Modeling and evaluation of magnetic fluid deformable mirror with dual-layer actuators

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\begin{abstract}
In this paper, a rectangular magnetic fluid deformable mirror (MFDM) with dual-layer actuators is proposed, which is designed to improve the correction performance for full-order aberrations. Compared with the conventional adaptive optics system that uses two mirrors to configure as a woofer–tweeter system, the proposed MFDM combines the two mirrors into one by using a two-layer layout design of the actuators. Firstly, based on the governing equations of the magnetic fluid, derived from the principles of conservation of fluid mass and magnetic field, the dynamics model of surface deflection of the MFDM is analyzed in Cartesian coordinates under the boundary conditions of the magnetic field and the kinematic conditions of magnetic fluid. Then, the analytical solutions of the surface movement of the mirror subject to the applied currents in the electromagnetic coils are obtained by properly separating the variables with truncated model numbers. Finally, the experimental results based on a fabricated prototype square MFDM show the effectiveness of modeling and the correction performance of the mirror for the full-order aberrations.
\end{abstract}

\begin{keywords}
Deformable mirror; dual-layer actuators; dynamics modeling; magnetic fluid; multiphysics coupling
\end{keywords}

1. Introduction

Adaptive optics (AO) technology has been used for a wide range of applications.\textsuperscript{[1]} The aim of this technology is to acquire the image with increased resolution or achieve precise beam control, which uses the wavefront corrector to control and correct optical wavefront in real-time. Currently, AO technology has been used in astronomical observations,\textsuperscript{[2]} retinal imaging,\textsuperscript{[3]} and other fields.\textsuperscript{[4–7]} The wavefront corrector plays a key role in the AO systems since its performance directly determines the wavefront correction capability of the AO system.

The wavefront aberrations caused by possible imperfections in the optical components, or more likely, by irregularity in the medium the light travels through traditionally are corrected with solid deformable mirrors or liquid crystal spatial light modulators. These devices have the common drawbacks of the high-cost per-channel, and the relatively low surface deflection stroke (normally less than 50 $\mu$m) and inter-actuator stroke (normally less than 10 $\mu$m)\textsuperscript{[8,9]} due to the limitation of materials and manufacturing technology. For example, the largest stroke and...
inter-actuator stroke of the popular commercial deformable mirrors produced by OKO are limited to 25 µm and 5 µm, respectively. The magnetic fluid deformable mirror (MFDM) can easily produce strokes of more than 100 µm both for the single actuator or inter-actuators\textsuperscript{[10−12]} which is due to the free surface movement constraint of the liquid. The deformable mirror with such large strokes can be used to correct the large defective aberrations of rotating liquid telescopes, which could reach an RMS peak of 36 µm and a peak-to-valley amplitude of more than 180 µm\textsuperscript{[13,14]} or actively correct the aberrations of large and lower optical quality primary mirrors held by simple support systems in astronomical telescopes, which could reach an RMS peak of 28 µm and a peak-to-valley amplitude of more than 150 µm\textsuperscript{[15]} Therefore, compared with the solid mirrors, the main advantages of the MFDM are the large stroke, low cost, easy scalability, and simple fabrication process, on account of which, they can be easily adapted to different applications.

In practice, studies have shown that in many applications such as beam shaping,\textsuperscript{[16]} microscopy\textsuperscript{[17]} and other optical imagining systems,\textsuperscript{[14,15,18]} AO system is required to effectively handle the low-amplitude high-order aberrations as well as high-amplitude low-order aberrations simultaneously. A set of strategies are to use two DMs with different capabilities that complement each other as the woofer–tweeter configuration.\textsuperscript{[19,20]} The woofer is usually equipped with a low number of actuators and can produce large surface deflection with low spatial frequencies for the low-order aberration correction, such as the defocus and astigmatism, etc. The tweeter could have a large number of actuators but only produce limited strokes with high spatial frequencies. The combination of these two DMs has allowed for the correction of the wavefront with full-order aberrations. However, the implementation of these two mirrors in the single AO system will result in a complex optical path, which restricts its practical applications.

