Relation Between Initial Cosmological Conditions and the Properties of Dark Matter Haloes

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Abstract. The core-cusp problem is one of the essential issues in modern cosmology. The Entropy Theory of haloes Evolution recently suggested by Lukash, Doroshkevich and Mikheeva is one of the possible solutions to this problem. This work compares some results of numerical simulation of Large-Scale Structure with the conclusions of the Entropy Theory in order to verify this theory. The numerical simulation was performed in a volume 100 Mpc/h in a side using ~17 million particles. Dark matter particles, which then form virialized haloes, were found in the initial perturbation field. This work investigates the distribution of these dark matter particles and measures the velocity dispersion profiles. It also traces evolution of haloes entropy profiles. On the whole, simulation results correspond to Entropy Theory of haloes evolution.

1. Introduction

In the standard (ΛCDM) cosmological model the energy density of dark matter is 5 times higher than that of baryon matter. Therefore, it is the dark matter that governs the processes of clustering and formation of the Large-Scale Structure of the Universe. Numerical simulation of the dark matter distribution is a powerful method to investigate the properties of the Universe structure on the range of scales and verify or exclude existing physical models. Even though nowadays results of numerical simulations are mainly in a good agreement with observations [1], some drawbacks exist. One of them is a divergence of dark matter haloes’ density behaviour near the center of halo in observations and numerical simulations, the so-called core-cusp problem. Observations of the velocity curves of galaxies with low surface brightness (LSB) show that there is constant density in dark matter haloes, whereas haloes in numerical simulations have singular density in the center ($\rho(r) \propto r^{-1}$).

There are several plausible solutions to this problem. First of all, cusps might be produced by inadequacy of the simulation technique itself, where one deals with the truncated power spectrum of initial perturbation due to limited size of cells and the box. It worth noting that cosmological numerical simulation is an intensively developing brunch of science. It has been being refined for several last decades. The computation power of modern supercomputers is becoming higher and higher. Nevertheless, the density profiles of dark matter haloes still have singular cusps. The other solution to the problem is to modify the theory. For instance, the core-cusp problem might be solved by including the exotic dark matter in the theory. However, authors of [2] believe that there is no reason in such significant modifications and the core-cusp problem can be solved within the ΛCDM model. They suggest the Entropy Theory of dark
matter halo formation. According to this theory cusps in numerical simulations are produced by the underestimating of initial small-scale entropy of dark matter particles.

In order to verify some conclusions of the Entropy Theory a numerical simulation of dark matter distribution was performed. This report provides the first results of this simulation.

2. The numerical simulation of the Large-Scale Structure of the Universe

To investigate properties of the inner structure of dark matter haloes the numerical simulation was run in a cubical volume 100 Mpc/h in a side. $253^3$ probe dark matter particles were used and corresponding mass resolution was $4.96 \cdot 10^9 \, M_\odot$. The simulation was performed with the aid of GADGET-2 code [3, 4]. The used values of the main parameters are specified in the table 1.

| Variable                  | Value         | Variable                  | Value         |
|---------------------------|---------------|---------------------------|---------------|
| Box Size                  | 100 Mpc/h     | Dark energy density ($\Omega_\Lambda$) | 0.7           |
| Number of particles       | $256^3$       | Matter density ($\Omega_0$)  | 0.3           |
| Mass of particles         | $4.96 \cdot 10^9 \, M_\odot$ | Hubble constant ($H_0$)      | 70 Mpc/(km/s) |
| Force resolution          | 7 kpc/h       | Spectrum index ($n$)       | 1             |
| Initial redshift ($z_{init}$) | 79            |                            |               |

To find haloes of dark matter widely applied spherical overdensity criterium [5] was used. The approach starts with finding of the densest clusters of particles via Minimal Spanning Tree Method. This method is very similar to “Friend-of-Friend” Method [6] and provides ability to find all clusters which are denser than given threshold density (specified by threshold length $l_{th}$) and threshold mass (i.e. the number of particles within a cluster $M$). The threshold length defines the boundary density of the cluster, where separation of particles is about $l_{th}$, whereas within the cluster separation is lower than $l_{th}$. In this work $l_{th}$ was selected to be about the length of force resolution. Apart from finding clusters, this choice of $l_{th}$ showed that the length of force resolution was reasonable. Found clusters were associated with the inner parts of dark matter haloes. It may be suggested that the center of the halo, that corresponds to some cluster, lays within this cluster, because found clusters are the densest parts of haloes.

On the first step the center of each halo can be roughly associated with the center of mass of the corresponding dark matter particles cluster. In order to define the size of the particular halo, one can place a probe sphere and choose the size of this sphere so that the average density inside the sphere would exceed the critical density by a factor of $\Delta_c \approx 200$. For virialized systems in standard cosmological model dependence of $\Delta_c$ on $z$ is given in [7].

In order to obtain appropriate profiles of density and entropy, one should accurately define the center of the halo. While there are several definitions of the halo’s center [8], in this paper the density maximum is used as one. All the haloes were further fitted with universal Navarro-Frenk-White (NFW) profile [9] with dependence of concentration on a halo’s mass proved to be very similar to the one obtained in [10].

3. Evolution of the individual halo

3.1. Initial clouds of particles

With data on the position and velocity of each particle in different moments of time, one can study in details processes in the evolution of the individual halo. This data also provides means
for investigating the distribution of particles on the early stages of evolution and even in the initial conditions. It is interesting to study initial properties of particles that form halo. These particles form extended “clouds” in the initial distribution of dark matter particles. However, the investigating of such clouds is complex due to entanglement of particles, which will form a halo, with particles, which will be thrown out during halo formation. This work shows that profile of a halo’s particle fraction is almost the same for all clouds and in the center of the cloud there are almost all particles will get into the halo. Furthermore, distribution of these particles from the central part of the cloud in the halo depends on process of the halo evolution. In case of monolithic collapse these particles are concentrated in the center of the halo. In more general case of hierarchical clustering these particles are spread over the halo.

