Minimal Flavour Violation and Beyond

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Abstract. We review the formulation of the Minimal Flavour Violation (MFV) hypothesis in the quark sector, as well as some “variations on a theme” based on smaller flavour symmetry groups and/or less minimal breaking terms. We also review how these hypotheses can be tested in $B$ decays and by means of other flavour-physics observables. The phenomenological consequences of MFV are discussed both in general terms, employing a general effective theory approach, and in the specific context of the Minimal Supersymmetric extension of the SM.

1 Introduction

The Standard Model (SM) is an effective theory valid up to some still undetermined cutoff scale $\Lambda$. The gauge hierarchy problem suggests that this scale should be in the TeV region currently being probed at LHC, where some new physics (NP) should appear. If this NP interacts directly or indirectly with the SM particles, it necessarily contribute to flavour-violating processes. Since there is no exact flavour symmetry in the SM (the flavour symmetry of the gauge sector is broken by the Yukawa interactions), one cannot assume an exact flavour symmetry in the NP model: some breaking would unavoidably appear at the quantum level.

On the other hand, the excellent agreement of flavour data with the SM predictions implies that, for generic flavour violation, the scale $\Lambda$ cannot be low. For example, $K^0-\bar{K}^0$ mixing alone sets a bound on $\Lambda$ of roughly 10$^4$ TeV for generic $\Delta S = 2$ flavour-violating effective operators of dimension six $^{[1]}$. Consequently, insisting on a solution to the hierarchy problem by TeV-scale NP, we are forced to conclude that the flavour structure of the NP is highly non-generic, especially in the quark sector. On general grounds, a NP flavour structure able to satisfy the existing tight constraints requires two main ingredients: i) a large flavour symmetry, and ii) small symmetry-breaking terms. Given the flavour-violating structure present in the SM, the most restrictive assumption to protect in a consistent way quark-flavour mixing beyond the SM is to assume that the flavour symmetry is the one present in the SM in the limit of vanishing Yukawa couplings (namely a $U(3)^3$ quark-flavour symmetry $^{[2]}$) and that the two quark Yukawa couplings are the only two irreducible symmetry-breaking terms. As a result of this hypothesis of Minimal Flavour Violation (MFV), which can naturally be formulated with the language of effective theories $^{[3]}$, non-standard contributions in flavour-changing neutral current (FCNC) transitions turn out to be suppressed to a level consistent with experiments even for $\Lambda \sim$ few TeV.

Similarly to the violation of flavour, the violation of the CP symmetry is a challenge for any NP theory. The non-observation of electric dipole moments (EDMs) of fundamental fermions puts stringent lower bounds on the scale $\Lambda$, assuming generic (flavour-conserving) CP-violating NP interactions. This problem persists even in theories with MFV since the minimal breaking of flavour does not preclude the presence of flavour blind CP-violating phases.

It should also be kept in mind that, while the MFV principle would naturally explain the absence so far of any signals of flavour violation beyond the SM, it is not a theory of flavour, in the sense that there is no explanation for the observed hierarchical structure of the Yukawa couplings. Finally, it is worth to stress that while the MFV hypothesis is the most efficient way to suppress NP contribution to flavour-violating processes, it is by no means the only symmetry + symmetry-breaking pattern allowed by present data.

This review is organized as follows. In section 2 we discuss the MFV principle in its effective field theory formulation, analyzing in detail the different implementations in theories with one or two Higgs doublets. In section 3 we discuss some possible variations around the MFV hypothesis, addressing in particular the issue of flavour-blind phases, emphasizing the special role of the third-generation Yukawa couplings (non-linear formulations), and addressing the compatibility with Grand Unified Theories. In section 4 we analyze the implementation of the MFV hypothesis in the specific context of the Minimal Supersymmetric extension of the SM (MSSM). In section 5 we discuss possible variations of the MFV hypothesis relevant in the so-called split-family SUSY framework: a MSSM with a large mass gap between the first two generations of squarks and the third one. We review in particular the split-family framework with a $U(2)^3$ flavour
symmetry, which provides an interesting attempt to explain (at least in part) the observed Yukawa hierarchies, addressing at the same the absence of large deviations from the SM in flavour- and CP-violating observables.

2 Minimal Flavour Violation

The largest set of unitary, global field rotations in the quark sector commuting with the SM gauge symmetry is

$$G_q = U(3)_{Q_L} \times U(3)_{U_R} \times U(3)_{D_R}.$$  

(1)

As mentioned, we cannot promote this global flavour symmetry to be an exact symmetry beyond the SM, since it is already broken within the SM by the Yukawa interactions,

$$- \mathcal{L}^{\text{SM}}_{\text{Yukawa}} = Y_{d}^{ij} \bar{Q}^{i}_{L} \phi D_{R}^{j} + Y_{u}^{ij} \bar{Q}^{i}_{L} \phi U_{R}^{j} + \text{h.c.}$$  

(2)

(where $\phi = i\gamma_2 \phi^\dagger$). The MFV hypothesis consists in assuming that $Y^d$ and $Y^u$ are the only sources of flavour symmetry breaking also in the NP model. To implement and interpret this hypothesis in a consistent way, we can assume that $G_q$ is a good symmetry and promote $Y^u, d$ to be non-dynamical fields (spurions) with non-trivial transformation properties under $G_q$:  

$$Y^u \sim (3, \bar{3}, 1), \quad Y^d \sim (3, 1, 3).$$  

(3)

If the breaking of the symmetry occurs at very high energy scales, at low-energies we would only be sensitive to the background values of the $Y$, i.e. to the ordinary SM Yukawa couplings. Employing the effective theory language, an effective theory satisfies the MFV criterion in the quark sector if all higher-dimensional operators, constructed from SM and $Y$ fields, are formally invariant under the flavour group $G_q$.

As mentioned in the introduction, not only flavour-violating interactions but also flavour-conserving CP-violating interactions provides a serious challenge to NP models at the TeV scale. Following the approach of Ref. [3], this problem can be circumvented enlarging the symmetry to $G_q \times \text{CP}$ and assuming that the Yukawa couplings are the only symmetry breaking terms of both $G_q$ and CP.

