Cosmological constraints from the masses and abundances of $L_*$ galaxies

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ABSTRACT
We place limits on the mean density of the Universe ($\Omega_m$) and the effective slope of the linear power spectrum around a megaparsec scale ($n_{\text{eff}}$) by comparing the universal mass function to the observed luminosity function. Numerical simulations suggest that the dark matter halo mass function at small scales depends only on $\Omega_m(n_{\text{eff}} + 3)$ independent of the overall power spectrum normalization. Matching the halo abundance to the observed luminosity function requires knowledge of the relation between the virial mass and luminosity (separately for early- and late-type galaxies) and the fraction of galaxies that resides in larger haloes such as groups and clusters, all of which can be extracted from the galaxy–galaxy lensing. We apply the recently derived values from the Sloan Digital Sky Survey and find $\Omega_m(n_{\text{eff}} + 3) = (0.15 \pm 0.05)/(1 - f_{\text{in}})$, where $f_{\text{in}}$ accounts for the possibility that some fraction of haloes may be dark or without a bright central galaxy. A model with $\Omega_m = 0.25$ and primordial $n = 0.8$ or with $\Omega_m = 0.2$ and $n = 1$ agrees well with these constraints even in the absence of dark haloes, although with the current data somewhat higher values for $\Omega_m$ and $n$ are also acceptable.

Key words: galaxies: general – galaxies: haloes – theory – cosmology: dark matter.

1 INTRODUCTION

The halo mass function has been long recognized as a powerful probe of cosmology since the seminal work by Press & Schechter (1974). Most of the applications so far have focused on clusters, which are easy to detect in X-rays and for which the observed X-ray temperatures correlate well with the cluster mass. While there is currently some uncertainty in the normalization of this relation, upcoming X-ray and weak lensing observations should provide empirical means to calibrate it. Because the clusters are the most massive haloes formed in the Universe, they lie on the exponential tail of the mass function, whose amplitude depends mainly on the overall normalization of the power spectrum and the density parameter $\Omega_m$.

On the other hand, for the halo mass below the non-linear mass, the mass function depends only on the density of the Universe and on the effective slope of the linear power spectrum at that scale and it is independent of the power spectrum normalization (Press & Schechter 1974; Sheth & Tormen 1999; Jenkins et al. 2001).

To constrain these cosmological parameters with the mass function, we must be able to determine masses and abundances of haloes below the cluster scale. Groups with masses between $10^{13} - 10^{14} M_\odot$ are difficult to observe directly, because they contain only a handful of galaxies, and their abundance is quite uncertain in this range. Below $10^{11} M_\odot$, many of the haloes may not host a bright galaxy because of effects that prevent either gas cooling or star formation, such as ultraviolet (UV) background radiation, feedback or insufficient surface density for star formation. Moreover, a significant fraction of galaxies corresponding to these halo masses may be satellites inside a larger halo and it is necessary to correct for that, because the halo mass function only counts isolated haloes. Estimating this fraction is difficult for small haloes.

In the mass range $10^{11} - 10^{12} M_\odot$, typical for $L_*$ galaxies, the luminosity function is well determined and theoretical models suggest that each of these haloes should host a bright galaxy at the centre; see, for example, Kauffmann et al. (1999) and Benson et al. (2000). The main challenge in this range is to determine the relation between a galaxy luminosity and a halo mass, and the fraction of these galaxies that belongs to larger haloes such as groups and clusters. Even though we have good dynamical probes of mass within the optical region of a galaxy, such as the Tully–Fisher (TF) relation for late-type galaxies and the Faber–Jackson (FJ) relation or strong lensing for early-type galaxies, the relation between optical masses and virial masses is more uncertain, because the virial mass depends on the adopted density profile and a typical virial radius is a factor of 10 larger than the optical radius. Virial masses thus cannot be observationally constrained from the optical or H i studies alone; this was used in previous work on the mass function determination on galactic scales (Gonzalez et al. 2000; Kochanek 2001).

An alternative approach adopted here is to use virial masses derived directly from the galaxy–galaxy (g–g) lensing. In this method, we use tangential distortions of background galaxies by the foreground galaxy to place limits on the mass distributions around galaxies. A recent Sloan Digital Sky Survey (SDSS) analysis has presented g–g lensing results using a much larger sample than previously available, allowing a detailed study of the relation between
mass and light for several luminosity bands and morphological types (McKay et al. 2001). The latter is particularly important as the relation between mass and luminosity differs significantly between early- and late-type galaxies and it is necessary to extract the relations separately before combining them together. The morphological classification is based on the colour information and light concentration index and the early-type sample is dominated by ellipticals, but also includes S0 and Sa galaxies, while the late-type sample is dominated by Sb/Sc galaxies; see Strateva et al. (2001) for a thorough discussion of the relation to the Hubble sequence. The data are most sensitive to 100–200 h⁻¹ kpc transverse separations, which is a typical virial radius of a 10¹¹ h⁻¹ M⊙ halo. The second complication is the fraction of galaxies that is not in isolated haloes. This can also be extracted from g–g lensing; if there is a significant lensing signal around the galaxies at separations above 200 h⁻¹ kpc then it signals the presence of groups and clusters around them (Guzik & Seljak 2002). This allows us to determine the fraction of galaxies of a given luminosity in these larger haloes. Thus, together g–g lensing provides all the necessary information needed for a quantitative study of the halo mass function on galactic scales.

