On the thermal footsteps of Neutralino relic gases

Luis G. Cabral-Rosetti\textsuperscript{a}, Xabier Hernández\textsuperscript{b} and Roberto A. Sussman\textsuperscript{a}

\textsuperscript{a}Instituto de Ciencias Nucleares, Universidad Nacional Autónoma de México (ICN-UNAM), Circuito Exterior, C.U., Apartado Postal 70-543, 04510 México, D.F., México.
\textsuperscript{b}Instituto de Astronomía, Universidad Nacional Autónoma de México, (IA-UNAM), Circuito de la Investigación Científica, C.U., Apartado Postal 70-264, 04510 México, D.F., México.

Abstract. Current literature suggests that neutralinos are the dominant cold dark matter particle species. Assuming the microcanonical definition of entropy, we examine the local entropy per particle produced between the “freeze out” era to the present. An “entropy consistency” criterion emerges by comparing this entropy with the entropy per particle of actual galactic structures given in terms of dynamical halo variables. We apply this criterion to the cases when neutralinos are mostly \(b\)-inos and mostly higgsinos, in conjunction with the usual “abundance” criterion requiring that present neutralino relic density complies with \(0.1 < \Omega_{\tilde{\chi}^0} < 0.3\) for \(h \simeq 0.65\). The joint application of both criteria reveals that a better fitting occurs for the \(b\)-ino channels, hence the latter seem to be favoured over the higgsino channels. The suggested methodology can be applied to test other annihilation channels of the neutralino, as well as other particle candidates of thermal gases relics.

There are strong theoretical arguments favouring lightest supersymmetric particles (LSP) as making up the relic gas that forms the halos of actual galactic structures. Assuming that \(R\) parity is conserved and that the LSP is stable, it might be an ideal candidate for cold dark matter (CDM), provided it is neutral and has no strong interactions. The most favoured scenario \([1, 2, 3, 4, 5, 6]\) considers the LSP to be the lightest neutralino \((\tilde{\chi}^0_1)\), a mixture of supersymmetric partners of the photon, \(Z\) boson and neutral Higgs boson \([2]\). Since neutralinos must have decoupled once they were non-relativistic, it is reasonable to assume that they constituted originally a Maxwell-Boltzmann (MB) in thermal equilibrium with other components of the primordial cosmic plasma. In the present cosmic era, such a gas is either virialized in galactic halos, in the process of virialization in halos of galactic clusters or still in the linear regime for superclusters and structures near the scale of homogeneity \([7, 8, 9]\).

The equation of state of a non-relativistic MB neutralino gas is \([7, 8, 9]\)

\[
\rho = m_{\tilde{\chi}^0_1} n_{\tilde{\chi}^0_1} \left( 1 + \frac{3}{2x} \right), \quad p = \frac{m_{\tilde{\chi}^0_1} n_{\tilde{\chi}^0_1}}{x}, \quad x \equiv \frac{m_{\tilde{\chi}^0_1}}{T},
\]

where \(m_{\tilde{\chi}^0_1}\) and \(n_{\tilde{\chi}^0_1}\) are the neutralino mass and number density. Since we will deal exclusively with the lightest neutralino, we will omit henceforth the subscript \(\tilde{\chi}^0_1\), understand-

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ing that all usage of the term “neutralino” and all symbols of physical and observational variables (i.e. $\Omega_0$, $m$, $\rho$, $n$, etc.) will correspond to this specific particle. As long as the neutralino gas is in thermal equilibrium, we have

$$n \approx n^{\text{eq}} = g \left[ \frac{m}{\sqrt{2}\pi} \right]^3 x^{-3/2} \exp(-x),$$  \hspace{1cm} (2)$$

where $g = 1$ is the degeneracy factor of the neutralino species. The number density $n$ satisfies the Boltzmann equation \[2, 7\]

$$\dot{n} + 3H n = -\langle \sigma | v \rangle \left[ n^2 - (n^{\text{eq}})^2 \right],$$  \hspace{1cm} (3)$$

where $H$ is the Hubble expansion factor and $\langle \sigma | v \rangle$ is the annihilation cross section. Since the neutralino is non-relativistic as annihilation reactions “freeze out” and it decouples from the radiation dominated cosmic plasma, we can assume for $H$ and $\langle \sigma | v \rangle$ the following forms

