Explaining the subpulse drift velocity of pulsar magnetosphere within the space-charge limited flow model

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ABSTRACT

We try to explain the subpulse drift phenomena adopting the space-charge limited flow (SCLF) model and comparing the plasma drift velocity in the inner region of pulsar magnetospheres with the observed velocity of drifting subpulses. We apply the approach described in a recent paper of van Leeuwen & Timokhin (2012), where it was shown that the standard estimation of the subpulse drift velocity through the total value of the scalar potential drop in the inner gap gives inaccurate results, while the exact expression relating the drift velocity to the gradient of the scalar potential should be used instead. After considering a selected sample of sources taken from the catalog of Weltevrede, Edwards & Stappers (2006) with coherently drifting subpulses and reasonably known observing geometry, we show that their subpulse drift velocities would correspond to the drift of the plasma located very close or above the pair formation front. Moreover, a detailed analysis of PSR B0826-34 and PSR B0818-41 reveals that the variation of the subpulse separation with the pulse longitude can be successfully explained by the dependence of the plasma drift velocity on the angular coordinates.

Key words: stars: neutron — plasma magnetosphere — subpulse drift — B 0818-41 — B0826-34

1 INTRODUCTION

The research on pulsar magnetospheres started 45 years ago from the pioneering paper of Goldreich & Julian (1969), where it was shown that the magnetic field, together with the fast rotation of the pulsar, generates strong electric fields tending to pull out charged particles from the surface of the neutron star and to form the plasma magnetosphere. However, the very next arising question - how many particles will actually leave the surface of the star under the action of this force - is still a subject of scientific debate. In fact, the magnetic field which generates the pulling out electric field also leads to the substantial increase in the cohesive energy of the surface charged particles (positive ions in larger degree than electrons), making the outer layer of the star very dense and strongly bound. Medin & Lai (2007) showed that for each magnetic field intensity there exists a critical surface temperature, above which the particles are able to freely escape from the surface of the star. The magnetic field of pulsars is typically inferred from observations under the assumption that it is strictly dipolar, thus providing $B_d = 2 \times 10^{12} \sqrt{PP} \times 10^{15} G$ (here $P$ is the period and $P$ is the period derivative of the pulsar). Using this formula, it turns out that the majority of pulsars satisfy the condition for the free particle outflow. However, in many works, starting from Ruderman & Sutherland (1975), it was suggested that the magnetic field close to the surface of the pulsar should have a multipole structure, with the surface magnetic field several orders of magnitude larger than the estimated $B_d$. This idea is supported by X-ray observations (Zhang, Sanwal & Pavlov 2005; Kargaltsev, Pavlov & Garmire 2006; Pavlov et al. 2009), and several studies have been performed searching for a mechanism of generation and maintenance of such small-scale strong magnetic fields [see Geppert, Gil & Melikidze (2013) and references therein].

The amount of charged particles extracted from the surface of the star by the rotationally induced electric field is a key aspect of any pulsar magnetosphere model. The model of Ruderman & Sutherland (1975), for example, assumes that no particle leaves the pulsar surface and there is a vacuum gap formed above the star with a huge difference in the scalar potential between the bottom and the top, i.e., $\sim 10^{12} V$. According to this idea, the gap will be periodically discharged and rebuilt, making it intrinsically "non-
stationary”. On the contrary, in the “stationary” space-charge limited flow (SCLF) model of Arons & Scharlemann (1979) there is a free flow of charged particles from the pulsar surface. The partially screened gap model of Gil, Melikidze & Geppert (2003) combines some features of both the previous models and is based on the assumption that the vacuum gap discharge will lead to the back flow of charged particles bombarding and heating the surface of the star, causing the thermal ejection of ions from the surface and partially screening the original accelerating electric field.

The choice among different models of pulsar magnetosphere should be made by accurate comparison of their predictions with the results of observations. A very interesting phenomena serving as a diagnostic tool is the subpulse drift. Pulsar radio emission comes to us in the form of pulses, which may look very different among each other and typically consist of individual subpulses. Although the average pulse profile is very stable and represents a unique fingerprint of each pulsar, a range of successive pulses plotted on top of each other in a so-called pulse stack, quite often shows an organized phase shift of the subpulses forming drift bands. This phenomenon was reported for the first time in Drake & Craft (1968), while the first systematic analysis of “drifting” pulsars dates back to Backus (1981) and Rankin (1986). So far, the largest statistical study of the phenomenon has been presented in the works of Weltevrede, Edwards & Stappers (2006) and Weltevrede, Stappers & Edwards (2007), who considered a sample of 187 pulsars, 55% of which show drifting subpulses. Usually the subpulse drift bands are characterized by the horizontal separation between them in subpulse longitude, \( P_2 \), and the vertical separation in pulse periods, \( P_3 \). The subpulse behavior of the individual pulsars may be rather complex and demonstrate smooth or abrupt change of drift direction, phase steps, longitude and frequency dependence of the separation \( P_2 \), or even presence of subpulses drifting in the opposite directions at the same time. For the graphical representation of the periods \( P_2 \) and \( P_3 \) we refer the reader to Fig. 1 of Weltevrede, Edwards & Stappers (2006), where one can find several examples of the different subpulse behavior of individual sources.

The vacuum gap model, and especially the partially screened gap model, has been widely used to explain the subpulse drift phenomena (Gil & Sendyk 2000; Melikidze, Gil & Pataraya 2000; Gil, Melikidze & Geppert 2003; Gupta et al. 2004; Bhattacharyya et al. 2007; Gil et al. 2008), while for a long time the SCLF model has been regarded unable to account for it. However, recent analytical (van Leeuwen & Timokhin 2012) and numerical (Timokhin 2010; Timokhin & Arons 2013) progresses have shown that the door can be left open even for the SCLF model. The main goal of this paper is to explain the subpulse drift velocity in the framework of the relativistic SCLF model, without addressing the question of the generation mechanism of the plasma features responsible for the appearance of the subpulses, while trying to compare the results with the available observational data.

The plan of the paper is the following. In Sect. 2 we give a brief review of the models used so far to explain the subpulse phenomena and motivate our choice to concentrate on the SCLF model. In Sect. 3 we present the basic equations to explain the subpulse drift velocity in the framework of the SCLF model. In Sect. 4 we consider a set of pulsars from the catalog of Weltevrede, Edwards & Stappers (2006) with coherently drifting subpulses, and try to deduce in which regions of the pulsar magnetosphere the SCLF model would predict the plasma with the observed velocities. In Sect. 5 we focus on two specific sources, PSR B0826-34 and PSR B0818-41, trying to account for their phenomenology. Finally, Sect. 6 is devoted to the summary of the results obtained and to the conclusions.

2 A BRIEF SURVEY OF EXISTING MODELS

The first theoretical explanation of the subpulses was provided by Ruderman & Sutherland (1975), who associated the subpulses to the spark discharges of the vacuum gap above the pulsar surface. In its original form, the model applied to the pulsars with anti-parallel angular velocity \( \Omega \) and magnetic moment \( \vec{\mu} \), and assumed that the charged particles (positive ions) are tightly bound to the surface of the star and cannot be pulled out by the rotationally induced electric field. This requirement leads to the formation of a vacuum gap in the region where the magnetic field lines are open and with a potential drop between the top and the bottom of the order of \( 10^{12} \) V for typical pulsar parameters. Due to the presence of strong curved pulsar magnetic fields, the gap will be unstable and periodically discharged by the photon induced pair creation process. The discharges will build up plasma columns, which are subject to the \( E \times B \) drift in the electromagnetic field of the magnetosphere.

Ruderman & Sutherland (1975) showed that, unless the potential drop of the gap is completely screened, the plasma columns will not exactly corotate with the star but always lag behind the rotation of the star, and this is responsible for the visual drift of the subpulses along the pulse longitude. The sparks are assumed to form rings and the so-called tertiary periodicity \( P_3 \) is the time needed for the spark carousel to make one full rotation around the magnetic axis. Although in this model the subpulses cannot outrun the rotation of the star, due to the effect of aliasing (Gil, Melikidze & Geppert 2003; Gupta et al. 2004), the apparent velocity of the subpulses may be both positive (from earlier to later longitudes) and negative (from later to earlier longitudes).

Ruderman & Sutherland (1975) estimated the subpulse drift velocity to be proportional to the full potential drop across the gap, resulting in excessively large values of the drift velocity compared with the observed ones. Later on, Gil & Sendyk (2000), Melikidze, Gil & Pataraya (2000), Gil, Melikidze & Geppert (2003) generalized this model to account for arbitrary inclination angles \( \chi \) between \( \Omega \) and \( \vec{\mu} \), and modified it to allow for the partial outflow of ions and electrons from the surface of the star, forming partially screened gap instead of the pure vacuum. It was argued that the favorable conditions for the spark discharge persist even if the original vacuum gap is screened up to 95% or more, making the velocity of drifting subpulses consistent with the observed values. However, the partially screened gap model requires surface values of the magnetic field of the order of \( 10^{14} \) G, much larger than those deduced when the magnetic field is dipolar (i.e. \( \sim 10^{12} \) G). The partially screened gap model has been used in a number of works to describe the subpulses of specific pulsars as well as their X-ray emission (Gupta et al. 2004; Bhattacharyya et al. 2007; Gil et al. 2008) and it has typically revealed a strong predictive power.

