Forecasting China’s per Capita Living Energy Consumption by Employing a Novel DGM (1, 1, $t^\alpha$) Model with Fractional Order Accumulation

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1. Introduction

With the advancement of urbanization, economic growth, and improvement of the living standards of residents, the demand for energy in China has been greatly increased, in which the energy consumption of residents shows the characteristics of rapid growth, accounting for a large proportion of China’s total energy consumption. Whether the future energy supply can support the sustainable growth of China’s economy has become a topic of concern at home and abroad. Therefore, it is of great practical significance to accurately predict the per capita living energy consumption in the future for maintaining the healthy, sustainable, and stable development of China’s social economy. At present, there are few studies on per capita living energy consumption [1, 2]. With the development of science and technology, various prediction methods emerge in endlessly. Among them, grey prediction models and regression prediction models are two commonly used prediction methods. The main difference between them is the different samples required for modeling. The former requires only a small amount of sample data, while the latter is based on large sample modeling. As we all know, some data in China has been lost due to various reasons. In this small sample case, the regression prediction model is obviously no longer applicable.

The grey prediction models play a vital role in the grey system theory, which were originally proposed by Deng [3].
The grey prediction models preprocess the original time series by the operation of accumulation generation to make it become a series with obvious exponential law and then construct differential equation and difference equation to establish the model, so as to realize the simulation and prediction of the original time series. Because of the widespread existence of poor information and uncertain system, grey system theory has a very broad application and development prospect. At present, grey prediction models have been applied in various fields of society due to its excellent forecasting performance [4–10]. Because this type of forecasting model is developed based on the grey system theory, it is usually called the grey forecasting model (GM).

The basic GM (1, 1) model is the most popular and important grey model. In order to further improve the prediction accuracy of the GM (1, 1) model, a number of scholars have improved the model from the cumulative order [11, 12], background values [13], discrete grey prediction model [14, 15], and time response function [16]. With the widespread application of the GM (1, 1) model, some scholars have found that the GM (1, 1) model cannot be applied to all time series. In fact, since the GM (1, 1) model is a model based on homogeneous exponential function, it is difficult to fit time series with approximately nonhomogeneous exponential law. In order to solve this problem, Cui et al. proposed the NGM (1, 1, k) model, which opens a new door for improving the GM (1, 1) model [17]. Qian et al. proposed the GM (1, 1, tα) model based on the research foundation of Cui et al. [18]. The GM (1, 1, tα) model can adapt to different time series by replacing the time power term α, so as to achieve the purpose of accurate prediction. It is a grey prediction model with strong adaptability. These improved methods have greatly expanded the scope of application of grey prediction models and enriched grey system theory.

The fractional accumulation operation was originally proposed by Professor Wu, and it is a measure that can effectively improve the accuracy of the grey prediction model [11]. It can improve the adaptability of the prediction model by expanding the search range of the cumulative order of the grey prediction model to achieve the purpose of accurate prediction. At present, the grey prediction model with fractional accumulation has been widely used in various fields of society [19–21]. In order to further expand the application of fractional accumulation operations in grey prediction models, many scholars are committed to combining the new fractal theory with grey prediction models. For example, Zhu et al. proposed a new fractional grey prediction model-adaptive grey score weighting model and used this model to predict the electricity consumption of Jiangsu Province in China [22], Chen et al. proposed the Fractional Hausdorff grey model [23], and Ma et al. established a conformable fractional grey system model [12]. These studies have greatly expanded the grey system theory and further promoted the combination of analysis theory. Using an operator is an effective way to improve the prediction accuracy of grey prediction models.

Admittedly, both the discrete grey model with time power term and fractional accumulation can improve the prediction performance of the grey models to some extent; however, few studies combine these two methods simultaneously. Based on the previous knowledge, the novel model considering the discrete grey model with time power term and fractional accumulation can not only improve the adaptability, but also fill knowledge gap of grey forecasting theory. After that, this mixed model is applied to predict China’s per capita living energy consumption, which provides a solid basis for policy makers to formulate the reasonable plans.

