Low-frequency gravitational waves from cosmological compact binaries

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ABSTRACT
We consider gravitational waves emitted by various populations of compact binaries at cosmological distances. We use population synthesis models to characterize the properties of double neutron stars, double black holes and double white dwarf binaries, and white dwarf–neutron star, white dwarf–black hole and black hole–neutron star systems.

We use the observationally determined cosmic star formation history to reconstruct the redshift distribution of these sources and their merging rate evolution.

The gravitational signals emitted by each source during its early spiralling in phase add randomly to produce a stochastic background in the low-frequency band with spectral strain amplitude between $10^{-18}$ and $5 \times 10^{-17} \text{Hz}^{-1/2}$ at frequencies in the interval $5 \times 10^{-6} - 5 \times 10^{-5} \text{Hz}$.

The overall signal, which at frequencies above $10^{-4} \text{Hz}$ is largely dominated by double white dwarf systems, might be detectable with LISA in the frequency range 1–10 mHz and acts like a confusion-limited noise component, which might limit the LISA sensitivity at frequencies above 1 mHz.

Key words: gravitation – gravitational waves – binaries: general – stars: formation.

1 INTRODUCTION

Binaries with two compact stars are the most promising sources for gravitational radiation. The final phase of spiral in may be detected with ground-based (LIGO, VIRGO, GEO and TAMA) and space-borne laser interferometers (LISA). This has motivated researchers to model gravitational waveforms and to develop population synthesis codes to estimate the properties and formation rates of possible sources for gravitational wave radiation.

Since there is not yet a single prescription for calculating the gravitational emission from a compact binary system, it is customary to divide the gravitational waveforms into two main pieces: the in-spiral waveform, emitted before tidal distortions become noticeable, and the coalescence waveform, emitted at higher frequencies during the epoch of distortion, tidal disruption and/or merger (Cutler et al. 1993).

As the binary, driven by gravitational radiation reaction, spirals in, the frequency of the emitted wave increases until the last three orbital cycles prior to complete merger.

With post-Newtonian expansions of the equations of motion for two point masses, the waveforms can be computed fairly accurately in the relatively long phase of spiral in (see, for a recent review, Rasio & Shapiro 2000 and references therein). Conversely, the gravitational waveform from the coalescence can only be obtained from extensive numerical calculations with a fully general relativistic treatment. Such calculations are now well underway (Brady, Creighton & Thorne 1998; Damour, Iyer & Sathyaprakash 1998; Rasio & Shapiro 1999).

In this paper, we consider the low-frequency signal from the early phase of the spiral in, which is of interest for space-borne antennas, such as LISA. For this purpose, we use the leading-order expression for the single-source emission spectrum, obtained using the quadrupole approximation. Our analysis includes various populations of compact binary systems: black hole–black hole (bh, bh), neutron star–neutron star (ns, ns), white dwarf–white dwarf (wd, wd) and mixed systems such as (ns, wd), (bh, bh) or (bh, ns).

For some of these sources [(ns, ns), (wd, wd) and (ns, wd)], statistical information on the event rate can be inferred from electromagnetic observations. In particular, there are several observational estimates of the (ns, ns) merger rate obtained from statistics of the known population of binary radio pulsars (Narayan, Piran & Shemi 1991; Phinney 1991; van den Heuvel & Lorimer 1996; Stairs et al. 1998; Kalogera & Lorimer 2000).

A rather large number of close white dwarf binaries have been found recently (see Maxted & Marsh 1999; Moran 1999). However, it is customary to constrain the (wd, wd) merger rate.
from the observed SNeIa rate (see Postnov & Prokhorov 1998). Also the population of binaries where a radio pulsar is accompanied by a massive unseen white dwarf may be considerably higher than hitherto expected (Portegies Zwart & Yungelson 1999).

Since most stars are members of binaries and the formation rate of potential sources of gravitational waves may be abundant in the Galaxy, the gravitational-wave signal emitted by such binaries might produce a stochastic background. This possibility has been explored by various authors, starting from the earliest work of Mironovskij (1966) and Rosi & Zimmermann (1976) until the more recent investigations of Hils, Bender & Webbink (1990), Lipunov et al. (1995), Bender & Hils (1997), Giaiottj, Bonazzola & Gourgoulhon (1997), Giampieri (1997), Postnov & Prokhorov (1998), Webbink & Han 1998 and Nelmans, Portegies Zwart & Verbunt (1999). This background, which acts like a noise component for the interferometric detectors, has always been viewed as a potential obstacle for the detection of gravitational wave backgrounds coming from the early Universe.

In this paper we extend the investigation of compact binary systems to extragalactic distances, accounting for the binaries which have been formed since the onset of galaxy formation in the Universe. Following Ferrari, Matarrese & Schneider (1999a,b; hereafter referred to as FMSI and FMSII, respectively), we modulate the binary formation rate in the Universe with the cosmic star formation history derived from observations of field galaxies out to redshift z ~ 5 (see, e.g., Madau, Pozzetti & Dickinson 1998a; Steidel el al. 1999).

The magnitude and frequency distribution of the integrated gravitational signal produced by the cosmological population of compact binaries is calculated from the distribution of binary parameters (masses and types of both stars, orbital separations and eccentricities). These orbital parameters characterize the gravitational-wave signal which we observe on Earth.

Detailed information for the properties of the binary population may be obtained through population synthesis. We use the binary population synthesis code seba to simulate the characteristics of the binary population in the Galaxy (Portegies Zwart & Verbunt 1996; Portegies Zwart & Yungelson 1998). The characteristics of the extragalactic population are derived from extrapolating these results to the local Universe.

The paper is organized as follows. In Section 2 we describe the population synthesis calculations. Section 3 deals with the energy spectrum of a single source followed, in Section 4, by the derivation of the extragalactic backgrounds for the different binary populations. In Sections 3 and 4 we also give details on the adopted astrophysical and cosmological properties of the systems. In Section 5, we compute the spectral strain amplitude produced by each cosmological population and investigate its detectability with LISA. Finally, in Section 6 we summarize our main results and compare them with other previously estimated astrophysical background signals.

