A Bayesian filtering approach for inclusion detection with ultrasound reflection tomography

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Abstract. Ultrasound is frequently used in nondestructive material testing due to its high sensitivity to inhomogeneities like cracks and gas inclusions. Tomographic approaches have the potential to attain higher resolution in terms of defect location and shape compared to conventional B-mode imaging. However, when commonplace reconstruction algorithms are used, the limited number of transducers available in realistic measurement setups leads to blurring and the presence of artifacts in the reconstructed image. This paper presents a novel reconstruction algorithm for ultrasound reflection tomography of binary material distributions. A key component is the Bayesian filtering approach based on particle filtering for the inverse problem solution. A parameterized model of material disturbances is employed. It allows the convenient incorporation of prior knowledge on the problem setup. Blurring and artifacts are inherently eliminated, allowing for more reliable and accurate reconstruction results.

1. Introduction
Ultrasound reflection tomography has been proposed for a variety of target applications like imaging industrial multiphase flows in process tomography [1], flaw detection in nondestructive material testing (NDT) [2], and breast tumor detection in medicine [3]. All applications have in common that acoustic impedance inhomogeneities, leading to the reflection of sound waves at interfaces, are to be imaged. The contrast between objects and background tissue in terms of reflectivity is lowest in medical applications. This allows the use of first order Born or Rytov approximations of the wave equation for image reconstruction under the corresponding assumptions of weak and isotropic scatterers [4, 5, 6].

In industrial applications, however, the acoustic impedance contrast is usually much higher than in biological tissue. The ratio of the acoustic impedance of steel and a gas inclusion in NDT, e.g., is about $10^5$, so that the simplifications of the first order approximations cannot be applied. The situation is similar in industrial process tomography, which is concerned with imaging multiphase material distributions in industrial environments. Typical applications are the determination of process parameters like component flow rates and material fractions in gas/liquid and gas/solid flows and monitoring and control of mixing and filtering processes [1, 7]. The approaches reported in the literature are mostly based on sensing the acoustic impedance distribution in discrete two-phase flows (see, e.g., [8, 9, 10, 11, 12]). At gas/liquid and gas/solid interfaces virtually all of the acoustic energy is reflected back due to the very small acoustic impedance of gases compared to liquids and solids if the acoustic wavelength is small compared to the object dimensions. Therefore first order approximations cannot be employed.
and specific reconstruction algorithms have been adopted for industrial ultrasound tomography. Most methods are based on the backprojection of transmission or reflection measurements. However, the reconstructed images suffer from blurring and artifacts. Postprocessing steps like thresholding or image segmentation are often used to filter the reconstructions.

In this work a general model-based approach to image reconstruction from reflection data in the case of infinite contrast is presented. Material inclusions in a background medium are modeled by closed contours. The curves are parameterized by B-spline functions or Fourier coefficients, allowing for a wide range of admissible shapes \[13, 14\]. The image reconstruction procedure consists of identifying the optimal set of model parameters and is performed through a Bayesian filtering approach. In particular we use a sequential Monte Carlo method in the form of particle filtering \[14, 15, 16\]. This allows the convenient incorporation of prior knowledge on the material distribution for the solution of the highly nonlinear ill-posed inverse problem. The model-based approach inherently eliminates common problems like blurring and artifacts.

In the subsequent Section the used problem setup is introduced, followed by a discussion of backprojection. Then the model-based reconstruction approach is developed by introducing the process model and the forward problem formulation. The key component of the method is the Bayesian inversion algorithm and is discussed in some detail. Reconstruction results are finally presented to show the validity of the proposed approach.

2. Problem Setup

The configuration assumed in this work is a closed circular aperture with a finite number of sensors. The imaging domain is the interior of the aperture. This is the configuration most frequently encountered in industrial process tomography, where the material distribution in the cross-section of a pipe is imaged. Similar setups are used for breast imaging \[3\]. The individual transducers can both transmit and receive acoustic waves. The totality of acquired data consists of multiple projections. For a single projection measurement one transducer is excited with a broadband pulse to transmit a wave. All transducers then act as receivers to collect fractions of the emitted wave that have either been directly transmitted across the domain or reflected at material interfaces. The desired information is basically coded into the travel times and peak amplitudes of the received pulses. Other projections are recorded by changing the transmitting transducer.

