Parameters and pitfalls in dark energy models with time varying equation of state

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ABSTRACT

Are geometrical summaries of the CMB and LSS sufficient for estimating cosmological parameters? And how does our choice of a dark energy model impact the current constraints on standard cosmological parameters? We address these questions in the context of the widely used CPL parametrization of a time varying equation of state $w$ in a cosmology allowing spatial curvature. We study examples of different behavior allowed in a CPL parametrization in a phase diagram, and relate these to effects on the observables. We examine parameter constraints in such a cosmology by combining WMAP5, SDSS, SNe, HST data sets by comparing the power spectra. We carefully quantify the differences of these constraints to those obtained by using geometrical summaries for the same data sets. We find that (a) using summary parameters instead of the full data sets give parameter constraints that are similar, but with discernible differences, (b) due to degeneracies, the constraints on the standard parameters broaden significantly for the same data sets. In particular, we find that in the context of CPL dark energy, (i) a Harrison-Zeldovich spectrum cannot be ruled out at 2\textsigma levels with our current data sets. and (ii) the SNe IA, HST, and WMAP 5 data are not sufficient to constrain spatial curvature; we additionally require the SDSS DR4 data to achieve this.

1. Introduction

A number of observations (Perlmutter et al. 1999; Riess et al. 1998; Garnavich et al. 1998; Knop et al. 2003; Tonry et al. 2003; Riess et al. 2004; Astier et al. 2006; Wood-Vasey et al. 2007; Hicken et al. 2009) have established that the expansion of the universe is accelerating. The cause is usually attributed to a currently dominant component called dark energy. Current data is consistent with a standard cosmological model called ΛCDM, with dark energy in the form of a cosmological constant Λ. However, dark energy might, in fact, be a dynamical component. Indeed, there exist numerous models of cosmology which produce the observed acceleration, either by postulating the existence of one or more otherwise unobserved fields, or as the effects of a departure of gravity from General Relativity at large scales that cannot be ruled out by current data. Therefore an important objective of current and future observational efforts is to study the acceleration of the universe in different ways and detect departures in the behavior from that expected in a standard ΛCDM model. To this end it is usual to parametrize dark energy as a ‘fluid’ with its equation of state (EoS), and the speed of sound in the fluid specified independently. Generally such a description encompasses a large variety of physical models if the equation of state is assumed to depend on the density, and the speed of sound depends on both the background density and the wavelength of perturbation. The idea is that constraining these phenomenological parameters of a fluid will narrow down the class of physical models causing the acceleration, and in particular, study
the differences with $\Lambda$CDM.

A specific time dependent parametrization of the EoS, and a constant speed of sound describes a subclass of these phenomenological models. A simple example is the CPL parametrization of the equation of state (Chevallier & Polarski 2001; Linder 2003)

$$w(a) \equiv w_0 + w_1(1 - a)$$

of a non-interacting dark energy, which has been adopted by the Dark Energy Task force (Albrecht et al. 2006) in determining the relative importance of future experiments studying dark energy. Conveniently, it includes the case of a constant EoS (wCDM) with $(w_0 = w, w_1 = 0)$, and the $\Lambda$CDM model $(w_0 = -1, w_1 = 0)$.

Parameter constraints on dark energy parameters with different time varying equation of state, including the CPL parametrization have been investigated in the context of similar recent data sets (Wright 2007; Wang 2008; Lazkoz et al. 2008; Davis et al. 2007). These analysis use certain summary parameters (the shift parameters for supernovae (Kowalski et al. 2008). We point out the differences in dark energy parameter constraints computed from the two approaches. Evidently, dark energy with a time varying equation of state behaves very differently from $\Lambda$CDM cosmology, where the fractional density of dark energy is tied to acceleration at a particular redshift. Even in the simple and comparatively benign CPL model this gives rise to counter-intuitive effects on general parameter constraints, which we study here. While we discuss the behavior of generic time varying EoS, we restrict our calculations to the specific case of the CPL parametrization. This is an appropriate example, not only because of the current consensus of using the CPL as a standard (Albrecht et al. 2006), but also because it is a fairly benign evolution. Model independent constraints one might hope to study will include much stiffer variation of the EoS, where the dynamical effects we describe (see Sec. 2) could be more pronounced possibly leading to larger differences with parameter constraints computed from summary parameters.

In Sec. 2 we discuss the behavior of different relevant regions of the CPL parameter space, their observable signatures and their possible consequences for parameter estimation. In Sec. 3 we describe the details of our method to investigate parameter constraints derived using summary parameters and power spectra. The results of our investigations are presented in Sec. 4. We discuss our conclusions and possible implications in Sec. 5.

2. Characteristics of Dark Energy with Time Varying Equation of State

The density of a dark energy component with a variable equation of state $w(a)$ evolves with scale factor as $\rho_{DE} \sim a^{-3(1+w_{eff}(a))}$, while the pressure $P(a) \sim (1 + w(a))\rho_{DE}(a)$, where

$$w_{eff}(a) = -1 + \int_{a_0}^{a} da \frac{(1 + w(a))/a}{ln(a)}$$

with the last expression being valid for a CPL EoS. Dark energy of current density (in units of critical density today) $\Omega_{DE}$ contributes an amount $\ddot{a} = aH_{0}^{2}\Omega_{DE}(1 + 3w(a))a^{-3w_{eff}(a)+1}$ towards the acceleration of the universe at a redshift $z = 1/a - 1$, while its density in units of matter density grows as $a^{-3w_{eff}}$. Thus, the presence of dark energy at a particular redshift modifies the background expansion. This (along with the presence of curvature) affects the CMB and the matter power spectrum (a) geometrically by altering the angular position of the peaks and (b) dynamically by altering the magnitude of the spectrum. Dark energy is believed to be smooth (ie. its density perturbations do not grow at scales smaller than the Hubble scale), leading to less clustering of density perturbations at a particular redshift if a significant fraction of the background density is made up of dark
energy. The gravitational potentials also evolve differently because the background evolves differently from a matter dominated universe. This leads to weaker sourcing of the growth of perturbations, thereby suppressing the number of galaxies formed at a particular redshift reducing the matter power spectrum inferred from galaxy surveys (Doran et al. 2001b). The change in gravitational potentials also affects the CMB power spectrum through the Integrated Sachs-Wolfe (ISW) effect. On a plot of the matter power spectrum or CMB power spectrum the geometrical effects show up as horizontal differences (change in scales), while the dynamical effects show up on the vertical differences (magnitudes).

