Neutrino Oscillation in Matter and Parameters $s_{13}, \delta_{CP}$

Leonard S. Kisslinger$^a$, Ernest M. Henley$^b$, and Mikkel B. Johnson$^c$

$^a$Department of Physics, Carnegie Mellon University, Pittsburgh, PA 15213
$^b$Department of Physics, University of Washington, Seattle, WA 98195
$^c$Los Alamos National Laboratory, Los Alamos, NM 87545

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Abstract

We estimate the dependence of $\nu_\mu$ to $\nu_e$ conversion on parameters $\theta_{13}$ and $\delta_{CP}$ for several experimental facilities studying neutrino oscillations. We use the S-Matrix theory to estimate $\bar{\nu}_e$ disappearance and compare estimates based on an older theory being used to extract $\theta_{13}$ from the Double Chooz, Daya Bay, and RENO data, to assist in extracting an accurate value for $\theta_{13}$ from these projects. We use values of $\theta_{13}$ within known limits, and estimate the dependence of $\nu_\mu - \nu_e$ CP violation (CPV) probability on $\delta_{CP}$ in order to suggest new experiments to measure CPV for neutrinos moving in matter.

1 Introduction

In our present work we study $\nu_\mu$ to $\nu_e$ neutrino conversion, $\bar{\nu}_e$ disappearance, and CP violation (CPV) measurements using the S-Matrix method for neutrino oscillations. The study of CP violation is essential for understanding weak interactions. Almost half a century ago CP violation in weak interactions was found in the decay of $K^0_L$ into $\pi^+ + \pi^-$ [1] and $2\pi^0$ [2], with branching ratios of the order of .001. The decay $K^0_L \rightarrow \pi^0 + \nu + \bar{\nu}$ is almost entirely CP violating [3] but requires accurate determination of the CKM matrix [4] and
accurate measurements. See Ref[5] for a review of this experiment and references. There have many other studies of CP asymmetries in weak decays: see Ref[6] for a recent study of $\bar{B}$ radiative decay with references to earlier work on CP violation in various weak decays.

In recent years there have been a number of experimental studies of neutrino oscillations using neutrino beams from accelerators and reactors, and important objectives of these experiments are the measurements of $\nu_\mu$ to $\nu_e$ conversion and CPV. The first study in our present work is an estimate the $\nu_\mu$ to $\nu_e$ conversion probability using parameters for the baseline and energy corresponding to MiniBooNE[7], JHF-Kamioka [8], MINOS[9], and CHOOZ[10]-Double Chooz[11], which are on-going projects, although the CHOOZ project does not have a beam of $\nu_\mu$ neutrinos.

There have been many recent studies of CP and T symmetries via neutrino oscillations for future facilities, e.g., see Refs[12, 13], which also give references to earlier publications, and the ISS report[14] on future neutrino facilities. The two main parameters of interest in the present work are $\delta_{CP}$, which is essentially unknown, and the angle $\theta_{13}$. One possible future facility for studying CPV and the $\delta_{CP}$ parameter is the LBNE Project, where neutrino beams produced at Fermilab would have a baseline of $L \simeq 1200$ km, being detected with deep underground detectors[15, 16]. With the methods used in the present work, described below, predictions of CPV with the baseline and energies of the LBNE Project have recently been made for $\delta_{CP}$ from 90 to 0 degrees[17].

Although the angle $\theta_{13}$ is not well known, there are limits on its value. Recently the T2K collaboration [18] published limits, and found a best-fit value of $\theta_{13} \simeq 11$ degrees, or $\sin\theta_{13} \equiv s_{13} \simeq 0.19$, which is one of the values we use in the present work on neutrino transition probability in Sec. 2.

The angle $\theta_{13}$ will be measured by the Daya Bay experiment[19, 20] in China, the Double Chooz project[21, 22] in France, and RENO[23] in Korea, via $\bar{\nu}_e$ disappearance. A very recent result from the Daya Bay project[24] concludes that $s_{13} \simeq 0.15$, and in our study (Sec. 3) of $\bar{\nu}_e$ disappearance we use the the Daya Bay parameters to test the theory. Our results will also be useful for the Double Chooz and RENO projects.

Finally, in Sec 4, using the expected range of values for $\theta_{13}$, we estimate CPV for $\mu - e$ neutrino oscillation for the entire range of $\delta_{CP}$ to help in the planning for future CPV experiments.

