Duality between simple-group gauge theories and some applications

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Abstract

In this paper we investigate N=1 supersymmetric gauge theories with a product gauge group. By using smoothly confining dynamics, we can find new dualities which include higher-rank tensor fields, and in which the dual gauge group is simple, not a product. Some of them are dualities between chiral and non-chiral gauge theories. We also discuss some applications to dynamical supersymmetry breaking phenomena and new confining theories with a tree-level superpotential.

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1 Introduction

$\mathcal{N} = 1$ supersymmetric gauge theories in four dimensions are very attractive and have been intensively investigated. Recent developments in understanding non-perturbative effects reveal the strong dynamics of supersymmetric gauge theories. It has been applied to constructing various models beyond the standard model, dynamical supersymmetry breaking models [1], composite models of quark and lepton [2], and so on. Especially, phenomena of duality in supersymmetric gauge theories appear in many different gauge and superstring theories. So, the study of duality itself may be important.

We know that $\mathcal{N} = 1$ supersymmetric gauge theories have various phases of dynamics in the infrared region. For example, consider the $SU(N)$ gauge theory with $N_F$ flavors of fundamental matter fields [3]. For $N_F < N$ the theory has no stable supersymmetric vacuum (runaway behavior) because the Affleck-Dine-Seiberg superpotential is dynamically generated. When $N_F = N$ the theory confines with chiral symmetry breaking (i-confinement), and when $N_F = N + 1$ the theory confines without chiral symmetry breaking (s-confinement). In case $N_F > N + 1$ the theory has a dual description which has the same infrared behavior as the original theory.

So far, various models have been known to have dual descriptions. Among these we know several examples of duality in theories with matters in the higher-rank tensor representation. When using the deconfinement method [4] to expand these higher-rank tensor fields, in general, the gauge group of the dual theory is the product group [4]–[9]. On the other hand, the Kutasov-Schwimmer-type dualities [10, 11] and the dualities in spin gauge theories with matters in spinorial representation [12]–[16] have simple dual gauge groups. It is, however, difficult to find these types of dualities because no systematic method has been discovered as of yet. In this paper, by applying the deconfinement method, we propose a systematic way to find examples of such duality. With this method, we can construct many examples of duality including chiral to non-chiral dualities and a duality for the exceptional $G_2$ gauge theory.

This paper is organized as follows. In Section 2 we shortly review the deconfinement method and discuss our idea for finding duality. In Section 3 we show several examples and check the consistency in an example, and we show some more examples and applications to dynamical supersymmetry breaking and confining theories in Section 4. Section 5 is devoted to summary and conclusion.

2 The idea

In this section we describe our method for finding duality by using a variant of the deconfinement method. In the deconfinement method, we apply the confining dynamics to expanding higher-rank tensor fields in order to obtain the deconfined
theory with only fundamental representation matters. We first summarize the sorts of confining theories.

In the absence of tree-level superpotential, there are four types of confining theory which are formally called s-confinement [17], i-confinement [18], c-confinement [18] and affine-confinement [19, 20], respectively. The s-confinement is defined as a theory for which the low-energy effective theory is described by gauge invariant operators everywhere in the moduli space. In addition, the structure of quantum moduli space is the same as that of the classical theory. Therefore the effective superpotential is certainly generated to reproduce the classical constraints as equations of motion. In i- (c-) confining theories the classical moduli is deformed by quantum mechanics. The global symmetry is broken in i-confining theories. In c-confining theories the origin of moduli space is allowed and global symmetries are not broken at that point. The affine-confinement is the confining theory without any constraints between the composite states which describe the low-energy dynamics. We summarize the types of confining theories in Table 2.1.

| type             | example                                      |
|------------------|----------------------------------------------|
| s-confinement    | $SU(N)$ with $(N + 1)$                       |
| i-confinement    | $SU(N)$ with $N$                            |
| c-confinement    | $SU(4)$ with $3$ + $(\square + \square)$    |
| affine-confinement | $SO(N)$ with $(N - 4)$                        |

Table 2.1: The classification of the confining theories.

2.1 The deconfinement method

Before we discuss our idea, we shortly review the deconfinement method proposed by Berkooz [4]. The deconfinement method is used to find the dual theories (the magnetic theories) of the theories (the electric theories) containing the two index tensor representation (antisymmetric, symmetric or adjoint representation) matters.

The electric theory is expanded into the theory (the deconfined theory) as the two index tensor fields replaced by a composite operator. For example the $SU(N)$ symmetric tensor $S^{ab}$ can be replaced by

$$S^{ab} \rightarrow [x^a x^b]$$

(2.1)

where the field $x^a$ belongs to the vector representation of $SO(N + 5)$ gauge symmetry which now confines without chiral symmetry breaking. Here and through-
out this paper, we write composite states as $[\cdots]$. The matter content and superpotential are shown in Table 2.2. In this table, we omit the $SU(N)$ gauge anomaly which is usually compensated by more antifundamental fields. When

|      | $SU(N)$ | $SO(N+5)$ |
|------|---------|-----------|
| $x_i^a$ | □       | □         |
| $y_i$  | 1       | □         |
| $z_a$  | □       | 1         |
| $u$    | 1       | 1         |

$W = xyz + y^2u$

Table 2.2: The deconfined theory for $SU(N)$ theory with a symmetric tensor field.

this $SO(N+5)$ gauge group confines before the $SU(N)$ gauge coupling becomes strong, i.e., $\Lambda_{SU} \ll \Lambda_{SO}$ (the dynamical scale $\Lambda_{SU}$ ($\Lambda_{SO}$) means that it is the scale when $SO$ ($SU$) gauge coupling turns off), no dynamical superpotential is generated. The resultant $SU(N)$ theory is described by the $SO(N+5)$ gauge singlet composite states,

$$[x^a x^b], \ [x^a y], \ [yy].$$  \hspace{1cm} (2.2)\

The superpotential makes the fields $[x^a y]$ and $[yy]$ massive. We integrate out these massive modes by using their equations of motion and then we obtain the $SU(N)$ gauge theory with a symmetric tensor field $S^{ab}$ (the electric theory).

A way to find the magnetic description of this electric theory is apparent. The holomorphy of moduli space tells that the equivalence of the infrared limit is valid for all value of $\Lambda_{SU}/\Lambda_{SO}$ \cite{21}. So, when taking the dual of the $SU(N)$ gauge symmetry in the above deconfined theory at first, i.e., in the case of $\Lambda_{SU} \gg \Lambda_{SO}$, we can obtain a magnetic description of the electric theory. Moreover, if we dualize the $SO(N+5)$ gauge symmetry in this magnetic theory, we obtain the second magnetic theory. In this way we can find the sequence of the dual magnetic theories. We can easily check the ’t Hooft anomaly matching conditions and the correspondence of the gauge invariant operators between the electric and magnetic theories because they have been confirmed between the deconfined theory and each theory.

\section{2.2 The idea}

With the deconfinement method described in the last subsection, one can find duality between the electric theory including the two index tensor representation matters and the magnetic theory whose gauge group is necessarily a product. We


\[
\begin{align*}
G_1 \times G_2 \text{ (duality } \times \text{ confinement)} \\
\downarrow \quad \downarrow \\
G_1 \text{ with tensors} \quad \tilde{G}_1 \times G_2 \\
\text{the electric theory} \quad \text{the magnetic theory}
\end{align*}
\]

Table 2.3: The deconfinement method.

display this procedure in Table 2.3. The label “duality \times confinement” means that when the \( G_2 \) gauge coupling turns off the theory has other descriptions which describe the same infrared physics, and when \( G_1 \) gauge coupling turns off the theory confines. Therefore if we want to obtain dualities in which both gauge groups are simple, we take the deconfined theory with the label “confinement \times confinement” (see Table 2.4).

\[
\begin{align*}
G_1 \times G_2 \text{ (confinement } \times \text{ confinement)} \\
\downarrow \quad \downarrow \\
G_1 \text{ with tensors} \quad G_2 \text{ with tensors} \\
\text{the electric theory} \quad \text{the magnetic theory}
\end{align*}
\]

Table 2.4: Our method.

