On the role of the time scale $\Delta t$
 in Bose–Einstein correlations

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Abstract

The time scale parameter $\Delta t$, which appears in the Bose-Einstein Correlations (BEC) treated in terms of the Heisenberg uncertainty relations, is reexamined. Arguments are given for the role of $\Delta t$ to be a measure of the particles' emission time rather than representing the strength property of the correlated particles. Thus in the analyses of the $Z^0$ hadronic decays, the $\Delta t$ given value of $\sim 10^{-24}$ seconds, is the particles' emission time determined by the $Z^0$ lifetime. In heavy ion collisions $\Delta t$ measures the emission time of the particles produced in a nucleus of atomic number $A$. This emission time is shown here to be equal to $\Delta t = \frac{m_e a^2}{\hbar c} A^{2/3}$ that is, proportional to the nucleus surface area. This relation agrees rather well with the experimental $\Delta t$ values deduced from the BEC analyses of heavy ion collisions.

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1 Introduction

Bose-Einstein Correlations (BEC) of identical pairs of spin zero hadrons, produced in a variety of particle interactions, have been studied over more than four decades. From those studies which were carried out in one dimension, a single length parameter $R_{1D}$ was extracted which is taken to represent the radius of a spherical symmetric Gaussian interaction volume. Further on these studies were extended to the spin 1/2 baryon pairs, like the proton-proton and the $\Lambda\Lambda$ systems, by using the Fermi-Dirac Correlations (FDC) procedure [1]. Experimentally it has been observed that in the $Z^0$ gauge boson hadronic decays the $R_{1D}$ value decreases as the hadron mass $m$ increases [1]. To account for this feature two main approaches have been proposed. The first [2] rests on the Heisenberg uncertainty relations which correlate the dimension $R_{1D}$ with $m$ via a time scale parameter $\Delta t$. In the same approach one has also considered the three dimensions (3D) BEC analyses having the axes $R_{\text{long}}$, $R_{\text{side}}$ and $R_{\text{out}}$, defined in the Longitudinal Center of Mass System (LCMS) [3]. In this case, similar to the relation of $R_{1D}$ to $m$ and $\Delta t$, $R_{\text{long}}$ was found to depend on $\Delta t$ and $m_T$ [4]. Here $m_T$ is defined as the average transverse mass given by

$$m_T = \frac{1}{2} \left( \sqrt{m^2 + p_{T1}^2} + \sqrt{m^2 + p_{T2}^2} \right),$$

where $p_{T1}$ and $p_{T2}$ are the transverse momenta of the two identical particles [1]. The second proposed approach [5] rests mainly on the Bjorken-Gottfried relation [6] which associates $R_{\text{long}}$ and $R_{T}$, the longitudinal and transverse dimensions, with $m_T$.

Here our study is confined to the first approach with the aim to gain further insight to the BEC and FDC time scale $\Delta t$ associated with $R_{1D}$. In addition to the $Z^0$ hadronic decays we also examine the BEC results of the identical hadron pairs produced in central heavy ion collisions. Finally we draw the attention to the interest in analyzing the BEC of the $Z^0Z^0$ di-gauge bosons expected to be produced copiously in the recently constructed 14 TeV $pp$ collider, the LHC at CERN, in its future high luminosity setup.

2 Physics background

A compilation of the measured one dimension $R_{1D}$ values, obtained from BEC and FDC studies of identical hadron pairs present in the $Z^0$ decays at LEP, are shown in Fig. 1a as a function of the outgoing hadron mass $m$. The error bars attached to the $R_{1D}$ values include the statistical and systematic uncertainties. To note is the significant spread of the $R_{1D}(m_{\pi0})$ values between two of the experiments. This spread occurs often in the BEC and FDC measurements which can be traced back to the different adopted procedures and choices of the reference sample. Notwithstanding this deficiency, $R_{1D}$ is seen to decrease with the increase of the particle mass. That this behavior of $R_{1D}(m)$ is not only limited to the $Z^0$ hadronic decays is demonstrated
in Fig. 1b. This figure shows the $R_{1D}$ results obtained from BEC and FDC analyses of the outgoing hadrons, from pion to deuteron pairs, produced in the central $Pb + Pb$ collisions at 158/A GeV [7].

