Lagrangian quantum turbulence model based on alternating superfluid/normal fluid stochastic dynamics

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Abstract – Inspired by recent measurements of the velocity and acceleration statistics of Lagrangian tracer particles embedded in a turbulent quantum liquid we propose a new superstatis-
tical model for the dynamics of tracer particles in quantum turbulence. Our model consists of random sequences S/N/S/... where the particle spends some time being trapped on vortex lines of the superfluid (S) and some time in the normal liquid (N). This model leads to a superposition of power law distributions generated in the superfluid and Gaussian distributions in the normal liquid, in excellent agreement with experimental measurements. We include memory effects into our analysis and present analytic predictions for probability densities and correlation functions.

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The statistics of tracer particles embedded in a turbulent quantum liquid is an interesting field of current turbulence research [1–21], for recent review papers see [22–24]. Measurements of Paoletti et al. [1] have shown that the measured velocity distributions asymptotically exhibit a power law with exponent approximately given by −3. This is in contrast to classical fully developed turbulence, where velocities are approximately Gaussian (but velocity differences and accelerations are strongly non-
Gaussian [25–32]). Recent measurements of La Mantia and Skrbek [2] have illustrated that, depending on the spatial scale probed, there can be a mixture of quantum and classical features in the distributions. The measured velocity distributions in [2] again exhibit asymptotic power laws with exponent −3, but superimposed to that there appears a Gaussian core for low velocities. In [3], for the first time, also histograms of measured accelerations for particles embedded in turbulent quantum flow were presented, which appear to be of similar shape as those of accelerations in classical Lagrangian turbulence [25]. Numerical simulations of the velocity statistics based on the Gross-Pitaevskii equation have been performed in [4], also providing evidence for power law tails. A simple model for the dynamics of test particles based on χ²-superstatistics (a method of nonequilibrium statistical mechanics [33–37]) was proposed in [5]. This model predicts power law tails with exponent −3, and in addition yields further information on the shape of the probability density near the maximum. Effectively it leads to so-called q-Gaussian distributions [38–40] with entropic exponent q = 5/3.

In this paper we introduce a rather general but powerful stochastic model for the dynamics of a test particle in a turbulent quantum liquid, which, contrary to previous work, takes into account the fact that there is a mixture of superfluid and normal liquid. Consequently, this new model is based on a mixture of two different statistics: one is related to the trapped particle movement on vortex lines of the superfluid (S), leading to power law distributions, and one is based on the movement in the normal liquid (N), leading to a superimposed Gaussian. For a given test particle both phases alternate in a random sequence S/N/S/... If we attribute to each symbol a fixed time scale, then repetitions of the same symbol occur as well, e.g. SSNSNNNSS.... From a statistical mechanics point of view our model has the form of a superstatistical stochastic differential equation [29,32,40] combined with a symbolic dynamics [41,42]. From a condensed-matter physics point of view the model has some analogy with regarding the quantum liquid as a spatial random collection of infinitely many S/N/S Josephson junctions [43,44]. The symbol sequences generated by the test particle that moves through the quantum liquid have the ability to encode complex memory effects in the dynamics generated by the quantum turbulent flow.

We present results for the probability distributions generated by this model and compare with recent...
experimental data presented in [2,3]. Excellent agreement is found. The model is simple enough to allow for analytical calculations of temporal correlation functions, and we present some results for the case of a generalized dynamics that also contains a memory kernel.

Consider a tracer particle (a small particle visualizing the flow) embedded in a quantum liquid that consists of a mixture of superfluid (S) and normal liquid (N). Consider a sequence (e.g. SNSNSSSSN...) where the local dynamics of the particle is different depending on whether it is in phase S or N. In phase S we consider a superstatistical local dynamics [5,40] where the velocity \( \mathbf{v} \) of the tracer particle satisfies

\[
\dot{\mathbf{v}} = -\gamma(t)\mathbf{v} + \omega(\mathbf{e}(t) \times \mathbf{v}) + \sigma(t)L(t). \tag{1}
\]

