The Phillips - Barger model for the elastic cross section and the Odderon

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Inspired by the recent TOTEM data for the elastic proton – proton (pp) scattering at √s = 8 and 13 TeV, we update previous studies of the differential cross sections using the Phillips – Barger (PB) model, which parametrizes the amplitude in terms of a small number of free parameters. We demonstrate that this model is able to describe the recent pp data on a statistically acceptable way. Additionally, we perform separate fits of the pp data for each center - of - mass energy and propose a parametrization for the energy dependence of the parameters present in the PB model. As a consequence, we are able to present the PB predictions for the elastic proton - proton cross section at √s = 546 GeV and 1.8 TeV, which are compared with the existing antiproton – proton (¯pp) data. We show that the PB predictions, constrained by the pp data, are not able to describe the ¯pp data. In particular, the PB model predicts a dip in the differential cross section that is not present in the ¯pp data. Such result suggests the contribution of the Odderon exchange at high energies.

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The LHC measurements for the total, elastic and differential cross sections at high energies of 8 and 13 TeV [1][10] provide very interesting results, which challenge the description of the theory of strong interactions at high energies. In particular, the high-precision measurement of the elastic differential cross section data (dσ/dt) in the Coulomb-Nuclear interference region in proton-proton (pp) scattering, performed by the TOTEM Collaboration [8], has allowed the determination of the ρ parameter, which is defined as the ratio between the real and imaginary part of the elastic scattering amplitude in the forward direction. The value obtained experimentally is lower than those predicted by different Regge inspired models that take into account the Reggeon and Pomeron exchange contributions for the scattering amplitude, whose predictions for the total cross section (σtot) are compatible with the measurements [11].

Such result has motivated an intense debate about the possible contribution of the Odderon [12–19]. The existence of an Odderon is a natural prediction of the Quantum Chromodynamics (QCD), has a C-odd parity and determines the hadronic cross section difference between the direct and crossed channel processes at very high energies (for a review see Ref. [20]). If the Odderon exchange contributes for the hadronic scattering at high energies we will have σtot(√s) ≠ σtot(√s). Moreover, the squared momentum transfer (t) dependence of dσ/dt for |t| ≠ 0 is predicted to be different for pp and ¯pp scattering at a fixed center - of - mass energy (√s) [21][22]. Namely, a dip is expected to be present in pp and absent in ¯pp collisions, which is directly associated to the fact that the Odderon contribution, being a C-odd term in the elastic scattering amplitude, enters with an opposite sign in each case, resulting in a different t – behaviour for dσ/dt. Consequently, a comparison between the data for the pp and ¯pp distributions at the same energy can be considered a direct probe of the Odderon. Such comparison have only been performed using the data at low energies (√s = 53 GeV), but due to the low statistics of the ¯pp data, the difference between the pp and ¯pp data is not very large and depends on only a few data points. Therefore, the interpretation of the difference as a evidence of the Odderon remains a subject of debate [20].

Another important aspect of the recent TOTEM data for the transverse momentum distributions is its high precision and the large range of t values spanned at high energies. The description of the t - dependence of dσ/dt in the full kinematical range is still a theoretical challenge, since the model should be able to describe the optical point, the forward peak, the dip and the tail for a fixed energy, as well as to predict the change of these structures with the energy. As the elastic process is dominated by non – perturbative physics, such process have been described using phenomenological models based on different assumptions. Our goal in this letter is to update the Phillips – Barger (PB) model, proposed originally in Ref. [23] and modified in Ref. [24], which describes the scattering amplitude in terms of a small number of free parameters. As demonstrated in Ref. [24], this model is able to describe the LHC data at √s = 7 TeV. In what follows we extend the analysis for √s = 8 and 13 TeV and demonstrate that the PB model also allows to successfully describe the recent (preliminary) TOTEM data. A shortcoming of the PB model is

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TABLE I: Results for the fits of the differential cross section data considering $pp$ scattering at ISR energies. The reduced $\chi^2$ and degrees of freedom, $\nu$, for each fit are also presented.