A major complication for the determination of T, CP, and CPT violation is the interaction of neutrinos with matter as they travel along the baseline. These matter effects have been studied by a number of theorists. See, e.g., Refs[25, 26, 27]. One main objective of the present research is to estimate matter effects for the MiniBooNE, JHF-Kamioka, MINOS, and CHOOZ facilities, to help find the values of $\theta_{13}$ and $\delta_{CP}$.
For the basic interactions, which are CPT invariant for local theories, CP and T violation have the same magnitude. With matter effects T and CP are not directly related. Our present research is an extension of our recent work on T reversal violation\cite{28}. In that study we used the formalism of Ref\cite{29} for $\nu_e \leftrightarrow \nu_\mu$ TRV, and that of Ref\cite{30} for $\nu_e \rightarrow \nu_\mu$ conversion probability to calculate the effects of neutrinos moving through matter. In the present work we use the notation and formalism of Jacobson and Ohlsson\cite{31}, who studied possible matter effects for CPT violation.

CP violation in the $a-b$ sector is given by the transition probability, denoted by $P(\nu_a \rightarrow \nu_b)$, for a neutrino of flavor $a$ to convert to a neutrino of flavor $b$; and similarly for antineutrinos $\bar{\nu}_a, \bar{\nu}_b$. The CPV probability differences are defined as

$$\Delta P_{ab}^{CP} = P(\nu_a \rightarrow \nu_b) - P(\bar{\nu}_a \rightarrow \bar{\nu}_b). \tag{1}$$

In our present work we study $P(\nu_\mu \rightarrow \nu_e)$ and $P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ since the neutrino beams at MiniBooNE, JHF-Kamioka, and MINOS, are muon or anti-muon neutrinos. We then calculate $\bar{\nu}_e$ disappearance probability for Daya Bay baseline and energy, comparing our S-Matrix theory to the formula used by Daya Bay, Double Chooz and RENO\cite{19, 20, 21, 22}.

2 Transition Probability $P(\nu_\mu \rightarrow \nu_e)$ and $\bar{\nu}_e$ Disappearance

In this section we review $\nu_\mu$ to $\nu_e$ oscillation probability derived from standard S-matrix theory and then compare the probability of $\bar{\nu}_e$ disappearance derived from this theory to that used by the Double Chooz experimental project\cite{21, 22}.

2.1 $P(\nu_\mu \rightarrow \nu_e)$ Derived Using S-Matrix Theory

In this subsection we review the derivation of the probability of a muon neutrino to convert to an electron neutrino, $P(\nu_\mu \rightarrow \nu_e)$, using the notation of Ref\cite{31}. We then make an estimate of the transition probabilities for sample accelerator and reactor experiments. Although at the present time no experiments for CPV are possible, this can serve as a basis for future experiments. In the next section we give somewhat more accurate calculations for CPV for the same set of experimental facilities.

As in Refs\cite{29, 31} we use the time evolution matrix, $S(t, t_0)$ to derive the transition probabilities. For neutrino oscillations the initial neutrino beam is emitted at time $t_0$, usually taken as $t_0 = 0$, and the neutrino or converted neutrino is detected at baseline length $= L$ at time $= t$. Since the
neutrinos move with a velocity near that of the speed of light, at the end of our derivation we take $t - t_0 \to L$, with the units $c=1$.

Given the Hamiltonian, $H(t)$, for neutrinos, the neutrino state at time $t$ is obtained from the state at time $t_0$ from the matrix, $S(t, t_0)$, by

$$|\nu(t)\rangle = S(t, t_0)|\nu(t_0)\rangle$$

and

$$\frac{d}{dt}S(t, t_0) = H(t)S(t, t_0).$$

Neutrinos (and antineutrinos) are produced as $\nu_a$, where $a$ is the flavor, $a = e, \mu, \tau$. However, neutrinos of definite masses are $\nu_\alpha$, with $\alpha = 1, 2, 3$.

