Manual of BlackMax

A black-hole event generator with rotation, recoil, split branes, and brane tension

Version 2.00

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This is the users manual of the black-hole event generator BlackMax [1], which simulates the experimental signatures of microscopic and Planckian black-hole production and evolution at proton-proton, proton-antiproton and electron-positron colliders in the context of brane world models with low-scale quantum gravity. The generator is based on phenomenologically realistic models free of serious problems that plague low-scale gravity. It includes all of the black-hole gray-body factors known to date and incorporates the effects of black-hole rotation, splitting between the fermions, non-zero brane tension and black-hole recoil due to Hawking radiation (although not all simultaneously). The main code can be downloaded from [2].

1 Introduction and New Features

BlackMax is a very versatile semi classical and quantum black-hole generator which simulates a number of different extra dimension models and black-hole evolution scenarios. It also gives the user the possibility to set many of the parameters which influence the formation and decay of black holes. The manual intends to explain the various parameters the user can set.

After the release of BlackMax version 1.01 we received comments, bug-reports and requests for new features from the user’s community, which we tried to incorporate as much as possible. The new features of BlackMax version 2.00 are listed below and will be discussed in the upcoming sections in more detail.

1. BlackMax is able to simulate black-hole production in proton-proton, proton-anti-proton and electron-positron collisions.
2. The user is able to choose among different Planck scale conventions or use his/her own convention.
3. Graviton emission can be simulated.
4. The user can define if baryon, lepton numbers or flavour should be conserved or not. In case the user does not conserve baryon numbers BlackMax events cannot be hadronised with Pythia or HERWIG.
5. The parton density functions of the LHAPDF package [3] are now included into BlackMax.
6. The user can choose to use the Yoshino-Rychkov cross section enhancement factors and simulate the energy loss before the formation of the event horizon as described in [4]. These corrections to the cross section and the energy loss are only applicable for non-tension models and BlackMax will not apply any corrections or modifications to the energy loss if the user turns the Yoshino-Rychkov corrections on for a positive tension brane scenario.
7. The default BlackMax output is in the LHA format [5].

8. Graviton and photon emission is now also included into the balding phase before the black-hole is formed. If the user has turned on graviton emission and choses to use the Yoshino-Rychkov suppression factors or sets the mass loss factor to a non-zero value, BlackMax sheds this energy/mass by emitting two gravitons before the formation of the black-hole. If the user has disabled graviton emission BlackMax will shed this energy by emitting two photons.

9. If the user simulates the split-fermion scenario BlackMax takes into account the position of the black-hole remnant when it calculates which particles it should decay into during the final burst phase.

10. Users should note that the output format has changed with respect to the description in [1] and is described in section 8.2.

2 Installation

The most up-to-date source code and TarBall can be downloaded from:

http://projects.hepforge.org/blackmax/

Having downloaded the zipped tar file you will need to unzip it, then extract the files and make the executable:

```
gunzip BlackMax-2.00.tar.gz
tar -xvf BlackMax-2.00.tar
```

Before compilation you will need to check the compiler version of gcc you are using. This is because the latest gcc compiler version (4.1.2) has changed the names of some system libraries needed to compile Fortran with C code. The download is configured to use gcc version 4. If you have an older gcc version e.g. 3.4.6 then you will need to modify the BlackMax Makefile. You can check your compiler version by doing the following:

```
gcc --version
```

Which will generate output like this:

```
gcc (GCC) 3.4.6 20060404 (Red Hat 3.4.6-10)
Copyright (C) 2006 Free Software Foundation, Inc.
...
```

You will need to change the Fortran system library names in the Makefile in case your compiler version is an older one. Do this by uncommenting the following lines in the Makefile:

```
F77LIB =g2c
F77COMP=g77
```

You are now ready to compile BlackMax.

There are three different ways to run BlackMax:

1. in standalone mode - no additional libraries required
2. accessing PDFs from LHAPDF
3. accessing PDFs from LHAPDF and simultaneous hadronisation from Pythia
To produce the executable for each method simply requires a different compilation/linking step and
is described below. In all three options the default format of the event output is the Les Houches
Accord format [5]. This text file can be used as input to HERWIG/Pythia to hadronise the BlackMax
events at a later date.

2.1 To Run in Standalone Mode

In this version the proton parton densities are taken from CTEQ6m which are packaged with Black-
Max. After unpacking simply do:

```
gmake BlackMaxOnly
```

modify parameter.txt to pick one of the 41 CTEQ6m PDF sets that has been bundled with Black-
Max, for example:

```
choose_a_pdf_file(200_to_240_cteq6)Or_ >10000_for_LHAPDF
```

200

Then run the executable

```
BlackMax > &! out
```

2.2 To Run with LHAPDF

This version of the executable uses the proton parton densities from the LHAPDF library which you
will need to download and install from here:

```
http://projects.hepforge.org/lhapdf/
```

For LHAPDF please ensure you install the package in a directory where you have write permission.
You can do this by specifying an installation directory - for more info see the LHAPDF manual.
Edit the BlackMax Makefile and insert the library locations. Ensure that your `LD_LIBRARY_PATH`
environment variable includes the location of your newly built LHAPDF library:

```
export LD_LIBRARY_PATH = $LD_LIBRARY_PATH:/data/rizvi/atlas/lhapdf-5.3.0/lhapdf/lib
export LHAPATH = /data/rizvi/atlas/lhapdf-5.3.0/lhapdf/share/lhapdf/PDFsets
```

Ensure you have chosen a valid PDF set in parameter.txt, for example to choose the LHAPDF
partons from the H1 PDF2000 fit of HERA data:

```
choose_a_pdf_file(200_to_240_cteq6)Or_ >10000_for_LHAPDF
```

70050

After unpacking the source files do:

```
gmake BlackMax
```

Once the executable is created you can run the program:

```
BlackMax > &! out
```

\[1\] Only if baryon numbers are conserved, please see section 8.1 bullet item 29.
2.3 To Run with Simultaneous Pythia Hadronisation

In order to hadronise the events during the generation job BlackMax comes with an interface to Pythia. The user can only run BlackMax simultaneously with Pythia if s/he is conserving baryon numbers (see section 8.1, bullet item 29). In order to generate fully hadronised events you will need to download and install the latest versions of LHAPDF and PYTHIA. They are available at:

http://www.hepforge.org/downloads/pythia6
http://www.hepforge.org/downloads/lhapdf

