**RESEARCH ARTICLE**

**DARCY- AND PORE-SCALE ISSUES ASSOCIATED WITH MULTI-PHASE FLUID FLOW THROUGH A PETROLEUM RESERVOIR**

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**ARTICLE DETAILS**

**ABSTRACT**

This manuscript primarily focuses on the constraints associated with the extended version of Darcy’s law that is used to describe the multiphase flow through a porous media; and in particular, a petroleum reservoir. This manuscript clearly brings out the basics associated with the usage of Darcy’s law, and reasons out the inapplicability of the Navier-Stokes Equation in order to describe the momentum conservation in a typical petroleum reservoir. Further, this work highlights the essence of continuum-based Darcy’s macrosopic-scale equation with that of Navier-Stokes’s microscopic-scale equation. Further, the absence of capillary forces in original Darcy’s equation and extending the same by considering the concept of ‘capillary pressure’ in order to accommodate the multi-phase flow has several critical constraints associated with it. In this manuscript, all these constraints or limitations have been posed in the form of a list of basic queries that need to be addressed or at least to be understood with clarity, when applying the multi-phase fluid flow equations associated with a petroleum reservoir. This study is limited to an oil-water two-phase system.

**KEYWORDS**

Hydrocarbon reservoir, oil-water system, capillary pressure, reservoir up-scaling, Darcy’s equation.

1. INTRODUCTION

Fluid flow through a porous medium is of great significance in various applications that includes groundwater flow; contaminant hydrogeology; enhanced geothermal energy system; enhanced oil recovery; CO2 sequestration; mass transport in vadose zone; flow through shale gas reservoirs; flow through tight gas reservoirs, and onshore oil spill (Nitha et al., 2018; Nitha et al., 2019; Kumar and Rakesh, 2018; Kumar 2019a, 2019b; Manojkumar and Kumar, 2020; Kumar and Sekhar, 2005; Kumar et al., 2006; Sekhar and Kumar, 2006; Sekhar et al., 2006; Kumar, 2008; Kumar et al., 2008; Kumar, 2009; Natarajan and Kumar, 2010; Natarajan and Kumar, 2011a; Natarajan and Kumar, 2012a, 2012b; Renu and Kumar, 2012; Kumar, 2014a, 2014b; Natarajan and Kumar, 2014a, 2014b; Renu and Kumar, 2014; Nihkl and Kumar, 2015a, 2015b; Nihkl and Kumar, 2015; Kumar and Rakesh, 2015; Kumar et al., 2016; Renu and Kumar, 2016a, 2016b; Kumar, 2016; Rakesh and Kumar, 2016; Nihkl and Kumar, 2016; Natarajan and Kumar, 2016a, 2016b; Abhishek et al., 2016; Nihkl and Kumar, 2017; Renu and Kumar, 2017a, 2017b; Natarajan and Kumar, 2018; Bagalkot et al., 2018a, 2018b; Naga babu et al., 2018; Kumar and Ghassemi, 2005; Kumar and Ghassemi, 2006; Ghassemi and Kumar, 2007; Reddy and Kumar, 2014; Sharma et al., 2014a, 2014b; Sivasanker et al., 2014; Kannu et al., 2014; Reddy and Kumar, 2015a, 2015b; Sharma et al., 2015a, 2015b; 2015c; Abhishek et al., 2015; Kidambi and Kumar, 2016; Sivasanker et al., 2016; Kumar and Reddy, 2017; Sivasanker and Kumar 2017a, 2017b; Sivasanker and Kumar, 2018, 2019; Vivek and Kumar, 2016; Vivek et al., 2017, 2019; Vivek and Kumar, 2020; Mohanasundaram et al., 2013a, 2013b; Berlin et al., 2013; Berlin et al., 2014a, 2014b; Berlin et al., 2015a, 2015b, 2015c; Omkar et al., 2016a, 2016b; Mohanasundaram et al., 2017; Berlin et al., 2018a, 2018b; Omkar et al., 2019a, 2019b; Berlin and Kumar, 2019; Mohanasundaram et al., 2019, 2020; Patwardhan et al., 2014; Patwardhan et al., 2015; Patwardhan et al., 2016; Patwardhan et al., 2017a, 2017b; Kidambi et al., 2017; Vasudevan et al., 2014a, 2014b; Vasudevan et al., 2015; Vasudevan et al., 2016a, 2016b, 2016c; Vasudevan et al., 2017; Renu and Kumar 2018a, 2018b, 2018c; Rajasekhar et al., 2018; Renu and Kumar, 2019).

"Fluid flow through a porous medium" is fundamentally different from the concept of ‘fluid flow through pipes’ in the sense that the flow through a porous medium is associated with a hydraulic head with an insignificant kinetic head. All the kinetic head in a porous medium gets dissipated as it passes through the complex network of solid grains. The presence of these solid grains or solid phase within the flow regime is fundamental to the concept of ‘porosity’ that makes distinct from the concept of ‘fluid flow through pipes’. Since, these solid grains do not form a regular solid boundary (unlike as seen in a fluid flow through pipes with a geometrically well-defined and mathematically well-placed solid-boundary along the
pipe-walls), deducing a continuous and a closed-form (solid) boundary condition at the microscopic-scale remains nearly impossible in order to describe the fluid flow through a porous medium at the macroscopic-scale, despite the fluid remaining continuous at the microscopic-scale itself.

