Spin wave surface states in 1D planar magnonic crystals

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Abstract

We have investigated surface spin wave states in 1D planar bi-component magnonic crystals, localized on the surfaces resulting from the breaking of the periodic structure. The two systems have been considered: the magnonic crystal with periodic changes of the anisotropy field in an exchange regime and the magnonic crystal composed of Fe and Ni stripes in a dipolar regime with exchange interactions included. We chose the symmetric unit cell for both systems to implement the symmetry related criteria for existence of the surface states. We investigated also the surface states induced by the presence of perturbation of the surface areas of the magnonic crystals. We showed that the system with modulated anisotropy is a direct analog of the electronic crystal. Therefore, the surface states in both systems have the same properties. For the surface states existing in magnonic crystals in dipolar-exchange regime we demonstrated that the spin waves show distinct differences in comparison to the electronic crystals, which are due to long-range dynamic dipolar interactions. We found that tuning of the strength of magnetization pinning, resulting from the surface anisotropy or dipolar effect, is vitally important for the existence of surface states in magnonic crystals.

Keywords: spin waves, surface magnetism, magnetization dynamics, magnetic properties of nanostructures, surface states, magnonic crystals, magnonics

(Some figures may appear in colour only in the online journal)

1. Introduction

Surfaces are the natural limitation of any real physical system. The physical processes are determined and described not only by the particular form of differential equations but also by the geometry of surfaces and the boundary conditions on the surfaces [1].

Surfaces play an important role in magnonics [2, 3, 4, 5]. Constraints of the systems influence both the static magnetic configuration and the dynamic properties of the systems. For the magnetization dynamics both the static effective field describing the landscape perceived by all spin waves (SWs) (independent on magnitude and the direction of the wave vector), and the dynamical field determining the dynamical coupling of precessing magnetic moments are important. Even for the systems in the saturation state, for which the magnetization is collinear and constant in magnitude, the (nonlocal) static demagnetizing field can appear as a result of the presence of surfaces and interfaces. Also, the magneto-crystalline anisotropy field can be induced (locally) by the presence of surfaces/interfaces [6]. This field determines the strength of magnetization pinning on the surfaces. The dynamical demagnetizing field also can contribute to magnetization pinning on the surface [7, 8]. The surface anisotropy introduces additional torque acting on magnetic moments on the surface and can lead to the magnetization pinning or additional freedom depending on its orientation [9, 10]. The surface/interface induced dynamical demagnetizing field is also crucial for ensuring the coupling between precessing magnetic moments, which is necessary for the SW propagation with non-zero group velocity dependent on the direction and magnitude of the wave vector [11, 12]. Summarizing, the surfaces are responsible for the following features of magnonic system: (i) formation of the static magnetic configuration for the given external magnetic field due to spatial constraints and induction of the static demagnetizing field [12, 13], (ii)
forcing boundary conditions on the magnetization dynamics at the surfaces (external interfaces between magnetic and nonmagnetic material) due to surface anisotropy or dipolar pinning [10, 14] (iii) coupling of the precession of magnetic moments and tuning the group velocity of SWs due to dynamical dipolar field [2, 3].

In magnonics the terms surface states (waves) or edge states (modes) have usually specific meaning. The first one refers usually to the magnetostatic waves (in planar system) propagating perpendicularly to the in-plane applied magnetic field—i.e. for the so-called Damon–Eschbach (DE) configuration [15, 16, 17]. These waves are localized on the top or at the bottom face of the planar structure, depending on the direction of their wave vector. Magnetostatic surface waves decay exponentially inside the magnetic layer with the rate inversely proportional to their wavelength. On the other hand, the magnonic edge modes are the states localized inside the wells of demagnetizing field, located close to the surface of magnetic material, in the vicinity of the interface between two magnetic materials. The wells appear inside the material of higher saturation magnetization when the static magnetization has a non-zero component normal to the surface/interface and can confine the SWs of the frequencies below the ferromagnetic resonance frequency of a bulk material [16, 17].

In this paper we are going to investigate the surface states in their original meaning known from solid state physics. The surface states in solid state physics were considered initially in electronic systems as defect states, resulting from the termination of the infinite periodic structure of the ideal crystal [18–21].

The main goal of this paper is to discuss magnonic surface states, using the mentioned approach, and to show the similarities and differences between the electronic and magnonic surface states. Such discussion will be useful for the sake of clarification of the complex issue of spin wave surface states. The conditions for existence of surface states, localized on the surfaces terminating the periodic medium, depend both on the bulk parameters (determining periodic structure) and surface parameters (describing the perturbation introduced by the surface). We will investigate the existence of magnonic surface states, taking into account both the impact of bulk and surface parameters.

The solution of the Schrödinger equation for electronic waves in infinite periodic structures (crystals) has a form of Bloch waves [22]. In the energy ranges corresponding to the energy gaps, the wave vector is complex and solutions expand and decay exponentially for opposite directions of the wave vector. Such solutions are rejected because they cannot be normalized in an infinite periodic system. However, for the system limited by surfaces, we can accept the solution which decay exponentially inside crystal and match them to the solutions in vacuum, which also decay exponentially for the energies lower than the reference potential in vacuum determined by the work function. Such states, bounded at the surfaces of periodic medium (e.g. crystal lattice), are called surface states. The described theory of wave excitations in periodic systems can be generalized for any kind of physical system that can be described by the set of linear differential equations with spatially dependent and periodic coefficients [23]. Therefore, in linear regime, we can describe the wave excitation (electromagnetic waves, elastic waves, SWs) in artificial crystals (photonics crystals [24], phononic crystals [25], magnonic crystals (MCs) [26–28], formed by periodic modulation of (electric, elastic, magnetic) material parameters, in the form of Bloch waves. In these systems we can also observe the surface states localized on the boundary between the periodic and homogeneous medium. These states have the frequencies in the ranges of the gaps forbidden for Bloch waves.