It is worth comparing this approach to the traditional mass-to-light ratio (M/L) method to obtain the density parameter Ωm; see, for example, Bahcall, Lubin & Dorman (1995) and Carlberg, Yee & Ellingson (1997). This often assumes that M/L extracted from some type of objects, such as groups or clusters, can be applied to the global M/L. Because we can measure the total luminosity density in the Universe, we can obtain the total matter density by multiplying the two. However, M/L depends on both the halo mass and scale on which it is measured. While light is concentrated to the inner parts of the clusters, mass continues to increase, so M/L for any given object increases with radius. Even adopting the virial masses, defined so that only baryons within that radius can condense to make stars, there is no reason why M/L should not depend on L. Theoretical models predict M/L to be at the minimum at galactic masses, where cooling and star formation are most efficient; see, for example, Kauffmann et al. (1999) and Benson et al. (2000). It is possible that there are many small haloes that have no light at all (e.g. Jimenez et al. 1997), which is why we cannot see them, but which contribute to the mass of the Universe. Moreover, there may be mass in the Universe that is not associated with any collapsed structures at all. For current generation of simulations only about 40 per cent of the total mass has been resolved into haloes above 10¹¹ h⁻¹ M⊙ (Jenkins et al. 2001). It is not obvious that as the resolution of simulations increases this fraction converges to unity, which is what is commonly assumed in the forms of mass function (Press & Schechter 1974; Sheth & Tormen 1999), because some fraction of the mass could remain in a diffuse form. All this makes any extrapolation of M/L to the total luminosity density highly uncertain as a method to deduce the density of the Universe. We can attempt to correct for this using simulations (e.g. Bahcall et al. 2000), but this relies on the ability of these to reproduce the light distribution in the Universe across a large dynamic range of masses and scales. This is one direction to pursue in the future as simulations and modelling of physical processes improve.

The alternative way is to measure mass and light over a much larger volume, so that it becomes representative for the whole Universe. Currently, there are no reliable methods that can measure the mean mass directly on such large scales. The closest method with which to achieve this goal is gravitational lensing, which measures ellipticity distortions of background galaxies. This method, however, cannot measure the mean component of the matter, which does not produce shear. So instead we must look at fluctuations around the mean by comparing the relation between galaxy light and convergence (Wilson, Kaiser & Luppino 2001). However, if galaxies are a biased tracer of dark matter then only a combination Ωm/b can be determined, and if b > 1 this will underestimate Ωm. Because this is a luminosity weighted statistic, it will give a larger weight to brighter galaxies, which are known to be biased relative to fainter galaxies; see, for example, Norberg et al. (2001a) and Zehavi et al. (2001).

Because it is difficult to detect a signal on large scales with gravitational lensing, where linear biasing applies, we must compare the fluctuations in light and mass at smaller scales. At any given scale, haloes of a certain mass dominate the fluctuations; see, for example, Seljak (2000) for a discussion of this in the context of halo models. For example, on a megaparsec scale the dominant fluctuations come from groups and clusters, while on a 50 kpc scale the galactic haloes dominate the fluctuations. This means that, even if the galaxies are unbiased, the relation between light and mass will be appropriate only for the haloes that dominate the fluctuations at that smoothing scale. As discussed above, this M/L relation may not be the universal one, because most of the mass may be in smaller objects or in diffuse structures, which have signals that are too weak to be detected through the weak lensing fluctuations. There is no preferred scale at which we should evaluate M/L to multiply it with the total galaxy light in order to derive the mean density of the Universe. If M/L increases with halo mass above L*, as suggested by observations (Girardi et al. 2002; Guzik & Seljak 2002), then the convergence–light correlation function will be more extended than that of light itself and there is some observational evidence for this effect (Wilson et al. 2001). It is not clear, however, whether this leads to an underestimate or overestimate of Ωm, since the trend of M/L to increase with mass is likely to be reversed below L*, and much mass associated with either diffuse structures or small haloes may not be associated with any light at all.

The above arguments suggest that the global M/L ratio method of determining the density of the Universe cannot be derived without making additional assumptions. In the approach presented here, we instead limit the analysis only to the galaxies around L*, which are well studied. We combine the virial mass to luminosity relations extracted from the SDSS data with the SDSS luminosity function of L*, galaxies and we compare this to the universal mass function to place limits on cosmological models. The analysis is carried out entirely within the SDSS data set, which reduces the uncertainties related to the photometric calibrations and colour transformations, which usually plague luminosity function comparisons.