In general, the primary contributions of this paper can be summarized as follows:

1. A novel discrete GM (1, 1, tα) model with fractional order accumulation is proposed
2. The final optimal nonlinear parameters are determined by the whale algorithm
3. The novel grey prediction model based on the metabolic thought is used to predict China’s per capita living energy consumption from 2018 to 2029

The rest of paper is organized as follows. Section 2 introduces the prerequisite knowledge of this article, including the fractional accumulation operation, the basic GM (1, 1, tα) model, and the discrete GM (1, 1, tα) model with fractional accumulation proposed in this article. Section 3 introduces the method of solving the FDGM (1, 1, tα) model by using the whale algorithm. The application of the FDGM (1, 1, tα) to predict China’s per capita living energy consumption is presented in Section 4, including the comparison with the other eight grey prediction models, and the conclusions are drawn in Section 5.

2. Methods

2.1. Discrete GM (1, 1, tα) Model. As an extensive version of the traditional discrete grey model (DGM (1, 1) for short), the discrete grey model with time power term (referred as GM (1, 1, tα)) further expands the applicable scope by virtue of adjustable time-power coefficient. The detailed steps can be seen as follows.

Assume a nonnegative sequence to be 

\[ X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)) \]

and the first-order accumulated generating operator sequence of \( X^{(0)} \) is given as

\[ X^{(1)} = (x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)), \]

where

\[ x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i), k = 1, 2, \ldots, n. \]

On this basis, the differential equation of the GM (1, 1, tα) model is constructed as

\[ \frac{dx^{(1)}(t)}{dt} + ax^{(1)}(t) = bt^{\alpha} + c, \]

where \((a, b, c)^{t^{\alpha}}\) are the parameters of the GM (1,1, tα) model. It can be inferred that the discrete formula of equation (1) is calculated as
\[ x^{(0)}(k) + az^{(1)}(k) = \frac{b(k^{\alpha+1} - (k-1)^{\alpha+1})}{\alpha + 1} + c, \]  
\[ z^{(1)}(k) = 0.5 \cdot (x^{(1)}(k) + x^{(1)}(k-1)). \]  

By the least square method, the model parameters can be obtained as follows:

\[ (a, b, c)^T = (B^T B)^{-1} B^T Y. \]  

where

\[
B = \begin{pmatrix} 
-2^{1+\alpha} - 1 \\
-3^{1+\alpha} - 2^{1+\alpha} \\
\vdots \\
-n^{1+\alpha} - (n-1)^{1+\alpha} 
\end{pmatrix}, \quad Y = \begin{pmatrix} 
 x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(n) 
\end{pmatrix} \]

According to equation (2), it can be seen that the GM (1, 1, \(t^\alpha\)) model has no specific solution formula (in general, the prediction formula of the grey prediction model is the analytical formula of the differential equation of the model). The introduction of the power term not only makes the model more adaptable but also makes the model difficult to solve. In fact, when the literature proposed the GM (1, 1, \(t^\alpha\)) model, in order to facilitate the solution, the power term \(a\) of the GM (1, 1, \(t^\alpha\)) model was set to 2, which not only restricted the prediction accuracy of the GM (1, 1, \(t^\alpha\)) model but also violated the original intention of improving the adaptability of the model. Discretization of the grey prediction model can avoid the steps of solving differential equations. In order to give full play to the performance of the GM (1, 1, \(t^\alpha\)) model, this article will establish a discrete GM (1, 1, \(t^\alpha\)) model with fractional accumulation.

### 2.2. Discrete GM (1, 1, \(t^\alpha\)) Model with Fractional Order Accumulation

As previously suggested, the fractional accumulated generating operator is crucial for processing original sequences that are affected by nonlinearity and uncertainty, thus generating satisfactory results in many applications. Therefore, we introduce the fractional order accumulation into the GM (1, 1, \(t^\alpha\)) model to improve the performance of the existing grey models.

