Gapless formation in the $K^0$ condensed color-flavor locked quark matter: a model-independent treatment

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The electric/color neutral solution and the critical conditions for gapless formation are investigated in the $K^0$ condensed color-flavor locked matter. We point out that there exist no longer gapless modes for down-strange quark pairing while the gapless phenomenon for up-strange one is dominated in the $K^0$ condensed environment. In a model-independent way, the phase transition to the resulting gapless phase is found to be of first-order. The novel phase structure implies that the chromomagnetic instability happens in the previous-predicted gapless phase might be removed at least partly.

PACS number(s): 25.75.Nq, 12.39.Fe, 12.38.-t

I. INTRODUCTION

In recent years, the studies of dense matter become a "hot" topic since quark color superconducting phases were proposed for high-density QCD. For three flavors, the original color and flavor $SU(3)_{color} \times SU(3)_L \times SU(3)_R$ symmetries of QCD are broken down to the diagonal subgroup $SU(3)_{color+L+R}$ at very high baryon density [1]. The quark matter with such a particular symmetry pattern is called color-flavor locked (CFL) matter which is widely believed to be the densest phase of strongly interacting matter [2]. In the situation where the strange quark mass $m_s$ is large and/or the quark chemical potential $\mu$ is small, the so-called gapless color-flavor locked phase (gCFL) has been predicted [3, 4]. Similar as the gapless phase of two-flavor color superconductor [5], gCFL is triggered by mismatches between the chemical potentials for paired quasiquarks. There, the mismatch in the flavor (say $d$ and $s$) chemical potentials is characterized by $m_s^2/2\mu$ directly while that in the color (say $b$ and $g$) chemical potentials is related with $m_s^2/2\mu$ via the electric/color neutrality in CFL matter [4]. As long as $m_s^2/2\mu$ is large enough, therefore, there exist the unpaired (gapless) modes for $bd$-$gs$ pairing and the resulting gCFL phase becomes energetically favorable. Recently the Meissner masses for some gluons in this phase were found to be imaginary so that gCFL is actually unstable in the chromomagnetic sense [6, 7, 8]. The presence of chromomagnetic instability is a serious problem so that the gapless phase of CFL matter especially its needs to be further examined.

On the other hand, another kind of less-symmetric phase of CFL matter was predicted...
in the situation of \( m_s \neq 0 \) also. At the leading order, it is practical to regard the effect of \( m_s^2/2\mu \) as an effective chemical potential associated with strangeness, namely

\[
\mu_S = \frac{m_s^2}{2\mu}.
\]  

(1)

As one of the pseudo Goldstone bosons related to the CFL symmetry pattern, the neutral kaon mode \( K^0 \) obtains its chemical potential \( \mu_{K^0} = \mu_S \) due to the chemical equilibrium. When \( \mu_S \) exceeds the mass of kaon-mode, \( K^0 \) becomes condensed in CFL matter \([9, 10]\). If ignoring the instanton contribution to the kaon mass, it was found that the \( K^0 \) condensed phase of CFL matter (CFLK\(^0\)) becomes possible as long as \( m_s \) is larger than the critical value \( \sim m_q^{1/3} \Delta_0^{2/3} \) \([9]\). Because the light quark mass \( m_q \) is far smaller than \( m_s \) and the CFL gap \( \Delta_0 \) has order of tens MeV, CFLK\(^0\) is usually favored over CFL. Starting at the ideal situation of \( m_s = 0 \) and raising \( m_s^2/2\mu \) gradually, one expects that CFL matter to be disrupted, at first by the presence of \( K^0 \) condensation and then by the gapless formation. In the CFL matter with \( K^0 \) condensation, the gapless formation has been reexamined by using an effective theory \([11]\) and the NJL model \([12, 13]\). It was found the gapless formation is delayed with respect to that in the conventional CFL matter. More recently, it has been suggested that the chromomagnetic instability in the gapless phase can be resolved by the formation of the so-called kaon supercurrent state \([14]\).