The used sensor configuration is sketched in Fig. 1, where 16 transducers T1–T16 are evenly spaced around the imaging domain. Compared to medical imaging only a limited number of spatially separated transducers is usually employed for industrial tomography due to time and cost constraints. The wide fan-shaped beam of the transducers in the imaging plane is sketched by the dashed lines emerging from T2. The received projection data depend on the position and shape of material inclusions. As an example a circular object is depicted in Fig. 1. The acoustic wavelength in the background medium is commonly assumed to be large compared to the inclusion dimensions, so that scattering effects are negligible and specular reflection occurs \[9, 10\]. Our approach is aimed at discrete gas-liquid or gas-solid material distributions, which allows to assume total reflection at material interfaces. At a water-air interface, e.g., the pressure reflection coefficient is 99.8\%. The basic cases of signal formation, through transmission and simple reflection, are illustrated Fig. 1 for the case of T2 acting as transmitter. Direct through transmission can be detected, e.g., by transducers T7, T8, and T9. The transmissions can be distinguished from reflected pulses because their travel times are known from the fixed geometry of the setup if the sound velocity in the background medium is known. T3 to T6 are outside the beam region of T2, so that no direct transmission occurs for these transducers. Reflection signals are, e.g., detected by T3 and T16 as the emitted wave is partially reflected back at material interfaces.
Figure 1. Typical configuration of ultrasound reflection tomography with a closed circular aperture. The transducers T1–T16 are evenly spaced around a pipe and feature fan-shaped beam profiles in the imaging plane. Different pulse travel times for reflected and directly transmitted pulses can be recorded as one transducer after the other is used as transmitter. The travel times of reflected pulses define ellipsoidal loci in the imaging domain.

3. Backprojection Reconstruction
The most commonly used method for reconstructing images from projection measurements in industrial ultrasound tomography is backprojection. The pixel-based approach basically consists of combining the contributions of individual measurements to form the pixel values in the image plane. It is typically assumed that the recorded time series contain isolated echoes. Then discrete echo loci can be extracted from the signals using thresholding or cross-correlation. In transmission tomography only directly transmitted pulses are used and backprojected onto straight lines connecting the transmitting and receiving transducer. In the case of discrete two-phase material distributions binary backprojection can be applied [10, 17], where individual backprojections are multiplied element-wise. This approach avoids the blurring usually associated with backprojection reconstructions. However, the minimum detectable object size is strictly limited by the transducer spacing.

In reflection tomography the backprojections take the form of ellipsoidal loci in general [8, 9, 18]. A pulse transmitted and received by the same transducer gives rise to a circular arc centered at the transducer location. If transmitter and receiver are different the backprojection is an ellipsoidal locus with the foci of the ellipse at the transducer positions. The images obtained from backprojection generally suffer from blurring. For a point reflector it shows a $1/r$ form with the distance from the reflector $r$ under idealized conditions [18]. With realistic conditions for industrial systems further background noise and artifacts are introduced through limited sampling of the aperture and multiple reflections.

The reconstructed images can be improved by using a filtered backprojection algorithm [18], where the projections are convolved with a deblurring filter before backprojection. The method yields good results for full apertures with dense sampling. However, the background noise level is increased through the filtering operation if only a limited number of projections is sampled. This restricts the use of filtered backprojection in industrial tomography, where usually only a limited number of transducers and measurements are available due to real-time and cost constraints.

An attempt to improve the interpretability of the blurred images is the application of threshold filters [9]. The thresholded image contains only two types of pixels, interface pixels
corresponding to material boundaries, and background pixels corresponding to bulk of either material. The nonuniform sensitivity distribution of reflection tomography can be accounted for with a nonlinear threshold function. Reconstructed edge intensities tend to be higher in the central region of the imaging plane as this area is interrogated by more transducers then regions close to the pipe walls. So the optimal threshold value is higher in the center of the pipe. In an ideal situation the interface pixels would give a closed contour and allow the unambiguous calculation of further parameters like material fractions. However, the reconstructed contours are not closed in many cases and noise pixels present in the image additionally complicate image interpretation.

