Shear strength parameter determination for the single plane of weakness criterion: Implications for wellbore stability

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Abstract. Wellbore stability is a classic rock mechanics problem encountered during drilling and completion, which costs the drilling industry more than $100 million per year worldwide. The failure criterion is a very important aspect of the wellbore stability analysis. Accordingly, for shale rich in bedding, fissures, joints, and cuts, we often use the single plane of weakness criterion to describe the compressive shear strength. The anisotropic strength of a rock depends on the strength of the rock matrix and the weak plane (i.e., cohesive strength of the matrix, friction angle in the matrix, cohesion of the weak plane, and friction angle in the weak plane). However, directly and accurately determining the compressive shear strength parameters of the rock matrix and the weak plane using the conventional least square and molar stress circle methods is difficult because of the difference between the rock matrix and the weak plane. This study uses the grid search algorithm to quickly match the optimal matrix cohesion, matrix internal friction angle, weak plane cohesion, and weak plane internal friction angle through the published triaxial test data of two sets of anisotropic rocks. The four parameters of the compressive shear strength materials obtained by the grid search algorithm are used to predict the results, which are very consistent with the experimental results. The correlation coefficient ($R^2$) above 0.98 is enough to meet the engineering requirements. Taking a shale gas well in the Sichuan Basin as an example, a collapse pressure prediction model is finally established by combining the grid search method to determine the material parameters of the compressive shear strength, shear wellbore instability model, and SWP criterion. Consequently, the model results are found to be in good match with the engineering practice.

1. Introduction

Borehole stability is a very important rock mechanical problem encountered in petroleum drilling engineering [1–4]. A reasonable drilling fluid density can effectively maintain the borehole stability and prevent a series of complicated down-hole accidents, such as borehole collapse and pipe sticking and burying. Most of the formations with wellbore instability are shale formations, and the cost of dealing with borehole instability accounts for more than 15% of the entire investment and development [5].

The major step for the wellbore stability analysis includes the determination of the stress distribution around the borehole wall and the selection of a failure criterion [5–6]. The failure criterion for predicting the borehole collapse is generally divided into three categories: stress failure criterion;
strain failure criterion; and energy failure criterion. Among which, the application of the stress failure criterion is the most commonly used for the wellbore stability analysis [3, 6]. These criteria consider some material parameters. The least squares (LS) method [7, 8] is often used to determine these material parameters, especially for the nonlinear isotropic criterion; however, it requires a very high quality of experimental data under the triaxial compression condition. Accordingly, the grid search method combined with the failure criterion can be used to determine the optimal material parameters from the overall experimental data to obtain the optimal material parameters of rocks. Colmenares and Zoback [9] used the grid search method combined with the isotropic criteria to find the optimal material parameters with the smallest average standard deviation of all experimental data. They then discussed the applicability of different criteria to different rock types.

Banz and Schwab [10] used the grid search method to determine the parameters of six 3D criteria. Consequently, they found that the Hoek–Brown–Matsuoka–Nakai criterion fitting accuracy is the best one. Bahrami [11] used the grid search and axis changing methods to compare various failure criteria. The obtained results indicated that the modified Lade criterion has the best match with the experimental data. Meng [12] considered the effect of temperature on rock strength and compared the difference between the grid search method and Molar’s stress circle to obtain the material parameters. As a result, the modified Lade criterion was proven to be the most suitable for considering the rock strength at high-temperature conditions.

The grid search method has been widely applied to determine the material parameters of different isotropic failure criteria for several kinds of rock materials. However, almost all related studies did not use the grid search method to determine the material parameters of the anisotropic strength criterion. Shale has the following characteristics: hard and brittle lithology; well-developed bedding; and in-situ stress complex. These characteristics result in a well-developed plane of weakness in shale rock. Shale is a typical kind of anisotropic rock because of its well-developed plane of weakness. The single plane of weakness (SPW) criterion is often used to characterize its strength variation with angle. Therefore, this study aims to determine the shear strength parameters of anisotropic rocks using the SPW criterion and the grid search method. First, we introduce the major theoretical formulas for the SPW criterion and the grid search method. Next, we identify two sets of published triaxial test data [11] to verify the accuracy of the grid search method and the SWP criterion. Finally, the present method is used to provide the wellbore stability implications of shale formations.

