sin(2\(\beta\)): Status and Prospects

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An overview of the observation of CP violation in the neutral B system, and the measurements of the CP-violating asymmetry \(\sin(2\beta)\) with \(B \rightarrow \text{charmonium } K^{*}_{0}\) events, performed by the Babar and Belle experiments at the SLAC and KEK B factories is given. In addition, the measurements of \(\sin(2\beta)\) with several other modes are described, including \(B \rightarrow f\bar{K}^{0}\), which, as the leading contribution is from a loop diagram, could be sensitive to physics beyond the Standard Model.

1 Introduction

CP violation has been a central concern of particle physics since its discovery in 1964 in the decays of \(K^{0}\) decays [1]. An elegant explanation of the CP-violating effects in these decays is provided by the CP-violating phase of the three-generation Cabibbo-Kobayashi-Maskawa (CKM) quark-mixing matrix [2]. However, existing studies of CP violation in neutral kaon decays and the resulting experimental constraints on the parameters of the CKM matrix [3] do not provide a stringent test of whether the CKM phase describes CP violation [4]. In the CKM picture, large CP-violating asymmetries are expected to occur in the time distributions of \(B^0\) decays to charmonium final states.

In general, CP-violating asymmetries are due to the interference between amplitudes with a weak phase difference. For example, a state initially produced as a \(B^0(\bar{B}^0)\) can decay to a CP eigenstate, such as \(J/\psi K^{*}_0\), either directly, or it can first oscillate into a \(\bar{B}^0(\bar{B}^0)\) and then decay to \(J/\psi K^{*}_0\). With little theoretical uncertainty in the Standard Model, the phase difference between these two amplitudes is equal to twice the angle \(\beta = \arg \left[V_{ub}/V_{tb}\right]\) of the Unitarity Triangle [5]. The measurement of the CP-violating asymmetry in this decay allows a direct determination of \(\sin(2\beta)\), and can thus provide a crucial test of the Standard Model.

Initial measurements of the CP asymmetry in \(B^0 \rightarrow J/\psi K^{*}_0\) were performed at LEPI by Aleph and Opal, and at the Tevatron by CDF [6], but the small branching ratio of this decay made it difficult for the experiments to obtain sufficient events for a statistically significant measurement. The KEK and SLAC-based B factories, running at the \(\Upsilon(4S)\) resonance, were designed to provide the required high luminosity to perform this measurement. Although the measurements from the Babar and Belle experiments, at SLAC respectively KEK, after the first year of running, shown in summer of 2000, were not yet conclusive, only a year later both experiments were able to claim the observation of CP violation in the B meson system. And in 2002

\[ f_\beta(\Delta t) = \frac{e^{-|\Delta t|/\tau_\beta}}{4\tau_\beta} \left[ 1 \pm S \sin(\Delta m_d \Delta t) \pm C \cos(\Delta m_d \Delta t) \right], \]

where \(\Delta t = t_{\text{rec}} - t_{\text{tag}}\) is the difference in proper decay times of the reconstructed B meson (\(B_{\text{rec}}\)) and the tagging B meson (\(B_{\text{tag}}\)), \(\tau_\beta\) is the \(B^0\) lifetime, and \(\Delta m_d\) is the \(B^0\) oscillation frequency. The sine coefficient, which is given by \(S = 25.1/(1 + |\lambda|^2)\), is due to the interference between direct decay and decay after flavour change, and the cosine coefficient, \(C = (1 - |\lambda|^2)/(1 + |\lambda|^2)\), is due to the interference between decay amplitudes with different strong and weak phases. In the Standard Model, \(\lambda_f = \eta_f e^{-2i\beta}\) for \(f \neq u, d, s\) with 

\[ \eta_f = \frac{1}{\sqrt{2}} \left( \frac{m_f}{m_u m_d} \right)^{1/2}, \]

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charm-producing $b \rightarrow c(\tau s)$ decays, where $\eta_f$ is the CP eigenvalue of the final state $f$.

At asymmetric $e^+e^-$ colliders such as PEP-II at SLAC and KEK-B at KEK [10], resonant production of the $\Upsilon(4S)$ provides a copious source of $B^0\bar{B}^0$ pairs moving along the beam axis ($z$ direction) with an average Lorentz boost $(\beta \gamma)$ of 0.56 and 0.43 respectively. Therefore, the proper decay-time difference $\Delta t$ is, to an excellent approximation, proportional to the distance $\Delta z$ between the two $B^0$-decay vertices along the axis of the boost, $\Delta t \approx \Delta z/c(\beta \gamma)$.