2 RELATION BETWEEN GALAXY AND HALO ABUNDANCES

The halo mass function describes the number density of haloes as a function of mass. It can be written as

$$\frac{dn}{d\ln M} = \frac{\bar{\rho}}{M} f(\sigma) \frac{d\ln \sigma^{-1}}{d\ln M},$$

(1)

where \(\bar{\rho}\) is the mean matter density of the Universe, \(M\) is the virial mass of the halo, and \(n(M)\) is the spatial number density of haloes of a given mass \(M\). We have introduced a function \(f(\sigma)\), which has a universal form independent of the power spectrum, matter density, normalization or redshift if written as a function of rms variance of linear density field

$$\sigma^2(M) = 4\pi \int P(k) W_p(k^2) \, dk.$$

(2)
significant mass function, where $\ln$ is remarkable that the mass function in this form is almost constant virial radius where overdensity is 200 in units of mean density. It which they argue is universal if mass is expressed in terms of the slope

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Here $W_R(k)$ is the Fourier transform of the spherical top hat window with radius $R$, chosen such that it encloses the mass $M = 4\pi R^3 \rho/3$ and $P(k)$ is the linear power spectrum.

The relation between mass and rms variance depends on the linear power spectrum. For a smooth power spectrum, we can locally approximate it as $P(k) \propto k^{n_{eff}}$ and the relation is

$\frac{d \ln \sigma^{-1}}{d \ln M} = \frac{n_{eff} + 3}{6}. \tag{3}$

For cold dark matter (CDM) models on galactic scales, the effective slope $n_{eff}$ ranges between $-2.0$ and $-2.5$ (Fig. 1).

The universality of the mass function has been recently investigated by a number of authors (Sheth & Tormen 1999; Jenkins et al. 2001; White 2001). It has been shown that the mass function is indeed universal for a broad range of cosmological models. Jenkins et al. (2001) propose the following form

$f(\sigma) = 0.315 \exp[-(\ln \sigma^{-1} + 0.61)^3], \tag{4}$

which they argue is universal if mass is expressed in terms of the virial radius where overdensity is 200 in units of mean density. It is remarkable that the mass function in this form is almost constant for all haloes with $\ln \sigma > 0$. Here, we are interested in haloes on galactic scales where $\ln \sigma \sim 1$, so this limit applies and we can take $f \sim 0.3$, a universal value which varies only weakly with halo mass, as shown in Fig. 1 (dashed curve).

To relate the theoretical halo abundance with the observed galaxy abundance we must assume a relation between the galaxy luminosity and its halo mass. Here we use a direct probe of the halo virial mass obtained from the $g$–$g$ lensing of SDSS (McKay et al. 2001). Detailed modelling of CDM profiles show that these observations are best fitted with $L_*$ galaxies (which dominate the luminosity distribution of that sample) having a virial mass around $10^{12} h^{-1} M_\odot$ (Guzik & Seljak 2002). The virial mass strongly depends on the morphology or colour; for a given luminosity, early-type galaxies can be

Figure 1. $n_{eff} + 3$ for models with $\Omega_m = 0.3$, $\Omega_b = 0.04$, $h = 0.7$, $n = 1$ (solid curve, top), $n = 0.9$ (solid curve, middle) and $\Omega_m = 0.25$, $\Omega_b = 0.04$, $h = 0.65$, $n = 0.8$ (solid curve, bottom). All the transfer functions were computed using CMBFAST. Also shown is $f(\sigma(M))$ (dashed curve), which is essentially 0.3 over this range of masses.

An alternative and more model-dependent estimate of virial masses comes from the TF relation for late-type galaxies (Giovanelli et al. 1997) or FJ relation for early-type galaxies (Bernardi et al. 2001). These observe galaxy properties in the inner $10 h^{-1}$ kpc and do not directly measure the virial mass of the halo in which the galaxy sits. At these radii, the dynamical effect of baryons on the rotation curves (as well as on the dark matter distribution itself) is important and modifies the relation between the rotation velocity and virial mass. We can, however, model this assuming the stellar $M/L$ ratio and dark matter profile. The first comes from the stellar population synthesis models and is rather uncertain because of age, metallicity and initial mass function (IMF) (Bruzual & Charlot 1993), while the latter is obtained from the cosmological dark matter simulations and depends on the assumed cosmological model (Bullock et al. 2001). In addition, the response of dark matter to baryon cooling must be included and is often modelled with adiabatic contraction (Blumenthal et al. 1986). Adopting standard values for the dark matter profiles and the stellar $M/L$ ratios, we find that at $L_*$, the rotation velocity decreases by 1.8 from optical to virial radius both for early- and late-type galaxies, in good agreement with g–g lensing results (Seljak 2002). This agreement is encouraging, but we should be careful not to overinterpret it. Both virial masses and stellar $M/L$ ratios still carry considerable uncertainties and must be confirmed with future data. Note that throughout the paper we use rotation velocity $v$ to describe the density profile, $GM(r)/r = v^2$, where $M(r)$ is the mass within radius $r$. A flat profile describes the isothermal case $\rho \propto r^{-2}$, while a decrease in $v$ with $r$ indicates that the profile is steeper than the isothermal case.

