The formation of voids in a universe with cold dark matter 
and a cosmological constant

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\textbf{ABSTRACT}

A spherical Lagrangian hydrodynamical code has been written to study the formation of cosmological structures in the early Universe. In this code we take into account the presence of collisionless non-baryonic cold dark matter (CDM), the cosmological constant and a series of physical processes present during and after the recombination era, such as photon drag resulting from the cosmic background radiation and hydrogen molecular production. We follow the evolution of the structure since the recombination era until the present epoch. As an application of this code we study the formation of voids starting from negative density perturbations which evolved during and after the recombination era. We analyse a set of \textit{COBE}-normalized models, using different spectra to see their influence on the formation of voids. Our results show that large voids with diameters ranging from $10^\text{h}^{-1}$Mpc up to $50^\text{h}^{-1}$Mpc can be formed in a universe model dominated by the cosmological constant ($\Omega_{\Lambda} \sim 0.8$). This particular scenario is capable of forming large and deep empty regions (with density contrasts $\bar{\delta} < -0.6$). Our results also show that the physical processes acting on the baryonic matter produce a transition region where the radius of the dark matter component is greater than the baryonic void radius. The thickness of this transition region ranges from about tens of kiloparsecs up to a few megaparsecs, depending on the spectrum considered. Putative objects formed near voids and within the transition region would have a different amount of baryonic/dark matter when compared with $\Omega_b/\Omega_d$. If one were to use these galaxies to determine, by dynamical effects or other techniques, the quantity of dark matter present in the Universe, the result obtained would be only local and not representative of the Universe as a whole.

\textbf{Key words:} hydrodynamics – galaxies: formation – cosmology: observations – cosmology: theory – dark matter – large-scale structure of Universe.

\section{INTRODUCTION}

In the 1970s a series of studies showed the existence of filamentary distribution of galaxies in clusters and superclusters and large void regions among such filaments. Gregory & Thompson (1978) found in galaxy redshift surveys, in the direction of the Coma cluster, a filamentary structure that is part of the nowadays denominated ‘Great Wall’ (Geller & Huchra 1989). This supercluster is encompassed by a large region of low mass density (or void).

Additional investigations by Gregory, Thompson & Tifft (1981) and Chincarini, Rood & Thompson (1981) showed similar results for the Hercules and Perseus-Pisces superclusters and, later, other redshift surveys showed that the superclusters are boundaries of underdense regions (Giovanelli, Haynes & Chincarini 1986; de Lapparent, Geller & Huchra 1986; da Costa et al. 1988).

A redshift survey in the Boötes region (Kirshner et al. 1981) discovered a large void structure whose estimated diameter is $\sim 60^\text{h}^{-1}$Mpc. Later, observations showed that this region in Boötes is not totally empty of matter - it contains some galaxies (Thuan, Gott & Schneider 1987; Dey, Strauss & Huchra 1990). In spite of this, the void in Boötes has a lower mass density as compared to the mass density of the Universe.

In particular, Dey et al. (1990) found 21 galaxies (of which 13 are \textit{IRAS} sources) in Boötes, which led the authors to estimate the average density contrast of this region to be $-0.84 \leq \bar{\delta} \leq -0.66$. More recently, Szomoru et al. (1996a,b) analysed, through the VLA (Very Large Ar-
ray) technique, \( \sim 1 \) per cent of the volume of the void in Boötes (\( \sim 1100 h^{-3} \text{Mpc}^3 \)). These authors found that galaxies in Boötes are systems rich in gas, composed by late-type galaxies. The study of their optical properties shows that these systems are very similar to field galaxies of the same morphological type.

From the theoretical point of view, some models of galaxy formation (Dekel & Silk 1986) that studied the cold dark matter (CDM) scenarios including a bias parameter, predict that dwarf galaxies should originate from the 1\( \sigma \) fluctuations, thus being more smoothly distributed than the rare high-density peaks that form the more massive galaxies. In these scenarios, the dwarf galaxies could trace the dark matter and also fill up the voids. On the other hand, Popescu, Hopp & Elsässer (1997), using results of a search for emission-line galaxies (ELGs), did not find that the voids are occupied by a homogeneous population of dwarf galaxies. They found in their sample a few galaxies near the void regions, but the number of void galaxies is not significant as compared to the field galaxies.

It is worth stressing that, although the Boötes void has been one of the first underdense region studied, the distribution of galaxy clusters present in the Abell catalogue also reveals the existence of large empty regions with diameters of \( \sim (20 – 50) h^{-1} \text{Mpc} \) (Bahcall & Soneira 1983; Huchra et al. 1990).

Some authors (Goldwirth, da Costa & Van de Weygaert 1995) argue that the typical size of underdense regions (voids) is related to the first zero of the correlation function \( \xi(r) \), which gives \( \sim 20 h^{-1} \text{Mpc} \). This length is less than the void region sizes found in studies of the large-scale structure of the Universe.

Recently, El-Ad, Piran & da Costa (1996), using a particular way to define voids, have found an average diameter of 37 \( \pm 9 h^{-1} \text{Mpc} \). In their particular method to define voids they allow the existence of field galaxies therein, this being a very reasonable assumption because galaxies could in principle be formed therein or escape to there, as a result of mutual galaxy interactions.

Also, quite recently, Müller et al. (2000) studied the distribution of void regions in the two-dimensional slices of the Las Campanas Redshift Survey (LCRS; the deepest redshift survey presently available). They found that the distribution of void sizes scales with the mean galaxy separation, \( \lambda \). In particular, they found that the size of voids covering half of the area is given by \( D_{\text{med}} \approx \lambda + (12 \pm 3) = (20 – 30) h^{-1} \text{Mpc} \), where \( D_{\text{med}} \) is the median void diameter.

