QED RADIATIVE CORRECTIONS
TO THE DECAY $\pi^0 \rightarrow e^+e^-$

George Triantaphyllou
Department of Physics, Yale University, New Haven, Ct. 06520

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Abstract

In view of the recent interest in the decays of mesons into a pair of light leptons, a computation of the QED radiative corrections to the decay of $\pi^0$ into an electron-positron pair is presented here. The results indicate that the peak value of the differential decay rate is reduced by about 50%, because of soft-photon radiation. The number of $e^+e^-$ pairs having invariant masses in an energy bin of 0.5 MeV centered around $m_\pi$ is found to be about 20% smaller than the one it would be if radiative corrections were neglected.
The decay of $\pi^0$ into a pair of electrons via two virtual photons (see Fig.1) was studied long ago [1]. The authors of Ref.[1] calculated the absorptive part of that decay amplitude, in order to derive a unitarity bound for the branching ratio

$$B \equiv \frac{\Gamma(\pi^0 \rightarrow e^+e^-)}{\Gamma(\pi^0 \rightarrow \gamma\gamma)},$$

using as input the experimentally measured value for $\Gamma(\pi^0 \rightarrow \gamma\gamma)$. They found that $B \geq 4.7 \times 10^{-8}$, an exact result. Similar exact unitarity bounds can be derived for the decays of heavier mesons into lepton-antilepton ($l^+l^-$) pairs.

To complete the above calculation though, one would also need the dispersive part of the amplitude. The knowledge of the exact branching ratio is essential, because it could provide us with a test of new physics [2]. To be more precise, the decay of a meson to a $l^+l^-$ pair could be influenced by the existence of point-like effective 4-Fermi interactions between the constituent quarks of the meson and the final two leptons. Such interactions exist in technicolor models, in models of composite quarks and leptons, etc. In technicolor models, for example, extended-technicolor forces couple not only technifermions to ordinary fermions, but also ordinary fermions to themselves.

The dispersive parts of the two photon contribution to the mesonic decay amplitudes are difficult to calculate, however, since in that case the two intermediate photons are off-shell; there exists presently no exact model describing the non-perturbative meson dynamics needed to make this calculation. In addition, the problem is even more complex in the case of the pion. This is because this meson plays also the role of a Goldstone-boson, arising from the spontaneous breakdown of the $SU(2)_L \times SU(2)_R$ chiral symmetry of QCD, and the assumption that it is
Figure 1: The decay $\pi^0 \rightarrow e^+e^-$. The 4-momenta of the various particles are given inside parentheses.

just a bound state of two quarks must be viewed with caution.

There have been several attempts to estimate the dispersive part of the decay amplitude, using a vector dominance model [3], leading to $B = 1 \times 10^{-7}$, a nucleon loop model [4], leading to $B = 1.4 \times 10^{-7}$, an extreme-non-relativistic model [4, 5], leading to $B = 6.2 \times 10^{-8}$, a relativistic bag model [5], leading to $B \approx 1 \times 10^{-7}$, and a chiral Lagrangian approach [7], leading to $B \approx 7 \pm 1 \times 10^{-8}$. Early experimental results [8] gave $B = 1.8 \pm 0.6 \times 10^{-7}$, which was somewhat large, and caused some controversy over the theoretical predictions [9]. Nevertheless, the latest experiments in Fermilab [10] and Brookhaven [11], which give $B = 6.9 \pm 2.8 \times 10^{-8}$ and $B = 6.0 \pm 1.8 \times 10^{-8}$ respectively, indicate that the dispersive part of the amplitude is indeed smaller than the absorptive part.

Another issue relevant to the decay $\pi^0 \rightarrow e^+e^-$ is the one related to final-state
QED radiative corrections. The fact that $m_\pi \gg m_e$ makes the two electrons in the final state quite energetic, and radiative effects can therefore be quite important. Similar instances, where QED radiative corrections are large, have already been observed in different mesonic decays into leptons [12]. Radiation of soft photons can alter substantially the decay profile of the pion, reducing considerably the peak differential decay rate of this process, and obviously affecting the branching ratio B. This paper is focused on these corrections, and their treatment is done independently of the method used to calculate the non-radiatively-corrected decay rate.

