Neutrino Mixing from the Charged Lepton Sector with Sequential Right-Handed Lepton Dominance

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Abstract

We systematically analyze the possibility that bi-large lepton mixing originates from the charged lepton sector in models with sequential dominance for the right-handed charged leptons. We derive analytical expressions for the mixing angles and CP phases of the MNS matrix for the case of zero mixing from the neutrino sector, which is arranged for with the help of sequential dominance for the right-handed neutrinos. For small $\theta_{13}$, the two large mixing angles $\theta_{12}$ and $\theta_{23}$ are determined by the Yukawa couplings to the dominant right-handed tau. The mixing angle $\theta_{13}$ is then governed by the subdominant right-handed muon Yukawa couplings. Naturally small $\theta_{13}$ and sequential right-handed lepton dominance can be realized in type I see-saw models and their type II upgrades via spontaneously broken SO(3) flavour symmetry and real vacuum alignment. We discuss the prediction for $\theta_{13}$ in this scenario and corrections to it including the dependence on the neutrino mass scale.

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1 Introduction

The mixing matrix in the lepton sector, the MNS matrix $U_{\text{MNS}}$, is defined by the charged electroweak current $\overline{e_L} \gamma^\mu U_{\text{MNS}} \nu^f_L$ in the mass basis. It is of course possible to choose a basis where all the lepton mixing stems entirely from the neutrino sector or from the charged lepton sector. From the perspective of model building, the goal is to identify symmetries and mechanisms for understanding the observed fermion masses and mixings. Here it makes a big difference whether the mixing is generated in the neutrino sector, the charged lepton sector or maybe partly in both sectors. For a review on neutrino mass models, see e.g. [1]. Experimentally, it has been found that the lepton mixing angles $\theta_{12}$ and $\theta_{23}$ are both large whereas $\theta_{13}$ is comparably small.

Recently, there has been some interest in the possibility of generating the large neutrino mixings in the charged lepton sector. In [2, 3], bi-large mixing and small $\theta_{13}$ from the charged lepton mass matrix have been achieved by a kind of see-saw mechanism in the charged lepton sector. In an earlier study [4], a texture for bi-large mixing and small $\theta_{13}$ from the charged leptons and its realization by effective operators in the SO(10)-GUT framework have been considered.

In this work, we systematically analyze the possibility that the lepton mixing originates from the charged lepton sector in models with sequential dominance for the right-handed charged leptons as well as for right-handed neutrinos [5, 6]. We find that for several reasons, such a generalized sequential dominance is a useful foundation for models of this type: It implies the desired hierarchy for the charged lepton masses and small right-handed charged lepton mixings. In addition, it can naturally lead to small mixings from the neutrino sector, even if the neutrino Yukawa matrix has a similar structure to the charged lepton Yukawa matrix. For investigating the requirements for bi-large mixing with naturally small $\theta_{13}$, we derive analytic expressions for the mixing angles and CP phases of the MNS matrix for the case of zero mixing coming from the neutrino sector. We find that for small $\theta_{13}$, the two large mixing angles $\theta_{12}$ and $\theta_{23}$ are governed by the Yukawa couplings to the dominant right-handed tau. The mixing angle $\theta_{13}$ is then determined by the subdominant right-handed muon Yukawa couplings. We find a condition for naturally small $\theta_{13}$, which is different from the vanishing determinant condition of [2, 3].

Sequential right-handed lepton dominance and lepton mixing originating from the charged lepton sector can be realized via spontaneously broken SO(3) flavour symmetry and real vacuum alignment [7]. Three zero entries in the neutrino and charged lepton Yukawa couplings, leading to small mixing from the neutrino sector in combination with small lepton mixing $\theta_{13}$, can arise with real alignment of the SO(3)-breaking vacuum. Type I see-saw models of this type can be upgraded to type II see-saw models, where hierarchical neutrino masses are as natural a partially degenerate ones. We discuss the prediction for $\theta_{13}$ in this scenario and how corrections to it depend on the neutrino mass scale.
2 Our Conventions

For the mass matrix of the charge leptons \( M_\ell = Y_\ell v_d \) defined by \( \mathcal{L}_\ell = -M_\ell e^T \ell_R + \text{h.c.} \) and for a neutrino mass matrix \( m^\nu_{LL} \) of Majorana-type defined by \( \mathcal{L}_\nu = -\frac{1}{2} m^\nu_{LL} \nu_L \nu^C_L + \text{h.c.} \), the change from flavour basis to mass eigenbasis can be performed with the unitary diagonalization matrices \( U_{\ell L}, U_{\ell R} \) and \( U_{\nu L} \) by

\[
U_{\ell L} M_\ell U_{\ell R}^\dagger = \begin{pmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{pmatrix}, \quad U_{\nu L} m^\nu_{LL} U_{\nu L}^T = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}.
\]