$$H = 1.66 g_*^{1/2} \frac{T^2}{m_p},$$  \hspace{1cm} (4)$$

$$\langle \sigma | v \rangle = a + b \langle v^2 \rangle,$$  \hspace{1cm} (5)$$

where $m_p = 1.22 \times 10^{19}$ GeV is Planck’s mass, $g_* = g_*(T)$ is the sum of relativistic degrees of freedom, $\langle v^2 \rangle$ is the thermal averaging of the center of mass velocity (roughly $v^2 \propto \frac{1}{x}$ in non-relativistic conditions) and the constants $a$ and $b$ are determined by the parameters characterizing specific annihilation processes of the neutralino (s-wave or p-wave) \[2\]. The decoupling of the neutralino gas follows from the condition

$$\Gamma \equiv n \langle \sigma | v \rangle = H,$$  \hspace{1cm} (6)$$

leading to the freeze out temperature $T_f$. Reasonable approximated solutions of (6) follow by solving for $x_f$ the implicit relation \[2\]

$$x_f = \ln \left[ \frac{0.0764 m_p c_0 (2 + c_0) (a + 6b/x_f) m}{(g_* x_f)^{1/2}} \right],$$  \hspace{1cm} (7)$$

where $g_* = g_*(T_f)$ and $c_0 \approx 1/2$ yields the best fit to the numerical solution of (3) and (6). From the asymptotic solution of (3) we obtain the present abundance of the relic neutralino gas \[2\]

$$\Omega_0 h^2 = Y_\infty \frac{S_0 m}{\rho_{\text{crit}}/h^2} \approx 2.82 \times 10^8 Y_\infty \frac{m}{\text{GeV}},$$  \hspace{1cm} (8)$$

$$Y_\infty \equiv \frac{n_0}{S_0} = \left[ 0.264 g_*^{1/2} m_p m \left\{ a/x_f + 3(b - 1/4a)/x_f^2 \right\} \right]^{-1},$$  \hspace{1cm} (9)$$

where $S_0 \approx 4000 \text{ cm}^{-3}$ is the present radiation entropy density (CMB plus neutrinos), $\rho_{\text{crit}} = 1.05 \times 10^{-5} \text{ GeV cm}^{-3}$. 
Since neutralino masses are expected to be in the range of tens to hundreds of GeV's and typically we have $x_f \sim 20$ so that $T_f < \text{GeV}$, we can use $g^*_f \sim 106.75 \ [3]$ in equations (7) – (9). Equation (7) shows how $x_f$ has a logarithmic dependence on $m$, while theoretical considerations \[1, 2, 3, 4, 5, 6\] related to the minimal supersymmetric extensions of the Standard Model (MSSM) yield specific forms for $a$ and $b$ that also depend on $m$. Inserting into (8)–(9) the specific forms of $a$ and $b$ for each annihilation channel leads to a specific range of $m$ that satisfies the “abundance” criterion based on current observational constraints that require $0.1 < \Omega_0 < 0.3$ and $h \approx 0.65 \ [9]$.

Suitable forms for $\langle \sigma | v \rangle$ can be obtained for all types of annihilation reactions \[2\]. If the neutralino is mainly pure bino, it will mostly annihilate into lepton pairs through t-channel exchange of right-handed sleptons. In this case the cross section is p-wave dominated and can be approximated by (5) with \[3, 10, 11\]

$$a \approx 0, \quad b \approx \frac{8 \pi \alpha_1^2}{m^2 [1 + m_l^2/m_f^2]^2}, \quad (10)$$

where $m_l$ is the mass of the right-handed slepton and $\alpha_1^2 = g_1^2/4\pi \approx 0.01$ is the fine structure coupling constant for the $U(1)_Y$ gauge interaction. If the neutralino is Higgsino-like, annihilating into W-boson pairs, then the cross section is s-wave dominated and can be approximated by (5) with \[3, 10, 11\]

$$b \approx 0, \quad a \approx \frac{\pi \alpha_2^2 (1 - m_w^2/m_f^2)^{3/2}}{2m^2 (2 - m_w^2/m_f^2)^2}, \quad (11)$$

where $m_w = 80.44 \ \text{GeV}$ is the mass of the W-boson and $\alpha_2^2 = g_2^2/4\pi \approx 0.03$ is the fine structure coupling constant for the $SU(2)_L$ gauge interaction.

In the freeze out era the entropy per particle (in units of the Boltzmann constant $k_B$) for the neutralino gas is given by \[7, 9, 8\]

$$s_f = \frac{\rho + p}{nT} = \frac{5}{2} + x_f, \quad (12)$$

where we have assumed that chemical potential is negligible and have used the equation of state \[14\]. From (7) and (12), it is evident that the dependence of $s_f$ on $m$ will be determined by the specific details of the annihilation processes through the forms of $a$ and $b$. In particular, we will use (10) and (11) to compute $s_f$ from (7)-(12).