Usually either transmission or reflection tomography is employed although projections may give rise to transmitted as well as reflected pulses. So some of the acquired information would not be used in the reconstruction process. A possibility to incorporate transmission measurements in reflection tomography is to backproject them with negative amplitude [9, 11]. So the pixel intensities are actually reduced along through-transmission paths. However, image degradations may occur where object contours are located close to these paths. Then the transmission backprojection may reduce the values of boundary pixels, leading to reduced contrast [1].

A postprocessing approach to overcome the limitation that reconstructed contours may be disconnected was presented in [19]. It is based on the Hough transform which is a well known tool in image processing. The Hough transform is able to identify parameterized objects, like circles of variable position and radius, buried in incomplete and noisy images. It maps the probability of occurrence of an object in the image space to a parameter space, where a single point corresponds to the parameter set defining a certain instance of the object. Point clusters in the parameter space suggest the existence of the corresponding object in the image. It is able to treat multiple objects without further modifications. However, the computational complexity is exponentially increasing with the number of parameters since the dimension of the parameter space increases accordingly. Consequently only circles of variable radius were considered in [19].

A direct model-based approach to image reconstruction without resorting to backprojection was proposed for a specialized application in [12]. The position and diameter of the air core of a hydrocyclone were to be determined for varying configurations of the device. Tailored to this specific application, the air core was approximated by a circle. The diameter and the position of the circle were fitted to the reflection data from several transducers using a simplified model assuming only small deviations from the central position. It was found that the achievable accuracy was superior to backprojection reconstruction. The iterative model-based reconstruction approach introduced in the subsequent Sections is based on a general formulation suitable for arbitrary object shapes and positions. In addition, it allows a unified treatment of reflection as well as transmission data.

4. Forward Model
A crucial component of iterative reconstruction algorithms for time-critical applications is the efficient solution of the forward problem. In the present case it consists of simulating the pulse arrival times for a given material distribution in the measurement plane. Since it has to be solved multiple times it has a major impact on the execution time of the reconstruction algorithm. Full numerical solutions of the wave equation, e.g. using the finite element or boundary element method, are far too time-consuming for the intended real-time-approaching applications.

One possibility to simulate the forward problem under the mentioned assumptions is ray-casting [20, 21]. It yields a high frequency asymptotic solution of the wave equation by applying the principles of geometrical optics. The sound beam emitted by a transducer is finely discretized into rays that are individually traced through the imaging region. Every ray is followed from the source until an intersection with a reflecting obstacle or the pipe occurs. The reflected ray is then determined using the law of reflection and propagated further. Rays that hit receiver
positions are recorded. This approach is able to cope with arbitrary obstacle configurations and can account for multiple reflections. However, the simulation is rather time-consuming due to the need for fine discretization in order to obtain reliable results.

To allow for fast image reconstruction with iterative algorithms a simpler approach based on Fermat’s principle is proposed in this work. Fermat’s principle in acoustical terms states that the acoustic path length between two transducers must be minimal. An acoustic ray reflected at an arbitrary contour on its way from the transmitter T to the receiver R is sketched in Fig. 2. Without loss of generality a local Cartesian coordinate system \((x, y)\) is assumed, with the axes along the surface tangent and normal at an arbitrary point \((x_c, y_c)\) on the contour \(c\). The total travel time \(t_t\) of the ray is determined by the sound velocity \(V_0\), the transmitter location \((x_T, y_T)\), receiver position \((x_R, y_R)\), and the location of the point under consideration,

\[
t_t = \frac{1}{V_0} (s_1 + s_2) = \frac{1}{V_0} \left( \sqrt{(x_c - x_T)^2 + (y_c - y_T)^2} + \sqrt{(x_R - x_c)^2 + (y_R - y_c)^2} \right). \tag{1}
\]

The partial derivative of the travel time with respect to a displacement of the candidate reflection point \((x_c, y_c)\) along the contour is

\[
\frac{\partial t_t}{\partial c} = \frac{\partial t_t}{\partial x_c} = \frac{1}{V_0} \left( \frac{x_c - x_T}{s_1} - \frac{x_R - x_c}{s_2} \right) = \frac{1}{V_0} (\sin \alpha_1 - \sin \alpha_2). \tag{2}
\]

