The nucleon thermal width due to pion-baryon loops and its contribution in Shear viscosity

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In the real-time thermal field theory, the standard expression of shear viscosity for the nucleonic constituents is derived from the two point function of nucleonic viscous stress tensors at finite temperature and density. The finite thermal width or Landau damping is traditionally included in the nucleon propagators. This thermal width is calculated from the in-medium self-energy of nucleon for different possible pion-baryon loops. The dynamical part of nucleon-pion-baryon interactions are taken care by the effective Lagrangian densities of standard hadronic model. The shear viscosity to entropy density ratio of nucleonic component decreases with the temperature and increases with the nucleon chemical potential. However, adding the contribution of pionic component, total viscosity to entropy density ratio also reduces with the nucleon chemical potential when the mixing effect between pion and nucleon components in the mixed gas is considered. Within the hadronic domain, viscosity to entropy density ratio of the nuclear matter is gradually reducing as temperature and nucleon chemical potential are growing up and therefore the nuclear matter is approaching toward the (nearly) perfect fluid nature.

I. INTRODUCTION

The recent hydrodynamical [1, 2] as well as some transport studies [3, 4] have indicated about an (nearly) ideal fluid nature of nuclear matter, which may be produced in the experiments of heavy ion collisions (HIC) like Relativistic Heavy Ion Collider (RHIC) at BNL. The hydrodynamical calculations became very successful in explaining the elliptical flow parameter, \( v_2 \) from RHIC data [5–7] only when they assumed a very small ratio of shear viscosity to entropy density, \( \eta/s \), at RHIC data [5–7] only when they assumed a very small ratio of shear viscosity to entropy density, \( \eta/s \), at RHIC. The viscosity to entropy density ratio \( \eta/s \) may reach a minimum in the vicinity of a phase transition, then some special attentions are drawn to the smallness of this minimum value with respect to its lower bound \( \eta/s = \frac{1}{4\pi} \), commonly known as the KSS bound [8]. In this context, the temperature \( T \) dependence of \( \eta/s \) is taken into account in some recent hydrodynamical calculations [14–17] instead of its constant value during the entire evolution. Niemi et al. [14] have interestingly observed that the \( v_2(\rho_T) \) of RHIC data is highly sensitive to the temperature dependent \( \eta/s \) in hadronic matter and almost independent of the viscosity in QGP phase. This work gives an additional boost to the microscopic calculations of \( \eta/s \) of the hadronic matter in the recent years [18–34], though historically these investigations are slightly old [35–40].

Except a few [24, 31, 32], most of the microscopic calculations are done in zero baryon or nucleon chemical potential \( (\mu_N = 0) \). Along with the \( T \) dependence of \( \eta \) or \( \eta/s \), their dependence on the baryon chemical potential should also be understood in view of the future experiments such as FAIR. In the work of Itakura et al. [23] and Denicol et al. [51], we notice that the \( \eta/s \) is reduced at finite baryon chemical potential, whereas Gorenstein et al. [31] observed an increasing nature of \( \eta/s \) with \( \mu_N \). Itakura et al. have obtained \( \eta \) by solving the relativistic quantum Boltzmann equation, where phenomenological amplitudes of hadrons are used in the collision terms. Denicol et al. have calculated the \( \eta \) at finite \( T \) and \( \mu_N \) by applying Chapman-Enskog theory in Hadron Resonance Gas (HRG) model, whereas Gorenstein et al. have taken a simplified ansatz of \( \eta \) to estimate \( \eta/s \) in the van der Waals excluded volume HRG model. Similar to the ansatz of \( \eta(T) \) taken by Gorenstein et al., the \( \eta \) itself increases with increasing temperature in Ref. [24], but their \( \eta/s \) are exhibiting completely opposite nature of \( T \) dependence. Therefore, the behavior of the \( \eta/s \) may largely be influenced by the \( T \) dependence of entropy density \( s \).

Motivating by these delicate issues of shear viscosity at finite \( \mu_N \), the present manuscript is concentrated on the matter with nucleon degrees of freedom at finite \( T \) and \( \mu_N \). The nucleons in the medium can slightly become off-equilibrium because of their thermal width or Landau damping, which can be originated from the nucleon thermal fluctuations into different baryons and pion. The inverse of nucleon thermal width measures the relaxation time of nucleon in the matter from which one can estimate its corresponding shear viscosity contribution.

In the next section, the one-loop expression of \( \eta \) for nucleon degrees of freedom is derived from the Kubo relation, where a finite thermal width is traditionally included in the nucleon propagators. This standard expression of \( \eta \) can also be deduced from relaxation time approximation of kinematic theory approach. In the real-time thermal field theory, the nucleon thermal width from the different pion-baryon loops are calculated in Sec. 3, where their interactions are determined from the effective hadronic model. In Sec. 4, the numerical results are discussed followed by summary and conclusions in Sec. 5.
II. KUBO RELATION FOR SHEAR VISCOSITY OF NUCLEAR MATTER

From the simple derivation of Kubo formula [41, 42], let us start with the expression of shear viscosity for nucleonic constituents in momentum space [24, 46],

\[ \eta_N = \frac{1}{20} \lim_{q_1, q_2 \to 0} \frac{A_{\eta}(q_0, \tilde{q})}{q_0}, \]

(1)

where

\[ A_{\eta}(q_0, \tilde{q}) = \int d^4x e^{iq \cdot x} \langle [\pi_{\mu\nu}(x), \pi_{\mu\nu}(0)] \rangle, \]

