Geometric constraint-based modeling and analysis of a novel continuum robot with Shape Memory Alloy initiated variable stiffness

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Abstract
Continuum robots exhibit promising adaptability and dexterity for soft manipulation due to their intrinsic compliance. However, this compliance may lead to challenges in modeling as well as positioning and loading. In this paper, a virtual work-based static model is established to describe the deformation and mechanics of continuum robots with a generic rod-driven structure, taking the geometric constraint of the drive rods into account. Following this, this paper presents a novel variable stiffness mechanism powered by a set of embedded Shape Memory Alloy (SMA) springs, which can make the drive rods become ‘locked’ on the body structure with different configurations. The resulting effects of variable stiffness are then presented in the static model by introducing tensions of the SMA and friction on the rods. Compared with conventional models, there is no need to predefine the actuation forces of the drive rods; instead, actuation displacements are used in this new mechanism system with stiffness being regulated. As a result, the phenomenon that the continuum robot can exhibit an S-shaped curve when subject to single-directional forces is observed and analyzed. Simulations and experiments demonstrated that the presented mechanism has stiffness variation of over 287% and further demonstrated that the mechanism and its model are achievable with good accuracy, such that the ratio of positioning error is less than 2.23% at the robot end-effector to the robot length.

Keywords
Soft continuum robot, variable stiffness, statics model, geometric constraint

1. Introduction
Inspired by biological tentacles or snakes, soft continuum robots possess theoretically infinite degrees of freedom (DOFs), intrinsic compliance, and splendid adaptability, which extends the capabilities of traditional rigid robots in areas such as detection in unstructured environments and grasping non-cooperative targets with few sensors equipped (Deimel and Brock, 2016; Rone and Ben-Tzvi, 2014b; Simaan et al., 2009). Since the concept was first proposed by Robinson and Davies (1999), various types of continuum robots have emerged in the past years, which can be categorized roughly in terms of their actuation mechanisms as the cable-driven (rod-driven) type (Catalano et al., 2014; Dong et al., 2017; Kato et al., 2015; McMahen et al., 2005; Rone and Ben-Tzvi, 2014b; Simaan et al., 2009; Zhang et al., 2016), the pneumatically driven type (Hawkes et al., 2017; Kang et al., 2013; Kim et al., 2014a; Walker et al., 2005), and the concentric-tube type (Dupont et al., 2010; Rucker et al., 2010; Webster et al., 2009). However, their flexible structures make it difficult for all types of soft continuum robots to withstand large forces and keep motion precision at the same time (Dai and Ding, 2005). These defects have driven researchers to focus on the design of...
variable stiffness mechanisms (Cianchetti et al., 2014). In recent years, several stiffness control methods for continuum manipulators have been studied, and can be classified into two categories: algorithm-based and mechanism-based methods.

Using active control algorithms at the actuation level to change the stiffness characteristics of continuum manipulators originates from the impedance control or hybrid motion/force control in traditional rigid robots, but the strain energy was also taken into account (Bajo and Simaan, 2016; Mahvash and Dupont, 2011). These methods were usually performed by detecting/estimating the position and contact force at the arm tip and calculating the corresponding control variables in a closed-loop fashion, which is complex and time-consuming for the controller.

Mechanism-based methods have also been used in numerous prior works, as summarized in Table 1. The idea of antagonistic actuators is to increase the internal stress of the structure by applying a pair of opposing forces. Stilli et al. (2014) implemented pneumatic and tendon-driven actuators to control the stiffness of soft manipulators. Kim et al. (2014b) proposed a continuum manipulator for minimally invasive surgery, which can adjust its stiffness by tensioning all the cables along the robot simultaneously, but high tension may cause structural buckling. A slide-linkage locking mechanism was proposed by Yagi et al. (2006) for a flexible endoscopic manipulator that increases rigidity by meshing the racks embedded in adjacent segments. Sun et al. (2020) proposed a hybrid continuum robot based on pneumatic muscles with embedded elastic rods, which can enhance stiffness by locking the rods to the robot base. Note that these mechanisms are relatively large in size or heavy in weight. A central-cable-tensioning mechanism was designed by Degani et al. (2006) for a continuum endoscope whose stiffness can intensify under the tension of a built-in cable passing through the central axis of the robot. A similar design was used in a tension-stiffening continuum catheter made of a series of spherical joints connected end to end (Chen et al., 2010). These mechanisms are feasible to implement but it remains difficult to evenly distribute the tension of the built-in cable along the arm, which means that the stiffness change on the joints might be unequal, especially when the arm bends. Moreover, such mechanisms will occupy the central channel of the continuum manipulators and make it difficult to route tubing to the end-effector. Jamming-based mechanisms can stiffen the robots without affecting their position and shape. Cheng et al. (2012) and Ranzani et al. (2015) combined soft continuum robots with a granular jamming-based stiffening mechanism. In addition, several tension cables were further used to strengthen the jammed body (Cheng et al., 2012). However, the above approaches may make the robots less compact. Kim et al. (2013), Moses et al. (2013), and Santiago et al. (2016) presented layer jamming mechanisms that cover the surface of snake-like and tail-like manipulators. The thin layers can keep the

| Stiffness regulation approach                  | Total length (mm) | Self-weight (g) | Payload (N) | Deflection (mm) | Stiffness (N/mm) | Percentage change in stiffness |
|-----------------------------------------------|-------------------|----------------|-------------|-----------------|-----------------|------------------------------|
| Antagonistic actuation                        | 200               | –              | 15          | 0.093           |                  | 156% (Stilli et al., 2014)   |
| Rack-locking mechanism                       | 87                | About 1.6      | About 16   | 0.049           |                  | 165% (Kim et al., 2014b)     |
| Drive-rod-locking mechanism                  | 290               | 98             | 102         | 0.042           |                  | About 156% (Yagi et al., 2006) |
| Central-cable-tensioning mechanism           | 34                | About 7       | About 3.8  | 0.049           |                  | About 190% (Degani et al., 2006) |
| Layer jamming                                | 400               | About 1.5     | About 1.5  | 0.049           |                  | About 925% (Moses et al., 2013) |
| Granular jamming                             | 250               | –              | About 90   | 0.049           |                  | About 90% (Cheng et al., 2012) |

This paper also used additional tension cables to further strengthen the manipulator.

– Not reported.
internal passage free of obstruction but a vacuum pump was still required as an extra power source.