In the face of a growing demand for deformable mirrors with large stroke and high spatial frequencies, a rectangular MFDM with dual-layer actuators is proposed in this paper. The upper-layer actuators with small size and high density are used to compensate for small-amplitude high-order aberrations. The lower-layer actuators with big size and low density are used to correct large-amplitude low-order aberrations. One of the major advantages of MFDM is the simple fabrication process and hence the mirror can be easily custom designed for different applications. Therefore, in order to obtain an optimal desired performance of MFDM, the accurate dynamical model analysis plays an important role for each design instance.

A circular MFDM with a two-layer layout of actuators is proposed by Wu et al.\textsuperscript{[21]} and the dynamical model is analyzed in the polar coordinate with Maxwell equation and Bessel equations. However, the modeling method used by Wu et al.\textsuperscript{[21]} cannot be applied to the MFDM with a rectangular layout of actuators. In the following, the dynamics of MFDM with a rectangular two-layer deployment of actuators was newly studied in Cartesian coordinates and the derivation of the surface response is formulated within Cartesian boundary conditions using Laplace equations. In the study by Iqbal and Amara,\textsuperscript{[22]} the modeling of rectangular MFDM with a single-layer of actuators was studied, where the actuators are simply considered to be exactly beneath the bottom of the magnetic fluid. Since for the MFDM with a two-layer layout of actuators, the distance between the bottom of the magnetic fluid and the top of the actuators should be optimally designed, therefore, this parameter has been explicitly considered in the modeling process. In the following sections, the dynamics model of surface deflection of the rectangular MFDM is first analyzed under the Cartesian boundary conditions of the magnetic field and the kinematic conditions of magnetic fluid. Then, based on the derived analytical model, an optimal design of a prototype MFDM to obtain the required performance, i.e. the largest stroke and inter-actuator stroke of the mirror, as well as the coupling coefficient of the influence function, is presented. Finally, the accuracy of the model and the wavefront aberration correction performance of MFDM are verified by experiment results in the AO system.
2. Modeling of magnetic fluid deformable mirror with dual-layer actuators

2.1. Structure of MFDM

As shown in Figure 1(a), an MFDM is represented by a cuboid horizontal layer of magnetic fluid. The mirror shape is depicted as the deflection of the top surface measured with respect to its flat state. Aiming to achieve the correction of full-order aberrations with a high spatial resolution, the arrangement of dual-layer actuators is proposed. Figure 1(b) shows the arrangement of the dual-layer electromagnetic coils, where the actuators with small dimensions and high density are placed in the upper layer to correct the high-order small-amplitude aberrations, while the actuators with big dimensions and low density are placed in the lower layer to compensate the low-order large-amplitude aberrations.

To linearize the response of the MFDM, Iqbal, Ben Amara, and Wu used a Helmholtz coil in their setup to produce a large uniform magnetic field and superposes to the magnetic field generated by the miniature actuators.[23,24] In this paper, a Maxwell coil is used to instead of the Helmholtz coil, which produces a uniform magnetic field up to the 6th order derivative with respect to the position near the center of the assembly.[25]

2.2. Modeling of MFDM in cartesian coordinates

As shown in Figure 1, $\zeta(x, y, t)$ is the deflection of the magnetic fluid surface, which produced by the cumulative magnetic potentials $\psi_j$, where $i = 1, 2$ is the $i$th layer of actuators, and $j = 1, 2, 3, \ldots, I_i$ is the $j$th coil of each layer. $\psi_j$ is generated by a dual-layer electromagnetic coil located underneath the magnetic fluid layer. $h_1$ and $h_2$ represent the height of the dual-layer actuators to the surface of MFDM, respectively. The magnetic field itself is governed by Maxwell’s equations that are applied to all three sub-domains marked in Figure 1 as (1), (2) and (3). According to, the perturbation part of the surface dynamic governing equations can be written as follows:

$$\nabla^2 \phi = 0, -d \leq z \leq \zeta \quad (1)$$

$$\nabla^2 \psi_{ij} = 0, l = 1, 2, 3, i = 1, 2 \quad (2)$$

$$-\rho \frac{\partial \phi}{\partial t} + \rho g \zeta + \chi B_0 \frac{\partial \psi_{ij}^{(2)}}{\partial z} - \sigma \left( \frac{\partial^2 \zeta}{\partial x^2} + \frac{\partial^2 \zeta}{\partial y^2} \right) = 0 \text{ at } z = \zeta \quad (3)$$
where \( \rho \) is the density and viscosity of the fluid, \( \phi \) and \( \psi_i^l, l = 1, 2, 3, i = 1, 2 \) are the perturbation components of the velocity potential and the magnetic potential, respectively, \( \sigma \) is the surface tension, \( \mu \) is the magnetic permeability of the magnetic fluid, \( \chi = (\mu/\mu_0 - 1) \) is the susceptibility of the fluid, which is considered to be constant, \( B_0 \) represents the magnetic flux density.

Based on the two physic kinematic conditions of MFDM, the solution of \( \phi \) can be obtained as follows:

\[
\phi(x, y, z, t) = -\frac{1}{k} \cosh(k(z + d)) \frac{d\zeta(t)}{dt} E(x, y)
\]

where \( k \) is the separation constant, \( \zeta = \zeta(t)E(x, y) \) and \( E(x, y) \) satisfy the following ordinary differential equations,

\[
\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} = - \left( k_x^2 + k_y^2 \right) E(x, y)
\]

\( k_x \) and \( k_y \) are the mode numbers, and \( k = \sqrt{k_x^2 + k_y^2} \), which is an infinite number of solutions to be obtained for \( \phi \).

Based on the magnetic field boundary conditions between the three different materials and consider the magnetic potential sources of input magnetic coils as

\[
\psi_i^l(x, y, z, t) = \sum_{j=1}^{l} \psi_j(t) \delta^2(x - x_j)(y - y_j)f(z - h_i),
\]

the \( \psi_i^l, l = 1, 2, 3, i = 1, 2 \), in Equation (2) are then solved as:

\[
\psi_i^1(x, y, z, t) = C_i(t)e^{-kx}E(x, y)
\]

\[
\psi_i^2(x, y, z, t) = \left[ C_i(t) \left( \cosh(kz) - \frac{\mu_0}{\mu} \sinh(kz) \right) - \chi B_0 \tilde{\zeta}(t) \cosh(kz) \right] E(x, y)
\]

\[
\psi_i^3(x, y, z, t) = \left( C_i(t)V(t) - W(t)B_0 \tilde{\zeta}(t) \right) E(x, y)
\]

\[
V(kz) = - \left[ \left( 2 \frac{\mu}{\mu_0} + \frac{\mu_0}{\mu} \right) - \left( 1 + \frac{\mu_0}{\mu} \right) \frac{\beta}{\alpha} \right] \cosh(kz)
\]

\[
- \left[ \left( 2 - \frac{\mu_0}{\mu} \right) - \left( 1 + \frac{\mu_0}{\mu} \right) \frac{\beta}{\alpha} \right] \sinh(kz), \ k \neq 0
\]

\[
W(kz) = \frac{1}{\alpha} \frac{\chi}{\mu} \left( \beta \cosh(kz) + \chi \sinh(kz) \right) , \ k \neq 0
\]

\[
\alpha = - \frac{1}{\sinh(kd) \cosh(kd)} = \tanh(kd) - \coth(kd)
\]

\[
\beta = \frac{\mu_0}{\mu} \tanh(kd) - \coth(kd)
\]

and \( E(x, y) \) satisfy the following equation:

\[
E(x, y) = \cos(k_x x) \cos(k_y y)
\]

