EXPLAINING THE GAMMA RAY BURST
LAG/LUMINOSITY AND VARIABILITY/LUMINOSITY
RELATIONS

Bradley E. Schaefer

University of Texas, Department of Astronomy, C-1400, Austin TX 78712

Received ________________; accepted ________________
ABSTRACT

Two different luminosity indicators have been recently proposed for Gamma Ray Bursts that use the lag between light curve peaks with hard and soft photons as well as the variability (‘spikiness’) of the burst light curve. Schaefer, Deng, & Band have proven that both of these luminosity indicators are valid by finding a lag/variability relation for 112 BATSE bursts. Here, I provide simple and general explanations for both indicators. The lag/luminosity relation is a consequence of conservation of energy when radiative cooling dominates. High luminosity bursts cool rapidly so the lag between when hard and soft photons peak is short, while low luminosity bursts cool slowly so the lag is long. The variability/luminosity relation arises within internal shock models because the luminosity and variability both scale as strong functions of the bulk relativistic motion of the jet, $\Gamma$. With luminosity roughly proportional to $\Gamma^5$ and variability scaling as $\Gamma^2$, we get the luminosity being correlated to variability just as the observed power law.

*Subject headings: gamma rays: bursts*
1. INTRODUCTION

One of the most fundamental problems of Gamma-Ray Burst (GRB) studies is to establish the distance scale. The recent discoveries of x-ray, optical, and radio transients have allowed only a dozen or so red shifts to be measured. At the Fifth Huntsville Gamma-Ray Burst Symposium, two luminosity (and hence distance) indicators were proposed that make use of gamma ray light curves only and hence can be used for large numbers of long-duration bursts (Norris, Marani, & Bonnell 1999; Fenimore & Ramirez-Ruiz 2000). The first indicator is $\tau_{\text{lag}}$, which is the time delay between peaks in the light curve as viewed with hard and soft photons. More specifically, the lag is the delay of the maximum cross-correlation between the burst light curves in BATSE channels 1 (25-50 keV) and 3 (100-300 keV). The second indicator is the variability, $V$, which is the ‘spikiness’ of its light curve, taken as the normalized variance of the light curve around a smoothed light curve of the burst. Both indicators were calibrated by only 6 or 7 bursts with known red shifts; however, Schaefer, Deng, & Band (2001) have proven that both the lag/luminosity and the variability/luminosity relation are correct by finding the required lag/variability relation with 112 BATSE bursts.

Before the two luminosity indicators can be used with confidence, we should have some physical understanding of why they both work. This Letter gives a simple explanation for both relations with standard assumptions.

2. BACKGROUND

I will presume that GRBs are internal shocks produced by narrow jets expanding with a relativistic $\Gamma$ (roughly 100-1000) with a total mass $M_{\text{jet}}$ (with a kinetic energy that does not vary greatly between bursters) with an opening angle $\Theta_{\text{jet}}$ (such that $\Theta_{\text{jet}} < \Gamma^{-1}$) for which the cooling is dominated by radiation.

The luminosity is calculated as

$$L = 4\pi D_L^2 \cdot P_{256} \cdot < E >,$$

where $P_{256}$ is BATSE’s observed peak flux (from 50-300 keV evaluated on the 256 ms time scale), $D_L$ is the burst’s luminosity distance (for $H_0 = 65 km \cdot s^{-1} \cdot Mpc^{-1}$, $\Omega_M = 0.3$, and $\Omega_{\Lambda} = 0.7$), and $< E >$ is the average energy of a photon for an $E^{-2}$ spectrum ($1.72 \times 10^{-7} erg \cdot photon^{-1}$). This formula includes a K-correction for an $E^{-2}$ spectrum as appropriate for average bursts (Schaefer et al. 1994, 1998). These luminosities implicitly represent the case when the radiation is emitted isotropically.

The existence of the fairly tight lag/luminosity relation (and the less tight variability/luminosity relation) suddenly makes GRBs into standard candles. This is not in the sense that all bursts have the same luminosity, but in the sense that they have a light curve property that can be used to uniquely determine their luminosity. In a similar sense, Cepheids are standard candles not because they all have the same luminosity but since the period/luminosity relation allows their luminosity to be known from a light curve property. Similarly, Type Ia supernovae are standard candles with their luminosity determined by their decline rate.
With GRBs as standard candles, there must be something 'standard' imposed by the physics of the problem. Within the general picture of internal shocks, the luminosity will scale something like the energy of the jet \((\Gamma M_{jet}c^2)\) perhaps with additional factors of \(\Gamma\). This is supported by Kulkarni (2000) who claims that the total energy budget of bursts is constant. It might not be surprising to have a constant quantity between GRBs, since bursts may be a critical phenomenon such that they are possible only when this quantity is above some threshold value and the frequency of that value falls off sharply above that threshold hence producing a near constant value. In any case, some such quantity must be roughly a constant from burst-to-burst for the lag/luminosity relation to be true.

