On Non-singlets in Kaon Production in Semi-inclusive DIS reactions

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Abstract. We consider semi-inclusive unpolarized DIS for the production of charged kaons and the different possibilities, both in LO and NLO, to test the conventionally used assumptions $s - \bar{s} = 0$ and $D_{K^+ - K^-} = 0$. The considered tests have the advantage that they do not require any knowledge of the fragmentation functions.

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INTRODUCTION

It is well known that inclusive deep inelastic scattering (DIS) yields information only about quark plus antiquark parton densities. Semi-inclusive DIS (SIDIS), where some final hadron $h$ is detected, plays an essential role to obtain separate knowledge about quark and antiquark densities, but it requires a knowledge of the fragmentation function (FF) for a given parton to fragment into the relevant hadron. As pointed out in [1] and more recently in [2] a precise knowledge of the FFs is vital.

When the spin state of the detected hadron is not monitored, it is possible to learn about the FFs from, both, $e^+ e^- \rightarrow hX$ and unpolarized SIDIS $l + N \rightarrow lhX$. In the case of pion production $SU(2)$ plays a very helpful role in reducing the number of independent FFs needed. For charged kaon production, which is important for studying the strange quark densities, $SU(2)$ is less helpful, and even a combined analysis of $e^+ e^-$ and SIDIS data on both protons and neutrons does not allow an unambiguous determination of the kaon FFs [3].

It is thus conventional to make certain reasonable sounding assumptions about the strange quark densities and the kaon FFs. The usually made assumptions in analyzing the data are $s(x) = \bar{s}(x)$ and $D_{K^+ - K^-}(z) = D_{K^+ - K^-}(z)$. In this paper we discuss to what extend these assumptions can be justified and tested experimentally in, both, LO and NLO in QCD. We examine possible tests for the reliability of a leading order (LO) treatment of the considered processes.

UNPOLARIZED SIDIS $e + N \rightarrow e + h + X$

In semi-inclusive deep inelastic scattering we consider the non-singlet difference of cross-sections $\bar{\sigma}_N^{h-h}$. The measurable quantity is the ratio of semi-inclusive $\bar{\sigma}_N^{h-h}$ and
inclusive $\bar{\sigma}^{DIS}_N$ deep inelastic lepton-nucleon scattering:

$$R_N^{h-\bar{h}} = \frac{\bar{\sigma}_N^{h-\bar{h}}}{\bar{\sigma}^{DIS}_N}, \quad \bar{\sigma}_N^{h-\bar{h}} = \bar{\sigma}_N^h - \bar{\sigma}_N^{\bar{h}}. \quad (1)$$

For simplicity, we use $\bar{\sigma}_N^h$ and $\bar{\sigma}^{DIS}_N$ in which common kinematic factors have been removed \cite{4}. For $\bar{\sigma}^{DIS}_N$ any of the parametrizations for the structure functions $F_2$ and $R$ or, equivalently, any set of the unpolarized parton densities can be used.

As shown in \cite{3}, the general expression for the cross section differences in NLO, is:

$$\hat{\sigma}_{qq}(x,z) = \frac{1}{9} \left[ 4uv \otimes D^{h-\bar{h}}_u + dv \otimes D^{h-\bar{h}}_d + (s-\bar{s}) \otimes D^{h-\bar{h}}_s \right] \otimes \hat{\sigma}_{qq}(\gamma q \rightarrow qX)$$

$$\hat{\sigma}_N^{h-\bar{h}}(x,z) = \frac{1}{9} \left[ 4dv \otimes D^{h-\bar{h}}_u + uv \otimes D^{h-\bar{h}}_d + (s-\bar{s}) \otimes D^{h-\bar{h}}_s \right] \otimes \hat{\sigma}_{qq}(\gamma q \rightarrow qX). \quad (2)$$

Here $\hat{\sigma}_{qq}$ is the perturbatively calculable, hard partonic cross section $q\gamma^* \rightarrow q + X$:

$$\hat{\sigma}_{qq} = \hat{\sigma}_{qq}^{(0)} + \frac{\alpha_s}{2\pi} \hat{\sigma}_{qq}^{(1)}, \quad (3)$$

normalized so that $\hat{\sigma}_{qq}^{(0)} = 1, D^{h-\bar{h}}_q \equiv D^h_q - D^{\bar{h}}_q$.

