A quantum hydrodynamics approach to the formation of new types of waves in polarized two-dimension systems of charged and neutral particles

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In this paper we explicate a method of quantum hydrodynamics (QHD) for the study of the quantum evolution of a system of polarized particles. Though we focused primarily on the two-dimension physical systems, the method is valid for three-dimension and one-dimension systems too. The presented method is based upon the Schrödinger equation. Fundamental QHD equations for charged and neutral particles were derived from the many-particle microscopic Schrödinger equation. The fact that particles possess the electric dipole moment (EDM) was taken into account. The explicated QHD approach was used to study dispersion characteristics of various physical systems. We analyzed dispersion of waves in a two-dimension (2D) ion and hole gas placed into an external electric field which is orthogonal to the gas plane. Elementary excitations in a system of neutral polarized particles were studied for 1D, 2D and 3D cases. The polarization dynamics in systems of both neutral and charged particles is shown to cause formation of a new type of waves as well as changes in the dispersion characteristics of already known waves. We also analyzed wave dispersion in 2D exciton systems, in 2D electron-ion plasma and 2D electron-hole plasma. Generation of waves in 3D system neutral particles with EDM by means of the beam of electrons and neutral polarized particles is investigated.

I. INTRODUCTION

A two-dimension electron gas (2DEG) is in the focus of many studies. Sometimes it is considered to be an element of the spin-field effect transistor [1]. Ions and holes are distinct from electrons in that they can polarize in an external electric field or arrange in the line of the external field in the case they are rigid dipoles. EDM of ions and holes may cause changes in the dispersion of known eigenwaves as well as the onset of waves of novel types.

Dispersion characteristics of 2DEG have been analyzed in works of [2–4]. Magnetoplasma waves in 2DEG have been studied in [2, 3]. Excitation of charge and spin density in 2DEG has been addressed in [4] where spin-orbital interaction has been taken into account and the 2DEG has been supposed to locate in an external magnetic field. Not only wave processes attract researchers’ interest, but 2D magneto-transport also does. Studies of the effect of spin-orbital interaction on the 2D magneto-transport are also important. An equation set has been presented in [5] that describes charge and spin diffusion in 2DEG with account of spin-orbital interaction [6]. 2D hole gas along with 2DEG is used for construction of transistors, for example Atomic Layer Doping-Field Effect Transistor (ALD-FET) [7]. Collective modes in 2D bilayer graphene are considered in article [8].

In recent years attention is paid to the effect of the intrinsic magnetic moment (IMM) on the characteristics of charged particle systems. The propagation of perturbation of IMM and EDM does not require much energy as it occurs without mass transfer. Both processes may be used in the information transfer. In biological systems, for example, polarization processes, i.e. EDM propagation, are the predominant way of signal transfer.

There are modern studies which are focused primarily on the EDM dynamics [9], [10]. They deal with quasi-2D and multilayered systems of both charged and neutral polarized particles. There are articles where take into account of the EDM influence on the Bose-Einstein condensate (BEC) [11], [12]. In a particular work a contribution of polarization to the dispersion of Bogoliubov’s mode has been analyzed.

Influence of EDM on dynamic of charged particles is not large, analogously with the influence of IMM. But, evolution of IMM lead to existence of new physical effects which described below. In this way we can expect the existence new physical process which are the consequence of EDM dynamics. IMM dynamics causes changes in the wave dispersion in magnetized plasma [13–15] as well as the existence of novel branches of dispersion in systems of such kind [14]. Interaction between IMM of neutron beam and magnetized plasma leads to the generation of waves in plasma [12], [15]. This effect is due to spin-spin, spin-current [15] and spin-orbital interactions [16] between IMM of the beam and IMM in plasma. A lot of works is also devoted to the dynamics of IMM in BEC.

Classical methods were used previously to create a description of the collective dynamics of charged particles that takes into account the EDM that arises in the medium as a consequence of charged particles’ movement [17].

A quantum mechanics description for systems of N interacting particles is based upon the many-particle
Schrödinger equation (MPSE) that specifies a wave function in a 3N-dimensional configuration space. As wave processes, processes of information transfer, diffusion and other transport processes occur in the three-dimensional physical space, a need arises to turn to a mathematical method of physically observable values which are determined in a 3D physical space. To do so we should derive equations that determine dynamics of functions of three variables, starting from MPSE. This problem has been solved with the creation of a method of many-particle quantum hydrodynamics (MPQHD). Therefore, the reason for development such method is analogous with the motivation of the density functional theory \cite{13}. In our work we propose a further development of the MPQHD method. Here we consider the EDM dynamics in systems of charged and neutral particles. The MPQHD method has been applied to systems of charged \cite{19–21, 22} and neutral \cite{23} particles before. Many researchers \cite{13, 24, 25, 26} e.g. Madelung \cite{27} and Takabayashi \cite{28} consider derivation of QHD equations from a Schrödinger equation for a single particle in an external field. Marklund et. al. \cite{13} suggests the method of generalization of one-particle QHD equation for description of many-particle systems. In article \cite{26} method proposed in \cite{13} is used for derivation of QHD equations for system of relativistic particles. Both classical and quantum dynamics of a plenty of interacting particles in a configuration space are discussed in \cite{29}.

Bibliography contains many examples of QHD application for the analysis of various processes and phenomena. Ion-acoustic waves in dusty plasma \cite{30–32} and the distribution of non-linear electrostatic solitary excitations in \cite{33} have been analyzed. QHD method is also used to study wave processes in electron-postion-ion plasma \cite{34–36}. A system of QHD equations can be applied to the analysis of instability of quantum plasma \cite{37–41}, and in particular of the modulation instability of electron cyclotron waves \cite{38} and magnetoacoustic waves \cite{39}. Dispersion characteristics of magnetoacoustic waves have been analyzed in this way too \cite{42–44}. The dispersion of electrostatic waves with frequencies below the electron plasma frequency has also been studied using QHD \cite{45}. The dispersion relation for ion acoustic waves and the absorption coefficient for Landau damping are obtain in paper \cite{38} and the quantum electrodynamical short wavelength correction on plasma wave propagation for nonrelativistic quantum plasma is investigated in article \cite{46}.

Some of the results that have been obtained using QHD equations derived from a single-particle Schrödinger equation are presented in the review \cite{47}.

The MPQHD approach to systems of neutral particles, e.g. of ultracold boson-fermion mixtures, has been developed in \cite{23} and applied later to describe linear and non-linear characteristics of BEC and of boson-fermion mixtures \cite{23, 48, 50}.

In this work we extend MPQHD approach to a system of EDM-having particles. The method we developed here may be an effective tool for the investigation of static and dynamic behavior and transport characteristics of in graphene \cite{51} and nanofluidics \cite{52}, the last one deals with the dynamics of ions in water, a substance with large EDM of molecules.

In this article, using the QHD approach we calculate the dispersion of polarization waves and spatial charge waves. We show the EDM dynamics to cause novel branches of dispersion in quasi-2D plasma-like media and in systems of neutral particles. We also accomplish analytical calculations of dispersion characteristics for the waves we discovered.

Electrically polarized particle system can interact with the beam of charged and polarized particles by means charge-dipole and dipole-dipole interaction. Such interaction could lead to transfer energy from beam to medium and, consequently, to generation of waves. In plasma physics the effect of generation of waves by means electron \cite{53}, \cite{54} or magnetized neutron \cite{16} beam is well-known. In presented article we consider similar effects in system neutral polarized particles.

