Andreev reflection resonant tunneling through a precessing spin

Xiufeng Cao, Yaoming Shi * and Xiaolong Song

Department of Physics, Shanghai University, Shanghai 200436, People’s Republic of China

Hao Chen

Department of Physics, Fudan University, Shanghai 200433, People’s Republic of China

Abstract

We investigate Andreev reflection (AR) resonant tunneling through a precessing spin which is coupled to a normal metallic lead and a superconducting lead. The formula of the AR conductance at zero temperature is obtained as a function of chemical potential and azimuthal angle of the spin precessing by using the nonequilibrium Green function method. It is found that as the local spin precesses in a weak external magnetic field at Larmor frequency $\omega_l$, the AR tunneling conductance exhibits an oscillation at the frequency $2\omega_l$ alone. The amplitude of AR conductance oscillation enhances with spin-flip tunneling coupling increasing. The study also shows that spin-orbit interaction in tunneling barriers is crucial for the oscillations of AR conductance. The effect of spin-flip tunneling coupling caused by spin-orbit interaction and local spin precessing on resonant behavior of the AR conductance are examined.

key word: Andreev reflection, precessing spin, spin-orbit interaction

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1. Introduction

In recent years, the technique that is capable of single spin detection is developed very quickly in theoretical and experimental regime. Manassen et al.\textsuperscript{1,2} carried out scanning tunneling microscopy (STM) measurement of the tunneling current while scanning the surface of Si in the vicinity of a local spin impurity (Fe cluster) or imperfection (oxygen vacancy in Si-O) in an external magnetic field. Durkan and Welland\textsuperscript{3} performed a similar STM experiment on organic molecules. Above experiments detected a small signal in the current power at the Larmor frequency. Balatsky and Martin\textsuperscript{4} proposed a new mechanism for the spin-detection technique – electron spin precessing-STM. They found that in the presence of a external magnetic field, the local spin precessing and the tunneling current are modulated at the Larmor frequency, and the spin-flip scattering between the injected unpolarized electron current and the local spin produces the nodal structure of the spatial single profile. Zhu et al.\textsuperscript{5} studied the electronic quantum transport through a local spin precessing in an external magnetic field in adiabatic condition. It is found that when the spin is precessing very slowly at Larmor frequency $\omega_L$, the conductance develops the oscillation with the frequency of both $\omega_L$ and $2\omega_L$ components. The authors of Ref.\textsuperscript{[4,5]} have pointed out that spin-orbit interaction of the conduction electron in the tunneling barriers can result in a spin-flip tunneling coupling between the precessing spin, and the leads and the spin-flip tunneling is crucial for electronic conductance oscillations versus the spin precessing. On the other hand, there has been a growing interest in spin-dependent electronic transport in mesoscopic ”hybrid” systems\textsuperscript{6–12}. When one of the leads is a superconductor, an important transport process–Andreev reflection (AR) tunneling will occur, in which an incident electron picks up another electron to form Cooper pair and enters the superconductor with a hole reflected. Hence it is an interesting subject to study resonant AR tunneling through a precessing spin.

In this letter, we mainly study AR tunneling current through a local precessing spin (PS), which is weakly coupled to a normal metallic lead and a superconducting lead. The schematic layout of the normal metal-PS-superconductor (N-PS-S) system is depicted in Fig. 1, which is different from that in Ref. [4,5]. We assume that the spin-orbit interaction
is confined in the barrier between the metallic lead and the spin site only. It shows that the
AR conductance oscillates with frequency of twice of Larmor frequency. The amplitude of
the conductance oscillations is dependent on not only the spin-orbit interaction, but also the
equilibrium chemical potential of the system. We found that the spin-flip tunnelling coupling
caused by spin-orbit interaction plays a crucial role for the conductance oscillations and it
always enhances the oscillation amplitude of the AR conductance.