The formulation of the conditions for existence of the surface states is an interesting problem. We should specify both the bulk parameters (describing the periodic structure and determining the dispersion relation of bulk modes) and surface parameters (describing the way how the surface is introduced in the system), to find the conditions for which surface states appear in the system. This problem was studied in depth for electronic systems, both for simple models [1] and for electronic superlattices [29–31]. Such studies were also conducted for photonic [32–34] and phononic systems [35]. The very interesting approach to the problem of existence of surface states was presented in the works of Artmann [36] and later Zak [37], in which two different mechanisms of induction of surface states were proposed. Both authors considered the model of electron scattering in periodic potential of atomic lattice with symmetric wells. They distinguished between Shockley surface states existing in the crystal in which the external potential wells are not (or are only slightly) deformed (in reference to the potential wells in the bulk region of crystal) and Tamm surface states induced by the deformation of external potential wells. For the Shockley surface states we can introduce symmetry related criteria [37]. These criteria allow us to point out the (energy) gaps allowed and forbidden for Shockley surface states (for the given values of bulk parameters). By the deformation of the external wells (described by the surface parameters) the Tamm states can be introduced in the gaps forbidden for the Shockley states.

The investigations of the surface states in magnonics are difficult because of the presence of two general mechanisms of localization [38–41]: (i) strong inhomogeneity of static demagnetizing field on the interfaces of the nanostructure—evoking the localization of edge modes and (ii) time reversal symmetry breaking due to magnetic dipolar interactions for partially confined geometries (planes, wires)—inducing the localization of magnetostatic surface modes/waves. These effects can be present also in the magnonic systems without periodicity. By reference to electronic and photonic systems, we consider as magnonic surface states only the states of frequencies in the ranges of frequency gaps forbidden for Bloch modes, which are localized on the surfaces breaking the periodic structure.

Let us consider the MC for which the inhomogeneous demagnetizing field on interfaces can lead to strong localization of SWs. The edge modes, confined in periodically distributed wells of demagnetizing field, will form almost flat magnonic band(s) of the frequency shifted, in the spectrum, outside the bands of propagating bulk modes [17]. However, such modes cannot be regarded as surface modes because they...
are not localized on the surface of MC with decaying amplitude in the bulk region of the structure. On the other hand, we can consider (Tamm) surface states, induced by the demagnetizing field—note that the termination of periodic structures results in the landscape of demagnetizing field, which is substantially altered in the surface regions. In our studies we will not investigate this case. For simplicity we will consider the geometry in which the static demagnetizing field is absent and perturbation of the surface regions (necessary for Tamm states) will be introduced by structural changes.

For MC, for which the external field is tangential to the infinitely extended surfaces terminating the periodicity [42], the localization resulting from the presence of periodicity and the localization characteristic for the magnetostatic surface modes may coexist with each other (be intermixed), which aggravate the study of surface states. In our studies we avoid this ambiguity by (a) considering the planar structure of finite thickness and (b) neglecting the SW component tangential to the surfaces terminating the periodicity. This component (of nonzero number wave) could be responsible for the localization of magnetostatic surface modes on the edges of the first and the last stripes, regardless on the periodicity of the system, in the case of the structures of large thickness (when the area of side faces of edge stripes is large).

In this work we plan to adapt the classification of electronic surface states (making a distinction between Shockley and Tamm states) to magnetic surface states [43]. The magnonic periodic condition can be formed both by modulation of structural [44] and material [52] parameters. Here, we are going to consider 1D planar MCs, which are finite sequences of two different kinds of stripes. The SW dynamics in these systems will be investigated in the saturation state with external magnetic field applied along the stripes. This magnetic configuration allows us to cancel static demagnetizing field and to avoid inducing edge modes on surfaces and interfaces. For the considered structures, the localization resulting from periodicity (which is essential for the appearance of Shockley and Tamm states) and the localization occurring due to time reversal symmetry breaking (all modes propagating from stripe-to-stripe are DE modes) takes place in orthogonal direction. Shockley and Tamm states have significant amplitude in initial (or/and final) periods of the structure, whereas the DE modes are localized in out-of-plane direction.

The first considered by as system is a direct counterpart of the electronic system—magnetic layer with periodically modulated (from stripe to stripe) in-plane anisotropy in an exchange dominated regime. The latter one is a more general system, composed of stripes differing in magnetic material parameters (in saturation magnetization and exchange length), with both exchange and dipolar interactions taken into account. This study will be the significant extension of the research presented in [44], in which we investigated two semi-infinite, 1D MCs composed of two kinds of magnetic layers. Two semi-infinite MCs were placed on opposite sides of wide magnetic layer, which ensured the coupling between them. In the work [44] we have not been forced to face the problem of boundary conditions on surfaces (interfaces between magnetic material and air), which result from the strength of magnetization pinning/releasing. We are going to investigate this issue in the presented paper by considering the finite MCs [45] with an assumed strength of the surface anisotropy (the first system) and MCs with dipolar pinning (the latter system).

The manuscript is organized as follows. Section 2 describes the geometry of the investigated structures and their material parameters. We present also the theoretical model which we use to calculate the frequencies and the spatial profiles of SW eigenmodes. In section 3, we compare the condition of existence of surface states in electronic system and its magnonic counterpart. Afterwards, in section 4, we discuss the magnonic system with dipolar interactions taken into account and the spatial modulation of the saturation magnetization and exchange length, which cannot be directly compared to the electronic system. In both studies we pay special attention to the role of the strength of surface pinning of magnetization (determined by the surface anisotropy or dipolar interactions)
on the conditions of existence of the surface states. The systems are investigated numerically with the aid of the plane wave method (PWM) and the finite element method (FEM) (performed in COMSOL Multiphysics package). The main outcomes of our research are summarized in section 5.

2. The model

We investigate the surface states in 1D MCs. We consider two systems: (i) the MC with periodic change of the anisotropy field in an exchange regime, being the counterpart of electronic system (figure 1(a)), (ii) the MC composed of the Fe and Ni stripes in a dipolar regime with an exchange interactions included (figures 1(b) and (c)). The MCs are terminated in symmetry points, i.e. in the middle of the stripes of low or high anisotropy (or the stripes made of Fe or Ni), to implement the symmetry related criteria for existence of Shockley surface states. We investigate also Tamm surface states, induced (i) by replacing Ni or Fe wires in the middle of edge cells with Py wires (green stripes) (figures 1(d) and (e)) and (ii) by the presence of perturbation of the surface areas of MCs (figures 1(f) and (g)).

