The existence of deconfined quark matter in the superdense interior of neutron stars is a key question that has drawn considerable attention over the past few decades. Quark matter can comprise an arbitrary fraction of the star, from 0 for a pure neutron star to 1 for a pure quark star, depending on the equation of state of matter at high density. From an astrophysical viewpoint, these two extreme cases are generally expected to manifest different observational signatures. An intermediate fraction implies a hybrid star, where the interior consists of mixed or homogeneous phases of quark and nuclear matter, depending on surface and Coulomb energy costs, as well as other finite size and screening effects. In this brief review article, we discuss what we can deduce about quark matter in neutron stars in light of recent exciting developments in neutron star observations. We state the theoretical ideas underlying the equation of state of dense quark matter, including color superconducting quark matter. We also highlight recent advances stemming from re-examination of an old paradigm for the surface structure of quark stars and discuss possible evolutionary scenarios from neutron stars to quark stars, with emphasis on astrophysical observations.

Keywords: Quark matter; Neutron stars; Strange quark stars.

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1. Introduction

In the aftermath of a core-collapse supernova of massive stars 8 solar masses (8 M⊙) and above, the iron core of the progenitor star implodes from a radius of ~ 1000 km to a compact object of radius ~ 15 km that is neutron-rich and bound by gravitational forces. A neutron star (NS) is thus born. In 1934, Baade and Zwicky hypothesized the supernova-neutron star connection, but did not specify any observational signals associated to the formation of a neutron star. Thirty years later,
Bell and Hewish made the serendipitous discovery of the first radio pulsar, interpreted shortly thereafter by Gold as a rotating neutron star. Since then, over 1500 similar “rotation-powered” neutron stars have been identified with spin periods ranging from few milliseconds to several seconds. In addition, less common categories such as the anomalous X-ray pulsars (AXPs), soft gamma-repeaters (SGRs) and X-ray dim isolated neutron stars (XDINS) now exist and their relation to the “garden-variety” pulsars is not well-understood. In some cases, by monitoring the pulsed and thermal radiation from these diverse sources, important physical quantities such as mass, radius, spin-period, magnetic fields, age and surface black-body temperature for the neutron star can be inferred.

The main aim of neutron star observations is to understand the underlying structure of the neutron star and its observed thermal and magnetospheric emissions. From a theoretical standpoint, the study of neutron star structure involves understanding the nature of matter over an enormous density range from $\rho \sim 10^2 - 10^{15}$ g/cm$^3$. The neutron star atmosphere and envelope, having density $\rho \leq 10^4$ g/cm$^3$ constitute a tiny fraction of the star’s volume, but can cause significant deviation from the emergent Planckian photon flux depending on the atmospheric composition and magnetic field strength. The outer crust of the neutron star, from $\rho \sim 10^4 - 10^{11}$ g/cm$^3$, is composed chiefly of increasingly neutron-rich nuclei starting with $^{56}$Fe, that are embedded in an ordered lattice structure (at zero temperature) that minimizes the Coulomb interaction energy. The presence of free electrons that form a degenerate Fermi sea ensures charge neutrality and stability against $\beta$-decay. In the inner crust, somewhat above $\rho \sim 10^{11}$ g/cm$^3$, neutrons drip out of nuclei, leading to a mixture of the ordered phase with a degenerate gas of neutrons. As densities approach the saturation density ($\rho_0 \sim 2.6 \times 10^{14}$ g/cm$^3$) of nuclear matter, the fraction of free neutrons increases while the nuclei may form extended non-spherical shapes leading to rod/slab-like “pasta” phases (2-D voids) and “Swiss cheese” phases (3-D voids). The outer core of the neutron star, from $\rho \sim 0.5 - 2\rho_0$, consists of an admixture of neutrons, protons and electrons in accordance with charge neutrality and $\beta$-equilibrium. Other negatively charged leptonic or hadronic species can appear when their ground state energy becomes lower than their respective chemical potential dictated by $\beta$-equilibrium. In the inner core ($\rho \geq 2\rho_0$), with increasing density, negatively charged bosons can form Bose-condensates and quarks may be deconfined, resulting in a free quark-gluon phase where the quark pairs may also condense. These ideas are summarized in several existing reviews. Currently, our constraints in performing ab initio calculations of strongly interacting dense matter severely limits accurate formulations of the equation of state (EoS) at higher densities, leading to theoretical uncertainties that preclude a definitive statement about the phase of matter in the interior of neutron stars. However, models of the EoS that address pure or mixed phases can be constructed and its predictions compared to astrophysical observations to judge the viability of the model. This brief review is aimed at assessing the evidence for the existence of quark matter.
inside neutron stars, given the current status of neutron star observations. We set the stage by recounting recent progress in theoretical studies of dense quark matter.