For a dark energy characterized by a constant EoS, \( w_{\text{eff}}(a) = w(a) = w \). To explain the acceleration observed today from SN IA data, \( w < -1/3 \) and is close to \(-1\) if all data sets are considered. This also implies that the dark energy density (in units of matter density) grows as \( \approx a^3 \), so that it is negligible at earlier times. Hence, its dynamical effect on the matter power spectrum is small, and its effect on the CMB power spectrum is limited to a late time ISW effect, which only affects the low multipoles of the spectrum. Since the cosmic variance at the low multipoles is high, the observable imprints of a redshift independent equation of state are limited to the geometrical effects. For a redshift dependent equation of state, \( w(a) \) and therefore \( w_{\text{eff}}(a) \) can be quite different from \( w(0) \) for smaller values of \( a \). Hence, it is possible to simultaneously have the observed acceleration due to very negative values of \( w(a = 1) \), while the ratio of densities of dark energy to matter \( \sim a^{3w_{\text{eff}}(a)} \) remains significant at early times, if \( w_{\text{eff}}(a) \approx 0 \). Thus dark energy with redshift dependent EoS can cause acceleration today, and also be non-negligible at early times, thereby suppressing the growth of structures, and affecting the CMB by an Early ISW Effect. It is possible for models with a time dependent EoS to have regions in parameter space which are relatively indistinguishable in terms of the geometric effects, but distinguishable in terms of dynamical effects. This kind of geometrical degeneracy is likely to be more pronounced for parametrizations which allow a strong variation of \( w(a) \), as that would facilitate a quicker transition from an equation of state around \(-1\) to \( 0 \). Here, we will work out the consequences

![Phase Diagram of the CPL model](image)
in the specific example of the widely used CPL parametrization in Eq. (1) which is a fairly gentle variation.

The CPL parametrization is an ad-hoc parametrization which allows the dark energy EoS $w(a)$ (and $w_{\text{eff}}(a)$) to asymptote between two constant values $w_0 + w_1$ and $w_0$ at early and late times. In order to understand the behavior of this parametrization and the physical models it may represent, we can study a phase diagram on the $w_0$, $w_1$ plane. In the upper panel of Fig. 1 we show the evolution of the CPL EoS $w$ (solid lines) and $w_{\text{eff}}$ (dashed). It is useful to study this parametrization as it captures general features of dynamical dark energies, and CPL models that correspond to $(w_0, w_1)$ different from $(-1, 0)$ are definitely dynamical models distinct from a cosmological constant. From the asymptotic behavior of the upper panel of Fig. 1 we can study different phases of the CPL EoS, where the evolution of dark energy can be asymptotically similar to different models of dark energy. This is studied in the lower panel of Fig. 1. Firstly we can separate the regions where the dark energy is in the Phantom phase with equation of state ($w(a) < -1$) which occurs for scalar field theories with tachyonic instabilities, or with non canonical kinetic energy terms inspired by higher derivative theories (Caldwell 2002, Carroll et al 2003). The two black lines (solid) bound the regions where the dark energy behaves like a phantom model ($w(a) < -1$): the parameter region left of the vertical black line is currently in the phantom phase, while the parameter region below the diagonal black line was in a phantom phase at early times. The region above the horizontal dot-dashed blue line will eventually become a phantom model, resulting in a ‘Big Rip’. The region below the horizontal dot-dashed line will eventually become ‘normal’ with $w > -1$. The region between the dot-dashed blue line and the diagonal black line (labeled quintessence), to the right of the vertical black line is the only region where $w(a) > -1$ at all times. Thus this is the only phase which may be similar to models of a single, non-interacting stable canonical scalar field. The North Western and South Eastern quadrants (shaded yellow) marked out by the solid black lines exhibit the crossing behavior from the ‘phantom phase’ to the ‘normal phase’ where $w$ crosses $-1$; this leads to instabilities in the evolution of perturbations.

Fig. 2.— The upper panel shows the ratio of the densities of dark energy to the density of dark matter in models selected from different parts of the phase diagram in Fig. 1. The middle panel shows that even though dark energy dominates at large redshifts for the (Big Rip, Phantom) kind of dark energy models (blue line), it does not accelerate the universe at early times. The lower panel shows the ratio of the total density of components (matter, dark energy, relativistic components) in units of the total density assuming $(w_0, w_1)=(-1, 0)$ of the models selected in Fig. 1.
at the crossing (see Sec. 3.2 and references therein for details). The South Eastern quadrant shows a rapid change of $w(a)$ at recent times, but decays rapidly with redshift. The dashed red line marks the boundary at which $w(a) = 0$ at early times; the regions above this line have significant dark energy density at early times like recombination. In order to study the distinct effects, we mark five points on this phase diagram. The brown point represents a cosmological constant with $\Omega_m = 0.3, \Omega_{DE} = 0.7, H_0 = 72 \text{km}/\text{s}/\text{Mpc}$. The other points were chosen from our chains representing the posterior of current CMB, HST and SNe data using summary parameters (see Sec. 3.1 for details). The red filled circle is a ‘Phantom’, the black diamond is ‘Burst DE’, the green triangle is similar to a ‘Quintessence’ till now, while the blue star will have a ‘Big Rip’ and is chosen to have significant early dark energy. We will stick with this color code in discussing effects, and use these names to label the plots when possible.

