^{\nu} \langle F_{\mu \nu}^+ \rangle \langle F^{+\mu \nu} \rangle$ | $\gamma N \to \gamma N$ |
| 91  | $\langle F_{\mu \nu}^+ \rangle \langle F^{+\mu \nu} \rangle$ | $\langle F_{\mu \nu}^+ \rangle \langle F^{+\mu \nu} \rangle$ | $\gamma N \to \gamma N$ |

Table 5:
Here, shown is the physical process with the least number of particles to which these operators contribute.

Table 5: Independent dimension four operators for the relativistic and the HB Lagrangian. Also shown is the physical process with the least number of particles to which these operators contribute. Here, $N, \pi, \gamma(*)$, $W$ denote nucleons, pions, real (virtual) external photons and the $W$–boson external sources, respectively. Electromagnetic (weak) form factors are abbreviated as “em. (weak) ff.” and $m$ is the nucleon mass. The last four terms have no pion matrix elements.
| $i$ | $8m^3(\dot{e}_i - e_i)$ |
|-----|------------------|
| 1   | $-\frac{3}{16}g_A^2 + mc_2 - \frac{1}{8}mc_4$ |
| 2   | $\frac{1}{32}g_A^2 - mc_2 + \frac{1}{8}mc_4$ |
| 3   | $\frac{1}{4}(4g_A^2 + g_A^4) - \frac{5}{8}g_A^2mc_3 + \frac{1}{4}(2 + g_A^2)mc_4 + (mc_1)^2$ |
| 4   | $-\frac{1}{16}g_A^2 - \frac{1}{4}(2 + g_A^2)mc_4 - (mc_4)^2 - g_Am^2d_{10} - 2m^2(d_{14} - d_{15})$ |
| 5   | $-\frac{1}{64}g_A^4 - \frac{1}{8}g_A^2mc_2 - g_Am^2(d_{12} + d_{13})$ |
| 6   | $-\frac{1}{32}g_A^2 + \frac{1}{4}mc_3$ |
| 7   | $\frac{1}{32}(g_A^2 + g_A^4) + \frac{1}{4}mc_2 + \frac{3}{4}g_A^2mc_4$ |
| 8   | $-\frac{1}{16}(2 + 4g_A^2 + g_A^4) - (1 + g_A^2)mc_4 - 2(mc_4)^2$ |
| 9   | $-\frac{1}{16}(g_A + g_A^3) + \frac{1}{2}g_Amc_4$ |
| 10  | $-\frac{1}{16}(2g_A + g_A^3) - \frac{1}{2}g_Amc_4$ |
| 11  | $\frac{1}{16}(2g_A + g_A^3) + \frac{3}{4}g_Amc_4$ |
| 12  | $-\frac{1}{16}(3g_A + g_A^3) + \frac{1}{2}g_Amc_4$ |
| 13  | $-\frac{1}{4}mc_2$ |
| 14  | $-\frac{1}{32}g_A^2 - \frac{1}{2}m^2(d_{14} - d_{15})$ |
| 15  | $\frac{1}{32}g_A$ |
| 16  | $-\frac{1}{16}(2 + g_A^2) - \frac{1}{2}mc_4$ |
| 17  | $-\frac{1}{8}g_A^2mc_1 - g_Am^2d_{16}$ |
| 18  | $\frac{1}{4}mc_1$ |
| 19  | $\frac{1}{4}g_Amc_5$ |
| 20  | $-\frac{1}{4}g_Amc_5$ |
| 21  | $-g_Amc_5$ |
| 22  | $-\frac{1}{32}g_A^2 - \frac{1}{8}mc_4 - \frac{1}{32}(c_6 + 2c_7)$ |
| 23  | $\frac{1}{8}g_A + \frac{1}{4}mc_4 + \frac{1}{16}(2 + g_A^2)(c_6 + 2c_7) + \frac{1}{2}mc_4(c_6 + 2c_7)$ |
| 24  | $\frac{1}{32}g_A^2 - \frac{1}{4}mc_3$ |
| 25  | $\frac{1}{8}g_A^2 + \frac{1}{8}g_A^2(c_6 + 2c_7)$ |
| 26  | $-\frac{1}{8}g_A + \frac{1}{8}g_A(c_6 + 2c_7)$ |
| 27  | $\frac{3}{8}g_A + \frac{1}{8}g_A(c_6 + 2c_7)$ |
| 28  | $\frac{1}{4}g_A(1 + c_6 + 2c_7)$ |
| 29  | $-\frac{1}{4}g_A + \frac{1}{4}g_A(c_6 + 2c_7)$ |
| 30  | $\frac{1}{4}g_A(c_6 + 2c_7)$ |
| 31  | $-\frac{1}{32}g_A^2 + mc_2 - \frac{1}{8}mc_4 - \frac{1}{32}c_6$ |
| 32  | $\frac{1}{16}g_A^2 + \frac{1}{4}mc_4 + \frac{1}{16}(2 + g_A^2)c_6 + \frac{1}{4}mc_4c_6$ |
| 33  | $+2m^2(d_{14} - d_{15}) + g_Am^2d_{21}$ |
| 34  | $\frac{1}{16}g_A^2 - \frac{1}{2}mc_3$ |
| 35  | $-\frac{1}{16}g_A^2 - \frac{1}{2}mc_2 - \frac{1}{16}g_A^2c_6$ |

