CP Violation from Scatterings with Gauge Bosons in Leptogenesis

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Abstract

We present an explicit computation of the CP asymmetry in scattering processes involving the heavy right-handed neutrinos of the type I seesaw framework and the Standard Model gauge bosons. Compared to CP violation in two–body decays and in scatterings with top quarks there are new contributions at one loop in the form of new type of vertex corrections as well as of box diagrams. We show that their presence implies that, unlike the CP asymmetry in scatterings with top quarks, the CP asymmetry in scatterings with gauge bosons is different from the two-body decay asymmetry even for hierarchical right-handed neutrinos. This also holds for the L-conserving CP asymmetry in scatterings with U(1)\textsubscript{Y} gauge bosons.

Key words: leptogenesis, CP asymmetry, gauge boson scatterings

The discovery of neutrino oscillations makes leptogenesis a very attractive solution to the baryon asymmetry problem \cite{1,2}. In the standard framework the tiny neutrino masses are generated via the (type I) seesaw mechanism \cite{3,4,5,6} and since the new singlet neutral leptons have heavy lepton number (L) violating Majorana masses they can produce dynamically a lepton asymmetry through out of equilibrium processes. Eventually, this lepton asymmetry is partially converted into a baryon asymmetry due to fast sphaleron processes.

In this framework the CP asymmetry can be generated in the $N_i$ decays \cite{7}, i.e. $N_i \rightarrow l_j H$ \cite{8} as well as in scattering processes of the $N_i$ with particles in the plasma. In perturbation theory, to lowest order, the CP asymmetry in
any of these processes arises from the interference between the tree level and one-loop amplitudes. The CP violating asymmetry in scatterings with top quarks has been considered in Refs. [9–12]. In Refs. [9–11] it was argued that an approximate equality between the scattering and decay asymmetries should hold for hierarchical right-handed neutrinos (RHNs). Ref. [12] presented an explicit computation of the CP asymmetry in top quarks scatterings (TQS) and directly showed that this was correct. It is important to note that there is a one to one correspondence between one-loop diagrams in decays and TQS. Then, as discussed in Ref. [11], the “factorization” of a common CP asymmetry can be easily understood in terms of an effective field theory in which all but the lightest of the RHNs have been integrated out and their effect appears in a dim-5 operator $(H\ell)(H\ell)$. In this approximation only “bubble-like” diagrams contribute to the one-loop amplitudes of both decays and TQS.

As for the CP asymmetry in scatterings with gauge bosons, its effect was estimated in Ref. [12] under the assumption that it also factorizes in terms of the decay CP asymmetry. However, the same argument that leads to the understanding of the equality between the CP asymmetries in decay and TQS does not hold for gauge boson scatterings (GBS) due to the presence of additional contributions to the amplitude at one loop (which in the effective field theory approximation contain three rather than two particles in the loop). With this motivation in this note we carry out an explicit calculation of the CP asymmetry in GBS.

For a given heavy neutrino species (called here $N_i$) there are three different types of GBS, namely

$$N_i A \rightarrow \ell_j H, \quad N_i \bar{\ell}_j \rightarrow H A, \quad \bar{\ell}_j A \rightarrow N_i H,$$

and each one has an associated CP conjugate process. Here $A = W_a$ or $B_Y$ for SU(2)$_L$ and U(1)$_Y$ bosons, respectively. The Lagrangian for the Yukawa and gauge interactions relevant for the computation of these processes reads

$$L_{Y+A} = -Y_{N_{ij}} \bar{\ell}_j P_R N_i \bar{H} - \frac{ig_2}{2} (\partial_\mu H) \dagger W_\mu^a \sigma_a H - \frac{ig_1}{2} (\partial_\mu H) \dagger B_Y^\mu H + \text{h.c.}$$

$$+ \sum_j \left[ \frac{g_2}{2} \bar{\ell}_j \gamma_\mu W_\mu^a \sigma_a P_L \ell_j - \frac{g_1}{2} \bar{\ell}_j \gamma_\mu B_Y^\mu P_L \ell_j \right],$$

where $i, j$ are generation indices of RHNs and lepton doublets $\ell_j^T = (\nu_j, \ell_j^-)$.

\footnote{In principle one must consider all processes at a given order in the coupling constants. In particular the three body decays $N_i \rightarrow \ell_j H A$ should be included together with the gauge boson scatterings. However the three body decay rate is phase space suppressed, thus the processes $N_i \rightarrow \ell_j H A$ and their CP asymmetries can be safely neglected (see e.g. the appendix of [12]).}
respectively, \( P_{L,R} = \frac{1}{2} (1 \mp \gamma_5) \), and \( \tilde{H} = i\tau_2 H^* \) with \( H^T = (h^+, h^0) \) being the Higgs doublet.

