Initial state of the QGP from perturbative QCD + saturation

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The production of the initial state of the QGP in very high-energy AA collisions is discussed within the framework of perturbative QCD and saturation. The next-to-leading order computation of the transverse energy of minijets is reviewed. Saturation of parton production, conjectured to occur at a dynamically determinable perturbative scale, leads to estimates of the initial densities. The final state multiplicities are predicted by assuming an isentropic hydrodynamical further evolution. Comparison with RHIC data is shown.

1. INTRODUCTION

Particle production at the early stages, $\tau \sim 1/p_T$, in ultrarelativistic heavy ion collisions is dominated by processes of large transverse momentum scales $p_T$. The partonic multiplicities $N_{AA}(p_T \geq p_0, \sqrt{s}, \Delta y)$ and transverse energies $E_{T}^{AA}(p_0, \sqrt{s}, \Delta y)$ produced in processes above some scale $p_0 \gg \Lambda_{QCD}$ into a rapidity window $\Delta y$ are calculable \cite{1-3} in perturbative QCD (pQCD) through the formalism of collinear factorization. Subsequently, the number and energy densities due to the pQCD quanta can be estimated.

Towards smaller transverse momenta, i.e. later in formation time, the number of produced gluons is expected to grow to the extent that non-linearities start to dominate: a new dynamically generated scale appears, further growth in the number of gluons becomes inhibited, and gluon production “saturates” \cite{1,4-15}. For sufficiently high cms-energies and large nuclei, the saturation scale becomes perturbative, $\sim 1...2$ GeV at RHIC...LHC in central AA collisions of $A \sim 200$ \cite{1,4-15}.

Gluon saturation can be viewed to take place already in the initial wave functions of the colliding nuclei \cite{1,4-15}, and gluon production around the dominant saturation scale describable in terms of classical fields \cite{1,4-15}. As non-perturbative gluon production cannot be computed from truly first principles, one may equally well try to estimate the bulk production of gluons in an AA collision by using the pQCD component alone but by extending the perturbative computation from large-$p_T$ down to a dynamically determined saturation scale \cite{1,4-15}. I shall focus on the latter, the pQCD+final-state saturation approach, in the following. The initial-state saturation and classical gluon fields are discussed by L. McLerran, R. Venugopalan and D. Kharzeev in these proceedings. See also ref. \cite{1} for self-screening in parton production, and \cite{1} for other studies of particle production with saturation.

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In the final-state saturation approach the formation time of the system can be estimated from the saturation scale \( p_{sat} \) as \( \tau_i = 1/p_{sat} \). This makes the computation of the initial number and energy densities possible: identifying the spatial and the momentum rapidities, and focusing on a volume element \( \Delta V_i \approx \pi R_A^2 \frac{1}{\tau_i} \Delta y \) around \( \eta = y = 0 \), we get the average initial densities

\[
n_i(\tau_i, z \approx 0) = \frac{N_{AA}(p_{sat}, \sqrt{s}, \Delta y)}{\pi R_A^2 \tau_i \Delta y}, \quad \epsilon_i(\tau_i, z \approx 0) = \frac{E_{T}^{AA}(p_{sat}, \sqrt{s}, \Delta y)}{\pi R_A^2 \tau_i \Delta y}.
\]  

These densities serve as initial conditions for the further evolution of the system. I shall first discuss how the quantities \( N_{AA} \) and \( E_{T}^{AA} \) are computed in pQCD with the conjecture of saturation, and then return to the multiplicity predictions for RHIC and LHC.

