Nonsymmetric Gravitational Theory as a String Theory

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Abstract

It is shown that the new version of nonsymmetric gravitational theory (NGT) corresponds in the linear approximation to linear Einstein gravity theory and antisymmetric tensor potential field equations with a non-conserved string source current. The Hamiltonian for the antisymmetric field equations is bounded from below and describes the exchange of a spin $1^+$ massive vector boson between open strings. The non-Riemannian geometrical theory is formulated in terms of a nonsymmetric fundamental tensor $g_{\mu
u}$. The weak field limit, $g_{\mu\nu} \to 0$, associated with large distance scales, corresponds to the limit to a confinement region at low energies described by an effective Yukawa potential at galactic distance scales. The limit to this low-energy confinement region is expected to be singular and non-perturbative. The NGT string theory predicts that there are no black hole event horizons associated with infinite red shift null surfaces.
I. INTRODUCTION

A new version of nonsymmetric gravitational theory (NGT) has recently been published \cite{1,2}, which was shown to have a linear approximation free of ghost poles and tachyons with a Hamiltonian bounded from below. The expansion to linear order in $g_{\mu\nu}$ about a fixed GR background also has a ghost-free Lagrangian with physical asymptotic behavior. The theory produces good fits to galaxy rotation curves and can explain gravitational lensing and cluster dynamics without appreciable amounts of dark matter \cite{7}.

In the following, we shall show that the linear approximation for weak fields corresponds to the spin $2^+$ graviton linear equations of general relativity (GR) and to massive spin $1^+$ field equations. The source for the graviton field is the standard point particle energy-momentum tensor of GR, while the source for the antisymmetric tensor field is a string source current for open strings which is not conserved. The rigorous nonlinear action of NGT is a nontrivial non-Riemannian geometrical unification of GR and string theory, which has as one of its predictions that black hole event horizons are not expected to form during gravitational collapse \cite{6,9}. The predictions of the new NGT at cosmological scales is expected to produce a novel dynamical scenario, as an alternative to the standard inflationary model \cite{8}.

Clayton has developed a canonical Hamiltonian formalism for NGT, and shown that the rigorous theory possesses six degrees of freedom \cite{10}. He also showed that the limit to the weak field linear theory for $g_{\mu\nu} \to 0$ may be singular, i.e., a Lagrange multiplier associated with the skew fields behaves as $\sim 1/g_{\{0i\}}$ as $g_{\{0i\}} \to 0$ ($i = 1, 2, 3$), so that the very low-energy limit of NGT, going from six degrees of freedom of the rigorous theory to the three degrees of freedom of the linear theory, may be a singular limit. In the following, this limit is interpreted physically to be the weak field galactic scale limit of NGT, for which the Newtonian and GR predictions fail to be valid. We shall argue that this is a low-energy string confinement limit for large distance scales of NGT, which is a non-perturbative sector of the theory in which three degrees of freedom are not excited.
II. NGT ACTION AND STRING THEORY

The nonsymmetric gravitational theory (NGT) is based on the decomposition of the fundamental tensor $g_{\mu\nu}$:

$$g_{\mu\nu} = g_{(\mu\nu)} + g_{[\mu\nu]},$$  \hspace{1cm} (1)

where

$$g_{(\mu\nu)} = \frac{1}{2}(g_{\mu\nu} + g_{\nu\mu}), \quad g_{[\mu\nu]} = \frac{1}{2}(g_{\mu\nu} - g_{\nu\mu}).$$

The connection $\Gamma^\lambda_{\mu\nu}$ also has the decomposition:

$$\Gamma^\lambda_{\mu\nu} = \Gamma^\lambda_{(\mu\nu)} + \Gamma^\lambda_{[\mu\nu]}.$$  

The Lagrangian density takes the form:

$$\mathcal{L}_{NGT} = \mathcal{L}_R + \mathcal{L}_M,$$  \hspace{1cm} (2)

where

$$\mathcal{L}_R = \mathcal{g}^{\mu\nu} R_{\mu\nu}(W) - 2\Lambda \sqrt{-g} - \frac{1}{4} \mu^2 \mathcal{g}^{\mu\nu} g_{[\mu\nu]} - \frac{1}{6} \mathcal{g}^{\mu\nu} W_\mu W_\nu,$$  \hspace{1cm} (3)

where $g = \text{Det}(g_{\mu\nu})$, $\Lambda$ is a cosmological constant and $\mu$ is a “mass” associated with $g_{[\mu\nu]}$. Moreover, $\mathcal{L}_M$ is the matter Lagrangian density ($G = c = 1$):

$$\mathcal{L}_M = -8\pi g^{\mu\nu} T_{\mu\nu},$$  \hspace{1cm} (4)

