Fluids in Cosmology

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Abstract We review the role of fluids in cosmology by first introducing them in General Relativity and then by applying them to a FRW Universe’s model. We describe how relativistic and non-relativistic components evolve in the background dynamics. We also introduce scalar fields to show that they are able to yield an inflationary dynamics at very early times (inflation) and late times (quintessence). Then, we proceed to study the thermodynamical properties of the fluids and, lastly, its perturbed kinematics. We make emphasis in the constrictions of parameters by recent cosmological probes.

1 Introduction

Modern cosmology is understood as the study of fluids and geometry in the Universe. This task involves the development of theoretical ideas about the nature of fluids and gravity theories, both to be compared with current observations that cosmic probes have been undertaking. The present understanding is condensed in the standard model of cosmology, that incorporates the material content of the standard model of particle physics and Einstein’s theory of General Relativity (GR) with a cosmological constant. These two schemes, the fluid and gravity parts, have made predictions that have been tested and confirmed, albeit there are still some issues that remain open. Certainly, we have really no firm knowledge of what dark mat-
ter and dark energy are, as well as their nature and detailed properties. Still we are confident of some specific roles that these dark components play in cosmology and astrophysics. Their influence is at least gravitational, as so far we know from cosmic measurements. This knowledge allows us to build a picture of fluids in the background and perturbed geometry in the history of the Universe and this is what we deal with in the present work.

The purpose of the present review is to provide the reader with a panorama of the role that fluids play in the standard model of cosmology. Tracking the recent history, in the late 1940’s George Gamow [1,2] predicted that the Universe should have begun from a very dense state, characterized by a huge density at very high temperatures, a scenario dubbed the Big Bang, that was conjectured by George Lemaître in the early 30’s. This scenario predicts that matter and light were at very high energetic states in thermal equilibrium and described by a Planckian blackbody. As the Universe expanded, it cooled down, and eventually matter and light decoupled. The image of the last scattering of light is a fingerprint of the initial state and remains today imprinted in the Cosmic Microwave Background Radiation (CMBR). Gamow’s scenario predicted that this primeval radiation would be measured at a temperature of only a few Kelvin’s degrees; since the expansion of the Universe cools down any density component.

The CMBR was for the first time measured by A. A. Penzias and R. W. Wilson in 1965 [3]. Later on, in the early 1990s Smoot et al. [4] and Mather et al. [5] measured further important properties of this radiation: its tiny anisotropies for large angular scales and its blackbody nature. The first property – also imprinted in the matter distribution – accounts for the perturbed fluids in the Universe that led to structure formation in the cosmos. The second property is a distinctive sign of the equilibrium thermodynamic properties of the primeval plasma – composed of photons, electrons, and baryons, plus decoupled (but gravitationally coupled) neutrinos, dark matter, and dark energy. The evolution and effects of these fluids is the main concern of the present review.

We begin our work by explaining the context of fluids in GR, and especially in cosmology. We then analyze the evolution of perfect fluids - since real fluids allow them to be described as such- and their background dynamics. We explain that the main cosmic components are baryons, photons, neutrinos, dark matter, and dark energy. We also introduce scalar fields since they are ubiquitous in modern cosmology because they enable to model different cosmic dynamics, from inflation [6,7] and dark energy [8,9] to dark matter [10,11]. Then, we proceed to study the thermodynamical properties of the fluids (as in Ref. [12]) and, lastly, its perturbed kinematics. We make emphasis on the constraints of parameters as imposed by recent cosmological probes.

In this work, we use “natural” units $\hbar = c = k_B = 1$ and our geometrical sign conventions are as in Ref. [13].
2 Fluids in general relativity

The GR theory is based on the Einstein-Hilbert Lagrangian density

$$L = \frac{1}{16 \pi G} (R + L_m) \sqrt{-g},$$  \hspace{1cm} (1)

where $R$ is the Ricci scalar, $G$ the Newton constant, $g = |g_{\mu \nu}|$ is the determinant of the metric tensor, and $L_m$ is the material Lagrangian that will give rise to the fluids. By performing the metric variation to this equation, one obtains the well known Einstein’s field equations

$$R_{\mu \nu} - \frac{1}{2} R g_{\mu \nu} = 8 \pi G T_{\mu \nu},$$  \hspace{1cm} (2)

where $R_{\mu \nu}$ is the Ricci tensor and $T_{\mu \nu}$ is the stress energy–momentum tensor whose components are given through $T_{\mu \nu} \equiv -\frac{1}{2} \frac{\partial L_m}{\partial g_{\mu \nu}}$. Tensors in Eq. (2) are symmetric which is a requirement of the theory. Being space-time four dimensional the imposed symmetry implies that Eq. (2) represents a collection of ten coupled partial differential equations. However, the theory is diffeomorphism invariant, and one adds to them a gauge condition, implying in general four extra equations to Eq. (2) that reduce the physical degrees of freedom. Thus, symmetries and gauge choice determine the fluid properties allowed by the theory.

The stress energy–momentum tensor $T$ encodes the information of the fluid, and all kinds of energy types contribute to curve space-time: density, pressure, viscosity, heat, and other physical quantities. But before introducing them, one needs other elementary concepts.

Giving some reference frame, one defines the four-velocity $u \equiv dx/d\tau$ as the vector tangent to the worldline of a particle, with $x$ being the local coordinates and $\tau$ the proper time along the worldline; its four-momentum is $p = mu$, where $m$ is the rest mass of the particle. Now, given a space-time surface, $x^\alpha = \text{const.}$, one defines its associated one-form as $dx^\alpha$, to obtain the components $T(\tilde{dx}^\alpha, \tilde{dx}^\beta) = T^{\alpha \beta}$, which is interpreted as the flux of momentum $\alpha$, $p^\alpha = \langle \tilde{dx}^\alpha, p \rangle$, passing through the surface $x^\beta = \text{const.}$ In this way, $T^{00}$ is the energy density, which is the flux of momentum ($p^0 =$particle’s energy) that crosses the surface $x^0 = t = \text{const.}$ and $T^{0i}$ is the flux of energy that crosses the surface $x^i = \text{const.}$, where latin labels run from 1 to 3 and greek labels from 0 to 3. Given the symmetry of the tensor, $T^{0i} = T^{0i}$, that is, energy fluxes are equal to momentum densities since mass equals relativistic energy. Finally, the components $T^{ij}$ denote the momentum flux $i$ crossing the surface $x^j = \text{const.}$, and again symmetry implies that $T^{ij} = T^{ji}$, avoiding a net intrinsic angular momentum.

The left-hand side of Eq. (2) is known as the Einstein tensor ($G_{\mu \nu}$) and, giving the symmetries of the theory, it happens to fulfill the Bianchi identities, that is, its covariant derivative is null. This in turn implies, on the right-hand side, a conservation law for any fluid within this theory. The conservation law reads:
\[ T^v_{\mu} = 0. \]  

(3)

As we shall see, this equation is the most important since it encodes the thermodynamic laws of matter.

3 Fluids in cosmology

The kinematical properties of a fluid element are determined by its velocity, acceleration, shear, and vorticity. All these quantities are defined in the space-time, and for convenience one uses comoving coordinates, that is Lagrangian coordinates that follow the flow motion. We refer the reader to standard gravity textbooks for details, e.g., Refs. [13, 14]. One splits the space-time structure into surfaces of simultaneity to rest frame observers, with a projected metric on the surface \( h_{\mu \nu} = g_{\mu \nu} + u_\mu u_\nu \); where \( u^\mu \) are the components of the four velocity \( u \). In this frame it is natural to define an expansion tensor, \( \Theta_{\mu \nu} = \Theta(\mu \nu) = \nabla(u_\mu u_\nu) \), and the vorticity tensor, \( \omega_{\mu \nu} = \omega(\mu \nu) = \nabla(u_\mu u_\nu) \), where \( \nabla \) operates on the projected 3-dimensional space. The trace of the expansion tensor is a scalar measure of the volume expansion, given by \( \Theta = \nabla u^\nu \), and the shear tensor is the projected symmetric free-trace part of \( \Theta_{\mu \nu} \), such that \( \Theta_{\mu \nu} = \sigma_{\mu \nu} + \frac{1}{3} \Theta h_{\mu \nu} \) (see Ref. [15]).

Accordingly, the energy-momentum tensor associated to the fluid can be separated into components parallel and orthogonal to the four velocity as:

\[ T_{\mu \nu} = \rho u_\mu u_\nu + q_\mu u_\nu + q_\nu u_\mu + P h_{\mu \nu} + \pi_{\mu \nu}, \]  

(4)

where \( \rho = T_{\alpha \beta} u^\alpha u^\beta \) is the energy density that includes rest masses and possibly the internal energy, such as the chemical energy; \( P = h^{\alpha \beta} T_{\alpha \beta} / 3 \) is the pressure; \( q_\mu = -h^{\alpha \beta} T_{\alpha \mu} u^\nu \) is the momentum density or energy flux due to either diffusion or heat conduction; and \( \pi_{\mu \nu} = [h^{\alpha \beta} h_{\mu \nu} - \frac{1}{3} h_{\mu \nu} h^{\alpha \beta}] T_{\alpha \beta} \) is the trace-free anisotropic stress tensor due to viscosity.

A perfect fluid is an inviscid fluid with no heat conduction, that is, \( q_\mu = 0 \) and \( \pi_{\mu \nu} = 0 \). It is analogous to an ideal gas in standard thermodynamics. In terms of the full metric, it is a standard practice to represent it as:

\[ T^{\mu \nu} = (\rho + P) u^\mu u^\nu + P g^{\mu \nu}, \]  

(5)

in comoving coordinates, \( u^\mu = \delta^\mu_0 \). Equation (5), is the energy-momentum tensor that correctly describes fluids in the background geometry of the Universe.
4 Fluids in the standard model of cosmology

The Universe is described by its material components and geometry. The former is fed with microscopic or thermodynamic information about the fluids and the latter is determined by Eq. (2). In the following, we explain the features of the geometry and the properties of the fluids that have governed the evolution of the standard model of cosmology.

