We investigate supersymmetric $SU(4)_c \times SU(4)_{L+R}$ theory in 5 dimensions whose compactification on a $S^1/Z_2$ orbifold yields $N = 1$ supersymmetric $SU(4)_c \times SU(2)_L \times SU(2)_R$ supplemented by a $	ilde{U}(1)$ gauge symmetry. We discuss how the $\mu$ problem is resolved, a realistic Yukawa sector achieved, and a stable proton realized. Neutrino masses and oscillations are also briefly discussed.

PACS numbers: 12.60.-i, 11.10.Kk, 11.30.-j, 14.60.Pq

I. INTRODUCTION

The minimal supersymmetric standard model (MSSM) provides an elegant, albeit only partial resolution of the gauge hierarchy problem. The apparent unification at $M_G(\sim 2 \cdot 10^{16}$ GeV) of the three MSSM gauge couplings [1] hints at the existence of an underlying supersymmetric grand unified theory (SU(5) GUT). Recent excitement about higher supersymmetric GUT's is primarily sparked by the observation [2] that the notorious doublet-triplet (DT) splitting problem can be resolved through suitable boundary conditions on an appropriately chosen orbifold [2], [3]. The magnitude of $M_G$ also seems consistent with the scale of lepton number violation, thereby providing the possibility of explaining the atmospheric [4] and solar [5] neutrino anomalies through neutrino oscillations.

In the absence of extra dimensions, the choice of SUSY GUT is normally dictated by the requirement that the known matter multiplets fall into chiral families [6]. This imposes limitations on possible gauge groups, and in 4D, the simplest choices are $SU(5)$, $SO(10)$, etc. However, in the presence of extra dimensions, dimensional reduction can yield chiral 'zero' modes on a boundary subspace, even if the initial 5 (or higher) dimensional theory is vector-like. This opens up the possibility of discussing new unified groups in higher dimensions.

In this paper we follow this reasoning and consider $SU(4)_c \times SU(4)_{L+R}$ gauge theory which, in turn, can be embedded say in $SO(12)$ in five dimensions. Through compactification on a $S^1/Z_2$ orbifold, we obtain four dimensional $N = 1$ SUSY $SU(4)_c \times SU(2)_L \times SU(2)_R$ [7], supplemented by an apparently anomalous $\tilde{U}(1)$ symmetry. The desired 'matter' as well as 'scalar' supermultiplets emerge after proper selection of parities under $Z_2$. At the 4D level, the anomalies due to $\tilde{U}(1)$ are canceled by the contribution from bulk Chern-Simons term [8]-[13] and additional states on fixed point(s) are crucial. Let us note that there are wide class of string constructing models, which in 4D low energetical limit give anomalous $U(1)$ factors. The latter can be canceled by the Green-Schwarz mechanism [14]. The 5D scenario considered here is transparent field-theoretical example of GUT model, which after compactification breaking gives anomalous $\tilde{U}(1)$. Let us note also that in [15], [12] the 5D $SU(3)_W$ unified models, involving gauge symmetry and SUSY breaking, were considered and got discussed extensively consistency of these type of models from the viewpoint of anomaly cancellation [11]-[13]. Together with Chern-Simons term, the selected additional states on the fixed points can play crucial role for anomaly cancellation [12], [13]. In difference from $SU(3)_W$ model, considered $SU(4)_{L+R}$ scenario unifies left-right $SU(2)_L \times SU(2)_R$ group (of Pati-Salam [7]) in a single $SU(4)_{L+R}$. Being broken at the low energies, the latter could give interesting phenomenological implications.

We discuss how realistic phenomenology emerges in 4D. An extra triplet state decouples at a high mass scale, while the DT splitting problem does not exist at all. The breaking of $\tilde{U}(1)$ symmetry is guaranteed by the Fayet-Iliopoulos D-term, with the consequence that a realistic Yukawa sector can be realized. After reduction of the 5D theory, there are some $\mathcal{R}$-symmetries in 4D which can provide for an automatic baryon number conservation, thereby guaranteeing a stable proton. The same $\mathcal{R}$-symmetry is also crucial for avoiding an unacceptably large $\mu$ term in unbroken $N = 1$ SUSY limit. We also briefly consider neutrino masses as well as oscillations.

