Development of a moving model rig with a large Reynolds number and measurement of aerodynamic drag

Qiansuo Yang

Key Laboratory for Mechanics in Fluid Solid Coupling Systems, Institute of Mechanics, Chinese Academy of Sciences, Beijing, China

Correspondence
Qiansuo Yang, Key Laboratory for Mechanics in Fluid Solid Coupling Systems, Institute of Mechanics, Chinese Academy of Sciences, Beijing 100190, China.
Email: qsyang@imech.ac.cn

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Abstract
The principles and techniques for developing a moving model rig (MMR) driven by compressed air were introduced for moving model tests at large Reynolds numbers ($\geq 10^6$). A corresponding methodology for the accurate measurement of the aerodynamic drag was developed. The main functions of the MMR included the acceleration of the trailer and the model, the deceleration of the model, and the deceleration of the trailer. The acceleration power was provided by the expansion of the compressed air in a pipe, and the deceleration of the model came from the relative motion between the permanent magnet and the static iron deceleration floor. The braking of the trailer was supplied by either a magnetic damping force or the compression of the air in the pipeline by the trailer piston. The free motion of the model in the test section followed the Davis equation. Based on this, a method for the measurement of the model aerodynamic drag using the measured acceleration data was proposed and experimentally demonstrated in an MMR developed in this work.

KEYWORDS
aerodynamic drag, compressed air, Davis equation, magnetic inducing friction, moving model rig

1 | INTRODUCTION

The optimization of the profiles of high-speed trains and aircraft leads to significant reductions in the energy consumed during movement in the atmosphere, which is one of the ongoing pursuits in aerodynamics.\textsuperscript{1–4} At present, high-speed trains are an important means of passenger land transportation. The aerodynamic drag of a high-speed train accounts for 80\% of the total resistance when the speed reaches 300 km/h.\textsuperscript{5,6} As the speed increases, this ratio increases further since the aerodynamic drag is usually proportional to the square of the speed. Therefore, to ensure safe operation and low energy consumption, the key to reducing the aerodynamic drag is mainly the selection and optimization of the three-dimensional aerodynamic shape.\textsuperscript{1,7,8} For an aircraft in the air, especially for large passenger airliners, shape optimization is also vital to lowering the transportation costs. Therefore, the development of test devices for the simulation of the various movements of the vehicles in an atmospheric environment and the measurement of the aerodynamic parameters has great significance to the scientific progress and the engineering design of high-speed trains and aviation.\textsuperscript{1,2,8} Although some advantages of moving model test (MMT) over wind tunnel test in the simulation of the movement patterns are gradually
emerging,3,9–16 the complete principle and technology of a moving model rig (MMR) with high speed, strong driving power, and corresponding methodologies for aerodynamic drag assessment are still lacking in the literature.

In the investigation of the aerodynamic characteristics of high-speed trains and fixed-wing aircraft, the tests are typically performed with relative motion between the model and the air.1,9–11,15–18 To apply the model data to an actual vehicle, it is usually required to correct the errors in the measured data caused by the differences of the Reynolds number \((Re)\).8,17,18 If the model data will be directly extrapolated to the full scale, not only must the shapes of the model and full-scale vehicles be as similar as possible but the corresponding \(Re\) values must be nearly equal, or the \(Re\) of the model must be in the supercritical region (i.e., the self-simulation region).2,3,6,18 Furthermore, the accuracy of the corresponding parameters depends on the test process. The feasibility and rationality of the method on the processing of the measured data are particularly important.

In an MMT, the movement of the model occurs as follows. First, the model is accelerated in the acceleration section. In the test section, some relevant tests are carried out during this free motion. Finally, the braking force in the deceleration section slows the model to a stationary state.3,9 The key technologies of an MMR are related to the acceleration and deceleration of the model and its relevant components. The effective acceleration gives an MMR a strong driving ability, allowing the model to reach a high speed to achieve a large \(Re\). Reliable deceleration ensures that the moving parts, including the model, can be decelerated to a standstill safely. Therefore, the excellent integration of these two functions is required for an advanced MMR.

