Temporal evolution of radiation from a plasma blob with constant deceleration: application to $\gamma$-ray bursts

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Abstract

It has been widely accepted that relativistic effects have to be used for modelling the wide classes of events associated with active galactic nuclei and galactic superluminal sources. Here we propose a simple analytical model of decelerated motion which is able to explain the shape of double-peaked light curves of $\gamma$-ray bursts. We show that the differences between individual bursts of this class may be due to the angle between the line-of-sight and direction of the outflow, and that the duration of the burst is intimately connected with the deceleration parameter. Assuming special relativity formulae and the orientation of the observer with respect to the outflow, we are able to model observed double-peaked light curves and obtain good quality fits. We propose that such smooth double peaked bursts could be produced by new born galactic superluminal sources.

Keywords: Gamma-rays: bursts — gamma rays: theory — relativity: kinematics and dynamics

1 Introduction

Only a small fraction of $\gamma$-bursts detected by BATSE (onboard the Compton Gamma Ray Observatory) possess an optical identification. Therefore, most of the registered bursts do not have predictions for their total energy and we can expect that among unidentified bursts may exist some which happened close to us. Many active galactic nuclei (AGN) exhibit relativistic jets, and the Galactic jet sources, or microquasars, also exhibit outflow in the form of jets (Mirabel & Rodríguez [11]). We can expect that lower energetic type of $\gamma$-ray bursts, may be connected with a class of new born galactic superluminal sources as it may be inferred from a recent review by Ghisellini and Celotti ([4] see also [12]). However, regardless of whether the radiation from a burst is isotropic or anisotropic, an observer would see only a small fraction of the radiation emitted in the burst, even if the observer is looking directly at the beam which has finite angular width, he still sees only a fraction of the beam. The majority of radiation detected by the observer will be that having the maximum relativistic intensity boosting in the direction of the observer. Thus, the shape of the burst afterglow light curve should depend on beaming effects (Rhoads [17]).

The Lorentz time dilation between the frame of reference of the burst and that of the observer, modifies the time-evolution of the observed flux of photons. It is known that radiating plasma eventually decelerates (Piran [16]) Such a situation requires that we should derive a simple and fully analytical model, which allows us to investigate the effects produced in light curves by deceleration. We show below that the simplest model having constant deceleration can explain particular light curves obtained with BATSE. The bursts detected by BATSE consist of either single, double or multiple peaks in the light curve, but we should realize that such a classification is determined by our resolution capabilities in the time domain. Because we are interested in the influence of deceleration we will avoid discussing any spatial structure of the source and we will assume that the temporal resolution of observational data is does not allow us to detect spatial extension of the emitting region. In the following, we show that light curves exhibiting double peaks can be explained in the framework of a simple model in a simple way. The double peaked light curve appears to be a generic property of models with a decelerated motion.

2 The model and its properties

Consider the motion of a single blob of plasma in a relativistic jet emerging from an active region (c.f. Dar & Rújula [2]). Suppose that this has the simplest form of decelerated motion, i.e. motion with constant deceleration $g$ in a straight line - the simplest assumption that could be made. The agreement we find with observations (see Sec. 3), supports this assumption although the physical reason for constant deceleration is not clear. One-dimensional analytical calculations give us (Rybicki & Lightman [13]) the time dependence of the Lorentz factor $\gamma$
where the time elapsed in the frame of the emitter received by the observer is:

\[ \gamma = \sqrt{1 + \left( \frac{\beta_0}{\sqrt{1 - \beta_0^2}} - \frac{gt}{c} \right)^2} \]  

(1)

\[ \beta = \left( \frac{\beta_0}{\sqrt{1 - \beta_0^2}} - \frac{gt}{c} \right) \frac{1}{\gamma(t)} \]  

(2)

where \( \gamma_0 = \gamma(t = 0) \), and \( \beta_0 = \beta(t = 0) \). We also use the Doppler correction of light travel times in the form:

\[ t - \frac{r}{c} \cos \theta = T_{\text{obs}} \]  

(3)

where \( \theta \) is the angle between direction of motion and the line of sight, \( t = (\gamma_0 \beta_0 - \gamma \beta)c/g \), and \( r = (\gamma_0 - \gamma)c^2/g \), are quantities in the observer’s frame, and \( T_{\text{obs}} \) is the time measured at the location of the observer.

