Comparison of Reference Heights of O/N₂ and \( \sum \text{O}/\text{N}_2 \) Based on GUVI Dayside Limb Measurement

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Abstract
We define a new thermospheric concept, the reference heights of O/N₂, referring to a series of thermospheric heights corresponding to the fixed ratios of O to N₂ number density. Here, based on Global Ultraviolet Imager (GUVI) limb measurement, we compare O/N₂ column density ratio (\( \sum \text{O}/\text{N}_2 \)) and the reference heights of O/N₂. We choose the transition height of O and N₂ (transition height hereafter), a special reference height at which O number density is equal to N₂ number density, to verify the connection with \( \sum \text{O}/\text{N}_2 \) during geomagnetically quiet periods. It is found that transition height and \( \sum \text{O}/\text{N}_2 \) have noticeable negative correlation with correlation coefficient of -0.887. An empirical model of transition height (O/N₂ model hereafter) is established based on nonlinear least-squares-fitting method. The considerable correlation (greater than 0.96), insignificant errors (less than 4%) and the great influencing weight of \( \sum \text{O}/\text{N}_2 \) to reference heights indicate the validity of O/N₂ model and the existence of quantitative relation between \( \sum \text{O}/\text{N}_2 \) and transition height. Besides, it is verified that the similar quantitative relation also exists between \( \sum \text{O}/\text{N}_2 \) and reference heights of other O/N₂ values. Namely, using the O/N₂ model coefficients, we can roughly get the whole altitude profiles of O/N₂ within 6% precision for any given \( \sum \text{O}/\text{N}_2 \).

1. Introduction
The thermosphere is a critical layer of the Earth’s atmosphere (Liu et al., 2013), extending from ~80 km to over 600 km and playing a significant role in atmospheric vertical coupling (Rishbeth, 2000). Thermospheric O/N₂ (ratio of atomic oxygen number density to molecular nitrogen number density) is an important indicative parameter in determining which composition predominates, and also frequently used to study the neutral compositional perturbation especially during geomagnetic storms (Meier et al., 2005; Stephan et al., 2008; Zhang et al., 2004). The thermosphere is tightly interactive and highly coupled to ionosphere and lower atmosphere, forming a complex coupled system (Liu et al., 2013; Ren et al., 2009). The electron production mainly attributes to the photoionization process of atomic oxygen (O) and its loss rate is primarily in proportion to the molecule nitrogen (N₂) number density. Namely, through affecting ionospheric chemistry, O/N₂ has an influence on the ionospheric electron density quantity (Fuller-Rowell et al., 1996; Strickland et al., 1999, 2001; Zhang et al., 2003, 2004; Crowley et al., 2006; Grigorenko et al., 2007; Kil et al., 2011), for instance, O/N₂ is known as a contributor on the ionospheric seasonal variations (e.g. winter/seasonal anomaly and semiannual/equinoctial anomaly) (e.g., England et al., 2010; Fuller Rowell, 1998; Luan et al., 2017; Mendillo et al., 2005; Qian et al., 2016; Qian et al., 2016; Rishbeth, 1998; Yu et al., 2004). Besides, O/N₂ is also used to probe the thermosphere-tide coupling process (He et al., 2010; Zhang et al., 2010).

O/N₂ at a given height is expressed as O/N₂ volume density ratio, and the O/N₂ at different heights in the thermosphere constitute the O/N₂ altitude profiles. O/N₂ column density ratio, \( \sum \text{O}/\text{N}_2 \), that is the ratio of height-integrated O to N₂ referenced to an N₂ depth of 10¹⁷ cm⁻² (Strickland et al., 1995, 2012), is widely applied to the above thermospheric studies based on the observations and simulations (e.g., Luan et al., 2017; Qian, Burns, Solomon, et al., 2016; Qian, Burns, Wang, et al., 2016). Note that the height at which O and N₂ number densities make major contribution to \( \sum \text{O}/\text{N}_2 \) is around 140 km, due to their densities’ exponential decay with heights, or \( \sum \text{O}/\text{N}_2 \) only represents the low thermospheric neutral composition measurements (Lei et al., 2010; Strickland et al., 1995; Yue et al., 2019). Kil et al. (2011) also demonstrated that
the disturbances of O/N$_2$ volume density ratio in F region during the storm period are consistent with $\Sigma$O/N$_2$ disturbance where $\Sigma$O/N$_2$ is depressed, while are inconsistent where $\Sigma$O/N$_2$ is enhanced. Obviously, $\Sigma$O/N$_2$ could not provide sufficient information on the vertical neutral composition distributions. Besides, the composition distributions are driven by solar- and geomagnetic-heating, upwelling/downwelling, atmospheric waves (e.g., tides) and so on, which have altitude dependency (Rishbeth, 1998). It means the distribution and variations of O/N$_2$ altitude profiles are more important for atmospheric indicating, assessment and forecasting accurately than $\Sigma$O/N$_2$. However, it is not extensively used due to the lack of altitude profiles of neutral composition from observations. It is necessary to find a method to specify the O/N$_2$ temporal and spatial distribution in different altitudes for ionospheric/thermospheric studies.