For aerodynamic tests of high-speed trains, MMRs that are capable of driving models with large masses to actual train operating speeds have been developed in the UK, Germany, and China.3,9,10 Other countries, including Japan, South Korea, and France, have carried out corresponding tests using simple moving model devices and smaller models.12–14,16,20 The driving energy for the acceleration is mainly from elongated elastic cables, flowing liquids, or compressed air. For the MMR developed by Institute of Mechanics, CAS, Beijing, the acceleration power is provided by compressed air, and models with large sizes and masses can be driven to reach high speeds of over 400–500 km/h.9

To assess the aerodynamic drag of the model using the Davis equation, the speed and acceleration of the model in the test section must be known.1 However, from our experimental results, it is easy to find that the model exhibits uniform deceleration in the test section,9,21 which means that the nonlinear change of the model speed is too small. Therefore, it is not easy to apply the Davis equation to estimate the aerodynamic drag accurately. Using the relation between the stagnation pressure and speed of the model and the Davis equation, Yang et al.22 measured the aerodynamic drag of the CRH380A model with a scale of 1/16.8. To achieve an accurate measurement of the aerodynamic drag using the Davis equation, the speed range of the model should be sufficiently large.

If an MMR can accelerate a model with a large-scale ratio to a required speed, which ensures \(Re\) of the model is in the self-simulation region,3,6,18 and the aerodynamic drag can be exactly estimated by an effective method, the aerodynamic drag of the corresponding real vehicle will be accurately assessed. For this purpose, these principles of acceleration and deceleration and the corresponding improvements made by ourselves are introduced and analyzed in this article on the basis of the developments and upgradation of the MMR,9,21 in which the acceleration was achieved using the adiabatic expansion of compressed air and the deceleration was achieved by the braking forces from the compression of air in the pipe and the relative motion between the permanent magnets and the iron plates. Using the integral form of the Davis equation and the measured acceleration data from MMT, a novel method for estimating the aerodynamic drag was proposed and demonstrated experimentally using our MMR with a scale of 1/8.9

### 2 | PRINCIPLE OF ACCELERATION AND DECELERATION

The methods for achieving acceleration and deceleration in MMRs around the world vary widely, and the corresponding drive capabilities are also greatly different.9,12–14,16,20 To build an MMR capable of driving a model with a large mass to a high speed and to decelerate all the moving components safely, the principles of acceleration by the expansion of the compressed air in the pipe9,21 and of deceleration by the air compression in the pipe and magnetically induced friction (MIF) had been completely established. These principles were experimentally verified during the development and improvement of our MMRs. In the following sections, these principles and their application in the development of MMRs are comprehensively described, and the corresponding properties are also discussed.
2.1 Principle and process of acceleration

During the operation of our MMRs, the acceleration, testing, and deceleration of the model all occurred on the upper track of the device. The acceleration power arose from the piston inside the pipe that was driven by the compressed air released suddenly from an air gun, as also described in References 9,21. A braided towrope with a low elasticity and a high tensile strength was used to connect this driving piston to a trailer outside the acceleration pipe, the power transmission from the trailer to the model was achieved by a rope loop. In addition, the trailer moved on a lower track parallel to the upper track. One end of the acceleration pipe was closed, a small hole on the sealing end was used to allow the towrope to pass through, and the other end was smoothly connected with the air leakage section, as shown in Figure 1. The whole operation process was as follows. The compressed air from the air gun entered the accelerating pipe and then pushed the driving piston forward in the pipe. The power of the piston was transferred to the trailer on the outside of the acceleration pipe through the towrope so that it drove the model on the upper track through the rope loop to a state of acceleration.

Because of this setup, the shape and dimension of the model were unconstrained. In addition, since the compressed air used for acceleration was confined to the inside of the pipe and the lower space of the MMR, the space of the test section was the same as that of a real atmospheric environment. Moreover, from an economic point of view, since the medium of energy storage was the compressed air and the moving parts were all generally low in weight compared with other driving methods, the conversion efficiency from the compressed air into the kinetic energy of the model was relatively high.9 Furthermore, the construction and operating costs of this driving method were lower than those of other driving methods.

After the driving piston passed through the acceleration pipe and entered the leaking section for the compressed air, the vent holes in the pipe allowed the compressed air to flow to the outside of the pipe. Therefore, the piston nearly lost all the thrust driven by the compressed air. This corresponds to the moment at which the trailer exactly entered the deceleration process, and both were decelerated due to the connection of the towrope. The model continued to move due to its inertia. As the trailer began to decelerate, the trailer and model automatically separated due to the difference in speed, and the latter would be tested after entering the test section. After passing through the test section, the model entered the deceleration section. The braking force, which was generated automatically in this section, allowed the model to decelerate safely to a stationary state, as shown in Figure 1.