2. model and formulation

We consider a precessing spin is coupled via tunnel barriers to a normal metallic lead
and a superconducting lead. The local spin precesses around the weak external magnetic
field applied along z-axis, which is set as the spin quantization axis of the system shown in
Fig. 1. The N-PS-S system under consideration can be modeled by the Hamiltonian:

\[ H = H_N + H_S + H_T + H_{PS} \]  

with

\[ H_N = \sum_{k \in (l), \sigma} \varepsilon_{k\sigma} a_{k\sigma}^\dagger a_{k\sigma} \]  

\[ H_S = \sum_{k \in (r), \sigma} \varepsilon_{k\sigma} s_{k\sigma}^\dagger s_{k\sigma} + \sum_{k \in (r)} (\Delta^* s_{k\uparrow}^\dagger s_{-k\downarrow} + \Delta s_{k\uparrow} s_{-k\downarrow}) \]  

\[ H_T = \sum_{k \in (l), \sigma, \sigma'} (T_{k\sigma,\sigma'} a_{k\sigma}^\dagger c_{\sigma'} + H.c.) + \sum_{k \in (r), \sigma} (T_{k\sigma,\sigma} s_{k\sigma}^\dagger c_{\sigma} + H.c.) \]  

and

\[ H_{PS} = J(\cos \theta c_{\uparrow}^\dagger c_{\uparrow} - \cos \theta c_{\downarrow}^\dagger c_{\downarrow} + \sin \theta e^{-i\phi} c_{\downarrow}^\dagger c_{\uparrow} + \sin \theta e^{i\phi} c_{\uparrow}^\dagger c_{\downarrow}) \]

where \( H_N \) and \( H_S \) are the Hamiltonians for the normal metallic lead and the superconducting
lead respectively. Under mean-field approximation, \( \Delta \) is energy gap of the superconducting
lead. \( H_T \) describes the tunneling part between the spin site and two leads with \( T_{k\sigma,\sigma'} \) denoting
the tunneling matrix. Spin-orbit interaction, which may cause the spin-flip scattering,
considered in the barrier of the metal side. The single electron in the region of the site is
coupled to local spin through a direction spin-exchange interaction \(-g\vec{\sigma} \cdot \vec{S}\). Comparing with
the energy of the exchange interaction, the Zeeman energy of the electrons on the spin site
in the external magnetic field \(\vec{B}\), is very small, so it can be neglected and for simplicity
the Coulomb interaction between electrons on the spin site is ignored as well. The motion
equation of the local spin is \(d\vec{\mu}/dt = \vec{\mu} \times \gamma \vec{B}\), in which \(\vec{\mu} = \gamma \vec{S}\) with \(\gamma\) the gyromagnetic ratio.
In the second quantization, the Hamiltonian of the spin-exchange interaction for electrons on
the site \(H_{PS}\) is written as the form in Eq.(5), in which \(J\) is the effective exchange energy, \(\theta\) is
the tilt angle between the local spin and the external magnetic field, and \(\phi = \phi_0 - \omega_l t\) is the
azimuthal angle with the Larmor frequency \(\omega_l\) and the initial azimuthal angle \(\phi_0\). Since the
energy associated with the spin precession, \(\hbar \omega_l \sim 10^{-6}\) eV, is much smaller than the typical
electronic energy on the order of 1 eV, the spin precession is very slow as compared with the
time scale of all conduction electron processes. This fact allows us to treat the electronic
transport processes adiabatically, as if the local spin is static for every instantaneous spin
orientation\(^5\). In this situation, the AR tunneling processes studied is treated as a kind of
time-independent transport problems.

In the generalized 4×4 Nambu representation, these Green’s functions of the site for
non-interacting electrons can be solved exactly in the terms of Dyson’s equation, \(G^{r,a} =
g^{r,a} + g^{r,a} \sum^{r,a} G^{r,a}\), in which \(\sum^{r,a}\) is the self-energy due to spin-dependent tunneling coupling
and the off-diagonal elements of the local spin processing \(J \sin \theta e^{\pm i\phi}\), and \(g^{r,a}\) is the Green
function without perturbation and spin-flip scattering on the spin site:

\[
(g^{r,a})^{-1} = \begin{pmatrix}
\omega - J \cos \theta \pm i\delta^+ & 0 & 0 & 0 \\
0 & \omega + J \cos \theta \pm i\delta^+ & 0 & 0 \\
0 & 0 & \omega - J \cos \theta \pm i\delta^+ & 0 \\
0 & 0 & 0 & \omega + J \cos \theta \pm i\delta^+
\end{pmatrix}
\] (6)