Suppose that a nonnegative sequence to be

\[ X^{(0)} = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)), \]

and the \(r\)-order accumulated generating operator sequence of \(X^{(0)}\) is given as

\[ X^{(r)} = (x^{(r)}(1), x^{(r)}(2), \ldots, x^{(r)}(n)), \]  

where

\[ x^{(r)}(k) = \sum_{i=1}^{k} (k - i + r - 1) x^{(0)}(i), k = 1, 2, \ldots, n. \]

Then, the whitening differential equation of the GM (1, 1, \(t^\alpha\)) model with fractional accumulation (FDGM (1, 1, \(t^\alpha\))) can be expressed as

\[
\frac{dx^{(r)}(t)}{dt} \bigg|_{t=k} \approx \lim_{\Delta t \to 1} \frac{x^{(r)}(k) - x^{(r)}(k - \Delta t)}{\Delta k} = x^{(r)}(k) - x^{(r)}(k - 1).
\]

Then, the left side of the whitening differential equation of the FDGM (1, 1, \(t^\alpha\)) model with fractional accumulation (FDGM (1, 1, \(t^\alpha\))) can be written as

\[
x^{(r)}(k) - x^{(r)}(k - 1) + ax^{(r)}(k) = (1 + a)x^{(r)}(k) - x^{(r)}(k - 1).
\]

Furthermore, we obtain

\[
(1 + a)x^{(r)}(k) - x^{(r)}(k - 1) = bk^a + c,
\]

which can also be written as

\[
x^{(r)}(k) = \frac{x^{(r)}(k - 1)}{1 + a} + \frac{bk^a}{1 + a} + \frac{c}{1 + a}.
\]

Denoting \(\eta_1 = (1/(1 + a)), \eta_2 = (b/(1 + a)), \) and \(\eta_3 = (c/(1 + a)), \) we have

\[
x^{(r)}(k) = \eta_1 x^{(r)}(k - 1) + \eta_2 k^a + \eta_3.
\]

Equation (11) is the expression of the FDGM (1, 1, \(t^\alpha\)) model [24] proposed in this paper. The parameters in equation (10) can also be solved using the least square method, namely,

\[
\hat{y} = [\tilde{a}, \tilde{b}, \tilde{c}]^T = (B^T B)^{-1} B^T Y, \]

where

\[
B = \begin{pmatrix} 
x^{(r)}(1) & 2^a & 1 \\
x^{(r)}(2) & 3^a & 1 \\
\vdots & \vdots & \vdots \\
x^{(r)}(n - 1) & n^a & 1 
\end{pmatrix}, \quad Y = \begin{pmatrix} 
x^{(r)}(2) \\
x^{(r)}(3) \\
\vdots \\
x^{(r)}(n) 
\end{pmatrix} \]

The relationship between the FDGM (1, 1, \(t^\alpha\)) and the other commonly used grey prediction models is obvious. When \(r = 1\) and \(a = 0\), the FDGM (1, 1, \(t^\alpha\)) yields the basic...
DGM (1, 1) model [14]. When \( r = 1 \) and \( \alpha = 1 \), the FDGM (1, 1, \( t^\alpha \)) yields the NDGM (1, 1) model [25]. When \( r \in (0, 1) \) and \( \alpha = 0 \), the FDGM (1, 1, \( t^\alpha \)) yields the FDGM (1, 1) model [26]. When \( r \in (0, 1) \) and \( \alpha = 1 \), the FDGM (1, 1, \( t^\alpha \)) yields the FNDGM (1, 1) model [27]. Thus, it can be seen that the FDGM (1, 1, \( t^\alpha \)) model is a highly adaptive grey prediction model.

The recursive formula of the FDGM (1, 1, \( t^\alpha \)) model is

\[
x^{(r)}(k) = \eta_1^{k-1} x^{(r)}(1) + \sum_{i=0}^{k-2} \eta_1^i (\eta_2(k-1-i) + \eta_3).
\]  

According to Section 2.1, the prediction formula of the FDGM (1, 1, \( t^\alpha \)) model can be obtained as

\[
\bar{X}^{(0)} = \{\bar{x}^{(0)}(1), \bar{x}^{(0)}(2), \ldots, \bar{x}^{(0)}(n)\} = \bar{X}^{(r)} A^{-r} = \{\bar{x}^{(r)}(1), \bar{x}^{(r)}(2), \ldots, \bar{x}^{(r)}(n)\} A^{-r}.
\]

3. Determination of Parameters for FDGM (1, 1, \( t^\alpha \))

Since the solving method of the FDGM (1, 1, \( t^\alpha \)) model is too complicated, this section is divided into three parts to explain the solution of the FDGM (1, 1, \( t^\alpha \)) model.

