Dependence of η-mode and anomalous transport on entropy drift in magnetoplasma

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Abstract. More than 90 years passed when name plasma was coined and more than 60 years in controlling plasma. It seems to be missing something, may be entropy that depends upon maximum constituents of plasma. Inculcating entropy in theoretical and experimental research may lead to the easy confinement. In this paper, Yaqub’s work is extended, transport phenomenon is studied with drift approximation inculcating entropy and η-mode depending upon entropy gradient drift is observed. New dispersion relation is derived by using Branginskii transport equations. Dependence of Anomalous transport on entropy is also observed. This work will be helpful in understanding different phenomena of space plasmas and tokamak specifically the pedestal H-mode.

1. Introduction
Although the concept of entropy plays an important role in diverse fields ranging from classical thermodynamics of Celsius, dissipation theorems, statistical thermodynamics of Boltzmann and Gibbs, Stochastic thermodynamics, Quantum mechanics and other fields of science, like plasma [1], there is no single definition of entropy, therefore, an attempt is made to define distribution function of entropy and entropy drift as well [2]. The reason is, the research on plasma is diverging continuously. Though, it is helpful to understand the different aspects of plasma in Tokamak and space plasmas. This trend is enhancing the problem of confining plasma. Therefore, it is better to converge the research to reduce the number of variables to single main variable. For this purpose, entropy can be taken as single variable. Because, maximum variables are connected to entropy. In this way, a minute change in plasma status can be observed on the canvas of entropy. Thus, confinement of plasma may be easier only by controlling entropy. Although, at this stage, it is not a simple task but, in the long run, entropy based devices to confine plasma will be made. Nuclear Fusion is a process in which light nuclei fuse to form a heavier nuclei and release energy.

Thus, Nuclear Fusion process is a large source of energy but, the control of this process is still not achieved. For this purpose, different schemes have been proposed such as, Magnetic Confinement Fusion and Inertial Confinement Fusion schemes etc. In magnetic confinement problems arise due to anomalous transport of particles and energy. In Classical, Neoclassical processes and Anomalous transport [3-6] triggered by Coulomb collisions have been studied extensively. Thus, Anomalous transport is the main problem to the confinement of plasma due to temperature-gradient driven drift modes [7-13]. Entropy fluctuations were observed experimentally first time in 1989 by Zhang et al.[14], two of the three peaks of spectrum were ion acoustic waves propagating oppositely, while, the third peak named as entropy
fluctuations at ω = 0. They used Braginskii’s model to calculate different coefficients of transport in low frequency, long-wavelength range. Muller et al. [15] investigated the ratio of shear viscosity η to entropy density S and suggested the lower bound of this ratio $\eta / S = (1 / 4\pi)(h / k_B)$, for a quantum fluid. Kobayashi et al. [16] suggested that entropy mode were able to generate robust, radially inward, particle transport comparable to the level observed in Levitated dipole experiment [17]. Brantov et al. [18] solved kinetic equations and found the entropy mode in entire range of frequencies with the help of Landau electron-electron, electron-ion and ion-ion collision integrals in fully ionized plasma. Therefore, it is important to investigate the role of entropy drift on electron-temperature-gradient driven plasma.

In this paper, entropy drift is added to drift approximation, a new dispersion relation is derived in which $\eta_e$ mode is observed depending on entropy and anomalous transport is also investigated. This investigation will help to understand the electrostatic turbulence in linear regime and electron energy transport in inhomogeneous tokamak as well as in space plasmas.

2. Basic Model Equations and Dispersion Relation

Taking the gradients of equilibrium quantities along the x-axis, our model equations which consists of electron continuity equation, momentum equation and energy balance equation along with entropy equation can be written as:

$$m_e n_e \left( \frac{\partial}{\partial t} + v_e \cdot \nabla \right) v_e = -e \left[ E + v_e \times B \right] - \nabla P$$  \hspace{1cm} (1)

$$\frac{\partial}{\partial t} n_e + \nabla \cdot (n_e v_e) = 0$$  \hspace{1cm} (2)

$$\left( \hat{\partial}_i + \frac{1}{2} v_{be} \cdot \nabla \right) T + n_e T_e (\nabla \cdot v_e) = -\nabla \left[ \frac{5c n_e T_e}{2eB_0} z \times \nabla T_e \right].$$  \hspace{1cm} (3)

