An Efficient Numerical Model for Squeeze Film Air Damping of Microplates in the Free Molecular Regime

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Abstract: The squeeze film air damping of microplates in the free molecular regime has been received particular attention for many years. Based on Energy Transfer Model (ETM) proposed by Bao et al and Monte Carlo (MC) model proposed by Hutcherson and Ye, this paper proposes an efficient numerical model, compared with experiment data and present models, the present model costs less computational work than the previous MC model and is more accurate than ETM.

1. Introduction

Squeeze film air damping has been identified as an important mechanism of energy dissipation in microplates, even encapsulated in rarefied air [1]. The accurate prediction of squeeze film air damping is important for the design of microplates [2-5].

Based the free molecular approach, by considering the squeeze film air damping effect, Bao et al [6] proposed a closed form Energy Transfer Model (ETM) in the free molecular regime, which based on some assumptions for simplicity. However, the deviation with experiments remains large. Hutcherson and Ye [7] developed a Monte Carlo (MC) model by releasing some assumptions in the ETM. The MC model is verified to be more accurate than the ETM, however, the process of the MC model costs a lot of computational work and is difficult to handle. Based on the ETM and MC model, this paper gives an efficient numerical model for squeeze film air damping in the free molecular regime, which shows relatively high accuracy and efficiency.

This paper will introduce the ETM and MC model in Section 2. In Section 3, the new numerical model is given. The comparison with experiment data is shown in Section 4. At last, a conclusion of this paper is given in Section 5.

2. Previous models

2.1 Process of ETM [4]

As the vibrating plate experiences a time dependent sinusoidal motion parallel to the substrate, the displacement of the plate is $z = A \sin \omega t$, $A$ and $\omega$ are the amplitude and vibration frequency of the plate. Bao et al [6] made several assumptions:

1. Collisions of molecule and wall are fully elastic and specular.
2. Velocity of molecules going through the gap is constant.
3. During the travel of a molecule staying in the gap, the velocity of the plate is assumed to be constant.
4. The gap length is assumed to be constant during the travel of the molecule.

The number of molecules entering the gap per unit time is $N = \frac{1}{4} n \dot{v} L d$, where $n$ is the...
concentration of molecules, $L=2(a+b)$ the peripheral length of the plate, $d$ the gap length. $v=\sqrt{8kT/(\rho m)}$ the average velocity of the molecules, $k$ the Boltzmann constant, $T$ and $m$ the air temperature and the molecule mass, The total energy loss of the plate in one cycle is:

$$\Delta E = \frac{1}{4} \pi L \sum_{i=1}^{M} \frac{z_i}{\rho \omega} \frac{L^2}{v^2} \frac{A^2}{v^2} \cos^2 \omega t \cdot d(\omega t) = \frac{\pi}{16} \frac{A^2}{v^2} \rho \omega L d$$

(1)

$\hat{I}$, $v_{y0}^2$ and $v_{c0}^2$ are approximated as the average traveling distance $\sum_{i=1}^{M} \frac{z_i}{\rho \omega} \frac{L^2}{v^2} \approx \frac{L^2}{v^2} = \frac{2ab}{\pi}$, the molecule velocity $\frac{v_{y0}^2}{v_{c0}^2}$ and $\frac{v_{c0}^2}{v_{c0}^2}$, meanwhile $\frac{v_{y0}^2}{v_{c0}^2} = \frac{2v_{y0}^2}{v_{c0}^2}$. The initial velocity of the molecule is assigned randomly on the basis of a uniform distribution, and the initial velocity is assigned as the average velocity $v=\sqrt{8kT/(\rho m)}$. By summing up over $M$ divisions, the quality factor of the MC model is obtained:

$$Q_{\text{ref}} = \frac{(2\pi)^2}{\rho \omega L d} \sqrt{R T \frac{1}{M_m}}$$

(2)

2.2 Process of MC model[7]

In the MC model, the assumptions (2), (3), (4) are released. The vibration period of the microplate is divided into $M$ time divisions with a length of $\Delta t$. At the $i$th time division, the number of molecules entering the gap is $1/4 \cdot nL(d - z_i)\Delta t$, $z_i = A \sin (\omega_i t)$. The energy change $\Delta E_i$ of a representative molecule entering at the $i$th time division is calculated by tracking the motion of this molecule and recording velocity changes after each collision with the plate. The initial position of the molecule is assigned randomly on the basis of a uniform distribution, and the initial velocity is assigned as the average velocity $v=\sqrt{8kT/(\rho m)}$. By summing up over $M$ divisions, the quality factor of the MC model is obtained:

$$Q_{\text{ref}} = \pi a b H \rho \omega A^2 / \Delta E \quad \text{and} \quad \Delta E = \sum_{i=1}^{M} \frac{1}{4} \rho \omega L(d - z_i)\Delta E_i \Delta t$$

(3)

3. Numerical model

As can be seen, the MC model discretizes the vibration period of the plate, the initial velocity and initial position of molecules entering the gap to release the assumptions of the ETM. However, these discretizations cost much computational work. Based on the ETM and MC model, this paper proposes a new numerical model, which just releases the assumption (3). The process of the model is shown as follows.

Like the MC model, the vibration period of the plate is divided into $M$ time divisions with a length of $\Delta t$. When the molecule going through the gap at the $i$th time division, the velocity of the plate is considered to be constant $z_i = A \omega \cos (\omega_i t)$. Due to the fully elastic and specula collisions of molecule and wall, the time of molecule staying in the gap is calculated as $\hat{I}/v_{y0}$. The energy change of a molecule in the gap can be calculated as $\Delta E_i = \sum_{i=1}^{M} \frac{1}{4} \rho \omega L(d - z_i)\Delta E_i \Delta t$.