Three angular scales are important for the internal shock scenario. First is the relativistic Doppler beaming where the radiation is sent into a forward cone with a characteristic opening angle of \(\Gamma^{-1}\). At least a substantial fraction of bursts must have \(\Gamma\) greater than \(\sim 100\) since otherwise it will be difficult to get prompt GeV photons out of the shock region. Second is the jet opening angle, \(\Theta_{jet}\), which might be perhaps 10 degrees or much smaller than 1 degree. Third is the characteristic angular size of the disconnected emitting regions as viewed from the central source, \(\Theta_{region}\). This angle must be very small since the majority of bursts have millisecond variability (Walker, Schaefer, & Fenimore 2000), the average light curve during decline does not show curvature effects (Fenimore 1999), and three bursts with good blackbody spectra have very small blackbody radii for their emitting regions. Quantitative analysis of these three constraints show that \(\Theta_{region}\) is as small as arc-minutes.

At a great distance from the burster, the beam pattern will be a convolution of the jet opening angle and the relativistic beaming. The characteristic angle of this will be roughly the larger of \(\Theta_{jet}\) and \(\Gamma^{-1}\), for a total solid angle of \(\Omega\). I would expect that \(\Theta_{jet}\) is not greatly larger than \(\Theta_{region}\), so then \(\Theta_{jet} < \Gamma^{-1}\) and \(\Omega = \pi\Gamma^{-2}\). In this case, an observer on Earth will be able to see the entire working surface of the jet and all the emitting regions, with none being hidden by relativistic beaming. Unless some arrangement of conspiring factors is invoked, the visibility of the entire jet is a requirement for the existence of any tight luminosity relation (such as the lag/luminosity relation). That is, otherwise the fraction of the available jet energy detected by BATSE will vary with \(\Gamma\) from burst-to-burst creating a large scatter in the lag/luminosity relation.

### 3. LAG/LUMINOSITY RELATION

The lag/luminosity relation (Norris, Marani, & Bonnell 1999) is

\[
L = 2.9 \times 10^{51}(t_{\text{lag}}/0.1s)^{-1.14\pm0.20}, \tag{2}
\]

where \(L\) is the observed peak luminosity in units of \(\text{erg} \cdot \text{s}^{-1}\) (Schaefer, Deng, & Band 2001).

The average cooling rate per particle in the emitting region of the jet will be the total luminosity over all directions \((L_{\text{tot}})\) of the jet divided by the number of emitting particles. The number of emitting particles will be roughly \(M_{\text{jet}}/m_{\text{proton}}\). The cooling rate will also be the time derivative of the mean particle energy, \(E_{\text{peak}}\). So

\[
dE_{\text{peak}}/dt = -L_{\text{tot}}/(M_{\text{jet}}/m_{\text{proton}}), \tag{3}
\]
which is really just a statement of the conservation of energy for when radiative cooling dominates. Equation 3 is valid both in the reference frame of the jet as well as the reference frame of BATSE (Liang 1997). The proportionality between $dE_{\text{peak}}/dt$ and $L$ has previously been discovered from BATSE data by Liang & Kargatis (1996) as a general law covering the decay of many GRB pulses.

Let us evaluate equation 3 at the time of peak flux. On the right-hand-side,

$$L_{\text{tot}} = L \cdot (\Omega/4\pi),$$

(4)

where $\Omega$ is the solid angle into which the radiation is beamed (roughly $\pi \Gamma^{-2}$ since $\Theta_{\text{jet}} < \Gamma^{-1}$). On the left-hand-side of equation 3, we can approximate the derivative as a finite difference evaluated between times when the light curves peak in BATSE channel 3 and channel 1;

$$dE_{\text{peak}}/dt = [E_{\text{peak}}(T_1) - E_{\text{peak}}(T_3)]/[T_1 - T_3].$$

(5)

The time when the light curve peaks in BATSE channel 3, $T_3$, is a competition between the turn on of the pulse versus the cooling of the emitting region and will be approximately when $E_{\text{peak}}$ is centered in channel 3 at $\sim 200\text{keV}$. Similarly, $E_{\text{peak}}(T_1)$ will be near the center energy of the BATSE channel 1 ($\sim 30\text{keV}$) at the time when the flux in this channel reaches maximum. The time difference between the peaks in channels 1 and 3 $(T_1 - T_3)$ is equal to $\tau_{\text{lag}}$, so

$$dE_{\text{peak}}/dt = -170\text{keV}/\tau_{\text{lag}}.$$  

(6)

With eqs 3, 4, and 6, we get

$$L = (4\pi M_{\text{jet}} \cdot 170\text{keV} \cdot m_{\text{proton}}^{-1} \Omega^{-1})\tau_{\text{lag}}^{-1}.$$  

(7)

Thus, we find that the observed luminosity should be inversely proportional to $\tau_{\text{lag}}$. To within the fairly small uncertainties in the exponent of eq. 2, this reproduces the empirical lag/luminosity relation.