2. Methodology

2.1. Theory of the SPW criterion

The SPW theory was proposed by Jaeger [13] to describe the shear failure conditions of a single or a group of parallel planes of weakness. Figure 1 illustrates that a group of planes of weakness AB is developed in the rock mass assuming that the angle between the AB plane (normal) and the direction of the maximum principal stress is β. According to Mohr’s stress circle theory and Coulomb’s criterion, the shear failure condition of the weakly planar rock mass can then be obtained as follows [13–16]:

\[
\sigma_1 = \sigma_3 + \frac{2(c_0 + \sigma_3 \tan \varphi_0)}{(1 - \tan \varphi_0 \cot \beta) \sin 2\beta} \quad \beta_1 \leq \beta \leq \beta_2
\]  

(1)

A rock body failure will occur if Eq. (1) is not satisfied:

\[
\sigma_1 = \sigma_3 + \frac{2(c_0 + \sigma_3 \tan \varphi_0)}{(1 - \tan \varphi_0 \cot \beta_0) \sin 2\beta_0} \quad \beta < \beta_1 \text{ or } \beta > \beta_2
\]

(2)

where, \(\beta_1\) and \(\beta_2\) are given as follows:

\[
\beta_1 = \frac{\varphi_0}{2} + 0.5 \arcsin \left[ \frac{(\sigma_1 + \sigma_3 + 2c_0 \cot \varphi_0 \sin \varphi_0)}{\sigma_1 - \sigma_3} \right]
\]

(3)
\[
\beta_2 = \frac{\pi}{4} + \varphi_w - \beta_1 \quad (4)
\]

where, \(\sigma_1\) is the maximum principal stress, MPa; \(\sigma_3\) is the minimum principal stress, MPa; \(c_w\) is the cohesion of the rock plane of weakness, MPa; \(\varphi_w\) is the internal friction angle of the rock plane of weakness, (°); \(c_0\) is the cohesion of the rock body, MPa; \(\varphi_0\) is the internal friction angle of the rock body, (°); \(\beta\) is the angle between the weak plane normal and the maximum principal stress, (°); \(\beta_0\) is the angle between the body failure plane and the maximum principal stress, (°), \(\beta_0 = \pi/4 + \varphi_0\) and \(\beta_1\) and \(\beta_2\) are the angle limitations during failure along weak planes, (°).

**Figure 1.** Theoretical analysis diagram of the rock strength with a single plane of weakness [14].

### 2.2. Grid search method

Meng [12] used the triaxial test data to predict the difference between the two types to test the failure criterion accuracy under static loading conditions. The grid search method was also used. The major process is as follows:

1. Set the ranges of the desired rock material parameters (i.e., matrix cohesion \(c_0\), matrix internal friction angle \(\varphi_0\), weak plane cohesion \(c_w\), and weak plane internal friction angle \(\varphi_w\)).
2. Mesh the grid to form nodes within the range.
3. Use each node of rock material parameters to calculate the predicted strength for each testing sample.
4. Utilize the objective function for the calculation. Draw a contour map for the objective function value. Find the rock material parameters corresponding to the minimum objective function value as the optimal material parameters. The objective function is a very important aspect, and the average error (\(\text{Diff}_{\text{ave}}\)) and the absolute average error (\(\text{Diff}_{\text{abs}}\)) can be used:

\[
\text{Diff}_{\text{ave}} = 100 \frac{\sum_{i=1}^{n} \left[ \left( (\sigma_i)_{\text{exp}} \right) - \left( (\sigma_i)_{\text{pre}} \right) \right]}{n \times \text{UCS}} \quad (5)
\]

\[
\text{Diff}_{\text{abs}} = 100 \sqrt{\frac{\sum_{i=1}^{n} \left( (\sigma_i)_{\text{exp}} - (\sigma_i)_{\text{pre}} \right)^2}{n \times \text{UCS}}} \quad (6)
\]

where, \([(\sigma_i)_{\text{exp}}]\) and \([(\sigma_i)_{\text{pre}}]\) are the experimental and predicted strengths of each testing sample, respectively, MPa; \(n\) is the number of triaxial test data; \(\text{UCS}\) is the uniaxial compressive strength, MPa; \(\text{Diff}_{\text{ave}}\) is the average error (dimensionless); and \(\text{Diff}_{\text{abs}}\) is the absolute average error (dimensionless).