2 POPULATION SYNTHESIS MODEL

In order to characterize the main properties of different compact binary systems, we use the binary population synthesis program seba of Portegies Zwart & Yungelson (1998). Details of the code can be found in Portegies Zwart & Yungelson (1998). Here, we simply recall the main assumptions of their adopted model B, which satisfactorily reproduces the properties of observed high-mass binary pulsars (with neutron star companions).

The following initial conditions were assumed: the mass of the primary star m1 is determined using the mass function described by Scalo (1986) between 0.1 and 100 M⊙. For a given m1, the mass of the secondary star m2 is randomly selected from a uniform distribution between a minimum of 0.1 M⊙ and the mass of the primary star. The semimajor axis distribution is taken flat in log a (Kraicheva et al. 1978) ranging from Roche lobe contact up to 103 R⊙ (Abt & Levy 1978; Duquennoy & Mayor 1991). The initial eccentricity distribution is independent of the other orbital parameters, and is Ξ(e) = 2e (Duquennoy & Mayor 1991). Neutron stars receive a velocity kick upon birth. Following Hartman et al. (1997), model B assumes the distribution for isotropic kick velocities (Paczyński 1990),

$$P(u) du = \frac{4}{\pi (1 + u^2)} du$$

with u = v/σ and σ = 600 km s⁻¹. Black holes are assumed to received a velocity kick upon birth taken from the same distribution but scaled with their mass via v ≈ 1.4 M⊙/mBH. Here mBH is the mass of the black hole, which equals the mass of the exploding star.

The birth rate of the various compact binaries is normalized to the type II + Ib/c supernova rate (see Portegies Zwart & Verbunt 1996). The supernova rate of 0.01 yr⁻¹ was assumed to be constant over the lifetime of the galactic disc (~10 Gyr). The current supernova rate in the Galaxy is uncertain by about a factor of 2, being between 1/50 and 1/100 yr, and it may have varied strongly in the past. This directly affects our normalization and therefore the total birth and merger rates. Normalizing to, for example, the star formation rate in the Galaxy hardly reduces the uncertainty in the normalization.

The main uncertainty in the binary population synthesis model hides in the selection of initial conditions, the normalization, the binary fraction and a few internal parameters which are understood rather poorly. The initial conditions are rather well established for G-dwarfs by Duquennoy & Mayor (1991), but more recent work including higher-mass primaries is painfully sparse. The results of our calculations, however, are quite sensitive to these parameters; in particular, the initial mass ratio distribution may affect the birth rate of various binary types by about a factor of 3 (see Portegies Zwart & Yungelson 1998; Kalgore 1999). Variation in the initial eccentricity distribution does not significantly affect the formation rates of compact binaries as these binaries are circularized by tidal interaction (see Portegies Zwart & Verbunt 1996). The shape and slope of the initial mass function affects the relative birth rates of various binary species; a rather steep initial mass function results in a higher birth rate of white dwarfs at the cost of neutron stars and black holes. We adopted the initial mass function as described by Scalo (1986) which matches the solar neigbourhood.

Uncertainties in the modelling are mainly a result of unknown physics. The main parameters are the efficiency of common envelope ejection, the amount of mass lost from a mass-transferring binary system and the amount of angular momentum carried with the lost material. These parameters affect the evolution of an individual binary quite strongly and the effect may be different for low-mass binaries compared with high-mass binaries.

Portegies Zwart & Yungelson (1998) discuss the effect of changing model parameters and initial conditions on the evolution of binaries with an initial primary mass ≥ 7 M⊙. The birth rates of...
the binaries with the highest-mass primaries, those which evolve into (bh, bh), are quite insensitive to the adopted model parameters. The progenitors of these binaries hardly ever experience a phase of mass transfer owing to the large mass-loss rates in the stellar winds, which tends to widen the binary orbit rather effectively. The birth rates of (ns, ns) and (bh, ns) binaries varies by at most a factor of 4 when the efficiency of the common-envelope ejection is varied by an order of magnitude (see Portegies Zwart & Yungelson 1998).

The uncertainties in the evolution of binaries with lower-mass primaries is discussed extensively by Nelemans et al. (2000). They use the binary population synthesis program seba to compute the formation and evolution of close white dwarf binaries. They compare the birth and merger rates of these binaries with the population synthesis calculations of Iben, Tutukov & Yungelson (1997) and Han (1998), and with the observed sample of 16 (wd, wd) binaries (see Maxted & Marsh 1999; Moran 1999). Their changes in the initial conditions and model parameters result in a variation of the birth rates of (wd, wd) binaries of less than a factor 2.

Variations in the initial conditions and model parameters affects binaries with different mass primaries in a slightly different way, causing the birth rates of one binary type to vary by ~50 per cent compared with another type; the relative birth rates are rather constant. Total birth rates are uncertain by a factor of 2–4, partially caused by uncertainties in the normalization, the initial conditions and the above discussed model parameters.

When computing the birth and merger rates we account for the time delay between the formation of the progenitor system and that of the corresponding degenerate binary, \( \tau_s \). Its value is set by the time it takes for the least massive of the two companion stars to evolve on the main sequence. For (bh, bh), (ns, ns) and (bh, ns) systems \( \tau_s \approx 50 \) Myr and it is negligible compared with the assumed lifetime of the galactic disc. Conversely, (wd, wd), (ns, wd) and (bh, wd) binaries follow a slower evolutionary clock and \( \tau_s \) can be considerably larger. The cumulative probability distribution, \( P(\tau_s) \), predicted by the population synthesis code is shown in Fig. 1. For these systems \( \tau_s \) can be as large as 10 Gyr, although all systems are predicted to have \( \tau_s \approx 10 \) Gyr.

