η′ Production in Proton-Proton Collisions near Threshold

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Abstract

Within the one-pion exchange model we calculate the near-threshold η′ production from pp collisions including the final state interaction between the protons. Since the description of the data is quite well we conclude that η, ρ or ω exchange currents either play no role or cancel each other to a large extent in this reaction.

Recently the COSY-11 [1] and SPES III [2] Collaborations have measured the η′ production in proton-proton collisions at excess energies \( \epsilon = \sqrt{s} - 2m_N - m_{\eta'} \leq 10 \) MeV. As found in Ref. [2] the calculation for the \( pp \to pp\eta' \) cross section within the one-pion-exchange model – based on a comparison between η and η′ production amplitudes – underestimates the experimental data by up to a factor of 2, such that the ρ and other heavy-meson exchange diagrams [3] should contribute substantially for η′ production. In this short note we will argue that this discrepancy vanishes when including the average amplitude \( |M_{\pi N \to \eta' N}| \) from experimental data while also taking into account the interaction between the protons in the final-channel.

Here we calculate the η′ production within the one-boson exchange model first neglecting the Final-State-Interaction (FSI). For the one-pion exchange the \( pp \to pp\eta' \) production amplitude [4] then reads

\[
M = g_{NN\pi} F(t) \bar{u}(p_1) \gamma_5 u(p_a) \frac{1}{t - \mu^2} M_{\pi^0 p \to p\eta'},
\]

where \( g_{NN\pi} = 13.59 \) [3] is the \( pp\pi^0 \) coupling constant, \( t = (p_a - p_1)^2 \) is the squared 4-momentum transfer from the initial to the final proton, \( \mu \) is the pion mass and \( F(t) \) is the form factor for the \( NN\pi \) vertex

\[
F(t) = \frac{\Lambda^2 - \mu^2}{\Lambda^2 - t}
\]

with a cut-off parameter \( \Lambda = 1.3 \) GeV in line with Ref. [3]. In principle, η, ρ and ω exchanges should also contribute as suggested by the analysis in Ref. [2], however, there

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are no data on the coupling of the η′ to the baryonic resonances, which are necessary to include the exchanges of other mesons in a reliable way.

In (1) \( M_{\pi N \rightarrow N \eta'} \) is the amplitude for the \( \pi N \rightarrow N \eta' \) reaction, which is related to the physical cross section as

\[
|M_{\pi N \rightarrow N \eta'}|^2 = 32\pi s_1 \frac{q_\pi}{q_{\eta'}} \sigma(\pi N \rightarrow \eta' N).
\]

Here \( s_1 \) is the squared invariant mass of the final \( \eta'p \) system and \( q_\pi \) and \( q_{\eta'} \) are the momenta of the corresponding particles in the center-of-mass. Since the \( \pi N \rightarrow \eta'N \) cross sections are known from the experimental data \([4]\) the amplitude \( |M_{\pi N \rightarrow \eta' N}| \) can be extracted from the data using (3). For the \( \pi^+n \rightarrow \eta'p \) and \( \pi^-p \rightarrow \eta'n \) reactions this amplitude is shown in Fig. 1; here the solid line corresponds to the average amplitude that will be used in our following calculations. Note that \( \sqrt{2}M_{\pi^0p \rightarrow \eta'p} = M_{\pi^-p \rightarrow \eta'n} = M_{\pi^+n \rightarrow \eta'p} \). For excess energies \( \epsilon \leq 10 \text{MeV} \) we need the \( \pi N \rightarrow \eta'N \) amplitude in the range \( m_N + m_\eta' \leq \sqrt{s_1} \leq m_N + m_\eta' + \epsilon \) where \( |M_{\pi N \rightarrow \eta' N}| \) is almost constant.

The \( pp \rightarrow pp\eta' \) cross section then can be obtained by integrating

\[
\frac{d^2 \sigma}{dt \, ds_1} = \frac{1}{2^9\pi^3 q_a^2 s \sqrt{s_1}} |M_{\text{prod}} - \text{exch.}|^2,
\]

where \( s \) is the squared invariant mass of the colliding protons and \( q_a \) is the momentum of the incident proton in their center-of-mass. In (4) the exchange term is given by interchanging the initial proton momenta. The \( pp \rightarrow pp\eta' \) cross section calculated within the model described above is shown in Fig. 2 by the dashed line and substantially underestimates the experimental data from Refs. \([1, 2]\).