For the anisotropic rock, Eqs. (1)–(4) show that only a few parts of the angles can meet the conditions of the plane of weakness failure. Thus, as rewards the calculation of the strength of each testing sample, if \(\beta_1 \leq \beta \leq \beta_2\), Eq. (1) should be used to calculate the rock strength. If \(\beta < \beta_1\) or \(\beta > \beta_1\), Eq. (2) should be used to calculate the rock strength.

### 2.3. Result evaluation method

On the basis of the grid search method, we must judge the degree of deviation of strength predicted from the optimal material parameters to evaluate and validate the fitting degree of the material parameters obtained from the grid search method. The reliability of the optimal material parameters
can then be evaluated. The coefficient of determination is cited as follows to evaluate and validate the optimal material parameters [8]:

\[
R^2 = 1 - \frac{\sum_{i=1}^{n} \left[ (\sigma_{1})_{exp,i} - (\sigma_{1})_{pre,i} \right]^2}{\sum_{i=1}^{n} \left[ (\sigma_{1})_{exp,i} - \sigma_{1\text{ave}} \right]^2}
\]

(7)

where, \(\sigma_{1\text{ave}}\) is the average triaxial strength (MPa), and \(R^2\) is the coefficient of determination (the closer this value is to 1, the better the reliability and the accuracy).

3. Results and discussions

Two groups, namely Green River (GR) shale 1 and GR shale 2, were used for the triaxial testing [17]. We identified these two sets of triaxial testing data to validate the present method. Combining the SPW criterion and the grid search method, we used the GR shale triaxial testing data to determine the optimal material parameters. We conducted a grid search to allow the cohesion and internal friction angle to vary within a specific range and meet the applicability of the criterion to the triaxial test data. We selected the optimal fitting parameters for the cohesion and internal friction angle of the rock body and the bedding plane by minimizing the absolute average error (Diffabs) from the data. The advantage of this method is that all triaxial test data are computed, and the quality of individual triaxial test data is not deliberately evaluated. The grid search method matched the optimal material parameters (Figure 2). The color bar in the legend of each subplot represents the absolute average error (Diffabs) indicating the accuracy of the prediction results. Table 1 lists the optimal material parameters. For GR shale 1, the cohesion of the bedding plane was 33.2 MPa; the internal friction angle of the bedding plane was 27.5°; the cohesion of the rock matrix was 49.2 MPa; and the internal friction angle of the bedding plane was 30.3°. For GR shale 2, the cohesion of the bedding plane was 29.0 MPa; the internal friction angle of the bedding plane was 17.7°; the cohesion of the rock matrix was 42.9 MPa; and the internal friction angle of the bedding plane was 19.7°. In summary, the cohesion and the internal friction angle of the bedding plane are smaller than those of the rock matrix for the same kind of rock.

![Plane of weakness of GR shale 1](image1)

![Rock matrix of GR shale 1](image2)
The obtained optimal material parameters were used to verify the accuracy of the grid search method and the SWP criterion. The predicted strengths calculated by the optimal material parameters were compared herein with the triaxial testing data (Figure 3). The solid line represents the predicted strength calculated by the optimal material parameters, while the dots with different colors represent the triaxial testing results. A dot below the solid line means that the SWP criterion underestimated the rock strength, while a dot above the solid line denotes that the SWP criterion overestimated the rock strength. Overestimation or underestimation is expressed by the average error (Diff_{ave}), where the average error (Diff_{ave}) is a positive means overestimation. The absolute average error (Diff_{ave}) indicates the accuracy of the optimal material parameter. The smaller the absolute average error (Diff_{ave}), the closer the test dots to the predicted values and the higher the accuracy for the corresponding failure criterion. Figure 3 depicts that the predicted strength is in a good agreement with the experimental results. Table 2 presents the average error (Diff_{ave}) and the absolute average error (Diff_{ave}) of these two rocks. Diff_{ave} and Diff_{ave} under the conditions of the optimal material parameters were very small for these two groups of rocks, especially for the rock body or matrix. In other words, the optimal material parameters obtained by the search method can successfully match with the triaxial testing results. $R^2$ was also used for the overall data evaluation. When predicting the triaxial testing results, the correlation coefficient $R^2$ of GR shale 2 can reach 0.9901, while that of GR shale 1 was greater than 0.98. These results indicate that the grid search method combined with the SPW criterion can accurately obtain the material parameters of the anisotropic rocks.

