The Effect of Draft Ratio of Side-By-Side Barges on
Fluid Oscillation in Narrow Gap

Linfeng Chen, Xueshen Cao, Shiyan Sun and Jie Cui *

School of Naval Architecture and Ocean Engineering, Jiangsu University of Science and Technology,
Zhenjiang 212003, China; chenlinfeng@just.edu.cn (L.C.); 182010010@stu.just.edu.cn (X.C.);
shiyan_sun@just.edu.cn (S.S.)
* Correspondence: cuijie@just.edu.cn

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Abstract: In the present study, the effects of the draft ratio of the floating body on the fluid oscillation in the gap are investigated by using the viscous fluid model. Numerical simulations are implemented by coupling wave2Foam and OpenFOAM. The Volume of Fluid (VOF) model is used to capture the free surface waves. It is verified that the numerical results agree well with the experimental and other results. It is firstly found that, within the water depth range investigated in the present study, the depth of the wave tank has a significant effect on the numerical results. As the depth of the wave tank increases, the oscillation amplitude of the narrow-gap fluid largely decreases and the resonant frequency of the fluid oscillation in the narrow gap increases. The results also reveal that the draft ratio of floating bodies has a significant nonlinear influence on the resonant frequency and on the oscillation amplitude of the fluid in the narrow gap. With an increase in the draft of either the floating body on the wave side or the one on the back wave side, the resonant frequency decreases. The increase in the draft of the floating body on the wave side causes an increase in the reflection wave coefficient and leads to a drop in the fluid oscillation amplitude, and the increase in the draft of the floating body on the back wave side triggers a decrease in the reflection wave coefficient and results in an increase in the fluid oscillation amplitude. Meanwhile, the viscous dissipation induced by the fluid viscosity synchronously increases with the oscillation amplitude of the fluid in the increasing gap. Moreover, it is found that the draft ratio mainly affects the horizontal force of the floating body on the back wave side and that the highest calculated force increases with the draft ratio.

Keywords: side-by-side barges; narrow-gap fluid resonance; draft ratio; oscillation amplitude; horizontal force

1. Introduction

With the shift of oil and gas exploitation to the deep sea, Floating Production Storage and Offloading (FPSO) and Floating Liquefied Natural Gas (FLNG) marine platforms are developing rapidly. In the process of unloading oil and gas to the shuttle ship, FPSO and FLNG platforms will form a multi floating body system with the shuttle ship. There is a narrow gap between the platform and the shuttle ship. Very large floating structures (VLFS) [1] contain a large number of individual modules. Within a large multi floating body system, there are many narrow gaps between these individual modules. The sea water in the narrow gap of a multi floating body system will oscillate or even resonate under certain incident wave conditions, exerting forces on the structures on both sides of the narrow gap and affecting the stability of the structures in the process of operation. This phenomenon is called the narrow gap resonance problem. There is also a narrow gap between the two hulls of the catamarans. When designing a catamaran, the narrow gap resonance problem should be considered.
In the presence of a complicated hydrodynamic interference between the wave and the multi floating body system, the fluid in the narrow gap normally oscillates [2]. For a specific incident wave, the amplitude of the fluid in the narrow gap will be higher than that of the incident wave, even up to five times the incident wave height [3]. The reason for this is that the fluid in the narrow gap absorbs more wave energy, when the frequency of the incident wave becomes close to the natural frequency of the fluid in there, and thus the amplitude of the wave motion in the narrow gap increases. The narrow gap resonance in the multi floating body system will affect its stability and operation, so it has been widely considered. Extensive theoretical, numerical and experiments studies have been carried out [4].