After the degenerate binary has formed, its further evolution is determined by the time it takes to radiate away its orbital energy in gravitational waves. The time between the formation of the degenerate system and its final coalescence, \( \tau_{m} \), depends on the orbital configuration and on the mass of the two companion stars. The predicted cumulative probability distribution is shown in Fig. 2 for the (wd, wd), (ns, ns) and (ns, wd) samples. We see from the figure that there is a significant fraction of the systems that do not merge in 10 Gyr. For (bh, bh) binaries and mixed systems with one black hole companion the population synthesis code predicts very long merger times. In particular, all (bh, bh) systems appear to have \( \tau_{m} \) greater than 15 Gyr. As explained above, binaries with a black hole companion are characterized by very large initial orbital separations (see, e.g., Fig. 3). In fact, bh progenitors are very massive stars and have a very strong stellar wind. For this reason they do not easily reach Roche lobe overflow and rarely experience a phase of mass transfer, which is required to reduce the orbital separation of the stars. Unfortunately, the evolution (especially the amount of mass loss in the stellar winds) of high-mass stars is rather uncertain (Langer et al. 1994). The result that we obtain at least indicates that it will be very rare to observe any of these bh mergers. Recently, Portegies Zwart &

\[
\frac{dE}{d\nu} = \frac{\pi}{3} \left( \frac{M}{\nu} \right)^{5/3},
\]

where \( M = m_1 + m_2 \) denotes the total mass and \( M = m_1 m_2 / M \) is the reduced mass.

The frequency \( \nu \) at which gravitational waves are emitted is

![Figure 1.](https://example.com/figure1.png)

Figure 1. The cumulative probability distribution for the time delay \( \tau_s \), (in Myr) between the formation of the progenitor system and the formation of the corresponding degenerate binary obtained for the (bh, wd), (ns, wd) and (wd, wd) samples.

![Figure 2.](https://example.com/figure2.png)

Figure 2. The cumulative probability distribution for the merger time \( \tau_{m} \) (in Myr) is shown for the (ns, ns), (ns, wd) and (wd, wd) samples.

McMillan (2000), however, identified a new channel for producing black hole binaries which are eligible to mergers on a relatively short time-scale.

In Table 1, we summarize the results for all binary types that we have investigated.

### 3 IN-SPIRAL ENERGY SPECTRUM OF SINGLE SOURCES

Assuming that the orbit of the binary system has already been circularized by gravitational radiation reaction, the in-spiral spectrum \( dE/d\nu \) emitted by a single source can be obtained using the quadrupole approximation (Misner, Thorne & Wheeler 1995). The resulting expression, in geometrical units \( (G = c = 1) \), can be written as

\[
\frac{dE}{d\nu} = \frac{\pi}{3} \left( \frac{M}{\nu} \right)^{5/3},
\]
The probability distribution for the orbital parameters of (bh, bh) systems (upper panel) is compared with that obtained for the (ns, ns) (lower panel) population. The semimajor axis is in R⊙.

Table 1. The galactic birth rates, \( R_{X,gal} \), and merger rates, \( R_{X,gal}^{mer} \), obtained for each compact binary type X using model B of Portegies Zwart & Yungelson (1998), see text. The rates are normalized to the core-collapse supernova rate of 0.01 yr⁻¹ and 100 per cent binarity. Merger rates are computed after 10 Gyr of the evolution of the Galaxy with a constant supernova rate.

| Binary type X | \( R_{X,gal} \) (yr⁻¹) | \( R_{X,gal}^{mer} \) (yr⁻¹) |
|---------------|----------------|----------------|
| (bh, bh)      | 6.3×10⁻⁵       | NA             |
| (bh, ns)      | 1.0×10⁻⁵       | 1.0×10⁻⁶       |
| (bh, wd)      | 4.4×10⁻⁵       | 10⁻⁵           |
| (ns, ns)      | 3.6×10⁻⁵       | 2.5×10⁻⁵       |
| (ns, wd)      | 3.6×10⁻⁴       | 1.6×10⁻⁴       |
| (wd, wd)      | 4.4×10⁻²       | 4.8×10⁻³       |

\[ (\pi \nu)^{-8/3} = (\pi \nu_{\text{max}})^{-8/3} + \frac{256}{5} \mathcal{M}_{5/3}(t_c - t), \]  

where \( t_c \) is the time of the final coalescence and we terminate the in-spiral spectrum at a frequency \( \nu_{\text{max}} \). The approximation of circular orbits is well justified for (wd, wd) systems, which are expected to be circular at the time of their formation. Other binary types may not be circular during the early spiralling in phase but their contributions to the overall stochastic signal is less significant (see the next section). It is customary to consider the in-spiral spectrum as a good approximation all the way up to \( \nu_{\text{LSCO}} \), i.e. the frequency of the quadrupole waves emitted at the last stable circular orbit (LSCO, see, e.g., Flanagan & Hughes 1998). Post-Newtonian terms lead to corrections of a few tens of per cent and, for the purposes of our study, might be neglected. However, to be conservative, we set the value of \( n_{\text{per cent}} \) and, for the purposes of our study, might be neglected.

The value \( n_{\text{per cent}} \) and, for the purposes of our study, might be neglected. For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \). For (wd, wd) binaries and binaries with one wd companion, the maximum frequency, which is more conservative for constraining the maximum frequency, is 0.022/M and \( \nu_{\text{max}} \approx 0.19 n_{\text{LSCO}} \).
where \( \nu \) is the observed frequency emitted by a system at time \( t(z) \)

\[
(\pi \nu)^{-8/3} = (\pi \nu_{\text{max}})^{-8/3}(1 + z)^{8/3} + \frac{256}{5} \mathcal{M}^{5/3} \\
\times [t(z) + \tau_m - t(z)](1 + z)^{8/3}
\]

and we have written the time of the final coalescence \( t_c = t(z) \) in terms of the time of formation \( t(z) \) and of the merger time \( \tau_m = t(z) - t(z) \).

4 Extragalactic backgrounds from different binary populations

Our main purpose is to estimate the stochastic background signal generated by different populations of compact binary systems at extragalactic distances.

