Baryon asymmetry of the Universe from evaporation of primordial black holes.

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(March 19, 2022)

The process of baryogenesis through the evaporation of black holes formed at the end of inflation phase is considered. The increase of black hole mass due to accretion from the surrounding radiation after the reheating is taken into account. It is shown that the influence of the accretion on the baryogenesis is important only in the case when the initial values of black hole mass are larger than \( \sim 10^4 \) g. The behavior of calculated baryon asymmetry, as a function of model parameters, is studied.

I. INTRODUCTION

First discussions about possible connection between baryon asymmetry and primordial black hole evaporation appeared at the middle of seventies, just after discovery of a phenomenon of the black hole evaporation. The possibility of an appearing of the excess of baryons over antibaryons in the process of evaporation of primordial black holes was noticed in works [1, 2]. The mechanism discussed in [1, 2] doesn’t require the non-conservation of baryon number in an underlying microscopic theory. Detailed calculations using this mechanism were done in works [3].

Later, this question was studied in the context of grand unified theories (GUTs) (i.e., theories in which baryon number is not conserved). The idea is simple: when black holes decay by the emission of Hawking radiation, they may emit baryon-number-violating Higgs particles (and/or leptoquarks) whose decays naturally generate baryon asymmetry.

In the work of J.Barrow et al. [6] baryogenesis via primordial black holes was considered using GUT and extended inflation scenario. The formation of very light primordial black holes (which disappear, as a result of evaporation, before nucleosynthesis, without any trace, except of the net baryon asymmetry) seemed to be most probable in inflationary models with first order phase transition. Recently, however, it was shown that in some modern variants of second order inflation models [4, 5] the formation of small black holes (and, consequently, baryon asymmetry production via black hole evaporation) is also quite possible.

In two recent works [7, 8] the baryogenesis through the evaporation of primordial black holes was studied quantitatively. Authors of these works argued that this scenario of baryogenesis can, in principle, explain the observed baryon number of the Universe (which is constrained by primordial nucleosynthesis data to be in range \((1.55 - 8.1) \cdot 10^{-11}\)). However, the distinct conclusion of works [7, 8] is that a sufficient number of black holes can survive beyond the electroweak phase transition and therefore, the baryon excess produced is not washed out by sphaleron transitions. These conclusions are in contradiction with qualitative statements of pioneering work of J.Barrow et al. [6]. Authors of [7, 8] claim that the difference between their result and those of J.Barrow et al. can be explained by taking into account the accretion (accretion term was omitted in formulas of work [6]).

Aside from the problem of the possible erasing of produced baryon asymmetry by sphaleron transitions, it is important to have the definite answer on the following question: is it possible to obtain by such mechanism (i.e., by primordial black hole evaporation) the cosmologically interesting value of baryon asymmetry?

In present work we try to answer just this question. We calculate the baryon asymmetry using the same assumptions, as in the work of J.Barrow et al. The main attention is paid to rigorous solution of kinetic equations (containing accretion term) and to comparison of exact results with predictions based on approximate formulas derived in [6].

II. BASIC ASSUMPTIONS

1. A generation of the black holes in early Universe may take place in many inflationary scenarios. The main assumption is that in the early Universe there was the period of inflationary expansion and the inflation is completed by a symmetry-breaking phase transition. The energy scale of symmetry breaking is \( \sigma_0 \sim 10^{16} \) GeV (GUT scale).

Clearly, the concrete mechanisms of black hole production are not the same in different inflationary models.

1a. Inflation of "old" type (extended inflation, first order inflation): the Universe exits from a false-vacuum state by bubble nucleation. Reheating and thermalization of the Universe proceeds through bubble collisions at the end of inflation [6]. False vacuum energy density is

\[
\rho_v \sim \xi \sigma_0^4 ; \quad \xi \sim 10^{-4} ,
\]

so

\[
\rho_v \sim 10^{60} \text{ GeV}^4 \equiv M^4 .
\]

Hubble parameter at the end of inflation is

\[
H_{end} = \sqrt{\frac{8 \pi}{3} \frac{\rho_v}{m_{pl}^2}} \sim 10^{12} \text{ GeV} ,
\]
so the time of the end of inflation is

$$t_{\text{end}} \sim \frac{1}{H_{\text{end}}} \sim 10^{-35} \text{ s.} \quad (2.4)$$

Usually one supposes that

$$t_{RH} \approx t_{\text{end}} \approx t_f \ll \tau_h \quad (2.5)$$

($t_{RH}$ is the time of reheating, $t_f$ is the time of formation of black holes, $\tau_h$ is life-time of black hole).

