Mathematical Modeling of a Linear Motion on a Deformable Bearing Surface of a Saddle-Type Road Train with Active Semi-Trailer Element

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Abstract. The paper introduces a mathematical modeling of linear motion on a deformable bearing surface (DBS) of a two-link road train consisting of four-axle truck with independent wheel drive and a three-axle semi-trailer. Predictive modeling based on application of pull-traction and energy efficiency obtained during the field research was used in the course of the model development to predict wheeled vehicle mover properties. Results of the theoretical research are given that relates to road train uphill motion with various wheel drives of the semi-trailer.

1. Introduction
The most useful technique to improve efficiency of transportation is to increase carrying capacity of vehicles, in particular by forming road trains. Under operational conditions typical of extractive and primary industries, the need often arises to transport cargo, heavy machinery, equipment along temporary roads or off-road (Fig. 1). At the same time, the most serious limitation for operation of 2-wheel road trains in the above industries is the low level of their cross-country performance [1–5].

![Fig. 1. Off-road operation of semi-trailer trucks](image)

Development and construction of road trains with a sufficiently high level of traction dynamics is thus a pressing scientific and practical issue, which may be effectively solved by introducing an active drive of link element wheels [6–8].
By their transmission gear, driving systems of link element active axles are divided into mechanical, hydrostatic and electric transmission types. Subject to its characteristics, a mechanical transmission, traditional for commercial multiple-purpose vehicles (TVs), is hardly suitable for multi-wheel cars or all-wheel drive road trains [9]. When designing multi-wheel cars, it is assumed most rational to use electromechanical transmission [10–13]. It is expedient to design link elements with electric wheel drive for future road tractors, which already have a sufficient capacity generator installed, featuring an electromechanical transmission [14]; it is also possible to use standard algorithms for an electromechanical transmission control.

To solve the issue of increased energy efficiency of road trains under various operational conditions, designers should have at their disposal a tool for simulation experiments and methods, which allows predicting their performance subject to different designs. Nowadays, the most efficient estimation method with regard to technical solutions and vehicle mobility characteristics prediction at the design stage is a computer-aided mathematical simulation. Complexity of the task of modeling a cross-country vehicle movement is due to vehicle interaction with deforming grounds featuring high heterogeneity and complex structure, which significantly impacts a vehicle dynamics [15]. Efficiency of mathematical models of wheeled vehicles movement depends greatly on the soil characteristics applied in modeling [16–18]. In most of the known models, wheel propeller interaction with soil base is described by different empirical equations, which requires rather wide range of test data [19]. Such formalization does not always provide high reproducibility of calculation results or full-scale experiments [20].

2. Theoretical Background

In this regard, it seems appropriate to use integrated test data, obtained in the course of bench or field tests, for modeling of a wheel mover interaction with DBL, which was offered and demonstrated in [21]. Within such approach, linear rolling parameters of a wheel under different motion conditions are defined by rate of energy loss $f_w$ (energy loss in rolling per a unit of distance covered by the wheel at a vertical unit load), free specific thrust $\varphi$ (axial force applied to the rolling wheel at vertical unit load at its axis), as well as by skid coefficient:

\[
S_b = \frac{\omega_k \cdot r_k - V_{sk}}{\omega_k \cdot r_{ks}} = 1 - \frac{r_k}{r_{ks}},
\]

where $\omega_k$ is the angular spin rate of a wheel; $r_{ks}$ is the free roll wheel radius; $r_k$ is the rolling radius ($r_k = V_{sk} / \omega_k$); $V_{sk}$ is the center-of-mass velocity of a wheel in $X$-direction.

Experiments should reveal values in the energy balance equation, which states that energy delivered to a rolling wheel with even motion is consumed for axial force work and losses caused by interaction with DBL:

\[
M_k \cdot \omega_k = P_x \cdot V_{sk} + f_w \cdot P_z \cdot V_{sk},
\]

where $M_k$ is the torque moment delivered to the wheel axis; $P_x$ is the axial force acting on the wheel axis; $P_z$ is the vertical force acting on the wheel axis.

Subject to test, $P_x = R_s$, and $P_z = R_z$, where $R_s$ is the force of interaction between a wheel and the bearing line; $R_z$ is the normal response in the contact spot of the wheel and the bearing line; the equation for calculation of the specific energy losses will be as follows:

\[
f_w = \frac{M_k}{P_z} \cdot \frac{\omega_k}{V_{sk}} - \frac{P_x}{P_z} = \frac{M_k}{P_z} \cdot \frac{\omega_k}{V_{sk}} - \varphi.
\]

The values included in the right side of equation (1) are found in the course of experiment. We can thus obtain pull and energy $f_w = f(\varphi)$ and traction $\varphi = f(S_b)$ capacity values for a set of road conditions (Fig. 2).
Fig. 2. Characteristic of interaction between a wheel and deforming soil: a – traction; b – pull and energy

