Neutrino Mass Spectrum and Future Beta Decay Experiments

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Abstract

We study the discovery potential of future beta decay experiments on searches for the neutrino mass in the sub-eV range, and, in particular, KATRIN experiment with sensitivity \( m > 0.3 \) eV. Effects of neutrino mass and mixing on the beta decay spectrum in the neutrino schemes which explain the solar and atmospheric neutrino data are discussed. The schemes which lead to observable effects contain one or two sets of quasi-degenerate states. Future beta decay measurements will allow to check the three neutrino scheme with mass degeneracy, moreover, the possibility appears to measure the CP-violating Majorana phase. Effects in the four neutrino schemes which can also explain the LSND data are strongly restricted by the results of Bugey and CHOOZ oscillation experiments: Apart from bending of the spectrum and the shift of the end point one expects appearance of small kink of (< 2\%) size or suppressed tail after bending of the spectrum with rate below 2 \% of the expected rate for zero neutrino mass. We consider possible implications of future beta decay experiments for the neutrino mass spectrum, the determination of the absolute scale of neutrino mass and for establishing the nature of neutrinos. We show that beta decay measurements in combination with data from the oscillation and double beta decay experiments will allow to establish the structure of the scheme (hierarchical or non-hierarchical), the type of the hierarchy or ordering of states (normal or inverted) and to measure the relative CP-violating phase in the solar pair of states.

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1 Introduction

The reconstruction of the neutrino mass spectrum is one of the fundamental problems of particle physics. The program includes the determination of the number of mass eigenstates, and of the values of masses, mixing parameters and CP-violating phases.

At present, the evidence for non-zero neutrino mass follows from oscillation experiments which allow to measure the mixing parameters \( |U_{\alpha j}| \), the mass squared differences and, in principle, the so called Dirac CP-violating phases. However, the absolute values of the neutrino masses cannot be determined. From the oscillation experiments one can only extract a lower bound on the absolute value of neutrino mass. Obviously, for a given \( \Delta m^2 \), at least one of the mass eigenvalues should satisfy inequality:

\[
m_i \geq \sqrt{|\Delta m^2|}.
\]

Thus, the oscillation interpretation of the atmospheric neutrino data \(^1\) gives the bound:

\[
m_3 \geq \sqrt{\Delta m^2_{\text{atm}}} \sim (0.04 - 0.07) \text{ eV}.
\]

Clearly, without knowledge of the absolute values of neutrino masses our picture of Nature at quark-lepton level will be incomplete. The knowledge of absolute values of neutrino masses is crucial for understanding the origin of the fermion masses in general, the quark-lepton symmetry and unification. The determination of the absolute mass scale of neutrinos is at least as important as the determination of other fundamental parameters such as the CP-violating phases and the mixing angles. Actually, it may have even more significant and straightforward implications for the fundamental theory. It is the absolute mass which determines the scale of new physics.

The absolute values of masses have crucial implications for astrophysics and cosmology, in particular, for structure formation in the Universe. In fact, the recent analysis of the latest CMB data (including BOOMERanG, DASI, Maxima and CBI), both alone and jointly with other cosmological data (e.g., galaxy clustering and the Lyman Alpha Forest) shows that \(^2\)

\[
m_\nu < 2.2 \text{ eV},
\]

for a single neutrino in eV range. Future observations can improve this bound. The Planck experiment will be sensitive to neutrino masses down to \( m_\nu \sim 1 \text{ eV} \) \(^3\). However, the cosmological data may not be conclusive. Even if some effects are found, it will be difficult to identify their origin. Modification of the original spectrum of the density fluctuations can mimic to some extent the neutrino mass effect. If no distortion is observed in the
spectrum, one can put an upper bound on the neutrino mass assuming, however, that there is no conspiracy which leads to cancellation of different effects [4]. Therefore independent measurements of the neutrino mass are needed and their results will be used in the analysis of the cosmological data as an input deduced from particle physics.

Several methods have been proposed to determine neutrino masses by using the supernova neutrino data. One method is based on searches for the energy ordering of events which has, however, rather low sensitivity [5]. The limits on the mass can be also obtained from observations of sharp time structures in the signals. It was suggested to study the time distribution of detected neutrino events emitted from supernova which entails to black hole formation [6]. By this method Super-Kamiokande can measure values of the $\nu_e$ mass down to 1.8 eV and SNO can put an upper bound 20 eV on the $\nu_\mu$ and $\nu_\tau$ masses [6]. (Clearly this bound on the $\nu_\mu$ and $\nu_\tau$ masses is much weaker than bounds implied by combined analysis of the solar and atmospheric neutrino data and direct measurements of the $\nu_e$ mass.) In this case one can check the still non-excluded possibility in which the solar neutrino problem is solved by the oscillations to sterile neutrino and the masses of $\nu_\mu$ and $\nu_\tau$ are in 20 eV range. (Such neutrinos should be unstable in cosmological time.) The absolute values of the neutrino masses can be determined in the assumption that the cosmic rays with energies above the GZK cutoff are produced in annihilation of the ultra-high energy neutrinos with the cosmological relic neutrinos [7, 8, 9]. The analysis of the observed energy spectrum of cosmic rays above $10^{20}$ eV gives the mass $m_\nu = (1.5 - 3.6)$ eV, if the power-like part of the ultra-high energy cosmic rays spectrum is produced in Galactic halo, and $m_\nu = (0.12 - 0.46)$ eV, if this part has the extragalactic origin [10].

Neutrinoless double beta decay ($2\beta0\nu$) searches are sensitive to the Majorana mass of the electron neutrino. However, in the presence of mixing the situation can be rather complicated: The effective Majorana mass of $\nu_e$ relevant for the $2\beta0\nu$-decay, $m_{ee}$, is a combination of mass eigenvalues and mixing parameters given by

$$m_{ee} = \left| \sum_i m_i U_{ei}^2 \right|.$$  \hspace{1cm} (2)

From this expression it is easy to find that if the $2\beta0\nu$-decay is discovered with the rate which corresponds to $m_{ee}$, at least one of the mass eigenvalues should satisfy the inequality [11]

$$m_i \geq \frac{m_{ee}}{n},$$  \hspace{1cm} (3)

where $n$ is the number of neutrino mass eigenstates that mix in the electron neutrino. This bound is based on the assumption that exchange of the light Majorana neutrinos is the only
mechanism of the $2\beta 0\nu$-decay and all other possible contributions are absent or negligible. Another uncertainty is related to $n$. We know only the lower bound: $n \geq 3$.

The best present bound on the $2\beta 0\nu$-decay obtained by Heidelberg-Moscow group gives

$$m_{ee} < 0.34 \ (0.26) \ eV, \quad 90 \% \ (68\%) \ C.L.$$

(4)

This bound, however, does not include systematic errors related to nuclear matrix elements.

A series of new experiments is planned with increasing sensitivity to $m_{ee}$: CUORICINO [13], CUORE ($m_{ee} \sim 0.1 \ eV$) [14], MOON ($m_{ee} \sim 0.03 \ eV$) [15] and GENIUS ($m_{ee} \sim 0.002 \ eV$) [16].

Although the knowledge of $m_{ee}$ provides information on the mass spectrum independent of $\Delta m^2$’s, from $m_{ee}$ one cannot infer the absolute values of neutrino masses without additional assumptions. Since in general the mixing elements are complex there may be a strong cancellation in the sum (2). Moreover, to induce the $2\beta 0\nu$ decay, $\nu_e$ must be a Majorana particle.

The information about the absolute values of the masses can be extracted from kinematic studies of reactions in which a neutrino or an anti-neutrino is involved (e.g., beta decays or lepton capture). The most sensitive method for this purpose is the study of the electron spectrum in the tritium decay:

$$^3\mathrm{H} \rightarrow ^3\mathrm{He} + e^- + \bar{\nu}_e.$$

(5)

In absence of mixing, the energy spectrum of $e^-$ in (5) is described by

$$\frac{dN}{dE} = R(E)[(E - E_0)^2 - m_{\nu}^{-2}]^{\frac{1}{2}},$$

(6)

(see, e.g., [17]) where $E$ is the energy of electron, $E_0$ is the total decay energy and $R(E)$ is a $m_{\nu}$-independent function given by

$$R(E) = G_F^2 \frac{m_{ee}}{2\pi^3} \cos^2 \theta_C |M|^2 F(Z, E) pE(E_0 - E).$$

(7)

Here $G_F$ is the Fermi constant, $p$ is the momentum of the electron, $\theta_C$ is the Cabibbo angle and $M$ is the nuclear matrix element. $F(Z, E)$ is a smooth function of energy which describes the interaction of the produced electron in final state. Both $M$ and $F(Z, E)$ are independent

\footnote{In what follows we will use the bound (4) in our estimations for definiteness. At the same time, one should keep in mind that due to uncertainties of nuclear matrix element the values of $m_{ee}$ up to $\sim 0.5 \ eV$ can not be excluded.}
of \( m_\nu \), and the dependence of the spectrum shape on \( m_\nu \) follows from the phase volume factor only. The bound on neutrino mass imposed by the shape of the spectrum is independent of whether neutrino is a Majorana or a Dirac particle.

The best present bound on the electron neutrino mass, (obtained in the assumption of no mixing) is given by Mainz tritium beta decay experiment [18]:

\[
\begin{align*}
    m_{\nu_e} & \leq 2.2 \text{ eV} \quad (95\% \text{ C.L.}) .
\end{align*}
\]

Analysis of the Troitsk results leads to the “conditional” (after subtraction of the excess of events near the end point) bound [19]

\[
\begin{align*}
    m_{\nu_e} & \leq 2.5 \text{ eV} \quad (95\% \text{ C.L.}).
\end{align*}
\]

The present spectrometers are unable to improve the bounds (8, 9) substantially. Further operation of Mainz experiment may allow to reduce the limit down to 2 eV. In this connection a new experimental project, KATRIN, is under consideration with an estimated sensitivity limit [20]

\[
\begin{align*}
    m_{\nu_e} & \sim 0.3 \text{ eV}.
\end{align*}
\]

In the case of negative result from the KATRIN searches one can get after three years of operation the bound \( m_{\nu_e} \leq 0.35 \) \((0.40)\) eV at 90 \% (95 \%) C.L. [20].

Note that with this bound KATRIN experiment can explore the range of neutrino mass which is relevant for the Z-burst explanation of the cosmic ray with super-GZK energies [8].

The aim of this paper is to study the discovery potential of the next generation tritium beta decay experiments with sensitivity in the sub-eV range and in particular, KATRIN experiment. We consider the effects of neutrino mass and mixing on the \( \beta \)-decay spectrum expected for specific neutrino schemes. We describe the three-neutrino schemes which are elaborated to explain the data on the solar and atmospheric neutrinos as well as the four-neutrino schemes which accommodate also the LSND result. We study the bounds that the present and forthcoming 2\(\beta\)0\(\nu\)-decay searches, as well as the oscillation experiments can put on possible tritium decay results. We also consider the implications of future beta decay measurements for the identification of the neutrino mass spectrum.

The paper is organized as follows. In section 2 we give a general description of the effect of massive neutrinos on the beta decay spectrum in the presence of mixing. In section 3 the three-neutrino schemes are explored. In section 4 we present a general discussion of predictions for the beta decay in the four-neutrino schemes which explain the LSND result. We emphasize the importance of the bounds on the beta decay parameters imposed by Bugey
and CHOOZ experiments. In section 5 we study the properties of the beta decay in the hierarchical four-neutrino schemes. In section 6, the non-hierarchical four-neutrino schemes are considered. In section 7 we summarize the role that future beta decay measurements will play in the reconstruction of the neutrino mass spectrum. Conclusions are given in section 8.

2 Neutrino mixing and beta-decay. The effects of degenerate states

In presence of mixing, the electron neutrino is a combination of mass eigenstates $\nu_i$ with masses $m_i$: $\nu_e = \sum_i U_{ei} \nu_i$. So that, instead of (6), the expression for the spectrum is given by

$$
\frac{dN}{dE} = R(E) \sum_i |U_{ei}|^2 [(E_0 - E)^2 - m_i^2]^{1 \over 2} \Theta(E_0 - E - m_i),
$$

where $R(E)$ is defined in (7). The step function, $\Theta(E_0 - E - m_i)$, reflects the fact that a given neutrino can be produced if the available energy is larger than its mass. According to eq. (11) the presence of mixing leads to distortion of the spectrum which consists of

(a) the kinks at the electron energy $E_e^{(i)} = E \sim E_0 - m_i$ whose sizes are determined by $|U_{ei}|^2$;

(b) the shift of the end point to $E_{ep} = E_0 - m_1$, where $m_1$ is the lightest mass in the neutrino mass spectrum. The electron energy spectrum bends at $E \sim E_{ep}$.

