Majorana CP phases in bi-pair neutrino mixing and leptogenesis

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We estimate Majorana CP phases for a given flavor neutrino mass matrix ($M_{ij}$) consistent with the bi-pair neutrino mixing, which is recently proposed to describe neutrino mixings given by $\sin^2 \theta_{13} = 0$ for the reactor neutrino mixing, $\sin^2 \theta_{12} = 1 - 1/\sqrt{2}$ for the solar neutrino mixing and either $\sin^2 \theta_{23} = \tan^2 \theta_{12}$ or $\sin^2 \theta_{23} = 1 - \tan^2 \theta_{12}$ for the atmospheric neutrino mixing. Sizes of Majorana CP phases are evaluated so as to generate the observed baryon asymmetry in the universe via a leptogenesis scenario within the framework of the minimal seesaw model, where $M_{ii}$ satisfies $\det(M_{ii}) = 0$ and one active Majorana CP phase ($\phi$) is present. Assuming the normal mass hierarchy for light neutrinos and one zero texture for a $3 \times 3$ Dirac neutrino mass matrix, we find that $\phi$ lies in the region of $0.69 \lesssim |\phi| \lesssim 0.92$ [rad], which is converted into allowed regions of $\alpha = \arg(M_{e\mu})$ and $\beta = \arg(M_{e\tau})$, where $M_{ij}$ (i, j = e, $\mu$, $\tau$) denote the $i$-$j$ matrix element of $M_{ii}$. The phases $\alpha$ and $\beta$ turn out to satisfy $0.31 \lesssim |\alpha| \lesssim 0.40$ [rad] and $-1.25 \lesssim \beta \lesssim -0.32$ [rad]. The approximate numerical equality of $|\phi| \approx 2|\alpha|$ is consistent with our theoretical estimation of $\phi = \phi_2 - \phi_3$ for $\phi_2 = -(\alpha + \beta)$ and $\phi_3 \approx \alpha - \beta$ valid for the normal mass hierarchy. We also find the following scaling property: $(M_{e\mu} - M_{e\tau}/|t_{12}|)/M_{e\tau}^\prime = M_{e\mu}^\prime/(M_{e\tau} - M_{e\mu}/|t_{12}|) = -M_{e\tau}^\prime/M_{e\mu}^\prime (|t_{12}| = \tan^2 \theta_{12} = \sqrt{2} - 1)$, where $M_{ij}^\prime$ stands for $M_{ij}$ evaluated on the basis of the Particle Data Group’s phase convention.

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I. INTRODUCTION

Experimental and theoretical studies of neutrino mixings have revealed various properties of neutrinos. The results from the atmospheric [1], solar [2], reactor [3, 4] and accelerator [5] neutrino oscillation experiments have provided us with robust evidence that neutrinos have tiny masses and their flavor states are mixed with each other [6]. The squared mass differences of solar and atmospheric neutrinos, respectively, defined by $\Delta m^2_2 = m_2^2 - m_1^2$ and $\Delta m^2_{atm} = m_3^2 - m_1^2$, where $m_i$ ($i = 1, 2, 3$) is the mass of the corresponding generation of neutrinos, are observed to be [7]:

$$\Delta m^2_2 = 7.65^{+0.23}_{-0.20} \times 10^{-5} \text{eV}^2,$$

$$|\Delta m^2_{atm}| = 2.46^{+0.12}_{-0.11} \times 10^{-3} \text{eV}^2.$$

The flavor mixing angles $\theta_{12}, \theta_{23}$ and $\theta_{13}$ are obtained as

$$\sin^2 \theta_{12} = 0.304^{+0.022}_{-0.016},$$

$$\sin^2 \theta_{23} = 0.50^{+0.07}_{-0.06},$$

$$\sin^2 \theta_{13} = 0.011^{+0.016}_{-0.011},$$

where $\theta_{12}, \theta_{23}$ and $\theta_{13}$ stand for solar neutrino mixing angle, atmospheric neutrino mixing angle and reactor neutrino mixing angle, respectively. These mixing angles describe the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) matrix $U_{PMNS}$ [8] that converts mass eigenstates of neutrinos into flavor neutrinos.

One of the important and unsolved problem in neutrino physics is to understand CP properties of neutrinos. There are two sources of CP violations arising from Dirac CP phase and Majorana CP phase [9]. For three flavor neutrinos, CP violation is induced by one Dirac CP phase and two Majorana CP phases. Since Dirac CP violation involves the factor $\sin \theta_{13}$, no CP violation is induced by Dirac CP phase if $\sin \theta_{13} = 0$. The current experimental data Eq. [2] are consistent with $\sin \theta_{13} = 0$. The latest data of $\sin \theta_{13}$ reported by T2K collaboration [10] seem to suggest that $\sin \theta_{13} \neq 0$ at 90% C.L., namely, $0.03(0.04) < \sin^2 2\theta_{13} < 0.28(0.34)$ for normal (inverted) mass hierarchy, giving $\sin^2 \theta_{13} \gtrsim 0.0075$. The MINOS Collaboration has also reported to disfavor $\sin \theta_{13} = 0$ [11]. Since these indications are not statistically sufficient, to get the definite confirmation of $\sin \theta_{13} \neq 0$ needs more data samples. Theoretically, it is useful to construct a model giving $\sin \theta_{13} = 0$, which can be regarded as a reference point to discuss effects of $\sin \theta_{13} \neq 0$.

If $\sin \theta_{13} = 0$, Dirac CP phase is irrelevant. The remaining Majorana CP phases completely disappear from the oscillation probabilities and cannot be measured by quite familiar oscillation experiments [12]. Although Majorana CP phases can enter in processes of neutrinoless double beta decay, the detection of Majorana CP violation has not been succeeded [13]. On the other hand, in the leptogenesis scenario [14], the baryon-photon ratio in the universe $(\eta_B)$ is generated if Majorana CP phases exist and sizes of Majorana CP phases can be evaluated such that the observed ratio by WMAP collaboration [15] is reproduced. There are theoretical discussions that predict $\sin \theta_{13} = 0$ [16, 20]. We have recently proposed a bi-pair neutrino mixing scheme [21] that also predicts $\sin \theta_{13} = 0$ as well as $\sin^2 \theta_{12} = 1 - 1/\sqrt{2}$ and either $\sin^2 \theta_{23} = \tan^2 \theta_{12}$ (referred to the case 1)
or $\sin^2 \theta_{23} = 1 - \tan^2 \theta_{12}$ (referred to the case 2).

