Gravitational collapse in braneworld models with curvature corrections

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Abstract. We study the collapse of a homogeneous braneworld dust cloud in the context of the various curvature correction scenarios, namely, induced gravity, Gauss–Bonnet, and combined induced gravity and Gauss–Bonnet. In accordance with the Randall–Sundrum model, and contrary to four-dimensional general relativity, we show in all cases that the exterior spacetime on the brane is non-static.

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In this paper, we discuss the Oppenheimer–Snyder-like collapse on a brane in the context of curvature correction terms. In the Randall–Sundrum scenario, this problem has been analysed in [1], and it was found that, in contrast to the general relativity case, the vacuum exterior of a spherical cloud is non-static. This is a result of modification of the effective Einstein equations on the brane with local and non-local terms representing high energy corrections to general relativity. The non-static nature of the exterior metric mainly arises because of the presence of bulk graviton stresses, which transmit effects non-locally from the interior to the exterior on the brane, and of the non-vanishing of the effective pressure at the boundary surface [2], which connects the interior with the exterior metric via the four-dimensional matching conditions.

We derive here the same result within the induced-gravity, the Gauss–Bonnet, and the combined Gauss–Bonnet and induced-gravity braneworld models. In all these models, the effective Einstein equations on the brane are modified by local and non-local terms, which are much more complicated than the corresponding terms in the Randall–Sundrum case, but nevertheless the non-staticity arises because of a mismatch of the interior with the exterior metric on the boundary collapsing surface.

We consider for convenience and without loss of generality the extra-dimensional coordinate $y$ such that the brane is fixed at $y = 0$. The induced metric $h_{\mu\nu}$ on this hypersurface is defined by $h_{AB} = g_{AB} - n_A n_B$, with $n^A$ the unit vector normal to the brane ($\mu, \nu = 0, 1, 2, 3; A, B = 0, 1, 2, 3, 5$). The total action of the system is taken to be

$$
S = \frac{1}{2\kappa_5^2} \int d^5x \sqrt{-g} \left\{ R - 2\Lambda_5 + \alpha \left[ R^2 - 4R_{AB}R^{AB} + R_{ABCD}R^{ABCD} \right] \right\} + \frac{r_c}{2\kappa_5^2} \int_{y=0} d^4x \sqrt{-h} (R - 2\Lambda_4) + \int_{y=0} d^4x \sqrt{-h} L_{\text{mat}},
$$

where $\mathcal{R}, R$ are the Ricci scalars of the metrics $g_{AB}$ and $h_{AB}$ respectively. The Gauss–Bonnet coupling $\alpha$ has dimensions (length)$^2$ and is defined as

$$
\alpha = \frac{1}{8g_s^2},
$$

with $g_s$ the string energy scale, while the induced-gravity crossover length scale $r_c$ is

$$
r_c = \frac{\kappa_5}{\kappa_4} = \frac{M_4^2}{M_5^2}.
$$
Here, the fundamental \( M_5 \) and the four-dimensional \( M_4 \) Planck masses are given by
\[
\kappa_5^2 = 8\pi G_5 = M_5^{-3}, \quad \kappa_4^2 = 8\pi G_4 = M_4^{-2}.
\]
The brane tension is given by
\[
\lambda = \frac{\Lambda_4}{\kappa_4^2},
\]
and is non-negative. (Note that \( \Lambda_4 \) is not the same as the cosmological constant on the brane.)

The collapse region has in comoving coordinates a Robertson–Walker metric
\[
ds^2 = -d\tau^2 + a(\tau)^2(1 + k\chi^2/4)^{-2}(d\chi^2 + \chi^2 d\Omega_2^2),
\]
where the scale factor \( a(\tau) \) is given by the modified Friedmann equation of the corresponding model, while the energy density is given by the usual dust law \( \rho = \rho_0(a_0/a)^3 \), with \( a_0 \) standing for the epoch when the cloud started to collapse. This Friedmann equation can also be written in terms of the proper radius from the centre of the cloud \( r(\tau) = a(\tau)\chi/(1 + k\chi^2/4) \) of the collapsing boundary surface at \( \chi = \chi_0 \).