The *cosmological principle* states that the Universe is both spatially homogeneous and isotropic on large scales, and this imposes a symmetry on the possible fluids present in it. Any departure from this symmetry in the fluid would be reflected in the geometry through Eq. (2). The symmetry assertion is compatible with observations made of the all-sky cosmic microwave background radiation from the last twenty years, through the satellites COBE [4] in the 1990s, the Wilkinson Microwave Anisotropy Probe (WMAP) [16, 17] in the 2000’s, and the PLANCK [18] nowadays, although some large scale CMBR anomalies in the isotropy have been detected [19] that require further investigation. On the other hand, homogeneity and isotropy have also been tested for the distribution of matter at large scales, see for instance Refs. [20, 21].

In GR, as in any other metric theory, symmetries of the physical system are introduced through the metric tensor. The homogeneous and isotropic space-time symmetry was originally studied by Friedmann, Robertson, and Walker (FRW) (see Refs. [22, 23, 24, 25, 26, 27]). The symmetry is encoded in and defines the unique form of the line element:

\[ ds^2 = g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a^2(t) \left[ \frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (6) \]

where \( t \) is the cosmic time, \( r, \theta, \) and \( \phi \) are polar coordinates, and the constant curvature can be adjusted to take the values \( k = 0, +1, \) or \( -1 \) for a flat, closed, or open space, respectively. \( a(t) \) is the unknown potential of the metric that encodes the size at large scales, and more formally, it is the *scale factor* of the Universe that measures how the model grows or shrinks as time evolves. Measurements show that it always grows, but a bounce in the very early or final stages is possible (see, e.g., Ref. [28]).

The beautiful symmetric FRW solutions to the Einstein Eqs. (2) represent a cornerstone in the development of modern cosmology, since with them it is possible to understand the expansion of the Universe. Although in the first years of relativity, Einstein sought for a static solution – since observations seemed to imply that – it was soon realized by E. Hubble and others in the mid 1920’s that the Universe is indeed expanding, following Hubble’s law [29].

Using the FRW metric and a perfect fluid, the GR cosmological field equations are,

\[ H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G}{3} \rho - \frac{k}{a^2} \quad (7) \]

and
\[
\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P),
\]
where \( H \) is the Hubble parameter that has dimensions of inverse of time, and therefore, it encodes the model’s expansion rate; \( H^{-1} \) is proportional to the age of the Universe. Moreover, \( \rho \) and \( P \) are the density and pressure that enter in Eq. (5). Dots stand for cosmic time derivatives.

As explained above, the energy-momentum tensor is covariantly conserved, as shown by Eq. (3). In the present case, it implies the continuity equation,

\[
\dot{\rho} + 3H(\rho + P) = 0.
\]

Equations (7), (8), and (9) involve three unknown variables \((a, \rho, p)\) for three equations, but the system is not mathematically closed, since the equations are not all linearly independent, but just only two of them. Thus, an extra assumption has to be made to solve the system. The answer comes from the micro-physics of the fluids considered. For the moment let us assume a barotropic equation of state that is characteristic for different cosmic fluids, i.e., \( w = \text{const} \).

\[
P = w = \begin{cases} 
\frac{1}{3} & \text{for radiation or relativistic matter,} \\
0 & \text{for dust,} \\
1 & \text{for stiff fluid,} \\
-1 & \text{for cosmological constant or vacuum energy,}
\end{cases}
\]

to integrate Eq. (9), yielding

\[
\rho = \frac{M_w}{a^{3(1+w)}} \quad \text{or} \quad \frac{\rho_i}{\rho_0} = \left(\frac{a_0}{a}\right)^{3(1+w)},
\]

where \( M_w \) is the integration constant and has different dimensions for different \( w \)-fluids. The equation on the right shows a different re-scaling of the integration constant, where the subscript \( i \) stands for the different \( i \)-fluids. Quantities with either a subscript or superscript “0” are evaluated at the present time. With this equation the system is mathematically closed and can be solved.

The system of ordinary differential equations described above needs a set of initial, or alternatively boundary conditions to be integrated. One has to choose a set of two initial values, say, \((\rho(t_\ast), \dot{a}(t_\ast)) \equiv (\rho_\ast, \dot{a}_\ast)\) at some (initial) time \( t_\ast \), in order to determine its evolution. A full analysis of this assumption can be found in many textbooks [13, 30, 31]. In order to show some physical consequences of the early Universe, we assume that \( k = 0 \). This is consistent with data from recent cosmological probes, as we shall explain shortly. This tell us that curvature has not played a role for most of the age of the Universe. On the other hand, this can be justified as follows: from Eqs. (7) and (11) we may see that the expansion rate, given by the Hubble parameter, is dominated by the density term as \( a(t) \to 0 \), since \( \rho \sim 1/a^{3(1+w)} > k/a^2 \) for \( w > -1/3 \), that is, the flat solution fits very well the very beginning of times. Therefore, taking \( k = 0 \), Eq. (7) implies that
which is composed of dark matter (DM) and baryons, and the argument given above to neglect $k$ prior to it GR has to be modified to include quantum effects. To obtain Eq. (13), the right-hand side of Eq. (8) is positive, $\ddot{a}/a = 3k/\Lambda^2$, otherwise $k$ cannot be ignored. Nevertheless if $\Lambda$ is present, it will eventually dominate over the other decaying components, this is the so-called cosmological no-hair theorem [32].

A general feature of all the above solutions is that they are expanding, at different Hubble rates, $H = \dot{a}/a = \frac{1}{3(1+w)}$ for Eqs. (12) and $H = \frac{c}{a}$ for Eq. (13).

From Eq. (12) one can immediately see that at $t = t_s$, $a_s = 0$ and from Eq. (11), $\rho_s = \infty$, that is, the solution has a singularity at that time, at the beginning of the Universe. This initial cosmological singularity is precisely the Big Bang singularity. As the Universe evolves the Hubble parameter goes as $H \sim 1/t$, i.e., the expansion rate decreases, whereas the matter-energy content acts as an expanding agent [cf. Eq. (9)]. It decelerates the expansion, however, by decreasing asymptotically [cf. Eqs. (8) and (11)]. In this way, $H^{-1}$ represents an upper limit to the longevity of the Universe; for instance, $H^{-1} = 2t$ for $w = 1/3$ and $H^{-1} = 3t/2$ for $w = 0$, $t$ being the age of the Universe.

The exponential expansion (13) possesses no singularity (at finite times), being the Hubble parameter a constant. A fundamental ingredient of this inflationary solution is that the right-hand side of Eq. (8) is positive, $\ddot{a} > 0$, and this occurs when $\rho + 3\rho = 0$, that is, one does not have necessarily to impose the stronger condition $w = -1$, but it suffices that $w < -1/3$, in order to have a horrible inflationary solution; for example, $w = -2/3$ implies $a = a_0t^2$: a mild power-law inflation.

It is convenient to define dimensionless density parameters as $\Omega_i \equiv \frac{8\pi G \rho_i}{3H^2}$. With them, Eq. (7) can be expressed as the constraint:

$$\Omega \equiv \Omega_R + \Omega_M + \Omega_\Lambda = 1 + \frac{k}{a^2H^2},$$

where $i$ labels the different components present in the Universe: $R$ stands for the radiation components (photons, neutrinos, and relativistic particles), $M$ for matter which is composed of dark matter (DM) and baryons, and $\Lambda$ for a cosmological constant. The actual values of the density parameters ($\Omega_R, \Omega_M, \Omega_\Lambda$) impose a value for the curvature term. If $\Omega > 1$, it turns out that $k$ is greater than zero, meaning a Universe with a positive, closed curvature. If $\Omega < 1$, then $k < 0$, which corresponds
to a negative, open curvature. Obviously, a critical value is obtained when $\Omega = 1$, then the spatial curvature is null, $k = 0$. The value of the energy density for which $\Omega = \left(\rho + \frac{\Lambda}{(8\pi G)}\right)/\rho_c = 1$ holds is known as the critical density, $\rho_c = 3H^2/(8\pi G)$. The last term in Eq. (14) can be defined as $\Omega_k = -k/(a^2H^2)$, and thus the Friedmann equation becomes a constraint for the density parameters, i.e., $\sum \Omega_i = 1$, and this expression holds at any time. It is worth mentioning that solutions $\Omega(a)$ are unstable in the presence of a curvature term (see Fig. 1). In fact, this is related to the flatness problem in the old cosmological picture: Why the Universe is nowadays close to a flat model? Inflation offered the solution to this issue.

![Fig. 1](image)

Fig. 1 The parameter $\Omega$ as a function of the scale factor, $a$, in a radiation dominated Universe (the dust model behaves similarly). For closed models, with $k = +1$, $\Omega$ diverges as the scale factor tends to its maximum value, whereas for open models, with $k = -1$, $\Omega$ tends asymptotically to zero as the Universe expands. Finally, for a flat metric, with $k = 0$, $\Omega$ always remains equal to one.

In (background) cosmology, typical times and distances are determined mainly by the Hubble parameter, and in practice measurements are often related to redshift, as measured from stars, gas, etc. It is then useful to express the individual density parameters in terms of the redshift ($z$), $1 + z = a_0/a(t)$, where $a_0$ is the scale factor at present and is set to unity by convention. Today $z_0 = 0$ and towards the early Universe the redshift grows. In terms of the redshift the density parameters are, from Eq. (11),

$$\Omega_i = \Omega_i(0) (1 + z)^{3(1+w_i)},$$

(15)

where $w_i$ is the equation of state parameter for each of the fluids considered. Now, the Hubble parameter can be put in terms of the density parameters. In the standard model of cosmology, considering baryons, photons, neutrinos, cold dark matter (CDM), and a cosmological constant ($\Lambda$) – termed $\Lambda$CDM –, one has:
\[ H^2 = H_0^2 \sum_i \Omega_i^{(0)} (1+z)^{3(1+w_i)} . \] (16)

As defined above, the density parameter depends on \( 1/H^2 \), so to avoid a bias with the expansion rate one defines the physical density parameter \( \omega_i \equiv \Omega_i h^2 \), where \( h \) is a dimensionless number given by the Hubble constant \( H_0 \equiv 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \). The physical density parameters of matter are important since they are directly determined from CMBR experiments. The current best-fit values for the physical density parameters from PLANCK are \([33]\): \( \omega_b = 0.022 \), \( \omega_{DM} = 0.120 \), from which one computes the best fits

\[ \Omega_b^{(0)} = 0.0492, \quad \Omega_{DM}^{(0)} = 0.267, \quad \Omega_\Lambda^{(0)} = 0.683, \quad h = 0.671. \] (17)

The Universe at present is dominated by dark energy, which accounts for 68% of the energy budget, dark matter for 27%, and in minor proportion baryonic matter only for about 5%, from which visible matter is made of. Photons and neutrinos contribute in a much less proportion at present. When one considers a curved model, the best fit for the curvature parameter is \( \Omega_k^{(0)} = -0.01 \) with an uncertainty of few percent \([33]\).