In this paper, we would like to estimate sizes of phases of flavor neutrino masses associated with the bi-pair neutrino mixing. To do so, we rely upon the seesaw mechanism \cite{22} to calculate $\eta_B$. Since $\eta_B$ depends on Majorana phases, to find constraints on phases of flavor neutrino masses, we have to derive direct relations between Majorana phases and phases of flavor neutrino masses. It is convenient to adopt the minimal seesaw model \cite{22,24,25}, where the number of physical phases associated with the seesaw mechanism are equal to that of CP phases of the flavor neutrino sector.

This paper is organized as follows. In the next section, we show a brief introduction to the bi-pair neutrino mixing and we also show phase structure of flavor neutrino masses leading to the bi-pair neutrino mixing. The direct relationship between Majorana phases and phases of flavor neutrino masses are derived. In section III we give an outline of the minimal seesaw model and leptonogenesis. We perform numerical calculations to show the allowed region of Majorana phases and of phases of flavor neutrino masses. The last section is devoted to summary and discussions.

II. BI-PAIR NEUTRINO MIXING

It is a good approximation that the reactor neutrino mixing angle is exactly zero \cite{49}. In this case, the PMNS matrix $U_{PMNS}$ given by the Particle Data Group \cite{23} to be $U_{PMNS} = U_0^{PDG} P^{PDG}$:

$$U_{PDG}^{0} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -c_{23}s_{12} & c_{23}c_{12} & s_{23} \\ s_{23}s_{12} & -s_{23}c_{12} & c_{23} \end{pmatrix},$$

$$P^{PDG} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi_{2}}/2 & 0 \\ 0 & 0 & e^{i\phi_{3}}/2 \end{pmatrix},$$

where $c_{ij} = \cos \theta_{ij}$, $s_{ij} = \sin \theta_{ij}$ ($i, j = 1, 2, 3$) and $\phi_{1,2}$ denote Majorana phases.

A. Texture

The bi-pair neutrino mixing $U_{BP}$ is determined by a mixing matrix $U_{BP}^{PDG}$ with two pairs of identical magnitudes of matrix elements. There are two possibilities of the bi-pair texture \cite{21}, both of which predict $\sin^2 \theta_{12} = 1 - 1/\sqrt{2}(0.293)$.

Case 1: The first possibility shows

$$| (U_{0}^{PDG})_{12} | = | (U_{0}^{PDG})_{22} |,$$

$$| (U_{0}^{PDG})_{22} | = | (U_{0}^{PDG})_{32} |.$$  \hspace{1cm} (4)

These relations in turn provide useful relationship among the atmospheric neutrino mixing and the solar neutrino mixing as

$$\sin^2 \theta_{23} = \tan^2 \theta_{12}, \quad \tan^2 \theta_{23} = \cos^2 \theta_{12}. \hspace{1cm} (5)$$

The bi-pair neutrino mixing in the case 1 is parameterized by only one mixing angle $\theta_{12}$:

$$U_{BP} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -t_{12}^2 & t_{12} & t_{12} \\ s_{12}t_{12} & -s_{12} & t_{12}/c_{12} \end{pmatrix},$$

where $t_{ij} = \tan \theta_{ij}$ ($i, j = 1, 2, 3$). Numerically, the mixing angles are predicted to be:

$$\sin^2 \theta_{12} = 1 - \frac{1}{\sqrt{2}} \sim 0.293,$$

$$\sin^2 \theta_{23} = \tan^2 \theta_{12} = \sqrt{2} - 1 \sim 0.414. \hspace{1cm} (7)$$

The bi-pair neutrino mixing well describes the observed solar neutrino mixing ($0.288 \leq \sin^2 \theta_{12} \leq 0.326$); however, the atmospheric neutrino mixing is slightly inconsistent with the $1\sigma$ data ($0.44 \leq \sin^2 \theta_{23} \leq 0.57$). It is expected that additional contribution to the atmospheric neutrino mixing angle is produced by the charged lepton correction if a non-diagonal matrix element of charged lepton mass matrix only arises from a $\mu$- $\tau$ mixing mass so that $\theta_{23}$ can be shifted to the $1\sigma$ region without affecting the value of $\theta_{12,13}$.

Case 2: The second possibility shows

$$| (U_{0}^{PDG})_{12} | = | (U_{0}^{PDG})_{22} |,$$

$$| (U_{0}^{PDG})_{22} | = | (U_{0}^{PDG})_{32} |.$$  \hspace{1cm} (8)

The atmospheric neutrino mixing is related to the solar neutrino mixing as

$$\cos^2 \theta_{23} = \tan^2 \theta_{12}, \quad \tan^2 \theta_{23} = 1/\cos^2 \theta_{12}. \hspace{1cm} (9)$$

The bi-pair neutrino mixing in the case 2 is parameterized by

$$U_{BP} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12}^2t_{12} & s_{12} & t_{12}/c_{12} \\ t_{12}^2 & -t_{12} & t_{12} \end{pmatrix}. \hspace{1cm} (10)$$

Numerically, the mixing angles are predicted to be

$$\sin^2 \theta_{12} = 1 - \frac{1}{\sqrt{2}} \sim 0.293,$$

$$\sin^2 \theta_{23} = 1 - \tan^2 \theta_{12} = 2 - \sqrt{2} \sim 0.586. \hspace{1cm} (11)$$

Same as in the case 1, the atmospheric neutrino mixing is slightly inconsistent with the $1\sigma$ data.

B. General discussion on phase structure

One may wonder what kind of flavor structure of a neutrino mass matrix $M_{\nu}$ is associated with the bi-pair neutrino mixing. To find phase structure of $M_{\nu}$ for the bi-pair neutrino mixing, we start our discussion with most general form of the PMNS mixing matrix $U_{PMNS} = U_{0}P$ with
After redundant phases are removed from $U_{\text{mixing}}$, it becomes:

$$U_0 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\gamma} & 0 \\ 0 & 0 & e^{-i\gamma} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} e^{i\rho} & 0 \\ -s_{12} e^{-i\rho} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

After appropriate redefinition of Majorana phases, Eq. (3) moves these phases by the redefinition of flavor neutrinos denoting three Majorana phases [26].

