Supersymmetry Breaking at Finite Temperature in a Susy Harmonic Oscillator with Interaction.

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Supersymmetry breaking of a supersymmetric harmonic oscillator with polynomial interaction is analyzed. Some thermal effects are studied in the context of Thermo Field Dynamics (TFD). The restored supersymmetry results in non vanishing energy at finite temperatures due the additivity of the thermal effects, while at $T = 0$ the energy is zero.

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I. INTRODUCTION

Supersymmetry has different characteristics than other internal symmetries. One of these characteristics is that in contrast with the internal symmetries, supersymmetry can be broken for theories with finite fermionic and bosonic degrees of freedom. Therefore we can think of spontaneous symmetry breaking in supersymmetric quantum mechanics. Some clear motivations and significant progress about the mechanisms of SUSY breaking in quantum field theory could be seen at [9, 10, 11, 12, 13, 14, 15]. The interest in supersymmetry breaking in quantum mechanics becomes obvious, when we verify that some results from the breaking of supersymmetric quantum mechanics can be generalized and applied to quantum field theory. In the limit of low energy, the underlying field theory should approach a Galilean invariant supersymmetric field theory and, by the Bergmann superselection rules, such a field theory would be equivalent to a supersymmetric Schrödinger equation in each particle number sector of the theory.

Supersymmetry at finite temperature has been studied in refs [10, 11, 12, 13, 14, 15]. Nevertheless, the issue of whether supersymmetry is broken at finite temperature has raised some controversy. In [10, 11] it is argued that supersymmetry (SUSY) is broken at positive temperature when unbroken at $T = 0$. Regarding this it has been suggested [11], that when a change of an operator under SUSY transformation at finite temperature is considered, one should take into account the Klein operator. When this operator is incorporated, the author of ref. [11] shows that the thermal average of the operator variation is zero for all $T$, thereby maintaining supersymmetry at finite temperature. On the other hand, considering this issue within the context of Thermo Field Dynamics (TFD) [14, 16] the SUSY is broken at finite temperature. The vacuum expectation value of the SUSY Hamiltonian at $T = 0$, in the thermal vacuum $|\theta(\beta)\rangle$ (where $\beta = 1/kT$, $k$ being the Boltzmann constant) is non-zero at finite temperature. By evaluating the statistical average of the SUSY Hamiltonian at $T = 0$ as its vacuum expectation value in the thermal vacuum $|0(\beta)\rangle$ (where $\beta = 1/kT$, $k$ being the Boltzmann constant) and showing that it is non-zero at finite temperature.

In this paper following TFD, in $T = 0$ we show that for some conditions the supersymmetric harmonic oscillator is broken getting a bosonic and fermionic harmonic oscillator, with $\omega_1$ and $\omega_2$ respectively the bosonic and fermionic frequencies, and the opposite way is possible. Later we show that in agreement with the former results, the supersymmetry is broken at finite temperatures.

Many quantum systems can be considered as an application of this work. A direct application is in the Searching Neutrino-Nucleus interaction [17]. For example the two level system of neutron valence vibrations considered in the Coherent Neutrino Nucleus Scattering.

II. SUSY BREAKING OF THE FREE SUSY HARMONIC OSCILLATOR AT FINITE TEMPERATURE.

Consider a Hamiltonian in supersymmetric quantum mechanics in the component form

$$H = \frac{1}{2}[p^2 + W^2(x) + \sigma_3 W'(x)]. \quad (1)$$

The boson-boson interaction is represented by the term $W^2(x)$, and the boson fermion is represented by $\sigma_3 W'(x)$. Both are determined by the same function $W(x)$. This property is found in all supersymmetric models.

As shown in [1], $W(x)$ varies from the superpotential $V(x)$, where $W(x) = V(x)$ and $V'(x)$ is the derivative of $V(x)$. Sometimes $W(x)$ is also called the superpotential.
The double degeneration that occurs in all levels of energy with \( E > 0 \), follows directly from the supersymmetric algebra; it does not depend on \( W(x) \). In addition we find that the energy of the ground state is non-negative. Similarly this follows from the supersymmetric algebra and it is independent of the function \( W(x) \).

Considering any internal symmetry \( S \) with generator \( G_S \). This symmetry \( S \) is exact and is not spontaneously broken if \( [H, G_S] = 0 \) and the ground state \( |0\rangle \) is invariant, \( G_S|0\rangle = 0 \). In the case of supersymmetry, an example with exact supersymmetry is the free supersymmetric harmonic oscillator, where \( W = \omega_1 x \) [13, 12].