Here \( L(t) \) is vector-valued Gaussian white noise, and \( \omega \) is a parameter. We assume that the effective damping constant \( \gamma \) and the noise strength \( \sigma \) are functions of \( t \), and so is \( \omega \) and the direction of the unit vector \( \mathbf{e} \), which is uniformly distributed. The unit vector \( \mathbf{e} \) and the noise strength \( \sigma \) evolve stochastically on a large time scale \( T_e \) and \( T_\sigma \), respectively. In the superstatistics approach, one consider the parameters of a local stochastic differential equation to be random variables [40]. That is, the parameters in eq. (1) can take on very different values during time evolution. A very small \( \gamma \) corresponds to nearly undamped motion for a limited amount of time. A very small \( \omega \) corresponds to almost no rotation, i.e. straight movement for a limited amount of time. All these cases are included as possible local dynamics and averaged over in the superstatistical approach. Due to the integration over probability densities in \( \gamma, \omega, \sigma \), eq. (1) yields non-Gaussian behavior represented by some non-Gaussian density function \( p_S(v) \) (see later sections for concrete calculations).

The above dynamics is taken as a model if the test particle is trapped on a vortex line of the superfluid component. In fact, the phenomenon of vortex trapping has been studied in detail in several recent papers, see, e.g., [45]. Vortex lines induce a radial pressure gradient which exhibit a radial attracting force on test particles in the liquid. In the trapped state, the dynamics of the test particle essentially represents the dynamics of the vortex on which it is trapped. It is clear that such a dynamics is very different from that of a free particle. In fact, as shown in numerical simulations, this dynamics leads to power law distributions [6]. Similar power law distributions are actually also observed in other systems, for example in defect turbulence [46]. We denote this phase of the particle dynamics in the quantum turbulent flow by the symbol S.

Contrary to that, if the particle is not trapped on a vortex, then its behavior is dominated by the normal liquid (phase N), and we then assume that its dynamics is described by an ordinary Langevin equation with constant parameters \( \gamma_0 \) and \( \sigma_0 \), leading to a standard type of Gaussian behavior, as expected for the velocity of a test particle in classical liquids:

\[
\dot{\mathbf{v}} = -\gamma_0\mathbf{v} + \omega(\mathbf{e}(t) \times \mathbf{v}) + \sigma_0L(t). \tag{2}
\]

This simple linear equation just yields Gaussian behaviour with inverse variance parameter \( \beta_0 = 2\gamma_0/\sigma_0^2 \) during phase N. If the variance is rescaled to 1, meaning that we consider the velocity in units of its variance, then the stationary probability density in this case is simply

\[
p_N(v) = \frac{1}{\sqrt{2\pi}v} e^{-\frac{v^2}{2}}. \tag{3}
\]

In our model we assume that both phases are relevant and for a given tracer particle occur in random (but not necessarily uncorrelated) sequences. Switching from one phase to the other simply means that the tracer particle is hopping from a vortex-trapped state to a non-trapped state and vice versa. Each tracer particle in the quantum liquid produces a temporal sequence of symbols S and N, which, depending on parameters of the quantum turbulent flow environment, the size of the test particle, and the time scales involved, will have different stochastic properties.

An important remark is in order. Clearly our stochastic differential equations used to model state S and N are models and they should not be regarded as the exact theory. It is well known that a realistic dynamics for a particle trapped on a vortex line is essentially Schwarz’s equation [6,47]. For an untrapped particle influenced by the normal liquid it is essentially Newton’s law with a Stokes friction arising from the normal liquid and inertial forces, as discussed in [12]. It was observed in [6] by numerical simulation that Schwarz’s equation produces power law tails, whereas the dynamics studied in [12] essentially generates Gaussian behavior. Our model equation are chosen in such a way that they reproduce precisely the same probability densities in a given state S or N, and allow hopping between these states. Still our model equations are simple enough that they allow for some analytical calculations.