(2)

is the spectral representation of two point function for nucleonic viscous-stress tensor, \( \pi_{\mu\nu} \) and

\[ \langle \hat{O} \rangle_{\beta} = \text{Tr} e^{-\beta H} \hat{O} Z \]

(3)

is denoting the thermodynamical ensemble average. The energy momentum tensor of free nucleon is

\[ T_{\rho\sigma} = -g_{\rho\sigma} \mathcal{L} + \frac{\partial \mathcal{L}}{\partial (\partial^\gamma \psi)} \partial_\gamma \psi = -g_{\rho\sigma} \mathcal{L} + i \psi \gamma_\rho \partial_\sigma \psi, \]

(4)

and hence the viscous stress tensor will be

\[ \pi_{\mu\nu} = t_{\mu\nu}^\rho T_{\rho\nu} = t_{\mu\nu}^\rho \psi \gamma_\rho \partial_\sigma \psi \] (since \( t_{\mu\nu}^\rho g_{\rho\sigma} = 0 \)),

(5)

where

\[ t_{\mu\nu}^\rho = \Delta_\mu^\rho \Delta_\nu^\sigma - \frac{1}{3} \Delta_{\mu\nu} \Delta^\rho^\sigma, \quad \Delta^\mu^\nu = g^{\mu\nu} - u^\mu u^\nu. \]

(6)

In real-time formalism of thermal field theory, the ensemble average of any two point function always becomes a \( 2 \times 2 \) matrix structure. Hence, for viscous-stress tensor, the matrix structure of two point function becomes

\[ \Pi_{ab}(q) = i \int d^4x e^{iq \cdot x} \langle T_{c} \pi_{\mu\nu}(x) \pi_{\mu\nu}(0) \rangle_{\beta}^{ab}, \]

(7)

where the superscripts \( a, b (a, b = 1, 2) \) denote the thermal indices of the matrix and \( T_c \) denotes time ordering with respect to a symmetrical contour [43, 44] in the complex time plane.

The matrix can be diagonalized in terms of a single analytic function, which can also be related with the retarded two point function of viscous-stress tensor. The retarded function \( \Pi_R(q) \), diagonal element \( \Pi(q) \) and the spectral function \( A_{\eta}(q) \) are simply related to any one of the components of \( \Pi_{ab}(q) \). Their relations with 11 component is given below

\[ A_{\eta}(q) = 2\text{Im} \Pi_R(q) = 2\epsilon(q_0) \text{Im} \Pi(q) = 2 \text{tanh}(\beta q_0/2) \text{Im} \Pi_{11}(q). \]

(8)

Hence, Eq. (1) can broadly be redefined as

\[ \eta_N = \frac{1}{10} \lim_{q_0, \beta \to 0} \frac{\text{Im} \Pi_R(q_0, \tilde{q})}{q_0} = \frac{1}{10} \lim_{q_0, \beta \to 0} \frac{\epsilon(q_0) \text{Im} \Pi(q)}{q_0} = \frac{1}{10} \lim_{q_0, \beta \to 0} \frac{\text{Im} \Pi_R(q_0, \tilde{q})}{q_0}. \]

(9)

Using [5] in the 11 component of [17] and then applying the Wick’s contraction technique, we have

\[ \Pi_{11}(q) = t_{\mu\nu}^\rho t_{\alpha\beta} \text{Tr} i \int d^4x e^{iq \cdot x} \langle T_{c} (x) \gamma_\rho \partial_\alpha \psi(x) \psi(0) \gamma_\beta \partial_\nu \psi(0) \rangle_{\beta} \]

\[ = i \int \frac{d^4k}{(2\pi)^4} N(q, k) D_{11}(k) D_{11}(p = q + k), \]

(10)

where

\[ N(q, k) = -N \frac{t_{\mu\nu}^\rho}{t_{\alpha\beta}^\rho} \text{Tr} \frac{[\gamma^\mu (q + k)^\nu, \gamma^\nu (q + k + m_N) \gamma^\beta (q + k + m_N) \gamma^\alpha (q + k + m_N)^\beta]}{4} \]

(11)

This self-energy function, \( \Pi_{11}(q) \) for \( NN \) loop can diagrammatically be represented by Fig. [1][A]. In the co-moving frame, i.e., for \( u = (1, 0) \), the \( N(q, k) \) becomes

\[ N(q, k) = -N \left[ \frac{32}{3} \right] \frac{k_0(q_0 + k_0)}{k \cdot (q + k)} \left[ -\frac{\bar{k} \cdot (q + k)}{3} \right]^2 \]

(12)

In the above equations, \( N = 2 \) is the isospin degeneracy of nucleon.

In Eq. (11), \( D_{11} \) is scalar part of 11 component of the nucleon propagator at finite temperature and density. Its form is

\[ D_{11}(k) = \frac{1}{k^2 - m_N^2 + i\eta} - \frac{1}{2\pi i} F_k(k_0) \delta(k^2 - m_N^2) \]

with

\[ F_k(k_0) = n^+_k + \delta(k_0 - k_0) \]

\[ - \frac{1}{2\pi} \frac{1}{k_0 + \omega_N} - n^+_k + \delta(k_0 + \omega_N + i\eta) \]

\[ - \frac{1}{k_0 + \omega_N + i\eta} - \frac{n^-_k}{k_0 + \omega_N + i\eta}, \]

