LETTER

Accurate Doppler radar-based heart rate measurement using matched filter

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Abstract To enhance the vital sign radar’s detection accuracy of heartbeat rate (HR), which is heavily affected by the harmonics of respiration frequency, this paper proposes a waveform-driven matched filtering method based on polynomial fitting extraction. The merit of this approach lies in that, the quasi-ideal matching impulse approximating the actual heartbeat signal can be readily retrieved by subtracting the polynomial fitted waveform from a received signal, which provides high adaptability for individual subjects. Essentially, this extraction greatly removes the harmonic interference originated from respiration, thus considerably improving the HR detection accuracy. Simulations are performed to acquire the proper fitting order and the effective impulse duration. Guided by these two parameters, a 10-GHz non-contact continuous-wave (CW) Doppler radar system with typical microwave laboratory instruments is constructed. Experimental results confirmed the effectiveness of the proposed method, showing that the average errors of HR can be reduced from 46.21% to 0.96% under different subjects and distances, and from 38.05% to 0.90% in continuous measurement for one subject.

key words: heartbeat rate, matched filter, polynomial fitting, respiration harmonics cancellation

Classification: Devices, circuits and hardware for IoT and biomedical applications

1. Introduction

Non-contact vital sign detection has drawn considerable attention in the past few decades [1, 2, 3, 4, 5, 6]. Traditional contact detection sensors, such as photoplethysmography (PPG) or electrocardiography (ECG), need to place electrodes on patients’ body, which will make people feel uncomfortable for long-term monitoring [7]. What’s more, contact sensors can’t be used for some special patients, such as infants or psychiatric patients who have sensitive skin, or burned patients who have damaged skin [8]. These problems of contact sensors can be overcome well by non-contact vital sign detection sensors [9]. For example, these non-contact detection sensors can achieve long-term detection of human without making them uncomfortable [10, 11]. Among various non-contact vital sign detection methods, continuous-

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method more practical in real applications. What’s more, the harmonics of respiration won’t affect the extraction of the heartbeat signal, which will improve the detection accuracy of HR.

Simulations using MATLAB are performed to verify the method. In addition, experiments are performed with a 10 GHz non-contact CW Doppler radar vital sign detection system, which is con-structed using the typical microwave laboratory instruments. The measurement results with and without matched filter are compared.

2. Doppler radar and harmonics problem

![Fig. 1. Block diagram of the non-contact CW Doppler radar.](image1)

Fig. 1 shows the block diagram of a typical CW Doppler radar structure. In general, the transmitting antenna radiates a CW to illuminate a human being. The transmitted signal $T(t)$ is an unmodulated sinusoidal signal and can be expressed as

$$T(t) = \cos(2\pi ft + \phi(t))$$

(1)

where $f$ is the carrier frequency, $t$ is the elapsed time and $\phi(t)$ is the total phase noise of the transmitter. Assume that a human being stands at an initial distance $d_0$ away from the transmitting antenna with time-varying displacements (i.e., chest-wall movements). After hitting the human body, the phase of the transmitted signal is modulated by the periodic physiological movement $x(t)$, which includes the chest-wall displacements due to heartbeat and respiration. And the $x(t)$ can be represented as [26]

$$x(t) = x_h(t) + x_r(t) = m_h \sin \omega_h t + m_r \sin \omega_r t$$

(2)

where $m_h$ and $\omega_h$ are the magnitude and angular frequency of the respiration signal, $m_r$ and $\omega_r$ are the magnitude and angular frequency of the heartbeat signal. After modulated by the periodic physiological movement $x(t)$, the reflected signal captured by the radar receiver is shown as

$$R(t) = \cos\left[\frac{4\pi d_0}{\lambda} - \frac{4\pi x(t)}{\lambda} + \phi(t) - \frac{2d_0}{c}\right]$$

(3)

where $c$ is the propagation speed of the signal (the speed of light) and $\lambda = c/f$ is the signal’s wavelength in the free space. In the formula (3), the amplitude variation of $R(t)$ is neglected, owing to the valuable information what we are interested in is included in the phase term [27]. Then, the received signal is mixed with the LO signal which originates from the same source as the transmitted signal. Finally, the output I and Q channel baseband signals are sampled and low-pass filtered to remove the high frequency noises. Based on the Bessel function, the filtered I and Q channel baseband signals can be expressed as follows [28]:

$$B_i(t) = \cos\left[\frac{4\pi x_h(t)}{\lambda} + \frac{4\pi x_r(t)}{\lambda} + \phi(t)ight]$$

