On Optimal Sensing and Capacity Trade-off in Cognitive Radio Systems with Directional Antennas

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Abstract—We consider a cognitive radio system, in which the secondary users (SUs) and primary users (PUs) coexist. The SUs are equipped with steerable directional antennas. In our system, the secondary transmitter (SUtx) first senses the spectrum (with errors) for a duration of τ, and then transmits data to the secondary receiver (SUrx) if spectrum is sensed idle. The sensing time as well as the orientation of SUtx’s antenna affect the accuracy of spectrum sensing and yield a trade-off between spectrum sensing and capacity of the secondary network. We formulate the ergodic capacity of secondary network which uses energy detection for spectrum sensing. We obtain optimal SUtx transmit power, the optimal sensing time τ and the optimal directions of SUtx transmit antenna and SUrx receive antenna by maximizing the ergodic capacity, subject to peak transmit power and outage interference probability constraints. Our simulation results show the effectiveness of these optimizations to increase the ergodic capacity of the secondary network.

I. INTRODUCTION

The explosive rise in demand for high data rate wireless applications has turned the spectrum into a scarce resource. Cognitive radio (CR) technology is a promising solution which alleviates spectrum scarcity problem by allowing an unlicensed (secondary) user to access licensed bands in a such way that its imposed interference on the primary users (PUs) is limited [1]. The focus of most literature is optimizing spectrum sensing and transmission strategies for opportunistic spectrum access of secondary users (SUs), when the SUs are equipped with omni-directional antennas [2]–[7]. In those works, spectrum sensing seeks spectrum holes in the time domain so that SUs exploit them for transmitting their data. Different from the bulk of the literature, in this paper we assume the SUs are equipped with steerable directional antennas which allow them to use spatial spectrum holes [8]–[11] to increase spectrum utilization, especially in cognitive satellite networks [12]. The directional antennas can identify and enable transmission and reception across spatial domain and further enhance spectrum utilization, compared with omni-directional antennas.

In this paper, the SU transmitter (SUtx) first senses the spectrum and transmits data only when the spectrum is sensed idle. Since all spectrum sensing methods, including the energy detection method we use, are prone to sensing errors their false alarm and detection probabilities should be incorporated in the design and performance analysis [13]. Suppose, the SUtx employs a frame with duration T seconds, depicted in Fig. 1a, for spectrum sensing and data transmission. Each frame consists of a sensing time slot with duration τ seconds and the SUtx uses this time to decide whether spectrum is idle or busy. The remaining frame of duration T − τ seconds is used for data transmission if the spectrum is sensed idle. As τ increases, the false alarm probability decreases and detection probability increases. Thus, the result of spectrum sensing will be more accurate. On the other hand, the available time for data transmission decreases. Therefore, a trade-off exists between the sensing time and the capacity of our CR network.

We assume that the SUtx knows only the channel state information (CSI) of link between the SUtx and the secondary receiver (SUrx), and the statistics of the other links. Also, we assume that the SUtx knows the geometry of CR network. The orientation of the SUtx’s antenna with respect to the direction of primary transmitter (PUtx) affects the spectrum sensing accuracy. During spectrum sensing, to increase the detection probability and to receive the maximum power, the SUtx’s antenna should be pointed to the PUtx’s direction. On the other hand, the SUtx’s antenna should be pointed to the SUrx’s direction to maximize the transmission capacity. Thus, in addition to sensing-capacity trade-off in terms of sensing time τ, there is another sensing-capacity trade-off in terms of the SUtx’s antenna orientation. In this work, we establish the ergodic capacity of the channel between the SUtx and the SUrx, when spectrum sensing is imperfect and find the optimal directions of the SUtx and the SUrx antennas, the optimal SUtx transmit power and the optimal sensing time τ such that the ergodic capacity is maximized, subject to two constraints, namely, peak transmit power and outage interference probability constraints.
II. SYSTEM MODEL

A. Network Geometry

Our CR system model is shown in Fig. 1b. The SUs are equipped with direction antennas. The orientation of primary receiver (PU rx) with respect to SU ix is denoted by \( \theta_{pr} \). Also, the orientation of PU ix and SU rx with respect to SU ur are denoted by \( \theta_{ru} \) and \( \theta \), respectively and the orientation of PU ur with respect to SU ix is denoted by \( \theta_{up} \). The orientation of SU ix and SU ur antennas in their local coordinate are denoted by \( \phi_{ru} \) and \( \phi_{pr} \), respectively (to be optimized). We assume \( \theta_{ru} \), \( \theta_{up} \) and \( \theta_{pr} \) are known or can be estimated [14].

