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PLACE_SUBST Transformation of P/T Petri Process 
Nets and Its Properties

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Abstract. Petri nets is one of mathematical modeling languages for the description of all kind of parallel systems. Property-preserving Petri net process algebras (PPPA) were originally designed for the specification and verification of Petri net processes representing the manufacturing systems. PPPA does not need to verify the composition of Petri net processes because all their algebraic operators preserve the specified set of the properties. These original PPPA are generalized for the newly introduced class of the P/T Petri process nets (PTPN) in this article. The only one PLACE_SUBST transformation is defined for the class of PTPN and its chosen properties are presented. The PTPN can be with the support of the PLACE_SUBST transformation easily applied into the area of design, simulation and verification of multithreading programming systems executed in parallel or distributed environment. This fact is demonstrated on the simple example of the client-server distributed programming system.

Keywords: P/T Petri process net, well-formed process net, property preservation, PLACE_SUBST transformation, parallel systems modeling.

1 Introduction

Petri nets [1, 2, 3, 4] is one of mathematical modeling languages for the description of all kind of parallel systems and they represent a popular formalism connecting advantages of the graphic representation of a modeled system with the possibilities of its simulation and the formal analyzability. Property-preserving Petri net process algebras (PPPA) [6] were originally designed for the specification and verification of manufacturing systems. PPPA also follows many ideas that can be originally seen in the class of workflow nets [5]. The elements of PPPA are the Petri net processes, an ordinary connected Petri nets with a unique entry place, a unique exit place and a set of places for handling resource sharing with their initial marking allowed. Among other features, PPPA does not need to verify composite components because all of their operators preserve many properties. PPPA have five types of operators: extensions, compositions, refinements, reductions and place-merging. All the operators can preserve about twenty properties (some under additional conditions), such as liveness, boundedness, reversibility, traps, siphons, proper termination, etc. Hence, if the primitive modules satisfy

The only one PLACE_SUBST transformation is defined for the class of PTPN and its chosen properties are presented. The PTPN can be with the support of the PLACE_SUBST transformation easily applied into the area of design, simulation and verification of multithreading programming systems executed in parallel or distributed environment. This fact is demonstrated on the simple example of the client-server distributed programming system.

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1 Introduction

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the desirable properties, each of the composite components, including the system itself, also satisfies these properties.

In this article, we will generalize PPPA for the special class of the P/T Petri process nets (PTN), define the only one PTPN transformation called PLACE_SUBST and present its base characteristic of the well-formedness and purely-formedness properties preservation. This class of PTPN with the support of the PLACE_SUBST transformation can be then successfully used in the area of design, simulation and verification of programming systems executed in distributed or parallel environment. This fact is then demonstrated on the simple example of the distributed programming system based on the client-server architecture.

2 P/T Petri process nets and their properties

Let \( N \) denotes the set of all natural numbers, \( N := \{1, 2, \ldots\} \), \( N_0 \) the set of all non-negative integer numbers, \( N_0 := \{0, 1, 2, \ldots\} \), \( \emptyset \) the empty set, \( \neg \) the logical negation operator, \(|A|\) the cardinality of the given set \( A \), where \(|N| := N_0\).

Let \( A \) be a non-empty set. By the (non-empty finite) sequence \( \sigma \) over the set \( A \) a function \( \sigma \) can be understood, \( \sigma : \{1, 2, \ldots, n\} \rightarrow A \), where \( n \in N \). The function \( \varepsilon : \emptyset \rightarrow A \) is called the empty sequence over the set \( A \). The sequence \( \sigma : \{1, 2, \ldots, n\} \rightarrow A \) is usually represented by the notation \( \sigma = a_1, a_2, \ldots, a_n \) of the elements of the set \( A \), where \( a_i = \sigma(i) \) for \( 1 \leq i \leq n \).

Let \( A = \{a_1, a_2, \ldots, a_n\} \), where \( n \in N \), is a finite non-empty set. Vector \( V \) is a function \( V : A \rightarrow N_0 \). We will denote a vector \( V \) by the statement \( V = (V(1), V(2), \ldots, V(n)) \).