Since the driving force is from the adiabatic expansion of compressed air, the theoretical formula of the accelerate process is as follows:

$$\frac{1}{2}(M + M_T)V_{i0}^2 = nMg]\left\{P_0V_0 \frac{\gamma}{\gamma - 1} \left[1 - \left(\frac{V_0}{V_0 + SL}\right)^{\gamma - 1}\right] - SLP_{am}\right\}$$

where $P_0$ and $V_0$ are the initial pressure and volume of the compressed air, respectively. The compressed air expands in the acceleration pipe to drive the piston to move. The trailer assembly has a mass $M_T$, and the trailer itself, towrope, and driving piston are accelerated together with the model with a mass $M$. When the driving piston advanced to the end of the acceleration section, the volume occupied by the compressed air became $V_0 + SL$, where $S$ and $L$ are the cross-sectional area and length of the acceleration pipe, respectively. The energy from the compressed air was converted to kinetic energy of the model and the trailer assembly with a certain ratio (i.e., the energy conversion efficiency, $n_M$). $V_{i0}$ and $g$ are the

![Figure 1](image-url)
speed as the piston completes the acceleration process and the gravitational constant, respectively. \( \gamma = 1.4 \) is the specific heat ratio of air, and \( P_{am} = 10^4 \text{ kg m}^{-2} \) is the atmospheric pressure.

The process described above was successfully applied to the MMRs we built. Test results showed that more than 50% of the energy from the expansion of the compressed air can be converted into the kinetic energy of the trailer assembly and the model in this kind of process. This conclusion is crucial to the design and size of the main framework of the MMR. Based on the operating parameters and the length of the construction site or the position of the MMR, the main parameters, such as the volume of the tank for the compressed air in the air gun, the inner diameter and length of the acceleration pipe, and the initial pressure of the compressed air for the highest test speed, will be determined and balanced. The greater the initial pressure of the compressed air is, the higher the speed of the model becomes. However, the operating range of the MMR is finally limited by the deceleration of the model and the trailer, which determine the maximum values of the kinetic energy and speed of the model.

For the purpose of safe and reliable deceleration of the model and trailer assembly, MIF and damping by air compression were developed and examined. For the trailer assembly and the model, the use of the deceleration method mainly depends on their respective deceleration lengths, and the corresponding principles are described in the following sections.

### 2.2 Braking force from relative movement between magnets and iron floor

After passing through the test section, the model entered the deceleration section to begin to decelerate. Whether the model was decelerated to a standstill safely depends on the magnitude of the damping force. Therefore, the deceleration method determines the highest test speed of the model. In our MMR, the deceleration of the model relies on the MIF. The principle of this kind of damping is described and analyzed below.

MIF results from the relative motion of permanent magnets and iron plates. The components of the MIF include the electromagnetic damping force and the friction caused by the strong attraction between the magnet and the iron medium. A bulk conductor is considered to contain multiple closed conductor loops. When it moves in the magnetic field, all these circuits create motion of the cutting magnetic lines, thus forming many closed induced currents in the loop called eddy currents. According to Lenz’s law, all these eddy currents are affected by the magnetic field, which hinders the movement of the bulk conductor. That is, the force between the eddy current and the magnetic field is opposite to the relative motion direction, which is called the electromagnetic damping. Therefore, when a relative motion between the metal frame with the magnet and the iron plane takes place along the direction perpendicular to the magnetic lines of force, the electromagnetic damping occurs. In addition, when the metal frame above is in contact with the iron plane, a huge attraction force is generated between the moving magnet and the iron plane, and this attraction force causes corresponding friction between both of them, which is directly proportional to the attraction. The superposition of the friction and the electromagnetic damping constitutes the MIF of the permanent magnet and the iron plate. The simplest structural form is shown in Figure 2A.

