Measurement of $\sin 2\beta$ in tree-dominated $B^0$-decays and ambiguity removal

H. Lacker$^a$

On behalf of the BaBar and Belle Collaborations

$^a$Institute of Nuclear and Particle Physics, Technical University Dresden, D-01062 Dresden, Germany

The most recent results from the $B$-factories on the time-dependent $CP$ asymmetries measured in $B^0$-decays mediated by $b \to c \bar{c} s$ quark-transitions are reviewed. The Standard Model interpretation of the results in terms of the parameter $\sin 2\beta$ leads to a four-fold ambiguity on the unitarity triangle $\beta$ which can be reduced to a two-fold ambiguity by measuring the sign of the parameter $\cos 2\beta$. The results on $\cos 2\beta$ obtained so far are reviewed.

1. INTRODUCTION

Within the Standard Model, quark flavor-mixing is described by the $3 \times 3$ unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$. The size of $CP$ violation carried by the CKM matrix is quantified by the imaginary part of the apex, $\Im (V_{us}^* V_{ub})$, of the unitarity triangle.

Constraints on the parameter $\sin 2\beta$ ($\beta = \arctan(\frac{\eta}{1-\eta})$) can be obtained from the measurement of time-dependent decay rates of neutral $B$-mesons into $CP$ eigenstates $f_{CP}$ mediated by $b \to c \bar{c} s$ quark transitions. The time-dependent rate asymmetry between $B^0$ and $B^0$ decays is given by

$$A_{CP}(t) = \frac{2\Re (V_{ts}^* V_{tb}) \sin (\Delta m_d \cdot t) - \frac{1-|\lambda|^2}{1+|\lambda|^2} \cos (\Delta m_d \cdot t)}{1+|\lambda|^2}$$

where $\Delta m_d$ is the mass difference between the heavy ($B_H = pB^0 + qB^0$) and the light ($B_L = pB^0 - qB^0$) neutral mass eigenstate, and

$$S = \frac{2\Re (V_{ts}^* V_{tb})}{1+|\lambda|^2}$$

and

$$C = -A = \frac{1-|\lambda|^2}{1+|\lambda|^2}$$

where the parameter $C$ is used by the BaBar collaboration and $A$ by the Belle collaboration.

The parameter $\lambda$, given by $\eta_{CP} \frac{A_{CP}}{|A_{CP}|}$, $\eta_{CP}$ being the $CP$ eigenvalue of the final state, is a phase-convention-independent quantity which contains possible sources of $CP$ violation in mixing ($|\frac{\eta}{\rho}| \neq 1$), direct $CP$ violation ($|\frac{\eta}{\rho}| \neq 1$), and $CP$ violation in the interference between decay with and without mixing ($3\lambda \neq 0$). In the $B^0 - {\bar{B}}^0$ system, to a very good approximation $\frac{\eta}{\rho} = \frac{V_{ts}^* V_{tb}}{V_{td}^* V_{tb}}$ and $CP$-violation in mixing is expected to be small, which is confirmed by the experimental constraints: $|\frac{\eta}{\rho}| - 1 = 0.0015 \pm 0.0039$. For the decay $B^0/\bar{B}^0 \to (c\bar{c})K^0/\bar{K}^0$, the dominant amplitude is a tree-mediated $b \to c\bar{c}s$ quark-transition. There are also contributions from penguin amplitudes. In the case of the $c$- and $t$-penguin, the CKM phases are equal or almost equal, respectively, to the tree-decay phase ($\propto V_{ts} V_{cb}^* \bar{V}_{ub}$). Only in the case of the $u$-quark penguin the CKM phase ($\propto V_{us} V_{ub}^*$) differs significantly from the tree amplitude. However, it is doubly-Cabibbo suppressed with respect to the tree amplitude. An additional effect comes from $CP$-violation in the $K^0 - \bar{K}^0$ system, however which changes the relative phase by only a small amount. Hence, for the $CP$ asymmetry, one expects to a very good approximation $A_{CP}(t) = \eta_{CP} \sin 2\beta \sin (\Delta m_d \cdot t)$.