Meier and Picone (1994) used limb-viewing images of emission intensities in the OI 135.6 nm and N$_2$ Lyman-Birge-Hopfield short filter (LBHS) profiles to retrieve altitude profiles of O and N$_2$ by the inversion technique. The $\Sigma$O/N$_2$ could be computed from the retrieved O and N$_2$ altitude profiles. The Global Ultraviolet Imager (GUVI) limb measurements can be utilized to retrieve the O and N$_2$ altitude profiles by the inversion procedure, and the O/N$_2$ altitude profiles and $\Sigma$O/N$_2$ are computed from these composition profiles. Strickland et al. (1995) used nadir-viewing (disk-viewing) images of the same airglow spectrum to infer the unique relation between the ratio of OI 135.6 nm and N$_2$ LBHS bands column emission rate ($I_{135.6}/I_{LBHS}$) and the ratio of the column O to N$_2$ ($\Sigma$O/N$_2$). Zhang et al. (2004) used the method and combined the Mass-Spectrometer-Incoherent-Scatter (MSIS) model and Atmospheric Ultraviolet Radiance Integrated Code (AURIC) simulations (Strickland et al., 1999) to acquire the GUVI disk $\Sigma$O/N$_2$ through the GUVI 135.6 nm and LBHS dayglow observations, and such data contain no altitude information. It is noticeable that the way to acquire $\Sigma$O/N$_2$ from GUVI limb and disk measurements is different, but the definition or significance of $\Sigma$O/N$_2$ are both the column density ratio of O and N$_2$ down to a fixed N$_2$ depth (Kil et al., 2011). The N$_2$ depth of $10^{17}$ cm$^{-2}$ is chosen in deriving the disk $\Sigma$O/N$_2$ values due to the least uncertainty or best correlation between the $\Sigma$O/N$_2$ and the $I_{135.6}/I_{LBHS}$ in this N$_2$ depth (Strickland et al., 1995). Strickland et al. (2004) found $\Sigma$O/N$_2$ observed from GUVI disk measurements and that integrated from limb measurements is nominal consistent under the LBH scaling factor of 1.4 for the lookup table. Stephan et al. (2008) also found them in strong agreement during a geomagnetic storm. Yue et al. (2019) demonstrated their consistent seasonal variations from GUVI limb and disk $\Sigma$O/N$_2$ during geomagnetically quiet periods.

Emmert et al. (2006) presented the first validation of mass density from GUVI limb retrievals by comparing with that from satellite orbit data. Yu et al. (2019) demonstrated that GUVI daytime observations are in good conformity with the Challenging Minisatellite Payload (CHAMP) satellite observations. Those all represent the validation of GUVI limb neutral density profiles.

In this paper, by making use of altitude profiles of O and N$_2$ number density and $\Sigma$O/N$_2$ derived from TIMED/GUVI limb FUV dayglow observations, we calculate a series of heights, corresponding to fixed ratios of O to N$_2$ number density (reference heights of O/N$_2$ different values). First, we choose one height, namely the height at which O number density is equal to N$_2$ number density (transition height hereafter), to describe its relationship with $\Sigma$O/N$_2$ during geomagnetically quiet periods. Second, we establish an empirical model and obtain the quantitative relation between $\Sigma$O/N$_2$ and transition height, and then evaluate the fitting results. Finally, we fit the similar models to all reference heights of different O/N$_2$ values with $\Sigma$O/N$_2$ and make an assessment. To our knowledge, it is innovative that we try to find the corresponding relation between O/N$_2$ in disparate altitudes and $\Sigma$O/N$_2$ in detail.

2. Instrument and Data

The GUVI is an imaging spectrograph, measuring the upper atmospheric airglow in 115−180 nm spectrum from both the Earth limb and disk viewing. GUVI was equipped on the Thermosphere Ionosphere Mesosphere Energy and Dynamics (TIMED) satellite, which was launched on 7 December 2001 into a 630 km circular orbit with an orbital inclination of 74.1° (Christensen et al., 2003; Paxton et al., 1999). GUVI data cover five major airglow emission bands: H 121.6 nm, OI 130.4 nm and 135.6 nm, two N$_2$ Lyman-Birge-Hopfield (LBH) bands: LBH short band (LBHS) between 140 and 150 nm and LBH long band (LBHL) between 165 and 180 nm. Where, the OI 135.6 nm and LBHS dayglow intensities from limb scan are
inverted to produce altitude profiles of atmospheric neutral concentration (O, N₂, O₂) using the forward model, that is to calculate the maximum likelihood solutions by adjusting model parameters (Meier & Picone, 1994). The altitude range of profiles is from 110 km to 667 km, but the densities outside ~300 km are viewed as extrapolations due to the altitude limitation of the dayglow sensitivity. Then the O and N₂ altitude profiles are directly integrated to get \( \sum \frac{O}{N_2} \) where \( N_2 \) column abundance equals \( 10^{17} \text{cm}^{-2} \) (a changed altitude of approximate 140 km) (He et al., 2010). The limb data used in our study are Neutral Density Profile (NDP) that contain the O/N₂ altitude profiles and the \( \sum \frac{O}{N_2} \) (Meier et al., 2015; Yu et al., 2019). For the disk-viewing, 135.6 nm and LBHS are directly used to derive the \( \sum \frac{O}{N_2} \) described in the introduction. Strickland et al. (1995) and Zhang et al. (2004) provided the full details about deducing, analysis and comprehension of the disk data.

