A new high-precision Fourier frequency measurement algorithm based on modified sampling sequence

Jingjing Ju¹, Guozheng Han*¹

¹ School of Electrical Engineering and Automation, Shandong Academy of Sciences, Jinan, Shandong, 250353, China
*Corresponding author’s e-mail: hgz@qlu.edu.cn

Abstract. Aiming at the accuracy of frequency measurement algorithms in power systems, this paper proposes a new high-precision Fourier frequency measurement algorithm based on modified sampling sequence. Through the improvement of the traditional Fourier algorithm, the theoretical error of the traditional algorithm is eliminated and the speed of the algorithm is improved. By modifying the sampling sequence and iterative methods, the original sampling sequence is as close to the ideal sequence as possible, so that the sampling frequency and the fundamental frequency of the system tend to be synchronized, thereby reducing the problem of spectrum leakage and the fence effect. Through simulation analysis, the algorithm effectively improves the frequency measurement accuracy.

1. Introduction
Frequency is an important parameter that reflects the operating conditions of the power system, and it is also one of the indicators to measure power quality. Fast and accurate frequency measurement provides indicators for power system operation and monitoring. After a lot of research on frequency measurement at home and abroad, a variety of algorithms have been accumulated in terms of measurement speed and accuracy, mainly including: Zero-crossing detection method, Kalman filter algorithm [1], DFT algorithm [2-3], Wavelet transform algorithm.

In recent years, with the construction of smart grids, more and more distributed power sources have been connected to the grid, and the requirements for grid frequency measurement have become stricter. Therefore, this paper proposes a new high-precision Fourier frequency measurement algorithm based on modified sampling sequence.

2. Traditional Fourier algorithm
Assuming that only the fundamental frequency component of the original voltage signal is considered, the signal is

\[ u(t) = U_m \sin(2\pi f t + \varphi_0) = U_m \sin(2\pi f_0 t + 2\pi \Delta f t + \varphi_0) \]  

(1)

In equation (1), \( U_m \) is the amplitude of the voltage signal, \( \varphi_0 \) is the initial phase angle of the voltage signal, \( f_0 \) is the true frequency, \( \Delta f \) is the frequency deviation, \( f = f_0 + \Delta f \). Suppose the system sampling frequency is \( f_s \), the sampling period is \( T_s \), and \( U_m \) and \( f \) are always unchanged during the frequency measurement.
This paper adopts the rotating phasor notation in the Technical Specification for Power System Real-time Dynamic Monitoring System [4], let the phase angle be \( \varphi(t) = 2\pi ft + \varphi_0 \) at time \( t \), if after the time window of \( \Delta t \), the phase angle difference of the voltage signal is \( \Delta \varphi \), then

\[
\Delta \varphi = 2\pi f \Delta t
\]

(2)

It can be seen from equation (2) that if the phase angle difference \( \Delta \varphi \) can be measured, the true frequency \( f \) can be obtained accurately. Therefore, the key link of frequency measurement is the measurement of phase angle difference [5].

The basic idea of the traditional Fourier algorithm is to perform discrete Fourier transform on the signal, and obtain the real part modulus \( U_R \) of the voltage signal and imaginary part modulus \( U_I \) of the voltage signal. According to the voltage phase angle formula:

\[
\varphi = \arctan \frac{U_I}{U_R}
\]

(3)

Since the true frequency \( f \) is unknown, when calculating for the first time, set the initial preset frequency \( f_0 \) as the rated frequency \( (f_0 = 50Hz) \). Therefore, in the first period \( T_0 \) \( (T_0 = 1/f_0) \), the time window is \([0, T_0]\), and the Fourier analysis of the voltage signal can calculate the real modulus \( U_{R1} \) and the imaginary modulus \( U_{I1} \).

\[
U_{R1} = \frac{2}{T_0} \int_{0}^{T_0} u(t) \sin(2\pi f_0 t)dt = \frac{2U_0 f_0}{\pi T_0(f^2 - f_0^2)} \cdot \cos(\pi \Delta f T_0 + \varphi_0) \sin(\pi \Delta f T_0)
\]