The infinite number of discrete values of \( k_x \) and \( k_y \) can be written in the series form as follows:

\[
k_x = \frac{(m - 1)\pi}{L_x}, m = 1, 2, 3 \ldots
\]
\[
\begin{equation}
\frac{\partial^2 \tilde{r}_{\text{imn}}(t)}{\partial t^2} + \omega_{dnn} \frac{\partial \tilde{r}_{\text{imn}}(t)}{\partial t} + \omega_{\text{imn}}^2 \tilde{r}_{\text{imn}}(t)
= R_m R_n B_0 \frac{\chi}{\rho} \frac{k_m^2 R_m}{L_x L_y} \cdot \sum_{j=1}^{l_j} \psi_j(t) \cos(k_{mx_j}) \cos(k_{ny_j})
\end{equation}
\]

where 

\[
\omega_{\text{imn}} = -\chi \cdot \frac{B_0^2}{\rho} \tanh(k_{mn}d)k_{mn}^2 \frac{\mu_0}{\mu} W(-k_{mn}h_1)
- \frac{\sigma}{\rho} \tanh(k_{mn}d)k_{mn}^3 \frac{\mu_0}{\mu} V(-k_{mn}h_1)
+ 4 \frac{\eta}{\rho} k_{mn}^2
\]

For convenience, the solution is truncated to a finite number of modes such that \(m = 1, 2, 3, \ldots, M\) and \(n = 1, 2, 3, \ldots, N\). The second-order differential Equation (18) has the capability to obtain the generalized displacements \(\tilde{r}_{\text{imn}}(t)\), and the corresponding mode shapes \(E_{mn}\), evaluated at any desired location \((x, y)\), give the total surface displacement at the location as

\[
\zeta(x, y, t) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{r}_{1mn}(t) \cos(k_{mx}) \cos(k_{ny})
+ \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \tilde{r}_{2mn}(t) \cos(k_{mx}) \cos(k_{ny})
\]

The set of Equations (18) and (19) illustrate that the surface response \(\zeta(x, y)\) is linearly dependent on the input \(\psi_j\) introduced by each electromagnetic coil. Then using Equations (18) and (19), the static surface response model of the mirror relative to the disturbing magnetic field generated by each actuator can be obtained.
Based on Equations (18) and (19), the state-space model of the mirror can be written as follows:

\[
P: \begin{cases}
    \dot{x}_1 = A_s x_1 + B_s i_1 \\
    \dot{x}_2 = A_s x_2 + B_s i_2 \\
    y_s = C_s x_1 \\
\end{cases}
\]

where \( x_i = [\hat{\zeta}_{i1}, \hat{\zeta}_{i2}, \ldots, \hat{\zeta}_{iMN} ]^T, \ i = 1, 2 \) is the vector of the generalized displacements and velocities, \( i_i = [i_1, i_2, \ldots, i_J ]^T, \ i = 1, 2 \) is the vector of input currents, and \( y_s = [\zeta_1, \zeta_2, \ldots, \zeta_K ]^T \) is the vector of wavefront produced by the deformable mirror at \( K \) sampling points. \( A_s, B_s, \) and \( C_s \) are the corresponding system matrices.

3. Experimental evaluation of the MFDM

3.1. MFDM design

Based on the aforementioned schematic diagram (Figure 1), a prototype of a square MFDM has been built. As shown in Figure 2, the MFDM mainly consists of a Maxwell coil, the magnetic fluid filled in a container, and dual-layer electromagnetic coils. The magnetic fluid (EMG 304, Ferrotec Corporation, USA) has a saturation magnetization of 27.5 mT, a relative magnetic permeability of 5.03, a viscosity of 5 cP, and a density of \( 1.24 \times 10^3 \) kg/m\(^3\).

Magnetic fluids typically show low reflectance to light and must be coated with thin metal liquid-like films (MELLFs) that exhibit reflective properties like liquid metals, but are thin enough not to have any significant effect on the deformation of the substrate magnetic fluid. Consequently, a silver liquid-like film has been prepared for the MFDM, which is assembled by a series of processes using encapsulated silver nano-particles as raw material.[11]