2. MEASUREMENTS OF $\sin 2\beta$

The most precise determinations of $\sin 2\beta$ are coming currently from the $B$-factory experiments BaBar and Belle thanks to the excellent performance of their accelerators PEP II and KEKB. The new peak luminosity records achieved in 2006 are $1.207 \times 10^{34}$ cm$^{-2}$s$^{-1}$ for PEP II and $1.6517 \times 10^{34}$ cm$^{-2}$s$^{-1}$ for KEKB. By summer 2006, PEP II/BaBar had collected 390 fb$^{-1}$ of data while KEKB/Belle had collected a data sam-
ple of 630 fb$^{-1}$. This results in a sum of $10^9 \bar{B}B$ events written to tape. Both experiments have updated their sin $2\beta$ analyses recently [31, 32].

At the $B$-factories, the $T(4S)$ resonance is produced in $e^+e^-$ collisions with asymmetric electron and positron beam energies resulting in a boost factor of $\beta\gamma = 0.56$ at PEP II and $\beta\gamma = 0.425$ at KEKB, respectively. The $\Upsilon(4S)$ decays in about 50$\%$ of the cases to neutral $B$-meson pairs, produced in a coherent quantum state. The time-dependent $CP$ asymmetry is measured by determining the decay-time difference $\Delta t$ between the decay of one $B$ to a $CP$ eigenstate ($B_{CP}$) and the decay of the other $B$ meson ($B_{tag}$) to flavour-specific final states. These final states are not reconstructed exclusively and the flavor of $B_{tag}$ is determined (“tagged”) only on a statistical basis by determining the sign of the charge mainly from high-energy leptons, and kaons and low-energy pions mutually stemming from $D^*$ decays. The figure of merit for the tagging performance is given by the quality factor $Q = \sum \epsilon_i (1 - 2 \times w_i)^2$, where $\epsilon_i$ is the tagging efficiency and $w_i$ the mistag fraction in tagging category $i$. $Q$ enlarges the statistical uncertainty of the decay-rate asymmetry measurements and has been measured by $\text{BaBar}$ (Belle) to be $Q = (30.4 \pm 0.3)/\%$ ($Q = (29.0 \pm 1.0)/\%$). The decay-time difference $\Delta t$ is estimated from the distance between the two $B$-decay vertices $\Delta z$ in the beam-direction $z$. Due to the $\Upsilon(4S)$ boost, $\Delta z$ can be measured with sufficient precision by means of silicon vertex detectors. The average $\Delta z$ is of order 200 $\mu$m at Belle and about 250 $\mu$m at $\text{BaBar}$. The $\Delta z$ resolution is dominated by the $B_{tag}$ vertex resolution and is of order 190 $\mu$m.

The most recent $\text{BaBar}$ analysis [33] was performed on a sample of $348 \times 10^6 \bar{B}B$ events. The $CP$-odd final states ($\eta_{CP} = -1$) used are $B^0/\bar{B}^0 \rightarrow (c\bar{c})K_S^0$ where $c\bar{c} = J/\psi \rightarrow e^+e^-, \mu^+\mu^-, \psi(2S) \rightarrow e^+e^-, \mu^+\mu^-, J/\psi\pi^+\pi^-$, $\chi_{c1} \rightarrow J/\psi\gamma$, and $\eta_c \rightarrow K_0^0K^+\pi^-$. The $CP$-even final state $B^0/\bar{B}^0 \rightarrow J/\psi K^0_S$ is also reconstructed. Compared to the high-purity sample $B^0/\bar{B}^0 \rightarrow (c\bar{c})K_S^0$, this final state suffers from a significantly higher background level.

In addition, the vector-vector final state $B^0/\bar{B}^0 \rightarrow J/\psi K^{*0}$ with $K^{*0} \rightarrow K_0^0\pi^0$ is used. A recent angular analysis performed by $\text{BaBar}$ finds an effective $CP$ eigenvalue of $\eta_{CP} = 0.504 \pm 0.033$ for this final state [5]. For the $CP$-odd final states the result is $\sin 2\beta = +0.713 \pm 0.038_{\text{stat}}$, for the final state $B^0/\bar{B}^0 \rightarrow J/\psi K^0_S$, it is $\sin 2\beta = +0.716 \pm 0.080_{\text{stat}}$ (see Fig. 1). The average of all modes including $B^0/\bar{B}^0 \rightarrow J/\psi K^{*0}$ results in $\sin 2\beta = +0.710 \pm 0.034_{\text{stat}} \pm 0.019_{\text{sys}}$. The main systematic uncertainties are due to the

