ε expansion analysis of very weak first-order transitions in the cubic anisotropy model, Part II

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(December 7, 2017)

Abstract

A companion article analyzed very weakly first-order phase transitions in the cubic anisotropy model using ε expansion techniques. We extend that analysis to a calculation of the relative discontinuity of specific heat across the transition.
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I. INTRODUCTION

In this work, we use $\epsilon$ expansion methods to compute the universal ratio $C_+/C_-$ of specific heats for arbitrarily weak first-order phase transitions in the cubic anisotropy model. $C_+$ and $C_-$ are the specific heats of the disordered and ordered phases at the transition temperature. This work is a direct follow up to the computations of other universal ratios in ref. [1], and we shall eschew any discussion of motivation or review of method and notation; instead, we jump directly into the calculation. The reader should read ref. [1] first.

In the next section, we review the leading-order calculation of the ratio, which was first performed by Rudnick in ref. [2]. Our result differs by a factor of 4. In section III, we proceed to next-to-leading order in $\epsilon$. Our final results are displayed in section IV.

II. LEADING-ORDER ANALYSIS OF $C_+/C_-$

Recall from the introduction of ref. [1] that, in the three-dimensional theory, the square $m^2$ of the scalar mass plays the roll of the reduced temperature near the transition and the effective potential $V$ represents the free energy of the system. The specific heat can be extracted from the effective potential as

$$C \propto \frac{d^2V}{d(m^2)^2}. \tag{1}$$

The proportionality constant will not matter to our determination of the ratio $C_+/C_-$.\footnote{In particular, the reduced temperature is proportional to $m^2(\mu_0)$ for some fixed renormalization scale $\mu_0$, but we will instead apply (1) with $m^2(\mu)$ where $u\mu$ is roughly the order of the correlation length and varies as we approach the transition. [Specifically, we choose $\mu$ so that $u(\mu) = -v(\mu)$.] However, $m^2(\mu)$ is related to $m^2(\mu_0)$ by multiplicative renormalization which, even though $\mu$ dependent, cancels in the ratio.}

Because of this relationship, it is useful to begin by summarizing the leading-order form of the effective potential discussed in ref. [1].
A. Summary of leading-order potential

At one-loop order, the effective potential along an edge \( \vec{\phi} = (\phi, 0, 0, \ldots) \), evaluated at the tree-level instability line \( u = -v \), is [eq. (3.9) of ref. [1]]:

\[
\begin{align*}
N \mu^\epsilon (V_0 + V_1) = & \Lambda + 3u^{-1}m^2 M^2 + \frac{1}{4}m^4 \left[ \ln \left( \frac{m^2}{\mu^2} \right) - \frac{C_{10}}{C_{11}} \right] \\
& + C_{11}(m^2 + M^2)^2 \left[ \ln \left( \frac{m^2 + M^2}{\mu^2} \right) - \frac{C_{10}}{C_{11}} \right] + O(\epsilon), \\
\end{align*}
\]

(2)

where \( M^2 \equiv \frac{1}{6} N u \phi^2 \), \( C_{11} = \frac{1}{4}(n-1) \), \( C_{10} = -\frac{3}{2}C_{11} \), and the normalization \( N \) is

\[
N = (4\pi)^{d/2} \Gamma \left( \frac{d}{2} - 1 \right)
\]

(3)
in \( d = 4 - \epsilon \) spatial dimensions.

A first-order transition occurs as \( m^2 \) is varied. At the transition, in the asymmetric phase, \( m^2/M^2 \sim O(\epsilon) \). In the asymmetric phase, the potential then reduces to

\[
\begin{align*}
N \mu^\epsilon (V_0 + V_1) = & \Lambda + 3u^{-1}m^2 M^2 + M^4 \left[ C_{11} \ln \left( \frac{M^2}{\mu^2} \right) + C_{10} \right] + O(\epsilon^2 V)_{\text{asym}}. \\
\end{align*}
\]

(4)

The transition occurs when

\[
m^2 = m_1^2 [1 + O(\epsilon)].
\]

(5)

and the asymmetric minimum is at

\[
M^2 = M_1^2 [1 + O(\epsilon)].
\]

