New Viewpoint to the Source of Weak CP Phase

Jing-Ling Chen\textsuperscript{1}, Mo-Lin Ge\textsuperscript{1,2}, Xue-Qian Li\textsuperscript{3,4}, Yong Liu\textsuperscript{1}

\textbf{1. Theoretical Physics Division}  
Nankai Institute of Mathematics  
Nankai University, Tianjin 300071, P.R.China

\textbf{2. Center for Advanced Study}  
Tsinghua University, Beijing 100084, P.R.China

\textbf{3. CCAST(World Laboratory)} P.O.Box 8730, Beijing 100080, P.R.China

\textbf{4. Department of Physics}  
Nankai University, Tianjin 300071, P.R.China

\textbf{Abstract}

The relation between CP-violation phase angle and the other three mixing angles in Cabibbo-Kobayashi-Maskawa matrix is postulated. This relation has a very definite geometry meaning. The numerical result coincides surprisingly with that extracting from the experiments. It can be further put to the more precise tests in the future.

PACS number(s): 11.30.Er, 12.10.Ck, 13.25.+m
New Viewpoint to the Source of Weak CP Phase

Although more than thirty years have elapsed since the discovery of CP violation [1], our understanding about the source of CP violation is still very poor. In the Minimal Standard Model (MSM), CP violation is due to the presence of a weak phase in the Cabibbo-Kabayashi-Maskawa (CKM) matrix [2][3]. It is generally believed to be independent of the three mixing angles.

Up to now, all the experimental results are in good agreement with MSM. Nevertheless, the correctness of CKM mechanism is far from being proved. On other hand, the MSM which possesses many free parameters is not fully satisfactory. People have tried to reduce the number of the free parameters, but searching for the source of CP violation is more profound in high energy physics [4] [5][6][7][8]. Fritzsch [9][10] noticed that because the eigenstates of the weak interaction are not the quark mass-eigenstates, there should be a unitary transformation to connect the two bases. It would establish a certain relation between the quark masses and the weak interaction mixing angles, while a weak CP phase is embedded explicitly.

From the general theory of Kabayashi-Maskawa [2], we know that there can exist a phase factor in the three-generation CKM matrix and it cannot be rotated away by redefining the phases of quarks, but we can ask whether there is an intrinsic relation between the phase and the three rotation angles.

Concretely, supposing $V_d$ and $V_u$ diagonalize the mass matrices for d-type and u-type quarks respectively [11], $V_{KM} = V_u^* V_d$ is the CKM matrix and can be written as

$$V_{KM} = \begin{pmatrix} c_1 & -s_1 c_3 & -s_1 s_3 \\ s_1 c_2 & c_1 c_2 c_3 - s_2 s_3 e^{i\delta} & c_1 c_2 s_3 + s_2 c_3 e^{i\delta} \\ s_1 s_2 & c_1 s_2 c_3 + c_2 s_3 e^{i\delta} & c_1 s_2 s_3 - c_2 c_3 e^{i\delta} \end{pmatrix}$$  \hspace{1cm} (1)

with the standard notations $s_i = \sin \theta_i$ and $c_i = \cos \theta_i$.

It is noted that here we adopt the original form of the CKM parametrization. There are some other parametrization ways, for example the Wolfenstein’s [12][13][14] and that recommended by the data group [15][16] [17], but it is believed that physics does not change when adopting various parametrizations.

It is well known that the KM parametrization can be viewed as a product of three Eulerian rotation matrices and a phase matrix [11]

$$V_{KM} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_2 & -s_2 \\ 0 & s_2 & c_2 \end{pmatrix} \begin{pmatrix} c_1 & -s_1 & 0 \\ s_1 & c_1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -e^{i\delta} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_3 & s_3 \\ 0 & -s_3 & c_3 \end{pmatrix}. \hspace{1cm} (2)$$
People have noticed that the weak CP phase $\delta$, which cannot be eliminated in the three generation CKM matrix by any means, is introduced artificially and seems to have nothing to do with the three "rotation" angles. Anyway, such a fact does not seem to be natural.