| Energy | 23 GeV | 31 GeV | 45 GeV | 53 GeV | 63 GeV |
|--------|--------|--------|--------|--------|--------|
| $A$    | 24.42(18) | 26.09(10) | 28.328(40) | 29.658(40) | 30.706(68) |
| $B$    | 1.80(26) | 1.48(14) | 0.772(50) | 1.101(52) | 0.990(95) |
| $C(\times 10^{-3})$ | 2.089(55) | 2.070(35) | 1.837(24) | 1.929(14) | 1.910(38) |
| $\phi$ | 3.296(11) | 3.225(12) | 3.449(12) | 3.446(11) | 3.459(26) |
| $t_0$  | 1.245 | 1.218 | 3.689 | 2.864 | 1.245 |
| $\chi^2/\nu$ | 1.24 | 1.218 | 3.689 | 2.864 | 1.245 |
| $\nu$  | 128 | 167 | 202 | 200 | 119 |

TABLE II: Results for the fits of the differential cross section data considering $pp$ scattering at LHC energies. The reduced $\chi^2$ and degrees of freedom, $\nu$, for each fit are also presented.

| Energy | 7 TeV | 8 TeV | 13 TeV |
|--------|-------|-------|--------|
| $A$    | 177.42(53) | 201.1(6.2) | 226.83(85) |
| $B$    | 8.69(17) | 5.94(63) | 8.92(19) |
| $C(\times 10^{-3})$ | 406(19) | 114.9(45) | 714(40) |
| $D$    | 4.609(39) | 5.20(36) | 4.927(40) |
| $\phi$ | 3.5366(76) | 3.393(31) | 3.441(10) |
| $t_0$  | 0.727(12) | 0.602(22) | 0.722(13) |
| $\chi^2/\nu$ | 2.626 | 0.809 | 1.312 |
| $\nu$  | 156 | 84 | 218 |

that the energy dependence of the parameters present in this model is not predicted. In this letter we will overcome this limitation using the $pp$ data for ISR [25, 26] and LHC [1, 2, 5, 6] energies to determine the energy dependence of the BP parameters. As a consequence, this procedure allows us to predict $d\sigma/dt$ for $pp$ scattering at FERMILAB energies, where $pp$ data exist. As we will show below, the PB model, constrained by $pp$ data, is not able to describe the FERMILAB data at $\sqrt{s} = 546$ GeV and 1.8 TeV. Such result suggests the presence of the Odderon in the description of the hadronic scattering at high energies.

Initially let’s present a brief review of the modified version of Phillips – Barger model proposed in Ref. [24]. In this model the elastic scattering amplitude is parametrized as follows

$$A_{PB}(s,t) = i[F_2^p \sqrt{A} e^{i\phi} + e^{i\phi} \sqrt{C} e^{i\phi}],$$

(1)

where $F_2^p(t) = 1/(1-t/t_0)^4$ is the proton form factor and $t_0$, $\sqrt{A}$, $\sqrt{C}$, $B$, $D$, $\phi$ are free parameters. The simple functional form of Eq. (1) allows to identify the role of each parameter: $t_0$ is important for the description of the data at small values of $|t|$ (excluding the Coulomb-Nuclear interference region), $\sqrt{A}$ and $B$ are relevant in the diffraction peak region, while $\sqrt{C}$ and $D$ are important to describe the data beyond the dip. Finally, the phase $\phi$, which enters in the interference term appearing in $|A_{PB}|^2$, controls the position and depth of the dip. As pointed out before, such model does not predict the energy dependence of the parameters. In order to determine this dependence we will assume the following strategy in what follows: we will fit the $d\sigma/dt$ data separately for each energy using Eq. (1) and the normalization $d\sigma/dt = \pi |A_{PB}|^2$. After we will fit the parameters appearing in the modified PB model by empirical functions of $s$. We will include in our analysis the $pp$ data at ISR energies ($\sqrt{s} = 23, 31, 45, 53$ and 63 GeV) [25, 26] and the LHC data for $\sqrt{s} = 7$ TeV, 8 and 13 TeV obtained by the TOTEM Collaboration [1, 2, 5, 6]. In order to cover a larger range of values of $t$, we also include the preliminary TOTEM data for 8 TeV [5] and 13 TeV [6] in the dip region. For all energies considered, the optical point was included in the dataset, so that our fits are constrained by the data for the total cross sections. Finally, we have constrained the $|A_{PB}|^2$ to be in the 3rd quadrant in order to obtain the correct signal of the real part of the amplitude and, consequently, to be in accordance with the signal of the experimental information available for the $\rho$ parameter. This constraint does not affect the other parameters or the description of $d\sigma/dt$ data, since $\phi$ appears inside a cosine function in the interference term of $|A_{PB}|^2$.