The GUVI limb data are available from 2002 to 2007 on the website, http://guvimed.jhuapl.edu/data_products. The scan mirror failed since December 2007, leading to the lack of limb data after 2007. Data under geomagnetically quiet conditions (ap<=12) are used in the present work to exclude the effects of the geomagnetic disturbances. The reference heights of different O/N₂ values (heights at which log\(_{10}(O/N_2)\) equals from -1 to 2 with step of 0.1) including the transition height (height at which log\(_{10}(O/N_2)\) equals zero, namely the height that O number density equals N₂ number density) are calculated. The changes of reference heights of O/N₂ are equivalent to the changes of the logarithm of O/N₂ volume density ratio at a fixed height, and the changes of reference heights is more linear with the height. Besides, the values of O/N₂ volume density ratio are smaller, basically between zero to two. The reference heights with larger orders of magnitude can reflect relatively obvious differences of parameter values, and the altitude change features of the parameters are showed more intuitively. Therefore, we choose the reference heights to investigate relationship with \( \sum \frac{O}{N_2} \) instead of O/N₂ volume density ratio. Here, we mainly utilize GUVI limb data products to present the verification of the relation between them.

## 3. Results

### 3.1. Preliminary Data Comparison

Figure 1a shows the comparison between the daily mean values of the transition height (blue line) and \( \sum \frac{O}{N_2} \) (black line) during 2002-2007. The data gaps result from the algorithmic limitations in GUVI inversion technique (Meier et al., 2015). The transition height has antipodal variation tendency with \( \sum \frac{O}{N_2} \), rising of \( \sum \frac{O}{N_2} \) corresponding to dropping of transition height, or vice versa. Besides, they both have obvious seasonal variations, for example, dominant annual and semiannual variations. Figure 1b and 1c depicts the local time versus latitude distribution of \( \sum \frac{O}{N_2} \) and transition height averaged over all longitudes, respectively. The semidiurnal variations are reflected in both panels, and are connected with latitudes. But above all, their contrary spatial distribution pattern is clearly revealed, high (low) values of \( \sum \frac{O}{N_2} \) corresponding to the low (high) values of transition height.

Figure 2 depicts their scatter plot of daily mean values for 2002-2007, and shows the non-linear negative correlation. The correlation coefficient reaches -0.887. In view of the seasonal variation reflected in Figure 1a, the scatter plots distinguishing four seasons (spring: March to May; summer: June to August; fall: September to November; winter: December to February) are presented in Figure 3, exhibiting the obvious differences of variation distribution in different seasons. The slopes of the scatter distribution in spring and fall are larger than those in summer and winter. However, the changes of correlation coefficients are not distinct relative to that of all data in Figure 2. The annual and semiannual variations in thermospheric compositions are usually explained by the annual insolation variation caused by the change of Sun-Earth distance (Lei et al., 2013) and the large-scale interhemispheric circulation respectively (Fuller Rowell, 1998; Paetzold & Zschorner, 1961), and are also noticed in previous thermospheric modeling works (Lei et al., 2012; Liu et al., 2013; Yu et al., 2019).

\( F_{10.7p} \) is used to describe solar activity level in the unit of solar flux unit (sfu) (1 sfu=10\(^{-22}\)W/m\(^2\)/Hz), defined as \( F_{10.7p} = (F_{10.7a}+F_{10.7})/2 \), where \( F_{10.7} \) is the daily solar flux at 10.7 cm, and \( F_{10.7a} \) is the 81-day average (Liu et al., 2013). Figure 4 shows scatter plots of transition height and \( \sum \frac{O}{N_2} \) in four scopes of limited \( F_{10.7p} \) (\( F_{10.7p} < 100 \) sfu, 100 sfu < \( F_{10.7p} < 140 \) sfu, 140 sfu < \( F_{10.7p} < 180 \) sfu and \( F_{10.7p} > 180 \) sfu). There are not many differences in variation trends between them, but their correlation coefficients increase significantly
due to the limited F10.7p. Figures 3 and 4 indicate that the effects of solar activity and seasonal variations on the relation between transition height and $\Sigma O/N_2$ are both important and should be considered.

3.2. Mathematical Formulation and Model Results

On the basis of the figures 1–4, we take the three variables, $\Sigma O/N_2$, solar activity and seasonal variations as well as non-linearity into account to construct an empirical model (denoted as O/N2 model hereafter) based on the multivariable nonlinear least-squares-fitting method as done in the previous works (e.g., Marinov et al., 2004; Kutiev et al., 2006; Kakinami et al., 2009; Liu et al., 2009; Yu et al., 2019). For a specific longitude-latitude grid at a fixed local time, we express the transition height $H$ using three parameters as below,

$$H = f_1(\text{Con}_2)f_2(\text{DOY})f_3(P)$$

(1)

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We first collect all transition height data falling within a bin, which is centered at specified local time, latitude, longitude. Before fitting the model, we smooth all transition height and $\sum O/N_2$ data using a seven-point moving average for reducing singularities. A nonlinear least squares fitting is then applied to the data points in each bin to extract a set of coefficients using equation (1). To provide sufficient points for a reliable fitting, the size of the bins was chosen to be 2 hours in local time, 20° in latitude, and 30° in longitude.

