The pseudoscalar meson and baryon octet interaction with strangeness zero in the unitary coupled-channel approximation

Bao-Xi Sun,¹ Si-Yu Zhao,¹ and Xiang-Yu Wang¹

¹College of Applied Sciences, Beijing University of Technology, Beijing 100124, China

The interaction of the pseudoscalar meson and the baryon octet is investigated by solving the Bethe-Salpeter equation in the unitary coupled-channel approximation. In addition to the Weinberg-Tomozawa term, the contribution of the $s-$ and $u-$ channel potentials in the S-wave approximation are taken into account. In the sector of isospin $I = 1/2$ and strangeness $S = 0$, a pole is detected in the reasonable region on the complex energy plane of $\sqrt{s}$ in the center of mass frame by analyzing the behavior of the scattering amplitude, which is higher than the $\eta N$ threshold and lies on the third Riemann sheet. Thus it can be regarded as a resonance state and might correspond to the $N(1535)$ particle in the review of the Particle Data Group(PDG). The coupling constants of this resonance state to the $\pi N$, $\eta N$, $K\Lambda$ and $K\Sigma$ channels are calculated, and it is found that this resonance state couples strongly to the hidden strange channels. Apparently, the hidden strange channels play an important role in the generation of the resonance state with strangeness zero. The interaction of the pseudoscalar meson and the baryon octet is repulsive in the sector of isospin $I = 3/2$ and strangeness $S = 0$, therefore, no resonance state can be generated dynamically.

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I. INTRODUCTION

The pion-nucleon interaction is an interesting topic and has attracted more attentions of the nuclear society in the past decades. There are two very closed excited states of the nucleon in the $S_{11}$ channel, $N(1535)$ and $N(1650)$, which are difficult to be described within the framework of the constituent quark model[3]. However, in the unitary coupled-channel approximation of the Bethe-Salpeter equation, most of the excited states of the nucleon are treated as resonance states of the pseudoscalar meson and the baryon in the $SU(3)$ flavor space, so are these two particles. In Ref. [1], it is pointed out that the hidden strange channels of $K\Lambda$ and $K\Sigma$ might play an important role in the dynamically generation of the $N(1535)$ particle.

The $N(1535)$ particle is generated dynamically in the unitary coupled-channel approximation with the final state interaction of a three-body $N\pi\pi$ channel considered[2]. However, the inclusion of the $N\pi\pi$ as a final state in the calculation is complex, especially there are six arbitrary constants in the real part of the three-body $N\pi\pi$ loop function, and thus it must be treated as a free function consistent to the experiment. This work is studied again by including the $\rho N$ and $\pi\Delta$ channels in a non-relativistic approximation besides the pseudoscalar meson -baryon octet channels[2]. Actually the the elastic scattering process of $\rho N \rightarrow \rho N$ mainly gives a contribution to the generation of the $N(1560)$ resonance dynamically, just as done in Ref. [6]. In the processes of $\rho N \rightarrow \pi N$ and $\rho N \rightarrow \pi\Delta$, the Kroll-Ruderman term supplies a constant potential and plays a dominant role, while the $\pi$-exchange potential is trivial and proportional to the square of the three-momentum of the final state in the center of mass frame. Moreover, the structure of the $N(1535)$ and $N(1650)$ particles are also studied in the unitary coupled-channel approximation in [3, 4], where a loop function of the intermediate pseudoscalar meson and the baryon in the on-shell approximation is taken into account when the Bethe-Salpeter equation is solved.

In Ref. [5], the $N(1535)$ and $N(1650)$ resonance states are studied in the unitary coupled-channel approximation with a Lagrangian of the pseudoscalar meson and the baryon octet up to the next-to-leading-order term. By fitting the $S_{11}$ partial wave amplitude with experimental data up to the energy $\sqrt{s} = 1.56$GeV, the resonance state corresponding to the $N(1535)$ particle is generated dynamically. In addition, it is amazing that the $N(1650)$ resonance state can also be produced at higher energies at the same time.

The property of the $N(1535)$ particle has also been studied by solving the relativistic Lippmann-Schwinger equation, where the corresponding Hamiltonian is divided into two parts, a non-interacting part and an interacting part, and the couplings and the bare mass of the nucleon are determined by fitting the experimental data. This method is
named as Hamiltonian effective field theory by the authors. Recently, the different partial wave phase shifts are analyzed by calculating the K-matrix of the pion-nucleon interaction. It is more interesting that the internal wave functions of the \( \Delta(1232) \), \( N(1535) \) and \( N(1650) \) resonance states are investigated, and it is announced that the \( \pi N \), \( \eta N \), \( K \Lambda \) and \( K \Sigma \) components are negligible in these resonance states. It is apparent that the conclusion made in Ref. 10 is inconsistent with the previous principles due to chiral unitary models.

