Grand Unification without Proton Decay

Andreas Mütter,1 Michael Ratz,1 and Patrick K.S. Vaudrevange1,2

1Physik Department T30, Technische Universität München,
James–Franck–Straße, 85748 Garching, Germany
2Excellence Cluster Universe, Technische Universität München,
Boltzmannstr. 2, D-85748, Garching, Germany
(Dated: Thursday 9th June, 2016)

It is commonly believed that grand unified theories (GUTs) predict proton decay. This is because
the exchange of extra GUT gauge bosons gives rise to dimension 6 proton decay operators. We show
that there exists a class of GUTs in which these operators are absent. Many string and supergravity
models in the literature belong to this class.

INTRODUCTION

Grand unified theories (GUTs) are a hypothetical framework that unifies three out of four known
forces, electromagnetism, the weak force and strong interactions in a larger symmetry group
$G_{GUT}$. The perhaps greatest virtue of GUTs is that they provide a compelling explanation of
the structure and quantum numbers of standard model (SM) matter. In SU(5), the lepton doublets
$\ell$ and $d$–type quarks $d$ get combined in 5–plets while the remaining three representations of a
generation, i.e. $q$, $\bar{u}$ and $\bar{e}$, transform as a 10–plet. It is commonly believed that the most compelling
“smoking gun” signature of GUTs is proton decay. This is because GUTs are endowed with extra “X”
gauge bosons from the coset SU(5)/$G_{SM}$, where $G_{SM} = SU(3)_C \times SU(2)_L \times U(1)_Y$ is the SM
gauge group. These gauge bosons mediate transitions between $\ell$ and $d$ as well as $q$, $\bar{u}$ and $\bar{e}$, thus
inducing dimension 6 proton decay operators (cf. Figure 1).

This mode can be eliminated by either the Babu–Barr mechanism [7] or by going to higher
dimensions. Specifically, in the so–called orbifold GUTs [8, 9], this mode is automatically absent
[10]. As is well known, the dimension 4 operators can be forbidden by $R$ parity [11]. One is then
left with the dimension 6 proton decay, which is induced by gauge bosons transforming in the coset
SU(5)/$G_{SM}$, and thus believed to be the smoking gun signature of unification. The purpose of this
Letter is to show that these dimension 6 operators are also absent in a class of grand unified
theories in which the GUT symmetry gets broken non–locally in extra dimensions.

HIGHER–DIMENSIONAL MODELS OF
GRAND UNIFICATION

Higher–dimensional models of grand unification decompose into two classes, models with local
and models with non–local GUT symmetry breaking. Models with local GUT symmetry breaking

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{proton_decay_operator_dim6}
\caption{Proton decay operator of dimension 6.}
\end{figure}
cations of the heterotic string [12–21] (see 21–
23 for reviews) as well as orbifold GUTs. On
the other hand, models with non–local GUT sym-
metry breaking include Calabi–Yau compactifica-
tions (see 24–25) for models that come very close
to the SM as well as field–theoretic constructions 30–33 and some orbifold compactification
of the heterotic string [34, 35], which have only
been explored more recently.

To illustrate our main points, let us start by
looking at one extra dimension, which is
parametrized by $y \in F = [0, 2\pi R]$. Take an SU(5)
GUT where the gauge bosons are free to propa-
gate in a fifth dimension. Furthermore, we assume
$n_G$ generations of quarks and leptons transform-
ing as $\mathbf{5} \oplus \mathbf{10}$. In our discussion, we focus on
the matter $\mathbf{5}$–plets of the first two generations and
denote them by $\Psi_i(x, y) = \left( \ell_i(x, y), \bar{d}_i(x, y) \right)$ $(i = 1, 2)$. (1)

Local GUT breaking

We first discuss models with “local breaking”,
where $n_G = 3$, using the example of an $\mathbb{S}^1/(\mathbb{Z}_2 \times
\mathbb{Z}_2)$ orbifold [8, 9]. Here the SM matter is local-
ized at fixed points with a GUT symmetry (see
Figure 3). Therefore, the fields have to appear in
complete GUT multiplets. The GUT symmetry
gets broken at the other fixed point, and the zero
modes of the $X$ bosons get projected out. There-
fore, the profile of the $X$ bosons is non–trivial at
the points at which matter is localized. As a con-
sequence, the Kaluza–Klein modes of the extra
gauge bosons $X \subset \text{SU}(5)/G_{\text{SM}}$ mediate between
d–type quarks and lepton doublets within each
$\Psi_i(x, y)$ field. That is, there are effective interac-
tions of the form

$$\mathcal{L}_{\text{eff}} \supset \int dy g_{\text{5D}} \sum_{i=1}^{n_G} \bar{t}_i(x, y) \gamma^\mu X_\mu(x, y) t_i(x, y),$$ (2)

where $g_{\text{5D}}$ denotes the 5D gauge coupling and $\gamma^\mu$
the 4D $\gamma$–matrices. Here the $X$ bosons have a
mass of the order of $M_{\text{GUT}}$.