The MNS matrix is then given by

\[
U_{\text{MNS}} = U_{\ell L} U_{\nu L}^\dagger.
\]

We use the parameterization

\[
U_{\text{MNS}} = R_{23} U_{13} R_{12} P_0 \quad \text{with} \quad R_{23}, U_{13}, R_{12} \quad \text{and} \quad P_0 \quad \text{being defined as}
\]

\[
R_{12} := \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad U_{13} := \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta} & 0 & c_{13} \end{pmatrix},
\]

\[
R_{23} := \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}, \quad P_0 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\beta_2} & 0 \\ 0 & 0 & e^{i\beta_3} \end{pmatrix}
\]

and where \( s_{ij} \) and \( c_{ij} \) stand for \( \sin(\theta_{ij}) \) and \( \cos(\theta_{ij}) \), respectively. The matrix \( P_0 \) contains the possible Majorana phases \( \beta_2 \) and \( \beta_3 \). \( \delta \) is the Dirac CP phase relevant for neutrino oscillations.

3 Sequential Dominance for Right-Handed Leptons

We first generalize the notion of sequential right-handed neutrino dominance (RHND) \cite{5, 6}, which is an extension of single right-handed neutrino dominance \cite{8, 9}, to all right-handed leptons. Let us therefore write the Yukawa couplings for the charged leptons and the neutrinos \( Y_\ell \) and \( Y_\nu \) and the heavy Majorana mass matrix for the charged leptons as

\[
Y_\ell = \begin{pmatrix} p & d & a \\ q & e & b \\ r & f & c \end{pmatrix}, \quad Y_\nu = \begin{pmatrix} p' & d' & a' \\ q' & e' & b' \\ r' & f' & c' \end{pmatrix}, \quad M_{RR} = \begin{pmatrix} M_{R1} & 0 & 0 \\ 0 & M_{R2} & 0 \\ 0 & 0 & M_{R3} \end{pmatrix}.
\]

We can choose a basis where \( M_{R1}, M_{R2} \) and \( M_{R3} \) are real and positive. The entries in the Yukawa matrices are in general complex. In the case of a type I see-saw scenario \cite{10, 11, 12, 13}, the neutrino mass matrix is given by

\[
m^\nu_{LL} = m^\nu_{LL} = -v^2 Y_\nu M_{RR}^{-1} Y^T_\nu.
\]
We will assume sequential dominance among the right-handed neutrinos and charged leptons, respectively. In our notation, each right-handed charged lepton couples to a column in $\nu_e$ and each right-handed neutrino to a column in $\nu_e$. For the charged leptons, the sequential dominance conditions are

$$ |a|, |b|, |c| \gg |d|, |e|, |f| \gg |p|, |q|, |r| .$$

(6)

They imply the desired hierarchy for the charged lepton masses $m_\tau \gg m_\mu \gg m_e$ and small right-handed mixing of $U_{eR}$. In the neutrino sector, the sequential dominance conditions determine which column of $\nu_e$ in combination with the corresponding mass eigenvalues of the right-handed neutrinos $M_{Ri}$ contributes dominantly and which sub-dominantly to $m_{\nu LL}$. Usually, sequential RHND \cite{5, 6} is viewed as a framework for generating large solar mixing $\theta_{12}$ and large atmospheric mixing $\theta_{23}$ in the neutrino mass matrix. However, given sequential dominance in the neutrino sector, one can easily find the conditions for small mixing from the neutrinos as well. Small mixing from the neutrino sector requires three small entries in $\nu_e$. In particular, we shall require $\nu_e$ and $M_{RR}$ to have the approximate form

$$ Y_e \approx \begin{pmatrix} a' & 0 & 0 \\ b' & q' & 0 \\ c' & r' & f' \end{pmatrix}, \quad M_{RR} \approx \begin{pmatrix} X' & 0 & 0 \\ 0 & X & 0 \\ 0 & 0 & Y \end{pmatrix},$$

(7)

with the sequential dominance conditions being

$$ \frac{|f'|^2}{Y} \gg \frac{|q'|^2, |r'|^2}{X} \gg \frac{|a'|^2, |b'|^2, |c'|^2}{X'} .$$