BlackMax has been tested with Pythia 6.4.10 and LHAPDF 5.3.0. Install Pythia and LHAPDF according to the instructions. With Pythia, create the libraries, but remove the following dummy routines from Pythia:

upinit.f
upevnt.f
pdfset.f
structm.f

You will also need to remove the mention of the pdfset.f routine from the Pythia Makefile. The four routines above are all dummy routines which actually exist in LHAPDF. Edit the BlackMax Makefile and insert the library locations. Ensure that your LD_LIBRARY_PATH environment variable includes the location of your newly built Pythia and LHAPDF libraries:

export LD_LIBRARY_PATH = $LD_LIBRARY_PATH:/data/rizvi/atlas/lhapdf−5.3.0/lhapdf/lib
export LHAPATH = /data/rizvi/atlas/lhapdf−5.3.0/lhapdf/share/lhapdf/PDFsets

Then create the BlackMax executable using the target "all" which will link to the Pythia and LHAPDF libraries:

gmake all

Ensure you have chosen a valid PDF set in parameter.txt, for example:

choose_a_pdf_file(200_to_240_cteq6)Or_ > 10000_for_LHAPDF
10050

then run the executable:

BlackMax > &! out

3 Black-Hole Production

We assume that the fundamental quantum-gravity energy scale $M_*$ is not too far above the electroweak scale. Consider two particles colliding with a center-of-mass energy $E_{CM}$. They will also have an angular momentum $J$ in their center-of-mass (CM) frame. By the hoop conjecture [6], if the impact parameter, $b$, between the two colliding particles is smaller than the diameter of the horizon of a $(d + 1)$-dimensional black-hole (where $d$ is the total number of space-like dimensions) of mass $M = E_{CM}$ and angular momentum $J$,

$$b < 2r_h(d, M, J),$$

(1)
then a black-hole with \( r_h \) will form. The cross section for this process is approximately equal to the interaction area \( \pi (2r_h)^2 \).

In Boyer-Lindquist coordinates, the metric for a \((d+1)\)-dimensional rotating black-hole (with angular momentum parallel to \( \hat{\omega} \) in the rest frame of the black-hole) is:

\[
\begin{align*}
    ds^2 &= \left(1 - \frac{\mu r^{4-d}}{\Sigma(r, \theta)}\right) dt^2 \\
    &\quad - \sin^2 \theta \left( r^2 + a^2 \left( + \sin^2 \theta \frac{\mu r^{4-d}}{\Sigma(r, \theta)} \right) \right) d\theta^2 \\
    &\quad + 2a \sin^2 \theta \frac{\mu r^{4-d}}{\Sigma(r, \theta)} dt d\phi - \frac{\Sigma(r, \theta)}{\Delta} dr^2 \\
    &\quad - \frac{\Sigma(r, \theta) dr^2 - r^2 \cos^2 \theta d\Omega_{d-3}}{\Delta},
\end{align*}
\]

where \( \mu \) is a parameter related to mass of the black-hole, while

\[
\begin{align*}
    \Sigma &= r^2 + a^2 \cos^2 \theta, \\
    \Delta &= r^2 + a^2 - \mu r^{4-d}.
\end{align*}
\]

The mass of the black-hole is

\[
M = \frac{(d-1)A_{d-1}}{16\pi G_d} \mu,
\]

and

\[
J = \frac{2Ma}{d-1}
\]

is its angular momentum. Here,

\[
A_{d-1} = \frac{2\pi^{d/2}}{\Gamma(d/2)}
\]

is the hyper-surface area of a \((d-1)\)-dimensional unit sphere. The higher-dimensional gravitational constant \( G_d \) is defined as

\[
G_d = \frac{(2\pi)^{d-4}}{4M^{d-1}}.
\]

(The user is able to choose other Planck scale conventions than the one in equation 8, see Section 8)

The horizon occurs when \( \Delta = 0 \). That is at a radius given implicitly by

\[
\begin{align*}
    r_h^{(d)} &= \left[ 1 + \frac{1}{(a/r_h^{(d)})^2} \right]^{-\frac{1}{d-2}} \\
    &= \left[ 1 + \frac{1}{(a/r_{s}^{(d)})^2} \right]^{-\frac{1}{d-2}},
\end{align*}
\]

Here

\[
r_{s}^{(d)} \equiv \mu^{1/(d-2)}
\]

is the Schwarzschild radius of a \((d+1)\)-dimensional black-hole, i.e. the horizon radius of a non-rotating black-hole. Equation 10 can be rewritten as:

\[
r_{s}^{(d)}(E_{CM}, d, M_s) = k(d)M_s^{-1}[E_{CM}/M_s]^{1/(d-2)},
\]

where

\[
k(d) \equiv \left[ 2^{d-3} \pi^{(d-6)/2} \Gamma[d/2] \right]^{1/(d-2)}.
\]

The Hawking temperature of a black-hole is

\[
T_H = \frac{M^d}{4\pi M^{d-1}}.
\]
If two highly relativistic particles collide with center-of-mass energy \(E_{CM}\), and impact parameter \(b\), then their angular momentum in the center-of-mass frame before the collision is \(L_{in} = bE_{CM}/2\). Suppose for now that the black-hole that is formed retains all this energy and angular momentum. Then the mass and angular momentum of the black-hole will be \(M_{in} = E_{CM}\) and \(J_{in} = L_{in}\). A black-hole will form if:

\[
b < b_{\text{max}} \equiv 2r_{h}^{(d)}(E_{CM}, b_{\text{max}}E_{CM}/2).
\]  

We see that \(b_{\text{max}}\) is a function of both \(E_{CM}\) and the number of extra dimensions.

We can rewrite condition (14) as

\[
b_{\text{max}}(E_{CM}; d) = \frac{2r_{s}^{(d)}(E_{CM})}{1 + \left(\frac{d-1}{2}\right)^{2}}\]  

There is one exception to this condition. In the case where we are including the effects of the brane tension, the metric (and hence gray-body factors) for a rotating black-hole are not known. In this case we consider only non-rotating black-holes. For the model with with non-zero tension brane, the radius of the black-hole is defined as

\[
r_{h} = \frac{r_{s}}{B^{1/3}},
\]  

with \(B\) the deficit-angle parameter which is inverse proportional to the tension of the brane \(B = 1 - \frac{\lambda}{2\pi M_{pl}}\) (see equation 8 in [5]). Therefore, for branes with tension

\[
b_{\text{max}}^{(d)}(E_{CM}, d) = 2r_{h}^{(d)}(E_{CM}).
\]  

Also, for branes with positive tension only the \(d = 5\) metric is known.

4 Simulated Scenarios in BlackMax

BlackMax is able to simulate several different extra dimension models, black-hole scenarios.