Our paper is organized as follows. In Sect. 2 we present the derivation of the momentum balance equation for EDM-having charged particles from MPSE in a self-consistent field approximation. An explicit form of the quantum part of the pressure tensor is also presented. In Sect. 3 we obtain equations of polarization evolution and polarization current. The self-consistent field approximation is used. In Sect. 4 a calculation is accomplished of eigenwaves in a 2D system of EDM-having charged particles located in uniform external electric field. New dispersion branch of \(\omega(k)\) is shown to exist and the contribution of polarization into the dispersion of 2D Langmuire wave is estimated. In Sect. 5 we show the existence of a polarization wave along with acoustic wave in a system of neutral polarized particles for 1D, 2D and 3D cases. In Sect. 6 dispersion characteristics of a two-sort 2D system of charged particles are discussed. An assumption is made that particles of one of the sorts bear EDM. We show that polarization dynamics here leads to the existence of a new dispersion branch. Analytical relation for \(\omega(k)\) is constructed. Deep analysis of quantum magnetoacoustic waves is performed. In Sect. 7 we apply the results presented in sections 4-6 to the analysis of excitations in 2D electron plasma and in a system of excitons. In Sect. 8 we show that there is instability at interaction of beam of polarized particles with the polarized medium. The increment of instabilities is calculated. In Sect. 9 we obtain the increment of instabilities which arise at electron beam propagation through system of neutral particles with the EDM. In sect. 10 brief summary of obtained results are presented.
II. CONSTRUCTION OF FUNDAMENTAL EQUATIONS AND THE MODEL ACCEPTED

In this section we derive the MPQHD equations from MPSE. Here we present the key steps of getting the MPQHD equations. We receive the equations for the system of charged particles with EDM. Obtaining equations could be used for neutral particles with the EDM as well. Method of MPQHG allow to present dynamic of system of interacting quantum particles in terms of functions defined in 3D physical space. It is important at investigation of wave process, which take place in 3D physical space.

Starting from MPSE

\[-i\hbar \partial_t \psi = \hat{H} \psi\]

with the Hamiltonian

\[\hat{H} = \sum_i \left( \frac{1}{2m_i} D_i^2 + e_i \varphi_{i,ext} - d_i^\alpha E_i^\alpha_{i,ext} \right) + \sum_{i,j \neq i} \left( \frac{1}{2} e_i e_j G_{ij} + e_i d_j^\alpha C_{ij}^\alpha - \frac{1}{2} d_i^\alpha d_j^\beta G_{ij}^{\alpha\beta} \right),\]  \hspace{1cm} (1)

we construct a system of QHD equations for charged particles that have EDM \(d_i^\alpha\). Equations constructed in this way are also valid for neutral polarized particles. The following designations are used in the Hamiltonian (1): \(D_i^\alpha = -i\hbar \partial_i^\alpha - e_i A_i^\alpha_{ext}/c, \varphi_{i,ext}, A_i^\alpha_{ext}\) the potentials of external electromagnetic field, \(E_i^\alpha_{ext}\), is the electric field, quantities \(e_i, m_i\)-are the charge and mass of particles, \(\hbar\)-is the Planck constant, and \(G_{ij} = 1/r_{ij}, C_{ij}^\alpha = -\delta_i^\alpha 1/r_{ij}, G_{ij}^{\alpha\beta} = \delta_i^\alpha \delta_j^\beta 1/r_{ij}\) - the Green functions of the Coulomb, charge-dipole, dipole-dipole interactions, respectively.

The first step in the construction of QHD apparatus is to determine the concentration of particles in the neighborhood of \(r\) in a physical space. If we define the concentration of particles as quantum average of the concentration operator in the coordinate representation \(\hat{n} = \sum_i \delta(r - r_i)\) we obtain:

\[n(r, t) = \int dR \sum_i \delta(r - r_i) \psi^*(R, t) \psi(R, t)\]  \hspace{1cm} (2)

where \(dR = \prod_{p=1}^N dr_p\).

Differentiation of \(n(r, t)\) with respect to time and applying of the Schrödinger equation with Hamiltonian (1) leads to continuity equation

\[\partial_t n(r, t) + \nabla j(r, t) = 0\]  \hspace{1cm} (3)

where the current density takes a form of

\[j^\alpha(r, t) = \int dR \sum_i \delta(r - r_i) \frac{1}{2m_i} \left( \psi^*(R, t) (D_i^\alpha \psi(R, t)) + (D_i^\alpha \psi(R, t))^* \psi(R, t) \right),\]  \hspace{1cm} (4)

The velocity of \(i\)-th particle \(v_i(R, t)\) is determined by equation

\[v_i(R, t) = \frac{1}{m_i} \nabla_i S(R, t) - \frac{e_i}{m_i c} A_{i,ext}.\]  \hspace{1cm} (5)

The quantity \(v_i(R, t)\) describe the current of probability connected with the motion of \(i\)-th particle, in general case \(v_i(R, t)\) depend on coordinate of all particles of the system \(R\), where \(R\) is the totality of 3N coordinate of N particles of the system \(R = (r_1, ..., r_N, t)\).

The \(S(R, t)\) value in the formula (5) represents the phase of the wave function

\[\psi(R, t) = a(R, t) \exp\left(\frac{i S(R, t)}{\hbar}\right).\]

Velocity field \(v(r, t)\) is the velocity of the local centre of mass and determined by equation:

\[j(r, t) = n(r, t) v(r, t).\]  \hspace{1cm} (6)

This means that \(u_i(r, R, t) = v_i(R, t) - v(r, t)\) is a quantum equivalent of the thermal speed.

A moment balance equation can be derived by differentiating current density (4) with respect to time:

\[\partial_t j^\alpha(r, t) + \frac{1}{m} \partial^\beta \Pi^{\alpha\beta}(r, t) = F^\alpha(r, t),\]  \hspace{1cm} (7)

where \(F^\alpha(r, t)\) is a force field and

\[\Pi^{\alpha\beta}(r, t) = \int dR \sum_i \delta(r - r_i) \times \]
\[\times \frac{1}{4m_i} \left( \psi^+(R, t) (\hat{D}_i^\alpha \hat{D}_i^\beta \psi)(R, t) + \right.\]
\[\left. + (\hat{D}_i^\alpha \psi)^+(R, t) (\hat{D}_i^\beta \psi)(R, t) + c.c. \right)\]  \hspace{1cm} (8)

represents the momentum current density tensor.

Let’s now perform explicit separation of particles’ thermal movement with velocities \(u_i(r, R, t)\) and the collective movement of particles with velocity \(v(r, t)\) in equations of continuity (3) and of the momentum balance (7). We can see now that the tensor \(\Pi^{\alpha\beta}(r, t)\) takes the form

\[\Pi^{\alpha\beta}(r, t) = m n(r, t) u^\alpha(r, t) u^\beta(r, t)\]
\[+ p^{\alpha\beta}(r, t) + T^{\alpha\beta}(r, t).\]

In this formula

\[p^{\alpha\beta}(r, t) = \int dR \sum_{i=1}^N \delta(r - r_i) a^2(R, t) m_i u_i^\alpha u_i^\beta\]  \hspace{1cm} (9)

is the tensor of kinetic pressure. This tensor tends to zero by letting \(u_i \to 0\).