However, for a naive $O(3)$ rotation group, a geometric phase can automatically arise while two non-uniaxial successive rotation transformations being performed. For instance, $R_x(\theta_1)$ denotes a clockwise rotation about the $x$-axis by $\theta_1$, while $R_y(\theta_2)$ is about the $y$-axis by $\theta_2$. Supposing on a unit-sphere surface, the positive $z$-axis intersects with the surface at a point A, after performing these two sequential operations $R_y(\theta_2)R_x(\theta_1)$, the point-A would reach point-B via an intermediate point-C, by contrast, one can connect A and B by a single rotation $R_{\xi}(\theta_3)$, where $R_{\xi}(\theta_3)$ denotes a clockwise rotation about the $\xi$—axis by $\theta_3$. The geometric meaning can be depicted in a more obvious way is that if one chooses an arbitrary tangent vector $\hat{\alpha}$ at point-A which would rotate to $\hat{\alpha}'$ and $\hat{\alpha}''$ by $R_y(\theta_2)R_x(\theta_1)$ and $R_{\xi}(\theta_3)$ respectively, then one can find that $\hat{\alpha}'$ does not coincide with $\hat{\alpha}''$, but deviates by an extra rotation. Concretely, if one writes down the rotation in the adjoint representation of $O(3)$, he can find

$$R_{\hat{\eta}}(\delta)R_{\xi}(\theta_3) = R_y(\theta_2)R_x(\theta_1),$$

where $R_{\hat{\eta}}(\delta)$ represents a counterclockwise rotation about the $\hat{\eta}$—axis by $\delta$. The $\hat{\eta}$—axis is a vector from the center of the sphere to the point-B and $\delta$ satisfies a relation

$$\cos \frac{\delta}{2} = \frac{1 + \cos \theta_1 + \cos \theta_2 + \cos \theta_3}{4 \cos \frac{\theta_1}{2} \cos \frac{\theta_2}{2} \cos \frac{\theta_3}{2}}.$$  

Here $\delta$ is fully determined by the three rotation angles. Geometrically, it is the solid angle which $\theta_1$, $\theta_2$ and $\theta_3$ span, or in other words, this solid angle corresponds to an area of the spherical triangle constructed by $\theta_i$ ($i = 1, 2, 3$), or the excess of the spherical triangle.

Eq.(3) can be recast in the form

$$R_{\hat{\eta}}(\delta) = R_y(\theta_2)R_x(\theta_1)R_{\xi}^{-1}(\theta_3)$$

(5)

to emphasize that the product of two finite rotations about different axes cannot be cancelled out by a third rotation but leads to a residual phase described by $R_{\hat{\eta}}(\delta)$. The appearance of this phase is due to the non-commutativity of finite three-dimensional rotations. The same conclusion can be drawn for $SU(2)$, $SU(3)$ group and the Lorentz group $O(3,1)$ etc.[18][19][20][21][22]. It is worthy mentioning that the Wigner angle can be interpreted as a geometric phase (or anholonomy) associated with a triangular circuit in rapidity space in the theory of special relativity [23][24].
When a group is an Abelian one, the phase factor will not emerge. Indeed, for the two-generation quarks, the mixing corresponds to a planar rotation, all operations are commutative, there does not exist such an extra factor, correspondingly, we know that there is no a CP phase if there were only two generations. This stimulates us to reach an understanding that for the three-generation case, the weak CP phase which cannot be eliminated arises from a hidden $O(3)$ rotation symmetry in the flavor space. Accepting this understanding, the value of the mysterious weak phase $\delta$ will be determined by the simple relation Eq.(4) and has a definite geometric meaning: it is just the solid angle spaned by the three mixing angles $\theta_1$, $\theta_2$ and $\theta_3$ in the flavor space. In fact, it has been recognized that the CP violation parameter $\epsilon$ is related to a certain area [4][25] more than ten years before, but the geometric relation between this area and the three mixing angles and CP violation phase has not been recognized yet.