For the F-PS-S system, the \(\sum^{r,a}\) is written as \(\sum^{r,a} = \sum_{ps} + \sum_{n}^{r,a} + \sum_{s}^{r,a}\). Here the off-
diagonal term of \(H_{PS}\) is considered by self-energy \(\Sigma_{ps}\) with:
Within the wide bandwidth approximation, the self-energy $\Sigma^{r,a}_{n}$ coupling to the normal metallic lead is evaluated from $\Sigma^{r,a}_{n} = \mp \frac{i}{2} \Gamma_{n}$ with

$$\Gamma_{n} = \Gamma_{0} \begin{pmatrix}
1 + \lambda^2 & 0 & 2\lambda & 0 \\
0 & 1 + \lambda^2 & 0 & 2\lambda \\
2\lambda & 0 & 1 + \lambda^2 & 0 \\
0 & 2\lambda & 0 & 1 + \lambda^2
\end{pmatrix}$$

(8)

where $\lambda$ is defined as a ratio of the spin-flip and spin-unflip tunneling amplitude, $\lambda = \frac{|T_{k\sigma,\bar{\sigma}}|}{|T_{k\sigma,\sigma}|}$. $\Gamma_{0} = 2\pi T_{k\sigma,\sigma}^{*} \rho_{n} T_{k\sigma,\sigma}$ is the tunneling coupling without spin-flip scattering. Thus the tunneling couplings associated with the spin-flip tunneling amplitude, are expressed as $\lambda \Gamma_{0} = 2\pi T_{k\sigma,\bar{\sigma}}^{*} \rho_{n} T_{k\sigma,\sigma}$ or $2\pi T_{k\sigma,\sigma}^{*} \rho_{n} T_{k\sigma,\bar{\sigma}}$, and $\lambda^2 \Gamma_{0} = 2\pi T_{k\sigma,\bar{\sigma}}^{*} \rho_{n} T_{k\sigma,\bar{\sigma}}$, due to spin-orbit interaction in barrier of metal side. The self-energy coupling to the S-lead is:

$$\Sigma^{r,a}_{s} = \mp \frac{i}{2} \rho_{s}^{*}(\omega) \Gamma_{0} \begin{pmatrix}
1 & -\frac{\Delta}{\omega} & 0 & 0 \\
-\frac{\Delta}{\omega} & 1 & 0 & 0 \\
0 & 0 & 1 & \frac{\Delta}{\omega} \\
0 & 0 & \frac{\Delta}{\omega} & 1
\end{pmatrix}$$

(9)

where $\rho_{s}^{*}(\omega)$ is the dimensionless BCS density of states:

$$\rho_{s}^{*}(\omega) = \frac{|\omega| \theta(|\omega| - \Delta)}{\sqrt{\omega^2 - \Delta^2}} + \frac{|\omega| \theta(\Delta - |\omega|)}{i \sqrt{\Delta^2 - \omega^2}}$$

(10)

For convenience, we introduce the linewidth function matrix coupling to the S-lead:

$$\Gamma_{s} = \rho_{s}^{<}(\omega) \Gamma_{0} \begin{pmatrix}
1 & -\frac{\Delta}{\omega} & 0 & 0 \\
-\frac{\Delta}{\omega} & 1 & 0 & 0 \\
0 & 0 & 1 & \frac{\Delta}{\omega} \\
0 & 0 & \frac{\Delta}{\omega} & 1
\end{pmatrix}$$

(11)
with \( \rho_s^\ell(\omega) = |\omega| \theta(|\omega| - \Delta)/\sqrt{\omega^2 - \Delta^2} \). After a straightforward calculation, the normal electron tunneling conductance and the Andreev reflection conductance are obtained in the linear response regime as follows:

\[
G_N = \frac{e^2}{\hbar} \int d\omega \left[-\frac{\partial f}{\partial \omega}\right] \sum_{i=1,3} [G^r_s \Gamma_s G^a \Gamma_n]_{ii} \tag{12}
\]

and

\[
G_A = 2\frac{e^2}{\hbar} \int d\omega \left[-\frac{\partial f}{\partial \omega}\right] \sum_{i=1,3} \sum_{j=2,4} G^r_{ij} (\Gamma_n G^a \Gamma_n)_{ji} \tag{13}
\]

Since normal linear conductance is zero, \( G_N = 0 \), at zero temperature, only the Andreev reflection process contributes to electronic transport of the system. So the total conductance \( G \) is equivalent to \( G_A \).

