WEAK PHASE $\gamma$ FROM $B_s(t) \to K^+K^-$

Michael Gronau

Physics Department, Technion – Israel Institute of Technology
32000 Haifa, Israel

Jonathan L. Rosner

Enrico Fermi Institute and Department of Physics
University of Chicago, Chicago, Illinois 60637

We re-examine the time-dependent rates of $B_s(t) \to K^+K^-$ and $\overline{B}_s(t) \to K^+K^-$, including a lifetime difference between neutral $B_s$ mass eigenstates. The two rates, normalized by the rate of $B_s \to K^0\overline{K}^0$, are used to obtain ambiguity-free information on a strong phase and on the weak phase $\gamma$. We discuss the sensitivity of extracting $\gamma$ to the measured quantities, and find that an error of $\pm 10^\circ$ in $\gamma$ is possible for a sample of several thousand $B_s(t) \to K^+K^-$ decays. This study is complementary to a recent similar analysis of the U-spin related decays $B^0(t) \to \pi^+\pi^-$ and $\overline{B}^0(t) \to \pi^+\pi^-$. 

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I Introduction

Time-dependent CP asymmetry measurements in neutral $B$ meson decays provide useful tests for the standard model, which attributes CP violation to a single phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix. The potential variety of $B^0$ and $B_s$ decay modes permits one to overconstrain the CKM unitarity triangle involving the three angles $\alpha = \phi_2$, $\beta \equiv \phi_1$ and $\gamma = \phi_3$ \cite{1}. The observation of CP violation in $B^0(t) \to J/\psi K_S$ \cite{2,3}, interpreted in terms of $\sin 2\beta$, passed this test successfully. First results of a CP asymmetry measurement in $B^0(t) \to \pi^+\pi^-$, consistent with a zero asymmetry, though involving large errors, were reported in \cite{4}. Updates, implying world average asymmetries which are nonzero at about a $2.3\sigma$ level, were presented very recently \cite{5,6}. The two measured quantities, coefficients of terms involving $\sin \Delta m t$ and $\cos \Delta m t$ in the time-dependent asymmetry, were studied recently \cite{7} as functions of relevant strong and weak phases. It was shown how discrete ambiguities in a strong phase can be resolved in order to learn the weak phase $\alpha$.

Approximate flavor SU(3) symmetry of strong interactions can be used to relate a variety of $B$ and $B_s$ decays to two charmless pseudoscalar mesons \cite{8}. Thus, the process $B^0 \to \pi^+\pi^-$ is related to $B_s \to K^+K^-$ in the limit of U-spin symmetry.
involving an interchange of $d$ and $s$ quarks. This symmetry implies equal CP rate differences in any pair of U-spin related processes \[1\], and in particular in this pair of decay modes. The U-spin relation between $B^0 \to \pi^+\pi^-$ and $B_s \to K^+K^-$ was used in \[10\] to suggest a simultaneous determination of the two weak phases $\beta$ and $\gamma$ by combining information from the asymmetries in these two processes. By measuring for both channels a $\cos \Delta m t$ and a $\sin \Delta m t$ term one has four equations for four unknowns: $\beta$ (which can also be considered as given from direct measurements), $\gamma$, the ratio of penguin to tree amplitudes in $B^0 \to \pi^+\pi^-$, and the relative strong phase between these two amplitudes. It was noted \[10\] that factorized U-spin breaking effects cancel in the double ratio of penguin to tree amplitudes and in the difference between the two relative strong phases occuring in the two processes. This reduces the intrinsic theoretical uncertainty of this method.

Several earlier suggestions for measuring weak phases in $B_s \to K^+K^-$ were made in \[11, 12\], by combining this decay within flavor SU(3) with information from other processes, involving $B_s \to K^0\bar{K}^0$, $B_s \to K^-\pi^+$, $B^0 \to K^+\pi^-$ and $B^+ \to K^0\pi^+$. Usually, these methods require some assumption about SU(3) breaking. Also, as a result of the large number of required measurements, which are expressed in terms of trigonometric functions of strong and weak phases, one encounters several discrete ambiguities in the solutions for weak phases.