In the past, the models addressed to explain the formation of voids in the distribution of galaxies considered their formation as a result of the process of clustering (Aarseth & Saslaw 1982; Peebles 1982; Vettolani et al. 1985) or as a result of the evolution, after the recombination era, of negative density perturbations (Hausman, Olson & Roth 1983; Hoffman & Shalam 1983; Filmore & Goldberg 1984; Bertschinger 1985; de Araujo & Opher 1990, 1993).

The formation of voids as a result of explosions of primordial objects, such as quasars or other pre-galactic objects (Ikeuchi, Tomisaka & Ostriker 1983) was also considered. This scenario, however, might have some problems because of the distortions that could be caused on the cosmic background radiation. On the other hand, this explosive scenario was revisited by Miranda & Opher (1996, 1997) and the formation of voids through such a mechanism may be consistent, within some limits, with the COBE satellite data.

In an interesting study, Srianand (1997) has reported the detection of a large void \( \sim 7 h^{-1} \text{Mpc} \) centred near the quasi-stellar object (QSO) Tol 1037-2704, where this void could be produced by an excess of ionization due to the QSO, thereby constituting another mechanism to produce voids of several megaparsecs.

Also, de Araujo & Opher (1997) considered an unusual possibility for the formation of void regions, namely, through putative primordial magnetic fields.

However, after many years of studies it is not completely clear how voids are really formed, particularly the largest ones, such as those found by El-Ad et al. (1996) and Müller et al. (2000). It is possible to have their formation through any of the mechanisms mentioned above.

It is worth mentioning that, although the dimensions of voids depend on the particular technique employed to find them (see e.g. Lindner et al. 1995, 1999; El-Ad et al. 1996; EL-Ad & Piran 1997; Müller et al. 2000), their very existence is widely accepted.

Concerning the presence of non-baryonic dark matter in the structures of the Universe, there is considerable evidence for its existence. It has been argued that non-baryonic dark matter can dominate the density and the dynamics of the Universe (for reviews see Kolb & Turner 1990; Primack 1997). Thus, there is no reason to believe that dark matter is not present inside voids; therefore, it is important to take into account this kind of matter in the study of the formation of underdense regions, in order to verify its role.

It is also important to stress that a calculation taking into account the non-baryonic dark matter and a series of effects during and after the recombination era, focused on void formation, has so far not been performed.

In the present work we use a spherical Lagrangian hydrodynamical code, written by one of us (ODM), which allowed us to study the formation of cosmological structures. Besides the presence of the non-baryonic dark matter, we consider in our calculations the expansion of the Universe, the photon drag due to the cosmic background radiation, the recombination processes and molecular hydrogen formation. We also included, in our model, the presence of the cosmological constant to see its influence on the dimension of void regions.

In Section 2 we describe the model, in Section 3 we present the numerical results and the discussions, and in Section 4 we present the conclusions.

### 2 THE MODEL

We assume spherically symmetric negative density perturbations which produce clouds of baryonic and non-baryonic dark matter with densities lower than the density of the Universe. We treat the dark matter and the baryonic matter as two fluids coupled by gravity. We assume, as an initial condition, a top-hat density profile for the void region.

The hydrodynamic equations that describe the dynamics of the baryonic density perturbations are

\[
\frac{\partial \rho_b}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \rho_b v_b \right) = 0, \tag{1}
\]
(the mass conservation equation to the baryonic matter written in spherical coordinates), where $\rho_b$ is the baryonic mass density, $v_b$ is the velocity and $r$ is the radial coordinate.

The equation of motion is given by

$$\frac{Dv_b}{Dt} = -\frac{1}{\rho_b} \frac{\partial P}{\partial r} - \nabla \Phi - \frac{4}{3} \frac{\sigma T^4 x_e}{m_p c} [v_b - H(t)r],$$

(2)

where $P$ is the pressure of the gas, $x_e$ is the degree of ionization, $b = 4\sigma T^4 / c$ (with $\sigma T^4$ the Stefan-Boltzmann constant), $T_r$ is the temperature of the radiation, $m_p$ is the proton mass, $c$ is the velocity of light and $H(t)r$ is the Hubble flow, which takes into account the expansion of the Universe. The last term on the right-hand side of equation (2) describes the photon drag and $D/Dt \equiv \partial / \partial t + v \partial / \partial r$ is the convective derivative.

The field equation (Poisson’s equation) is

$$\nabla^2 \Phi = 4\pi G(\rho_b + \rho_d) - \Lambda c^2,$$

(3)

where $\rho_d$ is the density of the non-baryonic dark component and $\Lambda$ is the cosmological constant.

The density parameters of the Universe read

$$\Omega_b + \Omega_d + \Omega_k + \Omega_\Lambda = 1,$$

(4)

where

$$\Omega_b = \frac{\Lambda c^2}{3H_0^2} \quad \text{and} \quad \Omega_k = \frac{k c^2}{R^2 H_0^2}.$$  

(5)

In the equations (4) and (5), $\Omega_b$ ($\Omega_d$) is the baryonic (non-baryonic dark matter) density parameter, $R$ is the scalefactor of the Universe, $H_0$ is the Hubble Parameter (the zero subscript stands for its present value) and $k$ is the curvature of the Universe, that is, $k = -1$ for an open universe, $k = 0$ for a flat universe and $k = +1$ for a closed universe (thus $\Omega_k$ is the curvature parameter).

The energy equation is written as

$$\frac{DE}{Dt} - \frac{P}{\rho_b} \frac{D\rho_b}{Dt} = L,$$

(6)

where $E$ is the thermal energy per gram and $L$ is the cooling-heating function of the gas.

