6-dimensional Kaluza-Klein Theory for Basic Quantum Particles and Electron-Photon Interaction

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By extending original Kaluza-Klein theory to 6-dimension, the basic quantum field equations for 0-spin particle, 1-spin particle and 1/2 spin particle with mass >0 are directly derived from 6-dimensional Einstein equations. It shows that the current quantum field equations of basic particles become pure geometry properties under 6-dimension time-space. The field equations of electron and photon can be unified in one 6-dimensional extended Maxwell equation. The equations containing interactions between electron and photon will be derived from Einstein equation under 6-dimension time-space. It shows that the interactions in QED can be considered as the effect of local geometry curvature changing instead of exchange virtual photons.

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I. INTRODUCTION

Kaluza [1] first developed his method in 1919 in an attempt to unify Electromagnetism and General Relativity. In fact, Kaluza’s idea showed the possibility to unify not only gravity and electromagnetism, but also matter and geometry [2]. It gave physicists the hope of extending Einstein’s vision of nature as pure geometry to quantum level. The original Kaluza metric can be written as follows:

\[
g_{AB} = \left( \begin{array}{cc} g_{\alpha\beta} - \phi A_{\alpha} A_{\beta} & -\phi A_{\alpha} \\ -\phi A_{\beta} & -\phi \end{array} \right) \] (1)

where the \(\alpha\beta\)-part of \(g_{AB}\) with \(g_{\alpha\beta}\) (the four-dimensional metric tensor), the \(\alpha\)-part with \(A_{\alpha}\) (the electromagnetic potential), and the \(\beta\)-part with \(\phi\) (a scalar field). The four-dimensional metric signature is taken to be \((+ - - -)\).

The metric above contains a massless scalar field and a massless 1-spin vector field. The method did not include the mass part of quantum field (although people believe that the metric describe a spin-2 graviton, a spin-1 photon and a spin-0 boson which is thought to be connected with how particles acquire mass.); it also can not be used to describe the particle with half integer spin. To completely extend Einstein’s vision of nature as pure geometry to quantum level, we need include quantum fields for particles of mass > 0, and the particles of 1/2 spin into the theory. In other efforts, a possible “theory of everything”– Superstring theory extended Kaluza’s idea to ten-dimension, which contains all kinds of fields. But superstring theory is much complicate, it lost the beauty of simplicity in original Kaluza-Klein theory.

In this paper, one will find that by extending Kaluza-Klein theory to 6-dimension time-space, the field equations of 0-spin particle with mass > 0 (Klein-Gordon equation), equations of 1-spin particle with mass > 0 and equations of half integer spin particle (Dirac equation) can be derived directly from 6-dimensional Einstein equations under new 6-dimensional metric, especially the equations of electron and photon can be unified in one 6-dimensional Maxwell equation when we expand the components of fields of electron and photon to 6-dimension. It indicates that the current quantum field equations of basic particles are pure geometry properties of 6-dimension time-space. The coupling equations between electron and photon will be obtained from Einstein equation under 6-dimension time-space. It shows that the interactions in QED can be considered as the effect of local geometry curvature changing instead of exchange virtual photons.

II. EQUATIONS OF 0-SPIN FREE PARTICLE

Throughout this paper, the four-dimensional metric signature is taken to be \((+ - - -)\), and the indices for 6-dimensional time-space to be 0,1,2,3,4,5.

The 6-dimensional Einstein equations keep the same format as 4-dimensional:

\[
\hat{G}_{AB} = \kappa \hat{T} AB \ ,
\] (2)

where \(\hat{T} AB\) is 6-dimensional energy momentum tensor, \(\hat{G}_{AB} \equiv \hat{R}_{AB} - \hat{R} \hat{g}_{AB}/2\) is the Einstein tensor, \(\hat{R}_{AB}\) and \(\hat{R}\) are the 6-dimensional Ricci tensor and scalar respectively, and \(\hat{g}_{AB}\) is the 6-dimensional metric tensor, A, B.. run over 0,1,2,3,4,5.