Therefore, new methods are still required to achieve effective and reliable stiffness adjustment for a continuum robot with slender structure. In the above prototypes, rod-driven type soft continuum robots are compact, compliant, and easy to control with bidirectional actuation properties. In this paper, we select this solution to develop a continuum robot with variable stiffness for potential use in soft manipulation under unstructured environments, such as rescue, space exploration, and medical devices.

Mathematical models are essential for the design and application of soft continuum robots when facing variable conditions, including varying their stiffness. Nevertheless, most previous studies focused on the kinematics and dynamics of the robot body, while few models were concerned with the variable stiffness mechanisms. The kinematics models of continuum robots are generally based on the piecewise constant-curvature assumption (Jones and Walker, 2006) and use in-plane bending angles and the angles of the associated bending planes to characterize the configurations (Simaan et al., 2009). These models established mappings between the actuator space, configuration space, and task space (Mahl et al., 2014; Webster and Jones, 2010). However, pure kinematic models usually ignore the effects of external loads, notably gravity, which limits their application.

The statics of continuum robots, taking the external forces into account, can be approximately analyzed by beam theory. Desired configurations can be transformed into tendon inputs utilizing a Euler–Bernoulli beam model (Camarillo et al., 2008) or Cosserat beam model (Renda et al., 2012). Beam models can also be represented by quaternions as configuration variables, which naturally incorporate inflation/extension, bending, twisting, extension, and shear deformations of extensible continuum manipulators (Tunay, 2013). These beam models usually ignore the interactions between individual robotic segments, including friction and geometric constraints. Xu and Simaan (2009) combined elliptic integrals and optimization to obtain the desired external loads of a multi-segment continuum robot. For more general cases, the curve routing paths of the tendons have been considered (Rucker and Webster, 2011), and the influence of internal friction as well as the variation in curvature were captured (Rone and Ben-Tzvi, 2014a, 2014b). The force–deflection relationships of multi-segment continuum robots can also be derived by lumped-mass approaches (Kang et al., 2013) or via a compliance matrix (Qi et al., 2016) where screw theory (Dai, 2012) and Rodrigues’ formula (Dai, 2015) have been used. It was found that in the above static models, whether using the Newton method (Qi et al., 2016) or the virtual work principle (Rone and Ben-Tzvi, 2014a), the configuration parameters were related to Young’s modulus of the robot, as well as applied forces including actuation and external forces. However, it remains difficult to measure and control the non-uniformly distributed actuation forces along the drive cables/rods, especially when configuration and stiffness can vary in such flexible robots.

In this paper, a static model and variable stiffness mechanism are developed based on a general class of rod-driven continuum robots. The contributions of this work include the following. (1) Using the geometric constraint and the principle of virtual work, a static model considering the influence of actuation displacement, rod elasticity, gravity, friction, and external load is established. Compared with previous static models, it uses the actuation displacements derived from geometric constraint equations, rather than the actuation forces on the drive rods, as the inputs to predict and control body deformation under external pay-loads. Therefore, the problem that the non-uniformly distributed actuation forces of such continuum robots cannot usually be measured is solved. In addition, the phenomenon that the robot body can show an S-shaped curve subject to only a single-directional force is revealed. The relationship among friction, room temperature, and the amplitude of current was also revealed. The signal function was used to judge if the friction forces do the virtual work. (2) A new variable stiffness mechanism with built-in Shape Memory Alloy (SMA) springs is proposed. Such a mechanism can ‘lock’ the robot body by tuning the internal friction between the drive rods and constraint disks fixed on the central backbone.

The paper is organized as follows: Section 2 describes the statics model of a general type of rod-driven continuum robot. In Section 3, the working principle of the variable stiffness mechanism and its model are revealed. Based on the above methods, a prototype of the soft continuum robot is proposed in Section 4. Section 5 validates the statics model through experiments and presents the relationship between the variable friction and stiffness. Section 6 summarizes the paper and discusses future work.

2. Geometric constraint-based modeling and analysis of the generalized rod-driven continuum robot

The generalized design of rod-driven continuum robots has a central backbone and $M$ modules (sections) (Rone and Ben-Tzvi, 2014b; Simaan et al., 2009), as shown in Figure 1. Each module is composed of three drive rods and $N$ constraint disks, allowing for two DOF bending motion. The backbone and drive rods of the robot discussed in this paper are made of a hyperelastic alloy (e.g. NiTi alloy) so that the drive rods are able to pull and push bidirectionally. The drive rods, fixed to the end disk of the corresponding modules and moving through other constraint disks freely, can control the configuration of the continuum robot with different displacements.
2.1. Kinematic configurations

We define the constraint disk \( n = 1, 2, 3, \ldots, N \) of module \( m = 1, 2, 3, \ldots, M \) as \( \text{disk}_{m,n} \). Thus, \( \text{disk}_{m,0} \) represents the base disk of module \( m \), which is also the end disk of module \( m-1 \), \( \text{disk}_{m-1,N} \). Next, the segment between \( \text{disk}_{m-1,n} \) and \( \text{disk}_{m,n} \) is abbreviated as \( \text{Seg}_{m,n} \). Here, we introduce a new variable \( u (u = 1, 2, \ldots, M) \) to describe the situation in which the drive rod \( j \) \((j = 1, 2, 3)\) is fixed to the end disk of module \( u \), which is marked as \( \text{rod}_{u,j} \).

The spatial frame \( O_0 \) is fixed to the base of the continuum robot and a series of local frames \( O_{m,n} \) is established at each segment, as shown in Figure 2, with all of the \( x \)-axes point to the \( \text{rod}_{1,1} \). Assuming each segment is a constant-curvature mini-arc, the shape of \( \text{Seg}_{m,n} \) can be represented by the two sets of configuration parameters \( \theta_{m,n} \) and \( \varphi_{m,n} \), which make up the vector \( q_{m,n} \) as

\[
q_{m,n} = \begin{bmatrix} \theta_{m,n} & \varphi_{m,n} \end{bmatrix}^T
\]  

Integrating all the vectors, the overall robot configuration can be uniquely described by the generalized coordinate vector \( q \) as

\[
q = [q_{1,1}^T \cdots q_{1,N}^T \cdots q_{2,1}^T \cdots q_{2,N}^T \cdots q_{M,1}^T \cdots q_{M,N}^T]^T
\]