We denote three Majorana phases $\gamma$, $\delta$, $\rho$ where $\gamma = \text{CP}$, $\delta = \theta$, $\rho = \delta$.

The phases $\gamma$ and $\rho$ in $U_0$ are redundant and we can remove these phases by the redefinition of flavor neutrinos resulting from the phase ambiguities present in charged leptons. This redefinition can be expressed by a rotation matrix $R$ which has three phases $\theta_\ell$, $\theta_\mu$ and $\theta_\tau$:

$$R = \begin{pmatrix} e^{i\theta_\ell} & 0 & 0 \\ 0 & e^{i\theta_\mu} & 0 \\ 0 & 0 & e^{i\theta_\tau} \end{pmatrix}. \quad (14)$$

After redundant phases are removed from $U_{\text{PMNS}}$, the mixing matrix becomes

$$U_{\text{PMNS}} \rightarrow U'_{\text{PMNS}} = RU_{\text{PMNS}}. \quad (15)$$

and accordingly the $3 \times 3$ symmetric flavor neutrino mass matrix

$$M_\nu = \begin{pmatrix} M_{ee} & M_{e\mu} & M_{e\tau} \\ M_{e\mu} & M_{\mu\mu} & M_{\mu\tau} \\ M_{e\tau} & M_{\mu\tau} & M_{\tau\tau} \end{pmatrix}. \quad (16)$$

is shifted as follows:

$$M_\nu \rightarrow M'_\nu = R^\dagger M_\nu R. \quad (17)$$

For example, to obtain the Particle Data Group’s form of the mixing matrix, we take $\theta_\ell = -\rho$, $\theta_\mu = -\gamma$ and $\theta_\tau = \gamma$, which give $R$ for the PDG version, $R_{\text{PDG}}$:

$$R_{\text{PDG}} = \begin{pmatrix} e^{-i\rho} & 0 & 0 \\ 0 & e^{-i\gamma} & 0 \\ 0 & 0 & e^{i\gamma} \end{pmatrix}. \quad (18)$$

We, then, find the PDG version of $U_0$, $U_{0\text{PDG}}$:

$$U_{0\text{PDG}} = \begin{pmatrix} c_{12}c_{13} & s_{12} c_{13} e^{i\delta_{CP}} \\ -c_{23} s_{12} - s_{23} c_{12} c_{13} e^{i\delta_{CP}} \\ s_{23} s_{12} - c_{23} c_{12} c_{13} e^{i\delta_{CP}} \end{pmatrix} \begin{pmatrix} s_{12} c_{13} & c_{23} c_{12} - s_{23} s_{12} c_{13} e^{i(\rho+\delta)} \\ c_{23} s_{12} - s_{23} s_{12} s_{13} e^{i(\rho+\delta)} \\ -s_{23} c_{12} - c_{23} s_{12} s_{13} e^{i(\rho+\delta)} \end{pmatrix}. \quad (19)$$

and the PDG version of $P$, $P_{0\text{PDG}}$:

$$P_{0\text{PDG}} = \begin{pmatrix} e^{i\phi_1'} & 0 & 0 \\ 0 & e^{i\phi_2'} & 0 \\ 0 & 0 & e^{i\phi_3'} \end{pmatrix}. \quad (20)$$

where

$$\delta_{CP} = \delta + \rho, \quad \phi_1' = \varphi_1 - \rho, \quad \phi_{2,3}' = \varphi_{2,3}. \quad (21)$$

After appropriate redefinition of Majorana phases, Eq. (3) turns out to be:

$$U_{\text{PMNS}}^D = U_0^D P_{\text{PDG}}.$$

where

$$\phi_2 = 2(\phi_2' - \phi_1')$$
$$\phi_3 = 2(\phi_3' - \phi_1'). \quad (23)$$

C. Phase structure in the bi-pair neutrino mixing

For any type of neutrino mixings that give $\sin \theta_{13} = 0$, it can be argued that Eq. (10) becomes the following mass
matrix \cite{27}: 
\[
M_{\nu}^{\rho_{13}=0} = \begin{pmatrix}
M_{ee} & e^{i\alpha} |M_{e\mu}| & -t_{23} e^{i\beta} |M_{e\tau}|
M_{\mu\mu} & M_{\mu\tau} & M_{\mu\tau}
M_{\tau\mu} & M_{\tau\tau} & M_{\tau\tau}
\end{pmatrix},
\]
(25)
with 
\[
M_{\tau\tau} = e^{i\gamma} M_{\mu\mu} + \frac{1 - t_{23}^2}{t_{23}} e^{i\gamma} M_{\mu\tau},
\]
(26)
and 
\[
\gamma = \frac{\beta - \alpha - \frac{\pi}{2}}{2}.
\]
(27)
The neutrino masses \(m_1, m_2, m_3\) defined by 
\[
U_{PMNS}^T M_{\nu}^{\rho_{13}=0} U_{PMNS} = \begin{pmatrix}
m_1 & 0 & 0 
0 & m_2 & 0 
0 & 0 & m_3
\end{pmatrix},
\]
(28)
are calculated to be 
\[
m_1 e^{-i\phi_1} = e^{2\rho} M_{ee} - t_{12} \frac{e^{i\xi} |M_{e\mu}|}{c_{23}},
m_2 e^{-i\phi_2} = e^{2\rho} M_{ee} + \frac{1 - \xi}{t_{12}} \frac{e^{i\xi} |M_{e\mu}|}{c_{23}},
m_3 e^{-i\phi_3} = e^{2\gamma} M_{\mu\mu} - \frac{1}{t_{23}} M_{\mu\tau},
\]
(29)
where 
\[
\xi = \rho + (\alpha + \beta)/2.
\]
(30)
The mixing angle \(\theta_{12}\) is given by 
\[
\tan 2\theta_{12} = \frac{2 e^{i\xi} c_{23}^{-1} |M_{e\mu}|}{e^{2\gamma} M_{\mu\mu} - t_{23} M_{\mu\tau} - e^{2\rho} M_{ee}}.
\]
(31)
From Eq. (25), we obtain \(M_{\nu}^{PDG}^{\rho_{13}=0} = \) 
\[
M_{\nu}^{PDG}^{\rho_{13}=0} = \begin{pmatrix}
e^{2\rho} M_{ee} & e^{i\xi} |M_{e\mu}| & -t_{23} e^{i\beta} |M_{e\tau}|
e^{2\gamma} M_{\mu\mu} & M_{\mu\tau} & M_{\mu\tau}
e^{-2\gamma} M_{\tau\tau} & M_{\tau\tau} & M_{\tau\tau}
\end{pmatrix},
\]
(32)
Furthermore, by using (31) to eliminate \(e^{2\gamma} M_{\mu\mu}\), we find the following flavor structure: 
\[
M_{\nu}^{PDG}^{\rho_{13}=0} = e^{2\rho} M_{ee} I
+ \begin{pmatrix}
0 & 1 & -t_{23} 
\frac{2}{\tan 2\theta_{12}} \frac{1}{c_{23}} & 0 & 1 \frac{2}{\tan 2\theta_{12}} \frac{1}{c_{23}}
0 & t_{23} & 1 \frac{1}{t_{23}}
\end{pmatrix} e^{i\xi} |M_{e\mu}|
+ \begin{pmatrix}
0 & 0 & 0 
0 & 0 & 0 
0 & 0 & 0
\end{pmatrix} |M_{\mu\tau}|.
\]
(33)
For the bi-pair mixing scheme, the mixing angles in Eq. (33) are fixed to be \(\tan^2 2\theta_{12} = 2(\sqrt{2} - 1)\) together with \(\tan^2 \theta_{23} = 1/\sqrt{2}\) for the case 1 and \(\tan^2 \theta_{23} = \sqrt{2}\) for the case 2. A more transparent form of its flavor structure can be obtained when either \(m_1\) or \(m_3\) vanishes as in the minimal seesaw model to be discussed in the next section.

Let us consider neutrinos exhibiting \(m_1 = 0\), which corresponds to the normal mass hierarchy. Since there is the phase ambiguity in the charged lepton sector, we can choose three phases associated with flavor neutrino masses to be any values. One may assign a specific value to \(M_{\mu\tau}\) to be consistent with \(m_1 = 0\) and take \(M_{ee}\) and \(M_{\mu\mu}\) to be real. Accordingly, Eqs. (32), (26), (29) and (31) turn out to be 
\[
M_{\nu}^{PDG}^{\rho_{13}=0} = \begin{pmatrix}
e^{2\rho} |M_{e\mu}| & e^{i\xi} |M_{e\mu}| & -t_{23} e^{i\beta} |M_{e\tau}|
e^{2\gamma} |M_{\mu\mu}| & M_{\mu\tau} & M_{\mu\tau}
e^{-2\gamma} |M_{\tau\tau}| & M_{\tau\tau} & M_{\tau\tau}
\end{pmatrix},
\]
(34)
\[
M_{\mu\tau} = e^{i\gamma} |M_{\mu\mu}| + \frac{1 - t_{23}^2}{t_{23}} e^{i\gamma} M_{\mu\tau},
\]
(35)
\[
m_1 e^{-i\phi_1} = e^{2\rho} |M_{e\mu}| - t_{12} \frac{e^{i\xi} |M_{e\mu}|}{c_{23}},
\]
(36)
\[
\tan 2\theta_{12} = \frac{2 e^{i\xi} c_{23}^{-1} |M_{e\mu}|}{e^{2\gamma} |M_{\mu\mu}| - t_{23} M_{\mu\tau} - e^{2\rho} |M_{ee}|},
\]
(37)
where the sign of \(M_{ee}\) is taken care of by \(\kappa_\nu = \pm 1\) for 
\(-\pi/2 \leq 2\rho \leq \pi/2\). From \(m_1 = 0\) in Eq. (35), we find \(\xi = 2\rho\) leading to 
\[
\xi = 2\rho = \alpha + \beta.
\]
(38)
We then find that, from (37), \(M_{\mu\tau}\) is given by 
\[
M_{\mu\tau} = \frac{1}{t_{23}} \left( e^{i(\beta - \alpha)} |M_{\mu\mu}| - \frac{e^{i(\alpha + \beta)} |M_{e\mu}|}{c_{23} t_{12}} \right).
\]
(39)
The phase of \(M_{\mu\tau}\) should be adjusted to satisfy Eq. (39).

The neutrino masses turn out to be: 
\[
m_2 e^{-i\phi_2} = \frac{e^{i(\alpha + \beta)} |M_{e\mu}|}{c_{12} s_{12} c_{23}},
m_3 e^{-i\phi_3} = \frac{1}{s_{12}^2} \left( e^{i(\beta - \alpha)} |M_{\mu\mu}| - \frac{e^{i(\alpha + \beta)} c_{23} |M_{e\mu}|}{t_{12}} \right),
\]
(40)
The Majorana phase \(\phi_2\) is simply given by 
\[
\phi_2 = -(\alpha + \beta).
\]
(41)
The phase \(\beta - \alpha\) can also be calculated from the equivalent relation of 
\[
\arg (\beta - \alpha) = \arg \left( t_{23}^2 m_3 e^{-i\phi_3} + c_{12}^2 m_2 e^{-i\phi_2} \right).
\]
(42)
The condition of $m_3^2 \gg m_2^2$ leads to

$$\phi_3 \approx \alpha - \beta.$$  \hfill (43)

For the rest of paper, we use $\phi$:

$$\phi = \phi_2 - \phi_3.$$  \hfill (44)