So, in general the effect of mixed massive neutrinos on the spectrum cannot be described by just one parameter. In particular, for the three-neutrino scheme, five independent parameters are involved: two mixing parameters and three masses.

Substantial simplification, however, occurs in the schemes which explain the solar and atmospheric neutrino data and have the states with absolute values of masses in the range of sensitivity (10). The simplification appears due to existence of sets of quasi-degenerate states. Indeed, in these schemes there should be eigenstates with mass squared differences $\Delta m^2_{\odot} < 2 \cdot 10^{-4}$ eV$^2$ and $\Delta m^2_{\text{atm}} \sim 3 \cdot 10^{-3}$ eV$^2$. If the neutrino masses, $m_i$, are larger than 0.3 eV (10), the mass differences

$$
\Delta m \sim \frac{\Delta m^2}{2m}
$$

In what follows we will use the terminology elaborated for the ideal Kurie plot without background.
turn out to be smaller than $5 \times 10^{-3}$ eV. Moreover, $\Delta m/m \sim \Delta m^2/2m^2 \ll 1$, that is, the states are strongly degenerate. Since the detectors cannot resolve such a small mass split, different masses will entail just to one visible kink with certain effective mass and mixing parameter. As a consequence, the number of relevant parameters which describe the distortion of the beta spectrum is reduced to one or three, depending on the type of the scheme (see sections 3 - 6).

In the Ref. \cite{21} it has been shown that for energies $E_{\nu_e} \gg m_{\nu_i}$ the distortion of the electron energy spectrum in the $\beta$-decay due to non-zero neutrino mass and mixing is determined by the effective mass

$$m_{\text{eff}} = \frac{\sum_i m_i |U_{ei}|^2}{\sum_i |U_{ei}|^2}.$$  \hspace{1cm} (13)

However, the highest sensitivity to the mass of $\nu_i$ appears in the energy range close to the end point where $E_{\nu} \sim m_i$ and therefore the approximation used in \cite{21} to introduce $m_{\text{eff}}$ does not work. In what follows we show that still it is possible to use the mass parameter (13) for a set of quasi-degenerate states.

In general, the neutrino mass spectrum can have one or more sets of quasi-degenerate states. Let us consider one such a set which contains $n$ states, $\nu_j$, $j = i, i + 1, ..., i + n - 1$ with $\Delta m_{ji} \ll m_j$. We define $\Delta E$ as the smallest energy interval that the spectrometer can resolve. (Note that $\Delta E$ may be smaller than the width of resolution function, and the latter is about 1 eV in KATRIN experiment.) We assume that $\Delta m_{ij} \ll \Delta E$.

Let us introduce the coupling of this set of the states with the electron neutrino as

$$\rho_e \equiv \sum_j |U_{ej}|^2,$$  \hspace{1cm} (14)

where $j$ runs over the states in the set. We will show that the observable effect of such a set on the beta spectrum can be described by $\rho_e$ and the effective mass $m_\beta$ which can be introduced in the following way. Let us consider the interval $\Delta E$ in the region of the highest sensitivity to the neutrino mass, that is, the interval of the electron energies

$$(E_0 - m_i - \Delta E) - (E_0 - m_i),$$  \hspace{1cm} (15)

where $m_i$ is the mass of the lightest state in the set. The number of events in this interval, $\Delta n$, is given by the integral

$$\Delta n = \int_{E_0 - m_i - \Delta E}^{E_0 - m_i} \frac{dN}{dE} dE.$$  \hspace{1cm} (16)

We will define the effective mass $m_\beta$ in such a way that the number of events calculated for the approximate spectrum with single mixing parameter $\rho_e$ and mass $m_\beta$, $\Delta n(\rho_e, m_\beta)$,
reproduces, with high precision, the number of events calculated for exact neutrino mass and mixing spectrum $\Delta n(U_{ej}, m_j)$. That is,

$$R \equiv \frac{\Delta n(\rho_e, m_\beta)}{\Delta n(\rho_e, m_\beta)} - \frac{\Delta n(U_{ej}, m_j)}{\Delta n(U_{ej}, m_j)} \ll 1. \quad (17)$$

Expanding $\Delta n$ (see Appendix) in powers of $\Delta m_j/\Delta E \ll 1$, where

$$\Delta m_j \equiv m_j - m_\beta, \quad (18)$$

we obtain:

$$R \propto \sum_j |U_{ej}|^2 \frac{\Delta m_j}{\Delta E} + O \left( \frac{(\Delta m_j/\Delta E)^2}{(\Delta m_j/\Delta E)^2} \right). \quad (19)$$

It is easy to see that the first term vanishes if

$$m_\beta = \frac{\sum_j m_j |U_{ej}|^2}{\rho_e}. \quad (20)$$

So, for this value of $m_\beta$, $R$ is of the order of $(\Delta m_j/\Delta E)^2$. Note that if we set $m_\beta$ to be equal to the value of any mass from the set or the average mass, the difference of the number of events would be of the order of $\Delta m/\Delta E$. If $\Delta E$ is relatively small, this correction may be significant. The expression (20) is similar to (13), but in (20) $j$ runs over a quasi-degenerate set (not over all the states.) Moreover, provided that $\Delta m \ll \Delta E$, the approximation works for all energies.

In reality the background should be taken into account. However it is easy to see that if the change of the background with energy in the interval $\Delta E$ is negligible, our analysis will be valid in the presence of the background, too.

If the scheme contains more than one set of quasi-degenerate states with the corresponding effective masses $m^q_{\beta}$ and mixing parameters $\rho^q_e$, the observable spectrum can be described by the following expression

$$\frac{dN}{dE} = R(E) \sum_q \rho^q_e \left[ (E_0 - E)^2 - (m^q_{eff})^2 \right]^{\frac{1}{2}} \Theta(E_0 - E - m^q_{eff}), \quad (21)$$

where $q$ runs over the sets. Each set of quasi-degenerate states will produce a single kink at the electron energy $E^q \sim E_0 - m^q_{\beta}$ with the size of the kink determined by $\rho^q_e$. The set with the lightest masses leads to bending of spectrum and the shift of the end point.

### 3 Three neutrino scheme

Let us consider the three-neutrino schemes which explain the solar and atmospheric neutrino results. In the case of mass hierarchy, $m_1 \ll m_2 \ll m_3$, the largest mass, $m_3 \simeq \sqrt{\Delta m^2_{atm}} =$
\((4 - 7) \times 10^{-2} \text{ eV}\), is too small to produce any observable effect in the planned tritium decay experiments (see eq. (10)).

If \(m_3\) is in the sensitivity range of KATRIN experiment \((m_3 \geq 0.3 \text{ eV})\), the mass spectrum should be quasi-degenerate. Indeed,
\[
\frac{\Delta m_{31}}{m_3} \simeq \frac{\Delta m_{atm}^2}{2m_3^2} \leq 0.03.
\]
Moreover, from the unitarity condition we get the coupling parameter
\[
\rho_e = \sum_{j=1,2,3} |U_{ej}|^2 = 1. \quad (22)
\]
Therefore the effect of non-zero neutrino masses and mixing on the \(\beta\)-decay spectrum is characterized by unique parameter - the effective mass
\[
m_\beta = \sum_{j=1,2,3} m_j |U_{ej}|^2 \simeq m_3. \quad (23)
\]
Correspondingly, the distortion of the \(\beta\)-decay spectrum consists of a bending of the spectrum and shift of the end point determined by \(m_\beta (E_0 \rightarrow E_0 - m_\beta)\), as in the case of \(\nu_e\) with definite mass and without mixing. Let us consider the bounds on \(m_\beta\) imposed by the \(2\beta0\nu\)-decay searches and the oscillation experiments. (The \(2\beta0\nu\)-decay in schemes with three degenerate neutrinos has been extensively discussed before [22], [23]). Assuming that neutrinos are Majorana particles we get from (2) and (20) the relation between the effective masses in the beta decay and the double beta decay:
\[
m_{ee} \simeq m_\beta ||U_{e1}|^2 + e^{i\phi_2} |U_{e2}|^2 + e^{i\phi_3} |U_{e3}|^2 ||,
\]
where \(\phi_2\) and \(\phi_3\) are the relative CP-violating phases of the contributions from the second and the third mass eigenstates.

According to the CHOOZ bound [24] which is confirmed by the slightly weaker bound obtained in Palo Verde experiment [25], one of the squared mixing elements (let us take \(|U_{e3}|^2\) for definiteness) must be smaller than 0.05. The other two elements are basically determined by the mixing angle \(\theta_\odot\) responsible for the solution of the solar neutrino problem, so that the eq. (24) can be rewritten as
\[
m_{ee} = m_\beta \left| (1 - |U_{e3}|^2)(\cos^2 \theta_\odot + e^{i\phi_2} \sin^2 \theta_\odot) + e^{i\phi_3} |U_{e3}|^2 \right|.
\]
(25)
From this equation we find the following bounds on the beta decay mass (see also [11]):
\[
m_{ee} < m_\beta < \frac{m_{ee}}{|| \cos 2\theta_\odot (1 - |U_{e3}|^2) - |U_{e3}|^2 ||},
\]
(26)
where the upper bound corresponds to the maximal cancellation of the different terms in (25).

The bounds (26) are shown in fig. 1. (See also [22].) The following comments are in order:

1) For zero value of $U_{e3}$ the weakest bound on $m_\beta$ from the double beta decay appears at maximal mixing: $\tan^2 \theta_\odot = 1$. For non-zero $U_{e3}$ the points of the weakest bound shift to $\tan^2 \theta_\odot \simeq 1 \pm 2|U_{e3}|^2$. In the vicinity of these points, the upper bound on $m_\beta$ is given by the present beta decay result (see eq. (8)).

2) Taking the best fit values of $\theta_\odot$ from the various large mixing solutions of the solar neutrino problem [26] we find from eq. (26) the following bounds:

$$m_\beta < \begin{cases} 
0.67 - 0.74 \text{ eV LMA} \\
1.6 - 2.2 \text{ eV LOW} \\
1.0 - 1.3 \text{ eV VAC}
\end{cases}$$

(27)

where we have used $m_{ee} \leq 0.34 \text{ eV}$ [1] and the two numbers in each line correspond to $|U_{e3}|^2 = 0$ and 0.05, respectively. Note that already existing data on the $2\beta0\nu$-decay give bounds (at the best fit points) which are stronger than the present bound from direct measurement.

Moreover, for $m_{ee} < 0.07 \text{ eV}$ which can be achieved already by CUORE experiment [14], the bound on $m_\beta$ from $2\beta0\nu$-decay in the LMA preferable region of $\tan^2 \theta$ is below the sensitivity of KATRIN experiment. Therefore, the positive result of KATRIN experiment (and identification of the LMA solution of the solar neutrino problem) will lead to exclusion of such a $3\nu$-scheme.

3) For the SMA solution of the solar neutrino problem we get $m_\beta \simeq m_{ee}$, and consequently, according to the bound (4): $m_\beta \leq 0.34 \text{ eV}$. So, the expected range of $m_\beta$ only marginally overlaps with the KATRIN sensitivity region. Thus, if the SMA solution is identified and the LSND result is not confirmed, favoring the three neutrino scheme, the chance for observation of the beta spectrum distortion in KATRIN experiment is rather small.

4) A positive signal in the $2\beta0\nu$-decay searches will have important implications for the tritium decay measurements:

a). According to (23), it gives a lower bound on $m_\beta$ independently of the solution of the solar neutrino problem: $m_{ee} \leq m_\beta$.

b). If the values of $m_{ee}$, $m_\beta$ and $|U_{e3}|^2$ are measured, we will be able to determine the CP-violating phase $\phi_2$ in (25):

$$\sin^2 \frac{\phi_2}{2} = \frac{1}{\sin^2 2\theta_\odot} \left[ 1 - \left( \frac{m_{ee}}{m_\beta} \right)^2 - 2|U_{e3}|^2 \left( \frac{m_{ee}}{m_\beta} \right)^2 \pm m_{ee} \right]$$
\[ \frac{1}{\sin^2 2\theta_\odot} \left[ 1 - \left( \frac{m_{ee}}{m_\beta} \right)^2 \right], \]

where (±) sign of the last term reflects an uncertainty due to the phase of \( U_{e3}^2 (\phi_3 \text{ in (24)}) \).

c). If \( m_\beta \) turns out to be smaller than \( m_{ee} \), we will conclude that there are some additional contributions to the \( 2\beta 0\nu \)-decay unrelated to the Majorana neutrino mass.

If future \( \beta \) decay measurements with sensitivity (11) give a negative result, the largest part of the allowed mass range of the 3\( \nu \)-scheme with strong degeneracy will be excluded. Still a small interval \( (m_\nu \sim 0.1 - 0.3 \text{ eV}) \) will be uncovered. This will have important implications for the theory of the neutrino masses.

