SPIN EFFECTS IN RADIATING COMPACT BINARIES

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We review and summarize our results concerning the influence of the spins of a compact binary system on the motion of the binary and on its gravitational reaction. We describe briefly our method which lead us to compute the secular changes in the post-Newtonian motion and the averaged radiative losses. Our description is valid to 1.5 post-Newtonian order. All spin-orbit and some spin-spin effects are considered which contribute at this accuracy. This approach enabled us to give both the evolutions of the constants of the nonradiative motion and of the relevant angular variables under radiation reaction.

1 Introduction

Gravitational radiation, predicted by Einstein’s theory, has long been unavailable to experimentalists because of the low power of laboratory sources. However, a new generation of earth-based gravitational-wave detectors is approaching its final stage of construction (LIGO\textsuperscript{4}, VIRGO\textsuperscript{5}, GEO600 and TAMA300). A strong hope is that in the next decade direct experimental evidence for this brilliant theoretical prediction will be obtained.

Coalescing binary neutron star systems are certainly among the most promising sources for earth-based detectors with the frequency range ($1 - 10^4$ Hz). Neutron star-black hole and black hole-black hole binaries are also significant sources\textsuperscript{6} for the frequency range ($10^{-4} - 1$ Hz) of the Laser Interferometer Space Antenna (LISA). For data reduction, signal templates as well as the knowledge of the reaction effects of the gravitational radiation emitted by these compact binary systems are needed to a high precision. There are indications that computations up to the third post-Newtonian (3PN) approximation will ensure the required accuracy. The computations have almost reached this level; there are several generic treatments at 2PN accuracy\textsuperscript{7,8,9} and a recent one\textsuperscript{10} at 2.5PN, however in most cases spin effects were not taken into account. Binary systems do, in many cases, have a non-negligible spin. In a series of papers\textsuperscript{11,12,13} we have considered the influence of the spin on radiation reaction.

Spin-orbit and spin-spin effects appear at 1.5PN and 2PN orders, respectively. The instantaneous losses in the constants characterizing the nonradiative motion (the energy and the total angular momentum vector) and also the
wave forms in the presence of these spin effects were given by Kidder for
generic eccentric orbits. He has used the Blanchet-Damour-Iyer formalism for evaluating the symmetric trace-free moments, the covariant spin supplementary condition (SSC) and employed a description of the binary motion following Barker and O’Connell, Thorne and Hartle and Kidder, Will and Wiseman. He gave also the averaged losses of the dynamical quantities for circular orbits.

Despite the classical result on the circularization of the orbits due to gravitational radiation reaction, eccentric orbits can be relevant in various physical scenarios, as emphasized by several authors. Such binaries are likely to be formed, for example, in galactic nuclei by capture events, in which time is insufficient for circularization before plunging.

Averaging over eccentric orbits however turns out to be difficult for binary systems with spins. We are content to include leading order spin effects, which appear at 1.5PN order. For a test particle, this is a good approximation either for a black hole-neutron star binary or for the debris particle orbiting about a massive spinning central body. The averaged losses in the constants characterizing the nonradiative motion on eccentric orbits were given by Ryan. His analysis lead to the same results as our approach based on the Lense-Thirring picture.

2 The method and the results

We have generalized our description for the case when the masses of the two bodies are comparable, but one spin dominates over the other and, more recently, for comparable masses and spins.

For a full description to the 1.5PN order we have introduced additional angle variables (Fig.1), which are not constant even in the absence of radiation, and computed their radiative changes. These angles subtended by the directions of the Newtonian orbital angular momentum $\hat{L}_N$ and spin vectors $\hat{S}_i$ can be important in monitoring the relative orientation of the binary with respect to the detector. For circular orbits, the evolution of these angles has been discussed in recent works. For eccentric orbits we have given both the instantaneous and averaged evolution equations. We have found that eccentricity speeds up the evolution.

In order to carry out these computations we had to appeal to the Burke-Thorne potential, since in the angular losses, the radiative losses of the spins give contributions.

A striking feature of these losses was that (although these Burke-Thorne potential terms are present in the instantaneous losses) they average out to
zero in the corresponding secular expressions. Our results concerning the losses in the constants of motion are in agreement with results of Rieth and Schäfer\textsuperscript{20}, which were obtained in an other SSC. When complemented with our equations for the angles in terms of the semimajor axis $a$, mass ratio $\eta = m_2/m_1$ and eccentricity $e$,

$$\langle d\kappa_1 \rangle = \frac{G^{7/2}m^{3/2}\mu}{30c^2a^{11/2}(1-e^2)^4} \left\{ (285e^4+1512e^2+488)(S_1 \sin \kappa_1 + S_2 \sin \kappa_2 \cos \Delta \psi) \\
+ (221e^4+1190e^2+384)(\eta S_1 \sin \kappa_1 + \eta^{-1} S_2 \sin \kappa_2 \cos \Delta \psi) \\
+ (156e^4+240e^2)(S_1 \sin \kappa_1 \cos(2\psi_1-2\psi_0) + S_2 \sin \kappa_2 \cos(\psi_1+\psi_2-2\psi_0)) \\
+ (119e^4+193e^2)(\eta S_1 \sin \kappa_1 \cos(2\psi_1-2\psi_0) + \eta^{-1} S_2 \sin \kappa_2 \cos(\psi_1+\psi_2-2\psi_0)) \right\}$$

we have a complete dynamical system describing the evolution of the radiating binary\textsuperscript{10}.

Figure 1: The angles characterizing the angular momenta, the position $r$, the direction of the periastron and the node line.

In obtaining our results we were much helped by the averaging method on quasi-elliptical orbits developed by us. This method relies on the application of the residue theorem for various integrands. When written in terms of a suitably chosen parameter, the only pole is in the origin. This feature obviously simplifies the computations. The „suitably chosen parameter” for most of the integrands is a generalization of the true anomaly parameter $\chi$ of the Kepler orbits, defined by:

$$\frac{2}{r} = \frac{1 + \cos \chi}{r_{\text{min}}} + \frac{1 - \cos \chi}{r_{\text{max}}}$$

where $r_{\text{max}}$, $r_{\text{min}}$ are the values of the radial distance $r$ at the turning points $\dot{r} = 0$.

We need another parameter when computing the period. This is provided by a generalization to the spinning binary case of the eccentric anomaly parametrization of Kepler orbits. This type of parameter was employed previously by Damour and Deruelle\textsuperscript{25} to quasi-Keplerian systems with 1PN perturbations and by Damour, Schäfer and Wex\textsuperscript{26} to the 2PN order of accuracy. Currently we investigate under which conditions and parametrizations do the advantageous properties of the integrands continue to remain valid\textsuperscript{30}. 

3
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