II. 5D SUSY $SU(4)_c \times SU(4)_{L+R}$ MODEL AND ITS ORBIFFOLD BREAKING

We consider a $SU(4)_c \times SU(4)_{L+R} \equiv G_{44}$ supersymmetric model in 5D dimension. In 4D notation we have $N = 2$ SUSY, where the chiral supermultiplet $\Phi_{N=2} = (\Phi, \Phi^c)$ contains two $N = 1$ chiral supermultiplets $\Phi$, $\Phi^c$ transforming as $p$ and $\Phi^c$-plets respectively under the gauge group. Under $\Phi$ we denote all 'matter' and/or 'scalar' superfields of the model, while $\Phi^c$ indicate their
mirrors. The $N = 2$ gauge superfield is $\mathbf{V}_{N=2} = (V, \Sigma)$, where $V$ and $\Sigma$ are $N = 1$ vector and chiral superfields in the adjoint representation of the gauge group. In terms of $N = 1$ components, the 5D action includes [8]:

$$S^{(5)} = \int d^5x (L_V^{(5)} + L_{\Phi}^{(5)}),$$

where

$$L_V^{(5)} = \frac{1}{g^2} \int d^4 \theta \left( (\sqrt{2} \partial_5 V + \Sigma^+) e^{-V} (-\sqrt{2} \partial_5 V + \Sigma)e^V + \frac{1}{4g^2} \int d^2 \theta W^\alpha W_\alpha + \text{H.c.} \right),$$

$$L_{\Phi}^{(5)} = \int d^4 \theta \left( \Phi^+ e^{-V} \Phi + \bar{\Psi} e^V \Psi \right) + \int d^2 \theta \bar{\Phi} \left( M_\Phi + \partial_5 - \frac{1}{\sqrt{2}} \Sigma \right) \Phi + \text{H.c.},$$

and $W_\alpha$ are the supersymmetric field strengths. The action in (1) is invariant under the gauge transformations:

$$e^V \rightarrow e^{\Lambda} e^V e^{\Lambda^+}, \quad \Sigma \rightarrow e^{\Lambda} (\Sigma - \sqrt{2} \partial_5)e^{-\Lambda},$$

$$\Phi \rightarrow e^{\Lambda} \Phi, \quad \bar{\Psi} \rightarrow \bar{\Psi} e^{-\Lambda}.$$  

(4)

Next we introduce one matter $N = 2$ supermultiplet $\mathbf{F}_{N=2} = (\mathbf{F}, \bar{\mathbf{F}})$ per generation where, under $G_{44}$, $\mathbf{F}$ and $\bar{\mathbf{F}}$ transform as

$$\mathbf{F} \sim (4, 4) \equiv (\mathbf{F}^c, F), \quad \bar{\mathbf{F}} \sim (\bar{4}, \bar{4}) \equiv (\mathbf{F}^c, \bar{F}),$$

(5)

with

$$\mathbf{F}^c = \begin{pmatrix} \bar{u}_1, & d_1 \\ \bar{u}_2, & d_2 \\ \bar{u}_3, & d_3 \\ \nu, & e \end{pmatrix}, \quad \mathbf{F} = \begin{pmatrix} u_1, & d_1 \\ u_2, & d_2 \\ u_3, & d_3 \\ \nu, & e \end{pmatrix},$$

$$\bar{\mathbf{F}}^c = \begin{pmatrix} \bar{u}_1^c, & d_1^c \\ \bar{u}_2^c, & d_2^c \\ \bar{u}_3^c, & d_3^c \\ \bar{\nu}^c, & e^c \end{pmatrix}, \quad \bar{\mathbf{F}} = \begin{pmatrix} u_1^c, & d_1^c \\ u_2^c, & d_2^c \\ u_3^c, & d_3^c \\ \bar{\nu}, & e \end{pmatrix}.$$  

(6)

It is clear that (6) is just the chiral content of $SU(4)_c \times SU(2)_L \times SU(2)_R \equiv G_{422}$ model [7], supplemented by mirrors. It will turn out that mirrors can be projected out after compactification of the fifth dimension on the orbifold $S^{(1)}/Z_2$. The $G_{44}$ symmetry is broken by orbifolding via the channel $SU(4)_c \times SU(4)_{L+R} \rightarrow SU(4)_c \times SU(2)_L \times SU(2)_R \times \tilde{U}(1)$, where the generator of $\tilde{U}(1)$ is

$$Y_{\tilde{U}(1)} = \frac{1}{\sqrt{8}} \cdot \text{Diag} \left( 1, 1, -1, -1 \right),$$