(4)

\[
U_{I1} = \frac{2}{T_0} \int_{0}^{T_0} u(t) \cos(2\pi f_0 t)dt = \frac{2U_0 f_0}{\pi T_0(f^2 - f_0^2)} \cdot \sin(\pi \Delta f T_0 + \varphi_0) \sin(\pi \Delta f T_0)
\]

(5)

Get the initial phase angle of the voltage phasor in the time window \([0, T_0]\):

\[
\varphi_1 = \arctan \frac{U_{I1}}{U_{R1}} = \arctan \frac{\sin(\pi \Delta f T_0 + \varphi_0) \cdot f}{\cos(\pi \Delta f T_0 + \varphi_0) \cdot f_0}
\]

(6)

Similarly, the time window is advanced by one sampling point, that is, within the time window \([T_s, T_0+T_s]\). Get the initial phase angle of the voltage phasor in the time window \([T_s, T_0+T_s]\):

\[
\varphi_2 = \arctan \frac{U_{I2}}{U_{R2}} = \arctan \frac{\sin(\pi \Delta f T_0 + \varphi_0 + 2\pi f T_s) \cdot f}{\cos(\pi \Delta f T_0 + \varphi_0 + 2\pi f T_s) \cdot f_0}
\]

(7)

According to equation (6) and equation (7), the phase angle difference of two consecutive time windows can be obtained by the traditional Fourier algorithm:

\[
\Delta \varphi = \varphi_2 - \varphi_1 = \arctan \frac{\sin(\pi \Delta f T_0 + \varphi_0 + 2\pi f T_s) \cdot f}{\cos(\pi \Delta f T_0 + \varphi_0 + 2\pi f T_s) \cdot f_0} - \arctan \frac{\sin(\pi \Delta f T_0 + \varphi_0) \cdot f}{\cos(\pi \Delta f T_0 + \varphi_0) \cdot f_0} \neq 2\pi f T_s
\]

(8)

Equation (8) shows that when \( f \neq f_0 \), the phase angle difference calculated by the traditional Fourier algorithm is not equal to the true phase angle difference, and there is a theoretical error.

3. Improvement of traditional Fourier algorithm

3.1. Algorithm improvement principle

(1)Due to the above theoretical error, this paper improves the traditional Fourier algorithm by introducing an improvement coefficient \( r \) to eliminate the theoretical error:

\[
r = \frac{f_0}{f}
\]

(9)
(2) Multiplying \((U_1/U_{R1})\) in equation (6) by the improvement coefficient \(r\), the improvement of the initial phase angle of the voltage phasor in the time window \([0, T_0]\) is:

\[
\bar{\phi}_1 = \arctan \frac{U_{1f}f_0}{U_{R1}f} - \pi \Delta fT_0 + \varphi_0
\]

(10)

Similarly, multiply \((U_2/U_{R2})\) in equation (7) by the improvement coefficient \(r\) to obtain the improved phase angle of the voltage phasor in the time window \([T_s, T_0+T_s]\):

\[
\bar{\phi}_2 = \arctan \frac{U_{2f}f_0}{U_{R2}f} - \pi \Delta fT_0 + \varphi_0 + 2\pi fT_s
\]

(11)

Then the phase angle difference between two consecutive time windows after improvement is obtained:

\[
\bar{\phi}_2 - \bar{\phi}_1 = \arctan \frac{U_{2f}f_0}{U_{R2}f} - \arctan \frac{U_{1f}f_0}{U_{R1}f} = 2\pi fT_s
\]

(12)

It can be seen that by introducing the improved coefficient \(r\), the phase angle difference calculated by the Fourier improved algorithm is equal to the true phase angle difference, eliminating the theoretical error of the traditional algorithm.

3.2. Specific improvement process

(1) Calculation of improvement coefficient \(r\) for the first time: Since \(f\) in the improvement coefficient \(r\) is an unknown quantity, it cannot be directly improved. This article uses the method of two advance time windows to make the first improvement.