Based on the analytical model, in order to achieve the stroke requirement of 100 \( \mu \)m, the uniform magnetic field intensity is determined to be \( B_0 = 7.4 \) mT and the maximum perturbed...
magnetic field intensity measured at the fluid surface is calculated to be \( b_z = 0.5 \text{mT} \). According to the Biot–Savart law, the magnetic field at the center of the Maxwell coil is given by \( B_0 = 15\mu_0 N_m I / 16 R_m \), where \( N_m \) is the number of turns of the middle coil, \( R_m \) is the radius of the middle coil, and \( I \) is the current supplied to the Maxwell coil. Then, based on the desired magnetic field of \( B_0 = 7.4 \text{mT} \), the radius of the middle coil is set at \( R_m = 100 \text{mm} \), the current is set at \( I = 500 \text{mA} \), and the turns of the three coils with AWG25 magnet wires are set at 1152, 883, and 1152, respectively. It should be noted that the parameters of the MFDM can be modified with respect to different application cases.

According to Equation (20), for the fixed input currents, the resulting wavefront provided by MFDM is as follows:

\[
y_0^s = P_0 i_0
\]

(21)

where the matrix \( P_0 = -\mathbf{C}_s \mathbf{A}_s^{-1} \mathbf{B}_s \) is the DC gain of the system model. Based on Equation (21), to obtain a given desired wavefront \( y_s^r \), an optimal control signal \( i \) can be calculated as follows:

\[
i = (P_0^T P_0)^{-1} P_0^T y_s^r
\]

(22)

The residual wavefront RMS is then obtained as follows:

\[
\sigma_{\text{RMS}} = \sqrt{\frac{(y_s - y_s^r)^T (y_s - y_s^r)}{K}}
\]

(23)

where \( K \) is the number of the sampling points of deformable mirror surface. Then based on the maximum perturbed magnetic field intensity \( (b_z = 0.5 \text{mT}) \) needed at the fluid surface of MFDM and the theory of magnetic field distribution of spiral coil,[26] through the Taguchi method, the length, the number of turns, the inner radius and outer radius of the lower-layer coils are designed as \( L_l = 8 \text{ mm}, N_l = 496, r_l = 2 \text{ mm} \) and \( r_{ol} = 4 \text{ mm} \), respectively. In order to correct high order aberrations, similarly, the length, the number of turns, the inner radius and outer radius of the upper-layer coils are designed as \( L_u = 1 \text{ mm}, N_u = 38, r_u = 1 \text{ mm} \) and \( r_{ou} = 2 \text{ mm} \), respectively. The physical parameters of upper-layer and lower-layer are given in Table 1. According to the requirement that the design criteria of the residual wavefront RMS value to the normalized Zernike mode \( Z_1 \) to \( Z_{21} \) aberrations is less than 50 nm and the best coupling coefficients of the MFDM locates between 15% and 35%,[1] the upper-layer coils are radially spaced at 2.1 mm from center to center and the lower-layer coils are radially spaced at 4.2 mm, respectively. The upper-layer holds 81 custom made actuators arranged in a 9 \( \times \) 9 square array with a size of 20 \( \times \) 20 mm². For the lower-layer actuators, a 5 \( \times \) 5 square array is created with 25 actuators with a size of 21 \( \times \) 21 mm².

Figure 3 illustrates the schematic layout of the experimental setup of the adaptive optics system for the performance evaluation of MFDM. Components of the setup have been labeled along with the path of an aberration-free laser beam. The 635 nm laser beam is dilated through the first and second optic relays \( R_1, R_2 \) and a square optic aperture until it is deflected down to the

| Table 1. Parameters of miniature electromagnetic coil. |
|-----------------------------------------------|
| **Electromagnetic coil Parameters**          | **Parameters** |
| **Position**                                 | Upper         | Lower         |
| Core-type                                    | Air-cored     | Air-cored     |
| Material                                     | Copper        | Copper        |
| Wire gauge                                   | AWG37         | AWG36         |
| Internal diameter                            | 1 mm          | 2 mm          |
| External diameter                            | 2 mm          | 4 mm          |
| Length                                       | 1 mm          | 8 mm          |
| Resistance                                   | 6.7 \( \Omega \) | 16.7 \( \Omega \) |
horizontal MFDM by the folding mirror. The reflected beam will reflect directly back onto the
folding mirror and the third optic relay $R_3$ which de-magnifies the diameter of the laser beam.
The laser is divided by a beam splitter to fit properly into the Shack–Hartman wavefront sensor
and the CCD camera.