- **Non-rotating black-hole on a tensionless brane:** For a non-rotating black-hole, we used previously known gray-body factors for spin 0, \(\frac{1}{2}\) and 1 fields in the brane, and for spin 2 fields (i.e. gravitons) in the bulk.

- **Rotating black-hole on a tensionless brane:** For rotating black-holes, we used known gray-body factors for spin 0, \(\frac{1}{2}\) and 1 fields on the brane. The correct emission spectrum for spin 2 bulk fields is not yet known for rotating black-holes, we currently do not allow for the emission of bulk gravitons from rotating black-holes.

- **Non-rotating black-holes on a tensionless brane with fermion brane splitting:** In the split-fermion models, gauge fields can propagate through the bulk as well as on the brane, so we have calculated gray-body factors for spin 0 and 1 fields propagating through the bulk, but only for a non-rotating black-hole for the split-fermion model [9].

- **Non-rotating black-holes on a non-zero tension brane:** The bulk gray-body factors for a brane with non-zero tension are affected by non-zero tension because of the modified bulk geometry (deficit angle). We have calculated gray-body factors for spin 0, 1 and 2 fields propagating through the bulk, again only for the non-rotating black-hole for a brane with non-zero tension and \(d = 5\).

- **Two particle final states:** We use the same gray-body factors as a non-rotating black-hole to calculate the cross section of two-particle final states (excluding gravitons according to [10]).
The formation of the black-hole is a very non-linear and complicated process. BlackMax gives the user the possibility to set parameters which effect the formation phase of the black-hole by hand or use corrections calculated by Yoshino and Rychkov [4].

If the user would like to set the formation parameters her/himself BlackMax assumes that, before settling down to a stationary phase, a black-hole loses some fraction of its energy, linear and angular momentum. These losses are parameterize by three parameters: $1 - f_E$, $1 - f_P$ and $1 - f_L$. Thus the black-hole initial state that we actually evolve is characterized by

$$
E = E_{in} f_E; \\
P_z = P_{z, in} f_P; \\
J' = L_{in} f_L;
$$

where $E_{in}$, $P_{z, in}$ and $L_{in}$ are initial energy, momentum and angular momentum of colliding partons, while $f_E$, $f_P$ and $f_L$ are the fractions of the initial energy, momentum and angular momentum that are retained by the stationary black-hole.

Black-Hole Evolution in BlackMax

The Hawking radiation spectra are calculated for the black-hole at rest in the center-of-mass frame of the colliding partons. The spectra are then transformed to the laboratory frame as needed. In all cases we have not (yet) taken the charge of the black-hole into account in calculating the emission spectrum, but have included phenomenological factors to account for it as explained below.

6.1 Electric and Color Charge Suppression

A charged and highly rotating black-hole will tend to shed its charge and angular momentum. Thus, emission of particles with charges of the same sign as that of the black-hole and angular momentum parallel to the black-hole’s will be preferred. Emission of particles that increase the black-hole’s charge or angular momentum should be suppressed. The precise calculation of these effects has not as yet been accomplished. Therefore, to account for these effects we allow optional phenomenological suppression factors for both charge and angular momentum.

The following charge-suppression factors can currently be used by setting parameter `charge_suppression` (cf. section 8) equal to 2.

$$
F^Q = \exp(\zeta_Q Q^{bh} Q^{em}) \\
F^3_a = \exp(\zeta_3^{bh} c^{em}_a) \quad a = r, b, g.
$$

$Q^{bh}$ is the electromagnetic charge of the black-hole, $Q^{em}$ is the charge of the emitted particle; $c^{bh}_a$, is the color value for the color $a$, with $a = r, b, g$, of the black-hole, and $c^{em}_a$, is the color value for the color $a$, with $a = r, b, g$, of the emitted particle. $\zeta_Q$ and $\zeta_3$ are phenomenological suppression parameters that are set as input parameters of the generator.

We estimate $\zeta_Q = \mathcal{O}(\alpha_{em})$ and $\zeta_3 = \mathcal{O}(\alpha_s)$, where $\alpha_{em}$ and $\alpha_s$ are the values of the electromagnetic and strong couplings at the Hawking temperature of the black-hole. Note that we currently neglect the possible restoration of the electroweak symmetry in the vicinity of the black-hole when its Hawking temperature is above the electroweak scale. Clearly, since $\alpha_{em} \simeq 10^{-2}$ we do not expect electromagnetic (or more correctly) electroweak charge suppression to be a significant effect. However, since $\alpha_s(1 \text{ TeV}) \simeq 0.1$, color suppression may well play a role in the evolution of the black-hole.
6.2 Angular Momentum Suppression

Since the TeV black-holes are quantum black-holes, the gray-body factors should really depend on both the initial and final black-hole parameters. The calculation of the gray-body spectra on a fixed background can cause some problems. In particular, in the current case, the angular momentum of the emitted particle (as indeed the energy) may well be comparable to that of the black-hole itself. There should be a suppression of particle emission processes in which the black-hole final state is very different from the initial state. We therefore introduce a new phenomenological suppression factor, parameter $L_{\text{suppression}}$, to reduce the probability of emission in which the angular momentum of the black-hole changes by a large amount.

If parameter $L_{\text{suppression}}$ is equal to 1 (cf. section 8), BlackMax does suppress the increase the angular momentum of the black-holes. There are three angular momentum suppression models BlackMax is able to simulate.

1. $\Delta$Area suppression: If the user sets $L_{\text{suppression}}$ equal to 2 the $\Delta$Area suppression model is used where the suppression factor is defined as
   \[ F^L = \exp(\zeta_L r_h^{bh}(t + \Delta t)^2/r_h^{bh}(t)^2 - 1)), \]
   (21)
   here $\Delta t$ is the next time step in the simulation.

2. $J_{bh}$ suppression: By setting $L_{\text{suppression}}$ equal to 3 the $J_{bh}$ suppression model is used and the angular suppression factor is defined as
   \[ F^L = \exp(-\zeta_L |J_{bh}(t + \Delta t)|). \]
   (22)

3. $\Delta J_{bh}$ suppression: For the $\Delta J_{bh}$ suppression model $L_{\text{suppression}}$ has to be equal to 4 and
   \[ F^L = \exp(-\zeta_L |J_{bh}(t + \Delta t) - J_{bh}(t)|). \]
   (23)

We might expect $\zeta_L \sim 1$, however there is no detailed theory to support this; as indeed there is no detailed theory to choose among these three phenomenological suppression scenarios.