In summary, in models with local GUT sym-
metry breaking the structure of SM matter gets
explained by a local GUT symmetry. Like con-
ventional four–dimensional GUT models, these
constructions predict dimension 6 proton decay

$$p \rightarrow e^+ \pi^0.$$ However, unlike in most of the con-
ventional SUSY GUTs, here the dimension 5 pro-
ton decay mode is absent.

Non–local GUT breaking

Let us now switch to settings in which the SU(5)
symmetry gets broken non–locally. To illustrate
our points, matter fields are now assumed to be lo-
calized in the fifth dimension, see Figure 4. Later,
when we present a stringy completion, we will dis-
cuss an orbifold compactification of the heterotic
string, where the corresponding states are local-
ized at some fixed planes. We start in the “up-
stairs picture” with a setting exhibiting $n_G = 6$
generations of quarks and leptons, and focus on
two generations of matter $\mathbf{5}$–plets $\Psi_i(x, y)$ for
$i = 1, 2$.

In the next step, we break the SU(5) GUT
group non–locally to $G_{\text{SM}}$ by a so–called freely–
acting $\mathbb{Z}_2$ with associated Wilson line. In our ex-
ample, the freely–acting $\mathbb{Z}_2$ acts as a translation
that, from a 5D point of view, identifies points
in the $y$-direction which differ by $\tau = \pi R$, i.e. $y \sim y + \tau$.

In addition to the geometrical $\mathbb{Z}_2$ action, due to the presence of the Wilson line, the SU(5) gauge bosons $A^\mu(x,y)$ are subject to the non-trivial boundary condition
\[ A^\mu(x,y) \mapsto A^\mu(x,y + \tau) \frac{1}{P} P A^\mu(x,y) P^{-1}. \]  
Here $A^\mu(x,y) = A^\mu_a(x,y) T_a$ with SU(5) generators $T_a$ and
\[ P = \text{diag}(-1,-1,1,1,1) \]  
with $P^2 = 1$. This $\mathbb{Z}_2$ boundary condition projects out the zero modes of the extra gauge bosons $X \subset SU(5)/G_{SM}$, and, hence, breaks SU(5) to $G_{SM}$.

In addition, the freely-acting translation by $\tau$ identifies the two fields $\Psi_1(x,y)$ and $\Psi_2(x,y)$. We therefore obtain a non-trivial $\mathbb{Z}_2$ boundary condition for the matter $\mathbf{5}$-plets $\Psi_i(x,y)$, i.e.
\[ \Psi_1(x,y) \sim \Psi_1(x,y + \tau) \frac{1}{P} P \Psi_2(x,y). \]  
Then, the $d$-type quark and the lepton doublet of the first SM generation are given by the $\mathbb{Z}_2$ invariant linear combinations
\[ \ell(x,y) = \frac{1}{\sqrt{2}} [\ell_1(x,y) - \ell_2(x,y)], \]  
\[ \overline{d}(x,y) = \frac{1}{\sqrt{2}} [\overline{d}_1(x,y) + \overline{d}_2(x,y)]. \]  
The orthogonal field directions
\[ \ell^{(\perp)}(x,y) = \frac{1}{\sqrt{2}} [\ell_1(x,y) + \ell_2(x,y)], \]  
\[ \overline{d}^{(\perp)}(x,y) = \frac{1}{\sqrt{2}} [\overline{d}_1(x,y) - \overline{d}_2(x,y)], \]  
are projected out in 4D. Thus, two “upstairs” generations of matter $\mathbf{5}$-plets are combined to the $d$-type quark and the lepton doublet of the first SM generation. Repeating these steps for $n_G = 6$ generations of $\mathbf{5} \oplus \mathbf{10}$ yields the SM with three generations. In particular, the matter still furnishes complete SU(5) representations!

