Thick de Sitter 3-Branes, Dynamic Black Holes and Localization of Gravity

Anzhong Wang
Department of Physics and Astronomy, Brigham Young University, Provo, Utah, 84602
and
Departamento de Física Teórica, Universidade do Estado do Rio de Janeiro
Rua Sao Francisco Xavier 524, Maracanã, CEP. 20550-013, Rio de Janeiro, RJ, Brazil

March 27, 2022

Abstract

The embedding of a thick de Sitter 3-brane into a five-dimensional bulk is studied, assuming a scalar field with potential is present in the bulk. A class of solutions is found in closed form that can represent a thick de Sitter 3-brane interpolating either between two dynamical black holes with a $R \times S_4$ topology or between two Rindler-like space-times with a $R_2 \times S_3$ topology. The gravitational field is localized in a small region near the center of the 3-brane. The analysis of graviton fluctuations shows that a zero mode exists and separates itself from a set of continuous modes by a mass gap. The existence of such a mass gap is shown to be universal. The scalar perturbations are also studied and shown to be stable.

PACS Numbers: 98.80.Cq, 97.60.Lf, 04.20.Jb
1 Introduction

The idea that our universe is embedded in a higher dimensional world has received a great deal of renewed attention over the last couple of years (see, for example, [1] and references therein). This interest is motivated by the possibility of resolving the hierarchy problem, namely the large difference in magnitudes between the Planck and electroweak scales [2, 3], in addition to possibly solving the long-standing cosmological constant problem [4]. According to this scenario, Standard Model physics is confined to a three (spatial) dimensional hypersurface (often referred to as a 3-brane) in a larger dimensional space. While most physics is on the 3-brane, gravity propagates in the whole bulk spacetime. In previous considerations of such models, it was shown that the bulk propagation of gravity is in contradiction with the observational fact that four-dimensional gravity satisfies an inverse-square Newtonian law. However, in a model proposed by Randall and Sundrum (RS) [3], this problem was solved by relaxing one of the commonly used assumptions that our four-dimensional universe is independent of the coordinates defining the extra dimensions. When one does this, one can show that (even when the extra dimensions are infinitely large) gravity can be localized near the 3-brane, and Newtonian gravity can be restored at long distances. In particular, RS showed that a single massless graviton can be localized on the brane. This mode is responsible for producing 4D gravity on the 3-brane, while additional massive modes only introduce small corrections to the Newtonian law.

Considering our four-dimensional universe as an infinitely thin 3-brane is an idealization, and in more realistic models the thickness of the brane should be taken into account. It is for this reason that various thick 3-brane models were considered [5, 6]. However, these models tend to have either naked singularities appearing at a finite distance from the center of
the brane or the scalar field has an unusual potential. While the nature of these singularities is still unclear [5], one might like to try to avoid them by starting with a potential induced from some high energy theory, such as Superstring or Supergravity. However, once a scalar field appears in the bulk, curved four-dimensional spacetimes, rather than Minkowski, can appear.

In this paper, we consider the embedding of a thick de Sitter 3-brane in a five-dimensional spacetime described by the metric,

$$ds^2 = e^{2A(z)} \left\{ dt^2 - e^{2\alpha t} dx^2 - dz^2 \right\}, \quad (1)$$

where $z$ denotes the conformal coordinate of the extra dimension with $z \in (-\infty, \infty)$, and $x^\mu = \{t, x\}$ ($\mu = 0, 1, 2, 3$) are the usual four-dimensional Minkowskian coordinates. The potential of the scalar field to be considered here is assumed to take the form,

$$V(\phi) = V_0 \cos^{2(1-n)} \left( \frac{\phi}{\phi_0} \right), \quad (2)$$

where $V_0$ and $n$ are arbitrary constants, subject to $0 < n < 1$, and $\phi_0 \equiv [3n(1-n)]^{1/2}$. This form for the potential is quite similar to that proposed in [7] for weakly interacting pseudo-Goldstone bosons.

