Dynamical instabilities in density-dependent hadronic relativistic models

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Unstable modes in asymmetric nuclear matter (ANM) at subsaturation densities are studied in the framework of relativistic mean-field density-dependent hadron models. The size of the instabilities that drive the system are calculated and a comparison with results obtained within the non-linear Walecka model is presented. The distillation and anti-distillation effects are discussed.

PACS number(s):24.10.Jv, 21.30.Fe, 21.65.+f, 26.60.+c

I. INTRODUCTION

Many efforts are recently being done in order to understand the supernova evolution. Of particular interest is the scenario in the aftermath of a core bounce, where a large number of neutrinos is produced and radiated out towards the infalling matter from the outer layers onto the core. The mean free path of the neutrinos and their interaction with matter can be an explanation for the mantle ejection during the explosion. In [1] the opacity of the nuclear non-uniform neutron-rich matter is calculated in a semiclassical approach to describe neutrino scattering. In the present work we will investigate some general properties of the non-uniform matter at the crust of a compact star within different relativistic models. In particular we will investigate the dynamical collective unstable modes and study the isospin content of the non-homogeneous phase of asymmetric nuclear matter.

In two previous works [2, 3] we have investigated the influence of the electromagnetic interaction and the presence of electrons on the unstable modes of npe matter at zero and finite temperature within the NL3 parametrization of the non-linear Walecka model (NLWM) [4]. This parametrization describes the ground-state properties of both stable and unstable nuclei.

Models with density-dependent meson-nucleon couplings are an alternative approach for the description of nuclear matter and finite nuclei [5]. Non-linear self-interactions of the mesons in constant coupling models are substituted by density-dependent meson-nucleon coupling parameters and are motivated by Dirac-Brueckner calculations of nuclear matter.

The parametrization introduced by Typel and Wolter, which we will refer as TW [6], describes finite nuclei and nuclear matter with similar quality as non-linear parametrizations and has a more reasonable extrapolation to extreme conditions: high density and large charge asymmetry. In [7] a parametrization denoted DD-ME1 was used the same density dependence of TW for the coupling parameters, but adjusted the parameters in a different way. More recently the parametrization DD-ME2 [8] has been developed as an improvement of DD-ME1 in order to obtain better fittings to excitation energies of isoscalar monopole and isovector dipole giant resonances. Other possibilities for density dependent parametrizations are found in the literature [9].

In [10] it was shown that the thermodynamical instabilities at subsaturation densities of NLWM parametrizations with constant couplings differ from the behavior of relativistic nuclear models with density-dependent parameters. In particular, in the last models the distillation effect is not so strong and follows more closely the behavior of non-relativistic nuclear models. In the present work we will study the dynamical instabilities within density-dependent relativistic models (DDRMs) and will compare them with the results obtained with the NL3 parametrization of NLWM.

This investigation will be performed in the framework of the Vlasov formalism [11, 12, 13]. We will study the role of isospin and the modification of the distillation phenomenon due to the presence of the Coulomb field and electrons.

In section II we review the Vlasov equation formalism for nuclear neutral matter including electrons and the electromagnetic field. In section III the dispersion relation is displayed and in section IV the numerical results are shown and discussed. Finally, in the last section the most important conclusions are drawn.

II. THE VLASOV EQUATION FORMALISM

We start from the lagrangian density of the relativistic TW model [6] including electrons interacting with the electromagnetic field

$$\mathcal{L} = \bar{\psi} \left[ \gamma_\mu \left( i \partial^\mu - \Gamma^\mu - \frac{\Omega_\mu}{2} F^\mu_{\nu} \cdot b^\nu - e A^\mu \frac{1 + \tau_3}{2} \right) \right]$$

$$- \left( M - \Gamma_s \phi \right) \psi + \frac{1}{2} \left( \partial_\mu \phi \partial^\mu \phi - m_s^2 \phi^2 \right) - \frac{1}{4} \Omega_{\mu\nu} \Omega^{\mu\nu}$$

$$+ \frac{1}{2} m_e^2 V^\mu \partial_\mu V^\nu - \frac{1}{4} B_{\mu\nu} \cdot B^{\mu\nu} + \frac{1}{2} m_e^2 \partial_\mu b^\nu \cdot b^\mu$$

$$- \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \bar{\psi} \left[ \gamma_\mu \left( i \partial^\mu + e A^\mu \right) - m_e \right] \psi$$  \hspace{1cm} (1)

where $\Omega_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $B_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu - \Gamma^\rho (b_\mu \times b_\nu)$.