Motivated by these results, we would like to investigate the gapless formation in the CFLK\(^0\) environment through a model-independent approach. Firstly, we consider the electric/color neutral solution of CFLK\(^0\) in the presence of electron chemical potential and point out that the above-mentioned delay is a direct consequence of the deviation of the CFLK\(^0\) neutral solution from the CFL one. Then, it is found that there exist no longer the gapless modes for \( bd-gs \) pairing while the gapless phenomenon for \( bu-rs \) pairing becomes dominated in the \( K^0 \) condensed environment. Based on this feature, the resulting gapless phase (termed as gCFLK\(^0\) below) including its electric/color neutrality and gap variation are studied. As a consequence, a first-order phase transition from CFLK\(^0\) to gCFLK\(^0\) is obtained. Finally, the stability of the gCFLK\(^0\) phase is examined qualitatively and it is argued that the gCFL instability might be removed at least partly.

II. ELECTRIC/COLOR NEUTRALITY IN THE \( K^0 \) CONDENSED ENVIRONMENT

The stable, bulk matter must be electrically neutral and must be a color singlet, i.e. its free energy \( \Omega \) satisfies

\[
\frac{\partial \Omega}{\partial \mu_e} = \frac{\partial \Omega}{\partial \mu_3} = \frac{\partial \Omega}{\partial \mu_8} = 0,
\]  

(2)
where $\mu_e$, $\mu_3$ and $\mu_8$ are the chemical potentials coupled to the negative electric charge, the color generators $T_3$ and $2T_8/\sqrt{3}$ respectively. For the CFL matter, the electric/color neutral solution has been obtained from the derivative of $\Omega_{CFL}(\mu_e, \mu_3, \mu_8)$ in a model-independent way and it can be expressed by [15]

$$\mu_3 = \mu_e, \quad \mu_8 = \frac{\mu_e}{2} - \frac{m_s^2}{2\mu} \equiv \frac{\mu_e}{2} - \mu_S,$$

(3)

to order $m_s^2$. Another important feature involving CFL is that its free energy is independent of $\mu_\tilde{Q} = -\frac{4}{3}(\mu_e + \mu_3 + \mu_8/2)$, (4)
since all the CFL pairings do not break the rotation $\tilde{Q} = Q - T_3 - \frac{1}{\sqrt{3}}T_8$. In this sense, CFL is not merely an electric insulator [16] but also a $\tilde{Q}$ insulator [15].

For the matter with $K^0$ condensation, its free energy $\Omega_{CFLK^0}$ makes up of the CFL free-energy contribution and the contribution from $K^0$ condensation. Since the condensation does not bring in any additional electric and color charges, the electric/color neutral solution in CFL$K^0$ may be obtained from the derivative of the former contribution. Superficially, it seems obvious that the CFL$K^0$ solution has the same form as Eq.(3). However, this is not correct yet. In the presence of $K^0$ condensation, the ground state of Goldstone bosons takes non trivial values so that the corresponding vacuum should be changed with respect to the conventional CFL vacuum. By treating the gauge fields as dynamical degrees of freedom, Kryjevski has pointed out that there exist nonzero vacuum expectation values for the gluon and photon fields in the CFL$K^0$ matter [17]. It was found that the electric charge neutrality leads to that the time components of two color-diagonal gluon fields are

$$\langle A_3^0 \rangle = \frac{m_s^2}{4g\mu}, \quad \langle A_8^0 \rangle = \frac{m_s^2}{4\sqrt{3}g\mu},$$

(5)

where $g$ is the QCD coupling coefficient. Due to the non-vanishing vacuum, the CFL free-energy contribution in the $K^0$ condensed environment is not equal to $\Omega_{CFL}(\mu_e, \mu_3, \mu_8)$ defined in CFL matter. In this case, the electric/color neutral solution in CFL$K^0$ obtained from Eq.(2) is expected to be different from the well-known solution Eq.(3).

In order to yield the CFL$K^0$ neutral solution, we need to take such a non-vanishing vacuum as Eq.(5) into account self-consistently. Note that introducing Eq.(5) means that there exist terms like

$$g\langle A_3^0 \rangle \psi^+_3 \psi = \frac{\mu_S}{2} \psi^+_3 T_3 \psi,$$

$$g\langle A_8^0 \rangle \psi^+_8 \psi = \frac{\mu_S}{4} \psi^+_8 \frac{2}{\sqrt{3}} T_8 \psi,$$

