Influence of spatial structure of the field on a signal-to-interference ratio in communication systems with spatial signal separation

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Abstract. This article presents research results of the signal-to-interference ratio dependence (S/I) on the spatial structure of the radiation field of a three-element linear sparse antenna array in communication systems with spatial signal separation. As a result of the research, it was stated that when the receiving linear antenna is located in the region of the zero order maximum of the field of the useful signal, the formation of the S/I ratio does not depend on whether the field distribution is considered on a straight line or on a circular arc. The location of the receiving antenna in the region of the first-order maximum of the field of the useful signal gives the best results in the formation of the S/I ratio if the field distribution is used on a circular arc and not on a straight line.

1. Introduction
The current stage of scientific research in the field of radio communications is characterized by studies of a large range of complex signals developed for specific applications and methods of processing these signals. One of these methods is the spatial-temporal signal processing (STSP) [1-3]. Its main purpose is to combat interference in information and radio communication channels. The predicted increase in the signal-to-interference ratio is 10 to 20 dB when using STSP.

One of the promising areas of application of STSP in radar, radio navigation and communications is taking into account the curvature of the phase front of an electromagnetic wave when processing various signals. Realization of this possibility allows reusing the radio frequency spectrum in radio lines of stationary radio communication systems and thereby increasing the throughput capacity of the radio line [4].

In [5], as a result of research based on mathematical models, conditions for reliable spatial separation in the reception area of coherent signals with different curvature of phase fronts were obtained, while the calculated degree of the signal excess over the noise reaches 60 dB and more. In this case, the maximum number of transmitted signals is equal to the number of radiating elements of the transmitting linear sparse antenna array (SAA). The presented results were obtained in the approximation of signal satisfaction with the conditions of spatial-temporal narrowband [1].

One of the important questions to be investigated when assessing the efficiency of communication systems with spatial separation of signals is the following: how does the signal-to-interference ratio (S/I) depend on the structure of the interference field in the receiving region (in the approximation of spatial-temporal narrowband signals)?
In communication systems with spatial separation of signals, several information signals can be transmitted in one frequency channel. In the part of the receiving area where conditions are created for effective reception of one of the signals, all other transmitted signals are interference. It is logical to call such interference intra system, in contrast to external interference and noise, which, with one exception, will not be taken into account in this work.

In [5], the structure of the signal field distributed along a straight line in the receiving region is studied. In this paper, various field structures are considered and the optimal structure is determined, which provides the best ratio of S/I. The field of the optimal structure must necessarily contain zeros in the distribution of the field amplitude along a straight line, and the phase of the field at these zeros must undergo a 180° jump. In [5], the conditions are given under which the optimal structure of the signal field is realized.

2. Purpose of the study
The purpose of this work was to calculate and compare the dependence of the S/I ratio on the length of the receiving linear array (AA) for two types of spatial separation of the field in the receiving region: 1) on a straight line; 2) on the arc of a circle tangent to a straight line. The calculations were carried out for two positions of the linear receiving AA: 1) the axis of the AA is parallel to the field distribution line, its center coincides with the point of that maximum of the signal field amplitude, which is at the origin of coordinates; 2) the antenna center spatially coincides with the first lateral maximum of the signal field amplitude distribution.

3. Construction of mathematical models of the radiation field
3.1 Straight line field distribution model
The mathematical model of signal field distribution along a straight line is based on the model given in [5]. Figure 1 shows the parameters of this model. Here B0, B1, B2 denote the transmitting antennas and points in their phase centres simultaneously; C – is current X-axis point where the field is calculated; d – is communication distance (distance between the phase centre B0 and the origin point 0); djk – is a distance between point C and the phase centre of the transmitting antenna with the number j; θjk – are angles of directions from phase centres to point C (j = 0, 1, 2); L – is a step of AA. The subscript x means the dependence of the parameter on this coordinate.

In the constructed model, the carrier frequency of the signals was \( f = 15 \text{ GHz} \) (\( \lambda = 2 \text{ cm} \)). As a transmitting line, a linear SAA was used, consisting of three reflector parabolic antennas with a diameter of 1 m with in-phase and equal-amplitude excitation of the aperture. It was assumed that the field analysis area lies in the far zone of any of the transmitting antennas. This made it possible to use the radiation pattern of a circular in-phase aperture with constant excitation amplitude when calculating the radiation field of each antenna

\[
F(\theta) = (1 + \cos\theta) \cdot \frac{J_1(\beta \sin\theta)}{\beta \sin\theta}
\]

where \( a \) – is the radius of the aperture, \( \beta = 2\pi/\lambda \) – is a phase factor, \( J_1(\ldots) \) – is Bessel function of the first kind of the first order.