The second assumption made is that the temporal evolution of the flux of photons emitted during the burst is described by a power law \( F \sim \nu^{-\alpha} T^{-\mu} \) in the rest frame of the blob. Such a power law description of flux evolution is widely discussed and frequently (assumed) obtained as an approximation of various emission mechanisms in the literature (van Paradijs et al. [21]). Therefore the flux of photons received by the observer is:

\[ F(t) = F_0 \cdot D^{3+\alpha} \cdot \tau^{-\mu} \]  

(4)

where the time elapsed in the frame of the emitter \( \tau \) and the Doppler boosting factor \( D \) are given by:

\[ \tau = \frac{c}{g} \ln \left( \frac{(\beta_0 + 1)\gamma_0}{(\beta + 1)\gamma} \right) \]  

(5)

\[ D = \frac{1}{\gamma(t) \cdot (1 - \beta(t) \cos \theta)} \]  

(6)

The shape of the light curve implied by (4) is determined by the values of \( \cos \theta, \alpha, \mu, \) and \( \gamma_0 \). The deceleration parameter \( (g/c) \) has no influence on the curve shape but only on the burst duration. For any set of these parameters we can simply calculate that there is a \( T_{\text{obs}}^{\text{max}} \) after which the blob stopped its motion. From (2), \( \beta = 0 \) implies that \( T_{\text{obs}}^{\text{max}} = \frac{\gamma_0 - (\gamma - 1)\cos \theta}{\gamma_0 - (\gamma - 1)\cos \theta/c} \), providing the value of \( T_{\text{obs}}^{\text{max}} \). When the blob has stopped (\( T_{\text{obs}} > T_{\text{obs}}^{\text{max}} \)) we adopt (4) to describe the burst simply by putting \( D = 1 \). At this point, times intervals measured in the source and observer frames are equal. Thus we are able to model light curves in terms of the parameters: \( \cos \theta, \alpha, \mu, \) and \( \gamma_0 \).

Figure 1 shows the properties of our model and the influence of various parameters on the shape of the light curve, including the angle between the velocity vector of the blob and the line of sight. In the case of small angles, we obtain a single burst with a ‘broken power law’ shape of light curve in the observer’s frame, studied by several authors (e.g. Sari at al. [20], Rhoads [17], Panaitescu & Kumar [15]). For larger angles we obtain no break in the light curve slope but simply a brightening due to the debeaming effect (of the radiation) at later times in the decay leading to the formation of a second observed peak. Thus, a double-peaked burst can be produced by a single blob. The value of critical angle demarcating these effects depends on the spectral index \( \alpha \) of the burst and its decay index \( \mu \). One can also see that a larger \( \gamma_0 \) leads to a larger ratio between the second peak intensity and the minimum intensity between the two peaks.

## 3 Two examples of fitting

To test the model, we selected two double-peaked \( \gamma \)-ray light curves from the BATSE archive (Mallozzi & Six [7]) which are thus consistent with being produced by a single blob. We chose the bursts which took place on 1999, May 31 and 2000, May 19 designated GRB 990531 and GRB 000519. We fitted the data in each case with the above model defined by (4) using a fitting procedure in which each of the 4 free parameters is assigned random values by a Monte Carlo procedure, this being repeated many times until an acceptable fit is obtained (i.e. \( \text{RMS}_{\text{fit}} < 3 \times \text{RMS} \) of the fluctuation before the burst). Because we adopted the simplest model of evolution with a singularity at \( \tau = 0 \), (i.e. \( T_{\text{obs}} = 0 \), - the trigger time) we were unable to calculate the growth phase of the burst and we fitted only its descending phase from the first peak (from \( T_{\text{obs}} = T_0 > 0 \)) and the entire second peak. The results of fitting are shown in Figure 2 & 3 and in Table 1.

| GRB   | 990531 | 000519 |
|-------|--------|--------|
| BATSE trigger | 7975 | 8111 |
| \( \gamma_0 \) | 15.2 (119.) | 7.0 (35.8) |
| \( \cos \theta \) | 0.183 (.985) | 0.070 (.847) |
| \( \alpha \) | 1.42 (1.97) | -0.15 (1.36) |
| \( \mu \) | 3.11 (3.18) | 1.46 (1.65) |
| \( g/c \) | 0.489 (.067) | 1.113 (.747) |
| \( T_0 \) | 2.071 (2.02) | 0.679 (.611) |

Table 1: Fitted values of parameters. In parentheses are values obtained by independent fitting of the model with the assumption \( \gamma_{\text{end}} = 10 \) (see text).
Figure 1: Properties of the model i.e. the dependence of a light curve shape on parameters $\mu$, $\Gamma$, $\cos \theta$, and $\alpha$. Heavy curves are those with $\mu = 1.2$, $\alpha = 0.7$, $\gamma_0 = 7.2$, $\cos \theta = 0.5$. The curves end at $T = T_{\text{obs}}^{\text{max}}$ i.e. at the time when the blob stopped its motion. For different values of $\gamma_0$ and $\cos \theta$ we change the value of $g/c$ in order to make the same value of $T_{\text{obs}}^{\text{max}}$ for each curve.

4 Discussion

Our model describes a point-like element of plasma moving with relativistic speed. Using a Lagrangian description of fluid motion we could construct a model of more complicated flow similar to a supernova shell during its explosion (Colgate [11], Galama et al. [3]). In this case we would observe different parts of the shell possessing different values of the parameter $\cos(\theta)$ and the observed light curve shape would be a superposition of the single burst light curve models. Since the light curve data are fitted well using a model involving only a single blob of plasma, we do not have to assume that the shape of the moving volume influences the observed light curve as proposed by Moderski et al. ([11]).