The Dirac neutrino mass matrix and

$$\begin{pmatrix}
A e^{i(\alpha + \beta)} | M_{e\mu}| & \frac{A}{\sqrt{2}} e^{i(\alpha + \beta)} | M_{e\mu}| & -\frac{A}{\sqrt{2}} e^{i(\alpha + \beta)} | M_{e\mu}| \\
\frac{A}{\sqrt{2}} e^{i(\alpha + \beta)} | M_{e\mu}| & M_{\mu\tau} & M_{\mu\tau} \\
-M_{\mu\tau} & M_{\mu\tau} & M_{\mu\tau}
\end{pmatrix},$$  \hfill (45)

where $(A, B) = (c_{12}, 1)$ for the case 1 while $(A, B) = (1, c_{12})$ for the case 2. It should be mentioned that the mass matrix $M^{PDG}_{\nu}|_{\theta_{13}=0}$ exhibits the following scaling property:

$$\frac{M'_{ee}}{M'_{e\mu}} = A,$$

$$\frac{M'_{\mu\tau}}{M'_{e\mu}} = \frac{M'_{e\mu}}{M'_{\mu\tau}} = -\frac{M'_{\mu\tau}}{M'_{e\mu}} = A/B.$$  \hfill (46)

where $M'_{ij}$ ($i,j=e, \mu, \tau$) stand for the matrix elements of $M^{PDG}_{\nu}$ as in Eq. (23).

III. LEPTOGENESIS

In this section, first, we give an outline of the minimal seesaw model and leptogenesis, then, we show the allowed region of Majorana phases from a numerical calculation.

A. Minimal seesaw model

In the minimal seesaw model [22], we introduce two heavy neutrinos $N_1$ and $N_2$ into the standard model. We obtain a symmetric $3 \times 3$ light neutrino mass matrix by the relation of $M_\nu = -m_D M_R^{-1} m_D^T$, where $m_D$ is a $3 \times 2$ Dirac neutrino mass matrix and $M_R$ is a $2 \times 2$ heavy neutrino mass matrix. We assume that the mass matrix of the heavy neutrinos as well as of the charged leptons is diagonal and real. For the heavy neutrinos, $M_R$ takes the form of

$$M_R = \begin{pmatrix} M_1 & 0 \\ 0 & M_2 \end{pmatrix} \quad (M_2 > M_1).$$  \hfill (47)

The Dirac neutrino mass matrix $m_D$ can be expressed in terms of 6 parameters $a_1, a_2, a_3, b_1, b_2, b_3$ and two heavy neutrino masses $M_1, M_2$ as [24]

$$m_D = \begin{pmatrix} \sqrt{M_1 a_1} & \sqrt{M_2 b_1} \\ \sqrt{M_1 a_2} & \sqrt{M_2 b_2} \\ \sqrt{M_1 a_3} & \sqrt{M_2 b_3} \end{pmatrix},$$  \hfill (48)

where one zero texture is assumed and the light neutrino mass matrix is obtained from $M^{PDG}_{\nu} = -m_D M_R^{-1} m_D^T$ as

$$M^{PDG}_{\nu} = \begin{pmatrix} M'_{ee} & M'_{e\mu} & M'_{e\tau} \\ M'_{\mu\mu} & M'_{\mu\tau} & M'_{\mu\tau} \\ M'_{\tau\tau} & M'_{\tau\tau} & M'_{\tau\tau} \end{pmatrix}$$

$$= \begin{pmatrix} a_1^2 + b_1^2 & a_1 a_2 + b_1 b_2 & a_1 a_3 + b_1 b_3 \\ a_2^2 + b_2^2 & a_2 a_3 + b_2 b_3 \\ a_3^2 + b_3^2 \end{pmatrix},$$  \hfill (49)

whose phase structure is determined by Eq. (23), which is described by Eq. (33). The condition Eq. (26) giving $\sin \theta_{13} = 0$ now reads

$$M'_{\tau\tau} = M'_{\mu\mu} + 1 - \frac{c_{23}^2}{\sin^2 \theta_{23}},$$  \hfill (50)

The mass matrix Eq. (49) contains 5 parameters because of the condition of $\det(M^{PDG}_{\nu}) = 0$. Since the Dirac mass matrix $m_D$ is one zero texture, we can analytically express the Dirac mass matrix elements in $m_D$ in terms of the light neutrino masses in $M^{PDG}_{\nu}$. For instance, if $a_2 = 0$, the solution consists of

$$a_1 = -\sigma_1 \sqrt{M'_{\mu\mu} M'_{ee} - M'_{e\mu}^2},$$

$$a_3 = -\sigma_3 \sqrt{M'_{\mu\tau} M'_{e\mu} - M'_{e\tau}^2},$$

$$b_1 = \frac{M'_{e\mu}}{\sqrt{M'_{e\mu}^2}}, \quad b_2 = \frac{M'_{e\tau}}{\sqrt{M'_{e\tau}^2}}, \quad b_3 = \frac{M'_{\mu\tau}}{\sqrt{M'_{\mu\tau}^2}}.$$  \hfill (51)

with

$$M'_{\tau\tau} M'_{\mu\mu} = \sigma_1 \sigma_3 \sqrt{(M'_{e\mu} M'_{e\mu} - M'_{e\tau}^2) (M'_{e\mu} M'_{e\tau} - M'_{e\tau}^2)}$$

$$+ M'_{e\mu} M'_{\mu\tau},$$  \hfill (52)
due to \( \det(M_{\nu}^{\text{PDG}}) = 0 \), where \( \sigma_{1,3} = \pm 1 \) and the sign of \( \sigma_{1,3} \) can be calculated. Similarly, other solutions with \( a_{1,3} = 0 \) or \( b_{1,3} = 0 \) can be obtained.

According to the condition of \( \det(M_{\nu}^{\text{PDG}}) = 0 \), at least one of the neutrino mass eigenvalues \( (m_1, m_2, m_3) \) must be zero \[23\]. We obtain the two types of hierarchical neutrino mass spectrum in the minimal seesaw model. One is the normal mass hierarchy \( (m_1, m_2, m_3) = \left(0, \sqrt{\Delta m^2_{\odot}}, \sqrt{\Delta m^2_{\text{atm}}} \right) \) and the other is the inverted mass hierarchy \( (m_1, m_2, m_3) = \left(\sqrt{-\Delta m^2_{\odot}}, \sqrt{\Delta m^2_{\odot}} - \Delta m^2_{\text{atm}}, 0 \right) \). The matrix elements of \( m_D \) can be reconstructed in terms of two mixing angles \( \theta_{12,23} \), two neutrino masses \( m_{2,3} \), one CP phase \( \phi \) and two heavy neutrino masses \( M_{1,2} \).