Figure 5a shows the scatter plot of the original transition heights and the corresponding model predictions including all bins. The diagonal line represents the perfect prediction. It could be seen that the $O/N_2$ model predictions distribute closely to the diagonal line with a high correlation coefficient of 0.9878. This demonstrates $O/N_2$ model based on multivariable functional nonlinear fitting is successful and it reproduces the original transition heights fairly well. Figures 5b and 5c portrays the local time versus latitude distribution of their correlation coefficients and relative standard errors (RSE) averaged over all longitudes. The definition of RSE refers to Yu et al. (2019). The correlation coefficients are basically greater than 0.96 except in low latitudes at 7-8 local time. The RSE are all less than 4%. These further proves their high consistency between original transition heights and model predictions. The $O/N_2$ model probably can describe the transition heights with a function of $\sum O/N_2$, DOY, and solar flux levels reasonably at a fixed location and local time.

The weights of the variables in every bin are computed by the linear regression analysis. The variables are the first three order of $\sum O/N_2$, $(Con_2)$, $(Con_2)^2$, $(Con_2)^3$, $F_{10.7p}$, and DOY respectively, and they are normalized before the fitting. The correlation coefficients of linear regression fitting basically reach 0.9 except some bins at low latitudes, and the RSE are all less than 6%, so the weights that we compute are credible. The detailed descriptions and figures involving in the linear regression refer to Appendix A. The weight values averaging all bins of the $Con_2$, $(Con_2)^2$ and $(Con_2)^3$ are approximately 0.29, 0.38 and 0.18, which indicate the great influence of $\sum O/N_2$ on transition height. The weights of $F_{10.7p}$ and DOY are 0.02 and 0.13, which are relatively small, maybe because the variation of $F_{10.7p}$ and DOY are partly reflected in $\sum O/N_2$. The detailed analysis on these factor weights will be discussed in the next work. In addition, we did a test to

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**Figure 3.** Scatter plots of daily mean transition height and $\sum O/N_2$ in four seasons for 2002-2007. The correlation coefficients are respectively -0.8656, -0.8595, -0.8654 and -0.882.
decompose $H$ with $f_1(\text{Con}_2)$ (Eq. (2)), namely take no account of the variation of $F_{10.7p}$ and DOY. The deviation of fitting results is evident, and the total correlation coefficient of original transition heights and the model predictions declined from 0.9878 to 0.9388, so Eq. (1) is still selected to fit the model and extract fitted coefficients eventually. We draw a conclusion that there is a high correlation between $\sum O/N_2$ and transition height, and it can be quantified by $O/N_2$ model. The number of coefficients obtained from the Eq. (1) are almost fewer than the number of observations in a bin (see the Figure S1 in

**Figure 4.** Scatter plots of daily mean transition height and $\sum O/N_2$ in four scopes of limiting $F_{10.7p}$. The correlation coefficients are respectively -0.9203, -0.9336, -0.9410 and -0.9684.

Figure 5. a: Scatter plot of model predictions versus original transition heights. The correlation coefficient is 0.9878. b (c): Latitude versus local time distribution of the correlation coefficients (RSE) averaged over all longitudes.
Appendix B), ensuring the overdetermined condition of the equation. Therefore, the transition height can be unique calculated for any given \( \sum O/N_2 \), \( F_{10.7p} \) and DOY using \( O/N_2 \) model coefficients.

4. Discussion and Conclusion

Our validation of \( O/N_2 \) model and great influence weight of \( \sum O/N_2 \) demonstrate the good quantifiable correlation between \( \sum O/N_2 \) and the transition height. Next, the relationships between \( \sum O/N_2 \) and the reference heights of different \( O/N_2 \) values will be discussed. We acquire the high-resolution data by altitudinal interpolation scheme in the present work, and utilize the \( O/N_2 \) values in the base-10 \( O/N_2 \) logarithm \( (\log_{10}(O/N_2)) \) from -1 to 2 with the step width of 0.1 to calculate the corresponding reference heights (31 reference heights in total), from \( \sim 110 \) km to above 500 km altitude, covering the whole thermosphere. The \( O/N_2 \) model described by Eq. (1) in section 3.2 are constructed to predict these reference heights. The predicted values are further compared to the original reference heights.

Figure 6 shows the correlation coefficients between the reference heights and original ones in different \( \log_{10}(O/N_2) \)/reference heights. The reference height of red dotted line is about 140 km.