The antenna gain is modeled as \( A(\phi) = A_1 + A_0 \exp(-B(\phi_{\text{null}})^2) \), where \( B = \ln(2) \phi_{\text{null}} \) is the half-power beamwidth, \( A_1 \) and \( A_0 \) are two constant parameters [8]. Let \( d_{su}, d_{ps}, d_{sp}, d_{sr} \) and \( d_{sp} \) be the distances between SU ix, SU ur, PU ur and SU ur, and SU ur, respectively.

The fading coefficients from SU ur to SU ix, PU ur to SU ur, SU ur to PU ur and SU ur to PU ur are denoted by \( g_{ss}, g_{ps}, g_{sp}, g_{sr} \) and \( g_{sp} \), respectively. We assume \( g_{ss}, g_{ps}, g_{sp}, g_{sr} \) and \( g_{sp} \) are independent exponential random variables with means \( \gamma_{ss}, \gamma_{ps}, \gamma_{sp}, \gamma_{sr} \), and \( \gamma_{sp} \), respectively. The path-loss is \( L = (d_0/d)\nu \), where \( d_0 \) is the reference distance, \( d \) is the distance between users, and \( \nu \) is the path loss exponent. We assume there is no cooperation between SUs and PUs and hence, SU ur and SU ur only do not know the realizations of \( g_{sp} \) and \( g_{ps} \) and only know their statistics. On the other hand, SU ur knows \( g_{ss} \).

B. Spectrum Sensing

The SU ix employs a frame with duration \( T \) seconds. Each frame consists of a sensing time slot with duration \( \tau \) seconds (to be optimized) and SU ur uses this time to decide whether spectrum is idle or busy. The remaining frame of duration \( T - \tau \) seconds is used for data transmission if the spectrum is sensed idle. It is clear that for a given \( T \), if we increase the sensing time \( \tau \), the spectrum sensing will be more accurate. On the other hand, the available time for data transmission decreases. Therefore, a trade-off exists between the sensing time and the transmission capacity of our CR network.

We formulate the spectrum sensing at the SU ur as a binary hypothesis testing problem where the received signal in SU ur can be written as

\[
\begin{align*}
H_0 : r[k] = w[k], \\
H_1 : r[k] = \sqrt{g_{sp}} A(\phi_{ru} - \theta_{ru}) L_{sp} \cdot p[k] + w[k]
\end{align*}
\]

for \( k = 1, \ldots, N_s \). The two hypotheses \( H_0 \) and \( H_1 \) with probabilities \( \pi_0 \) and \( \pi_1 = 1 - \pi_0 \) denote the spectrum is truly idle and truly busy, respectively. The term \( w[k] \sim \mathcal{N}(0, \sigma_n^2) \) is the additive white Gaussian noise (AWGN) at the SU ur and \( p[k] \) is the transmitted symbol from the PU ur with average power \( P_p \). We assume the SU ur knows \( P_p \). We note \( N_s = \tau f_s \) is the number of signal samples available for spectrum sensing and \( f_s \) is the sampling frequency. Let \( \hat{H}_1 \) and \( \hat{H}_0 \) with probabilities \( \hat{\pi}_0 \) and \( \hat{\pi}_1 \) denote that the result of spectrum sensing is busy and idle, respectively. Considering energy detection as our spectrum sensing method, the decision statistics at the SU ur can be written as

\[
Z = \frac{1}{N_s} \sum_{k=1}^{N_s} |r[k]|^2.
\]