Thus, the very concept of Representative Elementary Volume (REV) in a porous media involves a bunch of both the fundamental entities namely ‘solid-grains’ as well as the ‘pore-spaces’, while the REV associated with ‘fluid flow through pipes’ has only a cluster of fluid molecules called the ‘fluid particle’ in the absence of any solid-grains. And, this is the reason why the Navier-Stokes Equation (the momentum conservation equation) associated with the fluid flow through pipes remained isolated and unused from the applications relating to ‘fluid flow through a porous medium’.

And, Darcy in 1856 provided us an equivalent momentum conservation equation that describes fluid flow through a porous medium, not at the macroscopic-scale, but at the macroscopic-scale. Thus, the Darcy-based single-phase fluid flow equation through a porous medium is associated with the macroscopic-scale. In addition, it should be noted that Darcy’s equation is an empirical relation; and it has not been deduced based on classical (fundamental) laws, unlike an NSE.

However, the same Darcy’s law was later extended in order to accommodate the multi-phase fluid flow through the same porous medium. While describing multi-phase fluid flow, the concept of single-phase ‘absolute permeability’ gets replaced with the multi-phase ‘effective permeability’. In addition, in the case of multi-phase fluid flow, the concept of ‘capillary pressure’ comes into the picture, which is the difference between the wetting and non-wetting phase pressures. It should be noted that Darcy deduced his empirical relation that describes the one-dimensional fluid flow through a porous medium in the absence of both capillary and gravity forces. However, for the multi-phase fluid flow, the capillary force is the fundamental phenomenon, while gravity plays a sensitive role. In essence, the extended law for multi-phase fluid flow has several limitations; and in fact, they violate very severely from the original Darcy’s law in some instances. Thus, the objective of the present paper is to bring out a list of critical queries, which need to be addressed, when applying the extended form of Darcy’s law for describing the multi-phase fluid flow in a petroleum reservoir.

This work is primarily focused on addressing the various fundamental queries in the context of Petroleum Engineering; and thus, the queries posed are directly pertaining to a petroleum reservoir, where the migration of oil and water takes place simultaneously; and hence, limited to an oil-water system. It should be clearly noted that the queries posed in this manuscript will make the reservoir or petroleum engineers to better understand the fundamentals of fluid flow through a petroleum reservoir; and subsequently, these queries will help the petroleum engineers to discuss the applications such as enhanced oil recovery with greater care. This manuscript clearly brings out the limitations associated with the usage of data deduced directly from the laboratory-scale measurements, and which are applied as such in the macroscopic Partial Differential Equations in the absence of integrating those measurements over the REV meant for the concerned petroleum reservoir. It is believed by the author that the queries raised in this manuscript will particularly help the research scholars who have just commenced their research work related to the characterization of multi-phase fluid flow through a petroleum reservoir.

In particular, the basic concepts that need to be understood during a water-flooding and during the application of enhanced oil recovery (particularly, pore-scale displacement efficiency and its relevance with the macroscopic Darcy’s law) have been discussed in detail. More importance has been on the usage of measurements taken from the laboratory-scale (such as capillary pressure, interfacial tension, wettability through contact angle, oil viscosity, water viscosity, oil compressibility, and water compressibility); and applying the same directly in the macroscopic extended form of Darcy’s law that describes the multi-phase fluid flow through a porous medium. Since commercial software packages are widely used by the petroleum industry, the fundamental queries posed in this manuscript are expected to be very useful for both the research scholars as well as the field-engineers who just have begun their work on multi-phase fluid flow through a petroleum reservoir.

2. METHODS: MATHEMATICAL MODEL

The conventional fluid flow equations associated with a saturated subsurface system are based on mass, momentum, and energy conservation equations. By assuming the physical system to be under isothermal conditions, only mass and momentum conservation equations are focused on ignoring the energy conservation equation. Regarding the mass conservation equation, it represents the difference between the fluid mass fluxes that enter and leave a particular element (within the reservoir); and this difference is related to the time rate of change of fluid mass within the same element.

Since the physical system is a porous medium, the volume of the fluid mass pertains only to the pore-volume and not to the bulk volume; and hence, the parameter ‘porosity’ comes into the picture. It should be clearly noted that the difference between the fluid mass fluxes that enter and leave the element takes the differential form under the assumption that the cell width Δx→0 along the one-dimensional horizontal flow direction. In other words, the conversion of ‘fluid mass flux’ into ‘mass flow rate’ is feasible only when, Δx→0. Thus, the mass conservation equation that describes fluid flow through a porous medium is written as given in Eqn. (1).

\[ \frac{\partial}{\partial x} (\rho A u) = \frac{\partial}{\partial t} (\rho \phi) \]

In Eqn. (1), it should be clearly noted that all the four parameters need to be differentiated either as a function of space (x) or time (t). In other words, all these four parameters (density, velocity, porosity, and area) remain as a function of independent variables, either ‘x’ or ‘t’ or both. Thus, the original mass conservation equation neither allows ‘larger cell widths’ nor the ‘non-uniform flow’ (fluid flow through varying cross-sectional area). In the radial coordinate system, the cross-sectional area keeps reducing (varying) as we proceed towards the center of the well; and thus, the fluid flow associated with the radial system is non-uniform fluid flow; and the original mass conservation does not support such flow.

