A New Estimation Method of Coke Strength by Numerical Multiscale Analysis

Yusuke ASAKUMA, Munetaka SOEJIMA, Tsuyoshi YAMAMOTO,1) Hideyuki AOKI1) and Takatoshi MIURA1)
Graduate Student, Tohoku University, Aza-Aoba, Aramaki, Aobaku, Sendai 980-8579 Japan.
1) Graduate School of Engineering, Tohoku University, Aza-Aoba, Aramaki, Aobaku, Sendai 980-8579 Japan.
(Received on December 4, 2002; accepted in final form on March 6, 2003)

The homogenization method is proposed as a new numerical approach to understand the fracture mechanism of the microscopic behavior in coke because the evaluations of the strength has been still difficult for the improvement of coke qualities in coke making process. The conditions for the stress concentration and relaxation effect are investigated by homogenization method. These results show that the distribution of micro cracks and pores in coke is important factor for the microscopic fracture. Also, the digital image technique is applied to the actual coke with the complicated microstructure and the improvement mechanism of coke strength by the carbon deposition is clarified by using homogenization method. The effect of carbon deposition on the coke strength is illustrated from the stress distribution while calculating the homogenized elastic modulus. Although the stress concentration between large pores is caused in the case of carbon deposited coke, the maximum stress becomes smaller than the tensile strength of coke because of large homogenized elastic modulus. As a result, the microscopic fracture does not easily occur for carbon deposited coke.

KEY WORDS: coke strength; multiscale analysis; homogenization method; stress intensity factor; microstructure.

1. Introduction

Pulverized Coal Injection (PCI) in the blast furnace is carried out to reduce the amount of coke production for the deterioration of coke. However, the increase of the PCI raises the breakage and the degradation of coke by crushing between coke particles in the raceway or the lower part of the blast furnace. Therefore, size and mechanical strength of coke are important factors for the microscopic fracture. Also, the digital image technique is applied to the actual coke with the complicated microstructure and the improvement mechanism of coke strength by the carbon deposition is clarified by using homogenization method. The effect of carbon deposition on the coke strength is illustrated from the stress distribution while calculating the homogenized elastic modulus. Although the stress concentration between large pores is caused in the case of carbon deposited coke, the maximum stress becomes smaller than the tensile strength of coke because of large homogenized elastic modulus. As a result, the microscopic fracture does not easily occur for carbon deposited coke.

For improvement of these coke qualities in coke making process, the evaluations of coke size and strength have been still difficult. The consideration of the microscopic behavior by analytical method is required for the evaluation of coke because the characteristics of coke are affected by their own microstructure. The fracture behavior of coke caused by the stress concentration between micro pores and cracks is very important for coke size and strength. In order to prevent the bulk breakage and the degradation in the blast furnace, it is required to estimate various microstructures of coke, such as porosity, pore shape, inert and deposited carbon. Fundamental researches are hardly performed in regard to the effect of these microstructures on coke size and strength. Recently, the homogenization method, which enables us to analyze macroscopic and microscopic behavior simultaneously, was applied for the understanding of the fracture mechanism in coke, and then the finite element method was used to solve the partial differential equations derived by the homogenization method. Dealing with the microstructure such as pores, their distribution and micro cracks was not enough because the finite element cell to express the microstructure was too simple. In this study, Delaunay Triangulation method is introduced for more strict estimation of the complex microstructures. The effect of stress relaxation and concentration can be expressed by this method, considering the complex structure with micro pores and cracks.

Furthermore, The digital image analysis based on the finite element model by the homogenization method is introduced to reflect the actual microstructure of coke. Carbon deposition by the pyrolyzed gas of tar is one of the phenomena in oven chamber. This phenomenon is known to make coke strong. However, the improving mechanism of the coke strength by the carbon deposition is not clear completely. In this study, the effects of the carbon deposition could be evaluated by using the digital image analysis based on the homogenization method quantitatively.

