Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
On equity market inefficiency during the COVID-19 pandemic

Robert Navratil a, Stephen Taylor a,b,∗, Jan Vecer a,c

a Charles University, Department of Probability and Statistics, Faculty of Mathematics and Physics, Sokolovska 83, 18675 Praha 8, Czech Republic
b School of Management, New Jersey Institute of Technology, Newark, NJ 07102, United States of America
c Frankfurt School of Finance and Management, Adickesallee 32-34, 60322 Frankfurt am Main, Germany

ARTICLE INFO

Keywords:
Utility maximization
Merton’s optimal portfolio
Efficient market hypothesis

ABSTRACT

We show that during the weeks following the initiation of the COVID-19 pandemic, the United States equity market was inefficient. This is demonstrated by showing that utility maximizing agents over the time period ranging from mid-February to late March 2020 can generate statistically significant profits by utilizing only historical price and virus related data to forecast future equity ETF returns. We generalize Merton’s optimal portfolio problem using a novel method based upon a likelihood ratio in order to construct a dynamic trading strategy for utility maximizing agents. These strategies are shown to have statistically significant profitability and strong risk and performance statistics during the COVID-19 time-frame.

1. Introduction

The COVID-19 pandemic has disrupted global financial markets in a manner seldom considered in typical risk scenarios. In order to minimize virus transmission, governments imposed stringent stay-at-home orders which had the collateral effect of suddenly halting a large portion of economic activity. This resulted in rapid and steep declines in international equity markets; for example, the S&P 500 lost approximately one third of its value in the span of only one month starting on February 20, 2020, c.f. Baker et al. (2020). We show that during the weeks following this date that the United States equity market became inefficient and certain simple utility maximizing trading strategies had statistically significant profitability.

The effect of COVID-19 and subsequent interventions on properties of American equity markets including performance and volatility has been the subject of a number of recent works. In Azimli (2020), the authors demonstrate how tail dependence structures between equity sectors were altered during the pandemic period. Market efficiency during the COVID-19 and an associated comparison to the global financial crisis are given in Choi (2021). Efficiency issues are also explored in Dima et al. (2021) in the context of the VIX index response to the crisis. The impact of news coverage during COVID-19 on different quantile ranges of equity indices is examined in Cepoi (2020). Cross country studies of the equity market impact are considered in Frezza et al. (2021). Here the authors consider the effect of COVID-19 on fifteen equity markets using tools of fractal analysis to find that while Asian markets have regained efficiency, the European and US markets still have inefficient components and have been slower to rebound to pre-pandemic efficiency. In related work, Nguyen et al. (2021) examine international equity market effects of COVID-19. In particular, they study how volatility spillover effects propagated from United States and Chinese equity markets to other major international analogues. Structural changes to volatility and their resulting impact on returns is examined in Baek et al. (2020), Just and Echaust (2020). A thorough performance analysis of American equity sectors and associated connections with asymmetric volatility is studied in Mazur et al. (2020). We confirm and further the results of several of these authors by considering the market inefficiency problem from the perspective of utility maximizing agents, showing that it naturally extends the Merton optimal portfolio framework to the dynamic trading setting, c.f. Merton (1975). Namely, we construct such portfolios that trade only in a single risk security, taken to be a broad based ETF, and a treasury bill. We note that dynamic generalizations of the Merton optimal portfolio have been previously considered, c.f. Campbell and Viceira (1999). This offers a new approach to defining and testing market efficiency. Namely, if a portfolio produces a statistically significant profit over a benchmark, then the market is inefficient. However, we note that the market may not necessarily offer any statistically efficient means to monetize such inefficiency via trading.

This article presents several novel contributions. We develop a likelihood based derivation of optimal trading rules for utility maximizing agents, thereby extending the Merton optimal portfolio problem. This result it utilized to document market inefficiency during the 2020 COVID-19 pandemic. The central theme is that returns of index securities became predictable to the point that the profitability of certain
utility maximizing trading strategies was statistically significant. This inefficiency is demonstrated numerically in the case of broad based market and GICS sector ETFs. We note that one may monetize market inefficiency by executing a trading strategy which is optimal for a power utility maximizing agent.

This article is organized into two parts. First in Section 2, we develop an optimal trading strategy for power utility maximizing agents that naturally extend Merton model trading rules utilizing a likelihood ratio argument. We also develop a novel market efficiency test based upon these ideas. Next, we demonstrate the equivalence of the Kelly Criterion and optimal Merton portfolio as specific cases of a more general optimal distributional trading gain framework. Second, we show that such optimal trading strategies have strong Sharpe ratios, desirable risk statistics, and statistically significant profitability during the weeks following the initial spread of COVID-19.