2. MINIJETS AND THEIR TRANSVERSE ENERGY FROM PQCD

In the leading twist approximation the semihard partonic collisions with \( p_T \sim 1...2 \) GeV are independent of each other. For an \( AA \) collision at an impact parameter \( b \) the average number of minijets produced into \( \Delta y \) with \( p_T \geq p_0 \) and the transverse energy carried by them can be computed as

\[
N_{AA}(b, p_0, \sqrt{s}, \Delta y) = T_{AA}(b)\sigma_{pQCD} \langle N \rangle(p_0, \sqrt{s}, \Delta y, A) \quad (2)
\]

\[
E_{T}^{AA}(b, p_0, \sqrt{s}, \Delta y) = T_{AA}(b)\sigma_{pQCD} \langle E_T \rangle(p_0, \sqrt{s}, \Delta y, A). \quad (3)
\]

The nuclear overlap function \( T_{AA}(b) = \int d^2 s T_A(s)T_A(b-s) \) takes care of the collision geometry. With the Woods-Saxon nuclear densities \( T_{AA}(0) \approx A^2/\pi R_A^2 \). The first moment \( \sigma_{pQCD} \langle E_T \rangle \) of the transverse energy distribution and \( \sigma_{pQCD} \langle N \rangle \) of the number distribution of the minijets in \( \Delta y \), are the quantities computable in pQCD.

2.1. Leading order

In leading order (LO), the semihard partonic collisions are \( 2 \rightarrow 2 \) processes. Their inclusive cross section can be obtained through collinear factorization,

\[
\frac{d\sigma^{AA \rightarrow kl+X}}{dp_T^2dy_1dy_2} = \sum_{ij} x_1 f_{i/A}(x, Q)x_2 f_{j/A}(x_2, Q) \sum_{kl} \frac{d\hat{\sigma}^{ij \rightarrow kl}}{dt}
\]

with the momentum fractions \( x_{1,2} = \frac{E_{T}}{\sqrt{s}(e^{\pm y_1} + e^{\pm y_2})} \), the sub-cross sections \( \frac{d\hat{\sigma}^{ij \rightarrow kl}}{dt} \sim \alpha_s^2 \) and the factorization/renormalization scales \( Q \sim p_T \). For the 1...2 GeV minijets at mid-rapidity \( (y_1 \text{ or } y_2 \sim 0) \) at \( \sqrt{s} = 200...5500 \) GeV of interest here, typically \( x \sim 10^{-2}...10^{-4} \). Due to the dominance of gluons at small values of \( x \), the gluons also clearly dominate the production of minijets.

Nuclear effects in Eq. (1) are taken into account through nuclear shadowing of parton distributions: \( f_{i/A}(x, Q) \neq f_{i}(x, Q) \). For this purpose, we apply the EKS98 nuclear modifications which are based on a DGLAP analysis with data from deep inelastic \( IA \) scattering and the Drell-Yan process and sum rules as constraints.

The transverse energy distribution of the minijets with \( p_T \geq p_0 \) and \( y \in \Delta y \) is

\[
\frac{d\sigma}{dE_T}(p_0, \Delta y) = \int dp_T dy_1 dy_2 S_2^{E_T}(p_1, p_2) \frac{1}{2 \int dp_T dy_1 dy_2} \frac{d\sigma}{dp_T dy_1 dy_2}. \quad (5)
\]
The key element above is the “measurement function” for $E_T$ in $\Delta y$,

$$S_2^{E_T}(p_1, p_2) \equiv \delta(E_T - [\epsilon_1 + \epsilon_2]p_T)\Theta(p_T \geq p_0), \quad (6)$$

where $\epsilon_i \equiv \Theta(y_i \in \Delta y)$. This measurement function on one hand keeps track of which partons fall in $\Delta y$ and on the other hand ensures that the collisions are hard enough to allow for a perturbative treatment. The first moment of the $E_T$ distribution becomes then

$$\sigma_{pQCD}(E_T)(p_0, \sqrt{s}, \Delta y) \equiv \int dE_T E_T \frac{d\sigma}{dE_T}_{p_0, \Delta y} = \int_{p_0} d\epsilon_1 \int_{\Delta y} d\epsilon_2 \int_{\Delta y} d\epsilon_3 \int_{\Delta y} d\epsilon_4 \frac{d\sigma}{d\epsilon_1 d\epsilon_2 d\epsilon_3 d\epsilon_4}. \quad (7)$$