The existence of the lag/luminosity relation implies that some quantity must be fairly constant from burst-to-burst, and equation 7 shows that this quantity is $M_{\text{jet}}/\Omega$. For $\Theta_{\text{jet}} < \Gamma^{-1}$, $\Omega$ will be $\pi \Gamma^{-2}$ and so the nearly constant quantity scales as $\Gamma^2 M_{\text{jet}}$.

A comparison of equations 2 and 7 requires that the constants are approximately equal. This gives a value for the emitting mass in the jet;

$$M_{\text{jet}} = 0.89 M_{\odot} (\Omega/4\pi).$$  

(8)

For $\Gamma = 100$ (and hence $\Omega \sim 3 \times 10^{-4}$), $M_{\text{jet}}$ is $2.2 \times 10^{-5} M_{\odot}$. This is just 7 Earth masses.

To recap, my explanation of the lag/luminosity relation is that the rate of cooling of the emitting region (and hence the lag) will depend on the rate at which the energy is radiated away (and hence the luminosity). When the burst has a high luminosity, then the individual particles in the emitting region are radiating their energy rapidly and cooling quickly so the time from when $E_{\text{peak}} \sim 200\text{keV}$ until the time when $E_{\text{peak}} \sim 30\text{keV}$ (i.e., the lag time) is short. When the burst has a low luminosity, the particles are cooling slowly and the lag time to cool from 200 keV to 30 keV is long.

GRB980425 (associated with SN1998bw [Galama et al. 1999]) has a measured peak luminosity ($2 \times 10^{46}\text{erg} \cdot \text{s}^{-1}$) which falls roughly five orders of magnitude below the
luminosity of typical bursts. The observed very high energy photons from ordinary GRBs implies that $\Gamma \sim 100$ or somewhat higher, so with $L \propto \Gamma^5$ (see below) the $\Gamma$ for GRB980425 must be 10 or somewhat higher. The lag of GRB980425 is 4 seconds, whereas its luminosity implies a lag of 3400 seconds from equation 2. This suggests that the emitting region cooled 850 times faster than if radiative cooling dominated. (Radiative cooling certainly dominates under normal conditions since individual bursts obey equation 3 [Liang & Kargatis 1996; Liang 1997].) Adiabatic cooling from the expansion of the material in the jet is a promising mechanism to provide this cooling (Liang 1997) and might allow the lag/luminosity relation to be a broken power law with a downturn to low luminosities. Indeed, Daigne & Mochkovitch (1998, Eq. 30) show that adiabatic cooling will dominate for $\Gamma$ values below some threshold.

4. VARIABILITY/LUMINOSITY RELATION

The variability/luminosity relation (Fenimore & Ramirez-Ruiz 2000) is

$$L = 10^{52}(V/0.01)^{2.5 \pm 1},$$

(Schaefer, Deng, & Band 2001). My explanation for this correlation between $L$ and $V$ is that both quantities are strong functions of the jet’s bulk relativistic expansion $\Gamma$ and hence are themselves correlated.

Burst light curves are presumed to be composed of emission from disconnected regions each of which generates a sub-pulse that combine to produce the pulses in the burst light curve (Fenimore et al. 1999). Throughout the time of the burst, the total number of emitting regions is $N_{\text{total}}$. Within the internal shock scenario, the time at which BATSE sees the light from each region is dictated by events in the collapsing object and hence does not depend on the jet’s $\Gamma$. So the observed duration of the burst or peak pulse ($\Delta T$) will also be independent of $\Gamma$. However, the observed duration of emission from each region ($\tau$) will scale as $\Gamma^{-2}$ (Fenimore, Madras, & Nayakshin 1997).