In fig. 1 we show also the upper bound on \( m_\beta \) from data on the large scale structure of the Universe obtained in [41] for values of total matter contribution to the energy density of the Universe, \( \Omega_m = 0.4 \) and the reduced Hubble constant, \( h = 0.8 \).

Without \( \beta \)-decay measurements the absolute value of the neutrino mass can be determined and the scheme can be identified provided that all the following conditions are satisfied:

- no effect of sterile neutrinos is observed,
- the SMA solution is established as the solution of the solar neutrino problem,
- the neutrinoless double beta decay gives a positive result with \( m_{ee} \) close to the present upper bound.

In this case \( m_\nu = m_{ee} \). However, not too much room is left for such a possibility keeping in mind that recent solar neutrino data disfavor the SMA solution. For all large mixing solutions of the solar neutrino problem, \( m_{ee} \) gives only the lower bound on the absolute scale of masses.

Let us stress that, even in the case of the SMA solution one should make the assumption that there are no additional contributions to the \( 2\beta 0\nu \)-decay apart from the exchange of light Majorana neutrinos. In this scheme there are no “test equalities”, that is, the relations between \( m_{ee} \) and the oscillation parameters, \( \Delta m^2, \theta \), which could allow to check this assumption independently. Furthermore, the determination of \( m_{ee} \) and therefore \( m_\nu \) will be restricted by uncertainties in the nuclear matrix elements. Thus, the study of the beta spectrum is the only way to measure the absolute scale of neutrino mass without ambiguity.

Clearly, if the LSND result is confirmed the scheme will be excluded.
4 4-ν schemes: Bugey, CHOOZ and LSND bounds

Four-neutrino schemes, which explain the LSND result in terms of oscillations, have two sets of mass eigenstates separated by $\Delta m_{LSND}^2$ (see fig. 2). Hereafter, we call them the light set of states and the heavy set of states. Let us consider the heavy set. The masses of states in this set are equal or larger than $\sqrt{\Delta m_{LSND}^2}$. The mass differences are equal or smaller than $\Delta m^2_{\text{atm}}$ or $\Delta m^2_{\odot}/2\sqrt{\Delta m_{LSND}^2}$. Both splits are much smaller than the energy resolution $\Delta E$ as well as masses themselves. So, the states in the heavy set are quasi-degenerate and their effect on the beta spectrum can be characterized by $m^h_\beta$ and $\rho^{h,e}$ given in eqs. (14) and (20).

In the (2 + 2) schemes both the heavy and light sets contain two states, whereas in the (3 + 1) scheme one set contains 3 states, while the other set consists of only one state (see fig. 2).

The $\nu_e$ oscillation disappearance experiments, Bugey [27] and CHOOZ [24], impose a direct and very strong bound on $\rho^{h,e}$, and therefore on the expected effects in $\beta$-decay in all 4ν-schemes. Since Bugey and CHOOZ experiment do not resolve small mass squared differences, $\Delta m_{\text{atm}}^2$ and $\Delta m_{\odot}^2$, their results can be described by 2ν-oscillations with a unique mass squared difference $\Delta m^2 \simeq \Delta m_{LSND}^2$ and the effective mixing parameter

$$\sin^2 2\theta_{\text{eff}} = 4 \sum_i |U_{ei}|^2 (1 - \sum_i |U_{ei}|^2),$$

where the sum runs over the heavy (or light) set. Using the definition of $\rho^{h,e}$ in eq. (14) we can rewrite the mixing parameter as

$$\sin^2 2\theta_{\text{eff}} = 4\rho^h_e(1 - \rho^h_e).$$

Thus, the negative results of the oscillation searches in Bugey and CHOOZ experiments give immediate bound on $\rho^h_e$ as a function of $\Delta m_{LSND}^2$ (see figs. 3 - 7).

For the range of masses relevant for LSND experiment ($\Delta m^2 > 0.2 \text{ eV}^2$) two possibilities follow from (28) and the Bugey or CHOOZ bounds:

1). Small $\rho^h_e$:

$$\rho^h_e < 0.027.$$  

That is, the admixture of $\nu_e$ in the heavy set is very small and the electron flavor is distributed mainly in the light set. This corresponds to the schemes with normal mass hierarchy (or to
normal ordering of states in the non-hierarchical schemes). Let us recall that in the schemes with normal hierarchy (order of states) the light set contains the pair of states which are separated by $\Delta m_{\odot}$. This pair is responsible for the conversion of the solar neutrinos, and for brevity, we will call it "solar pair". According to (29), in this class of schemes one expects small kink at $E_e \sim E_0 - m_\beta$, where $m_\beta \geq \sqrt{\Delta m_{LSND}^2}$. Here, the inequality corresponds to the non-hierarchical case (see sect. 6). Also in the case of non-hierarchical scheme one predicts an observable shift of the end point associated to the masses of the light set.

2). Large $\rho^h_e$:

$$1 - \rho^h_e < 0.027.$$  

The admixture of $\nu_e$ in the heavy set is close to one: the electron flavor is mainly distributed in the heavy set. This corresponds to the schemes with inverted mass hierarchy or inverted ordering of states. The effect in the beta spectrum consists of a "large" kink with size close to 1 at $E_e \sim E_0 - m_\beta$. Above $E_e \sim E_0 - m_\beta$ the spectrum has a tail related to emission of the neutrinos from the light set. The rate of events in the tail is suppressed by factor given in eq. (30). If the tail is unobservable, the whole effect will look like the effect of the electron neutrino with a unique definite mass $m_\beta$. (And it is similar to the effect in the scheme with three degenerate neutrinos.)

We will further discuss these possibilities in the sections 5-7.

Let us consider implications of the LSND result itself for the $\beta$-decay searches. Apart from providing the mass scale in the range of sensitivity of future $\beta$-decay experiment, it imposes an important bound on the relevant mixing parameters.

In the $4\nu$-schemes under consideration the oscillations in LSND experiment are reduced to two neutrino oscillations with $\Delta m^2 \simeq \Delta m_{LSND}^2$ and the effective mixing parameter

$$\sin^2 2\theta_{LSND} = 4 \left| \sum_{j \in h} U_{\mu j} U_{ej}^* \right|^2 = 4 \left| \sum_{j \in l} U_{\mu j} U_{ej}^* \right|^2,$$

where summations run over the states of the heavy set in the first equality and of the light set in the second equality. Using Schwartz inequality we get

$$\sin^2 2\theta_{LSND} \leq 4 \left( \sum_{i \in h} |U_{ei}|^2 \right) \left( \sum_{j \in h} |U_{\mu j}|^2 \right) = 4 \rho^h_e \rho^h_{\mu},$$

or equivalently,

$$\sin^2 2\theta_{LSND} \leq 4 \left( \sum_{i \in l} |U_{ei}|^2 \right) \left( \sum_{j \in l} |U_{\mu j}|^2 \right) = 4 \left( 1 - \rho^h_e \right) \left( 1 - \rho^h_{\mu} \right),$$

where

$$\rho^h_{\mu} = \sum_{j \in h} |U_{\mu j}|^2$$
is the coupling of the heavy set with the muon neutrino. The upper bound on $\rho_{\mu}^h$ (or upper bound on $1 - \rho_{\mu}^h$, depending on the scheme) follows from CDHS experiment at high $\Delta m^2$ \cite{28}, and from the atmospheric neutrino studies at low $\Delta m^2$ \cite{29}. From eq. (32) we get a lower bound on $\rho_{e}^h$:

$$\rho_{e}^h > \frac{\sin^2 2\theta_{LSND}}{4\rho_{\mu}^h},$$

where both $\sin^2 2\theta_{LSND}$ and $\rho_{\mu}^h$ are functions of $\Delta m^2$. Implications of this bound for the $\beta$-decay measurements strongly depend on specific scheme, and we will discuss them in sections 5 and 6.

The character of the distortion of the spectrum and the sizes of the effects depend on
1) the structure of the scheme: hierarchical or non-hierarchical;
2) the type of hierarchy (ordering of levels): normal or inverted;
3) the number of states in the heavy and light sets.

In what follows we consider possible 4$\nu$-schemes in order.

5 Four-neutrino schemes with mass hierarchy

In the hierarchical schemes, the masses of the states from the light set are much smaller than the sensitivity limit 0.3 eV \cite{11}. In the (2 + 2) schemes they are restricted by the atmospheric ($m^l \leq \sqrt{\Delta m^2_{atm}}$) or solar ($m^l \leq \sqrt{\Delta m^2_\odot}$) neutrino mass scales. Therefore the observable distortion of the spectrum is only due to effect of the heavy set with the effective mass

$$m_\beta \approx \sqrt{\Delta m^2_{LSND}}.$$  

As we have mentioned in the previous section, the character of distortion of the beta decay spectrum depends, first of all, on the type of hierarchy.

5.1 Schemes with normal mass hierarchy

In the schemes with the normal hierarchy (both in the (2 + 2) and (3 + 1) cases) the electron flavor is in the light set and $\rho_{e}^h$ is strongly restricted by the Bugey result (see eq. (29) and figs. 3, 4). The beta decay spectrum has only a “small” kink with $\rho_{e}^h < 0.027$ at $E = E_0 - \sqrt{\Delta m^2_{LSND}}$. 

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Additional restrictions on the $\beta$-decay parameters may appear depending on whether the scheme is of the (2 + 2) or (3 + 1) type.

1). The (3 + 1) scheme. In this scheme one gets a substantial lower bound on $\rho^h_e$ from the LSND result (see eqs. (32), (33)). Indeed, in this scheme $\rho^h_e$ is restricted from above by the CDHS result [28]. Inserting the bound on $\rho^h_e$ into eq. (35) we get the lower bound on $\rho^h_e$ which is close to the upper Bugey bound or, for certain ranges of $\sqrt{\Delta m^2_{LSND}}$, even above it. So that only certain ranges of $m_\beta$ are allowed (see fig. 3). This is a manifestation of the fact that in the (3 + 1) scheme an explanation of the LSND result requires the $\nu_e$ admixture in the isolated state to be at the level of the upper Bugey bound [30].

Let us consider implications of the $2\beta0\nu$-decay search. The contribution to $m_{ee}$ from the fourth (isolated) state dominates. It can be estimated as:

$$m_{ee}^{(4)} = \sqrt{\Delta m^2_{LSND}} |U_{e4}|^2 \sim (0.005 - 0.05) \text{ eV}.$$  \hspace{1cm} (37)

In the hierarchical case with $m_2 = \sqrt{\Delta m^2_\odot}$, the contributions from other mass eigenstates can be estimated as $m_{ee}^{(3)} = \sqrt{\Delta m^2_{atm}} |U_{e3}|^2 < 3.5 \cdot 10^{-3} \text{ eV}$ and $m_{ee}^{(2)} \approx \sqrt{\Delta m^2_\odot} \sin^2 \theta_\odot < 7 \cdot 10^{-3}$ eV. Hence

$$m_{ee} \approx m_{ee}^{(4)} = m_\beta \rho^h_e,$$  \hspace{1cm} (38)

or $m_\beta = m_{ee}/\rho^h_e$.

A version of the scheme is possible in which the mass hierarchy in the light set is inverted, so that the states which contain the electron flavor have masses $m_2 \simeq m_3 \simeq \sqrt{\Delta m^2_{atm}}$ and $m_1 \ll m_2$. In this case we have

$$m_{ee} \simeq \left| m_4 |U_{e4}|^2 e^{i\delta} + \sqrt{\Delta m^2_{atm}} \left( \cos^2 \theta_\odot e^{i\alpha} + \sin^2 \theta_\odot \right) \right|,$$  \hspace{1cm} (39)

and the contribution from the light set can be comparable to the contribution from the 4th state. The corresponding lines are shown in fig. 3. According to the figure one expects $m_{ee}$ to be substantially below the present bound: We find $m_{ee} \sim 0.005 \text{ eV}, \sim 0.015 \text{ eV}, \sim 0.015 \div 0.03 \text{ eV}$ and $\sim 0.06 \text{ eV}$ for the allowed “islands” of $m_\beta$ and $\rho^h_e$ (from smallest to largest $m_\beta$). Clearly, the observation of $m_{ee}$ near its present experimental bound will exclude the scheme.

For the SMA solution of the solar neutrino problem, eq. (39) reduces to

$$m_{ee} \simeq \left| \sqrt{\Delta m^2_{LSND}} \rho^h_e e^{i\delta} + \sqrt{\Delta m^2_{atm}} \right|.$$  \hspace{1cm}

In principle, using this equation one can determine the relative phase $\delta$. 

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For large angle solutions of the solar neutrino problem still significant contributions can come from the solar pair of states and therefore two different phases ($\delta$ and $\alpha$) are involved in the determination of $m_{ee}$ (see (39)).