2. Optimal behavior of utility maximizing agents

The problem of specifying and deriving optimal strategies for utility maximizing agents has been studied widely in many forms. Historically, its origins may be found in a 1738 article of Bernoulli, later republished in Bernoulli (1954). A more modern formulation of a related problem appeared in Kelly (1956) whose main result is commonly referred to as the Kelly criterion. This problem is typically understood in the specific context of a logarithmic utility maximizing agent trading on a binary outcome, where the subjective belief of the agent differs from that of the broader market. The binary outcomes appear in betting markets, and the Kelly criterion determines the optimal bet size, given the agent’s bankroll, on both available outcomes. In the financial setting, the digital outcomes can be understood as Arrow–Debreu securities and the corresponding prices as state prices introduced in Arrow (1964) and Debreu (1959). We also note that market inefficiency in the binary setting was applied to prediction markets in Richard and Vecer (2021); one of our main aims below is to extend this work to the continuous case.

As the Kelly criterion is limited to discrete outcomes, it may not be applied in a straightforward fashion to continuous price distributions that commonly appear in continuous time finance. The continuous analogue of the Kelly criterion was developed in Merton (1975), who found the optimal trading behavior for power utility maximizing agents in the case of normally distributed asset prices. It is not directly evident that the Kelly criteria and Merton optimal portfolio solve identical problems and our theoretical contribution is to demonstrate this relationship. In recent work, Vecer (2020) introduces the problem of “Optimal Distributional Trading Gain” that generalizes both these problems in a unifying framework. We also show that such an approach allows one to directly determine the optimal trading strategy in Merton’s setup in the form of a likelihood ratio of the subjective probability measure $P$ and the risk neutral measure $Q$, simplifying the original work in Merton (1975) which utilizes techniques of stochastic optimal control.

Consider an agent who seeks to maximize a general utility function $U$ of a random variable $X$ with probability measure $P$ where the utility function encodes the subjective opinion of the agent about the distribution of $X$. Here, $X$ may be continuous or discrete. The market quotes prices of the realizations of the random variable $X$ in terms of a risk neutral distribution with measure $Q$. We seek to determine the optimal payoff function $F$ for the utility maximizing agent.

Specifically, the goal of the utility maximizing agent is to maximize the expected value $E^P[U(F)]$. However, the agent can construct only replicable payoffs $F$ available from the market. The set of replicable payoffs satisfies the constraint $E^Q[F] = 0$, so the market only allows payoffs $F$ that have zero expectation from the perspective of the market. This condition has not appeared explicitly in the previous literature, but it has been used implicitly in the sense that the discounted price increments have zero expectation under the risk neutral measure $Q$.

The explicit condition on the zero expectation of the payoff $F$ allows one to solve the problem of the optimal trading gain explicitly, c.f. Vecer (2020). Stated formally,

**Theorem 2.1.** Let $U(x)$ be a utility function that is increasing $U'(x) > 0$ and concave $U''(x) < 0$. Let $p(x), q(x)$ denote the probability density functions associated with measures $P, Q$, respectively. The random variable $F^{p,q}$ that maximizes

$$E^P[U(F)] \text{ subject to } E^Q[F] = 0,$$

is given by

$$F(x) = I \left( \lambda \cdot \frac{q(x)}{p(x)} \right),$$

where $I(x) = [U'(x)]^{-1}$,

and where $\lambda$ solves

$$\int I \left( \lambda \cdot \frac{q(x)}{p(x)} \right) q(x) dx = 0.$$

**Proof.** Consider the following Lagrange-type functional

$$J[F] = \int [U(F(x))p(x) - \lambda F(x)q(x)] dx.$$

The optimal $F$ is given by the gradient $\frac{\partial J}{\partial F} = 0$, or equivalently,

$$U'(F(x))p(x) - \lambda q(x) = 0,$$

Solving, one finds that

$$F(x) = I \left( \lambda \cdot \frac{q(x)}{p(x)} \right),$$

and where $\lambda$ solves the integral equation

$$\int I \left( \lambda \cdot \frac{q(x)}{p(x)} \right) q(x) dx = 0.$$

Related results have appeared in the literature, c.f. Kramkov and Schachermayer (1999); however, their approach is based upon a more complicated Legendre transform based optimization technique. Note that the above result gives the optimal payoff on every Arrow–Debreu security. If one is limited to trading in the underlying, then one can replicate payoffs in the case of complete markets, such as in the geometric Brownian motion model studied by Merton (1975).