(8)

Small entries in $\nu_e$ have been neglected. Note that equations (7) and (8) are completely general, since a simultaneous reordering of the columns of $\nu_e$ and of $Y, X$ and $X'$ leads to the same type I neutrino mass matrix. As shown in \cite{4}, three zero entries in $\nu_e$ might stem from a spontaneously broken SO(3) flavour symmetry and real vacuum alignment. Other realizations might by found via Abelian or discrete symmetries. Note that the structure of equation (7) implies $\theta_{e12} = \theta_{e13} = 0$ and a small mixing $\theta_{e23} = \mathcal{O}(m_2^2/m_3^2)$ from the neutrino mass matrix. $m_2^1$ and $m_3^1$ are the neutrino mass eigenvalues in a type I see-saw model. For a hierarchical mass spectrum, they correspond to the square roots of the solar and atmospheric mass squared differences. We will analyze the consequences of this perturbation for the total lepton mixing angles $\theta_{13}, \theta_{12}$ and $\theta_{23}$ in an example in section 5. The small $\theta_{e23}$ will in particular contribute to the corrections for the $\theta_{13}$-mixing generated in the lepton sector, which we will discuss in section 6.

### 4 Bi-Large Mixing from the Charged Lepton Sector

We now derive analytic expressions for the mixing angles and CP phases of the MNS matrix in the limit of zero mixing originating from the neutrino sector. We assume
sequential right-handed charged lepton dominance $|a|, |b|, |c| \gg |d|, |e|, |f| \gg |p|, |q|, |r|$, as in equation (6). Zero mixing from the neutrino sector corresponds to

$$U_{\nu L} = \begin{pmatrix} 1 & 0 & 0 \\ e^{-i\beta_2^R} & 0 & 0 \\ 0 & e^{-i\beta_3^R} & 0 \end{pmatrix},$$  \tag{9}$$

and the MNS matrix is then given by $U_{\text{MNS}} = U_{\nu L} \cdot \text{diag}(1, e^{i\beta_2^R}, e^{i\beta_3^R})$. Note that sequential right-handed charged lepton dominance implies small right-handed mixings of the unitary diagonalization matrix $U_{\nu R}$ defined in equation (11) and, in particular, hierarchical charged lepton masses. With sequential right-handed charged lepton dominance, the lepton mixing matrix $U_{\text{MNS}}$ is determined by the requirement that $U_{\nu L} \cdot Y_e$ has triangular form, i.e.

$$U_{\nu L} \cdot Y_e = R_{23} U_{13} R_{12} P_0 \cdot \begin{pmatrix} |p| e^{i\phi_p} & |d| e^{i\phi_d} & |a| e^{i\phi_a} \\ |q| e^{i\phi_q} & |c| e^{i\phi_c} & |b| e^{i\phi_b} \\ |r| e^{i\phi_r} & |f| e^{i\phi_f} & |c| e^{i\phi_c} \end{pmatrix} = \begin{pmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & * \end{pmatrix},$$  \tag{10}$$

with $P_0$ given by

$$P_0 := \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\beta_3^R} & 0 \\ 0 & 0 & e^{i\beta_3^R} \end{pmatrix},$$  \tag{11}$$

and with $R_{23} (\theta_{23}), U_{13} (\theta_{13}, \delta)$ and $R_{12} (\theta_{12})$ defined as in equation (12). From equation (10), together with equation (9), we obtain for the mixing angles $\theta_{12}, \theta_{13}$ and $\theta_{23}$

$$\tan(\theta_{12}) \approx \frac{\frac{a}{c} e^{i(\phi_a - \phi_c)} + \frac{d}{f} e^{i(\phi_d - \phi_f)}}{\frac{b}{c} e^{i(\phi_b - \phi_c + \beta_2^R)} - \frac{c}{f} e^{i(\phi_c - \phi_f + \beta_2^R)}},$$  \tag{12a}$$

$$\tan(\theta_{13}) \approx \frac{s_{12} |c| e^{i(\phi_c + \beta_2^R + \delta)} + c_{12} |d| e^{i(\phi_d + \delta)}}{|f| e^{i(\phi_f + \beta_2^R)}},$$  \tag{12b}$$