2. Dense Quark Matter

The critical question regarding quark matter in neutron stars is whether the density in the interior of neutron stars is large enough so that hadronic matter is deconfined and quarks become the relevant degrees of freedom. This question remains unanswered because the underlying theory of strong interactions, Quantum Chromodynamics (QCD), is still not sufficiently well understood at neutron star densities.

2.1. Color Superconductivity

Recently, a lot of progress has been made in understanding QCD at asymptotically high densities. In that regime, where pertubative studies are reliable, quark matter is believed to be in a color-flavor-locked (CFL) phase, characterized by quark pairing with a completely gapped spectrum for single particle excitations. Such a phase is an electromagnetic insulator in bulk and admits no electrons, even when stressed by small quark masses. If dense quark matter indeed exists inside neutron stars, where densities are well above nuclear matter density but well below the density where perturbative QCD is expected to be valid, the ground state of quark matter is uncertain. Nevertheless, in such a “hybrid star”, attractive interactions between quarks will lead to the formation of a color superconducting state, characterized by quark pairing and superfluidity. The singlet pairing gaps could be as large as 100 MeV and transport properties are strongly modified by the presence of collective excitations below the scale of the gap.

Many phases can intervene at these intermediate densities, such as the crystalline color superconducting phase, where quarks with different Fermi surfaces pair at non-zero momentum, resulting in an inhomogenous but spatially periodic order parameter. This phase spontaneously breaks translation and rotational symmetries, and the free energy of the system is minimized when the gap varies spatially in accordance with the residual discrete symmetries of this phase. Interestingly, the crystalline structure may also serve as sites for pinning rotational vortices formed in the superfluid as a result of stellar rotation, and could generate the observed glitch phenomena in neutron star spin-down.

2.2. Strange Quark Stars

The natural domain of physical applicability for color superconductivity is the dense interior of neutron stars, where quark matter may exist in a deconfined state. How would such a state arise? Over thirty years ago, it was conjectured that at sufficiently high density, macroscopic quark matter composed only of up and down \((u,d)\) quarks might be stabilized by the introduction of strange \((s)\) quarks, and
constitute the true ground state of matter, as it would be more bound than nuclear matter\textsuperscript{19,20,21}. This conjecture has not yet been decisively ruled out by experiment or observation. The introduction of strangeness reduces Pauli repulsion by increasing the flavor degeneracy and ensures a lower charge-to-baryon ratio for strange quark matter compared to nuclear matter. The latter fact can render even a large lump of strange matter stable against fission, although it may decay by other means. On the other hand, the stability of small lumps or “nuggets” of quark matter depends on energy costs associated with surface tension and curvature energy. In the absence of concrete results from lattice studies of QCD at finite density and zero temperature, we rely on simple model-dependent studies\textsuperscript{21} that admit a parameter window (the parameters being the strange quark mass, the strong coupling constant and a phenomenological Bag constant) within which bulk strange quark matter is stable, even at zero pressure. This implies that, if central densities inside neutron stars are large enough to create two-flavor ($u, d$) quark matter, or if a small “nugget” of cosmological/cosmic-ray origin is present, the entire neutron matter inside the star will convert to strange quark matter by absorbing neutrons and equilibrating strangeness. If temperatures are below the critical temperature for color superconductivity, the strange quark star will be a giant color superconductor.

2.3. Nature of the Crust

If strange quark matter is not absolutely stable at zero pressure, then the crust of neutron stars that contain quark matter in their core consists of hadronic matter. The gross spectral features of such hybrid stars tend to be very similar to neutron stars which do not contain quark matter. Hybrid stars can be somewhat cooler than normal neutron stars of the same age, since quark matter opens up the direct Urca process for rapid cooling, but this effect is suppressed by quark pairing\textsuperscript{22}. On the other hand, if strange quark matter is absolutely stable at zero pressure, then there are three possibilities for the nature of the crust\textsuperscript{a} of the quark star.

