Comparative analysis of designing wall panels with hole based on one-dimensional and two-dimensional computer models

To cite this article: Zh S Nuguzhinov et al 2018 IOP Conf. Ser.: Mater. Sci. Eng. 456 012082
Comparative analysis of designing wall panels with hole based on one-dimensional and two-dimensional computer models

Zh S Nuguzhinov¹, S K Akhmediyev¹, B S Donenbayev¹, A Yu Kurokhtin¹ and O Khabibolda²

¹Kazakhstan Multidiscipline Institute of Reconstruction and Development (KazMIRR) at Karaganda State Technical University, Kazakhstan
²Kazakh National University n.a. al-Farabi, Kazakhstan
kazmirr@mail.ru

Abstract. The operation of reinforced concrete wall panels with holes used in civil engineering has been studied. Under conditions of plane stress wall panels operate in vertical ($q_y$) and horizontal ($q_x$) evenly distributed loads conditioned by the own weight of the panels and pressure of the overlying overlap and covering structures. The effect of wind load on the building develops the flexural state of wall panels, i.e. loading the structures with the transverse evenly distributed load takes place. To study the stress-strain state of the above-mentioned structures, the work proposes a rod approximation in the form of a rod of a step-variable cross section, taking into account the presence of a hole in the body of the wall panel. Using the numerical finite-difference method there one solved the initial fourth-order differential equation for the compressed-bent state of the deformed rod. The results are presented in the form of the diagrams of transverse, axial and angular displacements, bending moments, longitudinal and shear forces diagrams. The results of the studies are of scientific and practical importance.

1. Introduction

In practical construction external wall panels that present reinforced concrete thin plates with holes for window openings are widely used [1, 2, 3]. The issue of designing variable-thickness plates with holes was already dealt by a number of researchers. In a number of works [4, 5] there were studied the effects of pliability and local strength and stability loss of building structures joint connections that, as it is well known, by virtue of the Saint-Venant principle, are allowed being ignored in modeling directly shell and beam structures.

When designing such wall panels the problems of integrating differential equations of the 4th order arise. In these cases there are used analytical and numerical methods of calculation [6, 7] including the ones in variation setting [8].

In the process of numerical calculation, the initial wall panel with an opening (Figure 1) is approximated (displayed) by a design diagram in the form of a rod with variable values of bending moments with hinge-supported ends (Figure 3) [9].

To achieve convergence of the solution using the numerical method, it is necessary to break the model into finite elements with a certain step. In accordance with this, to ensure sufficient accuracy of calculation, there can be adopted a grid with a number of divisions (density) 9 that results in the engineering accuracy required of the proposed designing results [10].
2. Methods of designing
From the point of view of the solid deformable body mechanics, the external wall reinforced concrete panels of a large-panel housing construction are a multi-layered plate with holes for window openings (Figure 1).

In this paper use was made of the numerical method of finite differences using a "linear" grid. The advantages of this method is that the solution is obtained in a closed (numerical) form.

![Figure 1. Geometrical and structural diagram of a reinforced concrete wall panel: 1 – reinforced concrete; 2 – heat insulation; 3 – ceramic concrete; 4 – window opening.](image1)

When erecting a residential house using large elements, panels on the exterior facade of the building are joined together by horizontal and vertical seams of embedment [11]; through these seams, respectively vertical ($q_y$) and horizontal ($q_x$) loads are transmitted to the body of the panel. In addition, there acts the wind load on the surface of the panels.

According to the constructive solution, the work of the wall panel with a hole can be approximated in various ways; in general case it is a multilayered and doubly linked system.

Considering that the thickness and specific weight of heat insulation (soft mineral wool) are insignificant, its effect on the panel operation can be neglected; in this regard the reinforced concrete panel can be considered uniform in thickness.

To perform primary "machine-free" calculations in the process of preliminary (variant) design in order to reduce complexity of computing processes, it is proposed to replace the two-dimensional model of the wall panel with a hole (Figure 1) by a one-dimensional model in the form of a step-variable-thickness rod (Figure 2a) that is hinge-supported at the ends.

3. Results and discussion
Acting to the rod loads are:

a) Wind load $q_y$ [11]: $q_y = 198.4 \text{kg/m.}$

b) Axial load for the first-floor panels:

$P = 14.3t = 143 \text{kN}$ (of the weight of the overlying wall panels of the five-floor residential building 4 floors. The wall panel material is class B20 concrete.

To reduce the two-dimensional double-linked problem of mechanics (Figure 1) to the one-dimensional (rod) system (Figure 2 (a)), one adopted the calculation approach through the reduced moment of inertia of the cross section $J_0$.