![Figure 1](image-url)
functions, the background in $B^0/\bar{B}^0 \rightarrow J/\psi K^0_S$, the differences in the mistag fractions between $B^0$ and $\bar{B}^0$, and the knowledge of the event-by-event beam spot position. For the final state $B^0/\bar{B}^0 \rightarrow (c\bar{c}) K^0_S$, which has the highest purity (92%), $\bar{B}Ar$ fits $\sin 2\beta$ and $|\lambda|$ simultaneously and finds $C = +0.070 \pm 0.028^{\text{stat}} \pm 0.018^{\text{sys}}$, translating into a 2 $\sigma$ deviation from zero. The dominant systematic uncertainty in this case (0.014) is due to the interference between the suppressed $b \rightarrow \bar{u}cd$ amplitude and the $b \rightarrow c\bar{u}d$ amplitude for certain hadronic $B_{taq}$ decays.

The Belle analysis [4] uses a sample of 532 million $B\bar{B}$ events where only the CP-odd final state $B^0/\bar{B}^0 \rightarrow J/\psi K^0_S$ and the CP-even final state $B^0/\bar{B}^0 \rightarrow J/\psi K^0_L$ are considered. Both results for the $S$ and $C$ coefficients are consistent: $\sin 2\beta(J/\psi K^0_S) = +0.643 \pm 0.038^{\text{stat}}$ and $\sin 2\beta(J/\psi K^0_L) = -0.641 \pm 0.057^{\text{stat}}$, respectively, $C(J/\psi K^0_S) = -0.018 \pm 0.021^{\text{stat}}$ and $C(J/\psi K^0_L) = -0.045 \pm 0.033^{\text{stat}}$. The averages between both results are: $\sin 2\beta = +0.642 \pm 0.031^{\text{stat}} \pm 0.017^{\text{sys}}$ and $C = -0.018 \pm 0.021^{\text{stat}} \pm 0.014^{\text{sys}}$. The dominant systematic uncertainties on $\sin 2\beta$ come from the vertex reconstruction, the resolution function, the background fraction, flavor tagging, and the effect of the tag-side interference. The systematic uncertainty on $C$, as in the case of $B_{taq}$, is dominated by the tag-side interference (0.009).

The world average computed by the Heavy Flavor Averaging Group (HFAG) [2] is $\sin 2\beta = +0.675 \pm 0.023^{\text{stat}} \pm 0.012^{\text{sys}}$, where most of the systematic uncertainties have been treated as uncorrelated. Correlated error sources like the $B$-lifetime, the mixing frequency $\Delta m_d$ or the effect of the tag-side interference play only a minor role at this stage. This suggests that on the time scale of 2008, when an integrated luminosity of order 2 ab$^{-1}$ is expected from $B_{taq}$ and Belle together, the total uncertainty on $\sin 2\beta$ will fall below 0.020. Theoretical estimates for the difference between the measured $S$ coefficient and the true $\sin 2\beta$ value are of order 0.01 or below [6]. The average value of $B_{taq}$ and Belle for the cosine coefficient is $C = 0.012 \pm 0.017^{\text{stat}} \pm 0.014^{\text{sys}}$ and hence consistent with zero as expected in the Standard Model at this level of precision. The systematic uncertainty is dominated by the tag-side interference effect. Under the assumption that both measurements have estimated their uncertainties correctly and that the uncertainties are Gaussian-like, the p-value (denoted CL by HFAG [2]) to find a deviation between the two experiments as large as or larger than the one observed is 0.02.

