Spin observables in the two-nucleon capture and dissociation processes at low energies

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Abstract Spin observables in radiative neutron capture on a proton and its inverse process, photodisintegration of the deuteron are calculated using a pionless effective field theory with di-baryon fields. Good agreement with the results of existing standard nuclear physics approach is obtained at very low energies. As energy increases, however, the discrepancy between the effective field theory and the standard nuclear physics approach becomes substantial. We discuss the origin of the difference.

Keywords Spin observables · Effective field theory · \( np \rightarrow d\gamma \) · \( d\gamma \rightarrow np \)

1 Introduction

The importance of spin observables in nuclear physics is in the fact that they can give more detailed information on the dynamics of the system. Because spin observables are sensitive to the transition amplitudes, they can be good testing grounds to check the accuracy of a theory. Traditional standard nuclear physics approach (SNPA) uses phenomenological potential models and current operators which satisfy the current conservation. SNPA could explain many nuclear phenomena including total cross sections of radiative neutron capture on a proton. Recently effective field theory (EFT) approach has been widely used. EFT approach is model-independent, provides theoretical error estimation, and its accuracy can be improved in a systematic way. EFT calculations showed good agreement with experiments and SNPA calculations at low energies with relatively less number of parameters for many observables. However, most of the calculations are focused on unpolarized observables which are dominated by a few partial wave amplitudes. It would be interesting to check whether the same amplitudes can explain spin observables which are sensitive to the interference between the amplitudes.

It was observed that there exist discrepancies between SNPA calculation and experiments in the induced neutron polarization, \( P_y' \), in photo-disintegration of the deuteron \([1]\). Recent pionless dibaryon effective field theory (dEFT) calculation up to next-to-leading order showed good agreement with SNPA at low energies, but showed conspicuous difference with both theoretical SNPA calculation and measurements at energies larger than 8 MeV \([2]\). Though it is not yet clear whether the differences in experiments and theory are genuine or not, it is important to check whether the same amplitudes used for \( P_y' \) can explain other spin observables and whether the difference between SNPA and dEFT at high energies can be explained by increasing the accuracy of dEFT.

In this work, we calculate the left-right asymmetry in the polarized neutron capture on a proton, and the linear polarization of the photon in the disintegration of the deuteron at low energies using...
the dEFT. We find good agreement with the results obtained from SNPA at low energies, but the discrepancy becomes significant as the energy increases.

2 Formalism

We relegate details of the calculation to Ref. [2] and only summarize the results. Relevant transition amplitude in the c.m. frame for $\gamma d \to np$ is

$$A = \chi_1^+ \sigma_2 \tau_2 \chi_2^+ \cdot \left\{ [\epsilon_{(d)} \times (\hat{k} \times \epsilon_{(\gamma)})] X_{MS} + \epsilon_{(d)} \epsilon_{(\gamma)} \cdot \hat{p} Y_{EV} \right\}$$

$$+ \epsilon_{(d)} \epsilon_{(\gamma)} \cdot \hat{p} X_{EV} + [\epsilon_{(d)} \times (\hat{k} \times \epsilon_{(\gamma)})] Y_{MV}$$

$$+ \chi_1^+ \sigma_2 \tau_2 \chi_2^+ \cdot \left\{ [\epsilon_{(d)} \epsilon_{(\gamma)}] \cdot \hat{p} X_{EV} + [\epsilon_{(d)} \times (\hat{k} \times \epsilon_{(\gamma)})] Y_{MV} \right\}$$

$$= \chi_1^+ \sigma_2 \tau_2 \chi_2^+ \cdot \left\{ [\epsilon_{(d)} \times (\hat{k} \times \epsilon_{(\gamma)})] X_{MS} + \epsilon_{(d)} \epsilon_{(\gamma)} \cdot \hat{p} Y_{EV} \right\}$$