In the present paper we wish to re-examine more carefully the simplest among all these suggestions \[11\], which does not require full SU(3) but only isospin symmetry. The method is based primarily on the time-dependent rate of $B_s(t) \to K^+K^-$ and its charge-conjugate, normalized by the time-integrated rate of $B_s \to K^0\bar{K}^0$. We will also include effects due to a lifetime difference between neutral $B_s$ mass eigenstates which were neglected in \[11\].

A major purpose of this study will be to show how to resolve certain discrete ambiguities in strong and weak phases in order to determine unambiguously the weak phase $\gamma$. One of the outputs of the above measurements is the ratio of tree to penguin amplitudes in $B_s \to K^+K^-$. This ratio is related by U-spin to the corresponding ratio in $B^0 \to \pi^+\pi^-$, for which some information already exists. We will use this information to demonstrate the sensitivity of this method.

We introduce notations for $B_s \to K^+K^-$ and $B_s \to K^0\bar{K}^0$ in Sec. II, where we explain some necessary assumptions and ways of testing these assumptions. The general time-dependence of $\Gamma(B_s(t) \to K^+K^-)$, normalized by $\Gamma(B_s \to K^0\bar{K}^0)$, involves four measurable quantities $R_{KK}$, $C_{KK}$, $S_{KK}$ and $D_{KK}$, for which expressions are derived in Sec. III in terms of a ratio of tree to penguin amplitude, a strong phase difference and the weak phase $\gamma$. We discuss the resolution of ambiguities associated with the strong phase. Solutions for $\gamma$ in terms of the measurables are studied in Sec. IV in order to evaluate the sensitivity of the method. We conclude in Sec. V.
II Notation and assumptions

In Ref. [7], studying $B^0 \rightarrow \pi^+\pi^-$ in terms of tree and penguin amplitudes,

$$A(B^0 \rightarrow \pi^+\pi^-) = -(|T| e^{i\delta_T} e^{i\gamma} + |P| e^{i\delta_P}) ,$$  \hspace{1cm} (1)

we used the convention in which top quark contributions are integrated out in the short-distance effective Hamiltonian, and the unitarity relation $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} = -V_{tb}^* V_{td}$ is used. The $V_{ub}^* V_{ud}$ piece of the penguin operator is included in the tree amplitude. We adopt the same convention for $B_s \rightarrow K^+K^-$, such that

$$A(B_s \rightarrow K^+K^-) = -(|T'| e^{i\delta'_T} e^{i\gamma} - |P'| e^{i\delta'_P}) .$$ \hspace{1cm} (2)

In the U-spin symmetry limit one has [9, 10]

$$\frac{|T'|}{|T|} = \frac{V_{ub}^* V_{us}}{V_{ub}^* V_{ud}} = \frac{\lambda}{1 - \lambda^2/2} , \quad \frac{|P'|}{|P|} = \frac{-V_{cb}^* V_{cs}}{V_{cb}^* V_{cd}} = \frac{1 - \lambda^2/2}{\lambda} , \quad \delta'_T = \delta_T , \quad \delta'_P = \delta_P ,$$ \hspace{1cm} (3)

where $\lambda = 0.22$ [11].

We note in passing that Eq. (2) by itself is a general expression of the decay amplitude in the CKM framework, and involves no assumption. The amplitude for $B_s \rightarrow K^+K^-$ is simply obtained by changing the sign of the weak phase $\gamma$.

The amplitude for $B_s \rightarrow K^0\bar{K}^0$ will be assumed to have the simple form

$$A(B_s \rightarrow K^0\bar{K}^0) = -|P'| e^{i\delta'_P} .$$ \hspace{1cm} (4)

The QCD penguin terms in $B_s$ decays to charged and neutral kaons are equal by isospin. However, the form (4) which assumes equal overall penguin contributions in the two processes and no tree amplitude (with weak phase $\gamma$) in $B_s \rightarrow K^0\bar{K}^0$, involves two approximations. First, we neglect a small color-suppressed electroweak penguin contribution in $B_s \rightarrow K^+K^-$ [8]. Using factorization, such contributions are estimated to be at a level of a few percent [13]. Tests for the magnitude of the electroweak penguin contribution in $B_s \rightarrow K^+K^-$ were proposed in [12]. In Eq. (4) one also disregards the possibility of enhanced rescattering effects in $B_s \rightarrow K^0\bar{K}^0$, typically of order $|V_{ub}^* V_{us}/V_{cb}^* V_{cs}| \approx 2\%$ without such enhancement. Such enhanced effects could, in principle, induce a sizable term with phase $\gamma$. One test for such a term would be observing a CP asymmetry between this process and its change conjugate. Other indirect tests involve rate measurements of hadronic decays such as $B_s \rightarrow \pi^+\pi^-$ and $B^0 \rightarrow K^+K^-$, which are very rare in the absence of rescattering [14].

