Development of deep sea ARV cables physical characteristics

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Abstract. Aiming at the challenging frontier problem of umbilical cable properties theory, which has been puzzled by this kind of robot for many years but has not been solved yet, this paper proposes an umbilical cable properties modelling method based on Kirchhoff elastic bar theory. This method analyzes the force on the umbilical cable member in the equilibrium state, obtains the Kirchhoff equation of the mobile cable in the equilibrium state, and then establishes the physical characteristics model of the mobile cable, and obtains the form of umbilical cable and the stress on the end point by solving the model. The new modelling theory research on the physical characteristics of deep-sea umbilical cable carried out in this paper is expected to solve the problem of "unclear" problems of deep-sea slender umbilical cable, get rid of the dilemma of "blind man and elephant", and provide guarantee for the safe and efficient operation of deep-sea robots.

1. Introduction

The deep sea contains treasures that are far from being recognized and exploited on earth. To obtain such treasures, key technologies must be mastered in terms of deep sea access. The deep-sea underwater vehicle is an essential tool to realize deep-sea entry, exploration and development, and an important equipment to carry out underwater intelligent covert warfare. Umbilical cable is the only effective means of real-time signal transmission and remote control in deep-sea environment, but it is the most vulnerable link, which is always facing the risk of fracture. The safety of umbilical cable is the prerequisite for the normal operation of underwater vehicle with cable. Therefore, it is of great significance and value to carry out research on deep-sea underwater robot system and innovative technology to meet the country's major needs.

Many scholars at home and abroad have done a lot of research on the nonlinear mechanical model of flexible cables. Mireille et al. [1] and Lv et al. [2] based on the particle spring model, the flexible parts such as wire harness, cable and hose are analyzed in the computer virtual simulation assembly
process. Zhu et al. [3] and Park et al. [4] established multi-body systems model of umbilical cable of underwater robot, and analyzed the strongly nonlinear coupling motion between umbilical cable and underwater robot. Quan et al. [5] and Alexander et al. [6] established the cable dynamic model of the underwater towing system based on the finite element method, which improved the directional stability, maneuverability, safety and control characteristics of the cable towing body. Mass spring model, multi-body model and finite element model focus on the solution of cable shape, while Kirchhoff theory. Not only can the shape of flexible body such as cable be solved, but also the mechanical properties of discrete points of flexible body such as cable can be solved. The nonlinear mechanics theory of elastic bar mainly includes Kirchhoff theory [7,8,9] and Cosserat theory [10,11]. Compared with Cosserat theory, Kirchhoff theory ignores complex factors such as shear deformation of flexible body sections such as cables when using, so Kirchhoff theory is a special case of Cosserat theory [12]. Liu et al. [13] first established the constraint theory model of space cable surface based on Kirchhoff theory, and solved the joint disturbance moment of the cable to space robot, providing an important theoretical reference for the stability study of space robot system. Liu et al. [14, 15] proposed Kirchhoff elastic rod nonlinear mechanics model, introducing the condition of fixed length, ignore environmental forces, using the method of discretization numerical solution under the restriction of both ends of the cable shape, and successfully applied to the virtual reality technology, virtual assembly can be accurately completed cable, to avoid the interference between cable and mechanical and electrical system.

![Figure 1. General structure of deep-sea ARV umbilical cable system](image)

Above all, Kirchhoff theory is mainly applied to the virtual assembly of cables at present. There are few researches on deep-sea umbilical cables, and none of them takes into account the influence of environment on the shape of flexible cables. To solve this problem, this paper establishes a deep-sea umbilical cable physical properties modeling method considering ocean currents, which provides a guarantee for the safety operation of deep-sea umbilical cable.