The experimental results showed that the deceleration force is also proportional to the mass of the magnet on the moving parts and is slightly affected by the relative motion between them, which means that the electromagnetic damping force accounts for a small proportion of the total resistance, because this damping force is proportional to the relative speed. From this result, the engineering calculation describing the deceleration of the MIF is simplified as follows:

\[
\alpha_{LD}m_m = \frac{1}{2}(m_m + M_n)V_{i0}^2 = \frac{1}{2}M_mV_{i0}^2,
\]

where \( m_m \) is the mass of the magnets embedded in the model, \( L_D \) is the longest braking distance in the deceleration section of the model, \( M_m \) is the total mass of the model, \( M_n \) is the mass of the part unrelated to the deceleration magnets, and \( \alpha \) is the braking force generated per unit mass of the permanent magnet. The left-hand side of Equation (2) expresses the work done by the MIF, and the right-hand is the kinetic energy of the model.

When the permanent magnets are embedded on the surfaces of moving objects, the relative motion between the two surfaces encounters a huge damping force due to MIF. Several arrangements of the magnet inlay were examined. The test results showed that the parameters in Equation (2) mainly depended on the arrangement of the magnets, the shapes of the magnets, and the distances of the surfaces of the magnets to those of the iron plates.

In the MMR with a model scale of 1/8 for the aerodynamics of the high-speed train, the layout of the magnets shown in Figure 2A was utilized in the bottom of the train model, and a detailed description for the magnet inlay was given in
FIGURE 2  Schematic diagram of (A) a simplest MIF layout and (B) a modified MIF where some magnets are embedded in the braking floor and other magnets are embedded in the bottom iron frame of the model.

FIGURE 3  Two acceleration, $a_0$ and $a_{m_r}$, curves of models for a simple braking floor and the floor with the permanent magnets inlay.

Reference 9. This configuration produced an MIF with approximately 440 $N$ per kilogram of magnets, and it used only one pole of these magnets.

Based on the mechanism of MIF, by strengthening the magnetic field between the lower surface of the model and the upper surface of the iron braking floor, the attractive force between them is intensified, which leads to a rise in the friction. To achieve this, some magnets were placed on the upper part of the braking floor. From the measured results for the different inlays of the magnets on the floor and the bottom of the model, an optimized structure was obtained, as shown in Figure 2B. This structure is advantageous for protecting the magnets embedded in the bottom of the model and the braking floor. Meanwhile, the use of iron on the bottom of the model helped to enhance the attractive force between the model and the floor, since the magnets in the braking floor are attracted to not only the upper magnets in the model but also the lower iron frame of the model. The permanent magnets inlay on the braking floor had been implemented in our MMR in Huairuo District, Beijing. The experimental results showed that the average deceleration capability of the MIF increased by more than 60% compared to the case without magnets in the braking floor, as shown in Figure 3.

The maximum speed of the model is not only limited by the braking capability of the model; it also cannot exceed the highest speed at which the trailer can be decelerated to a standstill safely, since both the trailer and the model have the same speed during the acceleration process. Therefore, the range of the test speed for the MMR also depends on the deceleration ability of the trailer.
For a specific trailer braking distance, the MIF can provide a safe deceleration process for the trailer with a corresponding maximum speed. Figure 4 shows a schematic diagram of the deceleration of the trailer using the two poles of the magnets embedded in the trailer. This method of deceleration of the trailer mechanism had been successfully used on the MMR built in Beijing. The trailer was composed of two sliders and a trailer frame made from aluminum alloy. Some cylindrical magnets were installed from both sides of the frame. The frame effectively held these magnets because the axes of each pair of holes were shifted relative to each other, and there was a strong attraction between every pair of magnets. A special glue was also required to bond the magnets in the holes during the installation of the magnets. Two grooved sliders constrained the trailer to slide on two parallel trailer tracks.

In the acceleration section, there were no metal components within about 10 cm of the two sides of the trailer to avoid the effect of electromagnetic damping induced by the relative motion between the metal components and the magnets. In the trailer deceleration section, the L-shaped iron braking plates were securely mounted on the platform of the MMR. As the trailer entered the middle of the two braking plates, relative motion occurred between the magnets in the trailer and the braking plates. Therefore, the MIF from the two contacted surfaces between the trailer frame and the braking plates slowed the whole trailer assembly, including the towrope and driving piston, to rest. Since the two poles of the magnets in the trailer are all used to decelerate, the average braking force per kilogram of magnets is about 820 N, which is almost twice that of single pole deceleration in Figure 2A. This braking force increases further if additional magnets are mounted in these L-type braking plates.