The parameters of the model are: the nucleon mass $M$, the electron mass $m_e$, the masses of the mesons $m_s$, $m_v$, $m_\rho$, the electromagnetic coupling constant $e = \sqrt{4\pi/137}$,
and the density-dependent coupling parameters $\Gamma_s$, $\Gamma_v$, and $\Gamma_\rho$, which are adjusted in order to reproduce some of the nuclear matter bulk properties, using the following parametrization:

$$\Gamma_i(\rho) = \Gamma_i(\rho_{sat}) g_i(x), \quad i = s, v$$  \hspace{1cm} (2)

with

$$g_i(x) = a_i \frac{1 + b_i(x + d_i)^2}{1 + c_i(x + d_i)^2},$$  \hspace{1cm} (3)

where $x = \rho/\rho_{sat}$ and

$$\Gamma_\rho(\rho) = \Gamma_\rho(\rho_{sat}) \exp[-a_\rho(x - 1)].$$  \hspace{1cm} (4)

In the sequel we will present results obtained with TW and DD-ME2. The values of the parameters $m_i$, $\Gamma_i$, $a_i$, $b_i$, $c_i$ and $d_i$, $i = s, v, \rho$ are given in Table I.

|        | TW [6] | DD-ME2 [8] |
|--------|--------|------------|
| $m_s$  (MeV) | 550    | 550.1238   |
| $m_v$  (MeV) | 783    | 783.0000   |
| $m_\rho$ (MeV) | 763    | 763.0000   |
| $\Gamma_s(\rho_{sat})$ | 10.72854 | 10.5396    |
| $\Gamma_v(\rho_{sat})$ | 13.29015 | 13.0189    |
| $\Gamma_\rho(\rho_{sat})$ | 7.32196 | 7.3672     |
| $a_s$  | 1.365469 | 1.3881     |
| $b_s$  | 0.226061 | 1.0943     |
| $c_s$  | 0.409704 | 1.7057     |
| $d_s$  | 0.901995 | 0.4421     |
| $a_v$  | 1.402488 | 1.3892     |
| $b_v$  | 0.172577 | 0.9240     |
| $c_v$  | 0.344293 | 1.4620     |
| $d_v$  | 0.983955 | 0.4775     |
| $a_\rho$ | 0.515   | 0.5647     |

TABLE I: Parameters of the density-dependent models.

Notice that in these density-dependent models the non-linear terms are not present, in contrast with the usual non-linear Walecka model (NLWM). For comparison we summarize in Table II the nuclear matter properties at saturation calculated for the models we will use. For the NL3 parametrization of the NLWM the lagrangian saturation is obtained in the same way as NLWM and the corresponding one-body hamiltonian $h = \text{diag}(h_p, h_n, h_e)$, with

$$h_i = \sqrt{(p - V_i)^2 + M^* - V_0}, \quad i = p, n$$

and

$$h_e = \sqrt{(p + eA)^2 + m_e^2 - eA_0},$$

where $M^* = M - \Gamma_s\phi$ denotes the effective baryon mass and

$$V_0 = \Gamma_v V_0 + \frac{\Gamma_\rho}{2} \tau_1 b_0 + eA_0 \left(1 + \frac{\tau_1}{2}\right) + \Sigma^R,$$

$$\Sigma^R = u^\mu \left(\frac{\partial \Gamma_s}{\partial \rho} \phi^{\nu} V_\nu + \frac{\partial \Gamma_v}{\partial \rho} \phi^{\nu} b_0,\nu - \frac{\partial \Gamma_\rho}{\partial \rho} \phi \right),$$

due to the density dependence of the coupling parameters $\Gamma_i$.

The time evolution of the distribution function is described by the Vlasov equation

$$\frac{\partial f_i}{\partial t} + \{f_i, h_i\} = 0, \quad i = p, n, e,$$  \hspace{1cm} (5)

where $\{,\}$ denotes the Poisson brackets. It has been argued in [14, 15] that (5) expresses the conservation of the number of particles in phase space and is, therefore, covariant.