The tensor

\[T^{\alpha\beta}(r, t) = -\frac{\hbar^2}{2m} \times\]
\[ \times \int dR \sum_{i=1}^{N} \delta(r - r_i) a^2(R, t) \frac{\partial^2 \ln a(R, t)}{\partial x_{\alpha i} \partial x_{\beta i}} \]  

(10)

is proportional to \( h^2 \) and has a purely quantum origin. For the large system of noninteracting particles, this tensor is

\[ T^{\alpha \beta}(r, t) = -\frac{\hbar^2}{4m} \partial^\alpha \partial^\beta n(r, t) \]

\[ + \frac{\hbar^2}{4m n(r, t)} (\partial^\alpha n(r, t)) (\partial^\beta n(r, t)). \]  

(11)

This term is named Bohm quantum potential.

As the particles of the system under consideration interact via long-range forces the approximation of a self-consistent field is sufficient to analyze collective processes. With the use of this approximation two-particle functions in the momentum balance equation can be split into a product of single-particle functions. Taken in the approximation of self-consistent field, the set of QHD equation, continuity equation and momentum balance equation has a form:

\[ \partial_t n(r, t) + \nabla (n(r, t) v(r, t)) = 0, \]  

(12)

\[ mn(r, t)(\partial_t + v \nabla) v^\alpha (r, t) + \partial_\beta (p^{\alpha \beta} (r, t) + T^{\alpha \beta}(r, t)) \]

\[ = en(r, t) E^{\alpha}_{ext}(r, t) + P^{\beta}(r, t) \partial^\alpha E^{\beta}_{ext}(r, t) \]

\[ + en(r, t) e^{\alpha \beta \gamma} v^\beta (r, t) B^{\gamma}_{ext}(r, t) \]

\[ - e^2 n(r, t) \partial^\alpha \int d\mathbf{r}' G(r, \mathbf{r}') n(\mathbf{r}', t) \]

\[ - en(r, t) \partial^\alpha \partial^\beta \int d\mathbf{r}' G(r, \mathbf{r}') P^{\beta}(\mathbf{r}', t) \]

\[ + e P^{\beta}(r, t) \partial^\alpha \partial^\beta \int d\mathbf{r}' G(r, \mathbf{r}') n(\mathbf{r}', t) \]

\[ + P^{\beta}(r, t) \partial^\alpha \int d\mathbf{r}' G^{\beta \gamma}(r, \mathbf{r}') P^{\gamma}(\mathbf{r}', t). \]  

(13)

Let’s discuss the physical significance of terms on the right side of (13). The first three terms describe the interaction with external electromagnetic field. The first term represents the effect of external electric field on the charge density. The second term is the effect of non-uniform external electric field on the polarization density. The form of this term is similar to that of the force field which affects the magnetic moment in a magnetic field \[20, 28]. It should be noted that the form of this term is distinct from the expression that describes the force affecting a single dipole. The third term of the equation (13) is the Lorentz force. Other terms in (13) describe a force field that represents interactions between particles, namely the Coulomb interaction of charges, the effect of dipoles on charges, charges on dipoles and dipole-dipole interactions.

Note, that for a 3D system of particles the momentum balance equation (13) may be written down in terms of electrical intensity of the field that is created by charges and dipole moments of the particle system:

\[ mn(r, t)(\partial_t + v \nabla) v^\alpha (r, t) + \partial_\beta (p^{\alpha \beta} (r, t) + T^{\alpha \beta}(r, t)) \]

\[ = en(r, t) E^{\alpha}(r, t) + P^{\beta}(r, t) \partial^\alpha E^{\beta}(r, t) \]

\[ + en(r, t) e^{\alpha \beta \gamma} v^\beta (r, t) B^{\gamma}_{ext}(r, t), \]  

(14)

where: \[ E^{\alpha}(r, t) = E^{\alpha}_{ext}(r, t) + E^{\alpha}_{int}(r, t) \] and \[ E^{\alpha}_{int}(r, t) = E^{\alpha}_{q}(r, t) + E^{\alpha}_{d}(r, t) \]. These variables meet equations \[ div E_{q}(r, t) = 4 \pi \rho \] and \[ div E_{d}(r, t) = -4 \pi \nabla \mathbf{P}(r, t) \] where \[ \rho = \sum a e a n_a(r, t) \]. This leads to a field equation

\[ div E_{int}(r, t) = 4 \pi \rho - 4 \pi div \mathbf{P}. \]  

(15)

The method we develop in this work is valid both for bosons and fermions. The type of statistics that particles are subject to affects the calculation of many-particle functions (correlations) that evolve in the momentum balance equation (13) and are neglected in the self-consistent field approximation. A method for the calculation of correlations in QHD equations has been developed in works \[19–21\].

Polarization evolves in the momentum balance equation (13) or (14):

\[ P^\alpha(r, t) = \int dR \sum_i \delta(r - r_i) \psi^*(R, t) \hat{d}^\alpha \psi(R, t). \]  

(16)

The EDM operator \[ d^\alpha_i \] there affects coordinates of the i-th particle. The polarization thus affects the evolution of concentration and of the velocity field. To calculate \[ P^\alpha(r, t) \] may be interesting itself. It is, therefore necessary to derive an equation that describes the evolution of polarization.

### III. EQUATIONS FOR THE EVOLUTION OF POLARIZATION

To close the QHD equations set (12), (13) we derive equation for the polarization evolution. If we differentiate the definition for polarization (16) with respect to time and apply the Schrödinger equation, the required equation for the polarization evolution can be obtained:

\[ \partial_t P^\alpha(r, t) + \partial^\beta R^{\alpha \beta}(r, t) = 0. \]  

(17)

A polarization current density

\[ R^{\alpha \beta}(r, t) = \int dR \sum_i \delta(r - r_i) \frac{d^\alpha}{2m_i} \times \]

\[ \times \left( \psi^*(R, t)(D^\beta_i \psi(R, t)) + (D^\beta_i \psi(R, t))^* \psi(R, t) \right) \]  

(18)
occurs in this equation.

We have two ways to close the QHD equations set. The first one is to express $R^{\alpha\beta}(r,t)$ in terms of $n(r,t)$, $v^\alpha(r,t)$ and $P^\alpha(r,t)$ using additional assumptions or experimental data. The other way is to derive the equation for evolution $R^{\alpha\beta}(r,t)$ in the same fashion it was accomplished previously for other material fields, for example quantities considered above $n(r,t)$, $v^\alpha(r,t)$ and $P^\alpha(r,t)$. Now the evolution equation $R^{\alpha\beta}(r,t)$ occurs in the form of

\[ \partial_t R^{\alpha\beta}(r,t) + \frac{1}{m} \partial^\gamma R^{\alpha\beta\gamma}(r,t) = \frac{e}{m} P^\alpha(r,t) E_{ext}^\beta(r,t) \]

\[ + \frac{e \epsilon_{\gamma\delta}}{mc} R^{\alpha\gamma}(r,t) B_{ext}^\delta(r,t) + \frac{1}{m} (\partial^\beta E_{ext}(r,t)) \int dR \sum_p \delta(r - r_p) d_p^\alpha d_p^\gamma \psi^\ast(R,t) \psi(R,t) \]

\[ - \frac{e^2}{m} \int dR' (\partial^\beta G(r,r')) \int dR \sum_p \delta(r - r_p) \delta(r - r_n) d_p^\alpha \psi^\ast(R,t) \psi(R,t) \]

\[ - \frac{e}{m} \int dR' (\partial^\beta G^\ast(r',r)) \int dR \sum_p \delta(r - r_p) \delta(r - r_n) d_p^\alpha d_p^\gamma \psi^\ast(R,t) \psi(R,t) \]

\[ + \frac{e}{m} \int dR' (\partial^\beta G^\ast(r',r)) \int dR \sum_p \delta(r - r_p) \delta(r - r_n) d_p^\alpha d_p^\gamma \psi^\ast(R,t) \psi(R,t) \]

\[ + \frac{1}{m} \int dR' (\partial^\beta G'^\ast(r',r)) \int dR \sum_p \delta(r - r_p) \delta(r - r_n) d_p^\alpha d_p^\gamma \psi^\ast(R,t) \psi(R,t). \tag{19} \]

Now we consider the physical meaning of the terms in right side of this equation. The first three terms describe the interaction of particles with external electromagnetic field. The first term represents the effect of external electric field on the charge density. The second term is the effect of non-uniform external electric field on the polarization density. The form of this term is similar to that of the force field which affects the magnetic moment in a magnetic field. The third term of the equation (19) is an analog of the Lorentz force. Other terms in (19) describe a tensor field that represents interactions between particles, namely the Coulomb interaction of charges, the effect of dipoles on charges, charges on dipoles and dipole-dipole interactions.