Let us discuss whether the dynamically generated superpotential changes when global symmetries are gauged and whether we can expect that there is a dual description for the electric (magnetic) theory. We comment on these two points below and will check more non-trivially for some examples in detail in the next section.

The matter content of the deconfined theory can be generally written as in Table 2.5. Here we omit the non-abelian global flavor symmetries. \( R_i \) and \( r_i \) \((i = 1, \cdots, l)\) denote the representations of the field \( \Phi_i \) under the gauge group \( G_1 \) and \( G_2 \). Each \( U(1) \) flavor symmetry has gauge anomaly, so both the dynamical scales \((\Lambda_{b_1}^{1}, \Lambda_{b_2}^{2})\) have nonzero charge under these symmetries. The exponents \( b_1 \) and \( b_2 \) are the coefficients of the one-loop beta function of \( G_1 \) and \( G_2 \) gauge couplings. The \( U(1)_R \) is the R-symmetry in \( \mathcal{N} = 1 \) supersymmetric theories. We now take the deconfined theory with the label ”s-confinement \times s-confinement”, so the \( U(1)_R \) charges of these dynamical scales are \(-2\). We take the tree-level
superpotential

\[ W = 0. \]

We now consider the case for \( \Lambda_1 \ll \Lambda_2 \) and examine the effective theory near below the scale \( \Lambda_2 \), i.e., we now take into account the effect of the \( G_2 \) gauge dynamics. Using the symmetries and considering the classical limit, the superpotential of the electric \( G_1 \) theory takes the same form as that when the \( G_1 \) gauge coupling turns off;

\[ W = \frac{1}{\Lambda_2^{b_2}} \left( \Phi_1^{\mu(r_1)} \cdots \Phi_1^{\mu(r_l)} \right). \]

Therefore it is found that the superpotential does not change even when flavor symmetries are gauged. It should be noted that from the argument of symmetries and the classical limit, the terms which include the scale \( \Lambda_1 \) are also allowed. However, these terms are important only when we consider the low-energy behavior of the electric \( G_1 \) theory. A similar analysis can be made in the magnetic \( G_2 \) theory in the case of \( \Lambda_1 \gg \Lambda_2 \).

The \( \mathcal{N} = 1 \) supersymmetric asymptotically free gauge theories without classical superpotential are roughly classified into three types of theories: runaway, confinement, and duality. Using symmetries and taking the classical limit, it is found that the relationship between these types and the Dynkin index constraints arises. The relation is represented in terms of the sum of indices of matter fields \( \sum \mu_i \) and the index of the adjoint representation \( \mu_G \) (see Table 2.6).

Since we take a deconfined theory with the label “s-Confinement \( \times \) s-Confinement,” the sum of indices of each theory is equal to \( \mu_G + 2 \). We now want to study how the index of the \( G_1 \) theory changes as a consequence of the \( G_2 \) confining dynamics. Applying the results in Ref. [23], we obtain the following expression for the \( G_1 \) index after the confinement,

\[
\sum_{i' \in G_1} \mu_{i'} \geq \sum_{i \in G_1} \mu_i + \sum_{i \in G_2} \mu_i - \mu_G = (\mu_G + 2) + 2,
\]

Table 2.5: The matter content and the charge assignment of the deconfined theory. \( (\mu(R)) \) is the Dynkin index of representation \( R \).
Table 2.6: The rough classification of $\mathcal{N} = 1$ gauge dynamics in the absence of the tree-level superpotential.

| type      | index constraint                        |
|-----------|------------------------------------------|
| runaway   | $\sum \mu_i < \mu_G$                    |
| confinement| $\sum \mu_i = \mu_G, \mu_G + 2$        |
| duality   | $\sum \mu_i \geq \mu_G + 4$            |

where $i'$ on the left-hand side denotes the kind of fields in the low-energy $G_1$ theory after $G_2$ confinement. Note that the inequality (2.5) is valid only in confining theories except for affine-confining theories. From this we expect that the electric theory has a dual which describes the same infrared behavior. We can obtain the magnetic theory in another limit $\Lambda_1 \gg \Lambda_2$. In this way we obtain dualities between the electric and magnetic theories.

From the above expression for the Dynkin index, it can be easily seen that we can construct other types of low-energy $G_1$ gauge theories by taking $G_2$ gauge theories with other indices. We will show such examples in subsection 4.3.

### 3 Example and consistency check

In this section, we present several examples of the new dual pair which are obtained by the method explained in the previous section. We can obtain these examples easily since the s-confining theories without a tree-level superpotential have been completely studied \[17\]. The massless composite fields are often in higher-rank tensor representation under the flavor symmetries. Then we have dualities with fields in higher-rank tensor representation. In addition, it sometimes occurs that both electric and magnetic theories have already been known to have other dual descriptions in the absence of the superpotential. In that case, we can check that these pairs of theories are indeed connected by duality transformations and describe the same low-energy dynamics.

The first example is a theory based on the gauge group of $SU(2N)$ and $Sp(2N)$. We take the following theory with the tree-level superpotential $W = 0$ as the deconfined theory (Table \[3.1\]). Both the $SU(2N)$ and $Sp(2N)$ theories are supposed to confine \[24\] at the scales of $\Lambda_1$ and $\Lambda_2$, respectively. First, let us consider the case for $\Lambda_1 \ll \Lambda_2$, in which case the strong coupling dynamics of the $Sp(2N)$ theory is important below the scale of $\Lambda_2$. The resultant confined theory, which we call the electric theory, is the $SU(2N)$ gauge theory, and has the following matter content and the dynamically generated superpotential (Table \[3.2\];
This electric theory surely has the index which satisfies $\sum \mu_i - \mu_G > 2$. Therefore we expect there to be a dual description. As mentioned earlier, a dual description of this electric theory can be obtained when one considers another limit of $\Lambda_1 \gg \Lambda_2$. In this limit, the $SU(2N)$ gauge group in the deconfined theory smoothly confines and the field content of the resultant magnetic theory is given in Table 3.3. This theory has a dynamically generated superpotential

$$W = \frac{1}{\Lambda_2^{2N+1}} \left( V_{\mu_2} V_{\mu_2}^{\prime 2} + V_{\mu_1} V_{\mu_2}^{\prime 2} V'' + V_{\mu_2}^{-2} V_{\mu_4}^{\prime 2} \right). \quad (3.1)$$

This electric theory surely has the index which satisfies $\sum \mu_i - \mu_G > 2$. Therefore we expect there to be a dual description. As mentioned earlier, a dual description of this electric theory can be obtained when one considers another limit of $\Lambda_1 \gg \Lambda_2$. In this limit, the $SU(2N)$ gauge group in the deconfined theory smoothly confines and the field content of the resultant magnetic theory is given in Table 3.3. This theory has a dynamically generated superpotential

$$W = \frac{1}{\Lambda_1^{4N-1}} \left( M^{2N} M' + B M' \overline{B} + B' M \overline{B} \right). \quad (3.2)$$
These two theories based on simple gauge groups are certainly dual in the senses that the moduli space of vacua is identical in a consequence of the holomorphy of couplings in the moduli space, and the ’t Hooft anomaly matching conditions are trivially satisfied between the electric and magnetic theories because this is guaranteed to work originally between the deconfined theory and each theory.

One more support of duality is the consistency against deformations added in both theories. Especially, in the above case, two theories have other dual descriptions obtained from the known dualities, in which the consistency for deformations has already been shown. Therefore, we can corroborate the above obtained duality by applying the known dualities to each theory.