These gravitational sources have been forming since the onset of galaxy formation in the Universe and for each binary type \( X \) [(ns, ns); (wd, wd); (bh, bh); (ns, wd); (bh, wd); (bh, ns)] we should think of a large ensemble of unresolved and uncorrelated elements, each characterized by its masses \( m_1 \) and \( m_2 \) (or \( M \) and \( \mu \)), by its redshift and by its time delays \( \tau_s \) and \( \tau_m \) (see equations 6 and 7).

Thus, in order to consider all contributions from different elements of the ensemble \( X \), we must integrate the single-source emission spectrum over the distribution functions for the masses \( M \) and \( \mu \), time delays and redshifts.

The distribution functions for \( \tau_s \) and \( \tau_m \) depend on the binary type \( X \) and have been derived from the population synthesis code discussed in the previous section. The distribution function for \( M \) can be similarly estimated.

However, \( \tau_s \), \( \tau_m \) and \( M \) are not independent random variables. In fact, \( \tau_s \) depends on the masses of the two progenitor stars and \( \tau_m \) and \( M \) are correlated because \( M \) defines the rate of orbital decay, once the degenerate system has formed.

Thus, for each binary population \( X \), we consider the joint probability distribution for \( \tau_s \), \( \tau_m \) and \( M \),

\[
p_X(\tau_s, \tau_m, M) \, d\tau_s \, d\tau_m \, dM.
\]

Conversely, the redshift distribution function, i.e. the evolution of the formation rate for each binary type \( X \), can be deduced from the observed cosmic star formation history out to \( z \sim 5 \).

In the following subsections, we illustrate the procedure we have followed to derive the birth and merger rates for all binary populations and to compute the spectra of the corresponding stochastic gravitational backgrounds.

4.1 Cosmic star formation rate

Over the last few years, the extraordinary advances attained in observational cosmology have led to the possibility of identifying actively star-forming galaxies at increasing cosmological look-back times [for a thorough review see Ellis (1997)]. Using the rest-frame ultraviolet–optical luminosity as an indicator of the star formation activity and integrating on the overall galaxy population, the data obtained with the Hubble Space Telescope (HST, Madau et al. 1996; Connolly et al. 1997; Madau et al. 1998a) Keck and other large telescopes (Steidel et al. 1996, 1999) together with the completion of several large redshift surveys (Lilly et al. 1996; Gallego et al. 1995; Treyer et al. 1998) have enabled us, for the first time, to derive coherent models for the star formation rate evolution throughout the Universe. A collection of some data obtained at different redshifts is shown in the upper panel of Fig. 4 for a flat cosmological background model with \( \Omega_m = 1 \), \( \Omega_{\Lambda} = 0 \), \( H_0 = 50 \text{ km} \, \text{s}^{-1} \, \text{Mpc}^{-1} \) and a Scalo (1986) IMF. Upper panel, the data points correspond to UV, H\( \alpha \) and IR observations of field galaxies. Full circles, UV observations of Treyer et al. (1998), Lilly et al. (1996), Connolly et al. (1997), Steidel et al. (1999); asterisks, H\( \alpha \) observations of Gallego et al. (1995), Gronwall (1999), Tresse & Maddox (1998), Glazebrook et al. (1999), Yan et al. (1999); triangles, ISO IR observations (Flores et al. 1999) and the lower limit of SCUBA data (Hughes et al. 1998). Lower panel, the dust-corrected SFR density as derived from UV data in the two models most favoured by observations predicted by: asterisks, Calzetti & Heckman (1999); full circles, Pei et al. (1999) (see text).

Although the strong luminosity evolution observed between redshift 0 and 1–2 is believed to be quite firmly established, the behaviour of the star formation rate at high redshift is still relatively uncertain. In particular, the decline of the star formation rate density implied by the \( z \sim 4 \) point of the Hubble Deep Field (HDF; see Fig. 4) is now contradicted by the star formation rate density derived from a new ground-based sample of Lyman break galaxies with \( z \sim 4 \) (Steidel et al. 1999) which, instead, seems to indicate that the star formation rate density remains substantially constant at \( z > 1 \). It has been suggested that this discrepancy might be caused by problems of sample variance in the HDF point at \( z = 4 \) (Steidel et al. 1999).

Because dust extinction can lead to an underestimate of the real
ultraviolet–optical emission and, ultimately, of the real star
formation activity, the data shown in the upper panel of Fig. 4
need to be corrected upwards according to specific models for the
evolution of dust opacity with redshift. In the lower panel of Fig.
4, the data have been dust-corrected according to factors obtained
by Calzetti & Heckman (1999) and by Pei, Fall & Hauser (1999).
Using different approaches, these authors have recently investigat-
gated the cosmic histories of stars, gas, heavy elements and dust in
galaxies using as inputs the available data from quasar absorption-
line surveys, optical and UV imaging of field galaxies, redshift
surveys and the COBE DIRBE and FIRAS measurements of the
cosmic IR background radiation. The solutions they obtain appear
to reproduce remarkably well a variety of observations that were
not used as inputs, among which the SFR at various redshifts
from Hα, mid-IR and submillimetre observations and the mean
abundance of heavy elements at various epochs from surveys of
damped Lyman-α systems.

As we can see from the lower panel of Fig. 4, spectroscopic and
photometric surveys in different wavebands point to a consistent
picture of the low-to-intermediate redshift evolution: the SFR
density rises rapidly as we go from the local value to a redshift of
between ~1 and 2, and remains roughly flat between redshifts of
~2–3. At higher redshifts, two different evolutionary tracks seem
to be consistent with the data: the SFR density might remain
substantially constant at z ~ 2 (Calzetti & Heckman 1999) or it
might decrease again out to a redshift of ~4 (Pei et al. 1999).
Hereafter, we always indicate the former model as the ‘monolithic
scenario’ and the latter as the ‘hierarchical scenario’, although this
choice is only meant to be illustrative. In fact, preliminary
considerations have pointed out that a constant SFR activity at
high redshifts might not be unexpected in hierarchical structure
formation models (Steidel et al. 1999).

Thus, we have updated the star formation rate model that we
have considered in previous analyses (FMSI, FMSII), even though
the gravitational wave backgrounds are more contributed by low-
to-intermediate redshift sources than by distant ones. In addition,
if a larger dust correction factor should be applied at intermediate
redshifts, this would result in a similar amplification of the
gravitational background spectra.