We used the denotation in (19) for $R^{\alpha\beta\gamma}(r,t)$:

\[ R^{\alpha\beta\gamma}(r,t) = \int dR \sum_i \delta(r - r_i) \frac{d^\alpha_i}{4m_i} \left( \psi^\ast(R,t)(\hat{D}_i^\beta \hat{D}_i^\gamma \psi)(R,t) + \right. \]

\[ \left. + (\hat{D}_i^\beta \psi)^\ast(R,t)(\hat{D}_i^\gamma \psi)(R,t) + c.c. \right). \tag{20} \]

Equation (19) show the evolution of polarization current $R^{\alpha\beta}(r,t)$ in consequence of interaction of particles with external and internal force field. Equation (19) contain two-particle correlation function. The two-particle function found in the four last term of equation (19). In articles [19], [21] the method of calculation of correlation is developed. Direct calculation of many-particle function using methods developed in [19], [21] show that statistics affect on correlations only and have no influence on terms arisen in self-consistent field approximation. Below, in this article we bound our self by self-consistent field approximation.

We can see that the equation (19) contains information about the effect of interaction on the transformation of $R^{\alpha\beta}(r,t)$ and, as a consequence, on the polarization evolution $P^\alpha(r,t)$.

In the approximation of the self-consistent field, the equation (19) takes the form

\[ \partial_t R^{\alpha\beta}(r,t) + \frac{1}{m} \partial^\gamma R^{\alpha\beta\gamma}(r,t) = \frac{e}{m} P^\alpha(r,t) E_{ext}^\beta(r,t) \]

\[ + \frac{e \epsilon_{\gamma\delta}}{mc} R^{\alpha\gamma}(r,t) B_{ext}^\delta(r,t) + \frac{1}{m} (\partial^\beta E_{ext}(r,t)) \int dR \sum_p \delta(r - r_p) d_p^\alpha d_p^\gamma \psi^\ast(R,t) \psi(R,t) \]

\[ - \frac{e^2}{m} \int dR' (\partial^\beta G(r,r')) \int dR \sum_p \delta(r - r_p) \delta(r - r_n) d_p^\alpha \psi^\ast(R,t) \psi(R,t) \]

\[ - \frac{e}{m} \int dR' (\partial^\beta G^\ast(r',r)) \int dR \sum_p \delta(r - r_p) \delta(r - r_n) d_p^\alpha d_p^\gamma \psi^\ast(R,t) \psi(R,t) \]

\[ + \frac{1}{m} \int dR' (\partial^\beta G'^\ast(r',r)) \int dR \sum_p \delta(r - r_p) \delta(r - r_n) d_p^\alpha d_p^\gamma \psi^\ast(R,t) \psi(R,t). \tag{19} \]
The physical meaning of terms on the right side of this equation is similar to that of terms in the equation (19).

We used the denotation
\[ D^{\alpha\beta}(r, t) = \int dR \sum_i \delta(r - r_i) d_i^\alpha d_i^\beta \psi^*(R, t) \psi(R, t) \]
in the equation (21).

It can be assumed that
\[ D^{\alpha\beta}(r, t) \simeq \sigma \frac{P^\alpha(r, t) P^\beta(r, t)}{n(r, t)}. \]
where \( \sigma \) is the nondimension constant.

Likewise the previous section we can reformulate the equation (21) for the three-dimension case in terms of the electrical field (15). The equation (21) in this situation has the form
\[ \partial_t R^{\alpha\beta}(r, t) + \frac{1}{m} \partial^\gamma R^{\alpha\beta\gamma}(r, t) = \frac{e}{m} P^{\alpha}(r, t) E^{\beta}(r, t) \]
\[ + \frac{e}{mc} \varepsilon^{\beta\gamma\delta} R^{\alpha\gamma}(r, t) B^{\delta}_{\text{ext}}(r, t) + \frac{1}{m} D^{\alpha\beta}(r, t) \partial^\delta E^{\gamma}(r, t). \]
(24)

If we put a velocity field into the equation (21) the tensor \( R^{\alpha\beta\gamma}(r, t) \) transforms into
\[ R^{\alpha\beta\gamma}(r, t) = r^{\alpha\beta\gamma}(r, t) + T^{\alpha\beta\gamma}(r, t) \]
\[ + m R^{\alpha\beta}(r, t) v^{\gamma}(r, t) + m R^{\alpha\gamma}(r, t) v^{\beta}(r, t) \]
\[ - m P^{\alpha}(r, t) v^\beta(r, t) v^\gamma(r, t). \]
(25)

In this representation the contribution of thermal movement
\[ r^{\alpha\beta\gamma}(r, t) = \int dR \sum_{i=1}^N \delta(r - r_i) d_i^\alpha a^2(R, t) m_i u_i^\beta u_i^\gamma \]
becomes explicit and the analog for the Bohm quantum potential occurs
\[ T^{\alpha\beta\gamma}(r, t) = -\frac{\hbar^2}{2m} \int dR \sum_{i=1}^N \delta(r - r_i) \times \]
\[ \times d_i^\alpha a^2(R, t) \frac{\partial^2 \ln a}{\partial x_{\beta_i} \partial x_{\gamma_i}}. \]
(27)

Approximate connection of \( T^{\alpha\beta\gamma}(r, t) \) with the concentration and polarization of particles is presented in work [12].

The major contribution into alterations of polarization in a system of charged particles is from charge interactions and from the effect that the external electrical field has on charges. As a result
\[ \partial_t R^{\alpha\beta}(r, t) = \frac{e}{m} P^{\alpha}(r, t) E^{\beta}_{\text{ext}}(r, t) \]
\[ + \frac{e^2}{m} P^{\alpha}(r, t) \partial^\beta \int dr' G(r, r') n(r', t) \]
(28)
can be derived from the equation (21).

Later in this paper the equation (28) is used to analyze the elementary excitations spectrum in a two-dimensional system of EDM-having charged particles.

Method of QHD allows derive an one-particle nonlinear Schrodinger equation (NLSE). Corresponding one-particle wave function describes evolution of many-particle system in 3D physical space. Most famous NLSE are the Ginzburg-Landau equation in superconductivity [55] and the Gross-Pitaevskii equation in the physics of ultracold gases and Bose-Einstein condensate [56]. In this article we consider the derivation of NLSE. We do not use NLSE for studying of wave dispersion and wave generation. Because of it we present NLSE in Appendix. In this paper we receive NLSE for the charged particle having EDM. Interesting consequence of derivation of NLSE is the obtaining of London equation. Derivation of the London’s equation from QHD also is presented in appendix.