Here, $g^{\mu\nu} = \sqrt{-g} g^{\mu\nu}$ and $R_{\mu\nu}(W)$ is the NGT contracted curvature tensor:

$$R_{\mu\nu}(W) = W^\beta_{\mu\nu,\beta} - \frac{1}{2} (W^\beta_{\mu\beta,\nu} + W^\beta_{\nu\beta,\mu}) - W^\beta_{\alpha\nu} W^\alpha_{\mu\beta} + W^\beta_{\alpha\beta} W^\alpha_{\mu\nu},$$  \hspace{1cm} (5)

defined in terms of the unconstrained nonsymmetric connection:

$$W^\lambda_{\mu\nu} = \Gamma^\lambda_{\mu\nu} - \frac{2}{3} \delta^\lambda_{\mu} W_\nu,$$  \hspace{1cm} (6)

where
\[ W_\mu = \frac{1}{2}(W^{\lambda\mu} - W^{\lambda\mu}). \]

Eq. (6) leads to the result:

\[ \Gamma_\mu = \Gamma^{\lambda}_{[\mu\lambda]} = 0. \]

We shall assume that \( \Lambda = 0 \) and expand \( g_{\mu\nu} \) about Minkowski spacetime:

\[ g_{\mu\nu} = \eta_{\mu\nu} + (1) h_{\mu\nu} + \ldots, \]

where \( \eta_{\mu\nu} \) is the Minkowski metric tensor: \( \eta_{\mu\nu} = \text{diag}(-1, -1, -1, +1) \). We also expand \( \Gamma^{\lambda}_{\mu\nu} \) and \( W^{\lambda\mu}_{\mu\nu} \) in a similar manner, and we adopt the notation: \( \psi_{\mu\nu} = (1) h_{[\mu\nu]} \). Then, to first order of approximation, we find that \([1,2]\):

\[ \psi_\mu = \frac{1}{2} W_\mu = \frac{16\pi}{\mu^2} T^{\mu\nu}_{\mu\nu}, \]

where

\[ \psi_\mu = \psi_\mu^{\nu}\beta^{\beta} = \eta^{\beta\sigma} \psi_\mu^{\beta,\sigma}. \]

The symmetric and antisymmetric field equations decouple to lowest order; the symmetric equations are the usual Einstein field equations in the linear approximation, while the skew equations are given by

\[ F_{\mu\nu\lambda}^{\lambda} + \mu^2 \psi_{\mu\nu} = 16\pi T_{\mu\nu}, \quad (7) \]

where

\[ F_{\mu\nu\lambda} = \psi_{\mu\nu,\lambda} + \psi_{\nu\lambda,\mu} + \psi_{\lambda\mu,\nu}. \]

Eq. (7) can be written as

\[ (\Box + \mu^2) \psi_{\mu\nu} = 16\pi (T_{\mu\nu} + \frac{2}{\mu^2} T_{[\mu\sigma],\nu}^{\sigma}). \quad (8) \]

The action has the form:
\[ S = \int d^4x \left( \frac{1}{12} F_{\mu\nu\lambda}F^{\mu\nu\lambda} - \frac{1}{4} \mu^2 \psi_{\mu\nu}\psi^{\mu\nu} + 8\pi \psi^{\mu\nu} T_{\mu\nu} \right). \]  

(9)

The form of the field equation, Eq. (8), is the same as that derived by Kalb and Ramond from a string action [11]:

\[ I = -\Sigma_a \mu_a^2 \int (d\sigma_a \cdot d\sigma_a)^{1/2} + \Sigma_{a,b} \mu_a \mu_b \int d\sigma_a^{\mu\nu} d\sigma_{b\mu} \Delta(s_{ab}^2), \]  

(10)

where \( \Delta(s_{ab}^2) \) is a Green’s function describing time-symmetric interactions, and

\[ s_{ab}^2 = (x_a - x_b) \cdot (x_a - x_b). \]

