The sensitivity of the zero position of the forward–backward asymmetry to new physics effects in the $B \to K^* \mu^+ \mu^-$ decay

Altuğ Arda and Müge Boz
Hacettepe University, Department of Physics,
06532 Ankara, Turkey

ABSTRACT

Starting with the most general effective Hamiltonian comprising scalar and vector operators beyond the standard model, we discuss the impact of various operators on the zero of the forward–backward asymmetry in the dileptonic $B$ decay $B \to K^* \mu^+ \mu^-$. We find that, zero of the asymmetry is highly sensitive to the sign and size of the vector–vector operators and opposite chirality counterparts of the usual operators. The scalar–scalar four–fermion operators, on the other hand, have mild effect on the zero of the asymmetry. Our results are expected to be checked in the near future experiments.
1 Introduction

The flavour-changing transitions, which generally arise at one and higher loop orders provide an excellent testing ground for the standard model (S.M). Moreover, it is with such decays that the new physics effects can be probed via the loops of the particles beyond the S.M spectrum. Therefore, there are sound theoretical and experimental reasons for studying the flavour changing neutral current (FCNC) processes. Among all the FCNC phenomena, the rare $B$ decays are especially important, since one can both test the SM and search for possible NP effects, by confronting the theoretical results with the experiment.

In addition to having already determined the branching ratio of $B \to X_s \gamma$ and the CP asymmetry of $B \to J/\psi K$, experimental activity in $B$ physics, has begun to probe FCNC phenomena in semileptonic $B$ decays, and these experiments are expected to give precise measurements in semileptonic decays in the near future.

Concerning the semileptonic $B$ decays, $B \to X_s \ell^+\ell^-$ ($X_s = K, K^*$, $\ell = e, \mu, \tau$) decay is an example having both theoretical and experimental importance. The forward–backward asymmetry $A_{FB}$ of these decays is a particularly interesting quantity, since it vanishes at a specific value of the dilepton invariant mass in a hadronically clean way. In the recent literature, the dilepton invariant mass spectra, and the forward backward asymmetry in $B \to X_s \ell^+\ell^-$ decays has been analyzed in the detailed work of using the large energy effective theory (LEET) approach, and a simple analytic expression for the zero position of the $A_{FB}$ in the S.M has been derived. It has been found that the value of the dilepton invariant mass for which $A_{FB}$ may become zero provides a quite simple relation between the electric dipole coefficient $C_7$ and $C_9$, which is nearly free of hadronic uncertainties. Furthermore, the next to leading order (NLO) corrections to the exclusive decay has been carried out in. It is known that in the S.M, and in many of its extensions, the $B \to X_s \ell^+\ell^-$ decay is completely determined by the Wilson coefficients of only three operators evaluated at the scale $\mu = m_b$. On the other hand, the most general analysis of the $B \to X_s \ell^+\ell^-$ decay, based on the general four Fermi interaction, include new operators beyond the usual set. The new structures in the effective Hamiltonian make these decays quite interesting as an alternative testing platform for the S.M, and provide clues about the nature of the physics beyond the S.M. In the literature, the general model independent analysis of the inclusive $b \to s \ell^+\ell^-$ decay, in terms of 10 types of local four–Fermi interactions, has been performed in Ref. and, a systematic analysis of the exclusive $B \to K^* \ell^+\ell^-$ decay has been presented in. Moreover, a detailed study of the lepton polarization asymmetries in $B \to X_s \ell^+\ell^-$ decay has been carried out in a rather general model in, and, in the exclusive decay $B \to K^* \ell^+\ell^-$ in.
In this work, based on the work of [15], our aim is to analyze the possible new physics effects stemming from the new structures in the effective Hamiltonian in the $B \to K^*\mu^+\mu^-$ decay. Among the several works on $B \to K^*\ell^+\ell^-$ decays in the existing literature, there are several papers, which discusses the subject either using the zero mass approximation (for instance, see [18]), or, by taking into account the lepton mass effects [15, 19]. In our analysis, we include the lepton mass effects, and we study the influence of the new operators to the value of the dilepton invariant mass for which $A_{FB}$ vanishes, by taking into account of its value in the S.M. It will be seen that the position of the zero shifts in accord with the new physics contributions to the Wilson coefficients.

The organization of the work is as follows. In Sec. 2, starting from the differential decay width of the exclusive $B \to K^*\ell^+\ell^-$ decay, we calculate the numerator of the forward backward asymmetry whose intersectional value with the zero axes will determine the zero-position of the forward backward asymmetry. In Sec. 3, we carry out the numerical analysis to study the dependence of the zero position of the forward–backward asymmetry on the new coefficients. We conclude in Sec. 4.

\section{The Model}

The matrix element of the $B \to K^*\ell^+\ell^-$ decay at quark level is described by $b \to s\ell^+\ell^-$ transition for which the effective Hamiltonian at $O(\mu)$ scale can be written as:

$$\mathcal{H}_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{tb} V^*_{ts} \sum_{i=1}^{10} C_i(\mu) O_i(\mu),$$

where the full set of the operators $O_i(\mu)$ and the corresponding expressions for the Wilson coefficients $C_i(\mu)$ in the SM are given in [21, 22].