$$\tan(\theta_{23}) \approx \frac{s_{12} |a| e^{i\phi_a} - c_{12} |b| e^{i(\phi_b + \beta_2^R)}}{c_{13} |c| e^{i(\phi_c + \beta_3^R)} - s_{13} (c_{12} |a| e^{i\phi_a} + s_{12} |b| e^{i(\phi_b + \beta_2^R)})},$$  \tag{12c}$$

where the CP phases $\beta_2^c, \beta_3^c$ and $\delta$ are determined by the requirement that the mixing angles in equation (12) are real and in the range $[0, \frac{\pi}{2}]$. The phases are thus given by

$$\beta_2^c \approx \text{Arg} \left( \frac{\frac{a}{c} e^{i(\phi_a - \phi_c)} + \frac{d}{f} e^{i(\phi_d - \phi_f)}}{\frac{b}{c} e^{i(\phi_b - \phi_c + \beta_2^R)} - \frac{c}{f} e^{i(\phi_c - \phi_f + \beta_2^R)}} \right),$$  \tag{13a}$$

$$\beta_3^c - \delta \approx \text{Arg} \left( \frac{s_{12} |c| e^{i(\phi_c + \beta_2^R)} + c_{12} |d| e^{i\phi_d}}{|f| e^{i\phi_f}} \right),$$  \tag{13b}$$

$$\delta \approx \text{Arg} \left( \frac{s_{12} |a| e^{i\phi_a} - c_{12} |b| e^{i(\phi_b + \beta_2^R)}}{c_{13} |c| e^{i(\phi_c + \beta_3^R)} - s_{13} (c_{12} |a| e^{i\phi_a} + s_{12} |b| e^{i(\phi_b + \beta_2^R)})} \right).$$  \tag{13c}$$
We can extract the parameters in the following sequence: First we calculate $\beta^e_2$ and, using this result, we determine $\theta_{12}$. This allows to calculate $\beta^e_3 - \delta$ and subsequently $\theta_{13}$. Then, $\beta^e_3 - \delta$ has to be eliminated in equation (13c) in order to obtain $\delta$. Using all the previous results allows to calculate $\theta_{23}$. The mass eigenvalues of the charged leptons are approximately given by

\[ m_\tau \approx \left( |a|^2 + |b|^2 + |c|^2 \right)^{\frac{1}{2}} v_d, \]  

\[ m_\nu \approx \left( |d|^2 + |e|^2 + |f|^2 - \frac{|d^* a + e^* b + f^* c|^2}{m_7^2} \right)^{\frac{1}{2}} v_d, \]  

\[ m_e \approx \mathcal{O}(|p|, |q|, |r|) v_d. \]  

The mixing angle $\theta_{13}$ has a present experimental upper bound of roughly $15^\circ$. From equation (12b), we see that a natural possibility for obtaining a small $\theta_{13}$ is

\[ |d|, |e| \ll |f|. \]  

In leading order in $|d|/|f|$ and $|e|/|f|$, the formulae of equation (12) and (13) simplify considerably. For the mixing angles $\theta_{12}, \theta_{23}$ and $\theta_{13}$, we obtain

\[ \tan(\theta_{12}) \approx \frac{|a|}{|b|}, \]  

\[ \tan(\theta_{23}) \approx \frac{s_{12} |a| + c_{12} |b|}{|e|}, \]  

\[ \tan(\theta_{13}) \approx \frac{s_{12} |e| e^{i(\phi_a - \phi_c + \phi_e + \delta)} - c_{12} |d| e^{i(\phi_d + \delta)}}{|f| e^{i(\phi_a - \phi_c + \phi_f)}}, \]  

where the Dirac CP phase $\delta$ is determined such that $\theta_{13}$ is real, which requires

\[ \tan(\delta) \approx \frac{c_{12} |d| \sin(\phi_a - \phi_c - \phi_d + \phi_f) - s_{12} |e| \sin(\phi_b - \phi_c - \phi_e + \phi_f)}{c_{12} |d| \cos(\phi_a - \phi_c - \phi_d + \phi_f) - s_{12} |e| \cos(\phi_b - \phi_c - \phi_e + \phi_f)}. \]  

Given $\tan(\delta)$, $\delta$ has to be chosen such that $\tan(\theta_{13}) \geq 0$ in order to match with the usual convention $\theta_{13} \geq 0$. The phases $\beta^e_2$ and $\beta^e_3$ from the charge lepton sector are given by

\[ \beta^e_2 \approx \phi_a - \phi_b + \pi, \]  

\[ \beta^e_3 \approx \phi_a - \phi_c. \]

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1 If $a/b = d/e$, which implies that the two zero entries in the first row of equation (10) can be achieved by a $R_{12}$-rotation alone and thus no $U_{13}$ rotation is required, $\theta_{13}$ is approximately zero. This condition, which is equivalent to $a \cdot e = d \cdot b$, has been used in the model proposed in [2, 3]. Note that our condition (15) does not in general satisfy the condition $a \cdot e = d \cdot b$. 