### B. Leptogenesis

In the leptogenesis scenario, the baryon-photon ratio is obtained from the baryon asymmetry \( Y_B = (n_B - n_{\bar{B}})/s \) as

\[
Y_B = 7.04 Y_L, \quad (53)
\]

where \( s \) is entropy density in the universe. Via the sphaleron process, this baryon asymmetry is related to the lepton asymmetry \( Y_L \):

\[
Y_B = \frac{8 N + 4 m}{14 N + 9 m} Y_L, \quad (54)
\]

where \( N \) is the number of generation of fermions and \( m \) is the number of Higgs doublets. In the particle contents of the standard model, we have \( Y_B \approx -0.549 Y_L \).

The lepton asymmetry \( Y_L \) is parameterized by three terms as

\[
Y_L = d \frac{\epsilon}{g_*} g_* \quad , \quad (55)
\]

where \( d, \epsilon \) and \( g_* \) are generally called the dilution factor, CP-asymmetry parameter and effective number of the relativistic degree of freedom, respectively. We use the following estimates:

1. The dilution factor \( d \) should be determined by solving the Boltzmann equation. In the present analyses, however, we use the good analytical approximation proposed by Nielsen and Takanishi \[28\]:

\[
d \sim \begin{cases} 
\frac{1}{2 \sqrt{0.3 K^{1/4}}} & 0 \leq K \leq 10, \\
\frac{1}{K^{(m K)^{1/4}}} & 10 \leq K \leq 10^6, 
\end{cases} \quad (56)
\]

where

\[
K = \frac{M_{\nu}}{1.66 \sqrt{g_* (8 \pi v^2)}} \frac{(m_D m_D)_{11}}{M_1} \approx \frac{1}{10^{-3} \text{eV}} (|a_1|^2 + |a_2|^2 + |a_3|^2), \quad (57)
\]

with the Plank mass \( M_\nu \approx 1.22 \times 10^{19} \) GeV and the vacuum expectation value of the Higgs field \( v \approx 174 \) GeV.

2. The CP-asymmetry \( \epsilon \) is generated by the decay processes of the heavy neutrinos. If we assume a hierarchical mass spectrum of the heavy neutrinos \( M_1 \ll M_2 \), the interactions of \( N_1 \) can be in thermal equilibrium when \( N_2 \) decays and the asymmetry caused by the \( N_2 \) decay is washed out by the lepton number violating processes with \( N_1 \). Thus, only the decays of \( N_1 \) are relevant to the generation of the final lepton asymmetry. In this case, the CP-asymmetry parameter is calculated to be \[14\]

\[
\epsilon = \frac{M_2}{8 \pi v^2} \frac{\text{Im}[(a_1^* b_1 + a_2^* b_2 + a_3^* b_3)^2]}{|a_1|^2 + |a_2|^2 + |a_3|^2} f \left( \frac{M_2}{M_1} \right), \quad (58)
\]

where the function \( f(x) \) is given by

\[
f(x) = x \left[ 1 - (1 + x^2) \ln \frac{1 + x^2}{x^2} + \frac{1}{1 - x^2} \right] \approx -\frac{3}{2x} \quad \text{for} \ x \gg 1, \quad (59)
\]

3. The effective number of the relativistic degree of freedom \( g_* \) is calculated as \[30\]

\[
g_* = \sum_{i=\text{bosons}}^{} g_i \left( \frac{T_i}{T} \right)^4 + \sum_{i=\text{fermions}}^{} g_i \left( \frac{T_i}{T} \right)^4, \quad (60)
\]

where \( T \) is thermal equilibrium temperature of the universe, \( T_i \) and \( g_i \) are temperature and number of internal degrees of freedom of the relativistic particle species \( i \). For \( T \geq 300 \) GeV, all the species in the standard model are relativistic and we have \( g_* \approx 106.75 \).

We note that, in the case of \( M_1 \ll M_2 \), the baryon-photon ratio \( \eta_B \) is nearly proportional to the mass of heavy neutrino \( M_1 \) \[28\]. This \( M_1 \) dependence to \( \eta_B \) can be understand by the following rough estimation. For \( x = M_2/M_1 \gg 1 \), the CP-asymmetry parameter \( \epsilon \) Eq.\(58 \) becomes

\[
\epsilon \sim \frac{3}{16 \pi v^2} M_1 \hat{\epsilon}, \quad (\hat{\epsilon} = \frac{\text{Im}[(a_1^* b_1 + a_2^* b_2 + a_3^* b_3)^2]}{|a_1|^2 + |a_2|^2 + |a_3|^2}). \quad (61)
\]

Since \( \hat{\epsilon} \) is independent of \( M_1 \), \( \epsilon \) is nearly proportional to the mass of the heavy neutrino \( M_1 \).

### C. Numerical analysis

**Assumptions:** We have performed the numerical calculation with the following assumptions:
FIG. 1: Majorana CP phase $\phi$ vs the baryon-photon ratio $\eta_B$ for $a_2 = 0$ (upper figure) and $a_3 = 0$ (lower figure) in the case $1 (\sin^2 \theta_{23} = \tan^2 \theta_{12})$ of the bi-pair neutrino mixing. The horizontal rectangles show the allowed region of $\eta_B$ and the thick black curves show our predictions.

1. The light neutrino mass spectrum is the normal mass hierarchy.

2. $\Delta m^2_\odot = 7.65 \times 10^{-5}$eV$^2$ and $|\Delta m^2_{\text{atm}}| = 2.40 \times 10^{-3}$eV$^2$.

3. The heavy neutrino masses $M_{1,2}$ lie between electroweak scale $\sim 10^2$ GeV and GUT scale $\sim 10^{15}$ GeV. We take $M_1 = 5 \times 10^{10}(\ll M_2)$ GeV. In order to ensure the thermal leptogenesis to be the source of the baryon asymmetry in the universe, the reheating temperature after inflation must have been greater than the mass scale of the lightest heavy Majorana neutrino [31]. Hence the lower bound on the reheating temperature must be greater than $\sim 10^{10}$ GeV. However, this high reheating temperature is not suitable for supersymmetric (SUSY) theories because it may lead to an overproduction of light supersymmetric particles, such as a gravitino after inflation [32]. We are not considering this problem here and are limiting discussion on non-SUSY cases.

