On The Experience Of Constructing A Vented Under-Floor Space With Heat-Insulated Fences Under The Buildings Of The Lightweight Steel-Framed Constructions On Permafrost Soils

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Abstract. The main reason of the violation of the thermal regime of buildings from lightweight steel-framed constructions (LSFC) are established. Considering the low efficiency of the additional thermal insulation of the outer envelopments it is proposed to construct a cold under-floor space with heat-insulated fences. Such constructive approach reduces the influence of air infiltration in the winter period and thereby improves the thermal regime of low-rise frame buildings. A mathematical model of the thermal interaction of the buildings with permafrost soils has been developed considering the climate, thermophysical parameters of the foundation soils, the size and configuration of buildings, and the thermal characteristics of the external fencing. The results of numerical calculations of soils thawing depth and under-floor air temperature are given when enveloping them with heat insulated panels. The most optimal parameters of the outer panels of the vented under-floor space are proposed for the preservation of the soils of buildings in the frozen state during exploitation.

1. Introduction

In the construction of residential and public buildings in the remote settlements of the Far North, the technology of frame buildings using light steel thin-walled structures (LSFC) is increasingly being used. The main advantages of this technology lies in the low weight of structures, the combination of load-bearing and enclosing functions of thin-walled profiles, rapid assembly of structures [1, 2].

The experience of the exploitation of these buildings has shown that their basement part is the most vulnerable from the point of view of ensuring the thermal protection of buildings [3, 4]. LSFC buildings have numerous heat-conducting elements in the form of steel sections, which significantly reduce the given resistance to heat transfer of external enclosing structures [5-8]. Results of numerical analyzes [9] have shown that any rends and insufficiency of abutment of heat-insulating materials to structural frame elements or poor performance of various joints lead to the intrusion of cold air, which has a big influence on the temperature regime of the building. In buildings with a vented under-floor
space, this negative effect in the basement of the building is enhanced by increased air infiltration during the winter period, due to the difference of the external and internal air pressure.

One of the first experiments using LSFC technology in the Far North was the construction number houses in the village of Zhatai, Republic of Sakha (Yakutia). Since the beginning of the operation of these houses, it had been discovered that the internal air temperature and the internal surface of the outer envelopes do not fit to requirements of thermal protection of buildings according to construction rules (SP) 50.13330.2012. "Thermal performance of the buildings”. Based on the results of thermal imaging studies at the nodes of conjugation of the basement overlap with the wall structure, pronounced "thermal bridging" (also known as "cold bridging") were revealed. At an outside temperature of -34 °C in the thermograms of the floor surface of the first floor, the lower temperature limit is fixed at -2.0 °C and the upper limit is fixed at +11 °C. In the current situation, additional heat insulation of external walls, attic floor and basement overlap was performed to restore the temperature of the buildings in use. After the above works, the actual temperature regime of the buildings from the LSFC has not practically improved. Calculation of the temperature fields of the basement overlap, taking into account additional thermal insulation, shows that the line with zero temperature at the locations of the vertical wall profiles, as in the original solution, is located close to the inner surface of the wall (Figure 1).

Figure 1. The distribution of temperature in the basement overlap after additional insulation at an outside temperature of -54 °C: a - basement overlap; b - Layout of connection with outer envelope
1 – basement’s double-tee girder; 2 – corrugated sheets H75; 3 – fabric-reinforced sand cement screed; 4 – expanded polystyrene M-35; 5 – rack steel section; 6 – mineral wool slab П-75; 7 – 2 layers of gypsum board of 12.5 mm. thickness; 8 – heat insulation “Knauf insulation”; 9 – ventilated facade; 10 – additional heat insulation (cellulose wool)

In the current situation and taking into account the impossibility of resettlement of tenants, it was proposed to improve the temperature conditions of buildings from the LSFC by reducing the effect of air infiltration through under-floor space with thermally insulated fences (Figure 2).
Figure 2. The constructing a vented under-floor space with heat-insulated fences under the buildings