Table 6:
| i  | $8m^2(\hat{e}_i - e_i)$                                              |
|----|---------------------------------------------------------------|
| 60 | $\frac{1}{4}g_A^2c_6$                                         |
| 64 | $\frac{1}{8}g_A^2c_6$                                         |
| 65 | $-\frac{1}{2}(1 + g_A^3) - 2mc_4 - \frac{1}{4}(2 + g_A^3)c_6 - 2mc_4c_6$ |
| 66 | $\frac{1}{4}(2 + g_A^3) + 2mc_4 + \frac{1}{4}(2 + g_A)^3c_6 + 2mc_4c_6$ |
| 67 | $\frac{3}{8}g_A(c_6 - 1)$                                     |
| 69 | $\frac{3}{8}g_A + \frac{1}{8}g_Ac_6$                          |
| 70 | $-\frac{1}{32}g_A(1 + c_6)$                                    |
| 71 | $\frac{1}{4}g_A(1 + c_6)$                                      |
| 72 | $-\frac{1}{4}g_A(1 + c_6)$                                     |
| 73 | $\frac{1}{4}g_Ac_6$                                           |
| 75 | $\frac{1}{32}(-g_A + g_A^3) + \frac{1}{2}g_Amc_4 - \frac{1}{16}g_Ac_6$ |
| 76 | $\frac{1}{32}(g_A + g_A^3) + \frac{1}{4}g_Amc_4 + 2g_Am^2d_{23}$ |
| 77 | $-\frac{1}{32}(g_A + 3g_A^3) - \frac{3}{8}g_Amc_4 + \frac{1}{8}g_Ac_6$ |
| 78 | $\frac{1}{16}g_A^3 + \frac{1}{2}g_Amc_4 - \frac{1}{8}g_Ac_6$    |
| 79 | $-\frac{1}{16}g_A(2 + g_A^3) - \frac{3}{8}g_Amc_4 - \frac{1}{8}g_Ac_6$ |
| 80 | $\frac{1}{16}(4g_A + g_A^3) + \frac{3}{16}g_Amc_4 + \frac{1}{16}g_Ac_6$ |
| 81 | $m^2d_{15}$                                                   |
| 83 | $-\frac{1}{8}(1 + g_A^3) - \frac{1}{2}mc_4$                   |
| 84 | $\frac{1}{16}(1 + g_A^3) + \frac{1}{4}mc_4$                   |
| 86 | $g_Am^2d_{22} + 2m^2d_{15}$                                    |
| 89 | $-\frac{1}{2}mc_2 - \frac{1}{8}(c_6 + 2c_7)$                   |
| 90 | $-\frac{1}{8}g_A^3 + \frac{1}{8}(c_6 + 6c_7) + \frac{1}{16}(c_6 + 2c_7)^2 - \frac{1}{4}m^2(d_{14} + d_{15})$ |
| 91 | $-\frac{1}{8}(c_6 + c_7)$                                      |
| 92 | $\frac{1}{2}(c_6 + c_7) + \frac{3}{4}c_6(c_6 + 2c_7)$          |
| 93 | $-\frac{1}{2}mc_2 - \frac{1}{16}c_6$                          |
| 94 | $-\frac{1}{32}g_A^2 + \frac{1}{8}(2c_6 + c_6^2) - \frac{1}{2}m^2(d_{14} + d_{15})$ |
| 96 | $-\frac{1}{4}(1 + 2c_6 + c_6^2)$                               |
| 98 | $-\frac{1}{16}g_A^2$                                          |
| 99 | $-\frac{1}{4}g_A(c_6 + 2c_7)$                                  |
| 100| $\frac{1}{8}g_A(1 + c_6 + 2c_7)$                               |
| 101| $-\frac{1}{4}g_Ac_6$                                          |
| 102| $\frac{1}{8}g_A(1 + c_6)$                                     |
| 103| $-\frac{1}{16}g_A(1 - c_6)$                                   |
| 104| $-\frac{1}{32}g_A(1 + 3c_6)$                                  |
| 105| $-\frac{1}{8}mc_1$                                            |
| 106| $-mc_1$                                                        |
| 107| $-\frac{1}{8}mc_5$                                            |
| 108| $-\frac{1}{8}mc_5$                                            |
| 109| $\frac{1}{2}g_Amc_5$                                          |

Table 6:
Table 6: $1/m$ corrections to the dimension four LECs $\hat{c}_i$ in the heavy nucleon basis. Terms which are not subject to such corrections are not listed.

| $i$ | $W_i$ | $8m^3c_i$ |
|-----|-------|-----------|
| 1   | $i \langle [S \cdot u, v \cdot u] h_\mu^\nu \rangle$ | $\frac{1}{16} (g_A + g_A^2) + \frac{2}{3} g_A mc_4$ |
| 2   | $\langle h_\mu^\nu h_\nu^\mu \rangle$ | $-\frac{1}{2} mc_2$ |
| 3   | $v^\mu v^\nu \langle [h_\mu^\nu, h_\nu^\mu] \rangle$ | $-\frac{1}{16} g_A^2 + \frac{1}{2} m^2 (d_{14} - d_{15})$ |
| 4   | $\langle u_\mu [D_\mu, h_\nu^\nu] \rangle$ | $-2 mc_2$ |
| 5   | $\langle v \cdot u [v \cdot D, h_\mu^\mu] \rangle$ | $-\frac{1}{16} g_A^2 - m^2 (d_{14} - d_{15})$ |
| 6   | $v^\mu v^\nu [v \cdot D, h_\mu^\nu]$ | $-\frac{1}{3} g_A^2 + m^2 (d_{14} - d_{15})$ |
| 7   | $v^\mu v^\nu \langle v \cdot u [v \cdot D, h_\mu^\nu] \rangle$ | $\frac{1}{4} g_A^2$ |
| 8   | $[S^\nu, S^\rho] v^\rho [u_\nu, [v \cdot D, h_\mu^\rho]]$ | $\frac{1}{4} (1 + g_A^2) + mc_4$ |
| 9   | $iv \cdot u [v \cdot D, \langle \chi_- \rangle]$ | $-g_A m^2 (d_{18} + 2d_{19})$ |
| 10  | $i \langle v \cdot u [v \cdot D, \chi_-] \rangle$ | $-g_A m^2 d_{18}$ |
| 11  | $v^\mu S^\nu \langle F_{\mu \nu}^+ h_\rho^\rho \rangle$ | $-\frac{1}{3} g_A (c_6 + 2c_7)$ |
| 12  | $S^\mu u^\nu [v \cdot D, \langle F_{\mu \nu}^+ \rangle]$ | $-\frac{1}{3} g_A (c_6 + 2c_7)$ |
| 13  | $v^\mu S^\nu v \cdot u [v \cdot D, \langle F_{\mu \nu}^+ \rangle]$ | $\frac{1}{4} g_A (c_6 + 2c_7)$ |
| 14  | $i[S^\nu, S^\rho] \langle v \cdot D, [v \cdot D, \langle F_{\mu \nu}^+ \rangle] \rangle$ | $\frac{1}{4} (1 + c_6 + 2c_7)$ |
| 15  | $v^\mu S^\nu \langle F_{\mu \nu}^+ h_\rho^\rho \rangle$ | $-\frac{1}{3} g_A c_6$ |
| 16  | $S^\mu \langle v^\nu [v \cdot D, F_{\mu \nu}^+] \rangle$ | $-\frac{1}{4} g_A c_6$ |
| 17  | $v^\mu S^\nu \langle v \cdot u [v \cdot D, F_{\mu \nu}^+] \rangle$ | $\frac{1}{4} g_A c_6$ |
| 18  | $i\varepsilon^{\lambda \mu \rho \nu} v_\nu [u_\lambda, [v \cdot D, \bar{F}_{\mu \nu}^+]]$ | $-\frac{1}{16} g_A c_6$ |
| 19  | $i[S^\nu, S^\rho] \langle v \cdot D, [v \cdot D, F_{\mu \nu}^+] \rangle$ | $\frac{1}{4} (1 + g_A^2) + mc_4$ |
| 20  | $i[S^\nu, S^\rho] v^\rho \langle v \cdot D, F_{\mu \nu}^+ \rangle$ | $\frac{1}{4} (1 - g_A^2) + 2 mc_4 - c_6$ |
| 21  | $v^\mu \langle v^\nu [v \cdot D, F_{\mu \nu}^-] \rangle$ | $\frac{1}{8} g_A^2 - m^2 (d_{14} - d_{15})$ |
| 22  | $[S^\nu, S^\rho] \langle v \cdot u, [v \cdot D, F_{\mu \nu}^-] \rangle$ | $-\frac{1}{8} - \frac{1}{2} mc_4$ |
| 23  | $\varepsilon^{\mu \nu \rho \tau} v_\tau [v \cdot D, [D_\mu, F_{\nu \rho}^-]]$ | $-\frac{1}{8} g_A$ |