2. Analytical Method

2.1. Application of Homogenization Method to Coke

Numerical analysis of microscopic mechanical behavior is becoming possible by recent rapid progress in computer technology. In spite of the importance of understanding mi-
crosscopic behaviors, the objective still has been performed against a large-scale structure. It is common that macrostructure is regarded as a homogeneous material because the correlations between macrostructure and heterogeneous microstructure are not necessarily clear. Generally, both the generation of micro-voids in material and the macro cracks propagation cause damage and fracture of materials by the growth and coalescence of the void. These micro-voids lead to not only the generation of the macro crack and the final fracture but also the decrease of the strength, rigidity and toughness. Because this dynamic behavior of micro-voids is treated simply as the continuum field in damage mechanics, the field equalized all micro-voids is analyzed without considering detail damage condition at the microscopic region. In this way, it is easy for the damage dynamics to be applied to numerical simulation comparatively, although the fracture estimation is not enough due to simplification of microstructure. Accordingly, the development of the damage mechanics at the microscopic region is required against the complex compounds such as coke including micro cracks, pore and inert etc. In this study, the homogenization method is proposed to evaluate the microstructure for the development of the damage mechanics.

The homogenization method is becoming more and more important for micro mechanics of composite materials as theoretical and numerical technique to solve macro–micro coupling problems. One of the merits of the homogenization method is strict dynamics theory and the method can reflect the correlations between macro and micro scale. To analyze the microscopic fracture of composite materials such as coke, the correlations must be taken into consideration. The microstructure of actual materials deforms by macroscopic load or thermal expansion and these microscopic behaviors affects on macroscopic behavior as macroscopic property. The difficulty in treating the homogeneous field is the fracture decision at microscopic region, that is, whether micro-cracks propagate or not. Since many micro cracks and pores exist at microstructure level in coke, whether fractures from micro cracks and pore wall are caused or not must be considered as the fracture condition. Actually, main feature of this method is to evaluate the fracture condition from the stress distribution at the microstructure level.

When micro cracks are distributed in the complex compounds like coke, their distribution causes some correlations such as stress concentration and relaxation behaviors that influence crack propagation. These microscopic behaviors must be considered in numerical analysis of macro homogeneous field. The region ahead of the macro crack, where micro cracks generate and grow, is called ‘process zone’ for the correlation between macro and micro cracks. The propagation of the macro crack is controlled in this area because the elastic property at the zone decreases and the amount of the stress becomes small by the micro-crack generation. This effect is called ‘crack tip shielding’. Therefore, it is important for dynamics mechanics to consider damage conditions at microstructure.

Although more precise analysis around the crack tip is required, the latest homogenization analysis for the group of micro cracks is limited to parallel, periodic and simple micro cracks due to the difficulty of finite element mesh formation. For more strict estimation of the complex microstructure, Delaunay Triangulation method and the digital image analysis based on the homogenization method are used to make finite element mesh of the complex geometry.

2.2. Formulations of Homogenization Method

An assembly of periodic microscopic unit cells is shown in Fig. 1. The microstructure is assumed to have a representative volume element and to be spatially distributed in locally periodic manner. The macroscopic coordinate \( x \) and the microscopic one are defined as this equation, \( y = x/\varepsilon \), where \( \varepsilon \) is the scale ratio and is a very small positive number. The multiscale method can expand the displacements asymptotically as follows:

\[
u_i = u_i^0(x, y) + \varepsilon u_i^1 \left( \frac{x}{\varepsilon} \right) \quad \ldots \ldots (1)
\]

\( u_i^0 \) and \( u_i^1 \) are macro and microscopic displacements, respectively. In other words, macroscopic displacement, \( u_i^0 \) is a kind of averaged smooth displacement and the real displacement, \( u_i \), is rapidly oscillating due to the microscopic inhomogeneity. \( u_i^1 \) is the perturbed displacement according to the microstructure. Hence, the scale factor \( \varepsilon \) is multiplied to the microscopic displacement. Macroscopic deformation is recognized as the first order term of the expanded displacement.

The variational formulation of a boundary value problem for an elastic problem is given by:

\[
\int_\Omega E_{ijkl} \frac{\partial u_k}{\partial x_i} \frac{\partial \tilde{\sigma}_l}{\partial x_j} \, d\Omega = \int_{\Gamma} \bar{t}_i \, d\Gamma \quad \ldots \ldots (2)
\]

\( E_{ijkl} \) is an elastic tensor and \( \bar{t}_i \) is the traction vector applied on the boundary \( \Gamma_n \), respectively. \( \tilde{u}_i \) is the virtual function. Here, the body force is neglected.