We shall show that, when the scalar field is the only source to the five-dimensional Einstein field equations $R_{MN} - R_{\gamma MN}/2 = T^\phi_{MN}$, thick brane solutions exist. Note that in this paper we shall choose units such that $M^3 = 1/4$, where $M$ is the five dimensional Planck scale. In these thick brane models, the naked singularities which often appear in other brane world models are replaced by event horizons. The extension of the spacetime beyond these horizons gives rise to either dynamical back holes with a $R \times S_4$ topology or Rindler-like spacetimes with a $R_2 \times S_3$ topology. This is very much in the same spirit as the so-called asymmetrically warped model proposed in [8]. However, a fundamental difference between their model and
ours is that in [8] the induced energy-momentum tensor (EMT) on the brane violates all three energy conditions, while in the present case the weak and dominant (but not the strong) energy conditions are satisfied in the whole bulk. We also find that the spectrum of graviton fluctuations has a mass gap that separates the massless mode from a set of continuous modes. The existence of such a mass gap is shown to be universal. In addition, we also study scalar perturbations and show that all the corresponding modes are stable.

2 Solutions of Thick de Sitter 3-Branes and Dynamical Black Holes

To show explicitly the above claims, let us start with the Einstein field equations and the corresponding Klein-Gordon equation \( \phi_{;MN} \gamma^{MN} = V'(\phi) \). It can be shown that there are now only two independent equations [3],

\[
A'' + A'^2 - \alpha^2 = -\frac{1}{6} \left( \phi'^2 + 2e^{2A}V(\phi) \right),
\]

\[
A'^2 - \alpha^2 = \frac{1}{12} \left( \phi'^2 - 2e^{2A}V(\phi) \right),
\]

where the semicolon denotes the covariant derivative with respect to the bulk metric \( \gamma_{MN} \), and a prime denotes ordinary differentiation with respect to \( z \). Integrating these equations, we find the solutions,

\[
A = -n \ln \left[ \cosh(\beta z) \right], \quad \phi = \phi_0 \sin^{-1} \left[ \tanh(\beta z) \right],
\]

where, in terms of \( V_0 \) and \( n \), the constants \( \alpha \) and \( \beta \) are given via the relations \( \alpha^2 = n^2 \beta^2 = 2nV_0/[3(1 + 3n)] \). As is well-known, the EMT for a scalar field is energetically equivalent to an anisotropic fluid, \( T_{MN} = \rho(\gamma_{MN} + z_M z_N) + pz_M z_N \), where \( z_M = e^A \delta^2_M \) and

\[
\rho = \frac{1}{2} \left[ (\nabla \phi)^2 + 2V(\phi) \right] = \frac{2(1 + n)V_0}{(1 + 3n) \cosh^{2(1-n)}(\beta z)},
\]
\[ p \equiv \frac{1}{2} \left[ (\nabla \phi)^2 - 2V(\phi) \right] = -\frac{4nV_0}{(1 + 3n) \cosh^{2(1-n)}(\beta z)}, \quad (6) \]

which shows clearly that the thick 3-brane is localized in the region \(|z| \approx 0\), and the corresponding EMT satisfies the weak and dominant energy conditions, but not the strong one. On the other hand, restoring the units, one can show that the reduced four-dimensional Planck mass on the brane, \(M_{Pl}\), is given by

\[ \frac{M^2_{Pl}}{M^3} = \int_{-\infty}^{\infty} e^{3A(z)} dz = \frac{2^{1+3n} \Gamma^2(1 + 3n/2)}{3n \beta \Gamma(1 + 3n)}, \quad (7) \]

where \(\Gamma(m)\) denotes the usual gamma function. Note that this is finite for any \(n \in (0, 1)\).

