Calculation of the controlled parameters of the 6-coordinate robot in the process of additive forming of products

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Abstract: The article is devoted to the investigation of the surface layer formation accuracy of engineering products by additive methods. The analysis of advantages and disadvantages of layered products synthesis technologies was carried out. It was revealed that, in additive shaping, the exact characteristics of the surface layer differ significantly from the accuracy characteristics of the surface layer of products obtained by the traditional methods. The analysis of domestic and foreign works on the topic of research was carried out. It is revealed that to increase the accuracy characteristics of products obtained by additive methods, it is necessary to realize dynamically the spatial orientation of the working element of the additive installation in the process of shaping. To control the spatial orientation of the working organ of additive equipment, a method is proposed. According to the proposed method, the controlled parameters of the additive installation are calculated on the basis of a 6-coordinate robot. The proposed methodology will allow to calculate the controlled parameters of the process equipment, to provide the required orientation of the working element of the additive installation to reduce the error of shaping (approximation), using 6-axis positioning mechanisms.

1. Introduction
At present, additive technologies are positioned as technologies capable to replace the traditional approaches of complex geometric shapes parts manufacturing for a variety of purposes. However, the accuracy characteristics of parts obtained by additive methods are not identical to the characteristics of parts obtained by computational methods [1].

Formation of parts by additive methods is characterized by high values of the static component of the processing error, in particular, the magnitude of the error in shaping (approximation). This is due to the fact that the formation of the surface layer of a complex shape detail occurs in a row, and the orientation of the final element of the forming system of the additive installation is thus unchanged and independent of the curvature of the surface being formed.

Increasing the accuracy of additive methods of shaping is an actual task, one of the solutions of which is to ensure the spatial orientation of the final link of the forming system of additive equipment along the normals at the point of the nominal surface of the formed part, which requires the creation of a new or expansion of the technological capabilities of existing additive equipment.

2. Analysis of domain research
A large number of works have been devoted to improving the accuracy of additive methods of shaping [2], for example, in [6] to reduce the error in shaping (approximation), it is proposed to use the static orientation of the surface to be formed. The questions of the dynamic orientation of the final element of the formative system of additive equipment in the process of shaping by additive methods remain insufficiently studied. The use of dynamic orientation will reduce the magnitude of the error in shaping (approximation) of additive methods by reducing the curvature of the surface of the applied elementary layer at the point of contact with the nominal surface of the part [7–10].

In [4], a method of additive shaping using the Stewart platform was proposed, in which the orientation of the final link of the forming system of additive equipment will be provided in such a
way that when it approaches the surface of the part to be formed, the platform will change the orientation of the part, ensuring the coincidence of the axis of the final link of the forming system and the normal to the surface of the part at the point being formed.

The use of mechatronic systems with parallel kinematics is justified by the need to ensure high accuracy in the functional control of the displacement and orientation of the output link of the basic component of the mechatronic device in the three-dimensional working space, the stiffness and compactness of this device under the action of dynamic loads. The disadvantages of mechanisms with parallel kinematics include a smaller working space, in comparison with classical sequential structures, a relatively small manipulation and a more complex mechanism design. In the process of operation of mechatronic devices with a parallel structure, the internal connections arise that limit their working spaces and can lead to loss of controllability by the mechanism. The presence of forces and moments of frictional forces in the kinematic pairs of the Stewart platform leads to the fact that in the vicinity of special positions the mechanism may become jammed, thus the mechanism will not be operable in the most special position, but in some neighborhood of this position [15].

Thus, to solve the problems of increasing the accuracy of additive form-building methods, the mechanisms with a sequential structure – multi-position robots – can be used as an alternative to mechanisms with parallel structure, the use of which will expand the working space of technological equipment, as well as the ranges of technological parameters control, which is a necessary condition for shaping of complex shape surfaces.

3. Construction of the forming system model of the 6-coordinate robot

To implement the dynamic spatial orientation of the final link of the formative system of additive equipment, consider the use of a 6-axis industrial robot. To do this, it is necessary to solve the problem of calculating the controlled parameters of the robot, under which the necessary spatial orientation of its final link will be obtained (Fig. 1).