Table 1

| \( \log_{10}(O/N_2)/rh \) | RSE (%) |
|--------------------------|---------|
| -1 (112 km)              | -0.35   |
| -0.5 (132 km)            | -0.44   |
| 0 (187 km)               | -0.41   |
| 0.5 (264 km)             | -0.38   |
| 1 (349 km)               | -0.42   |
| 1.5 (437 km)             | -0.49   |
| 2 (520 km)               | -0.57   |

Figure 7 depicts a case of the \( O/N_2 \) altitude profile from GUVI limb original reference heights (solid line) and \( O/N_2 \) model fitting results (dotted line) for given \( F_{10.7p} \) (70), doy (80) at latitude of 16°S, at longitude of...
90°E, 16 LT. The altitude profile of O/N₂ model fitting is obtained using model coefficients and limb \( \sum \) O/N₂ in above conditions, which is in good agreement with the original profile. It further demonstrates that the O/N₂ model is capable to unfold the altitude profiles of O/N₂ with \( \sum \) O/N₂ effectively.

As mentioned above, \( \sum \) O/N₂ is primarily determined by the O and N₂ volume density around 140 km, as shown in red dotted line of Figure 6. However, the good model fitting results are shown from 140 km to higher altitudes of above 500 km, not only limited in ~140 km. The unfolded altitude profile using O/N₂ model is highly consistent with that from original data as shown in Figure 7. These all emphasize the existence of quantitative relation between \( \sum \) O/N₂ and O/N₂ in almost all thermospheric heights, as well as the availability of O/N₂ model. According to the ideal gas law, pressure equation and molecular diffusion flux equation, the neutral compositions in different heights are controlled by the temperature profiles and species' scale heights (Yue et al., 2019). The temperature profile (or scale height) is mainly controlled by solar radiation, which are well related with DOY (or distance between Sun and Earth) as well as F10.7. Therefore, the internal physical mechanisms of the compositions in high thermosphere and in low thermosphere are reflected in our model including both DOY and F10.7, resulting in the good fitting results in the whole thermosphere. The establishment of the quantitative equations between them are significant to the whole thermospheric studies. For example, the unfolded altitude profiles of O/N₂ could provide more precise information of neutral composition disturbances especially in the F region. Lei et al. (2010) demonstrated the consistent distances between \( \sum \)O/N₂ and O/N₂ on the Z = 3 constant-pressure level.
(~400 km) during geomagnetic storm by the National Center for Atmospheric Research (NCAR) Thermosphere-Ionosphere-Electrodynamics General Circulation Model (TIE-GCM) simulation. However, as mentioned in the introduction, Kil et al. (2011) found there are inconsistent distances between $\sum O/N_2$ and $O/N_2$ in the F region (246 km and 472 km) during the positive geomagnetic storm. Therefore, the single measurement of $\sum O/N_2$ cannot determine the composition disturbances in different heights and their effects on the ionosphere, which strengthen the significance of $O/N_2$ altitude profiles unfolding.

$\sum O/N_2$ from GUVI disk-viewing is extensively applied, and compared with models and observations from other satellites (Hecht et al., 2012; Meier et al., 2005; Zhang et al., 2004). $\sum O/N_2$ from GUVI limb-viewing are obtained by integration based on their intrinsic O and N$_2$ altitude profiles, satisfying the logic of self-consistent. Besides, the definition or significance of $\sum O/N_2$ from limb- and disk-viewing are uniform mentioned in the introduction. So, the O/N$_2$ model should be appropriate to GUVI disk $\sum O/N_2$ to derive the O/N$_2$ altitude profiles in theory, and their variation trends are consistent mentioned above. However, the disk $\sum O/N_2$ values are ~45% lower than limb values (Meier et al., 2015; Zhang & Paxton, 2011), and the systematic errors may result in the system discrepancy of their unfolded altitude profiles. The sources of the discrepancy remained to be solved. The difference of swept viewing, the mirror steps, the inversion algorithm between disk and limb measurements may all lead to their deviation (Meier et al., 2015; Strickland et al., 2004). In addition, Global-scale Observations of the Limb and Disk (GOLD) geostationary satellite is also based on an ultraviolet imaging spectrograph, measuring densities and temperatures in Earth’s thermosphere and ionosphere. The level 2 $\sum O/N_2$ measurements are available now, which may also provide the dataset for our altitude profile unfolding. The detailed comparison will be discussed in a future publication.

To conclude, $\sum O/N_2$ and transition height have obvious negative correlation during geomagnetically quiet periods. Thus, an empirical O/N$_2$ model of transition height is constructed from $\sum O/N_2$. The considerable correlation (correlation coefficients greater than 0.96), insignificant errors (relative standard errors less than 4%) and the great influencing weight of $\sum O/N_2$ to the reference heights reveal that O/N$_2$ model could establish their quantitative formula successfully. The quantitative relation between $\sum O/N_2$ and the transition height really exists. Besides, the relationship can be extended to $\sum O/N_2$ and all reference heights of O/N$_2$ values. And then, we can roughly get the whole altitude profiles of O/N$_2$ within 6% precision using the obtained coefficients for any given $\sum O/N_2$. It is meaningful for upper atmospheric assessment and forecasting precisely.