From our MMT, one of the advantages of the MIF for the trailer deceleration described above is the easy separation of the soft ring rope of the trailer and the model after the acceleration process. This rope transfers power from the trailer to the model. After the trailer enters the deceleration section, the braking effect from the MIF is immediately evident, and a speed difference between the trailer and model occurs within a short distance. Finally, the braking force of the trailer is nearly unchanged during the whole deceleration process since most of the force is from the friction between the model frame and the braking floors.

The inlay of the magnets in the trailer leads an increasing of the trailer mass. Although both the increases in the trailer braking distance and the weight of the magnets on the trailer increase the maximum speed, the limits of this type of method are also evident. The length of the deceleration section of the trailer is limited by its mass and the length of the MMR or the site. Increasing the number of magnets in the trailer also requires a higher initial pressure of compressed air from the air gun. Moreover, the improvement of the deceleration effect of the MIF will be weakened gradually as the mass of the magnet increases, as described by Equation (2). To allow the trailer to decelerate safely in a short distance, an aerodynamic braking method was developed and applied in our MMR device. In this method, the air was compressed by a trailer with a piston shape in a pipe (ACTPP). Compared with the MIF, the advantage of the ACTPP is that the total weight of the trailer is relatively small, and the deceleration section for the trailer is shorter. Therefore, the speed range for the safe deceleration of the trailer is greatly increased for the same deceleration length.
2.3 | Damping force from compressing of air in trailer deceleration pipe

Figure 5 shows the schematic diagram of the trailer braking using the ACTPP. An important requirement is that the acceleration track and the deceleration pipe of the trailer must be accurately connected for smooth passing of the trailer. During the acceleration process, the trailer and the model accelerated together in the individual tracks. After the acceleration was over, the trailer was separated from the model and entered the deceleration pipe. Since the trailer acted as a piston with an inner diameter that matched that of the deceleration pipe, it compressed the air inside the pipe to obtain a damping force. Due to the tightness of the deceleration pipe, the braking force acting on the trailer became larger and larger until it stopped the motion of the trailer at a certain position. In this process, the force from the ACTPP and the deceleration distance depend on the initial pressure of the air in the pipe as the trailer enters and the gap between the pipe wall and trailer piston. The better the tightness of the pipe is and higher the initial pressure is, the stronger the braking force and the shorter the deceleration distance become. In our improved MMR, the trailer acceleration track was a pipe with an upper straight crack parallel to the pipe axis, and the inner diameter of this pipe was equal to that of the deceleration pipe for the smooth passage of the trailer. Through the crack, the soft connection transferred the power from the trailer to the model during the acceleration process.

In the structure above, the initial pressure of the air being compressed is either standard atmospheric pressure or raised by the injection of compressed air into the pipe through the small hole at the right end, as shown in Figure 5. The ultimate goal is to achieve the safe braking of the trailer by making the trailer compress the air in the pipe. The deceleration distance of the trailer is limited to the extent that it does not collide with the sealing device of the pipe. The compressed air can be injected either by the main air gun, which pushes the driving piston, or by other compressed air sources. At the injection hole, the check valve device, which allows only compressed air to enter the pipe, may be utilized. Thus, as the pressure in the pipe is lower than the pressure of the injected air, it is opened, and the compressed air is injected through it. Because the air in the pipe is compressed by the trailer, the check valve will close automatically and rapidly if the pressure in the pipe is higher than that of the injected air.

Thus, the highest speed for the trailer to be safely decelerated to a standstill mainly depends on the pressure of the air in the deceleration pipe and the cross-sectional area of the injection hole. A higher pressure of the compressed air injected and a larger total cross-sectional area of the injection hole increase the braking capability of the trailer further. After the specific structure for the ACTPP has been determined, the maximum speed limit for the safe deceleration of the trailer is adjusted by changing the amount of compressed air injected into the pipe. If the injected compressed air is from the main air gun, the pressure of this gas source is directly proportional to the speed of the trailer and the model. Thus, this feature is more suitable for ACTPP, that is, the higher the trailer speed is, the greater the pressure of the main air gun

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**Figure 5** Schematic diagram of the aerodynamic damping structure of the trailer. The air in the pipe is compressed by the trailer with a piston shape and the holes on the right end are for the injection of the compressed air from the acceleration pipe in Figure 1 or the other sources of compressed air.
is, and the greater the amount of the air injected becomes. Thus, the safe limit of the trailer speed will also rise with the initial pressure of the main air gun.

