Magnetic moments of heavy $J^P = \frac{1}{2}^+$ baryons in light cone QCD sum rules

T. M. Aliev *,†, T. Barakat ‡, M. Savcı §

Physics Department, Middle East Technical University, 06531 Ankara, Turkey
‡ Physics and Astronomy Department, King Saud University, Saudi Arabia

Abstract

The magnetic moments of heavy sextet $J^P = \frac{1}{2}^+$ baryons are calculated in framework of the light cone QCD sum rules method. Linearly independent relations among the magnetic moments of these baryons are obtained. The results for the magnetic moments of heavy baryons obtained in this work are compared with the predictions of the other approaches.

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* e-mail: taliev@metu.edu.tr
† permanent address: Institute of Physics, Baku, Azerbaijan
‡ e-mail: tbarakat@KSU.EDU.SA
§ e-mail: savci@metu.edu.tr
1 Introduction

The quark model predicts the existence of heavy baryons composed of single, double and triple quarks. Essential improvement has been achieved in heavy baryon spectroscopy in recent years. All baryons with a single charm quark that are predicted by the quark model have been observed in experiments. Moreover, heavy baryons with a single bottom quark, such as $\Lambda_b$, $\Sigma_b$, $\Xi_b$ and $\Omega_b$ have also been discovered (for a review see [1]).

These progresses in experiments stimulated future investigation for the properties of these baryons at LHC, as well as further theoretical studies on this subject.

Remarkable information about the internal structure of baryons can be gained by studying their magnetic moments. Magnetic moments of the heavy baryons have long been under the focus of theoretical physicists, and they have been studied in framework of various approaches. So far, magnetic moments of charmed heavy baryons have been calculated in framework of the naive quark model [2, 3], relativistic quark model [4], chiral perturbation model [5], hypercentral model [6], soliton model [7], skyrmion model [8], bag model [9], QCD sum rules method [10], nonrelativistic quark model [11], phenomenological relativistic model [12], quark model with confinement law potential [13], and chiral constituent quark model, respectively.

Magnetic moments of heavy hadrons have also been studied in numerous works within the QCD sum rules method. Magnetic moments of the $\Lambda_c$ and $\Sigma_c$ baryons have been calculated in framework of the traditional QCD sum rules method in [10]; of the $\Lambda_Q$ and $\Xi_Q$ ($Q = c$ or $b$) baryons within the light version of the QCD sum rules method in [15, 16], respectively. It should be noted here that magnetic moments of the spin-3/2 heavy baryons have already been analyzed within the same approach in [17].

The goal of the present work is calculation of the magnetic moments of heavy $\Sigma_Q$, $\Xi_Q$ and $\Omega_Q$ baryons within the light cone QCD sum rules method.

The paper is organized as follows. In section 2, sum rules for the magnetic moments of the above-mentioned baryons are constructed. Numerical results of our calculations are presented in section 3. This section further contains comparison of our results with the predictions of other approaches, and concluding remarks.

2 Light cone sum rules for the sextet heavy baryons

We start this section by giving a brief summarizing of the classification of heavy baryons in SU(3) symmetry. According to SU(3) symmetry, heavy baryons with single heavy quark belong to either symmetric sextet or antisymmetric anti-triplet flavor representations. As has already been noted, magnetic moments of the $\Lambda_Q$ and $\Xi_Q$ baryons belonging to anti-symmetric flavor representations of SU(3) are calculated within the light QCD sum rules method in [15] and [16], respectively. In the present work we calculate magnetic moments of the heavy baryons belonging to sextet representation of SU(3) group.

In order to calculate magnetic moments of the heavy sextet baryons, we start by considering the following correlation function:

$$\Pi(p, q) = i \int d^4xe^{ipx} \langle \gamma(q) | T \{ \eta_Q(x) \bar{\eta}_Q(0) \} | 0 \rangle ,$$

(1)
where $\eta_Q$ is the interpolating current of the heavy spin-1/2 baryon. The main task in constructing the sum rules for magnetic moments of the heavy sextet baryons is the calculation of the correlation function in terms of the photon distribution amplitude by using the operator product expansion (OPE) over twist from one side, and in terms of the hadrons from the other side, and then equating both representations. Calculation of the correlation function from the hadronic side is accomplished by inserting a complete set of hadrons carrying the same quantum numbers of the interpolating current $\eta_Q$. Isolating the the ground state’s contribution, we obtain,

$$\Pi = \frac{\langle 0 | \eta_Q | B_Q(p_2) \rangle}{p_2^2 - m_{B_Q}^2} \frac{\langle B(p_2) \gamma(q) | B_Q(p_1) \rangle}{p_1^2 - m_{B_Q}^2} \frac{\langle B_Q(p_1) | \eta_Q | 0 \rangle}{p_1} + \cdots,$$

