Optimisation of screw spindle vacuum pumps with variable rotor pitch regarding load-lock operation

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Abstract. Screw spindle vacuum pumps are characterised by high suction speed and the ability to achieve high pressure ratios in a dry running process. If the dimensions of the pump are comparable to the volume of the system’s receiver, evacuation only takes a few seconds. For this reason, a typical application for screw spindle vacuum pumps is the evacuation of a load-lock chamber in a predefined time during a clocked production process. In order to reduce power consumption, screw spindle vacuum pumps with an internal change of volume are used almost exclusively. These pumps are commonly with variable pitch rotors. In this article, a new approach for optimisation of variable pitch rotors in consideration of the requirements of the load-lock process is presented. A dimensionless energetic efficiency is defined based on an idealised load-lock process, this parameter is maximised by the optimisation algorithm. The rotor pitch progression, an important factor in pump performance, is approximated by segments of constant pitch at the suction and discharge sides with a linear decrease in between. The overall wrap angle, the minimum rotor pitch, the crown circle diameter, the root circle diameter, and the circumferential speed are constant for the optimisation, during which the rotor length and load-lock pressure are varied. Optimisations are carried out for double-lobed cycloid profiles. The optimisations result in selection of internal volume ratios which are significantly below a theoretical calculated value for the idealised lock-lock process. This is explained by the internal clearances and details of compression process. The studies reveal a positive influence of the rotor length on the load-lock efficiency, this is because the proper rotor length leads to an uniform compression. It is also shown that the wrap angle of the segment on the suction side should be kept low.

1. Introduction
Evacuation of a load-lock chamber during a clocked production process is a common application for vacuum pumps. In such a process a given volume is evacuated until the target pressure in the receiver is reached. The operating point of the vacuum pump is changing continuously from atmospheric pressure to the target pressure. Therefore, the whole pressure range needs to be taken into account in the design process. In recent years dry-running screw spindle vacuum pumps have replaced multi-stage-roots pumps and oil-lubricated rotary vane pumps because they are tolerant to dirt and small amounts of liquid [1]. In order to reduce compression power, screw spindle vacuum pumps with an internal change of volume are used almost exclusively. In this context, an ideal volume ratio can be identified for every operating point. The ideal volume ratio depends on the pressure ratio, internal leakage and heat exchange within the positive displacement machine. The most common solution is an change of volume during the transport phase, which can be achieved in various ways. This article deals with the solution based on
a variable rotor pitch, an option now available due to advances in manufacturing [2]. In [3] variable rotor pitch and other options affecting internal compression are discussed. It is shown that a continuous, uniform progression of chamber volume is advantageous since this leads to an uniform heat distribution and a low maximum component temperature. Although there are disadvantages regarding to the volumetric efficiency and the ultimate pressure. Pfaller [4] presented an evolutionary multi-criteria optimisation approach to optimise variable pitch rotors for mass flow and power consumption, when only a single operating point is taken into account. The influence of manufacturing methods on the mismatch of the profile shape of vacuum pumps with variable rotor pitch and therefore on the volumetric efficiency of these pumps is discussed in [5]. Utri [6] investigated the potential of a variable pitch for dry running screw compressors and identified an opportunity to increase the efficiency for fluids with low isentropic exponents and in applications with high pressure ratios. In [7] Utri uses a Nelder-Mead-simplex optimisation to identify the best rotor pitch and internal volume ratio for dual pitch rotors. The aim of work the current work is to present a suitable approach to optimise the rotor pitch of a screw spindle vacuum pump for a load-lock process.

2. Working principle and characteristics of screw spindle vacuum pumps

Screw spindle vacuum pumps are two shaft rotary positive displacement machines with a large wrap angle, which transport the trapped gas axially from the inlet to the outlet by a rotation of the rotors (Figure 1). A large wrap angle is necessary to reduce the internal clearance back flow and therefore to achieve the high pressure ratio, common for such vacuum pumps [2].

