Study on the reduction of the general / overall torsion on multi–story, rectangular, reinforced concrete structures

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Abstract: This paper researches the reduction of the general / overall torsion of multi–story, rectangular, reinforced concrete structures. The general torsion is caused by the distance (eccentricity) between the centre of stiffness and the centre of mass of the structure. During the seismic shaking of the structural systems, the inertia force acts through the centre of mass, while the resistive force acts through the centre of stiffness. Under this coupled lateral - torsional motion, structural members located along the perimeter of the buildings develop increased deformations and stresses due to buildings' twisting, which results in a higher risk of collapse (brittle, non - ductile failure, damage). This leads to the torsional behaviour of buildings, which is one of the most frequent sources of structural damage and failure during strong ground motions. This paper investigates the influence of the cross sectional properties of the structural components / members, to reduce the above mentioned effect as much as possible. The reduction of the eccentricity can be obtained by modifying cross sectional properties of some rectangular reinforced concrete walls, such as the moment of inertia (length and width of the reinforced concrete walls), which are asymmetrically arranged relative to the central reinforced concrete core. The position of these rectangular reinforced concrete walls is known. Finding the optimal height and width of these walls can be done by using a MatLab function, which minimizes the distance between the centre of stiffness and the centre of mass. The obtained results are then verified in a structural software (ETABS 2016), using modal analysis, or the mode – superposition method, to determine the mode shapes of the structure.

1. Introduction

Buildings are mainly classified into regular and irregular. Asymmetric - plan buildings, namely buildings with in - plan asymmetric mass and strength distributions, are systems characterized by a coupled torsional - translational seismic response.

Asymmetric building structures are almost unavoidable in modern construction, due to various types of functional and architectural requirements. Even in symmetric structures, the asymmetric position of the structural components (non - symmetric distributions of mass and stiffness) tends to produce an effective asymmetric structure. Such an asymmetry, even if it is small, can produce a torsional response, coupled with a translational response.
The presence of structural irregularities has an adverse effect on the seismic response of the structure. Structural irregularities can be broadly classified as plan irregularities and vertical irregularities. In the present study, the effect of plan irregularity on the seismic response of a structure is studied [1].

1.1. Plan Irregularity
The condition of being non-uniform in the plan of a structure is called plan irregularity. These can be characterized by five different types such as torsional, reentrant corners, diaphragms discontinuity, out of plane offset and non–parallel system for plan irregularity [1].

1.2. Vertical Irregularity
Structures having significant physical discontinuities in a vertical configuration or in their lateral force resisting systems are termed as vertically irregular structure. The vertical irregularities in structures are Stiffness irregularity, Mass irregularity, Vertical geometric irregularity, Discontinuity in capacity [1].

1.3. Reinforced concrete (R.C.) cores
Reinforced concrete (R.C.) cores are used in many residential multi-story buildings as the primary seismic force resisting system. To minimize the effect of the structural system on the architectural layout, shear walls often act as stairway and elevator shafts in many R.C. buildings. Seismic damage surveys and analyses conducted on modes of failure of building structures during past severe earthquakes concluded that the most vulnerable building structures are those which are asymmetric in nature [2].

The lateral-torsional coupling due to the eccentricity between the centre of mass (C.M.) and the centre of stiffness (C.S.) in asymmetric building structures (in their floor plans or along their height) generates torsional vibration even under purely translational ground shaking, with significant swaying and twisting.

During the seismic shaking of the structural systems, the inertia force acts through the centre of mass, while the resistive force acts through the centre of stiffness, as shown in figure 1 [3].

![Figure 1. Generation of torsional moment in asymmetric structures during seismic excitation.](image)

The natural eccentricity is generally defined as the distance between the centre of mass (C.M.) and the centre of stiffness (C.S.) for a considered floor, while accidental eccentricity generally accounts for factors such as the rotational component of ground motion about the vertical axis, the difference between computed and actual values of the mass and stiffness, and an unfavourable distribution of live load mass.

1.4. Types of eccentricities
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difference between computed and actual values of the mass and stiffness and an unfavourable distribution of live load mass.

1.5. Centre of mass
The centre of mass is a position defined as the average position of all the parts of the system, weighted according to their masses. The distribution of mass is balanced around the centre of mass and the average of the weighted position coordinates of the distributed mass defines its coordinates.

