Masses and mixing of neutrinos in grand unified SO(10) model

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1 Introduction

Non-vanishing but extremely small masses for neutrinos are deduced from the measurements of the solar neutrino deficit and the atmospheric neutrino anomaly. If the neutrino masses do really not vanish, the existence of physics beyond the standard model is immediately implied. Then, grand unified theories (GUTs) are most plausible. However, it has also been observed that the Maki-Nakagawa-Sakata (MNS) matrix, which describes the generation mixing among the leptons similarly to the Cabibbo-Kobayashi-Maskawa (CKM) matrix for the quarks, has unsuppressed off-diagonal elements. The off-diagonal elements of the CKM matrix are small. This difference between the quarks and the leptons poses a severe constraint on GUT models, since the Higgs couplings of the quarks and the leptons are closely related to each other.

We propose a GUT model based on SO(10) and supersymmetry [NO1]. The masses and mixings of the quarks and the leptons are well described simply by introducing a Higgs superfield belonging to 120 representation, in addition to ordinarily contained 10 and 126 representations. Below the GUT energy scale, the model is the same as the minimal supersymmetric standard model except the inclusion of dimension-5 operators. These new terms lead to small Majorana masses for the left-handed neutrinos after electroweak symmetry is broken down.

2 Model

The grand unified group of our model is SO(10). Its spinor 16 representation contains all the quark and lepton superfields of one generation, both left-handed and right-handed components. The right-handed neutrinos are thus naturally incorporated. For the Higgs superfields which give masses to the quarks and leptons, one superfield is introduced for each of 10, 120, and 126 representations. These three representations are all that can couple to the direct product of 16 and 16.

The representation 10 contains SU(2) doublets $H_{10}^5$ and $H_{10}^5$ with hypercharges $Y = -1/2$ and $Y = 1/2$, respectively, where upper indices denote transformation properties under SU(5). In the representation 120 there are four SU(2) doublets: $H_{120}^5$ and $H_{120}^5$ with $Y = -1/2$ and $H_{120}^5$ and $H_{120}^{45}$ with $Y = 1/2$. The representation 126 involves SU(2) doublets $H_{126}^{55}$ with $Y = -1/2$ and $H_{126}^{55}$ with $Y = 1/2$, an SU(2) triplet $H_{126}^{45}$ and an SU(2) × U(1) singlet $H_{126}^1$.

The Higgs doublet superfields $H_1$ with $Y = -1/2$ and $H_2$ with $Y = 1/2$ responsible for electroweak symmetry breaking are given by linear combinations of possible SU(2)-doublets with the same hypercharges, which we can write as

\[
H_1 = (C_1^{1})_{11}H_{10}^5 + (C_1^{1})_{12}H_{120}^5 + (C_1^{1})_{13}H_{120}^{45} + (C_1^{1})_{14}H_{126}^{55} + \ldots,
\]

\[
H_2 = (C_2^{1})_{11}H_{10}^5 + (C_2^{1})_{12}H_{120}^5 + (C_2^{1})_{13}H_{120}^{45} + (C_2^{1})_{14}H_{126}^{55} + \ldots,
\]

where $C_1$ and $C_2$ represent unitary matrices. Some components of $H_1$ and $H_2$ may belong to the representations different from 10, 120, and 126, which are expressed by the ellipses. The other linear combinations of SU(2) doublets should be sufficiently heavy below the GUT energy scale to satisfy the unification of the gauge coupling con-
SU(3) lepton masses are given, in the framework of enough not to develop a non-vanishing VEV. The superpotential relevant to the quark and lepton masses are given, in the framework of SU(3) × SU(2) × U(1), by

\[
W = \eta_D^i H_1 Q^i D'^j + \eta_U^i H_2 Q^j U'^c j + \eta_L^c H_1 L^j E'^c j + \frac{1}{2} \kappa_i^j H_2 L_i^j H_2 L^j + \text{H.c.,}
\]

where \(Q^i, U'^c j, \) and \(D'^j\) denote the quark superfields and \(L^j\) and \(E'^c j\) represent the lepton superfields in self-explanatory notations, with \(i\) being the generation index. The dimension-5 terms proportional to \(\kappa\) are induced by exchanging the heavy right-handed neutrinos and sneutrinos.

