The developed method for forecasting the survivability indicators of the executive element of a special-purpose system based on analytical-stochastic simulation of a conflict situation is presented. The purpose of the method was to solve the problem of preserving and rational use of the resource of the executive elements to achieve the desired effect of the functioning of special-purpose systems.

The method is sensitive to the description of the patterns of changes in the survivability and efficiency indicators of the system. It is supposed to compare the predicted value of the survivability indicator with its criterion value and forecast the time when the system loses the ability to effectively perform tasks.

The survivability indicator is the mathematical expectation of the number of executive elements of a special-purpose system, which retained their ability to perform tasks as intended during a conflict situation.

Based on the results of the study, the values of the time characteristics of a conflict situation were obtained, in particular, the duration of the corresponding states of the executive element: preparation, waiting, implementation.

Graph-analytical simulation provides a solution to the problem of forecasting the time when the loss of executive elements leads to the system’s inability to effectively perform tasks.

Checking of the adequacy of the method showed that the confidence interval of the discrepancy between the calculation results of other methods with a confidence level of 0.9 does not exceed 0.095, and no contradictions between the methods were found. The proposed method provides an increase in the efficiency of determining the corresponding indicators within 8–11% and reliability by 22%. The possibility to determine the required reserve and the time for introducing executive elements into the system can provide a justification for how they are used to maintain the required level of efficiency of a special-purpose system.

Keywords: forecasting method, survivability indicators, system efficiency, analytical-stochastic simulation, conflict situation
system performance. This fact was confirmed by the experience of military conflicts in Libya (2011 – to date), Syria (2013 – to date), Nagorno-Karabakh (2020). Due to the imbalance of combat capabilities, the loss of forces and means of the warring parties reached a critical level, which led to the inability of the system to perform tasks in the conflict.

In such conditions, the need to preserve and efficiently use resources of executive elements of special-purpose systems is formed. Substantiation of procedures (measures) for preserving survivability to achieve the desired effect of a special-purpose system requires the use of adequate methods of forecasting and determining the survivability indicators of the executive elements during a conflict situation.

This fact demonstrates unresolved survivability issues and emphasizes the relevance of this work.

2. Literature review and problem statement

The monograph [1] deals with the systematization of modern methods of modeling and evaluation of military operations of air defense forces and information support of control processes. The provisions for constructing mathematical models of military operations and evaluating their effectiveness are stated. A generalized set of efficiency indicators and criteria of combat operations is presented. Methods of modeling and assessing the consequences of conflicts for forecasting the dynamics and results of system elements functioning to justify decisions of governing bodies are disclosed.

The methods proposed in [1] are not sensitive to the description of the impact of maneuvering methods of performing tasks by executive elements on the efficiency and survivability indicators of air defense forces. This complicates forecasting the consequences of a conflict situation and reduces the validity of decisions.

In [2], a methodical approach to estimating losses of the executive elements of the air defense system is stated. The procedure for forecasting survivability indicators takes into account the resistance of weapons and military equipment to expected destructive effects of three degrees: strong, medium, weak. However, the forecasting procedure is not sensitive to the description of the influence of the application methods of the executive elements of the system on the stochastic survivability indicators of the system during a conflict situation.

The approach proposed in [3] provides calculation of probabilities of negative consequences of cyberattacks on the executive elements of the information system. Diagnostics of the survivability of the information system is based on the theory of delay differential equations and the mechanism for constructing a fuzzy logic function. At the same time, forecasting the security of the information system does not provide a procedure for taking into account the impact of the executive element preparation on the quality of system survivability measures.

The monographs [4, 5] focus on the description of mathematical models for analyzing conflict processes based on the provisions of game theory and procedures for describing conflict situations. Decision-making procedures for using executive elements covered by the control loop reflect only the limit values of loss indicators. Conflict models are not sensitive enough to the procedure of forecasting the dynamics of changes in system survivability during a conflict situation. Therefore, they cannot be used to predict the moment of critical losses, when performance of tasks by the system is not expedient.

[6] focuses on the description of the methodology of studying the process of confrontation between the executive elements of the airspace protection system and air objects under mutual destructive influence. The basis for forecasting survivability indicators is a model of the situation with the threat of mutual loss of ability to perform tasks as intended by all parties to the conflict.

At the same time, the proposed method does not allow describing the components of the conflict at the microlevel and is not sensitive to taking into account the probabilistic indicators of the respective risks for the components of a conflict situation. In particular, it does not take into account the ability of the executive element (EE) to create favorable conditions for solving a typical task, and accordingly scenarios of situations in which the EE loses functionality due to destructive effects.

