Gravitational diffusion in the intracluster medium

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ABSTRACT
We revisit the process of gravitational sedimentation of helium and heavy elements in the intracluster medium. We find that helium applies an inward drag force on heavy elements, boosting their sedimentation speed to nearly half its own. This speed is almost independent of the mass and the electric charge of heavy elements. In the absence of small-scale magnetic fields, helium sedimentation can increase the He/H abundance ratio in the cores of hot clusters by three orders of magnitude. It also steepens the baryonic density profile yielding a higher X-ray luminosity, which offers an explanation of the observed luminosity–temperature relation.

If the primordial He/H ratio is assumed, then the gas density inferred from the observed X-ray luminosity might be underestimated by 30 per cent in the cores of clusters and overestimated by 7 per cent in the outer regions. The dark matter density, on the other hand, might be overestimated by a factor of 2.3 in the cores and underestimated by 18 per cent in the outer regions.

Key words: galaxies: clusters: general – cosmology: theory – dark matter – X-rays: galaxies.

1 INTRODUCTION
In the equilibrium state of a multicomponent plasma, the number density, \( n_i \), of particles of mass \( m_i \), is \( n_i \propto e^{-\phi(r)/kT} \), where \( \phi(r) \) is the gravitational potential. In the high-temperature intracluster medium (ICM), light elements such as helium (He) and hydrogen (H) can diffuse fast enough to reach their equilibrium distribution in a Hubble time. Larger frictional drag forces work on heavier elements, but we still expect at least partial segregation as a result of diffusion.

Diffusion can have important consequences. Fabian & Pringle (1977) suggested diffusion as a possible explanation for the observed gradients in the iron abundance inside galaxy clusters. In their calculations iron ions sediment by a distance comparable to the radius of the cluster within a Hubble time. For other elements they predicted the diffusion velocity to be proportional to \( A Z^2 \), where \( A \) is the atomic number and \( eZ \) is the charge of the ion. These calculations assumed that only protons apply appreciable drag forces on heavy elements. Rephaeli (1978) claimed that, by neglecting helium drag, Fabian & Pringle overestimated the iron diffusion rate.

Helium sedimentation can potentially change the global observational properties of X-ray emission from rich clusters. As has been noted by Qin & Wu (2000), current estimates of cluster masses from X-ray observations, which rely on the assumption of constant mean molecular weight, can be off by \( \pm18 \) per cent if helium is concentrated inside the core. Here we also point out that, by steepening the baryonic density profile, helium sedimentation increases the X-ray emissivity in hot clusters.

In this letter we revisit the calculation of element sedimentation in the ICM. We confirm the claim made by Rephaeli that helium drag on iron is comparable to that of protons. However, instead of hindering the sedimentation of iron and other heavy elements we show that helium acts as a catalyst. Our analysis includes electric fields that are inevitably generated by segregation of charged elements. Although these fields reduce the diffusion rate, the sedimentation time-scales of heavy elements are still several times shorter than previous estimates.

The paper is organized as follows. In Section 2 we present the basic equations and estimate the sedimentation speeds and time-scales. In Section 3 we discuss the equilibrium distribution as a limiting case of element sedimentation. We conclude in Section 4.

2 THE EQUATIONS OF ELEMENT DIFFUSION
We write the equations governing the evolution of individual species in an ICM of any composition. We do not consider magnetic fields in this paper, but include electric fields which must exist in any ionized plasma in a gravitational field (Eddington 1926). Let the ICM be made of any number of species each made of particles characterized by mass \( m_i \) and electric charge \( q_i = eZ_i \). We denote the local number density and velocity of a patch of matter of each species by \( n_i \) and \( V_i \), respectively. Then, the mass density is \( \rho_i = n_i m_i \) and the pressure is \( P_i = n_i kT \), where we have assumed that the ICM is in local thermal equilibrium so that all species share the same temperature, \( T \). With this notation the equations are

\[
\frac{\partial V_i}{\partial t} + V_i \cdot \nabla V_i = -\frac{\nabla P_i}{\rho_i} + g + \frac{q_i E}{m_i} + \sum_j \frac{(V_i - V_j)}{\tau_{ij}},
\]

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where \( E \) is the electric field, and \( g \) is the gravitational field (force per unit mass). The constants \( \tau_{ij} \) are the time-scales for the drag forces acting on the species \( i \) as a result of Coulomb interactions with species \( j \). Momentum conservation implies that \( \tau_{ij} = (\rho_j / \rho_i) \tau_{ji} \). If \( m_i \gg m_j \), then \( \tau_{ij} \) can be approximated by (Spitzer 1968)

\[
\tau_{ij} = \frac{2(3\pi/4)^{1/2}(kT)^{1/2}}{8 \pi Z_i^2 e^2 n_j \ln \Lambda \ m_j^{1/2}} \frac{m_j}{m_i} \frac{1}{(m_i / m_j)^{1/2}} \frac{n_j}{(10^5 \text{ m}^{-3})^{-1}} \times \left( \frac{m_j}{m_p} \right)^{1/2} (Z_i Z_j)^{-2} \text{ yr},
\]