Table 7: Explicit form of the monomials $W_i$ and their coefficients.
| $i$ | $X_i^\lambda$ | $8m^3e'_i$ |
|-----|----------------|--------------|
| 1   | $i\langle S \cdot u v \cdot u \rangle u^\lambda$ | $\frac{1}{4} (g_A + g_A^3) + 2g_Amc_4 + 4m^2d_{13}$ |
| 2   | $iS \cdot u (v \cdot uu^\lambda)$ | $-\frac{1}{4} (g_A + g_A^3) - 2g_Amc_4 + 8m^2d_{12}$ |
| 3   | $iS^\lambda \langle u \cdot u \rangle v \cdot u$ | $\frac{1}{8} (2g_A + g_A^3) - g_Am (c_3 - c_4) - 4m^2d_{10}$ |
| 4   | $iS^\lambda \langle (v \cdot u)^2 \rangle v \cdot u$ | $-\frac{1}{8}g_A - g_Amc_2 - 4m^2(d_{12} + d_{13})$ |
| 5   | $iS^\lambda \langle v \cdot uu_\mu \rangle u^\mu$ | $-\frac{1}{4}g_A - g_Amc_4 - 4m^2d_{11}$ |
| 6   | $i(S \cdot uu^\lambda)v \cdot u$ | $4m^2d_{13}$ |
| 7   | $iS \cdot u (\langle (v \cdot u)^3 \rangle v^\lambda$ | $\frac{3}{8} (2g_A + g_A^3) + 3g_Amc_4$ |
| 8   | $i(S \cdot uu \cdot u)v \cdot uu^\lambda$ | $-\frac{1}{8} (3g_A + 2g_A^3) - 3g_Amc_4$ |
| 9   | $\varepsilon_{\mu \nu \rho \sigma} \langle v \cdot u [u_\mu, u_\nu] \rangle$ | $\frac{1}{16} (2g_A + g_A^3) + \frac{3}{8}g_Amc_4$ |
| 10  | $\varepsilon_{\mu \nu \rho \sigma} \langle u_\mu u_\nu u_\rho \rangle$ | $4m^2d_4$ |
| 11  | $[h^\mu_\nu, u^\lambda]$ | $\frac{1}{16}g_A$ |
| 12  | $v^\lambda [h^\mu_\nu, v \cdot u]$ | $\frac{1}{16} (g_A^2 + \frac{3}{2}mc_4$ |
| 13  | $[h^\mu_\nu, u_\mu]$ | $\frac{1}{16} (1 + 2g_A^2) - \frac{1}{2}mc_4$ |
| 14  | $v_\mu [h^\lambda_\mu, v \cdot u]$ | $-\frac{1}{16}g_A^2 - \frac{1}{4}mc_4 + 2m^2 (d_1 + d_2)$ |
| 15  | $v^\mu [u^\sigma, v \cdot u]$ | $\frac{1}{16} (g_A^2 - 1) + 4m^2d_3$ |
| 16  | $v^\mu [u^\sigma, v \cdot u]$ | $\frac{1}{16} + 2m^2d_3$ |
| 17  | $[S^\lambda, S^\mu] [h^\lambda_\mu u^\nu]$ | $\frac{1}{8}g_A^2 - mc_3$ |
| 18  | $[u^\mu, u^\sigma] [h^\mu_\nu, v^\lambda]$ | $\frac{1}{16} (1 + 2g_A^2) + \frac{7}{4}mc_4$ |
| 19  | $v^\mu [u^\sigma, v \cdot u]$ | $-\frac{1}{8}g_A^2$ |
| 20  | $[S^\lambda, S^\mu] [h^\lambda_\mu u_\nu]$ | $2m^2 (d_{14} - d_{15})$ |
| 21  | $[S^\lambda, S^\mu] [h^\lambda_\mu v \cdot u]$ | $-\frac{1}{8}g_A^2 - mc_2 + 2m^2 (d_{14} - d_{15})$ |
| 22  | $[S^\lambda, S^\mu] [v^\sigma, u^\nu] [u_\mu h^\nu_\rho]$ | $\frac{1}{8}g_A^2 - 2m^2 (d_{14} - d_{15})$ |
| 23  | $[S^\lambda, S^\mu] [v^\sigma, h^\nu_\rho u^\lambda]$ | $\frac{1}{8}g_A^2$ |
| 24  | $iS^\lambda [v \cdot D, h^\mu_\nu]$ | $\frac{1}{4}g_A$ |
| 25  | $i[S \cdot D, h^\mu_\nu] v^\lambda$ | $\frac{1}{4}g_A$ |
| 26  | $iS^\nu [D^\mu, h_\mu \nu] v^\lambda$ | $-\frac{1}{4}g_A$ |
| 27  | $iS^\nu [v \cdot D, h_\mu \nu] v^\lambda$ | $-\frac{1}{4}g_A$ |
| 28  | $iS^\lambda v^\mu [v \cdot D, h_\mu \nu] v^\lambda$ | $-\frac{1}{4}g_A$ |
| 29  | $iS^\lambda (\lambda_+ v \cdot u$ | $-2g_Amc_1 - 4m^2d_{16}$ |
| 30  | $[S^\lambda, S^\mu] [D^\mu, (\lambda_+)]$ | $-2mc_1$ |
| 31  | $iS^\lambda (\lambda_+ v \cdot u$ | $-g_Amc_5 - 4m^2d_{17}$ |
| 32  | $[S^\lambda, S^\mu] [D^\mu, (\lambda_+) v \cdot u$ | $-2mc_5$ |
| 33  | $[S^\lambda, S^\mu] [v \cdot D, (\lambda_+)]$ | $2m^2 (d_{18} + d_{19})$ |
| 34  | $i[\lambda_-, u^\lambda]$ | $4m^2d_5$ |
| 35  | $S^\lambda [v \cdot D, (\lambda_-)]$ | $4m^2d_{18}$ |