When constructing the considered mathematical model, the following basic assumptions were made: the propagation medium is homogeneous and isotropic; the attenuation of radio wave amplitudes in the propagation area was considered the same; the multipath effect was not taken into account; for radio signals in the field of field analysis, the condition of spatial-temporal narrowband is satisfied.

Based on all accepted assumptions and excluding all common constant amplitude and phase coefficients, the mathematical model of the distribution of the total field of all three SAA antennas on a straight line (X-axis) in the reception area has the following form:
\[ E_x(x) = g_0 F_0(\theta_{0x}) \cdot \exp[-i\beta(d_{0x} + \delta)] + g_1 F_1(\theta_{1x}) \cdot \exp[-i(\beta d_{1x} + \varphi)] + g_2 F_2(\theta_{2x}) \cdot \exp[-i(\beta d_{2x} - \varphi)], \]  

\[ \text{(1)} \]

**Figure 1.** Block diagram of the system for studying the field distribution of linear SAA along the X axis

**Figure 2.** Block diagram of the system for studying the distribution of the field of linear SAA along the arc of a circle

where \( g_n \) – are amplitude coefficients which are proportional to the signal levels at the inputs of transmitting antennas; \( F_n(\ldots) \) – are directional characteristics of transmitting parabolic antennas; the meaning of the parameters \( d_n \) and \( \theta_n \) is clear from the figure (\( j = 0, 1, 2 \)); \( \delta = \sqrt{d^2 + L^2} - d; \varphi \) – is additional phase (of different sign) to the fields of the extreme antennas B1 and B2 to control the displacement of the interference field pattern along the X-axis. An additional phase shift \( \beta \delta \) on the central antenna B0 is performed by a phase shifter at the input of this antenna (it is not shown in Figure 1). The presence of this phase shift ensures the common-mode fields of all three SAA antennas at the point \( x = 0 \). This is the first necessary condition for the presence of zeros in the spatial separation of the amplitude of the total signal field. The second necessary condition is the following relations between the amplitude coefficients [5]

\[ g_1 = g_2, \quad g_0/g_2 \leq 2. \]  

\[ \text{(2)} \]

The above stated requirements for the amplitudes and phases of signals automatically entail a 180° jump in the phase of the total field at the zero point of the amplitude. The consequence of this jump is a significant suppression of interference at the output of a receiving antenna of finite dimensions and a significant increase in the S/I ratio. Between the zeroes of the amplitude, the phase changes according to the quadratic law.

### 3.2 Model of field separation on a circular arc

To construct a mathematical model of the field separation on a circle arc, refer to Figure 2. In this case, the phase centres of the transmitting antennas A0, A1 and A2 lie on the arc of the circle (the left dotted arc) with the centre at point 0, which automatically ensures that all radiated fields are in common mode at this point. The centre of the circle of the right arc, at the points where the field is calculated, is the phase centre of the antenna A0. All angles and distances in Figure 2 have the same meaning as in Figure 1.

Taking into account the structure of the field in formula (1) and taking into account all the assumptions and conditions used in the output of this formula in paragraph 3.1, we can write an expression for the field at the current point D of the right arc in Figure 2.
\[ E_A(\theta) = F_0(\theta) \cdot k \cdot \exp(-i\beta d) + F_1(\theta) \cdot \exp[-i(\beta d_1 + \varphi)] + F_2(\theta) \cdot \exp[-i(\beta d_2 - \varphi)], \]  

where \( k \) – is the adjusting factor for setting when modelling permissible ratios (2) between the amplitudes of the fields of the radiating antennas; the designations of the distances between the phase centers of the antennas and a point on the arc in the receiving area correspond to Figure 2; the designations of all other parameters have already been stated above.

A comparative analysis of the graphs of the distribution of the amplitudes and phases of the fields, constructed using expressions (1) and (3), shows a complete coincidence of the distributions of the fields on a straight line and on an arc of a circle. The field phase distribution on the arc contains phase jumps at the points of the amplitude zeroes (as well as on the straight line), and between the zeroes it retains a constant value.

4. Procedure and results of modelling

4.1 Notes on preparing for modelling

The field amplitudes in formulas (1) and (2) after simple transformations, discarding constant coefficients insignificant for analysis and taking into account the first of conditions (2) can be reduced to one general formula.

\[ |E_{x,\theta}| = |k + 2\cos(\beta b_{x,\theta})|/3, \]

where \( b_1 = Lx/d, b_0 = L\theta, 1/3 \) – is normalization factor. On the graphs constructed in accordance with the last formula, three characteristic systems of points can be noted: large amplitude maxima, small maxima and amplitude zeroes. For convenience, large maxima will be called shortly - maxima. The maximum coinciding with the origin of the coordinate \((x \theta)\) will be called the maximum of the zero order. The two closest maxima, located on opposite sides of the origin, are first-order maxima, etc. By the width of the maximum we mean the length of the interval between the two closest zeros of the amplitude.