The planar structure presented in figure 1(a) is made of Py layer finite in x-direction (800 nm width) and in y-direction (c1 = 15 nm thickness) but infinite in z-direction. We assumed that the static magnetic configuration of the considered structure is saturated by an external magnetic field $\mu_0H_0 = 100$ mT applied in the z-axis. We then introduced the periodical modulation of anisotropy field along the x-axis [46-48]. The anisotropy field has got the same direction as the external magnetic field—it is directed along the z-axis. By this we have introduced eight symmetric unit cells, each of $a_1 = 100$ nm width and $c_1 = 15$ nm thicknesses, characterized by periodically modulated magnetic anisotropy field $H_{a1} = 200$ mT, or $H_{a2} = 0$, periodically changing every $d_1 = 50$ nm width (along x-axis). The material parameters of Py, that we used, are: saturation magnetization $M_{Py} = 0.7 \times 10^6$ A m$^{-1}$, exchange constant $A_{Py} = 1.1 \times 10^{-11}$ J m$^{-1}$, gyromagnetic ratio $\gamma = 176$ GHz T$^{-1}$.

The planar structures (as before—finite in x- and y-directions, infinite in z-direction), shown in figures 1(b)-(g), are composed of ferromagnetic wires arranged in periodic sequences. The infinitely long wires of the same dimensions are made of Ni, Fe and Py. We investigate MCs consisting of eight symmetric unit cells, made of wires of $d_2 = 250$ nm widths (along the x-axis), $c_2 = 30$ nm thicknesses (along the y-axis) and infinite length along the z-axis. The wires are in direct contact, which ensures the exchange coupling between the successive wires. We have also considered dipolar interactions between those wires. We assumed that the static magnetic configuration of considered structures is saturated by an external magnetic field $\mu_0H_0 = 100$ mT applied in the direction of the wire’s axis. The material parameters of Ni, Fe and Py are: saturation magnetizations $M_{Ni} = 0.484 \times 10^6$ A m$^{-1}$, $M_{Fe} = 1.752 \times 10^6$ A m$^{-1}$, $M_{Py} = 0.86 \times 10^6$ A m$^{-1}$, exchange constants: $A_{Ni} = 0.858 \times 10^{-11}$ J m$^{-1}$, $A_{Fe} = 2.0992 \times 10^{-11}$ J m$^{-1}$, $A_{Py} = 1.299774 \times 10^{-11}$ J m$^{-1}$. We used a slightly different value of material parameters for Py in MC with dipolar interaction included to observe the coexistence of Tamm and Shockley states (see figure 3(e) and discussion below). The gyromagnetic ratio $\gamma = 176$ GHz T$^{-1}$ is assumed the same for all materials.

To describe the magnetization dynamics we solve the Landau–Lifshitz equation (LLE), which is the equation of motion for the magnetization vector $\mathbf{M}$:

$$\frac{d\mathbf{M}}{dt} = \gamma \mu_0 \left[ \mathbf{M} \times \mathbf{H}_{\text{eff}} + \alpha \frac{\mathbf{M} \times (\mathbf{M} \times \mathbf{H}_{\text{eff}})}{M_S^2} \right],$$

where: $\alpha$—is damping coefficient $\mu_0$—is permeability of vacuum, $M_S$—is saturation magnetization and $\mathbf{H}_{\text{eff}}$—is effective magnetic field. The first term in LLE describes precessional motion of the magnetization around the direction of the effective magnetic field and the second term enriches that precession with damping. We neglect damping in these calculations, putting $\alpha = 0$. The effective magnetic field in general can consist of many terms, but in this paper we will consider: external magnetic field $\mathbf{H}_0$, nonuniform exchange field $\mathbf{H}_ex$, dipolar field $\mathbf{H}_d$ and anisotropy field $\mathbf{H}_a$: $\mathbf{H}_{\text{eff}} = \mathbf{H}_0 + \mathbf{H}_ex + \mathbf{H}_d + \mathbf{H}_a$.

We consider the linear regime of magnetization dynamics. Therefore, it enables us to clearly discuss the SW motion on the background of the static magnetic configuration in a saturation state and investigate the SW eigenmodes in the system characterized by harmonic dynamics in time: $e^{i\omega t}$, where $\omega$ is an angular eigenfrequency. The LLE (1) for the magnetization vector $\mathbf{M}$ can be linearized in the form of a set of two differential equations for complex amplitudes of dynamical components of magnetization $m_x$ and $m_y$ [49]:

$$i \frac{\omega}{\mu_0} m_x(r) = (H_0 + H_{ex}(r)) m_x(r) + \frac{2}{\mu_0} \nabla \cdot \left( \frac{A(r)}{M_S(r)} \nabla m_y(r) + M_S(r) \frac{\partial}{\partial y} \varphi(r) \right),$$

$$i \frac{\omega}{\mu_0} m_y(r) = -(H_0 + H_{ex}(r)) m_y(r) + \frac{2}{\mu_0} \nabla \cdot \left( \frac{A(r)}{M_S(r)} \nabla m_x(r) - M_S(r) \frac{\partial}{\partial x} \varphi(r) \right).$$

(2)

The anisotropy field and homogeneous external field have only z-component: $\mathbf{H}_a = [0, 0, H_a]$, $\mathbf{H}_0 = [0, 0, H_0]$. Exchange field for the saturated system in DE geometry has only dynamical components: $H_{ex}(r, t) = [h_{ex_x}(r)e^{i\omega t}, h_{ex_y}(r)e^{i\omega t}, 0]$. The relation between the dynamical exchange field and the amplitudes of dynamical magnetization $\mathbf{m}(r, t) = \mathbf{m}(r)e^{i\omega t} = [m_x(r)e^{i\omega t}, m_y(r)e^{i\omega t}, 0]$ can be expressed in linear approximation as [50]:

$$H_{ex}(r, t) = \frac{2}{\mu_0 M_S(r)} \nabla \cdot \left( \frac{A(r)}{M_S(r)} \right) \nabla \mathbf{m}(r)e^{i\omega t}.$$

(4)

The second terms on the right-hand side of equations (2) and (3) have exchange origin and result directly from equation (4). Our calculations are based on FEM in which the model of continuous medium is investigated. The exchange interactions between the successive stripes is included by
appropriate boundary conditions at the Ni/Fe interfaces. We use the natural boundary conditions, resulting from an exchange operator (4) implemented in the linearized LLE equations (2) and (3). As was pointed out in [51] these boundary conditions are: (i) continuity of dynamical components of magnetization and (ii) continuity of first derivatives of the dynamical components of magnetization multiplied by factors: $A/M_S$.