3. MEASUREMENTS OF $\cos 2\beta$

Since only the parameter $\sin 2\beta$ is measured, there are four different solutions for $\beta$. Two solutions correspond to $\cos 2\beta > 0$, $\beta = 21.2^\circ$ or $\beta = 21.2^\circ + 180^\circ$, while the ones corresponding to $\cos 2\beta < 0$ are $\beta = 68.8^\circ$ or $\beta = 68.8^\circ + 180^\circ$. The constraints on the unitarity triangle excluding $\sin 2\beta$ as an input result is $\beta = (27.7^{+0.8}_{-3.9})^\circ$ [7], suggesting that $\beta = 21.2^\circ$ is the correct solution. However, this argument relies on the validity of the Standard Model. If one allows for New Physics contributions in $B^0 - \bar{B}^0$ mixing a new phase, $2\theta_d$, is introduced. As a result, the measured parameter is $\sin (2\beta + 2\theta_d)$ and the new degree of freedom invalidates the argument given above. Only the measurement of a positive sign of $\cos (2\beta + 2\theta_d)$ allows a restriction of the possible solutions to $\beta + \theta_d = 21.2^\circ$ or $\beta + \theta_d = 21.2^\circ + 180^\circ$. Even then the allowed space for $\beta$ and $\theta_d$ is quite large and can only be reduced by adding other experimental inputs (see e.g. Ref. [7]).

3.1. $B^0/\bar{B}^0 \rightarrow J/\psi K^{*0}$

The first constraints on $\cos 2\beta$ have been obtained from a time-dependent angular analysis of the final state $B^0/\bar{B}^0 \rightarrow J/\psi K^{*0}$. Since this is a vector-vector mode there are three amplitudes contributing to this final state: two $CP$-even amplitudes, $|A_0|e^{i\delta_0}$ and $|A_1|e^{i\delta_1}$, and a $CP$-odd amplitude $|A_\perp|e^{i\delta_\perp}$, where the $\delta_i$ represent strong phases. The sizes and relative strong phases of these amplitudes can be measured from the angular distributions (usually described in the transversity basis) of the decay products of the $J/\psi$ and $K^{*0}$. The measured time-dependent decay-rate asymmetry in the three-dimensional phase space of the transversity angles is sensitive to $\cos 2\beta$ due to the interference between the
CP-even amplitudes $A_0$ and $A_{||}$, and the CP-odd amplitude $A_\perp$. However, there is an ambiguity in the solution of the strong phase differences $(\delta_{||} - \delta_0, \delta_{\perp} - \delta_0) \rightarrow (\delta_0 - \delta_{||}, \pi + \delta_0 - \delta_{\perp})$ resulting in a sign ambiguity \( \cos 2\beta \rightarrow -\cos 2\beta \) which seems to spoil the \( \cos 2\beta \) extraction.

**B\(\bar{B}\)** has performed such a time-dependent angular analysis on a sample of $88 \times 10^9 \, B\(\bar{B}\)$ pairs and finds \( \sin 2\beta = 0.10 \pm 0.14_{\text{stat}} \pm 0.57_{\text{sys}} \), and $\cos 2\beta = 3.32 \pm 0.06_{\text{stat}} \pm 0.27_{\text{sys}}$. When fixing $\sin 2\beta = 0.731$ as their best measured value, **B\(\bar{B}\)** finds $\cos 2\beta = 2.72 \pm 0.50_{\text{stat}} \pm 0.27_{\text{sys}}$. The sign ambiguity in $\cos 2\beta$ has been resolved in **B\(\bar{B}\)**'s analysis by taking advantage of the interference in the $K\pi$ system between the $P$-wave coming from the $K^{*0}$ decay and the underlying $S$-wave.

**Belle** has also performed a time-dependent angular analysis on a sample of $275 \times 10^6 \, B\(\bar{B}\)$ pairs and finds $\sin 2\beta = 0.24 \pm 0.31_{\text{stat}} \pm 0.03_{\text{sys}}$ and $\cos 2\beta = 0.56 \pm 0.11_{\text{stat}}$. When fixing $\sin 2\beta = 0.726$, $\cos 2\beta = 0.87 \pm 0.74_{\text{stat}} \pm 0.12_{\text{sys}}$ is found. The sign ambiguity in $\cos 2\beta$ has not been resolved. Instead, **Belle** uses the theoretical argument of s-quark helicity conservation to select the solution for the strong phase difference. This solution is consistent with **B\(\bar{B}\)**'s experimental result on the strong phase difference and leads to the positive sign for $\cos 2\beta$.