In Clemens & Rosen (2004); Rosen & Clemens (2008); Rosen & Demorest (2011), the drifting subpulses are instead explained by non-radial oscillations of the surface of the star. This model gives a very natural explanation to the subpulse phase shift, relating it to the intersection of the observer’s line of sight with the nodal line. The empirical model of Wright (2003) relates the formation of

\[ \text{Stationary}\]
drifting subpulses to the interaction between electron and positron beams traveling up and down between the inner and the outer gaps of the pulsar magnetosphere.

Kazbegi, Machabeli & Melikidze (1991) and later Gogoberidze et al. (2005) proposed a model where the subpulses are formed due to the modulation of the emission region by large-scale “drift waves”, generated by oppositely directed curvature drifts of electrons and positrons. Finally, Fung, Khechinashvili & Kuipers (2006) proposed a model to explain drifting subpulses that is based on the diocotron instability in the pair plasma on the open field lines. Detailed description of all these models may be found in the review of Kuipers (2009) and references therein.

Recently van Leeuwen & Timokhin (2012) have shown that the order of magnitude estimation of the subpulse drift velocity used in Ruderman & Sutherland (1975), and subsequent works, can be replaced by a more precise expression, relating the velocity to the radial derivative of the potential instead of the absolute value.

The main issue that we try to address is whether it is possible, in the framework of the SCLF model, to explain the observed subpulse drift velocities, and if so, to infer in which part of the magnetosphere they should be produced. In the second part of the paper we will instead apply our arguments to two specific sources.

3 SUBPULSE DRIFT VELOCITY IN THE FRAMEWORK OF THE SCLF MODEL

Scharlemann, Arons & Pawley (1979) and Arons & Scharlemann (1979) were the first to show analytic solutions for the scalar potential in the vicinity of the pulsar polar cap and in the framework of the SCLF model. In their analysis, the accelerating electric field parallel to the magnetic field of the pulsar is due to the curvature of magnetic field lines and to the inertia of particles. Later, Muslimov & Tsygan (1992) have shown that, due to the effect of dragging of inertial frames in general relativity, it is possible to obtain accelerating electric fields which are two orders of magnitude larger than those normally expected. This approach has further been elaborated in Harding & Muslimov (1998, 2001, 2002). For convenience, in this subsection we present the main results found by Muslimov & Tsygan (1992) as well as the expressions for the subpulse drift velocity that we obtained, i.e., using Eq. (1).

In general relativity the dipole-like magnetic field in the exterior spacetime close to the surface of a slowly rotating neutron star described by the metric

$$ds^2 = -N^2 c^2 dt^2 + N^{-2} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 - 2\omega r^2 \sin^2 \theta dt d\phi$$

is given by the expressions

$$\bar{B}_r = B_0 \frac{f(\bar{r})}{f(1)} \bar{r}^{-3} \cos \theta,$$

$$\bar{B}_\theta = \frac{1}{2} B_0 N \left[ -2 \frac{f(\bar{r})}{f(1)} + \frac{3}{(1 - \varepsilon/\bar{r}) f(1)} \right] \bar{r}^{-3} \sin \theta.$$  

Here the spherical coordinates $(r, \theta, \phi)$ are used with the polar axis oriented along the magnetic moment of the pulsar, $\bar{r} = r/R$, $R$ is the radius of the neutron star, $B_0 = 2\mu/R^3$ is the value of the magnetic field at the pole, $N = (1 - 2GM/rc^2)^{1/2}$ is the lapse function of the metric, $G$ is the gravitational constant, $M$ is the mass of the star, $\omega$ is the frequency of dragging of inertial frames, $\varepsilon = 2GM/Rc^2$ is the compactness parameter, while the function $f(\bar{r})$ is given by

$$f(\bar{r}) = -3 \left( \frac{\bar{r}}{\varepsilon} \right)^3 \left[ \ln \left( 1 - \frac{\varepsilon}{\bar{r}} \right) + \frac{\varepsilon}{\bar{r}} \left( 1 + \frac{\varepsilon}{2\bar{r}} \right) \right].$$

The polar angle of the last open magnetic field line $\Theta$ is equal to

$$\Theta \approx \sin^{-1} \left\{ \left[ \frac{\bar{r} f(1)}{f(\bar{r})} \right]^{1/2} \sin \Theta_0 \right\},$$
where

$$\Theta_0 = \sin^{-1}\left(\frac{R\Omega}{c\sqrt{(1)}}\right)^{1/2}$$  \hspace{1cm} (7)$$

is the polar angle of the last open magnetic field line at the surface of the star.

The scalar potential \(\Phi\) in the polar cap region of the inner pulsar magnetosphere is obtained from the solution of the equation

$$\Phi_{\text{low}} = 12\frac{\Phi_0}{r} \sqrt{1 - \varepsilon k \Theta_0^{\cos \chi} \sum_{i=1}^{\infty} \left[ \exp\left(\frac{k_i(1 - \tilde{r})}{\Theta_0 \sqrt{1 - \varepsilon}}\right) - 1 + \frac{k_i(1 - \tilde{r})}{\Theta_0 \sqrt{1 - \varepsilon}} \right] \frac{j_0(k_i \xi)}{k_i^3 j_1(k_i)} + 6\frac{\Phi_0}{r} \sqrt{1 - \varepsilon k \Theta_0^\chi} \frac{H(1) \delta(1)}{\sin \chi} \sum_{i=1}^{\infty} \left[ \exp\left(\frac{k_i(1 - \tilde{r})}{\Theta_0 \sqrt{1 - \varepsilon}}\right) - 1 + \frac{k_i(1 - \tilde{r})}{\Theta_0 \sqrt{1 - \varepsilon}} \right] \frac{j_0(k_i \xi)}{k_i^3 j_1(k_i)}},$$

while at the distances \(\Theta_0 << \tilde{r} - 1 << c/\Omega r\) one gets

$$\Phi_{\text{high}} = \frac{1}{2} \Phi_0 \Theta_0^\chi \left(1 - \xi^2\right) \cos \chi + \frac{3}{8} \left(\frac{\Phi_0}{\Theta_0^\chi} \right) \frac{H(1) \delta(1)}{\sin \chi} \sum_{i=1}^{\infty} \left[ \exp\left(\frac{k_i(1 - \tilde{r})}{\Theta_0 \sqrt{1 - \varepsilon}}\right) - 1 + \frac{k_i(1 - \tilde{r})}{\Theta_0 \sqrt{1 - \varepsilon}} \right] \frac{j_0(k_i \xi)}{k_i^3 j_1(k_i)} \cos \chi \sin \chi \phi.$$  \hspace{1cm} (9)

Here \(\Phi_0 = \Omega B_0 R^2/c, \kappa \equiv \varepsilon \beta\), while \(\beta = 1/I_0\) is the stellar moment of inertia in units of \(I_0 = M R^2\). The parameter \(\xi = \theta / \Theta\) changes from 0 to 1 inside the polar cap region, \(J_\nu\) is the Bessel function of order \(\nu\), \(k_i\) and \(\tilde{k}_i\) are the positive zeroes of the Bessel functions \(J_0\) and \(J_1\), arranged in ascending order. Moreover,

$$H(\tilde{r}) = \frac{1}{\tilde{r}} \left(\frac{\varepsilon - \frac{\kappa}{\tilde{r}}}{2}\right)^{1/2} \left(\frac{1 - \frac{3}{2} \frac{\varepsilon}{\tilde{r}} + \frac{\kappa}{\tilde{r}^2}}{2}\right)^{1/2},$$

and \(\delta(\tilde{r}) = d \ln(\Theta(\tilde{r}) H(\tilde{r})) / d \tilde{r}\).