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5
Note that in the case that the neutrino sector induces Majorana phases, the total Majorana phases $\beta_2$ and $\beta_3$ of the MNS matrix are given by

$$\beta_2 \approx \beta_2^e + \beta_2^\nu,$$

$$\beta_3 \approx \beta_3^e + \beta_3^\nu.$$  \hspace{1cm} (19a)

$$\theta_{13}$$ only depends on $d/f$ and $e/f$ from the Yukawa couplings to the subdominant right-handed muon and on $\theta_{12}$. We find that in the limit $|d|, |e| \ll |f|$, the two large mixing angles $\theta_{12}$ and $\theta_{23}$ approximately depend only on $a/c$ and $b/c$ from the right-handed tau Yukawa couplings. Both mixing angles are large if $a, b$ and $c$ are of the same order.

5 A Model with SO(3) Flavour Symmetry and Real Vacuum Alignment

Spontaneously broken SO(3) flavour symmetry and a real alignment mechanism for the SO(3)-breaking vevs \[7\] naturally allows for sequential dominance for the right-handed charged leptons as well as for the right-handed neutrinos. It furthermore leads to three zero entries in the neutrino and charged lepton Yukawa matrices in a special basis. The motivation for choosing such a basis would come from a full theory beyond the scope of this framework. D-brane models leading to textures with a dominant third column have been discussed e.g. in \[14\]. A classification of the textures arising from SO(3)-breaking with real vacuum alignment can be found in table 4 of \[7\] and a particular interesting class of models there is type C2, in which the neutrino and charged lepton Yukawa matrices have the form

$$Y_e = \begin{pmatrix} 0 & 0 & ae^{i\delta_3} \\ q e^{i\delta_1} & 0 & be^{i\delta_3} \\ re^{i\delta_1} & fe^{i\delta_2} & ce^{i\delta_3} \end{pmatrix}, \quad Y_\nu = \begin{pmatrix} 0 & 0 & a'e^{i\delta'_3} \\ q'e^{i\delta'_1} & 0 & f' e^{i\delta'_2} \\ r'e^{i\delta'_1} & f' e^{i\delta'_2} & c'e^{i\delta'_3} \end{pmatrix}. \hspace{1cm} (20)$$

with real parameters \{a, b, c, f, q, r, \delta_1, \delta_2, \delta_3\} and \{a', b', c', f', q', r', \delta'_1, \delta'_2, \delta'_3\}.

5.1 The Charged Lepton Sector

In the charged lepton sector the sequential dominance condition

$$|a|, |b|, |c| \gg |f| \gg |q|, |r|$$  \hspace{1cm} (21)

which is similar to equation (19) leads to the desired hierarchy among the charged lepton masses and to small right-handed mixing angles of $U_{eR}$. A characteristic feature of the Yukawa matrices, which stems from real alignment of the SO(3)-breaking vacuum, is that each column of the Yukawa matrices has a common complex phase $\delta_I$ or $\delta'_I$ ($I \in \{1, 2, 3\}$).
The lepton mixing angles from the charged lepton sector can be extracted using equation (16), which reduces to
\[ \tan(\theta_{e 12}^c) \approx \frac{|a|}{|b|} , \]  
\[ \tan(\theta_{e 13}^c) \approx 0 , \]  
\[ \tan(\theta_{e 23}^c) \approx \frac{\sqrt{|a^2| + |b^2|}}{|c|} . \]  

Approximately zero mixing \( \theta_{e 13}^c \) is a consequence of the two zero Yukawa couplings \((Y_e)_{12}\) and \((Y_e)_{22}\) to the right-handed muon. A texture for \( Y_e \) with a structure similar to the one we have found from spontaneous SO(3)-breaking has been considered in [4]. Note that due to the specific structure of \( Y_e \), the complex phases \( \delta_i' \) can be absorbed by \( U_{e R} \) and they thus have no effect on the CP phases of the MNS matrix. With zero \( \theta_{13} \) at leading order, the Dirac CP phase \( \delta \) is undefined. We now turn to the parameters of the neutrino sector and to corrections from small mixing in the neutrino sector for the lepton mixing angles and for the Dirac CP phase.