4.2 Birth and merger rate evolution

Following the method we have previously proposed (FMSI,
FMSII), for each binary type X the birth and merger-rate evolution
could be computed from the observed star formation rate
evolution. However, this procedure proves to be unsatisfactory
because it fails to provide a fully consistent normalization. Its
main weakness is that, even if we assume 100 per cent binarity,
i.e. that all stars are in binary systems, the star formation histories
that we have described above are not corrected for the presence
of secondary stars. For the mass distributions that we have
considered, secondary stars are expected to give a significant
contribution to the observed UV luminosity as they account for
~1/3 of the fraction of mass in stars more massive than 8M☉.

In order to circumvent the necessity of extrapolating the UV
luminosity indication of massive star formation to the full range of
stellar masses predicted by the model, we could directly normalize
to the rate of core-collapse supernovae. This is consistent with the
adopted normalization for galactic rates.

The core-collapse supernova rate can be derived directly from
the observed UV luminosity at each redshift, as stars that
dominate the UV emission from a galaxy are the same stars
that, at the end of the nuclear burning, explode as type II + Ib/c
supernovae. Moreover, the supernova rate is observed independ-
ently of the SFR. Therefore, it can be used as an alternative
normalization.

Figure 5. The rest-frame frequency of core-collapse SNe versus redshift
predicted by the monolithic and hierarchical models. The predictions are
consistent with the observed value for the present-day galaxy population
(see text).

Figure 6. The formation rate of (bh, bh) (asterisks), (ns, ns) (dots) and (bh,
ns) (triangles) binaries as a function of z is shown for hierarchical (upper
panel) and monolithic (lower panel) scenarios for a flat cosmological
background.
Low-frequency GWs from compact binaries

The rates of core-collapse supernovae predicted by the models shown in Fig. 4 are shown in Fig. 5, assuming a flat cosmological background model with zero cosmological constant and $h = 0.5$.

In the same figure, we have plotted the available observations for the core-collapse supernova frequency in the local Universe (Evans et al. 1989, Tamman et al. 1994, Cappellaro et al. 1997; see also Madau, Della Valle & Panagia 1998b).

The binary birth rate per entry per year and comoving volume $\eta(z)$ can be related to the core-collapse supernova rate $n_{\text{SNaeII+Ib/c}}(z)$ shown in Fig. 5 in the following way:

$$\eta(z) = \frac{n_{\text{SNaeII+Ib/c}}(z)}{N_{\text{SNaeII+Ib/c}}}$$

where $N_{\text{SNaeII+Ib/c}}$ is the total number of core-collapse supernovae that we find in the simulation.

In order to estimate, from $\eta(z)$, the birth and merger-rate evolution of a degenerate binary population $X$, we need to multiply equation (9) by the number of type $X$ systems in the simulated samples, $N_X$, and we also need to properly account for both $\tau_d$ and $\tau_m$.

We shall assume that the redshift at the onset of galaxy formation in the Universe is $z_f = 5$ and that a zero-age main-sequence binary forms at a redshift $z_f$. After a time interval $\tau_d$ the system has evolved into a degenerate binary. Then, the redshift of formation of the degenerate binary system, $z_f$, is defined as $\tau(z_f) = \tau(z_d) + \tau_d$.

The system then evolves according to gravitational wave reaction until, after a time interval $\tau_m$, it finally merges. Thus, the redshift at which coalescence occurs, $z_c$, is given by $\tau(z_c) = \tau(z_f) + \tau_m$.

This simple picture implies that the number of $X$ systems formed per unit time and comoving volume at redshift $z_t$ is

$$\dot{n}_X(z_t) = \int d\tau_m d\mathcal{M} \frac{\eta_f N_X}{(1 + z_c)} \cdot \delta(\tau_m - \tau_c) \delta(\mathcal{M} - \mathcal{M}_i),$$

where $f_X = N_X/N_{\text{SNaeII+Ib/c}}$ and $z_c$ is defined by $\tau(z_c) = \tau(z_f) - \tau_d$.

If we write,

$$p_X(\tau_m, \mathcal{M}) = \frac{1}{N_X} \sum_{i} \delta(\tau_m - \tau_{m,i}) \delta(\mathcal{M} - \mathcal{M}_i),$$

where $\tau_{m,i}$ and $\mathcal{M}_i$ indicate the time delays and the chirp mass for the $i$th element of the ensemble $X$, the birth rate reads,

$$\dot{n}_X(z_t) = \frac{1}{N_{\text{SNaeII+Ib/c}}} \sum_{i} \frac{n_{\text{SNaeII+Ib/c}}(z_c)}{(1 + z_c)} \cdot \Theta[\tau(z_c) - \delta(z_c) - \tau_{d,i}],$$

where $\Theta(x)$ is the step function.
Similarly, the number of $X$ systems per unit time and comoving volume which merge at redshift $z_c$ is,

$$n_{\text{mrg}}^X(z_c) = \frac{1}{N_{\text{SNaeII}} \Delta t_{\text{hs}}} \sum_{t_{\text{m}}} \frac{dM_{\text{X}}(t_{\text{m}}, t_{\text{m}, i})}{(1 + z_c)}$$

where $z_s$ is defined by $t(z_s) = t(z_c) - t_{\text{m}} - t_{\text{sc}}$. If we apply equation (11), we can write the merger rate in a form similar to equation (12), i.e.

$$n_{\text{mrg}}^X(z_c) = \int_{t(z_c)}^{t(z_{c})} d\tau_{\text{m}} \frac{dM_{\text{X}}(\tau_{\text{m}}, \tau_{\text{m}, i})}{(1 + z_c)}$$

where $z_s$ is defined by $t(z_s) = t(z_c) - t_{\text{m}} - t_{\text{sc}}$. If we apply equation (11), we can write the merger rate in a form similar to equation (12), i.e.

$$n_{\text{mrg}}^X(z_c) = \frac{1}{N_{\text{SNaeII}} \Delta t_{\text{hs}}} \sum_{t_{\text{m}}} \frac{dM_{\text{X}}(t_{\text{m}}, t_{\text{m}, i})}{(1 + z_c)}$$

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where $z_s$ is defined by $t(z_s) = t(z_c) - t_{\text{m}} - t_{\text{sc}}$. If we apply equation (11), we can write the merger rate in a form similar to equation (12), i.e.