Let \( V = ((V(1), V(2), \ldots, V(n)) | V(i) \in N_0 \text{ for } 1 \leq i \leq n \) and \( W = ((W(1), W(2), \ldots, W(m)) | W(j) \in N_0 \text{ for } 1 \leq j \leq m \). Then

\[
V \otimes W = \{(V(1), \ldots, V(n), W(1), \ldots, W(m)) | V(i) \in N_0 \text{ for } 1 \leq i \leq n, W(j) \in N_0 \text{ for } 1 \leq j \leq m \}, \quad (V \otimes W) \setminus \emptyset = V \otimes W
\]

It is clear that the \( \otimes \) is an associative operation (i.e., \( (V \otimes W) \otimes U = V \otimes (W \otimes U) \)).

Process net (PN) is an ordered 9-tuple \( PN := (P, T, A, AF, TP, IP, OP, RM) \), where

- \( P \) is a finite non-empty set of places,
- \( T \) is a finite set of transitions, \( P \cap T = \emptyset \),
- \( A \) is a finite set of arcs, \( A \subseteq (P \times T) \cup (T \times P) \),
- \( AF \) is the arc function, \( AF : (P \times T) \cup (T \times P) \rightarrow N_0 \), \( AF(x, y) \in N \) iff \( (x, y) \in A \), \( AF(x, y) = 0 \) iff \( (x, y) \not\in A \),
- \( TP \) is a transition priority function, \( TP : T \rightarrow N \),
- \( RP \) is a finite set of resource places, \( RP \subseteq P \),
- \( IP \) is the input place, \( IP \in (P \setminus RP) \), it is the only one place such that \( IP \otimes P = \emptyset \),
- \( OP \) is the output place, \( OP \in (P \setminus RP) \), it is the only one place such that \( OP \otimes P = \emptyset \),
\begin{itemize}
  \item RM is the set of all allowed resource places static markings, \( RM = \mathbb{M} \), where \( \mathbb{M} \) is the set of all the static markings \( M \) of the PN (see below).
  \item \((P, T, A)\) is a connected net.
\end{itemize}

We will denote the class of all the PNs by the symbol \textbf{PNET}. The given PN is then described with a bipartite graph containing a finite non-empty set \( P \) of places used for expressing of the conditions of a modeled process (we usually use circles for their representation), a finite set \( T \) of transitions describing the changes in the process (we usually draw them in the form of rectangles), a finite set \( A \) of arcs being principally oriented while connecting the place with the transition or the transition with the place and we usually draw them as lines with arrows, the arc function \( AF \) assigning each arc with a natural number (such number has the default value of 1, if not explicitly indicated in the PN diagram) expressing the number of removed or added tokens from or to the place associated with that arc when executing a particular transition. Transition priority function \( TP \) assigns with each transition the natural number value expressing its priority (with the default value of 1). The finite set \( RP \) of resource places is used for expressing conditions of a modeled process containing some initial resources and we use circles with the double line for their representation. The input place \( IP \) is the only one non-resource place of PN \( PN \) with no input arc(s) and the output place \( OP \) is the only one non-resource place of PN \( PN \) with no output arc(s). \( RM \) is the set of all the allowed resource places static markings that is subset of the set \( \mathbb{M} \) of all the static markings \( M \) of the PN \( PN \) (the term static marking \( M \) of PN \( PN \) will be explained below). The net \((P, T, A)\) must be connected.

Some commonly used notations for PNs are \( \bullet y = \{ x \mid (x, y) \in A \} \) for the preset and \( \bullet y = \{ x \mid (y, x) \in A \} \) for the postset of a PN element \( y \) (i.e., place or transition).

\textbf{Marking} \( M \) of the PN \( PN \) is a mapping \( M: P \rightarrow \mathbb{N}_0 \). Marking \( M \) then expresses the current status of a modeled process. Marking \( M \) can be written as a \([P]\)-vector \( M := (M(IP), M(P_1), M(P_2), ..., M(P_n), M(R_1), M(R_2), ..., M(R_m), M(OP)) \), where \( P := (IP, P_1, P_2, ..., P_n, R_1, R_2, ..., R_m, OP) \), \( RP := \{R_1, R_2, ..., R_m\}, \) \( n \in \mathbb{N}_0, m \in \mathbb{N}_0 \).