5.2 The Neutrino Sector

In leading order, the Majorana mass matrix of the heavy right-handed neutrinos has a diagonal structure from SO(3)-symmetry,
\[ M_{RR} = \begin{pmatrix} X & 0 & 0 \\ 0 & Y & 0 \\ 0 & 0 & X' \end{pmatrix} . \]  

In order to obtain small lepton mixings from the neutrino sector, we impose the sequential RHND condition
\[ \frac{|f'|^2}{Y} \gg \frac{|q'|^2, |r'|^2}{X} \gg \frac{|a'|^2, |b'|^2, |c'|^2}{X'} . \]  

The dominant and the subdominant columns of \( Y_\nu \) are equivalent to the form of equation (7). The type I version of the model implies a strongly hierarchical neutrino mass spectrum. We can however generalize the above type I see-saw model to a type II see-saw model (see e.g. [15, 16]) where the neutrino mass matrix is given by
\[ m_{\nu LL}^\nu = m_{LL}^{II} + m_{LL}^I . \]  

\( m_{LL}^{II} \) corresponds to an additional direct mass term for the neutrinos. The SO(3) flavour symmetry forces the direct mass term to be proportional to the unit matrix
\[ m_{LL}^{II} \approx m^{II} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} . \]
at leading order in the effective theory. This allows to upgrade the type I scenario of section 5 to natural models for hierarchical well as for partially degenerate neutrino masses \[7\]. The neutrino mass matrix is approximately given by

\[
m^\nu_{LL} = \begin{pmatrix}
0 & 0 & 0 \\
0 & m^{I\!I} - |q'| \frac{v^2}{X} e^{i2\delta'_1} & -|q'| \frac{v^2}{X} e^{i2\delta'_1} \\
0 & -|q'| \frac{v^2}{X} e^{i2\delta'_1} & m^{I\!I} - |f'| \frac{v^2}{Y} e^{i2\delta'_2} - |r'| \frac{v^2}{X} e^{i2\delta'_1}
\end{pmatrix}, \tag{27}
\]

where we have only considered \( m^{I\!I}_{LL} \) and the dominant and the subdominant part of \( m^{I\!I}_{LL} \). From the neutrino sector, we obtain corrections to the total lepton mixings due to a small mixing \( \theta^{\nu}_{23} \) of \( U^\dagger_{\nu L} \) determined by

\[
\theta^{\nu}_{23} \approx \frac{\text{sign}(q' r') |q'| |r'| \frac{v^2}{X}}{2 m^{I\!I} \sin(2\delta'_1) \sin(\delta) + |f'|^2 \frac{v^2}{Y} \cos(2\delta'_1 - 2\delta'_2 + \delta)} , \tag{28}
\]

where \( \delta \) is given by

\[
\tan(\delta) \approx \frac{|f'|^2 \frac{v^2}{Y} \sin(2\delta'_1 - 2\delta'_2)}{2 m^{I\!I} \cos(2\delta'_1) - |f'|^2 \frac{v^2}{Y} \cos(2\delta'_1 - 2\delta'_2)} \tag{29}
\]

and, given \( \tan(\delta) \), \( \delta \) is chosen such that \( \theta^{\nu}_{23} > 0 \). The latter leads to a positive induced total lepton mixing angle \( \theta_{13} \). \( \delta \) is the Dirac CP phase, which is induced from the complex phases in the neutrino sector. The smallness of \( \theta^{\nu}_{23} \) is guaranteed by equation (24). Including the corrections from the neutrino sector in leading order in \( \theta^{\nu}_{23} \), the total mixing angles of the MNS matrix are given by

\[
\tan(\theta_{12}) \approx \tan(\theta_{12}^e) , \tag{30a}
\]

\[
\theta_{13} \approx \theta^{\nu}_{23} \sin(\theta_{12}^e) , \tag{30b}
\]

\[
\tan(\theta_{23}) \approx \frac{\sin(\theta_{23}^e) + \theta^{\nu}_{23} \cos(\theta_{12}^e) \cos(\theta_{23}^e) \cos(\delta)}{\cos(\theta_{23}^e) - \theta^{\nu}_{23} \cos(\theta_{12}^e) \sin(\theta_{23}^e) \cos(\delta)} . \tag{30c}
\]

