Numerical investigation of dynamic behaviour of single and two hinged articulated towers in random sea

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Abstract. The present paper deals with comparative studies on hydrodynamic behaviour of Single and two hinged articulated towers in Random sea. The dynamic behaviour of towers is investigated under long crested Uni directional random sea and the wave force calculations are performed using Modified Morison equation. A visual based C ++ program is developed to evaluate the response of the towers by taking into account non linearities like time variation of submergence, instantaneous tower orientation and hydrodynamic loading. Finally, the equations of dynamic motion are solved using New Mark integration scheme followed by Fast Fourier transform to obtain Power spectral densities. From results, it is observed that presence of intermediate hinges reduced the spectral energy coming on Two hinged articulated tower indicating its significance over Single hinged towers.

Keywords: Frequency, Hydrodynamic, Response, Spectrum

1. Introduction

For deep water oil exploration and exploitation, articulated towers belonging to the class of compliant towers have been found quite suitable for deep water applications. Compliant offshore platforms move along with the waves along its degrees of freedom and offers flexibility to structural framework to disperse hydrodynamic forces. As the natural frequency of these platforms is lower than wave force frequency it reduces the magnification of wave forces acting over the tower length. As a result, the structure responds not only to first order wave forces, but also to higher order wave forces. These higher order wave forces are due to nonlinear behaviour of the wave – structure system which produces sub harmonics and super harmonics resulting in large displacement and non-linear drag force. The nonlinear drag force is generated due to interaction between water particle velocity and structural velocity that adds to wave forces at every displaced position of the structure. Numerous researches have adopted several numerical approaches to investigate the non-linear behaviour of articulated towers under different loading conditions. [1] and [2] carried out dynamic stability analysis of Single hinged Articulated Tower (SHAT) under different categories of wave loads and earthquake loads. [3] carried out stochastic responsive studies on double hinged articulated towers and highlighted wind induced dynamic responses of the towers. [4] examined the behaviour of a Multihinged towers under earthquake loads in the influence of varying wave load frequencies. [5] conducted a numerical study based on hydrodynamic analysis of Single and Multihinged articulated tower using ANSYS and concluded that the location of intermediate should be decided based on structural stability and buoyancy. [6] examined the behaviour of articulated towers in varying water depth under the effect of aerodynamic and wind loads and concluded that towers in shallow waters developed more oscillations than the towers in deep waters. [7] investigated the behaviour of bi-articulated tower under wind, wave and current forces and concluded that current force modified the peaks of response spectrum of various parameters like bending moment, shear at hinge, base rotation and surging actions. [8] developed a theoretical modeling to examine the vibrational behaviour of articulated towers under random wave and wind conditions.
Though there are several works carried on articulated towers, only very few researchers examined and compared the behaviour of Single and two hinged towers in Uni-directional random sea conditions. So, the current study is focused on developing numerical model of towers using finite element method and carry out comparative forced vibration analysis to investigate the wave structure interactive behaviour of Single and Two hinged articulated towers in Uni-directional random sea conditions.

2. Numerical modelling

2.1 Assumptions and structural idealisations

For finite element modelling of compliant platforms, two nodded beam column elements with one translational and one rotational degree of freedom is selected. The displacement or geometric location of the element is described by translational degree of freedom. The assumptions adopted for physical idealizations of dynamic model system are listed as below:

- Column is divided into an arbitrary number of beam column finite elements having two degree of freedom.
- Elements are composed of linearly elastic material; significant deviation from the linearity of modulus of elasticity in such elements is usually an indication of yielding a condition desirable for a practical design.
- Although beam column elements may undergo large displacements elastic deformation are assumed to be small enough that small deflection beam theory is used within limit.
- Shear deformation are neglected so that transverse deformation due to shear are negligible compared to those due to bending.
- Time dependent Hydrodynam coefficients are taken into account.
- The fluid particle velocities and accelerations are found from linear wave theory and Modified Morison’s equation is used.
- Tower is subjected to Uni directional random waves.

3. Solutions of dynamic equilibrium equations

The dynamic equation of tower motion in terms of inertia force, damping force, restoring and exciting forces are expressed as given in Equation (1):

\[
[M]\ddot{X} + [C]\dot{X} + [K]X = \{F(t)\}
\]

\{\ddot{X}\}, \{\dot{X}\}, \{X\} are the horizontal acceleration, velocity and displacement vectors. \([M]\), \([C]\), \([K]\) are the structural mass, damping and stiffness matrices of the tower.

\(F(t)\) is called as Hydrodynamic forcing vector. Mass matrix is the sum of structural mass and added mass which is due to water present near structural interface as represented in Equation (2).

\[
[M] = \begin{bmatrix}
156 & 22L & 54 & -13L \\
22L & 4L^2 & 13L & -3L^2 \\
54 & 13L & 156 & -22L \\
-13L & -3L^2 & -22L & 4L^2 \\
\end{bmatrix} + [M]_{\text{added mass}}
\]

(2)

The added mass matrix, \([M]_{\text{added mass}} = \rho_w(C_M - 1)\pi \frac{D^4}{4}\)

Where \(\rho_w\) the mass density of fluid, \(D\) is the diameter of Articulated tower and \(C_M\) is the inertia coefficient.