The kinetic energy of the trailer assembly will be dissipated by the work from the adiabatic compression of the air in the trailer deceleration pipe in the atmospheric environment, which is described as follows:

$$\frac{1}{2} M T V_t^2 = n_i g \left\{ \frac{P_{i0} S_i L_{i0}}{y - 1} \left[ \left( \frac{L_{i0}}{L_{i\text{,min}}} \right)^{y-1} - 1 \right] - P_{am} S_i (L_{i0} - L_{i\text{,min}}) \right\}$$

where $P_{i0}$ and $S_i$ are the initial pressure and the cross-sectional area of the trailer deceleration pipe, respectively, $L_{i0}$ and $L_{i\text{,min}}$ are the length of the trailer deceleration pipe and the minimum distance of the trailer to the end of the deceleration pipe as the trailer is decelerated to a standstill, respectively, and $n_i$ is the corresponding efficiency.

3 | METHODOLOGY FOR MEASUREMENT OF AERODYNAMIC DRAG

In an MMT, the motion of the model in the test section is similar to the coasting movement of an actual train on a track in open air or a long tunnel. The Davis equation describing the rolling or sliding of a model on a track indicates that the three sources of movement resistance are the friction, a damping force that varies linearly with the speed, and the aerodynamic drag, which is proportional to the square of the speed. Therefore, if the aerodynamic parameters of the model are directly estimated according to the Davis equation, the speed and acceleration of the model must be measured simultaneously. However, from the results of the MMT we obtained, the model seems to undergo uniform deceleration in the relative short test section, that is, different speeds correspond to the same acceleration, which means that the nonlinear evolution of the model speed is very small in a small speed range.

In an actual MMT, the data that can be continuously measured over the whole process is the acceleration of the model, which also includes the data in the acceleration section and the deceleration section. From the complete acceleration data, the entire speed information of the model can be obtained through integration. Meanwhile, a large range of speed variations is fundamental for high-accuracy aerodynamic drag measurements. To use the acceleration data measured in the MMT more effectively, the integral form of the Davis equation in the time domain is utilized.

3.1 | Measurement principle

From the Davis equation, $aM = A + BV + CV^2$, the following equation can be obtained:

$$M(V_t - V_0) = A \Delta T + B \int_{t_1}^{t_2} V dt + C \int_{t_1}^{t_2} V^2 dt,$$

where $M$ is the mass of the model, $V_0$ and $V_t$ are the speeds of the model at the two moments $t_1$ and $t_2$, respectively, which correspond to the moments at the beginning and end of the test as the model passed through the test section. $\Delta T = t_2 - t_1$ is defined as the interval between the two moments.

With the discrete data obtained from the measurement, Equation (4) becomes the following:

$$M(V_t - V_0) = A \Delta T + B \sum_i V(t_i) \Delta t + C \sum_i V^2(t_i) \Delta t$$

where $\Delta t$ is the time interval of the two adjacent measured data points. Using the acceleration integration formula and the measured acceleration data, the speed evolution of the entire MMT process can be obtained. The acceleration integration formula is as follows:

$$V(t_{i+1}) = V(t_i) + a(t_i) \Delta t$$

With the measured acceleration data for multiple MMTs, the method of the least squares is used to determine the three parameters of the Davis equation: $A$, $B$, and $C$. The values of $V_0$, $V_t$, $\Delta T$, $\sum_i V(t_i) \Delta t$, and $\sum_i V^2(t_i) \Delta t$ can be obtained
using the measured data for one MMT. The corresponding total value of the mean squared error between the theoretical description and the experimental result for \(N\) MMTs, \(Q\), is written as follows:

\[
Q = \sum_{j=1}^{N} \left( M(V_{t,j} - V_{0,j}) - A\Delta T_j - B \sum_i V_{ji} \Delta t - C \sum_i V_{ji}^2 \Delta t \right)^2. \tag{7}
\]

To minimize the mean squared error, the following conditions are imposed:

\[
\frac{\partial Q}{\partial A} = \frac{\partial Q}{\partial B} = \frac{\partial Q}{\partial C} = 0. \tag{8}
\]