(c) Plane of weakness of GR shale 2

(d) Rock matrix of GR shale 2

**Figure 2.** Optimal material parameters of the GR shale determined by the grid search method.

**Table 1.** Optimal material parameters of the GR shale.

| Rock types | Sample size (mm) | Rock matrix | Plane of weakness |
|------------|-----------------|-------------|-------------------|
| GR shale 1 | Φ25 × 50        | 49.2        | 30.3              |
| GR shale 2 | Φ25 × 50        | 42.9        | 19.7              |

The obtained optimal material parameters were can be used to verify the accuracy of the grid search method and the SWP criterion. The predicted strengths calculated by the optimal material parameters were compared herein with the triaxial testing data (Figure 3). The solid line represents the predicted strength calculated by the optimal material parameters, while the dots with different colors represent the triaxial testing results. A dot below the solid line means that the SWP criterion underestimated the rock strength, while a dot above the solid line denotes that the SWP criterion overestimated the rock strength. Overestimation or underestimation is expressed by the average error (Diff_{ave}), where the average error (Diff_{ave}) is a positive means overestimation. The absolute average error (Diff_{ave}) indicates the accuracy of the optimal material parameter. The smaller the absolute average error (Diff_{ave}), the closer the test dots to the predicted values and the higher the accuracy for the corresponding failure criterion. Figure 3 depicts that the predicted strength is in a good agreement with the experimental results. Table 2 presents the average error (Diff_{ave}) and the absolute average error (Diff_{ave}) of these two rocks. Diff_{ave} and Diff_{ave} under the conditions of the optimal material parameters were very small for these two groups of rocks, especially for the rock body or matrix. In other words, the optimal material parameters obtained by the search method can successfully match with the triaxial testing results. $R^2$ was also used for the overall data evaluation. When predicting the triaxial testing results, the correlation coefficient $R^2$ of GR shale 2 can reach 0.9901, while that of GR shale 1 was greater than 0.98. These results indicate that the grid search method combined with the SPW criterion can accurately obtain the material parameters of the anisotropic rocks.

**Figure 3.** Comparison of the testing and theoretical results for the GR shale.

**Table 2.** Diff_{ave} and Diff_{ave} with the optimal material parameters for the GR shale.

| Rock types | Sample size (mm) | Diff_{ave} | Diff_{ave} | Diff_{ave} |
|------------|-----------------|------------|------------|------------|
| GR shale 1 | Φ25 × 50        | 2.1402     | 0.0043     | 5.4053     |
| GR shale 2 | Φ25 × 50        | 2.1186     | 0.0637     | 2.9174     |

4. Implications for the wellbore stability

4.1. Material parameter determination

The experimental samples were taken from the Silurian Longmaxi shale formation in the Southern Sichuan Basin. The Longmaxi shale lithology is mainly dominated by black carbonaceous and
calcarenous shale with parallel interactions of thin and thick layers. The Longmaxi shale is mainly composed of quartz, feldspar, calcite, and clay minerals mixed with a few dolomite and pyrite [18]. A standard rock sample measuring Φ25 × 50 mm was prepared along different angles from 0° to 90°. The triaxial compression tests were conducted for Longmaxi shale. Table 3 presents the testing results. The shale rock reached the smallest strength when β = 60°.

Table 3. Testing results of the triaxial compression for the Longmaxi shale.

| β (°) | σ1 = 0 MPa | σ1 = 10 MPa | σ1 = 20 MPa | σ1 = 30 MPa | σ1 = 40 MPa | σ1 = 50 MPa |
|-------|------------|-------------|-------------|-------------|-------------|-------------|
| 0°    | 64.3       | 110.5       | 156.7       | 210.3       | 255.4       | 301.5       |
| 15°   | 60.2       | 103.4       | 152.0       | 202.2       | 248.7       | 296.7       |
| 30°   | 58.6       | 98.7        | 143.2       | 183.4       | 238.8       | 280.8       |
| 45°   | 23.6       | 60.9        | 98.2        | 135.2       | 170.5       | 210.8       |
| 60°   | 18.2       | 52.1        | 80.1        | 110.2       | 144.6       | 180.8       |
| 75°   | 24         | 62.3        | 110.2       | 140.7       | 175.4       | 230.9       |
| 90°   | 65.2       | 116.7       | 160.3       | 208.2       | 265.4       | 310.7       |