(4.a)

$$= DC_i - 2C_{01}\sin(\omega_i t) \cdot \sin \phi + 2[C_{20}\cos(2\omega_i t) + C_{02}\cos(2\omega_h t) + \cdots] \cdot \cos \phi$$

$$B_q(t) = \sin\left[\frac{4\pi x_h(t)}{\lambda} + \frac{4\pi x_r(t)}{\lambda} + \phi(t)\right]$$

(4.b)

$$= DC_q + 2C_{01}\sin(\omega_i t) \cdot \cos \phi + 2[C_{20}\cos(2\omega_i t) + C_{02}\cos(2\omega_h t) + \cdots] \cdot \sin \phi$$

where $DC_i$ and $DC_q$ are the $DC$ components in the I and Q channels, respectively. $\phi = 4\pi d_0/\lambda + \theta_0 + \Delta \phi(t)$ is the total residual phase, where $4\pi d_0/\lambda + \theta_0$ is the constant phase shift and $\Delta \phi(t) = \phi(t) - \phi(t - 2d_0/c)$ is the residual phase noise which can be ignored according to the range correlation theory [29]. $C_{ij} = J_i(4\pi m_j/\lambda) \cdot J_i(4\pi m_i/\lambda)$ determines the amplitude of every frequency component and $J_n$ is the Bessel function of the first kind.

It has been reported that the most important limitation using CW Doppler radar for vital sign detection is the presence of null detection points [30]. To eliminate the null point problem, the complex signal demodulation (CSD) method is used to extract the phase information, which can be expressed as follows [31]:

$$S(t) = B_i(t) + j \cdot B_q(t)$$

$$= \exp\left\{j \cdot \left[\frac{4\pi x_h(t)}{\lambda} + \frac{4\pi x_r(t)}{\lambda} + \phi(t)\right]\right\}$$

(5)

$$= DC_i + 2j[C_{01}\sin(\omega_i t) + C_{10}\sin(\omega_h t) + \cdots]e^{j\phi} + 2[C_{20}\cos(2\omega_i t) + C_{02}\cos(2\omega_h t) + \cdots]e^{j\phi}$$

Fig. 2 shows the simulated baseband waveforms of demodulated signal in frequency-domain about how harmonics of the respiration interfere with the heartbeat. In general, most of the respiration is typically within a frequency range of less than 0.6Hz, whereas the heartbeat lies within a frequency range of 0.8–3Hz. The body movement caused by respiration is typically less than 1 cm and by heartbeat is less than 2mm movement [32]. In the simulation, the frequency
and amplitude of the heartbeat are 1.4Hz and 0.5mm, the frequency of respiration is 0.3Hz, and the amplitudes of respiration are 7mm, 8mm and 9mm, respectively. First, it can be seen from Fig. 2 that the amplitudes of respiratory harmonics increase as the respiratory amplitudes increase. Second, it’s obvious that the second harmonic of respiration has larger amplitude than the heartbeat signal in frequency-domain, and the third harmonic of respiration has comparable amplitude as the heartbeat signal. Therefore, the heartbeat peak is not easy to be identified when harmonics of respiration exist.

3. Proposed method for harmonics cancellation

The matched filter has been widely used in impulse radar systems to detect the presence of template signals hidden in input signals where additive noises exist [33]. In our experiment, the respiration signals and its corresponding harmonics can be considered as additive noises embedded in baseband signals, so the matched filter can be used to detect the heartbeat signals hidden in baseband signals. The discretized baseband signals of (5) containing the heartbeat signal $x_h[n]$ and the additive noise $x_{\text{noise}}[n]$ can be rewritten as

$$S[n] = x_h[n] + x_{\text{noise}}[n]$$  \hspace{1cm} (6)

Then, the $x_h[n]$ can be retrieved using the matched filter by convolving $S[n]$ with a conjugated, time-reversed version of the template signal $h[n]$ shown as

$$x_h[n] = S[n] \ast h^*[−n]$$  \hspace{1cm} (7)