Black hole production proceeds 1) via the gravitational instability of inhomogeneities formed during the thermalization phase (i.e., during the bubble wall collisions) or 2) via the appearance of trapped regions of false vacuum caught between bubbles of true vacuum.

1b. "New" or "slow-roll-over" inflation: In this case black holes are produced through collapses of the overdense regions in space. So, for large probability of primordial black hole production one must exist large amplitudes of primordial density fluctuations at small scales.

Such fluctuations appear during inflation. The overdense region of mass $M$ can produce black hole when this fluctuation crosses horizon inside. At this time $M$ is equal to horizon mass $M_h$, and black hole produced has mass $M_{BH}$ which is close to $M_h$.

$$M_{BH} \sim 0.1 M_h \quad (2.6)$$

The time of PBH formation is

$$t_f \sim \frac{8M_h}{m_{pl}} \sim 10^2 \frac{M_{BH}}{m_{pl}} t_{pl}. \quad (2.7)$$

If, e.g., $M_{BH}$ is about $10^3$ g, then

$$t_f \sim 10^{-33} \text{ s.} \quad (2.8)$$

At this time, the corresponding scale factor is

$$a_f \sim (H_0^{3/2}(2.4 \times 10^4)^{-3/4}f_f^{3/2})^{1/3} \sim 10^{-27} \quad (2.9)$$

and the comoving length scale of the perturbation is

$$\lambda_f \sim \frac{ct_f}{a_f} \sim 10^{-14} \text{ pc.} \quad (2.10)$$

Further, we use the known formula

$$N_{\lambda_f} = 45 + \ln \frac{\lambda_f}{1 \text{ Mpc}} + \frac{2}{3} \ln \frac{M}{10^{15} \text{ GeV}} + \frac{1}{3} \ln \frac{T_{RH}}{10^{10} \text{ GeV}}. \quad (2.11)$$

Here, $M \sim 10^{15}$ GeV (it is $\sqrt{\rho_{\text{crit}}}$), $T_{RH}$ is reheating temperature; $N_{\lambda_f}$ is a number of e-folds before the end of inflation beginning from the moment when the scale crosses horizon outside.

If, e.g., $T_{RH} \sim 10^{11}$ GeV, one has $N_{\lambda_f} \sim 1$. Now it is clear that the fluctuations responsible for $M_{BH} \sim 10^3$ g are formed just near the end of inflation. So, in this sense, black holes with $M_{BH} \sim 10^2 - 10^3$ g are lightest ones.

The large amplitudes of density perturbations at small scales corresponding $N_{\lambda_f} \sim 1$ are naturally obtained in the hybrid inflation model of refs. [7,8].

If $T_{RH} \sim 10^{11}$ GeV, the reheating time is

$$t_{RH} \sim 0.3 \frac{1}{\beta^2 T_{RH}} \sim 10^{-29} \text{ s.} \quad (2.12)$$

So, black holes can be produced even at a time before reheating, when the Universe is dominated by the oscillations of the inflation field.

2. We assume that all produced primordial black holes have the same mass. It will be approximately so in hybrid inflation-type models, where the sharp maximum of density fluctuation-type models exists at some definite scale.

3. We assume that at $t_{RH}$ the part of energy density is in black holes:

$$\rho(t_{RH}) = \beta \rho_{BH}(t_{RH}) + (1 - \beta) \rho_R(t_{RH}). \quad (2.13)$$

$\beta$ is the free parameter of our model.

In the following we will consider only black holes having the life-time $\tau_h$, which is much larger than $t_{RH}$,

$$\tau_h \gg t_{RH}. \quad (2.14)$$

We will see that the most interesting predictions for baryon asymmetry don’t depend on $t_{RH}$ and are the same for both inflation scenarios provided we use in both cases the parameter $\beta$. 
III. KINETIC EQUATIONS

Evolution of black hole mass is described by the equation

$$\dot{M}_{BH} = AM_{BH}^2 - \frac{\alpha(M_{BH})}{M_{BH}}.$$  (3.1)

Accretion term $AM_{BH}^2$ is equal to $\sigma_{abs} \rho_R c$, where $\sigma_{abs}$ is cross section of absorption of relativistic particles by a black hole,

$$\sigma_{abs} = \frac{27\pi M_{BH}^2}{c^4},$$  (3.2)

$\rho_R$ is an energy density of the radiation. Evaporation term is $-\frac{\alpha(m)}{m^2}$, where $\alpha(m)$ counts the degrees of freedom of the black hole radiation ($m$ is an instantaneous value of the black hole mass).