However, the area remains as a constant in a Cartesian coordinate during the fluid flow between injection and production wells; and thus, the parameter ‘A’ is taken out of the derivative function from Eqn. (1); and it is considered as a constant-coefficient as given in Eqns. (2). In addition, in the radial coordinate system, as we approach the center of the well, since, the cross-sectional area gets reduced, the fluid velocity gets increased accordingly in order to maintain the constant fluid flow rate. Thus, there is a possibility that the fluid flow regime in the vicinity of the well may have inertial effects also, despite the flow being laminar; and Darcy’s law does not support such flows with inertial effects. Because Darcy’s law is based on the assumption that the hydraulic head results only from the combination of datum (or potential) head and pressure head with an insignificant kinetic or velocity head.

\[ \frac{\partial}{\partial x} (\rho u v) = \frac{\partial}{\partial t} (\rho \phi) \]

Since Darcy’s law provides a relation for Darcy-velocity (Darcy flux) as a linear function of the pressure gradient (with the constant of proportionality being equal to the ‘mobility’) as given in Eqns. (3), the velocity term in Eqn. (2) maybe replaced with Darcy’s equation. Darcy’s equation represents the equivalent momentum conservation equation for describing fluid flow through a porous media. It should be noted that Darcy’s law has only ‘permeability’ and not ‘porosity’ as Darcy’s law represents the volumetric fluid flow through the entire cross-sectional area. However, the actual fluid flow occurs only through the hydraulically connected pore spaces; and hence, the concept of ‘effective porosity’ may be brought, in order to convert, the Darcy velocity into the ‘mean fluid velocity’ (Darcy velocity/Effective porosity).

\[ q_{\text{Darcy}} = \frac{Q}{A} = \frac{q}{\mu} = \frac{k}{\mu} \frac{\rho}{\mu} = \frac{k}{\mu} \nabla p \]

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The fluid velocity ‘u’ found in Eqn. (2) can thus be related as a function of (average reservoir) ‘pressure’ as given in Eqn. (3). Since ‘u’ has been related with ‘pressure’, the other two parameters namely density and porosity are to be related with the same ‘pressure’ so that the resulting equation will have ‘pressure’ as the primary dependent variable. For this purpose, we bring in the concept of ‘compressibility’ into the picture. So, fluid density is expressed as a function of pressure as in Eqn. (4); and subsequently, by rearranging, we get Eqn. (5). Similarly, rock porosity is expressed as a function of ‘pressure’ as given in Eqn. (6); and we get Eqn. (7) by rearranging Eqn. (6).

\[
c_r = \frac{1}{V} \left( \frac{dV}{dp} \right)_r = \frac{1}{\rho} \left( \frac{dp}{d\phi} \right)_r
\]

From Eqn. (4), it can be seen that ‘fluid density’ is a function of pressure \( \rho = \rho(p) \).

\[
\left( \frac{d\phi}{dp} \right)_r = c_r \phi
\]

From Eqn. (6), it can be seen that the ‘rock porosity’ is a function of pressure \( \phi = \phi(p) \).

\[
\frac{d\phi}{dp} = c_r \phi
\]

From Eqn. (8), it is interesting to note that ‘porosity’ is a function of time, while ‘permeability’ is time-independent. In any porous medium, porosity and permeability are highly interrelated; and it is the ‘hydraulic connectivity’ that decides the resultant ‘permeability’.

Now, applying chain rule on the RHS of Eqn. (8), and making use of the relations in Eqs. (5) and (7) yields Eqn. (9).

\[
\frac{\partial \rho}{\partial t} = \frac{k}{\phi (c_r + c_j)} \frac{\partial p}{\partial x}
\]

In Eqn. (9), the summation of ‘rock-compressibility’ \( (c_r) \) and ‘fluid-compressibility’ \( (c_j) \) correspond to the ‘total compressibility’ \( (c_s) \), and the coefficient attached with the second-order spatial derivative of pressure \( \left( \frac{k}{\phi (c_r + c_j)} \right) \) is called ‘hydraulic diffusivity’ (provides the details on the speed with which the diffusion of the ‘pressure pulse’ that takes place within the reservoir).

Eqn. (9) represents the single-phase, single-continuum, one-dimensional fluid flow equation along the horizontal direction in a Cartesian coordinate system. Eqn. (9) has been deduced by substituting an equivalent momentum conservation equation (Darcy’s equation) in the mass conservation equation meant for a porous medium. Eqn. (9) provides the spatial and temporal distribution of average reservoir pressure all the coefficients associated with Eqn. (9) act as a (scalar) constant, which remains independent of both dependent as well as independent variables. Eqn. (9) is a simple linear Partial Differential Equation (PDE) of parabolic nature. A parabolic PDE tends to reach the (asymptotic) steady-state condition after a very long time, following the initial disturbances or perturbations or noises.

Thus, solving a parabolic PDE will be much simpler with reference to solving a hyperbolic dominant PDE. Eqn. (9) requires two boundary conditions as the equation has the second-order spatial derivative (highest order associated with the spatial derivative), while the same equation requires only one initial condition as the equation has the first-order temporal derivative (highest order associated with the temporal derivative). Further, it is interesting to note that from Eqn. (6), porosity is a function of ‘pressure’, while the same ‘porosity’ becomes a constant in Eqn. (9). In addition, from Eqns. (4) and (6), it can be seen that the fluid and rock compressibilities independently depend on fluid pressure, while from Eqn. (9), it is observed that the same rock and fluid compressibilities remain independent of pressure. It is also interesting to note that the famous ‘diffusivity equation’ in the context of ‘petroleum engineering’ (as seen in Eqn. [9]) has been deduced by expanding only the RHS of Eqn. (2), while on the LHS of Eqn. (2), ‘density’ remains untouched, and Darcy’s relation has been substituted for ‘velocity’ in the absence of expanding the LHS.

