Imprint of SUSY in radiative $B$-meson decays

Helmut Eberl$^1$, Keisho Hidaka$^2$, Elena Ginina$^{1,3}$, Akimasa Ishikawa$^{4,5,6}$

$^1$ Institut für Hochenergiephysik der Österreichischen Akademie der Wissenschaften, A-1050 Vienna, Austria
$^2$ Department of Physics, Tokyo Gakugei University, Koganei, Tokyo 184-8501, Japan
$^3$ VRVis Zentrum für Virtual Reality und Visualisierung Forschungs-GmbH, A-1220 Vienna, Austria
$^4$ Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Ibaraki 305-0801, Japan
$^5$ The Graduate University for Advanced Studies (SOKENDAI), Hayama 240-0193, Japan
$^6$ International Center for Elementary Particle Physics, University of Tokyo, Tokyo 113-0033, Japan

Abstract

We study supersymmetric (SUSY) effects on $C_7(\mu_b)$ and $C_7'(\mu_b)$ which are the Wilson coefficients (WCs) for $b \to s\gamma$ at b-quark mass scale $\mu_b$ and are closely related to radiative $B$-meson decays. The SUSY-loop contributions to $C_7(\mu_b)$ and $C_7'(\mu_b)$ are calculated at leading order (LO) in the Minimal Supersymmetric Standard Model (MSSM) with general quark-flavour violation (QFV). For the first time we perform a systematic MSSM parameter scan for the WCs $C_7(\mu_b)$ and $C_7'(\mu_b)$ respecting all the relevant constraints, i.e. the theoretical constraints from vacuum stability conditions and the experimental constraints, such as those from $K$- and $B$-meson data and electroweak precision data, as well as recent limits on SUSY particle masses and the 125 GeV Higgs boson data from LHC experiments. From the parameter scan we find the following: (1) The MSSM contribution to Re($C_7(\mu_b)$) can be as large as $\sim \pm 0.05$, which could correspond to about $3\sigma$ significance of New Physics (NP) signal in the future LHCb and Belle II experiments. (2) The MSSM contribution to Re($C_7'(\mu_b)$) can be as large as $\sim -0.08$, which could correspond to about $4\sigma$ significance of NP signal in the future LHCb and Belle II experiments. (3) These large MSSM contributions to the WCs are mainly due to (i) large scharm-stop mixing and large scharm/stop involved trilinear couplings $T_{U23}$, $T_{U32}$ and $T_{U33}$, (ii) large strange-sbottom mixing and large strange-sbottom involved trilinear couplings $T_{D23}$, $T_{D32}$ and $T_{D33}$ and (iii) large bottom Yukawa coupling $Y_b$ for large
tan \beta \text{ and large top Yukawa coupling } Y_t. \text{ In case such large NP contributions to the WCs are really observed in the future experiments at Belle II and the LHCb Upgrade, this could be the imprint of QFV SUSY (the MSSM with general QFV) and would encourage to perform further studies of the WCs } C_7'(\mu_b) \text{ and } C_7^{MSSM}(\mu_b) \text{ at higher order (NLO/NNLO) level in this model.}
1 Introduction

Our present knowledge of elementary particle physics is very successfully described by the Standard Model (SM) of electroweak and strong interactions. This model has, however, several essential problems, such as naturalness and hierarchy problems. Moreover, it cannot explain observed phenomena like the neutrino masses and mixings, the matter-antimatter asymmetry in our universe, and the origin of dark matter. Hence, it is necessary to search for New Physics (NP) theory that solves these problems. The theory of Supersymmetry (SUSY) is still the most prominent candidate for such a NP theory solving the SM problems.

Here we study the influence of SUSY on $C_7(\mu_b)$ and $C'_7(\mu_b)$ which are the Wilson coefficients (WCs) for the quark flavour changing transition $b \to s\gamma$ at the b-quark mass scale $\mu_b$. They are closely related to radiative $B$-meson decays. We calculate the SUSY-loop contributions to $C_7(\mu_b)$ and $C'_7(\mu_b)$ at leading order (LO) in the Minimal Supersymmetric Standard Model (MSSM) with general quark-flavour violation (QFV) due to squark generation mixing. In the numerical computation of the WCs, we perform a MSSM parameter scan respecting all the relevant theoretical and experimental constraints, such as those from vacuum stability conditions, those from $K$- and $B$-meson data, the 125 GeV Higgs boson data from LHC, and electroweak precision data, as well as recent limits on SUSY particle (sparticle) masses from LHC experiments.

On the experimental side, the WCs $C_7(\mu_b)$ and $C'_7(\mu_b)$ can be measured precisely in the ongoing and future experiments at Belle II and LHCb Upgrade [1–4]. There are many papers studying the radiative $B$-meson decays in the SM [5–12], the 2HDMs (Two-Higgs Doublet models) [13–15] and the MSSM [16–20].

However, there is no systematic numerical study on the SUSY-loop contributions to $C_7(\mu_b)$ and $C'_7(\mu_b)$ even at LO in the MSSM with general QFV [1]. In this paper we thoroughly perform such a systematic study with special emphasis on the importance of SUSY QFV in order to clarify a possibility that an imprint of SUSY can be found in radiative $B$-meson decays, focusing on the WCs $C_7(\mu_b)$ and $C'_7(\mu_b)$.

In the phenomenological study of the MSSM, usually quark-flavour conservation (QFC) is assumed, except for the quark-flavour violation stemming from the Cabibbo-Kobayashi-Maskawa (CKM) matrix. However, in general there can be SUSY QFV terms in the squark mass matrix. Especially important QFV terms are the mixing terms between the 2nd and the 3rd squark generations, such as $\tilde{c}_{L,R} - \tilde{t}_{L,R}$ and $\tilde{s}_{L,R} - \tilde{b}_{L,R}$ mixing terms, where $\tilde{c}$, $\tilde{t}$, $\tilde{s}$ and $\tilde{b}$ are the charm-, top-, strange- and bottom-squark, respectively. In this study we put special emphasis on the influence of the SUSY QFV due to $\tilde{c}_{L,R} - \tilde{t}_{L,R}$ and $\tilde{s}_{L,R} - \tilde{b}_{L,R}$ mixings on the WCs $C_7(\mu_b)$ and $C'_7(\mu_b)$.

\footnote{To our knowledge, there is no complete next to leading order (NLO) computation of WCs $C_7(\mu_b)$ and $C'_7(\mu_b)$ in the MSSM with general QFV in the present literature. In [21] gluino-squark loop contributions to the WCs $C_{7,8}(\mu_W)$ and $C'_{7,8}(\mu_W)$ at the weak scale $\mu_W$ are calculated at NLO of SUSY-QCD in the MSSM with general QFV. However, they did not perform a complete NLO computation of $C_7(\mu_b)$ and $C'_7(\mu_b)$. Here we remark that in [13,14] the charged Higgs boson loop contributions to the WCs $C_{7,8}(\mu_W)$ and $C_7(\mu_b)$ are calculated at NLO of QCD in the 2HDMs.}
In our analysis we assume that there is no SUSY lepton-flavour violation. We also assume that R-parity is conserved and that the lightest neutralino $\tilde{\chi}_0$ is the lightest SUSY particle (LSP). We work in the MSSM with real parameters, except for the CKM matrix.

In the following section we introduce the SUSY QFV parameters originating from the squark mass matrices. Details about our parameters scan are given in Section 3. In Section 4 we define the relevant WC's and analyze their behaviour in the MSSM with QFV. The conclusions are in Section 5. All relevant constraints are listed in Appendix A.

## 2 Squark mass matrices in the MSSM with flavour violation

In the super-CKM basis of $\tilde{q}_{\alpha\gamma} = (\tilde{q}_{1L}, \tilde{q}_{2L}, \tilde{q}_{3L}, \tilde{q}_{1R}, \tilde{q}_{2R}, \tilde{q}_{3R})$, $\gamma = 1, \ldots, 6$, with $(q_1, q_2, q_3) = (u, c, t)$, $(d, s, b)$, the up-type and down-type squark mass matrices $\mathcal{M}_{\tilde{u}}$, $\tilde{q} = \tilde{u}, \tilde{d}$, at the SUSY scale have the following most general $3 \times 3$ block form [22]:

$$
\mathcal{M}_{\tilde{q}} = \begin{pmatrix}
\mathcal{M}_{\tilde{q},LL} & \mathcal{M}_{\tilde{q},LR}
\mathcal{M}_{\tilde{q},RL} & \mathcal{M}_{\tilde{q},RR}
\end{pmatrix}, \quad \tilde{q} = \tilde{u}, \tilde{d}.
\tag{1}
$$

Non-zero off-diagonal terms of the $3 \times 3$ blocks $\mathcal{M}_{\tilde{u},LL}$, $\mathcal{M}_{\tilde{u},RR}$, $\mathcal{M}_{\tilde{d},LL}$ and $\mathcal{M}_{\tilde{d},RR}$ in Eq. (1) explicitly break quark-flavour in the squark sector of the MSSM. The left-left and right-right blocks in Eq. (1) are given by

$$
\mathcal{M}_{\tilde{u},LL}^2 = M_{u}^2 + D_{\tilde{u},LL} + \tilde{m}_u(d),
\tag{2}
\mathcal{M}_{\tilde{u},RR}^2 = M_{u}^2 + D_{\tilde{u},RR} + \tilde{m}_u(d),
\mathcal{M}_{\tilde{d},LL}^2 = M_{d}^2 + D_{\tilde{d},LL},
\mathcal{M}_{\tilde{d},RR}^2 = M_{d}^2 + D_{\tilde{d},RR} + \tilde{m}_d(d),
\mathcal{M}_{\tilde{d},RL}^2 = M_{d}^2 + D_{\tilde{d},RL} + \tilde{m}_d(d),
\mathcal{M}_{\tilde{d},LR}^2 = M_{d}^2 + D_{\tilde{d},LR} + \tilde{m}_d(d),
$$

where $M_{Q_{\alpha}}^2 = V_{\text{CKM}} M_{Q}^2 V_{\text{CKM}}^\dagger$, $M_{Q_{\alpha}}^2 \equiv M_{Q_{\alpha}}^2$, $M_{Q,U,D}$ are the hermitian soft SUSY-breaking mass matrices of the squarks, $D_{\tilde{u},LL}$, $D_{\tilde{d},RR}$ are the D-terms, and $\tilde{m}_u(d)$ are the diagonal mass matrices of the up(down)-type quarks. $M_{Q_{\alpha}}^2$ is related with $M_{Q_{\alpha}}^2$ by the CKM matrix $V_{\text{CKM}}$ due to the $SU(2)_L$ symmetry. The left-right and right-left blocks of Eq. (1) are given by

$$
\mathcal{M}_{\tilde{u},RL}^2 = \mathcal{M}_{\tilde{u},LR}^2 = \frac{v_2(v_1)}{\sqrt{2}} T_{U(D)} - \mu^* \tilde{m}_u(d) \cot \beta (\tan \beta),
\tag{3}
$$

where $T_{U,D}$ are the soft SUSY-breaking trilinear coupling matrices of the up-type and down-type squarks entering the Lagrangian

$$
\mathcal{L}_{\text{int}} \supset -(T_{U_{\alpha\beta}} \tilde{u}_{\alpha L}^\dagger \tilde{u}_{\beta L} H_2^0 + T_{D_{\alpha\beta}} \tilde{d}_{\alpha R}^\dagger \tilde{d}_{\beta R} H_1^0),
\mu \text{ is the higgsino mass parameter, and } \tan \beta = v_2/v_1 \text{ with } v_{1,2} = \sqrt{2} \langle H_{1,2}^0 \rangle.
\text{The squark mass matrices are diagonalized by the } 6 \times 6 \text{ unitary matrices } U_{\tilde{q}}, \tilde{q} = \tilde{u}, \tilde{d}, \text{ such that}
\end{equation}

$$
U_{\tilde{q}} \mathcal{M}_{\tilde{q}}(U_{\tilde{q}})^\dagger = \text{diag}(m_{\tilde{q}_1}^2, \ldots, m_{\tilde{q}_6}^2),
\tag{4}
\text{with } m_{\tilde{q}_1} < \cdots < m_{\tilde{q}_6}. \text{ The physical mass eigenstates } \tilde{q}_i, i = 1, \ldots, 6 \text{ are given by } \tilde{q}_i = U_{\tilde{q}} \tilde{q}_{i\alpha}, \tilde{q}_{0\alpha}.$$

4
In this paper we focus on the $\tilde{c}_L - \tilde{t}_L, \tilde{c}_R - \tilde{t}_R, \tilde{c}_R - \tilde{t}_L, \tilde{s}_L - \tilde{b}_L, \tilde{s}_R - \tilde{b}_R, \tilde{s}_R - \tilde{b}_L$, and $\tilde{s}_L - \tilde{b}_R$ mixing which is described by the QFV parameters $M_{Q_{u3}}^2 \approx M_{Q_{23}}^2, M_{U_{23}}^2, T_{U_{23}}, T_{U_{32}}, M_{D_{23}}^2, T_{D_{23}}$, and $T_{D_{32}}$, respectively. We will also often refer to the QFC parameter $T_{U_{33}}$ and $T_{D_{33}}$ which induces the $\tilde{t}_L - \tilde{t}_R$ and $\tilde{b}_L - \tilde{b}_R$ mixing, respectively, and plays an important role in this study.