(6)
in the effective Lagrangian describing quasiquarks. By combining Eq.(6) with the regular terms involving $\mu_3$ and $\mu_8$, it is convenient to define the fictional chemical potentials

$$
\mu'_3 = \mu_3 + \frac{\mu_S}{2}, \quad \mu'_8 = \mu_8 + \frac{\mu_S}{4},
$$

which are associated with the new gluon fields with vanishing vacuum. With the help of Eq.(7), the CFL free-energy contribution in the $K^0$ condensed environment behaves as the function of $\mu'_3$, $\mu'_8$ and $\mu_e$ formally. In analogy with the treatment in Ref.[15], it is easily found that

$$
\frac{\partial \Omega_{CFLK^0}(\mu_e, \mu'_3, \mu'_8)}{\partial \mu'_3} = \frac{\mu^2}{4\pi^2}(\mu_e - \mu'_3),
$$

$$
\frac{\partial \Omega_{CFLK^0}(\mu_e, \mu'_3, \mu'_8)}{\partial \mu'_8} = \frac{\mu^2}{6\pi^2}(\mu_e - 2\mu'_8),
$$

(8)

at the leading order. By requiring Eq.(8) to be zero (color neutrality), the factual color chemical potentials become

$$
\mu_3 = \mu_e - \frac{\mu_S}{2}, \quad \mu_8 = \frac{\mu_e}{2} - \frac{\mu_S}{4},
$$

(9)

where Eq.(7) has been used. In addition, Eq.(9) makes the derivative

$$
\frac{\partial \Omega_{CFLK^0}(\mu_e, \mu_3, \mu_8)}{\partial \mu_e} = \frac{\mu^2}{12\pi^2}(2\mu_S + 3\mu_3 + 2\mu_8 - 4\mu_e),
$$

(10)

to be zero automatically since the electric neutrality has been considered in Eq.(5). As expected, the CFL$K^0$ solution Eq.(9) is deviated from the CFL solution. Interestingly, the $\mu_e$-dependence in the two solutions remain to have the same forms, which manifests that $K^0$ condensation is independent of electric charge and CFL$K^0$ is still an insulator. Eq.(9) can be understood as the extrapolation of the solution $\mu_3 = -\mu_S/2$ and $\mu_8 = -\mu_S/4$ obtained in Ref.[12] to the case of $\mu_e \neq 0$.

Ignoring the diquark condensates which are symmetric in color, we consider the pairing ansatz in the form of

$$
\langle \psi_i^\alpha C \gamma_5 \psi_j^\beta \rangle \sim \Delta_1 \epsilon^{\alpha \beta 1} \epsilon_{ij1} + \Delta_2 \epsilon^{\alpha \beta 2} \epsilon_{ij2} + \Delta_3 \epsilon^{\alpha \beta 3} \epsilon_{ij3},
$$

(11)

where $(i, j)$ and $(\alpha, \beta)$ denote the flavor indices $(u, d, s)$ and the color indices $(r, g, b)$ respectively and the gap parameters $\Delta_1$, $\Delta_2$ and $\Delta_3$ are approximately equal to $\Delta_0$ in the absence of gapless phenomena. Based on Eq.(11), the $bd - gs$, $bu - rs$ and $rd - gu$ pairings are described by the gaps $\Delta_1$, $\Delta_2$ and $\Delta_3$ respectively (termed as I, II and III channels for short), while the $ru$, $gd$ and $bs$ quarks pair among each other in a way involving all three gaps. For the
three channels, the mismatches between the chemical potentials for paired quarks are given by

\[
\begin{align*}
\delta \mu_I &= \frac{\mu_{bd} - \mu_{gs}}{2} = \frac{\mu_3}{4} - \frac{\mu_8}{2} + \frac{\mu_S}{2}, \\
\delta \mu_{II} &= \frac{\mu_{bu} - \mu_{rs}}{2} = -\frac{\mu_e}{2} - \frac{\mu_3}{4} - \frac{\mu_8}{2} + \frac{\mu_S}{2}, \\
\delta \mu_{III} &= \frac{\mu_{rd} - \mu_{gu}}{2} = \frac{\mu_e}{2} + \frac{\mu_3}{2},
\end{align*}
\]

(12)

respectively, where Eq.(1) has been considered for the strange quark. It is well known that gapless formation becomes possible when the mismatch exceeds the gap. By inserting the solutions Eqs.(3) and (9) into Eq.(12), we will investigate the critical conditions for gapless formations in CFL and CFLK\(^0\), respectively.