These results allow one to find the plasma drift velocity

$$\vec{v}_D = c \frac{\vec{E} \times \vec{B}}{B^2},$$  \hspace{1cm} (11)

in the polar cap region of the magnetosphere with the electric field \(\vec{E} = -\nabla \Phi\) and the magnetic field \(\vec{B}\). One can easily show that the largest contribution to the azimuthal drift in the corotating frame of the star is due to the term \(-c E_\theta B_r / B^2\) \(\phi\), which, after proper transformations (see subsection 2.2 of van Leeuwen & Timokhin [2012] for the details), leads to the subpulse drift velocity in degrees per period as given by (11). The final expressions for the drift velocity, obtained from (8) and (9) using \(J'_0(x) = -J_1(x)\) and \(J'_1(x) = (J_0(x) - J_2(x))/2\) look like

$$\omega_{D\text{ low}} = \frac{180^\circ}{\xi} ~ 12 \sqrt{1 - \frac{\Theta_0}{\tilde{r}}} \left\{ -2 \kappa \cos \chi \sum_{i=1}^{\infty} \left[ \exp\left(\frac{k_i(1 - \tilde{r})}{\Theta_0 \sqrt{1 - \varepsilon}}\right) - 1 + \frac{k_i(1 - \tilde{r})}{\Theta_0 \sqrt{1 - \varepsilon}} \right] \frac{j_1(k_i \xi)}{k_i^3 j_1(k_i)} H(1) \delta(1) \sin \chi \cos \phi \right\},$$

and

$$\omega_{D\text{ high}} = \frac{180^\circ}{\xi} \left[ -2 \kappa \cos \chi \left(1 - \frac{1}{\tilde{r}^2}\right) \sin \chi \cos \phi \right] \left(\frac{\Theta(\tilde{r}) H(\tilde{r})}{\Theta_0 H(1)} - 1\right) \sin \chi \cos \phi,$$  \hspace{1cm} (12)

(13)

For inclination angles \(\chi < 90^\circ\) (except for the almost orthogonal pulsars) the scalar potential in the polar cap region is positive, has a maximum close to the magnetic axis and goes to zero at the last open magnetic field lines, so that the value of \(\omega_{D}\) is negative almost everywhere. From the point of view of observations it means that the SCLF model predicts negative drift (from larger to smaller longitudes) in case of the outer line-of-sight geometry and positive drift in case of the inner line-of-sight geometry. In the rest of our work we claim that the velocities (12) and (13) represent the true drift velocities of whatever features are responsible for the subpulses in a specific portion of the magnetosphere [see also van Leeuwen & Timokhin [2012]]. Moreover, the expressions (12) and (13) predict the longitude dependent (not constant) apparent drift velocity of the subpulses along any observer’s line of sight, unless the inclination angle of the pulsar is exactly zero and the line of sight is exactly concentric with the magnetic field axis. In section 5 we will use this fact to explain the longitude dependence of the subpulse separation in case of two individual pulsars.
Tables 1-3. Coherent drifters from the catalog of Weltevrede, Edwards & Stappers (2006) for which the inclination angle $\chi$ is known from previous studies.

\footnotesize

| Pulsar name   | $P$ (s) | $P_2$ (21 cm) | $P_3$ (21 cm) | $P_2$ (92 cm) | $P_3$ (92 cm) | $\chi$ (°) | $\beta$ (°) |
|--------------|---------|---------------|---------------|---------------|---------------|------------|-------------|
| B0148-06     | 1.4647  | $-12.5 \times 10^{-16}$ | $14.2 \pm 0.2$ | $-14.5 \pm 0.5$ | $14.4 \pm 0.1$ | 14.5        | 1.9$^a$, 2.1$^b$ |
| B0149-16     | 0.3827  | $-9.4 \pm 12$    | $5.8 \pm 0.5$  | $-13.2 \pm 0.4$ | $5.7 \pm 0.2$  | 84          | 1.9$^a$, 1.9$^b$ |
| B0230-39     | 3.0321  | $-18.4 \pm 3$    | $8.4 \pm 0.1$  | $6.4 \pm 0.3$   | $8.46 \pm 0.01$| 69          | 2.3$^a$     |
| B0621-04     | 1.0391  | $25.16 \pm 14$   | $20.655 \pm 0.001$ | $-$           | $-$           | 32          | 0$^a$       |
| B0809+74     | 1.2922  | $-16.1 \pm 11$   | $11.1 \pm 0.1$ | $-13.2 \pm 0.1$ | $11.12 \pm 0.01$| 9           | 4.5$^a$     |
| B0818-13     | 1.2381  | $-6.6 \pm 2.6$   | $4.7 \pm 0.2$  | $-5.1 \pm 0.1$  | $4.74 \pm 0.01$| 15.5        | 5.1$^a$, 2$^b$ |
| B1702-19     | 0.2990  | $-16.7 \pm 0.7$  | $11.0 \pm 0.4$ | $-90.5 \pm 0.4$ | $10.8 \pm 0.2$ | 85          | $-4.10^b$, 4.1$^b$ |
| B1717-29     | 0.6204  | $-9.6 \pm 0.6$   | $2.45 \pm 0.02$| $-10.9 \pm 0.4$ | $2.46 \pm 0.001$| 28.9        | 4.6$^b$     |
| B1844-04     | 0.5978  | $5.2 \times 10^{-14}$ | $80 \pm 10$    | $-26 \pm 4$    | $3 \pm 0.1$   | 36          | $1.10^a$, 1.1$^b$ |
| B2045-16     | 1.9616  | $17 \pm 1.8$    | $3.2 \pm 0.1$  | $-26 \pm 4$    | $3 \pm 0.1$   | 36          | $1.10^a$, 1.1$^b$ |
| B2303-30     | 1.5759  | $15 \pm 2$      | $2.1 \pm 0.1$  | $10.6 \pm 0.8$ | $2.06 \pm 0.02$| 20.5        | 4.5$^a$     |
| B2310-42     | 0.3494  | $60 \pm 20$     | $2.1 \pm 0.1$  | $13 \pm 6$     | $2.1 \pm 0.05$| 56          | 6.8$^a$     |
| B2319+60     | 2.2565  | $7 \times 10^{-15}$ | $7.7 \pm 0.4$  | $80 \pm 30$    | $5 \pm 3$    | 18          | $2.20^a$, 2.3$^b$ |

4 COMPARISON WITH THE OBSERVED VELOCITIES OF THE DRIFTING SUBPULSES

Equations (12) and (13) give the subpulse drift velocity within the SCLF model. In order to check whether these expressions predict numbers in agreement with observations, we have used the data from the catalog of Weltevrede, Edwards & Stappers (2006) and Weltevrede, Stappers & Edwards (2007).

These authors present the results of the observations of 187 pulsars in the northern hemisphere at wavelengths of 21 cm and 92 cm. Pulsars revealing the drifting subpulses phenomenon are divided into three classes, depending on the character of their Two-Dimensional Fluctuation Spectrum (2DFS). Coherent drifters (marked as Coh) have narrow pronounced feature in their 2DFS spectra, meaning that $P_3$ has a stable value through the observations. Diffuse drifters of the classes Dif and Dif* have a broader feature in 2DFS spectra, which for Dif pulsars is clearly separated from the alias borders of the spectra ($P/P_3 = 0$ and $P/P_3 = 0.5$), while for Dif* pulsars is not (see Weltevrede, Edwards & Stappers, 2006 for more details and examples). For our purposes we considered the coherent drifters from Tab. 2 of Weltevrede, Edwards & Stappers (2006) (corresponding to the observations at 21 cm), selecting only those with known inclination angle $\chi$. The values of the inclination angles were taken from Rankin (1993) and, if absent, from Lyne & Manchester (1988). The resulting sample is reported in Table 1. When two values of the periods $P_2$ and $P_3$ were given in Weltevrede, Edwards & Stappers (2006), we chose the first one.

We assumed all pulsars to have the typical numbers for compactness $e = 0.4$, $\kappa = 0.15$, and stellar radius $R = 10^6$ cm. A clear picture of drifting subpulses is observed when the line of sight of the observer grazes the emission cone, so that it is reasonable to take $\xi = 0.9$. One may also notice that the second term in (12) and (13), containing the dependence on $\phi$, is smaller than the first term (it depends on a higher degree of the small angle $\Theta_0$) and plays a role mostly for the pulsars with large inclination angles. Hence, for the purposes of this subsection we fix $\phi = \pi$. Under these assumptions, the drift velocities (12) and (13) for each individual pulsar depend only on the radial coordinate $\vec{r}$. So, by solving numerically the equation

$$\omega_D \text{low/high} = \frac{P_2}{P_3}$$

for each pulsar of Table 1 we can find the altitude $r_0 - 1$ of the plasma features that are responsible of the subpulses. When solving the equation (14) in the low altitude approximation we took the first 30 terms of the expansion (6), which reduces the error to less than one percent.

In Table 1, we have pulsars with both positive and negative subpulse drift velocities, and no preferred direction of the drift (sign of $P_2$) was found in Weltevrede, Edwards & Stappers (2006) and Weltevrede, Stappers & Edwards (2007). For comparison, we report in the table also the values of the impact angle $\beta$, taken from Rankin (1993) and Lyne & Manchester (1988), which is the angle of the closest approach between the magnetic axis and the line of sight. As we already mentioned, the SCLF model predicts negative drift for the outer line of sight (positive $\beta$) and positive drift for the inner line of sight (negative $\beta$). However, from Table 1, we don't see a correlation between the signs of $\beta$ and $P_2$. According to the vacuum/partially screened gap model, the discrepancy between the predicted and the observed direction of the drift is usually explained in terms of aliasing (Gupta et al. 2004), but an alternative explanation is possible. Expressions (12) and (13), depending on the inclination angle $\chi$, give negative drift velocities for pulsars with inclination angle in a range from zero to almost $90^\circ$ and positive velocities for the inclination angles from $\sim 90^\circ$ to $\pi$ (pulsars with inclination angles close to $90^\circ$ need to be considered separately). This suggests that pulsars with positive subpulse drift velocities may have $\vec{\Omega} \cdot \vec{B} < 0$, while those with negative velocities have $\vec{\Omega} \cdot \vec{B} > 0$ (provided that the line of sight geometry is outer in all cases).