2). The (2 + 2) scheme. In the (2 + 2) scheme with normal mass hierarchy the mass difference of the heavy set is given by $\Delta m^2_{\text{atm}}$, and $\nu_{\mu}$ is distributed, mostly, in the heavy set. According to the CHDS bound, $\rho^h_\mu$ should be close to 1 in this scheme. Hence, the LSND bound is less restrictive:

$$\rho^h_e > \frac{1}{4} \sin^2 2\theta_{\text{LSND}},$$

and $\rho^h_e$ can be as small as $(2 - 3) \cdot 10^{-4}$ (see fig. 4). The effect of the kink is unobservable for such a small $\rho^h_e$.

Let us consider bounds from the 2$\beta$0$\nu$-decay searches. The Majorana mass of the electron neutrino can be written in the following form

$$m_{ee} \simeq m_\beta \left| (U_{e3}^2 + U_{e4}^2) + \frac{\sqrt{\Delta m^2_{\odot}}}{m_\beta} \sin^2 \theta_{\odot} (1 - |U_{e3}|^2 - |U_{e4}|^2) \right|. \quad (41)$$

Neglecting the contribution of light neutrinos ($\sqrt{\Delta m^2_{\odot}} \sin^2 \theta_{\odot} < 7 \cdot 10^{-3}$ eV), we obtain from (11) the following bound on the effective $\beta$-decay mass:

$$\frac{m_{ee}}{\rho^h_e} < m_\beta < \frac{m_{ee}}{||U_{e3}|^2 - |U_{e4}|^2||}. \quad (42)$$

If the neutrinoless double beta decay is discovered, this inequality will put a strong lower bound on $m_\beta$:

$$m_\beta > 25m_{ee}, \quad (43)$$

where we have used the bound on $\rho^h_e$ from the Bugey experiment. For $m_{ee} = 0.1$ eV we get $m_\beta = 2.5$ eV which is in the upper allowed region of the LSND experiment. For this reason, one cannot expect $m_{ee} > 0.1$ eV in this scheme.

5.2 Schemes with inverted mass hierarchy

In the schemes with inverted hierarchy, the electron flavor is distributed mainly in the heavy set (see fig. 3). Therefore the $\beta$-decay spectrum should have a “large” kink with parameters $\rho^h_e \simeq 1$ and $m^h_\beta \simeq \sqrt{\Delta m^2_{\text{LSND}}}$. The light set leads to appearance of the tail at $E_e > E_0 - m^h_\beta$. 

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with the suppressed rate determined by \((1 - \rho^h_e) < 0.027\). To detect the signal from the tail one may search for the integral effect above \(E_0 - m_4\).

Note that for the LSND region with \(\Delta m^2_{LSND} = (6 - 8)\ eV^2\) allowed at 99 % C.L. one gets \(\sqrt{\Delta m^2_{LSND}} \simeq 2.5\ eV\) which is already excluded by the Mainz result at 95 % C.L. (see fig. 4). Hence, for the schemes with inverted hierarchy the preferable range of mass would be

\[
0.4\ eV < m_\beta < 1.75\ eV. \tag{44}
\]

1). The \((3 + 1)\) scheme. The upper Bugey bound and the lower LSND bound on \(1 - \rho^h_e\) are shown in fig. 5. They are similar to the bounds on \(\rho^h_e\) in schemes with normal mass hierarchy.

Let us consider the implications of the 2\(\beta0\nu\)-decay searches. The contribution of the light states to \(m_{ee}\) is negligible. As a consequence, the bounds imposed by the 2\(\beta0\nu\)-decay searches are similar to the bounds in the three-neutrino scheme, and the only difference is that in the latter scheme, \(m_\beta\) is a free parameter, whereas in the \((3+1)\) scheme it equals to \(\sqrt{\Delta m^2_{LSND}}\).

For the SMA solution of the solar neutrino problem we have \(m_{ee} \approx \sqrt{\Delta m^2_{LSND}} > 0.4\ eV\) which is already larger than the present upper bound. So, keeping in mind the uncertainties of the nuclear matrix element, we can say that this possibility is disfavored.

In fig. 5, we show the upper bounds on \(m_\beta\) \((m_\beta < m_{ee}^{max}/\cos 2\theta_\odot)\) which correspond to \(|U_{e3}| = 0\) and \(m_{ee}^{max} = 0.34\ eV\). The bounds are shown for different values of \(\sin^2 2\theta_\odot\) from the large mixing solution regions. Note that for the LMA solution the 99 % upper bound, \(\sin^2 2\theta_\odot \simeq 0.95\), gives \(m_\beta < 1.5\ eV\), which is already below the present kinematic bound. The best fit region \(\sin^2 2\theta_\odot \sim (0.6 - 0.8)\) leads to \(m_\beta \simeq (0.55 - 0.75)\ eV\) well in the range of the KATRIN sensitivity. For the LOW solution the maximal mixing is possible for which \(\cos 2\theta_\odot = 0\), and the whole region of \(m_\beta\) up to the present kinematic bound \((8)\) is accepted.

2). The \((2 + 2)\) scheme. Here the effects are similar to those in the \((3 + 1)\) scheme with two differences:

i). As it is shown in fig. 5, the LSND result is less restrictive and the rate in the tail can be substantially lower.

ii). The bound on \(m_\beta\) from the neutrinoless double beta decay coincides with the bound in the \((3 + 1)\) scheme at \(|U_{e3}| = 0\):

\[
m_{ee} < m_\beta < \frac{m_{ee}}{\cos 2\theta_\odot}. \tag{45}
\]
6 Non-hierarchical schemes

Let us consider schemes in which the masses of states from the light set are also in the range of sensitivity of KATRIN experiment: \( m_1 > 0.3 \) eV. Clearly, these states are quasi-degenerate, and their effect on the \( \beta \)-decay spectrum can be characterized by the effective mass, \( m_\beta \), and the total coupling with the electron neutrino, \( \rho_\nu \). From the unitarity condition we have \( \rho_\nu^l + \rho_\nu^h = 1 \).

In the non-hierarchical schemes, the effect of neutrino mass on the \( \beta \)-decay spectrum consists of the kink and the shift of the end point. The kink is at

\[
E_{e}^{kink} = E_0 - m_\beta^h,
\]

where

\[
m_\beta^h = \sqrt{\Delta m_{LSND}^2 + (m_\beta^l)^2},
\]

and its size is determined by \( \rho_\nu^h \). In contrast to the hierarchical case, the mass of the heavy set is not fixed by \( \sqrt{\Delta m_{LSND}^2} \). The tail above the kink, \( E_e > E_{e}^{kink} \), is described by \( \rho_\nu^l \) and the end point is shifted to

\[
E_{e}^{ep} = E_0 - m_\beta^l.
\]

Note that if \( m_\beta^l \approx 0.5 \) eV, from (47) we obtain the effective mass of the heavy set \( m_\beta^h \approx 2.5, 1.5 \) and \( 0.8 \) eV for \( \Delta m_{LSND}^2 = 6, 1.5 \) and \( 0.4 \) eV\(^2 \) correspondingly. For large values of \( \Delta m_{LSND}^2 \) both the structures (the kink and the bending of the spectrum at the end point) can in principle be separately detected by KATRIN. For \( \Delta m_{LSND}^2 \) at the lower allowed end the difference of the two effective masses becomes small: \( m_\beta^h - m_\beta^l < 0.3 \) eV, and these two structures may not be resolved.

With increase of \( m_\beta^l \) the scheme transforms into the scheme with four degenerate neutrinos. Already at \( m_\beta^l = 1 \) eV, we get the mass of the heavy set \( m_\beta^h = 2.7, 1.5, \) and \( 1.2 \) eV, for \( \Delta m_{LSND}^2 = 6, 1.5, \) and \( 0.4 \) eV\(^2 \). For the smallest mass, \( m_\beta^h =1.2 \) eV, the difference of masses, \( m_\beta^h - m_\beta^l < 0.2 \) eV, is too small to be resolved. With the increase of \( m_\beta^l \) the two structures in the spectrum, the kink and the bending, merge.

The parameters of the kink and of the tail depend on specific properties of the scheme. We will call it the scheme with normal ordering of states when the electron neutrino is distributed mainly in the light set. And we will refer to the opposite situation, when \( \nu_e \) is in the heavy set, as to the scheme with inverted order of states.

In the non-hierarchical schemes the mass spectrum is shifted to larger values of mass. The oscillation pattern, however, is not changed and the oscillation bounds are the same as
in the hierarchical schemes described above (see fig. 3, 6). However, the implications of the $2\beta 0\nu$-decay searches are changed.

6.1 Schemes with normal order of states

The electron neutrino is mainly in the light set. The beta decay spectrum should have a small kink at $E_0 - m^h_\beta$ with the size restricted by Bugey experiment: $\rho^h_\beta < 0.027$, and a strong bending of spectrum at $E_0 - m^l_\beta$.

In the fig. 7 we show the bounds on the $\beta$-decay parameters $m^l_\beta$ and $\rho^h_\beta$ for two representative values of the mass squared difference: $\Delta m^2_{\text{LSND}} = 1.75$ eV$^2$, which corresponds to the weakest bound on $\rho^h_\beta$ from Bugey experiment, and $\Delta m^2_{\text{LSND}} = 0.227$ eV$^2$ from the lowest (in $\Delta m^2$ scale) LSND region.

For $\Delta m^2_{\text{LSND}} = 1.75$ eV$^2$ the allowed region of the $\beta$-decay parameters is between the vertical lines at $\rho^h_\beta = 0.013$ (dashed) and $\rho^h_\beta = 0.026$ (solid) for the $(3 + 1)$ scheme (shadowed), while for the $(2 + 2)$ scheme the valid region stays between $\rho^h_\beta = 0.004$ (dash-dotted) and $\rho^h_\beta = 0.026$ (solid).

For $\Delta m^2_{\text{LSND}} = 0.227$ eV$^2$ the allowed regions are substantially smaller: for the $(3 + 1)$ scheme the region is between lines at $\rho^h_\beta = 0.0095$ (dashed) and 0.010 (solid). For the $(3 + 1)$ scheme the region (shadowed) is restricted by lines at 0.0002 (dash-dotted) and 0.010 (solid). All the regions are bounded from above by the Mainz result.

The implications of the $2\beta 0\nu$-decay searches depend on the specific arrangements of levels.

1. $(3 + 1)$ scheme. The contribution to the effective Majorana mass, $m_{ee}$, from different mass eigenstates can be evaluated in the following way. The solar pair of states ($\nu_e$ is mainly in these states) yields the contribution:

$$m_{ee}^{(1+2)} \approx m_{ee}^{\text{sun}} = m^l_\beta (1 - |U_{e3}|^2 - |U_{e4}|^2)(\cos^2 \theta_\odot + e^{i\delta} \sin^2 \theta_\odot).$$  \hspace{1cm} (49)

The contributions from the two other states are

$$m_{ee}^{(3)} = m^l_\beta U_{e3}^2, \quad m_{ee}^{(4)} = m^h_\beta U_{e4}^2.$$  \hspace{1cm} (50)

In general these contributions may have arbitrary relative phases and cancel each other in the sum. The contribution from the third level, which belongs to the light set, is restricted by the CHOOZ bound: $m_{ee}^{(3)} < 0.05$ eV for $m^l_\beta < 1$ eV. In turn, the fourth contribution (from the isolated level) is restricted by the Bugey result: $m_{ee}^{(4)} < 0.05$ eV. The contribution of the
solar pair (49) depends on the solution of the solar neutrino problem and, in most of the cases, dominates over other contributions. Indeed, for the SMA solution we get $m_{\text{sun}}^{\text{ee}} \approx m_{l\beta}^l > 0.3$ eV (if masses of the light set are in the range of sensitivity of KATRIN experiment). For the LMA solution the cancellation of the two terms in (49) may occur but typically, $m_{\text{sun}}^{\text{ee}} \sim (0.2 - 1)m_{l\beta}^l > 0.1$ eV is large enough to be detected in the forthcoming $2\beta 0\nu$-decay experiments. For other solutions with large mixing angle (LOW, VO) the cancellation can be stronger. The solar pair contribution can be comparable with two others if the mixing is close to maximal: $\sin^2 2\theta_\odot > 0.98$ and the two terms in (49) have opposite signs.

So, in general we expect the effective mass of the Majorana neutrino to be $m_{\text{ee}} \sim 0.1$ eV, that is, not too far from the present experimental bound. The identification of the solution of the solar neutrino problem can clarify a situation leading to more definite predictions.

If the solar pair gives the dominant contribution, the measurements of $m_{\text{ee}}, m_{l\beta}^l$ and $\theta_\odot$ will allow to determine the relative phase of masses in the solar pair, $\delta$ (see eq. (49)). Otherwise, due to the presence of three different phases we cannot determine values of these phases, and $m_{\text{ee}}$ will give only a lower bound on the mass scale.