The slepton parameters are defined analogously to the squark ones. All the parameters in this study are assumed to be real, except the CKM matrix $V_{CKM}$.

3 Parameter scan

In our MSSM-parameter scan we take into account theoretical constraints from vacuum stability conditions and experimental constraints from $K$- and $B$-meson data, the $H^0$ mass and coupling data and electroweak precision data, as well as limits on SUSY particle masses from recent LHC experiments (see Appendix A). Here $H^0$ is the discovered SM-like Higgs boson which we identify as the lightest $CP$ even neutral Higgs boson $h^0$ in the MSSM. Concerning squark generation mixings, we only consider the mixing between the second and third generation of squarks. The mixing between the first and the second generation squarks is strongly constrained by the $K$- and $D$-meson data [23, 24]. The experimental constraints on the mixing of the first and third generation squarks are not so strong [25], but we don’t consider this mixing since its effect is essentially similar to that of the mixing of the second and third generation squarks. We generate the input parameter points by using random numbers in the ranges shown in Table 1, where some parameters are fixed as given in the last box. All input parameters are $\overline{DR}$ parameters defined at scale $Q = 1$ TeV, except $m_A(pole)$ which is the pole mass of the $CP$ odd Higgs boson $A^0$. The parameters that are not shown explicitly are taken to be zero. The entire scan lies in the decoupling Higgs limit, i.e. in the scenarios with large $\tan \beta \geq 10$ and large $m_A \geq 1350$ GeV (see Table 1), respecting the fact that the discovered Higgs boson is SM-like. It is well known that the lightest MSSM Higgs boson $h^0$ is SM-like (including its couplings) in this limit. We don’t assume a GUT relation for the gaugino masses $M_1, M_2, M_3$.

All MSSM input parameters are taken as $\overline{DR}$ parameters at the scale $Q = 1$ TeV, except $m_A(pole)$, and then are transformed by RGEs to those at the weak scale of $Q = \mu_W$ for the computation of the WCs $C_{7,8}(\mu_W)$ and $C'_{7,8}(\mu_W)$ in the MSSM. The masses and rotation matrices of the sfermions are renormalized at one-loop level by using the public code SPheno-v3.3.8 [26, 27] based on the technique given in [28].

From 8660000 input points generated in the scan 72904 points survived all constraints. These are 0.84%. We show these survival points in all scatter plots in this article.
Table 1: Scanned ranges and fixed values of the MSSM parameters (in units of GeV or GeV$^2$, except for tan $\beta$). The parameters that are not shown explicitly are taken to be zero. $M_{1,2,3}$ are the U(1), SU(2), SU(3) gaugino mass parameters.

| tan $\beta$ | $M_1$ | $M_2$ | $M_3$ | $\mu$ | $m_A$(pole) |
|-------------|-------|-------|-------|-------|-------------|
| 10 ± 80     | 100 ± 2500 | 100 ± 2500 | 2500 ± 5000 | 100 ± 2500 | 1350 ± 6000 |
| $M_{Q_{22}}^2$ | $M_{Q_{33}}^2$ | $|M_{Q_{23}}^2|$ | $M_{U_{22}}^2$ | $M_{U_{33}}^2$ | $|M_{U_{23}}^2|$ |
| 2500$^2$ ± 4000$^2$ | 2500$^2$ ± 4000$^2$ | < 1000$^2$ | 1000$^2$ ± 4000$^2$ | 600$^2$ ± 3000$^2$ | < 2000$^2$ |
| $M_{D_{22}}^2$ | $M_{D_{33}}^2$ | $|M_{D_{23}}^2|$ | $|T_{U_{23}}|$ | $|T_{U_{32}}|$ | $|T_{U_{33}}|$ |
| 2500$^2$ ± 4000$^2$ | 1000$^2$ ± 3000$^2$ | < 2000$^2$ | < 4000 | < 4000 | < 5000 |
| | | | | | |
| < 3000 | < 3000 | < 4000 | < 500 |

| $M_{Q_{11}}^2$ | $M_{Q_{11}}^2$ | $M_{D_{11}}^2$ | $M_{D_{11}}^2$ | $M_{L_{22}}^2$ | $M_{L_{33}}^2$ | $M_{E_{11}}^2$ | $M_{E_{22}}^2$ | $M_{E_{33}}^2$ |
|-------------|-------|-------|-------|-------|-------|-------|-------|-------|
| 4500$^2$ | 4500$^2$ | 4500$^2$ | 1500$^2$ | 1500$^2$ | 1500$^2$ | 1500$^2$ | 1500$^2$ |

4 WCs $C_7(\mu_b)$ and $C'_7(\mu_b)$ in the MSSM with QFV

The effective Hamiltonian for the radiative transition $b \to s\gamma$ is given by

$$H_{eff} = \frac{-4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i O_i,$$

where $G_F$ is the Fermi constant and $V_{tb} V_{ts}^*$ is a CKM factor. The operators relevant to $b \to s\gamma$ are

$$O_2 = \bar{s}_L \gamma_\mu c_L \bar{c}_L \gamma^\mu b_L,$n
$$O_7 = \frac{e}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} b_R F_{\mu\nu},$$
$$O_8 = \frac{g_s}{16\pi^2} m_b \bar{s}_L \sigma^{\mu\nu} T^a b_R G_{\mu\nu}^a,$$

and their chirality counterparts

$$O'_2 = \bar{s}_R \gamma_\mu c_R \bar{c}_R \gamma^\mu b_R,$$
$$O'_7 = \frac{e}{16\pi^2} m_b \bar{s}_R \sigma^{\mu\nu} b_L F_{\mu\nu},$$
$$O'_8 = \frac{g_s}{16\pi^2} m_b \bar{s}_R \sigma^{\mu\nu} T^a b_L G_{\mu\nu}^a,$$

where $m_b$ is the bottom quark mass, $e$ and $g_s$ are the electromagnetic and strong coupling, $F_{\mu\nu}$ and $G_{\mu\nu}^a$ the $U(1)_{em}$ and $SU(3)_c$ field-strength tensors, $T^a$ are colour generators,
and the indices L,R denote the chirality of the quark fields. Here note that the SM contributions to $C'_{2,7,8}(\mu_W)$ are (almost) zero at LO. The WCs $C_7(\mu_b)$ and $C'_7(\mu_b)$ at the bottom quark mass scale $\mu_b$ can be measured precisely in the experiments at Belle II and LHCb Upgrade [1–4]. We compute $C_7(\mu_b)$ and $C'_7(\mu_b)$ at LO in the MSSM with QFV and study the deviation of the MSSM predictions from their SM ones. Following the standard procedure, first we compute $C_{7,8}(\mu_W)$ and $C'_{7,8}(\mu_W)$ at the weak scale $\mu_W$ at LO in the MSSM and then we compute $C_7(\mu_b)$ and $C'_7(\mu_b)$ by using the QCD RGEs for the

\[ P(b \to s\gamma) = \frac{|C'_7(\mu_b)|^2 - |C_7(\mu_b)|^2}{|C'_7(\mu_b)|^2 + |C_7(\mu_b)|^2}. \]  

(9)

In the SM $C'_7(\mu_b)$ is strongly suppressed by a factor $m_s/m_b$ and hence the photon in $b \to s\gamma$ decay is predominantly left-handed. In principle, the photon polarization can be extracted from the measurement of radiative $B$-meson decays in the experiments such as Belle II and LHCb Upgrade [1, 2, 29–38].
We take the NLO formula with 5 flavours for the strong coupling constant $\alpha$ In Ref. [39] it has been pointed out that the gluino contribution to the WCs $C_i$ scale evolution at leading log (LL) level [8]:

\[ C_7(\mu_b) = \eta \frac{16}{3} C_7(\mu_W) + \frac{8}{3}(\eta \frac{14}{23} - \eta \frac{16}{23}) C_8(\mu_W) + \sum_{i=1}^{8} h_i \eta^{a_i} C_2(\mu_W) \]

\[ C'_7(\mu_b) = \eta \frac{16}{3} C'_7(\mu_W) + \frac{8}{3}(\eta \frac{14}{23} - \eta \frac{16}{23}) C'_8(\mu_W) + \sum_{i=1}^{8} h_i \eta^{a_i} C'_2(\mu_W), \tag{10} \]

where

\[ \eta = \frac{\alpha_s(\mu_W)}{\alpha_s(\mu_b)} \]

\[ h_i = \left( \begin{array}{cccc}
\frac{626126}{272277} & -\frac{56281}{51730} & -\frac{3}{14} & -0.6494, -0.0380, -0.0186, -0.0057
\end{array} \right) \]

\[ a_i = \left( \begin{array}{cccc}
\frac{14}{23}, \frac{6}{23}, -\frac{12}{23}, 0.4086, -0.4230, -0.8994, 0.1456
\end{array} \right). \tag{11} \]

We take the NLO formula with 5 flavours for the strong coupling constant $\alpha_s(\mu)$ for $\mu_b \lesssim \mu \lesssim \mu_W$ [8]:

\[ \alpha_s(\mu) = \frac{\alpha_s(m_Z)}{v(\mu)} \left[ 1 - \frac{\beta_1}{\beta_0} \frac{\alpha_s(m_Z)}{4\pi} \frac{\ln(v(\mu))}{v(\mu)} \right], \tag{12} \]

where

\[ v(\mu) = 1 - \frac{\alpha_s(m_Z)}{2\pi} \frac{\ln(m_Z)}{\mu}, \tag{13} \]

---

Here we comment on the RG running of the WCs at LL level.

In footnote 5 of Ref. [18], it is argued as follows:

In Ref. [39] it has been pointed out that the gluino contribution to the WCs $C_7(\mu)$ is the sum of two different pieces, one proportional to the gluino mass and one proportional to the bottom mass, which have a different RG evolution (i.e. Eqs.(40) and (41) of [39], respectively). However, it has been found that at LO this is equivalent to the usual SM RG-evolution (i.e. Eqs.(13,14) of [18] which correspond to Eq.(10) of the present paper) once the running bottom mass $m_b(\mu_0)$ is used instead of the pole mass $m_b(\text{pole})$ in the WCs $C_i(\mu_0)$, where $\mu_0$ is the high-energy matching scale (e.g. the electroweak scale $\mu_W$).

We have also confirmed this point (fact) independently of Ref. [18]. Here, note that we have used the public code SPheno-v3.3.8 [26,27] in the computation of the WCs $C_7(\mu_0 = 160\text{GeV})$, and that SPheno-v3.3.8 uses the running b-quark mass $m_b(\mu_0 = 160\text{GeV})$ (not the pole mass $m_b(\text{pole})$) in the computation of the $C_7(\mu_0 = 160\text{GeV})$.

Therefore, Eqs.(40) and (41) of [39] are equivalent to the usual SM RG-evolution (i.e. Eq.(10) of the present paper) at LO.

Moreover, just after Eq.(41) in Ref. [39] it is clearly stated that the terms $R_{7b,\tilde{b}}(\mu_b)$ and $R_{8b,\tilde{b}}(\mu_b)$ turn out to be numerically very small with respect to the other terms on the right-hand sides of Eq.(41) for the RG running of the WCs. Here $R_{7b,\tilde{b}}(\mu_b)$ and $R_{8b,\tilde{b}}(\mu_b)$ are linear combinations of the WCs (such as $C_{7,8}(\mu_W)$, $e.g.$ $i=15,16,19,20$) of the additional four-quark operators in Eq.(15) of [39], all of which are operators at NLO of QCD. Hence, the effects of the additional four-quark operators onto the RG running of $C_7(\mu)$ are numerically very small.

Therefore, the contributions of the WCs of the new four-quark operators mentioned in [39] (which are all at NLO of QCD) to the RG scale evolution (RG running) are numerically very small and hence the presence of the mentioned new four-quark operators can not change Eq.(10) in the present paper practically (essentially).
Figure 1: The SM and MSSM one-loop contributions to the WCs $C_{7,8}(\mu_W)$ and $C'_{7,8}(\mu_W)$ at the weak scale $\mu_W$ for the transitions $b_R \rightarrow s_L \gamma_L, g_L$ and $b_L \rightarrow s_R \gamma_R, g_R$, respectively (see Eqs. (5, 6, 7)). Here $\gamma_L, g_L$ and $\gamma_R, g_R$ are left-handed photon, gluon and right-handed photon, gluon, respectively. The photon is emitted from any electrically charged line and the gluon from any colour charged line. For the SM one-loop contributions $(X, Y) = (t/c/u, W^+)$. For the MSSM one-loop contributions $(X, Y) = (\text{stop/scharm/sup}, \text{chargino}), (\text{sbottom/sstrange/sdown}, \text{gluino}), (\text{sbottom/sstrange/sdown}, \text{neutralino})$ and $(t/c/u, H^+)$, where stop/scharm/sup denotes top-, charm-, , up-squark mixtures and so on.