The simplest case are just statistically independent random sequences of symbols S an N. The probability density \( p(v) \) of velocity of the tracer particle is then given in the mixed form

\[
p(v) = w_S p_S(v) + w_N p_N(v), \tag{4}
\]

where \( w_S \) and \( w_N \) describe the probability of the symbol S, respectively N, to occur \( (w_s + w_N = 1) \). In other words, the relative duration of the two different phases is relevant. The probabilities \( w_S \) and \( w_N \) will depend on external parameters of the quantum liquid, as well as on the spatial and temporal scale on which the measurements are done. If the spatial scale probed by the test particle is larger than the average distance of vortex filaments, then we expect \( w_N \approx 1 \) to dominate, leading basically just to Gaussian behaviour. Non-trivial quantum turbulence properties are probed by very small test particles at very low temperature, leading to \( w_S \approx 1 \).
The assumption of independent random sequences of symbols will usually be too simple for a quantum liquid. For example, if the particle is firmly trapped near a vortex core, then long-lasting sequences SSSSSS... will occur, similarly to the laminar phase for intermittent chaotic maps near a tangent bifurcation [41]. We propose to condition the probabilities \( w_S \) and \( w_N \) on the actual velocity of the test particle observed. This means we may consider conditioned probabilities \( w_S|v \) and \( w_N|v \), conditioned on a given observation of the velocity \( v \), in the sense that given a large observed velocity \( v \), \( w_S|v \) is close to 1, and in case a small velocity \( v \) is observed, \( w_N|v \) conditioned on that velocity is close to 1. This is plausible, because given a large velocity \( v \) it is very likely that this happened during a phase where the particle was trapped on a rapidly moving vortex and close to another vortex filament with rapid rotation. In this latter model, which has strong correlations between the observed value of \( v \) and the quantities \( w_S \) and \( w_N \) conditioned on the observed velocity, the probability density is given in good approximation by

\[
p(v) = \begin{cases} 
  p_N(v), & |v| \leq v_c, \\
  p_S(v), & |v| > v_c,
\end{cases}
\]

(5)

because large velocities are almost sure to occur in phase S, and hence above a given threshold are distributed according to the tails given by \( p_S(v) \). \( v_c \) is a critical velocity, whose value depends on the relative occurrence probabilities \( w_N \) and \( w_S \): the larger \( w_S \), the smaller \( v_c \). For \( w_S \to 1, v_c \to 0 \). For \( w_N \to 1, v_c \to \infty \).

Let us now work out more details on \( p_S(v) \) in phase S. As in the experiments, we restrict ourselves to the statistics \( p_S(v_i) \) of a single component \( v_i \). For ease of notation, we suppress the index \( i \) in the following. Following the same argument as in ref. [5], we consider an effective friction dependent on the distance to the nearest vortex filament. One obtains by integration for a \( \chi^2 \)-distributed effective friction \( \gamma(t) \) of \( n \) degrees of freedom (and under the assumption of a uniform distribution of rotation axis vectors \( \vec{c} \))

\[
p_S(v) = \frac{\Gamma(\frac{n}{2} + \frac{1}{2})}{\Gamma(\frac{n}{2})} \left( \frac{\beta_0}{\pi n} \right)^{\frac{n}{2}} \frac{1}{\left( 1 + \frac{\beta_0}{\pi n} v^2 \right)^{\frac{n}{2} + \frac{1}{2}}}
\]

(6)

The typical velocity of the tracer particle in the S phase depends on the perpendicular distance between the particle and the nearest (and sometimes merging) vortex filament. Therefore, the relevant degrees of freedom are \( n = 2 \) for 3-dimensional quantum turbulence, since the distance to a 1-dimensional vortex line has two components. In experiments there is often a drift velocity in the system that gives a non-zero mean velocity \( c \) to the test particle. In this case one has to replace \( v \) by \( v - c \) in eqs. (6) and (3), and for \( n = 2 \) one ends up with

\[
p_S(v) = \frac{\sqrt{\pi n_0}}{(2 + \beta_0(v - c)^2)^{\frac{n}{2}}}.
\]

(7)

For the above distribution \( p_S \), the variance does not exist, so in practice one introduces a cutoff \( v_{\text{max}} \), so that the variance is well defined [5]. This cutoff is also physically motivated, one typically observes in experiments values of the velocity up to about 15 standard deviations [2].