Table 8:
| $i$ | $X_i^\lambda$ | $8m^3\epsilon_i''$ |
|-----|----------------|------------------|
| 36  | $ie^{\lambda\mu\nu}(F^+_{\mu\nu})u_\rho$ | $-4m^2d_9$ |
| 37  | $i\varepsilon^{\lambda\mu\nu\rho}u_\rho \langle F^+_{\mu\nu} \rangle v \cdot u$ | $-\frac{1}{16}g_A(c_6 + 2c_7)$ |
| 38  | $i\varepsilon^{\lambda\mu\nu\rho}u_\rho v^\tau u_\mu \langle F^+_{\nu\tau} \rangle$ | $-\frac{1}{8}g_A(1 + c_6 + 2c_7)$ |
| 39  | $i\varepsilon^{\lambda\mu\nu\rho}v_\tau (F^+_{\mu\nu})u_\rho v^\lambda$ | $-\frac{1}{16}g_A$ |
| 40  | $i[D_\mu, \langle F^+\mu\lambda \rangle]$ | $-\frac{1}{8}(c_6 + 2c_7) + 4m^2d_7$ |
| 41  | $iv_\mu[u \cdot D_\nu, \langle F^+\mu\lambda \rangle]$ | $i\frac{1}{8}$ |
| 42  | $iv^\nu[D_\mu, \langle F^+\nu\lambda \rangle]v^\lambda$ | $i\frac{1}{8} (c_6 + 2c_7)$ |
| 43  | $v_\mu \left[ F^+\lambda\mu, S \cdot u \right]$ | $\frac{1}{4}g_A (1 + c_6)$ |
| 44  | $S_\mu \left[ F^+\lambda\mu, v \cdot u \right]$ | $\frac{1}{4}g_A c_6 + 4m^2d_{20}$ |
| 45  | $v^\mu S^\nu \left[ F^+_{\mu\nu}, u^\lambda \right]$ | $\frac{1}{4}g_A (1 + c_6) + 4m^2d_{20}$ |
| 46  | $S^\lambda v^\mu \left[ F^+_{\mu\nu}, u^\nu \right]$ | $\frac{1}{4}g_A (1 + c_6) + 4m^2d_{21}$ |
| 47  | $S^\mu v^\nu \left[ F^+_{\mu\nu}, v \cdot u \right] v^\lambda$ | $\frac{3}{4}g_A (1 + c_6)$ |
| 48  | $i\varepsilon^{\lambda\mu\nu\rho} \langle F^+_{\mu\nu}u_\rho \rangle$ | $-4m^2d_8$ |
| 49  | $i\varepsilon^{\lambda\mu\nu\rho}v_\rho \left[ F^+_{\mu\nu}, v \cdot u \right]$ | $-\frac{1}{16}g_A c_6$ |
| 50  | $i\varepsilon^{\lambda\mu\nu\rho}v_\rho v^\tau \left[ u_\mu F^+_{\nu\tau} \right]$ | $-\frac{1}{8}g_A (1 + c_6)$ |
| 51  | $i\varepsilon^{\lambda\mu\nu\rho}v_\tau \left[ F^+_{\mu\nu}u_\rho \right] v^\lambda$ | $-\frac{1}{16}g_A$ |
| 52  | $i[D_\mu, F^+\mu\lambda]$ | $-\frac{1}{4}c_6 + 4m^2d_6$ |
| 53  | $iv_\mu[v \cdot D_\nu, F^+\lambda\mu]$ | $i\frac{1}{4}$ |
| 54  | $iv^\nu[D_\mu, F^+\nu\lambda] v^\lambda$ | $\frac{1}{4} + \frac{1}{2}c_6$ |
| 55  | $[F^-\lambda\mu, u_\mu]$ | $\frac{1}{16}g_A^2 + \frac{1}{4}mc_4 + 2m^2 (d_1 - d_2)$ |
| 56  | $v_\mu[F^-\lambda\mu, v \cdot u]$ | $\frac{1}{16} (1 - g_A^2)$ |
| 57  | $v^\mu[F^-_{\mu\nu}, u^\nu] v^\lambda$ | $-\frac{1}{16} (1 + 2g_A^2) - \frac{1}{4}mc_4$ |
| 58  | $[S_\mu, S_\nu] \langle F^-\lambda\mu u^\nu \rangle$ | $2m^2 (d_{14} + d_{15})$ |
| 59  | $[S^\lambda, S^\mu] \langle F^-_{\mu\nu}u^\nu \rangle$ | $\frac{1}{8}g_A^2 - mc_3$ |
| 60  | $[S^\lambda, S^\mu] u^\nu \langle F^-_{\mu\nu} v \cdot u \rangle$ | $-\frac{1}{8}g_A^2 - mc_2 - 2m^2 (d_{14} + d_{15})$ |
| 61  | $[S^\mu, S^\nu] v^\rho \langle F^-_{\rho\mu}u_\nu \rangle v^\lambda$ | $-\frac{1}{8}g_A^2$ |
| 62  | $[S^\mu, S^\nu] \langle F^-_{\mu\nu} v \cdot u \rangle v^\lambda$ | $\frac{1}{8}g_A^2$ |
| 63  | $iS^\lambda \epsilon^{\mu\nu\rho\tau} v_\tau \langle F^-_{\mu\nu}u_\rho \rangle$ | $4m^2d_{23}$ |
| 64  | $iS_\mu \left[ v \cdot D_\nu, F^-\lambda\mu \right]$ | $-\frac{1}{2}g_A$ |
| 65  | $iS^\mu [D_\nu, F^-_{\mu\nu}] v^\lambda$ | $\frac{1}{4}g_A$ |
| 66  | $iS^\lambda v^\mu [D_\nu, F^-_{\mu\nu}]$ | $-4m^2d_{22}$ |
| 67  | $iS^\mu v^\nu \left[ v \cdot D_\nu, F^-_{\mu\nu} \right] v^\lambda$ | $-\frac{5}{7}g_A$ |

Table 8: Explicit form of the monomials $X_i^\lambda$ and their coefficients.
We briefly comment on the fourth order heavy baryon Lagrangian recently presented in ref. [15].
First, it is entirely based on the HB projection and contains no information concerning the relativistic formulation. Furthermore, strong isospin breaking terms as well as external sources (apart from the quark mass matrix) are entirely omitted. The tables in that work contain considerably more terms than found here, which could be reduced by some of the identities spelled out in this paper.

