Early structure formation with cold plus hot dark matter — a success of strings plus inflation models

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Quantum fluctuations created during inflation can account for the observed matter distribution in the linear regime if the universe has two components of dark matter, one which is cold and collisionless, and the other which is hot and free streams on small scales. However, this free streaming property of the hot component prevents early structure formation, and since objects, such as damped Lyman-α systems, have been observed at high redshift, it is necessary to produce more power on small scales. Here, we show that the situation can be improved substantially in models where cosmic strings are formed at the end of inflation, and in which both inflation and strings participate in the generation of structure.

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During the next decade various high precision measurements will enhance our understanding of the universe immeasurably. Two satellites (MAP and PLANCK) will map the cosmic microwave background (CMB) over a wide range of scales and ground-based redshift surveys (for example, Sloan Digital Sky Survey and 2Df) should measure the clustering of luminous matter. Using these complementary measurements — as well as many others (for example, those of Type Ia supernovae and light element abundances) — it is hoped that robust estimates can be made for a large number of cosmological parameters. However, underlying this ambitious enterprise is the origin of the primordial fluctuations, which is usually assumed to be a near scale invariant spectrum of adiabatic density perturbations created by quantum effects during some de-Sitter phase associated with cosmic inflation.

Under various assumptions as to the matter content of the universe three broad classes of models based on inflation have been suggested to fit the current observations of galaxy clustering in the linear regime ($λ > 30h^{-1}$Mpc) when normalized to the amplitude of anisotropies in the CMB detected by the COsmic Background Explorer (COBE) satellite (see, for example, ref. [2]). Each of these models is based primarily on a universe whose matter density is dominated by cold dark matter (CDM) particles ($Ω_m ∼ Ω_c$), such as neutralinos or axions, with a much smaller component of baryons as predicted by Big Bang Nucleosynthesis ($Ω_b ≈ 0.05 − 0.1$). At present, possibly the most popular of these classes of models have a matter density which is less than critical ($Ω_m ≈ 0.3 − 0.5$), with in one class ($Λ$CDM) the shortfall from critical being made up by a non-zero cosmological constant ($Ω_Λ + Ω_m = 1$) and the other (OCDM) having an open topology. The final class of models (CHDM) [3] have $Ω_m = 1$ with an extra relativistic or hot dark matter (HDM) component ($Ω_α ≈ 0.2 − 0.3$), such as massive neutrinos, which, since they free-stream, reduce the amount of power on intermediate and small scales, allowing the model to fit the shape of the observed power spectrum in the linear regime. Reconciling the predictions of this cold plus hot dark matter model with observations at high redshift is the subject of this letter.

The relevant observations concerning us are those of damped Lyman-α (DLYA) systems [3] which are thought to probe structure as it was for $z > 2$ in the linear regime, and on scales smaller than otherwise possible. There seems to be a conflict between the measured neutral gas fraction, particularly at very high redshifts ($z ≈ 4$), and the predictions made for CHDM models [4,5], which — due to the free-streaming property of the HDM component — tend to lack of power on such small scales. This is one of the main reasons why, in spite of the superior achievements of CHDM models on linear scales, OCDM and ACDM models are now preferred.

For many years the most popular alternative to the standard adiabatic scenarios discussed above has been to form structure by accretion of matter onto topological defects (for a review, see ref. [6]). After much recent work [7–10], it appears that, at least when $Ω_m = 1$, these models appear to suffer from the converse problem; they have a tendency to produce insufficient power on large scales to match the current observations when normalized to COBE, but are known to produce copious amounts of small scale power [8,11], even in scenarios with a HDM component [12]. This is particularly true in the case of cosmic strings and one may wonder if the complementary failings of CHDM based on inflation and string models could be erased in a mixed string plus inflation CHDM model.

