SOLVING THE FACILITY LOCATION AND FIXED CHARGE SOLID TRANSPORTATION PROBLEM

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Abstract. In this paper, a new variant of the Solid Transportation Problem (STP) that incorporates both facility location and Fixed Charge Solid Transportation Problem (FCSTP) is presented with significant applications in logistics. It integrates decisions of diverse planning horizons: operational, tactical and strategic. The problem is termed Fixed Charge Solid Location and Transportation Problem (FCSLTP). Benchmark data obtained from the literature was extended for experimentation purposes. Solution to the FCSLTP was obtained using CPLEX commercial optimization solver. A Lagrange Relaxation Heuristic (LRH) was developed as an alternative solution for users not possibly having access to CPLEX. We further define an equivalent FCSLTP in the main paper and termed this as FCSTP-EQ. The FCSTP-EQ was compared to our FCSLTP to investigate possible cost savings with both formulations. Results obtained showed CPLEX outperforming the Lagrange relaxation heuristic developed both in the upper bound and lower bound generation for the problem sizes considered. Additionally, the cost savings obtained using the FCSLTP was consistently better than the FCSTP-EQ. The upper bound generation capability of Lagrange relaxation could possibly be improved by using better search methods such as metaheuristics. Under certain conditions, the FCSTP could feasibly be used as a starting solution to solve the FCSLTP.

1. Introduction. A transportation problem from source to destination which takes into account conveyance or transport capacities during distribution decisions has been termed the solid transportation problem. The Solid Transportation Problem (STP), described by [15] as multi-index simple transportation problem and solved using an extended transportation solution model, has lately been receiving attention from several authors. Recent application of the solid transportation problem has been creating a shipment plan that guides operational managers on the number of products to move from production or warehouse locations while selecting from different transportation sources to decrease the total of fixed and variable transportation costs. Researchers have continued to explore distribution problems such as this, and most especially those that involve merging various individual optimization problems. These optimization decisions could range from operational,

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i.e. involving day to day business decisions of fulfilling customer orders, to tactical policies of establishing distribution channels, and then the more strategic decisions such as facility location. When these optimization problems are holistically considered, they result in an integrated distribution problem with different planning horizons, requiring a global solution in order to avoid suboptimal solutions.

Facility Location Problem (FLP) is quite strategic in distribution planning and basically involves selecting, from a set of possible locations, the best locations from which to ship out products (or to offer services) to customers. This is an important decision in modern-day logistic planning. One of the basic forms of plant location models (simple plant location problem) as reported by [34, 2] has been transformed into complex variants today. An extension to the simple plant location model was done by [1, 17]. They considered capacitated plant location models of the multi-objective, multi-product type and large scale single-source capacitated models. Several authors have since researched the FLP. [20, 8] researched discrete facility location models providing both exact and heuristics solution approaches, while [25, 32, 3] studied facility location under uncertainty.

Similar to the simple plant location problem, the simple Transportation Problem (TP), usually solved using the transportation tableau of which no fixed charges are considered has also been extended into several variants to capture real-world distribution scenarios such as indicated by [31, 23]. Among the variants which are currently being explored by researchers are the Fixed Charge Transportation Problem (FCTP), Fixed Charge Solid Transportation problem (FCSTP) studied by [33, 5, 39] and recently the Step Fixed Charge Solid transportation problem (SFCTP) as captured in the works of [35]. Furthermore, the raw material blending and transportation problem modeled by [21] is also another variant of STP and a practical application in the field of manufacturing. Both the FCSTP and SFCTP are distribution problems which involve \(m\)-sources, \(n\)-destinations and \(k\)-conveyances. [35] Indicated that both FCSTP and SFCTP seek to determine the quantities that the chosen capacitated \(k\)-conveyances will be able to ship from the \(m\)-sources to \(n\)-destinations under a mix of route fixed charges at minimum cost. They also showed that the FCSTP and the SFCTP were NP-hard problems in which exact methods may be deficient in tackling certain problem sizes and structure in a reasonable time. They presented the LRH as an attempt to solve the SFCTP.

There has been a growing emphasis on integrating facility location into other distribution planning models. This integration has been seen in the location inventory problem of [30], FLP with plant size selection of [38], location inventory and routing models of [16] and transportation location problem of [7]. Some other integrated facility location models include the works of [24] who studied a model of integrating customer behavior into facility location model, [36] who considered optimizing facility location and routing problems simultaneously in health care logistics and [29] who developed perturbation heuristics to solve the problem of facility location and step fixed charges. Also, [4] considered integrating time windows in their off/on shore supply vessel while determining the location of warehouse and marine transportation simultaneously.

In order to solve these problems that have integrated facility location problem with some other problem types, several solution techniques ranging from exact solutions to heuristics have been considered in the literature. Exact methods such as branch and bound and branch and cut algorithms can either be manually computed or simply implemented from standard solvers such as in CPLEX, AMPL as seems
to be the norm in operations research literature. Heuristics have also received much attention due to the inefficiency of exact methods in solving large problem sizes in a reasonable time. Heuristics such as the Linear Programming Relaxation (LPR), Lagrange Relaxation (LR), Lagrange Relaxation Heuristics (LRH), local search heuristics and metaheuristics are being developed to tackle both small to large problem sizes. [38, 26, 28, 6, 27] have demonstrated how the different heuristic methods work with different upper bound and lower bound results when using these methods.