In conditions of permafrost soils, the main principle of building multi-storey buildings is the preservation of the frozen state of the foundation soils (according to principle I). The traditional way of building multi-storey houses is practicing the frame buildings with a cold and ventilated cellar under-floor. The method of constructing foundations according to principle I guarantees the absence of a thawing bowl under the building and thereby prevents undesirable deformations of the foundation. But this method involves expensive and technically complicated insulation of the basement overlap. This is largely justified for multi-storey houses, but leads to an undesirable rise of the cost for low-rise constructions. At the same time, the Institute of Permafrost SB RAS has carried out a series of observations of the studies confirming the possibility of frame low-rise buildings with warm undergrounds with a small width of buildings and the presence of permafrost soils. According to data collected by Saltykov N.I. in Yakutsk under the wooden buildings with a width of up to 10-12 meters with double-layered floors and underfloor space with a height of up to 0.3 ... 0.4 meters, there is no thawing of frozen soil [10]. Melnikov P.I. and Shamshura V.Ya. concluded that in the construction of residential buildings, we can confine ourselves to the arrangement of a double-layer warm floor and an underground with a height of 0.25-0.5 meters [11]. According to the observations of Porkhayev, under many pre-revolutionary buildings in Yakutsk, the depth of thawing for 20-30 years of operation reached only 2.5-3.5 meters [12].

The authors of [13, 14] substantiated the possibility of regulating the temperature regime of the soils, which allows thawing of the ground beneath the building to a depth less than the depth of seasonal thawing of the grounds outside the building. The method assumes the construction of a thermal fence around the perimeter of the under-floor space and the thermal insulation of the basement overlap, while the parameters of thermal insulation are determined by mathematical modeling. The method assumes the installation of a thermal fence around the perimeter of the under-floor space and the thermal insulation of the basement floor, while the parameters of thermal insulation are determined by mathematical modeling. By the appropriate selection of the insulation parameters of the fence along the perimeter of the under-floor space and the basement, it is possible to provide the necessary temperature regime of the ground, while the cost of thermal insulation is comparable or less than in the case of using a cold underground. An additional advantage in this method is the favorable heat exchange regime through the basement floor, due to the increase of the underground air temperature of the building.

Modeling of heat exchange processes of freezing and thawing soils has a relatively long history, at present a whole series of mathematical models have been developed that describe the process of freezing-thawing of permafrost [13-19]. The process of thawing of permafrost soils, taking into account the dynamics of changes of the atmosphere temperature, total solar radiation, surface albedo, snow cover thickness, and convective heat exchange coefficient using mathematical modeling methods, was studied in [20, 21]. In [22], the authors found that the depth of thaw, calculated by the construction rules SP 25.13330.2012 “Soil bases and foundations on permafrost soils”, does not take into account the thermal resistance of the basement floor in buildings of small dimensions. The currently accepted methods for constructing foundations of buildings on permafrost soils are based on the results of studies [10, 11, 12], which give overestimated depths of thawing under the buildings.
Miscalculation are due to the use of empirical dependencies that do not take into account the dynamics of the change in temperature conditions due to imperfections in the methods of calculating heat and mass transfer processes used in those years.

The most practical mathematical model of the process of freezing-thawing of soils is the classical Stefan problem [23, 24]. Currently it is possible to more accurately calculate the temperature field of soils in the foundations of buildings using the increased computing capabilities and the broad development of mathematical modeling. It became possible to develop numerical models with a high degree of detail and accuracy, considering most of the factors of heat exchange of buildings and soil [14].

The purpose of this paper is to justify a technical solution for improving the floors temperature of the ground floor of low-rise buildings from the LSF by building an unventilated underground with heat-insulated fences. This solution will reduce the effect of cold air infiltration during the winter period due to an increase in the temperature of the air in the underground and will reduce the effect of cold air infiltration during the winter period due to an increase in the temperature of the air in the underground space.

To do this, it is necessary to perform a numerical analysis of the temperature regime of the soils of the under buildings considering unventilated underground with heat-insulated fences, air temperature in the underground space for a long period of operation and other various factors.

2. Methods

The main task is to determine the thickness of the thermal insulation of the underground fence, with the exception of the formation of a bowl of thawing, in connection with an increase in the temperature of the underground during the winter period.

To solve this problem, a mathematical model of heat exchange of building that have underground space, with permafrost soils has been developed, taking into account the thermal flows through the basement floor and the walls of the underground.

