Open inflationary universe with tachyonic matter

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Abstract.

We study a one-field open inflationary model in a universe dominated by tachyon matter [1, 2]. We work in the scenario of the one-bubble universe where bubble formation is described by the Coleman-De Lucia (CDL) instanton [3]. The conditions of existence and basic characteristics of the CDL instanton were determined for the tachyon model using analytical and numerical approaches. Also, we study the Lorentzian regime, that is, the period of inflation after tunnelling has occurred. In particular we focus on the viability of the model in order to generate a consistent inflationary scenario, compatible with observations.

1. Introduction

The recent observations from WMAP [4] are entirely consistent with a universe having a total energy density that is very close to its critical value, where the total density parameter has the value $\Omega = 1.02 \pm 0.04$. Most people interpret this value as corresponding to a flat universe. But, according to this result, we might take the alternate point of view of a marginally open or closed universe model [5], which at early times in the evolution of the universe presents an inflationary period of expansion.

In this work we study a single-field open inflationary universe, where inflation is driven by a tachyon field. In particular, in our model the tachyon begins at the false vacuum and then decays to the value $\phi_T$ via quantum tunnelling through the potential barrier (see Fig.1), generating during this process an open inflationary universe [3]. The mechanism of tachyon decay via quantum tunnelling has been studied in the context of brane-antibrane and dielectric brane decays [6], where tachyon potential with similar characteristics of our potential have been derived and studied. In our work we are particularly interested in the cosmological implication of this process and its application to open inflationary models.

2. The Euclidean cosmological equations in tachyon models

We consider the Euclidean effective action of the tachyon field given by [1]

$$S = \int d^4x \sqrt{-g} \left[ \frac{1}{2\kappa} R + V(\phi) \sqrt{1 + \partial_{\mu}\phi \partial^{\mu}\phi} \right], \quad (1)$$
where $R$ is the Ricci scalar curvature, $\kappa = 8\pi G$, the cosmological constant term is set to zero and $V(\phi)$ is an effective scalar potential associated with the tachyon field

$$V(\phi) = V_0 e^{-\lambda \phi} \left[ 1 + \frac{\alpha^2}{\beta^2 + (\phi - v)^2} \right].$$  

We are going to consider $\lambda > 0$ and $V_0 > 0$ as free parameters, and $\alpha$, $\beta$ and $v$ as arbitrary constants, which will be set by phenomenological considerations. The first term of Eq. (2) controls inflation after quantum tunnelling has occurred and the second term controls the bubble nucleation [7]. After the quantum tunnelling, we should note that the stable vacuum to which the tachyon condenses, is at $\phi \rightarrow +\infty$, when $V(\phi) \rightarrow 0$. The $O(4)$- invariant Euclidean spacetime metric is described by $ds^2 = d\tau^2 + a^2(\tau) (d\psi^2 + \sin^2(\psi) d\Omega_2^2)$, where $a(\tau)$ is the scale factor of the universe and $\tau$ represents the Euclidean time. The tachyon field and the scale factor obey the equations of motions:

$$\frac{\phi''}{1 + \phi'^2} = -3 \frac{a'}{a} \phi' + \frac{1}{V} \frac{dV}{d\phi},$$  

$$a'' = -\frac{\kappa}{3} a \frac{V(\phi)}{\sqrt{1 + \phi'^2}} \left( 1 + \frac{3}{2} \phi'^2 \right).$$

where the primes denote derivatives respect to $\tau$. Follow the scheme of Ref. [7] we found that a CDL instanton could exist in our model only if the condition

$$m^2 = \frac{V,\phi}{V} \left( \frac{V,\phi}{V} \right) 2 \left( 3 - \frac{V^2}{\rho^2} \right) > H^2,$$

is satisfied during the tunnelling process. Here $m^2$ correspond to the mass of the tachyon [8], $\rho$ is the energy density of the tachyon field and $H^2 = \kappa V/3$.

We use the COBE normalized value for the amplitude of the scalar density perturbations in order to estimate $\lambda$ and $V_0$ [9]. Thus we have $\lambda = 10^{-5}\kappa^{-1/2}$ and $V_0 = 10^{-7}\kappa^{-2}$. We also consider $\beta^2 = 2\alpha^2$ with $\beta = 6.67 \times 10^3 \kappa^{1/2}$ and $v = 3 \times 10^5 \kappa^{1/2}$, which, as we will see, will provide about 46 e-folds of inflation after the tunnelling. The lower value of the e-folding is not a problem, since in the context of the tachyonic-curvaton reheating, it could be of the order 45 or

Figure 1. Scalar potential associated to the tachyon field. All values are given in units where $\kappa = 1$. 