\( T_{m,n} \), defined as Equation (25) in Appendix B, represents the transformation matrix from frame \( O_{m,n-1} \) to frame \( O_{m,n} \). Therefore, \( 1,0 T_{m,n} \) can be derived as shown in Equation (3) to obtain the pose \( R_{m,n} \) and position \( P_{m,n} \) of frame \( O_{m,n} \)

\[
1,0 T_{m,n} = T_{1,1} T_{1,2} \cdots T_{1,n} T_{2,1} \cdots T_{m,n} = \begin{bmatrix} 1 & 0 & P_{m,n}^T \\ 0 & 1 \end{bmatrix}
\]

2.2. Geometric constraint

For most slender continuum robots (Camarillo et al., 2008; Xu and Simaan, 2009), their bending stiffness is lower than their torsional stiffness, justifying the assumption that the twisting deformation can be ignored in comparison to the bending deformation. Therefore, the central backbone and the drive rods are always assumed parallel, as shown in Figure 2(b). The geometric constraint mentioned above means that the length change of \( \text{rod}_{u,j} \) within the \( \text{Seg}_{m,n} \) is a function of the configuration parameters, which can be expressed as

\[
\eta_{u,j,m,n} = -r \theta_{m,n} \cos (\omega_{u,j} - \varphi_{m,n})
\]  

where \( r \) and \( \omega_{u,j} \) are the distribution radius and the angle relative to the \( x \)-axis of drive \( \text{rod}_{u,j} \) respectively. The three drive rods of each module are evenly distributed on a circle with radius \( r \), which means \( \omega_{u,j} = \frac{2\pi}{3} \).

In addition, the relative sliding displacement between \( \text{rod}_{u,j} \) and \( \text{disk}_{m,n} \) is denoted as \( D_{u,j,m,n} \) and obtained by

\[
D_{u,j,m,n} = \sum_{n=0}^{N} \sum_{m=0}^{M} \eta_{u,j,m,n} \quad (u = m)
\]

\[
D_{u,j,m,n} = \sum_{n=0}^{N} \sum_{m=0}^{M} \eta_{u,j,m,n} + \sum_{n=0}^{N} \eta_{u,j,m,n} \quad (u > m)
\]

where \( D_{u,j,m,n} = 0 \) \((m = u)\) since the drive rods are fixed at the end disks of each module. When \( \text{disk}_{1,0} \) is considered, the relative sliding displacement \( D_{u,1,0} \) is simplified as \( D_{u,1} \), representing the actuation displacement of the \( \text{rod}_{u,1} \), that is, the total length displacement of the drive rod. Thus, the geometric constraint is also reflected by Equation (5), showing that the sum of length change in individual segments equals the actuation displacement. If there are three drive rods for each of the two DOF modules, the redundancy of the third drive rod \( D_{u,3} \) can be expressed as

\[
D_{u,3} = a_{u,3} D_{u,1} + b_{u,3} D_{u,2}
\]  

where \( a_{u,3} \) and \( b_{u,3} \) are the coefficients, which can be obtained by Equations (26) and (27) in Appendix B. The above-described geometric constraint not only reflects the relationship between each actuation displacement and the resulting configuration of the continuum robot but also...
reduces the dependence of the static model on the actuation forces, as shown in the following sections.

2.3. Statics analysis based on the principle of virtual work

A typical configuration of the continuum robot under static equilibrium is shown in Figure 3, and the virtual work of the system $\delta W$ under the virtual displacement $\delta q$ is established as

$$
\delta W = \delta W_{el} + \delta W_{ac} + \delta W_{gr} + \delta W_{lo} = 0
$$

where $\delta W_{el}$, $\delta W_{ac}$, $\delta W_{gr}$, and $\delta W_{lo}$ represent the virtual works of elastic force, actuation force, gravity, and external load, respectively.

2.3.1. The elastic force. The central backbone and the drive rods made of the hyperelastic alloy are considered as Euler–Bernoulli beams with linear and isotropic relations between strain and stress (Nemat-Nasser and Guo, 2006). The virtual work of internal elastic forces, $\delta W_{el}$, can be derived by summing the virtual work of the backbone and drive rods of each module as

$$
\delta W_{el} = - \sum_{m=1}^{M} \sum_{n=1}^{N} M_{m,n} \delta \kappa_{m,n} - \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{u=1}^{3} \sum_{j=1}^{3} M_{u,j,m,n} (l + \eta_{u,j,m,n}) \delta \kappa_{u,j,m,n}
$$

where $M_{m,n}$ and $M_{u,j,m,n}$ indicate bending moments of the backbone and the rod $u,j$ within $Seg_{m,n}$, and $\kappa_{m,n}$ and $\kappa_{u,j,m,n}$ are the corresponding curvatures. All of these quantities are functions of $\theta_{m,n}$, as Equations (28)–(31) show in Appendix B. $l$ is the backbone length of every segment.

2.3.2. The Actuation Force. As mentioned in Section 2.2, there are just two independent drive rods for each module. Therefore, the actuation force of the robot can be equivalent to the tension or pressure of two drive rods in module $u$ marked as $\tau_{u,1}$ and $\tau_{u,2}$. The corresponding actuation displacements $D_{u,1}$ and $D_{u,2}$ are obtained from Equation (5).

The virtual work of the actuation force, $\delta W_{ac}$, accumulates the virtual work of equivalent actuation forces in two drive rods as

$$
\delta W_{ac} = \sum_{u=1}^{M} (\tau_{u,1} \delta D_{u,1} + \tau_{u,2} \delta D_{u,2})
$$

Note that, in this paper, although the actuation force of each drive rod is considered in the model, its value does not need to be known because of the introduction of geometric constraints.