During an earthquake, acceleration-induced inertia forces will be generated at each floor level and they will act at a point where the mass of the entire story may be assumed to be concentrated. In a building having a symmetrical distribution of mass, the positions of the centres of floor masses will not differ from floor to floor. However, irregular mass distribution over the height of a building may result in variation in the centres of masses at various floors [1].

1.6. Centre of stiffness
The centre of stiffness is a point at a particular story, as the location of application of lateral load at that point will not produce rotation of that story. This definition is valid when the slab is modelled with an infinite in-plane floor behaviour. An infinite in-plane floor behaviour causes all of its constrained joints to move together as a planar diaphragm that is rigid against membrane deformation. As a function of structural properties, the centre of stiffness is independent of loading [1].

2. Building details
The structural analysis of a six story R.C. building structure has been done with the help of ETABS 2016 [4] structural software. The building is assumed as commercial complex. A linear static analysis has been performed. The regular grid plan of the structure is shown in figure 2. The structure is assumed to be located in Vrancea. The building contains a highly "accentuated" plan irregularity.

![Figure 2. Building plan “before”](image-url)
Figure 3. Isometric view of the structure “before”.

2.1. Structural data

| Structural data               |       |
|-------------------------------|-------|
| Number of stories             | 6     |
| Ground story height           | 3.50 [m] |
| Intermediate story height    | 3.00 [m] |
| Total number of columns       | 32    |
| Slab thickness                | 15 [cm] |
| Core wall thickness           | 35 [cm] |
| Outer wall thickness          |       |
| $W_1 = 60$ [cm]               |       |
| $W_2 = 70$ [cm]               |       |
| Beam size                     |       |
| $B_1 = 30 \times 65$ [cm]     |       |
| $B_2 = 25 \times 55$ [cm]     |       |
| Column size                   |       |
| $C_1 = 70 \times 60$ [cm]     |       |
| Grade of concrete             | C 20/25 |
| Grade of steel                | BST 500S |
| Density of concrete           | 25 [kN/m$^2$] |
| Super dead load               | 1.3 [kN/m$^2$] |
| Live load                     | 3.0 [kN/m$^2$] |
| Roof load                     | 2.0 [kN/m$^2$] |
Super dead loads are basically superimposed dead loads, which are applied on a structure; the load of any finished, partitioning, cladding, false ceiling are all super dead loads.

The live load is the load superimposed by the use or occupancy of the building, not including the environmental loads, such as wind load, rain load, earthquake load or dead load.

| Location, Country | Vrancea, Romania |
|-------------------|------------------|
| \(a_g\)           | 3.924 \([\text{m/s}^2]\) |
| \(T_e\)           | 1.0 [s]          |
| \(T_B\)           | 0.20 [s]         |
| \(T_D\)           | 3.00 [s]         |
| Importance factor  | 1.00             |
| Framing type      | D.C.H.           |
| Behaviour factor  | 6.75             |
| Base shear coefficient, \(c\) | 0.10 |
| \(\nu\)           | 0.45             |
| Damping           | 5 [%]            |

3. Analysis

The building is modelled in ETABS 2016; an infinite in-plane floor behaviour is assigned at every story level, as shown in figure 4. Supports are assigned as fixed supports, neglecting soil - structure interaction. A linear static analysis was performed for earthquake cases: X Dir.; X Dir. ± Eccentricity; Y Dir.; Y Dir. ± Eccentricity.

![Figure 4. Prescribed diaphragm classification [6].](image)

\[
\Delta_{\text{dia}} \leq 0,50 \cdot \Delta_{L.F.R.S} \rightarrow \text{Rigid diaphragm}
\]  
\[
0,50 \cdot \Delta_{L.F.R.S.} < 0,50 \cdot \Delta_{\text{dia}} \leq 2,0 \cdot \Delta_{L.F.R.S.} \rightarrow \text{Stiff diaphragm}
\]  
\[
\Delta_{\text{dia}} > 2,0 \cdot \Delta_{L.F.R.S.} \rightarrow \text{Flexible diaphragm}
\]

An infinite in - plane floor behaviour is considered for this study. In this case, its midpoint displacement, under lateral load, is less than half the average displacement of the lateral resisting system. The rigid diaphragm distributes the horizontal forces to the lateral resisting elements, in direct proportion to their relative stiffness. It is based on the assumption that the diaphragm does not deform itself and will cause each vertical element to deflect the same amount. Rigid diaphragms capable of transferring torsional and shear deflections and forces are also based on the assumption that the
diaphragm and shear walls undergo rigid body rotation, and this produces additional shear forces in the shear wall.