The analysis of the data proceeds in the following steps:

1. selection of events where one $B$, referred to as $B_{rec}$ is fully reconstructed;
2. determination of the vertex of the other $B$ decay, $B_{tag}$, and computation of $\Delta t$;
3. determination of the flavour of $B_{tag}$ from its charged decay products.

Both experiments determine their $\Delta t$ resolution and the mistag rate of the flavour tagging algorithms from control samples, obtained from the data itself.

### 2.1 Data samples and $B$ reconstruction

Both experiments have so far published their sin2$\beta$ measurements on samples obtained as of July 2002. In the case of $\text{BABAR}$, this implies a sample of $88 \cdot 10^6 \Upsilon(4S)$ decays, whereas Belle collected a sample of $85 \cdot 10^6$ decays. As the branching ratios of decays of $B$ mesons to CP eigenstates are small, e.g. a few times $10^{-4}$ for $J/\psi K^0_S$, both experiments increase the size of the event sample by reconstructing several final states: $J/\psi K^0_S, \psi(2S)K^0_S, K^0, K^0_L, \bar{K}^0, J/\psi K^0_L(K^0_{\pi^0})$ and $J/\psi K^0_S$. In addition, to determine the performance of the $\Delta t$ reconstruction and the flavour tagging, control samples of fully reconstructed decays of $B$ mesons to self-tagging flavour eigenstates are selected\(^2\): $B^0 \rightarrow D^{(*)-}\pi^+$, $D^{(*)-}\rho^+$, $D^{(*)-}a_1^+$ and $J/\psi K_{LL}(K^+\pi^-)$.

In addition, semileptonic decays into $D^{*-}\ell^+\nu$ are selected. The main selection criteria of the fully reconstructed decays are the energy difference, $\Delta E$, between the energy of the reconstructed candidate and the beam-energy in the $\Upsilon(4S)$ center-of-mass system, and the beam-energy substituted mass, $m_{ES}$, also known as the beam-constrained mass, defined as $m_{ES} = \sqrt{s}/4 - p^*$, where $s$ is the square of the center-of-mass energy and $p^*$ is the momentum of the $B$ candidate in the center-of-mass.

In the case of signal events, these variables are distributed according to Gaussian distributions, centered at $\Delta E = 0$ and $m_{ES} = m_B$ respectively. The distributions of $m_{ES}$ for charmonium $K^0_S$ events are shown in Figure[1]. In the case of $J/\psi K^0_S$, only the direction of the $K^0_S$ is measured, and, to determine its momentum, both experiments constrain the mass of the candidate to the $B$ mass. Next, they plot either the $p^*$ of the candidate, or $\Delta E$. These distributions are also shown in Figure[1].

### 2.2 Determination of $\Delta t$

The time difference $\Delta t$ can be related to the distance $\Delta z$ along the boost axis between the decay points of the two $B$ mesons. Approximating the unmeasured sum of the proper times by the average $B^0$ lifetime, $\tau_B$, yields $\Delta z = \beta \gamma \tau_B c(\Delta t + \sqrt{\Delta t^2 + m_{ES}^2})$, where $\tau_B$, $\beta \gamma$, and $\Delta t$ are the polar angle with respect to the boost direction, the velocity and the boost of the reconstructed $B$ candidate in the $\Upsilon(4S)$ frame. Whereas $\text{BABAR}$ solves the above equation for $\Delta t$, Belle makes an approximation which only keeps the first term: $\Delta t = \Delta z/c(\beta \gamma)$. As one of the $B$ mesons, $B_{rec}$ is fully reconstructed, its decay vertex position is well known. The decay vertex of the other $B$, $B_{tag}$, is inferred from the charged particle tracks remaining after the decay products of $B_{rec}$ are removed. To remove tracks from secondary decays, both experiments first remove tracks from $K^0_S$ and $\Lambda$ candidates as well as photon conversion, and then perform an iterative fit procedure, rejecting those tracks with the large contribution to the $\chi^2$.