Using this procedure, we compute the birth and merger rates for all the synthetic binary populations. The results are presented in Figs 6, 7 and 8.

Owing to their relatively small $t_s$ compared with the cosmic time, the birth rates of (bh, bh), (ns, ns) and (bh, ns) systems closely trace the UV-luminosity evolution, although with different amplitudes. Our simulation suggests that (bh, bh) systems are more numerous than (ns, ns) or (bh, ns) (see Fig. 6).

Conversely, Fig. 7 shows that the birth rates of (wd, wd), (ns, wd) and (bh, wd) systems misrepresent the original UV-luminosity evolution consequently, of their large $t_s$. The largest is the characteristic time delay $\tau_{s}$, the more the maximum is shifted versus lower redshifts because the intense star formation activity observed at $z \approx 2$, especially for monolithic scenarios, boosts the formation of degenerate systems at $z \leq 2$. For hierarchical

Figure 9. The spectral energy density of the gravitational background produced by various extragalactic populations of degenerate binaries in monolithic and hierarchical scenarios assuming a flat cosmological background with zero cosmological constant.

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4.3 Stochastic backgrounds

Having characterized each ensemble $X$ by the distribution of chirp mass and time delays, $p_X(\tau, \tau_m, M)$, and by the birth rate density evolution per entry $\eta(z)$, we can sum up the gravitational signals coming from all the elements of the ensemble. The spectrum of the resulting stochastic background, for a binary type $X$ and at a given observation frequency $\nu$, is given by the following expression:

$$\frac{dE}{d\nu} = \int_0^\nu d\nu' \sum_{i=1}^8 \int_0^{\tau(z_i)} d\tau \frac{N_X \eta(z_i)}{(1 + z_i^*)} \times \int_0^\infty dM d\tau_m p_X(\tau, \tau_m, M) \frac{dV}{dz_c} f(\nu, z_c^*),$$

(15)

where $z_F$ is the redshift of the onset of star formation in the Universe, $z_I$ is the redshift of formation of the degenerate binary systems, $z_c^*$ is the redshift of formation of the corresponding progenitor system defined by $f(\nu, z_c^*) = f(\nu, z_c) - f(\nu, z_c^*)$ is given by equation (5) and $z_c^*$ is the redshift of emission that an element of the ensemble must have in order to contribute to the energy density at the observation frequency $\nu$.

It follows from equation (7) that, for a given observation frequency $\nu$, $z_c^*$ is a function of $z_F$, $\tau_m$, $M$ and $\nu_{\text{max}}$. In principle, an in-spiralling compact binary system emits a continuous signal from its formation to its final coalescence, thus $z_c \equiv z_c^* \equiv z_t$.

However, in equation (15) we do not restrict ourselves to systems which reach their final coalescence at $z_c \equiv 0$ as we are interested to any source between $z = 0$ and $z = z_F$ emitting gravitational waves during its early spiralling in phase. Therefore, the signals which contribute to the local energy density at observation frequency $\nu$ might be emitted anywhere between $\sup[0, z_c] \leq z_c \leq z_t$, provided that,

$$(\pi \nu)^{-8/3} = (\pi \nu_{\text{max}})^{-8/3} (1 + z_c^*)^{8/3} + \frac{256}{5} M^5$$

$$(1 + z_c^*)^{8/3}.$$  

(16)

Substituting equation (11) in equation (15), we can write the background energy density generated by a population $X$ in the form,

$$\frac{dE}{d\nu} = \int_0^\nu d\nu' \sum_{i=1}^8 \int_0^{\tau(z_i)} d\tau \frac{N_X \eta(z_i)}{(1 + z_i^*)} \times \int_0^\infty dM d\tau_m p_X(\tau, \tau_m, M) \frac{dV}{dz_c} f(\nu, z_c^*) \text{erg cm}^{-2} \text{s}^{-1} \text{Hz}^{-1},$$

(17)

where $z_c^*$ satisfies equation (16).

The predicted spectral energy densities for the populations of degenerate binary types that we have considered are plotted in Fig. 9. For each binary type, we show the results obtained assuming both monolithic and hierarchical scenarios for the evolution of the underlying galaxy population.

The spectral energy densities are characterized by the presence of a sharp maximum which, depending on the binary population, has an amplitude spanning about two orders of magnitudes, in the frequency range $10^{-5}$ to $10^{-4}$ Hz. In the following, we refer to this part of the signal as the ‘primary’ component. At higher frequencies, a ‘secondary’ component appears for all but (bh, bh) systems. The frequency which marks the transition between primary and secondary components, and also their relative amplitudes depend sensitively on the population.

The reason why (bh, bh) systems do not show a secondary component is that this is entirely contributed by sources that merge before $z = 0$. Conversely, the low-frequency part of the spectrum is dominated by systems with merger times greater than the Hubble time. These sources are observed at very low frequencies because the value of the minimum frequency (which is emitted at formation, $z_F$) is set by the amplitude of the merger time (see equation 16). The larger the merger time is, the smaller the minimum frequency at which the in-spiral waves are emitted. Moreover, equation (6) shows that the flux emitted by each source decreases with frequency. This explains the larger amplitude of primary components with respect to secondary ones. For systems with merger times greater than the Hubble time, the largest frequency is emitted at $z = 0$ by binaries which form at $z_c \approx z_F$.

No contribution from such objects can be observed above this critical frequency and the primary component falls rapidly to zero.