Galaxy–galaxy lensing can also be used to determine the slope $\beta$ of the relation between mass and luminosity

$M = \left( \frac{L}{L_*} \right)^{\beta}, \tag{5}$

where $M_*$ is the mass associated with the $L_*$ galaxy. Using SDSS data, we find $\beta = 1.4 \pm 0.2$ in red bands, which becomes $\beta^\prime = 1.2 \pm 0.2$ after correcting for a luminosity-dependent fraction of early-type galaxies in the sample (Guzik & Seljak 2002). This is valid only for early-type galaxies between $L_*$ and $7L_*$, which dominate the g–g lensing signal. Note that this differs significantly from the FJ relation, which would predict $\beta \sim 2/3$, but agrees well with the detailed modelling of rotation curves for early-type galaxies (Seljak 2002). The main reason for the difference in these models, which are based on constant stellar $M/L$ and include adiabatic response of dark matter to baryon cooling, is that for more massive haloes the contribution of dark matter at the optical radius is reduced, so we cannot use stellar velocity dispersion directly as a halo mass estimator. To determine $\beta$ around $L_*$ for late-type galaxies we
cannot use g–g lensing, because the signal is too weak to be detected as a function of luminosity. Instead we use the TF relation, which gives \( L_i \propto T_{\text{opt}}^{3/4} \) (Giovanelli et al. 1997). We model the contributions from the stellar disc and adiabatic response of dark matter to disc formation to relate between the rotation velocity at the optical radius and virial velocity (or mass). We assume a Navarro, Frenk and White (NFW) profile with \( c_{\text{200}} = 10 \) and we use \( \gamma_T = 1.5h \), which were shown to reproduce well the virial velocity constraint from g–g lensing at \( L_e \). (Seljak 2002). We find \( \beta^* \sim 1 \) around \( L_e \), which is the value we adopt below.

The third parameter that we need is the fraction \( \gamma \) of galaxies that are at the centres of isolated galactic haloes, as opposed to larger haloes such as groups and clusters. Universal mass functions from \( N \)-body simulations only count isolated haloes, so it is necessary to correct for the fraction of galaxies at a given luminosity that is in larger groups and clusters. This fraction can be determined from the relative contribution to g–g lensing at small and large separations.

For \( L > \approx 300 h^{-1} \) kpc, the signal is dominated by groups and clusters, while below it is dominated by individual galactic haloes. An analysis of SDSS g–g lensing data finds \( \gamma = 0.72 \pm 0.1 \) for early-type galaxies and \( \gamma = 0.93 \pm 0.1 \) for late-type galaxies around \( L_e \) (Guzik & Seljak 2002). The error includes various systematic uncertainties, of which the galaxy occupation as a function of group and cluster mass is the most important. Thus, about 10–30 per cent of \( L_e \) galaxies reside in groups and clusters as defined in \( N \)-body simulations and we must reduce the galaxy abundance by this fraction when relating it to the halo abundance.

The abundance of galaxies of a given luminosity can be extracted from the luminosity function \( dN(\mathcal{L})/d \ln \mathcal{L} \), which determines the abundance per logarithmic interval of luminosity. It is often fitted to the Schechter form:

\[
\Phi(\mathcal{L}) = \frac{dn}{d \ln \mathcal{L}} = \phi_s \left( \frac{\mathcal{L}}{\mathcal{L}_*} \right)^{\alpha+1} \exp(-L/L_*) \exp(-\beta_0 - \beta L).
\]

For \( L_e \) galaxies, the abundance is \( \Phi(L_e) = \phi_e/e, \) where \( e = 2.718 \) is the natural logarithmic base constant. From the early SDSS analysis, the values for \( \phi_0 \) are \( 1.46 \times 10^{-2}, 1.28 \times 10^{-2} \) and \( 1.27 \times 10^{-2} \) (with a 10 per cent error) in units of \( h^3 \) Mpc\(^{-3} \) for \( r', i' \) and \( z', \) respectively, while the values for \( \alpha \) are around \( 1.2 \) to \( 1.25 \) with a few per cent error (Blanton et al. 2001). We choose these three bands because they show least variation in virial \( M/L \) ratio between early- and late-type galaxies, minimizing the colour dependence of the signal. In addition, the redshift evolution corrections in red bands are smaller than in \( g' \) or \( u' \). These have been suggested as one possible reason for the discrepancy between the 2D luminosity function in \( b_j \) and SDSS in \( g' \) (Norberg et al. 2001b).