After decoupling, particle numbers are conserved and the neutralinos constitute a weakly interacting and practically collisionless self gravitating gas. This gas initially expands with the cosmic fluid and eventually undergoes gravitational clustering forming stable bound virialized structures \[9, 8, 12, 13\]. The virialization process involves a variety of dissipative effects characterized by collisional and collisionless relaxation processes \[12, 13, 14\]. However, instead of dealing with the details of this complexity, we will compare the initial and end states of this gas with the help of simplifying but general physical assumptions.

Consider the microcanonical ensamble definition of entropy per particle for a diluted, non-relativistic gas of weakly interacting particles, given in terms of the volume of phase
\[ s = \ln \left[ \frac{(2mE)^{3/2}V}{(2\pi\hbar)^3} \right], \tag{13} \]

where \( V \) and \( E \) are local average values of volume and energy associated with a macrostate that is sufficiently large as to contain a large number of particles, but sufficiently small so that macroscopic variables are approximately constant. For a gas characterized by non-relativistic velocities \( v/c \ll 1 \), we have \( V \propto 1/n \propto m/\rho \) and \( E \propto m v^2/2 \propto m/x \). Assuming as the initial and final states, respectively, the decoupling \((s_f, x_f, n_f)\) and the values \((s_f^b, x_f^b, n_f^b)\) that correspond to a suitable halo structure, the change in entropy per particle that follows from (13) is

\[ s_f^b = \frac{5}{2} + x_f + \ln \left[ \frac{n_f}{n_0} \left( \frac{x_f^b}{x_f^b} \right)^{3/2} \right], \tag{14} \]

where we have used (12) to eliminate \( s_f \) in terms of \( x_f \). Considering the halo gas as a roughly spherical, inhomogeneous and self-gravitating system that is the end result of the evolution and gravitational clustering of a density perturbation at the freeze out era (the initial state), the microcanonical description is an excellent approximation for gas particles near the symmetry center of this system where the density enhancement is maximum but spacial gradients are negligible. We will consider then current halo macroscopic variables (the end state) as evaluated at the center of the halo: \( s_c^b, x_c^b, n_c^b \).

Bearing in mind that the density perturbations at the freeze out era were very small \((\delta n_f/n_f < 10^{-4}, [7, 8, 9])\), the density \( n_f \) is practically homogeneous and so we can estimate it from the conservation of particle numbers: \( n_f = n_0 (1 + z_f)^3 \), and of photon entropy: \( g_s S_f = g_{s0} S_0 (1 + z_f)^3 \), valid from the freeze out era to the present for the unperturbed homogeneous background. Eliminating \((1 + z_f)^3\) from these conservation laws yields

\[ n_f = n_0 \frac{g_{s0}}{g_s} \left( \frac{T_f}{T_0^{\text{CMB}}} \right)^3 \simeq 27.3 n_0 \left( \frac{x_0^{\text{CMB}}}{x_f} \right)^3, \tag{15} \]

where \( x_0^{\text{CMB}} \equiv \frac{m}{T_0^{\text{CMB}}} = 4.29 \times 10^{12} \frac{m}{\text{GeV}} \) \( \tag{16} \)

where \( g_{s0} = g_s(T_0^{\text{CMB}}) \simeq 3.91 \) and \( T_0^{\text{CMB}} = 2.7 \text{ K} \). Since for present day conditions \( n_0/n_0^{\text{CMB}} = \rho_0/\rho_c^{\text{CMB}} \) and \( \rho_0 = \rho_{\text{crit}} \Omega_0 h^2 \), we collect the results from (15) and write (14) as

\[ s_c^b = x_f + 81.60 + \ln \left[ \left( \frac{m}{\text{GeV}} \right)^3 \frac{h^2 \Omega_0}{(x_f x_0^{\text{CMB}})^{3/2} \rho_c^{\text{CMB}}} \frac{\text{GeV/cm}^3}{\text{GeV/cm}^3} \right], \tag{17} \]