2.3.3. The gravity effect. The gravity of the constraint disk is $G$, and the density of the hyperelastic alloy used as the backbone and the drive rod is $p$ with their center of gravity $C_{m,n}$ approximately located in the middle of $Seg_{m,n}$, which can be expressed as

$$
C_{m,n} = P_{m,n} - P_{m,n-1}
$$

where $P_{m,n}$, obtained from Equation (3), represents the position of disk $k$, $d_{n,m}$ and $d_{m}$ represent the diameter of the backbone and the drive rods, respectively. Then, the virtual work of gravity, $\delta W_{gr}$, including the virtual work of the gravity of constraint disks, backbone, and rods, can be derived as

$$
\delta W_{gr} = \sum_{m=1}^{M} \sum_{n=1}^{N} \left( G \delta P_{m,n} + \frac{\pi \rho (d_{n,m}^2 + 3d_{m}^2)}{4} + \frac{\pi \rho d_{m}^2}{4} \sum_{k=m}^{M} \sum_{j=1}^{3} \eta_{u,j,m,n} \delta C_{m,n} \right)
$$

2.3.4. The external load. The virtual work of the external load, $\delta W_{lo}$, accumulates the virtual work of external loads and moments as

$$
\delta W_{lo} = \sum_{m=1}^{M} \sum_{n=1}^{N} \left( F_{c,m,n} \delta P_{m,n} + M_{c,m,n} \delta A_{m,n} \right)
$$

where $F_{c,m,n}$ and $M_{c,m,n}$, as shown in Figure 3, represent the external load and the external couple acting on disk $m,n$, respectively. The term $\delta A_{m,n}$ represents the virtual angular displacement and $A_{m,n}$ can be obtained from Equation (32) in Appendix B.

2.4. The equilibrium equation of the generalized rod-driven continuum robot

The virtual displacements in Equations (8)–(12) are functions of the generalized coordinates $q$. Equation (7) can be transformed into the product of generalized force $Q$ and generalized coordinate virtual displacement $\delta q$ as
\[ \delta W = \sum_{t=1}^{\text{Len}(q)} (Q_{el,t} + Q_{ac,t} + Q_{gr,t} + Q_{lo,t}) \delta q(t) = 0 \quad (13) \]

where \( \text{Len}(q) \) is the length of \( q \). \( Q_{el,t} \), \( Q_{ac,t} \), \( Q_{gr,t} \), \( Q_{lo,t} \) are generalized elastic forces, the actuation force, gravity, and external load, respectively, as Equation (33) shows in Appendix B. However, except \( q \), there are still \( 2 \times M \) unknowns, the actuation forces \( \tau_{u1} \), \( \tau_{u2} \) (\( u = 1, 2, \ldots, M \)) in Equation (13). Unlike the actuation forces given by the sensors in the conventional static model, this paper takes the actuation displacement of the drive rod that is derived from the geometric constraint as the input. Considering the redundancy, only two inputs are required for each module, so an additional \( 2 \times M \) equations can be derived from Equations (4) and (5), as shown in Equation (14). By combining Equation (14) with Equation (13), the configuration parameters \( \theta_{m,n} \) and \( \varphi_{m,n} \) in each local frame can be solved for

\[ D_{u,j} = -\sum_{m=1}^{u} \sum_{n=1}^{N} r \theta_{m,n} \cos(\omega_{u,j} - \varphi_{m,n}), (j = 1, 2), \]

\[ (u = 1, 2, \ldots, M) \quad (14) \]

3. Modeling and analysis with Shape Memory Alloy initiated variable stiffness

3.1. Variable stiffness mechanism

Based on the generalized rod-driven continuum robot design, we propose a stiffness regulation mechanism that can change the friction between the drive rods and constraint disks. Unlike the antagonistic actuators or central-cable-tensioning methods, which apply a longitudinal force at the tip to stiffen the entire manipulator, possibly leading to uneven internal forces at individual segments/joints, our design uses distributed mechanisms along the continuum manipulator to achieve local stiffness regulation for individual segments.

Fig. 4. Leverage mechanism for stiffness adjustment. SMA: Shape Memory Alloy.

As shown in Figure 4(a), there are three groups of levers assembled on a pair of adjacent constraint disks. SMA springs are utilized to drive the levers due to their high power-weight ratio, flexibility, and compactness making them suitable for implementation in such a slender, narrow space. Each group of levers is pulled by one SMA spring through a thread tension mechanism, as shown in Figure 4(b). In the unlocked mode, when the SMA spring and thread are loose, there is no force applied to the corresponding lever on the upper and lower constraint disks and the drive rods can move freely. In the locked mode, the SMA spring is heated by electrical current, \( I \), and therefore pulls two threads at both ends with a tension of \( F_t \). The thread goes through the constraint disk and lever through a curved hole. Once it is tensioned, it will apply a force \( F_1 \) to one end of the corresponding lever at point B. So, the lever will rotate about the pivot, C, and apply a force \( F_2 \) to the other end, A, and therefore clamp the drive rods to the corresponding constraint disk, as shown in Figure 4(c). In this way, we can use three SMA springs to lock the movement of the rod on two adjacent constraint disks. By increasing the current to the SMA springs, the contact force to the drive rods and the friction between the rods and constraint disks will be increased until they are locked together, which means the overall stiffness of the continuum robot increases.

3.2. The tensions of the SMA

When the stiffness of the proposed continuum robot changes, new variables such as the tension of the SMA and the resulting frictional force will be introduced. The SMA spring between \( \text{disk}_{m,n-1} \) and \( \text{disk}_{m,n} \) is abbreviated as \( \text{SMA}_{m,n} \), as shown in Figure 5. Here, \( n \) can only take even numbers since every two adjacent constraint disks share
one SMA. Then, the position of the upper fixed point \(H_{j,m,n}\) of \(\text{SMA}_{j,m,n}\) in \(\text{disk}_{m,n}\) can be deduced by the following equation

\[
H_{j,m,n} = R_{m,n} \begin{bmatrix} r \cos \Omega_{u,j} & r \sin \Omega_{u,j} \end{bmatrix}^T + P_{m,n}
\]

(15)

where \(r\) is the distribution radius of the SMA spring, \(\Omega_{u,j}\) is the distribution angle of \(\text{SMA}_{j,m,n}\) relative to the x-axis and \(\Omega_{u,j}^2 \cos \phi = 2\pi/3\). \(l_{j,m,n}\) is the length of \(\text{SMA}_{j,m,n}\), which can be obtained from Equation (16)

\[
l_{j,m,n} = \|H_{j,m,n} - H_{j,m,n-1}\|\]

(16)

In addition, the tension \(F_{j,m,n}\) of the SMA spring is also related to the temperature \(T\) as

\[
F_{j,m,n} = \frac{G_T d_s}{8 k N_{s}} (l_{j,m,n} - l_{\text{SMA}})
\]

(17)

\[
F_{j,m,n} = -F_{j,m,n-1} = \frac{H_{j,m,n} - H_{j,m,n-1}}{l_{j,m,n}} F_{j,m,n}
\]