The fit results are shown in the Tables I and II for $pp$ collisions at ISR and LHC energies, respectively, as well as the reduced $\chi^2$ and degrees of freedom, $\nu$, for each fit. Parameters $A$ and $C$ are given in mb$^2$GeV$^{-4}$, $B$ and $D$ in
GeV$^{-2}$, $t_0$ in GeV$^2$ and $\phi$ in radians. The quoted uncertainties correspond to 1σ of confidence level. The comparison between the data and the curves calculated with these parameters is shown in Fig. 1. Such results indicate that the PB model is able to describe the $p\bar{p}$ data, including the dip region. Using the values of the parameters for the different energies presented in the Tables I and II, we will fit $\sqrt{A}$, $B$, $\sqrt{C}$ and $D$ assuming that they can be expressed by a function given by: $p_0 + p_1 \ln s + p_2 \ln^2 s$. On the other hand, for the parameters $\phi$ and $t_0$ we assume two empirical functions that become constant at large energies [24]. We obtain that:

$$\sqrt{A}(s) = 3.548 - 0.255 \ln s + 0.0297 \ln^2 s,$$  
$$B(s) = 3.50 - 0.049 \ln s + 0.0181 \ln^2 s,$$  
$$\sqrt{C}(s) = 0.253 - 0.0604 \ln s + 0.00391 \ln^2 s,$$  
$$D(s) = 3.512 - 0.405 \ln s + 0.0258 \ln^2 s,$$  
$$\phi(s) = 3.506 \frac{1 + \exp(-0.435 \ln s)}{1 + 0.00124ln^2 s},$$  
$$t_0(s) = \frac{0.997}{1 + 0.00124ln^2 s}.$$  

We note that, in contrast with the values presented in the Tables I and II, $\sqrt{A}$ and $\sqrt{C}$ are given in mb in the above equations. The Eqs. (2)-(7) allow us to calculate $d\sigma/dt$ for the $pp$ collisions at other center-of-mass energies. In particular, we can estimate the PB predictions for the energies of 546 GeV and 1.8 TeV and compare with the existing experimental data for $p\bar{p}$ collisions at these energies. This comparison is performed in Fig. 2, where we also present the separate fits of the $p\bar{p}$ data for $\sqrt{s} = 546$ GeV [27-29], 1.8 TeV [29, 30] and 1.96 TeV [31]. As in previous studies, the 1.8 TeV and 1.96 TeV data were considered as being of the same energy (1.8 TeV). The corresponding parameters are presented in the Table III. We have that the PB predictions are not able to describe the $pp$ data, with the corresponding values of the reduced $\chi^2/\nu$ being 85.796 and 5.998 for $s = 546$ GeV and 1.8 TeV, respectively. In contrast, if the data are separately fitted using the PB model, the description is very good, as verified in the Table III. Moreover, we have that the PB prediction implies a dip at $|t| \sim 0.75$ GeV$^2$ for $\sqrt{s} = 546$ GeV and $|t| \sim 0.6$ GeV$^2$ for 1.8 TeV, followed by a bump, in contrast with the $pp$ data. The large difference between the PB predictions, constrained by $pp$ data, and the experimental $pp$ data implies that the scattering amplitude for the $pp$ and $p\bar{p}$ processes is different for a given energy, which can be interpreted as being due to the Odderon contribution. Therefore, our results suggest the presence of the Odderon in the description of the hadronic collisions at high energies. We stress that in order to properly estimate the Odderon contribution, it is fundamental to have data of both $pp$ and $p\bar{p}$ scattering at the same energy. Our analysis only provides more one indication of its presence at high energies.