To study the above solutions further, let us first note that all the scalars built from the Riemann tensor are finite in the whole bulk, \(-\infty < x^M < \infty, \ (M = 0, 1, \ldots, 4)\). Thus, the spacetime described by the above solutions are free of scalar singularities [10]. However, for the cases where \(1/2 < n < 1\), the hypersurfaces \(|z| = \infty\) actually represent non-scalar spacetime singularities, as the tidal forces experienced by a freely falling observer become unbounded there. To show this explicitly, let us consider the time-like geodesics perpendicular to the 3-brane, which can be shown to allow the first integral

\[ E^M_{(0)} \equiv i\delta^M_t + \dot{z}\delta^M_z = \cosh^{2n}(\beta z) \left\{ E\delta^M_t \pm \left[ E^2 - \cosh^{-2n}(\beta z) \right]^{1/2} \delta^M_z \right\}, \quad (8) \]

where \(E\) denotes the total energy of the test particles and \(\tau\) their proper time. From \(E^M_{(0)}\) we can construct four other space-like unit vectors that are parallelly transported along the geodesics,

\[ E^M_{(i)} = \dot{z}\delta^M_i + i\delta^M_z, \quad E^M_{(i)} = e^{\alpha t} \delta^M_i, \quad (i = 1, 2, 3) \]

which satisfy the relations,

\[ E^M_{(A),N} E^N_{(B)} = 0, \quad E^M_{(A)} E^N_{(B)} \gamma_{MN} = \eta_{AB}, \quad (10) \]
Projecting the Riemann tensor into this orthogonal frame, we find that some of its components, which represent the tidal forces, become unbounded at \(|z| = \infty\) for \(1/2 < n < 1\). As one example, consider

\[
R_{(0)(2)(0)(2)} = n \beta^2 \cosh^{2(2n-1)}(\beta z) \left\{ (1 - n) E^2 - \cosh^{-2n}(\beta z) \right\}.
\]  

(11)

It is interesting to note that the distortion, which is equal to twice the integral of the tidal forces with respect to \(d\tau\), is finite as the hypersurfaces \(|z| = \infty\) approach. In this sense these singularities are weak [1]. When \(0 < n \leq 1/2\), these surfaces represent horizons. This can be seen, for example, by calculating the proper distance in the perpendicular direction to the brane, which is finite. Thus, in this latter case the spacetime needs to be extended beyond these surfaces. Because of the reflection symmetry of the spacetime, it is sufficient to consider the extension across the surface \(z = \infty\). To this end, let us first consider the coordinate transformations

\[
v = \alpha^{-1} e^{-\alpha(t+z)}, \quad u = \alpha^{-1} e^{\alpha(t-z)},
\]

(12)
in terms of which the above solutions take the form

\[
ds^2 = 2^{2n} \left[ 1 + (-\alpha^2 uv)^{1/n} \right]^{-2n} \left[ du dv - (\alpha v)^2 d\mathbf{x}^2 \right],
\]

\[
\phi = \phi_0 \sin^{-1} \left[ \frac{1 - (-\alpha^2 uv)^{1/n}}{1 + (-\alpha^2 uv)^{1/n}} \right].
\]

(13)
The corresponding Kretschmann scalar is given by

\[
\mathcal{R} = 8 \alpha^2 \left( 3 \alpha^2 + 2 \beta^2 \right) \left[ \frac{2 (-\alpha^2 uv)^{1/2n}}{1 + (-\alpha^2 uv)^{1/n}} \right]^{4(1-n)}.
\]

(14)
The coordinate transformations (12) hold only in the region \(v \geq 0, \ u \leq 0\). Thus, in the whole \(uv\)-plane we obtain, in general, three extended regions, \(I', \ II\) and \(II'\), as shown by Figs. 1 and 2. The singular properties of
the spacetime in these extended regions depend on the free parameter \( n \).
In particular, when \( n^{-1} \) is an integer, the extension is analytic across the
hypersurfaces \( u = 0 \) and \( v = 0 \). Otherwise, it is at best maximal. As a
matter of fact, when \( n = (2l)/(2m+1) \), where \( l \) and \( m \) are positive integers,
the metric in the extended regions becomes complex, which indicates that
the above extension is not even applicable to this case. However, in the rest
of this paper, we shall consider only the case where the extension is analytic,
that is, \( n^{-1} \) is an integer. This can, in turn, be divided into two subcases,
\( n = (2l+1)^{-1} \) and \( n = (2l)^{-1} \).

