Detection of Parametric Roll Resonance using Bayesian Discrete-Frequency Model Selection

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Abstract: This paper explores the use of two discrete-frequency models and a probabilistic Bayesian model selection procedure to detect the inception of parametric resonance in ships. We exploit knowledge of the coupling between roll and pitch due to the restoring forces arising from the shape of the hull to propose a single and a double discrete-frequency model. These models are then used within a Bayesian framework to compute the posterior distribution of the frequencies and amplitudes of the two models and this information is used for model selection. The latter is based on the computation of the probability that each one of the models is correct given the data analysed within a past window of samples. The algorithm is tested with data from a scale-model experiment for both regular and irregular sea states. The results indicate the proposed detector is effective in accusing the inception of parametric resonance.

Keywords: Parametric Roll; Detection Algorithms; Frequency Estimation; Bayesian Inference; Statistical Inference;

1. INTRODUCTION

The issue of parametric roll resonance in surface ships has gained significant attention in the last two decades due to events that resulted in cargo damage and losses, for example, the APL China incident (France et al., 2001). This effect is manifested by a rapid build-up of roll motion when ships are sailing in head seas. It arises due to a combination of several effects. Hull shapes with significant bow flare, raised sterns, and low roll damping contribute. Wave lengths comparable to the ship length and wave height above a threshold value relative to the sheer freeboard (see Watson (1998)) also contribute. Likewise, encounter wave frequencies inducing a dominant frequency in pitch motion that is twice the roll natural frequency also contribute.

Consequently, recent research efforts have been dedicated not only to understand the dynamics of transverse ship stability in waves (Neves and Rodríguez, 2008; Holden et al., 2007a) but also in terms of detection and control measures (Galeazzi, 2009; Holden, 2011; Breu, 2013). This paper develops a novel detection algorithm based on Bayesian inference techniques. The objective is to identify a non-dominant frequency component in the pitch and heave motion of a vessel under parametric roll resonance conditions, and exploit this identification for detection.

Due to the shape of the hulls that experience parametric roll, there is a coupling in the ship restoring forces between different degrees of freedom as evidenced in the parametric models of Neves and Rodríguez (2008) and Holden et al. (2007a). As a result of this coupling, a sub-harmonic of the dominant frequency in pitch motion, and also in heave motion, appears early in the inception of parametric roll resonance. The phenomenon has been observed in experimental data (see Holden et al. (2007a)) in regular and irregular waves, and the proposed detection algorithms are verified using this data. We therefore investigate the use of a Bayesian inference technique that has been shown to be effective in discrete-frequency estimation, and use the framework to choose between two competing models to identify parametric roll resonance. We also compare the proposed detector with that of Holden et al. (2007b), which adapts a simplified roll model with a Kalman Filter and detects the model instability through its eigenvalues.

2. SHIP DYNAMICS

2.1 Seakeeping models of Ship-wave Interactions

In deep water, the sea-surface elevation, and hence the pressure field they create, can be modelled as a realisation
of a zero-mean correlated Gaussian stochastic process (Ochi, 1998). Under the assumption of stationarity, the sea-surface elevation is characterised by a power-spectral density $S_{ww}(\omega)$. When a vessel is subjected to waves, pressure variations on the wetted surface induce forces and thus motion. Within seakeeping theory of ship motion, the vessel response to wave excitations is modelled as a linear dynamical system. In this context, the power-spectral density of stationary ship motion in all six degrees of freedom $\ell = 1, \ldots, 6$ can be expressed as

$$S_{\eta\eta,\ell}(\omega) = |H_\ell(j\omega)|^2 S_{ww}(\omega)$$

(1)

where $S_{\eta\eta,\ell}(\omega)$ is the power spectral density of the pitch motion, $\omega$ denotes the encounter frequency and $H_\ell(j\omega)$ is known as the motion response amplitude operator in the naval architecture literature.