The 6-dimensional Ricci tensor and Christoffel symbols are defined in terms of the metric exactly as in four dimensions:

\[
\hat{R}_{AB} = \partial_{C} \hat{\Gamma}^{C} AB - \partial_{B} \hat{\Gamma}^{C} AC + \hat{\Gamma}^{C} AB \hat{\Gamma}^{D} CD - \hat{\Gamma}^{C} AD \hat{\Gamma}^{D} BC \ ,
\]

\[
\hat{\Gamma}^{C} AB = \frac{1}{2} \hat{g}^{CD} (\partial_{A} \hat{g}_{DB} + \partial_{B} \hat{g}_{DA} - \partial_{D} \hat{g}_{AB}) \]

where A, B.. run over 0,1,2,3,4,5.

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For 0-spin particle, we choose the 6-dimensional time-space metric as:

\[ (\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} & \phi \\ \phi & -1 \end{pmatrix} \]  

(4)

where metric elements \( g_{\alpha\beta} \) is 4-dimensional metric. We concentrate on quantum field equations in this paper, so ignore gravity field, then \( g_{\alpha\beta} = \delta_{\alpha\beta} \), \( g_{55} = -1 \), \( g_{44} = \phi \), with conditions:

\[ \partial_4 \phi = 0, \quad \partial_5 \phi = \frac{m_0}{\hbar} \phi \]  

(5)

where \( \hbar \) is Planck constant, \( m_0 \) is rest mass of particle, or equivalently

\[ \phi = \phi_k e^{-im_0 x_5} \]  

(6)

where \( \phi_k \) is \( \phi \) in Kaluza metric \( i \). Start from here, throughout this paper, capital Latin indices \( A,B,C \) .. run over 0,1,2,3,5 \((AB \neq 4)\), Greek indices \( \alpha, \beta \) ... run over 0, 1, 2, 3, and small Latin indices \( a, b \) ... run over 1, 2, 3. Also we have \( g_{AB} g^{AB} = \delta_{AB} \), that makes \( g^{44} = \frac{1}{\phi} \).

Using equations (5), (6), the \( \alpha \beta = \alpha 5, \alpha \beta = 55, \alpha \beta = \alpha 4 \), and 44-components of Einstein equations (2) become:

\[ \frac{1}{\phi} \partial^\alpha \partial_\beta \phi = \kappa \hat{T}_{\alpha\beta} \]  

(7)

\[ -\frac{im_0}{\hbar \phi} \partial_\alpha \phi = \kappa \hat{T}_5 \phi = \kappa \hat{T}_5 \phi \]  

(8)

\[ -\left(\frac{m_0}{\hbar}\right)^2 = \kappa \hat{T}55 \]  

(9)

\[ \kappa \hat{T}44 = \kappa \hat{T}_4 \phi = 0 \]  

(10)

\[ \Box \phi + \left(\frac{m_0}{\hbar}\right)^2 \phi = 0 \]  

(11)

Here \( \hat{T}44 = 0 \), i.e. no 5th dimensional energy momentum tensor. Equation (11) is Klein-Gordon Equation for free 0-spin particle with mass \( > 0 \). Notice that for free single particle:

\[ \phi = e^{-\frac{im_0}{\hbar \phi} (p^\alpha x_\alpha - m_0 x_5)} \]  

(12)

Then equation (11) become:

\[ -\frac{1}{\hbar^2} p^\alpha p_\beta = \kappa \hat{T}_\alpha \beta \]  

(13)

If we let \( \kappa = \frac{\hbar}{\sqrt{2}} \), then \( \hat{T}_\alpha \beta = p^\alpha p_\beta \), i.e. energy momentum tensor is the products of two 4-dimensional momentum. Let \( p_5 = m_0, p_4 = 0 \), then 6-dimensional energy momentum tensor is: \( \hat{T}AB = p^A p_B \). One can also see that as we describe quantum field in pure geometry, \( \frac{\hbar}{\sqrt{2}} \) plays the similar geometry role as \( \sqrt{8\pi G} \)'s role in 4-dimensional Einstein equation, where \( G \) is gravitational constant.

