A refined cellular automata method for predicting failure patterns of masonry wall panels

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Abstract: In view of the inaccuracy of the existing cellular automata (CA) technique in predicting failure patterns of masonry wall panels, this paper proposed a refined CA method which contains two measures to promote the precision of results. The optimization of matching criterion based on Taylor expansion updates the definition of state values for the CA model of wall panels and improves the accuracy of predicting the failure patterns of wall panels with opening. Besides, the post-processing of noise reduction explains the discontinuity of cracks in wall panels from the perspective of noise based on two proposed theorems, which improves the prediction of the failure patterns in appearance, continuity and clarity of cracks. Meanwhile, the difference functions and similarity of wall panels are introduced to evaluate the predicting results. Moreover, the combination of two measures shows the brilliant capacity in enhancing the accuracy and efficiency of prediction.

1. Introduction
The internal state of stress for masonry wall panel is complex which makes the accurate prediction of its failure pattern difficult [1]. In recent years, the development of various artificial intelligence prediction methods has improved the prediction accuracy of traditional methods, but in many cases it still has not reached the acceptable accuracy [2]. Especially, these methods are not suitable for wall panels with openings and cracks. Hence, it still needs to develop new measures to improve the exiting methods.

The CA method for predicting the failure patterns of wall panels is based on the similar characteristics of the basic wall panels (the failure patterns are known) and the predicted wall panels. And the matching criteria are the keys to improve the prediction results, among which the minimum error matching criteria for von Neumann neighborhood and Moore neighborhood models are most widely applied [3]. However, these matching criteria do not reflect the location information adequately and apply the state values of each region indiscriminately, which enlarges the effect of region similarity and weakens the difference information.

In view of the drawbacks of the CA methods above, this paper put forward two improvement measures and formed a refined CA method to enhance the accuracy of prediction.
2. Refined CA method
The refined CA method for predicting the failure patterns of wall panels includes two measures: improvement of matching criteria and post-processing of noise reduction:

**Measure 1** Optimization of existing matching criteria based on Taylor expansion. New criterion transforms state values of nine cells of Moore neighbors with overlapping information into six independent variables, from where four similarity influencing factors are extracted. By continuously adjusting the weight of each factor, the failure pattern can be predicted more accurately.

**Measure 2** Post-processing of noise reduction for prediction of failure pattern. After applying the matching criterion to predict the failure patterns of predicted wall panels, noise reduction is carried out according to the discernibility function to improve the continuity of the predicted cracks.

2.1. Matching criterion based on Taylor expansion
In measure 1, a quadric function is used to fit the state values of nine cells in a Moore neighborhood according to the continuous characteristic of state values in the digital model. The conicoid has six independent coefficients, where the zero-order coefficient is the state value of the central cell, the two linear coefficients are the gradients of the local cell’s state value and the three quadratic coefficients are Hessian matrix of the local cell’s state value. In general, the second-order Taylor expansion [4] of the binary function \( f(x, y) \) is carried out around the element cell \((x_0, y_0)\), as expressed in equation (1):

\[
f(x_0 + \Delta x, y_0 + \Delta y) \approx f(x_0, y_0) + \nabla f(x_0, y_0) \cdot \left( \frac{\Delta x}{\Delta y} \right) + \frac{1}{2} \left( \frac{\Delta x}{\Delta y} \right) \cdot \nabla^2 f(x_0, y_0) \cdot \left( \frac{\Delta x}{\Delta y} \right)
\]

From the right side of the equation, we can get three parameters of \( f(x, y) \) when it is near the element cell location: function value \( f(x_0, y_0) \), gradient \( \nabla f(x_0, y_0) = \begin{pmatrix} f_x(x_0, y_0) \\ f_y(x_0, y_0) \end{pmatrix} \) and Hessian matrix. For simplicity, \( f, f_1 \) and \( f_2 \) are introduced to represent \( f(x, y) \), \( f_1(x_1, y_1) \) and \( f_2(x_2, y_2) \), respectively. Equation (2) represents the absolute value of the difference of two functions’ values and Equation (3) represents the difference of two gradient vectors in Euclidean space:

\[
\Delta_1 = \|f_1 - f_2\| = |f_1 - f_2|
\]
\[
\Delta_2 = \|\nabla f_1 - \nabla f_2\| = \left[ (f_{xx} - f_{2x})^2 + (f_{xy} - f_{2y})^2 \right]^{1/2}
\]