As discussed in Perez (2005), the roll motion response amplitude operator ($\ell = 4$) can be approximated by a second-order filter of the form

$$H_4(j\omega) = \frac{2\zeta_4\omega_4(j\omega)}{(j\omega)^2 + 2\zeta_4\omega_4(j\omega) + \omega_4^2}$$

(2)

where $\omega_4$ is the natural frequency and $\zeta_4$ is the damping coefficient in a degree of freedom. These parameters depend on the hull shape, ship loading, forward speed, and the direction in which the waves approach the vessel.

2.2 Characteristics and Detection of Parametric Roll

Consider the experimental data shown in Figure 1 with regular waves. This data is part of a set of wave tank experiments with a 1:45 scale-model container ship described in Holden et al. (2007a). From this figure, we can see the build-up of roll motion with a clear frequency signature from about 2 minutes into the experiment, where the magnitude of the roll motion grows from close to 0 degrees to greater than 20 degrees in 12 roll periods. In the time series of the pitch, shown in the bottom plot, it is apparent towards the end of the experiment that there are multiple frequency components, and this motivates the work in this paper.

From the coupled restoring model developed in Holden et al. (2007a), it is proposed that this effect in the pitch and heave motion is related to the increasing roll motion magnitude, and the frequency of the roll motion may be present in the pitch and heave motion.

To investigate this, we compute the power spectral density $S_{\eta\eta,\ell}(\omega)$ of the pitch motion (degree of freedom $\ell = 5$), using the Bartlett method. We split the time series into cases where pre and post parametric resonance is developing, and compute the power spectral density on these two time series. Pre parametric resonance, time up to 2.5 minutes, is shown as the solid blue line, and post parametric resonance is the dashed red line in Figure 2.

We note that the power spectral density of the heave motion (degree of freedom $\ell = 3$), is similar to the pitch density shown. The power spectral density of the roll motion (degree of freedom $\ell = 4$), has a single peak at approximately 0.3 rad/sec and a large increase in magnitude is present as parametric resonance builds.

As we can see, this experiment fits the traditional measure of parametric roll resonance. The main pitch frequency content is close to the wave-encounter frequency, and the main roll frequency is approximately half of the encounter frequency. However, we also observe an interesting effect in the pitch frequency spectrum. At the inception of the parametric resonance, another peak in the frequency spectrum develops at the same frequency as the main roll frequency. This effect was also observed in the heave density.

This leads to the questions of whether the roll frequency, approximately half of the wave-encounter frequency, develops in the pitch and heave motion at the inception of parametric resonance events. Likewise, it questions whether a Bayesian discrete frequency detection algorithm with proven performance in other applications can provide a suitable detection mechanism for this effect. These questions motivate the study in this paper.

3. PARAMETRIC ROLL RESONANCE DETECTORS

A number of algorithms have been proposed to detect or warn of parametric roll resonance conditions. One example is the algorithm by Holden et al. (2007b), which we review in Section 3.1. This algorithm will be used as a benchmark for comparison with the new algorithm proposed in this paper.
paper that is detailed in Section 3.2 and its use as a
detector detailed in Section 3.3.

3.1 Roll Stability through identified Second-order Models

The algorithm proposed by Holden et al. (2007b) consists
of modelling the wave-induced roll motion as a linear
time invariant (LTI) system and using a Kalman Filter
to estimate the parameters. As discussed in (Perez, 2005,
Ch 12.4) the motion spectrum in (1) and (2) can be
approximated in state-space form by

\[
\begin{bmatrix}
\phi_k^{W+1} \\
p_k^{+1}
\end{bmatrix} = \begin{bmatrix}
1 & T_s & -\omega_\phi^2 T_s^2 \\
-2\zeta_\phi \omega_\phi T_s & 1 - 2\zeta_\phi \omega_\phi T_s & \phi_k^W
\end{bmatrix} \begin{bmatrix}
\phi_k^W \\
p_k
\end{bmatrix} + \begin{bmatrix}
w_{1,k} \\
w_{2,k}
\end{bmatrix}
\] (3)

where \(\phi^W\) is the roll motion due to waves, \(p^W\) is the
roll rate, \(\omega_\phi\) is the roll frequency, \(\zeta_\phi\) is the roll
damping, \(T_s\) is the sampling time, and \(w \equiv [w_1, w_2]^T\) are
independent additive white noise processes. This model
relates to the power spectrum in (1).