By substituting Eq. (1) into Eq. (2) and taking the limit of \( \varepsilon \rightarrow 0 \) while using the averaging principle, the following relation over a microscopic unit cell \( Y \) is obtained as:

\[
\int_Y E_{ijkl} \frac{\partial \chi_{kl}}{\partial y_i} \frac{\partial \tilde{\sigma}_l}{\partial y_j} \, dY = \int_Y E_{ijkl} \frac{\partial \tilde{\sigma}_l}{\partial y_j} \, dY \quad \ldots \ldots (3)
\]

This equation can be solved with the periodic boundary condition. \( \chi_{kl} \) is the characteristic displacement which is a periodic function of \( y \). The characteristic displacement has three modes of the microstructural displacement that reflect the mismatch of the mechanical properties of the con-
stituents and the geometrical configuration of the constituents. The microscopic displacement depends on the macroscopic boundary conditions and the macroscopic deformation. Hence, the macroscopic strain can connect the macroscopic and microscopic behaviors. Therefore, the macroscopic strain can be derived as follows:

\[ \varepsilon_i^m = -\chi_i^m(y) \frac{\partial \phi_j^m}{\partial x_j} \]  

Equation (4) proves the existence and the uniqueness of the solution of Eq. (3).3)

Weak forms for the homogenized macroscopic body can be derived as follows:

\[ \int_{\Omega} E^H \frac{\partial \varepsilon_i^m}{\partial x_j} \frac{\partial \varepsilon_j^m}{\partial x_l} d\Omega = \int_{\Gamma} t_i \tilde{n}_j d\Gamma \]  

Finally, microscopic stresses are obtained as:

\[ \sigma_i^m = \left( E_{ijkl}^{iH} - E_{ijkl}^{ijkl} \right) \frac{\partial \varepsilon_j^m}{\partial x_j} \frac{\partial \varepsilon_k^m}{\partial x_l} \]  

From these equations, the macroscopic strains, \( \partial \varepsilon_i^m / \partial x_j \), connect macro–micro behaviors. Hence, accurate calculation of macroscopic strains is essential.

By using the finite element discretization to solve the partial differential equations, Eq. (3) for microscopic unit cell can be written as follows:

\[ \left( \int_B B^T E^H B \right) \chi^H = \int_B B^T E^i dY \]  

where \( E \) is the stress–strain matrix of the constituents of the composite materials and \( B \) is the displacement–strain matrix. \( E^i \) is a vector of column \( ij \) (\( ij = 11, 22, 12 \)) of the stress–strain matrix \( E \). \( \chi^H \) is the characteristic displacement vector associated with the \( i \)th mode. The following matrix is defined as:

\[ \chi^H = (\chi_1^{H11}, \chi_2^{H11}, \chi_2^{H12}) \]  

Then, Eq. (6) can be expressed as follows:

\[ E^H = \frac{1}{|Y|} \int_B (E - B \chi) dY \]  

Here \( l \) is unit vector. The macroscopic Eq. (5) can be described as follows:

\[ \left( \int_B B^T E^i B d\Omega \right) \phi^i = \int_{\Gamma} N^T t d\Gamma \]  

where \( N \) is the shape function of the finite element and \( t \) is the traction vector. Microscopic stresses, Eq. (7), can be expressed as follows:

\[ \sigma = E(l - B \chi) B^H d^\phi \]  

Equations (8) and (10) are the homogenized elastic tensor that can be calculated by Eq. (6) after solving the microscopic Eq. (3) subjected to the periodic boundary condition.

From this dimensionless values and the homogenized elastic modulus at microscopic region. This equation means that composite material is treated as continuum field by using \( E^i \). Therefore, \( E^H \) is index of strength for heterogeneous material.

3. Analytical Conditions

3.1. Analysis at Macroscopic Region

An overall analysis is carried out by the homogenized elastic modulus. The Schematic diagram of macro analysis is shown in Fig.2. Tensile load are applied at the top and bottom of the specimen. The value of the stress is 4 MPa that corresponds to the tensile strength of coke. Elastic modulus of matrix and Poisson’s ratio are 17.5 GPa and 0.24, respectively. Those are the mechanical properties of ultra-high-density carbon. The average strain at point A in Fig. 2 is obtained by macroscopic analysis. Based on these results, the localization, that is, the calculation of Eq. (12) can be performed.

