Mathematical Modeling on Levitating Forces of Droplets with Electromagnetic Levitation

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Abstract. This research focuses on studying levitating forces of droplets by utilizing a refining process known as electromagnetic levitation (EML). With the mathematical predictions, the levitating forces were described by a mathematical model in this article. Combined with theory analysis, the balancing phenomenon was also investigated in silicon-iron systems. The research found that the relationship between droplets position and levitation forces was linear. As EML system was a multi-induction, the levitating forces generation was affected by the properties of droplet, quality and size. The electromagnetic properties of the droplets were mainly the reasons for the levitation force to change from heating process before melting. EML droplet getting its melting point, electromagnetic properties of EML droplet were fixed due to the droplets absorbing latent heat. The results of this investigation will be used for vacuum electromagnetic levitation technology, and solar grade silicon samples will be prepared from relative inexpensive raw material, metallurgical grade silicon.

1. Introduction
The electromagnetic levitation (EML) technology has a long history, and people have performed many metallurgical experiments with it\cite{1,2}. Today EML is used to assist experiments on melting, purifying, undercooling, and measuring properties\cite{1}, such as removing phosphorus from metallurgical grade silicon or ferrosilicon using EML method to provide solar photovoltaic industry with low cost silicon materials, hence bring the cost of solar energy down\cite{3}. This research focuses on studying levitating forces of droplets during phosphorus removal from ferrosilicon alloys by utilizing a refining process known as EML with subjecting the levitated alloy to an argon-hydrogen gas flow. With the mathematical predictions\cite{2}, the levitating forces were described by a mathematical model in this article. Combined with theory analysis, the balancing phenomenon was also investigated in ferrosilicon systems.

2. Theoretical analysis
2.1. The heating Mechanism of the EML
Figure 1 below depicts the principle of electromagnetic levitation. Levitation coil design for levitation melting is essential to provide adequate lifting force, induction heating, and droplet stability.
The generator of the EML generates a non-uniform high frequency alternating current passes through the levitation coil with subjecting the levitated alloy to an argon-hydrogen gas flow[1]. When a conductive sample such as iron is placed in between the non-uniform, high frequency electromagnetic field generated by the levitation coil, the eddy current is induced on the material surface. The eddy current then generate heat according to the relationship[3,4]:

\[ H = I^2R \]  

Where \( H \) is the heat, \( I \) is the current density, and \( R \) is the electrical resistance. Normally, the eddy current will flow at a direction opposite to the current flow direction of the levitation coil. It will also generate its own field in opposition to the field generated by the coil and thereby prevent the field from penetrating to the center of the sample.

2.2. The levitation Mechanism of the EML

A conductor can be made to levitate by balancing the electromagnetic body force (Lorentz force) and the gravity force which can be described using the governing equation [3-5]:

\[ J \times B = \rho g \]  

Where \( J \) is the eddy current induced by the alternating magnetic field \( B \), \( \rho \) is the density of the levitated material and \( g \) is the gravitational constant. The penetration depth of the eddy current, \( d \), is given by [3-6]:

\[ d = \frac{\rho}{\pi\mu_0\mu_f^2} \]  

Where \( \rho \) is the resistivity of the sample; \( \mu_0 \) is the magnetic permeability of the vacuum; \( \mu \) is the relative magnetic permeability of the sample; and \( f \) is the frequency of the alternating current. According to the above relation the higher the frequency the thinner or more shallow the heating. On the same time, the interaction between the eddy currents and applied field will generate a lifting force to hold the sample [7-9]. When the currents in the upper and lower coils flow in opposite directions, a field is produced in which the magnetic field is zero at the center point between the coils and increases in every direction outward. This magnetic well is capable of supporting the sample in the space without contacting any holder.

2.3. Numerical Modeling

The levitation model of EML equipment was shown in Figure 2. The levitation system was shown in Figure 2 (a) and Figure 2 (b) was simulated in the present study[2]. The calculations were performed by ANSYS soft, and the variation of physical properties while heating was considered. Electromagnetic field was calculated by Maxwell’s Equation 4 to Equation 7, and the Lorentz force of
EML was calculated by Equation 8[10-12].

\[
\nabla \times H = J + \frac{\partial D_1}{\partial t} = J_S + J_E + J_V + \frac{\partial D_1}{\partial t} \\
\n\nabla \times E = -\frac{\partial B}{\partial t} \\
\n\n\nabla \cdot B = 0 \\
\n\n\nabla \cdot D = \rho_1 \\
\nF = \frac{1}{2} \int_{vol} N^T (J^* \times B) d(vol)
\]

Where \( H \) is magnetic field intensity vector, \( J \) is Total current density vector, \( D_1 \) is electric flux density vector, \( t \) is time, \( J_S \) is applied source current density vector, \( J_E \) is induced eddy current density vector, \( J_V \) is velocity current density vector, \( E \) is electric field intensity vector, \( B \) is magnetic flux density vector, \( \rho_1 \) is electric charge density, \( F \) is levitation force, \( N \) is vector of shape functions, \( J^* \) is complex conjugate of \( J \).