In the early research, the narrow gap resonance problem has been mainly studied by theoretical analysis based on the linear potential flow theory. In 2001, Molin used a linear potential flow theory model to find a solution formula for the narrow gap resonance frequency of multi floating body systems. He found that there are two oscillation modes of fluids trapped between simple bodies: piston mode and longitudinal sloshing mode. Piston mode is found to be more crucial than the longitudinal sloshing mode with respect to fluid flow amplification. The gap resonance discussed in this study refers to the piston mode unless otherwise specified. Saitoh et al. [5], Kristiansen et al. [6] and others gradually found that the linear potential flow theory has defects in solving the narrow gap resonance problem of multi floating body systems. Linear potential flow models tend to overestimate the fluid oscillation response amplitudes in the narrow gap resonance according to comparisons with the results of experimental tests. In order to correct this defect of linear potential flow theory, Buchner et al. [7] and Huïjsmans et al. [8] proposed to apply a rigid cover to the narrow gap between floating bodies, Newman [9] proposed to apply a flexible cover to the narrow gap, and Chen [10] proposed to introduce artificial damping into the linear potential flow model. Both a flexible cover and artificial damping can effectively inhibit the fluid response amplitude, but there is no reasonable physical justification for these approaches. For the rigorous potential flow theory, the introduction of artificial damping has no corresponding physical significance, and, in some cases, it is difficult to obtain a unique value for the damping parameter.

To better understand narrow gap resonance and to verify the theoretical analysis, a large number of physical model tests have been carried out in two-dimensional and three-dimensional wave fields. Tan et al. [11] conducted a number of tests on the influence of the shape of the floating body bilge on the gap resonance and found that the energy dissipation of a sharp shaped bilge is higher than that of a smooth one. Zhao et al. [12] studied 3D narrow gap resonance by using transient waves and analyzed the linear damping coefficient in the gap resonance. Later, Wang [13] repeated Zhao’s experiments and found that a transient wave group is more suitable to study the narrow gap resonance than the normal incident wave.

In recent years, computational fluid dynamics (CFD) numerical simulation has become a new and effective method to investigate the narrow gap resonance of multi floating systems. Previous studies have shown that the CFD results are in good agreement with the experimental results. At present, most of the numerical studies of narrow gap resonance are based on the classical potential flow model, which adopts the boundary element method and the proportional finite element method. Moradiet et al. [14] studied the two-dimensional narrow gap resonance of side-by-side barges by numerical simulation considering viscosity. It was found that the resonant response amplitude decreased with an increase in the gap inlet radius. Lu et al. [15] studied two-dimensional narrow gap resonance by a modified potential flow model and by means of a viscous fluid method. By optimizing the artificial damping term of the potential flow solver, the numerical results of the potential flow theory are basically consistent with those of the turbulent viscosity solver in terms of resonant response amplitude and hydrodynamic force. With the continuous growth of computational resources, some researchers, e.g., Lu et al. [16], began to use viscous CFD flow models to compute the highly nonlinear hydrodynamic load on the bodies for the narrow gap resonance problem. Jin et al. [17], based on the unsteady Reynolds averaged Navier–Stokes (URANS) equation, evaluated the hydrodynamic load of FLNG-LNG platforms alongside a floating body model. Through comparisons with the potential flow
calculation, it was found that the results of URANS simulations are more accurate when calculating
hydrodynamic load, narrow gap oscillation amplitude and wave diffraction problems.

In most of the previous gap resonance studies, only identical floating bodies were considered
for the multi floating body system. To make the research resemble practical problems more closely,
the drafts of floating bodies in the multi floating body system considered are different in the present
work. Five kinds of examples are set up, in which the minimum draft of the floating body is 0.25 m, and
the ratio \( h_2/h_1 \) (the ratio of draft of the body on the back wave side to draft of those on the wave side)
is varied. The five examples considered for the draft ratios are 2, 1.5, 1.0, 0.667 and 0.5, respectively.
This work aims to explore the effect of barge draft on the gap resonant response amplitude and resonant
frequency, illustrate the damping mechanism at the gap resonance and investigate the effect of the
floating body draft on the flow structures and energy dissipation around two 2D side-by-side barges.
Following the argumentation of Zhang et al. [18] that turbulent dissipation does not play a significant
role as far as wave damping in the narrow gap problem is concerned, laminar CFD simulations are
performed in a 2D wave tank by coupling the Waves2Foam package with the code OpenFOAM in the
present study, and the Volume of Fluid (VOF) model is used to capture the free surface wave during
the simulation.