In CFL matter, the mismatches for the three channels are

\[
\begin{align*}
\delta \mu_I &= \mu_S, \\
\delta \mu_{II} &= \mu_S - \mu_e, \\
\delta \mu_{III} &= \mu_e,
\end{align*}
\]

(13)

respectively, while the average chemical potentials defined by \(\bar{\mu}_{\alpha i-\beta j} = (\mu_{\alpha i} + \mu_{\beta j})/2\) have the same value, i. e. \(\bar{\mu}_I = \bar{\mu}_{II} = \bar{\mu}_{III} = \mu - \mu_S/3\). When \(\delta \mu_I = \mu_S \geq \Delta_0\) the I-channel gapless formation occurs, since it precedes the phase transition from CFL to unpaired quark matter [15, 18]. Note that not merely the pairings, but also the I-channel involved modes (i. e. \(bu\) and \(gs\)) themselves are \(\tilde{Q}\) neutral. If only the I-channel gapless phenomenon exists, thus, the gapless phase remains to be a \(\tilde{Q}\) insulator, as shown in the region between the dot and dashed lines in Fig. 1. On the other hand, the II-channel gapless formation becomes possible when \(\delta \mu_{II} = \mu_S - \mu_e \geq \Delta_0\). Once the II-channel modes with nonzero \(\tilde{Q}\) charge exist, the resulting phase is no longer an insulator. In this sense, the so-called gCFL phase is a \(\tilde{Q}\) conductor and the additional electron density is required to guarantee its \(\tilde{Q}\) neutrality. As stressed in Ref.[4], the gCFL location in the \((\mu_S, \mu_e)\) plane is limited on a single line that very close to the dashed line of Fig. 1. Because the dashed line comes from the critical condition \(\mu_S = \mu_e + \Delta_0\), the gapless strength in II channel is almost infinitesimal while the I-channel gapless phenomenon actually dominates the physics of gCFL.

Now we consider the mismatches in the CFLK\(^0\) environment to demonstrate how the modified neutral solution Eq.(9) works. Different from Eq.(13), the mismatches for the three channels are

\[
\begin{align*}
\delta \mu_I &= \frac{\mu_S}{2}, \\
\delta \mu_{II} &= \frac{3\mu_S}{4} - \mu_e, \\
\delta \mu_{III} &= \mu_e - \frac{\mu_S}{4},
\end{align*}
\]

(14)

respectively, while the average chemical potentials are

\[
\begin{align*}
\bar{\mu}_I &= \mu - \frac{\mu_S}{3}, \\
\bar{\mu}_{II} &= \mu - \frac{7\mu_S}{12}, \\
\bar{\mu}_{III} &= \mu - \frac{\mu_S}{12}.
\end{align*}
\]

(15)
According to Eq.(14), the critical condition for the I-channel gapless formation is $\mu_S = 2\Delta_0$. When $\mu_S$ is close to $2\Delta_0$, however, the phase transition to unpaired quark matter occurs and the I-channel gapless phenomenon no longer appears accordingly.  

On the other hand, the II-channel gapless formation is still possible, but the critical condition is modified as

$$\mu_S = \frac{4\Delta_0}{3} + \frac{4\mu_e}{3}. \quad (16)$$

If substituting $\mu_e = 0$, Eq.(16) means that there exist gapless modes when $\mu_S \geq 4\Delta_0/3$. In comparison with $\mu_S \geq \Delta_0$ in the gCFL case, the gapless formation in the $K^0$ condensed environment is delayed by a factor $4/3$, which is just the conclusion drawn in Ref.[11]. Keeping in mind that the corresponding gapless modes are $\bar{Q}$ charged, the $\bar{Q}$ neutrality in the resulting gCFL$K^0$ phase needs to be realized by taking electrons into account explicitly. Therefore, effects of nonzero $\mu_e$ and electron density on the gCFL$K^0$ phase must be studied seriously. This issue was not mentioned in the above-cited literature and might be relevant for understanding the gapless phase.