Rewriting the system in terms of four unknown parameters
\(\beta \equiv [\beta_1, \beta_2, \beta_3, \beta_4]^T\), we obtain:

\[
\begin{bmatrix}
\phi_k^{W+1} \\
p_k^{+1}
\end{bmatrix} = \begin{bmatrix}
\beta_1 & \beta_2 & \beta_3 & \beta_4
\end{bmatrix} \begin{bmatrix}
\phi_k^W \\
p_k
\end{bmatrix} + \begin{bmatrix}
w_k
\end{bmatrix}
\] (4)

where \(w_k \sim N(0, R_w)\). By assuming that the parameters
are approximately constant, i.e. \(\beta \approx 0\), we can
rearrange the system to be in a form suitable for a Kalman Filter
with the state and measurement equations given as:

\[
\beta_{k+1} = \beta_k + \epsilon_k \\
C_k \beta_k + w_k
\]

where \(\epsilon \sim N(0, R_\beta)\), and

\[
C_k = \begin{bmatrix}
\phi_{k-1}^W p_{k-1}^W & 0 & 0 \\
0 & \phi_{k-1}^W p_{k-1}^W & 0
\end{bmatrix}.
\]

The prediction update:

\[
\hat{\beta}_{k|k-1} = \beta_{k-1|k-1} + R_\beta.
\]

With measurement update:

\[
\hat{\beta}_{k|k} = \hat{\beta}_{k|k-1} + L_k \left( \begin{bmatrix}
\phi_k^W \\
p_k
\end{bmatrix} - C_k \hat{\beta}_{k|k-1} \right)
\]

\[
L_k = P_{k|k-1} C_k (C_k P_{k|k-1} C_k + R_w)^{-1}
\]

\[
P_{k|k} = (I - L_k C_k) P_{k|k-1}
\]

where \(R_\beta\) and \(R_w\) are design parameters, and \(P_{0|0}\) is
the initial covariance of the parameter estimates. See (Perez,
2005, Ch 12.5) for further details, design parameters, and
initial conditions.

The process is stable when the eigenvalues of the system
matrix in (4) are within the unit circle, and marginally
stable when equal to 1. Parametric roll is declared in the
system when the eigenvalues are greater than 1 + \(\epsilon\), where
\(\epsilon > 0\) is a tolerance parameter. The reported alarm time
is when the system first becomes unstable.

3.2 Bayesian Discrete-Frequency Estimation

We propose an algorithm to infer a discrete set of fre-
bquencies by modifying the offline Bayesian approach to

frequency estimation as developed by Brethorst (1988)
and further developed by Ó Ruanaidh and Fitzgerald
(1996).

The problem consists of estimating the parameters of a
linear-combination signal model of the form

\[
f(t) = \sum_{i=1}^{N} a_i \cos(\omega_i t) + b_i \sin(\omega_i t),
\]

with \(N\) frequencies, and \(2N\) amplitudes. We consider a
measured data set of \(W\) noisy samples:

\[
d_t = f_t + \epsilon_t,
\]

where \(t = k, k-1, \ldots, k-W+1, k > W, f_t = f(T_s t)
\)
with sampling period \(T_s\) and measurement noise \(\epsilon_t\) are
i.i.d. with each \(\epsilon_t \sim N(0, \sigma^2)\) and mutually independent
from \(f_t\), Brethorst’s technique then reduces to computing
the marginal posterior distributions:

\[
p(\omega_1|D), \quad p(a_1|D), \quad \text{and} \quad p(b_1|D)
\]

which are conditional on the data represented by \(D\).
Brethorst (1988) shows that the marginal posterior
distribution of \(p(\omega_1|D)\) is related to the Schuster Periodogram.