The model is based on the solution of the three-dimensional heat conduction problem:

\[
\left[ c \rho(T) + m_p L \frac{d \rho}{dT} \right] \frac{\partial T}{\partial t} = \frac{\partial}{\partial x} \left( \lambda(T) \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( \lambda(T) \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( \lambda(T) \frac{\partial T}{\partial z} \right),
\]

\( x, y, z \in \Omega, t > 0; \)

\( \lambda(T) \frac{\partial T}{\partial x} = 0, x = \pm \infty; \)

\( \lambda(T) \frac{\partial T}{\partial y} = 0, y = \pm \infty; \)

\( \lambda(T) \frac{\partial T}{\partial z} = \alpha^* (T - T_\infty) + (1 - \Lambda) Q, z = 0; \)

\( T(x, y, z) = T_\infty(x, y, z); \)

The temperature and heat transfer coefficients on the surface are determined as follows

\[
T_\infty = \begin{cases} 
T_u, & \text{outside the building} \\
T_{\infty u}, & \text{under the building} 
\end{cases};
\]

\[
\alpha^* = \begin{cases} 
\alpha_u, & \text{outside the building} \\
\alpha_{\infty u}, & \text{under the building} 
\end{cases};
\]

\[
\alpha^* = \frac{\alpha_u + \frac{S_u}{S_{\infty u}} \alpha_2}{1 + \frac{\alpha_u}{\alpha_{\infty u}} + \frac{S_u}{S_{\infty u}} \alpha_{\infty u}};
\]
Where \( \alpha_1, \alpha_2, \alpha_n \) - coefficients of heat exchange on the ceilings of the basement overlap, walls of the underground and on the floor; \( T_B, T_H \) - the temperature of the indoor and outdoor air, \( S_B = ab \) and \( S_* = 2(b+a)H_u \) - the floor area and the total area of the walls of the underground, with length \( a \), width \( b \) of the building and height of the underground \( H_u \). Thus, the temperature in the underground in an explicit form is not included into the heat balance.

The problem is solved numerically by the method of total approximation. When constructing difference schemes, the solution of the three-dimensional problem can be represented as a sequential solution of two-dimensional problems for horizontal layers with the subsequent solution of a set of one-dimensional problems along vertical columns that coincide with the nodes of the horizontal grid to account for heat transfer in the vertical direction. The main effect of heat transfer along the vertical to the temperature field is characteristic near the upper boundary, where heat exchange occurs on the surface and the main seasonal phase transitions are localized. This allows you to apply a non-uniform grid spacing vertically, which makes it possible to speed up the counting time without reducing the accuracy of calculations. The area in horizontal section has the form of a square with a side of 60 m. The building is located in the center. On the vertical direction area is 50 m deep.

The three-dimensional model, unlike two-dimensional models, can take into account the influence of the ratio of the size of the building and the height of the underground.

For calculations, the values of the thermal physical parameters of the soil were used, which were close to those given in [12]. Climatic conditions were set for the city of Yakutsk, since Zhatai is located at a distance of 21.5 km. The annual course of the outdoor air temperature was consistent with long-term observations, the influence of solar radiation on Set of Rules 23-101-2004 "Thermal performance design of buildings" and albedo change of the surface was taken into account. The temperature inside the building was accepted as meeting the sanitary standards and equaled +21 ° C.

\[
T = A \cos\left(\frac{2\pi t - t_o}{n}\right) + B
\]

where \( A = -31.8 \) °C, \( B = -10.2 \) °C -respectively, the amplitude and average annual temperature of the outside air for the latitude of Yakutsk; \( n = 365.2449 \) days - the average duration of the Gregorian year; \( t_o = 9 \) days. (January 9 is the day when the minimum negative temperature is observed); \( t \) - is the time in days.

The objects of numerical research are the soils of two dwelling houses from LSFC of various configurations. The building No.1 has a rectangular configuration of 30x15 m (46 apartments house in the village of Zhatai), building No.2 - L-shaped of 20x30x15 m (44 apartment house in the village of Zhatai). The height of the underground in both buildings is 1.2 meters. R-value of the basement overlap of buildings is taken equal to 8.4 m² °C / W according to the results of the engineering calculation of the overlap after additional heat insulation with green fiber. As a thermal insulation of the envelopments of the underground of buildings, a mineral wool board with a coefficient of thermal conductivity \( \lambda = 0.042 \) W / m °C has been adopted. Calculations were carried out for two types of buildings using two types of the enclosure of the underground with a thickness of the thermal insulation layer of 0.05 and 0.1 m, respectively.