One may assume that in reality any of the three proposed explanations for the visible direction of the drift (aliasing, line of sight, inclination angle) may be applicable depending on the individual pulsar properties. There are pulsars showing different sign of the drift velocity in different modes, or in the same mode, like B0826-34 or J0815+09, which obviously cannot be explained by the change of the inclination angle. However, keeping this in mind
values of the altitude above the surface of the star. We can perform the following analysis. In case when the subpulse drift velocity of the pulsar is positive, we can use the inclination angle $\chi$ reported in Table 1 for the period $T$ to solve the equation $\dot{r} = \chi$ for the radial coordinate $r$. The pulsar B2045-16 in Table 1 breaks this assumption, having opposite sense of the subpulse drift for the observations at 21 cm and 92 cm. At least six more pulsars among those reported by Weltevrede, Edwards & Stappers (2006) and Weltevrede, Stappers & Edwards (2007) show a similar phenomenology. However, all of them are diffuse drifters (class Diff*) at 92 cm (including B2045-16 itself), suggesting that aliasing is very likely to occur for them. Changing the inclination angle from $\chi$ to $\pi - \chi$ in our calculations for the majority of pulsars (apart from the most orthogonal ones, where the term with $\sin \chi$ in expressions (12), (13) starts to play a role) is essentially analogous to considering the inner line of view instead of the outer one. If we simply put negative sign in front of all drift velocities from Table 1 we get the same values of $\dot{r}_0$, solving Eq. (14).

The results of our analysis are schematically represented in Fig. 1. The red points represent the values of the altitude above the surface of the star in units of stellar radii ($\bar{r}$) obtained from Table 1. No solution is found for the pulsars of Table 1. Red points correspond to the observing wavelength at 21 cm, green points correspond to the observing wavelength at 92 cm. The obtained altitudes are compared to the PFF altitudes from Hibschman & Arons (2001) (blue shadowed regions) and Harding & Muslimov (2001, 2002) (blue points). Black stars show the angular radii of the polar cap at $\Theta_0$. Special cases are indicated as described in the text. The figure does not report the two sources B1702-19 and B2045-16. The first one does not admit a solution of Eq. (14) for any radial coordinate, while the second one has opposite sense of the subpulse drift for the observations at 21 cm and 92 cm.

The expressions for the subpulse drift velocity (12) and (13) derived in the framework of the SCLF model naturally contain a dependence on $\xi$ and $\phi$, and they predict different velocities for different regions of the polar cap, in contrast to the estimations of the vacuum gap model. In this subsection we attempt to exploit these additional degrees of freedom to explain the variability of the subpulse velocities along the pulse longitude in case of two specific pulsars, i.e., PSR B0826-34 and PSR B0818-41. Although not included in the catalog of Weltevrede, Edwards & Stappers (2006), both of them have been repeatedly investigated at several observing frequencies, and since they have wide profiles allowing to track several subpulse drift bands at a time, they can be regarded as ideal test cases.

5 DISCUSSION OF SPECIFIC SOURCES

The expressions for the subpulse drift velocity (12) and (13) derived in the framework of the SCLF model naturally contain a dependence on $\xi$ and $\phi$, and they predict different velocities for different regions of the polar cap, in contrast to the estimations of the vacuum gap model. In this subsection we attempt to exploit these additional degrees of freedom to explain the variability of the subpulse velocities along the pulse longitude in case of two specific pulsars, i.e., PSR B0826-34 and PSR B0818-41. Although not included in the catalog of Weltevrede, Edwards & Stappers (2006), both of them have been repeatedly investigated at several observing frequencies, and since they have wide profiles allowing to track several subpulse drift bands at a time, they can be regarded as ideal test cases.

5.1 PSR B0826-34

5.1.1 Basic parameters

The pulsar B0826-34, with spin $P = 1.8489$ s and $\dot{P} = 1.0 \times 10^{-15}$ s, has an unusually wide profile, extending through the whole pulse period. The pulsar emits in its strong mode for 30% of the time. For the rest of the time, the pulsar stays in the weak mode, with an average intensity of emission which is $\sim 2\%$ of the emission of the strong mode (Esamdin et al. 2005; Serylak et al. 2011; Esamdin et al. 2012). Because of its weakness, the very existence of the weak mode was confirmed only very recently and for a long time.
time it was thought to be a null pulsar (Durdin et al. 1979; Biggs et al. 1985) [Bhattacharyya, Gupta & Gil 2008].

The average pulse profile of B0826-34 consists of the main pulse (MP) and the interpulse (IP), separated by regions of weaker emission. The intensity of the MP is much larger than the intensity of the IP at the frequencies 318 MHz and 606 MHz, while at frequencies larger than ~1 GHz the IP starts to dominate. The MP itself has a double peaked structure with a separation between the peaks decreasing at higher frequencies, following the common trend described by the radius-to-frequency mapping model of Kijak & Gil (2003). A detailed description of the average profile evolution with frequency may be found in Gupta et al. (2004), Bhattacharyya, Gupta & Gil (2008).

Additional relevant physical parameters are those related to the observing geometry of PSR B0826-34, i.e., the values of the inclination angle $\chi$ and of the impact angle $\beta$. Usually the values of these angles are determined by fitting the polarization profile of the pulsar. According to the “rotating vector model” of Radhakrishnan & Cooke (1969), the polarization angle of the pulsar radio emission is equal to

$$\phi_{PA} = \tan^{-1}\left(\frac{\sin\chi \sin l}{\sin(\chi + \beta) \cos \chi - \cos(l + \beta) \sin \chi \cos l}\right)$$

(15)

where $l$ is the pulse longitude, related to the azimuthal coordinate $\phi$ by means of standard theorems of spherical geometry (Gupta et al. 2004) as

$$\sin l = \sin\phi \sin(\Theta) / \sin(\chi + \beta).$$

However, in many cases this method does not give a unique value for the inclination and for the impact angles, but rather a wide range of possible combinations (Miller & Hamilton 1993). For example, early estimations of Biggs et al. (1985) for PSR B0826-34 based on the polarization measurements suggested a large range for $\chi$ and $\beta$ with the best fit of $\chi = 53^\circ \pm 2^\circ$ and $\beta + \chi = 75^\circ \pm 3^\circ$, a fact which does not agree with the large width of the profile. In Gupta et al. (2004), these angles were estimated from the polarization information together with the frequency evolution of the profile and we found to lie in the range $1.5^\circ \leq \chi \leq 5.0^\circ$ and $0.6^\circ \leq \beta \leq 2.0^\circ$.

5.1.2 Subpulse phenomenology

The drifting subpulses are seen almost along the whole range of longitudes, showing from 5-6 up to 9 visible tracks at a time. The character of the observed subpulse drift is irregular, the apparent velocity reveals an oscillatory behavior, changing sign with a periodicity of the order of tens to several hundred periods of the pulsar. The values of the apparent subpulse drift velocities were measured in a number of papers for different observing frequencies (Gupta et al. 2004; Biggs et al. 1985; Esamdin et al. 2005; van Leeuwen & Timokhin 2012) and are reported in Table 2. In all cases the declared velocity range is not symmetric with respect to zero, having a positive average velocity (see Figs. 5 and 6 of Esamdin et al. 2005) for an example of a positive average drift. This phenomenon suggests the interpretation that the average drift velocity reflects some “undisturbed” characteristic of the pulsar, while the oscillatory change of the velocity is determined by some sort of perturbation (see the discussion at the end of this subsection for possible explanations).

Another interesting property of PSR B0826-34 is the longitude dependence of the period $P$, Esamdin et al. (2005) reports the values for $P_2$ between 26.8$^s$ and 28$^s$ in the MP region (average 27.5$^s$) and between 19$^s$ and 23.5$^s$ in the IP region (average 22.2$^s$). In order to explain this behaviour the authors proposed a model of spark carousel consisting of two rings of 13 sparks each, with the separation between the sparks in the outer ring (responsible for the MP) 27.5$^s$/22.2$^s$ ≈ 1.2 larger than in the inner ring (responsible for the IP). In Gupta et al. (2004) $P_2$ was found to vary between 21.5$^s$ and 27$^s$ with the mean value of 24.9$^s$. As a possible explanation of the observed phenomena the authors proposed a scheme where the ring of sparks is centered not around the dipolar axis of the pulsar magnetic field, but around the “local magnetic pole”, shifted with respect to the global dipole one. This agrees well with the models suggesting that the pulsar magnetic field has a multipole structure near the surface of the neutron star (Gil & Sendyk 2000), which arises due to the dynamo mechanism in the newborn stars (Urpin & Gil 2004) or, more probably, due to the Hall drift (Geppert, Rhenhardt & Gil 2003; Geppert, Gil & Melikidze 2013).