Let's describe the basic equation of the process of additive shaping using this robot, providing a spatial orientation of the final element of the forming system of the additive installation. The equation has the form:

$$\overrightarrow{r_0}(u, v) = A_0 \cdot A_x \left( B_{x_1}, B_{x_2}, B_{x_3}, B_{x_4}, B_{x_5}, B_{x_6} \right) \overrightarrow{e},$$

(1)

where

- $A_0$ – matrix for setting the coordinate system of the detail in the working space of the technological equipment;
- $A_x \left( B_{x_1}, B_{x_2}, B_{x_3}, B_{x_4}, B_{x_5}, B_{x_6} \right)$ – matrix of the forming system of the technological equipment, corresponding to the values of the angles of rotation of the corresponding links of the 6-coordinate robot;
- $B_{x_1}$ – angle of the 1st link rotation relative to the axis $OZ_1$;
- $B_{x_2}$ – angle of the 2nd link rotation relative to the axis $OZ_2$;
- $B_{x_3}$ – angle of the 3rd link rotation relative to the axis $OZ_3$;
- $B_{x_4}$ – angle of the 4th link rotation relative to the axis $OZ_4$;
- $B_{x_5}$ – angle of the 5th link rotation relative to the axis $OZ_5$;
- $B_{x_6}$ – angle of the 6th link rotation relative to the axis $OZ_6$;
- $\overrightarrow{e}$ – radius vector of the point of origin;
- $u, v$ – curvilinear coordinates of the formed detail.
Figure 1. Scheme of calculation of controlled parameters for formative element spatial orientation providing of the additive installation using a 6-coordinate robot.

The matrix for setting the coordinate system of the detail in the working space of the 6-axis robot has the form (see Fig. 1) [18]:

$$A_{01} = A_{0,1}^{(3)}(l_{02}) \cdot A_{1,1}^{(2)}(l_{01}),$$  \hspace{1cm} (2)

where

- \(A_{0,1}^{(3)}\) – matrix for setting the coordinate system of the detail in the working space of the 6-coordinate robot along the axis \(OZ\);
- \(A_{1,1}^{(2)}\) – matrix for setting the coordinate system of the detail in the working space of the 6-coordinate robot along the axis \(OY\).

The matrix of the forming system of the 6-coordinate robot has the form [18]:

$$A_{32} = A_{3,5}^{(2)}(B_{x_z}) \cdot A_{4,4}^{(1)}(l_{1}) \cdot A_{5,6}^{(3)}(B_{x_z}) \cdot A_{6,6}^{(3)}(B_{x_z}) \cdot A_{7,8}^{(3)}(l_{2}) \cdot A_{8,6}^{(3)}(B_{x_z}) \cdot A_{9,9}^{(3)}(l_{3}) \cdot A_{10,11}^{(3)}[\pi]\cdot A_{11,12}^{(2)}(B_{x_z}) \cdot A_{12,12}^{(2)}(l_{6}) \cdot A_{\alpha,\alpha'}^{(1)}(\pi),$$  \hspace{1cm} (3)

where

- \(A_{3,5}^{(2)}, A_{4,4}^{(1)}\) – displacement matrix of the coordinate systems of the corresponding links relative to their initial position along the axis \(OY\);
- \(A_{5,6}^{(3)}, A_{6,6}^{(3)}, A_{7,8}^{(3)}, A_{8,6}^{(3)}\), \(A_{9,9}^{(3)}\), \(A_{10,11}^{(3)}\), \(A_{11,12}^{(2)}\) – displacement matrix of the coordinate systems of the corresponding links relative to their initial position along the axis \(OZ\);
- \(A_{12,12}^{(2)}, A_{\alpha,\alpha'}^{(1)}\) – rotation matrix of the coordinate systems of the corresponding links relative to their initial position about the axis \(OZ\);
$A_{l_1}^{[k]}, A_{l_2}^{[k]}, A_{l_3}^{[k]}, A_{l_4}^{[k]}$ – rotation matrix of the coordinate systems of the corresponding links relative to their initial position about the axis $OX$.

