Mathematical modeling of processes of heat and mass transfer in channels of water evaporating coolers

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Abstract. The variety of cooling systems is dictated by a wide range of demands placed on them. This is the price, operating costs, quality of work, ecological safety, etc. These requirements in a positive sense are put into correspondence by water evaporating plate coolers. Currently, their widespread use is limited by a lack of theoretical base. To solve this problem, the best method is mathematical modeling.

1. Introduction
The climate of any operational building consists of a number of parameters, among which an important place is occupied by temperature and humidity. In the hot period of time, these coolers are able to bring these parameters close enough to the optimal one. But due to the lack of the theoretical base, their use is rather limited [1].

The maximum possible depth of cooling at minor air flow will not allow one to effectively fight with external heat penetration. On the other hand, an excessive air flow will increase velocity in the airway tracts providing a negative impact on the depth of cooling. To select the most optimal geometric parameters of the cooler, it is necessary to understand the process of change of parameters of the treated air while passing through the channels [2], [3]. Mathematical modeling helps to solve this task efficiently [4].

2. Mathematical modeling
Cooling capacity \( Q = C \cdot \rho \cdot G \cdot \Delta t \) is an indicator informing about the performance of the water evaporating cooler. As one sees, it depends on such indicators as \( C \), which is specific heat of air, joule/(kg \* K); \( \rho \) - air density, kg/m\(^3\); \( G \), m\(^3\)/s - volumetric air flow in the channels of the evaporative pads; \( \Delta t \), °C - the depth cooling value, which is the difference between the temperature at the inlet and outlet of the cooler. The last two indicators directly depend on the geometric parameters of the cooler. The highest value of refrigerating capacity proves the most efficient operation of the cooler.

The model of the plate water evaporating cooler has the following form:

Channel cross section is \( H=2h \). The equations of heat and mass transfer, respectively, have the form:
\[
\rho \cdot V(y) \cdot C \cdot \frac{dt}{dx} = \frac{d}{dy} \left( \lambda \frac{dr}{dy} \right) \tag{1}
\]

\[
V(y) \cdot \frac{dw}{dx} = \frac{d}{dy} \left( D \frac{dw}{dy} \right) \tag{2}
\]

\(w(x, y) = \Phi(x, y) \cdot w_H(t),\) where \(D\) is the diffusion coefficient; \(w\) is the humidity.

\(w_H(t) = 10^{-5} (3,5t^2 - 40,6t + 1090,5).\tag{3}\)

On the surface of the porous plate:

\(RJ_a = \frac{\lambda}{dy} \bigg|_{y=0},\)

\(w \bigg|_{y=0} = w_H(t_{surface}).\)

where \(R=(2500,6-2.372 \cdot t) \cdot 10^3, t_{surface} - \) the surface temperature of the plate and on the axis of symmetry of the channel of the condition of the parity:

\(\frac{dt}{dy} \bigg|_{y=h} = 0, \quad \frac{dw}{dy} \bigg|_{y=h} = 0.\)

The initial conditions are the conditions at the entrance to the channel:

\(t \bigg|_{x=0} = t_{\text{entrance}}, \quad \Phi \bigg|_{x=0} = \Phi_{\text{entrance}}.\)

Numerical implementation of this mathematical model, which is called complete, allows determining the temperature and humidity along the length of the evaporative pads \([5]\). Unfortunately, this model has no analytical solution, and for engineering calculations the formula to determine the above-mentioned temperature is desired.

Let us take the integral of equation (1) in the \(y\) direction from 0 to \(h\):

\[
\int_0^h \rho \cdot V(y) \cdot C \cdot \frac{dt}{dx} dy = \int_0^h \frac{d}{dy} \left( \lambda \frac{dr}{dy} \right) dy. \text{ Then:}
\]

\[
\rho C \frac{d}{dx} \int_0^h V(y) \cdot t \cdot dy = \lambda \frac{dr}{dy} \bigg|_{y=0}^h.
\]

One should remember that \(h\) is half the channel cross section, \(V\) is the air velocity. Taking into account the condition of "adhesion" \(V \bigg|_{y=0} = 0,\) the known definition of heat flow average temperature \(T = \frac{1}{hV_{\text{average}}} \int_0^h V(y) \cdot t \cdot dy,\) the condition of parity of temperature and flow on the axis of symmetry of the channel, there will be:

\[
\rho CV_{\text{average}} \frac{dT}{dx} = -\lambda \frac{dr}{dy} \bigg|_{y=0},
\]

or

\[
\rho Cv_h \frac{dT}{dx} = -J. \tag{5}
\]

Similarly, let us obtain the equation for heat flow average steam density \(W:\)
\[ V_{\text{average}} \cdot h \cdot \frac{dW}{dx} = -D \frac{dw}{dy} \bigg|_{y=0}, \quad \text{or} \]
\[ \nu h \frac{dW}{dx} = J_n. \]  
\( (6) \)