5.1.3 Analysis of the subpulse drift

We start our analysis of the subpulse drift by estimating the altitude above the surface of the star, which would correspond to the average subpulse drift velocities of PSR B0826-34 [cf. Sec. 4.]. Since the average drift velocities are positive at all the observing frequencies, while our model predicts negative values of the velocity for small angles $\chi$ and outer line of sight, we are lead to infer that PSR B0826-34 is anti-parallel, with $\chi$ close to 180$^\circ$. Assuming for the moment $\chi = 180^\circ$ and $\xi = 0.5$, we have computed from Eq. (14) the altitudes corresponding to the values of the average drift velocities and we have reported them in the last column of Table 2. The results depend very weakly on the chosen $\xi$ and $\chi$, provided the latter is close to 180$^\circ$. For comparison, when the temperature of the polar cap is $T = 3 \times 10^6 K$, the altitude of the PFF $h^{HA}$ lies in the range 0.0335 – 0.036 for the different values of $f_P$ and $h^{HM} = 0.062$ (for the definitions of $h^{HA}$ and $h^{HM}$ see the Appendix). On the contrary, when the temperature is $T = 4 \times 10^6 K$, the corresponding values for the $h^{HA}$ are 0.0258 – 0.029, with the same $h^{HM}$. Hence, the values of $R$ = 1 that we have obtained lie within the corresponding range of values deduced through the PFF approach, and they are typically closer to its lower boundary.

A closer look at the resulting $\rho_0$ = 1 shows that they satisfy the frequency scaling suggested by the radius-to-frequency mapping, i.e., $\rho_0 \sim \nu^{-0.26 \pm 0.09}$ (Kijak & Gil 2003), where $\rho_0$ is the height of the radio emission generation and $\nu$ is the observed frequency. To demonstrate it, we have fitted the four data points of Table 2 by the function $A\nu^{-\beta}$, getting $A = 0.026$. In Fig. 5 we plot the four considered data points with the error bars calculated under the assumption that the accuracy of the measurements of the apparent subpulse drift velocity is of the order of $\sim 0.05^s/P$ together with the obtained fit (solid blue line).
Table 2. Subpulse drift velocity ranges of PSR B0826-34 measured at different observing frequencies and the altitudes corresponding to the average drift velocities within the SCLF model.

| Observing frequency (MHz) | Measured drift velocity (°/P) | Reference | Average drift velocity (°/P) | $r_0 - 1$ |
|--------------------------|-------------------------------|-----------|-----------------------------|---------|
| 318                      | $-0.8 \pm 1.9$               | Gupta et al. (2004) | 0.55                        | 0.00662 |
| 645                      | $-1.5 \div 2.1$              | Biggs et al. (1985) | 0.3                         | 0.00465 |
| 1374                     | $-3.2 \div 3.6$              | Esamdin et al. (2005) | 0.2                         | 0.00372 |
| 1374                     | $-1 \div 1.5$                | van Leeuwen & Timokhin (2012) | 0.25                        | 0.0042  |

The two dashed lines correspond to the fits $A\nu -0.26+0.09$, with $A_1 = 0.014$, and $A_2 \nu -0.26-0.09$ with $A_2 = 0.046$. As we already mentioned, it is commonly accepted that the generation of radio emission takes place in the upper region of the magnetosphere at altitudes $\approx (10 - 60) R$ (Kramer et al. 1997; Kijak 2001; Kijak & Gil 2003). Our results tend to confirm the existence of a radius-to-frequency scaling, although the lack of more robust observational data does not allow us to draw stronger conclusions.

Fig. 2 is devoted to the illustration of the geometry of PSR B0826-34. In the upper panel we show a sketch of the polar cap, whose boundary is represented with a black circle. At the altitude $r_0 = 0.0044$, the polar cap has an angular radius $\Theta \sim 0.5^\circ$ [cf. Eq. (1)]. It should be stressed that different observations of the angular size of the polar cap do not provide consistent conclusions. According to Lyne & Manchester (1988), for instance, $\Theta \sim 13^\circ P^{-1/3}/2$, which, for PSR B0826-34, gives $\Theta \approx 5.3^\circ$, i.e., an order of magnitude larger compared to what we have found. On the contrary, the angular size of the polar cap deduced from X-ray observations is much smaller and close to the values that we have obtained through Eq. (7).

Another relevant quantity is the trajectory of the observer’s line of sight, which is given by Manchester & Taylor (1977).

$$\xi = \frac{1}{\Theta(\vec{\nu})} \sin^{-1} \left[ \frac{\cos(\chi + \beta) \cos \chi - \sin \chi \cos \phi \sqrt{\sin^2(\chi + \beta) - \sin^2 \phi \sin^2 \chi}}{1 - \sin^2 \chi \sin^2 \phi} \right],$$

where $0 \leq \phi < 2\pi$, is represented with a green line in the upper panel of Fig. 2. The range of the coordinate $\xi$ along the line of sight approximately coincides with that estimated in Gupta et al. (2003) as $0.25 \div 0.85$. In agreement with the tiny size of the polar cap, we have chosen the inclination angle and the impact angle as $\chi = 180^\circ + 0.15^\circ$ and $\beta = 0.09^\circ$. The resulting ratio is consistent with the available observational data, which uses the relation $\chi/\beta \approx \sin \chi/\sin \beta = (d\psi/d\phi)_{max}$, where $(d\psi/d\phi)_{max}$ is the value of the steepest gradient of the polarization angle curve, estimated to be $2.0^\circ \pm 0.5^\circ/\nu$ in Gupta et al. (2004) and $1.7^\circ/\nu$ in Lyne & Manchester (1988). The lower panel of Fig. 3 reports the polarization profile computed through Eq. (13). It should be compared with Fig. 6d of Biggs et al. (1985) and with the upper panel of Fig. 1 of Gupta et al. (2004).

An additional quantity that our model can potentially reproduce is the ratio between the velocities at $\phi = 0$ and $\phi = \pi$ along the line of sight, which, for the considered geometry, is equal to 1.22. This value is very close to the observed ratio of the subpulse separation $P_2$ in the MP to the subpulse separation $P_2$ in the IP, namely $27.5/22.2 \approx 1.239$ (Esmadin et al. 2005). This suggests us that $P_2$ is proportional to the velocity $\omega$ at any given point of the polar cap. Intuitively, if one imagines the carousel of sparks (or any other feature responsible for the subpulse phenomena) moving around the polar cap with the longitude dependent velocity, it seems plausible that in the regions with lower velocity the subpulse tracks will tend to look closer, while in the regions with larger velocity they will look farther away from each other. Thus, in our model we associate the region around $\phi = 0$ with the MP, while the region around $\phi = \pi$ with the IP. Concerning the value of $P_3$,

$^6$ This makes our picture for the observed geometry a bit different from Fig. 10 of Esmadin et al. (2005), where the angle $\beta$ is apparently larger than $\chi$.

$^7$ Note that the plots of the polarization angle and of apparent drift velocity in this subsection are shifted in longitude in order to make them easier comparable with the corresponding plots in the literature, associating zero of the pulse longitude with the bridge region before the IP. In our analysis the zero of the azimuthal angle and of the pulse longitude is instead associated with the peak of the MP. The shift is taken to be $105^\circ$ to match the distance between the second zero of the position angle curve and the end of the pulse from the top panel of Fig. 1 of Gupta et al. (2004).
Polarization angle

\[ H \text{deg} \]

\[ L \text{w proj} \]

\[ -0.4 \quad 0.2 \quad 0.4 \quad -0.4 \]

\[ 0.48 \quad 0.50 \quad 0.52 \quad 0.54 \quad 0.56 \quad 0.58 \quad 0.60 \quad 0.62 \]

\[ 40 \quad 40 \]

\[ 0 \quad 0.2 \quad 0.4 \]

\[ \text{cap} \]

\[ (12) \]

Projected on the trajectory of the line of sight across the polar are reported in Fig. 4. The solid blue curve shows the drift velocity characteristic property of the individual pulsar.

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Figure 3. Upper panel: observing geometry of PSR B0826-34 (the coordinate axes show the values of the angular coordinate \( \theta \) in degrees). The black circle indicates the polar cap, while the green circle indicates the trajectory of the line of sight of the observer. Lower panel: Polarization angle as a function of longitude in our model. The plot is shifted in longitude for easier comparison with previous results from the literature (see discussion in the text).

which, we recall, also enters the expression for the apparent drift velocity, we could not find it in the literature for PSR B0826-34, possibly because the subpulse tracks are irregular. However, from the observations of other pulsars we know that the values of \( P_3 \) are essentially the same at different observing frequencies for a given pulsar (Weltevrede, Edwards & Stappers 2006; Weltevrede, Stappers & Edwards 2007; Bhattacharyya, Gupta & Gil 2009), suggesting that this subpulse parameter is more “stable”. We therefore argue that the longitude dependence of \( P_2 \) is explained in terms of the longitude dependence of the drift velocity of the features that generate the subpulse, while \( P_3 \) could represent a specific characteristic property of the individual pulsar.