Since the scale factor evolves as a smooth function of time, one is able to use it as a variable, instead of time, in such a way that \( d/da = aH \, da/dt \). This change of variable helps to integrate the continuity equation for non-constant \( w(a) \) to obtain:

\[ \rho(a) = \rho_0 e^{-3 \int [1+w(a)] da/a} . \] (18)

If, for instance, one parameterizes dark energy through an analytic function of the scale factor, \( w(a) \), one immediately obtains its solution in terms of

\[ t = \int \frac{1}{\sqrt{8\pi G \rho(a)/3}} \frac{da}{a} . \] (19)

From Eq. (19) one obtains the age of the Universe in terms of the redshift, \( H_0 \), and the density parameters:

\[ t_0 = H_0^{-1} \int_0^\infty \frac{dz}{(1+z)H(z)} . \] (20)

When combining different cosmological probes one obtains for the \( \Lambda \)CDM model an age of \( t_0 = 13.81 \pm 0.06 \) Gyr \([33]\).

In general, if dark energy is a function of the redshift, from Eq. (18) one can generalize the Friedmann equation to:

\[ H(z)^2 / H_0^2 = \Omega_m^{(0)} (1+z)^3 + \Omega_r^{(0)} (1+z)^4 + \Omega_k^{(0)} (1+z)^2 + \Omega_{DE}^{(0)} f(z) , \] (21)

where \( DE \) stands for dark energy, and

\[ f(z) = \exp \left[ 3 \int_0^z \frac{1+w(z')}{1+z'} dz' \right] . \] (22)
Different DE models can be directly parametrized through \( w = w(z) \). The most popular one is perhaps the Chevalier-Polarski-Linder’s \([34, 35]\) formula
\[
  w = w_0 + w_a \left( \frac{z}{1 + z} \right),
\]
where \( w_0 \) and \( w_a \) are constants.

We would like to remark that the first strong evidence for the existence of dark energy, and hence for a present accelerated expansion of the Universe, came from fits of supernovae luminosity curves to data \([36]\). Two different supernova groups \([37, 38, 39]\) found a clear evidence for \( \Lambda \) in the late 90’s. The presence of a cosmological constant makes the Universe not only expanding, but also accelerating and, in addition, its age is older, and not in conflict with the globular cluster ages \([40,41]\). In the course of the years, various supernova groups have been getting more confident that the data is compatible with the presence of dark energy, dark matter, and a high value of the Hubble parameter. By moving a little beyond the standard model of cosmology and letting \( w \) be a constant (but not necessarily \(-1\)), one of the latest data released, the Union2 compilation \([42]\), reports that the flat concordance \( \Lambda \)CDM model remains an excellent fit to the data, with the best fit to the constant equation-of-state parameter being
\[
  w = -0.997^{+0.050}_{-0.054}
\]
for a flat Universe, and
\[
  w = -1.035^{+0.055}_{-0.059}
\]
for a curved Universe. Also, they found that \( \Omega_{M}^{(0)} = 0.270 \pm 0.021 \) (including baryons and DM) for fixed \( \Omega_{k}^{(0)} = 0 \). That is, \( \Omega_{\Lambda}^{(0)} = 0.730 \pm 0.021 \). Using CMB PLANCK data, these numbers change a few percent, having little less DE and more DM, as shown by Eq. (17).

### 4.1 Fluids’ chronology

The standard model of cosmology is described by a set of periods in which different fluids dominated the dynamics. We first consider a period of inflation in which the Universe experienced an accelerated expansion rendering enough \( e \)-folds to explain the horizon and flatness problems of the old Big Bang theory (see, for instance, Ref. \([12]\)). This very early epoch is well described by an exponential expansion characterized by an equation of state \( w = -1 \). This is achieved through a scalar field that slowly rolls its potential, as we will see in section 5.1. Eventually, the scalar field steps down the potential hill and begins to oscillate, to behave as a fluid of dust \((w = 0)\) \([43]\). This period is thought to be short to let particle production and to heat the Universe in a period of reheating \([44,45,46]\) and/or preheating \([47,48]\), for a modern review see Ref. \([49]\). This is needed since after inflation the Universe is cooled down exponentially and it is deprived of particles. The new, produced particles, generically lighter than the scalar field mass, are relativistic \((T \gg m, m\) being its rest mass), and therefore they are well described by \( w = 1/3 \). This epoch is important because it marks the beginning of the hot Big Bang theory. In this very early epoch particle physics theories (such as grand unification schemes) should describe the details of particle interactions to eventually reach the lower energies of the well tested standard model of particle physics. Then, the material content of the Universe consisted of a hot plasma with photons, protons, neutrons, neutrinos,
electrons, and possibly other particles with very high kinetic energy. After some cooling of the Universe, some massive particles decayed and others survived (protons, neutrons, electrons, and DM) whose masses eventually dominated over the radiation components (photon, neutrinos, and possibly dark radiation; the latter being any other relativistic degree of freedom present at that epoch) at the equality epoch \( \rho_{\text{eq}} = \rho_m \) at \( z_{\text{eq}} \sim 3402 \). From this epoch and until recent e-folds of expansion \( (z_{\text{DE}} \sim 0.8) \), the main matter component produced effectively no pressure on the expansion and, therefore, one can accept a model filled with dust, \( w = 0 \), to be representative for the energy content of the Universe in the interval \( 3402 < z < 0.8 \). The dust equation of state is then representative of inert CDM. DM does not (significantly) emit light and therefore it is dark. Another possibility is that dark matter interacts weakly, which is generically called WIMP (Weakly Interacting Massive Particle); the neutralino being the most popular WIMP candidate. Another popular dark matter candidate is the axion, a hypothetical particle postulated to explain the conservation of the CP symmetry in quantum chromodynamics (QCD). Back to the Universe evolution, from \( z \sim 0.8 \) until now the Universe happens to be accelerating with an equation of state \( w \approx -1 \), due to some constant energy that yields a cosmological constant, \( \Lambda = 8\pi G \rho = \text{const} \). The cosmological constant is the generic agent of an inflationary solution (see the \( k = 0 \) solution in Eq. 13). The details of the accelerated expansion are still unknown and it is possible that the expansion is due to some new fundamental field (e.g., quintessence) that induces an effective \( \Lambda(t) \sim \text{const} \). We call (as M. Turner dubbed it) dark energy (DE) this new element. Dark energy does not emit light nor any other particle, and as known so far, it simply behaves as a (transparent) media that gravitates with an effective negative pressure. The physics behind dark energy or even the cosmological constant is unclear since theories of grand unification (or theories of everything, including gravity) generically predict a vacuum energy associated with fundamental fields, \( < 0/T_{\mu\nu} > = < \rho > g_{\mu\nu} \), that turns out to be very large. This can be seen by summing the zero-point energies of all normal modes of some field of mass \( m \), to obtain \( < \rho > \approx M^4 / (16\pi^2) \), where \( M \) represents some cut-off in the integration, \( M \gg m \). Then, assuming that GR is valid up to the Planck (\( Pl \)) scale, one should take \( M \approx 1/\sqrt{8\pi G} \), which gives \( < \rho > \approx 10^{11} \text{ GeV}^4 \). This term plays the role of an effective cosmological constant \( \Lambda = 8\pi G < \rho > \approx M_{Pl}^2 \sim 10^{21} \text{ GeV}^2 \), which must be added to Einstein’s Eqs. (2), or directly to Eqs. (7) and (8), yielding an inflationary solution as given by Eq. (13). However, since the cosmological constant seems to dominate the dynamics of the Universe nowadays, one has that

\[
\Lambda \approx 8\pi G \rho_0 = 3H_0^2 \sim 10^{-83} \text{GeV}^2,
\]

(23)

which is very small compared to the value derived above on dimensional grounds. Thus, the cosmological constraint and the theoretical expectations are rather dissimilar, by about 121 orders of magnitude! Even if one considers symmetries at lower energy scales, the theoretical \( \Lambda \) is indeed smaller, but never as small as the cosmological constraint: \( \Lambda_{\text{GUT}} \approx 10^{21} \text{ GeV}^2 \), \( \Lambda_{\text{SU}(2)} \approx 10^{-29} \text{ GeV}^2 \). This problem has been reviewed many decades ago \([51, 52]\) and still remains open.
Scalar fields are ubiquitous in cosmology since they allow for modelling different cosmic dynamics, from inflation \[6,7\] and dark energy \[8,9\] to dark matter \[10,11\]. The full characterization of scalar fields is not describable in terms of perfect fluids, but its background dynamics allows for that. A scalar field with mass, \(m_\phi\), has an associated Compton wavelength, \(\lambda_C = 1/m_\phi\). Thus, one can conceive the fluid picture as a collection of scalar particles with a typical size of \(\lambda_C\). For if \(\lambda_C = H^{-1}_0\) the corresponding scalar field mass is of course very light, \(m_\phi = 10^{-33}\) eV. If \(\lambda_C < H^{-1}\) the particle is localizable within the Hubble horizon, otherwise its mass is too light and counts effectively as a massless particle.

A canonical scalar field \((\phi)\) is given by the Lagrangian density

\[
\mathcal{L} = \frac{1}{2} \partial^\mu \phi \partial_\mu \phi - V(\phi),
\]

(24)

where the first term accounts for the kinetic energy and \(V(\phi)\) is its potential.