Appendix A.: Linear Regression Analysis

We analyze the weights of variables by the linear regression analysis, and the equation is given as:

$$f = a_1 (\text{Con}_2) + a_2 (\text{Con}_2)^2 + a_3 (\text{Con}_2)^3 + \sum_{j=1}^{2} \left( b_1(j) \sin \left( \frac{2 \text{DOY} \pi}{365.25} \right) + b_2(j) \cos \left( \frac{2 \text{DOY} \pi}{365.25} \right) \right) + c_1 + D$$

The above coefficients ($a_i$ (i = 1, 2, 3), $b(j)$ (i = 1, 2; j = 1, 2), $c_0$ and D) could reflect the weights of variables. In addition, we make the variables normalizations before the fitting by the formula:

$$X_{\text{norm}} = \frac{X - X_{\text{min}}}{X_{\text{max}} - X_{\text{min}}}$$

$X_{\text{min}}$ and $X_{\text{max}}$ are maximum and minimum of the corresponding variable X, and X refer to $\text{Con}_2$, $(\text{Con}_2)^2$, $(\text{Con}_2)^3$, $F_{10.7p}$ and DOY.
Appendix B.: GUVI Data Volume in Fixed Bins

We show the local time versus latitude distribution of GUVI data volume averaged over all longitudes as shown in Figure S2. The size of the bins is 2 hours in local time, 20° in latitude, and 30° in longitude. The number of observations are sufficient except some bins in high latitudes, in twilight (<6 and >18 LT), and at high noon (11–13 LT). The issue of GUVI data volume has been analyzed in detail in Yu et al., (2019). The coefficients of obtained from the Eq. (1) are the same combination of $a_i (i = 0, 1, 2, 3), b_0, b_i (i = 1, 2, j = 1, 2)$ and $c_i (i = 0, 1)$, and is 40 in total. Therefore, the number of coefficients is almost fewer than the number of observations in a bin, ensuring the overdetermined condition of the equation.

References

Cai, X., Yuan, T., & Liu, H.-L. (2017). Large Scale Gravity Waves perturbations in mesosphere region above northern hemisphere mid-latitude during Autumn-equinox: A joint study by Na Lidar and Whole Atmosphere Community Climate Model. Annales de Geophysique, 35, 181–188. https://doi.org/10.5194/angeo-35-181-2017

Cai, X., Yuan, T., Zhao, Y., Pautet, P.-D., Taylor, M. J., & Pendleton, W. R. Jr. (2014). A coordinated investigation of the gravity wave breaking and the associated dynamical instability by a Na lidar and an Advanced Mesosphere Temperature Mapper over Logan, UT (41.7°N, 111.8°W). Journal of Geophysical Research: Space Physics, 119, 6852–6864. https://doi.org/10.1002/2014JA020131

Christensen, A. B., Paxton, L. J., Avery, S., Craven, J., Crowley, G., Humm, D. C., et al. (2003). Initial observations with the Global Ultraviolet Imager (GUVI) in the NASA TIMED satellite mission. Journal of Geophysical Research, 108(A12), 1451. https://doi.org/10.1029/2003JA009918

Crowley, G., Hackert, C. L., Meier, R. R., Strickland, D. J., Paxton, L. J., Pi, X., et al. (2006). Global thermosphere-ionosphere response to onset of 20 November 2003 magnetic storm. Journal of Geophysical Research, 111, A10S08. https://doi.org/10.1029/2005JA011518

Emmert, I. T., Meier, R. R., Picone, J. M., Lean, J. L., & Christensen, A. B. (2006). Thermospheric density 2002–2004: TIMED/GUVI dayside limb observations and satellite drag. Journal of Geophysical Research, 111, A10S16. https://doi.org/10.1029/2005JA011495

England, S. L., Inmel, T. J., Huba, J. D., Hagan, M. E., Maute, A., & DeMaistre, R. (2010). Modeling of multiple effects of atmospheric tides on the ionosphere: An examination of possible coupling mechanisms responsible for the longitudinal structure of the equatorial ionosphere. Journal of Geophysical Research, 115, A05308. https://doi.org/10.1029/2009JA014894

Forbes, J. M., Roble, R. G., & Fesen, C. G. (1993). Acceleration, heating, and compositional mixing of the thermosphere due to upward propagating tides. Journal of Geophysical Research: Space Physics, 98(A1), 311–321. https://doi.org/10.1029/92JA00442

Fuller Rowell, T. (1998). The “thermospheric spoon” - A mechanism for the semianual density variation. Journal of Geophysical Research, 103(A3), 3951–3956.