For the gCFL$K^0$ phase, the electric/color neutrality condition Eq.(2) needs to be considered if it exists as a bulk matter. Note that the total free energy consists of $\Omega_{CFLK^0}$, $\Omega_g$ and $\Omega_e$, where $\Omega_g$ denotes the contribution from gapless modes (see Sec. III) and $\Omega_e = -\mu^4_e/12\pi^2$ from electrons. Without losing generality, we suppose that the values of $\mu_3$ and $\mu_8$ are deviated from the CFL$K^0$ neutral solution Eq.(9) once gapless phenomenon has occurred. In this case $\Omega_{CFLK^0}$, as one part of the gCFL$K^0$ free energy, no longer satisfies Eq.(2) and its derivatives Eqs.(8) and (10) are usually nonzero. By considering Eqs.(8) and (10), the model-independent results of the electric/color neutrality in gCFL$K^0$ read

$$\frac{\mu^2}{12\pi^2}(2\mu_S + 3\mu_3 + 2\mu_8 - 4\mu_e) + \frac{\partial\Omega_g(\mu_e, \mu_3, \mu_8)}{\partial\mu_e} \frac{-\mu^3_e}{3\pi^2} = 0, \quad (17)$$

$$\frac{\mu^2}{8\pi^2}(2\mu_e - 2\mu_3 - \mu_S) + \frac{\partial\Omega_g(\mu_e, \mu_3, \mu_8)}{\partial\mu_3} = 0, \quad (18)$$

$$\frac{\mu^2}{12\pi^2}(2\mu_e - 4\mu_8 - \mu_S) + \frac{\partial\Omega_g(\mu_e, \mu_3, \mu_8)}{\partial\mu_8} = 0, \quad (19)$$

1Strictly speaking, the phase transition to unpaired matter is slightly delayed in the presence of $K^0$ condensation. By taking into account the $K^0$ condensed free energy as well as the changes in the common Fermi momenta Eq.(15), we have reexamined this transition in the similar way as Ref.[15]. To order $m^4_{\pi}$, it is found that the critical value of $\mu_S$ is $2\Delta_0/\sqrt{1 - 2f_\pi^2/3\mu^2} \approx 2\Delta_0$, where the in-CFL-medium decay constant $f_\pi$ is about 0.2$\mu$ [19].
which correspond to the derivatives of the gCFL\(^0\) free energy with respect to \(\mu_e\), \(\mu_3\) and \(\mu_8\) respectively. Obviously, the gCFL\(^0\) neutral solution is deviated from that of CFL\(^0\) unless \(\partial \Omega_g/\partial \mu_3\) and \(\partial \Omega_g/\partial \mu_8\) are zero.

Besides electric/color neutrality, gCFL\(^0\) is required to be \(\tilde{Q}\) neutral. Note that the free energy \(\Omega_{CFLK}^0\) is independent of \(\tilde{Q}\) charge and the \(\tilde{Q}\) charge derived from the gapless phenomenon should be canceled by that from electrons. By using Eq.(4), the neutrality condition may be expressed by

\[
\frac{\partial \Omega_g(\mu_e, \mu_3, \mu_8)}{\partial \mu_e} + \frac{\partial \Omega_g(\mu_e, \mu_3, \mu_8)}{\partial \mu_3} + 2 \frac{\partial \Omega_g(\mu_e, \mu_3, \mu_8)}{\partial \mu_8} = \frac{\mu_3^3}{3\pi^2},
\]

(20)

where the left-hand side comes from the complete-derivative of \(\Omega_g\) with respect to \(\tilde{Q}\). Combining Eqs.(17), (18) and (19) with (20), it is found that the derivative \(\partial \Omega_g/\partial \mu_8\) is equal to zero so that \(\mu_8 = \mu_e/2 - \mu_S/4\) actually holds unchanged. Physically, the reason lies in the facts that only the II-channel gapless phenomenon occurs in our concerned phase and the combination \(Q + 2T_S/\sqrt{3}\) becomes zero for the unpaired \(ub\) and \(rs\) quarks. As a result, the \(\tilde{Q}\) neutrality condition Eq.(20) can be simplified as

\[
\frac{\partial \Omega_g(\mu_e, \mu_3, \mu_8)}{\partial \mu_e} + \frac{\partial \Omega_g(\mu_e, \mu_3, \mu_8)}{\partial \mu_3} = \frac{\mu_3^3}{3\pi^2}.
\]

