A Quantum Approach to Gravitational Waves

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Abstract

In light of the Gravitational Waves observation and the discrete structure of spacetime at high scales $M_{QG}$, we investigate the behavior of Gravitational Waves and the spacetime deformation patterns in terms of gravitons and derive the effective results with respect to photons. Using the GW150914 data and related signals, we probe the underlying quantum scale $M_{QG} \sim 10^6\text{GeV}$.

Key words: General Relativity; Graviton; Quantum Gravity.

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1 Introduction

The existence of gravitational field, satisfying Einstein’s equations of General Relativity (GR), as well as Gravitational Waves (GWs) traveling at the speed of light is widely agreed [1, 2]. Recently, after the discovery of (GWs) announced by the LIGO and VIRGO detectors in the event GW150914 [3], a new era in physics such as tests of GR, heavy black holes and measurements of astrophysical processes that have been inaccessible to observations with electromagnetic waves is now open [4, 5, 6, 7, 8]. In particular, the LIGO detectors have observed GWs from the merger of two stellar-mass black holes. The detected waveform matches the predictions of GR for the inspiral and merger of a pair of black holes and the ringdown of the resulting single black hole. These observations support the existence of binary stellar-mass black hole systems. This is the first direct detection of GWs and the first observation of a binary black hole merger. Fundamentally, these distortions of the neighbouring spacetime may shed light on the fundamental laws governing our universe such as quantum aspects of gravity and related models. In particular, an upper limit on the graviton mass $m_g < 10^{-22}$ eV has been reported by the LIGO Collaboration [7]. Gravitons are the field quanta of the gravitational field, in analogy with photons being the quanta of the electromagnetic field; they are the ultimate constituents of GWs. New studies of graviton propagation has been encouraged by the recent thrust of interest for GWs in which the single-quantum behaviour play a central role [7, 9, 10, 11, 12]. With these continuous efforts, especially theoretical, the search for a quantum description of gravity has rapidly grown [13, 14], and there is currently a large interest in experimental tests hoping to derive related effects or unconventional spacetime structure [16, 16, 17]. Such quantum signatures of spacetime could be observable only in high energy interactions over the actual accelerator scales or in very long baseline experiments.

In this work, we are concerned with the question whether we can study the single-quantum of the GWs, or in other words, the propagation of the individual gravitons in the fundamental spacetime fabric. We consider for that the effective discrete spacetime structure at quantum scales $\sim M_{Planck}$ in terms gravitons of to determine the spacetime deformation patterns and the graviton propagation behaviour within the possible dispersive effect. Seen that such effect would be naturally very small and thus only observable at long baselines experiments, we refer to the GW150914 data and the related signals to derive the propagation behavior and the underlying mass scale $M_{QG}$.
2 Classical description

According to GR the spacetime geometry is determined by the energy matter distribution, and the curvature of the spacetime metric is linked to the energy content of the universe by the equation

\[ G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu} \]  

where \( G_{\mu\nu} \) is the Einstein tensor describing the geometry of the 4D spacetime as the sum of the Ricci tensor \( R_{\mu\nu} \) and the metric tensor \( g_{\mu\nu} \), and \( T_{\mu\nu} \) is the stress-energy tensor representing the source of the GWs. The metric \( g_{\mu\nu} \) describing the geometry of spacetime links the spacetime coordinate \( dx^\mu \) to the spacetime interval \( d\ell^2 \) through the relation

\[ d\ell^2 = g_{\mu\nu} dx^\mu dx^\nu. \]

By the fact that GWs will always be very weak at the earth, the background curvature can be ignored and the metric can be approximated as that of the Minkowski flat metric \( \eta_{\mu\nu} \) and the equation (1) can be linearized in the case of small perturbations. These perturbations can then be expressed by an approximation as

\[ g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \]

where \(| h_{\mu\nu} | \ll 1 \) is the small perturbation induced by the GWs. The small changes \( \delta\ell^2 \) in the spacetime interval \( d\ell^2 \) reads

\[ \delta\ell^2 = h_{\mu\nu} dx^\mu dx^\nu \]

with \( \pm d\ell^2 \) is the small changes, i.e., contractions and dilatations in the length \( \ell \) of the spacetime fabric. In the weak field limit and with the most useful gauge choice\(^2\) the solution of equation (1) wherein the equation of GR becomes a system of linear equations, specifically a system of wave equations\(\text{[17]}\), corresponds to a 3D wave equation traveling at the speed of light \( c = 1 \). In the sinusoidal case and with the symmetry of \( h_{\mu\nu} \), the physical part of these waves can be expressed here in the z-direction by the equation

\[ h_{\mu\nu} = \delta\ell e_{\mu\nu} \cos(\omega t - kz) \]

\(^1\)Here for simplicity we work in natural units with \( \hbar = c = 1 \).