![Figure 1. Operating principle of a screw vacuum pump](image)

Screw spindle vacuum pumps are commonly used as forepumps and are therefore compressing against atmospheric pressure (\( p_{out} = p_{at} \)). The inlet pressure ranges from \( 10^{-3} \text{ hPa} \) to \( 10^3 \text{ hPa} \) for the most of the time. The suction speed \( S \) is the pump’s delivery volume flow rate, referred to the thermodynamic state at the inlet. Another important characteristic is the inner power consumption \( P_i \), which is strongly influenced by the volume ratio \( v_i \), defined as the ratio of the volume of the first closed working chamber at the inlet to the volume of the last closed working chamber at the outlet.

\[
v_i = \frac{V_{in}}{V_{out}}
\]

The theoretical process of a single operating point is defined by assuming no internal leakages and the following change of state:

(i) isobaric suction at inlet pressure
(ii) isothermal compression
(iii) isochoric adjustment to the outlet pressure
(iv) isobaric discharge at outlet pressure

The density work is defined as the ratio of inner power consumption $P_i$ and the suction speed $S$:

$$w_i = \frac{P_i}{S} \quad (2)$$

It is calculated for the theoretical process by

$$w^t_i = p_{in} \cdot \left( \frac{\Pi}{v_i} + (\ln(v_i) - 1) \right) \quad (3)$$

which is minimised by choosing the volume ratio equal to the pressure ratio ($v_i = \Pi$), which corresponds to an adjusted operating point. The minimal value is given by:

$$w^t_{i, ideal} = p_{in} \cdot \ln(\Pi) \quad (4)$$

A theoretical work similar to Eq. 3 is presented by Dreifert [1], who assumes an isentropic internal compression.

In order to evaluate the efficiency of an operating point of a screw spindle vacuum pump, two dimensionless quantities are commonly used. The volumetric efficiency $\lambda_v$, given by the ratio of the suction speed $S$ to the suction speed $S_{th}$ of a theoretical ideal machine with no leakage:

$$\lambda_v = \frac{S}{S_{th}} \quad (5)$$

The isothermal efficiency $\eta_T$ is defined by the ideal theoretical density work (Eq. 4) divided by the real density work $w^r_{i,LL}$ at the operating point:

$$\eta_T = \frac{w^t_{i, ideal}}{w^r_{i,LL}} \quad (6)$$

3. Load-lock operation and quantities

In a load-lock process, like it is shown in Fig. 2, a receiver of volume $V$ is evacuated from atmospheric pressure $p_{at}$ to a desired load-lock pressure $p_{LL}$. The time required - the load-lock time $t_{LL}$ - is normally required to be short. The inner power consumption $P_i$ of the vacuum pump depends on the current inlet pressure and defines the work $W_{i,LL}$ which is needed for the evacuation. An efficient load-lock process is characterised by a low inner work $W_{i,LL}$. This process parameters are evaluated as follows where the temperature inside the receiver is assumed to be constant $T = 273 \, K$.

![Figure 2. Sketch of a load-lock process](image-url)
The load-lock time is calculated for a given suction speed characteristic $S(p)$ as shown below, assuming an ideal gas:

$$t_{LL} = \int_{0}^{t_{LL}} dt = -V \int_{p_{at}}^{p_{LL}} \frac{1}{S(p) \cdot p} dp$$

(7)

Using this equation the load-lock work $W_{i,LL}$ can be calculated using the known characteristics of inner power consumption $P_{i}(p)$ and suction speed $S(p)$:

$$W_{i,LL} = \int_{0}^{t_{LL}} P_{i}(t) dt = -V \int_{p_{at}}^{p_{LL}} \frac{P_{i}(p)}{S(p) \cdot p} dp = -V \int_{p_{at}}^{p_{LL}} \frac{w_{i}(p)}{p} dp$$

(8)

The density load-lock work is defined as the ratio of the load-lock work $W_{i,LL}$ to the volume $V$ of the receiver:

$$w_{i,LL} = \frac{W_{i,LL}}{V}$$

(9)

If the theoretical process (Eq. 3) is assumed at every operating point, Eq. 8 can be rearranged to

$$w_{i,LL,\text{th}} = \frac{W_{i,LL,\text{th}}}{V} = p_{at} \left( \frac{\ln (\Pi_{LL})}{v_{i}} + (\ln(v_{i}) - 1) \left( 1 - \Pi_{LL}^{-1} \right) \right)$$