3.2. Stress Intensity Factor at Microscopic Region

The stress intensity factor, \( K_i \), is used as criterion of the microscopic fracture, that is, degree of the stress concentration. For example, the stress intensity factor is calculated for the center crack in the unit cell of Fig. 3(a) by deformations of singular points located at the crack tip as shown in Figs. 3(b), 3(c).13,14) Furthermore, the factor is translated into the dimensionless values as follows:

\[ K_i^* = \frac{K_i}{\sigma \sqrt{\pi a}} \]  

where \( \sigma \) and \( a \) are the tensile load in Fig. 2 and crack length in Fig. 3(a), respectively.

From this dimensionless values and the homogenized elastic modulus in Eq. (10), the estimation of the fracture mechanism is performed against complex microstructure like coke.
4. Analysis Result of Stress Concentration and Relaxation Effect at Microscopic Region

4.1. Validity of This Model
First, the validity of this model is confirmed before showing the results. Used unit cell with simple crack, whose length is 2\(a\), is shown in Fig. 4(a) and the stress intensity factor, \(K_I^*\), is calculated against the crack length in Fig. 4(b). The curve and plots represent the analytical solution\(^\text{15}\) of periodic cracks and calculations from this model, respectively. Good agreements prove the validity of this analytical method.

4.2. Effect of Pore Shape on Strength
To evaluate the relation between the anisotropy of pore and the mechanical strength, unit cells as shown in Fig. 5(a), which have the ellipsoidal pore of oblateness, \(r\) are used. Figures 5(b) and 5(c) show the homogenized elastic modulus, \(E_{H11}\) and \(E_{H22}\) against the porosity of the ellipsoidal pore. The homogenized elastic modulus become small as the porosity becomes large. This means the mechanical strength decreases because of the large strain by the stress concentration at the pore wall. The dependence of the porosity becomes large when the oblateness is large in the case of \(E_{H11}\). The other hand, the results show the opposite tendency in the case of \(E_{H22}\). From these results, the stress is easy to concentrate at the pore wall for flatter pore than circular one because the curvature is small. Accordingly, it is desirable that shape of pore is circle and both the porosity and the anisotropy of pore become important factor for the estimation of coke quality.

4.3. Effect of Pore Distribution on Strength
Unit cells as shown in Fig. 6(a) are used and then the homogenized elastic modulus, \(E_{H22}\) and the stress intensity factor, \(K_I^*\) of the center crack are calculated by homogenization method against the dimensionless crack length, \(a\) in the case of porosity = 20%. That is, the effect of the pore distribution on the mechanical strength is investigated by changing the number of pore. Figure 6(b) shows that the homogenized elastic modulus decreases due to the existence of...
pore. This decline means the degradation of the mechanical strength. Also, the effect of pore distribution is small for the strength as macroscopic property.

From these results, the stress concentration can be predicted due to the decline of the homogenized elastic modulus. However, Fig. 6(c) shows that the stress intensity factor, which is criterion of the stress concentration, is oscillating as compared with the case of no pore. This figure means that microscopic behavior shows the stress concentration effect when the value is larger than the line of no pore, and the stress relaxation effect when the value is smaller than the line of no pore case. Positional relation between pore and crack causes these oscillating behaviors. In order to understand the stress concentration or relaxation behaviors, stress distributions around the crack tip and at unit cell are shown in Fig. 7 in the case of \( a = 0.3 \). The stress relaxation effect becomes small when pores do not exist just above or just below the crack tip (Fig. 7(a)). To the contrary, the stress relaxation effect becomes more dominant when pores exist in the perpendicular direction of the crack tip (Fig. 7(b)). Calculations of the stress intensity factor show that the strength for the microscopic fracture depends on the pore distributions. However, it is required for more strict research of the stress relaxation and concentration effect to remove the stress concentration effect at pore wall. Therefore, analysis in the case of small porosity, that is, estimation of micro cracks was performed in the next section.

4.4. Effect of Ambient Cracks on Strength

Using unit cells with center crack and four ambient
cracks as shown in Fig. 8(a), the stress intensity factor, $K_{I*}$, of the center crack are calculated for microscopic fracture by the homogenization method. By changing distances between center crack and ambient crack, $d$, cell length, $b$ (four times of vertical distance between center crack and ambient crack), the center crack length, $a$, and the ambient crack length, $c$, the stress concentration and relaxation effect are investigated. Microscopic behavior represents the stress concentration effect when the stress intensity factor in Fig. 8(b) or 8(c) is larger than the values in the case of $b/H = 0.5$. The reason for $b=2$ is obtained from calculations that the stress intensity factor becomes the same value when $b$ is larger than 2. This means that there are no correlations between center crack and ambient crack. On the other hand, the stress relaxation effect arises when it is smaller than the values in the case of $b=2$. The conditions of the stress concentration effect are following three cases. First, quarter of vertical distance between center crack and ambient crack, that is, the cell length, $b$, is smaller. Second, parallel distance, $d$, is larger. Third, ambient crack length, $c$, are larger.