$$+ \epsilon_{(d)} \epsilon_{(\gamma)} \cdot \hat{p} X_{EV} + [\epsilon_{(d)} \times (\hat{k} \times \epsilon_{(\gamma)})] Y_{MV}$$

where $\epsilon_{(d)}$ and $\epsilon_{(\gamma)}$ are spin polarization vectors for the incoming deuteron and photon, respectively, while $\chi_1^+$ and $\chi_2^+$ are the spinors of the outgoing nucleons. The coefficients of the terms in Eq. (1) are given as

$$X_{MV} = \frac{\pi \gamma}{1 - \gamma \rho d \gamma + ip - \frac{1}{2} \rho d (\gamma^2 + p^2) 2m_N} \left\{ \mu V \left( \sqrt{\frac{m_N}{(m_N + \frac{1}{2} \omega)^2 - p^2}} \right) + i \ln \left( \frac{m_N + \frac{1}{2} \omega + p}{\sqrt{(m_N + \frac{1}{2} \omega)^2 - p^2}} \right) \right\}$$

$$X_{MS} = \frac{\pi \gamma}{1 - \gamma \rho d \gamma + ip - \frac{1}{2} \rho d (\gamma^2 + p^2) 2m_N} \left\{ \mu S \left( \sqrt{\frac{m_N}{(m_N + \frac{1}{2} \omega)^2 - p^2}} \right) + i \ln \left( \frac{m_N + \frac{1}{2} \omega + p}{\sqrt{(m_N + \frac{1}{2} \omega)^2 - p^2}} \right) \right\}$$

$$X_{EV} = \sqrt{\frac{\pi \gamma}{1 - \gamma \rho d m_N^2 \omega}} \frac{p}{F^+}, \quad Y_{EV} = \sqrt{\frac{\pi \gamma}{1 - \gamma \rho d m_N^2 \omega}} \frac{p}{F^-},$$

$$Y_{MV} = \sqrt{\frac{\pi \gamma}{1 - \gamma \rho d m_N^2 \omega}} \frac{\mu V}{F^-}, \quad Y_{MS} = \sqrt{\frac{\pi \gamma}{1 - \gamma \rho d m_N^2 \omega}} \frac{\mu S}{F^-},$$

with

$$2F^\pm = \frac{1}{1 + \frac{\omega}{2m_N} - \frac{k}{m_N}} \pm \frac{1}{1 + \frac{\omega}{2m_N} + \frac{k}{m_N}}$$

where $p = |p|$, and $\omega$ is the incoming photon energy in the c.m. frame. Low energy constant $L_1$ is fitted to reproduce the total cross section of a thermal neutron capture on a proton, and $L_2$ is determined with the magnetic moment of the deuteron [3]. The process $\gamma d \to np$ and its inverse process $np \to d\gamma$ share the same amplitude structure with only changing kinematics. Keeping in mind this kinematic difference, we can apply the transition amplitude of $\gamma d \to np$ to the calculation of the amplitude of $np \to d\gamma$. It is noteworthy that kinematics makes the $M1$ transition as a dominant contribution to $np \to d\gamma$ while $E1$ transition to $\gamma d \to np$. 
Square of the amplitude with no polarization then reads
\[ S^{-1} \sum_{\text{spin}} |A|^2 = 16 \left( |X_{MS}|^2 + |Y_{MV}|^2 \right) + 8 \left( |X_{MV}|^2 + |Y_{MS}|^2 \right) + 12 \left[ 1 - (\hat{p} \cdot \hat{k})^2 \right] \left( |X_{EV}|^2 + |Y_{ES}|^2 \right), \] (7)
where the symmetry factor \( S \) is equal to 2 in the present case.