The amplitude of secondary components reflects the number of systems with moderate merger times. The maximum frequency which might be observed is emitted by systems that are very close to their coalescence at $z = 0$. Since $\nu_{\text{max}}$ is larger for (ns, ns) than for (wd, wd) or (ns, wd), the secondary component produced by double neutron stars extends up to $\sim 10^5$ Hz. However, because of the smaller number of (ns, ns) systems compared with (wd, wd) or (ns, wd), the amplitude of the cumulative signal emitted by double neutron stars is smaller.

It is interesting to note that monolithic scenarios predict a maximum amplitude that is a factor of $\sim 20$ to $25$ per cent larger than the hierarchical case. This difference is much larger than...
what has been previously obtained for other extragalactic backgrounds (see, e.g., FMSI), indicating that the energy density produced by extragalactic compact binaries is substantially contributed by sources which form at redshifts $z \approx 1–2$. It is quite difficult to unveil the origin of this effect because of the large number of parameters that determine the appearance of the final energy density. However, a plausible explanation might be that, depending on its specific time delays $\tau_s$ and $\tau_m$, each system emits the signal at redshifts that can be substantially smaller than the formation redshift of the corresponding progenitor system. Thus, although the background signal is mostly emitted at low-to-intermediate redshifts, the sources that produce these signals might have been formed at higher redshifts and reflect the state of the Universe at earlier times, when the differences among hierarchical and monolithic scenarios are more significant.

For the same reason, in spite of the fact that most of the signal is emitted at moderate redshifts, the background generated by compact binaries is sensitive to the assumed cosmological background model. In Fig. 10 we show the spectral energy density produced by (wd, wd) systems for a flat cosmological background model with a non-zero cosmological constant.

Comparing the different panels of Fig. 9, we conclude that the background produced by (bh, bh) binaries has the largest amplitude but it is concentrated at frequencies below $2 \times 10^{-5}$ Hz. At higher frequencies, which are more interesting from the point of view of detectability, the dominant contribution comes from (wd, wd) systems. This is consistent with what has already been found for the galactic populations (Hils et al. 1990).

From the background spectrum it is possible to compute the closure density $\Omega_{gw} h^2$ and the spectral strain amplitude of the

Figure 11. The strain amplitude of the gravitational background produced by various extragalactic populations of degenerate binaries in monolithic and hierarchical scenarios assuming a flat cosmological background model with zero cosmological constant.
The strain amplitude of the backgrounds has a maximum amplitude of between $10^{-18}$ and $10^{-17}$ Hz$^{-1/2}$ at frequencies in the interval $\sim 5 \times 10^{-5} - 5 \times 10^{-3}$ Hz. The function $S_b$ is more sensitive to the low-frequency part of the energy density. Therefore, its shape reflects mainly the primary components of the corresponding energy density. In all but the (bh, bh) population, the presence of a tail at frequencies above the maximum is evident, which is the secondary component of the energy density: in the next section we compare this part of

The results are shown in Figs 11 and 12 for all binary types within monolithic and hierarchical scenarios.

Table 2. The closure density at 1 mHz obtained for (wd, wd) and (ns, wd) extragalactic binary populations investigated. These values are larger than or comparable with the minimum detectable value of $\Omega_{gw} h^2(1 \text{ mHz})$ predicted by the LISA team for a $S/N = 5$ and after 1 yr of observation.

| Frequency (Hz) | (wd, wd) | (ns, wd) |
|---------------|----------|----------|
| $\Omega_{gw} h^2$ | $6 \times 10^{-12}$ | $1.1 \times 10^{-12}$ |
the background signal with the LISA sensitivity to assess the possibility of a detection. Still, it is clear that the prominent part of the background signals produced by extragalactic populations of degenerate binaries could be observed with a detector sensitive to smaller frequencies than LISA.

Conversely, $\Omega_{gw}h^2$ is mostly dominated by secondary components. We can compare the predictions for (bh, bh), (wd, wd) and (ns, ns) systems. In contrast to what has been found for the spectral energy density or for the strain amplitude of the signal, the largest $\Omega_{gw}h^2$ is produced by (ns, ns), consequently, of the high amplitude of the secondary component. In particular, no significant contribution from the primary component appears. For (wd, wd), instead, the contribution of the primary component is relevant, although its amplitude is roughly half that of the secondary component. Finally, for (bh, bh) no secondary component is produced and thus the amplitude of the closure density is very low and at very low frequencies. Mixed binary types have different properties, depending on the relative importance of the above effect. For instance, (bh, wd) produce a secondary component but the amplitude is so small to be comparable with that of the primary.

We stress that the maximum frequency at which the quadrupole approximation should still be considered accurate is quite uncertain. Our choice appears to be sufficiently conservative to cut-off the region where tidal distortions and other non-linear interactions between the two stars start to be noticeable.

5 CONFUSION NOISE LEVEL AND DETECTABILITY BY LISA

To have some confidence in the detection of a stochastic gravitational background with LISA it is necessary to have a sufficiently large signal-to-noise ratio (S/N). The standard choice made by the LISA collaboration is $S/N = 5$ which, in turn, yields a minimum detectable amplitude of a stochastic signal of (see Bender 1998 and references therein),

\[(h^2 \Omega_{gw}(v = 1 \text{mHz}) = 10^{-12}. \quad (20)\]

This value already accounts for the angle between the arms ($60^\circ$) and the effect of LISA motion. It shows the remarkable sensitivity that would be reached in the search for stochastic signals at low frequencies. Table 2 shows that the backgrounds generated by (wd, wd) and (ns, wd) extragalactic binary populations exceed this minimum value and LISA might be able to detect these signals.

We plot in Fig. 13 the predicted sensitivity of LISA to a stochastic background after 1 yr of observation (Bender 1998). On the vertical axis it is shown $h_{\text{rms}}$, defined as

\[h_{\text{rms}} = [2\pi S_0(v)]^{1/2} \left(\frac{\Delta v}{v}\right)^{1/2}, \quad (21)\]

where $S_0(v)$ is the predicted spectral noise density and the factor $(\Delta v/v)^{1/2}$ is introduced to account for the frequency resolution $\Delta v = 1/T$ attained after a total observation time $T$.

