Parametrizing the Neutrino sector of the seesaw extension in tau decays

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Abstract

The Standard Model includes neutrinos as massless particles, but neutrino oscillations showed that neutrinos are not massless. A simple extension of adding gauge singlet fermions to the particle spectrum allows normal Yukawa mass terms for neutrinos. The smallness of the neutrino masses can be well understood within the seesaw mechanism. We analyse two cases of the minimal extension of the standard model when one or two right-handed fields are added to the three left-handed fields. A second Higgs doublet is included in our model. We calculate the one-loop radiative corrections to the mass parameters which produce mass terms for the neutral leptons. In both cases we numerically analyse light neutrino masses as functions of the heavy neutrinos masses. Parameters of the model are varied to find light neutrino masses that are compatible with experimental data of solar $\Delta m^2_{\odot}$ and atmospheric $\Delta m^2_{\text{atm}}$ neutrino oscillations for normal and inverted hierarchy.
1 The model

We extend the Standard Model (SM) by adding a second Higgs doublet and right-handed neutrino fields. The Yukawa Lagrangian of the leptons is expressed by

\[
\mathcal{L}_Y = - \sum_{k=1}^{2} \left( \Phi_k^+ \ell_R \Gamma_k + \bar{\Phi}_k^R \bar{\nu}_R \Delta_k \right) D_L + \text{H.c.} \tag{1.1}
\]

in a vector and matrix notation, where \( \Phi_k^R = i \tau_2 \Phi_k^+ \). In expression (1.1) the \( \ell_R, \nu_R \), and \( D_L = (\nu_L \ell_L)^T \) are the vectors of the right-handed charged leptons, of the right-handed neutrino singlets, and of the left-handed lepton doublets, respectively, and \( \Phi_\alpha, \alpha = 1, 2 \) are the two Higgs doublets. The Yukawa coupling matrices \( \Gamma_k \) are \( n_L \times n_L \), while the \( \Delta_k \) are \( n_R \times n_L \).

In this model, spontaneous symmetry breaking of the SM gauge group is achieved by the vacuum expectation values \( \langle \Phi_k \rangle_{\text{vac}} = \left( \begin{array}{c} 0 \\ v_k / \sqrt{2} \end{array} \right) \), \( k = 1, 2 \). By unitary rotation of the Higgs doublets we can achieve \( \langle \Phi_1 \rangle_{\text{vac}} = v / \sqrt{2} > 0 \) and \( \langle \Phi_2 \rangle_{\text{vac}} = 0 \) with \( v \simeq 246 \text{ GeV} \). The charged-lepton mass matrix \( M_\ell \) and the Dirac neutrino mass matrix \( M_D \) are

\[
M_\ell = \frac{v}{\sqrt{2}} \Gamma_1 \quad \text{and} \quad M_D = \frac{v}{\sqrt{2}} \Delta_1 , \tag{1.2}
\]

with assumption that \( M_\ell = \text{diag} (m_e, m_\mu, m_\tau) \). The mass terms for the neutrinos can be written in a compact form with an \((n_L + n_R) \times (n_L + n_R)\) symmetric mass matrix

\[
M_\nu = \begin{pmatrix} 0 & M_D^R \\ M_D & M_R \end{pmatrix} , \tag{1.3}
\]

where the hat indicates that \( \hat{M}_R \) is a diagonal matrix. \( M_\nu \) can be diagonalized as

\[
U^T M_\nu U = \tilde{m} = \text{diag} (m_1, m_2, \ldots, m_{n_L+n_R}) , \tag{1.4}
\]