Parameters included in the matrix $A_x$, can be divided into controllable and uncontrolled (constructive) parameters of technological equipment.

To ensure the process of shaping, on the uncontrollable parameters of the 6-coordinate robot included in the shaping equation, it is necessary to impose a relationship of the form:

$$q_i = \text{const}$$  \hspace{1cm} (4)

Uncontrolled parameters include the lengths of the robot's links, namely the elements of the matrix of the forming system: $l_1, \ldots, l_8$.

To ensure the process of shaping, on six controlled parameters of the robot, it is necessary to impose a relationship of the form:

$$q_i = q_i(u,v);$$  $q_2 = q_2(u,v);$  ...  $q_6 = q_6(u,v)$  \hspace{1cm} (5)

The controlled parameters include the rotation angles of the links around the corresponding axes: $B_{Z_1}, B_{Z_2}, B_{Z_3}, B_{X_1}, B_{X_2}, B_{X_3}$.

When the $j$-th point of the surface of the detail is formed, equation (5) can be determined by solving the matrix equation

$$A_{0j} (u_j, v_j) \cdot A_{02} \cdot A_{k} [B_{Z_1}, B_{X_1}, B_{X_2}, B_{X_3}, B_{Z_2}, B_{Z_3}] = E,$$  \hspace{1cm} (6)

where $A_{0j} (u_j, v_j)$ – matrix of the transition from the coordinate system of the formed detail to the coordinate system of the $j$-th point on the surface of the detail $\tilde{r}_{0j} (u_j, v_j)$;

$E$ – identity matrix.

Matrix $A_{0j}$ of the transition from the coordinate system of the formed detail to the coordinate system of the $j$-th point on the surface of the detail $\tilde{r}_{0j} (u_j, v_j)$ is calculated according to the method described in [16-19], along the vectors which define the positive direction of the axis $Z_j$

$$k_{j0} = \frac{\partial \tilde{r}_{0j}}{\partial u} \times \frac{\partial \tilde{r}_{0j}}{\partial v};$$  \hspace{1cm} $Y_j \cdot j_{j0} = \frac{\partial \tilde{r}_{0j}}{\partial u} \times \frac{\partial \tilde{r}_{0j}}{\partial v}$, or $j_{j0} = \frac{\partial \tilde{r}_{0j}}{\partial v} \times \frac{\partial \tilde{r}_{0j}}{\partial u}$, where: $\frac{\partial \tilde{r}_{0j}}{\partial u}, \frac{\partial \tilde{r}_{0j}}{\partial v}$ – partial derivatives of the vector $\tilde{r}_{0j}$ with respect to the parameters $u, v$, as well as the vector $\tilde{r}_{0j}$, specifying the origin of the coordinate system $X_j Y_j Z_j$.

4. The results of the controlled parameters calculation

Let's consider an example of calculation of controlled parameters of a 6-coordinate robot forming the hemispherical surface by the additive methods (Fig. 2).

The equation of the hemisphere has the form:

$$r_0(\theta, z) = \begin{bmatrix} \sqrt{R^2 - z^2} \cdot \cos(\theta) \\ \sqrt{R^2 - z^2} \cdot \sin(\theta) \\ z \end{bmatrix}$$  \hspace{1cm} (7)

where $\theta, z$ – curvilinear coordinates of the surface;

$R$ – radius of the sphere.
It should be noted that solving the equation (6), several solutions can be obtained. It means that it is possible to perform the transition of the final link of the formative system of the additive installation from the point \( j \) to the point \( j+1 \) of the detail by different ways, and, in addition to the problem of calculating the controlled parameters, it is also necessary to solve the problem of choosing rational parameters from the set of possible ones.