One of the heuristics earlier mentioned and discussed in this paper is the LRH. The LRH utilizes Lagrangian Relaxation. Lagrangian multipliers are used in dualizing difficult constraints in the Original Problem (OP) to be added to the objective function. The type of constraints selected in the LR also affects the strength of the Lower Bound (LB) derived [9]. Following relaxation of the hard constraints, the new model obtained which is the Lagrange Relaxation of the Original Problem (LR of OP) is thus decomposed into simpler Sub-Problems (SP) based on either the continuous or integer decision variables of the OP. Solving the SP separately and subsequently adding their objective values result in a lower bound (LB) for a minimization problem as shown by [35]. Moreover, as indicated by [38, 26] the methods of coupling the decomposed solutions would determine how the optimal solutions or upper bounds of the OP can be formed. An iterative procedure then follows the sub-problem coupling in which the Lagrange multipliers, the LB, the Upper Bound (UB) and other optimization methods are utilized in arriving at a good solution. Among the best-known optimization methods utilized during the application of the Lagrangian heuristics for both the FLP and FCSTP is the sub-gradient optimization method. It has been argued by [11], and supported by [18, 13] that using the sub-gradient method possibly gave a strong lower bound and upper bound to the Original Problem.

In this paper, we present a distribution problem that simultaneously optimizes facility location and fixed charge solid transportation problem. We have termed this problem as Fixed Charge Solid Location and Transportation Problem (FCSLTP) as shown in figure 1. The objective of the FCSLTP is to minimize total transportation and location costs by determining the optimal allocations from open locations through open routes by a set of conveyances. In order to solve this problem we utilize the CPLEX mixed-integer program dynamic solver which utilizes the branch and cut algorithm to search for optimality. Furthermore, we attempt to compare the performance of an alternative solution method against the CPLEX by extending [35] LRH feasibility resolution pattern. We also compare the FCSLTP and FCSTP in terms of the total cost. This is done to determine if there are any possible cost savings and the magnitude of cost savings obtained when an optimal number of facilities are opened (FCSLTP) compared to opening all facilities (FCSTP).

2. Mathematical model of FCSLTP. Our FCSLTP is formulated as a Mixed-Integer Programming (MIP) problem, with \( m \)- sources, \( n \)- destinations, and \( a \)- conveyances. We have extended the model of FCSTP as presented by [35] to include fixed costs of facility location. In their FCSTP a single product is to be shipped through a set of locations to a number of demand points using a set of transport mediums. The capacity of each location to supply products in FCSTP is simply determined by the route fixed and variable costs, and also the problem capacities. However, in our FCSLTP, fixed location costs, route fixed and variable
route costs and problem capacities are simultaneously used in determining whether the locations will be opened or closed for shipment.

2.1. Model assumptions for the FCSLTP. The following basic assumptions are made in our model formulation:
1. Deterministic costs
2. Two echelon distribution problem.
3. Fixed location costs, route variable and fixed charge costs.
4. One planning period.
5. Single item distribution problem.

2.2. Model development parameters and variables for FCSLTP. The parameters and variables used in the model formulation are given below.

**Parameters**
- $i$: Index for sources or locations (plants, warehouses, depots etc.).
- $j$: Index for destinations (customers, other warehouses etc.).
- $r$: Index for conveyances (or Transportation medium).
- $m$: Number of sources.
- $n$: Number of destinations.
- $a$: Number of conveyances.
- $c_{ijr}$: Variable cost of shipment on route $(i, j)$ using conveyance $r$.
- $S_i$: Capacity of source $i \forall i = 1 \ldots m$.
- $D_j$: Demand at Destination $j \forall j = 1 \ldots n$.
- $T_r$: Capacity for the conveyance $r \forall r = 1 \ldots a$.
- $F_i$: Fixed cost of opening the facility at location $i$.
- $H_{ijr}$: Fixed charge cost incurred for shipping through route $(i, j)$ based on the conveyance $r$.

**Decision Variables:**
- $x_{ijr}$: Quantity of products transported from source $(i)$ to destination $(j)$ using conveyance $(r)$.
- $y_i$: Location variable for setting source $(i)$ as either opened or closed.
$z_{ijr}$: Fixed charge variable in selecting whether conveyance($r$) is utilized or not on route($i,j$).

A mathematical model of the FCSTP is described below.

\[
\min \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} c_{ijr}x_{ijr} + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} H_{ijr}z_{ijr} \quad (1)
\]

Subject to

\[
\sum_{j=1}^{n} \sum_{r=1}^{a} x_{ijr} \leq S_i \quad \forall \ i = 1 \ldots m \quad (2)
\]

\[
\sum_{i=1}^{m} \sum_{r=1}^{a} x_{ijr} = D_j \quad \forall \ j = 1 \ldots n \quad (3)
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijr} \leq T_r \quad \forall \ r = 1 \ldots a \quad (4)
\]

\[
x_{ijr} \geq 0 \quad (5)
\]

\[
z_{ijr} = \begin{cases} 
1 & x_{ijr} > 0 \\
0 & Otherwise 
\end{cases} \quad (6)
\]

Equation 1 is the objective function. The first term is the route variable cost per conveyance type and the second term is the route fixed charge cost per conveyance type. Equation 2 is the supply capacity constraint ensuring no supply preference for selected locations. Equation 3 is the demand constraint to be met at each destination. Equation 4 is the conveyance capacity constraint. Equation 5 refers to the non-negativity constraint for the continuous variables and Equation 6 refers to the binary constraints for the route fixed charge requirement.