The cooling-heating function $L$ is given by the summation of four mechanisms:

$$L = L_R + L_C + L_{H_2} + L_\alpha.$$  

(7)

(i) The cooling from recombination $L_R$ (see, e.g., Schwarz, McCray & Stein 1972) is given by:

$$L_R = -kN_eT_m \frac{Dx_e}{Dt}.$$  

(8)

The ionization fraction is given by

$$\frac{Dx_e}{Dt} = C \left[ \beta e^{-(b_1 - b_2)/kB_T} (1 - x_e) - \frac{a \rho_b x_e^2}{m_p} \right] + I,$$

(9)

where $T_m$ is the temperature of the matter, $N_e$ is the Avogadro number, $B_1$ is the bound energy of the ground state and $B_2$ is the bound energy of the first excited state. The $C$ and $\beta$ parameters present in equation (9) are given by

$$C = \frac{\Lambda_{2s,1s}}{\Lambda_{2s,1s} + \beta} \quad \text{and} \quad \beta = a \left( \frac{2m_kkB_1}{h^3} \right)^{3/2} e^{-B_2/k_BT_m}.$$  

(10)

In the above equations $\Lambda_{2s,1s} = 8.227 \, s^{-1}$ is the decay rate from the 2s state to the 1s state, $a = 2.84 \times 10^{-11} T_m^{1/2} \, \text{cm}^3\text{s}^{-1}$ is the recombination rate, $m_k$ is the electron mass and $k_B$ is the Boltzmann constant. The collisional ionization rate $I$ (see, e.g., Defouw 1970) is given by

$$I = 1.23 \times 10^{-5} N_e x_e (1 + x_e) \frac{k_B T_m^{1/2}}{B_1} e^{-B_1/k_BT_m} \, s^{-1}.$$  

(11)

(ii) The Compton cooling-heating mechanism $L_C$ (Peebles 1968) is

$$L_C = \frac{4k_B \sigma T^4 x_e}{m_e c} (T_m - T_r).$$  

(12)

(iii) For the cooling by molecular hydrogen $L_{H_2}$, we have

$$L_{H_2} = \frac{\Lambda M}{\rho_b}.$$  

(13)

The equation for $\Lambda M$ and the complete set of equations for the formation and destruction of $H_2$ molecules can be obtained in de Araujo & Opher (1988, 1989) and Oliveira et al. (1998a).

(iv) The Lyman $\alpha$ cooling $L_\alpha$ (Carlberg 1981) is given by

$$L_\alpha = 1.25 \times 10^{-11} C_{12} \frac{A_{2s}}{A_{2\gamma} + C_{21}},$$  

(14)

where $A_{2\gamma} = 8.227 \, s^{-1}$ is the two-photon emission rate, $C_{21} = 1.2 \times 10^{-6} T_m^{-1/2} x_e n_H \, s^{-1}$ is the collisional de-excitation rate, $n_H$ is the numerical density of neutral hydrogen and $C_{12} = 2C_{21} e^{-1.19 x_e^{0.5}/T_m}.$

The equation of state, to complete the system of equations to the baryonic matter, is

$$P = k_B N_e T_m \rho_b (1 + x_e).$$  

(15)

It is worth stressing that the temperatures in our calculations are never greater than $\sim 5000K$; in this way the cooling and heating mechanisms considered here account for the thermal history of the baryonic matter present in the underdense regions that we study.

The equations of conservation of mass and momentum for the dark matter are, respectively,

$$\frac{\partial \rho_d}{\partial t} + \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho_d v_r) = 0,$$

(16)

$$\frac{Dv_d}{Dt} = -\nabla \Phi.$$  

(17)

We consider that the negative density fluctuations evolve within a medium of density $\rho_t = \rho_0 (1 + z)^3$, where $\rho_0 = \rho_{ba0} + \rho_{ba0}$ (the subscript ‘u’ stands for the physical parameters of the Universe). Thus, in our model we take into account the influence of the external medium on the evolution of the underdense region, and therefore as external boundary condition we have the pressure of the Universe, namely

$$P_{ba} = k_B N_e T_m \rho_{ba} (1 + x_{ba}).$$  

(18)

It is also worth stressing that in many studies of structure formation the influence of the external medium on the structure under study is not taken into consideration.

In order to calculate the temperature of the matter of the Universe we consider in the cooling function only the contributions due to the Compton cooling-heating and the
cooling due to the recombinant processes. The other phys-
ical processes considered for the evolution of the baryonic
density perturbation are not relevant for the evolution of
the matter temperature of the Universe. The energy equa-
tion thus reads

\[ \frac{DE_{\text{bu}}}{Dt} = \frac{P_{\text{bu}}}{\rho_{\text{bu}}} \frac{D\rho_{\text{bu}}}{Dt} - L_{\text{r}} - L_{\text{c}}. \] \tag{19} \]

Similarly, the collisional ionization is not important for
the matter of the Universe. Thus, we use the study of Peebles
(1968), where the degree of ionization is given by

\[ \frac{Dx_{\text{bu}}}{Dt} = c \left[ \beta e^{-(B_{1} - B_{2})/k_{h}T_{\text{r}}} \left( 1 - x_{\text{bu}} \right) - \frac{n_{\text{bo}}x_{\text{bu}}^{2}}{m_{p}} \right]. \] \tag{20} \]

where now

\[ a = 2.84 \times 10^{-11} T^{-1/2}_{\text{r}} \text{ cm s}^{-1}. \] \tag{21} \]

Equations (19) and (20) are, respectively, the equations of
conservation of thermal energy and ionization balance, with
$T_{\text{bu}}$ the temperature of the Universe and $x_{\text{bu}}$ its ionization fraction.