Since in the above approximation $B_s \rightarrow K^0\bar{K}^0$ involves a single weak phase, we expect no CP asymmetry in this decay. We denote by $R_{KK}$ the ratio of charge averaged rates for $B_s$ (or $\bar{B}_s$) to $K^+K^-$ and $B_s$ (or $\bar{B}_s$) to $K^0\bar{K}^0$,

$$R_{KK} \equiv \frac{B(B_s \rightarrow K^+K^-) + B(\bar{B}_s \rightarrow K^+K^-)}{B(B_s \rightarrow K^0\bar{K}^0) + B(\bar{B}_s \rightarrow K^0\bar{K}^0)} .$$ \hspace{1cm} (5)
The time-dependent decay rate of \( B_s |_{\text{initial}} \rightarrow K^+ K^- \) is given by

\[
\Gamma(B_s(t) \rightarrow K^+ K^-) \propto e^{-\Gamma_s t} \times \left[ \cosh(\Delta\Gamma_s t/2) - D_{KK} \sinh(\Delta\Gamma_s t/2) \right. \\
\left. + C_{KK} \cos(\Delta m_s t) - S_{KK} \sin(\Delta m_s t) \right], \tag{6}
\]

where \( \Gamma_s \equiv (\Gamma_H + \Gamma_L)/2 \), and \( \Delta m_s \equiv m_H - m_L \) and \( \Delta\Gamma_s \equiv \Gamma_L - \Gamma_H \) are the mass and width differences between the two \( B_s \) mass eigenstates. In the standard model one expects \( \Delta\Gamma_s > 0 \), and estimates imply that \( \Delta\Gamma_s/\Gamma_s \) can be as large as about 15\% [10]. An expression similar to (6), in which the \( C_{KK} \) and \( S_{KK} \) terms occur with opposite signs, describes the rate of \( B_s |_{\text{initial}} \rightarrow K^+ K^- \).

The coefficients \( C_{KK}, S_{KK} \) and \( D_{KK} \), obeying

\[
C_{KK}^2 + S_{KK}^2 + D_{KK}^2 = 1, \tag{7}
\]

are defined by

\[
C_{KK} \equiv \frac{1 - |\lambda_{KK}|^2}{1 + |\lambda_{KK}|^2}, \quad S_{KK} \equiv \frac{2\text{Im}(\lambda_{KK})}{1 + |\lambda_{KK}|^2}, \quad D_{KK} \equiv \frac{2\text{Re}(\lambda_{KK})}{1 + |\lambda_{KK}|^2}, \tag{8}
\]

where

\[
\lambda_{KK} \equiv \frac{A(B_s \rightarrow K^+ K^-)}{A(B_s \rightarrow K^+ K^-)}. \tag{9}
\]

A very small overall phase from \( B_s \rightarrow \overline{B}_s \) mixing, \( \arg(V_{tb}^* V_{ts}/V_{cb}^* V_{cs})^2 \sim \lambda^2 \), was neglected.

Defining \( \delta' \equiv \delta'_{P} - \delta'_{T} \), where we use a convention in which \( -\pi \leq \delta' \leq \pi \), one finds

\[
\lambda_{KK} = \frac{1 - |T'/P'| e^{-i(\delta'+\gamma)}}{1 - |T'/P'| e^{-i(\delta'-\gamma)}}. \tag{10}
\]

This expression simplifies to a pure phase in the two special cases when \( \delta' = 0 \) or \( \pi \)

\[
\lambda_{KK} = e^{2\zeta}, \quad \zeta = \begin{cases} \arctan \frac{|T'/P'| \sin \gamma}{|T'/P'| \cos \gamma} & (\delta' = 0) \\
-\arctan \frac{|T'/P'| \sin \gamma}{|T'/P'| \cos \gamma} & (\delta' = \pi) \end{cases}. \tag{11}
\]

In such cases \( C_{KK} = 0, \ S_{KK} = \sin 2\zeta, \ D_{KK} = \cos 2\zeta \).