7 Final Burst of Black-Holes in BlackMax

In the absence of a self-consistent theory of quantum gravity, the last stage of the evaporation cannot be described accurately. Once the mass of black-hole becomes close to the fundamental scale $M_*$, the classical black-hole solution can certainly not be used anymore. We adopt a scenario in which the final stage of evaporation is a burst of particles which conserves energy, momentum$^2$ and all of the gauge quantum numbers. For definiteness, we assume the remaining black-hole will decay into the lowest number of Standard-Model particles that conserve all quantum number, momentum and energy.

Black-holes do not conserve global quantum numbers, like flavor, baryon or lepton numbers [11]. If the user would like to conserve global quantum numbers s/he can set this in the parameter-file (see section 8.1).

8 Input and Output

8.1 Input

The input parameters for the generator are read from the file parameter.txt, see Fig. In the following bulleted list an explanation is given for each of the input parameters in parameter.txt.

---

$^2$Black holes are not elementary particles and do not conserve spin and angular momentum separately. Hence spin is not separately conserved in BlackMax, only the total angular momentum is.
Figure 1: parameter.txt is the input file containing the parameters that the user can set.
1. **Number of simulations**: sets the total number of black-hole events to be simulated;

2. **incoming particle(1:pp, 2:ppbar, 3:ee+)**: sets the type of incoming collision particles:
   - 1 = proton-proton collisions,
   - 2 = proton-antiproton collisions,
   - 3 = electron-positron collisions.

3. **Center of mass energy of protons**: sets the center-of-mass energy of the colliding protons in GeV;

4. **M_pl**: sets the value of the fundamental quantum-gravity scale ($M_*$) in GeV;

5. **definition of M_pl(1:M_D, 2:M_p, 3:M_DL, 4:put in by hand)**: this sets the definition of the used fundamental quantum-gravity scale:
   - 1 = Convention according to [12] and first reference of [13] (PDG definition).
   - 2 = Convention according [14], which is useful in quoting experimental bounds (Giddings and Thomas).
   - 3 = Convention given by Dimopoulos and Landsberg; $M_D^{D-2} = 1/G_D$, with $D$ the total number of dimensions and $G_D$ the $D$-dimensional Newton gravity constant.
   - 4 = User defined convention. In this case BlackMax will read the value of the next parameter **if definition==4** to set the scale. The value of **if definition==4** is $k(d)$ in equation [11]. Note that $k(d)$ is depending on the number of total space dimensions and the user has to make sure to that the value of this parameter is appropriately updated when the user simulates black holes in scenarios with different number of extra dimensions.

6. **Choose a case**: defines the extra dimension model to be simulated:
   - 1 = non-rotating black-holes on a tensionless brane with possibility of fermion splitting,
   - 2 = non-rotating black-holes on a brane with non-zero positive tension,
   - 3 = rotating black-holes on a tensionless brane with $d=5$,
   - 4 = two-particle final-state scenario;

7. **number of extra dimensions**: sets the number of extra dimensions; this must equal 2 for branes with non-zero positive tension (**Choose a case==2**);

8. **number of splitting dimensions**: sets the number of extra split-fermion dimensions (**Choose a case==1**);

9. **extradimension size**: sets the size of the mini-bulk in units of $1/\text{TeV}$ (**Choose a case==1**);
   Please also refer to the discussion under item number 28.

10. **tension**: sets the deficit-angle parameter $B$ [8, 15]; typical values for branes with tension are from 1 to 0.9. (**Choose a case==2**);

11. **choose a pdf file(200 to 240 cteq6G or >100 for LHAPDF)**: The user can choose to run BlackMax with the bundled CTEQ6.1m PDFs [16, 17] which come with the BlackMax release (see subdirectory cteq_pdf) or to run BlackMax with the PDFs of the LHAPDF package. In case the user is running BlackMax in standalone mode s/he has to set this variable in parameter.txt between 200 and 240 to choose one of the 41 CTEQ6.1M PDF sets. It is recommended to use 200 - the others correspond to PDF uncertainties. If the user wants to use the PDFs of the LHAPDF package the user must link the BlackMax executable to the LHAPDF library (see section [2.2]). The input parameter to be given here then is the chosen LHAPDF ID number. The definition of the LHAPDF ID numbers can be found at [3].

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3 This is the distance between fermion branes where only gauge bosons and Higgs field can propagate in split-fermion brane scenario.
12. **Minimum mass**: sets the minimum mass $M_{\text{min}}$ in GeV of the initial black-holes;

13. **Maximum mass**: sets the maximum mass $M_{\text{max}}$ in GeV of the initial black-holes;

14. **fix time step**:
   - In case the user has chosen to simulate black-holes in a split fermion scenario BlackMax will simulate the production and evaporation of the black-holes in time steps. This variable influence the way the size of the time steps are calculate in the split fermion scenario. If set equal to 1, the code uses the parameter `time step` to determine the time interval between events; if set equal to 2 then code tries to optimize the time step, keeping the probability of emitting a particle in any given time step below 10%.
   - In case the user has chosen to simulate black-holes in a non-split fermion scenario BlackMax will simulate the production and evaporation of black-holes in fix time steps if its set to equal 1 using the input of `time step` to set the size of the time steps. If the user would like to save computation time s/he can turns this off by setting this parameter to equal 2. In this case the code will not calculate the location of the black-hole for each time step, which speeds up the event generation.

15. **time step**: defines the time interval $\Delta t$ in GeV$^{-1}$ which the generator will use for the black-hole evolution if `fix_time_step` = 1;

16. **do_yo_shino(1:do_0:no)**: If the user would like to include the Yoshino-Rychkov cross section enhancement factors and the energy loss before the event horizon formation [4] this parameter needs to be set equal 1 otherwise it should be set to 0. In case it is set to 1 the `Mass_loss_factor`, `momentum_loss_factor` and `Angular_momentum_loss_factor` will be ignored by BlackMax.

17. **Mass_loss_factor**: sets the energy loss factor $0 \geq f_E \geq 1$ as defined in equation 18. Recommended values are between 10% to 15%. This depends also on the minimum black-hole mass the user wished to simulate. If the energy loss factor is too big the probability to create a black-hole will be to low and BlackMax would stop the generation.

18. **momentum_loss_factor**: defines the loss factor $0 \leq f_p \leq 1$ for the momentum of initial black-holes as defined in equation 18. Recommended values are 10% to 15%.

19. **Angular_momentum_loss_factor**: sets the loss factor $0 \leq f_L \leq 1$ for the angular momentum of initial black-holes as defined in equation 18. Recommended values are 10% to 15%.