Taking into account of the most general form of the effective Hamiltonian, there are ten independent local four–Fermi interactions which may contribute to the process\cite{14}. Neglecting the tensor type interactions $C_T$ and $C_{TE}$ [15], the explicit form of the matrix element $\mathcal{M}$ of the $b \to s\ell^+\ell^-$ transition can be written as a sum of the SM and new physics contributions, including eight independent local four–Fermi interactions.

$$\mathcal{M} = \frac{G_F}{\sqrt{2\pi}} V_{tb} V^*_{ts} \left\{ \left( C^\text{eff}_{9} - C_{10} \right) \bar{s}_L\gamma_\mu b_L \bar{\ell}_L\gamma_\mu L + \left( C^\text{eff}_{9} + C_{10} \right) \bar{s}_L\gamma_\mu b_L \bar{\ell}_R\gamma_\mu R \right\}$$

\begin{equation}
\begin{aligned}
&- 2C^\text{eff}_{7} \bar{s}_i \sigma_{\mu\nu} \frac{q^\nu}{8} (\bar{m}_s L + \bar{m}_b R) b_L \bar{\ell}_R \gamma_\mu L + C_{LL} \bar{s}_L\gamma_\mu b_L \bar{\ell}_L\gamma_\mu L + C_{LR} \bar{s}_L\gamma_\mu b_L \bar{\ell}_R\gamma_\mu R \\
&+ C_{RL} \bar{s}_R\gamma_\mu b_R \bar{\ell}_L\gamma_\mu L + C_{RR} \bar{s}_R\gamma_\mu b_R \bar{\ell}_R\gamma_\mu R + C_{LRL} \bar{s}_L b_R \bar{\ell}_L \bar{\ell}_R + C_{RLR} \bar{s}_R b_L \bar{\ell}_L \bar{\ell}_R \\
&+ C_{LRL} \bar{s}_R b_R \bar{\ell}_L \bar{\ell}_R + C_{RLR} \bar{s}_R b_L \bar{\ell}_L \bar{\ell}_R \right\}
\end{aligned}
\end{equation}
Here, $C_{LL(RL)}$, $C_{RR(RL)}$, $C_{LRLR(LRL)}$, $C_{RLLR(RLRL)}$, are the coefficients of the four Fermi interactions. $R = (1 + \gamma_5)/2$ and $L = (1 - \gamma_5)/2$, and

$$\hat{\mathbf{s}} = q^2/m_B^2, \quad \hat{m}_b = m_b/m_B, \quad q = (p_B - p_K),$$

The expression for $C_9^{eff}(\hat{s})$ in Eq. (2) is given by:

$$C_9^{eff}(\hat{s}) = C_9 + g(z, \hat{s})(3C_1 + C_2 + 3C_3 + C_4 + 3C_5 + C_6)$$

$$- \frac{1}{2}g(1, \hat{s})(4C_3 + 4C_4 + 3C_5 + C_6)$$

$$- \frac{1}{2}g(0, \hat{s})(C_3 + 3C_4) + \frac{2}{9}(3C_3 + C_4 + 3C_5 + C_6),$$

where $z = \frac{m_s}{m_b}$, the values of $g(z, \hat{s}), g(1, \hat{s}), g(0, \hat{s})$ can be found in [20, 21], and the values of $C_i$ in the SM are given in the numerical analysis.

As mentioned in the Introduction, our aim is to find the value of the dilepton invariant mass for which $\Lambda_{FB}$ vanishes. Therefore, in order to determine this quantity, following the work of [15], we first concentrate on the decay width of the $B \rightarrow K^* \mu^+\mu^-$ decay. The matrix elements, $K^* | \bar{s} \gamma_\mu(1 \pm \gamma_5)b | B)$, $K^* | \bar{s} \sigma_{\mu\nu} q^\nu(1 + \gamma_5)b | B)$, $K^* | \bar{s}(1 \pm \gamma_5)b | B)$ have been calculated in [15]. Using the matrix elements, and the helicity amplitude formalism [12, 22, 23], the decay width of the $B \rightarrow K^* \mu^+\mu^-$ decay is given by:

$$\frac{d\Gamma}{dq^2 du} = \frac{G_F^2 \alpha^2}{214 \pi^3 m_B} |V_{tb} V_{ts}^*|^2 v \lambda^{1/2}(1, r, s) \sum_{i=1}^{6} \mathcal{M}_i^2,$$

where, $v$ is the velocity of $\mu$, and

$$v = \sqrt{1 - 4\hat{m}_\mu^2/\hat{s}}, \quad \hat{m}_\mu = \frac{m_\mu}{m_B},$$

$$\lambda(1, \hat{m}_K^*, \hat{s}) = 1 + \hat{m}_K^* + \hat{s}^2 - 2\hat{m}_K^* \hat{s} - 2\hat{m}_K^* - 2\hat{s},$$

$$\hat{m}_K^* = \frac{m_K^*}{m_B^2}.$$

Here, $\mathcal{M}_i^2$ are the the combinations of the matrix elements which can be written in the following form:

$$\mathcal{M}_1^2 = |\mathcal{M}_1^+|^2 + |\mathcal{M}_{++}|^2,$$

$$\mathcal{M}_2^2 = |\mathcal{M}_1^-|^2 + |\mathcal{M}_{--}|^2,$$

$$\mathcal{M}_3^2 = |\mathcal{M}_2^+|^2 + |\mathcal{M}_{+-}|^2,$$

$$\mathcal{M}_4^2 = |\mathcal{M}_2^-|^2 + |\mathcal{M}_{-+}|^2,$$

$$\mathcal{M}_5^2 = |\mathcal{M}_3^+|^2 + |\mathcal{M}_{0+}|^2,$$

$$\mathcal{M}_6^2 = |\mathcal{M}_3^-|^2 + |\mathcal{M}_{0-}|^2,$$
where, the superscripts correspond to the helicity of the $K^*$ meson, and the subscripts denote the helicities of the muons. The combinations of the matrix elements $M_i^2$ include the Wilson coefficients $C_7, C_9, C_{10}$ and the new operators beyond the usual set. Therefore, for convenience, we define:

$$ \tilde{C}_7 = 4C_{\text{eff}} , $$

and, the combinations of the new coefficients as:

$$ \begin{align*}
  \tilde{C}_{RR+} &= C_{RR} + C_{RL} , \\
  \tilde{C}_{RR-} &= C_{RR} - C_{RL} , \\
  \tilde{C}_{LL+} &= C_{LL} + C_{RL} , \\
  \tilde{C}_{LRLR} &= C_{LRL} - C_{RLRL} , \\
  \tilde{C}_9 &= 2C_{\text{eff}}^9 , \\
  \tilde{C}_{10} &= 2C_{\text{eff}}^{10} ,
\end{align*} $$