**Objective Function for FCSTP:**

Original Problem (OP)

Min (OP):

\[
\sum_{i=1}^{m} F_i y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} c_{ijr}x_{ijr} + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} H_{ijr}z_{ijr} \quad (7)
\]

Subject to

\[
\sum_{j=1}^{n} \sum_{r=1}^{a} x_{ijr} \leq S_i y_i \quad \forall \ i = 1 \ldots m \quad (8)
\]

\[
\sum_{i=1}^{m} \sum_{r=1}^{a} x_{ijr} = D_j \quad \forall \ j = 1 \ldots n \quad (9)
\]

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijr} \leq T_r \quad \forall \ r = 1 \ldots a \quad (10)
\]

\[
x_{ijr} \leq M_{ijr} z_{ijr} \quad \forall \ i = 1 \ldots m, \ j = 1 \ldots n, \ r = 1 \ldots a \quad (11)
\]

\[
M_{ijr} = \min (S_i, D_j, T_r) \quad (12)
\]

\[
x_{ijr} \geq 0 \quad (13)
\]

\[
z_{ijr} = \begin{cases} 
1 & x_{ijr} > 0 \\
0 & Otherwise 
\end{cases} \quad (14)
\]

\[
y_i = 0 \ or \ 1 \quad (15)
\]
Equation 7 is the objective function. The first term is the facility location cost, the second term is the route variable cost per conveyance type and the third term is the route fixed charge cost per conveyance type. Equation 8 is the supply capacity constraint of each location or source with preference given to selected locations. Equation 9 is the conveyance capacity constraint. Equation 10 is the demand constraint to be met at each destination. Equation 11 refers to upper bound limit on the continuous variables. It also represents a valid inequality for hopefully improving the solution to the FCSLTP. Equation 12 refers to the non-negativity constraint for the continuous variables. Equations 13 and 14 refer to the binary constraint for route fixed charge and facility location requirement.

2.3. Solution approaches. As earlier indicated, our FCSLTP and the FCSTP are optimization problems with the presence of fixed charges. Optimization problems with fixed charges have been noted by [8] to be classified under NP-hard network design problems. Usually, these NP-hard problems have a time complexity which makes computational time increase exponentially as problem size increase. In order to solve these NP-hard network design problems, solution techniques such as the branch and bound, branch and cut can be implemented from commercial optimization programmes such as CPLEX, LINGO, AMPL etc. These solution techniques are generally known in optimization literature to possess significant capacity in obtaining optimal solutions. We make an attempt to seek optimal solutions to our FCSLTP using the CPLEX optimization tool. Unfortunately, MIP optimization tools such as CPLEX may be costly to acquire for some category of users and practitioners such small business owners and start-ups. In some cases, the commercial solver may not be quickly available to some others requiring urgent solutions to combinatorial problems such as our FCSLTP. These categories of users will mostly desire a solution technique which may not guarantee optimal solution but can help obtain good solutions within appreciable bounds. As a result, we have developed a solution technique known as the Lagrange relaxation heuristic to provide a solution to the FCSLTP. This technique can also help provide the user with an understanding of how feasible solutions to such combinatorial problems are achieved.

2.3.1. Solving the FCSLTP using CPLEX. According to [19] the IBM ILOG CPLEX is a commercial development platform for modelling and solving combinatorial problems. Some combinatorial problems such as Linear Programming (LP), MIP, and Mixed-Integer Quadratic Problem (MIQP) have been noted to be solvable using CPLEX [22]. The quality of solutions provided by CPLEX has been noted by [22] to depend on the problem type and size being solved. Our FCSLTP is formulated as an MIP and solved using the MIP dynamic optimizer tool of the CPLEX. The dynamic optimizer tool has the capacity to serially launch a variety of exact solutions to solve an MIP. In order to solve a minimization MIP, the dynamic optimizer uses the continuous relaxation of integrality constraints to obtain a lower bound from which different cuts are applied to improve on the lower bounds obtained. Some cuts used are the mixed-integer rounding cuts, cover cuts and Gomory fractional cut [19]. We refer readers to [37] on the formulation and application of some of these cuts. The branch and cut algorithm is essentially used by CPLEX to obtain its solution to combinatorial problems.
2.3.2. Solving the FCSLTP using Lagrange relaxation heuristic method. We also make an attempt in this section to develop an alternative solution method to using CPLEX known as the Lagrange relaxation heuristic. Equation 11 is included as a valid inequality to possibly arrive at a good solution to the Lagrange relaxation. In order to use the Lagrange relaxation heuristic, constraints 8 and 11 have been selected for the application of the Lagrangian multipliers \( \lambda_i \) i.e \( \sum_{i=1}^{m} \lambda_i \) and \( \beta_{ijr} \) i.e \( \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} \beta_{ijr} \) respectively. These constraints are similar to that used by [6] to give a strong lower bound. Moreover, dualizing such constraints as in equation 8 and 11 have been noted by [26] to leave the OP with a structure that is easy to exploit in finding a solution. In addition, we have used non-negative Lagrangian multipliers \( \lambda_i \geq 0 \) and \( \beta_{ijr} \geq 0 \) to help generate a lower bound. The Lagrangian Relaxation of the Original Problem (LR of OP) is given as:

\[
LR \text{ of OP } (\lambda, \beta) = \min \sum_{i=1}^{m} F_i y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} c_{ijr} x_{ijr} + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} H_{ijr} z_{ijr}
\]

\[
+ \sum_{i=1}^{m} \lambda_i (\sum_{j=1}^{n} \sum_{r=1}^{a} x_{ijr} - S_i y_i) + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} \beta_{ijr} (x_{ijr} - M_{ijr} z_{ijr})
\]

This can be equivalently written as:

\[
\sum_{i=1}^{m} (F_i - \lambda_i S_i) y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} (c_{ijr} + \beta_{ijr}) x_{ijr} + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} (H_{ijr} - \beta_{ijr} M_{ijr}) z_{ijr} + \sum_{i=1}^{m} (\lambda_i \ast \sum_{j=1}^{n} \sum_{r=1}^{a} x_{ijr})
\]

Subject to Constraints 9, 10, 12, 13 and 14

1. Decomposition Method for LR of OP \( (\lambda, \beta) \)

The solution to the original problem (OP) starts with the decomposition of the Lagrangian relaxation of the original problem. The decomposition is done based on the separation of the continuous variable \( x_{ijr} \) and the binary variables \( y_i \) and \( z_{ijr} \). This is to utilize the easy problem structures created. This decomposition allows for an easy solution to the original problems through simpler methods of solving the individual sub-problems and aggregating them into one piece. The decomposition into two major sub-problems is given below.