In the adaptive optics system, the relationship between the corrected wavefront $G$ and mirror
response matrix (Influence function) $R$ can be written as follows:

$$G = RU$$

(24)

where $U$ is the current vector of actuators of the deformable mirror. Since the wavefront $G$ can
also be represented as $G = Za$, where $Z$ is the matrix of Zernike mode shapes and $a$ is the
Zernike coefficient vector, the control current of the actuators $U$ can then be calculated as:

$$U = R^\dagger Za$$

(25)

where $R^\dagger$ is the pseudo-inverse matrix of $R$. Therefore, the control input of the lower-layer actuators
and upper-layer actuators can be calculated as:

$$U_1 = R_1^\dagger Za_1; U_2 = R_2^\dagger (Za - R_1 R_1^\dagger Za_1)$$

(26)

where $a_1$ is the coefficient of low order Zernike modes, $R_1^\dagger$ and $R_2^\dagger$ are the pseudo-inverse of
response matrixes of the lower-layer actuators and upper-layer actuators, respectively.

3.2. Surface response of MFDM

In this section, the experimental results of the surface response of the mirror are first presented
to validate the linear characters of the MFDM within the overall large stroke range. The surface
deflection at the top of the central miniature electromagnetic coil with respect to different input
currents is measured using Laser Doppler Vibrometer (LDV) (OFV-552/5000, Polytec) as shown
in Figure 4. The measured point on the surface was accurately positioned using a Polytec

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**Figure 3.** Schematic layout of the experimental setup.
alignment test-bed VIB-A-T31. A 7.4 mT uniform magnetic field inside of the Maxwell coil is generated by a constant current of 500 mA driven in the Maxwell coil.

In Figure 5(a) the points marked as ‘*’ signify the peak surface deflections of the MFDM when the central coil (#13 as shown in Figure 2) in the lower layer is active. The experimental peak surface deflections for the case when the central coil (#41 as shown in Figure 2) in the upper layer is active are marked as ‘*’ as shown in Figure 5(b). It shows that the surface deflections vary linearly with the increasing currents, moreover, it’s also able to generate both negative and positive deflections and the stroke of the surface can easily achieve 100 μm.

In order to explore the response performance of the MFDM surface response in 3D, a wavefront sensor will be used to obtain the 3D surface deflection. Figure 6 shows the layout of the AO system setup. The Shack–Hartmann-type wavefront sensor (WFS150-5C, Thorlabs) is used to record the surface deflection. The CCD camera (DCU223C, Thorlabs) is used to image the geometric profile and measure the intensity profile of the beam. Because the detection range of the
WFS150-5C wavefront sensor is less than 60 μm. The magnetic field generated by the actuators is limited to the range of $-0.2 \text{mT}$ to $0.2 \text{mT}$.

The magnetic coil arrangement has a layout of 81 smaller actuators that sit atop of 25 larger actuators. Figure 7 illustrates the response of twelve different actuators, including six upper-layer coils and six lower-layer coils, respectively. A current of 20 mA was applied for each of the lower-layer coils, while a current of 30 mA was applied for the upper-layer coils. As can be seen from the Figure 7, the lower-layer actuators can produce a stroke of about 40 μm with an applied

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**Figure 6.** Layout of the experimental system setup of the adaptive optics system.

**Figure 7.** The experimental responses of the MFDM (color scale in nanometers). Top: the surface responses under different actuators in the upper layer with a current of 30 mA; Down: the surface responses under different actuators in the lower layer with a current of 20 mA.
current of 20 mA, while the upper-layer actuators produce a stroke of about 4 μm with an applied current of 30 mA. The lower-layer actuators create a much larger response influence function area with a lower density compared to the upper-layer actuators of MFDM.

