Thermal radiation of various gravitational backgrounds

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We present a simple and general procedure for calculating the thermal radiation coming from any stationary metric. The physical picture is that the radiation arises as the quasi–classical tunneling of particles through a gravitational barrier. We study three cases in detail: the linear accelerating observer (Unruh radiation), the non-rotating black hole (Hawking radiation), and the rotating/orbiting observer (circular Unruh radiation). For the linear accelerating observer we obtain a thermal spectrum with the usual Unruh temperature. For the non-rotating black hole we obtain a thermal spectrum, but with a temperature twice that given by the original Hawking calculations. We discuss possible reasons for the discrepancies in temperatures as given by the two different methods. For the rotating/orbiting case the quasi–classical tunneling approach indicates that there is no thermal radiation. This result for the rotating/orbiting case has experimental implications for the experimental detection of this effect via the polarization of particles in storage rings.

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I. INTRODUCTION

One of the most surprising results of doing quantum field theory on a curved gravitational background was the theoretical discovery that certain gravitational backgrounds gave rise to thermal radiation from the vacuum. The first, prototypical example is the Hawking radiation of a Schwarzschild black hole [1]. The original derivation given by Hawking is lengthy and complex making it difficult to connect a concrete physical picture with the calculation. Recently [2] [3] [4] [5] [6] [7] a simple derivation of Hawking radiation was presented. In this note we present a similar calculation based on the Hamilton-Jacobi equation. This calculation is based on the physical picture of Hawking radiation as a tunneling process – particles tunnel out from behind the horizon. The advantage of the present approach is that it can easily be applied to any stationary metric. Using this procedure we are able to make some interesting observations concerning the Hawking temperature of Schwarzschild black hole. We find that the temperature as calculated in the semi-classical tunneling approach is twice that of the original Hawking calculations [1]. In addition we apply our procedure to the Rindler [8] and rotating/orbiting system [9] and find that the semi-classical tunneling approach indicates that one does not observe thermal, Unruh-like radiation for circular motion. This has experimental implications for the experimental observation of this effect using the polarization of particles in circular particle accelerators as “thermometers” [10]. If we take into account the considerations of [11] along with the calculations of this note, we can conclude that the quasi–classical tunneling approach gives correct answers only in the case of stationary backgrounds with event horizons.

In this note we do not take into account the back–reaction of gravity on the quantum fluctuations of all other fields. In this context, to determine whether a particular gravitational background radiates or not we solve the Klein–Gordon

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equation in a curved background:

$$\left[-\frac{\hbar^2}{\sqrt{-g}} \partial_{\mu} g^{\mu\nu} \sqrt{-g} \partial_{\nu} + m^2\right] \phi = 0. \quad (1)$$

The signature of the metric is (-1,1,1,1) and $ds^2 = g_{\mu\nu}(x) dx^\mu dx^\nu$. $g_{\mu\nu} g^{\nu\alpha} = \delta_{\mu}^{\alpha}$. We are looking for the solutions of (1) having the form: $\phi(x) \propto \exp \{ -\frac{1}{\hbar} S(x) + \ldots \}$. Inserting this into (1) and taking the limit $\hbar \to 0$ we find to order $\hbar^0$ the following equation

$$g^{\mu\nu} \partial_{\mu} S \partial_{\nu} S + m^2 = 0, \quad (2)$$

which is just the relativistic Hamilton–Jacobi equation for the classical action of a relativistic particle in the curved background. The condition under which our approximation is valid is worked out in [12].

The metrics which we consider are stationary and, hence, have time–like Killing vectors. We are going to look for the particle–like solutions of (2) which behave as $\phi(x) \propto e^{-\frac{1}{\hbar} \text{Im} S_0}$, where $x_\mu = (t, \vec{x})$. $E$ is the energy of the particle. The wave function for such a solution behaves as $\phi \propto e^{-\frac{1}{\hbar} \text{Re} S_0}$ and corresponds to a state with definite energy. It is these states which are supposed to be observed by detectors [8].