The effect of dark energy parameters of these distinctive types can be seen in Fig. 2, where we show the impact on the background for the parameter points marked out in Fig. 1. The upper panel shows the ratio of dark energy density to dark matter density as a function of redshift. We notice that the dark energy density decays with redshift, except for the model ‘Big Rip’, which had significant early dark energy (cf lower panel Fig. 1). The middle panel shows the acceleration of the universe for dark energy with the same parameters. Clearly, the ‘Phantom’ and ‘Big Rip’ models with very low values of $w_0$ exhibit a super-accelerated phase. It is interesting to note that even for the ‘Big Rip’ where dark energy density is significant at early times, the universe is decelerating before a redshift of 2 because its EoS (Blue solid curve in the upper panel of Fig. 1) keeps increasing. Hence for these models with significant early dark energy, the universe can be ‘dark energy dominated’ without accelerating, with the effect of dark energy on background expansion being similar to matter. The lower panel shows the evolution of the total density of matter, dark energy and relativistic components. $\rho_{tot} = \rho_m + \rho_{DE} + \rho_r$ with redshift, in units of the corresponding quantity assuming that dark energy was a cosmological constant.

The effect of these parameters on observables is

![Fig. 3.— Effect on observables for models with different values of dark energy parameters. The parameters are the points marked in the phase diagram of Fig. 1 chosen from different regions marked on the phase diagram.: The upper panel shows the CMB power spectrum for the parameters shown. The middle panel shows the matter power spectrum as a function of wave-number in units of $h^{-1} \text{Mpc}$. The lower panel shows the change $\Delta m$ in the apparent magnitude with redshift $z$. However, we use a likelihood which is analytically marginalized over the absolute magnitude of the supernovae; hence we only show $\Delta m$ up to a constant for each cosmology. The horizontal dashed black lines are at $\pm 0.15$ which is the usual intrinsic dispersion for supernovae.](image-url)
studied in Fig. 3 for the models chosen in Fig. 1. Since we are interested in comparing the effect on the magnitudes of the spectra due to dark energy, we set the amplitude of the primordial power spectrum to be the same for each model. This does not change the value of the posterior (using Likelihood B, see Sec. 3.1) probability of the model plotted, as the posterior is independent of the amplitude. The upper panel shows the CMB power spectrum, the middle panel shows the matter power spectrum, and the lower panel shows the apparent magnitude of the SNe IA, which is a purely geometrical effect. The difference in positions of the peaks in both the CMB and the oscillations in matter power spectrum, as also the differences in the redshift-magnitude plot is due to geometrical effects. The difference in the magnitudes of the power spectrum is due to dynamical effects that are not captured in summary parameters (see Sec. 3.1). It is interesting to note that the models for which the CMB power spectrum is enhanced have suppressed matter power spectrum.

The comparison of the difference of the CMB and Matter Power Spectra inferred from the data with their theoretically computed counterparts with the scale of expected deviations leads to constraints on cosmological parameters. It is possible that a subset of this information pertaining to specific features in the spectra is almost as useful in constraining parameters. For example, many features of CMB power spectra depend on the cosmological parameters through very specific functions of the these parameters (Hu et al. 2001, Hu & Dodelson 2002, Doran et al. 2001a, Doran & Lilley 2002). This motivates the idea of using CMB shift parameters to summarize the information content of the CMB anisotropies, introduced in Bond et al. (1997) in the context of forecasting for standard cosmologies. Further work on this subject (Elgarøy & Multamäki 2007, Wang & Mukherjee 2007) suggests that one can summarize the information the in the spectra efficiently in terms of the summary parameters $R, l_a$, which relate to the position of the first peak of the CMB spectrum, and the spacing of the peaks due to acoustic oscillations. Recently such summary parameters have become popular in studying the constraints on dark energy parameters. If the effect of dark energy is mostly geometrical, it is tempting to speed up the calculation by summarizing the CMB/LSS data in terms of a few geometrical summary parameters that describe these effects, instead of going through the time consuming process of theoretically computing the angular power spectrum.

This approach of using summary parameters has been studied in Wang & Mukherjee (2007), Elgarøy & Multamäki (2007), Doran & Lilley (2002), and the procedure nicely outlined in Komatsu et al. (2008). One finds samples of the posterior distribution of the data set concerned for cosmological models with a specific form ($\Lambda$CDM) of dark energy. One then uses these samples to estimate the set of summary parameters over the posterior distribution as well as the covariance over these summary parameters. Wang & Mukherjee (2007) also show that the summary parameters are comparatively weakly correlated to the other cosmological parameters. One then approximates the likelihood of the CMB data set, by a Gaussian distribution over these summary parameters. It can be seen that there are three crucial assumptions in this procedure:

1. The Likelihood of the CMB spectra are well described by CMB summary parameters.
2. The mean and covariance of the summary parameters for a particular data set do not change appreciably when the model space is enlarged, and do not develop correlations.
3. The summary parameters are relatively weakly correlated with the other cosmological parameters.

We examine these by studying the differences in the parameter constraints in comparison of the power spectra and the use of the summary parameters.