The Lagrangian relaxation of the Original problem i.e. LR of OP \( (\lambda, \beta) \) is decomposed into two sub-problems (SP1 and SP2).

**First Sub-problem** i.e. SP1 of OP \( (\lambda, \beta) \):

Minimize

\[
\sum_{i=1}^{m} (F_i - \lambda_i S_i) y_i + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} (H_{ijr} - \beta_{ijr} M_{ijr}) z_{ijr}
\]

Subject to

\[
y_i = 0 \text{ or } 1, \quad z_{ijr} = 0 \text{ or } 1
\]

**Second Sub-problem** i.e. SP2 of OP \( (\lambda, \beta) \):
Minimize
\[\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} (c_{ijr} + \beta_{ijr}) x_{ijr} + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} \lambda_{i} \ast \sum_{j=1}^{n} \sum_{r=1}^{a} x_{ijr} \] (18)

Subject to:
\[\sum_{i=1}^{m} \sum_{r=1}^{a} x_{ijr} = D_{j} \quad \forall \ j = 1 \ldots n \] (19)
\[\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} x_{ijr} \leq T_{r} \quad \forall \ r = 1 \ldots a \] (20)
\[x_{ijr} \geq 0 \] (21)

2. Aggregation Method for SP1 of OP \((\lambda, \beta)\) and SP2 of OP \((\lambda, \beta)\)

Following the decomposition we couple the sub-problems SP1 of OP \((\lambda, \beta)\) and SP2 of OP \((\lambda, \beta)\) created. The aggregation is based on methods that can generate the lower bounds and upper bounds necessary for utilization in the iterations of the Lagrange relaxation heuristic. The first step is to determine the lower bound to be used in the Lagrange relaxation. Thereafter the upper bound is formulated through identified infeasibility resolutions.

**Lower bound formulation for LRH**

The continuous nature of the variable of LR of OP \((\lambda, \beta)\) i.e SP2 of OP gives an LP problem which could be easily solved to optimality using a general-purpose solver such as TORA, Microsoft Excel solver and CPLEX. The integer variable aspect of SP1 of OP gives a pure integer programming problem, and is solved using mathematical deductions and scenarios to arrive at possible values \(y_{i}^{*}\) and \(z_{ijr}^{*}\) for these integers.

**Scenario 1**: This considers the fact that a minimization problem is being solved and thus makes efforts to obtain the best location and route fixed charge under the following conditions listed:
1. If the term \((F_{i} - \lambda_{i} S_{i}) < 0\), which imply a negative term, the best integer variable will be obtained when \(y_{i}^{*} = 1\).
2. If the term \((H_{ijr} - \beta_{ijr} M_{ijr}) < 0\), which also imply a negative term, the best integer variables will be obtained when \(z_{ijr}^{*} = 1\).

**Scenario 2**: Similarly to scenario1, when considering a minimization problem, the listed conditions are also observed.
1. If the term \((F_{i} - \lambda_{i} S_{i}) > 0\) this imply positive values, therefore, the best integer variables will be obtained when \(y_{i}^{*} = 0\).
2. If the term \((H_{ijr} - \beta_{ijr} M_{ijr}) > 0\), this also imply positive values, therefore, the best integer variables will be obtained when \(z_{ijr}^{*} = 0\).

To arrive at the LRH Lower bound, we follow the procedure below:
1. Compute SP2 of OP to generate the optimal \(x_{ijr}^{*}\) and SP2* of OP \((\lambda, \beta)\):
2. For SP1 of OP, For all \(i = 1 \ldots m\), if \((F_{i} - \lambda_{i} S_{i}) < 0\) then \(y_{i}^{*} = 1\)
   Else \(y_{i}^{*} = 0\)
3. For SP1 of OP, For all \(i = 1 \ldots m, j = 1 \ldots n, r = 1 \ldots a\), if \((H_{ijr} - \beta_{ijr} M_{ijr}) < 0\) then \(z_{ijr}^{*} = 1\)
   Else \(z_{ijr}^{*} = 0\)
4. Compute the optimal value for SP1 of OP i.e. $SP1^*$
   Where $SP1^* = \sum_{i=1}^{m} (F_i - \lambda_i S_i)y_i^* + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} (H_{ijr} - \beta_{ijr} M_{ijr}) z_{ijr}$ \hspace{1cm} (22)

5. The LB of OP is given as $SP1^* + SP2^*$

**Upper bound formulation for LRH**

In order to determine the upper bound to be used in the LRH, we extend the method of resolving feasibility contradictions as used also by [35]. In the solution of SP1 of OP to arrive at $SP1^*$ according to equation 22, since the $x_{ijr}^*$ values are not directly considered in the selection of $y_i^*$ and $z_{ijr}^*$ some possible contradictions or infeasibilities would have to be resolved. This will ensure feasibility as noted by [11]. The resolution of the contradictions are used in generating an upper bound to be used in the LRH.