According to the MFV criterion one should in principle consider operators with arbitrary powers of the (dimensionless) Yukawa fields. However, a strong simplification arises by the observation that all the eigenvalues of the Yukawa matrices are small, but for the top (and possibly the bottom one, see later), and that the off-diagonal elements of the CKM matrix are very suppressed. Working in the basis

$$Y^d = \lambda_d, \quad Y^u = V^\dagger \lambda_u,$$  

(4)

where $\lambda_{d,u}$ are diagonal,

$$\lambda_d = \text{diag}(y_d, y_s, y_t), \quad \lambda_u = \text{diag}(y_u, y_c, y_t).$$  

(5)

and neglecting the ratio of light quark masses over the top mass, we have

$$[Y^u (Y^u)^\dagger]_{i\neq j} \approx y^u_{i} V_{ti}^* V_{tj}.$$  

(6)

As a consequence, including high powers of the the Yukawa matrices amounts only to a redefinition of the overall factor in (6) and the the leading $\Delta F = 2$ and $\Delta F = 1$ FCNC amplitudes get exactly the same CKM suppression as in the SM:

$$\mathcal{A} (d \to d')_{\text{MFV}} = (V_{ti}^* V_{tj})^2 \mathcal{A}_{\text{SM}}^{(\Delta F = 1)} \left[ 1 + a_{1} \frac{16 \pi^2 M_W^2}{A^2} \right],$$

$$\mathcal{A} (d \to d')_{\text{MFV}} = (V_{ti}^* V_{tj})^2 \mathcal{A}_{\text{SM}}^{(\Delta F = 2)} \times \left[ 1 + a_{2} \frac{16 \pi^2 M_W^2}{A^2} \right],$$  

(7)

where the $\mathcal{A}_{\text{SM}}^{(i)}$ are the SM loop amplitudes and the $a_i$ are $\mathcal{O}(1)$ parameters. The $a_i$ depend on the specific operator considered but are flavour independent. This implies the same relative correction in $s \to d$, $b \to d$, and $b \to s$ transitions of the same type. In the minimal set-up, where CP is also a good symmetry of the theory in absence of Yukawa couplings, the $a_i$ are real parameters.

2.1 Universal UT and MFV bounds on the effective operators

As originally pointed out in Ref. [5], within the MFV framework several of the constraints used to determine the CKM matrix (and in particularly the unitarity triangle) are not affected by NP. In this framework, NP effects are negligible not only in tree-level processes but also in a few clean observables sensitive to loop effects, such as the time-dependent CP asymmetry in $B_d \to J/\psi K_{L,S}$. Indeed the structure of the basic flavour-changing coupling in Eq. (2) implies that the weak CP phase of $B_d \to B_d$ mixing is arg[$(V_{td} V_{tb})^2$], exactly as in the SM. The determination of the unitarity triangle using only these clean observables (denoted Universal Unitarity Triangle) is shown in Fig. 1. This construction provides a natural (a posteriori) justification of why no NP effects have been observed in the quark sector: by construction, most of the clean observables measured at $B$ factories are insensitive to NP effects in this framework. In Table 1 we report a few representative examples of the bounds on the higher-dimensional operators in the

$^2$ As pointed out in [4], this statement is not fully correct since the $a_i$ could have non-vanishing flavour-blind phases proportional the Jarlskog invariant $J_{CP} = \text{det}[Y^u (Y^u)^\dagger, Y^d (Y^d)^\dagger]$. However, the smallness of $J_{CP}$ implies that these phases play a negligible role in flavour-violating observables (unless enhanced by unnaturally large coefficients).

$^3$ The Unitarity Triangle shown on the left plot of Fig. 1 includes also the $\Delta M_d/\Delta M_s$ constraint, assuming this ratio is not modified with respect to the SM. This condition holds only in the so-called constrained MFV scenario of Ref. [5] (see Sect 2.2).
MFV framework. As can be noted, the built-in CKM suppression leads to bounds on the effective scale of new physics not far from the TeV. These bounds are very similar to the bounds on flavour-conserving operators derived by precision electroweak tests. This observation reinforces the conclusion that a deeper study of rare decays is definitely needed in order to clarify the flavour problem: the experimental precision on the clean FCNC observables required to obtain bounds more stringent than those derived from precision electroweak tests (and possibly discover new physics) is typically in the 1% – 10% range.

Although MFV seems to be a natural solution to the flavour problem, it should be stressed that we are still far from having proved the validity of this hypothesis from data (in the effective theory language we can say that there is still room for sizable new sources of flavour symmetry breaking beside the SM Yukawa couplings [9]). A proof of the MFV hypothesis can be achieved only with a positive evidence of physics beyond the SM exhibiting the flavour-universality pattern (same relative correction in $s \to d$, $b \to d$, and $b \to s$ transitions of the same type) predicted by the MFV assumption. While this goal is quite difficult to be achieved, the MFV framework is quite predictive and could easily be falsified. For instance, no significant enhancement over the SM predictions is expected in direct CP asymmetries in singly-Cabibbo suppressed charm decays: as in the SM case, the asymmetries are suppressed by $\Im(V_{ub}^*V_{cb}/V_{ub}V_{cb})$ [10]. As a result, if the recent evidence of direct-CP violation in the charm system [11] will be established to be incompatible with the SM expectation, this would rule out not only the SM, but would also be a clear signal against MFV models.

### 2.2 Comparison with other approaches

The idea that the CKM matrix rules the strength of FCNC transitions also beyond the SM has become a very popular concept in the recent literature and has been implemented and discussed in several works (see e.g. Refs. [5][12]).

It is worth stressing that the CKM matrix represents only one part of the problem: a key role in determining the structure of FCNCs is also played by quark masses, or by the Yukawa eigenvalues. In this respect, the MFV criterion illustrated above provides the maximal protection of FCNCs (or the minimal violation of flavour symmetry), since the full structure of Yukawa matrices is preserved. At the same time, this criterion is based on a renormalization-group-invariant symmetry argument. Therefore, it can be implemented independently of any specific hypothesis about the dynamics of the new-physics framework. The only two assumptions are: i) the flavour symmetry and its breaking sources; ii) the number of light degrees of freedom of the theory (identified with the SM fields in the minimal case).

This model-independent structure does not hold in most of the alternative definitions of MFV models which can be found in the literature. For instance, the definition of Ref. [13] (denoted constrained MFV, or CMFV) contains the additional requirement that only the effective FCNC operators which play a significant role within the SM are the only relevant ones also beyond the SM. This condition is realized within weakly coupled theories at the TeV scale with only one light Higgs doublet, such as the MSSM with small $\tan \beta$ and small $\mu$ term. However, it does not hold in other frameworks, such as technicolour/composite models (see e.g. [14][15]) or the MSSM with large $\tan \beta$ and/or large $\mu$ term, whose low-energy phenomenology can still be described using the general MFV criterion discussed in Sect. 2.

### 2.3 MFV at large $\tan \beta$

The flavour group in Eq. [1] can be decomposed as

$$\mathcal{G}_q = U(1)^3 \times SU(3)_Q \times SU(3)_U \times SU(3)_D \times SU(3)_L$$

$$= U(1)^3 \times SU(3)_Q^3.$$  

(8)

If the Yukawa Lagrangian contains more than one single Higgs field it is natural to treat separately the breaking of the $U(1)$ groups and that of $SU(3)_Q^3$. To this purpose, we note that two of the $U(1)$ charges can be identified with the baryon number and the (quark) hypercharge, which are not broken by the Yukawa interaction even in the SM case. The third Abelian group, $U(1)_{PQ}$, can be identified as a group under which only the three $D_R$ fields are charged (with the same charge), while $U_R^1$ and $Q_L^1$ are neutral (Peccei-Quinn symmetry). Only this Abelian symmetry is explicitly broken by the Yukawa interaction in the one-Higgs doublet case.