A necessary condition for a minimum of the travel time of a ray reflected at \((x_c, y_c)\) is that (2) equals zero. Since it holds that \(\alpha_1 < \pi/2\) and \(\alpha_2 < \pi/2\), \(\partial t_t/\partial c = 0\) is only satisfied for \(\alpha_1 = \alpha_2\). It can easily be shown that the second derivative \(\partial^2 t_t/\partial x_c^2 > 0\) for all cases.

This result is in agreement with the law of reflection and offers an efficient way of determining the travel times of singly reflected pulses. In the numerical implementation the reflecting contour is discretized pointwise and the path lengths are calculated for all points on the contour. The desired path between a transmitter and a receiver with reflection at the contour is the minimum length path. As an additional constraint it has to be considered that there are no intersections of the path with a reflecting contour. This simulation of the forward problem can be easily solved for arbitrary reflectors since only simple algebraic operations and a minimum search are necessary. Multiple reflections are not considered in our case because this would exponentially increase the required number of investigated ray paths. If such multiple reflections occurred in the measurement they would lead to a remaining residual in the solution of the inverse problem. They are also a nuisance in backprojection reconstruction since they cannot be resolved and lead to backprojection artifacts.
5. Parameterized Contour Model

The process model used in the present approach, where discrete two-phase distributions are assumed, represents material inclusions by means of a closed boundary contour \( c \in \mathbb{R}^2 \). The regions are distinguished by their acoustic impedance, leading to total reflection of sound waves at the boundary. The contour is a function \( c(s) : \mathbb{R} \to \mathbb{R} \times \mathbb{R} \) with the parameter \( s \in [0, 1] \). \( c \) is closed if \( c(0) = c(1) \). Different representations of contours have been proposed in the literature, e.g. chain codes, Fourier series, and B-splines (see [22] for an overview). The common denominator of these contour models is the representation of \( c \) by as few parameters as possible while still meeting the requirements of the given application in terms of versatility. The set of parameters needed to fully describe the contour at any instance in time is referred to as the state of the model. Thus the reconstruction problem can be recast in the form of a state estimation problem.

The B-spline contour model is particularly well suited for the present application since it gives smooth contours, does not oscillate between sampling points, and allows for a simple representation of occurring shapes in industrial tomography and NDT. \( c(s) \) is represented by piecewise polynomial curves parameterized by a set of \( N \) control points. B-splines composed of third-order polynomial basis functions \( b_n(s) \) with \( n = 0, \ldots, (N - 1) \) are most commonly used. Such cubic B-splines feature continuous first and second derivatives. With \( b(s) = (b_0(s), b_1(s), \ldots, b_{N-1}(s))^T \) and the vector of control points \( q \in \mathbb{R}^{2N} \) the B-spline contour model can be written as

\[
\hat{c} = \begin{bmatrix} b(s) & 0 \\ 0 & b(s) \end{bmatrix} \begin{bmatrix} q^x \\ q^y \end{bmatrix} = U(s)q.
\]

The number of parameters required to fully parameterize the shape can be reduced by introducing the shape space representation of B-splines [13]. The shape space is given by an affine transformation that maps a shape space vector \( x \) to a spline vector \( q \) using a reference spline \( q_0 \),

\[
q = Wx + q_0.
\]

The shape space vector is used as state vector for the subsequent parameter estimation problem. Compared to the dimension of the spline vector the low-dimensional state vector allows for a considerable reduction in the complexity of the shape representation. The reference contour can be freely chosen, offering the flexibility to incorporate various levels of prior knowledge. The shape matrix \( W \) enforces that deviations from the reference spline are bound to certain classes of meaningful deformations. We apply the affine transformation with five degrees of freedom, allowing for translation, rotation, scaling, and shear of 2D splines, as sketched in Fig. 3, with the transformation matrix [14]

\[
W = \begin{pmatrix} 1 & 0 & q_0^x & 0 & 0 & q_0^y \\ 0 & 1 & 0 & q_0^x & q_0^y & 0 \end{pmatrix}.
\]