Similarly, for the number distribution of the semihard minijets in $\Delta y$, the measurement function is $S_3^N(p_1, p_2) \equiv \delta(N - [\epsilon_1 + \epsilon_2])\Theta(p_T \geq p_0)$, and the first moment

$$\sigma_{pQCD}(N)(p_0, \sqrt{s}, \Delta y) \equiv \int dN \frac{d\sigma}{dN}_{p_0, \Delta y} = \int_{p_0} d\epsilon_1 \int_{\Delta y} d\epsilon_2 \int_{\Delta y} d\epsilon_3 \frac{d\sigma}{d\epsilon_1 d\epsilon_2 d\epsilon_3 d\epsilon_4}. \quad (8)$$

### 2.2. Next-to-leading order

In next-to-leading order (NLO), where $\hat{\sigma} \sim \alpha_s^3$, one must include both virtual corrections to $2 \to 2$ processes and real emissions, i.e. $2 \to 3$ processes. The squared matrix elements for the different subprocesses were computed in $4 - 2\epsilon$ dimensions first by R.K. Ellis and Sexton [17]. The subtraction procedure of getting from the squared matrix elements in $4 - 2\epsilon$ dimensions to the physical (jet) cross sections was introduced by S. Ellis, Kunszt and Soper [18]. A detailed documentation can be found in [19].

For the $2 \to 3$ contributions with massless partons, singularities arise when the momentum of an external leg becomes soft, or when any two external particles become collinear. In $4 - 2\epsilon$ dimensions, these singularities appear as terms $\sim 1/\epsilon$ and $1/\epsilon^2$. Similar terms arise also from the 1-loop graphs (interfered with the LO $2 \to 2$). In the sum of the different contributions, formed for computing a cross section of a physical observable, the terms singular at the limit $\epsilon \to 0$ cancel, provided that one properly defines

- infra-red safe generalization of the measurement function $S_2$ to $S_3$ for the 3-particle final state: $S_3 \to S_2$ when any final state parton becomes collinear with any other parton in the final or initial state, or, when any final state parton becomes soft [18].

- NLO PDFs to absorb one $1/\epsilon$ singularity in the initial state (MS scheme),

- infra-red safe choice of the renormalization and factorization scales.

In the computation of the minijet $E_T$ production in NLO in $pp$ and $AA$ collisions [20,21], we have (see also [22]) adopted the subtraction method [19]. Although the minijet measurement functions differ from those for observable jets, the computational procedure itself is similar to that of inclusive jet production [19], see [21] for details. For e.g. the $2 \to 3$ terms we need to numerically evaluate 6-dimensional integrals with various kinematical cuts.

The infra-red safe measurement functions $S_2^{E_T}$ for the $2 \to 2$ terms and $S_3^{E_T}$ for the $2 \to 3$ terms are designed to answer the following question: How much $E_T$ is carried into $\Delta y$ by minijets produced in partonic collisions where at least an amount $2p_0$ of $E_T$ is produced? They are

$$S_2^{E_T}(p_1^\mu, p_2^\mu) = \delta(E_T - [\epsilon_1 p_{T1} + \epsilon_2 p_{T2}])\Theta(p_{T1} + p_{T2} \geq 2p_0)$$
\[ S^E_T(p^1_t, p^2_t, p^3_t) = \delta(E_T - [\epsilon_1 p_{T1} + \epsilon_2 p_{T2} + \epsilon_3 p_{T3}])\Theta(p_{T1} + p_{T2} + p_{T3} \geq 2p_0) \]  

with \( y_i \) and \( p_{Ti} \) referring to the rapidities and transverse momenta of the final state partons.