Assuming the maximal cancellation of contributions in $m_{\text{ee}}$ we find from (49) and (50), an implicit upper bound on $m_{l\beta}^l$ as a function of $\rho_e^h$:

$$|m_{l\beta}^l(1 - \rho_e^h)| \cos 2\theta_\odot| - \sqrt{(m_{l\beta}^l)^2 + \Delta m_{\text{LSND}}^2 \rho_e^h - m_{l\beta}^l(1 - |\cos 2\theta_\odot|)|U_{e3}|^2}| < m_{\text{ee}}. \quad (51)$$

We show this bound on $m_{l\beta}^l$ for different values of $\sin^2 2\theta_\odot$ in the fig. [7]. Note that for $\sin^2 2\theta_\odot \geq 0.95$, the effect of non-zero $|U_{e3}|^2$ (the last term in the left hand side of eq. (51)) is nonnegligible but it decreases with $\sin^2 2\theta_\odot$. In fig. [7] the line marked by “3+1” shows the $2\beta 0\nu$-decay bound for the (3 + 1) scheme at $|U_{e3}|^2=0.05$ and $\sqrt{\Delta m_{\text{LSND}}^2} =0.477$ eV. The lower line in the pair marked by “2+2” is the corresponding bound for the (3 + 1) scheme at $|U_{e3}|^2 = 0$. At $|U_{e3}|^2 = 0$ the bounds for the (2 + 2) and (3 + 1) schemes coincide (see below). The pair of lines marked by “2+2” illustrates dependence of bound on $\Delta m_{\text{LSND}}^2$. The upper line (a) is for $\sqrt{\Delta m_{\text{LSND}}^2} = 1.32$ eV while the lower one is for 0.477 eV. For smaller values of $\sin^2 2\theta_\odot$, the lines have been calculated at $|U_{e3}|^2=0$ and $\sqrt{\Delta m_{\text{LSND}}^2}=1.32$ eV.

2. (2 + 2) scheme. Now the heavy set contains two states, $\nu_3$ and $\nu_4$, and the solar pair is in the light set. The contribution to the Majorana mass, $m_{\text{ee}}$, from the light set is described by eq. (13). The heavy set gives

$$m_{\text{ee}}^{(3+4)} = m_{l\beta}^h |U_{e3}|^2 + e^{i\delta'}|U_{e4}|^2 | < m_{l\beta}^h \rho_e^h \quad (52)$$
which is restricted by the Bugey bound: \( m_{ee}^{(3+4)} < 0.04 \) eV for \( m_\beta^h < 0.1 \) eV. Again, the solar pair gives the dominating contribution unless the solar mixing is very close to maximal and the phase in \((49)\) is close to \(\pi\).

Assuming the maximal cancellation of contributions in \( m_{ee} \) we find from \((52)\) and \((49)\), an implicit upper bound on \( m_\beta^l \) as a function of \( \rho_h^e \):

\[
\left| m_\beta^l (1 - \rho_h^e) \cos 2\theta_\odot - \sqrt{(m_\beta^l)^2 + \Delta m_{LSND}^2 \rho_h^e} \right| < m_{ee}.
\]  

We show this bound on \( m_\beta \) for different values of \( \sin^2 2\theta_\odot \) in the fig. 7. The pair of lines marked by “2+2”, corresponds to two different values of \( \sqrt{\Delta m^2_{LSND}} \): 1.32 eV (upper line) and 0.477 eV (lower line). For other values of \( \sin^2 2\theta_\odot \), \( \sqrt{\Delta m^2_{LSND}} \) is taken to be 1.32 eV. Note that for the \((3+1)\) scheme with \( |U_{e3}|^2 = 0 \), the bounds from the \( 2\beta 0\nu \)-decay searches are the same as for the \((2+2)\) scheme.

6.2 Schemes with inverted order of states

The electron neutrino is mainly distributed in the heavy set, so that one expects a large kink at \( E_0 - m_\beta^h \) with the size close to 1 and the suppressed tail with the end point at \( E_0 - m_\beta^l \). In the tail the rate is restricted by the Bugey bound: \( \rho^e_\ell < 0.027 \). As for the case of the spectrum in the schemes with inverted mass hierarchy, we can conclude that \( \Delta m^2_{LSND} < 3 \) eV.2

In fig. 8, we show the bounds on the relevant beta decay parameters: \( m_\beta^h \), the effective mass of the heavy set, and \( (1 - \rho_h^e) \) which determines small admixture of the electron neutrino in the light set. Clearly,

\[
m_\beta^l = \sqrt{(m_\beta^h)^2 - \Delta m^2_{LSND}}
\]

and

\[
m_\beta^h \geq \sqrt{\Delta m^2_{LSND}}.
\]

We show the bounds for two representative values of \( \sqrt{\Delta m^2_{LSND}} \): 1.32 and 0.447 eV (as for the scheme with normal ordering). The allowed ranges for \( (1 - \rho_h^e) \) determined by the oscillation experiments are the same as the ranges of \( \rho_h^e \) for normal ordering (fig. 8). The allowed regions for the \((3 + 1)\) scheme are shadowed. The allowed values of \( m_\beta^h \) are restricted by the Mainz limit from above and by the LSND result from below (see eq. \((53)\)).

Let us consider the bounds from the \( 2\beta 0\nu \)-decay searches. The effective Majorana mass can be immediately obtained from the results for the scheme with normal ordering by the
interchange: $m^h_\beta \leftrightarrow m^l_\beta$. This leads to a stronger dominance of the solar pair contribution to the effective Majorana mass.

1. (2 + 2) scheme. The contribution from the heavy set is given by

$$m_{ee}^h \approx m_{ee}^{\text{sun}} = m^h_\beta (1 - |U_{e3}|^2 - |U_{e4}|^2)(\cos^2 \theta_\odot + e^{i\delta} \sin^2 \theta_\odot),$$

(56)

whereas the contribution from the light states can be written as

$$m_{ee}^l = m^l_\beta |U_{e1}^2 + e^{i\delta'} U_{e2}^2| < m^l_\beta \rho_e^l,$$

(57)

and according to the Bugey result: $\rho_e^l < 0.027$. Implications of the $2\beta0\nu$ searches are similar to those in the hierarchical case. However now $m^h_\beta$, and consequently $m_{ee}$, can be even larger than $\sqrt{\Delta m_{\text{LSND}}^2}$.

From eqs. (56, 57) we find the lower bound on $m_{ee}$:

$$m_{ee} > m^h_\beta (1 - \rho_e^l) \cos 2\theta_\odot| - m^l_\beta \rho_e^l.$$ 

(58)

Using this inequality and the upper experimental bound on $m_{ee}$, we get an implicit upper bound on $m^h_\beta$ as a function of $\rho_e^h$:

$$m^h_\beta \rho_e^h | \cos 2\theta_\odot| - \sqrt{(m^h_\beta)^2 - \Delta m_{\text{LSND}}^2(1 - \rho_e^h)} \leq m_{ee}^\text{max} = 0.34 \text{ eV}.$$ 

(59)

The bounds for different values of the solar mixing parameter are shown in fig. 8. The identification of the solution of the solar neutrino problem and measurements of $\sin^2 2\theta_\odot$ as well as mild improvement of the bound on the Majorana mass will have strong impact on this scheme. For instance, as follows from the fig. 8, the possible bounds: $\sin^2 2\theta_\odot < 0.9$ and $m_{ee} < 0.1 \text{ eV}$ would exclude whole the region of parameters of the scheme down to the KATRIN sensitivity limit.

2. (3 + 1) scheme. The contributions to $m_{ee}$ from the heavy and the light sets are equal to:

$$m_{ee}^h = m^h_\beta \left[ (1 - |U_{e3}|^2 - |U_{e4}|^2)(\cos^2 \theta_\odot + e^{i\delta} \sin^2 \theta_\odot) + U_{e3}^2 \right]$$

(60)

and

$$m_{ee}^l = m^l_\beta (1 - \rho_e^h),$$

(61)

respectively. In (60) $|U_{e3}|^2$ is restricted by the CHOOZ results and $m_{ee}^l$ is restricted by the Bugey results: $m_{ee}^l \lesssim 0.03 \text{ eV}$ taking $m^l_\beta \leq 1 \text{ eV}$. The contribution from the heavy set is
similar to the one in the scheme with three degenerate neutrinos. For the largest part of the allowed parameter space the contribution of the solar pair to $m_{ee}$ dominates. Still significant cancellation is not excluded which can cause the two contributions in \( (60) \) (from the solar pair and the third mass eigenstate) to be comparable. Note that when $|U_{e3}|^2$ is smaller than 0.015 – 0.05, $m_{ee}'$ can be as large as $m_3^h|U_{e3}|^2$.

For the SMA solution we have $m_{ee} > 0.6$ eV for $m_1 > 0.3$ eV, so that such a possibility is excluded by the present bound \( \text{(2)} \).

Assuming the maximal cancellation of contributions in $m_{ee}$, we find from \( (60) \) and \( (61) \) an implicit upper bound on $m_3^h$ as a function of $\rho_e^h$:

\[
m_3^h \rho_e^h \cos 2\theta_{\odot} - \sqrt{(m_3^h)^2 - \Delta m^2_{\text{LSND}}(1 - \rho_e^h) - m_3^h(1 - |\cos 2\theta_{\odot}|)|U_{e3}|^2} < m_{ee}.
\]

We show this bound on $m_3^h$ for different values of $\sin^2 2\theta_{\odot}$ in the fig. 8. Note that for $\sin^2 2\theta_{\odot}=0.95$, the effect of non-zero $|U_{e3}|^2$ is non-negligible but for smaller values of $\sin^2 2\theta_{\odot}$ we can neglect $|U_{e3}|^2$. In the fig. 8 for $\sin^2 2\theta_{\odot}=0.95$, we have taken $|U_{e3}|^2=0.05$ and for other values of $\sin^2 2\theta_{\odot}$ we have set $|U_{e3}|^2$ equal to zero.

6.3 4ν- schemes without LSND

Apart from the LSND result, there is a number of other motivations to introduce new neutrino mass eigenstates. In particular, the sterile neutrino in the eV-range has been discussed in connection to the supernova nucleosynthesis \( (r\text{-processes}) \) \( [31] \). The mixing of the keV-mass sterile neutrino with the active neutrinos can provide a mechanism of the pulsar kicks \( [32] \). The keV sterile neutrinos may compose the warm dark matter of the Universe \( [33] \). Small mixing of the sterile neutrino with the active neutrinos can induce the large mixing among the active neutrinos \( [34] \).

Light \( (SU(2) \times U(1)) \) singlet fermions ("sterile neutrinos") can originate from some new sectors of the theory beyond the standard model. Neutrinos, due to their neutrality are unique particles which can mix with these fermions. So, searches for the effects of the sterile neutrinos are of fundamental importance even if these fermions do not solve directly any known problem and thus their existence is not explicitly motivated.

In this connection we will consider a general four neutrino scheme in which three (dominantly active) neutrinos are light and the fourth neutrino (dominantly sterile) has a mass in the eV - keV range. The three light neutrinos may form a hierarchical structure with the heaviest component being below 0.07 eV, so that their masses will not show up in the plan-
ning beta decay experiments. The fourth neutrino has a small mixing with active neutrinos, and in particular, with the electron neutrino.

The scheme is similar to the (3 + 1) schemes with normal mass hierarchy. The difference is that now the mass $m_4$ and the mass squared differences $m_4^2 - m_i^2$ ($i = 1, 2, 3$) are not restricted by the LSND result so that $m_4$ can be larger or much larger than 2.5 eV.

Let us consider the possible effect of this fourth neutrino in the beta decay. We concentrate on the range of masses $\sim (0.5 - 5)$ eV which satisfy the cosmological bounds. The fourth state produces the kink in the beta decay spectrum at $E = E_0 - m_4$ with the size

$$\rho_e = |U_{e4}|^2.$$  \hfill (63)

Let us evaluate the allowed range of $\rho_e$ for different values of $m_4$. At $m_4 > 1.5$ eV the strongest bound follows from the CHOOZ result: $\rho_e < 0.027$ (we assume that other neutrinos are much lighter, but generalization to the non-hierarchical case is straightforward). This bound does not depend on the mass for $m > 1.6$ eV.

Another direct restriction comes from the $2\beta 0\nu$-decay searches provided that neutrino is the Majorana particle:

$$m_4 < \frac{m_{ee}^{\text{max}}}{\rho_e},$$ \hfill (64)

where $m_{ee}^{\text{max}}$ is the upper bound on the effective Majorana mass of the electron neutrino. Taking the present bound $m_{ee}^{\text{max}} = 0.34$ eV and the maximal allowed value of $\rho_e$ we find that $m_4 < 12.6$ eV. Thus, one may see the kink of the 3% size (or smaller) in the energy interval $(0 - 13)$ eV below the end point. The recent cosmological bound \footnote{1} shrinks substantially this interval.