3.1. Analysis results

Table 3. Modal participating mass ratios “before”.

| Case | Mode | Period [sec] | UX     | UY     | RZ     | Sum UX | Sum UY | SumRZ  |
|------|------|--------------|--------|--------|--------|--------|--------|--------|
| Modal| 1    | 0.508        | 0.2387 | 0.2083 | 0.3861 | 0.2387 | 0.2083 | 0.3861 |
| Modal| 2    | 0.337        | 0.2822 | 0.4757 | 0.0089 | 0.5209 | 0.684  | 0.395  |
| Modal| 3    | 0.255        | 0.2546 | 0.074  | 0.4586 | 0.7755 | 0.758  | 0.8535 |
| Modal| 4    | 0.159        | 0.0535 | 0.0387 | 0.0242 | 0.829  | 0.7966 | 0.8777 |
| Modal| 5    | 0.095        | 0.0642 | 0.0959 | 0.0007 | 0.8932 | 0.8925 | 0.8784 |

Figure 5. Mode shape 1 – $T_1 = 0.508$ s.

Figure 6. Mode shape 2 – $T_2 = 0.337$ s.
The coupled lateral - torsional motion appears already in the first mode shape, the structure is subjected to a predominant torsional motion (table 3), after that a lateral coupled motion can be observed in the following mode shapes (figure 5, figure 6 and figure 7).

4. Solution

As expected, in the first three mode shapes a coupled lateral - torsional motion appears. To avoid this torsional behaviour, two R.C. structural walls are placed in different positions, with different sectional properties (length and width of the reinforced concrete walls), as shown in figure 6 and 7. The position of these rectangular reinforced concrete walls is known.

To optimize the length of these elements, the Nelder - Mead algorithm or downhill simplex method or amoeba method is used, which is a commonly applied numerical method, used to find the minimum or maximum of an objective function in a multidimensional space. It is applied to nonlinear optimization problems, for which derivatives may not be known.

For the given system of equations:

\[
\begin{cases}
    f_1(x_1, x_2, x_3, x_4, \ldots, x_n) = 0 \\
    f_2(x_1, x_2, x_3, x_4, \ldots, x_n) = 0 \\
    \ldots \\
    \ldots \\
    f_{n-1}(x_1, x_2, x_3, x_4, \ldots, x_n) = 0 \\
    f_n(x_1, x_2, x_3, x_4, \ldots, x_n) = 0
\end{cases}
\] (4)

the following function is constructed:

\[
M(X) = f_1^2(x_1, x_2, \ldots, x_n, x_{n-1})^2 + \ldots + f_n^2(x_1, x_2, \ldots, x_n, x_{n-1})^2 = \sum_{i=1}^{n} f_i(x_1, x_2, \ldots, x_n, x_{n-1})^2
\] (5)

Thus the system of equations becomes:

\[
\begin{cases}
    x_{CM} - x_{CS} = 0 \\
    y_{CM} - y_{CS} = 0
\end{cases}
\] (6)

where:

\(x_{CM}, y_{CM}, x_{CS}, y_{CS}\) are the coordinates of the centre of the mass, respectively of the stiffness of the studied structure.

Observation:
During this preliminary phase, the additional accidental eccentricity, according to P100 / 1 – 2013 [5], was not taken into account:

\[ e_{a_i} = \pm 0.05 \cdot L_i \]  

(7)

where:

- \( e_{a_i} \) is the accidental eccentricity of the mass, at level \( i \), relative to the calculated position of the centre of the masses, applied to the same direction at all levels.
- \( L_i \) is the dimension of the floor, perpendicular to the direction of the seismic action.

\[
\begin{align*}
\sum_{i=1}^{n} A_i \cdot x_i - \sum_{i=1}^{n} L_i x_i &= 0 \\
\sum_{i=1}^{n} A_i \cdot y_i - \sum_{i=1}^{n} L_i y_i &= 0
\end{align*}
\]