The equations of motion for the fields are obtained from the Lagrangian and are given by

$$\frac{\partial^2 \phi}{\partial t^2} - \nabla^2 \phi + m_\phi^2 \phi = \Gamma_s \rho_s(r, t),$$  \hspace{1cm} (6)

$$\frac{\partial^2 V^\mu}{\partial t^2} - \nabla^2 V^\mu + m_v^2 V^\mu = \Gamma_v j^\mu(r, t) + \partial^\mu (\partial_v V^\nu),$$  \hspace{1cm} (7)

$$\frac{\partial^2 b^\mu}{\partial t^2} - \nabla^2 b^\mu + m_b^2 b^\mu = \frac{\Gamma_\rho \rho}{2} j^\mu(r, t) + \partial^\mu (\partial_b^{-\nu}),$$  \hspace{1cm} (8)

and the values of the parameters $m_i$, $\Gamma_i$, $a_i$, $b_i$, $c_i$ and $d_i$, $i = s, v, \rho$ are given in Table II.

|        | NL3 [4] | TW [6] | DD-ME2 [8] |
|--------|--------|--------|------------|
| $B/A$ (MeV) | 16.3   | 16.3   | 16.14      |
| $\rho_0$ (fm$^{-3}$) | 0.148  | 0.153  | 0.152      |
| $K$ (MeV) | 272    | 240    | 250.89     |
| $\epsilon_{sym}$ (MeV) | 37.4   | 32.0   | 32.3       |
| $M^*/M$ | 0.60   | 0.56   | 0.572      |

TABLE II: Nuclear matter properties.
\[ \frac{\partial^2 A^\mu}{\partial t^2} - \nabla^2 A^\mu = e \left[ j^\mu_p(r, t) - j^\mu_e(r, t) \right], \quad (9) \]

where the scalar density is

\[ \rho_s(r, t) = 2 \sum_{i=p, n} \int_0^{\frac{d^3p}{(2\pi)^3}} f_i(r, p, t) \frac{M^*}{\varepsilon_i}. \quad (10) \]

The components of the baryonic four-current density are

\[ j_0(r, t) = 2 \sum_{i=p, n} \int_0^{\frac{d^3p}{(2\pi)^3}} f_i(r, p, t) = \rho_p + \rho_n, \quad (11) \]

\[ j(r, t) = 2 \sum_{i=p, n} \int_0^{\frac{d^3p}{(2\pi)^3}} f_i(r, p, t) \frac{p - \mathbf{V}_i}{\varepsilon_i}, \quad (12) \]

where \( \rho_p, \rho_n \) are the proton and neutron densities. The electron four-current density has components

\[ j_{0e}(r, t) = 2 \sum_{i=p, n} \int_0^{\frac{d^3p}{(2\pi)^3}} e f_i(r, p, t) = \rho_e, \quad (13) \]

\[ j_e(r, t) = 2 \sum_{i=p, n} \int_0^{\frac{d^3p}{(2\pi)^3}} e f_i(r, p, t) \frac{p - \mathbf{V}_e}{\varepsilon_e}, \quad (14) \]

where \( \rho_e \) is the density of electrons and the components of the isovector four-current density are

\[ j_{3,0}(r, t) = 2 \sum_{i=p, n} \int_0^{\frac{d^3p}{(2\pi)^3}} f_i(r, p, t) \tau_i = \rho_p - \rho_n, \quad (15) \]

\[ j_3(r, t) = 2 \sum_{i=p, n} \int_0^{\frac{d^3p}{(2\pi)^3}} f_i(r, p, t) \frac{p - \mathbf{V}_i}{\varepsilon_i} \tau_i, \quad (16) \]

with \( \varepsilon_i = \sqrt{(p - \mathbf{V}_i)^2 + M^2}, \ i = p, n \) and \( \varepsilon_e = \sqrt{(p + eA)^2 + m_e^2} \).

The four-currents \( j^\mu, j^\mu_e \) and \( j_3^\mu \) satisfy the continuity equations [13] \( \partial_\mu j^\mu = 0, \partial_\mu j_3^\mu = 0 \) and \( \partial_\mu j^\mu_e = 0 \). Substituting these continuity equations into (7) and (8), the following relations between the components of the vector mesonic fields are obtained:

\[ m_0^2 \partial_\mu V^\mu = j^\mu \partial_\mu \Gamma_v, \quad m_0^2 \partial_\mu b^\mu = j_3^\mu \partial_\mu \Gamma_p. \]

For constant coupling parameters the above relations reduce to the usual relations between the components of a vector field

\[ \partial_\mu V^\mu = 0, \quad \partial_\mu b^\mu = 0. \]

