Dynamics of glass windows in mining vehicles under the impacts of rock pieces: numerical and analytical comparison for computational models validation

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Abstract. This study represents the results of linear dynamics analysis of glass plates subjected to rock pieces impacts occurring in underground machines’ windows. The aim of the work is to provide analytical and numerical solutions, obtain frequencies and plate displacement, and compare results of stress calculation for different models. The work performs finite element method (FEM) computations within a modal analysis in 3D statement including a mesh-size convergence analysis. Given approach is a basement for estimation of safety work conditions for operators in cabins of underground mining vehicles when glass windows are subjected to rock bursts and damages.

1. Introduction
Estimation of dynamic processes of structural elements under impact loading is an important task in modern branches of mechanical engineering and industry. Exploitation conditions of transport, aviation and industrial machinery are characterized by impulsive loads and impacts [1–4]. Studying the elastic response of a plate is important for understanding the dynamic response of monolithic structures under impact loading [5–7]. Despite the fact that more and more underground mining processes have become fully automated, for some physically demanding manual tasks, it is required the presence of a worker in a hazardous environment.

In this paper, it is taken into consideration the hazard of rock bursts and pieces falling in an underground mine environment that is a common dangerous factor, when the worker has been injured [8]. Despite the usage of high strength glass or additional shields, it is one of the major risks affecting workers, which cannot be completely minimized. Rock impacts occurrence is not foreseeable, because primarily it comes from underground rock bursts whose occurrence probability is very high. Since it happens, there should be a strategy to mitigate and manage that risk due to glasses destroying, reducing possible fatalistic results to the acceptable level [9–10]. Not only geo-mechanical events...
could endanger underground workers. Rock fall and glass damages can also be triggered by mechanical contact of mining vehicles with uneven walls in the confined space of underground tunnels [11] during the transportation of excavated material.

There is a lack of dynamic research of glass windows in mining vehicles under the impacts of rock pieces in the literature. This is due to usage of window grills for driver’s cabin housing. However, additional non-transparent elements are restricting the view field of the driver. Thus, in-depth analysis of glass strength under dynamic impacts is vital for enhancing occupational safety and health of underground activities thanks to some kind of models with specific physical parameters.

In addition to the theoretical description, experimental studies have been conducted over the last century to obtain data on the properties of impact displacement and material fracture, to verify the calculated results. To determine the impact properties of materials, special laboratory installations are used, in which the basic schemes for impact are implemented [12–14]. The vast majority of the ball impact studies have been performed for rectangular plates, which are most common in practice. Seregin's work [15] contains a description of approaches for impact testing. The description of means for carrying out impact tests, measuring equipment, means of amplification signals etc. is contained in the DeSilva directory [16]. In the works of Ugrimov, Smetankina et al. [17–18], experimental studies of impact on thin-walled structural elements – shells and plates of non-canonical geometry, that includes multilayer material. Experimental investigation on the damage and wave propagation characteristics in laminated glass panels subjected to impact loading was conducted in the article by Nassr et al. [19]. The study of Zhang et al. [20] investigates the influence of the interlayer materials on the low velocity impact performance of laminated glass.

According to the work review, it can be concluded that the study of thin-walled structural elements under impact loading was and remains a relevant topic. Real problems in practice have many limitations: complex boundary conditions, problems with convergence, multilayer structure, curvilinear geometry etc. To solve them, numerical approaches are used, however, the assessment of the reliability of computational models and schemes requires validation, at least for some special cases in which experimental data or analytical approximations can be obtained. One of such important elements of the test evaluation of the numerical procedures reliability is the comparison of the numerical impact interaction solutions with the linearized setting of the plate vibrations under impact loading.