1) CFL stars: If the CFL phase is the ground state at zero pressure, then the CFL phase is likely the ground state at all densities. CFL matter extends to the star’s surface and there is no crust in the traditional sense. There is no known mechanism for pulsar glitches in a CFL star, so not all neutron stars can be CFL stars. Further, pure CFL stars are likely unstable to gravitational r-modes\textsuperscript{24}. If the CFL phase is not the ground state of matter at the surface of the strange quark star, then quark matter is positively charged, and requires electrons to make the system charge neutral. As explained below, this leads to two interesting possibilities for the nature of the crust.

2) The traditional paradigm: At the surface, positively charged quark matter is compensated by a thin layer of electrons, termed the “electrosphere”, which is integrated to the quark surface by Coulomb forces. Solving the Poisson equation in the

\textsuperscript{a}We are referring here to the “quark” crust as opposed to the nuclear crust discussed in other work\textsuperscript{23}.
plane-parallel approximation yields the profile for the electrostatic potential in the
electrosphere. In natural units $\hbar = c = 1$, the profile just outside the star’s surface
($z > 0$) is given (at low temperatures $T < 10^{10}$K) by \(^{(25)}\)

$$
\phi = \frac{\phi_0}{(1 + z/z_0)}, \quad z_0 = \frac{\pi \sqrt{6}}{e^2 \phi_0} = 501.3 \left( \frac{30 \text{ MeV}}{\phi_0} \right) \text{ fm},
$$

where $\phi_0$ is determined by the discontinuity in the electric field (net charge dens-
ity) at the surface. The large electric field that binds these electrons to quark matter
leads to the Schwinger instability of the vacuum \(^{(26)}\), resulting in electron-positron
pair-emission, which can be an additional source of photons in the electrosphere,
which normally radiates photons via $2 \rightarrow 3$ processes of Quantum Electrodynamics
(QED). Therefore, the light curves of quark stars, determined by this surface photon
emission, should be very different from neutron stars, which have vanishing electric
fields at surface.

Photon cooling calculations of bare quark stars including these electrospheric
effects as well as color superconductivity (which can alter the specific heat and
thermal conductivity of quark matter) have been performed \(^{(27)}\) and are being in-
vestigated further. The conclusion is that bare quark stars will display Super-
Eddington photon luminosities at surface temperatures $T > 10^9$K, with a hard
spectrum that distinguishes it from thermally radiating neutron stars. At lower
temperatures $6 \times 10^8$K$< T < 10^9$K, the bulk of the luminosity comes from electron-
positron pairs which subsequently imprint a wide annihilation line on the non-
thermal spectrum. Thermal emission is much suppressed owing to plasma frequency
effects in quark matter and in the electrosphere \(^{(28)}\). Further cooling to temperatures
$T \sim 10^8$K results in a non-thermal spectrum dominated by bremsstralhing pho-
tons from electron-electron collisions in the electrosphere \(^{(29)}\). At temperatures just
below $T < 10^8$K, $2 \rightarrow 3$ QED processes in the electrosphere dominate resulting in
a thermal spectrum, even though radiation from the underlying quark matter is
cutoff for frequencies below the plasma frequency of dense quark matter ($\omega_p \sim 20$
MeV). At very low temperatures $T \ll 10^8$K, the luminosity of the electrosphere is
exponentially suppressed.

3) A new picture: This picture of the quark star surface, involving homogenous
quark matter and electrons, has been recently challenge \(^{(30)}\). Matter may satisfy
charge neutrality globally rather than locally, provided surface and Coulomb costs
are not prohibitively large in a heterogenous mixed phase. In effect, relaxing the
condition of local charge neutrality provides freedom to reduce the strangeness
fraction in quark matter and thereby lower its free energy. This mixed phase would
then be qualitatively similar to the mixed phase of nuclei and electrons in the crust
of normal neutron stars and would share several features with the mixed phase of
quark drops and nuclear matter in hybrid stars \(^{(31)}\).