(8)

with $SU(4)_{L+R}$ normalization. In terms of $G_{422} \times \tilde{U}(1)$, the fermionic fragments read

$$\bar{F}^c = (4, 1, 2)_{-1}, \quad F = (4, 2, 1)_1,$$

$$F^c = (\tilde{4}, 1, 2)_1, \quad \bar{F} = (\tilde{4}, 2, 1)_{-1}.$$  

(9)

where subscripts denote $\tilde{U}(1)$ charges in units of $1/\sqrt{8}$ [see (8)]. Decomposition of $SU(4)_{L+R}$ adjoint yields

$$15_{L+R} = (3, 1)_0 + (1, 1)_0 + (\bar{2}, 2)_0 - 2(2, \bar{2}) + (2, 2) + (\bar{2}, 2)_2.$$  

(10)

Under $Z_2$ the fifth coordinate $y$ changes sign $y \rightarrow -y$. Because of this, states with positive and negative parities $\Phi_+, \bar{\Phi}_-$ can be written in factorized forms

$$\Phi_+ = \sum_{n=0}^{n=\infty} \Phi_n(x) \cos \left( \frac{ny}{R} \right),$$

$$\bar{\Phi}_- = \sum_{n=1}^{n=\infty} \bar{\Phi}_n(x) \sin \left( \frac{ny}{R} \right).$$  

(11)

As expected, $\bar{\Phi}_-$ in (11) lacks zero modes on $S^{(1)}/Z_2$ orbifold. The fixed point $y = 0$ will be identified with a 3-brane corresponding to our 4D world. The $Z_2$ parities of the various fragments of the gauge ($\mathbf{V}_{N=2}$) and matter ($\mathbf{F}_{N=2}$) superfields are shown in Table 1. Under $Z_2$, the D-terms in (2), (3) involving $\mathbf{V}_{N=2}$ and $\mathbf{F}_{N=2}$ states are invariant. The $F$-terms in (3) can be written schematically as

$$F^c \left( \partial_5 - \frac{1}{\sqrt{2}} \Sigma(15_c) + \Sigma(1, 3) + \Sigma(1, 1) \right) \mathbf{F}^c +$$

$$\bar{F} \left( \partial_5 - \frac{1}{\sqrt{2}} \Sigma(15_c) + \Sigma(3, 1) + \Sigma(1, 1) \right) F$$

$$- \frac{1}{\sqrt{2}} \bar{F} \Sigma(2, 2) \mathbf{F}^c - \frac{1}{\sqrt{2}} F^c \Sigma(2, 2) F + \text{H.c.}.$$  

(12)

Note that bare mass terms are forbidden by $Z_2$ symmetry.
From Table 1 the gauge fields with positive $Z_2$ parity are just those of $G_{422} \times U(1)$. The $\Sigma$ states in the same representations have opposite parities, so that compactification gives 4D $N = 1$ SUSY. By the same token, on $S^{(1)}/Z_2$ orbifold we only have $F$, $F^c$ chiral states.

