Vacuum fluctuations and moving atoms/detectors: From Casimir-Polder to Unruh Effect

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Abstract.
In this note we report on some new results [1] on corrections to the Casimir-Polder [2] retardation force due to atomic motion and present a preliminary (unpublished) critique on one recently proposed cavity QED detection scheme of Unruh effect [3]. These two well-known effects arise from the interaction between a moving atom or detector with a quantum field under some boundary conditions introduced by a conducting mirror/cavity or dielectric wall.

The Casimir-Polder force is a retardation force on the atom due to the dressing of the atomic ground state by the vacuum electromagnetic field in the presence of a conducting mirror or dielectric wall. We have recently provided an improved calculation by treating the mutual influence of the atom and the (constrained) field in a self-consistent way. For an atom moving adiabatically, perpendicular to a mirror, our result finds a coherent retardation correction up to twice the stationary value.

Unruh effect refers loosely to the fact that a uniformly accelerated detector feels hot. Two prior schemes have been proposed for the detection of ‘Unruh radiation’, based on charged particles in linear accelerators and storage rings. Here we are interested in a third scheme proposed recently by Scully et al [4] involving the injection of accelerated atoms into a microwave or optical cavity. We analyze two main factors instrumental to the purported success in this scheme, the cavity factor and the sudden switch-on factor. We conclude that the effects engendered from these factors are unrelated to the Unruh effect.
1. Introduction

In this short note we report on some new results and present some thoughts on two well-known effects arising from the interaction between a moving atom or detector with a quantum field. The Casimir-Polder force \[2\] is a retardation force on the atom due to the dressing of the atomic ground state by the vacuum electromagnetic field in the presence of a boundary (e.g., a mirror or a cavity). We set forth to improve on existing calculations by treating the mutual influence of the atom and the (constrained) field in a self-consistent way. This is in order to maximally preserve the coherence of the combined system, which is a highly desirable if not critically demanded criteria for quantum computer designs. Since a self-consistent treatment requires backreaction considerations, the atom’s motion should be included to account for the full effect of the field on the atom. For an atom moving adiabatically, perpendicular to a mirror, our recent result finds a coherent retardation correction up to twice the stationary value \[1\].

The Unruh effect \[3\] described colloquially states that a uniformly accelerated detector (UAD) feels hot at the Unruh temperature. There are at least three classes of detection schemes proposed. One based on charged particles in linear accelerators purports to measure at a distance the radiation emitted from the UAD \[5\]. Researchers have commonly agreed that a uniformly accelerated detector does not emit radiation. (See, e.g., \[6, 7, 8, 9, 10, 11, 12\] and references therein). If there were emitted radiation it would have to be from nonuniform acceleration and treated by nonequilibrium quantum field theory concepts and techniques \[13, 14\]. But these are not the Unruh effect. Detection of Unruh effect in storage rings has earlier been proposed \[15\]. Even though acceleration in the circular case shares some features with linear acceleration which is what Unruh effect entails, there are basic differences between these two cases. For one, in circular motion acceleration comes from changes in the direction, not in the speed. Controversies of its nature and viability remain \[16\].

Recently a detection scheme has been proposed by Scully et al \[4\] involving the injection of accelerating atoms into a microwave or optical cavity. It is this scheme which we wish to discuss here. We first identify the main working parts in this scheme and then analyze their effects and relevance to the Unruh effect. The main factors we shall consider here are a) the effect of cavity fields on the accelerating atom in the emission of photons and b) the effect of sudden switching on of the atom-field interaction from the injection of atoms into the cavity. For the cavity factor we find that a thermal distribution of the photons in the cavity should not be identified as a manifestation of the Unruh effect. As for the sudden switch-on effect, photons are produced from the nonadiabatic amplification of vacuum fluctuations (similar to cosmological particle creation \[17\]), but not from the atom’s uniform acceleration (analogous to Hawking radiance in black holes \[18\]).

Note that in both effects there are three dynamical variables: the internal degrees of freedom (dof) of a detector – a two level system, the external degrees of freedom – the center of mass (COM) of the detector/atom in motion, and the quantum field (scalar
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or electromagnetic) under some constrained condition imposed by the presence of the mirror or cavity. (One can easily extend this class of problems to cases where the mirror is in motion [19, 11] giving rise to dynamical Casimir effects [20] and analogs of Hawking effect [18] in quantum black holes.) In Casimir-Polder effect it is the external dof – the COM – of the atom interacting with the field modes altered by the mirror which are of interest, whereas in the Unruh effect, as the trajectory is prescribed, it is the internal dof of the detector/atom and the field correlations which are of interest. A more obvious difference is, of course, the atom in Casimir-Polder undergoes nonrelativistic motion, whereas it is the uniform acceleration engendering an event horizon which gives the distinct thermal feature in the Unruh radiation. But ultimately these are only different regimes and manifestations of the same set of problems related to the kinematic and boundary effects on the quantum vacuum. We deliberately bring these two effects into the same discussion so as to stimulate stronger cross fertilization between these two apparently disparate fields, namely, quantum/atom optics and quantum field theory in curved spacetime [17]. It is the belief of one of us that many fundamental effects in the latter discovered theoretically in the 70's may find verifications in the experimental schemes of the former. What is more interesting is that there is plenty of room in their intersection for asking new sets of questions pertaining to the (nonequilibrium) statistical mechanical properties of the quantum vacuum as influenced by moving atoms, detectors, mirrors or background fields such as vacuum viscosity due to particle creation [21, 22, 23], vacuum friction due to atom motion [24, 25], quantum noise under exponential red-shifting [26, 27], field correlations in spacetimes with event horizons [12, 28] and entanglement and teleportation with non-inertial observers [29, 10].