3.1. Method for Determining Parameters \( \alpha \) and \( r \) of the FDGM (1, 1, \( t^\alpha \)) Model. It can be seen that the FDGM (1, 1, \( t^\alpha \)) model is established based on the given conditions of parameters \( \alpha \) and \( r \). Therefore, how to determine the optimal values of parameters \( \alpha \) and \( r \) is also a problem. The best parameter values should enable the model proposed in this paper to have the highest accuracy under the given sample. Therefore, we should establish a programming problem that aims to minimize the errors of the proposed model by replacing the values of parameters \( \alpha \) and \( r \) and follows the modeling steps of the proposed model. This article chooses the mean absolute percentage error (MAPE) as the standard for evaluating model errors, and then, this planning problem can be written as

\[
\text{min } f(\alpha, r) = \frac{1}{n-1} \sum_{t=1}^{n} \left| \frac{x^{(0)}(t) - \bar{x}^{(0)}(t)}{\bar{x}^{(0)}(t)} \right| \times 100%,
\]

\[
\begin{align*}
\begin{bmatrix}
\bar{x}^{(0)}(1) \\
\bar{x}^{(0)}(2) \\
\vdots \\
\bar{x}^{(0)}(n)
\end{bmatrix}
&= (B^T B)^{-1} B^T Y,
\end{align*}
\]

\[
\begin{align*}
\begin{bmatrix}
x^{(r)}(1) \\
x^{(r)}(2) \\
\vdots \\
x^{(r)}(n)
\end{bmatrix}
&= \begin{bmatrix} 2^\alpha & 1 \\ 3^\alpha & 1 \\ \vdots & \vdots \\ n^\alpha & 1 \end{bmatrix} \begin{bmatrix}
\bar{x}^{(0)}(1) \\
\bar{x}^{(0)}(2) \\
\vdots \\
\bar{x}^{(0)}(n)
\end{bmatrix},
\end{align*}
\]

s.t.

\[
\begin{align*}
\bar{x}^{(r)}(k) &= \eta_1^{k-1} x^{(r)}(1) + \sum_{i=0}^{k-2} \eta_1^i (\eta_2(k-1-i) + \eta_3),
\end{align*}
\]

\[
\bar{X}^{(0)} = \{\bar{x}^{(0)}(1), \bar{x}^{(0)}(2), \ldots, \bar{x}^{(0)}(n)\} = \bar{X}^{(r)} A^{-r} = \{\bar{x}^{(r)}(1), \bar{x}^{(r)}(2), \ldots, \bar{x}^{(r)}(n)\} A^{-r}.
\]

3.2. The Whale Optimization Algorithm. In the past, people often use genetic algorithm (GA) or particle swarm optimization algorithm (PSO) to solve grey forecasting models, but this method has certain defects [28, 29]. GA and PSO are prone to the phenomenon of too many iterations, slow
convergence, and falling into local extreme value, but whale algorithm (WOA) does not have such a situation. Therefore, this paper chooses to use WOA to solve the model proposed in this paper. Inspired by the social behavior of humpback whale groups, Mirjalili and Lewis proposed the whale optimization algorithm (WOA) in 2016 [30]. At present, WOA has been widely used in bioinformatics [31], image processing [32], and other fields due to its excellent performance. At the same time, WOA is also used to solve the nonlinear programming problem 14 [33]. Therefore, this paper chooses WOA to solve the nonlinear programming problem 14. The main idea of WOA is as follows.

When whales prey, they move in a spiral to surround the school of fish currently considered the best target. Then, these whales update their positions based on the candidate target. This behavior can be expressed by a mathematical formula, namely,

$$\bar{P}(i+1) = \begin{cases} P^*(i) - (2f(i) \cdot \bar{r} - f(i)) \cdot \bar{D}, & \text{if } \xi < 0.5(a), \\
\bar{P}(i) - \bar{P}(i) \cdot e^{\beta t} \cdot \cos(2\pi t) + P^*(i), & \text{if } \xi \geq 0.5(b), \end{cases}$$

(17)

where $$\bar{P}(i)$$ represents the current position of the whales, $$P^*(i)$$ represents the current best position of the whales, $$\bar{r}$$ is a random number in the interval $$[0, 1]$$, $$l$$ is a stochastic number in the interval $$[-1, 1]$$, $$\beta$$ is an arbitrary constant which determines the shape of the spiral movement, $$T$$ is the maximum number of iterations of the algorithm, and $$\xi$$ is a probability to choose a movement strategy from encircling and spiral moving behaviors. When the norm of $$\bar{D}$$ is greater than 1, the position of all whales is updated based on the position of a whale randomly selected. This model can also be expressed by mathematical formulas, namely,

$$\bar{P}(i+1) = \bar{P}_r(i) - C \cdot \left| 2\bar{P}_r(i) \cdot \bar{r} - \bar{P}(i) \right|,$$

(18)

where $$\bar{P}_r(i)$$ is the position of a randomly selected whale in the herd.