The baryonic ambient density, i.e., the baryonic density
of the Universe, is given by

\[ \rho_{\text{bu}} = \rho_{\text{b}} \left( 1 + z \right)^{3} = \frac{3H_{0}^{2}\Omega_{b}}{8\pi G} \left( \frac{T_{\text{r}}}{T_{0}} \right)^{3}, \] \tag{22} \]

where $z$ is the redshift and $T_{0}$ is the present cosmic back-
ground radiation temperature.

The equation for the time evolution of the radiation
temperature and, therefore, the time evolution of the Hubble
parameter is given by

\[ \left( \frac{T_{r}}{T_{0}} \right) = -H(t) \]

\[ = -H_{0} \left[ \Omega_{m} \left( \frac{T_{r}}{T_{0}} \right)^{3} + \left( 1 - \Omega_{m} - \Omega_{\Lambda} \right) \left( \frac{T_{r}}{T_{0}} \right)^{2} + \Omega_{\Lambda} \right]^{1/2}, \] \tag{23} \]

where $\Omega_{m} = \Omega_{b} + \Omega_{d}$ is the density parameter of the matter.

To solve the above set of equations, a hydrodynamical
Lagrangian numerical code has been written, which basi-
cally follows the method developed by Rybicki & Morton
(1967). It is important to stress that the total amount of
baryonic matter and non-baryonic dark matter are taken
to be constant throughout the calculations.

We use in our models 9000 shells for the baryonic matter
and 9000 shells for the dark matter. Models with 45000
shells for each component showed no considerable difference
in the results.

We also test our code, with good agreement, against
the one-zone code of de Araujo & Opher (1991) for the col-
lapse of baryonic and non-baryonic dark matter Population
III objects (positive density perturbation). Also, as an addi-
tional test we reproduce the results of de Araujo & Opher
(1993) for voids formed from negative density perturbations,
where a one-zone model was used. The program conserves
total mass and total energy of the system to at least one part in $10^{6}$.

### 3 NUMERICAL RESULTS AND DISCUSSIONS

In the present paper we use different power spectra of den-
sity perturbations to study their role on the scale of voids
formed and to see if cosmological parameters could be con-
strained.

First we take a typical CDM power spectrum of pertur-
bations given by (Padmanabhan 1993)

\[ | \delta_{k} |^{2} = \frac{A_{k}^{(4 - 2n)}}{(1 + B_{k} + C_{k}^{1/2} + D_{k}^{2})^{2}}. \] \tag{24} \]

where $A$ is the spectrum amplitude, $B = 1.7(\Omega h^{2})^{-1}\text{Mpc}$,
$C = 9.0(\Omega h^{2})^{-3/2}\text{Mpc}^{3/2}$ and $D = 1.0(\Omega h^{2})^{-2}\text{Mpc}^{-2}$. In
the above equation the power index is related to $\alpha$ by the
relation $n = (4 - 2\alpha)$. The variance of fluctuations on scale
$r_{0}$ reads

\[ \Delta^{2}(r_{0}) = \frac{1}{2\pi^{2}} \int_{0}^{\infty} dk k^{2} | \delta_{k} |^{2} W^{2}(k r_{0}). \] \tag{25} \]

We analysed two different window functions $W(k r_{0})$:
Gaussian and top-hat filters. Our results are very weakly
dependent on $W(k r_{0})$ so, for convenience and to compare
our results with previous studies, we present here the results
using a top-hat window function. Then, we have

\[ W_{k} = 4\pi r_{0}^{3} \frac{\sin(k r_{0})}{(k r_{0})^{3}} - \frac{\cos(k r_{0})}{(k r_{0})^{2}}, \] \tag{26} \]

with the volume of the top-hat window given by

\[ V_{W} = \frac{4}{3} \pi r_{0}^{3}. \] \tag{27} \]

The mass fluctuation in a radius $R$ reads

\[ \left( \frac{\delta \rho}{\rho} \right)_{R}^{2} = \frac{1}{2\pi^{2}} \int_{0}^{\infty} k^{2} dk | \delta(k)|^{2} \frac{W_{k}^{2}}{V_{W}}. \] \tag{28} \]

The input parameters that describe the cosmological sce-
narios are taken from a recent study performed by Cen
(1998a,b), who analysed mass and autocorrelation functions
of rich clusters of galaxies from linear density fluctuations.
Cen presents 32 different cosmological models to determine
the fitting parameters in his Gaussian peak method, using
them to obtain the cluster mass function. In particular, we
take four of these models that present different characteris-
tics related to the present radii and present density contrasts
of the voids. The models chosen have their parameters de-
scribed in Table 1 (see also Cen 1998a,b).

All models considered here have ‘excess power’ $EP = 1.30 \pm 0.15$. (‘Excess power’ is defined as $EP \equiv 3.4 \sigma_{25}/\sigma_{8}$,
where $\sigma_{25}$ is the linear rms density fluctuation in a
$25 h^{-1}\text{Mpc}$ top-hat sphere at $z = 0$, and $\sigma_{8}$ has a simi-
lar definition in an $8 h^{-1}\text{Mpc}$ top-hat sphere.) This para-
meter describes the shape of the power spectrum on scales
$\sim 3 - 300 h^{-1}\text{Mpc}$ and its values range from 1.15 to 1.45 to
fit the COBE data.

