E-LEARNING MATHEMATICS

HASAN ALMANASREH

Abstract. This study concerns the use of e-learning in the educational system shedding the light on its advantages and disadvantages, and analyzing its applicability either partially or totally. From mathematical perspectives, theories are developed to test the courses tendency to online transformation. This leads to a new trend of learning, the offline-online-offline learning (fnf-learning), it merges e-learning mode with the traditional orientation of education. The derivation of the new trend is based on the learning approaches and the study levels, this makes the new trend flexible and applicable for all mathematical courses.

1. Introduction.

Technology rapidly continues transforming our aspects of life, improving the health care, the military developments, the financial affairs, and even the educational system, etc., ending without end. Many advantages, profits, and benefits for both users and developers are a consequence of the technology revolution, also, to that, many interesting and important (and some time frightening) problems were solved or being arisen as a side-resulted of deploying new strategies and equipments. Thus, careful investigations should intensively be devoted to test the efficiency of these technologies and to determine to which extents and scopes they can be applied.

Like others, education is affected by technology, and learning is bit by bit departing from the traditional mode, letting the online electronic-learning (e-learning) occupy a large place of the educational process. The information and communication technology plays a crucial (and even, in some situations, an irreplaceable) role nowadays, where most attentions are being devoted to transform the conventional learning system to (what is expected and hoped to be a peer orientation) e-learning by a help of the technology sagacity.

In a topic field of education, using e-learning as an alternative to the conventional learning is not simple per se. Indeed, it needs deep study to determine the efficiency of the new method and its consequences. Also, for a course in a given field, further study is needed to investigate the readiness of that course for the online transformation either entirely (which exempts the offline strategy) or partially, at the same time to determine whether all courses admit the same portion of e-learning applicability.

In this work, we study e-learning from mathematical point of view, showing the efficiency and deficiency of the method, criticizing and commenting on some views on e-learning in general and particularly e-learning mathematics. Also, we will discuss the capability of mathematics courses to e-learning applicability. Finally, we will analyze which parts of a course admit e-learning and which parts can only be treated by the conventional technique. This leads to an alternative proposed learning strategy, the so-called offline-online-offline learning (fnf-learning). For this

Key words and phrases. e-Learning, mathematics, blended learning, fnf-learning, learning approach levels, study levels, theories of e-learning, online transformation, gap.

Mathematics Department, Hebron University, P.O: Box 40, Hebron, Palestine.
approach, a mathematical formula is developed to control the time period for the online mode in teaching a mathematical course. This formula is not in its final shape and needs to be more studied and improved, but it is still a good starting step towards controlling the parts of the online and the traditional modes in teaching mathematics courses. The derived formula depends on the study level and the learning approach level of each course, which makes it applicable for any mathematics course.

The paper is arranged as follows; In Section 2, we will discuss the advantages and disadvantages of e-learning dwelling upon the excessive use of e-learning from mathematical perspectives. Theories of e-learning mathematics are developed in Section 3. In Section 4, the approach full-learning is discussed, and a mathematical formula is modelled for this approach which naturally provides theorems about e-learning mathematics for any course.

2. Advantages and Disadvantages of e-learning

e-Learning is a form of distance learning, where the time and geographical flexibility are preserved. Students can any time any where pursue their study, which has great beneficial effects especially for those who have part-time jobs or have families. Beside that, the students can take as many courses as they can manage. This flexibility works for teachers as well, the teacher can instruct many courses at the time without geographical restrictions. Moreover, e-learning gives the students more self-independence and self-confidence. Over that, it gives opportunities for shy students to ask questions where they hesitate to deliver questions directly to the lecturer in the face-to-face (f2f) interaction, and others else.

Nevertheless, despite of the above positive features, where many of pro-e-learning are mentioning them repeatedly, there are also disadvantages of this trend. Many students learn better in the traditional class which is a result of live and direct interaction with the tutors and other students. In e-learning, the need arises to specific level of skills in order to follow the course, which varies from one student to another; hence a weak knowledgeable student of these skills will get himself lost in some parts of the course. Also, e-learning needs continuous accessibility to the online updated instructions, i.e., every student should have a computer with internet connection to follow the course.

These are not only the disadvantages of the system, below we will dwell upon some others, where it is worthwhile to mention each of which separately as critics for some views. Also we will elaborate on some of them from a generic point of view and some others from mathematical perspectives.