The interactions of the first generation’s $d$-type quark and the lepton doublet with the $X$ boson, Equation (2), reads in the new field basis
\[ \mathcal{L}_{\text{eff}} \supset \int dy g_5 D \left[ \overline{\ell}(x,y) \gamma^\mu X_\mu(x,y) \ell^{(\perp)}(x,y) + \overline{d}^{(\perp)}(x,y) \gamma^\mu X_\mu(x,y) \overline{d}(x,y) \right]. \]  
There is no interaction of the physical $d$ quark and the lepton doublet, i.e. $\mathcal{L}_{\text{eff}} \not\supset T \gamma^\mu X_\mu \overline{d}$. Hence, the dimension 6 proton decay operator, which usually arises from integrating out the $X$ bosons in Figure 1, does not appear.

The absence of the dimension 6 proton decay operator can also be understood in terms of a $\mathbb{Z}_2$ symmetry that acts on quarks, leptons and the extra gauge bosons as
\[ \ell(x,y) \xrightarrow{\mathbb{Z}_2} + \ell(x,y), \]  
\[ \overline{d}(x,y) \xrightarrow{\mathbb{Z}_2} + \overline{d}(x,y), \]  
\[ X^\mu(x,y) \xrightarrow{\mathbb{Z}_2} - X^\mu(x,y). \]  
However, this symmetry does not imply that the profiles of the $X$ bosons vanish in the regions where SM matter lives.

**A STRINGY COMPLETION**

Let us now study a string-derived setup that realizes the scenario of non–locally broken GUTs discussed above. We consider a $\mathbb{T}^6/(\mathbb{Z}_2 \times \mathbb{Z}_2)$ orbifold compactification of the heterotic string as discussed in [37]. A more detailed description of the model will be presented elsewhere [38]. The actual orbifold model has six compact dimensions, but it is sufficient to study three of the extra dimensions $\vec{y} = (y_2, y_4, y_6)^T \in \mathbb{R}^3$ in order to understand the non-local breaking.

The $\mathbb{Z}_2 \times \mathbb{Z}_2$ twists $\theta$ and $\omega$ act as
\[ (y_2, y_4, y_6) \xrightarrow{\theta} (y_2, -y_4, -y_6), \]  
\[ (y_2, y_4, y_6) \xrightarrow{\omega} (-y_2, y_4, -y_6). \]  
In a first step, these twists act on an orthogonal three–torus $\mathbb{T}^3$ spanned by
\[ \vec{e}_2 = (2\pi R_2, 0, 0)^T, \]  
\[ \vec{e}_4 = (0, 2\pi R_4, 0)^T, \]  
\[ \vec{e}_6 = (0, 0, 2\pi R_6)^T. \]  
In a second step, we include a freely–acting translation
\[ \vec{\tau} = \frac{1}{2} (\vec{e}_2 + \vec{e}_4 + \vec{e}_6), \]  
which renders $\mathbb{T}^3$ non–factorizable, and yields a non–trivial fundamental group $\pi_1 = \mathbb{Z}_2$ for the resulting orbifold.

The model is constructed in such a way that prior to the action of $\vec{\tau}$ the SU(5) gauge bosons
\( A^\mu \) survive the orbifold projection. This amounts to requiring the boundary conditions

\[
A^\mu(x, \vec{y} + n_i \vec{e}_i) = A^\mu(x, \vec{y}),
\]

\( k = 0, \ell = 1 \) and \( n_2, n_6 \in \{0, 1\} \),

\[
A^\mu(x, \theta^k \omega^\ell \vec{y}) = A^\mu(x, \vec{y}),
\]

where \( n_i \in \mathbb{Z} \) and \( k, \ell \in \{0, 1\} \). Next, SU(5) is broken to \( G_{\text{SM}} \) by the freely–acting translation. In order to achieve this, we choose a gauge embedding of \( \vec{y} \), i.e. the Wilson line, such that the boundary condition for the extra gauge bosons \( X \subset \text{SU}(5)/G_{\text{SM}} \) reads

\[
X^\mu(x, \vec{y} + \vec{\tau}) = -X^\mu(x, \vec{y}),
\]

cf. Equation (3). In particular, this removes the zero modes of the \( X \) bosons.

The combined action of lattice translations and twists leaves planes fixed. These fixed planes are determined by

\[
\theta^k \omega^\ell \vec{y}_i + n_i \vec{e}_i = \vec{y}_i.
\]

There are twelve solutions of this equation,

\[
k = 0, \ell = 1 \quad \text{and} \quad n_2, n_6 \in \{0, 1\}, \quad (16a)
\]

\[
k = 1, \ell = 0 \quad \text{and} \quad n_4, n_8 \in \{0, 1\}, \quad (16b)
\]

\[
k = 1, \ell = 1 \quad \text{and} \quad n_2, n_4 \in \{0, 1\}, \quad (16c)
\]

which we label by \( \vec{y}^{(\alpha)} \), \( \alpha = 1, \ldots, 12 \). On these planes, one finds six generations of localized SM matter sitting in SU(5) multiplets. The \( X \) boson wave function, which satisfies (13) and (14), does not vanish on the fixed planes where the matter is located. Therefore, one may naively expect that interaction terms of the form of (2) induce gauge–mediated proton decay.