3. The results and discussion

We only concentrate here on the case of the spin-exchange interaction strength \( J \) is restricted in the range of energy gap of the superconductor \( \Delta \) (\( J \leq \Delta \)). In the following calculation, \( \Delta \) is taken as energy unit and the spin-exchange interaction strength is chosen as \( J = 0.5 \).

In order to examine resonant behaviors of the AR conductance versus the chemical potential \( \mu \), we first consider AR tunneling processes through a static spin in zero magnetic field. In Fig. 2, we plot the conductance versus chemical potential \( \mu \) with some different values of the ratio of spin-dependent tunneling amplitude, \( \lambda = 0.0 \) (solid line), 0.4 (dashed line), 0.8 (dotted line). The curves shown in panels (a), (b) and (c) correspond to three different orientations of the local spin: \( (\theta, \phi) = (0, 0), (\pi/4, 0), (\pi/2, 0) \), in which the tunneling coupling without spin-unflip scattering is taken as \( \Gamma_0 = 0.1 \). There are several generic features of resonant AR conductances in (a), (b) and (c) of Fig. 2. It is clearly seen that two resonant peaks of the conductance appear symmetrically at the two sides of \( \mu = 0 \), due to the Andreev reflection is determined by spin minority population. Moreover the position of every resonant peak is almost independent on the relative spin-dependent tunneling amplitude \( \lambda \), and has a small deviation from \( \mu = J(-J) \). This is different from previous
results in normal electron tunneling conductance in Ref.[5]. The spin-orbit interaction in tunneling barrier influences resonant amplitude of the AR conductance, which is different from Ref.[5]. Comparing Fig. 2(a)-2(c), it is clearly seen that in the case of $\theta = 0$, the spin-orbit interaction strongly suppresses the conductance not only at $\lambda = 0.4$, but also $\lambda = 0.8$. In the case of $\theta \neq 0$, however, the AR conductance is efficiently suppressed only for strong spin-orbit interaction $\lambda = 0.8$. For $\theta = 0$, spin-flip scattering is dominated by spin-orbit interaction, but for $\theta \neq 0$, spin-orbit interaction is only one part of the spin-flip scattering, which also involves the component of precessing spin, $J \sin \theta e^{\pm i\phi}$.

Fig. 3 presents some curves of the AR conductance oscillations of as a function of the phase $\phi$ in units of $2\pi$, which a tunneling electron accumulates from the precession of the local spin, with various values of $\lambda = 0.0$ (solid line), 0.4 (dashed line), 0.8 (dotted line) for given parameters $\Gamma_0 = 0.1$, $\theta = \pi/2$, and $\phi_0 = 0$. The results shown in the left (a), (b) and (c) panels of Fig.3 correspond to three different chemical potentials, (a) $\mu = 0.1$, (b) 0.4, and (c) 0.7, respectively. The Fourier spectrums of the AR conductance oscillations with $\lambda = 0.4$ are presented in the right three corresponding panels of Fig.3, in which other parameters are the same as in the left panels of Fig.3. The calculated result exhibits obviously that oscillation of the AR conductance occurs only in the case of the spin-flip tunneling coupling and its oscillation amplitude enhances with spin-flip tunneling coupling. The same conclusion was given in Ref. [4,5].

It is well known, in the presence of magnetic field, the local spin will precess with the Larmor frequency $\omega_l$. The question is how will this Larmor precession influence the conductance of electron transported through the local spin. We find that the AR conductance will oscillate at double Larmor frequency $2\omega_l$ alone, because the Andreev reflection conductance is usually expressed by the off-diagonal terms of the Green functions, such as $T^A = \Gamma^2_l |G^r_{12}(\omega)|^2$. Moreover the off-diagonal terms of Green functions often satisfy the relation of $G_{ij}(\phi + \pi) = G_{ji}(\phi)^5$, which results in the change of the oscillation period from $2\pi$ to $\pi$. The occurrence of non-oscillatory conductance is understood as follow. The spin-flip scattering of transited electrons is determined only by the scattering terms of the exchange
interaction, \( J \sin \theta e^{\pm i\phi} \), on the spin-site, if there is no spin-orbit interaction in the tunneling barriers. Due to the above two terms, \( J \sin \theta e^{i\phi} \) and \( J \sin \theta e^{-i\phi} \), are out-phase for \( \phi \), the tunneling conductance, which is proportional to the absolute value squared of transmission amplitude, should carry no information of the spin precessing i.e. the azimuthal angle \( \phi \). However, in the presence of spin-orbit interaction in the tunneling barriers, the spin-flip scattering amplitude of electrons should be expressed as \( J \sin \theta e^{\pm i\phi} + 2\lambda \Gamma_n \), so the information of \( \phi \) can be contained in the multiplication of \( (J \sin \theta e^{\pm i\phi} + 2\lambda \Gamma_n) \) and \( (J \sin \theta e^{\pm i\phi} + 2\lambda \Gamma_n)^* \), and in the tunneling conductance of the system.