In this work, the interaction of the pseudoscalar meson and the baryon octet is studied in the unitary coupled-channel approximation, and the contribution of the \( s- \) and \( u- \) channel potentials in the S-wave approximation is taken into account besides the Weinberg-Tomozawa contact term. Furthermore, a revised loop function of the Bethe-Salpeter equation is used in the calculation, where the relativistic correction is included.

By adjusting the subtraction constants for different intermediate particles of the loop function in the sector of isospin \( I = 1/2 \) and strangeness \( S = 0 \), a pole at \( 1518 - i\times 46\text{MeV} \) on the complex energy plane is detected, which might be a counterpart of the \( N(1535) \) particle in the PDG data.

This article is organized as follows. In Section II the potential of the pseudoscalar meson and baryon octet is constructed, where the Weinberg-Tomozawa contact term, the \( s- \) channel and \( u- \) channel interactions are all taken into account in the S-wave approximation. In Section III a basic formula on how to solve the Bethe-Salpeter equation in the unitary coupled-channel approximation is shown. The cases of isospin \( I = 1/2 \) and \( I = 3/2 \) are discussed in Section IV and Section V respectively. Finally, the summary is given in Section VI.

II. FRAMEWORK

The effective Lagrangian of the pseudoscalar meson and the baryon octet can be written as

\[
L = \langle \bar{B}(i\gamma_{\mu}D^{\mu} - M)B \rangle + \frac{D/F}{2}\langle \bar{B}\gamma_{5}[u^{\mu}, B]_{\pm} \rangle. \tag{1}
\]

In the above equation, the symbol \( \langle \ldots \rangle \) denotes the trace of matrices in the \( SU(3) \) flavor space, and \( D^{\mu}B = \partial^{\mu}B + \frac{i}{2}[u^{\mu}, B] \) with \( u^{2} = U = \exp(i\frac{f_{0}}{\sqrt{6}}) \) and \( u^{\mu} = i\bar{u}\gamma^{\mu}u - i\bar{u}\partial^{\mu}u \), where \( f_{0} \) is the meson decay constant in the chiral limit.

The matrices of the pseudoscalar meson and the baryon octet are given as follows

\[
\Phi = \sqrt{2}\begin{pmatrix}
\frac{1}{\sqrt{2}}\pi^{0} + \frac{1}{\sqrt{6}}\eta \\
\pi^{-} \\
K^{-} \\
\frac{1}{\sqrt{2}}\pi^{0} + \frac{1}{\sqrt{6}}\eta \\
\frac{1}{\sqrt{2}}\pi^{+} + \frac{1}{\sqrt{6}}\eta \\
K^{0} \\
-\frac{2}{\sqrt{6}}\eta
\end{pmatrix}, \tag{2}
\]

and

\[
B = \begin{pmatrix}
\frac{1}{\sqrt{2}}\Sigma^{0} + \frac{1}{\sqrt{6}}\Lambda \\
\Sigma^{-} \\
\Xi^{-} \\
-\frac{1}{\sqrt{2}}\Sigma^{0} + \frac{1}{\sqrt{6}}\Lambda \\
\Sigma^{+} \\
\Xi^{0} \\
-\frac{2}{\sqrt{6}}\Lambda
\end{pmatrix}. \tag{3}
\]

The first term in the Lagrangian in Eq. (1) supplies the contact interaction of the pseudoscalar meson and the baryon octet, which is usually called as Weinberg-Tomozawa term, while the other terms which are relevant to the coefficients \( D \) and \( F \) give a contribution to the \( s- \) and \( u- \) channel interactions, as shown in Fig. II.

According to Feynmann rules, The Weinberg-Tomozawa contact potential of the pseudoscalar meson and the baryon octet can be written as

\[
V_{ij}^{\text{con}} = -C_{ij}\frac{1}{4f_{i}f_{j}}\bar{U}(p_{j}, \lambda_{j})\gamma_{\mu}U(p_{i}, \lambda_{i})(k_{i}^{\mu} + k_{j}^{\mu}), \tag{4}
\]

where \( p_{i}, p_{j}(k_{i}, k_{j}) \) are the momenta of the initial and final baryons(mesons), and \( \lambda_{i}, \lambda_{j} \) denote the spin orientations of the initial and final baryons, respectively. For low energies, the three-momenta of the incoming and outgoing mesons can be neglected, and thus the potential in Eq. (4) is simplified as

\[
V_{ij}^{\text{con}} = -C_{ij}\frac{1}{4f_{i}f_{j}}\bar{U}(p_{j}, \lambda_{j})\gamma_{0}U(p_{i}, \lambda_{i})(k_{i}^{0} + k_{j}^{0}). \tag{5}
\]
FIG. 1: Feynman diagrams of the pseudoscalar meson-baryon octet interaction. (a) contact term, (b) \( u \)-channel and (c) \( s \)-channel.