In Ref. [2] it has been shown that from the Heisenberg uncertainty relations one can derive a connection between $R_{1D}$ and the non-zero particle mass $m$, namely

$$R_{1D}(m) = \frac{c\sqrt{\hbar \Delta t}}{\sqrt{m}} ,$$

where $\Delta t$ is a time scale parameter. This time scale has been taken in [2] to be equal to $10^{-24}$ seconds representing the strong interactions, and thus independent of the hadron mass and its identity. As a result, the $R_{1D}$ behavior on the hadron mass was fairly well reproduced by Eq. (1) which is represented by the continuous line in Fig. 1a. Furthermore, this line essentially coincides with the one deduced from a general QCD potential [2]. Here we point out that the data given in the figure are the decay product of the $Z^0$ gauge boson, the lifetime of which is of the order of $10^{-24}$ seconds. Thus the success of the choice of $\Delta t=10^{-24}$ seconds, may in fact be, as we further argue, due to the decay particles’ emission time which is prescribed by the $Z^0$ boson lifetime.
3 The time scale $\Delta t$

It is clear that the $R_{1D}(m_{\pi})$ values obtained from BEC analyses of pion pairs emerging from collisions like the electron-nucleon [13] and neutrino-nucleon [14] are very similar to those obtained in $e^+e^-$ and $pp$ collisions and thus exclude the possibility that $\Delta t$ is related to the interaction strength of the incoming particles. As for the association of $\Delta t$ with the interaction strength of the outgoing identical correlated particles, it is instructive to examine Fig. 2 where, using Eq. (1), the expectation of $R_{1D}(m)$ are plotted against $m$ for three $\Delta t$ values of $10^{-24}$, $10^{-19}$ and $10^{-12}$ seconds standing for strong, electro-magnetic and weak interactions. As can be seen, if indeed $\Delta t$ is representing the interaction strength of the outgoing particles then for weak interacting particles the $R_{1D}$ measured by BEC or FDC should reach unreasonable high values as compared to those measured in $Z^0$ hadronic decays.

![Figure 2](image)

Figure 2: The expected $R_{1D}$ dependence on the particle mass for three $\Delta t$ values of $10^{-24}$, $10^{-19}$ and $10^{-12}$ seconds, representing respectively strong, electro-magnetic and weak interactions of the outgoing particles. The data in the figure are the LEP measured $R_{1D}$ values from $Z^0$ decays.

An additional evidence against the association of $\Delta t$ with the interaction strength of the outgoing particles is coming from the BEC measurement of the non-zero transverse momenta photon pairs, directly produced in the central $Pb + Pb$ interactions at $158/A$ GeV [15]. In this case Eq. (1) cannot be applied since $m_{\gamma} = 0$. However, from the Heisenberg uncertainty relations one can also derive [4] a relation between the longitudinal dimension $R_{long}$, defined in the LCMS, and the average transverse mass $m_T$, namely

$$R_{\text{long}}(m_T) = \frac{c\sqrt{\hbar \Delta t}}{\sqrt{m_T}},$$

(2)
which is applicable to photons with a non-zero transverse momentum. The BEC analysis of the directly produced photon pairs of the WA98 collaboration [15] was divided into two transverse momentum $P_T$ regions which yielded the following $R_{1D}$ and their associated chaoticity $\lambda_{1D}$ parameter values:

$$R_{1D}^I = 5.9 \pm 1.2 \text{ fm}; \quad \lambda_{1D}^I = 0.0028 \pm 0.0007; \quad \text{for } 100 < P_T < 200 \text{ MeV/c}$$

$$R_{1D}^{II} = 6.1 \pm 1.4 \text{ fm}; \quad \lambda_{1D}^{II} = 0.0029 \pm 0.0017; \quad \text{for } 200 < P_T < 300 \text{ MeV/c}$$