First, we take a dual of the electric theory. Though the dual description of $SU(2N)$ theory with an antisymmetric tensor is known only in the case of the critical number of flavors by using the deconfinement method, a dual description of the case of five flavors has been discussed by Terning [9] in terms of a theory with gauge group $SU(2) \times SU(2)$ (see Table 3.4). When applying this dual transformation, some terms in the superpotential (3.1) become mass terms for elementary fields in the dual $SU(2) \times SU(2)$ theory. By integrating out these fields, we arrive at the following theory. The superpotential is

\[ W = q^4x^2 + M qxy + M'ylx^2 + Py^2 + P'qxI + P^N M'. \] (3.3)

Under this dual transformation, the mapping of the gauge invariant operators in the chiral ring is

\[ V'Q \leftrightarrow M, \quad Q'Q \leftrightarrow M', \quad VQ^2 \leftrightarrow P, \]
\[ V^{N-1}V'Q'' \leftrightarrow P', \quad V^N \leftrightarrow x^2, \quad Q'^2N \leftrightarrow yl. \] (3.4)

Hereafter we omit various scale factors which appear in the superpotential and duality mapping for simplicity. In the above superpotential (3.3), we added the
last term which is allowed by all the flavor and gauge symmetries and holomorphy resulting in a superpotential which is the most generic one. This type of terms is important in considering the relevant description of the low-energy physics. Generally, in confining theories, its existence may be interpreted as the non-perturbative (instanton) effects in the completely broken dual gauge group. It should be noted that even when there are terms allowed by symmetries and holomorphy arguments in the deconfined theories, they are not generated in the low-energy theories unless they can be nonsingularly composed by the products of composite operators.

Then, the $SU(2)_1$ theory has three flavor quarks and smoothly confines. After the confinement of the $SU(2)_1$ theory, finally, the field content of the resultant dual theory is given in Table 3.5 and the superpotential is

$$ W = T^2 T'' + TT'^2 + MT'y + M'T''yl + Py^2 + P'T' + P^N M'. \quad (3.5) $$

On the other hand, the $Sp(2N)$ magnetic theory also has a known dual description. In this dual picture, again, some of the terms in the superpotential (3.2) become mass terms and some fields are decoupled from the low-energy dynamics. Integrating out these fields, we have the matter content shown in Table 3.6, and the superpotential is given by

$$ W = Z^N M' + M'Bmb' + Zm^2 + Z'mq' + Z''q'b' + Z'''q'^2 + BZ'''. \quad (3.6) $$

We also added the last term which is allowed by all symmetries and holomorphy, and which may concern the non-perturbative dynamics in the broken dual gauge group. The above superpotential is the most general one with respect to all the symmetries and holomorphy. The operator mapping in the chiral ring between the magnetic and dual theories is as follows:

$$ M^2 \leftrightarrow Z, \quad MQ' \leftrightarrow Z', \quad B'Q' \leftrightarrow Z'', \quad Q'^2 \leftrightarrow Z'''. \quad (3.7) $$

|   | $SU(2)_2$ | $SU(4)$ | $SU(2N + 1)$ | $U(1)_1$ | $U(1)_2$ | $U(1)_R$ |
|---|-----------|----------|--------------|----------|----------|----------|
| $y$ | □         | 1        | □            | 0        | 1        | $1 - \frac{1}{N}$ |
| $l$ | □         | 1        | 1            | $-2N$    | $-2N - 1$ | $-1 + \frac{1}{N}$ |
| $M$ | 1         | □        | □            | $-\frac{N}{2}$ | $-1$ | $\frac{1}{N}$ |
| $M'$ | 1         | 1        | □            | 0        | $2N$    | 0        |
| $P$ | 1         | 1        | □            | 0        | $-2$    | $\frac{2}{N}$ |
| $P'$ | 1         | □        | □            | $\frac{N}{2}$ | $2N + 1$ | $2 - \frac{1}{N}$ |
| $T = [qq]$ | 1        | □        | □            | $-N$    | 0        | 0        |
| $T' = [qx]$ | □        | □        | □            | $N$    | 0        | 1        |
| $T'' = [xx]$ | 1        | 1        | 1            | $2N$    | 0        | 2        |

Table 3.5: The low-energy description of a dual of the electric theory.
The gauge singlet meson \([MB']\) is decoupled by the mass term with \(B\). Remarkably enough, this theory is the same as the above SU(2)_2 theory which is the dual of the electric SU(2N) theory. The one-to-one correspondence of the elementary fields are easily seen from the tables:

\[
y \leftrightarrow m, \quad l \leftrightarrow b', \quad M \leftrightarrow Z', \\
M' \leftrightarrow M', \quad P \leftrightarrow Z, \quad P' \leftrightarrow Z'', \\
T \leftrightarrow Z''', \quad T' \leftrightarrow q', \quad T'' \leftrightarrow B. \tag{3.8}
\]

From this, we can explicitly confirm that the electric and magnetic theories are certainly dual.

It should be noted that the above dual pair gives a new example of chiral to non-chiral dualities. So far, this type of duality has been obtained only in the restricted type of theories; that is, the dualities between SO gauge theory with spinors in the real representation [12, 13, 14] and SU gauge theory with a second rank symmetric tensor, which is a chiral theory. In addition, those chiral to non-chiral dualities are highly nontrivial and less easily found. However, it is interesting that with our method, we can easily discover many new dualities of this type by only setting product gauge groups as one wants. More examples of new duals of this type are presented below.

We show the next example of dualities constructed by using our method. As with the previous example, we can check the consistency of the duality in a non-trivial manners which incorporate the non-perturbative dynamics. However, we here concentrate on describing new dual pairs. Instead of the SU(2N) gauge group, we can consider SU(2N − 1) theory which may be interesting from phenomenological points of view. The matter content of the deconfined theory which we now consider is shown in Table 3.7, and the tree-level superpotential \(W = 0\).

We can obtain a dual pair by confining each gauge group in this example as well as in the previous one. After the Sp(2N) group confines, we have the electric
SU(2N − 1) theory and the matter content is given in Table 3.8. This theory has

\[ W = V^{N-1}V'V'^2 + V^{N-2}V'^3V'' + V^{N-3}V'^5. \]  

(3.9)

On the other hand, by confining the SU(2N − 1) theory at first, we have the magnetic theory as shown in Table 3.9 with the superpotential

\[ W = M^{2N} + BMB. \]  

(3.10)

This is also an example of chiral to non-chiral duality based on simple gauge groups.

Another interesting feature of this method for finding dualities is that the mapping of gauge invariant operators is constructed very easily. This can be
done by considering gauge invariant operators in the deconfined theory. By decomposing gauge invariant operators in both electric and magnetic theories, we can see the operator mapping between the chiral rings of those theories without comparing the transformation properties under flavor symmetries.

In addition, these dual pairs are also certainly stable with deformations of two theories as well as the known dual pairs. By some deformations, it may occur that one theory flows out of the non-abelian Coulomb phase and at the same time the dual gauge group does not completely broken. But in this situation, we can also expect that two theories describe the same infrared physics; confinement, chiral symmetry breaking, dynamical supersymmetry breaking, etc. We will actually see such deformations in the next section.

We can find many other dual pairs based on other gauge groups by altering the way to gauge flavor symmetries in various s-confining theories. For instance, we can straightforwardly construct dual pairs of theories based on the gauge groups $SU(2N)$, $SU(2N − 1)$ versus $Sp(2N − 2)$, $Sp(2N − 4)$ as in the previous examples. Especially, the magnetic $Sp(2N − 4)$ theory (versus $SU(2N)$ or $SU(2N − 1)$) can be found from the known duality which is obtained by the deconfinement method [4, 5]. We can obtain these dualities by setting the number of flavors in the electric theory to the critical one such that one of the dual gauge groups, which is of product form, becomes trivial and the remaining dual gauge group is simple. In this way, some of the dual pairs obtained by our method can be interpreted as the critical situations of the known dualities obtained by the deconfinement method. However, among theories which have more flavors than s-confining theories, there are only a few theories whose magnetic descriptions have been known. Therefore, our method may be a more powerful tool than the usual deconfinement method in order to find new dual pairs of theories. Conversely, we may construct new dual theories based on the “product” gauge group as the duals of theories which contain more fields than the electric theories obtained by our method.