(2)

where the \( \ln \Lambda \) is the Coulomb logarithm. The equations (1) must be supplied by additional relations that specify the electric field. To create an electric force comparable with gravitational and pressure forces a tiny fractional charge excess \((Gm_e^2/e^2 \sim 10^{-50})\) is sufficient. The additional relations can then be obtained by assuming charge neutrality and zero electric currents, i.e.

\[
\sum_i n_i q_i = 0, \quad \text{and} \quad \sum_i n_i q_i V_i = 0.
\]

(3)

### 2.1 Sedimentation speeds and time-scales

Multiplying the equations (1) by \( n_i m_i / \sum n_i m_i \) and summing over all species yields

\[
\nabla \rho \frac{\rho}{\rho} = \vec{g},
\]

(4)

where \( \rho = \sum n_i m_i \) is the total local density of all species, \( V = \sum \rho_i V_i / \rho \) and \( P = \sum P_i \) are, respectively, the mass-weighted mean velocity and the total pressure, and \( \vec{g} = \vec{g} - \vec{g} \cdot \nabla V / \partial t - \nabla \cdot \vec{V} \) is the gravitational field in the frame of reference moving with the velocity \( \vec{V} \). The terms involving the electric and drag forces have disappeared by virtue of charge neutrality and momentum conservation, respectively. As \( |V_i - V| \) is much smaller than the thermal velocities, the velocity dispersion term \( \sum \rho_i (V_i - V) \nabla (V_i - V) / \rho \) is negligible compared to the pressure and gravity terms, so we have not included it in the equation (4).

At the initial stage of the diffusion process the dominant light species have the same distribution, which gives \( \nabla (n_i kT) / (n_i m_i) = (\mu m_p / m_i) \vec{g} \), where \( \mu = (1 / m_\rho) \sum n_i m_i / \sum n_i \) is the mean molecular weight. Thus equation (1) yield

\[
-(\mu m_p / m_i - 1) \vec{g} + \frac{q_i E}{m_i} + \sum_j (V_i - V_j) / \tau_{ij} = 0.
\]

(5)

Using equations (3) and (5) we find that the electric field is \( E = \mu \vec{g} / e \). From (2), \( \tau_{ij} \propto n_j^{1/2} \mu_i^{1/2} \), so because of the low abundance of heavy ions and the low mass of the electrons, only the drag terms from protons and helium ions is important. Thus we are left with a single equation for each element

\[
\left( 1 + \frac{Z_i}{A_i} \mu - 1 \right) \vec{g} + \frac{V_p - V_i}{\tau_{ip}} + \frac{V_o - V_i}{\tau_{io}} = 0.
\]

(6)

We can now estimate the velocity of each species relative to \( V_p \), the velocity of the proton fluid at each point. The advantage of using velocities relative to protons is the independence of the results from physical processes other than diffusion (radiative cooling, heating by supernovae, and so on). Even though these processes can have important dynamical effects on the cluster, they do not affect the expressions we develop for the relative velocities and element abundance.

From (6), the helium velocity is

\[
V_o - V_p = (3 \mu / 4 - 1) \vec{g} \tau_{ip} \approx -0.56 \vec{g} \tau_{ip}.
\]

(7)

The relative sedimentation speed of heavy elements, \( V_i - V_p \), can easily be related to that of helium \( V_o - V_p \). Heavy elements experience an upward proton drag and inward helium drag forces. According to (2), \( \tau_{ij} \propto Z_i^2 \), and for large \( Z_i \) these drag terms dominate all others in (6). Thus we are left with

\[
(V_p - V_i) / \tau_{ip} \approx (V_i - V_o) / \tau_{ip}.
\]

(8)

Substituting \( \tau_{ip} \) and \( \tau_{ip} \) from (2) in the last equation yields

\[
\frac{V_p - V_i}{V_o - V_p} \approx 1 + \frac{n_p}{8n_o} \left( \frac{1}{Z_i} + \frac{1}{Z_i} \right).
\]

(9)

where we have taken \( n_o / n_p = 0.08 \). Thus, contrary to the prediction of Fabian & Pringle (1977), who by neglecting helium drag obtained \( \tau_{ij} \propto A_i Z_i^2 \), the diffusion speeds are approximately the same for all heavy elements. Including all forces in the calculation increases this result slightly (by a factor of \(~1 + 4 / Z_i\)):}

\[
\frac{V_p - V_i}{V_o - V_p} = 0.4 \left[ 1 + \frac{3A_i}{Z_i} - 1.8 \left( \frac{1}{Z_i} + \frac{1}{Z_i} \right) \right].
\]

(10)

There is also a correction of a similar magnitude if the initial distribution of heavy elements is different from that of light elements.