As the energy dependence of the parameters present in PB model have been determined, we also can estimate the total $pp$ cross sections for other values of the center-of-mass energy. In Table IV we present our predictions for $\sigma_{tot}$.
for the values of $\sqrt{s}$ in which experimental $\bar{p}p$ data are available. The uncertainties quoted in our predictions have been obtained by means of error propagation from the uncertainties and correlation coefficients of the parameters present in Eqs. (2)-(7). We note that our prediction for 546 GeV is compatible with the experimental $\bar{p}p$ data [32]. We obtain $84^{+113}_{-37}$ mb and the value reported by TOTEM [7] is $84^{+113}_{-37}$ mb. Moreover, the differential cross section presents a dip around \(|t|_\text{dip} \approx 0.55 \text{GeV}^2\). At $\sqrt{s} = 14 \text{ TeV}$, we predict $\sigma_{\text{tot}}^{\bar{p}p} = 113^{+32}_{-27}$ mb and $|t_{\text{dip}}| \lesssim 0.5 \text{ GeV}^2$.

In summary, we have analyzed the experimental data for the differential cross section in elastic $pp$ collisions using the model proposed by Phillips and Barger in the 1970’s and recently improved by the inclusion of the proton form factor in order to describe data in the low $|t|$ region. We have demonstrated that the PB model is able to describe the recent LHC data and derived the energy dependence of its parameters. Predictions for $d\sigma/dt$ and $\sigma_{\text{tot}}$ considering $pp$ collisions at different values of $\sqrt{s}$ have been presented and a comparison with the available $\bar{p}p$ data was performed. We have verified that the PB model implies $\sigma_{\text{tot}}^{\bar{p}p} > \sigma_{\text{tot}}^{pp}$ and the presence of a dip in $pp$ collisions at

| Energy         | $546 \text{ GeV}$ | $1.8 \text{ TeV} + 1.96 \text{ TeV}$ |
|----------------|--------------------|----------------------------------------|
| $A$            | $67.51^{(29)}$     | $88.51^{(1.3)}$                        |
| $B$            | $5.103^{(23)}$     | $13.01^{(1.9)}$                        |
| $C(\times 10^{-3})$ | $59.1^{(7.2)}$     | $24^{(14)}$                            |
| $D$            | $3.67^{(10)}$      | $2.26^{(58)}$                          |
| $\phi$         | $3.6087^{(94)}$    | $4.08^{(12)}$                          |
| $t_0$          | $0.7520^{(80)}$    | $2.7^{(18)}$                           |
| $\chi^2/\nu$  | $1.097$            | $2.365$                                |
|                | $130$              | $91$                                    |

TABLE III: Results for the fits of the differential cross section data considering $\bar{p}p$ collisions. The reduced $\chi^2$ and degrees of freedom, $\nu$, for each fit are also presented.
\begin{table}[h]
\centering
\begin{tabular}{|c|c|c|}
\hline
Energy & $\sigma_{\text{tot}}$ (pp) & Experimental value (\bar{pp}) \\
\hline
546 GeV & 62.1 ± 0.2 mb & 61.26 ± 0.93 mb (CDF [32]) \\
900 GeV & 68.1 ± 0.2 mb & 65.3 ± 1.6 mb (UA5 [33]) \\
1.8 TeV & 77.6 ± 0.2 mb & 71.42 ± 2.42 mb (E-811 [35]) \\
\hline
\end{tabular}
\caption{Predictions of the PB model for the total cross section considering pp collisions at different values of the center-of-mass energy. For comparison, the experimental results for \bar{pp} scattering are also presented.}
\end{table}

\sqrt{s} = 546 GeV and 1.8 TeV, which is not present in the \bar{pp} data. These results indicate that, for a fixed energy, the scattering amplitudes for the two crossed channels are different, which can be interpreted as being due to the Odderon contribution. Therefore, our result can be considered more one indication that the Odderon should be considered in the description of the hadronic collisions at high energies. The future LHC data at $\sqrt{s} = 900$ GeV probably will allows to establish a definitive conclusion.

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