In the two-Higgs doublet case, assigning different $U(1)_{PQ}$ charges to the two Higgs fields ($\phi_U$ neutral and $\phi_D$ with

| Operator | Bound on $A$ | Observables |
|----------|--------------|--------------|
| $H_i^+D_R^aY_i^aY_i^aY_j^a\sigma_{\mu\nu}Q_L^a$ ($eF_{\mu\nu}$) | 6.1 TeV | $B \to X_s\gamma$, $B \to X_t\ell^+\ell^-$ |
| $i(Q_i^L Y_i^a\gamma^a_{\mu}Q_L^a) (\bar{B}_R^j D_R^a H_U)$ | 5.9 TeV | $\epsilon_K$, $\Delta m_{B_s}$, $\Delta m_{B_d}$ |
| $H_i^i \left(\bar{D}_R^a Y_i^a Y_i^a\sigma_{\mu\nu}T^a Q_L^a\right) (g_s G_{\mu\nu})$ | 3.4 TeV | $B \to X_s\gamma$, $B \to X_t\ell^+\ell^-$ |
| $i(Q_i^L Y_i^a\gamma^a_{\mu}Q_L^a) (\bar{D}_L^a D^a_{\mu\nu}E_R)$ | 2.7 TeV | $B \to X_s\gamma$, $B \to X_t\ell^+\ell^-$, $B_s \to \mu^+\mu^-$ |
| $i(Q_i^L Y_i^a\gamma^a_{\mu}Q_L^a) (\bar{D}_L^a D^a_{\mu\nu}L_L)$ | 2.3 TeV | $B \to X_s\gamma$, $B \to X_t\ell^+\ell^-$, $B_s \to \mu^+\mu^-$ |
| $i(Q_i^L Y_i^a\gamma^a_{\mu}Q_L^a) (eD^a_{\mu\nu})$ | 1.7 TeV | $B \to X_s\gamma$, $B \to X_t\ell^+\ell^-$ |
| $i(Q_i^L Y_i^a\gamma^a_{\mu}Q_L^a) (eD^a_{\mu\nu})$ | 1.5 TeV | $B \to X_s\gamma$, $B \to X_t\ell^+\ell^-$ |

Table 1. Bounds on the scale of new physics (at 95% C.L.) for some representative $\Delta F = 1$ [6] and $\Delta F = 2$ [7] MFV operators (assuming effective coupling $\pm 1/A^2$), and corresponding observables used to set the bounds.
opposite charge to $D_R$), we can write a $U(1)_{\text{PQ}}$-invariant Yukawa interaction:

$$-\mathcal{L}_{\text{Yukawa}}^{2\text{HDM}} = Y_{ij}^u \bar{Q}_{Li} \phi_D D_{Rj} + Y_{ij}^d \bar{Q}_{Li} \phi_U U_{Rj} + \text{h.c.}$$  \hspace{1cm} (9)

This interaction prevents tree-level FCNCs and implies that $Y^{u,d}$ are the only sources of $SU(3)_C^3$ breaking appearing in the Yukawa interaction (similar to the one-Higgs-doublet scenario). Consistently with the MFV hypothesis, we can then assume that $Y^{u,d}$ are the only relevant sources of $SU(3)_C^3$ breaking appearing in all the low-energy effective operators. This is sufficient to ensure that flavour-mixing is still governed by the CKM matrix, and naturally guarantees a good agreement with present data in the $\Delta F = 2$ sector. However, the extra symmetry of the Yukawa interaction allows us to change the overall normalization of $Y^{u,d}$ with interesting phenomenological consequences in specific rare modes. These effects are related only to the large value of bottom Yukawa, and indeed can be found also in other NP frameworks where there is no extended Higgs sector, but the bottom Yukawa coupling is of order one.

Assuming the Lagrangian in Eq. (9), the normalization of the Yukawa couplings is controlled by the ratio of the vacuum expectation values of the two Higgs fields, or by the parameter

$$\tan \beta = \langle \phi_U \rangle / \langle \phi_D \rangle .$$  \hspace{1cm} (10)

For $\tan \beta \gg 1$ the smallness of the $b$ quark and $\tau$ lepton masses can be attributed to the smallness of $1/\tan \beta$ rather than to the corresponding Yukawa couplings. As a result, for $\tan \beta \gg 1$ we cannot anymore neglect the down-type Yukawa coupling. In this scenario the determination of the effective low-energy Hamiltonian relevant to FCNC processes involves the following three steps:

- construction of the gauge-invariant basis of dimension-six operators (suppressed by $A^{-2}$) in terms of SM fields and two Higgs doublets;
- breaking of $SU(2)_L \times U(1)_Y$ and integration of the $O(M_H^2)$ heavy Higgs fields;
- integration of the $O(M_H^2)$ SM degrees of freedom (top quark and electroweak gauge bosons).

These steps are well separated if we assume the scale hierarchy $A \gg M_H \gg M_W$. On the other hand, if $A \sim M_H$, the first two steps can be joined, resembling the one-Higgs-doublet scenario discussed before. The only difference is that now, at large $\tan \beta$, $\lambda_D$ is not negligible and this leads to enlarge the basis of effective dimension-six operators. From the phenomenological point of view, this implies the breaking of the strong MFV link between $K$- and $B$-physics FCNC amplitudes occurring in the one-Higgs-doublet case [3].

A more substantial modification of the one-Higgs-doublet case occurs if we allow sizable sources of $U(1)_{\text{PQ}}$ breaking. It should be pointed out that the $U(1)_{\text{PQ}}$ symmetry cannot be exact: it has to be broken at least in the scalar potential in order to avoid the presence of a massless pseudoscalar Higgs. Even if the breaking of $U(1)_{\text{PQ}}$ and $SU(3)_C^3$ are decoupled, the presence of $U(1)_{\text{PQ}}$ breaking sources can have important implications on the structure of the Yukawa interaction, especially if $\tan \beta$ is large [3,16–18]. We can indeed consider new dimension-four operators such as

$$\epsilon \bar{Q}_{Li} Y^d D_R \tilde{\nu} U + \epsilon \bar{Q}_{Li} Y^u Y^d Y^d D_R \tilde{\nu} U ,$$  \hspace{1cm} (11)

where $\epsilon$ denotes a generic MFV-invariant $U(1)_{\text{PQ}}$-breaking source. The effective Yukawa Lagrangian then assumes the
and \( Y_1 \) and \( Y_2 \) are the remaining non-trivial polynomials in powers of \( X \) structures appearing in the effective Yukawa interaction in Eq. (12).