We denote by \( \gamma^A_B \equiv \gamma(A \rightarrow B) \) the thermally averaged density rate for a state \( A \) to go into the state \( B \) (summed over initial and final spin and gauge degrees of freedom), and by \( \Delta \gamma^A_B \equiv \gamma^A_B - \gamma^\bar{A}_\bar{B} \) the CP difference between the rates for particle and antiparticle processes. Neglecting the thermal motion with respect to the plasma, the relevant thermal average rates for \( 1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \) processes can be written as

\[
\gamma_{34}^{12} = \frac{T}{64\pi^4} \int_{s_-}^{\infty} ds \sqrt{s} \hat{s}_{34}^{12}(s) K_1 \left( \frac{\sqrt{s}}{T} \right) = \frac{T}{2^9\pi^5} \int_{t_-}^{t_+} dt \frac{|M_{34}^{12}|^2}{\sqrt{s}} K_1 \left( \frac{\sqrt{s}}{T} \right),
\]

where \( \hat{s}_{34}^{12}(s) \) is the reduced cross section, \( K_1(x) \) is the modified Bessel function of second kind of order 1, and \( T \) is the temperature. We have also introduced the Mandelstam variables in terms of the momenta \( p_i \) of the four particles in the process, \( s = (p_1 + p_2)^2 \), \( t = (p_1 - p_3)^2 \), and \( u = (p_1 - p_4)^2 \), with \( p_i^2 = m_i^2 \). Thus for each process \( s_- = \text{Max}[(m_1 + m_2)^2, (m_3 + m_4)^2] \) and

\[
t_+ = \frac{(m_1^2 - m_2^2 - m_3^2 + m_4^2)^2}{4s} - \left[ \sqrt{\frac{(s+m_1^2-m_2^2)^2}{4s}} - m_1^2 \right] \pm \sqrt{\frac{(s+m_3^2-m_4^2)^2}{4s} - m_3^2}.\]

For the sake of simplicity in our evaluation of the CP asymmetries we neglect the thermal mass of the gauge bosons, and we include the thermal masses of leptons \( m_\ell \) and the Higgs boson \( m_H \) only in the propagators (but not in the loops), in order to regularize the infrared divergences that appear when these states are exchanged in a \( t \)- or \( u \)-channel.
In this approximation the tree level amplitudes for the three processes in (1) are related by crossing symmetry and are given by

\[ |M(N_i A \to \ell_j H)|^2 = |M(N_i A \to \bar{\ell}_j \bar{H})|^2 = 2G_A |Y_{Nij}|^2 I_0(s, t, u), \]

\[ |M(N_i \bar{\ell}_j \to HA)|^2 = |M(N_i \ell_j \to \bar{H}A)|^2. = -2G_A |Y_{Nij}|^2 I_0(u, s, t), \]

\[ |M(\ell_j A \to N_i H)|^2 = |M(\ell_j A \to N_i \bar{H})|^2 = 2G_A |Y_{Nij}|^2 I_0(u, t, s), \]

where \( G_A = 3g^2/2 \) or \( g^2/2 \) for SU(2)_L and U(1)_Y GBS, respectively.

The CP asymmetry in GBS arises from the interference between the tree-level and one–loop amplitudes in Figs. 1 and 2, respectively. In Fig. 2 we show as a “waving” line labeled \( C_1, C'_1 \) or \( C_2 \) the possible branch cuts in which the particles in the propagators can be on–shell and therefore give a contribution to the imaginary part of the corresponding amplitude. As seen in Fig. 2 besides the \( N_i \) self-energy (wave) corrections (diagrams (a) and (b)), and the vertex corrections (diagrams (c) and (d)), which are also present at one loop in two–body \( N_i \) decays and in TQS, there are new type of vertex diagrams (labeled (e) and (f)), as well as contributions from boxes (diagrams (g) and (h)). Furthermore for process \( \ell_j A \to N_i H \) additional imaginary parts appear from the last two diagrams 3.(i) and 3.(j).

For each process the sum of amplitudes of wave diagrams (a) and (b) is gauge invariant. The sum of amplitudes for vertex diagrams (e) and (f) is also gauge invariant, while the amplitudes of vertices (c), (d) and boxes (g), (h) are not separately gauge invariant but the sum of the four amplitudes is. Finally for process \( \bar{\ell}_j A \to N_i H \) the sum of amplitudes 3.(i) and 3.(j) is gauge invariant.