This paper is organized as follows. Governing equations of a viscous fluid flow model, VOF
model definition, boundary conditions of the numerical wave tank and numerical implementation
of wave generation and absorption are described in Section 2. In Section 3, the viscous fluid flow model
is validated by using the available experimental data and the results of a previous gap oscillation study.
The effects of numerical tank depth on the numerical simulation results are discussed by using the
multi floating body system with a draft ratio of 1. The influence of the draft ratio of floating bodies
in a multi floating body system on the response amplitudes and resonance frequencies of narrow
gap oscillation problems as well as viscous dissipation are discussed. The horizontal hydrodynamic
forces of a side-by-side floating body system with different draft ratios are compared for different wave
frequencies. Concluding remarks are drawn in Section 4.

2. Numerical Models

According to the viscous fluid flow model [19], a two-phase flow with air and water is governed
by the incompressible Navier–Stokes (N-S) equations. A volume of fluid (VOF) method [20] is used
to capture the free surface of the wave. A waves2Foam package [21,22] based on the OpenFOAM
solver is used to build the numerical wave tank, in which a regular wave is generated using the wave
generation utility of waves2Foam and the explicit relaxation zone technique is used to remove spurious
wave reflection in the inlet zone.

2.1. Viscous Fluid Flow Model

The governing equations for the incompressible viscous fluid flow model [21] read

\[
\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p_{rgh}}{\partial x_i} + \nu \frac{\partial^2 u_i}{\partial x_j^2} - \frac{g}{\rho} (x_2 - x_r) \nabla \rho \tag{2}
\]

where \( u_i \) and \( x_i \) represent the velocity components and the Cartesian coordinates in the \( i \)th direction,
respectively; \( \rho, \nu \) denote the fluid density and kinetic viscosity coefficient, respectively. \( p_{rgh} \) is the
pressure in excess of the hydrostatic pressure and \( p_{rgh} = \rho - \rho g (x_2 - x_r) \), \( g \) is the gravity acceleration;
\( x_r \) is the reference coordinate defined at sea level.
2.2. Volume of Fluid Method

The volume of fluid (VOF) method proposed by Hirt and Nichols [20] is used to capture the free surface of a wave in the viscous fluid flow model. A fractional function of the VOF method denoted by \( \alpha \), for a computational cell, is defined as

\[
\alpha = \begin{cases} 
0 & \text{in air} \\
0 < \alpha < 1 & \text{on free surface} \\
1 & \text{in water} 
\end{cases}
\]

(3)

The transport equation of the water volume fraction follows an advection equation:

\[
\frac{\partial \alpha}{\partial t} + \nabla \cdot (\alpha \mathbf{v}) + \nabla \cdot (\alpha (1-\alpha) \mathbf{v}_r) = 0
\]

(4)

where \( \mathbf{v}_r = \mathbf{v}_{water} - \mathbf{v}_{air} \) represents the relative velocity between the water and air, termed “compression velocity”. The contour of the VOF function with \( \alpha = 0.5 \) is used to represent the interface between the air and water phases. In the computations, the fluid density and the effective viscosity are average by using the available VOF function

\[
\rho = \alpha \rho_w + (1-\alpha) \rho_a, \quad \mu_e = \alpha \mu_{ew} + (1-\alpha) \mu_{ea}
\]

(5)

where the subscripts \( w \) and \( a \) represent the water phase and the air phase, respectively.

2.3. Problem Setup

The influence of the draft ratio on the narrow-gap fluid resonant response (the phenomenon of the maximum amplitude of narrow-gap fluid under certain frequencies of incident wave) is examined by using the viscous fluid [23,24] flow model. Numerical simulations are performed in an \( 18.09 \text{ m} \times 1.5 \text{ m} \) rectangular wave tank. A sketch of the viscous numerical wave tank used in the study is presented in Figure 1. Two fixed floating rectangular bodies in the numerical water tank are denoted by floating body \( A_1 \) and floating body \( A_2 \), respectively. The two floating bodies have the identical breadth \( B \), \( B = 0.5 \text{ m} \). A narrow gap with the width \( B_g \) is present between the two bodies, \( B_g = 0.05 \text{ m} \). The draft depths of two floating bodies are denoted by \( h_1 \) and \( h_2 \), respectively. The minimum value of draft depth \( h \) is 0.25 m. The range of the ratio \( (h_2/h_1) \) of draft depth considered in the study is 2.0, 1.5, 1.0, 0.67, 0.5. The depth \( H \) of the numerical water tank is 1 m. The space of fluid under the floating bodies and in the narrow gap is defined as a control volume, which is denoted by \( CV \).