Moreover,

\[ d\sigma_a^{\mu\nu} = d\tau_a d\xi_a \sigma_a^{\mu\nu}, \]

where

\[ \sigma_a^{\mu\nu} = \dot{x}_a^{\mu} x_a^{\nu} - x_a^{\mu} \dot{x}_a^{\nu} \]

and

\[ \dot{x}_a^{\mu} = \frac{\partial x_a^{\mu}}{\partial \tau_a}, \quad x_a^{\mu} = \frac{\partial x_a^{\mu}}{\partial \xi_a}. \]

The coupling constants \( g_a \) have the units of mass, \( \mu_a \) is chosen to make the action dimensionless in natural units and the sums are over all strings. A string is a one-dimensionally extended object, \( x_a^{\mu}(\tau_a, \xi_a) \), which is traced out by a world sheet in spacetime by the invariant parameters \( \tau_a \) and \( \xi_a \). The action is manifestly parametrization invariant.

The current density \( T_{a[\mu\nu]} \) has the form:

\[ T_{a[\mu\nu]} = g_a \int d\sigma_{a\mu\nu} \delta^{(4)}(y - x_a(\tau, \xi)). \]  

(11)

For the open string interactions the current is not conserved:

\[ T_{a[\mu\nu]}(y) = g_a \int_{\tau_i}^{\tau_f} d\tau [\dot{x}_a^{\mu}(\tau, \xi) \delta^{(4)}(y - x_a(\tau, \xi))]_{\xi=0} \]  

(12)
where \( l \) is a constant with dimensions of a length. The non-conservation of the source is due to the dependence of the right-hand side on the end points of the string.

The Hamiltonian obtained from the action, Eq.(9), is bounded from below for reasons similar to those that apply to the point particle action of the massive Maxwell-Proca theory.

A more general string action in spacetime can be written \[12,13\]:

\[
I = -\frac{1}{4\pi\alpha'} \int d\xi d\tau g_{\mu\nu} \sqrt{-g} g^{ab} \partial_a X^\mu \partial_b X^\nu,
\]

where \( g_{\mu\nu} \) is the nonsymmetric fundamental tensor, \( g = \text{Det}(g_{ab}) (a, b = 1, 2) \) and \( \alpha' \) is related to the string tension \( T \) by \( T = (2\pi\alpha')^{-1} \).

### III. THE LOW-ENERGY CONFINEMENT SECTOR OF THE NGT STRING THEORY

The vibrations of the strings in the theory generate modes of excitation corresponding to different field degrees of freedom. Clayton has used a canonical Hamiltonian formulation of NGT to demonstrate that the full non-linear theory possesses six degrees of freedom \[10\]. The diffeomorphism invariance of the action, Eq.(2), reduces the number of degrees of freedom of \( g_{\mu\nu} \) from 16 to 12; their is no further reduction of the degrees of freedom in the rigorous theory, owing to a lack of further gauge invariance constraints in the antisymmetric sector. However, the linear approximation for weak fields is characterized by field equations with only 3 degrees of freedom, since the three \( g_{[0i]} \) components can be gauged away due to the gauge invariance of the kinetic energy term: \( \frac{1}{12} F_{\mu\nu\lambda} F^{\mu\nu\lambda} \) in the action, Eq.(3), with \( \delta \psi_{\mu\nu} = \xi_{\mu,\nu} - \xi_{\nu,\mu} \). The Lagrange multiplier associated with \( \Gamma^\lambda_{[\mu\nu]} \), determined by the \( g \) and \( \Gamma \) compatibility equation in the Hamiltonian formulation, has a singular limit as \( g_{[0i]} \rightarrow 0 \), due to a transition from a derivative coupled kinetic energy in the rigorous theory to the gauge invariant kinetic energy of the linear approximation \[10\].

This feature of NGT can be understood by viewing the vibrations of the strings as generating different energy scales. A basic scale is set by \( r_0 = \mu^{-1} \); it is interpreted as the
onset of the “confinement” distance scale. It corresponds to the low-energy scale for very weak $g_{[\mu\nu]}$ fields, and is taken to be of galactic dimensions, $r_0 \sim 25$ kpc. The transition from the non-confining to the confining energy region is characterized by a reduction in the number of degrees of freedom in the “effective” NGT field theory description of the string theory. Thus, an increase in energy as the strings vibrate, excites the additional three degrees of freedom associated with $g_{[0i]}$, although these degrees of freedom are only measurable “locally” in the spacetime structure, due to the short-range nature of the antisymmetric field $g_{[\mu\nu]}$. The singular limit of the theory, as $g_{[\mu\nu]} \to 0$, is due to the non-perturbative nature of the confinement region.