IV. THE EIGENWAVES IN A 2D SYSTEM OF CHARGED PARTICLES WITH EDM

One of the primary goals of this article is to derive dispersion characteristics of eigenwaves in 2D systems of neutral and charged particles that take account of the EDM collective dynamics. In this section we consider 2D systems of charged particles. To do that let’s analyze small perturbations of physical variables from the stationary state. The ionic or holes gas is assumed here to localize on a \( x, y \) -plane
\[ n = n_0 + \delta n, \quad v^\alpha = 0 + \delta v^\alpha, \quad P^{\alpha} = P_0^{\alpha} + \delta P^{\alpha}, \]
(29)

\[ E^{\text{ext}}_{\text{ext}} = E_0^{\delta\alpha z}, \quad P_0^{\alpha} = \kappa E_0^{\alpha}, \quad \kappa E_0^{\delta\alpha z}, \]
where \( n \) is a surface concentration.

We can mark, in this section and below, all physical quantity is presented in the form of sum of equilibrium part and small perturbations
\[ f = f_0 + \delta f. \]

In this case if we assume that linear excitations \( \delta f \) are proportional to \( e^{i\omega t} + e^{-i\omega t} \) a linearized set of equations (12, 13, 17, 28) gives us the dispersion equation
\[ \frac{m \omega^2}{m} = n_0^2 k^2 + \frac{\hbar^2}{4m} k^4 \]
\[ + 2\pi e^2 n_0 k - 2\pi P_0^\beta(k) e^2 m \omega^2 k^4, \]
(30)
where

\[ \beta(k) = 2\pi \int_\xi^\infty dr \frac{f_0(r)}{r^2}, \]  

(31)

\[ \beta = \frac{2\pi}{k_{\max}} > \frac{2r_0}{k_{\max}} \]  

and \( \xi = r_0 \beta_k \). (32)

\[ \omega^2 = \frac{1}{2} \left( \frac{\beta^2}{\kappa^2} k^2 + \frac{\hbar^2}{m^2} k^4 + \omega_{L,12}^2 \right) \]

(32)

\[ \omega_{L,12} = \frac{2\pi\kappa^2 E_0^2 \beta \epsilon^2 k^4}{m^2 v_s^2 k^2 + \frac{1}{2} \beta \kappa^2 k^4 + m^2 \omega_{L,12}^2}, \]

(33)

\[ \omega = \frac{2\pi\kappa^2 E_0^2 \beta \epsilon^2 k^4}{m^2 v_s^2 k^2 + \frac{1}{2} \beta \kappa^2 k^4 + m^2 \omega_{L,12}^2}. \]

(34)

The relation (33) expresses the dispersion of 2D Langmuir waves with the account of dipole-dipole interactions. The solution (34) expresses the dispersion of waves that emerge as a result of polarization dynamics. Plots at Figs. 2 and 3 represent relationships (33), (34), respectively.

\[ \beta \]

FIG. 1. The figure presents the dependence of the variable \( \beta(k) \) on the wave vector \( k \). Ionic or atomic radius \( r_0 \) is assumed to equal 0.1 nm.

\[ \omega, \text{s}^{-1} \]

FIG. 2. The figure shows the dispersion characteristic of the Langmuir wave frequency \( \omega \) versus the wave vector \( k \), which is described by the equation (33). The ionic radius \( r_0 \) is supposed to be 0.1 nm. Equilibrium polarization has form \( P_0 = \kappa E_0 \). Static electric permeability \( \kappa \) is defined by the equation \( \kappa = \kappa_0 r_0^2/(3k_B T) \). \( p_0 \) - is a dipole moment of an ion or atom, \( T \) - temperature of the medium, \( k_B \) - Boltzmann constant. System parameters are assumed to be as follows: \( r_0 = 10^8 \text{sm}^{-2}, \) \( p_0 = 3 \times 10^{-20} \text{Cm}, T = 100K, E_0 = 3 \times 10^4 \text{V/m}, m_i = 10^{-23} \text{g}. \)

\[ k, \text{sm}^{-1} \]

FIG. 3. The figure shows the dispersion characteristic of the quantum ionic polarization wave frequency \( \omega \) versus the wave vector \( k \), which is described by the equation (34). The ionic radius \( r_0 \) is supposed to be 0.1 nm. Physical conditions of the system are supposed to be the same as described in the legend to Fig. 2.

V. EIGENWAVES IN A SYSTEM OF POLARIZED NEUTRAL PARTICLES

In this section we consider 1D, 2D and 3D systems of EDM-having neutral particles. To do that we use the equation (21). Since the system of neutral particles resides in a uniform electromagnetic field, there is only one term on the right side of the equation (21). It is also assumed that interactions make the largest contribution into the changes in \( R^{\alpha\beta}(r, t) \).

If so then the equation (21) and (23) transforms into

\[ \partial_t R^{\alpha\beta}(r, t) = \frac{P_0}{mn(r, t)} \times \]

\[ \times \partial^\beta \int d\mathbf{r}' G^{\gamma\delta}(r, r') P^{\delta}(r', t). \]  

(35)
FIG. 4. The figure shows the dispersion characteristic of the 2D quantum atomic polarization wave frequency $\omega(k)$ versus the wave vector $k$, which is described by the equation (36). The atomic radius is supposed to be 0.1 nm. System parameters are the same as in Fig. 2.

If taken in a linear approximation, the set of equations (12), (13), (17), (35) for neutral particles is closed. This allows the analysis of polarization waves in a system of neutral particles. Equations of continuity (12) and of the momentum balance (13) herein describe the dynamics of acoustic wave. If we derive a solution for eigenwaves in a 2D system the dispersion equation has a form of

$$\omega = \sqrt{\frac{\beta(k)}{mn_0}} |\kappa| E_0 k^{3/2},$$  

(36)

where $\beta(k)$ is defined by the relation (31). The dispersion dependence (36) is presented on Fig. 4.

In 1D case $\omega(k)$ occurs as

$$\omega = \sqrt{\frac{\beta_1(k)}{mn_0}} |\kappa| E_0 k^2,$$  

(37)

where

$$\beta_1(k) = 2 \int_0^\infty dr \frac{\cos(r)}{r^3}.$$  

(38)

The quantity (38) and 1D dispersion dependence (37) are presented on Figs. (5) and (6) correspondingly.

The set of equations (17), (35), (23) may be applied to analyze 3D systems, otherwise the electric field $E(r, t)$ that dipoles create may be introduced explicitly (15), (17), (24), (25).

As this is done the dispersion equation $\omega(k)$ transforms into

$$\omega = \sqrt{\frac{4\pi\sigma}{mn_0} P_0 k_z},$$  

(39)

Equations (36) and (37) differ in the power coefficient by the wave vector $k$ and also with the coefficients $\beta(k)$ and $\beta_1(k)$, which depend on the wave vector, occur in 1D and 2D cases, respectively.

VI. EIGENWAVES IN A 2D TWO-SORT SYSTEM OF CHARGED PARTICLES

A 2D system comprised by charged particles of two sorts can be modeled either in a thin film of metal and semiconductors or in a thin, nano-sized ionic crystal. Only ions and holes can have EDM in the former case. Both sorts of ions can have EDM in the case of ionic crystal. In our analysis we deal with a material where one sort of ions has much greater EDM than the other and investigate elementary excitations in such system.