TABLE I: $Z_2$ parity numbers of gauge, matter and scalar superfields

| $V_{N=2}$ | Multiplets in terms of $G_{422} \times U(1)$ | $Z_2$par. |
|-----------|------------------------------------------|-----------|
| $V(15c)$, $V(3,1)_0$, $V(1,3)_0$, $V(1,1)_0$, $\Sigma(2,2)_{-2}$, $\bar{\Sigma}(2,2)_2$ | + |
| $\bar{\Sigma}(6,1)_2$, $\bar{\Sigma}(3,1)_0$, $\bar{\Sigma}(1,5)_0$, $\Sigma(1,1)_0$, $\bar{\Sigma}(2,2)_{-2}$, $\bar{\Sigma}(2,2)_2$ | - |
| $F_{N=2}$ | $F(4,2,1)_1$, $F^c(4,1,2)_1$ | + |
| $F(4,2,1)_{-1}$, $F^c(4,1,2)_{-1}$ | - |
| $H_{N=2}$ | $H(4,1,2)_1$, $H^c(4,1,2)_{-1}$ | + |
| $H(4,2,1)_1$, $H^c(4,2,1)_{-1}$ | - |
| $\Omega_{N=2}$ | $D_1(6,1,1)_{+2}$, $D_2(6,1,1)_{-2}$, $P^c(6,2,2)_0$ | + |
| $D_1(6,1,1)_{-2}$, $D_2(6,1,1)_{+2}$, $P^c(6,2,2)_0$ | - |
| $\Psi_{N=2}$ | $S^c(1,1,1)_{-2}$, $S(1,1,1)_{2}$, $h(1,2,2)_0$ | + |
| $S^c(1,1,1)_{2}$, $S(1,1,1)_{-2}$, $h(1,2,2)_0$ | - |

As $N_6 \times (6,1,1)_{-2} + N_2 \times (1,\bar{2},\bar{2})_{-2} + N_1 \times (1,1,1)_2$ (where $N_6$, $N_2$, $N_1$ denote number of appropriate representations), one can check that for $(N_6, N_2, N_1) = (3,3,24)$ we will have

$$A_{br}[SU(4)^2_c - \bar{U}(1)] = A_{br}[SU(2)_L^c - \bar{U}(1)] = A_{br}[SU(2)_R^c - \bar{U}(1)] = -6 \ , \ A_{br}[\bar{U}(1)^3] = -48 . \quad (15)$$

Since, with $3 \times (F + F^c)$ states we have

$$A[SU(4)_c^2 - \bar{U}(1)] = A[SU(2)_L^c - \bar{U}(1)] = A[SU(2)_R^c - \bar{U}(1)] = 6 \ , \ A[\bar{U}(1)^3] = 48 , \quad (16)$$

using (15), (16) the relation (14) is indeed satisfied, which is sufficient to cancel anomalies by the bulk CS term. Of course for the latter to work, there are a variety of possibilities. Namely, one can introduce additional states either on both fixed points, or in the bulk, or simultaneously on the fixed points and in the bulk [13]. We have presented just one example to demonstrate how the anomaly cancellation mechanism works out and the theory becomes selfconsistent.

For $G_{422}$ breaking we need additional scalar superfields. For this purpose, and also for decoupling some unwanted colored triplets [16], in 5D we introduce $N = 2$ supermultiplets $H_{N=2} = (H, \bar{H})$ and $\Omega_{N=2} = (\Omega, \bar{\Omega})$. Under $G_{44}$,

$$H = (4,4) = (\bar{H}^c, H) , \quad \bar{H} = (4,4) = (H^c, \bar{H}) , \quad \Omega = (6,6) , \quad \bar{\Omega} = (\bar{6},\bar{6}) . \quad (17)$$

In terms of $G_{422} \times \bar{U}(1)$ the $H, \bar{H}$ decompose as $F$ and $F^c$ [see (5), (6), (9)], while for $\Omega, \bar{\Omega}$ we have

$$\Omega = (6,6) = D_1(6,1,1)_{+2} + D_2(6,1,1)_{-2} + P^c(6,2,2)_0 , \quad \bar{\Omega} = (\bar{6},\bar{6}) = D_1(6,1,1)_{-2} + D_2(6,1,1)_{+2} + P^c(6,2,2)_0 . \quad (18)$$

The transformation properties of fragments from $H_{N=2}$, $\Omega_{N=2}$ under $Z_2$ orbifold symmetry are given in Table 1. In 4D we are left with $H^c$, $\bar{\Omega}$, $D_1$, $D_2$, and $P$, with the remaining states projected out. The state $P$ is self conjugate and neutral under anomalous $\bar{U}(1)$, and at 4D level it gains mass through the superpotential coupling $M^F_2$ ($M \sim M_{G_2}$) and decouples. The states $(\nu^c + \bar{\nu}^c)_H$ from $H^c$ and $\bar{\Omega}$ respectively, develop non zero VEVs ($\sim M_{G_2}$) so that the symmetry $G_{422}$ is broken down to $SU(3)_c \times SU(2)_L \times U(1)_Y \equiv G_{321}$. For correct symmetry breaking we have to avoid the mass term $M_H \bar{\Omega}^c H^c$
[which otherwise would cause unacceptable SUSY breaking in 4D]. This is easily achieved by introducing $Z_2$ symmetry (not to be confused with $Z_2$ orbifold symmetry), under which $H \rightarrow -H$, $\mathbb{H}^c \rightarrow -\mathbb{H}^c$ (e.g., $H \rightarrow -H$). All D-terms in (3) involving $H_{N=2}$ and $\Omega_{N=2}$ states are invariant under $Z_2$.