(2)

where dots correspond to the contributions of higher states and continuum, $p_1 = p_2 + q$. In further analysis we shall make the replacement $p_2 = p$. The matrix elements in Eq. (2) are determined as,

$$\langle 0 | \eta_Q | B_Q(p) \rangle = \lambda_Q u_{B_Q}(p),$$

(3)

$$\langle B_Q(p) | B_Q(p_1) \rangle = \varepsilon^\mu \bar{u}_{B_Q}(p) \left[ f_1 \gamma_\mu - f_2 \frac{i \sigma_{\mu \nu}}{2m_{B_Q}} q^\nu \right] u_{B_Q}(p_1),$$

(4)

where $\varepsilon^\mu$ is the photon polarization vector, $f_1$ and $f_2$ are the form factors. Using the equation of motion, Eq. (4) can be written as,

$$\langle B_Q(p) | B_Q(p_1) \rangle = \bar{u}_{B_Q}(p) \left[ (f_1 + f_2) \gamma_\mu + \frac{(p + p_1) \mu}{2m_{B_Q}} \right] u_{B_Q}(p_1) \varepsilon^\mu.$$

(5)

Since the photon involved in these transitions is a real photon, we set $q^2 = 0$, and hence in calculation of the magnetic moments of the heavy sextet baryons the values of the form factors are needed only at $q^2 = 0$.

Substituting Eqs. (3) and (5) in Eq. (2), and performing summations over spins of the heavy baryons, for the hadronic of the correlation function we get,

$$\Pi = \lambda_{B_Q}^2 \varepsilon^\mu \frac{(p + m_{B_Q})}{p^2 - m_{B_Q}^2} \left[ (f_1 + f_2) \gamma_\mu + \frac{(p + p_1) \mu}{2m_{B_Q}} \right] \frac{p_1 + m_{B_Q}}{p_1^2 - m_{B_Q}^2}.$$

(6)

We observe from Eq. (6) that the correlation function contains many structures, any of them can be chosen in calculating magnetic moments of the sextet baryons, and in this respect we choose the structure $\frac{p_1 + \hat{q}}{p_1^2 - (p + q)^2}. This structure envelopes the magnetic form factor $f_1 + f_2$, and at $q^2 = 0$ it gives the magnetic moment $\mu_{B_Q}$ of the heavy baryons in units of $e\hbar/2m_{B_Q}$. As a result, the correlation function can be written in terms of the magnetic moment of heavy baryons as,

$$\Pi = \lambda_{B_Q}^2 m_{B_Q} \frac{1}{m_{B_Q}^2 - p^2} \frac{1}{m_{B_Q}^2 - (p + q)^2}.$$  

(7)

In order to calculate the correlation function in terms of quark and gluon degrees of freedom and photon distribution amplitudes, the expressions of the interpolating currents
of the heavy baryons are needed. The general form of the interpolating currents of the heavy spin-1/2 positive parity baryons is given as (see for example [18]),

$$\eta_{BQ} = -\frac{1}{\sqrt{2}}e^{abc}\left\{ (q_1^aT C Q^b)\gamma_5 q_2^c + t(q_1^a T C \gamma_5 Q^b)q_2^c - (Q^a T C \gamma_5 q_2^b)\gamma_5 q_1^c - t(Q^a T C \gamma_5 q_2^b)q_1^c \right\}, (8)$$

where \(a, b, c\) are the color indices, \(C\) is the charge conjugation operator, and \(t\) is an arbitrary parameter whose value at \(t = -1\) gives the so-called Ioffe current. The quark contents of the sextet heavy sextet baryons are given in Table 1.

| \(\Sigma^+_{(++)}\) | \(\Sigma^0_{(++)}\) | \(\Sigma^-_{(0-)}\) | \(\Xi^{0(+)\uparrow}_{(0-)}\) | \(\Omega^0_{(0-)}\) |
|---|---|---|---|---|
| \(q_1\) | \(u\) | \(u\) | \(d\) | \(s\) |
| \(q_2\) | \(u\) | \(d\) | \(s\) | \(s\) |

Table 1: Quark contents of the heavy sextet baryons.