In [7], methods and technologies for modeling the functioning of critical infrastructures in conflict situations are considered. The main attention is paid to the methods of assessing the viability of the technical components of the automated organizational control system as a property of the system to implement a certain set of tasks and achieve the goals. The method takes into account threats to critical infrastructure, predicts the moment when the system and its components go into a dangerous state.

However, the description of the states of the system components is limited only to defining their numerical values as functions of deterministic specific indicators of efficiency and stability. This does not allow detailing the process at the microlevel and reduces the objectivity of predicting the results of conflict simulation.

In [8], the method of forecasting the results of searching for routes of a moving object as a separate element of the executive system is considered. The described method allows building a model with different levels of detail, which allows adjusting the duration of solution search in certain organizational areas of system application. This is due to the problems of multi-criteria optimization using a vector of indicators and requires significant time and computational resources to predict the consequences of individual situations. At the same time, the forecast of the consequences of a conflict situation does not take into account the possibility of forecasting the dynamics of critical values of the survivability indicators of a moving object, as well as their allowable threshold values.

In [9, 10], the method of forecasting the consequences of joint search and detection of objects by individual elements of executive surveillance systems is considered. The calculations of forecasting indicators take into account the differential characteristics of the Bayesian criterion of the minimum average risk of reduced performance, a priori probabilities of hypotheses about the absence and presence of an object. The forecasting indicator is the absolute probability of detecting the observation object during joint search and detection of objects in technical surveillance systems.

At the same time, the method is not sufficiently adapted to take into account the threat of losing the ability to perform tasks as intended due to the success of object search and detection. The process of creating favorable object search conditions and their absence during forecasting are not taken into account. There is a need to substantiate additional hypotheses and assumptions, which requires additional study of the adequacy to the real process.

[11] discusses the method of determining the number of groups of heterogeneous elements in the executive system.
to ensure the necessary effectiveness of repelling enemy attacks on objects and troops using the Lagrange multiplier method. This method allows justifying a set of elements that will ensure maximum performance of the executive system. At the same time, the relationship between performance and survivability indicators during a conflict situation is not described. The analytical relationships, described in the paper, are not sensitive to the reflection of the situation patterns in the context of the dynamics of changes in combat capabilities and survivability.

The analysis of the works [1–11] allows asserting that the existing scientific achievements do not fully provide for forecasting the dynamics of changes in stochastic survivability indicators over time. The described modeling methods only partially describe the relationship between survivability and efficiency indicators and do not provide their evaluation in the dynamics of relevant events. All this does not make it possible to reliably predict the results of a conflict, determine the moment of survivability loss and the size of the required reserves of the system EE. This can lead to unexpected consequences of destructive impact on the system EE and failure to perform tasks.

Thus, in the theory of conflict research, there is an unsolved problem of the imperfection of existing methods for EE survivability forecasting. This fact demonstrates the need to improve methods for conflict consequences forecasting and emphasizes the relevance of the research topic.

### 3. The aim and objectives of the study

The aim of the study is to develop a method for forecasting the survivability indicators of EE of a special-purpose system based on analytical and stochastic simulation of a conflict situation. This will make it possible to justify the size of the required reserve and predict the time of EE introduction into the system to preserve performance.

To achieve the aim, the following objectives were set:

- to formalize the process of EE functioning in a conflict situation;
- to substantiate the mechanism for forecasting the EE survivability in a conflict situation;
- to build a graph-analytical model for interpreting the results of the forecasting method;
- to check the adequacy of the method.

### 4. Materials and methods of the study

The study of the survivability of EE of a special-purpose system is based on a systematic approach.

It is important to note that the executive element of a special-purpose system in the study refers to the minimum required set of functionally related special and technical means of the system that provide autonomous performance of typical tasks in a conflict situation. Examples of EE can be fire and special means of weapons systems, elements of information security systems, socio-economic systems operating under the destructive influence of external and/or internal factors.

The method consists of four stages (Fig. 1):  

1) Stage I – «Formalization of the process of EE functioning in a conflict situation» includes procedures for determining hypotheses and assumptions, the formal consequences of which are the results of a conflict situation. Next, the substantiation of the intensities of changes in the EE functional states under the influence of external and internal factors inherent in a conflict situation is performed.