Therefore, given \( m \) and a specific form of \( \langle |\sigma|v| \rangle \) associated with \( a \) and \( b \), the entropy per particle of the neutralino halo gas depends on the initial state given by \( x_f \) in (7) and (12), on observable cosmological parameters \( \Omega_0, h \) and on state variables associated to the halo structure.
If the neutralino gas in present halo structures strictly satisfies MB statistics, the entropy per particle, \( s^\text{c} \), in terms of \( \rho^\text{c} = m n^\text{c} \) and \( x^\text{c} = m c^2 / ( k_B T^\text{c} ) \), follows from the well known Sackur–Tetrode entropy formula [15]

\[
\ln \left[ \frac{m^4 c^3}{\hbar^3 (2 \pi x^\text{c})^{3/2} \rho^\text{c} } \right] = 94.42 + \ln \left[ \left( \frac{m}{\text{GeV}} \right)^4 \left( \frac{1}{x^\text{c}} \right)^{3/2} \frac{\text{GeV/cm}^3}{\rho^\text{c} } \right]
\]

(18)

Such a MB gas in equilibrium is equivalent to an isothermal halo if we identify [16]

\[
\frac{c^2}{x^\text{c}} = \frac{k_B T^\text{c}}{m} = \sigma^2_\text{c},
\]

(19)

where \( \sigma^2_\text{c} \) is the velocity dispersion (a constant for isothermal halos).

However, an exactly isothermal halo is not a realistic model, since its total mass diverges and it allows for infinite particle velocities (theoretically accessible in the velocity range of the MB distribution). More realistic halo models follow from “energy truncated” (ET) distribution functions [13, 16, 17, 18, 22] that assume a maximal “cut off” velocity (an escape velocity). Therefore, we can provide a convenient estimate of the halo entropy, \( s^\text{c} \), from the microcanonical entropy definition [13] in terms of phase space volume, but restricting this volume to the actual range of velocities (i.e. momenta) accessible to the central particles, that is up to a maximal escape velocity \( v_e(0) \). From theoretical studies of dynamical and thermodynamical stability associated with ET distribution functions [17, 18, 19, 22, 21, 23, 20] and from observational data for elliptic and LSB galaxies and clusters [24, 25, 26, 27, 28], it is reasonable to assume

\[
v_e^2(0) = 2 |\Phi(0)| \simeq \alpha \sigma^2_\text{c}(0), \quad 12 < \alpha < 18,
\]

(20)

where \( \Phi(r) \) is the newtonian gravitational potential. We have then

\[
\ln \left[ \frac{m^4 v_e^3}{(2 \pi \hbar)^3 \rho^\text{c} } \right] = 89.17 + \ln \left[ \left( \frac{m}{\text{GeV}} \right)^4 \left( \frac{\alpha}{x^\text{c}} \right)^{3/2} \frac{\text{GeV/cm}^3}{\rho^\text{c} } \right],
\]

(21)

where we used \( x^\text{c} = c^2 / \sigma^2_\text{c}(0) \) as in (19). As expected, the scalings of (21) are identical to those of (18). Similar entropy expressions for elliptic galaxies have been examined in [29].

Comparison between \( s^\text{c} \) obtained from (21) and from (17) leads to an “entropy consistency” criterion. Since (21) scales with \( \ln m^4 \), while (17) does so approximately with \( \ln m^3 \), we have a weak logarithmic dependence of \( s^\text{c} \) on \( m \). Therefore, the fulfillment of the “entropy consistency” criterion identifies a specific mass range for each dark matter particle. This allows us to discriminate, in favour or against, suggested dark matter particle candidates and/or annihilation channels by verifying if the standard abundance criterion (8) is simultaneously satisfied for this range of masses. It is interesting to notice that both equations, (17) and (21), display an identical functional dependence of \( s^\text{c} \) on the present day halo parameters, \( \rho^\text{c} \) and \( x^\text{c} \). This implies that a given dark matter particle candidate, characterized by \( m \) and by specific annihilation channels given by \( x_f \) through (7), will pass or fail to pass this consistency test independently of the details one
FIGURE 1. Figures (a) and (b) respectively correspond to the higgsino and b-ino channels. The figures display \( s_c^{\alpha} \) as a function of \( \log_{10} m \), obtained from (20)–(21) (gray strip), from (18) (crosses) and from (17) for \( h = 0.65 \) and \( \Omega_0 = 0.2 \) (thick curve), together with its uncertainty strip \( \Omega_0 = 0.2 \pm 0.1 \). The vertical strip marks the range of values of \( m \) that follow from (8)–(9) for the same values of \( \Omega_0 \) and \( h \). It is evident that only the b-ino channels allow for a simultaneous fitting of both the abundance and the entropy criteria.

assumes regarding the present day dark halo structure. However, the actual values of \( s_c^{\alpha} \) for a given halo structure, whether obtained from (21) or from (17), do depend on the precise values of \( \rho_c^{\alpha} \) and \( x_c^{\alpha} \).