The linear model (6) can be expressed in matrix form as

\[
D = F + E = G(\Omega) B + E,
\]

where \(F = G(\Omega) B, D = [d_1, \ldots, d_{W-1}]^T\),

\[
B = [a_1, a_2, \ldots, a_N, b_1, b_2, \ldots, b_N]^T, \quad E = [\epsilon_1, \epsilon_2, \ldots, \epsilon_{W-1}]^T, \quad \Omega = [\omega_1, \ldots, \omega_N]^T, \quad \text{and}
\]

\[
G(\Omega) = \begin{bmatrix}
\begin{bmatrix}
c_{1,1} & \cdots & c_{1,N}
\end{bmatrix} & \cdots & \begin{bmatrix}
c_{2,1} & \cdots & c_{2,N}
\end{bmatrix} \\
\vdots & \ddots & \vdots \\
\begin{bmatrix}
c_{W,1} & \cdots & c_{W,N}
\end{bmatrix} & \cdots & \begin{bmatrix}
s_{W,1} & \cdots & s_{W,N}
\end{bmatrix}
\end{bmatrix},
\]

with \(c_{i,j} = \cos(\omega_j (t-i))\) and \(s_{i,j} = \sin(\omega_j (t-i))\).

The likelihood function can be shown to be:

\[
p(D|\Omega, B, \sigma) = (2\pi\sigma^2)^{-W/2} \exp \left[ \frac{-E^T E}{2\sigma^2} \right]
\]

As the noise standard deviation \(\sigma\) is a scale parameter
a Jeffrey’s prior is chosen, and for the amplitudes \(B\) a
uniform (improper) prior (Brethorst, 1988).

Using Bayes’ theorem, and marginalising over the
amplitude parameters and the noise standard deviation, the posterior distribution of the parameters is:

\[
p(\Omega|D) = \frac{1}{\sqrt{2\pi\sigma^2}} \int_{\mathbb{R}^N} p(\Omega, B, \sigma|D) dB d\sigma,
\]

\[
\propto \left[ D^T D - D^T G (G^T G)^{-1} G^T D \right]^{-\frac{(W-2N)}{2}}.
\]

Note that there is an equivalent marginalisation for the
amplitudes:

\[
p(B|D) = \int_{\mathbb{R}^N} p(\Omega, B, \sigma|D) d\Omega d\sigma.
\]

The maximum of the marginal posteriors is chosen as a
point estimate of the parameters and amplitudes:

\[
\hat{\Omega} = \text{sup}_\Omega p(\Omega|D), \quad \hat{B} = \text{sup}_B p(B|D).
\]

Due to the diffused distribution assumed for the amplitudes,
the point estimate based on the maximum of the posterior
coincides with the ML estimate, which due to
the linearity of the measurement results is the least squares estimate (Ó Ruanaidh and Fitzgerald, 1996):

\[ \hat{B} = (G^T G)^{-1} G^T D. \]  

(11)

This analysis framework can also be used to choose a model from a set of potential models. Given a set of \( M \) models, we denote the \( k \)th model as \( M_k \), and encode the choice of parameters, amplitudes, and priors into this model. An example of two models could be a single frequency model, and a two-frequency model. Using Bayes’ Theorem, the posterior probability of a model given the data set can be written as:

\[ p(M_k|D) = \frac{p(D|M_k)p(M_k)}{p(D)}, \quad k = 1, \ldots, M \]  

(12)

where \( p(D|M_k) \) is the model evidence, and

\[ p(D) = \sum_{j=1}^{M} p(D|M_j)p(M_j). \]

Assuming the models are equally likely, i.e. \( p(M_k) = \frac{1}{M} \), the posterior probability of a model given the data reduces to the relative evidence:

\[ p(M_k|D) = \frac{p(D|M_k)}{\sum_{j=1}^{M} p(D|M_j)}. \]  

(13)

The model with the largest posterior probability is the one that provides the best fit to the data.

Bretthorst (1988) shows that by assuming a diffuse Gaussian prior for a set of \( K \) amplitudes:

\[ p(B|\Delta, M_k) = (2\pi\Delta)^{-K/2} \exp \left[ -\frac{B^T B}{2\Delta^2} \right] \]

with a Jeffreys’ prior for the so-called hyperparameter \( \Delta \), it is possible to analytically compute the model evidence. While a uniform prior was chosen for the amplitudes above in the parameter estimation, Bretthorst (1988) showed that the estimates obtained under a uniform or Gaussian prior are identical in the limit as \( \Delta \) approaches infinity.

