Solving the Schwinger-Dyson Equations for Gluodynamics in the Maximal Abelian Gauge*

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We derive the Schwinger-Dyson equations for the SU(2) Yang-Mills theory in the maximal Abelian gauge and solve them in the infrared asymptotic region. We find that the infrared asymptotic solutions for the gluon and ghost propagators are consistent with the hypothesis of Abelian dominance.

1. Introduction

The Schwinger-Dyson (SD) equation is one of the most popular approaches to investigate the non-perturbative features of quantum field theory. The analyses by making use of the SD equation for quark propagator are well-known. Recently, the coupled SD equations for the gluon and ghost propagators in Yang-Mills theory have been studied mainly in the Lorentz (Landau) gauge.[1,2] In this paper, we derive the SD equations for the SU(2) Yang-Mills theory in the maximal Abelian (MA) gauge and solve them analytically in the infrared (IR) asymptotic region. The MA gauge is useful to investigate the Yang-Mills theory from the viewpoint of the dual superconductivity.

In the MA gauge, in contrast to the ordinary Lorentz gauge, we must explicitly distinguish the diagonal components of the fields from the off-diagonal components. This is indeed the case even in the perturbative analysis in the UV region. Therefore, we must take account of the four propagators for the diagonal gluon, off-diagonal gluon, diagonal ghost and off-diagonal ghost. Numerical behaviors of gluon propagators in the MA gauge are also investigated on a lattice simulation.[4]

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2. SD equations in the MA gauge

First, we derive the SD equations from the SU(2) Yang-Mills action in the MA gauge[3]. The graphical representation of SD equations are shown in Figure 1. For the diagonal gluon propagator, we adopt the Landau gauge so that the diagonal gluon propagator $D_{\mu\nu}(p^2) = F_d(p^2) p_{\mu\nu}$ where we defined the form factor $F_d(p^2)$. While, the off-diagonal gluon propagator $D_{ab\mu\nu}(p^2)$ has both the transverse and longitudinal parts

$$D_{ab\mu\nu}(p^2) := \left[ F_T(p^2) P_{\mu\nu} + \alpha F_L(p^2) P_{\mu\nu} \right] \delta_{ab},$$

where we defined the form factors $F_T(p^2)$ and $F_L(p^2)$. The form factor $G(p^2)$ for the off-diagonal ghost propagator $\Delta_{ab}(p^2)$ is defined

$$\Delta_{ab}(p^2) := \frac{G(p^2)}{p^2} \delta_{ab}.$$
3. Truncation and Approximations

We employ the following approximations.

- Instead of the full vertex functions, we adopt modified vertex functions which are compatible with the ST identities. We adopt approximations for vertex functions as
  \[ F_{\mu}^{ab}(p, q) \sim F_{\mu}^{(0)ab}(p, q)\partial_{\rho}\{p^2G^{-1}(p^2)\}, \quad (9) \]
  and
  \[ F_{\mu\rho\sigma}^{ab}(p, q) \sim F_{\mu\rho\sigma}^{(0)ab}(p, q)\partial_{\rho}\{p^2F_{T}^{-1}(p^2)\}. \quad (10) \]
  Here, we adopt the Feynman gauge for the off-diagonal gluon for simplicity, that is, \( \alpha = 1 \) and \( F_T(p^2) = F_L(p^2) \). Substituting the bare form factors, which are \( G(p^2) = F_T(p^2) = 1 \), into the right hand side of the ansatz \( 9 \) and \( 10 \), we obtain the bare vertex functions. Moreover, these ansatz are compatible with the ST identities \( 7 \) and \( 8 \) in the limit of \( p \to q \).

- In the momentum integration, we use the Higashijima-Miransky approximation\( 11 \) as
  \[ F((p - q)^2) = F(\max\{p^2, q^2\}) . \quad (11) \]

4. Solving the SD equations

Now we adopt the ansatz for the form factors in the IR region:

\[ F_{\delta}(p^2) = A(p^2)^u + \cdots, \]
\[ G(p^2) = B(p^2)^v + \cdots, \]
\[ F_T(p^2) = C(p^2)^w + \cdots. \quad (12) \]

Substituting the ansatz \( 12 \) for the form factors, and the ansatz \( 9 \) and \( 10 \) for vertex func-
Table 1
The relation $u$, $v$ and $w$ (and $v'$, $w'$) in the Feynman gauge ($\alpha = 1$).

|        | $w > 1$                | $w = 1$                |
|--------|------------------------|------------------------|
| $v > 1$| $u = -\min\{v, w\}$   | $u = -\min\{v, w\}$   |
| $v = 1$| $u = -\min\{v', w\}$  | $u = -\min\{v', w\}$  |

Table 2
The relation $u$, $v$ and $w$ (and $v'$, $w'$) in the gauge $\alpha \neq 1$.

|        | $w > 1$                | $w = 1$                |
|--------|------------------------|------------------------|
| $v > 1$| $u = -\min\{v, w\}$   | $u = -1$               |
| $v = 1$| $u = -\min\{v', w\}$  | $u = -1$               |

5. Conclusion

In the IR limit, the form factors of each propagator behave as
\[
F_{d}(p^2) \sim A(p^2)^u + \cdots \quad (u \leq -1),
\]
\[
G(p^2) \sim B(p^2)^v + \cdots \quad (v \geq 1),
\]
\[
F_{t}(p^2) \sim C(p^2)^w + \cdots \quad (w \geq 1).
\]

Therefore the solution shows that the diagonal gluon propagator is enhanced in the IR limit, while the off-diagonal gluon and off-diagonal ghost propagators are suppressed in the IR region. Our results are compatible with a hypothesis of Abelian dominance.\[7

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