Since the useful signal and the interference signal from the aperture to the output of the receiving antenna undergo the same transformations, when calculating the S/N ratio, the amplitudes of both signals can be estimated by the following formulas

\[ S(s) = \left| \int_{m-s}^{m+s} E_c(x)dx \right|, \quad I(s) = \left| \int_{m-s}^{m+s} E_n(x)dx \right|, \]

where \( \pm s \) – are symmetric integration interval limits within the signal maximum width; \( m \) – is the coordinate of the maximum point of the signal field of the first (or higher) order; \( E_c(x) \) is \( E_o(x) \) – are complex signal fields and interference, respectively; \( S(s) / I(s) \) – are values of signal modules and noise at the output of the receiving antenna.

To estimate the S/I ratio as a function of the length of the integration interval, the following formula was used

\[ SI(s) = 20 \log[S(s) / I(s)]. \]

If the total length of the integration interval \( 2s \) is understood as the length of the receiving linear AA, then formula (5) determines the dependence of the S/I ratio on the length of such receiving antenna. Studies of this dependence are presented below.

In modelling, the S/I ratio was achieved by aligning the coordinates of the zero amplitude of the interference and the maximum of the signal by selecting the value of the phase addition \( \varphi \) in equation (1) or (2) for the interference signal.

During the simulation, the parameters were unchanged: communication range was \( d = 50 \text{ km} \); the step of the transmitting SAA was \( L = 20 \text{ m} \); wavelength was \( \lambda = 2 \text{ cm} \) (frequency \( f = 15 \text{ GHz} \)); radius of the transmitting parabolic reflector antenna was \( a = 0.5 \text{ m} \). The amplitude coefficient \( k \) was given discrete values from the range \( 0 \leq k \leq 2 \). The phase addition \( \varphi \) (denoted as \( \Phi \) on the graphs) was selected using the amplitude distribution graph from the condition of matching the coordinates of the points of the signal maximum and zero noise. The coordinates of the first-order maximum - linear \( m \) (m) or angular \( \gamma \) (rad.) - were determined from the amplitude distribution of the field on a straight line and on an arc, respectively. The S/I function was considered as a function of either a linear variable \( s \) or a
function of an angular variable \( \theta \). Both variables changed continuously within the limits of the maximum width of the useful signal.

Comparative analysis of the graphs of the \( S/I(s) \) and \( S/I(\theta) \) functions for various values of the parameters \( k \) and \( \phi \) showed that, in general, their properties can be divided into two groups: graphs constructed for the value \( k = 0 \) and for \( 0 < k \leq 2 \). The graphs constructed for the values \( k = 0; 1 \) are presented below for discussion.

4.2 Simulation results for a maximum of zero order

Graphs constructed for the distribution of the signal field and interference on a straight line in the region of the maximum signal of the zero order for various values of the amplitude coefficient \( k \) are shown in Figures 3 and 4. The same graphs for the distribution of the field along the arc have exactly the same form and therefore are not presented in the work.

Figure 3 shows three graphs of the dependence of the \( S/I \) ratio on the length of the integration interval \( s \) for the value \( k = 0 \). Each graph corresponds to a specific value of the "noise" parameter \( t \), shown in the window in the upper right corner of the field in Figure 3. This parameter is artificially introduced in the second formula (4) as the level of external noise for calculating the value of the interference signal \( I(s) \), since the calculation of its own internal interference gives a zero value as a result of integration. This approach avoids the zero value of the interference signal when calculating the \( S/I \) ratio.

The shape of the graphs in Figure 3 shows the independence of the \( S/I \) ratio from the length of the integration interval, i.e. from the horizontal size of the receiving antenna. Thus, in this case, the horizontal size of the antenna can be selected not from the condition of achieving the maximum \( S/I \) ratio, but from other conditions, for example, to obtain the highest possible level of the useful signal.

The graphs of the \( S/I(s) \) function in Figure 4 at \( k = 1 \) correspond to the case of radiation of each of two coherent signals by three antennas with the same amplitude. The red graph, built according to the parameters of the corresponding amplitude distribution, is the border between the two forms of the graphs and corresponds to the phase addition \( \Phi_{\text{max}} = 120,02^\circ \). One group of graphs is represented by smooth monotonically decreasing lines to the side of large values of the variable \( s \) - these are green solid, crimson dash-dotted and red solid lines. The graph in red gives the maximum values of the \( S/I \) ratio from all graphs of the first group. The rest of the graphs of the first group correspond to phase additions \( \Phi < \Phi_{\text{max}} \). The second group of graphs for \( \Phi > \Phi_{\text{max}} \) contains "resonant" bursts of the \( S/I \) ratio.
values, which, with an increase in the phase addition above $\Phi_{\text{max}}$, shift smoothly away from the origin. The formation of "resonances" on the graphs is due to the competition between two anti-phase components of the interference signal. When these components, being summed at the output of the receiving antenna, almost completely suppress each other at a certain value of the integration interval length $s$, then the interference signal tends to zero and a "resonance" occurs.