(7)

Here, the coefficients $\tilde{C}_9$ and $\tilde{C}_{10}$ describe the contributions from the S.M, and the new physics. The combinations $\tilde{C}_{LRLR}$, and $\tilde{C}_{RLRL}$ describe scalar type interactions. Then, using the explicit forms of the matrix elements [15], we calculate $M_1^2$, and $M_2^2$ as:

$$ \begin{align*}
  M_1^2 &= 2m_\mu^2(1-u^2) f_1 , \\
  M_2^2 &= \mathcal{M}_2^2 ,
\end{align*} $$

(8)

where,

$$ u = \cos \theta , $$

(9)

and $\theta$ is the angle between $K^*$, and $\mu^-$. Here, $f_1$ read as:

$$ \begin{align*}
  f_1 &= (H_+^2 + H_-^2) |\tilde{C}_9|^2 + \left( \frac{m_b}{s_{MB}} \right)(H_+ H_+ + H_- H_-)2Re[\tilde{C}_9 \tilde{C}_7^*] \\
  &+ (H_+ h_+ + H_- h_-)2Re[\tilde{C}_9 \tilde{C}_{RR+}^*] + \left( \frac{m_b}{s_{MB}} \right)(h_+ H_+ + h_- H_-)2Re[\tilde{C}_9 \tilde{C}_{RR+}^*] \\
  &+ \left( \frac{m_b}{s_{MB}} \right)^2(\mathcal{H}_+^2 + \mathcal{H}_-^2) \left| \tilde{C}_7 \right|^2 + \left( h_+^2 + h_-^2 \right) \left| \tilde{C}_{RR+} \right|^2 ,
\end{align*} $$

(10)

The expressions of $\mathcal{M}_3^2$ and $\mathcal{M}_4^2$ are given by:

$$ \mathcal{M}_3^2 = \frac{\hat{s}_{MB}^2}{2} \left\{ (1+u^2)f_3^{(+)} - 2uf_3^{(-)} \right\} , $$

(11)
\[
\mathcal{M}_4^2 = \frac{\hat{s}m_B^2}{2} \left\{ (1 + u^2)f_3^{(+)}(v \to -v) + 2uf_3^{(-)}(v \to -v) \right\},
\]
where
\[
f_3^{(\pm)} = \left(\frac{m_B}{\hat{s}m_B}\right)(H_+\mathcal{H}_+ \mp H_-\mathcal{H}_-) (2Re[\hat{C}_9\hat{C}_7^*] + v2Re[\hat{C}_9\hat{C}_7^*])
+ (H_+h_+ \mp H_-h_-)(2Re[\hat{C}_9\hat{C}_{RR+}^*] + v2Re[\hat{C}_9\hat{C}_{RR-}^*])
+ v2Re[\hat{C}_{10}\hat{C}_{RR+}^*] + v^22Re[\hat{C}_{10}\hat{C}_{RR-}^*])
\]
\[
+ \frac{m_B}{\hat{s}m_B}2(H_+h_+ \mp H_-h_-)(2Re[\hat{C}_7\hat{C}_{RR+}^*] + v2Re[\hat{C}_7\hat{C}_{RR-}^*])
\]
\[
+ \frac{m_B}{\hat{s}m_B}2(H_+h_+ \mp H_-h_-)(2Re[\hat{C}_7\hat{C}_{RR+}^*] + v2Re[\hat{C}_7\hat{C}_{RR-}^*])
\]
\[
+ (\frac{m_B}{\hat{s}m_B})^2(H_+^2 \mp H_-^2) \left| \hat{C}_7 \right|^2
+ (H_+^2 \mp H_-^2)(\left| \hat{C}_{RR+} \right|^2 + v2Re[\hat{C}_{RR+}\hat{C}_{RR-}^*] + v^22Re[\hat{C}_{RR-}]^2),
\]
In Eq. (10), and Eq. (13) the functions \(H_+, h_-\) are the helicity amplitudes, and in terms of the form factors, they have the following structures:
\[
H_+ = m_B \left[ \pm \lambda^{1/2} \frac{V(\hat{s})}{1 + \hat{m}_K} + (1 + \hat{m}_K)A_1(\hat{s}) \right],
\]
\[
\mathcal{H}_+ = 2m_B^2 \left[ \pm \lambda^{1/2}T_1(\hat{s}) + (1 - \hat{m}_K^2)T_2(\hat{s}) \right],
\]
\[
h_+ = H_+(A_1 \to -A_1, A_2 \to -A_2)
\]
Finally, \(\mathcal{M}_5^2\), and \(\mathcal{M}_6^2\) can be calculated as:
\[
\mathcal{M}_5^2 = 2\hat{s}m_B^2 (1 - u^2) f_5,
\]
where
\[
f_5 = \hat{H}_0^2(\left| \hat{C}_9 \right|^2 + v^2\left| \hat{C}_{10} \right|^2) - \left(\frac{m_B}{\hat{s}m_B}\right)H_0\mathcal{H}_0 (2Re[\hat{C}_9\hat{C}_7^*])
+ H_0h_0(2Re[\hat{C}_9\hat{C}_{RR+}^*] + v^22Re[\hat{C}_{10}\hat{C}_{RR-}^*]) + \left(\frac{m_B}{\hat{s}m_B}\right)^2H_0^2(\left| \hat{C}_7 \right|^2)
\]
\[
- \left(\frac{m_B}{\hat{s}m_B}\right)H_0h_02Re[\hat{C}_7\hat{C}_{RR+}^*] + h_0^2(\left| \hat{C}_{RR+} \right|^2 + v^2\left| \hat{C}_{RR-} \right|^2),
\]
and,
\[
\mathcal{M}_6^2 = 4m_\mu^2u^2f_6^{(1)} + 4m_\mu u_6f_6^{(2)} + 4m_\mu^2f_6^{(3)},
\]
with
\[
f_6^{(1)} = 2(H_0)^2(\left| \hat{C}_9 \right|^2 - 2(\frac{m_B}{\hat{s}m_B})H_0\mathcal{H}_0(2Re[\hat{C}_9\hat{C}_7^*])
\]
\[ f_6^{(2)} = m_\mu H_0^0 h_0 (2 \text{Re}[\tilde{C}_{\tilde{R}R}^{*} \tilde{C}_{10}^{*}] - 2 \text{Re}[\tilde{C}_{L L}^{*} \tilde{C}_{10}^{*}]) \]
\[ + \left( \frac{\hat{s}}{m_{\mu} m_b} \right) H_s^0 h_0 \left\{ (1 - v) \left( 2 \text{Re}[\tilde{C}_{L L}^{*} \tilde{C}_{L L R R}^{*}] - 2 \text{Re}[\tilde{C}_{R R}^{*} \tilde{C}_{L L R R}^{*}] \right) \right\} \]
\[ + h_0^2 \left( |\tilde{C}_{L L}^{*}|^2 + |\tilde{C}_{R R}^{*}|^2 \right), \quad (18) \]
\[ f_6^{(3)} = 2(h_s^0)^2 |\tilde{C}_{R R}^{*}|^2 + 2 h_s^0 H_s^0 (2 \text{Re}[\tilde{C}_{10}^{*} \tilde{C}_{R R}^{*}]) \]
\[ + \left( \frac{\hat{s}}{m_{\mu} m_b} \right) h_s^0 H_s^0 \left( 2 \text{Re}[\tilde{C}_{R R}^{*} \tilde{C}_{L L R R}^{*}] - 2 \text{Re}[\tilde{C}_{R R}^{*} \tilde{C}_{L R R L}^{*}] \right) \]
\[ + \left( H_s^0 \right)^2 \left\{ 2 \tilde{C}_{10}^{*} \left( 1 - v^2 \right) \left( |\tilde{C}_{L L R R}^{*}|^2 + |\tilde{C}_{L R R L}^{*}|^2 \right) \right\} \]
\[ - (1 - v^2) 2 \text{Re}[\tilde{C}_{L R R L}^{*} \tilde{C}_{L L R R}^{*}] \right\}, \quad (19) \]