(13)

where \( n^+_k(\omega_N) = 1/\{e^{(\omega_N^+ \tau_{1/2})} + 1 \} \) is Fermi-Dirac distribution function for energy \( \omega_N^+ = \sqrt{k^2 + m_N^2} \). Here the \( \pm \) signs in the superscript of \( n_k \) stand for nucleon and anti-nucleon respectively. Among the four terms in Eq. (13), the first and the second terms are associated with the nucleon propagation above the Fermi sea and the propagation of its hole in the Fermi sea respectively, while the third and fourth terms represent the corresponding situations for anti-nucleon. The full relativistic nucleon propagator, thus, treats the particle and anti-particle on an equal footing and all possible singularities (nucleon, hole of the nucleon, anti-nucleon and hole of the anti-nucleon) are automatically included.
FIG. 1: Diagrammatic representation of $NN$ loop is shown in (A), where double lines stand for effective $N$ propagators, which contain their thermal widths $\Gamma$. The diagrammatic representation of nucleon self-energy for \(\pi B\) loop is shown in (B) from where $\Gamma$ can be determined.

After doing the $k_0$ integration of Eq. (10) and then using it in Eq. (9), we have

$$\eta_N = \frac{1}{10} \lim_{q_0, q \to 0} \int \frac{d^3 k}{(2\pi)^3 4\omega_k^0 \omega_p^0} \left[ \frac{-n_k^+(\omega_k^N) + n_k^-(q_0 + \omega_k^N)}{q_0 + \omega_k^N - \omega_p^N} \right] \frac{\delta(q_0 - \omega_k^N + \omega_p^N)}{q_0 + \omega_k^N - \omega_p^N} + \ldots$$

where $N = N(k_0 = \pm \omega_k^N, \vec{k}, q)$ and $\omega_p^N = \sqrt{(\vec{q} + \vec{k})^2 + m_N^2}$.

The two $\delta$-functions will be responsible for generating the Landau cuts ($-q < q_0 < q$), whereas the $\text{Im} \Pi^R(q)$ will be non-zero. However, there will be two more $\delta$-functions (not written explicitly), which are not important for the limiting point $q_0, q \to 0$ since they will generate unitary cuts ($-\infty < q_0 < -\sqrt{q^2 + 4m_N^2}$ and $\sqrt{q^2 + 4m_N^2} < q_0 < \infty$).

Using the identity

$$-\pi \delta(x) = \text{Im} \left[ \lim_{\Gamma_N \to 0} \frac{1}{x + i\Gamma_N} \right]$$

in Eq. (14), we have

$$\eta_N = \frac{1}{10} \lim_{q_0, q \to 0} \text{Im} \left[ \int \frac{d^3 k}{(2\pi)^3 4\omega_k^0 \omega_p^0} \lim_{\Gamma_N \to 0} \left\{ \frac{-n_k^+(\omega_k^N) + n_k^-(q_0 + \omega_k^N)}{q_0 + \omega_k^N - \omega_p^N} \right\} \frac{\delta(q_0 - \omega_k^N + \omega_p^N)}{q_0 + \omega_k^N - \omega_p^N} \right].$$

We will continue our further calculation for finite value of $\Gamma_N$ to get a non-divergent contribution of $\eta_N$. Including thermal width $\Gamma_N$ for constituent particles (here nucleons) of the medium is a very well established technique in Kubo approach to remove the divergence of $\eta_N$ as well as to incorporate the interaction scenario, which is very essential for a dissipative system. The interaction scenario is coming into the picture by transforming the delta functions to the spectral functions with finite thermal width. The thermal width (or collision rate) $\Gamma_N$ of the constituent particles reciprocally measures the shear viscosity coefficient, which is approximately equivalent to the quasi particle description.

In the limiting case of $q_0, q \to 0$, we get $\omega_p^N \to \omega_p^N$ and therefore Eq. (16) is transformed to

$$\eta_N = \frac{1}{10} \int \frac{d^3 k}{(2\pi)^3 4\omega_k^0 \omega_p^0} \left[ I_2 + I_3 \right],$$

where

$$N_0 = \lim_{q_0, q \to 0} N(k_0 = \pm \omega_k^N, \vec{k}, q)$$

and

$$I_{2,3} = \lim_{q_0 \to 0} \frac{d}{dq_0} \left\{ \pm n_k^+(\omega_k^N) \pm n_k^+(q_0 + \omega_k^N) \right\}.$$

In the above Eq. (19), one can notice that the limiting value of $I_{2,3}$ is of the $0/0$ form. Therefore, we can apply the L’Hospital’s rule, i.e.,

$$I_{2,3} = \lim_{q_0 \to 0} \frac{d}{dq_0} \left\{ \pm n_k^+(\omega_k^N) \pm n_k^+(q_0 + \omega_k^N) \right\} = \beta [n_k^+(1 - n_k^+)],$$

since

$$\lim_{q_0 \to 0} \frac{d}{dq_0} \left\{ \pm n_k^+(\omega_q = \mp q_0 + \omega_k^N) \right\} = \frac{\beta}{(e^{\beta(\omega_q + \mu_N)} + 1)^2}.$$

Again, in the limiting value of $q_0, q \to 0$, Eq. (12) can be simplified to

$$\frac{d}{dq_0} \frac{\pm n_k^+(\omega_q = \mp q_0 + \omega_k^N)}{\omega_k^N} = \frac{\beta}{(e^{\beta(\omega_q + \mu_N)} + 1)^2}.$$

