Dark Radiation from Modulated Reheating

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We show that the modulated reheating mechanism can naturally account for dark radiation, whose existence is hinted by recent observations of the cosmic microwave background radiation and the primordial Helium abundance. In this mechanism, the inflaton decay rate depends on a light modulus which acquires almost scale-invariant quantum fluctuations during inflation. We find that the light modulus is generically produced by the inflaton decay and therefore a prime candidate for the dark radiation. Interestingly, an almost scale-invariant power spectrum predicted in the modulated reheating mechanism gives a better fit to the observation in the presence of the extra radiation. We discuss the production mechanism of the light modulus in detail taking account of its associated isocurvature fluctuations. We also consider a case where the modulus becomes the dominant component of dark matter.

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1 Introduction

There have been accumulating evidences for the existence of extra radiation. Recent combined analysis with the data from cosmic microwave background (CMB), large scale structure (LSS) and so on has given a constraint on the effective number of light neutrino species as $N_{\text{eff}} = 3.86 \pm 0.42$ (1σ C.L.) [1], which suggests the presence of extra radiation at about 2σ level [1, 2, 3]. Also, it is known that the $^4\text{He}$ mass fraction $Y_p$ is sensitive to the expansion rate of the Universe during the BBN epoch [2], and the authors of Ref. [11] claimed an excess of $Y_p$ at the 2σ level, $Y_p = 0.2565 \pm 0.0010$ (stat) $\pm 0.0050$ (syst), which can be understood in terms of the effective number of neutrinos, $N_{\text{eff}} = 3.68^{+0.80}_{-0.70}$ (2σ) [3]. Although the precise determination of the Helium abundance is still limited by systematic uncertainties, it is intriguing that the CMB and LSS data as well as the Helium abundance from BBN are showing hints for the presence of additional relativistic species, $\Delta N_{\text{eff}} \sim 1$, while they are sensitive to the expansion rate of the Universe at vastly different times.

The extra radiation may be dark radiation composed of unknown particles (see e.g. Refs. [13, 14, 15, 16, 17, 18] for particle physics models). Such particles must be light and have only very weak interactions with the standard model particles. Although one can just add such a light degree of freedom by hand to explain the data, it still remains a puzzle why

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1 Recent another analysis gives $N_{\text{eff}} = 4.08^{+0.71}_{-0.68}$ at 95% C.L. [2].
2 $Y_p$ is also sensitive to large lepton asymmetry, especially of the electron type, if any [4, 5, 6, 7, 8, 9, 10].
3 The authors of Ref. [12] estimated the primordial Helium abundance with an unrestricted Monte Carlo taking account of all systematic corrections and obtained $Y_p = 0.2561 \pm 0.0108$ (68%CL), which is in broad agreement with the WMAP result.
there is such a light particle at all, and how it is produced. In this paper, we show that there is a well motivated candidate in a scenario with the modulated reheating mechanism [19, 20].

The origin of the density perturbation is one of the unresolved issues in modern cosmology. While the inflaton is the prime candidate, there are several alternatives proposed so far. The modulated reheating scenario is one of them, and has been extensively discussed in various contexts, especially in connection with non-Gaussianity [21, 22, 23]. The key feature of the modulated reheating scenario is that the decay rate of the inflaton depends on a light scalar field, $\sigma$, which acquires quantum fluctuations extending beyond the horizon scale during inflation. In this paper we focus on the cosmological fate of the scalar $\sigma$, which was not studied in detail so far. We find that some amount of $\sigma$ is necessarily produced by the inflaton decay, since $\sigma$ must have couplings with the inflaton to make the decay rate fluctuate. Also, the $\sigma$ has only very suppressed couplings with the standard model particles since otherwise it would acquire a thermal mass spoiling the modulated reheating mechanism. We therefore argue that the modulus $\sigma$ can naturally account for the extra radiation. Interestingly, in the presence of the extra radiation, observational data favors a larger scalar spectral index $n_s$, and so, the modulated reheating mechanism which generically predicts a very flat power spectrum\footnote{Even though the modulus dynamics itself hardly contributes to the spectral tilt, we should also note that special classes of inflationary models (such as large-field ones) giving large time-variation to the Hubble parameter during inflation as large as $|\dot{H}/H^2| \sim 0.01$, can source large tilt for the resulting perturbation power spectra.} gives a better fit to observations. In addition, a (classical) oscillating background of the modulus can also play a role of dark matter. We evaluate their abundances and give some explicit values for an example model. In particular, we show that the amount of dark radiation can be predicted as $\Delta N_{\text{eff}} \sim 1$ in the model.

In fact, in this kind of model, (correlated-type) isocurvature fluctuations could be generated in dark radiation and/or dark matter sectors. Since too large isocurvature fluctuations are inconsistent with observations such as CMB, we discuss the conditions to evade cosmological constraints for the isocurvature modes.

2 Modulated Reheating Mechanism

The modulated reheating mechanism proposed in [19, 20] generates cosmological perturbations through a modulated decay of the inflaton. This can happen when the couplings between the inflaton and the Standard Model (SM) particles are determined by a vacuum expectation value of a modulus field, such as

$$\mathcal{L} \supset \phi f(\sigma) \mathcal{O}_{\text{SM}},$$

(2.1)
where \( \phi \) is the inflaton, \( \sigma \) is the modulus, and \( O_{\text{SM}} \) represents a SM gauge invariant operator.\(^5\) Given that the modulus is light during inflation, the field acquires (almost) scale-invariant quantum fluctuations at length scales that are stretched to super-horizon sizes by the end of inflation, as

\[
\delta \sigma \sim \frac{H_{\text{inf}}}{2\pi},
\]

where \( H_{\text{inf}} \) denotes the Hubble parameter during inflation. This results in the inflaton possessing different decay rates among different patches of the universe, producing cosmological perturbations. As for perturbations generated by the inflaton itself, we ignore them (or consider them to be negligibly tiny) throughout this paper.