where the \( m_i \) are real and non-negative. In order to implement the seesaw mechanism we assume that the elements of \( M_D \) are of order \( m_D \) and those of \( M_R \) are of order \( m_R \), with \( m_D \ll m_R \). Then the neutrino masses \( m_i \) with \( i = 1, 2, \ldots, n_L \) are of order \( m_R^2 / m_R \), while those with \( i = n_L + 1, \ldots, n_L + n_R \) are of order \( m_R \). It is useful to decompose the \((n_L+n_R) \times (n_L+n_R)\) unitary matrix \( U \) as \( U = \left( \begin{array}{cc} U_L^T \\ U_R \end{array} \right) \), where the submatrix \( U_L \) is \( n_L \times (n_L+n_R) \) and the submatrix \( U_R \) is \( n_R \times (n_L+n_R) \) \([3, 4]\). With these submatrices, the left- and right-handed neutrinos are written as linear superpositions of the \( n_L + n_R \) physical Majorana neutrino fields \( \chi_i \); \( \nu_L = U_L P_L \chi \) and \( \nu_R = U_R P_R \chi \), where \( P_L \) and \( P_R \) are the projectors of chirality.

We can express the couplings of the model in terms of mass eigenfields, where three neutral particles are coupling to neutrinos. Interaction of the \( Z \) boson with the neutrinos is given by

\[
\mathcal{L}_\text{uc}^{(\nu)} = \frac{g}{4c_w} Z_{\mu} \tilde{\chi}^\dagger \mu \left[ P_L \left( U_L^T U_L \right) - P_R \left( U_L^T U_R \right) \right] \chi , \tag{1.5}
\]

where \( g \) is the SU(2) gauge coupling constant and \( c_w \) is the cosine of the Weinberg angle.
The full formalism for the scalar sector of the multi-Higgs-doublet SM is given in Refs. [3, 4]. The Yukawa couplings of the Higgs bosons $H_0^b$ to the neutrinos are given by

$$L(\nu) Y(H_0) = -\frac{1}{2\sqrt{2}} \sum_b H_0^b \bar{\chi} \left[ \left( U_R^b \Delta_b U_L^T + U_L^b \Delta_b^T U_R^T \right) P_L + \left( U_L^b \Delta_b U_R^T + U_R^b \Delta_b^T U_L^T \right) P_R \right] \chi,$$

(1.6)

with $\Delta_b = \sum_k b_k \Delta_k$, where $b$ are two-dimensional complex unit vectors. The neutral Goldstone boson $G_0^b$ is given by the vector $b_{Z}$ with $b_Z = (i, 0)$.

Once the one-loop corrections are taken into account the neutral fermion mass matrix is given by

$$M^{(1)}_{\nu} = \delta M_L \delta M_D + \delta M_D^T \delta M_R + \delta M_R^T \delta M_L,$$

(1.7)

where the $0 \times 3$ matrix appearing at tree level (1.3) is replaced by the contribution $\delta M_L$. This correction is a symmetric matrix, it dominates among all the sub-matrices of corrections. Neglecting the sub-dominant pieces in (1.7), one-loop corrections to the neutrino masses originate via the self-energy function $\Sigma_L^0(0) = \Sigma_L^{S(Z)}(0) + \Sigma_L^{S(G)}(0) + \Sigma_L^{S(H)}(0)$, where the $\Sigma_L^{S(Z,G,H)}(0)$ functions arise from the self-energy Feynman diagrams and are evaluated at zero external momentum squared. In the calculation of the self energies the neutrino couplings are determined by eqs. (1.5) and (1.6). Each diagram contains a divergent piece but when summing up the three contributions the result turns out to be finite.

The final expression for one-loop corrections is given by [5]

$$\delta M_L = \sum_b \frac{1}{32\pi^2} \Delta_b^T U_R^T \hat{m} \left( \frac{\hat{m}^2}{m_{H_0}^2} - 1 \right)^{-1} \ln \left( \frac{\hat{m}^2}{m_{H_0}^2} \right) U_R^T \Delta_b$$

$$+ \frac{3g^2}{64\pi^2 m_W^2} M_D^T U_R^T \hat{m} \left( \frac{\hat{m}^2}{m_Z^2} - 1 \right)^{-1} \ln \left( \frac{\hat{m}^2}{m_Z^2} \right) U_R^T M_D,$$

(1.8)

where the sum $\sum_b$ runs over all neutral physical Higgses $H_0^b$.

2 Case $n_R = 1$

First we consider the most minimal extension of the standard model by adding only one right-handed field $\nu_R$ to the three left-handed fields contained in $\nu_L$.