4 Summary

In this work, we have constructed the minimal effective chiral pion–nucleon two–flavor Lagrangian at fourth order in the chiral expansion. To arrive at this minimal form, we have studied in detail all strictures arising from the equations of motion, trace relations and other algebraic identities. The so constructed Lagrangian contains 118 in principle measurable terms. These are accompanied by the so–called low–energy constants, which can be obtained by a fit to data, from some models or maybe from lattice gauge theory. Note that four of these operators are special since they have no pion matrix elements. Based on the fact that a consistent power counting can be set up in the relativistic as well as in the heavy baryon formulation of the effective field theory, we have worked out the effective Lagrangian in both schemes. In the heavy baryon case, it is mandatory to calculate the various \(1/m\) corrections at a given chiral order. Here, these are explicitly spelled out up–to–and–including fourth order.

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A EOM eliminations

The number of independent terms in the effective \(\pi N\) chiral Lagrangian is considerably reduced by the use of the so-called EOM (equation of motion) eliminations. One can view them simply as replacements of \(\not D \Psi\) by \(-im\Psi\) (a consequence of the fact that \((\not D + im) \Psi\) is of higher order), or as a consequence of the lowest order EOM (defining the classical solution, around which the loop expansion is performed), or as a result of specific field transformations (leaving the S–matrix elements untouched). However, the simple replacements \(\not D \Psi \rightarrow -im\Psi\) do not exhaust the EOM eliminations. The point is that also terms (or combinations of terms) not containing \(\not D \Psi\) explicitly can be brought into a form that includes \(\not D \Psi\). Such terms can then also be eliminated. To systematically obtain all such terms, one has to inspect all the possible terms with \(\Gamma_{\ldots} \not D \Psi\) and rewrite them as combinations of terms without \(\not D \Psi\). To achieve this, we expand any product \(\Gamma_{\ldots} \gamma_{\lambda}\) in the form (the last \(\gamma\)–matrix originates from \(\not D\))

\[
\Gamma_{\ldots} \gamma_{\lambda} = \sum_i \Gamma'_{i,\ldots} + \sum_j \Gamma''_{j,\ldots}, \quad c_{\Gamma''} = c_{\Gamma'} + 1 = c_{\Gamma}, \tag{A.1}
\]

where \(\Gamma'\) and \(\Gamma''\) contain only elements from the standard basis of the Clifford algebra, multiplied by appropriate combinations of \(g\)- and \(\varepsilon\)-tensors. When contracted with \(D_{\lambda}\) and inserted into \(2.13\).
instead of $\Gamma_\ldots$, this yields (after the replacement $\bar{D} \Psi \rightarrow -im \Psi$)
\[
\sum_i \bar{\Psi} \Gamma_{i,\mu\nu\ldots}^\alpha A^{\mu\nu\ldots\alpha\beta\ldots} D_{\alpha\beta\ldots}^{n} \Psi + \text{h.c.} = -im \bar{\Psi} A^{\mu\nu\ldots\alpha\beta\ldots} D_{\alpha\beta\ldots}^{n-1} \Psi + \text{h.c.},
\]
\[
\sum_j \bar{\Psi} A^{\mu\nu\ldots\alpha\beta\ldots} \Gamma_{j,\mu\nu\ldots}^\alpha D_{\alpha\beta\ldots}^{n} \Psi + \text{h.c.} = 0,
\]
(A.2)

where the symbol $\doteq$ stands for "equal up-to higher orders".

For $\Gamma = 1$, one obtains the so-called "master relation"
\[
\bar{\Psi} A^{\alpha\beta\ldots} \gamma^5 \Psi + \text{h.c.} \doteq -im \bar{\Psi} A^{\alpha\beta\ldots} D_{\alpha\beta\ldots}^{n-1} \Psi + \text{h.c.},
\]
(A.3)

but letting $\Gamma$ run through the rest of the standard basis of the Clifford algebra, a couple of additional relations is generated
\[
0 \doteq -im \bar{\Psi} A^{\alpha\beta\ldots} \gamma_5 D_{\alpha\beta\ldots}^{n-1} \Psi + \text{h.c.},
\]
(A.4)

\[
\bar{\Psi} A^{\alpha\beta\ldots} \gamma^5 \gamma^\lambda \Psi + \text{h.c.} \doteq 0,
\]
(A.5)

\[
\bar{\Psi} A^{\mu\alpha\beta\ldots} D_{\mu\alpha\beta\ldots}^{n} \Psi + \text{h.c.} \doteq -im \bar{\Psi} A^{\mu\alpha\beta\ldots} \gamma_5 D_{\mu\alpha\beta\ldots}^{n-1} \Psi + \text{h.c.},
\]
(A.6)

\[
\bar{\Psi} A^{\mu\alpha\beta\ldots} \gamma_5 \gamma^\lambda D_{\mu\alpha\beta\ldots}^{n} \Psi + \text{h.c.} \doteq m \bar{\Psi} A^{\mu\alpha\beta\ldots} \gamma_5 D_{\mu\alpha\beta\ldots}^{n-1} \Psi + \text{h.c.},
\]
(A.7)

\[
\bar{\Psi} A^{\mu\alpha\beta\ldots} \gamma_5 \gamma^\mu D_{\mu\alpha\beta\ldots}^{n} \Psi + \text{h.c.} \doteq 0,
\]
(A.8)

\[
\bar{\Psi} A^{\mu\alpha\beta\ldots} \gamma_5 \gamma^\mu D_{\mu\alpha\beta\ldots}^{n} \Psi + \text{h.c.} \doteq 0,
\]
(A.9)

\[
\bar{\Psi} A^{\mu\alpha\beta\ldots} \varepsilon_{\mu\nu} \gamma_5 \gamma^\mu D_{\mu\alpha\beta\ldots}^{n} \Psi + \text{h.c.} \doteq im \bar{\Psi} A^{\mu\alpha\beta\ldots} \varepsilon_{\mu\nu} D_{\mu\alpha\beta\ldots}^{n-1} \Psi + \text{h.c.},
\]
(A.10)

\[
\bar{\Psi} A^{\mu\alpha\beta\ldots} (\gamma_5 D_{\nu\alpha\beta\ldots}^{n} - \gamma_5 D_{\nu\alpha\beta\ldots}^{n-1}) \Psi + \text{h.c.} \doteq 0.
\]
(A.11)