### 3.2. $B^0/\bar{B}^0 \rightarrow D^{(*)0}/\bar{D}^{(*)0}h^0$

It has been proposed recently that the sign of $\cos 2\beta$ can also be extracted from a time-dependent Dalitz plot analysis of the decay $B^0/\bar{B}^0 \rightarrow D^{(*)0}/\bar{D}^{(*)0}h^0$ where $h^0$ denotes a light neutral meson. These color-suppressed decay topologies are mediated by tree diagrams. If one neglects doubly-Cabibbo suppressed diagrams, the leading relevant quark transitions are $b \rightarrow u\bar{c}d$ for $B^0 \rightarrow D^{(*)0}h^0$, and $b \rightarrow c\bar{u}d$ for $\bar{B}^0 \rightarrow D^{(*)0}\bar{h}^0$, respectively. Interference between these amplitudes is obtained by reconstructing the neutral $D$ mesons in the common final state $D^{(*)0}/\bar{D}^{(*)0} \rightarrow K^{0}_{S}\pi^{+}\pi^{-}(\pi)$. We assume no CP violation in the $D^{(*)0}/\bar{D}^{(*)0}$ system and denote $f_{+-} = f(m^2_{K^{0}_{S}\pi^{+}}, m^2_{K^{0}_{S}\pi^{-}})$ for the $D^{(*)0}$ and $f_{-+} = f(m^2_{K^{0}_{S}\pi^{-}}, m^2_{K^{0}_{S}\pi^{+}})$ for the $D^{(*)0}$ decay amplitudes, respectively. Then, the time-dependent $B$ decay amplitudes are given by:

\[
M_{FP}(t) = f_{-+} \cos (\Delta m_d \cdot t/2) - ie^{-2i\beta} \eta h(-1)^{t} f_{+-} \sin (\Delta m_d \cdot t/2)
\]

\[
M_{FB}(t) = f_{-+} \cos (\Delta m_d \cdot t/2) - ie^{+2i\beta} \eta h(-1)^{t} f_{+-} \sin (\Delta m_d \cdot t/2),
\]

where also direct CP violation as well as CP violation in mixing in the neutral $B$ system has been neglected. Here, $\eta$ is the CP eigenvalue of $h^0$ and $\ell$ is the relative orbital angular momentum in the $D^{(*)0}h^0$ system. Due to the interference between $f_{+-}$ and $f_{-+}$ over the Dalitz plane, the time-dependent decay-rate asymmetry allows us to extract simultaneously $\sin 2\beta$ and $\cos 2\beta$ once the model for the Dalitz plot amplitudes is fixed.

A time-dependent Dalitz plot analysis on a sample of 386 million $B\(\bar{B}\)$ pairs has been first performed by Belle by reconstructing $B^0/\bar{B}^0 \rightarrow D^0/\bar{D}^0h^0$ with $h^0 = \pi^0, \eta, \omega$ and $B^0/\bar{B}^0 \rightarrow D^{*0}/\bar{D}^{*0}h^0$ with $D^{*0}/\bar{D}^{*0} \rightarrow D^0/\bar{D}^0\pi^0$ and $h^0 = \pi^0, \eta$. The measured Dalitz plot is shown in Fig. 2. A simultaneous fit results in $\sin 2\beta = 0.78 \pm 0.44_{\text{stat}} \pm 0.22_{\text{sys+Dalitz}}$ and $\cos 2\beta = 1.87 \pm 0.53_{\text{stat}} \pm 0.32_{\text{sys+Dalitz}}$. In a similar **B\(\bar{B}\)** analysis on a sample of 311 million $B\(\bar{B}\)$ pairs, the decays $B^0/\bar{B}^0 \rightarrow D^0/\bar{D}^0h^0$ with $h^0 = \pi^0, \eta, \eta', \omega$ and $B^0/\bar{B}^0 \rightarrow D^{*0}/\bar{D}^{*0}h^0$ with $D^{*0}/\bar{D}^{*0} \rightarrow D^0/\bar{D}^0\pi^0$ and $h^0 = \pi^0, \eta$ have been reconstructed. **B\(\bar{B}\)** finds $\sin 2\beta = 0.45 \pm 0.30_{\text{stat}} \pm 0.07_{\text{Dalitz}}$ and $\cos 2\beta = 0.94 \pm 0.54_{\text{stat}} \pm 0.18_{\text{Dalitz}}$. In this fit also $|\lambda|$ has been determined and found to be consistent with 1 (0.98 ± 0.09). This is as expected due to the smallness of the neglected doubly-Cabibbo suppressed tree-amplitudes carrying a different CKM phase with respect to the dominant tree amplitudes. In principle, the neglected amplitude could be taken into account in the analysis but this is likely to be impractical since the ratio of amplitudes $\frac{|A[\bar{B}^0 \rightarrow \bar{D}^{*0}h^0]|}{|A[B^0 \rightarrow D^{*0}h^0]|} \sim \frac{|V_{cb}V_{ub}^*|}{|V_{cb}V_{ud}^*|}$ is expected to be of order 0.02. With higher statistics, the precision on $\cos 2\beta$ might be dominated by the systematic uncertainty on the Dalitz plot model which is already the largest systematic error.
5