If the solution $S_0(\vec{x})$ of (2) has a non–zero imaginary part for some particle trajectory this implies that the gravitational background in question is unstable with respect to radiation of such a definite energy states. What we show via our procedure is that quantum mechanically if there is a particle behind the horizon it can tunnel through it with the given rate. It seems that if there are no particles behind the horizon to begin with then we will not see any flux of particles from behind the horizon. In quantum field theory, however, the situation is different, because the number of particles is not conserved. In general if the corresponding calculation in the quantum mechanical limit shows a tunneling, then in quantum field theory one sees a flux of particles. In this case the wave function behaves as: $\phi \propto e^{-\frac{1}{\hbar} \text{Im} S_0}$, which describes tunneling of the particle through the gravitational barrier and leads to the decay rate of the background in question as follows: $\Gamma \propto |\phi|^2 \propto e^{-\frac{1}{\hbar} \text{Im} S_0}$. Now we are going to solve (2) for various well known gravitational backgrounds.

### II. Schwarzschild Black Hole Radiation

We first look at the radiation coming from a Schwarzschild black hole with mass $M$. Using the Schwarzschild background

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{2M}{r}\right)} + r^2 d\Omega^2, \quad (3)$$

the Hamilton–Jacobi equation becomes

$$-\frac{1}{\left(1 - \frac{2M}{r}\right)} \left( \frac{\partial S}{\partial t} \right)^2 + \left(1 - \frac{2M}{r}\right) \left( \frac{\partial S}{\partial r} \right)^2 + m^2 = 0. \quad (4)$$

We are interested in radial trajectories which are independent of the angles $\theta, \varphi$.

For the definite energy state we obtain

$$-\frac{E^2}{\left(1 - \frac{2M}{r}\right)} + \left(1 - \frac{2M}{r}\right) \left( \frac{dS_0}{dr} \right)^2 + m^2 = 0. \quad (5)$$

Despite the fact that the Schwarzschild metric has two disjoint parts separated by $r = 2M$, we can nevertheless consider solutions of (5) in these two regions and glue them by going around the pole in the complex $r$-plane. The solution is

$$S_0 = \pm \int_0^{+\infty} \frac{dr}{\left(1 - \frac{2M}{r}\right)} \sqrt{E^2 - m^2 \left(1 - \frac{2M}{r}\right)}, \quad (6)$$

where the limits of integration are chosen such that the particle goes through the horizon $r = 2M$. We focus on the integration through $r = 2M$ since this is exactly where the complex part of $S_0$ comes from. The $+$ ($-$) sign in front of this integral indicates that the particle is ingoing (outgoing). Although there is no classical path that crosses $r = 2M$ there are well established methods for dealing with differential equations with singularities as in (5).
review article by Brout and Spindel [13] or more recent work [14] [15] [16] shows, in the present Schwarzschild case, how to deal with the paths which cross \( r = 2M \).

Because there is a pole at \( r = 2M \) along the path of integration the integral will just be the Cauchy principle value. The imaginary part of the principle value of (6) is given by the contour integral over a small half–loop going around the pole from below from left to right. To explicitly take the imaginary part of the principle value we make the change of variables \( r - 2M = \epsilon e^{i\theta} \).

Then

\[
\text{Im} S_0 = \pm \lim_{\epsilon \to 0} \int_0^\pi (2M + \epsilon e^{i\theta}) \frac{e^{e^{i\theta}} i d\theta}{\epsilon e^{i\theta}} \sqrt{E^2 - m^2} \left(1 - \frac{2M}{2M + \epsilon e^{i\theta}}\right) = \pm 2\pi M E. \tag{7}
\]

All the above calculations can be easily performed for Reissner-Nordstrom and Kerr black holes. All relevant formulas for the latter case can be found in [17].

Using this result for \( \text{Im} S_0 \) for the outgoing particle the decay rate of the black hole is \( \Gamma \propto e^{-\frac{4\pi M E}{\hbar}} \). This is just the Boltzmann weight with the temperature \( T = h/4\pi M \). However, this disagrees with Hawking’s value of the temperature by the factor of 2. We now discuss this disagreement by examining different frames. In turn we will work in the isotropic frame and the Painlevé frame.

