A NEW INTERPRETATION OF THE PROTON-NEUTRON BOUND STATE.
THE CALCULATION OF THE BINDING ENERGY.

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Abstract

We treat the old problem of the proton-neutron bound state (the deuteron). Using a new concept of incomplete (partial) annihilation process we derive a formula for the binding energy of the deuteron, which does not contain any new constant. Some implications of this new approach are discussed.

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1 Introduction.

Since the strongest interaction between two particles is the particle-antiparticle annihilation process, we must take into account this process when we treat the problem of the elementary nuclear bound state. In an annihilation process a maximum quantity of energy is released and we know that the strength of a bound state is proportional to the energy released to the exterior when the bound state is build up.

The concrete fact from which we start in this paper is the analysis of the contradictory characteristics of the neutron. Indeed, the neutron which has a zero total electric charge, seems to have very strange charge structure, having perhaps no parallel example in whole of the hadron physics [1].

A new concept of incomplete (limited) annihilation process is used to explain the formation of the proton-neutron bound state (paragraph 2). The formula derived for the binding energy of this state does not contain any new constant (paragraphs 3 and 4). Some comments are given in paragraph 5.

2 The incomplete annihilation process.

The magnetic form factors of the neutron and proton have the same functional dependence, given by a scaling law determined experimentally, which proves that the two particles have similar structures. But the magnetic moment of the neutron ($\mu = -1.91 \mu_N$) is of opposite sign to that of the proton ($\mu_p = +2.79 \mu_N$).

In spite of the fact that the electric charge of the neutron is zero, the mean-square charge radius of the neutron, determined experimentally, is different from zero ($< r_E^2(\text{neutron}) > = -0.112 fm^2$) and again opposite in sign to that of the proton ($< r_E^2(\text{proton}) > = +0.67 fm^2$).

While the ratio of the magnetic moment of the neutron to that of the proton was derived using the SU(6) symmetry, the calculated value (-2/3) being in a good agreement with the experimental one, the value and the sign of $< r_E^2(\text{neutron}) >$ are difficult to explain [1]. Further more in [2] was derived a theorem proving a contradiction between the experimental and theoretical data on the $< r_E^2(\text{neutron}) >$; more elaborate theoretical treatments do not elucidate complete the problem [3].
The data on the neutron magnetic moment and $< r_E^2 (\text{neutron}) >$, in comparison with that of the proton, suggest that the neutron consists of two hadronic masses: a larger one $m_{h^-}$ which carries a negative electric charge and dominates into the neutron structure, and, to keep the charge neutrality of the neutron, a smaller one $m_{h^+}$ which carries a positive electric charge. We postulate that these two constituent hadronic masses of the neutron obey properties similar to the asymptotic freedom and infrared confinement.

The fact that the neutron decay is intermediate by the vectorial boson $W^-$ suggests also that into the neutron structure the negatively charged mass is dominant.

The sum of these two constituent masses must be equal to the neutron mass:

$$m_{h^-} c^2 + m_{h^+} c^2 = m_{\text{on}} c^2 \quad (1)$$

In the case of a perfect particle-antiparticle symmetry, for instance proton-antiproton pair, the annihilation process of the particle with the antiparticle is "complete" and a maximum quantity of energy is released. It is the well known particle-antiparticle annihilation process.

In the case of an imperfect symmetry the annihilation process is incomplete and the released energy is correspondingly smaller.

Otherwise spoken, in the case of complete annihilation the two partners, which have opposite electrical charges and equal masses, have a maximum interaction compatibility. In the case of incomplete annihilation, since one of the partners has a mass smaller than its partner, the interaction compatibility decreases.

Based on these we propose a new interpretation of the formation of the proton-neutron bound state: an incomplete (limited) annihilation process takes place between the proton and the negatively charged mass from neutron ($m_{h^-}$) which is smaller than the mass of the proton (see the next paragraph). The binding energy of the proton-neutron bound state (the deuteron) is equal with the energy released in this incomplete annihilation process.

In the case of a symmetry with a higher degree of "imperfection", which means that the two partners have very different masses, the annihilation process will be weak, the released energy being much smaller. The process of formation of the proton-electron bound state (the hydrogen atom) can be compared with a weak annihilation process, the proton and the electron having opposite electric charges but very different masses (a very low degree
of symmetry, the electron mass being of leptonic origin only).

3 The calculation of the negatively charged mass \( m_{h^-} \) from neutron.

We start from the relativistic expression of the energy \( E \) for a particle of rest mass \( m_o \):

\[
E^2 \equiv m^2 c^4 = m_o^2 c^4 + p^2 c^2 \tag{2}
\]

where \( m \) is the ”dynamical” mass.

It is well known from quantum mechanics that when such a particle evolves with negative kinetic energy, this means imaginary impulse, the energy equation becomes [4]:

\[
E^2 \equiv m^2 c^4 = m_o^2 c^4 - p^2 c^2 \tag{3}
\]

where \( p \) was replaced in (2) by \( ip \); \( m \) is now smaller than the rest mass.