The relation (10) is independent of the physical state of the ICM. To obtain the relative velocities we have to know the temperature, the density and the gravitational acceleration. Taking in (2) \( \ln \Lambda \approx 40 \) as the typical value for the ICM, we find the helium velocity relative to protons to be

\[
V_o - V_p = 5 \times 10^4 \frac{\vec{g}}{g_{10^{-5}}} n_o^{-1} T_{\odot} \text{ m s}^{-1},
\]

(11)

where \( \vec{g}_{10^{-5}} = \vec{g} / (10^{-5} \text{ m s}^{-2}) \), \( n_o = n_o / (10^3 \text{ m}^{-3}) \), and \( T_{\odot} = T / 10^6 \text{ K} \). This is lower than the result of Qin & Wu (2000) by ~40 per cent; the main difference is the inclusion of electric field in our calculation. As seen from equation (11), the diffusion speeds depend on the local density and temperature. However, a single diffusion time-scale is obtained if the gas and dark matter both follow an isothermal spherical density profile \( \rho(R) \propto R^{-2} \) and the gas is in hydrostatic equilibrium \( \vec{g} = g \) with constant \( T \). In this case, \( V_i - V_p \propto g / n_p \rho \propto R \), which together with continuity equation, \( d n_i / d t = -R^{-2} \partial_{\rho} (R^2 V_i) \), yields

\[
\frac{d}{d t} \ln \frac{n_i}{n_o} = \frac{3(V_i - V_p)}{R}.
\]

(12)

This motivates us to define the time-scale for diffusion between two species as

\[
\tau_{D} = \frac{R}{3(V_i - V_p)}.
\]

(13)

which for helium relative to protons gives

\[
\tau_D = 3 \times 10^9 \left( \frac{f_b}{0.1} \right) \left( \frac{T}{10^6 \text{ K}} \right)^{3/2} \text{ yr}.
\]

(14)

where \( f_b \) is the baryonic mass fraction in the cluster.

### 3 The equilibrium distribution

The diffusion time-scale of helium as seen from equation (14) is comparable to the Hubble time. It is thus prudent to examine in

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detail the equilibrium distribution of hydrogen and helium, which is the final product of diffusion.

We assume a spherically symmetric cluster in which the dark mass inside a radius \( r \) is given by (Navarro, Frenk & White 1997)

\[
M_{\text{dm}}(r) = 4\pi \rho_s r_s^3 \left[ \ln (1 + r/r_s) + \frac{1}{1 + r/r_s} \right],
\]

(15)

where \( \rho_s \) and \( r_s \) are constants. The gas density profile, \( \rho \), is determined by the equation of hydrostatic equilibrium (see equation 4),

\[
\frac{kT}{\mu m_p} \frac{d}{dr} \ln \left( \frac{\rho}{\mu} \right) = -\frac{G M_{\text{dm}}(r)}{r^2}.
\]

(16)

where we have neglected the contribution of baryons to gravity and assumed constant temperature, \( T \), throughout the ICM. In the absence of sedimentation, \( \mu \) is constant, and the solution to (16) is (Gilfanov & Syunyaev 1984).

\[
\rho(r) = \rho(0)e^{-\mu \eta (1 + r/r_s)^{\beta/m_p}/r}.
\]

(17)

where \( \eta = 4\pi G m_p \rho_s r_s^2/kT \). Sedimentation introduces a dependence of \( \mu \) on \( r \) and the above analytical solution is no longer valid. The abundances of the various elements are then determined by the equation of hydrostatic equilibrium for each element separately and the condition for local charge neutrality:

\[
\frac{kT}{m_i} \frac{d\ln n_i}{dr} = -\frac{G M_{\text{dm}}(r)}{r^2} - \frac{q_i E(r)}{m_i},
\]

(18)

\[
\sum_i n_i(r) q_i = 0.
\]

(19)

Taking \( n_a = 4m_p \) and neglecting the electron mass, these equations have the solution

\[
n_p = 6C_1 [f(r) + f^{-1}(r) - 1]/h^2(r),
\]

(20)

\[
n_a = C_1 [f(r) + f^{-1}(r) - 1]^2/h^2(r),
\]