This paper is evenly divided into two parts: Sections 2, 3 deal with the correction to the Casimir-Polder effect due to atom motion, Sec. 4 summarizes the main features of Unruh effect, Sec. 5 discusses the scheme of Scully et al for the detection of Unruh effect by accelerating atoms in a cavity.

**Part I: Casimir-Polder Effect**

Recent experiments [30, 31] have reported on the measurement of the Casimir-Polder (C-P) retardation force [2], which is a retardation force arising from the quantum modification of the electrostatic attraction of a polarizable atom to its image in the wall. In its usual interpretation the retardation force arises from the dressing of the atomic ground state by the vacuum electromagnetic field (EMF) in the presence of a boundary. With this interaction, the ground state of the combined atom-EMF system is no longer a product of the separate free ground states, but is instead an entangled atom-EMF state. The retardation force originates from the quantum correlation between two interacting systems (or, if one of them is of less concern in the observation, one system S – such as the atom, and its environment E – the quantum field). Another manifestation is colored noise from the coarse-grained quantum fields. To follow the quantum correlations between S and E, we need to adopt a method which can maximally preserve the quantum
coherence in treating the S and E interaction consistently.

The state-of-the-art derivation [32, 33, 34] of the Casimir-Polder retardation force is by calculating the gradient of a spatially dependent dressed ground state. Here we report on a rederivation [1] of the C-P retardation force given in terms of the recoil associated with the emission and reabsorption of virtual photons. While the gradient calculation of the force assumes a stationary atom, our method treats an atom that moves adiabatically.

Our result in the stationary atom limit is in exact agreement with the Casimir-Polder force. In the case of an adiabatically moving atom, we find a coherent retardation correction up to twice the stationary value. The additional correction due to atomic motion can be understood as indicating that the dressed ground states for stationary and moving atoms are not the same, the difference arising from the Doppler shift of the EMF modes with respect to the conducting wall. That is, a moving atom is in a Doppler shifted vacuum, so its dressed ground state is altered from the stationary one. In addition, what is usually pictured physically in terms of a gradient force is explained here in terms of the recoil associated with emission and absorption of virtual photons.

‡ This provides a quantum field theory interpretation of the dipole force [35].

This work is relevant to applications in which atoms are trapped on the order of a resonant atomic wavelength near a surface. Examples include evanescent wave gravito-optical [36], microlens array [37], and magnetic chip trapping [38]. Recent experiments have demonstrated the measurable effects of retardation on atomic motion near a surface [30, 31]. Those effects will become more important as such applications become more refined. That is especially true when precision control over the motion of atoms is needed, such as the implementation of two-qubit gates in AMO quantum computer designs.

‡ After our work [1] was published Professors Paul Davies and Gerard Milburn brought to our attention Pendry’s work [24] on vacuum friction due to atoms moving near an imperfectly conducting surface. Pendry’s interpretation of the force in his case is similar to ours here for the Casimir-Polder retardation force arising from the recoil of Doppler shifted emitted and absorbed virtual photons. Note atoms are moving parallel in Pendry’s case whereas in ours they are perpendicular to the wall.
2. Approach

We assume an atom placed near a conducting wall is initially in a factorizable state with the EMF vacuum. The combined system evolves according under the minimal coupling QED Hamiltonian in the dipole approximation, but without the rotating wave approximation

\[
H = \frac{P^2}{2M} + \hbar \omega_0 S_+ S_- + \hbar \sum_k \omega_k b_k^\dagger b_k + H_I = H_0 + H_I. \tag{1}
\]

The operators \(S_\pm\) are the up and down operators of the atomic qubit and \(\omega_0\) is the atomic transition frequency. The operators \(b_k\) and \(b_k^\dagger\) are the EMF mode annihilation and creation operators, and \(\omega_k\) are the frequencies of the EMF modes. The interaction Hamiltonian is made up of two parts \(H_I = H_{I1} + H_{I2},\)

\[
H_{I1} = \hbar \sum_{k \in \epsilon} \frac{g}{\sqrt{\omega_k}} [p_{eg} S_+ + p_{ge} S_-] \cdot [u_k b_k + u_k^\dagger b_k^\dagger] \tag{2}
\]

\[
H_{I2} = \hbar \sum_{k \in \epsilon} \frac{\lambda^2}{\sqrt{\omega_k}} [u_k \cdot u_k b_k b_k^\dagger + u_k^\dagger \cdot u_k (\delta_{kl} + 2b_l^\dagger b_l) + u_k^\dagger \cdot u_k^\dagger b_k b_k^\dagger]. \tag{3}
\]

The vector \(p_{eg} = \langle e|p|g\rangle = -im\omega_0\langle e|f|g\rangle\) is the dipole transition matrix element between the ground (\(g\)) and excited (\(e\)) states. The vectors \(u_k(X) = \hat{\epsilon}_k f_k(X)\) contain the photon polarization vectors \(\hat{\epsilon}_k\) and the spatial mode functions \(f_k(X)\). The coupling constants are \(g = -\sqrt{8\pi^2\alpha c/m^2}\) and \(\lambda = \sqrt{4\pi^2\hbar c/m},\) with \(\alpha\) being the fine structure constant.