Geomorphologically, the site of the investigated objects is located within the first supra-flood terrace of the Lena River. In the geological structure of the construction site, to the depth of 10 m,
there are upper-caliper alluvial deposits, represented by a stratum of stratified silty, small and medium-sized sand. The thickness of sand from the surface is covered by a cover of clayey soils-loam and sandy loam, with a thickness of up to 2.3 m. To the depth of 0.3...0.7 m in loam and sandy loam contain fragments of broken brick, gravel and pebbles. The temperature of the beginning of freezing of medium saline sands, depending on the concentration of the pore solution, is minus 0.41-0.58 °C, for slightly saline clayey soils is minus 0.43-0.72 °C. The measured temperature at a depth of 10 m is minus 2.6-3.5 °C. The thickness of the seasonal thawing layer is 3.0 m.

3. Formatting the text
The calculations were carried out using the application developed by the authors based on the above mathematical model of the thermal interaction of buildings with permafrost soils, taking into account the elements of the climate, the thermophysical parameters of the soil, the size and configuration of buildings, and the thermal characteristics of the fences. A two-dimensional version of the calculation program was used for the calculations. Taking into account the establishment of the long-term temperature regime of soil grounds, the estimated time is taken up to 10 years.

Numerical studies and long-term observations show that the soil’s top layers base has greatest influence to the process of heat transfer in permafrost. So, to simplify the task, the ground base is considered as consisting from two layers: loam to a depth of 1 m, sand below 1 m.

The monthly calculation of all climate elements for Yakutsk is used in the program describing the heat transfer to permafrost soils: the monthly average values of outdoor temperature, wind speeds, parameters of the snow cover, heat of evaporation and melting of snow, total solar radiation on the horizontal surface, albedo of the active surface, effective radiation. The internal air temperature is assumed to be constant at +21 °C. As an example, the mean monthly values of the outside air temperature are given in Table 1.

| Table 1. The average monthly temperatures of outdoor air in Yakutsk |
|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|------------------|
| January          | February         | March            | April            | May              | June             | July             | August           | September        | October          | November         | December         |
| -39,6            | -35,0            | -20,8            | -5,2             | 7,3              | 16,1             | 19,1             | 15,1             | 5,9              | -8,0             | -28,2            | -38,1            |

The results of calculations of the air temperature in the underground space and the depth of thawing of permafrost under the buildings are shown in Fig. 3-4. With a thermal insulation thickness of 0.05 m of the envelopment of the unventilated underground of Building No. 1, the temperature of the air in the coldest days of the winter period is -7.0 °C in the first year of operation and after stabilization in subsequent years -5.0 °C... -4 °C, in the warmest days of the summer period +1.8 °C...+2 °C. In this case, the depth of thawing under the building No. 1 in the first year after the warm up period is predicted to be 0.9 m and in subsequent years 1 m, when the depth of thawing outside the building is calculated to be 2.4 m.

With an increase in the thickness of the insulation of the fence of the underground to 0.1 m, no special changes in the temperature of the air in the underground are observed. In winter, the temperature of the air in the underground rises by 0.1-0.2 °C in comparison with the previous version of the thermal insulation of the underground. At the same time, stabilization of the thawing depth of soils under building No. 1 with the thickness of the underground fence taking place in 4 years. In the first year, the depth of thawing under the building is 0.8 m, and gradually increases, reaching a constant value by the fourth year - 1 m.

The results of calculating the depth of the ground thawing and air temperature in the underground of the L-shaped building No. 2 showed that the air temperature in the underground under building No. 2 with different thickness of insulation of the fence is higher than under building No. 1. With a thermal insulation thickness of 0.05 m, the minimum air temperature in the underground in the first year is -6.5 °C and in subsequent years after stabilization an average of -3 °C. In the warm period of the year, the air temperature in the underground does not exceed +2 °C. For this type of building No. 2, an increase in the thickness of the thermal insulation of the enclosure of the underground to 0.1 m more influences...
the change in the air temperature in the underground than for building No. 1. So the air temperature in
the underground in the first year is -6.5 °С and in subsequent years after stabilization on the average -1
°С.