The down-type quarks to the heavy neutral Higgs fields couplings \([20–22]\). Moreover, sizable FCNC couplings for \( \epsilon \) as discussed first in specific supersymmetric frameworks, for \( \epsilon \times \tan \beta = O(1) \) the U(1)\( PQ \)-breaking terms induce \( O(1) \) corrections to the down-type Yukawa couplings \([16] \), the CKM matrix elements \([17] \), and the charged-Higgs couplings \([20] [22] \). Moreover, sizable FCNC couplings of the down-type quarks to the heavy neutral Higgs fields are allowed \([18] [23] [24] \). All these effects can be taken into account to all orders with a proper re-diagonalization of the effective Yukawa interaction in Eq. (12).

Since the b-quark Yukawa coupling becomes \( O(1) \), the large-\( \tan \beta \) regime is particularly interesting for helicity-suppressed observables in \( B \) physics, such as the rare decays \( B_{s,d} \to \ell^+ \ell^- \), whose decay rates could be substantially enhanced over the SM expectations even taking into account all the tight constraints from the other observables. The channels of this type where the experiments reach the best sensitivity compared to the SM expectations are \( B_{s,d} \to \mu^+ \mu^- \). Despite the recent improved limits from CDF \([27] \), LHCb \([25] \), and CMS \([26] \), there is still substantial room for enhancements over the SM predictions \([19] \).

\[
\begin{align*}
\mathcal{B}(B_d \to \mu^+ \mu^-)_{SM} &= (1.1 \pm 0.15) \times 10^{-10} , \\
\mathcal{B}(B_s \to \mu^+ \mu^-)_{SM} &= (3.7 \pm 0.4) \times 10^{-9} .
\end{align*}
\]

The correlation of \( \mathcal{B}(B_s \to \mu^+ \mu^-) \) and \( \mathcal{B}(B_d \to \mu^+ \mu^-) \) in a generic model with Higgs-mediated FCNC amplitudes respecting the MFV hypothesis is shown in Fig. 2. An enhancement of both modes respecting the MFV relation \( \Gamma(B_d \to \ell^+ \ell^-)/\Gamma(B_d \to \ell^+ \ell^-) \approx |V_{td}/V_{ts}|^2 \) would be an unambiguous signature of MFV at large tan \( \beta \) \([9] \).

As far as \( B \to \ell \nu \) is concerned, in the limit of exact U(1)\( PQ \) symmetry the tree-level charged-Higgs amplitude interfere destructively with the SM one \([28] \): assuming \( m_H > m_W \times \tan \beta \), this implies a suppression (typically in the 10 – 50\% range) of the decay rate with respect to its SM prediction. However, for sizable U(1)\( PQ \)-breaking terms (\( |\epsilon \times \tan \beta| \gtrsim 1 \), the sign of the correction is not unambiguously determined and rate enhancements up to 50\% cannot be excluded \([29] \). Potentially measurable effects in the 10 – 30\% range are expected also in \( B \to X_s \gamma \) \([20] \).

\( 3 \) Beyond the minimal set-up

3.1 MFV with flavour-blind phases

The breaking of the flavour group \( G_f \) and the breaking of the discrete CP symmetry are not necessarily related, since generic MFV models can still contain flavour blind phases \([4,14,30] \). Because of the experimental constraints on electric dipole moments (EDMs), which are generally sensitive to such flavour-diagonal phases \([4,31] \), in this

Fig. 2. Correlation between \( \mathcal{B}(B_s \to \mu^+ \mu^-) \) and \( \mathcal{B}(B_d \to \mu^+ \mu^-) \) in presence of Higgs-mediated FCNC amplitudes respecting the MFV hypothesis. The continuous red line indicates the central value of the correlation, while the green points take into account the uncertainties in \( |V_{ts}| \) and \( |V_{td}| \). The black cross denotes the SM prediction. The horizontal dashed line indicates the combined 95\% CL upper limit from LHCb \([25] \) and CMS \([26] \).
more general case the bounds on the scale of new physics are substantially higher with respect to the “minimal” case, where the Yukawa couplings are assumed to be the only breaking sources of both symmetries [3].

However, there are concrete examples of MFV models where flavour blind phases lead to observable effects in $B_s$ and $B_d$ mixing, or CP asymmetries in $B$ decays, without violating the EDM bounds. These include the MSSM with complex trilinear couplings of third generation squarks [31,32] and a two Higgs doublet model with flavour blind phases in Yukawa interactions or the Higgs potential [19,53].

The correlation of CP-violating effects in the three observable $\Delta F = 2$ systems ($B_{d,s}$ and $K^{\pm}$ meson mixing) is a powerful test of possible flavour-blind phases in a MFV framework. At small $\tan \beta$ there is only one relevant $\Delta F = 2$ operator:

$$\left( Q_L^* Y_u Y_u^* \gamma_\mu Q_L \right)^2$$

(21)

Since this operator is Hermitian, its coupling must be real and no deviations are expected in the $\Delta F = 2$ sector compared to the case without flavour-blind phases. The situation changes if $\tan \beta$ is large. In this case, thanks to the large bottom Yukawa coupling, additional operators with the insertion of $Y_d$ break the universality between $K$ and $B$ systems. In the limit where we can neglect the strange-quark Yukawa coupling, the extra CPV induced by flavour blind phases is equal in $B_s$-$\bar{B}_s$ and $B_d$-$\bar{B}_d$ mixing and does not enter $K^0-L^0$ mixing [14]. This can be understood from an approximate $U(2)^3$ symmetry in the large $\tan \beta$ case (see section 5.2).

Neglecting the strange-quark Yukawa coupling is usually a good approximation, but for the specific case of Higgs-mediated amplitudes with sizable U(1)$_{\text{PQ}}$-breaking terms. In this case operators containing right-handed light quarks cannot be neglected and break the correlation between CPV in the $B_s$ and $B_d$ systems. In the limit where these scalar FCNCs are dominant, the modification of the $B_s$ mixing phase is $m_s/m_d$ larger with respect to the corresponding effect in the $B_d$ system [19].

While sizable corrections to $B_s$ and $B_d$ mixing phases are possible in general, this is not the case in the specific realization of MFV with flavour-blind phases in the MSSM (even at large $\tan \beta$) [34]. This can be traced back to the suppression in the MSSM of effective operators with several Yukawa insertions. Sizable couplings for these operators are necessary both to have an effective large CP-violating phase in $B_s$-$\bar{B}_s$ mixing and, at the same time, to evade bounds from other observables, such as $B_s \rightarrow \mu^+\mu^-$ and $B \rightarrow X_s \gamma$.