The two forms are connected by a 3 by 3 unitary transformation matrix, $U$: $\nu_a = U\nu_\alpha$, where $\nu_a, \nu_\alpha$ are 3x1 column vectors and $U$ is given by ($\sin \theta_{ij} \equiv s_{ij}$)

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta_{CP}} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta_{CP}} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}s_{23}s_{13}e^{i\delta_{CP}} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta_{CP}} & c_{23}c_{13} \end{pmatrix},$$

similar to the CKM matrix for quarks. We use the best fit value\[12\] $s_{23} = 0.707$. $\theta_{13}$ and $s_{12} = 0.56$, $c_{12} = 0.83$. We use $s_{13} = 0.19$ and $s_{13}$=0.095, as discussed above, to determine the dependence of $\nu_\mu \to \nu_e$ conversion, and CPV on this parameter. We calculate the dependence of $P(\nu_\mu \to \nu_e)$ and $\Delta P^\mu_e$ on $\delta_{CP}$, as discussed below. In the vacuum the $S(t, t_0)$ is obtained from

$$S_{ab}(t, t_0) = \sum_{j=1}^{3} U_{aj}e^{iE_j(t-t_0)}U_{bj}^*.$$  

Since neutrino beams in neutrino oscillation experiments travel through matter, and the main neutrino-matter is scattering from electrons, we must include potential, $V = \sqrt{2}G_F n_e$, for neutrino electron scattering in the earth: where $G_F$ is the universal weak interaction Fermi constant, and $n_e$ is the density of electrons in matter. Using the matter density $\rho=3$ gm/cc, the neutrino-matter potential is $V = 1.13 \times 10^{-13}$ eV.

The transition probability $P(\nu_\mu \to \nu_e)$ is obtained from $S_{12}$:

$$P(\nu_\mu \to \nu_e) = |S_{12}|^2 = Re[S_{12}]^2 + Im[S_{12}]^2,$$

with
\[
S_{12} = c_{23}\beta - is_{23}ae^{-i\delta_{CP}}A
\]
\[
a = s_{13}(\Delta - s_{12}^2\delta)
\]
\[
\delta = \delta m_{12}^2/(2E)
\]
\[
\Delta = \delta m_{13}^2/(2E)
\]
\[
A \simeq f(t)I_{\alpha}^*
\]
\[
I_{\alpha}^* = \int_0^t dt' \alpha^*(t')f(t')
\]
\[
\alpha(t) = \cos \omega t - i\cos 2\theta \sin \omega t
\]
\[
f(t) = e^{-i\Delta t}
\]
\[
2\omega = \sqrt{\delta^2 + V^2} - 2\delta V \cos(2\theta_{12})
\]
\[
\beta = -isin2\theta \sin \omega L
\]
\[
\bar{\Delta} = \Delta - (V + \delta)/2
\]
\[
sin2\theta = s_{12}c_{12}\frac{\delta}{\omega}
\]

where the neutrino mass differences are \(\delta m_{12}^2 = 7.6 \times 10^{-5}(eV)^2\) and \(\delta m_{13}^2 = 2.4 \times 10^{-3}(eV)^2\). Note that \(\delta \ll \Delta\), and \(t \to L\) for \(v_\nu \simeq c\). From Eqs.(5 to 16):

\[
Re[S_{12}] = s_{23}a[\cos(\bar{\Delta}L + \delta_{CP})Im[I_{\alpha}^*] - \sin(\bar{\Delta}L + \delta_{CP})Re[I_{\alpha}^*]
\]
\[
Im[S_{12}] = -c_{23}sin2\theta \sin \omega L - s_{23}a[\cos(\bar{\Delta}L + \delta_{CP})Re[I_{\alpha}^*]
+\sin(\bar{\Delta}L + \delta_{CP})Im[I_{\alpha}^*]]
\]

Using \(\delta, \omega \ll \Delta\) one can show that
\[
Re[I_{\alpha}^*] \simeq \frac{\sin \bar{\Delta}L}{\Delta}
\]
\[
Im[I_{\alpha}^*] \simeq \frac{1 - \cos \bar{\Delta}L}{\Delta}.
\]

From Eqs(18,19) we find
\[
\mathcal{P}(\nu_\mu \to \nu_e) \simeq (c_{23}s_{12}c_{12}(\delta/\omega)\sin \omega L)^2 + 2(s_{23}s_{13})^2(1 - \cos \bar{\Delta}L)
\]
\[
+2s_{13}s_{12}c_{12}s_{23}^2c_{23}(\delta/\omega)\sin \omega L
\]
\[
(cos(\bar{\Delta}L + \delta_{CP})\sin \bar{\Delta}L + \sin(\bar{\Delta}L + \delta_{CP})(1 - \cos \bar{\Delta}L))
\]

