LCFT and Sine-Liouville theory

A O Shishanin
Bauman Moscow State Technical University, Moscow, Russia
E-mail: shishandr@rambler.ru

Abstract. LCFT is special conformal field theory with logarithmic terms in operator product expansion OPE. It is well-known that it is possible to construct operators with logarithmic terms in OPE at Liouville model. Here we discuss this topic for some generalization of Liouville model, such as Sine-Liouville model.

1. Introduction
Logarithmic conformal field theories LCFT are two-dimensional conformal field theories with logarithmic terms in operator product expansion OPE [1]. Logarithmic terms appear because of some primary operators have degeneration for conformal dimensions. Most famous example of such theory is $c = -2$ model. This model describes the system of ghosts with conformal dimensions 0 and 1. Also LCFT appear at WZWN model for some (super)groups and some levels $k$. Caux, Kogan and Tsvelik [2] had remarked that in Liouville model some primary operators generate LCFT. We will discuss LCFT for some generalization of Liouville model Sine-Liouville theory. Remarkable that Sine-Liouville theory [3-4] is dual to two-dimensional black hole (the Witten cigar) [5,6].

Let us consider WZWN model with $SL(2,\mathbb{R})_k/U(1)$-coset in level $k$. The Witten solution [5,6] is described by the sigma-model with target metric

$$ds^2 = k (dr^2 + \tanh^2 r d\theta^2).$$

This model has dilaton field $\Phi = \log(\cosh^2 r)$. Also this theory is CFT with central charge

$$c = \frac{3k}{k-2} - 1.$$  \hspace{5cm} (2)

There is another parametrization of metric (1)

$$ds^2 = \frac{dud\nu}{1-uv}, \quad \Phi = \log(1-uv).$$

Here spectrum of primary fields has form

$$\Delta_{P,m,n} = -\frac{1}{k-2} + P^2 + \frac{(m \pm nk)^2}{4k},$$

where $m, n$ are integers and $P$ is continuous.
2. Liouville and Sine-Liouville theories

The Liouville model has the following action

$$S_L = \frac{1}{8\pi} \int \left( \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - QR^{(2)} \varphi - \mu e^{b\varphi} \right) \sqrt{g} d^2 x,$$

where integration is carried out over some Riemann surface with metric $g$ and the Ricci scalar $R^{(2)}$, $\mu$ is a cosmological constant and $b$ is a coupling constant. For simplicity we will discuss case of disk with $R^{(2)} = 0$. For conformal invariance is necessary to

$$Q = b + \frac{2}{b}.$$  

Here the energy-momentum tensor is

$$T_L = -\frac{1}{4} \partial \varphi \partial \varphi + Q \partial^2 \varphi$$

and the central charge

$$c = 1 + 3Q^2.$$  

The primary field : $e^{\alpha \varphi}$ : has a dimension

$$\Delta_\alpha = \frac{\alpha (Q - \alpha)}{2}.$$  

The Sine-Liouville theory on disk has the following action

$$S_{SL} = \frac{1}{8\pi} \int \left( \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \mu \cos (a \phi_L) e^{b\varphi} \right) \sqrt{g} d^2 x$$

The field $\phi$ is sum of $\phi_L$ and $\phi_R$. Here the stress-energy tensor is

$$T_{SL} = -\frac{1}{4} \partial \varphi \partial \varphi - \frac{1}{4} \partial \phi \partial \phi + \frac{1}{4b} \partial^2 \varphi.$$  

For matching of the parameters for both models [4] we need coincidence of central charges

$$2 + \frac{3}{b^2} = \frac{3k}{k-2} - 1.$$  

Matching of another parameters is given by

$$b^2 = \frac{k-2}{4}, \quad a^2 = \frac{k}{4}.$$  

Also there is requirement that last term in action has conformal dimension 1

$$\Delta_{SL} = 2a^2 - 2b^2 = 1.$$  

3. Logarithmical conformal field theories

Logarithmical terms [1,2,7] appear in operator product expansion when $L_0$ component of stress-energy tensor acts by mixing the primary fields. Simplest example with two fields is given by

$$L_0|C\rangle = \Delta|C\rangle, \quad L_0|D\rangle = \Delta|D\rangle + |C\rangle.$$  

The fields $C$ and $D$ form Jordan cell. The operators $C$ and $D$ for Liouville theory were presented in [2]. Let’s consider primary field $V_\alpha = : e^{\alpha \varphi(z,\bar{z})} : = C$, where $\alpha = Q/2$. There is another operator with the same conformal dimension $\Delta = Q^2/8$

$$V_\alpha = \frac{\partial V_\alpha}{\partial \alpha} = : \varphi e^{\alpha \varphi(z,\bar{z})} : .$$  

Here operator $D$ can be find from $V_\alpha$

$$D = \frac{2}{\alpha} : \varphi e^{\alpha \varphi(z,\bar{z})} : .$$
4. LCFT in Sine-Liouville theory

Let us consider primary fields of Sine-Lioville model [6]

\[ \Psi_{\alpha,n,m} = \exp \left( \alpha \phi + i an \phi_L + \frac{i}{4a} m \phi_R \right) :. \]  

Here right and left dimensions of these operators are

\[ \Delta_{\alpha,n,m}^\pm = \alpha \left( \frac{1}{2b} - \alpha \right) + \frac{1}{4k} (m \pm nk)^2. \]  

Matching with spectrum of $SL(2, \mathbb{R})_k/U(1)$ model (3) if one put $iP = \alpha - \frac{1}{4b}$.

Let $\Psi_{\alpha,n,m}$ be $C$ operator. Then like Liouville theory $D$ operator is

\[ D = \frac{b}{2} : \phi \exp \left( \alpha \phi + i an \phi + \frac{i}{4a} m \phi \right) :. \]  

There are not another operators with logarithmical terms in OPE because of stress-energy tensor (10).

Acknowledgments

The author would like to thank Alexei Litvinov for discussions.

References

[1] Gurarie V 1993 Logarithmic operators in conformal field theory Nucl. Phys. B 410 535 (Preprint hep-th/9303160)
[2] Caux J S, Kogan I I and Tsvelik A M 1996 Logarithmic Operators and Hidden Continuous Symmetry in Critical Disordered Models Nucl. Phys. B 466 444 (Preprint hep-th/9511134)
[3] Kazakov V A, Kostov I I and Kutasov D 2002 A matrix model for two-dimensional black hole Nucl. Phys. B 622 141 (Preprint hep-th/0101011)
[4] Fateev V A 2017 Integrable deformations of Sine-Liouville conformal field theory and duality SIGMA 13 080 (Preprint hep-th/1705.0642)
[5] Witten E 1991 String theory and black holes Phys. Rev. D 44 314
[6] Dijkgraaf R, Verlinde H and Verlinde E 1992 String propagation in a black hole geometry Nucl. Phys. B 371 269
[7] Kogan I I and Lewis A 1998 Origin of Logarithmic Operators in Conformal Field Theories Nucl.Phys. B 509 687 (Preprint hep-th/9705240)