Now \([K] = [K_s] + [K_g]\) where \([K_s]\) is the structural stiffness matrix and \([K_g]\) is geometric stiffness matrix as given in Equation (3).
\[ [K] = \begin{bmatrix} 12EI/L^3 & -6EI/L^2 & -6EI/L & -12EI/L \\ -6EI/L^2 & 6EI/L & 2EI/L & -6EI/L \\ -6EI/L & 6EI/L & 2EI/L & -6EI/L \\ -12EI/L & 2EI/L & 6EI/L & 4EI/L \end{bmatrix} \times \frac{-\rho}{30L^2} \begin{bmatrix} 36 & 3L & -36 & 3L \\ 3L & 4L^2 & -3L & -L^2 \\ -36 & -3L & 36 & -3L \\ 3L & -L^2 & -3L & 4L^2 \end{bmatrix} \] (3)

The dynamic buoyancy force is obtained by integration of dynamic pressure at the bottom of the cylinder, \( f = \int PdA \), where ‘\( P \)’ is effective axial compressive force in the member [4].

Damping force is considered proportional to a mass and stiffness matrices is given in Equation (4)

\[ [C] = a_0 [M] + a_1 [K] \] (4)

Where \( \{\alpha_0\} = \frac{2\xi}{\omega_n + \omega_m} \{\omega_m \omega_n \} \) where \( \omega_m \) is the natural frequency of multi degree freedom system, \( \omega_n \) is the higher frequencies of modes that contribute to dynamic response and \( \xi \) is Damping ratio.

The fluid forces are computed using Morison’s equation as given in Equation (5) [9]:

\[ \begin{align*}
    \{f_x\} = & \frac{1}{2} \rho C_D \times D \times |W_{nr}| |N| \begin{bmatrix} u - \ddot{x} \\ \nu - \ddot{y} \\ w - \ddot{z} \end{bmatrix} + C_M \times \rho \times \pi D^2 \times \frac{1}{4} \times |N| \begin{bmatrix} \ddot{u} - \dddot{x} \\ \ddot{v} - \dddot{y} \\ \ddot{w} - \dddot{z} \end{bmatrix} \\
\end{align*} \] (5)

where \( \{f_x\} \) are the wave forces in the global X,Y,Z directions, \( |W_{nr}| \) is the magnitude of relative normal velocity, \( C_D \), \( C_M \) are drag and inertia coefficients, \( \rho \) is mass density, \( \ddot{x}, \ddot{y}, \ddot{z} \) are the velocities of towers in X,Y,Z direction and \( \{u, \nu, w\} \) are the wave velocities. The components of water component velocities (horizontal and vertical) are given as in Equation (6) and (7).

\[ \begin{align*}
    u &= \sum_{n=1}^{N} \frac{1}{2} H_n \omega_n \frac{\cosh k_n y \cos (k_n x - \omega_n t + \tau_n)}{\sinh k_n (h+\eta)} \\
    v &= \sum_{n=1}^{N} \frac{1}{2} H_n \omega_n \frac{\sinh k_n y \cos (k_n x - \omega_n t + \tau_n)}{\sinh k_n (h+\eta)} \\
\end{align*} \] (6,7)

Where \( k_n \) and \( \omega_n \) denotes wave number and wave frequency of the \( n^{th} \) interval wave [9].

Finally, response of the dynamic motion is expressed in terms of the random amplitude operator (RAO) which is the square root of power spectral densities of the response and wave surface elevation as shown in Equation (8).

\[ \text{RAO} (f) = \frac{S_{RR}(f)}{S_{\eta}(f)} \] (8)

Where \( S_{RR}(f) \) is power spectral density (PSD) of response and \( S_{\eta}(f) \) is PSD of wave surface elevation \( \eta \).

4. Program development and approach for solutions

A computer code for the analysis of articulated tower has been developed in Visual C++. In the present work, eigen values and corresponding eigen vectors are computed using Jacobi’s method and damping matrix is calculated through Rayleigh’s method. The loads due to gravity, added mass buoyancy and Morison are first computed at a point in an element and then they are integrated over element using Gaussian quadrature. Then these forces are converted into equivalent nodal forces and moments and finally equations of motion are solved by New mark - \( \beta \) method. Once response of the tower is
obtained, a respective power spectral density of tower is obtained by Fast Fourier Transform Algorithm. Figure. 1 shows the finite element discretisation for Single (One) and Two hinged towers.