Based on the provisions of probability theory, a system of differential equations is determined, the solution of which under initial conditions ($t=0$) is an analytical-stochastic model of EE participation in a conflict situation. Within analytical-stochastic simulation:

- systems of differential equations of probabilities of possible states of EE of a special-purpose system in a conflict situation are formed and investigated;
- internal regularities of a conflict situation are described and analytical expressions for forecasting the values of integral characteristics of the results of performing typical tasks by EE of a special-purpose system are defined. The method meets the requirements for efficiency and reliability of the results.

![Fig. 1. Structural-logical diagram of the method of forecasting the survivability indicators of the executive element of a special-purpose system](image-url)
2) Stage II – «Assessment of survivability, efficiency indicators and required reserve» involves determining the following indicators: EE survivability – mathematical expectation of the number of executive elements that retained their ability to perform the task as intended in a conflict situation; efficiency – mathematical expectation of the number of typical tasks performed by a special-purpose system during a conflict situation.

Substantiation of the survivability criterion of a special-purpose system is carried out on the basis of compliance of the survivability indicator with the mathematical expectation of the required number of EE necessary for the effective performance of the task.

3) Comparison of the calculated values with the criteria is carried out at Stage III – «Forecasting the consequences of a conflict situation». During the stage, a graph-analytical model of changes in the survivability and efficiency indicators in a conflict situation is built. The consequences of a conflict situation are described by the values of the corresponding efficiency and survivability indicators at the time when the system operation results become unacceptable, i.e. the survivability indicator is reduced to the value of the required EE reserve.

The method of graph-analytical simulation allows analyzing the development of a conflict situation and forecasting the time when the efficiency and survivability indicators of a special-purpose system reach limit values. The advantages of the method are simplicity and clear physical meaning of the simulation results.

4) Stage IV – «Generalization of forecasting results» involves the generalization of calculation results and, based on the appropriate analysis, justification of ways to ensure the survivability of EE of a special-purpose system in a conflict situation.

5. Results of studying the survivability of the executive element of a special-purpose system

5. 1. Formalization of the process of executive element functioning in a conflict situation

Formal description consists in constructing the analytical-stochastic model and calculating the values of indicators and parameters of the functioning of EE of a special-purpose system when solving typical tasks.

Given the uncertainty of information about the parties to the conflict, the randomness of events, the main indicators of the consequences of a conflict situation are mathematical expectations of the loss of ability to perform tasks as intended by the parties.

The criterion of survivability can be the requirement of minimum losses at the required (allowable) values of the efficiency indicators of the executive element of a special-purpose system during a conflict situation.

To obtain the forecasting results of a conflict situation, the level of detail must be lower than the described process by at least two ranks [1]. A conflict situation consists of sequences of contacts with objects of influence, which are one rank below the formalized process. Changes in states are internal elements of the process of EE functioning under destructive effects [1]. Therefore, the EE states during a conflict situation are chosen as the minimum elements of a conflict situation [1].

The simulation cycle from the beginning of data preparation to obtaining the results of combat effectiveness assessment should be 10–100 times less than the probable time required for decision-making [1].

The reliability of calculations should be determined by a confidence level and interval of at least 0.95 and 0.01, respectively, for the entire range of possible conditions for performing a combat task [1].

Factual description. To perform tasks in a conflict, the system EE occupies an initial position in a limited space within the area of interest of a special-purpose system. In this case, preliminary information about the parties to the conflict, the conditions for performing tasks and current information on EE management are known [12].

The EE functioning process includes a set of measures to prepare for the performance of a typical task, waiting for the moment of a conflict situation, performing a typical task to achieve the goal. The main recurring phenomenon is the subprocess of performing a typical task. Each of them begins and ends at a previously unknown (random) time and has a previously unknown (random) duration [12].

The features of performing typical tasks are transience, antagonism and unpredictability. Therefore, each of the results of a conflict situation can only be predicted with some probability.

The results of each contact may be:
1) performance of the task;
2) non-performance of the task;
3) loss of EE ability to perform tasks;
4) preservation of EE ability to perform tasks;
5) combinations of items 1–4.

Thus, the EE functioning consists of three characteristic states-subprocesses (SS): preparation, waiting, implementation [12].

The preparation SS is aimed at creating conditions that provide the advantages of a typical EE in performing tasks as intended. An example can be a change in both one’s own position and that of the corresponding means in space and time. The initial data for this SS are the state of the environment, information about the features of typical tasks [1, 12].