Hence, using the above results, the Eq. (17) becomes

$$\eta_N =\frac{8\beta \Gamma_N}{15} \int \frac{d^3 k}{(2\pi)^3 4\omega_k^0 \omega_p^0} \left[ n_k^+(1 - n_k^+) + n_k^+(1 - n_k^+) \right] \frac{1}{\omega_k^0 \omega_p^0} \left[ n_k^+(1 - n_k^+) + n_k^+(1 - n_k^+) \right].$$

This is the one-loop expression of shear viscosity for the matter with nucleon degrees of freedom in the Kubo approach. Though there are possibility of infinite number of ladder-type diagrams, which are supposed to be of same order of magnitude ($O(1/\Gamma_N)$) like the one-loop, they
TABLE I: From the left to right columns, the table contain the baryons, their spin-parity quantum numbers $J^P_B$, isospin $I_B$, total decay width $\Gamma_{\text{tot}}$, decay width in $N\pi$ channels $\Gamma_{B\rightarrow N\pi}$ or $\Gamma_B(m_B)$ in Eq. (22) (brackets displaying its Branching Ratio) and at the last coupling constants $f/m_\pi$.

| Baryons | $J^P_B$ | $I_B$ | $\Gamma_{\text{tot}}$ | $\Gamma_{B\rightarrow N\pi}$ (B.R.) | $f/m_\pi$ |
|---------|---------|-------|-----------------------|-------------------------------------|-----------|
| $\Delta(1232)$ | $\frac{1}{2}^+$ | $3/2$ | 0.117 | 0.117 (100%) | 15.7 |
| $N^*(1440)$ | $\frac{1}{2}^+$ | $1/2$ | 0.300 | 0.195 (65%) | 2.5 |
| $N^*(1520)$ | $\frac{3}{2}^-$ | $1/2$ | 0.115 | 0.069 (60%) | 11.6 |
| $N^*(1535)$ | $\frac{1}{2}^-$ | $1/2$ | 0.150 | 0.068 (45%) | 1.14 |
| $\Delta^*(1600)$ | $\frac{3}{2}^+$ | $3/2$ | 0.320 | 0.054 (17%) | 3.4 |
| $\Delta^*(1620)$ | $\frac{1}{2}^-$ | $3/2$ | 0.140 | 0.035 (25%) | 1.22 |
| $N^*(1650)$ | $\frac{1}{2}^-$ | $1/2$ | 0.150 | 0.105 (70%) | 1.14 |
| $\Delta^*(1700)$ | $\frac{3}{2}^-$ | $3/2$ | 0.300 | 0.045 (15%) | 9.5 |
| $N^*(1700)$ | $\frac{1}{2}^-$ | $1/2$ | 0.100 | 0.012 (12%) | 2.8 |
| $N^*(1710)$ | $\frac{1}{2}^+$ | $1/2$ | 0.100 | 0.012 (12%) | 0.35 |
| $N^*(1720)$ | $\frac{3}{2}^+$ | $1/2$ | 0.250 | 0.028 (11%) | 1.18 |

will be highly suppressed [46]. As we increase the number of loops, the number of extra thermal distribution functions will also appear in the shear viscosity expression and hence their numerical suppression will successively grow. On this basis, the one-loop results may be considered as a leading order results. One can derive exactly same expression from relaxation time approximation in kinetic theory approach.

III. CALCULATION OF NUCLEON THERMAL WIDTH

Now, our next aim is to calculate the thermal width of nucleon $\Gamma_N$, which can be estimated from the retarded component of nucleon self-energy ($\Sigma^R$) at finite temperature and density. Their relation is given by

$$\Gamma_N(\vec{k}, T, \mu_N) = -\text{Im} \Sigma^R(k_0 = \omega_\vec{k}, \vec{k}, T, \mu_N) .$$

During the propagation in the hot and dense nuclear matter, nucleon may pass through different $\pi B$ loops, where $B$ stand for different higher mass baryons including nucleon itself. In this work, all possible 4-star baryon resonances with spin 1/2 and 3/2 are considered. These are $N(980), \Delta(1232), N^*(1440), N^*(1520), N^*(1535), \Delta^*(1600), \Delta^*(1620), N^*(1650), \Delta^*(1700), N^*(1700), N^*(1710)$ and $N^*(1720)$, where masses (in MeV) of the baryons are given inside the brackets. The nucleon self-energy for $\pi B$ loop is shown in diagram [1(B) and its 11 component can be expressed as

$$\Sigma^{11}(k, T, \mu_N) = -i \int \frac{d^4l}{(2\pi)^3} L(k, l) D_{11}(l, m_\pi, T) ,$$

where $D_{11}(l, m_\pi, T), D_{12}(u = k-l, m_B, T, \mu_N)$ are scalar part of pion and baryon propagators at finite temperature and density. The $L(k, l)$ contains vertices and numerator parts of the propagators. The chemical potential of all baryons are assumed to be the same as nucleon chemical potential $\mu_N$. Similar to Eq. (5), this 11 component is also related with retarded self-energy as

$$\text{Im} \Sigma^R(k) = \coth \left( \frac{\beta(k_0 - \mu_N)}{2} \right) \text{Im} \Sigma^{11}(k) .$$

Performing the $l_0$ integration in (25) and then using the relation (20), we get the imaginary part of retarded self-energy,

$$\text{Im} \Sigma^R(k) = \pi \int \frac{d^4l}{(2\pi)^3} \frac{1}{4\omega_l^4} [L(l_0 = \omega_\vec{k}, \vec{l}, k)

\{[1 + n_l(\omega^B_l)] - n^+_l(k_0 - \omega_\vec{k})\} \delta(k_0 - \omega^0_\vec{k} - \omega^B_k) + \{-n_l(\omega^B_l) - n^-_l(-k_0 + \omega^0_\vec{k})\} \delta(k_0 - \omega^0_\vec{k} + \omega^B_k)

