Possibility of Additional Intergalactic and Cosmological Dark Matter

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Abstract

The entropies of the known entities in the universe add to a total which is some twenty orders of magnitude below the holographic limit. Based on an assumption that the entropies should saturate the limit, we suggest that there exists dark matter, in the form of extremely massive primordial black holes, in addition to the dark matter known to exist inside galaxies and clusters of galaxies.

Keywords: Dark matter; Entropy of the universe; Black holes.

1 Introduction

In particle theory, the concept of entropy is usually not regarded as fundamental. Particle theorists rarely even use the word entropy. For one elementary particle, entropy is neither defined nor useful.

In general relativity and cosmology, the situation is different. For black holes, entropy is a central and useful concept. We shall in this talk argue that the origin and nature of cosmological dark matter can be best understood by consideration of the entropy of the universe.
We have made such an argument some years ago but that discussion was perhaps too diluted by considering simultaneously dark matter being made from elementary particles such as WIMPs and axions. In this talk, we dispose of microscopic candidates in one paragraph. The standard model of particle theory (SM) has two examples of lack of naturalness, the Higgs boson and the strong CP problem. Our position is that to understand these we still need to understand better the SM itself. Regarding the strong CP problem, it is too ad hoc to posit a spontaneously broken global symmetry and consequences which include an axion. Concerning the WIMP, the idea that dark matter experiences weak interactions arose from assuming TeV-scale supersymmetry which is now disfavoured by LHC data. To identify the dark matter, we therefore instead look up at the night sky.

Assuming dark matter is astrophysical, and that the reason for its existence lies in the Second Law of Thermodynamics, we shall be led uniquely to the dark matter constituent as the Primordial Black Hole (PBH). We must admit that there is no observational evidence for any PBH, but according to our discussion PBHs must exist. In the ensuing discussion, we shall speculate that they exist in abundance in three tiers of mass up to and including extremely high masses which are far greater than the masses of galaxy clusters and approach closely to the mass of the visible universe.

Because PBH entropy goes like mass squared, we are mainly interested in masses satisfying \( M_{PBH} > 100 M_\odot \). Within the Milky Way, we use the acronym PIMBH for intermediate mass PBHs in the mass range \( 10^2 M_\odot < M_{PIMBH} < 10^3 M_\odot \). Outside the Milky Way we entertain all masses \( 10^2 M_\odot < M_{PBH} < 10^{22} M_\odot \). Of these, we use PSMBH for supermassive PBHs in the mass range \( 10^6 M_\odot < M_{PSMBH} < 10^{11} M_\odot \) and PEMBH for extra massive PBHs with \( 10^{12} M_\odot < M_{PEMBH} < 10^{22} M_\odot \).

Although the visible universe (VU) is not a black hole, its Schwartzschild radius is about 68% of its physical radius, 30 Gly versus 44 Gly, so it is remarkably close. This curious fact seems to have no bearing on the nature of dark matter. Acronyms will be useful: CMB is the familiar cosmic microwave background while CIB refers to its counterpart, cosmic infrared background.

2 Entropy

We begin with the premise that the early universe be regarded in an approximate sense as a thermodynamically-isolated system for the purposes of our discussion. It certainly contains a number of particles, \( \sim 10^{80} \), vastly
larger than the numbers normally appearing in statistical mechanics, such as Avogadro’s number, $\sim 6 \times 10^{23}$ molecules per mole.

No heat ever enters or leaves and it can be considered as though its surface were covered by a perfect thermal insulator. It is impracticable to solve all the Boltzmann transport equations so it is mandatory to use thermodynamic arguments, provided that we may argue that the system is proximate to thermal equilibrium.

Making the then-unsupported assumption in 1872 of atoms and molecules, Boltzmann discovered the quantity $S(t)$ in terms of the molecular momentum distribution function $f(p, t)$

$$S(t) = -\int dp f(p, t) \log f(p, t)$$

which satisfies

$$\left(\frac{dS(t)}{dt}\right) \geq 0$$

and can be identified with the thermodynamic entropy. The crucial inequality, Eq.(2), the Second Law, was derived for non-equilibrium systems assuming only the Boltzmann transport equations and the ergodic hypothesis.

Ascertaining the nature of the dark matter can be regarded as a detective’s mission and there are useful clues in the visible universe. We can made an inventory of the entropies of the known objects in the visible universe, using a venerable source, Weinberg’s 1972 book.