(6)

where

\[
\begin{align*}
m_1^2 &\equiv \frac{C_{11} u \mu^2}{3} \exp \left( -1 - \frac{C_{10}}{C_{11}} \right) = \frac{n-1}{12} u \mu^2 \epsilon^{1/2}, \\
M_1^2 &\equiv \mu^2 \exp \left( -1 - \frac{C_{10}}{C_{11}} \right) = \mu^2 \epsilon^{1/2}.
\end{align*}
\]

(7)

(8)

We will also need the variation of the asymmetric minimum as one varies \( m^2 \) slightly away from the transition:

\[
\frac{d(M^2)}{d(m^2)} \bigg|_{m^2=m_1^2(1+O(\epsilon))} = -\frac{3}{C_{11} u} + O(\epsilon^0),
\]

(9)
Finally, we will need the renormalization group flow of the various couplings, previously presented in ref. [1]. At leading order,

$$\mu \partial_\mu m^2 = (\beta^{(1)}_{m^2}(u, v) + O(u^2, v^2))m^2,$$

(10)

$$\mu \partial_\mu u = -\epsilon u + \beta^{(1)}_u(u, v) + O(u^3, v^3),$$

(11a)

$$\mu \partial_\mu v = -\epsilon v + \beta^{(1)}_v(u, v) + O(u^3, v^3),$$

(11b)

where

$$\beta^{(1)}_{m^2} = n + \frac{2}{3} u + v,$$

(12)

$$\beta^{(1)}_u = u \left( \frac{n + 8}{3} u + 2v \right),$$

(13)

$$\beta^{(1)}_v = v(4u + 3v).$$

(14)

It is also convenient to introduce $f \equiv u/v$ and

$$\mu \partial_\mu f = \beta^{(1)}_f(u, f) + O(u^2, v^2),$$

(15)

$$\beta^{(1)}_f = u \left( \frac{n - 4}{3} f - 1 \right).$$

(16)

### B. Specific heats

The most non-trivial task in computing the specific heat ratio at the transition will be to handle the renormalization group flow of the constant term $\Lambda$ in the potential. Before investigating this, we first write down the expression for $C_+$ and $C_-$ given the potential. From (3),

$$C_+ \propto \left( \frac{d}{d(m^2)} \right)^2 \mathcal{N}_i V(M^2 = 0)|_{m^2 = m_i^2(1 + O(\epsilon))}$$

$$= \left( \frac{d}{d(m^2)} \right)^2 \Lambda + \frac{n}{2} \left[ \ln \left( \frac{m_i^2}{\mu^2} \right) \right] + O(\epsilon)$$

$$= \left( \frac{d}{d(m^2)} \right)^2 \Lambda + 2 \left( \frac{1}{4} + C_{11} \right) \left[ \ln (2C_{11} u) + \frac{1}{2} \right] + O(\epsilon),$$

(17)
where the second derivative of $\Lambda$ is understood to be evaluated at the transition $m^2 = m_1^2(1 + O(\epsilon))$. Using (4) for the asymmetric phase, and remembering that $\partial_M V(M) = 0$ at the asymmetric minimum,

$$C_- \propto \left( \frac{d}{d(m^2)} \right)^2 N \mu' V(M) \big|_{m^2 = m_1^2(1 + O(\epsilon))}$$

$$= \left( \frac{d}{d(m^2)} \right)^2 \Lambda + \frac{3 d(M^2)}{u \, d(m^2)} \big|_{m^2 = m_1^2} + O(1/\epsilon)$$

$$= \left( \frac{d}{d(m^2)} \right)^2 \Lambda - \frac{9}{u^2} \frac{1}{C_{11}} + O(1/\epsilon), \quad (18)$$

where $M$ has been evaluated at the transition using (3) and (3). $C_+$ will be shown to be $O(1/\epsilon)$ while $C_-$ is $O(1/\epsilon^2)$.