### 3. Method

We study parameter constraints using a suitably modified version of the Markov Chain Monte Carlo (MCMC) engine CosmoMC (Lewis & Bridle 2002) to explore the Bayesian posteriors for the cosmological parameters. We assume an isotropic and homogeneous universe with dynamics dictated by standard general relativity with the densities of the background components to be determined, take the non-baryonic dark matter to be
Table 1: Parameters used in the MCMC: Cosmological parameters used and the lower and upper limit on the flat priors on these parameters. For a $w$CDM model, $w_1$ is fixed to be 0.

| $\Omega_b h^2$ | $\Omega_c h^2$ | $\theta$ | $\tau$ | $\Omega_k$ | $w_0$ | $w_1$ | $n_s$ | $\log(10^{10} A_s)$ |
|-----------------|-----------------|---------|--------|------------|--------|--------|--------|------------------|
| (0.005,1)       | (0.01,0.99)     | (0.5,10)| (0.01,0.6) | (-0.2,0.2)| (-3.1,5)| (-7.0,4.5)| (0.5,1.5) | (2.7,4) |

...entirely cold and neglect effects of neutrino mass, assuming $N_{\text{eff}} = 3.04$ species of massless neutrinos. The current background density of photons is also assumed to be fixed, by neglecting any error-bars associated with the measurement of the CMB temperature. We then explore possible values for the background densities of other components (baryons, cold dark matter, curvature) with broad, flat priors over $\omega_b, \omega_c, \Omega_k$, where $\Omega_i$ is the density of the component in units of the current critical density, and $\omega_i = h^2\Omega_i$, with the Hubble constant $H_0 = 100 h$ Km/s/Mpc. The primordial perturbations are assumed to be adiabatic and Gaussian distributed with a power spectrum $P_l(k) = A_l(k/k_p)^{(n_l(k)-1)}$. We neglect the effect of tensor perturbations setting the ratio $r$ of its amplitude $A_s$ to that of the scalar perturbations $A_s$ to be zero, and $n_t = 0$. The scalar spectral index $n_s$ is assumed to be scale independent (running of spectral index) $n_r = 0$ and we choose a pivot scale $k_p = 0.05/Mpc$. We also ignore the effect of the the SZ amplitude. We allow for a single re-ionization taking place at an optical depth of $\tau$, and $\theta$ parametrizing the angle subtended by the sound horizon at the surface of last scattering instead of the Hubble constant $H_0$. Therefore, our model space has the fixed values $\omega_\nu = n_t = n_r = r = 0$, and priors on the parameters $\{\Omega_b h^2, \Omega_c h^2, \theta, \tau, \Omega_k, w_0, w_1, n_s, \log(10^{10} A_s)\}$ summarized in Table 1. The Markov chains are assumed to have converged when the $R - 1$ statistic had been below 0.03 for a few tens of thousand chain steps; this results in a final $R - 1$ statistic of about $\sim 4 \times 10^{-3} - 1 \times 10^{-2}$.

3.1. Data and Likelihoods

We summarize our usage of different data sets and likelihoods in Table 2 and explain it in detail below.

CMB data: We use the WMAP 5 year data in two different ways:

(A) By using the publicly available WMAP likelihood code (version 3) [Dunkley et al. 2008, Hinshaw et al. 2008, Nolta et al. 2008], which compares the observation to our theoretical computation of the CMB power spectrum. and

(B) By using a Gaussian likelihood in the summary parameters $\{l_A, R, z_\star\}$ as recommended by Komatsu et al. (2008)

\[ l_A = (1 + z_\star) \frac{\pi D_A(z_\star)}{r_s(z_\star)} \]

\[ R = \sqrt{\Omega_m H_0^2 (1 + z_\star) D_A(z_\star)} \]  (2)

where $z_\star$ is the redshift of the surface of the last scattering computed from the fitting formula [Hu & Sugiyama 1996] in terms of only densities of baryons ($\omega_\nu$) and matter ($\omega_m$) with the mean of the distribution taken to be the maximum Likelihood values in Table 10, and the inverse covariance matrix in Table 11 of Komatsu et al. (2008). There are slight differences in the literature about how the parameters $l_A, R$ are best defined, and we adopt the definition of Komatsu et al. (2008) as we use their numerical values for the likelihood.

Galaxy Power Spectrum Data: We use the galaxy power spectrum data from the Luminous Red Galaxy (LRG) sample of the Sloan Digital Sky Survey DR4 (SDSS)

(A) By using a modified version of the publicly available likelihood code which compares the matter power spectrum inferred from the data with the theoretical computation after analytic marginalization over a scale independent linear bias [Tegmark et al. 2006]. We modified the code in order to recomputed the geometric scaling at the redshift of the sample related to

\[ D_V = \left( (1 + z)^2 D_A^2(z) cz/H(z) \right)^{1/3} \]

for the CPL model, and also to compute the growth function to account for the scale independent change in the matter power spectrum at
the mean redshift 0.35 of the LRG sample from the matter power spectrum at a redshift of $z = 0$. (B) By using only the geometric distance measure $r_s(z_d)/D_V(z)$, where $r_s$ is the sound horizon at the redshift of drag epoch $z_d$ where the baryons were released from the photons. Following Komatsu et al. (2008), the redshift $z_d$ is computed through a fitting function (Hu & Eisenstein 1998) which again depends only on $\omega_b$ and $\omega_m$. As recommended, we use a Gaussian Likelihood with a mean of 0.1094 and a standard deviation of 0.0033.

**Supernovae Data:** We use the Union data set [Kowalski et al. 2008] which is a compilation of 307 supernovae IA discovered in different surveys. This combines high redshift supernovae by the ESSENCE, SNLS and HST Goods surveys, with low redshift ones ($z \approx 0.02 - 0.1$) Astier et al. (2006); Perlmutter et al. (1999); Riess et al. (2004) using a weighting scheme to take into account the heterogeneous sources. The Likelihood is Gaussian distributed in the magnitude space and includes covariance contributions due to systematic effects. However, we ignore lensing of supernovae.