50, since the inflationary scale can be lower [10]. We solve the Equations (3) and (4) considering the following boundary conditions: \( \phi = \phi_T, \phi' = 0, a = 0, a' = 1 \), at \( \tau = 0 \) and working in units where \( \kappa = 1 \). A numerical solution which corresponds to the CDL instanton was found. In that case tunnelling occurs from \( \phi_F \approx 2.95 \times 10^5 \) to \( \phi_T \approx 3.043 \times 10^5 \). Now, we calculate the instanton action for the quantum tunnelling between the values \( \phi_F \) and \( \phi_T \) in the effective tachyon potential. The bounce-action is given by \( B = S_B - S_F \), i.e. the difference between the action associated with the bounce solution and the false vacuum. This action determines the probability of tunnelling for the process. Under the approximation that the bubble wall is infinitesimally thin we obtain:

\[
B = \frac{6\pi^2}{\kappa} R_b S_1 + \frac{12\pi^2}{\kappa^2} \left[ \frac{1}{V_T} \left( 1 - \kappa V_T R_b^2 / 3 \right)^{3/2} - 1 \right] - \frac{1}{V_F} \left[ (1 - \kappa V_T R_b^2 / 3)^{3/2} - 1 \right], \tag{6}
\]

where \( R_b \) is the radius of the bubble and \( S_1 \) correspond to the surface tension of the wall

\[
S_1 = \int_{\phi_T}^{\phi_b} d\phi \left( \frac{V(\phi)}{V_T} \right)^2 (1 - 1)^{1/2}. \tag{7}
\]

The radius of curvature of the bubble could be obtained demanding that the bounce-action (6) is an extremum. Then, the radius of the bubble is determined by setting \( dB/dR_b = 0 \), which gives

\[
\frac{S_1}{2R_b} = \left[ \left( 1 - \kappa V_T R_b^2 / 3 \right)^{1/2} - \left( 1 - \kappa V_F R_b^2 / 3 \right)^{1/2} \right]. \tag{7}
\]

Thus, the radius of the bubble, \( R_b \), is found by solving this equation.

3. Inflation after tunnelling and scalar perturbation spectra

Let us study what happens after the tunnelling. In order to do that we make an analytical continuation to the Lorentzian space-time and study the time evolution of the tachyon field \( \phi(t) \) and scale factor \( a(t) \), where \( t \) represents the cosmological time. Numerical solution to the Lorentzian version of equations (3) and (4) are show in Fig. 2 and Fig. 3, where we have consider the parameters of the potential defined in the previous section and the following boundary conditions \( \phi(0) = \phi_T \approx 3.043 \times 10^5 \) (the value obtained from the Euclidean numerical solution), \( \phi(0) = 0 \), \( a(0) = 0 \), \( a(0) = 1 \). We note in Fig. 2 that after the tunnelling, during the period \( 0 \leq t < t_f \), the tachyon field satisfies the slow roll condition \( \dot{\phi}^2 < 2/3 \). Then, we conclude that after the tunnelling our model generates a consistent inflationary era. On the other hand, the scale factor expands approximately \( e^{46} \) times during this period, see Fig. 3.

Even though the study of scalar density perturbations in open universes is quite complicated [7], it is interesting to give an estimation of the standard quantum scalar field fluctuations inside the bubble for our scenario. In particular, the spectra of scalar perturbations for a flat space, generated during tachyon inflation, expressed in terms of the slow-roll parameters defined in Ref. [11] is \( \frac{\delta \rho}{\rho} = [1 - 0.11 \epsilon_1 + 0.36 \epsilon_2] \frac{\kappa H}{2\pi \sqrt{2\epsilon_1}} \) [12]. Certainly, in our case, this expression is an approximation and must be supplemented by several different contribution in the context of an open inflationary universe [7]. However, one may expect that the flat-space expression gives a correct result for \( N > 3 \). Figure 4 shows the magnitude of the scalar density perturbations \( \delta \rho/\rho \) for our model as a function of the \( N \) e-folds of inflation. Note that our values coincide with the result of COBE normalized value \( \delta H \sim 2 \times 10^{-5} \) [13]. From Fig. 4 we see that \( \delta \rho/\rho \) has a deep minimum at \( N \leq 3 \) (mechanism of suppression of large scale density perturbation) and then approaches to its maximum at \( N \sim 7 \). The shape of the graph showed in Fig. 4 seems to be a generic feature of the one-field open inflation models based on tunnelling.
Figure 2. The behavior of the tachyon field after the tunnelling as a function of the cosmological time.

Figure 3. The growth of the logarithm of the scale factor as a function of the cosmological time. Here $H_0^2 = V_T/3$.

and bubble formation [7, 14]. If $N = O(1)$ corresponds to density perturbations on the horizon scale $\sim 10^{28}$ cm, then the maximum of the spectrum appears on a scale which is about three orders of magnitude smaller $\sim 10^{25}$ cm, in our model. In tachyon inflationary models the scalar spectral index and the tensor spectral index are given by $n = 1 - 2\epsilon_1 - \epsilon_2$ and $n_T = -2\epsilon_1$, in the slow-roll approximation [12]. From the numerical solution we can obtain their values. In particular for $N \sim 7$ we have $n \approx 0.98$ and $n_T \approx -0.033$. Notice that those indices are very closed to the Harrison-Zel’dovich spectrum [15].

4. Conclusion and Final Remarks
We have studied a one-field open inflationary universe model, where inflation is driven by a tachyonic field. In particular we focus on the viability of the model in order to generate a consistent inflationary scenario, compatible with observations (number of e-folding, density perturbations, spectral and tensor spectral indices). The idea of consider a tachyon field to drive inflation is a natural choice since the tachyon is a unstable particle. On the other
hand the possibility that this tachyon generates an open universe via bubble nucleation by quantum tunnelling appears as an interesting possibility to explore, given that, in the context of nonstandard brane decays [6], tachyon potentials with similar characteristics as our potential appear.

In this way, we have shown that one-field open inflationary universe models can be realized in the tachyonic theory in the context of the Standard Einstein General Relativity. We hope, that the most natural study for tachyonic theory would be realized for the Brane World Cosmology scenarios and we leave this task for a near future.

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