From Equation (8), the following linear equations for determining \(A\), \(B\), and \(C\) can be obtained:

\[
\sum_{j=1}^{N} \left[ M(V_{t,j} - V_{0,j}) - A \Delta T_j - B \sum_i V_{ji} \Delta t - C \sum_i V_{ji}^2 \Delta t \right] \Delta T_j = 0, \tag{9a}
\]

\[
\sum_{j=1}^{N} \left[ M(V_{t,j} - V_{0,j}) - A \Delta T_j - B \sum_i V_{ji} \Delta t - C \sum_i V_{ji}^2 \Delta t \right] \sum_i V_{ji} \Delta t = 0, \tag{9b}
\]

\[
\sum_{j=1}^{N} \left[ M(V_{t,j} - V_{0,j}) - A \Delta T_j - B \sum_i V_{ji} \Delta t - C \sum_i V_{ji}^2 \Delta t \right] \sum_i V_{ji}^2 \Delta t = 0. \tag{9c}
\]

Therefore, using \(N\) sets of measured data from the MMT, the three parameters, \(A\), \(B\), and \(C\) of the Davis equation will be determined. The values in Equation (9) are all the time-integrated values of the acceleration values measured by the accelerometer. Therefore, the background noise of the measurement system and the acceleration fluctuation due to some vibrations are all naturally filtered out.

### 3.2 Protective cover test in MMT

To verify the feasibility and effectiveness of the aerodynamic drag test method proposed above, a protective cover, which is usually on the top of some high-speed trains and for the protection of the monitor camera of the pantograph system, was measured according to the method and the corresponding process mentioned above. This test demonstration was completed with our high-speed train MMR with a scale of 1/8, as shown in Figure 6. The protective cover was installed on the test plate with an accelerometer to record the acceleration during the whole process for each MMT. The measurement range of the accelerometer above was \(\pm 40 G\), the data acquisition frequency was 2.0 KHz, and the total weight of the tested device was 39.1 kg. Acceleration measurements of the reverse movement of the protective cover (glass window backwards) were collected.

![Protective cover for the monitoring camera on the top of some high-speed trains](image)
3.3  Measured results and data process

For an MMT, the acceleration evolution of the protective cover was recorded by an on-model accelerometer, and the corresponding speed curve was obtained by numerical integration of the acceleration data. From the acceleration results of the protective cover, as disclosed in Reference 9 and Figure 3, it was easy to discern the three processes. Having been integrated over time, the acceleration data were translated into speed histories over the whole process, and the corresponding curves were all smooth because the integral of the acceleration data over time naturally filtered out the irregular fluctuations around the zero point. The adjustment of the speed was realized by changing the initial pressure of the compressed air from the air gun.

For the solution of the Davis equation, the measured data from the accelerometer and the corresponding integrating data were displayed in Table 1. By using Equation (9), the three parameters in the Davis equation were obtained as follows: $A = -113.7 \text{ N}$, $B = 1.7491 \text{ N s m}^{-1}$, and $C = -0.0418453 \text{ N s}^2 \text{ m}^{-2}$. In the Davis equation, $A$ corresponds to the dynamic friction of the protective cover in the test section and is written into $A = mg\mu$ where $m$ and $\mu$ are the mass of the protective cover and the track friction coefficient. In addition, the positive value of $B$ means that the dynamic friction was weakened as the protective cover speed increased.

From the values of the three parameters, it can be found that $4AC - B^2 > 0$. Thus, based on the Davis equation, the speed evolution versus time in the test section can be expressed as follows:

$$V(t) = \frac{\sqrt{4AC - B^2}}{2C} - \frac{B}{2C} t + \frac{\sqrt{4AC - B^2}}{2C} t_0 - \frac{B}{2C} t_0$$ \hspace{1cm} (10a)

$$V(t) = 47.7531 \times t_0\left[0.0511059(23.8037 - t)\right] + 20.8996$$ \hspace{1cm} (10b)

Figure 7 shows the curves of the speed evolution in the test section and the theoretical curve from Equation (10b). The two types of curves agree closely. In Figure 7, one of the motions of the protective cover in the test section corresponds to the Davis equation with the calculated parameters. The longer the test section of the MMR is, the longer the corresponding mapping of each test on the speed trajectory of the Davis equation (Equation (10)) becomes. It is fundamental that the friction coefficient and the aerodynamic environment in the test section should be as constant as possible. Moreover, the greater the number of effective experimental tests and the larger the speed range is, the more accurate the corresponding aerodynamic drag coefficient becomes.