The energy-momentum tensor of the \(\phi\)-field is

\[
T_{\mu\nu}(\phi) = \frac{\partial \mathcal{L}}{\partial (\partial_\mu \phi)} \partial_\nu \phi - \mathcal{L} g_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} \partial_\lambda \phi \partial^\lambda \phi \ g_{\mu\nu} + V(\phi) g_{\mu\nu}.
\]

(25)

The field energy density and pressure are, by associating \(\rho(\phi) = T_{00}(\phi)\) and \(P(\phi) = T_{\mu\nu}(\phi)/a^2\) (no \(i\)-sum),

\[
\rho(\phi) = \frac{1}{2} \dot{\phi}^2 + V(\phi) + \frac{1}{2a^2(t)} (\nabla \phi)^2 \approx \frac{1}{2} \dot{\phi}^2 + V(\phi),
\]

\[
P(\phi) = \frac{1}{2} \dot{\phi}^2 - V(\phi) - \frac{1}{6a^2(t)} (\nabla \phi)^2 \approx \frac{1}{2} \dot{\phi}^2 - V(\phi),
\]

(26)

where the gradient terms (in comoving coordinates) are neglected. This typically occurs for the background cosmology and the reason for this is that the Universe is assumed to be sufficiently homogeneous within a horizon distance.

The equation of state associated to a scalar field is

\[
w = \frac{P}{\rho} = \frac{\dot{\phi}^2 - V(\phi)}{\dot{\phi}^2 + V(\phi)},
\]

(27)

with \(w\) taking values in the interval \(-1 \leq w \leq 1\).

The conservation of energy, Eq. (9), yields, using Eq. (26), the equation of motion for the \(\phi\)-field,

\[
\ddot{\phi} + 3H \dot{\phi} + V'(\phi) = 0,
\]

(28)

where the prime stands for the scalar field derivative. The expansion term plays the role of a friction, whereas the potential contribution depends upon the scalar field model at hand.
In what follows, we present the main features of two applications of the scalar field dynamics: inflation and quintessence. We will refer the reader to recent reviews on these subjects for a more profound account of these topics, cf. [53, 54, 55].

### 5.1 Inflation

The scalar field responsible for the inflationary dynamics is dubbed the *inflaton*. There are hundreds of models of inflation and several theoretical aspects related to perturbations (see section [7], non-Gaussianities, etc; for a recent review see Ref. [53]. The basics of the dynamics is as follows: the inflaton evolves from an initial value \( \phi^* \) down the hill of the potential, but typically in a *slow roll-over* way, to a final state in which reheating takes place.

In order to get enough \( e \)-folds of inflation the scalar field should stay long time, compared to the cosmic time, in a potential ‘flat region’ where the potential is almost constant \( V(\phi) \approx V(0) \). To construct such a flat curvature for the potential and to permit the \( \phi \)-field to evolve slowly, one has to impose the slow roll-over conditions, namely, that \( \dot{\phi} \approx 0 \). From Eq. (28), it implies that \( \ddot{\phi} \approx -V' / 3H \), which in turn means that [56]:

\[
\frac{\ddot{\phi}}{3H\phi} = -\frac{V''}{9H^2} + \frac{1}{48\pi G} \left( \frac{V'}{V} \right)^2 \ll 1 ,
\]

or in terms of the dimensionless potential slow-roll parameters, \( \epsilon \equiv 1/(16\pi G)(V'/V)^2 \ll 1 \) and \( \eta \equiv 1/(8\pi G)(V''/V) \ll 1 \).

This condition also ensures that \( \rho(\phi) \approx V(\phi) > \frac{1}{2}\phi^2 \), and so from Eq. (27) one has \( w \approx -1 \), which guarantees an accelerated expansion. However, if the initial conditions are such that at the outset \( \frac{1}{2}\dot{\phi}^2 \gg V(\phi) \), then the solution takes the form, \( \dot{\phi}^2 = \text{const.}/t^2 \) and \( \phi = \phi_0 - A \ln(1 + Bt) \), where \( A \) and \( B \) are constants. Then, the kinetic terms fall faster than the logarithmic decrease of a polynomial potential. Therefore, after some asymptotic time the Universe will be dominated by its potential and thus, inflation follows [7]. However, in other gravity theories the kinetic terms play an important role and could prevent the Universe from inflation [57, 58].

The scalar field solution with \( w \approx -1 \), considered in Eq. (10), emulates a vacuum energy term or a cosmological constant. Given the slow roll-over of the \( \phi \)-field this behaviour happens for a minimum of \( N \) \( e \)-folds of expansion in which the Hubble rate is effectively given by

\[
H^2 = \frac{8\pi G}{3} V(\phi \approx \text{const.}) .
\]

In this way, \( H \approx \text{const.} \) and the scale factor exhibits an exponential behaviour, as given by Eq. (13). Strictly speaking, during inflation \( \phi \) is an increasing function of time, since \( V' < 0 \) in Eq. (28). However, under slow roll-over conditions its characteristic evolution time will be much greater than the cosmological time. Therefore, \( H \) will be a very slow, monotonically decreasing function of time.
Inflation lasts for a sufficient number of $N$ e-folds to solve the horizon and flatness problems in cosmology, and this depends very much on the energy scale of inflation. In standard inflationary scenarios $N \sim 60$. This ensures that a possible curved model will look like a flat one for all the expansion history, including today (see Fig. 2 and compare with Fig. 1).

![Diagram](image)

*Fig. 2* The parameter $\Omega$ as a function of the scale factor, $a$, during inflation and thereafter in a radiation/matter dominated Universe. Inflation makes the space to look like as flat, even if it is initially curved. If there are enough e-folds of inflation to solve the horizon problem, it implies that the Universe nowadays is still flat. Later on, the behaviour is as in Fig. 1.

Among the multiple inflaton potentials considered in the literature, the most favoured models by the PLANCK CMBR temperature map fits [59] are those having potentials with $V'' < 0$. Exponential potential models, the simplest hybrid inflationary models, and monomial potential models of degree $n \geq 2$ do not provide a good fit to the data. The most favoured models are Hill-top models, a simple symmetry breaking potential, natural inflation, $R^2$ inflation, and non-minimal coupled to gravity with a Mexican-hat potential; see Ref. [59] for details.

### 5.2 Dark energy: quintessence

Dark energy is a generic name for an energetic “fluid” that has had little or no evolution in the past few giga-years of the cosmic expansion. Since then dark energy dominates the total density of the Universe over all other components (dark matter, baryons, photons, and neutrinos). During dark energy domination, the Hubble pa-
rameter, as given by Eq. (7), is basically a constant. Thus, a cosmological constant added to the gravitational theory is the simplest candidate for dark energy that fits the data from the different cosmic probes. There are at least seven independent observations that imply the presence of dark energy: the ages of some globular clusters surpasses the age of the Universe in models without dark energy [40, 41]; the supernovae best fits to distance moduli [37, 38, 39]; the dynamics of clusters of galaxies [60]; the combination of the CMBR lensing deflection power spectrum with temperature and polarization power spectra [61]; the measurements of the integrated Sachs-Wolf effect [62]; the measurements of Baryon Acoustic Oscillations (BAO) [63]; and the change of the Hubble rate behaviour from galaxy surveys [50].

Another possible candidate for dark energy is a canonical scalar field, dubbed quintessence [8]. The equations governing the scalar field dynamics in a cosmological background are those displayed in section 5. Basically, the FRW equations, i.e., Eqs. (7) and (8), are now fulfilled with the density and pressure terms given by Eqs. (26). To complete the whole picture, we add the rest of the known four material elements (dark matter, baryons, photons, and neutrinos) to the scalar field.

In a similar fashion to inflation, one demands that \( V(\phi) > \frac{1}{2} \dot{\phi}^2 \) has a flat potential and allows for an accelerated behaviour. One may again use the slow roll-over parameters (\( \varepsilon, \eta \)) to ensure an accelerated dynamics, but here we have the other four components that may spoil the exact accelerated dynamics. Still, this approach works well.

Originally, runaway potentials were considered, but nowadays there is a vast set of models that achieve the desired accelerated dynamics, including non-standard kinetic terms [9, 28] or scalar fields interacting with matter [64], among many others. To avoid the over dominance of the scalar field during the early stages of the cosmic dynamics, one looks for scaling properties (of tracker nature) of the scalar field dynamics in which the field energy density (\( \rho_\phi \)) evolves proportionally to the material fluid energy density (\( \rho_m \)) with \( \rho_\phi < \rho_m \), and only until recently the scalar field turns to dominate. Depending on the evolution of the scalar-field equation of state, Eq. (27), quintessence models can be freezing or thawing [65]. The former class is when the scalar field gradually slows down to eventually freeze in a constant value. The latter class implies that the scalar field has recently started to change from a past constant value. These behaviours can in principle be tested (see Ref. [55] for a recent review on the subject).

6 Thermodynamics in the early Universe

In the early Universe one considers a plasma of particles and their antiparticles, as was done originally by Gamow [11], who first considered a physical scenario for the hot Big Bang model as a description of the beginning of the Universe. Later on, with the development of modern particle physics theories in the 70’s it was unavoidable to think about a physical scenario which should include the “new” physics for the early Universe. It was also realized that the physics described by GR should not be
applied beyond Planckian initial conditions, because there the quantum corrections to the metric tensor become very important, a theory which is still in progress.

After preheating/reheating, one assumes that the Universe is filled with a plasma of relativistic particles which include quarks, leptons, and gauge and Higgs bosons, all in thermal equilibrium at a very high temperature, $T$, with some gauge symmetry dictated by a particlephysics theory.