2. Problem formulation

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The scope of the work is to study the linear transient dynamics of glass to consider frequencies and displacements. The following tasks are set in this study:
1) To describe the analytical solution for a glass plate under impact loading.
2) To develop a computer mathematical model that allows investigating the impact force with a glass model and carry out numerical studies.
3) To compare the results of numerical with analytical studies, assess the level of accuracy and give recommendations on the parameters of the calculation model;

3. Simulation model
In current work, the glass plate of sizes \( a = 500 \text{ mm}, \ b = 750 \text{ mm} \) and thickness \( h = 6.38 \text{ mm} \) is considered (Figure 1). The material properties of the glass is taken from manufacturer’s data, verified by the initial properties considered by Zhang et al. [21] and shown in Table 1.

| Material | Density, \( \rho \) (kg/m\(^3\)) | Young’s modulus, \( E \) (GPa) | Poisson ratio, \( \nu \) | Hardness |
|----------|---------------------------------|--------------------------------|------------------------|-----------|
| Glass    | 2500                            | 70                             | 0.23                   | 1550 HB   |

A hit on a plate with a solid body of mass 1 kg that moves with velocity 8 m/s was considered. The theory of elastic bodies hit will be used. The function \( w \) describes the displacement of the plate, which is due to the concentrated contact force \( P(t) \), at the point of contact in the center of the plate with the coordinates \((x_0, y_0)\). It can be determined using the differential equation of constrained vibrations of the plate [22], that is under the action of a surface load \( p(x, y, t) \), that depends on time and coordinates:

\[
D \nabla^4 w + 2\rho h \frac{\partial^2 w}{\partial t^2} = p
\]  

where \( D = \frac{2h^3E}{3(1-\nu^2)} \);  
\( 2h \) – plate thickness; \( w \) – displacement; \( \rho \) – material density.

4. Modal analysis

The solution to equation (1) can be found analytically in a linear statement. For this, the series expansion in its natural modes was applied. The natural frequency equations for plates with orthogonal properties are presented in the formulas:

\[
\omega_{nk}^2 = \left( \frac{n\pi}{a} \right)^2 + \left( \frac{k\pi}{b} \right)^2
\]

\[
f_{nk} = \frac{h}{2} \omega_{nk} \sqrt{\frac{E}{3\rho(1-\nu^2)}}
\]

where \( f_{nk} \) – natural frequencies of plate vibrations.

Solution (3) is sought in the form of decomposition by natural functions \( H_{nk} \) at the point of contact \((x_0, y_0)\).

\[
H_{nk} = \sin \left( \frac{n\pi x}{a} \right) \sin \left( \frac{k\pi y}{b} \right)
\]  

The spectrum of natural frequencies and vibration modes can be obtained, naturally, by numerical approaches within the framework of three-dimensional computer modeling. Comparison of numerical and analytical results for a given particular case (straight rectangular single-layer simple supported plate) makes it possible to conduct a primary analysis of the completeness of the numerical model, as
well as obtain recommendations on the calculated parameters of the model – the required FE mesh density.

Modal analysis in 3D modelling and finite element method (FEM) was carried out. Hexagonal FE with 20 nodes with 3 degrees of freedom in each was used. As boundary conditions, the glass was simply supported on all sides. The plate was modeled in a three-dimensional physical linear-elastic statement. The FE meshes with elements of different size have been created for the investigated model.

Numerical solutions will be approximate values that were compared with the analytical solution to determine the accuracy of the calculation. The FE mesh model is shown in Figure 2.

![Figure 2 Mesh size of the model: (a) – 95.7 (mm), (b) – 12.76 (mm), (c) – 3.19 (mm)](image)

To determine computational accuracy for each mesh size, natural frequencies were calculated. A comparison of analytical and numerical results has a good consistency with the values. The relative error between the frequency values of mesh sizes 95.7 mm and 12.76 mm is more than 2 %, and between mesh sizes 12.76 mm and 3.19 mm is small. So, in this article, the FEM model is assumed with a mesh size 12.76 mm.