In the first column of Table 1 we present the number of
the model [we use here the same numbers presented by Cen
(1998a,b)]; in the second and third columns we present the
density parameter for the matter and that associated with
the cosmological constant, respectively; the fourth column
presents the rms density fluctuation in an $8 h^{-1}\text{Mpc}$ top-hat
sphere at $z = 0$; in the fifth column we present the power
index $n$; in the sixth column we present the value of the
Table 1. The cosmological models chosen from Cen (1998a,b). The model numbers correspond to those presented in Cen’s table 1. We consider a baryonic component \( \Omega_b = 0.05 \) and for the dark component we have \( \Omega_i = \Omega_m - 0.05 \). Models 23 and 26 include the cosmological constant (\( \Lambda \)).

| Model | \( \Omega_m \) | \( \Omega_{\Lambda} \) | \( \sigma_8 \) | \( n \) | \( H_0 \) (km s\(^{-1}\) Mpc\(^{-1}\)) | Comment |
|-------|---------------|-----------------|-------------|-------|-------------------------------|---------|
| 14    | 0.35          | 0               | 0.80        | 1.0   | 70                            | Open CDM|
| 20    | 0.60          | 0               | 0.73        | 1.0   | 60                            | Open CDM|
| 23    | 0.40          | 0.60            | 0.79        | 0.95  | 65                            | \( \Lambda \) CDM |
| 26    | 0.20          | 0.80            | 1.5         | 0.95  | 100                           | \( \Lambda \) CDM |

Hubble constant; and finally the seventh column indicates the type of power spectrum used.

For the normalization of the power spectra studied here we followed the usual methodology found in the literature, where the value of \( \sigma_8 \) permits one to find the constant \( A \) of the power spectra. Our normalized spectra follow the same shape seen in fig. 2 of Cen (1998a).

Thus, for a given perturbation with mass \( M_C \), and for the models described in Table 1, we can obtain the present radius and the present density contrast. Then, to determine the initial conditions at the beginning of the recombination era, we used the general formula given by Opher, Pires & de Araujo (1997) for the amplification of the density perturbations (their study incorporates both growing and decaying modes).

We start the calculations at the beginning of the recombination era, at redshift \( z_{\text{rec}} \sim 1000 \), where the ionization degree begins to be significantly lower than unity. For \( z_{\text{rec}} > 1000 \) the matter is completely ionized and density perturbations (positive or negative) cannot evolve due to the photon drag that inhibits the growth of any peculiar velocity. We start our calculations assuming that the underdense regions evolve initially with a velocity field given by the Hubble flow.

In the present study we assume that initially

\[
\delta_i = \frac{\delta \rho_i}{\rho_i} = \frac{\delta \rho_b}{\rho_b} = \frac{\delta \rho_d}{\rho_d},
\]

(29)

where \( \rho_i \) is the total density (dark matter and baryons), \( \rho_b \) is the baryon density, \( \rho_d \) is the dark matter density, and \( \delta \rho_i \), \( \delta \rho_b \), and \( \delta \rho_d \) are the respective density perturbations.

The baryonic mass contained within the underdense region is obtained from

\[
M_b = M_C \Omega_b / \Omega_m
\]

(30)

and, analogously, the collisionless dark matter within the underdense region is

\[
M_d = M_C \Omega_d / \Omega_m.
\]

(31)

The initial radius of the underdense region is given by

\[
R_i = \left[ \frac{3}{4 \pi} \frac{M_C}{\rho_0} \frac{1}{1 + \delta} \right]^{1/3} (1 + z_{\text{rec}})^{-1}.
\]

(32)

With such an initial profile, the density at the void boundary does not fall smoothly to the density of the Universe. Thus, the interface (between the external medium and the last shell of the computational grid) is a membrane-like one. This membrane is maintained throughout the void time evolution, because the mass inside the void region is taken to be constant throughout the void evolution.

Table 2. Principal results for model 14 described in Table 1. The final diameter of the baryonic component is \( D_{\text{BF}} \), while \( D_{\text{DF}} \) is the final diameter of the dark matter component. The density contrasts for the baryonic and dark matter components are, respectively, \( \delta_{\text{BF}} \) and \( \delta_{\text{DF}} \).

| \( M_C (\text{M}_\odot) \) | \( D_{\text{BF}} (\text{h}^{-1}\text{Mpc}) \) | \( \delta_{\text{BF}} \) | \( D_{\text{DF}} (\text{h}^{-1}\text{Mpc}) \) | \( \delta_{\text{DF}} \) |
|----------------|----------------|----------|----------------|----------|
| \( 10^{12} \) | 3.82           | -0.755   | 3.89           | -0.755   |
| \( 10^{13} \) | 7.18           | -0.619   | 7.28           | -0.619   |
| \( 10^{14} \) | 14.2           | -0.453   | 15.7           | -0.455   |

Table 3. Principal results for model 20 described in Table 1. See Table 2 for notation.

| \( M_C (\text{M}_\odot) \) | \( D_{\text{BF}} (\text{h}^{-1}\text{Mpc}) \) | \( \delta_{\text{BF}} \) | \( D_{\text{DF}} (\text{h}^{-1}\text{Mpc}) \) | \( \delta_{\text{DF}} \) |
|----------------|----------------|----------|----------------|----------|
| \( 10^{12} \) | 3.07           | -0.770   | 3.11           | -0.770   |
| \( 10^{13} \) | 5.72           | -0.609   | 5.80           | -0.609   |
| \( 10^{14} \) | 11.3           | -0.360   | 11.5           | -0.360   |

In Tables 2-5 we present the main results for the models described in Table 1. We see that the spectra of perturbations used here produce different results concerning the final diameter of the voids (and for their density contrasts) for the same mass perturbation \( M_C \).

In particular, if we consider that voids are regions with density contrasts \( \delta \leq -0.6 \), then large voids with diameters of \( D > 20 - 30 \text{h}^{-1}\text{Mpc} \) [the sizes found by El-Ad et al. (1996) and Müller et al. (2000)] could be produced only for the spectrum 26 or similar one (see Tables 1 and 5).