The semi-inclusive minijet \( E_T \)-distributions can then be defined in NLO as

\[
\frac{d\sigma}{dE_T(p_0, \Delta y)} = \int d[PS]_2 \frac{d\sigma^{2\rightarrow 2}}{d[PS]_2} S^{E_T}_2(p^1_t, p^2_t) + \int d[PS]_3 \frac{d\sigma^{2\rightarrow 3}}{d[PS]_3} S^{E_T}_3(p^1_t, p^2_t, p^3_t),
\]

where \( d[PS]_2 \) (\( d[PS]_3 \)) is the 2(3)-particle phase space volume element. The first term now includes both \( \sim \alpha_s^2 \) and \( \sim \alpha_s^3 \) contributions. The first \( E_T \) moment becomes now

\[
\sigma_{pQCD}(E_T)(p_0, \sqrt{s}, \Delta y) \equiv \int_{0}^{\sqrt{s}} dE_T \frac{d\sigma}{dE_T(p_0, \Delta y)} = \sigma(E_T)^{2\rightarrow 2}_{\Delta y, p_0} + \sigma(E_T)^{2\rightarrow 3}_{\Delta y, p_0},
\]

where

\[
\sigma(E_T)^{2\rightarrow 2}_{\Delta y, p_0} = \int d[PS]_2 \frac{d\sigma^{2\rightarrow 2}}{d[PS]_2} [\epsilon_1 + \epsilon_2] p_{T2}\Theta(p_{T2} \geq p_0)
\]

\[
\sigma(E_T)^{2\rightarrow 3}_{\Delta y, p_0} = \int d[PS]_3 \frac{d\sigma^{2\rightarrow 3}}{d[PS]_3} [\epsilon_1 p_{T1} + \epsilon_2 p_{T2} + \epsilon_3 p_{T3}] \Theta(p_{T1} + p_{T2} + p_{T3} \geq 2p_0)
\]

For the renormalization and factorization scales we use \( Q = n(\sum_i p_{Ti})/2 \) with \( n \sim 1 \).

Figure 1 shows the first moment of the minijet \( E_T \)-distribution as a function of the minimum \( p_T \)-scale \( p_0 \), computed for \( pp \) collisions at \( \sqrt{s} = 200 \text{ GeV} \) and \( \sqrt{s} = 5500 \text{ GeV} \). The solid curves are the full NLO result with 2-loop \( \alpha_s \) and NLO PDFs, GRV94 [23] (left) and CTEQ5 (right) [24]. The dotted curves (LO) show the lowest order results \( \sim \alpha_s^2 \) computed with the corresponding LO sets of PDFs and 1-loop \( \alpha_s \), and the dashed curves are the LO results with NLO PDFs and 2-loop \( \alpha_s \).

Figure 1. \( \sigma_{pQCD}(E_T)(p_0, \Delta y, \sqrt{s}) \) vs. \( p_0 \), from [21]. For the details, see the text.

We see that even towards the perturbatively small scales of 1 GeV at RHIC and 2 GeV at the LHC, the NLO results do not grow rapidly relative to the LO (or LO') results. This speaks for the applicability of pQCD at the semihard scales. If a \( K \)-factor is defined as NLO/LO, we notice the stability of \( K \) within \( p_0 = 1\ldots2 \text{ GeV} \) but that quite obviously
the $K$-factor depends on $\sqrt{s}$ and on the PDF set used. It is also interesting to observe that $\sigma(E_T)$ at the LHC is insensitive to the choice of PDFs.

Even though the NLO results are stable relative to the ones in LO, the $K$-factors are rather large, and one should be concerned of the convergence of the perturbation series. The dependence of $\sigma(E_T)$ on the scale choice is very similar, quite weak, both in NLO and in LO [21]. The large NLO contributions mainly come from a kinematical domain $E_T < p_0$ which is not included at all in LO. The new region is due to processes where two hard partons fall outside $\Delta y$ and one parton with $p_T < p_0$ falls inside. To compare the NLO and LO results in the common kinematical region $E_T \geq p_0$, we have excluded the new region $E_T < p_0$ in Fig. 2 (left). The $K$-factors now reduce dramatically: at the scales 1..2 GeV $K \sim 1$ relative to LO. This also speaks in favour of the applicability of pQCD at the semihard scales. At the same time it shows, however, that to search for a convergence of the perturbative series, a NNLO computation is needed.