The coefficients \(\eta_d\) and \(\eta_u\) become symmetric concerning generation indices, if the \((\sqrt{3} H_{120}^5 + H_{120}^5)/2\) component in \(H_1\) and the \(H_{120}^5\) component in \(H_2\) can be neglected. Taking a generation basis in which the coefficient matrix for the up-type quarks is diagonal, they are written as

\[
\eta_u = \eta_u^D, \quad \eta_d = V_{CKM}^\dagger \eta_d V_{CKM},
\]

where \(\eta_u^D\) and \(\eta_d^D\) represent diagonal matrices, and \(V_{CKM}\) stands for the CKM matrix. The coefficients for the leptons are given by

\[
\eta_e = -\frac{3r_1 + r_4}{r_1 - r_4} \eta_d + \frac{4}{r_1 - r_4} \eta_u + 4\tilde{c}, \quad \kappa = -\eta_p (M_{\nu_R})^{-1} \eta_D, \\
\eta_\nu = -\frac{4r_1 r_4}{r_1 - r_4} \eta_d + \frac{r_1 + 3r_4}{r_1 - r_4} \eta_u + 2r_2 \tilde{c}, \\
M_{\nu_R} = \frac{2\sqrt{3} v_S}{(C_1)_{41}} \left[ \frac{r_1}{r_1 - r_4} \eta_d - \frac{1}{r_1 - r_4} \eta_u \right], \\
\tilde{c} = (C_1)_{21} \tilde{c}, \\
r_1 = (C_2)_{11} (C_1)_{11}, \quad r_2 = (C_2)_{21} (C_1)_{21}, \quad r_4 = (C_2)_{41} (C_1)_{41},
\]

where \(\epsilon\) denotes the coefficients for the couplings \(120 \times 16 \times 16\). Owing to the group structure, \(\epsilon\) is antisymmetric. The parameters for the leptons are expressed by those for the quarks and the additional six parameters \(r_1, r_2, r_4, \) and \(\tilde{c}\).

### 3 Numerical results

The independent parameters at the GUT energy scale are given by the diagonal matrices \(\eta_u^D\) and \(\eta_d^D\), the CKM matrix \(V_{CKM}\), the ratio \(r_1, r_2,\) and \(r_4\), the antisymmetric matrix \(\epsilon\), and the right-handed neutrino mass scale \(v_S/(C_1)_{41}\). At the electroweak energy scale, the eigenvalues of \(\eta_u, \eta_d,\) and \(\eta_e\) are known experimentally, if the ratio \(\tan \beta\) of the VEVs for \(H_1\) and \(H_2\) is given. The CKM matrix has been measured. The quantities obtained experimentally for the neutrinos are the mass-squared differences and the MNS matrix.

The matrices \(\eta_u, \eta_d, \eta_e,\) and \(\kappa\) evolve depending on the energy scale. To discuss phenomena at the electroweak energy scale, we make analyses with the aid of the renormalization group equations for \(\eta_u^D, \eta_d^D, \eta_e^D, \kappa^D, V_{CKM},\) and \(V_{MNS}\). The observed quantities at the electroweak energy scale have to be accommodated by suitable values of the parameters at the GUT energy scale.

The numerical analyses show that the measurements of the neutrino masses and the MNS matrix are explained well in certain regions of the parameter space \([NO2]\). The experimental results for the quark and charged lepton masses and those for the CKM matrix, though they stringently constrain the model parameters, can also be satisfied. The Higgs bosons in \(120\) make the mixing structures different between the quarks and the leptons. The \(126\) representation is the origin of the extreme smallness of the neutrino masses. Without invoking contrived schemes, the masses and mixings of the quarks and the leptons can be accommodated consistently by a simple extension of the minimal SO(10) model.

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### References

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[NO2] N. Oshimo, hep-ph/0305166 OCHA-PP-204 (2003).