During $G_{122}$ breaking, the states $u^c$, $e^c$ and $\bar{u}^c$, $\bar{e}^c$ from $H^c$ and $\mathbb{H}^c$ are absorbed by the appropriate gauge fields, while $d^c + \mathbb{H}^c$ are still massless. However, they can acquire masses through mixings with appropriate states from $D_{1,2}$. The relevant 4D superpotential couplings are given by

$$
\lambda_1 H^c H^c D_2 + \lambda_2 \mathbb{H} \mathbb{H}^c D_1 + M_D D_1 D_2 ,
$$

(19)

where $\lambda_{1,2}$ are order unity dimensionless couplings, and $M_D \sim M_G$. After substituting appropriate VEVs in (19), mass matrix for color triplets is given by

$$
M_T = \begin{pmatrix}
\vec{d}_{H}^c & \vec{d}_{D_1}^c & \vec{d}_{D_2}^c \\
0 & \lambda_2 V & \lambda_1 V \\
0 & 0 & M_D
\end{pmatrix},
$$

(20)

where $V$ is the VEV of $H^c$, $\mathbb{H}^c$ along the $G_{321}$ singlet direction. This shows that the triplets from $H^c$, $\mathbb{H}^c$ gain the masses through mixings with the triplets of $D_{1,2}$.

### III. YUKAWA SECTOR

Turning to the Yukawa sector, recall that from $V_{N=2}$ supermultiplets, the $N = 1$ chiral superfields $\Sigma(2,2)_{-2} \equiv \Sigma_{-2}$ and $\Sigma(2,2)_2 \equiv \Sigma_2$ have positive $Z_2$ parities (see Table 1) and are therefore not projected out. These states are bi-doublets and $\Sigma_{-2}$ has coupling with matter even at 5D level [last term in (12)]:

$$
FF^c \Sigma_{-2}.
$$

(21)

However, $\Sigma_{-2}$ cannot contain the MSSM higgs doublets because it forms a massive state with $\Sigma_2$.

$$
M_2 \Sigma_{-2} \Sigma_2.
$$

(22)

In order to build a realistic Yukawa sector we introduce a new state $\Psi_{N=2} = (S, \bar{S})$, where

$$
\Psi = (1,6) = h(1,2,2)_0 + \bar{S}'(1,1,1)_2 + S'(1,1,1)_{-2},
$$

$$
\bar{\Psi} = (1,\bar{6}) = h'(1,\bar{2},\bar{2})_0 + S(1,1,1)_2 + S(1,1,1)_{-2}.
$$

(23)

With the transformation properties for the fragments of $\Psi_{N=2}$ presented in Table 1, the states $h$, $\bar{S}$, $S$ are not projected out. These states turn out to be crucial for realistic Yukawa sector. The D-terms in (3), involving fragments of $\Psi_{N=2}$, are invariant under orbifold symmetry, while the relevant allowed F-terms are $\bar{S}\Sigma_{-2}h$ and $S\Sigma_2h$. Combining these two terms with (22), the relevant superpotential couplings are

$$
\bar{S}\Sigma_{-2}h + S\Sigma_2h + M_2 \Sigma_{-2} \Sigma_2.
$$

(24)