The waiting SS characterizes the EE ability to perform a typical task. An important feature of this state is waiting time, consisting of the duration of observation and assessment of the situation within the areas of responsibility.

The implementation SS is a set of consecutive or simultaneous simple operations reproduced by EE to perform typical tasks. The implementation SS is the main executive process that is implemented.

The EE can be in any SS of unknown duration, and the features of SS are antagonism and unpredictability, which forms the stochastic nature of executive operations [1, 12].

Hypotheses: the consequences of performing operations characteristic of the state-subprocess by EE are unknown in advance and may lead to the loss of their ability to perform typical tasks [12];
– the change of SS during the EE functioning in time develops as a random process, the objects fall into the area of EE interest (influence) at random intervals and distributed exponentially [12];
– objects of influence when falling to the area of EE interest tend to leave it due to limited resources in these areas [12];
– flows of events are stationary homogeneous without aftereffect [12];
– detection and identification of relevant events within the areas of EE influence occur in a timely manner with deterministic probabilities.

Assumptions [12]. The impact on the objects in the area of EE interest occurs under approximately identical conditions and does not change over time.
Based on the analysis of the requirements [1] and the degree of compliance with the described hypotheses and assumptions, an analytical-stochastic model is chosen to describe the SS of EE functioning in a conflict situation, the state graph of which is shown in Fig. 2:

- $S_0$ – the state of the executive element, in which a set of measures is carried out to achieve advantages (create favorable conditions) when performing a typical task.
- $S_1$ – the state of the executive element, which characterizes the degree of its readiness to perform a typical task.
- $S_2$ – the state of the executive element, in which the object is actively affected by performing a sequence of elementary operations.
- $S_3$ – the state of the inability of the executive element to perform typical tasks due to the destructive influence of participants in a conflict situation.

Transitions from state to state occur with appropriate intensities characterizing the influence of the main factors on the functional state of the EE when solving typical tasks in a conflict situation [12]:

- $I_0$ – the intensity of transition from $S_0$ to $S_1$, determined by the duration of preparation measures to create favorable conditions for performing a typical task;
- $I_3$ – the intensity of transition from $S_1$ to $S_3$, determined by the dynamics (frequency) of changes in the situation, which requires additional measures to prepare for performing a typical task;
- $I_2$ – the intensity of transition from $S_1$ to $S_2$, determined by the degree of EE functionality while in readiness;
- $I_0$ – the intensity of transition from $S_3$ to $S_2$, determined by the efficiency of the executive element in performing elementary operations;
- $I_1$ – the intensity of transition from $S_2$ to $S_3$, determined by the time of loss of EE functionality when performing typical elementary operations;
- $I_0$ – the intensity of transition from $S_3$ to $S_0$, determined by the degree of EE functionality during preparation measures to create favorable conditions;
- $I_1$ – the intensity of transition from $S_1$ to $S_0$, determined by the time of loss of EE functionality while in readiness.

Taking into account the Markov nature of the process of performing typical tasks, the system of differential equations for determining the EE state probabilities is as follows [12]:

\[
\frac{dP_1(t)}{dt} = - (I_0 + I_0) P_1(t) + I_0 P_2(t);
\]

\[
\frac{dP_2(t)}{dt} = I_1 P_1(t) - (I_0 + I_0 + I_1) P_1(t) + I_0 P_2(t);
\]

\[
\frac{dP_3(t)}{dt} = I_1 P_1(t) - (I_0 + I_0 + I_1) P_2(t) + I_0 P_3(t);
\]

\[
\frac{dP_4(t)}{dt} = I_0 P_0(t) + I_0 P_1(t) + I_0 P_3(t);
\]

\[
P_0(t) + P_1(t) + P_2(t) + P_3(t) = 1.
\]

To solve the system, we define the initial integration conditions. At the beginning of functioning ($t=0$), when there are no objects of influence in the area of interest and the EE is ready to solve typical tasks, we assume that $P_0(t=0)=0$; $P_1(t=0)=1$; $P_2(t=0)=0$; $P_3(t=0)=0$.