$\beta_0 = \frac{23}{3}, \beta_1 = \frac{116}{3}$ and $m_Z$ is the Z boson mass. We take $m_Z = 91.2$ GeV and $\alpha_s(m_Z) = 0.1179$ [24]. The SM and MSSM contribution to $C_2(\mu_W) = C'^{\text{SM}}_2(\mu_W) + C'^{\text{MSSM}}_2(\mu_W)$ is 1 and 0 at LO, respectively. The SM and MSSM contributions to $C'_2(\mu_W) = C'^{\text{SM}}_2(\mu_W) + C'^{\text{MSSM}}_2(\mu_W)$ are 0 at LO. In our numerical analysis, we take $\mu_W = 160$GeV and $\mu_b = 4.8$GeV [3].

We use the numerical results for $C_{7,8}(\mu_W)$ at LO in the MSSM obtained from the public code SPheno-v3.3.8 [26, 27], which takes into account the following one-loop contributions to $C_{7,8}(\mu_W)$ at the weak scale $\mu_W$ (see Fig. 1):

1) SM one-loop contributions:
   - up-type quark - $W^+$ loops

2) MSSM one-loop contributions:
   - up-type squark - chargino loops
   - down-type squark - gluino loops
   - down-type squark - neutralino loops
   - up-type quark - $H^+$ loops

Here the chargino $\tilde{\chi}_{1,2}^\pm$ is a mixture of charged wino $\tilde{W}^\pm$ and charged higgsino $\tilde{H}^\pm$, the neutralino $\tilde{\chi}_1^0$ is a mixture of photino $\tilde{\gamma}$, zino $\tilde{Z}$ and two neutral higgsinos $\tilde{H}_1^0$, and $H^+$ is the charged Higgs boson.
Figure 2: Schematic illustration of important parts of the up-type squark - chargino loop contributions to $C_{7,8}(\mu_W)$ in terms of the mass-insertion approximation.

Figure 3: Schematic illustration of an important part of the down-type squark - gluino loop contributions to (a) $C_{7,8}(\mu_W)$ and (b) $C'_{7,8}(\mu_W)$ in terms of the mass-insertion approximation.

Figure 4: Schematic illustration of the top quark - $H^+$ loop contribution to $C_{7,8}(\mu_W)$.

Before we show the results of the full parameter scan, we comment on the expected qualitative behavior of the MSSM one-loop contributions to $C_{7}^{\ell}(\mu_b)$ at the bottom mass scale $\mu_b$. We find that large squark trilinear couplings $T_{U23}^{T}$, $T_{U32}^{T}$, $T_{D23}^{T}$, large $M_{Q23}^2$, $M_{U23}^2$, $M_{D23}^2$, large bottom Yukawa coupling $Y_b$ for large $\tan \beta$, and large top Yukawa
coupling $Y_t$ can lead to large MSSM one-loop contributions to $C_{7,8}(\mu_W)$ at the weak scale $\mu_W$, which results in large MSSM one-loop contributions to $C_{7,s}(\mu_b)$ at the bottom mass scale $\mu_b$ (see Eq.(10)). This is mainly due to the following reasons:

- The lighter up-type squarks $\tilde{u}_{1,2,3}$ are strong $\tilde{c}_{L,R}$ - $\tilde{t}_{L,R}$ mixtures for large $M_{Q23}^2$, $M_{U23}^2$, $T_{23,32,33}$. The lighter down-type squarks $\tilde{d}_{1,2,3}$ are strong $\tilde{s}_{L,R}$ - $\tilde{b}_{L,R}$ mixtures for large $M_{Q23}^2$, $M_{D23}^2$, $T_{D23,32,33}$. Here note that $|T_{U23,32,33}|$ can be large due to large $Y_t$ (see Eqs.(14,16)) and that $|T_{D23,32,33}|$ can be large due to large $Y_b$ for large tan $\beta$ (see Eqs.(15,17)). In the following we assume these setups.

- As for the up-type squark - chargino loop contributions to $C_{7}(\mu_W)$ and $C_{8}(\mu_W)$ which is the effective coupling for the transition $b_R \rightarrow s_L \gamma$ and $b_R \rightarrow s_L g$, respectively;

  The $b_R$ - $\tilde{u}_{1,2,3}$ - $\tilde{\chi}_L^{\pm}$ vertex which contains the $b_R$ - $\tilde{t}_L$ - $\tilde{H}^{\pm}$ coupling can be enhanced by the large bottom Yukawa coupling $Y_b$ for large tan $\beta$. The $s_L$ - $\tilde{u}_{1,2,3}$ - $\tilde{\chi}^{\pm}$ vertex contains the $s_L$ - $\tilde{c}_L$ - $\tilde{\tilde{W}}^{\pm}$ coupling which is not CKM-suppressed. This vertex contains also the $s_L$ - $\tilde{t}_R$ - $\tilde{H}^{\pm}$ coupling which is enhanced by the large top Yukawa coupling $Y_t$ despite the suppression due to the CKM factor $V_{ts}^\ast$. Hence, the up-type squark - chargino loop contributions to $C_{7,s}(\mu_W)$ can be enhanced by the large $Y_b$ for large tan $\beta$ and the large $Y_t$, and further by the large $\tilde{c}_L$ - $\tilde{t}_L$ mixing term $M_{Q23}^2$ and the large $\tilde{t}_L$ - $\tilde{t}_R$ mixing term $T_{U33}$ for which $\tilde{u}_{1,2,3}$ contain a strong mixture of $\tilde{c}_L$, $\tilde{t}_L$, and $\tilde{t}_R$. Important parts of this squark - chargino loop contributions to $C_{7,s}(\mu_W)$ are schematically illustrated in terms of the mass-insertion approximation in Fig. 2.

- As for the down-type squark - gluino loop contributions to $C_{7,8}(\mu_W)$;

  The $b_R$ - $\tilde{d}_{1,2,3}$ - $\tilde{g}$ vertex which contains the $b_R$ - $\tilde{b}_R$ - $\tilde{g}$ coupling can be enhanced by the sizable QCD coupling. The $s_L$ - $\tilde{d}_{1,2,3}$ - $\tilde{g}$ vertex which contains the $s_L$ - $\tilde{s}_L$ - $\tilde{g}$ coupling can also be enhanced by the QCD coupling. Furthermore, absence of the CKM-suppression factor in this loop diagram results in additional strong enhancement. Therefore, the down-type squark - gluino loop contributions to $C_{7,8}(\mu_W)$ can be enhanced by the sizable QCD coupling, and further by the large $\tilde{b}_R$ - $\tilde{s}_L$ mixing term $T_{D32}$ for which $\tilde{d}_{1,2,3}$ contain a strong mixture of $\tilde{b}_R$ and $\tilde{s}_L$. Moreover, $|T_{D32}|$ can be large due to large $Y_b$ for large tan $\beta$ (see Eq.(17)). An important part of this squark - gluino loop contribution to $C_{7,8}(\mu_W)$ is schematically illustrated in terms of the mass-insertion approximation in Fig. 3(a).

- As for the down-type squark - neutralino loop contributions to $C_{7,8}(\mu_W)$;

  The $b_R$ - $\tilde{d}_{1,2,3}$ - $\tilde{\chi}^0_{1,2,3,4}$ vertex which contains the $b_R$ - $\tilde{b}_R$ - $\tilde{\gamma}/\tilde{Z}$ and $b_R$ - $\tilde{b}_L$ - $\tilde{H}^0_1$ couplings with the latter coupling being proportional to $Y_b$ can be enhanced by large $Y_b$ for large tan $\beta$. The $s_L$ - $\tilde{d}_{1,2,3}$ - $\tilde{\chi}^0_{1,2,3,4}$ vertex contains the $s_L$ - $\tilde{s}_L$ - $\tilde{\gamma}/\tilde{Z}$ couplings. The absence of the CKM-suppression factor in this loop diagram results in additional strong enhancement. Hence, the down-type squark - neutralino loop

\[4\text{Note that the CKM-suppression factor } V_{ts}^\ast \text{ is factored out from WCs } C_t \text{ in their definition (see Eq.(5)). Therefore, absence of the CKM-suppression factor in the one-loop diagram results in strong enhancement of the loop contribution to the WCs } C_t.\]
contributions to $C_{7,8}(\mu_W)$ can be enhanced by large $Y_b$ for large $\tan\beta$, and further by the large $\tilde{b}_R-\tilde{s}_L$ and $\tilde{b}_L-\tilde{s}_L$ mixing terms ($T_{D32}$ and $M_{Q23}^2$), for which $d_{1,2,3}$ contain a strong mixture of $\tilde{b}_R-\tilde{s}_L$ and $\tilde{b}_L-\tilde{s}_L$. Moreover, $|T_{D32}|$ controlled by $Y_b$ can be large for large $\tan\beta$ (see Eq.(17)).

- As for the up-type quark - $H^+$ loop contributions to $C_{7,8}(\mu_W)$; The $b_R- t - H^+$ vertex which contains the $b_R- t_L - H^+$ coupling can be enhanced by large $Y_b$ for large $\tan\beta$. The $s_L - t - H^+$ vertex which contains the $s_L - t_R - H^+$ coupling can be enhanced by the large top-quark Yukawa coupling $Y_t$ despite the suppression due to the CKM factor $V_{ts}^*$. Hence $t - H^+$ loop contributions to $C_{7,8}(\mu_W)$ can be enhanced by large $Y_b$ for large $\tan\beta$ and large $Y_t$. The top quark - $H^+$ loop contribution to $C_{7,8}(\mu_W)$ is schematically illustrated in Fig. 4.

- As for the up-type squark - chargino loop contributions to $C'_7(\mu_W)$ and $C'_8(\mu_W)$ which are the effective couplings for the transition $b_L \rightarrow s_R \gamma$ and $b_L \rightarrow s_R g$, respectively; From a similar argument one finds that these loop contributions to $C'_{7,8}(\mu_W)$ should be small due to the very small s-quark Yukawa coupling $Y_s$.

- As for the down-type squark - gluino loop contributions to $C'_{7,8}(\mu_W)$; The $b_L - \tilde{d}_{1,2,3} - \tilde{g}$ vertex which contains the $b_L - \tilde{b}_L - \tilde{g}$ coupling can be enhanced by the sizable QCD coupling. The $s_R - \tilde{d}_{1,2,3} - \tilde{g}$ vertex which contains the $s_R - \tilde{s}_R - \tilde{g}$ coupling can also be enhanced by the QCD coupling. Absence of the CKM-suppression factor in this loop diagram results in additional strong enhancement. Therefore, the down-type squark - gluino loop contributions to $C'_{7,8}(\mu_W)$ can be enhanced by the sizable QCD couplings, and further by large $\tilde{b}_L-\tilde{s}_R$ mixing term $T_{D23}$ for which $d_{1,2,3}$ contain a strong mixture of $\tilde{b}_L$ and $\tilde{s}_R$. Moreover, $|T_{D23}|$ can be large due to large $Y_b$ for large $\tan\beta$ (see Eq.(17)). An important part of this squark - gluino loop contribution to $C'_{7,8}(\mu_W)$ is schematically illustrated in terms of the mass-insertion approximation in Fig. 3(b).

- As for the down-type squark - neutralino loop contributions to $C'_{7,8}(\mu_W)$; The $b_L - \tilde{d}_{1,2,3} - \tilde{\chi}_{1,2,3,4}^0$ vertex which contains the $b_L - \tilde{b}_L - \tilde{\gamma}/\tilde{Z}$ and $b_L - \tilde{b}_R - \tilde{H}_1^0$ couplings with the latter coupling being proportional to $Y_b$ can be enhanced by large $Y_b$ for large $\tan\beta$. The $s_R - \tilde{d}_{1,2,3} - \tilde{\chi}_{1,2,3,4}^0$ vertex contains the $s_R - \tilde{s}_R - \tilde{\gamma}/\tilde{Z}$ coupling. Absence of the CKM-suppression factor in this loop diagram results in additional strong enhancement. Hence, the down-type squark - neutralino loop contributions to $C'_{7,8}(\mu_W)$ can be enhanced by large $Y_b$ for large $\tan\beta$, and further by large $\tilde{b}_L-\tilde{s}_R$ and $\tilde{b}_R-\tilde{s}_R$ mixing terms $T_{D23}$ and $M_{D23}^2$, for which $d_{1,2,3}$ contain strong mixtures of $\tilde{b}_L-\tilde{s}_R$ and $\tilde{b}_R-\tilde{s}_R$. Moreover, $|T_{D23}|$ controlled by $Y_b$ can be large for large $\tan\beta$ (see Eq.(17)).