III. EQUATIONS OF 1-SPIN FREE PARTICLE

For 1-spin particle with mass \( m_0 > 0 \), we let:

\[ \hat{A}_\alpha = A_\alpha e^{im_0 x_5} \]  

(14)

where where \( m_0 \) is rest mass of particle, \( x_5 \) is 6th dimension coordinate, and start from this section, we always choose \( \hbar = 1 \). Let

\[ \hat{A}_5 = 0 \]  

(15)

6-dimensional metric for 1-spin free particle become:

\[ (\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} + \hat{A}_\alpha \hat{A}_\beta & \hat{A}_\alpha \\ \hat{A}_\beta & 1 \end{pmatrix} \]  

(16)

Let \( \hat{F}_{AB} \equiv \partial_\alpha \hat{A}_B - \partial_\beta \hat{A}_A \), and \( A,B \) runs over 0,1,2,3,5, energy momentum tensor

\[ \hat{T}_{AB} \equiv g_{AB} \hat{F}_{CD} \hat{F}^{CD} / 4 - \frac{1}{2} \hat{F}_{AB} \hat{F}^{AB} \]  

(17)

where \( A,B \) run over (0,1,2,3,5). so the \( \alpha \beta = \alpha 4, \alpha \beta = 45, \alpha \beta = \alpha 5 \), and 44-components of 6-dimensional Einstein equations (2) become:

\[ G_{\alpha\beta} = \frac{1}{2} \hat{F}_{\alpha\beta}, \quad \nabla^\alpha \hat{F}_{\alpha\beta} - m_0^2 \hat{A}_\alpha \hat{A}_\beta = 0 \]  

(18)

This is equations for 1-spin single particle with mass \( > 0 \). As we see above, the particle of 1-spin obtains its mass from derivative of 6th dimension.

When \( m_0 = 0 \), metric (14) becomes usual Kaluza metric at the case of \( \phi = -1 \). The second and third equation of equations (15) become

\[ \nabla^\alpha \hat{F}_{\alpha\beta} = 0, \quad \frac{1}{4} \hat{F}_{\alpha\beta} \hat{F}^{\alpha\beta} = 0 \]  

(19)

The first equation of (19) is Maxwell equation in vacuum and the second equation of (19) is true for free photon (plane-wave).

IV. EQUATIONS OF \( \frac{1}{2} \)-SPIN FREE PARTICLE

In section III for 1-spin particle, we choose \( \hat{A}_5 = 0 \), now we will see that if we let \( \hat{A}_5 <> 0 \), we will get field equations for \( \frac{1}{2} \) particle. We choose the 6-dimensional metric for \( \frac{1}{2} \) particle as:

\[ (\hat{g}_{AB}) = \begin{pmatrix} g_{\alpha\beta} + \hat{K}_\alpha \hat{K}_\beta & \hat{K}_\alpha \\ \hat{K}_\beta & 1 \end{pmatrix} \]  

(20)

where \( \hat{K} \) is 6-dimensional vector field in 6-dimensional time-space, with conditions

\[ \partial_5 \hat{K}_A = im \hat{K}_A, \quad \hat{K}_5 <> 0, \quad \hat{K}_4 = 0, \quad \partial_4 \hat{K}_A = 0 \]  

(21)
\[ (\hat{g}^{AB}) = \begin{pmatrix} g^{\alpha\beta} & -\hat{K}^\alpha \\ -\hat{K}^\beta & 1 + K_A K^A - K^5 \end{pmatrix} \] (22)