We use \(s_{13}=.19\) and \(.095\) to show the effect of \(s_{13}^2\).
Figure 1: The ordinate is $P(\nu_\mu \rightarrow \nu_e)$ for MINOS ($L=735$ km), MiniBooNE ($L=500$ m), JHF-Kamioka ($L=295$ km), and CHOOZ ($L=1.03$ km). Solid curve for $s_{13}=0.19$ and dashed curve for $s_{13}=0.095$. The curves are almost independent of $\delta_{CP}$. 
From Eq(20) we obtain the results for \( \mathcal{P}(\nu_\mu \rightarrow \nu_e) \) shown in Fig.1. These results can provide guidance for future experiments on CPV via \( \nu_\mu \leftrightarrow \nu_e \) oscillation. Note that in Ref[8] \( \mathcal{P}(\nu_\mu \rightarrow \nu_e) \) was calculated for the 295 km JHF-Kamioka project for \( E=0-2 \) GeV, and our calculation based on the theory developed in Refs.[31, 29], finds \( \mathcal{P}(\nu_\mu \rightarrow \nu_e) \) is in agreement for \( E=0.4-1.0 \) GeV with this earlier estimate. We calculated \( \mathcal{P}(\nu_\mu \rightarrow \nu_e) \) for \( \delta_{CP} \) from \(-\pi/2\) to \( \pi/2\), and the results are almost independent of \( \delta_{CP} \). The results for CHOOZ are shown in preparing for the following subsection on \( \bar{\nu}_e \) disappearance, even though Double Chooz, Daya Bay, and RENO projects have beams of \( \bar{\nu}_e \) rather than \( \nu_\mu \) neutrinos.

### 3 \( \bar{\nu}_e \) Disappearance Derived Using S-Matrix Theory Compared to Daya Bay Evaluation

In this subsection we derive \( \bar{\nu}_e \) disappearance, \( \mathcal{P}(\bar{\nu}_e \rightarrow \bar{\nu}_e) \), defined as

\[
\mathcal{P}(\bar{\nu}_e \rightarrow \bar{\nu}_e) \equiv 1 - \mathcal{P}(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) - \mathcal{P}(\bar{\nu}_e \rightarrow \bar{\nu}_\tau), \tag{21}
\]

using the S-matrix method (see previous subsection), and compare it to the expression for \( \mathcal{P}(\bar{\nu}_e \rightarrow \bar{\nu}_e) \) used by the Daya Bay, Double Chooz, and RENO, which is (see, e.g., Refs[11, 21])

\[
\mathcal{P}^{DB}(\bar{\nu}_e \rightarrow \bar{\nu}_e) \simeq 1 - 4(s_{13}c_{13})^2\sin^2\left(\frac{\Delta L}{2}\right) \tag{22}
\]

where \( \Delta \equiv \delta m^2_{13}/(2E) \), Eq(8), and \( s_{13}, c_{13} = \sin \theta_{13}, \cos \theta_{13} \). A third term with a factor of \( \sin^2(\delta L/2) \)[22] was dropped[11, 21] because \( \delta m^2_{12} \ll \delta m^2_{13} \) and \( \sin^2(\delta L/2) \ll \sin^2(\Delta L/2) \) for Daya Bay baseline \( L=1.9 \) km and energy \( E=4 \) Mev.

In the S-matrix method (see Ref[31]) the probability of \( \bar{\nu}_e \) oscillation to \( \bar{\nu}_\mu \) and \( \bar{\nu}_\tau \) is given by (see, e.g., Ref[31])

\[
\begin{align*}
\mathcal{P}^{SM}(\bar{\nu}_e \rightarrow \bar{\nu}_\mu) &= |\bar{S}_{21}|^2 \\
\mathcal{P}^{SM}(\bar{\nu}_e \rightarrow \bar{\nu}_\tau) &= |\bar{S}_{31}|^2.
\end{align*} \tag{23}
\]

We take \( \delta_{CP} = 0 \), since \( |S_{12}|^2 \) is essentially independent of \( \delta_{CP} \), so \( A = C \) (see Ref[29] for definition and proof). Therefore \( |\bar{S}_{21}|^2 = |S_{12}(V \rightarrow -V)|^2 \), and \( |\bar{S}_{31}|^2 = |S_{12}|^2(V \rightarrow -V, c_{23} \rightarrow s_{23}, s_{23} \rightarrow -c_{23}) \). Using \( s^2_{23} = c^2_{23} = 1/2 \) we find

\[
\mathcal{P}^{SM}(\bar{\nu}_e \rightarrow \bar{\nu}_e) = 1 - \left[ (0.46\delta \sin \omega \bar{L}/\bar{\omega})^2 + 2(s_{13})^2(1 - \cos \bar{\Delta}L) \right], \tag{24}
\]

with \( \bar{\Delta} = \Delta + (V - \delta)/2 \) and \( 2 \bar{\omega} = \sqrt{\bar{\delta}^2 + \bar{V}^2 + 2\bar{\delta} \bar{V} \cos (2\theta_{12})} \)