2. Static analysis of umbilical cable system

The solution of the armored cable dynamic model based on Kirchhoff elastic bar theory is mainly carried out in three coordinate systems, namely world coordinate system \( O - \xi \eta \zeta \), Frent coordinate system \( P - NBT \) and spindle coordinate system \( P - xyz \). The origin \( O \) of the world coordinate system \( O - \xi \eta \zeta \) is fixedly connected with the initial point of the armoring center line. In this coordinate system, the position of any point \( P_0 \) on the curve on the curve can be determined by arc coordinate \( s \), and by the vector diameter \( r \) of point \( P_0 \) relative to point \( O \). \( r \) is a single-valued continuous differentiable function of arc coordinate \( s \).
As shown in Fig. 2, statics analysis is carried out on the micro arc segment of armored cable in the world coordinate system \( O - \xi \eta \zeta \). The vector diameter of \( P_0 \) and \( P \) relative to point \( O \) are \( r \) and \( r + \Delta r \), and the arc coordinates of \( P_0 \) and \( P \) relative to point \( O \) are \( s \) and \( s + \Delta s \). It is stipulated that the section of the outer normal vector at \( P \) and \( P_0 \) at the same direction of increase in arc coordinates is a positive section, and vice versa. Let the internal forces and internal torques received by the negative section of point \( P_0 \) be \(-F\) and \(-M\), and the internal forces and internal torques received by the positive section of point \( P \) be \((F + \Delta F)\) and \((M + \Delta M)\), and the distributed forces received by this arc section be \(f\). The force balance and moment balance of micro element arc can be obtained.

\[
\frac{dF}{ds} + \omega \times F + f = 0 \quad \frac{dM}{ds} + \omega \times M + e_3 \times F = 0
\]  

(1)

Where \( \omega \) is the rate of change of section angle \( \phi \) with respect to of arc-coordinate \( s \). The wavy line is the derivative with respect to the spindle coordinate system.

\[
\omega = \lim_{\Delta s \to 0} \frac{\Delta \phi}{\Delta s}
\]  

(2)

The cable section torque can be expressed as

\[
M_x = k_x \omega_x \quad M_y = k_y \omega_y \quad M_z = k_z \omega_z
\]  

(3)

Where \( k_x \) is the flexural stiffness of the section around the coordinate axis \( x \), \( k_y \) is the flexural stiffness of the section around the coordinate axis \( y \), and \( k_z \) is the torsional stiffness of the section around the coordinate axis \( z \).

The relationship between \( \omega \) and Euler parameters can be obtained from the infinitesimal rotation theory of cylinder block

\[
\begin{align*}
\omega_x &= -q_2 \frac{dq_1}{ds} + q_1 \frac{dq_2}{ds} + q_4 \frac{dq_3}{ds} - q_3 \frac{dq_4}{ds} \\
\omega_y &= -q_3 \frac{dq_1}{ds} - q_4 \frac{dq_2}{ds} + q_1 \frac{dq_3}{ds} - q_2 \frac{dq_4}{ds} \\
\omega_z &= -q_4 \frac{dq_1}{ds} + q_3 \frac{dq_2}{ds} - q_2 \frac{dq_3}{ds} + q_1 \frac{dq_4}{ds}
\end{align*}
\]  

(4)
Euler parameter satisfies
\[ y_1 = q_1^2 + q_2^2 + q_3^2 + q_4^2 - 1 = 0 \] (5)

Write equation (1) as a scalar form to get
\[ y_2 = \frac{dF_x}{ds} + 2(\omega_y F_z - \omega_z F_y) + f_x = 0 \] (6)
\[ y_3 = \frac{dF_y}{ds} + 2(\omega_z F_x - \omega_x F_z) + f_y = 0 \] (7)
\[ y_4 = \frac{dF_z}{ds} + 2(\omega_x F_y - \omega_y F_x) + f_z = 0 \] (8)
\[ y_5 = k_x \frac{d\omega_x}{ds} + (k_z - k_y)\omega_y \omega_z - F_y = 0 \] (9)
\[ y_6 = k_y \frac{d\omega_y}{ds} + (k_x - k_z)\omega_z \omega_x + F_z = 0 \] (10)
\[ y_7 = k_z \frac{d\omega_z}{ds} + (k_y - k_x)\omega_x \omega_y = 0 \] (11)

Equation (5) ~ (11) can be written as matrix form
\[ y = [y_1 \ y_2 \ y_3 \ y_4 \ y_5 \ y_6 \ y_7]^T = 0 \] (12)

Equation (12) determines a closed system of equations.