Similar to equation (11), if the time window is advanced by two sampling points, the improved phase angle of the voltage phasor in the time window \([2T_s, T_0+2T_s]\) can be obtained:

\[
\bar{\phi}_3 = \arctan \frac{U_{3f}f_0}{U_{R3}f} - \pi \Delta fT_0 + \varphi_0 + 4\pi fT_s
\]

(13)

Then the improved phase angle difference of the last two time windows is obtained:

\[
\bar{\phi}_3 - \bar{\phi}_2 = \arctan \frac{U_{3f}f_0}{U_{R3}f} - \arctan \frac{U_{2f}f_0}{U_{R2}f} = 2\pi fT_s
\]

(14)

According to equations (12) and (14), there are obviously:

\[
\bar{\phi}_3 - \bar{\phi}_2 = \bar{\phi}_2 - \bar{\phi}_1 = 2\pi fT_s
\]

(15)

\[
\arctan \frac{U_{3f}f_0}{U_{R3}f} - \arctan \frac{U_{2f}f_0}{U_{R2}f} = \arctan \frac{U_{2f}f_0}{U_{R2}f} - \arctan \frac{U_{1f}f_0}{U_{R1}f}
\]

(16)

According to the triangular identity transformation, the formula (16) is solved to obtain the first improved frequency formula:

\[
f = f_0 \left( \frac{|U_{12}(2U_{1i}U_{1i}U_{i2} - U_{1i}U_{1i}U_{i3} - U_{i2}U_{i3}U_{i1})|}{U_{R2}(2U_{Ri}U_{Ri}U_{i2} - U_{Ri}U_{Ri}U_{i3} - U_{i2}U_{i3}U_{i1})} \right)^{1/2}
\]

(17)

The \(f\) in equation (17) is inserted into \(r\), so that the first improved improvement coefficient \(r\) is obtained.
(2) Seek the true frequency for the second improvement: Bring the specific improvement coefficient \( r \) into equation (10) and equation (13), and calculate the phase angle difference between time window \([0, T_0]\) and time window \([2T_s, T_0+2T_s]\):

\[
\phi_{0}-\phi_{r} = \arctan \frac{U_{1s}f_{0}}{U_{k3}f} - \arctan \frac{U_{1s}f_{0}}{U_{m}f} = 4\pi fT_s
\]

(18)

(3) According to the phase angle difference calculated by the improved algorithm, the real frequency \( f \) can be accurately obtained:

\[
f = \frac{\phi_3 - \phi_1}{4\pi T_s}
\]

(19)

4. Modified sampling sequence

In practical applications, due to the frequency offset phenomenon, the sampling frequency is not strictly equal to an integer multiple of the fundamental frequency of the signal. Spectrum leakage and fence effects will occur, causing errors in calculations and failing to meet the requirements of power system frequency measurement. Therefore, this paper proposes the idea of correcting the sampling sequence, re-correcting the original sampling points, so that the adjusted sampling sequence is as close to the ideal sampling sequence as possible, ensuring the synchronization of sampling.

4.1. Modified sampling sequence idea

Assuming that the system sampling frequency \( f_s \) is constant, \( N_0 \) represents the number of sampling points under the initial preset frequency \( f_0 (f_0 = 50Hz, f_i = N_0 f_0 = 50N_0) \), \( f_i \) represents the estimated frequency calculated last time, the new number of sampling points \( N = \text{Round}(f_i/f_k) \), \( \text{Round} \) is the rounding function, so that the new number of sampling points \( N \) can be obtained in advance[6]. Then use cubic spline interpolation to modify the original sampling sequence \( u(i) \), and the new sequence \( \bar{u}(i) \) is obtained. The new sequence satisfies \( f_0/f_i = f_i/T_s0 = N \), \( f_0 \) is the ideal sampling frequency, \( T_s0 \) is the ideal sampling period.

4.2. Cubic spline interpolation function

According to the definition of cubic spline interpolation function [7], this paper uses the three-moment algorithm to express the cubic spline interpolation function. The cubic spline interpolation curve is connected by segmented cubic curves. There are second-order continuous derivatives at each node, which not only ensures the smoothness of the curve at the node, but also it retains the low-order characteristics of the piecewise interpolation function [8], and the interpolation algorithm has high precision, which is one of the most widely used interpolation methods.