Concerning the exterior of the collapse region, the most general static spherically symmetric metric is written in standard coordinates as
\[
ds^2 = -F(r)^2A(r)\,dt^2 + A(r)^{-1}\,dr^2 + r^2\,d\Omega_2^2.
\]
In order for a metric of the form (7) to be the exterior of the interior metric (6), the metric and the extrinsic curvature have to be continuous across the collapsing boundary surface. Following the method appearing in [1], we first write the standard radial geodesic motion of the freely falling boundary surface for the exterior metric
\[
r^2 = -A(r) + \frac{\dot{E}}{F(r)^2},
\]
where the dot denotes the derivative with respect to proper time \( \tau \), and \( \dot{E} \) is a constant. Secondly, transforming to null coordinates \((v, r)\), where \( dv = dt + dr/[F(r)A(r)] \), the exterior metric (7) takes the form
\[
ds^2 = -F(r)^2A(r)\,dv^2 + 2F(r)\,dv\,dr + r^2\,d\Omega_2^2,
\]
while the interior metric (6) becomes
\[
ds^2 = -\frac{a^2 - (k + a^2)r^2}{a^2 - kr^2}\,\tau^2\,dv^2 + \frac{2a\tau}{\sqrt{a^2 - kr^2}}\,dv\,dr + r^2\,d\Omega_2^2.
\]
Comparing equations (9), (10), we obtain
\[
A = 1 + E - r^2,
\]
where \( E = -k\chi_0^2/(1 + k\chi_0^2/4)^2 \). Then, from equations (8), (11) we obtain that \( F(r) \) is a constant, and by choosing \( E = 1 + E \), we take \( F(r) = 1 \). Finally, the candidate exterior metric (7) becomes
\[
ds^2 = -A(r)\,dt^2 + A(r)^{-1}\,dr^2 + r^2(d\theta^2 + \sin^2\theta\,d\phi^2),
\]
where \( A(r) \) is given by equation (11), with \( r^2 \) provided by the Friedmann equation of the interior region.
1. Induced gravity

The scale factor $a(\tau)$ is given by the modified Friedmann equation [3]–[9] of induced gravity ($\alpha \to 0$) [10]

$$\left( \frac{\dot{a}}{a} \right)^2 = \frac{\kappa^2}{3} \left( \rho + \lambda \right) + 2 \frac{k}{r_c^2} \pm \frac{1}{\sqrt{3} r_c} \left[ 4 \kappa^2 \left( \rho + \lambda \right) - 2 \Lambda_5 + \frac{12}{r_c^2} - \frac{12 C_1}{a^4} \right]^{1/2}, \quad (13)$$

where $C$ is the integration constant related to the mass of the bulk black hole. We can write equation (13) in terms of the proper radius $r(\tau)$ as

$$r^2 = \left( \frac{\kappa^2 \lambda}{3} + \frac{2}{r_c^2} \right) r^2 + \frac{\kappa^2 m 1}{3} + E \pm \frac{1}{r_c} \left[ \frac{2}{3} \left( 2 \kappa^2 \lambda - \Lambda_5 + \frac{6}{r_c^2} \right) r^4 + \frac{4 \kappa^2 m}{3} r - 4q \right]^{1/2}, \quad (14)$$

where $m = \rho_0 \phi_0 \chi_0^3 / (1 + k \chi_0^2 / 4)^3$, $q = C \chi_0^4 / (1 + k \chi_0^2 / 4)^4$. Thus, from equation (11), we obtain

$$A(r) = 1 - \frac{\kappa^2 m 1}{3} - \left( \frac{\kappa^2 \lambda}{3} + \frac{2}{r_c^2} \right) r^2 + \frac{1}{r_c} \left[ \frac{2}{3} \left( 2 \kappa^2 \lambda - \Lambda_5 + \frac{6}{r_c^2} \right) r^4 + \frac{4 \kappa^2 m}{3} r - 4q \right]^{1/2}. \quad (15)$$

For the Randall–Sundrum model, the generic four-dimensional effective equations were derived in [11]. For the induced gravity model, such effective braneworld equations were derived in [12] in the form

$$G^\mu_\nu = \kappa^2 T^\mu_\nu - \left( \kappa^2 \lambda + \frac{6}{r_c^2} \right) \delta^\mu_\nu + \frac{2}{r_c} \left( L^\mu_\nu + \frac{L}{2} \delta^\mu_\nu \right), \quad (16)$$

where the quantities $L^\mu_\nu$ are given by the algebraic equation

$$L^\mu_\nu L^\nu_\lambda - \frac{L^2}{4} \delta^\mu_\nu = T^\mu_\nu - \left( \frac{3}{r_c^2} + \frac{L}{2} \right) \delta^\mu_\nu, \quad (17)$$