Fig. 2 for \( L=1.9 \) km and Fig. 3 for \( L=10 \) km are discussed below.
From Fig. 2 the ratios $R1, R2$, of $1 - P(\bar{\nu}_e \rightarrow \nu_e)$ for $P^{DB}$ (Eq. (22)) to $P^{SM}$ (Eq. (24)) for $s_{13} = .15$, and for $s_{13} = .15$ for $P^{DB}$ and $s_{13} = .147$ for $P^{SM}$, for $E \simeq 4.0 \text{MeV}$ and $L=1.9 \text{ km}$ are

\[
R1 = \frac{1 - P^{DB}(s_{13} = .15)}{1 - P^{SM}(s_{13} = .15)} = 1.04
\]

\[
R2 = \frac{1 - P^{DB}(s_{13} = .15)}{1 - P^{SM}(s_{13} = .147)} = 1.00 ,
\]

which demonstrates that using the S-Matrix formulation for $L=1.9 \text{ km}$ and $E \simeq 4.0 \text{ MeV}$ one would extract $s_{13} = .147$ from the data for which the older formalism finds $s_{13} = .15$. This is a 2\% correction. We use the notation $E \simeq 4.0 \text{ MeV}$ as there is uncertainty in the antineutrino energy.

Figure 3: $P(\bar{\nu}_e \rightarrow \nu_e)$. For $s_{13} = .15$ (a) $P^{DB}$ and (b) $P^{SM}$
Fig. 3 is the same as Fig. 2, except the baseline is L=10 km, as future
might use a longer baseline for a larger effect given $s_{13}$. For $E \simeq 4.0$ MeV and
L=10 km the ratios are

$$R1 = \frac{1 - P^{DB}(s_{13} = .15)}{1 - P^{SM}(s_{13} = .15)} = 1.47$$

$$R2 = \frac{1 - P^{DB}(s_{13} = .15)}{1 - P^{SM}(s_{13} = .095)} = 1.00.$$

Thus using the S-Matrix formulation for L=10 km and $E \simeq 4.0$ MeV one
would extract $s_{13} = .095$ from the data for which the older formalism finds
$s_{13} = .15$. This is a 35% correction.

We have carried out similar calculations for the T2K project, with $E=0.6$
GeV, L=295 km[18]. With both a larger L and larger E than Daya Bay, we
find a correction of 2.4%.

It is also important to note that our SM method gives $P(\bar{\nu}_e \rightarrow \bar{\nu}_e) \neq 1.0$
even for $s_{13}=0$.

4 CP Violation $\Delta P_{\mu e}^{CP}$

In this section we shall extend the derivation of the transition probability
$P(\nu_\mu \rightarrow \nu_e)$ of the previous section to derive the CPV probability

$$\Delta P_{\mu e}^{CP} = P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$$

$$= |S_{12}|^2 - |\bar{S}_{12}|^2$$

(27)

with $S_{12}$ defined in Eq(8) and

$$\bar{S}_{12} = c_{23}\bar{\beta} - is_{23}ae^{i\delta_{CP}}A,$$  

(28)

with $\bar{\beta} = \beta(V \rightarrow -V)$ and $\bar{A} = A(V \rightarrow -V)$. For example (see Eqs(16,18))

$$2\varpi = \sqrt{\delta^2 + V^2 + 2\delta V \cos(2\theta_{12})}$$

and $\bar{\Delta} = \Delta + (V - \delta)/2$. Using conservation
of probability[31], $|\bar{A}|^2 = |A|^2$.

$$\Delta P_{\mu e}^{CP} = c_{23}^2(|\beta|^2 - |\bar{\beta}|^2) - 2c_{23}s_{23}ai(Im[\beta e^{i\delta_{CP}} A^*] - Im[\bar{\beta} e^{i\delta_{CP}} \bar{A}^*]).$$