We realize that the low value of the Lorentz factors derived in this paper are inconsistent with all but the very softest gamma-ray spectra (Woods & Loeb [22], Lithwick & Sari [6]). However, our model may be interpreted from the viewpoint of an internal shock model scenario (Kobayashi at al. [5], Piran [16], Dar & Rújula [2]) which assumes that moving blobs (or shells) possess different velocities which lead to collisions between them. Each collision generates a relativistic shock which radiates and decelerates continually. In order to compare this scenario with our model we need to introduce a new parameter $\gamma_{\text{end}}$ (which is the Lorentz factor after the deceleration is switched off), but we are unable to determine its value because the corrections produced by $\gamma_{\text{end}}$ take the form of a scaling factor (relative to $g$). This means that we are able to determine it only when we have some predictions of total (peak) power or the true time scale of the burst phenomenon. In the case of a burst having an afterglow, it is obvious that $\gamma_{\text{end}}$ has a value much greater than 1. Such a value may be calculated as the initial value in the model of a particular afterglow. Therefore values given in Table 1 should be scaled in the case of comparison with true physical conditions inside the shock and $\gamma_0$ represents the difference in Lorentz factors between two collided blobs. In particular, this should be done in the majority of GRBs which could not be explained by a low value of $\gamma_0$. As an example, in Table 1 we put in parentheses the values obtained during independent
fitted of the model in the case of $\gamma_{\text{end}} = 10$.

Given that GRBs will have a distribution in space, it is clear that some bursts will take place relatively close to us. If we accept that the bursters produce an extreme relativistic outflow, it is more likely that the outflows will move in a direction different from the line of sight to the observer, compared with a more distant burst which we only see if there is alignment (Paczyński [13]). In such a case, we will see the burst light curves that are strongly modified by relativistic effects, but with small boosting in the luminosity. Because we obtain small values of $\cos \theta$ and $\gamma_0$ by fitting, we deduce that the two examples of light curves presented in Figures 2 & 3 belong to this type of burst and they are much closer to us than bursts for which redshifts have been determined (Mészáros [8]).

Since we obtain low values of $\cos \theta$, therefore the outflow direction is far from the line of sight and we have no Doppler boosting of radiation in the observer’s frame. Energy considerations lead us to the conclusion that the examples of light curves discussed here, were produced by objects close to us than those detected at cosmological distance. If we have a burst event at a distance of a few kpc in comparison to $Gpc$, the flux can be ($10^{12}$ times larger which allows us to see an event which is not boosted but dimmed by relativistic effects (or may be also less energetic). A scaling factor of $10^{12}$ allows us to calculate the upper limit for the Lorentz factor as: $10^{12} > (\gamma^2)^2$; i.e. $\gamma < 10^{1.5} \sim 30$. We use $\gamma^4$ twice because firstly we have no Doppler boosting and secondly because we have Doppler dimming.

It is well known that galaxy microquasars (like GRS1915+105) eject blobs with relativistic speed (Mirabel & Rodriguez [10]). If we assume that similar sources are born in a much more dense environment and work in a similar manner ejecting blobs which are abruptly stopped leading to an explosive transfer of kinetic energy into radiation, we have a physical interpretation of the results obtained.

5 Summary

This paper discusses the simplest model of the influence of deceleration on the observed shape of the light curves of $\gamma$-ray bursts. Taking into account the deceleration, we obtain important modification of the power law decaying light curve. The transformation of times between the proper frame of a blob and the observer’s frame, i.e. a logarithmic dependence (Eq. 5), may be interpreted as an amplification of the time interval, i.e. at the beginning of the burst we can observe processes lasting few milliseconds as events lasting a few seconds.

In any case of emission, a spatially extended region could be modelled as a superposition of point-like sources. Therefore the shape of the light curve of a point-like source give us the limit for the fastest temporal variations of the flux. This fact will lead us to the prediction that all spatially extended flows should possess more smooth light curves than predicted by our model. In the framework of a decelerated motion model, we are able to produce shorter bursts (i.e. with lower value of $T_{\text{obs}}^{\text{max}}$) if we assume stronger deceleration. All events shorter than the pulse of radiation discussed here, i.e. induced by a deceleration process, probably come from single events of coherent emission, but for discussion of such an event we should first separate the influence of special relativity effects from observational data. Therefore when one postulates a spatial shape of the
outflow (e.g. jet type), the temporal properties of the emitting region should be discussed first with inclusion that the decelerated motion gives us the possibility of seeing the millisecond events expanded to more than a few seconds.

Values of $\gamma_0$ obtained by fitting suggest that the events under consideration belong to a lower energetic type of $\gamma$-ray burst, which may be connected with a class of new born galactic superluminal sources (c.f. [4, 12]).

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