The final ingredient that is needed is the fraction of early-type (\( \xi^e \)) and late-type (\( \xi^l \)) galaxies as a function of magnitude. As shown in Strateva et al. (2001), this fraction depends on luminosity, so that the early-type galaxies dominate at the bright end and the late-type galaxies dominate at the faint end. This statement is only valid for red bands and a reverse trend is seen in \( u' \), while \( g' \) shows comparable fractions almost independent of luminosity. In red bands around \( L_e \), the fraction is somewhat higher for the early-type galaxies. While the transition between the early- and late-type galaxies is not well defined, the transitional types (S0–Sa) do not dominate the counts, so here we simply assume a bimodal early/late distribution (\( \xi^e + \xi^l = 1 \)), rather than attempting to model the whole range of stellar ages and morphologies. This could be improved in the future as larger statistical samples are obtained and it is particularly important for spirals, which have a larger scatter in the stellar ages. Early-type galaxies are more homogeneous and older, so their scatter in the stellar \( M/L \) ratio should be small.

We can now combine the above equations to relate halo and galaxy abundances. This gives

\[
\frac{dn}{d \mathcal{L}} = \frac{dn}{d \beta dL} = \frac{\gamma \Phi(L) \beta}{6} = \frac{n_{\text{eff}} + 3}{\hat{\rho} M_{200m}} f. \tag{7}
\]

Using \( \hat{\rho} = 3 \Omega_m \rho_c = 2.77 \times 10^{11} \) h\(^{-1} \) M\(_{\odot}\) Mpc\(^{-3} \) we find that the minimum mean density at a given luminosity is given by

\[
\Omega_m(n_{\text{eff}} + 3) = 0.3 \frac{\xi L}{\hat{\rho}} \left( \frac{\Phi(L) M_{200m}}{1.4 \times 10^{10} M_{\odot} Mpc^{-3}} \right). \tag{8}
\]

Note that we must add up the contribution from both early- and late-type galaxies separately, where the two contributions have to be evaluated at equal mass, not luminosity. If there are dark haloes without a bright galaxy at the centre, the above expression becomes inequality and we can only place a lower limit on \( \Omega_m(n_{\text{eff}} + 3) \). We parametrize this uncertainty with the fraction of dark haloes \( f_{\text{d}} \), which in general is a function of halo mass.

We can evaluate this expression at different values for halo mass. The virial mass of an early-type galaxy at \( L_e = 2.1 \times 10^{10} h^{-2} \) L\(_{\odot}\) in \( i' \) is \( M_{200m} = 1.2 \times 10^{12} h^{-1} M_{\odot} \) (Guzik & Seljak 2002). Using \( \beta^* = 1.2, \gamma^* = 0.72 \) and \( \xi^* = 0.6 \) we find \( \Omega_m(n_{\text{eff}} + 3) = 0.14/(1 - f_{\text{d}}) \). To this we must add the contribution from late-type galaxies at the same mass. At \( L_e \), their virial mass is \( M_{200m} = 4.3 \times 10^{11} h^{-1} M_{\odot} \). Using \( \beta = 1 \) we find that for \( M_{200m} = 1.2 \times 10^{12} h^{-1} M_{\odot} \) the corresponding luminosity is \( L_\gamma \). At this luminosity, the fraction of late-type galaxies in the sample is already small, \( \xi^* \sim 0.2 \). In addition, from the luminosity function in equation (6) we can see that the abundance of \( L_\gamma \) galaxies has decreased by a factor of 10 relative to that of \( L_e \). So, late-type galaxies do not actually add much to the limit above and we also find \( \Omega_m(n_{\text{eff}} + 3) = 0.15/(1 - f_{\text{d}}) \). These constraints are evaluated in \( i' \), but we find similar constraints also in \( r' \) and \( z' \). The error budget is dominated by the errors on \( M_{200m} \), \( \gamma \) and \( \beta \), which combined give about 30 per cent uncertainty. We note that the value of \( \beta^* \) is determined between \( L_e \) and \( L_\gamma \) and we are assuming here that it does change significantly over this range. This is an assumption that cannot be directly verified at present, but with upcoming SDSS observations we should be able to verify it observationally.

We can repeat the same analysis one magnitude above and below \( L_e \). At \( L = 2.5 L_e = 5.2 \times 10^{10} h^{-2} M_{\odot} \), the sample is dominated by early-type galaxies, so \( \phi^e \sim 0.8, \beta^* \sim 1.2 \) and \( M = 3.6 \times 10^{12} h^{-1} M_{\odot} \). The fraction of these galaxies in isolated galactic haloes is not well determined, but it is likely to be larger than \( L_e \) (Guzik & Seljak 2002), so we assume \( \phi^e = 0.9 \). This gives \( \Omega_m(n_{\text{eff}} + 3) > 0.13/(1 - f_{\text{d}}) \), where we have ignored the very small contribution from the late-type galaxies. The effective slope is about 5–10 per cent higher than at \( 1.2 \times 10^{12} h^{-1} M_{\odot} \) (Fig. 1), so the obtained value is about 20 per cent lower than the value obtained at \( L_e \). If we assume that \( \phi^e \) does not differ from that at \( L_e \), we find that the two estimates are in perfect agreement. This is quite impressive given that the masses and abundances change significantly over this range. The error is comparable to the error at \( L_e \) and is of the order of 30 per cent. It is dominated by the error in \( \beta^* \), which also affects the virial mass, since the pivot point of the \( M/L \) relation is at \( L_e \).