In the case of Belle, the constraint that the vertex of $B_{tag}$ is consistent with the beamspot is applied. $\text{BABAR}$ instead requires that the $B_{tag}$ vertex is consistent with the line of flight computed from the location of the beamspot, the momentum of $B_{rec}$ and the known $\Upsilon(4S)$ boost. The resolution obtained on $\Delta t$, determined from the fully reconstructed flavour samples, is 1.1 ps for $\text{BABAR}$ and 1.4 ps for Belle, partly due to the difference in the $\Upsilon(4S)$ boost.

### 2.3 Flavour tagging

After the daughter tracks of the $B_{rec}$ are removed from the event, the remaining tracks are analyzed to determine the flavour of the $B_{tag}$, and this ensemble is assigned a flavour tag, either $B^0$ or $\bar{B}^0$. For this purpose, flavour tagging information carried by primary leptons from semileptonic $B$ decays, charged kaons, soft pions from $D^*$ decays, and more generally by high momentum charged particles is used.

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\(^2\)Throughout this paper, charge-conjugate modes are implied.

| Mode          | $\text{BABAR}$ | | $\text{Belle}$ | |
|---------------|----------------|-----------------|----------------|----------------|
|               | $N_{sig}$ | $\mathcal{P}$(%) | $N_{sig}$ | $\mathcal{P}$(%) |
| $J/\psi K^0_S(\pi^+\pi^-)$ | 1429 | 96 | 1116 | 96 |
| other ($\bar{\psi}c\bar{\psi}$) | 721 | 85 | 523 | 86 |
| $J/\psi K_{LL}(K^0_{\pi^0})$ | 283 | 73 | 89 | 84 |
| flavour eigenstates | 32700 | 83 | 18045 | 82 |

Table 1. Number of selected events in the signal region ($N_{sig}$) and the corresponding purities ($\mathcal{P}$).
Belle uses the likelihood ratios of the properties of these particles to estimate the mistag rate for each individual event, and then ranks events into six mutually exclusive groups based on their estimated mistag rate. \( \text{BABAR} \) uses neural networks, trained according to each of the physics processes mentioned above, and classifies events into four mutually exclusive categories according to the underlying physics process, combined with performance criteria based on the neural network output.

As the amplitude of the observed \( CP \) asymmetries will be reduced by a factor \( 1 - 2w \), where \( w \) is the mistag rate, it is crucial for the experiments to determine the mistag rates of the various tagging categories from data. This can be done by considering decays to flavour eigenstates, where the deviation of the observed mixing asymmetry from unity is also given by \( 1 - 2w \). \( \text{BABAR} \) uses fully reconstructed events in the modes \( D^{*-} h^+ (h^+ = \pi^+, \rho^+, \omega^*) \) and \( J/\psi K^{*0} (K^{*0} \rightarrow K^\pi) \), whereas Belle uses fully reconstructed events in modes \( D^{(*)} \pi^+ \) and \( D^{(*)} \rho^* \), complemented by \( B^0 \rightarrow D^{*-} \ell^+ \nu_\ell \) events. In the case of Belle, the mistag rates are determined by fitting the control samples separately, and then propagating the obtained values to the fit on the \( CP \) sample. The statistical uncertainty on the mistag rates due to the finite size of the control samples is accounted for in systematic errors. \( \text{BABAR} \) proceeds differently, performing a simultaneous fit to both the control samples and the \( CP \) sample. This automatically insures that the statistical error on the mistag rates is propagated into the statistical error on the \( CP \) asymmetries. Even though the flavour tagging algorithms are somewhat different between the experiments, their performance is very similar: the total effective tagging efficiency \( Q \), which is given by \( Q = \sum_i e_i (1 - 2w_i)^2 \), is measured to be \( 28.6 \pm 0.6\% \) for Belle, and \( 28.1 \pm 0.6\% \) for \( \text{BABAR} \).