In fact, it is worth mentioning that even if \( (2 - 3)\sigma \) negative density perturbations are considered for the models 14, 20 and 23 or similar ones, large void regions cannot be accounted for. For example, for model 23 (see Tables 1 and 4) a 2\( \sigma \) negative density perturbation of \( 10^{14} \text{M}_\odot \) would produce a void with baryonic (dark matter) diameter of 14.7h\(^{-1}\)Mpc (16.7h\(^{-1}\)Mpc) with \( \delta_{\text{BF}} = -0.639 \). If instead a 3\( \sigma \) negative perturbation is considered, a void with baryonic (dark matter) diameter of 17.2h\(^{-1}\)Mpc

Table 4. Principal results for model 23 described in Table 1. See Table 2 for notation.

| \( M_C (\text{M}_\odot) \) | \( D_{\text{BF}} (\text{h}^{-1}\text{Mpc}) \) | \( \delta_{\text{BF}} \) | \( D_{\text{DF}} (\text{h}^{-1}\text{Mpc}) \) | \( \delta_{\text{DF}} \) |
|----------------|----------------|----------|----------------|----------|
| \( 10^{12} \) | 3.80           | -0.800   | 3.90           | -0.801   |
| \( 10^{13} \) | 6.84           | -0.632   | 6.93           | -0.632   |
| \( 10^{14} \) | 13.4           | -0.369   | 14.2           | -0.372   |
This result is a lower limit since it depends only on the expansion rate of the Universe (the above result is taken for an Einstein-de Sitter universe) and on the gravity of the void. For the cases in which other physical processes are relevant for the evolution of the cloud, the turnaround should occur for

$$\left(\frac{\delta \rho}{\rho}\right) > 4.6.$$  

(35)

In particular, we refer the reader to the papers by Oliveira et al. (1998a,b) who studied the collapse of Population III objects and consider such issues in detail.

Concerning the void regions, their density contrast is always negative and they never stop expanding. As a result the condition given by equation (35) is never fulfilled and so the evolving shells do not collapse.

Density perturbations that could be present in the evolving shells, however, could in principle collapse and fragment if they had the time to stop expanding and evolve. Note that this issue concerning the collapse and eventual fragmentation in the evolving shells is by itself so interesting that it deserves to be investigated in detail. We leave, therefore, such an issue to be considered in another paper to appear elsewhere.

An interesting result concerning the distribution of dark matter and baryonic matter can be seen in Tables 2-5. The final diameter of the dark matter is always greater than the final diameter of the baryonic matter, when all physical processes considered in the present paper are included in the calculations.

To see the influence of the several physical processes on the evolution of a given negative density perturbation, in particular on the final diameters of the voids, we performed some calculations studying individually the effects of all the four cooling-heating mechanisms and also the influence of the photon drag (Table 6).

Note that the dissipative baryonic processes do not define a particular characteristic void scale; their main effect is to segregate the dark matter and baryonic matter.

From Tables 4 and 6 we note that the baryonic diameter is a little bit lower than the dark matter diameter when all physical processes are included, and this result remains when we disregard each one of the cooling-heating mechanisms.

Another interesting result is obtained when we disregard all the processes. In this case, the final diameter of the baryonic matter is almost the same as the dark matter component. When we disregard all the processes and the pressure of the Universe, the final diameters are the same for both components, as expected, since the two fluids in these circumstances are subject only to gravity.

In Table 6 we see the influence of each physical process one at a time. When we disregard the photon drag, but maintain all the other processes and the pressure of the Universe, the final radius of the baryonic (dark) component goes from 6.84 (6.94) h⁻¹Mpc to 6.93 (6.99) h⁻¹Mpc. This occurs because the photon drag acts against the expansion of the negative density perturbations, inhibiting the growth of peculiar expansion velocities.

The cooling mechanisms $L_{Rb}$, $L_\alpha$ and $L_{H_2}$ have very similar effects on the void evolution, and their principal influence is related to the thermal evolution of the matter inside the void regions. The Compton heating-cooling when disre-

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### Table 5. Principal results for model 26 described in Table 1. See Table 2 for notation.

| $M_0$ (M☉) | $D_{BF}$ (h⁻¹Mpc) | $\delta_{BF}$ | $D_{DF}$ (h⁻¹Mpc) | $\delta_{DF}$ |
|------------|------------------|--------------|------------------|--------------|
| 10^{12}    | 8.30             | −0.932       | 8.40             | −0.934       |
| 10^{13}    | 13.4             | −0.853       | 13.5             | −0.853       |
| 10^{14}    | 23.9             | −0.709       | 25.0             | −0.709       |
| 10^{15}    | 48.7             | −0.622       | 53.5             | −0.628       |

(18.9h⁻¹Mpc) with $\delta_{BF} = −0.776$ ($\delta_{DF} = −0.776$) would be produced. Although large, these voids are not enough to explain those reported in the literature.

It is worth noting that some authors define void regions as having a density of 0.2 times the mean galaxy density (Schmidt, Ryden & Melott 2001). The density contrast in clusters of galaxies is around 1.5-3.0, which means that the density contrast in void regions would amount to $\delta = −(0.5 \pm 0.2)$, which is less conservative than our definition. Even considering that regions with $\delta \sim −0.3$ are voids, it is easily seen from our results that the largest voids in the Universe cannot be accounted for by models 14, 20 and 23 or similar ones.

A possible explanation for the largest voids could be related to the interaction and eventual merger of two or more small voids. We refer the reader to a paper by Dubinski et al. (1993), who investigated the evolution, interaction and merger of voids formed from negative density perturbations. Then, the merger of small voids could be a way to build up large voids from the spectra of density perturbations related to models 14, 20 and 23.