Additional restrictions on the possible effects appear if there is a substantial admixture of the muon flavor in the fourth state. In this case one predicts the existence of the $\nu_\mu - \nu_e$ oscillations with the effective mixing parameter

$$\sin^2 2\theta_{e\mu} = 4|U_{e4}|^2|U_{\mu4}|^2$$ \hfill (65)

and $\Delta m^2 \approx m_4^2 = (1 - 100)$ eV$^2$. For $\Delta m^2 > 7$ eV$^2$ the stronger bound, $\sin^2 2\theta_{e\mu} < 1.3 \cdot 10^{-3}$, is given by KARMEN experiment \footnote{35}. Clearly, these bounds are satisfied, if $|U_{\mu4}|^2$ is small enough. However, if $|U_{\mu4}|^2 \geq |U_{e4}|^2$ (which might be rather natural assumption) the bound from KARMEN experiment becomes important. Taking $|U_{\mu4}|^2 = |U_{e4}|^2$ we get from \footnote{35}

$$\rho_e < \frac{1}{2} \sqrt{\sin^2 2\theta_{e\mu}},$$ \hfill (66)

and for $m_4^2 > 7$ eV$^2$, it follows from \footnote{35} that $\rho_e < 1.7 \cdot 10^{-2}$ which is stronger than the Bugey bound. For heavy neutrinos (in keV range) the neutrinoless double beta decay gives
very strong bound on the size of the kink: \( \rho_e < 3 \cdot 10^{-4} (1\text{keV}/m_4) \) which will be very difficult to observe. This bound does not exist if the keV neutrino is the Dirac (or pseudo-Dirac) particle.

7 Beta decay measurements and the neutrino mass spectrum

In this section we consider the astrophysical and cosmological bounds on the neutrino mass. We will discuss possible future developments in the field.

7.1 Beta decay measurements and supernova neutrinos

Studies of the supernova neutrinos open unique possibility to test the schemes of neutrino mass and mixing \([45]\). Therefore they may have an important impact on predictions for future beta decay measurements.

Considering the level-crossing patterns \([45]\) and the adiabaticity conditions in various resonances it is easy to show that in all the \((2 + 2)\) schemes with inverted mass hierarchy (or ordering of the states) the originally produced \(\bar{\nu}_e\)-flux is almost completely converted to some combination of \(\bar{\nu}_\mu, \bar{\nu}_\tau\) and \(\bar{\nu}_\tau\)-fluxes at high densities in the resonance associated to \(\Delta m^2_{LSND}\). The mixing parameter in this resonance, given by \(\sin^2 2\theta_{LSND}\), is large enough to guarantee the adiabaticity of the conversion. In this case the \(\bar{\nu}_e\) survival probability equals to

\[
P = \sin^2 2\theta_{LSND} < 10^{-2}.
\]

At the same time, the \(\bar{\nu}_e\)-flux observed at the Earth appears as a result of conversion

\[
\bar{\nu}_\mu, \bar{\nu}_\tau \rightarrow \bar{\nu}_e
\]

at high densities inside the star. Therefore the spectrum at the Earth will practically coincide with the hard original spectrum of \(\bar{\nu}_\mu\) and \(\bar{\nu}_\tau\):

\[
F_{\bar{\nu}_e}(E) \approx F_{\bar{\nu}_\mu}(E), \quad (68)
\]

and moreover, this result does not depend on the solution of the solar neutrino problem. Such a hard spectrum of \(\bar{\nu}_e\) is strongly disfavored by the SN1987A data \([46]\). Future detections of the Galactic supernovae can exclude the conversion \((67)\), and consequently the schemes will be excluded, completely.

In the \((3+1)\) scheme with inverted mass hierarchy (ordering) the result of conversion depends on the solution of the solar neutrino problem \([30]\). As in the \((2+2)\) scheme, the
original $\bar{\nu}_e$-flux is converted to a combination of $\bar{\nu}_\tau$ and $\bar{\nu}_s$-fluxes. For the SMA solution no opposite conversion (that is, $\bar{\nu}_\tau$ and $\bar{\nu}_\mu$ to $\bar{\nu}_e$) occurs. Therefore, in this scheme the transitions lead to practically complete disappearance of the $\bar{\nu}_e$-flux. The suppression factor, $\sin^2\theta_{LSND} < 10^{-2}$, can not be compensated by the allowed increase of the original flux. The disappearance of $\bar{\nu}_e$ contradicts the data from SN1987A, so that the scheme is excluded. Notice that this scheme is also practically excluded by the present bound from the neutrinoless double beta decay.

In the case of the LMA solution some part of the original $\bar{\nu}_\mu$- and $\bar{\nu}_\tau$- fluxes will be transformed to the $\bar{\nu}_e$- flux at low densities, so that at the surface of the Earth one expects:

$$F_{\bar{\nu}_e}(E) \approx \sin^2\theta_{\odot} F_{\bar{\nu}_\mu}^0(E).$$

(69)

Thus, $\bar{\nu}_e$ will have hard spectrum suppressed by factor $1/3 - 1/2$. This is again disfavored by the SN1987A data.

Notice that in all these schemes the electron neutrino flux is not eliminated from the region proposed for the $r$-processes [31]. So that the mechanism of production of the heavy elements will not work.

In contrast, the non-hierarchical 4$\nu$-schemes with normal ordering are well consistent with the SN1987A data and they predict an observable effect in the $\beta$-decay spectrum, as was discussed in sect. 6.

### 7.2 Beta decay and forthcoming experiments

The results of the forthcoming oscillation as well as non-oscillation experiments can substantially influence the both predictions of the effects of neutrino mass and mixing in the beta decay spectrum and the significance of future beta decay measurements. In particular,

(i) the identification of the solution of the $\nu_{\odot}$-problem and measurements of relevant oscillation parameters,

(ii) the MiniBooNE result,

(iii) further searches for the neutrinoless double beta decay

will have crucial impact. Also further improvements of the bound on $|U_{e3}|$ will be important. Cosmology can give a hint for the absolute scale of neutrino mass.

Let us analyze consequences of possible results from these experiments.

1). The solution of the solar neutrino problem can be identified in the forthcoming experiments: SNO [36, 37], KamLAND [38], BOREXINO [39]. The identification will not influence the predictions for the $\beta$-decay immediately. Indeed, from these experiments we
will get specific values (ranges) of $\Delta m^2_\odot$ and mixing angle $\theta_\odot$, i.e., the distribution of the electron flavor in the solar pair of states will be determined. But, the effects in the $\beta$-decay are not sensitive to a particular value of $\Delta m^2_\odot$, since for all possible solutions $\Delta m^2_\odot$ can not be resolved in $\beta$-decay searches. Also the effects are not sensitive to the distribution of the electron flavor since they are determined by the sum over states in the solar pair: $|U_{e1}|^2 + |U_{e2}|^2 \sim 1$. However, the identification of the solution of the solar neutrino problem will influence substantially the bounds on the $\beta$-decay effects from the $2\beta0\nu$-decay searches. A number of schemes discussed here will be excluded and for other schemes the possible effects in the $\beta$-decay spectrum will be strongly restricted.

The key issues are whether the correct solution of the solar neutrino problem is the small mixing solution or the large mixing solution, and if it is the large mixing how large the deviation from maximal mixing is.

Suppose that the SMA solution will be identified, then the following information can be obtained from the $2\beta0\nu$-decay searches in the assumption that the Majorana neutrino exchange is the only mechanism of the decay:

- For the $3\nu$-scheme this will imply that $m_\beta < 0.34$ eV and further moderate improvement of the $2\beta0\nu$-decay bound will exclude the scheme.

- According to present data: $\sqrt{\Delta m^2_{LSND}} > 0.38$ eV (at 99 % C.L.) [10]. Therefore in the schemes with inverted mass hierarchy or inverted order of states we get $m_{ee} \geq \sqrt{\Delta m^2_{LSND}} > 0.38$ eV. On the other hand, the bound from the $2\beta0\nu$-decay is $m_{ee} < 0.34$ eV (at 90 % C.L.). Therefore these schemes are excluded at stronger than 90 % C.L. One should, however, keep in mind the uncertainties of the nuclear matrix elements. Future double beta decay measurements will be able to confirm and improve the bound. Similar conclusion can be made for non-hierarchical schemes with normal ordering of states and $m_l > 0.34$ eV.

Thus, the schemes which will survive after the identification of the SMA solution, are the schemes with normal hierarchy as well as the non-hierarchical schemes with normal order of states and $m_l < 0.34$ eV.

- If $m_{ee}$ turns out to be close to the present bound (e.g., $\sim 0.2$ eV) and the LSND result is confirmed, the only possibility will be the non-hierarchical scheme with normal ordering of states and $m_l = m_{ee}$.

Suppose now that one of the large mixing solutions of the solar neutrino problem will
be identified. In this case a possible cancellation between various contributions to $m_{ee}$ will relax the bounds from the $2\beta 0\nu$-decay.

- The important point is that the present upper bound on $m_{ee}$ is already smaller than $\sqrt{\Delta m^2_{LSND}}$:

$$m_{ee}^{\text{max}} < \sqrt{\Delta m^2_{LSND}}$$

(although, one should keep in mind the uncertainties of the nuclear matrix element).

At the same time, $m^h_\beta > \sqrt{\Delta m^2_{LSND}}$. Consequently, the schemes with inverted mass hierarchy or inverted ordering of states require cancellation between the contributions from the solar pair states, and therefore, there should be CP-violating phase difference in the corresponding mass eigenvalues. The scheme with the SMA solution of the solar neutrino problem is practically excluded.

The upper bound on $\sin^2 2\theta_\odot$ (lower bound on the deviation from maximal mixing) restricts a possible cancellation of contributions to $m_{ee}$. This, in turn, gives an upper bound on the mass $m^h_\beta$. Therefore, further improvements of the bounds on $m_{ee}$ and $\sin^2 2\theta_\odot$ can strongly restrict the parameter space of the schemes and even exclude them.

Similar conclusions hold for the non-hierarchical schemes with $m^l_\beta > m_{ee}^{\text{max}} \sim 0.34$ eV.

- As we discussed in section 5, the Majorana mass $m_{ee}$ gives the lower bound on the mass of the solar pair. So, if $m_{ee}$ turns out to be close to the present bound (4), we can exclude the schemes with normal hierarchy (in which $m_{ee} < m^l < 0.07$ eV).

- If $m_{ee}$ is much smaller than the present bound (e.g., $\sim 0.01$ eV) the scheme should have the normal mass hierarchy or should lead to a strong cancellation of contributions to the $2\beta 0\nu$-decay.

2). MiniBooNE experiment will give strong discrimination among the possibilities.

If MiniBooNE does not confirm the LSND result, a large class of schemes discussed here will be excluded. There will be no strong motivation to consider the $4\nu$ schemes (see, however, sec. 6.3).

Also further improvements of bounds on the involvement of a sterile neutrino in the solar and in the atmospheric neutrino conversions may give independent confirmation of the $3\nu$-schemes.
We will be left with the three neutrino scheme with strong degeneracy or with schemes having more than three mass eigenstates without observable signal in MiniBooNE (see sect. 6).

So, in this case the searches of the neutrino mass in the sub-eV range will basically test the $3\nu$ scheme with strong degeneracy. The observation of the shift of end point to $E_0 - m_\beta$ in KATRIN experiment will be the proof of the scheme with $m_1 \simeq m_2 \simeq m_3 \simeq m_\beta$.

Further insight can be obtained confronting the results of the $\beta$-decay and the $2\beta0\nu$ decay measurements, as we have discussed in sect. 3.

If MiniBooNE confirms the LSND result we will be forced to consider the $4\nu$-schemes. In this case we get for the absolute value of the mass of heavy set:

$$m_\nu \geq \sqrt{\Delta m_{LSND}^2},$$

where the equality corresponds to the scheme with the mass hierarchy ($m^h_\beta \gg m^i_\beta$).

MiniBooNE experiment will not only check the LSND result but also further restrict the oscillation parameters. Moreover, it may allow to disentangle the (3+1) and (2+2) schemes. Further searches of the sterile neutrinos in the solar and atmospheric neutrino fluxes should discriminate the (2+2) and (3+1) schemes [30].

3). Let us consider implications of future cosmological measurements. The present and future cosmological bounds on the mass scale of neutrinos are summarized in the table 1. The bound on $m_\nu$ taken from [1] corresponds to the energy density of matter $\Omega_m = 0.4$ and the reduced Hubble constant $h = 0.8$.

Note that by chance the best cosmological bound [1] coincides numerically with the best laboratory limit [5]. However, in contrast to the latter, the cosmological bound is valid for any flavor including the sterile neutrino, provided that this neutrino had been equilibrated in the Early Universe.

The bound [1] (obtained for one neutrino in the eV range) applies immediately to the (3+1) scheme with normal hierarchy. In this scheme the heaviest (isolated) state can produce only a small kink in the $\beta$-decay spectrum which will be difficult to detect. So the cosmological bound being confronted with the value of $\sqrt{\Delta m_{LSND}^2}$ will play important role in checking the scheme. The bound is even stronger for the non-hierarchical (3+1) schemes.