where,
\[ H_0 = \frac{m_B}{2m_K \sqrt{\hat{s}}} \left[ (1 - \hat{m}_K^2 - \hat{s})(1 + \hat{m}_K) A_1(\hat{s}) + \lambda \frac{A_2(\hat{s})}{1 + \hat{m}_K} \right], \]
\[ H_s^0 = - \frac{m_B \lambda^{1/2}}{\sqrt{\hat{s}}} A_0(\hat{s}), \]
\[ H_+ = 2m_B^2 \left[ \pm \lambda^{1/2} T_1(\hat{s}) + (1 - \hat{m}_K^2) T_2(\hat{s}) \right], \]
\[ H_0 = \frac{m_B}{2m_K \sqrt{\hat{s}}} \left\{ (1 - \hat{m}_K^2 - \hat{s})(1 - \hat{m}_K^2) T_2(\hat{s}) - \lambda \left[ T_2(\hat{s}) + \frac{\hat{s}}{1 - \hat{m}_K^2} T_3(\hat{s}) \right] \right\}, \]
\[ h_0 = H_0(A_1 \to -A_1, \ A_2 \to -A_2), \quad (20) \]

As mentioned in the Introduction, our aim is to determine the zero position of the forward-backward asymmetry,
\[ \frac{d}{dq^2} A_{FB}(q^2) = \frac{\int_0^1 dx \frac{d\Gamma}{dq^2 dx} - \int_{-1}^0 dx \frac{d\Gamma}{dq^2 dx}}{\int_0^1 dx \frac{d\Gamma}{dq^2 dx} + \int_{-1}^0 dx \frac{d\Gamma}{dq^2 dx}}, \quad (22) \]
which can predict possibly the new physics contributions. Indeed, existence of the new physics


can be confirmed by the shift in the zero position of the forward backward asymmetry\(^9\). Therefore, using Eqs.(8)-(20), we calculate the the numerator of the forward backward asymmetry as:

\[
\mathcal{N} = \frac{G^2 \alpha^2}{24 \pi^5} |V_{tb}V_{ts}^*|^2 \mathcal{R},
\]

where

\[
\mathcal{R} = \frac{1}{m_B} v \lambda^{1/2} \left[ 2\sin^2 \theta \left( \frac{h_2^2 - h_2^2}{1} \right) 2Re[\tilde{C}_{RR+} \tilde{C}_{RR-}^*] + (H_2^2 - H_2^2) \ 2Re[\tilde{C}_9 \tilde{C}_{10}^*] \right.
\]

\[
+ (H_+ - H_+ - H_+ - H_+)(2Re[\tilde{C}_9 \tilde{C}_{RR-}^*] + 2Re[\tilde{C}_{10} \tilde{C}_{RR+}^*])
\]

\[
+ \left( \frac{m_b}{\sin^2 \theta B} \right) \left( (H_+ - H_+ - H_+ - H_+)(2Re[\tilde{C}_7 \tilde{C}_{RR-}^*] + (H_+ - H_+ - H_+ - H_+)(2Re[\tilde{C}_7 \tilde{C}_{10}^*]) \right)
\]

