Variable-Speed-of-Light Cosmologies

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Variable-Speed-of-Light (VSL) cosmologies are currently attracting much interest as a possible alternative to cosmological inflation. We discuss the fundamental geometrodynamical aspects of VSL cosmologies, and provide several alternative implementations. These implementations provide a large class of VSL cosmologies that pass the zeroth-order consistency tests of being compatible with both classical Einstein gravity and low-energy particle physics. While they solve the “kinematic” puzzles as well as inflation does, VSL cosmologies typically do not solve the flatness problem since in their purest form no violation of the strong energy condition occurs. Nevertheless, these models are easy to unify with inflation.

1. INTRODUCTION

Variable-Speed-of-Light (VSL) cosmologies have recently generated considerable interest as alternatives to the inflationary framework. They serve both to sharpen our ideas concerning falsifiability of the standard inflationary paradigm, and also to provide a contrasting scenario that is itself amenable to observational test. In this presentation we wish to assess the internal consistency of the VSL framework, and ask to what extent it is compatible with geometrodynamics (Einstein gravity). This will lead us to propose a particular class of VSL models that implement this idea in such a way as to inflict minimal “violence” on GR, and which at the same time are “natural” in the context of one-loop QED. For a detailed discussion of all these issues we refer to the paper \textsuperscript{[1]} which has inspired the present talk.

The question of the intrinsic compatibility of VSL models with GR is not a trivial one: Ordinary Einstein gravity has the constancy of the speed of light built into it at a deep and fundamental level; \( c \) is the “conversion constant” that relates time to space. Even at the level of coordinates we need to use \( c \) to relate the zeroth coordinate to Newtonian time: \( dx^0 = c \, dt \). Thus, simply replacing the constant \( c \) by a position-dependent variable \( c(t, \vec{x}) \), and writing \( dx^0 = c(t, \vec{x}) \, dt \) is a highly suspect proposition.

If this substitution is performed at the level of the metric, it is difficult to distinguish VSL from a mere coordinate change (under such circumstances VSL has no physical impact). Apparently more attractive, (because it at least has observable consequences), is the possibility of replacing \( c \to c(t) \) directly in the Einstein tensor itself. \[ \text{This is the route chosen by Barrow–Magueijo \textsuperscript{[2–4]}, by Albrecht–Magueijo \textsuperscript{[5]}, and by Avelino–Martins \textsuperscript{[6]}.} \]

If one does so, the modified “Einstein tensor” is \emph{not} covariantly conserved (it does \emph{not} satisfy the contracted Bianchi identities), and this modified “Einstein tensor” is not obtainable from the curvature tensor of \emph{any} spacetime metric. Indeed, if we define a timelike vector \( V^\mu = (\partial/\partial t)^\mu = (1, 0, 0, 0) \) then a brief computation yields

\[
\nabla_\mu \mathcal{G}^{\mu\nu}_{\text{modified}} \propto \dot{c}(t) \, V^\nu.
\]

Thus violations of the Bianchi identities for this modified “Einstein tensor” are part and parcel of
this particular way of trying to make the speed of light variable. If one now couples this modified "Einstein tensor" to the stress-energy via the Einstein equations, then the stress-energy tensor (divided by \( c^4 \)) cannot be covariantly conserved either, and so it cannot be variationally obtained from any action. To our minds, if one really wants to say that it is the speed of light that is varying, then one should seek a theory that contains two natural speed parameters, call them \( c_{\text{photon}} \) and \( c_{\text{gravity}} \), and then ask that the ratio of these two speeds is a time-dependent (and possibly position-dependent) quantity. To implement this idea, it is simplest to take \( c_{\text{gravity}} \) to be fixed and position-independent. So doing, \( c_{\text{gravity}} \) can be safely used in the usual way to set up all the mathematical structures of differential geometry needed in implementing Einstein gravity.

2. TWO–METRIC VSL COSMOLOGIES

Based on the preceding discussion, we feel that the first step towards making a VSL cosmology “geometrically sensible” is to write a two-metric theory in a form where the photons couple to a second electromagnetic metric, distinct from the spacetime metric that describes the gravitational field. [Somewhat related two-metric implementations of VSL cosmology are discussed by Clayton and Moffat \[12\] and by Drummond \[13\]. See \[14\] for details on how those implementations differ from our own.] This permits a precise physical meaning for VSL: If the two null-cones (defined by \( g \) and \( g_{\text{em}} \), respectively) do not coincide at some time, one has a VSL cosmology.