To understand this mixed phase, we note that since the electron chemical poten-
tial, $\mu_e$, is significantly smaller than the quark chemical potential $\mu$ for all known
models of quark matter, a general parameterization of the EoS can be obtained by expanding in powers of $\mu/\mu_0$:

$$p_{QM} = p_0(\mu, m_s) - n_Q(\mu, m_s)\mu + \frac{1}{2}\chi_Q(\mu, m_s)\mu^2_e + \ldots$$  \hspace{1cm} (2)

where $p_0$, $n_Q$, and $\chi_Q$ are well-defined and calculable functions of $\mu$ and the strange quark mass, $m_s$. This second-order expansion, which neglects the electron pressure $p_e \sim \mu^4_e$, can be used for any model EoS or for that predicted by QCD.

The structure of droplets in the crust of a strange quark star can be obtained from the Poisson equation. At zero temperature and pressure, the Gibbs free energy per quark for droplets can be compared with the Gibbs free energy per quark for homogeneous matter. If the surface tension, i.e. the energy cost of creating a droplet surface, is small enough, then the crustal phase is preferred over homogeneous quark matter. The critical surface tension is

$$\sigma_{crit} = \frac{0.8n_Q^2}{12\sqrt{\pi}\alpha^{3/2}}.$$  \hspace{1cm} (3)

In the context of the Bag model for dense quark matter, the condition for forming a mixed phase becomes

$$\sigma \lesssim 12 \left( \frac{m_s}{150 \text{ MeV}} \right)^3 \frac{m_s}{\mu} \text{ MeV/fm}^2.$$  \hspace{1cm} (4)

Using two estimates of the surface energy of strangelets: (i) $\sigma \simeq 8$ MeV/fm$^2$ for $m_s = 150$ MeV and $\mu \simeq 300$ MeV; and (ii) $\sigma \simeq 5$ MeV/fm$^2$ for $m_s = 200$ MeV at $\mu \simeq 300$ MeV, the condition in Eq. 4 implies that a homogeneous phase is marginally favored for $m_s = 150$ MeV while the structured mixed phase is favored for $m_s = 200$ MeV. The sensitivity to $m_s$ in Eq. 4 and uncertainty in other finite size effects can alter these quantitative estimates. If the structured phase is favored, it will be composed of quark nuggets immersed in a sea of electrons. The size of the quark nuggets in this phase is determined by minimizing the surface, Coulomb and other finite size contributions to the energy. At low temperature, this mixed phase will be a solid with electrons contributing to the pressure while quarks contribute to the energy density - much like the mixed phase with electrons and nuclei in crust of a conventional neutron star. This modified picture of the strange star surface has a much reduced density gradient and negligible electric field unlike the old paradigm. In the modern viewpoint, there is no need for the electrosphere, or large photon luminosities thereof, since matter at the surface is globally neutral. The observed photon spectrum from such a surface will be very different than from an electrosphere.

Neutron star observations can help in distinguishing between different equations of state (EoS), and also between differing models for the crust as discussed above. In the following section, we explain the importance of such observations and their potential for advancing our knowledge of dense matter.
3. Neutron Star Observations:

Neutron star observations can be broadly classified into two categories: (i) those that provide information about the structural aspects of the star such as its mass and radius; and (ii) those that provide information about transport and cooling processes. Observations of mass and radius provide constraints on the EoS which typically involves physics at the energy scale set by the baryon chemical potential ($\mu_B \simeq 1$ GeV). In contrast, transport phenomena probe the low energy response properties of the dense interior at an energy scale set by the temperature ($T \simeq$ keV-MeV). This complementarity proves crucial in inferring the phase structure of matter residing in neutron stars. We discuss these in some detail below, and also address the role of transient phenomena in determining the crustal parameters of neutron stars.