At zero temperature, the state which minimizes the energy of asymmetric nuclear matter is characterized by the Fermi momenta \( P_{Fi}, \ i = p, n \), \( P_{Fe} = P_{Fp} \) and is described by the distribution function \( f_0(r, p) = \text{diag} \left( \Theta(P_{Fp}^2 - p^2), \Theta(P_{Fe}^2 - p^2) \right) \) and by the constant mesonic fields (defined with a (0) superscript) which obey the following equations \( m_0^2 \partial_\mu \phi(0) = \Gamma_s \rho_s(0), \ m_0^2 \phi(0) = \Gamma_v \rho_v(0), \ \phi(0) = 0, \ m_0^2 \rho_v(0) = \frac{1}{2} \Gamma_s(0), \ \phi(0) = 0, \ A_v(0) = 0, \ A_p(0) = 0. \)

Collective modes in the present approach correspond to small oscillations around the equilibrium state, and they are described by the linearized equations of motion [11]. We take for the distribution function \( f = f_0 + \delta f \) and, as in [11] we introduce a generating function \( S(r, p, t) = \text{diag} \left( S_p, S_n, S_e \right) \), defined in isospin space such that the variation of the distribution function is

\[ \delta f_i = \{ S_i, f_0 \} = -\{ S_i, p^2 \} \delta (P_{Fi}^2 - p^2). \quad (17) \]

In terms of this generating function, the linearized Vlasov equations for \( \delta f_i \) are equivalent to the following time evolution equations

\[ \frac{\partial S_e}{\partial t} + \{ S_e, h_{0e} \} = -e \left[ \delta A_0 - \frac{p \cdot \delta A}{\varepsilon_e} \right], \quad (18) \]

\[ \frac{\partial S_i}{\partial t} + \{ S_i, h_{0i} \} = -\Gamma_s \delta \frac{\phi(0)}{\varepsilon_0} + \delta V_{0i} - \frac{p \cdot \delta V_i}{\varepsilon_0}, \quad (19) \]

for \( i = p, n \), where we have taken linear variations for the fields. In equation (13) \( \varepsilon_{0e} = \sqrt{p^2 + m_e^2} \) and in equation (19) \( h_{0i} = \sqrt{p^2 + M^2 + V_{0i}} = \varepsilon_0 + V_{0i} \). The linearized equations of the fields are obtained using the procedure already presented in [13].

### III. SOLUTIONS FOR THE NORMAL MODES AND DISPERSION RELATION

The longitudinal normal modes of the system, with momentum \( \mathbf{k} \) and frequency \( \omega \) are well described by the ansatz

\[ \mathbf{F}_i = \mathbf{F}_i, \omega \exp \left[ i(\omega t - \mathbf{k} \cdot \mathbf{r}) \right] \]

for the fields and

\[ \mathbf{S}_j(r, p, t) = \mathbf{S}_j^l(\cos \theta) \exp \left[ i(\omega t - \mathbf{k} \cdot \mathbf{r}) \right], \ j = e, p, n, \]

for the generating functions, where \( \theta \) is the angle between \( \mathbf{p} \) and \( \mathbf{k} \). A different choice of the generating function would allow to study the transverse modes [12]. This, however, will not be carried out in the present work. For the longitudinal modes \( \delta V_{0x} = \delta V_{0y} = 0, \ \delta b_{0x} = \delta b_{0y} = 0 \) and \( \delta A_{0x} = \delta A_{0y} = 0 \).

Equations (18) and (19) are written in terms of the amplitudes \( A_{\omega i} \) related to the transition densities by \( \delta \rho_i = \frac{1}{2} \rho_i \rho_0 \rho_{0i} A_{\omega i} \), and they read

\[ \begin{pmatrix} 1 + F^p L_p & F^{pn} L_p & C^p \rho_v L_p \\ F^{np} L_n & 1 + F^{np} L_n & 0 \\ C^p \rho_v L_e & 0 & 1 - C^e \rho_v L_e \end{pmatrix} \begin{pmatrix} A_{\omega p} \\ A_{\omega n} \\ A_{\omega e} \end{pmatrix} = 0, \quad (20) \]
with $A_{ij} = \int_{-1}^{1} x S_{ij}(x) dx$, $L_i = 2 - s_i \ln \left( \frac{s_i + 1}{s_i - 1} \right)$ where $s_i = \omega/\omega_o i$ for solutions of the dispersion relation with imaginary frequency. These modes are obtained by replacing $i\beta$ for $i\epsilon$, $\omega_F = \frac{p_F}{\sqrt{2}}$ being the Fermi velocity of particle $i$, $\epsilon_F = \sqrt{P_F^2 + m_i^2}$, $i = p, n$, $\epsilon_F = \sqrt{P_F^2 + m_e^2}$. We also have