In terms of the creation and annihilation operators the Hamiltonian becomes,

\[
H = \omega_1 (a^{\dagger}a + b^{\dagger}b)
\]

where \( a^{\dagger} \) and \( b^{\dagger} \) are respectively the bosonic and fermionic creation operators and the dual \( a \) and \( b \), are respectively the annihilation operators. The algebra is:

\[
a^{\dagger} \wedge a = -1/2; \quad b^{\dagger} \bullet b = 1/2
\]

where, \( \wedge \) and \( \bullet \) are related the commutator and anticommutator respectively. Following the algorithm of TFD, we double the Hilbert Space writing,

\[
\hat{H} = \omega_1 (a^{\dagger}a + b^{\dagger}b) - \omega_1 (\overline{a^{\dagger}}\overline{a} + \overline{b^{\dagger}}\overline{b})
\]

Then we introduce a thermal vacuum so that the statistical average of any operator is given in the thermal vacuum. Then all Feynman techniques of zero temperature field theory can be used.

The thermal energy given in [12], shows that the thermal vacuum \( |0(\beta)\rangle \) of the free supersymmetric harmonic oscillator has non-vanishing energy for a positive temperature. In addition the interaction will not lead to null energy, therefore supersymmetry will not be restored.

III. SUPERSYMMETRIC OSCILLATOR WITH INTERACTION

Consider a general Hamiltonian defined by:

\[
H = F(a^{\dagger}, b, b^{\dagger})a + G(a^{\dagger}, a, b^{\dagger})b
\]

where \( a^{\dagger}, a \) are bosonic fields and \( b, b^{\dagger} \), are fermionic fields, and \( F \) is a bosonic polynomial of \( a^{\dagger}, b, b^{\dagger} \) and \( G \) is a fermionic polynomial of \( a^{\dagger}, a, b^{\dagger} \). The following transformation preserves the structure of the Hamiltonian,

\[
a_2 = a + \beta_1 b^{\dagger}b; \quad a_2^{\dagger} = a^{\dagger} + \beta_1 b^{\dagger}b; \quad b_2 = (\exp[\beta_2(a^{\dagger} - a)])b; \quad b_2^{\dagger} = b^{\dagger}(\exp[\beta_2(a - a^{\dagger})]);
\]

where \( \beta_1 \) and \( \beta_2 \) are real parameters. This transformation defines the Bogoliubov transformation [21].

Restricting the model, eq. (1), in order to consider a oscillator model with interactions, we define the polynomials \( F = \omega_1 a^{\dagger} \) and \( G = \omega_2 b^{\dagger} = \alpha_1 a^{\dagger}b^{\dagger} - \alpha_2 ab^{\dagger} \) where \( \omega_1, \omega_2, \alpha_1, \alpha_2 \) are real parameters. The definition of perpendicularity and parallelism in the extended Fock space are:

\[
a^{\dagger} \wedge a = -1/2; \quad a^{\dagger} \bullet a = n_b + 1/2 \quad (7)
\]

\[
b^{\dagger} \bullet b = 1/2; \quad b^{\dagger} \wedge b = n_f - 1/2
\]

\( n_b \in \mathbb{N} \) and \( n_f \in \{0, 1\} \). This algebra is invariant by duality.

It is possible to establish some conditions over the oscillator \( H = F(a^{\dagger}, b, b^{\dagger})a + G(a^{\dagger}, a, b^{\dagger})b \), in the way that it will be a supersymmetric oscillator. To accomplish this we define \( \alpha_1 = \alpha_2; \ \omega_2 = (\omega_1^2 + (\omega_2)^2)/\omega_1 \), turning \( H \) a supersymmetric oscillator, that will produce the supercharges and the supersymmetric transformations.

A. Supercharge and Supersymmetric Transformations

Supercharge follows from the condition \( [H, G_S] = 0 \). The supersymmetric oscillator with interactions Eq. (11) has the following supercharges \( G_S = a^{\dagger}b \exp \frac{1}{\omega_1} (\alpha_2 a^{\dagger} - \alpha_2 a)b \) and \( G'_S = \left[ \exp -\frac{1}{\omega_1} (\alpha_2 a^{\dagger} - \alpha_2 a) \right] b^{\dagger}a \).

The supersymmetric transformations of the component fields are through \( G_S \):

\[
\delta_{\text{susy}} a = (\exp \frac{1}{\omega_1} (\alpha_2 a^{\dagger} - \alpha_2 a))b
\]

\[
+ \frac{1}{\omega_1} \alpha_2 a^{\dagger} \left[ \exp \frac{1}{\omega_1} (\alpha_2 a^{\dagger} - \alpha_2 a) \right] b \epsilon;
\]

\[
\delta_{\text{susy}} b = 0;
\]

\[
\delta_{\text{susy}} a^{\dagger} = \frac{1}{\omega_1} \alpha_2 a^{\dagger} b \left[ \exp \frac{1}{\omega_1} (\alpha_2 a^{\dagger} - \alpha_2 a) \right] \epsilon;
\]

\[
\delta_{\text{susy}} b^{\dagger} = \frac{1}{\omega_1} a^{\dagger} \left[ \exp \frac{1}{\omega_1} (\alpha_2 a^{\dagger} - \alpha_2 a) \right] \epsilon;
\]

where \( \epsilon \) is a Grassmann parameter. The conjugation is direct from above.