3.1. Neutron Star Structure: Masses and Radii

The observation of orbital parameters in close binary systems containing neutron stars is the classic and by far the most accurate method of determining the component masses. Keplerian parameters typically determine only the reduced mass of the binary system. To infer the individual masses, general relativistic effects which lead to post-Keplerian corrections to the orbital evolution need to be measured. There are five known types of binary systems containing neutron stars: (1) double neutron star systems containing a pulsar (PSR) and a neutron star; (2) neutron star-white dwarf systems containing a PSR and a white dwarf (WD); (3) High mass X-ray binaries (HMXBs) containing a neutron star and a massive companion star ($M > 10M_\odot$); (4) Low mass X-ray binaries (LMXBs) consisting of a neutron star and a companion star with mass $M < 1M_\odot$; and (5) neutron star-black hole (BH) binaries. In compact binary systems, such as double-neutron star and PSR-WD systems, orbital decay due to gravitational wave-emission, advance of periastron and Shapiro time-delay have been measured (for an excellent review of general relativistic orbital effects in compact binaries see [33]). Once the post-Keplerian parameters are measured, it overdetermines the set of quantities needed to infer the individual masses and thereby allows a high precision neutron star mass measurement virtually free of any systematic error. In these systems, the neutron star masses lie in the range 1.18-1.44 $M_\odot$ and have errors of less than a tenth of a percent. This tight clustering of masses at a relatively low value $\sim 1.4M_\odot$ warrants an explanation. One possible explanation proposed by Bethe and Brown[34] is that the maximum mass of a neutron star is $\sim 1.5M_\odot$; any heavier and they would become black-holes. A more mundane explanation is based on the evolution of HMXBs which are suspected to be the progenitors of double neutron star systems. The lifetime of HMXBs is short because the massive star evolves rapidly on a time scale of a few million years - too short for significant accretion from the massive star to increase the mass of the neutron star significantly from its mass at birth, which is expected to lie in the range 1.2-1.5 $M_\odot$. 
Clearly, the discovery of a massive neutron star in systems where adequate mass accretion is possible with a mass close to $2\, M_\odot$ would disprove the Bethe-Brown scenario and lend credence to the evolutionary argument. PSR+WD systems are prime candidates for finding heavy neutron stars. These systems are thought to have evolved from LMXBs which have long lifetimes and where significant accretion is expected to occur. Indeed a candidate heavy neutron star called PSR J0751+1807 has been found in the NS+WD system. Measurement of the orbital decay and Shapiro delay have yielded a neutron star mass $M = 2.1 \pm 0.2\, M_\odot$. This along with the possible confirmation (at 95% confidence) of at least one pulsar with $M > 1.68\, M_\odot$, following the recent detection of 21 millisecond pulsars in the globular cluster Terzan 5, 13 of which are in binaries, adds to the emerging trend toward relatively large mass neutron stars.

Radius measurements of neutron stars have become even more crucial in pinning down the equation of state at high density, now that mass ranges have widened. In principle, determining the radius of a neutron star may seem fairly straightforward. If the neutron star radiated like a black body, then X-ray observations would be able to determine both the flux $f$ and the spectral temperature $T$. Further, if the distance to the object $d$ were known, the radius of the star $R$ can inferred from the relation between the observed flux and the temperature $f = 4\pi R^2 \sigma_{SB} T^4 / d^2$ where $\sigma_{SB}$ is the Stefan-Boltzmann constant. In practice however there are several complications that make radius measurements a challenging task: (i) even for isotropic black bodies, the observed flux, temperature and apparent radius are all modified due to the effects of gravitational red-shift and it is only possible to infer the radius at infinity which is related to the true radius of the star through the relation $R_\infty = (1 + z) R$ with the red-shift factor $(1 + z)^{-1} = \sqrt{1 - 2GM/Rc^2}$. Consequently, instead of measuring a radius we can only infer a relation between mass and radius as shown in Fig.1 where curves corresponding to different values of $R_\infty$ are depicted as dashed curves; (ii) the assumption that neutron stars radiate isotropically with a black body spectrum is seldom true because most neutron stars are characterized by magnetic fields and atmospheres; (iii) it is not possible to measure the luminosity or flux of the object directly due to interstellar absorption and hence modeling the atmosphere and the role of magnetic fields become crucial; and (iv) although the distance measurements can in principle be obtained through parallax, accurate measurements have been difficult to obtain.