These possible eight 9 contradictions are identified and given below:

1. Given that $x_{ijr}^* > 0$, $y_i^* = 0$ and $z_{ijr}^* = 0$
2. Given that $x_{ijr}^* > 0$, $y_i^* = 0$ and $z_{ijr}^* = 1$
3. Given that $x_{ijr}^* > 0$, $y_i^* = 1$ and $z_{ijr}^* = 0$
4. Given that $x_{ijr}^* = 0$, $y_i^* = 1$ and $z_{ijr}^* = 1$
5. Given that $x_{ijr}^* = 0$, $y_i^* = 0$ and $z_{ijr}^* = 1$
6. Given that $x_{ijr}^* = 0$, $y_i^* = 1$ and $z_{ijr}^* = 0$
7. Given that $x_{ijr} > M_{ijr}$
8. $\sum_{i=1}^{n} \sum_{r=1}^{a} x_{ijr} > S_i y_i \forall \ i = 1 \ldots m$

The following procedure can be used in resolving these contradictions respectively:

For all $i = 1 \ldots m$, $(y_i^*)$ and For all $i = 1 \ldots m$, $j = 1 \ldots n$, $r = 1 \ldots a$, $(z_{ijr}^*)$

1. If $x_{ijr}^* > 0$, and $y_i^* = 0$ and $z_{ijr}^* = 0$ then set $y_i^* = 1$ and $z_{ijr}^* = 1$
2. If $x_{ijr}^* > 0$, and $y_i^* = 0$ and $z_{ijr}^* = 1$ then set $y_i^* = 1$
3. If $x_{ijr}^* > 0$, and $y_i^* = 1$ and $z_{ijr}^* = 0$ then set $z_{ijr}^* = 1$
4. If $x_{ijr}^* = 0$, and $y_i^* = 1$ and $z_{ijr}^* = 1$ then set $y_i^* = 0$ and $z_{ijr}^* = 0$
5. If $x_{ijr}^* = 0$, and $y_i^* = 0$ and $z_{ijr}^* = 1$ then and $z_{ijr}^* = 0$
6. If $x_{ijr}^* = 0$, and $y_i^* = 1$ and $z_{ijr}^* = 0$ then set $y_i^* = 0$
7. To resolve this contradiction we add a constraint to SP2 of OP $(\lambda, \beta)$ i.e $x_{ijr}^* \leq M_{ijr}$
8. We simply attempt to resolve any possible supply infeasibility by load shifting from open locations with capacity overload into open locations with capacity under-load in the order of increasing relaxation cost given as:

$$\left( \frac{F_i}{S_i} + \sum_{j=1}^{n} \sum_{r=1}^{a} \left( \frac{H_{ijr}}{M_{ijr}} + c_{ijr} \right) \right) \forall \ i = 1 \ldots m$$

If there are no open locations with available capacities, new locations are opened to receive load shifting in the order of increasing relaxation cost.

The load shifting is done between locations within the same conveyance capacity constraints (randomly selected) to maintain the feasibility of equation 20.

A high relaxation cost shows a possibility of high supply cost from that location and a high possibility of closing the location.

After resolving the eight (8) possible infeasibilities as indicated above, we use the values of $y_i^*$, $z_{ijr}^*$, and $x_{ijr}^*$ obtained in equation 7 to arrive at our upper bound.
UB of OP = \sum_{i=1}^{m} F_i g_i^t + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} c_{ijr} x_{ijr}^t + \sum_{i=1}^{m} \sum_{j=1}^{n} H_{ijr} z_{ijr}^t \quad (23)

2.3.3. Lagrange relaxation heuristic using the sub-gradient optimization procedure.

We briefly review the workings of the sub-gradient optimization for developing the Lagrange relaxation heuristic using the sub-gradient optimization procedure. The parameters required during the computations of this heuristic. The parameters required for the sub-gradient optimization and termination procedure are listed below:

1. A value (\( \varepsilon \)) that is user-determined (or pre-specified) for algorithm termination. It is usually small-sized positive number such that (UB\(_{Best}\) ) – (LB\(_{Best}\) ) \( \leq \varepsilon \). The term UB\(_{Best}\) refers to the best Upper Bound (UB) and while LB\(_{Best}\) is the best Lower Bound (LB).

2. Step size for Lagrange multipliers (\( \lambda \) and \( \beta \)) generation is given as \( \theta^t \). The symbol \( t \) refers to the iteration number.

\[
\theta^t = \frac{\delta \left[ \left( UB_{Best} \right) - \left( LB_{Best} \right) \right]}{\sum_i \left( \sum_{j=1}^{n} \sum_{r=1}^{a} x_{ijr}^t - S_{ijy_i^t} \right)^2 + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{r=1}^{a} \left( x_{ijr}^t - M_{ijr} z_{ijr}^t \right)^2}
\]

3. A value (\( \delta \)) is chosen from the interval (0, 2) as normally used in the sub-gradient procedure. When \( t \geq t_{max} \), \( \delta = \delta / 2 \). \( t_{max} \) is the number of maximum iterations allowed per \( \delta \) used. In this paper, we have also included a termination condition of \( \delta = 0 \) for the LRH.