Before solving the Differential equation, the equation should be discretized. In this paper, DQM (Differential Quadrature Method) is applied to discretization of the equation. DQM algorithm discretizes the equation in spatial domain, and the discrete format is
\[ \frac{d^2x}{ds^2} \bigg|_{s=s_i} = \sum_{k=1}^{N} A^T_k x_k \] (13)

After spatial dispersion, equation (12) becomes a nonlinear algebraic system of equations about \([q_1, q_2, q_3, q_4, F_x, F_y, F_z]\). Write the system of equations as \( \mathbf{h}(x) = [h_1(x), ..., h_n(x)]^T = 0 \). The problem is transformed into a nonlinear least squares problem by using the nonlinear least squares method, and the objective function is
\[ \begin{cases} \min H(x) \\ H(x) = \frac{1}{2} \sum_{i=1}^{N} h_i^2(x) \end{cases} \] (14)

We were used Matlab optimization toolbox to solve the problem.

The Euler parameter form of the projection in the world coordinate system is
\[ \begin{align*}
\xi(s) &= 2\int_0^s (q_x(\sigma) q_x(\sigma) + q_4(\sigma) q_3(\sigma)) \, d\sigma \\
\eta(s) &= 2\int_0^s (q_3(\sigma) q_4(\sigma) - q_4(\sigma) q_3(\sigma)) \, d\sigma \\
\zeta(s) &= \int_0^s (2(q_1^2(\sigma) + q_2^2(\sigma)) - 1) \, d\sigma
\end{align*} \] (15)
The spatial form of umbilical cable can be obtained by substituting Euler parameters into equation (15) through equation (14).

3. Calculation result
In this paper, the steady current, the existence of undercurrent vortex and other conditions were calculated theoretically.

It is assumed that there is only one direction current, the distribution force of the current is 1.0 N/m, and the end point is located at (-100,-80,-500).

![Figure 3. Schematic diagram of umbilical cable shape and force under steady current](image1)

The shape of umbilical cable under the steady current is shown in Fig. 3(a). The umbilical cable bends to some extent under the action of the current. The force diagram of umbilical cable is shown in Fig. 3(b). The initial point of umbilical cable bears the maximum stress, which is the most easily pulled off position.

(2) Undercurrent
Suppose an undercurrent exists at a depth of 300m, opposite to the current direction. The distribution force of the current is 1.0 N/m, and the distribution force of the undercurrent is 0.8 N/m. The terminal point is located at (-100,-80,-500).

![Figure 4. Schematic diagram of umbilical cable shape and force with an undercurrent](image2)

The form of umbilical cable in the presence of undercurrent is shown in Fig. 4(a). Under the combined action of ocean current and undercurrent, the umbilical cable bends greatly. As shown in
Fig. 4(b), the umbilical cable has a peak internal force in the undercurrent section. When the force in the undercurrent section increases, the peak internal force will increase too. When the allowable internal force is exceeded, the umbilical cable will break.

4. Conclusion
The umbilical cable of deep-sea robot is still in the blank stage of theoretical research. In this paper, based on Kirchhoff elastic bar theory, the nonlinear mechanical model of umbilical cable of deep-sea robot is established. The influences of ocean current and undercurrent on umbilical cable properties are considered, and the morphology and mechanical characteristics of umbilical cable under different sea conditions are obtained, which provides a strong guarantee for the safe operation of deep-sea robot.

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