For considered geometry the static components of the demagnetizing field are equal to zero, nonzero are only $x$- and $y$-components of the dynamical dipolar field. Using the magnetostatic approximation, the demagnetizing field can be expressed as a gradient of the scalar magnetostatic potential:

$$H_d(r, t) = [h_{d,x}(r)e^{i\omega t}, h_{d,y}(r)e^{i\omega t}, 0] = - \nabla \varphi(r)e^{i\omega t}. \quad (5)$$

With the aid of the Gauss equation, we obtain the following equation, which relates magnetization and magnetostatic potential:

$$\nabla^2 \varphi(r) - \frac{\partial m_x(r)}{\partial x} - \frac{\partial m_y(r)}{\partial y} = 0. \quad (6)$$

Equations (5) and (6) can be used to find dynamic components of demagnetizing field, implemented already in equations (2) and (3), i.e. last terms on the right-hand side of these equations.

We solve the linearized LLE in the form of the eigenvalue problem (2) and (3) by use of FEM with the aid of COMSOL Multiphysics software.

3. The surface states in anisotropy modulated MC

The MC in an exchange regime for which the saturation magnetization $M_S$ is spatially homogeneous is the direct counterpart of the electronic system. For a 1D system of this type, the linearized LLE (2) and (3) can be written in the form of the Schrödinger equation, using effective mass approximation [44]:

$$-\frac{d}{dx}\lambda_x^2(x)\frac{d}{dx}m_x(x) + v(x)m_x(x) = \Omega m_x(x), \quad (7)$$

where

$$v(x) = \frac{H_0 + H_s(x)}{M_S}, \quad \Omega = \frac{1}{\gamma \mu_0 M_S} \quad (8)$$

play the role of fictitious potential and energy by comparison to the electronic system. The inverse of the squared exchange length $1/\lambda_x^2$ is the counterpart of effective mass. If an exchange length is also spatially constant, then equation (7) is mathematically equivalent to the 1D stationary Schrödinger equation. The spatial changes of fictitious electrostatic potential $v(x)$ are then expressed by an anisotropy field $H_s(x)$. It is worth noting that for purely exchange waves the precession is circular and the complex components of dynamical magnetization are related by a simple relation: $m_x(x) = \pm im_y(x)$, which means that their amplitudes are the same but their harmonic oscillations $e^{i\omega t}$ are shifted by the temporal phase $\pi/2$.

Due to the mathematical form of equation (7), the Bloch functions (spin wave eigenmodes) and frequency spectrum of considered MC share properties of the electronic Bloch functions (electronic eigenmodes) and energy spectrum for 1D potential. Therefore, we can almost straightforwardly apply the formalism describing the condition of the existence of electronic surface states, proposed by Zak [31, 37], to the magnonic surface states in 1D MCs with modulated anisotropy field in an exchange regime. To find the electronic eigenmodes in a (semi-)finite 1D crystal, we can match the logarithmic derivatives at the surface to the solutions in the vacuum and at the crystal. The logarithmic derivative (of electronic wave function) in vacuum $\rho_V$ has constant sign—negative (positive) on the left(right) surface. When the crystal is terminated in the symmetry point of the periodic potential (in the middle of barrier or well) then the logarithmic derivative of Bloch function $\rho_B$ is real and monotonous function of energy in every energy gap, reaching zero and pole on opposite edges of each gap. The sign of $\rho_B$ is then constant for a given gap—the sign of $\rho_B$ is also (similarly like for $\rho_V$) flipped after swapping between left and right surface. Therefore, by matching logarithmic derivatives $\rho_B$ and $\rho_V$, and then comparing their signs, we can point out the gaps forbidden or allowed for the surface states. We can also prove that for semi-infinite crystal terminated in symmetry point, only one surface state can appear in one gap. In order to use this formalism for exchange SWs in MCs we have to impose the boundary conditions on the surfaces. SWs cannot propagate in nonmagnetic medium (e.g. vacuum) and it is not possible to extend there the solution of LLE. Hence, instead of matching $\rho_B$ and $\rho_V$, we have to compare the $\rho_B$ to the ratio of dynamical magnetization and its spatial derivative on the surface, resulting from the magnetization pinning in an exchange regime. The arbitrary strength of pinning can be imposed by application of the so-called Rado–Weertman boundary conditions [9]:

$$m_x(\mp p \frac{dm_x}{dx}) = 0, \quad (9)$$

where the $\mp$ sign in (9) refers to the left and right surface and index $\alpha = x, y$ denotes different components of the dynamical magnetization.

From equation (9), we can notice that the inverse of pinning parameter $1/\rho_p$ is nothing else than the logarithmic derivative of the dynamical magnetization, and it can be matched directly to the logarithmic derivative of the Bloch function.

The strength of pinning depends on the state of the surface and is determined by the surface magnetocrystalline anisotropy. Hence, the pinning parameter is defined by the ratio of surface anisotropy $K_S$ to an exchange stiffness constant $A = \mu_0 M_S^2 \lambda_x^2$. It means that the value of logarithmic derivative on the surface is fixed and does not depend on the frequency. This behavior is different for electronic surface states in which logarithmic derivative in vacuum decreases with increasing energy $\rho_V \propto \sqrt{E - E_0}$. It means that the pinning of an electronic surface state is increasing (with increasing energy), and it is reaching the limit of an ideal pinning for $E = E_0$ (i.e. the maximum energy for which the electronic surface state can be bound at the surface).