3.2 Non-linear representation, discrete subgroups, and gauging of $U(3)^3$

As stressed above, the MFV expansion relies on the smallness of the off-diagonal elements of the CKM matrix and the hierarchies between the Yukawa eigenvalues. It does not suffer from the fact of $y_t$ (and possibly $y_{t^\prime}$ at large $\tan \beta$) being sizable. As explicitly shown in Eq. (6), the effect of considering high powers in $y_t$ only modify the overall strength of the basic flavour-violating spurion

$$(V^\dagger \lambda_{\delta}^s V)_{i \neq j}. \quad (22)$$

An elegant implementation of the MFV hypothesis, taking into account explicitly the special role the diagonal third-generation Yukawa couplings is obtained with a non-linear realization of the flavour symmetry [13,35]. Particularly interesting is the so-called GMFV case, where both $y_t$ and $y_{t^\prime}$ are assumed to be of order one and their effects are re-summed to all orders [14]. As shown in [14], the flavour symmetry group surving after this resumma-

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3.3 MFV in Grand Unified Theories

The notion of MFV can also be extended to the lepton sector. However, in this case there is not a unique way to define the minimal sources of flavour symmetry breaking if one wants to accommodate non-vanishing neutrino masses. Indeed different versions of Minimal Lepton Flavour Violation (MLFV) have been proposed in the literature, depending on how the irreducible breaking terms in the neutrino sector are identified [43–46]. As for the quark sector, the key tool to possibly test these assumptions relies on the observation of possible correlations in the rates of neutral-current LFV processes, such as $\tau \rightarrow \mu \gamma$ and $\mu \rightarrow e\gamma$ [43–46].

Once we accept the idea that flavour dynamics obeys a MFV principle, it is interesting to ask if and how this is compatible with Grand Unified Theories (GUTs), where quarks and leptons sit in the same representations of a unified gauge group. This question has been addressed in Ref. [47], considering the exemplifying case of SU(5)$_{gauge}$.

Within SU(5)$_{gauge}$, the down-type singlet quarks ($D_R^i$) and the lepton doublets ($L^c_L$) belong to the 5 representation; the quark doublet ($Q^c_L$), the up-type ($U^c_R$) and lepton singlets ($E^c_R$) belong to the 10 representation, and finally the right-handed neutrinos ($\nu_R^i$) are singlet. In this framework the largest group of flavour transformation commuting with the gauge group is

$$G_{GUT} = U(3)_5 \times U(3)_{10} \times U(3)_1,$$

which is smaller than the direct product of the quark and lepton flavour groups compatible with the SM gauge sector: $G_q \times G_l$, where $G_q = U(3)_E \times U(3)_L$. We should therefore expect some violations of the MFV predictions, either in the quark sector, or in the lepton sector, or in both.

A phenomenologically acceptable description of the low-energy fermion mass matrices requires the introduction of at least four irreducible sources of $G_{GUT}$ breaking. From this point of view the situation is apparently similar to the non-unified case: the four $G_{GUT}$ spurions can be put in one-to-one correspondence with the low-energy spurions $Y^{u,d,e}$ plus the neutrino Yukawa coupling $Y^\nu$ (which is the only low-energy spurion in the neutrino sector assuming an approximately degenerate heavy $\nu_R$ spectrum). However, the smaller flavour group does not allow the diagonalization of $Y^d$ and $Y^e$ (which transform in the same way under $G_{GUT}$) in the same basis. As a result, two additional mixing matrices can appear in the expressions for flavour changing rates [47]. The hierarchical texture of the new mixing matrices is known since they reduce to the identity matrix in the limit ($Y^\nu)^T = Y^d$. Taking into account this fact, and analysing the structure of the allowed higher-dimensional operators, a number of reasonably firm phenomenological consequences can be deduced [47].

There is a well defined limit in which the standard MFV scenario for the quark sector is fully recovered: $|Y_u| \ll 1$ and small tan $\beta$. The upper bound on the neutrino Yukawa couplings implies an upper bound on the heavy neutrino masses ($M_\nu$). In the limit of a degenerate heavy neutrino spectrum, this bound is of about $10^{12}$ GeV. For $M_\nu \sim 10^{12}$ GeV and small tan $\beta$, deviations from the standard MFV pattern can be expected in rare $K$ decays but not in $B$ physics. Ignoring fine-tuned scenarios, $M_\nu \gg 10^{12}$ GeV is excluded by the present constraints on quark FCNC transitions. Independently from the value of $M_\nu$, deviations from the standard MFV pattern can appear both in $K$ and in $B$ physics for tan $\beta \gtrsim m_t/m_b$.

Contrary to the non-GUT MFV framework for the lepton sector, the rate for $\mu \rightarrow e\gamma$ and other LFV decays cannot be arbitrarily suppressed by lowering the mass of the heavy $\nu_R$. This fact can easily be understood by noting that the GUT group allows also $M_\nu$-independent contributions to LFV decays proportional to the quark Yukawa couplings. The latter become competitive for $M_\nu \lesssim 10^{12}$ GeV and their contribution is such that for $A \lesssim 10$ TeV the $\mu \rightarrow e\gamma$ rate is above $10^{-13}$ (i.e. within the reach of MEG).

Within this framework improved experimental searches on $\tau \rightarrow \mu\gamma$ and $\tau \rightarrow e\gamma$ are a key tool: they are the best observables to discriminate the relative size of the non-GUT MFV contributions with respect to the GUT ones. In particular, if the quark-induced terms turn out to be dominant, the $B(\tau \rightarrow \mu\gamma)/B(\mu \rightarrow e\gamma)$ ratio could reach values of $O(10^{-4})$, allowing $\tau \rightarrow \mu\gamma$ to be just below the present exclusion bounds.

4 The MFV Hypothesis in the MSSM

Low-energy supersymmetry provides an elegant solution to the gauge hierarchy problem, allows for precision gauge coupling unification and accommodates candidates for dark matter (see [48] for a comprehensive introduction). These virtues make the minimal supersymmetric extension of the SM (MSSM) one of the most extensively studied avenues of new physics and suggest supersymmetric partners to be present around the TeV scale (see [49]–[51] for recent global fits of SUSY scenarios). However, it is worth to recall that the adjective minimal in the MSSM acronym refers to the particle content of the model and not to its flavour structure. In general, the MSSM contains a huge number of free parameters, most of them related to the flavour structure of the model (fermion masses and trilinear couplings). As long as we are ignorant about the mechanism of SUSY breaking, some assumptions about the soft SUSY breaking terms are necessary to avoid excessive flavour violation. The most restrictive — and, arguably, most successful — assumption of this type is MFV.