($L \equiv L^\mu_\mu$) with

$$T^\mu_\nu = (\kappa_4^2 \lambda - \frac{1}{2} \Lambda_5) \delta^\mu_\nu - \kappa_4^2 T^\mu_\nu - \mathcal{E}^\mu_\nu. \quad (18)$$

$T^\mu_\nu$ is the braneworld matter content, if any, while the electric part $\mathcal{E}^\mu_\nu = C^\mu_{\ AB} n^A n^B$ of the five-dimensional Weyl tensor $C^A_{\ BCD}$ carries the influence of non-local gravitational degrees of freedom in the bulk onto the brane, making the brane equations (16) not, in general, closed [13]. Additionally, the energy–momentum tensor was shown to satisfy the usual conservation equations

$$T^\mu_\nu_{\ ;\mu} = 0, \quad (19)$$

and thus the Bianchi identities on the brane give from equation (16) differential equations among $L^\mu_\nu$:

$$L^\mu_\nu_{\ ;\nu} + \frac{L}{2} = 0, \quad (20)$$

(the semicolon means covariant differentiation with respect to $h_{\mu\nu}$).
For spherically symmetric braneworld metrics of the form (12), the system of equations (16)–(20) was fully integrated in vacuum in [14, 15], and the results are as follows. For $\mathcal{E}_{\nu}^\mu = 0$ on the brane, the solution is Schwarzschild-(A) $dS_4$

$$A(r) = 1 - \frac{\gamma}{r} - \sigma r^2,$$  \hspace{1cm} (21)

where $\gamma$ is the integration constant and $\sigma = 6/3 + 2/r_c^2 - 2\sqrt{2\kappa^2 \lambda - \Lambda} + 6/r_c^2/\sqrt{6}\Lambda$. For $\mathcal{E}_{\nu}^\mu \neq 0$, there are two classes of solutions, given in parametric form. The first one (with constant $\mathcal{E}_{\nu}^\mu$) is

$$A = 1 - \frac{\gamma}{r} - \sigma r^2 + sg(\zeta)\frac{\delta}{r}\left[\frac{128}{105} F_1\left(\frac{15}{8}, \frac{23}{8}; sg(\zeta)z\right) + \frac{9}{8}\left(1 - sg(\zeta)\frac{8}{7}\right) e^{sg(\zeta)z}\right]z^{7/8},$$

$$r = (\delta/\sqrt{\zeta})^{1/3} z^{1/8} e^{sg(\zeta)z/3},$$  \hspace{1cm} (22)

where $\delta > 0$, $\gamma$ are integration constants, and $\sigma = 6/3 + 2/r_c^2$, $\zeta = 8(2\kappa^2 \lambda - \Lambda) + 6/r_c^2/9r_c^2$. The second solution (with non-constant $\mathcal{E}_{\nu}^\mu$) is

$$A = 1 - \frac{\gamma}{r} - \sigma r^2 \pm \frac{\delta}{r} \int \frac{|v - \sqrt{3}|^{-3(\lambda + 3)/8}(v + \sqrt{3})^{-3(\lambda + 3)/8}}{|v - 3|^{7/4}} \frac{v \, dv}{(\sqrt{3}|v + \sqrt{3}|^{(\lambda - 1)/8} |v - \sqrt{3}|^{(\lambda + 3)/8})},$$  \hspace{1cm} (24)

$$r = \left(\frac{\delta}{\sqrt{\zeta}}\right)^{1/3} |v - \sqrt{3}|^{(\lambda + 1)/8} |v + \sqrt{3}|^{(\lambda - 1)/8}$$  \hspace{1cm} (25)

where $\delta > 0$, $\gamma$ are integration constants, and $\sigma = 6/3 + 2/r_c^2$, $\zeta = 9(2\kappa^2 \lambda - \Lambda) + 6/r_c^2/r_c^2$. (The $\pm$ sign of equation (24) is independent of that in equation (15).) Note that the solutions (21)–(25) are the generic braneworld solutions and have been obtained without any assumption for the bulk space. It is now obvious that the only possible static exterior solution (15), surrounding the collapsing region, cannot take one of the permissible forms (21), (22), or (24) of induced-gravity theory, which means that the no-go theorem for induced gravity has been proved.