An early calculation of QED radiative corrections to the decay of a meson (a kaon) into two leptons via two virtual photons, to first order in the fine structure constant $\alpha$, appeared in Ref.[13]. Later, a similar calculation appeared [14] for $\pi^0 \rightarrow e^+e^-$, the decay studied here. That calculation was also limited to first order in $\alpha$. In such a process, however, $e^+e^-$-invariant-mass cuts must be very strict, as much as experimental resolution allows, in order to minimise the background effects, an issue that will be discussed later in the paper. And it was made clear in Ref.[14] that the higher orders in the perturbative expansion in $\alpha$ influence considerably the differential decay rate, if the $e^+e^-$-invariant-mass resolution becomes smaller than about 1% of the pion mass. This is so because, in that case, the first-order result already gives a larger-than-20% correction. The present experimental resolution for the $e^+e^-$ invariant mass being on the order of 0.5 MeV $\approx 0.4\% \ m_\pi$ [15], an improved calculation of the radiative corrections for this decay is needed, in order to include higher orders in $\alpha$.

The following analysis is similar to the one usually applied to calculate the initial-state radiative corrections to meson production via electron-positron colli-
The differential decay rates, denoted by $P(x)$ below, where $x$ is the invariant mass of either the pion or the $e^+e^-$ pair, are defined here to be equal to $\frac{d\Gamma(\pi^0 \rightarrow e^+e^-)}{dx}$. Assuming a relativistic Breit-Wigner shape for the pion resonance, the result for $P(s)$ to zeroth order in the fine structure constant $\alpha$ is

$$P_0(s) = P_0(m_{\pi}^2) = \frac{s\Gamma_{\pi}^2}{(s - m_{\pi}^2)^2 + \Gamma_{\pi}^2 m_{\pi}^2},$$

(2)

where $\sqrt{s}$ is the invariant mass of $\pi^0$, $m_{\pi} = 134.9$ MeV is its mass, and $\Gamma_{\pi} \approx 8$ eV its natural width. The pion natural width is so small, that any possible dependence of $P_0$ on $s$ coming from the form factor associated to the decay $\pi^0 \rightarrow e^+e^-$ is neglected.

Generally, if the experimentally measured quantity is the invariant mass $\sqrt{s'}$ of the $e^+e^-$ pair, the non-corrected and corrected differential decay rates, denoted by $P_0(s')$ and $P_{\text{rad}}(s')$ respectively, can be both expressed as follows:

$$P_{0,\text{rad}}(s') = \int_s^{2s'} ds P_0(s) f_{0,\text{rad}}(s - s')$$

(3)

The distribution $f_{0,\text{rad}}(s - s')$ is subject to the requirement that

$$\int_s^{2s'} ds f_{0,\text{rad}}(s - s') = 1.$$  

(4)

In a reference frame where the two final electrons have equal and opposite 3-momenta, the relation $s - s' = 4m_e\omega$ holds, where $\omega$ is the total energy of the radiated photons, and $m_e$ is the mass of the electron. The upper bound ($2s'$) of the above integration is placed in order to avoid the pair creation of $e^+e^-$ pairs having invariant mass $s'$ and originating from the radiated photons, since they could be

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1This is a different approach from the one in Ref. [14], where $\frac{d\Gamma(\pi^0 \rightarrow e^+e^-)}{ds}$, the quantity corresponding to $P_0(s)$, is taken to be proportional to $\delta(s - m_{\pi}^2)$. 

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misidentified as being $e^+e^-$ pairs having the same invariant mass and originating from the pion. More on this upper bound will be discussed shortly.

The differential decay rate $P_0(s')$ is associated to the distribution

$$f_0(s - s') = \delta(s - s')$$

(5)

because, to zeroth order in $\alpha$, $s = s'$. The effect of the radiation of soft photons is the "smearing" of this $\delta$-function, even though the resulting differential decay rate still retains an integrable singularity at $s = s'$. The convolution of the Breit-Wigner shape of Eq.(2) with the adequate "smearing" distribution $f_{\text{rad}}(s - s')$, the calculation of which is the main purpose of the present paper, gives us the radiatively corrected differential decay rate $P_{\text{rad}}(s')$.

The pion is assumed to be produced hadronically, as given, for instance, by the hadronic reaction $K^+ \rightarrow \pi^0\pi^+$, and therefore only final-state QED radiative corrections are considered. Furthermore, it is assumed that all radiated photons, no matter what their energy is, escape detection. The calculation presented here is restricted to the soft-photon resummation technique, since this is known to give the bulk of the radiative corrections. The final result, applicable only to $e^+e^-$ invariant masses very close to the pion mass, is non-perturbative, since the leading logarithms of diagrams of all orders in $\alpha$ are summed, in a process well known as "exponentiation" \cite{17}.