To define the difference between two Hessian matrices, stress state expressed by Mohr’s circle (Figure 1) is introduced to transform Hessian matrix into vector and numerical value. The value obtained by linearly adding the difference of the two quantities is expressed as the difference of the Hessian matrix. Supposing \( f \) is a planar state stress function [5], Hessian matrix can be expressed as \( \nabla^2 f = \begin{pmatrix} f_{xx} & f_{xy} \\ f_{xy} & f_{yy} \end{pmatrix} = \begin{pmatrix} \sigma_x & -\tau_{xy} \\ -\tau_{xy} & \sigma_y \end{pmatrix} \). Thus, the abscissa of Mohr circle center (hydrostatic pressure) and the deviatoric stress in the circle can represent the stress state of any point in the plane, as shown in Figure 1.

![Mohr circle of planar stress state](image)

Figure 1. Mohr circle of planar stress state
The hydrostatic pressure $A$ in Figure 1 is:

$$A = \frac{1}{2} (\sigma_x + \sigma_y) = \frac{1}{2} (f_{xx} + f_{yy})$$  \hspace{1cm} (4)

Deviatoric stress tensor $\hat{B}$ is:

$$\hat{B} = \left( \frac{1}{2} (\sigma_x - \sigma_y), \tau_{xy} \right) = \left( \frac{1}{2} (f_{xx} - f_{yy}), -f_{xy} \right)$$  \hspace{1cm} (5)

And the variance of hydrostatic pressure is expressed by the absolute value of its difference:

$$\Delta_1 = \|A_1 - A_2\| = \frac{1}{2} \left\| \left( f_{1xx} + f_{1yy} \right) - \left( f_{2xx} + f_{2yy} \right) \right\|$$  \hspace{1cm} (6)

Then, the variance of deviatoric stress tensor is expressed as the distance between two deviatoric stress vectors in Euclidean space:

$$\Delta_4 = \|\hat{B}_1 - \hat{B}_2\| = \left[ \frac{1}{4} \left( (f_{1xx} - f_{1yy}) + (f_{2xx} - f_{2yy}) \right)^2 + (f_{1xy} - f_{2xy})^2 \right]^{1/2}$$  \hspace{1cm} (7)

The difference between two Hessian matrices is calculated by linearly adding the difference of hydrostatic pressure $A$ and deviatoric stress tensor $\hat{B}$. Therefore, the difference between $f_i(x,y)$ near $(x_1, y_1)$ and $f_j(x,y)$ near $(x_2, y_2)$ can be expressed by difference function $\Delta(x_1, y_1, x_2, y_2)$, which is the sum of the difference of Hessian matrix, functions’ value and gradient vectors, as shown below:

$$\Delta(x_1, y_1, x_2, y_2) = \sum_{i=1}^{4} k_i \Delta_i = k_1 |f_i - f_j| + k_2 \left[ (f_{xx} - f_{xx})^2 + (f_{yy} - f_{yy})^2 \right]^{1/2}
+ k_3 \left| (f_{ixx} + f_{1yy}) - (f_{2xx} + f_{2yy}) \right| + k_4 \left[ \frac{1}{4} \left( (f_{1xx} - f_{1yy}) - (f_{2xx} - f_{2yy}) \right)^2 + (f_{1xy} - f_{2xy})^2 \right]^{1/2}$$  \hspace{1cm} (8)

where $k_i$ is weight coefficient.

For CA model, the state values of Moore neighborhood cells are discrete, as shown in Figure 2. And the interval between cells is assumed to be 1.

| $(x-1,y+1)$ | $(x,y+1)$ | $(x+1,y+1)$ |
|-------------|-----------|-------------|
| $(x-1,y)$  | $(x,y)$   | $(x+1,y)$   |
| $(x-1,y-1)$| $(x,y-1)$ | $(x+1,y-1)$ |

Figure 2. Moore neighborhood

Assuming that $f_i(x_1, y_1)$ is the digital pattern of basic wall panel, one cell is positioned as $(x_1, y_1)$. Similarly, $f_j(x_2, y_2)$ is the digital pattern of predicted wall panel and one cell of it is positioned as $(x_2, y_2)$. Applying the difference function, gradient, Hessian matrix, the state value of the cell as well as the difference function $\Delta(x_1, y_1, x_2, y_2)$ can be calculated. Thus a matching criterion based on Taylor expansion is established through difference function, as expressed by Equation (9):

$$(\hat{x}_1, \hat{y}_1) = \arg \min_{x_1,y_1} \Delta(x_1, y_1, x_2, y_2) \big| x_1 = \hat{x}_2, y_1 = \hat{y}_2$$  \hspace{1cm} (9)

2.2. Post-processing of noise reduction

Post-processing of noise reduction can enhance the continuity of predicted wall cracks, which is a reprocessing technology when the wall panel failure pattern is obtained. Besides, this technology is applicable to all criteria, which has wide application and universal significance.