To calculate the moments of inertia of the sections of the rod system through the $J_0$ value, the surface of the rod system in the $x_0y_0$ plane is conventionally divided into two parts: the first part is
the surface of the reinforced concrete; the second part is the surface of the hole under the window opening (Figure 3).

Figure 3. For calculating reduced moments of inertia \((C\) is the panel center of gravitation).

Let us calculate the moments of inertia relative to the panel center of gravitation \(C\):

a) for the entire panel

\[
J_{X_c} = \left( \frac{bh^3}{12} \right)_{II} - \left( \frac{bh^3}{12} \right)_{III} = 4.597 m^4; \quad J_{Y_c} = \left( \frac{hb^3}{12} \right)_{II} - \left( \frac{hb^3}{12} \right)_{IV} = 5.904 m^4.
\]

b) for the panel parts

\[
J_{X} = J_{X_0} + a^2 A; \quad J_{X_{c,A}} = 1.835 m; \quad J_{X_{c,B}} = 0.3914 m^4; \quad J_{X_{c,D}} = 1.455 m^4.
\]

Taking into account the panel and its parts geometrical dimensions, there have been calculated the moments of inertia relative to the central axis \(x_c\) (point \(C\) in Figure 3); at this \(J_0 = 0.3914 m^4\);

Let us adopt the following values of the reduced moments of inertia:

\[
J_{X_{c,B}} = 0.3914 m^4 = J_0; \quad J_{X_{c,A}} = 4.69 J_0; \quad J_{X_{c,D}} = 3.72 J_0
\]

Thus, through the \(J_0; 3.72 J_0; 4.69 J_0\) value the two-dimensional problem is reduced to the one-dimensional one.

\[
A = 3.18 \cdot 2.7 - 2.2 \cdot 1.5 = 5.286 m^2 \text{ is the wall panel area (without considering the hole).}
\]

The wall panel center of gravitation in the \((x_0, 0, y_0)\) plane is:

\[
y_c = \frac{S_{x_0}}{A} = \frac{3.18 \cdot 2.7 \cdot 1.35 - 2.2 \cdot 1.5 \cdot 1.65}{5.286} = 1.163 m; \quad x_c = \frac{3.18}{2} = 1.59 m.
\]

The differential equation of the compressed-bent rod with variable bending rigidity, has the form (under the axial load \(N=\text{const}\)), according to [9]:

\[
EJy'''' + 2EJ'y''' + EJ''y'' + Ny'' = P(y)
\]

where \(E\) is the material elasticity module; \(N\) is the axial force (load); \(EJ = EJ(x)\) is variable bending rigidity; \(P = P(y)\) is the function of transverse distributed load; \(y = y(x)\) is the function of the rod axis deflections.
To realize equation (2) use is made of the finite difference method (FDM) [10, 13, 14].

The design diagram of the finite-difference method for a step-variable thickness rod under the effect of axial and transverse load is presented in Figure 2, b.

For a grid model for the i-th grid point (Figure 4) equation (2) will take the form:

\[ a_{ii}y_i + a_{ij}y_j + a_{ik}y_k + a_{jl}y_l + a_{si}y_s = P(y) \frac{\Delta x^4}{EJ_0} \]  

(3)

Where the coefficients \( a_{ii}, a_{ij}, \ldots, a_{si} \) taking into account the grid parameters (Figure 2, b) and equation (1) of variable values of the moments of inertia will take the form:

\[ a_{ii} = 6 - 2(-2\alpha_i + \alpha_k + \alpha_l) - 2N EJ_0 \Delta x^2; \quad a_{ij} = -4 + 0.5(\alpha_i - \alpha_k) - 2\alpha_i + (\alpha_k + \alpha_l) + 2N EJ_0 \Delta x^2; \]

\[ a_{ik} = -4 - 0.5(\alpha_i - \alpha_k) - 2\alpha_i + (\alpha_k + \alpha_l) + 2N EJ_0 \Delta x^2; \]

\[ a_{jl} = 1 - 0.25(\alpha_i - \alpha_k); \quad a_{sl} = 1 + 0.25(\alpha_i - \alpha_k). \]  

(4)

Here: \( \alpha_i = J_i / J_0; \quad \alpha_k = J_k / J_0; \quad \alpha_l = J_l / J_0 \); is the grid points moments of inertia ratio (Figure 4) to the reduced moment of inertia \( J_0(J_0 = 4.69 m^4) \).