6. Particle Filtering

Using (4), a dynamic state space representation of a contour can be introduced,

\[
\begin{align*}
x_k &= f(x_{k-1}, v_{k-1}) \\
z_k &= h(x_k, w_k),
\end{align*}
\]

where \( f(\cdot) \) represents the transition of the state \( x \) from time step \( k - 1 \) to \( k \) subjected to process noise \( v \). A measurement based on the current state \( x_k \) subjected to measurement noise \( w \) is modeled by \( h(\cdot) \). With (6) the reconstruction problem can be formulated as the problem of estimating the state of a dynamic system. In addition to approaches for reconstructing static
setups, the use of dynamic models allows to extend the algorithm towards dynamically changing environments. A powerful class of algorithms that are able to estimate the unknown state vector of dynamic systems given the measurements $z_l, l = 0, 1, \ldots, k$ are the Bayesian estimators. A well known example for the linear case is the Kalman Filter (KF).

The Particle Filter (PF), which is adopted in this work, covers both nonlinear state transitions and measurements, as present in (6), as well as non-Gaussian and even multimodal probability densities of the state variables [23]. This makes the method even applicable to problems with multiple inclusions. However, in the following only the single inclusion case is considered. The PF is a sequential Monte Carlo method based on a point mass representation of the state densities, consisting of particles $x$ in state space and their corresponding weights $w$,

$$f_X(x) \approx \{x^{(m)}, w^{(m)}\}_{m=1,\ldots,M}.$$  

(7)

Based on the set of particles the recursive PF consists of two main steps, prediction and measurement update, related to the state transition and measurement equations of the state space model (6).

In the prediction step the transition model, formulated as conditional density $p(x_k|x_{k-1})$, is used to obtain the predicted state $p(x_k|Z_{k-1})$ at time $k$, where $Z_k = \{z_1, \ldots, z_k\}$. The calculation follows the Chapman-Kolmogorov equation

$$p(x_k|Z_{k-1}) = \int_{\Omega} p(x_k|x_{k-1}) p(x_{k-1}|Z_{k-1}) \, dx_{k-1}.$$  

(8)

The transition encompasses a deterministic drift and a stochastic diffusion of the states. The number of samples used to represent the state density is crucial for real-time operation. In general a trade-off between the quality of approximation and processing time is required. A residual resampling step is applied to ensure a good approximation using as few particles as possible [24].

The second step of the PF iteration uses the measurement model to estimate the posterior density $p(x_k|Z_k)$ from the current measurement and the available prior knowledge by applying Bayes’ theorem,

$$p(x_k|Z_k) = \frac{p(z_k|x_k)p(x_k|Z_{k-1})}{p(Z_k)}.$$  

(9)
The measurement likelihood \( p(z_k|x_k) \) incorporates the outcome of the nonlinear forward problem to give the probability of occurrence of a state \( x_k \) given the actual noisy measurement \( z_k \). A modified multivariate Gaussian distribution is used as likelihood function in this work,

\[
p(z_k|x_k) \propto \exp \left( -\frac{1}{2\sigma_0^2} \left\| \sqrt{u_k} \otimes (z_k - h(x_k)) \right\|^2 \right),
\]

where \( u_k \) is the vector of measured pulse amplitudes and \( \otimes \) denotes element-wise multiplication. The pulse amplitudes are included in the likelihood to express the degree of confidence in a measurement, as small amplitude signals have a higher probability to be corrupted by nuisance effects like multiple reflections, diffraction, and cross-coupling. \( \sigma_0^2 \) relates to a generalized variance with contributions from the measurement noise covariance \( \text{cov}(w) \) and the model error \( \text{cov}[h(x_k) - h^d(x_k)] \), with the discretized mapping \( h^d \) that is actually used in the algorithm. Model errors may have serious impact on the solution of ill-posed inverse problems [25, 26] and are therefore included in the proposed approach, especially as the used forward model is highly simplified due to computational constraints. Experimental justification for the choice of (10) will be given in the subsequent Section.

