Lidar-based wake tracking for closed-loop wind farm control

Steffen Raach, David Schlipf, Po Wen Cheng
Stuttgart Wind Energy (SWE), University of Stuttgart, Allmandring 5B, 70569 Stuttgart, Germany
E-mail: raach@ifb.uni-stuttgart.de

Abstract. This work presents two advancements towards closed-loop wake redirecting of a wind turbine. First, a model-based estimation approach is presented which uses a nacelle-based lidar system facing downwind to obtain information about the wake. A reduced order wake model is described which is then used in the estimation to track the wake. The tracking is demonstrated with lidar measurement data from an offshore campaign and with simulated lidar data from a SOWFA simulation. Second, a controller for closed-loop wake steering is presented. It uses the wake tracking information to set the yaw actuator of the wind turbine to redirect the wake to a desired position. Altogether, this paper aims to present the concept of closed-loop wake redirecting and gives a possible solution to it.

1. Introduction
In recent years, the focus of control applications in wind energy has shifted more and more to issues in the wind farm. Since wind turbines are growing in size and the available areas for installations are limited, the interactions between wind turbines in a wind farm array are becoming more important. The wind speed in the wake of a wind turbine is reduced with respect to the free stream wind speed. Additionally, the turbulence in the wake is increased. If a wind turbine is hit by a wake from a wind turbine located upwind, the wind turbine produces less power and is faced with higher structural loads because of the increased turbulence, see [1]. Describing the wake effects and quantifying the decay has been of interest for years. Different models have been developed to address different phenomena, such as the velocity deficit and the increased turbulence intensity. There are empirical models, data driven models, or models which describe the physical behavior in the wake, all varying in complexity and computational effort. Mainly, models with low complexity are steady state models which means they describe the interaction in a static manner and no wake propagation is modeled. Further research is needed to develop control oriented dynamic wake models.

In relation to wind turbine control, the same two goals are valid for wind farm control: 1) maximization of the total power and 2) reduction of the structural loads. These goals were addressed in research with different approaches: 1) axial induction based wind farm control is proposed and investigated and 2) an approach was introduced to redirect the wake. Axial induction control aims at manipulating the axial induction by the blade pitch or torque actuator and steering the wind turbine to a lower production level. This results in a weaker wake deficit and aims at minimizing structural load effects on the downwind wind turbines. The effects on the overall energy capture of the wind farm is not clear yet, see [2].

The idea of redirecting the wake by the yaw actuator instead of trying to mitigate its intensity has been discussed in different publications, see [3–5]. In simulation studies it was shown that the wake
is redirected up to 0.54 times the rotor diameter (in seven diameter downwind distance) by yawing the
turbine up to 40 deg. Different investigations have shown promising results using this method in open-
loop approaches, see [5] and [4].

A major barrier for wind farm control applications is the lack of measurement devices to measure
the flow interactions between wind turbines. Further, modeling the three dimensional flow field is not a
straight forward approach since the flow is usually described by the Navier-Stokes equations. Lidar can
be a useful tool to address the measurement problem in wind farm applications although the limitations
of a lidar system always remain and assumptions are necessary to extract the information and exploit the
lidar measurement data.

This paper addresses the wind farm control concept of wake redirecting. It aims to enable a closed-
loop wake redirecting using lidar measurements to obtain the wake position. First, it presents a model-
based estimation approach to obtain important quantities for wake redirecting using a nacelle-based lidar
system facing downwind. Furthermore, a closed loop controller is In summary, this work presents an
entire concept for lidar-based closed-loop wake redirecting.

2. Methodology

In order to enable a lidar-based closed-loop wake redirecting within a wind farm, the problem can be
divided into two main tasks: 1) the measurement task and 2) the control task. This work focuses mainly
on the measurement task but gives also a summary of a solution to the control task, which was presented
in [6]. Fig. 1 presents the general concept of the closed-loop wake redirecting and the link between
measurement task and control task.

Figure 1: The conceptual idea of a closed-loop wake redirecting and its two main tasks: 1) the estimation
task and 2) the control task.