The transition \( t \in T \) is enabled in the marking \( M \) of the PN \( PN \) if at each input place of the transition \( t \) are in the marking \( M \) at least as many tokens as required by the value of the arc function \( AF \) of the particular input arc of the transition \( t \), i.e., if \( \forall p \in \bullet t: M(p) \geq AF(p, t) \). If the transition \( t \) is enabled in the marking \( M \) of the PN \( PN \), we denote that fact in the form of \( t \text{ en } M \). \textbf{Firing of the transition} \( t \in T \) itself consists in the removal of as many tokens from each preset place of the transition \( t \) as required by the value of the arc function \( AF \) of the particular input arc of the transition \( t \), and adding of as many tokens into each of the postset places of the transition \( t \) as required by the value of the arc function \( AF \) of the particular output arc of the transition \( t \), i.e., it results in changing the marking \( M \) into the marking \( M' \), where \( \forall p \in P: M'(p) = M(p) - AF(p, t) + AF(t, p) \), that is denoted by \( M \{ t \} M' \).

We say that the marking \( M' \) is reachable from the marking \( M \) iff there exists a finite sequence \( \sigma := t_1 t_2 ... t_n, n \in \mathbb{N}, \) of the transitions \( t_1, t_2, ..., t_n \), such that \( M \{ t_1 t_2 ... t_n \} M' \).

The set of all the PN’s markings reachable from its given marking \( M \) will be denoted by the symbol \( [M] \), the set of all the transition sequences \( \sigma := t_1 t_2 ... t_n, n \in \mathbb{N}, \) of the transitions that are fireable from the marking \( M \) will be denoted by the symbol \( (M) \), i.e., \( (M) := \{ t_1 t_2 ... t_n \} \exists M'' \in [M]: M \{ t_1 t_2 ... t_n \} M'', n \in \mathbb{N} \).
Let \( k \in \mathbb{N} \). The following special markings of the PN \( PN := (P, T, A, AF, TP, RP, IP, OP, RM_i) \) are defined:

- **static marking** \( M_s : \forall p \in RP: M_s(p) \geq 0; \forall p \notin RP: M_s(p) = 0; \forall t \in T: \neg(t \text{ en } M_s) \),
- **entry marking** \( M_e(IP) = k; \forall p \in (P \setminus \{IP\}): M_e(p) = M_s(p) \),
- **exit marking** \( M_e(IP) = 0; M_e(OP) = M_s(IP) = k; \forall p \notin (RP \cup \{OP\}): M_s(p) = 0; \forall t \in T: \neg(t \text{ en } M_e) \).

For the given PN \( PN \) we will denote the set of all its static markings \( M_s \) by the symbol \( \textbf{M_s} \), the set of all its entry markings \( M_e \) by the symbol \( \textbf{M_e} \) and the set of all its exit markings \( M_e \) by the symbol \( \textbf{M_e} \).

Fig. 1 illustrates the PN \( PROC := (P, T, A, AF, TP, RP, IP, OP, RM_i) \), where \( P = \{IP, P1, R1, OP\}, T = \{T1, T2, T3\}, A = \{(IP, T1), (IP, T2), (T1, P1), (T2, P1), (R1, T1), (P1, T3), (T3, R1), (T3, OP)\}, AF = \{(IP, T1), 1\}, (IP, T2), 1\}, ((T1, P1), 1\}, ((T2, P1), 1\}, ((R1, T1), 1\}, (P1, T3), 1\}, (T3, R1), 2\}, (T3, OP), 1\}\}, TP = \{(T1, 1\}, (T2, 1\}, (T3, 1\}\), IP = OP, OP = OP, RM_i = \{(RM_i(R1))\} = \{(n) | n \in N\}, \) in its static \( M_s \), entry \( M_e \) and exit \( M_e \) markings where \( k = 3 \) (note the resource place R1 with the initial token in the static marking \( M_s \), and the fact that no transition must be enabled in any static \( M_s \) or exit \( M_e \) markings) where the notation \( RM_i = \{(RM_i(R1))\} = \{(n) | n \in N\} \) means that in any allowed resource static marking \( M_s \) the resource place R1 must contain at least one token.