\[
+ 4m_b \left( h_0 h_0^0(2Re[\tilde{C}_{RR+} \tilde{C}_{RR-}^*] - 2Re[\tilde{C}_{LL+} \tilde{C}_{LL-}^*])
\]

\[
+ h_0 H_0^0(2Re[\tilde{C}_{RR+} \tilde{C}_{10}^*] - 2Re[\tilde{C}_{LL+} \tilde{C}_{10}^*]) \right)
\]

\[
+ \frac{8m_b \sin^2 \theta B}{m_B} \left( \frac{m_b}{\sin^2 \theta B} \right) \left( h_0 H_0^0(2Re[\tilde{C}_9 \tilde{C}_{LRLR-}^*] + 2Re[\tilde{C}_7 \tilde{C}_{LRLR-}^*]) \right)
\]

\[
- H_0 H_0^0(2Re[\tilde{C}_9 \tilde{C}_{LRLR-}^*] + 2Re[\tilde{C}_7 \tilde{C}_{LRLR-}^*]) \right)
\]

\[
+ \frac{4m_b \sin^2 \theta B}{m_B} h_0 H_0^0 \left( (1 - v)(2Re[\tilde{C}_{RR+} \tilde{C}_{LRLR-}^*] + 2Re[\tilde{C}_{LL+} \tilde{C}_{LRLR-}^*]) \right)
\]

\[
- (1 + v)(2Re[\tilde{C}_{RR+} \tilde{C}_{LRLR-}^*] - 2Re[\tilde{C}_{LL+} \tilde{C}_{LRLR-}^*]) \right),
\]

Then, using Eq. (13), and Eq. (21), \(\mathcal{R}\) can be written in the following form:

\[
\mathcal{R} = \frac{1}{m_B} v \lambda^{1/2} \left( R_1 \sin \lambda^{1/2} A_1 V - R_2 v \lambda^{1/2} T_2 V + R_3 v \lambda^{1/2} A_1 T_1 \right.
\]

\[
- R_4 \lambda^{1/2} A_0 A_1 (1 - \tilde{m}_K^2 - \tilde{s})(R_5 \frac{R_5}{s} + R_6 v + R_7) + R_8 \lambda^{3/2} A_0 A_2 (\frac{R_5}{s} + R_6 v + R_7)
\]

\[
- R_9 v \lambda^{1/2} A_0 T_2 (1 - 3 \tilde{m}_K^2 - \tilde{s}) + R_{10} v \lambda^{1/2} A_0 T_3 \right)
\]

Here,

\[
\mathcal{R}_1 = 8 m_B^4 \left( 2Re[\tilde{C}_{RR+} \tilde{C}_{RR-}^*] - 2Re[\tilde{C}_9 \tilde{C}_{10}^*] \right),
\]

\[
\mathcal{R}_2 = 8 m_B^4 \tilde{m}_b (1 - \tilde{m}_K) \left( 2Re[\tilde{C}_7 \tilde{C}_{10}^*] + 2Re[\tilde{C}_7 \tilde{C}_{RR-}^*] \right),
\]

\[
\mathcal{R}_3 = 8 m_B^4 \tilde{m}_b (1 + \tilde{m}_K) \left( - 2Re[\tilde{C}_7 \tilde{C}_{10}^*] + 2Re[\tilde{C}_7 \tilde{C}_{RR-}^*] \right),
\]
\[ R_4 = \frac{4m_B^4}{m_{K^*}}(1 + \hat{m}_{K^*}), \]
\[ R_5 = \frac{\hat{m}_\mu^2}{2m_b} \left( 2\text{Re}[\tilde{C}_{RR^+} \tilde{C}_{\gamma 10}^*] + 2\text{Re}[\tilde{C}_{RR^+} \tilde{C}_{RR^-}] - 2\text{Re}[\tilde{C}_{LL^+} \tilde{C}_{10}^*] - 2\text{Re}[\tilde{C}_{LL^+} \tilde{C}_{RR^-}] \right), \]
\[ R_6 = \frac{\hat{m}_\mu^2}{2m_b} \left( 4\text{Re}[\tilde{C}_9 \tilde{C}_{\gamma LLRL}^*] + 4\text{Re}[\tilde{C}_9 \tilde{C}_{RLRL}^*] - 2\text{Re}[\tilde{C}_{LL^+} \tilde{C}_{LRRL}^*] \right) - 2\text{Re}[\tilde{C}_{LL^+} \tilde{C}_{LRRL}^*] - 2\text{Re}[\tilde{C}_{RR^+} \tilde{C}_{LRRL}^*] - 2\text{Re}[\tilde{C}_{RR^+} \tilde{C}_{LRRL}^*] \right), \]
\[ R_7 = \frac{\hat{m}_\mu^2}{2m_b} \left( 2\text{Re}[\tilde{C}_{RR^+} \tilde{C}_{RLRL}^*] + 2\text{Re}[\tilde{C}_{LL^+} \tilde{C}_{LRRL}^*] \right) - 2\text{Re}[\tilde{C}_{RR^+} \tilde{C}_{LRRL}^*] - 2\text{Re}[\tilde{C}_{LL^+} \tilde{C}_{LRRL}^*] \right), \]
\[ R_8 = \frac{4m_B^4}{m_{K^*}(1 + \hat{m}_{K^*})}, \]
\[ R_9 = 8m_B^4 \frac{\hat{m}_\mu}{m_{K^*}} \left( 2\text{Re}[\tilde{C}_7 \tilde{C}_{\gamma LLRL}^*] + 2\text{Re}[\tilde{C}_7 \tilde{C}_{RLRL}^*] \right), \]
\[ R_{10} = 8m_B^4 \frac{\hat{m}_\mu}{m_{K^*}(1 - \hat{m}_{K^*})} \left( 2\text{Re}[\tilde{C}_7 \tilde{C}_{\gamma LLRL}^*] + 2\text{Re}[\tilde{C}_7 \tilde{C}_{RLRL}^*] \right). \]

Naturally, to find the zero position of \( A_{FB} \), it is reasonable to find the roots of the function \( R \). However, due to the effective coefficient \( \tilde{C}_9 \), the computation is quite complicated. Therefore, in determining the zero position of \( A_{FB} \), we analyze the variation of function \( R \) with the dilepton invariant mass. The intersectional value of \( R \) with the zero axes will determine the zero position of \( A_{FB} \), which can be interesting as an alternative testing platform for the S.M, and provide clues about nature of the new operators beyond the S.M.