A promising candidate class for radius measurement is quiescent neutron stars in LMXBs situated in globular clusters. Their magnetic fields are small ($B < 10^{10}$ G), and their atmosphere is very likely to be composed of hydrogen accreted from the companion. Since distances to the globular clusters are typically well-known, uncertainties in determining the radius are minimal. This is exemplified by the recent determination of $R_\infty$ in the quiescent LMXB called X7 in the globular cluster 47 Tucanae. A model hydrogen atmosphere with consistent surface gravity was employed to obtain the 90% confidence contours in the mass-radius plane shown in Fig.1.
It has recently been realized that compact objects in LMXBs which exhibit X-ray bursting behavior may provide a promising new avenue to determine, simultaneously, both the mass and radius of a neutron star. In these objects, there is the potential to observe, in addition to the quiescent luminosity that can be used to infer $R_{\infty}$, the Eddington luminosity during the burst and the gravitational red-shift through direct observation of the shift in identifiable atomic absorption lines in the atmosphere. The peak luminosity of the burst can be identified with the Eddington luminosity if it remains constant over time and over several bursts (Eddington luminosity $L_{\text{Edd}} = (4\pi cGM/\kappa)(1+z)$ is essentially determined by the mass of the object and is the maximum radiation possible in equilibrium wherein the radially outward radiation force in the shell exactly counterbalances gravity). In her recent article, Ozel proposes that observations of bursting behavior in the low-mass X-ray binary EXO 0748-676 already provide detailed information about all three aforementioned quantities. Since these quantities have different dependences on mass and radius their simultaneous determination in turn allows for the independent determination of mass ($2.10 \pm 0.28 \, M_\odot$) and radius ($13.8 \pm 1.8 \, \text{km}$). This is also shown in Fig. 1.
where it has been the assumed that the gas accreted from the companion has solar composition.

In Fig. 1, the mass radius predictions for different model EoS are shown. We have broadly classified models into three classes/regions in the mass-radius plot: (1) Nucleonic stars; (2) Hybrid stars characterized by a soft EoS in the interior (due to a phase transition to an exotic state) surrounded by a nuclear shell; and (3) Strange quark stars made entirely of stable strange quark matter. The heavy neutron star candidates and the rather large inferred radii disfavor the scenario in which significant softening due to a phase transition at high density occurs. Strange stars and possibly even hybrid stars with a fairly stiff high density EoS remain viable.

3.2. Thermal Evolution

Observations of thermally emitting neutron stars provide another handle, besides mass and radius measurements, to probe the interior of neutron stars. The cooling history of thermally emitting neutron stars in the first million years of their life is governed by neutrino emission from the dense interior. The low lying excitation spectrum of quasiparticles and phase structure of matter plays a role in the neutrino emission rates, enabling observational constraints on the neutron star cooling rate to constrain the interior physics. As mentioned previously, x-ray observations of neutron star surface temperatures are complicated by atmosphere and magnetic field considerations. The other key ingredient needed is the age of the neutron star. There are typically three methods used to determine the age: (i) direct association with a historic supernova; (ii) association with a supernova remnant’s measured expansion rate and radius of the nebula or association with a measured neutron star velocity and distance; and (iii) spin-down age as inferred from the measurement of the period and period derivative assuming spin-down is due to magnetic dipole radiation. There are three young neutron stars which are associated with historic supernova: (1) PSR B0531+21 (the Crab Pulsar); (2) CXO J232327.8+584842 (Cas A); and (3) PSR J0205+6449 (or 3C58). In these young systems 300-1000 years old, there are significant sources of non-thermal emission and the thermal emission itself is not detectable. Consequently, only upper limits on the thermal radiation can be inferred. As we shall discuss later these limits already provide useful constraints. In about 10 other sources thermal radiation from the surface has been observed. In these cases the inferred luminosity or temperature depends on the assumptions made about the NS atmosphere but they nonetheless provide valuable data to constrain NS cooling at late times ($10^3 - 10^7$ yrs).

Through detailed modeling efforts by several groups it is possible to relate the interior cooling directly to the surface temperature (for a recent review see 39,40). These studies have shown that surface temperature depends sensitively on the composition of the envelope in the uppermost regions of the star just below the photon emitting region. This region can support the largest temperature gradients due to
their small thermal conductivity. Typically, light elements like H, He, C or O have a larger conductivity relative to the heavier Fe-like elements and thereby result in higher luminosity at early times. The change in luminosity due to the compositional changes can be as large as a factor of 10 both at early and late times. With this observational background and caveats in mind, we now turn to a discussion of how neutrino emission rates in different high density phases can impact neutron star cooling.