One complication has recently received attention, partly due to its relation to the measurement of \( \sin(2\beta + \gamma) \): when decays of the type \( B \rightarrow DX \) are used to infer the flavour of the parent \( B \) mesons, one suffers from an intrinsic mistag rate due to the contribution of CKM suppressed \( B \rightarrow u(\bar{c}\bar{d}) \) decays. This effect is put to good use in the measurement of \( \sin(2\beta + \gamma) \), as the suppressed mode can, once \( B^0 \bar{B}^0 \) oscillations are taken into account, interfere with the favoured \( B^0 \rightarrow \bar{c}(u\bar{d}) \) amplitude. As the relative weak phase between these decay amplitudes is given by \( \gamma \), the results is a time-dependent \( CP \) asymmetry, depending on \( \sin(2\beta + \gamma) \), albeit with a magnitude which is suppressed by
$|V_{ub}^* V_{cd}/V_{cb}^* V_{ud}| \approx (0.02)^2$. This same interference, when applied to the tagging decay effectively results in a mistag rate which is not constant as a function of $\Delta t$, and thus is not accounted for in the experimental determined mistag rate which is assumed to be independent of $\Delta t$. However, because the two $B$ mesons produced by $\Upsilon(4S)$ decays are correlated until one of them decays, $B_{rec} - B_{tag}$ interference terms involving favoured and suppressed amplitudes are only suppressed by a factor of about 0.02. The result is that for $|\lambda_{B^0 K^0_s}| = 1$ the $S_{B^0 K^0_s}$ and $C_{B^0 K^0_s}$ coefficients are now given by [11]:

\[
C_{B^0 K^0_s} = -2r' \sin \gamma \sin \delta'
\]

\[
S_{B^0 K^0_s} = \sin 2\beta \left[ 1 - 2r' \cos \delta' \left( \cos 2\beta \cos (\beta + \gamma) + \kappa \sin 2\beta \sin (\beta + \gamma) \right) \right]
\]

where $\delta'$ and $r'$ are the effective strong phase and ratio of the suppressed to favored amplitudes obtained when all final states contributing to a particular tagging category are combined, and $\kappa$, an empirical constant which depends on the values of $\beta$ and $\gamma$, is approximately 0.3. Fortunately, lepton tags are unaffected by this effect, and, as lepton tags represent about 1/3 of the effective tagging efficiency, this effect is suppressed by a factor of 2/3. As can be seen from equations above the largest effect is present for $C_{B^0 K^0_s}$, whereas the extraction of $\sin 2\beta$ from $S_{B^0 K^0_s}$ is not very much affected. However, this effect currently dominates the systematic uncertainty on the extraction of $|\lambda_{B^0 K^0_s}|$ from $C_{B^0 K^0_s}$.

2.4 Current measurements with $b \to c(\bar{c} s)$ transitions

The value of $\sin 2\beta$ is determined from unbinned maximum-likelihood fits to the $\Delta t$ distributions, taking into account the $\Delta t$ resolution and the mistag rates. The projections of the likelihood fits onto the observed $\Delta t$ distributions is shown in Figure 2. A clear difference in the $\Delta t$ distributions for $B^0$ and $\bar{B}^0$ tagged events is visible. The values measured by the two experiments are

$\sin 2\beta = 0.741 \pm 0.067 \pm 0.034$ (Babar),

$\sin 2\beta = 0.719 \pm 0.074 \pm 0.035$ (Belle),

in good agreement with each other. Combining the two measurements yields

$\sin 2\beta = 0.734 \pm 0.055$.

The constraint of this measurement on the parameters of the CKM matrix can be visualized in the $(\rho, \eta)$ plane, as shown in Figure 3. In addition the constraints derived from CP-conserving measurements and the observed CP violation in the neutral kaon system are included [8].

**Figure 3.** Constraints on the position of the apex of the Unitarity Triangle in the $(\rho, \eta)$-plane, including the direct measurement of $\sin 2\beta$.

2.5 Extrapolation to larger samples

Both $B$-factories are performing above expectations, having accumulated well over 100 fb$^{-1}$ each in their first four years of operation. Currently, PEP-II is capable of routinely delivering more than 300 pb$^{-1}$ per day, whereas KEK-B has recently set a record for daily integrated luminosity of 500 pb$^{-1}$. As a result, both experiments are well on their way to collecting on the order of 500 fb$^{-1}$ by 2006. Looking into the past, comparing how the statistical error on $\sin 2\beta$ has improved versus the integrated luminosity, both experiments have been able to perform better than $\sigma_{\text{stat}}^2 \propto \int dt L$ by improving their reconstruction, calibrations and selections. It is however clear that the impact of future improvements, other than increased sample size, on the statistical error will be less and less pronounced. As a result one can expect a statistical error on $\sin 2\beta$ of approximately $\pm 0.03$ given a 500 fb$^{-1}$ sample. The main effort will have to be focused on reducing the systematic error. Currently the measurement of $\sin 2\beta$ is still dominated by the statistical error, but the current systematic uncertainty, even though it is partly driven by the available sample size, will reach parity with the statistical error at the level of about 500 fb$^{-1}$. It is expected that with a combination of additional improvements to selections, vertexing and tagging, and further studies of the data with improved control samples, the systematic error can be reduced sufficiently such that the measurement on 500 fb$^{-1}$ will still be limited by the statistical accuracy.
3 Approximations in the determination of sin2β