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3. Results and discussions

3.1. levitation forces of EML
Mathematical modeling on levitation forces was shown in Figure 3 [2].

(a) levitation forces of difference current (b) levitation forces of 100A current

Figure 3. Mathematical modeling on levitation forces of EML (Coil design, D=6mm, f=320KHz)
Figure 3 revealed that around droplet balanced position the relationship between the droplet position and the levitation force and is linear. Since the relationship between the shape change of a spring and elastic force fits Equation 9, the levitation force of droplet can be described as a spring. A modified mathematical model of spring was built, it was used to describe the EML system, and it was shown in Figure 4. The resultant upward force exerted on droplet and levitation force was recorded. The droplet can be lifted if the $F_{\text{resultant}}$ showed in Equation 10 was positive. The static deformation on the spring $\lambda$ can be recorded as the distance between positions where $F_{\text{resultant}}$ and $F$ become zero by moving the droplet through initial position $Z=0$. The gravity $G$ and the static deformation of the spring $\lambda$ will follow Hooke’s Law as described in Equation 6, where $c$ is the spring stiffness[13-15].

\[
G = c\lambda \quad (9)
\]

\[
F_{\text{resultant}} = F - G \quad (10)
\]

Where $G$ is gravity, $c$ is stiffness of the spring, $\lambda$ is the static deformation of the spring, $F_{\text{resultant}}$ is resultant upward force.

The levitation force will push droplet back, and droplet may be lead to droplet oscillation if the droplet was not placed at its position of balanced. The frequencies on the oscillation with free vibration as well as the with damping is defined as $\omega_0$ and $\omega'$, in Equation 11 and Equation 12 respectively[16-19].

\[
\omega_0 = 2\pi f_s = \sqrt{\frac{c}{m}} \quad (11)
\]

\[
\omega' = \sqrt{\omega_0^2 - \beta^2 (\beta < \omega_0)} \quad (12)
\]

Where $\beta = \frac{\gamma}{2m}$, $\omega_0$ is angular frequency under free vibration, $f_s$ is frequency, $\omega'$ is angular frequency under under damping, $\gamma$ is damping coefficient, $m$ is the mass of the droplet.

The functions on oscillation of the droplet with free damping and vibration are as follows, in Equation 13 and Equation 14 respectively [20-23]:

\[
y = A\sin(\omega_0 + \alpha) \quad (13)
\]

\[
y = Ae^{\beta t}\cos(\omega + \alpha) \quad (14)
\]

Where $\alpha$ is the initial phase. $A$ is the maximum amplitude, $\alpha$ and $A$ are related to the initial oscillation condition $x_0$. 
In the modified mathematical model of spring, oscillation frequencies of the droplet will be affected by the spring stiffness. The spring stiffness is the spring intrinsic property, and is connected with the generation of levitation forces. EML system being a multi-induction, the magnitude of levitating forces is affected by the properties of droplet, quality of droplet and size of droplet.

3.2. Oscillation frequency of EML
Oscillation frequency with current change was shown in Figure 5.

Figure 5 indicated that EML droplets will oscillate under frequencies between 9.37Hz to 22.23Hz, the increasing current will lift the droplet, and EML currents to lift metals is increased. The increasing current will lift the levitation of metals, and lead to the frequency increase of oscillation. The droplet cannot be levitated successfully if the levitating force cannot counter the gravity of the droplet. Metals can be lifted in the settled EML system, and these stiffness are in the range of 4 to 6.

3.3. Levitation force under difference temperature
Levitation force of droplets under difference temperature was shown in Figure 6. The specimens used in the preliminary levitation mathematical modeling are available ferrosilicon with analysis shown in Table 1.
### Table 1. Chemical analysis of charge materials (wt%)

| Element | Si  | F   | P   | S   | C   |
|---------|-----|-----|-----|-----|-----|
| Content of Ferrosilicon | 76% | balance | 0.04% | 0.02% | 0.2% |

Figure 6 indicated that the properties electromagnetic of the droplets were mainly the reasons for the levitation force to change during heating process before melting. EML droplet getting its melting point, electromagnetic properties of EML droplet are fixed due to the droplets absorbing latent heat. The drastic variation on the levitating forces of the melting point is contributed by the change in volume during melting, and leading to a balance position movement with an obvious oscillation on the levitated droplet.

### 4. Conclusion

With the mathematical predictions, the levitating temperature and levitating forces were described by a mathematical model. Combined with theory analysis, the balancing phenomenon was also investigated in silicon-iron systems. The relationship between the droplet position and the levitation force is linear. EML system being a multi-induction, the magnitude of levitating forces is affected by the properties of droplet, quality of droplet and size of droplet. The electromagnetic properties of the droplets were mainly the reasons for the levitation force to change from heating process before melting. When EML droplet got its melting point, electromagnetic properties of EML droplet were fixed since the droplets had to absorb latent heat.

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