We want to stress that the basic idea of a quantum-physics-induced effective metric, differing from the spacetime metric (gravity metric), and affecting only photons is actually far from being a radical point of view. This concept has gained in the last decade a central role in the discussion of the propagation of photons in non-linear electrodynamics. In particular we stress that “anomalous” (larger than \( c_{\text{gravity}} \)) photon speeds have been calculated in relation with the propagation of light in the Casimir vacuum \[10\], as well as in gravitational fields \[11\].

Within our own framework, alternative approaches can be (I) to couple just the photons to \( g_{\text{em}} \) meanwhile all the other matter and gravity couple to \( g \), or (II) to couple all the gauge bosons to \( g_{\text{em}} \), but couple everything else to \( g \), or (III) to couple all the matter fields to \( g_{\text{em}} \), keeping gravity as the only field coupled to \( g \). A particularly simple EM metric is

\[
\left[ g_{\text{em}} \right]_{\alpha\beta} = g_{\alpha\beta} - \left( A M^{-4} \right) \nabla_\alpha \chi \nabla_\beta \chi.
\] (2)

Here we have introduced a dimensionless coupling \( A \), and taken \( \hbar = c_{\text{gravity}} = 1 \), in order to give the scalar field \( \chi \) its canonical dimensions of mass-energy. The normalization energy scale, \( M \), is defined in terms of \( \hbar, G_{\text{Newton}} \), and \( c_{\text{gravity}} \). Provided \( M \) satisfies \( M_{\text{Electroweak}} < M < M_{\text{Pl}} \), the EM lightcones can be much wider than the standard (gravity) lightcones without inducing a large backreaction on the spacetime geometry due to the scalar field \( \chi \). The presence of this dimensionfull constant implies that \( \chi \) VSL models will automatically be non-renormalizable. \( M \) is then the energy at which the non-renormalizability of the \( \chi \) field becomes important. So these models should be viewed as “effective field theories” valid for sub-\( M \) energies. In this regard \( \chi \) VSL implementations are certainly no worse behaved than many of the models of cosmological inflation and/or particle physics currently extant.

The evolution of the scalar field \( \chi \) will be assumed to be governed by some VSL action \( S_{\text{VSL}} \). Then the complete action for the first of the models proposed above is

\[
S_I = \int d^4x \sqrt{-g} \left[ R(g) + \mathcal{L}_{\text{matter}} \right] + \int d^4x \sqrt{-g_{\text{em}}} g_{\text{em}}^{\alpha\beta} F_{\beta\gamma} F_{\alpha}^{\gamma} + \int d^4x \sqrt{-g} \mathcal{L}_{\text{VSL}}(\chi).
\] (3)

Let us suppose the potential in this VSL action has a global minimum, but that the \( \chi \) field is displaced from this minimum in the early universe: either trapped in a meta-stable state by high-temperature effects or displaced due to chaotic initial conditions. The transition to the global minimum may be either of first or second order and during it \( \nabla_\alpha \chi \neq 0 \), so that \( g_{\text{em}} \neq g \). Once the
true global minimum is achieved, $g_{em} = g$ again. Since one can arrange $\chi$ today to have settled to the true global minimum, current laboratory experiments would automatically give $g_{em} = g$.

Since $V(\chi) \geq 0$ in the early universe, $\chi$ could drive an inflationary phase. While this is true we stress instead the more interesting possibility that, by coupling an independent inflation field to $g_{em}$, $\chi$ VSL models can be used to improve the inflationary framework by enhancing its ability to solve the cosmological puzzles.

3. COSMOLOGICAL PUZZLES

The general covariance of General Relativity means that the set of models consistent with the existence of the apparently universal class of preferred rest frames defined by the Cosmic Microwave Background (CMB) is very small and non-generic. Inflation seeks to alleviate this problem by making the flat Friedmann–Lemaître–Robertson–Walker (FLRW) model an attractor within the set of almost–FLRW models, at the cost of violating the strong energy condition (SEC) during the inflationary epoch. VSL cosmologies by contrast typically sacrifice Lorentz invariance, again thereby making the flat FLRW model an attractor.

Our own approach, while is able to solve the “kinematic” puzzles as well as inflation does, cannot solve the flatness problem since in its purest formulation (no inflation driven or enhanced by $\chi$) violations of the SEC do not occur, and because our models do not lead to an explicit “hard” breaking of the Lorentz invariance like other VSL models do. [Our class of VSL models exhibit a “soft” breaking of Lorentz invariance, which is qualitatively similar to the notion of spontaneous symmetry breaking in particle physics.]

We will now consider some of the major cosmological puzzles, directing the reader to the discussion.