In the next section we will study the dependence of the measurables defined in Eqs. (3) and (8) on \( |T'/P'|, \ \delta' \) and \( \gamma \).

### III \ Dependence of observables on \( \delta' \) and \( \gamma \)

Expressions for \( R_{KK}, C_{KK}, S_{KK} \) and \( D_{KK} \) in terms of \( |T'/P'|, \ \delta' \) and \( \gamma \) are readily derived:

\[
R_{KK} = 1 - 2|T'/P'| \cos \delta' \cos \gamma + |T'/P'|^2, \tag{12}
\]

\[
C_{KK} = -2|T'/P'| \sin \delta' \sin \gamma / R_{KK}, \tag{13}
\]

\[
S_{KK} = [2|T'/P'| \cos \delta' \sin \gamma - |T'/P'|^2 \sin 2\gamma] / R_{KK}, \tag{14}
\]

\[
D_{KK} = [1 - 2|T'/P'| \cos \delta' \cos \gamma + |T'/P'|^2 \cos 2\gamma] / R_{KK}. \tag{15}
\]

There are some interesting differences between the present case of \( B_s \rightarrow K^+ K^- \) and that of \( B^0 \rightarrow \pi^+ \pi^- \) discussed in Ref. [10].
1. Assuming \( \text{sign}(\sin \delta') = \text{sign}(\sin \delta) \), the quantities \( C_{KK} \) and \( C_{\pi\pi} \) are opposite in sign. In the U-spin symmetry limit, the corresponding CP rate differences are equal in magnitude and have opposite signs.

2. In contrast to the case of \( S_{\pi\pi} \), which measures \( \sin 2\alpha \) in the limit \( |P/T| \to 0 \), there is no term independent of the ratio of the smaller amplitude to the larger amplitude; \( S_{KK} \) vanishes in the limit \( |T'/P'| \to 0 \).

3. Whereas \( D_{\pi\pi} \) is expected to be large and negative, \( D_{KK} \) is predicted to be large and positive. Its maximal value is +1.

4. While one expects \( R_{\pi\pi} > 1 \) for \( \cos \gamma > 0 \) and \( \cos \delta \approx 1 \), one expects \( R_{KK} < 1 \) for \( \cos \delta' \approx 1 \).

5. All equations involve only the CKM phase \( \gamma \), while for \( B_0 \to \pi^+\pi^- \) the relations are not as simple because they contain both \( \alpha \) and \( \gamma \) (or \( \beta \) and \( \gamma \)). This should not be considered a disadvantage, since \( \beta \) is known rather precisely from direct measurements.

The first three expressions for \( R_{KK} \), \( C_{KK} \) and \( S_{KK} \) agree with results obtained in [11] using different notations. One may regard these three equations as specifying the three unknowns \( |T'/P'| \), \( \delta' \) and \( \gamma \). The quadratic equations for \( |T'/P'| \) and the trigonometric expressions in terms of strong and weak phases yield solutions which involve several discrete ambiguities.

Whereas the magnitude of \( D_{KK} \) is specified by \( C_{KK} \) and \( S_{KK} \), which are likely to be more easily measured, its sign could in principle resolve some of these ambiguities. \( D_{KK} \) may change sign as a function of \( \delta' \) and \( \gamma \) if \( |T'/P'| \) is sufficiently large. For instance, in [11] this ratio was taken to be 8/9, which would permit a change of sign in \( D_{KK} \). As we will show now, there exists already indirect information on \( |T'/P'| \) from nonstrange \( B \) decays which implies \( |T'/P'| = 0.184 \pm 0.043 \), which is much smaller than assumed in [11]. With such a small value of \( |T'/P'| \), \( D_{KK} \) must be positive and large in the CKM framework. A negative value would signify new physics.