In the presence of a conducting plane boundary, the spatial mode functions of the EMF which satisfy the imposed boundary conditions are the TE and TM polarization modes \(\mathcal{E}\),

\[
u_{k1}(X) = \sqrt{\frac{2}{L^3}} \hat{Z} \sin(k_Z Z) e^{ik \parallel \cdot X} \tag{4}
\]

\[
u_{k2}(X) = \sqrt{\frac{2}{L^3}} \frac{1}{k_\parallel} [k_\parallel \hat{Z} \cos(k_Z Z) - ik_Z \hat{K}_\parallel \sin(k_Z Z)] e^{ik \parallel \cdot X}, \tag{5}
\]

and their complex conjugates.

The transition amplitude for the atom to move from some initial to final position, while remaining in the atomic ground state with the EMF in the vacuum is,

\[
K[X_f; t + \tau, X_i; t] = \langle \phi(X_f; t + \tau) | \exp[\frac{-i}{\hbar} \int_{t}^{t+\tau} H(s) ds] | \phi(X_i; t) \rangle \tag{6}
\]

for which the expectation value of the momentum operator is derived.

\[
\langle \hat{P} \rangle(t + \tau) = \frac{\hbar}{N} \int \frac{dP_f}{(2\pi)^3} \int dX_i dX_i' K[P_f; t + \tau | X_i; t] \Psi(X_i) \Psi^*(X_i') K^*[P_f; t + \tau | X_i'; t], \tag{7}
\]

with the normalization factor

\[
N = \int \frac{dP_f}{(2\pi)^3} \int dX_i dX_i' K[P_f; t + \tau | X_i; t] \Psi(X_i) \Psi^*(X_i') K^*[P_f; t + \tau | X_i'; t]. \tag{8}
\]
The transition amplitude is evaluated as a path integral in which Grassmannian and bosonic coherent states are used to label the atomic and EMF degrees of freedom, respectively \[40, 41, 42, 43\]. The position and momentum basis are used for the atom’s center of mass degree of freedom. The major approximation applied is a resummed 2nd order vertex approximation. The 2nd order vertex approximation preserves the coherence of the combined system at long and short times, as it is a partial resummation of all orders of the coupling. The resulting transition amplitude is

\[
K[X_f; t + \tau, X_i; t] = \left( \frac{M}{2\pi i\hbar} \right)^{3/2} \exp \left\{ i \int_t^{t+\tau} \left[ \frac{M\dot{X}_c^2}{2\hbar} \right. \right.
+ \left. \left. \frac{g^2}{\omega_k} \int_s^t \sum_k \frac{g^2}{\omega_k} e^{-i(\omega_k+\omega_0)(s-r)} \mathbf{u}_k(X_c^0(s)) \cdot \mathbf{u}_k(X_c^0(r)) \right. \right.
- \sum_k \frac{\lambda^2}{\omega_k} \left. \frac{\mathbf{u}_k(X_c^0(s)) \cdot \mathbf{u}_k(X_c^0(s))}{\omega_k} \right] ds \right\}. \tag{9}
\]

Assuming the position wavefunction of the atom is a Gaussian wavefunction centered at \((R, P_0)\) with the standard deviations \((\sigma, 1/\sigma)\), the force on the atom, which is the time derivative of the momentum expectation value is found to be

\[
F_c(R, v, t + \tau) = -2\pi i\alpha_0\hbar\omega_0^2 \sum_k \frac{k_z \cos^2 \theta}{\omega_k} e^{-2ik_z \cdot (R + V\tau)}
+ \frac{\pi\alpha_0\hbar\omega_0^3}{L^3} \sum_k \frac{k_z \cos^2 \theta}{\omega_k} \int_t^{t+\tau} ds e^{-ik_z \cdot (2R + V(\tau + s - t))}
\times \left[ e^{-i(\omega_k+\omega_0)(t+\tau-s)} - e^{i(\omega_k+\omega_0)(t+\tau-s)} \right], \tag{10}
\]

where \(\alpha_0\) is the static ground state polarizability. The mass of the atom has been taken to infinity and its extension to a point, while finite terms are retained to their effect on the dynamics.