The bound is also immediately applied to the schemes with additional heavy neutrino (sect. 6.3). Even for very small admixture of the electron neutrino in this state ($|U_{e4}|^2 \ll 0.05$) this, predominantly sterile, neutrino will have equilibrium concentration in the Universe.

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Table 1: Cosmological bounds on the neutrino mass scale. All the four-neutrino schemes are considered to be hierarchical.

| Mass scheme          | present $m_\nu$ eV; ref. | future $m_\nu$ eV, ref. |
|----------------------|---------------------------|-------------------------|
| 3 $\nu$ or (3+1)    | 1.8 [41]                  | 0.41 [12]               |
| inverted             | 2.5                       |                         |
| (2+2) inverted       | 3.0 [13]                  | 0.57 [12]               |
| or normal            | 3.8                       |                         |
| (3+1) normal         | 5.5 [13]                  | 0.99 [12]               |
|                      | 7.6                       |                         |
|                      | 2.2                       |                         |

The bound on mass of two or three heavy degenerate neutrinos will be stronger than (1). However, the decrease of the limit is weaker than just $n^{-1}$, where $n$ is the number of the degenerate neutrinos. We estimate the bounds for two and three neutrinos performing rescaling of the bound (1) according to results for 1, 2, and 3 neutrinos in [12]. Thus, for two degenerate neutrinos we get from [13] and [2] $m < 1.3$ eV. In the $(2 + 2)$ schemes with normal mass hierarchy or with normal ordering this limit excludes the upper “island” of parameters allowed by LSND (see fig. 4). In the case of inverted hierarchy it confirms the Mainz result.

For the three degenerate neutrinos we find $m < 0.9$ eV. This bound excludes significant parts of the otherwise allowed regions of the $3\nu$-scheme and the $(3 + 1)$ schemes with inverted hierarchy or order of the states.

Forthcoming cosmological data can further substantially improve the bounds. In the last column of the table 1 we show the bounds which can be obtained using data from the Sloan Digital Sky Survey (SDSS) [12] for $\Omega_m h^2 < 0.17$.

### 7.3 Beta decay measurements and the neutrino mass spectrum

The identification power of the $\beta$-decay studies depends on whether future measurements will be able to observe the small kink and the tail after large kink or not.

If the $\beta$-decay measurements are not able to identify the “small” kinks and the suppressed tail, the only expected distortion effect is a shift of the end point and the corresponding bending of the spectrum.

Suppose that the LSND result will not be confirmed and the effects of sterile neutrinos will not be found neither in the solar, nor in the atmospheric neutrino fluxes. This will
be the evidence for the three neutrino scenario. (The existence of additional sterile neutrino which mixes weakly with the block of active neutrinos does not change the conclusion.) As we have discussed, in this case the $\beta$-decay parameter, $m_\beta$, will determine the scale of three degenerate neutrinos.

If the LSND result is confirmed and $\Delta m^2_{LSND}$ is measured, the important conclusions will be drawn from the comparison of the values of $m_\beta$ and $\sqrt{\Delta m^2_{LSND}}$.

Let us remind that experiments which are insensitive to the small kinks and tails will find $m_\beta = 0$ for the schemes with normal mass hierarchy, and $m_\beta = \sqrt{\Delta m^2_{LSND}}$ for the schemes with inverted mass hierarchy. Any type of relation is possible for the non-hierarchical scheme with normal ordering of states: (i) $m_\beta < \sqrt{\Delta m^2_{LSND}}$; (ii) $m_\beta > \sqrt{\Delta m^2_{LSND}}$ in the case of relatively small $\Delta m^2_{LSND}$ or (iii) $m_\beta \approx \sqrt{\Delta m^2_{LSND}}$ (which looks as an accidental coincidence since this will imply the equality $m^l = \sqrt{(m^h)^2 - (m^l)^2}$). For the non-hierarchical schemes with inverted order one has $m_\beta > \sqrt{\Delta m^2_{LSND}}$.

Therefore:

- A negative result of the neutrino mass measurements will be the evidence of the scheme with normal hierarchy or the non-hierarchical scheme with normal order and $m^l$ below the sensitivity limit: $m^l < 0.3$ eV (see eq. (10)).

- If it is established that $m_\beta < \sqrt{\Delta m^2_{LSND}}$, the non-hierarchical scheme with normal order of states should be selected. Moreover, the mass scales will be completely determined: $m^l = m_\beta$, and $m^h = \sqrt{m^2_\beta + \Delta m^2_{LSND}}$.

- The inequality $m_\beta > \sqrt{\Delta m^2_{LSND}}$ will testify for the non-hierarchical scheme with inverted order of states. In this case: $m^h = m_\beta$ and $m^l = \sqrt{m^2_\beta - \Delta m^2_{LSND}}$. The inequality can correspond also to the non-hierarchical scheme with normal order, so that $m_\beta = m^l$ and $m^h = \sqrt{m^2_\beta + \Delta m^2_{LSND}}$.

- If $m_\beta$ coincides (within the error bars) with $\sqrt{\Delta m^2_{LSND}}$, one of the following possibilities will be realized:
  a) the schemes with inverted mass hierarchy and $m_\beta \simeq m_h \simeq \sqrt{\Delta m^2_{LSND}}$,
  b) the non-hierarchical scheme with inverted order of levels and relatively small mass of the light set, $m^l$, so that $m^h$ is only slightly (within the error bars) larger than $\sqrt{\Delta m^2_{LSND}}$.
  c) non-hierarchical schemes with normal order of levels and $m_\beta = m^l$. In this case the equality $m^l \sim \sqrt{\Delta m^2_{LSND}}$ is accidental.
Recently, possible implications of results from LSND and KATRIN experiments have been discussed also in [47].

If future $\beta$-decay experiments detect the “small” kinks and the suppressed tail, we will be able to measure $\rho_e^h$ and $\rho_e^l$ and unambiguously discriminate the hierarchical schemes from the non-hierarchical ones and the inverted scenarios from the normal ones even without using the LSND result. This will be independent test of the 4$\nu$-scheme. Indeed:

- The spectrum with the small kink at $m_\beta$ and the tail without bending, will be the evidence of the scheme with normal mass hierarchy.

- The spectrum with the large kink at $m_\beta$ and the tail without bending will testify for the schemes with inverted mass hierarchy.

- The spectrum with the small kink and the bending at higher energies will correspond to the non-hierarchical scheme with normal order of the states.

- The spectrum with the large kink and the suppressed tail with shifted end point (the latter will probably be impossible to establish) will indicate the non-hierarchical scheme with inverted order of states.

Notice that all these results are the same for the (3+1) and (2+2) schemes. The study of the $\beta$-decay spectrum cannot distinguish these schemes. The (3+1) and (2+2) schemes can be discriminated by studies of effects of the sterile neutrinos in the solar and atmospheric neutrino oscillations, by MiniBooNE experiment, by searches for the oscillations in the $\nu_e$ and $\nu_\mu$ disappearance channels, etc.

The detection of the small kinks will also open a possibility to search for the mixing of the sterile neutrinos which are not associated with the LSND result.

### 7.4 Measuring the absolute mass scale

Without direct kinematic measurements, the absolute scale of neutrino mass can be established only in certain exceptional cases. This will be possible if solar neutrino data are explained by the SMA solution and the $2\beta0\nu$-decay is observed. In this case $m_{ee}$ will give the mass of the solar pair: $m_{\text{sun}} = m_{ee}$. Then using the oscillation results one can reconstruct the whole spectrum.

In the 3$\nu$-scheme $m_{ee}$ will immediately determine the mass of all the three quasi-degenerate neutrinos.
In 4ν-schemes the mass reconstruction will depend on the type of the scheme:

In the scheme with normal mass hierarchy \( m_{ee} \) is expected to be small: \( m_{ee} \leq \sqrt{\Delta m_{atm}^2} < 0.07 \text{ eV} \). So, if a small \( m_{ee} \) is detected or a strong bound on \( m_{ee} \) is obtained, the mass of the heavy set will be given by the LSND result: \( m^h \approx \sqrt{\Delta m_{LSND}^2} \). Establishing the scale of light masses still will be rather problematic. Moreover, if the \( 2\beta 0\nu \)-decay searches give only the upper bound on \( m_{ee} \), we should assume that neutrinos are the Majorana particles to make conclusion on the mass.

In the non-hierarchical scheme with normal ordering of states, \( m_{ee} \) can be as large as the present upper bound. Thus, \( m_{ee} \) will determine the mass of the light set: \( m^l = m_{ee} \), and for the heavy set we find \( m^h = \sqrt{m_{ee}^2 + \Delta m_{LSND}^2} \).

The schemes with inverted mass hierarchy or inverted ordering are almost excluded by the fact that already present data indicate inequality \( (m_{ee}^{max})^2 < \Delta m_{LSND}^2 \).

For the large mixing solutions of the solar neutrino problem (LMA, LOW, VAC) the absolute mass scale can not be restored from \( m_{ee} \) due to possible cancellation which depends on unknown CP-violating phase. Inversely, the data from the \( \beta \)-decay measurements can be used to determine this phase. If \( m_{ee} \) is measured, we will be able to put both the lower and the upper bounds on the absolute scale of masses: \( m_\beta \leq m_{ee} \cos 2\theta_\odot \) and \( m_\beta > m_{ee} \).

Even in those special cases where the determination of the absolute scale is possible there are two problems:

- Uncertainties of the nuclear matrix elements will lead to significant uncertainty in the determination of the absolute scale.

- We should assume that the exchange of the light Majorana neutrinos is the only mechanism of the \( 2\beta 0\nu \)-decay.

In view of this, and keeping also in mind that the SMA solution is disfavored by the latest solar neutrino data, we can conclude that developments of the direct methods of determination of the neutrino mass (and KATRIN may be only the first step) is unavoidable if we want to reconstruct neutrino mass spectrum completely.

8 Conclusions

1. We have studied effects of the neutrino mass and mixing on the \( \beta \)-decay spectrum in three neutrino schemes which explain the solar and atmospheric neutrino data as well as in all possible 4ν-schemes which explain also the LSND result.
We find that schemes which can produce an observable effect in the planned sub-eV measurements should contain the sets (one or more) of quasi-degenerate states. The only exception is the \((3 + 1)\) scheme with normal mass hierarchy. However it leads to a small kink which will be difficult to observe. We show that the effects in the \(\beta\)-decay spectrum are described by the effective masses of the quasi-degenerate sets, \(m_{\beta}^{(q)}\), and their coupling with the electron neutrino, \(\rho_{\beta}^{(q)}\).

2. At present, a rather wide class of realistic schemes exist which can lead to an observable effect in the sub-eV studies of the \(\beta\)-decay spectrum. We show however that future oscillation experiments and \(2\beta0\nu\)-decay searches can significantly restrict these possibilities.

3. The three neutrino schemes which explain the solar and atmospheric neutrino anomalies in terms of neutrino oscillations can lead to an observable effect in future \(\beta\)-decay measurements only if all three mass eigenstates are quasi-degenerate. The \(\beta\)-decay measurements give unique possibility to identify these schemes. Even if SMA solution is established and the neutrinoless double beta decay is discovered, so that \(m_{ee}\) sets the scale of the neutrino masses, the question will rise whether the neutrino exchange is the only mechanism of the neutrinoless double beta decay. Only comparison of the \(m_{ee}\) with results of the beta decay measurements will give the answer. In the case of large mixing solutions of the solar neutrino problem, simultaneous measurements of the \(m_{ee}\) and \(m_{\beta}\) as well as the solar mixing angle open the possibility to determine the relative CP-violating phase of the mass eigenvalues.

4. In the four neutrino schemes which explain also the LSND result, observable effects in the \(\beta\)-decay are strongly restricted by the Bugey and CHOOZ bounds. Four types of spectrum distortion are expected depending on the type of mass hierarchy (ordering of levels) in the scheme:

a). The spectrum with the large kink and suppressed tail above the kink. This type of distortion realizes in the scheme with inverted mass hierarchy.

b). The spectrum with small kink. This type of distortion is expected in the scheme with normal mass hierarchy.

c). The spectrum with small kink and “strong” bending at the shifted end point. Such a distortion corresponds to the non-hierarchical spectrum with normal ordering of states.

d). Spectrum with large kink and strongly suppressed tail above the kink and a shift of end point. This type of distortion realizes in the non-hierarchical scheme with inverted order of states.

The rates in the suppressed tails and sizes of small kinks are determined by \(\rho_{\beta}^h\) or \((1 - \rho_{\beta}^h)\) and the latter quantities are restricted by the Bugey or CHOOZ bounds: \(\rho_{\beta}^h \leq 0.027\).
The lower bound on $\rho_h^e$ appears from the LSND result. This bound is close to the Bugey upper bound in the (3+1) schemes and it can be much weaker for the (2+2) schemes.