4. UB\(_{initial}\) and LB\(_{initial}\) are the initial values for the Upper Bound and Lower bound respectively.

5. \( \lambda_i^t \) and \( \beta_{ijr}^t \) refer to the Lagrange multipliers at the iteration number \( t \). The initial Lagrange multiplier chosen are \( \lambda_i^t = \lambda_i^* \) and \( \beta_{ijr}^t = \beta_{ijr}^* \)

Iterative steps for LRH:

Step 1. Initialize using the parameters (\( \varepsilon \), \( t \), \( t_{max} \), \( \lambda_i^t = \lambda_i^* \), \( \beta_{ijr}^t = \beta_{ijr}^* = 1 \), \( UB^t = UB_{initial} \), \( LB^t = LB_{initial} \), \( \delta = 2 \))

\[
\left( LB \text{ of } OP \right)^t = \left( LB \text{ of } OP \right)^* = SP1^* + SP2^*
\]

\[
\left( UB \text{ of } OP \right)^t = \left( UB \text{ of } OP \right)^*
\]

Step 2. Solve \( \left( LB \text{ of } OP \right)^t = SP1^t + SP2^t \)

\[
LB_{Best} = \max \left[ \left( LB \text{ of } OP \right)^t, \ LB^t \right]
\]

Step 3. Find a feasible solution to the Upper bound from \( \left( LB \text{ of } OP \right)^t \) i.e. \( \left( UB \text{ of } OP \right)^t \)

\[
UB_{Best} = \min \left[ \left( UB \text{ of } OP \right)^t, \ UB^t \right]
\]

Step 4. if \( \left( UB_{Best} \right) - \left( LB_{Best} \right) \leq \varepsilon \) Terminate the heuristic. Else

Step 5. Update the Lagrange Multipliers

\[
\lambda_i^{t+1} = \lambda_i^t + \theta^t \left( \sum_{j=1}^{n} \sum_{r=1}^{a} x_{ijr}^t - S_{ijy_i^t} \right) \quad \forall \ i \in m
\]

\[
\lambda_i^{t+1} = \max \left( \lambda_i^{t+1}, 0 \right)
\]

\[
\beta_{ijr}^{t+1} = \beta_{ijr}^t + \theta^t \left( x_{ijr}^t - M_{ijr} z_{ijr}^t \right) \quad \forall \ i \in m, \ j \in n, \ r \in a
\]
\[ \beta_{ijr}^{t+1} = \text{Max} (\beta_{ijr}^{t+1}, 0) \]

**Step 6.** if no improvement in \( LB_{Best} \) at \( t \geq t_{\text{max}} \), then \( \delta = \delta/2 \)

set \( t = 0 \) (termination and restart condition)

**Step 7.** if \( \delta = 0 \) (we terminate the Heuristic and select \( UB_{Best} \))

**Step 8.** Else set \( t = t + 1 \) and return to Step 2.

3. **Computational study.** In order to assess the effectiveness (objective value of OP) of using CPLEX and the LRH developed, we conducted a number of experiments using reference problems obtained in the literature. The Original problem (OP) and the LRH were coded on ECLIPSE development platform using Java and IBM ILOG Concert technology Library. IBM CPLEX 12.8 was used as both the MIP solver, and LP solver. In addition, we have used a windows 8.1 operating system with 6GB Random Access Memory (RAM) for the computation experiments.

The original problem (OP) was solved using the MIP dynamic solver of CPLEX which basically uses the Branch and Cut algorithm to search the solution space for optimality. In addition, we present the lower and upper bounds and optimality gap obtainable by both CPLEX and the LRH.

3.1. **Data generation for the Lagrange relaxation Heuristic and benchmark data.** The following values have been randomly selected for the Lagrange Heuristic initial parameters: \( \epsilon = 0.01, UB_{\text{initial}} = 1\exp9, LB_{\text{initial}} = 0, t_{\text{max}} = 10, t = 2. \)

As we are not aware of any benchmark data in the literature for this particular facility location and fixed solid transportation problem, we have extended the Benchmark data used for the fixed solid transportation problem [35] by including the facility location fixed cost in the data supplied.

3.2. **Data generation for the problem sizes.** The Benchmark data used by [35] basically considers uniformly distributed data randomly generated as integers in a unit square coordinate \( U[a, b] \). The uniform distribution is used to ensure a constant probability of selecting values randomly within the interval. The letter “a” refers to the lower cost limit and “b” is termed the upper-cost limit. They specified the supply capacity, demand capacity, conveyance capacity, unit costs and route fixed charge. For the facility location cost, we have used the method of generating facility location cost instances from the supply capacities considered in facility location literature as used by [12, 10, 14]. In this method, the facility location cost is calculated using \( F_i = U(0, 90) + \sqrt{S_i} \ U(100, 110) \). A total of 50 problem instances were solved across 10 different problem sizes generated as presented in 1. The data distribution used in generating the parameters is given in 2 below.

4. **Experimentation and results.** The following tests were conducted in our computational study

1. Using the problem sizes 1 to 6 in 1, we computed the mean lower bounds, mean upper bounds, and gap% obtainable by both methods. A selection of the better solution method as regards best optimality gap (gap%) is done and used for further comparing the FCSLTP and FCSTP.

\[
gap\% = \left( \frac{UB_{Sm} - LB_{Sm}}{LB_{Sm}} \right) \times 100
\]
Table 1. Problem sizes and number of instances used for experimentation

| Problem Size No. | Problem Size $m \times n \times a$ | No of instances |
|------------------|-----------------------------------|-----------------|
| 1                | $5 \times 5 \times 2$            | 5               |
| 2                | $5 \times 8 \times 2$            | 5               |
| 3                | $7 \times 10 \times 2$           | 5               |
| 4                | $8 \times 8 \times 2$            | 5               |
| 5                | $10 \times 10 \times 3$          | 5               |
| 6                | $10 \times 20 \times 3$          | 5               |
| 7                | $15 \times 30 \times 4$          | 5               |
| 8                | $20 \times 20 \times 5$          | 5               |
| 9                | $25 \times 38 \times 8$          | 5               |
| 10               | $35 \times 42 \times 9$          | 5               |