Because \( U(p_i, \lambda_i) \) and \( \bar{U}(p_j, \lambda_j) \) stand for the wave functions of the initial and final baryons, respectively, the matrix \( \gamma_0 \) in Eq. (5) can be replaced by the unit matrix \( I \) at the low energy region, i.e., \( \gamma_0 \rightarrow I \). Finally, the Weinberg-Tomozawa contact term of the pseudoscalar meson and the baryon octet takes the form of

\[
V_{ij}^{\text{con}} = - C_{ij} \frac{1}{4f_i f_j} (2\sqrt{s} - M_i - M_j) \left( \frac{M_i + E}{2M_i} \right)^{\frac{1}{2}} \left( \frac{M_j + E'}{2M_j} \right)^{\frac{1}{2}},
\]

(6)

where \( \sqrt{s} \) is the total energy of the system, \( M_i \) and \( M_j \) denote the initial and final baryon masses, respectively, while \( E \) and \( E' \) stand for the initial and final baryon energies in the center of mass frame, respectively. The coefficient \( C_{ij} \) for the sector of strangeness zero and charge zero is listed in Table I. Moreover, we assume the values of the decay constants are only relevant to the pseudoscalar meson with \( f_\eta = 1.3 f_\pi, f_K = 1.22 f_\pi \) and \( f_\pi = 92.4 \text{MeV} \), as given in Ref. [2, 7].

| \( C_{ij} \) | \( K^+ \Sigma^- \) | \( K^0 \Sigma^0 \) | \( K^0 \Lambda \) | \( \pi^- p \) | \( \pi^0 n \) | \( \eta n \) |
|---|---|---|---|---|---|---|
| \( K^+ \Sigma^- \) | 1 | \(-\sqrt{2}\) | 0 | 0 | \(-\sqrt{2}\) | \(-\sqrt{2}\) |
| \( K^0 \Sigma^0 \) | 0 | 0 | \(-\sqrt{2}\) | \(-\sqrt{2}\) | \(-\sqrt{2}\) | \(-\sqrt{2}\) |
| \( K^0 \Lambda \) | 0 | \(-\sqrt{2}\) | \(-\sqrt{2}\) | \(-\sqrt{2}\) | \(-\sqrt{2}\) | \(-\sqrt{2}\) |
| \( \pi^- p \) | 1 | \(-\sqrt{2}\) | 0 | 0 | 0 | 0 |
| \( \pi^0 n \) | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE I: The coefficients \( C_{ij} \) in the pseudoscalar meson and baryon octet interaction with strangeness \( S = 0 \) and charge \( Q = 0 \), \( C_{ji} = C_{ij} \).

The second term in Eq. (1) supplies antibaryon-baryon-meson vertices, and can be rewritten as

\[
L = A_{lmm} \bar{N}_l \gamma_\mu \gamma_5 \partial^\mu M_m N_n,
\]

(7)

with \( N = \{ \Sigma^+, \Sigma^-, \Sigma^0, p, \Xi^-, n, \Xi^0, \Lambda \} \) and \( M = \{ \pi^+, \pi^-, \pi^0, K^+, K^-, K^0, \bar{K}^0, \eta \} \).

The coefficient \( A_{lmm} \) in Eq. (7) takes the form of

\[
A_{lmm} = - \frac{1}{2f_0} [(D + F) C_{lmm} + (D - F) C_{lmm}]
\]

(8)
where
\[ C_{lmn} = \frac{1}{2} \sum_{i,j,k=1}^{8} X_{il}^{\dagger} X_{jm} X_{kn} \langle \lambda_i \lambda_j \lambda_k \rangle, \]  
(9)

with \( \lambda \) the matrix of the SU(3) generator and
\[ X = \frac{1}{\sqrt{2}} \begin{pmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \sqrt{2} & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & i & -i & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & i & -i \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2}
\end{pmatrix}. \]  
(10)

Thus the \( s \)- and \( u \)-channel interaction of the pseudoscalar meson and the baryon octet can be constructed according to the vertices in Eq. (7).

If the three-momenta of the incoming and outgoing particles are neglected in the calculation, the \( s \)-channel potential of the pseudoscalar meson and the baryon octet can be written as
\[ V^s_{ij} \approx A A' \frac{(\sqrt{s} - E)(\sqrt{s} - E')}{\sqrt{s} + M}, \]  
(11)

approximately, where \( M \) denotes the mass of the intermediate baryon, \( A \) and \( A' \) represent the coefficients depicted in Eq. (8), respectively.

