We introduce a new approach to analyze the phase diagram of QCD at finite chemical potential and temperature, based on the definition of a generalized QCD action. Several details of the method will be discussed, with particular emphasis on the advantages respect to the imaginary chemical potential approach.

In 1997, at the end of the previous century, the general wisdom on the possibility of understanding the behavior of matter in extreme conditions from first principles was moderately optimistic. The determination of the grand canonical partition function coefficients of QCD by the Glasgow-Illinois group [1] using the Glasgow reweighting method allowed for the detection of a first order phase transition at high baryonic density in the strong coupling regime.

One year later, in 1998, we published a paper containing very surprising results. Indeed using the microcanonical fermion average approach [2] and taking the modulus of the determinant of the Dirac operator in the integration measure, we were able to reproduce with very good agreement [3] the results obtained with the Glasgow method. These results were actually surprising since it was well known that the phase of the fermion determinant should play a fundamental role in the dynamics of finite density QCD.

In the same year we published another paper [4] in which we did a detailed analysis of the grand canonical partition function and detected how rounding effects in the standard routine used by both groups to calculate the coefficients of the grand canonical partition function from the eigenvalues of the quark propagator matrix, were the responsible for the appearance of a phase transition. We proposed also in [4] a solution to avoid the rounding problem and the main conclusion was that the phase transition observed by both groups was missing. In other words, the apparent progress in the understanding of finite density QCD was fictitious.

From our previous experience we learned that, at large fermion masses, the region of the chemical potential in which the phase of the fermion determinant becomes almost irrelevant enlarges, and then numerical approaches can be successfully applied. In this way we were able to predict a tentative phase diagram for fat QCD [5] which confirmed the theoretical expectations.

This was more or less the general situation of three color QCD until two years ago, when an important step forward in the field was possible by using two alternative schemes. The first scheme, based on extracting information on the phase structure of the model at finite $\mu$ from numerical simulations at imaginary chemical potential, was implemented in [6]. The second scheme [7] relies on a two-dimensional generalization of the Glasgow reweighting method, and has also been combined with a Taylor expansion in the chemical potential which allows simulations at large lattice sizes in the small $\mu/T$ region [8].

We want to introduce now a new approach [9] to simulate QCD at finite temperature and baryon density. Even if the theory scheme can resemble in some aspects the imaginary chemical potential approach, its range of applicabil-
ity seems much wider. The results of a test of our method using as toy model the Gross-Neveu model at finite $\mu$, as well as those for four flavor-QCD, can be found in the talk by Angelo Galante at this conference.

Our theory scheme is based on the introduction of a generalized QCD action

$$S(x, y) = S_{PG} + S_{\tau}(x, y) +$$

$$\frac{1}{2} \sum_{n,i=1}^{3} \bar{\psi}_n \eta_i(n) (U_{n,i} \psi_{n+i} - U_{n-i,i}^+ \psi_{n-i})$$

(1)

which depends on the two independent parameters $x, y$ through

$$S_{\tau}(x, y) =$$

$$\frac{1}{2} x \sum_{n} \bar{\psi}_n \eta_0(n) (U_{n,0} \psi_{n+0} - U_{n-0,0}^+ \psi_{n-0})$$

$$+ \frac{1}{2} y \sum_{n} \bar{\psi}_n \eta_0(n) (U_{n,0} \psi_{n+0} + U_{n-0,0}^+ \psi_{n-0})$$

(2)

A natural question now is, what should we expect for the phase diagram of this model in the $x, y$ plane?. The point $x = 1, y = 0$ would correspond to standard QCD at vanishing chemical potential. Now assume we are in the scaling region but at a physical temperature $T$ lower than the deconfining critical temperature. The point $x = 1, y = 0$ will be in the confined phase. If we increase now the inverse gauge coupling $\beta$, the physical temperature increases and for $\beta$ values large enough the point $x = 1$ will be in the unconfined phase. This strongly suggests the presence of a phase transition point in the $y = 0$ line approaching the $x = 1$ point in this line by increasing $\beta$ and eventually crossing it for $\beta$ values large enough. This argumentation drives us to conjecture the minimal phase diagram shown in Fig. 1. The solid line would be a line of phase transitions that crosses the $y = 0$ axis at values of $x$ larger than 1 for small $\beta$ values. By increasing $\beta$ and keeping fixed the temporal lattice extent $L_t$, the critical point on the $y = 0$ axis moves toward $x = 1$ and eventually crosses it. The discontinuous line in this figure stands for the physical line $x^2 - y^2 = 1$ along which one recovers standard QCD. The intersection of the solid line with the discontinuous one will give us therefore the critical chemical potential of QCD at a given temperature.

The numerical analysis of this model for real values of $x, y$ is not possible because we find again the sign problem. However if we take the $y$ parameter as a pure imaginary number, numerical simulations are feasible. Assuming that the phase transition line of Fig. 1 continues to imaginary $y = i \bar{y}$, the expected phase diagram in the $x, \bar{y}$ plane will show the qualitative structure of Fig. 2, where we have incorporated the $\frac{2\pi}{3} L_t$ periodicity of the model. We have also included in this figure the line $x^2 + \bar{y}^2 = 1$ (dotted line) which contains the only points accessible to numerical simulations of QCD at imaginary chemical potential. As stated in the introduction, one can see now how this approach has in principle more potentialities than the imaginary chemical potential approach. Indeed by increasing the inverse gauge coupling $\beta$, the phase transition line of Fig. 2 moves approaching more and more the origin of coordinates. In some interval $(\beta_m, \beta_M)$ the transition line intersects the dotted line and then a phase transition will appear at imaginary chemical potential. In such a situation, the physical temperature is so high that the system is in an unconfined phase for any real value of the chem-

Figure 1. Minimal phase diagram in $(x, y)$ plane.
Figure 2. Phase diagram in $(x,\bar{y})$ plane.

ical potential. This is in contrast with the fact that, within our approach, simulations can be performed at any value of $T$.

The practical implementation of our method consists therefore in performing numerical simulations at imaginary $y$ to determine the phase transition line of Fig. 2, and after that, to do analytical extensions to real $y$. This can be done mainly in two different ways. In the first scheme we fix the inverse gauge coupling and the number of temporal lattice points i.e., we fix the physical temperature, and determine the equation for the critical line of Fig. 1 as an expansion in powers of $y^2$. The intersection point of this line with the physical line will give us the corresponding critical value of the chemical potential. In the second scheme we fix the parameter $x$ and the gauge coupling, and look for the intersection of the critical line in the $\beta, y$ plane, obtained again from analytical extension of the critical line in the $\beta, \bar{y}$ plane, with the physical line. This scheme corresponds to fix the value of $\mu/T$, and the intersection point will give us the critical temperature at this value of $\mu/T$.

The main points to remark in the imaginary chemical potential approach are: i. the sign problem is avoided and standard numerical simulations can be performed, ii. simulations have to be performed at high temperature (large $\beta$) i.e., in the deconfined phase for real values of $\mu$, and iii. the analytical extension is limited to the range $\frac{\mu}{T} < \frac{\pi}{2}$.

Our approach shares the first point with the imaginary chemical potential case. Furthermore, and this is a relevant feature, simulations can be performed at any temperature and any value of $\mu/T$. The price to pay for that is one more parameter in the numerical simulations.

A last suggestion, concerning the Glasgow reweighting procedure in two parameters space [7], is the possibility of using the fermion determinant at the critical point of the $y = 0$ line instead of the determinant at $\mu = 0$ in the integration reweighted measure. We suspect that hereby one could improve the overlaps.

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