20. **turn_on_graviton(0:off 1:on)**: If this parameter is set to equal 1 BlackMax produces gravitons. Otherwise, BlackMax does not produce any gravitons.

21. **Seed**: This sets the seed for the random-number generator (9 digit positive integer);

22. **Write_LHA_Output_Record(0=NO, 1=Yes, 2=more detailed output)**: If the user sets this variable to 1 BlackMaxLHA.txt is written containing the full event info and output.txt will contain input parameters and cross section information only. If the user sets this variable equal to 2 BlackMaxLHA.txt is written containing full event information and output.txt will contain information about the chosen input parameters, the cross section, the particles which were emitted and the black-holes produced (see section 8.2).

23. **L_suppression**: This chooses the model for suppressing the accumulation of large black-hole angular momenta during the evolution phase of the black-holes (cf. discussion surrounding equations 21,23):
   - 1 = no suppression;
   - 2 = $\Delta$ Area suppression;
   - 3 = $J_{bh}$ suppression;
\[ 4 = \Delta J \] suppression;

24. **angular_momentum_suppression_factor**: defines the phenomenological angular-momentum suppression factor, \( \zeta_L \) (cf. discussion surrounding equation [21,23]); Recommended value is 0.2.

25. **charge_suppression**: turns the suppression of accumulation of large black-hole electromagnetic and color charge during the black-hole evolution process on or off (cf. discussion surrounding equation [19]).
   - 1 = charge suppression turned off;
   - 2 = charge suppression turned on;

26. **charge_suppression_factor**: sets the electromagnetic charge suppression factor, \( \zeta_Q \), in equation [19] Recommended values is 0.2.

27. **color_suppression_factor**: sets the color charge suppression factor, \( \zeta_3 \) in equation [19] Recommended values is 0.2.

28. **split_fermion_width(1/Mpl) and_location(from-15to15)(up_to_9extradimensions)** In the first line after the name of the fermion, e.g. **u_quark** _Right_, the user can set the width of fermion wave functions (in \( M_{-1} \)* units). In the next line the user can set the centers of fermion wave functions (in \( M_{-1} \)* units) in split-brane models, represented as 9-dimensional vectors (for non-split models, set all entries to 0).

   To satisfy the suppression of proton decay, the quark brane and lepton brane must be separated by at least 10\( W \) (with \( W \) the width of fermion wavefunction in extra dimension) for one extra dimension. For higher extra dimensions, the separation can be smaller. It can be less than 10\( W/n \) with \( n \) the number of extra dimension.

   To suppress n-nbar oscillation, the quarks have to be separated. The recommended value needs to be at least 3\( W/n \) to 5\( W/n \) with \( n \) the number of extra dimensions.

   There is no way to satisfy all the constraints at the same time for the split-fermion case. But there are several papers in which the possible position for each fermion from the CP violation constraint are calculated. [18,19] which we would like to refer the user to.

29. **number_of_conservation**: During the balding and evaporation phase, the black hole will emit a certain number of quarks and leptons and acquire global quantum numbers [11]. These quantum numbers are recorded by BlackMax as it reaches the final burst step, at which point it will generate further particle emissions so that the whole process, from beginning to end, will obey certain user-specified global fermion number conservation rules.

   The **number_of_conservation** parameter specifies the number of conservation rules which are to follow the subsequent \( d,s,b,u,c,t,e,m,u,\tau,e,n,u,m,u,\nu,\tau,e \) line in the parameter file. The individual conservation rules are specified as an ordered set of integral coefficients \( \{ a_f \} \) (conservation matrix) such that the quantity \( \sum_f a_f N_f \) is a constant, where \( N_f \) is the number of particular fermion flavor.

   For example, baryon number \( B \) can be conserved in BlackMax by making sure that in the final burst stage, the sum of the number of emitted \( u \) and \( d \)-type quarks compensate for the number \( u \) and \( d \)-type quarks emitted in previous stages. The conservation rule is thus

   \[
   N_u + N_d + N_c + N_s + N_t + N_b = 3B \tag{24}
   \]

   (recalling that quarks have a baryon number of 1/3). If this is the only additional rule, then the following lines would be entered into the parameter file:

   \[4\] It will also acquire angular momentum and electric and color charges, but this is not relevant for this discussion.
The zeroes for the leptons indicate that they play no role in baryon number conservation. If, on the other hand, the user wishes to conserve lepton number $L$, the quark and lepton coefficients are swapped:

\[
\begin{align*}
\text{number_of_conservation} & \quad 1 \\
d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau & \quad 0,0,0,0,0,1,1,1,1,1,1
\end{align*}
\]

The two rules can be combined in the parameter file to conserve $B$ and $L$ separately:

\[
\begin{align*}
\text{number_of_conservation} & \quad 2 \\
d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau & \quad 1,1,1,1,1,0,0,0,0,0,0 \\
 & \quad 0,0,0,0,0,1,1,1,1,1,1
\end{align*}
\]

Conservation of $B - L$ takes the following form:

\[
\frac{1}{3}(N_u + N_d + N_s + N_c + N_t + N_b) - (N_e + N_{\nu_e} + N_{\mu} + N_{\nu_{\mu}} + N_{\tau} + N_{\nu_{\tau}}) = B - L \quad (25)
\]

from which parameters follow:

\[
\begin{align*}
\text{number_of_conservation} & \quad 1 \\
d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau & \quad 1,1,1,1,1,-3,-3,-3,-3,-3,-3
\end{align*}
\]

Finally, if all fermion flavors are to be conserved individually, twelve equations need to be specified:

\[
\begin{align*}
\text{number_of_conservation} & \quad 12 \\
d,s,b,u,c,t,e,mu,tau,nu_e,nu_mu,nu_tau & \quad 1,0,0,0,0,0,0,0,0,0,0,0 \\
 & \quad 0,1,0,0,0,0,0,0,0,0,0,0 \\
 & \quad 0,0,1,0,0,0,0,0,0,0,0,0 \\
 & \quad 0,0,0,1,0,0,0,0,0,0,0,0 \\
 & \quad 0,0,0,0,1,0,0,0,0,0,0,0 \\
 & \quad 0,0,0,0,0,1,0,0,0,0,0,0 \\
 & \quad 0,0,0,0,0,0,1,0,0,0,0,0 \\
 & \quad 0,0,0,0,0,0,0,1,0,0,0,0 \\
 & \quad 0,0,0,0,0,0,0,0,1,0,0,0 \\
 & \quad 0,0,0,0,0,0,0,0,0,1,0,0 \\
 & \quad 0,0,0,0,0,0,0,0,0,0,1,0 \\
 & \quad 0,0,0,0,0,0,0,0,0,0,0,1
\end{align*}
\]

It should be noted that not all the possibilities the user can enter as rules are realized in BlackMax. If the user specifies a rule which BlackMax cannot implement, it will print an error message and halt.