(8)

\[
\begin{align*}
 f_1(L_1, L_2) &= 21.2491 - \left( \frac{0.6^3 L_1}{12} \cdot 15.35 + \frac{0.7^3 L_2}{12} \cdot 0.35 + (41.39219 \cdot 26.60) + 7.12530 \right) = 0 \\
 f_2(L_1, L_2) &= 15.9379 - \left( \frac{0.6 L_1}{12} \cdot 0.30 + \frac{0.7 L_2}{12} \cdot 5.30 + (80.54802 \cdot 20.4569) + 7.22015 \right) = 0
\end{align*}
\]

(9)

where:

- \( L_1, L_2 \) are the optimal length of the reinforced concrete (R.C.) walls.

\[
M(L_1, L_2) = \sum_{i=1}^{2} f_i^2(L_1, L_2) = f_1^2(L_1, L_2) + f_2^2(L_1, L_2)
\]

(10)

Using the MatLab [3] function fminsearch (‘func’, [1 1]), with the initial solution vector \( X_0 = [1 1] \) the following solutions are obtained:

\[
L_1 = 7.731 \text{ m} \approx 7.75 \text{ m}; \ L_2 = 5.649 \text{ m} \approx 5.65 \text{ m}
\]

(11)

To verify the results, another numerical method was used (genetic algorithm). The genetic algorithm (G.A.) is a metaheuristic method inspired by the process of natural selection that belongs to the larger class of evolutionary algorithms (E.A.). Genetic algorithms are commonly used to generate high-quality solutions to optimization and search problems by relying on bio-inspired operators such as mutation, crossover and selection.

The obtained results are:

\[
L_1 = 7.736 \text{ m} \approx 7.75 \text{ m}; \ L_2 = 5.647 \text{ m} \approx 5.65 \text{ m}
\]

(12)

After finding the optimal length of the reinforced concrete walls, a linear static analysis was performed again to see the differences between the two models (with and without reinforced concrete walls).
Figure 8. Building plan “after”.

Figure 9. Isometric view of the structure “after”.
Table 4. Modal participating mass ratios “after”.

| Case | Mode | Period [sec] | UX    | UY    | RZ    | Sum UX | Sum UY | Sum RZ |
|------|------|--------------|-------|-------|-------|--------|--------|--------|
| Modal 1 | 1    | 0.304        | 0.0001 | 0.7422 | 0.0003 | 0.7422 | 0.0003 |
| Modal 2 | 2    | 0.270        | 0.7419 | 0.00003687 | 0.0109 | 0.7423 | 0.0112 |
| Modal 3 | 3    | 0.228        | 0.0171 | 0.00003025 | 0.751 | 0.7591 | 0.7423 | 0.7622 |
| Modal 4 | 4    | 0.079        | 0.0042 | 0.1755 | 0.0039 | 0.7633 | 0.9177 | 0.7661 |
| Modal 5 | 5    | 0.075        | 0.1536 | 0.0076 | 0.0053 | 0.9168 | 0.9254 | 0.7714 |

Figure 10. Mode shape 1 – $T_1 = 0.304$ s.

Figure 11. Mode shape 2 – $T_2 = 0.270$ s.
It is clearly visible now that, after optimizing the R.C. walls, the structure has a smooth orthogonal motion in the first two mode shapes (figure 10, figure 11) and the coupled lateral - torsional motion has been reduced to the minimum (table 5). The torsional motion appears only in the third mode shape (figure 12).

5. Conclusions
From the above mentioned results it can be seen that now the structure has a proper mode shape in the first three modes, thus preventing the coupled lateral - torsional motion, increased deformations and stresses due to buildings’ twisting.

Asymmetric buildings are no longer a problem to create / design, due to this method, applied with numerical methods, as the distance between the centre of mass (C.M.) and the centre of stiffness (C.S.) can be reduced to 0.

For the structure, a much more favourable behaviour can be obtained, such as post-yield behaviour and energy dissipation when subjected to actual earthquake ground motions, but with the following important observation:

Such a building / structure must have only an elastic structural behaviour, because, in a postelastic response, if the structural walls are damaged, the whole building will get a strong torsion behaviour (and a possible structural collapse) is expected, due to the additional mass of these R.C. walls, which will not work as structural members. Hence, supplementary safety measures for the seismic structural behaviour must be applied if a post-elastic design is considered.

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