We are going to give now another way of showing the no-go theorem of induced gravity, without using the above static solutions of the model. This is necessary in the case where such exact solutions are not known, as e.g. in the Gauss–Bonnet or in the combined Gauss–Bonnet and induced-gravity braneworld. For the induced-gravity scenario, alternatively to equation (16), generic covariant effective braneworld equations have been derived in [9] in the form (for the vacuum case)

$$\left(1 + \frac{\lambda}{6} r_c^2\kappa^2\right) G^\mu_\nu = \frac{1}{2} \left(\Lambda + \lambda \kappa^2 - 6\kappa^2\right) \delta^\mu_\nu + r_c^2 \pi^\mu_\nu - \mathcal{E}^\mu_\nu,$$  \hspace{1cm} (26)

where

$$\pi^\mu_\nu = -\frac{1}{6} G^\mu_\lambda G^\nu_\lambda + \frac{1}{12} G^\lambda_\lambda G^\mu_\nu + \frac{1}{6} G^\rho_\rho G^\mu_\nu \delta^\lambda_\nu - \frac{1}{24} (G^\lambda_\lambda)^2 \delta^\mu_\nu.$$ 

It is obvious that by taking the trace of equation (26), the electric components of the Weyl tensor, which from the braneworld viewpoint are not intrinsic quantities, disappear, and the resulting equation is

$$R^\mu_\nu R_\mu^\nu - \frac{1}{3} R^2 + \frac{4}{r_c^2} \left(1 + \frac{\lambda}{6} r_c^2\kappa^2\right) R - \frac{8}{r_c^2} \left(\Lambda + \lambda \kappa^2 - 6\kappa^2\right) = 0.$$  \hspace{1cm} (27)
We can now check by substituting the candidate solution (15) into equation (27) that this is not satisfied, which means that the interior solution cannot match to any static exterior.

Note that, while the Friedmann equation (13) was initially derived in [3] under the assumption of a particular bulk ansatz inducing, of course, the metric (6) on the brane, the derivation of this Friedmann equation was also given in [9] in a bulk-independent way, thus showing the full generality of the cosmology (13).

2. Gauss–Bonnet term

The cosmology of the braneworld Gauss–Bonnet model \( r_c \to 0 \) is given by the Friedmann equation [16, 17]

\[
\left( \frac{\dot{a}}{a} \right)^2 = -\frac{k}{a^2} + \frac{1}{8\alpha} \left( -2 + \frac{64I^2}{J} + J \right),
\]  

(28)

where the dimensionless quantities \( I, J \) are given by

\[
I = \frac{1}{8} \left[ 1 + \frac{4}{3} \alpha \Lambda_5 + \frac{8\alpha}{a^4} \right]^{1/2},
\]  

(29)

\[
J = \left[ \frac{\kappa_5^2 \sqrt{\alpha}}{\sqrt{2}} \left( \rho + \lambda \right) + \frac{\kappa_5^2 \alpha}{2} \left( \rho + \lambda \right)^2 + (8I)^3 \right]^{2/3}.
\]  

(30)

Similarly to the induced gravity case, we obtain

\[
A(r) = 1 + \frac{r^2}{8\alpha} \left[ 2 - \frac{64I(r)^2}{J(r)} - J(r) \right],
\]  

(31)

where

\[
I(r) = \frac{1}{8} \left[ 1 + \frac{4}{3} \alpha \Lambda_5 + \frac{8\alpha q}{r^4} \right]^{1/2},
\]  

(32)

\[
J(r) = \left[ \frac{\kappa_5^2 \sqrt{\alpha}}{\sqrt{2}} \left( \frac{m}{r^3} + \lambda \right) + \frac{\kappa_5^2 \alpha}{2} \left( \frac{m}{r^3} + \lambda \right)^2 + (8I(r))^3 \right]^{2/3}.
\]  

(33)