Figure 1. Sketch of flow oscillation in the narrow gap between two floating bodies.
In the numerical wave tank, the first-order Stokes regular head wave is generated at the inlet. The wave elevation \( \zeta \), the incident wave potential \( \varphi \) and the velocity components \((u, w)\) are written as follows:

\[
\begin{align*}
\zeta &= A \cos(kx - \omega t) \\
\varphi &= \frac{Ag \cosh(z + H)}{2} \cos(kx - \omega t) \\
u &= A\omega \frac{\cosh(z + H)}{\sinh kH} \cos(kx - \omega t) \\
w &= A\omega \frac{\sinh(z + H)}{\sinh kH} \sin(kx - \omega t)
\end{align*}
\]

where \( A \) represents the wave amplitude (\( A \) is set to a constant 0.012 in the study), \( k \) is the wave number, \( \omega \) omega denotes the natural circular frequency. In the simulations, Equations (8) and (9) are then used to define the boundary condition of the velocity, and the normal zero-gradient is set for the pressure at the inlet. At the outlet, a normal zero-gradient condition is applied for the velocity and the pressure is specified as zero. At the bottom of the numerical wave tank and the solid walls of the bodies, “no-slip” boundary conditions are applied. In the upper part of the numerical wave tank, the atmosphere boundary is adopted, which allows the air to flow in and out of the domain. The total pressure (sum of static pressure and dynamic pressure) at the top boundary is constant at zero.

To eliminate the unphysical effect of the reflection wave, two numerical wave absorption zones are arranged on the two ends of the numerical wave tank to absorb the reflection waves. The wave absorption is accomplished using an explicit relaxation zone technique, which is given as

\[
\phi = (1 - w_R)\phi_{\text{target}} + w_R\phi_{\text{computed}}
\]

where \( w_R \) is the weighting function between the computed solution of the velocity field and the indicator field with a target solution. The method corrects the field \( \alpha \) and \((u, w)\) according to Equation (10) prior to the solution to the pressure–velocity coupling. The sizes of both the absorption zones are set to two wavelengths, which are sufficient to remove the spurious reflection.

Unstructured meshes including quadrilateral and triangular cells were generated using the blockMesh tool of OpenFOAM. The meshes were set so that 1120 cells were distributed in the wave-length direction and 220 in the wave height direction for each case, which is sufficient to accurately capture the interface between the water and air. To take the effect of the fluid viscosity around the bodies into account, the cells around the floating bodies are locally refined. A zoom-in sketch of the refined mesh is shown in Figure 2.

![Figure 2. Mesh refinement of numerical water tank.](image-url)

Continuing along the previous study on the fluid motion at the narrow gap between double floating bodies with the same draft, the various draft ratios of front to rear floating bodies are concerned with exploring the influence of the variation of the draft ratio on the motion characteristics of the narrow-gap fluid. Five different draft ratios are considered in this study. The detailed parameters of

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the five sets are shown in Table 1. For each draft ratio, seven incident waves with a constant amplitude of 0.024 m and different frequencies are numerically measured; their corresponding periods are 1.28 s, 1.2197 s, 1.2114 s, 1.187 s, 1.1762 s, 1.1505 s, 1.1108 s.

| Draft Ratio ($h_2/h_1$) | 1  | 0.667 | 0.5 | 1.5 | 2  |
|-------------------------|----|-------|-----|-----|----|
| Draft $h_1 = h_2 = 0.25$ m | 2  | 3  | 4  | 5  |
| $h_1 = 0.375$ m          | 6  | 7  |    |    |
| $h_2 = 0.25$ m           |    |    |    |    |
| $h_2 = 0.25$ m           |    |    |    |    |
| $h_2 = 0.375$ m          |    |    |    |    |
| $h_2 = 0.5$ m            |    |    |    |    |

3. Results and Discussion

3.1. Verification of Numerical Simulations

In the numerical wave tank, four wave gauge probes are used to record the wave elevations. The four gauge probes are located at $x = 2.705$, $3.205$, $4.73$ and $6.255$ (the inlet is at $x = -4.02$). The third probe is located at the center of the gap. Simulations were performed for sets 1–5 to investigate the influence of floating body draft on the gap resonance of the multi floating system; discussions were carried out in terms of the surface elevation amplitudes in the gap, resonance frequency, the energy dissipation and energy conversion around the floating bodies and in the gap.