In the determination of sin2β described above some very reasonable assumptions are made about both B\(\bar{B}\) mixing and the decay amplitudes \(B^0 \rightarrow J/\psi K^0\) and \(B^0 \rightarrow J/\psi K^{0*}\). The evolution of the \(B^0\) and \(\bar{B}^0\) states prior to their decay is described by oscillations \(B^0 \rightarrow \bar{B}^0\) and \(\bar{B}^0 \rightarrow B^0\) with a frequency given by the mass difference \(\Delta m_d = m_H - m_L\) of the \(B_d\) mass eigenstates, multiplied by factors \(|q/p|\) and \(|q/p|\), respectively. In the measurement of sin2β, it is assumed that \(|q/p| = 1\), which, given the Standard Model expectation of \(|q/p| = 1 = (2.5 - 6.5) \times 10^{-4}\) for the \(B_d\) system [12], is a very good approximation. If \(|q/p| = 1\), the rate of \(B^0 \rightarrow \bar{B}^0\) and \(\bar{B}^0 \rightarrow B^0\) should be equal, unlike the case for the neutral Kaon system. This possible rate difference can be determined by measuring the like-sign lepton asymmetry, \(A_{\Delta t} = (N_{\ell^+\ell^-} - N_{\ell^--\ell^+})/(N_{\ell^+\ell^-} + N_{\ell^--\ell^+}) = (1 - |q/p|^2)/(1 + |q/p|^2)\). Several measurements of this asymmetry are available [13], and recently this asymmetry has also been measured by BABAR [14] to be \(A_{\Delta t} = (0.5 \pm 1.2 \pm 1.4) \times 10^{-2}\), which corresponds to \(|q/p| = 0.998 \pm 0.006 \pm 0.007\).

Recently BABAR has also determined \(|q/p|\) using samples of flavour tagged, fully reconstructed decays of \(B_d\) mesons to either CP or flavour eigenstates [15]. Although the sensitivity to \(|q/p|\) is less than for a like-sign dilepton analysis, these samples allow one to also set a limit on the lifetime difference \(\Delta \Gamma\) between the mass eigenstates and on the complex CPT violating parameter \(\zeta\), which is proportional to the mass- and lifetime differences between \(B^0\) and \(\bar{B}^0\) states. In the Standard Model, CPT is conserved, and \(\Delta \Gamma/\Delta m\) is expected to be \(O(m^2_B/m^2_\chi)\) [16], and thus both effects are neglected in the extraction of sin2β. Within the limited uncertainties of this measurement, no deviations from the Standard Model expectations of \(\zeta\) and \(\Delta \Gamma\) are observed.

An additional assumption made in identifying the sine coefficient of the time-dependent CP asymmetry in \(B^0 \rightarrow J/\psi K^0\) as sin2β is that the decay itself is dominated by a single weak phase. This is an excellent approximation as the leading penguin contributions have the same weak phase as the CKM favoured tree diagram. This assumption can to some extent be tested by considering the decay \(B^+ \rightarrow J/\psi K^0_s\), which is related to \(B^0 \rightarrow J/\psi K^0\) by exchange of the spectator quark. In case there would
be a sizable contribution from diagrams with a different weak phase, there might be a non-zero charge asymmetry: \( \mathcal{A}_{\pm}(J/\psi K^\pm) = \left( N_{J/\psi K^\pm} - \bar{N}_{\bar{J}/\bar{\psi} K^{\mp}} \right) / \left( N_{J/\psi K^+} - \bar{N}_{\bar{J}/\bar{\psi} K^-} \right) \). This asymmetry has been measured by both \( \text{BABAR} \) and Belle [17], and the values obtained are consistent with zero:

\[
\mathcal{A}_{+}(J/\psi K^+) = +0.003 \pm 0.030 \pm 0.004 \quad (\text{BABAR}) \\
\mathcal{A}_{-}(J/\psi K^+) = -0.042 \pm 0.020 \pm 0.017 \quad (\text{Belle})
\]

