Analysis of student’s proof construction on logarithms

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Abstract. This research is a descriptive study that aims to describe the construction of student's proof on logarithmic material. Proof construction is the ability to construct direct or indirect proof using information provided such as definitions, principles, theorems, and assumptions in a logical and detailed manner. The research subjects were 40 students of the X-grade students of MIA 6 in Palembang 10 Public High School. The learning process is carried out with direct learning steps. The data collection technique used is a written test consisting of 3 questions. The data obtained will be corrected, grouped and analysed using evaluation of proof consisting of five categories to identify students' understanding of the construction of proof for each question. The results of the study stated that the construction of proof by students on logarithmic material in the X-grade students of MIA 6 in Palembang 10 Public High School was already in the K1 category. This is due students are able to provide valid general arguments and use them to construction of proof.

1. Introduction

Proof construction is an important ability for anyone involved with mathematics [1 - 3]. Proof is an important component in curriculum and mathematics learning [4]. An important ability that must be possessed by students and teachers is the ability of representation and the ability to prove [5]. Proof is one of the 5 goals of mathematics learning [6]. Also, proof is important in mathematics because it functions to find out whether or not a mathematical statement is true [4, 7, 8].

Proof is part of the curriculum at every level of education [6]. Reinforced by [9], the experience of high school students in the proof construction would have an impact on the ability to do proof construction at a higher level when in college related to mathematics [9]. Therefore, proof is very important to be applied at every level, including secondary school, so that it can add to the experience of students so that it will have an impact on the next level.

One of the material learned at the secondary school level and related to proof is logarithms. As in research [10] that is needed to present the proof of logarithmic properties in class X high school mathematics textbooks. Not only that, logarithms can also be associated with everyday life and are material related to other materials such as logarithmic functions [11, 12]. So that when students experience difficulties with logarithmic material, students will also experience difficulties with material related to logarithms. Therefore, logarithms are very important material and are related to proof.

Research related to proof has often been done on geometry [13 - 17]. In addition to geometry materials, there is also research on proving to on real analysis, the theory of number and integration [18 - 21]. Not only that, research on proof in students in addition to geometry material is also found in the set material, research [8]. The research on the proof especially the proof construction on geometry and trigonometry material [9, 22]. Research [9] focuses on measuring the ability of proof construction by analyzing using five indicators of proof construction capability, namely: identifying what is the data
from the statement (M1), identifying what is the conclusion of the statement (M2), stating the relationship between the data and conclusions by showing a warrant (M3), making guesses about key concepts that bridge the data and conclusions (M4) and evaluating the rules of drawing conclusions from the facts given or obtained critically (M5). Based on the five indicators of proof construction ability, there are criteria for achieving indicators of proof construction ability of students in research [9], as follows: excellent criteria if students are able to fulfill five indicators of proof construction ability (M1, M2, M3, M4, and M5), good criteria if students are able to meet four indicators of proof construction ability (M1, M2, M3, and M4), sufficient criteria if students are able to meet three indicators of proof construction ability (M1, M2, and M3) and criteria are not able to read proof if students are unable to fulfill either one or more than the indicators M1, M2, and M3. Research [22] also focuses on measuring the ability mathematical proofs construction but only by analyzing using two indicators mathematical proofs construction, namely: manipulating facts to show the truth of a statement and making connections between facts and the elements of conclusions to be proven. Meanwhile, this research will focus on the construction of student’s proof and will be analyzed using proof evaluation consisting of five categories to identify students’ understanding of the construction of proof for each question rather than based on indicators of proof construction ability. Five categories of proof evaluation, namely: valid general argument and proof (K1), valid general argument but not proof (K2), unsuccessful attempt for a valid general argument (i.e., invalid or unfinished general argument) (K3), empirical arguments (K4) and non-original arguments (K5) [23].

Therefore, researchers are interested in researching the construction of proof on students in logarithmic material which is included in important material in algebra. This is because to previous studies there has been no research on the construction of student’s proof on logarithmic material, there is only research [24] about understanding student’s property noticing in terms of gender differences using the matter of proof of logarithmic properties. Although the results are not to find out the extent of the construction of proof owned by students, but only want to compare the understanding of male and female students in the property noticing layer by using the matter of proving the properties of logarithms. Meanwhile, the results of this study will relate to how the construction of student’s proof so that if from the secondary school level students can do proof construction well then for a higher level will certainly get better results. However, if from the secondary school level students are not able to do proof construction properly then this will be input to find out how the construction of student’s proof so that henceforth we can find out what needs to be done to improve the construction of student’s proof that will have a good impact at a higher level.

2. Method
This research is a descriptive research that aims to describe the construction of student’s proof of logarithmic material. The research subjects were 40 students of the X-grade students of MIA 6 in Palembang 10 Public High School consisting of 5 groups based on the types of answers and proof evaluation categories. Each group is chosen 1 student as the subject to represent each category of proof evaluation. The first subject was TTA, the second subject was TC, the third subject was ODC, the fourth subject was RS and the fifth subject was NM. The study was conducted from August 19, 2019 to August 28, 2019. The data collection technique used was a written test consisting of 3 questions. The research procedure consisted of the stages of preparation, implementation, and analysis of data. In the preparation stage, researchers prepare instruments in the form of lesson plans and test questions. At the implementation stage, the study was conducted as many as 3 meetings with 2 meetings conducting learning conducted with direct learning steps and 1 meeting conducting a written test. After that, the test results obtained were analysed by evaluating the proof of student work results consisting of 5 assessment categories.