Ó Ruanaidh and Fitzgerald (1996) derive an analytical expression for the integration of a generalised linear model to produce an approximation for the model evidence:

\[
p(D|M_k) = \int p(D|B, \sigma, M_k)p(\sigma)p(B|\Delta, I_k)p(\Delta|I)dBd\Delta d\sigma
\]

\[ \approx \frac{\pi^{W/2}}{4R_\Delta R_\sigma} \Gamma \left( \frac{M}{2} \right) \Gamma \left( \frac{W-M}{2} \right) \left| G(\tilde{\Omega})^T G(\tilde{\Omega}) \right|^{-1/2} \]

\[ \left( \frac{B^T \tilde{B}}{M/2} \right)^{M/2} \left( \frac{D^T D - \tilde{F}^T \tilde{F}}{W-M/2} \right)^{(W-M/2)/2} \]

where \( M \) is the model order, \( \Gamma(x) \) is the Gamma function computed on \( x \), and \( R_\Delta, R_\sigma \) are normalisation parameters arising from the integration.

The model evidence calculation inherently penalises higher order models to prevent the fitting of noise. This means a two-frequency model would only have a larger posterior probability when there is significant evidence of the second frequency compared to a single frequency model.

We modify Bretthorst’s approach by setting a lag window of \( W \) samples, and computing the parameter estimates and model evidence at each new time sample.

### 3.3 Detection using Bayesian Inference

We now consider a discrete approximation of the wave-induced motion spectrum:

\[ S_{\eta\eta}(\omega) \approx \sum_{i=1}^{N} \frac{c_i^2 \delta(\omega - \omega_i)}{2}, \]

which leads to the following time-domain representation:

\[ \eta(t) = \sum_{i=1}^{N} c_i \cos(\omega_i t + \kappa_i) = \sum_{i=1}^{M} a_i \cos(\omega_i t) + b_i \sin(\omega_i t), \]

where \( \kappa_i \) are uniformly distributed in \([0,2\pi]\), and \( c_i^2 = a_i^2 + b_i^2 \), and \( \tan \kappa_i = -b_i/a_i \). With this approximation, we can then apply the Bayesian estimation technique described in the previous section.

Let us denote the data of pitch and heave motion as \( D_\theta \) and \( D_z \), respectively. Then, the dominant frequency in the pitch and heave motion of a vessel corresponds to the dominant wave encounter frequency. We can then consider two estimates of the first frequency as an estimate of the wave encounter frequency based on the data available:

\[ \hat{\omega}_{\theta,e} = \sup_{\omega} p(\omega|D_\theta), \]

\[ \hat{\omega}_{z,e} = \sup_{\omega} p(\omega|D_z). \]

During parametric roll resonance the roll frequency is approximately half of the wave encounter frequency, and based on the discussion in Section 2.2, it is proposed that the roll frequency is present in the pitch motion during these conditions. Our proposed detector compares the probability of the pitch motion being represented by a model of just the encounter frequency, which we denote \( M_1 \):

\[ \theta(t) = c \cos(\hat{\omega}_{\theta,e} t + \kappa), \]  

(14)

to a model of the encounter frequency and the roll frequency, which we denote \( M_2 \):

\[ \theta(t) = c_1 \cos(\hat{\omega}_{\theta,e} t + \kappa_1) + c_2 \cos(\hat{\omega}_{z,e} t + \kappa_2), \]  

(15)

where \( \hat{\omega}_{\theta,e} \) is the dominant frequency estimated from the pitch motion and the \( \omega_r \) is the roll frequency.