Figure 1 shows an example where \texttt{number_of_conservation} is set to 2 and there are 6 lines in the conservation matrix. Only the first two lines of the conservation matrix in Figure 1 will be read and the rest will be ignored.
8.2 Output

The BlackMax code produces 3 types of output: basic information printed to screen, the BlackMaxLHAreCORD.txt file, and the output.txt file shown in Figs. 2, 3, and 4. This is controlled by the parameter \texttt{WriteLHARedict} which can be set to 0, 1, or 2 for increasingly detailed output.

The screen dump provides basic version and input settings, cross section and timing information and can be redirected to a file. This information is always given. For parameter value 0 output.txt is written and includes a complete list of all input parameters and the calculated cross section only. For parameter value 1 (default) the BlackMaxLHAreCORD.txt file is additionally created containing all input parameters, followed by an event-by-event record in LHA format \cite{LHA}. This file may be used as input to any other LHA compliant MC e.g. for further hadronisation by Pythia or HERWIG. Finally for parameter value 2 the output.txt file then also contains much more detailed event-by-event information for each emission step in the black-hole decay. The information is tagged by an ID word (Begin,Parent, Pbhh, trace, Pcm, Pem, or Elast). Note that the energy in the output.txt file is the bulk energy of the particles where as the energy in BlackmaxLHAreCORD.txt is the observed energy as defined in equation 57 in \cite{1}.

- **Parent**: identifies the partons whose collision resulted in the formation of the initial black-hole (see Fig. 5).
  - column 1: identifies the black-hole;
  - column 2: PDGID code of the parton;
  - column 3: electric charge of parent parton in $3Q$;
  - column 4-6: color-charge vector components of the parent parton;
  - column 7: energy of the parton in GeV;
  - columns 8-10: brane momenta of the parton in GeV;
  - column 11: mass of the parton in GeV.

- **Pbhh**: contains the evolution of the charge, color, momentum and energy of the black-holes, and, for rotating black-holes, their angular momentum (cf. Fig. 6).
  - column 1: identifies the black-hole;
  - column 2: time at which the black-hole emitted a particle;
  - column 3: PDGID code of a black-hole;
  - column 4: three times the electromagnetic charge of the black-hole;
  - columns 5 to 7: color-charge vector components of the black-hole;
  - columns 8: energy of the black-hole in the laboratory frame;
  - columns 9 to 11: brane components of the black-hole momentum in the laboratory frame;
  - columns 12 to (8+d): bulk components of the black-hole momentum;
  - column (9+d): mass of the black-hole;
  - column (10+d): angular momentum of the black-hole, in the case of rotating black-holes; empty otherwise.

- **trace**: contains the evolution history of the black-holes’ positions (cf. Fig. 7):
  - column 1: identifies the black-hole;
  - column 2: the times at which the black-hole emitted a particle;
  - columns 3 - 5 are the brane components of the black-hole position vector when the black-hole emitted a particle.
Figure 2: This is the basic information which is printed to screen and which can be redirected to a file. It contains information about version numbers, dates, input parameters and run time.
Printout of chosen input parameters:

Number_of_simulations 10
incoming_particle(1:pp_2:ppbar_3:ee+)
Center_of_mass_energy_of_incoming_particle 14000.000000

Information about cross section:

total cross section = 6.745298e-08 (GeV^-2) = 2.623921e-11 b
cross section error = 6.402274e-10 (GeV^-2) = 2.490485e-13 b

Information about generated black holes and emitted particles:

Figure 3: output.txt: There are three parts to this file. The first part is a copy of parameter.txt followed by the information on the black-hole production cross section as inferred from the events in this generator run and is produced for WriteLHAOutputRecord equal to 0 and 1. The third part includes information about the black-hole and the emitted particles and is produced for WriteLHAOutputRecord equal to 2. The first column identifies what type of information each row is supplying: lines starting with “Begin” contain information about the emitted particles before the formation of the black-hole; lines beginning with “parent” have information about the two incoming partons; lines beginning “Pbh” contain information on the energy and momenta of the produced black-holes; lines starting with “trace” describe the location of the black-holes; rows beginning with “Pem” contain information about the emitted particles in the lab frame; lines headed by “Pemc” have the information about the emitted particles in the center-of-mass frame; rows starting with “Elast” describe the final burst.
Figure 4: BlackMaxLHRecord.txt: This file has the output of BlackMax in LHA format.

| BH ID | PDGID | 3Q | Colours | Energy | Px   | Py   | Pz   | Mass  |
|-------|-------|----|---------|--------|------|------|------|-------|
| Parent 1 | 4     | 2  | 0      | 0      | 1.61349e+03 | 0.00000e+00 | 0.00000e+00 | 0.00000e+00 | 0.00000e+00 | 1.61349e+03 | 1.20000e+00 |
| Parent 1 | 4     | 2  | 0      | 1      | 0.00000e+00 | 4.40436e+03 | 0.00000e+00 | -4.40436e+03 | 3.20000e+03 | 3.20000e+03 |
| Parent 2 | 21    | 0  | 1      | 0      | -2.5880e+03 | 0.00000e+00 | 0.00000e+00 | 0.00000e+00 | 0.00000e+00 | 2.5880e+03 | 0.00000e+00 |
| Parent 2 | 21    | 0  | -1     | 1      | 0.00000e+00 | 0.00000e+00 | 2.7369e+03 | 0.00000e+00 | 0.00000e+00 | -2.7369e+03 | 0.00000e+00 |
| Parent 3 | 2     | 2  | 0      | 0      | 5.17716e+03 | 0.00000e+00 | 0.00000e+00 | 0.00000e+00 | 0.00000e+00 | 5.17716e+03 | 3.20000e+03 |

Figure 5: Lines in the output file headed by the ID = Parent contain information about the initial partons which formed the black-hole.