Equation (9) has been deduced based on the expansion of RHS in Eqn. (2) in the absence of expanding the LHS component. Now, expansion of LHS in Eqn. (2) yields the following:

\[
\frac{\partial (\rho u)}{\partial t} = -\rho \frac{\partial (\phi \rho)}{\partial x} - \mu \frac{\partial (\rho \phi \rho)}{\partial x}
\]

\[
\frac{\partial (\rho \phi)}{\partial t} = -\rho \frac{\partial (k \frac{\partial p}{\partial x})}{\partial x} \frac{\partial (\rho \phi \phi)}{\partial x}
\]

\[
\frac{\partial (\rho \phi \rho)}{\partial t} = \frac{k}{\phi (c_r + c_j)} \frac{\partial \rho \phi \rho}{\partial x}
\]

\[
\frac{\partial \rho}{\partial t} = \frac{k}{\phi (c_r + c_j)} \frac{\partial \rho}{\partial x}
\]

Expansion of RHS of Eqn. (2) yields the following:

\[
\frac{\partial (\rho \phi \rho)}{\partial x} = \rho \frac{\partial \phi}{\partial x} + \phi \frac{\partial \rho}{\partial x}
\]

\[
\frac{\partial (\rho \phi \phi)}{\partial x} = \rho \frac{\partial \phi}{\partial x} \left( \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial t} \right)
\]

\[
\frac{\partial \phi}{\partial t} = \rho \frac{\partial \phi}{\partial x} \left( \phi \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial t} \right)
\]

\[
\frac{\partial \rho}{\partial t} = \rho \frac{\partial \rho}{\partial x} \left( \frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial t} \right)
\]

Substituting Eqs. (14) and (19) in Eqn. (2) yields Eqs. (20) and (21).

\[
\rho \left( k \frac{\partial \rho}{\partial x} + c_r \frac{\partial p}{\partial x} \right) = \rho \phi (c_r + c_j) \frac{\partial \rho}{\partial x}
\]

\[
\left( \frac{k}{\phi (c_r + c_j)} \frac{\partial \rho \phi \rho}{\partial x} + c_r \frac{\partial \rho}{\partial x} \right) = \frac{\partial \rho \phi \rho}{\partial x}
\]

From Eqn. (21), it can be seen that the LHS consists not only of a parabolic nature of PDE term (1st term within the square parenthesis) but a hyperbolic characteristic PDE term (2nd term within the square parenthesis) as well. Also, the hyperbolic nature term is characterized not by a linear variation but by a non-linear quadratic variation. Thus, by expanding the LHS of Eqn. (2), the relatively simple parabolic dominant linear diffusivity equation becomes a relatively complex, coupled parabolic and hyperbolic dominant (sometimes, a pure hyperbolic dominant by suppressing the parabolic term), non-linear diffusivity equation. It will be difficult to numerically solve Eqn. (21), when it is
completely hyperbolic dominant. In general, by assuming the coefficient of the non-linear quadratic pressure gradient term to be insignificant, the non-linear part of the PDE in Eqn. (21) is avoided.

Such assumption holds well, only when, the compressibility of the fluid remains to be insignificant; and when the pressure-gradient associated with the reservoir formation remains smaller than the ‘threshold pressure gradient’ (the minimum pressure gradient required by the fluid for its continuous mobility within the hydraulically connected pores by overcoming the frictional resistive forces acting in the opposite direction of fluid flow; and that either ceases or mitigates the propagation of pressure (drop) pulse, any further).

However, in a producing oil reservoir, both the conditions are not relevant (oil remains compressible, unlike water, while the formation pressure gradient remains very significant during the oil production phase); and subsequently, we cannot afford to lose this non-linear quadratic pressure gradient term. In addition, this non-linear term becomes very sensitive for a reservoir, which is neither homogeneous nor isotropic.

For example, the distribution of the pressure within the reservoir is supposed to be smooth and continuous in the absence of any steep gradient. Also, the pressure gradient will not be smooth but large for low permeable reservoirs with very high injection flow fluid flow rate. A steep pressure gradient resulting from reservoir heterogeneity mathematically would lead to ‘discontinuity’ of the pressure distribution within the reservoir, and such scenarios require the use of special numerical techniques. On top of it, by simply solving the linear diffusivity equation, we will not be able to detect any issues associated with the upscaling of the rock and fluid parameters involved in Eqn. (9). Equation (9) is essentially parabolic in nature; and subsequently, it has the characteristic of suppressing all kinds of errors associated with that equation with a larger time.

In other words, any parabolic equation would tend to achieve a steady-state condition after a long time, despite having any perturbations or noises or fluctuations during its initial period. However, when a PDE is hyperbolically dominant (as against parabolic dominant), it will conserve its properties in the absence of any changes during its simulation period. In other words, any errors associated with a hyperbolic dominant PDE, especially, during its initial stages will not get vanished with time (as observed in a parabolic dominant PDE); but it will get accumulated with time; and hence, solving a hyperbolic dominant problem will have dominant convergence and stability issues. Hence, a hyperbolic dominant PDE will require numerical solution techniques such as finite volume, which are based on mass conservation principles. By using principles based on mass conservation, the issues related with the upscaling can significantly be avoided.