2.1. Lack of controlling student-student interaction. Once the online course has been started, the instructor needs to encourage certain range of social talk, to give his course a slot of attraction to interaction; this is suggested by L. Jonsson and R. Säljö [2]. They caution also from the dominancy of the social interaction in the communications and they stipulate the instructor to control that. As teachers, we fully agree that we should allow, from time to time, some sociality in our course to reactivate and attract the students. We do believe this can be done and easily be controlled in the traditional courses, where the lecturer can manage that. But, in e-learning, the teacher is not always there (on the course webpage) to get the students under his eyes. Even more, if we controversially assume that the teacher all the time is available online, then how he can control the students social interactions and in which way. As a result, there is no clear strategy to be used to efficiently control the initiated social talk. We believe here (in the online mode) that, there is no need to create social aspects where it is known there is no
efficient way to get them controlled, taking into account that there are many social websites for this purpose. Also the students of the online mode are not sitting, as those who are continuously sitting for two or more hours where they need to be refreshed to keep them awake and active, in the classroom.

2.2. **What is learning mathematics.** What is needed is, to adapt the technology for the purpose of learning, not the opposite. To this end, learning is not about gathering information, it is about interpreting it. We would exactly repeat, and completely agree, what Andrew Hart [5, Ch.9, p. 151] said in this issue

"Becoming rich in information but poor in knowledge. The spread of information is dangerously entropic. It may lead to uncertainty and insecurity rather than confidence and self-assurance. What we need from educational technology is forms of knowledge which may lead to understanding, rather than information overload and confusion."

As mathematicians, we teach the students how and why to choose this method or that, to interpret the facts, to relate them to real-life problems, and to teach them the way, and even the most powerful and comprehensible way, of proving theorems. In other words, we teach them the way of thinking and the hidden logic of mathematics, which the students need most to acquire rather than just collecting information and remembering facts which, with time passage, are susceptible to oblivion. We do not down the latter, but in mathematics, not like most other fields, it is of the least importance. To this end, we do not see how students achieve, gain, and develop the properties above if the course is given completely online. Of course, they can learn the ways of solving exercises, they can learn the proofs of theorems, and they can remember facts and ideas (which might be sufficient learning outcomes of a course in some other fields, but not mathematics). But they will miss the most crucial part, the mathematical logical thinking.

In the traditional class, the teacher tries not only to solve exercises or to prove theorems, but also, through out that, he provides the students ways of thinking, makes them challenge the problems and be enthusiastic, gives them the opportunity to comment on some obstacles (that naturally arise or deliberately created as a result of the direct interaction and discussion) and on some new and further assumptions. We do not want to overstate the words here, but one more thing can be addressed; for a practiced teacher, a direct feedback can be obtained from the students reactions (simply by reading their faces), so that he can judge if they get interested or bored, whether they do or do not understand the topic, and if they are keeping up with or getting lost.

2.3. **Poor deep understanding.** For an online mathematics course, the claim of being providing deep understanding is stripped of truth. Some authors [1, 6] claim that e-learning may induce deep understanding and strong retention. They support their views by a study run at the University of Helsinki in fall 2004: The basic course in calculus was offered in the traditional way and as a fully online course. The students of the online trend did all of their study through the web, but the examination was only as usual, traditionally. The results of the studentss achievement are given in Table 1 below

From our point of view, the result is not surprising. On the contrary, the online students might be expected to perform even better. The courses in calculus focus on collecting information, solving exercises, verifying facts, and at most proving some simple theorems. So, as much the students are acquainted with the materials (reading and practicing) as much they perform better,
Table 1. The result of the basic course in calculus taken by two groups, online and offline at the University of Helsinki in fall 2004.

|                        | Online | Offline |
|------------------------|--------|---------|
| Retention Rate         | 66%    | 62%     |
| Average of students passing the course (out of 24) | 12.74  | 11.74   |

which is the main feature of the online courses where more study materials can be distributed. Besides that, the students have more time compared to those who are enrolled in the traditional courses to devote to the course materials. To be more precise, let us classify the calculus course corresponding to learning approaches. In the figure below, we divide the approaches of learning in an inverted hierarchy diagram. The first two from above are what we call the surface levels, the third is the first intermediate level, the two coming after is the second intermediate level, and the last two are the deep levels. The calculus course is at the surface levels, and if it extends deeper it mostly touches the first intermediate level. The students there are not required to interpret or analyze, instead they are assumed to remember and practice, and as much they do so, as much they perform better. So, as a result, the major performance of such course is proportional to how much time it is being devoted. The role of the teacher here, in the courses with surface levels of learning, is to solve as many examples as he can, rather than going deeper in the material, simply because there are no goals of such courses that touch the second intermediate level.

Figure 1. The levels of learning approaches according to mathematical perspective.