\(^2\)Where there is no stress-energy source term \( T = 0 \) and with the most useful gauge i.e., the TT gauge in which the coordinates are defined by the geodesics of freely falling test bodies.
where $\epsilon_{\mu\nu}$ are the so-called unit polarization tensors $\epsilon_{\mu\nu}^+, \epsilon_{\mu\nu}^\times$ with the signs $+, \times$ mean that there are just two possible independent polarization states such as

$$\delta \epsilon_{\mu\nu} = \delta \epsilon^{+,\times}$$  \hspace{1cm} (6)

are the strain amplitudes of each polarization, and $\omega$ and $k$ being the angular frequency and the wave vector respectively. These classical GWs have now been confirmed by the recent successful detection in the LIGO experiment. After this confirmation, the hope for the detection of an individual quantum of gravity is now desired.

3 Quantum approach

Theoretically, the quantum picture of GWs is accepted because although any quantum field theory that describes gravity is non-renormalizable, the energy scale at which we expect quantum gravity effect to be detectable is extremely large $\sim M_{\text{Planck}}$. Hence, an effective theory valid at low energies can be used to describe GWs in terms of particles, even if the exact theory of gravity valid at an arbitrary scale is somewhat different conceptually. In this context, at the scale where quantum gravitational effects are expected to be felt $M_{\text{QG}} \sim M_{\text{Planck}}$, the classical perception of the spacetime continuum must be altered by a granular structure. As a result, the vacuum will act as a non trivial medium characterized by gravitons of mass $\sim m_g$ spaced by elementary length $\ell_{\text{QG}} \geq \ell_{\text{Planck}}$ as pictured in the figure 1.

Figure 1: 2D discrete spacetime at quantum scale $M_{\text{QG}}$.

According to this discretisation, the propagating spacetime perturbation (5) can be viewed as coherent superposition of a large number of gravitons spaced by

$$\ell_{\text{QG}} \pm \delta \ell_{\text{QG}} \geq \ell_{\text{Planck}}$$  \hspace{1cm} (7)
such as $\pm \delta \ell_{QG}$ are now the small perturbations induced by GWs, i.e., contractions and dilatations of the length $\ell_{QG}$ in the spacetime fabric. Such deformation patterns are shown in the figure 2 and the figure 3.

In flat spacetime (Fig.1) or in a spacetime with small curvature (Fig.2 and Fig.3) the propagation of gravitons can be described by equations which are similar to those encountered in electromagnetism. Concretely, in this quantum description where GWs are viewed as large number of gravitons, a propagating graviton of energy (frequency) $E_g \equiv \omega_g$ will be not
immune to the quantum gravity effect, i.e., to the surrounding gravitons. In general, the conservation of energy-momentum during the scattering process implies that such interaction is of the order

$$\delta \omega_g \sim \pm \frac{\omega_g^2}{M_{\text{QG}}}. \quad (8)$$

In this direction, the wave equation (5) can be translated into a particle equation where the phenomenology can be discussed from a generic modification of the graviton dispersion relation as

$$\omega_g^2 \simeq k_g^2 + m_g^2 - \frac{\omega_g^4}{M_{\text{QG}}^2}, \quad (9)$$

where we see that the existence of such quantum gravity regime phenomena at a scale $\sim M_{\text{QG}}$ manifests itself through a retardation effect $-\omega_g^2/M_{\text{QG}}$ in the dynamics of the moving gravitons themselves. At this stage, it is worth mentioning that the effect $\omega_g^2/M_{\text{QG}}$ in the particle description (9) should be physically related somehow to the perturbation amplitude $\delta \ell_{\text{QG}}$ in the wave description (9), since they are already dimensionally related as $[\delta \ell_{\text{QG}}] \simeq \left[\omega_g^2/M_{\text{QG}}\right]^{-1}$; I believe this requires deep investigations. Here, by neglecting the effect of the extreme small graviton mass $m_g \ll \omega_g$ and using the fact that $\omega_g \ll M_{\text{QG}}$, straightforward calculations from (9) lead to the corresponding effective energy-dependent graviton velocity

$$v_g(\omega_g) \simeq \frac{1}{1 + \omega_g^2/M_{\text{QG}}^2}, \quad (10)$$

which appear slightly deviated from the velocity of light, and decreases with increasing energy. In this sense, the higher-energy graviton propagates slower than the low-energy graviton because of the fact that it undergoes a spacetime attenuation that grows with its energy. This could be then experienced from (10), with respect to a light ray propagating with speed $c = 1$ and emitted by the same source at a distance $d$ from the detector, by the velocity shift

$$\delta v_g \simeq - \frac{\omega_g^2}{\omega_g^2 + M_{\text{QG}}^2}, \quad (11)$$

and the temporal shift

$$\delta t_g \simeq \frac{d}{M_{\text{QG}}} \quad (12)$$

being the velocity difference and the time delay of the propagating graviton.
4 GW150914 and results

