Multiplicity fluctuations in high energy hadronic and nuclear collisions

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The showers of cosmic rays entering the Earth’s atmosphere are main sources of information on cosmic rays and are also believed to provide information on elementary interactions at energies not accessible to accelerators. In this context we would like first to remind the role of inelasticity $K$ and elementary cross section $\sigma$ and then argue that similar in importance are fluctuations of different observables. The later will be illustrated by multiplicity fluctuations in hadronic and nuclear collisions.

1. INTRODUCTION

The EAS (Extensive Air Showers) initiated by cosmic rays (CR) entering the Earth’s atmosphere are our main tool to investigate the CR as such and they are believed to remain our only possible window to look at the elementary interactions at energies not accessible to accelerators\textsuperscript{[1]}. Agreeing fully with the first part of this statement we would like to remind here that EAS are themselves macroscopic stochastic processes of variable length and this fact sometimes leads to results, which are unexpected from the elementary particle interactions point of view, the apparent intermittency seen there and connected solely with stochasticity of EAS only\textsuperscript{[2]} being the best example. Other example worth to mention at this point is the apparent self-organized character of EAS depending in a visible way on their length and clearly seen in data\textsuperscript{[3]}. Both phenomena do not depend on details of elementary interactions used in describing EAS. Actually, as it was widely appreciated some time ago\textsuperscript{[4]} and is being rediscovered also at present\textsuperscript{[5]}, out of numerous parameters of models of elementary interactions used in MC codes describing production and development of EAS the most important are: inelasticity $K$ and elementary cross section $\sigma$ (actually, the hadron - air nucleus cross section). Out of these two, the inelasticity $K$, i.e., the fraction of energy used for production of secondaries and therefore not available for the subsequent interactions in developing EAS, is nowadays not really a parameter as it used to be before (cf.\textsuperscript{[4]}) but rather a number which must be reproduced by combination of all parameters used in the present models\textsuperscript{[5]}.

2. FLUCTUATIONS

However, there are some data (cf.\textsuperscript{[7]}) which seem to demand more information. In particular, in\textsuperscript{[8]} we have shown that the phenomenon of the so called \textit{long-flying-component} can be most naturally explained by allowing for the fluctuations in the cross section (which at that time were also widely discussed in the literature in other context). This conjecture was later quantify by using the so called nonextensive statistical approach (essentially corresponding to using Tsallis entropy characterized by parameter $q$ such that for $q \rightarrow 1$ it becomes the usual Boltzmann-Gibbs entropy)\textsuperscript{[9]}. In such approach parameter $q$ measures, in a sense, the amount of fluctuations.

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For the purpose of this presentation it is enough to say that whenever in the exponential formula, \( \exp(-X/\Lambda) \), parameter \( 1/\Lambda \) fluctuates according to gamma distribution then one should use instead expression

\[
\exp_q(-X/\Lambda_0) = [1 - (1 - q)X/\Lambda]^{1/(1-q)} \tag{1}
\]

where \( 1/\Lambda_0 = \langle 1/\Lambda \rangle \) and \( q = \langle (1/\Lambda)^2 \rangle / \langle 1/\Lambda \rangle^2 \) (averages are over the above gamma distribution). We first applied this approach to the above mentioned data on long flying component \(^\text{12}\) and found that fluctuations they correspond to are given by \( q = 1.3 \).

In Figs. 1-3 we provide results on inelasticity \( K \) and fluctuations seen in the rapidity distributions, which originate from the multiplicity fluctuations \(^\text{13,14}\). They were obtained using nonextensive version of the information theory approach to multiparticle production processes based on the Tsallis \( q \)-entropy. Given the mean multiplicity \( \langle n \rangle \) and mean transverse momentum \( \langle p_T \rangle \) one gets the most probably and least biased rapidity distribution, being of the \( \exp_q(\ldots) \) type, i.e., depending on the nonextensivity parameter \( q \), which further depends only on the amount of energy available for production of secondaries, \( W = K \cdot \sqrt{s} \), i.e., on the inelasticity \( K \). In this way inelasticity \( K \) and nonextensivity parameter \( q \) specifying entropy used as measure of information are the only parameters to be fitted. Fig. 1 shows the energy dependence of inelasticity, \( K = K(s) \), whereas Fig. 2 the energy dependence of fluctuations as given by \( q = q(s) \). In the case when one knows rapidity distributions also for a given multiplicity (or, at least, for some multiplicity bins) this method allows to deduce also inelasticity distribution, as can be seen in Fig. 3 where it was done for \( \sqrt{s} = 200 \) GeV (black symbols, \( K = 0.52 \)) and 900 GeV (open symbols, \( K = 0.38 \)). They can be fitted either by gaussians (full lines) or, better, by lorentzian (dotted lines) shapes.