For the sake of completeness, we have written down both relations (A.2) for every $\Gamma \in (\gamma_5, \gamma_5 \gamma_5 \gamma_\mu, \gamma_\mu \gamma_\mu, \gamma_\mu \gamma_\mu)$, in spite of the fact that they are not all independent (e.g. eqs. (A.4) and (A.9) are equivalent, and (A.11) follows from (A.6)).

For a general $\Gamma$ one obtains, as a rule, some linear combinations of the above relations. But there are exceptions, namely for $\varepsilon$–tensors contracted with elements of the standard basis. The first such case is $\Gamma = \gamma_5 \sigma_{\mu\nu} = (1/2i)\varepsilon_{\mu\nu\rho\tau} \sigma^{\rho\tau}$, which yields two new relations,
\[
\bar{\Psi} A^{\mu\alpha\beta\ldots} (\gamma_5 \gamma_\mu D_{\nu\alpha\beta\ldots}^{n} - \gamma_5 \gamma_\nu D_{\mu\alpha\beta\ldots}^{n}) \Psi + \text{h.c.} \doteq -m \bar{\Psi} A^{\mu\alpha\beta\ldots} \gamma_5 \sigma_{\mu\nu} D_{\alpha\beta\ldots}^{n-1} \Psi + \text{h.c.},
\]
(A.12)

\[
\bar{\Psi} A^{\mu\alpha\beta\ldots} \varepsilon_{\mu\nu} \gamma^\mu D_{\mu\alpha\beta\ldots}^{n} \Psi + \text{h.c.} \doteq 0.
\]
(A.13)
For $\Gamma = \varepsilon_{\mu\nu\rho\tau}\gamma^\tau$, one obtains

$$
\bar{\Psi} A^{\mu\nu\rho\alpha\ldots} \varepsilon^\lambda_{\mu\nu\rho} D^\mu_{\lambda\alpha\ldots} \Psi + \text{h.c.} = -im\bar{\Psi} A^{\mu\nu\rho\alpha\ldots} \varepsilon^\lambda_{\mu\nu\rho} \gamma_5 D^\mu_{\lambda\alpha\ldots} \Psi + \text{h.c.}, \quad (A.14)
$$

$$
\bar{\Psi} A^{\mu\nu\rho\alpha\ldots} \gamma_5 \left( g^\lambda_{\mu\nu\rho} + g^\lambda_{\nu\rho\mu} + g^\lambda_{\rho\mu\nu} \right) D^\mu_{\lambda\alpha\ldots} \Psi + \text{h.c.} \triangleq 0 , \quad (A.15)
$$

and $\Gamma = \varepsilon_{\mu\nu\rho\tau}\gamma_5\gamma^\tau$ gives

$$
\bar{\Psi} A^{\mu\nu\rho\alpha\ldots} \left( g^\lambda_{\mu\nu\rho} + g^\lambda_{\nu\rho\mu} + g^\lambda_{\rho\mu\nu} \right) D^\mu_{\lambda\alpha\ldots} \Psi + \text{h.c.} = im\bar{\Psi} A^{\mu\nu\rho\alpha\ldots} \varepsilon_{\mu\nu\rho} \gamma_5 \gamma_\tau D^\mu_{\lambda\alpha\ldots} \Psi + \text{h.c.}, \quad (A.16)
$$

$$
\bar{\Psi} A^{\mu\nu\rho\alpha\ldots} \gamma_5 \varepsilon^\lambda_{\mu\nu\rho} D^\mu_{\lambda\alpha\ldots} \Psi + \text{h.c.} \triangleq 0 . \quad (A.17)
$$

The relations (A.4, A.17) can be used to eliminate some terms from the list of all the possible invariants, but it is preferable to use them already at the stage of generating such a list. This was exactly what we did, e.g. when counting $\gamma_5$ as a quantity of first chiral order in Table 1, or when considering the special structure of $\Theta_{\mu\nu\ldots}$ in eq. (2.11). In this way, the relations (A.3, A.5) and (A.7, A.9) were taken into account. The relations (A.6, A.11) and (A.14) are also easily accounted for, namely by ignoring all structures $\Theta_{\mu\nu\ldots}$ with an explicit $\gamma_\mu$ (but not with an explicit $\gamma_5\gamma_\mu$). The relations (A.10) and (A.12, A.13) are simultaneously embodied in the simple requirement that $\Theta_{\mu\nu\ldots}$ can contain an $\varepsilon$-tensor with at most one index contracted within $\Theta_{\mu\nu\ldots}$. The relation (A.13) is just a simple consequence of eq. (A.12), and the relation (A.16) allows to ignore $\Theta_{\mu\nu\rho\ldots}$ containing $\varepsilon_{\mu\nu\rho\tau}\gamma_5\gamma^\tau$.