Using the expression for the interpolating current and Wick’s theorem, the theoretical part of the correlation function for the \(\Sigma^0_b\) can be written as,

$$\Pi^{\Sigma^0}_b = -3\left\{ \gamma_5 S_d(x)S'_b(x)S_u(x)\gamma_5 + \gamma_5 S_u(x)S'_u(x)S_d(x)\gamma_5 + \gamma_5 S_u(x)\gamma_5 Tr[S_b(x)S'_d(x)] \\
+ \gamma_5 S_d(x)\gamma_5 Tr[S_u(x)S'_u(x)] + t(\gamma_5 S_d(x)\gamma_5 S'_b(x)S_u(x) + \gamma_5 S_u(x)\gamma_5 S'_b(x)S_d(x)) \\
+ S_d(x)S'_b(x)\gamma_5 S_u(x)\gamma_5 + S_u(x)S'_u(x)\gamma_5 S_d(x)\gamma_5 + tS_d(x)\gamma_5 S'_b(x) + tS_u(x)\gamma_5 S'_u(x) \right\} ,$$

where \(S' = CS^TC\), \(T\) symbolizes transposition operator, and \(S\) is the quark (light or heavy) propagator. The corresponding expressions of the correlation functions for the other members of the sextet baryons can be found from the \(\Sigma_{BQ}\) by making the following replacements:

$$\Pi^{\Sigma^0}_{\Sigma^+} = \Pi^{\Sigma^+}_{\Sigma^0} (u \rightarrow d) ,$$

$$\Pi^{\Sigma^0}_{\Sigma^+} = \Pi^{\Sigma^+}_{\Sigma^0} (d \rightarrow u) ,$$

$$\Pi^{\Sigma^+}_{\Sigma^-} = \Pi^{\Sigma^-}_{\Sigma^+} (u \rightarrow d) ,$$

$$\Pi^{\Sigma^+}_{\Sigma^-} = \Pi^{\Sigma^-}_{\Sigma^+} (d \rightarrow u) ,$$

$$\Pi^{\Xi^{0(+)\uparrow}}_{\Xi^{0(-)\downarrow}} = \Pi^{\Xi^{0(-)\downarrow}}_{\Xi^{0(+)\uparrow}} (d \rightarrow s) ,$$

$$\Pi^{\Xi^{0(+)\uparrow}}_{\Xi^{0(-)\downarrow}} = \Pi^{\Xi^{0(-)\downarrow}}_{\Xi^{0(+)\uparrow}} (u \rightarrow d, d \rightarrow s) ,$$

$$\Pi^{\Xi^{0(+)}_{\Xi^{0(-)}}} = \Pi^{\Xi^{0(-)_{\Xi^{0(+)}}}} (d \rightarrow s) ,$$

$$\Pi^{\Xi^{0(+)}_{\Xi^{0(-)}}} = \Pi^{\Xi^{0(-)_{\Xi^{0(+)}}}} (u \rightarrow d, d \rightarrow s) ,$$

$$\Pi^{\Omega^0_{\Omega^+}} = \Pi^{\Omega^+_{\Omega^0}} (u \rightarrow s) ,$$

$$\Pi^{\Omega^0_{\Omega^+}} = \Pi^{\Omega^+_{\Omega^0}} (u \rightarrow s, d \rightarrow s) .$$
The correlation function described by Eq. (9) contains three different parts: 
a) Perturbative part, when a photon is radiated from short distance (perturbative contribution); 
b) a photon is radiated from short distance from the quark propagators, and light quarks form 
a condensate; c) a photon is radiated from long distance (nonperturbative contribution).