However, a universal feature of models with non–local GUT breaking is that there are \( 3n \) copies of matter in the upstairs picture. These get identified by the freely acting symmetry of order \( n \), thus reducing the number of generations to three. Specifically, in our construction, there are three pairs of matter fields sitting on three pairs of distinct fixed planes. Under the action of the freely–acting translation \( \vec{\tau} \), the fixed plane \( \vec{y}^{(\alpha)} \) gets mapped to \( \vec{\tilde{y}}^{(\alpha)} \),

\[
\vec{y}^{(\alpha)} \xrightarrow{\vec{\tau}} \vec{\tilde{y}}^{(\alpha)} \iff \vec{\tilde{y}}^{(\alpha)} = \vec{y}^{(\alpha)} + \vec{\tau}.
\]

Thus, \( \vec{y}^{(\alpha)} \) and \( \vec{\tilde{y}}^{(\alpha)} \) get identified, and, analogous to (6) only the \( \vec{\tau} \)–invariant linear combinations of the fields localized at \( \vec{y}^{(\alpha)} \) and \( \vec{\tilde{y}}^{(\alpha)} \) survive the projection conditions. This reduces the number of SM generations from six to three. The Wilson line is such that it reproduces \( \vec{\tau} \) for the SU(5) representations. Hence, the \( \vec{\tau} \)–invariant linear combinations for quarks and leptons come either with + or −, cf. (6). Consequently, as in the scenario discussed in the 5D toy–example, the dimension 6 proton decay operators do not get induced.

**SUMMARY**

We have discussed grand unified models in which the GUT symmetry gets broken non–locally, and found that there the dimension 6 proton decay operators are absent. Nevertheless, these settings do explain the structure of matter, i.e. the SM fermions are guaranteed to appear in complete GUT multiplets. That is to say, the constructions exhibit the main virtues of grand unified theories but do not predict proton decay, which was believed to be the smoking gun signal of grand unification.

As we have mentioned, many existing string and supergravity models belong to this class. In other words, the absence of proton decay does not concern some exotic type of constructions, but some of the most promising string compactifications known to date. Turning this around, one could say that proton decay experiments can give us invaluable insights on how the SM is completed in the ultraviolet. If the mode \( p \to K^+ \pi \) was observed, this would point towards four–dimensional SUSY GUTs. On the other hand, if this mode gets even further constrained, one is led to higher–dimensional models of grand unified theories or non–supersymmetric GUTs. If one would see the decay \( p \to e^+ \pi^0 \) with a rate of about \( 1/(10^{35} \text{yr}) \), this would strongly favor settings in which the GUT symmetry gets broken locally. However, if one does not observe this decay, this would point towards models of non–local grand unification.

**Acknowledgments**

We would like to thank Mu–Chun Chen, Stefan Groot Nibbelink and Andreas Trautner for useful discussions. M.R. would like to thank the Aspen Center for Physics and UC Irvine, where part of this work was done, for hospitality. This work was partially supported by the DFG cluster of excellence “Origin and Structure of the Universe”
www.universe-cluster.de). This research was done in the context of the ERC Advanced Grant project “FLAVOUR” (267104).