2.1. The estimation task

Measuring flow quantities is crucial for enabling a closed-loop controller which want to manipulate
the wake quantities. The task of the measurement problem is to provide the necessary quantities for the
controller. This means using a measurement device, a lidar, and processing the measurement data in such
a way that they are useful for the controller. Since the lidar measurement principle has several limitations
in providing wind field information an adequate estimation technique is used. This estimation approach is crucial in estimating parameters of the wake and is described in Sec. 3.

2.2. The control task
The second task towards a closed-loop wake redirecting is the control task. Its main challenge is to convert the estimated wake position information and the demanded position to a demanded yaw signal. A feedback controller has to be designed which steers the wake center to the desired position and compensates uncertainties in the models. Since the reaction of a change in the yaw can be measured with a delay, which is due to the wake propagation time, the controller has to be designed in such a way that it can overcome this limitation.

In the following, first, the measurement problem is addressed. A method is presented to estimate wake information from lidar measurement data using a nacelle-based lidar system. Second, the controller problem is addressed in Sec. 4. A wake redirecting controller is presented which uses the obtained wake information, namely the wake center position, and steers the wake center using the yaw actuator to a desired position.

The goal of this paper is to present a concept for closed-loop wake redirecting. The in the following described tasks present a solution to the problem. Therefore, the models can be replaced, modified, or improved but the general concept remains for closed-loop wake redirecting.

3. The estimation task - a model-based wake tracking approach
The estimation task addresses the processing and estimation of useful information and provides them to the controller. Since a lidar system has several limitations, the desired quantities, like the wake position, or the wake deficit, are not measurable and have to be estimated from the measurement data. One main limitation of a lidar system is that it only returns the projection of the wind speeds along the direction of the laser beam. This means that a lidar system only provides scalar information of the actual wind vectors. Further, the wind speed is not measured at a certain point but in a volume around the desired measurement location. A solution to this limitations is to implement model-based wind field reconstruction. Wind field reconstruction methods have been developed and used for different applications of lidar system usage in wind energy, for example static two- and three-dimensional, [7], dynamic three dimensional wind field reconstruction methods, see [8], and approaches for floating lidar systems, [7]. Here, the concept of wind field reconstruction is used to obtain information about the wake.

The general approach of wind field reconstruction from lidar data is to estimate wind field characteristics from an internal model by fitting simulated lidar measurement data to the measured ones. In Fig. 2 the basic idea of model-based wind field reconstruction is shown. An optimizer is used to find the best fit for a model of the assumed wind field with the defined lidar configuration. The optimizer

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Figure 2: The general concept of model-based wind field reconstruction: Estimating the wind field characteristics by fitting simulated lidar measurement data to the measured ones.
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minimizes the square error of the modeled (simulated) $v_{los,a}$ and the measured $v_{los,m}$ lidar line-of-sight velocities and returns the estimated wind field parameter.

For the model-based wind field reconstruction an adequate wind field model is crucial. For estimating wake information and tracking the wake, the wind field model has to include a model for the wake in the wind field. Thus, a wake model is necessary which includes the main wake effects: wake deficit, wake evolution, and wake center displacement. Further, an underlaying wind field model is used. The models are presented in the following section.

3.1. Wind field model

Fig. 3 shows the subparts of the wind field model: 1) the underlaying wind field, and 2) the wake model.

Figure 3: The submodels of the wind field model (in the wind coordinate system $W$): 1) the underlaying wind field, and 2) the wake model.

The wind field model is described in a wind coordinate system which is denoted by the subscript $W$. It is rotated horizontally with respect to the global inertial coordinate system $I$. The wind speed vector in the $W$-system is transformed in the $I$-system by

$$
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix}_I = \begin{bmatrix}
  \cos \alpha & -\sin \alpha & 0 \\
  \sin \alpha & \cos \alpha & 0 \\
  0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix}_W
$$

where $\alpha$ is the horizontal rotation of the wind field. The underlaying wind field includes the rotor effective wind speed $v_0$ and vertical linear shear $\delta V$. It is assumed that the wind field has only a $u$ component. Thus in the $W$ coordinate system, the underlaying wind field is

$$
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix}_{i,W} = \begin{bmatrix}
  v_0 + z_i \delta V \\
  0 \\
  0
\end{bmatrix},
$$

where $z_i$ is the height above the ground. This is illustrated in Fig. 3 on the left. Further, the wind field is linearly overlaid with the wake model $\Psi$ for the $u$ and $v$ component. Thus, this yields

$$
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix}_{i,W} = \begin{bmatrix}
  v_0 + z_i \delta V + \Psi_{u,i} \\
  \Psi_{v,i} \\
  0
\end{bmatrix}.
$$