Fuller-Rowell, T. J., Codrescu, M. V., Rishbeth, H., Moffet, T. J., & Quegan, S. (1996). On the seasonal response of the thermosphere and ionosphere to geomagnetic storms. Journal of Geophysical Research, 101(A2), 2343–2353. https://doi.org/10.1029/95JA01614

Grigorenko, E. I., Lysenko, V. N., Pazyura, S. A., Taran, V. I., & Chernogor, L. F. (2007). Ionospheric disturbances during the severe magnetic storm of November 7–10, 2004. Geomagnetism and Aeronomy, 47(6), 720–738. https://doi.org/10.1134/S0016793207060059

He, M., Liu, L., Wan, W., Lei, J., & Zhao, B. (2010). Longitudinal modulation of the O/N2 column density retrieved from TIMED/GUVI measurement. Geophysical Research Letters, 37, L20108. https://doi.org/10.1029/2010GL045105

Hecht, J. H., Mulligan, T., Correia, J. T., Clemons, J. H., Strickland, D. J., Walterscheid, R. L., & Conde, M. G. (2012). A multyear (2002–2006) climatology of O/N2 in the lower thermosphere from TIMED GUVI and ground-based photometer observations. Journal of Geophysical Research, 117, A03508. https://doi.org/10.1029/2011JA017146

Kakinami, Y., Chen, C. H., Liu, J. Y., Oyama, K.-I., Wang, W. H., & Abe, S. (2009). Empirical models of total electron content based on functional fitting over Taiwan during geomagnetic quiet condition. Annales de Geophysique, 27, 3321–3333.

Kil, H., Kwak, Y.-S., Paxton, L. J., Meier, R. R., & Zhang, Y. (2011). O and N2 disturbances in the F region during the 20 November 2003 storm seen from TIMED/GUVI. Journal of Geophysical Research, 116, A02314. https://doi.org/10.1029/2010JA016227

Kutiev, I. S., Marinov, P. G., & Watanabe, S. (2006). Model of topside ionosphere scale height based on topside sounder data. Advances in Space Research, 37, 943–950. https://doi.org/10.1016/j.asr.2005.11.021

Lei, J., Dou, X., Burns, A., Wang, W., Luan, X., Zeng, Z., & Xu, J. (2013). Annual asymmetry in thermospheric density: Observations and simulation. Journal of Geophysical Research: Space Physics, 118, 2503–2510. https://doi.org/10.1002/jgra.50253

Lei, J., Matsuo, T., Dou, X., Sutton, E., & Luan, X. (2012). Annual and semianual variations of thermospheric density: EOF analysis of CHAMP and GRACE data. Journal of Geophysical Research, 117, A01310. https://doi.org/10.1029/2011JA017324

Lei, J., Thayer, J. P., Burns, A. G., Lu, G., & Deng, Y. (2010). Wind and temperature effects on thermosphere mass density response to the November 2004 geomagnetic storm. Journal of Geophysical Research, 115, A05303. https://doi.org/10.1029/2009JA014754

Liu, H., Hirano, T., & Watanabe, S. (2013). Empirical model of the thermospheric mass density based on CHAMP satellite observation. Journal of Geophysical Research: Space Physics, 118, 843–848. https://doi.org/10.1002/jgra.50144

Liu, L., Zhao, B., Wan, W., NING, B., Zhang, M.-L., & He, M. (2009). Seasonal variations of the ionospheric electron densities retrieved from Constellation Observing System for Meteorology, Ionosphere, and Climate mission radio occultation measurements. Journal of Geophysical Research, 114, A02023. https://doi.org/10.1029/2008JA013819

Luan, X., Wang, W., Burns, A., & Dou, X. (2017). Solar cycle variations of thermospheric O/N2 longitudinal pattern from TIMED/GUVI. Journal of Geophysical Research: Space Physics, 122, 2605–2618. https://doi.org/10.1002/2016JA023696