Thus generically the CP asymmetry from any of the three processes in (1) can be written as

\[ \epsilon_{34}^{12} \equiv \sum_j \frac{\gamma_{34}^{12} - \gamma_{34}^{12}}{\gamma_{34}^{12} + \gamma_{34}^{12}} \equiv \epsilon_{34}^{12(w)} + \epsilon_{34}^{12(v1)} + \epsilon_{34}^{12(v2)} + \epsilon_{34}^{12(b)} + \Delta \epsilon_{N_i H}^{\ell_j A}, \]

where \( \epsilon_{34}^{12(w,v1,v2,b)} \) is generated by the interference between the tree-level amplitudes and the self-energy amplitudes (a) and (b), the vertex amplitudes (c) and (d), the vertex amplitudes (e) and (f), and the box amplitudes (g) and (h), respectively. We denote by \( \Delta \epsilon_{N_i H}^{\ell_j A} \) the extra asymmetry for the process \( \bar{\ell}_j A \to N_i H \) from the diagrams 3.(i) and 3.(j). Notice that each of the diagrams (a), (b), (e) and (f) represents two amplitudes depending on whether the internal lepton line is a lepton or an anti-lepton, which leads respectively to total L-conserving or L-violating contributions.
It is straightforward to show that for any of the processes in (11) we have

\[
\epsilon_{\ell j H}^{N_i A(u)} = \epsilon_{AH} = \epsilon_{N_i H} = \epsilon_{\ell j H}^{(w)} = \frac{\sum_{k \neq i} \left[ \frac{C_{jk} - \sqrt{a_k}}{a_k - 1} + \tilde{C}_{jk} \frac{1}{a_k - 1} \right]}{8\pi (Y_N Y_N^*)_{ii}}, \quad (7)
\]

\[
a_k = \frac{M_k^2}{M_i^2}, \quad C_{jk} = -\text{Im} \left[ Y_{N_{kj}}^* (Y_N Y_N^*)_{ik} \right], \quad \tilde{C}_{jk} = -\text{Im} \left[ Y_{N_{kj}}^* (Y_N Y_N^*)_{ik} \right].
\]

In the above, \( \epsilon_{\ell j H}^{N_i (w)} \) is the contribution to the CP asymmetry in \( N_i \to \ell j H \) from the \( N_i \) self-energy diagrams [8] and the first (second) term arises from the L-violating (L-conserving) diagrams.

The contributions to the CP asymmetries from the vertex (labeled with superscripts \((v1),(v2)\)), box (superscript \((b)\)), and the extra piece from diagrams \(3.(i) \) and \(3.(j) \) (superscript \((\Delta)\)) read

\[
\left[ |M(12 \to 34)|^2 - |M(\bar{1}2 \to \bar{3}4)|^2 \right]^{(v1),(v2),(b),(\Delta)} = -\sum_{k \neq i} \left\{ \frac{G_A C_{jk} M_k M_i}{2\pi} \left[ I_{34}^{12(v1),(b)}(s,t,u) + I_{34}^{12(v2)}(s,t,u) + \theta(s - M_k^2) J_{34}^{12(v1),(b)}(s,t,u) \right] \right.
\]

\[
\left. + \frac{F_A C_{jk} M_k M_i}{2\pi} \left[ K_{34}^{12(v2)}(s,t,u) + K_{34}^{12(v2)}(s,t,u) + \tilde{C}_{jk} M_i^2 \right] \right\}
\]

where \( I_{34}^{12(v1),(b)}(s,t,u) \equiv I_{34}^{12(v1)}(s,t,u) + I_{34}^{12(b)}(s,t,u) \) comprises the contribution from the \( C_1 \) cuts of the vertex diagrams \((c) \) and \((d) \) (given by \( I_{34}^{12(v1)}(s,t,u) \)), and box diagrams \((g) \) and \((h) \) (given by \( I_{34}^{12(b)}(s,t,u) \)). \( I_{34}^{12(b)}(s,t,u) \) contains the contribution from the \( C_1 \) cuts present only in the box diagrams of \( N_i A \to \ell j H \). \( J_{34}^{12(v1),(b)}(s,t,u) \) has the contribution from the \( C_2 \) cuts, which are kinematically allowed when \( s > M_k^2 \) and they are possible only in the processes \( N_i \bar{7} j \to HA \) and \( \bar{7} j A \to N_i H \). All these contributions are L-violating and they are present for both scatterings with SU(2)\(_L\) and U(1)\(_Y\) gauge bosons.

2. When evaluating \( \epsilon_{34}^{12(v1)} \) and \( \epsilon_{34}^{12(b)} \) one must specify a gauge since, as mentioned above, the one–loop vertex \((v1)\) and box amplitudes are not separately gauge-invariant. In what follows we will work in the unitary gauge.

3. Since the resulting expressions are somewhat cumbersome, to double check our results we have computed the asymmetries both by explicit evaluation of the corresponding loop integrals as well as by using the Cutkosky rules that give directly the absorptive part of the Feynman diagrams.
$J_{34}^{12(\Delta)}(s,t,u)$ has the $-L$-conserving contribution from the $C_2$ cuts in the diagrams 3.(i) and 3.(j) present only for $\bar{7}_j A \rightarrow N_i H$.