Since WOA is designed to solve unconstrained programming problems, we cannot directly use it to solve the model in this paper. Therefore, we need to establish a fitness function to calculate the fitness of each whale agent. According to the nonlinear programming problem described in Section 4.1, the fitness function can be described as

$$\text{fitness} = \frac{1}{n-1} \sum_{i=1}^{n} \frac{x_1^{(0)}(t) - x_1^{(0)}(t)}{x_1^{(0)}(t)} \times 100\%.$$  

(19)

The revised WOA is presented in detail in Algorithm 1.

### 3.3. The Computational Steps

#### Step 1: calculate the parameters $$\alpha$$ and $$r$$ of the model according to the method described in Section 3.1

#### Step 2: bring the parameters $$\alpha$$ and $$r$$ obtained according to Step 1 into the FDGM (1, 1, $$r^k$$) model, and then, calculate the parameters $$a$$, $$b$$, and $$c$$ of the FDGM (1, 1, $$t^e$$) model according to equation (12)

#### Step 3: put the parameters obtained in Step 2 into equations (14) and (15) to get the predicted results of the model

### 4. Application

#### 4.1. Raw Data Collection

The raw data of the per capita living energy consumption of China are collected from the official website National Bureau of Statistics of China (http://www.stats.gov.cn/english/), as shown in Table 1. The points from 2002 to 2011 are used for building the prediction models, and the last 6 points are used for testing the prediction accuracy of the models.

#### 4.2. Evaluating Indicator

In this section, we describe several metrics that are commonly used to evaluate the performance of a prediction model, as shown in Table 2.

#### 4.3. Numerical Results

The numerical results of the FDGM (1, 1, $$t^e$$) model are compared to the commonly used prediction models, including the GM (1, 1) model, DGM (1, 1) model [14], FDGM (1, 1) model [17], NDGM (1, 1, k) model [25], FNDGM (1, 1, $$k^e$$) model [27], ARGM (1, 1) model [15], and DGM (1, 1, $$t^e$$) model. The results of these models are shown in Table 3. The metrics of the models for fitting and prediction are listed in Table 4 and plotted in Figure 1. According to the data shown in Table 3, we can see that the GM (1, 1), DGM (1, 1), and FDGM (1, 1, $$t^e$$) models overestimated the actual values, and the other six prediction models underestimated the actual values. According to the prediction indicators shown in Table 4, we can see that the six indicators of the FDGM (1, 1, $$t^e$$) model are the best of the five prediction models. Therefore, the FDGM (1, 1, $$t^e$$) model shows the best performance in this case. Calculation formulas of nine prediction models are listed as follows.

(1) GM (1, 1) model:
Input: the raw data $X^{(0)}$ and lower and upper bound of $a$ and $r$

Output: the optimal value of the nonlinear parameters $a$ and $r$

1. Initialize the maximum number of iterations $T$ and the number of humpback whales
2. Initialize the locations $\overrightarrow{P}$ of the humpback population
3. Compute the fitness of each humpback by equation (19)
4. Determine the best candidate $\overrightarrow{P}$ based on fitness of each whale agent
5. for $k = 1; k < T; k = k + 1$ do
   6. for each humpback whale do
      7. Update the parameters $r, p, l, \alpha$
      8. if $x(\overrightarrow{P}) < 0.5$ then
      9. if $|\overrightarrow{P}| < 0.5$ then
         10. Update the location of each humpback by equation (17);
      11. else
         12. Determine $\overrightarrow{P}$ by randomly choosing a whale;
         13. Update the location of each humpback by equation (18);
      14. end
      15. else
         16. Update the location of each humpback by equation (17);
      17. end
      18. Compute the fitness of each humpback by equation (19);
      19. end
      20. Update $\overrightarrow{P}$ if a better solution exists;
      21. end
   22. return the optimum value $\overrightarrow{P}$;

Algorithm 1: Algorithm of WOA to search for the nonlinear parameters $a$ and $r$ of the FDGM $(1, 1, t^r)$ model.