4. The Dirac neutrino mass matrix is one zero texture. However, if either $a_1 = 0$ or $b_1 = 0$ is chosen, we can prove that $\epsilon \propto a_1^* b_1 + a_2^* b_2 + a_3^* b_3 = 0$ and $\eta_B \propto \epsilon = 0$. The case with $a_1 = 0$ or $b_1 = 0$ is excluded.

5. All the particle species in the standard model were relativistic when the leptonic CP-asymmetry was generated by the decay process of the lightest heavy neutrino $N_1$. However, $N_1$ was heavy enough to be non-relativistic itself. We take $g_* = 106.75$.

6. We use $\eta_B = (6.2 \pm 0.15) \times 10^{-10}$ as the upper and lower bound of the baryon-photon ratio from the WMAP observation [15].

Predictions: Our results are summarized in five figures FIG.1 ~ FIG.5. Basically, since the case with $a_2 = 0$ ($b_2 = 0$) is identical to the case with $a_3 = 0$ ($b_2 = 0$) if the $\mu$-$\tau$ symmetry is exact in $M_{\nu}^{PDG}$. The
FIG. 3: The same as in FIG.1 but for the phase $\alpha$ for $a_2 = 0$ (upper figure) and $a_3 = 0$ (lower figure).

FIG. 4: The same as in FIG.1 but for the phase $\beta$ for $a_2 = 0$ (upper figure) and $a_3 = 0$ (lower figure).

experimental results are consistent with the approximate $\mu$-$\tau$ symmetry and we expect that our predictions based on $a_2 = 0$ ($b_2 = 0$) are quite similar to those based on $a_3 = 0$ ($b_3 = 0$). We discuss implications from these figures in the followings:

1. **Case 1:** This case corresponds to the bi-pair neutrino mixing with $\sin^2 \theta_{23} = \tan^2 \theta_{12}$. FIG.1 and FIG.2 show our predictions on $\eta_B$ in the case of $a_2 = 0$ or $a_3 = 0$ (FIG.1) and $b_2 = 0$ or $b_3 = 0$ (FIG.2). From FIG.1 we observe that Majorana CP phase $\phi$ lies in the following regions:

$$|\phi| \sim 0.69 - 0.86 \; \text{[rad]},$$

(62)

for the case of $a_2 = 0$, and

$$|\phi| \sim 0.76 - 0.92 \; \text{[rad]},$$

(63)

for the case of $a_3 = 0$, where $0 \leq \phi \leq \pi/2$. The differences between the cases of $a_2 = 0$ and of $a_3 = 0$ are not so large (less than 10%). On the other hand, FIG.2 shows too small $\eta_B$ and no consistent regions of $\phi$ with the observed $\eta_B$. This is naturally expected because $\eta_B$ gets larger for the smaller denominator of Eq. (61) for $\epsilon$, which prefers the case of $a_2 = 0$ or $a_3 = 0$ compared with the case of $b_2 = 0$ or $b_3 = 0$.

The black regions in FIG.3 and FIG.4 show our predictions of $\alpha$ and $\beta$, which, respectively, stand for the phases of $M_{\mu\tau}$ and $M_{e\tau}$. The horizontal narrow bands in each figures are the allowed regions of the phases, which give

$$0.31 \lesssim |\alpha| \lesssim 0.37 \; \text{[rad]},$$

$$-1.25 \lesssim \beta \lesssim -0.32 \; \text{[rad]},$$

(64)

for the case of $a_2 = 0$, and

$$0.34 \lesssim |\alpha| \lesssim 0.40 \; \text{[rad]},$$

$$-1.23 \lesssim \beta \lesssim -0.35 \; \text{[rad]},$$

(65)

for the case of $a_3 = 0$. Depicted in FIG.5 is the direct relation of $\alpha$ and $\beta$, where the range of $\beta$ is restricted to the allowed region. From these figures, we observe that
for $a_2 = 0$. As a result, $a_1 b_1^* + a_2 b_2^* + a_3 b_3^*$ and $|a_1|^2 + |a_2|^2 + |a_3|^2$ to calculate $\epsilon$ for $a_2 = 0$ in the case 2 are same as those for $a_3 = 0$ in the case 1. Therefore, $\eta_B$ for $a_2 = 0$ in the case 2 is equal to $\eta_B$ for $a_3 = 0$ in the case 1. Similarly for $a_3 = 0$ in the case 2.

IV. SUMMARY AND DISCUSSIONS

We have discussed Majorana CP violation in the recently proposed bi-pair neutrino mixing scheme that predicts $\sin \theta_{13} = 0$ as well as $\tan^2 \theta_{12} = 2/(\sqrt{2} - 1)$ with either $\tan^2 \theta_{23} = 1/\sqrt{2}$ (the case 1) or $\tan^2 \theta_{23} = \sqrt{2}$ (the case 2). Within the minimal seesaw model, where $m_1 = 0$ is chosen, we have found that the Majorana CP phase $\phi$ is constrained to be:

$$|\phi| \sim 0.69 - 0.86 \text{ [rad]},$$

(67)

for the case of $a_2 = 0$, and

$$|\phi| \sim 0.76 - 0.92 \text{ [rad]},$$

(68)

for the case of $a_3 = 0$, in order to reproduce the observed WMAP baryon-photon ratio.

The theoretical study on the flavor structure of the mass matrix giving $\sin \theta_{13} = 0$ further reveals that two Majorana phases $\phi_{2,3}$ are estimated to be:

$$\phi_2 = -(\alpha + \beta),$$

$$\phi_3 \approx \alpha - \beta,$$

(69)

where $\phi = \phi_2 - \phi_3 \approx -2\alpha$. The phases $\alpha$ and $\beta$, respectively, specify the phases of $M_{\mu\mu}$ and $M_{\tau\tau}$ for a given neutrino masses $M_{ij}$. We have estimated that

- $\alpha$ satisfies $0.31 \lesssim |\alpha| \lesssim 0.40$ [rad],

- $\beta$ lies in the broad range of $-1.25 \sim -0.32$ [rad].