Table 2. Parameter distribution used for experimentation

| Parameter Distribution |
|------------------------|
| $S_i \sim U(200, 400)$ |
| $D_j \sim U(50, 100)$  |
| $T_r \sim U(800, 1800)$|
| $c_{ijr} \sim U(20, 150)$ |
| $H_{ijr} \sim U(200, 600)$ |
| $F = U(0, 90) + \sqrt{S_i} U(100, 110)$ |
| $M_{ijr} = \min(S_i, D_j, T_r)$ |

$UB_{Sm} =$ Best upper bound found by CPLEX and LRH  
$LB_{sm} =$ Best lower bound found by either CPLEX or LRH

2. Comparative study of FCSLTP and FCSTP to investigate possible cost savings resulting from either formulation. Comparing both the FCSLTP and the FCSTP seems to provide a biased comparison because they both have different objective functions. A FCSTP makes an assumption that all locations are opened for shipment, while this may not hold for the FCSLTP. A FCSTP with total supply matching total demand is termed as balanced with all locations opened. However, when solving an unbalanced FCSTP with total supply capacity greater than total demand requirement, the possibility of having locations without any allocation exits. Unbalanced transportation problems seem to show more real-world applications than the balanced transportation problems. This may be due to the competition of resources in meeting demand requirements, demand uncertainties that require inventory keeping at various locations or unplanned disruptions that limits supply capacities. Therefore, when in-active locations of a FCSTP are closed, possible cost savings are made and the assumption that all locations are opened does not hold.
Based on these observations, we define an equivalent FCSLTP from a FCSTP and termed it as FCSTP-EQ. This is done by computing normally the FCSTP without the facility location constraint and the facility location costs. Subsequently, the load distributions obtained are used to compute the equivalent facility location costs and added to the FCSTP objective function. This procedure is shown in Figure 2 below.

Under experimentation 1, results from table 3 below show that for the Six (6) problems considered, the CPLEX solution obtained better mean lower bounds per problem size compared to the LRH. In addition, the mean upper bounds obtained by CPLEX solution were superior compared to the LRH. This is also supported by the gap% representing the optimality gap obtained in table 4. The superior lower bounds achieved by CPLEX have been used to compute the optimality gap as shown in table 4. The worst gap% obtained by CPLEX solution for the problem sizes considered was 0.1% while that of the LRH was 15.98%. The superior performance as displayed by CPLEX is likely connected to the several cutting planes solutions of improving the lower bounds obtained from the initial linear relaxations as earlier indicated in section 2.3.1. On the other hand, our LRH is an alternative option
we developed but have not obtained significant results when compared with the quality of solutions obtainable using CPLEX. The best optimality gap obtained using our LRH for the problem sizes considered was 3.62%. Therefore, based on the superior performance of CPLEX solution for the problem sizes considered, we use the CPLEX optimization tool for a comparative study between the FCSTP-EQ and FCSLTP.

Table 3. Mean values for best lower bound and upper bound computation per solution method

| Problem Size No. | Problem Size $m \times n \times a$ | Total Problem Instances | mean $LB_{LRH}$ (best) | mean $UB_{LRH}$ (best) | mean $LB_{CPLEX}$ (best) | mean $UB_{CPLEX}$ (best) |
|------------------|-----------------------------------|-------------------------|------------------------|------------------------|-------------------------|-------------------------|
| 1                | $5 \times 5 \times 2$             | 5                       | 10879.80               | 18505.79               | 17859.01                | 17860.29                |
| 2                | $5 \times 8 \times 2$             | 5                       | 17322.20               | 28333.22               | 25534.45                | 26333.42                |
| 3                | $8 \times 8 \times 2$             | 5                       | 15736.80               | 29614.83               | 25534.45                | 25667.43                |
| 4                | $7 \times 10 \times 2$            | 5                       | 22063.60               | 35494.88               | 33925.34                | 33925.34                |
| 5                | $10 \times 10 \times 3$           | 5                       | 18764.20               | 34423.95               | 29758.67                | 29758.67                |
| 6                | $10 \times 20 \times 3$           | 5                       | 39061.60               | 62065.47               | 58664.97                | 58813.86                |

Table 4. Mean Gap% of each solution method using the best mean lower bound (CPLEX)

| Problem Size No. | Problem Size $m \times n \times a$ | mean $LB_{CPLEX}$ (best) | Gap% LRH | Gap % CPLEX |
|------------------|-----------------------------------|--------------------------|-----------|-------------|
| 1                | $5 \times 5 \times 2$             | 17859.01                 | 3.62%     | 0.007%      |
| 2                | $5 \times 8 \times 2$             | 25534.45                 | 7.72%     | 0.01%       |
| 3                | $8 \times 8 \times 2$             | 25534.45                 | 15.98%    | 0.04%       |
| 4                | $7 \times 10 \times 2$            | 33925.34                 | 4.63%     | 0.00%       |
| 5                | $10 \times 10 \times 3$           | 29758.67                 | 15.68%    | 0.00%       |
| 6                | $10 \times 20 \times 3$           | 58664.97                 | 5.8%      | 0.03%       |
Under experimentation 2 we compared the equivalent FCSLTP developed (using the FCSTP) which we have described as FCSTP-EQ (Figure 2) and the original formulation (equations 7-14) which we have termed as the FCSLTP. We placed a computation time limit of 9000 seconds to obtain solutions. Table 5 below shows the results obtained. It was observed that the original formulation (FCSLTP) performed considerably better than the FCSTP-EQ through 10 different problem sizes considered. Problems Size (10) in particular showed a 25% reduction in total costs when using the FCSLTP formulation. Figure 3 shows a trend of the increase in cost savings as the problem size increased, with the lowest cost savings at 3% in the smallest sized problem (1) and 25% in the largest problem size (10). This trend is possibly connected to the increase in solution search space of the FCSTP-EQ formulation when compared with the FCSLTP as problem size increased. The feasible solution search space of FCSTP-EQ is wider due to less limitation on the amount of locations to open for shipment. In addition, the FCSTP-EQ obtains its load distribution assuming all locations are opened for shipment. Although during actual allocations, there are possibilities of some locations not shipping to any destinations, therefore requiring the need for closure of such in-active location as done in the FCSTP-EQ computation.