### 3. Results and Interpretation

#### 3.1. stationary atom

The total force on a stationary atom is given by the sum of the correction force, while setting \(v = 0\), and the electrostatic force. The stationary atom force exhibits a transient behavior when the atom first "sees" itself in the wall. Then, on a timescale of several atom-wall round trip light travel times it asymptotes to the following constant steady state value

\[
F_{sa}(R, \tau >> 2R/c) = \hat{e}_z \frac{\alpha_0\hbar\omega_0^2}{8\pi} \left( \frac{d}{dR} \right)^3 1 \int_0^\infty \frac{dx}{x^2 + \omega_0^2} e^{-2Rx/c}. \tag{11}
\]

From Eq. (11) the potential which a stationary atom feels is easily found to be

\[
U_{sa}(R) = -\frac{\alpha_0\hbar\omega_0^2}{8\pi} \left( \frac{d}{dR} \right)^2 1 \int_0^\infty \frac{dx}{x^2 + \omega_0^2} e^{-2Rx/c}, \tag{12}
\]
with asymptotic limits
\[
U_{sa}(R) \to -\frac{\alpha_o \hbar \omega_0}{8 \pi R^3} \quad \text{for } R << \frac{c}{\omega_0},
\]
\[
U_{sa}(R) \to -\frac{3\alpha_o \hbar c}{8\pi R^3} \quad \text{for } R >> \frac{c}{\omega_0},
\]
which exactly reproduces the results from the energy gradient approaches \[2\].

3.2. moving atom

The retardation force for a moving atom can be determined from Eq. (10) by applying a separation of short time scale dynamics from long time scale dynamics analogous to standard methods for determining the dipole force on an atom in a laser beam \[35\]. There, assuming that the atom’s position is constant on short timescales, the optical Bloch equations are solved for the steady state values of the internal state density matrix elements. On long time-scales the matrix elements are replaced by their steady state values and put into the Heisenberg equation of motion for the atomic COM momentum. Such a procedure is justified when the internal and external dynamics evolve on vastly different timescales. The analogous separation here will be of the short timescale describing the self-dressing of the atom-EMF system and the long timescale describing the motion of the atom.

After evaluating adiabatically and combining the retardation correction force with the electrostatic force, the atom-wall force is found to be
\[
F_{am}(R) = \hat{e}_z \frac{\alpha_o \hbar \omega_0^2}{8\pi} \left( \frac{d}{dR} \right)^3 \left( \frac{R}{x^2 + \omega_0^2} \right)^R \int_0^\infty \frac{dx}{x^2 + \omega_0^2} e^{-2Rx/c}
\]
\[
- \hat{e}_z \frac{\alpha_o \hbar \omega_0^2}{4\pi} \left( \frac{d}{dr} \right)^3 \int_0^\infty \frac{dk}{k + \omega_0} \sin(2kr) \bigg|_R^{R_0}.
\]
The first term is the stationary atom-wall force and the second term is a residual force which pulls the atom back to its original point of release. The force can easily be turned into the potential which the atom feels:
\[
U_{am}(R) = -\frac{\alpha_o \hbar \omega_0^2}{8\pi} \left( \frac{d}{dR} \right)^2 \left( \frac{R}{x^2 + \omega_0^2} \right)^R \int_0^\infty \frac{dx}{x^2 + \omega_0^2} e^{-2Rx/c}
\]
\[
+ \frac{\alpha_o \hbar \omega_0^2}{4\pi} \left( \frac{d}{dr} \right)^2 \int_0^\infty \frac{dk}{k + \omega_0} \sin(2kr) \bigg|_{R_0}^R.
\]
Since the first term in the potential is the stationary atom-wall potential, in the regions near and far from the wall it will have the expected inverse powers of distance dependence, as shown in Eq. (13). The second term is the residual potential due to the motion.

3.3. Conclusion

In the traditional (energy gradient) approach, one interprets the force between a polarizable atom and a wall as arising from the Lamb shift in the atomic ground state energy. Spatial variation of the ground state energy is expected to generate a force
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which pushes the atom perpendicular to the wall, but the mechanism for such a force is not given explicitly. Our approach provides an interpretation of how a net force arises from the emission-reabsorption processes in the presence of a boundary. It goes beyond Lamb shift calculations in that it incorporates the effect of slow atomic motion.

The extra correction from our coherent QED calculation makes a verifiable prediction. The alteration of the force has its best chance of being measured in experiments involving cold atoms bouncing off the evanescent field of a laser beam totally internally reflected in a crystal. In those experiments the laser is blue detuned, which imposes a repulsive potential to counter the attractive potential of the wall thus creating a barrier for cold atoms moving toward the crystal to bounce against. As the intensity of the evanescent laser field is lowered the height of the barrier is lowered. At some threshold value the barrier height will fall below the classical tunnelling height and no atoms will be reflected. The van der Waals, Casimir-Polder, and our coherent QED (corrected Casimir-Polder) forces all give different predictions for that threshold laser intensity. The calculations done here are for a perfect conductor, not a dielectric boundary, so the modifications predicted here should not be applied directly to the case of a dielectric boundary. However, a general statement can be made that a coherent QED correction will cause a lowered prediction for the threshold laser power, since it will tend to decrease the atom-wall attraction. If one naively applies a dielectric factor to our result for the conducting plate to compensate for the difference, the present prediction for the threshold energy in units of the natural line width (14.8 Γ) is closer to the measured value (14.9 \pm 1.5 Γ), compared to the previously predicted value of (15.3 Γ) [31].