Before further simulations are performed, a comparison of the wave elevations obtained using numerical simulation and linear wave theory is carried out to validate the wave generation utility. Figure 3 shows time histories of the wave elevations by the numerical simulation and linear wave theory in good agreement with the theoretical result after the simulation reaches a steady state. For the gap resonance problem, three meshes with 186,450 (Mesh A), 246,400 (Mesh B) and 302,300 (Mesh C) cells are tested for the mesh convergence study. Figure 4 shows time histories of the wave elevations in the gap for $h_2/h_1 = 1$ and $T = 1.1108$ s. The results show that good convergence is obtained using a mesh with 246,400 cells (Mesh B). The meshes at this level are used for further simulations in the study.

![Figure 3](image_url). Time histories of the wave elevations obtained by the numerical simulation and by the linear wave theory.

![Figure 4](image_url). Time histories of the wave elevations obtained using a mesh with 246,400 cells (Mesh B).
In all the simulations, the computation duration is 50 s, which is more than 40 wave periods of the incident wave. Time histories of the wave elevation of the wave at the gap for \( h_2/h_1 = 1 \) are shown in Figure 5. It is shown that it takes a long time for the motion of the fluid in the gap to reach a dynamic steady state. The motion almost reaches a dynamic steady state after 20 s. The time history shows that the motion of the fluid in the gap almost exhibits periodicity when it reaches a steady state.

In set 1, the draft ratio of the floating body is 1. We compared the calculation results of set 1 with Saitho’s [5] experimental data and others’ calculation results [18], as shown in Figure 6. It shows that the results of set 1 are in good agreement with the experimental results and others’ data.

Since Saitho’s experiments were carried out in a 0.5 m deep water tank, the depth of the numerical water tank of the set 1 is also 0.5 m. The good agreement between the data of example 1 and the experimental data shows that the example based on the viscous fluid flow theory established by OpenFOAM can accurately simulate the narrow gap oscillation of the multi floating body system; the results of simulation are reliable. In order to compare the data better, the frequency \( \omega \) of the incident wave and the response amplitude \( \eta_A \) of the wave in the narrow gap are post processed; the treatment follows the Equations (11) and (12).

\[
\eta^* = \eta_A/A_i
\]  
\[\omega^* = \omega/\sqrt{\gamma B}
\]
3.2. Fluid Oscillation Amplitudes

Owing to the deeper draft of floating body in the subsequent computations, the water depth of the numerical wave tank is changed to 1m in the subsequent calculation example. The results for the draft ratio 1 under the condition of water depths of 0.5, 1 and 1.5 m are compared, as shown in Figure 7. The result shows that the water depth of the numerical wave tank has a significant influence on the numerical simulation results of the cases, which not only changes the wave amplitude of the gap resonance but also changes the resonant frequency of the gap resonance. As the water depth of the numerical wave tank increases from 0.5 to 1.0 m, the wave amplitude at resonance of the narrow-gap fluid drops obviously. It can be observed that the resonance frequency of narrow-gap fluid in shallow water is different from that in deep water, and that the resonance frequency of narrow-gap fluid in deep water is higher than that in shallow water. As the water depth continues to increase from 1.0 to 1.5 m, the wave amplitude at resonance of the narrow-gap fluid reduces slightly, and the resonance frequency stays unchanged.

Figure 6. Fluid oscillation amplitudes against angular frequency of incident wave.

Figure 7. Comparison of fluid oscillation amplitudes for water depths of 0.5, 1 and 1.5 m.