Figure 8 illustrates a cross-section of the surface deformation of the MFDM by using the Laser Doppler Vibrometer (Polytec OFV 5000/552 and VIB-A-T31). The plots marked ‘x’ in Figure 8(a) represent the surface deflection when the currents of 20 mA and −10 mA are applied to the #13 coil (as shown in Figure 7) and its nearest left neighbor #14 coil (as shown in Figure 7) in the lower-layer, independently. When both currents are applied simultaneously, the surface deflection is represented in the red line, which matches the arithmetic addition result (green dashed line) of the individual surface deflection of each coil very well. In Figure 8(b) the plots marked ‘o’ represent the surface deflection when the currents of 15 mA and −30 mA are applied to the #41 coil (as shown in Figure 7) and its nearest left neighbor #42 coil (as shown in Figure 7) in the upper-layer, independently. The black dashed line illustrates when both currents are applied simultaneously, and the red line is the arithmetic addition result of the individual surface deflection of each coil. In Figure 8(c), the plots marked ‘x’ represent the surface deflection when the currents of 6 mA and −15 mA are applied to the central miniature electromagnetic coils of each layer (#13 coil in lower-layer and #41 coil in upper-layer) independently. The black and red lines represent the measured and the arithmetic addition results, respectively. As can be seen from the graphs in Figure 8, there is a very small discrepancy between the measured and the theoretical addition results, which proves that the magnitude of the surface deformation of the magnetic fluid is linearly dependent on the applied input current and that the deformation can easily
be estimated. It should be noted that this good linear surface response property of MFDM is experimentally verified within the overall stroke range and still holds up to more than 100μm.

Figure 9 shows the Bode plots of the experimental identified model of the MFDM along with the analytical model. It can be seen from Figure 9 that the analytical model agrees with the identified model very well. Figure 10 displays the deformation of the MFDM with a sinusoidal input.
applied to the central upper-layer actuator and lower-layer actuator, respectively, paired with the amount of error between the analytical and experimental results. It is evident from the comparison of the response given by the analytical model to the experimental results that there exists a good agreement between them. It can be seen that the dynamic performance of the mirror subject to the actuators in the lower-layer or upper-layer is similar. Based on the analysis of the derived dynamical model of MFDM and the corresponding experimental evaluation, it can be concluded that the longer stroke will not affect the dynamic performance of the mirror if the vertical uniform magnetic field is large enough.

### 3.3. Aberration correction

Since the MFDM has a linear response, it should be possible to produce wavefronts, expressed as a combination of Zernikes, by linearly adding the current vector of each Zernike scaled by the appropriate amount of each coefficient.

Before the aberration correction, the flatness of the initial surface will be corrected. Because the nonuniformity of the magnetic field of the Maxwell coil does produce a curvature in the initial surface of the fluid, moreover, any misalignment in the vertical axis of the Maxwell coil and the gravitational vector, as well as the imperfections in the optical components and non-common-path aberrations, could result in a non-flat initial wavefront surface as measured by the wavefront sensor. Generally, these initial imperfections in the system can be eliminated by presetting the actuators, which ensures a planar initial wavefront surface. The surface plot of the initial surface map of the MFDM is given in Figure 11(a) and the surface plot of the resulting wavefront shape obtained after applying the bias currents is given in Figure 11(b), which show a drop of the RMS value from 0.078 μm to 0.016 μm.

Since the typical aberrations in the liquid telescopes\cite{13} or the retinal imaging systems\cite{27} are below the 5th order, the correction of a targeted wavefront with a combination of Zernikes $Z_1$ to $Z_{21}$ which represent the 1st order to the 5th order aberrations is evaluated. The corresponding Zernike mode numbers and the resulting wavefront are shown in Figure 12 and Figure 13(a), respectively. The residual wavefront for the case of correction only by the lower layer coils is shown in Figure 13(b) with an RMS value of 1.411 μm and the residual wavefront for the case of correction only by the upper layer coils is shown in Figure 13(c) with an RMS value of 4.409 μm. When the actuators in both layers are actuated, the resulting residual wavefront is shown in Figure 13(d) and the RMS value has been lowered to 0.208 μm correspondingly.