\section*{C. The running of $\Lambda$}

Now we turn to the contribution to the specific heat from the term $\Lambda$. The renormalization group equation for $\Lambda$ at one-loop order is

$$\mu \partial_\mu \Lambda = \epsilon \Lambda + \frac{n}{2} m^4 (1 + O(u, v)) \ . \quad (19)$$

The solution, in terms of $m^2(\mu)$, is

$$\Lambda(\mu) = \left( \frac{\mu}{\mu_0} \right)^\epsilon \Lambda(\mu_0) + \frac{n}{2} \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left( \frac{\mu}{\mu'} \right)^\epsilon [m^2(\mu')]^2 (1 + O(u, v)) \ , \quad (20)$$

where $\mu_0$ is some initial scale. The running (14) of $m^2$ yields

$$m^2(\mu) = E^{(1)}(\mu, \mu_0) m^2(\mu_0) (1 + O(u, v)) \ , \quad (21)$$

where

$$E^{(1)}(\mu, \mu_0) = \exp \left( \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \beta^{(1)}(\mu') \right) \quad (22)$$

and the integral is to be understood as evaluated along the leading-order solution for the coupling constant trajectory. The contribution of $\Lambda$ to the specific heat is then
\[
\left(\frac{d}{d(m^2)}\right)^2 \Lambda = n \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left(\frac{\mu}{\mu'}\right)^\epsilon \left[E^{(1)}(\mu', \mu)\right]^2 \left(1 + O(u, v)\right) .
\]  

(23)

\( E^{(1)} \) can be easily evaluated by changing variables from \( \mu' \) to \( f \) using (15) and noting that \( \beta_m^{(1)}/\beta_f^{(1)} \) depends only on \( f \):

\[
E^{(1)}(\mu, \mu_0) = \exp \left\{ \int_{f_0}^{f} \frac{d f'}{f} \left( \frac{f_0 + \lambda}{f + \lambda} \right)^{(n\lambda-1)/2} \right\}
\]

where

\[
\lambda \equiv \frac{3}{4-n} .
\]

(25)

The remaining integral in (23) can be performed similarly if we make use of the following relation ((A10a) of ref. [1]) for the solution to the leading-order RG flow equations (11):

\[
\left(\frac{\mu}{\mu_0}\right)^\epsilon = \frac{u_0}{u} \left(\frac{f_0}{f}\right)^2 \left(\frac{f_0 + \lambda}{f + \lambda}\right)^{n\lambda} .
\]

(26)

This gives

\[
\left(\frac{d}{d(m^2)}\right)^2 \Lambda = -\frac{n\lambda}{u(f + \lambda)} \int_{f_0}^{f} df' \left(\frac{f'}{f}\right)^2 \left(\frac{f' + \lambda}{f + \lambda}\right)^{n\lambda-1} \left[E^{(1)}(f', f)\right]^2 \left(1 + O(u, v)\right) \\
= -\frac{n\lambda}{u(f + \lambda)} \int_{f_0}^{f} df' \left(1 + O(u, v)\right) .
\]

(27)

So the result is

\[
\left(\frac{d}{d(m^2)}\right)^2 \Lambda = -\frac{n\lambda}{u(\mu)} \frac{[f(\mu) - f(\mu_0)]}{(f(\mu) + \lambda)} \left[1 + O(\epsilon)\right] .
\]

(28)

The only other elements we need are the values of \( f(\mu_0) \), \( f(\mu) \), and \( u(\mu) \) for the desired trajectory. As discussed in ref. [1], we can obtain the universal ratios of interest by studying the trajectory that flows away from the cubic fixed point at \( f(\mu_0) = -\lambda \) to the line of classical instability \( u= -v \) at \( f(\mu) = -1 \) and \( u = u_\ast \), where, at leading order,

\[
u_\ast = \frac{3(n^2 + 5n + 3)}{n(n + 2)(n + 8)} \epsilon .
\]

(29)

For this trajectory,

\[
\left(\frac{d}{d(m^2)}\right)^2 \Lambda = -\frac{n\lambda}{u_\ast} + O(\epsilon^0) .
\]

(30)
This is $O(1/\epsilon)$ and dominates $C_+ \, [17]$, but it does not contribute at leading order to $C_- \, [18]$. Putting it all together,

$$\frac{C_+}{C_-} = \frac{n\lambda C_{11}}{9} u_\ast + O(\epsilon^2)$$

$$= \frac{(n-1)(n^2+5n+3)}{4(n+2)(n+8)(4-n)} \epsilon + O(\epsilon^2)$$

$$= \frac{17}{320} \epsilon + O(\epsilon^2) \quad \text{for } n = 2. \quad (31)$$

This result is 4 times smaller than the result originally quoted by Rudnick [2].