One magnitude below \( L_e \) the sample is dominated by late-type galaxies, for which we use \( \beta^* \sim 1, M = 1.7 \times 10^{11} h^{-1} M_{\odot} \) and \( \phi^e \sim 0.9 \). Adopting \( \xi^* = 0.8 \) leads to \( \Omega_m(n_{\text{eff}} + 3) = 0.11 \). To this we have to add the contribution from early-type galaxies at \( M = 1.7 \times 10^{11} h^{-1} M_{\odot} \). If \( \beta^* = 1.2 \) extends to this mass range,

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this mass corresponds to $L \sim 0.1L_*$ and at this luminosity the fraction of early-type galaxies is about 10 per cent. This increases the above estimate by 20 per cent, so $\Omega_m(\eta_{500} + 3) = 0.13/(1 - \eta_{500})$. This estimate is more uncertain, because both $\beta^{1/2}$ and $y^{1/2}$ have not been directly measured over this range. In addition, the large scatter in stellar ages for late-type spirals leads to a scatter in the mass to luminosity relation. Note that at this mass we expect $\eta_{500} + 3$ to decrease by about 20 per cent relative to $1.2 \times 10^{12} h^{-1} M_\odot$ (Fig. 1), so this constraint is actually very similar to that at $L_*$ based on early-type galaxies, even though the masses differ by almost an order of magnitude.

From the above analysis, we find that over the range of masses between $1.6 \times 10^{11} h^{-1} M_\odot$ to $3.6 \times 10^{12} h^{-1} M_\odot$ the cosmological constraints on $\Omega_m/(1 - \eta_{500})$ are all very similar:

$$\Omega_m(\eta_{500} + 3)(1 - \eta_{500}) = 0.15 \pm 0.05. \tag{9}$$

The good agreement found over a wide range of mass suggests that the shape of the mass function agrees well with that predicted from cosmological simulations, assuming the fraction of dark haloes $f_{\rm dm}$ if different from zero, does not vary over this mass range.

3 DISCUSSION

In this paper, we propose a new method to derive cosmological constraints from the virial masses and the number density of $L_*$ galaxies, and we make a preliminary application to early SDSS observations. The method differs from other methods using the $M/L$ ratio in that it only uses this information around $L_*$ galaxies and not the overall luminosity density. This sidesteps the uncertainties related to the variation of $M/L$ with luminosity. The main observational inputs are the relation between virial mass and luminosity around $L_*$ as a function of morphological type and the fraction of these galaxies in isolated haloes (as opposed to groups and clusters), all of which can be extracted from $g$–$g$ lensing. Another essential ingredient is the luminosity function of galaxies around $L_*$ as a function of morphological type, which can be obtained from the same data as $g$–$g$ lensing information. The main theoretical input is the halo mass function, which has been shown to be universal by a number of studies (Sheth & Tormen 1999; Jenkins et al. 2001; White 2001).

The abundance of haloes depends only on the density parameter $\Omega_m$ and the effective slope of the linear power spectrum $n_{\text{eff}}$ through the combination $\Omega_m(n_{\text{eff}} + 3)$.

In the absence of dark haloes, the obtained constraint $\Omega_m(n_{\text{eff}} + 3) = 0.15 \pm 0.05$ is low compared to the predictions of the $\Lambda$CDM model with $\Omega_m = 0.3$, $h = 0.7$ and $n = 1$, which gives $\Omega_m(n_{\text{eff}} + 3) = 0.28$ at these scales. This is excluded by the current constraints, unless a significant fraction of haloes is dark over this range of masses. To bring the models into a better agreement with this constraint, we can either lower the effective slope or the mean density. The former can be lowered by reducing the primordial spectrum slope $n$, decreasing the shape parameter $\Gamma$, which depends on $\Omega_m$, $\Omega_\Lambda$ and the Hubble constant $H_0$ (see, for example, Eisenstein & Hu 1998), or by introducing warm dark matter (Bode, Ostriker & Turok 2001). For example, we find that a model with $\Omega_m = 0.25$, $h = 0.65$, $\Omega_\Lambda = 0.04$ and $n = 0.8$ gives good agreement with the observational constraints, but somewhat lower values of $\Omega_m \sim 0.2$ with $n = 0.9$–1.0 are also acceptable. For warm dark matter models, which suppress power on small scales, we compute transfer functions using CMBFAST (Seljak & Zaldarriaga 1996) and we find that the effective slope at the galactic mass scale remains almost unchanged for $m_s > 500$ eV and drops to $n_{\text{eff}} = -2.5$ at $m_s = 250$ eV. Such low masses are probably excluded from $Ly - \alpha$ forest studies (Narayan et al. 2000), although this must be confirmed with a more careful error analysis of $Ly - \alpha$ forest constraints.