The generic effective braneworld equations for the Gauss–Bonnet model have been derived in [18] in the form

\[
\frac{2}{5} (M_{\mu \nu} + \mathcal{E}_{\mu \nu}) - \frac{1}{4} M h_{\mu \nu} + \alpha \left[ H^{(1)}_{\mu \nu} + H^{(2)}_{\mu \nu} + H^{(3)}_{\mu \nu} \right] + \frac{\alpha (10 \Lambda_5 - \alpha l)}{2(3 + \alpha M)} \left( M_{\mu \nu} - \frac{1}{4} M h_{\mu \nu} \right) = \frac{\Lambda_5}{4} h_{\mu \nu},
\]  

(34)

where

\[
H^{(1)}_{\mu \nu} = 2 M_{\mu \alpha \beta \gamma} M_{\nu}^{\alpha \beta \gamma} - 6 M^\rho_{\sigma \tau} M_{\mu \rho \sigma \tau} + 4 M M_{\mu \nu} - 8 M_{\mu \rho} M_{\nu}^{\rho} - \frac{1}{8} h_{\mu \nu} (7 M^2 - 24 M_{\alpha \beta} M^{\alpha \beta} + 3 M_{\alpha \beta \gamma} M^{\alpha \beta \gamma}),
\]  

(35)

\[
H^{(2)}_{\mu \nu} = -6 (M_{\mu \rho} \mathcal{E}^{\rho}_{\nu} + M_{\nu \rho} \mathcal{E}^{\rho}_{\mu} + M_{\mu \rho \sigma} \mathcal{E}^{\rho \sigma} + M_{\nu \rho \sigma} \mathcal{E}^{\rho \sigma} + \frac{9}{2} h_{\mu \nu} M_{\rho \sigma} \mathcal{E}^{\rho \sigma} + 3 M \mathcal{E}_{\mu \nu},
\]  

(36)
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$$H^{(3)}_{\mu\nu} = -4N_{\mu}N_{\nu} + 4N^{\rho}(N_{\rho\mu} + N_{\rho\nu}) + 2N_{\rho\sigma\mu\nu}N^{\rho\sigma} + 4N_{\mu\rho\sigma}N^{\rho\sigma}$$

$$+ 3h_{\mu\nu}(N_{\alpha}N^{\alpha} - \frac{1}{2}N\alpha\beta\gamma N^{\alpha\beta\gamma}),$$

$$I = M^2 - 8M\alpha\beta M^{\alpha\beta} + M\alpha\beta\gamma M^{\alpha\beta\gamma} - 8N_{\mu}\rho N^{\rho} + 4N_{\rho\sigma\mu\nu}N^{\rho\sigma\mu\nu} - 12M_{\mu\nu}\mathcal{E}^{\mu\nu}.$$  

These equations contain the quantities $M_{\alpha\beta\gamma}$ and $N_{\mu\rho\sigma}$, which are expressed in terms of the induced metric $h_{\mu\nu}$ and the extrinsic curvature $K_{\mu\nu}$ as follows:

$$M_{\alpha\beta\gamma} = R_{\alpha\beta\gamma} - K_{\alpha\beta}K_{\gamma} + K_{\alpha\beta\gamma},$$

$$M_{\alpha\beta} = h^{\gamma\delta}M_{\gamma\beta\delta},$$

$$N_{\mu\rho\sigma} = K_{\mu\rho\sigma} - K_{\mu\rho\sigma},$$

where $K_{\mu\nu}$ satisfies the matching conditions [19] (for the vacuum case)

$$K_{\mu}^{\nu} + \frac{2\alpha}{3}[3J_{\mu}^{\nu} - 2J_{\delta}^{\nu} - 2(3P^{\mu\rho\nu\sigma} + \delta_{\nu}^{\mu}G_{\sigma})K_{\rho}^{\sigma}] = -\frac{K_{3}^{2}\Lambda}{\delta_{\mu}^{\nu}},$$

with

$$3J_{\mu}^{\nu} = 2KK_{\mu}^{\rho}K_{\rho}^{\nu} + K_{\rho}^{\mu}K_{\sigma}^{\nu}K_{\rho}^{\sigma} - 2K_{\mu}^{\rho}K_{\sigma}^{\nu}K_{\mu}^{\sigma} - K_{2}K_{\mu}^{\nu},$$

$$P_{\mu\rho\sigma} = R_{\mu\rho\sigma} + 2\delta_{\mu}[\sigma]R_{\nu}^{\sigma} + 2\delta_{\rho}[\nu]R_{\sigma}^{\rho} + R_{\nu}^{\nu}[\sigma]$$