4 Model and application

4.1 Dynamical supersymmetry breaking

Dynamical supersymmetry breaking is a very interesting phenomenon for theoretical and phenomenological meanings and has been intensively investigated for a long time [1]. In chiral gauge theories, many models of dynamical supersymmetry breaking have been constructed. In these models, there is no classical flat direction and some of the global symmetries are broken by strong coupling dynamics. That is, by the quantum effects, supersymmetry is broken dynamically. Among these theories, one famous model is based on $SU(5)$ gauge theory with an antisymmetric tensor and an antifundamental field [23]. This model has no gauge invariant operator, so the classical moduli space is only at the origin of
the field space. Since this model is asymptotically free, at the origin it is very strongly coupled in the infrared region and it is plausible that global \((U(1)_R)\) symmetry is broken by non-perturbative dynamics. Therefore, supersymmetry is considered to be broken according to the argument of the Nambu-Goldstone theorem or the Konishi anomaly \[26\]. So far, this probable supersymmetry breaking has been confirmed by considering the flavor decoupling from calculable models \[27\] or the weakly coupled magnetic descriptions of models which contain more fundamental flavors than this \(SU(5)\) model \[5\]. But in this dual description, the magnetic gauge group is completely broken through the Higgs mechanism or confined and then becomes trivial due to the effects corresponding to the mass decoupling effects. So, this argument is only equivalent to considering the flavor decoupling from the s-confining theory in the electric side, which is only a Wess-Zumino model for the composite gauge singlets. In this subsection, we present another check of this supersymmetry breaking scenario by directly analyzing the non-abelian dual description of this \(SU(5)\) model. A similar analysis has been done in the different context \[28\]. As is seen below, though this dual theory has non-chiral field content, the dynamical supersymmetry breaking indeed occurs when taking into account the non-perturbative effects properly.

Let us consider the following deconfined theory with zero tree-level superpotential \(W\) (Table 4.1). Both gauge groups of \(SU(5)\) and \(SU(4)\) confine at \(\Lambda_1\) and \(\Lambda_2\), respectively. After confinement of each gauge group at each limit, \(\Lambda_1 \ll \Lambda_2\) or \(\Lambda_1 \gg \Lambda_2\), we have the following dual pair of theories. The field content of the electric theory is shown in Table 4.2 and a dynamically generated superpotential is given by

\[
W = M^5 + BM\overline{B}.
\]  

The magnetic theory becomes as shown in Table 4.3 and has a superpotential

\[
W = V^3 PR + V P^2 T + RTU.
\] 

Here let us consider the deformation of this duality. This can be done by adding a tree-level superpotential in the deconfined theory. We add the following super-

|     | \(SU(5)\) | \(SU(4)\) | \(SU(5)\) | \(SU(5)\) | \(U(1)_1\) | \(U(1)_2\) | \(U(1)_R\) |
|-----|---------|---------|---------|---------|----------|----------|----------|
| \(A\) | | 1 | 1 | 1 | 1 | 4 | −6/5 |
| \(Q\) | | | | 1 | 1 | 0 | −3 | −3/5 |
| \(Q'\) | | | | | | 1 | −3/5 | 0 | 8/5 |
| \(Q'\) | | 1 | | | | | 3 | 1 |

Table 4.1: The deconfined theory.
potential:

\[ W = \sum_{a,i=1}^{5} h^{ai} Q_{a} Q_{i}' . \]  

(4.3)

With this term, the flavor symmetries \( SU(5) \times SU(5) \times U(1)_1 \) are broken to \( SU(5)_{\text{diag}} \). This term becomes the mass term of the elementary fields in both theories after conﬁnements. In the electric \( SU(5) \) theory, integrating out these massive modes, we arrive at the \( SU(5) \) model with matter fields consisting one antisymmetric tensor and one antifundamental tensor. Since this theory has no gauge invariant operator, we also have no superpotential. This is just the above-mentioned theory (except for a singlet field \( B \) which has no interaction and is physically irrelevant in this case). In this theory supersymmetric vacuum is probably lifted by the \( SU(5) \) strong dynamics.

On the other hand, after we integrate out the massive fields, the low-energy description of the magnetic theory is an \( SU(4) \) gauge theory with one flavor quarks and singlet fields with the following superpotential:

\[ W = RTU + 3 \left( \frac{\tilde{\Lambda}^{11}_{2}}{RT} \right)^{1/3} . \]  

(4.4)
where $\tilde{\Lambda}_2$ is the dynamical scale of the above $SU(4)$ theory (Table 4.4). The second term is induced by the non-perturbative effects in the $SU(4)$ theory. Without the second term this model has a classical flat direction associated with the singlet field $U$. (The flat direction along $P$ is not concerned with any dynamics, so we neglect this direction in the following.) However, as seen in the following, this flat direction is stabilized in the full quantum theory. For this purpose, it is convenient to consider the theory with $U$ as a fixed parameter \[29\]. In this theory, clearly supersymmetry is unbroken and the meson $m = RT$ has a vacuum expectation value $m \sim U^{-3/4}$ in the vacuum. Therefore we find a runaway property of the full theory potential $V$:

$$V = \left| \frac{\partial W}{\partial U} \right|^2 \sim U^{-3/2}. \quad (4.5)$$

However, it is noted that this picture is valid only for $U \ll \Lambda_1$, i.e., only when $U$ is sufficiently smaller than the $SU(5)$ confinement scale. In the case of $U \gg \Lambda_1$, the relevant description is the deconfined theory. In this picture, the classical flat direction is parametrized by $U = \tilde{Q}^5 = v^5$. Reconsidering the behavior of the potential, we find

$$V = \left| \frac{\partial W}{\partial \tilde{Q}} \right|^2 \sim U^{-3/2} v^{4/2} \sim v^{1/2}. \quad (4.6)$$

Thus the classical flat direction is stabilized by the quantum effects. In the finite size region of the moduli space, supersymmetry is surely broken by the contradictory F-flatness conditions of $U$ and $m$.

In this way, we can directly confirm the probable dynamical supersymmetry breaking scenario of the $SU(5)$ chiral model in terms of its non-chiral dual description in a consistent manner by using the non-perturbative dynamics. Similar analyses can be made for the $SU(2N + 1)$ theory with one antisymmetric tensor and $(2N - 3)$ antifundamental fields. With the relevant tree-level terms to lift the classical flat direction, it is known that this theory has a stable supersymmetry breaking vacuum \[30, 3\]. We can also check this supersymmetry breaking phenomenon by considering the dual gauge dynamics obtained in the previous section which has a non-abelian $Sp$ gauge group and non-chiral field content.

### Table 4.4: The deformed magnetic theory.

| $SU(4)$ | $SU(5)$ | $U(1)_R$ |
|---------|---------|---------|
| $P$     | 1       | 1       |
| $R$     | 2       | 1       |
| $T$     | 1       | 3       |
| $U$     | 1       | 8       |
4.2 New confining theories

In this section we present an example of new confining theories with a tree-level
superpotential as an application of our method. For supersymmetric gauge theo-
ries without tree-level superpotential, it is known that we can classify the sort of
theories according to the Dynkin index arguments. Especially, any theories which
reveal the s-confinement behavior have matter contents satisfying the index con-
straint $\sum \mu_i = \mu_G + 2$ and have been already listed completely \[17\]. This index
condition severely restricts the varieties of possible s-confining theories, conse-
quently the possibilities of their applications for model building. However, we do
not know what kind of theory confines in the presence of a tree-level superpoten-
tial. In this section, we show that some examples of this type of confining model
can be obtained by adding the relevant superpotential to the deconfined theories
so that one of the electric and magnetic theories can flow to the known confining
theories when taking into account the effects of this perturbation. A similar type
of confining theories has recently been obtained \[31\] by considering the case when
the dual gauge group reduces to trivial. New confining theories obtained in this
section are interpreted as the same type of theory as theirs, since our method can
be regarded as considering the cases in which the dual gauge group reduces to a
trivial one by confinement dynamics and/or the Higgs mechanism.