The total times of collisions of a single molecule and the plate at the $i$th time division is $K_i$, the time cost is

$$\frac{2d}{v_{y0}} + \frac{2d}{v_{c0} + 2z_i} + \frac{2d}{v_{c0} + 2 \cdot 2z_i} + \cdots + \frac{2d}{v_{c0} + K_i \cdot 2z_i}$$

or equal to $\hat{I}/v_{y0}$:

$$\frac{2d}{v_{y0}} + \frac{2d}{v_{c0} + 2z_i} + \frac{2d}{v_{c0} + 2 \cdot 2z_i} + \cdots + \frac{2d}{v_{c0} + K_i \cdot 2z_i} \leq \hat{I}/v_{y0}$$

(4)

In order to calculated the value of $K_i$, we start $K_i$ from 1, if equation (4) is true, $K_i = K_i + 1$ until equation (4) is false, which means the time cost of $(K_i + 1)$ collisions would be larger than $\hat{I}/v_{y0}$. Then we can get the value of $K_i$ at the $i$th time division. By calculating all values of $K_i$ at all time...
divisions, the quality factor of the new numerical model is obtained:

\[ Q_{\text{New}} = \pi abH \rho_p A^2 \omega^2 / \Delta E_{\text{New}} \] and \[ \Delta E_{\text{New}} = \sum_{i=1}^{N} \left[ \frac{1}{4} \nu Ld \cdot \frac{1}{2} m(4K_i z_i v_{i,0} + 4(K_i z_i)^2) \delta t \right] \] (5)

4. Experiments

To validate the new numerical model, Zook’s experiment data is used [1]. The parameters of Zook’s experiment are shown in Table 1. Firstly, the quality factor is calculated at different time discretizations of the vibration period and shown in Figure 1. As can be seen, the simulation results have converged to within 0.01% with less than 1000 time divisions, and the computational time is less than 0.001s, which shows high efficiency. As a consequence, the time division \( M \) is set as 1000, which can meet the requirement of convergence.

Figure 2 gives the comparison of quality factors. It is obvious that at \( Kn>10 \), the results of HY’s MC model and the new model are both more accurate than those of Bao’s ETM. The cure of the new model are very close to those of the MC model at the whole range of \( Kn \). Table 2 lists the detailed values of MC and the new models at \( Kn>10 \).

Table 1. Parameters of Zook’s experiment [1]

| Parameters         | Value |
|--------------------|-------|
| Plate density \( \rho_p \) (kg/m\(^3\)) | 2330  |
| Length \( a \) (µm)             | 200   |
| Width \( b \) (µm)              | 40    |
| Thickness \( H \) (µm)          | 1.8   |
| Gap length \( d \) (µm)         | 1.1   |
| Frequency \( f \) (kHz)         | 550   |
| Amplitude \( A \) (µm)          | <0.1  |
| Temperature \( T \) (K)         | 293   |

![Figure 1](image1.png)  
**Figure 1.** Convergence of simulation results at \( Kn=561 \)

![Figure 2](image2.png)  
**Figure 2.** Comparison of quality factors

Table 2. Quality factors at \( Kn>10 \)

| \( Kn \) | \( Q_{\text{Exp}} \) | \( Q_{\text{HY}} \) | \( Q_{\text{New}} \) | \( (Q_{\text{New}}-Q_{\text{Exp}})/Q_{\text{Exp}}\) % | \( (Q_{\text{New}}-Q_{\text{HY}})/Q_{\text{HY}}\) % |
|--------|---------------------|---------------------|---------------------|---------------------------------|---------------------------------|
| 1320   | 10000              | 16724              | 15901              | 59.01                            | 4.92                            |
| 561    | 5000               | 7107               | 6758               | 35.16                            | 4.91                            |
| 189    | 2400               | 2369               | 2252               | -6.14                            | 4.91                            |
| 83     | 1000               | 1035               | 1000               | -0.01                            | 5.05                            |
| 41     | 600                | 526                | 494                | -17.68                           | 6.10                            |
| 19     | 330                | 236                | 229                | -30.64                           | 3.02                            |
| 10     | 240                | 132                | 120                | -49.81                           | 8.74                            |

It can be seen in Table 2, the results of the new model are slightly less than those of the MC model and are slightly closer to the results of experiment. The largest deviation of the new model and the MC
model is less than 10%. HY [7] pointed that the main factor that influences the accuracy of the ETM is the assumption (2), which may take 90% compared with the MC model. The assumptions (3), (4) may take 10%. The new numerical model has just released the assumption (2) and the deviation is less than 10%, which verifies the assumption (2) is the main factor that influences the accuracy pointed by HY. It may be confusing that the results of the new model are slightly closer to experiment data than those of the MC model. It should be noted that, the assumption (3) considers the travelling time of each molecules entering the gap is smaller than the vibration period, which means during the vibration period, all molecules entering the gap can go through the gap. Actually, some of the molecules cannot go through the gap within the vibration period. As a result, the assumption (3) would slightly overestimate the energy change of molecules in the gap and underestimate the quality factor, which can explain that the new model is “more accurate” than the MC model.

5. Conclusion

For predicting squeeze film air damping in the free molecular regime, Bao et al proposed an analytical ETM [6]. However, the deviation with experiment remains large. HY developed a MC model more accurate than ETM by releasing some assumptions in ETM [7]. However, the MC model costs a lot computational work. Based on ETM and MC model, this paper proposes an efficient numerical model, which just releases the assumption (2)—the main factor that influences the accuracy. Compared with experiment, the new model is more accurate than the ETM, the deviation of the new model and the MC model is less than 10%, while the simulation process is simple and the computational time is reduced to 0.01s.

In general, owing to relatively high accuracy and efficiency, the new numerical model can have a practical value in design applications of MEMS devices.

Acknowledgments

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