In order to calculate the perturbative contribution of the correlation function, it is 
easy enough to make the replacement

\[ S \rightarrow \int d^4y S^{\text{free}}(x - y) A(y) S^{\text{free}}(y), \]

for one of the quark propagator, where \( S^{\text{free}} \) is the free quark operator; and the other 
two are being the free propagators. The expressions of the free light and heavy quarks in 
coordinate representation are given as,

\[
S^{\text{free}}_q = \frac{i\not\! q}{2\pi^2 x^2} - \frac{m_q}{4\pi^2 x^2},
\]

\[
S^{\text{free}}_\bar{q} = \frac{m_{Bq}^2 K_1(m_{Bq} \sqrt{-x^2})}{4\pi^2 \sqrt{-x^2}} + \frac{m_{Bq}^2 \not\! K_2(m_{Bq} \sqrt{-x^2})}{(\sqrt{-x^2})^2},
\]

where \( K_1 \) are the Bessel functions.

The contribution of part (c) can easily be calculated by replacing one of the light quark 
operator with,

\[ (S^{\text{non}}_q)_{\alpha\beta} \rightarrow -\frac{1}{4} \delta^{ab} \not\! q\not\! \Gamma_i \not\! q (\Gamma_i)_{\alpha\beta}, \]

where \( \Gamma_j \) are the full set of Dirac matrices, and other quark operators that involve pertur- 
bative, as well as nonperturbative terms. The explicit forms of the “full” quark propagators 
can be found in [19] and [20].

Nonperturbative contribution is realized as the matrix element of the nonlocal operator 
\( \not\! q \Gamma_i \not\! q \) between the vacuum and one-photon states. These matrix elements are described in 
terms of photon distribution amplitudes, as are given below (see [21]):

\[
\langle\gamma(q)|\bar{q}(x)\sigma_{\mu\nu}q(0)|0\rangle = -ie_q\bar{q}q(\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int_0^1 du e^{i\bar{u}qx} \left( \chi\gamma(u) + \frac{x^2}{16} A(u) \right)
\]

\[
-\frac{i}{2(qx)}e_q\langle\bar{q}q\rangle \left[ x_\nu \left( \varepsilon_\mu - q_\mu \frac{x}{qx} \right) - x_\mu \left( \varepsilon_\nu - q_\nu \frac{x}{qx} \right) \right] \int_0^1 du e^{i\bar{u}qx} h_\gamma(u)
\]

\[
\langle\gamma(q)|\bar{q}(x)\gamma_\mu q(0)|0\rangle = e_q f_{3\gamma} \left( \varepsilon_\mu - q_\mu \frac{x}{qx} \right) \int_0^1 du e^{i\bar{u}qx} \psi_\nu(u)
\]

\[
\langle\gamma(q)|\bar{q}(x)\gamma_\mu \gamma_5 q(0)|0\rangle = -\frac{1}{4} e_q f_{3\gamma} \varepsilon_{\mu\nu\alpha\beta} \bar{q}^\alpha x^\beta \int_0^1 du e^{i\bar{u}qx} \psi_\nu(u)
\]

\[
\langle\gamma(q)|\bar{q}(x)g_\mu G_{\mu\nu}(vx)q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int D\alpha_i e^{i(\alpha_q + \gamma vx)qx} S(\alpha_i)
\]

