Numerical Modelling of Interaction between Solitary Wave and Sloped Seawall

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Abstract. A numerical model for solving interaction between solitary wave and a sloped seawall is established. The CIP (Constrained Interpolation profile) scheme is employed to solve the flow field. THINC (Tangent of Hyperbola Interpolation for Interface Capturing) method is applied for capturing the free surface. Hydrodynamic forces is calculated by integrating the pressure and friction along the seawall surface. First, convergence tests with respect to mesh resolution and time step are examined, and the solitary wave profile is validated by experimental data from the published paper. Then, time series of the free surface elevation at specific location with different heights of incident solitary wave are presented. Finally, hydrodynamic forces acting on the weather side of the seawall are analyzed. The conclusion can be drawn that this numerical model has applicability for strongly nonlinear wave-body interaction problems.

1. Introduction
Seawall has long been concerned about the application in coastal engineering, for its protection mechanism for shore structures. Sloped seawalls are more proper energy dissipaters compared with vertical seawalls, because the considerable wave reflection amplifies the kinematics in front of vertical seawall. This paper employed a type of trapezoid seawall on the sloping beach proposed by Hsiao and Lin [1]. One of the key issue in solving this problem is that the nonlinear phenomenon occurs when solitary waves interacting with ocean structures. Analytical solutions are limited to idealized conditions or simplified geometries. Ma et al [2], Wu and Hsiao [3] investigated solitary waves interacted with vertical obstacles through experiments. Recent numerical studies have investigate plate structure as seawall such as [4,5], and vertical seawalls such as [6]. Scholars also carried out experiments to investigate solitary wave interacting with vertical seawalls [7]. More work is need to investigate the solitary wave interacting with sloped seawalls.

The present study establishes a numerical model based on a CIP scheme. The CIP is first developed by Yabe et al [8] to solve hyperbolic equation, and then is employed to marine engineering problems by Hu and Kashiwagi [9,10]. To simulate the large surface deformations with high accuracy, THINC (Tangent of Hyperbola Interpolation for Interface Capturing) [11] method is employed. The purpose of the present paper is to examine the effective performance of CIP-based numerical model in solving solitary waves impinging on a trapezoid seawall, and to analyse the difference of free surface due to different incoming wave height. This paper is organized as follows. In section 2, we briefly describe the numerical method employed in this paper. Section 3 describes the numerical experimental setup and the validation of the numerical model. Results and discussion are presented in section 4.

2. Numerical Method
This section briefly describes the numerical method we used to construct the wave-body interaction numerical model. It mainly contains three parts, one part is the CIP based flow solver, one part is the interface capturing method, and the last one is the calculation method for hydrodynamic forces.

2.1. Flow Solver

For incompressible flow, governing equations including continuity equations and Navier-Stokes equations can be expressed as:

\[
\frac{\partial u}{\partial t} + u_j \frac{\partial u}{\partial x_j} = - \frac{1}{\rho} \frac{\partial p}{\partial x_i} + \frac{1}{\rho} \frac{\partial}{\partial x_j} (2\mu S_{ij}) + f_i
\]

(2)

Where \( u, p, t \) and \( f \) denote velocity, density, pressure, time and body force, respectively; and \( S_{ij} = \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) / 2 \). The flow solver divides the Navier-Stokes equation into three fractional steps:

\[
\frac{u_i^* - u_i^n}{\Delta t} + u_j^* \frac{\partial u_i^n}{\partial x_j} = 0
\]

(3)

\[
\frac{u_i^{**} - u_i^*}{\Delta t} = \frac{2\mu}{\rho} \frac{\partial}{\partial x_j} \left( S_{ij} - \frac{1}{3} \delta_{ij} S_{kk} \right) + f_i
\]

(4)

\[
\frac{u_i^{n+1} - u_i^{**}}{\Delta t} = - \frac{1}{\rho^*} \frac{\partial p^{n+1}}{\partial x_i}
\]

(5)

Where, \( n \) denotes the present time level, \( n+1 \) for the new time level. Equation (3) is the advection step, which is solved by the CIP method. Equation (4) and Equation (5) are the Non-advecton steps, diffusion term in Equation (4) is treated by central difference method. In Equation (5), the pressure coupled with velocity is solved by SOR (Successive Over Relaxation) iteration.

2.2. Interface Capturing Method

THINC employs a hyperbolic tangent function which makes it a suitable interpolation for the flux computation of a VOF (Volume of Fluid) function [12]. The competitive accuracy compared with other method has been verified by Xiao et al [10]. The hyperbolic tangent function is written as:

\[
F_i(x) = \frac{\alpha}{2} \left[ 1 + \gamma \tanh \left( \beta \left( \frac{x - x_i - u_i \Delta t}{\Delta x_i} - \delta \right) \right) \right]
\]

(6)

Where \( \alpha, \beta, \gamma, \delta \) are parameters can be specified.

2.3. Hydrodynamic Forces

After the calculation of pressures, velocities and density functions, the hydrodynamic forces acting on the solid body can be calculated by integrating the pressure and skin friction along the body surface as follows:

\[
F_i = F_i^{(p)} + F_i^{(v)} = \iint_A (-p \delta u) n_x dA + \iint_A 2\mu S_{uk} n_k dA
\]

(7)

Where \( F_i^{(p)} \) and \( F_i^{(v)} \) are the forces due to pressure and friction, respectively. \( A \) denotes the surface of solid body, and \( n_i \) presents the \( i \)-th component of the outward unit normal vector.