(10)

which can be minimised with the following volume ratio

$$v_{i,LL,\text{ideal}}^{th} = \ln \left( \Pi_{LL} \right) \cdot \left( 1 - \Pi_{LL}^{-1} \right)^{-1},$$

(11)

where $\Pi_{LL}$ is the ratio of the atmospheric pressure $p_{at}$ to the load-lock target pressure $p_{LL}$. The minimised density load-lock work $w_{i,LL,\text{ideal}}^{th}$ for a theoretical load-lock process is given by the combination of Eq. 11 and Eq. 10.

$$w_{i,LL,\text{ideal}}^{th} = p_{at} \cdot \left( 1 - \Pi_{LL}^{-1} \right) \cdot \ln \left( \frac{\ln (\Pi_{LL})}{1 - \Pi_{LL}} \right)$$

(12)

Results from application of equation 10 are shown for a range of load-lock pressures in Figure 3 where the minima are marked. Starting at an volume ratio of $v_{i} = 1$, the density load-lock work at first strongly decreases to a minimum with increasing volume ratio, then more gradually increases. Especially in the case of low load-lock pressures, the volume ratio can be chosen less than the optimal parameter without having a great impact on the load-lock work. A reduced volume ratio reduces the maximum power consumption in case of over-compression and improves the volumetric efficiency and ultimate pressure of the pump.

The theoretical optimal evacuation work for a given load-lock pressure is known, and this can be used to define a dimensionless load-lock efficiency $\eta_{T,LL}$. This efficiency is defined as the ratio of the ideal work to the real density inner load-lock work. In the following, this quantity is optimised.

$$\eta_{T,LL} = \frac{w_{i,LL,\text{ideal}}^{th}}{w_{i,LL}^{\text{real}}}$$

(13)
4. Geometric characteristics of cycloid profile rotors

For the zero-dimensional simulation of rotary displacement pumps, it is necessary to define the working chamber and the adjacent clearances. Nadler [8] describes an approach based on the analysis of different transverse planes of a screw machine. The analysis requires the clearance points and chamber areas, which need to be defined numerically by searching the points of minimal distance between the rotor flanks for various rotor orientations. The results are combined with the rotor pitch to create a phase-depending progression of chamber volume and clearance dimensions.

The front section of a typical pump rotor pair is shown in Figure 4. The cycloid geometry used on these rotors is well known in conjunction with tooth flanks of transmissions [9]. The segments of a cycloid flank are the curves traced by a point on the rim of a roll circle rolls on the inside (hypocycloid) and the outside (epicycloid) of a pitch circle. In the case of rotary positive displacement pumps, the tooth flank needs to be modified at the root to ensure a closed interlobe line. Therefore, an envelope of the tooth tip is created [10]. Since the interlobe line of cycloid profiles is well known from transmission technology [9], the analysis of the front section can be simplified using the analytical solution. For every front section position up to three interlobe clearance (ILC) points can be identified. The radial point (1) is the contact between the crown and the root circle. The flank-to-tip point (2) occurs when the tooth tip is contacting the envelope of the opposite rotor. The flank-to-flank point (3) results from the contact of the profile flanks. The combination of all clearance points for several front section positions results in the complete interlobe line, which is highlighted in bold in Figure 4. The housing clearance (4) is defined in the middle of the tooth tip.

Figure 3. Density load-lock work $w_{th,i,LL}$ for theoretical load-lock processes as function of the internal volume ratio $v_i$
5. Simulation approach
In consideration of the optimisation task a computationally efficient thermodynamic model is needed. The approach used here is taken from Rohe [11], who approximates the spindle pump as series of pumping stages (Figure 5). Rohe used this model to investigate the effects of internal leakages, external leakages and internal volume ratio on the suction speed characteristic of a screw spindle vacuum pump [11]. Later, Pfaller used the same approach to optimise the rotor pitch of screw spindle vacuum pumps for a single operating point [4]. In this model, every pumping stage $i$ transports the trapped mass $m_i$ of the chamber $i$ to the next chamber $i+1$, while the internal leakages are lead to back flow $\dot{m}_{cl}$ from higher pressure chambers to lower pressure ones. Based on the conservation of mass and energy and the ideal gas law, the distribution of pressure $p_i$ and the mass flow rate $\dot{m}_d$ can be calculated. The volumes $V_i$ of the working chambers are determined for a constant rotor position (see [4] and [11]).