The curve labelled with LISA refers to both the predicted instrumental noise and the confusion noise owing to close white dwarf binaries computed by Bender & Hils (1997). On the same plot we show the equivalent $h_{\text{rms}}$ levels predicted for different extragalactic binary populations and for the galactic population of close white dwarfs binaries considered by Nelemans et al. (1999).

The latter calculation is based on the same population synthesis code adopted in the present analysis and appears to be in good agreement with previous calculations.

The plot shows that, according to our analysis, previous estimates have underestimated the contribution from extragalactic systems. These background signals represent additional noise components to the LISA sensitivity curve when searching for signals from individual sources.

In particular, backgrounds from unresolved astrophysical sources represent a confusion-limited noise. In fact, unless the signal emitted by an individual source has a much higher amplitude, the background signal prevents the individual source from being resolved. Clearly, the magnitude of this effect depends on the frequency resolution of the instrument, i.e. on the observation time. The $h_{\text{rms}}$ noise levels produced by extragalactic compact binaries shown in Fig. 13 have been computed assuming $T = 1$ yr. For the same total observation time we show, in Fig. 14, the number of extragalactic (wd, wd) and (ns, ns) observed in each frequency resolution bin. At frequencies where these backgrounds might be relevant (between 1 and 10 mHz), the number of sources per bin is $\gtrsim 1$, representing a relevant confusion-limited noise component. The critical frequency above which the number of

**Figure 13.** The sensitivity of LISA to a stochastic background of gravitational waves after one year of observation is plotted together with the extragalactic backgrounds from (wd, wd), (ns, ns) and (ns, wd) predicted by the present analysis and the (wd, wd) galactic component estimated by Nelemans et al. (1999, see text).

**Figure 14.** The number of extragalactic (wd, wd) and (ns, ns) binaries per resolution bin after a total observation time of 1 yr.
sources per bin is lower than 1 occurs at ~0.1 Hz for (wd, wd) and outside the LISA sensitivity window for (ns, ns). However, at these frequencies the dominant noise component is the instrumental noise.

6 CONCLUSIONS

In this paper we have obtained estimates for the stochastic background of gravitational waves emitted by cosmological populations of compact binary systems during their early spiralling in phase.

Since we have restricted our investigation to frequencies well below the frequency emitted when each system approaches its last stable circular orbit, we have characterized the single-source emission using the quadrupole approximation.

Our main motivation was to develop a simple method to estimate the gravitational signal produced by populations of binary systems at extragalactic distances. This method relies on three main pieces of information:

(i) the theoretical description of gravitational waveforms to characterize the single-source contribution to the overall background;

(ii) the predictions of binary population synthesis codes to characterize the distribution of astrophysical parameters (masses of the stellar components, orbital parameters, merger times, etc.) among each ensemble of binary systems;

(iii) a model for the evolution of the cosmic star formation history derived from a collection of observations out to z ~ 5 to infer the evolution of the birth and merger rates for each binary population throughout the Universe.

The stochastic background signals produced by (wd, wd) and (ns, ns) might be observable with LISA and add as confusion-limited noise components to the LISA instrumental noise and to the signal produced by binaries within our own Galaxy. The extragalactic contributions are dominant at frequencies in the range 1–10 mHz and limit the performance expected for LISA in the same range, where the previously estimated sensitivity curve was attaining its minimum.

Finally, in Fig. 15 we show the spectral densities of the extragalactic backgrounds that have been investigated so far. The high-frequency band appears to be dominated by the stochastic signal from a population of rapidly rotating neutron stars via the r-mode instability (see FMSII). For comparison, we have shown the overall signal emitted during the core-collapse of massive stars to black holes (see FMSI). In this case, the amplitude and frequency range depend sensitively on the fraction of progenitor star that participates to the collapse. The signal indicated with BH corresponds to the assumption that the core mass is ~10 per cent of the progenitor’s (see FMSI). Recent numerical simulations of core-collapse supernova explosions (Fryer 1999) appear to indicate that for progenitor masses >40 M⊙ no supernova explosion occurs and the star collapses directly to form a black hole. The final mass of this core depends strongly on the relevance of mass loss caused by stellar winds (Fryer & Kalogera 2000). If massive black holes are formed the resulting background would have a larger amplitude and the relevant signal would be shifted towards lower frequencies, i.e. more interesting for ground-based interferometers.

In the low-frequency band, we have plotted the backgrounds produced by (bh, bh), (wd, wd), (ns, wd) and (ns, ns) binaries because their signals largely overwhelm those from other degenerate binary types. The curve labelled with Pop III represent a recent estimate made by Schneider et al. (2000) of the gravitational signal produced by the first stars in the Universe.

We find that in both the low- and high-frequency bands, extragalactic populations generate a signal that is comparable to and, in some cases, larger than the backgrounds produced by populations of sources within our Galaxy (Giampieri 1997; Giazotto et al. 1997; Postnov 1997; Hils et al. 1990; Bender & Hils 1997; Postnov & Prokhorov 1998; Nelemans et al. 1999). It is important to stress that even if future investigations reveal that the amplitude of galactic backgrounds might be higher than presently conceived, their signal could still be discriminated from that generated by sources at extragalactic distance. In fact, the signal produced within the Galaxy shows a characteristic amplitude modulation when the antenna changes its orientation with respect to fixed stars (Giampieri 1997; Giazotto et al. 1997).

The same conclusions can be drawn when the extragalactic backgrounds are compared with the stochastic relic gravitational signals predicted by some classical early Universe scenarios. The relic gravitational backgrounds suffer from the many uncertainties that characterize our present knowledge of the early Universe. According to the presently conceived typical spectra, we find that their detectability might be severely limited by the amplitude of the more recent astrophysical backgrounds, especially in the high-frequency band.

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Figure 15. The predicted strain amplitude of the stochastic backgrounds produced by extragalactic populations of gravitational sources. In the high-frequency band, we show the estimates for the background produced by rotating neutron stars via r-mode instability, and the signal emitted by massive stars collapsing to black holes (see text). In the low-frequency band, we plot the background predicted for different populations of binary systems and the signal emitted by Population III stars (see text).
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