- As for the up-type quark - $H^+$ loop contributions to $C'_{7,8}(\mu_W)$; These contributions turn out to be very small due to the very small $Y_s$. 
In the following we will show scatter plots in various planes obtained from the MSSM parameter scan described above (see Table 1), respecting all the relevant constraints (see Appendix A).

In Fig. 5 we show scatter plots for $C_7^{\text{MSSM}}(\mu_b)$ and $C_7'(\mu_b)$. In Fig. 5(a) we show a scatter plot in the $\text{Re}(C_7'(\mu_b))$-$\text{Im}(C_7'(\mu_b))$ plane. We see that the MSSM contribution to $\text{Re}(C_7'(\mu_b))$ can be as large as $\sim -0.07$, which could correspond to an about $4\sigma$ New Physics (NP) signal significance in the combination of the future LHCb Upgrade (Phase III) and Belle II (Phase II) experiments (see Figure A.13 of [3]). Note that $|\text{Im}(C_7'(\mu_b))|$ is very small ($\lesssim 0.004$) and that $C_7'(\mu_b) \approx 0$ in the SM.

In Fig. 5(b) we show the scatter plot in the $\text{Re}(C_7^{\text{MSSM}}(\mu_b))$-$\text{Im}(C_7^{\text{MSSM}}(\mu_b))$ plane. We see that the MSSM contribution to $\text{Re}(C_7(\mu_b))$ can be as large as $\sim -0.05$, which could correspond to an about $3\sigma$ NP signal significance in the combination of the future LHCb Upgrade (50 $fb^{-1}$) and Belle II (50 $ab^{-1}$) experiments (see Figure 8 of [3]). Note that $|\text{Im}(C_7^{\text{MSSM}}(\mu_b))|$ is very small ($\lesssim 0.003$) and that the MSSM contribution $C_7^{\text{MSSM}}(\mu_b)$ can be quite sizable compared to $C_7^{\text{SM}}(\mu_b) \approx -0.325$.

In Fig. 5(c) we show a scatter plot in the $\text{Re}(C_7^{\text{MSSM}}(\mu_b))$-$\text{Re}(C_7'(\mu_b))$ plane. We see that the $\text{Re}(C_7(\mu_b))$ and $\text{Re}(C_7^{\text{MSSM}}(\mu_b))$ can be quite sizable simultaneously.

Here we comment on the errors of the data on $C_7'(\mu_b)$ and $C_7^{NP}(\mu_b)$. The errors of the data on $C_7'(\mu_b)$ and $C_7^{NP}(\mu_b) \equiv C_7(\mu_b) - C_7^{\text{SM}}(\mu_b)$ from the future $B$-meson experiments shown in Figure A.13 and Figure 8 of [3] stem from experimental and theoretical errors. In general, $B$-meson observables are functions of the relevant WCs such as $C_7'(\mu_b)$ and $C_7(\mu_b)$. Hence, from the observed data on relevant $B$-meson observables one can determine (extract) the values of the WCs. The WCs thus determined (extracted) have two types of errors, one is the experimental error stemming from the (systematic and statistical) errors of the observable data and the other is the theoretical error due to the uncertainties of input parameters, such as the CKM matrix elements ($V_{ts}$, $V_{tb}$ ...), hadronic form factors and meson-decay constants, in the computation (prediction) of the observables by using the WCs (i.e. the effective couplings).

Here we remark the following points: (i) As for the determination of $C_7(\mu_b)$ one can get much more precise information from the fully-inclusive $B(B \to X_s\gamma)$ measurement than from the measurement of the exclusive observables such as $B(B \to K^*\gamma)$ since the theoretical predictions for the exclusive observables involve hadronic form factors which have large theoretical uncertainties. (ii) The fully-inclusive observable $B(B \to X_s\gamma)$ can be measured reliably and precisely at Belle II whereas its measurement is very difficult at LHCb. (iii) As a result, Belle II plays a specially important role in the precise determination (extraction) of $C_7(\mu_b)$ in the near future.

As for the experimental errors of the WCs $C_7'(\mu_b)$ and $C_7^{NP}(\mu_b)$ obtained (extracted) from the future $B$-meson experiments, Belle II is now planning to upgrade to accumulate about 5 times larger data (up to $\sim 250 ab^{-1}$) [40]. If this is realized, the (statistical) uncertainty of the observable data from Belle II could be reduced by a factor of about $\sqrt{5}$.

\footnote{Here note that $B(B \to X_s\gamma) \approx B(b \to s\gamma)$ is proportional to $|C_7(\mu_b)|^2 + |C_7'(\mu_b)|^2$ at LO.}
As for the theoretical errors of the WCs $C_7^r(\mu_b)$ and $C_7^{NP}(\mu_b)$ obtained (extracted) from the $B$-meson experiments, there is a sign of promising possibility of significant reduction of the theoretical errors in the future: Very recently M. Misiak et al. performed a new computation of $B(B \to X s \gamma)$ in the SM at the NNLO in QCD [12]. Taking into account the recently improved estimates of non-perturbative contributions, they have obtained $B(B \to X s \gamma) = (3.40 \pm 0.17) \cdot 10^{-4}$ for $E_c > 1.6 GeV$. Compared with their previous SM prediction $B(B \to X s \gamma) = (3.36 \pm 0.23) \cdot 10^{-4}$ [11], the theoretical uncertainty is now reduced from 6.8% to 5.0%. Note here that the Figure A.13 and Figure 8 of [3] showing expected errors of $C_7^r(\mu_b)$ and $C_7^{NP}(\mu_b)$ obtained (extracted) from the future $B$-meson experiments were made in the year 2017.

Hence, in case the significant reduction of the experimental and theoretical errors is achieved in the future, the NP signal significances for $Re(C_7^r(\mu_b))$ and $Re(C_7^{NP}(\mu_b))$ in the MSSM could be significantly higher than those mentioned above which are about $4 \sigma$ NP significances for $Re(C_7^r(\mu_b))$ and about $3 \sigma$ significance for $Re(C_7^{MSSM}(\mu_b))$.

Thus, it is very important to improve the precision of both theory and experiment on $B$-meson physics by a factor about 1.5 or so in view of NP search (such as SUSY search). Therefore, we strongly encourage theorists and experimentalists to challenge this task.

In Fig. 6 we show scatter plots in the $T_{U32}-Re(C_7^r(\mu_b))$, $T_{U32}-Re(C_7^r(\mu_b))$, $T_{U32}-Re(C_7^r(\mu_b))$, and $\tan \beta- Re(C_7^r(\mu_b))$ planes. From Fig. 6(a) we see that $Re(C_7^r(\mu_b)) \simeq Re(C_7^{MSSM}(\mu_b))$ can be sizable ($-0.07 \lesssim Re(C_7^r(\mu_b)) \lesssim 0.05$) for large $T_{U23}$ ($\gtrsim 3$ TeV). From Fig. 6(b) we see that $Re(C_7^r(\mu_b))$ can be large for large $|T_{U32}|$: $-0.07 \lesssim Re(C_7^r(\mu_b)) \lesssim 0.025$ for $T_{U32} \lesssim -2$ TeV and $-0.04 \lesssim Re(C_7^r(\mu_b)) \lesssim 0.045$ for $T_{U32} \gtrsim 2$ TeV. A significant correlation between $Re(C_7^r(\mu_b))$ and $T_{U32}$ can be seen. From Fig. 6(c) we see that $Re(C_7^r(\mu_b))$ can be large for large $|T_{U33}| \gtrsim 3$ TeV. The fewer scatter points around $T_{U33} = 3.5$ TeV is due to the fact that the $m_{\tilde{b}_{0}}$ bound tends to be violated around this point. From Fig. 6(d) we see that $Re(C_7^r(\mu_b))$ can be large for large $\tan \beta$: $-0.07 \lesssim Re(C_7^r(\mu_b)) \lesssim 0.05$ for $\tan \beta \gtrsim 40$. All of these features are consistent with our expectation from the argument above.

In Fig. 7 we show scatter plots in the $T_{D23}-Re(C_7^r(\mu_b))$, $T_{D32}-Re(C_7^r(\mu_b))$, and $T_{D33}-Re(C_7^r(\mu_b))$ planes. From Fig. 7(a) and Fig. 7(b) we see that $Re(C_7^r(\mu_b)) \simeq Re(C_7^{MSSM}(\mu_b))$ can be large ($-0.07 \lesssim Re(C_7^r(\mu_b)) \lesssim 0.05$) for large $T_{D23}$, $T_{D32}$ ($\lesssim -1.5$ TeV). An appreciable correlation between $T_{D23}$ and $Re(C_7^r(\mu_b))$ can be seen in Fig. 7(a). From Fig. 7(c) we see that it can be large for large $|T_{D33}| \gtrsim 2$ TeV. These behaviors are also consistent with our expectation.

In Fig. 8 we show scatter plots in the planes of $T_{U23}-Re(C_7^{MSSM}(\mu_b))$, $T_{U32}-Re(C_7^{MSSM}(\mu_b))$, and $\tan \beta- Re(C_7^{MSSM}(\mu_b))$. From Fig. 8(a) and Fig. 8(b) we see that $Re(C_7^{MSSM}(\mu_b))$ can be sizable (up to $\pm 0.05$) compared with $Re(C_7^{MSSM}(\mu_b)) \simeq -0.325$ for large $T_{U23}$ and $T_{U32}$ ($\gtrsim 2$ TeV). From Fig. 8(c) we see that it can be large for large $|T_{U33}|$: $-0.03 \lesssim Re(C_7^{MSSM}(\mu_b)) \lesssim 0.045$ for $T_{U33} \lesssim -2$ TeV and $-0.05 \lesssim Re(C_7^{MSSM}(\mu_b)) \lesssim 0.035$ for $T_{U33} \gtrsim 2$ TeV. There is a significant correlation between $Re(C_7^{MSSM}(\mu_b))$ and $T_{U33}$, which can be explained partly by the important contribution of Fig. 2(b) (see Eq.(10)). The fewer scatter points around $T_{U33} = 3.5$ TeV is
Figure 5: The scatter plot of the scanned parameter points within the ranges given in Table 1 in the planes of (a) $\text{Re}(C'_7(\mu_b)) - \text{Im}(C'_7(\mu_b))$, (b) $\text{Re}(C_{7}^{\text{MSSM}}(\mu_b)) - \text{Im}(C_{7}^{\text{MSSM}}(\mu_b))$, and (c) $\text{Re}(C_{7}^{\text{MSSM}}(\mu_b)) - \text{Re}(C'_7(\mu_b))$.

again due to the fact that the $m_{h^0}$ bound tends to be violated around this point. From Fig. 8(d) we see that it can be large (up to $\sim \pm 0.05$) for large $\tan \beta$ ($\gtrsim 40$). These behaviors are also consistent with our expectation.

In Fig. 9 we show scatter plots in the $T_{D23} - \text{Re}(C_{7}^{\text{MSSM}}(\mu_b))$ plane. We see $\text{Re}(C_{7}^{\text{MSSM}}(\mu_b))$
Figure 6: The scatter plots of the scanned parameter points within the ranges given in Table 1 in the planes of (a) $T_{U23} - \text{Re}(C_7'(\mu_b))$, (b) $T_{U32} - \text{Re}(C_7'(\mu_b))$, (c) $T_{U33} - \text{Re}(C_7'(\mu_b))$ and (d) $\tan \beta - \text{Re}(C_7'(\mu_b))$.

can be sizable (up to $\sim \pm 0.05$) for any values of $T_{D23}$. We have found that scatter plots in the $T_{D32} - \text{Re}(C_7^{\text{MSSM}}(\mu_b))$ and $T_{D33} - \text{Re}(C_7^{\text{MSSM}}(\mu_b))$ planes have similar behavior to that in the $T_{D23} - \text{Re}(C_7^{\text{MSSM}}(\mu_b))$ plane.
Figure 7: The scatter plots of the scanned parameter points within the ranges given in Table 1 in the planes of (a) $T_{D23} - \text{Re}(C_7^u(\mu_b))$, (b) $T_{D32} - \text{Re}(C_7^u(\mu_b))$ and (c) $T_{D33} - \text{Re}(C_7^u(\mu_b))$.

In order to see the relevant parameter dependences of $\text{Re}(C_7^\text{MSSM}(\mu_b))$ and $\text{Re}(C_7^u(\mu_b))$ in more detail, we take a reference scenario P1 where we have sizable $\text{Re}(C_7^\text{MSSM}(\mu_b))$ and $\text{Re}(C_7^u(\mu_b))$ and then variate the relevant parameters around this point P1. All MSSM input parameters for P1 are shown in Table 2, where one has $(\text{Re}(C_7(\mu_b)), \text{Im}(C_7(\mu_b))) = (-0.370, -5.13 \cdot 10^{-4})$, $(\text{Re}(C_7^\text{SM}(\mu_b)), \text{Im}(C_7^\text{SM}(\mu_b))) = (-0.325, 5.63 \cdot 10^{-7})$. 
Figure 8: The scatter plot of the scanned parameter points within the ranges given in Table 1 in the planes of (a) $T_{U23} - \text{Re}(C_7^{\text{MSSM}}(\mu_b))$, (b) $T_{U32} - \text{Re}(C_7^{\text{MSSM}}(\mu_b))$, (c) $T_{U33} - \text{Re}(C_7^{\text{MSSM}}(\mu_b))$ and (d) $\tan \beta - \text{Re}(C_7^{\text{MSSM}}(\mu_b))$.