The decays \( B^0 \to J/\psi K^0 \) and \( B^0 \to J/\psi K^\mp \) both proceed through CP-favoured, colour suppressed tree diagrams \( B^0 \to J/\psi K^0 \), followed by \( K^0 \to K^0 \) and \( K^0 \to K^\mp \) respectively. As a result, neglecting the tiny amount of CP violation in neutral kaon mixing, the time dependent asymmetries in \( B^0 \to J/\psi K^0 \) and \( B^0 \to J/\psi K^\mp \) should be equal in magnitude, but opposite in sign, \( \mathcal{A}_{\text{BF}}(J/\psi K^\mp) = -\mathcal{A}_{\text{BF}}(J/\psi K^0) \). It can be shown that to generate a deviation of more than a few times \( 10^{-3} \), interference between the favoured decay and a so-called wrong flavour decay, \( B^0 \to J/\psi K^0 \), is required [18]. By considering the related decay \( B^0 \to J/\psi K^0 \), with \( K^0 \) decaying to \( K^+ \pi^- \), one can tag the kaon flavour in the decay, and by performing a time-dependent analysis \( \text{BABAR} \) measures the following ratios of wrong-flavour to favoured amplitudes [19]:

\[
\Gamma(B^0 \to J/\psi K^0)/\Gamma(B^0 \to J/\psi \bar{K}^0) = -0.022 \pm 0.028 \pm 0.016 \\
\Gamma(B^0 \to J/\psi \bar{K}^0)/\Gamma(B^0 \to J/\psi \bar{K}^0) = 0.017 \pm 0.026 \pm 0.016.
\]

Again, no evidence for a deviation from the Standard Model expectations is observed.

### 4 Measurement of \( \cos 2\beta \) with \( B^0 \to J/\psi K^\pm \)

The decay of \( B^0 \to J/\psi K^0 \), \( K^\pm \to K^0 \pi^0 \) proceeds through two CP-even amplitudes \( (A_0, A_\parallel) \) and one CP-odd amplitude \( (A_\perp) \). This implies that, unless one takes into account the angular dependence of the contributing amplitudes, the magnitude of the CP asymmetry is diluted by an additional factor \( 1 - 2R_\tau \), where \( R_\tau \) is the fraction of CP-odd decay rate. The simplest way to extract sin\( 2\beta \) from these decays is to measure \( R_\tau \) and insert the additional dilution \( 1 - 2R_\tau \) in the time dependent analysis. Both \( \text{BABAR} \) and Belle have measured \( R_\tau = 0.179 \pm 0.030 \). One can improve the sensitivity by taking into account the dependence of CP-even and odd amplitudes on \( \cos(\theta_\tau) \), where \( \theta_\tau \) is the angle in the \( J/\psi \) rest-frame between the positive lepton and the normal to the decay plane of the \( K^0 \); the CP-even components are proportional to \( 1 - \cos^2 \theta_\tau \), and the CP-odd component is proportional to \( (1 + \cos^2 \theta_\tau) / 2 \). A further refinement can be obtained by including all three angles that describe this decay. Denoting the three observable angles in this decay by \( \tilde{\phi} \), the decay rate is given by [21, 22]:

\[
f_{\pm}(\Delta \tilde{\phi}, \tilde{\phi}_N, \tilde{\phi}_L) \propto \frac{\exp(-|\Delta \tilde{\phi}| \tau_{\beta}) \times}{4 \tau_{\beta}} \\
\left[ I \left( \tilde{\phi}, \tilde{A} \right) \mp C \left( \tilde{\phi}, \tilde{A} \right) \cos \Delta m_\beta \Delta t \right. \\
\pm \left. \left[ S_{\sin} \left( \tilde{\phi}, \tilde{A} \right) \sin 2\beta \right. \\
+ \left. S_{\cos} \left( \tilde{\phi}, \tilde{A} \right) \cos 2\beta \right] \cos \Delta m_\beta \Delta t \right]
\]