As a further step, we have tried to account for the longitude dependence of \( P_2 \) reported in (Gupta et al. 2004) and our results are reported in Fig. 4. The solid blue curve shows the drift velocity projected on the trajectory of the line of sight across the polar cap

\[ \omega_{\text{proj}} = \frac{\omega}{\sqrt{1 + \left( \frac{d \phi}{d \xi} \right)^2}} \]  

(18)

as a function of the pulse longitude \( \xi \). The value of \( \bar{r} \) taken for this plot corresponds to the average drift velocity 0.55"/\( P \), as from Gupta et al. (2004). The grey and black sets of data points have been obtained in the following way. We first started from the observed values of \( P_2 \) given in the top panel of Fig. 8 of Gupta et al. (2004) along the pulse profile and shifted them for 10\( ^{\circ} \) to bring in correspondence with our longitude scale. Then, since the value of \( P_3 \) is not available for PSR B0826-34, we divided the data values (including the error bars) by a number (44 in this specific case) which brings the data points and the blue curve in better visual overlap, and we plotted them in grey color. The black points are the same as the grey ones, but shifted by 50\( ^{\circ} \) in longitude. Although the value of this shift is also chosen “ad hoc” and not the result of a fitting procedure, there is a positive uncertainty in our choice of the origin of the longitude (see the footnote 7), as well as a possible longitude shift between the 318MHz data of Gupta et al. (2004) and the 1520MHz data of Esamdin et al. (2005), that we have chosen as a reference. In spite of the observational uncertainties, the correspondence that we have found between the data points and the analytical curve is very promising and needs further investigation and comparison with more data.

Finally, as already mentioned before, the measured subpulse drift velocity of PSR B0826-34 reveals an irregular behavior on the timescales of tens to hundreds of period, for which a firm explanation is still lacking. Gupta et al. (2004) explained these variations within the partially screened gap model by invoking small fluctuations of the polar cap temperature around the mean value\(^8\) (Gil, Melikidze & Geppert 2003). On the contrary, van Leeuwen & Timokhin (2012) argued that the potential drop in the polar cap region may be determined not only by the local physical conditions, but by the global structure of the magnetosphere (Li, Spitkovsky & Tcheikovskoy 2012; Kalapotharakos et al. 2012; Timokhin 2010a,b). As a result, the long term changes of the observed drift rate may be related to the evolution of the magnetospheric current.

\( ^8 \) The key assumption of this interpretation is that the visible subpulse velocity is in fact the aliased value of the true (higher) one, so that the periodic change of the apparent drift direction corresponds to the slowing down and speeding up of the intrinsic drift rate with respect to its average value.

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Figure 4. Comparison of the variation of the plasma drift velocity along the pulse longitude (solid blue curve) with the corresponding variation of the period \( P_2 \) of PSR B0826-34, measured in Gupta et al. (2004). In the absence of the measured \( P_2 \) for this pulsar, the values of \( P_2 \) from the upper panel of the Fig. 8 of Gupta et al. (2004) are divided on 44 (grey data points) and shifted on 50\( ^{\circ} \) (black points), to bring them in better visual correspondence (these parameters are not a result of fit, but estimated by eye).
density distribution, for example, due to switches between metastable magnetosphere configurations [Timokhin 2010a].

The alternative explanation that we propose is that the observed variations are related to stellar oscillations. In our preceding research [Morozova, Ahmedov & Zanotti 2010; Zanotti, Morozova & Ahmedov 2012] we have studied the influence of the non-radial stellar oscillations on the scalar potential of the polar cap region of the magnetosphere. The oscillation velocity at the stellar surface modulates the linear velocity of the pulsar rotation, introducing a new term in the charge density, in the scalar potential and in the accelerating electric field above the surface of the star. We also showed that oscillations may increase the electromagnetic energy losses of the pulsar, causing its migration above the death-line in the $P - \dot{P}$ diagram. Taking into account that PSR B0826-34 is located relatively close to the death-line ($\tau = 3 \times 10^7$ yr, $B_d = 1.4 \times 10^{13}$G) and that it is very intermittent, staying in the ON state for only 30% of the time, it is likely that this pulsar is visible only when it oscillates, which may also determine the character of variation of the observed subpulse velocity.

5.2 PSR B0818-41

5.2.1 Basic parameters

The pulsar B0818-41, with the main parameters $P = 0.545$, $\tau = 4.57 \times 10^8$ yr, $B_d = 1.03 \times 10^{13}$G, was discovered during the second pulsar survey [Manchester et al. 1978; Hobbs et al. 2004]. The width of the average pulse profile of PSR B0818-41 is close to 180° with a pronounced subpulse drift along a wide range of longitudes. The typical subpulse drift pattern (see Fig. 2 of Bhattacharyya et al. 2007) at the frequency of 325 MHz consists of an inner region with slower apparent drift velocity, surrounded by an outer region with larger intensity of subpulses and steeper drift. Multiperiod observations of Bhattacharyya, Gupta & Gil (2009) at 157, 244, 325, 610 and 1060 MHz show that at lower frequencies the subpulses become weaker and may be seen only in the outer regions at 244 MHz and only in the trailing outer region at 157 MHz. The observing geometry, i.e., the values of the inclination angle and of the impact angle, is not uniquely determined for this pulsar and the polarization profile admits several interpretations. However, based on the average polarization behaviour, Qiao et al. [1995] concluded that the inclination angle of PSR B0818-41 should be small. Unique nulling properties of PSR B0818-41 are studied in Bhattacharyya, Gupta & Gil (2010).

5.2.2 Subpulse phenomenology

The value of $P_3 = 18.3 \pm 1.6P$ was found in Bhattacharyya et al. (2007), observing at the frequency 325 MHz, by means of the fluctuation spectrum analysis, and the same value was confirmed later in Bhattacharyya, Gupta & Gil (2009) for all other observing frequencies. In the inner part of the subpulse drift region, where several (typically 3 to 4) subpulse tracks are observed within one pulse, the value of $P_2$ was found from the second peak of the autocorrelation function to be $17.5^\circ \pm 1.5^\circ$ (Bhattacharyya et al. 2007). In the outer regions of the profile the value of $P_2$ is larger (already from the visual inspection of the subpulse tracks) and not easily measurable, because typically no more than one subpulse per pulse is seen in these regions. Estimations for the different observing frequencies and different parts of the profile can be found in Bhattacharyya, Gupta & Gil (2009), while the average value of $P_2$ may be taken around 28° [Bhattacharyya et al. 2007].

As indicated in Gupta et al. (2004), the measured values of the periods $P_2$ and $P_3$ are not necessarily equal to the true intrinsic ones. The value of $P_3$ may be affected by aliasing, which starts to play a role when $P_2 < 2P_3$. The value of $P_2$ is affected by the finite time required for the line of sight to traverse the polar cap as well as by the difference between the longitude $l$ along the pulse (in which we measure $P_2$) and the azimuthal coordinate $\phi$ around the magnetic axis. However, if we assume that there is no aliasing, the correction to the measured value of $P_2$ due to the finite traverse time is given by a factor of $1/\{1 + (P_3/3P_2)\}$ [derived from the equation (5) of Gupta et al. (2004)], which in our case is $0.997$. As long as we are not concerned with the structure of the carousel as a whole and we don’t consider the possibility of aliasing in our calculations, we assume everywhere that the measured values of $P_2$ and $P_3$ are equal to the intrinsic ones.

5.2.3 Analysis of the subpulse drift

As in the previous subsection, we start the analysis from the determination of $\bar{r}$, corresponding to the measured subpulse drift velocity of PSR B0818-41. Choosing for this estimation $\chi = 0$ and $\xi = 0.5$ and using the other known parameters of the pulsar, we get $\bar{r}_0 = 1.011$ for $\omega_D = -0.956/P$ (this value of $\bar{r}_0$ depends very weakly on the chosen $\xi$ and $\chi$, provided the inclination angle is small). For comparison, assuming that the temperature of the polar cap is $T = 3 \times 10^7$K, the altitude of the PFF $h_{\text{HA}}$ lies in the range $0.013 - 0.106$ for the different values of $f_p$ and $h_{\text{HM}} = 0.182$, while for the temperature $T = 4 \times 10^6$K the corresponding values for the $h_{\text{HA}}$ are $0.0097 - 0.0796$ with the same $h_{\text{HM}}$. So, the estimated value for the altitude lies close to the lower boundary obtained through the PFF approach.