Note that the optimal solution $F^{p,q}$ depends on the likelihood ratio $p(x)/q(x)$. More specifically, the optimal payoff $F^{p,q}$ in the case of the power (isoelastic) utility function $U$:

$$U(x) = \left( 1 + \frac{1}{a} \right)^{1-a} - 1, \\ for \text{ risk aversion parameter } a > 0 \text{ and agent bankroll } B > 0 \text{ is given by}$$

$$F^{p,q}(x) = B \left( \frac{p(x)}{q(x)} \right)^{\frac{1}{a}} \left( \frac{1}{\int \left( \frac{p(x)}{q(x)} \right)^{\frac{1}{a}} q(x) dx} - 1 \right).$$

The limiting case $a \to 1$ corresponds to the logarithmic utility function and the optimal payoff simplifies to

$$F^{p,q}(x) = B \left( \frac{p(x)}{q(x)} - 1 \right).$$

The choice of the logarithmic utility seems to be canonical as logarithmic utility maximizers in the absence of a market maker will arrive at an equilibrium given by a mixture of their distributions. Their resulting wealth follows a Bayesian updating relation, for more details, c.f. Vecer (2020). Note that the choice of the logarithmic utility and the case of a binary random variable leads to the Kelly criterion, a result that we restate in the following remark.
Remark 2.2 (Kelly Criterion). Assuming that the subjective probability $P$ assigns a value $P(X = 1) = p$ to an event $X$ and the market probability, given by $Q$, assigns $P(X = 1) = q$, the optimal payoff $F$ is

$$F = \begin{cases} \frac{B - (\frac{q}{p} - 1)}{\frac{q}{p} - 1} & X = 1 \\ \frac{B - (\frac{q}{p} - 1)}{-1} & X = 0. \end{cases}$$

The Kelly criterion is typically stated in terms of the fraction of the bankroll that is lost on the outcome of $X = 0$, or in other words, the value $1 - \frac{1}{\frac{q}{p} - 1} = \frac{p}{q} = \frac{2 \ln(1 + p)}{2}$, where $b = \frac{1}{\frac{q}{p} - 1}$.

We now develop a connection between the optimal distributional trading gain and Merton’s optimal portfolio problem.

Example 2.3 (Merton’s Optimal Portfolio). The problem of the optimal distributional trading gain in the context of a stock market can be formulated as follows. Suppose an agent believes that a stock price evolves according to

$$dS(t) = S(t)(\mu dt + \sigma dW(t)),$$ (2.4)

while the market in terms of the risk neutral measure is given by

$$dS(t) = S(t)(\mu dt + \sigma dW^Q(t)).$$ (2.5)

In terms of the discounted price process, we have

$$dX(t) = \frac{d(e^{-rt}S(t))}{e^{-rt}S(t)} = (\mu - r)dt + \sigma dW(t) = \sigma dW^Q(t).$$ (2.6)

The agent believes that the market increment $dX$ has distribution

$$dX \sim N((\mu - r)dt, \sigma \sqrt{dt}),$$ (2.7)

while the market holds the risk neutral view

$$dX \sim N(0, \sigma \sqrt{dt}).$$ (2.8)

The optimal payoff $P$ corresponds to the discounted value of the optimal portfolio $P$ of the agent maximizing power utility is given by

$$e^{-rT} P(T) = \frac{1}{\int \frac{1}{\sqrt{2\pi \sigma^2 T}} q(x)dx} \exp\left( \frac{\mu - r}{\sigma^2} X(T) \right) \left( 1 - \frac{(\mu - r)^2}{2(\sigma^2)^2} T \right).$$ (2.9)

An immediate observation is that the optimal terminal wealth is a geometric Brownian motion and in this situation, we have a complete market and the optimal payoff can be replicated by trading in the underlying asset $S$. Thus the evolution of the discounted price of the optimal portfolio $P$ should follow

$$d(e^{-rt} P(t, X(t))) = \frac{\mu - r}{\sigma^2} dX(t),$$ (2.10)

meaning that the proportion $\frac{\mu - r}{\sigma^2}$ should be invested in the risky asset.

Remark 2.4. Note that Theorem 2.1 can be used for any distributional opinion of the agent $P$ and any distributional opinion $Q$ of the market. Thus it provides a more general approach to determine the optimal trading behavior of utility maximizers than that considered in Merton (1975) which is restricted to the normal distribution.