### 3 Numerical analysis

In the following we will perform a numerical analysis, to study the sensitivity of the zero position of the forward backward asymmetry to new physics effects, and discuss its phenomenological implications. The zero position of the \( A_{FB} \) has been calculated in the S.M [3],

\[ \text{Re}[\tilde{C}_9^{eff}(\hat{s})] = -\frac{\hat{m}_b}{\hat{s}} \frac{T_2(\hat{s})}{A_1(\hat{s})} \left( 1 - \hat{m}_{K^*} \right) + \frac{T_1(\hat{s})}{V(\hat{s})} \left( 1 + \hat{m}_{K^*} \right) \],

which depends on the ratio of the form factors, as well as the other quantities. Thus, in principle the expression Eq. (27) is affected by the presence of the hadronic form factors making it a more uncertain relation. However, using the large energy expansion theory (LEET), it has been shown in [3] that both ratios of the form factors have no hadronic uncertainty since the dependence of the intrinsically non-perturbative quantities cancels, and the position of zero in \( B \to K^* \ell^+ \ell^- \) is predicted simply in terms of the short distance Wilson coefficients \( \tilde{C}_9^{eff} \) and \( \tilde{C}_7^{eff} \).

\[ \text{Re}[\tilde{C}_9^{eff}(\hat{s})] = -2\frac{\hat{m}_b}{\hat{s}} \frac{C_9^{eff}}{C_7^{eff}} \frac{1 - \hat{s}}{1 + \hat{m}_{K^*}^2 - \hat{s}}, \]
Therefore, they have shown in [9] that the form factor dependence in $\hat{s}$ cancels in the large energy expansion approximation. Moreover, they have found the value of the zero position as $\hat{s} = 0.1\text{GeV}$, with the numerical values of the coefficients at $\mu = m_b$ within the S.M,

\begin{align*}
C_1 &= -0.248, & C_2 &= 1.107, & C_3 &= 0.011, \\
C_4 &= -0.026, & C_5 &= 0.007, & C_6 &= -0.031, \\
C_7 &= -0.313, & C_9 &= 4.344, & C_{10} &= -4.669,
\end{align*}

and for $m_b = 4.4\text{ GeV}$. To find a reasonable agreement with the results of S.M, we use the same input parameters for $m_b$, and $C_i$, in our analysis. However, we choose light cone QCD sum rules method predictions for the form factors [12]. Thus, using the results of [12], in which the form factors are described by a three parameter fit, the $\hat{s}$ dependence of any of the form factors appearing in Eq. (14) could be parametrized as:

$$F(\hat{s}) = \frac{F(0)}{1 - a_F \hat{s} + b_F \hat{s}^2},$$

(29)

The parameters for $F_0$, $a_F$, and $b_F$ for each form factor are given by:

\begin{align*}
A_0 &= 0.47, & a_F &= 1.64, & b_F &= 0.94, \\
A_1 &= 0.35, & a_F &= 0.54, & b_F &= -0.02, \\
A_2 &= 0.30, & a_F &= 1.02, & b_F &= 0.08 \\
V_1 &= 0.47, & a_F &= 1.50, & b_F &= 0.51, \\
T_1 &= 0.19, & a_F &= 1.53, & b_F &= 1.77, \\
T_2 &= 0.19, & a_F &= 0.36, & b_F &= -0.49, \\
T_2 &= 0.13, & a_F &= 1.07, & b_F &= 0.16.
\end{align*}

Moreover, in the numerical analysis, we use the the kinematical range for the normalized dilepton invariant mass in terms of the lepton and pseudo scalar masses:

$$4m_\ell^2/m_B^2 \leq \hat{s} \leq (1 - m_K^*/m_B)^2.$$ 

(30)

In what follows, we will analyze the variation of function $\mathcal{R}$ with the dilepton invariant mass. In forming the scatter plots, we first consider the case where all the new coefficients are zero to investigate whether one can find a reasonable agreement with the results of the S.M, and then let $C_{10}$, and $-C_{10}$ values for each of the new coefficients, setting all the others zero, to analyze the shift in the zero position of $\mathcal{R}$, as compared to the S.M. We would like to note that, in each scatter plot, the function $\mathcal{R}$ is divided by $10^3$, for convenience.
Figure 1: The dependence of $\mathcal{R}$ on $\hat{s}$ which corresponds to the cases: $C_{LL} = -C_{10}$ (top curve), $C_{LL} = C_{10}$ (bottom curve), and all the new coefficients are zero (middle curve).

In Fig. 1, we show the dependence of $\mathcal{R}$ on $\hat{s}$ when $C_{LL} = -C_{10}$ for the top curve, when $C_{LL} = C_{10}$ for the bottom curve, and when all the new coefficients are set to zero for the middle curve (as mentioned before, in plotting the top and the bottom curves, we have set all the other new coefficients to zero). As we can see from the figure that, when all the new coefficients are set to zero, the zero position of the forward-backward asymmetry occurs at $\hat{s} \sim 0.1$, which is consistent with the results of S.M [9]. Naturally, for non-zero values of the new operators, we expect the zero position of $\mathcal{R}$ to shift from its value of S.M. Indeed, when $C_{LL} = -C_{10}$ and for all the other coefficients set equal to zero, $\mathcal{R}$ crosses zero around $\hat{s} \sim 0.06$. On the other hand, when $C_{LL} = C_{10}$, and for all the other coefficients set equal to zero, the position of zero gradually shifts to $\sim 0.2$. A closer comparative look at the figure suggests that when $C_{LL} = -C_{10}$, the position of the zero shifts to left, and when $C_{LL} = C_{10}$, it shifts to right, as compared to its value of S.M.