In Fig. 3, the graphs of the controlled parameters change of the robot obtained by solving the equation (6) in the section of the detail \( z = 0 \) are shown, in the range of the controlled parameters values [22]:

\[
0' \leq B_{x_i} \leq 360'; \\
0' \leq B_{z_i} \leq 360'; \\
0' \leq B_{x_i} \leq 360'; \\
-90' \leq B_{x_i} \leq 90'; \\
-180' \leq B_{x_i} \leq 180'; \\
-180' \leq B_{x_i} \leq 180'.
\]

The choice of rational variants of the transition of the final link of the forming system of the additive installation from the \( j \) point to the \( j+1 \) point of the detail is possible on the basis of calculating the value of the total increment of the controlled parameters of the robot:

\[
S_{i,j}^{[n]} = \Delta S_{Z_{i,j}(j+1)}^{[n]} + \Delta S_{Z_{i,j}(j+1)}^{[n]} + \Delta S_{X_{i,j}(j+1)}^{[n]} + \Delta S_{X_{i,j}(j+1)}^{[n]} + \Delta S_{X_{i,j}(j+1)}^{[n]} + \Delta S_{X_{i,j}(j+1)}^{[n]} = \left| B_{x_{i,j}} - B_{x_{i,j}}^{[n]} \right| + \left| B_{z_{i,j}} - B_{z_{i,j}}^{[n]} \right| + \left| B_{x_{i,j}} - B_{x_{i,j}}^{[n]} \right| + \left| B_{x_{i,j}} - B_{x_{i,j}}^{[n]} \right|, \\
\]

where

\[
\Delta S_{Z_{i,j}(j+1)}, \Delta S_{Z_{i,j}(j+1)}, \Delta S_{X_{i,j}(j+1)}, \Delta S_{X_{i,j}(j+1)}, \Delta S_{X_{i,j}(j+1)} \quad \text{— increment of the values of the corresponding controllable parameters during the transition of the final link of the forming system from the point \( j \) to the point \( j+1 \);} 
\]
We consider the rational variant with the minimum value of the total increment of the parameters $S^{(e)}_{(j+1)}$.

To calculate the controlled parameters of the 6-coordinate robot, we determine the discreteness of the angular displacement of the final link of the forming system from the $j$ point with coordinates $(10;180)$ to the point $j+1$ with the coordinates $(179.5;10)$.

In Fig. 4, the results of the graphical modeling of the position of the robot links during the shaping of the spherical surface are presented. Two variants of displacements obtained solving the equation (6) are considered.

According to the method presented above, the values of the total increment of the controlled parameters of the robot were calculated for two possible variants.

| Table 1. The results of the controlled parameters calculation of the 6-coordinate robot |
|---|---|---|---|
| Controlled Point $j$ | Controlled Point $j+1$ | $\Delta S^{(e)}_{(j+1)}$ |

**Figure 4.** Simulation of the positions of the links of the 6-coordinate robot: a) at the $j$ point; b) at $j + 1$ point (variant 1); in the $j+1$ point (variant 2).
According to the calculation results, as well as graphical modeling, the rational variant of moving the final link of the formative system of additive equipment from the \( j \) point to the \( j+1 \) point is the variant with the smallest value of the increment of the controlled parameters - variant 2.

5. Conclusion

The proposed method will allow to calculate the controlled parameters of the 6-coordinate robot in additive shaping of products, to provide the dynamic spatial orientation of the final element of the forming system of additive equipment, to choose a rational variant of the displacement of the final element of the forming system in case of multivariance of the solution of the problem, which will reduce the magnitude of the error in shaping (approximation), increase the productivity of the shaping process.

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| parameter | Var.1          | Var.2          | Var.1          | Var.2          |
|-----------|----------------|----------------|----------------|----------------|
| \( B_{X_1} \) | -45.61         | 55.26          | -45.73         | 100.87         |
| \( B_{X_2} \) | -89.49         | 130.09         | -89.19         | 219.58         |
| \( B_{X_3} \) | -89.20         | -57.17         | -89.52         | 32.03          |
| \( B_{Z_1} \) | 146.39         | 326.64         | 146.568        | 180.25         |
| \( B_{Z_2} \) | 46.14          | 238.75         | 45.86          | 192.60         |
| \( B_{Z_3} \) | 216.11         | 174.87         | 216.04         | 41.23          |
| \( \Sigma \Delta S_{(j,i)} \) | | 766.58         | | 1.27          |
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