Figure 2. Dalitz plot distribution \( m_2^- = m_{K_S^0\pi^-}^2 \) versus \( m_2^+ = m_{K_S^0\pi^+}^2 \) as measured in the Belle \( B^0/\bar{B}^0 \rightarrow D^0/\bar{D}^0 h^0 \) analysis.

3.3. \( B^0/\bar{B}^0 \rightarrow D^{*+}D^{*-}K_S^0 \)

Another possible way to extract \( \sin 2\beta \) and \( \cos 2\beta \) simultaneously has been proposed in Ref. [14] and has then been studied in more detail in Ref. [15]. The method uses the final state \( B^0/\bar{B}^0 \rightarrow D^{*+}D^{*-}K_S^0 \). The time-dependent \( CP \) asymmetry is given by:

\[
A_{CP}(t; \sin 2\beta, \cos 2\beta) = \eta_y \frac{J_c}{J_0} \cos (\Delta m \cdot t) - \frac{2J_{s1}}{J_0} \sin 2\beta + \eta_y \frac{2J_{s2}}{J_0} \cos 2\beta \sin (\Delta m \cdot t).
\]

The \( J_i \) are integrals of functions of the amplitudes \( A = A(B^0 \rightarrow D^{*+}D^{*-}K_S^0) \) and \( \bar{A} = \bar{A}(\bar{B}^0 \rightarrow D^{*+}D^{*-}K_S^0) \) over the half Dalitz space in the variables \( s^+ = m_{D^{*+}K_S^0}^2 \) and \( s^- = m_{D^{*-}K_S^0}^2 \):

\[
J_{0(c)} = \int_{s^+<s^-} (|A|^2 + (-)|\bar{A}|^2) \, ds,
\]

\[
J_{s1} = \int_{s^+<s^-} \Re(\bar{A}A^*) \, ds, J_{s2} = \int_{s^+<s^-} \Im(\bar{A}A^*) \, ds.
\]

The parameter \( \cos 2\beta \) can be measured if \( J_{s2} \neq 0 \). This could be realized if a broad intermediate resonance contributed. In this case, one expects that \( J_c \) becomes large.

The \( \text{BaBar} \) collaboration has performed such an analysis [16]. In the invariant \( m_{D^{*\pm}K_S^0} \) spectrum there is a 4.6 \( \sigma \) evidence for the \( D_{S1}^*(2536) \) narrow width resonance. In addition, there is evidence for a broad structure below 2.9 GeV/c² (Fig. 3). The results of the time-dependent anal-

Figure 3. The \( m_{D^{*\pm}K_S^0} \) distribution as measured in the \( \text{BaBar} B^0/\bar{B}^0 \rightarrow D^{*+}D^{*-}K_S^0 \) analysis.
the $D^\pm K_S^0$ threshold. In this case, the nature of this possible broad resonance is unclear.