In other words, if there is a significant departure of the rock and fluid properties observed at Darcy’s scale from that of the pore-scale, then, it will get reflected in the accumulation of errors in the fluid mass with time. For the selected numerical scheme to remain stable, the error estimated at the current level should be less than or equal to the error estimated at the previous time level. Thus, if the rock and fluid properties remain nearly the same between the pore-scale and the Darcy-s-scale, then, the accumulated errors will exponentially decline with time; and the generated results will be more reliable. For gas reservoirs, the non-linear quadratic pressure gradient term will remain very sensitive as gas is significantly compressible. It should be noted that the concept of ignoring the non-linear quadratic pressure gradient term will not work well for a multi-phase fluid flow system such as flow-through oil or gas reservoirs as both oil and gas cannot be treated to remain to be incompressible, unlike water. The magnitude of the coefficient of non-linear quadratic pressure gradient term remained insignificant for the applications such as flow through a saturated groundwater system, while the magnitude of the ‘fluid compressibility’ becomes larger when the associated reservoir fluid is either oil or gas (unlike water).

3. DISCUSSION: Basic Queries on Multi-phase Fluid Flow Dynamics

It is well known that the primary forces causing migration of hydrocarbons, in a typical petroleum reservoir include buoyant and capillary forces. If so, whether Darcy’s law can be used to describe fluid flow through a petroleum reservoir in the absence of capillary and gravity forces? Even if both gravity and capillary forces are included in Darcy’s equation in order to describe the multi-phase fluid flow through a petroleum reservoir, why are these two forces are NOT coupled and are treated independently? It should be clearly noted that in an actual reservoir, the gravity forces tend to segregate the immiscible fluids elevation wise (gas/oil at the top with water at the bottom), while the capillary forces tend to transmit the wetting fluid into the pores of the fluid regime with the non-wetting fluid, and thus the capillary forces act against the segregation process (unlike the gravity force); and thus, gravity and capillary forces are strongly coupled with each other.

What will be the typical magnitude of the range of Representative Elementary Volume (REV) associated with an oil-water flow of an Oil Reservoir? In other words, what will be the approximate range of the ‘Characteristic Volume’ (also known either as the Physical point or Material point of the pore-fluid at the given Mathematical point) in terms of either mm3 or number of mean pore-diameters – associated with the typical multi-phase fluid flow? Will it be feasible to delineate the oil-water interface (the required phenomena for estimating the capillary pressure) at the pore-scale?

Unlike a single-phase fluid flow system, where fluid density depends on ‘mean reservoir pressure’ & ‘mean reservoir temperature’; in an oil reservoir with the multi-phase oil-water flow system; the resulting density of oil and water becomes a relatively complex function (apart from its dependence on pressure and temperature), and this function requires to be defined clearly both at the microscopic- and macroscopic continua. For (real) gas reservoirs, the equation of state for fluid density becomes further complex as it requires the details on the specific heat capacity of gas at constant-pressure and constant-volume (from which Universal gas constant per molecular weight of the fluid can be estimated) apart from the details on the gas compressibility factor; and, in such cases, more clarity is required with reference to the scales associated with the measurement of this Equation of State (EoS) parameters with reference to the macroscopic parameters such as gas-pressure and gas-saturation.

In the context of multi-phase fluid flow through a porous medium using continuum mechanics, the continuum concept should satisfy the condition that the perpendicular component of the immiscible fluid velocities on either side of the discontinuous phase should remain continuous. In fact, in a typical water flooding (during secondary oil recovery), both fluid pressure and fluid velocity of the immiscible fluid phases on either side of the flooding front is supposed to remain continuous for the continuum concept to be valid. However, in reality, it remains discontinuous as there is a definite jump in velocity/pressure at the flooding front. With a clear discontinuity at the flooding front, will it be feasible to delineate a reasonable REV in the vicinity of the flooding front that will make the multi-phase fluid flow system to be treated at the continuum scale that includes the location of flooding front interface, where both oil pressure and water pressure can be treated to remain as a continuous phase? It is also known that the discontinuity of the immiscible fluid phase pressures on either side of the flooding front not only depends on the magnitude of the capillary pressure but also depends on the complex hysteresis of the drainage-inhibition capillary pressure curves. If so, will it be feasible to deduce the corresponding REV associated with the flooding-front location so that the concept of ‘continuum’ be ensured?

It is well known that when the dimensionless Knudsen number hangs around unity, then, it pertains to the slip-flow regime, while for the cases with Knudsen number greater than unity, the fluid flow is characterized by ‘Knudsen flow’ or ‘free molecular flow’. However, when the flowing fluid needs to be treated as a continuum, where the macroscopic assumptions remain valid, the respective Knudsen number should remain

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It is well known that the overall oil displacement efficiency associated with a petroleum reservoir not only depends on the larger, macroscopic, volumetric sweep efficiency but also on the smaller, microscopic, pore-scale displacement efficiency. Since macroscopic displacement efficiency is associated with the volume of injected fluids contacting the oil zone; and bulk reservoir volume, both of which are much greater than the REV, macroscopic-based Darcy's law can be applied without any problem. However, while applying microscopic displacement efficiency, utmost care is required as it involves the mobilization of trapped oil by the displacing fluid at the pore-scale; and not at the macroscopic Darcy's scale.