In this context, $P_{\text{rad}}$ can be expressed as an infinite sum of individual differential decay rates $P(\pi^0 \rightarrow e^+e^- + n\gamma)$, each having $n$ real soft photons radiated from the $\pi^0$:
\[ P_{\text{rad}}(s') = \sum_{n=0}^{\infty} P(\pi^0 \to e^+ e^- + n\gamma). \]  

Intuitively, this infinite sum has the meaning that in the soft-photon (classical) limit, where radiation loses its particle-like properties, two final states containing \( n \) and \( n + 1 \) photons respectively are indistinguishable, so a summation over all possible numbers of photons is required. In such an analysis, each of the quantities \( P(\pi^0 \to e^+ e^- + n\gamma) \) contain the virtual photon corrections needed to cancel the infrared divergencies. Moreover, the \( n \) soft photons are considered to be independent from each other. For this to be true, their emission must not influence considerably the motion of the electrons, so they must have an energy much smaller than the electron. Therefore, the relation \( s - s' \ll 4m_e^2 \) must hold, and then the logarithms of the form \( \ln \left( \frac{s}{s'} - 1 \right) \) dominate the QED perturbative expansion, and are summed to all orders in \( \alpha \).

The appearance of these logarithms is closely connected to the cancellation of the infrared divergencies. In order to see this, it is assumed that the emission of soft real photons, of energy less than \( \omega_0 \), is used in order to cancel the infrared divergencies associated to the virtual photons. Then the calculation involving soft real and virtual photons contains logarithms of the form \( \ln \left( \frac{\omega_0}{\sqrt{s'}} \right) \). On the other hand, the hard-real-photon part of the calculation, involving photons of energy larger than \( \omega_0 \), contains logarithms of the form \( \ln \left( \frac{s - s'}{\sqrt{s'\omega_0}} \right) \). The addition of these two contributions cancels the dependence of the radiative corrections on \( \omega_0 \), leaving logarithms of the form \( \ln \left( \frac{s}{s'} - 1 \right) \) in the final result.

The soft photons are assumed to be radiated only by the two real final
electrons. This is because the pion decay cannot proceed via the emission of three photons, because of C-conservation, so no photon can come from there. Moreover, the effect of photons emitted by the virtual electron, denoted by \( e^* \) on the right side of the ”\( \gamma \gamma e^* \)” triangle (see Fig.1), is neglected. Intuitively, this happens because the radiated soft-photons, which give the largest corrections, correspond to large spatial scales, and are therefore associated to the two final real electrons. The virtual electron, however, is far off-shell, since the typical momenta flowing in the triangle of Fig.1 are on the order of \( m_\pi \), and it is associated to small spatial scales, so its contribution to these corrections is negligible.

According to the previous discussion, only radiative corrections pertinent to the two final electrons are considered, and the result is, according to the analysis of Ref. [17, 18]

\[
 f_{\text{rad}}(s - s') = \beta \frac{(s - s')^{\beta - 1}}{s'\beta} \Delta_{1\text{rad}}^{1} + \frac{\Delta_{2\text{rad}}}{s'} 
\]

(7)

with

\[
 \beta = \beta(s') = \frac{2\alpha}{\pi} \left( \ln \left( \frac{s'}{m_e^2} \right) - 1 \right). 
\]

(8)

This expression gives a reasonable estimate only in the relativistic limit \( s' \gg m_e^2 \), a relation that obviously holds for \( s' \) near the pion mass. Moreover, this estimate is even more accurate when \( \beta \ll 1 \). Here \( \beta(m_\pi^2) \approx 0.047 \).

The quantities \( \Delta_{1,2\text{rad}}^{1,2} \) correspond to radiative corrections that can be treated perturbatively. Among various other terms, they contain powers of \( \beta \), i.e. basically terms of the form \( (\alpha \ln (s'/m_e^2))^n \), that were left out of the exponentiation procedure.
The effects of vacuum polarization of the two virtual photons are also assumed to have been absorbed in $\Delta_{\text{rad}}^{1/2}$. To zeroth order in $\alpha$, $\Delta_{\text{rad}}^1 = 1$ and $\Delta_{\text{rad}}^2 = 0$. In the following, the approximation

$$\Delta_{\text{rad}}^1 \approx 1 \quad (9)$$

and

$$\Delta_{\text{rad}}^2 \approx 0 \quad (10)$$

is used, i.e. only the factor multiplying $\Delta_{\text{rad}}^1$, the non-perturbative part of the calculation, is considered, because it is expected to give the bulk of the radiative corrections. In view of the magnitude of $\beta$, the error induced by this approximation should be on the order of 5%.