We proceed the analysis by taking the $SU(5) \times SU(4)$ model in the previous
section as an example, for simplicity. We use the same notation for the matter
fields as the previous one. Let us consider adding the following perturbative
superpotential in the deconfined theory:

$$ W = A^2QQ'_5 + AQ'_X, $$

where $Q'_5$ is the fifth component of $Q'$ and we introduced a new field $X$ which
is a gauge singlet but has the indices of the flavor symmetries. As one can see
below, the field $X$ is needed to guarantee that the low-energy magnetic theory
has no tree-level superpotential and surely confines. This type of singlet field is
generally used in the deconfinement method (see Section 2). By the first term
one of the flavor $SU(5)$ symmetry is broken to $SU(4)$ but other symmetries are
not broken (by changing the $U(1)$ quantum number assignment of the field $Q'$).

First, we see the magnetic $SU(4)$ theory deformed by these perturbations. In
this theory, these two terms in Eq. (4.7) correspond to the mass terms of $P, R, Q'_5$
and $X$. By integrating out these fields we have the following matter content as
shown in Table \[4.3\] ($Q'$ denotes the first four components of $Q'$) with the zero
tree-level superpotential. Exactly speaking, an additional $U(1)$ flavor symmetry
is enhanced. However, it rotates only $U$ field and therefore is irrelevant to the
following analysis. If one is anxious about this fact, one can introduce one more
singlet field into the deconfined theory and can integrate out $U$ away from the
low-energy physics. The above resultant $SU(4)$ theory is a supersymmetric QCD
with five flavor quarks and then s-confines. We show the confining spectra in
Table 4.5: The deformed magnetic theory.

Table 4.6 and the dynamically generated superpotential is given by

\[ W = ZZ'A + FZ\overline{F} + FZ\overline{F}'. \] (4.8)

This model certainly describes a low-energy limit of the deconfined theory with

Table 4.6: The low-energy effective theory of the magnetic theory.

the superpotential (4.8). The 't Hooft anomaly matching conditions and the equivalence of the quantum moduli space are satisfied between two theories. In this case, a new confining theory can be constructed by considering the electric side. By taking into account the SU(4) dynamics at first in the deconfined theory, we have the following electric SU(5) theory deformed by the perturbations terms (Table 4.7) with a tree-level superpotential;

\[ W = M^4M' + BM\overline{B} + BM'\overline{B}' + A^2M' + A\overline{Q}^2X. \] (4.9)

At a glance, this SU(5) theory is complicated. However, we have already known from the above argument that this theory must confine since it can be found that SU(5) gauge symmetry breaking does not occur with the tree-level superpotential. One interesting feature of this method is that the confining massless spectra which satisfy the 't Hooft anomaly matching conditions and their confining superpotentials are already given through another different pass in the above.
\[
M = [Q\hat{Q}'], \quad M' = [QQ'_4], \quad B = [Q^4], \\
\mathcal{B} = [\hat{Q}^aQ'_5], \quad \mathcal{B}' = [\hat{Q}'^a]
\]

Table 4.7: The deformed electric theory which is expected to confine.

|     | \(SU(5)\) | \(SU(5)\) | \(SU(4)\) | \(U(1)_1\) | \(U(1)_2\) | \(U(1)_R\) |
|-----|-----------|-----------|-----------|-----------|-----------|-----------|
| \(A\) | \(1\) | \(1\) | \(1\) | \(1\) | \(4\) | \(-6/5\) |
| \(\overline{Q}\) | \(1\) | \(1\) | \(-3/5\) | \(0\) | \(8/5\) |          |
| \(X\) | \(1\) | \(1/5\) |          | \(-4\) |          |          |
| \(M = [Q\hat{Q}']\) | \(1\) | \(1/2\) | \(2\) | \(-3/5\) |          |          |
| \(M' = [QQ'_4]\) | \(1\) | \(1\) | \(-2\) | \(-8\) | \(22/5\) |          |
| \(B = [Q^4]\) | \(1\) | \(1\) | \(0\) | \(-12\) | \(-12/5\) |          |
| \(\mathcal{B} = [\hat{Q}^aQ'_5]\) | \(1\) | \(1\) | \(-1/2\) | \(10\) | \(5\) |          |
| \(\mathcal{B}' = [\hat{Q}'^a]\) | \(1\) | \(1\) | \(2\) | \(20\) | \(0\) |          |

That is, they can be fixed from the viewpoint of the magnetic theory, and all we have to do is to translate them into the electric theory by pursuing the operator mapping. This operator mapping is also easily identified by decomposing the gauge invariant operators into the preon fields (namely, into the deconfined theory). The confining spectra can be written as follows in terms of the \(SU(5)\) theory;

\[
Z = [A\overline{Q}B], \quad Z' = [\overline{Q}M], \quad F = [\overline{Q}'B], \\
\mathcal{F} = \mathcal{B}', \quad \mathcal{F}' = [AM^3], \quad U = [\overline{Q}^3].
\] (4.10)

Here we used the same notation as in Table 4.6. The dynamically generated confining superpotential for these massless composite states is also given by Eq. (4.8) as previously.

It should be noted that in general, the consistency against deformations only says that quantum moduli spaces of two theories are still equivalent under these deformations. With our method in which we use smoothly confining theories without a superpotential, even the classical moduli spaces of the electric and magnetic theories are identical to each other. However, when we add perturbation terms to the superpotential, the equivalence of the classical moduli is no longer guaranteed but of course the quantum moduli are still equivalent. This situation certainly takes place in the above electric \(SU(5)\) theory between high- and low-energy descriptions. In the \(SU(5)\) theory, there exist some gauge invariant operators such as \([A\overline{Q}^2]\), \([A^2M]\), \([A^2M']\), \([AM^2M']\), \([\overline{Q}M']\), \([MB]\), \([M'B]\), \([M'M']\), \(\mathcal{B}\) and \(X\) in addition to the above composite states (4.10). Some of these operators do not span the classical moduli space due to the presence of the tree-level superpotential. However, there are surely remaining operators describing the classical moduli space of vacua in addition to the moduli space spanned by the operators (4.10). That is, the classical moduli space is not the same between the high-energy electric theory (Table 4.7) and the low-energy confined
theory (Table 4.6). Therefore this confinement dynamics is not s-confinement to be exact. However, additional classical flat directions are certainly stabilized quantum mechanically. Along such directions, the tree-level superpotential may change the dynamics of the electric theory into another phase. As a result of this dynamics, these moduli are not massless degrees of freedom in the low-energy physics as expected.

We can surely confirm these confining spectra and superpotentials from another dual theory in a nontrivial way. This other dual theory has an $SU(2) \times SU(2)$ gauge group and is obtained by applying the deconfinement method to the above electric theory and by considering the tree-level superpotentials as perturbation terms. The field content of this dual theory is shown in Table 4.8 and the superpotential is given by

$$W = \sum_{i=1}^{\text{number of terms}} M_i q_i \bar{q}_i + M' q' \bar{q}' + M'' q'' \bar{q}'' + M''' q''' \bar{q}''' + B_1 p \bar{q} + B'_1 p \bar{q}' + x l q + x l' q' + X l^2 + x^2 + M' \bar{B} + M'' \bar{B} + B'_1,$$  (4.11)

The tree-level superpotential (4.9) in the electric theory induces mass terms for some fields in the dual theory which are the classical moduli in the absence of this superpotential. After integrating out these fields, the $SU(2)_2$ gauge theory
has three flavor quarks and then confines. As a result, we have an $SU(2)_1$ theory whose matter content is displayed in Table 4.9. The superpotential is

$$W = M q \tilde{q} + M' q' \tilde{q}' + B_1 p \tilde{q} + B'_1 p q' + V q^2 + V' q q' + B'_1 + V X. \quad (4.12)$$

The $SU(2)_2$ confining dynamics also generates mass terms for some fields. Moreover, by the presence of the linear term $B'_1$, the $SU(2)_1$ gauge symmetry is broken and then some fields become massive. These massive modes just correspond to the classical moduli in the electric $SU(5)$ theory which are absent in the low-energy description (Table 4.6). Thus we finally arrive at just the same theory as that in Table 4.6 when we include a superpotential term induced by the one-instanton effect in the broken $SU(2)_1$ gauge group. In this way, in the product dual gauge theory the effects of the tree-level perturbation terms ensure that the extra classical moduli are surely decoupled from low-energy physics and the low-energy theory is described by a Wess-Zumino model. In other words, in the electric side, the finally obtained confining superpotential (4.8) produces the quantum constraints satisfied by the massless composite states which arise from taking into account the effects resulting from the $F$-flatness conditions of the tree-level superpotential and other quantum mechanics. We do not know the actual origin of this mechanics yet.