**III. NEXT-TO-LEADING-ORDER ANALYSIS OF $C_+/C_-$**

Because the contribution of $\Lambda$ dominated $C_+$, our formula (17) for $C_+$ is adequate at next-to-leading order provided we extend our analysis of $\Lambda$ to next-to-leading order. Before doing so, let us first consider $C_-.$

**A. The asymmetric phase: $C_-$**

The two-loop effective potential near the asymmetric phase is given in ref. [1] (see eqs. (4.1), (4.2), and (4.4)):

$$N \mu^\epsilon (V_0 + V_1 + V_2) = \Lambda + 3u^{-1} m^2 M^2 + M^4 \left[ C_{11} \ln \left( \frac{M^2}{\mu^2} \right) + C_{10} \right]$$

$$+ N \mu^\epsilon \delta V_1 + N \mu^\epsilon V_2 + O(\epsilon^2 V)_{\text{asym}}, \quad (32)$$

where

$$N \mu^\epsilon \delta V_1 = 2C_{11} m^2 M^2 \left[ \ln \left( \frac{M^2}{\mu^2} \right) - 1 \right]$$

$$+ \epsilon M^4 \left[ -\frac{1}{4} C_{11} \ln^2 \left( \frac{M^2}{\mu^2} \right) - \frac{1}{2} C_{10} \ln \left( \frac{M^2}{\mu^2} \right) + C_{e10} \right] \quad (33)$$

$$N \mu^\epsilon V_2 = u M^4 \left[ C_{22} \ln^2 \left( \frac{M^2}{\mu^2} \right) + C_{21} \ln \left( \frac{M^2}{\mu^2} \right) + C_{20} \right], \quad (34)$$

and
\[ C_{22} = \frac{n+2}{6} C_{11}, \quad C_{21} = -\frac{(n+6)}{3} C_{11}. \]  

(35)

As discussed in ref. [1], we shall not need to know \( C_{10}^{\varepsilon} \) and \( C_{20}^{\varepsilon} \). The transition takes place at

\[ m^2 = m_2^2 \left[ 1 + O(\varepsilon^2) \right], \]  

(36)

with the asymmetric minimum at

\[ M^2 = M_2^2 \left[ 1 + O(\varepsilon^2) \right], \]  

(37)

where

\[ m_2^2 = m_1^2 \left[ 1 + \left( -\frac{5}{16} - \frac{C_{10}}{C_{11}} \right) \varepsilon + \left( \frac{(n-4)}{24} - \frac{1}{2} C_{21} \frac{C_{20}}{C_{11}} \right) u \right], \]  

(38)

\[ M_2^2 = M_1^2 \left[ 1 + \left( -\frac{13}{16} - \frac{C_{10}}{C_{11}} \right) \varepsilon + \left( -\frac{(3n+2)}{8} - \frac{3}{2} \frac{C_{21}}{C_{11}} \frac{C_{20}}{C_{11}} \right) u \right]. \]  

(39)

At the transition,

\[ \left. \frac{d(M^2)}{d(m^2)} \right|_{m^2 = m_2^2 (1 + O(\varepsilon^2))} = -\frac{3}{C_{11} u} \left[ 1 + (C_{11} - 3 C_{22} \frac{C_{20}}{C_{11}}) + O(\varepsilon^2) \right]. \]  

(40)