$K_{34}^{12(v2)}(s,t,u)$ ($\widetilde{K}_{34}^{12(v2)}(s,t,u)$) contains the contribution from the $C_1$ cuts of
the L-violating (L-conserving) vertex diagrams in graphs (e) and (f). Similarly, 
\( K_{34}^{(v2)} (s, t, u) (K_{34}^{(v2)} (s, t, u)) \) involves the contribution from the \( C_1 \) cuts of the L-violating (L-conserving) diagrams in the graphs (e) and (f) of process \( N_i A \rightarrow \ell_j H \). Notice that the diagrams (e) and (f) correspond to amplitudes where the gauge boson is coupled to the one-loop self energy \( N_i - N_k \). Since the RHNs are SU(2)\( _L \) singlets, the sum over the SU(2)\( _L \) degrees of freedom cancel in each of these graphs. This, however, is not the case for the U(1)\( _Y \) gauge boson. Correspondingly the coupling factors \( \mathcal{F}_A = 0 \) for \( A = W_a \) while \( \mathcal{F}_X = g_A^2 / 2 \) for \( A = B_Y \).

The \( C_1 \) cuts are kinematically allowed for all processes and their corresponding contributions to the amplitudes are related by crossing symmetry. For the amplitudes of diagrams (c) and (d) this reads: \( I_{ij}^{N_i A (v1)} (s, t, u) = I^{(v1)} (s, t, u) \), \( I_{AH}^{(v1)} (s, t, u) = -I^{(v1)} (u, s, t) \), and \( I_{N_i}^{A (v1)} (s, t, u) = I^{(v1)} (u, t, s) \), with

\[
I^{(v1)} (s, t, u) = \\
D^2_{\mu \ell} M_i^2 \left[ \frac{-i \bar{u}^2}{(1 - \bar{u})^2} \left( -1 + \frac{a_k}{1 - \bar{u}} L_2 (\bar{u}) \right) + \frac{-i \bar{u}}{(1 - \bar{u})^2} \left( 1 - \frac{a_k + 1 - \bar{u}}{1 - \bar{u}} L_2 (\bar{u}) \right) \right] \\
+ D_{tH} D_{\mu \ell} M_i^2 \left[ \frac{-i \bar{s}}{1 - \bar{t}} \left( 1 - \frac{a_k + 1 - \bar{t}}{1 - \bar{t}} L_2 (\bar{t}) \right) + \frac{-i \bar{s}}{1 - \bar{u}} \left( 1 - \frac{a_k + 1 - \bar{u}}{1 - \bar{u}} L_2 (\bar{u}) \right) \right].
\]

The \( C_2 \) contributions to the vertex asymmetry from diagrams (c) and (d) are

\[
J_{AH}^{N_i A (v1)} (s, t, u) = D_{sH} D_{\mu \ell} M_i^2 \frac{-i \bar{u}}{1 - \bar{s}} \left[ 1 - \frac{a_k}{s} - \frac{a_k + 1 - \bar{s}}{1 - \bar{s}} L_4 (\bar{s}) \right], \\
J_{ij}^{(v1)} (s, t, u) = D_{\mu \ell} D_{sH} M_i^2 \frac{-i \bar{s}}{1 - \bar{s}} \left[ a_k - \bar{s} + \frac{a_k + 1 - \bar{s}}{1 - \bar{s}} L_4 (\bar{s}) \right] \\
+ D_{sH}^2 M_i^2 \frac{-i \bar{s}}{(1 - \bar{s})^2} \left[ (\bar{s} - \bar{u})(a_k - \bar{s}) + \frac{\bar{s} (a_k + 1 - \bar{s}) - a_k \bar{s} \bar{u}}{1 - \bar{s}} \right] L_4 (\bar{s}),
\]

and \( J_{ij}^{N_i A (v1)} (s, t, u) = 0 \). In Eqs.(9)-(10) and the following we define \( \bar{s} = \frac{s}{M_t^2}, \bar{t} = \frac{t}{M_t^2}, \bar{u} = \frac{u}{M_t^2} \), and

\[
L_1 (x) \equiv \ln \frac{a_k + x}{x}, \quad L_2 (x) \equiv \ln \frac{a_k + 1 - x}{a_k}, \quad L_4 (x) \equiv \ln \frac{x (a_k + 1 - x)}{a_k}, \\
L_3 (x, y) \equiv \ln \frac{x y + a_k (1 - x)}{a_k (1 - x)}, \quad L_5 (x, y) \equiv \ln \frac{x y + a_k (1 - x)}{a_k}.
\]