Let us begin by computing the curvature perturbations \( \zeta \) in the modulated reheating mechanism. Using the \( \delta N \)-formalism [25, 26, 27, 28], \( \zeta \) can be computed as the difference in the e-folding number \( N \) on a constant energy density hypersurface, among different patches of the universe with different inflaton decay rates \( \Gamma \). We consider an inflaton \( \phi \) which undergoes harmonic oscillations after inflation, so that its energy density redshifts as that of matter, i.e. \( \rho_\phi \propto a^{-3} \) (here \( a \) is the scale factor of the universe). Assuming a sudden decay of the inflaton into radiation (\( \rho_r \propto a^{-4} \)), then the number of e-folds obtained between some time before the inflaton decay when the energy density of the universe was \( \rho_i \), and after decay when the energy density is \( \rho_f \), is

\[
N = \int_{\rho_i}^{\rho_f} \frac{H d\rho}{\dot{\rho}} = \frac{1}{3} \log \frac{\rho_i}{\rho_{\text{dec}}} + \frac{1}{4} \log \frac{\rho_{\text{dec}}}{\rho_f},
\]

where an overdot denotes a time derivative, and \( \rho_{\text{dec}} \) is the energy density of the universe at the inflaton decay. Considering the inflaton to decay when \( H = \Gamma \) with \( \Gamma \) being the decay rate for the inflaton, and since \( \rho_i \) and \( \rho_f \) can be chosen independently of \( \Gamma \), (2.3) can further be written as

\[
\mathcal{N} = \text{const.} - \frac{1}{6} \log \Gamma,
\]

where we have denoted terms that are independent of \( \Gamma \) by const. Since \( \Gamma \) is a function of the modulus field \( \sigma \), the curvature perturbations can be computed by differentiating \( \mathcal{N} \) in terms of \( \sigma_* \), where the subscript \( * \) denotes values at the time when the CMB scale exits the horizon during inflation. Using (2.2), one obtains the power spectrum of the curvature perturbations as

\[
P_\zeta = \left( \frac{\partial \mathcal{N}}{\partial \sigma_*} \right)^2 \mathcal{P}_{\delta \sigma_*} = \left( \frac{\Gamma'}{(6\Gamma)} \right)^2 \left( \frac{H_*}{2\pi} \right)^2,
\]

where the prime denotes a derivative with respect to \( \sigma_* \). In this mechanism, primordial non-Gaussianity can be large and the non-linearity parameter \( f_{\text{NL}} \), which characterizes the

\(^5\)In [24], a light modulus which is coupled with the mass term of the inflaton in a specific inflation model was considered. However in this case, even when such a modulus can play a role of dark radiation to some extent, it is not possible to satisfy the isocurvature constraint on dark radiation.
amplitude of three point correlation function for $\zeta$, is given by \cite{21, 22, 23},
\[
f_{\text{NL}} = \frac{5}{6} \frac{\partial^2 N}{\partial \sigma^2} \left( \frac{\partial N}{\partial \sigma} \right)^{-2} = 5 \left( 1 - \frac{\Gamma \gamma'}{\Gamma' \gamma} \right).
\]

(2.6)

3 Dark Radiation from Modulated Reheating

As we already stressed, the modulus necessarily couples to the inflaton in order to affect the inflaton decay rate. Further when the modulus is light enough to be frozen at some field value until the inflaton decay, its super-horizon field fluctuations induce the modulated decay and produce cosmological perturbations. (If the modulus mass was heavier than the Hubble parameter at the inflaton decay, then the modulus would oscillate about its potential minimum and the super-horizon fluctuations would be suppressed.) Since such modulus should be much lighter than the inflaton which already has been oscillating prior to its decay, and in addition, it only has highly suppressed couplings with the SM particles because otherwise it would receive thermal mass from them and hence would start its oscillation before inflaton decay, the modulus serves as a natural candidate for dark radiation. In this section we lay out the conditions for the modulus to be the dark radiation. Here we discuss the case of a rather general coupling (2.1). Then in the next section we present an explicit example for the function $f(\sigma)$. In order to avoid isocurvature perturbations, we assume that the inflaton decay and the production of the SM particles/modulus to proceed only through the interaction (2.1), although this assumption is not a sufficient condition for vanishing isocurvature perturbations, as we will soon see. All the decay products, i.e. the SM particles and the modulus, are assumed to be relativistic at the inflaton decay.

We divide the modulus field as
\[
\sigma = \sigma_c + \hat{\sigma},
\]
(3.1)

where $\sigma_c$ is the classical background, and $\hat{\sigma}$ denotes the quantum fluctuations around it. The interaction term (2.1) can now be expanded as
\[
\phi f(\sigma) \mathcal{O}_{\text{SM}} = \phi f(\sigma_c) \mathcal{O}_{\text{SM}} + \phi f^{(1)}(\sigma_c) \hat{\sigma} \mathcal{O}_{\text{SM}} + \phi f^{(2)}(\sigma_c) \frac{\hat{\sigma}^2}{2} \mathcal{O}_{\text{SM}} + \cdots ,
\]
(3.2)

where in the right hand side the term with an $i$-th derivative $f^{(i)}(\sigma_c)$ represents the inflaton decay channel into the SM particles and $i$ modulus particles. Since $\sigma_c$ possesses super-horizon field fluctuations $\delta \sigma_c$ as in (2.2), that is, $\sigma_c = \sigma_0 + \delta \sigma_c$ ($\sigma_0$ : homogeneous mode),\footnote{Note that even though the fluctuations were originally sourced as quantum fluctuations during inflation, they can be considered as classical after stretched to super-horizon sizes.} the decay rates of all the channels fluctuate as well. For quantities (say, $x = x_0 + \delta x$) discussed in this paper, we assume that their fluctuations always satisfy $|x| \gg |\delta x|$, so that $x_0$ can be considered as the spatially averaged value in the universe. This is in many cases expected to hold as long as the required tuning of $\sigma_c$ to specific values are no less than $\delta \sigma_c$. 