Part II. Detection of Unruh Effect by Accelerating Atoms in Cavity

4. Unruh Effect: Main Features and Common Misunderstandings

Unruh effect attests that a particle detector following a uniformly accelerated trajectory § in Minkowski spacetime perceives the vacuum of the field as a thermal bath. It is a purely quantum mechanical effect since it originates from the vacuum quantum fluctuations of the field and is nonexistent for a classical field. We define a detector as any quantum system with some internal degree of freedom which couples to a quantum environment. Unruh used a monopole detector interacting with a scalar field in his exposition. It could also be a two-level atom interacting with an electromagnetic field. In this section we briefly discuss several fundamental features of Unruh effect whose lack of understanding has caused some confusion in some experimental proposals for its detection. We then focus on the Scully et al scheme involving accelerating atoms in a cavity field.

§ In special relativity, an object is considered to be uniformly accelerated if the acceleration always remains the same as measured at each instant of time by the observers in the inertial frame where the object is at rest at that time.
4.1. Interaction between a Moving Detector and a Quantum Field

We model the moving detector by a harmonic oscillator locally coupled to a massless real scalar field $\phi$ in $1 + 1$ dimensions and follow the discussions of [10, 11]. The total action $S[Q, \varphi] = S_{\text{osc}}[Q] + S_{\text{f}}[\varphi] + S_{\text{int}}[Q, \phi]$ is the sum of three contributions: The action for a free harmonic oscillator of mass $M$ and frequency $\Omega$ is

$$S_{\text{osc}}[Q] = \int d\tau \left[ \frac{1}{2} M \dot{Q}^2(\tau) - \frac{1}{2} M \Omega^2 Q^2(\tau) \right],$$

where $Q(\tau)$ is the internal coordinate of the oscillator which follows a trajectory $x^\mu = (t(\tau), x(\tau))$ parametrized by its proper time $\tau$. The action for the free massless field $\phi$ is

$$S_{\text{f}}[\phi] = \int dt dx \left[ \frac{1}{2} \left( \partial_\tau \phi(x^\mu) \right)^2 - \frac{1}{2} \left( \partial_x \phi(x^\mu) \right)^2 \right].$$

The interaction action is

$$S_{\text{int}}[Q, \phi] = -\lambda \int d\tau Q(\tau) \frac{d\phi}{d\tau}(x^\mu(\tau)),$$

where $\lambda$ is the coupling constant between the field and the detector, represented by the harmonic oscillator. The detector couples locally to the field at the spacetime points along its trajectory. (For details in the transform between $t$ and $\tau$ in the integration limits, see [10].)

The dynamics of the detector (our system S) can be studied by using the influence functional formalism [44] for quantum open systems, viewing the field as its environment (E). The state of the detector is described by the reduced density matrix $\rho_r(Q_i, Q'_i, t_i)$. At the initial time $t_i$ assume that the field and the detector are uncorrelated. The reduced density matrix for the detector at a later time $t_f$ is given formally by

$$\rho_r(Q_f, Q'_f, t_f) = \int dQ_i dQ'_i \int_{Q_i}^{Q_f} DQ \int_{Q'_i}^{Q'_f} DQ' e^{i(S[Q] - S[Q'] + S_{\text{IF}}[Q, Q'])/\hbar} \rho_r(Q_i, Q'_i, t_i).$$

The influence action $S_{\text{IF}}$ has the following expression:

$$S_{\text{IF}}[Q, Q'] = -2 \int_{\tau_i}^{\tau_f} d\tau \int_{\tau_i}^{\tau_f} d\tau' \Delta(\tau) D(\tau, \tau') \Sigma(\tau')$$

$$+ \frac{i}{2} \int_{\tau_i}^{\tau_f} d\tau \int_{\tau_i}^{\tau_f} d\tau' \Delta(\tau) N(\tau, \tau') \Delta(\tau'),$$

where $\Delta(\tau) \equiv Q'(\tau) - Q(\tau)$, $\Sigma(\tau) \equiv \frac{1}{2} [Q'(\tau) + Q(\tau)]$ and the dissipation and noise kernels are given respectively by (note these simple expressions are valid only for linear S-E coupling, see, [11]):

$$D(\tau, \tau') = \frac{i}{2} \left\langle \left[ \frac{d\phi}{d\tau}(\tau), \frac{d\phi}{d\tau'}(\tau') \right] \right\rangle,$$

$$N(\tau, \tau') = \frac{1}{2} \left\langle \left\{ \frac{d\phi}{d\tau}(\tau), \frac{d\phi}{d\tau'}(\tau') \right\} \right\rangle.$$
Here $[,]$ and $\{,\}$ denote the commutator and anti-commutator respectively, and the expectation values are taken with respect to the initial state of the field, assumed to be the Minkowski vacuum.