Similarly, the \( u \)-channel potential can be obtained as
\[ V^u_{ij} \approx A A' \frac{(\sqrt{s} - E)(E + E' - \sqrt{s} - M')(\sqrt{s} - E')}{u - M^2}, \]  
(12)

with the Mandelstam variable \( u = (p_i - k_j)^2 \).

In the calculation of Eqs. (11) and (12), a physical baryon mass is adopted so as to obtain the \( s \)-channel and \( u \)-channel interaction potentials of the pseudoscalar meson and the baryon octet. The mass renormalization of baryons has been assumed to be accomplished before the tree-level diagrams in the interaction of the pseudoscalar meson and the baryon octet are studied. In the chiral unitary model, the loop function of the intermediate pseudoscalar meson and baryon is considered in an on-shell approximation when the Bethe-Salpeter equation is solved, which will be iterated in Sect. III, so that the whole interaction chain is taken into account without a cutoff. Therefore, we can examine whether a resonance state can be generated dynamically or not.

The Weinberg-Tomozawa term and the \( s \)-channel potential of the pseudoscalar meson and the baryon octet are only related to the Mandelstam variable \( s \), therefore, they only give a contribution to the S-wave amplitude in the scattering process of the pseudoscalar meson and the baryon octet.

Since a function can be expanded with the Legendre polynomials, i.e.,
\[ f(x) = \sum_{n=0}^{+\infty} c_n P_n(x), \]  
(13)

with \( P_n(x) \) the \( n \)th Legendre polynomial and the coefficient
\[ c_n = \frac{2n + 1}{2} \int_{-1}^{1} f(x) P_n(x) dx. \]  
(14)

In the S-wave approximation, only the coefficient \( c_0 \) is necessary to be considered.

The denominator \( u - M^2 \) in Eq. (12) can be written as
\[ u - M^2 = M_i^2 + m_j^2 - M^2 - 2(p_i^0 k_j^0 - \vec{p}_i \cdot \vec{k}_j) \]
\[ = (M_i^2 + m_j^2 - M^2 - 2p_i^0 k_j^0) \left( 1 - \frac{2|\vec{p}_i||\vec{k}_j| \cos \theta}{M_i^2 + m_j^2 - M^2 - 2p_i^0 k_j^0} \right), \]  
(15)
where \( \theta \) is the angle between the three-momenta of incoming and outgoing mesons, and \( \bar{p}_i(k_i) \) and \( M_i(m_i) \) are the three-momentum in the center of mass frame and the mass of the initial baryon (final meson), respectively. Supposing \( \alpha = \frac{2|\bar{p}_i(k_i)|}{M_i^2 + m_i^2 - M^2 - 2E(\sqrt{s} - E')} \), and \( x = \cos \theta \), we can obtain

\[
\frac{1}{2} \int_1^0 \frac{1}{1 - \alpha x} dx = -\frac{1}{2} \ln \left( \frac{1 - \alpha}{1 + \alpha} \right).
\]

Thus the \( u- \) channel potential of the pseudoscalar meson and the baryon octet in the S-wave approximation can be calculated easily

\[
V_{ij}^u(S) = \frac{\alpha}{2} \ln \left( \frac{1 - \alpha}{1 + \alpha} \right) = \frac{1}{2} \ln \left( \frac{1 - \alpha}{1 + \alpha} \right) = \frac{1}{2} \ln \left( \frac{1 - \alpha}{1 + \alpha} \right).
\]

Therefore, the S-wave potential of the pseudoscalar meson and the baryon octet can be written as

\[
V_{ij} = V_{ij}^{\text{on}} + V_{ij}^s + V_{ij}^u(S).
\]

### III. BETHE-SALPETER EQUATION

The Bethe-Salpeter equation can be expanded as

\[
T = V + VGT = V + VGV + VGVGV + \ldots.
\]

When the Bethe-Salpeter equation in Eq. (19) is solved, only the on-shell part of the potential \( V \), in Eq. (19) gives a contribution to the amplitude of the pseudoscalar meson and the baryon octet, and the off-shell part of the potential can be reabsorbed by a suitable renormalization of the decay constants of mesons \( f_i \) and \( f_j \). More detailed discussion can be found in Refs. [12, 13]. Therefore, if the potential in Eq. (19) is adopted, the second term \( VGV \) in Eq. (19) can be written as

\[
V_{jl}G_lV_{li} \sim \bar{U}(p_j, \lambda_j)G_lU(p_i, \lambda_i)(k_i^0 + k_j^0)^2.
\]

If the relativistic kinetic correction of the loop function of the pseudoscalar meson and the baryon octet is taken into account, the loop function \( G_l \) can be written as

\[
G_l = i \int \frac{d^4q}{(2\pi)^4} \frac{\hat{q} + M_l}{q^2 - M_l^2 + i\epsilon (P - q)^2 - m_i^2 + i\epsilon}.
\]

with \( P \) the total momentum of the system, \( m_i \) the meson mass, and \( M_l \) the baryon mass, respectively.