The optimal material parameters were obtained using the grid search method and the SPW criterion. Figure 4 depicts the results. Table 4 lists the optimal material parameters. The cohesion of the bedding plane was 5.7 MPa. The internal friction angle of the bedding plane was 30.3°. The cohesion of the rock matrix was 13.7 MPa. The internal friction angle of the bedding plane was 40.7°. Table 5 presents the Diff ls and Diff ave values with the optimal material parameters. The difference between the predicted and actual strengths of the matrix and weak plane failures was very small. The predicted strength and testing results were drawn in the same figure to validate the optimal material parameters (Figure 5). The predicted strength data were in a good agreement with the experimental results. The overall data evaluation, where R² was 0.9921, indicated that the predicted strength was very close to the actual triaxial test strength.

(a) Plane of weakness of the Longmaxi shale  
(b) Rock matrix of the Longmaxi shale

Figure 4. Optimal material parameters of the Longmaxi shale determined by the grid search method.

Figure 5. Comparison of the testing and theoretical results for the Longmaxi shale.

Table 4. Optimal material parameters for the GR shale.
is the direction of the minimum horizontal principal stress. However, if the influence of the drilling azimuth is N109°E, the predicted result of the critical equivalent density of the collapse pressure is approximately 1.5 g/cm³ that deviates by approximately 90° from the direction of the maximum horizontal stress, whereas the most unstable well path should be a horizontal well (1.64 g/cm³). The stability of the deviated and horizontal wells is much better than that of the vertical well. Moreover, the optimal drilling direction for the wellbore stability is the direction of the minimum horizontal stress. However, if the influence of the plane of weakness is involved, the most stable well path should be a horizontal well (1.15 g/cm³) close to the direction of the maximum horizontal stress, whereas the most unstable well path should be a horizontal well (2.02 g/cm³) close to the direction of the minimum horizontal stress. The optimal drilling direction for the wellbore stability is the direction of the minimum horizontal stress.

The horizontal section of this well is drilled along the direction of the minimum horizontal principal stress; the drilling azimuth is N19°E; and the horizontal section is inclined by approximately 30°. The predicted result of the critical equivalent density of the collapse pressure is approximately 1.50 g/cm³ if the influence of the plane of weakness is ignored (Figure 6(a)). However, the predicted result of the critical equivalent density of the collapse pressure is approximately 2.02 g/cm³ if the influence of the plane of weakness is involved (Figure 6(b)). That is, the required drilling fluid density should be higher.

### 4.2. Wellbore stability analysis

We identified a horizontal well in the Silurian Longmaxi shale formations in the Southern Sichuan Basin to provide the implications of the present method for the wellbore stability. The bedding planes and the micro-fractures in the Silurian Longmaxi shale formations were well developed. Table 4 shows the basic parameters. We only utilized these basic data to determine the critical equivalent density of the collapse pressure considering the serious borehole collapse problem in the horizontal drilling process. We utilized the analytical method published by several studies to calculate the critical equivalent density of the collapse pressure [15–16, 19–20].

**Table 5.** Diff-abs and Diff-ave with the optimal material parameters for the Longmaxi shale.

| Rock types         | Rock matrix | Plane of weakness |
|--------------------|-------------|-------------------|
|                    | c₀ (MPa)    | φ₀ (°)            |
|                    | cₑ (MPa)    | φₑ (°)            |
| Longmaxi shale     | 13.7        | 40.7              |
|                    | 5.7         | 30.3              |