The scenario P1 satisfies all present experimental and theoretical constraints, see Appendix A. The resulting physical masses of the particles are shown in Table 3. The

$$(\text{Re}(C_7^{\text{MSSM}}(\mu_b)), \text{Im}(C_7^{\text{MSSM}}(\mu_b)) = (-0.0441, -5.13 \cdot 10^{-4})$$ and $$(\text{Re}(C_7'(\mu_b)), \text{Im}(C_7'(\mu_b)) = (-0.0472, -1.81 \cdot 10^{-3})$$ with $C_7(\mu_b) = C_7^{\text{SM}}(\mu_b) + C_7^{\text{MSSM}}(\mu_b)$. The scenario P1 satisfies all present experimental and theoretical constraints, see Appendix A. The resulting physical masses of the particles are shown in Table 3. The
We obtain \( \kappa \) level in the MSSM with QFV by using the code developed by us [41]. For the coupling \(|\kappa|\) width \( \Gamma(B) \) and \( K \) meson observables we get; – about -0.07 in the allowed region. We also see that it is large (quickly with the increase of \( T \)).

In Figs. 10 and 11 we show contours of \( \text{Re}(C_7^{\text{MSSM}}(\mu_b)) \) around the benchmark point P1 in various parameter planes. Fig. 10(a) shows contours of \( \text{Re}(C_7'(\mu_b)) \) in the \( T_{U23}-T_{U32} \) plane. We see that \( \text{Re}(C_7'(\mu_b)) \) is sensitive to both \( T_{U23} \) and \( T_{U32} \), especially to \( T_{U23} \), increases quickly with the increase of \( T_{U23} \) and \( T_{U32}(< 0) \), as is expected, and can be as large as about -0.07 in the allowed region. We also see that it is large (-0.07 \( \lesssim \text{Re}(C_7'(\mu_b)) \lesssim -0.04 \) respecting all the constraints in a significant part of this parameter plane. From Fig. 10(b) we see that \( \text{Re}(C_7'(\mu_b)) \) is also fairly sensitive to \( T_{U33} \) and can be as large as \( \sim -0.08 \). From Fig. 10(c) we find that \( \text{Re}(C_7'(\mu_b)) \) is very sensitive to \( \tan \beta \), especially for

flavour decompositions of the lighter squarks \( \tilde{u}_{1,2,3} \) and \( \tilde{d}_{1,2,3} \) are shown in Table 4. For the calculation of the masses and the mixing, as well as for the low-energy observables, especially those in the B and K meson sectors (see Table 5), we use the public code SPheno v3.3.8 [26, 27]. For the calculation of the coupling modifier \( \kappa_b = C(h^0b\bar{b})/C(h^0b\bar{b})_{\text{SM}} \) (or equivalently the deviation \( \text{DEV}(b) \equiv \Gamma(h^0 \to b\bar{b})/\Gamma(h^0 \to b\bar{b})_{\text{SM}} - 1 (= \kappa_0^2 - 1) \) of the width \( \Gamma(h^0 \to b\bar{b}) \) from its SM value) we compute the width \( \Gamma(h^0 \to b\bar{b}) \) at full one-loop level in the MSSM with QFV by using the code developed by us [41]. For the coupling modifier \( \kappa_x = C(h^0xx)/C(h^0xx)_{\text{SM}} \) with \( x = g \) or \( \gamma \) (or the deviation \( \text{DEV}(x) \equiv \Gamma(h^0 \to xx)/\Gamma(h^0 \to xx)_{\text{SM}} - 1 \)) we compute the width \( \Gamma(h^0 \to xx) \) according to [42]. We obtain \( \kappa_0 = 1.03 \) (or \( \text{DEV}(b) = 0.0686 \)), \( \kappa_g = 0.994 \) (or \( \text{DEV}(g) = -0.0120 \)) and \( \kappa_\gamma = 1.0018 \) (or \( \text{DEV}(\gamma) = 0.0036 \)), which satisfy the LHC data in Table 5. For the B and K meson observables we get; \( B(b \to s\gamma) = 3.764 \cdot 10^{-4} \), \( B(b \to sl^+l^-) = 1.589 \cdot 10^{-6} \), \( B(B_s \to \mu^+\mu^-) = 2.5930 \cdot 10^{-9} \), \( B(B_s \to \tau^+\nu) = 9.942 \cdot 10^{-5} \), \( \Delta M_{Bs} = 17.180[ps^{-1}] \), \( |\epsilon_K| = 2.201 \cdot 10^{-3} \), \( \Delta M_K = 2.304 \cdot 10^{-15} \) (GeV), \( B(K_L^0 \to \pi^0\nu\bar{\nu}) = 2.295 \cdot 10^{-11} \), and \( B(K^+ \to \pi^+\nu\bar{\nu}) = 7.771 \cdot 10^{-11} \), all of which satisfy the constraints of Table 5.

Figure 9: The scatter plot of the scanned parameter points within the ranges given in Table 1 in the \( T_{D23}\text{-Re}(C_7^{\text{MSSM}}(\mu_b)) \) plane.

Figure 10: (a) Contours of \( \text{Re}(C_7'(\mu_b)) \) in the \( T_{U23}-T_{U32} \) plane. (b) Contours of \( \text{Re}(C_7'(\mu_b)) \) in the \( T_{U33}-T_{U32} \) plane. (c) Contours of \( \text{Re}(C_7'(\mu_b)) \) in the \( T_{U23}-T_{U33} \) plane.
Table 2: The MSSM parameters for the reference point P1 (in units of GeV or GeV$^2$ expect for $\tan\beta$). All parameters are defined at scale $Q = 1$ TeV, except $m_A(\text{pole})$. The parameters that are not shown here are taken to be zero.

| $\tan\beta$ | $M_1$ | $M_2$ | $M_3$ | $\mu$ | $m_A(\text{pole})$ |
|--------------|-------|-------|-------|-------|------------------|
| 70           | 910   | 1970  | 2795  | 800   | 4970             |
| $M^2_{Q_{22}}$ | $M^2_{Q_{33}}$ | $M^2_{Q_{23}}$ | $M^2_{U_{22}}$ | $M^2_{U_{33}}$ | $M^2_{U_{23}}$ |
| 3630$^2$     | 3365$^2$ | -740$^2$ | 2755$^2$ | 1510$^2$ | -1705$^2$ |
| $M^2_{D_{22}}$ | $M^2_{D_{33}}$ | $M^2_{D_{23}}$ | $T_{U_{23}}$ | $T_{U_{32}}$ | $T_{U_{33}}$ |
| 2985$^2$     | 1270$^2$ | -1820$^2$ | 2700  | -260  | 4995             |
| $T_{D_{23}}$ | $T_{D_{32}}$ | $T_{D_{33}}$ | $T_{E_{33}}$ |       |                 |
| -2330        | -335  | 3675  | -335  |       |                 |

| $M^2_{Q_{11}}$ | $M^2_{U_{11}}$ | $M^2_{D_{11}}$ | $M^2_{L_{11}}$ | $M^2_{L_{22}}$ | $M^2_{L_{33}}$ | $M^2_{E_{11}}$ | $M^2_{E_{22}}$ | $M^2_{E_{33}}$ |
|---------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|--------------|
| 4500$^2$      | 4500$^2$     | 4500$^2$     | 1500$^2$     | 1500$^2$     | 1500$^2$     | 1500$^2$     | 1500$^2$     | 1500$^2$     |

large $T_{U_{23}} > 0$, as expected, and can be as large as $\sim -0.07$. As can be seen in Fig. 10(d), Re($C_7^L(\mu_b)$) is sensitive to $M^2_{U_{23}}$, especially for large $T_{U_{23}} \gtrsim 2.5$ TeV, as expected, and is large ($-0.08 \lesssim \text{Re}(C_7^L(\mu_b)) \lesssim -0.04$) respecting all the constraints in a significant part of this parameter plane. Fig. 11(a) shows contours of $\text{Re}(C_7^L(\mu_b))$ in the $T_{D_{23}}$-$T_{D_{32}}$ plane. It is fairly sensitive to $T_{D_{23}}$ and mildly dependent on $T_{D_{32}}$ as is expected partly from the contribution of Fig. 3(b) (see Eq.(10)), can be as large as $\sim -0.06$ in the allowed region, and is large ($-0.058 \lesssim \text{Re}(C_7^L(\mu_b)) \lesssim -0.046$) respecting all the constraints in a significant part of this parameter plane. From Fig. 11(b) we see that Re($C_7^L(\mu_b)$) is also rather sensitive to $T_{D_{33}}$ and can be as large as $\sim -0.06$ in the allowed region. As can be seen in Fig. 11(c), Re($C_7^L(\mu_b)$) is very sensitive to $\tan\beta$ and also sensitive to $T_{D_{23}}$ for large $\tan\beta \gtrsim 70$, as expected, and is sizable ($-0.05 \lesssim \text{Re}(C_7^L(\mu_b)) \lesssim -0.04$) respecting all the constraints in a significant part of this parameter plane. From Fig. 11(d) we find that Re($C_7^L(\mu_b)$) is very sensitive to $M^2_{D_{23}}$, and is sizable ($-0.05 \lesssim \text{Re}(C_7^L(\mu_b)) \lesssim -0.04$) respecting all the constraints in a significant part of this parameter plane.

In Figs. 12 and 13 we show contour plots of $\text{Re}(C_7^{\text{MSSM}}(\mu_b))$ (i.e. the MSSM contributions to Re($C_7^L(\mu_b)$)) around the benchmark point P1 in various parameter planes. Fig. 12(a) shows contours of $\text{Re}(C_7^{\text{MSSM}}(\mu_b))$ in the $T_{U_{23}}$-$T_{U_{32}}$ plane. We see that Re($C_7^{\text{MSSM}}(\mu_b)$) is sensitive to $T_{U_{23}}$ and $T_{U_{32}}$: $|\text{Re}(C_7^{\text{MSSM}}(\mu_b))|$ quickly increases with the increase of $T_{U_{23}}$ and $T_{U_{32}}$ as is expected. We find also that Re($C_7^{\text{MSSM}}(\mu_b)$) can be as large as about -0.05 in the allowed region and is sizable ($-0.05 \lesssim \text{Re}(C_7^{\text{MSSM}}(\mu_b)) \lesssim -0.04$) respecting all the constraints in a significant part of this parameter plane. From Fig. 12(b) we see that Re($C_7^{\text{MSSM}}(\mu_b)$) is very sensitive also to $T_{U_{33}}$ (see Fig. 8(c) also), quickly in-
Table 3: Physical masses in GeV of the particles for the scenario of Table 2.

|          | $m_{\tilde{\chi}_1}^0$ | $m_{\tilde{\chi}_2}^0$ | $m_{\tilde{\chi}_3}^0$ | $m_{\tilde{\chi}_4}^0$ | $m_{\tilde{\chi}_1}^+$ | $m_{\tilde{\chi}_2}^+$ |
|----------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|--------------------------|
|          | 800                      | 812                      | 925                      | 2030                     | 809                      | 2030                     |

|          | $m_{h^0}$ | $m_{H^0}$ | $m_{A^0}$ | $m_{H^+}$ |
|----------|-----------|-----------|-----------|-----------|
|          | 124.9     | 4970      | 4970      | 4997      |

|          | $m_{\tilde{u}}$ | $m_{\tilde{u}_1}$ | $m_{\tilde{u}_2}$ | $m_{\tilde{u}_3}$ | $m_{\tilde{u}_4}$ | $m_{\tilde{u}_5}$ | $m_{\tilde{u}_6}$ |
|----------|-----------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|          | 2934            | 1231              | 2986              | 3431              | 3656              | 4491              | 4493              |

|          | $m_{\tilde{d}}$ | $m_{\tilde{d}_1}$ | $m_{\tilde{d}_2}$ | $m_{\tilde{d}_3}$ | $m_{\tilde{d}_4}$ | $m_{\tilde{d}_5}$ | $m_{\tilde{d}_6}$ |
|----------|-----------------|-------------------|-------------------|-------------------|-------------------|-------------------|-------------------|
|          | 836             | 3272              | 3416              | 3654              | 4489              | 4492              |

|          | $m_{\tilde{\nu}}$ | $m_{\tilde{\nu}_1}$ | $m_{\tilde{\nu}_2}$ | $m_{\tilde{\nu}_3}$ | $m_{\tilde{\nu}_4}$ | $m_{\tilde{\nu}_5}$ | $m_{\tilde{\nu}_6}$ |
|----------|-------------------|----------------------|----------------------|----------------------|----------------------|----------------------|----------------------|
|          | 1506              | 1507                 | 1582                 | 1495                 | 1496                 | 1509                 | 1564                 | 1652                 |