**HST:** We also incorporate the results of the Hubble Space Telescope Survey measurements of the Hubble constant [Freedman et al. 2001] as a Gaussian prior on the value of the Hubble constant $H_0 = 72 \pm 8 \text{km/s/Mpc}$.

**Big Bang Nucleosynthesis:** The primordial abundance of light nuclei, determined at the time of Big Bang Nucleosynthesis (BBN) depends on the baryon to photon ratio, as well as the expansion rate at this time (Amsler et al. 2008; Cyburt et al. 2002). Since, in time varying equations of state dark energy models, dark energy can be non-negligible around the time of BBN ($z \sim 10^7$), we need to examine the effect of BBN on both these parameters. Among the abundances of light elements, the abundance of primordial Helium is most sensitive to the expansion rate during BBN; the abundance of primordial Deuterium, while almost insensitive to the expansion rate, is extremely sensitive to the baryon to photon ratio. We ignore the abundance of $^7\text{Li}$ even though it is extremely sensitive to the baryon photon ratio, because of the controversies regarding systematic uncertainties in determining the observed abundances of $^7\text{Li}$, stemming from uncertainties in measurement of effective temperature of stars, or an unaccounted correlation between the metallicity and estimated abundance [Cyburt et al. 2008]. We constrain the dark energy density at the time of BBN by using the fit equations for the primordial abundance of Deuterium, and Helium from (Simha & Steigman 2008) in terms of the baryon-to-photon ratio $\eta_{10}$, and the ratio $\eta$ of the Hubble parameter at times of BBN ($z \sim 10^7$) with a Hubble parameter for a universe completely dominated by relativistic degrees of freedom (photons and massless neutrinos). The parameter $\eta_{10}$ depends on the mass fractions of the light nuclei, but the dependence is extremely weak. We adopt the values of $\eta_{10} = 273.9 \omega_b$ for our Likelihood calculations. We ignore the theoretical errors in the fitting functions, and write a Gaussian likelihood using the error estimates of observed abundances for De and $^4\text{He}$. We also note that since the $^4\text{He}$ fraction monotonically increases with time, the lowest values detected are a robust upper bound to the Helium fraction. We impose a much weaker hard prior on the equation of state parameters, as will be described and further justified in the Sec. 3.

| Data Set I                  | Data Set II                  | Likelihood A                  | Likelihood B                  |
|----------------------------|----------------------------|----------------------------|----------------------------|
| WMAP5 + SNE + HST          | WMAP5 + SNE + HST + SDSS + BBN | Summary Parameters           | Power Spectra               |
|                            |                            | $l_A, R_A, z_\star, D_A$      |                            |

3.2. Theoretical Computation of Power Spectra

The Likelihoods (A) for the CMB and Matter Power Spectra data require theoretically computed values of these power spectra. We evaluate these by modifying the background expansion, and the perturbation equations for this Dark Energy model using CAMB [Lewis et al. 2000],

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**Table 2: Definitions of data sets I and II and Likelihoods A and B**

| Data Set I                  | Data Set II                  | Likelihood A                  | Likelihood B                  |
|----------------------------|----------------------------|----------------------------|----------------------------|
| WMAP5 + SNE + HST          | WMAP5 + SNE + HST + SDSS + BBN | Summary Parameters           | Power Spectra               |
|                            |                            | $l_A, R_A, z_\star, D_A$      |                            |
which uses RECFAST \cite{Seager2000, Wong2007} to compute the recombination history. The CPL parametrization allows for the case where the dark energy equation of state is less than $-1$, and allows crossing of $-1$ during the evolution. For a single non-interacting scalar field, $w \geq -1$. However, lower values of $w$ occur for scalar fields with non-canonical kinetic energy terms, or due to interaction between more than one field \cite{Hu2003, Huey2006, Zhang2006, Li2005, Fang2008}. Thus, we follow the standard practice and do not rule out values of $w(a) < -1$. It is known, that in the fluid model perturbation theory of dark energy, there is a runaway problem associated with the models which allow crossing of $-1$. We follow the prescription of \cite{Huey2004, Caldwell2005}, which essentially assumes that this is a numerical artifact. While, a negative dark energy sound speed would lead to large clustering of structure that is unobserved in data, we find that current data is insufficient to put any meaningful constraints in the open interval $(0,1)$ (in natural units) on the speed of sound $c_s$ for dark energy. Therefore, we fix the value of the speed of sound to a reasonable choice of $c_s^2 = 1$ as would be the case for quintessence \cite{Weller2003, Caldwell2005}.

4. Results

We study the posterior distributions on the cosmological parameters and the joint posterior distributions of pairs of cosmological parameters. Use of the maximal data set available today exploits the complementarity of different probes to obtain the tightest constraints on the parameters. On the other hand, it is useful to examine meaningful constraints from different subsets of the maximal data set. Constraints from subsets allow one to check for self-consistency of cosmological models since different data sets actually constrain different aspects of physics; and also to separate the constraints from assumptions inherent to different data sets. We use two sets of data. (I) WMAP 5 yr data, the Supernovae Union data set, the HST constraints on the Hubble constant, and (II) additionally BBN constraints, and the SDSS LRG power spectrum. The data combination (II) represents the maximal data set we use here. We present the constraints obtained for our maximal data set in Fig. 4.