For advanced (high levels) courses, the picture is different, the intermediate and deep levels of learning are emphasized, and the tutor is required to come up with the students to higher levels of thinking which can not be achieved by distributing the course materials. As we mentioned before, the lecturer role here is not only to solve or to prove some facts, but also to provide ways of thinking and strategies of treatment as he can, e.g., assume some artificial obstacles and let
the students provide their own views before he directs them to the right approach, which may
be not unique. Consequently, the students are challenged and become enthusiastic when they
share and directly examine their views with the teacher in an interesting environment using
the simplest and the most preferable materials, the shocks and the board. Parallel to that,
the teacher can change, modify, and simplify his way of explanation depending on the direct
feedback he obtains by noting the students reactions.

3. THEORIES OF E-LEARNING MATHEMATICS COURSES

There is no resort of avoiding the technology facilities in the educational system; they ease
handling many processes and provide simpler and faster trends. Meanwhile, careful awareness
should be stressed from passive usage of them. So, the use of technology in the learning system
should be in a fluent way, keeping it as simple as possible while providing advanced functional-
ities. Given that, taking into account the individual differences of the students and freezing all
other factors of learning, still a question remains: Do all courses undergo online transformation,
totally, partially?

Concerning mathematics courses, the capability of the online applicability varies depending
on the levels of learning approaches and the study levels. Below are some theories formulated
according to our perspectives.

**Theory 1:** There are courses that can be completely transformed to online mode.

All courses that are not exceeding the first intermediate level of learning approaches can be
completely given online. The academic achievement is proportional to the time devoted to read-
ning and practicing the course materials. Such as courses are Calculus (I, II, III), the introductory
courses to probability, and the first course in ordinary differential equations.

**Theory 2:** Problem-based courses and project-oriented courses can be fully given online.

These courses are based on the fact that learning mathematics is achieved by interpreting not
by absorbing information.

In the above two theories, the teacher should convert his role from a lecturer to a coach.

**Theory 3:** Most courses have no tendency to totally online transformation.

This is actually the feature of mathematics courses, which arises from the fact that the learn-
ing approaches of most mathematics courses are exceeding the second intermediate level.

**Theory 4:** There exists no course that does not admit at least online partial transformation.

This fact is obvious per se.

The most powerful way to let the online strategy play a role in the learning process is to apply
it partially. Major parts of most mathematics courses undergo the advanced levels of learning
approaches, where the role of the traditional lectures is crucial and can not be replaced. To this
end, the lecturer can vary (or more appropriately can merge) between the two modes in the
same course, i.e., using blended-learning (b-learning), see e.g. [1, 8].

4. What is Blended Learning

The educational system is part of our life that can not be annexed from the revolution of
technology and of one of the most benefited from the technological achievements. Thus, it is
not possible to ignore the crucial role of the rapid acceleration of the technological advancement
in the learning system. Consequently, it is wise to mix between the two modes of the learning
system, the online and offline, which is the well-known blended learning. According to Singh [7]

"Blended learning mixes various event-based activities, including face-to-face classrooms, live
e-learning, and self-paced learning. This often is a mix of traditional instructor-led training,
synchronous online conferencing or training, and asynchronous self-paced study."

By using blended learning (online partial transformation), the disadvantages of both learning
modes can be get rid of, at the same time, most of their advantages are preserved ; whereas "a
single mode of instructional delivery may not provide sufficient choices, engagement, social con-
tact, relevance, and context needed to facilitate successful learning and performance" according
to Singh [7] . More about blended learning can be found in ,e.g., [3, 4, 8]

For online partially transformed courses, it is quite important to decide when to use the tradi-
tional classes (lecturing) and when to convert to the online mode. Also, to meet the beneficial
goals of the online transformation, it is not recommended to over alternate between the two
strategies. For that, we recommend to start the course with the conventional mode, then switch
to e-learning, finally switch back to the starting mode. We will abbreviate it as fnf-learning
(offline-online-offline learning). Before proceeding, to study this system in appropriate way we
will introduce some notations and definitions. By the gap, we mean the middle stage of fnf-
learning, i.e., the part of the course that is given online. Let the size of the gap, denoted by
$S_{gap}$, be the size of the course part that is given online. Let also $L$ denote the movement from
the lowest level towards the deepest level of the learning approaches, and $T_L$ be the transforma-
tion of the interval (lowest level , deepest level) into the interval $(0, \infty)$, so that the movement
from the lowest level to the deepest is equivalent to the movement from 0 to $\infty$. Further, let
$E$ be the length measure, clearly that $E(T_L) \to \infty$ as $L \to$ deepest level. In the same way we
introduce the variable $s$ to denote the movement toward higher levels in the study (i.e., say
$s$ moves from the first year of the Bachelor program and moving higher). Let $T_s$ be another
transformation (with functionality similar to $T_L$) connects the interval of $s$ with $[0, \infty)$. One
can also find that $E(T_s) \to \infty$ as $s \to$ highest study level.