The loop function in Eq. (21) can be calculated in the dimensional regularization (See Appendix 1 of Ref. [11] for details), and thus the loop function takes the form of

\[
G_l = \frac{2\mu P^\mu}{32\pi^2} \left[ (a_i + 1)(m_i^2 - M_l^2) + (m_i^2 \ln \frac{m_i^2}{\mu^2} - M_l^2 \ln \frac{M_l^2}{p^2}) \right]
\]

\[
+ \left( \frac{2\mu P^\mu (P^2 + M_l^2 - m_i^2)}{4P^2M_l} \right) + \frac{1}{2}\right) G_l',
\]

where \( a_i \) is the subtraction constant and \( \mu \) is the regularization scale, and \( G_l' \) is the loop function in Ref. [15],

\[
G_l'(s) = \frac{2M_l}{16\pi^2} \left\{ a_i(\mu) + \ln \frac{m_i^2}{\mu^2} + \frac{M_l^2 - m_i^2 + s}{2s} \ln \frac{M_l^2}{m_i^2} + \frac{\bar{q}_i}{\sqrt{s}} \left[ \ln(s - (M_l^2 - m_i^2) + 2\bar{q}_i \sqrt{s}) + \ln(s + (M_l^2 - m_i^2) + 2\bar{q}_i \sqrt{s}) \right] \right\},
\]

with \( \bar{q}_i \) the three-momentum of the meson or the baryon in the center of mass frame.
Since the total three-momentum $\vec{P} = 0$ in the center of mass frame, only the $\gamma_0P^0$ parts remain in Eq. (22). Similarly, this matrix $\gamma_0$ can be replaced by the unit matrix $I$ since the $U(p_i, \lambda_i)$ and $\bar{U}(p_j, \lambda_j)$ denote the wave functions of the initial and final baryons, respectively. Thus the loop function of the intermediate pseudoscalar meson and baryon octet becomes

$$G_I = \frac{\sqrt{s}}{32\pi^2} \left[ \left( a_l + 1 \right) \left( m_l^2 - M^2 \right) + \left( m_l^2 l_n \frac{m_l^2}{\mu^2} - M^2 l_n M^2 \right) \right]$$

$$+ \left( \frac{s + M^2 - m_l^2}{4M\sqrt{s}} + \frac{1}{2} \right) G'_I.$$  \hspace{1cm} (24)

When the $s-$ channel and $u-$ channel interaction are supplemented, the loop function in Eq. (24) is still suitable. However, the off-shell part of the potential is reabsorbed by a renormalization, so the decay constants of mesons, the masses of intermediate baryons all take physical values when the Bethe-Salpeter equation is solved.

In the calculation of the present work, we make a transition of

$$\tilde{V} = V \sqrt{M_i M_j},$$

$$\tilde{G}_l = G_l / M_i,$$

so the scattering amplitude

$$\tilde{T} = [1 - \tilde{V} \tilde{G}]^{-1} \tilde{V}$$ \hspace{1cm} (26)

becomes dimensionless.

**IV. $I = \frac{1}{2}$ AND $S = 0$**

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**FIG. 2:** Potentials of the pseudoscalar meson and the baryon octet as functions of the total energy of the system $\sqrt{s}$ in the sector of isospin $I = 1/2$ and strangeness $S = 0$. Left: $s-$ channel, Middle: $u-$ channel in the S-wave approximation, Right: The solid lines denote the Weinberg-Tomozawa contact interaction, while the dash lines stand for the total S-wave potential in Eq. (13).

In the sector of isospin $I = \frac{1}{2}$ and strangeness $S = 0$, the wave function in the isospin space can be written as

$$|\pi N; \frac{1}{2}, \frac{1}{2} \rangle = -\sqrt{\frac{2}{3}}|\pi^- p\rangle + \sqrt{\frac{1}{3}}|\pi^0 n\rangle,$$  \hspace{1cm} (27)

$$|\eta N; \frac{1}{2}, \frac{1}{2} \rangle = |\eta n\rangle,$$  \hspace{1cm} (28)

$$|K \Lambda; \frac{1}{2}, \frac{1}{2} \rangle = |K^0 \Lambda\rangle.$$  \hspace{1cm} (29)
respectively. Thus the coefficients $C_{ij}$ in the Weinberg-Tomozawa contact potential of the pseudoscalar meson and the baryon octet can be obtained in the isospin space, which are summarized in Table II.

| $C_{ij}$ | $\pi N$ | $\eta N$ | $K\Lambda$ | $K\Sigma$ |
|----------|--------|---------|---------|---------|
| $\pi N$  | 2      | 0       | $\frac{2}{3}$ | $\frac{1}{3}$ |
| $\eta N$ | 0      | $\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ |
| $K\Lambda$ | 0 | 0 | 0 | 2 |
| $K\Sigma$ | 0 | 0 | 0 | 2 |

TABLE II: The coefficients $C_{ij}$ in the pseudoscalar meson and baryon octet interaction with isospin $I = \frac{1}{2}$ and strangeness $S = 0$, $C_{ji} = C_{ij}$.