$(J = J_{\mu}^{\nu})$. It is remarkable that despite the complexity of the above equations, by taking the trace of equation (34) the quantities $\mathcal{E}_{\mu\nu}$ disappear (as well as the quantities $N_{\mu\rho\sigma}$ containing covariant derivatives of $K_{\mu\nu}$ with respect to $h_{\mu\nu}$), and the resulting equation is purely four dimensional

$$M + \alpha(M^2 - 4M\alpha\beta M^{\alpha\beta} + M\alpha\beta\gamma M^{\alpha\beta\gamma} = 2\Lambda_5.$$  

After solving the cubic system (42) for $K_{\mu}^{\nu}$, one has equation (43) constructed solely out of the induced metric $h_{\mu\nu}$.

In order to do so, it is convenient to write the metric (12) in a form where its angular part appears as a two-dimensional conformally Euclidean space

$$ds^2 = -A(r)dt^2 + \frac{dr^2}{A(r)} + r^2f(x_1, x_2)(dx_1^2 + dx_2^2),$$

where $f(x_1, x_2) = [1 + (x_1^2 + x_2^2)/4]^{-2}$, and $x_1 = 2\tan(\theta/2)\sin\phi$, $x_2 = 2\tan(\theta/2)\cos\phi$. Due to the symmetry of the metric (44), the components of $K_{\mu}^{\nu}$ in the coordinates $(t, r, x_1, x_2)$ take the form

$$K_{\mu}^{\nu} = \text{diag}(K_1, K_2, K_3, K_3).$$

Subtracting, now, the $tt$-equation from the $rr$-equation of the system (42), we obtain the separable equation

$$[4\alpha\tau^2K_2^3 + 4\alpha(A - 1) - r^2](K_2 - K_1) = 0,$$

with solutions

$$K_3 = \pm \frac{1}{\sqrt{8\alpha}} \left[ \frac{64(I(r))^2}{J(r)} + J(r) \right]^{1/2},$$

$$J_2 = \frac{64I(r)^2}{J(r)} + J(r).$$
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or

\[ K_2 = K_1. \] (48)

For the solution (47), plugging back into the system (42), we obtain a system of two equations for \( K_1, K_2 \), from which the following equation arises

\[ 2\alpha A' - r \mp \sqrt{\alpha \kappa_5^2 \lambda r^2 / 2r^2 + 4\alpha (1 - A)} = 0, \] (49)

(prime means differentiation with respect to \( r \)). This equation is easily seen to be inconsistent with \( A(r) \) given by equation (31). For the solution (48), plugging back into the system (42), the situation is more complicated, and a polynomial equation of fifth degree on \( K_1 \) arises:

\[
K_3 = \frac{4(1 - 2\alpha^{-1}A')K_1 + \kappa_5^2 \lambda}{8\alpha K_1^2 + 2(2\alpha' - 1)},
\]

(50)

\[ a_5 K_1^5 + a_4 K_1^4 + a_3 K_1^3 + a_2 K_1^2 + a_1 K_1 + a_0 = 0, \] (51)

where

\[
a_5 = 2\alpha[r^2 + 4\alpha(1 - A)], \quad a_4 = \alpha\kappa_5^2 \lambda r^2,
\]

\[
a_3 = (2\alpha A'' - 1)[r^2 + 4\alpha(1 - A)],
\]

\[
a_2 = \kappa_5^2 \lambda r[\alpha A' + r(\alpha A'' - 1)],
\]

\[
2a_1 = 1 - (2\alpha A'' - 1)[4\alpha A' + A(2\alpha A'' - 1) - 4\alpha A^2] + (2\alpha' - 1)[r^2 + 4\alpha(2\alpha + 4\alpha A' - 4\alpha A'')]
\]

\[
- (3 + 4\alpha \kappa_5^2 \lambda^2) r^2 / 4\alpha,
\]

\[
a_0 = \kappa_5^2 \lambda r(2\alpha A'' - 1)(2\alpha A'' + r - 4\alpha A') / 16\alpha.
\]