Theoretically, one introduces some thermodynamic considerations necessary for the description of the physical content of the Universe, which we would like to present here. Assuming an ideal-gas approximation, the number density $n_i$ of particles of type $i$, with a momentum $p$, is given by a Fermi or Bose distribution [66]:

$$n_i = \frac{g_i}{(2\pi)^3} \int \frac{d^3p}{e^{(E_i - \mu_i)/T} \pm 1},$$

(31)

where $E_i = \sqrt{m_i^2 + p^2}$ is the particle energy, $\mu_i$ is the chemical potential, the sign (+) applies for fermions and (−) for bosons, and $g_i$ is the number of spin states. One has that $g_i = 2$ for photons, quarks, baryons, electrons, muons, taus, and their antiparticles, but $g_i = 1$ for neutrinos because they are only left-handed. For the particles existing in the early Universe one usually assumes that $\mu_i = 0$: one expects that in any particle reaction the $\mu_i$ are conserved, just as the charge, energy, spin, and lepton and baryon number are. For a photon, which can be created and/or annihilated after some particle’s collisions, its number density, $n_\gamma$, must not be conserved and its distribution with $\mu_\gamma = 0$, $E = p = \omega$, reduces to the Planckian one. For other constituents, in order to determine the $\mu_i$, one needs $n_i$. Note from Eq. (31) that for large $\mu_i > 0$, $n_i$ is large too. One does not know $n_i$ in advance. However, the WMAP data constrains the baryon density at nucleosynthesis such that [67]:

$$\eta \equiv \frac{n_B}{n_\gamma} \equiv \frac{n_{\text{baryons}} - n_{\text{anti-baryons}}}{n_\gamma} = 6.14 \pm 0.25 \times 10^{-10}.$$  

(32)

The smallness of the baryon number density, $n_B$, relative to the photon’s, suggests that $n_{\text{leptons}}$ may also be small compared to $n_\gamma$. Therefore, one takes for granted that $\mu_i = 0$ for all particles. The ratio $n_B/n_\gamma$ is very small, but not zero. The reason of why matter prevailed over antimatter is one of the puzzles of the standard model of cosmology called baryogenesis [66]. There are some attempts to achieve baryogenesis at low energy scales, as low as few GeV or TeV [58, 68, 69, 70, 71]. Recent attempts to solve this problem are looking for prior to lepton asymmetry, leptogenesis, generated in the decay of a heavy sterile neutrino [72], to then end with baryogenesis.

The above approximation allows one to treat the density and pressure of all particles as a function of the temperature only. According to the second law of thermodynamics, one has [30]:

$$dS(V, T) = \frac{1}{T} [d(\rho V) + PdV],$$

(33)
where $S$ is the entropy in a volume $V \sim a^3(t)$, with $\rho = \rho(T)$ and $P = P(T)$ in equilibrium. Furthermore, the following integrability condition $\frac{\partial^2 S}{\partial T \partial V} = \frac{\partial^2 S}{\partial V \partial T}$ is also valid, which turns out to be

$$\frac{dP}{dT} = \frac{\rho + P}{T}. \quad (34)$$

On the other hand, the energy conservation law, Eq. (9), leads to

$$\frac{d}{dt} \left[ a^3(T) \rho (\rho + P) \right] = 0, \quad (35)$$

after using Eq. (34). Using Eq. (34) again, the entropy equation can be written as

$$dS(V, T) = \frac{1}{T} \left[ \frac{\rho + P}{V} \right] \frac{dV}{dt}.$$

These last two equations imply that the entropy is a constant of motion:

$$S = \frac{a^3}{T}[\rho + P] = \text{const.} \quad (36)$$

Moreover, the density and pressure are given by

$$\rho \equiv \int E_i n_idp, \quad P \equiv \int \frac{p^2}{5E_i} n_idp. \quad (37)$$

For photons or ultra-relativistic fluids, $E = p$, and the above equations become $P = \frac{1}{3} \rho$, thus confirming Eq. (10) for $w = 1/3$. After integration of Eq. (34), it comes out that

$$\rho = bT^4, \quad (38)$$

where $b$ is a constant of integration. In the real Universe there are many relativistic particles present, each of which contributes like Eq. (35). By including all of them, $\rho = \sum_i \rho_i$ and $P = \sum_i P_i$, where the summations are over all relativistic species, one has that $b(T) = \frac{2}{3T} (N_B + \frac{7}{8} N_F)$, which depends on the effective relativistic degrees of freedom of bosons ($N_B$) and fermions ($N_F$). Therefore, this quantity varies with the temperature. Different $i$-species remain relativistic until some characteristic temperature $T \approx m_i$ and after this point $N_{F_i}$ (or $N_{B_i}$) no longer contributes to $b(T)$. The factor 7/8 accounts for the different statistics of the particles [see Eq. (31)]. In the standard model of particle physics $b \approx 1$ for $T \ll 1$ MeV and $b \approx 35$ for $T > 300$ GeV [66]. In particular, one accounts for the effective number of neutrinos ($N_{\text{eff}}$) in terms of photons’ degrees of freedom as

$$\frac{\rho_{\nu}}{\rho_T} = \frac{7}{8} \left( \frac{4}{11} \right)^{4/3} N_{\text{eff}}, \quad (39)$$

with $N_{\text{eff}} = 3.046$ for standard model neutrino species [73]. Extra neutrino-type relativistic species – dark radiation – should augment $N_{\text{eff}}$, as was recently suggested from measurements of different cosmological probes. Combining PLANCK with previous CMB data and Hubble Space Telescope measurements, it has been concluded that $N_{\text{eff}} = 3.6 \pm 0.5$ with a 95% confidence level [74].
For relativistic particles, we obtain from Eq. (31) that

\[ n = cT^3, \quad \text{with} \quad c = \frac{\zeta(3)}{\pi^2} (N_B + \frac{3}{4}N_F). \]  

(40)

where \( \zeta(3) \approx 1.2 \) is the Riemann zeta function of 3. Nowadays, \( n_\gamma \approx 411T_{3.73}^3 \) cm\(^{-3} \), where \( T_{3.73} \equiv T_\gamma / (2.73 \text{K}) \). The precise measured value is \( T_\gamma = 2.72548 \pm 0.00057 \text{K} \) [75]. The mean energy per photon is \( 6.34 \times 10^{-4} \) eV which corresponds to a wavelength of 2 millimetres, and hence it is called cosmic “microwave” background radiation.

Using the relativistic equation of state given above (\( w = 1/3 \)), From Eq. (36) it follows that \( T \sim 1/a(t) \). From its solution in Eq. (12) one has

\[ T = \sqrt[3]{\frac{M_1}{b} \frac{1}{a(t)}} = \sqrt[3]{\frac{3}{32\pi G b} \frac{1}{(t - t_\gamma)^{2/3}}} \],

(41)

which predicts a decreasing temperature behaviour as the Universe expands. Then, initially at the Big Bang, \( t = t_\gamma \) implies that \( T_\gamma = \infty \), and so the Universe was not only very dense but also very hot. As time evolves the Universe expands, cools down, and its density diminishes.

The entropy for an effective relativistic fluid is given by Eq. (36) together with its equation of state and Eq. (38), i.e.,

\[ S = \frac{4}{3} \frac{b}{a(T)^3} \text{const}. \]

Combining this with Eq. (41), one can compute the value of \( M_1 \) to be \( M_1 = \left( \frac{3}{4}S \right)^{4/3}/b^{1/3} \approx 10^{116} \), since \( b \approx 35 \) and the photon entropy \( S_0 = \frac{4}{3} b (a_0 T_0)^3 \approx 10^{88} \) for \( a_0 \rightarrow d_H(t_0) = 10^{28} \) cm and \( T_\gamma = 2.73 \) K, as evaluated at the present time. One defines the entropy per unit volume, entropy density, to be \( s = S/V = \frac{4}{3} \frac{S^2}{30} (N_B + \frac{3}{4}N_F) T_\gamma T_0 \), then at the present time \( s \approx 7n_\gamma \). The nucleosynthesis bound on \( \eta \), Eq. (32), implies that \( n_B/s \approx 10^{-11} \).

We now consider particles in their non-relativistic limit (\( m \gg T \)). From Eq. (31) one obtains for both bosons and fermions that

\[ n = g \left( \frac{mT}{2\pi} \right)^{3/2} e^{-m/T}. \]  

(42)

The abundance of equilibrium massive particles decreases exponentially once they become non-relativistic. This situation is referred to as in equilibrium annihilation. Their density and pressure are given through Eqs. (37) and (42) by \( \rho = nm \) and \( P = nT \ll \rho \). Therefore, using these last two equations, the entropy for non-relativistic particles, given by Eq. (36), diminishes also exponentially during the in equilibrium annihilation. The entropy of these particles is transferred to that of the relativistic components by augmenting their temperature. Hence, the constant total entropy is essentially the same as the one given above, but the \( i \)-species contributing to it are just those which are in equilibrium and maintain their relativistic behaviour, that is, particles without mass such as photons.

Having introduced the abundances of the different particle types, we would like to comment on the equilibrium conditions for the constituents of the Universe as it
evolves. This is especially important in order to have an idea of whether or not a given \(i\)-species disappears or decouples from the primordial brew. To see this, let us consider \(n_i\) when the Universe temperature, \(T\), is such that (a) \(T \gg m_i\), during the ultra-relativistic stage of some particles of type \(i\) and (b) \(T \ll m_i\), when the particles \(i\) are non-relativistic, both cases in thermal equilibrium. From Eq. (40), one has that for the former case \(n_i \sim T^3\) and the total number of particles, \(\sim n_i a^3\), remains constant, whereas for the latter case, using Eq. (42), \(n_i \sim T^{3/2} e^{-m_i/T}\), i.e., when the Universe temperature goes down below \(m_i\), the number density of the \(i\)-species significantly diminishes; it occurs an in equilibrium annihilation. Let us take as an example the neutron-proton annihilation. Then we have

\[
\frac{n_n}{n_p} \sim \exp\left(\frac{m_p - m_n}{T}\right) = \exp\left(-\frac{1.5 \times 10^{10} \text{K}}{T}\right),
\]