It is seen that $\hat{\sigma}_N^{h-\bar{h}}$ involves only NS parton densities and fragmentation functions, implying that its $Q^2$ evolution is relatively simple. Eq.\,(2) is sensitive to the valence quark densities, but also to the completely unknown combination $(s-\bar{s})$. The term $(s-\bar{s})D^{h-\bar{h}}_s$ plays no role in pion production, since, by SU(2) invariance, $D^{\pi^+\pi^-}_s = 0$. However it is important for kaon production, for which $D^{K^+K^-}_s$ is a favoured transition, and thus expected to be big.

Up to now all analyses of experimental data assume $s = \bar{s}$. In the next Sections we shall consider the production of charged kaons, $h = K^\pm$ and show how this assumption, and the assumption $D^{K^+K^-}_d = 0$, can be tested without requiring knowledge of the FFs.

SU(2) symmetry is of little help if only charged kaons are measured. However, it is well known that charged and neutral kaons are combined into SU(2) doublets. This relates the FFs of $K^0$ to those of $K^\pm$, which implies that no new FFs appear in $K^0$-production. In \cite{5} we examine to what extend detecting neutral as well as charged kaons can help to determine the kaon fragmentation functions. We carry out the analysis in LO and NLO. We show that the non-singlet combination $(D_u - D_d)K^+K^-$ can be measured directly both in $e^+e^-$ and in SIDIS without any influence of the strange and gluon parton densities or any other FFs and this allows tests of the factorization of SIDIS into parton densities and fragmentation functions in any order in QCD.

PRODUCTION OF CHARGED KAONS

As seen from \cite{2}, in $R_N^{K^+K^-}$ both $s - \bar{s}$ and $D^{K^+K^-}_d$ appear. They are expected to be small, and the usual assumption is that they are equal to zero. Here we examine to what extent one can test these assumptions experimentally in SIDIS.
Up to now, all analysis of experimental data have been performed assuming both \( s = \bar{s} \) and \( D_d^{K^+ - K^-} = 0 \). Note, that from the quark content of \( K^\pm \), the assumption \( D_d^{K^+ - K^-} = 0 \) seems very reasonable if the \( K^\pm \) are directly produced. However, if they are partly produced via resonance decay this argument is less persuasive.

**LO approximation**

In LO we have:

\[
\sigma_p^{K^+ - K^-} = \frac{1}{9}[4uV D_u^{K^+ - K^-} + dv D_d^{K^+ - K^-} + (s - \bar{s}) D_s^{K^+ - K^-}], \tag{4}
\]

\[
\sigma_n^{K^+ - K^-} = \frac{1}{9}[4dv D_u^{K^+ - K^-} + uv D_d^{K^+ - K^-} + (s - \bar{s}) D_s^{K^+ - K^-}]. \tag{5}
\]

From a theoretical point of view it is more useful to consider the following combinations of cross-sections, which, despite involving differences of cross-sections, are likely to be large:

\[
(\sigma_p - \sigma_n)^{K^+ - K^-} = \frac{1}{9}[(uV - dV)(4D_u - D_d)]^{K^+ - K^-} \tag{6}
\]

\[
(\sigma_p + \sigma_n)^{K^+ - K^-} = \frac{1}{9}[(uV + dV)(4D_u + D_d) + 2(s - \bar{s})D_s^{K^+ - K^-}] \tag{7}
\]

We define:

\[
R_+(x, z) \equiv \frac{(\sigma_p + \sigma_n)^{K^+ - K^-}}{uV + dV}, \quad R_-(x, z) \equiv \frac{(\sigma_p - \sigma_n)^{K^+ - K^-}}{uV - dV}. \tag{8}
\]

From a study of the \( x \) and \( z \) dependence of these we can deduce the following:

1) if \( R_-(x, z) \) is a function of \( z \) only, then this suggests that a LO approximation is reasonable.