Figure 6 shows the curve of the wave resonance amplitude in the narrow gap changing with the incident wave frequency at different draft ratios of multi floating body systems. In Figure 8, the curves
with draft ratios \((h_2/h_1)\) of 2.0 and 1.5 are the oscillation amplitudes of narrow-gap fluid with larger drafts of the floating body on the back wave side. Compared with the curve with a draft ratio of 1, it is found that the oscillation amplitudes of the narrow-gap fluid increase with the draft of the floating body on the back wave side, especially at low frequencies. However, the frequency at resonance (associated with the largest oscillation amplitude) decreases when the draft of the floating body on the back wave side increases. The curves with draft ratios \((h_2/h_1)\) of 0.67 and 0.5 are the oscillation amplitudes of narrow-gap flow with larger draft of the floating body on the wave side. Compared with the curve with a draft ratio of 1, it is found that, with the draft of the floating body on the wave side increasing, the oscillation amplitude of the narrow-gap fluid decreases, and the frequency at resonance decreases. Such an observation is consistent with the experimental results in [24]. Since the control volumes for \(h_2/h_1 = 2.0\) and 1.5 are similar to those for \(h_2/h_1 = 0.5\) and 0.67, the added mass (induced by the equivalent piston) for calculating the natural frequency might be similar. Consequently, it can be found that the variation in the frequency at resonance from \(h_2/h_1 = 1.0\) to 2.0 behaves similarly to that from \(h_2/h_1 = 1.0\) to 0.5. The reasons for the variation in the oscillation amplitude are given subsequently through the variation in the transmission wave and reflection wave coefficients.

![Graph](image.png)

**Figure 8.** Fluid oscillation amplitude profiles against angular frequency of incident wave for different draft ratios.

The frequency \(\omega_f\) of fluid oscillation in the narrow gap of multi floating body systems with different draft ratios under different incident waves is obtained by Fast Fourier Transform (FFT). Figure 9 shows that the frequency of fluid oscillation in the narrow gap increases with the frequency \(\omega'\) of incident wave, while the amplitude of fluid oscillation in the narrow gap does not increase with the frequency of fluid oscillation. The amplitude of fluid oscillation in the narrow gap first increases, and, after reaching the maximum value at a specific frequency of incident wave, the amplitude of fluid oscillation begins to decrease, which shows that the narrow-gap fluid resonance phenomenon also occurs in the multi floating body systems with different draft ratios under a specific incident wave frequency.
3.3. Transmission and Reflection Wave Coefficients

Figure 10 shows the transmission ($K_t$) and reflection ($K_r$) wave coefficient curves against the incident wave frequency for three different draft ratios. The transmission wave coefficient ($K_t$) is defined as the ratio of transmitted wave height ($H_t = 2A_t$) to incident wave height ($H_i = 2A_i$), $K_t = H_t / H_i$, and the ratio of reflected wave height ($H_r = 2A_r$) to the incident wave height is defined as the reflection wave coefficient, $K_r = H_r / H_i$. The transmitted wave heights can be measured by the gauge behind the floating bodies (Gauge 4). The reflection wave coefficients are calculated using the method proposed by Sun et al. [25]:

$$K_r = \frac{A_r}{A_i} = \frac{\| e^{-ik\Delta x} \xi(x_1, t) - \xi(x_2, t) \|}{\| e^{ik\Delta x} \xi(x_1, t) - \xi(x_2, t) \|}$$  \hspace{1cm} (13)

where $\Delta x$ represents the distance between G1 and G2; $\xi(x_1, t)$ and $\xi(x_2, t)$ are complex surface elevations measured at G1 and G2, respectively, which can be calculated using Hilbert transform. Please see the details in [25].
The shear stress around the floating bodies with the increase in the draft of the body on the wave side or on the back wave side, the transmission viscosity may lead to energy dissipations, which may reduce the energy transferred from the wave to the fluid oscillation in the gap. For the floating system with different draft ratios, the response of the fluid in the narrow gap may lead to shear stress of different magnitude, which allows for a variation in the viscous dissipation. In order to interpret the influence of the draft of the floating body on the fluid oscillation in the narrow gap, contours of vorticity magnitude at the resonant frequency are computed and shown in Figure 11a–c.

3.4. Viscous Dissipation

The viscous dissipation also has an effect on the fluid oscillation in the narrow gap. When the fluid oscillation occurs in the narrow gap, the shear stress around the floating bodies owing to the fluid viscosity may lead to energy dissipations, which may reduce the energy transferred from the wave to the fluid oscillation in the gap. For the floating system with different draft ratios, the response of the fluid in the narrow gap may lead to shear stress of different magnitude, which allows for a variation in the viscous dissipation. In order to interpret the influence of the draft of the floating body on the fluid oscillation in the narrow gap, contours of vorticity magnitude at the resonant frequency are computed and shown in Figure 11a–c.