For the \( C_1 \) contributions to the asymmetry from diagrams (e) and (f) the crossing symmetry reads \( K_{ij}^{N_i A (v2)} (s, t, u) = K^{(v2)} (s, t, u) \), \( K_{AH}^{N_i (v2)} (s, t, u) = K_{AH}^{N_i (v2)} (s, t, u) = K_{AH}^{N_i (v2)} (s, t, u) = K_{AH}^{N_i (v2)} (s, t, u) = \)
\[ K^{(v2)}(u, s, t), \quad K_{N_iH}^{(v2)}(s, t, u) = K^{(v2)}(u, t, s), \] and equivalently for the \( \tilde{K} \) functions, with

\[
\begin{align*}
K^{(v2)}(s, t, u) & = \frac{2\tilde{s}}{(\tilde{s} - a_k)(t + \tilde{u})^3} \left\{ D_{tH}\tilde{t}\left[ (\tilde{t} - 2\tilde{u}) + \tilde{u}(1 - \tilde{s}) \right] - D_{u}\tilde{u}\left[ 3\tilde{t} + \tilde{u}(1 - \tilde{s}) \right] \right\}, \\
\tilde{K}^{(v2)}(s, t, u) & = \frac{2\tilde{s}\tilde{u}}{(\tilde{s} - a_k)(t + \tilde{u})^3} \left\{ D_{tH}\tilde{t}\left[ 3 - 2(\tilde{t} + \tilde{u}) \right] + D_{u}\tilde{u}\left[ 2\tilde{t} - \tilde{u} - \tilde{t}^2 + \tilde{u}^2 \right] \right\}. \quad (12)
\end{align*}
\]

The contribution from the \( C_1' \) cuts in diagrams (e) and (f) (which are only present for \( N_iA \to \ell_jH \)) is

\[
K_{\ell_jH}^{N_iA(v2)}(s, t, u) = -K^{(v2)}(s, t, u), \quad \tilde{K}_{\ell_jH}^{N_iA(v2)}(s, t, u) = -\tilde{K}^{(v2)}(s, t, u), \quad (13)
\]

and hence the contributions from vertices (e) and (f) to \( N_iA \to \ell_jH \) cancel exactly.

For the box diagrams: \( I_{N_iA}^{(b)}(s, t, u) = I^{(b)}(s, t, u) \equiv I_{b1}(s, t, u) + I_{b2}(s, t, u), \)

\[
I_{A}^{(b)}(s, t, u) = -I^{(b)}(u, s, t), \quad \text{and} \quad I_{N_iH}^{(b)}(s, t, u) = I^{(b)}(u, t, s), \]

with

\[
\begin{align*}
I_{b1}(s, t, u) &= \quad D_{tH} \left\{ \frac{\tilde{s}\tilde{u}}{(1 - \tilde{s})^2} - \frac{\tilde{s}(a_k + 1 - \tilde{u})}{\tilde{u}(1 - \tilde{t})}L_2(\tilde{u}) + \left[ \frac{a_k + \tilde{s}}{\tilde{u}} + \frac{a_k(a_k + 1 - \tilde{t})}{\tilde{s}t + a_k(1 - \tilde{s})} \right] L_3(\tilde{s}, \tilde{t}) \right\} \\
&+ D_{u}\tilde{u}\left[ -1 - \frac{\tilde{s}\tilde{t}}{(1 - \tilde{s})^2} + \frac{a_k + \tilde{s}}{1 - \tilde{t}}L_2(\tilde{u}) + \left( \frac{a_k + \tilde{s}}{\tilde{s}t + a_k(1 - \tilde{s})} \right) \right] L_3(\tilde{s}, \tilde{t}), \quad (14)
\end{align*}
\]

\[
\begin{align*}
I_{b2}(s, t, u) &= \quad D_{tH} \left\{ \frac{\tilde{s}\tilde{t}}{(1 - \tilde{s})^2} + \frac{a_k + 1 - \tilde{u}}{\tilde{t}} \left[ -\tilde{s}L_2(\tilde{u}) + \frac{(1 - \tilde{s})(a_k + \tilde{s}) + a_k\tilde{t}}{\tilde{s}\tilde{u} + a_k(1 - \tilde{s})} \right] \right\} \\
&+ D_{u}\tilde{u}\left\{ \frac{\tilde{s}\tilde{t}}{(1 - \tilde{s})^2} - \frac{\tilde{u}}{t(1 - \tilde{s})} - \frac{a_k + 1 - \tilde{u}}{t^2(1 - \tilde{u})}L_2(\tilde{u}) \right\}L_3(\tilde{s}, \tilde{u}) \quad (15)
\end{align*}
\]