Referring to the decay channel which produces $i$ modulus particles as the $i$-channel, its decay rate follows
\[
\Gamma_i \propto \epsilon^i \left[ (m_\phi)^i f^{(i)}(\sigma_c) \right]^2,
\] (3.3)
where $\epsilon \sim 10^{-3}$ represents the phase space factor and $m_\phi$ is the mass of the oscillating inflaton. The $i$-th derivative $f^{(i)}(\sigma_c)$ is compensated by powers of $m_\phi$, thus fixing the overall mass dimension of the decay rate (only the $m_\phi$-dependence relevant to $i$ is given in the expression). Hence the fluctuation of the decay rate at the linear order in $\delta \sigma_c$ is
\[
\delta \Gamma_i \propto 2\epsilon^i (m_\phi)^{2i} f^{(i)}(\sigma_0) f^{(i+1)}(\sigma_0) \delta \sigma_c.
\] (3.4)
Here one clearly sees that the decay rate of each channel can fluctuate in different manners, thus sourcing adiabatic as well as isocurvature perturbations. See Appendix A for detailed discussions on this issue. In order to suppress the isocurvature perturbations, we consider the case where one of the channels (say, the $i$-channel) is dominant over others, i.e. has the largest decay rate. (This corresponds to Solution 2 at the end of the appendix.) In the following subsection we will discuss how dominant the $i$-channel should be in order to satisfy observational constraints.

Before moving on, let us briefly comment on another way to avoid isocurvature perturbations: When all the decay channels are modulated in a similar fashion, i.e. $\delta \Gamma_i/\Gamma_i$ being independent of $i$, then isocurvature perturbations vanish at the linear order. (This is discussed as Solution 1 in the appendix.) This can be realized by a function $f(\sigma)$ satisfying
\[
f^{(1)}/f = f^{(2)}/f^{(1)} = f^{(3)}/f^{(2)} = \ldots,
\] (3.5)
at $\sigma = \sigma_0$. For example, a function with the following form satisfies the requirement:
\[
f(\sigma) = \exp \left( \frac{\sigma}{M} \right)
\] (3.6)
with $M$ being a constant. However, in order to generate dark radiation corresponding to $\Delta N_{\text{eff}} \sim 1$, the 0-channel producing only the SM particles and the other channels should distribute the inflaton energy to the SM particles and the modulus at the same order. In such case the decay rates of all the channels are expected to be of the same order, which invalidates the expansion (3.2). Since channels with sufficiently large $i$ produce more modulus particles than the SM particles, at the end the production of the modulus may dominate over that of the SM particles. It would be interesting to further investigate possibilities of a model with (3.5), however in the following sections we focus on the case where there is effectively only one decay channel.

3.1 Adiabatic and Isocurvature Perturbations

As we have discussed above, we consider the inflaton to decay into radiation mainly through the $i$-channel, whose decay rate $\Gamma_i$ is proportional to the square of $f^{(i)}(\sigma_0)$ (see (3.3)). We
suppose that the energy density of the classical background $\sigma_c$ is negligibly tiny compared to the total energy density of the universe. Since the modulus is assumed to be sufficiently light so that it is frozen to a certain field value at least until the inflaton decay, we can set the modulus field value to be the same at the inflaton decay and at the CMB scale horizon exit, i.e. $\sigma_0 = \sigma_*$, and also $\delta \sigma_c = \delta \sigma_* = H_*/2\pi$. Hence the power spectrum of the adiabatic perturbations (2.5) and its non-linearity parameter (2.6) are obtained as

$$P_\zeta = \left(\frac{1}{3} \frac{f^{(i+1)}(\sigma_0)}{f^{(i)}(\sigma_0)}\right)^2 \left(\frac{H_*}{2\pi}\right)^2 \approx 2.4 \times 10^{-9},$$

$$f_{NL} = \frac{5}{2} \left(1 - \frac{f^{(i)}(\sigma_0)f^{(i+2)}(\sigma_0)}{f^{(i+1)}(\sigma_0)^2}\right).$$

In (3.7), the WMAP normalization value [29] is also shown.

In fact, when the modulus produced from the inflaton decay plays the role of dark radiation, isocurvature fluctuations in the dark radiation sector could possibly be generated, whose size is characterized by

$$S_{\text{DR}} = 3(\zeta_{\text{DR}} - \zeta_r).$$

Here $\zeta_\alpha$ is the curvature perturbation on the uniform energy density hypersurface of species $\alpha$. Too large isocurvature fluctuations would be inconsistent with the CMB observations and thus severely constrained [30]. By using the data from CMB, BAO and the direct measurement of the Hubble constant, for the case where adiabatic and isocurvature perturbations are totally (anti-)correlated, which is relevant to our case \(^7\), the 2$\sigma$ limit on the power spectrum for $\zeta_{\text{DR}}$, denoted as $P_{S_{\text{DR}}}$, is roughly given by [30]

$$\frac{P_{S_{\text{DR}}}}{P_\zeta} \lesssim 0.01,$$

when $\Delta N_{\text{eff}} \sim 1$.