(21)

Further inserting Eq.(21) into (17), we find that no more result than Eq.(18) can be obtained. It means that the relation \(\mu_3 = \mu_e - \mu_S/2\) may change as the gapless phenomenon occurs. In the gCFL\(^0\) phase, the value of \(\mu_3\) must be solved from Eq.(18) ( and Eq.(17) equivalently ) numerically. It is worthy being stressed that the above results are still obtained in a model independent way and hold strictly valid at leading order. In a model-dependent treatment, color chemical potentials are usually given "by hand" to guarantee electric/color neutrality ( see e.g. Ref.[4] ). To examine the II-channel gapless phenomenon self-consistently, we take the possible change of \(\mu_3\) into account and thus adjust the relative and average chemical potentials to be

\[
\delta \mu = -\frac{3\mu_e}{4} - \frac{\mu_3}{4} + \frac{5\mu_S}{8},
\]

(22)

and

\[
\bar{\mu} = \mu - \frac{\mu_e}{4} + \frac{\mu_3}{4} - \frac{11\mu_S}{24},
\]

(23)

for II channel, which make sense only if \(\delta \mu_{II} \geq \Delta_0\) is satisfied; otherwise, no gapless formation occurs and \(\mu_3\) holds unchanged.

III. PHASE TRANSITION TO THE gCFL\(^0\) PHASE
As pointed out in Refs.[4, 5], the gapless phase corresponds to the common solution of electric/color neutrality and gap equation. By using Eqs.(22) and (23), let’s consider the gap variation and the gap equation in the gCFL\textsuperscript{K}\textsubscript{0} phase. When gapless modes appear, the gaps for various pairings separate from each other and their values should be solved from three gap equations, namely \( \partial \Omega / \partial \Delta_1 = \partial \Omega / \partial \Delta_2 = \partial \Omega / \partial \Delta_3 = 0 \), in principle. In view of the fact that the II-channel gapless phenomenon dominates the physics of gCFL\textsuperscript{K}\textsubscript{0}, the variation of \( \Delta_2 \) from the original value \( \Delta_0 \) is obvious while the changes in \( \Delta_1 \) and \( \Delta_3 \) are small relatively. Now we ignore the latter variations as well as the \( \mu_S \) dependence of \( \Delta_0 \) so as to simplify three gap equations into one equation.

For the II-channel pairing, the dispersion relation of quasiquarks takes the form of

\[
E(p) = |\delta \mu \pm \sqrt{(p - \bar{\mu})^2 + \Delta_2^2}|. 
\]

(24)

It is easily found that the excitation energy for quasiquarks becomes zero at the momenta

\[
p^\pm = \bar{\mu} \pm \sqrt{\delta \mu^2 - \Delta_2^2}. 
\]

(25)

When \( \delta \mu \geq \Delta_2 \) Eq.(25) makes sense and there exist gapless modes in the blocking region \( p \in (p^-, p^+) \). As a consequence, the gapless modes provide

\[
\Omega_g = -\int_{p^-}^{p^+} \frac{p^2 dp}{2\pi^2} [\delta \mu - \sqrt{(p - \bar{\mu})^2 + \Delta_2^2}], 
\]

(26)

to the total free energy for gCFL\textsuperscript{K}\textsubscript{0}. Besides the contribution \( \Omega_g \), the free energy from the CFL pairings is influenced by the gap variation. In the CFL matter with the common gaps, it is well known that such a free energy is equal to \(-3\Delta_0^2 \mu^2 / \pi^2\) at the leading order [15]. Since only the binding energy from the II-channel pairing is influenced within our assumption, the gap variation leads to the additional contribution

\[
\frac{(\Delta_0^2 - \Delta_2^2) \bar{\mu}^2}{\pi^2}, 
\]

(27)

to the pairing free energy. Together with Eqs.(26) and (27), the gap equation can be expressed by

\[
- \frac{2\Delta_2 \bar{\mu}^2}{\pi^2} + \frac{\partial \Omega_g}{\partial \Delta_2} = 0, 
\]