Let’s write similar equations for the two constituents of the neutron \( m_{h^-} \) and \( m_{h^+} \). Like rest masses are used the mass of the lightest baryon (the proton) for the negatively charged mass which dominates into the neutron, and the mass of the lightest meson (the pion) for the positively charged mass from neutron:

\[
E_{h^-}^2 \equiv m_{h^-}^2 c^4 = m_{op}^2 c^4 - p_{h^-}^2 c^2 \tag{4}
\]

\[
E_{h^+}^2 \equiv m_{h^+}^2 c^4 = m_{on}^2 c^4 - p_{h^+}^2 c^2 \tag{5}
\]

We postulate that:

\[
p_{h^-} = p_{h^+} \tag{6}
\]

taking into account the universality of the impulse conservation law.

Now is straightforward to calculate \( m_{h^-} \) and \( m_{h^+} \). From (2), (4), (5) and (6) it results:

\[
m_{h^-} c^2 = \frac{m_{oh^-}^2 c^2 - m_{on}^2 c^2 + m_{on}^2 c^2}{2 m_{on}} \tag{7}
\]

and an analogously relation for \( m_{h^+} c^2 \).
Replacing the rest mass values:

\[ m_{\text{op}}c^2 = 938.259\,\text{MeV} \]
\[ m_{\pi^+}\pm c^2 = 139.568\,\text{MeV} \]
\[ m_{\text{on}}c^2 = 939.552\,\text{MeV} \]  \hspace{1cm} (8)

we obtain the values of the negatively charged and positively charged masses from neutron:

\[ m_{h^-}c^2 = 927.893\,\text{MeV}, \]
\[ m_{h^+}c^2 = 11.659\,\text{MeV}. \]  \hspace{1cm} (9)

If like in [4] we take \( E_{h^+} \equiv m_{h^+}c^2 \simeq 0 \), it results from (8) that \( p_{h^+} = m_{\pi^+}c \) and we obtain directly from (4) the value of \( m_{h^-} \):

\[ m_{h^-}c^2 = \sqrt{m_{\text{op}}^2 - c^4} - m_{\pi^+}^2c^4 = 927.820\,\text{MeV} \]  \hspace{1cm} (10)

The relation (1) is in this case satisfied by the contribution of other degrees of freedom than \( m_{h^+}c^2 \).

The smallness of the positively charged mass from neutron (less than 12 MeV) means that it is ”empty” of hadronic mass (contains only ”current” mass) and for this reason does not participate at the strong interaction, in contrast with the negatively charged mass from neutron.

### 4 The calculation of the binding energy of the deuteron.

Now, we will treat quantitatively the incomplete annihilation process. The system proton-antiproton \( \left( \frac{m_{\text{op}}}{m_{\text{op}^-}} = 1 \right) \) has a maximum interaction compatibility; the distance of ”approach” between the particle and the antiparticle at ”complete” annihilation is \( a_{\text{ann}} \), where the annihilation probability is maximum [4].

The system proton-negatively charged mass from neutron \( \left( \frac{m_{\text{op}^-}}{m_{h^-}} > 1 \right) \) has a smaller interaction compatibility, it annihilates incompletely, the distance of approach being \( x \) (larger).
For a mass ratio much higher than 1 ($\frac{m_{op}}{m_-} >> 1$), where $m_-$ is the mass of the particle with negative charge, the system has a much smaller interaction compatibility, the annihilation being weak. Since in this case $m_-$ is "empty" of hadronic mass we postulate that the distance ($a_{m_-}$) and the binding energy ($E_{m_-}$) are given by the known formulae from quantum mechanics for the Coulomb potential (Bohr formulae).

Taking as parameter-the mass ratio, we systematize in the following table the above presented dependences:

| $\frac{m_{op}}{m_-}$ | $a_{ann}$ | $\frac{1}{2m_{op}^2c^2}$ | $\frac{m_{op}}{m_+} > 1$ | $x$ | $\frac{1}{E_{m_-}}$ |
|-----------------------|-----------|------------------------|------------------------|-----|---------------------|
| $\frac{m_{op}}{m_-} > 1$ | $a_{m_-}$ | $\frac{1}{E_{m_-}}$ |

In the same table are presented also the energies released in the complete, incomplete and weak annihilation process respectively, taken with the sign minus, this means just the binding energies of the states formed by these processes. Since the interaction compatibility, and by this also the binding energy, increases when the characteristic distance of approach (interaction) decreases, in analogy also with the Bohr formulae, we have taken the binding energies invers proportional to the distances. For distance $a_{ann}$ (complete annihilation), the value of the released energy is $2m_{op}c^2$, and this was taken as the "binding energy".

Assuming linear dependences, from the above table (if you represent graphically the distances in function of the parameter-mass ratio-from like triangles) the following relation can be drawn for distance $x$:

$$\frac{m_{op}}{m_-} - 1 = \frac{a_{m_-} - a_{ann}}{x - a_{ann}}$$

where $a_{m_-} = \frac{\hbar^2}{m_-e^2}$.