We assume that the ratio between the energy densities for the SM particles and modulus produced through the dominant $i$-channel is of order unity (so that the weights introduced in Appendix A satisfy $c_{\text{SM}i}/c_{\sigma i} = O(1)$), and also that such ratios in all channels are constants without any spatial fluctuation. Then based on the discussions in Appendix A (and especially from (A.5)), the isocurvature constraints are satisfied if the decay rates and their fluctuations for the subdominant channels $j \neq i$ are suppressed as

$$\left|\frac{\Gamma_j}{\Gamma_i}\right|, \left|\frac{\delta \Gamma_j}{\delta \Gamma_i}\right| \lesssim 0.1.$$  

\(^7\) In this paper we consider $\delta \sigma$ to be the only source of density perturbations, and other contributions (e.g. from the inflaton) to be negligibly tiny. Hence any isocurvature perturbations are correlated with the curvature perturbations. The correlation can be positive or negative, depending on the function $f(\sigma)$.
Hence from (3.3) and (3.4), the isocurvature constraint (3.10) is satisfied under
\[e^{i j - i} \left( \frac{f(j)(\sigma_0)}{f(i)(\sigma_0)} m^j_{\phi} \right)^2, e^{i j - i} \left| \frac{f(j)(\sigma_0)f(j+1)(\sigma_0)}{f(i)(\sigma_0)f(i+1)(\sigma_0)} m^2_{\phi} (j-i) \right| \lesssim 0.1, \] (3.12)
for every \(j\) with \(j \neq i\).

### 3.2 Modulus as Dark Radiation

The abundance of \(\sigma\) particles (dark radiation) in terms of the extra effective number of neutrinos is given by
\[
\frac{\rho_{\hat{\sigma}}}{\rho_{\nu}} = \Delta N_{\text{eff}}, \tag{3.13}
\]
where \(\rho_{\hat{\sigma}}\) and \(\rho_{\nu}\) denote the energy densities of \(\sigma\) particles (dark radiation) and one generation of neutrino, respectively. Note that, after the neutrino decoupling, both the energy densities of dark radiation and neutrino decrease in the same way, i.e. proportional to \(a^{-4}\), hence the extra effective number \(\Delta N_{\text{eff}}\) is fixed. At the neutrino decoupling (at temperature \(T = T_D\)), the ratio of the energy densities of one-generation neutrino and the relativistic SM particles is estimated as
\[
\frac{\rho_r(T_D)}{\rho_{\nu}(T_D)} = \frac{43}{7}. \tag{3.14}
\]
On the other hand, the energy densities of the relativistic SM particles at the neutrino decoupling \((T = T_D)\) and reheating \((T = T_R)\) are related as
\[
\frac{\rho_r(T_R)}{\rho_r(T_D)} = \left( \frac{g_*(T_R)}{g_*(T_D)} \right)^{-\frac{1}{4}} \left( \frac{a(T_R)}{a(T_D)} \right)^{-4}, \tag{3.15}
\]
where \(g_*(T)\) is the relativistic degree of freedom of the SM particles and \(a(T)\) is the scale factor at \(T\).

Setting the number of the SM particles produced in the dominant \(i\)-channel to be \(k\) (being the same order as \(i\)), the resulting energy density ratio between the modulus and SM particles at the decay of the inflaton is
\[
\left. \frac{\rho_{\hat{\sigma}}}{\rho_r} \right|_{\text{inflaton decay}} \simeq \frac{i}{k}. \tag{3.16}
\]
Here for simplicity we have assumed that the inflaton energy is equally distributed to each decay particle, and neglected further complication such as the momentum distribution of the decay products. Also, energy input through the other channels are omitted (which give uncertainty of order 10% at most, cf. (3.11)). Assuming the SM particles are quickly thermalized after the inflaton decay, \(\rho_{\text{SM}}|_{\text{inflaton decay}} \simeq \rho_r(T_R)\) and \(\rho_{\hat{\sigma}}|_{\text{inflaton decay}} \simeq \rho_{\hat{\sigma}}(T_R)\).
Then, the extra effective number $\Delta N_{\text{eff}}$ of the produced modulus (dark radiation) after the neutrino decoupling can be estimated as

$$
\Delta N_{\text{eff}} = \frac{\rho_\sigma(T_D)}{\rho_\nu(T_D)} = \frac{\rho_\sigma(T_R) \rho_\nu(T_R) \rho_\sigma(T_D)}{\rho_\sigma(T_R) \rho_\nu(T_R) \rho_\nu(T_D)} = \left( \frac{a(T_D)}{a(T_R)} \right)^{-4} \left( \frac{g_*(T_R)}{g_*(T_D)} \right)^{-\frac{1}{4}} \left( \frac{a(T_R)}{a(T_D)} \right)^{-4} \frac{43}{7} = 43 \frac{a(T_R)}{a(T_D)} \left( \frac{g_*(T_R)}{g_*(T_D)} \right)^{-\frac{1}{4}}. \tag{3.17}
$$

An explicit number will be given when we discuss a concrete model in the next section.

We also lay out the condition for the modulus to serve as dark radiation at least until the last scattering.\(^8\) The modulus particle produced through the $i$-channel obtains momentum of $m_\phi/(i+k)$, thus in order for this particle to stay relativistic until last scattering, the modulus mass $m_\sigma$ is required to be as small as

$$
m_\phi \frac{a_{\text{dec}}}{i+k} \frac{a_{\text{ls}}}{3(i+k)} \frac{H_{\text{eq}}}{\Gamma_i} \sim \frac{1}{2} m_\sigma, \tag{3.19}
$$

where $a_{\text{dec}}$ denotes the scale factor at the inflaton decay, $a_{\text{ls}}$ the scale factor at last scattering, and $H_{\text{eq}}$ the Hubble parameter at matter-radiation equality.

### 3.3 Oscillating Modulus as Dark Matter

So far we have not cared about the fate of the classical background $\sigma_c$. In this subsection, we further investigate the possibility that the classical background of the modulus starts sinusoidal oscillations after the inflaton decay and becomes the dark matter. Note that still in such case, the quantum fluctuation $\hat{\sigma}$ can serve as the dark radiation, given that it obtains enough momentum at the inflaton decay. This sets an lower bound on the inflaton mass (cf. (3.19)), but as we will see shortly, the bound is satisfied for almost all the models since the mass of $\sigma$ required to explain the dark matter abundance is extremely small.\(^9\)

\(^8\) If the dark radiation (which corresponds to the extra neutrino numbers $\Delta N_{\text{eff}}$) turns into (a small fraction of) dark matter at redshift $z_\gamma$ after last scattering (at $z_{\text{ls}}$), then its present abundance is written in terms of that of the photons as

$$
\Omega_{\text{DR}} \rightarrow \Omega_{\gamma} 0.01 \times \Delta N_{\text{eff}}. \tag{3.18}
$$

Hence as long as $\Delta N_{\text{eff}} = O(1)$, their present abundance is tiny compared to the total dark matter abundance, $\Omega_{\text{DM}} \approx 0.22$.