Thus having a subjective opinion about the drift $\mu$ gives an optimal trading strategy of a power utility maximizing agent in terms of the well known Merton ratio. We note that one significant difference between the likelihood and Merton approaches is that in the likelihood approach allows for dynamic updating of the drift as opposed to taking this to be a constant parameter as in the Merton model. Moreover, we note that the optimality of repeated updating of the drift parameter is justified by the likelihood method which provides a further extension of Merton’s model. The aim of the remainder of this article is to estimate $\mu$ on a daily basis, rebalance a portfolio consisting of a single risky and a riskless asset daily, and check whether the associated trading strategy has statistically significant profits. Note that we have two representations of the optimal profit, one that is based on the likelihood (2.9) and one that is based on the replication of the optimal portfolio (2.10). We finally note that the optimal portfolio representation in terms of the likelihood ratio is exact. The two representations should be identical in the situation of the complete market, but we can see some small discrepancies from discrete hedging, where we rebalance the positions on a daily basis rather than continuously.

3. Statistical model and data

We now discuss a regression based model used to estimate the drift and volatility parameters $\mu$ and $\sigma$ in Merton’s formula (2.10). Specifically, we describe a mechanism to generate out of sample predictions for the return $\mu$ and volatility $\sigma$ of several ETFs by combining multiple univariate linear regressions that utilize other liquid securities and virus related data as predictors. We then discuss the end of day price dataset and associated time period on which these models are estimated.

3.1. Estimating the drift and volatility

We utilize a combination of univariate ordinary least squares linear regression models to estimate the drift parameter on a daily basis. Here, the target excess return of the $j$th ETF is denoted by $\mu_j^{\delta t}$ and the $i$th predictor is $x_{ij}^t$ where the subscripts indicate that all predictors are lagged one day prior to the target time series. We consider linear models of the form

$$\mu_j^{\delta t} = \beta_i x_{ij}^t + \epsilon_{ij}^{\delta t}, \quad \text{for } t = 1, \ldots, T - 1,$$ (3.1)

where here we assume that the residuals $\epsilon_{ij}^{\delta t}$ are i.i.d. draws from a random variable that satisfies $E(\epsilon_{ij}^{\delta t}) = 0$ and $\text{Var}(\epsilon_{ij}^{\delta t}) = \sigma^2 < \infty$. Here the excess returns are defined in terms of the difference of an ETF and the three month treasury bill yield.

We re-estimate $\mu_j^{\delta t}$ on a rolling basis when testing market efficiency during the COVID-19 crisis. That is to say, at time $t$ we train the model on $N$ consecutive prior observations $\{(x_{ij}, \mu_j^{\delta t}) : \text{for } i = 1, \ldots, N\}$ yielding an estimate $\hat{\mu}_j^{\delta t}$ of the regression coefficient. Then we create a single out of sample prediction $\hat{\mu}_j^{\delta t} = \hat{\beta}_i x_i^t$ of the unobserved variable $\mu_j^{\delta t}$. During the next trading day, we observe the actual value $\mu_j^{\delta t}$ and refit the model by adding the observation $(x_i^t, \mu_j^{\delta t})$ and omitting the first observation $(x_{ij}, \mu_j^{\delta t})$ from the training set. Repeating this procedure we obtain a vector of sample forecasts $\{\hat{\mu}_j^{\delta t} : \text{for } i = 1, \ldots, T\}$. In addition, the volatility parameter $\sigma_t$ is estimated directly from the empirical standard deviation of the historical excess returns $\{\mu_j^{\delta t} : \text{for } i = 1, \ldots, T\}$.

3.2. Combining predictions

Multiple economic indicators have been shown to be a useful tool for enhancing the predictive power of the equity risk premium, c.f. Neely et al. (2014). In particular, the combination approach of utilizing the predictive power of several different models has proven successful in this application, e.g. Dangl and Halling (2012) and Rapach et al. (2010). In order to improve the robustness of the forecast and out of sample performance, we utilize a similar model combination technique by fitting several individual univariate linear regression models which comprise a single forecasting model for future excess returns. Mathematically, this method is specified by taking a weighted sum of individual drifts to construct an aggregate model,

$$\hat{\mu}_j^{\delta t} = \sum_j w_j^{\delta t} \hat{\mu}_j^{\delta t}.$$ (3.2)