Using Eq. (3), we find to leading order in \( \lambda^2 \),
\[
\frac{|T'|}{|P'|} = \frac{\lambda^2 |T|}{1 - \lambda^2 |P|} \quad . \tag{16}
\]

This relation, which is precise in the U-spin symmetry limit, may be affected by U-spin breaking. However, as noted in [11], such effects are expected to be small in the ratio of ratios \( |T'/P'| : |T/P| \) if one assumes approximate factorization. We will neglect nonfactorizable corrections in our evaluation of \( |T'/P'| \). The ratio \( |P/T| \) was estimated in Refs. [17] and [18], applying SU(3) with SU(3) breaking to experimental data on \( B^+ \to K^0\pi^+ \) (a process dominated by the penguin amplitude), and applying data on \( B \to \pi l \nu \) related to the tree amplitude in \( B^0 \to \pi^+\pi^- \) in the factorization approximation. We shall use the result of Ref. [17], \( |P/T| = 0.276 \pm 0.064 \). Ref. [11].
based on explicit calculations in QCD-improved factorization, found a very similar value \(|P/T| = 0.285 \pm 0.076\). Thus, we find from Eq. (10)
\[
\frac{|T'|}{|P'|} = 0.184 \pm 0.043
\]  
As mentioned, this value allows only positive values of \(D_{KK}\), which cannot provide new information for reducing discrete ambiguities.

Let us discuss ambiguities related to the strong phase \(\delta'\), which can be positive or negative, and can lie either in the range \(0 < |\delta'| \leq \pi/2\) or in \(\pi/2 < |\delta'| \leq \pi\). Calculations based on QCD-improved factorization [13] may indicate that this phase (related by U-spin to the phase \(\delta \equiv \delta_p - \delta_r\) defined in [1]) is small and has a definite sign; a value \(\delta \simeq 10^\circ\) is found in Ref. [13]. Other perturbative QCD calculations [20] imply a larger negative phase \(\delta\); correspondingly, Ref. [21] finds \(\delta' \simeq -27^\circ\) when the phase is expressed in our convention. It will be important to check these predictions.

In order to resolve ambiguities in \(\delta'\), we note that \(R_{KK}, S_{KK}\) and \(D_{KK}\) are even in \(\delta'\) while \(C_{KK}\) is odd in \(\delta'\). First, consider \(R_{KK}\). Since current constraints on CKM parameters [22, 23, 24] imply \(\gamma < \pi/2\) or \(\cos \gamma > 0\), a value \(R_{KK} < 1\) would imply \(\cos \delta' > 0\), namely \(0 < |\delta'| < \pi/2\). Furthermore, a value of \(R_{KK}\) below 1 permits one to set a significant bound on \(\gamma\) which is independent of \(\delta'\) [13, 17],
\[
\sin^2 \gamma \leq R_{KK} \quad . \tag{18}
\]

Second, consider \(C_{KK}\). Within the CKM framework \(\sin \gamma > 0\). Therefore, a nonzero measurement of \(C_{KK}\) will specify the sign of \(\delta'\). There exists an absolute bound on \(C_{KK}\), \(|C_{KK}| \leq 2|T'/P'|/(1 + |T'/P'|^2)\), which becomes stronger for a given value of \(\gamma\):
\[
|C_{KK}| \leq \frac{2|T'/P'| \sin \gamma}{\sqrt{(1 + |T'/P'|^2)^2 - 4|T'/P'|^2 \cos^2 \gamma}} \quad . \tag{19}
\]

Examining \(S_{KK}\), we note that its dominant term is \(2|T'/P'| \cos \delta' \sin \gamma\). Hence, neglecting the term quadratic in \(|T'/P'|\), \(S_{KK} \geq 0\) for \(0 < |\delta'| \leq \pi/2\), and \(S_{KK} < 0\) for \(\pi/2 < |\delta'| \leq \pi\), which resolves the same ambiguity in \(\delta'\) as in \(R_{KK}\). In this approximation, \(R_{KK} S_{KK}\) is bound by \(|R_{KK} S_{KK}| \leq 2|T'/P'| \sin \gamma\) for a given value of \(\gamma\).

### IV Solutions for \(\gamma\)

Combining the equations for \(R_{KK}\) and \(C_{KK}\), one can eliminate the strong phase \(\delta'\)
\[
R_{KK} = 1 + |T'/P'|^2 \pm \sqrt{4|T'/P'|^2 \cos^2 \gamma - (R_{KK} C_{KK})^2 \cot^2 \gamma} \quad . \tag{20}
\]
This equation can be inverted to obtain a quadratic equation for \(\sin^2 \gamma\) in terms of \(R_{KK}\) and \(C_{KK}\) [25]:
\[
4|T'/P'| \sin \gamma = \pm \{[(1 + |T'/P'|^2 - R_{KK}(1 + C_{KK})(R_{KK}(1 - C_{KK}) - (1 - |T'/P'|^2))]^{1/2} \\
\pm \{[(1 + |T'/P'|^2 - R_{KK}(1 - C_{KK})(R_{KK}(1 + C_{KK}) - (1 - |T'/P'|^2))]^{1/2}(1)
\]
As mentioned, only positive solutions of $\sin \gamma$ apply within the CKM framework.