where the statistical and systematic errors are added in quadrature. These $R_{1D}(\gamma)$ values are consistent with the $R_{1D}(m_\pi)$ values obtained in the same $Pb+Pb$ collision experiment at 158/A GeV. Since in general the $R_{1D}(m)$ values are similar within 10 to 20% to the corresponding $R_{long}(m_T = m)$, it is also instructive to note that the value $R_{1D}(\gamma) \simeq 5.6 \text{ fm}$ measured at an average $m_T$ of 200 MeV is consistent with the value of $R_{long}(m_T = 200 \text{ MeV}) \simeq 5.9 \text{ fm}$ both measured in the $Pb+Pb$ reactions [7]. From this one can safely infer that the $\Delta t$ associated with the directly produced photons is in any case away by a few orders of magnitude from the $\Delta t$ region that represents the electro-magnetic interaction strength (see Fig. 2). Following these observations we assign $\Delta t$ to be the particles’ emission time.

In the $Z^0$ decay, as well as in hadron interactions, like in $pp$ reactions, the particle’s collision and emission times are practically of the same order of magnitude. This apparently is not the case in heavy ion collisions. In the $Pb + Pb$ collisions at 158/A GeV, measured by the WA98 collaboration [7], the $R_{1D}$ values obtained from identical hadron pairs are seen in Fig. 1b to be described very well, apart from the slight departure of $R_{1D}(m_\rho)$, by the continuous line. This line is the result of a fit of Eq. (1) to the data yielding $\Delta t = (1.28 \pm 0.03) \times 10^{-22}$ seconds, much longer than the particles’ collision time of $\sim 10^{-24}$ seconds. Here it is important to note that the success of this fit is taken by the WA98 collaboration as an indication for a common emission duration of the various particle pairs produced in the $Pb+Pb$ reactions. Furthermore, in the same experiment the $R_{long}$ dependence on $m_T$ is very well reproduced by Eq. (2) with a fit result of $\Delta t = (1.61 \pm 0.05) \times 10^{-22}$ seconds.

Inasmuch that Eq. (1) can also describe successfully the $R_{1D}(m)$ results of other heavy ion reactions, apart from the $Pb + Pb$ collisions, one is naturally led to combine Eq. (1) evaluated at $m = m_\pi$, that is

$$R_{1D}(m_\pi) = \frac{c\sqrt{\hbar \Delta t}}{\sqrt{m_\pi}}, \quad (3)$$

with the known expression (see e.g. [1]) that relates the measured $R_{1D}(m_\pi)$ of pion pairs with the atomic number $A$ (see Fig. 3a), namely

$$R_{1D}(m_\pi) = aA^{1/3}, \quad (4)$$
where \( a \) is a constant of the order of 1.0 \( fm \). From these two last equations one obtains

\[
\Delta t = \frac{m_\pi a^2}{\hbar c^2} A^{2/3}.
\]

(5)

Moreover, BEC analyses of pion pairs emerging from central \( Pb + Pb \) collisions at the energies 20/A, 30/A, 40/A, 80/A and 158/A GeV show that the radii values measured in the LCMS reference frame are only very little, if at all, dependent on the collision energy [16]. As a consequence, Eq. (5) is expected to be independent of the collisions energy so that \( \Delta t \) is essentially only proportional to \( A^{2/3} \), the surface area of the nucleus.

The relation between \( \Delta t \) and the atomic number \( A \) is presented in Fig. 3b where the dotted, continuous and dashed lines are calculated from Eq. (5) respectively for \( a = 0.8, 1.0 \) and 1.2 \( fm \). The data used for the figure were taken from the well measured \( R_{1D} \) values of central heavy ion collision experiments, in the atomic number range of 12 to 207, reported in Refs. [17–20]. From these \( R_{1D} \) data the \( \Delta t \) values shown in the figure were calculated using \( \Delta t = m_\pi R_{1D}^2(m_\pi)/(\hbar c^2) \) as derived from Eq. (1). As can be seen, the \( \Delta t \) data are accounted for by Eq. (5) in or near the range defined by \( a = 0.8 \) and \( a = 1.2 \) \( fm \) lines. To note is that the spread of the \( \Delta t \) values increases, as expected, with \( A \). As seen from Fig. 3b the common particle emission time is predominantly proportional to the nucleus surface area. These particle emission times, in the range of \( A \) covered by Fig. 3b, are of the order of \( 10^{-23} \) seconds to be compared with the typical estimates inferred from hydrodynamical-like and other models applied to heavy ion collisions [21].