Let us model the visible universe as containing $10^{11}$ galaxies each of mass $10^{12} M_\odot$ and each containing one central SMBH with mass $10^7 M_\odot$. We recall the dimensionless entropy of a black hole $S/k(M_{BH} = \eta M_\odot) \sim 10^{78} \eta^2$. Then the inventory is

- SMBHs $\sim 10^{103}$
- Photons $\sim 10^{88}$
- Neutrinos $\sim 10^{88}$
- Baryons $\sim 10^{80}$

We regard this entropy inventory as a **first clue**. From the point of view of entropy the universe would be only infinitesimally changed if everything
except the SMBHs were removed. This suggests that more generally black holes totally dominate the entropy, as we shall find in the sequel.

A second remarkable fact about the visible universe is the near-perfect black-body spectrum of the CMB which originated some 300,000 years after the beginning of the present expansion era, or after the Big Bang in a more familiar language. We are not tied to a Big Bang which we believe will eventually be replaced by a bounce from contraction to expansion in cyclic cosmology.

The precise CMB spectrum is a second clue about dark matter. It suggests that the plasma of electrons and protons prior to recombination is in excellent thermal equilibrium, and hence the matter sector was in thermal equilibrium for the first 300,000 years. This, combined with the thermal isolation mentioned already, underwrites the use of entropy, and the second law, during this period.

A third clue and final one about dark matter lies with the holographic principle [5] which provides an upper limit on the entropy of the visible universe, the area of its surface in units of the Planck length. Given it present co-moving radius 44 Gly this requires $S_{\text{Universe}}/k \leq 10^{123}$. The entropy of the contents which is so bounded might nevertheless equal this limit which is many orders of magnitude higher than the total entropy in the limited inventory listed above. That this may be the case is based only on our cosmological intuition that the universe is beautiful.

In this talk we investigate the possibility that PBHs saturate the holographic entropy bound [1] and entertain the possibility of extremely high masses up to $10^{22} M_\odot$. Just a few of these could saturate the maximum entropy bound but here we discuss such a situation more generally.

One might reasonably be concerned that such objects might be inconsistent with existing knowledge in astronomy and cosmology? To our knowledge, there is no serious contradiction. There are at least three places where such a theory might be challenged. Firstly, there are limits on the cosmological principle which asserts the large-scale homogeneity and isotropy of the universe. Secondly, if PBHs are formed during the era of Large-Scale Structure formation, they might conceivably play a deleterious rôle. Thirdly and finally, one might also worry about whether the theory leaves inviolate the precise thermal spectrum and isotropy of the CMB? On the third point, consistency requires only that additional non-thermalised photons are absent or sufficiently suppressed.
It is appropriate to refer to the high mass objects as additional dark matter because they are not associated with specific galaxies or clusters of galaxies but are located elsewhere in the universe. The total mass of additional dark is expected to be comparable in order of magnitude to that of dark matter inside galaxies and clusters but its total entropy is extremely much greater. Indeed, we would say that the possibility of equalling the holographic bound can be uniquely achieved only with the presence of additional dark matter in the form of extremely massive black holes.

Another relevant consideration is PBH production. The mass governed by the horizon size at cosmic time \( t \) is

\[
M_{PBH} = 10^5 M_\odot \left( \frac{t}{1 \text{ second}} \right)
\]

so that taking \( t < 300,000 \text{ y} \) to precede recombination when the CMB originates would require that

\[
M_{PBH} < M_{cmb} \sim 10^{18} M_\odot
\]

However, it is possible that more massive PBHs may be formed later provided they do not produce photons which disturb the CMB spectrum.

As examples of higher masses we take \( 10^7 M_\odot \) and \( 10^{14} M_\odot \) respectively. According to Eq.(3) these are produced at \( t = 100 \text{ s} \) and \( t = 30 \text{ y} \). For the largest mass mentioned above, \( 10^{22} M_\odot \), the formation time in Eq.(3) is \( t = 3 \text{ Gy} \) which is quite recent cosmologically.

In reality, we might expect a smoother PBH mass spectrum than suggested by these monochromatic examples. However, at least one PBH formation model \[\text{[6]}\] does suggest quasi-monochromatic formation so we must consider all possibilities.