While the value of $|T'/P'|$ in Eq. (17) was obtained indirectly from nonstrange $B$ decays, a direct cross check in $B_s \to K\bar{K}$ can be achieved by using information from $S_{KK}$. One obtains

$$R_{KK}(1 + S_{KK} \cot \gamma) = 1 - |T'/P'|^2 \cos 2\gamma \ .$$

(22)

Alternatively, neglecting the term in $S_{KK}$ quadratic in $|T'/P'|$, one gets a useful approximation,

$$4|T'/P'|^2 \sin^2 \gamma \approx R_{KK}^2 (C_{KK}^2 + S_{KK}^2) \ .$$

(23)

Graphical solutions of Eqs. (20) and (22) can be used to demonstrate the sensitivity of the extracted value of $\gamma$ to the measured quantities. In Fig. 1 we use these two equations to plot $R_{KK}$ as a function of $\gamma$ for several values of $R_{KK}C_{KK}$ and
for several values of $R_{KK}S_{KK}$. For $|T'/P'|$ we adopt the central value in Eq. (17), $|T'/P'| = 0.184$. Figs. 2 and 3 show the effects of varying $|T'/P'|$ by $\pm 1\sigma$.

The curves in Figs. 1–3 indicate that $\gamma$ can be specified by measurements of $|T'/P'|$, $R_{KK}$, $S_{KK}$, and $C_{KK}$. The equations are overconstrained, allowing the elimination of some discrete ambiguities. Usually, measurements of $R_{KK}$ and $C_{KK}$ are seen to leave a twofold ambiguity in the solution for $\gamma$ within the physical CKM range, $0 < \gamma < \pi/2$, and another twofold ambiguity outside this range. This fourfold ambiguity is resolved by $S_{KK}$.

As an example, assume that $|T'/P'| = 0.184$, and let us imagine that we observe $R_{KK} = 0.8$ and $|R_{KK}C_{KK}| = 0.1$. Acceptable solutions are $\gamma \simeq \gamma_1 \equiv 22^\circ$, $\gamma_2 \equiv 47^\circ$, $\pi - \gamma_1$, and $\pi - \gamma_2$. These values correspond, respectively, to $R_{KK}S_{KK} \simeq 0.07$, $0.22$, $-0.07$, $-0.22$. Thus, even given perfect measurements of (say) $R_{KK}$ and $C_{KK}$, one will need to determine $R_{KK}S_{KK}$ to an accuracy of $\pm 0.05$ or better in order either to resolve the discrete ambiguity or to encounter an inconsistency. For $|T'/P'| = 0.141$ these measurements of $R_{KK}$ and $C_{KK}$ lead to no solution of $\gamma$, and for $|T'/P'| = 0.227$ the ambiguity in $\gamma$ can be resolved with a less accurate measure-
Figure 3: Dependence of $R_{KK}$ on $\gamma$ for fixed values of $|R_{KK}C_{KK}|$ (solid curves) and $R_{KK}S_{KK}$ (dashed curves). Same as Fig. 1 except $|T'/P'| = 0.227$.

Figure 3: Dependence of $R_{KK}$ on $\gamma$ for fixed values of $|R_{KK}C_{KK}|$ (solid curves) and $R_{KK}S_{KK}$ (dashed curves). Same as Fig. 1 except $|T'/P'| = 0.227$.

ment of $S_{KK}$. This indicates that at least several hundred flavor tagged $B_s \rightarrow K^+K^-$ decays would be needed for the present method to be useful. This estimate ignores background and resolution factors. A less optimistic estimate is based on extrapolation of the BaBar result for $B^0 \rightarrow \pi^+\pi^-$ [4], in which a sample of 65 flavor tagged events led to errors of about $\pm (0.5, 0.6)$ in $(C_{\pi\pi}, S_{\pi\pi})$. If these errors scale as the inverse square root of the number of events, it may be necessary to obtain several thousand tagged $B_s \rightarrow K^+K^-$ decays to achieve errors of $\pm 0.05$ in $C_{KK}$ and $S_{KK}$.