Still considering our calculations, note that, using model 26 with 3σ perturbations for 10^{15}M☉, void regions with diameters of up to 62h⁻¹Mpc (baryonic component) with density contrast $\delta_{BF} = −0.756$ could be produced. In this case, the dark matter void would have a diameter of $D_{DF} = 69h^{-1}\text{Mpc}$ (with $\delta_{DF} = −0.765$). This result is roughly that inferred to the diameter of the Boötes void.

An interesting issue has to do with the fact that the post-recombination Jeans mass is $\sim 10^9\text{M}_\odot$ at recombination and it decreases later on, and that the mass scales studied here are in the range of 10^{12}\text{M}_\odot to 10^{15}\text{M}_\odot. Thus, one could argue that collapse and eventual fragmentation should take place in the evolving (expanding) shells. To address this issue properly one should study in detail how density perturbations evolve in the expanding shells.

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Note that it is not enough that a perturbation be Jeans unstable to guarantee that it will collapse and fragment. Consider the following example. Let us think of the evolution of primordial clouds after the recombination era (that is, the evolution of positive density perturbations). Even the clouds that are already Jeans unstable at the recombination era will initially expand before collapsing. For a cloud that stops expanding, its mean density ($\bar{\rho}$) relative to the background is given by (see, e.g., Peebles 1993; Coles & Lucchin 1995)

$$\left(\frac{\bar{\rho}}{\rho}\right) = \left(\frac{3\pi}{4}\right)^2 \simeq 5.6,$$  

(33)

which corresponds to

$$\left(\frac{\delta \rho}{\rho}\right) \simeq 4.6.$$  

(34)
Voids in a universe with CDM and $\Omega_\Lambda$

Table 6. The influence of the physical processes on the evolution of the final diameter and density contrast of a void formed from a perturbation with mass $10^{13} M_\odot$ in model 23. The cooling-heating mechanisms for the baryonic matter are: $L_R$, the cooling from recombination, $L_B$ the Lyman $\alpha$ cooling, $L_C$ the Compton cooling-heating, and $L_{\text{H}_2}$ the cooling by molecular hydrogen. In the second and third columns we present the results without all the physical processes. The superscript $+U$ ($-U$) in these columns means that we keep (remove) the pressure of the Universe acting on the ‘void boundary’.

| Without all processes (+U) | Without all processes (−U) | Without photon drag | Without $L_R$ | Without $L_B$ | Without $L_C$ | Without $L_{\text{H}_2}$ |
|---------------------------|---------------------------|-------------------|-------------|-------------|-------------|-------------|
| $D_{\text{BF}} (h^{-1}\text{Mpc})$ | 6.97 | 7.03 | 6.94 | 6.86 | 6.86 | 6.83 | 6.86 |
| $\delta_{\text{BF}}$ | −0.647 | −0.648 | −0.643 | −0.640 | −0.640 | −0.639 | −0.640 |
| $D_{\text{DF}} (h^{-1}\text{Mpc})$ | 7.02 | 7.03 | 6.99 | 6.93 | 6.93 | 6.90 | 6.93 |
| $\delta_{\text{DF}}$ | −0.648 | −0.648 | −0.643 | −0.641 | −0.641 | −0.640 | −0.641 |

...garded reduces slightly the dimension of the void, because at high redshifts this physical process maintains the temperature of the matter inside the void close to the radiation temperature.

The above results also appear when we perform the present analysis for other models described in Table 1. In particular, independent of the spectra used, the physical processes produce very similar results concerning the final diameters of the baryonic and dark matter. We note that even using other spectra, like those used by de Araujo & Opher (1993) for example, this difference in diameter due to the presence of physical processes is maintained.

With all the physical processes included, there exists a transition region with thickness of up to $\sim 2.5 h^{-1}\text{Mpc}$, which depends on the mass of the perturbation, the normalization of the spectrum and the respective density parameters for the dark and baryonic components as well as the cosmological constant. These transition regions have sizes comparable to the diameters of galaxies and clusters of galaxies.

The model 26, for example, has a transition region of $\sim 2.5 h^{-1}\text{Mpc}$ thickness for a perturbation with mass $10^{13} M_\odot$. Within this region the density contrast of the cold dark matter is $\delta_{\text{BF}} = -0.628$ (its diameter is $53.5 h^{-1}\text{Mpc}$). Thus, the mass density of the dark matter is $\rho_{b} = 0.372 \rho_{\text{amb}}$, where $\rho_{\text{amb}}$ is the ambient mass density of the dark matter (that is, the mass density of the dark matter present in the Universe).

As the boundary of the baryonic matter is at $48.7 h^{-1}\text{Mpc}$, within the transition region the density contrast of the baryonic matter is $\delta_{b} = 0$, and so $\rho_{b} = \rho_{\text{amb}}$. The relation between the baryonic and dark components present within this transition region is given by

$$\frac{\rho_{b}}{\rho_{\text{amb}}} = 2.70 \times \left(\frac{\rho_{\text{amb}}}{\rho_{\text{amb}}}\right) = 2.70 \times \left(\frac{\Omega_{b}}{\Omega_{\Lambda}}\right) = 0.90$$ (36)

Thus, in this region the amount of baryonic matter is approximately equal to the amount of non-baryonic dark matter, although this universe model has three times more dark matter ($\Omega_{b} = 0.15$) than baryonic matter ($\Omega_{b} = 0.05$).

Consequently, galaxies formed near voids and within the transition region could, during their formation process, have a lower content of cold dark matter than those galaxies formed in regions far from voids. This result leads us to conclude that, if one uses an analysis of the content of dark to baryonic matter for the galaxies in different regions, one could infer very different results, reflecting therefore only local values.