It can be clearly noted that the treatment of oil and water as continua at the pore-scale is reasonable as the fluids are to be treated as a ‘particle’ that corresponds to the clustering of many similar fluid molecules over the REV. Thus, at the pore-scale, the ‘continua’ concept based ‘particle’ size of oil and water will be much larger than the mean free path of the respective individual oil and water molecules, while the same ‘particle’ size will be much smaller than the characteristic length of the flow domain.

However, liquid flow through a porous medium is generally described by Darcy’s law (Equivalent momentum conservation equation). Thus, there is a contradiction between the required 'pore-scale' displacement efficiency; and the mathematically applied, Darcy based fluid flow at the macroscopic-scale. Given this background, how will it be feasible for us to correlate pore-scale phenomena with a Darcy based fluid flow? Otherwise, there should have been a mechanism in order to upscale the pore-scale (displacement efficiency) phenomena to the Darcy based fluid flow. Do we have one? Directly coupling between the increased oil recovery efficiency resulting from the micro-scale (pore-scale displacement efficiency) phenomena with macroscopic-scale Darcy flow deserves special attention.

In a typical chemical EOR process, the fluid-fluid interaction; and the rock-fluid interaction essentially gets modified as the chemicals are injected into the reservoir along with the water slug. As a result, there can also be (a) a change (generally, reduction) in the Inter-Facial-Tension (IFT) between the displacing fluid and displaced fluid; (b) a change in reservoir wettability (that will increase the mobility of the displaced fluid). What are the scales associated with concepts such as ‘IFT’ and ‘Wettability Alteration’? Whether the volumes associated with ‘IFT’ and ‘Wettability Alteration’, in reality, will be really greater than the REV so that the Darcy based fluid flow concept can be applied comfortably? It should be noted that IFT reduction (between oil and water) associated with the surfactant solutions requires the details on interfacial film solubilization as well as its associated emulsification, both of which are associated at a scale much smaller than Darcy-scale.

The reduction in capillary forces resulting from IFT reduction does not ensure that the flow is viscous dominant; and hence, the applicability of Darcy’s law for the fluid regimes with dominant capillary forces (during the application of chemical EOR either using alkali or surfactant) deserves special attention as the concept of 'flow through porous medium based on Darcy's law’ does not accommodate ‘chemical and/or electrochemical surface phenomena’. In addition, since, the foam has the capability of diverting the injected fluid from high permeable thief zones to low permeable zones, again the validity of Darcy’s law in such low permeable zone deserves special attention. It should be clearly noted that the concept of ‘flow through porous media’ represents a geological formation, where the fluid flow takes place through ‘relatively large pores’; and it ‘does not include flow through micro-pores or colloid-sized pores and/or clay’.

It is known that the concept of flow through flow porous medium with the given pore at the pore-scale - would have exceeded the so-called ‘Characteristic Volume’ or ‘REV’? Generally, Capillary Pressure is estimated at the laboratory-scale; and then, this value is converted into an equivalent field-scale value using the average reservoir porosity and permeability values. If so, will it be fair enough to extend the single-phase macroscopic Darcy's law to be applied for characterizing the pore-scale multiphase fluid flow (when a fundamental multi-phase fluid flow parameter like ‘Capillary Pressure’ Can Not even be measured in a real field)?

In the context of REV for a porous medium, it is well known that the size of REV should be much smaller than the size of the entire flow domain, while the same REV should be much greater than the size of a single pore, in order to provide the meaningful average values of the dependent variables over the REV of interest and in turn, the spatial distribution of the considered dependent variables will be smooth and continuous. Since the minimum size of REV is supposed to be larger than the size of a pore, will it be fair enough to look at any associated physics that is less than a pore-size. These measured quantities from the laboratories at a scale either equal to or lesser than the microscopic-scale would remain meaningful only when, all such measurements at these smaller scales have been averaged over the (respective) REV. However, this is not being done in practice due to the associated complexities, and hence, requires further attention.

It is known that the single-phase fluid flow through a porous medium remains continuous even at the microscopic-scale (and we could have directly used Navier-Stokes Equation [NSE] in the absence of any new law such as Darcy’s law in order to describe the fluid flow at the microscopic-scale itself). However, since, the associated boundary conditions emerging from the complex pore geometry are highly tortuous, it is not practically feasible to deduce a mathematical model that will represent the fluid continuum at the microscopic-scale itself. For this purpose, to make use of relatively simpler boundary conditions and (getting rid-off complex boundary conditions), the mathematical model is deduced at the larger macroscopic-scale, where the concept of continuum remains valid. At this macroscopic-scale, a simple linear Darcy’s law is issued by avoiding the non-linear NSE in order to describe the fluid flow through a porous medium at the macroscopic-scale.

However, in the case of multi-phase fluid flow, we have an additional boundary emerging from the immiscible fluid-fluid interface (unlike the conventional solid-fluid interface, where the concept of vanishing fluid velocity can be applied as the boundary condition) within the pore-space at the pore-scale. What happens to this immiscible fluid-fluid interface boundary at the larger macroscopic-scale, where the Darcy-based continuum concept is applied? In other words, how does the additional ‘transfer coefficient’ namely ‘relative permeability’ treat the ‘transfer phenomena of the immiscible fluid-fluid interface, either from molecular-scale or from microscopic-scale to the Darcy-based continuum-scale? Also, will it be feasible to define any physical, chemical, or biological properties of the multi-phase fluids – exactly over the immiscible fluid-fluid interface – that is applicable at the Darcy-based continuum-scale?