3 Results

3.1 Asymmetry of photon direction in \( np \rightarrow d\gamma \)

Fig. 1 shows the coordinate system we use in the calculation. We are interested in the asymmetry of out-going photons with respect to the axis \( x \), which is proportional to \( \hat{k} \cdot (\hat{p}_n \times \hat{s}_n) = \sin \theta \sin \phi \). Since the asymmetry is to the left and the right of the neutron spin, and conserves the sign under the parity conversion, we call it parity-conserving (PC) left-right asymmetry, \( A_{LR}^\gamma \). There is another PC asymmetry, which is parallel and anti-parallel to the neutron spin. Consideration of this asymmetry will be reported elsewhere \([5]\). Retaining the term proportional to \( \sin \phi \) in the differential cross section, we have
\[ \frac{d\sigma}{d\Omega} = I_0(\theta) [1 + P_n B(\theta) \sin \phi], \] (8)
where \( P_n \) is a transverse polarization of the neutron. At low energies, \( P \)-wave contribution will dominate, and thus we can approximate as \( B(\theta) \approx A_{LR}^\gamma \sin \theta \). Then we can obtain \( A_{LR}^\gamma \) with an approximate relation
\[ A_{LR}^\gamma \approx \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-} \bigg|_{\phi = \frac{\pi}{2}}, \] (9)
where \( \sigma_{\pm} \) denote the differential cross sections with \( \phi = \pm \frac{\pi}{2} \), respectively. With low-energy neutrons, we obtain the numerical results
\[ A_{LR}^\gamma (3 \text{ meV}) = 6.10 \times 10^{-9}, \quad A_{LR}^\gamma (10 \text{ meV}) = 2.03 \times 10^{-8}. \] (10)
Results at 3 meV obtained in SNPA are \( A_{LR}^\gamma = 0.607 \times 10^{-8}, \quad 0.668 \times 10^{-8}, \quad 0.665 \times 10^{-8} \) with RSC, Av14 and Nijmegen93 potentials, respectively \([4]\). Our result agrees to that with RSC, but there is about 10 % suppression to those with Av14 and Nijmegen93.

3.2 Linear polarization asymmetry in \( \gamma d \rightarrow np \)

Another polarization observable \( \Sigma^l(\theta) \), the linear polarization asymmetry in \( \gamma d \rightarrow np \) is defined as
\[ \frac{d\sigma}{d\Omega} = \frac{d\sigma_0}{d\Omega} \left( 1 + P_\gamma \Sigma^l(\theta) \cos(2\phi) \right), \]
\[ \Sigma^l(\theta) = \frac{\sigma_{||}(\theta) - \sigma_{\perp}(\theta)}{\sigma_{||}(\theta) + \sigma_{\perp}(\theta)}, \] (11)
where the linear polarization is parallel or perpendicular to the reaction plane. Choosing \( \hat{k}_\gamma = \hat{z} \), \( \hat{p}_n = (\sin \theta, 0, \cos \theta) \), \( \epsilon_n(\gamma) = \hat{x} \) for parallel photons, and \( \epsilon_n(\gamma) = \hat{y} \) for perpendicular photons, we obtain

\[
\Sigma^l(\theta) = \frac{3 \sin^2 \theta(|Y_{ES}|^2 + |X_{EV}|^2)}{4|X_{MS}|^2 + 2|X_{MV}|^2 + 4|Y_{MV}|^2 + 2|Y_{MS}|^2 + 3 \sin^2 \theta(|Y_{ES}|^2 + |X_{EV}|^2)}.
\] (12)

\( \Sigma^l(\theta) \)'s with the incident photon energies 10, 30, and 60 MeV are shown in Fig. 2. Comparing the results with those reported in Ref. [6], the results at \( \omega = 10 \) MeV agree well, but there are significant discrepancies at higher energies.

4 Summary

We studied spin-dependent observables for \( np \leftrightarrow d\gamma \) processes. At very low energies pionless EFT agrees with other theoretical calculations. Comparison with experiment will confirm the accuracy of the theory because spin-dependent observables are more sensitive to the amplitudes than the total cross sections. However, our results disagree with other theoretical calculations at energies \( E > 10 \) MeV. This may be explained by the lack of higher partial waves or higher order operators in our calculation. Thus, more involved calculation is necessary.

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