The result for \( C_- \) is

\[ C_- \propto \left( \frac{d}{d(m^2)} \right)^2 N \mu^4 V(M) \mid_{m^2 = m_2^2 (1 + O(\varepsilon^2))} = \left( \frac{d}{d(m^2)} \right)^2 \Lambda + \left. \frac{d(M^2)}{d(m^2)} \right|_{m^2 = m_2^2} \left[ \frac{3}{u} + 2 C_{11} \ln \left( \frac{M^2}{\mu^2} \right) \right] + O(\varepsilon^0) = \left( \frac{d}{d(m^2)} \right)^2 \Lambda - \frac{9}{u^2} C_{11} \left[ 1 + \left( \frac{4}{3} C_{11} - 3 C_{22} \frac{C_{20}}{C_{11}} \right) u \right] + O(\varepsilon^0). \]  

(41)

Since the contribution of \( \Lambda \) is sub-leading, the leading-order result (30) for the contribution is adequate here.

**B. The NLO running of \( \Lambda \)**

Now we are left with calculating \( d^2 \Lambda / d(m^2)^2 \) to next-to-leading order. The two-loop renormalization group equation is
\[ \mu \partial_\mu \Lambda = \epsilon \Lambda + \frac{n}{2} (m^2)^2 (1 + O(u^2, v^2)) , \]  
(42)

which has the same form as the one-loop equation. It's solution then also has the same form,

\[
\left( \frac{d}{d(m^2)} \right)^2 \Lambda = n \int_{\mu_0}^{\mu} \frac{d\mu'}{\mu'} \left( \frac{\mu}{\mu'} \right)^\epsilon [E(\mu', \mu)]^2 (1 + O(u^2, v^2)) ,
\]  
(43)

where

\[ E(\mu, \mu_0) = \exp \left( \int_{\mu_0}^{\mu} d\mu' \beta (m^2) \right) , \]
(44)

but now both the \( \beta \)-functions and renormalization group trajectories should be evaluated at two loops. The two-loop \( \beta \)-functions are given in Sec. VI A of ref. [1]:

\[ \beta = \beta^{(1)} + \beta^{(2)} + O(u^4, v^4) , \]
(45)

where

\[ \beta^{(2)}_m = -\frac{5}{6} \left[ \frac{(n+2)}{3} u^2 + 2uv + v^2 \right] , \]
(46)

\[ \beta^{(2)}_u = -\frac{(3n+14)}{3} u^2 - \frac{22}{3} u^2 v + \frac{5}{3} uv^2 , \]
(47)

\[ \beta^{(2)}_v = -\frac{(5n+82)}{9} u^2 v - \frac{46}{3} uv^2 - \frac{17}{3} v^3 . \]
(48)

To evaluate the integrals, we again change variables from \( \mu \) to \( f \), and we shall treat the two-loop effects on \( \beta \)-functions and trajectories perturbatively. Following sec. V ID of ref. [1], it is helpful to make the \( \epsilon \) dependence explicit by rewriting \((u, v) = \epsilon (\bar{u}, \bar{v})\), and the expansion of the trajectory gives

\[ \bar{u}(f) = \bar{u}^{[1]}(f) + \epsilon \delta(f) + O(\epsilon^2) , \]
(49)

where \( \bar{u}^{[1]}(f) = f R(f, c) \) is the one-loop result described in sec. IV C of ref. [1]. The solution for \( \delta(f) \) is given by eq. (6.23) of ref. [1].

To change variables from \( \mu \) to \( f \), we use

\[ \frac{d\mu}{\mu} = \frac{df}{\beta_f} = \left[ \beta_f^{(1)} + \epsilon \beta_f^{(2)} + O(\epsilon^2) \right]^{-1} \bigg|_{\bar{u}^{[1]}(f), f} = \frac{df}{\beta_f^{(1)}} \left[ 1 - \frac{\epsilon}{\beta_f^{(1)}} \left( \delta(f) \partial_f \beta_f^{(1)} + \beta_f^{(2)} \right) + O(\epsilon^2) \right] \bigg|_{\bar{u}^{[1]}(f), f} . \]
(50)
The subscript \(\bar{u}^{[1]}(f), f\) at the end of this equation means that the \(\beta\)-functions in the expression are to be evaluated with \(u \rightarrow \bar{u}^{[1]}(f)\) and \(v \rightarrow \bar{u}^{[1]}(f)/f\). Expanding the definition \((44)\) in \(\epsilon\) then gives