In a broad sense, neutrino cooling in dense matter can be either fast or slow. Fast cooling neutrino processes are those that can occur at the one (quasi-) particle level, while slow cooling is due to two particle processes such as Bremsstrahlung reactions. In nucleonic matter, the one-particle process would be $\beta$-decay of neutrons and its inverse reaction ($n \rightarrow p e^- \bar{\nu}_e$ and $e^- p \rightarrow n \nu_e$). This reaction, which is called the direct-URCA (DURCA) reaction, when kinematically allowed, leads to a rapid energy loss rate $\dot{\epsilon}_\nu \simeq 10^{26} T^6_9 \text{ergs/cm}^3/\text{s}$ for typical densities characteristic of neutron star interiors. However, several models of dense nuclear matter predict a relatively small proton fraction which in turn forbids the DURCA reaction because the discrepancy between the neutron and proton Fermi momenta is too large to satisfy momentum conservation. Under these conditions a spectator nucleon is required to satisfy momentum conservation. Such reactions which are similar to Bremsstrahlung reactions lead to a significantly slower rate of energy loss $\dot{\epsilon}_\nu \simeq 10^{21} T^8_9 \text{ergs/cm}^3/\text{s}$. Whether or not DURCA reactions can occur in neutron stars with nucleonic matter remains an open issue; in particular it appears likely that even if it is forbidden in light neutron stars with mass $\sim 1.4M_\odot$ it may occur in heavier stars. This large range of allowed emissivities even in the standard case is unfortunate from the point of view of constraining novel high density phases.

In nuclear as well as quark matter, superfluidity plays a crucial role as it suppresses conventional single-particle beta decay rates, at the same time opening up new pathways for neutrino emission through pair-breaking and recombination or via other correlations in novel ground states of dense matter. Thus, admixtures of completely different phases can explain the cooling history of neutron stars equally well. For example, hybrid stars with color superconducting quark cores and a mantle of normal nuclear display cooling curves that are consistent with present surface temperature observations of neutron stars. Nevertheless, at present, the bulk of the data is consistent with standard neutron star cooling models which have superfluid effects included but disallow DURCA. One exception is the observation of the coldest neutron star PSR J0205+6449 (3C58) which is only marginally consistent with cooling curves based on these models since it relies on rather finely tuned singlet proton pairing and weak triplet neutron pairing. Interesting new luminosity limits are coming from studies of supernova remnants (SNR) reported in Ref. 46. These limits come from the non-observation of thermal flux from the as yet unidentified neutron star and are compelling only in a statistical sense since we expect a fair number of the SNR to contain neutron stars. Nonetheless the cold...
neutron star in 3C58 and the growing number of SNR with low observed luminosity may well suggest that some neutron stars require some type of rapid cooling mechanism.

3.3. Transient Phenomena

Additional observed phenomena such as glitches, quasi-periodic oscillations in accreting neutron stars, thermal radiation from quiescent LMXBs and seismic vibrations during magnetar flares (SGRs) can potentially yield valuable information about the neutron star interior and constrain the EoS. Glitches refer to the sudden spin-up of pulsars that otherwise gradually spin down. Although detailed modeling of the glitch behavior is still quite uncertain, they are thought to arise from the catastrophic unpinning of superfluid vortices in the neutron star crust. The observed $\Delta \dot{\Omega}/\dot{\Omega} \simeq 10^{-3} - 10^{-2}$ and the slow slow post-glitch relaxation of the NS spin down rate favors the existence of a superfluid component or at the very least some component that decouples and subsequently couples to the neutron star spin. The inner crust of the neutron star is a particularly favored location for the glitch since here a neutron superfluid coexists with a lattice of nuclei. Although the mechanism for the coupling to the superfluid is not fully understood quantitatively, it offers a natural explanation for glitch dynamics. Strange stars with homogeneous quark matter cannot support a nuclear crust with a coexisting superfluid. It has been argued that the very phenomenon of glitches disfavors the strange star scenario. However, recent advances in understanding the phase structure of strange quark matter suggest that structured ground states where a lattice and a superfluid co-exist can occur. These developments indicate that further work is necessary before we can rule out strange stars on the basis of glitches.

Giant flares observed in soft gamma repeaters (SGRs), now believed to be highly magnetized neutron stars called magnetars, show quasi-periodic oscillations (QPOs) in the tail of the burst at frequencies of few tens to few hundred Hz. There is exciting preliminary evidence that these are seismic in origin and are due to shear modes excited in the neutron star crust. If confirmed, it provides a direct means to measure both the composition and the radial extent of the crust. The detection of a QPO at 626.5 Hz in the 2004 Hyperflare of SGR 1806-20 is of particular interest since it is likely to be an $n=1$ mode that is sensitive to the the radius of the crust. If this identification is secure it restricts the crust thickness to be $\sim 1$ km, which is much larger than the crust atop a strange star ($\sim 0.1$ km).