Let us now compare our model predictions with recent experimental measurements performed by La Mantia et al. [2, 3]. The experimental data obtained in [3] are to a certain extent different from previous seminal measurements published by Paoletti et al. [1] due to the fact that the investigated flows are different. La Mantia et al. conducted their experiment on steady-state thermal counterflow, while in [1] it was mainly decaying thermal counterflow. Moreover, the used techniques in both experiments were different, for example, different tracer particles were employed and the probe heat flux range was also not the same. Therefore, one expects similar results for the measured \( p(v) \) rather than exactly the same results.

Our model can describe the experimental data obtained by both groups, assuming that for the experiment [1] \( w_N = 4 \) (see [5] for a fit), whereas for the measurements in [2, 3] larger values of \( w_N \) are relevant, depending on the scale probed.

The probability density functions (PDFs) of velocities of tracer particles measured in [2, 3] have power law tails but superimposed there is also a near-Gaussian distribution in the central part. These measurements agree well with the prediction of our S/N/S model with strongly conditioned \( w_S|v \) and \( w_N|v \), leading to formula (5). An excellent agreement is obtained, as shown in fig. 1. The critical velocity \( v_c \) (in units of the standard deviation) is about \( v_c \approx 4 \), whereas for the data of Paoletti et al. [1] it is close to zero (see fit in fig. 1 in [5]), meaning that in those measurements the particle is much more frequently in the S phase.

Let us now move on from Lagrangian velocities to Lagrangian accelerations. La Mantia et al. [3] also measured

![Fig. 1: (Colour on-line) Experimental data of La Mantia et al. [3] and a fit using eqs. (7), (5) and (3) with \( \beta_0 = 4.5 \) and \( c = 0.3 \) for the tail part (blue solid line), as well as a Gaussian with mean 0.3 and variance 1 for the central part. Clearly, for large \( v \) this implies power law tails]

\[
p_S(v) \propto v^{-3}.
\]

(8)
the probability density function of vertical particle accelerations in the quantum turbulent flow, see also [48] for a recent update and [49] for a numerical simulation in this context. They report that the tails of the density function become more pronounced as the temperature decreases and as the heat flux of the thermal counterflow increases. Again we expect an interplay between different statistics for the normal phase N and the superfluid phase S of the embedded test particle. However, for the acceleration statistics the normal phase N will play a more important role than the phase S since the frictionless superfluid does not exhibit strong forces on the test particle. The cascade of energy dissipation is also similar in quantum turbulence as compared to classical turbulence [20], so that the acceleration environment of a Lagrangian test particle is quite similar to the case of classical turbulence, at least on scales of the same order of magnitude as the average distance between quantized vortices. Hence, we basically expect a similar acceleration statistics in the quantum turbulent flow as for classical Lagrangian turbulence since quantum effects on the acceleration are small. For the acceleration statistics we may thus essentially consider superstatistical models that have been successfully applied before to reproduce acceleration statistics in classical turbulent flow, such as [32]. These are based on lognormal superstatistics [34]. The prediction of these types of models is that the PDF of acceleration components is given by

\[ p(a) = \frac{1}{2\pi s} \int_0^\infty \beta^{-1/2} \exp \left[ -\frac{\left( \log \frac{a}{a_0} \right)^2}{2s^2} \right] \frac{1}{\beta} \, d\beta. \]  

(9)

The above probability density function given by eq. (9) is in excellent agreement with the experimental measurements of La Mantia et al. [3], as shown in fig. 2. Note that distributions of type (9) have been previously discussed and introduced in [50]; they re-occur here in a Lagrangian turbulence setting.