In the following we will use the value 4

$$\alpha(m) = \text{const} = 80 \cdot 10^{25} \frac{\text{GeV}}{s}.$$  (3.3)

Evolution of the radiation energy density is given by the equation

$$\dot{\rho}_R = -\frac{\dot{a}}{a} \rho_R - \dot{M}_{BH} n_{BH},$$  (3.4)

$$n_{BH}(t) = \frac{\rho_{BH}(t_{RH})}{M_0} \cdot \frac{a^3(t_{RH})}{a^3(t)},$$  (3.5)

$$\rho_{BH}(t_{RH}) = \beta \rho(t_{RH}).$$  (3.6)

Here, $n_{BH}$ is number density of primordial black holes, $M_0$ is initial value of black hole mass, $\alpha(t)$ is scale factor.

Evolution of scale factor is given by Friedmann - Einstein equation:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3} (\rho_R + M_{BH} \cdot n_{BH}).$$  (3.7)

In this paper we use, for simplicity, the Einstein - de Sitter model.

Equation for evolution of the baryon number is

$$\dot{n}_B = -\frac{\dot{M}_{BH}^2}{3kT_{BH}^2} \cdot n_{BH} \cdot \epsilon f_H - \frac{3}{a} \dot{a} n_B.$$  (3.8)

We assume here, approximately, that for $dM_{BH}$ change of black hole mass we have $dM_{BH} \cdot c^2 / 3kT_{BH}$ particles produced ($T_{BH}$ is temperature of black hole connected with $M_{BH}$ by Hawking relation $kT_{BH} = \frac{\beta^3}{8\pi G M_{BH}}$); $f_H$ is the fraction of $X$ particles decaying with violation of B. Typically, $f_H \sim \frac{1}{9} \sim 10^{-2}$. Baryon number is obtained by introducing the factor $\varepsilon$ which is given by the relation

$$\varepsilon \equiv \sum_i B_i \frac{\Gamma(X \rightarrow f_i) - \Gamma(X \rightarrow f_i)}{\Gamma_X}.$$  (3.9)

$\varepsilon \neq 0$ if C,CP are not conserved. For obtaining baryon asymmetry one must divide $n_B$ on entropy $s$ which is given by

$$s(t) = \frac{2\pi^2}{45} g_* T^4(t) \left(\frac{k}{\hbar c}\right)^3,$$  (3.10)
It is very convenient to use dimensionless variables:

\[ m = \frac{M_{BH}}{M_0} \; ; \; \alpha = \frac{a(t)}{a(t_{RH})} \; ; \; R = \frac{\rho_R \alpha^4}{(1 - \beta) \rho(t_{RH})}, \]

\[ (3.12) \]

\[ \tau = \sqrt{G \rho(t_{RH}) t}, \]

for which we have simple initial conditions

\[ m(\tau = \tau_{RH}) = \alpha(\tau = \tau_{RH}) = R(\tau = \tau_{RH}) = 1, \]

\[ (3.13) \]

\[ \tau_{RH} = \sqrt{G \rho(t_{RH}) t_{RH}}. \]

We assume, approximately, that \( X \)-particles are evaporated when the following condition holds:

\[ 3kT_{BH} = 3k \frac{\hbar c^3}{8\pi G M_{BH}} \geq M_X c^2, \]

\[ (3.14) \]

\[ M_{BH}^{th} = \frac{3m_{pl}^2}{8\pi M_X}. \]

\[ (3.15) \]

Here, \( M_X \) is a mass of \( X \) particles.

Finally, we have the following free parameters:

\[ M_0 , \; \rho(t_{RH}) , \; \beta , \; M_X , \; \varepsilon. \]

\[ (3.16) \]

Some results of the solution of the system of kinetic equations (3.1), (3.4), (3.7) and (3.8) are presented on Fig.1-4. All calculations are carried out with \( \rho(t_{RH}) = 10^{55} \) GeV. Such value of \( \rho(t_{RH}) \) corresponds to \( t_{RH} \sim 10^{-33} \) s and reheating temperature \( T_{RH} \sim 10^{13} \) GeV.