Table 1: The raw data of the per capita living energy consumption of China (kg ce).

| Year | Data | Year | Data | Year | Data | Year | Data |
|------|------|------|------|------|------|------|------|
| 2002 | 146  | 2006 | 230  | 2010 | 274  | 2014 | 346.1|
| 2003 | 166  | 2007 | 250  | 2011 | 294  | 2015 | 365.4|
| 2004 | 191  | 2008 | 254  | 2012 | 313  | 2016 | 393.2|
| 2005 | 211  | 2009 | 264  | 2013 | 335  | 2017 | 415.6|

Table 2: Metrics for evaluating effectiveness of the models.

| Indicator | Formula |
|-----------|---------|
| The absolute percentage error (APE) | $APE(k) = |(\bar{x}^{(0)}(k) - \tilde{x}^{(0)}(k))/x^{(0)}(k)| \times 100\%$ |
| The mean absolute percentage error (MAPE) | $MAPE = (1/n) \sum_{k=1}^{n} |(\bar{x}^{(0)}(k) - \tilde{x}^{(0)}(k))/x^{(0)}(k)| \times 100\%$ |
| The mean absolute error (MAE) | $MAE = (1/n) \sum_{k=1}^{n} |\bar{x}^{(0)}(k) - \tilde{x}^{(0)}(k)|$ |
| The mean squares error (MSE) | $MSE = (1/n) \sum_{k=1}^{n} (\bar{x}^{(0)}(k) - \tilde{x}^{(0)}(k))^2$ |
| The root mean squares percentage error (RMSPE) | $RMSPE = \sqrt{(1/n) \sum_{k=1}^{n} (\bar{x}^{(0)}(k) - \tilde{x}^{(0)}(k))/x^{(0)}(k))^2 \times 100\%}$ |
| The index of agreement (IA) | $IA = 1 - (\sum_{k=1}^{n} (\bar{x}^{(0)}(k) - \tilde{x}^{(0)}(k))^2) / (\sum_{k=1}^{n} (\bar{x}^{(0)}(k) - \bar{x})^2 + \sum_{k=1}^{n} (\tilde{x}^{(0)}(k) - \bar{x})^2)$ |
| The correlation coefficient (R) | $R = (Cov(x^{(0)}(k), \tilde{x}^{(0)}(k))) / (\sqrt{Var(x^{(0)}(k))} \sqrt{Var(\tilde{x}^{(0)}(k))})$ |

\[
\hat{x}^{(1)}(t) = 2914.9837e^{0.061064(t-1)} - 2768.9837, \quad t = 1, 2, \ldots, 16,
\]
\[
\hat{x}^{(0)}(1) = \hat{x}^{(1)}(1); \hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1), \quad t = 2, 3, \ldots, 16.
\]
(2) DGM (1, 1) model:

\[
\hat{x}^{(1)}(t) = 146 \cdot 1.062862^t + 174.534876 \cdot \left(1 - 1.062862^t\right), \quad t = 1, 2, \ldots, 16,
\]

(3) FDGM (1, 1) model:

\[
\hat{x}^{(0.602843)}(t) = 146 \cdot 0.968297^t + 94.79946 \cdot \left(1 - 0.968297^t\right), \quad t = 1, 2, \ldots, 16.
\]

(1) NDGM (1, 1, k):

\[
\hat{x}^{(1)}(t) = 1471.193656 \cdot 0.86313337^t + \frac{46.625869}{0.136866} - 1464.5028, \quad t = 1, 2, \ldots, 16,
\]

\[
\hat{x}^{(0)}(1) = \hat{x}^{(1)}(1); \hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1), \quad t = 2, 3, \ldots, 16.
\]

(2) FNDGM (1, 1, k):

\[
\hat{x}^{(0.817046)}(t) = 366.7525 \cdot 0.8003855^t + \frac{40.211239}{0.19961447} - 348.98792767, \quad t = 1, 2, \ldots, 16.
\]