+ L(l_0 = -\omega_\vec{k}, \vec{l}, k) [n_l(\omega^B_l) + n^+_l(k_0 + \omega^0_\vec{k}) \delta(k_0 + \omega^0_\vec{k} - \omega^B_k) + \{-1 -n_l(\omega^B_l) + n^-_l(-k_0 - \omega^0_\vec{k})\} \delta(k_0 + \omega^0_\vec{k} + \omega^B_k)] ,$$

where $\omega^B_k = \sqrt{(k - l)^2 + m_B^2}$, $n^+_l$ and $n^-_l$ are respectively Fermi-Dirac and Bose-Einstein distribution functions. The regions of different branch cuts in $k_0$-axis are $-\infty$ to $-\sqrt{k^2 + (m_\pi + m_B)^2}$ for unitary cut in negative $k_0$-axis, $-\sqrt{k^2 + (m_B - m_\pi)^2}$ to $\sqrt{k^2 + (m_B - m_\pi)^2}$ for Landau cut and $\sqrt{k^2 + (m_\pi + m_B)^2}$ to $\infty$ for unitary cut in positive $k_0$-axis. These are representing the different kinematic regions where the imaginary part of the nucleon self-energy becomes non-zero because of the different $\delta$ functions in Eq. (24). The $\Gamma_N$ for all $\pi B$ loops (except the $\pi N$) are coming from the Landau cut contribution associated with the third term of Eq. (27), which can be simplified as

$$\Gamma_N = \frac{1}{16\pi k} \int_{\omega_{\min}}^{\omega_{\max}} d\omega \{n_l(\omega) + n^+_l(\omega_\vec{k} + \omega)\}

L(l_0 = -\omega, \vec{l} = \sqrt{\omega^2 - m_\pi^2}, k_0 = \omega_\vec{k}, \vec{k}) ,$$

(28)
where \( n_1(\bar{\omega}) = 1/\{e^{\bar{\omega}} - 1\} \), \( n_2^\pi(\omega_N^N + \bar{\omega}) = 1/\{e^{\bar{\omega}}(\omega_N^N + \bar{\omega}) + 1\} \), 
\( \bar{\omega} = \frac{R^2}{2m_N^2}(-\omega_N^N \pm \bar{k}W) \) with 
\( W = \sqrt{1 - \frac{4m_N^2m_s^2}{R^2}} \) and \( R^2 = m_N^2 + m_s^2 - m_s^2 \).

The effective Lagrangian densities for \( BN\pi \) interactions are given below \( \text{(47)} \)

\[
\mathcal{L} = \frac{f}{m_\pi} \psi_B \frac{\gamma^\mu}{\bar{\pi}} \left\{ i \gamma^5 \mathbb{I} \right\} \psi_N \partial_\mu \pi + \text{h.c.} \text{ for } J_B^P = \frac{1}{2}^\pm ,
\]

\[
\mathcal{L} = \frac{f}{m_\pi} \psi_B \frac{\gamma^\mu}{\bar{\pi}} \left\{ i \gamma^5 \mathbb{I} \right\} \psi_N \partial_\mu \pi + \text{h.c.} \text{ for } J_B^P = \frac{3}{2}^\pm , \quad \text{(29)}
\]

where coupling constants \( f/m_\pi \) for different baryons have been fixed from their experimental vacuum widths in \( N\pi \) channel. With the help of the above Lagrangian densities, one can easily find

\[
L(k, l) = -\left( \frac{f}{m_\pi} \right)^2 \left( \frac{R^2}{2} - m_s^2 \right) l_0 \text{ for } J_B^P = \frac{1}{2}^\pm ,
\]

\[
L(k, l) = \left( \frac{f}{m_\pi} \right)^2 \left( \frac{R^2}{2} - m_s^2 \right) \left\{ \frac{2}{3m_B^2} \right\} \left( \frac{R^2}{2} - m_s^2 \right) \text{ for } J_B^P = \frac{3}{2}^\pm ,
\]

\[
(31)
\]

For simplification the coefficients of \( \gamma^0 \) and \( \mathbb{I} \) are taken as in Ref. \( \text{(48)} \) and their addition gives

\[
L(k, l) = -\left( \frac{f}{m_\pi} \right)^2 \left\{ \frac{R^2}{2} - m_s^2 \right\} l_0 - Pm_s^2m_B \text{ for } J_B^P = \frac{1}{2}^\pm ,
\]

\[
L(k, l) = -\left( \frac{f}{m_\pi} \right)^2 \left\{ \frac{2}{3m_B^2} \right\} \left( \frac{R^2}{2} - m_s^2 \right) \left( \frac{R^2}{2} - m_s^2 \right) \text{ for } J_B^P = \frac{3}{2}^\pm .
\]

\[
\quad \text{(30)}
\]

The isospin part of the Lagrangian densities are not written in the Eq. \( \text{(29)} \). The isospin structure for \( J_B^P = \frac{1}{2}^\pm \) and \( J_B^P = \frac{3}{2}^\pm \) should be \( \bar{T}_{11} \) and \( \bar{T}_{12} \) respectively, where \( \bar{T} \) is the spin 3/2 transition operator and \( \bar{T}_{12} \) is the Pauli operator. This issue is managed by multiplying appropriate isospin factors with the expressions of corresponding loop diagrams. The isospin factor for \( \pi N \) or \( \pi N^* \) loop is \( I_{N=+N,N^*} = 3 \) whereas for the \( \pi \Delta \) or \( \pi \Delta^* \) loop, \( I_{N=+\Delta,N^*} = 2 \).