Here $\hat{\mu}_j^{\delta t}$ is a predictor of $\mu_j$ for the $j$th model, and $w_j^{\delta t}$ is the weight of this model at observation $t$ for the $j$th ETF whose excess returns are being estimated.
Common choices for the weights include the mean, median, and trimmed mean, c.f. Zhang et al. (2018) or Balcilar et al. (2015). We select uniform weights \( w_{ij} = \frac{1}{M} \), where \( M \) is the number of statistical models. In addition, we note that there exist many additional techniques to further improve the out of sample forecasting performance, such as constraining predictors as in Pan et al. (2020) or adding new low correlation predictors, c.f. Zhang et al. (2019). However, our main purpose is to demonstrate market inefficiency during the COVID-19 crisis and we found using a simple uniform weighting scheme is sufficient for this task.

3.3. Dataset construction

We aim to focus on examining market inefficiency during the period shortly after the emergence of COVID-19 and hence restrict our dataset to include daily data from February 2020 through May 2020. We consider excess returns of several ETFs including an S&P ETF SPY, as well as eleven ETFs which cover each of the GICS sectors; namely, VCR, VDC, VDE, VGT, VNM, XLB, XLC, XLF, XLI, XLV, and XL. These will all serve as target variables within the regressions considered below.

In addition, we construct a dataset of one day time lagged predictors which consist of daily returns from highly liquid securities as well as information related to the severity and spread of the virus. Specifically, we consider daily excess returns of the VIX volatility index, gold futures, and bitcoin. A fixed income component is incorporated with daily two year US treasury data and market loss risk aversion is captured through the short interest index (c.f. Rapach et al., 2010) of SPY. We also incorporate both the United States daily COVID-19 related case count and death rates into the predictor dataset.

Data was obtained from multiple public sources. Specifically, end of day ETF and VIX data was obtained from yahoo finance, treasury data was obtained from the United States Department of the Treasury website (treasury.gov), bitcoin prices were obtained from coinmarketcap.com, gold futures data was gathered from investing.com, and the data used in the SSI index was downloaded from http://regsho.finra.org/regsho-Index.html. In addition, the COVID-19 data was downloaded from ourworldindata.org. We utilized Python APIs when available to download data and the pandas package to align and prepare data for subsequent modeling.

We restrict our focus to the period starting in late February 2020, when the equity market begin to react to the global spread of COVID-19. During this time, equity markets in the United States exhibited strong mean reversion as can be seen for example by fitting an AR(1) process to the excess daily return time series of the SPY ETF. Specifically, assume that excess daily returns follow mean reversion process \( y_t = \alpha_{t-1} + \xi_t \). Then over the two month period under consideration, we estimate \( \hat{\alpha} = -0.40 \) with an in sample \( R^2 \) of 13.7%. Noting that the market exhibited extreme volatility during this time, the \( R^2 \) for this simple mean reversion indicator is quite strong. We next seek to understand if similar behavior is present in the out of sample dataset.

4. Empirical results

We now consider a simple trading technique based upon determining the optimal investment strategy of an agent who wishes to maximize the utility \( E[U(F)] \), where we take \( U \) to be the previously described power utility \( U(x) = \frac{x^{a}}{a} \) for \( a \geq 0 \) and where \( F \) is the final portfolio value. The agent assumes that the equity market or ETF price evolves according to a geometric Brownian motion with parameters \( \mu \) and \( \sigma \). Using the combination approach described above, the agent estimates the drift \( \mu \) and the volatility \( \sigma \) parameters from historical data. The optimal position \( \pi_t \) at time \( t \) in the risky asset is then defined via Merton’s fraction in Eq. (2.10) while the remainder of capital is invested in the three month treasury bill. We also impose a single trading constraint; specifically, we do not allow for leveraged portfolios, i.e. \( \pi_t \in [-1, 1] \). We will assume that the agent rebalances this position on a daily basis at the close and that there are no transaction costs given the strong liquidity of the securities under consideration.

To demonstrate the inefficiency of the entire market as well as individual sectors, we suppose the agent invests independently in each of the target ETFs. We first note that, the agent realizes a positive bankroll for all ETFs under consideration. The final bankroll is dependent upon the specific choice of hyperparameters; \( N \) for the training window size and the utility risk aversion parameter \( a \). While the parameter \( N \) requires some statistical insight to properly select, the parameter \( a \) is given according to the personal preference of the agent. We examine the trading strategy performance in more detail below as a function of \( N \) and \( a \), and initially select \( N = 10 \) and \( a = 0.8 \) in the examples below to demonstrate market inefficiency.