One more interesting feature is that the above confining $SU(5)$ theory is not restricted by the index constraint $\sum \mu_i = \mu_G + 2$ (but $\sum \mu_i > \mu_G + 2$) due to the presence of the tree-level superpotential. This is a general feature of confining
theories constructed by this method as mentioned in Section 2. In this way, we can construct this type of confining theory with various gauge groups, matter contents, and their tree-level superpotential by altering the deconfined theories (gauge and flavor symmetries) and perturbation terms in various ways. The wide possibility of this kind of confining theory may be interesting in considering phenomenological models, such as grand unified models, composite models of quarks and leptons, etc.

4.3 More examples with various labels

In this section we examine more examples of new duality in which both theories have a simple gauge group by considering deconfined theories with various types of labels. The low-energy effective behavior of the product gauge theory are also discussed in Refs. [21, 22]. According to Eq. (2.3) of the Dynkin index, we can discuss some feature of low-energy effective theories with various labels.

4.3.1 s-confinement \times s-confinement

One interesting application of our method is to investigate dualities for exceptional gauge groups. It is known that the only exceptional gauge group which can s-confine is $G_2$. By using this $G_2$ gauge theory in the deconfined theory with the label “s-confinement \times s-confinement”, we can find a dual description of an exceptional $G_2$ gauge theory. The duality in the exceptional $G_2$ gauge theory with $N_f$ fundamental representation matter has been discussed by Pouliot [12] in terms of the $SU(N_f - 3)$ chiral gauge theory. In this section we give a dual description of the $G_2$ gauge theory with an adjoint representation field. The matter content of the deconfined theory which we should now consider is shown in Table 4.10 and the tree-level superpotential is zero.

|       | $G_2$ | $Sp(4)$ | $U(1)$ | $U(1)_R$ |
|-------|-------|---------|--------|---------|
| $Q$   | $\square$ | $\square$ | 1      | 1/5     |
| $Q'$  | $\square$ | 1       | $-4$   | 1/5     |
| $q$   | 1     | $\square$ | $-7$   | 3/5     |

Table 4.10: The deconfined theory.

We can easily obtain the electric and magnetic theories in the same way as before. When we consider the $Sp(4)$ gauge dynamics first, we obtain the electric $G_2$ gauge theory. The matter content of this theory is given in Table 4.11. The superpotential of the electric $G_2$ gauge theory is

$$W = F^3F' + F^2\Phi F' + F\Phi^2F' + \Phi^3F'.$$

(4.13)
Table 4.11: The electric $G_2$ gauge theory.

On the other hand, when we consider the $G_2$ gauge dynamics before $Sp(4)$ gauge dynamics should be considered, we obtain the magnetic $Sp(4)$ theory. The matter content is shown in Table 4.12 and the superpotential of the magnetic

Table 4.12: The magnetic $Sp(4)$ gauge theory.

\[
\begin{array}{|c|c|c|c|}
\hline
  & S_{p}(4) & U(1) & U(1)_{R} \\
\hline
  M = [Q^{2}] & \text{adj.} & 2 & 2/5 \\
  M' = [QQ'] & \Box & -3 & 2/5 \\
  M'' = [Q'Q'] & 1 & -8 & 2/5 \\
  A = [Q^{3}] & \Box & 3 & 3/5 \\
  A' = [Q^{2}Q'] & \Box & -2 & 3/5 \\
  A'' = [Q^{2}Q'] & 1 & -2 & 3/5 \\
  B = [Q^{4}] & 1 & 4 & 4/5 \\
  B' = [Q^{3}Q'] & \Box & -1 & 4/5 \\
  q & \Box & -7 & 3/5 \\
\hline
\end{array}
\]

$Sp(4)$ gauge theory is given by

\[
W = M^4M'' + M^3M'^2 + A^2M^2 + A'A''M^2 + A''^2M^2 + AA'MM' + AA''MM' + A^2M'^2 + MB'^2 + M'B'B' + M''BB' + A^2B + A'^2B + A'AB' + A''AB'.
\] (4.14)

Let us consider the deformation of these theories. We add the following superpotential in the deconfined theory:

\[
W = QqQ'.
\] (4.15)

This superpotential breaks $U(1)$ flavor symmetry and the remaining flavor symmetry is R-symmetry. Then we obtain the superpotential of the electric theory as follows:

\[
W = F^3F' + F^2\Phi F' + F\Phi^2F' + \Phi^3F' + F'Q',
\] (4.16)
and this superpotential makes $F'$ and $Q'$ massive. After we integrate out these massive modes using their equations of motion, we have the following deformed electric theory which has a zero tree-level superpotential. This theory really has

\[
\Phi = [QQ] \quad \text{adj.} \\
F = [QQ] \quad 1/5
\]

Table 4.13: The deformed $G_2$ gauge theory.

the flavor symmetries $U(1) \times U(1)_R$. Therefore one $U(1)$ symmetry is missed. However, we expect that the duality is preserved by this deformation.

On the other hand, we can calculate the deformed magnetic theory including the superpotential (4.15). In the end we obtain the deformed magnetic theory whose field content is shown in Table 4.14 and a superpotential is

\[
W = M^4 M'' + A'^2 M^2 + A'' A'' M^2 + A'' M^2 + M'' B B + A^2 B + A'^2 B \\
+ A' A B' + A'' A B'.
\]

Table 4.14: The deformed magnetic theory.

This theory is expected to be a dual description of the $G_2$ gauge theory with one adjoint and one fundamental matter fields without superpotential.

This duality can be interpreted as the critical situation of the duality obtained by the deconfinement method. In this case the $G_2$ gauge theory is realized in the moduli of the $Spin(7)$ gauge theory with an adjoint and a spinor representation matters. If we give a VEV to the spinor field, the $Spin(7)$ gauge group is broken to $G_2$ gauge group and the components of the spinor field except for the moduli are absorbed by the Higgs mechanism. The adjoint representation in $Spin(7)$ gauge group is decomposed into an adjoint and a fundamental representation in $G_2$ gauge group. Therefore we obtain the electric $G_2$ gauge theory with an
adjoint and a fundamental matters. The duality in this $Spin(7)$ gauge theory can be obtained by the deconfinement method. When we consider the deconfined theory as the adjoint matter field is replaced by a composite state, the deconfined theory has $Spin(7) \times Sp(4)$ gauge group. The dual description of this $Spin(7)$ gauge theory with a spinor and some fundamental matters has been studied \[16\] and we obtain the $SU(2) \times Sp(4)$ as a dual gauge group. In order to obtain a dual description of the $G_2$ gauge theory, we need to deform the $SU(2) \times Sp(4)$ dual theory by taking into account the effects which correspond to giving a VEV to the spinor field in the electric $Spin(7)$ gauge theory. In the course of this deformation, several matter fields become massive and the $SU(2)$ gauge theory confines. Finally, we can exactly obtain the deformed magnetic theory in Table 4.14.

4.3.2 \textit{s-confinement} × \textit{runaway}

So far, we have discussed the deconfined theories with label “s-confinement × s-confinement”. In this section we consider the deconfined theories with other labels. Since the moduli space is holomorphic under couplings and furthermore theories which i-confine or have runaway behaviors can be obtained by mass deformations of s-confining theories, we expect to have consistent results in these cases when the relative ratio of two dynamical scales is exchanged.

We first consider the low-energy effective description of the deconfined theory with gauge group $G_1 \times G_2$ labeled by “s-confinement × i-confinement”. When the $G_1$ gauge theory confines (s-confinement) at first, the sum of indices of the resultant $G_2$ gauge theory is more than $\mu_{G_2} + 2$ from the inequality (2.5) and then the theory is expected to confine or have a dual description which describes the same infrared behavior. On the other hand, when first the $G_2$ gauge theory confines (i-confinement), the sum of indices of the resultant theory is more than $\mu_{G_1} + 2$, and the resultant theory is similarly expected to confine or have a dual description. Therefore we can expect that the “s-confinement × i-confinement” theory confines or has dual a description which is more relevant to describe the low-energy physics.