$$F^{ij} = \left[ G^{s}_s W_s - G^{\rho}_\rho W_\rho - G_v W_v \right] + \frac{1}{2\pi^2} \left( G_{sD, s} + \tau_i G_{D, D} + G_v D \right) - \phi_0 \frac{M^s \partial \Gamma_s}{\partial \rho} + \frac{b_0 G_{vD}}{2 \partial \rho} + \frac{1}{4}(1 + \tau_i)(1 + \tau_j)C^{pp}_{A} \frac{P^{F}_F \rho^{2}}{P^{F}_F \epsilon_F}$$

and

$$C^{ij}_{A} = -\frac{e^2}{2\pi^2 k^2} \frac{P^{F}_F}{V_F}$$

with

$$W_j = \frac{1}{2\pi^2 (\omega_s^2 - \omega_j^2)}$$

and $j = s, \rho, v$, and $\omega_j^2 = k^2 + m^2_{s,ff}, \omega_v^2 = k^2 + m^2_v, \omega_\rho^2 = k^2 + m^2_{\rho}$. With $m^2_{s,ff} = m^2_s + \Gamma_s^2 (\partial \rho_s / \partial M^s)$, all the other quantities are defined in the Appendix.

From (20) we get the following dispersion relation

$$\left[ 1 - C^{\rho \rho}_{A} L_e \right] \left[ 1 + L_e F^{pp} + L_n F^{nn} + L_p L_n (F^{pp} F^{nn} - F^{pn} F^{np}) \right] - C^{pp}_{A} C^{pc}_{A} L_e L_p (1 + L_n F^{nn}) = 0.$$

In order to study the instabilities of the system, we look for solutions of the dispersion relation with imaginary frequencies. These modes are obtained by replacing $s$ with $i\beta$ in the expression for $L_i$.

### IV. RESULTS AND DISCUSSION

In the present section we compare the dynamical spinodals, direction of instability and most unstable modes obtained with NL3, TW and DD-ME2. For reference, in Fig. [1] we compare the symmetry energy of the three models and the $\beta$-equilibrium equation of state (EoS) at low densities. It is known that NL3 symmetry energy grows nearly linearly with density and has a quite high value at saturation in comparison with TW and DD-ME2. Thus in $\beta$-equilibrium matter the proton fraction increases very quickly and reaches values which allow for the direct URCA process, and therefore predict a too fast cooling of neutron stars, already at density values close to the saturation density. The two models we will consider with density-dependent couplings have very similar symmetry energies and predict similar proton fractions. While npe matter is thermodynamically stable within TW and DD-ME2 models, for NL3 there is still a small unstable region [2]. Even being thermodynamically stable, npe matter is unstable with respect to perturbations with certain wavelengths. The region of instability is limited by the spinodal surface which, for a given $k$ transfer, is obtained from the dispersion relation and corresponds to the surface on which the eigenmode is zero. In Fig. [2] we plot the spinodal for three values of $k$: 11, 75 and 150 MeV. The value $k = 75$ MeV defines, except for small corrections, the envelope of the spinodals for $k$ values. In Fig. [2b] we include the spinodals obtained in three different situations: only neutron-proton (np) matter excluding the Coulomb field felt by the protons, together with np matter and neutron-proton-electron (npe) matter including the Coulomb interactions. While the first situation is not realistic, but allows a comparison with the thermodynamical limit, the second describes neutron-proton matter and the third one, stellar matter. For $k = 75$ and 150 MeV the results for np matter with Coulomb interaction essentially coincide with those for npe. Again for np matter, now with no Coulomb field for $k = 11$ MeV, the results practically coincide with the thermodynamical spinodal (which corresponds to $k = 0$ MeV). At $k = 150$ MeV the effect of the electrons and the Coulomb field is very small, as expected from the $1/k^2$ behavior of the Coulomb field [2].

The following conclusions may be taken: for $k = 11$ MeV the spinodal for np matter with Coulomb is much smaller than the corresponding spinodals for npe matter due to the attractive force between protons and electrons in the last case; for npe symmetric matter ($\rho_p = \rho_n$) the three models considered have similar results but differences occur for asymmetric matter, DD-ME2 showing instabilities at larger densities for the largest asymmetries.