For a more detailed understanding it is important to measure not only histograms of velocities and accelerations, but also 2-point and higher correlation functions. Depending on time scale, and the spatial scale probed by the test particles, as well as the temperature of the quantum liquid, the symbol sequences SNSSSS... will have memory effects, as mentioned before. Under simple model assumptions our approach allows us to calculate the two-point correlation function analytically. First, let us slightly generalize eq. (1) by introducing a memory kernel. This allows for non-Markovian effects in the quantum liquid, represented by memory effects in the symbol sequences. The more general local dynamics including memory reads

\[ \dot{v} = -\gamma v + \alpha \int_{-\infty}^t dt' e^{-\eta(t-t')} v + \omega [e(t) \times v] + \sigma(t) L(t). \]  

(10)

Taking \( \eta \to \infty \) in eq. (10), one can obtain the previous local dynamics without memory kernel. Equivalently, the same dynamics can be obtained by putting \( \alpha = 0 \). The dynamics described by eq. (10) without rotational term has been studied by Van der Straeten et al. [51] and it was found to be very useful to reproduce experimental data of turbulent Taylor-Couette flow.

For simplicity, let us first consider the direction of rotation to be represented by \( e = (0, 0, 1) \). Then eq. (10) reduces to

\[ \dot{v}_x = -\gamma v_x - \omega v_y + \alpha \int_{-\infty}^t dt' e^{-\eta(t-t')} v_y(t') + \sigma L_x(t), \]

\[ \dot{v}_y = -\gamma v_y + \omega v_x + \alpha \int_{-\infty}^t dt' e^{-\eta(t-t')} v_y(t') + \sigma L_y(t), \]

\[ \dot{v}_z = -\gamma v_z + \alpha \int_{-\infty}^t dt' e^{-\eta(t-t')} v_z(t') + \sigma L_z(t). \]  

(11)

Introducing a complex variable by defining \( u(t) = v_x(t) + iv_y(t) \) and \( g(t) = L_x(t) + iL_y(t) \), the first two equations of eq. (11) can be written as

\[ \frac{du}{dt} = -(\gamma - i\omega) u + \alpha \int_{-\infty}^t dt' e^{-\eta(t-t')} u(t') + \sigma g(t). \]  

(12)

To ease our calculations we introduce another variable by defining \( Y = \alpha \int_{-\infty}^t dt' e^{-\eta(t-t')} u(t') \) and then rewrite eq. (12) as a two-dimensional system of differential equations as follows:

\[ \dot{u} = -(\gamma - i\omega) u + Y + \sigma g(t), \]

\[ \dot{Y} = -\eta Y + \alpha u(t). \]  

(13)

This form of the equations is helpful to perform numerical simulation as well as to calculate the stationary distribution of the system.

In two-dimensional classical turbulent point vortex dynamics correlation behaviour that is well fitted by a sum of two exponentials has been observed (fig. 4 in [52]).
Correlation functions in three-dimensional quantum turbulence are constrained by the quantization condition and expected to be of similar complexity as in the two-dimensional point vortex case. One may thus conjecture that similar shapes of correlation functions are relevant. This we will derive now.

Using Fourier transformations in eq. (13), we obtain

\[ \langle u^*(t)u(0) \rangle = \int \frac{\sigma^2(\eta^2 + z^2)}{\pi[(\gamma - i\omega - iz)(\eta - iz) - \alpha]} \frac{dz e^{izt}}{[(\gamma + i\omega + iz)(\eta + iz) - \alpha]} \]  

Here, \( u^*(t) \) is the complex conjugate of \( u(t) \). As a special case for \( \alpha = 0 \) and \( \omega = 0 \) one can easily obtain the correlation function \( C_v(t) = \langle v_z(t + \tau)v_z(\tau) \rangle \) from eq. (14) as \( C_v(t) = \frac{\sigma^2}{2\gamma} e^{-\gamma t} \). Similarly for \( \alpha = 0 \) one obtains \( C_v(t) = \frac{\sigma^2}{2\gamma} e^{-\gamma t} \cos(\omega t) \).