Fig. 5 is devoted to the illustration of the geometry of PSR B0818-41, with the inclination angle between the magnetic and the rotational axes $\chi = 0.34^\circ$ and the impact angle $\beta = 0.51^\circ$. The considered geometry corresponds to an outer line of sight, which, within the SCLF model, naturally results in negative value of the subpulse drift velocity. The chosen inclination and impact angles are much smaller than those suggested before in the literature. These small values are required by the fact that the size of the polar cap is very small $\Theta = 0.94^\circ$ at the considered altitude. However, one may notice that the G-2 geometry of Bhattacharyya, Gupta & Gil (2009), which gives the best fit to the polarization angle profile of PSR B0818-41, effectively corresponds to an outer line of sight with $\chi = 180^\circ - 175.4^\circ = 4.6^\circ$ and $\beta = 6.9^\circ$, so that $\sin \beta/\sin \chi = 1.5$. Based on this, we chose the inclination and impact angles of our geometry to satisfy $\beta/\chi = 1.5$ in order to match the polarization profile of the pulsar. The polarization angle, calculated through the “Rotating Vector Model”, is plotted in the lower panel of the Fig. 5 as a function of the longitude and reproduces well the observational data [cf., Fig. 6 of Bhattacharyya, Gupta & Gil (2009)].

We can use the geometry of Fig. 5 to model the observed behavior of the subpulse tracks. Starting from the trailing end of the

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9 The observed periodicity of hundreds of seconds may be reached by core g-modes of the neutron star [McDermott, van Horn & Hansen 1988].

10 The two geometries proposed by Bhattacharyya, Gupta & Gil (2009) have $\chi = 11^\circ$, $\beta = -5.4^\circ$ (G-1) and $\chi = 175.4^\circ$, $\beta = -6.9^\circ$ (G-2).
Figure 5. Upper panel: observing geometry of PSR B0818-41 (the coordinate axes show the values of the angular coordinate $\theta$ in degrees). The black circle indicates the polar cap, while the green circle indicates the trajectory of the line of sight of the observer. Lower panel: Polarization angle as a function of longitude in our model.

profile (because the drift velocity is negative) we evolve the azimuthal coordinate $\phi$ with time using the projected velocity (15). At each time step we calculate the corresponding pulse longitude using relation (16) and add a new subpulse track every $P_s = 18.3P$. The resulting pattern of the subpulse motion for 200 pulses as a function of pulse longitude is shown in Fig. 6 through blue solid lines, which may be directly compared to Fig. 2 of Bhattacharyya et al. (2007). The solid vertical lines indicate the region, where the subpulse drift is actually observed. We estimated its width as $120^\circ$, corresponding to the pulse longitudes $140 - 260^\circ$ of Fig. 2 of Bhattacharyya et al. (2007). Note that the center of the profile there is slightly shifted with respect to $180^\circ$. When compared to the observational data, we find that the SCLF model is able to reproduce the curved subpulse tracks reasonably well.

The pattern of the subpulse tracks obtained with our model is symmetric by construction. However, according to the observations of Bhattacharyya, Gupta & Gil (2009), the drift bands of the trailing outer region of the pulse appear to be steeper than the drift bands of the leading outer region. Although a clean explanation for this effect is still missing, we argue that some degree of asymmetry can be due to the effects of retardation, aberration and refraction of the signal in the outer magnetosphere (Gangadhara & Gupta 2001; Gupta & Gangadhara 2003; Petrova 2000; Weltevrede et al. 2003).

We emphasize that the original spark model of Gil & Sendyk (2000), on the basis of very general arguments, predicts that the polar cap space may be densely filled by equidistant equal-size sparks. On the contrary, the carousel pattern proposed later in Esamdin et al. (2012), Bhattacharyya et al. (2007), and Bhattacharyya, Gupta & Gil (2009) has larger and wider separated sparks in the outer ring in order to explain the observed subpulse behavior (compare Fig. 1 of Gil & Sendyk (2000) with Fig. 10 of Esamdin et al. (2005)). We find it interesting that, within the SCLF model, it is possible to explain the observed curved subpulse tracks by means of the velocity variations only, without breaking the assumption that the features responsible for the subpulses are equal in size and equidistant. This supports the argument, already presented in the previous subsection, according to which the variations of $P_s$ with the pulse longitude, observed for many pulsars, may be completely explained by the variability of the subpulse drift velocity across the polar cap, while the value of $P_s$, which seems to be independent of the pulse longitude and even of the observing frequency, should reflect an intrinsic characteristic property of the individual pulsar.

One interesting observation made in Bhattacharyya, Gupta & Gil (2009) is that the leading and trailing outer regions of the pulse profile maintain a unique phase relationship, with the maximum of the energy in the trailing component being shifted in time by ~9$P$ with respect to the maximum of the energy in the leading component. Based on this observation, the authors propose an elegant solution to the aliasing problem, arguing that the considered shift may not be explained without aliasing and suggesting the model of 20 sparks ring with first-order alias and a true drift velocity of 19.05$P$ as the most plausible description of the system. As an alternative, we propose that the position of the picks of the pulse profile is not strictly determined by the position of the sparks in the outer ring, but modulated by the outer regions of the magnetosphere. One may assume, as it is customary, that the major radio emission mechanism of the pulsar is due the formation of the secondary plasma from the energetic photons emitted by the primary particles. Near the axis this process is negligible due to the large curvature radius of the magnetic field lines, while on the edges of the polar cap region the acceleration potential itself drops to zero. This produces the “hollow cone” distribution of the secondary plasma.
plasma in the magnetosphere above the PFF (see Petrova (2006), Weltevrede et al. (2003), where this model is used for the study of magnetospheric refraction, also Fung, Khechinashvili & Kuijpers (2006)). We therefore believe that such a distribution modulates the emission profile and changes the position of the maxima with respect to the position of the outer ring of sparks. However, this issue is beyond the scope of the present paper and we leave it for a future study.

6 CONCLUSIONS

The phenomena of drifting subpulses are typically explained by resorting to the vacuum or to the partially screened gap models of pulsar magnetospheres. For example, the partially screened gap model allows for the formation of a spark carousel due to discharges of the large potential drop through the inner polar gap and these sparks are thought to be responsible for the appearance of subpulses. However, it has been recently shown by van Leeuwen & Timokhin (2012) that the expression used for the estimation of the subpulse drift velocities, both in the vacuum and in the partially screened gap model, is not accurate enough.

On the other hand, considering the pulsar magnetosphere as a global object, one can propose alternative mechanisms for the formation of distinct emitting features representing subpulses and in this paper we have reconsidered the ability of the Space-Charge Limited Flow model to explain this phenomenology. The SCLF model provides analytical solutions for the scalar potential in the polar cap region of the pulsar magnetosphere in case of free outflow of the charged particles from the surface of the star. Hence, the drift velocity of subpulses along the pulse can be interpreted in terms of the plasma drift velocity, which in turn depends on the gradient of the scalar potential rather than on its absolute value.

After considering a selected sample of sources taken from the catalog of Weltevrede, Edwards & Stappers (2006), we have found the following conclusions:

- the SCLF model predicts the subpulse drift velocities compatible to the observed ones at heights above the surface of the star close to the pair formation front;
- the angular dependence of the plasma drift velocity in the SCLF model provides a natural explanation for the variation of the subpulse separation $P_2$ along the pulse. In particular it may explain the curved subpulse driftbands of PSR B0818–41 and the difference between the values of $P_2$ in the main pulse and interpulse of PSR B0826–34.

These results suggest that the role of the SCLF model in explaining the drifting subpulse phenomena has been underestimated, calling for additional investigations and systematic comparisons with all available observations.