The interactions terms follow from the polynomial \( G(a^{\dagger}, a, b^{\dagger}) \) giving \( H = H_0 + H_{\text{int}} \) where

\[
H_0 = \omega_1 a^{\dagger}a + \omega_2 b b^{\dagger},
\]

and

\[
H_{\text{int}} = \alpha_2 a^{\dagger}b b^{\dagger} - \alpha_2 ab^{\dagger} b.
\]

Although \( H \) could be a supersymmetric oscillator, \( H_0 \) is not a supersymmetric harmonic oscillator due to the fact that \( \omega_1 \neq \omega_2 = (\omega_1^2 + (\omega_2)^2)/\omega_1 \).

From the Eq.(5)-Eq.(6), after we perform the Bogoliubov transformation with \( \alpha_1 = \alpha_2 \), that preserves the algebra, Eq.(7), we obtain the harmonic oscillator with
the bosonic and fermionic frequencies $\omega_1$ and $\omega_3$, respectively.

$$H = \omega_1 a^\dagger a + \omega_3 b^\dagger b$$

The condition to be supersymmetric harmonic oscillator is $\omega_3 = \omega_1$, that leads the solution bellow

$$\omega_1 \pm = \frac{1}{2}(\omega_2 \pm (\omega^2_2 - 4\alpha^2_2)^{1/2});$$
$$\omega_2 = 2\alpha_2 + \xi$$

$\xi$ parametrize $\omega_2$ at phase space.

\textbf{IV. SUPERSYMMETRY BREAKING AT FINITE TEMPERATURE FOR THE SUPERSYMMETRIC OSCILLATOR WITH INTERACTION}

Supersymmetry will be broken if the thermal vacuum $|0(\beta)\rangle$, from the supersymmetric oscillator with interaction Eq.(4) has non vanishing energy for a positive temperature. To introduce the temperature, we double the Hilbert space following the algorithm of TFD. Which will allow us to calculate the thermal vacuum and then the statistical average of the Hamiltonian operator of the supersymmetric oscillator with interaction,

$$H = F(a^\dagger b^\dagger a + G(a^\dagger a b^\dagger b))$$

where: $F = \omega_1 a^\dagger a$ and $G = \omega_2 b^\dagger b + \alpha_2 a^\dagger b^\dagger - \alpha_2 a b^\dagger$; and $\omega_2 = (\omega^2_1 + 2\alpha^2_2)^{1/2}; \omega_1$ and $\alpha_2$ are real parameters. With the algebra

$$a^\dagger \wedge a = -1/2; \quad a^\dagger \bullet a = n_b + 1/2;$$
$$b^\dagger \bullet b = 1/2; \quad b^\dagger \wedge b = n_f - 1/2;$$

$n_b \in \mathbb{N}$ and $n_f \in \{0,1\}$. That will produce the energy.

From the Hamiltonian Eq.(9) and using thermal field dynamics, temperature is introduced in the system, doubling the boson and fermion creation and annihilation operators. The statistical average of the Hamiltonian is given by its vacuum expectation value in the thermal vacuum. The tilde operators are defined in a similar way as the tilde Hamiltonian bellow, where $\tilde{H}$ is the generator of time translation:

$$\tilde{H} = H - \bar{H} = F(a^\dagger b^\dagger a + G(a^\dagger a b^\dagger b)$$
$$-F(a^\dagger b^\dagger a - G(a^\dagger a b^\dagger b)).$$

But an elegant way to obtain the results is to use Eq.(10), and the Bogoliubov transformation that preserves the algebra given in Eq.(10). The Bogoliubov transformations are

$$a_2 = a + \frac{\alpha_2}{\omega_1} b^\dagger b; \quad a_2^\dagger = a^\dagger + \frac{\alpha_2}{\omega_1} b^\dagger b;$$
$$b_2 = \langle \exp\left[\frac{\alpha_2}{\omega_1}(a^\dagger a)\right]b; \quad b_2^\dagger = b^\dagger \exp\left[\frac{\alpha_2}{\omega_1}(a - a^\dagger)\right].$$