All baryon resonances have finite vacuum width in \( N\pi \) decay channel. The calculations of these decay widths are very essential in the present work for two reasons. First is to fix the coupling constants \( f/m_\pi \) for different \( BN\pi \) interaction Lagrangian densities and second is to include the effect of these baryon widths (\( \Gamma_B \)) on the nucleon thermal width \( \Gamma_N \). Using the Lagrangian densities, the vacuum decay width of baryons \( B \) for \( N\pi \) channel can be obtained as

\[
\Gamma_B(m_B) = \frac{I_{N^*-\pi N}}{2J_B^P + 1} \left( \frac{f}{m_\pi} \right)^2 \frac{|\hat{\rho}_{cm}|}{2m_B} \frac{|\hat{\rho}_{cm}|^2}{2m_B} \text{ for } J_B^P = \frac{1}{2}^\pm ,
\]

\[
\Gamma_B(m_B) = \frac{I_{\Delta N^*-\pi N}}{2J_B^P + 1} \left( \frac{f}{m_\pi} \right)^2 \frac{|\hat{\rho}_{cm}|}{3m_B} \frac{|\hat{\rho}_{cm}|^3}{3m_B} \frac{|\hat{\rho}_{cm}|^2}{2m_B} \text{ for } J_B^P = \frac{3}{2}^\pm , \quad \text{(32)}
\]

where \( |\hat{\rho}_{cm}| = \sqrt{\frac{\omega_{cm}^N - (m_N + m_s)^2}{m_N^2 - (m_N + m_s)^2}} \) and \( \omega_{cm}^N = \sqrt{|\hat{\rho}_{cm}|^2 + m_B^2} \). The isospin factors are \( I_{N^*-\pi N} = 3 \) and \( I_{\Delta N^*-\pi N} = 1 \) for the \( N\pi \) decay channels of \( N^* \) and \( \Delta^* \) (or \( \Delta \)) respectively.

Now, the \( \Gamma_N \) in Eq. \( \text{(28)} \) can be convoluted (see e.g. Refs. \( \text{(19, 51)} \)) as

\[
\Gamma_N(m_B) = \frac{1}{N_B} \int \frac{dM_B A_B(M_B) \Gamma_N(M_B)}{\sqrt{m_B^2 - 2\Gamma_{\Delta N^*}(m_B)}} \text{ for } J_B^P = \frac{1}{2}^\pm ,
\]

\[
N_B = \int \frac{dM_B A_B(M_B)}{\sqrt{m_B^2 - 2\Gamma_{\Delta N^*}(m_B)}} \text{ for } J_B^P = \frac{3}{2}^\pm , \quad \text{(33)}
\]

where

\[
A_B(M_B) = \frac{1}{\pi} \text{Im} \left[ \frac{1}{M_B - m_B + i\Gamma_B(M_B)/2} \right] \quad \text{(34)}
\]

is vacuum spectral function of baryons for their vacuum decay width in \( N\pi \) channel. Replacing baryon mass \( m_B \) by its invariant mass \( M_B \) in Eq. \( \text{(32)} \), one can get the mass shell expression of \( \Gamma_N(M) \). The values of coupling constants \( f/m_\pi \), which are fixed from the experimental values of baryon decay width in \( N\pi \) channels \( \text{(51)} \), are shown in a Table (I).

### IV. RESULTS AND DISCUSSION

Let us first take a glance at the invariant mass distribution of imaginary part of nucleon self-energy for different \( \pi B \) loops. Fig. \( \text{(2)} \) shows the results for baryons \( B = N(940), \Delta(1232) \) (upper panel) and \( B = \Delta^*(1620), \Delta^*(1650), \Delta^*(1720) \) (lower panel), whereas Fig. \( \text{(3)} \) displays the results for baryons \( B = N^*(1440), N^*(1520), \Delta^*(1600) \) (upper panel) and \( B = N^*(1535), \Delta^*(1700) \) (lower panel). The numerical strengths for \( B = N^*(1700) \) and \( N^*(1710) \) are too low to display with the other baryons. These results are obtained by replacing \( \omega_N^N = \sqrt{\bar{k}^2 + m_s^2} \) by \( \omega_N = \sqrt{\bar{k}^2 + M^2} \) in Eq. \( \text{(28)} \) (dashed line) and \( \text{(33)} \) (solid line) for the fixed values of \( \bar{k} = 0, \mu_N = 0 \) and \( T = 0.150 \) GeV. From the sharp
FIG. 2: Imaginary part of nucleon self-energy for different \( \pi B \) loops are individually shown before (dashed line) and after (solid line) folding by corresponding baryon spectral functions. \( B = N^*(940), \Delta^*(1232) \) are in upper panel whereas \( B = \Delta^*(1620), N^*(1650), N^*(1720) \) are in lower panel for fixed values of three momentum of \( \vec{k} = 0 \), temperature \( T = 0 \) and baryon chemical potential \( \mu_N = 0 \).