![Finite element discretisation](image)

**Figure 1.** Finite element discretisation of Single (One) hinged tower and Two hinged tower

5. **Free vibration analysis**

The first three modes of Single and Two hinged articulated towers are shown in Figure 2. The time periods associated with the three modes of the Single (One) hinged tower are 23.2 s, 1.2s and 0.64s and that for the Two hinged tower are 26.7 s, 0.54 s and 0.24 s respectively.
Hydrodynamic tower responses of Single and Two hinged articulated towers under random waves with significant heights $H_s = 15m$ are calculated. Table 1 and 2 shows the properties of Articulated towers. To highlight the importance of Multi hinged tower, quantities like bending moment, tip level displacement and axial forces are studied and compared with Single hinged tower. For two hinged towers, a hinge is introduced at height of 200 m from the base. The data for tower specification are taken from [10].

**Table 1. Tower properties of Single and Two Hinged Articulated tower**

| Data             | Properties       |
|------------------|------------------|
| Tower height     | 430 m            |
| Tower diameter   | 15m ,14.8m       |
| Modulus of elasticity | 2.07X10^{11}N/m² |
| Ballast Mass/length | 6371.04kg/m   |
| Deck weight      | 10000kN          |
| Structural damping | 9%              |

**Table 2. Fluid properties of Single and Two Hinged Articulated tower**

| Data             | Properties       |
|------------------|------------------|
| Mean water level | 400 m            |
| Water density    | 1025kg/m³        |
| Wave frequency   | 0.3696rad/s      |
| Significant wave height | 15m               |
6. Forced vibration analysis

6.1 Tip displacement spectrum

The tip displacement of tower is an important parameter, because the intensity of restoring forces generated during the oscillation of structure around the universal joint depends on the forced displacement of buoyancy chambers when the structure is tilted. Table 3 shows the significant values of tip displacement of Single (One) and two hinged articulated towers and by comparing it is found that significant value is decreased by 21% for two hinged towers. Figure 3 shows the PSD for tip displacement. The PSDF’s of tip displacement of towers is characterized by three peaks, one occurring at structure’s fundamental frequency (or period T), second occurring at vicinity of the peak frequency (Tp) and third at wave’s peak frequency of the respective power spectrum as shown in figure. Figure 4 shows the RAO of response of Two hinged articulated tower where tower peaks are near to fundamental frequency or periods of structures and at multiples of wave periods.

![Tip level displacement spectrum](image)

**Figure 3.** Tip level displacement spectrum
Table 3. Significant values - Tip level Displacement

| Data                  | Single (One) hinged tower | Two hinged tower |
|-----------------------|---------------------------|------------------|
| Tip level displacement(m) | 0.371                     | 0.294            |

Figure 4. RAO of tower displacement

6.2 Bending moment spectrum
Figure 5 shows the PSD of bending moment for both towers. Table 4 shows the bending moment values and from the Table it can be seen that bending moment values of Two hinged articulated tower is decreased by about 60% when compared to Single (One) hinged towers which is due to the presence of intermediate hinge which reduced the spectral energy which is coming on each member, which is one of the main parameters deciding the importance of Multi hinged towers.

Table 4. Significant values - Bending moment

| Data                  | Single (One) hinged tower | Two hinged tower |
|-----------------------|---------------------------|------------------|
| Bending moment (Nm)   | $1.88 \times 10^6$        | $0.758 \times 10^6$ |
From the Figure 6 and Table 5 shows the comparison of axial force of Single (One) and Two hinged articulated tower, and it is found that significant value of two hinged decreased by about 13% with that of Single (One) hinged articulated tower which is due to presence of intermediate hinge, which reduces the axial forces along the tower. The axial force component is the important parameter that decides design of underwater hinged connection at seabed level.

**Table 5. Significant values - Axial force spectrum**

| Data            | Single (One) hinged tower | Two hinged tower  |
|-----------------|----------------------------|-------------------|
| Axial force (N) | $7.491 \times 10^6$       | $6.504 \times 10^6$ |
7. Conclusions

- The Power spectral density function (PSDF’s) of tip displacement have three distinct three peaks. The highest peak occurred at natural frequency and corresponding peaks occurred at second and third wave frequency.
- Occurrence of peaks at second significant wave frequency are generated by instantaneous surface elevation, rather than the non-linear drag force, as Morison’s drag force can introduce only odd harmonics.
- The maximum bending moment along the height of two hinged articulated tower is reduced by about 60% of that of the Single (One) tower which indicates the importance of Multi hinged tower for deeper water conditions as intermediate hinge decreased the maximum bending moment along the height of the tower. Since the structural displacement is significant, the moment due to drag force which is proportional to relative velocity decreases, resulting in overall decrease in exciting moment. As the moment valve decreases, rotation of the tower decreases producing stiffer and stiffer tower rotation.
- It can be also deduced that only first mode of natural frequency contributes to spectral peaks for displacement, bending moment and axial force as they are outside the frequency range of spectrum.

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