(28)

for the given values of \( \mu_e, \mu_3 \) and \( \mu_8 \). As long as the II-channel gapless strength is not-very-large, the simplified gap equation is valid for reflecting the gap variation in gCFL\textsuperscript{K}\textsubscript{0}. Also, it is compatible with the gCFL\textsuperscript{K}\textsubscript{0} neutrality condition since Eqs.(22) and (23) have been considered in the free energy \( \Omega_g \).
With the help of the gCFL\(^0\) neutral solution and the gap equation, we turn to examine the location of gCFL\(^0\) in the \((\mu_S, \mu_e)\) plane. Different from the gCFL case, there is no reason to anticipate that the gCFL\(^0\) location is limited in the vicinity of the critical line obtained from Eq.(16) (the dashed line in Fig. 2). In fact, if gCFL\(^0\) located near this line, the free energy \(\Omega_g\) would be almost infinitesimal (remember only the II-channel gapless phenomenon exists). In that case, the resulting gapless phase might be hard to become energetically favorable with respect to the CFL\(^0\) phase. As the matter satisfying the neutrality and gap equation, gCFL\(^0\) becomes actually possible as long as it has a lower free energy (a higher pressure) than CFL\(^0\). Thus, the CFL\(^0\)-gCFL\(^0\) phase transition line is obtained from the Gibbs condition of pressure equilibrium, i.e.

\[
\delta P = -\frac{(\Delta^2_0 - \Delta^2_2)\mu^2}{\pi^2} - \Omega_g + \frac{\mu_e^4}{12\pi^2} = 0. \tag{29}
\]

As shown in Fig. 2, the critical line for the gapless formation (the dashed line) and the phase transition line (solid) separate from each other obviously. When \(m^2_s/2\mu\) is large relatively, the gCFL\(^0\) phase with \(\delta P \geq 0\) is found to exist in the region below the solid line. Correspondingly, the system remains to be CFL\(^0\) in the region between the solid and dashed lines.

The Gibbs condition Eq.(29) implies that the phase transition is of first-order.\(^{2}\) This is very different from the previous conclusion that the CFL-gCFL phase transition is of second-order (at zero temperature)\(^{4}\). In fact, the gCFL phase with the I- and II-channel gapless phenomena is barely a conductor while gCFL\(^0\) with only the I-channel one is a conductor at all. In the gCFL\(^0\) case, no extra \(\tilde{Q}\) neutral condition (such as \(\mu_e^3 \sim (p^+ - p^-)\bar{\mu}^2\) in the gCFL case\(^{4}\)) is required. As a consequence, the \(\tilde{Q}\)-neutral gCFL\(^0\) matter appears in a finite region of the \((\mu_S, \mu_e)\) plane rather than just at a single line. Also, the quadratic dispersion relation predicted in gCFL\(^{4}\) no longer valid and the gapless modes in gCFL\(^0\) might have a linear dispersion relation as the conventional gapless phase.

Finally, we analyze the possibility of eliminating the chromomagnetic instability happens in gCFL qualitatively. In the gCFL case, the I- and II-channel gapless formations coincide at the point of \((\mu_e, \mu_S) = (0, \Delta_0)\), i.e. the gs-bd and rs-bu pairings become breached simultaneously. As stressed in Refs.\(^{6, 7}\), this feature is responsible for the instability occurs for \(A_1\) and \(A_2\) gluons: since the \(m_s\)-relevant self energy for \(A_{1,2}\) stems from the loop diagram composed of gs and rs quarks, the possible coexistence of gapless (unpaired) gs and rs

\(^{2}\)The physical picture is valid at leading order. Since \(m^2_s/2\mu\) has been treated as an external chemical potential, there do exist not only one chemical potential in our concerned system. Thus it is natural that the transition to the gapless phase is first order while the gapless formation itself is continuous (see e.g. Ref.\(^{20}\) for an analogy).
modes provides a singular contribution to the $A_{1,2}$ Meissner masses. In the gCFL$^0$ case, however, the I-channel gapless formation does not occur yet. Therefore, there should exist no longer singularities in the self-energy for $A_{1,2}$ Meissner masses and the corresponding instability should disappear in the gCFL$^0$ phase. On the other hand, the feature that the gCFL location is very close to its critical line has been pointed out to be responsible for the instability for $A_3, A_8$ gluons and $A_\gamma$ photon mainly [7]. For instance, the (almost) infinitesimal II-channel gapless strength in gCFL leads to the singularities in the $A_{3,8,\gamma}$ masses [7]. In the gCFL$^0$ case, however, the location is not limited in the vicinity of the critical line. Instead, a finite region in the $(\mu_S, \mu_e)$ plane is available for the gCFL$^0$ existence. Thus such kind of instability might no longer appear at least for somewhat region in the $(\mu_e, \mu_S)$ plane.