For a “wall” grid we have the grid parameters and the external transverse load: \( n = (1,2,\ldots,8) \) – and the design (internal) grid points.

\[ \Delta x = \frac{I}{9} = \frac{2.7}{9} = 0.3; \alpha_1 = 4.69; \alpha_2 = 2.865; \alpha_3 = \alpha_4 = 1.0; \alpha_5 = 3.26; \alpha_6 = \alpha_7 = 3.72. \]

\[ P = P_1 = P_2 = P_3 = P_4 = P_5 = P_6 = P_7 = P \]

(5)

The accuracy of expressions (4) makes the \( (\Delta x^4) \) value [11].

The grid points deflections on the supports and outside are excluded taking into account the boundary conditions at the beam ends (Figure 2, a).

According to Figure 2, b:

\[ y_A = y_B = 0; y_C = -y_B; y_D = -y_8 \]  

(6)

According to the type of equation (3) let us write the deflections of the contour and out-of-contour grid points the finite-difference equations (6) for the calculated \( n = 8 \) of the grid points calculated (Figure 2, b); as a result we obtain a system of linear algebraic equations (SLAE) of the 8th order, which have the following matrix form:

\[ Ay = \bar{P} \]  

(7)

\( A \) is the square matrix of the 8th order; \( \bar{y} = [y_1, y_2, \ldots, y_8] \) is the vector of the deflections being determined at the grid points (Figure 2,b); \( \bar{P} = [P_1, P_2, \ldots, P_8] \) is the vector considering the effect of the transverse load with intensity \( q \) (Figure 2,b); The \( P_i \) of this element is calculated by formula (5).

Matrix \( A \) for the rod shown in Figure 2 is in general form given in Table 1. The values of coefficients \( a_{ji} = (j = 1,2,\ldots,5; i = 1,2,\ldots,8) \) are taken according to (4).

Solving equation (7) gives the values of the \( \bar{y} \), vector, i.e.

\[ y = A^{-1} \bar{P} \]  

(8)
Solution of equation (8), according to Table 1 data, was obtained based on the standard MatCad program; the results of calculation are given in Figure 5.

| i=1 | a_{11} \cdot a_{41} & a_{31} & a_{51} | 0 & 0 & 0 & 0 & 0 & 2.4 \times 10^{-9} |
| i=2 | a_{22} & a_{12} & a_{32} & a_{52} | 0 & 0 & 0 & 0 & 2.4 \times 10^{-9} |
| i=3 | a_{43} & a_{23} & a_{13} & a_{33} & a_{53} | 0 & 0 & 0 & 2.4 \times 10^{-9} |
| i=4 | 0 & a_{44} & a_{24} & a_{14} & a_{34} & a_{54} | 0 & 0 & 2.4 \times 10^{-9} |
| i=5 | 0 & 0 & 0 & a_{45} & a_{25} & a_{15} & a_{35} & a_{55} | 0 & 2.4 \times 10^{-9} |
| i=6 | 0 & 0 & 0 & 0 & a_{46} & a_{26} & a_{16} & a_{36} & a_{56} | 2.4 \times 10^{-9} |
| i=7 | 0 & 0 & 0 & 0 & 0 & a_{47} & a_{27} & a_{17} & a_{37} | 2.4 \times 10^{-9} |
| i=8 | 0 & 0 & 0 & 0 & 0 & 0 & a_{48} & a_{28} & a_{18} & a_{38} | 2.4 \times 10^{-9} |

**Table 1.** Matrix $A$ and vector $P$.

Figure 5. Results of calculating the step-variable rigidity rod.

Figures 6, 7 show the nomograms of deflections and rotation angles at the grid points (Figure 2 (b)) depending on the acting transverse load $q_y$ that allows widening the range of the problems solved on the subject considered.

**Figure 6.** Nomograms for determining deflections at the $y_i (i=1, 2, \ldots, 8)$ points.

**Figure 7.** Nomograms for determining rotation angles at the $\theta_i (i=A,1,2,\ldots,8,B)$ points.
To assess the accuracy of the calculation results for the rod of the step-variable flexural rigidity using the FDM (Figure 5), this rod was calculated by the second method known in the material resistance as the “initial parameters method” [12, 13, 14]. In this case the deflection function “y” is calculated by the formula:

\[ y_i = -\frac{1.942z}{\alpha_i EJ_0} + \frac{1}{\alpha_i EJ_0} \left[ 0.533(z)^3 - 0.09875(z)^4 \right] \]  

Let us adopt for the reinforce concrete rod (B20 concrete class) at \( EJ_0 = B_1 = 0.8 \cdot 10^2 \text{kNm}^2 \) (reinforcement percent not less than 1 % with reinforcement of AIII class). Then, according to formula (9) at \( z = 0.9m; \alpha_i = 1.0 \) we have (Figure 2, b).

\[ y_{\text{max}} = y_4 = 17.99 \cdot 10^{-8}(m) \]  

(10)

By the diagram of deflections (Figure5) we have \( y_{\text{max}} = 18.1 \cdot 10^{-8}(m) \). The calculation error is 0.6 %.