A uniformly accelerated trajectory for the detector is characterized by

$$t(\tau) = \alpha^{-1} \sinh \alpha \tau, \quad x(\tau) = c\alpha^{-1} \cosh \alpha \tau,$$

(23)

where $c$ is the speed of light and $\alpha c$ is a constant that corresponds to the acceleration measured by the instantaneously comoving inertial observer at each instant of time. When such a trajectory is considered, both the dissipation and noise kernels are stationary, i.e. $D(\tau, \tau') = D(\tau - \tau')$ and $N(\tau, \tau') = N(\tau - \tau')$. This follows from the fact that both the uniformly accelerated trajectory given by equations (23) and the Minkowski vacuum state are invariant under Lorentz transformations. (A Lorentz transformation simply corresponds to a constant shift of $\tau$ in equations (23).) What is more, while the dissipation kernel remains the same as in the case of an inertial detector, the noise kernel is equivalent to the result that one would obtain for an inertial detector if a thermal state rather than the vacuum were chosen as the initial state for the field. ||

We have modeled our detector by a harmonic oscillator, but one could alternatively have considered a two-level system with natural frequency $\omega_0$ described by the free action

$$S_{\text{2LS}}[S_+, S_-] = \int d\tau \hbar \omega_0 S_+^{(\tau)} S_-^{(\tau)},$$

(24)

and an interaction term of the form

$$S_{\text{int}}[S_+, S_-; \phi] = \lambda \int d\tau \left( S_+^{(\tau)} + S_-^{(\tau)} \right) \phi(x^{\mu}(\tau)),$$

(25)

where $S_+$ and $S_-$ are the Grassmann variables that, when quantizing, give rise to the up and down operators for the two-level system, and $\hbar \omega_0$ is the energy difference between the excited state and the ground state.

### 4.2. Stationarity, Lorentz invariance and absence of real emitted radiation

The behavior of the accelerated detector has the following characteristic features (see [10] [11] [14], [9] [15] and related work quoted therein). During an initial transient time, owing to the term involving the dissipation kernel, all the information on the initial state of the detector is dispersed into the field, while the detector thermalizes and reaches an equilibrium condition with the vacuum fluctuations of the field, perceived as a thermal bath. After this transient time, the detector remains stationary, implying that the

|| Due to the singular behavior of the dissipation kernel, in most cases it is necessary to consider smeared trajectories for the detector [14]. In some of those cases the limit in which the size of the smearing goes to zero can be taken after renormalizing the frequency of the oscillator-detector. This smearing can be introduced in a way that respects Lorentz invariance. Simply multiply the right-hand side of equations (23) by a factor $(1 + c^{-2}ad)$, which gives rise to trajectories at a constant physical distance $d$ from the original trajectory as measured by the accelerated observers, and integrate over some smearing function $f(d)$ centered at $d = 0$, such as $\exp(-x^2/\sigma^2)$, where $\sigma$ is the characteristic smearing scale in this case.
combined system of detector plus field is invariant under the Lorentz transformations. Making use of that symmetry one can easily argue that the energy flux of the field must vanish.

Indeed, for the model above, the self-consistent dynamics of the detector and the field, including their interaction, can be solved exactly. One finds that indeed the expectation value of the stress tensor operator for the field vanishes. However, even though there is no flux of energy emitted from the detector, the vacuum polarization is altered due to the emission and reabsorption of virtual particles by the accelerated detector. This can be verified by computing the two-point quantum correlation function of the field (which is still Lorentz invariant) and comparing to the case without detector (the correlation function for the free field in the vacuum state) [12].

There are many subtle points underlying the absence of emitted radiation or energy flux in the field from a UAD. Unruh effect is predicated upon the condition that the uniform acceleration goes on for an indefinite amount of time, which requires an agent doing work on it. If the detector has been accelerating for a finite duration [11] or if the interaction lasts for a finite period of time but switched on adiabatically [45] there will be radiation emitted. It also makes a difference whether the backreaction of the field on the detector is accounted for. For a preliminary consideration including backreaction, see [46]. For a fully self-consistent backreaction calculation involving moving charges, see [14]. We hope to return to these issues in a future investigation [47].

5. Accelerated atoms in optical or microwave cavities

We now describe a scheme recently proposed by Scully et al [4] to detect phenomena related to the Unruh effect based on accelerating atoms inside microwave (or optical) cavities. We will only highlight the main features and discuss their relevance to Unruh effect.

5.1. Relevant factors in the detection scheme of Scully et al

In their theoretical analysis they consider a model consisting of a two-level atom and a single electromagnetic mode inside the cavity. They evaluated the change of the reduced density matrix for the electromagnetic mode when an atom is injected in a cavity with the electromagnetic field in its ground state by working in the interaction picture and treating the interaction term perturbatively. They gave an expression for an atom which is injected at the proper time $\tau_i$ with zero initial velocity to quadratic order (the lowest nontrivial order) as

$$\delta \hat{\rho}_i = -\frac{1}{\hbar^2} \int_{\tau_i}^{\tau_i+T} \int_{\tau_i}^{\tau_i+\tau'} T_{r2LA} \left[ \hat{H}_{\text{int}}(\tau'), \left[ \hat{H}_{\text{int}}(\tau''), \hat{\rho}_{2LA}(\tau_i) \otimes \hat{\rho}_{\text{em}}(t(\tau_i)) \right] \right] d\tau' d\tau'',$$