2.2.1. Principle of noise reduction. The difference of local region between basic wall panel and
predicted wall panel is calculated through the definition of difference function. In Figure 3, \( \Delta(a_1,a_2) \) is the difference between the local region \( A_1 \) of the central cell \( a_2 \) on the predicted wall panel and the local region \( A_1 \) of the central cell \( a_1 \) on the basic wall panel. Similarly, the difference \( \Delta(b_1,b_2) \) of central cells’ neighborhood \( b_2 \) and \( b_1 \) can also be calculated.

![Figure 3. Sketch map of local regions](image)

If the central cell \( a_2 \) of the local region \( A_2 \) is known, the difference function between the two regions is the smallest only when the central cell of the local region \( A_1 \) is \( a_1 \), then the two local regions are called the similar regions. Ideally, the local region of \( b_1 \) should correspond to the local region of \( b_2 \). However, owing to the essential defect of difference function, the results are discrete for influence of noise, which leads to a fact that the value of \( \Delta(b_1,b_2) \) is not minimum. Therefore, the similar region for local region of \( b_1 \) is no longer the local region of \( b_2 \), which is conflicting with the definition of similar region, leading to discrete or blurry prediction results of cracks.

To solve this problem, let \( r \) be the diameter of local regions \( A_1 \) and \( A_2 \), then define a set \( A \) which includes all the cells in one local region: \( A = \{(\Delta x, \Delta y) | |\Delta x^2 + \Delta y^2| \leq r^2; \Delta x, \Delta y \in Z \} \). Then let \((x_i, y_i)\) be the coordinate of \( a_i \) and \((x_j, y_j)\) be the coordinate of \( a_2 \). So difference function \( \Delta(x_i + \Delta x, y_i + \Delta y, x_j + \Delta x, y_j + \Delta y) \) is continuous in \((\Delta x, \Delta y) \in A\), for \( x_i, y_i, x_j, y_j \) remain unchanged. Based on the above properties, this paper proposed a method of Gaussian filtering for noise reduction to improve the prediction accuracy.

2.2.2. Gaussian filtering for noise reduction. Gaussian filtering [6] defines the function value of a point as the weighted average value of all function values in its neighborhood, while the weight is the Gaussian probability density function centered on this point. The weight function of the two-dimensional Gaussian filtering formula is:

\[
G(x,y) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left( -\frac{x^2 + y^2}{2\sigma^2} \right)
\]  

(10)

Let \( g(x,y) \) replace \( f(x,y) \) after noise reduction, thus the discrete form of Gaussian filtering formula is:

\[
g(x_0, y_0) = \sum_{i,j} G(x-x_0, y-y_0) \cdot f(x,y)
\]

(11)

2.2.3. Repeated noise reduction algorithm. The time complexity of one noise reduction by Gaussian filtering algorithm is \( O(n^2) \), so the time complexity of the whole process is \( O(n^6) \). To simplify the calculation, repeated noise reduction algorithm is introduced to let weight function gradually converge to Gaussian function as follows:

Firstly, \( f(x,y) \) is a continuous function with noise while \( g(x,y) \) satisfies
\(g_i(x, y) = g_i(-x, y) = g_i(x, -y)\) and \(\iiint g_i(x, y) ds = 1\). From mentioned above, the function value of a point after noise reduction is the weighted average value of all functional values in its neighborhood. So the convolution of \(f(x, y)\) and \(g(x, y)\) can output the function \(f_i(x, y)\) after noise reduction, as expressed below:

\[
f_i(x, y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(x-t, y-s) f(t, s) ds dt = g(x, y) * f(x, y)\]

(12)

Repeat the operation of convolution in Equation (12), \(f_n(x, y)\) can be obtained by \(n\) times noise reduction of \(f(x, y)\):

\[
f_n(x, y) = g(x, y)^{(n)} * f(x, y)\]