![Figure 2. Left: As Fig. 1 but with the region $E_T < p_0$ excluded. Right: The dependence of $\sigma \langle E_T \rangle$ on the rapidity interval $\Delta y$. The figures are from [21].](image)

The inclusive jet cross sections are known to depend on the jet cone radius $R$ as $d\sigma_{\text{jet}}/dp_T dy \sim CR^2 + B \log R + D$ [18]. If such a dependence of $\sigma(E_T)$ on $\Delta y$ were found, the initial densities in Eqs. (1) would not be well-defined but the local density would badly depend on $\Delta y$. Fig. 2 (right panel) verifies that this is not the case but that the NLO results for $\sigma(E_T)$ depend in a same, linear, way on $\Delta y$ as the LO results do. The initial energy density $\epsilon_i$ at $\tau = 1/p_0$ can thus be computed as in LO before but using the $E_T$ obtained in NLO. Unfortunately, the NLO situation is not as straightforward for the number density $n_i$. The number of partons in $\Delta y$ is not an infrared-safe quantity unless one introduces an additional resolution scale for two nearly collinear partons.

To conclude the pQCD part, let me note that both $N_{AA}(p_0)$ (LO) and $E_{T}^{AA}(p_0)$ (LO and NLO) depend strongly on the minimum transverse momentum scale $p_0$. Therefore we cannot compute the initial densities $\epsilon_i$ and $n_i$ from pQCD alone but some additional (phenomenological) element of QCD must be introduced to dynamically determine the relevant scale $p_0$. In the following, saturation of produced partons [11] is suggested as the mechanism which determines the dominant scale for gluon production.
3. SATURATION IN PARTON PRODUCTION

Saturation is a dynamical non-abelian feature of gluon fields. Saturation phenomena were first discussed by Gribov, Levin and Ryskin [1] in the context of particle production in \( pp \) collisions. Saturation in gluon production in \( AA \) collisions was first discussed by Blaizot and Mueller [1], then by McLerran and Venugopalan in describing the particle production in \( AA \) in terms of classical gluon fields [4], also studied by Mueller [8], Kovchegov [9] and Kharzeev et al [10]. These studies emphasize the saturation phenomena already in the initial wave functions of the colliding objects. Saturation of gluons at very small values of \( x \) then also saturates the actual production (liberation) of gluons.

Saturation of gluon production can also be conjectured to arise in the following way: Let us consider a nuclear disc of a transverse area \( \pi R^2 \), filled with \( N_g^A(Q) \) gluons, all produced at a scale \( Q \) and correlated over some longitudinal length. The average area density of gluons is \( \rho(Q) \sim N_g^A(Q)/\pi R^2 \). The average charge within some area \( \pi R^2 \) is then \( \langle q_s \rangle \sim g_s \sqrt{N} \sim g_s \sqrt{\rho R^2} \), and the (Coulomb-like) potential \( A \sim \langle q_s \rangle/R \sim g_s \sqrt{\rho} \). Saturation takes place when non-linearities in the field-strength tensor of QCD, \( F \sim \partial A - g_s A^2 \), become important, i.e. when \( \partial A \lesssim g_s A^2 \). Dimensionally, \( \partial A \sim A/R \sim AQ \), and we obtain a criterion for saturation as \( N_g^A(Q) \times g_s^2/Q^2 \gtrsim \pi R^2 \). Thus, if \( N_g^A(Q) \) can be computed, the saturation scale \( Q_{\text{sat}} \) can be obtained.

The above is, however, at best a scaling argument. In practise, in ref. [11], we have used a geometrical saturation criterion

\[
N_{AA}(p_0, \sqrt{s}, \Delta y) \times \frac{\pi}{p_0^2} = \pi R_A^2
\]

where we have neglected possible effects due the running of the coupling, and have not included any group theoretical factors explicitly. The correlation length in rapidity has been taken to be \( \Delta y = 1 \). In other words, we extend the computation of the number of minijets, \( N_{AA}(p_0) \) discussed in the previous section, down to scale \( p_0 = p_{\text{sat}}(\sqrt{s}, A) \), which is determined from Eq. (14), and at which the produced gluons start to overlap transversally. The multiplicity at saturation, \( N_{AA}(p_{\text{sat}}) \), should then give a good first estimate of the total parton (gluon) production in central \( AA \) collisions. Notice that it is the integral over \( p_T \) that matters and the contribution from \( p_T \leq p_{\text{sat}} \) to the integral is included through the saturation criterion.