The relation Eq. (69) is based on the calculations of $m_{2,3}$:

$$m_2 e^{-i\phi_2} = \frac{e^{i(\alpha + \beta)} |M_{\mu\mu}|}{c_{12} \sin 2 \phi_{23}},$$

$$m_3 e^{-i\phi_3} = \frac{1}{s_{23}^2} \left( e^{i(\beta - \alpha)} |M_{\mu\mu}| - e^{i(\alpha + \beta)} |M_{\mu\tau}| \right) t_{23},$$

(70)

subject to the condition $m_3^2 \gg m_2^2$. The flavor structure of $M_{ij}$ compatible with the PDG phase convention is given by $M_{\nu_{i}}^{PDG} |_{\theta_{13} = 0}$. 

![FIG. 5: The allowed regions of the phase $\alpha$ vs the phase $\beta$ indicated by two narrow bands for $a_2 = 0$ (upper figure) and $a_3 = 0$ (lower figure) in the case 1 ($\sin^2 \theta_{23} = \tan^2 \theta_{12}$) of the bi-pair neutrino mixing.](image)

- $\alpha$, the phase of $M_{\mu\mu}$, satisfies $0.31 \lesssim |\alpha| \lesssim 0.40$ [rad],
- $\beta$, the phase of $M_{\tau\tau}$, lies in the broad range of $-1.25 \sim -0.32$ [rad].

2. Case 2: This case corresponds to the bi-pair neutrino mixing with $\sin^2 \theta_{23} = 1 - \tan^2 \theta_{12}$. The predictions of $\eta_B$ for $a_2 = 0$ ($a_3 = 0$) are reproduced by plots for $a_3 = 0$ ($a_2 = 0$) in the case 1. This correspondence can be understand by the following way: The Dirac matrix elements for the case 2 ($a_1^{\text{case2}}, a_2^{\text{case2}}, \cdots$) can be expressed in terms of those for the case 1 ($a_1^{\text{case1}}, a_2^{\text{case1}}, \cdots$). Namely, we can find that

$$a_1^{\text{case2}} = -a_1^{\text{case1}}, \quad b_1^{\text{case2}} = -b_1^{\text{case1}},$$

$$a_2^{\text{case2}} = 0, \quad b_2^{\text{case2}} = b_2^{\text{case1}},$$

$$a_3^{\text{case2}} = a_3^{\text{case1}}, \quad b_3^{\text{case2}} = b_2^{\text{case3}},$$

$$\sigma_1 \sigma_3^{\text{case2}} = -\sigma_1 \sigma_2^{\text{case1}},$$

(66)
\[ M_{\nu}^{PDG}\mid_{\theta_{13}=0} = \begin{pmatrix} A e^{i(\alpha + \beta)} |M_{e\mu}| & \frac{1}{\sqrt{2}} e^{i(\alpha + \beta)} |M_{e\mu}| & -\frac{B}{2} e^{i(\alpha + \beta)} |M_{e\mu}| \\ \frac{1}{\sqrt{2}} e^{i(\alpha + \beta)} |M_{e\mu}| & \frac{1}{2} |M_{\mu\tau}| & \frac{B}{2} |M_{\mu\tau}| \\ -\frac{B}{2} e^{i(\alpha + \beta)} |M_{e\mu}| & \frac{B}{2} |M_{\mu\tau}| & \frac{A}{2} |M_{\mu\tau}| \end{pmatrix}, \] (71)

where \((A, B) = (c_{12}, 1)\) for the case 1 while \((A, B) = (1, c_{12})\) for the case 2. There exist the relations among masses in \(M_{\nu}^{PDG}\mid_{\theta_{13}=0} = M'_{ij}\), given by

\[
\begin{align*}
\frac{M'_{ee}}{M'_{e\mu}} &= A, \\
\frac{M'_{e\mu} - M'_{\mu\tau}}{M'_{\mu\tau}} &= -\frac{M'_{\mu\tau}}{M'_{e\mu}} = \frac{A}{B},
\end{align*}
\] (72)

which exhibit the characteristic scaling property. To measure the magnitude of \(M'_{ee,e\mu,\tau}\) in future, we will find that

\[
\begin{align*}
\frac{|M'_{ee}|}{|M'_{e\mu}|} &= \begin{cases} c_{12} = \frac{1}{\sqrt{2}} = 0.84 \cdots \text{the case } 1 \\ 1 \cdots \text{the case } 2 \end{cases}, \\
\frac{|M'_{e\mu}|}{|M'_{e\mu}|} &= \begin{cases} c_{12} = \frac{1}{\sqrt{2}} = 0.84 \cdots \text{the case } 1 \\ \frac{1}{c_{12}} = \sqrt{2} = 1.19 \cdots \text{the case } 2 \end{cases},
\end{align*}
\] (73)

if the bi-pair neutrino mixing is correct.

In our future study, we will discuss the detailed feature of the charged lepton corrections arising from the following type of a mass matrix:

\[
M_\ell = \begin{pmatrix} m_{ee} & \varepsilon_{e\mu} & \varepsilon_{e\tau} \\ \varepsilon_{e\mu} & m_{\mu\mu} & m_{\mu\tau} \\ \varepsilon_{e\tau} & m_{\mu\tau} & m_{\tau\tau} \end{pmatrix},
\] (75)

As effects on the neutrino mixings, the correction to \(\theta_{23}\) becomes larger for the larger magnitude of \(m_{\mu\tau}\) and \(\theta_{13}\) becomes nonvanishing for \(\varepsilon_{\mu,\tau} \neq 0\). If there is an approximate conservation of the electron number, we may have tiny corrections to \(\theta_{13}\) and the normal mass hierarchy is welcome to tiny magnitudes of the electron-number-breaking flavor neutrino masses: \(M_{ee,e\mu,\tau}\) \[33\]. Furthermore, since we have equipped with the general parameterization of \(M_\ell\) in Eq. (25) compatible with \(\sin \theta_{13} = 0\), we may discuss Majorana CP violation in more model-independent way or in a way based on other specific models giving \(\sin \theta_{13} = 0\) \[20\]. These subjects will be discussed elsewhere \[34\].

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