We will now evaluate (21) and (17) for the two cases of neutralino channels: the b-ino and higgsino, including numerical estimates for \( x_c^{\alpha} \) and \( \rho_c^{\alpha} \) that correspond to central regions of actual halo structures. Considering terminal velocities in rotation curves we have \( v_{\text{term}}^2 \simeq 2\sigma^2(0) \), so that \( x_c^{\alpha} \simeq 2(c/v_{\text{term}})^2 \), while recent data from LSB galaxies and clusters [27, 28, 30, 31, 32] suggest the range of values \( \rho_c^{\alpha} < 1 \, \text{M}_\odot/\text{pc}^3 < 0.01 \, \text{M}_\odot/\text{pc}^3 \). Hence, we will use in the comparison of (17) and (21) the following numerical values: \( \rho_c^{\alpha} = 0.01 \, \text{M}_\odot/\text{pc}^3 = 0.416 \, \text{GeV/cm}^3 \) and \( x_c^{\alpha} = 2 \times 10^6 \), typical values for a large elliptical or spiral galaxy with \( v_{\text{term}} \simeq 300 \, \text{km/sec} \) [30, 31, 32]. Figure 1a displays the \( s_c^{\alpha} \) as a function of \( \log_{10} m \), for the halo structure described above, for the case of a neutralino that is mostly higgsino. The shaded region marks \( s_c^{\alpha} \) given by (21) for the range of values of \( \alpha \), while the vertical lines correspond to the range of masses selected by the abundance criterion (8) for \( \Omega_0 = 0.1, 0.2, 0.3 \). The solid curves are \( s_c^{\alpha} \) given by (17) for the same values of \( \Omega_0 \), intersecting the shaded region associated with (21) at some range of masses. However, the ranges of coincidence of a fixed (17) curve with the shaded region (21) occurs at masses which correspond to values of \( \Omega_0 \) that are...
different from those used in (17), that is, the vertical lines and solid curves with same \( \Omega_0 \) intersect out of the shaded region. Hence, this annihilation channel does not seem to be favoured.

Figure 1b depicts the same variables as figure 1a, for the same halo structure, but for the case of a neutralino that is mostly b-ino. In this case, both the abundance and the entropy criterion yield consistent mass ranges, which allows us to favour this annihilation channel as a plausible dark matter candidate, with \( m \) lying in the narrow ranges given by this figure for any chosen value of \( \Omega_0 \). As noted above, the results of figures 1a and 1b are totally insensitive to the values of halo variables, \( x_c^\text{in} \) and \( \rho_c^\text{in} \), used in evaluating (21) and (17). Different values of these variables (say, for a different halo structure) would only result in a relabeling of the values of \( s_c^\text{in} \) along the vertical axis of the figures.

We have presented a robust consistency criterion that can be verified for any annihilation channel of a given dark matter candidate proposed as the constituent particle of the present galactic dark matter halos. Since we require that \( s_c^\text{in} \) of present dark matter haloes must match \( s_c^\text{in} \) derived from the microcanonical definition and from freeze out conditions for the candidate particle, the criterion is of a very general applicability, as it is largely insensitive to the details of the structure formation scenario assumed. Further, the details of the present day halo structure enter only through an integral feature of the dark halos, the central escape velocity, thus our results are also insensitive to the fine details concerning the central density and the various models describing the structure of dark matter halos. A crucial feature of this criterion is its direct dependence on the physical details (i.e. annihilation channels and mass) of any particle candidate.

We have examined the specific case of the lightest neutralino for the mostly b-ino and mostly higgsino channels. The joint application of the “entropy consistency” and the usual abundance criteria clearly shows that the b-ino channel is favoured over the higgsino. This result can be helpful in enhancing the study of the parameter space of annihilation channels of LSP’s in MSSM models, as the latter only use equations (7) and (8)–(9) in order to find out which parameters yield relic gas abundances that are compatible with observational constraints [1, 2, 3, 4, 5, 6]. However, equations (7) and (8)–(9) by themselves are insufficient to discriminate between annihilation channels. A more efficient study of the parameter space of MSSM can be achieved by the joint usage of the two criteria, for example, by considering more general cross section terms (see for example [2]) than the simplified approximated forms (10) and (11). This work is currently in progress.

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