The neutrino mass eigenvalues of the type I see-saw version of the scenario are given by

\[
m^I_1 = \mathcal{O} \left( \frac{|a'|^2, |b'|^2, |a'| |b'|}{X'} v_u^2 \right) \approx 0 , \tag{31a}
\]

\[
m^I_2 \approx \frac{|q'|^2}{X'} v_u^2 , \tag{31b}
\]

\[
m^I_3 \approx \frac{|f'|^2}{Y'} v_u^2 . \tag{31c}
\]
The mass eigenvalues of the complete type II neutrino mass matrix are given by

\begin{align}
  m_1 & \approx |m^{II}|, \\
  m_2 & \approx |m^{II} - m^I_2 e^{i2\delta_1}|, \\
  m_3 & \approx |m^{II} - m^I_3 e^{i2\delta_2}|.
\end{align}

(32a)  
(32b)  
(32c)

For \( m^{II} \neq 0 \), the Majorana phases \( \beta_2 \) and \( \beta_3 \) can be extracted by

\begin{align}
  \beta_2 & \approx \frac{1}{2} \text{arg} \left( m^{II} - m^I_2 e^{i2\delta_1} \right), \\
  \beta_3 & \approx \frac{1}{2} \text{arg} \left( m^{II} - m^I_3 e^{i2\delta_2} \right).
\end{align}

(33a)  
(33b)

The corrections from the leading order neutrino mass matrix in general result in a non-zero value for \( \delta \) given by equation (29). The additional corrections for \( \theta_{13} \), which we will discuss below, could also modify the value for the Dirac CP phase.

6 The Neutrino Mass Scale and Predictions for \( \theta_{13} \)

A prediction \( \theta_{13} \approx 0 \) from the charged lepton sector as in section 5.1 will be subject to corrections from e.g. next-to-leading order effective operators allowed by symmetry, from the small mixings of \( U_{\nu_L} \), and from the renormalization group (RG) running between the high energy scale where the models are defined and the low energy scale where experiments are performed [17]. The corrections to \( \theta_{13} \approx 0 \) are linked to the neutrino mass scale. We expect the corrections from next-to-leading order effective operators to be larger for a larger neutrino mass scale since the parameters of the neutrino mass matrix are more sensitive to small modifications for a larger mass scale. Because of the latter reason, the RG effects are generically enhanced for a larger neutrino mass scale as well. However, for \( \theta_{13} \), this is compensated in the type-II-upgrade scenarios [7] by the fact that the Majorana phase \( \beta_2 \) gets smaller with increasing neutrino mass scale, which has a damping effect on the running of \( \theta_{13} \) [17]. Using sequential right-handed neutrino dominance for the type I part \( m^I_{LL} \) of the neutrino mass matrix, a small mixing \( \theta_{23}^\nu = \mathcal{O}(m^I_2/m^I_3) \) is generated from the neutrino mass matrix, as pointed out in section 3. This induces a contribution \( \Delta \theta_{13} \) to the total lepton mixing \( \theta_{13} \) given by \( \Delta \theta_{13} \approx \theta_{23}^\nu \sin(\theta_{12}) \). For type I see-saw scenarios, this results in a correction \( \Delta \theta_{13} \lesssim 5^\circ \). Since in the type-II-upgrade scenarios \( m^I_2/m^I_3 \) decreases with increasing \( m^{II} \), this correction will get smaller for a larger neutrino mass scale. For the model discussed in section 5, \( \theta_{23}^\nu \) is given by equation (30b) [7].
7 Conclusions

We have investigated the possibility that bi-large lepton mixing originates from the charged lepton sector in models with sequential dominance for the right-handed charged leptons as well as for right-handed neutrinos. In the charged lepton sector, sequential dominance implies the desired hierarchy for the charged lepton masses and in the neutrino sector, it can naturally lead to small mixings from the neutrino mass matrix even if the neutrino Yukawa matrix has a similar structure to the charged lepton Yukawa matrix. We have derived analytical expressions for the mixing angles and CP phases of the lepton mixing matrix $U_{\text{MNS}}$ for this case. We find that for small $\theta_{13}$, the two large mixing angles $\theta_{12}$ and $\theta_{23}$ are determined by the Yukawa couplings to the dominant right-handed tau lepton. The mixing angle $\theta_{13}$ is then governed by the subdominant right-handed muon Yukawa couplings. We find a condition for naturally small $\theta_{13}$, which is different from the vanishing determinant condition of $[2, 3]$. We have discussed an example with sequential dominance realized within the framework of spontaneously broken SO(3) flavour symmetry. Real vacuum alignment can lead to additional zero entries in the lepton Yukawa matrices which predict small $\theta_{13}$. Type I see-saw models of this type can be upgraded to type II see-saw models where hierarchical neutrino masses are as natural a partially degenerate ones $[7]$. We have discussed the predictions for $\theta_{13}$ and corrections to it, depending on the neutrino mass scale. Although the mixing $\theta_{13}$ generated in the charged lepton sector is very small, corrections to it from next-to-leading order effective operators, small mixings from the neutrino mass matrix and renormalization group running could lift the prediction to $\theta_{13} \lesssim 5^\circ$ or $\sin^2(2\theta_{13}) \lesssim 3 \cdot 10^{-2}$, which might lie within reach of planned reactor experiments (see e.g. $[18]$). In the type-II-upgrade scenarios, $\theta_{13}$ is in general predicted to be smaller for a larger neutrino mass scale.