As more the level in the learning approaches getting higher as much we need to use the offline
mode of learning. This means that $S(gap)$ is inversely proportional to $E(T_L)$, which is denoted
by $S(gap) \propto \frac{1}{E(T_L)}$. Also, as the study levels getting higher, the learner will have acquired enough
techniques and ways of thinking to independently pursue his study. This means that we can
completely switch to the online mode regardless the learning levels, thus, $S(gap)$ is proportional
to $E(T_s))$. Taking into account the above formulations, together with Theory 4, one can derive
a formula for $S(gap)$. Using the notations $S_g := S(gap)$, $x = E(T_s)$, and $y = E(T_L)$, then such
formula might be in the form

$$S_g(x, y) = e^{rx} + k$$
The constant $c$ is a positive real number called the difficulty coefficient of mathematics, $k$ is a positive real number represents the minimum size of the online part that the course can admit, $m$ is a positive integer, and $r \in (0, 1)$. By the formula above we do not intend to give $S_g$ the exact form, but we want to indicate the possibility of giving it a mathematical expression that can be applied for all courses. Note that each mathematics course admits at least a partial online transformation that is the value $k$ and this is harmonizing with Theory 4, on the other hand, each mathematics course has a particular $k$. Also, the difficulty coefficient, $c$, of each mathematics course depends on the topics and contents of that course, which means that each mathematics course has a different value of $c$. The constants $r$ and $m$ are independently chosen for each mathematics course.

Based on the above formula for $S_g$, we can formulate the following theorems

**Theorem 1.** Regardless the learning level ($y$), if the study approaches the highest level ($x \to \infty$), then the course can completely be given online.

**Theorem 2.** Regardless the study level, if learning approaches the lowest level ($y \to \infty$), then the course can completely be given online.

**Theorem 3.** If learning tends to the lowest level, and if the study becomes closed to the lowest level, then the course admits the $k$ value of online transformation.

Therefore, fnf-learning is flexible and can be applied for almost all courses. It gives the opportunity to efficiently match the rapidly accelerated digital technology, while keeping a live and direct interaction in the learning system. Moreover, particularly in mathematics, fnf-learning maintains the traditional mode which is of the most importance to feed the students with the logical mathematical thinking, at the same time it gives the students opportunity to develop their own thinking, consequently providing them with self-dependence and confidence.

**Conclusion.**

For any course in the educational system, efforts and deep study are needed to investigate the capability of the online transformation. According to the study, for mathematics courses that can be completely given online, we do admit that it is not wise to void the course from the traditional orientation. Nevertheless, in applying online partial transformation, other aspects should be considered such as individual differences (the students preferential to online or traditional modes, and the student ability of using the required technology), the availability and accessibility of the technology, the time and cost, and the social aspects. The latter is of significant importance which is being neglected. In the regions of the world, where the social activities are dying out, we should be aware of the intense and numerous digital transformation of our system of life. Thus, by complete transformation to the online mode, we freeze, and even kill, the opportunity of students meeting, which is a purpose of the educational process that also aims to increase the social interactions (f2f and not online interactions) among the people. Thus we exaggerate the problem instead of solving it.

**References**

[1] S. Descamps, H. Bass, G.B. Evia, R. Seiler, and M. Seppälä, *e-Learning mathematics*, Conference of Spanish mathematics deans, 2006.

[2] L-E. Jonsson and R. Säljö, *The Online Seminar as Enacted Practice*, Conference paper, ECER, 2008, Göteborg.
[3] B. Khan, *Web-based training*, Educational Technology Pubns, 2001.
[4] B. Khan, *Web-based instruction*, Educational Technology Publications Inc, 1997.
[5] P. Ramsden, *Learning to teach in higher education*, Routledge Falmer, London and New York, 2003.
[6] M. Seppälä, O. Caprotti, and S. Descamps, *Using web technologies to teach mathematics*, Departments de MatemÃtica Aplicada. Journal papers, 2005.
[7] H. Singh, *Building effective blended learning programs*, Educational Technology, 43(2003), pp. 51-54.
[8] B. Yushau, *The effects of blended e-learning on Mathematics and computer attitudes in pre-calculus algebra*, The Montana mathematics enthusiast, 3(2006), pp. 176-183.