Marinov, P. G., Kutiev, I. S., & Watanabe, S. (2004). Empirical model of O+–H+ transition height based on topside sounder data. Advances in Space Research, 34, 2021–2025. https://doi.org/10.1016/j.asr.2004.07.012
Meier, R. R., Crowley, G., Strickland, D. J., Christensen, A. B., Paxton, L. J., Morrison, D., & Hackert, C. L. (2005). First look at the 20 November 2003 superstorm with TIMED/GUVI. Comparisons with a thermospheric global circulation model. Journal of Geophysical Research, 110, A09S41. https://doi.org/10.1029/2004JA010990
Meier, R. R., & Picone, J. M. (1994). Retrieval of absolute thermospheric concentrations from the far UV dayglow: An application of discrete inverse theory. Journal of Geophysical Research, 99(A4), 6307–6320. https://doi.org/10.1029/93JA02775
Meier, R. R., Picone, J. M., Drob, D., Bishop, J., Emmert, J. T., Lean, J. L., et al. (2015). Remote Sensing of Earth’s Limb by TIMED/GUVI: Retrieval of thermospheric composition and temperature. Earth and Space Science, 2(1), 1–37. https://doi.org/10.1002/2014EA00035
Mendillo, M., Huang, C., Pi, X., Rishbeth, H., & Meier, R. (2005). The global ionospheric asymmetry in total electron content. Journal of Atmospheric and Solar-Terrestrial Physics, 67, 1377–1387.
Paetzold, H. K., & Zschorner, H. (1961). An annual and a semiannual variation of the upper air density. Pure and Applied Geophysics, 41, 85–92.
Paxton, L. J., Christensen, A. B., Humm, D. C., Ogorzailek, B. S., Pardoe, C. T., Morrison, D., et al. (1999). Global ultraviolet imager (GUVI): Measuring composition and energy inputs for the NASA Thermosphere Ionosphere Mesosphere Energetics and Dynamics (TIMED) mission. Proceedings of SPIE, 3756, 265–276.
Qian, L., Burns, A. G., Solomon, S. C., Wang, W., & Zhang, Y. (2016). Solar cycle variations of thermospheric composition at the solstices. Journal of Geophysical Research: Space Physics, 121, 3740–3749. https://doi.org/10.1002/2016JA023390
Qian, L., Burns, A. G., Wang, W., Solomon, S. C., & Zhang, Y. (2016). Longitudinal variations of thermospheric composition at the solstices. Journal of Geophysical Research: Space Physics, 121, 6818–6829. https://doi.org/10.1002/2016JA028988
Ren, Z., Wan, W., & Liu, L. (2009). GCITEM-IGGCAS: A new global coupled ionosphere-thermosphere-electrodynamics model. Journal of Atmospheric and Solar-Terrestrial Physics, 71(17–18), 2064–2076. https://doi.org/10.1016/j.jastp.2009.09.015
Rishbeth, H. (1998). How the thermospheric circulation affects the ionosphere F2 layer. Journal of Atmospheric and Terrestrial Physics, 60, 1385–1402.
Rishbeth, H. (2000). The equatorial F-layer: Progress and puzzles. Annales de Geophysique, 18, 730–739.
Stephan, A. W., Meier, R. R., & Paxton, L. J. (2008). Comparison of Global Ultraviolet Imager limb and disk observations of column O/N2 during a geomagnetic storm. Journal of Geophysical Research, 113, A01301. https://doi.org/10.1029/2007JA012599
Strickland, D. J., Cox, R. J., Meier, R. R., & Drop, D. P. (1999). Global O/N2 derived from DE-1 FUV imaging dayglow data: Technique and examples from two storm periods. Journal of Geophysical Research, 104, 4251–4266. https://doi.org/10.1029/98JA02817
Strickland, D. J., Daniell, R. E., & Craven, J. D. (2001). Negative ionospheric storm coincident with DE-1 observed thermospheric disturbance on October 14, 1981. Journal of Geophysical Research, 106, 21,049.
Strickland, D. J., Evans, J. S., & Correia, J. (2012). Comment on “Long-term variation in the thermosphere: TIMED/GUVI observations” by T. Zhang and L. J. Paxton. Journal of Geophysical Research, 117, A07302. https://doi.org/10.1029/2011JA017350
Strickland, D. J., Evans, J. S., & Paxton, L. J. (1995). Satellite remote sensing of thermospheric O/N2 and solar EUV 1. Theory. Journal of Geophysical Research, 100(A7), 12,217–12,226.
Strickland, D. J., Meier, R. R., Walterscheid, R. L., Craven, J. D., Christensen, A. B., Paxton, L. J., et al. (2004). Quiet time seasonal behavior of the thermosphere seen in the far ultraviolet dayglow. Journal of Geophysical Research, 109, A01302. https://doi.org/10.1029/2003JA010220
Yu, T., Ren, Z., Yue, X., Yu, Y., & Wan, W. (2019). Comparison of thermospheric density between GUVI dayside limb data and CHAMP satellite observations: based on empirical model. Journal of Geophysical Research: Space Physics, 124, 2165–2177. https://doi.org/10.1029/2018JA026229
Yu, T., Wan, W. X., Liu, L. B., & Zhao, B. (2004). Global scale annual and semi-annual variations of daytime NmF2 in high solar activity years. Journal of Atmospheric and Solar-Terrestrial Physics, 66, 1691–1701.
Yu, Y., Wan, W., Ning, B., Liu, L., Wang, Z., Hu, L., & Ren, Z. (2013). Tidal wind mapping from observations of a meteor radar chain in December 2011. Journal of Geophysical Research: Space Physics, 118, 2321–2332. https://doi.org/10.1002/2012JA017976
Yue, J., Jian, Y., Wang, W., Meier, R. R., Burns, A., Qian, L., et al. (2019). Annual and semiannual oscillations of thermospheric composition in TIMED/GUVI limb measurements. Journal of Geophysical Research: Space Physics, 124. https://doi.org/10.1029/2019JA026544
Zhang, Y., England, S., & Paxton, L. J. (2010). Thermospheric composition variations due to nonmigrating tides and their effect on ionosphere. Geophysical Research Letters, 37, L17103. https://doi.org/10.1029/2010GL044313
Zhang, Y., & Paxton, L. J. (2011). Long-term variation in the thermosphere: TIMED/GUVI observations. Journal of Geophysical Research, 116, A08H02. https://doi.org/10.1029/2010JA016337
Zhang, Y., Paxton, L. J., Kil, H., Meng, C.-I., Mende, S. B., Frey, H. U., & Immel, T. J. (2003). Negative ionospheric storms seen by the IMAGE FUV instrument. Journal of Geophysical Research, 108(A9), 1343. https://doi.org/10.1029/2002JA009797
Zhang, Y., Paxton, L. J., Morrison, D., Wolsen, B., Kil, H., Meng, C.-I., et al. (2004). O/N2 changes during 1-4 October 2002 storms: IMAGE SF-13 and TIMED/GUVI observations. Journal of Geophysical Research, 109, A10308. https://doi.org/10.1029/2004JA010441