Let us first consider the case \( n = (2l+1)^{-1} \). Then, from Eqs. (13) and
(14) we can see that the spacetime becomes singular on the hypersurfaces
\( uv = \alpha^{-2} \) in the regions \( II \) and \( II' \), which are represented by the horizontal
lines \( AB \) and \( CD \) in Fig. 1. The center of the 3-brane is the vertical line
\( bd \), while the line \( ec \) represents an identical 3-brane in the extended region
\( I' \). In the right (left) hand side of the vertical line \( bd \) (\( ec \)), we have \( z \leq 0 \),
the extension of which across the hypersurface \( z = -\infty \) is identical to the
one across the surface \( z = \infty \). Thus, a geodesically maximal spacetime in
this case consists of infinite 3-branes in the horizontal direction, as shown in
Fig. 1.

Figure 1: The Penrose diagram for the case \( n = (2l + 1)^{-1} \).
When $n = (2l)^{-1}$, from Eqs. (13) and (14) we can see that the spacetime is free of any kind of singularities in all the $uv$-plane. Thus, the geodesically complete spacetime now consists of infinite diamonds, as shown by Fig. 2. The spacetime structure in each diamond is quite similar to that of Rindler spacetime.

![Penrose Diagram](image)

Figure 2: The Penrose diagram for the case $n = (2l)^{-1}$.

It is remarkable to note that the requirement that the extension be analytic automatically restricts the parameter $n$ to the range $0 < n \leq 1/2$, namely for that range in which we have shown that the surfaces $|z| = \infty$ represent event horizons.

If we further introduce the coordinates,

\[
T = \frac{1}{2} \left[ (u + v) + \alpha^2 v x^2 \right], \\
Z = \frac{1}{2} \left[ (u - v) + \alpha^2 v x^2 \right], \\
X = \alpha^2 v x,
\]

we find that the above solutions can be written in the form,

\[
ds^2 = 2^{2n} \left\{ 1 + \left[ \alpha^2 (R^2 - T^2) \right]^{1/n} \right\}^{-2n} \left( dT^2 - dR^2 - R^2 d\Omega_3^2 \right),
\]

8
\[ \phi = \phi_0 \sin^{-1} \left\{ \frac{1 - [\alpha^2(R^2 - T^2)]^{1/n}}{1 + [\alpha^2(R^2 - T^2)]^{1/n}} \right\}, \quad (16) \]

where \( d\Omega_3^2 \equiv d\psi^2 + \sin^2 \psi (d\theta^2 + \sin^2 \theta d\phi^2) \) denotes the metric on the unit three-sphere with intrinsic coordinates \( \theta, \varphi \) and \( \psi \), related to the Minkowskian coordinates \( X \) and \( Z \) in the usual way. In particular, from Eq. (15) we find that

\[ R^2 - T^2 = -uv, \quad (17) \]

from which we can see that the hypersurface \( z = 0 \) or \( uv = -\alpha^{-2} \) is a bubble with constant acceleration. When \( n = (2l + 1)^{-1} \), a spacetime singularity develops on the hypersurfaces \( R^2 - T^2 = -\alpha^{-2} \) in regions \( II \) and \( II' \), where the geometric radius vanishes. Thus, the spacetime in this case actually has \( R \times S_4 \) topology. When \( n = (2l)^{-1} \), no such singularities are formed, and the spacetime extends to the whole range \( R \in [0, \infty) \). Consequently, the spacetime has \( R_2 \times S_3 \) topology.

### 3 Gravitational and Scalar Perturbations

The analysis of the metric fluctuations in general is complicated because of their coupling to the scalar field fluctuations. This is particularly true when the four-dimensional spacetime is curved as in the present case. Following [12], let us first consider the gravitational perturbations,

\[ ds^2 = e^{2A(z)} \left((g_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu - dz^2 \right), \quad (18) \]

where \( g_{\mu\nu} \) denotes the 4D background metric, and \( h_{\mu\nu} \) the metric perturbations, satisfying the transverse-traceless (TT) condition,

\[ h_\lambda^\lambda = 0 = h_{\mu\nu;\lambda}g^{\mu\lambda}. \quad (19) \]