Figure 3 (left) shows the average number partons produced into \( \Delta y = 1 \) with \( p_T \geq p_0 \) in a central \( AA \) collision of \( A = 208 \) at \( \sqrt{s}/A = 20 \) GeV (SPS), 200 GeV (RHIC) at 5500 GeV (LHC). The curve labelled as “saturation” is \( p_0^2 R_A^2 \) from Eq. (14), and the saturation scale at each energy can be read off from the intersection points of the curves.

It is quite interesting to notice that at the scaling limit (neglecting the small-\( x \) rise of the gluon densities, and the effects of the phase space) \( \sigma_{\text{pQCD}}(p_0) \sim p_0^{-2} \), and the saturation criterion (14) results in \( p_{\text{sat}} \sim A^{1/3} \) and in the multiplicity as \( N_{AA}(p_{\text{sat}}) \sim A \) instead of the \( A^{4/3} \) scaling typical for hard processes.

With realistic gluon densities (GRV94LO [23]) including the EKS98 shadowing effects [22], and an overall \( K = 2 \) to roughly simulate the NLO effects, we have obtained in [11] the scaling laws for \( p_{\text{sat}}(A, \sqrt{s}) \), \( N_{AA}(p_{\text{sat}}, \Delta y) \) and \( E_T^{AA}(p_{\text{sat}}, \Delta y) \) presented in Table 1. Recently, we have also shown analytically how these scaling laws arise [13] from the initial gluon densities, and that \( N_{AA}(p_{\text{sat}}) \sim A \alpha_s(p_{\text{sat}}^2)xg_A(2p_{\text{sat}}/\sqrt{s}, p_{\text{sat}}^2) \), where the explicit
Figure 3. **Left:** Determination of the saturation scale. **Right:** Charged particle multiplicity $dN_{ch}/dy$ in central $AA$ collisions for $A = 208, 136, 64, 32$ and $12$, as a function of $\sqrt{s}$. See the text for details. The figures are from [1].

The power of $\alpha_s$ now depends on the one included explicitly in the saturation criterion (14). At the collider energies, we see that the saturation scales are $p_{sat} \approx 1 \ldots 2$ GeV $\gg \Lambda_{QCD}$ for the heavy nuclei $A \sim 200$.

Table 1
The scaling laws for the saturation scale $p_{sat}$, initial multiplicity $N_{AA}(p_{sat}, \Delta y)$ and transverse energy $E_{T}^{AA}(p_{sat}, \Delta y)$, and the subsequent initial conditions for QGP at $\tau_i$. $A = 208, \Delta y = 1$

| $\sqrt{s}/A GeV$ | RHIC | LHC | scaling in $A$ and $\sqrt{s}$ |
|------------------|------|-----|-------------------------------|
| $p_{sat}(A, \sqrt{s})/GeV$ | 1.13 | 2.13 | $0.208A^{0.128}(\sqrt{s})^{0.191}$ |
| $N_{AA}(p_{sat}, \Delta y)$ | 1440 | 5140 | $1.383A^{0.922}(\sqrt{s})^{0.383}$ |
| $E_{T}^{AA}(p_{sat}, \Delta y)/GeV$ | 2360 | 17000 | $0.386A^{1.043}(\sqrt{s})^{0.595}$ |
| $\tau_i = 1/p_{sat}$ (fm) | 0.17 | 0.093 |
| $n_i/fm^{-3}$ | 59.8 | 401 | $0.370A^{0.383}(\sqrt{s})^{0.574}$ |
| $\epsilon_i/GeV fm^{-3}$ | 98.2 | 1330 | $0.103A^{0.504}(\sqrt{s})^{0.786}$ |
| $T_i/GeV$ (w. gluon d.o.f.) | 0.62 | 1.19 | $0.111A^{0.126}(\sqrt{s})^{0.197}$ |