(18)

where \(G_T\) is the shear modulus that changes with temperature (An et al., 2012; Liang and Rogers, 1990), and it remains constant when \(T > 80^\circ\mathrm{C}\) (Ma et al., 2010; Salerno et al., 2016). \(d_s\) is the diameter of the SMA spring, \(k\) is the spring index of the SMA, \(N_s\) is the number of turns, \(l_{j,m,n}\) is the length of the SMA spring from Equation (16), and \(l_{\text{SMA}}\) is the original length of the SMA spring. When the SMA spring resistance \(R_s\) is fixed, the relationship between temperature \(T_w\) of the SMA and the electric current \(I\) is deduced at room temperature \(T_0\), as in Equation (19), where \(h_i\) is the coefficient of heat transfer

\[
T_w = \frac{I^2 R_s}{h_i N_s k d_s^2 \pi^2} + T_0
\]

(19)

The virtual work of the tension of the SMA spring, \(\delta W_{\text{sma}}\), can be derived from the sum of the products of the virtual displacement \(\delta H_{j,m,n}\) and the tension \(F_{j,m,n}\) of the SMA spring as

\[
\delta W_{\text{sma}} = \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{j=1}^{3} (F_{j,m,n} \delta H_{j,m,n})
\]

(20)

3.3. The friction

The tension of the SMA makes the friction between \(\text{rod}_{u,j}\) and \(\text{disk}_{m,n}\) increase rapidly, where \(u = m\) is considered because \(\text{rod}_{u,j}\) is not locked with the constraint disk in other modules. Therefore, the virtual work of internal friction, \(\delta W_{fr}\), accumulates all the virtual work of sliding friction between the drive rods and the constraint disks as

\[
\delta W_{fr} = \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{j=1}^{3} (-\text{sgn}(D_{u,j,m,n}) f_{j,m,n} \delta D_{u,j,m,n})
\]

(\(u = m\))

(21)

It is difficult to directly judge whether all friction forces are sliding friction to do virtual work, so \(\text{sgn}(D_{u,j,m,n})\) is used to estimate the utility and direction of the friction. If \(D_{u,j,m,n} = 0\), it shows that the drive rod and the constraint disk never slide at the contact point. Therefore, there is no virtual work of static friction and the virtual work is mapped into 0 by sign function \(\text{sgn}\). If \(D_{u,j,m,n} \neq 0\), there is still a trend of sliding with sliding friction rather than an ideal constraint so that the virtual work should be considered, and the sign function extracts the sliding direction.

If there is sliding friction, we can use \(f_{j,m,n-1} = f_{j,m,n} (n = \text{even number})\), which is the function of the tension of SMA \(F_{j,m,n}\) (presented as \(F_i\) in Figure 4) and can be obtained as

\[
f_{j,m,n-1} = f_{j,m,n} = 2 \mu \cos \alpha \frac{l_1}{l_2} F_{j,m,n}
\]

(22)

where \(l_1\) and \(l_2\) are the lever arms, \(\mu\) is the friction coefficient, and \(\alpha\) is the winding angle of the wire.

3.4. The equilibrium equation considering the SMA-based stiffness regulation

Like the equilibrium equation proposed in Section 2.4, here we introduce the virtual displacements in Equations (20) and (21), which are also functions of the generalized coordinates \(q\). Equation (13) can be rewritten as

\[
\delta W = \sum_{i=1}^{\text{len}(q)} (Q_{el,i} + Q_{ac,i} + Q_{fr,i} + Q_{fr,t} + Q_{sma,i} + Q_{fr,t})
\]

\[
\delta q(t) = 0
\]

(23)

where \(Q_{sma,t}, Q_{fr,t}\) are generalized tensions of the SMA springs and sliding friction, respectively, as Equation (34) shows in Appendix B.

4. The variable stiffness-based soft continuum robot

4.1. The new continuum manipulator

Based on the design mentioned above, a prototype is designed, which consists of a variable stiffness continuum manipulator, an end-effector, an actuation box, and an electric cabinet, as shown in Figure 6. The design aims to manipulate targets and tune the stiffness of the main body in unstructured environments. There are two modules included and each module is composed of a central backbone, three drive rods evenly distributed 120 degrees apart, and 11 constraint disks. The NiTi alloy is assigned as the backbone and the drive rods. The manipulator has a total length of 880 mm and an outer diameter of 38 mm, which is covered with a nylon mesh. Note that the nylon mesh covering the constraint disks will limit the twisting deformation of the arm. The end-effector is mounted on the top of the manipulator and is capable of being replaced.
for different tasks. In this paper, a three-fingered gripper driven by a linear motor is used. A camera (ZBS-001, EBOSI Corp., China) providing vision is embedded in the center of the gripper. The electromagnetic sensor (3D Guidance trakSTAR, Northern Digital Inc., Canada) can detect and return its position in the magnetic field, which was established by the transmitter. Since the point on the backbone cannot be directly measured, both sides of the cross-section of the constraint disk were measured, and the midpoint of them can be calculated and recognized as the centroid of the constraint disk on the backbone.

4.2. Rod-driven actuation

As shown in Figure 7, the entire actuation box can be divided into two parts. The right-hand half contains six screw slider mechanisms transmitting the rotational motion of the motors to the linear motion of the drive rods. In order to detect the tension and compression on the rods, force sensors (MBZY-1, ZN Corp., China) are installed on each slider at a symmetrical position to the drive rod through an equal-arm lever mechanism. This design makes external load detection feasible. The left-hand half is used to arrange the corresponding motors (RE25, Maxon Motor AG, Switzerland) and their drivers.

The control system consists of a PC and various control objects linked by a Controller Area Network (CAN) bus, as shown in Figure 8. The configuration and stiffness control routines are coded in the VC++ language and run on the PC. The controlled objects mentioned include six direct current (DC) brush motors, data acquisition cards (ICAN-4017, GHD Corp., China), and a linear motor (LC1574AQ, China) for the gripper. The data acquisition card can collect the analog signals of the force sensors and output the signal for SMA current regulation.

4.3. Specifications of the continuum robot

The geometric and material parameters of the presented continuum manipulator are detailed in Table 2.

5. Effect of variation of actuation displacement, external load, and stiffness

In this section, a series of simulations with different actuation inputs and external loads was implemented and compared against the experimental results. In addition, the
influence of the internal friction on the stiffness was demonstrated through simulation and experiment. To simplify the results, the deformation of the manipulator is represented by the curve of its backbone.