Transforming the supersymmetric oscillator, Eq.(9), using the Bogoliubov transformation, Eq.(11), we get

$$H = \omega_1 a_2^\dagger a_2 + \omega_1 b_2^\dagger b_2,$$

the algebra $a_2^\dagger \wedge a_2 = -1/2; \quad a_2^\dagger \bullet a_2 = n_b + 1/2; \quad b_2^\dagger \bullet b_2 = 1/2; \quad b_2^\dagger \wedge b_2 = n_f - 1/2$. Showing that the supersymmetric oscillator with interaction Eq.(4) could be studied as a supersymmetric harmonic oscillator with frequency $\omega_1$. In order to write the thermal vacuum $|0(\beta)\rangle$, we double the Hilbert space

$$|0\rangle = |0; 0\rangle = |0\rangle \times |0\rangle,$$
$$|n_b, n_f; n_b, n_f\rangle = |n_b, n_f\rangle \times |n_b, n_f\rangle.$$

Any operator $A(\beta)$ and the thermal vacuum $|0(\beta)\rangle$ at a finite temperature are obtained from the zero temperature respectively by

$$A(\beta) = e^{-iG} A(0) e^{iG}, \quad |0(\beta)\rangle = e^{-iG} |0\rangle,$$

where $G = -i\theta(\beta)(\bar{b} \bar{b} - \bar{b}^\dagger \bar{b}^\dagger) - i\theta(\beta)(\bar{a} \bar{a} - \bar{a}^\dagger \bar{a}^\dagger)$, $\beta = 1/\kappa T$, $\kappa$ is the Boltzmann constant with tan $\theta(\beta) = e^{-\beta \omega_1}/2$.

The thermal energy of the thermal vacuum is given by

$$E_0(\beta) = \langle 0(\beta)| \omega_1 a_2^\dagger a_2 + \omega_1 b_2^\dagger b_2 |0(\beta)\rangle =$$

$$\omega_1 \left( \frac{e^{-\beta \omega_1}}{1 - e^{-\beta \omega_1}} + \frac{e^{-\beta \omega_1}}{1 + e^{-\beta \omega_1}} \right).$$

This shows that the supersymmetry is broken at $T > 0$, Fig.1.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure1.png}
\caption{Plot of the vacuum energy $E_0/\omega_1$, as function of $T/\omega_1$.}
\end{figure}

The Witten index is:

$$\Delta(\beta) = \frac{1 - e^{-\beta \omega_1}}{1 + e^{-\beta \omega_1}}.$$
The action of the supersymmetric charges over the thermal vacuum is given by:

\[
G_{2S}|0(\beta)\rangle = a_1' b_2|0(\beta)\rangle = e^{-\beta \omega_1/2} [(1 - e^{-\beta \omega_1})(1 + e^{-\beta \omega_1})]^{1/2} |\chi_1(\beta)\rangle,
\]

\[
G_{2S}|0(\beta)\rangle = b_1' a_2|0(\beta)\rangle = e^{-\beta \omega_1/2} [(1 - e^{-\beta \omega_1})(1 + e^{-\beta \omega_1})]^{1/2} |\chi_2(\beta)\rangle,
\]

where |\chi_1(\beta)\rangle and |\chi_2(\beta)\rangle are the Goldstino states at finite temperature, that are produced from the vacuum by applying supersymmetric charges.

V. CONCLUSION

An oscillator model with polynomial interactions is analysed and we show conditions that lead the same to a supersymmetric harmonic oscillator with interaction. The temperature is introduced in this oscillator model with interaction through TFD formalism. Reducing the model to a supersymmetric harmonic oscillator, we analyze the supersymmetry breaking at finite temperature. The fermionic and bosonic quantum corrections in a supersymmetric theory tend to cancel. The thermal effects are additive and the thermal energies are positive. The statistical average of the Hamiltonian at finite temperature is not zero.

At \(T = 0\) from the same parameter \(\alpha_2\) of polynomial interaction and frequency \(\omega_2 > 2\alpha\) we obtain two bosonic frequencies \(\omega_1\) that gives supersymmetric harmonic oscillator \((\omega_1, \omega_1^-)\) and \((\omega_{1+}, \omega_{1+})\). For the case \(\omega_2 = 2\alpha_2\), we have only one supersymmetric harmonic oscillator for the same polynomial parameter of interaction, since \(\omega_{1-} = \omega_{1+}\).

An upcoming and possible application can be implemented in the work Searching Neutrino-Nucleus interaction in Mössbauer Spectroscopy [17]; The break of spherical symmetry perceived by the wave function of the valence neutron field, in some conditions (Woods-Saxon potential), could be thought as a bosonic harmonic oscillator with \(\omega\) frequency. The vibrations between two modes could be described here as a two level system of a narrow band of vibration. That vibration around the 3/2 and 5/2 spin states of the Fe nucleus could be thought as a fermionic harmonic oscillator with \(\omega_2\) frequency. If \(\omega_1 = \omega_2\) we have the supersymmetric case.

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