If we make a generally covariant transformation of the Schwarzschild coordinate frame (3) which involves only spatial coordinates this amounts to only a change of integration variables in (6) or the variables in (5). This does not change the result in (7). For example, if we make a change of variables to isotropic coordinates as in [6] [7]

\[ r = \rho \left(1 + \frac{M}{2\rho}\right)^2 \]

the Schwarzschild metric (3) becomes

\[
ds^2 = -\left(\frac{\rho - M}{\rho + M^2}\right)^2 dt^2 + \left(\frac{\rho + M}{\rho}\right)^4 (d\rho^2 + \rho^2 d\Omega^2).
\]

Now instead of (6) we find

\[ S_0 = \pm \int \frac{(\rho + M^2)^3}{(\rho - M^2)^3 \rho^2} \sqrt{E^2 - m^2} \left(\frac{\rho - M}{\rho + M^2}\right)^2 d\rho, \]

If one does the contour integration in the same manner as in (7) by making a semi-circular contour one apparently finds that \( \text{Im} S_0 = \pm 4\pi M E \). However one must also deform the contour from (7) using (8) and when this is done the semi-circular contour of (7) gets transformed into a quarter circle so that one gets \( i\pi \text{Residue} \) rather than \( \pi \text{Residue} \). One could already guess this because from (8) \( \rho \approx \sqrt{r} \) which for the contour in (7) means the semi-circular contour becomes a quarter circle. In detail

\[ r = 2M - \epsilon e^{i\theta} = \rho + M + \frac{M^2}{4\rho} \rightarrow \rho = \frac{1}{2} \left(M + \epsilon e^{i\theta} \pm (2M + \epsilon e^{i\theta})\sqrt{\epsilon e^{i\theta}/2}\right) \]

The leading order in epsilon is now \( \sqrt{\epsilon} \) so in the limit \( \epsilon \to 0 \) we find from the above equation \( \rho - \frac{M^2}{4\rho} = M\sqrt{\epsilon e^{i\theta}/2} \) instead of \( r - 2M = \epsilon e^{i\theta} \). Thus one sees that the semi-circular contour of the Schwarzschild frame gets transformed into a quarter circular in the isotropic coordinate frame so that the result of integrating (10) is \( i\pi \text{Residue} \) and we find again \( \text{Im} S_0 = \pm 2\pi M E \). Note that in isotropic coordinates the spatial part of the metric is no longer singular at the horizon. Thus both the Schwarzschild and isotropic frame give the same temperature for the thermal radiation, but this temperature is twice that given in the original quantum field theory inspired calculation [1].

The previous examples were related to one another via a transformation of the spatial coordinates. Mathematically this just corresponds to a change of variable between (6) and (10), and this can not change the results. However, one could consider a transformation which modifies the \( t \) coordinate. For example, consider the transformation

\[ dt' = dt + \frac{2M}{1 - \frac{2M}{r}} \frac{dr}{r}, \quad r' = r, \quad \Omega' = \Omega. \]

With this the Schwarzschild metric takes the Painlevé form

\[ ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + 2\sqrt{\frac{2M}{r}} dr dt + dr^2 + r^2 d\Omega^2, \]

\[ \text{Im} S_0 = \pm \lim_{\epsilon \to 0} \int_0^\pi (2M + \epsilon e^{i\theta}) \frac{e^{e^{i\theta}} i d\theta}{\epsilon e^{i\theta}} \sqrt{E^2 - m^2} \left(1 - \frac{2M}{2M + \epsilon e^{i\theta}}\right) = \pm 2\pi M E. \tag{7}
\]
where we have dropped the primes. The use of the Painlevé metric in the tunneling picture was first considered in [18] in the context of condensed matter “dumb” holes. This metric is regular (i.e. does not have the horizon for the incoming particles) at \( r = 2M \). However the notion of time is changed with respect to the Schwarzschild coordinates, so that the Hamiltonian–Jacobi equation for the definite energy state becomes

\[
- E^2 + \left( 1 - \frac{2M}{r} \right) \left( \frac{dS_0}{dr} \right)^2 + 2 \sqrt{\frac{2M}{r}} E \frac{dS_0}{dr} + m^2 = 0. \tag{14}
\]