While the conclusion that the density of the Universe and/or the slope of the power spectrum are lower than usually assumed is the most natural interpretation of the results, there are several possible sources of systematic uncertainty that can affect this conclusion. First, there is the possibility that the virial masses used here are too low. This is certainly possible for late-type galaxies, which have a weak signal in $g$–$g$ lensing and for which a factor of 2 increase in mass is less than a $2 - \sigma$ excursion. For early-type galaxies, the statistical error on the virial mass is 20 per cent, so a factor of 2 excursion is unlikely, although some systematic uncertainties remain in the $g$–$g$ lensing analysis (Guzik & Seljak 2002). The agreement between early- and late-type galaxies implies that the mass scale must be changed for both types, unless there are cancellations of errors. Furthermore, any increase in the virial mass would affect the agreement between $g$–$g$ lensing masses and the TF or FJ relation. An increase in virial mass by a factor of 2 would reduce the ratio of optical rotation velocity to virial rotation velocity from 1.8 to 1.4 (Seljak 2002). Within the context of adiabatic compression models, such a ratio is likely to be too small to be explained with CDM profiles and stellar $M/L$ ratios, as expected from stellar population synthesis models with IMFs discussed in the literature. This possibility would thus require us to give up a successful prediction of CDM models, that of the structure of dark matter haloes in the outer parts. We should nevertheless caution that this statement should be viewed as preliminary, because IMFs could still be significantly different from what is commonly assumed and other processes, such as feedback, could modify the predictions of adiabatic models. We plan to address some of these issues in the future with larger data sets from SDSS.

Another possibility is that the relation between the halo mass and luminosity $M \propto L^\beta$ is shallower than assumed here, i.e. $\beta \sim 0.6$ instead of $\beta \sim 1$–1.2. This would be the case if the virial $M/L$ ratio decreased with luminosity. Both $g$–$g$ lensing and modelling of optical relations are consistent with $\beta = 1$–1.5 at $L_*$ and above, and theoretical models also predict $\beta > 1$ in this regime; see, for example, Kauffmann et al. (1999) and Benson et al. (2000). While $M/L$ probably decreases with $L$ well below $L_*$, this is unlikely to be the case over the range of luminosities of interest here. If $\beta^* = 1$ instead of 1.2 is used here, this would increase the lower limit on $\Omega_m(n_{\text{eff}} + 3)$ by 20 per cent.

It seems similarly unlikely that the luminosity function can be wrong by more than 30 per cent, unless there is a large fraction of low surface brightness galaxies that are missed by the SDSS survey. There are still important calibration differences between different surveys, which can cause a mismatch in derived luminosity functions (Norberg et al. 2001b), but these are not relevant for our analysis as we derive the $g$–$g$ lensing relation between light and luminosity using the same sample that is used for luminosity function as well.

Yet another uncertainty is the division into early- and late-type galaxies and their associated fractions, which is somewhat artificial, because there is a continuous range of colours and light concentrations in the sample. What is really relevant is the stellar mass of the galaxy, which seems to correlate quite well with the virial mass (Guzik & Seljak 2002). For late-type galaxies, the range of stellar ages is large and this introduces a spread in the luminosity for a given stellar mass, so representing them with a single $M/L$ ratio may not be accurate. This is less of an issue for early-type galaxies, which are very old and for which the spread in the stellar $M/L$ ratio is small. On the other hand, early-type galaxies tend to reside in denser environments and the fraction of these in groups and clusters...
is larger. This correction is also somewhat uncertain, as we must assume how the groups and clusters are populated with these galaxies to determine it (Guzik & Seljak 2002). There is also the issue whether the mass profile of galaxies within groups and clusters differs from that of the same luminosity in the field within the inner 100 kpc. Observations (Guzik & Seljak 2002) and numerical simulations (Ghigna et al. 2000) suggest that they do not differ much, but the uncertainties are still large. We can find an upper limit to this effect by looking at the extreme case when there is no mass attached to individual galaxies inside groups and clusters. Then we should use $\gamma' = 1$, which would increase the estimated $\Omega_m (n_{\text{eff}} + 3)$ from early-type galaxies by 30 per cent.