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REFERENCES

Arons J., Scharlemann E. T., 1979, ApJ, 231, 854
Backus P. R., 1981, PhD thesis, University of Massachusetts
Bhattacharyya B., Gupta Y., Gil J., 2008, MNRAS, 383, 1538
Bhattacharyya B., Gupta Y., Gil J., 2009, MNRAS, 398, 1435
Bhattacharyya B., Gupta Y., Gil J., 2010, MNRAS, 408, 407
Bhattacharyya B., Gupta Y., Gil J., Sendyk M., 2007, MNRAS, 377, L10
Biggs J. D., McCulloch P. M., Hamilton P. A., Manchester R. N., Lyne A. G., 1985, MNRAS, 215, 281
Clemens J. C., Rosen R., 2004, ApJ, 609, 340
Drake F. D., Craft H. D., 1968, Nature, 220, 231
Durdin J. M., Large M. I., Little A. G., Manchester R. N., Lyne A. G., Taylor J. H., 1979, MNRAS, 186, 39P
Esamdin A., Abdurixit D., Manchester R. N., Niu H. B., 2012, ApJ, 759, L3
Esamdin A., Lyne A. G., Graham-Smith F., Kramer M., Manchester R. N., Wu X., 2005, MNRAS, 356, 59
Fung P. K., Khechinashvili D., Kuijpers J., 2006, A&A, 445, 779
Gangadharra R. T., Gupta Y., 2001, ApJ, 555, 31
Geppert U., Gil J., Melikidze G., 2013, MNRAS, 435, 3262
Geppert U., Rheinhardt M., Gil J., 2003, A&A, 412, L33
Gil J., Haberl F., Melikidze G., Geppert U., Zhang B., Melikidze, Jr. G., 2008, ApJ, 686, 497
Gil J., Melikidze G., Zhang B., 2007, MNRAS, 376, L67
Gil J., Melikidze G. I., Geppert U., 2003, A&A, 407, 315
Gil J. A., Lyne A. G., 1995, MNRAS, 276, L55
Gil J. A., Sendyk M., 2000, ApJ, 541, 351
Gogoberidze G., Machabeli G. Z., Melrose D. B., Luo Q., 2005, MNRAS, 360, 669
Goldreich P., Julian W. H., 1969, ApJ, 157, 869
Gupta Y., Gangadharra R. T., 2003, ApJ, 584, 418
Gupta Y., Gil J., Kijak J., Sendyk M., 2004, A&A, 426, 229
Harding A. K., Muslimov A. G., 1998, ApJ, 508, 328
Harding A. K., Muslimov A. G., 2001, ApJ, 556, 987
Harding A. K., Muslimov A. G., 2002, ApJ, 568, 862
Hibschman J. A., Arons J., 2001, ApJ, 554, 624
Hobbs G. et al., 2004, MNRAS, 352, 1439
Kalapotharakos C., Kazanas D., Harding A., Contopoulos I., 2012, ApJ, 749, 2
Kargaltsev O., Pavlov G. G., Garmire G. P., 2006, ApJ, 636, 406
Kazbegi A. Z., Machabeli G. Z., Melikidze G. I., 1991, Australian Journal of Physics, 44, 573
Kijak J., 2001, MNRAS, 323, 537
Kijak J., Gil J., 2003, A&A, 397, 969
Kramer M., Xilouris K. M., Jessner A., Lorimer D. R., Wielebinski R., Lyne A. G., 1997, A&A, 322, 846
Kuijpers J. M. E., 2009, in Astrophysics and Space Science Library, Vol. 357, Astrophysics and Space Science Library, Becker W., ed., p. 543
Li J., Spitkovsky A., Tchekhovskoy A., 2012, ApJ, 746, 60
Lyne A. G., Manchester R. N., 1988, MNRAS, 234, 477
Manchester R. N., Lyne A. G., Taylor J. H., Durdin J. M., Large M. I., Little A. G., 1978, MNRAS, 185, 409
Manchester R. N., Taylor J. H., 1977, Pulsars.

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McDermott P. N., van Horn H. M., Hansen C. J., 1988, ApJ, 325, 725
Medin Z., Lai D., 2007, MNRAS, 382, 1833
Melikidze G. I., Gil J. A., Pataraya A. D., 2000, ApJ, 544, 1081
Miller M. C., Hamilton R. J., 1993, ApJ, 411, 298
Morozova V. S., Ahmedov B. J., Zanotti O., 2010, MNRAS, 408, 490
Muslimov A. G., Tsygan A. I., 1992, MNRAS, 255, 61
Pavlov G. G., Kargaltsev O., Wong J. A., Garmire G. P., 2009, ApJ, 691, 458
Petrova S. A., 2000, A&A, 360, 592
Qiao G., Manchester R. N., Lyne A. G., Gould D. M., 2009, ApJ, 701, 119
Rankin J. M., 1986, ApJ, 301, 901
Rankin J. M., 1993, ApIS, 85, 145
Rosen R., Clemens J. C., 2008, ApJ, 680, 671
Rosen R., Demorest P., 2011, ApJ, 728, 156
Ruderman M. A., Sutherland P. G., 1975, ApJ, 196, 51
Scharlemann E. T., Arons J., Fawley W. M., 1978, ApJ, 222, 297
Serylak M., 2011, PhD thesis, University of Amsterdam
Stappers B. W., van den Horn L. J., Edwards R. T., van Leeuwen J., Timokhin A. N., 2012, ApJ, 752, 155
Thome R., 1992, ApJ, 389, 631
Timokhin A. N., 2010b, MNRAS, 408, 2092
Timokhin A. N., 2010a, MNRAS, 408, L41
Timokhin A. N., Arons J., 2001, ApJ, 562, 412
Timokhin A. N., Arons J., 2001, ApJ, 562, 412
Timokhin A. N., Arons J., 2002, ApJ, 577, 412
Timokhin A. N., Arons J., 2002, ApJ, 577, 412
Timokhin A. N., Rybka W., 2003, ApJ, 592, 952
Thorne K. S., 1968, ApJ, 155, 473
Wright G. A. E., 2003, ApJ, 592, 473
Weltevrede P., Stappers B. W., van den Horn L. J., Edwards R. T., 2003, A&A, 424, 473
Xia T., Lyne A. G., 2005, ApJ, 624, L109

APPENDIX A: PAIR FORMATION FRONT

All mechanisms proposed for the generation of radio emission in the pulsar magnetosphere require the presence of an electron-positron plasma. Within the SCLF model, primary particles, extracted from the surface of the star by the rotationally induced electric field, accelerate in the inner magnetosphere and emit high energy photons, which in turn produce electron-positron pairs in the background magnetic field. The three main processes responsible for the emission of photons by the primary particles are curvature radiation (CR), nonresonant inverse Compton scattering (NRICS) and resonant inverse Compton scattering (RICS). Copious pair production in the open field lines region leads to the screening of the accelerating electric field and stops the acceleration of the particles above the so-called pair formation front (PFF).

The determination and even the definition of the PFF is not a trivial task. It was mostly investigated in the framework of the SCLF model by Hibschman & Arons (2001) and Harding & Muslimov (2001, 2002), with slightly different approaches. Hibschman & Arons (2001) define the location of the PFF as the place where the number of pairs created per primary particle is equal to κ, which means that the space charge density is large enough to screen the accelerating component of the electric field. Harding & Muslimov (2001, 2002), instead, locate the PFF front where the first electron-positron pair is produced. In Harding & Muslimov (2001, 2002) it is shown that the full screening of the accelerating electric field is not even possible for many pulsars, while the pair formation front is still formed. The difference between these two approaches affects significantly the inverse Compton scattering, while the results for the curvature radiation are essentially the same.

The expressions for the height of the PFF in units of the stellar radius obtained in Hibschman & Arons (2001) are

\[ h_{\text{CR}}^{\text{HA}} = 0.678 B_{12}^{-5/6} P_{19/12}^{9/4} f_{\rho}^{1/2}, \]  
\[ h_{\text{NRICS}}^{\text{HA}} = 0.119 B_{12}^{1/2} P_{19/4}^{1/4} T_{6}^{1} f_{\rho}^{1/2}, \]  
\[ h_{\text{RICS}}^{\text{HA}} = 12.0 B_{12}^{-7/3} P_{-2/3}^{3/4} T_{6}^{1} f_{\rho}^{1/2}. \]

Here \( B_{12} = B/10^{12} \) and \( T_{6} = T/10^{6} \). The quantity \( f_{\rho} \), which describes the curvature of the field lines in the considered regions, changes from \( f_{\rho} = 0.011 P^{-1/2} \) for the multipolar field with radius of curvature equal to the stellar radius, to \( f_{\rho} = 1 \) for the dipolar field. In Fig. 1 these two cases correspond to the lower and upper boundaries of the blue shaded regions.

The expressions for the height of the PFF in units of the stellar radius obtained in Harding & Muslimov (2001, 2002) are

\[ h_{\text{CR}}^{\text{HM}} \approx 0.03 \begin{cases} 1.9 P_{0.1}^{1/4} B_{12}^{-3/4} f_{\rho}^{9/4} & P_{0.1} < 0.5 B_{12} \\ 3.0 P_{0.1}^{1/3} B_{12}^{-2} f_{\rho}^{9/4} & P_{0.1} > 0.4 B_{12} \end{cases}, \]  
\[ h_{\text{NRICS}}^{\text{HM}} \approx 0.01 \begin{cases} 3(P/B_{12})^{2/3} f_{\rho} & P \lessapprox 0.4 B_{12}^{4/7} \\ 4P^{5/4} B_{12} f_{\rho} & P \gtrapprox 0.4 B_{12}^{4/7} \end{cases}, \]  
\[ h_{\text{RICS}}^{\text{HM}} \approx 0.01 \begin{cases} 7P^{2/3} B_{12} f_{\rho} & P \lessapprox 0.1 P_{12}^{6/7} \\ 17P^{5/4} B_{12}^{3/2} f_{\rho} & P \gtrapprox 0.1 B_{12}^{4/7} \end{cases}. \]

The values for the PFF shown in Fig. 1 are \( \min(h_{\text{CR}}^{\text{HM}}, h_{\text{NRICS}}^{\text{HM}}, h_{\text{RICS}}^{\text{HM}}) \) for the blue points, \( \min(h_{\text{CR}}^{\text{HA}}, h_{\text{NRICS}}^{\text{HA}}, h_{\text{RICS}}^{\text{HA}}) \) with \( f_{\rho} = 0.011 P^{-1/2} \) for the lower boundaries of the blue shaded regions and \( \min(h_{\text{CR}}^{\text{HA}}, h_{\text{NRICS}}^{\text{HA}}, h_{\text{RICS}}^{\text{HA}}) \) with \( f_{\rho} = 1 \) for the upper boundaries of the blue shaded regions.

\[ \text{[1]} \] Here for NRICS we report only the values obtained in the Klein–Nishina regime, as it will dominate for typical pulsar parameters.