4.1. Features of the Posterior Distribution

Focusing attention on the two dimensional joint posterior on $w_0, w_1$, the left panel of Fig. 5 shows the posteriors of data set (I), while the right panel shows the posteriors for data set (II). The black (solid) contours are computed by the use of likelihood A, while the blue contours are computed using Likelihood B. The blue contours may be compared to Fig. 1 of Wang \cite{Wang2008}. We note that the posterior distribution is highly non-Gaussian. Firstly, the posterior contours are fairly banana-shaped rather than ellipsoidal without the SDSS data. With the addition of the SDSS data, the elongated banana shape gets pinched off. However, the distribution is still elongated, and even in the high posterior region the posterior peak does not seem to be well-centered. Further, the plots show that the posterior falls abruptly around the blue (dot-dashed) line $w_1 = -w_0$. The diagonal (solid) black line represents the $2\sigma$ limit from BBN constraints: $w_0, w_1$ values lying above the black line are unlikely due to the BBN constraints. The red diagonal line is a hard prior we used to limit the exploration. The use of both Likelihoods (A) and (B) result in a dramatically lower number of points beyond the blue (dot-dashed) line $w_0 + w_1 = 0$, though the change in B is less sharp. This edge in the likelihood, is entirely due to the CMB data and represents the edge of parameter space beyond which the ‘dark energy’ dominates at the redshift of the surface of last scattering, rather than being related to the BBN data. We can see this better in a histogram of asymptotically early equation of state in Fig. 6. It shows the drop in the posterior sample for both Likelihoods B as well as A for data set I (Upper Panel) and data set II (Lower Panel), even though data set I does not include the BBN constraints. The lack of points above the (dot-dashed) blue line, further justifies our use of the hard prior, since the posterior is well disconnected from the hard prior. In our contour plot, we therefore could cut off the posterior contours that we would have drawn by smoothing the posterior densities with a Gaussian kernel. This sharp edge in the 2D joint posteriors is unlikely to go away with the addition of further
Fig. 4.— Marginalized one dimensional and two dimensional joint posterior distributions of our maximal data set (WMAP5, SDSS, HST, SNe, and BBN) on the cosmological parameters. The solid lines show the posteriors, while the dotted lines show the mean likelihoods.

Fig. 5.— 2D Joint Posterior distributions given (left) data set I (WMAP5, SNe, HST) and (right) data II (WMAP5, SDSS, SNe, BBN, HST). The black (solid) lines were computed using the Likelihoods A comparing spectra, while the blue (dash-dot) lines were computed using summary parameters B.
Fig. 6.— Binned one dimensional posterior on the dark energy EoS at very early times $w(0) \rightarrow w_0 + w_1$ marginalized over all other parameters showing the sudden drop in the posterior for values of $w(0) + w(1) = 0$. The upper panel shows the posterior given the combination data set (I), while the lower panel shows the posterior given the combination of data sets (II). The blue histogram shows the distribution due to the likelihood (A) while the magenta histogram is computed with the use of the likelihood (B).

Next, we look at the constraints on the dark energy parameter $w_1$. In Fig. 7 we show the one dimensional marginalized posterior distribution on $w_1$ computed from the data combinations I (left) and II (right) described above, according to the likelihoods A (black solid) and B (dot-dashed blue). The posteriors computed using Likelihood B may be compared to the Fig. 2 of Wang (2003) where they use likelihoods using $z_*$ as a parameter; as expected these posteriors match to extremely small differences that may be attributed to the slightly different data or numerical procedures. We note that these posteriors are asymmetric, a result of the fact that significant early dark energy is essentially ruled out by the CMB data. From our one dimensional distributions, we can see that while the approximate posterior (B) computed by using the summary have the same shape as the posteriors (A) computed by comparison of power spectra, there are significant differences between likelihoods A and B in the extent of the tails: the posteriors using the power spectra are sharper and narrower than the posteriors.
using the summary parameters. For our maximal data set (II), this translates to the tails (computed by using \((B)\)) extending about twice as much as the tails using summary parameters \((A)\). From the one-dimensional distributions, we see that there is a sharp edge in the distribution of \(w_1\) at the higher tail, while low \(w_1\) values are allowed. The edge at the tail is related to the edge in the joint posterior. The use of summary parameters using the likelihood \((B)\) has a similar effect, except the distributions are broader, and the distinction more significant in the low \(w_1\) tails.

From both the one dimensional and two dimensional posteriors, we note (I) allow fairly low values of \(w_1\) at levels, which are ruled out by the SDSS data (II), or if the model is constrained to be flat. This is because (I) allows models with low values of \(w_1, H_0,\) and \(\Omega_k\), which are ruled out by the simultaneous use of the SDSS data. This can be seen by studying the correlations of \(w_0, w_1\) shown in Fig. 8.

### 4.2. Comparison of Power Spectra and Summary Parameters

The differences between the posterior distributions due to the use of different likelihoods \(A\) and \(B\), are most clearly studied in a binned (unsmoothed) density plot over all the chains, since various differences can arise due to smoothing prescriptions inherent in making contours. We choose to study the joint posterior in the \(w_0, w_1\) plane due to its importance in classifying dark energy experiments ([Albrecht et al. 2006](#)). We present the differences as a density plots for both data set combinations I (top panel) and II (bottom panel) in Fig. 9. We can see that the differences are most appreciable for models, where the equation of state is close to 0 (like matter) at asymptotically early times, and for the tail, where \(w_1\) has low values. The density plots on the left due to likelihood \(A\) are much sharper and narrower than the corresponding plots using \(B\) on the right.