(29)
From Eq(29), the definitions in the previous section, defining $s \equiv \sin(\omega L)$, $c \equiv \cos(\omega L)$ one finds

$$
\Delta P_{\mu e}^{CP} = c_{23}^2 s_{12}^2 c_{12}^2 \left(\frac{s^2}{\omega^2} - \frac{s^2}{\bar{\omega}^2}\right) + 2c_{23}s_{23}s_{12}c_{12}\delta(\Delta - \delta s_{12}^2) \quad (30)
$$

$$(sin\theta_{CP}(\frac{s}{\omega}(c - \cos\Delta L)\frac{\Delta - \omega\cos2\theta}{\Delta^2 - \omega^2} + \frac{s}{\bar{\omega}}(c - \cos\bar{\Delta} L)\frac{\bar{\Delta} - \bar{\omega}\cos2\bar{\theta}}{\bar{\Delta}^2 - \bar{\omega}^2}))

-cos\theta_{CP}(\frac{s}{\omega} sin\Delta L(\bar{\Delta} - \omega\cos2\theta) + sin\omega L(\omega + \bar{\Delta}\cos2\theta)\frac{\Delta - \omega\cos2\theta}{\Delta^2 - \omega^2}) \quad (31)

-\frac{s}{\bar{\omega}} sin\bar{\Delta} L(\bar{\Delta} - \bar{\omega}\cos2\theta) + sin\bar{\omega} L(\bar{\omega} + \bar{\Delta}\cos2\theta)\frac{\bar{\Delta} - \bar{\omega}\cos2\bar{\theta}}{\bar{\Delta}^2 - \bar{\omega}^2}) \quad (32)
$$

The results for $\Delta P_{\mu e}^{CP}$ for $s_{13} = .19$ are shown in Fig.4. Note that $\Delta P_{\mu e}^{CP}$ depends strongly on $\delta_{CP}$, which could lead to a measurement of this parameter. The large value of $\Delta P_{\mu e}^{CP}$ for CHOOZ is promising for future experiments. $\Delta P_{\mu e}^{CP}$ is so small (from about $10^{-10}$ to $10^{-18}$) for MiniBooNE, we do not show the results. Similar results for $\Delta P_{\mu e}^{CP}$ for $s_{13} = .095$ are shown in Fig.5.
Figure 4: The ordinate is $\Delta P(\nu_\mu \rightarrow \nu_e)$ for MINOS (L=735 km), JHF-Kamioka (L=295 km), and CHOOZ (L=1 km). $s_{13}=.19$, and a, b, c, d, e for $\delta_{CP} = \pi/2, \pi/4, 0.0, -\pi/4, -\pi/2$. 
Figure 5: The ordinate is $\Delta P(\nu_\mu \rightarrow \nu_e)$ for MINOS (L=735 km), JHF-Kamioka (L=295 km), and CHOOZ (L=1 km). $s_{13} = 0.095$, and a, b, c, d, e for $\delta_{CP} = \pi/2, \pi/4, 0.0, -\pi/4, -\pi/2$. s13=.095, and a, b, c, d, e for $\delta_{CP} = \pi/2, \pi/4, 0.0, -\pi/4, -\pi/2$. 
5 Conclusions

We have estimated CP violation for a variety of experimental neutrino beam facilities, for values of the parameter $s_{13} = 0.19$ and $0.095$, and for $\delta_{CP}$ from 90 to -90 degrees, since its value is not known. As our results show, the probability $P(\nu_{\mu} \rightarrow \nu_{e})$ is strongly dependent on $s_{13}$ and is essentially independent of $\delta_{CP}$ (see Fig. 1), and therefore the measurement of $P(\nu_{\mu} \rightarrow \nu_{e})$ should determine the value of the $s_{13}$ parameter, as has been known for many years.

Our new results for $\bar{\nu}_{e}$ disappearance, as is being measured the Daya Bay, Double Chooz and RENO projects, however, make use of a different theoretical formulation than that used by these projects to extract $s_{13}$ from the data. We have shown that the recent result from the Daya Bay collaboration[24] with $E \approx 4$ MeV and $L=1.9$ km, from which it was estimated that $s_{13} \approx 0.15$, by our analysis is $s_{13} \approx 0.147$, a 2\% correction. This is small, but the goal of these projects is 1\% accuracy for $s_{13}$. For a baseline of $L=10$ km, with $E \approx 4$ MeV, we find $s_{13} \approx 0.097$ using the S-Matrix method, rather than 0.15, a 35\% correction. Also, our SM method gives $P(\bar{\nu}_{e} \rightarrow \bar{\nu}_{e}) \neq 1.0$ even for $s_{13}=0$.