Supposing that equation (43) is valid, for the metric (44), and after substituting the values of \( K_2, K_3 \) from equations (48), (50), it becomes a polynomial equation of sixth degree on \( K_1 \)

\[ b_6 K_1^6 + b_4 K_1^4 + b_3 K_1^3 + b_2 K_1^2 + b_1 K_1 + b_0 = 0, \] (53)

where

\[
b_6 = 32\alpha^2[r^2 + 4\alpha(1 - A)],
\]

\[
b_4 = 16\alpha[(2\alpha A'' - 3)r^2 - 6\alpha - 12\alpha A'(\alpha A'' - r) + 3\alpha(r^2 + 4\alpha)A'' - 6\alpha A(2\alpha A'' - 1)],
\]

\[
b_3 = 32\alpha \kappa_5^2 \lambda r(2\alpha A' - r),
\]

\[
b_2 = 12\alpha - (3 + 3\alpha \kappa_5^2 \lambda^2 + 8\alpha \Lambda_5)r^2 - 12\alpha A(2\alpha A'' - 1)^2 + 4\alpha^2 A''[4(\Lambda_5 r^2 - 3)
\]

\[ + (2\alpha + 4\alpha)A''],
\]

\[
b_1 = (2\alpha A'' - 1)[2\alpha(r^2 + 4\alpha)A'' - A''[8\alpha + (1 - 4\alpha \Lambda_5) r^2]] - 2(2\alpha A'' - 1)
\]

\[ \times [A(2\alpha A'' - 1) + 2\alpha A'(\alpha A' - r)] + 2 - (2\Lambda_5 + \kappa_5^4 \Lambda^2 / 2)r^2].
\]

Equations (51) and (53) have to be satisfied simultaneously. After some algebraic manipulations, this system of equations is written equivalently as the following system:

\[ F_2 K_1^2 + F_1 K_1 + 1 = 0, \] (55)
From the above equations, we can write the candidate black hole metric as

$$K_1 = \frac{C_2 - (C_1 - F_1)F_1 - F_2}{(C_1 - F_1)F_2 - C_3},$$

where

$$(C_1, C_2, C_3) = (B_2(B_1 - p_1) + p_3 - B_4, B_3(B_1 - p_1) + p_4 - B_4),$$

$$(F_1, F_2) = ((C_1C_2 - C_3)(B_1 - p_1) - C_2(B_2 - p_2) + B_4 - p_3, C_1C_3(B_1 - p_1) - C_3(B_2 - p_2) - p_3)/(C_1^2 - C_2)(B_1 - p_1)$$

and the various $p_i, q_i$ are related to $a_i, b_i$ as

$$p_i = a_i/a_0, \quad q_i = b_i/b_0.$$ (57)

It is now straightforward to check that the value $K_1$ of equation (56) does not satisfy equation (55), which means that the no-go theorem of the Gauss–Bonnet braneworld has been proved.

3. Gauss–Bonnet and induced gravity

The cosmology of the combined Gauss–Bonnet and induced gravity braneworld is given by the Friedmann equation [20]

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{k}{a^2} + \frac{4 - 3\beta}{12\beta\alpha} - \frac{2}{3\beta\alpha} \sqrt{P^2 - 6Q} \cos \left(\Theta \pm \frac{\pi}{3}\right),$$

where the dimensionless quantities $\beta, P, Q, \Theta$ are given by

$$\beta = \frac{256\alpha}{9r_c^2},$$

$$P = 1 + 3\beta I,$$ (60)

$$Q = \beta \left[\frac{1}{4} + I + \frac{k^2\alpha}{3}(\rho + \lambda)\right],$$ (61)

$$\Theta(P, Q) = \frac{1}{3} \arccos \left[\frac{2P^3 + 27Q^2 - 18PQ}{2(P^2 - 6Q)^{3/2}}\right].$$ (62)

The $\pm$ sign in equation (58) is the same as that in equation (13). The region in $(P, Q)$-space for which equation (58) is defined is

$$1 \leq P < \frac{4}{3},$$ (63)

$$2[9P - 8 - (4 - 3P)^{3/2}] \leq 27Q \leq 3P[3 - \sqrt{3(3 - 2P)}].$$ (64)