The effect of center crack length, $a$ on the strength in the case of $c/H = 0.2$, is shown in Fig. 9. The stress concentration effect becomes superior when center crack length is small ($a=0.1$). However, The stress relaxation effect is shown in all distance, $d$ if center crack length becomes large ($a=0.4$).

Macrostructure analysis by the homogenization method could represent the correlation between micro-cracks from the stress concentration and relaxation effect. The homogenized elastic modulus, the stress intensity factor and the stress distribution obtained in this analysis, become available parameter. By further analysis of this method, the fracture conditions from the microscopic view will be clarified in detail.

5. Actual Coke Analysis by Using Digital Image Analysis

The microstructure of coke has irregular distribution and
complex shape of pore. This microscopic randomness of actual coke comes from the complicated phenomenon during carbonization process, such as devolatilization, softening, resolidification, evaporation, gas generation and carbon deposition. Due to the irregularity of microstructural geometry, the stress distribution in the actual microstructure is probably different from one obtained by simplified unit cell of geometry. In order to estimate the microscopic fracture in coke accurately while considering the microscopic mechanical behavior, unit cell must trace the actual geometric configuration for the finite element model.

Therefore, the digital image analysis based on modeling technique is introduced. The whole procedure of the digital image analysis can be divided into the following major four parts. 1. Scanning the image of microstructure. 2. Selecting the region of representative volume element. 3. Thresholding to determine the type of material. 4. Mesh generation by converting a pixel of image into a finite element.

Modeling of coke microstructure by using this technique is not only accurate but also automatic. Furthermore, finite element analysis by the homogenization method can reflect the effects of the original geometric configuration. In this section, this technique is introduced as new estimation method of coke qualities for the improvement mechanism of the mechanical strength by carbon deposition.

In order to clarify the influence of carbon deposition during the process of coking on its strength, original coke (non carbon deposited coke) and carbon deposited coke are investigated. The pyrolyzed gas of tar is intentionally deposited on pore wall of coke to make carbon deposited coke in the reactor. Figure 10 shows the micrograph of carbon deposited coke. The white part, gray part and black part in the figure are deposited carbon, matrix and pore, respectively. The digital image meshes of both original coke (see Fig. 11(a)) and carbon deposited coke (see Fig. 11(b)) are prepared and their strengths are compared by the homogenization method. Here, the white part, that is, deposited carbon is treated as pore for original coke in Fig. 11(a). On the other hand, white part is treated as matrix in the case of Fig. 11(b). The elastic modulus of deposited carbon is taken as the same as that of matrix. For mechanical property used in this study, elastic modulus of matrix is 17.5 GPa, Poisson’s ratio is 0.24 and tensile strength is 46.1 MPa.

Table 1 shows porosity and the homogenized elastic modulus, $E_{H}$, from Eq. (10) against the carbon deposited coke and the original coke. Differences between the calculated elastic modulus of direction ‘11’ and ‘22’ are found and the anisotropy is remarkable. This is due to the orientation of pores, that is, the anisotropy in used image as discussed in Figs. 5(b), 5(c). The homogenized elastic modulus of the carbon deposited coke becomes larger than that of the original coke due to the increase of porosity.

Figure 12 shows the Von-Mises stress distribution in the unit cells for (a) original coke and (b) carbon deposited coke at point A in Fig. 2. It is expected that microscopic fractures happen because the values of Von-Mises stress within white circles in Fig. 12(a) are larger than the tensile strength (46.1 MPa). The stress in unit cell for original coke of Fig. 12(a) does not concentrate on the edges of large pores but disperses into the surface of small pores. This stress relaxation effect is seen at point 1 as discussed in Fig. 7. However, the stress concentrates around the left side of the specimen (point 2) due to the increase of the total strain by the low homogenized elastic modulus as shown in Table 1. Since the total strain is too large, the stress concentration, which causes the microscopic fractures, appears around small pores. On the other hand, small stress concentration is caused for carbon deposited coke by the decrease of small pores at point 3 in Fig. 12(b), the maximum value of the stress distribution are smaller than the tensile strength due to the decrease of the total strain from the improvement of the mechanical strength as shown in Table 1. In spite of the stress concentration between large pores, the microscopic fracture is not predicted. As a result, the improvement of the mechanical strength by carbon deposition was shown from the maximum value of the stress distribution and the increase in the homogenized elastic modulus.