![Fig. 1. Static, input and output markings of the PN PROC](image)

A P/T Petri process net (PTPN) is an ordered couple \( PTPN = (PN, M_s) \), where \( PN \) is a PN and \( M_s \) is an entry marking of the PN \( PN \). We will then denote a PTPNs by an ordered 10-tuple \( PTPN := (P, T, A, AF, TP, RP, IP, OP, RM_i, M_e) \) and the class of all the PTPNs by the symbol \( \textbf{PTPN} \).

When enabling individual transitions of a given PTPN so called **conflicts** can originate in its certain markings (or **conflict transitions**). At the enabling of the transitions \( t_1 \) and \( t_2 \) of the given PTPN in its marking \( M \) the conflict occurs, if both transitions \( t_1 \) and \( t_2 \) have at least one input place, each of the transitions \( t_1 \) and \( t_2 \) is individually enabled in the marking \( M \), but the transitions \( t_1 \) and \( t_2 \) are not in the marking \( M \) enabled in parallel and enabling of one of them will prevent enabling the other (i.e., \( \{t_1 \cap t_2 \neq \emptyset \} \wedge (t_1 \text{ en } M) \wedge (t_2 \text{ en } M) \wedge \neg((t_1, t_2) \text{ en } M) \)). The term of conflict transitions can be obviously easily generalized for the case of a finite set \( t_1, t_2, ..., t_n (n \in \mathbb{N}) \) of the transitions of a given PTPN.
A typical example of the conflict transitions in the particular marking of the PTPN is shown in Fig. 2, where transitions T1 and T2 have a common input place IP, both are enabled, but they are not enabled in parallel. When solving such conflicts we will therefore follow the rule which determines, informally said, that from the set of conflict transitions the one will be enabled, whose value of the transition priority function TP is the highest. If such transition from the set of conflict transitions does not exist, the given conflict would have to be solved by other means. The transition T2 is then enabled on the basis of that rule in our example (because TP(T1) = 1 and TP(T2) = 2).

**Definition 1.** Let PN := (P, T, A, AF, TP, RP, IP, OP, RM) be a PN, M_e be its entry marking. Then:

- PN is said to be **safe** iff ∀p ∈ P ∀M ∈ [M_e]: M(p) ≤ 1,
- PN is said to be **k-bounded** iff ∃k ∈ N ∀p ∈ P ∀M ∈ [M_e]: M(p) ≤ k,
- PN is said to **terminate properly** iff
  (∀M ∈ [M_e] ∃M ∈ M; M_e ∈ [M]) . (|M_e|) ≠ ∞)!
- PN is said to be **well-formed** PN iff ∀M ∈ M; PN terminates properly,
- well-formed PN is said to be **purely-formed** PN iff
  ∀M ∈ M ∀M ∈ M; ((M_e(IP) = M_e(OP)) ⇒ (∀r ∈ RP: M_e(r) = M_e(r))).

**Example 1.** Fig. 3, shows the PNs NET1, NET2 and NET3 in their entry markings M_e, where:

- PN NET1 does not terminate properly, because there exists the infinite sequence of reachable markings M_e[T1] M_1[T1] M_2[T1] ..., and so the PN NET1 is not a well-formed PN,
- PN NET2 terminates properly for every of its entry markings M_e (and so it is a well-formed PN), but it is not purely-formed PN, because ∃M ∈ M; ∃M ∈ M; (M_e(IP) = M_e(OP)) . (∃R ∈ RP: M_e(R1) = M_e(R1)), where M_e = (1, 0, 1, 0) and M_e = (0, 0, 2, 1),
- PN NET3 is a purely-formed PN.

**Lemma 1.** Let PN := (P, T, A, AF, TP, RP, IP, OP, RM) be a PN. Then

(PN is well-formed PN) ⇒ (∀M ∈ M; ∃k ∈ N: PN is k-bounded).