Integrating the presented system of differential equations under certain initial conditions, we obtain analytical expressions describing the probabilities of EE states in the process of performing tasks as intended [12]:

\[
P_0(t) = \frac{I_{01}}{\lambda' + z} e^{-\lambda' t};
\]

\[
P_1(t) = e^{-\lambda' t};
\]

\[
P_2(t) = \frac{I_{11}}{\lambda + a} e^{-\lambda t};
\]

\[
P_3(t) = 1 - \left(1 + \frac{I_{01}}{\lambda' + z} + \frac{I_{11}}{\lambda + a}\right) e^{-\lambda' t},
\]

where intermediate values are as follows [12]:

\[
\lambda = \sqrt{\frac{lh + q}{2}} + \sqrt{\left(\frac{lh + q}{2}\right)^2 + \left(\frac{I}{3}\right)^3} + \frac{1}{2} \sqrt{\frac{lh + q}{2} - \sqrt{\left(\frac{lh + q}{2}\right)^2 + \left(\frac{I}{3}\right)^3}} - h;
\]

\[
l = ab + az + bz - I_{01} I_{10} - I_{11} I_{12} - \frac{a + b + z}{3};
\]

\[
q = (ab - aI_{01})I_{10} - zI_{21} I_{12} - \frac{a + b + z}{27};
\]

\[
a = I_{11} + I_{21};
\]

\[
b = I_{12} + I_{13} + I_{10};
\]

\[
z = I_{01} + I_{03}.
\]

The analytical equations (1)–(4) are an analytical-stochastic model of the participation of EE of a special-purpose system in a conflict situation.

5.2. Substantiation of the mechanism for assessing the survivability indicators of the executive element in a conflict situation

The idea of the method of forecasting the survivability indicators involves operations to determine the duration of the corresponding EE state and the probabilities of their (states)
change under the influence of factors described by the flow of corresponding events.

The threat of loss of the EE ability to perform tasks under the influence of destructive factors is derived from the product of mathematical expectations of the time of the EE validity states \( \mathcal{M}_t^I(t), \mathcal{M}_t^2(t), \mathcal{M}_t^3(t) \) and the corresponding intensities of their change \( (\lambda_{10}, \lambda_{13}, \lambda_{23}) \).

Therefore, the mathematical expectations of the time of the corresponding states of the executive element are determined by integrating equations (1)–(4) in the interval \( 0, T \).

Substituting analytical equations for the state probabilities and integrating (1) we obtain:

- mathematical expectation of the time of the EE state \( S_0 \):
  \[
  \mathcal{M}_t^0(t) = \int_0^T P_0(t) dt = \frac{I_{10}}{\lambda(\lambda + z)} (1 - e^{-\lambda t});
  \]  
  (5)

- mathematical expectation of the time of the EE state \( S_1 \):
  \[
  \mathcal{M}_t^1(t) = \int_0^T P_1(t) dt = \frac{1}{\lambda} (1 - e^{-\lambda t});
  \]  
  (6)

- mathematical expectation of the time of the EE state \( S_2 \):
  \[
  \mathcal{M}_t^2(t) = \int_0^T P_2(t) dt = \frac{I_{12}}{\lambda(\lambda + a)} (1 - e^{-\lambda t}).
  \]  
  (7)

The probabilities of loss of the EE ability to perform tasks as intended are determined by the product of the mathematical expectation of the duration of the corresponding state and the intensity characterizing the destructive impact from outside the system.

The probability of loss of the EE ability to perform tasks as intended when creating favorable conditions for solving a typical task (transition from state \( S_0 \) to state \( S_3 \)):

- \( \mathcal{M}_{03}^I(t) = \frac{I_{10}I_{13}}{\lambda(\lambda + z)} (1 - e^{-\lambda t}) \) (8)

The probability of loss of the EE ability to perform tasks as intended when the EE is ready to solve a typical task (transition from state \( S_1 \) to state \( S_3 \)):

\( \mathcal{M}_{13}^I(t) = \frac{1}{\lambda} (1 - e^{-\lambda t}) \) (9)

The probability of loss of the EE ability to perform tasks as intended during the impact on the object by performing a sequence of elementary operations (transition from state \( S_2 \) to state \( S_3 \)):

\( \mathcal{M}_{23}^I(t) = \frac{I_{12}}{\lambda(\lambda + a)} (1 - e^{-\lambda t}) \) (10)

Under such conditions, the probability of loss of the EE ability to perform tasks as intended is the sum of the probabilities of transition from states \( S_0, S_1, S_2 \) to \( S_3 \) and becomes sensitive to the corresponding methods of solving the tasks of a conflict situation:

\[
\mathcal{M}_3(t) = \frac{I_{10}I_{13} + I_{12}}{\lambda(\lambda + z)} (1 - e^{-\lambda t}).
\]  
(11)

The obtained analytical equations (8)–(10) have a clear physical meaning and describe the probable share of losses of a single EE, which fall on each component of a conflict situation (preparation, waiting and implementation).