3.1. Isotropy

One of the major puzzles of the standard cosmological model is that the isotropy of the CMB seems in conflict with estimates of the region of causal contact at last scattering. The basic mechanism by which VSL models solve this cosmological puzzle relies on the fact that the (coordinate) size of the horizon at the time of last scattering $t_s$ is modified by the time dependence of the photon speed $R_{hz} = \int_0^{t_s} c_{photon} dt/a(t)$. It is this quantity that sets the distance scale over which photons can transport energy and thermalize the primordial fireball. On the other hand, the coordinate distance out to the surface of last scattering is $R_{ls} = \int_{t_s}^{t} c_{photon} dt/a(t)$. Observationally, the large-scale homogeneity of the CMB implies $R_{hz} \geq R_{ls}$. Although this is a paradox in the standard cosmological framework (without inflation), it can be achieved by having $c_{photon} \gg c_{gravity}$ early in the expansion and keeping $c_{photon} \approx c_{gravity}$ between last scattering and the present epoch (as it should be for VSL models to be compatible with observations at low-redshift).

3.2. Flatness

The flatness paradox arises from the fact that the flat FLRW universe, although plausible from observation, appears as an unstable solution of GR. From the Friedmann equation, it is a simple matter of definition that

$$\epsilon \equiv \Omega - 1 = \frac{K c^2}{H^2 a^2} = \frac{K c^2}{a^2},$$

where $K = 0, \pm 1$. Again we have to deal with the basic point of our VSL cosmologies: Which $c$ are we dealing with? We cannot simply replace $c \rightarrow c_{photon}$ in the above (as done in other VSL implementations). As we have pointed out, the $c$ appearing here must be the fixed $c_{gravity}$, otherwise the Bianchi identities are violated and Einstein gravity loses its geometrical interpretation in terms of spacetime curvature. Thus we have to take $\epsilon = K c_{gravity}^2 \dot{a}/a^2$. Differentiating this equation, we see that purely on kinematic grounds

$$\dot{\epsilon} = -2K c_{gravity}^2 (\ddot{a}/\dot{a}^3) = -2\epsilon (\ddot{a}/\dot{a}).$$

Given the way we have implemented VSL cosmology in terms of a two-metric model, this equation is independent of the details in the photon sector. In particular, if we want to solve the flatness problem by making $\epsilon = 0$ a stable fixed point of this evolution equation (at least for some significant portion of the history of the universe), then
we must have $\ddot{a} > 0$, which is equivalent to SEC violation in FLRW. VSL effects by themselves are not sufficient. [Superficially similar VSL models are claimed to solve the flatness puzzle. See for a discussion of such an apparent discrepancy.]

3.3. Monopoles and Relics

The Kibble mechanism predicts topological defect densities that are inversely proportional to powers of the correlation length of the Higgs fields. These are generally upper bounded by the Hubble distance $c/H$. Inflation solves this problem by diluting the density of defects to an acceptable degree. We deal with the issue by varying $c$ in such a way as to have a large Hubble distance during defect formation. Thus we need the transition in the speed of light to happen after the SSB that leads to monopole production. [We also want good thermal coupling between the photons and the Higgs field, to justify using the photon horizon scale in the Kibble freeze-out argument.]

4. PRIMORDIAL FLUCTUATIONS

The inflationary framework owes its popularity not only to its ability to strongly mitigate the main cosmological puzzles, but also to its providing a plausible micro-physics explanation for the causal creation of primordial perturbations.

In the case of $\chi$VSL, the creation of primordial fluctuations is also generic. The basic mechanism can be easily understood by modelling the change in the speed of light as due to the effect of a changing “effective refractive index of the EM vacuum”: $n_{EM} = c_{\text{gravity}}/c_{\text{photon}} = \left( \sqrt{1 + (AM^{-4})(\partial t \chi)^2} \right)^{-1}$. Particle creation from a time-varying refractive index is a well-known effect, and many features of it are similar to those derivable for its inflationary and VSL counterparts (e.g., the particles are still produced in squeezed pairs). We must stress the fact that these mechanisms are not entirely identical. In $\chi$VSL cosmologies a thermal distribution of the excited modes (with a temperature approximately constant in time) is no longer generic, and likewise the Harrison-Zel’dovich (HZ) spectrum is not guaranteed. Nevertheless, approximate thermality, at fixed temperature, over a wide frequency range can be proved for suitable regimes, implying an approximate HZ spectrum of primordial perturbations only on a finite range of frequencies. We hope that this “weak” prediction of a HZ spectrum will be among the possible observational test of these implementations of the VSL framework.

5. CONCLUSIONS

Implementing VSL cosmologies in a geometrically clean way seems to lead almost inevitably to some version of a two-metric cosmology. We have indicated that there are several different ways of building two-metric VSL cosmologies and have discussed some of their generic cosmological features.

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