**Table 6.** Basic parameters of the wellbore stability analysis for the Longmaxi shale.

| No. | Parameter/unit | Value   |
|-----|----------------|---------|
| 1   | Vertical depth/m | 2450    |
| 2   | Maximum horizontal in-situ stress/MPa | 85.43 |
| 3   | Maximum horizontal in-situ azimuth/(°) | N109°E |
| 4   | Minimum horizontal in-situ stress/MPa | 45.98 |
| 5   | Vertical in-situ stress/MPa | 62.44 |
| 6   | Formation pressure/MPa | 26.6 |
| 7   | Poisson’s ratio | 0.25 |
| 8   | Biot’s constant | 0.8 |
| 9   | Cohesion of the rock body/MPa | 13.7 |
| 10  | Internal friction angle of the rock body/(°) | 40.7 |
| 11  | Cohesion of the bedding plane/MPa | 5.7 |
| 12  | Internal friction angle of the bedding plane/(°) | 30.3 |
| 13  | Dip angle of the bedding plane/(°) | 10 |
| 14  | Dip direction of the bedding plane/(°) | N132°E |

Figure 6 depicts the hemispherical projection of the critical equivalent density of the collapse pressure calculated for the Longmaxi shale, which presents two kinds of situations (i.e., with and without the influence of the plane of weakness). If the influence of the plane of weakness is ignored, the most stable well path should be a horizontal well (1.06 g/cm³) that deviates approximately 30° from the direction of the maximum horizontal stress, whereas the most unstable well path should be a vertical well (1.64 g/cm³). The stability of the deviated and horizontal wells is much better than that of the vertical well. Additionally, the optimal drilling direction for the wellbore stability is the direction of the minimum horizontal stress. However, if the influence of the plane of weakness is involved, the most stable well path should be a horizontal well (1.15 g/cm³) close to the direction of the maximum horizontal stress, whereas the most unstable well path should be a horizontal well (2.02 g/cm³) close to the direction of the minimum horizontal stress.
than 2.02 g/cm$^3$ to keep the wellbore stability. In other words, the presence of the plane of weakness greatly increases the required drilling fluid density that maintains the borehole stability. The original drilling fluid density in real drilling engineering of this well was 1.84 g/cm$^3$; however, the borehole collapse problem occurred when drilling reached 3066.6 m depth. The borehole collapse alleviated when an oil-based drilling fluid with 2.10 g/cm$^3$ density was used. Although the actual density of the drilling fluid used was slightly greater than the calculation results, the well partially collapsed during the horizontal section drilling. The shear failure of the weak planes may be the main mechanical mechanism of the borehole collapse during drilling. In addition, the formation around the wellbore is subjected to an over-balanced condition during drilling. Physicochemical effects will occur once the drilling fluid comes in contact with the shale rock, which may lead to the strength reduction of the shale rock. Consequently, the wellbore much easily collapses. Meanwhile, the filtrate intrudes the formation along the bedding planes and the micro-fractures, thereby resulting in an increase in the pore pressure around the wellbore and further reducing the effective stress and strength on the plane of weakness. This leads to a wellbore that can easily collapse. These factors may cause the required drilling fluid density that maintains the borehole stability to be slightly higher than the predicted results. Thus, the effect of multi-field coupling on the wellbore stability should be considered in the future study.

![Hemispherical projection of the collapse pressure for the Longmaxi shale.](image)

**Figure 6.** Hemispherical projection of the collapse pressure for the Longmaxi shale.

### 5. Conclusions

The following results were obtained herein:

1. Based on the SPW strength theory and the grid search method, we need not consider the quality of each triaxial test data; instead, we must consider the data integrity, divide the mesh of the material parameters, and quickly match the optimal rock material parameters of all triaxial testing data as a whole. The optimal rock material parameters of the rock matrix and the plane of weakness can be determined accordingly.

2. The SPW strength theory combined with the grid search method has advantages when determining the rock material parameters. Among these three groups of shale rocks, the optimum rock material parameters of the rock matrix were larger than those of the plane of weakness. All error indicators (i.e., average error ($\text{Diff}_{\text{ave}}$), average error ($\text{Diff}_{\text{ave}}$), and coefficient of determination ($R^2$)) can validate the reliability and accuracy of the present method.

3. The field analysis results of the wellbore stability indicated that the plane of weakness significantly affects the wellbore stability for the shale rock formations. The critical equivalent density of the collapse pressure was approximately 1.50 g/cm$^3$ when ignoring the influence of the plane of weakness and approximately 2.02 g/cm$^3$ when involving the influence of the plane of weakness. The predicted result was much closer to the real equivalent density of the collapse pressure (2.10 g/cm$^3$) when the
influence of the plane of weakness was involved. These results indicate that the presence of the plane of weakness greatly increases the required drilling fluid density that maintains the borehole stability.

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