The solution of this equation is

\[
S_0 = - \int_C \frac{dr}{1 - \frac{2M}{r}} \sqrt{\frac{2M}{r}} E \pm \int_C \frac{dr}{1 - \frac{2M}{r}} \sqrt{E^2 - m^2 \left( 1 - \frac{2M}{r} \right)}. \tag{15}
\]

This result can not be obtained from (6) via a change of integration variables because the transformation (12) does affect the time–like Killing vector. One can see that (15) differs from (6) by the first term. The first term in (15) arises from the coordinate change in (12), since

\[
\int E dt + S_0 = \int E dt' - \int \frac{dr}{1 - \frac{2M}{r}} \sqrt{\frac{2M}{r}} E + S_0, \tag{16}
\]

where \( S_0 \) is given by (6). Physically this new coordinate system corresponds to \( r \)-dependent, singular shift of the initial time. It can be shown that the observer corresponding to the Schwarzschild metric detects the modes which do not get converted into normalized modes in the metric (13) after the coordinate change (12).

If we choose the minus sign in (15) and the contour \( C \) as before the result is

\[
\text{Im} S_0 = -4 \pi M E. \tag{17}
\]

This is twice the result of (7) because the first integral in (15) gives the same contribution to the complex part of \( S_0 \) as the second one. We still see thermal radiation in this reference frame but with a smaller temperature by a factor of two: \( T = h/8\pi M \). This temperature agrees exactly with Hawking’s original result. In fact, Hawking in his calculation did not use the Schwarzschild frame but rather used a frame where the time \( t' \) was related to the Schwarzschild time via \( dt' = dt + dr/(1 - \frac{2M}{r}) \) which is similar to the transformation which takes one from the Schwarzschild metric to the Painlevé metric.

We now explore this apparent discrepancy between the Hawking temperature as calculated in the Schwarzschild and isotropic frames and the Painlevé frame. The decay rate can in general be written as

\[
\Gamma = e^{-2 \text{Im} S_0} = e^{-2 \int \frac{p_r dr}{\hbar}}. \tag{18}
\]

In other words the integrands in (6) (10) and (15) are simply the momentum in the radial direction. It was pointed out in [19] that \( \int p_r dr \) is not canonically invariant implying that the \( \Gamma \) given in (18) is not canonically invariant and not a proper observable. One can [19] define a canonically invariant \( \tilde{\Gamma} \) by using the canonically invariant quantity \( \oint p_r dr \) i.e. one can write the decay rate as

\[
\tilde{\Gamma} = e^{-\int \frac{p_r dr}{\hbar}}. \tag{19}
\]

Using (19) for \( \Gamma \) one can see that the Schwarzschild and isotropic frames still give the same result as before, and now the Painlevé frame also yields the same result. For the closed path in the integral of (19) consider a path which begins just outside the horizon, \( r_0 \), crosses to just inside the horizon, \( r_i \), and then returns i.e.

\[
\oint p_r dr = \int_{r_0}^{r_1} p_r^+ dr + \int_{r_1}^{r_0} p_r^- dr. \tag{20}
\]

where \( p_r^+ (p_r^-) \) are for the ingoing (outgoing) particles. For Schwarzschild and isotropic frames \( p_r^+ = -p_r^- \) so taking into account the negative sign from reversing the order of integration in (20) we find that for Schwarzschild and isotropic metric \( \oint p_r dr = 2 \int p_r dr \). So for these metrics \( 2 \int p_r dr \) is equivalent to the canonically invariant \( \oint p_r dr \). On the other hand for Painlevé metric \( p_r^+ = 0 \) while \( p_r^- \) is twice the value of \( p_r^- \) given by the Schwarzschild or isotropic metric, thus for the Painlevé metric the entire contribution to \( \text{Im} S_0 \) comes from the outgoing particle. The conclusion is that if one requires \( \Gamma \) be canonical invariant the tunneling calculation gives the same answer for all three forms of the non-rotating black hole metric. One finds thermal radiation, but with twice the temperature found in the original
Hawking calculation [1]. At this stage, without a more rigorous calculation, we can conclude the following: either (i) the tunneling calculations are not correct in detail as far as calculating the temperature of the thermal radiation, or (ii) the Hawking temperature for a non-rotating black hole really is twice as large as given by the original calculations.