More nontrivial examples compared with the deconfined theories labeled by “s-confinement × i-confinement” are theories labeled by “s-confinement × runaway” without a tree-level superpotential. When we first consider the non-perturbative dynamics of the $G_2$ gauge group, the Affleck-Dine-Seiberg superpotential is generated and a VEV of some field tends to go to infinity. Since theories which have runaway behavior are obtained by mass deformations of s-confining theories and we have explicitly seen the equivalence of moduli space between the electric and magnetic theories obtained from the deconfined theory labeled by “s-confinement × s-confinement” by giving some examples, we expect that same phenomenon is supposed to occur when we first consider the strong dynamics of $G_1$ gauge group. When the $G_1$ gauge theory confines (s-confinement),
we obtain the magnetic theory in which the sum of indices increased by more than 2. If we take the $G_2$ theory which has runaway behavior with the sum of indices $\mu_{G_2} - 2$ in the deconfined theory (for example, the $SU(N)$ gauge theory with $(N - 1)$ fundamental flavors), the $G_2$ gauge theory is expected to confine or have a dual description. The above fact that a VEV of some field must go to infinity in the electric $G_1$ theory tells that an expectation value of some field in the magnetic side must go to infinity when we solve the equations of motion at the low-energy limit though no dynamics which induces the runaway behavior seems to occur. Let us check this expectation by using the following example.

The field content of the deconfined theory is given in Table 4.15 and the tree-

|     | $SU(4)$ | $Sp(4)$ | $SU(5)$ | $U(1)$ | $U(1)_R$ |
|-----|---------|---------|---------|--------|----------|
| $Q$ | □       | □       | 1       | 0      | $-1/2$   |
| $q$ | □       | 1       | 1       | $-5$   | 4        |
| $Q'$| □       | □       | 1       | 1      | 0        |

Table 4.15: The deconfined theory.

level superpotential is zero. The sum of indices of the $SU(4)$ and the $Sp(4)$ gauge groups are $\mu_{SU(4)} + 2$ and $\mu_{Sp(4)} - 2$, respectively.

Let us consider the $Sp(4)$ gauge dynamics first. In the electric $SU(4)$ gauge theory, the Affleck-Dine-Seiberg superpotential is generated and the VEV of $Q$ goes to infinity:

$$\langle Q \rangle_\alpha^a = s \delta_\alpha^a, \quad s \to \infty,$$

(4.18)

where $\alpha$ and $a$ denote the index of the $SU(4)$ and $Sp(4)$ gauge group, respectively. Since we should consider the D-flatness condition of $SU(4)$ gauge group, the form of $\langle Q \rangle$ is restricted to diagonal form. Along this direction, the product gauge symmetry is broken to diagonal $Sp(4)$ gauge symmetry and the field content of this electric theory is given in Table 4.16 where $U(1)'_R$ is the remaining $R$-symmetry which has no gauge anomaly.

|     | $Sp(4)$ | $SU(5)$ | $U(1)$ | $U(1)'_R$ |
|-----|---------|---------|--------|-----------|
| $q$ | □       | 1       | $-5$   | 0         |
| $Q'$| □       | □       | 1      | 0         |

Table 4.16: The electric $Sp(4)$ theory.

Since this $Sp(4)$ gauge theory has six fundamental fields, the theory confines with chiral symmetry breaking (i-confinement) and becomes a Wess-Zumino
model. The matter content of the low-energy effective theory is shown in Table 4.17 and the superpotential is given by

\[
W = X(FV^2 - \Lambda_D^3),
\]

where \(X\) is an auxiliary field and \(\Lambda_D\) is the dynamical scale of the diagonal \(Sp(4)\) gauge group. The constraint obtained from the equation of the motion by \(X\) breaks the global symmetry \(SU(5)\) to \(Sp(4)\).

On the other hand, when we consider the \(SU(4)\) gauge dynamics first the \(SU(4)\) gauge symmetry confines without chiral symmetry breaking (s-confinement). The matter content of the magnetic \(Sp(4)\) gauge theory is given in Table 4.18 and the superpotential is

\[
W = M^4F + SF\overline{F} + NM\overline{F}.
\]

As discussed before, the sum of indices increases by just 2 in this case and the sum of indices of \(Sp(4)\) gauge group in the magnetic theory is equal to \(\mu_{Sp(4)}\). Since this \(Sp(4)\) gauge theory has six fundamental matters, this theory confines with chiral symmetry breaking (i-confinement) and the field content in the low-energy effective description of the magnetic theory is given in Table 4.19. The superpotential is

\[
W = A^2F + SF\overline{F} + P\overline{F} + X(A^2P - \Lambda_M^3),
\]

Table 4.17: The low-energy description of the electric theory.

|     | \(SU(5)\) | \(U(1)\) | \(U(1)_R\) |
|-----|-----------|-----------|-------------|
| \(F = [Q'q]\) | \(\Box\) | \(-4\) | \(0\) |
| \(V = [Q'Q']\) | \(\Box\) | \(2\) | \(0\) |

Table 4.18: The magnetic \(Sp(4)\) theory.
where $X$ is an auxiliary field and $\Lambda_M$ is the dynamical scale of the magnetic $Sp(4)$ gauge group. From the equations of motion we find the runaway behavior

$$\langle S \rangle \rightarrow \infty.$$  \hfill (4.22)

The superpotential makes a pair of $SU(5)$ fundamental and antifundamental representation matter fields massive. In addition, the constraint derived from the equation of motion by $X$ says that the global symmetry $SU(5)$ is broken to $Sp(4)$.

In this way, we finally obtain the consistent results between the electric and magnetic theories because the VEV $s$ in Eq. (4.18) surely corresponds to $\langle S \rangle$ in the magnetic theory, and the flavor symmetries and the massless spectra are same between two theories. Therefore, the expected runaway behavior is certainly observed in the magnetic side as $\langle S \rangle$ goes to infinity.

4.3.3 affine-confinement $\times$ s-confinement

As another type of confinement dynamics, it is known that there is the affine confinement $[19, 20]$ in which there is no constraint between gauge invariant operators and therefore the confining superpotential is not generated at the low-energy region. Among the known affine-confining theories, the only candidate having non-abelian flavor symmetries is the $SO(N)$ gauge theory with $N - 4$ vector representation matters. We try to consider the product gauge theory using this $SO$ affine-confining theory.

The field content of the deconfined theory is displayed in Table 4.20 and the tree-level superpotential is zero. This theory has no global symmetry. After the $Sp(2N)$ confinement, the matter content of the electric theory is given in Table 4.21.

This theory has a superpotential

$$W = \frac{1}{\Lambda_{sp}^{N-1}} \text{Pf} \varphi,$$ \hfill (4.23)

Table 4.19: The low-energy description of the magnetic theory.

|        | $SU(5)$ | $U(1)$ | $U(1)_R$ |
|--------|---------|--------|----------|
| $A = [MM]$ |         | 2      | -1       |
| $P = [MN]$ |         | -4     | 2        |
| $F$   |         | -4     | 4        |
| $S$   | 1       | 0      | -2       |
| $\overline{F}$ | 4     | 0      |          |
Table 4.20: The deconfined theory with the label "affine-confinement × s-confinement".

| $SO(2N + 4)$ | $Sp(2N)$ |
|---------------|-----------|
| $Q$           | $\square$ |

Table 4.21: The electric theory.

Table 4.22: The magnetic theory

where $\Lambda_{sp}$ is the dynamical scale of $Sp$ gauge group.

On the other hand, we first consider the $SO$ gauge dynamics. In this case, there is a branch where the $SO(2N + 4)$ gauge theory with $2N$ flavors confines and no dynamical superpotential is generated (affine-confinement). Then the matter content of the magnetic theory is given in Table 4.22 and the superpotential is zero.