We parametrize $\Delta_1 = \sqrt{2} m_\nu a_1^T$ and $\Delta_2 = \sqrt{2} m_\nu a_2^T$ with $|a_1| = 1$ and $|a_2| = 1$. The symmetric mass matrix $M_\nu$ (1.3) in block form is diagonalized by

$$U^T M_\nu U = U^T \begin{pmatrix} 0 & m_D a_1^T \\ m_D a_1 & M_R \end{pmatrix} = \begin{pmatrix} M_l & 0 \\ 0 & M_h \end{pmatrix}.$$

(2.9)
The non zero masses in $\hat{M}_l$ and $\hat{M}_h$ are determined analytically by finding eigenvalues of the hermitian matrix $M_\nu M_\nu^\dagger$. These eigenvalues are the squares of the masses of the neutrinos $\hat{M}_l = \text{diag}(0,0,m_l)$ and $\hat{M}_h = m_h$. The solutions $m_D^2 = m_h m_l$ and $m_R^2 = (m_h - m_l)^2 \approx m_h^2$ correspond to the seesaw mechanism.

We can construct the diagonalization matrix $U$ for the tree level from two diagonal matrices of phases and three rotation matrices $U_{\text{tree}} = U_{\phi}(\phi_i)U_{12}(\alpha_1)U_{23}(\alpha_2)U_{34}(\beta)U_i$, where the angle $\beta$ is determined by the masses $m_l$ and $m_h$. The values of $\phi_i$ and $\alpha_i$ can be chosen to cover variations in $M_D$.

For calculation of radiative corrections we use the following set of orthogonal complex vectors: $b_Z = (i,0)$, $b_1 = (1,0)$, $b_2 = (0,i)$ and $b_3 = (0,1)$. Diagonalization of the mass matrix after calculation of one-loop corrections is performed with a unitary matrix $U_{\text{loop}} = U_{\text{egv}}(\phi_1,\phi_2,\phi_3)$, where $U_{\text{egv}}$ is an eigenmatrix of $M_\nu^{(1)}M_\nu^{(1)\dagger}$ and $U_\rho$ is a phase matrix. The second light neutrino obtains its mass from radiative corrections. The third light neutrino remains massless.

It is possible to estimate masses of the light neutrinos from experimental data of solar and atmospheric neutrino oscillations assuming that the lightest $m_{l_3} = 0$. Considering the normal ordering of the light neutrinos we receive $m_{l_1} = 5.0 \pm 0.2 \times 10^{-11}$ GeV and $m_{l_2} = 8.7 \pm 0.3 \times 10^{-12}$ GeV. Numerical analysis shows that we can reach those values for a heavy singlet with a mass bigger than 830 GeV, see Fig. 1.

Figure 1: Calculated masses of two light neutrinos as a function of the heavy neutrino mass $m_h$. The mass of the third light neutrino is zero, when $n_R = 1$. Solid lines show the boundaries of the allowed neutrino mass ranges when the model parameters are constrained by the experimental data on neutrino oscillations. The purple arrows indicate the values of $m_{l_2}$ which do not satisfy the allowed experimental neutrino mass ranges. Due to the scale, the allowed $m_{l_1}$ and $m_{l_2}$ values form a band, their scattered values are shown separately in the right plots.
3 Case $n_R = 2$

If we add two singlet fields $\nu_R$ to the three left-handed fields $\nu_L$, the radiative corrections give masses to all three light neutrinos.