FIG. 3: Same as Fig. 2 for the rest of the baryons \( B = N^*(1440), N^*(1520), \Delta^*(1600) \) (upper panel) and \( B = N^*(1535), \Delta^*(1700) \) (lower panel).

ending of the dashed line, the Landau regions for different loops are clearly visible. As an example for \( \pi N \) loop the Landau region is \( M = 0 \) to \( m_N - m_\pi \), \( \text{i.e.} \), 0 to 0.8 GeV. Due to the folding of the baryon spectral functions, these sharp endings are smeared towards higher value of \( M \). Since \( \Sigma^R(M) \) also depends on \( T, \mu_N \) and \( \vec{k} \) therefore total contribution of \( \Sigma^R(M) \) from all the loops has been shown in Fig. 4 for different sets of \( T, \mu_N \) and \( \vec{k} \).

The nucleon thermal width \( \Gamma_N \) is basically the contribution of \( \text{Im} \Sigma^R \) at \( M = m_N \), which is marked by dotted line. Being an on-shell quantity, \( \Gamma_N \) is associated with the thermodynamical probability of different on-shell scattering processes instead of off-shell scattering processes as described by Weldon for the imaginary part of self-energy in Ref. \[52\]. Following Weldon’s prescription, forward and inverse scattering of nucleon can be respectively described as follows. During propagation of \( N \), it can disappear by absorbing a thermalized \( \pi \) from the medium to create a thermalized \( B \). Again \( N \) can appear by absorbing a thermalized \( B \) from the medium as well as by emitting a thermalized \( \pi \). The \( n_\pi (1 - n_\pi^+ \) ) and \( n_\pi^+ (1 + n_\pi) \) are the corresponding statistical probabilities of the forward and inverse scattering respectively \[52\], because just by adding them, we will get the thermal distribution part of Eq. (28), \text{i.e.}, \( (n_1 + n_1^+) \).

From Eq. (28) or (33), we see that \( \Gamma_N \) depends on temperature \( T \), baryon chemical potential \( \mu_N \) and three momentum \( \vec{k} \) of nucleon. The upper panels of Fig. 5
defined as $\lambda_s$ ($T, \mu$), this relevant $T$ out from the medium, the relevant $T$ ($\vec{k}, T, \mu$) monotonically increasing with $T$ and $\mu$ with high $\mu$ to decrease with increase of $\mu$.

Using the numerical function $\Gamma_N(\vec{k}, T, \mu_N)$ in Eq. (35), we get $\eta_N$ as a function of $T$ and $\mu_N$, which are shown in the upper panels of Fig. (7) and (8). Here we see $\eta_N$ is monotonically increasing with $T$ and $\mu_N$ both. Using the simple equilibrium expression of entropy density ($s_N$) for nucleons,

$$s_N = 4\beta \int \frac{d^3\vec{k}}{(2\pi)^3} \left( \frac{\omega_k^N + \vec{k}^2}{3\omega_k^N} - \mu_N \right) n_k^N (\omega_k^N)$$

the $\eta_N/s_N$ has been generated as a function of $T$ and $\mu_N$. From the lower panels of Fig. (7) and (8), we see the scenario after freeze out of the medium. From the lower panels of Fig. (7) and (8), we see $T_{\text{f.o.}}$ for different sets of $(\vec{k}, T, \mu)$ and $(\vec{k}, T, \mu_N)$ is that relevant region for baryon nucleon ($N$) for finite baryon chemical potential ($\mu_N = 0$). Whereas for finite baryon chemical potential (e.g. solid line of Fig. (5) at $\mu_N = 0.7$ GeV), this relevant $T$ region will be shifted slightly toward lower temperature (in addition, $T_c$ is also expected to decrease with increase of $\mu_N$). Since high momentum ($\vec{k}$) of constituent particles always helps them to freeze out from the medium, the relevant $T$ region for nucleon with high $\vec{k}$ is reduced by shifting towards the high $T$ region. This can be understood by comparing the solid and dotted lines in the lower panel of Fig. (5).

Using the numerical function $\Gamma_N(\vec{k}, T, \mu_N)$ in Eq. (35), we get $\eta_N$ as a function of $T$ and $\mu_N$, which are shown in the upper panels of Fig. (7) and (8). Here we see $\eta_N$ is monotonically increasing with $T$ and $\mu_N$ both. Using the simple equilibrium expression of entropy density ($s_N$) for nucleons,

$$s_N = 4\beta \int \frac{d^3\vec{k}}{(2\pi)^3} \left( \frac{\omega_k^N + \vec{k}^2}{3\omega_k^N} - \mu_N \right) n_k^N (\omega_k^N),$$

the $\eta_N/s_N$ has been generated as a function of $T$ and $\mu_N$. From the lower panels of Fig. (7) and (8), we see $\eta_N/s_N$ is also expected to shift slightly toward lower $\mu_N$ (but up to $T_c \approx 0.175$ GeV) is that relevant region for baryon free nuclear matter ($\mu_N = 0$). Whereas for finite baryon chemical potential (e.g. solid line of Fig. (5) at $\mu_N = 0.7$ GeV), this relevant $T$ region will be shifted slightly toward lower temperature (in addition, $T_c$ is also expected to decrease with increase of $\mu_N$). Since high momentum ($\vec{k}$) of constituent particles always helps them to freeze out from the medium, the relevant $T$ region for nucleon with high $\vec{k}$ is reduced by shifting towards the high $T$ region. This can be understood by comparing the solid and dotted lines in the lower panel of Fig. (5).
that $\eta_N/s_N$ can be reduced by increasing $T$ as well as by decreasing $\mu_N$.

In the left and right panels of Fig. 9, the contributions of different loops (dominating loops only) are individually shown in $\eta_N$ vs $T$ and $\eta_N$ vs $\mu_N$ graphs respectively. The $\pi \Delta$ loop plays a leading role to generate the typical values ($0.0001 - 0.01$ GeV$^3$) of $\eta_N$ for strongly interacting matter because the major part of the nucleon thermal width is coming from this loop only.