We investigated the planar MCs presented in figure 1(a) with spatially modulated anisotropy field. In our studies we solve numerically the general form of linearized LLE (2) and
in which we exclude dipolar interactions and assume the homogeneity of saturation magnetization \( M_s \) and exchange stiffness constant \( A \). We consider a planar system in which the anisotropy field varies abruptly each 50 nm distance in the \( x \)-direction, taking two values: \( H_{A1} = 200 \text{ mT} \) (presence of anisotropy field) or \( H_{A2} = 0 \) (anisotropy field not present) (see figure 1(a)). We assumed the large value of the ratio of lattice constant \( t_x \) (being the doubled width of the stripe) to the thickness \( t_l \) of the structure. This allowed us to avoid the SW quantization along the thickness in the low frequency range. This assumption, being not so far from reality, ensures that the magnonic states, originating from a few lowest magnonic bands, will have practically uniform SW profiles along the thickness. The spectrum of these states will be practically the same as a spectrum of the layered system (i.e. systems without confinement in \( x \)- and \( y \)-direction). To observe the surface states we have also limited our system in the direction of periodicity (\( x \)-direction) and took eight complete and symmetric unit cells. At the side faces (i.e. at the external edges of the first and the last cell) we applied the Rado–Weertman boundary conditions \([9]\), which allowed us to impose the different strength of magnetization pinning. In calculations, we assumed that the external magnetic field and anisotropy field are oriented along the stripes. However, this particular orientation is not important for exchange SWs. For the case in which the \( z \)-component of a wave vector (component along the stripes) is equal to zero, the considered system can be treated as 1D and can be discussed using the simplified equations (7) and (8). On the other hand, the geometry and an external field orientation for the considered system (see figure 1(a)) is the same as for the MCs operating in dipolar regime, which we will discuss later (see figure 1(b)). Therefore, the system presented in figure 1(a), in which only the exchange interactions are included, can be treated as a reference one.

Figure 2 presents the outcomes of calculations for the MC with modulated anisotropy field for the discussed above structure. We start our calculations with verifying the model (equations (2) and (3)) and the geometry (figure 1(a)) implemented in the FEM solver. Initially, we investigated the infinite MC to obtain the dispersion relation (figure 2(a)) and to mark the position of frequency gaps (gray areas in figure 2, separated by the bands B1, B2, ..., B4). We compare the dispersion relation calculated using FEM (red line) to the result obtained by PWM (black dashed line). In the FEM calculations we assumed the Bloch boundary conditions linking the edges of one unit cell, while the PWM computations are naturally designed for infinite periodic structures. We achieved good agreement between both methods. In figures 2(b)–(d) we show the spectra of eigenfrequencies for finite MC consisting of eight symmetric unit cells, terminated in the middle of the stripe without anisotropy field (\( H_{A2} = 0 \)). The spectra are presented in the form of integrated density of states (IDOS) \([52]\). Dependences of IDOS on frequency are set together with unfolded dispersion relation (figure 2(a)), which approaches to the quadratic dependence \( f \sim k^2 \). Due to this relation, the successive bands (delimited in the domain of wave number \( k \) by successive edges and centers of Brillouin zones) have increasing width and are characterized by the growing group velocity.

![Figure 2](image-url)  
**Figure 2.** The spectra of MC with periodically modulated anisotropy field (see figure 1(a)) in an exchange regime, being the direct counterpart of electronic system. The dispersion relation for infinite MC (a) was calculated with the aid of PWM (black dashed lines) and FEM (red lined) to cross-check both methods and to determine the position of frequency bands (B1–B4) and gaps (gray areas) for finite structures (b)–(d). For finite systems, composed of eight symmetric unit cells, assuming different boundary conditions (i.e. different strength of magnetization pinning taken for the external edges of the first and the last cell, \( p \) approaching to \( \infty \), and equal 2.82 nm\(^{-1} \) and 0 in (b)–(d), respectively), we calculated (using FEM) the IDOS. We found two (almost degenerate) Shockley surface states (marked by \( S \) labeled in (c)) only for partial pinning. The profiles of selected spin wave eigenmodes (\( m_x \), component of dynamical magnetization) were plotted in the insets.

In figures 2(b)–(d) we present the discussed dependences obtained for the different strength of pinning on the edges of the external cells. We chose three different values of pinning parameters: \( p \to \infty \) (figure 2(b)—corresponding to completely unpinned magnetization, \( p = 2.82 \text{ nm}^{-1} \) (figure 2(c))—for partial pinning and \( p = 0 \) (figure 2(d))—for pinned magnetization. For each of those values we calculated the dependence of IDOS on frequency. The plateaus in these dependences correspond to the ranges of frequency gaps. The distinctive step appearing in the gap between the first and the second band for partially pinned magnetization (figure 2(c)) denotes the presence of double degenerated surface state. We
plot the profile of dynamical component of magnetization $m_t$ to check the changes of magnetization pinning for different values of pinning parameter (compare e.g. profiles of the lowest mode in the insets of figures 2(b) and (d)) and to verify the surface or bulk character of the modes. We found that the 8th and 9th modes are localized at the surface, which means that they are surface modes (see the inset in figure 2(c)). Due to the mirror symmetry of the whole structure, we find the even and the odd surface mode (plotted by solid and dashed lines in the inset) with respect to the center of the MC.

These surface modes can be considered as Shockley surface modes because they appear in the structure terminated in its symmetry points (the center of stripe of lower anisotropy field). The mathematical form of differential equation (7), identical to the Schrödinger equation, ensures that all properties of electronic Shockley (and Tamm) states will be observed in this kind of magnonic system. We checked that the following effects are observed in MC with spatial modulation of anisotropy field:

- For every frequency inside a given magnonic gap, the logarithmic derivative, taken at one of two symmetry points of the structure (the centers of stripe of a different kind), has constant sign. The sign of logarithmic derivative determines whether the magnonic gap is allowed or forbidden for Shockley states.
- If we change the unit cell by swapping the areas of low and high anisotropy (and shifting the location of surfaces by $a_2/2$ to the other symmetry point of the structure), then the sign of logarithmic derivative will be reversed only for the gaps opened at the edge of Brillouin zone (the real part of complex wave number equals $\pi/a_0$). It means in our case that the Shockley states in the gap between the first and the second band disappear, but they will be induced in the gap between the third and the fourth band.
- Perturbing the geometry of the MC (or changing the value of anisotropy field) close to its surfaces, we can induce the so-called Tamm states.