Since $\frac{m_{op}}{m_-} >> 1$ and $a_{m_-} << a_{ann}$ ($\sim 1 fm$, see[5]), we obtain:

$$x = K_1\left(\frac{m_{op}}{m_+} - 1\right) + a_{ann}$$

where $K_1 = \frac{\hbar^2}{m_{op}e^2}$. 

5
Likewise, from the same table we obtain for the binding energy $E_x$:

$$\frac{m_{op}^+ - 1}{m_{op}^+} - 1 = \frac{1}{E_{m_-}} + \frac{1}{2m_{op}^+ c^2}$$

(13)

where $E_{m_-} = -m_- \frac{e^4}{2h^2}$.

Since $\frac{m_{op}^+}{m_-} \gg 1$, we obtain:

$$E_x = -\frac{1}{(\frac{m_{op}^+}{m_-} - 1)K_2 + \frac{1}{2m_{op}^+ c^2}}$$

(14)

where $K_2 = \frac{2h^2}{m_{op}^+ c^4}$.

Substituting in (14) the values of the constants and of $m_{h^-}$, calculated in the paragraph 3 (the relation 9), we obtain for the binding energy $E_x$ the value:

$$E_x = -2.233 MeV$$

(15)

which, in the proposed model, is just the binding energy of the deuteron and is in fairly good agreement with the experimental value:

$$E_{exp} = -2.224 MeV$$

(16)

Substituting in (12) the values of the constants and of $m_{h^-}$ from (9), we obtain:

$$x = 0.321 \times 10^{-13} cm + a_{ann} \simeq 1.3 fm$$

(17)

It is interesting to note that for the other value of $m_{h^-}$ (relation 10) we obtain for the binding energy of the deuteron the value:

$$E'_x = -2.188 MeV$$

(18)

which means that the experimental value (16) is placed between the two calculated values (relations 15 and 18).

It should be noted that (14), if the term $\frac{1}{2m_{op}^+ c^2}$ is neglected, which in the present case is a very good approximation, is identical with a Bohr formula, with the fundamental distinction that the reduced mass is of the form:

$$\mu = \frac{m_{op}^+ m_{h^-}}{m_{op}^+ - m_{h^-}}$$

(19)
Indeed, the expression of the binding energy (14) gets:

\[ E_x = -\frac{m_{op^-} m_{h^-}}{m_{op^+} - m_{h^-}} \frac{e^4}{2\hbar^2} \]  

(20)

The value of the mass term (19), 83.986GeV or 83.392GeV depending on the value of \( m_{h^-} \) we used (relations 9 or 10), is very near the mass value of the charged intermediate vector W-boson (approx. 80 GeV). Then the expression of the binding energy of the deuteron can be written, in a precision of 5 percents, in the surprisingly form:

\[ E_x \simeq -M_W \frac{e^4}{2\hbar^2} \]  

(21)

We could call this system formed by a negative charged vector W-boson which evolve in a positive, central coulombian field a "heavy atom".

5 Discussions.

It is important to note that in [5] it was proved that at the distance where the attraction is strong in the NN potential (\( \sim 1\, fm \)), the \( N\bar{N} \) potential is dominated by the annihilation.

It is also well known that the p-n triplet potential is different from the p-n singlet, p-p and n-n potentials which are characterized by an important hard-core repulsion [6]. On the contrary the p-n triplet potential, which represent the deuteron bound state, has a negligible hard-core repulsion.

On the other hand it is known that the \( N\bar{N} \) potential is characterized by the complete lack of hard-core repulsion, the core being strongly attractive [7]. This means that the triplet p-n bound state, described here by an incomplete annihilation process, has an intermediate position between the unbound nucleon-nucleon states (p-p, n-n and p-n singlet) and the \( N\bar{N} \) states, characterized by a "complete" annihilation process.

We stress that the formula derived for the deuteron binding energy, either (14) or (21), does not contain any new constant.

In particular the formula (21) is a relation between the strong interaction constant \( \frac{g^2}{\hbar c} \), characteristic to the deuteron bound state, the nucleon mass
and the constants of the electroweak interaction \((M_W, \frac{e^2}{\hbar c})\). Indeed from:

\[
E_x \simeq -M_W \frac{e^4}{2\hbar^2} \equiv -\mu_D \frac{g_N^4}{2\hbar^2} \tag{22}
\]

where \(\mu_D\) is the deuteron reduced mass \((\mu_D \simeq \frac{m_{\text{op}}}{2})\) it results:

\[
\frac{g_N^2}{\hbar c} = \sqrt{\frac{M_W e^2}{\mu_D \hbar c}} \tag{23}
\]

Another observation regarding the relation (21): the vectorial W-boson has the spin 1, like the triplet nuclear bound state.

It is to be underlined that the present approach of the proton-neutron bound state (in particular the derived formula 14) is based on the proportionality between the ”dynamical” mass and the strong interaction, racording the ”very” strong interaction (complete annihilation, \(\frac{m_{\text{op}+}}{m_{\text{op}-}} = 1\)) to the e.m. interaction (atomic bound state, \(\frac{m_{\text{op}+}}{m_{\text{op}-}} \gg 1\)).

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