**Proof.** Clear. (PN is well-formed PN) ⇒ (∀M ∈ M; PN terminates properly) ⇒ (∀M ∈ M; ((∀M ∈ [M_e] ∃M ∈ M; M_e ∈ [M]) . (|[M_e]|) ≠ ∞)) ⇒ (∀M ∈ M; ([|M_e|] is finite) . (∀M ∈ [M_e]: M is finite marking of PN)) ⇒ (∀M ∈ M; ∃k ∈ N: PN is k-bounded).
3 Base P/T Petri process nets and their properties

As it was mentioned in the introduction of this article we want to generalize the PPPA for the class of the PNs. From many properties of PNs we are especially interesting in their well-formedness, resp. purely-formedness, property (in the terminology of programming systems well-formedness property informally means that the programming system modeled by a given PN will not cause deadlock and memory or device overflow and purely-formedness property informally means that given well-formed programming system will not have any side effects). It is of course possible to inspect the properties of traps, siphons, P- and T-invariants etc., of a given PN but it is outside the scope of this article. We will then represent given multithreading programming system by the PN, i.e., programming systems that will be executed in parallel environment by \(k\) programming threads, where \(k \in \mathbb{N}\). These programming threads will be represented by \(k\) tokens in the input place \(IP\) of given PN in its entry marking \(M_e\).

We start our generalization of PPPA for the class of PNs by introduction of so called base PNs (BPN). BPNs represent the subclass of PNs that are elementary purely-formed, (i.e., purely-formedness of these PNs can be trivially proved).

Fig. 4, shows six simple BPNs in their static markings \(M_s\), (i.e., the letter \(m\) in every of their resource places denotes \(m\) tokens, \(m \in \mathbb{N}\)). The BPN BASE1 is the simplest purely-formed BPN at all and it contains only one place IOP that is simultaneously its input and output place (definition of PN allows this case). Purely-formed BPN BASE3 can be used for the modeling of the programming statement IF … THEN … ELSE … (note that the value of the transition priority function \(TP(T2) = 2\) and it helps to solve potential conflict in the firing of the transitions T1 and T2), purely-formed BPN BASE4 represents the model of the programming critical section, where typically \(m = 1\) in the resource place R1. Purely-formed BPN BASE5 then represents the synchronous programming method calling mechanism, where the called method is represented by the place P2. Purely-formed BPN BASE6 can be then used for the modeling of the programming barrier.
Fig. 4. Examples of BPNs

Fig. 5. Examples of BPNs

4 PLACE_Subst transformation and its properties

The only one transformation PLACE_Subst (i.e., place substitution transformation) is defined for the class of all PNs and it is shown to be preserving the well-formedness and purely-formedness properties. The well-formed, resp. purely-formed, PNs form a closed set (i.e., informally said, the result of an application of the PLACE_Subst transformation onto any two well-formed, resp. purely-formed, PNs will result into another well-formed, resp. purely-formed, PN). The design of a given programming system will typically starts with the BPN BASE1 and it then follows with the several PLACE_Subst transformations (i.e., substitutions of the selected BPN over the non-
resource place of the actual PN) that result into the complex well-formed, resp. purely-formed, PN that models the whole programming system.

We will denote the set of all ordered pairs \((PN, p)\) where \(PN \in \textsc{Pnet}. \ \text{PN} := (P, T, A, AF, TP, RP, IP, OP, RM)\), \(p \in (P \setminus RP)\), by the symbol \(\textsc{Pnetpl}\). We will then denote the members of the set \(\textsc{Pnetpl}\) (i.e., ordered pairs \((PN, p)\)) by the shortened notation \(PN.p\).

**Definition 2.** Let \(PN1 := (P1, T1, A1, AF1, TP1, RP1, IP1, OP1, RM1)\) and \(PN2 := (P2, T2, A2, AF2, TP2, RP2, IP2, OP2, RM2)\) are the PNs, let \(p \in (P \setminus RP)\) is a non-resource place of the PN \(PN1\). The transformation \(\textsc{Place} _\text{Subst}. \ \textsc{Pnetpl} \times \textsc{Pnet} \rightarrow \textsc{Pnet}\) of the PN \(PN2\) substitution over the place \(p\) of the PN \(PN1\) resulting into the new PN \(PN := (P, T, A, AF, TP, RP, IP, OP, RM)\) will be denoted by the statement \(PN := PN1.p \setminus PN2\), where:

- \(P := (P1 \setminus \{p\}) \cup P2\),
- \(T := T1 \cup T2\),
- \(A := A2 \cup \{(x, y) \in A1 | (x \neq p) \lor (y \neq p)\} \cup \{(x, IP2) | (x, p) \in A1\} \cup \{(OP2, y) | (p, y) \in A1\},\)
- \(AF := AF2 \cup \{(x, y), v \in AF1 | (x \neq p) \lor (y \neq p)\} \cup \{(x, IP2), v \in AF2\} \cup \{(OP2, y), v \in AF1\},\)
- \(TP := TP1 \cup TP2\),
- \(RP := RP1 \cup RP2\),
- \(IP := IP1, \text{if } p \neq IP1; IP = IP2, \text{if } p = IP1;\)
- \(OP := OP1, \text{if } p \neq OP1; OP = OP2, \text{if } p = OP1;\)
- \(RM := RM1 \otimes RM2.\)