Table 4: Flavour decompositions of the mass eigenstates $\tilde{u}_{1,2,3}$ and $\tilde{d}_{1,2,3}$ for the scenario of Table 2. Shown are the expansion coefficients of the mass eigenstates in terms of the flavour eigenstates. Imaginary parts of the coefficients are negligibly small.

|          | $\tilde{u}_L$ | $\tilde{c}_L$ | $t_L$ | $\tilde{u}_R$ | $\tilde{c}_R$ | $t_R$ |
|----------|---------------|---------------|-------|---------------|---------------|-------|
| $\tilde{u}_1$ | 0             | 0.0016        | 0.0992| 0             | -0.4090       | -0.9071|
| $\tilde{u}_2$ | -0.012        | -0.0070       | -0.0225| 0             | 0.9104        | -0.4130|
| $\tilde{u}_3$ | 0.0660        | 0.2921        | 0.9491| 0             | 0.0607        | 0.0770|

|          | $d_L$ | $\tilde{s}_L$ | $b_L$ | $d_R$ | $\tilde{s}_R$ | $b_R$ |
|----------|-------|---------------|-------|-------|---------------|-------|
| $d_1$    | 0     | 0             | 0.0059| 0     | 0.4057        | 0.9140|
| $d_2$    | 0     | 0.0059        | 0.0289| 0     | -0.9137       | 0.4054|
| $d_3$    | 0     | 0.2898        | 0.9566| 0     | 0.0245        | -0.0172|

Increases with increase of $T_{U33}$ as is expected partly from the important contribution of Fig. 2(b) (see Eq.(10)), and can be as large as about -0.05 in the allowed region. It is sizable ($-0.05 \lesssim Re(C^{{\text{MSSM}}}_7(\mu_b)) \lesssim -0.04$) respecting all the constraints in a significant part of this parameter plane. From Fig. 12(c) we find that $Re(C^{{\text{MSSM}}}_7(\mu_b))$ is very sensitive to $\tan\beta$ and $T_{U23}$ as expected, quickly increases with increase of $\tan\beta$ and $T_{U23}(>0)$, and can be as large as $\sim -0.05$ in the allowed region. As can be seen in Fig. 12(d), $Re(C^{{\text{MSSM}}}_7(\mu_b))$ is sensitive to $M^2_{U23}$ and $T_{U23}$ increasing with the increase of $M^2_{U23}(<0)$ and $T_{U23}(>0)$ as expected, and is large ($-0.05 \lesssim Re(C^{{\text{MSSM}}}_7(\mu_b)) \lesssim -0.04$) respecting all the constraints in a significant part of this parameter plane.
Fig. 13(a) shows contours of $Re(C_7^\text{MSSM}(\mu_b))$ in the $T_{D23}$-$T_{D32}$ plane. We see that $Re(C_7^\text{MSSM}(\mu_b))$ is mildly dependent on $T_{D23}$ and fairly sensitive to $T_{D32}$ around P1 as is expected partly from the contribution of Fig. 3(a) (see Eq. (10)). It can be as large as about -0.046 in the allowed region, and is sizable ($-0.046 \lesssim Re(C_7^\text{MSSM}(\mu_b)) \lesssim -0.044$) respecting all the constraints in a significant part of this parameter plane. From Fig. 13(b) we see that $Re(C_7^\text{MSSM}(\mu_b))$ is also fairly sensitive to $T_{D33}$ around P1, can be as large as about -0.045 in the allowed region, and is sizable ($-0.045 \lesssim Re(C_7^\text{MSSM}(\mu_b)) \lesssim -0.044$) respecting all the constraints in a significant part of this parameter plane. From Fig. 13(c) we find that $Re(C_7^\text{MSSM}(\mu_b))$ is very sensitive to $\tan\beta$ quickly increasing with the increase of $\tan\beta$ as expected, can be as large as $\sim -0.05$ in the allowed region, and is sizable ($-0.05 \lesssim Re(C_7^\text{MSSM}(\mu_b)) \lesssim -0.04$) respecting all the constraints in a significant part of this parameter plane. As can be seen in Fig. 13(d), $Re(C_7^\text{MSSM}(\mu_b))$ is mildly dependent on $M_2^2$ around this benchmark point P1, can be as large as about -0.044 in the allowed region, and is sizable ($-0.044 \lesssim Re(C_7^\text{MSSM}(\mu_b)) \lesssim -0.043$) respecting all the constraints in a significant part of this parameter plane.

As the gluino is very heavy ($\sim 3$ TeV) around the reference point P1 (see Table 3), the down-type squark - gluino loop contributions to $Re(C_7^\text{MSSM}(\mu_b))$ and $Re(C_7'^\text{MSSM}(\mu_b))$ are suppressed there, which partly explains the rather mild dependences of $Re(C_7'(\mu_b))$ and $Re(C_7'^\text{MSSM}(\mu_b))$ on the down-type squark parameters $T_{D23}$, $T_{D32}$, $T_{D33}$ around P1 as is seen in Fig. 11 and Fig. 13, respectively.

Before closing this section we comment on the renormalization scale dependence of the WCs $C_7^\text{MSSM}(\mu_b)$ and $C_7'(\mu_b)$. For the reference scenario P1 we have the following result at LO:

\[
(Re(C_7(\mu_b/2)), Im(C_7(\mu_b/2))) = (-0.405, -4.04 \cdot 10^{-4} ), \quad (Re(C_7^\text{MSSM}(\mu_b/2)), Im(C_7^\text{MSSM}(\mu_b/2))) = (-0.0379, -4.04 \cdot 10^{-4} ) \quad \text{and} \quad (Re(C_7'(\mu_b/2)), Im(C_7'(\mu_b/2))) = (-0.0350, -1.34 \cdot 10^{-3} ); \quad (Re(C_7(2\mu_b)), Im(C_7(2\mu_b))) = (-0.341, -6.19 \cdot 10^{-4} ) \quad \text{at} \quad (Re(C_7^\text{MSSM}(2\mu_b)), Im(C_7^\text{MSSM}(2\mu_b))) = (-0.0499, -6.20 \cdot 10^{-4} ) \quad \text{and} \quad (Re(C_7'(2\mu_b)), Im(C_7'(2\mu_b))) = (-0.0594, -2.28 \cdot 10^{-3} ),
\]

where $\mu_b = 4.8$ GeV. We see that the scale dependence of the WCs at the b-quark mass scale is significant at LO in agreement with Refs. [5–7] and hence that it is important to compute the WCs at higher order (NLO/NNLO) level in order to reduce this scale-dependence uncertainties. In [21] MSSM loop contributions to the WCs $C_7,(\mu_W)$ and $C_7'(\mu_W)$ are calculated at NLO in the MSSM with QFV. So far, however, there is no complete NLO computation of the WCs $C_7(\mu_b)$ and $C_7'(\mu_b)$ in the MSSM with QFV.

---

\footnote{In principle the MSSM loop contributions to $C_7(\mu_b)$ and $C_7'(\mu_b)$ at NLO can be obtained from $C_i(\mu_W)$ and $C_i'(\mu_W)$ ($i = 1-8$) calculated at NLO in the MSSM by using QCD RG scale evolution from the scale $\mu_W$ down to $\mu_b$ at NLL (next-to-leading log) level \[7\], where $C_i(\mu_W)$ and $C_i'(\mu_W)$ ($i = 1-6$) are the Wilson coefficients of the four-quark operators.}
Figure 10: Contour plots of $Re(C'_7(\mu_b))$ around the benchmark point P1 in the parameter planes of (a) $T_{U23} - T_{U32}$, (b) $T_{U23} - T_{U33}$, (c) $T_{U23} - \tan \beta$, and (d) $T_{U23} - M^2_{U23}$. The parameters other than the shown ones in each plane are fixed as in Table 2. The "X" marks P1 in the plots. The red hatched region satisfies all the constraints in Appendix A. The red solid lines, the blue dashed lines, the red dashed lines and the blue dash-dotted lines show the $m_{h^0}$ bound, the $B(b \to s\gamma)$ bound, the $B(B_s \to \mu^+\mu^-)$ bound, and the $m_{\tilde{d}_1}$ bound, respectively.
Figure 11: Contour plots of $Re(C'_7(\mu_b))$ around the benchmark point P1 in the parameter planes of (a) $T_{D23} - T_{D32}$, (b) $T_{D23} - T_{D33}$, (c) $T_{D23} - \tan\beta$, and (d) $T_{D23} - M_{D23}^2$. The parameters other than the shown ones in each plane are fixed as in Table 2. The "X" marks P1 in the plots. The red hatched region satisfies all the constraints in Appendix A. The definitions of the bound lines are the same as in Fig. 10. In addition to these the blue solid lines and the green solid lines show the $\Delta M_{B_s}$ bound and the vacuum stability bound on $T_{D23}$, respectively.
Figure 12: Contour plots of $Re(C_7^{\text{MSSM}}(\mu_b))$ around the benchmark point P1 in the parameter planes of (a) $T_{U23} - T_{U32}$, (b) $T_{U23} - T_{U33}$, (c) $T_{U23} - \tan\beta$, and (d) $T_{U23} - M_{U23}^2$. The parameters other than the shown ones in each plane are fixed as in Table 2. The "X" marks P1 in the plots. The red hatched region satisfies all the constraints in Appendix A. The definitions of the bound lines are the same as those in Fig. 10.
Figure 13: Contour plots of $Re(C_7^{\text{MSSM}}(\mu_b))$ around the benchmark point P1 in the parameter planes of (a) $T_{D23} - T_{D32}$, (b) $T_{D23} - T_{D33}$, (c) $T_{D23} - \tan\beta$, and (d) $T_{D23} - M^2_{D23}$. The parameters other than the shown ones in each plane are fixed as in Table 2. The "X" marks P1 in the plots. The red hatched region satisfies all the constraints in Appendix A. The definitions of the bound lines are the same as those in Fig. 11.

5 Conclusions

We have studied SUSY effects on $C_7(\mu_b)$ and $C'_7(\mu_b)$ which are the Wilson coefficients for $b \rightarrow s\gamma$ at $b$-quark mass scale $\mu_b$ and are closely related to radiative $B$-meson decays. The
SUSY-loop contributions to the \( C_7(\mu_b) \) and \( C'_7(\mu_b) \) are calculated at LO in the Minimal Supersymmetric Standard Model with general quark-flavour violation. For the first time we have performed a systematic MSSM parameter scan for the WCs \( C_7(\mu_b) \) and \( C'_7(\mu_b) \) respecting all the relevant constraints, i.e. the theoretical constraints from vacuum stability conditions and the experimental constraints, such as those from \( K^- \) and \( B^- \)-meson data and electroweak precision data, as well as recent limits on SUSY particle masses and the 125 GeV Higgs boson data from LHC experiments. From the parameter scan, we have found the following:

- The MSSM contribution to \( \text{Re}(C_7(\mu_b)) \) can be as large as \( \sim \pm 0.05 \) which could correspond to about 3\( \sigma \) significance of NP (New Physics) signal in future Belle II and LHCb Upgrade experiments.

- The MSSM contribution to \( \text{Re}(C'_7(\mu_b)) \) can be as large as \( \sim -0.08 \) which could correspond to about 4\( \sigma \) significance of NP signal in future Belle II and LHCb Upgrade experiments.

- These large MSSM contributions to the WCs are mainly due to (i) large scharm-stop mixing and large scharm/stop involved trilinear couplings \( T_{U23}, T_{U32} \) and \( T_{U33} \), (ii) large strange-bottom mixing and large strange-bottom involved trilinear couplings \( T_{D23}, T_{D32} \) and \( T_{D33} \), and (iii) large bottom Yukawa coupling \( Y_b \) for large \( \tan \beta \) and large top Yukawa coupling \( Y_t \).

Moreover, we have pointed out the following:

- It is very important to reduce the (theoretical and experimental) errors of the WCs \( C'_7(\mu_b) \) and \( C^{NP}_7(\mu_b) \) obtained (extracted) from the future experiments at Belle II and the LHCb Upgrade. An improvement in precision of both theory and experiment by a factor about 1.5 or so would be very important in view of NP search (such as SUSY search). Therefore, we strongly encourage theorists and experimentalists to challenge this task.

- On the other hand, it is also very important to reduce the theoretical errors of the MSSM contributions to the WCs \( C'_7(\mu_b) \) and \( C_7(\mu_b) \) by performing higher order computations such as those at NLO/NNLO level.