Method of experimental characteristic application for simulation modeling of single wheel linear dynamics is described in detail in [22]. Design model of single wheel dynamics in the drive mode is given in Fig. 3 and may be described by a system of equations (2).

\[
\begin{align*}
\dot{m} \cdot \ddot{V}_{sk} &= \varphi \cdot P_z - P_x ; \\
J_k \cdot \ddot{\omega}_k &= M_k - (1 - S_b)(f_w + \varphi) \cdot P_x \cdot r_{ks},
\end{align*}
\]

where \( m \) is the weight per wheel; \( \ddot{V}_{sk} \) is the longitudinal acceleration of center of mass of the wheel; \( J_k \) is the spin-axis inertia; \( \ddot{\omega}_k \) is the angular acceleration of the wheel.

3. Mathematical Model of Road Train Motion

To solve a wide range of tasks related to dynamics of traction and cross-country performance, a mathematical model of two-link semi-trailer truck motion has been created. Development of the mathematical model included the following principal assumptions:

- Linear motion of a road train on an even deformable bearing surface is considered.
- The system is symmetric to the road train longitudinal axis.
- The wheel is considered as a hard rotating mass interacting with the bearing surface with the coefficient shown in Fig. 2.
- Soil deformation is not set through its immediate physical features, but as a specific energy loss during wheel rolling on a bearing surface.

Mathematical modeling of a semi-trailer truck movement is reviewed through the example of a 4-axle tractor truck with independent wheel drive and an active 3-axle semi-trailer. Design model of the road train link elements given in Fig. 4 is taken subject to the current task nature and is described with corresponding equations of motion (3).
Fig. 4. Design model of tractor truck link elements motion on a deformable bearing surface: 
(a) all-wheel drive tractor; (b) active (all-wheel drive) semi-trailer

\[
\begin{align*}
\mathbf{m}_1 \cdot \dot{V}_1 &= 2 \cdot \sum_{i=1}^{4} P'_{xi} - G_1 \cdot \sin \alpha - P_w - P_{kxx}; \\
\mathbf{m}_2 \cdot \dot{V}_2 &= 2 \cdot \sum_{i=5}^{7} P'_{xi} - G_2 \cdot \sin \alpha + P_{kxx}; \\
J_{ki} \cdot \dot{\omega}_{ki} &= M_{ki} - (1 - S_{kx}) \cdot (f_{w1} + \phi_i) \cdot R_{ji} \cdot n_x; \\
m_{ki} \cdot \dot{X}_{ksi} &= \dot{V}_1 \cdot R_{x1} - P'_{xi} - m_{ki} \cdot g \cdot \sin \alpha. 
\end{align*}
\]

(3)

The following designations are introduced in the system of equations (3): \(i\) is the road train axle number; \(m_1, m_2\) are mass values of a tractor and a semi-trailer, correspondingly; \(V_1, V_2\) are the longitudinal acceleration of the center of mass of a tractor and a semi-trailer, correspondingly; \(P'_{xi}\) is the longitudinal component of force acting on a tractor body from the \(i^{th}\) axle side; \(G_1, G_2\) are weights of a tractor and a semi-trailer, correspondingly; \(P_{kxx}\) is the longitudinal component of force in the bolster hitch; \(P_w\) is air resistance; \(M_{ki}\) is the torque of \(i^{th}\) axle wheels; \(m_{ki}\) is the mass of \(i^{th}\) axle wheels; \(g\) is the acceleration of gravity; \(\alpha\) is the bearing surface slope.

Force acting on a tractor body from wheels in \(X\)-direction:

\[
P'_{xi} = (X_{ki} - L_{ki}) \cdot k_p + (V_{ksi} - V_1) \cdot B_p.
\]

(4)
where $k_0$ is the longitudinal stiffness ratio of suspension; $B_p$ is the longitudinal damping ratio of suspension; $X_{ki}$ is the distance between the center of mass and the $i$th axle; $L_{ki}$ is the distance between the center of mass and suspension attachment points in $X$-direction; $V_{azi}$ is the wheel speed in the $i$th axle direction; $V_i$ is the center-of-mass velocity of a tractor.

Normal response values from interaction of $i$th axle wheels and the bearing line are defined using the following equation:

$$ R_{ij} = P'_{ij} + 2 \cdot m_{ki} \cdot g \cdot \cos \alpha , $$

where $P'_{ij}$ is the normal force acting on a body from $i$th axle wheels.