Under the assumption that the vessel is in parametric roll resonance, the roll frequency would be approximately half of the encounter frequency:

\[ \omega_r = \frac{1}{2} \hat{\omega}_{\theta,e} \pm \gamma \]  

(16)

where \( \gamma > 0 \) is a small margin to allow for numerical and calculation inaccuracies. We choose our estimate of the roll frequency as the frequency that maximises the model evidence in this range:

\[ \hat{\omega}_r = \sup_{\omega} p(D_\theta|M_2). \]

Using the model evidence, we compute the posterior probability of model \( M_2 \) at each iteration \( k \), and declare that the system is in parametric roll resonance if the probability of \( M_2 \) is close to 1, i.e.:

\[ p(M_2|D_\theta) \geq 1 - \lambda \]

where \( \lambda > 0 \) is a small tolerance.

Similarly, the heave motion could be utilised, replacing the encounter frequency estimate from the pitch motion: \( \hat{\omega}_{\theta,e} \), with the heave motion estimate: \( \hat{\omega}_{z,e} \).
3.4 Illustrative Example

We consider the experimental data for regular waves R-1172 reported by Holden et al. (2007a), with time series shown in Figure 1. From the parameter estimates of the Kalman Filter we compute the roll frequency:

$$\hat{\omega}_{p,k} = \sqrt{\frac{-\beta_{12,k}}{T_s}}$$

and the roll damping:

$$\hat{\zeta}_{p,k} = 1 - \frac{\beta_{22,k}}{2\hat{\omega}_{p,k}T_s}$$

We subsequently smooth these parameters by applying a moving average filter of 25 samples. Denoting the roll motion as $D_p$, and computing the online frequency estimate using the online Bayesian Inference technique with a lag window of $W = 400$ samples, we estimate the roll frequency:

$$\hat{\omega}_{p,k} = \sup_{\omega} p(\omega|D_p).$$

The roll natural frequency is known to be 0.3 rad/sec. Figure 3 shows the estimate from the Bayesian Inference Technique in solid blue, and Kalman Filter in dashed red. Ignoring initial transient behaviour, both systems are able to estimate the roll frequency.

4. EXPERIMENTAL TOWING-TANK SHIP DATA

We now apply our two detectors to an experimental data set from (Holden et al., 2007a) and compare to the Kalman Filter Detector. Detection false alarms are shown in Table 1, and detection times for all 30 experiments are shown in Table 2. Experiments denoted R-* were conducted in regular waves, and L-* in irregular waves. An experiment is denoted as unstable if parametric roll resonance was observed. The Kalman Filter Detector uses the roll motion data, and the Bayesian Inference Detector is applied to both the pitch and heave motion data.

For the Kalman Filter, the design parameters and initial condition were set as per (Perez, 2005, Ch 12.5):

$$R_p = 10^{-6}I_{4\times4}, \quad R_w = 10^{-4}I_{2\times2}, \quad P_{0|0} = I_{4\times4},$$

and the tolerance to $\epsilon = 1 \times 10^{-3}$. As in (Holden et al., 2007b) the data was low-pass filtered and downsampled (to avoid aliasing) such that $T_s = 0.67$ sec, and the first 100 samples were ignored to mitigate transient effects in the data and the filter.

For the Bayesian Detector, the lag window was set to $W = 400$ samples, estimated roll frequency margin to $\gamma = 0.005$, and tolerance to $\lambda = 1 \times 10^{-3}$. Both the pitch and heave motion were low-pass filtered and downsampled such that $T_s = 0.27$ sec, as this was found to work well. Again, the initial transient of the detector is ignored. The probability of the two competing models $M_1$ and $M_2$ for the pitch motion of experiment R-1172 is shown in Figure 4. The probability of model $M_1$ becomes smaller over time as the roll frequency becomes more apparent in the pitch motion. Detection time is marked in as a black vertical line, and is also marked in Figure 1.

In the regular-wave experiments, the Bayesian Inference Detector using the pitch motion performed similarly to the Kalman Filter Detector with an average increase in detection time of 7.1 sec. However, detection using the heave motion was slower with an average decrease in detection time of 63 seconds. All experiments in regular waves with parametric roll resonance were flagged as unstable by all detectors.

In the irregular wave experiments, the Kalman Filter Detector performed significantly faster on the one unstable experiment. The Bayesian inference detector using the pitch motion had the same false alarm rate as the Kalman Filter Detector but alarmed on different experiments. False alarms were set on all stable experiments using the heave motion.