Let \( E_{AB} \equiv \partial_A K_B - \partial_B K_A \). Define energy momentum tensor for half spin particle:
\[ T_{AB} = g_{AB} \hat{E}_{CD} E^{CD} / 4 - E_A^C \hat{E}_{BC} \] (23)
so the AB-, A-, and 44-components of 6-dimensional Einstein equations (2) become:
\[ G_{AB} = \frac{1}{2} T_{AB} , \quad \partial_A(\partial^C \hat{K}_C) - \partial^C \partial_C \hat{K}_A = 0 \]
\[ \frac{1}{4} E_{AB} \hat{E}^{AB} = 0 \] (24)
To derive the equation above, we used the relation below:
\[ \partial_C g^{CC}(\partial_A \hat{K}_C - \partial_C \hat{K}_A) = \partial_A(\partial^C \hat{K}_C) - \partial^C \partial_C \hat{K}_A \] (25)
For free particle, it is reasonable to assume that each components of \( \hat{K} \) satisfied plane-wave condition:
\[ \partial^C \partial_C \hat{K}_A = 0 \] (26)
The second equation of 6-dimensional Maxwell equations (28) becomes:
\[ \partial_A(\partial^C \hat{K}_C) = 0 \quad \text{for all A} = 0, 1, 2, 3, 5 \] (27)
i.e. \( \partial^C \hat{K}_C \) does not depended on \( x_0, x_1, x_2, x_3, x_5 \), so it is reasonable to let \( \partial^C \hat{K}_C \) equals zero (Not likely equals a constant other than zero because \( \hat{K} \) contains a plane-wave function part of condition (26)). Together with the third equation of equations (24), we have:
\[ \partial^C \hat{K}_C = 0 \quad \hat{E}_{AB} \hat{E}^{AB} = 0 \] (28)
Now let
\[ \hat{K}_0 = C g_{00} \phi_0 e^{i m_0 x_5} \quad \hat{K}_1 = C g_{11} \phi_1 e^{i m_0 x_5} \quad \hat{K}_2 = i C g_{22} \phi_2 e^{i m_0 x_5} \quad \hat{K}_3 = C g_{33} \phi_3 e^{i m_0 x_5} \quad \hat{K}_5 = C g_{55} \phi_5 e^{i m_0 x_5} \] (29)
where \( C \) is constant to be determined, \( g_{\alpha\beta} \) is element of usual 4-dimensional metric \((1,-1,-1,-1)\), \( g_{55} = -1 \). First equation of (28) becomes
\[ \partial_0 \phi_0 + i \partial_2 \phi_3 - i \partial_3 \phi_2 + i m_0 \phi_0 = 0 \] (30)
It is first Dirac equation in \( x_3 \) representation. The two equations in (28) plus normalization condition of \( \phi_0, \phi_2, \phi_3 \) keep the solution unique, plus we need choose constant \( C \) to make the energy momentum tensor \( T_{AB} \) reasonable.

The solutions of (28) are:
\[ \hat{K}_0 = C \sqrt{m_0 + p_0} e^{-i p^+ x_5} \quad \hat{K}_5 = -\hat{K}_0 \]
\[ \hat{K}_1 = -C \sqrt{m_0 + p_0} p_1 + i p_2 e^{-i p^+ x_5} \]
\[ \hat{K}_2 = i C \sqrt{m_0 + p_0} p_1 + i p_2 e^{-i p^+ x_5} \]
\[ \hat{K}_3 = -C \sqrt{m_0 + p_0} p_3 e^{-i p^+ x_5} \] (31)
The corresponding \( \phi_0, \phi_1, \phi_2, \phi_3 \) in relation (24) is the first solution of Dirac equation of \( \frac{1}{2} \) spin particle with positive energy. We choose the constant \( C \) in equations (20) as
\[ C = \frac{\sqrt{(m_0 + p_0)^2 m_0}}{p_3 \sqrt{p_0}} \] (32)
Substitute (32), (31) into (28), then:
\[ \hat{T}_{ab} = -\frac{p_a p_b}{p_0} \frac{e^{-i p^+ x_c}}{p^+} \] (33)
The negative sign can be removed by adding a constant \( \kappa \) in metric and let \( \kappa = \frac{m_0}{p_0} \), so the result is just we expected: the energy momentum tensor becomes the 5-dimensional momentum vector \( (p_1, m_0) \) multiply speed and times the square of plane wave function.

If we let
\[ \hat{K}_0 = C g_{00} \phi_1 e^{i m_0 x_5} \quad \hat{K}_1 = C g_{11} \phi_2 e^{i m_0 x_5} \quad \hat{K}_2 = i C g_{22} \phi_3 e^{i m_0 x_5} \quad \hat{K}_3 = -C g_{33} \phi_3 e^{i m_0 x_5} \quad \hat{K}_5 = C g_{55} \phi_5 e^{i m_0 x_5} \] (34)
We can get local time-space metric of half spin free particle with spin \( \frac{1}{2} \) and positive energy. Easy to examine if the constant \( C \) in equations (24) is general field equations for particles with half integer spin. It is interesting seeing that: equation (28) together with plane-wave condition (26), we have:
\[ \nabla^A (\hat{E}_{AB}) = 0 \] (35)
where \( \hat{E} \equiv \partial_A \hat{K}_B - \partial_B \hat{K}_A \). It is similar to Maxwell equation. If we let \( \hat{K}_5 = 0 \), it becomes the second equation of (14) for 1-spin particle; if we also let \( m_0 = 0 \), then the above equation (35) turns into Maxwell equation. So 1-spin particle (both massless and mass \( > 0 \) ) half spin particle are satisfied the same equations – extended 6-dimensional Maxwell equation (35) in 6-dimensional time-space. The field of \( \frac{1}{2} \) particle becomes 6-dimensional vector field.
Compare the metrics of 1-spin particle with $\frac{1}{2}$-spin particle, the only difference is that one without 6-th component of vector and the other with 6-th component. The metric of 1-spin particle has symmetry of 4-dimensional time-space, the metric of $\frac{1}{2}$-spin particle has symmetry of 5-dimensional time-space $(0, 1, 2, 3, 5)$.