During the filtering, a critical step is to extract the appropriate template signal which includes as many heartbeat signal components as possible and as few respiratory signal and its corresponding harmonic components as possible. According to the matched filter principle, to extract the heartbeat signals from noises, the template signal should contain at least one complete heartbeat cycle. In addition, the more heartbeat signal cycles are contained in the template signal, the better of the extraction quality using the matched filter. As mentioned in Section II, the RR is typically less than 0.6Hz, and the HR lies within 0.8 – 3Hz. Therefore, the period of respiration is longer than 1.67s and the heart is within 0.33 – 1.25s. As a result, a 2s period can ensure a template signal containing 2–3 heartbeat cycles for most human subjects. The initial template of heartbeat is intercepted from the original I/Q channel baseband signals. But the initial template still carries some low frequency components (e.g., the respiratory signal and its second and third harmonic). To obtain a good result using the matched filter, these low frequency components including in the initial template should be removed as much as possible. In our work, this removal is implemented by polynomial fittings. In the following, we will show how to use the matched filter based on 2s heartbeat templates. Fig. 3 shows the demodulated baseband signals which are same as the simulation in Fig. 2 except that the amplitude of respiration is in the worst condition (i.e., 9mm). The 2s heartbeat templates are intercepted from 0.8 to 2.8s of the demodulated baseband signals, because this signal segment contains as many high frequency parts as possible, which is more in line with the characteristics of the heartbeat.

The left figure in Fig. 4(a) shows the 2s initial template intercepted from the original I channel as shown in Fig. 3. And the polynomial fittings with 5th-order and 10th-order are shown for comparison. Then, by subtracting the fitted signals from the initial template, the heartbeat template can be obtained and shown in the right figure in Fig. 4(a). The Fig. 4(b) is same as the Fig. 4(a) except that the template is intercepted from the Q channel. Comparing these two I and Q channels’ heartbeat templates with different polynomial fitting orders, it can be observed that the frequency of the heartbeat template obtained by the 10th-order fitting is larger than the real frequency (i.e., 1.4 Hz) of heartbeat owing to the overfitting effect. This implies that for the heartbeat template extraction, high order fitting is not appropriate.

Fig. 3. The baseband signal with respiration amplitude of 9 mm.

Fig. 4. Extraction of heartbeat templates. (a) I channel. (b) Q channel.
From Fig. 5(a), it can be first observed that without any polynomial fitting process to the initial template, HR can’t be extracted by the matched filter. It is worth noting that the 3rd-order polynomial fitting can’t extract the HR. This is because it is not enough to filter out the low frequency components using the 3rd-order fitting. On the contrary, it can be seen that the SNR of the heartbeat spectrum can be significantly enhanced with the 5th-order polynomial fitting. However, when the fitting orders are further increased, the SNR will become worse due to overfitting.

In order to explore the effect of different template lengths on HR extraction, Fig. 5(b) shows the simulation results of comparison among different template lengths after applying matched filter based on a 5th-order polynomial fitting. According to Fig. 5(b), it can be seen that all the obtained HR by the matched filter with different template lengths are 1.4Hz, which is consistent with the actual heartbeat rate. Based on Fig. 5(b), when the template length is 1.2s (containing nearly two heartbeat periods), the heartbeat spectrum can be observed. When the template length is 2s (containing nearly three heartbeat periods), the SNR of heartbeat is significantly increased. A conclusion can be drawn that with a longer template length, the matched filter can make a great significant SNR enhancement of heartbeat.

4. Experiments

In order to reliably extract weak heartbeat signals, a Doppler radar system is developed for experiments. The block diagram of the CW Doppler radar detection system is shown in Fig. 6. This system can be constructed using the commonly used microwave laboratory instruments, which include a vector network analyzer (N5242A PNA-X), a spectrum analyzer (R&S FSV30), and two antennas (RB-84SGAH20). The used horn antennas have an average gain of 20dB and beam width of 18°. The transmitted antenna is connected to the vector network analyzer (VNA), which acts as the transmitter with a frequency tuning range of up to 26.5GHz. With appropriate setup, the VNA is able to output microwave signals with different powers, and the transmitted frequency of VNA can also be independently changed. A 10-MHz reference signal is generated from the VNA as the clock for the whole system, so that the transceiver is synchronized. Therefore, coherent demodulation can be realized. The reflected microwave signals, which are modulated by the cardiopulmonary movement, are received by the antenna and sent into the spectrum analyzer to demodulate the microwave signals to baseband outputs. The baseband signals are sampled at a sampling rate of 100Hz and filtered in the MATLAB environment. Fast Fourier transform (FFT) is applied to obtain the spectrum of vital sign signals. Due to Doppler phases are proportional to the carrier frequency, the radar system operating at a higher frequency would have a higher sensitivity for detecting weak heartbeat signals. Considering this, the transmitted frequency is set as 10GHz. The transmission power in the overall experiments is fixed to 10 dBm. In addition, a finger-pressur pulse sensor YX303 is attached to the tester’s index finger to measure the pulse rate simultaneously, which is used as the reference for HR.