\[
\langle\gamma(q)|\bar{q}(x)g_\mu \tilde{G}_{\mu\nu}(vx)q(0)|0\rangle = -ie_q\langle\bar{q}q\rangle (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int D\alpha_i e^{i(\alpha_q + \gamma vx)qx} \tilde{S}(\alpha_i)
\]
\begin{align*}
\langle \gamma(q) | \bar{q}(x) g_s \tilde{G}_{\mu\nu}(vx) \gamma_\alpha \gamma_\beta q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int D\alpha_i e^{i(\alpha_q + v_\alpha q)x} A(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) g_s G_{\mu\nu}(vx) i^\gamma_\alpha q(0) | 0 \rangle &= e_q f_{3\gamma} q_\alpha (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int D\alpha_i e^{i(\alpha_q + v_\alpha q)x} V(\alpha_i) \\
\langle \gamma(q) | \bar{q}(x) \sigma_{\alpha\beta} g_s G_{\mu\nu}(vx) q(0) | 0 \rangle &= e_q \langle \bar{q} q \rangle \left\{ \left[ (\varepsilon_\mu q_\nu - \frac{1}{qx} (q_\alpha x_\nu + q_\nu x_\alpha) ) q_\beta \right. \\
& - (\varepsilon_\mu - \frac{1}{qx} (q_\beta x_\nu + q_\nu x_\beta) ) q_\alpha \\
& - (\varepsilon_\nu - \frac{1}{qx} (q_\alpha x_\mu + q_\mu x_\alpha) ) q_\beta \\
& + (\varepsilon_\nu - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) ) q_\alpha \right] \int D\alpha_i e^{i(\alpha_q + v_\alpha q)x} T_1(\alpha_i) \\
& + \left[ (\varepsilon_\alpha - \varepsilon_\alpha q_\beta - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) ) q_\nu \right. \\
& - (\varepsilon_\alpha - \varepsilon_\alpha q_\nu - \frac{1}{qx} (q_\beta x_\mu + q_\mu x_\beta) ) q_\nu \\
& - (\varepsilon_\beta - \varepsilon_\beta q_\nu - \frac{1}{qx} (q_\mu x_\alpha + q_\alpha x_\mu) ) q_\nu \\
& + (\varepsilon_\beta - \varepsilon_\beta q_\mu - \frac{1}{qx} (q_\nu x_\alpha + q_\alpha x_\nu) ) q_\mu \left. \right] \int D\alpha_i e^{i(\alpha_q + v_\alpha q)x} T_2(\alpha_i) \\
& + \frac{1}{qx} (q_\mu x_\nu - q_\nu x_\mu) (\varepsilon_\alpha q_\beta - \varepsilon_\beta q_\alpha) \int D\alpha_i e^{i(\alpha_q + v_\alpha q)x} T_3(\alpha_i) \\
& + \frac{1}{qx} (q_\alpha x_\beta - q_\beta x_\alpha) (\varepsilon_\mu q_\nu - \varepsilon_\nu q_\mu) \int D\alpha_i e^{i(\alpha_q + v_\alpha q)x} T_4(\alpha_i) \right\}.
\end{align*}

In the definitions given above, \( \chi \) is the magnetic susceptibility of the quarks, \( \varphi_\gamma(u) \) is the leading twist-2, \( \psi^u(u) \), \( \psi^s(u) \), \( A \) and \( V \) are the twist-3, and \( h_\gamma(u), A, T_i \) \((i = 1, 2, 3, 4)\) are the twist-4 photon DAs, respectively, whose explicit expressions are given in Appendix A. The measure \( D\alpha_i \) is defined as

\[ \int D\alpha_i = \int_0^1 d\alpha_q \int_0^1 d\alpha_q \int_0^1 d\alpha_q \delta(1 - \alpha_q - \alpha_q - \alpha_q). \]

In constructing the sum rules for the magnetic moments of heavy sextet baryons it is necessary to equate the coefficients of the structure \( \tilde{p} \tilde{q} \) to from the phenomenological and theoretical representations of the correlation function. The following steps in obtaining the final form of the sum rules for the magnetic moment are: Fourier transformation, Borel transformation over the variables \( p^2 \) and \( (p + q)^2 \) variables, and continuum subtraction in order to suppress the contribution of the higher states and continuum. After these procedures we get the sum rules for the magnetic moment of the sextet heavy baryons, which can schematically be written in the following form,

\[ \lambda_{BQi}^2, \mu_{BQi}^2, e^{m_{BQi}^2/M^2} = \Pi_{B,\text{theor}}^B, \]

where the function \( \Pi_{B,\text{theor}}^B \) contain perturbative and nonperturbative contributions.
The expression for $\Pi^{B,\text{theor}}$ is quite lengthy, and for this reason we do not present it here.

In order to determine the magnetic moments of heavy sextet baryons, the value of the overlap amplitude is needed, which can be found from the analysis of the two-point correlation function. The residues of the heavy sextet baryons are calculated in [22], and their expressions can be found in this work.

3 Results and discussion

Having obtained the sum rules for the magnetic moments of the heavy sextet spin-$1/2$ baryons, we are now ready to perform the numerical analysis. The main input parameters of the light cone QCD sum rules for the magnetic moments are the photon distribution functions DAs [21]. Their explicit expressions are presented in Appendix A. The values of the other input parameters are:

\[
\begin{align*}
\langle \bar{u}u \rangle_{\mu=1 GeV} &= 1 GeV = \langle \bar{d}d \rangle_{\mu=1 GeV} = 1 GeV, \\
\langle \bar{s}s \rangle_{\mu=1 GeV} &= 0.8 \langle \bar{u}u \rangle_{\mu=1 GeV} [23], \\
m_0^2 &= (0.8 \pm 0.2) GeV^2 [22], \\
\Lambda &= (0.5 \pm 0.1) GeV [24], \\
f_{3\gamma} &= -0.039 GeV^2 [21], \\
m_{s(\mu=2 GeV)} &= (111 \pm 6) MeV [25].
\end{align*}
\]