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4 The conclusion that $K$ decays are the most sensitive probes of possible deviations from the strict MFV ansatz follows from the strong suppression of the $s \rightarrow d$ short-distance amplitude in the SM [$|V_{us}V^{*}_{ub}| = O(10^{-5})$], and goes beyond the hypothesis of an underlying GUT. This is the reason why $K \rightarrow \pi\nu\bar{\nu}$ decays, which are the best probes of $s \rightarrow d \Delta F = 1$ short-distance amplitudes, play a key role in any extension of the SM containing non-minimal sources of flavour symmetry breaking.
Since the new degrees of freedom (in particular the squark fields) have well-defined transformation properties under the quark-flavour group $G_q$, the MFV hypothesis can easily be implemented in the MSSM following the general rules outlined in Sect. 2. We need to consider all possible interactions compatible with i) softly-broken supersymmetry; ii) the breaking of $G_q$ via the spurion fields $Y_{U,D}$. This allows to express the squark mass terms and the trilinear quark-squark-Higgs couplings as follows [32]:

\begin{align}
\tilde{m}^2_{Q_i} &= \tilde{m}^2 \left( a_i 1 + b_i Y_i Y_i^\dagger \right), \quad (24) \\
\tilde{m}_{U_R}^2 &= \tilde{m}^2 \left( a_2 1 + b_2 Y_2 Y_2^\dagger \right), \quad (25) \\
\tilde{m}_{D_R}^2 &= \tilde{m}^2 \left( a_3 1 + b_3 Y_3 Y_3^\dagger \right), \quad (26) \\
A_{Q_i} &= A \left( a_4 1 + b_4 Y_4 Y_4^\dagger \right), \quad (27) \\
A_D &= A \left( a_5 1 + b_5 Y_5 Y_5^\dagger \right). \quad (28)
\end{align}

where the dimensional parameters $\tilde{m}$ and $A$ set the overall scale of the soft-breaking terms. In Eqs. (24)–(28) we have explicitly shown all independent flavour structures which cannot be absorbed into a redefinition of the leading terms (up to tiny contributions quadratic in the Yukawas of the first two families). When $\tan \beta$ is not too large and the bottom Yukawa coupling is small, the terms quadratic in $Y_D$ can be dropped.

In a bottom-up approach, the adimensional coefficients $a_i$ and $b_i$ in Eqs. (24)–(28) should be considered as free parameters of the model. Note that this structure is renormalization-group invariant: the value of $a_i$ and $b_i$ change according to the Renormalization Group (RG) flow, but the general structure of Eqs. (24)–(28) is unchanged, as has been demonstrated explicitly in Refs. [33,54]. This is not the case if the $b_i$ are set to zero (corresponding to the so-called hypothesis of flavour universality). If this hypothesis is set as initial condition at some high-energy scale $M$, then non vanishing $b_\beta \sim (1/4\pi)^2 \ln M^2/\tilde{m}^2$ are generated by the RG evolution. This is for instance what happens in models with gauge-mediated supersymmetry breaking [55,57], where the scale $M$ is identified with the mass of the hypothetical messenger particles.

Using the soft terms in Eqs. (24)–(28), the physical squark $6 \times 6$ mass matrices, after electroweak breaking, assume the form shown in Table 2. The eigenvalues of these mass matrices are not degenerate; however, the mass splittings are tightly constrained by the specific (Yukawa-type) symmetry-breaking pattern.

If we are interested only in low-energy processes, we can integrate out the supersymmetric particles at one loop and project this theory onto the general EFT discussed in the previous sections. In this case, the coefficients of the dimension-six effective operators written in terms of SM and Higgs fields (see Table 1) are computable in terms of the supersymmetric soft-breaking parameters. We stress that if $\tan \beta \gg 1$ (see Sect. 2.3) and/or if $\mu$ is large enough [58], the relevant operators thus obtained go beyond the restricted basis of the CMFV scenario [13]. We also emphasize that the integration of the supersymmetric degrees of freedom may lead to sizable modifications of the renormalizable operators and, in particular, of the effective Yukawa interaction. As a result, in an effective field theory with supersymmetric degrees of freedom, the relations between $Y_{U,D}$ and the physical squark masses and CKM angles are potentially modified. As already pointed out in Sect. 2.3, this effect is particularly relevant in the large $\tan \beta$ regime.

Comparing the resulting operators to the bounds in Table 1 using the typical effective suppression scale (assuming an overall coefficient $1/A^2$)

$$
A \sim 4\pi \tilde{m} \ ,
$$

we conclude that if MFV holds the present bounds on FCNCs do not exclude squarks in the few hundred GeV mass range, i.e. in the region currently probed by LHC. Since the top Yukawa (and at large $\tan \beta$ also the bottom Yukawa) is large, the squark mass matrices in Table 2 show that the third generation squark masses, which are currently less constrained by collider searches, can be split from the first two generation ones. A large hierarchy between these mass scales however goes beyond MFV and is the subject of Sect. 3.

The flavour signatures of the MFV MSSM are largely identical to the universal MFV predictions discussed in Section 2. An additional peculiar feature is that the contributions to $\Delta M_{d,s}$ turn out to be always positive [58]. In the large $\tan \beta$ regime, the most sensitive channels are $B_s \to \mu^+\mu^-$ and $B \to \tau\nu$, as discussed in Section 2.3.

If CP violating phases beyond the CKM phase are added to the MFV MSSM, new contributions to EDMs and CP asymmetries arise. The irreducible phases in the MFV MSSM (ignoring the lepton sector) are given by the flavour blind phases

$$
\text{Arg}(M_1 \mu) , \quad \text{Arg}(a_{4,5} \mu) , \quad \text{Arg}(a_{4,5}^* M_1) ,
$$

where the $M_i$ are gaugino masses and the $a_i$ are defined in Eq. (25), as well as the phases in the complex coefficients $b_{3,4}$ and $b_{7,8}$ in Eqs. (24)–(28).

The phases in (30) are strongly constrained by the upper bounds on EDMs and thus have to be tiny in the MFV MSSM. The parameters $b_{3,4}$ are suppressed unless $\tan \beta$ is large. If the coefficients $b_{7,8}$ are complex, one essentially obtains complex third generation trilinear couplings and real ones for the first two generations. This setup has interesting implications for electric dipole moments and CP violation in $\Delta B = 1$ decays [52]. However, the $b_{7,8}$ and the $a_{4,5}$ mix under renormalization, so this scenario is not RG invariant and typically suffers from excessive EDMs if imposed at a high scale (unless only $b_7$ is real and $\tan \beta$ small) [31].

Interestingly, even with flavour blind phases, a significant modification of the SM predictions for the $B_s$ or $B_d$ mixing phases is impossible in the MFV MSSM, even at large $\tan \beta$, once $\Delta F = 1$ constraints are taken into ac-
count \[34\]. This restriction can be avoided only by introducing non-MFV sources of flavour violation or by extending the MSSM, e.g. by introducing higher-dimensional operators respecting MFV and carrying flavour blind phases \[59,60\].

### 5 Flavour symmetry breaking with split families

The MSSM with very heavy superpartners of the first two generation quarks, is a long-standing alternative to MFV. While the solution to the gauge hierarchy problem requires mostly the third generation squarks to be light, the tight constraints from FCNCs are loosened in presence of a squark mass hierarchy \[61,62\]. Moreover, such spectrum is favoured by LHC sparticle searches: while the bounds on first generation squark masses are approaching a TeV, the third generation ones can still be significantly lighter \[63\]. However, a hierarchical spectrum is not enough to suppress flavour violation for a generic soft SUSY breaking sector \[64\]. Combining split-family SUSY with MFV is thus natural, but not trivial since some of the $U(3)$ factors in $G_q$ are broken explicitly by the hierarchical squark mass terms. In the following, we discuss two possible generalizations.