We plot the agent’s bankroll assuming an initial unit amount of capital for all ETFs on left subplot of Fig. 1. On the right subplot, we display the evolution of the agent’s bankroll benchmarked to the respective ETF, i.e. the right panel is a comparison of Merton’s portfolio against a long-only buy and hold strategy. During the first month prior to the spread of COVID-19, the data supports the hypotheses that markets were efficient given that the bankroll oscillates around the starting capital value. Then, the inefficiency of the market becomes prominent during late February, independent of the specific choice of the ETF the agent realizes significant profit. During the final month and a half of the period under consideration, efficiency returns as ETF profits again resemble noise.

Trading strategy performance and risk statistics for all ETFs are given in Table 1. Notice, that the profit for all ETFs is positive. The greatest profit is found in VDE, the Vanguard Energy ETF, while the smallest profit is in XLU, the Utilities Select Sector ETF. The annualized Sharpe ratio of trading in SPY, the market ETF, is 3.02 with a final bankroll of 1.58. The maximum drawdown statistics range from 7.9% to 27.4%. The value and conditional value at risk statistics are calculated at the 95% level, and the out of sample \( R^2 \) is given according to

\[
R^2_{OS} = 100 \times \left(1 - \frac{(r - \bar{r})^2}{(r - \bar{r})^2}ight),
\]

where here \( \bar{r} \) denotes the mean excess daily return \( r \) of the respective ETF.

Benchmarking by the underlying ETF, we find the largest trading opportunity resides in the Vanguard Energy ETF and in the financial select sector ETF (XLF), while rest of the considered ETFs behaved similarly from the perspective of the utility maximizing agent. Using multifractal detrended fluctuation analysis, a method developed in Kantelhardt et al. (2002) was used to study sector-level efficiency, Choi (2021) found that during the COVID-19 pandemic the consumer discretionary (VCR) and energy sector (VDE) ETFs were the most efficient while the financial sector (XLF) and utilities sector (XLU) were the least efficient. Our results suggest that in an inefficient market the degree of market inefficiency of a utility maximizing agent plays a smaller role than, for example, the volatility of the traded asset.

To illustrate the behavior of Merton’s portfolio in more detail, we plot the estimate of the drift parameter \( \mu \) in Fig. 2. Note the relatively large estimated values during mid-March for all ETFs. Similarly, we plot the evolution of the agent’s position in the respective ETFs in Fig. 3. Notice that in this volatile period the agent’s position is, in most cases, either fully long or fully short. This resembles a bang–bang type strategy where it is always optimal to switch from one extreme to another. This implies that the influence of the risk aversion parameter \( a \) will have a small effect on the final value of the portfolio.

In order to further study the properties that allowed Merton’s portfolio to generated excess returns due to the market inefficiency, we now focus on the individual constituents of the S&P 500. We are interested in studying the effect of the market beta, leverage, P/E
positive effect on the final bankroll of the portfolio. Cash assets and
We find that the market beta is statistically significant and has a
ter and normalize the regressors. The results are summarized in Table 2.
Moreover, to easily compare estimated coefficients, we cen-
where $Y_i$ is the predicted final bankroll of Merton’s portfolio for $i$th
company. Moreover, to easily compare estimated coefficients, we cen-
ter and normalize the regressors. The results are summarized in Table 2.
We find that the market beta is statistically significant and has a
positive effect on the final bankroll of the portfolio. Cash assets and
leverage have a statistically negative effect on the final bankroll. Fi-
ally, we have not found statistical evidence for the effect of the P/E
ratio on the final value of Merton’s portfolio. The $R^2$ of the linear
regression is 0.152. This result corresponds to Ramelli and Wagner
(2020) who studied non-financial companies in the Russel 3000 index
and found statistical evidence between company’s leverage, cash hold-
ings and the cumulative return of the company during the COVID-19
pandemic.

We finally demonstrate that the Merton’s portfolio does not out-
perform during a regular efficient market. Specifically, consider the
one year period prior to the market reaction to the global spread of
COVID-19, i.e. February 2019 through May 2019. We follow the same
methodology as before to estimate the $\mu$ and $\sigma$ parameters of each ETF
with the exception that no COVID-19 related data be included. In Fig. 5,
we display the evolution of the agent’s bankroll again in both dollar
value and in ETF relative value. The average loss in dollar value is
approximately 10%, while the average loss against a simple buy and
hold strategy is, on average, 12%.