Few words about the magnetic susceptibility involved in the numerical analysis are in order. Its value is estimated to have the value $\chi(1 GeV) = -4.4 GeV^{-2}$ in [26]. In [21], using the vector dominance model ansatz and QCD sum rules its value is predicted to be $\chi(1 GeV) = -(3.15 \pm 0.15) GeV^{-2}$. Furthermore, from an analysis of the radiative decays of heavy mesons its value is found to be $\chi(1 GeV) = -(2.85 \pm 0.50) GeV^{-2}$ [27]. In numerical analysis we have used all these predicted values of the magnetic susceptibility.

It should also be remembered that, the sum rules for the magnetic moments involve the following auxiliary parameters: The arbitrary parameter $t$ appearing in the expressions of the interpolating currents, Borel mass parameter $M^2$, and the continuum threshold $s_0$. The magnetic moment must be independent of these parameters. In order to find “regions” of these parameters where magnetic moment exhibits good stability with respect to their variations we proceed as follows. Firstly, we try to find the upper and lower bounds of Borel mass parameter at fixed values of $s_0$ and $t$. The upper bound of $M^2$ can be determined by requiring that the contribution due to the continuum threshold should be less than half of the contribution coming from the perturbative part. The lower bound of $M^2$ can be found by demanding that the higher powers of $1/M^2$ contribute less than the leading twist contributions. Our numerical analysis shows that these two conditions are satisfied simultaneously if $M^2$ ranges in the following regions:

\[
\begin{align*}
2.0 GeV^2 &\leq M^2 \leq 3.0 GeV^2 \quad \Sigma_c \\
2.2 GeV^2 &\leq M^2 \leq 3.4 GeV^2 \quad \Xi_c' \\
2.5 GeV^2 &\leq M^2 \leq 4.0 GeV^2 \quad \Omega_c \\
5.0 GeV^2 &\leq M^2 \leq 6.0 GeV^2 \quad \Sigma_b \\
5.0 GeV^2 &\leq M^2 \leq 6.4 GeV^2 \quad \Xi_b' \\
5.2 GeV^2 &\leq M^2 \leq 7.0 GeV^2 \quad \Omega_b
\end{align*}
\]

In regard to the continuum threshold $s_0$ appearing in the sum rules, it is known that the values of this arbitrary parameter is related to the energy of first excited state. The
difference $\sqrt{s_0} - m_{\text{ground}}$, where $m_{\text{ground}}$ is the ground state mass of the baryon, is the energy needed to excite the particle to its first energy state. This quantity usually varies between $0.3 \, \text{GeV}$ and $0.8 \, \text{GeV}$. Analysis of the mass sum rules shows that, in order to reproduce the experimental mass of the sextet baryons the continuum threshold should vary in the following domain:

$$
\sqrt{s_0} = \begin{cases} 
(3.1 \pm 0.1) \, \text{GeV} & \Sigma_c \\
(3.2 \pm 0.1) \, \text{GeV} & \Xi_c \\
(3.3 \pm 0.1) \, \text{GeV} & \Omega_c \\
(6.6 \pm 0.2) \, \text{GeV} & \Sigma_b \\
(6.7 \pm 0.2) \, \text{GeV} & \Xi_b \\
(6.8 \pm 0.2) \, \text{GeV} & \Omega_b 
\end{cases}
$$

(9)