Then, it can be shown that the equation for \( h_{\mu\nu} \) is given by [8]

\[ h_{\mu\nu}'' + 3A'h_{\mu\nu}' - \Box h_{\mu\nu} - 2\alpha^2 h_{\mu\nu} = 0, \quad (20) \]
where $\Box \equiv g^{\alpha\beta} \nabla_\alpha \nabla_\beta$ and $\nabla$ denotes the covariant derivative with respect to the 4D metric $g_{\mu\nu}$. Since we are looking for a mode that corresponds to a 4D graviton, let us define the mass of such a spin two excitation by

$$
\Box h_{\mu\nu} + 2\alpha^2 h_{\mu\nu} = -m^2 h_{\mu\nu}.
$$

Then, following the standard procedure we introduce the polarization tensor $\epsilon_{\mu\nu}(x^\alpha)$ via the relations

$$
h_{\mu\nu}(x^\alpha, z) = e^{-3A/2} \epsilon_{\mu\nu}(x^\alpha) \Psi(z),
$$

where $\epsilon_{\mu\nu}$ satisfies the TT condition (19). Inserting Eqs.(21) and (22) into Eq.(20), we find that the equation for $\Psi(z)$

$$
\left(-\partial_z^2 + V_{QM}(z)\right) \Psi(z) = m^2 \Psi(z),
$$

with the effective potential $V_{QM}(z)$ given by

$$
V_{QM}(z) \equiv \frac{9}{4} A'^2 + \frac{3}{2} A'' = \frac{3n \beta^2}{4} \left(3n - \frac{3n + 2}{\cosh^2(\beta z)}\right).
$$

When $m = 0$, Eq.(23) has the solution

$$
\Psi_0(z) = e^{3A(z)/2} = \cosh^{-3n/2}(\beta z).
$$

Since $0 < n \leq 1/2$, it can be seen that this zero mode is normalizable. Following a similar argument as that in [3, 5], one can show that this zero mode will give rise to 4D gravity on the 3-brane.

On the other hand, from Eq.(24) we can see that $V_{QM}(z) \to 9\alpha^2/4$, as $|z| \to \infty$. Thus, similar to thin de Sitter 3-branes [12], there is also a mass gap in the present case. It is interesting to note that the existence of such a mass gap is actually a universal property for de Sitter 3-branes. To see

\footnote{I would like to thank A. Karch for pointing out an error in an earlier definition for the mass in the de Sitter background.}
this, let us first notice that for any given scalar potential $V(\phi)$, the effective potential $V_{QM}$ can be written as

$$V_{QM}(z) = \frac{9}{4} \alpha^2 - \frac{1}{8} e^{2A} (4\rho - p), \quad (26)$$

where $\rho$ and $p$ are defined by Eq.(8). Thus, as long as the gravitational field is localized in the region $|z| \sim 0$, the second term at the right-hand side of Eq.(26) goes to zero as $|z| \to \infty$, while the effective potential $V_{QM}$ goes to $9\alpha^2/4$.

To study the above problem further, let us introduce a new variable $x$ by $x = \tanh(\beta z)$, then we find that Eq.(23) becomes the standard Legendre differential equation,

$$\left\{ (1 - x^2)\partial_x^2 - 2x\partial_x + \left[ \nu(\nu + 1) - \frac{\mu^2}{1 - x^2} \right] \right\} \Psi(x) = 0, \quad (27)$$

with

$$\nu \equiv \frac{3}{2} n, \quad \mu^2 \equiv -\frac{1}{\beta^2} \left( m^2 - \frac{9}{4} \alpha^2 \right). \quad (28)$$