The $s-$ channel, $u-$ channel and Weinberg-Tomozawa contact potentials of the pseudoscalar meson and baryon octet in the S-wave approximation are depicted in Figs. 2 respectively. In Fig. 2 it is found that the $\pi N s-$ channel potential is repulsive and the other $s-$ channel potential are weaker than the $\pi N$ case, while the $u-$ channel potentials in the S-wave approximation are attractive. Although the curves for $\eta N$ and $K\Sigma$ cases are not smooth when $\sqrt{s} < 1300\text{MeV}$, it is far away from the energy region which we are interested in, and we assume that it would not give an effect on the pole position of the amplitude in the calculation. However, the contact interaction originated from the Weinberg-Tomozawa term is dominant in the pseudoscalar meson and the baryon octet potential, and the correction from the $s-$ channel potential and the S-wave $u-$ channel potential is not important.

The total potentials for different pseudoscalar meson and baryon systems with isospin $I = 1/2$ and strangeness $S = 0$ are depicted in the right figure of Fig. 2. It shows that the $\pi N$ and $K\Sigma$ potentials are attractive, while the $\eta N$ and $K\Lambda$ interactions are weak.

Although the $s-$ channel and $u-$ channel potentials are weaker than the Weinberg-Tomozawa contact interaction in the sector of isospin $I = 1/2$ and strangeness $S = 0$, the subtraction constants should be readjusted in the calculation when the contribution of the $s-$ and $u-$ channel potentials are taken into account.

According to the PDG data, the $N(1535)$ particle is assume to lie in the region of $Re(poleposition) = 1490 \sim 1530\text{MeV}$, and $-2Im(poleposition) = 90 \sim 250\text{MeV}$ on the complex energy plane of $\sqrt{s}$. When the Bethe-Salpeter equation is solved in the unitary coupled-channel approximation, we set the regularization scale $\mu = 630\text{MeV}$, just as done in most of works with this method. Moreover, all of subtraction constants change from $-3.2$ to $-0.5$ with a step of $0.3$, and we hope a resonance state can be generated dynamically in the reasonable energy region. In the previous works, the subtraction constant is usually chosen to be $\eta N$ when the contribution of the $K\Lambda$ threshold is close to the energy region where we are interested, thus the pole position is not sensitive to the value of the subtraction constant $a_{\pi N}$. The changes of the other three subtraction constants $a_{\eta N}, a_{K\Lambda}$ and $a_{K\Sigma}$ are not so large. Especially, the subtraction constant $a_{K\Lambda} = -3.2$ in 38 sets of parameters except the eighth set, where it takes the value of $-2.9$. The $K\Lambda$ threshold is close to the energy region where we are interested, thus the subtraction constant $a_{K\Lambda}$ is stable and plays an important role in the generation of the $N(1535)$ particle.

A pole is generated dynamically at $1518 - i46\text{MeV}$ on the complex energy plane of $\sqrt{s}$ by solving the Bethe-Salpeter equation in the unitary coupled-channel approximation with the 19th set of parameters, i.e., $a_{\pi N} = -2.0, a_{\eta N} = -1.7, a_{K\Lambda} = -3.2$ and $a_{K\Sigma} = -3.2$. The squared amplitude $|T|^2$ as a function of the total energy $\sqrt{s}$ for different channels with isospin $I = 1/2$ and strangeness $S = 0$ are depicted in Fig. 3. The real part of the pole position is higher than the $\eta N$ threshold, and lower than the $K\Lambda$ threshold, so we assume it might be a resonance state and correspond to the $N(1535)$ particle in the PDG data.

The couplings of the resonance state to the $\pi N$, $\eta N$, $K\Lambda$ and $K\Sigma$ channels are also calculated, and it is found that it couples strongly to the $\eta N$, $K\Lambda$ and $K\Sigma$ channels, which implies that these channels is important in the generation
FIG. 3: The values of the subtraction constant $a_{eN}$, $a_{nN}$, $a_{KN}$, $a_{KS}$ with the regularization scale $\mu = 630\text{MeV}$ fixed in the loop function in Eq. (24).

of the $N(1535)$ resonance state. If different sets of subtraction constants are chosen in the calculation, the changes of couplings are not significant, as shown in the table of the Appendix part.