Since $\tilde{U}(1)$ is anomalous, the Fayet-Iliopoulos term $\xi \int d^4\theta V_{\tilde{U}(1)}$ will be allowed in 4D, and we assume that $\xi > 0$. The singlet $S$ has negative $\tilde{U}(1)$ charge, so it can be used for its breaking. So, Cancelling the $\xi$-term, $\langle S \rangle \sim \sqrt{\xi}$, while $\langle \bar{S} \rangle = 0$. From all this and from (24), one can see that the light bi-doublet $\tilde{h}$ belongs with equal weights to both $\Sigma_{-2}$ and $h$ [if $\langle S \rangle \sim M_G$]. Taking into account (21), the Yukawa coupling

$$
F F^c \tilde{h},
$$

(25)

is generated which, however yields’ degenerate masses for up-down quarks and charged leptons. This drawback exists in all minimal versions of $G_{122}$, and for its resolution some additional mechanisms must be applied [17], [18]. We do not go through the details of this issue here and refer the reader to [17], [18], where realistic patterns of fermion masses and mixings are constructed. Let us note that the anomalous $\tilde{U}(1)$ also can be exploited for understanding and solving various puzzles such as mechanism of SUSY breaking, suppression of FCNC baryon number conservation etc. in the spirit of refs. [22], [23].

### IV. $\mu$-TERM AND BARYON NUMBER CONSERVATION

5D SUSY theories have global $R$-symmetries, some of which after compactification survive in our $G_{34}$ model. These symmetries can be successfully employed for lepton and baryon number conservations [19], [18]. Namely, terms in (25) have an accidental $R$-symmetry: $F \rightarrow e^{i\alpha} F$, $F^c \rightarrow e^{-i\alpha} F^c$. This symmetry automatically forbids all matter parity violating couplings as well as $d = 5$ operators such as $FFFF$, $F^c F^c F^c F^c$. Thus, in this case, the LSP is expected to be stable. Also, since gauge mediated nucleon decay is absent in our model, conservation of baryon number is guaranteed to all orders in perturbation theory. Note that lepton number is also conserved at this stage by the same $R$-symmetry [24].

If we wish to accommodate the atmospheric and solar neutrino data [4], [5] we should generate non-zero but tiny neutrino masses. In particular, lepton number must be broken. It turns out that by modification of $R$-symmetry, one can violate lepton number, but still conserve $B$. The appropriate fields transform as $F^c \rightarrow e^{i\alpha} F^c$, $F \rightarrow e^{3i\alpha} F$, $h \rightarrow e^{i3h}$, $S \rightarrow e^{i(4\alpha - \beta)} S$, $(\Sigma_{-2}, \Sigma_2) \rightarrow (\Sigma_{-2}, \Sigma_2)$, with the superpotential $W \rightarrow e^{3i\alpha} W$. The relevant couplings are
The MSSM doublet-antidoublet pair now resides in $\Sigma_{-2}$, while $\Sigma_2$ decouples with $h$ [second term in (26)] forming mass $\sim \langle S \rangle \sim M_G$.

Note also that $U(1) \times \mathcal{R}$-symmetry guarantee a zero $\mu$ term, and its generation should be achieved by some additional mechanism (one possibility is a non-minimal Kähler potential [20]).

The terms in (19) are consistent with this $\mathcal{R}$-symmetry, with transformation properties $(H^c, \overline{H}^c) \rightarrow e^{i\alpha} (H^c, \overline{H}^c)$, $D_{1,2} \rightarrow e^{2i\alpha} D_{1,2}$. Note that all the D and F-terms in (3) are still allowed by $\mathcal{R}$-symmetry with $\overline{\Psi} \rightarrow e^{i\alpha} \overline{\Psi}$, $\overline{\Psi} \rightarrow e^{i(4\alpha - \beta)} \overline{\Psi}$ (e.g. all fragments living in $\Psi$, $\overline{\Psi}$). Thus, $\mathcal{R}$-symmetry applies to the full theory. Since the $\mathcal{R}$-charges of $H^c$ and $F^c$ superfields are the same, we must impose ‘matter’ parity by hand in order to eliminate some undesirable couplings.

The couplings generating Majorana masses for the right handed neutrinos (in $F^c$), read

$$\frac{1}{M_P} (F^c \overline{F^c})^2 .$$

From (27), $M_R \sim M_G^2 / M_P$ and neutrino mass $m_\nu \sim \frac{\sqrt{2}}{M_P} M_P \sim 0.1 \text{ eV}$ is readily obtained (we have taken $M_P = 2.4 \times 10^{18}$ GeV, the reduced Planck mass, and $h_u$ denotes the VEV of the higgs doublet that gives rise to Dirac neutrino mass), just the scale needed for explaining the atmospheric anomaly. The scale for solar neutrinos can be obtained through a suppression of appropriate Yukawa couplings in the Dirac type neutrino mass matrix. For realizing desirable values for mixing angles within various oscillation scenarios, the mechanisms suggested in [18], [21] can be applied.