The corresponding contributions from \( C_1' \) cuts read \( I_{A}^{N_iA(b)}(s, t, u) = \)

\[
I_{N_iH}^{\ell_jA(b)}(s, t, u) = 0 \quad \text{and} \quad I_{\ell_jH}^{N_iA(b)}(s, t, u) = I^{(b)}(s, t, u) \equiv I_{b1}(s, t, u) + I_{b2}(s, t, u),
\]

with
\( I'_{b_1}(s, t, u) = \)
\[ D_{\ell H} \left\{ -\frac{\tilde{s} \tilde{u}}{(1-s)^2} + \frac{a_k + \tilde{s}}{\tilde{u}} L_1(\tilde{s}) - \left[ \frac{a_k + \tilde{s}}{\tilde{u}} + \frac{a_k (a_k + 1 - \tilde{t})}{\tilde{s} t + a_k (1-s)} \right] L_3(\tilde{s}, \tilde{t}) \right\} \]
\[ + D_{a\ell} \left[ \frac{1 + \tilde{s} \tilde{t}}{(1-s)^2} - \frac{a_k + \tilde{s}}{\tilde{s}} L_1(\tilde{s}) - \frac{(a_k + \tilde{s})(a_k + 1 - \tilde{t})}{\tilde{s} t + a_k (1-s)} L_3(\tilde{s}, \tilde{t}) \right] , \quad (16) \]
\[ I'_{b_2}(s, t, u) = \]
\[ D_{\ell H} \left\{ -\frac{\tilde{s} \tilde{t}}{(1-s)^2} + \frac{a_k + 1 - \tilde{u}}{\tilde{t}} \left[ L_1(\tilde{s}) - \frac{(1-s) (a_k + \tilde{s}) + a_k \tilde{t}}{[\tilde{s} \tilde{u} + a_k (1-s)]} L_3(\tilde{s}, \tilde{u}) \right] \right\} \]
\[ + D_{a\ell} \left[ \frac{\tilde{s}}{\tilde{t}} - \frac{1 - \tilde{s} + \tilde{s} \tilde{t}}{(1-s)^2} + \frac{(1-\tilde{u})^2 (a_k + \tilde{s})}{\tilde{s} \tilde{t}^2} + \frac{1 - \tilde{u}}{\tilde{t}^2} \frac{a_k + 1 - \tilde{u}}{\tilde{s}} \right] L_1(\tilde{s}) \]
\[ + \left[ \frac{1 - 3\tilde{s} - 2a_k}{\tilde{t}} + \frac{(1-\tilde{s})(a_k + \tilde{s})}{\tilde{t}^2} + \frac{a_k - \tilde{s}}{\tilde{s}} - \frac{a_k (a_k + \tilde{s})}{\tilde{s}^2 \tilde{u} + a_k \tilde{u} (1-\tilde{s})} \right] L_3(\tilde{s}, \tilde{u}) \right\} . \quad (17) \]

From the Eqs. \[14\]–\[17\], we see that for the scattering process \( N_i A \rightarrow \ell_j H \) the contributions from \( C_1 \) and \( C'_2 \) partially cancel each other \(^4\).

The \( C_2 \) contributions to the box asymmetry are \( J_{\ell_j H}^{N_i A(b)}(s, t, u) = 0 \) and
\[ J_{A H}^{N_i A(b)}(s, t, u) = \]
\[ D_{sH} \left\{ \frac{\tilde{u} (a_k + 1 - \tilde{s})}{\tilde{t}(\tilde{s} - 1)} L_4(\tilde{s}) + \left[ \frac{a_k + \tilde{u}}{\tilde{t}} + \frac{a_k (a_k + 1 - \tilde{s})}{\til{s} \tilde{u} + a_k (1-\tilde{u})} \right] L_5(\tilde{u}, \tilde{s}) \right\} \]
\[ + D_{s\ell} \left\{ \frac{(1-\tilde{t})(a_k - \tilde{s})}{\til{s}^2} - \frac{(a_k + \tilde{u})}{\til{s} - 1} L_4(\tilde{s}) + \frac{(a_k + 1 - \tilde{s})(a_k + \tilde{u})}{\til{s} \tilde{u} + a_k (1-\tilde{u})} L_5(\til{u}, \til{s}) \right\} , \quad (18) \]
\[ J_{N_i H}^{\tilde{f}, A(b)}(s, t, u) = D_{s\ell} \left\{ \frac{\tilde{s} - a_k}{\til{t}} - \frac{(a_k - \til{s})(a_k + \til{u})}{\til{s} \til{u} + a_k (1-\til{u})} L_5(\til{u}, \til{s}) \right\} \]
\[ + \frac{a_k + 1 - \til{s}}{\til{t}^2} \left[ \frac{\til{u} \til{s} - \til{s} (1-\til{s})}{1-\til{s}} L_4(\til{s}) + (\til{s} - \til{t}) L_5(\til{u}, \til{s}) \right] \right\} \]
\[ + D_{\ell H} \left\{ \frac{\til{u} (a_k + 1 - \til{s})}{\til{t}(1-\til{s})} L_4(\til{s}) - \frac{(a_k + 1 - \til{s}) \left[ (a_k + \til{u})(1-\til{u}) + a_k \til{t} \right]}{\til{t} \til{s} \til{u} + a_k (1-\til{u})} L_4(\til{s}) \right\} . \quad (19) \]