In order to calculate the internal leakages of the vacuum pump for the full range of pressure and referred to effects of gas rarefaction, the results of the experimental investigations of Wenderott [12] are used. Additionally, the influence of moving boundaries is taken into account by using the theoretical investigations of Stratmann [13]. Regarding the results of [11] and [13], the heat transfer between the transported gas and the pump components is modelled as complete.

Figure 4. Clearance points in the front section of a screw spindle vacuum pump

Figure 5. Sketch of the thermodynamic model (according to [11] and [4])
heat exchange. For this, an isentropic compression is calculated followed by a cooling of the gas to the inlet temperature $T_{\text{Inlet}}$ in every pumping stage. The inner power consumption of the screw spindle vacuum pump is therefore equal to the sum of all partial heat flows $\dot{Q}_i$ of every pumping stage [4]. In order to solve the integral for load-lock work (eq. 8) the characteristics of mass flow and inner power consumption are calculated for several discrete inlet pressures. In the context of the optimisation one pressure point per decade is calculated. The values between these points are interpolated linearly.

6. Optimisation approach

A generalised rotor pitch progression, shown in Figure 6, is used in the optimisation process. The rotor pitch $s$ is plotted as a function of the wrap angle, where the rotor is divided into three sections, inlet, compression and outlet. At the inlet, the rotor pitch $s_{\text{in}}$ is constant for the wrap angle $\varphi_{\text{in}}$. In the next section ($\varphi_{\text{compress}}$), the rotor pitch decreases linearly from the inlet $s_{\text{in}}$ to the outlet parameter $s_{\text{out}}$ and finally remains constant in the outlet section $\varphi_{\text{out}}$.

![Simplified rotor pitch progression for the optimisation task](image)

The rotor pitch progression needs to be optimised for the best dimensionless evacuation work (Eq. 13) while satisfying four constraints. The first constraint is that the specified total rotor wrap angle needs to be realised:

$$\varphi_{\text{in}} + \varphi_{\text{compress}} + \varphi_{\text{out}} = \varphi_{\text{total}}$$

(14)

The second constraint is the rotor length, which is equal to the integral of the rotor pitch progression:

$$\int_{0}^{\varphi_{\text{total}}} s(\varphi)d\varphi = L$$

(15)

The third constraint takes possibilities of manufacturing into account by requiring a minimum gap $h_{\text{chamber,min}}$ between the rotor flanks of one rotor. This is ensured by defining a minimum rotor pitch:

$$s(\varphi) \geq s_{\text{min}}$$

(16)

The last constraint defines a minimum wrap angle for the inlet and outlet side which should be greater than the wrap angle of load change. In the case of double-lobed rotors, this is given by:

$$\varphi_{\text{in}}, \varphi_{\text{out}} \geq 450^\circ$$

(17)

As a result of the last constraint the internal volume ratio can be calculated as the ratio of the inlet and the outlet pitch:

$$v_i = \frac{s_{\text{in}}}{s_{\text{out}}}$$

(18)
In conclusion, the optimisation problem can be characterised as single objective and constrained. A local minimum of an unconstrained problem with one objective can be determined by several algorithms. In this case the down-hill simplex algorithm [14] is used. This approach was used in the context of rotor pitch optimisation in [7]. A solution to handle the bound constraints is given in [15] and [16]. In order to handle the non-linear constraints an augmented lagrangian algorithm is used, as described in [17] and [18]. At least one global minima should be found, this can be achieved using the monotonic basin hopping algorithm [19]. All algorithms are provided by the pagmo [20] and NLopt [21] optimisation libraries. In the context of the optimisation some parameters are not varied. These are listed in Table 1. The geometry of the front section is kept constant. Additionally the ratios of crown circle diameter to clearance heights are assumed to be constant. For all rotor pitch progressions a constant total wrap is chosen. All simulations are done with a constant circumferential speed and with dry air. The optimisations are done for several ratios of rotor length and crown circle diameter and for several load-lock pressures (Table 2).