2) if \( R_+(x, z) \) is also a function of \( z \) only, then, since \( D_s^{K^+ - K^-} \) is a favoured transition, we can conclude that \( s - \bar{s} = 0 \).

3) if \( R_+(x, z) \) and \( R_-(x, z) \) are both functions of \( z \) only, and if in addition, \( R_+(x, z) = R_-(x, z) \), then both \( s - \bar{s} = 0 \) and \( D_d^{K^+ - K^-} = 0 \).

4) if \( R_+(x, z) \) and \( R_-(x, z) \) are both functions of \( z \) only, but they are not equal, \( R_+(x, z) \neq R_-(x, z) \), we conclude that \( s - \bar{s} = 0 \), but \( D_d^{K^+ - K^-} \neq 0 \).

5) if \( R_-(x, z) \) is not a function of \( z \) only, then NLO corrections are needed, which we consider below.

The above tests for \( s - \bar{s} = 0 \) and \( D_d^{K^+ - K^-} = 0 \) can be spoilt either by \( s - \bar{s} \neq 0 \) and/or \( D_d^{K^+ - K^-} \neq 0 \), or by NLO corrections, which are both complementary in size. That’s why it is important to formulate tests sensitive to \( s - \bar{s} = 0 \) and/or \( D_d^{K^+ - K^-} = 0 \) solely, i.e. to consider NLO.
NLO approximation

If an NLO treatment is necessary it is still possible to reach some conclusions, though less detailed than in the LO case. We now have:

\[
(\tilde{\sigma}_p - \tilde{\sigma}_n)^{K^+ - K^-} = \frac{1}{9} (u_V - d_V) \otimes (1 + \alpha_s C_{qq}) \otimes (4D_u - D_d)^{K^+ - K^-} \tag{9}
\]

\[
(\tilde{\sigma}_p + \tilde{\sigma}_n)^{K^+ - K^-} = \frac{1}{9} \left[ (u_V + d_V) \otimes (4D_u + D_d)^{K^+ - K^-} \right. \\
\left. + 2(s - \bar{s}) \otimes D_s^{K^+ - K^-} \right] \otimes (1 + \alpha_s C_{qq}) \tag{10}
\]

Suppose we try to fit both (9) and (10) with one and the same fragmentation function \(D(z)\):

\[
(\tilde{\sigma}_p - \tilde{\sigma}_n)^{K^+ - K^-} \approx \frac{4}{9} (u_V - d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D(z), \tag{11}
\]

\[
(\tilde{\sigma}_p + \tilde{\sigma}_n)^{K^+ - K^-} \approx \frac{4}{9} (u_V + d_V) \otimes (1 + \alpha_s C_{qq}) \otimes D(z). \tag{12}
\]

If this gives an acceptable fit for the \(x\) and \(z\)-dependence of both \(p - n\) and \(p + n\) data, we can conclude that both \(s - \bar{s} \approx 0\) and \(D_d^{K^+ - K^-} \approx 0\), and that \(D(z) = D_u^{K^+ - K^-}\).

Note that for all above tests, both in LO and NLO approximation, we don’t require a knowledge of \(D_u^{K^+ - K^-}\). This is especially important since the \(e^+ e^-\) total cross section data determine only the \(D_q^{K^+ + K^-}\), and these are relatively well known, while \(D_u^{K^+ - K^-}\) can be determined solely from \(A_{FB}\) in \(e^+ e^-\) or from SIDIS.

The results of the above tests would indicate what assumptions are reliable in trying to extract the fragmentation functions \(D_{u,d,s}^{K^\pm}\) from the same data.

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REFERENCES

1. S. Kretzer, E. Leader and E. Christova, *Eur.Phys.J.* C22 269-276 (2001)
2. D. de Florian, G.A. Navaro and R. Sassot, *Phys. Rev.* D71 (2005) 094018
3. E. Christova and E. Leader, *Proceedings of XI-th workshop on high energy physics, Dubna-SPIN-2005* (Russia), (Preprint: hep-ph/0512075)
4. E. Leader and E. Christova, *Nucl. Phys.* B607, 369-390 (2001)
5. E. Leader and E. Christova, *preprint* hep-ph/0612049