In Table 2, it is shown analytical and numerical results comparison of frequencies calculated with 12.76 mm mesh size. According to Table 2, the frequency values have small relative error. In Figure 3, different natural modes are given.

| Frequency number | Wave number | Analytical frequency (Hz) | Numerical frequency (Hz) | Relative error (%) |
|------------------|-------------|---------------------------|--------------------------|--------------------|
| 1                | 1           | 90.88                     | 90.46                    | 0.46               |
| 2                | 1           | 174.78                    | 173.84                   | 0.54               |
| 3                | 1           | 314.60                    | 313.16                   | 0.46               |
| 4                | 1           | 510.35                    | 508.14                   | 0.43               |
| 5                | 2           | 279.65                    | 278.81                   | 0.30               |
| 6                | 2           | 363.54                    | 361.41                   | 0.59               |
| 7                | 2           | 503.36                    | 499.74                   | 0.72               |
| 8                | 2           | 699.11                    | 693.78                   | 0.76               |
| 9                | 3           | 594.25                    | 592.22                   | 0.34               |
| 10               | 3           | 678.14                    | 674.31                   | 0.56               |
5. Transient analysis under impact loading

An equally important part of verifying the adequacy of numerical modeling is the comparison of the numerical calculation results of the impact connection of a plate with another solid body with an approximate but analytical calculation for impact loading of plate. The last statement will in fact be quite adequate if the time dependence of the force transmitting the impact load will correspond to the force of contact interaction. Therefore, in order to analytically solve the problem of unsteady vibrations of a plate under impact loading, it is necessary to determine the distribution of the contact force (pressure) impulse in time (Figure 4). In the literature [23], an example is given of our problem modeling with the similar characteristics.

Upon impact, the load function is determined at the point of application of the contact force \( P(\tau) \). The force of the interaction at the point of contact \( (x_0, y_0) \) is set:

\[
P = \begin{cases} 
P_{\text{max}} \sin \left( \frac{n\pi}{2t_m} \right), & \tau < t_m \\
P_{\text{max}} \sin \left( \frac{\pi(\tau-(2t_m-t_{\text{max}}))}{2(t_{\text{max}}-t_m)} \right), & t_m < \tau < t_{\text{max}} \\
0, & \tau > t_{\text{max}}
\end{cases}
\]

where \( \tau \) – impact time, \( t_m \) – the time at maximum force value, \( t_{\text{max}} \) – the end loading time, \( P_{\text{max}} \) – the maximum value of contact force.

The method and algorithm of calculation for a thin rectangular plate that was hinged on all sides were formulated. The choice of the problem statement is because the analytical approximate solution...
of the boundary value problem is known for such plates. The theory, which takes into account the
displacement at the point of impact and their effect on the deforming plate, was applied [1].

\[
w(x, y, t) = \frac{1}{2h} \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} \frac{H_{nk}(x,y)}{f_{nk}f_{nk0}} \int_0^t P(\tau) \sin \left( f_{nk}(t - \tau) \right) d\tau
\]

\[
D = \frac{Eh^3}{12(1-\nu^2)}
\]

\[
\sigma_x = -\frac{12Dh}{h^3} \left( \frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2} \right)
\]

A numerical solution is found using the method of expansion in a series in its natural modes, carried out in the FE complex. The behavior of glass samples was modelled by transient analysis in 3D modelling and finite element method (FEM). For our glass plate, the displacement at the different points was obtained (see Figure 5). It can be seen that both solutions are well coincided.

![Figure 5](image)

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6. Conclusions

Analytical and numerical solutions have been carried out for the glass plate and natural frequencies were obtained. For each mesh size, computational accuracy is determined. A comparison of analytical and numerical results shows good consistency with the mesh size of 12.76 mm.
A numerical solution is found using the method of expansion in a series in its natural modes. The behavior of the glass plate was modelled by transient analysis over the FEM 3D model. The displacement and $X$-component of stress in different plate points were obtained, which does not exceed the yield stress of glass for assumed impact parameters.

The developed dynamical model can be applied for the estimation of glass window strength in the underground mining vehicles under different impact conditions. Besides, designers can assess physical parameters of cabin windows under dynamical conditions in the harsh underground environment. This allows providing safety work for operators.

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