In Fig [2b] we include the $\beta$-equilibrium EoS for npe neutrino-free matter $Y_\nu = 0$ and for npe electron matter as in supernovae with a constant lepton fraction $Y_e = Y_\nu + Y_\tau = 0.4$ [10]. The crossing of these EoS with the spinodal tell us that there is a non-homogeneous region in the star, at low densities. The density at the inner edge of the crust, as predicted by the present calculation, is given in Table [II]. For neutrino trapped matter the values shown are only an upper limit because for $T \neq 0$ the size of the instability region is smaller. In this situation, the three models give similar results because the matter considered has a very high proton fraction ($y_p \sim 0.3$), therefore closer to symmetric matter, where parameter sets are expected to coincide. However, for neutrino free matter, the density value at the inner edge of the crust is very sensitive to the model because we are dealing with highly asymmetric matter where the largest differences between models arise.

We next analyze the direction of the instability defined by the ratio of the fluctuations, $\delta \rho_p / \delta \rho_n$, corresponding to the eigenmode that becomes imaginary. In Fig [3a] and [3b] we plot $\delta \rho_p / \delta \rho_n$ as a function of $k$ for two proton
5

FIG. 1: (a) symmetry energy and (b) proton fraction in $\beta$-equilibrium matter versus density, for the relativistic models considered.

![Graph](attachment:image.png)

FIG. 2: Dynamical spinodals for a) $k = 11$ MeV, b) $k = 75$ MeV, c) $k = 150$ MeV

![Graph](attachment:image.png)

| model  | $Y_e = 0$ | $Y_L = 0.4$ |
|--------|----------|------------|
| NL3    | 0.050    | 0.082      |
| TW     | 0.076    | 0.084      |
| DD-ME2 | 0.073    | 0.083      |

TABLE III: Density at the inner edge of the crust of a compact star

fractions $y_p = 0.1$, typical of neutrino free stellar matter, and $y_p = 0.35$ which, as quoted above, would be found in stellar matter with trapped neutrinos, and for two densities, $\rho = 0.15\rho_0$ and $0.3\rho_0$. We include, for reference, a dashed thin line which indicates the corresponding $\rho_p/\rho_0$ ratio. In Fig. 3 we fix $k$ and for the same proton fractions referred above we show the dependence of $\delta\rho_p/\delta\rho_n$ on the density. Some conclusions are in order: at low densities, the distillation effect, which corresponds to $\delta\rho_p/\delta\rho_n > \rho_p/\rho_n$, is similar for all the models, with NL3 slightly less efficient for larger asymmetries. This can also be observed from Fig. 3 and was also seen in the thermodynamical instability calculations at low densities. For larger densities, both DDRM are less effective than NL3 in the reposition of symmetry. Indeed, from Fig. 3 we clearly observe that for densities larger than the ones considered in Fig. 3(a) and b) NL3 becomes the model which more efficiently describes the distillation effect, while TW and DD-ME2 keep showing a behavior which is similar among themselves, and almost independent of the density. The differences between the two types of models are larger for larger asymmetries.

For the np calculation with no Coulomb field the $\delta\rho_p/\delta\rho_n$ ratio is almost constant with respect to $k$, though slightly increasing, specially for small values of $y_p$. In addition to it, if the Coulomb field is included this ratio becomes much smaller than for the no Coulomb case, crossing even the $\rho_p/\rho_0$ line for $k \leq 25$ MeV, for the largest proton fraction considered here. This is the anti-distillation effect already discussed in [2]. In Table IV we show, for several pairs of asymmetry-density, the maximum $k$ values for which the anti-distillation occurs in the three models. All models have similar values although they are slightly larger for NL3. These values are never very large: we get $k \leq 25$ MeV and decreasing values of $k$ with increasing proton fraction. As discussed before, the distillation effect is larger for matter with no electrons, for in this situation protons do not couple to the electrons.

This effect may have important consequences in stellar matter. In fact, in a supernovae explosion 99% of the energy is carried away by the neutrinos. Neutrinos interact strongly with neutrons (large weak vector charge of the neutron) and therefore the way neutrons clusterize is important to determine the neutrino mean free path. Neutrinos may couple strongly to the neutron-rich matter low-energy modes present in this explosive environment.
and revive the stalled supernovae shock.