(21)

where \( f(r) = [C_2 h^2(r) - 1 + \sqrt{(C_2 h^2(r) - 1)^2 - 1}]^{1/3} \), \( h(r) = (1 + r/r_s)^{\beta/m_p} \), and \( C_1 \) and \( C_2 \) are constants. The behaviour of the solution in the inner and outer regions is easily understood, as follows. In the inner regions helium is dominant, thus \( \mu = 4/3 \) and \( E = \mu g/e = 4g/3e \). From this it follows that \( n_p \propto (1 + r/r_s)^{-\eta/3} \) and \( n_a \propto (1 + r/r_s)^{\beta/3} \). Note that, because the total force felt by protons becomes repulsive \( (eE > mg) \), their density is falling towards the centre. The outer regions consist almost entirely of hydrogen plasma, thus \( \mu = 1/2 \) and \( E = \mu g/e = g/2e \). This gives \( n_p \propto (1 + r/r_s)^{\eta/2} \) and \( n_a \propto (1 + r/r_s)^{\beta/2} \), and so \( n_a \propto n_p^3 \), in agreement with Gilfanov & Syunyaev (1984).

The constants \( C_1 \) and \( C_2 \) in the analytic solution (21) are fixed by the boundary conditions imposed on the abundances. Here we require that the ratio of total helium to hydrogen abundances inside the virial radius is equal to the primordial value (~0.08). We will present results for a cluster with gas temperature such that \( \eta = 10 \) and a virial radius equal to \( 3r_s \), in agreement with observations and N-body simulations of massive clusters (NFW 1997; Ettori & Fabian 1999). In Fig. 1 we show as the solid line the baryonic density obtained from the analytic solution. For comparison, we also plot as the dashed curve the profile (17) which corresponds to equilibrium without diffusion. A proper estimation of the density profile from the observed X-ray emissivity \( \propto \left( \sum n_i Z_i \right) \frac{1}{2} \) should take into account variations of the He/H abundance ratio throughout the ICM. Assuming constant He/H ratio can yield a biased estimate of the profile. To demonstrate this we plot as the dotted line in Fig. 1 the estimated profile if a constant abundance ratio were assumed. In Fig. 2 we show the baryonic mass fraction as a function of radius for the same three cases as in Fig. 1. We see (solid line) that diffusion introduces distinct features in the behaviour of the baryonic fraction as a function of radius. Finally, Fig. 3 shows the number density of helium and hydrogen in the case with diffusion.

### 4 CONCLUSIONS

We have seen that diffusion steepens the gas density profile near the centre of hot clusters, increasing their X-ray luminosity (by factor of 5 in our example). In colder clusters \( (T \sim 10^7 \text{ K}) \), the diffusion time-scale is larger than the Hubble time and the luminosity remains unchanged. This may explain the observed discrepancy between the observed \( L \propto T^3 \) (Mushotzky 1984; Edge & Stewart 1991; David et al. 1993) and \( L \propto T^2 \) which is expected from self-similar arguments (e.g. Kaiser 1986).

If the inner regions of clusters are dominated by helium then the baryonic mass density as inferred from the X-ray emissivity can be underestimated by ~30 per cent if constant He/H abundance
ratio is assumed, while in the helium-deficient outer region it can be overestimated by $\sim 7$ per cent. Estimates of the dark matter density can be affected by an even larger factor. These estimates assume hydrostatic equilibrium (equation 16), so by taking $\mu = 0.59$ (cosmic abundance) instead of 0.5 (pure hydrogen plasma) in the outer regions, we underestimate the total mass by $\sim 18$ per cent (Qin & Wu 2000). In the helium-dominated core the mass would be overestimated by a factor of 2.3. Gravitational lensing of background galaxies by clusters leads to core masses that are a factor of 2–4 larger than estimates based on X-ray observations, assuming the primordial He/H ratio (Allen 1998). Helium sedimentation might further worsen this discrepancy. If sedimentation is efficient then X-ray estimates from hydrostatic equilibrium using a perfect equation of state for the gas are almost a factor of 5–9 less than estimates based on lensing.

We have neglected magnetic fields. Fields with a coherence length comparable to the size of the cluster force the ions to move on longer orbits, defined by the field lines. This can increase the sedimentation time-scales by a factor of a few. Small-scale magnetic fields, however, can increase the time-scales by a factor ranging from a few in some estimates (Narayan & Medvedev 2001; Malyshekin 2001) to 100–1000 in others (Chandran & Cowley 1998). We have also neglected turbulence in the ICM. Turbulent motion on a scale of $R_t$ is effective in washing out abundance gradients if $R_t V_t \gg R_c V_d$, where $V_t$ is the turbulence velocity over $R_t$, $R_c$ is the cluster radius and $V_d$ is the initial sedimentation speed. However, if the ICM has been enriched long ago ($z > \sim 1$), then metal gradients would not have survived very strong turbulent motions. Therefore the observed gradients of metal abundances suggest that turbulence does not play a dominant role. A similar argument can be made about merging in clusters.

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