Figure 10. Transmission and reflection wave coefficient profiles against the incident wave frequency for different draft ratios. Top: transmission wave coefficient, bottom: reflection wave coefficient.
Figure 11a shows the contour of vorticity magnitude around the two floating bodies with $h_1 = 0.5$ m at the resonant frequency. Since most of the incident wave energy is reflected in this case, the oscillation amplitude of the fluid in the gap is much lower. The weak response may just cause minor shear stress near the floating bodies. Therefore, it can be seen that only a few vortices are formed near the corner of the body on the back wave side in this case. This indicates that low viscous dissipation is present at $h_1 = 0.5$ m. For the floating bodies with $h_1 = h_2 = 0.25$ m (Figure 11b), the oscillation amplitude of the fluid in the gap increases. The stronger response of the fluid leads to larger shear stress near the floating bodies. Consequently, more vortices are formed below the two bodies, which reveals that more viscous dissipation is present at $h_1 = h_2 = 0.25$ m. When the draft of the floating bodies increases to $h_1 = h_2 = 2$ m (Figure 11c), the oscillation amplitude of the fluid in the gap becomes even larger. The stronger response of the fluid leads to even larger shear stress near the floating bodies. Consequently, more vortices are formed below the two bodies, which reveals that even more viscous dissipation is present at $h_1 = h_2 = 2$ m.
body on the back wave side increases to \( h_2 = 0.5 \) m (Figure 11c), the oscillation amplitude of the fluid in the gap increases to the maximum. Apparently, the strongest response of the fluid results in the largest shear stress near the floating bodies. Therefore, it can clearly be seen that much more vortices with high vorticity magnitude are present below the gap. That is to say, the most viscous dissipation is drawn in this case. This fact can also be demonstrated by the lowest transmission and reflection wave coefficient in Figure 8. As a result, when the largest oscillation amplitude of the fluid in the narrow gap increases with the draft ratio \( h_2/h_1 \), the relative motion of the fluid near the floating bodies strengthens and the viscous dissipation increases synchronously.

3.5. Horizontal Hydrodynamic Forces

The fluid oscillation in a narrow gap exerts forces on the floating bodies on both sides of the gap. Based on the previous study by Lu et al. [15], the vertical forces of the bodies are dominated by the static buoyancy and not significantly affected by the fluid oscillation in the gap. Therefore, attention is paid to the horizontal forces acting on the floating bodies.

Profiles of the dimensionless horizontal force amplitudes \( (F^*) \) of the two bodies (nondimensionalized by \( \rho g B A \)) against the incident wave frequency for three draft ratios \( (h_2/h_1 = 0.5, 1.0 \) and \( 2.0) \) are plotted in Figure 12. To better examine the influence of the floating body draft on the horizontal force acting on the floating body, the profile of the oscillation amplitude of the fluid in the narrow gap under different incident wave frequencies is also attached in each figure. \( F^*_1 \) and \( F^*_2 \) represent the dimensionless horizontal force amplitude of the floating body \( A_1 \) and \( A_2 \), respectively. In order to explain the mechanism of the horizontal forces, the theoretical estimation formula of the horizontal force \( F_{xg} \) by the fluid oscillation is derived here. It is assumed that the first-order free surface amplitude between the narrow gap is \( \eta(t) = \eta_A \cos(\omega t - \theta) \) and that the flow in the narrow gap is uniform. The first-order horizontal hydrodynamic force generated by the fluid oscillation in the narrow gap can be estimated by the Lagrange integral from the free surface to the target position:

\[
F_{xg} = \left[ \int_{-b_2}^{0} \rho g (\eta - y) dy - \int_{0}^{b_2} \rho g (-y) dy \right] + \int_{-b_2}^{0} (-\rho \frac{d\Phi}{dt})dy \\
= \left( \rho g h_2 \eta_A - \frac{\rho g h_2^2}{2} \right) \cos(\omega t - \theta)
\]

(14)

where \( \Phi = \eta y \) represents the fluid velocity potential of uniform flow. The dimensionless theoretical horizontal forces \( F_{xg}^* \) for three draft ratios are also plotted in Figure 12 for comparison.
(b) Horizontal hydrodynamic force profiles of floating bodies with draft ratio of 1.0.