In Ref. [7], Eq. (15) indicates that the $N(1535)$ particle couples more strongly to the $K^+\Lambda$ channel, which is different from the results listed in Table IV. The different values of the coupling constants might be relevant to the next-to-leading-order chiral Lagrangian used in Ref. [7], while it is not included in this manuscript.

V. $I = \frac{3}{2}$ AND $S = 0$

The wave functions with isospin $I = 3/2$ and strangeness $S = 0$ can be written as

$$|\pi N; \frac{3}{2}, -\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|\pi^0 n\rangle + \sqrt{\frac{1}{3}}|\pi^- p\rangle,$$

and

$$|K\Sigma; \frac{3}{2}, -\frac{1}{2}\rangle = \sqrt{\frac{2}{3}}|K^0 \Sigma^0\rangle + \sqrt{\frac{1}{3}}|K^+ \Sigma^-\rangle,$$

respectively. According to Eqs. (31) and (32), the coefficients $C_{ij}$ in the isospin space can be calculated and listed in Table III. Since the coefficients are all negative, the Weinberg-Tomozawa contact interaction between the pseudoscalar meson and the baryon octet is repulsive in the sector of isospin $I = 3/2$ and strangeness $S = 0$. Even if the correction from the $s-$ channel and $u-$ channel interaction is taken into account, as is shown in Fig. 5, the total potential of the pseudoscalar meson and the baryon octet is still repulsive. Thus no resonance state could be generated in the S-wave approximation.
FIG. 4: The squared amplitude $|T|^2$ as a function of the total energy $\sqrt{s}$ for different channels with isospin $I = 1/2$ and strangeness $S = 0$. The cases of $\pi N$, $\eta N$ and $K\Sigma$ channels are labeled on the figure, while the case of the $K\Lambda$ channel is drawn with the dash line.

| $C_{ij}$ | $\pi N$ | $K\Sigma$ |
|----------|---------|-----------|
| $\pi N$  | $-1$    | $-1$      |
| $K\Sigma$|         | $-1$      |

TABLE III: The coefficients $C_{ij}$ in the pseudoscalar meson and baryon octet interaction with isospin $I = \frac{3}{2}$ and strangeness $S = 0$, $C_{ji} = C_{ij}$.

VI. SUMMARY

In this work, the interaction of the pseudoscalar meson and the baryon octet is studied within a nonlinear realized Lagrangian. The $s-$, $u-$ channel potentials and the Weinberg-Tomozawa contact interaction are obtained when the three-momenta of the particles in the initial and final states are neglected in the S-wave approximation.

In the sector of isospin $I = 1/2$ and strangeness $S = 0$, a resonance state is generated dynamically by solving the Bethe-Salpeter equation, which might be regarded as counterparts of the $N(1535)$ particle listed in the PDG data. We find the hidden strange channels, such as $\eta N$, $K\Lambda$ and $K\Sigma$, play an important role in the generation of the resonance state when the Bethe-Salpeter equation is solved in the unitary coupled-channel approximation. The coupling constants of this resonance state to different channels are calculated, and it is found that it couples strongly to the hidden strange channels.

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FIG. 5: The potential of the pseudoscalar meson and the baryon octet as a function of the total energy of the system $\sqrt{s}$ in the sector of isospin $I = 3/2$ and strangeness $S = 0$. The solid lines denote the contact interaction, while the dash lines stand for the total S-wave potential in Eq. (18).

[1] N. Kaiser, P. B. Siegel and W. Weise, Phys. Lett. B 362, 23 (1995)
[2] T. Inoue, E. Oset and M. J. Vicente Vacas, Phys. Rev. C 65, 035204 (2002)
[3] E. J. Garzon and E. Oset, Phys. Rev. C 91, no. 2, 025201 (2015)
[4] A. Ramos and E. Oset, Eur. Phys. J. A 44, 445 (2010)
[5] J. Nieves and E. Ruiz Arriola, Phys. Rev. D 64, 116008 (2001)
[6] M. Doring and K. Nakayama, Eur. Phys. J. A 43, 83 (2010)
[7] P. C. Bruns, M. Mai and U. G. Meissner, Phys. Lett. B 697, 254 (2011)
[8] Z. W. Liu, W. Kamleh, D. B. Leinweber, F. M. Stokes, A. W. Thomas and J. J. Wu, Phys. Rev. Lett. 116, no. 8, 082004 (2016)
[9] Y. F. Wang, D. L. Yao and H. Q. Zheng, Eur. Phys. J. C 78, no. 7, 543 (2018)
[10] T. Sekihara, T. Arai, J. Yamagata-Sekihara and S. Yasui, Phys. Rev. C 93, no. 3, 035204 (2016)
[11] F. Y. Dong, B. X. Sun and J. L. Pang, Chin. Phys. C 41, no. 7, 074108 (2017)
[12] C. Patrignani et al. [Particle Data Group], Chin. Phys. C, 40, 100001 (2016)
[13] J. A. Oller and E. Oset, Nucl. Phys. A 620, 438 (1997)
[14] E. Oset and A. Ramos, Nucl. Phys. A 635, 99 (1998)
[15] J. A. Oller and U. G. Meissner, Phys. Lett. B 500, 263 (2001)
Appendix: Subtraction constants, pole position and couplings of the resonance state to different channels