We also observed that when solving the FCSLTP, the FCSTP formulation could possibly be used as an initial feasible solution. The percentage total cost savings obtained (Table 5) between FCSLTP and FCSTP-EQ could also be interpreted as an optimality gap when using the FCSTP as starting solution to compute the FCSLTP. In addition, the run time of FCSTP was much lower compared to the FCSLTP as shown in Figure 4. This was expected although the magnitude of difference may not be easily quantified unless experimentally conducted. The reduction in computation time of FCSTP is likely due to the reduced number of computations required to be performed compared to the FCSLTP formulation. Therefore, the reduced time could be an insight into the development of a hybrid improvement solution or heuristic using the FCSTP as a basis to solve the original formulation of FCSLTP. The run time of FCSLTP using CPLEX showed significant exponential time complexity than the FCSTP for larger problem sizes. Figure 4 presents a comparison between the mean run time of the FCSTP and the FCSLTP as problem size increase.

5. Conclusion and future direction. We have considered a new optimization problem that integrates facility location decisions into a distribution problem known as the Fixed Charge Solid Transportation Problem (FCSTP). This problem was termed Fixed Charge Solid Location and Transportation Problem (FCSLTP) and was solved using CPLEX optimization tool. A LRH was developed as an alternative solution for users who possibly may not have access to commercial optimization solvers for certain MIP due to costs or other constraints. The LRH was compared to the CPLEX solution. Results obtained presents CPLEX solution as more effective with regard to a lower optimality gap for all the problem sizes considered. However, the LRH could still give some solutions within an optimization gap of 4%.

We discussed a formulation of FCSLTP using the load distribution obtained from FCSTP and termed it an equivalent FCSLTP (FCSTP-EQ). The basic motivation for this formulation was stated as the possibility of some locations turning out to be in-active for shipping out when solving an unbalanced FCSTP and the cost savings obtained when such locations are closed. The FCSTP-EQ was compared to the original formulation of FCSLTP denoted as FCSLTP. Using total cost as a measure
Table 5. Comparison between the FCSTP EQ and FCSLTP using CPLEX under 9000secs computation time

| Problem No | Problem Size $m \times n \times a$ | Total no. of Instances | FCSTP EQ mean | FCSLTP mean | Cost Difference | % Cost Difference |
|------------|-----------------------------------|-------------------------|---------------|-------------|-----------------|------------------|
| 1          | 5×5×2                             | 5                       | 18480.39      | 17860.29    | 620.10          | 3%               |
| 2          | 5×8×2                             | 5                       | 28333.22      | 26333.42    | 1999.80         | 8%               |
| 3          | 8×8×2                             | 5                       | 29064.07      | 25667.43    | 3396.64         | 13%              |
| 4          | 7×10×2                            | 5                       | 35807.10      | 33925.34    | 1881.76         | 6%               |
| 5          | 10×10×3                           | 5                       | 32147.15      | 29758.67    | 2388.48         | 8%               |
| 6          | 10×20×3                           | 5                       | 62797.43      | 58813.86    | 3983.57         | 7%               |
| 7          | 15×30×4                           | 5                       | 85778.98      | 77653.71    | 8125.27         | 10%              |
| 8          | 20×20×5                           | 5                       | 61498.56      | 50054.12    | 11444.44        | 23%              |
| 9          | 25×38×8                           | 5                       | 106532.31     | 89098.73    | 17433.58        | 20%              |
| 10         | 35×42×9                           | 5                       | 120932.51     | 96508.73    | 24423.78        | 25%              |

of comparison, the FCSLTP formulation presented better cost savings compared to the FCSTP-EQ. This possibly was attributed to a narrower feasible solution search space used by our FCSLTP as compared to the FCSTP-EQ. The percentage total cost difference was also interpreted as an optimality gap between FCSLTP and FCSTP-EQ. Based on the cost savings or optimality gap obtained, we consider the load distribution of a FCSTP as a feasible starting solution to solving the FCSLTP for larger problem sizes. This is further supported by the very low solution time obtained when solving a FCSTP as compared to the FCSLTP for larger problem sizes. Suggested improvement for using the FCSTP as a starting solution might be the requirement of a hybrid improvement heuristic to obtain improved upper bounds within the solution time limit of the FCSLTP.

Possible extensions to improve the quality of solution of the LRH can be strengthenning the upper bound search using other heuristics or possible metaheuristics such as genetic algorithm and/or simulated annealing. Metaheuristics have the capacity to further increase the solution search space and make the hybrid solution become very competitive with solution methods obtainable in commercial optimization tools such as in CPLEX. As noted in the experiments, the time complexity of CPLEX solution is exponential as the problem size increase. Consequently, this might strengthen the reason for a search for an efficient alternative solution method.
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