### 5.1. Effect of change of actuation displacement with zero external load

In this case, four sets of actuation displacements are given without external loads as the inputs, which are $D_{u,j} = [-4, 2, -2, 4, -2]$ (Conf. 1.1), $D_{u,j} = [2, 2, -4, 4, -2]$ (Conf. 1.2), $D_{u,j} = [2, -4, 2, -2, -2, 4]$ (Conf. 1.3), and $D_{u,j} = [2, -4, 2, -2, -2, 4]$ (Conf. 1.4). As a reference, Conf. 0 is also presented, which is the initial state of the manipulator $D_{u,j} = [0, 0, 0, 0, 0, 0]$ but only under gravity, as shown in Figure 9. The red line, blue dashed line, and gray circles present the deformation of modules 1 and 2 in simulations and the results of experiments, respectively. If only kinematics are considered, Conf. 1.1 and Conf. 1.2 are symmetric about $(0, 0, 880)$, yet due to gravity, the endpoint positions are all shifted toward the negative direction of the $x$-axis, whose positions obtained by simulations and experiments are listed in Table 3. The results show that in this case, the ratio of the positioning error at the manipulator tip to its length, defined as the error ratio, is less than 1.83%.

### 5.2. Effect of change of external load with the stand-still actuation displacement

In this case, the actuation displacement of all the drive rods is given to be zero ($D_{u,j} = 0$) to obtain the deformation of the manipulator under identical external loads acting on different points in the $x$–$z$ plane. Firstly, $F = -3$ N is applied to the midpoint (Conf. 2.1) and the endpoint (Conf. 2.2) of the manipulator respectively, then $F = 3$ N is applied to the midpoint (Conf. 2.3) and the endpoint (Conf. 2.4) of the manipulator respectively, as shown in Figure 10. It can be seen that the closer the load force $F$ is to the end, the larger the deflection of the manipulator generated. In addition, the deformations of Conf. 2.1 and 2.3 as well as Conf. 2.2 and

### Table 2. Material and geometric parameters of the prototype.

| Property                | Value       | Property                | Value       |
|-------------------------|-------------|-------------------------|-------------|
| $E$ – Young’s modulus of NiTi alloy | 160 GPa | $h_y$ – A coefficient of heat transfer | 6.3 W/(m$^2$·°C) |
| $\rho$ – Density of NiTi alloy | 6.5 g/cm$^3$ | $\alpha$ – Turning angle of wire | $\pi/3$ |
| $I_{ru}$ – Backbone radial moment of inertia | 0.78 mm$^4$ | $\kappa$ – Spring index of SMA | 9 |
| $I_{rv}$ – Drive rod radial moment of inertia | 0.25 mm$^4$ | $N_i$ – SMA spring number of turns | 30 |
| $G$ – Gravity of constraint disk | $5 \times 10^{-3}$ kg | $D_i$ – SMA spring diameter | 4.5 mm |
| $\mu$ – Coefficient of friction | 1 | $d_{b_i}$ – The diameter of backbone | 2 mm |
| $G_T$ – The shear modulus of SMA spring | 35 GPa | $d_{r_o}$ – The diameter of drive rods | 1.5 mm |
| $r$ – Distribution radius of SMA and rods | 15 mm | $l$ – Length of Segment | 40 mm |
| $\omega_{1,1}$ – Angle between rod$_{1,1}$ and x-axis | 0 | $T_0$ – Room temperature | 20 °C |
| $\omega_{2,1}$ – Angle between rod$_{2,1}$ and x-axis | $\pi/3$ | | |

SMA: Shape Memory Alloy.

### Table 3. Results of endpoint under different actuation displacements.

| Configurations | Conf 0 | Conf 1.1 | Conf 1.2 | Conf 1.3 | Conf 1.4 |
|----------------|--------|----------|----------|----------|----------|
| **Experiments** | (-53.6, 0.0, 876.3) | (-220.6, 2.6, 856.2) | (126.4, 3.2, 873.4) | (-102.3, 110.2, 879.5) | (-105.3, -116.5, 880.4) |
| **Simulations** | (-58.9, 1.5, 891.5) | (-215.6, 0.0, 861.4) | (119.6, 0.0, 870.2) | (-110.4, 115.2, 875.4) | (-112.3, -113.2, 876.8) |
| **Deviation** | 16.1 | 7.6 | 8.16 | 10.4 | 8.5 |
| **Error ratio** | 1.83% | 0.86% | 0.93% | 1.18% | 0.97% |

Fig. 9. Deformations under different actuation displacements. (Color online only.)
2.4 are almost symmetric but with a slight difference due to gravity.

Next, we keep the actuation displacement at zero and change the value of the external loads where all of them are applied on the endpoint. \( F = 0.5 \) (Conf. 2.5), \( F = 2 \) (Conf. 2.6), \( F = 0.5 \) (Conf. 2.7), and \( F = 1 \) (Conf. 2.8) are exerted, as shown in Figure 11. With the increase of the value of \( F \), the displacement of the endpoint on the \( x \)-axis becomes larger, and the manipulator shows a double ‘S-shaped’ curve. Figure 11 not only reveals the effects of different loads on the deformation but also shows the phenomenon that each module of the manipulator presents an S-shaped curve rather than the C-shaped curve usually observed in a cantilever beam. As the external load increases, the S-shape becomes more obvious. This is because we limit all the actuation displacements \( Du,j = 0 \). According to the geometric constraint given by Equation (5), an S-shaped manipulator allows for different local length changes of a drive rod in individual segments, but the total length change of the drive rod remains constant. If the manipulator bends in a C-shape, the local length of a drive rod in all segments will simultaneously become larger or smaller depending on which side (dorsal or ventral) the drive rod locates on, which is not consistent with the geometric constraints.