We also study the marginalized one dimensional posteriors in Fig. 10 on all other parameters. From Fig. 7 we see that with the maximal data set, the posterior due to Likelihood \(B\) (black) extends to about twice the tail of Likelihood \(A\) (blue) in \(w_1\). From Fig. 10 we show the 1 dimensional marginalized distributions on all other parameters from the comparison of spectra (ie. by using likelihood \(A\) (blue) and the summary parameters (red) (ie. by using likelihood \(B\)). We see that the posterior distributions for \(w_0, H_0\) are well approximated by the summary parameters. On the other hand, comparing the blue and red curves in Fig. 10 we see that the posteriors on all other quantities \((\Omega_m, \Omega_k, \Omega_{DE}, \omega_b, H_0)\) are shifted significantly in comparison to the width of the distribution when the summary parameters used. As expected, the parameters related to the optical depth of re-ionization, scalar spectral index, and the amplitudes of the scalar fluctuations \((\tau, n_s, A_s)\) are not constrained at all by the use.
Fig. 9.— Density Plots showing the Joint Posterior Distribution on $w_0, w_1$ given the data set I (upper panels), and data set II (lower panels) for likelihoods A (left) and B (right).
of summary parameters resulting in a distribution about as broad as the prior, while the spectra constrain them reasonably well. This is not surprising as all of them affect the shape and normalization of the CMB power spectrum without affecting the distance to the surface of last scattering, or the angular position of the peaks.

4.3. Impact of Relaxing the Dark Energy Parametrization

In general, when a model is relaxed to a more general model, one expects that the constraints on the model parameters could become broader. This happens, when the model parameters are correlated to the parameters that are allowed to float in the more general model, but were fixed in the special model. In our example, $\Lambda$CDM and $wCDM$ are special cases of the CPL model. Hence, we look at how the constraints get broader when $w_1$ is allowed to vary. From Fig. 11 we note the differences in posteriors between a $wCDM$ model (black) and a CPL model (blue). We see that for parameters like $(\theta, \omega, \tau, A_s)$ and derived parameters like $(\Omega_m, H_0)$, the constraints from both a $wCDM$ model and a CPL model are similar. However, for the parameters $(w_0, \Omega_K, \omega_0, n_s)$ and derived parameters $(\Omega_K, \sigma_8)$, there are differences between the posteriors of the $wCDM$ model, and the CPL model. Of these, only the constraints on $w_0$ are well approximated by the summary parameters. We note one cannot study the impact of dark energy on the standard cosmological parameters using summary parameters. These differences are reflected in the joint two dimensional posteriors of the $wCDM$ model (blue) and the CPL model (black) in Fig. 11 which show that the joint constraints on the CPL model are significantly different.

Of these parameters, it is interesting to note that the values of $\Omega_K$ and $n_s$ allowed by the CPL model. Both these parameters are important theoretically as they are strongly related to parameters in inflation. The tail of $n_s$ using the data set (II) are usually considered outside of the 2$\sigma$ limits. Similarly the values of $\Omega_K$ shown in Fig 8 allowed by the data set (I) are usually considered ruled out by the CMB alone. The usual statements are on the basis of $\Lambda$CDM or $wCDM$ models, and this shows that relaxing constraints on the dark energy models can over-ride some of our intuition. The impact of the SDSS and BBN constraints again underline the importance of complementary data sets in this context.

5. Summary and Discussion

Dark energy alters the background expansion of the universe leading to geometric effects on the CMB and matter power spectrum. It also changes the growth of gravitational potentials leading to dynamical effects that modify the magnitudes of the power spectrum. For a constant EoS dark energy, the observed acceleration requires rapid decay of dark energy density with redshift, precluding dynamical effects. However while dynamical effects are relatively unimportant for constant EoS dark energy, they may be important for dark energy with time varying EoS. Hence, for such models, the comparison of power spectra may be able to distinguish between regions of parameter space that are degenerate to dynamical effects

blind summary parameters used in previous studies.

We study this by estimating parameter constraints in a CPL cosmology using two combinations of data sets I and II (see Table 2) WMAP five year data, SDSS LRG data, the Union compilation of supernovae data, the results of HST, and using the fits to BBN constraints. We do this in two different methods (see Table 2) (A) where we compare the observed CMB and matter power spectra to our theoretical computations of these quantities in a cosmology with a CPL parametrized dark energy using modified versions of the publicly available WMAP and SDSS Likelihood codes, and (B) where we use a Gaussian likelihood in the three summary parameters for CMB, and the BAO summary parameters as reported in previous studies in the literature.

Differences between Constraints: Qualitative features in the $w_0, w_1$ joint posteriors are similar when either likelihood (A) or likelihood (B) are used, though there are quantitative differences. The differences in the $w_0, w_1$ joint posterior which is used to compute the dark energy Figure of Merit recommended by the Dark Energy Task force to rank the importance of experiments are studied in Fig. 9. This is an un-smoothed density plot to clearly show the differences between using Likelihoods A and B. Clearly, likelihood B using sum-
Fig. 10.— Marginalized one dimensional constraints on cosmological parameters given all current data sets (II) (WMAP5, SDSS, SNe, HST, BBN) using Likelihoods (A) for a wCDM model (solid, black), (A) for a CPL model (dashed, blue), and (B) for a CPL model (dotted, red)
Fig. 11.— 2D marginalized joint posteriors of parameters not involving $w_1$ in a $wCDM$ model (blue) and a CPL model (black) from a full comparison of power spectra (Likelihood A) using data set II (WMAP5, SDSS, SNe, HST)
mary parameters is good as an approximate likelihood for the CPL model for the purpose of studying dark energy parameters; however the likelihood (A) gives sharper and narrower posterior distributions. We study the differences in one dimensional marginalized constraints on the cosmological parameters when one uses the Likelihoods A and B in Fig. 10. We find that the distributions due to B are mostly broader, but also include significant biases in some cases, further Likelihood B does not constrain the parameters related to $n_s, \tau, A_s$ at all. It is not surprising that the likelihood comparing power spectra is more informative than one using summary parameters, as the latter only use a subset of the information available in the former.