and at first sight one expects to be able to determine \( \cos 2\beta \). This would allow one to eliminate two of the four ambiguities in \( \beta \) from the measurement of \( \sin 2\beta \). Unfortunately the observable \( S_{\cos} \left( \tilde{\phi}, \tilde{A} \right) \cos 2\beta \) is invariant under the transformation \( (\tilde{\phi}_N, \tilde{\phi}_L, \cos 2\beta) \to (\pi - \tilde{\phi}_N, -\tilde{\phi}_L, -\cos 2\beta) \), where \( \tilde{\phi}_i \) are the relative phases between \( A_i \). As a result one can only determine the sign of \( \cos 2\beta \) if one could choose between the two possible solutions for the strong phases. The two experiments quote both ambiguities [21, 22], including the corresponding strong phases:

\[
\cos 2\beta = \left\{ \begin{array}{ll}
+3.3^{+0.6}_{-1.0} & (\text{BABAR}) \\
-3.3^{+1.0}_{-0.6} & (\text{Belle})
\end{array} \right.
\]

Thus reducing the number of ambiguities in \( \beta \) will require additional information on which strong phase solution to pick. For example, assuming \( s \)-quark helicity conservation [23], the positive solution seems preferred, but even then the current errors on \( \cos 2\beta \) are still too large to rule out negative values.

### 5 Modes with penguin contributions

#### 5.1 \( B^0 \to J/\psi \pi^0 \)

In the case of the tree diagram is CKM suppressed compared to \( B^0 \to J/\psi K^0 \). One has thus the possibility that this mode receives non-negligible contributions from penguin diagrams with a weak phase different from the tree diagram. Both \( B \)-factory experiments have observed this decay and determined \( S_{J/\psi \pi^0} \) and \( C_{J/\psi \pi^0} \) [24]:

\[
S_{J/\psi \pi^0} = 0.05 \pm 0.49 \pm 0.16 \quad (\text{BABAR}), \\
S_{J/\psi \pi^0} = -0.93 \pm 0.49 \pm 0.08 \quad (\text{Belle}), \\
C_{J/\psi \pi^0} = 0.38 \pm 0.51 \pm 0.09 \quad (\text{BABAR}), \\
C_{J/\psi \pi^0} = 0.25 \pm 0.39 \pm 0.06 \quad (\text{Belle}).
\]

The precision is such that more data is needed to draw a conclusion on the possible penguin contribution to the \( \text{CP} \) asymmetries in this channel.
5.2 \( B^0 \to \phi K^0 \)

There is considerable interest in decays where the leading contribution to the amplitude is due to loop diagrams, as new physics processes could provide significant contributions. An example are transitions of the type the \( b \to s(\bar{s}s) \) and \( b \to s(\bar{d}d) \), which are given by gluonic penguin decays, and for which the dominant penguin contribution has the same phase as \( b \to c(\bar{c}s) \). As a result, the process \( B^0 \to \phi K^0 \) should exhibit the same CP asymmetry as \( B^0 \to J/\psi K^0 \). However, even in the Standard Model there are diagrams with different weak phases which contribute to the decay \( B^0 \to \phi K^0 \), but one can set limits on their magnitude using isospin related decays such as \( B^+ \to \phi \pi^+ \) and \( K^{*0} K^+ \). As a result one expects that within the Standard Model the deviation of \( S_{\phi K^0} \) from \( \sin 2\beta \) should be less than 5\% [25]. Again both experiments have observed clear signals in this mode and measured the CP asymmetries [19, 26]:

\[
S_{\phi K^0} = -0.18 \pm 0.51 \pm 0.07 \quad \text{(BABAR),}
\]
\[
S_{\phi K^0} = -0.73 \pm 0.64 \pm 0.22 \quad \text{(Belle),}
\]
\[
C_{\phi K^0} = -0.80 \pm 0.38 \pm 0.12 \quad \text{(BABAR),}
\]
\[
C_{\phi K^0} = 0.56 \pm 0.41 \pm 0.16 \quad \text{(Belle).}
\]

In addition, Belle has measured the time-dependent asymmetries for the non-resonant \( K^+ K^- K^0 \) final state, and obtains

\[
S_{K K K^0} = 0.49 \pm 0.43 \pm 0.11 \pm 0.33 \pm 0.0,\]
\[
C_{K K K^0} = 0.40 \pm 0.33 \pm 0.10 \pm 0.26 .\]

Although the measurements show a trend for smaller or even negative values for \( S \), the difference with \( \sin 2\beta \) is not yet statistically significant.