The accuracy of our spectrum sensing method is characterized by false alarm probability \( P_f = \Pr\{\hat{H}_1 | H_0 \} \) and detection probability \( P_d = \Pr\{\hat{H}_1 | H_1 \} \). For large \( N_s \), we can use the central limit theorem and approximate the probability distribution function (PDF) of decision statistics \( Z \) as Gaussian distribution and \( P_f \) and \( P_d \) can be written as [4]

\[
P_f(\phi_t, \tau) = Q \left( \frac{\xi}{\sigma_n} - 1 \sqrt{\tau f_s} \right) \quad (2)
\]

\[
P_d(\phi_t, \tau) = Q \left( \frac{\xi}{\sigma_n} - 1 \sqrt{\frac{\tau f_s}{2\gamma + 1}} \right) \quad (3)
\]

where \( \gamma = P_p g_{sp} A(\phi_t - \theta_{ru}) L_{sp} / \sigma_n^2 \) is the signal-to-noise-ratio (SNR) at the SU ur and \( \xi \) is the decision threshold. The probabilities in (2) and (3) are functions of the optimization parameters \( \tau \) and \( \phi_t \). For the simplicity of the presentation, we drop the parameters \( \tau \) and \( \phi_t \) in the remaining of the paper.

The orientation of SU ur’s antenna (\( \phi_{ru} \)) with respect to direction of PU ur affects the spectrum sensing accuracy. To increase \( P_d \) during spectrum sensing, the SU ur’s antenna should be pointed towards PU ur’s direction to receive the maximum power. On the other hand, the SU ur’s antenna should be pointed towards SU ur’s direction to maximize the transmission capacity. Thus, there is a sensing-capacity trade-off in terms of the SU ur antenna orientation.

C. Data Communication Channel

When the spectrum is sensed idle, the SU ur uses power \( P \) (to be optimized) to transmit signal to SU ur. Let \( s[m] \) denote the transmitted signal by SU ur with power \( P \), and \( y[m] \) denote the corresponding received signal by SU ur given by

\[
y[m] = \sqrt{g_{ss}} L_{ss} G(\theta_{ru}, \phi_{ru}) s[m] + n[m],
\]

where \( n[m] \) is the AWGN with power \( \sigma_n^2 \) and \( G(\theta_{ru}, \phi_{ru}) = A(\phi_{ru} - \theta_{ru}) A(\phi_{ru} - \pi - \theta) \) is the product of SU ur and SU ur antennas’ gain. For the simplicity of presentation, we drop the parameters \( \theta, \phi_{ru}, \phi_{ru} \) from \( G(\theta, \phi, \phi_\text{ru}) \).

Our goal is to find the ergodic capacity of the channel between SU ur and SU ur and explore the optimal SU ur transmit power, optimal sensing time \( \tau \) and the optimal directions of SU ur and SU ur antennas, \( \phi_{ru} \) and \( \phi_{ru} \), such that this capacity maximized, subject to peak transmit power and outage interference probability constraints.

III. CONSTRAINED ERGODIC CAPACITY MAXIMIZATION

A. Capacity Expression

When spectrum sensing is imperfect, the ergodic capacity would depend on the true status of the PU and the spectrum sensing result. In our problem, the ergodic capacity becomes

\[
C = D \mathbb{E}\{\alpha_0 c_{0,0} + \beta_0 c_{1,0}\},
\]

where \( \mathbb{E}\{\cdot\} \) is the expectation operator, and \( c_{i,0} \) is instantaneous capacity corresponding to \( H_i \) and \( H_0 \) with probability \( \alpha_0 = \Pr\{H_0, H_0\} \) and \( \beta_0 = \Pr\{H_1, H_0\} \), given as

\[
c_{0,0} = \log_2 \left( 1 + \frac{g_{ss} L_{ss} G P}{\sigma_n^2} \right) \quad (4)
\]

\[
c_{1,0} = \log_2 \left( 1 + \frac{g_{ss} L_{ss} G P}{\sigma_n^2 + P_p g_{ps} L_{ps}} A(\phi - \theta_{pu}) \right) \quad (5)
\]
and $D = (T - \tau)/T$ is the fraction of time in which SU$_{tx}$ transmits data to SU$_{rx}$. It is easy to verify $\alpha_0 = \pi_0(1 - \rho_f)$ and $\beta_0 = \pi_1(1 - P_d)$. It is worth noting that, if spectrum sensing errors are not considered (i.e., spectrum sensing is assumed to be perfect) $\alpha_0 = \rho_s, \beta_0 = 0$. Also, it is important to emphasize that the optimal transmit power $P$, the optimal antenna directions and the optimal sensing time $\tau$ are functions of the fading coefficient $g_{ss}$. After taking expectation with respect to $g_{sp}$ and $g_{ps}$, $C$ can be written as