In the following section, the considered wake effects are described and the wake model is presented.
3.1.1. Wake deficit and wake evolution model. The rotor extracts energy from the wind and converts it into electrical energy. Therefore, the wind speed is reduced behind a wind turbine. Through mixing and energy flow from the surrounding the deficit is cleared over distance. The wake deficit is modeled with an initial wake deficit at the rotor disk with tip and root losses depending on the energy extraction. In order to get the initial deficit, the energy extraction is mapped by applying Prandtl’s root and tip loss function $\Gamma_{Prandtl}$. Applying the energy conservation assumption yields

$$\left(v_0 + s\Gamma_{Prandtl}\right)^2 - (1 - c_P)v_0 = 0,$$

with the power coefficient $c_P$. Solving this equation for $s$ gives the initial wake deficit

$$\Psi_{init} = s_{solution}\Gamma_{Prandtl}.$$ (5)

An exemplary initial wake deficit $\Psi_{init}$ is shown in Fig. 4.

The wake is evolving as it moves away from the wind turbine. New energy is flowing from the side and above and the flow is mixed. Physically these dynamics are described via the Navier-Stokes equations. These are partial differential equations and it would be a very complex task to estimate the wake using these equations. However, here an empirical model is used which models the impulse dissipation. In contrast to other wake models, however, the wake evolution is modeled by a Gaussian shape 2D filter. The 2D filter $\Xi$ depends on the distance $d$ behind the wind turbine

$$\Xi(d, y_i, z_i) = \exp\left(\frac{y_i^2 + z_i^2}{2\sigma_f(d)}\right),$$ (6)

with

$$\sigma_f(d) = \frac{d\varepsilon}{2\sqrt{2\log(2)}},$$ (7)

and $y_i$ and $z_i$ the grid points in distance $d$. With the parameter $\varepsilon$ the dissipation rate can be set.

Thus, for every distance behind the rotor, the wake can be evaluated using the initial wake deficit $\Psi_{init}$ and the filter (6). The wake deficit results from the convolution of the initial wake deficit $\Psi_{init}$ with the filter $\Xi(d, y_i, z_i)$ to

$$\Psi(d, y_i, z_i) = \Xi(d, y_i, z_i) * \Psi_{init}.$$ (8)

3.1.2. Wake deflection model. The wake deflection caused by a yaw misalignment $\gamma$ is additionally modeled. The relationship is derived in the study of [9] and was successfully used in an optimization of the yaw angles for a wind farm in [5]. The angle of the wake with respect to the main wind direction is

$$\xi(d, c_T, \gamma) = \frac{\xi_{init}(c_T, \gamma)}{\left(1 + \beta \frac{d}{R}\right)^2},$$ (9)
with the initial angle of the wake at the rotor

\[ \xi_{\text{init}}(c_T, \gamma) = \frac{1}{2} \cos^2(\gamma) \sin(\gamma) c_T \]  

and the model parameter \( \beta \), which defines the sensitivity of the wake deflection to yaw and is here assumed to be known in advance. Further, \( c_T \) is the thrust coefficient and \( D \) the rotor diameter. Thus, the yaw induced deflection at the downwind position \( d \) is according to [5]

\[ \delta_{\text{yaw}}(d, c_T, \gamma) = -\xi_{\text{init}}(c_T, \gamma) \frac{D}{30\beta} \left[ 15 \left( 1 - \frac{1}{1 + \frac{2Dd}{D^2}} \right) + \xi_{\text{init}}(c_T, \gamma)^2 \left( \frac{1}{1 + \frac{2Dd}{D^2}} \right)^5 \right]. \]  