Let us discuss the impact of the pinning strength on the conditions of existence of surface states in the considered system. We can see that for an ideal pinning (figure 2(b)) or complete unpinning (figure 2(d)) of the magnetization on the surface we do not observe Shockley states. It results from the fact that for an ideal pinning (unpinning) of magnetization, the logarithmic derivative of Bloch function at symmetry point $\rho_B$ is matched at the surface to the value $1/p \to \pm\infty$ ($1/p = 0$). For such values of logarithmic derivative $\rho_B$ at the edges of the gaps, the surface states will not be induced. We know also [37] that the logarithmic derivative $\rho_B$ (independently on its sign in given gap) is an increasing function of the frequency in each gap. This property implicates that the decrease of pinning parameter from $\infty$ (unpinned magnetization) to 0 (pinned magnetization) will shift the position of Shockley states from the bottom of the gap to its top, which is observed in figures 2(b)–(d). It is also worth noticing that by changing the sign of the surface anisotropy constant, we can reverse the sign of the pinning parameter. We do not have such freedom in the case of electronic states for which the logarithmic derivative in the vacuum must always be positive (negative) on the left (right) surface to ensure the exponential decay of electronic wave function in $\pm\infty$. The change of the sign of the pinning parameter swap all gaps allowed for Shockley states to forbidden ones and vice versa. Summarizing this discussion, we can conclude that to observe the Shockley states in the given magnonic gaps we need the partial pinning of magnetization on the surface, with the sign of pinning parameter matching the sign of the logarithmic derivative of Bloch function.

4. The surface states in bi-component MC

In the previous paragraph we have discussed the localization properties of SWs in magnonic systems made of homogeneous magnetic material with spatial modulation of anisotropy field and only exchange interactions taken into account—such a system is a good counterpart of electronic system. In this paragraph we consider the surface states in bi-component MCs in general form, in which both dipolar and exchange interactions are included. As above, we investigate samples in the DE geometry. This leads to the situation, in which there is no static demagnetizing field, whereas the dynamical components of demagnetizing field have got different values for the out-of-plane component and for the in-plane component. In the out-of-plane direction, the interfaces with the big contrast of $M_S$ are present, whereas in the in-plane direction the contrast of $M_S$ is smaller. As a result, the precession is elliptical. The out-of-plane component of magnetization, in general, is smaller.

In figure 3(a) we present the dispersion relation of infinite MC composed of Ni and Fe stripes in the dipolar-exchange regime. We can see that the dispersion relation exhibits the features characteristic for the magnetostatic SWs. The group velocity is constant, non-zero in the vicinity of the Brillouin zone center and it achieves the largest values there. With the increase of the wave vector $k$, the group velocity of purely magnetostatic spin waves decreases. Whereas for dipolar-exchange waves, the spin wave dispersion relation transforms into a parabolic dispersion for large $k$. It is the reason why each next band is thinner (which is just the opposite case as in the dispersion in the exchange regime). We have calculated the SW spectra using two methods: the PWM (marked as the dashed black lines) and FEM (marked as the red lines). Two methods were used to cross-check the obtained results and to determine the position of the frequency bands (marked as the B1–B4) and the bandgaps (marked as the grey areas). We noticed that PWM gives slightly lower position of some magnonic bands. We can attribute this effect to the assumption we use in PWM, in which ideally uniform distribution of SW amplitude in out-of-plane direction is considered.

Firstly, we analyze the SW spectra and the SW modes of the finite structure without any surface perturbation—figure 3(b). The structure consists of eight symmetric unit cells, containing the Ni stripe in the center and half of the Fe stripe on each side of the Ni stripe—see figure 1(b). Bulk modes have greater amplitude in the Ni stripes, i.e. in the material having
smaller saturation magnetization (which means lower FMR frequency). In the whole structure we have eight wells (Ni stripes), in which the spin wave amplitude is concentrating for a few of the lowest bands. There are eight states for each band due to the coupling between those wells. For this structure, we do not observe any surface states. Note that the magnetization is fairly well pinned at the edges of external stripes (see the zoomed part of the SW profile in figure 3(b)). This pinning emerges naturally as a consequence of the occurrence of the dipolar interactions in the finite structure [8] of larger ratio of the width to thickness. It results from the presence of the dynamical components of the dipolar field (it is so-called dipolar pinning).

Afterwards, we have introduced the perturbation of the surface regions of the MC presented in figure 3(b) (see also figure 1(b)) by replacing the Ni stripes in the cells next to the surface with Py stripes, see figure 3(c) (or figure 1(e)). Permalloy, which is the nickel-iron magnetic alloy, has got intermediate material parameters. So it has got higher than Ni, but lower than Fe, FMR frequency. The mentioned replacement results in the lack of concentration of the SWs in bulk modes at the first and at the last unit cells. As a result we obtain six wells, in which SWs of low frequencies concentrate. For each lower band B1–B4 we have only six modes because of that. Surprisingly, with introducing Py stripes, we obtain surface states in this perturbed system. Intuitively, we should expect that introducing Py, as a material of higher than Ni FMR frequency, will not lead to the localization of the low frequency SWs in the stripe made of this material. This observation shows that the mechanism of induction of surface states of the Tamm-type, known from electronic theory, is general. The crucial factor here is the perturbation of the surface regions in reference to the structure of symmetric cells (see figure 3(b)). The Tamm states, which we found, appear in the third bandgap (between bands B3 and B4). We will show later that by introducing some geometrical changes in the external cells of the MC, we can induce the Tamm states in the other frequency gaps as well.