Turning to the issue of baryon number conservation, the Planck scale $d = 5$ operators $\frac{1}{M_P} (FFFF + F^c F^c F^c F^c)$ are forbidden by $\mathcal{R}$-symmetry. As far as the couplings $qqT + qaT + u^c d^cT + u^c e^cT$ are concerned $(T, T$ indicate colored triplet states which could induce $d = 5$ operators, after they are integrated out), due to $\mathcal{R}$ charge prescriptions the couplings $qqT + qaT$ do not emerge at all. The coupling $u^c e^c \overline{\Psi} \overline{T}$ emerges from (27) after extracting from $\overline{\Psi} \overline{T}$ the triplet state $\overline{\Psi} \overline{T}$. The coupling $F^c F^c D_2$ yields $u^c d^c D_2$. However, looking at (20), we see that there is no mass insertion between $\overline{\Psi} \overline{T}$ and $d^c D_2$ states, so the appropriate $d = 5$ operators do not emerge. This means that baryon number is conserved in our model.

V. CONCLUSIONS

In conclusion, we note that the gauge group $G_{44}$ can itself be embedded in a higher dimensional $SO(12)$ model. The decomposition of appropriate $SO(12)$ representations in terms of $G_{44}$ are as follows:

$$12 = (6, 1) + (1, 6) , \quad 32 = (4, 4) + (4, \bar{4}) ,$$

$$66 = (6, 6) + (15, 1) + (1, 15) ,$$

We see on the right hand sides all of the $G_{44}$ multiplets involved in our 5D SUSY $G_{44}$ scenario. Therefore, it is reasonable to think about higher dimensional (say $D = 6$) unification of $G_{44}$ in $SO(12)$. The breaking of $SO(12)$ could occur through the steps $SO(12) \rightarrow G_{44} \rightarrow G_{422} \times U(1)$. Details of these and related issues and their phenomenological implications will be presented elsewhere.

Acknowledgments

Q.S. would like to acknowledge the hospitality of the Alexander von Humboldt Stiftung and the Theory Group at DESY, especially Wilfried Buchmüller, while this work was in progress. This work was supported in part by the DOE under Grant No. DE-FG02-91ER40626 and by Nato under Grant No. PST.CLG.977666.

[1] C. Giunti, C. Kim, U. Lee, Mod. Phys. Lett. A 6 (1991) 1745; P. Langacker, M. Luo, Phys. Rev. D 44 (1991) 817; J. Ellis, S. Kelley, D. Nanopoulos, Phys. Lett. B 260 (1991) 131; U. Amaldi, W. de Boer, H. Furstenau, Phys. Lett. B 260 (1991) 447.

[2] Y. Kawamura, hep-ph/0012125; G. Altarelli, F. Feruglio, hep-ph/0102301; A. Kobakhidze, hep-ph/0102323; L. Hall, Y. Nomura, hep-ph/0103125; M. Kakizaki, M. Yamaguchi, hep-ph/0104103; N. Maru, hep-ph/0108002; R. Dermisek, A. Mafi, hep-ph/0108139.

[3] R. Barbieri, L. Hall, Y. Nomura, hep-th/0107004; A. Hebecker, J. March-Russell, hep-ph/0107039; C. Csaki, G. Kribs, J. Terning, hep-ph/0107266; H. Cheng, K. Matchev, J. Wang, hep-ph/0107268; T. Asaka, W. Buchmüller, L. Covi, hep-ph/0108021; T. Li, hep-ph/0108120; M. Chaichian, J. Chikarevli, A. Kobakhidze, hep-ph/0108131; L. Hall, et al., hep-ph/0108161; Y. Nomura, hep-ph/0108170.

[4] Y. Fukuda et al., Phys. Rev. Lett. 82 (1999) 2644; Phys. Lett. B 467 (1999) 185; N. Fornengo et al., Nucl. Phys. B 580 (2000) 58.