4.1 Measurement of different human subjects

In the first experiment, two human subjects are asked to sit still and situate toward the transceiver at distances of 1m, 1.25m, 1.5m and 1.75m, respectively. After the experiment, sampled data are transferred to a computer and filtered by a low-pass filter with its cutoff frequency at 5Hz to remove the high frequency noises. Fig. 7 shows spectrums of demodulated baseband signals. As mentioned in the Section III, the heartbeat templates with a length of 2s are intercepted from the baseband signals containing as many high frequency parts as possible. And a 5th-order polynomial fitting is applied to remove the low frequency parts. From Fig. 7, it can be seen that it is difficult to distinguish the heartbeat spectrum from respiration harmonics without matched filter, because they have comparable amplitude. As a result, the detection accuracy of HR will be deduced. On the contrary, with the matched filter, respiration harmonics are suppressed severely so that the detection accuracy of HR will be improved greatly. Specifically, Table I shows a comparison of the HR measurement results with and without the matched filter. In the measurements without respiration harmonics cancellation, the second peak after the respiration peak is considered as the HR. The errors in Table I are calculated as

$$error = \frac{HR_{measured} - HR_{ref}}{HR_{ref}} \times 100\%$$ (8)

And the average errors in Table I are calculated as

$$Average\ error = \frac{1}{L} \sum_{i=1}^{L} |error_i|$$ (9)

where $L$ is the number of measurement conditions.
Fig. 7 Heartbeat spectra obtained using the matched filter. (a) Subject A at 1m. (b) Subject A at 1.25m. (c) Subject A at 1.5m. (d) Subject A at 1.75m. (e) Subject B at 1m. (f) Subject B at 1.25m. (g) Subject B at 1.5m. (h) Subject B at 1.75m.

Table I Error comparison of HR w/o and with matched filter.

| Human subject | Distance | w/o matched filter | With matched filter |
|---------------|----------|--------------------|---------------------|
| A             | 1.00m    | -43.58%            | 0.30%               |
|               | 1.25m    | -41.83%            | -3.63%              |
|               | 1.50m    | -42.22%            | -0.76%              |
|               | 1.75m    | -39.66%            | -0.24%              |
| B             | 1.00m    | -56.29%            | 0.73%               |
|               | 1.25m    | -56.50%            | -1.24%              |
|               | 1.50m    | -53.35%            | -0.79%              |
|               | 1.75m    | -56.24%            | -0.24%              |
| Average error |          | -46.21%            | 0.96%               |

From the Table I, it can be observed that the HR measurement errors with matched filter are lower than that without matched filter. Specifically, the average error can be reduced from 46.21% to 0.96%.

4.2 Continuous measurement at a fixed distance
In the second experiment, one human subject is continuously tested at a fixed distance of 1.5m and each measurement takes 30s. Fig. 8 shows the comparison of HR measurement results w/o and with the matched filter in twenty data sets. From Fig. 8(a), it can be observed that the extracted HR is far from the reference HR without the matched filter. However, with the matched filter, due to the respiration harmonics are suppressed, the detected HR is very close to the reference HR. More specifically, calculated with formula (8), Fig. 8(b) presents errors with and without the matched filter. When without the matched filter, all the absolute errors are larger than 31%. In comparison, with the matched filter, all the absolute errors are smaller than 2%. With the help of formula (9), Table II shows that the average errors are reduced from 38.05% to 0.90% in continuous measurement.

Table II Error comparison of HR w/o and with matched filter in continuous measurement.

| Signal processing | w/o matched filter | With matched filter |
|-------------------|--------------------|---------------------|
| Average error     | 38.05%             | 0.90%               |

4. Conclusion
In this paper, a matched filtering method based on polynomial fitting extraction is proposed to eliminate the negative influence of respiration harmonics. Simulation and experimental results show that the proposed method, drawing
support from a “template signal” with appropriate length of 2s and 5th order polynomial fitting, can remove the interference of respiration harmonics effectively. As the results shown, the matched filter can improve the HR extraction accuracy not only for different subjects under different distances, but also for the same subject under continuous measurement. Compared to the traditional techniques, this method requires neither RR information before harmonics cancellation nor external reference source, making this method more practical in real applications.

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