The distribution $f_{\text{rad}}(s - s')$ contains the exponentiated logarithms, which give the leading QED correction to the decay profile for $s \approx s'$. Since $\beta < 1$, $f_{\text{rad}}(s - s')$ has an integrable singularity at $s = s'$. It is worth noting that this form of $f_{\text{rad}}(s - s')$ implies the assumption that the logarithms $\ln \left( s/s' - 1 \right)$ dominate the QED perturbative expansion of the radiative corrections. This ceases to be true when, in the integral of Eq.(3), the integration variable $s$ approaches the upper bound $(2s')$ of the integration. Moreover, in that case, interference effects between this process and the main background process, which will be discussed shortly, become non-negligible. However, the present form of the "smearing" distribution $f_{\text{rad}}(s - s')$, and the smallness of $\beta$ in Eq.(8), makes the integral of Eq.(3) completely dominated by the region of $s \approx s'$. Therefore, effects coming from a large $s - s'$ difference are not expected to influence the final result considerably.

In Fig. 2, the quantity $\frac{2\sqrt{s}}{P_0(m_\pi^2)} P_0(s) f_{\text{rad}}(s - s')$, evaluated at $s' = m_\pi^2$, is
Figure 2: The integrable singularity of the quantity $\frac{2s^{1/2}}{P_0(m_{\pi}^2)} P_0(s) f_{\text{rad}}(s-s')$, evaluated at $s' = m_{\pi}^2$.

plotted as a function of the pion invariant mass minus $m_{\pi}$, i.e. $\sqrt{s} - m_{\pi}$. The singularity at $s = s'$ shows clearly. The area under the curve that is partially shown in Fig. 2, contained in the region $0 < \sqrt{s} - m_{\pi} < m_{\pi}$, is equal to $\frac{P_{\text{rad}}(m_{\pi}^2)}{P_0(m_{\pi}^2)}$.

It is not difficult to see from Eq.(3) and (8) that, for $\Gamma_{\pi} \ll m_{\pi}$ and $\beta \ll 1$,

$$P_{\text{rad}}(m_{\pi}^2) \approx P_0(m_{\pi}^2) \left( \frac{\Gamma_{\pi}}{m_{\pi}} \right)^{\beta}, \quad (11)$$
with $\beta$ evaluated at $s' = m^2_{\pi}$. The ratio $\frac{\Gamma_{\pi}}{m_{\pi}}$ appears naturally, as being the characteristic energy scale over which the pion differential decay rate varies considerably, divided by the largest energy scale of the process. The dependance of the right-hand side of Eq.(11) on $m_e$ comes only through the parameter $\beta$, which contains the term $\ln (s'/m^2_e)$, responsible for the collinear singularity in the limit $m_e \to 0$. The fact that $m_e$ does not appear anywhere else in the final result is attributed to the factorisation of collinear singularities, a well-known property of QCD, which is also shared by QED \cite{18}. For the given values of $\beta(m^2_{\pi})$, $\Gamma_{\pi}$ and $m_{\pi}$, Eq.(11) leads to the relation

$$P_{\text{rad}}(m^2_{\pi}) \approx 0.46 P_0(m^2_{\pi}).$$

Therefore, the peak value of the radiatively corrected differential decay rate is about 50% smaller than the one without radiative corrections \cite{4}. The magnitude of this effect shows that, for $e^+e^-$ invariant masses very close to $m_{\pi}$, a calculation of these corrections using perturbation theory up to first order in $\alpha$ would not give an accurate estimate.

The position of the maximum of $P_{\text{rad}}(s')$ lies approximately at

$$\sqrt{s'} \approx m_{\pi} - \frac{\pi \beta}{8} \Gamma_{\pi},$$

which corresponds to a shift of about 0.2 eV from $m_{\pi}$, and is therefore experimentally unobservable.

\footnote{Similar examples can be found in connection with the production cross-section of heavy mesons via $e^+e^-$ colliding beams, where the QED radiative corrections are predicted to have a 50% effect \cite{14}. In those cases, however, in contrast to the analysis performed in this paper, the characteristic energy scale over which the production cross-section varies considerably is determined not by the width of the produced resonance, but by the energy spread of the colliding beams, which is much larger than the width.}
The differential decay rates $P_0$ and $P_{\text{rad}}$ as functions of the invariant mass of the electron-positron pair minus $m_\pi$, i.e. $\sqrt{s'} - m_\pi$, given by the numerical evaluation of the integral in Eq.(3), are shown in Fig. 3. The effect of soft-photons radiated in the final state is apparent, not only in the maximum height of the radiatively corrected differential decay rate, but also in the radiative "tail" that appears for lower $e^+e^-$ invariant masses, as expected, since the photons are radiated in the final state. Indicatively, numerical results show that the radiatively corrected differential decay rate is roughly three times larger than the non-corrected one at
$e^+e^-$ invariant masses 150 eV below $m_\pi$, and two times smaller at 150 eV above $m_\pi$.