\(^9\) Such an extremely light modulus may be ubiquitous in the Axiverse scenario [31].

8

9
For simplicity, we consider the modulus potential to be of the quadratic type

\[ V(\sigma) = \frac{1}{2} m_{\sigma}^2 (\sigma - \sigma_{\text{min}})^2, \]  

(3.20)

where \( m_{\sigma} \) is the modulus mass and \( \sigma_{\text{min}} \) is the potential minimum. Hence the classical background and quantum fluctuation \( \hat{\sigma} \) both have the same mass \( m_{\sigma} \), and the background starts its sinusoidal oscillations from the field value \( \sigma_c \) when \( H \sim m_{\sigma} \). Here we introduce

\[ \sigma_{\text{osc}} \equiv |\sigma_c - \sigma_{\text{min}}|, \]  

(3.21)

which represents the initial oscillation amplitude of the modulus background. In the following we give rough estimations for the modulus to serve both as dark radiation as well as dark matter.

**Dark Matter Abundance**

The present abundance of dark matter from the oscillating modulus, whose value should be \( \Omega_{DM0} \approx 0.22 \) from current observations, is estimated as

\[ \Omega_{DM0} = \frac{1}{\rho_c s_{\text{osc}}} \frac{m_{\sigma}^2 s_{\text{osc}}^2}{2} \simeq 0.1 \left( \frac{m_{\sigma}}{10^{-27} \text{eV}} \right)^{1/2} \left( \frac{\sigma_{\text{osc}}}{M_p} \right)^2, \]  

(3.22)

where \( M_p \) denotes the reduced Planck mass, \( \rho_c \) is the present critical density, \( s_0 \) and \( s_{\text{osc}} \) are the entropy densities of the SM particles at present and when the oscillation starts. Upon estimating \( s_{\text{osc}} \) and obtaining the far right hand side, we have assumed the onset of the oscillation to be at around or after the neutrino decoupling, and that \( \Delta N_{\text{eff}} \lesssim 1 \). (However we note that the result is not sensitive to such assumptions, e.g. if \( g_* \sim 200 \) at the onset of the oscillation, then the numerical factor in the far right hand side is about 0.06 for \( \Delta N_{\text{eff}} = 1 \).)

Furthermore, we require dark matter to have been present at least by the time of matter-radiation equality. This sets the latest time the modulus background starts its oscillation, giving a lower bound on the modulus mass,

\[ m_{\sigma} > H_{eq} \sim 2 \times 10^{-28} \text{eV}. \]  

(3.23)

On the other hand, the upper bound on the modulus mass was given in (3.19) from the requirement that the quantum fluctuation \( \hat{\sigma} \) produced by the inflaton decay to serve as dark radiation at least until the last scattering. The condition (3.19) can be rewritten in terms of \( T_R \) and \( m_{\sigma} \) as

\[ m_{\phi} \gg 10^{-17} \text{GeV} (i + k) \left( \frac{g_*(T_R)}{228.75} \right)^{1/4} \left( \frac{T_R}{10^9 \text{GeV}} \right) \left( \frac{m_{\sigma}}{10^{-27} \text{eV}} \right). \]  

(3.24)

Here, \( g_* = 228.75 \) corresponds to the maximum value allowed in the MSSM, and we have used the approximation,

\[ \Gamma_i \simeq \left( \frac{\pi^2 g_*(T_R)}{90} \right)^{1/2} \frac{T_R^2}{M_p}. \]  

(3.25)
Therefore there is a wide range of the parameters where (3.23) and (3.24) are satisfied.

We note that the potential energy of the modulus is negligibly small compared to the total energy of the universe upon the inflaton decay, as long as the oscillation amplitude is smaller than the Planck scale. This validates our computations of the perturbations in the previous sections where we have neglected the energy density of $\sigma_c$. Note that this is not always the case especially if one considers a potential deviated from the simple quadratic potential.

**Dark Matter Isocurvature**

In addition to the adiabatic perturbations (3.7), the oscillating modulus obtains isocurvature perturbations of order

$$\frac{\delta \rho_{DM}}{\rho_{DM}} \simeq \frac{\delta \sigma_c}{\sigma_{osc}},$$

which is necessarily correlated with the curvature perturbation (see footnote 7). The correlation can be positive or negative, depending on the position of the potential minimum. To be concrete, let us adopt the observational constraint on the totally anti-correlated dark matter isocurvature perturbations [29], roughly given by

$$\frac{P_{S_{DM}}}{P_\zeta} \lesssim 0.01,$$

where $P_{S_{DM}}$ is the power spectrum of dark matter isocurvature fluctuations $S_{DM}$ defined in the same manner as in (3.9) by replacing $\zeta_{DR}$ with $\zeta_{DM}$.

Hence we require the initial oscillation amplitude of the modulus as

$$\frac{1}{\sigma_{osc}} \frac{f^{(i)}(\sigma_0)}{f^{(i+1)}(\sigma_0)} \lesssim 0.01$$

in order to suppress the dark matter isocurvature perturbations.