From Figure 7, it is also seen that the multiple data segment with different time regions can be extracted from the speed curve in the test section for one MMT. And each data segment all contains the values of $V_0$, $V_t$, $\Delta T$, $\sum V(t_i)\Delta t$, and $\sum V^2(t_i)\Delta t$, which is also reasonably regarded as the measured data from one MMT. On the other hand, since the accuracy of the aerodynamic drag coefficient is directly proportional to the speed range of the model, the measured data only with a long test time and a large speed scope can provide the sufficient data for the accurate determination of the aerodynamic drag coefficient. Thus, the acquisition of such type of measured data from one MMT requires the corresponding MMR

| No | $V_0$ (m s$^{-1}$) | $V_t$ (m s$^{-1}$) | $\Delta T$ (s) | $\sum i V(t_i)\Delta t$(m) | $\sum V^2(t_i)\Delta t$(m$^2$s$^{-1}$) |
|----|-----------------|-----------------|---------------|-----------------|-----------------|
| 1  | 37.14           | 28.67           | 3.2540        | 106.83          | 3532.34         |
| 2  | 51.72           | 44.65           | 2.2035        | 106.21          | 5127.91         |
| 3  | 62.68           | 56.38           | 1.7540        | 104.93          | 6282.39         |
| 4  | 72.52           | 65.18           | 1.5200        | 104.53          | 7193.18         |
| 5  | 80.41           | 72.36           | 1.3835        | 105.44          | 8041.63         |
| 6  | 87.045          | 78.928          | 1.2575        | 104.09          | 8620.06         |
| 7  | 92.94           | 84.56           | 1.1735        | 104.05          | 9229.23         |
| 8  | 98.64           | 89.40           | 1.1065        | 103.89          | 9757.79         |
with a long test section and a strong accelerating power. It is also conceivable that this method on the measurement of the aerodynamic drag coefficient can be extended to the case of the model movement in a tunnel, which requires the tunnel with a sufficient length.

During the free motion of the protective cover in the test section, the aerodynamic lift affects $C$ term of the Davis equation through the friction between the track and the support slider on the bottom of the protective cover. First of all, the aerodynamic lift induced from the motion makes the pressure of the model on the track surface deviate from its weight value. Secondly, this deviation causes the friction force to change, and this amount of the friction change is the product of the aerodynamic lift and the track friction coefficient. Since both the aerodynamic lift and the aerodynamic drag are all proportional to the square of the speed, $CV^2$ term of the Davis equation naturally includes not only the usual aerodynamic drag but also this friction changing from the aerodynamic lift. Since this component in $C$ term of the Davis equation equals to the product of the aerodynamic lift coefficient and the friction coefficient, when any one of them or their product is small, this influence of the aerodynamic lift on the aerodynamic drag can be ignored.

4 \ CONCLUSION

In summary, the fundamental principles and key techniques to develop an MMR with a large $Re$ number were supplied and summarized. The acceleration power was supplied by the expansion of compressed air in the pipe, and the deceleration of the model and the trailer were achieved by the relative motion between the magnets and the iron floor or the compression of air in the pipe. Meanwhile, a novel methodology for the measurement of aerodynamic drag in the MMT was also proposed and demonstrated. The adiabatic expansion of compressed air in the pipe pushed a driving piston to produce a forward force, and this force was applied to the trailer and the model outside the pipe with a towrope. The braking force resulting from MIF caused by the relative motion of the permanent magnets on the iron floor could safely decelerate the model and trailer. The successful deceleration of the trailer over a short distance could also be achieved with the compression of air by a piston-type trailer in the pipe. The method for measuring the aerodynamic drag using the Davis equation is based on the measurement of the acceleration of the model and the minimization of the mean squared error for the integrated form of the Davis equation.

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DATA AVAILABILITY STATEMENT
Data sharing is not applicable to this article as no new data were created or analyzed in this study.

ORCID
Qiansuo Yang https://orcid.org/0000-0002-2632-8472
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SUPPORTING INFORMATION
Additional supporting information may be found online in the Supporting Information section at the end of this article.

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