The average initial conditions for the QGP obtained for central $AA$ collisions with $A = 208$ for RHIC and LHC are also summarized in Table 1. The initial system is strongly gluon dominated, $\sim 90\%$ of partons produced at $p_{sat}$ are gluons. We also note that from the point of view of the computed average densities $\epsilon_i$ and $n_i$, the system looks thermal: the energy per gluon is as in an ideal gas, $E_{T}^{AA}(p_{sat})/N_{AA}(p_{sat}) \approx 2.7T_i$, where $T_i$ is the temperature of an ideal massless gluon gas at the computed energy density $\epsilon_i$. Therefore, it seems that for thermalization gluon multiplication is not necessary, and early thermalization, perhaps already at $\tau_i$, is possible (see also A. Mueller in these proceedings).
4. FROM INITIAL TO FINAL MULTIPLICITIES

In between the primary production stage and decoupling, there is an expansion stage. A possible description of this stage is given in terms of relativistic hydrodynamics. Encouraged by the initial conditions for the QGP computed from the pQCD+saturation approach above, we assume thermalization at \( \tau_i = 1/p_{\text{sat}} \). The rapidity density of entropy in a longitudinally boost-invariant system is \( S_i \approx 3.6 N_{AA}(p_{\text{sat}}) \), counting only the gluonic degrees of freedom. In an isentropic expansion entropy is conserved, so \( S_i = S_f \approx 4 N_f \), including only pions in the final state. To a first approximation, we can estimate the final state charged-particle multiplicity in central \( AA \) collisions directly from the initial conditions as

\[
N_{\text{ch}} \equiv dN_{\text{ch}}/dy \approx \frac{2}{3} N_f = \begin{cases} 
\frac{2}{3} 1.24 A^{0.922} (\sqrt{s})^{0.383}, & \text{if } S_i \text{ computed from } N_{AA}(p_{\text{sat}}) \\
\frac{2}{3} 1.16 A^{0.92} (\sqrt{s})^{0.40}, & \text{if } S_i \text{ computed from } E_T^{AA}(p_{\text{sat}}) 
\end{cases}
\]  

(15)

This prediction is shown in Fig. 3 (right). As seen in the figure, since the system looks thermal from the very beginning, the scaling laws (15) obtained are very close to each other. This illustrates the basic idea in multiplicity predictions from pQCD+saturation. For a more detailed study with \( \sqrt{s} \)-dependent \( K \)-factors, transverse profiles, transverse expansion effects, more detailed EoS, detailed treatment of decoupling, more complete list of hadrons, resonance decays, computation of particle spectra and centrality cuts, see refs. \[25, 26\], and V. Ruuskanen and U. Heinz in these proceedings.

5. COMPARISON WITH RHIC DATA

The above were predictions about one year before the first data from RHIC. The PHOBOS experiment measured the charged-particle multiplicity in Au-Au collisions both at \( \sqrt{s} = 56 \, \text{AGeV} \) and at \( \sqrt{s} = 130 \, \text{AGeV} \) in Summer 2000 \[27\]. Fig. 4 from \[28\] shows the first PHOBOS data (6\% centrality cut) by the filled circles, and the pQCD+saturation (EKRT) prediction \[11\] by the thick solid line. The HIJING predictions \[29\] with and without jet quenching are drawn with the solid lines. The EKRT prediction shown is from Eq. (15) with the number of participants \( N_{\text{part}} = 2 A \), and an approximate factor 0.9 to account for the conversion \( y \rightarrow \eta \) at \( y \sim 0 \).