Here we note that the dark matter isocurvature perturbation (3.26) can be absent if the modulus possesses a non-quadratic potential. As was indicated in [32] for the case of the curvaton mechanism [33, 34, 35, 36], a non-quadratic potential sources non-uniform onset of the field oscillation, which can cancel out the perturbation (3.26). We also remark that for such non-quadratic potentials the classical background and the quantum fluctuation around it can possess different masses. Moreover, the initial mass of the classical background (which sets the starting time of the oscillation) can be quite different from the final mass for the oscillation.
4 A Concrete Example

Let us illustrate in a toy example how the modulus serves as dark radiation with the abundance \( \Delta N_{\text{eff}} \sim 1 \). The coupling term studied in this section is of the form

\[
\mathcal{L} \supset \kappa \frac{\phi}{M_p} \frac{\sigma}{M} e^{\sigma/M} F_{\mu\nu} F^{\mu\nu},
\]

where \( \kappa \) is a dimensionless coupling constant, \( M \) a constant of mass dimension, and \( F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \) denotes the gauge field strength of \( U(1)_{\text{EM}} \). The inflaton \( \phi \) decays into \( i \) modulus particles and 2 photons \((k = 2)\), where the tree-level decay rates of the channels are

\[
\Gamma_0(\phi \to 2\gamma) = \frac{1}{4\pi} \left( \frac{\kappa \sigma_c}{M_p M} e^{\sigma_c/M} \right) \left( \frac{\sigma_c}{M} \right)^2 m_\phi^3,
\]

\[
\Gamma_1(\phi \to \hat{\sigma} + 2\gamma) = \frac{1}{768\pi^3} \left\{ \frac{\kappa}{M_p} \left( 1 + \frac{\sigma_c}{M} \right) e^{\sigma_c/M} \right\} \left( \frac{m_\phi}{M} \right)^5 \frac{m_\phi}{M^2},
\]

\[
\vdots
\]

When \(|\sigma_0|\) is sufficiently smaller than \( m_\phi \) and \( M \), and \( m_\phi \) not so large compared to \( M \), then the \( i = 1 \) channel becomes dominant over the others. (More generally, the \( n \)-channel can be made dominant in a similar fashion by couplings of the form \( \phi \sigma^n e^{\sigma/M} \mathcal{O}_{\text{SM}} \).) Then the adiabatic perturbation spectrum (3.7) is of the form

\[
P_{\zeta} \simeq \left( \frac{1}{3\pi M} \right)^2 \approx 2.4 \times 10^{-9},
\]

with order-unity non-linearity parameter (3.8),

\[
f_{\text{NL}} \simeq \frac{5}{8}.
\]

Moreover, the isocurvature constraints (3.11) (or (3.12)) are satisfied under

\[
\frac{|\sigma_0|}{m_\phi} \lesssim 10^{-2}, \quad \frac{|\sigma_0|}{M} \lesssim 10^{-4} \frac{m_\phi^2}{M^2}, \quad \frac{m_\phi^2}{M^2} \lesssim 10.
\]

Here we also note that \(|\sigma_0|\) is considered to be no less than the super-horizon fluctuations \( H_{\text{inf}}/2\pi \).

The abundance of the dark radiation \( \hat{\sigma} \) in terms of the effective number of neutrinos (3.17) is obtained as

\[
\Delta N_{\text{eff}} \simeq \frac{43}{7} \cdot \frac{1}{2} \cdot \left( \frac{g_*(T_R)}{g_*(T_D)} \right)^{-\frac{1}{3}},
\]

which yields \( \Delta N_{\text{eff}} \sim 1.1 \) for \( g_*(T_D) \simeq 10.75 \) and \( g_*(T_R) \simeq 228.75 \).

Conditions under which the oscillating classical background of the modulus serves as dark matter can be obtained straightforwardly following the discussions in Subsection 3.3. Let us just point out here that the isocurvature constraint (3.28) requires a rather large oscillation amplitude for the modulus such that \( \sigma_{\text{osc}} > 10M \).


5 Discussion and Conclusions

In this paper, we have discussed cosmological implications of the modulus in the modulated reheating mechanism, which is one of the alternative mechanism generating primordial fluctuation, and studied especially in connection with primordial non-Gaussianity. Since the modulus has to couple to the inflaton in order to modulate the inflaton decay rate, it is inevitably produced by the decay. It also should be rather light and have only very suppressed interactions with the SM particles for successful modulated mechanism, thus, the modulus is a natural candidate for dark radiation. The presence of the dark radiation has a lot of implications on BBN and CMB. It speeds up the expansion of the Universe and hence the Helium abundance is increased. As for the CMB power spectrum, there are mainly three effects [37, 38]: (A) The matter-radiation equality is delayed so that the early ISW effect is enhanced, which leads to the increase of the height of the first acoustic peak, then relative suppression of other higher peaks. (B) The free-streaming effects become more significant so that the amplitudes at small angle scales are suppressed. (C) The sound horizon becomes smaller due to the additional contribution to the Hubble parameter at the recombination, which shifts the peaks and troughs to smaller scales. Recent observations might have seen these kinds of effects, in particular on small scales, which suggest \( \Delta N_{\text{eff}} \sim 1 \). It is also interesting to note that when \( \Delta N_{\text{eff}} > 0 \), more scale-invariant or slightly bluer spectral index \( n_s \) is favored compared to the case with \( \Delta N_{\text{eff}} = 0 \) where slightly red-tilted spectrum is favored.\(^{10}\)

Since the modulus mass is required to be extremely light for modulated reheating to operate, the resulting perturbation spectrum tends to be very flat (except for cases with specific inflationary mechanisms, see also discussions in footnote 4 and Appendix B). The modulated reheating mechanism shows a good agreement with the current data in this respect as well, in addition to giving an explicit candidate for dark radiation.

In general, produced modulus particles (dark radiation) have isocurvature fluctuations, which are severely constrained from observations. We have derived general conditions to suppress isocurvature perturbations in the modulated reheating mechanism. Such conditions are easily satisfied in the following three cases: (i) The ratios of the decay rates to their fluctuations for all decay channels are almost the same. (ii) One decay channel dominates over the others. (iii) The weights \( c_{\sigma_i}, c_{\chi_i}, \ldots \) have special relations between them. As a concrete example, a toy model corresponding to the case (ii) has been discussed. In the model, the decay mode with one modulus particle indeed dominates over the other modes and the dark radiation with \( \Delta N_{\text{eff}} \sim 1 \) is realized. Though we have mainly focused on the case (ii) in this paper, it would be interesting to investigate other cases in detail.