From the above equations, we can write the candidate black hole metric as

$$A(r) = 1 - \frac{4 - 3\beta}{12\beta\alpha} r^2 + \frac{2r^2}{3\beta\alpha} \sqrt{P(r)^2 - 6Q(r)} \cos \left(\Theta(r) \pm \frac{\pi}{3}\right).$$ (65)
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where

\[ P(r) = 1 + 3\beta I(r), \quad (66) \]

\[ Q(r) = \beta \left[ \frac{1}{4} + I(r) + \frac{\kappa_5^2 \alpha}{3} \left( \frac{m}{r^3} + \lambda \right) \right] , \quad (67) \]

\[ \Theta(r) = \frac{1}{3} \arccos \left[ \frac{2P(r)^3 + 27Q(r)^2 - 18P(r)Q(r)}{2(P(r)^2 - 6Q(r))^{3/2}} \right] . \quad (68) \]

Similarly to the induced gravity case, in the pure Gauss–Bonnet model, the Friedmann equation (28) was obtained in [18] in a bulk-independent way, showing, thus, that this is the most general Robertson–Walker braneworld cosmology of the model. For the combined Gauss–Bonnet and induced gravity braneworld scenario, its Friedmann equation (58) has not been derived yet in a bulk-independent way, but we have no reason to expect any discrepancy in this case either.

In the present case, equations (34)–(41)—as well as equation (43)—of the pure Gauss–Bonnet case remain unchanged, since they are described by bulk information, while the matching condition (42) is now modified by setting its right-hand side equal to

\[ -\frac{\kappa_5^2}{6} \left[ \lambda \delta^\mu_v - 3\kappa_5 \left( R^\mu_v - \frac{R}{6} \delta^\mu_v \right) \right] . \quad (69) \]

Following the same steps as before, we find the same separable equation (46), with solutions now

\[ K_3 = \pm \frac{1}{\sqrt{3\alpha}} \left[ 1 - 2\sqrt{P(r)^2 - 6Q(r)} \cos \left( \Theta(r) \pm \frac{\pi}{3} \right) \right]^{1/2} , \quad (70) \]

or

\[ K_2 = K_1 . \quad (71) \]

Solution (70) leads to the equation

\[ 2\alpha A' - r \mp \sqrt{\alpha} \left[ \kappa_5^2 \lambda r^2 - r_c(1 - A - rA') \right]/2\sqrt{r^2 + 4\alpha(1 - A)} = 0 , \quad (72) \]

which is inconsistent with equation (65). Solution (71) leads to two equations of fifth and sixth degree on \( K_1 \) of the form (51) and (53) respectively, with the corresponding coefficients defined now as follows (denoted with primes):

\[ a'_5 = a_5, \quad a'_4 = a_4 - \alpha r_c(1 - A - rA'), \quad a'_3 = a_3, \]

\[ a'_2 = a_2 + r_c[1 + 2\alpha A^2 + A(2\alpha A' - 1) + rA'(3\alpha A'' - 2) - A''(2\alpha + r^2/2)]/2, \]

\[ a'_1 = a_1 - r_c(2A' + rA'')\left[ 4\kappa_5^2 \lambda r + r_c(2A' + rA'') \right]/32, \]

\[ a'_0 = a_0, \quad (73) \]

\[ b'_6 = b_6, \quad b'_4 = b_4, \]

\[ b'_3 = b_3 - 16\alpha r_c(r - 2\alpha A')(2A' + rA''), \]

\[ b'_2 = b_2 - 3\alpha r_c(2A' + rA'')\left[ 4\kappa_5^2 \lambda r + r_c(2A' + rA'') \right]/2, \]

\[ b'_0 = b_0. \]
Equations (55)–(57) remain the same for the above primed quantities $a_i', b_i'$, and thus their incompatibility can be easily checked, proving the no-go theorem of the combined Gauss–Bonnet and induced-gravity braneworld.

Our analysis of all the above considered models is based on four-dimensional solutions or four-dimensional effective braneworld equations, and we have not studied the bulk extension of the considered braneworld regions (static exterior and Robertson–Walker interior). We know however that for a given continuous boundary metric and extrinsic curvature the propagation of the field equations in five dimensions is a well defined initial value problem, solvable in principle.

In conclusion, we have studied the Oppenheimer–Snyder-like collapse on braneworld models with curvature corrections. In all cases considered, using the four-dimensional effective equations, and without making assumptions about the bulk, we have found that the exterior vacuum spacetime on the brane is non-static. We have not found the exterior metric, thus we are not in a position to know if the gravitational collapse on the brane leaves at late times a signature in the exterior, or if, on the contrary, the non-static exterior is transient, tending to a static geometry.

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