So far the only reliably measured CP violation quantity is $\epsilon$ in the K-system and the mechanism causing $K^0 - \bar{K}^0$ mixing has already been well studied in the framework of MSM. Except an unknown B-factor, one can evaluate $\epsilon$ in terms of the CP phase $\delta$ as [26][27][28]

$$|\epsilon| \approx \cos \theta_2 \sin \theta_2 \sin \theta_3 \sin \delta \left[ \frac{\sin^2 \theta_2 (1 + \eta \log \eta) - \cos^2 \theta_2 \eta (1 + \log \eta)}{\sin^4 \theta_2 + \cos^4 \theta_2 \eta - 2 \sin^2 \theta_2 \cos^2 \theta_2 \eta \log \eta} \right],$$

(6)

where $\eta = m_c^2/m_t^2$.

The numerical results is listed in Table 1. Where the inputs of $|V_{ij}|$ are taken from the data book [29] and

$$m_c = 1.5 \text{ GeV}, \quad m_t = 176 \text{ GeV}, \quad |\epsilon| = 2.3 \times 10^{-3},$$

with all the given errors. Here, $\sin \delta_{Th}$ is calculated by using Eq.(4), while $\sin \delta_{Ex}$ extracted from Eq.(6).

One can notice that considering the experimental error tolerance, the two obtained values are consistent. Since the extraction of $\sin \delta$ from the data of $\epsilon$ still depends on the evaluation of the concerned hadronic matrix elements which is not reliable so far, namely we cannot handle the non-perturbative QCD effects well, the deviation between the two $\delta$ values is reasonable.

In conclusion, we propose that the weak CP phase in the CKM matrix is the geometric phase for an $O(3)$ rotation in the flavor space, and determined by the three mixing angles according to Eq.(4), its value is consistent with that extracted from the measurement of $\epsilon$ in K-system.
| $V_{ud}$ | $V_{cd}$ | $V_{us}$ | $\sin\delta_{Th}$ | $\sin\delta_{Ex}$ |
|---------|---------|---------|-------------------|-------------------|
| 0.9745  | 0.218   | 0.219   | 0.022638          | 0.002658          |
| 0.9748  | 0.218   | 0.219   | 0.018978          | 0.002746          |
| 0.9751  | 0.218   | 0.219   | 0.014477          | 0.002939          |
| 0.9754  | 0.218   | 0.219   | 0.007791          | 0.003524          |
| 0.9745  | 0.220   | 0.219   | 0.019884          | 0.002248          |
| 0.9748  | 0.220   | 0.219   | 0.015635          | 0.002255          |
| 0.9751  | 0.220   | 0.219   | 0.009757          | 0.002416          |
| 0.9745  | 0.222   | 0.219   | 0.015439          | 0.001830          |
| 0.9748  | 0.222   | 0.219   | 0.009410          | 0.002100          |
| 0.9745  | 0.224   | 0.219   | 0.006480          | 0.003807          |
| 0.9745  | 0.218   | 0.221   | 0.018836          | 0.003345          |
| 0.9748  | 0.218   | 0.221   | 0.014269          | 0.003837          |
| 0.9751  | 0.218   | 0.221   | 0.007357          | 0.005573          |
| 0.9745  | 0.220   | 0.221   | 0.016697          | 0.002829          |
| 0.9748  | 0.220   | 0.221   | 0.011367          | 0.003151          |
| 0.9745  | 0.222   | 0.221   | 0.012755          | 0.002302          |
| 0.9748  | 0.222   | 0.221   | 0.003806          | 0.002934          |
| 0.9745  | 0.224   | 0.221   | 0.002658          | 0.004790          |
| 0.9745  | 0.218   | 0.223   | 0.012551          | 0.005214          |
| 0.9748  | 0.218   | 0.223   | 0.002690          | 0.019451          |
| 0.9745  | 0.220   | 0.223   | 0.011066          | 0.004410          |
| 0.9745  | 0.222   | 0.223   | 0.006865          | 0.003589          |

Average Values: $0.0119 \pm 0.0056$, $0.0040 \pm 0.0035$

Meanwhile, to make the three rotation angles enclose a solid angle, the following constraint must be satisfied

$$\theta_i + \theta_j \geq \theta_k, \quad (i \neq j \neq k \text{ and } i,j,k = 1,2,3).$$

It would provide a criterion for judging our postulation.