Normal response values are calculated using potential mismatch of trim angle of tractor and semi-trailer bodies ($\theta_1$ and $\theta_2$, correspondingly). $P'_{ij}$ is calculated using the system of equations (6):

$$ \begin{align*}
2 \cdot \sum_{i=1}^{4} P'_{ij} &= (G_1 - 8 \cdot m_{k1} \cdot g) \cdot \cos \alpha + P_{krz} ; \\
2 \cdot \sum_{i=1}^{4} P'_{ij} \cdot L_{ki} + (P_w + P_{ax1} + (G_1 - 8 \cdot m_{k1} \cdot g) \cdot \sin \alpha) \cdot h'_{1i} + & P_{krz} \cdot l_{kr1} + P_{krx} \cdot (h_{kr1} - l_{kr1} \cdot \theta_1) = 0 ; \\
(P'_{22} - P'_{21})/k_1 &= (L_{k1} - L_{k2}) \cdot \theta_1 ; \\
(P'_{32} - P'_{31})/k_1 &= (L_{k1} - L_{k3}) \cdot \theta_1 ; \\
(P'_{42} - P'_{41})/k_1 &= (L_{k1} - L_{k4}) \cdot \theta_1 ,
\end{align*} $$

where $h'_{1i}$ is the $Z$-distance from tractor wheel axle to the tractor center of gravity; $P_{ax1}$ is the tractor inertia ($P_{ax1} = m_1 \cdot V_1 \cdot h_{k1}$); $l_{kr1}$ is the distance between the tractor center of mass and the bolster hitch; $h_{kr1}$ is the distance between the tractor wheel axle and its fifth-wheel coupling in $Z$-direction.

Normal response acting on semi-trailer wheels is calculated in a similar way. Longitudinal component of the tractor and semi-trailer coupling is introduced through the force at the hook, which is calculated as follows:

$$ P_{krx} = (X_{C1} - X_{C2}) \cdot k_0 + (V_i - V_2) \cdot B_0, $$

where $k_0$ is the longitudinal stiffness ratio of the bolster hitch; $B_0$ is the coupler damper longitudinal resistance ratio; $X_{C1}$ and $X_{C2}$ are a coordinate on the $X$-axis of tractor and semi-trailer centers of mass, correspondingly.

4. Calculation Results
The developed mathematical model is used to estimate the effect from link element wheel drive at road train uphill movement on a deformable bearing surface. The routine of the simulation experiment comprises as follows: a road train starts moving after a preliminary stop on the uphill. The experiment is considered passed if the road train has managed to build on steady speed of ascent motion.

Two-link semi-trailer truck with a total weight of 115,000 kg (max. weight of the semi-trailer is 90,000 kg) was selected as the object of study. A road train with an inactive semi-trailer was chosen to be the basic option. Alternative options differ due to their semi-trailer wheels drive. Transmission of the tractor and the semi-trailer is electromechanical, made by “in-wheel electric motor” layout. In the basic option, each wheel drive is implemented using a traction electric motor with the capacity of 60 kW. The total capacity of all traction electric motors of the tractor is thus 480 kW. The capacity of active road train traction electric motors has been selected so as to have equal specific capacity of all the options. Partial parameters of traction electric motors were obtained by additional multiplication by the capacity factor.

Three options of capacity split between the road train link elements have been reviewed: in the first option, total capacity goes to the tractor wheels (basic option); in the second and third options, part of
the total capacity (25 and 50%, correspondingly) is distributed between the semi-trailer wheels. Simulation experiments were conducted for various weights of a semi-trailer. Angle of climb was defined subject to maximum possible weight of the road train under the given conditions. Moreover, maximum speed of the full weight road train was defined for climbing at different angles of the surface slope. Calculation results are given in Fig. 5 and 6.

Analysis of the obtained results allowed to draw a conclusion that activation of semi-trailer wheels while moving on the analyzed type of bearing surface leads to increased angle of climb. Full-weight road train in the basic option loses its mobility at the slope of 6°, in the second case (45/20) the angle of climb increased to 11°, in the third case (30/40) – to 13.5°. Moreover, it can be seen from Fig. 6 that the basic option of a road train builds up the minimum speed in comparison with the other two cases when moving both on horizontal surface and uphill.

5. Conclusions
A model and a design program have been developed that allow studying two-link semi-trailer truck dynamics in its linear motion on deformable bearing surfaces. The developed model can be used to predict traction and dynamic properties and determine rational balance of capacity split between tractor wheels and semi-trailer wheels achieving improved energy efficiency of a road train in cross-country conditions.

Based on the conducted simulation experiments and with the help of the developed model, it has been proved that worse traction dynamic performance of motion on deformable bearing surfaces is caused by insufficient trailing weight. From the calculation results, it is seen that the basic option of a road train with its full weight can lose mobility at a longitudinal slope of 6°, which is clearly insufficient for on ground motion. The tractor itself (without useful load) is able to climb slopes of up to 19°, while being a part of an active road train, it can climb the same slope with the useful load of up to 47 tons.

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