Following our analysis on the near threshold \( K^+ \)-meson production \([8]\) the latter discrepancy might be entirely due to FSI. Within the Watson-Migdal approximation \([9, 10]\) the total reaction amplitude can be factorized in terms of the production amplitude (1) and the FSI amplitude. As was shown by Fäl dt and Wilkin \([11]\) (see also \([12]\)) the FSI might be introduced by multiplying the cross section (4) by the correction factor

\[
FW(\epsilon) = \left[ \frac{2\beta}{\alpha + \sqrt{\alpha^2 + \epsilon m_N}} \right]^2,
\]

where the parameters \( \alpha \) and \( \beta \) are related to the scattering length \( a \) and the effective range \( r_0 \) for the S-wave \( pp \) scattering as

\[
a = \frac{\alpha + \beta}{\alpha \beta}, \quad r_0 = \frac{2}{\alpha + \beta}.
\]

Furthermore, within the effective range expansion the S-wave scattering amplitude \( T(q) \) and the phase shift \( \delta \) are given

\[
T(q) = \frac{1}{q \cot \delta - iq} = \left( -\frac{1}{a} + \frac{r_0q^2}{2} - iq \right)^{-1}.
\]

The squared \( ^1S_0 \) \( pp \) scattering amplitude \( |T|^2 \) calculated with the phase shift from the Nijmegen-93 partial wave analysis \([13]\) is shown in Fig. 3 in comparison to the effective
range approximation with parameters $\alpha = -21.67$ MeV and $\beta = 162.9$ MeV. Fig. 3 also shows the contribution from $^3P_0$ and $^3P_1$ partial waves and illustrates that the FSI is dominated by S-wave proton-proton scattering for relative momenta of the final protons below $\approx 100$ MeV/c.

The result obtained with the prescription (5) is shown by the dotted line in Fig. 2 and provides a reasonable description of the data.

Another way to account for the FSI between the protons is to multiply the production amplitude (1) by the inverse S-wave Jost function [14]

$$J_0(q) = \frac{q - i\alpha}{q + i\beta},$$

where $q$ is the relative momentum of the final protons. The solid line in Fig. 2 shows our calculation with FSI according to (8) which reasonably reproduces the data and can be compared with prescription (5).

Note, as discussed in [8], that the Jost function approaches unity for large $q$ while the Watson factor, i.e. the use of the scattering amplitude itself, is reasonable only at energies close to the reaction threshold. Actually at energies close to the reaction threshold both models for FSI corrections (8) and (5) give a factor $\beta^2/\alpha^2$ for the production cross section.

We also note that the difference between the $pp$ and $pn$ FSI [15], which might explain the large ratio of the $pp \rightarrow pp\eta$ and $pn \rightarrow pn\eta$ cross sections near threshold, will lead to a comparable ratio of the $pp \rightarrow pp\eta'$ and $pn \rightarrow pn\eta'$ cross sections for low excess energies.

Since the one-pion-exchange model with the inclusion of the interaction between the final protons reproduces the experimental data on near-threshold $\eta'$ production from $pp$ collisions, we conclude that other exchange currents in the primary production amplitude either play no role or cancel each other to a large extend.

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Figure 1: The amplitudes for the $\pi^- p \to \eta' n$ (circles) and $\pi^+ n \to \eta' p$ reactions (squares) as a function of the $\eta'N$ invariant mass. The solid line corresponds to the average amplitude used in our calculations. The symbols show the results extracted from the experimental data from Ref. [7] according to Eq. (3).
Figure 2: The $pp \rightarrow pp\eta'$ cross section as a function of the excess energy $\epsilon$. The experimental data are from Ref. [1] (circles) and Ref. [2] (squares). The dashed line corresponds to the pion-exchange calculation without FSI. The solid and dotted lines are obtained including FSI according to (8) and (5), respectively.
Figure 3: The squared $pp$ scattering amplitude as function of the relative momentum of the final protons. Full dots show the results for the $^1S_0$, $^3P_0$ and $^3P_1$ partial waves calculated with the Nijmegen-93 model [13], while the line is the effective range approximation with parameters as shown in the figure.