Shown in Fig. 2 is the dependence of $\mathcal{R}$ on $\hat{s}$ when $C_{LR} = -C_{10}$ (top curve), $C_{LR} = C_{10}$ (bottom curve), and when all the new coefficients are zero (middle curve). As we can see from the figure that, $\mathcal{R}$ takes the largest value when all the new coefficients are set to zero, for which case the zero position occurs at $\hat{s} \sim 0.1$. Considering the non-zero values of the new operators, for instance, when $C_{LR} = -C_{10}$, with all the other new coefficients set equal to zero, $\mathcal{R}$ crosses zero around $\hat{s} \sim 0.06$, and when $C_{LR} = C_{10}$, with all the other new coefficients set equal to zero, it crosses zero around $\hat{s} \sim 0.2$. Namely, for $C_{LR} = -C_{10}$, and $C_{LR} = C_{10}$, the position of the zero shifts to left, and right, respectively, as compared to S.M, in accord with the new physics contributions to the Wilson coefficients.

In Fig. 3, we show the dependence of $\mathcal{R}$ on $\hat{s}$ when $C_{RR} = -C_{10}$ (top curve), $C_{RR} = C_{10}$ (bottom curve), and when all the new coefficients are zero (middle curve). One notes that $\mathcal{R}$
is less sensitive to the change in the values of $C_{RR}$, as compared to the first two cases (Fig. 1 and Fig. 2). That is, both curves which correspond to $C_{RR} = -C_{10}$, and $C_{RR} = C_{10}$ cases, behave similarly, overlapping up to $\hat{s} \sim 0.3$, and then there is a gradual shift between these curves. Therefore, they cross zero at the same value of $\hat{s}$ ($\sim 0.08$), which is slightly different from that of the S.M. Namely, the position of the zero gradually shifts to right, as compared to its S.M value. Similar observations can be made for Fig. 4, when $C_{RL} = -C_{10}$ (top curve), $C_{RL} = C_{10}$ (bottom curve), and when all the new coefficients are zero (middle curve). One notes from the figure that the curves which correspond to $C_{RL} = -C_{10}$, and $C_{RL} = C_{10}$ behave oppositely with respect to the curve which corresponds to S.M (middle curve), as compared to Fig. 3. Therefore, unlike Fig. 3, the position of zero gradually shifts to left, as compared to the S.M.
Figure 4: The dependence of $\mathcal{R}$ on $\hat{s}$ which corresponds to the cases: $C_{RL} = C_{10}$ (middle curve), $C_{RL} = -C_{10}$ (bottom curve), and all the new coefficients are zero (top curve).

A comparative analysis of Figs 1-4 shows that when $C_{LL(LR)} = C_{10}$, and all the other coefficients are zero, the zero position shifts to left, and when $C_{LL(LR)} = -C_{10}$ it shifts to right, as compared to its value of S.M. However, for $C_{RR} = \pm C_{10}$, and $C_{RL} = \pm C_{10}$, with all the other coefficients set to zero, the shift in the zero position is to the right for the former, and to the left for the latter. One notes that although the lepton mass effects are included in our analysis, it can not give observable effects, as compared to S.M, since the mass of $\mu$ ($m_{\mu} = 0.105$ GeV) is quite small.

Figure 5: The dependence of $\mathcal{R}$ on $\hat{s}$ which corresponds to the cases: $C_{RLRL} = C_{10}$ (top curve), $C_{RLRL} = -C_{10}$ (bottom curve), and all the new coefficients are zero (middle curve).

We show the dependence of $\mathcal{R}$ on $\hat{s}$ on the scalar exchange operators, when $C_{RLRL} = C_{10}$ (top curve), $C_{RLRL} = -C_{10}$ (bottom curve) in Fig. 5, and $C_{LRRL} = C_{10}$ (top curve), $C_{LRRL} = -C_{10}$ (bottom curve) in Fig. 6. In both figures, the middle curves correspond to the case when all the new coefficients are equal to zero. A comparative look at both figures suggests that, when
$C_{LRLR(CLRL)} = C_{10}$ (top curves of Fig. 5 and 6), and $C_{LRLR(LRLR)} = -C_{10}$ (bottom curves of Fig. 5 and 6), the dependence of $\mathcal{R}$ on the scalar exchange operators is exactly the same, and, the positions of zeros shift to left and right, as compared to S.M in both of these cases.

![Graph](image)

Figure 6: The dependence of $\mathcal{R}$ on $\hat{s}$ which corresponds to the cases: $C_{LRLR} = C_{10}$ (top curve), $C_{LRLR} = -C_{10}$ (bottom curve), and all the new coefficients are zero (middle curve).

4 Conclusion

In this work, we have studied the sensitivity of the zero position of the forward backward asymmetry to the new physics effects. It is found that the position of zero shifts in accord with the new physics contributions to the Wilson coefficients. Among all the new coefficients, the zero of the asymmetry is highly sensitive to the sign and size of the vector–vector operators and opposite chirality counterparts of the usual operators. The scalar–scalar four–fermion operators, on the other hand, have mild effect on the zero of the asymmetry.