| BH ID | PDGID | 3Q | Colours | Energy | Px   | Py   | Pz   | Pextra | Mass  | Jbh  |
|-------|-------|----|---------|--------|------|------|------|--------|-------|------|
| Pbh   | 1     | 0.00000e+00 | 40 | 4      | 0    | 1    | 1    | 4.1707e+03 | 0.00000e+00 | 0.00000e+00 | 0.00000e+00 |
| Pbh   | 1     | 0.00000e+00 | 40 | 3      | 1    | 1    | 1    | 5.7436e+03 | -2.1770e+02 | 3.6077e+01 | -2.5732e+03 | 6.00000e+00 | 5.1936e+03 | 0.00000e+00 |
| Pbh   | 1     | 0.00000e+00 | 40 | 3     | 1    | 0    | 1    | 5.2136e+03 | 0.00000e+00 | 1.8001e+02 | 0.00000e+00 | 4.7913e+03 | 0.00000e+00 |
| Pbh   | 1     | 0.00000e+00 | 40 | 1     | 0    | 1    | 1    | 4.6442e+03 | -3.4324e+02 | -3.5724e+02 | 2.0214e+02 | 4.1356e+03 | 0.00000e+00 |
| Pbh   | 1     | 0.00000e+00 | 40 | 3     | 1    | 1    | 1    | 4.4733e+03 | -2.6842e+02 | -1.6841e+02 | 0.00000e+00 | 1.7473e+03 | 0.00000e+00 |

Figure 6: Lines in the output file headed by the ID = Pbh contain the energies and momenta of the black-holes for each emission step. In case of rotating black-holes, the last column in the line is the angular momentum.
– columns 6 to (2+d): the bulk components of the black-hole position vector, when the
black-hole emitted a particle.

• **Pem**: contains a list of particles the black-holes has emitted during its evaporation phase (cf.
Fig. 3):
  – column 1: identifies the black-hole;
  – column 2: the times at which the black-hole emitted a particle;
  – column 3: PDGID code of the emitted particle;
  – column 4: three times the charge of the emitted particle;
  – columns 5 to 7: color-vector components of the emitted particle;
  – columns 8: energy of the emitted particle in the laboratory frame in GeV;
  – columns 9 to 11: brane components of the momentum of the emitted particle, in the
labouratory frame in GeV;
  – columns 12 to (8+d): bulk components of the momentum of the emitted particle in GeV;
  – column 9+d: mass of the emitted particle in GeV.

• **Pemc**: contains the same information as Pem, but in the center-of-mass frame of the collision.

• **Begin**: contains the same information as Pem, but for the particles which are emitted before
the black-hole is formed. This happens if the user chooses to use the Yoshion and Rykhov
factors or sets the mass-loss-factor to a non-zero value.

• **Elast**: contains the same information as Pem for the particles emitted in the final decay burst
of the black-hole but column 12 is the mass of the particle. The is no information on the bulk
momentum since these particles have no bulk momentum.

A Comparison between BlackMax and Charybdis
In this section we compare the cross sections of BlackMax and Charybdis [20] which is a very com-
monly used black hole generator[3]. There are several differences between BlackMax and Charybdis
which needs to be considered.

For a direct comparison one has to make sure that both generators use the same convention for
the definition of the Planck Mass. In Charybdis the user can choose between the different con-
ventions by setting the parameter **MSSDEF**. In BlackMax the user can switch between the different
conventions with the help of the parameter **definition_of_M_pl**.

• **PDG definiton, MD**:

  **definition_of_M_pl** = 1 corresponds to MSSDEF = 3;

---

5For the comparison in this section MSRT98LO was used as the input PDF.

| BH ID | Time       | X          | Y          | Z          | Xextra... |
|-------|------------|------------|------------|------------|-----------|
| trace | 1 0.00000e+00 | -9.48767e-05 | -3.13192e-04 | 0.00000e+00 | -2.37725e-04 |
| trace | 1 0.00000e+00 | -9.48767e-05 | -3.13192e-04 | 0.00000e+00 | -2.37725e-04 |
| trace | 1 0.00000e+00 | -9.48767e-05 | -3.13192e-04 | 0.00000e+00 | -2.37725e-04 |
| trace | 1 0.00000e+00 | -9.48767e-05 | -3.13192e-04 | 0.00000e+00 | -2.37725e-04 |

Figure 7: Lines in the output file headed by the ID = **trace** contain the location of the black-hole
for each emission step.
The table below shows the emitted particles with their properties:

| BH ID  | Time         | PDGID | 3Q | Colors | Energy     | Px          | Py          | Pz          | Pextra... | Mass          |
|--------|--------------|-------|----|--------|------------|-------------|-------------|-------------|------------|---------------|
| Pem    | 1.00000e+00  | -5    | 1  | -1     | 0          | 2.51757e+02 | -1.17708e+02| -2.19573e+02| 0.00000e+00| 4.20000e+00  |
| Pem    | 1.00000e+00  | -6    | 2  | 1      | 0          | 5.34489e+02 | -6.33172e+02| -4.78582e+02| 0.00000e+00| 1.78100e+02  |
| Pem    | 1.00000e+00  | -16   | 0  | 0      | 0          | 5.67379e+02 | 2.88857e+02 | 4.87872e+02 | 0.00000e+00| 0.00000e+00  |
| Pem    | 1.00000e+00  | -6    | -2 | -1     | 0          | 2.56932e+02 | -7.48229e+01| -1.38850e+02| -9.69335e+01| 0.00000e+00 | 1.78100e+02  |

Figure 8: Lines in the output file headed by the ID = Pem contain the types of the emitted particles, their energies and momenta in the lab frame and the times of their emission.

The table below shows the emitted particles with their properties:

| BH ID  | Time         | PDGID | 3Q | Colors | Energy     | Px          | Py          | Pz          | Pextra... | Mass          |
|--------|--------------|-------|----|--------|------------|-------------|-------------|-------------|------------|---------------|
| Elast  | 1.00000e+00  | 2     | 2  | 1      | 0          | 1.69854e+02 | 1.49973e+02 | 5.22441e+01 | -6.02406e+01| 0.00000e+00 | 3.20000e+03  |
| Elast  | 1.00000e+00  | 4     | 2  | 1      | 0          | 1.73812e+02 | 1.32172e+02 | 4.97772e+01 | -1.01301e+02| 0.00000e+00 | 1.20000e+00  |
| Elast  | 1.00000e+00  | 1     | -1 | 0      | 1          | 8.97411e+02 | 1.86142e+02 | 4.63555e+01 | -8.76669e+02| 0.00000e+00 | 3.20000e+03  |
| Elast  | 1.00000e+00  | 5     | -1 | 0      | 1          | 2.06777e+02 | -1.58071e+01| 1.36863e+02 | -1.54136e+02| 0.00000e+00 | 4.20000e+00  |
| Elast  | 2.00000e+00  | 21    | 0  | 0      | 1          | 2.09713e+02 | -2.09011e+02| -1.42072e+01| 9.58863e+00 | 0.00000e+00 | 0.00000e+00  |

Figure 9: Lines in the output file headed by the ID = Elast contain the types, energies and momenta of particles of the final burst.