In case such large New Physics contributions to the WCs, i.e. such large deviations of the WCs from their SM values, are really observed in the future experiments at Belle II and the LHCb Upgrade, this could be the imprint of QFV SUSY (the MSSM with general QFV) and would encourage to perform further studies of the WCs \( C'_7(\mu_b) \) and \( C^{\text{MSSM}}_7(\mu_b) \) at NLO/NNLO level in this model.
Acknowledgments

We would like to thank W. Porod for helpful discussions, especially for the permanent support concerning SPheno. VRVis is funded by BMVIT, BMDW, Styria, SFG and Vienna Business Agency in the scope of COMET - Competence Centers for Excellent Technologies (854174) which is managed by FFG.

A Theoretical and experimental constraints

The experimental and theoretical constraints taken into account in the present work are discussed in detail in [43]. Here we list the updated constraints from $K$- and $B$-physics and those on the Higgs boson mass and couplings in Table 5. For the mass of the Higgs boson $h^0$, taking the combination of the ATLAS and CMS measurements $m_{h^0} = 125.09 \pm 0.24$ GeV [52] and adding the theoretical uncertainty of $\sim \pm 3$ GeV [53] linearly to the experimental uncertainty at 2$\sigma$, we take $m_{h^0} = 125.09 \pm 3.48$ GeV. The $h^0$ couplings that receive SUSY QFV effects significantly are $C(hbb)$ [41], $C(hcc)$ [56], $C(hgg)$ and $C(h\gamma\gamma)$ [42]. The measurement of $C(hcc)$ is very difficult due to huge QCD backgrounds at LHC; there is no significant experimental data on $C(hcc)$ at this moment. Hence, the relevant $h^0$ couplings to be compared with the LHC observations are $C(hbb)$, $C(hgg)$ and $C(h\gamma\gamma)$. Therefore, we list the LHC data on $C(hbb)$ ($\kappa_b$), $C(hgg)$ ($\kappa_g$) and $C(h\gamma\gamma)$ ($\kappa_\gamma$) in Table 5.

As the constraints from the decays $B \to D^{(*)} \tau \nu$ are unclear due to large theoretical uncertainties [56], we don’t take these constraints into account in our paper. As the issues of possible anomalies of $R(D^{(*)}) = B(B \to D^{(*)} \tau \nu)/B(B \to D^{(*)} \ell \nu)$ with $\ell = e$ or $\mu$ and $R_{K^{(*)}} = B(B \to K^{(*)} e^+ e^-)/B(B \to K^{(*)} \mu^+ \mu^-)$ are not yet settled [45,49], we don’t take the constraints from these ratios into account either. In [25] the QFV decays $t \to qh^0$ with $q = u, c$, have been studied in the general MSSM with QFV. It is found that these decays cannot be visible at the current and high luminosity LHC runs due to the very small decay branching ratios $B(t \to qh^0)$, giving no significant constraint on the $\tilde{c} - t$ mixing.

We comment on the very recent data on the anomalous magnetic moment of muon $a_\mu$ from the Fermilab experiment [59]. The Fermilab data has been combined with the previous BNL data [60] resulting in 4.2$\sigma$ discrepancy between the experimental data and the

7Precisely speaking, in principle, $C(htt)$ coupling could also receive SUSY QFV effects significantly. However, predicting the (effective) coupling $C(htt)$ at loop levels in the MSSM is very difficult since its theoretical definition in the context of $t\bar{t}h$ production at LHC is unclear [57].

8As pointed out in [58], the theoretical predictions (in the SM and MSSM) on $B(B \to Dl\nu)$ and $B(B \to D^*l\nu)$ ($l = \tau, \mu, e$) have potentially large theoretical uncertainties due to the theoretical assumptions on the form factors at the $BDW^+$ and $BD^*W^+$ vertices (also at the $BDH^+$ and $BD^*H^+$ vertices in the MSSM). Hence the constraints from these decays are unclear.
SM prediction\(^9\). In our scenario with heavy sleptons/sneutrinos with masses of about 1.5 TeV the MSSM loop contributions to \(a_\mu\) are so small that they cannot explain the discrepancy between the new data and the SM prediction. Therefore, in the context of our scenario, this discrepancy should be explained by the loop contributions of another new physics coexisting with SUSY.

In addition to these we also require our scenarios to be consistent with the following experimental constraints:

Table 5: Constraints on the MSSM parameters from the \(K\) - and \(B\)-meson data relevant mainly for the mixing between the second and the third generations of squarks and from the data on the \(h^0\) mass and couplings \(\kappa_b, \kappa_g, \kappa_\gamma\). The fourth column shows constraints at 95% CL obtained by combining the experimental error quadratically with the theoretical uncertainty, except for \(B(K^0_L \to \pi^0\nu\bar{\nu}), m_{h^0}\) and \(\kappa_{b,g,\gamma}\).

| Observable | Exp. data | Theor. uncertainty | Constr. (95%CL) |
|------------|-----------|-------------------|-----------------|
| \(10^3 \times |c_K|\) | 2.228 ± 0.011 (68% CL) [24] | ±0.28 (68% CL) [44] | 2.228 ± 0.549 |
| \(10^3 \times \Delta M_K\) [GeV] | 3.484 ± 0.006 (68% CL) [24] | ±1.2 (68% CL) [44] | 3.484 ± 2.352 |
| \(10^3 \times B(K^0_L \to \pi^0\nu\bar{\nu})\) | < 3.0 (90% CL) [24] | ±0.002 (68% CL) [24] | < 3.0 (90% CL) |
| \(10^{10} \times B(K^+ \to \pi^+\nu\bar{\nu})\) | 1.7 ± 1.1 (68% CL) [24] | ±0.04 (68% CL) [24] | 1.7 ± 2.16 |
| \(\Delta M_{B_s}\) [ps\(^{-1}\)] | 17.757 ± 0.021 (68% CL) [24,45] | ±2.7 (68% CL) [46] | 17.757 ± 5.29 |
| \(B(b \to s\gamma)\) | 3.32 ± 0.15 (68% CL) [24,45] | ±0.23 (68% CL) [11] | 3.32 ± 0.54 |
| \(B(b \to s l^+l^-)\) | 1.60 \(\pm 0.45\) (68% CL) [47] | ±0.11 (68% CL) [48] | 1.60 ± 0.97 |
| \((l = e\) or \(\mu)\) | 10^9\times B(B_s \to \mu^+\mu^-) | 2.69 \(\pm 0.37\) (68% CL) [49] | ±0.23 (68% CL) [50] | 2.69 \(\pm 0.85\) |
| \(|m_{h^0}|\) [GeV] | 1.06 ± 0.19 (68% CL) [45] | ±0.29 (68% CL) [51] | 1.06 ± 0.69 |
| \(\kappa_b\) | 125.09 ± 0.24 (68% CL) [52] | ±3 [53] | 125.09 ± 3.48 |
| \(\kappa_g\) | 1.06 \(\pm 0.37\) (95% CL) [54] | 1.17 \(\pm 0.35\) (95% CL) [55] | 1.06 \(\pm 0.37\) (ATLAS) |
| \(\kappa_\gamma\) | 1.03 \(\pm 0.12\) (95% CL) [54] | 1.18 \(\pm 0.31\) (95% CL) [55] | 1.03 \(\pm 0.12\) (ATLAS) |

- The LHC limits on sparticle masses (at 95% CL) [62–66]:

We impose conservative limits for safety though actual limits are somewhat weaker than those shown here. In the context of simplified models, gluino masses \(m_{\tilde{g}} \lesssim\)

---

\(^9\)It is worth noting that according to the recent computation of the leading order hadronic vacuum polarization contribution to \(a_\mu\) using lattice QCD [61], the discrepancy between the experimental data and the SM prediction is only about 1.6 \(\sigma\).
2.35 TeV are excluded for $m_{\tilde{q}_f} < 1.55$ TeV. There is no gluino mass limit for $m_{\tilde{\chi}_1^0} > 1.55$ TeV. The 8-fold degenerate first two generation squark masses are excluded below 1.92 TeV for $m_{\tilde{q}_f} < 0.9$ TeV. There is no limit on the masses for $m_{\tilde{\chi}_1^0} > 0.9$ TeV. We impose this squark mass limit on $m_{\tilde{u}_1}$ and $m_{\tilde{d}_1}$. Bottom-squark masses are excluded below 1.26 TeV for $m_{\tilde{\chi}_1^0} < 0.73$ TeV. There is no bottom-squark mass limit for $m_{\tilde{\chi}_1^0} > 0.73$ TeV. Here the bottom-squark mass means the lighter sbottom mass $m_{\tilde{b}_1}$. We impose this limit on $m_{\tilde{d}_1}$ since $d_1 \sim b_R$ (see Table 4). A typical top-squark mass lower limit is $\sim 1.26$ TeV for $m_{\tilde{\chi}_1^0} < 0.62$ TeV. There is no top-squark mass limit for $m_{\tilde{\chi}_1^0} > 0.62$ TeV. Here the top-squark mass means the lighter stop mass $m_{\tilde{t}_1}$. We impose this limit on $m_{\tilde{u}_1}$ since $u_1 \sim t_R$ (see Table 4). For sleptons/sneutrinos heavier than the lighter chargino $\tilde{\chi}_1^\pm$ and the second neutralino $\tilde{\chi}_2^0$, the mass limits are $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^0} > 0.74$ TeV for $m_{\tilde{\chi}_1^0} \lesssim 0.3$ TeV and there is no $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^0}$ limits for $m_{\tilde{\chi}_1^0} > 0.3$ TeV; For sleptons/sneutrinos lighter than $\tilde{\chi}_1^\pm$ and $\tilde{\chi}_2^0$, the mass limits are $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^0} > 1.15$ TeV for $m_{\tilde{\chi}_1^0} \lesssim 0.72$ TeV and there is no $m_{\tilde{\chi}_1^\pm}, m_{\tilde{\chi}_2^0}$ limits for $m_{\tilde{\chi}_1^0} > 0.72$ TeV. For mass degenerate selectrons $\tilde{e}_L,R$ and smuons $\tilde{\mu}_{L,R}$, masses below 0.7 TeV are excluded for $m_{\chi_1^0} < 0.41$ TeV. For mass degenerate staus $\tilde{\tau}_L$ and $\tilde{\tau}_R$, masses below 0.39 TeV are excluded for $m_{\tilde{\chi}_1^0} < 0.14$ TeV. There is no sneutrino $\tilde{\nu}$ mass limit from LHC yet. Sneutrino masses below 94 GeV are excluded by LEP200 experiment [24].

- The constraint on $(m_{A^0,H^+}, \tan \beta)$ (at 95% CL) from searches for the MSSM Higgs bosons $H^0$, $A^0$ and $H^+$ at LHC, [67–73], where $H^0$ is the heavier CP-even Higgs boson.

- The experimental limit on SUSY contributions on the electroweak $\rho$ parameter [74]:

$$\Delta \rho \text{ (SUSY)} < 0.0012.$$ 

Furthermore, we impose the following theoretical constraints from the vacuum stability conditions for the trilinear coupling matrices [75]:

$$|T_{U_{\alpha\alpha}}|^2 < 3 Y_{U_a}^2 (M_{Q_{\alpha\alpha}}^2 + M_{U_{\alpha\alpha}}^2 + m_2^2),$$  

$$|T_{D_{\alpha\alpha}}|^2 < 3 Y_{D_a}^2 (M_{Q_{\alpha\alpha}}^2 + M_{D_{\alpha\alpha}}^2 + m_2^2),$$  

$$|T_{U_{\alpha\beta}}|^2 < Y_{U_{\gamma}}^2 (M_{Q_{\beta\beta}}^2 + M_{U_{\alpha\alpha}}^2 + m_2^2),$$  

$$|T_{D_{\alpha\beta}}|^2 < Y_{D_{\gamma}}^2 (M_{Q_{\beta\beta}}^2 + M_{D_{\alpha\alpha}}^2 + m_2^2),$$

where $\alpha, \beta = 1, 2, 3, \alpha \neq \beta$; $\gamma = \text{Max}(\alpha, \beta)$ and $m_2^2 = (m_{H_+}^2 + m_Z^2 \sin^2 \theta_W) \sin^2 \beta - \frac{1}{2} m_2^2$, $m_1^2 = (m_{H_+}^2 + m_Z^2 \sin^2 \theta_W) \cos^2 \beta - \frac{1}{2} m_2^2$. The Yukawa couplings of the up-type and down-type quarks are $Y_{U_{\alpha}} = \sqrt{2} m_{u_{\alpha}}/v_2 = \frac{g}{\sqrt{2}} m_{u_{\alpha}}/m_W \sin \beta$ ($u_{\alpha} = u, c, t$) and $Y_{D_{\alpha}} = \sqrt{2} m_{d_{\alpha}}/v_1 = \frac{g}{\sqrt{2}} m_{d_{\alpha}}/m_W \cos \beta$ ($d_{\alpha} = d, s, b$), with $m_{u_{\alpha}}$ and $m_{d_{\alpha}}$ being the running quark masses at the scale $Q = 1$ TeV and $g$ being the SU(2) gauge coupling. All soft SUSY-breaking parameters are given at $Q = 1$ TeV. As SM parameters we take $m_Z = 91.2$ GeV and the on-shell top-quark mass $m_t = 172.9$ GeV [24].
References

[1] E. Kou et al. (Belle II collaboration), The Belle II Physics Book, PTEP 2019 (2019) 12, 123C01, PTEP 2020 (2020) 2, 029201 (erratum) [arXiv:1808.10567 [hep-ex]].