3. Numerical Experiment Setup

Figure 1 shows a two-dimensional numerical wave flume constructed in this paper. Left side of the wave flume is a wave maker for generating solitary waves, which is defined as the origin of the Cartesian coordinate system. Reference wave gauge is set at \( x = 5.900 \) m, before the starting point of the slope, to measure the incident wave height. The non-dimensional wave height is defined as \( \alpha = \frac{H_0}{\lambda} \).
$H/h$, where $H$ is the wave height, and $h = 0.256$ m is the water depth. Four incident wave heights of solitary waves are investigated, including $\alpha = 0.1, 0.2, 0.23$ and 0.3, in which $\alpha = 0.23$ is to keep accordance with the experimental data. The parameter of the sloping seawall is listed in table 1. The slope starts at $x_{s1} = 7.000$ m, and has a trapezoid obstacle composed by $x_{s2}$, $x_{s3}$, $x_{s4}$ and $x_{s5}$ on it. Weather side is indicated in figure 1; the hydrodynamic forces acting on it is discussed in section 4.

![Schematic view of the numerical wave flume.](image)

**Figure 1.** Schematic view of the numerical wave flume.

| Vertex point | $x_{s1}$ | $x_{s2}$ | $x_{s3}$ | $x_{s4}$ | $x_{s5}$ |
|--------------|---------|---------|---------|---------|---------|
| Distance (m) | 7.000   | 10.600  | 10.900  | 10.948  | 11.045  |

### Table 1. Locations of vertex point.

#### 4. Results and Discussion

**4.1. Convergence Test**

Both mesh convergence and time step convergence tests are examined. Figure 2 shows normalized free surface elevation measured at the wave gauge due to different mesh resolution. Three types of grid are employed; the first number in the legend denotes total grid number in $x$ direction, and the second number denotes total grid number in $y$ direction. For example, “Grid-1: 810 * 220” means Grid-1 has 810 and 220 grids in $x$ and $y$ direction, respectively. The figure indicates that Grid-2 and Grid-3 agree very well with each other, while Grid-1 differs from them. So the conclusion can be obtained that Grid-2 has enough grids for this problem. In the following parts, all data is calculated by utilizing Grid-2. Circle is the experimental data from Hsiao and Lin [1], the main wave crest agrees well with numerical results.

![Mesh convergence test](image)

**Figure 2.** Mesh convergence test. Grid 1 (dash), Grid 2 (solid), Grid 3 (dot); Experiment [1] (circle)
Figure 3 compares the free surface elevation according to different time step. $Dt$ denotes time step, $T$ denotes truncated solitary wave period. As an example, “$Dt = T / 1000$” means calculate 1000 time steps in one wave period. As $Dt = T / 1000$, there is a slight discrepancy from another three time step; another three curves reach accordance, we choose the middle one $Dt = T / 2000$ as the time step in our computational cases.

![Figure 3](image.png)

**Figure 3.** Time step convergence test. $Dt = T / 1000$ (dash), $T / 1500$ (dot), $T / 2000$ (solid), $T / 2500$ (dash-dot); Experiment [1] (circle).

### 4.2. Free Surface Elevation

Figure 4 shows time series of the free-surface elevation, in terms of different incident solitary wave height $\alpha = 0.1$, 0.2, and 0.3, which are measured at $x = 5.900$ m. The water depth remains constant which equals to 0.256 m. The abscissa is the time series, while the ordinate is the non-dimensional free surface elevation normalized by dividing the constant water depth. The solitary wave only has a wave crest above the still water level. From the figure it can be seen that with the increase of incident wave height, the main wave crest becomes steeper, and occurs earlier.

![Figure 4](image.png)

**Figure 4.** Time series of free surface elevation measured at wave gauge, with normalized incident wave height $\alpha = 0.1$ (dot), 0.2 (dash), and 0.3 (solid).

### 4.3. Hydrodynamic Forces
Time series of the horizontal force $F_x$ and vertical forces $F_z$ acting on the weather side of the seawall is presented in figure 5 and figure 6, respectively. Three different incident solitary wave height $\alpha = 0.1$, 0.2, and 0.3 is compared. Generally, vertical forces are large than horizontal force by almost one order of magnitude. The absolute value of both vertical forces and horizontal forces go up reaching a peak, then drop down. With the increase of incident wave height, hydrodynamic forces increase, and the force curve becomes steeper. The peak force occurred time is illustrated in the figures, for example, “$t = 7.413$ s” means when non-dimensional incident wave height is 0.3, maximum horizontal force occurs at $t = 7.413$ s. It can be found that peak force occurs early as incident wave height increase, this is mainly because phase velocity of solitary wave increase as a function of wave height. This phenomenon is accord with the solitary wave propagates faster when wave height increase in figure 4.

![Figure 5](image1.png)  
**Figure 5.** Horizontal forces ($F_x$) acting on weather side, with normalized incident wave height $\alpha = 0.1$ (dot), 0.2 (dash), and 0.3 (solid).

![Figure 6](image2.png)  
**Figure 6.** Vertical forces ($F_z$) acting on weather side, with normalized incident wave height $\alpha = 0.1$ (dot), 0.2 (dash), and 0.3 (solid).

### 5. Conclusion

This paper implemented a CIP based numerical model to investigate interaction between solitary waves and a trapezoid seawall on a sloping beach. The convergence test and the validation compared with the experimental data confirms that the present numerical model is efficient to calculating wave-structure interaction problems. Different incident wave height was tested to comparing the variation of the free surface elevation and hydrodynamic forces, and the conclusion can obtained that with the increase of incident wave height, the profile of solitary waves becomes steeper; the maximum hydrodynamic force occurs earlier.

### 6. Reference

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