It is found that the degrees of freedom of the moduli space correspond to each other. The gauge invariant operators of each theory are written as follows:

\[
\text{electric} \quad \text{magnetic} \quad (4.24)
\]

\[
\text{tr} \varphi^2, \text{tr} \varphi^4, \ldots, \text{tr} \varphi^{2N+2}, \text{Pf} \varphi, \quad \text{tr} \Phi^2, \text{tr} \Phi^4, \ldots, \text{tr} \Phi^{2N}.
\]

The gauge invariant operators $\text{tr} \varphi^{2N+2}$ and $\text{Pf} \varphi$ in the electric theory are set to be zero by the equation of motion obtained from the dynamically generated superpotential (4.23).

At a glance this magnetic $Sp(2N)$ gauge theory seems to be an $\mathcal{N} = 2$ supersymmetric theory. Of course, since we have little information on the Kähler potential, this theory may have only $\mathcal{N} = 1$ supersymmetry. However, there may exist somewhere in the moduli space where the low-energy effective theory becomes an $\mathcal{N} = 2$ supersymmetric theory. Similarly, the R-symmetry also seems to be enhanced. We may understand this fact in the same way as we understand the enhancement of supersymmetry.
It is interesting that if the magnetic theory has $\mathcal{N} = 2$ supersymmetry in the infrared region, the above obtained duality between the $SO$ and $Sp$ gauge theories is a duality between $\mathcal{N} = 1$ and $\mathcal{N} = 2$ theories. In other words, the electric $SO$ gauge theory with the superpotential (4.23) has $\mathcal{N} = 2$ supersymmetry in the infrared limit and the quantum moduli space is the same as that in the $Sp$ magnetic theory.

5 Summary and discussion

In this paper, we have studied the $\mathcal{N} = 1$ supersymmetric gauge theories with product gauge group with various labels, especially “s-confinement $\times$ s-confinement”. We can investigate the behaviors of low-energy effective theories by using the facts that the equivalence of moduli is valid for all value of two dynamical scales and the inequality for the Dynkin index between the elementary and the infrared matter fields. As a consequence, we can find many dualities in theories which include the higher-rank tensor representation fields. In this case, we generally obtain the dual theories based on a simple gauge group, not a product one, and both electric and magnetic theories have a tree-level superpotential. Furthermore, we find that some of them are new dualities between the chiral and non-chiral gauge theories. By giving a few examples, we have justified this duality assumption in a nontrivial way by using the known dualities and other strong gauge dynamics. In some cases, the dual pairs obtained by our method can be interpreted as the critical situations of the known dualities obtained by the deconfinement method. However, among theories which have more flavors than s-confining theories, there are only a few theories whose magnetic descriptions have been known. Therefore, our method can be a more powerful tool than the usual deconfinement method in order to find new dual pairs. Conversely, we may use this duality assumption as a hint to find dual theories with a product gauge group which contains matter in the higher-rank tensor representations.

It is noted that the obtained duality is not the so-called strong-weak duality. We expect that there may exist some other dual theory with weak gauge coupling which is relevant to describe the low-energy physics, and we have presented some examples.

It is interesting to consider some applications of new dualities by deforming these theories. With these deformations, in this paper we have discussed the dynamical supersymmetry breaking phenomena, new confining theories, and other interesting theoretical models. In every case, we can see that the consistent behaviors between the two theories are observed. New confining theories obtained in this paper have several characteristic features. These theories are not restricted by the constraint $\sum \mu_i = \mu_G + 2$ because of the existence of the tree-level superpotential. Moreover, some moduli in the classical theories disappear in the infrared region. However, some unknown physics may be needed to have a proper
understanding of the consistency between two theories in studying new confining theories and the affine-confinement dynamics. We leave these matters to future investigations.

Moreover, the applications to phenomenological issues may also be attractive. In this manner, we can construct various types of dualities and confining theories. They generally contain higher-rank tensor fields and have a nonzero superpotential. In addition, the obtained new confining theories are not restricted by the index argument and have wide possibilities for applying to model building. It is interesting that one may find dual or more microscopic descriptions of realistic models of our world.

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References

[1] For a recent review, see E. Poppitz and S.P. Trivedi, hep-th/9803107.

[2] A. Nelson and M.J. Strassler, Phys. Rev. D56 (1997) 4226; M.A. Luty and R.N. Mohapatra, Phys. Lett. 396B (1997) 161; D.B. Kaplan, F. Lepelitre and M. Schmaltz, Phys. Rev. D56 (1997) 7193; N. Kitazawa, hep-ph/9806229.

[3] N. Seiberg, Nucl. Phys. B435 (1995) 129, “Electric-Magnetic Duality in Supersymmetric Non-Abelian Gauge Theories”

[4] M. Berkooz, Nucl. Phys. B452 (1995) 513, “The Dual of Supersymmetric SU(2k) with an Antisymmetric Tensor and Composite Dualities”

[5] P. Pouliot, Phys. Lett. 367B (1996) 151

[6] M. Luty, M. Schmaltz and J. Terning, Phys. Rev. D54 (1996) 7815

[7] T. Sakai, Mod. Phys. Lett. A12 (1997) 1025

[8] W.C. Su, hep-th/9707076

[9] J. Terning, Phys. Lett. 422B (1998) 149

[10] D. Kutasov, Phys. Lett. 351B (1995) 230, “A Comment on Duality in N=1 Supersymmetric Non-Abelian Gauge Theories”
[11] D. Kutasov and A. Schwimmer, Phys. Lett. 354B (1995) 315; R.G. Leigh and M.J. Strassler, ibid. 356B (1995) 492

[12] P. Pouliot, Phys. Lett. 359B (1995) 108, “Chiral Duals of Non-Chiral SUSY Gauge Theories”

[13] P. Pouliot and M.J. Strassler, Phys. Lett. 370B (1996) 76

[14] P. Pouliot and M.J. Strassler, Phys. Lett. 375B (1996) 175

[15] T. Kawano, Prog. Theor. Phys. 95 (1996) 963

[16] P. Cho, Phys. Rev. D56 (1997) 5260

[17] C. Csáki, M. Schmaltz and W. Skiba, Phys. Rev. D55 (1997) 7840, “Confinement in N=1 SUSY Gauge Theories and Model Building Tools”

[18] B. Grinstein and D.R. Nolte, Phys. Rev. D57 (1998) 6471; ibid. D58 (1998) 045012

[19] G. Dotti and A. Manohar, Phys. Rev. Lett. 80 (1998) 2758, “Supersymmetric Gauge Theories With an Affine Quantum Moduli Space”

[20] G. Dotti, A. Manohar and W. Skiba, Nucl. Phys. B531 (1998) 507

[21] K. Intriligator, R.G. Leigh and N. Seiberg, Phys. Rev. D50 (1994) 1092, “Exact Superpotentials in Four Dimensions”
E. Poppitz, Y. Shadmi and S.P. Trivedi, Nucl. Phys. B480 (1996) 125, “Duality and Exact Results in Product Group Theories”

[22] K. Intriligator and S. Thomas, Nucl. Phys. B473 (1996) 121; hep-th/9608046

[23] G. Dotti and A.V. Manohar, Phys. Lett. 410B (1997) 167; Nucl. Phys. B518 (1998) 575

[24] N. Seiberg, Phys. Rev. D49 (1994) 6857; K. Intriligator and P. Pouliot, Phys. Lett. 353B (1995) 471

[25] I. Affleck, M. Dine and N. Seiberg, Phys. Lett. 137B (1984) 187

[26] K. Konishi, Phys. Lett. 135B (1984) 439

[27] H. Murayama, Phys. Lett. 355B (1995) 187; E. Poppitz and S.P. Trivedi, ibid. 365B (1996) 125

[28] M.A. Luty and J. Terning, Phys. Rev. D57 (1998) 6799

[29] Y. Shirman, Phys. Lett. 389B (1996) 287
[30] I. Affleck, M. Dine and N. Seiberg, *Nucl. Phys.* **B256** (1985) 557

[31] C. Csáki and H. Murayama, *Phys. Rev.* **D59** (1999) 065001