Research article

Convolutional neural network with group theory and random selection particle swarm optimizer for enhancing cancer image classification

Kun Lan\textsuperscript{1,2}, Gloria Li\textsuperscript{1,2}, Yang Jie\textsuperscript{1,2}, Rui Tang\textsuperscript{3}, Liansheng Liu\textsuperscript{4,*} and Simon Fong\textsuperscript{1,2}

\textsuperscript{1} Department of Computer and Information Science, Faculty of Science and Technology, University of Macau, Macau 999078, China
\textsuperscript{2} DACC Laboratory, Zhuhai Institutes of Advanced Technology of the Chinese Academy of Sciences, Zhuhai 519080, China
\textsuperscript{3} Department of Management and Science and Information System, Faculty of Management and Economics, Kunming University of Science and Technology, Kunming 650093, China
\textsuperscript{4} Department of Medical Imaging, First Affiliated Hospital of Guangzhou University of Chinese Medicine, Guangzhou 510405, China

* Correspondence: Email: llsjnu@sina.com; Tel: +8602036598876.

Supplementary

Definition (group). Given a set of elements \( G \) and a binary multiplication operation \( \otimes \), then the group \( G \) is defined if:

\begin{itemize}
  \item Closure: \( \forall g, h \in G, \ g \otimes h \in G \)
  \item Associativity: \( \forall g, h, j \in G, \ (g \otimes h) \otimes j = g \otimes (h \otimes j) \)
  \item Identity: \( \forall g \in G, \ \exists e \in G, \ g \otimes e = g \)
  \item Inverses: \( \forall g \in G, \ \exists g^{-1} \in G, \ g \otimes g^{-1} = e \)
\end{itemize}

Definition (abelian group). An abelian group embodies a commutative binary operation:

\( \forall g, h \in G, \ g \otimes h = h \otimes g \)

Definition (permutation). A permutation \( p \) of a given set \( X \) is a function that arranges its members into an ordered sequence. So it is a bijective mapping of \( f: X \rightarrow X \) from \( X \) to itself, \( p = \begin{pmatrix} x_1 & x_2 & \cdots & x_n \\ p(x_1) & p(x_2) & \cdots & p(x_n) \end{pmatrix} \).

Definition (permutation group). A permutation group \( G \) is a group with the elements of some permutations of a given set \( X \).
**Definition (symmetric group).** A symmetric group $S_n$ is a group with the elements of all permutations of a given set $X$, where $n$ is the number of letters in $X$ and $S_n$ has the cardinality of $n!$.

**Definition (cycle).** A cycle is a permutation of some elements in the given set $X$ or its subset $S$ that maps those elements to each other in a cyclic form, while keeping others fixed. A cyclic form is called a $i$-cycle if there are $i$ elements in the set $(a_1, a_2, \ldots, a_i)$, and it maps $a_1$ to $a_2$, $a_2$ to $a_3$, ..., $a_{i-1}$ to $a_i$ and $a_i$ back to $a_1$.

**Definition (group action).** A group action is the transformation from one element to another of a group on a set. Given a group $G$ and a set $X$, let $X = \{x, y, z, \ldots\}$, the group action of $G$ on $X$, is a bijective mapping of $f: X \rightarrow X$ so that $\forall x \in X, \ f(x) = gx = y \in X$ and there exists $f^{-1}, f^{-1}(y) = x$.

**Definition (orbit).** An orbit is the subset of a given set $X$ composed of the elements that can be reached by particular group actions of a given group $G$. For $x \in X$, $\text{Orbit}(g, x) = \{gx | g \in G\}$.

**Definition (orbital plane).** An orbital plane is the partition of a given set $X$ where different partition results have disjoint elements but share the same collections of element positions of cycles in order.

**Definition (conjugation).** For $f, g, h \in G$, define $f$ and $h$ are conjugate by $g$ if $f = ghg^{-1}$, and conjugation can be symmetric and transitive.

**Definition (conjugacy class).** The conjugacy class is a set that contains all conjugate elements of the generator element. For $f, g \in G$, the conjugacy class of element $f$ is $\text{CC}(f) = \{gf^g^{-1} | g \in G\}$. If $G$ is abelian, then $\text{CC}(f) = \{gf^g^{-1} | g \in G\} = \{gg^{-1}f | g \in G\} = \{f | g \in G\}$, the only conjugate element is $f$ itself in $\text{CC}(f)$.