For non-zero \( \alpha \) and \( \omega \) a longer calculation to evaluate the integral given by eq. (14) yields the following form of the correlation function:

\[
C_v(t) = \frac{\sigma^2 e^{-Dt}}{2\sqrt{\mathcal{R}QD}} \left[ \Pi \cos(\gamma t - \theta) + \Upsilon \sin(\gamma t - \theta) \right] + \frac{\sigma^2 e^{-Dt'}}{2\sqrt{\mathcal{R}QD'}} \left[ \Pi' \cos(\gamma t' - \theta) - \Upsilon' \sin(\gamma t' - \theta) \right].
\]

Here the parameters are given as follows:

\[
\mathcal{R} = \left[ (\gamma^2 - (\gamma - \eta)^2 - 4\alpha)^2 + 4\omega^2(\gamma - \eta)^2 \right]^{1/2}, \\
\theta = \frac{1}{2} \tan^{-1} \left[ \frac{2\omega(\gamma - \eta)}{\omega^2 - (\gamma - \eta)^2 - 4\alpha} \right], \\
C = \frac{1}{2} \left[ \omega + \sqrt{\mathcal{R}} \cos(\theta) \right], \\
D = \frac{1}{2} \left[ \gamma + \sqrt{\mathcal{R}} \sin(\theta) \right], \\
C' = \frac{1}{2} \left[ \omega - \sqrt{\mathcal{R}} \cos(\theta) \right], \\
D' = \frac{1}{2} \left[ \gamma - \sqrt{\mathcal{R}} \sin(\theta) \right], \\
Q = (\gamma + \eta)^2 + \mathcal{R} \cos^2(\theta), \\
\Phi = \eta^2 + C^2 - D^2, \\
\Phi' = \eta^2 + C'^2 - D'^2, \\
\Pi = \Phi \sqrt{\mathcal{R}} \cos(\theta) + 2CD(\gamma + \eta), \\
\Pi' = \Phi' \sqrt{\mathcal{R}} \cos(\theta) - 2CD'(\gamma + \eta), \\
\Upsilon = \Phi(\gamma + \eta) - 2CD \sqrt{\mathcal{R}} \cos(\theta), \\
\Upsilon' = \Phi'(\gamma + \eta) + 2CD' \sqrt{\mathcal{R}} \cos(\theta).
\]

The numerically simulated correlation function from the Langevin dynamics for non-zero parameters \( \omega, \gamma, \sigma \) and \( \eta \) and the analytic result (15) are plotted in figs. 3 and 4. As expected the results are in good agreement. Similar shapes of autocorrelation functions of velocity components are observed in atmospheric turbulence [52–54]. Our results for the shape of correlation functions remain valid if \( \sigma(t) \) fluctuates in a superstatistical way, in this case \( \sigma^2 \) is simply replaced by its average \( \langle \sigma^2 \rangle \). Similarly, if different dynamical parameters are assumed in the S and N phases, one can average the correlation function over these. It would be very interesting to check in future Lagrangian quantum turbulence measurements whether shapes of correlation functions similar to those predicted in figs. 3 and 4 are observed.

To conclude, in this paper we have constructed a model for the dynamics of tracer particles in quantum turbulent flow, based on a mixture of contributions from the superfluid (S) and the normal fluid (N), distinguishing between trapped and non-trapped states of the particle dynamics. We presented analytic results for probability density functions and our model predictions are in very good agreement with recent experimental data [1–3]. In fact, the two states S and N on which our model is based can be distinguished experimentally: In the experiments particles move in alternating directions, along the normal fluid away from the heater, and along the superfluid towards the heater.
depending on whether they are free or trapped. One therefore has (in principle at least) an experimental possibility to investigate the statistical properties of the symbolic dynamics generated by the symbols S and N. We also showed that the acceleration statistics is well fitted by lognormal superstatistics, in a similar way as observed for classical Lagrangian turbulence [25,32].

To better understand memory effects in the symbolic dynamics, we extended our model by introducing a memory kernel so that the two-point correlation function of the velocity component can capture more complicated dynamics. We presented analytic results for the correlation function so that the two-point correlation function of tracer particles could help to single out the optimum class of stochastic models, and to better understand the properties of the symbolic dynamics of quantum turbulent flow as a function of the external control parameters.

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