$$C = D \mathbb{E}_{g_{sp}} \left\{ \tilde{\beta}_0 \log_2 \left( 1 + \frac{1}{x} \right) + \frac{\beta_0}{\ln(2)} \left[ T(y) - T(y + \frac{y}{x}) \right] \right\}$$

where $T(z) = e^{\xi} Ei(-z)$ and $Ei(z)$ is the exponential integration [15]. In (6), $x = \sigma_n^2/\alpha P$, $a = g_{ss}L_{ss} G$, $y = \sigma_n^2/\sigma_p$. The term $\sigma_p = P_T A_{pp} (\theta_r - \theta_p)$ captures the interference on SU$_{rx}$ due to PU activities.

B. Constraints

Upon transmitting data, the SU$_{tx}$ generates an interference on the PU$_{tx}$. Similar to the outage concept developed in wireless communication community, we define the interference outage probability as the probability that the interference exceeds a maximum threshold $I_{pk}$. As a mechanism to control the interference generated by the SU$_{tx}$, we require that the interference outage probability to be smaller than a maximum value $\varepsilon$. In other constraint, we consider the following constraint

$$\Pr \left\{ D \beta_0 P g_{sp} L_{sp} A(\phi_t - \theta_{p_r}) > I_{pk} | g_{ss} \right\} \leq \varepsilon. \quad (7)$$

We can rewrite (7) as $F_{g_{sp}} \left( \frac{I_{pk}}{D \beta_0 P g_{sp} L_{sp} A(\phi_t - \theta_{p_r})} \right) \geq 1 - \varepsilon$, where $F_{g_{sp}}(\cdot)$ is the cumulative distribution function (CDF) of random variable $g_{sp}$, given as $F_{g_{sp}}(x) = 1 - \exp(-\frac{x}{\sigma_p})$ for $x \geq 0$. Finally, we can write the constraint in (7) as

$$D \tilde{\beta}_0 P \leq \frac{-I_{pk}}{\ln(\varepsilon)} \quad (8)$$

where $\tilde{\beta}_0 = \beta_0 g_{sp} L_{sp} A(\phi_t - \theta_{p_r})$. Let $P_{pk}$ indicate the maximum allowed instantaneous transmit power of SU$_{tx}$. To satisfy the peak transmit power constraint, we have

$$D \tilde{\beta}_0 P \leq P_{pk}. \quad (9)$$

In order to ensure that the SU$_{tx}$’s (SU$_{rx}$’s) orientation lies within the half-power beam-width of the SU$_{tx}$ (SU$_{rx}$) antenna, we constrain them as

$$|\phi_t - \theta| \leq \phi_{MB}, \quad (10a)$$

$$|\phi_r - \pi - \theta| \leq \phi_{MB}. \quad (10b)$$

C. Behavior of Capacity with respect to sensing time $\tau$

In this section, we examine the behavior of $C$ with respect to $\tau$ and we show that, in a certain condition, $C$ is a concave function of $\tau$. Consider $P_L, P_d$ in (2), (3). By taking the second derivative of $P_f$ with respect to $\tau$, one can easily verify that $P_f$ is a convex function of $\tau$ when $\xi > \sigma_n^2$. With the same argument we can show that $D \alpha_{0}$ is a concave functions with respect to $\tau$ for $\xi > \sigma_n^2$. Taking the first derivative of $C$ with respect to $\tau$, we get

$$\frac{\partial C}{\partial \tau} = \frac{\partial}{\partial \tau} \left\{ \left( \pi_0 \mathrm{e}^{(-1/2) \theta_{0}} + \pi_1 \mathrm{e}^{(-1/2) \theta_{1}} \right) x^2 \right\}$$