Having determined the sum rules, input parameters, and working regions of all auxiliary parameters, we perform numerical analysis to calculate the magnetic moment $\mu_{B_Q}$ of the heavy sextet spin-1/2 baryons, whose results we can summarize as follows. As examples, in Figs. 1 and 2 we present the dependence of $\mu_{\Sigma_Q}$ on $M^2$ at several fixed values of $t$ and at the fixed value of $s_0$ listed above for the baryons $\Sigma_c^{++}$ and $\Sigma_b^0$, respectively. We observe from these figures that the magnetic moments of $\Sigma_Q$ show good stability with respect to the variation of the Borel mass parameter in aforementioned domains. The next step in finding the values of the magnetic moments of the baryons under consideration is to determine working region of the arbitrary parameter $t$, where $\mu_{B_Q}$ is practically independent of its variation. For this purpose, in Figs. 3 and 4 the dependence of $\mu_{\Sigma_Q}$ on $\cos \theta$, where $\tan \theta = t$ for the $\Sigma_c^{++}$ and $\Sigma_b^0$ baryons, at several fixed values of the Borel mass parameter $M^2$ and at the predetermined value of the continuum threshold $s_0$, are presented, respectively. It follows from these figures that the magnetic moments of the heavy sextet spin-1/2 baryons seem to be practically independent of the arbitrary parameter $t$ in the domain $-0.9 \leq \cos \theta \leq -0.6$, and insensitive to the the chosen values of $s_0$. Our final results on the magnetic moments of the heavy sextet spin-1/2 baryons are presented in Table 2. For comparison, in the same Table we present the predictions of other approaches on the magnetic moments of the relevant baryons, such as nonrelativistic quark model [11], bag model [9], phenomenological relativistic quark model [12], quark model with confinement law potential [13], relativistic quark model [4], hypercentral model [6], chiral constituent model [14], and QCD sum rules method [10].

We observe from this table that almost all approaches give, more or less, similar predictions, except the results of [9] and [4] (especially for the charmed baryons), which are smaller.

Using our results one can easily deduce the following relations among the magnetic moments of heavy baryons:

$$
\begin{align*}
\mu_{\Sigma_c^{++}} + \mu_{\Sigma_c^0} & \simeq 2\mu_{\Sigma_c^+}, \\
\mu_{\Sigma_c^{++}} + \mu_{\Omega_c^0} & \simeq 2\mu_{\Xi_c^+}, \\
\mu_{\Sigma_c^{++}} + 2\mu_{\Xi_c^0} & \simeq \mu_{\Sigma_c^0} + 2\mu_{\Xi_c^+}, \\
\mu_{\Sigma_b^+} + \mu_{\Sigma_b^0} & \simeq 2\mu_{\Xi_b^0}, \\
\mu_{\Sigma_b^+} + \mu_{\Omega_b^0} & \simeq 2\mu_{\Xi_b^0}, \\
\mu_{\Sigma_b^+} + 2\mu_{\Xi_b^0} & \simeq \mu_{\Sigma_b^0} + 2\mu_{\Xi_b^0},
\end{align*}
$$

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Table 2: Magnetic moments of the heavy sextet, $J^P = \frac{1}{2}^+$ baryons in units of the nuclear magneton $\mu_N$.

The direct measurement of the magnetic moments of heavy baryons are unlikely in the near future. Therefore, any indirect estimations of the magnetic moments of the heavy baryons could be very useful. For example, it could help extracting information about the mass spectrum of the heavy baryons. As is well known, experimentally measured mass difference is attributed to the hyperfine splitting. Moreover, the magnetic moments of quarks are proportional to the chromomagnetic moments, which determine the hyperfine splitting in the baryon spectrum. Following this reasoning, the magnetic moments of the $\Lambda_c$ and $\Lambda_b$ baryons are estimated in [28]. Hence, this approach could be an essential tool in estimating the magnetic moments of the heavy baryons.

In conclusion, the magnetic moments of the heavy sextet $J^P = \frac{1}{2}^+$ baryons are calculated in framework of the light cone QCD sum rules method. Empirically, linearly independent relations among the magnetic moments of the sextet baryons are obtained. Comparison of our results with the predictions of other approaches is presented.
Appendix A: Photon distribution amplitudes

Explicit forms of the photon DAs [21].