(13)

where \(g(x, y)^{(n)}\) is defined as \(g(x, y)\) do convolution operations on themselves for \(n\) times. From Equation (13), it is obvious that \(g(x, y)^{(n)}\) is the weight function of \(f(x, y)\) during the noise reduction process. To obtain the relationship between \(g(x, y)^{(n)}\) and \(g(x, y)\) in terms of the mean, variance and so on, this paper proposed and proved two theorems:

**Theorem 1:** If \(g_1(x, y)\) and \(g_2(x, y)\) satisfy three conditions:

1. \(g_i(x, y) = g_i(-x, y) = g_i(x, -y)\);
2. \(\iiint g_i(x, y) ds = 1\);
3. \(\sigma_i^2 = \iiint g_i(x, y)(x^2 + y^2) ds\) while \(g(x, y) = g_1(x, y) * g_2(x, y)\).

And assume that \(g(x, y) = g(x, y_1) * g(x_2, y_2)\), then:

1. \(g(x, y) = g(-x, y) = g(x, -y)\);
2. \(\iiint g(x, y) ds = 1\);
3. \(\sigma^2 = \iiint g(x, y)(x^2 + y^2) ds = \sigma_1^2 + \sigma_2^2\),

where \(\sigma^2, \sigma_1^2\) and \(\sigma_2^2\) are the variance of \(g(x, y)\), \(g_1(x, y)\) and \(g_2(x, y)\), respectively.

**Proof:**

For property (1):
\(g(x, y) = g_1(x, y) * g_2(x, y) = g_1(-x, y) * g_2(-x, y) = g_1(x, -y)\), likewise, \(g(x, y) = g(-x, y)\).

For property (2):
\[\iiint g(x, y) ds = \iiint g_1(x-u, y-v)g_2(u, v) dudv | dxdy\]

Assume that \(p = x-u, q = y-v\), then
\[\iiint g_2(u, v)(\iiint g_1(x-u, y-v) dxdy) dudv = \iiint g_2(u, v)(\iiint g_1(p, q) dpdq) dudv\],

so \(\iiint g_2(u, v) dudv = 1\).

For property (3): since \(\iiint g(x, y)(x^2 + y^2) ds = \iiint g_1(x-u, y-v)g_2(u, v) dudv | dxdy = \iiint g_2(u, v)(\iiint g_1(x-u, y-v) dxdy) dudv\), assume that \(p = x-u, q = y-v\), then
\[\iiint g_1(x-u, y-v)(x^2 + y^2) dxdy = \iiint g_1(p, q)((p+u)^2 + (q+v)^2) dpdq\]
\[= \iiint g_1(p, q)(p^2 + q^2) dpdq + \iiint 2pug_1(p, q) dpdq + \iiint 2vqg_1(p, q) dpdq + \iiint (u^2 + v^2)g_1(p, q) dpdq\]
\[= \sigma_1^2 + \sigma_2^2 + 2u + 2v\]

Finally:
\[\iiint (\sigma_1^2 + u^2 + v^2)g_2(u, v) dudv = \iiint \sigma_1^2 g_2(u, v) dudv + \iiint (u^2 + v^2)g_2(u, v) dudv = \sigma_1^2 + \sigma_2^2\].

The proof is completed.

Applying theorem 1 to \(g(x, y)^{(n)}\) repeatedly can derive Equation (14), which denotes that the variance of \(g(x, y)^{(n)}\) is proportional to the times of noise reduction \(n\):

\[\sigma_n^2 = \iiint (x^2 + y^2) g(x, y)^{(n)} ds = n\sigma^2\]

(14)
Let the weight function \( g(x, y) \) be conical distribution:

\[
g(x, y) = \max \left( \frac{3}{8\pi} \left( 2 - \sqrt{x^2 + y^2} \right), 0 \right)
\]  

(15)

Repeat convolution operation on \( g(x, y) \) to get \( g_n(x, y) \), when \( n = 1, 2, 4, 9, 16 \), the shape of \( g_n(x, y) \) is a revolving body. For the sake of simplicity, only some examples are shown in Figure 4. Here, we selected the cross sections of \( y = 0 \) to compare the relationship between \( g_n(x, y) \) and \( n \) in Figure 5. It is obvious that with the increase of \( n \), the shape of \( g_n(x, y) \) becomes flat and gradually converge to the two dimensional Gaussian probability density function, which indicates that the variance of the function is larger and verifies the validity of Equation (14).