[1] H. Georgi and S. L. Glashow, Phys. Rev. Lett. 32, 438 (1974).
[2] H. Fritzsch and P. Minkowski, Ann. Phys. 93, 193 (1975).
[3] R. Dermišek, A. Mafi, and S. Raby, Phys. Rev. D63, 035001 (2001), hep-ph/0007213.
[4] H. Murayama and A. Pierce, Phys. Rev. D65, 055009 (2002), arXiv:hep-ph/0108104 [hep-ph].
[5] N. Sakai and T. Yanagida, Nucl. Phys. B197, 533 (1982).
[6] S. Dimopoulos, S. Raby, and F. Wilczek, Phys. Lett. B112, 133 (1982).
[7] K. S. Babu and S. M. Barr, Phys. Rev. D48, 5354 (1993), hep-ph/9306242.
[8] Y. Kawamura, Prog. Theor. Phys. 103, 613 (2000), hep-ph/9902243.
[9] Y. Kawamura, Prog. Theor. Phys. 105, 999 (2001), hep-ph/0012125.
[10] G. Altarelli and F. Feruglio, Phys. Lett. B511, 257 (2001), hep-ph/0102301.
[11] G. R. Farrar and P. Fayet, Phys. Lett. B76, 575 (1978).
[12] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Nucl. Phys. B261, 678 (1985).
[13] L. J. Dixon, J. A. Harvey, C. Vafa, and E. Witten, Nucl. Phys. B274, 285 (1986).
[14] L. E. Ibáñez, H. P. Nilles, and F. Quevedo, Phys. Lett. B187, 25 (1987).
[15] L. E. Ibáñez, J. E. Kim, H. P. Nilles, and F. Quevedo, Phys. Lett. B191, 282 (1987).
[16] J. A. Casas and C. Muñoz, Phys. Lett. B214, 63 (1988).
[17] A. Font, L. E. Ibáñez, F. Quevedo, and A. Sierra, Nucl. Phys. B331, 421 (1990).
[18] O. Lebedev, H. P. Nilles, S. Raby, S. Ramos-Sánchez, M. Ratz, P. K. S. Vaudrevange, and A. Wingrterer, Phys. Lett. B645, 88 (2007), hep-th/0611095.
[19] O. Lebedev, H. P. Nilles, S. Ramos-Sánchez, M. Ratz, and P. K. S. Vaudrevange, Phys. Lett. B668, 331 (2008), arXiv:0807.4384 [hep-th].
[20] H. P. Nilles and P. K. S. Vaudrevange, Mod. Phys. Lett. A30, 1530008 (2015), arXiv:1403.1597 [hep-th].
[21] F. Quevedo, (1996), 10.1063/1.49735 [AIP Conf. Proc.359,202(1996)], arXiv:hep-th/9603074.
[22] D. Bailin and A. Love, Phys. Rept. 315, 285 (1999).
[23] H. P. Nilles, S. Ramos-Sánchez, M. Ratz, and P. K. S. Vaudrevange, Eur. Phys. J. C59, 249 (2009), arXiv:0806.3905 [hep-th].
[24] V. Braun, Y.-H. He, B. A. Ovrut, and T. Pantev, JHEP 05, 043 (2006), hep-th/0512177.
[25] V. Bouchard and R. Donagi, Phys. Lett. B633, 783 (2006), hep-th/0512149.
[26] L. B. Anderson, J. Gray, Y.-H. He, and A. Lukas, JHEP 1002, 054 (2010), arXiv:0911.1569 [hep-th].
[27] L. B. Anderson, J. Gray, A. Lukas, and E. Palti, Phys. Rev. D84, 106005 (2011), arXiv:1106.4804 [hep-th].
[28] L. B. Anderson, J. Gray, A. Lukas, and E. Palti, JHEP 06, 113 (2012), arXiv:1202.1757 [hep-th].
[29] S. G. Nibbelink, O. Loukas, and F. Ruehle, Fortsch. Phys. 63, 600 (2015), arXiv:1507.07559 [hep-th].
[30] L. J. Hall, H. Murayama, and Y. Nomura, Nucl. Phys. B645, 85 (2002), arXiv:hep-th/0107245 [hep-th].
[31] A. Hebecker, JHEP 01, 047 (2004), hep-ph/0309313.
[32] A. Hebecker and M. Trappletti, Nucl. Phys. B713, 173 (2005), hep-th/0411131.
[33] M. Trappletti, Mod.Phys. Lett. A21, 2251 (2006), arXiv:hep-th/0611030 [hep-th].
[34] R. Donagi and K. Wendland, J.Geom.Phys. 59, 942 (2009), arXiv:0809.0330 [hep-th].
[35] M. Fischer, M. Ratz, J. Torrado, and P. K. Vaudrevange, JHEP 1301, 084 (2013), arXiv:1209.3906 [hep-th].
[36] W. Buchmüller, K. Hamaguchi, O. Lebedev, and M. Ratz, in Symposium GustavoFest (2005) pp. 143–156, arXiv:hep-ph/0512326 [hep-ph].
[37] M. Blaszczyk, S. Groot Nibbelink, M. Ratz, F. Ruehle, M. Trappletti, et al., Phys. Lett. B683, 340 (2010), arXiv:0911.4905 [hep-th].
[38] A. Mütter, M. Ratz, and P. K. Vaudrevange, (2016), in preparation.