We should stress at this point that even in the original Hawking derivation if one uses the Schwarzschild frame instead of the Kruskal frame, one obtains a temperature twice that given in [1]. As a final comment the factor of two difference between tunneling calculations and the original field-theoretic inspired calculation is not a new. In [21] a factor of two difference was found between the tunneling and field theoretic inspired calculations of the Hawking-Gibbons temperature in an expanding de Sitter Universe. Additionally in [22] it was argued that when one combines the equivalence principle with quantum mechanics that the Hawking temperature may in fact be twice that of the original calculation. This factor of two difference for the Schwarzschild black hole is investigated in more detail in [20].

III. UNRUH RADIATION

We now apply our procedure to obtain the thermal radiation seen by a permanently accelerating reference system, i.e. Unruh radiation [8]. Starting from Minkowski space we make a coordinate change to a permanently accelerating reference frame:

\[
    x_0 = \left( \frac{1}{a} + x \right) \sinh (at), \quad x_1 = \left( \frac{1}{a} + x \right) \cosh (at) - \frac{1}{a},
\]

where \(a\) is the acceleration. This gives the Rindler metric

\[
ds^2 = -(1 + a x)^2 dt^2 + dx^2 + dy^2 + dz^2.
\]

The coordinate change (21) has introduced a horizon at \(x = -1/a\). The physical meaning of the horizon is as follows: in a permanently accelerating system there are some points behind the accelerating observer from which a classical, free particle will never be able to catch up with the center of the accelerating reference system.

The Hamiltonian–Jacobi equation for this metric is

\[
    -\frac{1}{(1 + a x)^2} \left( \frac{\partial S}{\partial t} \right)^2 + \left( \frac{\partial S}{\partial x} \right)^2 + m^2 = 0.
\]

One can also obtain this by applying the coordinate transformation of (21) to the Hamiltonian–Jacobi equation in the Minkowski space. This coordinate change introduces a singularity at \(x = -1/a\) in (23). As a result the Hamiltonian–Jacobi equation for the constant energy state in the Rindler space becomes

\[
    -\frac{E^2}{(1 + a x)^2} + \left( \frac{dS_0}{dx} \right)^2 + m^2 = 0.
\]

For the rest of this section we consider solutions of the Hamiltonian–Jacobi equations which depend only on one space coordinate: \(x\). The solution of the Hamiltonian–Jacobi equations in Minkowski space does not have any imaginary part, while the one in the Rindler space does. This can be seen directly from the formula:

\[
    \text{Im} S_0 = \pm \int_C \frac{dx}{1 + a x} \sqrt{E^2 - m^2 (1 + a x)^2}.
\]

As in the Schwarzschild case we must be careful with our contour of integration since there is a pole at \(x = -1/a\) on the real axis. Also as in the Schwarzschild case we expect to get the imaginary contribution to \(S_0\) precisely from the integration around the pole. The contour \(C\) that we take is a half–loop encircling the pole from below. As before the result is given by the Cauchy principle value. Explicitly \(S_0 = \pm \frac{E}{a} \pi\). The decay rate is \(\Gamma \propto e^{-\frac{2 \pi E}{a} x}\), and is canonically invariant since one can write \(\Gamma = \exp(\text{Im} \int p_x dr/h)\), as for the Schwarzschild and isotropic metrics. This is the Boltzmann weight with a temperature equal to \(T = \frac{a}{2 \pi} \frac{E}{m}\) [8].

As in the previous case any coordinate changes which do not affect the time will not effect the temperature, since such transformations are simply changes of the integration variables in (25). Again the physical meaning of the effect can be described as follows: classically a free particle moving from the spatial infinity \(x = -\infty\) to \(x = +\infty\) can never catch up with an observer who is moving with a constant acceleration in the same direction. However, quantum mechanically the corresponding wave can tunnel through the gravitational barrier. This effect is described by the complex part of the classical action.