The general solution of the above equation is given by

$$\Psi(x) = C_1 P^\mu_\nu(x) + C_2 Q^\mu_\nu(x), \quad (29)$$

where $P^\mu_\nu(x)$ and $Q^\mu_\nu(x)$ denote the associated Legendre functions of the first and second kinds, respectively. When $\mu$ is real, or $m^2 < 9\alpha^2/4$, both $P^\mu_\nu(x)$ and $Q^\mu_\nu(x)$ are singular at $|x| = 1$ (or $|z| = \infty$). Thus, the regularity conditions at $|x| = 1$ for the perturbations exclude the case $m^2 < 9\alpha^2/4$. When $\mu = 0$ or $m^2 = 9\alpha^2/4$, the functions $P^\mu_\nu(x)$ and $Q^\mu_\nu(x)$ reduce to the Legendre polynomials of the first and second kinds, $P_\nu(x)$ and $Q_\nu(x)$, respectively, and the latter is still singular at $|x| = 1$. Thus, the regularity conditions at $|x| = 1$ force $C_2 = 0$, and we obtain $\Psi_0(x) = C_1 P_\nu(x)$, which is always finite and normalizable. The case where $m^2 > 9\alpha^2/4$ corresponds to a continuous spectrum of eigenfunctions that asymptote to plane waves in
the limit \(|z| \to \infty\). Similarly, one can show that these continuous modes will produce small corrections to the Newtonian law as in the flat 4D case [3, 4].

Now let us consider the scalar perturbations given by [6]

\[
\frac{ds^2}{e^{A}} \left\{ (1 + 2\phi)dz^2 + (1 + 2\psi)g_{\mu\nu}dx^\mu dx^\nu \right\}.
\]  

(30)

As shown in [6], the corresponding linearized 5D Einstein-scalar equations become

\[
\delta \phi = \frac{3}{\phi'} (\phi A' - \psi'), \quad \phi = -2\psi, \quad (31)
\]

\[
\left( -\partial^2_z + V_{\text{eff}}(z) \right) \chi(x^\alpha, z) = \Box \chi(x^\alpha, z), \quad (32)
\]

where \(\delta \phi\) denotes perturbations of the scalar field, and \(\chi(x^\alpha, z)\) and \(V_{\text{eff}}(z)\) are defined as

\[
\chi(x^\alpha, z) \equiv \frac{1}{\phi'(z)} e^{3A/2} \psi(x^\alpha, z),
\]

\[
V_{\text{eff}}(z) \equiv -\frac{5}{2} A'' + \frac{9}{4} A^2 + A' \frac{\phi''}{\phi'} - \frac{\phi'''}{\phi'} + 2 \left( \frac{\phi''}{\phi'} \right)^2 - 6\alpha^2
\]

\[
= \frac{\beta^2}{4 \cosh^2(\beta z)} \left[ 2 \left( 2 + 5n - 12n^2 \right) \right. \\
+ \left. \left( 4 + 4n - 15n^2 \right) \sin^2(\beta z) \right].
\]  

(33)

Because \(V_{\text{eff}}(z)\) is always positive for \(0 < n \leq 1/2\), following the arguments given in [6], it can be shown that in the present case all the corresponding perturbation modes are stable. This is to be contrasted with the case of thin 3-branes [13], but it is a similar result to that for thick 3-branes [3].

### 4 Conclusions

In this paper, we have considered the embedding of thick de Sitter 3-branes in a 5D bulk in which a scalar field with potential given by Eq.(2) is assumed...
to be present. A class of solutions has been obtained in closed form, some of which represent a thick de Sitter 3-brane interpolating between two dynamical black holes with a $R \times S_4$ topology, and some of which represent such a 3-brane interpolating between two Rindler-like spacetimes with a $R_2 \times S_3$ topology. The thick brane is localized in the region where $|z| \approx 0$. The analysis of graviton fluctuations shows that the spectrum of perturbations consists of a zero mode and a set of continuous modes. The massless mode is separated by a mass gap from the continuous modes. The existence of such a mass gap has been shown to be universal for all such de Sitter 3-branes.

The scalar perturbations of the solutions have also been studied and found to be stable.