2.2.4. Noise reduction of difference function. Here, \( S_1 \) and \( S_2 \) denote the sets of cell coordinates for basic wall panel and predicted wall panel, respectively. Assume that \( (x_1, y_1) \in S_1 \), \( (x_2, y_2) \in S_2 \), repeat discretization noise reduction \( n \) times to transform \( \Delta_n(x_1, y_1, x_2, y_2) \) into \( \Delta_{n+1}(x_1, y_1, x_2, y_2) \). Similar to Equation (15), the weight function is shown below:

\[
G(\Delta x, \Delta y) = \max \left( 2 - \sqrt{\Delta x^2 + \Delta y^2}, 0 \right)
\]  

(16)

After noise reduction, the difference function has changed to the function below:

\[
\Delta_{n+1}(x_1, y_1, x_2, y_2) = \sum_{(\Delta x, \Delta y) \in A} G(\Delta x, \Delta y) \cdot \Delta_n(x_1 + \Delta x, y_1 + \Delta y, x_2 + \Delta x, y_2 + \Delta y)
\]  

(17)

where \( A = \{(\Delta x, \Delta y)|\Delta x^2 + \Delta y^2 \leq r^2; (x_1 + \Delta x, y_1 + \Delta y) \in S_1; (x_2 + \Delta x, y_2 + \Delta y) \in S_2\} \).

3. Verification of the refined CA method by examples

The experimental data of laterally loaded wall panels with openings from Chong [7] is adopted to verify the refined CA method and noise reduction method. Figure 6 shows the experimental failure pattern of five typical wall panels named W01, W02, ..., W05, respectively. The dimension of the wall
panel is 5615mm × 2475mm, with a free top edge and the other three simply supported edges. Uniformly distributed lateral static loads are applied to the wall panels until their failure.

Figure 6. The experimental failure patterns of wall panels

3.1. Verification of new matching criterion

3.1.1 Parameter analysis. The four weight coefficients in Equation (8) can be used to adjust the influence of each error term on the final results. Here, the prediction results of W01 based on W04 is taken as an example to carry out parameter analysis. Similarity $\eta (\eta \in (0, 1])$ is introduced to judge the accuracy of prediction:

**Definition 3.1** Similarity of two numbers

If two numbers $a$ and $b$ satisfy $a \geq 0$, $b \geq 0$, $a + b > 0$, then their similarity $\eta$ is:

$$\eta(a, b) = 1 - \frac{|a - b|}{a + b}$$

(18)

when $a = b$, $\eta = 1$; when $a$ or $b$ is 0, $\eta = 0$.

**Definition 3.2** Similarity of two matrices

$A = (a_{ij})_{m \times n}$ and $B = (b_{ij})_{m \times n}$ are two matrices with the same numbers of rows and columns. If their elements satisfy $a_{ij} \geq 0$, $b_{ij} \geq 0$, $a_{ij} + b_{ij} > 0$, The similarity $\bar{\eta}$ is the mean of the similarity of each element of the two matrices:

$$\bar{\eta}(A, B) = \frac{1}{mn} \sum_{i=1}^{m} \sum_{j=1}^{n} \eta(a_{ij}, b_{ij})$$

(19)

Thus, through applying equations (18)-(19) to difference function $\Delta_s(x_1, y_1, x_2, y_2)$, the similarity $\eta$ of wall panels can be calculated by normalizing the sum of each cell’s similarity $\eta$. By changing only one weight coefficient at one time and normalizing the results, the relationship between $\eta$ and the weight coefficient can be obtained, as shown in Figure 7. It is obvious that $\eta$ decreases with the increment of $k_1$, $k_3$, $k_4$ throughout the process. While $k_2$ is special, $\eta$ increases swiftly and then becomes stable with the increment of $k_2$. Therefore, increase $k_2$ and keep other weight coefficients at a low level can enhance the prediction accuracy. After statistical analysis, it is discovered that the prediction accuracy is the highest when $k_1 = 0.05$, $k_2 = 1$, $k_3 = 0$, $k_4 = 0$.

Figure 7. The relationship between similarity $\eta$ and weight coefficients $k_i$

3.1.2 Prediction results. To be brief, the original minimum matching criterion is marked as MC 1 and
the new Taylor expansion matching criterion is marked as MC 2. The weight coefficients of MC 2 are taken as $k_1 = 0.05$, $k_2 = 1$, $k_3 = 0$, $k_4 = 0$. The prediction results are divided into two categories according to the basic wall panel with openings or not.