\[
E(f, f_0) = E(1)(f, f_0) [1 + \epsilon \delta E(f, f_0) + O(\epsilon^2)],
\]

where \(E(1)\) is the leading-order form of \((24)\) and

\[
\epsilon \delta E(f, f_0) = \int f \frac{df'}{f_0} \left\{ \frac{1}{\beta_f^{(1)}} \left[ \delta(f') \partial_a \beta_a^{(1)} + \beta_a^{(2)} \right] - \frac{\beta_m^{(1)}}{\left(\beta_f^{(1)}\right)^2} \left[ \delta(f') \partial_a \beta_a + \beta_f^{(2)} \right] \right\} \bigg|_{\bar{u}^{[1]}(f'), f'}.
\]

To do the final integral of \((43)\), we need an expansion of \((\mu/\mu')^\epsilon\). This can be obtained by writing

\[
\left(\frac{\mu}{\mu'}\right)^\epsilon = \exp \left( \epsilon \int_{\mu'}^{\mu} \frac{d\mu''}{\mu''} \right),
\]

and converting to \(f\) with the expansion \((51)\). Putting all the expansions together yields

\[
\left(\frac{d}{d(m^2)}\right)^2 \Lambda = -\frac{n \lambda}{u^{[1]}(f(\mu)) (f(\mu) + \lambda)} \times \int^{f(\mu)}_{f(\mu_0)} df' \left\{ 1 + \epsilon \left[ 2 \delta E(f', f(\mu)) + X_1(f') + X_2(f') \right] + O(\epsilon^2) \right\},
\]

where

\[
X_1(f') = -\frac{1}{\beta_f^{(1)}} \left. \left( \delta(f') \partial_a \beta_a + \beta_f^{(2)} \right) \right|_{\bar{u}^{[1]}(f'), f'},
\]

\[
X_2(f') = -\int^{f(\mu)}_{f'} \frac{df''}{\left(\beta_f^{(1)}\right)^2} \left. \left( \delta(f'') \partial_a \beta_a + \beta_f^{(2)} \right) \right|_{\bar{u}^{[1]}(f''), f''}.
\]

This integration does not seem to have a simple form for general \(n\). For \(n = 2\), we are able to obtain a simple result for the trajectory flowing away from the cubic fixed point to the classical instability line \(u = -v\):

\[
\left(\frac{d}{d(m^2)}\right)^2 \Lambda = -\frac{3}{u_*} \left\{ 1 + \epsilon \left[ \frac{49}{40} - \frac{3}{5} \ln \frac{3}{2} \right] + O(\epsilon^2) \right\},
\]

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Putting everything together, and using the next-to-leading order \(n=2\) result

\[ u_* = \frac{51}{80} \epsilon + \left( \frac{243}{80} \ln \frac{3}{2} - \frac{171}{200} \right) \epsilon^2 + O(\epsilon^3) \] (58)

for \(u_*\) from eq. (6.28) of ref. [1], our final result is then

\[ \frac{C_+}{C_-} = \frac{17}{320} \epsilon \left\{ 1 - \frac{17}{80} \epsilon \ln \epsilon + \epsilon \left[ \frac{17}{80} \ln \frac{320}{17} + \frac{354}{85} \ln \frac{3}{2} - \frac{4967}{5440} \right] + O(\epsilon^2) \right\} \] (59)

for \(n=2\).

**IV. DISCUSSION**

Evaluated numerically, the final result (59) for the ratio is

\[ \frac{C_+}{C_-} = 0.0531 \epsilon \left[ 1 + \epsilon (-0.2125 \ln \epsilon + 1.3993) + O(\epsilon^2) \right] . \] (60)

This ratio is compared against Monte Carlo simulations [3] in ref. [4]. The 140\% correction at next-to-leading order for \(\epsilon=1\) suggests that the \(\epsilon\) expansion will be at best marginally successful for this quantity.

This work was supported by the U.S. Department of Energy, grants DE-FG06-91ER40614 and DE-FG03-96ER40956. We thank Larry Yaffe for useful discussions.
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