Equations (20) and (22) can be used to obtain both $|T'/P'|$ and $\gamma$. Eliminating $|T'/P'|$, one expresses $\gamma$ in terms of measurable quantities,

$$R_{KK}S_{KK} = \frac{1}{2} \sin 4\gamma - (R_{KK} - 1) \sin 2\gamma$$

$$\pm \cos 2\gamma \sqrt{\sin^2 2\gamma + 4(R_{KK} - 1) \sin^2 \gamma - (R_{KK}C_{KK})^2}$$

while $|T'/P'|$ can be obtained from (22). Eq. (24) provides the essence of this method of determining $\gamma$. Although the $\pm$ sign in this equation seems to indicate a possible discrete ambiguity in the solution for $\gamma$, we have shown in the above numerical exam-
ple that the three observables $R_{KK}, C_{KK}$ and $S_{KK}$ determine the weak phase in a unique manner. This follows from the requirement that $|T'/P'|$ lies in the range (17).

Since the direct asymmetry $C_{KK}$ occurs quadratically in Eq. (24), $\gamma$ is relatively insensitive to small values of this observable. Small values correspond to small values of $\delta'$ as predicted in [19]. The quantity $R_{KK}$, defined in (5), is the ratio of two charged averaged rates, the measurements of which do not require flavor tagging of neutral $B_s$ mesons. The error in this quantity is expected to be much smaller than in the mixing induced asymmetry $S_{KK}$, which requires both flavor tagging and time-dependence. Therefore, for small values of $R_{KK}C_{KK}$, the dominant uncertainty in $\gamma$ originates in the error of $S_{KK}$.

As an illustrative example, suppose $R_{KK} = 0.85 \pm 0.05$, $R_{KK}C_{KK} \leq 0.1$, and $R_{KK}S_{KK} = 0.25 \pm 0.05$. We find that $\gamma$ is indeed insensitive to $R_{KK}C_{KK}$. The lowest value $\gamma = 50^\circ$ is attained when $R_{KK} = 0.8$ and $R_{KK}S_{KK} = 0.25$, while the highest value $\gamma = 70^\circ$ is attained when $R_{KK} = 0.9$ and $S_{KK} = 0.35$. It can be checked that the values of $|T'/P'|$ for these solutions are within the allowed range. Thus, an error on $\gamma$ of $\pm 10^\circ$, following from an error of $\pm 0.05$ in $R_{KK}S_{KK}$, requires a sample of several hundred $B_s \rightarrow K^+K^-$ decays. A realistic number of required flavor tagged events, including background and resolution factors, is more like a few thousand. Plans to measure $B_s(t) \rightarrow K^+K^-$ at the Tevatron Run II, with a comparable precision in $S_{KK}$, are described in Ref. [26].

V Conclusions

We have discussed time-dependent observables in the decay $B_s \rightarrow K^+K^-$ in a manner parallel to that [4] for $B^0 \rightarrow \pi^+\pi^-$. Measurements based on the $B_s$ decay alone (when combined with a measurement of the rate for $B_s \rightarrow K^0\bar{K}^0$) suffice to specify both the weak phase $\gamma$ and the strong phase $\delta'$ which is the difference between the penguin and tree strong phases. The measurements are capable of resolving several discrete ambiguities. In order to achieve an error on $\gamma$ of $\pm 10^\circ$, it will be necessary to obtain a sample of several thousand flavor tagged $B_s \rightarrow K^+K^-$ decays.

This method does not require full flavor SU(3) or discrete U-spin but only isospin symmetry. We neglected two small contributions, a color-suppressed penguin amplitude in $B_s \rightarrow K^+K^-$, and rescattering effects in $B_s \rightarrow K^0\bar{K}^0$, as well as a small phase in $B_s - B_s$ mixing. Each of these terms is expected to introduce an uncertainty in $\gamma$ of only a few percent.

The observable $D_{KK}$, whose measurement depends on the observability of the width difference between the strange $B$ mass eigenstates, is predicted to be large and positive. (Its maximum value is +1.) Any deviation from this prediction would be evidence for physics beyond the standard Kobayashi-Maskawa picture of CP violation.
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