TRANSVERSITY AND DRELL–YAN K-FACTORS

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The Drell–Yan $K$-factors for transversely polarised hadrons are examined. Since transverse spin is peculiar in having no DIS reference point, the effects of higher-order corrections on DY asymmetries are examined via a DIS definition for transversity devised using a hypothetical scalar vertex. The results suggest that some care may be required when interpreting experimentally extracted partonic transversity, particularly when comparing with model calculations or predictions.*

1. Motivation

Transversity is the last, leading-twist piece in the partonic jigsaw puzzle that makes up the hadronic picture; the theoretical framework (i.e., QCD evolution, partonic processes, radiative effects, etc.) is now rather solid\[1,2\] while a number of experiments aimed at its measurement are on-line or under development: HERMES, COMPASS and the RHIC spin programme. Moreover, transverse-spin effects are notoriously surprising.

2. Transversity

2.1. Chirality and Hikasa’s Theorem

QCD and electroweak vertices conserve quark chirality, so that transversity decouples from DIS (see Fig. 1a). Chirality flip is not a problem if the quarks connect to different hadrons, e.g., as in Drell-Yan (DY) processes (see Fig 1b). A caveat to accessing transversity in DY is Hikasa’s theorem: chiral symmetry requires that the lepton-pair azimuthal angle remain unintegrated. No simple proof exists; it has to do with $\gamma$-matrix properties.

*Following correction of an error in the code used for the numerical estimates, the results shown here are a little less dramatic than those actually presented at the symposium.
2.2. Higher-Order Corrections

Quark densities are usually defined in DIS, where the parton picture was first formulated and model calculations are performed. When translated to DY, large radiative $K$ factors appear $\sim O(\pi \alpha_s)$, enhancing the cross-section \cite{4}. At RHIC energies the correction is roughly 30\% while at EMC/SMC energies it becomes nearly 100\%. Since spin asymmetries are ratios of differences and sums of cross-sections for different spin-alignment combinations, any strong polarisation dependence in the $K$ factors could lead to dramatic variations in the asymmetries, with respect say to model predictions. For the $q\bar{q}$ annihilation contribution in the case of longitudinally polarised hadrons, this turns out not to be the case \cite{5}. A partial explanation may be found in the helicity-conserving nature of vector interactions: only a single helicity combination contributes, to next-to-leading order (NLO).

However, the case of transversity is peculiar: as noted above, no DIS definition exists, nor is it obvious that quark helicity-conservation should still afford any protection. For pure DY, the NLO coefficient functions are known in various schemes \cite{6,7}; surprisingly, a new term $\propto z \ln \frac{1-z}{1+z}$ appears, which is found neither for spin-averaged nor helicity-dependent DY.

Now, to study the $K$-factor problem, we need a DIS-like process to which transversity may contribute. We thus seek a DIS helicity-flip mechanism, which could be provided by either a quark mass (i.e., in a propagator) or a scalar vertex (e.g., a Higgs coupling). Although a quark mass does what is required, the contribution cancels via the equations of motion and gauge invariance (see, e.g., \cite{8}). However, a (single) Higgs-like vertex, replacing one of the photon vertices in Fig. \ref{fig:a}, allows a chiral-odd contribution to DIS \cite{9, from a suggestion by R.L. Jaffe}. Indeed, such a gedanken process may be used to calculate the anomalous dimensions, but care is needed.

An attempt at calculating transversity anomalous dimensions $\gamma$ via this
method led to an apparent contradiction, which was corrected by Blümlein [11]: the vector current $J_V$ is conserved so $\gamma_V = 0$ but the scalar current $J_S$ is not and $\gamma_S \neq 0$. The product of two currents may be expanded as
\[
J_V(\xi) \cdot J_S(0) = \sum_n C(n; \xi) \mathcal{O}(n; 0), \tag{1}
\]
where the RGE’s for the Wilson coefficients $C(n; \xi)$ are
\[
\left[ D + \gamma_J V(g) + \gamma_J S(g) - \gamma_O(n; g) \right] C(n; \xi) = 0. \tag{2}
\]

Thus, the “Compton” amplitude correction has coefficient
\[
\gamma_C(n; g) = \gamma_J V(g) + \gamma_J S(g) - \gamma_O(n; g) \tag{3}
\]
and therefore $\gamma_C \neq \gamma_C!$

Moreover, since the scalar current is not conserved, there is an extra UV contribution from the scalar vertex, which must be factorised into the Higgs coupling constant (or equivalently, the running quark mass). The results for the coefficient functions are (see [11])
\[
C_{q,DY}^f - 2C_{q,DIS}^f = \frac{\alpha_s}{2\pi} C_F \left[ \frac{3}{(1-z)_+} + 2 \left( 1 + z^2 \right) \left( \frac{\ln(1-z)}{1-z} \right)_+ \right. \\
- 6 - 4z + \left( 4 \pi^2 + 1 \right) \delta(1-z), \tag{4a}
\]
\[
C_{q,DY}^g - 2C_{q,DIS}^g = C_{q,DY}^f - 2C_{q,DIS}^f + \frac{\alpha_s}{2\pi} C_F 2(1+z), \tag{4b}
\]
\[
C_{q,DY}^h - 2C_{q,DIS}^h = C_{q,DY}^f - 2C_{q,DIS}^f \\
+ \frac{\alpha_s}{2\pi} C_F \left[ 7 - 6z \ln^2 z \right. \\
- \left. 1 - z - 2(1-z) \ln(1-z) \right]. \tag{4c}
\]
The origins of the larger differences in the last line may be traced to different phase-space restrictions in the transversity case. Fig. 2 shows a comparison of the Mellin moments of the above coefficients and a simple purely valence estimate of the effects on a transverse asymmetry. The correction is rather more than twice that of the helicity case, reaching about 15%.

It could be argued that it is the Higgs-like vertex that spoils the $K$-factor cancellation in the transversity case. However, DY processes can also be constructed in which an intermediate Higgs state produces the lepton pair. The presence of scalar (chirality-flip) vertices avoids Hikasa’s theorem and the final lepton-pair azimuth may be integrated out. Likewise, a purely Higgs-exchange DIS process exists. In these cases the large $K$-factors are “well-behaved”. Thus, model calculations might not fare too well at first sight if not suitably corrected for the transition from DIS to DY.
3. Summary and Conclusions

A full description of the nucleon must include transversity. On the theory side, the standard QCD picture is complete to NLO, but only for DY or more exotic processes. We have no experimental data, though the future is promising. The phenomenology, while not dissimilar to the other leading-twist densities, has interesting peculiarities. Hikasa’s theorem forces us to keep the lepton-pair azimuth unintegrated in DY, leading to a new term in the NLO correction, which then affects the $K$-factor. Thus, comparison with model predictions and even the Soffer bound \cite{Soffer} could be misleading.

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