Algorithm 1: Optimization Algorithm

Where $X = (\frac{\pi_0}{\sigma_n^2} - 1) \sqrt{T_s}$ and $Y = (\frac{\pi_1}{\sigma_n^2} - 1) \sqrt{T_s}$. One can easily verify that

$$\lim_{\tau \to T} \frac{\partial C/\partial \tau}{\partial \tau} < 0.$$

Recall that our goal is to maximize the ergodic capacity $C$ over $P, \phi_t, \phi_r$ and $\tau$ subject to constraints (8), (9) and (10). The capacity is concave with respect to $P$ and $\phi_r$. However, in general, it is not concave with respect to $\phi_t$ and $\tau$. In the following we present our approach for solving this optimization problem. The optimal power can be written as

$$P_{\text{opt}} = \min \left\{ \frac{P_{pk}}{D \tilde{\beta}_0 \ln(\varepsilon)}, \frac{-I_{pk}}{D \tilde{\beta}_0 \ln(\varepsilon)} \right\}. \quad (11)$$

The optimal $\phi_t$ and $\phi_r$ can be obtained by using searching methods like bisection method. We consider an initial value for $\phi_t$ which satisfies (10a) and $\tau \in (0, T)$, and obtain $P_{\text{opt}}$ using (11). Then, the optimal $\phi_r$ can be found by solving $\partial C/\partial \phi_r = 0$, subject to the constraint (10b). For any realization of $g_{ss}$, the first derivative of $C$ with respect to $\phi_t$ is equal to

$$\frac{\partial C}{\partial \tau} = \frac{D}{\ln(2)} \left\{ \frac{A' \phi_t - \pi - \theta}{A \phi_t - \pi + \theta} g_0(x, y) \right\} \quad (11)$$

and $A' = \partial A(\cdot)/\partial \phi_t$. Then, we find the value of $\phi_t$ and $\tau$ which maximizes $C$. Algorithm 1 summarizes our proposed approach to find the optimal solutions $\phi_{t_{\text{opt}}}, \phi_{r_{\text{opt}}}$ and $P_{\text{opt}}$.

IV. NUMERICAL RESULTS AND CONCLUSION

In this section, we illustrate how effectively the directional antennas can improve the capacity of our secondary network by Matlab simulations. Assume $\sigma_n^2 = 1, A_0 = 9.8, A_1 = 0.2,$
Let $C_{\text{opt}}^{\text{Dir}}$ denote the optimal capacity when SU$_{\text{rx}}$ and SU$_{\text{tx}}$ have omni-directional antennas and only transmit power $P$ and sensing time $\tau$ are optimized subject to constraints (8) and (9). Furthermore, let $C_{\text{opt}}^{\text{LOS}}$ be the optimal capacity when directional antennas of SU$_{\text{tx}}$ and SU$_{\text{rx}}$ are exactly pointed at each other and only $P$ and $\tau$ are optimized subject to constraints (8) and (9). Fig. 3c illustrates $C_{\text{opt}}^{\text{Dir}}, C_{\text{opt}}^{\text{LOS}}$ and $C_{\text{opt}}^{\text{Omn}}$ versus $\theta$. This figure shows the effectiveness of using the directional antennas as well as optimizing their orientation on the capacity of the secondary network. It demonstrates that directional antennas can improve the secondary network capacity for all values of $\theta$.

We define the capacity ratio $\Gamma_{\text{D2O}} = C_{\text{opt}}^{\text{Dir}} / C_{\text{opt}}^{\text{Omn}}$. This ratio is shown in Fig. 3d for $P_{\text{pk}} = 6,8$ dB. We see that directional antennas yields as much as 22% capacity gain for $P_{\text{pk}} = 8$ dB in comparison to omni-directional antennas. Also, we can see that the capacity gain decreases when maximum allowable transmit power ($P_{\text{pk}}$) decreases.

In summary, we considered a CR system, where the SU$_{\text{rx}}$ and SU$_{\text{tx}}$ have directional antennas and use energy detection method for spectrum sensing. The optimal SU$_{\text{tx}}$ transmit power, the optimal sensing time $\tau$ and the optimal directions of SU$_{\text{tx}}$ transmit antenna and SU$_{\text{rx}}$ receive antenna are obtained by maximizing the ergodic capacity, subject to peak transmit power and outage interference probability constraints. Our simulation results demonstrated the effectiveness of these optimizations on increasing the ergodic capacity of the secondary network.

**ACKNOWLEDGMENT**

This research is supported by NSF under grant ECCS-1443942.
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