The variability $V$ arises from fast fluctuations that will depend on the average number of regions emitting at any given time ($N$). The $N_{\text{total}}$ sub-pulses of duration $\tau$ will be distributed over a time $\Delta T$, so that

$$N = N_{\text{total}}(\tau/\Delta T).$$

Since $\tau$ is proportional to $\Gamma^{-2}$ while the other factors do not vary with $\Gamma$,

$$N \propto \Gamma^{-2}. \quad (11)$$

If there are few regions emitting at any instant, then the light curve will be spiky because individual regions will appear relatively isolated. Alternatively, if $N$ is large, then the many independent regions will blur together to create a smooth light curve. The rms scatter of the light curve will scale as the square root of $N$. Thus the variance of the light curve will vary as $N$. The variability is the variance divided by the square of the peak count rate (Fenimore & Ramirez-Ruiz 2000) which scales as the square of $N$, so $V$ will scale as

$$V \propto N^{-1}. \quad (12)$$
This equation just quantifies the idea that if many independent regions contribute at any given time then the light curve should be smooth. From equations 11 and 12,

\[ V \propto \Gamma^2. \quad (13) \]

As \( \Gamma \) decreases, the emission from each region will broaden in time and merge to form a smooth light curve.

The luminosity of a burst will also depend on the jet’s \( \Gamma \). The total energy available will be \( \Gamma M_{\text{jet}} c^2 \) for constant efficiency, and this will be seen by BATSE over a time of \( \Delta T \). For \( \Theta_{\text{jet}} < \Gamma^{-1} \), the entire front surface of the jet will be visible from Earth so the luminosity should be \( \Gamma M_{\text{jet}} c^2 / \Delta T \). However, three correction factors are needed. The first factor is \( 4\Gamma^2 \), which corrects for the relativistic beaming of the radiation into a cone of opening angle \( \Gamma^{-1} \). The second factor is \( (1 + z)^2 - \alpha \), which is a K-correction for the cosmological expansion. Here, the average burst spectrum is taken as a power law of \( E^{-\alpha} \), while \( \alpha \) is typically around 2 (Schaefer et al. 1994, 1998). So the second correction factor is approximately unity on average. The third factor is \( \Gamma^\alpha \), which corrects for the red shift of the radiation in the BATSE energy band from the relativistic motion of the jet towards Earth. (There is no time dilation correction for the relativistic motion of the jet towards us, since the times of sub-pulses from individual regions are governed by processes in the rest frame of the exploding source. That is, each sub-pulse will become narrower as \( \Gamma \) increases, but the sub-pulses are seen by BATSE at the same times so the overall pulse made of smeared sub-pulses will have the same peak luminosity.) So for \( \alpha \sim 2 \), the third factor is roughly \( \Gamma^2 \). Taken together,

\[ L = 4\Gamma^2 \times 1 \times \Gamma^2 \times (\Gamma M_{\text{jet}} c^2) / \Delta T. \quad (14) \]

In all, the luminosity scales as

\[ L \propto \Gamma^5. \quad (15) \]

So as \( \Gamma \) increases, the available energy and the fraction of that energy received by BATSE increases to a high power.

Both \( L \) and \( V \) depend on the \( \Gamma \) value, and hence are correlated through \( \Gamma \). From equations 13 and 15,

\[ L \propto V^{2.5}. \quad (16) \]

This compares favorably with the observed variability/luminosity relation, in that the exponent of equation 16 (2.5) is well within the uncertainties for the exponent of equation 9 (2.5 ± 1).

In summary, I have simple and general explanations for the lag/luminosity relation and the variability/luminosity relation. When combined with the successful prediction that a particular lag/variability relation should be seen for 112 independent BATSE bursts, we can have every confidence in the use of the two distance indicators. So now we can get distances to GRBs from gamma ray light curves alone (with no need for counterparts), and this opens up a wide range of demographic studies involving all the many BATSE bursts.
REFERENCES

Daigne, F. & Mochkovitch, R. 1998, MNRAS, 296, 275.

Fenimore, E. E. 1999, ApJ, 518, 375.

Fenimore, E. E., Cooper, C., Ramirez-Ruiz, E., Sumner, M. C., Yoshida, A., & Namiki, M. 1999, ApJ, 512, 683.

Fenimore, E. E., Madras, C., & Nayakshin, S. 1997, ApJ, 473, 998.

Fenimore, E. E. & Ramirez-Ruiz, E. 2000, ApJ, submitted (astro-ph/0004176).

Galama, T. J. et al. 1999, Nature, 398, 394.

Kulkarni, S. 2000, 20th Texas Symposium on Relativistic Astrophysics, Austin.

Liang, E. P. 1997, ApJ, 491, L15.

Liang, E. P. & Kargatis, V. 1996, Nature, 381, 49.

Norris, J. P., Marani, G., & Bonnell, J. 2000, ApJ, 534, 248.

Schaefer, B. E., Deng, M., & Band, D. L. 2001, ApJ, submitted.

Schaefer, B. E. et al. 1994, ApJSup, 92, 285.

Schaefer, B. E. et al. 1998, ApJ, 492, 696.

Walker, K. C., Schaefer, B. E., & Fenimore, E. E. 2000, ApJ, 537, 264.

This manuscript was prepared with the AAS \LaTeX\ macros v4.0.