The system is driven to the non-homogeneous phase by the mode with a larger growth-rate. In Fig. 4 we plot the growth-rate of the most unstable mode as a function of density for np matter without Coulomb interaction and npe matter. The wavelength associated with these modes is related to the size of the inhomogeneities formed. In Fig. 4(b) half of the wavelength, which corresponds to the size of the clusters formed, is plotted as a function of density for the proton fractions and models considered in Fig. 4(a). As expected, in all the cases the presence of electrons reduces the growth-rate and the size of the clusters; this effect is more pronounced for larger densities.

For very small densities ($\rho \leq 0.1 \rho_0$) all three models have a similar behavior, characterized by a large growth-rate. As density increases, all the curves have similar slopes, but considerable differences between the models arise. For symmetric matter TW behaves like NL3 with the largest values for the growth rate and the size of the associated clusters. As asymmetry increases TW still maintains the largest instability, but NL3 changes its behavior becoming closer to DD-ME2 with the smallest growth-rate.

The size of the instabilities that drive the system is of the order of $4 - 10$ fm. For small $k$ the unstable mode

![FIG. 3: Direction of instability as a function of the momentum transfer for $\rho = 0.15 \rho_0$ and $0.3 \rho_0$ for (a) $y_p = 0.1$ and (b) $y_p = 0.35$ and as a function of density (c) for $k = 100$ MeV and $y_p = 0.1, 0.35$, for np matter only.]

![FIG. 4: Most unstable modes: a) growth rates and b) associated size of clusters for asymmetries $y_p = 0.1, 0.3, 0.5$ and the models NL3, TW and DD-ME2. Results including electrons (thick lines) are compared with np matter results with no Coulomb interaction (thin lines).]
disappears due to the quenching of the instability: \( \frac{1}{k^2} \) divergence of the Coulomb energy. In the large \( k \) limit the effect of the Coulomb interaction goes to zero. Larger differences between NL3, TW and DD-ME2 occur at densities and proton fraction of interest for \( \beta \)-equilibrium stellar matter. In particular TW predicts larger clusters at densities above \( \sim 0.02 \) fm\(^{-3}\). The size of the clusters calculated agree with the results of a density functional with relativistic mean-fields coupled with the electric field [18].

| \( \gamma_p \) | \( \rho \) | NL3 | TW | DD-ME2 |
|---|---|---|---|---|
| 0.1 | 0.15\( \rho_0 \) | 6 | - | - |
| 0.3\( \rho_0 \) | - | - | - |
| 0.25 | 0.15\( \rho_0 \) | 15 | 13 | 12.7 |
| 0.3\( \rho_0 \) | 14 | 13.5 | - |
| 0.35 | 0.15\( \rho_0 \) | 25 | 22.4 | 21.8 |
| 0.3\( \rho_0 \) | 24.8 | 24.5 | 24.6 |

TABLE IV: Maximum value of \( k \) in MeV for which the anti-distillation effect occurs.

V. CONCLUSIONS

We have investigated the low densities instabilities in density-dependent relativistic hadronic models (DDRM) and compared them with previous results obtained within NLWM, namely with the NL3 parametrization.

The spinodal region shows that both DDRM used here present instability regions larger than NL3, except for large \( k \) in which case DD-ME2 has a smaller spinodal region. These differences occur mainly at larger isospin asymmetry. From the astrophysical point of view, this could mean differences in low density stellar matter, namely the crust properties of compact stars. In particular, we have seen that while the predicted inner crust edge density for stellar matter with trapped neutrinos is very similar in all models considered, in cold stellar matter with no neutrinos the differences are large. For DDRM this density is about 50\% larger than the one for NL3.

It was shown that except for the lowest values of density, density-dependent parametrizations predict lower distillation effects. At low densities this trend is no more true with DDRM showing results which are similar to NL3 or even slightly larger for small proton fractions.

For small \( k \) an anti-distillation effect is present in npe matter and np matter with Coulomb interaction. It is for the NL3 parametrization that this behavior sets on at larger \( k \) values and it is present in this model even for very large asymmetries (see Table IV). This will have an important effect on the scattering of neutrinos which escape the proto-neutron star: a large neutron fraction implies a larger weak force interaction.

We have predicted the formation of clusters with sizes ranging from \( \sim 4 \) fm to 10 fm. These limits depend on the proton fraction and larger clusters are formed in more asymmetric matter.

We finally conclude that different parametrizations of DDRM have similar properties and different from other models with constant couplings. Their predictive power will depend on their ability of satisfying constraints both having astrophysical origin or laboratory measurements [19].