![Figure 10](image1.png)

The image of carbon deposited coke (413×191 pixel).

![Figure 11](image2.png)

Finite element mesh (413×191 mesh).

| Table 1. Homogenized elastic modulus. |
|--------------------------------------|
| Porosity (%) | $E_{11}^H$ (GPa) | $E_{22}^H$ (GPa) | $E_{33}^H$ (GPa) |
|----------------|-----------------|-----------------|-----------------|
| (a) Original coke | 22 | 5.84 | 2.91 | 0.777 |
| (b) Carbon deposited coke | 15 | 9.27 | 6.16 | 1.45 |
by the homogenization method.

6. Conclusion

The homogenization method is proposed to understand the fracture mechanism of coke from the consideration of the microscopic behavior.

First, the conditions of the stress concentration and relaxation effect are investigated by the homogenization method. These results show that the distribution of both cracks and pores in coke and the anisotropy of pores are important factors for the microscopic fracture.

The complicated microstructure of coke is accurately analyzed by means of the digital image technique. The effect of carbon deposition on the coke strength is illustrated from the stress distribution while calculating the homogenized elastic modulus. The stress in original coke disperses around small pores. Since the total strain is large due to low homogenized elastic modulus, large stress concentration, which causes the microscopic fractures, appears around small pores. In the case of carbon deposited coke, small stress concentration between large pores is caused. However, the microscopic fracture does not easily occur because the maximum value of the stress distribution is smaller than the tensile strength due to the increase in the homogenized elastic modulus by the reduction in porosity.

Nomenclature

- $N$: Shape function
- $r$: Oblateness of ellipsoidal pore
- $t$: Traction vector
- $u$: Displacement
- $x$: Macroscopic coordinate
- $Y$: Region of macroscopic cell
- $y$: Microscopic coordinate

Greek symbol

- $\chi$: Characteristic displacement
- $\varepsilon$: Scale ratio
- $\Gamma$: Boundary
- $\sigma$: Stress
- $\sigma_*$: Tensile load
- $\Omega$: Region of macroscopic cell

Superscript

- $0$: Macro-scale
- $1$: Micro-scale
- $H$: Homogenized parameter
- $T$: Transposed matrix

REFERENCES

1) M. Nishimura, H. Matsudaira, T. Yokoyama and S. Asada: CAMP-ISIJ, 11 (1998), 709.
2) K. Uebo, K. Inoue and K. Nishioka: CAMP-ISIJ, 6 (1993), 73.
3) J. M. Guedes and N. Kikuchi: Comp. Meth. Appl. Eng., 83 (1990), 143.
4) P. J. Green and R. Sibson: The Computer Journal, 21 (1978), 168.
5) R. Sibson: The Computer Journal, 21 (1978), 243.
6) A. Bowyer: The Computer Journal, 24 (1981), 162.
7) K. Terada, T. Miura and N. Kikuchi: Comp. Meth., 20 (1997), 331.
8) S. Murakami and Y. Liu: Mater. Sci. Res. Int., 2 (1996), 131.
9) N. Takano, M. Zako and Y. Ohnishi: Mater. Sci. Res. Int., 2 (1996), 81.
10) N. Takano, M. Zako and N. Kikuchi: Mater. Sci. Res. Int., 1 (1995), 82.
11) L. X. Han and S. Suresh: J. Am. Ceram. Soc., 72 (1989), 1233.
12) M. Soejima, Y. Asakuma, T. Mori, H. Aoki, T. Yamamoto, T. Miura, S. Tanioka and S. Itagaki: Tetsu-to-Hagané, 87 (2001), 245.
13) J. Padovan and G. Traore: Eng. Fract. Mech., 40 (1993), 457.
14) J. H. Kuang and L. S. Chen: Eng. Fract. Mech., 46 (1995), 736.
15) M. Ishida: JCM, 1 (1972), 394.