V. EQUATIONS OF ELECTRON-PHOTON INTERACTION

The similarity between the metric of single electron and the metric of single photon makes us easier to combine electron and photon into one metric. The metric $\gamma_{AB}$ of the coupling of electron-photon is:

$$\begin{pmatrix} g_{\alpha\beta} + \kappa^2 B_\alpha B_\beta & \kappa \dot{B}_\alpha \dot{K}_5 \\
\kappa \dot{B}_\beta & 1 \kappa \dot{K}_5 \\
\kappa^2 \dot{K}_5 B_\beta & \kappa \dot{K}_5 + 1 + \kappa^2 \dot{K}_5 \dot{K}_5 \end{pmatrix}$$

(36)

where $\dot{B}_\alpha \equiv A_\alpha + \dot{K}_\alpha$; $A_\alpha$ for 4-dimensional photon vector field, $K$ for 6-dimensional vector field (spinor in 4-dimension) which we defined in previous section for electron. $\kappa$ is constant for coupling with gravity; $\kappa = 4\sqrt{\pi G}$ where $G$ is gravitational constant. To describe interactions between electron and photon, we implement Klein's idea 4 to make the derivative of 5th dimension $<> 0$, we can understand that in two ways: 1) the interaction of Electron-Photon changed curvature of local time-space. 2) An Electron always associate with electromagnetic fields which cause non-zero derivative of 5th dimension. Let

$$\dot{K}_A \rightarrow \dot{K}_A e^{in\gamma x_4}$$

(37)

where $\gamma$ is a very small constant, $n$ is integer, and there is no $K_4$. Then $\partial_1 \dot{K}_A = i\gamma \dot{K}_A$. Because the derivative of 5th dimension $<> 0$, 5th dimensional energy momentum tensor is not zero.

Using (36), (37), through considerable algebra, the AB-, and 44-components of 6-dimensional Einstein equations 2 become:

$$G_{AB} = \frac{\kappa^2}{2} F_{AB}, \quad \frac{1}{4} L_{AB} L^{AB} = T_{44}$$

(38)

and the A4-components of Einstein equations can be write into 3 separate pieces:

$$D^C \dot{K}_C = 0, \quad D^C D_C \dot{K}_A = 0, \quad \nabla^\alpha \dot{F}_{\alpha A} = 0$$

(39)

where $D_\alpha = \partial_\alpha - in\gamma A_\alpha$. We extended photon field to 6-dimension with $A_4 = 0, A_5 = 0$: $F_{AB} = \partial_\alpha A_B - \partial_B A_\alpha$, $L_{AB} = \partial_\alpha A_B - \partial_B A_\alpha + D_A \dot{K}_B - D_B \dot{K}_A$, energy momentum tensor of electron coupling with photon becomes:

$$\dot{T}_{AB} \equiv \frac{g_{AB}}{4} \dot{L}_{CD} \dot{L}^{CD} - \dot{L}_C \dot{L}_{BC} + +in\gamma (\dot{B}_B \dot{K}_C F_{AC} + \dot{B}_A \dot{K}_C F_{BC}) + in\gamma (D_A \dot{K}_B + D_B \dot{K}_A)$$

(40)

where we dropped $\gamma^2$ items in energy momentum tensor since $\gamma$ is small; $\dot{E}_{AB} \equiv \partial_A \dot{K}_B - \partial_B \dot{K}_A$.