Neutrino opacity plays a crucial role but it is not the only mechanism of energy accounting in a supernova. The plasmon-decay into neutrino-antineutrino pairs should also be considered in neutron star evolution. In [20], we have studied plasmons in stellar matter within constant-coupling relativistic models. Plasmons are currently being studied in DDRM, as we have shown in [21]. There, nuclear plasmon modes were found at zero temperature. We are also carrying out finite temperature calculations and we expect that this contribution will allow estimations of neutrino production, due to neutrino-antineutrino decay, in nuclear matter under neutron star conditions.

Acknowledgments

We would like to thank S. S. Avancini for his clarifying discussions and helpful suggestions on this work. This work was partially supported by FEDER and FCT (Portugal) under the grant SFRH/BPD/29057/2006, and projects POCI/FP/63918/2005, PDCT/FP/64707/2006, and by CNPq (Brazil).
APPENDIX: Dispersion relation coefficients

The expressions used in Eq. 21 read as:

\[ G^{ij}_{s} = G_{\phi_i} G_{\phi_j}, \]
\[ G_{\phi_i} = \Gamma_s \left[ \frac{M^*}{\varepsilon_{F_i}} - \phi_0 \left( \frac{\partial \rho_s}{\partial M^*} \right)_0 \left( \frac{\partial \Gamma_s}{\partial \rho} \right)_0 \right] + \rho_s^{(0)} \left( \frac{\partial \Gamma_s}{\partial \rho} \right)_0, \]
\[ G^{ij}_{\rho} = \frac{1}{4} G_{\rho ij} G_{\rho j i}, \]
\[ G_{\rho ij} = \tau_i \Gamma_{\rho} + \left( \frac{\partial \Gamma_{\rho}}{\partial \rho} \right)_0 \left( 1 - \frac{\omega^2}{m_i^2} \right) \rho_3^{(0)}, \]
\[ G_{\rho i i} = \tau_i \Gamma_{\rho} \left( 1 - \frac{\omega^2}{k_i^2} \right) + \left( \frac{\partial \Gamma_{\rho}}{\partial \rho} \right)_0 \rho_3^{(0)}, \]
\[ G_{\rho} = G_{\rho 1} G_{\rho 2}, \]
\[ G_{\rho 1} = \Gamma_{\rho} + \rho^{(0)} \left( \frac{\partial \Gamma_{\rho}}{\partial \rho} \right)_0 \left( 1 - \frac{\omega^2}{m_1^2} \right), \]
\[ G_{\rho 2} = \rho^{(0)} \left( \frac{\partial \Gamma_{\rho}}{\partial \rho} \right)_0 + \Gamma_{\rho} \left( 1 - \frac{\omega^2}{k_2^2} \right), \]
\[ G_{sD_i} = H_{\rho_i} + \phi_0^2 \left( \frac{\partial \rho_s}{\partial M^*} \right)_0 \left( \frac{\partial \rho_s}{\partial \rho} \right)_0^2, \]
\[ G_{\rho D} = \frac{1}{4m_\rho^2} \left( \frac{\omega}{k} \right)^2 \Gamma_{\rho} \left( \frac{\partial \Gamma_{\rho}}{\partial \rho} \right)_0 \rho_3^{(0)}, \]
\[ G_{eD} = \frac{1}{m_e^2} \left( \frac{\omega}{k} \right)^2 \Gamma_e \left( \frac{\partial \Gamma_e}{\partial \rho} \right)_0 \rho_3^{(0)}, \]
\[ H_{\rho_i} = -\phi_0 \left[ \frac{M^*}{\varepsilon_{F_i}} \left( \frac{\partial \Gamma_s}{\partial \rho} \right)_0 + \rho_s^{(0)} \left( \frac{\partial^2 \Gamma_s}{\partial \rho^2} \right)_0 \right] + V_0 \left[ 2 \left( \frac{\partial \Gamma_{\rho}}{\partial \rho} \right)_0 + \rho^{(0)} \left( \frac{\partial^2 \Gamma_{\rho}}{\partial \rho^2} \right)_0 \right] + \frac{b_0}{2} \left[ \Gamma_{\rho} \left( \frac{\partial \Gamma_{\rho}}{\partial \rho} \right)_0 + \rho_3^{(0)} \left( \frac{\partial^2 \Gamma_{\rho}}{\partial \rho^2} \right)_0 \right]. \]

In the previous expressions the zero in the superscripts and subscripts on \( \rho \) and derivatives, respectively, mean that they are calculated with respect to the static background on which the oscillations take place.