An additional virtue of the split-family framework is the solution to the SUSY CP problem: as mentioned in the introduction, the non-observation of EDMs is a challenge for any SUSY theory and is not addressed by MFV. Since the observable EDMs are the ones of first generation fermions, the one-loop contributions to these EDMs are strongly suppressed in the case of hierarchical fermions, where the corresponding superpartners are heavy.

This corresponds to a flavour symmetry
\[
U(1)_{U_1} \times U(1)_{U_2} \times U(1)_{U_3} \times U(3)_{D_R}
\]
broken by the spurion $Y_D$.

In this setup, the first and second generation sfermions are not necessarily degenerate, leading to sizable contributions to $\epsilon_K$. These contributions are ameliorated by the fact that the right-handed sbottom squark is heavy and can be in agreement with the experimental bounds for reasonable values of the parameters. The mixing amplitudes in the $B_d$ and $B_s$ system, on the other hand, remain virtually unaffected.

In Ref. \[66\], it has been demonstrated explicitly that with effective MFV, the EDM problem is solved even for sizable flavour-blind phases. The one-loop contributions to the electron and neutron EDMs are strongly mass-suppressed and under control for first and second generation sfermion masses around 10 TeV, for $O(1)$ phases and moderate $\tan \beta$. However, two-loop effects become relevant, in particular from Barr-Zee type diagrams not suppressed by heavy sfermion masses. These effects lead to EDM contributions in the ballpark of the present experimental bounds.

Interestingly, even without excessive EDMs, flavour-blind CP violating phases can lead to observable effects in CP asymmetries in $B$ decays. This is because $B$ physics involves the third generation quarks, whose superpartners can be light. As an example, the mixing induced CP asymmetry in $B \to \eta' K_S$ and the CP asymmetry ($A_{K^0}$) in the low dilepton invariant mass region of $B \to K^+ \mu^+ \mu^-$ are shown in the left and center panels of Fig. 3 for a scan with $\tan \beta < 5$ and an $O(1)$ flavour blind phase in the $\mu$ term (left) or the stop trilinear term (center).

#### 5.2 $U(2)^3$

If all the third generation squarks are heavy, the flavour symmetry in the absence of Yukawa couplings is
\[
G_q' = U(2)_{Q_L} \times U(2)_{U_R} \times U(2)_{D_R}
\]

Interestingly, this is also the symmetry preserved by the SM Lagrangian if one neglects the small first and second generation quark masses and the small CKM mixings. A $U(2)$ symmetry relating the first two generation quark and

\[\text{The combination of this flavour symmetry with the split-family hypothesis was dubbed “effective MFV” in Refs} \ 65,66\] but we stress that it goes beyond MFV in the sense of section 2.
squark fields has received a lot of attention because it can explain, at least in part, the hierarchies manifest in the Yukawa couplings and at the same time ameliorate the flavour and CP problems [68][70]. However, a single U(2) acting on left- and right-handed fields turns out not to provide enough protection from flavour violation.

Motivated by these observations, the symmetry $G'_y$ has been considered in Ref. [71] together with an appropriate breaking pattern as an alternative to MFV. Analogously to the MFV case, a pair of bi-doublet spurions is needed. The minimal choice compatible with the observed quark masses and mixings is a doublet transforming as

$$\Delta Y = (2, 2, 1) \text{ and } \Delta Y_d = (2, 1, 2).$$

To allow for communication between the third generation and the first two, at least one additional spurion is needed. The minimal choice compatible with the observed quark masses and mixings is a doublet transforming as $V = (2, 1, 1)$.

Combining the symmetry breaking terms, the $3 \times 3$ Yukawa matrices in generation space assume the following form,

$$Y_u = y_t \begin{pmatrix} \Delta Y_u & x_t V \\ 0 & 1 \end{pmatrix}, \quad Y_d = y_b \begin{pmatrix} \Delta Y_d & x_b V \\ 0 & 1 \end{pmatrix},$$

where $x_t, x_b$ are complex parameters of $O(1)$ and the $2 \times 2$ matrices $\Delta Y_u$ and $\Delta Y_d$ and the vector $V$ are the small symmetry breaking parameters of $G'_y$ with entries of order $\lambda^2$ or smaller, with $\lambda$ the sine of the Cabibbo angle. Analogous expressions hold for the squark soft mass matrices.

Of particular phenomenological relevance is the mixing matrix $W^d_L$, diagonalizing the mass matrix of left-handed down-type squarks and appearing in the interaction vertex of left-handed down-type squarks with gauginos. It can be written as

$$W^d_L = \begin{pmatrix} c_d & s_d e^{i(\delta + \phi)} - s_d s_L e^{i\gamma} e^{-i(\delta + \phi)} \\ -s_d e^{i(\delta + \phi)} & c_d - s_d s_L e^{i\gamma} \\ 0 & s_L e^{i\gamma} \end{pmatrix},$$

where $c_d, s_d, \delta$ and $\phi$ are fixed by the requirement of reproducing the correct quark masses and CKM matrix and $s_L$ and $\gamma$ are a new mixing angle and phase entering $\Delta F = 1, 2$ transitions and are responsible for the non-MFV effects of $U(2)^3$ in flavour physics.

For example, the absorptive parts of the meson mixing amplitudes in the $K, B_d$ and $B_s$ systems can be written as

$$M^B_{12} = (M^B_{12})_{SM} \left[1 + xe^{2i\gamma} \right],$$

where $x = \frac{\lambda^2}{v^2} |V_{ts}|^2$ and $F_0$ is a loop function whose numerical value, assuming the dominance of gluino contributions, is shown in the left panel of Fig. 3 as a function of the gluino and sbottom masses. We observe that

i. in all cases the size of the correction is proportional to the CKM combination of the corresponding SM amplitude (MFV structure);

ii. the proportionality coefficient is the same in $B_d$ and $B_s$ systems, while it may be different in the $K$ system;

iii. new CP-violating phases can only appear in the $B_d$ and $B_s$ systems (in a universal way);

iv. the contribution in the $K$ system interferes constructively with the SM.

i.–iii. are model-independent consequences of the $U(2)^3$ symmetry; ii. and iii. have also been discussed in the literature in the context of MFV in the large tan $\beta$ limit and GMFV [14], which can be viewed as a special cases of a $U(2)^3$ symmetry. A universal $B_d$ and $B_s$ mixing phase is also predicted in a two Higgs doublet model with MFV [19][33] and some MFV extensions of the MSSM [59][60] with flavour blind phases in the Higgs potential. iv. is a
prediction of $U(2)^3$ in supersymmetry with dominance of gluino contributions to the amplitudes. Interestingly, iv. leads to an unambiguously positive contribution to $\epsilon_K$, as is preferred by the data. Also the non-standard contribution to the $B_d$ mixing phase is welcome in view of tensions in the SM CKM fit among $\epsilon_K$, $\sin 2\beta$ and $\Delta M_d/\Delta M_s$.

