“QUANTUM GRAVITY”: AN OXYMORON

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Abstract. I prove that “quantum” and “Einsteinian gravity” are incompatible concepts. Accordingly, the graviton is a mere object of science fiction.

1. Introduction

The innumerable and learned efforts during seventy years to create a quantum formulation of general relativity have only beaten the air — et pour cause, as we shall see. On the other hand, it is evident to any unprejudiced scientist that definite reasons must be at the root of this failure.

First of all, whereas “particles and fields exist within space-time, gravity is, in essence, space-time” [1]. This implies, in particular, that the physical meaning of the so-called critical (or “Planckian”) quantities \( M_0 \equiv (\hbar c/G)^{1/2} \approx 10^{-5} \text{ g} \), \( L_0 \equiv (\hbar/M_0c) \approx 10^{-33} \text{ cm} \) and \( T_0 \equiv L_0/c \) is rather uncertain (“unsicher”), as it was emphasized by Rosenfeld many years ago [2]. Rosenfeld was specially qualified to formulate a judgment of that kind because the above constants came forth through an extension to the quantized linear approximation of general relativity (whose substrate is Minkowski spacetime — and this is an essential point) of a deep method, created by Bohr and Rosenfeld for the quantum electromagnetic field [3].

The current belief that below time \( T_0 \), length \( L_0 \), and mass \( M_0 \) the Einsteinian theory of gravitation loses its validity is fully unfounded. Indeed, its justification by means of a bold application of more or less sophisticated quantum techniques does not possess any sound basis. General relativity has nothing to do with the classical field theories in Minkowski spacetime, or in “rigid” Riemann-Einstein spacetimes.

Further, “there is no experiment that tells us that the quantization of gravity is necessary” [1].

Finally, the fictive nature of the so-called gravitational waves [4] is sufficient to render meaningless any quantization program of general relativity.

(The physical inconclusiveness of the theoretical approaches that make use of supplementary dimensions of spacetime curled up with a radius comparable to the “Planckian” length \( L_0 \), does not need to be emphasized. The opinion according to which the superstring theory provides a possibility for a consistent quantum theory of gravity is destitute of a rational foundation).

For a bibliography on quantum gravity see e.g. the References of the papers [5] and [6].

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2. Two observational results

Two recent papers of observational nature (see [5] and [6]) raise serious doubts on the existence of the quantum fluctuations of the metric tensor of general relativity at the "Planckian" scales, i.e. at the scales of the constants $L_0, M_0, T_0$ (see sect. 1).

Lieu and Hillman [5] remark that if the above fluctuations really existed,line could not be determined more accurately than a standard deviation $\sigma_t / t = a_0 (T_0 / t)^\alpha$, where $a_0$ and $\alpha$ are positive constants $\sim 1$. (Analogously, the distances should be subject to an ultimate uncertainty $c \sigma_\tau$.) As a consequence of a cumulative effect of this "Planck-scale phenomenology", we should have a complete loss of phase of the e.m. radiation emitted at large distances from the observer. The conclusion of the abstract of paper [5] runs as follows: “Since, at optical frequencies, the phase coherence of light from a distant point source is a necessary condition for the presence of diffraction patterns when the source is viewed through a telescope, such observations offer by far the most sensitive and uncontroversial test. We show that the HST [Hubble Space Telescope] detection of Airy rings from the active galaxy PKS1413+135, located at the distance of 1.2 Gpc secures the exclusion of all first order ($\alpha = 1$) quantum gravity fluctuations with an amplitude $a_0 > 0.003$. [. . .]"

Ragazzoni, Turatto and Gaessler [6] write: “[. . .] We elaborate on such an approach [i.e., the approach of [6], which was subject to some criticism] and demonstrate that such an effect would lead to an apparent blurring of distant point sources. Evidence of the diffraction pattern from the HST observations of SN1994D and the unresolved appearance of a Hubble Deep Field galaxy at $z = 5.34$ lead us to put stringent limits on the effects of Planck-scale phenomenology."

I shall now prove rigorously that, from a sound theoretical standpoint, there are no quantum fluctuations of the fundamental tensor of general relativity.