Finally the \( C_2 \) contributions from diagrams 3.(i) and 3.(j) read
\[ J_{N_i H}^{\tilde{f}, A(b)}(s, t, u) = -D_{s\ell} \frac{M^2(s - a_k)}{s} \left[ \frac{s - a_k}{s} + D_{s\ell} M^2(s + a_k) \right] \]
\[ \times [D_{\ell H} \til{u} + D_{s\ell} \til{s}(1 - \til{u})] . \quad (20) \]

\(^4\) By definition \( I_{34}^{12}(b) \), \( I'_{34}^{12}(b) \), and \( J_{34}^{12}(b) \) must be real. It can be easily verified that in their expressions the imaginary contributions from negative arguments in the logarithms in Eq. \[11\] always cancel out.
Fig. 3. Ratio between the L-violating asymmetry in scatterings $\epsilon^{\text{scat}} \equiv \epsilon^{12}_{34}$ for the different processes as labeled in the figure and the corresponding quantity for decays $\epsilon^D \equiv \epsilon^{N_i}_{\ell_j H}$. To illustrate we have used $m_\ell = 0.18T$ and $m_H = 0.34T$. The left panels show the ratio as a function of $T/M_1$ for three different hierarchies $M_2/M_1 = 2, 10, 100$. The right panels show the ratio as a function of $M_2/M_1$ for two values of $T/M_1 = 1, 5$.

The ratios between the L-violating asymmetries in GBS and the L-violating asymmetry in decays are shown in Fig. 3 as a function of $T/M_1$ (left panels) and of $M_2/M_1$ (right panels). For simplicity we have considered the virtual effects of only one neutrino species $N_k = N_2$ different from $N_i = N_1$. It is apparent that for $N_1 \bar{\ell}_j \to H A$ and $\bar{\ell}_j A \to N_1 H$ this ratio deviates from unity even for hierarchical neutrinos $^5$. In the relevant temperature range

$^5$ Unlike the CP asymmetry in decays, the CP asymmetry in scatterings depends on the temperature, thus when comparing both types of asymmetries at a given $T$, the RHNS are considered to be hierarchical enough if $M_k \neq 1 \gg M_1, T$. 
for leptogenesis $0.1 \lesssim T/M_1 \lesssim 10$ the deviation can be of several tens of percent, e.g. 50% for $\ell_j B_Y \to N_i H$ at $T = M_1$. Conversely for $N_1 A \to \ell_j H$ the ratio tends to one for hierarchical neutrinos, hence the asymmetry can be well approximated by the decay one if $M_2/M_1 \gg 10$, but corrections appear at high temperatures for milder hierarchies.

These results can be easily understood analytically by expanding to lowest order in $M_i/M_k$ the expressions in Eqs. (9), (12), and (14)–(17) which gives

$$I^{(v1)}(s, t, u) = -\frac{1}{2} \frac{I_0(s, t, u)}{M^2_k},$$

$$I^{(b)}(s, t, u) = \frac{M^2_i s}{M^2_k} \left\{ D_{ul} \bar{u} \left[ 3 \bar{t} + \bar{u} (1 - \bar{s}) \right] - D_{tH} \bar{t} \left[ (\bar{t} - 2 \bar{u}) + \bar{u} (1 - \bar{s}) \right] \right\}, \quad (21)$$

$$I^{(b)}(s, t, u) = -I^{(b)}(s, t, u), \quad (22)$$

$$K^{(v2)}(s, t, u) = 2I^{(b)}(s, t, u) = -K^{(v2)}(s, t, u), \quad (23)$$

$$\tilde{K}^{(v2)}(s, t, u) = -\frac{M^2_i s}{M^2_k} \left\{ D_{tH} \bar{t} \left[ 3 - 2(\bar{t} + \bar{u}) \right] + D_{ul} \left[ 2(\bar{t} - \bar{u}) - \bar{t}^2 + \bar{u}^2 \right] \right\}, \quad (24)$$

Including these results in Eq.(8) and using Eqs. (3)–(5) one finds that

$$\epsilon^{N_i A^{(v1)}}_{\ell_j H} = \epsilon^{N_i B_Y^{(v1)}}_{\ell_j H} = \epsilon^{N_i B_Y^{(v2)}}_{\ell_j H} = \sum_{k \neq i} C_{jk} \frac{M_i}{16 \pi (Y_N Y^\dagger_N)_{ii}} \frac{M_i}{M_k} = \epsilon^{N_i A^{(v)}}_{\ell_j H}. \quad (26)$$