| Table 1. Constant optimisation boundary conditions |
| name | symbol | value |
|---|---|---|
| Gas | | dry Air |
| Circumferential speed | $u$ | 60 m/s |
| Wrap angle | $\varphi_{\text{total}}$ | 2160° |
| Crown circle diameter | $D_{CC}$ | 150 mm |
| Number of lobes | $z$ | 2 |
| Root circle ratio | $D_{CC}/D_{RC}$ | 1.7 |
| Ratio of pitch and roll circle | $D_{PC}/D_{Roll}$ | 2 |
| Housing clearance height ratio | $D_{CC}/h_{HC}$ | 600 |
| Flank-to-flank clearance height ratio | $D_{CC}/h_{\text{ILC,FP}}$ | 700 |
| Flank-to-tip clearance height ratio | $D_{CC}/h_{\text{ILC,FT}}$ | 700 |
| Radial clearance height ratio | $D_{CC}/h_{\text{ILC,R}}$ | 800 |
| Min. gap between rotor flanks | $D_{CC}/h_{\text{chamber,min}}$ | 17.0 |

| Table 2. Varied optimisation boundary conditions |
| name | symbol | value |
|---|---|---|
| Dimensionless rotor lengths | $L/D_{CC}$ | $2 \ldots 3.5$ |
| Load-lock pressure | $p_{LL}$ | $10^{-3} \ldots 10^{2}$ hPa |

7. Results

Figure 7 shows the optimised internal volume ratio for different load-lock pressures $p_{LL}$ and various dimensionless rotor lengths $L/D_{CC}$. Results are compared with the solution for an ideal process given by Eq. 11. It can be seen that the optimised volume ratios for the real process are significant below the parameters for the ideal process. Additionally, it can be noticed that
the optimum volume ratios are increasing with decreasing load-lock pressure until they reach a maximum value. As discussed on the example of the ideal process (Figure 3) the internal volume ratio can be reduced relative to the theoretical best point without a considerable impact on the efficiency. However, an increasing volume ratio increases internal leakage and the ultimate pressure, which has been explained in detail by [11]. So the ideal volume ratio is always a compromise between volumetric efficiency and an ideal compression. Even if higher volume ratios could be realised for larger dimensionless rotor lengths, this turns out to be disadvantageous. For an increasing dimensionless rotor length the volume ratio converges to a maximum which is still clearly below the theoretical ideal value. Additionally, it should be noticed that the curve for \( L/D_{CC} = 2 \) is limited by its maximal possible volume ratio \( v_i = 3 \). With higher volume ratios, the desired load-lock pressure cannot be achieved, thus the volume ratio decreases below the maximum. Since machines with larger rotor lengths have better abilities for sealing, these have higher volume ratios for low load-lock pressures. A load-lock pressure of \( p_{LL} = 10^{-3} \) could not be achieved for \( L/D_{CC} = 2 \).

The optimised isothermal load-lock efficiency is shown in Figure 8. For all dimensionless rotor lengths the optimised efficiency is decreasing with a decreasing load-lock pressure. This is explained by the higher pressure ratios inside the pump and the resulting internal leakages. The isothermal load-lock efficiency is increasing with larger dimensionless rotor lengths. This is explained by the uniform compression of larger rotors which is discussed below.

The positive effect of a larger dimensionless rotor length is explained considering the characteristics of isothermal (Eq. 6) and volumetric (Eq. 5) efficiency in Figure 9 and 10, which is for several \( L/D_{CC} \)-ratios optimised for a load-lock pressure of \( p_{LL} = 1 hPa \). The characteristics of volumetric efficiency have a progression typical for screw spindle vacuum pumps with inner volume ratio. In the range of 1 \( hPa \) to 20 \( hPa \) the maxima occur. For higher inlet pressures the characteristic is decreasing. For lower inlet pressures the volumetric efficiency is decreasing to zero delivery. If the characteristics for different dimensionless rotor lengths are compared, an advantage of higher rotor length can be observed again. A change of the dimensionless rotor lengths by 0.5 effects the ultimate pressure by about half a decade. Additionally a high \( L/D_{CC} \)-ratio is advantageous for high inlet pressures > 10^1 \( hPa \). The biggest improvement occurs by an increase of the \( L/D_{CC} \)-ratio from 2 to 2.5.
For each dimensionless rotor length $L/D_{CC}$ the best point of isothermal efficiency (Fig. 10) occurs at an inlet pressure of about $3 \cdot 10^2 \text{hPa}$. Because of the effect of over-compression, the efficiency decreases rapidly to zero for the highest inlet pressures. Starting from the best point, the efficiency decreases with decreasing inlet pressure and is almost zero at $10^{-1} \text{hPa}$. An increase of the dimensionless rotor length $L/D_{CC}$ leads to an improvement of isothermal efficiency for almost all operating points. Only in the range of over-compression is a lower volume ratio advantageous, making use of short rotor lengths beneficial.