According to GW150914 data, the distance estimate to the source is $\sim 10^9$ Ly, the wave frequency is $\sim 100$ Hz and the apparently coincident flash of photons is $> 50$ keV observed 0.4 s later (because of the scattering process in the intergalactic medium in contrast to gravitons with their low interaction cross sections) by the Fermi GBM signal [18]. The plausibility of the GBM signal has been questioned [19, 20]. All these data can be summarized, along with the resulting graviton-photon velocity difference and time delay, in the table 1.

| Event                      | GW150914   |
|----------------------------|------------|
| distance to GW150914 : $d_{GW150914}$ | $\sim 10^9$ Ly |
| EM frequency: $\omega_{\gamma}^{GW150914}$ | $> 50$ keV   |
| GW frequency: $\omega_{g}^{GW150914}$ | $\sim 100$ Hz |
| time delay: $\delta t_{g}^{GW150914}$ | $\sim 0.4$s   |
| velocity difference: $\delta v_{g}^{GW150914}$ | $\lesssim 10^{-17}$ |

Table 1: Summary of the GW150914 data.

Based on the GW150914 data (tab. 2) and the fact that $M_{QG} \ll M_{Planck}$, and according to (11) and (12), we bound the graviton-photon velocity difference and time delay in the table 2.

| Event     | time delay: $\delta t_{g}^{GW150914}$ | velocity difference: $\delta v_{g}^{GW150914}$ |
|-----------|-------------------------------------|-----------------------------------------------|
| GW150914  | $\gtrsim 10^{-27}$ s                  | $\gtrsim 10^{-52}$                           |

Table 2: Lower bounds of the GW150914 graviton-photon velocity difference and time delay.

and estimate the underlying quantum gravity scale as

$$M_{QG} \simeq \omega_{g}^{GW150914} \sqrt{\frac{d_{GW150914}}{\delta t_{g}^{GW150914}}} \sim 10^6 GeV. \quad (13)$$

Being of the order of $\sim 10^3$ TeV, the underlying scale $M_{QG}$ appears then to be above the reach of the recent accelerator experiments LHC$\sim 10$ TeV, but exploration of signatures of the quantum nature of spacetime remains possible. This unrefined estimate would need to be precised by a detailed numerical analysis, but it reinforces the point that mergers of bigger objects can give more powerful constraints on the deep fabric of spacetime.
5 Concluding remarks

In this work, a quantum approach to GWs has been proposed based on a discrete perception of spacetime. Precisely, starting from the classical GR description and considering a granular structure of spacetime motivated at high scales $M_{QG}$ in terms of gravitons as its fundamental fabric (fig.1), a particle description of GWs where the spacetime deformation patterns and the graviton propagation behaviour have been described $\text{(9)}$. According to this effective vision, the energy-dependent graviton velocity along with the corresponding velocity difference and time delay with respect to the photon have been derived $\text{(11)}$ and $\text{(12)}$. Then with the analysis of the GW150914 data, lower bounds of these graviton-photon shifts have been given (tab.2) and the underlying high scale $\text{(13)}$ has been approached $M_{QG} \sim 10^6 GeV$. To date, unfortunately, it is impossible with current technology to observe or manipulate single gravitons in a lab, so gravitons are still theoretical. But in spite of the fact that they hav not been observed yet, neutral spin-2 particles as the vectors of the gravitational interaction, are assumed by most of the quantum gravity models as the one proposed in this work, and likely massive in contrast to the current dominant opinion as for the case of neutrinos many years ago. This idea is very speculative, and the model calculations that we have presented here require refinement. However, it is well motivated by the basic fact that gravity and spacetime are deeply related if they are not even one thing.

Finally, we note that if the LIGO Collaboration observes a new signal for a merger of another pair of black holes $\text{(21, 22, 23)}$, much greater range of information will be gathered, useful and deterministic than was possible in GW150914.

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