5.3. Effect of different actuation displacements with external loads

In this case, the performance of the manipulator under the same external load, \( F = -1 \) N at the end disk, but with different actuation displacement, is shown. To better compare the results under different conditions, all the actuation displacements make the robot only bend in the \( x-z \) plane. \( D_{u,j} = [0, 0, 0, 5, -10, 5] \) (Conf. 3.1), \( D_{u,j} = [5, -2.5, -2.5, 5, -10, 5] \) (Conf. 3.2), \( D_{u,j} = [0, 0, 0, -5, 10, -5] \) (Conf. 3.3), \( D_{u,j} = [-5, 2.5, 2.5, -7.5, 15, -7.5] \) (Conf. 3.4), and \( D_{u,j} = [-10, 5, 5, -10, 20, -10] \) (Conf. 3.5) are presented in Figure 12. For comparison, Conf. 2.6 is also introduced with \( D_{u,j} = [0, 0, 0, 0, 0] \). In addition, the actuation displacement of the first module of Conf. 3.1 and 3.3 remains also zero, so their first module shows an S-shaped curve. Figure 12 shows that if the bending direction of two modules is the same, the larger the actuation displacement input was given, the larger deflection of the manipulator exhibited. Moreover, \( D_{u,j} = [5, -2.5, -2.5, 5, -2.5] \) (Conf. 3.6), \( D_{u,j} = [10, -5, -5, -2.5, 5, -2.5] \) (Conf. 3.7), \( D_{u,j} = [10, -5, -5, 0, 0] \) (Conf. 3.8), \( D_{u,j} = [-5, 2.5, 2.5, 5, -5, 2.5] \) (Conf. 3.9), \( D_{u,j} = [-10, 5, 5, 2.5, -5, 2.5] \) (Conf. 3.10), and \( D_{u,j} = [-20, 10, 10, 2.5, -5, 2.5] \) (Conf. 3.11) are shown in Figure 13, which makes the first module and the second module bend in opposite directions. Similarly, the simulation results obtained by the proposed model are well matched with the experimental results. In particular, at Conf. 3.8, the actuation displacements of module 2 were \( D_{u,j} = [0, 0, 0] \), but it still presents an upward curve due to the downward curve of module 1.
It can be noted from the above three cases that the deformations of the simulations are close to the experiments, but there still are some errors, which may be caused by some of the assumptions mentioned in Section 2. The maximum deviation occurred in Conf. 3.5 with a position error at the endpoint of 19.6 mm, and the error ratio is 2.23%.

5.4. Effect of variable stiffness demonstration and validation

As mentioned in Section 3, the stiffness of the manipulator can be improved by increasing the internal friction, whose relationship can also be solved by using the proposed statistics model with actuation displacement inputs. According to Equation (22), the maximum sliding friction depends on the electric current $I$ of the SMA springs. Therefore, in this case, the deformations of the manipulator obtained by experiments and simulations under the same end load 100 g but different electric currents are shown in Figures 14 and 15, respectively. Conf. 0 (0 N, 0 A) is the initial state without payload. Then, if $F = 1$ N is exerted at the endpoint, its configuration changes to Conf. 2.6 (−1 N, 0 A) with a displacement in the $x$-axis of 158.2 mm in the experiment at the endpoint. However, if the currents are applied before the payload, the deformation will change to Conf. 4.1 (−1 N, 0.5 A), Conf. 4.2 (−1 N, 1 A), and Conf. 4.3 (−1 N, 2 A), with the corresponding displacements in the $x$-axis of 132.5, 91.6, and 68.7 mm, respectively. In addition, we found that the SMA springs have little effect on the configuration of the continuum manipulator because the tension of the SMA springs is local and applied to all drive rods.

Furthermore, keeping the initial condition as Conf. 0, we change the magnitude of the end load and plot the relationship of payload and displacement under different currents, as shown in Figure 16. It can be seen that the modulation of current does promote the ability of the manipulator to maintain the desired configuration. The quotient between the $\Delta F$ and the corresponding $\Delta x$, as shown in the Equation (24), is used to characterize the stiffness of the manipulator

$$K = \frac{\Delta F}{\Delta x} \quad (24)$$

When external loads are lower than 1.5 N, a nonlinear relationship can be observed and the stiffness will decrease slightly with the increase of external loads. This is because, firstly, the constitutive model of the NiTi alloy itself conforms to this trend. Another factor is that the internal static friction provided by the stiffening mechanism is not enough.
to resist the deformation of the manipulator. However, when the external loads are greater than 1.5 N, the stiffness of the manipulator increases again and approximates to a constant, indicating that the load has less influence on the stiffness under this situation. This is because, since the actuation displacements of all the drive rods are limited to zero, this geometric constraint converts into a motion constraint, which exerts a non-negligible influence on the mechanical properties of the manipulator. Under this condition, when the external load is 3 N, the currents of 2, 1, and 0.5 A can increase the stiffness of the manipulator by 287%, 192%, and 121%, respectively.

5.5. Discussion on the variable stiffness performance

The main structure of our continuum manipulator is composed of NiTi alloy with its intrinsic compliance and light weight. The total length and the weight are 880 mm and 137 g, respectively. Before its stiffness is changed, the deflection of the manipulator is 54 mm when under gravity (Conf. 0). Then it increases to 287 mm when subjected to an extra payload of 3 N (Conf. 2.2), and the stiffness is 0.0129 N/mm according to Equation (24). By heating the SMA springs with a current of 2 A, the deflection under the same payload becomes 135 mm, and the stiffness increases 287% to 0.0370 N/mm. In this situation, the external load is 2.2 times the weight of the manipulator and the deflection is 15.3% that the manipulator length. The 3 N payload is 287% to 0.0370 N/mm. In this situation, the external load is 2.2 times the weight of the manipulator and the deflection is 15.3% that the manipulator length. The 3 N payload is considered as the upper limit of this prototype, as shown in Figure 16. If the external load is increased further, the large deflection will make it difficult to control the manipulator.

Compared with the previous variable stiffness continuum robots summarized in Table 1, the percentage change in stiffness, 287%, of our manipulator is at a relatively high level. However, the absolute stiffness of our manipulator may initially appear to be not so impressive. This can, however, be explained by using beam theory (Boresi et al., 1985). If a continuum manipulator is approximately considered as a cantilever, the stiffness at the tip and load capacity are inversely proportional to the cubic of its total length. Considering our manipulator has a much greater length than others, we believe the results are good.

In addition, we observe that the SMA material has some limitations, such as the hysteresis, low control accuracy, and long cooling time. However, compared with position control requirements (usually at the millimeter level for positioning and second level for response time (Webster and Jones, 2010)) for this type of manipulator, the stiffness control does not require very high precision or speed. It is usually allowable for the continuum manipulator to spend some time stiffening its body after reaching the desired position. In this paper, the SMA springs can reach the phase-transition temperature within 1–2 seconds by applying a current of 2 A and the stiffness of the manipulator will soon have a significant increase in 5–6 seconds. On the other hand, at room temperature around 20ºC, the SMA spring will take about 22–29 seconds to cool down and unload the internal stress, which is also acceptable.