The presence of the S/I($s$) function of movable "resonances" on the graphs simplifies the design of receiving antennas, since it allows to provide a sufficiently large value of the S/I ratio at the output of the receiving antenna with the specified geometric dimensions (within the width of the maximum signal) by selecting the appropriate phase additive $\phi$. Using the red graph in Figure 4 also allows a high S/I ratio at the output of the receiving antenna, but only for antennas with small horizontal dimensions, from about 0.5 to 1.5 meters.

The graph of the S/I function ($s$) at $k = 2$ is similar to the red graph in Figure 4, and the maximum level of the S/I ratio is achieved at a point on the ordinate at $\Phi = 180$ and is 72 dB. In this case, it is also possible to obtain a high level of S/I only at the output of antennas with a small horizontal size.

4.3 Simulation results for the first order maximum

First, consider the graphs of the S/I($\theta$) function obtained from the distribution of the amplitudes and phases of the signal and noise fields on the circular arc in the region of the first order signal maximum.

In this case, the graphs of the S/I($\theta$) function corresponding to the values of the amplitude parameter $k = 0; 1$ are qualitatively similar to the graphs of the same function plotted for the zero-order maximum region (Figures 3 and 4). There are only quantitative differences between them. The resonances on the graphs for $k = 1$ have a smoother shape, which makes it easier to adjust them to the length of the receiving antenna. The graphs of the function S/I($\theta$) for the parameter $k = 2$ in the case of maxima of the zero and first orders completely coincide. Conclusions about the possibility of using graphs of the S/I($s$) function in the design of antennas with a large S/I ratio at the output, made for the case of a zero-order maximum, remain valid for the first-order maximum.

Now we will discuss the properties of the graphs of the S/I($s$) function, built on the basis of the distribution of signal fields and interference on a straight line in the region of the maximum signal of the first order. The graphs of the S/I($s$) function in the case under consideration for the parameter value $k = 0$ have the same form as the red graph in Figure 4. The maximum value of the S/I ratio is achieved at the point $s = 0$ and is 59 dB with a phase addition of $\Phi = 90^\circ$. At a point with an abscissa $s \approx 11.1$ m, the graph crosses the abscissa axis and goes into the area of negative values of the S/I ratio. Such a dependence of the S/I ratio on the length of the integration interval differs significantly from similar graphs for $k = 0$, considered earlier and shown in Figure 3. This type of graph of the S/I($s$) function allows obtaining a high S/I ratio only for antennas with a small size along the field distribution line. So for an antenna with a linear size of ~ 1 m, the S/I ratio will be ~ 50 dB. The graphs of the S/I($s$) function for the parameter $k = 1$ differ little from the considered graphs for $k = 0$. They differ only in the values of phase additives $\Phi$, leading to the achievement of the maximum values of the S/I ratio.

For the value $k = 2$, the graph of the function S/I($s$) is complex and has a non-monotonic behaviour. This type of graph of the S/I($s$) function can be explained by rapid changes in the signal phase and interference in the region of the first-order signal maximum.

It should be noted that during the transition from the zero-order signal maximum to the first-order maximum, the S/I ratio decreased from ~ 70…100 dB to ~59 dB. This indicates that the spatial channels of information transmission are not equal in terms of the S/I ratio parameter.

5. Conclusion

Summing up the results of the research, the following main results can be noted:

1) for the region of the zero-order signal maximum:
   - the independence of the S/I ratio from the choice of the shape of the spatial area of the field distribution - a straight line or a circle arc is stated;
the fact of complete suppression of intra-system interference in the information transmission system with two transmitting antennas was discovered;
2) for the maximum of the zero-order signal field and the field distribution on an arc or straight line, as well as for the maximum of the first-order signal field and the field distribution on the circular arc:
   – the fact of independence of the S/I ratio from the horizontal size of the receiving linear antenna was established;
   – for the value of the parameter k = 1, the effect of the appearance of "resonant" bursts of the S/I ratio at the values of phase additions \( \Phi > \Phi_{\text{max}} \) was found. The reason for these "resonances" has been established and a mechanism for their use in the design of communication systems with a high S/I ratio (60 ... 80 dB) has been proposed.

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