So far we have investigated the implications of the nucleon EOM on the structure of $\Theta_{\mu\nu\ldots}$. There are, however, also consequences for the structure of $A^{\mu\nu\ldots}$, namely for $A^{\mu\nu\ldots} = [D^\mu, B^\nu\ldots]$. The point is that $D^\mu$ can be reshuffled to act on nucleon fields (by partial integration and exclusion of total derivatives) and, in some cases, eliminated afterwards. Such an elimination is obviously possible if $D^\mu$ is contracted with $\gamma_\mu D^\mu_{\nu\ldots}$ and consequently also if it is contracted with $\gamma_\nu D^\mu_{\nu\ldots}$ (cf. eq. (A.11)) or $D^\mu_{\mu\nu\ldots}$ (cf. eq. (A.6)). For $D^\mu$ contracted with $\gamma_5\gamma_\mu D^\mu_{\nu\ldots}$ and $\gamma_5\gamma_\nu D^\mu_{\nu\ldots}$ one obtains in a similar way

$$
\bar{\Psi} \left[ D^\mu, B^\nu\ldots \right] \gamma_5 \gamma_\mu D^\mu_{\nu\ldots} \Psi + \text{h.c.} \triangleq 2im\bar{\Psi} \left[ D^\mu, B^\nu\ldots \right] \gamma_5 D^\mu_{\nu\ldots} \Psi + \text{h.c.}, \quad (A.18)
$$

$$
\bar{\Psi} \left[ D^\mu, B^\nu\ldots \right] \gamma_5 \gamma_\nu D^\mu_{\nu\ldots} \Psi + \text{h.c.} \triangleq 0 , \quad (A.19)
$$

and for $D^\mu$ contracted with $\sigma_{\nu\rho} D^\mu_{\nu\rho}$...

$$
\bar{\Psi} \left[ D^\mu, B^\nu\ldots \right] \sigma_{\nu\rho} D^\mu_{\nu\rho} \Psi + \text{h.c.} \triangleq \begin{array}{c} i\bar{\Psi} \left[ D^\mu, B^\nu\ldots \right] g_{\mu\nu} D^\mu_{\nu\rho} \Psi + \text{h.c.} \\ - \bar{\Psi} \left[ D^\mu, B^\nu\ldots \right] g_{\mu\rho} D^\mu_{\nu\rho} \Psi + \text{h.c.} \\ + im\bar{\Psi} \left[ D^\mu, B^\nu\ldots \right] \varepsilon_{\mu\nu\rho\tau} \gamma_5 \gamma_\tau D^\mu_{\nu\rho} \Psi + \text{h.c.} \end{array} . \quad (A.20)
$$

\(^{#8}\) In [1] and [2] implications of the nucleon EOM have been expressed in the form of six identities. The first five of them are equivalent to each other, and they are consequences of our relations (A.4, A.6, A.14) and (A.17). The sixth relation of [2] follows from our relations (A.3, A.5), and is not given explicitly (although accounted for) by [1]. The sixth relation of [1] is neither given, nor used by [2]. The relation follows from our relations (A.12, A.16), however, not for a general $A^{\mu\nu\ldots}$, but rather only for $A^{\mu\nu\ldots}$ antisymmetric in $\mu\nu$ (or $\mu\rho$ or $\nu\rho$).
Note that the omitted contraction of $D^\mu$ with $\sigma_{\mu\nu} D^\rho_{\alpha\beta...}$ yields nothing but a trivial identity. The relations (A.18–A.20) are used to ignore terms of the type written on the left–hand–side. However, in case of eq.(A.18) it is more advantageous to keep the structure on the left–hand–side and eliminate the right–hand–side, which does not contain the special structure $[D^\mu, B^\nu...]$, but rather a general structure $B^\nu...$.

To summarize: we utilize the nucleon EOM in the form of restrictions on the structure of $\Gamma_{\mu\nu...}$ and $A_{\mu\nu...}$:

- $\Gamma_{\mu\nu...}$ is a product of $g$-tensors, $\varepsilon$-tensors and one matrix from the following set \{1, $\gamma_5 \gamma_\mu, \sigma_{\mu\nu}\}$;
- within $\Gamma_{\mu\nu...}$, only $\varepsilon$-tensors may have an upper index (every $\varepsilon$ at most one), which has to be contracted with $D^n_{\alpha\beta...}$;
- for $A_{\mu\nu...} = [D^\mu, B^\nu...]$, derivative $D^\mu$ cannot be contracted with $D^n_{\alpha\beta...}$.

This is not the only way of implementing the nucleon EOM, but it seems to be the most economic one. It results in the following set of $\Theta_{\mu\nu...}$, sufficient for generating the complete chiral Lagrangian up to fourth order:

\begin{equation}
1;
\gamma_5 \gamma_\mu, D_\mu;
\gamma_5 \gamma_\mu D_\nu, D_{\mu\nu};
g_{\mu\nu}, \sigma_{\mu\nu}, \gamma_5 \gamma_\mu D_\nu, D_{\mu\nu};
g_{\mu\nu} \gamma_5 \gamma_\rho, g_{\mu\nu} D_\rho, \sigma_{\mu\nu} D_\rho, \varepsilon^\lambda_{\mu\nu\rho} D_\lambda, \gamma_5 \gamma_\mu D_{\nu\rho}, D_{\mu\nu\rho};
g_{\mu\nu} \epsilon_{\mu\nu\rho\tau}, \epsilon_{\mu\nu\rho\tau}, g_{\mu\nu} \sigma_{\rho\tau}, g_{\mu\nu} \gamma_5 \gamma_\rho D_\tau, g_{\mu\nu} D_{\rho\tau}, \sigma_{\mu\nu} D_{\rho\tau}, \epsilon^\lambda_{\mu\nu\rho} D_\lambda\tau, \gamma_5 \gamma_\mu D_{\nu\rho\tau}, D_{\mu\nu\rho\tau}.
\end{equation}

(A.21)

B Operators for the heavy baryon Lagrangian

In order to construct the $1/m$-corrections up-to-and-including fourth order, one needs the expressions for the operators $A^{(1,2,3,4)}$, $B^{(1,2,3)}$, and $C^{(0,1,2)}$, the latter allowing one to reconstruct the chiral expansion of the inverse of $C$. These are given by:

\begin{align*}
C^{(0)} &= 2m, \quad \text{(B.1)} \\
A^{(1)} &= i (v \cdot D) + g_A (S \cdot u), \quad \text{(B.2)} \\
B^{(1)} &= -\gamma_5 \left( 2i (S \cdot D) + \frac{g_A}{2} (v \cdot u) \right), \quad \text{(B.3)} \\
C^{(1)} &= i (v \cdot D) + g_A (S \cdot u), \quad \text{(B.4)}
\end{align*}
\[ A^{(2)} = c_1 \langle \chi_+ \rangle + \frac{c_2}{2} ((v \cdot u)^2) + \frac{c_3}{2} (u^2) + c_5 \tilde{\chi}_+ \]
\[ - 2i [S^\mu, S^\nu] \left( \frac{ic_4}{4} [u_\mu, u_\nu] + \frac{c_6}{8m} F^+_{\mu\nu} \right) + \frac{c_7}{8m} \langle F^+_{\mu\nu} \rangle, \]
\[ B^{(2)} = 2i \gamma_5 (v^\mu S^\nu - v^\nu S^\mu) \left( \frac{ic_4}{4} [u_\mu, u_\nu] + \frac{c_6}{8m} F^+_{\mu\nu} \right) + \frac{c_7}{8m} \langle F^+_{\mu\nu} \rangle, \]
\[ C^{(2)} = -A^{(2)}, \]
\[ A^{(3)} = \left( \frac{i}{2} \frac{c_2}{2m} \langle (v \cdot u) u_\mu, D^\mu \rangle + \text{h.c.} \right) + E + 2S^\mu G_\mu - 2i [S^\mu, S^\nu] H_{\mu\nu}, \]
\[ B^{(3)} = \gamma_5 (-v^\mu G_\mu + 2i (v^\mu S^\nu - v^\nu S^\mu) H_{\mu\nu}), \]
\[ A^{(4)} = -\frac{c_2}{8m^2} \langle u_\mu u_\nu \rangle \{ D^\mu, D^\nu \} + \text{h.c.} \]
\[ \left( -\frac{d_1}{2m} ([u_\mu, [D^\nu, u_\mu]] D^\nu + \text{h.c.}) - \frac{d_2}{2m} ([u_\mu, [D^\mu, u_\nu]] D^\nu + \text{h.c.}) \right) \]
\[ - \frac{d_3}{2m} ([v \cdot u, [v \cdot D, u_\mu]] D^\mu + [v \cdot u, [D^\mu, v \cdot u]] D^\mu + [u_\mu, [v \cdot D, v \cdot u]] D^\mu + \text{h.c.}) \]
\[ - \frac{d_4}{2m} (\epsilon^{\mu \nu \alpha \beta} (u_\mu u_\nu u_\alpha) D_\beta + \text{h.c.}) + \frac{d_5}{2m} \left( i [\chi_-, u_\mu] D^\mu + \text{h.c.} \right) \]
\[ + \frac{d_6}{2m} \left( i [D^\mu, \bar{F}^\mu_{\nu}] D^\nu + \text{h.c.} \right) + \frac{d_7}{2m} \left( i [D^\mu, \langle F^+_{\mu\nu} \rangle] D^\nu + \text{h.c.} \right) \]
\[ + \frac{d_8}{2m} \left( i \epsilon^{\mu \nu \alpha \beta} (\bar{F}^\mu_{\nu} u_\alpha) D_\beta + \text{h.c.} \right) + \frac{d_9}{2m} \left( i \epsilon^{\mu \nu \alpha \beta} (\langle F^+_{\mu\nu} \rangle u_\alpha) D_\beta + \text{h.c.} \right) \]
\[ + \frac{d_{10}}{m} \langle v \cdot u u_\mu \rangle S \cdot u D^\mu + \text{h.c.} \]
\[ + \frac{d_{11}}{2m} \langle u_\mu v \cdot u \rangle u_\nu D^\nu + i S^\mu \langle u_\mu u_\nu \rangle v \cdot u D^\nu + \text{h.c.} \]
\[ + \frac{d_{12}}{2m} \left( [S^\mu, S^\nu] [\langle D_\lambda, u_\mu \rangle u_\nu] D^\lambda + \text{h.c.} \right) \]
\[ + \frac{d_{13}}{2m} \left( [S^\mu, S^\nu] \langle u_\mu [D_\nu, u_\lambda] \rangle D^\lambda + \text{h.c.} \right) \]
\[ - \frac{d_{14}}{2m} \left( S^\mu v^\nu [\bar{F}^+_{\mu\nu}, u_\lambda] D^\lambda + S^\mu [\bar{F}^+_{\mu\nu}, v \cdot u] D^\nu + \text{h.c.} \right), \]
\[ (B.10) \]

with

\[ E = id_1 [u_\mu, [v \cdot D, u^\mu]] + id_2 [u_\mu, [D^\mu, v \cdot u]] + id_3 [v \cdot u, [v \cdot D, v \cdot u]] \]
\[ + id_4 \epsilon^{\mu \nu \alpha \beta} v_\beta \langle u_\mu u_\nu u_\alpha \rangle + d_5 [\chi_-, v \cdot u] + d_6 v^\nu [D^\mu, \bar{F}^\mu_{\nu}] \]
\[ + dv^\nu [D^\mu, \langle F^+_{\nu} \rangle] + d_8 \epsilon^{\mu \nu \alpha \beta} v_\beta \langle \bar{F}^+_{\mu\nu} u_\alpha \rangle + d_9 \epsilon^{\mu \nu \alpha \beta} v^\nu \langle F^+_{\mu\nu} \rangle u_\alpha, \]
\[ (B.11) \]
\[ G_\mu = \frac{d_{10}}{2} (u^2) u_\mu + \frac{d_{11}}{2} \langle u_\mu u_\nu \rangle u^\nu + \frac{d_{12}}{2} ((v \cdot u)^2) u_\mu + \frac{d_{13}}{2} \langle u_\mu (v \cdot u) \rangle (v \cdot u) \]
\[ + \frac{d_{14}}{2} \langle \chi_+ \rangle u_\mu + \frac{d_{15}}{2} \langle \chi_+ u_\mu \rangle + \frac{i d_{16}}{2} [D^\mu, \chi_-] + \frac{i d_{17}}{2} [D^\mu, \langle \chi_- \rangle] + \frac{i d_{18}}{2} [D^\mu, \langle \chi_- \rangle] + \frac{i d_{19}}{2} [D^\mu, \langle \chi_- \rangle] \]
\[ + \frac{i d_{20}}{2} \langle \bar{F}^+_{\mu\nu}, u^\nu \rangle + \frac{d_{21}}{2} D^\nu, F^+_{\mu\nu} + \frac{d_{22}}{2} \epsilon^{\mu \nu \alpha \beta} (u_\nu F^\alpha_{\beta} -), \]
\[ (B.12) \]
\[ H_{\mu\nu} = \frac{d_{14}}{2} \langle [v \cdot D, u_\mu] u_\nu \rangle + \frac{d_{15}}{2} \langle u_\mu [D_\nu, v \cdot u] \rangle, \]
\[ (B.13) \]
and
\begin{equation}
    C^{-1} = \frac{1}{2m} - \frac{iv \cdot D + g_{AS} \cdot u}{(2m)^2} + \frac{(iv \cdot D + g_{AS} \cdot u)^2}{(2m)^3} - \frac{C^{(2)}}{(2m)^2}.
\end{equation}

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