which drops with the temperature from near 1 at \(T \geq 10^{12}\) K to about 5/6 at \(T \approx 10^{11}\) K and 3/5 at \(T \approx 3 \times 10^{10}\) K \textsuperscript{[76]}. If this is valid forever, we then end up without massive particles and our Universe would have been consisted only of radiative components. However, our own existence prevents that! Therefore, eventually the in equilibrium annihilation had to be stopped. The quest is now to freeze out this ratio to \(n_n/n_p \approx 1/6\) (due to neutron decays) until the time when nucleosynthesis begins (i.e., when \(n_n/n_p\) reduces to 1/7) in order to leave the correct number of hadrons and achieve later successful nucleosynthesis. The answer comes from comparing the Universe expansion rate, \(H\), with the particle physics reaction rates, \(\Gamma\). Hence, for \(H < \Gamma\) the particles interact with each other faster than the Universe expansion rate, then equilibrium is established. For \(H > \Gamma\) the particles cease to interact effectively, then thermal equilibrium drops out. This is only approximately true; a proper account of that involves a Boltzmann equation analysis. For that analysis numerical integration should be carried out in which annihilation rates are balanced with inverse processes, see for example Ref. \textsuperscript{[77, 66]}. In this way, the more interacting the particles are, the longer they remain in equilibrium annihilation and, therefore, the lower their number densities are after some time, e.g., baryons vanish first, then charged leptons, neutral leptons, etc.; finally, the massless photons and neutrinos, whose particle numbers remain constant, as it was mentioned above (see Fig. 3). Note that if interactions of a given \(i\)-species freeze out while it is still relativistic, then its abundance will be significant at the present time and will account for dark radiation, as was recently suggested in Ref. \textsuperscript{[74]}. It is worth mentioning that if the Universe would expand faster, then the temperature of decoupling, when \(H \sim \Gamma\), would be higher and thus, the fixed ratio \(n_n/n_p\) would be greater and the \(^4\text{He}\) abundance would be higher, leading to profound implications in the nucleosynthesis of light elements. Thus, the expansion rate cannot be arbitrarily modified during the equilibrium era of some particles. Furthermore, if a particle species is still highly relativistic \((T \gg m_i)\) or highly non-relativistic \((T \ll m_i)\), when decoupling from the primordial plasma occurs, it maintains an equilibrium distribution; the former being characterized by \(T_r a = \text{const.}\) and the latter by \(T_m a^2 = \text{const.}\) [cf. Eq. (46)].
There are also some other examples of decoupling, such as neutrino decoupling: during nucleosynthesis there exist reactions, e.g. $\nu \bar{\nu} \leftrightarrow e^+ e^-$, which maintain neutrinos efficiently coupled to the original plasma ($\Gamma > H$) until about 1 MeV, since $\Gamma/H \approx T^3$ MeV$^{-3}$. The reactions are no longer efficient below 1 MeV and therefore neutrinos decouple and continue evolving with a temperature $T_{\nu} \sim 1/a$. Then, at $T > m_e = 0.51$ MeV the particles in equilibrium are photons (with $N_B = 2$) and electron-positron pairs (with $N_e = 4$), which contribute to the entropy with $b(T) = (11/2)(\pi^2/30)$. Later, when the temperature drops to $T \ll m_e$, the reactions are again no longer efficient ($\Gamma < H$) and, after the $e^\pm$ pair annihilation, there will be only photons in equilibrium with $b(T) = 2(\pi^2/30)$. Since the total entropy, $S = (4/3)b(aT)^3$, must be conserved, a decrease of $b(T)$ must be balanced with an increase of the radiation temperature so that $T_{\gamma}/T_{\nu} = (11/4)^{1/3}$, which should remain so until today, implying the existence of a cosmic background of neutrinos with a present temperature of $T_{\nu_0} = 1.95$ K. This cosmic relic has not been measured yet.

Another example is the gravitation decoupling, which should be also present if gravitons were in thermal equilibrium at the Planck time and then decouple. Today, the temperature background should be characterized at most by $T_{\text{grav}} = (4/107)^{1/3}$ K $\approx 0.91$ K.

For the matter dominated era we have stressed that effectively one has $P = 0$. Next we will see the reason for this. First, consider an ideal gas (such as atomic hydrogen) with mass $m$, then $\rho = nm + \frac{3}{2}nT_m$ and $P = nT_m$. From Eq. (35), one

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**Fig. 3** Evolution of the particle density for different $i$-species. If a given $i$-species is in equilibrium, its abundance diminishes exponentially after the particle becomes non-relativistic (solid line). However, interactions of an $i$-species can freeze out, then it decouples from equilibrium and maintains its abundance (dashed line). Figure adapted from Ref. [66].
equivalently obtains that
\[
\frac{d}{da} (\rho a^3(t)) = -3Pa^2(t),
\] (44)
which after substitution of \(\rho\) and \(P\), as given above, becomes
\[
\frac{d}{da} \left( nma^3(t) + \frac{3}{2} nT_m a^3(t) \right) = -3nT_m a^2(t),
\] (45)
where \(nma^3(t)\) is a constant. This equation yields
\[
T_m a^2(t) = \text{const}.
\] (46)

so that the matter temperature drops faster than the radiation temperature as the Universe expands [cf. Eq. (41)]. Now, if one considers both radiation and matter, one has that \(\rho = nm + \frac{3}{2} nT_m + bT_r^4\) and \(P = nT_m + \frac{1}{3} bT_r^4\). The source of the Universe expansion is proportional to \(\rho + 3P = nm + \frac{9}{2} nT_m + 2bT_r^4\), where the first term dominates over the second, precisely because \(T_m\) decreases very rapidly. The third term diminishes as \(\sim 1/a^4\), whereas the first does it as \(\sim 1/a^3\). After the time of density equalization, \(\rho_m = \rho_r\), the matter density term is greater than the others and this explains why one assumes a zero pressure for that era.

From now on, when we refer to the temperature, \(T\), it should be related to the radiation temperature. The detailed description of the Universe thermal evolution for the different particle types, depending on their masses, cross-sections, etc., is well described in many textbooks, going from the physics known in the early 70’s [30] to the late 80’s [66], and therefore it will not be presented here. However, we notice that as the Universe cools down a series of spontaneous symmetry–breaking (SSB) phase transitions are expected to occur. The type and/or nature of these transitions depend on the specific particle physics theory considered. Among the most popular ones are the Grand Unification Theories (GUT’s), which bring together all known interactions except for gravity. One could also regard the standard model of particle physics or some extensions of it. Ultimately, when constructing a cosmological theory, one should settle the energy scale that one wants to describe physically. For instance, at a temperature between \(10^{14}\) GeV and \(10^{10}\) GeV a transition to the \(SU(5)\) GUT should take place, if this theory would be valid, in which a Higgs field breaks this symmetry to \(SU(3)_C \times SU(2)_W \times U(1)_{HC}\), a process through which some bosons acquire their masses. Due to the gauge symmetry, there are color (C), weak (W), and hypercharge (HC) conservation, as the subscripts indicate.

Later on, when the Universe evolved to around 150 GeV the electroweak phase transition took place in which the standard model Higgs field broke the symmetry \(SU(3)_C \times SU(2)_W \times U(1)_{HC}\) to \(SU(3)_C \times U(1)_{EM}\); through this breaking fermions acquired their masses. At this stage, there were only color and electromagnetic (EM) charge conservation, due to the gauge symmetry. Afterwards, around a temperature of 200 MeV [78] the Universe should undergo a transition associated to the chiral symmetry-breaking and color confinement from which baryons and mesons were
formed out of quarks. Subsequently, at approximately 10 MeV the synthesis of light elements (nucleosynthesis) began and lasted until temperatures below 100 keV, when most of the today observed hydrogen, helium, and some other light elements abundances were produced. So far the nucleosynthesis represents the earliest scenario tested in the standard model of cosmology. After some thousands of years ($z \sim 3402$), the Universe became matter dominated, over the radiation components. At about 380,000 years ($z \sim 1090$) recombination took place, that is, the hydrogen ions and electrons combined to form neutral hydrogen atoms, then matter and electromagnetic radiation decoupled from each other. At this moment, the (baryonic) matter structure began to form. Since that moment, the surface of last scattering of the CMBR evolved as an imprint of the early Universe. This is the light that Penzias and Wilson first measured, and that, later on, was measured in more detail by BOOMERANG, MAXIMA, COBE, WMAP, and now PLANCK, among other probes.

7 Perturbed fluids in the Universe

In the previous sections, we have outlined how the evolution of a homogeneous Universe can be described by means of few equations and simple concepts such as the ideal perfect fluids. The next step is that of introducing in this scenario small inhomogeneities that can be treated as first order perturbations to those equations, the goal being the description of the structures we see today in the Universe. This perturbative approach is sufficient to accurately explain the small temperature anisotropies ($\Delta T / T \sim 10^{-5}$) observed in the CMBR today, but it can only describe the distribution of matter today at those scales that are still in the linear regime. At the present epoch, scales smaller than $\sim 30 \text{ Mpc} h^{-1}$ have already entered the non linear-regime ($\Delta \rho / \rho > > 1$) due to the fact that matter tends to cluster under the effects of gravity. These scales can therefore be described only by means of numerical or semi-numerical approaches.

The approach is quite straightforward. It involves a differential equation for the density perturbation of each individual constituent: scalar fields in inflation, or baryons, radiation, neutrinos, DM, and DE (usually treated as cosmological constant) in later times, and in general it needs to be solved numerically. In the context of the metric theories of gravity, and in particular GR, the metric is treated as the general expansion term $g_{\mu \nu}^{(0)}$ plus a perturbation $h_{\mu \nu}$:

$$g_{\mu \nu} = g_{\mu \nu}^{(0)} + h_{\mu \nu},$$

(47)

with $h_{\mu \nu} << g_{\mu \nu}^{(0)}$, where $^{(0)}$ indicates unperturbed homogeneous quantities.

Inhomogeneities in the distribution of the components of the Universe are a source of scalar perturbations of the metric. Nevertheless, vector or tensor perturbations can modify the metric as well. The standard cosmological model does not predict vector perturbations that would introduce off-diagonal terms in the metric.
tensor. These perturbations would produce vortex motions in the primordial plasma, which are expected to rapidly decay. Models with topological defects or inhomogeneous primordial magnetic fields instead predict a consistent fraction of vector perturbations [85][86][87].