The possibility that cosmic structure could be due to both strings and inflation [12] is a very real one which has been the topic of much recent research [13–14], since it could arise in a number of well motivated inflationary scenarios [16–20]. The details of this paper are only
weakly sensitive to the specifics of the inflationary model and then only in an essentially predictable way. However, to give an example, let us consider D-term inflation (see ref. [17] for a review). These models make use of supersymmetry in order to provide cosmology with a flat potential, as required by inflation, without the need for a finely tuned coupling constant. During inflation, the inflaton moves along this flat direction, which is a direct result of the U(1) symmetries in the model, and at the end of inflation the underlying symmetry becomes broken, producing cosmic strings via the Kibble mechanism during the subsequent phase transition.

In any string plus inflation scenario the large angle CMB anisotropies, which are normalized to COBE, are due both to strings and inflation with a model dependent ratio between the two; it being a non-trivial function of the number of e-foldings during inflation, the inflationary energy scale and that for the symmetry breaking phase transition which produced the strings. Given a specific model it would be possible to relate this to parameters of the model as was done in refs. [18,19]. In both cases it was shown that any value of this ratio was possible for sensible model parameters and therefore for the purposes of this paper we will leave it arbitrary. In particular we will use the notation of ref. [18], where the power spectra were added as, for example, \( P(k) = \alpha P^\text{inf}(k) + (1 - \alpha) P^\text{str}(k) \), where \( 0 \leq \alpha \leq 1 \) and \( P^\text{inf}(k), P^\text{str}(k) \) are individually computed spectra for the inflation and string models normalized to COBE. The specific spectra that we shall use have been computed using CMBFAST [20] and in the string case the model for the defect stress energy used in refs. [18,19]. This has been shown to represent many of the features of a realistic string network, although it may under estimate the amount of small-scale power produced [21]. Any extra small-scale power will further improve the situation relative the observations being discussed here.

The neutral gas fraction in DLYA systems has been computed using a variety of techniques for the standard CHDM scenario based on inflation. Initially [8,15] the Press-Schechter (PS) approximation [23] was used to make an estimate and more recently [24–27] hydrodynamical simulations have improved upon this, although the basic picture has remained the same. In attempting to estimate the size of the effect of including a string component one is faced with a number of obstacles to making a quantitative prediction, mainly related to the non-Gaussian nature of the fluctuations involved. This makes creating a realization of initial conditions for a hydrodynamic simulation almost impossible without an accurate simulation of the defect network and of course one of the basic premises of the PS formalism is that the fluctuations initially form a Gaussian random field. It has been suggested [28] that a simple generalization of the PS approximation can be made by replacing the Gaussian probability distribution function (PDF) with that computed from a simulation and, indeed, it has been shown that such a modified theory yields good predictions for particular PDFs using N-body simulations [29]. A PDF has been computed for the pure string component [29,30] showing that it has positive skewness. In the mixed scenarios considered here, the total PDF is the convolution of the individual components, and so the total cumulants are the sum of the individual cumulants. Hence, the combined PDF will also have positive skewness.

Here, we will repeat the standard PS calculation assuming Gaussian initial conditions following closely the calculation of ref. [31]. Since the measured skewness for the cosmic strings PDF is positive, one might expect that this calculation would act as lower bound on the gas fraction created in a given scenario. However, as we shall see the effects of non-Gaussianity are subtle and this is not always the case. To illustrate this we will use a simple non-Gaussian distribution with a PDF which is a log-normal distribution, as suggested in ref. [29].

DLYA systems are observed as wide absorption troughs in the spectra of distant quasars. These troughs indicate that the line of sight to the quasar intersects a region of neutral hydrogen (HI) with a column density \( \gtrsim 10^{20}\text{cm}^{-2} \). For high redshift objects (\( z \geq 2 \)) there are hints, that these absorption lines are produced in turbulent protospheroids [32], which are the natural progenitors of galaxies. The measured abundance of damped Lyman-\( \alpha \) systems at redshift \( z = 4 \) [33] implies that the neutral gas density \( \Omega_{\text{gas}} \) in units of the critical density is \( \Omega_{\text{gas}} (z \approx 4) h = (9.3 \pm 3.8) \times 10^{-4} \), in a universe which has \( \Omega_{\text{m}} = 1 \), where the Hubble constant is parametrized in the usual way \( H_0 = 100 h \text{ km sec}^{-1} \text{ Mpc}^{-1} \).