In the case of low-frequency $\omega \ll \omega_e$, where the cyclotron frequency [13] is $\omega_e = (eB_e / m_e c)$, charge of electron is e, electron mass is $m_e$ and speed of light is c) and using total drift-approximation:

$$v_e = v_{E_b} + v_{De} + v_{pe} + v_S + v_{ez} \hat{z},$$  \hspace{1cm} (4)

and equations (1)-(3), we get the reduced forms:

$$\left( \hat{\partial}_i + v_E \cdot \nabla + v_{be} \cdot \nabla + v_S \cdot \nabla \right) N - (v_{be} - v_e) \cdot \nabla \Phi + v_{be} \cdot \nabla T + \rho e^2 \nabla (v_{DI} \cdot \nabla) \nabla \cdot \Phi + m_e S_0 \left[ (1 + \eta_e) v_e - v_{be} \right] \nabla S - \rho e^2 \left( \hat{\partial}_i + v_E \cdot \nabla + v_{be} \cdot \nabla + v_{pe} \cdot \nabla + v_{ez} \hat{z} \right) N + r (\hat{\partial}_i + v_E \cdot \nabla + v_S \cdot \nabla) N + T \left( \frac{\partial}{\partial t} + \frac{1}{2} v_{be} \cdot \nabla \right) v_e = v_e^2 \left[ \Phi - (N + T) \right],$$  \hspace{1cm} (5)

$$\left( \hat{\partial}_i + v_E \cdot \nabla + v_{be} \cdot \nabla + v_S \cdot \nabla + v_{ez} \hat{z} \right) v_e = v_e^2 \left[ \Phi - (N + T) \right],$$  \hspace{1cm} (6)

$$\left( \hat{\partial}_i + v_E \cdot \nabla + v_{be} \cdot \nabla + v_S \cdot \nabla \right) T - \frac{1}{2} \left( \hat{\partial}_i + v_E \cdot \nabla + v_S \cdot \nabla \right) N + r (\hat{\partial}_i + v_E \cdot \nabla + v_S \cdot \nabla) N = 0.$$  \hspace{1cm} (7)

In equilibrium, the external magnetic field is $B_0 = B_0 \hat{z}$, $n_{e0}(x)$ is the electron number density, where the different drifts are, entropy drift $v_S = (c m_e / e B_0) \hat{z} \times \nabla S$, diamagnetic drift $v_{Db} = (c / eB_0 n_e) \hat{z} \times \nabla \left( n_e T_e \right)$, polarization drift $v_{pe} = -\left( c / B_0 \omega_e \right) \left[ \hat{\partial}_i + (v_E + v_{Db}) \cdot \nabla + v_{ez} \hat{z} \nabla \cdot \Phi \right]$, $v_{Be}$ drift, $v_{S}$ drift is $v_S = (c T_{e0} / e B_0) \hat{z} \times \nabla \ln n_{e0}$, $\nabla B_0$ drift is $v_B = (c T_{e0} / e B_0) \hat{z} \times \nabla \ln B_0$, $E \times B$ drift $v_E = \frac{E \times B}{B^2}$ and $v_{ez}$ is electron-fluid velocity along $z$-axis. The different notations are, $T_e$ the electron temperature and $n_e$ the electron number density, $\varphi$ the wave potential, $N = n_{e1} / n_{e0}$, $r = n_e \epsilon / (\epsilon - 1)$. 
\[ S = \frac{S_1}{S_0}, \Phi = e\phi / T_{e0} \text{ and } \rho_s = cT_{e0} / eB_e\omega_s. \] The first order \[ \nu_{d0} = (1 + \eta_s)\nu_n \] of electron-diamagnetic drift, with \[ \eta_s = \frac{L_i}{L_u} = \frac{1}{\phi} \ln n_0 / \phi \partial \ln n_0 / \phi \partial x, \] \[ \nu_c = (cT_{e0} / eB_e) \hat{z} \times \nabla \ln T_{e0} \text{ and } \nu_w = \sqrt{T_{0} / m_e} \] is the electron-thermal speed, \[ \gamma = \frac{\omega}{\omega} \] for three dimensional case and \[ T_{e0} \] is the electron temperature in equilibrium. It is assumed that perturbations are proportional to \[ \exp[(k \cdot r - \omega t)], \] where \[ k \] the wave vector and \[ \omega \] is the perturbation frequency. Using Fourier transformation, we get the dispersion relation,

\[
\left(\omega - \omega_0 + \omega_s\right)\left(\omega - \omega_0\right)\left(\omega - \frac{5}{3}\omega_0 - \omega_s\right) - \frac{k_0 T_0}{m_e} \left(\omega - \frac{5}{3}\omega_0 - \omega_s\right)
\]