The QHD equation set for the system in question is comprised by continuity equations (12) and momentum balance equations (6) for both sorts of particles and also by equations (17), (21), which describe the polarization dynamics of particles those having greater EDM. The field on the...
right side of equations (6) and (21) comes from particles of both sorts. Likewise previous sections the 2D system is assumed here to be located in an external electric field which is orthogonal to the plane of particle movement. Solution of the eigenwave problem in a two-sort system of charged particles leads to the dispersion equation

\[(\omega^2)^3 - \left(\omega^2_e + v_{se}^2 k^2 + \frac{\hbar^2 k^4}{4m_e^2} + \omega^2_i + v_{si}^2 k^2 + \frac{\hbar^2 k^4}{4m_i^2}\right)(\omega^2)^2 \]

\[+ \left(v_{se}^2 + \frac{\hbar^2 k^2}{4m_e^2}v_{se}^2 + \frac{\hbar^2 k^2}{4m_i^2}v_{si}^2\right)^2 \]

\[\pm \sqrt{(\omega_e^2 + \omega_i^2)^2 + (v_{qse}^2 k^2 - v_{qsi}^2 k^2)^2 + (\omega_e^2 - \omega_i^2)(v_{qse}^2 k^2 - v_{qsi}^2 k^2)}\]. \hspace{1cm} (41)

The following denotation is used here:

\[v_{qs} = v_{sa} + \frac{\hbar^2 k^2}{4m_a^2},\]

where \(a\) denotes \(e\) for electrons and \(i\) for ions.

The equation (41) contains dispersion characteristics for the Langmuir wave and ion-acoustic wave. In the quantum case we consider these characteristics take on the form of

\[\omega^2 = \omega_e^2 + v_{se}^2 k^2 + \frac{\hbar^2}{4m_e^2} k^4,\] \hspace{1cm} (42)

and

\[\omega = \frac{k v_s}{\sqrt{1 + \frac{\hbar^2 k^4}{4m_e^2 v_{se}}} + \sqrt{1 + \frac{\hbar^2 k^4}{4m_i^2 v_{si}}}}.\] \hspace{1cm} (43)

Here used designation \(a_e^2 = v_{se}^2 / \omega_e^2\), \(v_s = (m_e / m_i) v_{se}\).

In approximation:

\[v_{si}^2 + \frac{\hbar^2 k^2}{4m_i^2} \ll \omega^2/k^2 \ll v_{se}^2 + \frac{\hbar^2 k^2}{4m_e^2}\] \hspace{1cm} (44)

\[\omega^2 = \frac{1}{n_0 m_i} \frac{\beta(k) P_0^2 \omega_i^2 k^3 (v_{qse}^2 + \frac{\hbar^2 k^2}{4m_e^2})}{\omega_e^2 (v_{si}^2 + \frac{\hbar^2 k^2}{4m_i^2})^2 + \omega_i^2 (v_{se}^2 + \frac{\hbar^2 k^2}{4m_e^2}) + (v_{se}^2 + \frac{\hbar^2 k^2}{4m_e^2})(v_{si}^2 + \frac{\hbar^2 k^2}{4m_i^2}) k^2}.\] \hspace{1cm} (46)

If the approximation (44) is used then the equation (46) takes the form

\[\omega = \sqrt{\frac{\beta(k) P_0 k^{3/2}}{n_0 m_i}}.\] \hspace{1cm} (47)
The equation (45) for ion-acoustic waves occurs in the same approximation.

The polarization also causes additional contributions in solutions (41) to occur. Thus, a generalization of solutions (41) that takes account of polarization appears in the following form:

\[
\omega^2 = \omega_{0\pm}^2 \mp \frac{1}{\omega_{0+}^2 - \omega_{0-}^2} \times \\
\times \frac{\beta(k) \rho_0^2 k^2 \omega_i^2}{n_0 m_i} \left( 1 - k^2 \frac{v_{se}^2}{\omega_i^2} \right).
\] (48)

VII. THE EFFECT OF POLARIZATION ON THE DISPERSION CHARACTERISTICS OF 2D SEMICONDUCTORS

The simplest model of semiconductors is a system comprised by particles of three sorts, namely electrons, ions and neutral atoms. Atoms and ions in the semiconductor may bear significant EDM. Dynamics of semiconductors is usually dealt with as dynamics of electrons and holes. Electrons and holes can also form bound states - the excitons. As excitons have EDM, the dipole-dipole interaction plays a substantial role in a system of excitons.

The approach we present in this paper allows us to describe the interaction of excitons via their EDMs and interaction of their EDM with an external electric field. Since excitons are neutral EDM-having quasi-particles, the result we obtained in Section VI may be used to describe elementary excitations in such system. We can therefore suggest that acoustic-like waves and polarization waves may be excited in exciton systems and their dispersion is described by formulas (46), (47) and (49).

In the case electrons and holes move in a semiconductor without significant formation of bound state excitations in a semiconductor may be regarded to as excitations in a two-sort system of charged particles. The term "hole" regards to ions in the semiconductor, which may have EDM. This means that holes may be dealt with as EDM-having particles.

Using the model described above we can apply formulae from the Section VII to describe the dispersion of elementary excitations in 2D electron-hole plasma. Index \(i\) in formulas (40)-(48) regards to holes.

VIII. EXCITATION OF POLARIZATION WAVES BY BEAM OF NEUTRAL POLARIZED PARTICLES

We do not meet experimental works there were obtained the polarization waves received in our article. On example of 3D sample we show the way to obtain such waves in experiment. A possible way of generation of waves of polarization it is the propagation of beam of electrons or neutral polarized particles through the sample. Instead of the beam can be considered a current of particles through the sample, in context of next section it can be the electrical current. In this and next sections we show there are instabilities which led to generation of waves in the sample. The effect of waves generation by beam of electrons is well in plasma physics [53], [54].

In this section we use equations (12), (14), (17) and (24) for each sorts of particles and equation of field (15). In right side of equation (15) the polarization \(P\) is a sum of polarization of medium \(P_d\) and polarization of the beam \(P_b\)

\[ P = P_d + P_b. \]

The equilibrium state of system is characterized by following values of the medium parameters:

\[ n_d = n_{0d} + \delta n_d, \quad v_{d} = 0 + v_{d}^\alpha, \]

and values of the beam parameters:

\[ n_b = n_{0b} + \delta n_b, \quad v_{b}^\alpha = U \delta v_{b}^\alpha + \delta v_{b}^\alpha, \]

The polarization \(P_b^\alpha\) is proportional to external electric field \(E_0\). We consider the case then \(E_0 = [E_0^\sin \varphi, 0, E_0^\cos \varphi]\). In this case the tensor \(R_{ab}^{\alpha\beta}\) has only two unequal to zero elements: \(R_{0b}^{xx} = R_{0b}^{y\sin \varphi}\) and \(R_{0b}^{zz} = R_{0b}^{z\cos \varphi}\). And we consider the small perturbations of described here equilibrium state.
In the absence of medium we obtain a dispersion dependence of polarized beam modes
\[ \omega = k_z \pm \sqrt{\omega^2_{\text{Db}} + \frac{\hbar^2 k^4}{4m_e^2}}. \]

At the presence of medium, far from resonant condition \( k_z U \approx \omega_{\text{Db}} \), we obtain
\[ \omega \approx k_z U \pm \frac{\omega_{\text{Db}}}{\sqrt{1 - \frac{\omega^2_{\text{Db}}}{(k_z U)^2}}}. \]

For the case \( \omega_D \gg k_z U \) from (53) we derive
\[ \omega = k_z U \left( 1 \pm \frac{\omega_{\text{Db}}}{\omega_D} \right). \]

Further we consider resonant interaction neutral polarized beam with medium. In this situation we present frequency in the form
\[ \omega = k_z U + \eta. \]

At resonant condition \( \omega_D \approx k_z U \) and \( \eta \gg \frac{\hbar k^2}{m_b} \) we have
\[ \eta = \xi \sqrt{\frac{\omega^2_{\text{Db}} \omega^2_{\text{Le}}}{2 \left( \omega^2_D + \frac{\hbar^2 k^4}{4m_e^2} \right)}}. \]

where \( \xi \) presented by formula (66). In the limit \( \eta \ll \frac{\hbar k^2}{m_b} \) we obtain a formula which is analogous to (67). But in this case we need to do some changes, i.e. \( m_e \rightarrow m_b \) and \( \omega_{\text{Le}} \) to \( \omega_{\text{Db}} \).