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References

[1] S. F. King, *Neutrino mass models*, Rept. Prog. Phys. 67 (2004), 107–158, hep-ph/0310204.

[2] G. Altarelli, F. Feruglio, and I. Masina, *Can neutrino mixings arise from the charged lepton sector?*, (2004), hep-ph/0402155.

[3] A. Romanino, *Charged lepton contributions to the solar neutrino mixing and $\theta_{13}$*, (2004), hep-ph/0402258.
[4] K. S. Babu and S. M. Barr, \textit{Bimaximal neutrino mixings from lopsided mass matrices}, Phys. Lett. B525 (2002), 289–296, hep-ph/0111215.

[5] S. F. King, \textit{Large mixing angle MSW and atmospheric neutrinos from single right-handed neutrino dominance and U(1) family symmetry}, Nucl. Phys. B576 (2000), 85–105, hep-ph/9912492.

[6] S. F. King, \textit{Constructing the large mixing angle MNS matrix in see-saw models with right-handed neutrino dominance}, JHEP 09 (2002), 011, hep-ph/0204360.

[7] S. Antusch and S. F. King, \textit{From hierarchical to partially degenerate neutrinos via type II upgrade of type I see-saw models}, (2004), hep-ph/0402121.

[8] S. F. King, \textit{Atmospheric and solar neutrinos with a heavy singlet}, Phys. Lett. B439 (1998), 350–356, hep-ph/9806440.

[9] S. F. King, \textit{Atmospheric and solar neutrinos from single right-handed neutrino dominance and U(1) family symmetry}, Nucl. Phys. B562 (1999), 57–77, hep-ph/9904210.

[10] T. Yanagida, in \textit{Proceedings of the Workshop on the Unified Theory and the Baryon Number in the Universe} (O. Sawada and A. Sugamoto, eds.), KEK, Tsukuba, Japan, 1979, p. 95.

[11] S. L. Glashow, \textit{The future of elementary particle physics}, in \textit{Proceedings of the 1979 Cargèse Summer Institute on Quarks and Leptons} (M. Lévy, J.-L. Basdevant, D. Speiser, J. Weyers, R. Gastmans, and M. Jacob, eds.), Plenum Press, New York, 1980, pp. 687–713.

[12] M. Gell-Mann, P. Ramond, and R. Slansky, \textit{Complex spinors and unified theories}, in \textit{Supergravity} (P. van Nieuwenhuizen and D. Z. Freedman, eds.), North Holland, Amsterdam, 1979, p. 315.

[13] R. N. Mohapatra and G. Senjanović, \textit{Neutrino mass and spontaneous parity violation}, Phys. Rev. Lett. 44 (1980), 912.

[14] L. L. Everett, G. L. Kane, and S. F. King, \textit{D branes and textures}, JHEP 08 (2000), 012, hep-ph/0005204.

[15] G. Lazarides, Q. Shafi, and C. Wetterich, \textit{Proton lifetime and fermion masses in an SO(10) model}, Nucl. Phys. B181 (1981), 287.

[16] R. N. Mohapatra and G. Senjanović, \textit{Neutrino masses and mixings in gauge models with spontaneous parity violation}, Phys. Rev. D23 (1981), 165.
[17] S. Antusch, J. Kersten, M. Lindner, and M. Ratz, *Running neutrino masses, mixings and CP phases: Analytical results and phenomenological consequences*, Nucl. Phys. **B674** (2003), 401–433, hep-ph/0305273.

[18] P. Huber, M. Lindner, T. Schwetz, and W. Winter, *Reactor neutrino experiments compared to superbeams*, (2003), hep-ph/0303232.