**Example 2.** The result of the \(\textsc{Place} _\text{Subst}\) transformation \(\textsc{base}.P2 \setminus \textsc{base}2\) where the place \(P2\) of the BPN \(\textsc{base}5\) (see Fig. 4) was substituted by the BPN \(\textsc{base}2\) can be shown in Fig. 6.

![Fig. 6. Result of \(\textsc{Place} _\text{Subst}\) transformation \(\textsc{base}.P2 \setminus \textsc{base}2\)](image)

**Lemma 2.** Let \(PN1 := (P1, T1, A1, AF1, TP1, RP1, IP1, OP1, RM1)\), \(PN2 := (P2, T2, A2, AF2, TP2, RP2, IP2, OP2, RM2)\) and \(PN3 := (P3, T3, A3, AF3, TP3, RP3, IP3, OP3, RM3)\) are the PNs. Let \(p1 \in (P1 \setminus RP1)\) is a non-resource place of the PN \(PN1\) and \(p2 \in (P2 \setminus RP2)\) is a non-resource place of the PN \(PN2\). Then:

\(PN1.p1 \setminus (PN2.p2 \setminus PN3) = (PN1.p1 \setminus PN2).p2 \setminus PN3.\)

**Proof.** Follows directly from the **Definition 2.** \(\Box\)
Lemma 3. Let \( PN_1 \) := \((P_1, T_1, A_1, AF_1, TP_1, RP_1, IP_1, OP_1, RM_1)\) is a well-formed, resp. purely-formed, PN and \( PN_2 \) := \((P_2, T_2, A_2, AF_2, TP_2, RP_2, IP_2, OP_2, RM_2)\) is a well-formed, resp. purely-formed, PN. Let \( p \in (P_1 \setminus RP_1) \) is a non-resource place of the PN \( PN_1 \). Let \((\forall t \in p\mathord{*}, (p, t) \in A_1) \Rightarrow t \times p\mathord{*} \subseteq A_1)\) \& \((\forall t \in p\mathord{*}, AF_1(p, t) = 1))\). Then \( PN_1.p \not< PN_2 \) is a well-formed, resp. purely-formed, PN.

Proof. Clear. Because both PN \( PN_1 \) and \( PN_2 \) are well-formed, i.e., \((\forall M_i \in M; PN_1 \text{ terminates properly}) \& (\forall M_i \in M; PN_2 \text{ terminates properly})\), then for the PN \( PN_1 \) holds true that \( \forall M \in [M_i], M(p) > 0, \exists M_i; M_i \in [M] \), and for the PN \( PN_2 \) holds true that \( \forall M \in [M_i], M(IP_2) > 0, \exists M_i; M_i \in [M] \), then the whole PN \( PN_1.p \not< PN_2 \) must also terminate properly for any of its input markings \( M_i \). The property of purely-formedness of the resulting PN \( PN_1.p \not< PN_2 \) can be proved similarly.

The necessity of the condition \((\forall t \in p\mathord{*}, (p, t) \in A_1) \Rightarrow t \times p\mathord{*} \subseteq A_1\) fulfilment is presented in Fig. 7., where both \( PN_1 \) and \( PN_2 \) are purely-formed PN but the resulting PN \( PN_1.p \not< PN_2 \) is not any more purely-formed PN. This fact can be seen for instance from the transition sequence \((1, 0, 0, 0, 0) [T1] (0, 1, 1, 0, 0) [T2] (0, 0, 0, 1, 1)\) that causes deadlock. This problem follows from the fact that for the PN \( PN_1 \) is not satisfied the condition \((P_2, T_3) \in A_1 \Rightarrow (P_2, T_2) \in A_1\). Satisfaction of this condition that is well-known from the class of free choice Petri nets (see [8]) implies of the purely-formedness of the resulting PN \( PN_1.p \not< PN_2 \).