### 3.4. Summary of $\cos 2\beta$ determinations

The analyses reviewed here have not extracted a confidence level as a function of $\beta$. Instead, the $B\to J/\psi K^*$ BABAR analysis [8] has calculated a p-value for $\cos 2\beta_0 < 0$, where $\beta_0$ corresponds to the value obtained from the precise $\sin 2\beta$ measurement. For $\cos 2\beta_0 < 0$ the p-value is small (0.6%). However, since the BABAR-measured $\cos 2\beta$ value lies outside the physical region, also the p-value for $\cos 2\beta_0 > 0$ is small (5.7%). For this reason, also the likelihood ratio $h_-/(h_+ + h_-)$ has been considered where $h_\pm$ is the probability density function value for the measured $\cos 2\beta$ value if the true value is $\pm \cos 2\beta_0$. This likelihood ratio has then been interpreted in the framework of Bayesian statistics by assuming equal a-priori probabilities for the two hypotheses, $\cos 2\beta_0 < 0$ and $\cos 2\beta_0 > 0$, and denoted a confidence level. The other analyses discussed in this review followed this procedure and the confidence levels obtained are summarized in Table 1.

An average of all $\cos 2\beta$ measurements is beyond the scope of this review. In some cases the results have central values outside the physical region and large asymmetric uncertainties which makes an average cumbersome. The interpretation of the $B^0/\overline{B}^0\to D^{(*)+}D^{(*)-}K_S^0$ result relies on a theoretical assumption which requires further investigation, including the understanding of the Dalitz plot structure. The systematic uncertainty due to the Dalitz plot model in $B^0/\overline{B}^0\to D^{(*)+}/D^{(*)-}h_0$ should be reduced since more statistics will come in the future. This channel provides a good opportunity to check whether $\sin 2\beta$ is measured to be the same to a good approximation in $b\to c\bar{c}s$ and $b\to c\bar{u}d$ transitions.

Besides the above quantitative difficulties, the qualitative results suggest that the preferred solution is $\beta + \theta_d = 21.2^\circ$ or $\beta + \theta_d = 21.2^\circ + 180^\circ$, in agreement with the Standard Model. This finding removes half of the possible range in the New Physics parameter $2\theta_d$.

### Table 1

| Mode | BABAR | Belle |
|------|-------|-------|
| $B\to J/\psi K^*$ | 86% | Not quantified |
| $B\to D^{(*)+}/\overline{D}^{(*)-}h_0$ | 87% | 98.3% |
| $B\to D^{(*)+}D^{(*)-}K_S^0$ | 94% | Not measured |

### REFERENCES

1. N. Cabibbo, Phys. Rev. Lett. 10 (1963) 531; M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49 (1973) 652.
2. Heavy Flavour Averaging Group, E. Barberio et al., hep-ex/0603003 updated (ICHEP06); http://www.slac.stanford.edu/xorg/hfag/.
3. BABAR collaboration (B. Aubert et al.), hep-ex/0607107.
4. Belle collaboration (K.-F. Chen et al.), hep-ex/0608039.
5. BABAR collaboration (B. Aubert et al.), hep-ex/0607081.
6. See e.g. Y. Grossman, A. Kagan and Z. Ligeti, Phys. Lett. B538 (2002) 327; H. Boos, T. Mannel and J. Reuter, Phys. Rev. D71 (2004) 036006; M. Ciuchini, M. Pierini and L. Silvestrini, Phys. Rev. Lett. 95 (2005) 221804; H.-n. Li and S. Mishima, hep-ph/0610120.
7. The CKMfitter Group (J. Charles et al.), Eur. Phys. J. C41 (2005) 1; updated in http://ckmfitter.in2p3.fr/.
8. BABAR collaboration (B. Aubert et al.), Phys. Rev. D71 (2005) 032005.
9. Belle collaboration (R. Itoh et al.), Phys. Rev. Lett. 95 (2005) 091601.
10. M. Suzuki, Phys. Rev. D64 (2001) 117503.
11. A. Bondar, T. Gershon and P. Krokovny, Phys. Lett. B624 (2005) 1.
12. Belle collaboration (P. Krokovny et al.), Phys. Rev. Lett. 97 (2006) 081801.
13. BABAR collaboration (B. Aubert et al.), hep-ex/0607015.
14. J. Charles et al., Phys. Lett. B425 (1998) 375; Erratum-ibid B433 (1998) 441.
15. T.E. Browder et al., Phys. Rev. D61 (2000) 054009.
16. BABAR collaboration (B. Aubert et al.), hep-ex/0608016.