The choice of the virial radius and its associated mass, defined differently by different authors, has a minor effect on our results. From the simulations, it is usually defined by the friends-of-friends algorithm (assuming a specific value of linking parameter, e.g. 0.2), but it is not always clear how this relates to the specific mean density within the virial radius. For consistency with Jenkins et al. (2001) we define the virial radius as the value where the mean overdensity is $\delta = 200$, while the observed masses are usually expressed in terms of overdensity relative to the critical density. For a given halo profile, we can convert between the two, but the conversion depends on the assumed density parameter $\Omega_m$. Fortunately the differences are not very important on galactic scales, where haloes are highly concentrated and where the mass around the virial radius only slowly grows with radius (logarithmically in the limit of a large concentration where the slope at the virial radius approaches $-3$). For example, the difference in mass between $\delta = 200 \Omega_m$ for $\Omega_m = 0.4$ and $\Omega_m = 0.2$ in a halo of $c = 12$ is 10 per cent, so this is not the dominating source of error. This effect has been included in the above estimates.

The remaining uncertainty is the fraction of dark haloes in the Universe. Theoretically, we would not expect haloes to be dark around $L_e$, where the efficiency of cooling and star formation is high. This is supported by the fact that, for the haloes that we do see, a large fraction of the baryons within the virial radius has converted into stars (Guzik & Seljak 2002). At lower halo masses, below $10^{13} h^{-1} M_{\odot}$, the fraction of dark haloes may increase because some of the formed discs may be stable against star formation (Verde, Oh & Jimenez 2002). At higher halo masses above $10^{13} h^{-1} M_{\odot}$, we enter the group regime, cooling becomes less efficient, and a significant fraction of these haloes may not host a bright galaxy at the centre. For example, applying the same analysis as in this paper to 7 $L_e$ early-type galaxies, we find the abundance of haloes to be several times below that expected from the mass function, a consequence of exponentially decreasing luminosity function at the bright end. This is not surprising, because a large fraction of the groups in this range probably hosts several fainter galaxies rather than a single bright galaxy at the centre. It is difficult to determine the abundance of such groups directly from the optical data, as it is not obvious which groups are virialized to satisfy the halo definition in an N-body simulation and which are just a collection of galaxies approaching each other for the first time (as, for example, the local group). The fact that the constraints derived here agree from 1.6 to $10^{13} h^{-1} M_{\odot}$ to $3.6 \times 10^{12} h^{-1} M_{\odot}$ suggests that, over this range of masses, the simplest possibility with $f_{\text{halo}} = 0$ is consistent with the data.

The prospects to determine the fraction of dark haloes directly seem difficult. The only direct way to observe these is through gravitational lensing, but in the absence of a significant baryon condensation such haloes are inefficient strong lenses (Kochanek & White 2001). The haloes that do cool and form a disc without making stars could be somewhat more efficient as lenses for a given halo mass, but are expected to have lower halo masses. Most of the lenses are bright early-type galaxies, which both reside in massive haloes and have a significant baryonic contribution to the lensing cross-section. It is thus possible that even if some fraction of haloes around $10^{12} h^{-1} M_{\odot}$ exist without a central galaxy, they may not have been detected so far with strong lensing. Current surveys such as SDSS may provide better limits on the fraction of dark haloes as a function of halo mass.

It is clear from the above discussion that the errors in the current analysis are still quite large, although the fact that the constraints are consistent over a range of masses increases the confidence level of the final result. It is interesting that the constraints obtained are in good agreement with the cluster gas fraction determination of the matter density (Erdogdu, Ettori & Lahav 2002) and with the redshift distortions and bias determination from 2dF (Verde et al. 2001). They are also comparable or somewhat higher than those using the global $M/L$ ratio (Bahcall et al. 2000; Wilson et al. 2001), although this method, as discussed above, may well be biased. Similarly, a tilted CDM model may help alleviate some of the small-scale problems with CDM (Alam, Bullock & Weinberg 2002). This perhaps indicates the need for a somewhat lower $\Omega_m$ or $n_{\text{eff}}$ than the previously suggested $\Omega_m = 0.3-0.4$, $n = 1$ model.

While the error from the method presented here is still large, the prospects to improve it are good. Currently, the errors are dominated by observationally determined parameters $M_{\text{200m}}$, $\phi$, $\beta$ and $\gamma$. These errors are dominated by statistics and have been obtained by using only 5 per cent of the final SDSS sample. With the full sample, we can reduce the error on each of these significantly, as well as extending the observable range to the lower luminosity galaxies. With the full sample, we can also study the morphological dependence of the mass–luminosity relation in more detail, which as we have shown here plays an important role in the analysis. With spectroscopic information it should be possible to extract stellar mass information for each galaxy separately and to use $g$–$g$ lensing to relate the stellar mass to the virial mass directly without splitting the sample into morphological types, as done here. Theoretical errors are mainly caused by the accuracy of the mass function over this mass range, but the uncertainty is already at a 10 per cent level and can be further improved with simulations that cover a wider range of cosmological models. The remaining uncertainty is the fraction of dark haloes, which should also be determined with a comparable accuracy (or shown to be negligible). In this case, the method presented here may become an accurate test of matter density and slope of the power spectrum on a megaparsec scale.

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