Figure 14 displays the MFDM is able to correct the aberrations effectively in each Zernike mode. For each Zernike mode, the result of the fitting ability of the MFDM has been shown in the Y axis. The green bars in Figure 14 indicate that the upper-layer coils have a high correction.
performance for the high-order aberrations, however, due to they can just produce the small amplitude of the surface deformation, which decreases the performance for low-order aberrations. While the lower-layer coils have opposite characteristics to correct the aberrations. The red bars in Figure 14 show that the lower-layer coils have a perfect correction performance for low-order aberrations, but owing to the density of lower-layer actuators is low, thus the correction capability
drops down for the high-order aberrations. The blue bars in Figure 14 describe that the correction performance of the MFDM can be improved for all Zernike modes when both layers of the coils are activated. As mentioned above, the comparison results illustrate that the correction capability can be improved by the proposed MFDM with a two-layer layout of actuators, particularly for the cases to handle the aberrations with the characteristics of both high-amplitude low-order modes and low-amplitude high-order modes, simultaneously.

Figure 15 shows a three-dimensional parabolic surface produced by the MFDM for the laser focus and recorded by the wavefront sensor. The parabolic surface shape as shown in Figure 15(a) is produced by the MFDM only with the lower-layer actuators, and the resulting average RMS error is 0.266 µm. In Figure 15(b) the parabolic surface shape is produced by the MFDM with the two-layer actuators. Due to the additional correction of the upper-layer actuators, a more accurate parabolic surface shape is obtained and the average RMS error is decreased to 0.029 µm. Figure 15(c) shows the focal spot generated by the resulting parabolic shape of the MFDM.

In this section, the modeling approach and the design concept of the MFDM with a two-layer rectangular layout of actuators are verified with the designed prototype MFDM. For future practical applications, the following related technical issues need to be further considered.

The susceptibility of the magnetic fluid volume to mechanical vibrations is one of the most obvious concerns associated with the use of MFDMs. However, the effects of mechanical vibrations can be minimized by different ways. It has been found that the amplitudes of disturbances are less than 1/10 of a wave when a low-viscosity magnetic fluid layer with a thickness of 2 mm is put on an optical table. Increasing viscosity of the magnetic fluid is also validated to have a minimizing impact on the mirror surface vibrations resulting from the ambient noise. Nonetheless, the most effective remedy is presented by the applied magnetic field itself. The experimental results indicate that the magnetic field applied to control the surface profile of the mirror dampens the vibrations significantly.

The dynamic bandwidth of MFDM can be improved by properly choosing the physical parameters of the magnetic fluid. For example, by increasing the viscosity of the magnetic fluid along with an overdrive technique, experiments have shown a bandwidth of 1 kHz for MFDM with a viscosity of 494 cP in. Because magnetic fluids typically show low reflectance to light and for practical applications should be coated with thin metal liquid-like films that exhibit reflective properties like liquid metals. Different preparation methods have been proposed in the literature and the quality of the films, i.e. the stability, roughness, and reflectivity, were discussed correspondingly. The quality of the reflective films could affect the spectral and power handling ability of the MFDM. These related technical parameters will be further compared and studied in future works.

Figure 15. Parabolic surface shape produced by the MFDM. (a) With the actuators of the lower-layer; (b) With the actuators of both layers; (c) Focal spot generated by the parabolic shape of the MFDM.
4. Conclusion

The rectangular MFDM with the dual-layer layout of actuators is proposed to correct the high-amplitude low-order wavefront aberrations and the low-amplitude high-order wavefront aberrations simultaneously. The model of the surface response of the MFDM with dual-layer actuators is derived in Cartesian coordinates. Based on the derived analytical model, a prototype of a square MFDM is fabricated, which has 81 and 25 actuators in the upper layer and lower layer, respectively. Both the simulation and the experimental results show that the stroke of the surface can easily achieve more than 100 μm and the residual value for the typical aberrations can be corrected down to 5% of the original RMS value. Experimental results show the effectiveness of the modeling approach and the aberration correction performance of MFDM for adaptive optics systems. In the future work, the MFDM will be further evaluated in the real applications, such as the retinal imaging system as well as the astronomical imaging system with a conceptual rotating liquid mirror in our lab.

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