The right panel of Fig. 4 shows the result of a global fit of the CKM matrix, using the 4 Wolfenstein parameters as well as $x$, $F_0$ and $\gamma$ as inputs to Eqs. (35)–(37) and all relevant observables from tree-level and loop-induced processes as constraints (for details see [71]). One can observe a non-zero value favoured for the NP phase $\gamma$, driven by the tensions in the SM CKM fit among $\epsilon_K$, $\sin 2\beta$ and $\Delta M_d/\Delta M_s$.

Recently, the LHCb collaboration has measured the $B_s$ mixing phase in $B_s \rightarrow J/\psi \phi$ and $B_s \rightarrow J/\psi f_0$ decays and found agreement, within errors, with the SM prediction [72]. To confront the $U(2)^3$ prediction of a universal modification of $B_d$ and $B_s$ mixing phases with the data, we performed a further fit of the CKM matrix, allowing for a general modification of the $\Delta B = 2$ amplitudes,

$$M_{B_d}^{B_d} = (M_{B_d}^{B_d})_{SM} \left[ 1 - h_d e^{2i\sigma_d} \right], \quad \text{(38)}$$

$$M_{B_s}^{B_s} = (M_{B_s}^{B_s})_{SM} \left[ 1 - h_s e^{2i\sigma_s} \right], \quad \text{(39)}$$

assuming for simplicity no NP in $K$ mixing. As constraints, we used the same observables as above, but in addition the data on the $B_s$ mixing phase from LHCb, CDF [73] and D0 [74]. The fit results are shown in Fig. 5. The left and center plot show a preference for a non-standard contribution to $B_d$ mixing, while $B_s$ mixing is compatible with the SM at 68% C.L. The right plot shows the preferred region (cf. also [60]) for the NP phases defined as

$$\phi_q = \arg (1 - h_q e^{2i\sigma_q}). \quad \text{(40)}$$

In this plot, the origin is the SM point, while the $U(2)^3$ prediction $\phi_d = \phi_s$ is shown as a solid line. As already discussed, the $\phi_d = \phi_s$ prediction holds beyond the hypothesis of supersymmetry with split families. It was found for
instance in two Higgs doublet models with MFV \cite{19,33} and in the MFV MSSM with an extended Higgs sector with flavour blind phases in the Higgs potential \cite{60}. However, in the latter models a large breaking of the Peccei-Quinn symmetry and flavour-blind phases only in the effective Yukawa interaction lead instead to the prediction with flavour blind phases in the Higgs potential \cite{60}. How-
and in the MFV MSSM with an extended Higgs sector instance in two Higgs doublet models with MFV \cite{19,33} (3 stars).

Table 3. Maximal possible size of new physics effects for different flavour symmetries (and the associated flavour symmetry breaking as discussed in the text): only small effects possible (1 star), moderate effects possible (2 stars) or large effects possible (3 stars).

|                | $U(3)^3$ | $U(1)^3 \times U(3)$ | $U(2)^3$ | $U(3)^3$ | $U(1)^3 \times U(3)$ | $U(2)^3$
|----------------|----------|---------------------|----------|----------|---------------------|----------
| $\epsilon_K$  | **       | **                  | **       | **       | **                  | **       |
| $S_{\phi\phi}$ | *        | *                   | *        | *        | *                   | *        |
| $S_{\phi K_S}$ | **       | **                  | *        | **       | **                  | **       |
| $B \to K^* \ell^+ \ell^-$ [CPV asym.] | *        | *                   | **       | **       | **                  | **       |
| EDMs           | *        | *                   | *        | *        | **                  | **       |

$$
\Delta F = 2 \text{ sector, where the pattern of deviations from the SM is unambiguously dictated by the } \U(2)^3 \text{ symmetry, the predictions of } \Delta F = 1 \text{ observables are more model dependent. Within supersymmetry with a } \U(2)^3 \text{ symmetry, it has been shown in } \cite{67} \text{ that visible effects can be generated in particular in the mixing induced CP asymmetries in } B \to \phi K_S \text{ and } B \to \eta' K_S, \text{ in the CP asymmetry } \langle A \eta \rangle \text{ in } B \to K^* \mu^+ \mu^- \text{ and in the direct CP asymmetry in } B \to X_S \gamma. \text{ While these processes are also the golden modes in MFV or “effective MFV” with flavour blind phases, they can be generated in } U(2)^3 \text{ even in the absence of flavour blind phases by means of the new phase } \gamma. \text{ In the right panel of Fig. 3 we show the difference } S_{\eta' K_S} - S_{\phi K_S} \text{ vs. } \langle A \eta \rangle \text{ for a scan with } \tan \beta < 10 \text{ and positive } \mu. \text{ While the contribution to the mixing phase in } \U(3)^3 \text{ affects } S_{\eta' K_S} \text{ and } S_{\phi K_S} \text{ in the same way, the } \Delta F = 1 \text{ penguin contributions modify only } S_{\eta' K_S}.$$

\section{6 Conclusions}

As anticipated in the introduction, the MFV hypothesis and its variations discussed here do not represent a complete answer to the flavour problem. They provide only an efficient answer to the question of why we have not seen large deviations from the SM in flavour-changing processes so far, under the assumption of new physics close to the TeV scale. However, they also provide an efficient tool to derive phenomenological predictions about deviations from the SM in flavour-changing processes that, when confronted with data, could lead to model-independent conclusions about the irreducible sources of flavour symmetry breaking accessible at low energies.

In table 3 we summarise the possible maximal deviations from the SM, taking into account theoretical errors and near-future experimental sensitivity, in a series of particularly clean CP-violating observables. We compare the MFV expectations to those of the two other flavour symmetries discussed in section 5 distinguishing the case where CP violation originates from the Yukawa couplings only and the case with additional flavour-blind phases (for more quantitative details see \cite{1,14,67}). The different pattern of effects can become useful both to distinguish between the different frameworks, if a significant deviation from the SM expectations is observed in one (or several) of the observables, or even to proof the existence of flavour symmetry-breaking terms not related to the Yukawa sector.

\section{Acknowledgments}

We thank R. Barbieri, A.J. Buras, G. Giudice, G. Perez, and W. Altmannshofer for useful comments and discussions. GI acknowledges the support of the TU München - Institute for Advanced Study, funded by the German Excellence Initiative, and the EU ERC Advanced Grant FLAVOUR (267104). DMS is supported by the EU ITN “Unification in the LHC Era”, contract PITN-GA-2009-237920 (UNILHC).

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