The neutral gas fraction in DLYA systems is given by \( \Omega_{\text{gas}} = f_{\text{HI}} \Omega_{\text{b}} \text{erfc} \left( \delta_c/\sqrt{2} \sigma_R \right) \), which is the result of the integration over the tail of the underlying Gaussian PDF. In this expression \( 0 < f_{\text{HI}} \leq 1 \) is the fraction of hydrogen which is neutral, \( \Omega_{\text{b}} \) is the average baryon density relative to critical, \( \delta_c (\approx 1.69) \) is the density contrast required for collapse to take place and \( \sigma_R \) is the rms density fluctuation in a ball with size of the DLYA systems. To estimate this scale we assume that the systems are protospheroids which are about to collapse to rotationally supported gaseous discs. In this spherical collapse scenario [34] the comoving size of the system is related to the circular velocity, \( V_c \), by

\[
R = 86h^{-1}\text{kpc} \left( \frac{V_c}{50\text{km sec}^{-1}} \right) \left( \frac{5}{1 + z} \right)^{1/2}.
\]

A typical velocity is \( V_c \sim 50 \text{ km sec}^{-1} \), which corresponds to the minimal mass (\( \sim 10^{10}h^{-1}\text{M}_\odot \)) needed for spherical collapse to take place [35]. The rms fluctuations on this scale, \( \sigma_R \), are computed in the standard way by integration over a window function corresponding to a spherical top-hat of size \( R \).

In fig. 2 we have plotted \( \Omega_{\text{gas}}/f_{\text{HI}} \) at \( z = 4 \) computed
assumed Gaussian fluctuations (solid line) as a function of \( \alpha \) for models with \( \Omega_\nu = 0.2 \) and \( h = 0.5 \) — those parameters which were suggested to give the best fit to the observed galaxy clusterings on large scales in ref. [1]. Simulations suggest that realistically one might expect \( f_{\text{HI}} \approx 0.1 \) [25,26], but if one were wanting to be conservative in ruling out a scenario, \( f_{\text{HI}} = 1 \) can be used to compute an absolute upper bound on \( \Omega_{\text{gas}} \). If \( \alpha = 1 \), that is, just adiabatic fluctuations, then one sees that for all the values of \( V_c \) used, the upper bounds are compatible with the observations; the larger values of \( V_c \) producing more neutral hydrogen than lower ones. However, if \( f_{\text{HI}} = 0.1 \) none of these are compatible with the value of \( \Omega_{\text{gas}} \) detected and this can be seen to be true for a wide range of cosmological parameters [1]. Even more striking from this figure is that the observed value can be achieved very easily by the inclusion of only a very small string component \( \alpha \approx 0.8 \), and in fact the predictions could be made compatible with even smaller values of \( f_{\text{HI}} \).

A simple non-Gaussian distribution which has been suggested to represent the PDF in cosmic string models [23] is the log-normal distribution given by

\[
p_\lambda(y) = \frac{C}{\sqrt{2\pi}A^2} \exp\left[ -\frac{1}{2} x^2(y) - Ax(y) \right], \tag{2}
\]

for \( y > -C/B \) and zero otherwise, where \( Ax(y) = \log(Cy + B), B = \exp(A^2/2) \) and \( C = \sqrt{B^2 - B^2} \). This has a single parameter \( A(>0) \), with the limiting case \( A = 0 \) corresponding to a Gaussian distribution. In this case, one can deduce that the neutral gas fraction is given by

\[
\Omega_{\text{gas}} = f_{\text{HI}} \Omega_\nu \text{erfc}\left[ \frac{1}{A\sqrt{2}} \log\left( \frac{\delta}{\sigma_R} + B \right) \right] / \text{erfc}\left[ \frac{A}{2\sqrt{2}} \right]. \tag{3}
\]

It was suggested in ref. [29] that \( A \approx 0.17 \) on cluster scales \( (R \sim 10h^{-1}\text{Mpc}) \), but on the smaller scales under consideration here the value is likely to be somewhat larger [30], say \( A \approx 0.5 \). We have also included computations of \( \Omega_{\text{gas}} \) for a model using this PDF and \( A = 0.5 \) in fig. 1 to illustrate the effects of non-Gaussianity. Intuitively, one might have expected the effects of introducing non-Gaussianity to increase \( \Omega_{\text{gas}} \), but it appears this is not always the case. Using a non-Gaussian PDF does enhance the creation of more rare objects, that is \( \sigma_R \gg \delta_c \). However, if \( \sigma_R \sim \delta_c \), as can be the case when \( \alpha \) is small, then the overall normalization, which allows the integration over the whole PDF account for all the matter in the universe, is reduced.