1) Failure patterns prediction of W02, W03, W04, W05 based on W01

Four panels’ experimental failure patterns and prediction results are shown in Table 1. Overall, MC 2 has better predicting capacity than MC 1 and the similarity $\eta$ for four panels are all over 0.9, which are higher than MC 1. Besides, the clarity and continuity of cracks are also enhanced by MC 2, which is closer to experimental results. Especially for W02, W03 and W04, the accuracy of prediction has substantial promotion and there are almost no discrete points in prediction results. As for W05, the appearance of cracks is also more distinct and continuous.

Table 1. The failure patterns of wall panels predicted based on W01

|       | W02  | W03  | W04  | W05  |
|-------|------|------|------|------|
| Experimental results | ![Image](image1) | ![Image](image2) | ![Image](image3) | ![Image](image4) |
| MC 1 prediction results | $\eta=0.83$ | $\eta=0.87$ | $\eta=0.89$ | $\eta=0.94$ |
| MC 2 prediction results | $\eta=0.92$ | $\eta=0.95$ | $\eta=0.91$ | $\eta=0.96$ |

2) Failure patterns prediction of W02, W03, W05 based on W04

The experimental and prediction results are shown in Table 2. For basic wall panel W04 with openings, the prediction results of MC 1 and 2 show intermittent points in the same region of four wall panels. Comparing to W05 whose opening is located at right side, the similarity $\eta$ of prediction for W02 and W03 are better for their openings are located at middle of panels, which is similar to the basic wall panel. It can be seen that the prediction of failure patterns of wall panels with openings requires to consider the location and shape of openings. Overall, MC 2 still has better predicting capacity than MC 1 and the similarity $\eta$ for four panels are all higher than MC 1.

Table 2. The failure patterns of wall panels predicted based on W04

|       | W02  | W03  | W05  |
|-------|------|------|------|
| Experimental results | ![Image](image5) | ![Image](image6) | ![Image](image7) |
| MC 1 prediction results | $\eta=0.92$ | $\eta=0.90$ | $\eta=0.82$ |
| MC 2 prediction results | $\eta=0.94$ | $\eta=0.93$ | $\eta=0.89$ |

3.2. Verification of post-processing of noise reduction

In this section, noise reduction is applied on the difference function in MC 1 for several times, and then the new function is used to predict the failure pattern of W01 again based on W02. Figure 8
shows the final prediction results of different times of noise reduction. With the increase of noise reduction times, the shape of the weight function becomes wider and flat while the cracks become more continuous and the area of them decreases. Besides, the similarity \( \eta \) is also enhanced with the increase of noise reduction times. However, considering the computational complexity of noise reduction, appropriate noise reduction times should be chosen according to the actual situation.

In addition, as shown in Figure 9, matching criterion 2 (Figure 9c) and post-processing of noise reduction (Figure 9d) both reach higher accuracy than existing CA method in predicting failure patterns of wall panels (Figure 9b). And the combination of them (Figure 9e) can achieve the highest similarity, in other words, the optimal result is achieved.

\[ \eta = 0.87 \quad \eta = 0.90 \quad \eta = 0.91 \quad \eta = 0.93 \]

Figure 8. The failure patterns of W01 predicted after different noise reduction times

\[ \eta = 0.88 \quad \eta = 0.94 \quad \eta = 0.92 \quad \eta = 0.96 \]

Figure 9. The failure patterns of W01 predicted by different criteria and noise reduction method

4. Conclusion
This paper proposed two measures to update the existing CA technique for predicting the failure patterns of masonry walls with openings which provides a reference for relative forecasting problems.

The new matching criterion based on Taylor expansion defines the difference of the state value of central cell, the gradient and Hessian matrix of the local cells’ state values and then linearly combine these differences to describe the total difference between the two wall panels. The similar region determined by the refined matching criterion can predict the failure patterns of wall panels more accurately than the current minimum error matching criterion. In addition, the sensitivity analysis of four weight coefficients \( k_1, k_2, k_3, k_4 \) in the difference function of Taylor expansion matching criterion verifies the appropriate combination through investigating the relationship between the four coefficients and prediction accuracy.

For the discontinuity of cracks in the prediction results of wall panels, a rational explanation is given from the perspective of noise. The new noise reduction method can replace Gaussian filtering algorithm by introducing the repeated noise reduction algorithm, which effectively enhanced the computational efficiency and prediction accuracy of the failure patterns for wall panels.

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