Finally the optimised rotor pitch progression is considered. Some example progressions for several dimensionless rotor lengths are shown in Figure 11 where all progressions are optimised for $p_{LL} = 1 \text{hPa}$. All progressions start at high rotor pitch and decrease to a lower outlet pitch, which means an internal volume ratio. Notable is the wrap angle $\varphi_{in}$ of the inlet segment which is equal to the minimum constrained angle $\varphi_{in,\text{min}} = 450^\circ$ for all dimensionless rotor lengths. The wrap angle of the compression $\varphi_{\text{compress}}$ increases with the dimensionless rotor length. In the case of the lowest dimensionless rotor length $L/D = 2$ the upper limit of the volume ratio is reached and therefore no free rotor length is used for the compression phase. Based on the rotor pitch progressions the differences in the volumetric efficiency characteristics (Fig. 9) can be explained. A continuous change of the chamber volume leads to a low pressure gradient at the suction side which results in low leakage mass flow rates during the charging, leading therefore to a good volumetric efficiency. Additionally, a continuous compression more closely follows an isothermal path, this benefits the isothermal efficiency of longer rotors (Fig. 10). An isochoric transport phase, which means a high value of $\varphi_{in}$, does not occur since an isochoric pressure increase would increase the enclosed area of the $pv$-diagram, even though a high value of $\varphi_{in}$ would improve the volumetric efficiency.

Figure 12 shows the influence of the load-lock pressure on the optimised rotor pitch progression for the dimensionless rotor length $L/D_{CC} = 3.5$. Again, it can be noticed that the wrap angle of the inlet segment is equal to the minimum allowed value $\varphi_{in,\text{min}} = 450^\circ$ for all dimensionless rotor lengths. Comparable with the results for the variation of the rotor length, a large wrap angle for compression $\varphi_{\text{compress}}$ is preferred. For example, in the case of $p_{LL} = 10^2 \text{hPa}$ the optimisation results in the maximum possible wrap angle $\varphi_{\text{compress,\text{max}}} = 1260^\circ$. 

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**Figure 9.** Characteristics of volumetric efficiency $\lambda_v$ for various dimensionless rotor lengths $L/D_{CC}$ (optimised for $p_{LL} = 1 \text{hPa}$)

**Figure 10.** Characteristics of isothermal efficiency for various dimensionless rotor lengths $L/D_{CC}$ (optimised for $p_{LL} = 1 \text{hPa}$)
Figure 11. Influence of the dimensionless rotor length \( L/D_{CC} \) on the rotor pitch progression (\( p_{LL} = 1 \text{ hPa} \))

Figure 12. Influence of the load-lock pressure \( p_{LL} \) on the rotor pitch progression (\( L/D_{CC} = 3.5 \))

8. Conclusion

In the current article, a method to optimise the rotor pitch progression of screw spindle vacuum pumps for load-lock operations is presented. In this study an idealised load-lock process is used to define a dimensionless efficiencies. It is shown that the influence of the internal volume ratio on the load-lock efficiency decreases for high volume ratios. The optimisations of actual load-lock processes result in volume ratios below the theoretical values for the idealised process. This is explained by the negative influence of the volume ratio on the volumetric efficiency and the ultimate pressure. Additionally, a positive influence of the rotor length on the load-lock efficiency is identified. Based on the optimised rotor pitch progressions, it is shown that a uniform compression benefits the characteristics of isothermal and volumetric efficiencies over the entire range of suction pressure. In order to optimise the isothermal load-lock efficiency, the decrease of rotor pitch should start directly after the end of charging. All optimisations are performed for double-lobed rotors. Since single lobed rotors reveal a good potential for volumetric efficiency, these should be taken into account for future work.

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