**Figure 3.** The dynamics of air temperature change in the underground building of building No. 1 (red
line) and outside the building (green line) by years with the thickness of the heat-insulating fence 0,05
m.

**Figure 4.** The dynamics of the depth of thawing under the building No. 1 (red line) and outside the
building (green line) by years with the thickness of the heat-insulating fence 0,05 m.

**Figure 5.** The dynamics of air temperature change in the underground building of building No. 2 (red
line) and outside the building (green line) by years with the thickness of the heat-insulating fence 0,05
m.
Figure 6. The dynamics of the depth of thawing under the building No. 2 (red line) and outside the building (green line) by years with the thickness of the heat-insulating fence 0.05 m.

Table 2 shows the numerical results of calculating the air temperature in the underground and the depth of thawing of the ground beneath the buildings. From this table it can be seen that the depth of thawing under buildings during the construction of an unventilated underground with insulated fences does not exceed the value of the seasonally thawed layer of soils outside the building.

Table 2. The results of calculations of the air temperature in the underground space and the depth of thawing of the grounds under the buildings during the seasonal cycle.

| Thermal insulation thickness, m | Minimum depth of thawing, m | Maximum depth of thawing, m | Minimum negative air temperature in the underground space, °C | Maximum negative air temperature in the underground space, °C | Minimum positive air temperature in the underground space, °C | Maximum positive air temperature in the underground space, °C |
|--------------------------------|-----------------------------|----------------------------|-----------------------------------------------------------|----------------------------------------------------------|-----------------------------------------------------------|-----------------------------------------------------------|
| Building No. 1                 |                             |                            |                                                           |                                                          |                                                           |                                                          |
| 0.05                           | 0.93                        | 1.17                       | -7                                                       | -3.7                                                     | +1.37                                                    | +1.8                                                     |
| 0.1                            | 0.78                        | 1.02                       | -6.8                                                     | -4.3                                                     | +1.79                                                    | +1.2                                                     |
| Building No. 2                 |                             |                            |                                                           |                                                          |                                                           |                                                          |
| 0.05                           | 0.79                        | 1.41                       | -6.5                                                     | -2.4                                                     | +1.29                                                    | +1.23                                                    |
| 0.1                            | 0.76                        | 1.42                       | -3.8                                                     | -0.42                                                    | +1.23                                                    | +1.2                                                     |

Separately, calculations of the ground state and air in the underground with a decrease in the thermal resistance of the basement of the building to 4.4 m² °C / W. Such a value of thermal resistance is adopted taking into account the possible shrinking of green fiber in the basement envelopment. Reducing the thermal resistance of the basement overlap leads to a slight increase in the temperature of the air in the underground. In this case, the duration of the period with positive temperatures increases substantially. This leads to an increase in the retention time of the melted period under the center of the building. The depth of thawing of soils averages 1 m and does not exceed the value of the seasonally thawed layer of soils outside the building.

Conclusion
1. To improve the temperature regime of operated buildings from LSFC by reducing the effect of air infiltration, a cold underground implementation is proposed by protecting the underground of buildings with heat-insulated panels, with the formation of a bowl of thawing when the temperature in the underground is raised.
2. The results of calculations of the thermal interaction of buildings with permafrost soils showed that with an underground height of 1.2 m and a resistance level of the thermal transfer of the basement overlap of buildings from 4.4-8.4 m² °C / W, the influence of the share of heat flux transferred to the ground on the temperature regime the grounds of the base are insignificant. The depth of thawing of the ground beneath the buildings during the enclosure of the underground with panels with thermal insulation made of mineral fibers of 0.05 m thickness is about 1.0 m, and the air temperature in the underground ranges from +2 to -7 °C. The increase in the thickness of the thermal insulation of the fence of the underground to 0.1 m practically does not affect the depth of thawing, and the amplitude of temperature oscillations in the underground decreases.

3. Currently, there is ongoing monitoring of the air temperature in the underground and the grounds of the basement of the considered buildings with thermally insulated substructures, which in the following will allow us to clarify the mathematical model of the thermal interaction of buildings with permafrost soils.

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