I have also added into the figure the latest measurement by PHOBOS at \( \sqrt{s} = 200 \, \text{AGeV} \) (filled triangle) \[30\]. Considering the theoretical uncertainties, the data confirms the EKRT prediction amazingly well both in absolute magnitude and in the scaling with \( \sqrt{s} \) in the RHIC energy regime. It should be emphasized that once the effective constants in the saturation criterion \( (14) \) have been verified by comparison with the data at some cms-energy, the pQCD+saturation approach gives a definite prediction for the multiplicities at other energies, and also for central collisions of other nuclei. As observed in Fig. 4. HIJING1.35 with a soft and a hard \( (p_0 = 2 \, \text{GeV}) \) particle production components, predicts too large a multiplicity at \( \sqrt{s} = 200 \, \text{AGeV} \). For further discussion, see \[31\], and X.-N. Wang in these proceedings.

**Centrality dependence** of the charged-particle multiplicity has been suggested as a further challenge for the different models \[28\]. In the pQCD+saturation approach the dependence of the centrality of the \( AA \) collision can be studied by making the saturation
\( \frac{dN_{\text{ch}}}{d\eta/0.5N_{\text{part}}} \) as a function of \( \sqrt{s} \). The prediction from pQCD-saturation approach [11] is shown by the thick solid line. The HIJING1.35 [29] prediction is shown by the solid lines both for pp and for Au+Au with jet quenching (upper curve) and without (lower). The PHOBOS data [27] is shown by the filled circles. The figure is from [28], I have added the new PHOBOS data point at \( \sqrt{s} = 200 \text{ AGeV} \) [30] and emphasized the saturation curve. Right: The same quantity as a function of the number of participants for \( \sqrt{s} = 130 \) and 5500 AGeV as predicted by the pQCD+saturation approach [13,12]. The data is from PHENIX [32].

Criterion local in the transverse plane. Noticing in Eq. (14) that \( N_{\text{AA}}/\pi R_A^2 \) is the average transverse density of produced partons, the saturation criterion generalizes as [12]

\[
\frac{dN_{\text{AA}}}{d^2s} = T_A(b-s)T_A(s)\sigma_{\text{pQCD}}\langle N \rangle(p_0, \sqrt{s}, \Delta y, A) = p_0^2/\pi
\]

for an AA collision at an impact parameter \( b \). The saturation momentum needs to be determined at each transverse location \( s \), and the multiplicity of produced partons is then \( N_{\text{AA}}(b) = \int d^2s p_{\text{sat}}(b, s, \sqrt{s}, A) \). At sufficiently large values of \( s \) or \( b \), \( p_{\text{sat}} \sim \Lambda_{\text{QCD}} \), where we should not trust the pQCD calculation anymore. To avoid this, we have considered particle production only from the region where \( p_{\text{sat}} \geq 0.5 \text{ GeV} \) [12]. Clearly, the saturation model [12] is applicable in the region where dominantly \( p_{\text{sat}} \gg \Lambda_{\text{QCD}} \).

In Fig. 4 (right), we show the comparison of the prediction [12] against the PHENIX data [12]. Also a prediction for the LHC is shown. The RHIC data suggests that the pQCD-saturation model works for central and nearly central collisions. However, for non-central collisions at \( b \gtrsim R_A \) and \( N_{\text{part}} \gtrsim 200 \), one obviously moves outside the validity region of the model; the dominant saturation momenta become too small, and particle production becomes overestimated.

I would like to emphasize, however, that although the centrality dependence does provide constraints for different models, the effect is still fairly modest, \( \sim 20 \ldots 30\% \) in the data at \( N_{\text{part}} \gtrsim 100 \), and that in the absence of a direct measurement of \( N_{\text{part}} \) the systematic errors can be large. A more dramatic effect is predicted from the \( \sqrt{s} \)-dependence of the charged-particle multiplicity of central collisions: as seen in Fig. 4 (right), \( N_{\text{ch}}/0.5N_{\text{part}} \) at the LHC should be almost 4 times that at RHIC [13]. The \( A \)-scaling of different models will be best tested by data from central AA collisions with different \( A \) instead of varying \( b \) at fixed \( A \).
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