Finally, we note that by making the replacement, $(t, x^1, \alpha) \rightarrow (ix, it, -i\alpha)$ in the solutions given by (1) - (5), we can obtain solutions that represent thick anti-de Sitter 3-branes, given by

$$ds^2 = e^{2A(z)} \left\{dz^2 + dx^2 + e^{2\alpha x} \left[dt^2 + (dx^2)^2 + (dx^3)^2\right]\right\}, \quad (34)$$

with

$$V(\phi) = V_0 \cosh^{2(1-n)} \left(\frac{\phi}{\phi_0}\right),$$

$$A(z) = -n \ln |\cos(\beta z)|,$$

$$\phi(z) = \phi_0 \sinh^{-1} [\tanh(\beta z)],$$

$$\alpha^2 = n^2 \beta^2 = \frac{-2nV_0}{3(1 + 3n)}, \quad (35)$$

but now with $\phi_0 \equiv [3n(n - 1)]^{1/2}$. The constants $V_0$ and $n$ are again arbitrary. However, to have $\phi_0$ real, we must have $n \geq 1$ or $n \leq 0$. On the other hand, to have $\alpha$ and $\beta$ real, $V_0$ has to be negative for $n > 1$ or $n < -1/3$ and positive for $-1/3 < n < 0$. Studying these solutions is outside the scope of this paper, but we plan to return to this problem on another occasion.
ACKNOWLEDGMENTS

The author would like to thank E. W. Hirschmann and Andreas Karch for valuable suggestions and discussions on braneworld scenarios, and the Department of Physics and Astronomy, BYU, for hospitality. Financial assistance from BYU, CNPq and FAPERJ is gratefully acknowledged.

References

[1] V.A. Rubakov, Phys. Usp. 44, 871 (2001); S. Föste, Fortsch. Phys. 50, 221 (2002).

[2] N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett. B429, 263 (1998); I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G. Dvali, Phys. Lett., B436, 257 (1998).

[3] L. Randall and R. Sundrum, Phys. Rev. Lett. 83, 3370 (1999); ibid., 4690 (1999).

[4] N. Arkani-Hamed, S. Dimopoulos, N. Kaloper and R. Sundrum, Nucl. Phys. B480, 193 (2000); S. Kachru, M. Schulz, and E. Silverstein, Phys. Rev. D62, 045021 (2000).

[5] O. DeWolfe, D. Z. Freedman, S. S. Gubser and A. Karch, Phys. Rev. D62, 046008 (2000); M. Gremm, Phys. Lett. B478, 434 (2000); Phys. Rev. D62, 044017 (2000); C. Csáki, J. Erlich, T. J. Hollowood and Y. Shirman, Nucl. Phys. B581, 309 (2000); S. Ichinose, Class. Quantum Grav. 18, 5239 (2001); A. Kehagias and K. Tamvakis, Phys. Lett. B504, 38 (2001); “A Self-Tuning Solution of the Cosmological Constant Problem,” hep-th/0011006 (2000); K. Behrndt and G. Dall’Agata, Nucl. Phys. B627, 357 (2002).
[6] S. Kobayashi, K. Koyama, and J. Soda, Phys. Rev. D65, 064014 (2002).

[7] C.T. Hill, D.N. Schramm, and J.N. Fry, Comm. Nucl. Phys. 19, 25 (1989); G. Goetz, J. Math. Phy. 31, 2683 (1990); M. Mukherjee, Class. Quantum Grav. 10, 131 (1993).

[8] C. Csaki, J. Erlich, and C. Grojean, Nucl. Phys. B604, 312 (2001).

[9] D.Z. Freedman, S.S. Gubser, K. Pilch, and N.P. Warner, JHEP, 0007, 038 (2000).

[10] G.F.R. Ellis and B.G. Schmidt, Gen. Relativ. Grav. 8, 915 (1977).

[11] L.M. Burko and A. Ori, Phys. Rev. Lett. 74, 1064 (1995), and references therein.

[12] J. Garriga and M. Sasaki, Phys. Rev. D62, 043523 (2000); N. Alonso-Alberca, P. Meessen, and T. Ortin, Phys. Lett. B482, 400 (2000); A. Karch and L. Randall, JHEP, 0105, 008 (2001).

[13] U. Gen and M. Sasaki, Prog. Theor. Phys. 105, 591 (2001); Z. Chacko and P.J. Fox, Phys. Rev. D64, 024015 (2001).