On the other hand, the standard cosmological model predicts the production of gravitational waves during the epoch of inflation, when the Universe expanded exponentially. Gravitational waves induce tensor perturbations $h_{\mu\nu}^T$ on the metric of the type:

$$h_{\mu\nu}^T = a^2 \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & h_+ & h_x & 0 \\
0 & h_x & -h_+ & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}$$

where $h_+$ and $h_x$ are the polarization directions of the gravitational wave. This tensor is traceless, symmetric, and divergenceless, i.e. it perturbs the time space orthogonally to the direction of propagation of the wave. The amplitudes of these tensor perturbations are expected to be small compared to the scalar ones, and therefore negligible in a first approximation as far as we are interested in studying the perturbations of the metric tensor. Nevertheless, these waves are expected to leave an imprint in the polarization of the CMBR, and their eventual detection would unveil an extremely rich source of information about an epoch of the Universe that is very hardly observable otherwise.

It is important to underline that choosing to model the metric perturbations corresponds to choosing a gauge, i.e. a specific coordinate system in which the metric tensor is represented. Changing the coordinate system, of course, do not change the physics, but can remarkably vary the difficulty of the calculations and ease the understanding of the physical meaning of the different quantities. In order to solve the perturbed equations one chooses convenient gauges for the different expansion epochs and depending on whether the formalism is theoretical or numerical, as we will see below.

The presence of weak inhomogeneous gravitational fields introduces small perturbations in the metric tensor. The most general perturbation to the FRW metric is:

$$d\tilde{x}^2 = a^2(\eta) \left[ - (1 + 2A) d\eta^2 - B_i dx^i d\eta + [(1 + 2D)\delta_{ij} + 2E_{ij}] dx^i dx^j \right]$$

(48)

where $\eta$ and $x'$ are comoving coordinates in which the expansion factor $a(\eta)$ is factored out. Different choices of them imply different gauges. We refer to Refs. [88][89][90][91] for an account of the physical meaning of the metric potentials and a full treatment of the perturbations.

In correspondence to the above metric perturbations, the energy-momentum tensor is also perturbed. One has:

$$T^0_0 = -(\rho + \delta \rho),$$

$$T^0_i = (\rho + P)(v_i - B_i),$$

$$T^0_0 = -(\rho + P)v',$$
\[ T^i_j = (P + \delta P)\delta^i_j + \pi^i_j, \] (49)

where \( v^i = dr^i/dt \) is the velocity in local orthonormal coordinates \( [dt = a (1 + A) d\eta; \ dr^i = a dx^i] \) and \( \pi^i_j \) are the anisotropic stresses; if they are null the perturbed fluid is also a perfect fluid. Anisotropic stresses are important before last scattering, when the primordial plasma was coupled. Later on, when structure formation begins they are set to zero.

A convenient gauge choice is given through two scalar functions \( \Phi(\eta, x^i) \) and \( \Psi(\eta, x^i) \) as [88]:

\[ ds^2 = a^2(\eta) \left[ -d\eta^2 + [1 + 2\Phi(\eta, x^i)]d\eta^2 + [1 + 2\Psi(\eta, x^i)]dx^i dx^j \right], \] (50)

where the perturbed part of the metric tensor is:

\[ h_{00}(\eta, x^i) = -2\Phi(\eta, x^i), \quad h_{0i}(\eta, x^i) = 0, \quad h_{ij}(\eta, x^i) = a^2\delta_{ij}(2\Psi(\eta, x^i)). \] (51)

This metric is just a generalization of the well-known metric for a weak gravitational field usually described in the textbooks (e.g. chapter 18 of Ref. [13]) for the case of a static Universe \( [a(\eta) = 1] \). The function \( \Phi \) describes Newton’s gravitational field, while \( \Psi \) is the perturbation of the space curvature. The above gauge is the Newtonian conformal gauge, which has the advantage of having a diagonal metric tensor \( g_{\mu\nu} \) in which the coordinates are totally fixed with no residual gauge modes and therefore with a straightforward interpretation of the functions introduced.

Another example of a gauge that is particularly popular in the literature is the synchronous gauge, defined by:

\[ ds^2 = a^2(\eta)[-d\eta^2 + (\delta_{i,j} + h_{i,j})dx^i dx^j], \] (52)

which is especially used in numerical codes for calculations of the anisotropies and inhomogeneities in the Universe. It behaves well numerically by choosing that observers fall freely without changing their spatial coordinates.

The full perturbed equations are obtained by substituting the above expressions, for the chosen gauge, into the Einstein equations. Alternatively, one may obtain the continuity equation from the time (\( \mu = 0 \)) component of Eq. (3) and the Euler equation from its space sector (\( \mu = i \)). Here we do not write down the perturbed equations for any particular gauge, but rather refer the reader to standard textbooks [91], where these equations are fully described.

### 7.1 Perturbations during inflation

The primeval fluctuations are thought to be present at the very beginning of time, at the inflationary epoch. The perturbations are produced by quantum fluctuations of the \( \phi \)-field during the accelerated stage. These fluctuations are usually studied in
the comoving gauge in which the scalar field is equal to its perturbed value at any
given time during inflation and therefore, the perturbation information resides in the
metric components (see Refs. [88,12,91] for reviews on the subject).

To understand how perturbations evolve it is necessary to introduce the concept
of horizon [36]. There are two types of horizons in cosmology: the causal or particle
horizon \( d\mathcal{H} \) and the event horizon \( d\mathcal{E} \). The former determines the region of space
which can be connected to some other region by causal physical processes, at most
through the propagation of light with \( ds^2 = 0 \). For the radiation cosmological era,
one has that \( d\mathcal{H}(t) = 2t = H^{-1} \) and for the matter era one has \( d\mathcal{H}(t) = 3t = 2H^{-1} \);
\( H^{-1} \) is sometimes called the Hubble horizon. During inflation (under an exponential
expansion of the Universe) \( d\mathcal{H}(t) = H^{-1}(e^{Ht} - 1) \) \( (H = \text{const.}) \) and hence, the causal
horizon grows exponentially. The event horizon, on the other hand, determines the
region of space which will keep in causal contact (again complying with
\( ds^2 = 0 \)) after some time; that is, it delimits the region from which one can ever receive (up
to some time \( t_{\text{max}} \)) information about events taking place now (at time \( t \)). For the
matter/radiation dominated eras \( d\mathcal{E} \to \infty \) as \( t_{\text{max}} \to \infty \). However, during inflation one
has that \( d\mathcal{E} = H^{-1}(1 - e^{-(t_{\text{max}}-t)H}) \approx H^{-1} \), which implies that any observer will
see only those events that take place within a distance \( \leq H^{-1} \). In this respect, there
is an analogy with black holes, from whose surface no information can get away.
Here, in an exponentially expanding Universe, observers encounter themselves in a
region which is apparently surrounded by black holes [92,7], since they receive no
information located farther than \( H^{-1} \).

Now, we turn back to the perturbation discussion. During the de Sitter stage the
generation of perturbations, which is a causal microphysical process, is localized
in regions of the order of \( d\mathcal{E} = H^{-1} \) in which the microphysics operates coherently.
At this time, the wavelength of inhomogeneities grows exponentially (as the causal
horizon does) and eventually they cross outside the event horizon. Much later on,
they re-enter into the event horizon, at the radiation and matter dominated epochs,
to yield an almost scale invariant density perturbation spectrum (Harrison-Zel’dovich,
\( n_s = 1 \)), as is required for structure formation and measured by different cosmolog-
ical probes.

It was shown that the amplitude of inhomogeneities produced corresponds to the
Hawking temperature in the de Sitter space, \( T_H = H/(2\pi) \). In turn, this means that perturbations with a fixed physical wavelength of size \( H^{-1} \) are produced throughout
the inflationary era. Accordingly, a physical scale associated to a quantum fluctuation, \( \lambda_{\text{phys}} = \tilde{\lambda}a(t) \), expands exponentially and once it leaves the event horizon, it
behaves as a metric perturbation; its description is then classical, general relativistic.
If inflation lasts for enough time, the physical scale can grow as much as a galaxy
or horizon-sized perturbation. The field fluctuation expands always with the scale
factor and after inflation, it evolves according to \( t^n \) \( (n = 1/2 \text{ radiation or } n = 2/3 \text{ matter}) \). On the other hand, the Hubble horizon evolves after inflation as \( H^{-1} \sim t \).
This means that it will come a time at which field fluctuations cross inside the Hub-
ble horizon and re-enters as density fluctuations. Thus, inflation produces a gross
spectrum of perturbations, the largest scale ones being originated at the start of in-
flation with a size $H^{-1}$, and the smallest ones with size $H^{-1}$ at the end of inflation (see Fig. 4).

![Diagram of inflation and standard cosmology](image)

**Fig. 4** Quantum perturbations were initially subhorizon-sized. During inflation they grow exponentially ($\lambda_{\text{phys}} = \lambda a(t)$), whereas the event horizon remains almost constant. Then, eventually they cross outside $H^{-1}$ and evolve as classical perturbations. Later on, they re-enter the event horizon to produce an almost scale invariant, Harrison-Zel'dovich density perturbation spectrum. In the figure are depicted two physical perturbations scales: galaxy and horizon-sized. Figure adapted from Ref. [66].

The power spectra for scalar ($S$) and tensor ($T$) perturbations are given by:

$$P_S(k) \approx \left( \frac{H^2}{16\pi^3} \dot{\phi}_c^2 \right)_{k = aH}, \quad P_T(k) \approx \left( \frac{H^2}{4\pi^2 m^2_{\text{Pl}}} \right)_{k = aH},$$

(53)

where $\dot{\phi}_c$ is the classical scalar field velocity. The equations are evaluated at the horizon crossing ($k = aH$) during inflation. Each of the $k$-modes generate an anisotropy pattern in the CMBR that was measured for scalar perturbations by the COBE [4] and later probes. The PLANCK satellite may have the chance to detect the ratio of tensor to scalar amplitudes $r \equiv C_T^l / C_S^l < 0.12$ (95% limits) [33], since the tensor modes modulate CMBR photons coming from last scattering.
The power spectra above give rise to the observed curvature and tensor power spectra in terms of the wavenumber ($k$) in a power law manner [91]:

$$P_R(k) = A_S \left( \frac{k}{k_0} \right)^{n_S-1+\frac{1}{2}dnS/d\ln k(n/k_0)}, \quad P_t(k) = A_t \left( \frac{k}{k_0} \right)^{n_t}$$

(54)

that has been determined by recent CMBR probes, such as PLANCK to give a best fit of $n_S = 0.96$ and $dn_S/d\ln k \approx -0.0090$ [33]. One should also have a tensor spectral index $n_t$ that has not been measured yet.