The outcome of the measurement update, and thus an iteration of the PF, is a set of weighted samples which is used to approximate the posterior distribution. From these samples, any estimate of the system state such as the expectation can be calculated,

\[
\hat{x}_k = E\{x_k|Z_k\} \approx \sum_{m=1}^{M} w_k^{(m)} x_k^{(m)}.
\]

7. Simulation Results
The synthetic measurement data used in this Section was generated with a ray casting algorithm that additionally incorporates characteristics of physical measurement systems like impulse responses, instrumentum circuits, measurement noise, and beam profiles of the transducers. The ray casting simulation includes effects not considered in the fast forward problem solution of the proposed approach like multiple reflections and interference of partial wave fields, which consequently act as model errors. The measurement likelihood (10) is based on a modified Gaussian distribution. The difference between measured and simulated pulse travel times contains two contributions, the measurement noise and the model error. While travel times can usually be measured with small standard deviation, a numerical investigation of the distribution of the model error was performed. The model error was determined for multiple positions of a circular contour in the imaging domain, where the assumed true shape simulated through ray casting was additionally distorted using higher order contour terms. The histogram of the obtained amplitude-weighted error values is shown in Fig. 4. The density of the model error resembles a Gaussian distribution, which is therefore adopted in the PF algorithm.

The feasibility of the proposed model-based ultrasound tomography approach is assessed with the reconstruction of an inclusion that resembles an ellipsoid, but shows two contractions. The parameterized reference shape used in the reconstruction process is a circle, so that arbitrary ellipsoids can be matched in state space, but contractions add to the model error. The PF was initialized with 50 circular particles uniformly distributed in the imaging domain. In the presence of prior knowledge the initial prior distribution can be adjusted in the framework of Bayesian estimation. The result of the reconstruction after 50 iterations of the PF is shown in Fig. 5, where the true inclusion with the contractions is depicted as dashed black contour. The thick blue contour shows the expectation of the posterior state density. The thin contours refer to the individual particles with color according to their weights derived from the color-bar to the right. It can be seen that the expectation closely matches the convex parts of the true contour and presents a robust fit despite the present modeling limitations.
Figure 4. Histogram of the amplitude-weighted model error for uniformly distributed modeling errors in the whole imaging domain.

Figure 5. Reconstruction result of the particle filter algorithm after 50 iterations. The dashed black contour is the simulated true inclusion and the thick blue contour shows the expectation. The individual particles are plotted with colors according to their respective weights.

Fig. 6 shows the evolution of the states over the iterations of the PF by means of the state expectations. The baseline with values of zero for all states corresponds to the reference contour. The first two states, corresponding to the center of the particle, go through rapid changes in the early iterations. After the coarse position of the inclusion is found there are smaller adjustments in the higher order states to adapt to the true contour.

8. Conclusion
Ultrasound reflection tomography is, e.g., used for monitor industrial multi-phase processes and nondestructive material testing (NDT). These cases can be modeled as binary gas-liquid or gas-solid material distributions, which offer very high contrast in terms of acoustic impedance. This limits the application of reconstruction algorithms like filtered backprojection that have
Figure 6. Evolution of the expectations of the contour states over the iterations of the PF. The first two states correspond to the position of the contour, while the higher order states account for its deformations.

been successfully applied in other fields, because they are based on several assumptions like weak inhomogeneities. In this paper the forward problem is defined differently by assuming infinite contrast between the background and the material inclusions to be reconstructed. A possibility to simulate the forward problem in this case is ray casting. However, the application of Fermat’s principle leads to a sharp reduction of the computational demands in order to enable the application of iterative reconstruction algorithms. Material inclusions are modeled as closed contours based on B-splines. A low order parameterization of the contours allows the introduction of a nonlinear dynamic state space model, and consequently the formulation of the inverse problem as state estimation problem. The problem itself is solved using a particle filter in the framework of nonlinear Bayesian estimation. The algorithm allows for the use of arbitrary reference contours and the incorporation of available prior knowledge. Due to proper modeling it shows to be robust against unavoidable modeling errors. Blurring and artifacts, as present in backprojection reconstructions, are inherently avoided. Although only a static setup has been addressed the proposed algorithm can be readily applied to dynamical reconstruction problems.

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