| n  | $a_{\pi N}$ | $a_{\eta N}$ | $a_{K\Lambda}$ | $a_{K\Sigma}$ | Pole position (MeV) | $g_{\pi N}$ | $g_{\eta N}$ | $g_{K\Lambda}$ | $g_{K\Sigma}$ | $g_{K\Sigma}$ |
|----|----------|----------|-------------|-------------|--------------------|---------|---------|-------------|-------------|-------------|
| 1  | -3.2     | -1.7     | -3.2        | -3.2        | 1518-46i           | 3+ i    | 4       | -65+ 25i   | 70          | 41+ 0i      | 41       | 94-27i     | 98        |
| 2  | -3.2     | -1.4     | -3.2        | -3.2        | 1530-58i           | 4+ i    | 4       | -66+ 28i   | 72          | 40+ 3i      | 40       | 95-27i     | 99        |
| 3  | -2.9     | -2.3     | -3.2        | -2.9        | 1520-41i           | -3+ 5i  | 5       | -62+ 21i   | 66          | 47+ 0i      | 47       | 96-24i     | 99        |
| 4  | -2.9     | -2.0     | -3.2        | -2.9        | 1532-51i           | 4+ i    | 4       | -64+ 22i   | 68          | 45+ 3i      | 45       | 97-24i     | 100       |
| 5  | -2.9     | -1.7     | -3.2        | -3.2        | 1518-46i           | 3+ 2i   | 4       | -65+ 25i   | 70          | 42+ 0i      | 42       | 94-27i     | 98        |
| 6  | -2.9     | -1.4     | -3.2        | -3.2        | 1530-58i           | 4+ i    | 4       | -66+ 28i   | 72          | 40+ 4i      | 40       | 95-27i     | 99        |
| 7  | -2.6     | -2.3     | -3.2        | -2.9        | 1520-41i           | -2+ 6i  | 7       | -62+ 21i   | 66          | 47+ 1i      | 47       | 96-24i     | 99        |
| 8  | -2.6     | -2.3     | -2.9        | -2.9        | 1527-41i           | -2+ 4i  | 5       | -60+ 19i   | 64          | 48+ 0i      | 48       | 95-21i     | 98        |
| 9  | -2.6     | -2.0     | -3.2        | -2.9        | 1532-51i           | 3+ 6i   | 7       | -64+ 22i   | 68          | 45+ 4i      | 45       | 97-24i     | 100       |
| 10 | -2.6     | -1.7     | -3.2        | -3.2        | 1518-46i           | 3+ 2i   | 4       | -65+ 25i   | 70          | 42+ 0i      | 42       | 94-27i     | 98        |
| 11 | -2.6     | -1.4     | -3.2        | -3.2        | 1530-58i           | 4+ 1i   | 4       | -66+ 28i   | 72          | 40+ 3i      | 40       | 95-27i     | 99        |
| 12 | -2.3     | -2.3     | -3.2        | -2.9        | 1520-41i           | 3+ 2i   | 5       | -67+ 28i   | 73          | 40+ 4i      | 40       | 95-27i     | 99        |
| 13 | -2.3     | -2.0     | -3.2        | -2.9        | 1532-51i           | 3+ 1i   | 4       | -66+ 28i   | 72          | 40+ 3i      | 40       | 95-27i     | 99        |
| 14 | -2.6     | -1.7     | -3.2        | -3.2        | 1518-46i           | 3+ 2i   | 4       | -65+ 25i   | 70          | 42+ 0i      | 42       | 94-27i     | 98        |
| 15 | -2.6     | -1.4     | -3.2        | -3.2        | 1530-58i           | 4+ 1i   | 4       | -66+ 28i   | 72          | 40+ 3i      | 40       | 95-27i     | 99        |

TABLE IV: The values of the subtraction constant $a_{\pi N}$, $a_{\eta N}$, $a_{K\Lambda}$, $a_{K\Sigma}$, the pole position on the complex energy plane of $\sqrt{s}$ and couplings of the resonance state to different channels. The regularization scale $\mu = 630$ MeV is fixed in the loop function in Eq. (24).