The rotation is applied to the wake deficit and yields a \( u \) and \( v \) component of the wake model,

\[
\begin{bmatrix}
\Psi_{u,i} \\
\Psi_{v,i} \\
\Psi_{w,i}
\end{bmatrix}_W =
\begin{bmatrix}
\cos \xi(d, c_T, \gamma) & -\sin \xi(d, c_T, \gamma) & 0 \\
\sin \xi(d, c_T, \gamma) & \cos \xi(d, c_T, \gamma) & 0 \\
0 & 0 & 1
\end{bmatrix}_W
\begin{bmatrix}
\Psi_{u,i} \\
\Psi_{v,i} \\
\Psi_{w,i}
\end{bmatrix}_I.
\]

In Fig. 5 two different wake situations are shown, the first is a non yawed case and in the second case the turbine is yawed with \( \gamma = 25 \text{ deg} \). In both cases the underlaying wind field has a constant wind speed of \( v_0 = 16 \text{ m/s} \) and no vertical shear.

Figure 5: Visualization of two wake situations within a constant wind field of \( v_0 = 16 \text{ m/s} \), axial induction \( a = 0.15 \) and dissipation rate \( \varepsilon = 0.1 \).

### 3.2. Lidar model

The lidar measurements can be modeled by a point measurement in the wind field. In the inertial coordinate system this is done by a projection of the wind vector \([u_i \ v_i \ w_i]^T\) onto the normalized laser vector in the \( i \)-th point \([x_i \ y_i \ z_i]^T\) with focus distance \( f_i = \sqrt{x_i^2 + y_i^2 + z_i^2} \) by

\[ v_{\text{los},i} = \frac{x_i}{f_i} u_i + \frac{y_i}{f_i} v_i + \frac{z_i}{f_i} w_i. \]  

\[ (13) \]
3.3. Model-based wake tracking
As depicted in Fig. 2 the model based wind field reconstruction method estimates the model parameter by minimizing the error between measured line-of-sight wind speed $v_{\text{los,m}}$ and simulated line-of-sight wind speed $v_{\text{los,s}}$. A nonlinear optimization problem is formed for $n$ measurement points. This yields

$$\min_p f(x) = \min_p \left[ (v_{\text{los,m,1}} - v_{\text{los,s,1}})^2 \right. \quad : \quad \left. (v_{\text{los,m,n}} - v_{\text{los,s,n}})^2 \right]$$

(14)

where in $p$ all free model parameters are included. The free model parameters are listed in Tab. 1.

Fig. 6 shows one estimation step of the wake tracking from a measurement campaign at the alpha ventus offshore wind farm.

![Figure 6: A plot of one estimation step of the wake tracking. The simulated lidar measurements in the first row are compared to the measured lidar data in the second row for five distances from 0.6 to 1.4 times the rotor diameter (from left to right). The estimated wake center is marked with the black dot.](image)

4. The control problem
The following closed-loop controller was first presented in [6] and is recapped here. Then, in a second step having analyzed the wake center displacement from the wake tracking, the filtering is discussed.

As mentioned above, the reaction of the wake to a yaw action can only be measured with a time delay. To control a delayed system, the Smith Predictor approach has been derived and used in many applications. Internal model control is the basic idea of a Smith Predictor.

| underlaying wind field | wake model |
|------------------------|------------|
| $v_0$                  | $c_T$      |
| $\delta_V$             | $c_P$      |
| rotor effective wind speed | turbine yaw angle |
| vertical linear shear  | $\gamma$  |
|                        | $\epsilon$ |

$\epsilon$ wake dissipation coefficient

Table 1: The free model parameter for the wind field model which are estimated in the optimizer.
The presented controller follows the idea of internal model control in which the difference between the actual system output and a predicted output is used within the controller to regulate the system. Therefore, a model is necessary for describing the wake effects in a simplified but sufficient way. It consists of the controller which is a classical proportional-integral controller. Further, an internal model is used which approximates the real system behavior. The wake propagation which exists because the wake flow has to evolve until it reaches the measurement location of the lidar system is approximated with an delay. The time delay $\tau$ varies with respect to the mean wind speed. Finally, a filter is needed to cancel out controller actions which can not be observed because of the time difference between control action and measurement location. Fig. 7 shows the general concept of the controller.