[2] R. Aaij et al. (LHCb collaboration), "Physics case for an LHCb Upgrade II - Opportunities in flavour physics, and beyond, in the HL-LHC era", arXiv:1808.08865 [hep-ex].

[3] J. Albrecht, F. Bernlochner, M. Kenzie, S. Reichert, D. Straub and A. Tully, "Future prospects for exploring present day anomalies in flavour physics measurements with Belle II and LHCb", arXiv:1709.10308 [hep-ph].

[4] A. Paul and D.M. Straub, JHEP 04 (2017) 027 [arXiv:1608.02556 [hep-ph]].

[5] A. Ali and C. Greub, Phys. Lett. B293 (1992) 226.

[6] A. Ali, C. Greub and T. Mannel, "Rare B decays in the Standard Model", in Proc. ECFA Workshop on B-Meson Factory, edited by R. Aleksan and A. Ali (DESY, Hamburg, 1993).

[7] A. J. Buras, M. Misiak, M. Muenz and S. Pokorski, Nucl. Phys. B424 (1994) 374 [arXiv:hep-ph/9311345].

[8] K. Chetyrkin, M. Misiak and M. Munz, Phys. Lett. B400 (1997) 206 [Erratum-ibid. B425 (1998) 414] [arXiv:hep-ph/9612313].

[9] A.L. Kagan and M. Neubert, Eur.Phys.J. C7 (1999) 5 [arXiv:hep-ph/9805303].

[10] A. J. Buras, "Climbing NLO and NNLO Summits of Weak Decays", arXiv:1102.5650 [hep-ph] and references therein.

[11] M. Misiak et al., Phys. Rev. Lett. 114 (2015) 221801 [arXiv:1503.01789[hep-ph]].

[12] M. Misiak, A. Rehman and M. Steinhauser, JHEP 06 (2020) 175 [arXiv:2002.01548 [hep-ph]].

[13] F. Borzumati and C. Greub, Phys. Rev. D 58 (1998) 074004 [arXiv:hep-ph/9802391].

[14] M. Ciuchini, G. Degrassi, P. Gambino and G. F. Giudice, Nucl. Phys. B 527 (1998) 21 [arXiv:hep-ph/9710335].

[15] K. Kiers, A. Soni and Guo-Hong Wu, Phys. Rev. D62 (2000) 116004 [arXiv:hep-ph/0006280].

[16] T. Goto, Y. Okada, and Y. Shimizu, Phys. Rev. D58 (1998) 094006 [arXiv:hep-ph/9804294].
[17] T. Besmer, C. Greub and T. Hurth, Nucl. Phys. B609 (2001) 359 [arXiv:hep-ph/0105292].

[18] L. Everett, G.L. Kane, S. Rigolin, L.T. Wang and T.T. Wang, JHEP 01 (2002) 022 [arXiv:hep-ph/0112126].

[19] T. Hurth, E. Lunghi and W. Porod, Nucl. Phys. B704 (2005) 56 [arXiv:hep-ph/0312260].

[20] E. Lunghi and J. Matias, JHEP 04 (2007) 058 [arXiv:hep-ph/0612166].

[21] C. Greub, T. Hurth, V. Pilipp, C. Schüpbach and M. Steinhauser, Nucl. Phys. B 853 (2011) 240 [arXiv:1105.1330 [hep-ph]].

[22] B. C. Allanach et al., Comput. Phys. Commun. 180 (2009) 8 [arXiv:0801.0045 [hep-ph]].

[23] F. Gabbiani, E. Gabrielli, A. Masiero and L. Silvestrini, Nucl. Phys. B 477 (1996) 321 [arXiv:hep-ph/9604387].

[24] P.A. Zyla et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020).

[25] A. Dedes et al., JHEP 11 (2014) 137 [arXiv:1409.6546 [hep-ph]].

[26] W. Porod, Comput. Phys. Commun. 153 (2003) 275 [arXiv:hep-ph/0301101].

[27] W. Porod and F. Staub, Comput. Phys. Commun. 183 (2012) 2458 [arXiv:1104.1573 [hep-ph]].

[28] D. M. Pierce et al., Nucl. Phys. B 491 (1997) 3.

[29] I. S. Choi, S. Y. Choi and H. S. Song, Phys. Rev. D 41 (1990) 1695.

[30] Y. Grossman and D. Pirjol, JHEP 06 (2000) 029 [arXiv:hep-ph/0005069].

[31] D. Becirevic and E. Schneider, Nucl. Phys.B854 (2012) 321 [arXiv:1106.3283 [hep-ph]].

[32] R. Aaij et al. (LHCb collaboration), JHEP 04 (2015) 064 [arXiv:1501.03038 [hep-ex]].

[33] D. Atwood, M. Gronau and A. Soni, Phys. Rev. Lett. 79 (1997) 185 [arXiv:hep-ph/9704272]; D. Atwood, T. Gershon, M. Hazumi and A. Soni, Phys. Rev. D71 (2005) 076003 [arXiv:hep-ph/0410036].

[34] Y. Ushiroda et al. (Belle Collaboration), Phys. Rev. D74 (2006) 111104 [arXiv:hep-ex/0608017].

[35] B. Aubert et al. (BABAR Collaboration), Phys. Rev. D78 (2008) 071102 [arXiv:0807.3103 [hep-ex]].

32
[36] F. Muheim, Y. Xie and R. Zwicky, Phys. Lett. B664 (2008) 174 [arXiv:0802.0876 [hep-ph]].

[37] R. Aaij et al. (LHCb collaboration), Phys. Rev. Lett. 118 (2017) 021801 [arXiv:1609.02032 [hep-ex]].

[38] R. Aaij et al. (LHCb collaboration), JHEP 12 (2020) 081 [arXiv:2010.06011 [hep-ex]].

[39] F. Borzumati, C. Greub, T. Hurth and D. Wyler, Phys. Rev. D 62 (2000) 075005 [arXiv:hep-ph/9911245].

[40] T. Browder, "SuperKEKB/Belle II upgrades", talk at Snowmass Community Planning Meeting - Virtual, 5-8 October 2020: https://indico.fnal.gov/event/44870/contributions/199272/

[41] H. Eberl, E. Ginina, A. Bartl, K. Hidaka and W. Majerotto, JHEP 06 (2016) 143 [arXiv:1604.02366 [hep-ph]].

[42] H. Eberl, K. Hidaka and E. Ginina, Int. J. Mod. Phys. A34 (2019) 1950120 [arXiv:1812.08010 [hep-ph]].

[43] H. Eberl, E. Ginina, K. Hidaka, Euro Physical Journal C77 (2017) 189 [arXiv:1702.00348 [hep-ph]].

[44] J. Brod and M. Gorbahn, Phys. Rev. Lett. 108 (2012) 121801 [arXiv:1108.2036 [hep-ph]].

[45] Y. Amhis et al. (Heavy Flavor Averaging Group (HFLAV)), Eur. Phys. J. C 81 (2021) 226 [arXiv:1909.12524[hep-ex]].

[46] T. Jubb, M. Kirk, A. Lenz, and G. Tetlalmatzi-Xolocotzi, Nucl. Phys. B915 (2017) 431 [arXiv:1603.07770 [hep-ph]]; M. Artuso, G. Borissov, and A. Lenz, Rev. Mod. Phys. 88 (2016) 045002 [arXiv:1511.09466 [hep-ph]].

[47] J. P. Lees et al. [BABAR Collaboration], Phys. Rev. Lett. 112 (2014) 211802 [arXiv:1312.5364 [hep-ex]].

[48] T. Huber, T. Hurth and E. Lunghi, Nucl. Phys. B 802 (2008) 40 [arXiv:0712.3009 [hep-ph]].

[49] Y. Amhis, Proceedings of the 40th International Conference on High Energy Physics, virtual conference (ICHEP2020), 2020, Prague, Czech Republic, PoS ICHEP2020 (2021), Proceedings of Science, Trieste.

[50] C. Bobeth et al., Phys. Rev. Lett. 112 (2014) 101801 [arXiv:1311.0903 [hep-ph]].

[51] J. M. Roney, Proceedings of the 26th International Symposium on Lepton Photon Interactions at High Energies, San Francisco, USA, 2013, Int. J. Mod. Phys. A 29 (2014) 22, World Scientific, Singapore.
[52] ATLAS and CMS collaborations, Phys. Rev. Lett. 114 (2015) 191803 [arXiv:1503.07589 [hep-ex]].

[53] S. Borowka, T. Hahn, S. Heinemeyer, G. Heinrich and W. Hollik, Eur. Phys. J. C75 (2015) 424 [arXiv:1505.03133 [hep-ph]].

[54] ATLAS Collaboration, Phys. Rev. D 101 (2020) 012002 [arXiv:1909.02845 [hep-ex]].

[55] CMS Collaboration, Eur. Phys. J. C 79 (2019) 421 [arXiv:1809.10733 [hep-ex]].

[56] A. Bartl, H. Eberl, E. Ginina, K. Hidaka and W. Majerotto, Phys. Rev. D 91 (2015) 015007 [arXiv:1411.2840 [hep-ph]].

[57] P. Wu et al., Phys. Lett. B618 (2005) 209 [arXiv:hep-ph/0505086 [hep-ph]]; S. Dittmaier et al., Phys. Rev. D90 (2014) 035010 [arXiv:1406.5307 [hep-ph]].

[58] U. Nierste, S. Trine and S. Westhoff, Phys. Rev. D 78 (2008) 015006 [arXiv:0801.4938 [hep-ph]]; see also references therein.

[59] Muon g-2 Collaboration, Phys. Rev. Lett. 126 (2021) 141801 [arXiv:2104.03281[hep-ex]].

[60] G. Bennett et al. (Muon g - 2 Collaboration), Phys. Rev. D73 (2006) 072003 [arXiv:hep-ex/0602035[hep-ex]].

[61] S. Borsanyi et al., Nature 593 (2021) 51 [arXiv:2002.12347[hep-lat]].

[62] F. Moortgat, Proceedings of the 29th International Symposium on Lepton Photon Interactions at High Energies (LP2019), Toronto, Canada, 2019, PoS(LeptonPhoton2019), Proceedings of Science, Trieste.

[63] C. Botta, Proceedings of the 40th International Conference on High Energy Physics, virtual conference (ICHEP2020), 2020, Prague, Czech Republic, PoS ICHEP2020 (2021), Proceedings of Science, Trieste; S. Alderweireldt, Proceedings of the 40th International Conference on High Energy Physics, virtual conference (ICHEP2020), 2020, Prague, Czech Republic, PoS ICHEP2020 (2021), Proceedings of Science, Trieste.

[64] ATLAS Collaboration, ATLAS PUB Note, “SUSY May 2020 Summary Plot Update”, ATL-PHYS-PUB-2020-013; See also the following Web Page: “Summary plots from the ATLAS Supersymmetry physics group” https://atlas.web.cern.ch/Atlas/GROUPS/PHYSICS/CombinedSummaryPlots/SUSY/

[65] See the following Web Page: ”Run 2 Summary plots – 13 TeV” https://twiki.cern.ch/twiki/bin/view/CMSPublic/PhysicsResultsSUSY#Run_2_Summary_plots_13_TeV

[66] ATLAS Collaboration, JHEP 02 (2021) 143 [arXiv:2010.14293 [hep-ex]].
[67] ATLAS Collaboration, Phys. Rev. Lett. 125 (2020) 051801 [arXiv:2002.12223 [hep-ex]].

[68] CMS Collaboration, JHEP 09 (2018) 007 [arXiv:1803.06553 [hep-ex]].

[69] ATLAS Collaboration, JHEP 11 (2018) 085 [arXiv:1808.03599 [hep-ex]].

[70] ATLAS Collaboration, JHEP 06 (2021) 145 [arXiv:2102.10076 [hep-ex]].

[71] ATLAS Collaboration, JHEP 09 (2018) 139 [arXiv:1807.07915 [hep-ex]].

[72] CMS Collaboration, JHEP 01 (2020) 096 [arXiv:1908.09206 [hep-ex]].

[73] CMS Collaboration, JHEP 07 (2019) 142 [arXiv:1903.04560 [hep-ex]].

[74] G. Altarelli, R. Barbieri and F. Caravaglios, Int. J. Mod. Phys. A 13 (1998) 1031 [hep-ph/9712368].

[75] J. A. Casas and S. Dimopoulos, Phys. Lett. B 387 (1996) 107 [arXiv:hep-ph/9606237].