5. Algorithm implementation process

(1) Given the original voltage signal and performing discrete Fourier transform, use equation (19) to calculate the improved first estimated frequency \( f_i \), if the difference between the first estimated frequency \( f_i \) and the initial preset frequency \( f_0 \) \( (f_0 = 50Hz) \) is less than 0.001Hz, then \( f_i \) is the true frequency of the signal, and the calculation is over.

(2) If the difference between the first estimated frequency \( f_i \) and the initial preset frequency \( f_0 \) is greater than 0.001Hz, and the number of iterations is less than 20, take \( f_i \) as the new preset frequency, that is, \( f_0 \rightarrow f_i \), and go to step (3). If the number of iterations is equal to 20, the estimated frequency of the last iteration is taken as the true frequency of the signal, and the calculation ends.

(3) Assuming that the system sampling frequency \( f_s \) is constant, according to Section 3, using the estimated frequency \( f_k \) calculated last time \( (f_k = f_1, f_2, f_3...) \), the new number of sampling points \( N \) can be obtained in advance. Then the sampling sequence is modified by cubic spline interpolation to obtain the ideal sampling sequence.
(4) According to the new preset frequency $f_0$ and the new sampling sequence $\bar{u}(i)$, continue to use equation (19) to calculate the new frequency $f_2$. If the difference between the new frequency $f_2$ and the new preset frequency $f_0$ is less than 0.001Hz, then $f_2$ is the true frequency of the signal, and the calculation ends. Otherwise, take $f_2$ as the new preset frequency, that is, $f_0 = f_2$, and proceed to step (2). The algorithm flow chart is as follows:

Figure 1. Algorithm flow chart

6. Algorithm simulation
In order to verify the accuracy and adaptability of the algorithm, the influence of harmonic components is fully considered in the signal. Suppose the original voltage signal is:

$$u(t) = 10\sin(2\pi f_1 t + 60^\circ) + 0.1\sin(4\pi f_1 t) + \sin(6\pi f_1 t) + 0.02\sin(8\pi f_1 t) + 0.05\sin(10\pi f_1 t)$$  (20)

The initial preset frequency of the signal (the rated frequency) $f_0 = 50$Hz, the signal period $T_0$ is 20ms, and the number of sampling points for a cycle is $N_0 = 128$. The threshold for the end of the algorithm is: the difference between the frequency of two consecutive measurements is less than or equal to 0.001Hz, or the number of iterations reaches 20, the calculation ends, and the last measurement frequency is regarded as the true frequency of the signal. Assuming the signal frequency change range is 49.00~51.00Hz, the measurement results of this algorithm are as follows:

| True frequency (Hz) | Measuring frequency (Hz) | Absolute error (Hz) | Relative error (%) |
|---------------------|--------------------------|---------------------|-------------------|
| 49.00               | 49.0003                  | 0.0003              | 0.0006            |
| 49.30               | 49.2996                  | 0.0004              | 0.0008            |
| 49.70               | 49.7003                  | 0.0003              | 0.0006            |
| 49.80               | 49.7997                  | 0.0003              | 0.0006            |
| 49.90               | 49.8997                  | 0.0003              | 0.0006            |
| 50.00               | 50.0000                  | 0.0000              | 0.0000            |

The sampling sequence is modified by cubic spline interpolation to obtain the ideal sampling sequence.
From the simulation results in the above table, when the signal contains 2, 3, 4, and 5 harmonic components, the absolute error can still be kept below 0.001Hz, effectively suppressing the influence of harmonic components, and the algorithm has a high accuracy.

7. Conclusions
This paper proposes a new high-precision Fourier frequency measurement algorithm based on modified sampling sequence. Through the improvement of the traditional Fourier algorithm, the theoretical error of the traditional algorithm is eliminated, the length of the sampling sequence is reduced, and the speed of the algorithm is improved. By modifying the sampling sequence and iterative methods, the original sampling sequence is as close to the ideal sequence as possible, so that the sampling frequency and the fundamental frequency of the system tend to be synchronized, thereby reducing the problem of spectrum leakage and fence effect, and improving the accuracy of frequency measurement, while maintaining the characteristics of the Fourier algorithm to suppress harmonics. The algorithm is clear in principle, easy to implement by software and hardware, and is more suitable for frequency measurement of power grid.

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