4 Summary and conclusions

The role of the time scale \( \Delta t \) parameter, associated with the BEC and FDC of identical particle pairs, has been examined in the framework of the Heisenberg uncertainties relations. From the studies of the \( Z^0 \) decays, the \( \Delta t \) is seen to be associated with the particle pair emission time of the order of \( 10^{-24} \) seconds, as determined from the \( Z^0 \) lifetime, rather than being a measure of the particles interaction strength property.

In heavy ion collisions one should differentiate between the typical collision time of \( \sim 10^{-24} \) seconds for strong interactions, and the emission time of the produced particles. This is well illustrated by the WA98 experiment where the collision time is negligible in comparison to the particle’s emission time of about \( 10^{-22} \) seconds.

Merging the known dependence of \( R_{1D}(m_\pi) \) on the atomic number \( A \) namely, \( R_{1D}(m_\pi) = a A^{1/3} \), with the \( R_{1D} \) dependence on the particle mass as derived from the Heisenberg uncertain-
Figure 3: (a) The measured $R_{1D}$ from BEC of pion pairs produced in heavy ions collisions as a function of $A^{1/3}$ reproduced from reference [19]. The straight line represents the relation $R = aA^{1/3}$ with $a = 1.2\, fm$. (b) $\Delta t$ as a function of the atomic number $A$. The data points are extracted via Eq. (1) from the measured $R_{1D}$ values reported in Refs. [17–20]. The dotted, continuous and dashed lines represent Eq. (5) respectively for $a = 0.8$, 1.0 and 1.2 $fm$.

tainties, yields the equation

$$
\Delta t = \frac{m_\pi a^2}{\hbar c^2} A^{2/3}.
$$

This expression relates the particle emission time $\Delta t$ with the surface area of the nucleus which agrees with the experimental results as is illustrated in Fig. 3b.

Even though the BEC analysis of two directly produced photons in $Pb + Pb$ collisions supports the notion that $\Delta t$ is the particle emission time, a decisive answer to this issue should come from BEC and/or FDC of weak interacting particles. Presently no such information exists. The $\mu^\pm\mu^\pm$ pairs are in general the decay product of pions and/or kaons so that they are not produced simultaneously. As for the $e^\pm e^\pm$ system produced in particle reactions, it also has similar drawbacks. For this reason it may be worthwhile to consider a BEC analysis of the two weakly interacting $Z^0Z^0$ system even though they are expected at lower order to be dominated by a coherent production in $pp$ collisions. High order corrections may well introduced a small non-coherent contributions which will be sufficient to allow a BEC analysis as was the case in the di-photon BEC analysis [15] which succeeded with a chaoticity $\lambda$ value as small as $\approx 0.003$. In case that $\Delta t$ is determined by the interaction strength of the $Z^0$ gauge boson then its $R_{1D}$ value should be very high as seen from Fig. 2. Finally we note that the opportunity to carry out a BEC analysis of the $Z^0Z^0$ pair may be realized at the 14 TeV CERN Large Hadron $pp$ Collider in its upgraded luminosity configuration where sufficiently high event statistics can be accumulated.
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2. The expected $R_{1D}$ dependence on the particle mass for three $\Delta t$ values of $10^{-24}$, $10^{-19}$ and $10^{-12}$ seconds, representing respectively strong, electro-magnetic and weak interactions of the outgoing particles. The data in the figure are the LEP measured $R_{1D}$ values from $Z^0$ decays.

3. (a) The measured $R_{1D}$ from BEC of pion pairs produced in heavy ions collisions as a function of $A^{1/3}$ reproduced from reference [19]. The straight line represents the relation $R = aA^{1/3}$ with $a = 1.2 \text{ fm}$. (b) $\Delta t$ as a function of the atomic number $A$. The data points are extracted via Eq. (1) from the measured $R_{1D}$ values reported in Refs. [17–20]. The dotted, continuous and dashed lines represent Eq. (5) respectively for $a = 0.8$, 1.0 and 1.2 $\text{ fm}$.