If let $n = 1, \gamma = -e$, $e$ is charge of electron, and use relation from (29), then first equation of (39) becomes:

$$(\partial_0 + ie A_0)\phi_0 + (\partial_1 + ie A_1)\phi_3 - i(\partial_2 + ie A_2)\phi_3 + (\partial_3 + ie A_3)\phi_2 + im_0 \phi_0 = 0$$

(41)

This is Dirac equation for electron-photon interaction. The second equation of (39) is Klein-Gordon equation in electromagnetic field that each components of electron field should satisfied individually. The 3rd equation of (39) is Maxwell equation.

Now let us look at energy momentum tensor (40). The first two parts are combined energy momentum tensor of electron and photon. The third item $ie B_B K^C F_{AC}$ is related to Lorenz force. The third item is electric current density.

6-dimensional action is

$$S = -\int \sqrt{-g} R \, d^6 x$$

(42)

where $R = \dot{g}^A \dot{R}_{AB}$ (A, B runs over 0, 1, 2, 3, 4, 5) is 6-dimensional Ricci scalar. The metric determinant $\dot{g}$ reduces in the simple manner:

$$\dot{g} = det(\dot{g})_{AB} = -det(g_{AB})$$

(43)

Using (36), (38) and (39), we also assume $\int d\gamma d\delta_6 = 1$, then

$$S = -\int \sqrt{-g} (R + \frac{1}{4} \dot{L}_{AB} \dot{L}^{AB}) \, d^4 x$$

(44)

where $R$ is usual 4-dimensional Ricci scalar. The above equation contains a gravity part and kinetic energy part. for interaction between electron and photon.

VI. DISCUSSIONS AND CONCLUSIONS

The kinetic energy part of (14) $\frac{1}{2} \dot{L}_{AB} \dot{L}^{AB}$ contains the combination of electron field and photon field. Actually, metric (40) can be used to describe:

1) Electron associate with its own static electromagnetic field. Electron is always accompany with photon (static electromagnetic fields), that’s why it can absorb photon to gain energy, emits photon and lost energy, annihilation with positron and turns into two photons, so it is reasonable to say that there is always $A_\alpha$ field in electron’s local geometry metric.

2) Electron absorb a photon and gains energy. If a free photon $A_0$ at time $t_0$, location $X(x_{01}, x_{02}, x_{03})$ which local geometry metric is (10), it is absorbed by an electron at time t, then the local metric at time $t$ becomes metric (40) with $\dot{B}' = A_0 + A_e + \dot{K}$, where photon $A_0$ turns to $A_0'$ after interaction and $A_e$ is initial photon contained in electron, and finally electron gain the energy.
3) The intermediate state of a photon interacts with electron. If a free photon \( A_0 \) at time \( t_0 \), location \( X(x_{01}, x_{02}, x_{03}) \) which local geometry metric is (16), it interacts with electron at time \( t \), then the local metric at time \( t \) becomes metric (36) with \( \hat{B}' = A'_0 + A_e + \hat{K} \), after interaction, the photon and electron will be separate. Of course the energy and momentum of both particle changed.

In the discussion above, we only dealing with curvature changes. We do not need use any “virtual” photons.

One can see from section III, U(1) field obtained its mass from derivative of 6th dimension without losing symmetry, so we could avoid Higgs mechanism. If we treat interactions as changes of geometry curvature of local time-space, we could avoid so called “vacuum effects” in current QED theory, the “vacuum effects” could be interpreted as local curvature changes too.

In addition, here we see a symmetry between gravity and electromagnetism: Mass is the source of gravity, it comes from the derivative of 6th dimension; Charge is the source of electromagnetic interaction, it comes from the derivative of 5th dimension. We can also use Klein’s original idea that, the 5th and 6th have circular topology and periodic conditions, that can make quantization of electric charge and quantization of mass (or energy).

As the conclusion, in this paper, we derived above equations directly from Einstein equations under 6-dimensional time-space metric. The energy momentum tensor is the products of extended 6-dimensional momentum vector \( p_A \) (where \( p_5 = m_0 \), \( p_4 = 0 \), no 5 dimensional momentum); Planck constant \( \hbar \) plays the similar role as the role of gravitational constant \( G \) in gravity equations. The interactions between photon and electron can be treated as the effects of local time-space geometry curvature changing. It indicates the pure geometry nature of quantum fields. Indeed, by using metric (4), one can find that the geodesic path of single free particle in 6-dimensional time-space is exact plane-wave function, the interpretation of basic quantum physics by using 6-dimensional time-space is discussed in another paper [3].

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