\(^{10}\) The analysis with the data from WMAP+SPT shows \( n_s = 0.9663 \pm 0.0112 (1\sigma) \) for ΛCDM model with a fixed \( N_{\text{eff}} = 3.046 \), on the other hand, when \( N_{\text{eff}} \) is varied, the constraint becomes \( n_s = 0.9874 \pm 0.0193 (1\sigma) \) [1]. Thus the scale-invariant spectrum is consistent with observation within 1σ, in the presence of the extra radiation. It is worth noticing that the modulated reheating mechanism predicts \( n_s - 1 = -2\epsilon + 2\eta_{\sigma} \) where \( \epsilon = -\dot{H}/H^2 \) and \( \eta_{\sigma} = V''/3H^2 \), cf. (B.8).
While modulus particles generated by inflaton decay serve as dark radiation, coherent oscillations of modulus can behave like dark matter as well if its mass is adequately large and it can start oscillation before the matter-radiation equality. We have shown that both the dark radiation and the CDM can be attributed to the modulus by taking appropriate parameters.

Lastly we comment on a case where the modulus has a heavier mass, and does not account for dark radiation. Such modulus produced by the inflaton decay may also have an impact on cosmology. For the case of a stable modulus, its abundance should not exceed the observed dark matter abundance. If it is unstable and decays into the SM particles, the energetic decay products may significantly change the light element abundance in contradiction with observation. Thus, even if the modulus does not account for dark radiation, it will be worth studying the cosmological effects of the modulus, which is necessarily produced by the inflaton decay in the modulated reheating mechanism.

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A Isocurvature Perturbations in Modulated Reheating

In this appendix we give general discussions on isocurvature perturbations among particles created through modulated decay. We consider the universe to be initially dominated by a single matter-like component $\phi$ (e.g. the inflaton), which decays into multiple relativistic components $\sigma, \chi, \ldots$, through multiple decay channels. Each decay particle is produced by one or more of the decay channels, and the decay rates of (some of) the channels possess super-horizon fluctuations. Interactions between the decay components are assumed to be negligibly tiny, hence thermalization among the components are forbidden.

Now let us focus on one of the decay particles, say, $\sigma$, and estimate its energy density fluctuations $\delta \rho_{\sigma}$ due to the modulated decay. We represent the decay rate of channel $i$ by $\Gamma_i$, and its fluctuations (if any) by $\delta \Gamma_i$. The size of the fluctuations $|\delta \Gamma_i|$ are assumed to be much smaller than the values of $|\Gamma_i|$ themselves. Furthermore, we denote the fractional
energy density of $\sigma$ produced through channel $i$ as $\rho_{\sigma i}$ (hence $\rho_{\sigma} = \sum_i \rho_{\sigma i}$), and assume that the decay of $\phi$ suddenly happens when $H = \sum_i \Gamma_i = H_{\text{dec}}$. Then $\rho_{\sigma i}$ at the time when $\phi$ decays can be written in terms of the energy density of $\phi$ as

$$
\rho_{\sigma \text{dec}} = \frac{c_{\sigma i} \Gamma_i}{\sum_j \Gamma_j} \rho_{\phi \text{dec}}, \quad (A.1)
$$

where the subscript “dec” denotes values at the $\phi$ decay. The weight $c_{\sigma i}$ ($0 \leq c_{\sigma i} \leq 1$) represents the fraction of $\rho_{\phi}$ that goes into $\rho_{\sigma}$ through channel $i$. For simplicity, we assume that $c_{\sigma i}$’s are constants, which do not fluctuate.

At some time after the $\phi$ decay when the size of the universe is $a$, the energy density $\rho_{\sigma i}$ becomes

$$
\rho_{\sigma i} = \rho_{\sigma \text{dec}} \left( \frac{a_{\text{dec}}}{a} \right)^4 = \rho_{\sigma \text{dec}} \left( \frac{a_0}{a} \right)^4 \left( \frac{H_0}{H_{\text{dec}}} \right)^{8/3}. \quad (A.2)
$$

In the far right hand side we have introduced values $a_0$ and $H_0$ which are values at some time before the decay. Let us now consider energy density fluctuations on the spatially flat slicing, where the scale factors are constants. Note that $H_0$ is also homogeneous since it is prior to the modulated decay. Then the spatial density fluctuations are sourced by $\delta \Gamma$ through

$$
\rho_{\sigma i} \propto \Gamma_i \left( \sum_j \Gamma_j \right)^{-5/3}, \quad (A.3)
$$

giving, up to linear order in $\delta \Gamma_i$,

$$
\delta \rho_{\sigma i} \simeq \rho_{\sigma i} \left( \frac{\delta \Gamma_i}{\Gamma_i} - \frac{5}{3} \sum_j \frac{\delta \Gamma_j}{\Gamma_j} \right). \quad (A.4)
$$

Hence up to linear order in the decay rate fluctuations, we arrive at

$$
\frac{\delta \rho_{\sigma}}{\rho_{\sigma}} \simeq \frac{\sum_i c_{\sigma i} \delta \Gamma_i}{\sum_j c_{\sigma j} \Gamma_j} - \frac{5}{3} \sum_j \frac{\delta \Gamma_j}{\Gamma_j}. \quad (A.5)
$$

For example, when there is only one decay channel producing solely $\sigma$, then $\frac{\delta \rho_{\sigma}}{\rho_{\sigma}} \simeq -\frac{2}{3} \frac{\delta \Gamma}{\Gamma}$. Also, note that even if the decay rates of all the channels producing $\sigma$ do not have fluctuations, i.e. $\delta \Gamma_i = 0$ for all $i$ satisfying $c_{\sigma i} \neq 0$, still the energy density fluctuation $\delta \rho_{\sigma}$ is produced as long as (one of) the other decay channels are modulated.