**IV. APPROXIMATE FORMULAS FOR B**

One can easily show that for practically important case, when

\[ \beta \gg \sqrt{\frac{t_{RH}}{\tau_h}}, \]

\[ (4.1) \]

and when accretion is not important \( (M_0 \leq 10^3 \) g), there is an approximate solution of the kinetic equations for \( B \) which is given by the formula

\[ B = \left( \frac{30}{G_\ast} \right)^{1/4} \sqrt{\frac{\hbar}{c}} \frac{3}{4} \rho^{1/4}(t_{RH}) \times \]

\[ (4.2) \]

\[ \frac{M_{BH}^{th^2}}{m_{pl}^2 M_0} \frac{\varepsilon}{g_\ast} \left( \frac{t_{RH}}{\tau_h} \right)^{1/2}. \]
Using the relation
\[ \rho(t_{RH}) = \frac{3}{32} \frac{m_{pl}^2}{t_{RH}} \sim \frac{1}{t_{RH}}, \]  
(4.3)
one can see that \( B \) doesn’t depend on \( \rho(t_{RH}) \) or \( t_{RH} \).

The expression for \( \tau \) is obtained from the equation
\[ \dot{M}_{BH} = -\frac{\alpha}{M_{BH}^2} \]  
(4.4)
and is given by
\[ \tau_h = \frac{M_h^3}{3\alpha} . \]  
(4.5)
For \( M_0 \sim 10^2 - 10^3 \) g one has
\[ \tau_h \sim 10^{-21} - 10^{-18} \text{ s}. \]  
(4.6)

In our numerical calculations we used \( t_{RH} = 10^{-33} \) s. So one has
\[ t_{RH} \ll \tau_h . \]  
(4.7)

From here, one has the condition for parameter \( \beta \):
\[ \beta \gg \sqrt{\frac{t_{RH}}{\tau_h}} \sim (10^{-6} - 10^{-7}). \]  
(4.8)

For such values of \( \beta, t_{RH}, \tau_h \) one has the final formula:
\[ B \approx 10^{-14} \left( \frac{M_0}{10^3 \text{ g}} \right)^{-5/2} \times \]  
\[ \left( \frac{M_X}{10^{14}\text{GeV}} \right)^{-2} \left( \frac{\varepsilon}{1} \right). \]  
(4.9)

Condition (4.1) means [3] that the black hole energy density dominates at the time of evaporation. If the value of parameter \( \beta \) is such that
\[ \beta \ll \sqrt{\frac{t_{RH}}{\tau_h}} , \]  
(4.10)
the evaporation occurs while background radiation dominates the energy density of the Universe (with the entropy arising mainly from the radiation). In this case the approximate formula for \( B \) [3] can be obtained from Eq.(4.2) by substitution of last factor:
\[ \left( \frac{t_{RH}}{\tau_h} \right)^{1/2} \rightarrow \frac{\beta}{(1-\beta)^{3/4}}. \]  
(4.11)

V. CONCLUSION

The main conclusion of our work is the following: the predicted baryon asymmetry in the region \( M_0 \lesssim 10^4 \) g is well described by the approximate formulas (4.2) and (4.3) with substitution of (4.11). These formulas were obtained without taking into account the accretion, and it means that the accretion process is not important for small values of initial black hole masses. At larger masses accretion becomes to be important (the same was argued in the work [1]). One can see from Fig.4 that the accretion leads to a significant decrease of \( B \) (this decrease is a consequence of a significant growth of entropy after an evaporation of black holes enlarged due to accretion).

The resulting formula for \( B \) (Eq.(3) doesn’t depend on \( \rho(t_{RH}) \) and on \( t_{RH} \) and, therefore, is valid for both variants of inflationary scenario mentioned in Sec.IV (as far as the condition (1.1) is fulfilled). It is seen that, if \( M_0 \sim 10^3 \) g and \( M_X \sim 10^{14}\text{GeV} \), the predicted asymmetry is quite small (even for \( \varepsilon \sim 1 \)). This our calculation strongly disagrees with the corresponding results of work [1]. The effect (baryon asymmetry) can be large if \( M_0 < 10^3 \) g and/or \( M_X \ll 10^{14} \text{ GeV} \) (e.g., if \( M_X \sim 10^{11} \text{ GeV} \)).

It follows from Fig.3 that the temperature of the Universe at a moment of the evaporation is smaller than \( \sim 100 \text{ GeV} \) only in the case, when \( M_0 \gtrsim 10^5 \) g. For smaller values of \( M_0 \) (for which the value of \( B \) can be acceptably large) one has \( T_{ev} > 100 \text{ GeV} \), and the problem connected with the sphaleron transitions exists. Evidently, the baryon asymmetry produced by primordial black hole evaporation can survive only if nonzero \( (B - L) \)-value is generated in decays of Higgs particles of GUT.

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