It is important to note that a special-purpose system may include a number of EE \( n \), covered by the appropriate control loop and the system of ensuring participation in a conflict situation.

Under such conditions, it is advisable to determine the survivability indicator of EE of a special-purpose system – the mathematical expectation of the number of executive elements that retained their ability to perform the task as intended during a conflict situation \( \mathcal{M}_{EE}^n(t) \).

To do this, it is necessary to determine the mathematical expectations of the number of EE that lost the ability to perform tasks as intended [12]:

- when creating favorable conditions for solving a typical task:
  \[
  \mathcal{M}_{03}^n(t) = n \frac{I_{10}I_{13}}{\lambda(\lambda + z)} (1 - e^{-\lambda t});
  \]  
  (12)

- while being ready to solve a typical task:
  \[
  \mathcal{M}_{13}^n(t) = \frac{n}{\lambda} (1 - e^{-\lambda t});
  \]  
  (13)

- while performing a typical task:
  \[
  \mathcal{M}_{23}^n(t) = n \frac{I_{12}}{\lambda(\lambda + a)} (1 - e^{-\lambda t});
  \]  
  (14)

The mathematical expectation of the number of EE that lost the ability to perform tasks as intended in a conflict situation:

\[
\mathcal{M}_{EE}^n(t) = \mathcal{M}_{03}^n(t) + \mathcal{M}_{13}^n(t) + \mathcal{M}_{23}^n(t).
\]  
(15)

The predicted value of the mathematical expectation of the number of EE of a special-purpose system that retained their ability to perform the task as intended in a conflict situation at a certain time \( t \) is as follows:

\[
\mathcal{M}_{EE}^n(t) = n - \mathcal{M}_{EE}^n(t).
\]  
(16)

Each time \( t \) corresponds to the efficiency of a special-purpose system – the mathematical expectation of the number of typical tasks performed by a special-purpose system during a conflict situation:

\[
\mathcal{M}_{EE}^n(t) = nP_{EE}M^f(t),
\]  
(17)

where \( n \) is the number of executive elements; \( P_{EE} \) is the conditional probability of performing a typical task by the executive element.

The criterion of survivability of a special-purpose system is the compliance of the indicator \( \mathcal{M}_{EE}^n(t) \) with the mathematical expectation of the required number of EE to perform tasks by a special-purpose system with an efficiency not lower than the specified \( n^{req} \):

\[
\mathcal{M}_{EE}^n(t) \geq n^{req};
\]  
(18)

\[
n^{req} = \frac{\mathcal{M}_{EE}^n(t)}{P_{EE}M^f(t)}.
\]  
(19)
where $M_{task}^{req}$ is the mathematical expectation of the required number of typical tasks performed by EE of a special-purpose system in a conflict situation.

The physical meaning of the criterion (18) is a characteristic of the degree of system readiness to participate effectively in a conflict situation, which corresponds to the statement (17) — survivability is a function of the effectiveness of the defined number of executive elements.

5.3. Graph-analytical model for interpreting forecasting results in decision-making

It is important for the study to determine the time $t_{cr}$, when the survivability of the system decreases to a level at which the performance of tasks with a given level of efficiency is not possible.

To determine $t_{cr}$, at which the mathematical expectation of the number of preserved EE of a special-purpose system will not satisfy the criterion (18), it is advisable to use the graph-analytical simulation method.

Applying the proposed approach, it is possible to build a graph-analytical model of changes in the corresponding parameters of a special-purpose system, characterizing the effectiveness of the system in performing tasks (Fig. 3).

The initial data for the calculations are $N_{as}=150$ air objects; expected direction of action – in the direction of action of 6 EE; EE efficiency – 1 air object per minute; tactics of air objects – straightforward (full availability of graphs is achieved, the importance of time parameters is leveled), the duration of a conflict situation is unlimited in time. The selected input data are close to the capabilities of the method described in [1]. Given that the efficiency indicator is a function of the survivability indicator, it is enough to choose the mathematical expectation of the number of tasks solved by the executive element $M_{task}(t)$ in identifying air objects by the method developed and presented in [1].

The main task of the correlation method is to determine the $M_{task}^{req}(t)$, characterizing the effectiveness of the system in performing tasks (efficiency and survivability) and forecast the time of survivability loss and the introduction of reserves of the executive elements.