### 4.1. Hyperparameter selection

We next offer suggestions on how one may select the estimation
window size $N$ and the utility function risk aversion parameter $\alpha$. We
note that the fitting window length $N$ has a large effect on the final
portfolio value while the risk aversion parameter $\alpha$ has a relatively
small impact on performance. This is due to the fact that we do not
allow for leverage and Merton’s optimal portfolio allocation will usually
either be fully long or fully short.

We demonstrate the effect of $N$ and $\alpha$ by examining the performance
of the trading strategy for all combinations of $N \in \{5, 6, \ldots, 50\}$ and
Fig. 2. Evolution of the estimated drift parameter $\mu$ for the respective ETFs during the COVID-19 crisis period.

Table 2
This table shows results of OLS regression (4.1).

|               | coef  | std err | t     | P>|t|  | [0.025] | 0.975 |
|---------------|-------|---------|-------|------|---------|-------|
| Intercept     | 1.5930| 0.023   | 70.747| 0.000| 1.549   | 1.637 |
| P/E ratio     | -0.0301| 0.023   | -1.328| 0.185| -0.075  | 0.014 |
| Leverage      | -0.0612| 0.023   | -2.645| 0.008| -0.107  | -0.016|
| Cash assets   | -0.0499| 0.024   | -2.059| 0.040| -0.097  | -0.002|
| Market beta   | 0.2013| 0.025   | 8.158 | 0.000| 0.153   | 0.250 |
We are interested in the average final bankroll for all ETFs. In Fig. 6, note that varying the \( a \) parameter for a fixed \( N \) value only has a marginal effect on performance. In contrast, varying the \( N \) parameter significantly impacts the final value of the portfolio. Note that extremely low values of \( N \) underperform in comparison with other choices. For example, relatively small \( N \) values, i.e. \( N = \{8, \ldots, 15\} \) yield the strongest performing strategies as such values allow one to quickly capture the changes in the market and also provide sufficient data in the rolling window to estimate the drift and volatility parameters to a sufficient accuracy. Large values of \( N \) behave similarly given that they do not allow the model to react swiftly enough to market changes. Note that for all choices of \( N \) the final portfolio realizes a net gain. The biggest profit obtained with a final bankroll value of 1.79 has parameters \( N = 9 \) and \( a = 0.48 \).
Fig. 4. Evolution of the dollar value of Merton’s portfolio for constituents in the S&P 500 index. For each studied property, the constituents of the S&P 500 are divided to five equally sized groups based upon their quintile buckets. For each group we compute the Merton’s portfolio and plot the respective group mean.

Fig. 5. The left subplot displays the evolution of an agent’s bankroll in terms of the dollar value for each ETF under the Merton fraction portfolio described above over the COVID-19 crisis. The right subplot depicts the evolution of the agent’s bankroll in terms of the relative value of the respective ETF.

5. Conclusion

In this paper we have examined the United States equity market inefficiency during the initial spread of the COVID-19 pandemic during 2020. We have shown, that even the relatively simple Merton optimal portfolio trading strategy has strong out-of-sample performance during this period. We also provided an alternative simplified derivation of the Merton optimal portfolio ratio in the context of a geometric Brownian motion. The results were applied to an S&P 500 and eleven GICS sector ETFs and the profitability of the trading strategy was shown to be robust to the choice of utility function risk aversion parameter and lookback window size.

We finally note that it would be of interest to further examine the performance of asset allocation techniques and the multivariate extension of Merton’s optimal portfolio ratio during the COVID-19 timeframe. In addition, it would be of interest to develop extensions of these results in the case where the portfolio follows an extension of geometric Brownian motion, i.e. jump processes, that more closely reflect market price movements. In particular, the likelihood approach offers a considerably simplified framework over known stochastic control based methods to derive optimal trading rules in the case of more general stochastic processes.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.
Acknowledgment

This research was funded in part by Grant Agency of the Czech Republic under grant numbers 18-01137S and GAUK No. 420120 (Robert Navratil and Jan Vecer), 19-28231X (Stephen Taylor and Jan Vecer).

References

Arrow, K. J. (1964). The role of securities in the optimal allocation of risk-bearing. Review of Economic Studies, 31(2), 91–96.

Azimli, A. (2020). The impact of COVID-19 on the degree of dependence and structure of risk-return relationship: A quantile regression approach. Finance Research Letters, 36, Article 101648.

Baek, S., Mohanty, S., & Glambosky, M. (2020). Covid-19 and stock market volatility: An industry level analysis. Finance Research Letters, 37, Article 101748.