Let us consider another example. The transition zone of model 26, for a perturbation with mass $10^{13} M_\odot$, gives $\rho_{b}/\rho_{\text{amb}} \sim 7(\Omega_{b}/\Omega_{\Lambda}) \sim 2.3$, although in such a universe model the amount of dark matter is three times greater than the amount of baryonic matter.

It is worth stressing that galaxies formed out of the transition zone and that escape to there could have a different relation between the baryonic and dark components than that found in the transition zone.

It is important to have in mind that the process of structure formation of the Universe is a very complicated one. For the canonical model of cosmology, namely primordial Gaussian random perturbation field, the formation of voids and clusters of galaxies is part of one process of clustering. The formation of regions with positive density contrasts, like clusters of galaxies, contributes to the growth of underdense regions and vice versa.

The present study only considers a small part of the very complicated structure formation process. We study the evolution of negative density perturbations taking into account all relevant physical processes acting on them. However, the assessment of the role of these very physical processes not considered in the literature, besides including the presence of non-baryonic dark matter and cosmological constant, are the main contributions of the present study.

4 CONCLUSIONS

In the present investigation, we study the evolution of negative density perturbations taking into account a series of physical processes that are present during and after the recombination era. In order to perform such a study, a spherical Lagrangian hydrodynamical code has been written to follow the evolution of these density perturbations.

We analysed a set of COBE-normalized spectra obtained from a recent study performed by Cen (1998a,b). The results presented here show that negative density perturbations can directly create large voids, with diameters of up to $\sim 50 h^{-1}\text{Mpc}$, only for some universe models dominated by the cosmological constant ($\Omega_{\Lambda} \sim 0.8$).

Empty regions as large as the Boötes void could have been formed only from model 26 with $3\sigma$ perturbations (for a mass perturbation of $10^{15} M_\odot$). The diameter of the baryonic component, $D_{\text{BF}} = 62 h^{-1}\text{Mpc}$, and the depth obtained from our simulations ($\delta_{\text{BF}} = -0.76$) are similar to those determined by Dey et al. (1990) for that large void.

Other CDM spectra studied here cannot account for the large voids found by El-Ad et al. (1996) and Müller et
al. (2000) in their studies. In particular, models 14, 20 and 23 could not account for the large voids even considering $(2 - 3)\sigma$ negative density perturbations. The formation of the largest voids could come, for these spectra, only from mergers of two or more small voids as addressed, for example, by Dubinski et al. (1993).

Our models show that the dark matter void is greater than the baryonic matter void, this result being produced by the physical processes acting on the baryons. We find a difference between the radii of $\sim 1 - 10$ per cent, depending on the power spectra and masses of the perturbations, when all physical processes are included. This effect corresponds to the creation of a transition zone defined by the radii of the baryonic and dark components. We note that disregarding each one of the physical processes, the thickness of this transition zone decreases and it goes to zero when all processes and the pressure of the Universe are removed. The analysis of the physical processes lead us to conclude that the principal mechanism that acts on the evolution of voids is photon drag. When we disregard this mechanism the final diameter of the baryonic matter increases and the thickness of the transition zone becomes almost half of its initial value.

Although the thickness of the transition zone is small (in the range of $\sim 0.03 - 2.5h^{-1}$Mpc) when compared to the radii of the baryonic and dark components, interesting effects on the formation of galaxies could occur. Putative objects formed near voids and within the transition region would have a dark matter to baryonic matter ratio such that $\rho_d/\rho_b \neq \Omega_d/\Omega_b$. This means that the amount of dark matter is different in such regions as compared to that present in the Universe as a whole.

Then, if one uses these galaxies to determine by dynamical effects or other techniques the quantity of dark matter present in the Universe, the result obtained would be only local and not representative of the Universe as a whole. Note, however, that within voids the relation between baryonic matter and dark matter maintains the same value as that of the Universe.

For different spectra one would obtain different results as compared to those present here, but void regions with diameters of tens of megaparsecs and transition zones of non-negligible sizes would occur as well.

Owing to the fact that the baryonic matter undergoes a series of physical processes, such as those considered here, that the non-baryonic dark matter does not undergo, motions of matter on large scale due to the presence of a gravitational potential could induce a segregation between the two components, producing regions with different values of $\rho_b/\rho_d$ that are different from the $\Omega_b/\Omega_d$ relation of the Universe.

It is important to note that the voids studied here are formed only by the evolution of negative density perturbations. The voids formed in this scenario are, therefore, cold. We do not consider a possible energy injection, or excess of ionization (see, e.g., Srianand 1997), by quasars or other primordial objects near the voids. In this way, the temperatures of the matter in our model are never greater than the temperature of the radiation. The inclusion of an energy source, such as quasars, is an interesting possibility for a future extension of the scenario presented here.

Certainly the process of formation of the structures of the Universe is a very complicated one. As discussed above, the formation of voids and clusters of galaxies is part of the same process of clustering. The formation of regions with positive density contrasts, like galaxies and clusters of galaxies, contributes to the growth of underdense regions. On the other hand, the underdense regions push the matter around them during their evolution and growth, contributing to the clustering processes.

In our study we only consider the evolution of negative density perturbations, which is a small part of the very complicated structure formation process. However, the assessment of the role of a series of physical processes not considered in the literature, besides including the presence of non-baryonic dark matter and cosmological constant, are the main contributions of the present study.

Also, because our study shows that the physical processes considered here are relevant to the evolution of negative density perturbations, this leads us to conclude that in the structure formation of the Universe as a whole these very processes could be relevant as well. Therefore, it would be of interest to take into account, in more realistic modelling of the structure formation, the various processes considered here.

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Voids in a universe with CDM and $\Omega_\Lambda$

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