FIG. 1. The neutral gas density \( \Omega_{\text{gas}}/f_{\text{HI}} \) at \( z = 4 \) plotted as a function of \( \alpha \) for a CHDM model with \( \Omega_\nu = 0.2 \). The three curved shaded regions are bounded by the Gaussian PDF (solid line) and a non-Gaussian PDF with \( A = 0.5 \) (dotted line) and correspond to the region in which the actual value for the mixed scenario actually lies. The circular velocities of the three regions are given by \( V_c = 750\text{km sec}^{-1} \) (top), \( 500\text{km sec}^{-1} \) (middle) and \( 25\text{km sec}^{-1} \) (bottom). The parallel shaded regions correspond to the observational limits if \( f_{\text{HI}} = 1.0 \) (bottom) and \( f_{\text{HI}} = 0.1 \) (top). Recall that \( \alpha = 1 \) corresponds to pure adiabatic perturbations and \( \alpha = 0 \) to strings only. Note that the non-Gaussian PDF gives a lower value, albeit only slightly, than the Gaussian case for certain values of \( \alpha \). This non-intuitive effect is explained in the text.

FIG. 2. The same quantities as plotted in fig. 1 but for \( \Omega_\nu = 0.3 \). Note that none of the models are even compatible with observations of \( \Omega_{\text{gas}} \) for \( \alpha = 1 \), and that the inclusion of the effects of non-Gaussianity is now to increase \( \Omega_{\text{gas}} \) in contrast to the case of \( \Omega_\nu = 0.2 \).

Fig. 2 contains the equivalent information for the larger value of \( \Omega_\nu = 0.3 \) (and now \( \Omega_\nu = 0.6 \)). Clearly, such values of \( \Omega_\nu \) are incompatible with the observations if \( \alpha = 0 \) since even the upper bound on \( \Omega_{\text{gas}} \) is woefully short of the observed gas fraction. However, for string-inflation admixtures of around 50% each the upper bound on the gas fraction becomes within range of the observations if one assumes Gaussian fluctuations, albeit with \( f_{\text{HI}} > 0.1 \). The effects of non-Gaussianity are now more intuitive, since \( \sigma_R \ll \delta_c \) and the DLYA systems can be thought of as being rare objects once again.
Therefore, we have shown that CHDM model can be made consistent with high redshift observations, if we are prepared to resort to well motivated strings plus inflation scenarios. The physical origin of this result is the extra fluctuations on small scales, due to the strings, but also non-Gaussianity can play an important role. If the DLYA systems can be thought of as rare objects ($\delta_c >> \sigma_R$), then this can enhance their production relative to Gaussian. But if they are formed as 1$\sigma$ fluctuations then their production is slightly suppressed in non-Gaussian models. It is clear that the effects of non-Gaussianity will be more prevalent at early times when $\sigma_R$ is small and that less objects will formed later as $\sigma_R$ increases. This has interesting implications for the epoch of galaxy and cluster assembly [36].

To conclude we have suggested that the CHDM scenario, and more importantly $\Omega_m = 1$, can be resuscitated by the inclusion of a string component since this allows structure to be formed earlier than in the pure adiabatic case. Apart from the flaw of the CHDM models which we have discussed here, there are other observations which favour the $\Lambda$CDM model and OCDM models mainly because they have $\Omega_m < 1$. It would be interesting to understand how these observations would be modified for the non-Gaussian theories discussed here. It is clear that dynamical measures, such as the evolution of X-ray cluster abundance, could all be made compatible with $\Omega_m = 1$ is a sufficiently non-Gaussian model. Work on the observational status of this class of models continues.

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