Now we parametrize $\Delta_1 = \frac{\sqrt{2}}{v} \left( \begin{array}{c} m_D^2 \bar{a}_1^T \\ m_D^1 \bar{b}_1^T \end{array} \right)$ and $\Delta_2 = \frac{\sqrt{2}}{v} \left( \begin{array}{c} m_D^2 \bar{a}_2^T \\ m_D^1 \bar{b}_2^T \end{array} \right)$ with $|\bar{a}_1| = 1$, $|\bar{b}_1| = 1$, $|\bar{a}_2| = 1$ and $|\bar{b}_2| = 1$. Diagonalizing the symmetric mass matrix $M_\nu (1.3)$ in block form we write:

$$U^T M_\nu U = U^T \left( \begin{array}{cc} 0_{3 \times 3} & m_D^2 \bar{a}^T \\ m_D^1 \bar{b}^T & \tilde{M}_R \end{array} \right) U = \left( \begin{array}{ccc} \tilde{M}_l & 0 \\ 0 & \tilde{M}_h \end{array} \right). \quad (3.10)$$

The non zero masses in $\tilde{M}_l$ and $\tilde{M}_h$ are determined by the seesaw mechanism: $m_{D_i}^2 \approx m_{h_i} m_{l_i}$, and $m_{R_i}^2 \approx m_{R_i}^2$, $i = 1, 2$. Here we use the ordering of masses $m_1 > m_2 > m_3$. The third light neutrino is massless at tree level.

The diagonalization matrix for tree level $U_{\text{tree}} = U_{12}(\alpha_1, \alpha_2) U_{\text{egv}}(\tilde{\beta}_1) U_\varphi(\varphi_1)$ is composed of a rotation matrix, an eigenmatrix of $U_{12}^T M_\nu M_\nu^T U_{12}^*$ and a diagonal phase matrix, respectively.

For calculation of radiative corrections we use the same set of the orthogonal complex vectors $b_i$ as in the first case. Diagonalization of the mass matrix including the one-loop correction is performed with a unitary matrix $U_{\text{loop}} = U_{\text{egv}} U_\varphi(\varphi)$, where $U_{\text{egv}}$ is the eigenmatrix of $M_\nu (1) M_\nu^T (1)$ and $U_\varphi$ is a phase matrix.

In numerical calculations the model parameters as well as the derived masses of the light neutrinos are obtained in several steps. First, the diagonal mass matrix for the tree level is constructed. The lightest neutrino is massless, and the masses of other two light neutrinos are estimated from experimental data on solar and atmospheric neutrino oscillations. The masses of the heavy neutrinos are input parameters. This diagonal tree level matrix is used to constrain the parameters $\alpha_i$ and $\phi_i$ that enter the tree-level mass matrix $M_\nu$ and its diagonalization matrix $U_{\text{tree}}$. Then the diagonalization matrix is used to evaluate one-loop corrections to the mass matrix. Diagonalization of the corrected mass matrix yields masses for three light neutrinos. If the calculated mass difference is compatible with the experimental neutrino mass difference, the parameter set is kept. Otherwise, another set of parameters is generated. Figure 2 illustrates the obtained results. Both normal and inverted neutrino mass orderings are considered.
Figure 2: The masses $m_{l_i}$ of the light neutrinos as functions of the heaviest right-handed neutrino mass $m_{h_1}$, for the case $n_R = 2$. The value of the second heaviest right-handed neutrino mass is fixed at $m_{h_2} = 100$ GeV. The plot on the left represents the normal hierarchy, and the plot on the right represents the inverted hierarchy of the light neutrinos. The wide solid lines indicate the area of the most frequent values of the scatter data. The nearly degenerate masses $m_{l_1}$ and $m_{l_2}$ are shown separately in the lower right plot for the case of the inverted hierarchy.

4 Conclusions

For the case $n_R = 1$ we can match the differences of the calculated light neutrino masses to $\Delta m^2_\odot$ and $\Delta m^2_{\text{atm}}$ with the mass of a heavy singlet larger than 830 GeV. Only normal ordering of neutrino masses is possible.

In the case $n_R = 2$ we obtain three non vanishing masses of light neutrinos for normal and inverted hierarchies. The numerical analysis shows that the values of the light neutrino masses (especially of the lightest mass) depend on the choice of the heavy neutrinos masses. The radiative corrections generate the lightest neutrino mass and have a big impact on the second lightest neutrino mass.

In future we plan to apply our parametrization to study the $\tau$ polarization coming from the decay of a $W$ boson in the data of the CMS experiment at LHC and thus determine restrictions to the parameters of the neutrino sector.

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