- Giddings and Thomas, $M_P$:
  definition of $M_P = 2$ corresponds to MSSDEF = 1;

- Dimopoulos and Landsberg, $M_D_L$:
  definition of $M_P = 3$ corresponds to MSSDEF = 2.

One important difference between the two generators is the definition of cross sections. Charybdis uses the cross section definition for a non-rotating black-hole

$$\sigma_{ch} = \pi r_s^2.$$  \hspace{2cm} (26)

Here, $r_s$ is the Schwarzschild radius of the black-hole.

But BlackMax uses

$$\sigma_{bm} = b_d^2 \pi r_s^2,$$ \hspace{2cm} (27)

$$b_d = \frac{2}{(1 + (d-1)^2)^{\frac{d}{2}}}.$$ \hspace{2cm} (28)

which is the definiton of the cross section for rotating black-holes (see also equation (15)), where $d$ is number of space dimensions. The variable $b_d$ is in general larger than one. Therefore one expects that the cross section of BlackMax is larger than the cross section of Charybdis. $\sigma_{bm}/\sigma_{ch}$ is expected to be equal to $b_d^2$. In the tables, the value of $\sigma_{bm}/\sigma_{ch}$ is in general smaller than $b_d^2$, and in some cases it is even smaller than 1. This difference comes from the fact that BlackMax assumes that the width of the extradimension is $M_{Pl}^{-1}$, and Charybdis assumes that the width of extradimension is 0. Because of the finite width of extradimensions, the cross section of BlackMax is further reduced compared to Charybdis.

To make sure that the two generators indeed output the same cross section in the same situation, we manually put the size of extradimension to zero in BlackMax for the purpose of this comparison. The results are shown in the sixth column of the tables and they agree with the expected value well and there is a small difference between $b_d^2$ and $\sigma_{bm}/\sigma_{ch}$ of 3%.
Table 1: $M_D=1000$ GeV, $M_{bh} > 5000$ GeV, and $D$ is the total number of dimensions (space + time).

| $D$ | $\sigma_{ch}$ [pb] | $\sigma_{bm}$ [pb] | $\sigma_{bm}$ with $L_{extra} = 0$ [pb] | $\sigma_{bm}/\sigma_{ch}$ | $\sigma_{bm}/\sigma_{ch}$ with $L_{extra} = 0$ | $b_f^2$ |
|-----|---------------------|--------------------|-------------------------------|--------------------------|-----------------------------------------------|--------|
| 6   | $75.20 \pm 0.6968$  | $90.69 \pm 0.8407$ | $99.70 \pm 0.9128$            | $1.21$                   | $1.32$                                        | $1.36$ |
| 7   | $122.1 \pm 1.126$   | $161.9 \pm 1.502$  | $177.0 \pm 1.638$            | $1.32$                   | $1.45$                                        | $1.48$ |
| 8   | $172.6 \pm 1.590$   | $247.6 \pm 2.304$  | $266.2 \pm 2.449$            | $1.43$                   | $1.54$                                        | $1.59$ |
| 9   | $225.7 \pm 2.076$   | $352.7 \pm 3.149$  | $369.0 \pm 3.285$            | $1.56$                   | $1.63$                                        | $1.69$ |
| 10  | $280.7 \pm 2.579$   | $455.2 \pm 4.182$  | $484.8 \pm 4.419$            | $1.62$                   | $1.72$                                        | $1.78$ |

Table 2: $M_p=1000$ GeV, $M_{bh} > 5000$ GeV, and $D$ is total number of dimensions (space + time).

| $D$ | $\sigma_{ch}$ [pb] | $\sigma_{bm}$ [pb] | $\sigma_{bm}$ with $L_{extra} = 0$ [pb] | $\sigma_{bm}/\sigma_{ch}$ | $\sigma_{bm}/\sigma_{ch}$ with $L_{extra} = 0$ | $b_f^2$ |
|-----|---------------------|--------------------|-------------------------------|--------------------------|-----------------------------------------------|--------|
| 6   | $119.4 \pm 1.106$   | $149.1 \pm 1.375$  | $158.3 \pm 1.449$            | $1.25$                   | $1.33$                                        | $1.36$ |
| 7   | $172.6 \pm 1.593$   | $234.5 \pm 2.158$  | $250.4 \pm 2.316$            | $1.36$                   | $1.45$                                        | $1.48$ |
| 8   | $227.8 \pm 2.098$   | $334.2 \pm 3.090$  | $351.2 \pm 3.231$            | $1.47$                   | $1.50$                                        | $1.59$ |
| 9   | $284.4 \pm 2.626$   | $450.2 \pm 4.010$  | $464.8 \pm 4.139$            | $1.58$                   | $1.63$                                        | $1.69$ |
| 10  | $342.1 \pm 3.144$   | $561.5 \pm 5.186$  | $591.0 \pm 5.387$            | $1.64$                   | $1.73$                                        | $1.78$ |

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Table 3: $M_{DL}=1000$ GeV, $M_{bh} > 5000$ GeV, and D is the total number of dimensions (space + time).

| D | $\sigma_{ch}$ [pb] | $\sigma_{bm}$ [pb] | $\sigma_{bm}$ with $L_{extra} = 0$ [pb] | $\sigma_{bm}/\sigma_{ch}$ | $\sigma_{bm}/\sigma_{ch}$ with $L_{extra} = 0$ | $b'_c$ |
|---|---|---|---|---|---|---|
| 6 | 55.65 ± 0.5156 | 64.82 ± 0.6047 | 73.78 ± 0.6755 | 1.16 | 1.33 | 1.36 |
| 7 | 38.85 ± 0.3586 | 41.78 ± 0.3894 | 56.35 ± 0.5212 | 1.08 | 1.45 | 1.48 |
| 8 | 33.12 ± 0.3050 | 32.86 ± 0.3083 | 51.07 ± 0.4698 | 0.992 | 1.54 | 1.59 |
| 9 | 30.90 ± 0.2842 | 27.81 ± 0.2590 | 50.51 ± 0.4498 | 0.900 | 1.63 | 1.69 |
| 10 | 30.20 ± 0.2775 | 24.70 ± 0.2353 | 52.16 ± 0.4755 | 0.818 | 1.73 | 1.78 |

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