Each moment of a conflict situation $t_i$ corresponds to the value of the efficiency $M_{task}(t_i)$ and survivability $M_{ret}(t_i)$ indicators determined by projecting the indicator on the corresponding axis. As can be seen (Fig. 2), at the initial time $t=0$, the values of the efficiency $M_{task}(0)$ and survivability $M_{ret}(0)$ indicators are within the initial values. The composition of the system is maximum.

In the course of a conflict situation, at time $t_1$, $t_2$, $t_3$, $t_4$, ..., the survivability indicator is compared with its criterion value $M_{ret}(t_1)$, $M_{ret}(t_2)$, $M_{ret}(t_3)$, $M_{ret}(t_4)$, ..., the moment of their equality $M_{ret}(t_c)=M_{req}$ is defined as critical for the system effectiveness $M_{task}(t_c)$.

The relationships in Fig. 2 are easy to analyze and are, in fact, a graphical interpretation of the procedure for determining the time of critical losses of a special-purpose system during a conflict situation.

This graph-analytical model can be used when forecasting the value of the required EE reserve, which must be introduced into the system at time $t_{cr}$ in order to maintain its effectiveness in a conflict situation.

5.4. Checking the adequacy of the method

The adequacy of the results of the forecasting method is proved by the qualitative substantiation of the accepted hypotheses and assumptions, as well as by the application of the tested models of the corresponding processes.

The verification of the developed method for adequacy to real processes occurring during a conflict situation was carried out by comparing the calculation results with the data obtained on the tested models. The tested approach is described in [1] as a method of operational forecasting of the results of a conflict situation in the airspace between the EE performing reconnaissance tasks and air objects operating in the area of EE interest without taking into account non-full availability of a special-purpose system.

The initial data for the calculations are $N_{as}=150$ air objects; expected direction of action – in the direction of action of 6 EE; EE efficiency – 1 air object per minute; tactics of air objects – straightforward (full availability of graphs is achieved, the importance of time parameters is leveled), the duration of a conflict situation is unlimited in time. The selected input data are close to the capabilities of the method described in [1]. Given that the efficiency indicator is a function of the survivability indicator, it is enough to choose the mathematical expectation of the number of tasks solved by the executive element $M_{task}(t)$ in identifying air objects by the method developed and presented in [1] as a verification parameter (VP). The calculation results are summarized in Table 1.

In order to prove the adequacy of the proposed method, it is necessary to check the calculations for correlation.

The graph of mathematical expectations of the number of tasks solved by the executive elements of the system against the number of air objects in the impact is shown in Fig. 4.

The main task of the correlation method is to determine from a large number of observations how the average value of the efficiency parameter changes with the change of the factor feature, all other things being equal [1]. Two random variables are correlated if each value of one of these quantities corresponds to a specific probability distribution of the other [1].
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When assessing the reliability of the correlation coefficient, there is an inequality \(|R|/\sigma > 68\). Thus, the estimate of the correlation coefficient is considered significant, and the relationship between the calculation results is real. The correlation coefficient of the calculations showed a relationship of the calculation results close to linear (functional).

The confidence interval of the discrepancy between the results calculated by these methods with a confidence level of 0.9 will not exceed 0.085. Thus, it can be concluded that no serious contradictions between the methods were found. The simulated process does not contradict the real one.

6. Discussion of the results of studying the survivability of the executive element of a special-purpose system

The main result of the study is the developed method of forecasting the survivability indicators of the executive element of a special-purpose system based on analytical-stochastic simulation of a conflict situation (Fig. 1).

The method is sensitive to the description of the patterns of changes in the corresponding state probabilities of the executive element and provides a forecasting function of the consequences of its (executive element) participation in a conflict situation.

The proposed method is based on the analytical-stochastic model of a conflict situation, unlike existing ones it allows:

- determining the value of the survivability indicator of a special-purpose system \(M_{\text{req}}(t)\) – the mathematical expectation of the number of EE of a special-purpose system that retained their ability to perform typical tasks in a conflict situation at a certain time (16);

- determining the value of the efficiency indicator of a special-purpose system \(M_{\text{ned}}(t)\) – the mathematical expectation of the number of typical tasks performed by a special-purpose system in a conflict situation at a certain time (17);

- using the graph-analytical model for interpreting the forecasting results in the decision-making process, forecasting the time \(t_{cr}\), at which the system will lose the ability to perform typical tasks in a conflict situation with a given efficiency \(M_{\text{ned}}(t=t_{cr})<M_{\text{req}}\) and forming initial data to determine the required reserve of executive elements of the system in order to maintain the required level of its (system) efficiency \(M_{\text{req}}=n_{\text{req}}-M_{\text{ned}}(t=t_{cr})\) (Fig. 2).