4. Evolution of neutron stars and quark stars

Finally, we review recent interesting suggestions on evolutionary scenarios that can explain apparently different categories of neutron stars. Anomalous X-ray pulsars or AXPs, so named because their strong X-ray emission is surprising given their low spin frequency, and Soft gamma-ray repeaters (SGRs), are believed to be neutron stars which emit irregular bursts of low-energy gamma rays. Could it be that these
two types of objects are actually evolving quark stars with extremely large magnetic fields? Recent work based on magneto-hydrodynamic studies of a quark star’s magnetic field, goes further in positing an attractive evolutionary picture that connects AXPs/SGRs with XDINS. AXPs/SGRs are conjectured to be quark stars whose interiors are superfluid except for rotational vortices that entrain the magnetic fields. The misalignment between the magnetic field in the vortex and the external dipole field is removed in a short time period following the formation of the superfluid state ($t \sim 0.1$ sec) with rapid external magnetic reconnections that produce the energetic X-ray bursts seen from AXPs/SGRs. As these stars spin down rapidly due to their large magnetic fields, magnetic field lines are expelled along with the quantized vortices, thereby increasing the spin-period and decreasing the spin-down rate. The evolutionary track of such stars can explain the suggestive period clustering of XDINS, as well as the lack of radio emissions, provided the star is sufficiently compact ($R < 10$ km). However, this model cannot as yet explain the broad absorption line seen in the spectrum of some XDINS, or the latter’s slow pulsations, and the model assumes the interior superfluid to be an electromagnetic insulator, such as the CFL phase, which may not be the ground state at moderately high density.

The diversity of sources identified as neutron stars indicates that interior compositions may differ from one category to another, although the equation of state must be unique. This leads naturally to the question of which neutron stars are likely to contain quark matter in their interior. This likelihood question was addressed recently by Staff et al. who studied the role of spin-down of isolated neutron stars in driving quark deconfinement in their high density core. Assuming spin-down to be solely due to magnetic braking, they obtained typical timescales to quark deconfinement for neutron stars that are born with Keplerian frequencies. The minimum and maximum neutron star masses that allow for deconfinement (via spin-down only) were identified, based on plausible EoS. Their results suggest that neutron stars lighter than $1.5M_\odot$ can not reach a deconfined phase. Further, depending on the EoS, neutron stars of more than $1.5M_\odot$ can enter a quark phase only if they are spinning faster than about 6 milliseconds as observed now, whereas larger spin periods imply that they are either already quark stars or will never become one. Thus, quark deconfinement is more likely in light, rapidly spinning neutron stars, especially if the deconfinement threshold density is low ($< 5\rho_0$). In this context, given that EXO 0748-676 has a deduced spin-period of 47Hz (uncommonly slow for an LMXB) and its high mass, it is clearly not a quark star and is not likely to suffer quark deconfinement in the future, unless the deconfinement threshold is much lower than $5\rho_0$. A low deconfinement threshold is still not ruled out since very stiff equations of state can reproduce the observed mass and radius of EXO 0748-676 with a central density of just $2\rho_0$. Therefore, hybrid stars remain very much a possibility while bare quark stars, which are based on the absolute stability of strange quark matter at zero pressure, face serious difficulties in explaining these observations of high mass neutron stars. It is also worth pointing out that contrary
to popular belief, low-mass X-ray binaries (LMXBs) are not expected to contain hybrid stars since the mass increase from accretion is more than compensated for by the concomitant spin-up and magnetic field quenching (which greatly increases the spin-down time to deconfinement densities), so that central densities are actually lowered, not raised.

5. Summary

It is apparent that recent neutron star observations disfavor the appearance of soft exotic (non-hadronic) matter at high density. They do not as yet completely rule out all quark matter equations of state. Hybrid stars may exist only as a small population among neutron stars and should not be dismissed until long-term evolution of neutron stars and transient phenomena occurring at neutron star surfaces are understood within a consistent neutron star picture. In particular, it is premature to rule out quark matter in neutron stars on the basis of mass-radius measurements alone.

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