### IX. EXCITATION OF POLARIZATION WAVES BY ELECTRON BEAM

In previous section we consider the effect of generation of the polarization waves in 3D system of neutral particles with the EDM. Here we analyse an analogous effect. It is a generation of waves by means of a beam of neutral polarized particles.

Let’s analyze the interaction of a single velocity beam of electrons that moves along z-axis with infinite three-dimension medium comprised of neutral particles. Particles of the medium are in equilibrium state and polarized without any external electric field. This may occur if the electron beam moves through a crystal of piezoelectric or through a sample of ferroelectric that has residual polarization. The result of such analysis would also represent a physical mechanism of the interaction of an electron beam with spatially confined or low-dimension polarized systems. The most significant types of interaction in such systems are the Coulomb interaction of electrons in the beam, the dipole-dipole interaction in the medium and the charge-dipole interaction of dipoles in the medium with the electron beam. Let’s assume that the beam moves alongside the polarization vector and the beam’s velocity doesn’t change significantly while it moves through the sample.

In this section we use equations (12), (14), (17) and (24) for each sorts of particles and equation of field (15).

If so, solution of the QHD equation set leads to the dispersion equation
\[ 1 - \frac{\omega^2_{\text{Le}}}{(\omega - k_z U)^2 - \frac{\hbar^2 k^4}{4m_e^2}} = 0, \]

where
\[ \omega^2_D = \frac{4 \pi \sigma P^2_0 k^2}{m m_0}. \]

This equation has three solutions. If an account is taken of Bohm quantum potential and no electron beam is present, solution
\[ \omega^2 = \omega^2_D + \frac{\hbar^2 k^4}{4m_d^2} \]
occurs for the polarization wave, while two beam modes occur in the absence of the medium
\[ \omega = k_z U \pm \sqrt{\omega^2_{\text{Le}} + \frac{\hbar^2 k^4}{4m_e^2}}. \]

If the contribution from the Bohm quantum potential is neglected and the limit of a low-density beam is assumed the solution for beam mode looks like
\[ \omega \approx k_z U. \]

If we consider the effect of the dipole-comprised medium on the beam mode the following solution is obtained
\[ \omega \approx k_z U \pm \frac{\omega_{\text{Le}}}{\sqrt{1 - \frac{\omega^2_{\text{Le}}}{(k_z U)^2}}} = k_z U \pm \frac{\omega_{\text{Le}}}{\sqrt{\frac{\omega^2_{\text{Le}}}{(k_z U)^2}}}. \]

This solution is complex and leads to the instability if \( \omega_D > k_z U \) i.e. in the case of long wave approximation.

If \( \omega_D \gg k_z U \) then solution transforms to
\[ \omega = k_z U \left( 1 \pm \frac{\omega_{\text{Le}}}{\omega_D} \right). \]

Solution (59) is valid for a beam moving in a 3D system of dipoles if
\[ k_z U \neq \omega_D, \]
i.e. if there is no resonance between beam mode and polarization mode.

Let’s see how a beam affects the dispersion of polarization wave in the absence of resonance with beam mode. The dependence of frequency on the wave vector has the form
\[ k_z U = \sqrt{\omega^2_D + \frac{\hbar^2 k^4}{4m_d^2}}. \]
To analyze a resonance interaction that occurs between the electron beam and a system of electric dipoles, i.e. the situation of (62) we should seek for a solution that looks like

$$\omega = k_z U + \eta = \sqrt{\omega_D^2 + \frac{\hbar^2 k^4}{4m_e^2}} + \eta. \quad (63)$$

In this instance we obtain the following equation of a frequency shift $\eta$:

$$\eta^3 \pm \frac{\hbar k^2}{m_e} \eta^2 - \frac{\omega_L^2 \omega_D^2}{2} + \frac{\hbar^2 k^4}{4m_e^2} = 0. \quad (64)$$

If $\eta \gg \frac{\hbar k^2}{m_e}$ then it has the following solution:

$$\eta = \xi \sqrt{\frac{\omega_L^2 \omega_D^2}{2} + \frac{\hbar^2 k^4}{4m_e^2}} \quad (65)$$

where

$$\xi = \sqrt{1 + \left(1 \frac{1 + 2\sqrt{3}}{2}, -1 \frac{-1 + 2\sqrt{3}}{2}\right)} \quad (66)$$

In the opposite limit of $\eta \ll \frac{\hbar k^2}{m_e}$ the frequency shift occurs as

$$\eta = \mp \sqrt{\frac{m_e \omega_L \omega_D}{\hbar k^2}} \sqrt{\frac{\omega_L^2 \omega_D^2}{2} + \frac{\hbar^2 k^4}{4m_e^2}}. \quad (67)$$

So we have shown above in this section that the electron beam that moves in a system of dipoles polarized by external electric field induces instability and increases amplitude of the polarization mode. It is assumed that the electron beam moves alongside of the external electric field and the medium is infinite and homogeneous. It should be noted that while the electron beam accelerates infinitely in such conditions we assumed changes of the beam velocity to be small. This was done because our goal was to approximate a system of finite length.

X. CONCLUSIONS

In this paper we analyzed wave excitations caused by the EDM dynamics in systems of charged and neutral particles. Method of QHD was developed for EDM-having particles. QHD equations are a consequence of MPSE in which particles' interaction is directly taken into account. In our work we consider the Coulomb, charge-dipole and dipole-dipole interactions. The system of QHD equations we constructed is comprised by equations of continuity, of the momentum balance, of the polarization evolution and of the polarization current. In our studies of wave processed we used a self-consistent field approximation of the QHD equations.

Using QHD equations we analyzed elementary excitations in various physical systems in a linear approximation. Waves in a 2D gas of EDM-having charged particles and in a gas of neutral EDM-having particles in various physical dimensions were considered. A two-sort 2D system of charged particles where particles of one sort are assumed to have EDM was also analyzed.

Dispersion branches of a novel type that occurs due to polarization dynamics were discovered in those physical systems. Furthermore, the effect of polarization on the dispersion characteristics of already known wave modes was studied. The contribution of Bohm quantum potential which is of purely quantum origin was taken into account in the calculations. Waves of the electric polarization we discovered possess the following feature: their frequencies $\omega$ tends to zero provided that $k \to 0$.

The polarization modes that have been derived in our work should be taken into account in calculations of the thermal capacity and non-linear dynamics of acoustic and ion-acoustic waves. Polarization modes may also contribute together with phonons to the process of formation of Cooper pairs in superconductors. These polarization modes may be applied in the construction of devices that implement information transfer processes. Excitements in polarization may be used as an alternative to spin waves or together with them. It also should be taken into account that transfer of polarization disturbances plays a major role in the information transfer in biological systems. Such processes do not require particles of the medium to possess EDM as the dynamics of a system of charged particles leads to collective polarization $P^c(r, t)$.

The effect of polarization dynamics on the dispersion characteristics of exciton systems and of electron-hole plasma is discussed in section VIII. The polarization mode occurred in those systems too and its dispersion has been analyzed. The existence of the polarization mode may affect characteristics of spintronic and nano-electronic devices.