Features of the Posterior Distributions of the CPL model: Using the method (A), we show the features of the current constraints on the CPL dark energy parameters $w_0, w_1$ in Fig. 5. The main features are (i) a sharp drop in the posterior for models that allow significant early dark energy demonstrated in Fig. 6 and (ii) a long tail for the combination of data sets (I) in the direction of low $w_1$ values, for which the dark energy equation of state changes decreases in the past, resulting in a specific fractional density of dark energy causing more acceleration in the past; but also implying that dark energy itself decays away more rapidly, and (iii) the distribution is fairly non-Gaussian. The drop in the posterior for parameters, described in (i) that allow for significant dark energy happens for a combination of geometrical and dynamical effects. Dynamical effects come from an Early ISW effect, and a different sourcing of the growth of matter perturbations due to different growth behavior of the potential. In Fig. 1 dark energy parameters of the kind described in (ii) were called “burst dark energy” since they lead to short burst of acceleration as shown in Fig. 2. Studying the correlations of the dark energy parameters with other background parameters in Fig. 3 suggests that such models are allowed if the flatness and Hubble constant values are also low, which are almost ruled out when we include the SDSS data. The non-Gaussian banana-shaped posterior on $w_0, w_1$ due to the data sets (I) are pinched off when data set (II) is used, however the distribution is still not very ellipsoidal. This is typical of posteriors of ill-constrained parameters, and can change with the addition of higher quality data. However, in this case the distribution is unlikely to be ellipsoidal if it extends to early dark energy cutoff described in (i), which is not too far from the peak of the distribution. These features are inherited in the marginalized one dimensional posterior of $w_1$, where we see an asymmetric distribution with a sharp drop in the distribution at high values of $w_1$ and a long tail in the low values of $w_1$ in Fig. 4.

Impact on Standard Cosmological Parameters: We study the differences in one dimensional marginalized constraints on the parameters in a $wCDM$ model, when the model is relaxed to a CPL model in Fig. 10. We find that the constraints on $w_0, n_s, \sigma_8, \omega_b$ are different in a CPL model (blue) from their counterparts in a $wCDM$ model (Black). We also compare the two dimensional joint posteriors of these cosmological parameters for a $wCDM$ model and a CPL model in Fig. 11 where the posteriors also change when these parameters are involved. In particular, we note from Fig. 10 that in a CPL model, there is a tail on the higher side of the posterior distributions of our maximal data set. Values above unity are allowed, in contrast to the situation for a $\Lambda CDM$ model. In Fig. 8, we show that, in contrast to the situation in $\Lambda CDM$ model with spatial curvature, where data set I (WMAP5, HST, SNe) constrains the flatness parameters, this is not possible in CPL models; Addition of the SDSS and BBN constraints are crucial to pinching off the banana in Fig. 8. As discussed, the posteriors on these parameters using the summary are not very good; so these questions cannot be addressed by summary parameters.

While the analysis presented is for the CPL parametrization, one should remember that EoS of dark energy might be quite different. We note that the dynamical effects are likely to be more pronounced for equations of state that have a stiffer evolution with redshift. On the other hand, if the EoS has a functional form significantly different from a CPL parametrization, the computed constraints may be biased (Linden & Virey 2008). Attempts to circumvent this problem have been made in terms of increasing the number of parameters describing EoS to enlarge the model space further, with an ultimate goal of making the description ‘model independent’. This number of
parameters that can be added is limited by the absence of tracers of cosmic evolution at high redshifts (Linder & Huterer 2005; Sarkar et al. 2008) even with the addition of futuristic SNe data. Typically, these analyzes are made possible by the degeneracies between other background parameters and the EoS by either invoking CMB constraints at very high redshift in terms of the summary parameters (Sarkar et al. 2008; Wang & Mukherjee 2007) or by taking the cosmology to be flat (Fay & Tavako 2006; Alam et al. 2004; Shafieloo et al. 2006). While our results suggest that such a use of summary parameters may be safe for CPL like parametrizations, it is unclear how good they are for other models intended to be included in these enlarged sets. While the assumption of flatness can be justified on the basis of an inflation prior, we show that it cannot be justified using the data. The WMAP5 constraints on flatness for a universe with a cosmological constant significantly worsen in a CPL model and one intuitively expects them to be still broader for a dark energy with a parameter independent EoS.

In comparing the matter power spectrum, we use analytic marginalization over a linear scale independent bias; thus parameter estimation from the matter power spectrum might be biased if there is really a scale dependent bias (Rassat et al. 2008). It is clear from their analysis as well as our results, that there is information in the power spectrum that is not encoded in the Baryon Acoustic Oscillations. Hence, it is yet another reason to study the bias of galaxies, so that one may be able to extract the maximum information from galaxy surveys.

While we have showed that a likelihood comparing power spectra is more informative that likelihoods comparing summary parameters even for CPL models, the computation of power spectra involves running computationally intensive Boltzmann code repeatedly to obtain a large number of posterior samples, while the computation of summary parameters is extremely fast. To give a quantitative idea, it takes about six hours to get a thousand chain steps on a single processor using Likelihood A, while it takes about six minutes to do the same using Likelihood B. Since such computations are quite doable for a particular model, one should use a comparison of power spectra to do precision cosmology, while summary parameters can still be used to explore new models and get rough estimates. It might be possible to make the likelihood using summary parameters more informative by adding further summary parameters; an example of this approach has been followed in (Wright 2007), where several additional summary parameters have been used. However, if it might be expected that a particular parametrization (such as the CPL model today) will have repeated use, one can also train Boltzmann accelerators (Fendt & Wandelt 2007a,b) to efficiently and accurately compute the CMB and matter power spectrum. If the parametrization is studied enough, the work in training the Boltzmann accelerators would be offset by the gain in time for computation of accurate constraints.

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