![Fig. 7](image)

The necessity of the condition \((\forall t \in p\mathord{*}, AF_1(p, t) = 1\) fulfilment is presented in Fig. 8., where both \( PN_3 \) and \( PN_4 \) are purely-formed PN but the resulting PN \( PN_3.P_1 \not< PN_4 \) is not any more purely-formed PN. This fact can be seen for instance from the transition sequence \((1, 0, 0, 0, 1, 0) [T1] (0, 3, 0, 0, 1, 0) [T4] (0, 2, 1, 0, 0, 0) [T5] (0, 2, 0, 1, 1, 0) [T3] (0, 2, 0, 1, 1, 0)\) that causes impossibility of reaching of the exit marking \( M_i \) of the PN. This problem follows from the fact that for the PN \( PN_1 \) is not satisfied the condition \( AF(P_1, T_2) = 1\). Satisfaction of this condition implies of the purely-formedness of the resulting PN \( PN_3.P_1 \not< PN_4 \).

\(\square\)
5 An example of PLACE_SUBST transformation application at the modeling of distributed programming system

In the following simple example we will develop a design of a PN modeling the distributed programming system operating on the client and application server side and realizing reading and writing procedures from the given database system on the basis of a client requests. Individual client requests are executed via a finite set of programming threads on the server side (in our model of that programming system we assume availability of totally three server side programming threads) stored in the created pool of threads. At the moment of accepting the user request is for the needs of its responding a free thread randomly selected in pool which realizes such request, i.e., it will ensure execution of reading or writing data into the database system. In so doing it is required that in the case of realization of the request for the writing of the database no other thread can access the database environment and in the case of request for the reading any finite number of threads realizing reading of data can access the database.

Fig. 9a, shows the PN BASE7.P3 \textless PN5 that represents the client side of the whole distributed programming system. Client side will perform PARALLEL FOR cycles with two programming threads (the tokens in the input place IP) requesting the read and write server side database operations (firing of the transition T6). Server side database service is represented by the place P4 and will be requested by synchronous manner.

Fig. 9b, shows the PN BASE4.P1 \textless PN8 determined for the execution of user requirements on reading or writing into the database while using three common accessible programming threads located in the common programming pool (see the resource place R10). The respective threads realize either reading (firing of the transition T20, resp. T21) or writing (firing of the transition T22) of the data from or to the database via the mechanisms of BPN BASE8 (see Fig. 5) that models the access into the database environment with the support of multiple-readers/single-writer lock. The token in the resource place R20 represents single lock determined for obtaining the multiple-read-
ers/single-writer access of the database environment. Following realization of the relevant event the programming thread will be returned back into the common pool of threads represented by the resource place R10 and it will be ready for the execution of the next user request.

![Diagrams showing distributed programming system](image)

**Fig. 9a, b.** PNs modeling client and server side of distributed programming system

We will obtain the resulting PN that represents the whole distributed programming systems as shown in Fig. 10, by applying the following PLACE_SUBST transformation: \((\text{BASE7.P3} \langle \text{BASE5}.P2 \langle (\text{BASE4.P1} \langle \text{BASE8} \rangle) \rangle) \). This resulting PN is according to Lemma 3 (because the BPNs BASE4, BASE5, BASE7 and BASE8 are purely-formed PNs) purely-formed PN.

![Diagrams showing resulting PN](image)

**Fig. 10.** Resulting PN modeling distributed programming system
6 Conclusions

There is an opportunity to define additional transformations and operators for the class of PNs and to generalize it (for instance transformations for connecting two or more PNs). Selected PN then represents typically a method in the model of the programming system realized with the support of the Petri nets formalism. PNs can be thus successfully applied also in the area of the bi-relational P/T Petri nets [9] that represent an interesting modification of conventional P/T Petri nets.

The principles introduced in the class of PNs can be further generalized and implemented in the definition of the class of high-level Petri nets called the sequential object Petri nets [10, 11] which differ from PNs mainly by mutually differentiable tokens. The tokens alone are formed by non-empty sequences over the set of all integer numbers (this is also where the name of this Petri nets class came from). Sequential object Petri nets are then specially determined for the design, modeling and verification of distributed multithreading programming systems based on the object oriented and functional paradigms.

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