Since only $A_{1,2}$ and $A_{3,8,\gamma}$ exhibit imaginary Meissner masses in gCFL, the previous-predicted instability is very likely to be removed in the gCFL$^0$ phase. Physically, it can be understood from the viewpoint that chromomagnetic instability is perhaps eliminated by including the proper vacuum. It has been suggested that nonzero vacuum expectation values of gluon such as $\langle A_{3}^0 \rangle$ and/or $\langle A_{8}^0 \rangle$ are helpful for removing the gCFL instability [6]. In Ref.[21], the instability in two-flavor superconductor was argued to be resolved by gluon condensation. In the present work, the nonzero $\langle A_{3}^0 \rangle$ and $\langle A_{8}^0 \rangle$ (anisotropic vacuum) derived from the kaon condensation have been attributed to the change of the electric/color neutral solution and then been considered in the study of the gapless formation. In this sense, the conjecture that gCFL$^0$ is out of chromomagnetic instability partly is not very surprising. However, intrinsic links between the (p-wave) kaon supercurrent phase established in [14] and the present-discussed gCFL$^0$ phase still remain to be clarified. Also, we could rule out the possibility that instabilities especially these occur for $A_4, A_5, A_6$ and $A_7$ gluons arise again.

The quantitative calculation of the gluonic Meissner masses in gCFL$^0$ is still necessary which is beyond the scope of the present work.

In summary, we investigated electric/color neutrality and gapless formation in the CFL matter with $K^0$ condensation. By taking the CFL$^0$ neutral solution into account, we clarify why the gCFL$^0$ formation is delayed in comparison with the gCFL case. More importantly, it is found that the gapless phenomenon for down-strange pairing (I channel) is absent while that for up-strange one (II channel) becomes dominated. After the model-independent treatments of the gCFL$^0$ neutrality and the gap equation, we suggest that the CFL$^0$-gCFL$^0$ phase transition is first order. The novel phase structure implies that the previous-predicted gCFL instability might be removed at least partly. These conclusions are likely to be important for fully understanding the unconventional CFL phases in the presence
of $m_s$ and $\mu_e$. Even if the variations of three gaps are considered in a model-dependent calculation, the current results should be qualitatively correct. Of course, there are still some unanswered aspects in the present work. First of all, the $K^0$ condensed CFL matter has been treated as the background of gapless formation, which is safe only if $\mu$ is not too small (otherwise, the condensation can easily suppressed by instanton effect [22]). Due to the nature of kaon condensation, nevertheless, the gCFL$K^0$ phase with only II-channel gapless phenomenon is difficult to gain an obviously lower free energy than the gCFL phase. In this sense, our suggested gapless phase could not replace the role of gCFL, in particular its role in neutron star cores [23], if ignoring the gCFL instability. Secondly, more physics involving Goldstone-mode condensation is not discussed. In fact, we could not exclude possibilities of the charged kaon (and even other modes) condensations completely. For instance, if $\mu_e$ is large enough and the $K^-$ condensation occurs, the gapless phenomenon for up-down pairing (the III channel) needs to be included seriously. Some of the above-mentioned problems are being investigated.

Acknowledgements

This work was supported by National Natural Science Foundation of China (NSFC) grant 10405012 and DOE grant DE-FG02-87ER40328.

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Figure 1: Schematic phase structure of the gCFL formation in the $(\mu_e, \mu_S \equiv m_s^2/2\mu)$ plane, where the critical conditions for I- and II-channel gapless phenomena and the phase transition to unpaired quark matter are shown by the dot, dashed and dot-dashed line respectively. The $\tilde{Q}$-neutral gCFL phase locates below but very close to the dashed line [4].

Figure 2: Similar as Fig.1, but for the gCFL$^0$ formation. The I-channel critical line (dot) coincides with the line of the phase transition to unpaired matter approximately, while the solid line corresponds to the CFL$^0$-gCFL$^0$ phase transition (see text).