\[ \varphi_\gamma(u) = 6u\bar{u}\left[ 1 + \varphi_2(\mu)C_2^\gamma(u - \bar{u}) \right], \]
\[ \psi^V(u) = 3[3(2u - 1)^2 - 1] + \frac{3}{64}(15w^V_\gamma - 5w^A_\gamma)[3 - 30(2u - 1)^2 + 35(2u - 1)^4], \]
\[ \psi^A(u) = [1 - (2u - 1)^2][5(2u - 1)^2 - 1]\frac{5}{2}\left( 1 + \frac{9}{16}w^V_\gamma - \frac{3}{16}w^A_\gamma \right), \]
\[ \mathcal{A}(\alpha_i) = 360\alpha_q\alpha_q\alpha_q^2\left[ 1 + w^A_\gamma\frac{1}{2}(7\alpha_g - 3) \right], \]
\[ \mathcal{V}(\alpha_i) = 540w^V_\gamma(\alpha_q - \alpha_q)\alpha_q\alpha_q\alpha_q^2, \]
\[ h_\gamma(u) = -10(1 + 2\kappa^+)^2C_2^\gamma(u - \bar{u}), \]
\[ A(u) = 40u^2\bar{u}^2(3\kappa - \kappa^+ + 1) + 8(\zeta_2^+ - 3\zeta_2)|u\bar{u}(2 + 13u\bar{u}) + 2u^3(10 - 15u + 6u^2)\ln(u) \]
\[ + 2\bar{u}^3(10 - 15\bar{u} + 6\bar{u}^2)\ln(\bar{u})], \]
\[ T_1(\alpha_i) = -120(3\zeta_2 + \zeta_2^+)(\alpha_q - \alpha_q)\alpha_q\alpha_q\alpha_g, \]
\[ T_2(\alpha_i) = 30\alpha_g^2(\alpha_q - \alpha_q)[(\kappa - \kappa^+) + (\zeta_1 - \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)], \]
\[ T_3(\alpha_i) = -120(3\zeta_2 - \zeta_2^+)(\alpha_q - \alpha_q)\alpha_q\alpha_q\alpha_g, \]
\[ T_4(\alpha_i) = 30\alpha_g^2(\alpha_q - \alpha_q)[(\kappa + \kappa^+) + (\zeta_1 + \zeta_1^+)(1 - 2\alpha_g) + \zeta_2(3 - 4\alpha_g)], \]
\[ S(\alpha_i) = 30\alpha_g^2\{(\kappa + \kappa^+)(1 - \alpha_g) + (\zeta_1 + \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \]
\[ + \zeta_2[3(\alpha_q - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}, \]
\[ \tilde{S}(\alpha_i) = -30\alpha_g^2\{(\kappa - \kappa^+)(1 - \alpha_g) + (\zeta_1 - \zeta_1^+)(1 - \alpha_g)(1 - 2\alpha_g) \]
\[ + \zeta_2[3(\alpha_q - \alpha_q)^2 - \alpha_g(1 - \alpha_g)]\}]. \]

The parameters entering the above DA’s are borrowed from [21] whose values are \( \varphi_2(1 \text{ GeV}) = 0, \ w^{V}_\gamma = 3.8 \pm 1.8, \ w^{A}_\gamma = -2.1 \pm 1.0, \ \kappa = 0.2, \ \kappa^+ = 0, \ \zeta_1 = 0.4, \ \zeta_2 = 0.3, \ \zeta_1^+ = 0, \) and \( \zeta_2^+ = 0. \)
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Figure captions

**Fig. (1)** The dependence of the magnetic moment of the $\Sigma^+ c$ baryon on $M^2$, at several fixed values of $t$, and at $s_0 = 9.0 \ GeV^2$.

**Fig. (2)** The same as Fig. (1), but for the $\Sigma^0 b$ baryon, at $s_0 = 41.0 \ GeV^2$.

**Fig. (3)** The dependence of the magnetic moment of the $\Sigma^+ c$ baryon on $\cos \theta$, at several fixed values of $M^2$, and at $s_0 = 9.0 \ GeV^2$.

**Fig. (4)** The same as Fig. (3), but for the $\Sigma^0 b$ baryon, at $s_0 = 41.0 \ GeV^2$. 
$t = 5$
$t = 3$
$t = 1$
$t = -1$
$t = -3$
$t = -5$

$M^2 (GeV^2)$

Figure 1:

$g_{M}^{\Sigma_c^0} (\mu N)$

$s_0 = 9.0 GeV^2$

$M^2 (GeV^2)$

Figure 2:

$g_{M}^{\Sigma_0} (\mu N)$

$s_0 = 41.0 GeV^2$
Figure 3:

\[ M^2 = 2.6 \text{ GeV}^2 \quad \text{and} \quad M^2 = 3.0 \text{ GeV}^2 \]

\[ s_0 = 9.0 \text{ GeV}^2 \]

Figure 4:

\[ M^2 = 5.5 \text{ GeV}^2 \quad \text{and} \quad M^2 = 6.0 \text{ GeV}^2 \]

\[ s_0 = 41.0 \text{ GeV}^2 \]