Baker, S., Bloom, N., David, S., Kost, K., Sammon, M., & Viratyosin, T. (2020). The unprecedented stock market reaction to Covid-19. The Review of Asset Pricing Studies, 10, 742–758.

Balciar, M., Gupta, R., & Miller, S. M. (2015). Regime switching model of US crude oil and stock market prices: 1859 to 2013. Energy Economics, 49, 317–327.

Bernoulli, D. (1954). Exposition of a new theory on the measurement of risk. Econometrica, 22(1), 23–36.

Campbell, J., & Viceira, L. (1999). Consumption and portfolio decisions when expected returns are time varying. Quarterly Journal of Economics, 114, 433–495.

Cepoi, C.-O. (2020). Asymmetric dependence between stock returns and news during COVID-19 financial turmoil. Finance Research Letters, 36, Article 101658.

Choi, S.-Y. (2021). Analysis of stock market efficiency during crisis periods in the US stock market: Differences between the global financial crisis and COVID-19 pandemic. Physica A: Statistical Mechanics and its Applications, 574, Article 125988.

Dangl, T., & Halling, M. (2012). Predictive regressions with time-varying coefficients. Journal of Financial Economics, 106(1), 157–181.

Debreu, G. (1959). Theory of value: An axiomatic analysis of economic equilibrium, Number 17. Yale University Press.

Dima, B., Dima, S. M., & Ioan, R. (2021). Remarks on the behavior of financial market efficiency during the COVID-19 pandemic. The case of VIX. Finance Research Letters, Article 101967.

Frezza, M., Bianchi, S., & Fianese, A. (2021). Fractal analysis of market (in)efficiency during the COVID-19. Finance Research Letters, 38, Article 101851.

Just, M., & Echaust, K. (2020). Stock market returns, volatility, correlation and liquidity during the COVID-19 crisis: Evidence from the Markov switching approach. Finance Research Letters, 37, Article 101775.

Kandelhardt, J. W., Zechieger, S. A., Koscielny-Bunde, E., Havlin, S., Bunde, A., & Stanley, H. E. (2002). Multifractal detrended fluctuation analysis of nonstationary time series. Physica A: Statistical Mechanics and its Applications, 316(1–4), 87–114.

Kelly, J. (1956). A new interpretation of the information rate. The Bell System Technical Journal, 35, 917–926.

Kramkov, D., & Schachermayer, W. (1999). The asymptotic elasticity of utility functions and optimal investment in incomplete markets. Annals of Applied Probability, 904–950.

Mazur, M., Dang, M., & Vega, M. (2020). Covid-19 and the march 2020 stock market crash. Evidence from S&P1500. Finance Research Letters, Article 101690.

Merton, R. C. (1975). Optimum consumption and portfolio rules in a continuous-time model. Stochastic optimization models in finance (pp. 621–661). Elsevier.

Neely, C. J., Rapach, D. E., Tu, J., & Zhou, G. (2014). Forecasting the equity risk premium: the role of technical indicators. Management Science, 60(7), 1772–1791.

Nguyen, D. T., Phan, D. H. B., Chwee, M. T., & Long, N. V. K. (2021). An assessment of how COVID-19 changed the global equity market. Economic Analysis and Policy, 69, 480–491.

Pan, Z., Pettenuzzo, D., & Wang, Y. (2020). Forecasting stock returns: A predictor-constrained approach. Journal of Empirical Finance, 55, 200–217.

Ramelli, S., & Wagner, A. F. (2020). Feverish stock price reactions to COVID-19. The Review of Corporate Finance Studies, 9(3), 622–655.

Rapach, D. E., Strauss, J. K., & Zhou, G. (2010). Out-of-sample equity premium prediction: Combination forecasts and links to the real economy. Review of Financial Studies, 23(2), 821–862.

Richard, M., & Vecer, J. (2021). Efficiency testing of prediction markets: Martingale approach, likelihood ratio and Bayes factors analysis. Risks 9(2), 1–20.

Vecer, J. (2020). Optimal distributional trading gain: State price density equilibrium and Bayesian statistics. Available at SSRN.

Zhang, Y., Ma, F., Shi, B., & Huang, D. (2018). Forecasting the prices of crude oil: An iterated combination approach. Energy Economics, 70, 472–483.

Zhang, Y., Zeng, Q., Ma, F., & Shi, B. (2019). Forecasting stock returns: Do less powerful predictors help? Economic Modelling, 78, 32–39.