Naturally, with increasing data and statistics, the experimental activities in B-physics are expected to give precise measurements in semileptonic decays, and the $A_{FB}$ is one of the key physical quantities that can be measured. Therefore, it could be appropriate to provide an estimate about the number of events which is needed to measure the forward-backward asymmetry with the BaBar and Belle experiments. (i) Assuming that, BaBar will produce $(3-10) \times 10^7 \ b\bar{b}$ pairs in 1 year, for $\mathcal{L} = (3-10) \times 10^{33}\ cm^{-1}s^{-1}$ at $\sqrt{s} = 10$ GeV [24], and (ii) taking into account the experimentally relevant number of events required to measure an asymmetry $A$ of the decay with the branching ratio $Br(B \rightarrow K^* \mu^+\mu^-) = 1.4 \times 10^{-6}$ [12] at the $n\sigma$ level ($N = \frac{n^2}{BrA^2}$), the number of events to observe 10% $A_{FB}$ in the $B \rightarrow K^* \mu^+\mu^-$ decay (which is the average forward-backward asymmetry in the S.M) at 1$\sigma$ level can be estimated as $7 \times 10^7$. Therefore, 1 year running of BaBar at 1$\sigma$ level is sufficient to measure the zero of the $A_{FB}$, assuming that
the asymmetry remains around 10%.

The observation of the zero of the forward-backward asymmetry (as suggested by the S.M) as well as possible shift induced by the new physics effects both require the measurement of $A_{FB}$ at different values of the dilepton invariant mass. The estimate above is good for observing the sign change in the asymmetry. However, to observe the depletion of the asymmetry (asymmetry values much smaller than 10% ) requires much larger number of $b\bar{b}$ pairs to be produced.

M. B would like to thank the Turkish Scientific and Technical Research Council (TÜBİTAK) for partial support under the project, No:TBAG2002(100T108).
References

[1] K. G. Chetyrkin, M. Misiak and M. Munz, Phys. Lett. B 400, 206 (1997) [Erratum-ibid. B 425, 414 (1998)] [arXiv:hep-ph/9612313].

[2] M. S. Alam et al. [CLEO Collaboration], Phys. Rev. Lett. 74, 2885 (1995); S. Chen et al. [CLEO Collaboration], Phys. Rev. Lett. 87, 251807 (2001) [arXiv:hep-ex/0108032]; R. Barate et al. [ALEPH Collaboration], Phys. Lett. B 429, 169 (1998); K. Abe et al. [Belle Collaboration], Phys. Lett. B 511, 151 (2001) [arXiv:hep-ex/0103042].

[3] B. Aubert et al. [BABAR Collaboration], Phys. Rev. Lett. 87, 091801 (2001) [arXiv:hep-ex/0107013]; K. Abe et al. [Belle Collaboration], Phys. Rev. Lett. 87, 091802 (2001) [arXiv:hep-ex/0107061].

[4] K. Abe et al. [Belle Collaboration], arXiv:hep-ex/0107072; Phys. Rev. Lett. 88, 021801 (2002) [arXiv:hep-ex/0109026].

[5] B. Aubert et al. [BABAR Collaboration], arXiv:hep-ex/0107026.

[6] T. Affolder et al. [CDF Collaboration], Phys. Rev. Lett. 83, 3378 (1999) [arXiv:hep-ex/9905004].

[7] S. Anderson et al. [CLEO Collaboration], Phys. Rev. Lett. 87, 181803 (2001) [arXiv:hep-ex/0106060].

[8] G. Burdman, Phys. Rev. D 57, 4254 (1998) [arXiv:hep-ph/9710550].

[9] A. Ali, P. Ball, L. T. Handoko and G. Hiller, Phys. Rev. D 61, 074024 (2000) [arXiv:hep-ph/9910221].

[10] M. Beneke, T. Feldmann and D. Seidel, Nucl. Phys. B 612, 25 (2001) [arXiv:hep-ph/0106067].

[11] S. W. Bosch and G. Buchalla, Nucl. Phys. B 621, 459 (2002) [arXiv:hep-ph/0106081].

[12] P. Ball and V. M. Braun, Phys. Rev. D 58, 094016 (1998) [arXiv:hep-ph/9805422].

[13] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999) [arXiv:hep-ph/9905312].

[14] S. Fukae, C. S. Kim, T. Morozumi and T. Yoshikawa, Phys. Rev. D 59, 074013 (1999) [arXiv:hep-ph/9807254].
[15] T. M. Aliev, C. S. Kim and Y. G. Kim, Phys. Rev. D 62, 014026 (2000) [arXiv:hep-ph/9910501].

[16] S. Fukae, C. S. Kim and T. Yoshikawa, Phys. Rev. D 61, 074015 (2000) [arXiv:hep-ph/9908229].

[17] T. M. Aliev, A. Ozpineci and M. Savci, Phys. Lett. B 511, 49 (2001) [arXiv:hep-ph/0103261].

[18] C. S. Kim, Y. G. Kim, C. D. Lu and T. Morozumi, Phys. Rev. D 62, 034013 (2000) [arXiv:hep-ph/0001151].

[19] A. Faessler, T. Gutsche, M. A. Ivanov, J. G. Korner and V. E. Lyubovitskij, [arXiv:hep-ph/0205287].

[20] A. J. Buras and M. Munz, Phys. Rev. D 52, 186 (1995) [arXiv:hep-ph/9501281].

[21] M. Misiak, Nucl. Phys. B 393, 23 (1993) [Erratum-ibid. B 439, 461 (1995)].

[22] K. Hagiwara, A. D. Martin and M. F. Wade, Nucl. Phys. B 327, 569 (1989).

[23] J. G. Korner and G. A. Schuler, Z. Phys. C 46, 93 (1990).

[24] K. Lingel, T. Skwarnicki and J. G. Smith, Ann. Rev. Nucl. Part. Sci. 48, 253 (1998) [arXiv:hep-ex/9804013].