We show the possibility of the process of waves generation in the system of polarized particles by means of the monoenergetic beam of neutral polarized particles and the monoenergetic beam of electrons.

Hence, in this work we present the advancement of the QHD approach to systems of polarized particles and use it to show the occurrence of polarization waves in various physical systems comprised by charged and neutral particles.

APPENDIX

Derivation of the non-linear Schrödinger equation

In this section we discuss the derivation of NLSE for systems of charged particles. The derivation is generally similar to the derivation of the GP equation for systems of neutral bosons in the state of BEC [23]. The NLSE comes about from the continuity equation and the Cauchy integral...
of the momentum balance equation. The Cauchy integral exists provided that the velocity field has the form

$$v^\alpha(r, t) = \frac{1}{m} \partial^\alpha \theta(r, t) - \frac{e}{mc} A^\alpha(r, t)$$  \hspace{1cm} (68)

where $\theta(r, t)$ is a velocity field potential.

Starting from QHD equations we can derive an equation for evolution of the model function defined in terms of hydrodynamic variables. Thus a macroscopic single-particle wave function may be defined as

$$\Phi(r, t) = \sqrt{n(r, t)} \exp\left(\frac{i}{\hbar} m \theta(r, t)\right).$$  \hspace{1cm} (69)

If we differentiate this function with respect to time and apply QHD equations then given the absence of EDM the equation obtained is

$$\partial_t \Phi(r, t) = \left( -\frac{\hbar^2}{2m} \nabla - \frac{ie}{\hbar c} A_{ext}(r, t) \right)^2 + e \varphi(r, t)$$

$$+ e^2 \int d\mathbf{r}' \left| \frac{\Phi(r', t)}{|\mathbf{r} - \mathbf{r}'|} \right|^2 + \Gamma(r, t) \Phi(r, t),$$  \hspace{1cm} (70)

where

$$\Gamma(r, t) = \int_{r_0}^r ds \nabla p(r', t) \frac{1}{n(r', t)}.$$  \hspace{1cm} (71)

The equation obtained (70) has the form of NLSE.

NLSE (70) describes collective characteristics in a system of many charged particles. This follows from the derivation of NLSE and from the definition of a many-particle wave function $\Phi(r, t)$ (69). A nonlinear term that is proportional to $e^2$ depicts the Coulomb interaction. Function $\Gamma(r, t)$ (71) is the contribution of the kinetic pressure and does not contain any interaction.

Now let’s write down the NLSE that arise from QHD equations (12) and (13) taking into account the EDM of particles:

$$\partial_t \Phi(r, t) = \left( -\frac{\hbar^2}{2m} \nabla - \frac{ie}{\hbar c} A_{ext}(r, t) \right)^2 + \frac{e^2}{4} \int d\mathbf{r}' \left| \frac{\Phi(r', t)}{|\mathbf{r} - \mathbf{r}'|} \right|^2 + \Gamma(r, t) \Phi(r, t)$$

$$+ e \varphi_{ext}(r, t) + \Gamma(r, t) + C(r, t)$$

$$+ e^2 \int d\mathbf{r}' \frac{P^\beta(r', t)}{|\mathbf{r} - \mathbf{r}'|} \Phi(r, t),$$  \hspace{1cm} (72)

here we use designation $\Gamma(r, t)$ described by formula (71). Also in (72) exist action of electric field on polarization of particles. This effect is presented by function $C(r, t)$, which determined by following formula

$$C(r, t) = \int_{r_0}^r d\mathbf{r}' \frac{1}{n(r', t)} \times$$

$$\times \left( e P^\beta(r', t) \nabla^\gamma \partial^\beta \int d\mathbf{r}'' G(r', \mathbf{r}'') n(r'') \right)$$

$$+ P^\beta(r', t) \nabla^\gamma \int d\mathbf{r}'' G^\beta(r', \mathbf{r}'') P^\gamma(r'', t) \right).$$  \hspace{1cm} (73)

If we deal with a 3D system of particles we can introduce a self-consistent electric field into NLSE as that was done in equations (12), (14) and (15). In such case equations (72) and (73) transform into

$$\partial_t \Phi(r, t) = \left( -\frac{\hbar^2}{2m} \nabla - \frac{ie}{\hbar c} A_{ext}(r, t) \right)^2$$

$$+ e \varphi(r, t) + \Gamma(r, t) + C(r, t) \Phi(r, t).$$  \hspace{1cm} (74)

In this case $C(r, t)$ has form

$$C(r, t) = \int_{r_0}^r d\mathbf{r}' \frac{1}{n(r', t)} P^\beta(r', t) \nabla E^\beta(r', t),$$  \hspace{1cm} (75)

$$E = -\nabla \varphi$$

in equations (74), (75) and this field satisfies the equation (15).

Given the additivity of electromagnetic field the equations (74), (75) can be generalized and $A_{ext}$ substituted with a vector potential $A$ that is created by both external and internal sources. QHD equations (13), (14), (21) and (24) can be generalized in the same way.

**Some words about London equation**

In many books the London equation is derived from equation which describe the dynamics of one particle in external field. But, the London equation describes the properties of many-particle system, i.e. system of electrons in superconductors. In this section we present a new way of derivation of the London equation from QHD equations, which describe collective dynamics of particles. Also, we obtain generalization of the London equation.

Given the absence of a magnetic field the condition (68) becomes a condition of vortex-free flow. Assuming the approximation of a homogeneous medium $n = n_0 = \text{const}$ an equation

$$\text{rot}(n_0 \mathbf{v}(r, t)) = -\frac{e n_0}{mc} \text{rot}(\mathbf{A}(r, t))$$  \hspace{1cm} (76)

can be derived from (68).

If we apply equations $n_0 \mathbf{v} = \mathbf{j}$, $\text{rot} \mathbf{A} = \mathbf{B}$, $\text{rot} \mathbf{B} = 4\pi e / c j$, $\text{div} \mathbf{B} = 0$ a well-known London equation

$$\Delta \mathbf{B}(r, t) = \frac{4\pi e^2 n_0}{mc^2} \mathbf{B}(r, t)$$  \hspace{1cm} (77)

can be obtained where

$$\frac{4\pi e^2 n_0}{mc^2} = \frac{\omega_L^2}{c^2} = \frac{1}{\lambda_L^2}, \quad \lambda_L = \frac{c}{\omega_L},$$

where $\omega_L$ is the London frequency.
\( \omega_{Le} \) is the Langmuire frequency and \( \lambda_L \) is the Londons' constant \[57, 58\].

In the case of an inhomogeneous medium the following relationship appears from the equation (68)

\[
\text{rot} \mathbf{j}(\mathbf{r}, t) = -\frac{e}{m c} \text{rot} (n(\mathbf{r}, t) \mathbf{A}(\mathbf{r}, t))
\]

\[
= -\frac{e}{m c} n(\mathbf{r}, t) \mathbf{B}(\mathbf{r}, t) - \frac{e}{m c} [\nabla n(\mathbf{r}, t), \mathbf{A}(\mathbf{r}, t)].
\] (78)

This gives us a generalization for the equation (77):

\[
\triangle \mathbf{B}(\mathbf{r}, t) = \frac{4 \pi e^2 n(\mathbf{r}, t)}{m c^2} \mathbf{B}(\mathbf{r}, t) + \frac{e}{m c} [\nabla n(\mathbf{r}, t), \mathbf{A}(\mathbf{r}, t)].
\] (79)

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