Up to now, we have calculated the contribution of shear viscosity from nucleon thermal width, although a major contribution comes from the thermal width of pion. Hence, one should add the pionic contribution with nucleon contribution for getting total shear viscosity of nuclear matter at finite temperature and density. In our recent work [53], the shear viscosity, coming from pionic thermal width has already been addressed. The one-loop Kubo expression of shear viscosity and ideal expression of entropy density for pionic components are respectively given below,

$$\eta_\pi = \frac{\beta}{10\pi^2} \int \frac{d^3 k}{\pi \omega_k^3} \frac{\omega_k}{\sqrt{\delta^2 + m_\pi^2 + \omega_k^2}} n_k(\omega_k^3) \left[ 1 + \frac{\omega_k}{\omega_k^3} \right],$$

and

$$s_\pi = 3 \beta \int \frac{d^3 k}{(2\pi)^3} \left( \frac{\omega_k^3 + \omega_{\pi k}^3}{3 \omega_k^3} \right) n_k(\omega_k^3),$$

where $n_k(\omega_k^3) = 1/\{e^{\beta \omega_k^3} - 1\}$ is the Bose-Einstein distribution function of pion with $\omega_\pi^3 = (\vec{k}^2 + m_\pi^2)^{1/2}$, and $\Gamma_\pi$ is the thermal width of $\pi$ mesons in the medium due to $\pi\sigma$ and $\pi\rho$ fluctuations.

Now, adding that pion contribution with the nucleon, one can simply get the total shear viscosity of nuclear matter as

$$\eta_{tot} = \eta_\pi + \eta_N,$$

where $\eta_\pi$ and $\eta_N$ do not face any mixing effect of pion density, $\rho_\pi = 3 \int \frac{d^3 k}{(2\pi)^3} n_k(\omega_k^3)$ and nucleon density, $\rho_N = 4 \int \frac{d^3 k}{(2\pi)^3} n_k^+(\omega_N^3)$. However, viscosity of single component gas should be different from the viscosity of that component in a mixed gas [24, 54]. To incorporate this mixing effect for rough estimation, we follow the approximated relation [24, 54]

$$\eta_{tot}^{mix} = \eta_\pi^{mix} + \eta_N^{mix},$$

where

$$\eta_\pi^{mix} = \eta_\pi \frac{1 + \left( \frac{\rho_\pi}{\rho_N} \right) \left( \frac{\sigma_{\pi N}}{\sigma_{\pi N}} \right)}{1 + \left( \frac{\rho_\pi}{\rho_N} \right) \left( \frac{\sigma_{\pi N}}{\sigma_{\pi N}} \right) \frac{m_N}{m_\pi}}$$

and

$$\eta_N^{mix} = \eta_N \frac{1 + \left( \frac{\rho_N}{\rho_\pi} \right) \left( \frac{\sigma_{\pi N}}{\sigma_{n N}} \right)}{1 + \left( \frac{\rho_N}{\rho_\pi} \right) \left( \frac{\sigma_{\pi N}}{\sigma_{n N}} \right) \frac{m_\pi}{m_N}}.$$
The total shear viscosity to entropy density ratio appears to be very important. However, the viscosity to entropy density ratio without (middle panel) and with (lower panel) mixing effect are shown in Fig. (12) and (13) as a function of $T$.

Different possible pion baryon loops are accounted to calculate the total $\Gamma_N$, which depends on the three momentum of nucleons ($\vec{k}$) as well as the medium parameters $T$ and $\mu_N$. Using the numerical function $\Gamma_N(\vec{k}, T, \mu_N)$, $\eta_N$ and $\eta_N/s_N$ are numerically generated as functions of $T$ and $\mu_N$. Adding the pionic contribution taken from Ref. 53 with the numerical values of the nucleonic component, we have obtained the total shear viscosity, where a gross mixing effect of two component system has been implemented. Along the temperature axis, the shear viscosity of both pion and nucleon components appear as increasing function, whereas along the $\mu_N$ axis shear viscosity of pion component changes from its constant behavior to a decreasing function due to presence of mixing effect. The total shear viscosity to entropy density ratio ($\eta_{\text{tot}}/s_{\text{tot}}$) for the pion-nucleon mixed gas reduces with increasing $T$ as well as $\mu_N$ and quantitatively becomes very close to the KSS bound. This behavior indicates that $\eta_{\text{tot}}/s_{\text{tot}}$ tends to reach its minimum value near the transition temperature at vanishing as well as finite value of $\mu_N$. According to these results, the finite baryon chemical potential helps the nuclear matter to come closer to its (nearly) perfect fluid nature.

**V. SUMMARY AND CONCLUSION**

Owing to the Kubo relation, the shear viscosity can be expressed in terms of two point function of the viscous stress tensors at finite temperature. By using the real-time thermal field theoretical method, this two point function has been represented as $NN$ loop diagram when the nucleons are considered as constituent particles of the medium. A finite nucleon thermal width $\Gamma_N$ has been traditionally included in the nucleon propagators of the $NN$ loop for getting a non-divergent shear viscosity $\eta_N$. This nucleon thermal width is obtained from the one-loop self-energy of nucleon at finite temperature and density. A finite nucleon thermal width $\Gamma_N$ has been traditionally included in the nucleon propagators of the $NN$ loop for getting a non-divergent shear viscosity $\eta_N$.

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