The oscillation amplitude increases with the spin-orbit interaction and the oscillation frequency is twice of Larmor frequency. Comparing Fig. 3(a)-3(c), we obtain that the oscillation amplitude is modulated by the equilibrium chemical potential except spin-orbit interaction. When chemical potential trends towards exchange interaction strength \( J \) from two sides, the oscillation amplitude increases. These features can be seen more clearly from the Fourier spectrum.

In summary, we have studied AR resonant tunneling through a local spin precessing in the external magnetic field, which is coupling to normal metallic and superconducting leads. It is found that the spin-orbit interaction in the tunneling barriers between the spin site and metallic lead is crucial for the appearance of AR conductance oscillations versus the azimuthal angle of the spin precessing \( \phi \). The conductance oscillation is modulated by spin-orbit interaction and the equilibrium chemical potential of the system. The oscillation amplitude of AR conductance enhances with spin-flip tunneling coupling increasing. The study shows that the AR tunneling conductance exhibits a oscillation at the frequency double Larmor frequency \( 2\omega_l \) alone. The technique combining STM with superconductor must be a new test of the proposed mechanism for the conductance oscillation.

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* Corresponding author.

E-mail: ymshi@mail.shu.edu.cn

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FIGURES

FIG. 1 Schematics of the system investigated: normal metallic (N) and superconducting (S) lead attached to a local spin ($\vec{S}$), which precesses around the magnetic field ($\vec{B}$) with tilt angle $\theta$.

FIG. 2. The linear resonant AR conductance vs chemical potential $\mu$ with $\Gamma_0 = 0.1$ for different ratio of the spin-flip and spin-unflip tunneling amplitudes: $\lambda = 0.0$ (solid line), 0.4 (dashed line), 0.8 (dotted line). The curves shown in panels (a) through (c) correspond to three different precessing orientations: $(\theta, \phi) = (0, 0), (\pi/4, 0), (\pi/2, 0)$.

FIG. 3 The conductance versus the phase $\phi$ with various values of the ratio of the spin-flip and spin-unflip tunneling amplitudes, $\lambda = 0.0$ (solid line), 0.4 (dashed line), 0.8 (dotted line). The curves shown in the left panels (a) through (c) correspond to three different values of the chemical potential: $\mu = 0.1, 0.4, 0.7$. Also shown with the right panels are the Fourier spectrum for $\lambda = 0.4$ with the chemical potential same as the left panels. Other parameter values: $\Gamma_0 = 0.1, \theta = \pi/2$ and $\phi_0 = 0$. 
Fig. 1
Fig. 2

(a) Conductance ($e^2/h$) vs. Chemical potential $\mu$

(b) Conductance ($e^2/h$) vs. Chemical potential $\mu$

(c) Conductance ($e^2/h$) vs. Chemical potential $\mu$

For each graph:
- $\theta=0$
- $\lambda=0.0$
- $\lambda=0.4$
- $\lambda=0.8$

Graph (a) is for $\theta=0$, Graph (b) is for $\theta=\pi/4$, and Graph (c) is for $\theta=\pi/2$. Each graph shows the conductance as a function of the chemical potential $\mu$, with different values of $\lambda$.
Fig. 3

(a) Conductance ($e^2/h$) vs. $\phi / (2\pi)$

(b) Conductance ($e^2/h$) vs. $\phi / (2\pi)$

(c) Conductance ($e^2/h$) vs. $\phi / (2\pi)$

Fourier Spectrum

(a) Fourier Spectrum

(b) Fourier Spectrum

(c) Fourier Spectrum