Thereafter, we have investigated the structure (presented in figures 3(d) and 1(c)), in which the Fe and Ni stripes are exchanged (in comparison to figure 3(b)) (or figure 1(b)), so that the Fe stripe is at the center of the unit cell, having one half of the Ni stripe on each side. In this case MC is terminated by the halves of Ni stripes. Redefining the unit cell is indifferent for infinite MCs. However, such change in finite MCs causes shifts of the surfaces from one symmetry point (of infinite structure) to the other one, i.e. from the center of Fe stripe to the center of Ni stripe. According to the findings of Zak [37], the change of the symmetry point at which the surface is located, turns the gaps opened at the edge of the Brillouin zone from allowed (forbidden) to forbidden (allowed) ones for the Shockley states. It seems that this mechanism works also in magnonics for an exchange-dipolar waves. In the structure presented in figure 3(d), we observed the appearance of the Shockley states in the first- (between bands B1 and B2) and the third- (between bands B3 and B4) frequency gaps. These gaps are opened at the edge of the Brillouin zone, in which the real component of the complex wave number is equal to $\pi a_2$ (see figure 3(a)). Note that for the structure with halves of the Fe stripes on the edges of the structure (figure 3(b)), we have not found the surface states. The Shockley states that we found (in the structure shown in figure 3(d)) are strongly localized exactly at the surface of the MC, i.e. at the Ni stripes of half width.

We have then perturbed the surface regions of MC from figure 3(d) (or figure 1(c)) by replacing the Fe stripe in the
cells nearest to the surface by Py stripes—figure 3(e) (or figure 1(d)). We find additional surface states (of Tamm-type) appearing in the other gaps, while Shockley-type surface states are still present in perturbed structure (see figure 3(e)) in the gaps, in which they initially appeared in the absence of perturbation. The distribution of SW amplitude for Shockley states in the considered structure (figure 3(e)) is similar to those observed in an unperturbed MC (figure 3(d)). Some of the Tamm states are found in the same frequency gap as Shockley states (i.e. the gap between bands B3 and B4)—see the insets of figure 3(e). This property of surface states is different than in electronic crystals or MCs with only exchange and anisotropy included. For these systems (for fixed values of the structural and material parameters), the Tamm and Shockley surface states cannot exist at the same frequency gaps. Here we can find both kinds of surface states appearing at the same gap. There is also another peculiar feature of magnonic systems in dipolar-exchange regime, in which long-range interactions are allowed. For simple electronic systems of 1D periodicity, maximum two surface states can exist in one gap. Here, in the bandgap between B3 and B4 bands, we can find four surface states—two Shockley-type (one symmetric and the second one antisymmetric) and two Tamm-type (differing also in the symmetry with respect to the center of the structure). However, these two Tamm states are not localized exactly on the surface but on the first full Ni stripes next to the surface.

The Tamm states can also be induced by the introduction of structural perturbation to the system. In this approach we can change continuously the strength of perturbation in the surface regions of the structure. It was not possible in the previous study, in which we played with material parameters, replacing Fe or Ni stripes with Py in surface cells. Here, we have investigated the behavior of the surface states, while changing the ratio \( d_S/d_L \) of the widths of the stripes in the surface cells \( d_S \) to the widths of regular bulk stripes \( d_L \) in the considered MCs.

Firstly, we will study MC with Fe as a middle stripe and Ni as an edge stripe in the unit cell, thus we start with the structure having Ni on the edges, for which the Shockley surface states are present in the absence of perturbation: \( d_S/d_L = 1 \) (see figure 3(d)). For this structure, we found Shockley surface states at the first and the third gap for \( d_S/d_L = 1 \). It could also be seen in figure 4(a). Then we have made calculations for different values of the ratio: \( d_S/d_L \). We kept constant the width of the surface cells (while \( d_S \) is increasing, the width of the halves of stripes at the edges of cell are decreasing—compare figures 1(c) and (f)). We have investigated MCs with the described above ratio, going from 1 to 1.4. The Shockley surface states (labelled as ‘S’) travel up to the higher frequencies, reaching the frequencies of the above band. During this process we also observe other surface states appearing. We identify them as Tamm surface states (labelled as ‘T’), because they are induced by the presence of perturbation of the surface cells. Tamm states emerge from the bands in the gaps above them, and, as it is in the case of the Shockley surface states, they travel up to the next band with increasing ratio: \( d_S/d_L \). The increase of the frequency of both kinds of surface states with growing value of the ratio \( d_S/d_L \), results from reducing the width of Ni stripes in the surface cells. The surface states are localized mostly in Ni stripes and they become more confined with the increase of \( d_S/d_L \). Therefore, their frequencies are lifted up for the higher values of the ratio \( d_S/d_L \).

We then have plotted the profiles of selected modes for the ratio \( d_S/d_L = 1.2 \) in figures 4(c)–(i). In figures 4(c) and (d) the first and the second modes (from the lowest band) are presented. In the whole structure, these modes have zero and one node, respectively. Their amplitudes follow the long-scale oscillations, characteristic for the metamaterial regime. In figure 4(e) two Shockley states are presented: symmetric mode and antisymmetric one (marked with solid and dashed line, respectively). These modes are localized at the surface cells, mainly at the Ni edge stripes with decreasing amplitude in neighboring Fe stripes and smaller amplitude in the next Ni stripe. We can see the same behavior of the Shockley surface states from the higher bandgap, presented on the figure 4(h). The difference is that, the Shockley states having higher frequency (figure 4(h)) are more oscillating in space and decay faster inside the MC than the Shockley states from the lower gap (figure 4(e)).

The amplitudes of the Tamm surface modes are distributed through the structure differently than in the case of the surface states of the Shockley-type. We noticed that the SW amplitude, for all of the Tamm states we found, is mostly concentrated under the surface, at the first Ni stripe of the full width (see figures 4(f), (g) and (i)).