3.2. Evolution of central density

It is the central part of the halo where one has surplus of particles in comparison to the observation data. Therefore, it is interesting to study the evolution of the central density of the halo. Typical behaviour of mass within the sphere of physical radius equals to $r_s/2 \approx 100$ kpc/h in the center of the halo is shown on figure 1.

![Figure 1](image_url)

**Figure 1.** Solid line represents evolution of mass within the central region of size $r_s/2 \approx 100$ kpc/h with scale factor $a$.

Mass changes significantly during the first collapse and then remains almost constant. Actually, after formation of the central part smaller sub-haloes continue falling onto the main halo. However, this process cannot change central mass significantly due to high velocities of dark matter particles and sub-haloes. Falling particles come through the central part and it results in slight oscillations of the mass (figure 1, $a > 0.4$).

3.3. Spatial asymmetry of haloes

In analytical models of the spherical cloud collapse one can obtain density profile with quite a steep slope in the center: $\rho(r) \propto r^{-\alpha}$, where $\alpha \approx 1.6 - 2.0$ [11, 12]. However in numerical simulations haloes density profiles can be approximated with NFW profile with slope index $\alpha = 1$. The obvious difference between spherical initial conditions and real conditions in simulations of the Large-Scale Structure is the spatial asymmetry of real distribution of dark matter particles. In this research the criterion of asymmetry was chosen as the ratio of the largest eigenvalue of the inertia tensor to the lowest one. During the evolution this ratio increases because sub-haloes collapse and then decreases with falling of sub-haloes on the center. Therefore, maximum of this ratio for the individual halo can be associated with the moment of formation of sub-haloes.

As for initial and final values of asymmetry, the ratio of eigenvalues changes significantly. For the main part of initial clouds of particles this ratio covers the range from 1.2 to 1.6, while for the major part of final haloes this ratio lies between 1.3 and 3.0. It is worth noting that the
minimal values of the ratio for initial and final states are 1.12 and 1.25, respectively. It gives an indication of maximal symmetry in the simulation.

4. The Entropy Theory of haloes formation
The Entropy Theory suggested in [2] provides a possible solution to the core-cusp problem. It claims that cusps in numerical simulations appear due to underestimation of low-scale initial perturbation. To make the analysis clearer authors suggest to convert density profiles $\rho(r)$ into entropy profiles $E(M)$. Here $M$ is the mass within sphere of radius $r$ and $E$ is an entropy function, defined as $E = \sigma^2 \cdot \rho^{-2/3}$, where $\sigma$ is the dispersion of particles’ velocities. The entropy function behaviour close to the center can be approximated as $E \propto M^\beta$. Critical value $\alpha = 1$ corresponds to $\beta = 5/6$. In this research inner slopes of all haloes proved to be $\beta > 5/6$, with averaged inner slope being about $\beta = 1$.

4.1. Initial entropy profiles
In the initial state distribution of dark matter particles is almost uniform, thus entropy function depends only on velocities dispersion: $E \propto \sigma^2$. To obtain analytical shape of initial clouds entropy profiles one can consider the perturbation of the cloud as a set of plane waves falling into the center of the cloud. In this case velocities dispersion is given by:

$$\sigma^2(r) = \frac{1.7aH_0^2\Omega_m}{4\pi^2} \int_0^\infty P(k) \left[1 - \frac{\sin(2kr)}{2kr}\right] dk,$$

where $P(k)$ is a power spectrum of perturbation.

Article [2] provides solutions obtained with other methods. These solutions, the solution given above and numerical results are given in figure 2. The article also predicts a large scatter of $\sigma$, thus the results are in a good agreement.

![Figure 2. Dispersion profiles in the initial conditions. Solid line represents solution obtained in this research, dashed lines represent solutions from [2]. The averaged entropy profile of particles clouds in the initial conditions ($z = 79$) is given by error bars.](image)

4.2. Evolution of entropy profiles
Figure 2 shows that entropy profiles are initially plane, whereas in the haloes they have steep slope $\beta \approx 5/6$. Tracing of entropy profiles evolution shows the existence of two different stages. Initially, on the linear stage of evolution, entropy profiles remain plain and then radically change their shapes during the hierarchical collapse. It is the non-linear stage of evolution when entropy profiles attain the slope $\beta > 5/6$. For that reason density profiles were converted into entropy ones. It makes analysis clearer.

Unfortunately, the efforts to find direct dependence of slopes on initial entropy magnitudes were not successful (fig. 3). The case needs more profound and refine analysis. For instant, the next step might be distinguishing between different times of collapse.
Figure 3. Entropy profiles in the initial conditions (left panel, $z = 79$) and in the results of simulation (right panel, $z = 0$). Solid lines represent haloes with high initial entropy, while dashed ones represent haloes with low initial entropy. The dash-dotted line on the right panel shows the slope equals to $5/6$.

5. Following steps
Finding actual processes that govern formation of haloes inner structure is a complicated problem. Now we have variety of means to study evolution of haloes in details. There are still plenty of factors which need more profound analysis: minor/major merging, velocity anisotropy, angular momentum, etc. Furthermore, investigation of modified spectra with enhanced or suppressed perturbations of various ranges is very instructive, since it enables to examine how perturbations of different scales affect inner structure of dark matter haloes. Thus, this report contains merely intermediate results of extensive research aimed to shed light on the core-cusp problem and working upon the problem is going on.

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