(c) Horizontal hydrodynamic force profiles of floating bodies with draft ratio of 2.0

Figure 12. Comparisons of oscillation amplitude of narrow slot fluid and horizontal forces of floating bodies. (a) $h_2/h_1 = 0.5$, (b) $h_2/h_1 = 1.0$, (c) $h_2/h_1 = 2.0$.

When $h_2/h_1 = 0.5$ and 1.0 (the draft of floating body $A_1$ is not smaller than that of floating body $A_2$), the curve of $F_2^*$ almost overlaps with the curve of $F_{xg}^*$, indicating that the horizontal forces of the body are almost completely induced by the fluid oscillation in the gap. When $h_2/h_1 = 2$ (the draft of floating body $A_1$ is smaller than that of floating body $A_2$), it can be observed that the horizontal forces of the floating body $A_2$ are no longer close to but are slightly larger than those of the theoretical values, implying that the horizontal forces of the floating body $A_2$ are not completely due to the fluid oscillation in the gap in this situation. This is because, when the draft of the floating body on the wave side is smaller than the draft of floating body on the back wave side, the incident wave may pass under the body on the wave side and contribute to the horizontal forces of the body with the larger draft ($A_2$). As the horizontal forces of the body $A_2$ are dominated by the fluid oscillation in the gap, it can be clearly seen that the variation trend of $F_2^*$ is similar to that of the oscillation amplitude of the fluid in the gap.
The oscillation amplitude of the fluid in the gap increases when the draft ratio $h_2/h_1$ increases. Therefore, it can be clearly observed that the horizontal forces of the floating body $A_2$ increase with the draft ratio $h_1/h_2$. For the floating body $A_1$, both the incident wave and the fluid oscillation in the gap would contribute to its horizontal force. It can be seen that the horizontal forces of the body $A_1$ are much larger than that of the floating body $A_1$ at $h_2/h_1 = 0.5$. This is because the fluid oscillation amplitudes are low at this situation. When $h_2/h_1$ increases, it can be seen that $F_1^*$ and $F_2^*$ become comparable. In particular, the values of $F_2^*$ are even larger than those of $F_1^*$ at low frequencies.

4. Conclusions

This study presented an investigation of the effect of the draft ratio of floating bodies on the fluid oscillation in a narrow gap using the viscous fluid theory and VOF model. Numerical simulations were constructed by the Waves2Foam package based on the OpenFOAM solver. To verify the viscous fluid theory and VOF model in the Waves2Foam package, comparisons of the numerical results with the experimental data were made. It was found that, for two bodies at $h_2/h_1 = 1$, the numerical results agree well with the experimental and theoretical results, verifying the accuracy of the Waves2Foam codes.

The results revealed that, within the water depth range investigated in the present study, the depth of the wave tank has a significant effect on the numerical results. As the depth of the wave tank increases, the oscillation amplitude of the narrow-gap fluid largely decreases and the resonant frequency of the fluid oscillation in the narrow gap increases. It was also found that the draft ratio of floating bodies has a significant nonlinear influence on the resonant frequency and on the oscillation amplitude of the fluid in the narrow gap. With an increase in the draft of either the floating body on the wave side or the one on the back wave side, the resonant frequency decreases. The increase in the draft of the floating body on the wave side causes an increase in the reflection wave coefficient and lead to a drop in the fluid oscillation amplitude, and the increase in the draft of the floating body on the back wave side triggers a decrease in the reflection wave coefficient and results in an increase in the fluid oscillation amplitude. Meanwhile, the viscous dissipation induced by the fluid viscosity synchronously increases with the increasing oscillation amplitude of the fluid in the gap.

Moreover, the fluid oscillation in the narrow gap significantly affects the longitudinal forces of the floating bodies. The longitudinal forces of the floating body on the back wave side are mainly affected by the draft ratio and the largest force increases with the draft ratio. Finally, the vortex evolutions are analyzed to interpret the variation of the oscillation amplitude of the fluid in the gap.

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