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\(^6\)See \(^\text{10}\) for details. Generalization of this approach to other type of fluctuations is discussed in \(^\text{11}\).
In what follows we shall, however, concentrate on fluctuations described by the parameter \( q \) shown in Fig. 2. We argue that these fluctuations are due to fluctuations of mean multiplicity used as our input when deducing inelasticity \( K \). Namely, when there are only statistical fluctuations in the hadronizing system one expects Poissonian form of the corresponding multiplicity distributions. The existence of intrinsic (dynamical) fluctuations means then that one allows mean multiplicity \( \bar{n} \) to fluctuate. In the case when these fluctuations are given by gamma distribution with normalized variance \( D(\bar{n}) \) then, as a result, one obtains the Negative Binomial multiplicity distribution, which depends on two parameters: the mean multiplicity \( \langle n \rangle \) and the parameter \( k \) \((k \geq 1)\) affecting its width, \[
\frac{1}{k} = D(\bar{n}) = \frac{\sigma^2(\bar{n})}{(\bar{n})^2}.
\] That is because (see [14]):
\[
P(n) = \frac{\gamma^k \bar{n}^{k-1} e^{-\gamma \bar{n}}}{\Gamma(k)} \frac{\Gamma(k+n)}{\Gamma(1+n) \Gamma(k)} = \gamma^k \frac{\bar{n}^n e^{-\gamma \bar{n}}}{n!}
\]
where \( \gamma = \frac{k}{\langle n \rangle} \). According to our philosophy it is therefore natural to describe these fluctuations by the nonextensivity parameter \( q \) assuming that \( D(\bar{n}) = q - 1 \), i.e., that \( q = 1 + 1/k \). However, in the nonextensive approach of this type \( q \) is limited by \( q < 3/2 \) [14]. This condition would then impose constraint on the amount of fluctuation as it corresponds to \( k < 2 \), the saturation limit, which from the naive extrapolation of tendency presented in Fig. 2 would appear at energies \( \sim 33 \) TeV (or \( E_{LAB} = 0.5 \cdot 10^{18} \text{ eV} \)) range. Notice that this energy range of the ultra high energetic cosmic rays in which effects connected with the GZK cut-off starts to be important [1]. It means therefore that the knowledge of fluctuations will most probably turn out to be as important as that of inelasticity and cross section.

We would like to add to this one more observation. In Fig. 4 we have plotted experimental data show on mean charged multiplicity \( \langle n_{ch} \rangle \) at different energies versus corresponding total inelastic cross section \( \sigma \) (Fig. 4a) and the estimated volume of the interaction region \( V \) (taken as being proportional to \( \sigma^{3/2} \)). The linearity of \( \langle n_{ch} \rangle \) and \( \sigma \) is remarkable. This suggest that fluctuations of the multiplicity distributions should also be connected with fluctuations of the cross section but at the moment there are no investigations in this direction[7].

3. SUMMARY

To summarize: phenomenon of EAS is macroscopic and so complicated that it depends only on a small number of characteristics of elementary interactions. It seems that they are: inelasticity \( K \), cross section \( \sigma \) and its fluctuations. We have argued that although we do not measure directly the later we measure fluctuations of multiplicity \( \langle n_{ch} \rangle \) at energies of interest to CR as predictions vary in unacceptable way [15].

[7] Although constraints on the production cross section imposed by the observed behaviour of multiplicity distributions have been investigated already long time ago [24], this subject is not pursued at moment. In models so far there is problem with \( \langle n_{ch} \rangle \) at energies of interest to CR as predictions vary in unacceptable way [15].
tivity distributions, which influence such observables as rapidity distributions. It is argued that they should also provide us with some estimations of the fluctuations of the cross section, albeit we do not yet know the respective formulas. However, there are already some data, which warn us that to get such connection could be very difficult task. Namely, in Fig. 5 we see clearly that such fluctuations measured in nuclear collisions are nonmonotonic function of the number of participating nucleons, which in turn is proportional to the volume of interaction \(c^2\). This could suggest that monotonicity observed in behaviour of first moments (cf. Fig. 4) could be lost when going to higher moments.

![Figure 5](image-url)  
**Figure 5.** Results on the nonmonoticity of fluctuations represented by \(\text{Var}(n_{\text{neg}})/\langle n_{\text{neg}} \rangle\) as function of number of participating nucleons (cf. \cite{19}).

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\(^{8}\)As shown in \cite{20} for nuclear collisions fluctuations in \(n\) are directly connected with the equation of state parameter \(c = p/\varepsilon\) and \(q:\) \(\text{Var}(n) = (q-1)\cdot\langle n \rangle^2/(1+c)^2 \approx 0.28\cdot\langle n \rangle^2\) for \(c = 1/3\) and limiting value of \(q = 1.5\).