### 3 The Great Attractor

It was pointed out by Dressler\[3\] that the peculiar velocities of certain galaxies point to the existence of a specific mass overdensity which corresponds to what he called the Great Attractor with mass \( M_{GA} \sim 10^{18} M_\odot \). This the only such overdensity in a volume \( \sim (1 \text{ Gpc})^3 \) so assuming a uniform density within the visible universe there could be a few thousand of them. The approximate equality of \( M_{GA} \) with \( M_{cmb} \) in Eq.(4) is presumably accidental.
For our present purposes, we shall assume that the Great Attractor is a PBH, and use it as a jumping off point to posit the existence in the visible universe of truly cosmological size PBHs. The size of the GA is comparable to that of the Milky Way $\sim 100 \text{kpc}$. For a PBH with mass $10^{22} M_\odot$ the Schwarzschild radius is comparable to that of the visible universe and too large to be detectable on a plot like Fig 2 in Reference [3].

In Table 1, we summarise the entropy properties for two examples of intergalactic dark matter.

We see from Table 1 that they suggest an opportunity to approach the holographic upper bound because if we take the maximum allowed number of GAs their entropy adds to $S/k \sim 10^{117}$ just a million times less than the limit.

To put this in perspective, we recall the dark matter suggested in [4] and which could be the correct explanation for the dark matter inside of galaxies such as the Milky Way. If we take those intermediate-mass black holes to all have mass $100 M_\odot$ their entropy adds to only $S/k \sim 10^{103}$ which is approximately the same as the entropy of the supermassive black holes (SMBHs) known to reside at galactic centres. Of the known objects in the universe, SMBHs overwhelmingly dominate the entropy. Nevertheless, their entropy falls mysteriously short of the holographic limit by a huge factor of some twenty orders of magnitude.

We are here exploring the assumption that the content of the universe possesses entropy adding to the holographic limit. Initially our expectation was to find this possibility excluded but all we would say now is that it must involve a significant quantity of additional dark matter.

Looking at the universe from the viewpoint of entropy is very different from the viewpoint of mass. For example, it is well known that normal matter

\footnote{This assumption is not part of, but is additional to, the holographic principle as proposed first by ‘t Hooft in 1993. Our additional assumption may be necessary in order to describe Nature.}

Table 1: Values of Schwarzschild radius and Dimensionless entropy.

| Mass  | Schwarzschild radius | Entropy $S/k$ |
|-------|----------------------|---------------|
| $10^{18} M_\odot$ | 100 kpc | $10^{114}$ |
| $10^{22} M_\odot$ | 1 Gpc | $10^{122}$ |
is only 5% in a mass pie-chart. In an entropy pie-chart, however, with only the known objects, baryonic matter provides only $10^{-25}$ of total entropy which diminishes its importance much further.

In the context of the additional dark matter model being discussed in the present article, normal matter would provide only an infinitesimal fraction ($\sim 10^{-45}$) of the total entropy, the rest being dark matter.

4 Maximal Additional Dark Matter

From the entropy viewpoint, it is interesting that the visible universe is so close to being itself a black hole in the sense that its Schwarzschild radius $9Gpc$ is some two thirds of the co-moving radius $13.5Gly$.

In terms of mass-energy, this is merely a restatement of the fact that the present density is close to the critical density. But for entropy it is extremely puzzling, because the known content has only an infinitesimally tiny fraction of its maximum possible value. This suggests that there is something dominating the cosmological entropy which is being overlooked.

The only candidate to fill this rôle is, to our knowledge, extremely massive black holes, such as those in Table 1, which may be regarded as a straightforward extension of the dark matter in galaxies and supermassive black holes in galactic cores, all here assumed to be PBHs.

We have no prejudice about the mass function of extremely massive PBHs. It may be a smooth function or a series of almost monochromatic steps as suggested by some numerical work [6]. Here we discuss the latter possibility.

First, we reconsider the Great Attractor mass, $M_{GA}$, and the viable possibility of one thousand PBHs of this size. As we have seen, these can contribute $S/k \sim 10^{117}$ to the entropy of the universe, much more than the supermassive black holes, $\sim 10^{103}$.

The other, higher, mass scale mentioned ut supra was $M_{cp}$ which was taken to be the largest mass which is consistent with present evidence supporting the cosmological principle. Each such black hole provides a contribution to dimensionless entropy which is only one order of magnitude below the holographic limit. Therefore, no more than a few are allowed. If these exist, then saturating the maximum allowed entropy can easily be fulfilled.
5 James Webb Space Telescope

At large red-shifts $Z > 15$, a population of PBHs would be expected to accrete matter and emit in X-ray and UV radiation which will be redshifted into the CIB to be probed for the first time by the James Webb Space Telescope which could therefore provide support for PBH formation.