The CP violation probability (CPV), $\Delta P_{\mu e}^{CP}$, is strongly dependent on both of these important parameters. After the Double Chooz/Daya Bay/RENO determination of $s_{13}$, both the JHF-Kamioka and Double Chooz projects might be able to determine the value of $\delta_{CP}$, since for most of the values of $\delta_{CP}$ these projects would have nearly a 1\% CPV, as shown in Figs. 4 and 5. No experiments are possible now to test CPV via neutrino oscillations, since beams of both neutrino and antineutrino with the same flavor would be needed. However, in the future such beams might be available. Our results should help in planning such future experiments.

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References

[1] J.H. Christenson, J.W. Cronin, V.L. Fitch, and R. Turlay, Phys. Rev. Lett. 13, 138 (1964)

[2] J.W. Cronin, P.F. Kunz, W.S. Risk, and P.C. Wheeler, Phys. Rev. Lett. 18, 25 (1967)

[3] L.S. Littenberg, Phys. Rev. D 39, 3322 (1989)
[4] N. Cabibbo, Phys. Rev. Lett. 10, 531 (1963); M. Kobayashi and T. Maskawa, Prog. Theor. Phys. 49, 652 (1973); L. Wolfenstein, Phys. Rev. Lett. 51, 1945 (1983)

[5] A.J. Buras and S. Uhlig, Rev. Mod. Phys. 90, 965 (2008)

[6] M. Benzke, S.J. Lee, M. Neubert, and G. Paz, Phys. Rev. Lett. 106, 141801 (2011)

[7] The MiniBooNE Collaboration, Phys. Rev. Lett. 105, 181801 (2010)

[8] The JHF-Kamioka neutrino Project, arXiv:hep-ex/106019

[9] The Minos Collaboration, Phys. Rev. Lett. 103, 261802 (2009); Phys. Rev. D 81, 052004 (2010)

[10] CHOOZ Collaboration, M. Apollonio et al, Eur. J. C 27, 331 (2003)

[11] Double Chooz Collaboration, F. Ardellier et. al. hep-ex/0606025 (2006)

[12] H. Davoudiasl, H-S. Lee, and W. J. Marciano, Phys. Rev. D 84, 013009 (2011)

[13] M.C. Gonzalez-Garcia, M. Maltoni, and J. Salvado, arXiv:1103.4365/hep-ph; JHEP 1105: 075 (2011)

[14] The ISS Working Group, arXiv:0710.4947/hep-ph

[15] LBNE Project, lbne.fnal.gov/project/general-info.shtml

[16] V. Barger et al, Report of the US long baseline neutrino experiment study, arXiv:0705.4396/hep-ex

[17] L.S. Kisslinger, arXiv:1108.4062/hep-ph

[18] K. Abe et al, T2K Collaboration, Phys. Rev. Lett 107, 041801 (2011)

[19] Daya Bay Collaboration, arXiv:0701029/hep-ex (2007)

[20] Daya Bay Collaboration, C-J Lin, arXiv:1101.0261/hep-ex (2011)

[21] M. Kuze for Double Chooz Collaboration, arXiv:1109.0074/hep-ex (2011)

[22] P. Novella, arXiv:1105.6079/hep-ex

[23] RENO Collaboreration, J.K. Ahn, et al, arXiv:1003.1391/hep-ex (2010)

[24] Daya Bay Collaboration, http://dayawane.ihep.ac.cn/docs/DYB_{ate}_{rA}PS.pdf
[25] J. Arafune and J. Sato, Phys. Rev. D 55, 1653 (1997) fnal

[26] S.M Bilenky, C. Giunti, and W. Grimus, Phys. Rev. D 58, 033001 (1997)

[27] K. Kimura, A. Takamura, and H. Yokomakura, Phys. Rev. D 66, 073005 (2002)

[28] E.M. Henley, M.B. Johnson, and L.S. Kisslinger, arXiv:hep-ph/1102.5106; Int J. Mod. Phys. E 20, 2463 (2012)

[29] E. Akhmedov, P. Huber, M. Lindner, and T. Ohlsson, Nucl. Phys. B608, 394 (2001)

[30] M. Freund, Phys. Rev. D64, 053003 (2001)

[31] M. Jacobson and T. Ohlsson, Phys. Rev. D 69, 013003 (2004)