3. The oxymoron

As is was explicitly pointed out by Pauli [7], in quantum mechanics the time $t$ is a "gewöhnliche Zahl (\"c-Zahl\")", i.e. it coincides with the time of classical physics. Thus, time $t$ is not a dynamical variable represented by an operator of the Hilbert space of the physical states. Analogously, also the co-ordinates of the points of three-dimensional physical space are parameters, and not dynamical variables; only the co-ordinates $q_r, (r = 1, 2, \ldots, n)$ of the $n$ degrees of freedom of a holonomic system are dynamical variables represented by Hilbert operators.

In the customary (Lorentzian) quantum field theory, a given field is described by a set of $m$, say, operators $\varphi_s$, $(s = 1, 2, \ldots, m)$, that are functions of the spatial points and of the instants of time.

In general relativity the fundamental spacetime interval $ds$ is given by

\[(1) \quad ds^2 = g_{jk}(x^0, x^1, x^2, x^3)dx^jdx^k, \quad (j, k = 0, 1, 2, 3) \]
where the coefficients $g_{jk}$ of the metric do not represent a classical field in the conventional meaning, but characterize directly the spatiotemporal structure – in other terms, they “are” the spacetime itself. (The co-ordinates $x^0, x^1, x^2, x^3$ are mere labels of the spacetime points, fully devoid of any metrical meaning).

If we write the Minkowskian $ds^2$ of a Lorentzian quantum theory making use of a system of general co-ordinates $x^0, x^1, x^2, x^3$, we obtain obviously an expression of the following kind:

$$ds^2 = h_{jk}(x^0, x^1, x^2, x^3)dx^jdx^k, (j, k = 0, 1, 2, 3),$$

and we see that, according to the basic axiom previously emphasized \[7\], the functions $h_{jk}(x) = h_{kj}(x)$ are non-operators, i.e. they are (necessarily!) customary functions (“c-numbers”) of the co-ordinates $x^0, x^1, x^2, x^3$.

We realize now that the project of a theory such that the $g_{ik}$’s of the exact (non-approximate) formulation of general relativity are promoted to the role of operators of a function space implies a blatant contradiction with the above axiom of quantum theory.

“Quantum” and “[Einsteinian] gravity” are incompatible concepts, and thus the expression “quantum gravity” is actually an oxymoron.

4. Recapitulation

The classic spacetimes of quantum theories are the following: i) the Euclidean-Newtonian substrate of Galilean group of transformations; ii) the Minkowskian substrate of Lorentzian group of transformations; iii) any given, “rigid” Riemann-Einstein spacetime.

We have correspondingly: i) the nonrelativistic quantum mechanics of the systems with a finite number of degrees of freedom; ii) the Lorentzian quantum theories – and the quantized linear approximation of GR (Pauli, Rosenfeld); iii) Dirac’s equation for a particle in a fixed Riemann-Einstein spacetime.

The known quantum formalisms can have a definite physical sense only under the condition that the above spacetimes are described by the customary non-operator entities. Consequently, the meaning of any quantization program of GR is doomed to a whimsical arbitrariness, because it implies necessarily some operator characterization of spacetime itself.

APPENDIX

A puffing operation

As it has been recalled in sect.1, the constants $L_0, M_0, T_0$ pertain, rigorously speaking, only to the quantum linearized version of GR. In the current astrophysical literature they are denominated “Planck constants”. Why? The reason is simple. In 1899 Planck \[8\] remarked that with suitable combinations of the fundamental constants $G, c, h$, it is possible to obtain the following four “natural” units of measure:

- unit of length: $\sqrt{\frac{Gh}{c^4}}$,
- unit of mass: $\sqrt{\frac{ch}{G}}$. 

unit of time: $\sqrt{\frac{G}{c^5}}$

unit of temperature: $\frac{1}{k} \sqrt{\frac{e^3}{c^3 h}}$

where $k$ is Boltzmann’s constant. (Actually, in the paper of 1899 Planck wrote $b$ in lieu of $h$, and $a$ in lieu of $h/k$.)

Clearly, “measure units” and “physical constants” are distinct concepts. I suppose that the astrophysical community is perfectly aware of this trivial difference.

To qualify with Planck’s name the constants $L_0$, $M_0$, $T_0$ has been a tricking operation with the aim to dignify with a great name three constants having a very dubious meaning.

“– Warum willst du dich von uns allen
Und unserer Meinung entfernen?
Ich schreibe nicht euch zu gefallen,
Ihr sollt was lernen!”

J.W.v. Goethe

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