It is also interesting to make a comparison with the four-generation case. There a rotation in the flavor space should be four-dimensional, correspondingly, six rotation angles and three independent phases should be present. That is obviously coincide with the parametrization in four dimensional CKM matrix where the number of rotation angles is $N(N - 1)/2 = 6$ while the number of phases is $(N - 1)(N - 2)/2 = 3$.

Anyway, the relation shown in Eq.(4) postulated by us can give some strict limits on the free parameters presented in KM matrix, just so, it can be further put to the more
precise tests in the future. At least, Eq.(4) can be taken as a good parameterization form for the weak CP-violation phase.

**Acknowledgment**: This work is partly supported by National Natural Science Foundation of China. We would like to thank Dr. Z.Z. Xing for helpful comments.

**References**

[1] J.H.Christenson, J.W.Cronin, V.L.Fitch and R.Turlay, Phys.Rev.Lett. 13(1964)138.

[2] M.Kobayashi and T.Maskawa, Prog.Theor.Phys. 42(1973)652.

[3] N.Cabibbo, Phys.Rev.Lett. 10(1963)531.

[4] *CP Violation* Ed. C.Jarlskog. World Scientific Publishing Co.Pte.Ltd 1989.

[5] *CP Violation* Ed. L.Wolfenstein, North-Holland, Elsevier Science Publishers B.V. 1989.

[6] L.L.Chau, Phys.Rept. 95(1983)1.

[7] E.A.Paschos and U.Turke, Phys.Rept. 4(1989)145.

[8] A.Pich, Preprint CP violation, CERN-TH.7114/93.

[9] H.Fritzsch, Phys.Lett.B. 70(1977)436. Phys.Lett.B. 73(1978)317.

[10] H.Fritzsch, Nucl.Phys.B. 155(1979)189. Preprint MPI-PHT/96-32.

[11] J.F.Donoghue, E.Golowich and B.R.Holstein, *Dynamics of the Standard Model* Cambridge University Press, 1992. P.60~P.69.

[12] L.Wolfenstein, Phys.Rev.Lett. 51(1983)1945.

[13] Z.Z.Xing, Phys.Rev.D. 51(1995)3958.

[14] A.J.Buras, M.E.Ladtenbacher and G.Ostermaier, Phys.Rev.D. 50(1994)3433.

[15] L.-L.Chau and W.-Y.Keung, Phys.Rev.Lett. 53(1984)1802.

[16] L.Maiani, Phys.Lett.B 62(1976)183.

[17] H.Fritzsch and J.Plankl, Phys.Rev.D 35(1987)1732.
[18] Eds. A.Shapere and F.Wilczek, *Geometric Phases in Physics*, World Scientific, Singapore, 1989.

[19] G.Khanna, S.Mukhopadhyay, R.Simon and N.Mukunda, Ann.Phys. 253(1997)55.

[20] T.F.Jordan, J.Math.Phys. 29(1988)2042.

[21] R.Simon and N.Mukunda, J.Math.Phys. 30(1989)1000.

[22] S.Weinberg, *The Quantum Theory of Fields* Published by the Press Syndicate of the University of Cambridge, 1995. P.81~P.100.

[23] P.K.Arvind, Am.J.Phys. 65(1997)634.

[24] Jing-Ling Chen and Mo-Lin Ge, On the Winger angle and its relation with the defect of a triangle in hyperbolic geometry, J.Geom.Phys., in press.

[25] C.Jarlskog, Phys.Rev.Lett. 55(1985)1039.

[26] T.P.Cheng, L.F.Li, *Gauge Theory of Elementary Particle Physics*, Clarendon Press.Oxford.1984.

[27] J.Ellis, M.K.Gaillard and D.V.Nanopoulos, Nucl.Phys. B. 106(1976)292. Nucl.Phys.B. 109(1976)216.

[28] J.Ellis, M.K.Gaillard, D.V.Nanopoulos and S.Rudaz, Nucl.Phys.B. 131(1977)285.

[29] Particle Data Group, Phys.Rev.D. 54(1996)94.