It is worth noticing that number of nodal points (or spatial oscillations) of Tamm states increases in successive frequency gaps. The profiles of Tamm states, presented in figures 4(f), (g)

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**Figure 4.** (a) Dependence of frequencies of the spin wave eigenmodes in the finite MC (composed of Ni (in blue) and Fe (in red) stripes—see figure 1(c)) on the perturbation of the external cells (the change of the width of the Fe stripe in external cells \( d_S \) with respect to the width of Fe stripes in the bulk cells \( d_L \)). The variation of the ratio \( d_S/d_L \) affects significantly the frequencies of surface states (of Tamm- and Shockley-type marked by ‘T’ and ‘S’, respectively) located in energy gaps (gray areas) of infinite MC (the dispersion of infinite system is presented in (b), with red-solid and black-dashed lines, obtained from FEM and PWM, respectively). The profiles of the SW eigenmodes \( m_n \) component of dynamical magnetization) were plotted in (c)–(i) for fixed value of the ratio \( d_S/d_L = 1.2 \).
and (i), have one, two or three nodal points in Ni stripe in the gaps between the bands: B2–B3, B3–B4 and B4–B5, respectively. We can also see that the surface states of Tamm-type, similarly like Shockley states, are localized stronger in higher gaps. The strength of localization of surface state depends on the value of imaginary component \( k_I \) of the complex wave vector \( k = k_R + i k_I \) attributed to this state. It is known [1] that the localization is stronger for higher absolute values of \( k_I \). The imaginary component of the wave vector has constant sign in the gap and reaches zero on its edges. Moreover, for wider gaps, \( k_I \) reaches larger absolute values. Therefore, the strong localization of the surface state is observed for the modes existing in the center of the wide frequency gaps. We showed for exchange waves that the location of the magnonic surface mode inside the gap is related to the strength of the surface pinning. Similar observation for dipolar waves allows us to connect the increase of the localization strength of surface modes (both of Shockley- and Tamm-type) at higher frequencies to the changes of the dipolar pinning.

In the gaps of higher frequencies (above 11.5 GHz) we can find four surface states. In the gap between bands B3–B4, there are two Shockley states (symmetric and antisymmetric) and two Tamm states (symmetric and antisymmetric). For higher gaps all surface states are of Tamm type. Here, for the magnonic system in dipolar-exchange regime, the rule which is in force for the electronic systems, is broken—we found out that more than two surface states can appear in one bandgap, and both, Shockley and Tamm surface states, can exist in one bandgap simultaneously.

Now we will investigate the behavior of the surface states while changing the ratio \( d_5/d_2 \) for the finite MC, arising due to termination at the second magnetic material (Fe)—see figure 1(b). For this MC there are no surface states in the absence of surface perturbation \( d_5/d_2 = 1 \). It is interesting to know if it is possible to generate the surface states at this structure simply by changing (increasing) the width of the Ni stripe in the surface cells. It occurs that increasing the width of the middle (Ni) stripe at the surface cell, we can induce the Tamm surface states. For this case, they appear below the bands and will be shifted, with increasing \( d_5/d_2 \) ratio, to the lower frequencies, and finally reach the region of the lower bands. We can explain this effect in similar manner as for the system discussed in figure 4. The increase of the ratio \( d_5/d_2 \) extends the width of the Ni stripe in which the surface modes are mostly concentrated (see figures 5(e)–(i)). Extending the size of this confined area results in lowering the frequency of the modes.

The changes of the spectrum of MC for different values of ratio \( d_5/d_2 \), going from 1 to 1.4, are similar as in the previous case (see figure 4), the surface states have got a different number of nodal points in successive gaps. However, the lowest Tamm states we have found (of the frequency in the gap between bands: B1–B2), have already one nodal point in the extended Ni stripe. The number of nodal points in the mentioned Ni stripe increases systematically for further Tamm states (of the frequencies from successive gaps). In the considered range of structural changes we found for higher gaps (above 11.5 GHz) more than one pair of Tamm states in one gap, which breaks the rule being in force for the electronic systems.

5. Conclusions

In an exchange regime, the magnonic system made of homogeneous magnetic material with spatial modulation of the anisotropy field is only a direct counterpart of the electronic systems. The surface states in these systems exhibit the same properties as the electronic surface states in simple 1D models. We can also introduce the clear separation between Shockley and Tamm surface states for these kinds of magnonic systems. The following features of Tamm and Shockley states are sustained in the mentioned sort of MCs: (i) for the surface located in the symmetry point of the structure we can introduce the symmetry criteria for existence of Shockley surface states, related to the parity of the Bloch function and its derivative in the symmetry point—we can point out in which gaps the Shockley surface states can exist, (ii) shifting the surfaces between two kinds of symmetry points and redefining the (symmetric) unit cell, we can change the gaps opened at the edge of the Brillouin zone from the allowed ones for Shockley states to the forbidden ones and vice versa (transform the gaps forbidden for surface states into the allowed ones), (iii) Tamm and Shockley states cannot exist in the same frequency gaps (for the same values of structural and material parameters), (iv) in one frequency gap can exist maximum two surfaces states. For the MCs in general form, i.e. with spatial changes of an exchange length and saturation magnetization, in a dipolar regime (with an exchange interactions included), we checked numerically that the properties (iii) and (iv) do not hold any more. This makes the clear separation of Tamm and Shockley states in magnonics discursive.
The other difference between the magnonic and electronic surface states is that the SWs, in contrast to electronic waves, cannot penetrate the (nonmagnetic) region outside of the crystal. Therefore, the boundary conditions on the surfaces of MC result from the strength of the magnetization pinning. In an exchange regime the strength of the pinning at the surfaces results from the surface anisotropy which is a frequency independent material parameter. The pinning parameter plays a similar role as the logarithmic derivative of the electronic wave function in the homogeneous medium (vacuum). The difference is that the ‘pinning’ of the electronic function on the surface, expressed by the logarithmic derivative, depends on the energy of electronic state. In dipolar dominated regime, the long-range dynamical dipolar interactions inside of MC are decisive for the pinning of the magnetization at the MC surfaces.

The studies on strongly localized spin waves are important because of two reasons: (i) due to the concentration of the amplitude, the surface modes can enter into nonlinear regime easier than the bulk modes, (ii) the localization of spin waves on the surface potentially can increase the interaction with the other kinds of surface waves (e.g. elastic waves).

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