3. Result and Discussion
Data on the results of the construction of student’s proof on logarithmic material was obtained from the student's written test answer sheet. After that, the data are grouped according to the types of student's answers and analysed by evaluating proof based on the five categories shown in Table 1.
Table 1. Distribution of student’s proof construction.

| Question Number | Proof Evaluation Category | K1 | K2 | K3 | K4 | K5 | Total |
|-----------------|----------------------------|----|----|----|----|----|-------|
| 1               |                            | 6  | 30 | 0  | 4  | 0  | 40    |
| 2               |                            | 29 | 2  | 1  | 8  | 0  | 40    |
| 3               |                            | 22 | 5  | 3  | 0  | 10 | 40    |
| %               |                            | 47.5% | 30.9% | 3.3% | 10% | 8.3% | 100% |

Table 1 shows that the construction of student’s proof on the logarithms is mostly found in the K1 category. Category K1 shows that students have been able to do proof construction on the logarithmic material. This is because students can provide valid general arguments and can use these valid arguments to prove. Students can construction of proof if they can link definitions and theorems to solve given problems [21].

Following are some of the results of student’s answers related to proof construction based on the proof evaluation category:

**Figure 1. TTA’s answer results with the K1 category.**

Figure 1 shows that the results of the TTA’s answer contained no errors and the steps in constructing the proof were correct. In the results of the TTA’s answer in the 3rd row, the general argument given is valid “\( a^n \log b^m = c \iff a^m = b^m \)” and TTA has been able to use the argument to prove. It can be concluded that TTA has been able to do proof construction, so the results of TTA’s answers are included in the K1 category (valid general argument and proof).

**Figure 2. TC’s answer results with the K2 category.**

Figure 2 shows that the results of the TC’s answer contained a mistake in the 4th line, which was wrong in manipulating the mathematics “\( a^m = b \) with \( m = a \log b \)” which should be “\( a^m = b \)” even though the general argument given is valid “\( a \log b = m \iff a^m = b \)”. It can be concluded that the TC’s answer results are included in the K2 category (valid general argument but not a proof).
Figure 3. ODC’s answer results with the K₃ category.

Figure 3 shows that the ODC’s answer results in an error in the first line i.e. the general argument given is invalid but ODC convinces the argument by verifying the truth “\( a^{c \log b} = c \iff a^c = b \)” which should be “\( a^{\log b} = c \iff a^c = b \)” even though the steps in proof constructing are correct. It can be concluded that OCD has not succeeded in trying to become a valid general argument, so the results of OCD’s answers are included in the K₃ category (invalid / unfinished general arguments). General arguments can be given using logarithmic definitions and rules. Most students still do not understand what is known for example in order to form a valid general argument so that students experience errors when giving general arguments used in the proof which result in the arguments given are invalid. Students still have difficulty understanding what is known and what will be proven [25].

Figure 4. RS’s answer results with the K₄ category.

Figure 4 shows the results of the RS’s answer that there was a mistake in the 3rd row, namely the argument given was invalid and RS did not convince the argument by verifying the truth “\( a^n \log b^m = \frac{a^{n \log b^m}}{a^{n \log a^n}} \)” which should be “\( a^n \log b^m = \frac{a^{n \log b^m}}{a^{n \log a^n}} \)” even though the argument in the line 2nd is valid “\( a^n \log b^m = c \iff a^{nc} = b^{m\gamma} \)” which is the definition of logarithms. Therefore, it is clear that RS cannot use the definition as an argument to prove it. It can be concluded that the RS has not been able to do proof construction, so the results of the RS’s answers are included in the K₄ category (empirical argument). Students are still mistaken in using concepts and definitions to construction of proof [8].
3. Buatlah, jika $a$, $b$, dan $c$ bilangan real positif. $a 
eq 1, b 
eq 1$ maka berlaku $a \log b \times b \log c = a \log c$!

Penyelesaian:

![Image]

Figure 5. NM’s answer result with the $K_5$ category.

Figure 5 shows the results of NM's answer that there was a mistake in the first line that the general argument given was not original “$b \log c = x \log c$” which should be $b \log c = \frac{x \log c}{\log b}$. This shows that the argument given is irrelevant and has minimal involvement in proof so that the results of NM's answers are included in the $K_5$ category (non-original argument).

4. Conclusion
The results of the study stated that the construction of student’s proof on logarithms in the X-grade students of MIA 6 in Palembang 10 Public High School was already in the $K_1$ category. It can be concluded that overall students can do proof construction. Seen from some of the results of student’s answers analysed by evaluating proof-based on five categories. Students in the $K_1$ category have been able to do proof construction correctly. Students in category $K_2$ are still experiencing errors in mathematical manipulation which shows that it is not logical even though the general argument given is valid. $K_3$ category students are still not able to do proof construction because the arguments given are not valid, they cannot define the general form of logarithms correctly but can convince the arguments by verifying the truth. Students in the $K_4$ category are almost the same as students in $K_3$ category, only students in $K_4$ category cannot use the definition and cannot convince the argument by verifying the truth. While students in $K_5$ category are still not able to do proof construction because they are still wrong in defining the general form of logarithms and in using logarithmic rules.

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