It is noted that it is harder to classify stable motion from frequency spectrum analysis in irregular waves due to spreading in the frequency spectrum. Note: the detection times reported for the Kalman Filter Detector differ from (Holden et al., 2007b) as the raw data was processed differently.

This study suggests that our proposed Bayesian Inference Detector using the pitch motion could assist in identifying if a vessel is in parametric roll resonance under both regular and irregular wave conditions. Using our proposed Bayesian Inference Detector with the heave motion would only be suitable in regular wave conditions.
Table 1. Number of False Alarms of the Kalman Filter and Bayesian Inference (BI) Detectors on the stable experiments

| Detector  | Regular | Irregular |
|-----------|---------|-----------|
| Kalman filter | 0       | 3         |
| BI - pitch  | 3       | 3         |
| BI - heave | 1       | 7         |
| Total (stable) | 9       | 7         |

Table 2. Detection Times (in seconds) of the Kalman Filter (KF) and Bayesian Inference Technique (BI)

| Experiment | Stable | KF | BI - Pitch | BI - Heave |
|------------|--------|----|------------|------------|
| R-1172     | No     | 213.33 | 167.71 | 203.12 |
| R-1174     | No     | 381.03 | 401.69 | 473.06 |
| R-1175     | Yes    | 0     | 0          | 0          |
| R-1176     | No     | 644.52 | 0          | 0          |
| R-1177     | No     | 562.82 | 545.51 | 576.1    |
| R-1178     | No     | 486.34 | 459.38 | 489.97 |
| R-1179     | Yes    | 0     | 0          | 0          |
| R-1180     | No     | 238.81 | 246.594 | 300.26 |
| R-1181     | Yes    | 0     | 0          | 0          |
| R-1182     | No     | 134.16 | 187.29 | 244.72 |
| R-1183     | No     | 480.31 | 450.25 | 524.04 |
| R-1184     | No     | 245.52 | 236.93 | 248.74 |
| R-1185     | No     | 297.84 | 343.19 | 382.9 |
| R-1186     | No     | 375.66 | 394.17 | 557.85 |
| R-1187     | No     | 279.06 | 225.13 | 375.92 |
| R-1188     | No     | 213.32 | 177.63 | 315.82 |
| R-1189     | Yes    | 0     | 230.76 | 123.16 |
| R-1190     | Yes    | 0     | 0          | 0          |
| R-1191     | No     | 199.9 | 182.19 | 236.13 |
| R-1192     | Yes    | 0     | 0          | 0          |
| R-1193     | Yes    | 0     | 195.07 | 0         |
| I-1194     | Yes    | 316.63 | 472.53 | 131.21 |
| I-1195     | No     | 79.16 | 172.27 | 178.97 |
| I-1196     | Yes    | 179.78 | 0 | 145.17 |
| I-1197     | Yes    | 0     | 667.33 | 121.28 |
| I-1198     | Yes    | 132.82 | 224.32 | 686.12 |
| I-1199     | Yes    | 0     | 0          | 215.74 |
| I-1200     | Yes    | 0     | 0          | 107.33 |
| I-1201     | Yes    | 0     | 0          | 211.17 |

5. DISCUSSION AND CONCLUSIONS

This paper approaches the detection of parametric resonance with the use of Bayesian probability theory. An algorithm for detection is proposed based on Bayesian hypothesis testing about the selection of a single or a double-frequency model. The two models are proposed based on knowledge of the coupling between roll and pitch motion. The Bayesian model selection approach naturally encodes uncertainty since it provides the probability of each of the models in the set being considered as the best description of the observed data. These probabilities can then be used within a decision-theoretic framework to guide decision making courses of action for mitigating the effect of any further development of parametric roll resonance. The latter is, however outside the scope of this paper.

Our proposed detection algorithm was tested on an experimental data set. The results indicated that the use of pitch motion provides a better detection (faster and with fewer false alarms and misses detections) over the use of heave motion. This result was expected due to the stronger coupling between roll and pitch motion than between roll and heave motion.

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