(3) ARGM (1, 1):

\[
\hat{x}^{(0)}(t) = -253.6 \cdot 0.908097^t + 399.6, \quad t = 1, 2, \ldots, 16.
\]
4.5. Forecasting Results of China’s per Capita Living Energy Consumption from 2018 to 2029. As the FDGM (1, 1, \(t^o\)) model proposed in this paper shows the best prediction performance in the previous section, this section will use the FDGM (1, 1, \(t^o\)) model based on metabolism mechanism to predict China’s per capita living energy consumption from 2018 to 2029 (According to the prediction results shown in Section 3.3, we can see that when \(d=6\), the prediction accuracy of the FDGM (1, 1, \(t^o\)) model is the highest. Therefore, this section sets \(d=6\). The prediction results are shown in Table 5, and the corresponding results are also shown in Figure 2. It can be seen in Figure 2 that China’s per capita living energy consumption will still increase rapidly in the next 12 years, and the annual consumption will exceed 1000 (kg ce) by the year of 2028. This means that China’s demand for energy is still very large in the next few years; thus, the production ability should be enhanced to meet the highly growing demand. At the same time, we can see that the growth rate of per capita energy living consumption in the next nine years will still increase. Increasing energy consumption will bring huge pressure and adjustment to society. Facing the future of high energy consumption, China needs to continue to adopt effective policies to reduce the consumption of living energy and build an energy-saving society. At the same time, China should optimize the energy consumption structure and increase the proportion of clean and high-quality energy in the consumption structure. Developing a low-carbon economy and building a low-carbon life will be an important strategic choice for sustainable development in the future.

(4) NGM (1, 1, \(k\)):

\[
\begin{align*}
\hat{x}^{(1)}(t) &= 1325.1184e^{-0.142698(t-1)} + 343.6577t - 1522.77615, \quad t = 1, 2, \ldots, 16, \\
\hat{x}^{(0)}(1) &= \hat{x}^{(1)}(1); \quad \hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1), \quad t = 2, 3, \ldots, 16.
\end{align*}
\]

(5) DGM (1, 1, \(t^o\)):

\[
\begin{align*}
\hat{x}^{(1)}(t) &= 115.3365198 + 53.44286 \cdot (t-1)^{0.62504} + 0.98096 \hat{x}^{(1)}(t-1), \quad t = 2, 3, \ldots, 16, \\
\hat{x}^{(0)}(1) &= \hat{x}^{(1)}(1); \quad \hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1), \quad t = 2, 3, \ldots, 16.
\end{align*}
\]

(6) FDGM (1, 1, \(t^o\)):

\[
\begin{align*}
\hat{x}^{(0.35742)}(t) &= 87.95390053 + 0.025791068 \cdot (t-1)^3 + 0.9056291165 \hat{x}^{(1)}(t-1), \quad t = 2, 3, \ldots, 16, \\
\hat{x}^{(0)}(1) &= \hat{x}^{(1)}(1); \quad \hat{x}^{(0)}(t) = \hat{x}^{(1)}(t) - \hat{x}^{(1)}(t-1), t = 2, 3, \ldots, 16.
\end{align*}
\]

Note: DGM (1) = DGM (1, 1); DGM (2) = DGM ((1, 1, \(t^o\)).
Figure 1: Continued.
Figure 1: Six performance indicators of the nine prediction models (1 = GM; 2 = DGM; 3 = FDGM; 4 = NGM; 5 = NDGM; 6 = FNDGM; 7 = ARGM; 8 = DGM(2); 9 = FDGM (1, 1, tα)).

Table 4: Performance evaluation indicators of the nine prediction models.