In the last equality we have used that the vertex CP asymmetry from the decay $N_i \to \ell_j H$

$$\epsilon^{N_i A^{(v)}}_{\ell_j H} = -\frac{\sum_{k \neq i} C_{jk} \sqrt{a_k} \left[ 1 - (1 + a_k) \ln \frac{1 + a_k}{a_k} \right]}{8 \pi (Y_N Y^\dagger_N)_{ii}} \approx \frac{\sum_{k \neq i} C_{jk} \frac{M_i}{M_k}}{16 \pi (Y_N Y^\dagger_N)_{ii}}, \quad (27)$$

where the last approximation holds for $M_k/M_i \gg 1$. So, as in the case of TQS, both wave and vertex contributions to the CP asymmetry in scatterings with SU(2)$_L$ gauge bosons are equal to the corresponding contributions to the decay CP asymmetry in the hierarchical limit ($M_k/M_i \gg 1$).

For the process $N_i A \to \ell_j H$ Eqs. (22), (24) imply that in the hierarchical limit the CP asymmetries from vertices (e) and (f) with U(1)$_Y$ gauge bosons, and from boxes with either SU(2)$_L$ or U(1)$_Y$ gauge bosons vanish,

$$\epsilon^{N_i W_a^{(b)}}_{\ell_j H} = \epsilon^{N_i B_Y^{(b)}}_{\ell_j H} = \epsilon^{N_i B_Y^{(v2)}}_{\ell_j H} = 0. \quad (28)$$
However for the other two processes we find that the L-violating contributions to the CP asymmetries satisfy

\[ \epsilon_{N_iA}^{(b)} \neq 0, \quad \epsilon_{N_iH}^{(b)} \neq 0, \quad \epsilon_{N_i\bar{\ell}j}^{(b)} = \frac{1}{2} \epsilon_{N_i\bar{\ell}j}^{(v2)} \neq 0, \quad \epsilon_{N_iH}^{(b)} = \frac{1}{2} \epsilon_{N_iH}^{(v2)} \neq 0. \]  

(29)

We note that the vanishing of \( \epsilon_{N_iA}^{(b)} \) and \( \epsilon_{N_i\bar{\ell}j}^{(v2)} \) at leading order in \( M_i/M_k \) is due to the presence of the additional \( C'_1 \) cuts, which, as mentioned above, results in a partial or total cancellation of the contributions from the \( C_1 \) cuts so that only terms of higher order in \( M_i/M_k \) remain.

We have mentioned before that for each process the sum of amplitudes of wave diagrams is independently gauge invariant, as it is the sum of the amplitudes for the vertex \((v2)\) diagrams, but the amplitudes of vertex \((v1)\) and box diagrams are not separately gauge invariant. However at the leading order in \( M_i/M_k \), the box and vertex \((v1)\) diagrams are independently gauge invariant, thus the results in Eq.(26) and Eq.(29) are also gauge independent.

Summarizing, we find that even for very hierarchical heavy RHNs

\[ \epsilon_{N_iA} = \epsilon_{N_i\bar{\ell}j}^{(w)} \neq \epsilon_{N_i\bar{\ell}j}^{(v)} \neq \epsilon_{N_iH}^{(b)} = \epsilon_{N_iH}^{(v2)} \neq 0, \]  

(30)

where \( \epsilon_{N_iH} = \epsilon_{N_iH}^{(w)} + \epsilon_{N_iH}^{(v)} \). Hence, we conclude that the CP asymmetry in the scattering \( N_iA \rightarrow \ell_jH \) factorizes in the hierarchical limit. However, this is not the case for the scatterings \( N_i\bar{\ell}j \rightarrow HA \) and \( \bar{\ell}jA \rightarrow N_iH \).

Also, as mentioned above, in the processes \( N_i\bar{\ell}j \rightarrow HB_Y \) and \( \bar{\ell}jB_Y \rightarrow N_iH \) there are new L-conserving vertex contributions to the CP asymmetry. They imply that in the hierarchical limit the L-conserving CP asymmetry in these scatterings is not equal to the L-conserving CP asymmetry in decays. Note that although the L-conserving asymmetries are always suppressed by a higher power of \( M_i/M_k \) compared to the L-violating ones, they are the only source of CP violation in models with conservation of L [13–15].

Finally, a few comments about the impact of our results for the determination of the baryon asymmetry are in order. Although the CP asymmetry in GBS has more interesting features compared to the one in TQS, we have found that still the CP asymmetry per scattering is \((0.5 - 2) \times \) the CP asymmetry per decay. Therefore the generation of CP asymmetry through GBS is relevant at high temperatures \( T \gtrsim M_i \) when the scattering rates are larger than the decay rates. Hence it can have effects in models for leptogenesis that are sensible to the high temperature regime. This includes standard type I leptogenesis in the weak washout regime and low energy models in which the sphaleron processes
freeze out before the RHNs have decayed [14,16].

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