Analysis of a specific PBH formation model supports this idea that the JWST observations in the infrared could provide relevant information about whether PBHs really are formed in the early universe. This is important because although we have plenty of evidence for the existence of black holes, whether any of them is primordial is not known. The gravitational wave detectors LIGO, VIRGO and KAGRA have discovered mergers in black hole binaries with initial black holes in the mass range $3 - 85M_\odot$. We suspect that all or most of these are not primordial but that is only conjecture. The supermassive black holes at galactic centres, including Sgr A* at the centre of the Milky Way, are well established and are primordial in our toy model. Whether that is the case in Nature is unknown.

Because of the no-hair theorem that black holes are completely characterised by their mass, spin and electric charge (usually taken to be zero), there is no way to tell directly whether a given black hole is primordial or the result of gravitational collapse of a star. The distinction between a primordial and a non-primordial black hole can be made only from knowledge of its history. For example, if it existed before star formation, it must be primordial. The infra-red data from JWST will able to provide insight into the central question of PBH formation.

A second deep insight likely to be provided by the JWST is whether or not Population III stars existed at high red shifts. Their existence looks inevitable from metallicity arguments. Our Sun and other typical stars have a surprisingly high metallicity close to 2%. Such stars cannot be formed directly from the primordial gases which have vanishing metallicity so there must be, and is, an earlier generation of Population II stars with metallicities orders of magnitude below that of the Sun. Even this is insufficient to account for the existence of the Sun and therefore Population III stars are expected to have existed at $Z > 15$. These extremely low metallicity stars would have lifetimes of only about ten million years and have long ago disappeared. Evidence from the infra-red observations by the JWST could find evidence of Population III stars, if they really existed.

It is familiar to study a mass-energy pie-chart of the universe with approximately 5% baryonic normal matter, 25% dark matter and 70% dark energy.
The entropy pie-chart is very different if the toy model considered in this paper resembles Nature. The slices corresponding to normal matter and dark energy are extremely thin and the pie is essentially all dark matter.

In this talk we have attempted to justify better that entropy and the second law applied to the early universe provide a raison d’être for the dark matter. We propose that the dark matter constituents are PBHs with a very wide range of masses from $10^2 M_\odot$ to $10^{22} M_\odot$.

Since it has never been observed except by its gravity, it does seem most likely that dark matter has no direct or even indirect connection to the standard model of strong and electroweak interactions in particle theory. The three clues we have mentioned: the dominance of black holes in the entropy inventory, the CMB spectrum and the holographic entropy maximum all hint toward PBHs as the dark matter constituent.

Assuming that the maximum entropy limit suggested by holography is saturated the mass function for the PBHs must extend to maximally high mass values.

6 Testability

So far, our discussion has been highly speculative and has populated the visible universe with objects which may well be the most massive ever contemplated. The nearest may be [7] which considered almost as massive black holes. From the point of view of entropy, all these very massive black holes are a natural extension of the dark matter expected inside galaxies and clusters.

Thus, dark matter in this generalised sense permeates all of space not as condensed clumps of mass but spread out on all scales up to cosmological ones. This occurrence of such extremely massive black holes seems inevitable, if we adopt the hypothesis that the bulk contents of the universe possess an entropy which saturates the holographic limit.

An obvious question is how to test this novel view of the universe. Additional great attractors, along the lines of [8], if they exist, require better technology to observe galaxy distributions at larger distances. As for the most extreme black holes comparable to the size of the universe itself, we are unaware of any good and practicable observational test.
There is the important question of whether and how PBHs were formed. According to Eq. (3), masses $10^{18} M_\odot$ and $10^{22} M_\odot$ would be formed at, respectively, $t = 300ky$ and $3Gy$ so there can be natural concern about distorting too much the CMB and of adversely affecting the formation of large-scale structure.

One possibility would be to test the PBH theory by numerical dark matter simulations, similar to those pioneered in [8], but this seems very challenging because they give a qualitatively acceptable result for the Large Scale Structure independent of the mass of the dark matter constituents. It is conceivable that more powerful computers than presently available will be able to discriminate between the predicted LSS estimated both with and without such large PBHs. In a similar vein, more advanced technology in telescope construction, both terrestrial and in space, is necessary to make astronomical observations sensitive enough to detect the existence of more examples similar to the Great Attractor.

Discussion of the central assumption of this article, that the holographic entropy maximum is reached by summing the entropies of all the objects within the universe might prove fruitless but hopefully not. What prompted us to publish this discussion was partly the response ”pure cowardice” by Dirac when asked why he did not predict the positron in his 1928 paper which announced the discovery of his eponymous equation.

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