The concept of 'Capillary Pressure’ associated with multi-phase fluid flow in a porous medium represents the pressure difference between the immiscible fluids, i.e., the pressure difference between the wetting and non-wetting phase pore fluids. If such a scenario exists at the Darcy's-scale; then, where exactly the pressure of the wetting-phase and non-wetting phase fluids is supposed to be measured in a real field scenario? Are they NOT supposed to be measured along either side of the (adjoint) oil-water interface? If so, whether the volume occupied by the oil (where the oil pressure is supposed to be measured); and the volume occupied by the water (where the water pressure is supposed to be measured) – in a...
The Darcian approach is strictly valid for Newtonian fluids only whose viscosity remains independent of shear rate. It is also known that the polymer solutions experience a variety of shear rates as a complex function of fluid flow rate, pore size distribution, tortuosity, cross-linked / curvy-linear interconnectivity between pores, directional permeability; and non-uniform fluid flow resulting from varying cross-sectional area as a function of space (that significantly influencing the yield stress and extensional viscosity) – characteristic of non-Newtonian fluids (and thereby, making the in-situ rheology to be more complex than the bulk rheology).

Using such non-Newtonian fluids within a petroleum reservoir for describing (horizontal) fluid flow through a porous medium (between injection and production wells excluding the vertical flow in the wellbores) essentially violates the basic assumptions associated with the Darcian approach. In other words, the reservoir fluids either with pseudo-plastic behavior (shear dominant) or the dilatant behavior (extensional dominant) involving a shear rate of greater than 1 per second should never be included under Darcian approach. Characterizing ‘viscous fingering’ using non-Newtonian polymer solutions should be dealt with separately in the absence of coupling it with the Darcian approach.

The set of partial differential equations associated with the oil-phase and water-phase mass conservation equations have certain coefficients associated with each (spatial/temporal, first-order/second-order derivative) term. These coefficients represent the macroscopic medium parameters that have been deduced based on the phenomena observed at the microscopic-scale. It should be clearly noted that such parameters representative of the macroscopic-scale has been introduced as it is nearly impossible for us to deduce the same at the microscopic-scale. In essence, the value of these coefficients is supposed to be deduced from experiments. However, there is no clarity on the up-scaling of such measurements from its microscopic-scale to its existence at the macroscopic-scale.

In the strict sense, applying the values from the laboratory measurements either at microscopic-scale or at a scale lesser than macroscopic-scale, and then, directly substituting these values in the Darcy-based macroscopic-scale equations would really not represent the continuum formulation. This conflict arises from the fact that few parameters from the laboratory have been measured at a scale that is either equal to or lesser than the microscopic-scale, while it is not possible to get the value of the dependent variable (say, pressure and saturation) at the microscopic-scale. Thus, in the given Partial Differential Equations (pertaining to the mass conservation equations for oil and water phases), the coefficients have been accommodated either at the microscopic-scale or at the scale lesser than the microscopic-scale, while the spatial distribution of (dependent) variables are associated with the macroscopic-scale. This mismatch would make the resulting numerical solutions to have ‘no’ practical value. Mathematically, ‘pressure’ or ‘piezometric-head’ are no more potential functions for a reservoir with any finite degree of anisotropy, despite the reservoir being homogeneous and in turn, clearly projecting the gap that needs to be filled between the laboratory measurements taken at the microscopic-scale and applying the same value at the macroscopic-scale.

It is known that the fluids deform continuously under the application of shear stress, however small the shear stress maybe, unlike the time-independent deformation associated with the solid materials, while viscosity represents the property of a fluid, by virtue of which, the concerned fluid resists any kind of deformation; and thus, ‘fluid flow’ & ‘viscosity’ are exactly opposite in nature. However, in a multi-phase fluid flow system, say, in an oil-water system, how exactly the viscosity of oil (the measure of the reluctance of oil to yield to shear stress, when the oil-water system is in motion); and the viscosity of water (the measure of the reluctance of water to yield to shear stress, when the oil-water system is in motion) are estimated, when, more than one fluid is moving simultaneously? It is to be noted that these values would decide the resulting mobility (representing the ratio of the effective phase permeability to phase viscosity) of oil and water.

In addition, both the values of oil viscosity and water viscosity are required for the estimation of the ‘shock front mobility ratio’ associated with the characterization of the stability of the Buckley Leverett displacement. Also, it is not easy to deduce the values of shear stresses exerted along the flow direction (say, along the positive x-direction) on both oil and water surfaces explicitly, whose outer normal remains perpendicular to the flow direction. In other words, for the case of a single-phase fluid flow, it is possible to define the fluid shear stress acting along the flow direction at a particular point (resulting from its respective outer normal). However, multi-phase fluid flow through a porous medium has essentially two different immiscible layers of fluids flowing parallel and simultaneously within the pore-spaces. Thus, the point shear stress, in such cases, may either pertain to the oil phase or to the water phase depending on its distance from its outer normal. So, the distribution of ‘velocity gradients for oil and water phases’ \( \frac{\partial u_o}{\partial r} \) \( \frac{\partial u_w}{\partial r} \) will be significantly different (as the shear force per unit area for oil-phase will be different from that for the water phase; and in addition, they may also not be directly proportional to the local oil and water velocity gradients) for a multi-phase fluid flow system with reference from the conventional single-phase fluid flow system \( \frac{\partial u_o}{\partial r} \) \( \frac{\partial u_w}{\partial r} \).