In order for the linear-order isocurvature fluctuations to vanish among different decay products (which generally have different values of $c_i$’s), i.e. $\delta \rho_{\sigma}/\rho_{\sigma} \simeq \delta \rho_{\chi}/\rho_{\chi} \simeq \cdots$, the first term in the right hand side of (A.5) needs to take the same value for all decay particles. We end this appendix by laying out three possible solutions:

1. Given that $\delta \Gamma_i/\Gamma_i$ take the same value for all decay channels, all particle products obtain the same energy density fluctuations. An example of such case is briefly discussed before Subsection 3.1.
2. Another simple solution is to have only one decay channel for $\phi$. This case is considered in the main body of this paper.

3. The isocurvature perturbations vanish also when the parameters $c_{\sigma i}, c_{\chi i}, \ldots$ take specific values such that the first term in the right hand side of (A.5) becomes the same for all decay products. For e.g., $c_{\sigma j}/c_{\sigma i} = c_{\chi j}/c_{\chi i}$ for all decay products ($\sigma, \chi, \cdots$) and channels.

**B Spectral Index in Modulated Reheating**

In this appendix we derive the spectral index of the density perturbation spectrum obtained from modulated reheating. We carry out the computations without specifying the functional forms of the modulus potential and the inflaton decay rate, thus the results can be applied to generic modulated reheating scenarios.

In Section 2 we derived the perturbation spectrum generated from modulated reheating:

$$P_\zeta = \left(\frac{1}{6\Gamma \partial \sigma_*} \right)^2 \left(\frac{H_*}{2\pi}\right)^2. \quad \text{(B.1)}$$

Recall that the inflaton decay rate is a function of the modulus, i.e. $\Gamma = \Gamma(\sigma_{\text{dec}})$, and that the subscripts $*$ and “dec” denote values at the horizon exit of the CMB scale, and at the inflaton decay, respectively. Since $\sigma_{\text{osc}}$ is independent of the comoving wave number $k$, the spectral index of the spectrum is obtained as

$$n_s - 1 \equiv \frac{d \ln P_\zeta}{d \ln k} \simeq \frac{1}{H_*} \frac{d}{dt} \ln \left( \frac{d \Gamma(\sigma_{\text{dec}}) \partial \sigma_{\text{dec}}}{d \sigma_{\text{dec}} \partial \sigma_*} \right)^2 + 2 \frac{\dot{H}_*}{H_*^2}, \quad \text{(B.2)}$$

where an overdot denotes a time derivative, and we have assumed the Hubble parameter during inflation to be almost a constant.

Upon computing the first term in the right hand side, we make use of the following approximation for the modulus dynamics: Supposing the modulus potential $V(\sigma)$ to be a function only of $\sigma$ (thus does not explicitly depend on, say, time), and that the dominant component of the universe obeys the equation of state $p = w\rho$ with constant $w$, then the equation of motion of the modulus

$$\ddot{\sigma} + 3H\dot{\sigma} = -V' \quad \text{(B.3)}$$

is approximated by [39]

$$cH\dot{\sigma} \simeq -V' \quad \text{with} \quad c = \frac{3(w + 3)}{2}, \quad \text{(B.4)}$$

given that the effective mass of the modulus is as small as

$$\left| \frac{V''}{cH^2} \right| \ll 1. \quad \text{(B.5)}$$
Note that in this appendix, a prime denotes a derivative in terms of \( \sigma \) (which should not be confused with the primes in the main body of this paper denoting \( \sigma^\ast \)-derivatives). (B.5) is a necessary condition for the approximation (B.4) to hold, and one can further show that (B.5) actually guarantees (B.4) to be a stable attractor. See, e.g., Appendix A of [32] for detailed discussions on scalar field dynamics in an expanding universe.

Hence the dynamics of the modulus, whose effective mass is considered to satisfy (B.5) at least until the inflaton decay, is described by (B.4) with \( c = 3 \) during inflation (which is nothing but the slow-roll approximation), and \( c = 9/2 \) after inflation until the inflaton decay. This gives

\[
\int_{\sigma_\ast}^{\sigma_{\text{dec}}} \frac{d\sigma}{V'} \simeq -\int_{t_\ast}^{t_{\text{end}}} \frac{dt}{3H} + \frac{4}{27} \int_{H_{\text{end}}}^{H_{\text{dec}}} \frac{dH}{H^3},
\]

where the subscript “end” represents values at the end of inflation. Taking into account that the modulus has little effect on the expansion of the universe before the inflaton decay, and that \( H_{\text{dec}} = \Gamma(\sigma_{\text{dec}}) \), then after differentiating both sides of (B.6) with \( \sigma^\ast \), one obtains

\[
\frac{\partial \sigma_{\text{dec}}}{\partial \sigma^\ast} \simeq \left\{ 1 - \frac{4}{27} \frac{V''(\sigma_{\text{dec}}) \, d\Gamma(\sigma_{\text{dec}})}{\Gamma(\sigma_{\text{dec}})^3 \, d\sigma_{\text{dec}}} \right\}^{-1} \frac{V'(\sigma_{\text{dec}})}{V'(\sigma^\ast)}.
\]

Here one sees that only the \( V'(\sigma^\ast) \) in the denominator contributes to the spectral index in (B.2). Combining the above results, we arrive at

\[
n_s - 1 \simeq \frac{2}{3} \frac{V''(\sigma_\ast)}{H^2_\ast} + \frac{\dot{H}_\ast}{H^2_\ast}.
\]

Since the effective mass of the modulus in the modulated reheating is generically required to be very light so that (B.5) is satisfied at least until reheating, the resulting perturbation spectrum tends to be extremely flat, unless the inflationary mechanism gives a rather large \( \dot{H}_\ast/H^2_\ast \) (see also Footnote 4).
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