Multiplying equation (6) by \( R \) and adding to equation (5), taking into account the boundary conditions (4), let us receive:

\[ \rho C_{\nu h} \frac{dT}{dx} + \nu h R \frac{dW}{dx} = 0. \]

Integrating in \( x \) and decreasing to \( \nu h \), one gets:

\[ \rho C T + R W = C_1 = \text{const}. \]  
\( (7) \)

Constant \( C_1 \) is determined from the initial conditions:

\[ C_1 = C \rho \rho_{\text{entrance}} + R(t_{\text{entrance}}) \cdot w_{\nu}(t_{\text{entrance}}) \]

For sufficiently large values of \( x \), the air will completely saturate with moisture, and heat transfer will stop. As a result, the relative humidity reaches 100% and formula (7) takes the form:

\[ \rho C T_{\liminary} + R W_{\nu}(T_{\liminary}) = C_1. \]

Using (3), let us obtain a quadratic equation, from which one can determine \( T_{\liminary} \):

\[ C \rho \rho_{\text{surface}} + R \cdot 10^{-5} (3.5 T_{\text{surface}}^2 - 40.6 T_{\text{surface}} + 1090.5) = \]
\[ = C \rho \rho_{\text{entrance}} + R(t_{\text{entrance}}) \cdot \varphi_{\text{entrance}} 10^{-5} (3.5 t_{\text{entrance}}^2 - 40.6 t_{\text{entrance}} + 1090.5). \]  
\( (8) \)

Numerical solution of the full system using the implicit finite–difference schemes implemented by the standard single pass method demonstrated the following results.

First, the surface temperature of evaporator plate \( T_{\text{surface}} \) remains virtually unchanged and coincides with \( T_{\liminary} \). This is due to the fact that at the channel entrance, the intensive evaporation of the liquid occurs, whereby the surface temperature of the plate is dramatically reduced.

Second, heat transfer coefficient \( \alpha \) decreases in the initial phase and becomes stable in the steady state flow \([6], [7]\).

In this regard, to calculate the temperature of the air in the channels, equation \( J(x) = \alpha(x)(T - T_{\text{surface}}) \) can be applied which together with (5) provides:

\[ \rho C_{\nu h} \frac{dT}{dx} = -\alpha(x)(T - T_{\text{surface}}). \]  
\( (9) \)

The authors tried to obtain an approximate solution by breaking the length of the plate into 2 sections: initial \([0, L_{\text{initial}}]\) and steady \([-L_{\text{initial}}, x]\) state where \( L_{\text{initial}} = 0.055 \cdot \frac{C \rho \nu H^2}{\lambda} \).

In the initial part of the interval, the following approximation can be used:

\[ Nu(x) = Nu(0) - Nu_{\liminary} \frac{L_{\text{initial}}}{Nu(0) - Nu_{\liminary}} \text{exp}(\frac{-Nu_{\liminary} - Nu(0)}{Nu_{\liminary}} x) + Nu_{\liminary}. \]  
\( (10) \)
where, \( \text{Nu}(0)=17.51 \), and \( \text{Nu}_{\text{limitary}}=3.777 \), which corresponds to established interval \( [L_{\text{initial}}, x] \).

Considering \( \alpha(x) = \frac{\text{Nu}(x) \cdot \lambda}{H} \), after integration and simple transformations, when \( x \leq L_{\text{initial}} \) the solution of the equation (10) takes the form:

\[
T(x) = T_{\text{surface}} + 0.896 \cdot (t_{\text{entrance}} - T_{\text{surface}}) \cdot e^{-\frac{L_{\text{initial}}}{\rho C_v H^2}} \cdot e^{\frac{13.733 x}{L_{\text{initial}}}} + 3.777 x.
\]

at \( x > L_{\text{initial}} \)

\[
T(x) = T_{\text{surface}} + (t_{\text{entrance}} - T_{\text{surface}}) \cdot 0.896 \cdot e^{-\frac{7.554 \lambda}{\rho C_v H^2}}.
\] (11)

The dependence of change of temperature of the length of the evaporative pads, computed by these formulas, coincides with the results obtained for the full mathematical model. The following graph evaluates the results (Figure 1). Comparing these curves, one can conclude on the possibility of using the proposed formula. Indeed, the calculations show that the inaccuracy is not more than 2.4 %

3. Conclusion
From the foregoing, it can be concluded that the derived formula accurately describes the processes of heat mass transfer. Consequently, it enables one to determine quite accurately the optimal geometrical parameters of the coolers depending on the type of system of ventilation and characteristics of the fan used for approximation of parameters of the air environment to the optimum ones. It is possible to make both direct calculations, i.e. determining the dynamics of the temperature change along the length of the cooler, selecting its most efficient length, and the opposite calculations, i.e. the determination of the geometry of the cooler depending on the specified parameters of the cooled air. Under the character of the latter, both the temperature and relative humidity can be considered.

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