Thus, an observer who permanently accelerates (from \(t = -\infty\) to \(t = +\infty\)) with constant acceleration sees a thermal radiation bath. This naturally leads one to ask if one sees thermal radiation if the acceleration is for a finite time or if the acceleration is due to rotation/orbiting. Using our procedure we now address the rotating/orbiting system.
IV. ROTATING AND ORBITING FRAMES

We now move on to the case of the rotating/orbiting reference frame. Starting with the cylindrical coordinate system \( ds^2 = -dt^2 + dr^2 + r^2 d\varphi^2 + dz^2 \) we transform to a permanently rotating reference frame via \( t = t' \), \( z = z' \), \( r = r' \), \( \varphi' = \varphi - \omega t \), where \( \omega \) is the angular velocity. Then the metric becomes

\[
ds^2 = -(1 - \omega^2 r^2) dt^2 + 2 \omega r^2 d\varphi dt + dr^2 + r^2 d\varphi^2 + dz^2.
\]  

(26)

This metric has a light radius at \( \omega r = 1 \). In the rotating/orbiting cases the light radius plays a role similar to that of the horizons in the previous examples. Physically this light radius comes from the fact that in a rigidly rotating frame at some large enough radius points move with the velocity of light. The time of this metric coincides with the time of the original Minkowski metric.

We now solve the Hamiltonian–Jacobi equation for the metric (26). The equation is

\[-\left( \frac{\partial S}{\partial t} - \omega \frac{\partial S}{\partial \varphi} \right)^2 + \frac{1}{r^2} \left( \frac{\partial S}{\partial r} \right)^2 + \left( \frac{\partial S}{\partial \varphi} \right)^2 + m^2 = 0.\]

(27)

We are looking for solutions which are independent of \( z \). The action for the constant energy, \( E \), and angular momentum, \( \mu \), state is \( S = E t + \mu \varphi + S_0(r) \). With this (27) becomes

\[- (E - \omega \mu)^2 + \frac{\mu^2}{r^2} + \left( \frac{dS_0}{dr} \right)^2 + m^2 = 0.\]

(28)

We note that \( \mu \) is the angular momentum both in the Minkowski and in the rotating reference systems. However the energy in the rotating frame is \( E - \omega \mu \) while in the Minkowski frame it is \( E \).

We are looking for the imaginary part of

\[S_0 = \pm \int_{r_2}^{r_1} \frac{dr}{r} \sqrt{(E - \omega \mu)^2 - m^2 - r^2 - \mu^2}.\]

(29)

The limits are \( 0 < r_1 < 1/\omega < r_2 \), so that the particle trajectory crosses the light radius since this is where we expect to get the imaginary contribution if there is one. The integrand has the standard polar coordinate pole at \( r = 0 \), but it does not have a pole at, \( r = 1/\omega \), unlike the previous cases.

If we put \( \omega = 0 \), but keep \( \mu \) finite and take the limit of integration as \( r_1 < \mu/\sqrt{E^2 - m^2} \), then the integral in (29) has a non-zero imaginary part. The physical meaning of the imaginary part is that a classical particle with non-zero angular momentum can never reach the center around which it rotates. Classically it can not go beyond the limiting radius of \( \mu/\sqrt{E^2 - m^2} \), but quantum mechanically the corresponding wave can reach \( r = 0 \). This latter effect, which shows itself here through the imaginary part of the action, has nothing to do with the radiation we are looking for.

Next we consider \( \omega \neq 0 \), and \( \mu = 0 \). Under these conditions the integral in (29) does not have an imaginary part. Although the metric in (26) has a light radius, there is no obstacle for a classical particle at the spatial infinity \( r = +\infty \) to reach the rotational center \( r = 0 \), independently how fast it rotates.