where $T$ is the proper time of flight inside the cavity as measured by the atom and $T_{r2LA}$ denotes the partial trace with respect to the states of the two-level atom. Assuming
that the dipole approximation is valid in the instantaneous rest frame (IRF) for the
atom, they write the atom-field interaction Hamiltonian \( \hat{H}_{\text{int}} \) as
\[
\hat{H}_{\text{int}}(\tau) = \hbar \lambda(\tau) \left( a_k e^{-i\omega t} + a_k^\dagger e^{i\omega t} - i k_x x(\tau) \right) \left( \hat{S}_+ e^{-i\omega \tau} + \hat{S}_- e^{i\omega \tau} \right),
\]
where \( k_z = \nu/c \) (for the counter-propagating mode we would have \( k_z = -\nu/c \), but we
will concentrate here on the co-propagating mode), \( \lambda(\tau) = \mu E'(\tau)/\hbar \), \( \mu \) is the
atomic dipole moment and \( E'(\tau) = E \exp(-\alpha \tau) \) is the electric field in the IRF. The
spacetime coordinates \( t(\tau) \) and \( x(\tau) \) for the trajectory of the accelerated atom are given
by equations (23) with a redefined origin of proper time, i.e., \( \tau = \tau' - \tau_i \), such that the
injection time corresponds to \( \tau = 0 \); in those coordinates the cavity injection point is
located at \( x(0) = c/\alpha \).

The coarse-grained evolution equation for a large number of atoms injected at
random times with an average injection rate (number of atoms per unit of time) \( r \) is
governed by the following Pauli-type master equation:
\[
\frac{d\rho_{n,n}}{dt} = -R_2 \left[ (n + 1)\rho_{n,n} - n\rho_{n-1,n-1} \right] - R_1 \left[ n\rho_{n,n} - (n + 1)\rho_{n+1,n+1} \right],
\]
where \( n \) is the number of photons in the cavity and \( R_1 \) and \( R_2 \) are, respectively, the
absorption and emission coefficients computed for an empty cavity, whose precise form
is given below. The absorption and stimulated emission when there are \( n \) photons in the
cavity is already accounted by the appropriate \( n \) and \( (n + 1) \) factors in equation (28). If
\( R_1 > R_2 \), there is a stationary solution which corresponds to a thermal density matrix
at temperature \( T_c = (\hbar \nu/k_B) \ln(R_1/R_2) \).

From equation (26) for a single injected atom and taking into account that the
injection rate is \( r \), the transition coefficients \( R_{1,2} \) for an empty cavity are
\[
R_{1,2} = r \frac{\hbar}{\hbar} \left[ \int_0^T V_{1,2}(\tau) \right]^2 d\tau,
\]
with \( V_1(\tau) = \langle e, 0 | \hat{H}_{\text{int}}(\tau) | g, 0 \rangle \) and \( V_2(\tau) = \langle e, 1 | \hat{H}_{\text{int}}(\tau) | g, 0 \rangle \),
where \( |g\rangle \) and \( |e\rangle \) denote the ground state and the excited state of the atom respectively.
Introducing the constant \( \lambda \) such that \( \lambda(\tau) = \lambda \exp(-\alpha \tau) \), the transition coefficients can be written
as \( R_{1,2} = r \lambda^2 |I_{1,2}(\omega)|^2 \), with \( I_2(\omega) = I_1(-\omega) \) and \( I_1(\omega) \) given by
\[
I_1(\omega) = \left( \int_{-\infty}^{T} d\tau - \int_{-\infty}^{0} d\tau \right) \exp \left( i \frac{\mu e^{-\alpha \tau} + i \omega \tau - \alpha \tau}{\alpha} \right),
\]
where we decomposed the integral \( \int_{0}^{T} d\tau \) into two separate parts for later convenience
and some adiabatic switch on at \( \tau = -\infty \) (or an equivalent analytic continuation)
may be required to get a finite result. The emission coefficient corresponds to the
so-called counter-rotating terms that would have been discarded in the rotating wave
approximation (RWA).

If we just consider the first integral in equation (30), we get an exponentially
suppressed ratio for the transition coefficients: \( R_2/R_1 = \exp(-2\pi \omega/\alpha) \). This
corresponds to the case in which the atom-field interaction has been present for a long
time in the past. On the other hand, when the second integral in equation (30) is
also included, which corresponds to switching on the interaction instantaneously at 
\( \tau = 0 \) (the time at which the atom is injected into the cavity), the situation changes 
drastically. In particular, if we consider the regime \( \nu \gg \omega \gg \alpha \), the ratio becomes 
\( R_2/R_1 \approx \alpha/(2\pi\omega) \), which implies an enhancement of many orders of magnitude for 
realistic values of \( \nu, \omega \) and \( \alpha \). It is this enhancement factor which Scully et al claim 
to give a better sensitivity in the detection of the radiation which they regard as due to 
the accelerated motion. We will see that this process is not responsible for the Unruh 
effect.

5.2. Discussion: This scheme does not detect Unruh effect

We focus here just on two factors, that related to the presence of cavity, and the effect 
of sudden switching on of the interaction.