An effective model of the the low-energy confinement limit has been derived from the weak field point particle limit of NGT \cite{7,14}. The total radial acceleration on a test particle in the weak field limit has the form:

$$a(r) = -\frac{G_0 M}{r^2} \left\{ 1 + \sqrt{\frac{M_0}{M}} [1 - \exp(-r/r_0)(1 + r/r_0)] \right\},$$  

where $M_0$ is a constant mass parameter, and the gravitational constant at infinity $G_\infty$ is defined by

$$G_\infty = G_0 \left( 1 + \sqrt{\frac{M_0}{M}} \right),$$  

and $G_0$ is the Newtonian gravitational constant. Thus, the gravitational constant $G$ “runs” with energy in analogy with the coupling constant $\alpha_{qcd}$ in quantum chromodynamics. In the high-energy limit $M \to \infty$, we have $G_\infty \to G_0$. The effective potential has the form for a range of $r \leq r_0$:

$$V(r) \sim \frac{a}{r} + b \ln r,$$

where $a$ and $b$ are constants. This describes a phenomenological $1/r$ plus a confining string potential. The non-additive nature of the Yukawa contribution in (14), caused by the $\sqrt{M}$ factor, is related to the non-perturbative nature of the confinement limit.

Eq.(14) has been applied to galaxy dynamics and good fits were obtained for small and large spiral galactic rotation curves, without assuming appreciable amounts of dark matter...
It is also possible to explain gravitational lensing effects and cluster dynamics without significant amounts of dark matter.

At somewhat higher energies – corresponding to the scale of the solar system – the Newtonian law of gravity and the corrections due to GR are valid, and the standard tests of Newtonian and GR theories will be predicted, in agreement with observations. The weak equivalence principle is retained in the new version of NGT, although the strong equivalence principle, which states that the non-gravitational laws of physics will not be the same in different local frames of reference, will not be valid.

The string sources in NGT may also be responsible for the elimination of black hole event horizons in gravitational collapse. For strong fields near the Schwarzschild radius, \( r = \frac{2GM}{c^2} \), the static spherically symmetric vacuum solution does not possess any null surfaces in the range \( 0 < r \leq \infty \), which results in a coordinate invariant, finite red shift for collapsed astrophysical objects. Nonetheless, their can exist a collapsed, massive compact object with a large but finite red shift that simulates the putative observational evidence for black holes. Near the Schwarzschild event horizon, the string theory is described by a high energy or short distance scale, and the effects of the \( g_{[\mu\nu]} \) fields become of critical importance. The elimination of black hole event horizons would remove the potential information loss problem at the classical level. The possibility that string theory may eliminate black hole event horizons has also been suggested by Cornish.

### IV. CONCLUSIONS

We have shown that NGT has to be associated with string theory, because the source of the antisymmetric field \( g_{[\mu\nu]} \) is naturally described by strings as demonstrated by Kalb and Ramond. This means that NGT can be interpreted as a geometrical unification of Einstein gravity and bosonic string theory. The weak field linear approximation corresponds to a non-perturbative limit reached at galactic scales, where Newtonian and Einstein gravity fail to be valid. The transition from a Newtonian and Einstein potential energy to a
confinement potential energy, described by an effective point-like Yukawa potential, predicts radically different gravitational dynamics at the galactic scale, exhibited observationally by the flat rotational velocity curves of spiral galaxies. The gravitational constant runs with energy in analogy to the coupling constant in quantum chromodynamics, and the gravitational confinement region at low energy is non-perturbative.

Another interesting consequence of NGT is at the cosmological scale, when the $g_{\mu\nu}$ field can be a natural source of inhomogeneities in the early universe. The NGT field equations could dynamically evolve towards a solution close to the standard big bang model with a small effective cosmological constant at the present epoch, produced by the antisymmetric field. Thus, the standard big bang model with a cosmological constant would act as an attractor for the solutions of the NGT field equations.

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