[5] J. Bahcall, P. Krastev, A. Smirnov, hep-ph/0006078; J. Bahcall, hep-ex/0002018; M.C. Gonzalez-Garcia et al., Nucl. Phys. B 573 (2000) 3.
[6] H. Georgi, S. Glashow, Phys. Rev. D 6 (1972) 429; J. Banks, H. Georgi, Phys. Rev. D 14 (1976) 1159; P. Frahmpton, Phys. Lett. B 89 (1980) 352.

[7] J. Pati, A. Salam, Phys. Rev. D 10 (1974) 275.

[8] N. Arkani-Hamed et al., hep-th/0101233.

[9] C.G. Callan, J.A. Harvey, Nucl. Phys. B 250 (1985) 427.

[10] R. Jackiw, C. Rebbi, Phys. Rev. D 13 (1976) 3398; E. Weinberg, Phys. Rev. D 24 (1981) 2669; G. Lazarides, Q. Shafi, Phys. Lett. B 151 (1985) 123; E. Witten, Nucl. Phys. B 269 (1985) 557.

[11] N. Arkani-Hamed, A. Cohen, H. Georgi, hep-th/0103135; C. Scurruca, M. Serone, L. Silvestrini, F. Zwerine, hep-th/0110073; L. Pilo, A. Riottto, hep-th/0201144; R. Barbieri et al., hep-th/0203039; S. Nibbelink, P. Nilles, M. Olechowski, hep-th/0205012.

[12] H.D. Kim, J.E. Kim, H.M. Lee, hep-th/0204132; JHEP 0206 (2002) 048.

[13] C.A. Lee, Q. Shafi, Z. Tavartkiladze, hep-ph/0206258; Phys. Rev. D 66 (2002) 055010.

[14] M. Green, J. Schwarz, Phys. Lett. B 149 (1984) 117.

[15] R. Barbieri, L. Hall, Y. Nomura, Phys. Rev. D 63 (2001) 105007; T. Li, L. Wei, hep-ph/0202090; L. Hall, Y. Nomura, hep-ph/0202107; S. Dimopoulos, D.E. Kaplan, N. Weiner, hep-ph/0202136.

[16] I. Antoniadis and G.K. Leontaris, Phys. Lett. B 216 (1989) 333; I. Antoniadis, G.K. Leontaris and J. Rizos, Phys. Lett. B 245 (1990) 161.

[17] S. King, Phys. Lett. B 325 (1994) 129; S. Rafone, E. Papageorgiu, Phys. Lett. B 295 (1992) 79; B. Allanach, S. King, G. Leontaris, S. Lola, Phys. Lett. B 407 (1997) 275.

[18] Q. Shafi, Z. Tavartkiladze, Nucl. Phys. B 549 (1999) 3, hep-ph/9811282.

[19] S. King, Q. Shafi, Phys. Lett. B 422 (1998) 135.

[20] G.F. Giudice, A. Masiero, Phys. Lett. B 206 (1988) 480.

[21] R. Barbieri et al., hep-ph/9901228; Q. Shafi, Z. Tavartkiladze, Phys. Lett. B 451 (1999) 129; hep-ph/0101350; Phys. Lett. B 482 (2000) 145; G. Altarelli, et al., hep-ph/0007254; R.N. Mohapatra, hep-ph/0008232; see also references therein.

[22] G. Dvali and A. Pomarol, Phys. Rev. Lett. 77 (1996) 3738; P. Binetruy and E. Dudas, Phys. Lett. B 389 (1996) 503.

[23] M. Dine, A. Kagan, S. Samuel, Phys. Lett. B 243 (1990) 250; A. Nelson, D. Wright, Phys.Rev. D 56 (1997) 1598; Q. Shafi, Z. Tavartkiladze, Phys. Lett. B 473 (2000) 272; hep-ph/0207231; L. Everett et al., Phys. Lett. B 477 (2000) 233; J. Feng, K. Matchev, hep-ph/0011356; J. Hisano, K. Kurosawa, Y. Nomura, hep-ph/0002286; See also references therein.

[24] For avoiding large Dirac type neutrino masses $\nu^c \ell h_u$ from (25), one can introduce an additional singlet state $N$ with a suitable $R$ charge. Through the coupling $NF^c \overline{\Phi}$ the state $\nu^c$ decouples forming a massive ($\sim M_G$) state with $N$. 