6. Conclusions

This paper introduced an SMA-spring-based stiffness regulating mechanism for a new rod-driven continuum robot. The friction along the drive rods is then adjusted by electric current applied to the SMA springs, so that the desired robot configuration can be maintained.

In the analysis, a new static model based on both the virtual work and geometric constraint was established and took for the first time both adjustable friction force and tension of the SMA into account. Therefore, the performance before and after stiffness regulation can be obtained. The paper in particular demonstrated that the geometric constraints contained two parts. One is the length change of drive rods as function of two sets of configuration parameters within individual segments and the other is that the sum of the length changes in each segment is equal to the actuation displacement. The geometric constraints in such rod-driven type continuum robots help solve the statics without a measurement of actuation force. Further, it was found that the robot under an external load will exhibit an S-shaped curve to meet the above geometric constraints. The experimental validations showed that the maximum error ratio of the continuum robot is 2.23%.

The proposed stiffness regulation mechanism and the virtual work-based static model present a new way for design and analysis of continuum robots for use in detection and maintenance tasks in unconstructed environments, for example, turbine engines, satellites, nuclear plants, and so on. Future work will implement micro grooves to the portion of the drive rod that is in contact with the lever mechanism, so that high friction or even complete locking will be achieved between the lever and the drive rods. The effects of the torsional motion will be assessed and incorporated in the analytical model. These will further improve the variable stiffness performance of our prototype and the accuracy of the model.

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Appendix A: Index to Multimedia Extensions

Archives of IJRR multimedia extensions published prior to 2014 can be found at http://www.ijrr.org, after 2014 all videos are available on the IJRR YouTube channel at http://www.youtube.com/user/ijrrmultimedia

Table of Multimedia Extensions

| Extension | Media type | Description |
|-----------|------------|-------------|
| 1 Video   | Video      | Video shows that the continuum robot grasps a ping-pong ball avoiding the obstacle. |
| 2 Video   | Video      | Video shows the actuation, transmission, and control systems of the continuum robot. |
| 3 Video   | Video      | Video shows how the variable stiffness mechanism works. |
| 4 Video   | Video      | Video shows the double ‘S-shaped’ curve of the robot. |

Appendix B: Equation derivation of the variables in the text

The transformation matrices from frame $O_{m,n-1}$ to frame $O_{m,n}$ and $l$ is the backbone length of $Seg_{m,n}$

$$T_{m,n} = Rot_z(\theta_{m,n}) Tr_z \left(\frac{l}{\theta_{m,n}}\right) Rot_x(\theta_{m,n})$$

$$Tr_z \left(\frac{-l}{\theta_{m,n}}\right) Rot_x(-\theta_{m,n})$$

The coefficients of redundant actuation displacement

$$a_{u,3} = \frac{\tan \omega_{u,2}}{\cos \omega_{u,3}} - \frac{\sin \omega_{u,3}}{\cos \omega_{u,1}}$$

$$b_{u,3} = \frac{\tan \omega_{u,1}}{\cos \omega_{u,3}} - \frac{\sin \omega_{u,3}}{\cos \omega_{u,2}}$$

The bending moments of backbone and $rod_{u,j}$ within $Seg_{m,n}$

$$M_{u,n} = EI_{\theta_{m,n}}$$

$$M_{u,j,m,n} = EI_{\theta_{m,n}}$$

The curvatures of backbone and $rod_{u,j}$ within $Seg_{m,n}$

$$\kappa_{m,n} = \frac{\theta_{m,n}}{l}$$

$$\kappa_{u,j,m,n} = \frac{\theta_{m,n}}{l(1 - \cos(\omega_{u,j,m,n} - \theta_{m,n}))}$$

The Euler angles of frame $O_{m,n}$

$$A_{u,n} = \begin{bmatrix} \arctan\left(\frac{R_{m,n}^{1,1}}{R_{m,n}^{2,1}}\right) & \arctan\left(\frac{R_{m,n}^{2,1}}{R_{m,n}^{3,1}}\right) & \arctan\left(\frac{R_{m,n}^{3,1}}{\sqrt{R_{m,n}^{1,1}^2 + R_{m,n}^{2,1}^2}}\right) \end{bmatrix}^T$$

The generalized force $Q$
\[
\begin{align*}
Q_{cl,t} &= -\sum_{m=1}^{M} \sum_{n=1}^{N} \frac{\partial (M_{m,n}(q_{m,n}))}{\partial q(t)} + \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{j=1}^{3} \frac{\partial (M_{m,j,n}(q_{m,j,n}))}{\partial q(t)} \\
Q_{ac,t} &= \sum_{m=1}^{M} \left( \sum_{n=1}^{N} \tau_{n,1} \frac{\partial p}{\partial q(t)} + \tau_{n,2} \frac{\partial p_{n}}{\partial q(t)} \right) \\
Q_{gr,t} &= \sum_{m=1}^{M} \sum_{n=1}^{N} \left( \frac{\partial P_{n,1}}{\partial q(t)} + \frac{\pi p (d_{m,n}^2 + 3d_{m,n}^4)}{4} \frac{\partial A_{m,n}}{\partial q(t)} + \frac{\pi p_{n}}{4} \right) \frac{\partial C_{n,1} \sum_{n=1}^{N} \eta_{n,1,n} \cdot q_{m,j,n}}{\partial q(t)} \\
Q_{loc,t} &= \sum_{m=1}^{M} \sum_{n=1}^{N} \left( F_{m,n,1} \frac{\partial P_{m,n}}{\partial q(t)} + M_{m,n,1} \frac{\partial A_{m,n}}{\partial q(t)} \right) \\
Q_{sma,t} &= \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{j=1}^{3} \left( \frac{\partial (F_{m,n,1}H_{m,n})}{\partial q(t)} \right) \\
Q_{fr,t} &= \sum_{m=1}^{M} \sum_{n=1}^{N} \sum_{j=1}^{3} \left( -\text{sgn}(D_{m,j,n}) \frac{\partial (f_{m,n,1}H_{m,n})}{\partial q(t)} \right)
\end{align*}
\]