These scalar and metric perturbations are small, but still very important. We discuss in the next section how to include them so that the information contained can be recognized and exploited.

### 7.2 Perturbations inside the horizon

We explained that in the early Universe baryons were tightly coupled to photons in an expanding background. Baryonic and dark matter potential wells provoked the local collapse of density fluctuations up to a certain point, at which the radiation pressure was big enough to pull out the matter apart and smooth the potential wells. These oscillations of the plasma are in fact acoustic waves. As we know, any wave can be decomposed into a sum of modes with different wave numbers, $k = 2\pi/\lambda$. Since these modes are in the sky, their wavelengths are measured as angles rather than as distances. Accordingly, instead of decomposing the wave in a Fourier series, what is normally done is to decompose the wave in terms of spherical harmonics, $Y_{lm}(\hat{n})$, where $\hat{n}$ is the direction of a measured photon. The angular power spectrum can be expanded in Legendre polynomials, since there is no preferred direction in the Universe and only the angular separation $\theta$ is relevant. A mode $l$ plays the same role of the wavenumber $k$, thus $l \approx 1/\theta$. We are interested in the temperature fluctuations that are analyzed experimentally in pairs of directions $\hat{n}$ and $\hat{n}'$, where $\cos(\theta) = \hat{n} \cdot \hat{n}'$. We then average these fluctuations, obtaining the multipole expansions:

$$\frac{\Delta T}{T} = \sum_{l=1}^{\infty} \sum_{m=-l}^{l} a_{lm}(x, \eta) Y_{lm}(\hat{n}), \quad P_3(\theta) = \sum \frac{(2l+1)}{4\pi} C_l P_l(\cos \theta),$$

(55)

where $P_3(\theta)$ is the angular power spectrum, $P_l$ are the Legendre polynomials, and the $C_l$ are estimated as averages of the $a_{lm}$ over $m$. All this information can be used to determine the cosmological parameters $\Omega_i$. We will not discuss here the detailed calculations nor the curve that must be adjusted to obtain the best fit values for such parameters. The peak of the fundamental mode appears at approximately

$$l \approx \frac{200}{\sqrt{\Omega_i(0)}}$$

(56)
BOOMERANG \cite{81} and MAXIMA \cite{82} were two balloon-borne experiments designed to measure the anisotropies at scales smaller than the horizon at decoupling \((\theta_{\text{hor-dec}} \sim 1^\circ)\), hence measuring the acoustic features of the CMBR. The sensitivity of the instruments allowed for a measurement of the temperature fluctuations of the CMBR over a broad range of angular scales. BOOMERANG found a value of \(l = 197 \pm 6\) and MAXIMA-1 found a value of \(l = 220\). This implies that the cosmological density parameter \(\Omega^{(0)} \approx 1\) \cite{13}, suggesting that the Universe is practically flat, \(\Omega_k^{(0)} \approx 0\). These two experiments provided the first strong evidence for a flat Universe from observations. Happily, this result was expected from inflation since an accelerating dynamics effectively flattens the curvature of the event horizon, which we later identify with our Universe \cite{96}. These results were confirmed by WMAP in a series of data releases in the last decade, as well as by other cosmological probes: the Universe is flat or pretty close to be flat. The problem in the exact determination of the curvature is because the CMBR anisotropies show strong degeneracies among the cosmological parameters \cite{93, 94}. However, the satellite PLANCK offers results on the density parameters with uncertainties less than a percent level, \(\Omega_k^{(0)} = -0.0105 \) \cite{33}.

Since baryons and photons were in thermal equilibrium until recombination, also called last scattering (ls), the acoustic oscillations (BAO) were also imprinted in the matter perturbations, as they were in the CMBR anisotropies. The sound horizon, at the moment when the baryons decoupled from the photons, plays a crucial role in the determination of the position of the baryon acoustic peaks. This time is known as the \textit{drag epoch} which happens at \(z_d = 1/a_d - 1\). The sound horizon at that time is defined in terms of the effective speed of sound of the baryon-photon plasma, \(c_s^2 = \delta p_T / (\delta \rho_T + \delta \rho_b)\),

\[
    r_s(z_d) = \int_0^{\eta_d} d\eta c_s(\eta) = \frac{1}{3} \int_0^{a_d} \frac{da}{a^2 H(a) \sqrt{1 + (3\Omega_b/4\Omega_\gamma)a}}. \tag{57}
\]

Note that the \textit{drag epoch} does not coincide with the last scattering. In most scenarios \(z_d < z_{ls} \) \cite{35}. The redshift at the drag epoch can be computed with a fitting formula that is a function of \(\omega_m = \Omega_m^{(0)} h^2\) and \(\omega_b = \Omega_b^{(0)} h^2\) \cite{96}. The WMAP team, and recently PLANCK, computed these quantities for the \(\Lambda\)CDM model, obtaining \(z_d = 1059.29 \pm 0.65\) and \(r_s(z_d) = 147.53 \pm 0.64\) Mpc \cite{33}.

BAO can be characterized by the angular position and the redshift \cite{97, 98}:

\[
    \theta_b(z) = \frac{r_s(z_d)}{(1 + z) d_A(z)}, \tag{58}
\]

\[
    \delta z_b(z) = r_s(z_d) H(z), \tag{59}
\]

where \(d_A(z) = \frac{1}{H_0 \sqrt{|\Omega_k|^{1/2} (1+z)}} \sin_k \left( \sqrt{\Omega_k} \int_0^1 \frac{dy}{H(y)} \right) \) is the proper (not comoving) angular diameter distance to the redshift \(z\), with \(sin_k = \sin\) for \(\Omega_k < 0\) and \(sin_k = \sinh\) for \(\Omega_k > 0\); where \(H(z)\) is determined by Eq. \tag{24}. The angle \(\theta_b(z)\) corresponds to the direction orthogonal to the line-of-sight, whereas \(\delta z_b(z)\) measures the fluctua-
tions along the line-of-sight. Observations of these quantities are encouraging to determine both $d_A(z)$ and $H(z)$. However, from the current BAO data it is not simple to independently measure these quantities. This will certainly happen in forthcoming surveys [99]. Therefore, it is convenient to combine the two orthogonal dimensions to the line-of-sight with the dimension along the line-of-sight to define [100]:

$$D_V(z) \equiv \left( (1+z)^2 d_A(z)^2 \frac{z}{H(z)} \right)^{1/3},$$

(60)

where the quantity $D_M \equiv d_A/a = (1+z)d_A(z)$ is the comoving angular diameter distance. The BAO signal has been measured in large samples of luminous red galaxies from the SDSS [100]. There is a clear evidence ($3.4\sigma$) for the acoustic peak at a scale of $100 h^{-1}$ Mpc. Moreover, the scale and amplitude of this peak are in good agreement with the prediction of the ΛCDM as confirmed by the WMAP and PLANCK data. One finds that $D_V(z = 0.35) = 1370 \pm 64$ Mpc, and more recently new determinations of the BAO signal has been published [101] in which $\theta_s(z = 0.55) = 3.90^\circ \pm 0.38^\circ$ and $w = -1.03 \pm 0.16$ for the equation of state parameter of the dark energy, or $\Omega_{m}^{(0)} = 0.26 \pm 0.04$ for the matter density, when the other parameters are fixed. One also defines the BAO distance $d_c \equiv \theta_s(z_d)/D_V(z)$, which has been measured by surveys. For instance, an analysis of the BOSS survey gives $d(0.57) = 13.67 \pm 0.22$ [102], which is the current most precise determination of the BAO scale.

Measuring the BAO feature in the matter distribution at different redshifts will help break the degeneracy that exists in the determination of the cosmological parameters. By combining line-of-sight with angular determinations of the BAO feature one will constrain even more the parameter space. Furthermore, a complete combination of BAO, the full matter power spectrum, direct $H(z)$ measurements, supernovae Ia luminosities, and CMBR data will certainly help envisage the true nature of the mysterious, dark Universe.

8 Outlook

We have reviewed the role that fluids have played in the entire history of the Universe. Their components are relatively simple and behave as perfect fluids, at least at the background level. The fluids’ evolution is as follows: first, scalar fields governed a very early inflationary dynamics with an equation of state $w \sim -1$. After inflation, the Universe was deprived of particles and it had a very low temperature. Then, reheating/preheating took place to give rise to the hot Big Bang era, governed by a radiation period with $w = 1/3$. But the density of radiation and/or relativistic particles (photons, neutrinos) decayed faster than that of non-relativistic particles (protons, neutrons, DM) and eventually matter dominated over the relativistic components in a dust ($w = 0$) period of the evolution. More recently, but still seven
billion years ago, dark energy with $w \sim -1$ entered to dominate the dynamics and to inflate the Universe again.

Real fluids are in a perturbed state, and the five main components of the Universe (photons, neutrinos, baryons, dark matter, and dark energy) are not the exception. The plasma that composed the hot Big Bang era oscillated with the well-known kinematics of perturbed fluids, and as a consequence anisotropies in the CMB and inhomogeneities in the matter distribution left a unique fingerprint that we measure at present. On the other hand, if dark energy is the simplest candidate, the cosmological constant, its perturbations are null, since it is simply a geometrical term in the Enstein’s equations. But if it is a fluid, perturbations are to be computed to understand their effect on structure formation.

Cosmological and astrophysical observations, since the early 1990’s, have been playing a main role in the cosmological science, which was governed mainly by exact solutions and mathematical analyses. Indeed, we have just entered in a high precision era in which the observations demand to construct new theoretical observables, and vice versa. In the coming years, we expect not only to learn more about the fluids in cosmology, such as dark matter and dark energy, but also about the left-hand side of Einstein’s equations: is GR correct? or, are modified gravity schemes more properly fitted to the cosmic kinematics? These are quests that challenge our present knowledge and that should be answered in the coming years.

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