|          | GM   | DGM (1) | FDGM | NGM  | NDGM | FNDGM | ARGM | DGM (2) | FDGM |
|----------|------|---------|------|------|------|-------|------|---------|------|
| MAPE     | 3.5060 | 3.5080  | 3.5080 | 1.1360 | 2.250 | 1.1970 | 1.2190 | 1.563 | 1.1310 | 1.0720 |
| RMSPE    | 4.5960 | 4.6100  | 4.6100 | 1.4330 | 3.480 | 1.4670 | 1.4750 | 1.8540 | **1.3940** | 1.4010 |
| MAE      | 764.1110 | 764.0000 | 764.0000 | 296.2220 | 311.1110 | 311.2220 | 306.0000 | 378.7780 | 293.7780 | **257.6670** |
| MSE      | 931.7600 | 931.8150 | 931.8150 | 378.2150 | 392.9960 | 393.1800 | 393.1800 | 463.7160 | 396.0050 | **347.5530** |
| IA       | 0.9857 | 0.9857  | 0.9978 | 0.9976 | 0.9967 | 0.9967 | 0.9969 | 0.9959 | 0.9971 | 0.9974 |
| R        | 0.9814 | 0.9814  | 0.9970 | 0.9976 | 0.9967 | 0.9967 | 0.9969 | 0.9959 | 0.9971 | 0.9974 |
| Prediction|      |         |       |       |       |       |       |       |       |       |
| MAPE     | 2.9880 | 2.9640  | 2.9640 | 11.6790 | 14.0770 | 14.2990 | 14.2990 | 10.7190 | 10.2540 | 12.1170 | **2.3850** |
| RMSPE    | 3.2580 | 3.2310  | 3.2310 | 12.7210 | 15.2380 | 15.4650 | 15.4650 | 11.6640 | 11.4410 | 13.1280 | **2.7860** |
| MAE      | 1108.170 | 1099.000 | 1099.000 | 4395.330 | 5289.670 | 5372.000 | 4032.830 | 3881.500 | 4554.000 | **899.000** |
| MSE      | 1224.470 | 1213.720 | 1213.720 | 4948.640 | 5914.460 | 6000.425 | 4536.850 | 4481.360 | 5098.680 | **1049.950** |
| IA       | 0.9728 | 0.9732  | 0.9585 | 0.5177 | 0.5126 | 0.6206 | 0.6131 | 0.5767 | 0.9808 | 0.9808 |
| R        | 0.9961 | 0.9961  | 0.9869 | 0.9789 | 0.9783 | 0.9894 | 0.9845 | 0.9868 | 0.9868 | 0.9868 |

Note. DGM (1) = DGM (1, 1); DGM (2) = DGM ((1, 1, tα)).

Table 5: The predicted values of China’s per capita living energy consumption from 2018 to 2029.

| Year | Data | Growth rate (%) | Year | Data | Growth rate (%) |
|------|------|-----------------|------|------|-----------------|
| 2018 | 443.46 | 6.7036        | 2024 | 722.51 | 9.7773        |
| 2019 | 475.25 | 7.1686        | 2025 | 796.36 | 10.2213       |
| 2020 | 511.75 | 7.6802        | 2026 | 880.87 | 10.6120       |
| 2021 | 553.78 | 8.2130        | 2027 | 970.75 | 10.2029       |
| 2022 | 602.26 | 8.7544        | 2028 | 1073.64 | 10.5990      |
| 2023 | 658.16 | 9.2817        | 2029 | 1188.81 | 10.7271      |
5. Conclusion

In this paper, a novel FDGM (1, 1, $t^\alpha$) model based on the metabolism mechanism (abbreviated as FDGM (1, 1, $t^\alpha$) model) has been proposed to study China’s per capita living energy consumption. The FDGM (1, 1, $t^\alpha$) model is developed based on the grey system theory combined with fractional accumulation. The numerical results show that the FDGM (1, 1, $t^\alpha$) model is more suitable to predict China’s per capita living energy consumption than other grey prediction models.

The FDGM (1, 1, $t^\alpha$) model is used to predict China’s per capita living energy consumption from 2018 to 2029. According to the prediction results of the FDGM (1, 1, $t^\alpha$) model, we can see that China’s per capita living energy consumption will exceed 5000 (kg ce) in the next few decades, and the growth rate will gradually balance, which is obviously consistent with China’s national conditions. Because the new normal of the Chinese economy weakens the driving force of macroeconomic growth on energy demand which is the general trend of future development, the growth rate of energy consumption will also change, and energy consumption will enter a period of medium and low-speed growth. Therefore, it can be seen that the FDGM (1, 1, $t^\alpha$) model proposed in this paper can well describe the future development trend of China’s per capita energy living consumption and has certain practical significance.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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Figure 2: The predicted values and growth rate of China’s per capita living energy consumption from 2018 to 2029.
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