\[
- \frac{2}{3} \left(\omega - \omega_0\right)\left(\omega - \omega_0\right)\left(\omega - \frac{5}{3}\omega_0 - \omega_s\right) + k_0^2 T_0 \left(\omega - \omega_0 - \omega_s\right)
\]

\[
- m_s S_0 \left(\omega - \omega_s\right)\left(\omega - \frac{5}{3}\omega_0 - \omega_s\right) - \frac{k_0 T_0}{m_e} \left(\omega - \frac{5}{3}\omega_0 - \omega_s\right)
\]

\[
- \tau\left(\omega - \omega_0\right)\left(\omega - \omega_0\right)\left(\omega - \frac{5}{3}\omega_0 - \omega_s\right) - \tau\left(\omega - \omega_0\right)\left(\omega - \omega_0\right)\left(\omega - \omega_0\right).
\]

The notations used in above equation are, \[ \omega_s = k \cdot v_s, \omega_D = k \cdot v_B, k \cdot v_{d0} = (1 + \eta_s)\omega_s \] and \[ \omega_n = k \cdot v_n. \] We now discuss the case for \[ \omega_D = \omega_n = 0, \] then dispersion relation reduces to \[ \omega = \omega_s \] and

\[
\left(\omega - \omega_0\right)^2 + k_0^2 \rho_s^2 \omega_0 \left(\omega - \omega_0\right) - \frac{2}{3} k_0^2 T_0 \left(\omega - \omega_0\right) = 0,
\]

\[
\omega = \omega_s \pm \sqrt{\frac{2}{1 + k_0^2 \rho_s^2}} \left(\omega - \omega_0\right).
\]

For \[ \frac{2}{3} v_n^2 k_0^2 < k_0^2 \rho_s^2 \omega_0, \] entropy instability exists which can play important role in destabilizing plasma to understand anomalous transport.

3. Anomalous Transport

Now, it will be shown that Anomalous transport due to ETG-mode across the confining magnetic field is expressed as: \[ \Gamma = \{v_{EB} T_{e0}\} + c.c., \] where \[ T_{e0} \] is the complex conjugate of the perturbed electron temperature and the angular brackets represent the ensemble average, thus, we have:

\[
T = \frac{1}{\left(\omega - \frac{2}{3}\omega_0 - \omega_s\right)} \left[\omega - \frac{2}{3}\omega_0 - \omega_s\right] N - \tau\left(\omega - \omega_0\right) \Phi.
\]

Here, \[ \omega = \omega_r + i\omega_i, \] where \[ \omega_r \] is real while \[ \omega_i \] is imaginary part of \[ \omega \]. We find the electron energy flux \[ \Gamma_x \] as:

\[
\Gamma_x = -32 \sum_k \frac{eB_0 k^2 + \omega_s}{\omega_n + \omega_D - \omega_s} \frac{\omega_n}{\omega_n + \omega_D - \omega_s} \right) + \gamma^2.
\]
The well-known Fick’s law is, $\Gamma_x = -\chi_e \frac{dT_e}{dx}$. Thermal conductivity of electron is expressed as:

$$\chi_e = 32 \sum \frac{cT_{eo} \gamma \pi^2 \omega_c^2 \omega_e^2}{eB_0 k_s^2 k_{y} n_{th}^4} \left( \frac{\omega_c}{\omega} + \frac{10}{3} \frac{\omega_c}{\omega_e} - \frac{4}{3} \frac{\omega_e}{\omega} \right)^3 \left( \omega - \frac{\omega_c}{\omega_e} \right)^2 + \gamma^2.$$

With the help of Bohm diffusion coefficient, the above equation is, $\chi_e = \Delta D_B$, where Bohm diffusion coefficient is $D_B = cT_{eo} / 16eB_0$ and enhancement factor $\Delta$ is $\Delta = \frac{384}{5} \frac{eB_0 S_0 \omega_c^2 \omega_e^2 \omega_L^2}{k_s^2 k_{y} n_{th}^2}$.

4. Conclusions

In this paper, the effect of entropy drift on electron-temperature-gradient mode is investigated. It is found that entropy-drift plays an important role in energy and mass transport. In the linear limit, new dispersion relation is derived and specific cases are also discussed, the presence of entropy instability is seen and if density and magnetic field gradients are ignored, one root is $\omega = \omega_S$. Dependence of $\eta_e$ term on entropy is also observed. Anomalous transport coefficient has been rederived and its dependence on entropy has been found. This result is not only helpful to understand entropy in a better way and to understand anomalous transport of plasma as well. On the basis of entropy drift, new horizon will be opened for research. This work has relevance in magnetically confined plasma in the presence of temperature and density gradients for tokamak and space plasmas.

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