5.3 \( B^0 \to \eta' K^0 \)

A mode which is similar to \( B^0 \to \phi K^0 \) is \( B^0 \to \eta' K^0 \), but with the additional complication of a contribution of a CKM suppressed tree-level \( b \to u \) contribution. Several estimates of the relative magnitude of the penguin diagram exist, and the deviation of \( S_{\eta' K^0} \) from \( \sin 2\beta \) is expected to be less than \( O(5\%) \) [27]. Both experiments observe clear signals for this mode, and measure the time-dependent asymmetries [20, 28]:

\[
S_{\eta' K^0} = 0.02 \pm 0.34 \pm 0.03 \quad \text{(BABAR),}
\]
\[
S_{\eta' K^0} = 0.71 \pm 0.37 \pm 0.06 \quad \text{(Belle),}
\]
\[
C_{\eta' K^0} = 0.10 \pm 0.22 \pm 0.03 \quad \text{(BABAR),}
\]
\[
C_{\eta' K^0} = -0.26 \pm 0.22 \pm 0.04 \quad \text{(Belle).}
\]

Again, no statistically significant deviations from \( \sin 2\beta \) respectively zero are observed.

5.4 \( B^0 \to D^+ D^- \)

The dominant contribution to this decay is the transition \( b \to c(\bar{c}s) \), but the presence of penguin contributions could cause deviations of \( S_{D^+ D^-} \) from \( \sin 2\beta \) of about 2\% [29]. Similarly to \( B^0 \to J/\psi K^0 \), the decay \( B^0 \to D^+ D^- \) is a vector-vector decay which receives contributions from three partial waves, and either an angular analysis or a measurement of the CP-odd fraction \( R_c \) is required to interpret the CP asymmetry. From the distribution of \( \cos \theta_D \), BABAR determines \( R_c = 0.07 \pm 0.06 \pm 0.03 \), and Belle concludes that the decay is dominantly CP-even [30]. BABAR proceeds to measure the time-dependent CP asymmetry and finds

\[
S_{D^+ D^-} = 0.32 \pm 0.43 \pm 0.13,\]
\[
C_{D^+ D^-} = 0.02 \pm 0.25 \pm 0.09.\]

5.5 \( B^0 \to D^+ D^- \)

This decay, like \( B^0 \to D^+ D^- \), is a \( b \to c(\bar{c}s) \), but in this case the final state is not a CP eigenstate. However, it is still possible to determine the CP asymmetries [31]. BABAR has measured the following time-dependent asymmetries [19]:

\[
S_{D^+ D^-} = -0.82 \pm 0.75 \pm 0.14,\]
\[
S_{D^+ D^-} = -0.24 \pm 0.69 \pm 0.12,\]
\[
C_{D^+ D^-} = -0.47 \pm 0.40 \pm 0.12,\]
\[
C_{D^+ D^-} = -0.22 \pm 0.37 \pm 0.10.\]

In addition, the time-integrated charge asymmetry has been measured by BABAR to be \( \mathcal{A} = -0.03 \pm 0.11 \pm 0.05 \). Again, no significant deviation from the Standard Model expectation is observed.

6 Conclusion

The determination of time dependent CP-violating asymmetries at asymmetric energy \( B \) factories has reached maturity: the measurement of \( \sin 2\beta \) with \( B^0 \to J/\psi K^0 \) is dominated by Belle and BABAR. In a short time we have gone from the first observation of CP violation in the B system, to the point where the precision of the direct measurements of \( \sin 2\beta \) has exceeded the prediction from the indirect measurements. The \( B \) factory experiments have started measuring time-dependent asymmetries in rare modes such as \( B^0 \to \phi K^0 \). In the Standard Model, the asymmetries in these modes are, up to small corrections, equal to \( \sin 2\beta \). A summary of these measurements, averaged over the experiments [32] is shown in Figure 4. There is an intriguing trend for these measurements to be lower than expected, but the current experimental errors are such that no firm conclusion can be drawn yet. It will be interesting to see
whether these measurements will converge, as additional luminosity is collected, towards the value of $\sin 2\beta$ measured with $B^0 \to J/\psi K^{*0}$, or whether they will become significant deviations, indicating the presence of New Physics.

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