To evaluate reliability of the new results obtained from the calculation using the one-dimensional model (Figure 2, a), the authors calculated the wall panel (Figure 1) as a rectangular thin plate with a hole by the finite element method based on the Lira CAD program complex. Figure 8 shows the diagram of loading the plate both in its middle surface and across it.

![Figure 8](image1)

**Figure 8.** Design combinations of external effects under the load in.

![Figure 9](image2)

**Figure 9.** Values of transverse displacements (deflections) of the plate with division into finite elements.

Figure 9 shows the vertical movements (deflections) at the grid points of finite elements. The greatest deflection occurs at the point “K” \( z_{\text{max}} = 1.2 \cdot 10^{-7}(m) \) close enough to the value (10) obtained in the calculation using the one-dimensional model (Figure 2, a).

4. **Conclusions**

1. This paper deals with the work of a reinforced concrete wall panel with a hole for large-panel residential buildings (Figure 1).
2. In order to simplify the calculation and to reduce it to the applied (engineering) technique, the wall panel is replaced by a conventional (equivalent) rod of step-variable rigidity loaded with the vertical and horizontal load (Figure 2). The transition from the two-dimensional problem of mechanics to the one-dimensional problem is accomplished through reduced moments of inertia.
3. Studying the stress-strain state of a similar (equivalent) rod is performed by the numerical finite differences method (FDM) using a "linear" grid (the number of computed nodes is eight, see Figure 2, b).
4. Based on the FDM, one obtained the resolving matrix "A" that in the general (alphabetical) form is given in Table 1; it allows calculating wall panels with a hole for different geometric dimensions and external loads.
5. Reliability of the results obtained is confirmed by the correct coincidence of the ordinates of the diagram, M, Q, N with their corresponding values calculated by the classical theory of materials resistance (analytical method), and by the similarity of the results of calculations for one-dimensional and two-dimensional models; The calculation using the two-dimensional model has been performed by the finite element method.

6. Thus, the approximation of the two-dimensional reinforced concrete wall panel with a hole (Figure 1) operation, equivalent in longitudinal and flexural rigidity to a one-dimensional rod of step-variable rigidity (Figure 2) yields results acceptable from the standpoint of engineering accuracy and greatly simplifies the calculation procedure for a wall panel reducing complexity of calculations.

References
[1] Zhugutov V M 2012 Magazine of Civil Engineering 1 27 79–80
[2] Zhugutov V M 2010 Magazine of Civil Engineering 11 38–48
[3] Solovei N A, Krivenko O P and Malygina O A 2015 Magazine of Civil Engineering 1 53 56–69
[4] Trubina D, Abdulaev D, Pichugin E and Rybakov V 2014 Applied Mechanics and Materials 633–634 982–90
[5] Trubina D, Abdulaev D A, Pichugin E, Rybakov V, Garifullin M and Sokolova O 2015 Applied Mechanics and Materials 725–726 752–57
[6] Lalin V, Rybakov V and Sergey A 2014 Applied Mechanics and Materials 578–579 858–63
[7] Liu M and Xu Y 2013 Analysis of full scaffold construction continuous beam bridges based on different codes Proc. 4th Int. Conf. Transportation Engineering October 19-20, 2013, Chengdu, China pp 2136–42
[8] Lalin V V, Zdanchuk E V, Kushova D A and Rozin L A 2015 Magazine of Civil Engineering 56 54–65
[9] Umanskiy A A 1973 Reference Book of the Designer of Industrial, Residential and Public Buildings and Structures: Designing and Theoretical (Moscow: Stroiizdat) p 480
[10] Varvak P M and Varvak L P 1977 Method of Grids in the Problems of Calculating Building Structures (Moscow: Stroiizdat) p 154
[11] Dykhovichny Yu A 1991 Residential and Public Buildings: Brief Reference Book of the Design Engineer (Moscow: Stroiizdat) p 656
[12] Teregulov N G 1984 Strength of Materials (Moscow: Higher School) p 472
[13] Akhmediev S K, Filippova T S and Donenbayev B S 2016 Analytical and Numerical Methods of Designing Engineering and Transport Structures (Karaganda: KSTU) p 158
[14] Nuguzhinov Zh S, Akhmediev S K, Zhakibekov M E and Kurohtina I A 2015 Structural Mechanics and Analysis of Constructions Issue 2 28–33