The 4$\nu$-schemes with inverted mass hierarchy are disfavored by the data from SN1987A, leading to a hard spectrum of $\bar{\nu}_e$.

5. The identification power of the $\beta$-decay measurements will depend on the possibility to detect the suppressed tail and the small-size kinks in the spectrum. Even if small kinks or suppressed tails are unobservable, the important conclusions can be drawn from comparison of the values of $m_\beta$ and $\sqrt{\Delta m^2_{LSND}}$. This will allow one to identify the type of mass hierarchy (in 4$\nu$-schemes) and also to distinguish the hierarchical from non-hierarchical schemes. Note that if the small kink or suppressed tail are unobservable, the effect expected from the presently favored schemes of neutrino masses is the same as the effect of the electron neutrino with definite mass.

Observations of the small kinks or suppressed tails will allow us to measure the mixing parameters $\rho_h^e$ and $\rho_l^e$ and to make the independent identification of the scheme. The $\beta$-decay measurements can also distinguish the three and four neutrino schemes. However, this can be done by MiniBooNE and other experiments even before new $\beta$-decay results will be available.

6. Even if the LSND result is not confirmed, there are some motivations to search for the kinks in the energy interval $(1-10)$ eV below the end point. The kinks can be due to mixing of the active neutrinos with the light singlet fermions which originate from some other sectors of theory beyond the Standard model.

7. The important conclusions can be drawn from the combined analysis of results of the $\beta$-decay measurements, $2\beta0\nu$-searches and the identification of the solution of the solar neutrino problem.

8. Negative results of future $\beta$-decay experiments will have a number of important implications. In particular,

- Large part of the parameter space of the 3$\nu$-schemes with degenerate mass spectrum will be excluded;
- If the LSND result is confirmed, and the bound from the beta decay is $m_\beta < \sqrt{\Delta m^2_{LSND}}$, the schemes with inverted hierarchy (order) as well as the schemes with normal order and $m' > \sqrt{\Delta m^2_{LSND}}$ will be excluded.

If we want eventually to know the whole “story about neutrinos” we should measure the absolute values of their masses. From the point of view of implications for the fundamental
theory, for astrophysics and cosmology, the masses are at least as important as other neutrino parameters such as the mixing angles and CP-violating phases. As follows from our study, to reconstruct the absolute values of masses unambiguously and without additional assumptions one needs almost unavoidably to develop and to perform new direct (kinematic) measurements (or look for some new alternatives). The KATRIN experiment may be just the first step. There is no reason, why we should devote less time, effort and resources for determination of the absolute scale of neutrino mass than, e.g., we are going to devote to measurements of the oscillation parameters and the CP-violating phases.

Appendix: Effective mass of the set of quasi-degenerate states

The effect of the set of quasi-degenerate states on the $\beta$-decay spectrum can be described by the effective mass, $m_\beta$, and the coupling $\rho_e$ of the set with the electron neutrino. Let us evaluate the accuracy of such an approximation. The error is maximal in the energy interval close to $E_0 - m_i$, where $m_i$ is the mass of the lightest state from the quasi-degenerate set. Let us compare number of events (decays) due to states from this set in the interval: $(E_0 - m_i - \Delta E) - (E_0 - m_i)$, using (1) the effective parameters $\rho_e$ and $m_\beta$: $\Delta n(\rho_e, m_\beta)$, and (2) the precise parameters of states: $\Delta n(U_{ei}, m_i)$. Here $\Delta E$ is the energy interval which can be resolved by the detector. Let us calculate

$$R \equiv \frac{\Delta n(\rho_e, m_\beta) - \Delta n(U_{ej}, m_j)}{\Delta n(\rho_e, m_\beta)}.$$ 

Since in the energy range that we are interested, $F(E, Z)$ and $p$ in (7) are smooth functions of energy we can factor out them and estimate the relative error as follows

$$R \approx 1 - \frac{\sum_j |U_{ej}|^2 \int_{E_0 - m_i}^{E_0 - (E_0 - E) - (E_0 - E)^2 - m_\beta^2} \Theta(E_0 - E - m_j) dE}{\int_{E_0 - m_i}^{E_0 - E} \rho_e(E_0 - E)^2 - m_\beta^2} \Theta(E_0 - E - m_\beta) dE$$

$$= \frac{\rho_e[(m_i + \Delta E)^2 - m_\beta^2]^{1/2} - \sum_j |U_{ej}|^2[(m_i + \Delta E)^2 - m_j^2]^{1/2}}{\rho_e[(m_i + \Delta E)^2 - m_\beta^2]^{1/2}},$$

where in the sum $j$ runs over the set.

Note that although the derivative of spectrum at $E_0 - m_i$ is divergent, $\Delta n$ has finite derivative. If $|m_j - m_\beta| \ll m_\beta$ one can expand the relative error around $m_\beta$ over $\Delta m/((m_i + \Delta E)^2 - m_\beta^2)$ as follows

$$R = \frac{3 \sum_j |U_{ej}|^2 m_\beta \Delta m_j}{\rho_e[(m_i + \Delta E)^2 - m_\beta^2]} + \frac{3 \sum_j |U_{ej}|^2 [2m_\beta^2 - (m_i + \Delta E)^2](\Delta m_j)^2}{\rho_e[(m_i + \Delta E)^2 - m_\beta^2]^2},$$

(71)
where $\Delta m_j \equiv m_j - m_\beta$. Using the inequality $\Delta m \ll \Delta E$, we can rewrite (71) as

$$R = 3 \sum_j |U_{ej}|^2 \Delta m_j \rho_e (2m_i \Delta E + \Delta E^2) + 3 \sum_j |U_{ej}|^2 \left[ m_j^2 - 2m_i \Delta E - \Delta E^2 \right] (\Delta m_j)^2$$

(72)

If bending of the energy spectrum is observable, $\Delta E$ is of the order of $m_i$ or smaller. Consequently, the first term in (71) is of the order of $\Delta m/\Delta E$ and the second term is of order $(\Delta m/\Delta E)^2$. If we set

$$m_\beta = \frac{\sum_j m_j |U_{ej}|^2}{\sum_j |U_{ej}|^2},$$

the first term in (72) vanishes and $R$ will be of the order $(\Delta m/\Delta E)^2$.

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Figure 1: The bounds on the effective $\beta$-decay mass, $m_\beta$, in the $3\nu$-scheme with mass degeneracy. Shown are the upper bounds from the $2\beta0\nu$-decay as the functions of mixing angle relevant for the solution of the solar neutrino problem (see eq. (26)). The upper solid (dashed) line corresponds to the present bound $m_{ee} \leq 0.34$ eV and $|U_{e3}|^2 = 0$ ($|U_{e3}|^2 = 0.05$). The lower solid (dashed) line corresponds to $m_{ee} \leq 0.05$ eV and $|U_{e3}|^2 = 0$ ($|U_{e3}|^2 = 0.05$). These lines are drawn in assumption of strong degeneracy of neutrino masses. The vertical lines mark the 90 % C.L. borders of the LMA solution region. Shown also are the present upper bound on the neutrino mass from structure formation [41] and the sensitivity limit of KATRIN experiment.
Figure 2: The four neutrino schemes of mass and mixing. The boxes correspond to the mass eigenstates. The position of the box in the vertical axis determines the mass. The shadowed parts of the boxes indicate the amount of the electron flavor in the corresponding eigenstate $\nu_i$, that is, $|U_{ei}|^2$. The solar pair is formed by the two strongly degenerate states with $\Delta m^2_\odot$ and significant amount of the electron flavor. In the hierarchical schemes the effective mass of the light set is much smaller than the mass of the heavy set. In the non-hierarchical schemes these two masses are comparable. For definiteness we show distribution of the electron flavor in the solar pair which corresponds to the large mixing solution of the solar neutrino problem.
Figure 3: The bounds on $m_\beta$ and $\rho_e^h$ in the (3+1) scheme with normal mass hierarchy. The dashed curve and solid lines attached to it (from below and above) show the upper bound on $\rho_e^h$ from Bugey and CHOOZ experiments, respectively. The LSND lower bound (see eq. (35)) is shown by dot-dashed curves. The allowed regions are shadowed. The triplets of solid lines show the upper bounds on $m_\beta$ assuming that future $2\beta 0\nu$-decay searches will give $m_{ee} \leq 0.01, 0.03$ and 0.05 eV. The central line in each triplet corresponds to the contribution from the heaviest mass eigenstate (eq. (38)) and the other two lines show the uncertainty due to the contribution of light states in the modification of the scheme in which the mass hierarchy of the three light states is inverted (see eq. (39)).
Figure 4: The bounds on $m_\beta$ and $\rho^h_e$ in the (2+2) scheme with normal mass hierarchy. The dashed curve and solid lines attached to it show the upper bounds on $\rho^h_e$ from Bugey and CHOOZ experiments, respectively. The LSND lower bound (see eq. 35) is shown by dash-dotted line. The allowed regions of parameters are shadowed. The triplets of solid lines show the lower bounds on $m_\beta$ from a positive signal in future $2\beta 0\nu$-decay searches which would correspond to $m_{ee}=0.01$, 0.03 and 0.05 eV. The central lines correspond to contribution to $m_{ee}$ from the heavy set only.
Figure 5: The bounds on $m_\beta$ and $\rho_e^h$ in the (3+1) scheme with inverted mass hierarchy. The dashed curve and solid lines attached to it (from below and above) show the upper bound on $\rho_e^h$ deduced by Bugey and CHOOZ experiments, respectively. The LSND lower bound (see eq. (35)) is shown by dot-dashed lines. The allowed regions are shadowed. The horizontal dashed lines are the upper bounds on $m_\beta$ from the $2\beta 0\nu$-decay searches which correspond to $m_{ee} < 0.34$ eV, $U_{e3} = 0$, and different values of $\sin^2 2\theta_\odot$. 
Figure 6: The bounds on $m_\beta$ and $\rho^h_e$ in the (2+2) scheme with inverted mass hierarchy. The dashed curve and solid lines attached to it (from below and above) show the upper bound on $\rho^h_e$ deduced by Bugey and CHOOZ experiments, respectively. The LSND lower bound (see eq. (35)) is shown by dot-dashed lines. The allowed regions are shadowed. The horizontal dashed lines are the upper bounds on $m_\beta$ from the $2\beta0\nu$ searches which correspond to $m_{ee} < 0.34$ eV and different values of $\sin^22\theta_\odot$. We assume that solar pair of states gives the dominant contribution (see eq. (33)). The curve for the SMA solution (not shown here) practically coincides with the curve of KATRIN sensitivity.
Figure 7: The bounds on the effective mass of the light set $m^l_β$, and the coupling of the electron neutrino with heavy set, $ρ^h_β$, in the non-hierarchical schemes with normal order of levels. The vertical solid lines show the upper bounds on $ρ^h_β$ from Bugey experiment. The dashed and dash-dotted vertical lines show lower bounds on $ρ^h_β$ from LSND experiment in the (3+1) and (2+2) schemes, respectively (see eq. (35)). The allowed regions for (3+1) scheme are shadowed. The lines with different values of $\sin^2 2θ_⊙$ are the upper bounds from the $2β0ν$-decay searches which correspond to $m_{ee} < 0.34$ eV. The line denoted by “3+1” shows the upper bound for the (3+1) scheme with $|U_{e3}|^2 = 0.05$ (see eq. (51)), while the others are valid both for the (2+2) scheme and the (3+1) scheme with $|U_{e3}|^2 = 0$ (see eqs. (53), (51)). The lines marked by (a) and (b) are calculated for $\sqrt{Δm^2_{LSND}} = 1.32$ eV and 0.477 eV, respectively.
Figure 8: The bounds on the effective mass of the heavy set, $m^h_β$, and the coupling of the electron neutrino with light set, $1 - \rho^h_e$, in the non-hierarchical schemes with inverted order of states. The vertical solid lines show the upper bounds on $(1 - \rho^h_e)$ from Bugey experiment. The dashed and dash-dotted vertical lines show lower bounds on $(1 - \rho^h_e)$ from LSND experiment in the (3+1) and (2+2) schemes, respectively (see eq. (35)). The allowed regions for the (3+1) scheme are shadowed. The lines with different values of $\sin^2 2\theta^\odot$ are the upper bounds from the $2\beta0\nu$-decay searches which correspond to $m_{ee} < 0.34$ eV. The line denoted by “3+1” corresponds to the (3+1) scheme with $|U_{e3}|^2 = 0.05$ (see eq. (54)) while others are valid both for the (2+2) scheme and the (3+1) scheme with $|U_{e3}|^2 = 0$ (see eqs. (53), (59)). The lines marked by (a) and (b) are calculated for $\sqrt{\Delta m^2_{LSND}} = 1.32$ eV and 0.477 eV, respectively.