The effect we are studying is supposed to come from the imaginary part due to \( \omega r = 1 \). To see this clearly we make the following change of the integration variables in (29): \( r = 1/\omega + a \) so the part of the integral that crosses the light radius becomes

\[S_0 = \lim_{\varepsilon \to 0} \int_{-\varepsilon}^{+\varepsilon} \frac{da}{1 + a \omega} \sqrt{(E - \omega \mu)^2 - m^2 - (1 + \omega a)^2 - (\omega \mu)^2}.\]

(30)

Taking the integral and the limit gives 0. For finite \( \varepsilon \) this integral can be complex, which simply means that \( r = 1/\omega \) is in the classically forbidden zone \((1/\omega < \mu/\sqrt{E^2 - m^2})\), but the limit \( \varepsilon \to 0 \) is still zero. Thus, we see that quasi–classical tunneling calculation tells us that there is no Hawking-like radiation for the rotating system.

One may ask if an orbiting reference frame which is displaced from \( r = 0 \) by \( R \) will see thermal radiation, i.e. consider a coordinate system displaced from the origin by a radius \( R \), and with this radius orbiting around the origin with the angular velocity \( \omega \). The coordinate change from Minkowski space is \( \vec{r}' = \vec{r} - \vec{R}(t) \), where \( \vec{r}' = (r, \varphi') \), and \( \vec{R}(t) = (R, \omega t) \). All other coordinates are left unchanged. As a result all axes associated with the rotating particle stay always parallel to the ones of the original, Minkowski reference system.

Starting from Minkowski metric, we obtain:

\[
ds^2 = -(1 - R^2 \omega^2) dt^2 - 2 \omega R \sin(\omega t - \varphi) dt \, dr dt + 2 \omega r R \cos(\omega t - \varphi) d\varphi dt + dr^2 + r^2 d\varphi^2 + dz^2.
\]

(31)
This metric is time dependent. In order to apply to our method we need to transform to a time independent metric. This is done by applying the transformation \(\varphi' = \varphi - \omega t\) on (31). The result is

\[
ds^2 = - \left[ 1 - (R^2 + r^2 + 2Rr \cos \varphi) \omega^2 \right] dt^2 - 2 \omega R \sin \varphi \, dr \, dt + 2r \omega (R \cos \varphi + r) \, d\varphi \, dt + dr^2 + r^2 \, d\varphi^2 + dz^2. \tag{32}
\]

This metric is time independent and the axes are rigidly fixed to the orbiting observer. The question of what kind of radiation, if any, such a circularly moving observer will see was first studied in \[9\]. Recent work on the question of what kind of radiation will be detected by an observer moving along some general trajectory in Minkowski spacetime can be found in \[23\] [24]. The second reference in particular has a fairly complete list of previous work. As with the rotating metric of (26) this metric has a light radius at \(1 - (R^2 + r^2 + 2Rr \cos \varphi) \omega^2 = 0\), but as for (26) we can show that there is no imaginary contribution to \(S_0\), so again quasi-classical tunneling approach tells us that there is no thermal radiation. According to the detailed investigation performed in \[11\] we can claim that the quasi-classical tunneling approach gives correct results only for backgrounds which have event horizons.

V. CONCLUSIONS

In this article we have given a general, quasi-classical procedure based on the Hamilton-Jacobi equation for calculating whether or not a particular gravitational background gives rise to thermal radiation or not. This procedure works for backgrounds which are stationary or have a time-like Killing vector. We have not taken into account the back-reaction of the radiation on the gravitational background. In order to take into account the back-reaction one would have to solve coupled equations of gravity and matter fields. We have shown that the quasi-classical tunneling calculations in the Schwarzschild and isotropic frames give a temperature for the thermal radiation which is twice as large as the original field theoretical calculations. Furthermore, if one requires that the decay rate, \(\Gamma\), be canonically invariant then the Painlevé frame also gives a temperature twice as large as the original calculation. To decide between the quasi-classical tunneling and field theoretic calculations one could try to perform a non-perturbative Schwinger-like [25] calculation for particle creation in a strong (gravitational) field. In lieu of such a calculation we can conclude: either (i) the tunneling calculation is wrong in detail or (ii) the Hawking temperature of a non-rotating black hole really is twice that given in the original calculation.

An interesting question is how to relate our quasi-classical observations to the picture of a simple detector moving along a particular trajectory \[8\], i.e. we would like to see how the presence of an imaginary contribution to the action in the tunneling calculation is related to the clicking rate of the detector in question \[9\] [11] [23] [24].

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