Effect of cavity The Unruh effect corresponds to the fact that a uniformly accelerated 
particle detector (the two-level atom in this case) perceives the vacuum fluctuations as 
a thermal bath. The presence of the cavity is going to change that situation because, in 
contrast to free space, the mode spectrum of the electromagnetic fields inside the cavity 
is no longer Lorentz invariant. Therefore, the effect of the vacuum fluctuations on the 
accelerated atom (characterized by the noise and dissipation kernels introduced above) 
will not be stationary and cannot correspond to a thermal bath.

We emphasize again that, strictly speaking, the Unruh effect refers to the thermal 
bath perceived by the detector, not to possible radiation emitted by the detector from 
the point of view of inertial observers. In fact, as discussed in the prior section, in free 
space (and under some idealized conditions), there is no real radiation emitted by the 
accelerated detector, i.e. no energy flux, just a modification of the vacuum polarization.

In the scheme of Scully et al \([4]\), briefly summarized in the previous subsection, there 
is some probability for the cavity mode to become excited when an atom is accelerated 
inside the cavity. If the atom-field interaction is somehow switched on adiabatically, 
the ratio of the emission and absorption coefficients is exponentially suppressed by 
the Boltzmann factor for a temperature \( T_c = \hbar \alpha/(2\pi k_B) \), which coincides with the 
temperature of the thermal bath perceived by a UAD in free space with the same 
alowder acceleration. The reason for such a coincidence can be understood qualitatively as 
follows: in the “golden rule” limit (limit of large \( T \) with finite \( \lambda \)\(^2 T \)) one can show 
that the ratio of excitation and de-excitation of a two-level detector with characteristic 
frequency \( \omega \) induced by each inertial mode in free space is given by the same Boltzmann 
factor \( \exp(-2\pi \omega / \alpha) \). Nevertheless, the thermal distribution of photons in the cavity is 
not in one-to-one correspondence with the Unruh effect for several reasons. First, the 
atoms accelerated inside the cavity are not in thermal equilibrium because they do not 
perceive the vacuum fluctuations in the cavity as a thermal bath, as explained above.

\[ \text{Footnote: From equations (23) it follows straightforwardly that the conditions exp(-}\alpha \tau \ll 1 \text{ and } \nu \gg \alpha \text{ imply that the size of the cavity should be much larger than the wavelength of the cavity mode.} \]
Second, the thermal population of photons in the cavity results from a statistically independent events of injecting a sufficient number of atoms at random times. Third, an analogous thermal population can be obtained even by injecting atoms with constant velocity, because the sudden switching on of interaction can produce such a result, as described below.

**Effect of sudden switch-on**  
The fact that the atoms are injected into the cavity at some initial time is effectively equivalent to suddenly switching on the atom-field interaction. In that case, the ratio of emission and absorption is enhanced. In particular, in the regime $\nu \gg \omega \gg \alpha$, it is given by $R_2/R_1 \simeq \alpha/(2\pi\omega)$. As recognized in reference [4], this is entirely due to the nonadiabatic switching on of the interaction. What is more relevant to the issues at hand is that with non-adiabatic switch-on, the acceleration no longer plays a crucial role. Indeed, in that regime the emission rate is $\lambda^2 |I_2|^2 \simeq \lambda^2/\nu^2$ and is thus independent of the acceleration. It is true that the absorption coefficient still depends on the acceleration, which is a consequence of the resonance when the increasingly redshifted frequency $\nu \exp(-\alpha\tau)$ of the cavity mode as perceived by the atom coincides with the frequency $\omega$ of the atom, but this is not essential.

This important point can be illustrated by considering the case in which the atoms are injected with constant velocity into the cavity. As mentioned in reference [4], the ratio becomes then

$$R_2/R_1 = \frac{\left| \frac{\nu' - \omega}{\nu' + \omega} \right|^2}{\left| \frac{1 - e^{-i(\nu' + \omega)T}}{1 - e^{-i(\nu' - \omega)T}} \right|^2},$$

where $T$ is the time that the atom spends in the cavity and $\nu' = \nu (1 - v/c)^{1/2}/(1 + v/c)^{1/2}$ is the Doppler shifted frequency of the co-propagating cavity mode (the velocity sign should be changed for the anti-propagating mode).

Due to space restriction we can only raise two issues in this detection scheme. Even though this detection scheme, like a few earlier ones proposed, does not capture the essence of Unruh effect, it is still a good intellectual exercise to expound the physics of the core processes in the proposals. This is because Unruh effect brings up interesting new issues related to the vacuum fluctuations, polarizations and energy flux in the quantum field, as influenced by different motional states of the atoms or detectors.

**Acknowledgments**  
BLH gladly acknowledges Alpan Raval, Don Koks, Mei-Ling Tseng and Phil Johnson for earlier fruitful collaborations on the problems of emitted radiation, vacuum polarization and radiation damping from moving detectors and charges, which form the basis of our current investigation. He thanks Steve Fulling for introducing (enlisting) him to think about the Scully *et al* proposal. He also thanks Paul Davies and Gerard Milburn for mentioning Pendry’s work and their ideas on dropping atoms to measure vacuum friction. AR and BLH thank Stefano Liberati for discussions on features of this detection scheme and Luis Orozco, Bill Phillips and Steve Rolston for useful comments. This work is supported in part by grants from NSF PHY03-00710, NIST and contract from ARDA-LPS.
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