The advantages of the proposed method are high efficiency, reliability, controllability, modularity and consistency, as well as the possibility to perform calculations without special numerical methods and software [1].

The main difference between the method proposed by the authors and the existing ones is taking into account the state (Fig. 2), in which a set of measures are taken to create more favorable conditions for performing a typical task (Fig. 2). In addition, the method provides an opportunity to substantiate the value of the survivability criterion (18), taking into account the features of the destructive impact on the EE in a typical conflict situation.

The proposed method, in comparison with the considered approaches [1–11], provides an increase (gain) in the reliability of forecasting the survivability indicator (18) within
certain hypotheses and assumptions by an average of 22%, and the average deviation of the values is reduced by 17%.

Under such conditions, the informativeness of the developed method (completeness of forecasting results) increases threefold due to the contribution of significant factors in the management decision-making process to the loss indicator value (13).

The increase in the consistency of survivability indicators and criteria with characteristic factors for the components of a conflict situation (EE states-subprocesses), compared to the considered methods, determined in accordance with the method [1] is for [1–3, 6] – 15%; [4–5, 7] – 25%; [8–10] – 18%. The average consistency of the survivability forecasting results using the proposed method reaches 19%.

In addition, the forecasting efficiency compared to [1–3, 6] will increase by 8%, and compared to [4–10] – by 11%, which does not significantly affect the effectiveness of justifying and making management decisions.

The main limitations of this method are:
- the need for reliable determination of initial data and complexity of the procedure for substantiating the intensity of changes in the functional states of the executive element of the system;
- the impossibility of direct consideration of space-time parameters of a conflict situation and the need to introduce additional hypotheses and assumptions to ensure higher adequacy to the real process on the grounds of non-full availability and uncommonness of the research conflict [1].

The method was tested during scientific-practical conferences and seminars at the Ivan Chernyakhovsky National Defense University of Ukraine, as well as during the International Conference at the Academy of the Armed Forces of the Slovak Republic (Akadémia ozbrojených síl generála Milana Rastislava Štefánika) [12].

The practical implementation of this method was carried out during the military-strategic games and command post exercises, as well as studies conducted as part of research at the Ivan Chernyakhovsky National Defense University of Ukraine.

Further development of this method involves detailing the description of a conflict situation by deepening the decomposition of the states of executive elements. The introduction of additional assumptions and hypotheses requires studying formal consequences and proving the adequacy to the real process.

7. Conclusions

1. Based on the formulated hypotheses and assumptions, the formalization of the functioning process of the executive element in a conflict situation was performed. The state graph of the conflict situation model is constructed, the system of differential equations for probabilities of characteristic preparation, waiting, implementation and inoperable states is described. The analytical-stochastic model of states of the executive element during a conflict situation is described by analytical equations for the corresponding indicators. The model is universal and meets the requirements for efficiency and reliability of simulation results.

2. The mechanism for forecasting the survivability of EE in a conflict situation contains a procedure for determining the mathematical expectations of the number of the executive elements of the system that lost the ability to perform tasks as intended. The degree of system’s readiness to participate in a conflict situation is determined by the compliance of the survivability indicator with the criterion value – the mathematical expectation of the required number of executive elements to perform tasks. The value of the survivability criterion is a function of the system effectiveness criterion.

3. Based on the description of the patterns of changes in the survivability and efficiency indicators of a special-purpose system, the graph-analytical model of forecasting results is constructed. The model is easy to use and allows comparing the consequences of a conflict situation, as well as forecasting the time of critical survivability of the system. The model is adaptive to changes in the system parameters and can be used to forecast the results of a conflict situation depending on the loss of survivability and intensity of reserve introduction. It is recommended to use this approach as a basis for making decisions on the composition of the system and procedure for applying its executive elements.

4. The adequacy of the results of the forecasting method is proved by comparing the calculation results with the data of the tested models. The comparison results showed that the correlation coefficient of the calculations was 0.962, the reliability of the correlation coefficient is estimated as «significant», and the relationship between the calculation results is real.

The effect of the method is an increase in the reliability of the survivability forecast by 22% and the consistency of indicators and criteria by an average of 19%; an increase in forecasting efficiency by an average of 9.5%.

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