In both cases, these differential decay rates are roughly three orders of magnitude smaller than the ones at $s' \approx m_\pi^2$. This situation is to be contrasted to the case of initial-state radiative corrections, for processes like production of resonances via $e^+e^-$ collisions, where one sees a more pronounced radiative tail for invariant masses larger than the resonance mass.

The experimental resolution of the invariant mass of the $e^+e^-$ pairs being on the order of 0.5 MeV, the structure of the decay curve shown in Fig.3 is not experimentally observable. By numerically integrating the function $P_{\text{rad}}(s')$ over $s'$, it is found that the number of $e^+e^-$ pairs detected in an energy bin of width 0.5 MeV, centered around the pion mass, is approximately 80% of the number it would be without radiative corrections, i.e. we have a 20% correction. This happens because the radiative tail for $s' < m_\pi^2$ partly compensates the reduced number of $e^+e^-$ pairs at $s' \gtrsim m_\pi^2$. A complete analysis of this process should take into account this binning for energy regions further away from $m_\pi$. However, results obtained in this way should not be trusted for $\sqrt{s}$ too far away from $m_\pi$ (more than a few MeV), because of several reasons. First, because background effects become non-negligible. Moreover, in that case, one should add to this calculation the contribution of hard photons, which have an energy that is not small compared to the electron mass any more, and which become increasingly important for invariant masses further away from $m_\pi$. Last, the dependance of the form factor, associated to the decay $\pi^0 \rightarrow e^+e^-$, on $s'$ cannot be neglected any more.

It is furthermore expected that the effect of radiative corrections on the total number of $e^+e^-$ pairs, having all possible invariant masses, is dominated by
perturbative quantum effects, and that these are on the order of a few percent. In fact, a first-order in $\alpha$ prediction for this effect (Eq.(18) in Ref.[14]), which is a reasonable approximation, gives a 3% correction. It is therefore seen that the radiative tail of the differential decay rate, when considered throughout the whole range of allowable $e^+e^-$ invariant masses, compensates, to within a few percent, the reduction of the differential decay rate at $s' \approx m_{\pi}^2$. These radiative effects on the total number of $e^+e^-$ pairs are not going to be discussed further here, because they are completely overwhelmed by interference terms between this process and its background, and by the background itself, which is discussed here below, and they are therefore experimentally unobservable.

To first order in the fine structure constant, the process considered here involves a final state of an electron-positron pair and at most one photon. A large background to this process is the same final state coming from the process $\pi^0 \rightarrow \gamma\gamma$, and the subsequent decay of one of the final photons into an $e^+e^-$ pair [19]. This process is of lower order in $\alpha$, and it has a much larger decay rate than the one considered here. However, this background is easily distinguishable from the signal of interest, because its differential decay rate does not have a pronounced peak at $e^+e^-$ invariant masses $\sqrt{s'}$ near the pion mass. Moreover, the interference of this background with the process of interest in this paper is negligible in the $\sqrt{s'}$ region that is considered here, which is $\sqrt{s'} \approx m_{\pi}$ (to within a few MeV) [20].

To conclude, this paper has dealt with the shape of the $\pi^0 \rightarrow e^+e^-$ differential decay rate, when radiation of soft-photons from the two final electrons is taken into account. In particular, the peak value of the radiatively corrected differential decay rate $P_{\text{rad}}$ is smaller than the non-corrected one by about 50%. Moreover,
the radiative tail, emerging at invariant masses \( \sqrt{s'} \) smaller than \( m_\pi \), even though it has a considerably smaller height than the peak differential decay rate, it does not fall-off very rapidly with \( \sqrt{s'} \), and, when considered over the whole range of allowable electron invariant masses, it almost compensates the reduction of the number of \( e^+e^- \) pairs at \( \sqrt{s'} \approx m_\pi \). This form of the differential decay rate should be taken into consideration when comparison between experimental and theoretical results for this decay is made, especially when stringent \( e^+e^- \) invariant mass cuts are placed. A more involved analysis should include the effect of higher order terms in the quantities \( \Delta^{1,2}_\text{rad} \). Finally, a similar analysis can be done for the decay of other mesons, like the \( \eta^0 \), to a lepton-antilepton pair.

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