The migration of oil and water within the pores should not lead to tortuous paths, and the streamlines of oil and water phases are supposed to be parallel and horizontal as the shear stresses are exerted along the flow direction (x-axis) either on an oil surface or water surface having a ‘constant thickness’ in its normal direction (y-axis) with the region of the ‘constant thickness’ occupied either by ‘water’ for water-surface or by ‘water and oil’ for oil surface. Thus, we will have two different flow of x-momentum across any plane of ‘constant thickness’ in its normal direction (y-direction): one within the water-phase and the other within the oil phase for an oil-water system. Further, it should also be noted that in an oil reservoir under isothermal conditions, the in-situ viscosity of oil either drastically gets reduced (below the bubble point pressure) or marginally gets increased (above the bubble point pressure) with increasing reservoir pressure, while the viscosity of water only decreases with increasing pressure. All the above aspects make the plot of shear stress as a function of shear strain rate to remain more complex for a multi-phase fluid flow system.

It is known that the compressibility of fluid under isothermal condition represents the measure of either volume or density changes in response to the changes in their respective normal pressures (or tensions)

\[ \beta = \frac{1}{V} \left( \frac{DV}{DP} \right) . \]

However, in the case of a multi-phase fluid flow system, in the absence of having the explicit details on oil-volume and water-volume with reference to pore-volume and bulk-volume (or oil-density and water-density with reference to solid-grain-density and bulk density); will it be feasible to deduce the values of oil-compressibility or water-compressibility at the macroscopic-scale, and in turn, at Darcy-based macroscopic-scale? Further, for an oil-water system, the fluid density of interest cannot be considered to remain as a constant (due to the presence of varying saturations of oil and water within the pores); and subsequently, such oil reservoirs cannot be treated to be characterized by the presence of ‘homogenous fluids’. The problem becomes further complex during water flooding. During water flooding, the mass of oil and water do not remain as a constant; and subsequently, the ‘material derivative \( \frac{DV}{DP} \) [and not the local derivative \( \frac{\partial V}{\partial P} \)] associated with the expression of the ‘fluid compressibility’ cannot be defined with ease; and in turn, estimating the oil or gas compressibility for an oil-water system is not straightforward.

For single-phase fluid flow, when the difference between the actual and reference reservoir pressures \( (p-p_o) \) is large; and when the fluid compressibility \( \beta \) remains independent of pressure; then, the respective volume of fluid can be expressed as

\[ V_o = V_o \left[ (p - p_o) \right]^{\beta} . \]
However, when the same concept is extended for multi-phase fluid flow, will it be feasible to deduce the values of (a) oil and water compressibility ($\beta_{o}$ & $\beta_{w}$) and (b) oil and water pressure ($p_{o}$ & $p_{w}$) explicitly in order to estimate $V_{o}$ & $V_{w}$? Having found these values, will it be feasible to connect these individual oil and water pressures with the average reservoir pressure? However, in most cases, the equation of State remains simplified by assuming that the difference between ($p_{o}$-$p_{w}$) is small. Will such an assumption can be applied for a flow system with multi-phase fluid? On top of it, the varying quantities of dissolved gases in oil and water might cause significant deviation from its original assumption.

4. CONCLUSIONS

This work has made an attempt to provide an overview that describes a physical system with multi-phase fluid flow. This work has specifically addressed the multi-phase fluid flow through a petroleum reservoir system. The derivation of the mathematical model that describes the fluid flow through a petroleum reservoir system using the coupled mass and momentum conservation equations has been revisited, and the significance of the non-linear quadratic pressure gradient term has been discussed in the context of reservoir upsampling from pore-scale to Darcy’s scale. The manuscript has also brought out a list of fundamental queries that requires to be understood before applying the extended form of Darcy’s law.

This manuscript clearly brings out the fundamental principles associated with the usage of Darcy’s law at the macroscopic-scale, while projecting the inapplicability of microscopic-scale based Navier-Stokes equation in order to describe the momentum conservation in a typical petroleum reservoir. This study provides the distinction between the momentum conservation equation used to describe fluid flow through a porous medium with that of the fluid flow through pipes; and eventually, to bring out the fundamental differences associated with the microscopic-continuum based NSE, and macroscopic-continuum based Darcy’s equation. This particular work is very critical in the sense that this work clearly conveys that any extension of multi-phase fluid flow from a single-phase Darcy’s law cannot be taken for granted as the original Darcy’s law does not involve the concept of capillary pressure, while the same concept is fundamental to the multi-phase fluid flow problem.

Recommendations for future study:
1. It needs to be investigated on the resultant or average density of the fluid that can be used in the mass conservation equation in order to characterize the multi-phase fluid flow.
2. Future investigations are required on the nature of pressure (positive pressure differential or a capillary pressure) that is being used in estimating Darcy’s flux while describing the multi-phase fluid flow in a petroleum reservoir.
3. Further investigations are required in order to deduce the resultant viscosity and compressibility while estimating the hydraulic diffusivity associated with the multi-phase fluid flow.
4. A detailed investigation is required in order to deduce a correlation between pore-scale capillary pressure and a larger macroscopic-scale positive pressure differential (applicable at Darcy’s scale) associated with a petroleum reservoir.

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