Figure 7: The general structure of the wake steering controller: The controller, a simplified wake model and the wake propagation modeled with a delay, and the filter.

4.1. Internal wake model of the controller
As depicted in Fig. 7, the wake controller needs an internal model to predict the reaction of the wake to the demanded yaw angle. The internal wake model includes the yaw actuator and the yaw induces wake deflection. For the wake model the assumptions of a constant thrust coefficient, $c_T$, is made.

Altogether, this yields an internal controller model $\tilde{\Psi}$ of the reality $\Psi$:

$$\tilde{\Psi} : \begin{cases} \dot{\gamma} + 2D\omega \dot{\gamma} + \omega^2 \gamma = \omega^2 \gamma_{dem} & \text{yaw actuator dynamic} \\ \tilde{y} = \delta_{yaw}(d_{Lidar}, c_T, \text{const}, \gamma) & \text{wake deflection model} \\ \tilde{y}_L(t) = \tilde{y}(t - \tau) & \text{time delay} \end{cases} \quad (15)$$

4.2. Controller design
The primal goal of the wake controller is to steer the wake center to a desired point in a defined distance by yawing the wind turbine. As mentioned, this is done using a Smith Predictor. A Smith Predictor uses an internal model to predict the output reaction. Then the predicted wake center position and the filtered error between predicted and measured wake center position is fed back to the controller.

4.2.1. Controller. A standard proportional-integral (PI) controller is used. It is designed such that the closed-loop performance with the internal model (15) meets a phase margin of 60 deg and a closed-loop bandwidth of $\omega_{CL} = \frac{1}{2\pi}$. This yields a controller of the form

$$u = K_p \left( \Delta y_L + \frac{1}{T_i} \int \Delta y_L dt \right) \quad (16)$$

with the proportional gain $K_p$ and the time constant $T_i$. 

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4.2.2. **Filter.** The wake propagation and the caused delay disables a direct measure of a yaw change and because of that one has to filter the measured feedback to prevent non-observable yaw actions. Since the delay $\tau$ is time varying and depends on the mean wind speed the filter has to be adaptable. Therefore, the cutoff frequency of the butterworth low-pass filter is set to $\omega_{\text{filter}} = \frac{2}{8\tau}$.

5. Results

Fig. 6 has already shown that the model fits well for the application and can be applied with real measurement data. In the following a time series is shown which was obtained from a SOWFA wind turbine simulation. The SOWFA simulation was stored in snapshots and later scanned with a lidar simulator. The lidar scans with a 7x7 grid in five distances from 0.6 to 1.4 times the rotor diameter ($D = 126$ m). The method shows the ability of estimating the wake parameter and tracking the wake center. Fig. 8 presents the time series of the wake parameter and the estimation of the wake center position at the most far scanning distance of 1.4D = 176.4 m.
6. Conclusion
In summary, a method which uses lidar measurements to estimate wind field parameters and enable a tracking of the wake center displacement is introduced. Second, a controller is presented which uses this information to redirect the wake to a desired position.

The wake estimation shows promising results in estimating the wake center of measurement data from a lidar measurement campaign carried out at alpha ventus, the first German offshore wind farm. Further, the method also works well with simulated lidar measurements of a SOWFA simulation.

The challenges of a lidar-based wake redirecting control problem are discussed and an appropriate controller is designed to meet the desired requirements. This enables the next step towards a closed-loop wake redirecting in an high fidelity simulation tool which is aimed as a next step.

As an outlook, the presented framework of lidar-based closed-loop wake steering offers new possibilities for wind farm control. In a next step, it will be implemented and tested in a high fidelity simulation tool. For the control problem, different controller approaches will be investigated like Hinf controllers or robust controllers. Dynamic estimation techniques as well as other wake estimation models will be used for comparing the ability of tracking the wake and finding the most suitable approach for this task.

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